Unification of couplings and soft supersymmetry breaking terms in 4D superstring models

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Abstract

We consider the predictions for the hierarchy of mass scales, the fine structure constant, the radii of compactification and the soft SUSY breaking terms which follow if SUSY breaking is triggered by a gaugino condensate.

1 Introduction

The superstring [1] offers the exciting possibility of predicting all the parameters of the standard model in terms of a single parameter, the string tension. However in order to realise the full predictive power of the superstring it is necessary to determine the origin and effects of supersymmetry breaking. Only after SUSY is broken are the vacuum expectation values (vevs) of moduli determined and these determine the couplings of the effective low energy theory. Also SUSY breaking must be responsible for the splitting of supermultiplets allowing for the superpartners to be heavier than their standard model partners.

In supergravity theories soft SUSY breaking terms may be added to the lagrangian without introducing new divergences and thus need not spoil the supersymmetric solution to the hierarchy problem. If SUSY breaking is dominated by the F term of a specific field the soft terms may be related in a model independent way if supersymmetry breaking occurs in a hidden sector with coupling to the visible sector via gravitational effects only[2, 3, 4]. In this letter we will investigate the predictions for the moduli and soft terms in superstring theories in which the origin of SUSY breaking is through a gaugino condensate in the hidden sector.

Gaugino condensation [5, 6] offers a very plausible origin for SUSY breaking for it is very reasonable to expect such a condensate to form at a scale between the Planck scale and the electroweak breaking scale if the hidden sector gauge group has a (running)
coupling which becomes large somewhere in this domain. Non-perturbative studies in effective supergravity theories resulting from orbifold compactification schemes suggest the dynamics of the strongly coupled gauge sector is such that the gaugino condensate will form and trigger supersymmetry breaking.

The advantage of gaugino condensation as the underlying supersymmetry breaking trigger is twofold. In the first place it is generic in 4D-superstring theories to have an hidden sector with a gauge group given by $E(6)$ or smaller. For such asymptotically free gauge theories the coupling usually becomes strong at low scales so gaugino condensation may be expected - thus there is no need to postulate an additional source of supersymmetry breaking. Secondly the scale of gaugino condensation and the associated supersymmetry breaking scale are dynamically determined and thus the mechanism offers an explanation for the magnitude of the hierarchy of masses scales. Thus, when combined with the radiative breaking of the electroweak symmetry, the resultant theory not only protects the electroweak and supersymmetry breaking scales from receiving large radiative corrections proportional to the unification or Planck scale (the usual hierarchy problem) but also predicts the supersymmetry breaking and electroweak breaking scales in terms of the Planck scale and the multiplet content of the theory. In addition, after supersymmetry breaking, the moduli of the theory are fixed giving a prediction for the value of the gauge couplings and the Yukawa couplings.

In order to realise the full predictive power of the 4D-superstring theory, it is necessary to compute the non-perturbative effects leading to gaugino condensation. The complete solution is clearly beyond our present-day technology so we are forced to employ approximation methods. In this paper we will consider the implications for the soft SUSY breaking parameters in the case that gaugino condensation is the source of supersymmetry breaking. We parameterise the strong binding effects in the hidden sector by a simple effective (locally supersymmetric) 4-gaugino coupling. We then are able to use Nambu-Jona-Lasinio (N-J-L) techniques [7] to obtain non-perturbative information about the gaugino binding. This shows that a reasonable pattern of supersymmetry breaking is possible with a single gaugino condensate. We generalize the discussion of reference [8] to consider an orbifold model which includes different moduli fields and we determine the tree level potential and the one-loop radiative corrections due to strong gaugino binding. We show that minimization of the potential leads to vacuum expectation values (v.e.v.s) for the moduli which either take dual invariant values or have a common value related to the v.e.v. of the dilaton. This naturally allows for squeezed orbifolds which were found to be better candidates for minimal string unification [2]. We also include chiral matter fields in the visible sector and determine the soft SUSY breaking terms. Unlike results obtained previously we find that the radiative corrections stabilise the potential for vanishing v.e.v.s of the matter fields. We also find values for the soft parameters which lie outside the favoured range of “model” independent parameterisations of the soft terms due to our inclusion of large non-perturbative effects beyond string tree level in the effective potential. Of course, due to the simplicity of our model for strong binding, our results should be considered as only indicative of those to be expected in a complete gauge theory. However, they do show the importance of including the full nonperturbative effects of gaugino binding before extracting predictions for SUSY breaking phenomena.

We start with the effective theory following from a 4-D superstring orbifold model.
It is specified by the Kahler potential $G = K + \ln \frac{1}{4}|W|^2$ and the gauge kinetic function $f$ given by

$$K = -\ln(S + \bar{S} + 2\Sigma_i k_\alpha \delta^G_{i\alpha} \ln T_{ri}) - \Sigma_i \ln(T + \bar{T})_i + \Sigma_i K^i_i(T, \bar{T})|\varphi_i|^2,$$

$$W = W_0 + W_m,$$

and

$$f_0 = S + 2\Sigma_i (b^i_\alpha - k_\alpha \delta^G_{i\alpha}) \ln(\eta(T_i)^2)$$

where

$$b^i_\alpha = \frac{1}{16\pi^2} (C(G_\alpha) - \Sigma_R h_R T(R_a)(1 + 2n^i_R))$$

where $C(G_\alpha)$ is the quadratic Casimir operator of the gauge group $G_\alpha$, $k_\alpha$ is the level of the corresponding Kac-Moody algebra for the gauge group $G_\alpha$ and $\delta^G_{i\alpha}$ is the Green-Schwarz term needed to cancel the gauge independent part of the duality anomaly $\frac{3}{2}$. $W_0$ in eq.(2) is the scalar potential due to the gaugino condensate (see below) and $W_m$ the matter superpotential. The subscript, 0, on the gauge kinetic function $f$ indicates that it is given at the string scale and the form of eq.(2) includes the contribution from the massive Kaluza-Klein modes.

