Acoustic characterization of hydraulic systems: application to POGO Phenomenon

A.Simon\textsuperscript{1}, R. Fortes-Patella\textsuperscript{2}, J-J Martinez Molina\textsuperscript{3}, C. Rebattet\textsuperscript{4}, R. Brillault\textsuperscript{4} and A. Kernilis\textsuperscript{5}

\textsuperscript{1}Cnes, 52 rue Jacques Hillairet, 75012 Paris, France
\textsuperscript{2}LEGI, University of Grenoble Alpes, 38000 Grenoble, France
\textsuperscript{3}GIPSA-LAB, University of Grenoble Alpes, 38000 Grenoble, France
\textsuperscript{4}CREMHyG, 101 rue de la Passerelle, 38042 Saint Martin D’Hères, France
\textsuperscript{5}SNECMA, Forêt de Vernon, 27000 Vernon, France

Alexandre.simon@legi.grenoble-inp.fr

Abstract. This paper presents an experimental methodology for the evaluation of dynamic transfer matrices using three pressure sensors. The experimental study was carried out at the CREMHyG’s acoustic test rig. The speed of sound in the pipe, as well as discharge fluctuations were evaluated from the pressure fluctuations measured during tests carried out under excitation of a modulator (piston). The method applied to reconstitute flow rate fluctuations from pressure sensors were validated by comparisons with piston displacement measurements. The transfer matrices for straight pipes and POGO corrective devices were identified based on electrical analogy. The identification method was validated by comparing the transfer matrix components to theoretical values. The results can be applied in the future, in the case of space rocket.

1. Introduction
The evaluation of dynamic behavior of hydraulic circuits and machines is a very important aspect in risk analysis for different industrial process. Systems dynamic behaviors are frequently described and studied by means of transfer functions approaches in the case of pumps or hydraulic turbines ([1–4]). In the space industry, propulsion systems in rocket launchers are exposed to many unsteady and unstable phenomena, which may strongly excite mechanisms and structures, as for example the POGO phenomenon ([5–7]). This phenomenon impacts the interaction between the turbopump liquid circuit and the propulsion system. As a consequence for the whole system, POGO is a risk to be considered in the dynamic analysis of interaction between the fluctuation of the engine thrust and the structure of the rocket ([5, 8],[9]). To control and reduce the risks related to POGO phenomenon, different solutions can be applied, as for example:
- reducing the length of feeding lines,
- installing feeding lines perpendicularly to the structure,
- creating a small cavity on the feeding line to modify the line capacitance and dissipate energy,
- changing the compressibility of the fluid by gas injection.
- setting up apparatus to damp waves energy. By example, installing a POGO Suppressor Device (PSD), as illustrated in ref [9]. The objectives of the PSD are to absorb flow fluctuations and to increase the POGO stability.
In the scope of a scientific collaboration between the University of Grenoble Alpes, the French Space Agency CNES and Snecma (SAFRAN Group), studies are carried out to evaluate the dynamic transfer matrix of pumps operating under cavitating conditions. A hydro-acoustic test rig, presented in section 2 of this paper, has been developed at CREMHyG Lab (Centre d’Essais de Machines Hydrauliques de Grenoble). Some experimental methodologies allowing the acoustic characterization of hydraulic systems have been developed and tested. The transfer functions identification procedure consists in:
(a) evaluating the speed of sound in the hydraulic system
(b) reconstituting flow rate fluctuations by using pressure sensor measurements
(c) identifying dynamic transfer matrix of different hydraulic components.
A new estimation method of the speed of sound in water and water/air flows, based on the least mean squares approach and using three pressure transducer measurements, was proposed and validated in a previous paper [10]. The present study focuses on:
- the reconstitution of the flow rate oscillations from pressure fluctuation measurements (section 3)
- the identification of dynamic transfer matrix of a POGO Suppressor Device (section 4).

2. Description of the test rig and the POGO Suppressor Device (PSD)
A detailed description of the CREMHyG’s hydro-acoustic test rig and available apparatus and instrumentations are presented in [10]. A scheme of the test rig with the PSD installation is given in Figure 1.

