GRavitational Lensing of the Microwave Background by Galaxy Clusters

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ABSTRACT

Galaxy clusters will distort the pattern of temperature anisotropies in the microwave background via gravitational lensing. We create lensed microwave background maps using clusters drawn from numerical cosmological simulations. A distinctive dipole-like temperature fluctuation pattern is formed aligned with the underlying microwave temperature gradient. For a massive cluster, the characteristic angular size of the temperature distortion is a few arcminutes, and the characteristic amplitude a few microkelvins. We demonstrate a simple technique for estimating the lensing deflection induced by the cluster; microwave background lensing measurements have the potential to determine the mass distribution for some clusters with good accuracy on angular scales up to a few arcminutes. Future high-resolution, high-sensitivity microwave background maps will have the capability of detecting lensing by clusters; we discuss various systematic limitations on probing cluster masses using this technique.

Subject headings: cosmic microwave background — cosmology: theory — galaxies: clusters: general — gravitational lensing

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1. INTRODUCTION

In the post–Wilkinson Microwave Anisotropy Probe era of microwave background measurements, attention is quickly shifting to smaller angular scales. At scales below 4′, the temperature fluctuations are dominated not by primordial fluctuations associated with the last scattering surface but rather by secondary fluctuations induced by interactions with the matter distribution at lower redshifts. Most prominent among these effects are the Sunyaev-Zel’dovich (SZ) effect (Sunyaev & Zel’dovich 1972) and gravitational lensing.

This paper studies the gravitational lensing of the microwave background by galaxy clusters. Much recent effort has been devoted to the lensing of the microwave background by the large-scale distribution of matter (Okamoto & Hu 2003; Hirata & Seljak 2003; Kesden et al. 2003; Hu & Okamoto 2002), and the lensing of optical and radio sources by galaxies, galaxy clusters, and large-scale structure is a major theoretical and experimental industry. Curiously, little effort has thus far been put into the lensing of the microwave background by clusters; the only major work on the subject is the seminal paper by Seljak & Zaldarriaga (2000). The likely reason for this benign neglect is that only recently have microwave maps of sufficient resolution and sensitivity to detect cluster lensing become a realistic expectation (Kosowsky 2003; Wootten 2002). The subject has recently been taken up by Dodelson & Starkman (2003) as it relates to galaxies; Cooray (2003) explicitly showed the dependence of the lensing signal on cosmological parameters, and Bartelmann (2003) showed a map of a cluster lensing the microwave background as an application of numerical techniques for gravitational lensing.

Observing cluster lensing of the cosmic microwave background requires temperature variations on scales large compared to the cluster (so that we have something to lens) but minimal variations on scales comparable to the size of the cluster (so that the lensing signal can be cleanly separated from the intrinsic temperature fluctuations). Notably, this is exactly what the primary microwave background fluctuations offer. A microwave background temperature map is dominated by fluctuations on scales of 1° or larger, arising from density and temperature perturbations at the surface of last scattering at redshift $z \approx 1100$. At scales smaller than a degree, the power spectrum of temperature fluctuations begins to decline because of diffusion damping: perturbations on scales smaller than the thickness of the last scattering surface are exponentially suppressed. On typical cluster angular scales of a few arcminutes, the primordial temperature perturbations are negligible. Secondary, nonlinear temperature fluctuations will arise, but these generally either are tiny (e.g., gravitational lensing by large-scale structure) or have a frequency dependence different from the blackbody distortions due to lensing (e.g., the thermal SZ effect). The only source of significant near-blackbody fluctuations on scales of galaxy clusters besides lensing is the kinematic SZ ($k$SZ) distortion, mainly due to the peculiar motion of the cluster itself, as well as a small component from large-scale structure (Vishniac 1987). In general, the $k$SZ signal has a different morphology from the lensing signal, and in most cases the difference between the two will be clear. Furthermore, most of the $k$SZ signal should be strongly correlated with the thermal SZ effect. However, the $k$SZ signal will be a large source of contamination in the central regions, with a significant source of noise coming from bulk motions within the cluster due to objects that have recently been accreted.

