Ultra-Wideband Flat Metamaterial GRIN Lenses Assisted With Additive Manufacturing Technique

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Abstract—This article presents the designs of ultrawideband microwave flat gradient index (GRIN) lenses, which realizes over a 108% fractional bandwidth (12–40 GHz). The frequency-independent ray optics method is employed to determine the radially varying permittivity profile of the lenses. The challenge of realizing such a radially varying profile and the limitations in dielectric material choice can be overcome by two additive-manufacturing-aided approaches: 1) partially infilled dielectrics with a varied periodicity, which ensures the lens performance at the higher end of the frequency range and 2) artificially engineered dielectrics (AED) with subwavelength-scale metallic inclusions, which enables high permittivity dielectrics and leads to benefits of thickness and mass reduction for the GRIN lenses. Measured results demonstrate that the GRIN lenses improve the gain of open-ended waveguide sources by 8.7–15.6 dB over a wide frequency range from 12 to 40 GHz, with the realized gain of up to 23.6 dBi. Both the simulation and measurements of the presented design confirm the potential of implementing the proposed GRIN lens design in high directivity and beamforming antenna applications, across an ultrawideband frequency range.

Index Terms—Additive manufacturing (AM), effective permittivity, flat grin lenses, metamaterials, millimeter waves, 3-D-printing, ultrawideband.

I. INTRODUCTION

CONVEX lenses are well-known optical devices that converge impinging parallel rays to a spot at the focal point. Similarly, the lenses can be used as collimating devices that transform the wave from the focal point into a plane wave to realize a highly directive beam. Rather than having a conventional convex lens that is made from a homogenous material with a curvature of the surface profile, it is possible to fabricate these lenses with a flat profile to simplify their mounting. Such a design is commonly referred to as a gradient index (GRIN) lens [1], whose refractive index varies locally to manipulate the ray paths. The flat GRIN lenses have great potential for antenna applications in realizing high directivity [2]–[4] and beamsteering [5]–[7].

Compared with other flat lenses or beam-collimation devices, such as a Fresnel zone plate lens [8], [9], transmit array [10], [11], and superstrates [12], [13], whose designs are typically based on a specific target frequency and, thus, lead to a relatively narrow band, a GRIN lens can offer an extremely large bandwidth. The radially varied index profile can be designed based on transformation optics [14], [15] or ray optics [16] to ensure the broadband antenna gain improvement. However, practical demonstrations appearing in the literature only have shown up to 66% fractional bandwidth and only with an 8-dB gain improvement. Here, a GRIN lens design based on the ray optics method is presented, realizing broadband beam-collimation over a 108% fractional bandwidth, together with a gain improvement of 8.7–15.6 dB.

The GRIN lenses are generally designed with radially varying refractive indexes that are challenging to fabricate in practice. The most common approach to realizing a GRIN medium is to employ metamaterials realized by using stacked printed circuit boards (PCBs) [17]–[21], which is a complex process. Precise alignment and micromachining are involved for the multilayer stacked PCB structures, which prevents manufacturing on a large scale. Furthermore, some metamaterials with resonant-particle inclusions suffer from narrow bandwidth and losses. On the other hand, some authors have used additive manufacturing (AM) to realize GRIN lenses [22]–[24], which offers a cost-effective and repeatable approach. However, one of the challenges of widely deploying AM to fabricate RF components is the limited option of materials. Common AM materials, such as polymer-based thermoplastics, such as acrylonitrile butadiene styrene (ABS) and polyactic acid (PLA), and ultraviolet sintered resins, have relative permittivities ($\varepsilon_r$) that are on the order of 2.5–3 and tan $\delta$ of 0.004–0.03. The lack of choice of materials limits the popularity of AM in applications requiring a high refractive index.

This article presents a novel and low-cost solution to design and additively manufacture a flat GRIN lens, which offers ultrawideband performance and overcomes the limitation of material choices. To the best of our knowledge, the concept of
utilizing the ray optics method to realize ultrawideband GRIN lenses for microwave applications has not been presented elsewhere heretofore. Frequency coverage in the range of 12–40 GHz is demonstrated in this work although the design can be scaled to realize lenses in a higher frequency range. Furthermore, this article details two approaches for tailoring the radially varied \(\varepsilon_r\) for the GRIN lenses. The first of these is to use the partially infilled dielectric that is realized by using the low-cost fused filament fabrication (FFF) technique. The AM process enables embedded subwavelength internal air voids in the 3-D object without requiring machining. Unlike perforating a solid material by removing parts from the raw materials, AM designs can be easily modified and rapidly prototyped with minimal material waste. The volume fraction of the internal air voids in the 3-D-printed geometries is predetermined based on the desired volume of the antenna system and the nature of the feed. Furthermore, it is equally

However, the true ray paths inside the GRIN medium are curved tracks, which leads to a greater phase delay than that of a straight ray path. Therefore, it is necessary to consider the arc length of the ray path for an accurate estimation of the phase.