The test rig is composed of about 14 m of stainless steel and PVC ducts of \( \frac{1}{2} \). The operating conditions may be selected from 0.05 to \( 5 \times 10^5 \) Pa (absolute pressure) and from 0 to 5 l/s in discharge. The acoustic circuit part is closed by two tanks, one of them is equipped of a pressure accumulator to change pressure condition (at the left side on Figure 1). A variable speed pump controls the discharge conditions. The flow is excited by a modulator allowing flow rate fluctuations with amplitudes of about 5% of the mean discharge and frequencies from 0 to 50 Hz. The fluctuations are generated by the rectilinear movement of a piston acting a calibrate volume oscillation perpendicular to the main pipe. The movement of the piston is measured by a displacement sensor, which allows the evaluation of the generated global discharge fluctuation \( \bar{q}_n(t) \) by the following relationship:
\[
\bar{q}_n(t) = \frac{S_p \left( c(t_{n-1}) - c(t_n) \right)}{t_{n-1} - t_n} \quad (m^3/s)
\]
with \( S_p \) the section of the piston (in \( m^2 \)), \( c(t_{n-1}) \) the piston stroke (in meter (m)) at the moment \( t_{n-1} \) (in second (s)). In Figure 1, the modulator is placed in position “2”.

![Figure 1: Configuration of the CREMHyG's hydro-acoustic test rig](image)
Pressure fluctuations are measured by piezoelectric sensors PCB (Piezotronics model S112A22). There are three measurement pipes with three pressure sensors mounted at pipe surface at equidistance $L = 0.3$ m. Instrumented pipes (or measurement stations) will be called A, B and C. Station A is placed in location “1” indicated in the Figure 1. Stations B and C can be placed at positions “3”, “4”, “5” or “6” depending on the considered test. Each station is composed of three instantaneous measurement points P1, P2 and P3 illustrated by Figure 2. The sensors have a sensitivity of 14.5 mV/kPa, a resolution of 0.007 kPa and a measurement range of 345 kPa. The measurements were carried out with a sampling frequency $f_s = 1000$ Hz.

![Figure 2 View of the measurement pipe](image)

In the studied configuration (Figure 1), the PSD occupies the position “5”. The PSD is similar to an accumulator, and composed of an annular tank partially filled with gas and balancing pressure equilibrium by mean of liquid exchange with the main circuit pipe through several holes (Figure 3). Position is vertical, shape is cylindrical and annular surrounding the main flow pipe. The accumulator part communicates with the feeding line through “communication holes”, sized to provide the appropriate resistance and inductance. A passive level control concept is applied to provide adjustable gas fraction to the PSD tank. The system uses a level control vent array to maintain the accumulator gas volume and liquid level, which meet required capacity and inductance goals, respectively.

The liquid level has been modified with the size of the plunging pipe. The gas capacity increases with the length of the plunging pipe. The overflow pipe ejects the gas surplus in the main flow pipe.

Tests were carried out in water and air. In the case of space launchers, the PSD is usually installed in LOX feeding line and filled with helium gas. PSD system is presented below Figure 3.

![Figure 3 Schema of PSD on LOX piping of liquid propulsion circuit](image)

3. Evaluation of the flow rate fluctuations

3.1. Theoretical descriptions
A reliable evaluation of the flow rate fluctuations in different parts of the circuit is a very important step in the procedure of identification of transfer function. This evaluation can be made by measurements from two pressure sensors [1, 3, 4] or three pressure sensors [11–13]. In the present study, the flow rate fluctuations have been estimated from the data obtained by three pressure sensors. Flow rate fluctuations are related to pressure ones from Alievi’s equation [14].

\[
\begin{align*}
\frac{\partial \bar{p}}{\partial t} + \rho c^2 \frac{\partial \bar{u}}{\partial x} &= 0 \\
\frac{\partial \bar{u}}{\partial t} + \bar{p} \frac{\partial \bar{p}}{\partial x} &= 0
\end{align*}
\]  

(2)

In the case of sinusoidal wave, the equations system solution is given by [15, 16]:

\[
\bar{p}(x, t) = \text{Real} \left( p_+ e^{i \omega (t - \frac{L}{c})} + p_- e^{i \omega (t + \frac{L}{c})} \right)
\]

(3)

And

\[
\bar{u}(x, t) = \text{Real} \left\{ \frac{1}{\rho c} \left( p_+ e^{i \omega (t - \frac{L}{c})} - p_- e^{i \omega (t + \frac{L}{c})} \right) \right\}
\]

(4)