Seljak & Zaldarriaga (2000) considered idealized spherical clusters lensing a pure temperature gradient. They also provided an extensive list of potential systematic effects that must be overcome to observe the signal. In the space of 3 years, the sensitivity and resolution of envisioned microwave background...
measurements have increased dramatically, prompting a more detailed assessment of the cluster lensing signal. In this paper we use model mass distributions from clusters in a cosmological $N$-body simulation to lens a background Gaussian temperature field constructed from the temperature power spectrum in a realistic cosmological model. The cluster lensing signal is clear in the resulting map. Section 2 displays how the signal varies with cluster mass and cluster location on the sky, and § 3 considers how well the cluster lensing signal can be inferred from an ideal map using a correlation function technique for estimating the unlensed temperature distribution. More elaborate model-fitting techniques are likely straightforward but beyond the scope of the paper, requiring explicit and careful consideration of specific instruments and observing strategies. Finally, we discuss potential advantages and systematic limitations of this method for cluster mass determination in the context of realistic experiments.

2. ORDER-OF-MAGNITUDE ESTIMATES

The rms temperature gradient in the primary microwave background fluctuations is around 12 $\mu$K arcmin$^{-1}$ for standard cosmological models consistent with the measured microwave power spectrum; the magnitude of the lensing signal is on the order of the local temperature gradient times the characteristic angular deflection. Typical deflection angles (roughly the cluster gravitational potential in units of $c^2$, $\Phi/c^2 \approx 10^{-4}$) are on the order of tens of arcseconds. The fractional perturbation due to lensing is therefore on the order of a few microkelvins.

This is at least an order of magnitude larger than perturbations due to the transverse velocity of the cluster (Birkinshaw & Gull 1983), which is on the order of the cluster potential $\Phi/c^2$ times the transverse velocity $v_{\text{trans}}/c$. Typical peculiar velocities are a few hundred km s$^{-1}$, so this term should be at the level of roughly 0.1 $\mu$K. Similarly, anisotropy imprinted from the cluster potential changing over the photon travel time (the integrated Sachs-Wolfe effect in the nonlinear regime) should be roughly given by the potential times the photon travel time relative to the dynamical time. With travel times on the order of 1 Myr and dynamical times on the order of 1 Gyr, this term should be comparable to the transverse velocity effects but well below the expected level of lensing effects. Therefore, the signatures of moving or dynamically evolving clusters will be strongly suppressed compared to lensing of the microwave background, even if the thermal and kinematic SZ effects can be efficiently removed.

As discussed below, the kSZ effect is also expected to be on the order of a few microkelvins, with a similar spectral dependence on gravitational lensing. These two signals must be separated via their spatial morphology or other techniques.

3. LENSED MAPS

Figure 1 displays the lensing of a pure temperature gradient by a numerically simulated cluster of galaxies of mass $M = 6 \times 10^{14} h^{-1} M_\odot$ at a redshift of $z = 0.5$, while Figure 2 shows the same cluster lensing two realizations of a Gaussian random temperature field with the power spectrum of the microwave background for a flat $\Lambda$-dominated cold dark matter cosmological model with $n = 1$, $h = 0.7$, $\Omega_0 h^2 = 0.024$, and $\Omega_\Lambda = 0.7$. Equal-temperature contours are plotted at 5 $\mu$K separations as thin dotted lines. To display the lensing signal more clearly, we then high-pass filter the map with a 3$'$ filter scale. The temperature contours of the filtered map are plotted at 0.5 $\mu$K separations as thick lines.

![Figure 1. Gravitational lensing of a pure background gradient of 15 $\mu$K arcmin$^{-1}$ by a galaxy cluster of mass $7 \times 10^{14} h^{-1} M_\odot$, extracted from a numerical simulation. Background contours are spaced by 5 $\mu$K and show the unfiltered lensed signal, while solid contours, spaced by 0.5 $\mu$K, show maps that have been high-pass filtered, with all $k < 1150$ ($l < 7200$) removed. Each image is 5$'$ x 5$'$ and does not include kSZ effects. Thick contours indicate positive temperature differences, while thin contours indicate negative values. [See the electronic edition of the Journal for a color version of this figure.]