The normalization of the kinetic term for the chiral matter superfields is given by $K^i_i(T, \bar{T})$ which is a polynomial function of the moduli fields

$$K^i_i = \Pi_j a_i T_{Rj}^{n_{ij}}$$

where $a_i$ a constant (in most cases $a_i = 1$) and $n_{ij}$ is the modular weight of the superfield $\varphi_i$ with respect to the real part, $T_{Rj} = T_j + \bar{T}_j$, of the moduli $T_j$. Eq.(3) was derived for $K^i_i(T, \bar{T})|\varphi_i|^2 \ll 1$. Note that the Kahler potential includes the Green-Schwarz term and the dilaton transforms under duality in a non-trivial way ($S \to S + 2\Sigma_i k_\alpha \delta^G_{i\alpha} \ln(i\epsilon T_i + d)$) so that $Y \equiv S + \bar{S} + 2\Sigma_i k_\alpha \delta^G_{i\alpha} \ln T_{ri}$ is duality invariant.

The form of this effective theory is fixed by the requirement that it should correspond to the strongly coupled theory in the hidden sector. We have suggested a simple approximation to this structure which, we hope, captures the essentials of the strong binding effects leading to gaugino condensate. The basis of this approximation is to start with a (N=1 locally supersymmetric) four fermion description of the strong hidden sector interaction and to use Nambu-Jona-Lasinio techniques to compute the non-perturbative effects responsible for gaugino condensation. The model is defined by

$$W_0 = \Pi_i \eta^{2-i}_j (T_i) \Phi$$

$$f = f_0 + \xi \ln(\Phi/\mu)$$

where $\Phi$ is an auxiliary chiral superfield and $\xi = \frac{2π}{3}$. Duality invariance fixes the mass parameter $\mu$ in eq.(3) to be $\mu = m \Pi_i T_{ri}^{-3/2} a_i/2 d_{l\alpha}$ with $m$ duality invariant. We choose $m = M_3^3$ so that the parameter $\mu$ becomes, at the field theoretical limit, $\mu = M_3^3$, with $M_{com} = M_s \Pi_i T_{ri}^{-1/6}$ the compactification scale. Since $\mu$ is non-holomorphic, consistency with supersymmetry requires to interpret $\mu$ as wave function renormalization of the dilaton field. In such a case $\mu$ would appear in the Kahler potential and not in the kinetic function.
From the classical equation of motion the scalar component of the auxiliary field $\Phi$ is given in terms of the gaugino bilinear by

$$\phi = \frac{e^{-K/2\xi}}{2m^2\Pi_i\eta_i \xi} \bar{\lambda}_R \lambda_L$$

while the fermion component is given in terms of the gaugino and chiral fermions.

For a pure gauge theory in the hidden sector the tree level scalar potential is

$$V_0 = h_i (G^{-1})^i_j h^j - 3e^G$$

with $i = S, T$ and $h_i$ the F-term of the chiral superfields $S$ or $T_i$.

For the choice of $W$ and $f$ given in eq.(6) and eq.(7) the F-terms for the $S$ and $T_i$ fields are

$$h_S = -e^{G/2}G_S + \frac{1}{4} f_S \bar{\lambda}_L \lambda_R = \frac{1}{2} e^{K/2} W_0 \left(1 + \frac{Y}{\xi}\right)$$

and

$$h_{T_i} = \frac{1}{2} e^K W_0 \left[\frac{1}{T_{ii}} \left(1 - \frac{a_i}{\xi} \right) + 2 \frac{\eta_i}{\eta} \left(1 - \frac{a_i}{\xi}\right)\right]$$

where $a_i = 2(k_a \delta_i^{GS} - b_{ai})$. The tree level scalar potential is then

$$V_0 = m_{3/2}^2 B_0$$

with

$$B_0 = \left(1 + \frac{Y}{\xi}\right)^2 + \Sigma_i \frac{Y}{\xi} \left(1 - \frac{a_i}{\xi}\right)^2 \frac{T_{ii}^2}{4\pi^2} |\hat{G}_2|^2 - 3$$

and $\hat{G}_2$ the Eisenstein modular form with weight 1/2. The gravitino mass is given by

$$m_{3/2}^2 = \frac{1}{4} e^{K/2} \Pi_i |\eta(T_i)|^{-4} |\phi|^2.$$

Clearly SUSY will be broken and a non-zero gravitino mass will result only if there is a non-vanishing v.e.v for $\phi$ generating a non-zero $h_S$ (cf. eq.(10)). Assuming this to be the case and solving eq.(8) gives a parameterisation of gaugino condensation equivalent to that derived by other methods [13]-[14]. To this extent the form of eq.(12) is just a re-parameterisation of the usual model of gaugino condensation. Where we depart from this analysis is our inclusion of radiative corrections in the effective potential. Summing these effects to all orders shows that the gaugino condensate dynamically forms.

Using eq.(8) the scalar potential may be written in terms of the gaugino field

$$V_0 = \frac{\xi^2 H}{16(Ref)^2} |\bar{\lambda}_R \lambda_L|^2$$

where the factor of $(Ref)^2$ in the denominator in eq.(15) appears because we have rescaled the gaugino fields appearing in this equation to have canonical kinetic terms. Thus we see that the choice of $W$ and $f$ in eqs.(6) and (7) leads to a four-fermion interaction with strength proportional to $f^{-1}$. 
In a pure gauge theory the running gauge coupling is related to \( f \) by \( g_s^2 = (Re f)^{-1} \) with

\[
\frac{1}{g_s^2(\Lambda)} = k_a \frac{1}{g_s^2} + b_a ln\left(\frac{\Lambda^2}{M_a^2}\right) + \Delta_a
\]  

(16)

where \( k_a \) is the level of the corresponding Kac-Moody algebra for the gauge group \( G_a \), \( b_a = \frac{1}{16\pi^2}(3C(G_a) - \Sigma_{R_a} h_{R_a} T(R_a)) \) is the \( N=1 \) \( \beta \)-function coefficient and \( h_{R_a} \) the number of chiral fields in a representation \( R_a \). \( M_a \) is the renormalization scale below which the coupling constants begins to run

\[
M_a^2 = \Sigma_i (T_{ri})^{\alpha_{ai}} M_s^2
\]  

(17)

where \( T_{Ri} \) and the constants \( \alpha_{ai} \) is model and gauge dependent.