The ray optics method for the arc ray path was first presented in [16] and applied to 3-D-printed lens design in [26]. However, the derivations of the method were not included in the literature. This section will show the mathematical representation of determining the inhomogeneous permittivity profile to convert the radiation between a spot and a plane wave.

The cross section of a flat GRIN lens, whose axis of symmetry is the focal axis shown in Fig. 1, has its maximum relative permittivity (or refractive index) at the center, and this index gradually decreases along the radius. The purpose of having such a GRIN is to ensure that all the rays that pass through the lens have equal phase at the exit plane \(y = T\).

The ray path integral, which is based on Fermat’s principle and satisfies the condition of equal phase delay of the rays from the focal point to where they exit the lens, is given by

\[
F + n_{\text{max}} T = OP_1 + \int_{P_1}^{P_2} n ds \tag{1}
\]

where \(n_{\text{max}}\) is the refractive index at the center.

The arc ray path length inside the GRIN medium is determined as follows:

\[
\int_{P_1}^{P_2} n ds = \frac{T (3\varepsilon_{r1} - 2\sin^2\theta)}{3\sqrt{\varepsilon_{r1} - \sin^2\theta}}. \tag{2}
\]

The derivations of (2) are given in the Appendix. Combining (1) and (2), we obtain a frequency-independent design equation for determining the variation of the permittivity \(\varepsilon_r\) at any location on the flat GRIN lens

\[
\frac{T (3\varepsilon_{r} - 2\sin^2\theta)}{3\sqrt{\varepsilon_{r} - \sin^2\theta}} = T \sqrt{\varepsilon_{r_{\text{max}}}} + F - \frac{F}{\cos\theta} \tag{3}
\]

For a practical approach, it is necessary to know the maximum value of \(\theta_{\text{max}}\), which is determined by the diameter of the lens and its focal length. These parameters are usually predetermined based on the desired volume of the antenna system and the nature of the feed. Furthermore, it is equally
important to have knowledge of the minimum and maximum permittivities of the available materials. After determining the values of $\theta_{\text{max}}$ and $\epsilon_{\text{rmin}}$, (3) can be used to find the lens thickness $T$. Generally, a small $F$ and a low $\epsilon_{\text{rmax}}$ both result in a thick lens. The $\epsilon_{\text{r}}$ variation can be calculated for a series of given $\theta$-values to produce an $\epsilon_{\text{r}}$ profile with respect to the locations at the lens. In practice, we can approximate the ideal smooth profile of $\epsilon_{\text{r}}$ with a stepped function and divide the lens into a series of concentric zones each with a different relative permittivity.

In this work, all the lenses were designed to have six dielectric rings; the ring widths were set to 10 mm. The lens thickness and the permittivity profile were determined by the available 3-D-printing materials.

III. 3-D-PRINTED DIELECTRIC UNIT CELL WITH VARIED PERIODICITY

The entire lens can be fabricated by using the FFF with a single material and in a single process, without machining and assembling, by taking advantage of the AM process. The dielectric rings are printed by using a nonsolid internal structure with air voids, whose volumes can be used to adjust $\epsilon_{\text{reff}}$ of the medium [26]. Here, we use the material volume percentage ($v$), which is determined by the ratio of the volume of the 3-D-printing material in the nonsolid printed structure, to the volume of the entire structure. The required volume percentage $v$ for tailoring the effective permittivity $\epsilon_{\text{reff}}$ of the 3-D-printed dielectrics can be approximately determined by

$$v = \frac{\epsilon_{\text{reff}} - 1}{\epsilon_{\text{ref}} - 1}$$

(4)

where $\epsilon_{\text{ref}}$ is the relative permittivity of the 100% 3-D-printing material.