The cluster is drawn from the VIRGO$^1$ simulations (da Silva et al. 2000). Outputs (including gas) were available at $z = 0$, and for our lensing calculation we artificially placed the simulation volume at $z = 0.5$. To be conservative, we did not scale the box size to account for the expansion of the universe between $z = 0.5$ and 0. This scaling would lead to an overestimate of the central concentration of virialized objects at $z = 0.5$ in the simulation and therefore an overestimate of the lensing signature. The box was translated such that a massive cluster was at the center of the projected mass distribution, and the dark matter distribution was projected onto a 2048 x 2048 grid using a simple nearest grid point method. The gas was smoothed (in projection) over the mean distance to the 24th nearest neighbor, assuming a uniform disk-smoothing kernel. The resulting surface mass density map was used to generate deflection maps with a resolution of roughly 10$''$, using a fast Fourier transform method.

We compute the reduced angular deflection vector due to the cluster mass distribution in the thin-lens approximation:

$$\alpha(x) = \frac{1}{\pi} \int \kappa(x') \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^2} \, d\mathbf{x'},$$

(1)

where $\mathbf{x}$ is a two-dimensional vector describing the angular position on the sky and $\kappa(x)$ is the projected convergence related to the surface mass density $\Sigma$ by

$$\kappa(x) = \frac{4\pi G}{c^2} \frac{D_d D_s}{D_s} \Sigma(x),$$

(2)

$^1$ See http://virgo.sussex.ac.uk.
with $D_d$, $D_s$, and $D_{ds}$ the angular diameter distances from the observer to the lens, the observer to the source, and the lens to the source, respectively. The actual deflection vector $\hat{\alpha}$ is related to the reduced deflection by the distance scaling $\alpha = (D_{ds}/D_s)\hat{\alpha}$. (See, e.g., Schneider et al. [1992] for a detailed exposition of standard gravitational lensing theory.)

A lensed map can be constructed from an unlensed background map and a surface mass distribution $\Sigma(x)$ by solving the lens equation

$$\beta(x) = x - \alpha(x),$$

where $\beta$ is the source position in the image plane. For a source covering the entire sky, such as the microwave background, the lens equation is straightforward to solve by pixelating the image plane, then using the displacement vector to map each point in the image plane to a point of the source plane, and then assigning the source temperature at that point to the original point in the image plane. The resulting lensed maps in Figure 1 represent the signal that an ideal experiment with very high angular resolution and sensitivity might measure.

The microwave background maps were generated using a microwave background power spectrum generated by CMBFAST\(^2\) (Seljak & Zaldarriaga 1996). To be conservative, we use the lensed power spectrum, which has slightly more power on small scales, but assume the underlying microwave background anisotropies are a Gaussian random field. We construct unlensed maps over small regions of sky using the flat-sky approximation.

The cluster lensing signals are obvious in the filtered maps. It is clear that lensing induces structure in the maps on significantly smaller scales than those on which the microwave background has significant temperature fluctuations. For a cluster situated in front of a region of the microwave background that is approximately a temperature gradient, lensing produces a characteristic dipole-like pattern, with a cool and a hot peak. The peak-to-peak amplitude is proportional to the gradient magnitude but is generally on the order of 1–10 $\mu$K; the angular separation of the peaks is on the order of an arcminute, with noticeable lensing effects out to radii of several arcminutes. Also note that a vector from the hot to the cool lensing peak must be in the direction of the local microwave background gradient: this distinctive signature can be used to discriminate between a lensing signal and other effects local to the galaxy cluster, which will be physically uncorrelated with the microwave background.

