Early and Late-time Cosmic Acceleration in Non-minimal Yang-Mills-\(f(G)\) Gravity

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Abstract

In this paper we show that power-law inflation can be realized in non-minimal gravitational coupling of Yang-Mills field with a general function of the Gauss-Bonnet invariant in the framework of Einstein gravity. Such a non-minimal coupling may appear due to quantum corrections. We also discuss the non-minimal Yang-Mills-\(f(G)\) gravity in the framework of modified Gauss-Bonnet action which is widely studied recently. It is shown that both inflation and late-time cosmic acceleration are possible in such a theory.

Keywords: Inflation; Late-time acceleration; Non-minimal coupling; Gauss-Bonnet gravity.

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1 Introduction

Inflation in the early time and acceleration in the current expansion of the universe are two periods of accelerated expansion in our universe which are confirmed by cosmological observations [1-4]. There are two ways to explain the current accelerated expansion of the universe. The first one is introducing some unknown matter, which is called dark energy [5] in the framework of general relativity and the second one is modified gravity.

The modified gravity in the simplest type, uses an arbitrary function $f$ of the Ricci scalar instead of $R$ in Einstein-Hilbert action which is known as $f(R)$ gravity (see [6-8] for reviews). There are also other modified gravity models which are the generalizations of $f(R)$ gravity and among them, the modified Gauss-Bonnet (GB) gravity i.e. $f(G)$ gravity, is more interesting [9,10]. In order to play some role in the Friedmann equation, it is required that the GB combination $G$, a topological invariant in four dimensions, couples to a scalar field $\phi$ or the Lagrangian density be a function of $G$ i.e $f(G)$. The GB coupling with a scalar field appears in the low energy effective action of string/M-theory [11] and the cosmological solution in such a theory have been studied in great details [12]. It was shown that, if the GB term is responsible for DE, this model does not satisfy local gravity constraints easily [13].

Furthermore, the modified $f(G)$ gravity has the possibility to describe the inflationary era, a transition from a deceleration phase to an acceleration phase, crossing the phantom divide line and passing the solar system tests for a reasonable defined function $f$ [14-16]. The $f(G)$ models might be less constrained by local gravity constraints compared to the $f(R)$ models [17]. Hence modified $f(G)$ gravity represents a quite interesting gravitational alternative for dark energy (for a recent review see [8]).

The non-minimal coupling of the Ricci scalar and matter Lagrangian, can be seen as a source of inflation and late-time accelerated expansion of the universe [18-20]. Such a non-minimal coupling has been studied widely in the literature. For example, the non-minimal coupling between $f(R)/f(G)$ gravity and the kinetic part of Lagrangian of a massless scalar field has been investigated in Ref. [21]. Non-minimal coupling of a viable model of $f(R)$ gravity and electromagnetic Lagrangian has been considered in Ref. [22] and it has been shown that inflation and current cosmic acceleration can be explained in such a model. The coupling between scalar curvature and Lagrangian of the electromagnetic field arises in curved space-time due to one-loop vacuum polarization effects in Quantum Electrodynamics (QED) [23] and breaks the conformal invariance of the electromagnetic field, so that electromagnetic quantum fluctuations can be generated at the inflationary stage and they act as a source for inflation.

It has been shown that both inflation and late-time accelerated expansion of the universe can be realized in a modified non-minimal Yang-Mills-$f(R)$ gravity [24]. Also, this result can be realized in a non-minimal vector-$f(R)$ gravity in the framework of modified gravity [24]. Moreover, the conditions for the non-minimal gravitational coupling of the electromagnetic field in order to remove the finite-time singularities have been investigated in Ref. [25]. Ref. [26] has considered $f(R)$ gravity coupling to non-linear electrodynamics. The criteria for the validity of a non-minimal coupling between scalar curvature and matter Lagrangian have
been studied in [27-30].
Furthermore, realizing power-law inflation in non-minimal gravitational coupling of electromagnetic field with a general function of Gauss-Bonnet invariant has been done in [31]. Also, it has been demonstrated that both inflation and late-time acceleration of the universe can be realized in a modified Maxwell-\(f(G)\) gravity proposed in Ref. [32] in the framework of modified Gauss-Bonnet gravity.