The AM process typically slices the 3-D-geometry into a series of thin layers and fills them with periodic 2-D infill patterns. For ease of fabrication, the flat GRIN lens was 3-D-printed with the largest cross-sectional area sitting on the printing platform, which resulted in the periodical infill patterns in a plane with thin, sliced, circular layers. Fig. 2(a) shows the unit cell of the 3-D-printed infill pattern of the grid. The infill linewidth $w$, which is determined by the FFF machine extrusion nozzle diameter, is a constant parameter, and it is the minimum manufacturable feature size. The different material volume percentage $v$ was realized by the variable parameter $d$. Unlike conventional metamaterial unit cell designs that have a fixed periodicity, this unit cell has a variable period $d$ because, in order to achieve a very low permittivity value, the unit cell must be large due to the restriction in the minimum manufacturable feature size (i.e., infill linewidth). However, the increased unit cell size would result in a reduced maximum operating frequency when the unit cell size approaches the wavelength. Thus, if a fixed periodicity of the unit cell is used for the entire lens, the upper end of the operating frequency of the lens is primarily determined by the largest unit cells. According to (3), a GRIN lens only requires low permittivity values (i.e., a large unit cell) in the outermost region of the lens; therefore, the inner part of the lens can have a small unit cell to ensure the inner region still functions at high frequencies. Hence, our unit cell design utilizes the variable unit cell size to allow a variable upper end of the operation frequency, which depends on the periodicity of the unit cell. This design method enables us to gradually increase its size from the center to the outer region of the lens, which extends the maximum operation frequency compared with lenses based on the unit cells with fixed periodicity.

In this work, we employed the FFF machine (Makerbot 2X) equipped with a 0.4 mm extrusion nozzle. The extruded linewidth was set to $w = 0.48$ mm since the linewidth expanded by approximately 20% due to heat expansion and extrusion pressure. The simulated $\epsilon_{\text{reff}}$ of this unit cell as a function of material volume percentage (realized by varying parameter $d$) for different $\epsilon_{\text{ref}}$’s (different 3-D-printing filaments) is shown in Fig. 2. The simulation was carried out by using the periodic boundary condition in the CST Microwave Studio. $\epsilon_{\text{ref}} = 2.7$ and $\epsilon_{\text{reff}} = 4.4$ represented a normal ABS material ($\epsilon_{\text{r}} = 2.7$ and $\tan \delta = 0.01$) and a high-$\epsilon_{\text{r}}$, 3-D-printing filament PREMIX PREPERM TP20280 ($\epsilon_{\text{r}} = 4.4$ and $\tan \delta = 0.004$), respectively. The dielectric properties were also verified by the measured $\epsilon_{\text{reff}}$ at 10 GHz using an $X$-band waveguide and the Nicolson–Ross and Weir (NRW) method [27]. The photograph in Fig. 2(b) shows a 3-D-printed sample with the grid infill for the NRW measurement. Fig. 2 shows a close agreement between the simulated and measured results, and they all match those predicted by (18).

The polarization of the incident wave was also rotated from $0^\circ$ to $45^\circ$ in the plane with the unit cell, and the simulated $\epsilon_{\text{reff}}$ with different incident polarizations did not change. Since the fields illuminated by the source were always parallel to the flat lens plane, this unit cell had an isotropic $\epsilon_{\text{reff}}$ in the unit cell plane, and the 3-D-printed lens was insensitive to the impinged fields’ polarization. Thus, the lens can be mounted without perfectly aligning the unit cell with the emitted fields, which offers flexibility and practicability, and also the potential application for different polarization illuminations.

The lens performance depends on the number of dielectric rings and the maximum unit cell size. It is recommended
that the number of rings should be large enough to realize a smooth permittivity variation profile that reduces the aberrations. The ring width should be smaller than the wavelength at the maximum operating frequency to achieve the wideband performance. However, this results in very thin ring widths. If nonresonant metamaterials are used, the ring width should be greater than two unit cells to form an effective dielectric medium. This constraint is typically most relevant to the outermost ring as the outermost ring has the lowest permittivity and requires the largest unit cell. Therefore, the minimum ring width and \( \varepsilon_{\text{ref}} \) of the dielectric ring are based on FFF extrusion limitation on the value of parameter \( w \) and the choice of 3-D-printing material \( \varepsilon_{\text{ref}} \). In this work, the lowest permittivity for the lens was set to \( \varepsilon_{\text{ref}} = 1.24 \) (realized by ABS, \( \varepsilon_{\text{ref}} = 2.7 \)), and the ring width was 10 mm. A finer extrusion width \( w \) or smaller value of \( \varepsilon_{\text{ref}} \) both achieve lower \( \varepsilon_{\text{ref}} \) and allow a smaller unit cell size \( d \), which leads to a thinner ring width with a smoother index variation in the lens. Furthermore, since the unit cell sizes at the lens center are small, the smoothness of the index variation could also potentially benefit from a radially varying ring width, which has finer ring widths toward the lens center and larger ring widths toward the edge. In this work, the 10 mm constant ring width provides a good balance between design complexity and lens performance in the frequency range of 12–40 GHz.