The gross features of the cluster lensing signal depend on cluster mass, redshift, and sky location. The lensing effect depends on the angular diameter distance to the cluster; for massive clusters that are distant enough that their angular size is on the order of an arcminute, their angular diameter distance is only a weak function of redshift. The lensing deflection depends primarily on the total mass of the cluster. The other crucial factor in the cluster lensing pattern is sky position: the amplitude of the lensing signal is proportional to the local background temperature gradient. To extract mass information about the cluster, we must be able to infer the unlensed background temperature pattern. This is easiest when the cluster is situated in a sky region that has a fairly uniform temperature gradient; in this case the characteristic dipole lensing pattern is produced. If the cluster is located in a region of the sky where the temperature field is near an extremum or where the iso-temperature contours are strongly curved, extraction of the lensing mass is more uncertain, because the unlensed temperature field is not as well constrained by the lensed map.

Figure 3 shows clusters with a range of different masses lensing the microwave background. Here the simulated lensing mass distribution is a cubic volume $100 \ h^{-1}$ Mpc on a side drawn from the VIRGO simulations, placed at a redshift of $z = 0.5$. Both the amplitude of the lensing peaks and their angular separation increase with cluster mass. To gauge the effects of sky position, we have outlined with a square the location of all clusters in the simulated mass distribution with

\(^2\) See http://www.cmbfast.org.
\( M > 10^{14} M_{\odot} \). Some marked clusters do not obviously correspond to clean features in the filtered map, indicating that the lensing signature is very difficult to observe at these positions. Roughly half of the \( 10^{14} M_{\odot} \) clusters in Figure 3 are in sky positions leading to appreciable lensing signals. As a rule of thumb, the lensing signal will be easiest to estimate when \( |\nabla T| \) is large and \( \nabla^2 T \) is small. At the same time, occasional less massive objects that are particularly well placed at the positions of strong gradients display clear lensing signatures, as with the cluster near (−30°, 5°). Note that the positions of all clusters will be known from their thermal SZ distortions, which are at least an order of magnitude larger than the lensing signature.

In Figure 4 we show the problems that can be introduced by significant substructure in the kSZ signal. This case is chosen as an unfavorable kSZ signal, although it could be even less favorable if aligned exactly with the gradient. Some of the kSZ is removed by selecting only that component that is antisymmetric around the cluster center (as the lensing signal should be). Along the direction of the gradient, it is still possible to see the characteristic lensing distortion, but it is clear that the central region will be useless for the purpose of lensing reconstructions. We emphasize that this was not selected as a typical contamination, and the simulation physics (no feedback or preheating) led to gas cores that are much more concentrated than in observed clusters.

4. Deflection Estimation

The maps in §3 make clear that in many cases (but not all), some estimate of cluster mass within a certain radius can be made from the pattern of lensing in the microwave background. It is likely that some kind of model fitting for both the cluster mass profile and the unlensed temperature map will give the most accurate cluster mass recovery, but a complete parametric study is beyond the scope of this paper. Here we present a crude technique for estimating the lensing deflection in the inner region of the cluster that works well in many cases. This deflection angle can then be used as a constraint on the mass distribution.

4.1. Wiener Estimation of Unlensed Background

For the case of a source intensity distribution that is a pure gradient, it is straightforward to estimate the lensing deflection in the direction along the temperature gradient. At sufficiently large distances from the cluster center, the background gradient will be clear and can be robustly estimated. For the case of the microwave background, a pure gradient is generally not a good description of the background temperature distribution on the scales of a few arcminutes over which lensing deflections are significant. However, the background temperature distribution is close to a Gaussian random field, and the statistical nature of this distribution can be used to estimate the unlensed background temperature distribution in the region of a cluster: we mask out the central regions of the image, where lensing is known to be important, and interpolate an approximate unlensed temperature distribution using the region that is more than a few arcminutes away from the center.