\[
\alpha_{ai} = \frac{k_a \delta_i^{GS} - b_i^a}{b_a}
\]  

(18)

In the case of a single overall (1,1) moduli \( T_i = T, i = 1, 2, 3 \), with \( \alpha = \Sigma \alpha_i = -1 \) one obtains the “naive” field theoretical expression for \( M_a = (Re SRe T)^{-1/2} \). The existence of an infinite number of massive states (Kaluza and winding states), above the string scale, give rise to string threshold contribution \( \Delta_a \) [15, 16] which are relevant to the determination of the coupling constant at the string scale (cf. eq.(3)). In the effective four fermion theory defined by eqs.(6) to (15) \( f^{-1} \) sets the scale for the strength of the interaction (cf. eq.(15)). If the theory is to model the strong binding effects due to a gauge interaction it clearly should be related to the gauge coupling \( g_s^2 \). Thus \( f^{-1} \) does indeed run like \( g^2 \). However this running gives rise to an unphysical singularity in \( f^{-1} \) related to the Landau singularity which we will choose to regulate when applying eq.(8). Below the scale at which the running coupling becomes large the hidden sector states are confined and have a mass of \( O(\Lambda_c) \). Thus we expect the 4 Fermion effective interaction to be pointlike at scales below \( \Lambda_c \) and so we cutoff the growth implied by eq.(15) when the strength of the interaction reaches \( 1/\Lambda_c^2 \).

The tree level potential for \( \phi \), given by eqs.(12),(13) and (14), has no stable solution. This is consistent with the observation [18] that gaugino condensation as usually parameterised does not occur in models with a single hidden sector gauge group factor. However we have argued it is essential to go beyond tree level to include non-perturbative effects in the effective potential which may allow for a non-trivial minimum even in the simple case of a single hidden sector gauge group. This non-perturbative sum (equivalent to the NJL sum) is readily obtained simply by computing the one loop correction to \( V \). If these destabilise the potential the resultant minimum will correspond to a cancelation of tree level and one-loop terms which, as noted above, is necessarily non-perturbative in character [1].

The one-loop radiative corrections may be calculated using the Coleman-Weinberg one-loop effective potential [19, 20],

\[
V_1 = \frac{1}{32\pi^2} Str \int d^2 p p^2 ln(p^2 + M^2)
\]  

(19)

where \( M^2 \) represents the square mass matrices and \( Str \) the supertrace. As discussed above the 4 Fermion interaction has a form factor which falls rapidly above \( \Lambda_c \) effectively
giving a momentum space cutoff at the condensation scale defined as the scale where the gauge coupling constant (cf. eq.(14)) becomes strong.

The leading contribution to the one-loop potential $V_1$ comes from the supersymmetry breaking mass of the gaugino fields and keeping only these terms one has

$$V_1 = -\gamma \Lambda_c^4 J(x_g)$$

with

$$J(x) = x + x^2 \ln\left(\frac{x}{1 + x}\right) + \ln(1 + x)$$

$$\gamma = \frac{n_g}{32\pi^2} \left( n_g \text{ the dimension of the hidden gauge group} \right) \text{ and } x_g = \frac{m_g^2}{\Lambda_c^2}. \text{ The tree level gaugino mass is given by}$$

$$m_g = \frac{2}{Re f} \frac{\partial V_0}{\partial W_0} \frac{\partial^2 \bar{W}_0}{\partial \lambda_R \partial \lambda_L} = \frac{b_0}{3Re f} B_0 m_{3/2}$$

with $B_0$ given in eq.(13) and the factor $Re f$ in eq.(22) is due to non-canonical kinetic terms for the gauginos. In addition there should be a supersymmetric contribution to the mass of the hidden sector states, gauginos and gauge bosons, generated by the strong hidden sector forces which (in analogy with QCD) may be expected to be confining. Since the inclusion of these supersymmetric masses only results in a shift of the v.e.v. of the dilaton field we will not consider them in this paper.

We now consider the minimisation of the full effective potential, $V_0 + V_1$. As discussed in [8, 17], a supersymmetry breaking solution with non zero $\phi$ is found for a large four fermion coupling. To regulate the unphysical divergence coming from the vanishing of $Re f$ we cut off its evolution at $Re f = \Lambda_c$, the point at which the gauge coupling becomes large. This is done to give the dimensionally correct form for the effective four fermion interaction at low energies, $\frac{1}{\Lambda_c^4} (\bar{\psi}\psi)^2$. With this there is a well defined minimum with the condition $\frac{\partial V}{\partial \phi} = 0$ being satisfied through a cancelation of tree level with one loop terms. As such it is necessarily non-perturbative and shows that, within our four fermion approximation, gaugino condensation is dynamically generated. The condition $\frac{\partial V}{\partial T_i}$ can be satisfied either for $T_i$ at the dual invariant points ($T_i = 1, e^{i\pi/6}$) or for large values of $T_i$ corresponding to squeezed orbifolds. In the latter case only if $\alpha_{ai} = \alpha_{aj} = \alpha$ it is possible that different moduli $T_i$ and $T_j$ have a large v.e.v and their v.e.v.s will necessarily be the same (up to modular invariant transformations), i.e. $T_i = T_j$. We see that it is thus only possible to have a large overall moduli $T$ if all three $\alpha_i$ have the same value, as in the case for a $Z_3$ or $Z_7$ orbifold where all $\alpha_i$ vanish [10]. For an orbifold with two completely rotated planes then only one $\alpha_i$ is (possibly) different than zero in which case the v.e.v. for the moduli attached to this plane will be different than those of the other two. A squeezed orbifold may thus naturally be obtained[3]. The v.e.v. of $Y$ and $x_g$ are given in terms of the dimension of the hidden gauge group, its one-loop N=1 $\beta$-function coefficient and $\alpha$ by

$$Y = 8\pi \sqrt{\frac{1}{n_g} \frac{2\alpha - 1}{3\alpha - 1} \epsilon^{-1}}$$

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[3] These orbifolds were found to be better candidates for a minimal string unification [17, 2].
\[ \epsilon \equiv x_g \ln(x_g) = \frac{4b_0}{\Lambda^2}(3\alpha - 1) \]  