IV. AED Materials

Although new high index 3-D-printing materials (e.g., PREMIX PREPERM) offers an extra degree of freedom of AM material choice, the range of the fabricated index is still limited by the permittivity of the filament. The AEDs method enables tweaking \( \varepsilon_r \) of dielectric materials by using discrete square (other shapes can be used as well) metallic patches and print them on top (or bottom, or both on top and bottom) of the dielectric sheet to increase the desired \( \varepsilon_r \) values and, therefore, leads to a thin lens or a short focal length to facilitate compact and low-profile antennas. Alternatively, the metallic elements can be etched from a commercially available PCB laminate, and then, the etched laminate sheet is placed above another dielectric [see Fig. 3(a)].

This type of composite structure aims to provide \( \varepsilon_{\text{ref}} \) that is equivalent to the given \( \varepsilon_r \) from a standard equivalent dielectric [EQD, see Fig. 3(b)] material. This equivalence is evaluated using the transmission phase behavior, namely, the phase of S21 of the AED should be matched to that of the EQD. The property of the AED is tuned by the parameter patch length \( l \). The \( l \) value should be smaller than the periodicity \( p \) and preferably be small to stay below resonance, so it ensures a broadband performance. However, due to the small cross-sectional area of the patches (see Fig. 3(a), front view), the cross-sectional area is determined by patch length \( l \) and the PCB metallization thickness (0.035 mm in this case); a single-layered patch has a limitation on maximum achievable \( \varepsilon_r \). Therefore, having the patches etched on both sides and tuning the dimensions of the double-layered patches offer an extra degree of freedom in tailoring the permittivity over a wide range. The effective dielectric properties of the AED were investigated by using the periodical boundary condition in the CST Microwave Studio, with the periodicity \( p \). It was found that the unit cell with periodicity smaller than \( \lambda/5 \) provided satisfactory results while avoiding the issues of high loss, narrow bandwidth, and dispersion, typically associated with metamaterials. The \( < \lambda/5 \) periodicity condition also ensured that the patch size was small enough to keep it below its resonance, which was the key to realizing a wideband design.

An AED lens was designed to have the same focal length (150 mm) and diameter (120 mm) as the 3-D-printed GRIN lens in [26] but aimed to reduce the overall thickness by \( \sim 30\% \). This required increasing \( \varepsilon_r \) of the center rings. The lens was designed to have six dielectric rings, and according to (17), the required \( \varepsilon_r \) values for the three inner rings are 2.90, 3.25, and 3.46, respectively. The values are all higher than the relative permittivity of conventional thermoplastic FFF materials, e.g., PLA and ABS, whose \( \varepsilon_r \approx 2.7 \). Although the required \( \varepsilon_r \) could be realized by using the high-\( \varepsilon_r \) FFF filament, here, we aimed to demonstrate the benefit of utilizing the AED approach in a scenario—there were no commercial-off-the-shelf specialized high-\( \varepsilon_r \) FFF materials available. For comparison, the high-\( \varepsilon_r \) FFF filament (PREMIX PREPERM) was used to fabricate the dielectrics with the same permittivities to evaluate \( \varepsilon_{\text{ref}} \) of AEDs.

Rogers RO4350B, with \( \varepsilon_r \) of 3.48 and \( \tan \delta \) of 0.0037 at 10 GHz, was used as the top PCB. The design parameters of the unit cell were \( t_1 = 1.52 \text{ mm}, \ t_2 = 11.56 \text{ mm}, \ p = 2 \text{ mm}, \ \varepsilon_{r1} = 3.48 \) (RO4350B), \( \varepsilon_{r2} = 2.70 \) (ABS), and the patch size, \( l \), was variable depending on the desired permittivity value. CST Microwave Studio was used for simulating the AED and EQD unit cells. Floquet port excitations were set up in the positive and negative z-directions. The patches were arranged in a circular/Cartesian pattern, as shown in Fig. 4, which also shows its cross section. The dimensions of the patches were chosen to ensure the S21 phases of the AED structures matched those of the EQDs that have the desired \( \varepsilon_r \) values.

The simulated S21 of three AED unit cells are shown in Fig. 5 compared with the S21 of three EQD unit cells.
A close match of the S21 phases between the AEDs and EQDs was observed, which indicated that the AED could realize the same phase delays as those achieved by standard dielectrics. It is worth noting that, although the AED shows higher insertion loss compared with the EQDs, particularly when the patch size is close to the unit cell size (see Fig. 5(c) when $l = 1.72$ mm), the lens performance does not exactly correlate with the S21 magnitude of the unit cell. This is because the unit cell simulation is based on the infinite periodical condition, but, in practice, the unit cells are only placed in a small region of the lens, and they have gradually changed geometries in order to achieve the graded-index property. The measurement results of the AED lens will be presented in Section V.