The most reliable method we found is Wiener interpolation. We assume that the correlation function of the microwave background temperature is known from power spectrum measurements. Then we have a good estimate of \( \langle T_i T_j \rangle = C(\theta_i - \theta_j) \equiv C_{ij} \), the mean product of temperatures at two points \( \theta_i \) and \( \theta_j \) in the map; here \( C(\theta) \) is the temperature correlation function for two points with angular separation \( \theta \). The matrix \( C \) is just the theoretical Gaussian random correlation matrix constructed from the power spectrum. Given accurate estimates of the temperature map outside the cluster region, we can use these correlation functions to interpolate across the cluster.

The pixels of some observed map can be viewed as a single data vector \( d \). We then form the noise-weighted covariance
matrix $D$: this covariance matrix includes the theoretical covariance $C$ plus the covariance arising from pixel noise. We mask out given pixels in the region of a cluster by setting the diagonal elements of $D$ corresponding to those pixels to a value that is large compared to the other elements of $D$. We then estimate the temperature map $t_{est}$ defined by the theoretical correlation function that matches the measured pixels in the region outside the cluster as $t_{est} = CD^{-1}d$. This construction requires inversion of a large matrix. For a map with $N \times N$ pixels, the computation of the inverse covariance matrix scales as $N^3$, but pixels well beyond the cluster region do not contribute much to the cluster region. In addition, note that this inversion must be done only once for a given microwave background power spectrum, pixel scale, and mask geometry.

As a verification of the method, we start with a microwave background realization (with no cluster lensing), cut out the central $8'$, and use the Wiener method to reconstruct the missing pixels. Results are shown in Figure 5. The reconstruction is not perfect, but the reconstruction errors are on the order of $5''$, small compared to typical lensing deflection angles of $30''$--$40''$. For Figure 5 we used a map that was $64 \times 64$ pixels with a $10''$ pixel scale. This reconstruction took a few minutes on a desktop computer without any particular optimization effort.

We also experimented with fitting a simple gradient or two-dimensional splines of varying stiffnesses, with varying degrees of success on a case-by-case basis. For the Wiener interpolation technique, assuming the outer regions are unaffected by lensing introduces an unnatural boundary condition, so an iterative approach would improve this estimate. Ultimately, a simultaneous fit for the unlensed microwave background distribution, kSZ, and deflection angle will be required for the most reliable constraints and will use the corresponding thermal SZ map to provide a rough template for the kSZ signal.

We note that the Wiener estimation technique provides a general method for testing the Gaussianity of the microwave background temperature distribution.

4.2. Deflection Angle Reconstruction

Applying the Wiener estimation method, we show the reconstructed deflection angle along a slice through the cluster center in Figure 6. At the edge of the masked region, the deflection goes to zero by construction; it may be useful to employ an iterative approach to more accurately reconstruct the outer regions. The inner few arcminutes are accurately reconstructed, providing information on the cluster mass distribution.

A cluster will produce a second microwave background distortion with a near-blackbody spectrum in addition to lensing, namely, the kSZ effect (Sunyaev & Zel'dovich 1972). This distortion arises from photons scattered by the cluster gas moving with some bulk velocity relative to the microwave background rest frame. If clusters were balls of gas with no significant internal motions, the result would be either a positive or negative temperature distortion proportional to the gas density profile, for clusters with radial velocities toward or away from the observer. This is easily distinguishable from the dipole-like pattern induced by lensing. Furthermore, this spatial pattern would be matched by the thermal SZ signature and could be projected out. However, actual clusters possess significant internal coherent bulk velocities, particularly for higher redshift clusters undergoing frequent mergers, and these internal velocities result in more complex kSZ distortions (see, e.g., Nagai et al. 2003). Occasionally, the resulting kSZ maps can even mimic the dipole-like pattern seen in lensing, although alignment with the temperature gradient would be purely coincidental.

The degree to which the kSZ effect will degrade mass estimates will vary between individual clusters. Generally, the region of significant kSZ distortion is more spatially concentrated than the lensing signal and will have some correlation with the thermal SZ distortion, since both scale with the cluster’s total optical depth. Figure 6 also shows recovered deflection angles when the kSZ effect is included. Clearly, the lensing reconstruction is seriously degraded in the central cluster region where the kSZ signal is comparable to the lensing signal, but for regions farther from the center, the kSZ signal does not alter the reconstruction. In practice, the temperature fluctuations in an annulus in $1'$ to $3'$ away from the cluster will provide most of the information about lensing deflections.