From eq. (24) we obtain the value of the moduli

\[ \Sigma_i (1 - \alpha_i) T_{ri} = \frac{3Y}{\pi b_0} - \frac{6}{\pi} \ln(Q) \]  

where the sum in eq. (25) is over all moduli that acquire a v.e.v different from the dual invariant points and \( Q \equiv \frac{9Y^3}{64b_0^2} \frac{\prod T^{(a-i)M^2}_{x_g}}{x_g} \). If the gauge group is broken down from \( E_8 \) to a lower rank group such as \( SU(N) \) with \( 5 \leq N \leq 9 \) (as can be easily done by compactifying on an orbifold with Wilson lines) a phenomenologically interesting solution can be obtained with only one gaugino condensate. The value of the dilaton is mainly given by the dimension of the hidden gauge group and for a phenomenologically interesting solution one needs \( \frac{Y}{b_0(1-\alpha)} \simeq 58 \). In particular if the hidden sector gauge group is \( SU(6) \) the gauge coupling constant at the unification scale would be \( g_{\text{ut}}^2 = Y/2 \simeq 2.1 \). Using eq. (23), the fine structure constant at the unification scale is given by

\[ \hat{\alpha}_{\text{ut}}^{-1} = \left( \frac{g_{\text{ut}}^2}{4\pi} \right)^{-1} \simeq \frac{16\pi^2}{\sqrt{ng}}. \]  

For an \( SU(6) \) gauge group \( \alpha_{\text{ut}}^{-1} \simeq 26.7 \). This value is consistent with minimal supersymmetric standard model (MSSM) unification [21].

On the other hand, the v.e.v. of the moduli fields does depend on \( \alpha_{ai} \), the v.e.v. of the dilaton and \( b_0 \). The value of \( \alpha \) is approximately \(-1/3\) if either the modular weight of the hidden matter fields is \( n_{ij}^{R_a} = -1/3 \) (i.e. untwisted fields) or if the dominant term in the \( N=1 \) \( \beta \)-function coefficient for the hidden gauge group \( b_0 \) is given by the quadratic Casimir operator \( C(G_a) \), in which case from eq. (18) (taking \( \delta_{G_S} = 0 \))

\[ \alpha_i = -\frac{b_{0ij}}{b_{0i}} = -\frac{1}{3}(1 - \frac{2}{C} \Sigma_R h_{R_a} T(R_a)(\frac{1}{3} + n_{R_a}^i)) \simeq -\frac{1}{3} \]

If we only consider the three diagonal (1,1) moduli (which are always present in orbifold compactification) then, for a gravitino mass of order 1\( TeV \) and \( \alpha = -1/3 \), \( ReT \simeq 8, 12, 22 \) depending on whether three, two or one moduli get a “large” v.e.v., respectively. In the case discussed above of a large overall modulus \( ReT = ReT_1 = ReT_2 = ReT_3 \simeq 8 \). Note that this result is quite different from the values previously obtained \( T \simeq 1.3 \) [22]. Clearly the discrepancy is due to the inclusion of the strong gaugino binding effects in \( V_1 \) (cf. eq. (20)).

Interesting enough these large values are necessary for the consistent minimal unification scenario with the correct values of the weak angle \( \sin\theta_w \) and strong coupling constant \( \alpha_{\text{strong}} \) [23].

Up to now we have only considered the scalar potential for gauginos in the hidden sector, dilaton and moduli fields. We would now like to include the contribution from matter superfields. In general the matter superpotential is given as a power series expansion in the chiral superfields \( \phi \) suppressed by the Planck mass. In the absence of linear and quadratic terms, the leading term in the superpotential is cubic in the
matter superfields since one assumes that the matter fields at most get a small v.e.v. compared to the Planck mass to be consistent with the Kahler expansion in eq.(1), i.e. \(|\varphi|^2 K_i^i(T,\bar{T}) \leq 1\). The gaugino condensate term \(W_0\) will then give the dominant contribution to the scalar potential and the v.e.v. of the dilaton and moduli fields obtained in eqs. (23) and (25) will remain valid.

For a generic matter superpotential \(W_m\), using the Kahler potential of eq.(1), the tree level scalar potential is [17]

\[
V_0 = \frac{1}{4} e^K \left( (K^{-1})_S^K F_S W_0 + K_S W_m |^2 + \Sigma_i (K^{-1})_i^i |F_i W_0 + \beta_i|^2 - 3|W_0 + W_m|^2 \right) \tag{27}
\]

where

\[
F_S = K_S - \frac{3}{2b_0} f_S = -\frac{1}{Y} (1 + \frac{3Y}{2b_0}),
\]

\[
F_i = K_i + \frac{W_{0,i}}{W_0} - \frac{3}{2b_0} f_i
\]

\[
\beta_i = K_i W_m + W_{m,i},
\]

\(b_0\) is the one-loop beta function coefficient for the hidden gauge group, \(f_i\) is the derivative of the gauge kinetic function with respect to the i-field and the index i runs over all matter and moduli fields. The first term in eq.(27) is just the square of the auxiliary field of the dilaton \(h_S\) while the second term is a sum of the squares of the auxiliary fields of the moduli \(h_{T_i}\) and chiral matter fields \(h_{\varphi_i}\).

To determine the vacuum structure one has to include quantum corrections and the main contribution to the one-loop potential is again given by the gaugino-loops. We would like to emphasise again that, although we are just calculating the one-loop potential, the final result is necessarily non-perturbative, since by solving the mass gap equation one is stabilizing the tree level potential by the one-loop potential, and one is effectively summing an infinite number of gaugino-bubbles. One can easily determine the contribution of these loops by calculating the gaugino mass at tree level and then using eq.(20). Since all dependence on the gaugino bilinear in eq.(8) is given in \(W_0\) one has,

\[
m_g = \frac{b_0}{6Re f} e^{K/2} H \tag{28}
\]

with

\[
H \equiv F^S (K^{-1})_S^K (F_S W_0 + K_S W_m) + F^i (K^{-1})_i^i (F_i W_0 + \beta_i) - 3(W_0 + W_m). \tag{29}
\]