Fig. 6 shows the $\varepsilon_{ref}$ values of the AEDs that were retrieved by using the analytical method described in [28]. It is worthwhile to note that retrieving the $\varepsilon_{ref}$ values is not essential for using the AED approach. This is because the AEDs and their EQDs can be considered as equivalent to each other in a lens, to provide equal phase delays, as long as their transmission phases are matched.

The radially varying $\varepsilon_r$ values for the AED lens were calculated by using (17), and the required patch sizes of the AEDs are shown in Table I. ABS was used for printing the rest of the lens by employing the method described in Section III. The ABS infill factors for the three outer rings are shown in Table I. Note that the dielectric bases in the three inner rings of the lens were 3-D-printed with 100% ABS infill, and thus, there are no dielectric unit cells in these three rings (i.e., $d = 0$ mm). The fabricated lens is shown in Fig. 7.

In order to evaluate whether the AEDs can be truly used as the replacements of their EQDs for lens applications, a pure dielectric lens (named the EQD lens), which had the same thickness and permittivity profile as those of the AED lens,
was fabricated for comparison. The three inner rings of this lens were made by using the EQDs that are equivalent to the three inner rings of the AED lens. PREMIX PREPERM TP20280 ($\varepsilon_r = 4.4$) was used for fabricating the three inner rings since the required $\varepsilon_r$ could not be achieved by ABS. The ABS was used for the three outer rings. The reason for using normal ABS instead of the PREMIX material to fabricate the three outer rings was to ensure the EQD lens having the same outer three rings (in both material composition and infill volume percentage) as those of the AED lens.

The entire EQD lens was 3-D-printed in one single process by using a dual-extruder FFF printer (Makerbot 2X). The PREMIX material showed good compatibility and solid interface adhesion to the normal 3-D-printing ABS material. The radial variations of the $\varepsilon_r$ of the EQD lens were the same as those of the AED lens, and the volume fraction of the ABS and PREMIX material is shown in Table II.

The lens reported in [26] was also reproduced to compare with the AED and EQD lenses. Fig. 8 shows the photographs of the three prototyped lenses. The lens in Fig. 8(b) is the AED lens that has the same focal length as the lens (a) (reproduced from [26]) but with a 5.42-mm-thick reduction. The lens in Fig. 8(c) is the EQD lens that had white PREMIX material for the three inner rings and blue ABS for the three outer rings. All three lenses were designed to deliver the same collimating performance to improve the gain of the source located at their focal points. For ease of identification, three lenses were printed with different colors. Dielectric measurements showed that there were no significant differences in $\varepsilon_r$ ($\sim2.7$) and tan $\delta$ ($\sim0.01$) in the three lenses with different colors, which represented different fabrication samples based on an identical design.

### V. Results

The wideband performance of the designed lens was evaluated in the frequency range of 12–40 GHz. Ideally, the wideband GRIN lenses would need a wideband feed antenna. An ideal wideband feed antenna should have a consistent radiation pattern across the frequency band, a symmetric radiation pattern in E- and H-planes, and a stable phase center position [29]–[31]. In this work, three open-ended waveguides (OEWGs) were used as the feeds to cover the frequency bands $Ku$ (12–18 GHz), $K$ (18–26 GHz), and $K a$ (26–40 GHz), respectively, due to their relatively stable radiation patterns (the 3 dB beamwidth variation $<7^\circ$), and the gain (6–8 dB) was similar in the different frequency bands. CST Microwave Studio simulations were carried out to investigate the near-field electric field and the far-field radiation patterns of the lenses. For simplicity, the waveguide port excitation was used instead of the coaxial to waveguide transition for exciting the OWEG feed. Although the partial in-filled structure had been proven earlier (see Fig. 2) to have an effective permittivity and behave as though it were comprised of a homogeneous dielectric material, it was simulated under periodic boundary conditions (i.e., infinite number of unit cells). When the frequency approached 40 GHz, the 3-D printed largest unit cells in the outermost ring became potentially problematic since there were less than two unit cells in one ring width. Thus, the outer ring no longer provided the desired effective permittivity values. The unit cell size was less critical for the five inner rings since they were far smaller than the wavelength. In the simulations, the actual printed structure of the outermost ring of the lenses was modeled, and the five inner dielectric rings were simulated by using homogeneous effective properties to reduce the computational complexity. The permittivity and the loss tangent of the ABS and PREMIX materials were measured by using a split-post dielectric resonator at 2.4 GHz and the NRW method in the 8–12-GHz range. The measured dielectric properties were similar at 2.4 and 10 GHz. The relative permittivity of ABS materials and the PREMIX materials were 2.72 and 4.4, respectively, and the loss tangents of the ABS and PREMIX materials were 0.010 and 0.004, respectively. The measured dielectric properties were used in the simulation. The loss tangents of the homogeneous dielectrics ABS and the PREMIX were set to 0.010 and 0.004, respectively, regardless of the infill percentages. Consequently, the simulation results represent the worst case scenarios since, in reality, the loss tangent is reduced as we reduce the percentages of the in-fill.

