Nonlinear dynamic analysis in the $V^2C$-mode-controlled buck converter by improved mLCE

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Abstract: In this paper, an improved computation method of maximal Lyapunov character exponent (mLCE) is proposed to analyze the nonlinear dynamic behaviors in the $V^2C$-mode-controlled buck converter. In this method, a compensation algorithm of the Jacobi matrix at the non-differentiable point is given after analyzing the geometric relationship between the two subsystems on both sides of the switching surface. Besides, according to the fact that the chaos control can be realized by increasing the correlation of system state variables, the coupling strength $\varepsilon$ is introduced to the system of $V^2C$-mode-controlled buck converter and optimized by particle swarm optimization (PSO), in order to eliminate the recognized chaotic behaviors. Finally, the effectiveness of the given method is validated by a simulation and an experimental setup.

Keywords: nonlinear dynamic analysis, maximal Lyapunov character exponent, buck converter, bifurcation, chaos control

Classification: Power devices and circuits

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1 Introduction

DC-DC converter is a kind of typical switching system, which could exist many nonlinear dynamic behaviors with the variation of the circuit parameters\(^{[1, 2]}\). Thus it is essential to measure the undesired nonlinearity, in order to make sure the stable domain of the converter. Recently, a series of tools have become available for analyzing the dynamic behaviors in power electronics circuits, including Poincaré maps\(^{[3]}\), the Filippov method\(^{[4]}\), and the Lyapunov method\(^{[5]}\), where, the maximal Lyapunov character exponent (mLCE) is the most extensive way to distinguish chaotic regimes from periodic regimes in a continuous system\(^{[6, 7]}\). However, because of the non-differentiable point on the switching surface, the previous computation method of mLCE may cause error in the switching systems. In order to overcome the limitation, this paper studies the geometric relationship
between the subsystems on both sides of the switching surface, and constructs the compensation Jacobi matrix at the non-differentiable point, which improves the accuracy of mLCE in switching system. Besides, we take a V2C-mode-controlled buck converter as an example to recognize the nonlinear dynamic behaviors by the improved mLCE.

Additionally, for the recognized chaotic behaviors, many methods have been proposed to suppress them, such as Ott Grebogi Yorke (OGY) method\[^8\], time-delayed feedback control method (TDFC)\[^9\], slope compensation method\[^10\], and others\[^11\], but these all depend on circuit parameters of the Buck converter, and determine the desired targeting orbits in advance. In order to solve the problem, paper \[^12\] proposed a method only depends on an external parameter named the coupling strength after finding the fact that the chaos behavior can be suppressed by increasing the correlation of system state variables. On this basis, this paper optimize the coupling strength with particle swarm optimization (PSO) algorithm to further improve the system performance.

**2 The calculation method of LCEs in the switching system**

Lyapunov characteristic exponents (LCEs) are basic indexes in recognizing nonlinear dynamics behaviors, they are asymptotic measures characterizing the average rate of growth (or shrinkage) of small perturbations to the trajectory of a dynamical system.

In a continuous system \( \dot{y}(t) = f(y(t)), y(0) = y_0 \), where \( y \in \mathbb{R}^n \) and \( t \in \mathbb{R}^+ \). The LCEs can be computed based on the QR decomposition as: \( \lim_{t \to \infty} Y(t) = QR \), where

\[
\dot{Y}(t) = J(t)Y(t), Y(0) = I_n, Y \in \mathbb{R}^{mn}, J(t) \text{ is the Jacobi matrix of } f(y), [\lambda_1, \lambda_2, \ldots, \lambda_n]
\]

are the diagonal elements of \( R \), which are correspond to LCEs of order \( n \). In periodic motion or quasi-periodic motion, all LCEs are negative; in a chaotic system, one of them is positive. Consequently, mLCE is treated as an index to distinguish chaotic regimes from periodic regimes.