Where \( p_+ \) the progressive wave and \( p_- \) the retrograde wave, \( \omega = 2\pi f = k \cdot c, f \) is the excitation frequency (Hz) and \( c \) is the speed of sound (m/s). Equations (2) and (3) can be also written as

\[
\bar{p}(x, t) = A e^{i \omega (t - \frac{L}{c})} + B e^{i \omega (t + \frac{L}{c})}
\]

(5)

and

\[
\bar{u}(x, t) = \frac{1}{\rho c} \left( A e^{i \omega (t - \frac{L}{c})} - B e^{i \omega (t + \frac{L}{c})} \right)
\]

(6)

The flow rate fluctuation is estimated experimentally by multiplying \( \bar{u} \) and the area of the pipe. \( A \) and \( B \) coefficient values in frequency spaces are evaluated from pressure sensor measurements.

In this case, the equations become:

\[
\bar{p}(x, f) = A(f) e^{-ikx} + B(f) e^{ikx}
\]

(7)

\[
\bar{u}(x, f) = \frac{1}{\rho c} \left( A(f) e^{-ikx} - B(f) e^{ikx} \right)
\]

(8)

By considering the position of the sensors in the measurement sections (Figure 2), the equations (7) and (8) can be written for each position of the sensor

\[
\begin{cases}
\bar{P}_1(-L, f) = A(f) e^{ikL} + B(f) e^{-ikL} \\
\bar{P}_2(0, f) = A(f) + B(f) \\
\bar{P}_3(L, f) = A(f) e^{-ikL} + B(f) e^{ikL}
\end{cases}
\]

(1)

By combining \( A(f) \) and \( B(f) \):

\[
A(f) = \frac{\bar{P}_1(-L, f) e^{ikL} - \bar{P}_3(L, f) e^{-ikL}}{e^{2ikL} - e^{-2ikL}}
\]

(9)

\[
B(f) = \frac{\bar{P}_3(L, f) e^{ikL} - \bar{P}_1(-L, f) e^{-ikL}}{e^{2ikL} - e^{-2ikL}}
\]

(10)

The mass flow rate fluctuations are calculated in frequency space by:

\[
\bar{Q}(x, f) = \frac{S}{c} \left( A(f) e^{-ikx} - B(f) e^{ikx} \right)
\]

(11)

By replacing \( A(f) \) and \( B(f) \) in equation (11), the mass flow rate fluctuation becomes:

\[
\bar{Q}(0, f) = \left( \bar{P}_1(0, f) - \bar{P}_3(0, f) \right) \ast \left( \frac{1}{2i \sin(kL)} \right) \ast \frac{S}{c}
\]

(12)

For a given oscillation frequency (for example \( f=45 \) Hz), considering \( L=0.3m \) and the speed of sound varying between 150 and 1500 m/s, it could be established that:

\[0.5655 \geq \frac{2\pi fL}{c} > 0.0565\]
And consequently
\[ 0.5358 > \sin\left(\frac{2\pi f L}{c}\right) > 0.0565 \]

With this condition, when the length \( L \) is much lower than the acoustic wavelength, the equation (12) can be written as follow:
\[
\hat{Q}(0, f) = \left( \hat{P}_1(0, f) - \hat{P}_3(0, f) \right) \ast \frac{S}{4\pi n f L} \tag{13}
\]

Figure 4 compares the flow rate oscillations obtained from some carried out tests by using the complete formula (12), and the simplified formula (13). Figure 5 presents the percentage of error between the considered results. One observes that the formula give similar results for typical high speed of sound. In another hand, for small speed of sound (in the case of diphasic fluid, caused by gas injection), the difference between theory and results becomes higher and can reach around 5%. In this case, the simplified approach is not well appropriated and it is better to apply the equation (12).

In conclusion, in monophasic fluid, the flow rate fluctuations estimated by the three sensors method could be simplified, as proposed by Gibson’s model [17] and applied by Yamamoto [4]. Nevertheless, for speed of sound under 500m/s, we recommend the equation (12) to obtain more reliable evaluation of flow rate oscillations.

![Figure 4 Comparison between complete and simplified formula for excitation at 45 Hz, Q=2.5L/s pressure =3 bar and void ratio varying from 0 to 1%.