In this paper we study early and late-time cosmic acceleration in non-minimal Yang-Mills-\(f(G)\) gravity in which Yang Mills (YM) field couples to a function of Gauss-Bonnet invariant. Non-minimal coupling appears in some string compactification where extra curvature terms exist in front of YM Lagrangian. Also, since the energy scale of the YM theory is higher than the electroweak scale, the existence of YM field with a non-minimal gravitational coupling might have influence on models of grand unified theories (GUT) [24]. In the past studies, the effective YM condensation as a candidate for dark energy has been proposed in [33,34] and the possibility for cosmic acceleration driven by a field with an anisotropic equation of state has been studied in [35]. We show that power-law inflation can be realized in non-minimal gravitational coupling of the YM field in the Einstein frame. Additionally, we demonstrate that both inflation and late-time acceleration of the universe can be realized in YM-\(f(G)\) model in the framework of modified Gauss-Bonnet gravity in which the function for \(f(G)\) is consistent with the solar system tests.

An outline of this paper is as follows. In section 2 we examine power-law inflation in a model of non-minimally coupled YM field with \(f(G)\) gravity in the general relativity (GR) framework. In section 3 we show that both inflation and late-time cosmic acceleration can be realized in such a model but in the framework of modified Gauss-Bonnet gravity proposed in [32]. Section 4 is devoted to our conclusion.

## 2 Power-law inflation in non-minimal Yang-Mills gravity

Our starting action is as follows:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{4} \left( F_{\mu\nu}^a F^{\mu\nu} + f(G) F_{\mu\nu}^a F^{\mu\nu} \right) \left[ 1 + b g^2 \ln \left| \left( \frac{1}{2} \frac{F_{\mu\nu}^a F^{\mu\nu}}{\mu^4} \right) \right] \right],
\]  

(1)

where \(g\) is the determinant of metric tensor \(g_{\mu\nu}\), \(R\) is the Ricci scalar and \(f(G)\) is a general function of Gauss-Bonnet invariant, \(G\), which is non-minimally coupled with YM Lagrangian. The effective YM Lagrangian up to one-loop order is as follows [36, 37]:

\[
- \frac{1}{4} \left( F_{\mu\nu}^a F^{\mu\nu} + f(G) F_{\mu\nu}^a F^{\mu\nu} \right) \left[ 1 + b g^2 \ln \left| \left( \frac{1}{2} \frac{F_{\mu\nu}^a F^{\mu\nu}}{\mu^4} \right) \right] \right].
\]  

(2)

The field strength tensor of YM Lagrangian is \(F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c\), where \(A_\mu^a\) is the \(SU(N)\) YM field and \(f^{abc}\) is the structure constants and completely antisymmetric [38].
Moreover, \( b \) is a constant and \( \tilde{g} \) is also constant which is a function of field strength and \( \mu \) is the mass scale of the renormalization point [37]. We note that the action (1) without the third term, corresponds to usual Einstein-YM theory. Varying the action (1) with respect to \( A^a_\mu \) leads to the following equations of motion for \( SU(N) \) YM field,

\[
\frac{1}{\sqrt{-g}} \partial_\mu \left[ \sqrt{-g} \right. (1 + f(G)) \varepsilon F^{a\mu
u} \left( 1 + f(G) \right) \varepsilon f^{abc} A^b_\mu F^{c\mu\nu} = 0, \tag{3}
\]

where \( \varepsilon \) is the effective dielectric constant which depends on field strength [37]. Now variation of (1) with respect to the metric \( g_{\mu\nu} \) leads to

\[
0 = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \frac{1}{2\kappa^2} \left( 1 \frac{\partial_{\mu} R - R_{\mu\nu}}{2} + T^{eff}_{\mu\nu} \right). \tag{4}
\]