We have not included residual noise from large-scale structure (Vishniac 1987), which should be at the level of roughly $1 \mu$K at the resolution of these maps. While this noise makes maps less visually appealing at the lowest contour levels, it does not quantitatively affect the ability to reconstruct deflection angles. Assuming typical gradients of $12 \mu$K arcmin$^{-1}$, noise at the level of $1 \mu$K corresponds to fluctuations at the level of $5''$ in the deflection angle.

All of the above analysis has been for an idealized map, which here is equivalent to an angular resolution of $10''$ and a noise level below $1 \mu$K per resolution element. The next generation of microwave background maps at small scales (e.g., the Atacama Cosmology Telescope [ACT; Kosowsky 2003] or South Pole Telescope [SPT]) will attain resolutions of roughly $1''$ and noise levels of a few microkelvins per resolution element. The ability to detect actual lensing deflections at these resolutions and sensitivities will be lower than presented here; quantitative analysis of how much the signal is degraded is ongoing. Eventually, the Atacama Large Millimeter Array
4.3. Mass From Deflection

In an ideal case, lensing of the microwave background determines the component of the deflection angle in the direction of the local temperature gradient. In principle, this is not sufficient information to determine the mass distribution. The deflection \( \alpha(x) \) can be expressed as the gradient of a lensing potential: \( \alpha = \nabla \psi \); thus, the deflection field is curl-free: \( \nabla \times \alpha = 0 \). Thus, if we infer \( \alpha_x \) from a map, in principle this determines \( \partial \psi / \partial x \) as well (although of course no practical algorithm should differentiate measured data!). However, we cannot obtain \( \alpha_x \) itself by integrating with respect to \( x \), because a different integration constant obtains for each value of \( y \). Therefore, we cannot recover the complete deflection angle \( \alpha \) nor the convergence (mass density) \( 2\kappa = \nabla \cdot \alpha \) but rather one component of the deflection and the derivative of its orthogonal component.

In practice, a number of other reasonable assumptions or constraints can be imposed. Assuming the deflection to be zero beyond some given radius will break the degeneracy while biasing the cluster mass estimate. The assumption of a spherically symmetric lens will also break the degeneracy; more generally, assuming almost any parametric form for the lens will largely lift the degeneracy.

4.4. Errors in Cluster Catalogs

So far we have discussed galaxy clusters individually. Future high-resolution microwave maps will yield samples of hundreds to thousands of galaxy clusters selected by their thermal SZ spectral distortions. The lensing signals in these samples can be used to probe the cluster mass distribution in a statistical sense. In this case, it is important to distinguish statistical errors, which will average out in a large cluster sample, from systematic ones, which will not.

The major sources of error affecting the lensing observations and analyses described here are residual uncertainties from projecting out the thermal SZ spectral distortions and other nonblackbody foreground components such as dust or point sources, separation of the kSZ signal from the lensing plus microwave background signals, errors in reconstructing the deflection map from the lensed microwave background map, and other neglected sources of signal such as the moving-cluster effect. In order for an error to have a systematic impact on lensing mass estimates, it must correlate with the cluster lensing signal, which is aligned with the local temperature gradient. This means that no foreground emission is likely to contribute any systematic error to the lensing signal. The separation of the thermal SZ signal is also unlikely to give any systematic error, because of its roughly symmetric shape. The moving-cluster effect is both negligibly small for any single cluster and also uncorrelated with the background temperature gradient so it will not give any systematic effect over a significant survey area (i.e., a few square degrees). The kSZ effect can coincidentally mimic the lensing signal to some extent in the case of a cluster undergoing a merger, but the dipole-like morphology of the signal will only be aligned with the background temperature gradient by coincidence in a small fraction of these merging clusters; in addition, any kSZ dipole-shaped signature will be over smaller angular scales than the lensing signal and will mostly be eliminated by masking out the central portion of the cluster.