Figure 5 Percentage of error between complete and simplified formula for excitation at 45Hz

3.2. Method validation

To try to evaluate the accuracy of the proposed method, comparisons have been carried out between flow rate oscillations obtained by three sensors approach and the total discharge fluctuation evaluated by the piston movement (equation (1)). The fluid is cold water at 20 °C; estimated speed of sound is around 1350 m/s and density of water around 1000 kg/m³.

Different operating points, presented in Table 1, have been considered in the present study. The pipe measurement sections have been displaced at different locations on the circuit to obtain three different configurations as synthesized in Table 2 and Figure 1. For this validation tests, the PSD was not installed.
To achieve comparisons for different conditions, the sum of the flow rates estimated from pipe measurements carried out in upstream and downstream regions of the modulator ($\tilde{q}_i(A)$, and $\tilde{q}_i(B)$ or $\tilde{q}_i(C)$ respectively) are supposed to be equal to the flow rates estimated by the piston displacement sensor.

The Figure 6 and Figure 7 present the different results obtained. Theoretical flow rate estimations were estimated from equation (1) by supposing a pure sinusoidal signal for the piston displacement. One can observe a good repeatability of the results obtained by mean of three sensors method under similar operating points, but we note some disparities for frequencies around 30 Hz and 50 Hz. The discrepancies between the flow rate fluctuations estimated by the pressure measurements and by the piston displacement are about 30% in the case of 1.5 bar. These discrepancies can be partially attributed to the evaluation of the piston section $S_p$ and to the possible effect of vibrations, which were not taken into account [18].

Note that the discrepancies are smaller in the case of higher pressure (Figure 7).

Table 1 Operating points considered

| Test | Mean pressure in the pipe (bar) | Mean flow rate in the pipe (L/s) | Piston stroke (mm) | Configuration |
|------|---------------------------------|---------------------------------|--------------------|--------------|
| 1    | 1.5                             | 4.5                             | 0.7                | 1            |
| 2    | 1.5                             | 4.5                             | 0.7                | 2            |
| 3    | 1.5                             | 4.5                             | 0.7                | 3            |
| 4    | 1.5                             | 2                               | 0.7                | 1            |
| 5    | 3                                | 4.5                             | 0.75               | 3            |

Table 2 Configuration of the test bench

| Configuration | Position sensor pipe A | Position sensor pipe B | Position sensor pipe C |
|---------------|------------------------|------------------------|------------------------|
| 1             | 1                      | 3                      | -                      |
| 2             | 1                      | 4                      | 5                      |
| 3             | 1                      | 3                      | 4                      |

Figure 6 Flow rate fluctuations calculated for different excitation frequencies for 1.5 bar. (A+B) is the sum of measurements in pipe A and B. (A+C) is the sum of measurements in pipe A and C

Figure 7 Flow rate fluctuations calculated for different excitation frequencies for 3 bars. (A+B) is the sum of measurements in pipe A and B. (A+C) is the sum of measurements in pipe A and C
4. Transfer function identification

4.1. Straight pipe identification

To carry out the identification of straight pipes, we have applied results obtained in Test 5 (Table 1), and have considered the transfer matrix proposed in [4]:

\[
\begin{pmatrix}
\bar{P}_s \\
\bar{Q}_s
\end{pmatrix} = \begin{pmatrix}
1 & -R - Li \cdot p \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\bar{P}_e \\
\bar{Q}_e
\end{pmatrix}
\]

We have evaluated the resistance (R) and the inductance (Li) of the pipes. p represents the Laplace variable: \(p = j\omega = j \cdot 2\pi f\)

Theoretically the resistance of the pipe is given by:

\[
R = \frac{\lambda L}{D S^2}
\]

And the inductance by:

\[
Li = \frac{L}{S}
\]

where \(L\) is the length of straight pipe (m), \(D\) is the pipe diameter (m), \(S\) is the pipe section (m²), \(\lambda\) the head loss coefficient.

To identify the transfer matrix, we calculate the magnitude and the phase of the relationship [19]:

\[
H(p) = \frac{\bar{P}_s - \bar{P}_e}{\bar{Q}_s}.
\]

The experimental results are compared with the theoretical transfer function \(H'(p) = -R - Li \cdot p\). Figure 8 illustrates the comparisons and indicates a good agreement between the analyzed data.