Here the effective energy momentum tensor \( T^{eff}_{\mu\nu} \) is defined by

\[
T^{eff}_{\mu\nu} = \frac{1}{2} \left[ 1 + f(G) \right] \left( \varepsilon g^{\gamma\delta} F^a_{\mu\gamma} F^a_{\nu\delta} - \frac{1}{4} g_{\mu\nu} F^a_{\gamma\delta} F^{a\gamma\delta} \right) + \frac{1}{2} \left\{ f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B R_{\mu\nu} \right. \\
- 2 f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B R_{\mu} R_{\nu\rho} + f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B R_{\mu\rho\lambda} R_{\nu\sigma} + 2 f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B R_{\mu\rho\sigma\nu} R^{\mu\sigma} \\
+ \left. \left[ (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B \right] R + 2 \left[ \nabla^\rho \nabla_\mu \left( f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B \right) \right] R_{\nu\rho} \right. \\
+ 2 \left[ \nabla^\rho \nabla_\nu \left( f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B \right) \right] R_{\mu\rho} + 2 \left[ [f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B] R_{\lambda\rho} + 2 \left[ \nabla^\rho \partial^{\lambda} \left( f'(G) F^a_{\gamma\delta} F^{a\gamma\delta} B \right) \right] R_{\mu\rho\nu\lambda} \right\}, \tag{5}
\]

where

\[
B = \left[ 1 + \tilde{g}e^{2} \ln \left| \frac{-(1/2) F^a_{\gamma\delta} F^{a\gamma\delta}}{\mu^4} \right| \right]. \tag{6}
\]

and

\[
\varepsilon = 1 + \tilde{g}e^{2} \ln \left| e \left| \frac{-(1/2) F^a_{\gamma\delta} F^{a\gamma\delta}}{\mu^4} \right| \right|. \tag{7}
\]

In equation (5), \( f'(G) = \frac{df(G)}{dG} \), \( \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu \) is the d’Alembertian operator and \( e \approx 2.72 \) is the Napierian number.

In a flat Friedmann-Robertson-Walker (FRW) spacetime with the metric

\[
ds^2 = -dt^2 + a^2(t) (dr^2 + r^2 d\Omega^2), \tag{8}
\]

the components of the Ricci tensor \( R_{\mu\nu} \) and the Ricci scalar \( R \) are given by

\[
R_{00} = -3(\dot{H} + H^2), \quad R_{ij} = a^2(t) \left( \dot{H} + 3H^2 \right) \delta_{ij}, \quad R = 6(\dot{H} + 2H^2), \tag{9}
\]

where \( H = \frac{\dot{a}(t)}{a(t)} \) is the Hubble parameter and \( a(t) \) is the scale factor. Also the Gauss- Bonnet invariant in this background is

\[
G = 24(\dot{H}H^2 + H^4). \tag{10}
\]
The $(0, 0)$ component and sum of $(i, i)$ components of equation (4) in FRW spacetime have the following forms respectively

\[ H^2 = \frac{\kappa^2}{3} \left[ \frac{1}{2} (1 + f(G)) (b\dot{g}^2 \chi + \varepsilon \psi) + 6 \left( f'(G) \dot{H}H^2 + H^4 \right) \right. \]

\[ - 24H^3 (\dot{H}H^2 + 2\dot{H}^2 + 4\dot{H}H^3) f''(G) \left. \right] F^\alpha_{\gamma \delta} F^{\alpha \gamma \delta} \mathcal{B} - 6H^3 f'(G) \frac{\partial}{\partial t} \left( F^\alpha_{\gamma \delta} F^{\alpha \gamma \delta} \mathcal{B} \right) \right], \]

(11)

and

\[ 2\dot{H} + 3H^2 = \kappa^2 \left[ \frac{1}{2} (1 + f(G)) \chi ( - \frac{1}{3} \varepsilon + b\dot{g}^2) + \left( 6f'(G) \dot{H}H^2 + H^4 \right) \right. \]

\[ - 24 \left[ 2f''(G) (8\dot{H}H^3 + 6\dot{H}^3H^2 + 24\dot{H}^2H^4 + 6\dot{H}H^5 + 8\dot{H}^2H^6 + \ddot{H}H^4) \right. \]

\[ + f''(G) (\dot{H}H^2 + 2\dot{H}^2 + 4\dot{H}H^3)^2 \left] \right. F^\alpha_{\gamma \delta} F^{\alpha \gamma \delta} \mathcal{B} - 4 \left[ f'(G) (H \dot{H} + H^2) \right. \]

\[ + 24f''(G) (\dot{H}H^4 + 2\dot{H}^2H^3 + 4\dot{H}H^5) \left. \right] \frac{\partial}{\partial t} \left( F^\alpha_{\gamma \delta} F^{\alpha \gamma \delta} \mathcal{B} \right) - 2f'(G) H^2 \frac{\partial^2}{\partial t^2} \left( F^\alpha_{\gamma \delta} F^{\alpha \gamma \delta} \mathcal{B} \right) \right], \]

