Common envelope evolution of eccentric binaries

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ABSTRACT

Common envelope evolution (CEE) is believed to be an important stage in the evolution of binary/multiple stellar systems. Following this stage, the CE is thought to be ejected, leaving behind a compact binary (or a merger product). Although extensively studied, the CEE process is still little understood, and although most binaries have non-negligible eccentricity, the effect of initial eccentricity on the CEE has been little explored. Moreover, most studies assume a complete circularization of the orbit by the CE onset, while observationally such eccentricities are detected in many post-CE binaries. Here we use smoothed particle hydro-dynamical simulations (SPH) to study the evolution of initially eccentric (0 ≤ e ≤ 0.95) CE-systems. We find that initially eccentric binaries only partially circularize. In addition, higher initial eccentricity leads to a higher eccentricity following the end of the inspiral phase, with eccentricities as high as 0.18 in the most eccentric cases, and even higher if the initial peri-center of the orbit is located inside the star (e.g. following a kick into an eccentric orbit, rather than a smooth transition). CEE of more eccentric binaries leads to enhanced dynamical mass-loss of the CE compared with more circular binaries, and depends on the initial closest approach of the binary. We show that our results and the observed eccentricities of post-CE binaries suggest that the typical assumptions of circular orbits following CEE might potentially be revised. We expect post-CE eccentricities to affect the delay time distributions of various transients such as supernovae, gamma-ray bursts and gravitational-wave sources by up to tens of percents.

Key words: stars: evolution – hydrodynamics – stars: mass-loss – (stars:) binaries (including multiple): close – stars: kinematics and dynamics

1 INTRODUCTION

Binary common envelope is a key process in the evolution of close binaries. During a common envelope evolution (CEE) a (typically) evolved star fills its Roche-lobe ensuing an unstable mass-transfer. The binary then evolves to a CE phase, leading to the inspiral of the companion inside the envelope and eventually, the ejection of a significant part (if not all) of the CE (see Ivanova et al. 2013, for a review). The inspiral can give rise to mergers of the companion with the core, or leave behind a compact short-period binary, CEE is believed to play a key role in the evolution of compact systems; the production of X-ray sources and of gravitational wave sources and supernovae progenitors (e.g. Paczynski 1976; Izzard et al. 2012; Ivanova et al. 2013; Soker 2017, and references therein).

Due to its relatively short duration and low luminosity, direct detection of systems experiencing this process is almost impossible, with only one candidate system to be observed as an active CE event (Stepień 2011; Tylenda et al. 2011). Theoretical studies that carry hydro-dynamical simulations of CEE typically fail to fully eject the envelope, and/or to reproduce the orbital properties of post-CE binaries. Various additional physical processes have been suggested to play a role in the CEE and the envelope ejection, such as recombination energy (Ivanova et al. 2015, accretion and jets (Shiber et al. 2019; Schreier et al. 2019), long pulsations (Clayton et al. 2017) and dust driven winds (Glanz & Perets 2018). The role and importance of each of these various components are still debated. Some of these processes (or their combinations) can potentially be distinguished by their timescales (e.g. Michaely & Perets 2019; Igoshev et al. 2019) and the specific stage of the CE in which they have their strongest influence. The characteristics of the observed late stage outcome system (such as orbital separation and eccentricity) could then potentially identify which of the proposed processes could indeed have an important role in the evolution of the CE.

In particular, the physical processes and their related timescales and mass ejection could determine the final outcome of the surviving binaries, their period and eccentricity. If most of the mass loss happens in a dynamical time scale, we should expect to observe many post-CE systems with high eccentricities, whereas a low mass loss rate during the plunge in should lead to a fast circularization of the orbit in most cases, even-though an eccentricity may be developed later-on due to fast episodic mass loss (Soker 2000; Clayton et al. 2017).

Observational studies of short-period binary systems with a sdB component (which might be the result of a RG that lost its envelope during the CE) tend to assume zero eccentricity; although some show non-negligible eccentricities of up to 0.15 (Delfosse et al. 1999; Edelmann et al. 2005; Kawka et al. 2015; Kruckow et al. 2021). Moreover, eccentric short-period binary systems with an Asymptotic Giant Branch (AGB) component do exist (Bonačić Marinović et al. 2008), as well as eccentric long- period binary systems that contain sdB stars (Vos et al. 2018).

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Although CEE have been extensively studied numerically, the vast majority have initiated the binary system with a circular orbit. One reason to believe the orbit has already circularized prior to the onset of the CE, is due to tidal dissipation (Zahn 1977; Verbunt & Phinney 1995; Soker 2000). Nevertheless, as was recently studied by Vigna-Gómez et al. (2020); Vick et al. (2021), Roche-lobe overflow of an eccentric binary is not necessarily a special scenario. As described in Soker (2000); Kashi & Soker (2018), the eccentricity grows dramatically during each pericenter passage due to large mass loss; if the mass loss rate is high enough to overcome the circularization due to dynamical tides, the eccentricity may even increase. Eccentric binary systems can also be formed following velocity kicks imparted to neutron-stars and possibly black-holes upon formation in binary systems, or through violent dynamical interactions in destabilized triple systems (Perets & Kratter 2012; Michaely & Perets 2014; Glanz & Perets 2021) or through secular Lidov-kozai (Lidov 1962; Kozai 1962) or quasi-secular (Antonini & Perets 2012) evolution in triple stellar systems (e.g. Perets & Fabrycky 2009; Perets & Kratter 2012; Shappee & Thompson 2013; Michaely & Perets 2014; Toonen et al. 2016; Naoz 2016; Moe & Di Stefano 2017, and references therein). These can lead to shorter period binary systems, that can even experience the unstable Roche lobe overflow, leading to the CEE, with very small pericenters, even inside the expanding envelope.

In this paper we study the evolution of eccentric binary systems which experience a CE phase. Here we only follow a gravitational and hydro-dynamical evolution, without considering further processes which might have an important role in the evolution. We first describe our methods and tests, in section 3 we present our findings; in 4 we further discuss the implications of our results and summarize in 5.