The setups for measuring the radiation patterns are shown in Fig. 9. Fig. 10 shows the measured radiation patterns of the antenna at 15, 22, and 33 GHz and compares them with the simulated patterns. Close agreement was observed between the far-field patterns and the directional beams at $\theta = 0^\circ$ among all three lenses, across the measured frequency range, and the sidelobe levels were all below $-10$ dB with respect to the main lobe.

As expected, it was found that the radiation patterns narrowed as the frequency was increased. The measured half-power beamwidths (HPBWs) across the frequency are shown in Fig. 11, both for the E- and H-planes. All three
lenses had similar HPBW levels in the E-plane, which started at $\sim 11^\circ$ at 12 GHz and reduced to $\sim 3.6^\circ$ as the frequency was increased to 40 GHz. The measured HPBW in the H-plane was slightly larger (by $\sim 0.7^\circ$) compared with that in the E-plane. This result clearly confirms that all three lenses worked equally well and generated pencil beams and facilitated highly directional antennas.

The measured realized gain of the three OEWG-fed lenses is shown in Fig. 12 compared with simulations. All three lenses offered good wideband performance. The measured gain improvements from the three lenses show good agreement between each other, and this, in turn, serves to confirm the viability of using the AED approach to realize low-profile lenses in a low-cost and convenient manner. The root-mean-square (rms) differences between the measured and simulated gain over the frequency between 12 and 40 GHz of the thick lens, AED lens, and the EQD lens were 0.91, 0.85, and 0.93 dB, respectively. Meanwhile, the close match between the performance of the AED lens and the EQD lens demonstrates the possibility of using the AEDs to augment the properties of the dielectric materials and offer an extra degree of freedom in material selection. Both AED and EQD lenses had similar measured realized gains ranging from 15.1 dBi at 12 GHz to 23.6 dBi at 40 GHz, which implies that the gain of the feeds was enhanced by approximately 8.7–15.6 dB. The measured total antenna efficiency (efficiency = realized gain/directivity, including the mismatch) of the thick lens, AED lens, and the EQD lens had values of 82%, 83%, and 82%, respectively, when averaged over the frequency band from 12 to 40 GHz. All lenses had little effect on the S11 of the OEWG feeds, and all the OEWG feeds were well-matched; thus, the loss is mainly due to the dielectric loss. Simulations were carried out by using loss-less dielectrics, and the realized gain was improved up to 1.5 dB compared with the lossy dielectrics.

The results indicate that the gain of the thick lens is $\sim 2$ dB higher than the AED lens at the higher end of the
frequency range. The increased loss is partly attributable to the increased insertion loss of the AED when the patch size approached the periodicity [see Fig. 5(c)]. This could be overcome by using a high-permittivity PCB to allow smaller patches. The second reason for the increased loss is the increased mismatch between the free space and the dielectric media, caused by the increased permittivity of the inner rings of the AED lens. An impedance matching layer can be added to improve the gain [32]; furthermore, the matching layer can also be additively manufactured.

Fig. 13 shows the holographic phase profile displaying the phase distribution in the plane that is normal to the boresight direction at 15, 22, and 33 GHz. Good agreement is observed between the simulation and measured results, which confirms that uniform phase distribution is achieved in the effective apertures of all three lenses. This indicates that the three lenses function equally well when transforming the quasi-spherical waves (radiated by their feeds) into plane waves. It is worthwhile to note that the phase profile at 33 GHz [see Fig. 13(c)] shows a slight nonuniform phase distribution in the regions of the outer rings of the lenses. This indicates that the fabricated outermost rings did not have the desired effective permittivity profiles needed to satisfy the uniform phase condition. However, due to the varied periodicity unit cell, the inner rings still function as the homogeneous dielectrics with the desired effective permittivities, which maintains the uniform phase distribution. Also, small phase discrepancies begin to appear at each ring center, where the ring width becomes relatively large compared with the wavelength and begins to degrade the lenses’ performance. However, this nonuniform phase region remains small, and therefore, all the lenses still provide good gain improvement at the higher end of the frequency range. The performance of the fabricated lenses at high frequencies could be further improved if the same volume fraction of ABS is achieved with smaller unit cells, so the outer rings would maintain their effective medium values that they have at lower frequencies, and the small unit cell also permits us to use thinner rings for smoother variation of the permittivity profile. High-resolution AM facilities, which can fabricate objects with fine features, can be used to fabricate the reduced size of individual air voids or meshes in relatively small values and, therefore, produce GRIN lenses with a wider bandwidth.