However, in switching system, there exists a non-differentiable point on the switching surface, LCEs cannot be computed via the above procedure. To solve the problem, the Jacobi matrix at the non-differentiable point should be computed different from other points\[^13\]. According to the geometric relationship between the two subsystems on both sides of the switching surface, the method is shown below:

\[
J_s = \frac{\partial \dot{x}}{\partial x} = \frac{\partial \dot{r}}{\partial x} = I + (f_2/\|f_2\|e - e\tan \theta \cdot d)^T
\]

(1)

Where, \( I \) is a unit matrix, \( f_1, f_2 \) are the subsystems on both sides of the switching surface, \( e \) is the tangent vector of the trajectory \( x_1(t), e = f_1(x)/\|f_1(x)\|, d \) is the unit vector perpendicular to \( e \), \( d = (n \cdot e - n)/\|n \cdot e - n\|, n \) is the unit vector perpendicular to the switching surface, and \( \theta \) is the angle between \( n \) and \( e \), i.e., \( \theta = \arccos n \cdot e \).
3 The dynamic analysis of the $V^2C$-mode-controlled buck converter

Here, we take the $V^2C$-mode-controlled buck converter as an example to analyze the bifurcation and chaos behaviors with the improved mLCE. The capacitance voltage $u_c$ and inductor current $i_L$ dynamics are governed by the following variable structure real switched system

$$\begin{bmatrix} i_L & u_c \end{bmatrix}^T = A_k \cdot \begin{bmatrix} i_L & u_c \end{bmatrix}^T + B_k \cdot U_{in}, \quad k = 1, 2, 3$$

Where $k = 1, 2, 3$ correspond to the three operation modes, including mode 1 (Mosfet S is on and diode D is reverse biased), mode 2 (Mosfet S is off and diode D is forward biased), and mode 3 (Mosfet S is off and diode D is reverse biased), and

$$A_1 = A_2 = \begin{bmatrix} \frac{RR_c}{(R+R_c)L} & -\frac{R}{(R+R_c)C} \\ \frac{1}{(R+R_c)C} & -\frac{1}{(R+R_c)C} \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{(R+R_c)C} \end{bmatrix}; \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = B_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

According to the transition condition from mode 1 to mode 2, from mode 2 to mode 3, $n_{1-2}$ and $n_{2-3}$ are $[m_1,m_2]/[m_1,m_2]$ and $[-1,0]$, where $m_1 = RR_c\cdot k\cdot \omega/(R_c+R) + RR_c\cdot k\cdot G_f/(R+R)+k\cdot \omega_c$, $m_2 = R\cdot (k\cdot \omega_c+ k\cdot G_f)/(R+R)$.

The circuit parameters are as follows: Clock period $T=40\mu s$, capacitance $C=1400\mu F$, inductor $L=75\mu H$, equivalent series resistance $R_e=0.1\Omega$, load resistance $R=3\Omega$, reference voltage $U_{ref}=5V$ and the amplification factor $G=1$. The input voltage $U_{in}$ is taken as the change parameter varying 4 V to 6 V with a step of 0.002 V. In the following, we plot the bifurcation diagram and its corresponding mLCE, the differential equations is solved by a method of fourth-order Runge-Kutta in a variable step (ode45).

![Bifurcation Diagram](image)

(a) The bifurcation diagram  
(b) mLCE  
(c) mLCE(improved)

Fig. 2 The bifurcation diagram and corresponding mLCE with variation of $U_{in}$
In the bifurcation diagram, we plot the output voltage at the end of every clock period, and the distribution of the points may act as a symbol to judge stability. A route from period-1 to chaos can be observed in Fig. 2(a). When \( U_{in} = 6V \), all of the points are located in a same position, which indicates that the circuit operates in period-1. With the decrease of \( U_{in} \), the distribution of the points changes. When \( U_{in} = 5.654V \), a period-doubling bifurcation takes place, all the points are located in two positions, which indicates that period-1 bifurcates to period-2. After that, period-2 bifurcates to period-4 when \( U_{in} = 5.43V \), and then the circuit gradually enters the region of intermittent chaos and robust chaos. In the region of intermittent chaos, the circuit operates in periodic regime or chaotic regime and transmutes into each other, thus there exist small periodic windows, where all of the points are located in several same positions. In the region of robust chaos, all points randomly distribute in a bounded domain.