2 METHODS

We simulated the common envelope phase of systems with different mass ratios and different initial eccentricities. As in Glanz & Perets (2021), we used the AMUSE framework (Portegies Zwart et al. 2009) to integrate between the different codes and to analyze the results. We used MESA stellar evolution code (version 2208, Paxton et al. 2011) to create models of $1M_\odot$, $4M_\odot$ and $8M_\odot$ that were evolved from zero age main sequence into red giants, all have convective envelopes, with the corresponding core masses- $0.39M_\odot, 0.48M_\odot$ and $1.03M_\odot$ (detailed properties are given in tables 1, 2, 3). While the stages of our $1M_\odot$ and $8M_\odot$ were chosen as in Passy et al. (2012) and Glanz & Perets (2021), the model with $4M_\odot$ was evolved to the tip of its Red giant phase, where it remains for much longer than the dynamical timescale expected for the CE phase (see Fig. 1). We then converted the 1-dimensional models into 3-dimensional Smoothed Particles Hydro-dynamical (SPH) models with 250K particles (see Glanz & Perets 2021 for more details about the mapping). As we are not interested in resolving the evolution of the internal structure of the giant core, both the core and (later-on) the companion are modeled as gravitational only interacting particles (also termed dark matter particles in Springel 2005, due to their use in cosmological simulations). For each giant model, we chose the softening lengths of the point mass particles as in Glanz & Perets (2021), as long as those satisfy our stability tests as described in Subsection 2.2.

We used the SPH code GADGET2 (Springel 2005) to simulate a relaxation phase and finally the common envelope phase. Within GADGET2, we use the ideal gas EOS for the gaseous particles, including their internal (thermal) energy and gravitational potential, and excluding other energy sources such as radiation pressure and recombination. To initialize a stable model and avoid nonphysical perturbation cause by the change of equation of state and dimensional model, one need to operate a relaxation/damping phase after mapping the 1D MESA model into the SPH model. The model first run in isolation for a few dynamical timescales (we chose 130 days which in all giant models are longer than 20 dynamical times). After each step of this stage, the positions and velocities of the SPH particles are adjusted such that its center of mass does not change, and the velocities are damped in a decreasing power as follows:

$$r_{i} = r_{i}^{\prime} - r_{i}^{\prime\prime} + r_{i}^0,$$

$$v_{i} = v_{i}^{\prime} - v_{i}^{\prime\prime} + v_{i}^0.$$

Where $r_{i}$ and $v_{i}$ are the jth gas particle position and velocity at step i, $r_{i}^0$ and $v_{i}^0$ are the giant’s center of mass position and velocity at step i, nsteps is the total number of the damping steps. After each step the internal velocities are damped by multiplying with a factor which increases from 0 in the first step to 1 in the last one. Finally, we simulated the common envelope (CE) phase with various simulation times, up to 2850 days, and terminated the simulations in case of a core merger (see Glanz & Perets 2021 regarding the definition of a merger in our simulations). We note that we only simulate the dynamical phase of the CE, until the orbital parameters do not change for a few orbits. Therefore, the calculated system parameters we present here might be modified later-on by other processes that act in longer timescales. In Sec 4.2 and 4.3 we describe how these parameters can be affected in the latter evolution, and how one can compare our results to the observed values.

2.1 Common envelope initiation in eccentric binary systems

Highly eccentric systems can have negligible tidal interaction throughout a large part of their orbits, with an approximately Keplerian motion. The binary separation during a Keplerian orbit with semi-major axis (SMA) $a$ and orbital eccentricity $e$, is:

$$r_{1,2} = a \left(1 - e^2\right) \left(1 + e \cos(\theta)\right).$$

(1)

If we choose the y axis to be the one pointing towards the apocenter (see Fig. 2), the components of the positions and velocities can be written as follows:

$$x = r_{1,2} \sin(\theta)$$

$$y = -r_{1,2} \cos(\theta).$$

(2)
we can write-

\[ v_x = \sqrt{\frac{GM}{a(1-e^2)}} (e + \cos \theta) \quad v_y = \sqrt{\frac{GM}{a(1-e^2)}} \sin \theta \]  

(3)

\[ \theta = -\pi \text{ at the apocenter and 0 at the pericenter.} \]

The common envelope is induced by an unstable Roche lobe overflow of one of the components, most likely by its evolutionary growth. The Roche-lobe size of the binary system can be approximated by the following expression by Eggleton (1983):

\[ R_{RL} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})} \]  

(4)

Where \( q \) is the mass ratio between the primary and secondary. Therefore, the distance between both components where the primary fills its Roche lobe is:

\[ r_{1,2} = \frac{R_1}{R_{RL}} \]  

(5)

Using eq. 1 with the proper separation, one can derive the angels at which the orbit fills the giant’s Roche-lobe:

\[ \theta_{RL} = \arccos \left( \frac{a_{RL}}{e_R} \left( 1 - e^2 \right) - \frac{1}{e} \right) \]  

(6)

To reduce cumulative numerical errors and to save time, we can begin the simulations of the extreme cases at the point when their orbital separation is more than 2.5 times the one defined in eq. 5. More details regarding this method and its verification can be found in Subsection 2.2.