(12)

where we have neglected the second order spatial derivative of the quadratic quantity $F^\alpha_{\gamma \delta} F^{\alpha \gamma \delta}$. In addition, $\chi$ and $\psi$ in above equations are given by [24],

\[ \chi = |E^\alpha_i(t)|^2 - |B^\alpha_i(t)|^2, \]

(13)

\[ \psi = |E^\alpha_i(t)|^2 + |B^\alpha_i(t)|^2, \]

(14)

and

\[ F^\alpha_{\gamma \delta} F^{\alpha \gamma \delta} \mathcal{B} = -2\chi (\varepsilon - b\dot{g}^2). \]

(15)

Here $E^\alpha_i(t)$ and $B^\alpha_i(t)$ are proper electric and magnetic fields in the $SU(N)$ YM theory respectively.

As a simple case our interest is the generation of large scale (YM) magnetic fields instead of (YM) electric fields, so we neglect terms in (YM) electric fields from this point and hence we have, $\chi = -|B^\alpha_i(t)|^2$ and $\psi = |B^\alpha_i(t)|^2$. Furthermore, for more simplicity, we assume that only one component of (YM) magnetic fields is non-zero and the other two components are zero i.e. $B^\alpha_1 = B^\alpha_2 = 0$, $B^\alpha_3 \neq 0$. We note that by considering these conditions, the off-diagonal components of the last term on the right hand side of $T^{eff}_{\mu \nu}$ are zero and so, all off-diagonal components of $T^{eff}_{\mu \nu}$ are zero.

The amplitude of (YM) magnetic fields on a comoving scale $L = \frac{2\pi}{k}$ with the comoving wave number $k$ in the position space is given by

\[ |B^\alpha_i(t)|^2 = \frac{|B^\alpha_0|^2}{a^4}, \]

(16)
where $|B_0^0|$ is a constant. Now substituting (16) in (11) and (12) and using equations (13)-
(15) leads to
\[
H^2 = \kappa^2 \left[ \frac{1}{6} (1 + f(G))(\varepsilon - b\dot{g}^2) + 4 \left( f'(G)(\dot{H}H^2 + H^4) - 24\dot{H}^3(\ddot{H}H^2 + 2H\dot{H}^2 + 4\dot{H}H^3) f''(G)(\varepsilon - b\dot{g}^2) + 16H^4 f'(G)\varepsilon \right) \right] \frac{|B_0^0|^2}{a^4},
\]
and
\[
2\dot{H} + 3H^2 = \kappa^2 \left[ \frac{1}{6} (1 + f(G))(\varepsilon - 3b\dot{g}^2) + 2 \left( 6f'(G)(\dot{H}H^2 + H^4) - 24 \left[ 2f''(G)(8\ddot{H}\dot{H}H^3 + 6\dot{H}^3H^2 + 24\dot{H}^2H^4 + 6\ddot{H}H^5 + 8\dot{H}^2H^6 + \ddot{H}H^4) + f''(G)(\ddot{H}H^2 + 2H\dot{H}^2 + 4\dot{H}H^3)^2 \right](\varepsilon - b\dot{g}^2) + 32 f'(G)(\frac{3}{2}H^2\dot{H}^2 - H^4) + 24f''(G)H(3\ddot{H}H^4 + 2\dot{H}^2H^3 + 4\dot{H}H^5) \right] \varepsilon + 64f'(G)H^4b\dot{g}^2 \right] \frac{|B_0^0|^2}{a^4},
\]
respectively. Then from equations (17) and (18) one can obtain
\[
\dot{H} + \frac{\varepsilon}{\varepsilon - b\dot{g}^2}H^2 = \kappa^2 \left[ 4f'(G) \left\{ 7\dot{H}H^2\varepsilon - \left[ (\frac{5\varepsilon - 9b\dot{g}^2}{\varepsilon - b\dot{g}^2})\varepsilon - 8b\dot{g}^2 \right] H^4 \right\} + 48f''(G) \left\{ 3\ddot{H}H^5(\varepsilon + b\dot{g}^2) - 6H^4\dot{H}^2(\varepsilon - 3b\dot{g}^2) + 4\ddot{H}H^6(7\varepsilon - b\dot{g}^2) - (8\ddot{H}\dot{H}H^3 + 6\dot{H}^3H^2 + \ddot{H}H^4)(\varepsilon - b\dot{g}^2) \right\} + 24f''(G)(\ddot{H}H^2 + 2\dot{H}^2H^3)^2(\varepsilon - b\dot{g}^2) \right\} \frac{|B_0^0|^2}{a^4}.
\]
From this point, we are going to consider $\varepsilon$ as a constant. The reason for doing so, is that the dependence of $\varepsilon$ on the field strength and therefore on time, is logarithmic as one can see from equation (7), while this is not the case for the other quantities in above equation. Now we examine the following function for $f(G)$, which has been proposed in Ref. [21]:
\[
f(G) = \frac{G^n}{c_1G^n + c_2},
\]
where $c_1$ and $c_2$ are constants and $n$ is a positive integer. It is known [32] that such a model naturally leads to unification of the inflation with late-time acceleration being consistent with local tests and cosmological bounds. We can see in the late-time universe, the ordinary YM theory can be naturally recovered because the value of Gauss- Bonnet invariant in this
time goes to zero.