The only clear potential systematic error arises from the technique used to separate the lensing signal and the background fluctuations. In our sample technique in this paper, for example, we reconstruct the deflection over some finite-area region around the cluster, assuming zero deflection at the edges...
of the region. This leads to a systematic underestimate of the
deflection in the outer parts of the cluster, as seen in Figure 6.
The extent to which this systematic error biases ultimate mass
estimates depends on the details of the method used to re-
construct the deflection. This bias depends primarily on the
shape of the background fluctuations and for a given recon-
struction technique can probably be modeled with a fair degree
of certainty. Detailed estimates of the size of this systematic
error when constructing statistical estimates from cluster cat-
alogs is beyond the scope of this paper.

5. DISCUSSION

Cluster mass estimates based on the gas distribution, such as
those extracted from X-ray temperature maps or from the SZ
effect, have intrinsic uncertainties arising from the detailed
relationship between the gas distribution and the total mass
distribution, particularly at higher redshifts when clusters un-
dergo frequent mergers. The only direct way to measure clus-
ter masses is through gravitational lensing. Strong and weak
lensing of background galaxies by a cluster give measurements
of the shear field induced by the cluster mass distribution, and
elaborate techniques have been developed to reconstruct an
estimated mass distribution from lensing observations (Kaiser
& Squires 1993; Squires & Kaiser 1996). A number of cluster
masses have been measured this way (Fisher & Tyson 1997;
Tyson et al. 1998; Wittman et al. 2001; King et al. 2002).

However, cluster mass determination via lensing of back-
ground galaxies has a number of limitations. At the usual mag-
nitude limits, most clusters do not exhibit the arcs of strongly
lensed background galaxies, which arise because of chance
alignment of the cluster and background galaxy. Mass meas-
urements for large samples of galaxy clusters thus cannot rely
on strong lensing as a primary tool. Weak lensing of faint
background galaxies provides a more generic signal. How-
ever, weak lensing has a number of difficult systematic con-
ciderations: (1) Orientations of background galaxies possess
some degree of intrinsic alignment, which can bias the lensing
signal (Heavens et al. 2000; Pen et al. 2000; Croft & Metzler
2000; Catelan et al. 2001; Crittenden et al. 2001; Brown et al.
2002). (2) The redshift distribution of background galaxies is
generally not well determined, and photometric redshift dis-
tributions may induce significant systematic errors (Smith et al.
2001; McKay et al. 2002; Sheldon et al. 2004). (3) The lensing
signal becomes weaker for higher cluster redshifts because fewer
background galaxies are behind high-redshift clusters. (4) At a
given exposure depth, weak-lensing measurements are limited by
shot noise due to the finite number of background galaxies.

Lensing of the microwave background ameliorates these
limitations. The photons all originate from a single, precisely
determined redshift, and this surface of last scattering is well
behind any cluster, simplifying the redshift dependence of the
lensing signal. Furthermore, given sufficient resolution, the
lensed temperature pattern can be probed with arbitrary accu-
cy, circumventing the shot-noise limits. An additional po-
tential advantage, pointed out by Seljak & Zaldarriaga (2000),
is that the deflection angle drops off more slowly with dis-
tance from the cluster than does shear. However, as radius
increases, the comparatively larger lensing signal is counter-
tered by the larger noise (arising from the primary microwave
background anisotropies) from uncertain knowledge of the
unlensed background; in practice, this advantage over weak
lensing is uncertain.

The microwave background lensing signal will have its
own systematic limitations. Foremost among these are the
contribution of the kSZ signal and our limited knowledge
about the underlying primordial temperature fluctuation pat-
tern. As illustrated here, the impact of each of these varies
from cluster to cluster. It is safe to say that some nonnegligible
fraction of clusters will be good candidates for lensing mass
determinations, because of their location in front of a favor-
able area of temperature fluctuations and their lack of signif-
icant internal bulk velocities producing a complicated kSZ
contribution. In addition, which clusters will provide the most
reliable lensing deflection determination will be evident from
the maps themselves.