2.2 Stability tests

2.2.1 Testing the evolution of the SPH model in isolation

As some of our models are initiated when the binary components are at separations (apocenter) much larger than the Roche-radius and evolved for long times, before experiencing close-interactions it is necessary to verify the stability in terms of error accumulation of our system in isolation for at least the duration until the beginning of the fast inspiral stage. We used the same method described in Glanz & Perets (2021), by evolving the system up to \( \approx 50 \) dynamical times and compared the SPH velocities to the typical velocities of the system. We define our stability criteria such that less than 1 percent of the particles gain velocities which are comparable to the typical values (i.e. relative Keplerian velocity of the companion, the sound speed, and the escape velocity). Using our default softening length, we found that for the giant with \( 1M_\odot \), non-negligible particle velocities were excited after experiencing \( \approx 45 \) dynamical times. These timescales are sufficiently long for most of our simulations, however the cases with initial eccentricities of 0.9 and 0.95 have longer dynamical timescales and require longer evolution to study the full CEE. Beginning those simulations at a later position on the orbit, when the binary still avoids interactions (as defined in the next Subsec 2.2.2), in addition to a 10 times larger core softening length in the most extreme case of \( e_i = 0.95 \), induced much smaller artificial excitation of the velocities which then satisfied our stability criteria.

2.2.2 Testing the evolution during low interaction parts of the orbit

Due to the large part of the orbit in which our binary systems have negligible tidal interactions but might be affected by numerical error accumulating over time, we verified whether an artificial deviation of the orbit from the purely Keplerian orbit occurs. After each pericenter passage, when the separation becomes \( 2.5R_L \), i.e. the binary components effectively do not interact hydro-dynamically, but only due to gravitational Keplerian motion, we calculate the new Keplerian orbit corresponding to the current system parameters. For each orbit, we calculated the maximal deviation of the orbit in the part between the two points where the separation was about 2.5 times the Roche lobe radius. Since the tidal force goes like \( F_{Tides} \sim (R_1/r_{1,2})^5 \) (see (Hut 1981)), we can write- \( r_{1,2} = A \cdot R_1/R_{RL} \) and then

\[ F_{Tides} = F_{Tides}(\theta_{RL}) \cdot A^{-5} \]  

(7)

At \( 2.5R_{RL} \) distance between the binary components, the configuration is outside the Roche lobe radius, and is far enough such that tidal affects can be neglected. We consider a system to be stable when the maximal deviation from the Keplerian orbit is not more than 10% of the separation at the point, and is not more than 20% of the minimal separation of orbit. We find that the orbit of the most eccentric systems, if initialized at apocenter accumulates too much errors by the time they evolve to pericenter, and we therefore initialize them close to peri-center (but at \( > 2.5R_{RL} \)). Following the first pericenter...
approach these orbits dissipate and the their next apocenter is
significantly reduced (see Fig. 4). We find that at no time in the later
evolution do the error accumulation significantly affects orbits and
these systems then pass our criteria.

3 RESULTS
3.1 Grazing the envelope at the pericenter
Our first simulated system contained a mass ratio which, with 0 ini-
tial eccentricity, results in a short period stable binary (Sim. 1R06-0
in table 1). The CE evolution of this system has been studied ex-
tensively (Passy et al. 2012; Iaconi et al. 2017; Reichardt et al. 2019;
Glanz & Perets 2021, and more); but even though it was found and
indicated that a small eccentricity has been developed during the
CE phase, the affect of an initial eccentricity has not been studied
yet. During the CE the eccentricity is not well defined, but we can
still calculate the orbit’s "quasi eccentricity", in order to examine its
evolution. We define the quasi eccentricity as the eccentricity of an
orbit with the maximal orbital separation as the apocenter and the
minimal orbital separation as the pericenter, such that:
\[
\epsilon_{\text{orb}} = \frac{r_{\text{orb}}^{\text{max}} - r_{\text{orb}}^{\text{min}}}{r_{\text{orb}}^{\text{max}} + r_{\text{orb}}^{\text{min}}}
\]
where \(r_{\text{orb}}^{\text{min}} \approx r_p\) and \(r_{\text{orb}}^{\text{max}} \approx r_a\) are the minimal and maxi-
mal separations between the two cores throughout the orbit.

Fig. 3 presents the evolution of the separations and quasi eccen-
tricities of simulation 106-0,106P2-106P95. All of the orbits circular-
ized significantly, but retain larger final eccentricities for larger
initial eccentricities, up to \(e_f \approx 0.18\). In addition, as can be seen
in the zoomed-in part of the upper panel, systems with larger initial
eccentricities show larger final separation.

While simulation 106-0, 106P2-106P7 began with the orbit at the
apocenter, simulations 106P9 and 106P95, that have a large part of
low interaction between the companion and the gas, were initialized
at separations of \(4R_{1,RL}\) and \(2.5R_{1,RL}\) of the orbits (see Subsec. 2.1),
with larger softening lengths for the point mass particles. The
verification of these extreme eccentric orbits were done as described in
Subsec. 2.2.2, and is presented for 106P96 in Fig. 4.

Along the spiral-in, the tidal and dynamical friction effect lead to
angular momentum transfer from the binary to the gaseous envelope
(Fig. 5, upper panel). As a consequence, even when the separation
between the two cores increases (after a pericenter passage), the mass
located around the core up to the distance of the companion keeps
decreasing, while extracting angular momentum and energy from
the binary (see Fig. 5). When both the angular momentum of the
envelope and its potential energy become larger than the one of the
two cores, the system begins to synchronize, and when the inner mass
does not change anymore, the dynamical friction goes to zero, the
mass unbinding effectively terminates (during the simulation time).
The ejection of the material from the inner regions and the effective
termination of the orbital dissipation lead to the stabilization of the
orbit at a fixed SMA and orbital eccentricity. Density snapshots of
simulation 1R06P95 are shown in Fig. 6, where one can now identify
the episodic mass ejections during pericenter passages.

Due to the higher initial values of the energy and angular mom-
entum, systems with larger eccentricities show larger amount of
unbound mass, and as expected from the calculation done by Soker
(2000), retain larger eccentricities by the end of the spiral in, despite
the strong tidal circularization. Fig. 7 shows both final eccentricities
and mass loss percentage versus the initial eccentricities, even though
the calculation of mass loss is not accurate (see discussion in Sub-
sec. 4.2), we can identify a strong correlation between these values
and the initial eccentricities. We note that the envelope continues to
unbind after the termination of the fast inspiral, but this further mass
loss was not calculated due to higher resolution required for longer
simulations. In addition, simulation 1R06P95 had a slightly more
massive and bigger core, as well as a resulting different smoothing
length profile, which can affect the amount of ejected mass.

