Method of Monte Carlo simulation for the analysis of uncertainty for ultrasonic time-of-flight

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Abstract. The measurement of wind speed may be performed using ultrasonic transducers based on the principle of measuring the time-of-flight. However, due to the propagation of the ultrasonic signal in the medium of measurement, noises are added which can be modelled by different natures that affect the measurement accuracy of transit time and therefore the speed of the wind. In this paper, the method of Monte Carlo Simulation was used for the analysis of uncertainties in the estimation of the ultrasonic transit time when considering different noise distributions of probability density functions such as Gaussian, Triangular and Uniform.

1. Introduction
The principle of measuring the wind speed using ultrasonic transducers is based on the time-of-flight estimation, which is the travel time of the ultrasonic wave measured as the time elapsed from the emission to detection (considering a pair of ultrasonic transmitter and receiver). The estimation of the transit time named ToF (Time of Flight) presents an increased uncertainty of the estimation when the measurement medium is influenced by noise, which could come from several forms such as electric, additive and multiplicative noises, thermal and more. Therefore, the ToF uncertainty is directly propagated to the wind speed measurement. This way, in the design of measurement systems using ultrasonic transducers, it becomes relevant to analyse the propagation of uncertainties in the estimative and measurement process of ToF.

The measurement uncertainty can be evaluated and expressed in several ways; two of them are the analysis by the Guide to the Expression of Uncertainty in Measurement (GUM) and the Method of Monte Carlo Simulation (MCS) [1] [2] [3]. GUM is utilized to determine a mathematical expression of the uncertainties in a process of measurement. However, it establishes some limitations where the uncertainties must be from random sources and known models of distribution. Moreover, the systematic uncertainties must be eliminated or not even exist. On the other hand, the method of Monte Carlo Simulation (MCS) allows us to build a probability density function (pdf) acquiring the statistical parameters of the variables studied [4]. In addition, the variables could have different distribution and not necessarily just Gaussian.

Given this scenario, this article aims to build probabilistic models associated with the estimation of the ultrasonic ToF considering distinct noise types present in the medium of measurement, with known distributions such as: Gaussian, Uniform and Triangular. For this purpose, the method of Monte Carlo simulation (MCS) will be used, performing under a computational model built to model the measurement process of wind speed using ultrasonic transducers.
2. Measurement of wind speed using ultrasonic transducers

Figure 1 illustrates the configuration for estimating the wind velocity, using a pair of ultrasonic transmitter - receiver. The transmitter transducer operates at a frequency of 40 kHz and emits a mechanical wave that passes through the medium and interacts with the air flow (wind). The medium introduces a random additive noise and attenuates the signal due to its impedance. When the receiving transducer detects the ultrasonic signal, it performs a domain change of mechanical energy into electrical energy. Usually, an amplification is required at the receiving stage [5] [6].

From the configuration of measurement are related the process variables that define the measurement of wind speed by:

\[ \theta = \frac{1}{\cos \theta} \left( \frac{L}{ToF} - C \right) \]  

(1)

Where: \( L \) is the distance between the ultrasonic transducers, \( \theta \) is the alignment angle between the ultrasonic transducers and the wind direction, \( ToF \) is the ultrasonic transit time and \( C \) is the speed of sound. The speed of sound in dry air is influenced by temperature variations, which is expressed by:

\[ C = 20.074 \sqrt{273.15 + T} \]  

(2)

Where, \( T \) is the medium temperature of measurement in °C.

Figure 2 illustrates the electrical signals in the transmitter (TX) and receiver (RX) ultrasonic transducer. The mathematical model of the electrical signal in TX is expressed by:

\[ v_{TX}(t) = A_0 \sin(\omega \times t) \]  

(3)

Where: \( A_0 \) is the generator amplitude, \( \omega \) is the generator frequency, \( t \) is the time and \( s(t) \) is the transmitted signal.

The mathematical model of the received ultrasonic signal can be built considering the attenuation of the medium into the signal, a delay due to the interaction with the medium (ToF), a phase difference due to the impedance of the medium and an additive noise (\( \eta \)). So, the modeling is given by:

\[ v_{RX}(t) = A \sin(\omega(t - ToF) + \phi) + \eta \]  

(4)

From the received ultrasonic signal, the Threshold Detection method can be utilized for the ToF estimation. However, the estimation by this method can exhibit some changes in its probability density function (pdf) due to the model parameterization and the noise kinds present in the measurement medium, as illustrated in figure 3. ToF’s pdf analysis can be made using the method of Monte Carlo Simulation (MCS) which is utilized in the evaluation of propagation of uncertainties as indicated in the supplement 1 to the GUM, where it demonstrates some procedures to find possible results.
associated to a distribution along with its respective probabilities of occurrence and its statistical moments [3]. To the pdf determination of ToF were utilized 3 kinds of noise distributions: Gaussian, Uniform and Triangular.