The one-loop potential (considering the contribution from the gauginos-loops only) is (cf. eq.(24))

\[
V_1 = -\gamma A_c^4 J(x_g)
\]

\[
V_1 = -\gamma A_c^4 (2x_g + x_g^2 l n(x_g)) \tag{30}
\]

where we have expanded the function \(J\) since, as we showed in eq.(24), \(x_g = O(10^{-2})\) and it is now given by

\[
x_g = \frac{m_g^2}{\Lambda_c^2} = \frac{b_0^2}{36 \Lambda_c^4} |H|^2. \tag{31}
\]
If $W_m = W_{m,i} = 0$ one has $H = B_0W_0$ and eq.\,(28) reduces to eq.\,(22), as should be the case. The scalar potential $V = V_0 + V_1$ is then

\[
V = \frac{1}{4} e^K [(K^{-1})^S S] F_S W_0 + K_S W_m|^2 + \Sigma_i (K^{-1})_{i|} F_i W_0 + \beta_i|^2 \\
-3|W_0 + W_m|^2 - \frac{2\gamma b_0^2}{9}|H|^2(1 + \frac{1}{2} x_g \ln(x_g))]
\]

(32)

So far we have not discussed the cosmological constant at the minimum of the potential. In fact, in the absence of matter fields, it is negative, of order $\Lambda_\varepsilon^4$. It may be cancelled through the addition to the matter superpotential of a term linear in a chiral superfield $D$

\[
W_m = c D
\]

To obtain a more transparent expression for $V$ one can expand the scalar potential in powers of the gravitino mass ($m_{3/2}^2 = \frac{1}{2} e^K |W_0|^2$). Making use of the solutions to the extremum equations eq.\,(24) and eq.\,(25) the scalar potential is

\[
V = m_{3/2}^2 B + (m_{3/2} A + h.c.) + C
\]

(34)

with

\[
B = \frac{1}{2} [(K^{-1})^S F_S F_S + (K^{-1})_{i|} K_i K^i + K_D^D |D|^2 - 3)(\frac{\alpha - 1}{3\alpha - 1} - K_D^D |D|^2) \epsilon,
\]

\[
A = \frac{1}{4} e^{K/2} \left(D(K_D^D |D|^2 c + c) + F_S (K^{-1})^S S K_S c D - 3 c D) \right)(\frac{\alpha - 1}{3\alpha - 1} - K_D^D |D|^2) \epsilon
\]

(35)

and

\[
C = \frac{1}{4} e^K ((K^{-1})_{i|} K_i W_m + W_{m,i}|^2 + (K_D^D)^{-1} |K_D^D |D|^2 c + c|^2 - 3|c D|^2
\]

where the $i$-index in eqs.\,(32) runs over all chiral superfields but for $D$. The leading contribution from the superpotential of eq.\,(32) is through the auxiliary field of $D$ ($h_D = \frac{1}{4} e^{K/2} (K_D W + W_D)$). This contribution is semipositive definite and for vanishing v.e.v. of $D$ the only term that survives is the one proportional to $|W_{m,D}|^2 = c^2$ and the one-loop potential is independent of it. It is in fact this term that gives the main contribution to the scalar potential even for non-vanishing v.e.v. of $D$ and since it is positive it allows for the cancellation of the cosmological constant by fine tuning $c$. While such a tuning is unnatural, it does demonstrate that it is possible to have a zero cosmological constant with supersymmetry broken at a phenomenologically realistic value.

It is now straightforward to compute the soft supersymmetry breaking masses in the visible sector. The main contribution comes from the auxiliary field of the dilaton $h_S$ as can be seen from eqs.\,(34) and \,(35) since the v.e.v. of $h_S^2 \gg h_S^2$. The leading contribution to the scalar mass $m_0$ comes from the $e^K$ dependence implicit in $B$ and gives (assuming zero cosmological constant)

\[
m_0^2 = - \frac{1}{2} B_0 \epsilon \left(\frac{3 - 5\alpha}{(3\alpha - 2)(\alpha - 3)}\right) m_{3/2}^2
\]

(36)

Since $\epsilon < 0$ and $\alpha < 1/3$ eq.\,(36) is positive showing, contrary to analyses neglecting the effect of $V_1$, that the scalar potential has a minimum for $\phi_i = 0$. Note that the scalar
mass is the same for all matter superfields regardless of their modular weights. Since the auxiliary field of the dilaton is much bigger than the auxiliary field of the $D$ field, the scalar mass is only sensitive to the fact that one has no cosmological constant and not to the details of the cancelation of the vacuum energy. In the absence of v.e.v.s for the matter fields, the common supersymmetric mass is independent of the trilinear superpotential.

Using eqs. (9), (10) and (19) one obtains the leading contribution to the common gaugino mass, $m_{1/2}$, of the observable sector

$$m_{1/2} = -6\alpha \left( \frac{3 - 5\alpha}{(3\alpha - 2)(\alpha - 3)} \right) m_{3/2} \quad (37)$$

This comes from the $h_s$ auxiliary field contribution and is the same for all gaugino masses at the unification scale where the gauge coupling constants meet.

The remaining soft terms are proportional to the $A$ component of the matter superpotential, $W_m |_A$. They are given by

$$L = \Sigma_N R_N m_{3/2} W^N_m + h.c. \quad (38)$$

where $N$ is the degree of the normalized superpotential $\tilde{W}^N_m = \frac{1}{2} e^{K/2} W^N_m$ and

$$R = \frac{3(\alpha - 1)}{(3\alpha - 2)(\alpha - 3)} \left[ (3 - 5\alpha) + \frac{(\alpha - 1)}{2} (1 + N + K^T(K^{-1})^T h_{123,T}) \right] \quad (39)$$

For the leading trilinear terms, $\tilde{W}^3_m = \frac{1}{2} e^{K/2} h_{ij,k} \phi_i \phi_j \phi_k$, whose coupling, $h_{ij,k}$, is independent of the moduli $T$ this reduces to

$$A_t \equiv R_3 = \frac{3(\alpha - 1)(1 - 3\alpha)}{(3\alpha - 2)(\alpha - 3)} \quad (40)$$