When calculating the corresponding alpha and gamma CE
parameters corresponding to a complete ejection of the enve-
lope in these models (Livio & Soker 1988; Nelemans et al. 2000;
Nelemans & Tout 2005), and comparing the final eccentricities ac-
cording to same constants, we get a good agreement between simu-
lations which began with a giant not yet filling its Roche-Lobe at
semi-major axis (1R06P7-1R06P95). However, systems with lower
initial eccentricities, which ended with lower final eccentricities,
should have increased their eccentricities by this relation.

In Fig. 8, we show the final snapshots of some of these simulations,
suggesting that higher eccentricities lead to an extended shape of the nebula in the direction of the angular momenta. However, this is not the final shape since the material continues to expand after our simulations terminated.

The second system we test has an $8M_\odot$ Red giant as a primary, and $2M_\odot$ companion (simulations 8R2G0-8R2G7). All of these simulations ended with a core merger (as defined in Glanz & Perets 2021), but as shown in Fig. 9, still highly circularized prior to this merger. The mass loss corresponding to these points can be seen in Tab. 3, showing larger amount of ejected, unbound mass found for systems with larger initial eccentricities.

The shape of the final ejecta in these cases cannot be studied without modeling the actual merger. Any asymmetry in the mass ejection that is present during the inspiral may disappear later on, due to the energy that is released from the merger (Pejcha et al. 2016).

We simulated another giant with a middle range mass of about $4M_\odot$, which as shown in Fig. 1, has a maximal radius of only $42R_\odot$ during in Red giant stage. In simulations 4R06P0 - 4R06P95 in Tab. 1 we explore the CEE of this giant with a $0.6M_\odot$ companion with initial eccentricities ranging up to $e_i = 0.95$. Since the mass ratio between the primary and secondary stars is very large, all simulations, including the most extreme cases ended with a core merger. As was done to the previous systems with extreme initial eccentricities, simulations 4R06P9 and 4R06P95 were initialized at locations corresponding to separations of $3R_{1RL}$.

Due to the large binding energy of this giant, the amount of mass ejected prior to the merger was very low (see Fig. 10), and the ejecta were therefore almost axial symmetric by that time.

### 3.2 Systems with varying pericenters located inside the envelope

Typically, evolution to the CE phase occurs following the slow evolutionary growth of the primary star, which expands beyond it's Roche-lobe, and strong interactions occur though initially grazing encounters. However, in some systems dynamical evolution could give rise to evolution into close encounters and collisions (i.e. encounters at impact parameters closer the stellar radius). Such systems can form following velocity kicks imparted to neutron-stars/black-holes, through violent dynamical interactions in destabilized triple systems.
Figure 5. Upper panel: Mass enclosed within the region between the primary’s core and the companion (including the mass of the core) of simulation 1R06P95 with 1$M_\odot$ giant and 0.6$M_\odot$ companion, initialized with 0.95 initial eccentricity. The time is measured in respect to the initialization of the hydro simulation with separation of $2.5R_{1,RL}$ = 3000 days from the previous apocenter at the corresponding initial Keplerian orbit (see Fig 4). Middle panel: Angular momenta for same simulation. Lower Panel: Energy conservation for same simulation.

Perets & Kratter (2012); Michaely & Perets (2014); Glanz & Perets (2021) or through secular Lidov-kozai (Lidov 1962; Kozai 1962) or quasi-secular (Antonini & Perets 2012) evolution in triple stellar systems (e.g. Perets & Fabrycky 2009; Perets & Kratter 2012; Shappee & Thompson 2013; Michaely & Perets 2014; Toonen et al. 2016; Naoz 2016; Moe & Di Stefano 2017, and references therein).

In the following, we discuss systems in which the initial peri-center is inside the envelope, unlike the initially grazing systems discussed before. We consider systems with the same apocenter, but different peri-center inside the envelope.

Fig. 11 and 12 compare the separation between the primary core and the companion during simulations 1R06-0, 1R06A2-1R06A7 (see Tab. 2), with varying eccentricities between 0 to 0.7. Since all of these systems were initialized with the giant already filling its Roche-lobe, and had their pericenter deeply inside the envelope of the
Table 2. Initial configuration of the simulated systems with varying pericenter distances. $M_1$ is the mass of the primary at zero age in the main sequence, $M_2$ is the mass of the secondary, $M_{1,core}$ and $R_1$ are the core mass and radius of the primary at the beginning of the common envelope, $r_i^a$ is the initial distance between the giant core and the companion and is the apocenter of their mutual orbit, $a_i^c$ is the initial semi-major axis of the binary system, $e_i$ is the initial eccentricity and $R_{1,RL}$ is the Roche-Lobe radius of the giant.

| Sim     | $M_1$ ($M_\odot$) | $M_2$ ($M_\odot$) | $M_{1,core}$ ($M_\odot$) | $R_1$ ($R_\odot$) | $r_i^a$ ($R_\odot$) | $a_i^c$ ($R_\odot$) | $e_i$ | $R_{1,RL}$ ($R_\odot$) |
|---------|-------------------|-------------------|---------------------------|-------------------|---------------------|---------------------|------|-----------------------|
| 1R06-0  | 1                 | 0.6               | 0.388                     | 8                 | 83                  | 83                  | 0.0  | 34                    |
| 1R06A2  | 1                 | 0.6               | 0.388                     | 8                 | 83                  | 83                  | 69   | 0.2                   |
| 1R06A5  | 1                 | 0.6               | 0.388                     | 8                 | 83                  | 83                  | 55.3 | 0.5                   |
| 1R06A7  | 1                 | 0.6               | 0.388                     | 8                 | 83                  | 83                  | 49   | 0.7                   |
| 8R2-0   | 8                 | 2                 | 1.03                      | 110               | 216                 | 216                 | 0.0  | 108                   |
| 8R2A2   | 8                 | 2                 | 1.03                      | 110               | 216                 | 180                 | 0.2  | 108                   |
| 8R2A5   | 8                 | 2                 | 1.03                      | 110               | 216                 | 144                 | 0.5  | 108                   |
| 8R2A7   | 8                 | 2                 | 1.03                      | 110               | 216                 | 127                 | 0.7  | 108                   |

