IR Quantum Gravity solves naturally cosmic acceleration and its coincidence problem

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The novel idea is that the undergoing accelerated expansion of the universe happens due to infrared quantum gravity modifications at intermediate astrophysical scales of galaxies or galaxy clusters, within the framework of Asymptotically Safe gravity. The reason is that structures of matter are associated with a scale-dependent positive cosmological constant of quantum origin. In this context no extra unproven energy scales or fine-tuning are used. Furthermore, this model was confronted with the most recent observational data from a variety of probes, and with aid of Bayesian analysis, the most probable values of the free parameters were extracted. Finally, the model proved to be statistically equivalent with ΛCDM, and thus being able to resolve naturally the concept of dark energy and its associated cosmic coincidence problem.

I. INTRODUCTION

The idea that Quantum Gravity effects can be important at astrophysical and cosmological distances has recently attracted much attention. In particular the framework of Exact Renormalization Group (RG) approach for quantum gravity [1], also found in the literature under the names Asymptotic Safety (AS) or Quantum Einstein Gravity, has opened the possibility of investigating both the ultraviolet (UV) and the infrared (IR) sector of gravity in a systematic manner.

The key element is the Effective Average Action \( \Gamma_k[g_{\mu\nu}] \), a Wilsonian coarse-grained free energy, which defines an effective field theory appropriate for the momentum scale \( k \). This \( \Gamma_k \), evaluated at tree level, describes appropriately all gravitational phenomena, including all loop effects. The application to Einstein-Hilbert action generates RG flow equations [2], which have made possible the consistent study of the scaling behavior of Newton constant \( G \) and cosmological constant \( \Lambda \) at high energies [3], [4]. The initial idea was first demonstrated by Weinberg [5], where he suggested that a perturbatively divergent theory could be consistently defined in four dimensions at a nontrivial UV fixed point with the dimensionless \( g(k) = G(k)k^2 \) non-vanishing in the \( k \to \infty \) limit. In the framework of AS, one should also include \( \Lambda \), which becomes energy dependent and receives quantum contribution form vacuum fluctuations.

Recent works have also considered matter fields or a growing number of purely gravitational operators in the action. In particular, truncations involving quadratic terms in the curvature or higher powers of the Ricci scalar have been studied [6], [7]. In all the investigations the UV critical surface has turned out to be finite dimensional (\( d_{UV} = 3 \)), implying that the theory is nonperturbatively renormalizable.

A weakly coupled gravity at high energies is expected to generate important consequences in several astrophysical and cosmological contexts and in fact the RG flow of \( \Gamma_k \), obtained by different truncations of theory space, has been the basis of various investigations of “RG improved” black hole spacetimes [8], [9] and early Universe models [10]-[12].

II. IR POINT FROM AS APPROACH

The behavior of AS theory is more complicated at low energies, corresponding at cosmological or astrophysical scales. The problem arises because the \( \beta \)-functions of any local operator of the type \( \sqrt{g}R^n \) are singular in the IR.
due to the presence of a pole at $\lambda(k) = \Lambda(k)/k^2 = 1/2$. This pole indicates that the Einstein-Hilbert truncation is not trustable approximation and new relevant operators emerge in the $k \to 0$ limit. It was shown in [13] that the dynamical origin of these strong IR effects is due to the “instability driven renormalization”, a phenomenon well-known from other physical systems [14]-[16]. Fortunately, the low energy domain of the theory is regulated by an IR fixed point which drives the cosmological constant to zero. Out of that, some first astrophysical consequences and a possible explanation for the galaxy rotation curves without dark matter appeared [13], [17], [18], although a detailed analysis based on available experimental data is still missing.

Very recently, authors in [19], [20] showed that the coincidence problem of cosmic acceleration can be explained naturally and without introducing new energy scales or fine-tuning in the context of a Swiss cheese cosmological model which utilizes this IR behavior of AS gravity. The present essay focuses on this discovery. The importance of these works lies on the fact that no special “dark energy” field is required and the explanation uses only the concrete well-motivated framework of AS.