For a bilinear term, such as the MSSM term $\tilde{W}^2_m = \mu H_1 H_2$, with $\mu$ independent of $T$ we have

$$B' \equiv R_2 = \frac{3(\alpha - 1)(3 - 7\alpha)}{2(3\alpha - 2)(\alpha - 3)} \quad (41)$$

One problem associated with such a term is that there is no obvious reason why $\mu \leq O(M_W)$ as must be the case if the hierarchy problem is not to be re-introduced. To answer this it has been suggested that the $\mu$ term originates from a term in the Kahler potential proportional to $H_1 H_2$ [24]. After supersymmetry breaking this generates the $\mu H_1 H_2$ term in the superpotential with $\mu \propto m_{3/2}$. In this case the soft term coming from the B term in eq. (34) is again of the form of eq. (38) with

$$B'' = \left( \frac{m_0}{m_{3/2}} \right)^2 \quad (42)$$

$$= \left( \frac{3 - 5\alpha}{(3\alpha - 2)(\alpha - 3)} \right)^2 \frac{\epsilon B_0}{2} \quad (43)$$

Another suggestion is that the $\mu$ term comes from a term in the superpotential proportional to $W_0 H_1 H_2$. In this case we find from the B term the contribution of the form
of eq. (38) with

\[ B''' = \frac{(\alpha - 1)}{(3\alpha - 1)} \left( \frac{m_0}{m_{3/2}} \right)^2 = \frac{(\alpha - 1)}{(3\alpha - 1)} \left( \frac{3 - 5\alpha}{(3\alpha - 2)(\alpha - 3)} \right)^2 \epsilon B_0 \]

This completes the derivation of the soft terms arising from gaugino condensation. It is also interesting to calculate the fine tuning parameter \( \Delta \) defined as

\[ \left| \frac{a_i}{M_Z} \frac{\partial M_Z^2(a_i, h_i)}{\partial a_i} \right| < \Delta \]  

where \( a_i \) are the soft SUSY parameters and \( M_Z \) the mass of \( Z \) boson. This quantity measures the allowed degree of the fine tuning needed to get the correct mass \( M_Z \); for \( \Delta = 10 \) the value of \( a_i \) must be fine tuned to \( 1/10 \) of the \( Z \) mass. At a crude approximation \( \Delta \simeq m_1^2/M_Z^2 \) with \( m_1 \) evaluated at the electroweak scale and it is given by \( m_1^2 = m_0^2 + \mu^2 + km_1^2/2 \) with \( k \simeq 1/2 \).

It is illuminating to compute them for realistic examples with an \( SU(6) \) gauge group in the hidden sector with different matter content. They are shown in tables 1-3. Notice that the fine structure constant at the unification scale is of the right order of magnitude as required by MSSM. It is the dimension of the gauge group in the hidden sector that sets the value of \( \hat{\alpha}_{\text{gut}} \) (cf. eq. (26)).

These results suggest that \( m_0/m_{1/2} \) is large contrary to the analyses based on assuming supersymmetry breaking originates at tree level from the dilaton F term even though in this case too the dilaton F term dominates. There are two reasons for this. In the first place the mechanism explored here, being nonperturbative, necessarily requires that loop corrections to the potential be as important as the tree level contribution. Indeed there is a strong suppression of the \( R_N \) terms due to a cancelation of the tree and loop contributions. Secondly the cancelation of the cosmological constant in gaugino condensation requires a contribution from the matter super field sector. Both these effects are absent in [3] and explains the difference between the resulting soft terms.

For a wide range of values for \( \alpha \), a large hierarchy can be obtained. However, the soft supersymmetric terms are more sensitive to \( \alpha \). In fig.1 we show \( m_0/m_{3/2}, m_{1/2}/m_{3/2} \) and the trilinear term \( A_t \) as a function of \( \alpha \). For increasing \( \alpha \), \( m_0 \) and \( A_t \) become smaller. The common supersymmetric mass, \( m_0 \), is always larger than \( m_{3/2} \). On the other hand, the gaugino mass is always smaller than \( m_{3/2} \) and it approximates the gravitino mass for \( |\alpha| \to 1/3 \). The ratio of \( m_{1/2}/m_{3/2} \) is independent of the value of \( m_{3/2} \), but \( m_0/m_{3/2} \) is not. The reason is that \( m_0/m_{3/2} \) depends on \( Y/b_0 \), which sets the hierarchy.

Running the masses down to the electroweak scale, neglecting corrections due to possible large Yukawa couplings, the common supersymmetric mass remains the same while the gluino mass increases by a factor of three. If we demand all supersymmetric masses to be in the range \( 100 \text{TeV} \leq m_{ss} \leq 1000 \text{TeV} \), then \( m_0/m_{1/2}|_{\text{gut}} \leq 30 \).

From the range of values of the parameter \( \alpha \), the preferred value is for \( \alpha \) positive and close to \( 1/3 \). For this value of \( \alpha \) the difference between the supersymmetric masses
of the scalar and gaugino fields is smallest. This allows for having a small fine tuning parameter $\Delta$. At the same time, the trilinear term $|A_t|$ decreases. This is welcome since one-loop diagrams, associated to the complex phases of the trilinear terms, contribute to the electric dipole moment of the neutron. For $A_t = O(1)$, this contribution is about $10^2 - 10^3$ bigger than the experimental values [27]. Furthermore an $\alpha \simeq 0.3$ minimizes the unification scale [28] and allows for unifying the standard model gauge groups with the hidden sector one and the fine tuning parameter $\Delta$ is smallest. Thus, from all the range of possible values for the parameters $n_l$, $b_0$ and $\alpha$, consistency with the MSSM and electroweak breaking, selects a very narrow band. It is remarkable that the values of all soft supersymmetric terms improve within this band taking quite acceptable values.