Figure 7. Final eccentricities (black, left vertical axis) and the fraction of the unbounded envelope mass (orange, right vertical axis) versus the initial eccentricity of the system, of models 1R06-0, 1R09P2 to 1R06P95.

primary, the orbit could not completely circularize. This can be seen in Fig. 13, which shows that the final eccentricities extends up to $e_f = 0.4$, with smaller final separation for smaller initial pericenter. As a consequence of retaining such high eccentricities, the amounts of unbounded mass were higher, in comparison to simulations 1R06P2-1R06P95. Such results indicate the that amount of mass loss during the CEE relies strongly on the location of the pericenter.

In case of systems with same apocenter, an initially smaller eccentricity, corresponds to larger angular momenta \( \propto \sqrt{a_i (1 - e_i^2)} \propto \sqrt{1 - e_i^2} \) and larger orbital energy \( \propto -a_i^{-1} \propto -(1 + e_i) \); thus, the companion efficiently unbinds the upper layers of the giant prior to stabilization. However, as the transferred energies and angular momenta go mostly to the upper layers, where the escape velocities are low, the amount of mass loss over the entire CE is larger for systems with initially smaller pericenter (higher eccentricities). Figure 13 shows the calculated unbound mass throughout these simulations, showing that the ejection continue (mostly at pericenter passages) after the orbit has almost stabilized at fixed semimajor axis (same figure, upper panel) and fixed eccentricity (middle panel). Nonetheless, in addition to the computed mass loss defined in Sec. 4.2, even after the region below the pericenter distance from the core has been completely diluted, further interactions at the apocenter lead to the increase in the ejected mass, that should further continue on a longer timescale, when interacting with the remaining bound mass. However, due to computational limitations we did not continue our simulations to measure this effect completely.

Simulations 8R2A0-8R2A7 began with initial apocenter more than twice the radius of the primary, where only the pericenter of the last 2 is initially inside the envelope, but with most of the first orbit outside the envelope (since $a_i > R_1$). All these simulations result with a core merger, and even in this case, the orbit could not circularize before the companion has reached the core. Since there was no synchronization,
the amount of mass loss up to the merger point increases with the initial angular momenta and orbital energy of the system. Therefore, in these cases, the amount of mass loss increases for lower initial eccentricities. We can conclude, that the amount of mass loss is larger for larger initial angular momenta and orbital energy, unless the semi-major axis is initially located inside the envelope, leading to a much stronger unbinding, with smaller final separations.

Fig. 14 shows a comparison of the orbital separations evolution of a different mass ratio, and initial apocenter more than twice the radius of the primary. Even in this case, the orbit could not circularize before the companion has reached the core.
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Figure 12. Separation between the primary’s core and the companion of simulations with $1M_{\odot}$ giant and $0.6M_{\odot}$ companion, initialized with same distance (same apocenter distance) and different eccentricities.

Figure 13. Upper panel: Mass of the giant (gas + core) located in the region between the two cores throughout the simulations with $1M_{\odot}$ giant and $0.6M_{\odot}$ companion, initialized with same distance (same apocenter distance) and different eccentricities. Middle panel: quasi eccentricities. Lower panel: Calculated mass loss.

Figure 14. The orbit (distances between the two cores) of simulations with a $8M_{\odot}$ giant and a $2M_{\odot}$ companion, initialized with same distance (same apocenter distance) and different eccentricities.

Figure 15. Separation between the primary’s core and the companion of systems with an $8M_{\odot}$ giant and a $2M_{\odot}$ companion, initialized with same distance (same apocenter distance) and different eccentricities. Lower panel: calculated unbounded mass for same simulations.
we present the evolution of the separation between the two cores, although the definition of unbinding also differs between studies in which the thermal energy is included in the definition, i.e. assuming it effectively translates to kinetic energy, or not; see more below. Nevertheless, consistent modelling of the role of recombinatation is difficult, and its imprtance and overall role in envelope ejection is still debated (Han et al. 1994; Harpaz 1998; Han et al. 2002; Soker & Harpaz 2003; Ivanova et al. 2015; Sabach et al. 2017; Gricener et al. 2018; Reichardt et al. 2020; Sand et al. 2020). Note, that the definition of bound material includes the thermal energies in some cases, but not in others. This could pose a major problem, since remaining bound mass might infall on longer timescales and eventually lead to the merger of any post CE binaries, in contrast with observations of such binaries (Ivanova et al. 2013; Edelmamn et al. 2005; Siess et al. 2014; Kupfer et al. 2015; Kawka et al. 2015; Vos et al. 2017; Ratzloff et al. 2020). These results suggest the potential importance of some processes which were not considered in those simulations as mentioned in the introduction.

In this paper, we do not aim to solve the mass loss problem, but we do want to measure its connection to the orbital eccentricity. We calculate the amount of mass lost at any point of the simulation by summing the mass of all SPH particles with a positive energy, define as:

$$E_i = E_{kin,i} - E_{pot,i}$$  \hspace{1cm} (8)

where $E_{kin,i}$ and $E_{pot,i}$ are the kinetic, thermal and potential energies of the particle with index $i$. The potential energy is calculated as follows:

$$E_{pot,i} = \sum_{core,comp,j\neq i} \frac{G m_i m_j}{r_{i,j}}$$  \hspace{1cm} (9)

where $m_i$ is the mass of particle $i$ and $r_{i,j}$ is the distance between particles $i$ and $j$. One can see that the potential energy is calculated with respect to all simulated particles, both bound and unbound. While a true condition for being bound to a system is not affected by external potentials. As a consequence, when the unbound mass fraction is higher, the number of unbound particles which affect our calculations is larger, and thus the uncertainty of the actual mass loss is increased. In order to consider and compare the ejected massed,
Therefore, if most of the post-CE binaries to have even larger eccentricities, possibly inconsistent with current post-CE binaries.