More precisely, in the proposed scenario, including all the components of the metric fluctuation $h_{\mu\nu}$, quantum effects dynamically drive $\Lambda$ along the RG trajectories generated by the unstable infrared modes of the gravitational sector. Close to the IR fixed point, $\Lambda$ runs proportional to $k^2$ to avoid the singularity,

$$\frac{\Lambda(k)}{k^2} = \lambda^\text{IR} + h_2 k^{2\theta}, \quad k \to 0,$$

where $\lambda^\text{IR} < 1/2$ is an infrared fixed point of the $\lambda$-evolution, $h_2$ is a constant related to the eigenvalues of the stability matrix, and $\theta > 0$ is a critical exponent which parametrizes the subleading terms and ensures that the fixed point is attractive. According to (2.1), the renormalized $\Lambda$ vanishes at very large (cosmological) distances, $\Lambda(k \to 0) = 0$, regardless of its bare value. Furthermore, recent investigations based on a conformal reduction of Einstein gravity discovered a new IR fixed point suggesting the existence of the counterpart of the physical IR fixed point present in the full theory [21].

III. COSMOLOGY BY MATCHING LOCAL ASTROPHYSICAL METRICS WITH GLOBAL COSMOLOGICAL METRIC

In [19], the recent cosmic acceleration naturally emanated from the recent formation of structure. A Swiss cheese (Einstein-Strauss) model was constructed to derive the cosmology. The interior static spherically symmetric metric, modeling a galaxy or a galaxy cluster, matches smoothly to a cosmological exterior across a spherical boundary. A quantum improved Schwarzschild-de Sitter (SdS) interior metric was used, which contains the appropriate antigravity effect. This model uses dimensionless order one parameters of AS, the conventional Newton constant $G_N$ and the astrophysical length scale. It provides a recent passage to the sufficient acceleration, while the freedom of the order one parameters has to be constrained by observational data. To the best of our knowledge, this is the first solution of the dark energy problem without using fine-tuning or introducing add-hoc energy scales.

The exterior FRW metric is

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - \frac{\Lambda k}{R^2}} + r^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right) \right],$$

with $a(t)$ the scale factor. The interior metric has the form

$$ds^2 = -\left( 1 - \frac{2 G_k M}{R} - \frac{1}{3} \Lambda_k R^2 \right) dt^2 + \frac{dR^2}{1 - \frac{2 G_m M}{R} - \frac{1}{3} \Lambda_k R^2} + R^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right)$$

with the functions $G_k = G(k), \Lambda_k = \Lambda(k)$ determined by AS. The matching of the two patches occurs at the boundary with constant $r = r_\Sigma$, which at the same time experiences the universal expansion. Since astrophysical structures are still large compared to the cosmological scales, $\Lambda(k)$ is expected to differ slightly from its IR form $k^2$, so a power law $\Lambda_k = \gamma k^b$, with $\gamma > 0$ and $b$ close to the value 2, is a fair approximation of the running behavior (2.1). Additionally, it is set $G_k = G_N$ at observable macroscopic distances, in agreement with standard Newton law.

The energy scale $k$ is expected to be associated with a characteristic length scale $L$, $k = \xi/L$, where $\xi$ is an order one dimensionless number. In the Swiss cheese approach, only the value $R_S = ar_\Sigma$ of the Schucking radius (or $k_S$) enters the cosmic evolution. Although a simple option is $L = R$, a more natural one is to set as $L$ the proper distance

$$D(R) = \int_R^\infty dR \left( 1 - \frac{2 G_N M}{R} - \frac{1}{3} \Lambda_k R^2 \right)^{-1/2}.$$
FIG. 1: The evolution of $w_{DE}(z)$ with the corresponding $1\sigma$ and $2\sigma$ uncertainties. In the upper panel we use the best fit values from the combination $H(z)/\text{Pantheon/QSO}$, while in the lower panel we utilize those of $H(z)/\text{Pantheon/QSO/CMB shift}$. The dashed line corresponds to $\Lambda$CDM value $w = -1$.

The new cosmological constant term is $\frac{1}{3} \Lambda R^2 \sim \frac{1}{G_N} \left( \frac{\sqrt{G_N}}{r_\Sigma} \right)^b \left( \frac{r_\Sigma}{r_N} \right)^b R^2$, where $\tilde{\gamma} = \gamma G_N^{1-b}$ a dimensionless order one number. For $b$ close to the value 2.1, the quantity $\frac{1}{G_N} \left( \frac{\sqrt{G_N}}{r_\Sigma} \right)^b$ is very close to the order of magnitude of the standard cosmological constant $\Lambda \simeq 4.7 \times 10^{-84}\text{GeV}^2$ of the concordance $\Lambda$CDM model, while the factor $\left( \frac{r_\Sigma}{r_N} \right)^b$ contributes only a small distance-dependent deformation since the today value of $D_S$ should be of order $r_N$ in order to have the correct amount of dark energy. Therefore, the hard coincidence of the standard $\Lambda \sim H_0^2$, has been exchanged with a mild adjustment of the index $b$ close to 2.1.