In a continuous system, mLCE is treated as an index to distinguish chaotic regimes from periodic regimes, mLCE is negative in periodic system, while it is positive in chaotic system. However, as is shown in Fig. 2(b), because of the existence of non-differentiable point, mLCE in previous computation method is not effective. On the contrary, according to Fig. 2(c), the improved mLCE reaches zero from a negative value when \( U_{in} = 5.644V, 5.438V \), the above positions are close to the positions where period-1 bifurcates to period-2 and period-2 bifurcates to period-4. Moreover, when \( U_{in} \) is less than 5.438V, the improved mLCE keep positive, which indicates that the circuit enters the chaos regions. Consequently, the improved mLCE is available for analyzing the dynamic behaviors in power electronics circuits as an index to distinguish chaotic regimes from periodic regimes.

4 Chaos control

The intrinsic randomness is one of basic characteristics in the chaotic system, so applying a mutual-coupling control strategy can restrain the randomness, and eliminate the chaos. In this section, an external parameter \( \epsilon \) is introduced to the system of \( V^2C \)-mode-controlled buck converter, which is the coupling strength between \( i_L \) and \( u_C \).

\[
\begin{align*}
  i_L' &= (1 - \epsilon) i_L + \epsilon u_C \\
  u_C' &= \epsilon i_L + (1 - \epsilon) u_C
\end{align*}
\]

\( i_L, u_C \) in the equation set (2) is replaced by \( i_L', u_C' \). Combined with the relevance \( E(i_L' u_C') \) and \( E(i_L u_C) \), we can know when \( \epsilon > 0 \), \( E(x_1' x_2') > E(x_1 x_2) \); when \( \epsilon < 0 \), \( E(x_1' x_2') < E(x_1 x_2) \), so the chaos can be eliminated by adjusting \( \epsilon \). Afterwards, the off-line particle swarm optimization (PSO) algorithm is applied to optimize the value of \( \epsilon \). The objective function and constraint condition is defined as

\[
\min \ k_i t_i^2 + k_r r_p^2 \quad \text{s.t.} \quad 0 < \epsilon < 1, \ mLCE < 0
\]

\[
\text{Max}(v_y) < u_{max}, \text{Max}(i_L) < i_{max}
\]

Where \( t_i, r_p \) are the settling time and the peak-peak amplitude of ripple, and \( k_i, k_r \) are their weights. PSO is run for a maximum of 100 iterations, the population
size is 50, the inertia factor $\omega_1 = 1$, the learning factors $\eta_1 = 2, \eta_2 = 2$.

$$\omega = 1, \quad \eta_1 = 2, \eta_2 = 2.$$ 

Fig (4) The waveform of output voltage in simulation when $U_{in} = 4.5V$

After the optimization with PSO, $\varepsilon = 0.217$ is the optimal solution. As is shown in Fig (4), when increasing the coupling strength between $i_c$ and $u_c$, the chaotic behavior can be suppressed, the DC-DC converter can be controlled to the period-1 regime. Then the experimental prototype is built with the same design parameters used in the above simulation. Where, Power MOSFET IRF540 is used as switch S, MBR340 is used as diode D, a Texas digital signal processor (DSP) TMS320F28377S is used for building the V^2C mode controller. The waveforms of the output voltage and the inductor current are as shown in Fig (5).

Fig (5) The waveform of output voltage in experiment when $U_{in} = 4.5V$

Compared Fig (5) with Fig (4), we can detect that there exist slight deviations between the experiment results and simulation results because some parasite components existing in the circuit are ignored in the modeling.

### 5 Conclusion

In this paper, we proposed an improved computation method of mLCE for analyzing the nonlinear dynamic behaviors in the V^2C-mode-controlled buck converter. On this basis, For the recognized chaotic behaviors, an external parameter $\varepsilon$ is introduced to increase the correlation between $i_c$ and $u_c$, where $\varepsilon$ is optimized with PSO, in order to control the system from the chaotic regime to period-1 regime and improve the performance. Finally, a simulation and an experimental setup have been constructed to verify the effectiveness of the method.