| $n_l$ | $Y$ | $T_r$ | $m_{3/2}$ | $m_{0}/m_{3/2}$ | $m_{1/2}/m_{3/2}$ | $\Lambda_{\text{gut}}$ | $\hat{\alpha}_{\text{gut}}^{-1}$ | $A_t$ | $B'$ | $B''$ | $B'''$ | $\Delta$ |
|------|-----|------|----------|-----------------|-----------------|---------|-----------------|------|------|------|------|-------|
| 3    | 4.25 | 17.3 | 55       | 4.3             | 0.9             | 2.8     | 26.7            | -0.07| -0.32| 18.6 | 130.5| 6.7   |
| 2    | 4.22 | 24.2 | 82       | 4.7             | 0.9             | 2.8     | 26.5            | -0.07| -0.32| 21.8 | 153  | 17.5  |
| 1    | 4.15 | 44.5 | 147      | 5.3             | 0.9             | 2.8     | 26.1            | -0.07| -0.32| 26.1 | 196  | 71    |

Table 1  $SU(6)$ gauge group with $16\pi^2 b_0 = 15$ and $\alpha_0 = 0.3$.

| $n_l$ | $Y$ | $T_r$ | $m_{3/2}$ | $m_{0}/m_{3/2}$ | $m_{1/2}/m_{3/2}$ | $\Lambda_{\text{gut}}$ | $\hat{\alpha}_{\text{gut}}^{-1}$ | $A_t$ | $B'$ | $B''$ | $B'''$ | $\Delta$ |
|------|-----|------|----------|-----------------|-----------------|---------|-----------------|------|------|------|------|-------|
| 3    | 4.26 | 16.3 | 440      | 4.2             | 0.9             | 3.5     | 26.7            | -0.07| -0.32| 17.5 | 122.4| 403   |
| 2    | 4.23 | 22.5 | 555      | 4.4             | 0.9             | 3.5     | 26.6            | -0.07| -0.32| 19.9 | 140  | 720   |
| 1    | 4.17 | 40.3 | 900      | 5.1             | 0.9             | 3.5     | 25.8            | -0.07| -0.32| 26.3 | 184  | 2508  |

Table 2  $SU(6)$ gauge group with $16\pi^2 b_0 = 16$ and $\alpha_0 = 0.3$.

| $n_l$ | $Y$ | $T_r$ | $m_{3/2}$ | $m_{0}/m_{3/2}$ | $m_{1/2}/m_{3/2}$ | $\Lambda_{\text{gut}}$ | $\hat{\alpha}_{\text{gut}}^{-1}$ | $A_t$ | $B'$ | $B''$ | $B'''$ | $\Delta$ |
|------|-----|------|----------|-----------------|-----------------|---------|-----------------|------|------|------|------|-------|
| 3    | 4.62 | 17.9 | 632      | 15.4            | 0.9             | 2.0     | 29.0            | -0.8 | -1.06| 238  | 158  | $10^4$|
| 2    | 4.61 | 24.7 | 661      | 15.4            | 0.9             | 2.0     | 28.9            | -0.8 | -1.06| 238  | 158  | $10^4$|
| 1    | 4.57 | 44.0 | 849      | 15.4            | 0.9             | 2.0     | 28.7            | -0.8 | -1.06| 238  | 158  | $10^4$|

Table 3  $SU(6)$ gauge group with $16\pi^2 b_0 = 11$ and $\alpha_0 = -1/3$.

We show in tables 1-3, the values for phenomenologically relevant parameters determining the supersymmetry breaking. $n_l$ is the number of moduli, $T_r$, with large v.e.v., $g_{\text{gut}}^{-2} = Y/2$ is the unification coupling, $m_{3/2}$ is given in GeV, $\Lambda_{\text{gut}}$ in $10^{16}$ GeV and $\hat{\alpha}_{\text{gut}}$ is the fine structure constant at $\Lambda_{\text{gut}}$. $A_t$, $B'$, $B''$ and $B'''$ are explained in eqs. (40-44). $\Delta$ measures the tuning of the electroweak breaking (cf. eq.(45)). The unification scale is given for $\alpha_a = \alpha_0 = 0.3$ in tables 1 and 2, where $\alpha_a$ corresponds to the visible sector gauge groups. With this choice
the $SU(6)$ and the standard model gauge groups are unified [28]. In table 3 $\alpha_a = 0.32$ and the $SU(6)$ gauge group does not become unified with the visible sector ones.

To summarise, we have considered the phenomenological implications of SUSY breaking triggered by a gaugino condensate in the hidden sector. In a supergravity theory the strong binding effects play a role in all sectors of the theory and must be included. Here we have modelled these effects via a simple (N=1 locally supersymmetric) four gaugino interaction which allowed us to use N-J-L techniques to estimate the nonperturbative physics. We thus constructed the modular invariant effective potential which determines the magnitude of the gaugino condensate and also the moduli of the theory.

We have generalized the work in [8] by including several moduli fields. As a result of minimizing the scalar potential we found that the v.e.v. of the dilaton is mainly given by the dimension of the hidden sector gauge group. For reasonable hidden sector gauge groups a large mass hierarchy can arise together with an acceptable value for the fine structure constant at the string scale with only one gaugino condensate. We found that the extremum eqs. for the moduli are solved for a unique value of the parameter $\alpha$ (cf. eq.(18)) and thus the v.e.v. of the moduli take a common value ($T_i = T_j$) related to $b_0, \alpha$ and the v.e.v of the dilaton or they take the dual invariant values ($T = 1, e^{i\pi/6}$). This result naturally accommodates the squeezed orbifold solutions which are better candidates for minimal string unification.

We also showed that it is possible to fine tune the cosmological constant to zero while having SUSY broken at a phenomenologically interesting value. Incorporating chiral matter fields we computed the soft SUSY breaking terms. The ratio between the scalar and gaugino mass is given and it depends on $b_0, \alpha$ and the v.e.v. of the dilaton. The numerical values for phenomenologically interesting solutions are consistent with results obtained from requiring acceptable values for the gauge couplings, electroweak breaking, dark matter abundance and $m_6 = m_\tau$ assuming a minimal unification of the MSSM[21].
Figure 1: $m_0/m_{3/2}$, $m_{1/2}/m_{3/2}$ and the trilinear term $A_t$ as a function of $\alpha$. The short-dashed line represents $m_0/m_{3/2}$, the dot-dashed one $m_{1/2}/m_{3/2}$ while the long-dashed one $A_t$.

References

[1] For a review of string theories, see M. Green, J. Schwarz and E. Witten, Superstring Theory, Cambridge University Press, 1987.