VI. CONCLUSION AND DISCUSSION

This article has presented the design and fabrication of flat GRIN lenses with an ultrawideband behavior that spans over a 108% fractional bandwidth. The design equation has been derived via the frequency-independent GRIN ray optics theory. The GRIN lenses were additively manufactured, and the partially infilled printing realizes the dielectric rings with tailored permittivities to form the flat GRIN medium. The varied periodicity unit cell realizes a gradually increased unit cell size from the center to the outer region of the lens, so the inner region still functions at high frequencies even when the outer region does not provide the desired effective permittivity value. The dielectric property of the partially infilled geometries has been investigated and proven via dielectric property measurements. Besides using partially infilled structures, \( \varepsilon_{\text{reff}} \) can be further tailored by using the AED approach. The AEDs have been conveniently realized by combining 3-D-printed structures with commercially available PCB laminate materials. The equivalence between AED materials and their EQDs has been verified by the agreement in the transmission performance, which enables us to implement the AEDs in the flat GRIN lens application.

Measurement results have shown the advantages of the ultrawideband GRIN lens in beam collimating, and all three
lenses had realized gain from 15.1 to 23.6 dBi between 12 and 40 GHz, fed by OEWGs, which increased the gain of the OEWG feeds by 8.7 up to 15.6 dBi. Furthermore, all lenses offered high antenna efficiency of 82% averaged between 12 and 40 GHz. Additional time-domain measurement and the fidelity factor that quantifies the signal distortion introduced by the lens can be examined, if desired, to show the wideband true-time-delay operation [33]–[35]; however, fully fledged investigation of the time-domain performance of the lens is beyond the scope of this work. The equivalent performance characteristics of the three lenses clearly demonstrate the benefits of utilizing high-permittivity materials to reduce their thicknesses by ~30% and their mass by ~20%. Furthermore, the results also prove the concept of using the AED approach to replace the dielectric materials in an ultrawideband frequency range when the required permittivity values are not commercially available. The AED approach is scalable and can be easily combined with materials with different permittivity ranges. For instance, the AED approach can be applied to the PREMiX ($\varepsilon_r = 4.4$) materials to create new AED materials that have $\varepsilon_{reff} > 4.4$. The number of metallic layers can also be increased to realize the required permittivity values. The AED approach is also compatible with the multilayer PCB or low-pass frequency selective surfaces (FSS)-based approaches [34], [36] for the applications that require thin lenses or a small $f/D$ value to realize compact lens antenna systems. This gives us extra degrees of freedom to select dielectric materials, which is certain to benefit the task of RF component design.

The lenses in this work had $f/D = 1.25$. Generally, a smaller $f/D$ lens would require a thinner ring width to realize a smoother permittivity profile to ensure the wideband and high gain performance. This is because the permittivity variation in a lens with a small $f/D$ is greater than the variation in a lens with a large $f/D$ value. A lens with decreased $f/D$ or thickness leads to an increased permittivity variation range, which also increases the reflection due to the impedance mismatch between the free space and the high-index medium. This could be the potential limitation of the AED approach since a high-index medium requires large metallic patches, but it also introduces additional loss due to the mismatch. A low-profile wideband impedance matching layer that does not significantly increase the lens thickness is highly desirable.

The proposed AM approach is a low-cost technique, which is well-suited for fast automated repeatable design and manufacturing. It has the potential to significantly transform the manufacturing processes and to significantly shorten the time that it takes for innovative new technologies to reach consumers while reducing labor costs and waste.

**Appendix**

We assume that the angle between the path where the ray enters the lens and the focal axis is $\theta$ (see Fig. 1). At $P_1$, according to Snell’s law, it follows that

$$\sin \theta \cdot 1 = \sin \varphi_1 \cdot n_1$$  \hspace{1cm} (5)

where $n_1$ is the reflective index at $P_1$, $\varphi_1$ is the angle between the tangent to the ray and the $y$-axis.