To explore Power-Law inflation, we assume \( a = a_0 t^{h_0} \), where \( h_0 \) is a positive constant. Therefore, we have

\[
H = \frac{h_0}{t}, \quad \dot{H} = -\frac{h_0}{t^2}, \quad \ddot{H} = \frac{2h_0}{t^3}, \quad \dddot{H} = -\frac{6h_0}{t^4}.
\]  

(21)

Note that the Gauss-Bonnet invariant in this case is as follows:

\[
G = 24 \frac{h_0^3}{t^4} (h_0 - 1).
\]  

(22)

Also we use the following approximate relations which work at the inflationary epoch:

\[
f(G) \approx \frac{1}{c_1} \left( 1 - \frac{c_2}{c_1} G^n \right),
\]  

(23)

\[
f'(G) \approx \frac{nc_2}{c_1^2} G^{-(n+1)},
\]  

(24)

\[
f''(G) \approx \frac{-n(n+1)c_2}{c_1^2} G^{-(n+2)},
\]  

(25)

\[
f'''(G) \approx \frac{n(n+1)(n+2)c_2}{c_1^2} G^{-(n+3)}.
\]  

(26)

By substituting above approximate relations for \( f(G) \) and its derivatives and (21) in (19), one can obtain

\[
h_0 = \frac{2n+1}{2}.
\]  

(27)

\[
a_0 = \left\{ \frac{2}{3} \right\}^{(n+1)} \frac{2n}{(2n+1)^3(n+1)(2n-1)(n+2)c_1^2 c_2^2 \left( 4n^2 - 1 \right) \varepsilon + 2(2n+1)bg^2 |B_0|^2 \kappa^2} \right\}^{\frac{1}{4}},
\]  

(28)

where \( \alpha = 32n^5 + 384n^4 + \frac{2162}{3} n^3 + \frac{1685}{3} n^2 + \frac{607}{3} n + \frac{82}{3} \), \( \beta = -\left( 416n^5 + 1200n^4 + \frac{4564}{3} n^3 + \frac{3106}{3} n^2 + \frac{1088}{3} n + \frac{149}{3} \right) \) and \( \gamma = 160n^5 + 480n^4 + \frac{2066}{3} n^3 + \frac{1589}{3} n^2 + \frac{607}{3} n + \frac{88}{3} \).

We see that if \( n \gg 1 \) then \( h_0 \) becomes very large and so the power-law inflation can be realized. From this discussion, we conclude that non-minimally coupled YM field with \( f(G) \) gravity can be seen as a source of inflation in the early universe. This result is the same as that of non-minimal Maxwell- \( f(G) \) gravity [31].