[2] L.E. Ibanez and D. Lust, Nucl. Phys. B382 (1992) 305.

[3] V. Kaplunovsky and J. Louis, CERN-TH.6809/93; R. Barbieri, J. Louis and M. Moretti CERN-TH.6856/93. A. Brignole, L.E. Ibanez and C. Munoz, Universidad Autonoma de Madrid preprint FTUAM-26/93.

[4] A. Font, L.E. Ibáñez, D. Lust and F. Quevedo, Phys. Lett. B245 (1990) 401. M. Cvetic, A. Font, L.E. Ibanez, D. Lust and F. Quevedo, Nucl. Phys. B361 (1991) 194.

[5] J.P. Derendinger, L.E. Ibanez and H.P. Nilles, Phys. Lett. B155 (1985) 65; M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B156 (1985) 55.

[6] For a review see D. Amati, K. Konishi, Y. Meurice, G. Rossi and G. Veneziano, Phys. Rep 162 (1988) 169; H.P. Nilles, Int. J. Mod. Phys. A5 (1990) 4199; J. Louis, SLAC-PUB- 5645 (1991).

[7] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 231; D. Gross and A. Neveu, Phys. Rev. D10 (1974) 3235.
[8] A. de la Macorra and G. G. Ross, Nucl. Phys. B404 (1993) 321.

[9] J. P. Derendinger, S. Ferrara, C. Kounas and F. Zwirner, Nucl. Phys. B372 (1992) 145.

[10] V. Kaplunovsky, Nucl. Phys. B307 (1988) 145.

[11] L. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B329 (1990) 27

[12] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B212 (1983) 413.

[13] G. Veneziano and S. Yankielowicz, Phys. Lett. 113B (1984); T. R. Taylor, Phys. Lett. 164B (1985) 43 S. Ferrara, N. Magnoli, T. R. Taylor and G. Veneziano, Phys. Lett. B245 (1990) 409; P. Binetruy and M. K. Gaillard, Phys. Lett. 232B (1989); Nucl. Phys. B358 (1991) 121.

[14] H. P. Nilles and M. Olechowski, Phys. Lett. B248 (1990) 268; P. Binetruy and M. K. Gaillard, Phys. Lett. B253 (1991) 119; J. Louis, “Status of Supersymmetry Breaking in String Theory”, SLAC-PUB-5645 (1991); D. Läst and T. R. Taylor, Phys. Lett. B253 (1991) 335; B. Carlos, J. Casas and C. Muñoz, Phys. Lett. B263 (1991) 248; S. Kalara, J. Lopez and D. Nanopoulos, “Gauge and Matter Condensates in Realistic String Models”, preprint CTP-TAMU-69/91.

[15] L. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649; G. Lopes Cardoso and B. Ovrut, Nucl. Phys. B369 (1992) 351; “Sigma Model Anomalies, Non-Harmonic Gauge and Gravitational Couplings and String Theory,” preprint UPR-0481T (1991).

[16] I. Antoniadis, E. Gava and K. S. Narain, Nucl. Phys. B383 (1992) 93; Phys. Lett. B283 (1992) 209, I. Antoniadis, K. S. Narain and T. R. Taylor, Phys. Lett. B267 (1991) 37.

[17] A. de la Macorra and G. G. Ross, “Supersymmetry Breaking in 4D String Theory”, (preprint OUTP-31P).

[18] N. V. Krasnikov, Phys. Lett. 193B (1987) 37; L. Dixon, in Proceedings of the APS DPF Meeting, Houston, 1990; J. A. Casas, Z. Lalak, C. Munoz, and G. G. Ross, Nucl. Phys. B347 (1990) 243; T. Taylor, Phys. Lett. B252 (1990) 59; B. de Carlos, J. A. Casas and C. Munoz, CERN-TH.6436/92 and ref. therein.

[19] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1883.

[20] R. Barbieri and S. Cecotti, Z. Phys. C 17 (1983) 183; M. Srednicki and S. Theisen, Phys. Rev. Lett. 54, (1985) 278; P. Binetruy, S. Dawson, M. K. Gaillard and I. Hinchliffe, Phys. Rev. D 37 (1988) 2633.

[21] G. Costa, J. Ellis, G. L. Fogli, D. V. Nanopoulos and F. Zwirner, Nucl. Phys. B297 (1988) 244; J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B249 (1990) 441; Phys. Lett. B260 (1991) 131; P. Langacker, “Precision tests of the standard model”, Pennsylvania preprint UPR-0435T, (1990); U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. B260 (1991) 447; P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817; R. Roberts and G. G. Ross, preprint RAL-92-005 (1992).
[22] A. Font, L.E. Ibáñez, F. Quevedo and A. Sierra, Nucl.Phys. B331 (1990) 421.

[23] L.E. Ibáñez, D. Lüst and G.G. Ross, Phys. Lett. B272 (1991) 251.

[24] G. F. Giudice and A. Masiero, Phys. Lett. 206B (1988) 480; J.E. Kim and H.-P. Nilles, Phys. Lett. B 138B (1984) 150; J.L. Lopez and D.V. Nanopoulos, Phys. Lett. B B251 (1990) 73; E.J. Chun, J.E. Kim and H.-P. Nilles, Nucl. Phys. B370 (1992) 105.

[25] R. Barbieri and G.F. Giudice, Nucl. Phys. B302 (1988) 63.

[26] K. Inoue et al., Prog.Theor.Phys. 68 (1982) 927; L.E. Ibáñez, Nucl.Phys. B218 (1983) 514; L.E. Ibáñez and C. López, Phys. Lett. B126 (1983) 54; Nucl.Phys. B233 (1984) 511; L. Alvarez-Gaume, J. Polchinsky and M. Wise, Nucl.Phys. B221 (1983) 495; L.E Ibanez, C. Lopez and C. Munoz, Nucl. Phys. B256 (1985) 218.

[27] J.E. Kim, Phys. Rep. 150 (1987)1; H.Y. Cheng, Phys. Rep. 158 (1988) 1; L. E. Ibanez and D. Lust, Nucl. Phys. B267 (1991) 51.

[28] A. de la Macorra, “Unification scale in string theory”, (preprint OUTP-93-33P).
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