If the ray path in the GRIN medium has a function $y = f(x)$, the length of the ray path from $P_1$ to $P_2$ can be expressed as

$$\int_{P_1}^{P_2} n dx = \int_{x_1}^{x_2} n \sqrt{1 + \left(\frac{df}{dx}\right)^2} dx = F(x)dx.$$  \hspace{1cm} (6)

The lens can be considered as a quarter-pitch GRIN fiber, and therefore, the ray from $P_1$ to $P_2$ has the shortest optical path, and the path length satisfies the Euler–Lagrange equation:

$$\frac{\partial F}{\partial x} = \frac{d}{dx} \left( n \sqrt{1 + \left(\frac{df}{dx}\right)^2} \right) = \frac{n \cdot \frac{df}{dx}}{\sqrt{\left(\frac{df}{dx}\right)^2 + 1}} = c$$  \hspace{1cm} (7)

where $c$ is a constant. For $f(x)$ at $P_1$, $(df/dx)$ is the slope of $f(x)$ with respect to $x$. Thus, (7) can be rewritten as

$$\frac{\partial F}{\partial x} = \frac{n_1 \cdot \cot \varphi_1}{\sqrt{\cot^2 \varphi_1 + 1}} = n_1 \cdot \cos \varphi_1 = c.$$  \hspace{1cm} (8)

Similarly, we have $n_2 \cdot \cos \varphi_2 = c$ at $P_2$, where $\varphi_2$ is the angle between the ray path outside the lens and the normal vector of the lens plane. Since the ray path outside the lens is also normal to the lens plane, $\varphi_2 = 0$, and we have $n_2 = c$. Substituting this into (8) results in the relation $n_1 \cdot \cos \varphi_1 = n_2$, and combining with (5), we have

$$n_2^2 = n_1^2 - \sin^2 \vartheta = \varepsilon_{r1} - \sin^2 \vartheta$$  \hspace{1cm} (9)

where $\varepsilon_{r1}$ is the permittivity at $P_1$.

Meanwhile, substituting $n_2 = c$ into (7), $(df/dx)$ can be expressed as

$$\frac{df}{dx} = \frac{n_2}{\sqrt{n_1^2 - n_2^2}}.$$  \hspace{1cm} (10)

Therefore, (6) can be rewritten as

$$\int_{P_1}^{P_2} n dx = \int_{x_1}^{x_2} \frac{n^2}{\sqrt{n_1^2 - n_2^2}} dx = \int_{x_1}^{x_2} \frac{n_2}{\sqrt{n_1^2 - (\varepsilon_{r1} - \sin^2 \vartheta)}} dx.$$  \hspace{1cm} (11)

Since $P_1$ and $P_2$ are the two points on the ray path $y = f(x)$, they satisfy

$$T = f(x_2) - f(x_1) = \int_{x_1}^{x_2} \frac{df}{dx} dx = \int_{x_1}^{x_2} \frac{n_2}{\sqrt{n_1^2 - n_2^2}} dx$$
$$= \int_{x_1}^{x_2} \frac{\sqrt{\varepsilon_{r1} - \sin^2 \vartheta}}{\sqrt{n_1^2 - (\varepsilon_{r1} - \sin^2 \vartheta)}} dx.$$  \hspace{1cm} (12)

In order to solve (12), it requires determining the relationship between $x_1$, $x_2$, and $n$. We assume that the relative permittivity varies linearly in the short region from $x_1$ to $x_2$, and it satisfies

$$a = \frac{\varepsilon_{r1} - \varepsilon_{r2}}{x_2 - x_1} = \frac{\sin^2 \vartheta}{x_2 - x_1}$$  \hspace{1cm} (13)

$$x_2 = \frac{\sin^2 \vartheta + ax_1}{a}$$  \hspace{1cm} (14)
where $a$ is a constant. For a random point $x$ between $x_1$ and $x_2$, the relative permittivity at $x$ also satisfies

$$-a = \frac{\varepsilon_r - \varepsilon_{r1}}{x - x_1} \quad (15)$$

$$\varepsilon_r = \varepsilon_{r1} - a(x - x_1). \quad (16)$$

Since $n^2 = \varepsilon_r$, we first substitute (14) and (16) into (11) and then carry out the integration

$$\int_{P_1} n ds = \int_{x_1}^{x_2} \frac{\varepsilon_r - a(x - x_1)}{\sqrt{\varepsilon_r - a(x - x_1) - (\varepsilon_{r1} - \sin^2\theta)}} \, dx$$

$$= \frac{\sin \theta(6\varepsilon_{r1} - 4\sin^2\theta)}{3a}. \quad (17)$$

Similarly, substituting (14) and (16) into (12) and carrying out the integration, we get

$$a = \frac{2 \sin \theta \sqrt{\varepsilon_{r1} - \sin^2\theta}}{T}. \quad (18)$$

Next, we substitute (18) into (17), and we can obtain the expression of the path length inside the GRIN medium as follows:

$$\int_{P_2} n ds = \frac{T(3\varepsilon_{r1} - 2\sin^2\theta)}{3\sqrt{\varepsilon_{r1} - \sin^2\theta}}. \quad (19)$$

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