### 4.3 The time scales for envelope ejection

Soker (2000) showed that the orbital eccentricity of a binary system inside a CE increases significantly with the mass loss rate. However, as was showed in our results, the circularization of the orbit by tidal forces is very strong, leaving our simulated system with an initial pericenter outside the envelope, with a relatively small final eccentricity even for the largest initial eccentricity. Therefore, an additional efficient mass ejection during the spiral-in should lead to larger final eccentricities at its termination. Nevertheless, interactions of the post-CE system with the surrounding gas may cause an additional eccentricity amplification at apocenter interactions (Artymowicz et al., 1991; Kashi & Soker, 2011; Vos et al., 2018). Therefore, if most of the mass ejection is due to the potential energy dissipation during the spiral-in (with some additional mass loss occurring later-on, e.g. as in the case of dust driven winds), we expect to find most post-CE binaries located inside planetary nebulae to have low eccentricities, if any, whereas post-CE binaries with a complete unbound nebulae to have larger eccentricities. If additional processes cause a larger mass loss rate during the spiral-in, we should expect the majority of the post-CE binaries to have even larger eccentricities, possibly inconsistent with current post-CE binaries.

### 4.4 Implications for compact object mergers: supernovae, gamma-ray bursts and gravitational-wave sources

Our findings suggest that post-CE binaries could have non-negligible remnant eccentricities following the CE phase. Many of the mergers of compact object binaries are thought to occur following a CE phase which shrinks the binary orbits, which is then followed by a typically much slower inspiral due to gravitational-wave (GW) emission. The latter, GW inspiral is sensitive to the initial eccentricity of the binary, where, for a given SMA eccentric binaries merge faster than more circular ones (Peters, 1964). In other words, remnant eccentricities could potentially affect delay time distribution of mergers, and thereby affect the delay time distributions of type Ia supernovae that result from mergers of WD, short Gamma-ray burst resulting from neutron stars (and possibly black hole) mergers and GW sources from the mergers of WD, NSs and/or BHs. Current population synthesis models inferring the delay-time distributions of such transient explosive events typically assume CEE always leads to final circular orbits. Our results suggest that such assumptions might be erroneous and should potentially be corrected. The eccentricities we find are typically not extremely high, and therefore we expect the effects to

| Sim  | $M_{1,CEO}$ (M$_\odot$) | $M_2$ (M$_\odot$) | $M_{1,core}$ (M$_\odot$) | $R_1$ (R$_\odot$) | $r_i^a$ (R$_\odot$) | $a_i$ (R$_\odot$) | $e_i$ | $R_{1,RL}$ (R$_\odot$) | $r_f^p$ (R$_\odot$) | $e_f$ | $M_{\text{unbound}}$ (M$_\odot$) | $M_{\text{unbound}}/M_{\text{envelope}}$ |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------|-----------------|-----------------|-------|-----------------|------------------|
| 1R06-0 | 0.913 | 0.6 | 0.388 | 83 | 83 | 83 | 0.0 | 34 | 83 | 0.035 | 0.092 | 0.175 |
| 1R06P2 | 0.913 | 0.6 | 0.388 | 83 | 124.5 | 103.8 | 0.2 | 52 | 83 | 0.038 | 0.1 | 0.19 |
| 1R06P5 | 0.913 | 0.6 | 0.388 | 83 | 249 | 166 | 0.5 | 103 | 83 | 0.09 | 0.11 | 0.21 |
| 1R06P7 | 0.913 | 0.6 | 0.388 | 83 | 470.3 | 276.7 | 0.7 | 195 | 83 | 0.11 | 0.12 | 0.229 |
| 1R06P9 | 0.913 | 0.6 | 0.388 | 83 | 1577 | 830 | 0.9 | 668 | 83 | 0.14 | 0.146 | 0.279 |
| 1R06P95 | 0.913 | 0.6 | 0.392 | 83 | 3237 | 1660 | 0.95 | 1372 | 83 | 0.18 | 0.136 | 0.261 |
| 1R06A2 | 0.913 | 0.6 | 0.388 | 83 | 83 | 69 | 0.2 | 34 | 55.2 | 0.1 | 0.12 | 0.229 |
| 1R06A5 | 0.913 | 0.6 | 0.388 | 83 | 83 | 55.3 | 0.5 | 34 | 27.65 | 0.2 | 0.18 | 0.381 |
| 1R06A7 | 0.913 | 0.6 | 0.388 | 83 | 83 | 49 | 0.2 | 34 | 13.7 | 0.4 | 0.4 | 0.762 |
| 8R2G-0 | 7.986 | 2 | 1.03 | 110 | 115 | 115 | 0.0 | 58 | 115 | - | 0.4 | 0.058 |
| 8R2G2 | 7.986 | 2 | 1.03 | 110 | 172.5 | 144 | 0.2 | 86 | 115 | - | 0.4 | 0.058 |
| 8R2G5 | 7.986 | 2 | 1.03 | 110 | 345 | 230 | 0.5 | 173 | 115 | - | 0.46 | 0.069 |
| 8R2G7 | 7.986 | 2 | 1.03 | 110 | 651.7 | 383 | 0.7 | 326 | 115 | - | 0.72 | 0.104 |
| 8R2-0 | 7.986 | 2 | 1.03 | 110 | 216 | 216 | 0.0 | 108 | 216 | - | 0.62 | 0.089 |
| 8R2A2 | 7.986 | 2 | 1.03 | 110 | 216 | 180 | 0.2 | 108 | 144 | - | 0.6 | 0.086 |
| 8R2A5 | 7.986 | 2 | 1.03 | 110 | 216 | 144 | 0.5 | 108 | 72 | - | 0.46 | 0.066 |
| 8R2A7 | 7.986 | 2 | 1.03 | 110 | 216 | 127 | 0.7 | 108 | 381 | - | 0.35 | 0.05 |
| 4A06-0 | 3.967 | 0.6 | 0.698 | 112 | 115 | 115 | 0.0 | 62.7 | 115 | - | 0.14 | 0.043 |
| 4A06P5 | 3.967 | 0.6 | 0.698 | 112 | 345 | 230 | 0.5 | 188 | 115 | 0.1 | 0.19 | 0.058 |
| 4A06P7 | 3.967 | 0.6 | 0.698 | 112 | 651.7 | 383 | 0.7 | 355.3 | 115 | 0.1 | 0.21 | 0.064 |
| 4R06-0 | 3.992 | 0.6 | 0.478 | 42 | 43 | 43 | 0.0 | 23.4 | 43 | - | 0.13 | 0.037 |
| 4R06P2 | 3.992 | 0.6 | 0.478 | 42 | 64.5 | 54 | 0.2 | 35.1 | 43 | - | 0.144 | 0.041 |
| 4R06P5 | 3.992 | 0.6 | 0.478 | 42 | 129 | 86 | 0.5 | 70.3 | 43 | - | 0.177 | 0.05 |
| 4R06P7 | 3.992 | 0.6 | 0.478 | 42 | 243.7 | 143.3 | 0.7 | 133 | 43 | - | 0.18 | 0.051 |
| 4R06P9 | 3.992 | 0.6 | 0.478 | 42 | 817 | 430 | 0.9 | 445.4 | 43 | - | 0.24 | 0.068 |
| 4R06P95 | 3.992 | 0.6 | 0.478 | 42 | 1677 | 860 | 0.95 | 914.2 | 43 | - | 0.22 | 0.063 |