The Hubble evolution in terms of the redshift $z$ arises by the integration of the cosmological equations of the model which consider the Israel-Darmois matching conditions on the boundary radius,

$$H^2(z) = \Omega_m(1+z)^3 + \left[ \Omega_{DE0} - \frac{3}{(\xi\tilde{\gamma})^b} \left( \frac{G_N H_0^2}{r_N} \right)^b \frac{r_S a_0}{\sqrt{G_N}} \frac{z}{1+z} \right]^{-b} + \Omega_k(1+z)^2. \tag{3.4}$$

In addition, a preliminary analysis has shown that there is no obvious contradiction between the model discussed and the internal dynamics of the astrophysical object. The potential and the force due to the varying cosmological constant term are small percentages of the corresponding Newtonian potential and force, where the precise values depend on the considered structure and the considered point at the boundary of the object or inside.

IV. THROUGH TEST OF THE MODEL USING OBSERVATIONAL DATA

No matter what are the theoretical merits of a given model, it needs to be confronted with the observational data. In [20], authors used the most recent observational data sets, namely direct measurements of the Hubble rate $H(z)$ [22], Supernovae Ia (Pantheon data set [23]), Quasi-Stellar-Objects (QSO), Baryonic Acoustic Oscillations and direct measurements of the CMB shift parameters, to constraint the free parameters of the AS cosmological model [19]. It was found that this model is very efficient and in excellent agreement with observations. The energy density of matter that was calculated is compatible with the same quantity imposed by concordance cosmology from the CMB angular power spectrum. Another result is that the AS model supports a lower value of Hubble constant than the value derived from Cepheids, so the best fit value for $H_0$ is closer to the Planck value.

In addition, after reconstructing the effective dark energy equation-of-state parameter $w_{DE}(z)$ using the derived values of the free parameters, it was found that the today’s value $w_{DE0}$ is close to $w = -1$. Both the transition redshift and the current value of the deceleration parameter are in perfect agreement with the corresponding values calculated with model-independent techniques.

Finally, the comparison of $\Lambda$CDM model with the considered AS model in terms of the fitting properties, using a variety of information criteria, revealed that the AS model is statistically equivalent with that of $\Lambda$CDM. This is a significant conclusion since the AS model, unlike the majority of the cosmological models, does not include new fields in nature or has fine-tuning problems. Therefore, it must be considered as a viable and efficient alternative.
TABLE I: The observational constraints on the density parameters $\Omega_m^0$, $\Omega_b^0$ and the statistical criteria values $\chi^2_{\text{min}}$, AIC, $\Delta$AIC for the AS cosmology and ΛCDM.

| Model       | $\Omega_m^0$       | $h$     | $\Omega_b^0 h^2$ | $\chi^2_{\text{min}}$ | AIC    | $\Delta$AIC |
|-------------|--------------------|---------|------------------|------------------------|--------|-------------|
| AS model    | $0.270^{+0.019}_{-0.018}$ | $0.684^{+0.013}_{-0.012}$ | -                 | 84.463                | 92.889 | 0.937       |
| ΛCDM        | $0.281^{+0.016}_{-0.015}$ | $0.686 \pm 0.013$ | -                 | 85.700                | 91.952 | 0           |

$H(z)/\text{Pantheon/QSO}$

$H(z)/\text{Pantheon/QSO/CMB shift}$

| Model       | $\Omega_m^0$       | $h$     | $\Omega_b^0 h^2$ | $\chi^2_{\text{min}}$ | AIC    | $\Delta$AIC |
|-------------|--------------------|---------|------------------|------------------------|--------|-------------|
| AS model    | $0.303 \pm 0.001$  | $0.685 \pm 0.009$ | $0.0223 \pm 0.0002$ | 89.774                | 100.419 | 1.653       |
| ΛCDM        | $0.307^{+0.008}_{-0.007}$ | $0.679 \pm 0.006$ | $0.0223 \pm 0.0001$ | 90.340                | 98.766 | 0           |

cosmological scenario towards explaining the recent accelerated expansion of the universe. Interestingly, the two models are expected to have differences at the perturbation level.

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