![Figure 8 Comparison between theoretical and experimental transfer functions for a straight pipe](image)

4.2. PSD identification

The identification of the transfer function for the POGO Suppressor Device was based on electrical analogy ([11, 20]) and on surge shaft or accumulator matrix proposed by [8, 11, 15] and [21]:

\[
\begin{pmatrix}
\bar{P}_s \\
\bar{Q}_s
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
-R_psd C_{psd} & 1 + R_psd C_{psd} + L_psd C_{psd} D^2
\end{pmatrix}
\begin{pmatrix}
\bar{P}_e \\
\bar{Q}_e
\end{pmatrix}
\]

(14)

The considered PSD (Figure 3) made in Plexiglas and in stainless steel, has the following characteristic dimensions:

- external diameter = 80mm, internal diameter = 40mm, gas volume = \(2.39 \times 10^{-4} m^3\), diameter of the plunging pipe = 7mm.
- Three configurations considering different diameters and number of communication holes have been tested:
  - Model 1: 6 communications orifices of diameter = 5.5 mm
  - Model 2: 6 communications orifices of diameter = 2.2 mm
Model 3: 12 communications orifices of diameter = 2.2 mm

The theoretical values of $R_{psd}$, $L_{psd}$ and $C_{psd}$ have been estimated from the following equations proposed by [9] and [11], and from previous SNECMA results (see Table 3):

$$R_{psd} = \frac{\Delta P}{Q}$$

$$L_{psd} = \frac{L_{liquid}}{S_{orifice}}$$

And

$$C_{psd} = \frac{\rho_{liquid} \gamma_{gas}}{\gamma_{gas} \rho_{gas}}$$

Where $L_{liquid}$ is the liquid level (in meter (m)), $S_{orifice}$ the area of the communications holes (in $m^2$), $\rho_{liquid}$ the density of liquid ($kg\cdot m^{-3}$), $P_{gas}$ the pressure of gas, $\gamma_{gas}$ the Laplace constant, $\Delta P$ the pressure difference in orifice, $Q$ the flow rate in orifice.

| Model | R Value | L Value | C Value |
|-------|---------|---------|---------|
| 1     | 1       | 1       | 1       |
| 2     | 20      | 4       | 1       |
| 3     | 7       | 2       | 1       |

Table 3 $R_{scp}$, $L_{scp}$ and $C_{scp}$ dimensionless values for each model. Model 1 is the reference. The value indicated is a multiple of reference model.

To achieve the identification of the system response, the configuration of the test bench is illustrated in Figure 1. The PSD is located in position “5”, and pipe measurement sections are placed in positions “4” and “6”. From pressure fluctuation measurements, the flow rate oscillations were estimated by equation (12) at the inlet and outlet of the PSD. The amplitude and the phase of $\dot{Q}_{psd}/\dot{P}_{e}$ have been calculated for several frequencies [19], where $\dot{Q}_{psd} = \dot{Q}_{e} - \dot{Q}_{s}$.

To be coherent with the considered linear approach, the pressure and flow rate fluctuations were kept lower than 10% of the mean pressure and mean flow rate in the pipe, respectively. The tests have been performed with a piston stroke of ±1.5 mm and ±4 mm, by considering two flow rates 2.5 l/s and 5 l/s. The mean pressure in the pipe is around 3 bar. The estimated speed of sound outlet of the PSD is around 350 m/s for 5 l/s and 250 m/s for 2.5 l/s.

Figure 9, Figure 10 and Figure 11 illustrate the results of identification for, respectively, model 1, 2 and 3. Theoretical representations have been evaluated by the following transfer function, obtained from equation (14):

$$H_{scp}(p) = \frac{C_{psd} * p}{1 + R_{psd} C_{psd} * p + L_{psd} C_{psd} * p^2}$$

The experimental identifications concerning the amplitude and the phase analyses appear in good qualitative agreement with the theoretical approach as shown in Figure 9, Figure 10 and Figure 11.
5. Conclusion

A methodology to evaluate dynamic transfer matrix using flow rate fluctuations estimated from three pressure sensors was developed and applied on a hydro-acoustic test rig. The transfer functions for one straight pipe and three configurations of POGO Suppressor Device were identified under various operating points and excitation conditions. Experimental results were in good agreement with theoretical formulations presented in the literature, based on electrical analogy and linear assumption.

The methodology is now being applied to investigate the transfer functions of an inducer working under different cavitating conditions. The interaction between inducer and PSD would be a next interesting step.