We note that in this paper we considered only the case in which the values of the terms proportional to \( f'(G) \), \( f''(G) \) and \( f'''(G) \) in equations (17) and (18) are dominant to the values of the term proportional to \( (1 + f(G)) \). Among the terms proportional to \( f'(G) \), \( f''(G) \) and \( f'''(G) \), the term proportional to \( f'(G) \) is dominant and its value from equation (24) is of order \( f'(G)H^4 \approx \frac{nc_2}{c_1} H^4 G^{-(n+1)} \). Also using (21) and (22) one can see that \( G \) is of order \( 20H^4 \). Now the condition for dominance \( f'(G) \) in the source term would be \( (1 + f(G)) \left[ f'(G)H^4 \right] \approx \frac{20a_0}{nc_2} G^a \ll 1 \). This leads to extremely small \( \left( \frac{a_0}{c_2} \right) \) because at the
inflationary epoch $G \gg 1$ and $n \gg 1$. In such a case, in equation (19) the value of right hand side which is order $\kappa^2 f'(G)H^4 \frac{|B_0|^2}{a^4}$ can be order $H^2$. So, the right and left hand sides of equation (19) can balance each other and there is no contradiction between our result and equation (19). In the opposite case i.e. if the term proportional to $(1 + f(G))$ is dominant, the power-law inflation cannot be realized. Our reasoning is as follows: in this case the equations (17) and (18) can be written approximately as $H^2 \approx \frac{\kappa^2}{6} (1 + f(G)) \frac{|B_0|^2}{a^4}$ and $2\dot{H} + 3H^2 \approx \frac{\kappa^2}{6} (1 + f(G)) \frac{|B_0|^2}{a^4}$, respectively. Therefore one finds from equations (17) and (18) that $H^2$ and $2\dot{H} + 3H^2$ are of the same order and their difference $2\dot{H} + 2H^2$ must be much smaller than $H^2$. Then in equation (19) the quantity $\left( \dot{H} + \frac{\varepsilon}{\varepsilon - \beta g} H^2 \right)$ must balance with much smaller quantity than $\left( \kappa^2 (1 + f(G)) \frac{|B_0|^2}{a^4} \right)$, so $\left( \dot{H} + \frac{\varepsilon}{\varepsilon - \beta g} H^2 \right) / H^2 = \frac{\varepsilon}{\varepsilon - \beta g} - \frac{1}{h_0} \ll 1$ and this leads to $h_0 \ll 1$ because $\varepsilon, b > 0$. One can see that power-law inflation cannot be realized. From above discussion we see that power-law inflation can be realized due to not the term proportional to $(1 + f(G))$ but the term proportional to $f'(G)$ namely a non-minimal YM field coupling.

Finally, we note the following points. If one compares our result in equation (27) with those in non-minimal YM-$f(R)$ gravity in Ref. [24] can see that in our model when $n > \frac{3}{2}$ the universe is accelerating but in non-minimal YM-$f(R)$ gravity the expansion of the universe is accelerating for $n > 3$. In addition, $f(G)$ gravity is inspired from a fundamental theory such as string theory, so the non-minimal coupling of YM field with $f(G)$ may induce more interest than YM-$f(R)$ gravity.

### 3 Inflation and late-time acceleration in modified Gauss-Bonnet gravity framework

In this section, we consider a non-minimally coupled YM field in the framework of modified Gauss-Bonnet gravity proposed in Ref. [32].

We describe the model by the following action:

$$ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} (R + F(G)) - \frac{1}{4} \left( F^{a\mu\nu} F_{a\mu\nu} + f(G) F^{a\mu}_{a\mu} F_{a\mu} \right) \left[ 1 + b\tilde{g}^2 \ln \left| -\frac{1}{2} \frac{F^{a\mu}_{\mu\nu} F_{a\mu\nu}}{\mu^4} \right| \right] \right]. $$

(29)

Note that in this case $F(G)$ is the modified part of gravity and it is different from $f(G)$ in the last term in the action (1). By choosing FRW metric (8), the $(0,0)$ component and sum of $(i,i)$ components of equation of motion for $g_{\mu\nu}$, have the following forms respectively:

$$ H^2 - \frac{1}{6} (GF'(G) - F(G)) + 4H^3 \dot{G} F''(G) = \frac{\kappa^2}{3} T_{00}^{\text{eff}}, $$

(30)

and

$$ 2\dot{H} + 3H^2 + \frac{1}{2} (GF'(G) - F(G)) = -\kappa^2 T_{ii}^{\text{eff}}, $$