Finally, any lensing mass estimate probes not only the
cluster mass but all the mass projected along the line of sight to
the cluster. Generally, this total projected mass is dominated by
the galaxy cluster mass, but other masses will lead to a small
systematic bias toward overestimating cluster masses. This can
be addressed by comparing observations with full cosmologi-
cal simulations instead of with a simple cluster mass distri-
bution (Chen et al. 2003). Note that weak and strong lensing of
background galaxies also suffer from this difficulty, although
the larger redshift of the microwave background makes the
effect somewhat more important. The linear contribution due
to large-scale structure has been crudely included by our use of
the lensed microwave background power spectrum, but this
does not adequately include the effects of non-Gaussianity and
nonlinearity.

Of course, the usual slew of difficulties in making accurate,
high-resolution, high-sensitivity microwave background maps
will also need to be overcome, including those from fore-
ground emission, point sources, and the cluster’s comparati-
lively large thermal SZ distortion. Confinement-limited infrared
point sources, in particular, will be a serious problem at arc-
minute resolutions (Knox et al. 2003; Borys et al. 2003; Blain
1998). The cluster thermal SZ effect can be minimized by
observing near the frequency null around 220 GHz. Thermal
SZ and foreground separation at the microkelvin level will
require multifrequency observations, although both will be
uncorrelated with the specific lensing temperature distortions
around clusters. Accurate measurement of lensing signals with
maximum amplitudes of a few microkelvins in individual
clusters will likely require subarcminute resolution observa-
tions over a range of frequencies by an instrument such as
ALMA and may be marginally detectable with ACT or SPT.

Lensing of the microwave background polarization fluctua-
tions should be less susceptible to many sources of confusion
than the lensing of the temperature fluctuations considered in
this paper: the kSZ and thermal SZ polarization fluctuations are
significantly smaller than the corresponding temperature fluc-
tuations, and infrared point sources, which are actually the total
dust emission from high-redshift galaxies, are likely to show
little polarized emission. Furthermore, the primary microwave
background polarization pattern on small scales will have a
particular spatial property (curl-free) that is violated in the pres-
ence of lensing (Zaldarriaga & Seljak 1997; Kamionkowski
et al. 1997), providing a "smoking gun" of the presence of
lensing distortions. If the systematic limitations to measuring
lensing deflections from temperature maps prove difficult to
overcome, polarization maps provide a viable alternative (al-
though they require still greater experimental sensitivity be-
cause of the smaller polarization amplitude in the primary
microwave background fluctuations).

The importance of constraining galaxy cluster masses hardly
needs to be emphasized. The number density of clusters of a
given mass as a function of redshift is a highly sensitive probe
of the growth rate of structure, and in principle could strongly constrain the scale factor evolution at recent cosmological epochs (Haiman et al. 2001). The problem is that traditional cluster observations in optical and X-ray bands, or measurements of the SZ distortion, measure the distribution of baryons in the cluster, not its total mass. Connecting the distribution of baryons and dark matter in clusters is a difficult and complex problem, providing many possibilities for systematic errors in cluster mass estimates, particularly at larger redshifts where clusters are further from dynamical equilibrium because of frequent merging. Lensing measurements appear to be the only route to a direct, reliable cluster mass estimate. A clear understanding and characterization of cluster lensing is also essential for probing the cluster kSZ signal (Holder 2004; Nagai et al. 2003), in which the lensing acts as an unwanted complication. We hope that the images and arguments in this paper convincingly demonstrate that microwave background lensing is a viable alternative to weak lensing of background galaxies for learning about the distribution of mass in galaxy clusters.

Note added in manuscript.—Since submission of this work, two noteworthy papers of relevance have appeared in the literature (Vale et al. 2004; Dodelson 2004). These works confirm the results presented here and present interesting discussions of other important issues and directions for lensing of the microwave background by galaxy clusters.

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