Acknowledgments

This study has been financed by the Snecma and the French Space Agency CNES. The authors would like to express their sincere gratitude to J. Toutin (from Snecma) for the very interesting discussions around this work. The laboratory LEGI is part of the LabEx Tec 21 (Investissements d’Avenir—Grant Agreement No ANR-11-LABX-0030).
References

[1] Brennen C, Acosta AJ. 1976 The Dynamic transfer function for a cavitating inducer. *Journal of Fluids Engineering; 98*: 182.

[2] Cervone A, Tsujimoto Y, Kawata Y. 2009 Evaluation of the dynamic transfer matrix of cavitating inducers by means of a simplified ‘lumped-parameter’ Model. *Journal of Fluids Engineering; 131*: 41103

[3] Pace G, Torre L, Pasini A, Valentini D, d'Agostino L 2013 Experimental characterization of the dynamic transfer matrix of cavitating inducers. In: 49th AIAA/ASME/SAE/ASEE Joint Propulsion Conference. American Institute of Aeronautics and Astronautics 136-145

[4] Yamamoto K, Müller A, Ashida T, et al. 2015 Experimental method for the evaluation of the dynamic transfer matrix using pressure transducers. *Journal of Hydraulic Research; 53*: 466–477

[5] Dordain JJ, Lourme D, Estoueig C. 1974 Etude de l’effet Pogo sur les lanceurs EUROPA II et DIAMANT B. *Acta Astronautica; 1*: 1357–1384.

[6] About G, Hauguel P, Hrisafovic N, et al. La prévention des instabilités Pogo sur Ariane 1. *Acta Astronautica 1983; 10*: 179–188

[7] Rasmouoff A, Winje RA. 1971 The Pogo phenomenon: its causes and cure. In: Napolitano LG, Contensou P, Hilton WF (eds) *Astronautical Research*. Dordrecht: Springer Netherlands, 307–322

[8] Lewis W. 1969 Simplified analytical model for use in design of pump-inlet accumulators for the prevention of liquid-rocket longitudinal oscillation (Pogo). *Nasa Technical Note.*

[9] Swanson L, Giel T. 2009 Design analysis of the Ares I Pogo accumulator. American Institute of Aeronautics and Astronautics

[10] Simon A, Martinez-Molina J-J, Fortes-Patella R. 2016 A new process to estimate the speed of sound using three-sensor method. *Experiments in Fluids; 57*

[11] Marie-Magdeleine A. 2013 *Caractérisation des fonctions de transfert d’organes hydrauliques en régimes cavitant et non-cavitant*. PHD Thesis, University of Grenoble

[12] Marie-Magdeleine A, Fortes-Patella R, Lemoine N, et al. 2012 Application of unsteady flow rate evaluations to identify the dynamic transfer function of a cavitating Venturi. *IOP Conference Series: Earth and Environmental Science; 15*: 62021

[13] Lauro JF, Boyer A. 1998 Matrice de transfert et sources hydroacoustiques d’une pompe centrifuge à faibles charges. *La Houille Blanche; 128–133.*

[14] Wylie EB, Streeter VL, Suo L 1993. *Fluid transients in systems*. Englewood Cliffs, NJ: Prentice Hall

[15] Blommaert G. 2000 *Étude du comportement dynamique des turbines Francis: contrôle actif de leur stabilité de fonctionnement*. PHD Thesis, KU Leuven

[16] Shamsborhan H. 2009 Développement d’une méthode de mesure de la célérité du son en écoulement diphasique application aux écoulements cavitants. PHD Thesis, Arts et Metiers ParisTech

[17] Gibson NR. 1923 *The Gibson method and apparatus for measuring the flow of water in closed conduits*. American Society of Mechanical Engineers

[18] Charley J. 2004 *HDR: Dynamique de structures complexes, Hydroacoustique et couplage fluide structure*. University of Lille

[19] Isermann R, Münchhof M. 2011 *Identification of Dynamic Systems*. Berlin, Heidelberg: Springer Berlin Heidelberg

[20] Brennen CE 2005 *Fundamentals of multiphase flow*. Cambridge University Press

[21] Nicolet C. 2007 *Hydroacoustic modelling and numerical simulation of unsteady operation of hydroelectric systems*. PHD Thesis, EPFL