Table 3. Initial configuration of the simulated systems at the onset of the CE, and final values. $M_{1,CEO}$, $R_1$ and $M_{1,core}$ are the mass, radius and core mass of the primary at the beginning of the simulation, $M_2$ is the mass of the secondary, $r_i^a$, $a_i$ are the initial apoapsis and periapsron distances, $e_i$ are the initial semi-major axis of the binary system, $e_i$, $e_f$ are the initial and final eccentricities, $R_{1,RL}$ is the initial Roche-Lobe radius of the giant, and $M_{\text{unbound}}$ is the calculated unbounded mass.
change the inspiral times by a few up to ~ 20%, however it could differ by as much as a factor of two for binaries which initial pericenters are embedded deep inside the giant envelope.

In principle, remnant eccentricities might also lead to eccentric GW sources, but our derived eccentricities are likely too low. Although a GW inspiral circularizes a binary, an initially highly eccentric binary might retain some eccentricity up to its merger, potentially observable by GW detectors. However such cases typically require very close and very eccentric orbits \((1 - e \ll 1)\), higher than expected from our models.

5 SUMMARY

In this work, we used hydro-dynamical simulations of the common envelope evolution to study the effect of different initial eccentricities on the evolution throughout this stage. We used the AMUSE framework \(\text{(Portegies Zwart & McMillan 2018)}\) to combine the stellar evolution of the giants prior to the common envelope, simulated with MESA \(\text{(Paxton et al. 2011)}\), and the hydrodynamical evolution of the binary system, simulated with the SPH code GADGET2 \(\text{(Springel 2005)}\). We simulated 26 systems with different mass ratio of the binary components, and a large variety of different initial eccentricities, up to 0.95, which initiated with a very large apocenter, required even higher computational resources than those usually used for CE simulations. We find that CEE of initially eccentric binaries give rise to remnant post-CE binaries with non-negligible eccentricities, with up to 0.18 (0.4) for post-CE binaries with initial peri-centers outside (inside) the giant star. The post-CE eccentricities appear to be correlated with the initial eccentricities. We note that population synthesis studies typically assume post-CE binaries have circular orbits. Our results suggest that such modelling might be inconsistent both with our models and observed systems, and that accounting for remnant eccentricities could affect the typical timescales and delay time distributions for the mergers of compact binaries and their resulting explosive electromagnetic and GW transients, such as supernovae gamma-ray bursts and GW-sources.

Finally, we find that the amount of mass loss during the evolution and the structure of the post-CE nebulae strongly depend on the location of the pericenter and the initial orbital energy. All simulations with the initial pericenter located outside the envelope show relatively low final eccentricities, in agreement with current observed post-CE systems. This suggests that the still bound envelope that remain at the end of the CEE, is eventually ejected on much longer timescales than the CEE inspiral, as suggested by Michaely & Perets \(\text{(2019)}\); Igoshev et al. \(\text{(2019)}\).

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DATA AVAILABILITY

All data underlying this research is available upon reasonable request to the corresponding authors.