(31)
where $T^a_{\mu\nu}$ is given by equation (5). As the previous section, we neglect the contribution of electric field and spatial derivatives of $F^a_{\mu\nu}$. Therefore, from equations (30) and (31), one can obtain

$$
\dot{H} + \frac{\varepsilon}{\varepsilon - b g^2} H^2 + \frac{1}{6} (G F'(G) - F(G)) \left( \frac{2 \varepsilon - 3 b g^2}{\varepsilon - b g^2} \right) - 2 H^2 F''(G) \left[ \dot{G} + H \dot{G} \left( \frac{\varepsilon - 3 b g^2}{\varepsilon - b g^2} \right) \right]
$$

$$
-2 H^2 G^2 F'''(G) = \kappa^2 \left[ 4 f'(G) \left\{ 7 H^2 \varepsilon - \left[ \left( \frac{5 \varepsilon - 9 b g^2}{\varepsilon - b g^2} \right) \varepsilon - 8 b g^2 \right] H^4 \right\} + 48 f''(G) \left\{ 3 \ddot{H} H^5 (\varepsilon + b g^2) - 6 H^4 \dot{H}^2 (\varepsilon - 3 b g^2) + 4 \ddot{H} H^6 (7 \varepsilon - b g^2) - (8 \ddot{H} H^3 + 6 H^3 \dot{H}^2 + \dddot{H} H^4) (\varepsilon - b g^2) \right\} + 24 f'''(G) (\dddot{H} H^2 + 2 H \ddot{H}^2 + 4 \dddot{H})^2 (\varepsilon - b g^2) \right\} |B_0|^2 \frac{a^4}{a^4}.
$$

Here we take $F(G)$ from Ref. [32],

$$
F(G) = \frac{(G - G_0)^{2n+1} + G_0^{2n+1}}{c_3 + c_4 ((G - G_0)^{2n+1} + G_0^{2n+1})},
$$

where $c_3$, $c_4$ are constants and $n$ is a positive integer. $G_0$ correspond to the present value of the Gauss-Bonnet invariant. The typical property of such theory is the presence of effective cosmological constant epochs in such away that early-time inflation and late-time cosmic acceleration are naturally unified within single model.

Since $F'(G) = 0$ when $G = G_0$ and $G = \infty$, $F(G)$ can be regarded as an effective cosmological constant. We may consider $F(\infty)$ as the cosmological constant for the inflationary stage and $F(G_0)$ as that at the present time,

$$
\lim_{G \to \infty} F(G) = \frac{1}{c_4} = \Lambda,
$$

$$
F(G_0) = \frac{G_0^{2n+1}}{c_3 + c_4 G_0^{2n+1}} = 2G_0.
$$

From the above equations, we find

$$
c_3 = \frac{G_0^{2n}}{2} - \frac{G_0^{2n+1}}{\Lambda} \approx \frac{G_0^{2n}}{2}, \quad c_4 = \frac{1}{\Lambda},
$$

because $\frac{G_0}{\Lambda} \ll 1$.

Also, $f(G)$ is given by

$$
f(G) = - \frac{(G - G_0)^{2m+1} + G_0^{2m+1}}{c_5 + c_6 ((G - G_0)^{2m+1} + G_0^{2m+1})},
$$

(37)
where \( c_5, c_6 \) are constants and \( m \) is a positive integer. At the inflationary epoch we can use the following approximate relations:

\[
F(G) \approx \frac{1}{c_4} \left[ 1 - \frac{c_3}{c_4} (G)^{-2(n+1)} \right],
\]

(38)

and

\[
f(G) \approx -\frac{1}{c_6} \left[ 1 - \frac{c_5}{c_6} (G)^{-2(m+1)} \right].
\]

(39)

Because \( G \to \infty \) at the inflationary stage and also \( \lim_{G \to \infty} F(G) = \Lambda \) and \( \lim_{G \to \infty} f(G) = \text{const} \), equation (32) at this epoch is reduced to

\[
\dot{H} + \frac{\varepsilon}{\varepsilon - b\tilde{g}^2} H^2 = \frac{\Lambda}{6} \left( \frac{2\varepsilon - 3b\tilde{g}^2}{\varepsilon - b\tilde{g}^2} \right).
\]

(40)