![Figure 3. Model Parametrization.](image)

In this model, the output ToF is the main variable to be estimated while the other independent variables are the inputs of the model parametrization, such as wind speed (\(\vartheta\)), distance between the ultrasonic transducers (\(L\)), angle between the ultrasonic transducers and the wind direction (\(\theta\)), medium temperature of measurement (\(T\)) and an additive noise (\(\eta\)). This model parametrization allows us to determine the combined uncertainties propagation by the following expression:

\[
ToF = f(L, C, \vartheta, \theta) + \eta
\]

where:

\[
f = \frac{L}{C + \beta\cos\theta}
\]

3. Monte Carlo simulation to estimate the uncertainties of the ToF

The method of Monte Carlo Simulation is defined as a class of statistical methods based on random sampling mass (random number) to obtain probability distributions of the variables of the problem, thus obtaining information about the future performance of systems or processes.

From the metrological point of view, the Monte Carlo simulation is a tool used for evaluation and propagation of uncertainties, which is indicated in the GUM Supplement 1, providing a range of possible outcomes associated with a distribution of their respective probabilities occurrence, providing useful information for the determination of statistical moments, such as mean, variance, standard deviation, kurtosis, and skewness. Figure 4 illustrates the flowchart for the estimating of ToF based on Monte Carlo simulation using different probability density functions (PDFs) of the additive noise [3].

This procedure has as inputs the mathematical model of the problem, which in this case was represented by two electrical signals associated with ultrasonic transducers transmitter (TX) and receiver (RX). Also, in the same level, it has the characterization of the influence of the uncertainties associated with an additive noise that is produced by means which propagates the ultrasonic waves. It is noticed that the received signal presents an additive noise that could have a probability density function (PDF) known that may be Gaussian, triangular or uniform shape. Additionally, we use an \(M\) number of simulations that for this work was 10000.

At the stage of processing, it was set up a random number generator capable of generating the distribution types Gaussian, Triangular and Uniform with known mean value and standard deviation,
which are related to the properties of additive noise present in the propagation medium. To implement the method of Monte Carlo simulation, $M$ sets of random numbers (simulations) were generated, where each set represents a signal of additive noise $u_d$.

The evaluation of measurement uncertainty using the MCS technique is performed in two phases. The first is to establish the model for the measurement, and subsequently, the second involves the evaluation of that model. The differences between more traditional methods and MCS are related to the several types of information describing the input parameters and the way this information is processed to obtain the measurement uncertainty.

Based on this procedure, we can show a sequence of actions necessary to evaluate the measurement uncertainty using the MCS, according to the Supplement Guide 1 to the Expression of Uncertainty in Measurement, entitled “Numerical Methods for the Propagation of Distributions.” According to the GUM Supplement 1, from $M$ simulations by the Monte Carlo method, PDFs were constructed of ultrasonic transit time (TOF). From these PDF forms were observed certain characteristics and extracted fundamental statistical parameters such as mean value (best estimate) and standard deviation (standard uncertainty).

![Figure 4](image_url)

**Figure 4.** Procedure for the estimation of the PDFs of ToF for different PDF of the additive noise.
4. Evaluation of uncertainties of ToF using Monte Carlo Method

The system for measuring wind speed (figure 1) has been configured for a wind speed of 10 m/s, distance between the transducers $L = 0.1$ m, temperature $T = 20$ °C, alignment angle $\theta = \pi/4$, transmitted electrical sign of 1 V amplitude, medium attenuation in 10% of the maximum amplitude signal. Furthermore, it was considered the presence of an additive noise with different probability distributions. The detection of the received signal utilizing the Threshold Detection method was carried out considering a threshold of 0.8 V.

Thus, simulations were performed using the Monte Carlo Simulation procedure (MCS) with $M = 10000$ (number of samples) to meet the shape of the distribution ToF when considering the diverse types of noises for the model.

Figure 5 illustrates the signal-to-noise ratio (SNR) in function of standard deviation of the medium noise. Three kinds of noise probability density functions were considered in the simulations: Gaussian, Uniform and Triangular (according to figure 3). It was observed the Gaussian noises provide results with smaller SNR. On the other hand, a noise with uniform distribution provides results with bigger values for SNR. Meanwhile, triangular distribution produces values for SNR between the maximum and minimum of it.

Figures 6, 7 and 8 illustrate results for the ToF's PDF acquired using MCS considering 3 noise types of PDF with standard deviation, $\sigma = 0.3$ mV. For each scenario of simulation, normalization tests were utilized for the respective ToF's PDF and in all the cases the results for the tests were positive to normal distribution or Gaussian distribution.
5. Conclusions
In this work, the implementation of the Monte Carlo simulation method has been shown to determine the probability distributions of transit of flight (ToF) for applications to wind speed measurement using ultrasonic transducers, when considering various forms of additive noise in the propagation medium (Gaussian, Triangular and Uniform). The proposed method considers the procedures of Supplement 1 to the Guide for the Evaluation and Expression of Uncertainty in Measurement (GUM). It was observed that the method of Monte Carlo simulation provided the ToF’s PDFs which could be related to the shapes of the additive noise PDFs. Therefore, it is concluded that the shape of the noise PDF distribution influences the shape of the ToF’s PDF. This result can be used to enhance the model of the system for measuring wind speed when used ultrasonic transducers whose measuring principle is the ultrasonic transit time (ToF).

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