REFERENCES

Antonini F., Perets H. B., 2012, ApJ, 757, 27
Artymowicz P., Clarke C. J., Lubow S. H., Pringle J. E., 1991, ApJ, 370, L35
Bonacic Marinovic A. A., Glebbeek E., Pols O. R., 2008, A&A, 480, 797
Clayton M., Poidsiadowski P., Ivanova N., Justham S., 2017, mnras, 470, 1788
Delfosse X., Forveille T., Beuzit J. L., Udry S., Mayor M., Perrier C., 1999, A&A, 344, 897
Edelmann H., Heber U., Altmann M., Karl C., Lisker T., 2005, A&A, 442, 1023
Eggleton P. P., 1983, ApJ, 268, 368
Glanz H., Perets H. B., 2018, MNRAS, 478, L12
Glanz H., Perets H. B., 2021, MNRAS, 500, 1921
Grichener A., Sabach E., Soker N., 2018, MNRAS, 478, 1818
Han Z., Poidsiadowski P., Eggleton P. P., 1994, mnras, 270, 121
Han Z., Poidsiadowski P., Maxted P. F. L., Marsh T. R., Ivanova N., 2002, MNRAS, 336, 449
Harpet A., 1998, apj, 498, 293
Harris C. R., et al., 2020, Nature, 585, 357
Hunter J. D., 2007, Computing in Science & Engineering, 9, 90
Hut P., 1981, A&A, 99, 126
Iaconi R., Reichardt T., Staff J., De Marco O., Pasy J.-C., Price D., Wurster J., Herwig F., 2017, mnras, 464, 4028
Igoshev A. P., Perets H. B., Michaely E., 2019, arXiv e-prints, arXiv:1907.10068
Ivanova N., et al., 2013, aap, 21, 59
Ivanova N., Justham S., Poidsiadowski P., 2015, mnras, 447, 2181
Izzard R. G., Hall P. D., Tauris T. M., Tout C. A., 2012, in IAU Symposium. pp 95–102
Kashi A., Soker N., 2011, MNRAS, 417, 446
Kashi A., Soker N., 2018, MNRAS, 480, 3195
Kawka A., Vennes S., O’Toole S., Németh P., Burton D., Kotze E., Buckley D. A. H., 2015, MNRAS, 450, 3514
Kozai Y., 1962, AJ, 67, 591
Kramer M., Schneider F. R. N., Ohmann S. T., Geier S., Schaffenroth Y., Pakmor R., Röpke F. K., 2020, A&A, 642, A97
Kruckow M. U., Neunteufel P. G., Di Stefano R., Gao Y., Kobayashi C., 2021, arXiv e-prints, arXiv:2107.05221
Kupfer T., et al., 2015, A&A, 576, A44
Lidov M. L., 1962, Plan. Space Sci., 9, 719
Livio M., Soker N., 1988, ApJ, 329, 764
Michaely E., Perets H. B., 2014, ApJ, 794, 122
Michaely E., Perets H. B., 2019, MNRAS, 484, 4711
Moe M., Di Stefano R., 2017, ApJS, 230, 15
Nandez J. L. A., Ivanova N., Lombardi J. C., 2015, MNRAS, 450, L39
Naoz S., 2016, ARA&A, 54, 441
Nelemans G., Tout C. A., 2005, MNRAS, 356, 753
Nelemans G., Verbunt F., Yungelson L. R., Portegies Zwart S. F., 2000, A&A, 360, 1011
Paczynski B., 1976, in Eggleton P., Mitton S., Whelan J., eds, IAU Symposium Vol. 73, Structure and Evolution of Close Binary Systems. p. 75
Passy J.-C., et al., 2012, apj, 744, 52
Paxton B., Bildsten L., Dotter A., Herwig F., Lesaffre P., Timmes F., 2011, ApJS, 192, 3
Pejcha O., Metzger B. D., Tomida K., 2016, MNRAS, 461, 2527
Pelupessy F. I., Janes J., Portegies Zwart S., 2012, New Astron., 17, 711
Perets H. B., Fabrycky D. C., 2009, ApJ, 697, 1048
Common envelope evolution of eccentric binaries

Perets H. B., Kratter K. M., 2012, apj, 760, 99
Peters P. C., 1964, Phys. Rev., 136, B1224
Pontzen A., Roškar R., Stinson G. S., Woods R., Reed D. M., Coles J., Quinn T. R., 2013, pynbody: Astrophysics Simulation Analysis for Python
Portegies Zwart S., McMillan S., 2018, Astrophysical Recipes. 2514-3433, IOP Publishing, doi:10.1088/978-0-7503-1320-9, http://dx.doi.org/10.1088/978-0-7503-1320-9
Portegies Zwart S., et al., 2009, New Astron., 14, 369
Ratzloff J. K., et al., 2020, ApJ, 902, 92
Reichardt T. A., De Marco O., Iaconi R., Tout C. A., Price D. J., 2019, MNRAS, 484, 631
Reichardt T. A., De Marco O., Iaconi R., Chamandy L., Price D. J., 2020, MNRAS, 494, 5333
Ricker P. M., Taam R. E., 2012, ApJ, 746, 74
Sabach E., Hillel S., Schreier R., Soker N., 2017, MNRAS, 472, 4361
Sand C., Ohlmann S. T., Schneider F. R. N., Pakmor R., Röpke F. K., 2020, A&A, 644, A60
Schreier R., Hillel S., Soker N., 2019, MNRAS, 490, 4748
Shappee B. J., Thompson T. A., 2013, ApJ, 766, 64
Shiber S., Iaconi R., De Marco O., Soker N., 2019, MNRAS, 488, 5615
Siess L., Davis P. J., Jorissen A., 2014, Astronomy & Astrophysics, 565, A57
Soker N., 2000, A&A, 357, 557
Soker N., 2017, MNRAS, 471, 4839
Soker N., Harpaz A., 2003, MNRAS, 343, 456
Springel V., 2005, MNRAS, 364, 1105
Stepien K., 2011, A&A, 531, A18
Toonen S., Hamers A., Zwart S. P., 2016, The evolution of hierarchical triple star-systems (arXiv:1612.06172)
Tylenda R., et al., 2011, A&A, 528, A114
Verbunt F., Phinney E. S., 1995, A&A, 296, 709
Vick M., MacLeod M., Lai D., Loeb A., 2021, Tidal dissipation impact on the eccentric onset of common envelope phases in massive binary star systems (arXiv:2008.05476), doi:10.1093/mnras/stab850
Vigna-Gómez A., et al., 2020, Publ. Astron. Soc. Australia, 37, e038
Vos J., Østensen R. H., Vučković M., Van Winckel H., 2017, A&A, 605, A109
Vos J., Németh P., Vučković M., Østensen R., Parsons S., 2018, MNRAS, 473, 693
Zahn J. P., 1977, A&A, 500, 121

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