It follows from above equation that

\[
a(t) \propto \exp \left( \frac{\Lambda}{3} \right)^{\frac{3}{2}} t,
\]

(41)

Hence exponential inflation can be realized. Thus, we conclude that the terms in \( F(G) \) on the left hand side of Eq. (32) can be a source of inflation, in addition to \( f(G) \) on the right hand side of Eq. (32). Note that if we do not consider the contribution of the terms in \( F(G) \) to inflation, equation (32) is reduced to equation (19). In this case, substituting \( a = a_0 t^{h_0} \) and the approximate expressions of \( f'(G) \), \( f''(G) \) and \( f'''(G) \) derived from equation (39) into equation (32) leads to \( h_0' = \frac{4m+3}{2} \). Hence if \( m \gg 1 \), \( h_0' \) becomes much larger than unity and power-law inflation can be realized.

We emphasize that there are two sources of inflation in the present model, one from the modified part of gravity \( F(G) \) and the other from the non-minimal coupling of YM field with \( f(G) \). Indeed, in this model even if the value of \( \Lambda \) is so small that the modification of gravity cannot contribute to inflation then inflation can be realized due to the non-minimal gravitational coupling of the YM field. This is an important cosmological consequence of the present model.

At the present time, because \( G - G_0 \ll 1 \), if \( m > n \), \( f(G) \) becomes constant more rapidly than \( F(G) \) in the limit \( G \to G_0 \). For such a case, when \( G \to G_0 \) Eq. (32) leads to

\[
\dot{H} + \frac{\varepsilon}{\varepsilon - b\tilde{g}^2} H^2 = \frac{G_0}{3} \left( \frac{2\varepsilon - 3b\tilde{g}^2}{\varepsilon - b\tilde{g}^2} \right),
\]

(42)

so, from this equation one can obtain

\[
a(t) \propto \exp \left( \frac{2G_0}{3} \right)^{\frac{1}{2}} t,
\]

(43)

so that the late-time acceleration of the universe can be realized. These results are also in agreement with the results of Ref. [31] where non-minimal Maxwell- \( F(G) \) gravity has been investigated. We mention that even if the value \( G_0 \) is so small that the modification
of gravity cannot contribute to the late-time acceleration of the universe, the late-time accelerated expansion can be realized due to the non-minimal coupling of the YM field. Indeed our results are the generalization of the results for non-minimal Maxwell theory with the coupling of the electromagnetic field to a function of Gauss-Bonnet invariant [31]. In here we considered a non-Abelian gauge field (the YM field) non-minimally coupled with \( f(G) \) gravity. The YM fields are indispensable to particle physics, there is no room for adjusting the functional form of the Lagrangian as it is predicted by quantum field theory. As a model for the cosmic dark energy, it has no free parameters except the present cosmic energy scale and the cosmic evolution only depends on the initial conditions [33].

4 Conclusion

To summarize, the non-minimal gravitational coupling of YM field with Gauss-Bonnet invariant function, \( f(G) \), has been considered in Friedmann-Robertson-Walker background metric. Such a non-minimal coupling has been examined in the framework of general relativity. We have shown that power law inflation can be realized due to non-minimal coupling of YM field in this model which is described by action (1). We have also studied cosmology in non-minimally coupled YM field in the framework of modified Gauss-Bonnet gravity, \( F(G) \). It has been shown that both inflation and late-time acceleration of the universe can be realized in such a model proposed in Ref. [32].

Clearly, more checks of this theory such as stability/instability of inflation should be done in order to conclude if the model is realistic or not. The conditions for stability of \( f(G) \) rarity have been derived in [17]. It has been shown that the condition \( \frac{d^2 f}{dG^2} > 0 \) needs to be fulfilled in order to ensure the stability of a late-time de-sitter solution as well as the existence of standard radiation and matter dominated epochs [17]. Studying stability/instability conditions for our model and models of this kind such as the Maxwell-\( f(G) \) model [31] will be our plan for future works.

It is also interesting to extend our formulation for more complicated theories. For instance one can investigate non-minimal coupling of YM Lagrangian with non-local \( f(G) \) gravity proposed in [39] or one can study our model in the case that instead of \( f(G) \) gravity an action with higher order string loop corrections replaced. This kind of superstring inspired action has been considered in [40].

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