The Holographic Approach to Cosmology

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Abstract: We review the successes and challenges of the holographic approach to cosmology. The model predicts an exactly scale invariant fluctuation spectrum with long and short distance cut-offs. It can account for the observed fluctuations in the CMB and might explain the low power at large scales. We outline various cosmological histories compatible with holographic initial conditions. This paper is based on talks given by the authors at Cosmo 04 in Toronto, and the 2004 Tamura Symposium in Austin

Keywords: holography, cosmology.
1. Introduction

Holographic Cosmology[1] is a quantum mechanical approach to cosmological initial conditions. The covariant entropy bound [3] bounds the entropy in causal diamonds for early time observers in FRW cosmology by a function of cosmic time, which decreases as one approaches the Big Bang. The entropy of any density matrix is bounded by the logarithm of the dimension of the quantum Hilbert space, and we have conjectured that the covariant entropy bound should be interpreted in terms of this dimension. If we accept the postulate that the description of a causal diamond does not depend on physics outside the diamond, this bound shows that no conventional field or string theoretic description can describe observations in the early universe, because they have too many degrees of freedom.

We have, instead, provided an explicit quantum model of early causal diamonds in terms of a sequence of Hamiltonians drawn from a certain random ensemble of Hamiltonians in finite dimensional Hilbert spaces[2]. We showed that this system had the following properties

- It solved a set of plausible consistency conditions[2] for any quantum description of a generally covariant system.
- In the limit when the Hilbert space dimension was large, it obeyed all of the scaling laws of Friedmann Robertson Walker (FRW) cosmology with equation of
state $p = \rho$ and flat spatial sections. The equation of state, and flatness, follow from the quantum dynamics, and are exact properties of the model. In addition, the full quantum dynamics had an exact symmetry under the conformal Killing transformation of the $p = \rho$ geometry.

- This enables one to prove a scaling law for fluctuations around the homogeneous $p = \rho$ background. We will review this below.

- All the features of our heuristic description of the $p = \rho$ cosmology as a dense black hole fluid\cite{4} were exhibited as properties of the mathematical model.

- All of these features follow for generic initial conditions in the model.

Holographic cosmology is based on the assumption that our own universe began as a close relative of the model cosmology of \cite{2}. That is, it was a somewhat less entropic choice of the initial conditions. We picture it as a more or less normal FRW region embedded as a subset of the coordinate space of the $p = \rho$ universe. Here, as always, we work in coordinates where the causal diamond reaching from each point on a time slice, back to the Big Bang, has the same area. We can think of this in terms of phases of a black hole fluid. In conventional cosmology a dilute black hole fluid has equation of state $p = 0$. Our model provides evidence for another phase, with equation of state $p = \rho$, which has larger entropy density. Our universe begins as an amorphous region of the $p = 0$ phase, embedded in the $p = \rho$ phase. We call this region the cosmic emulsion.

The Israel junction condition gives us some idea of the constraints on this region. We first apply it to a sphere. Matching the geometry, on equal area time slices, we find that the coordinate size of a $p = w\rho$ ($w < 1$) sphere must shrink\footnote{de Sitter space is an exception. We can match the horizon of a causal patch to the horizon of a black hole in the $p = \rho$ universe, with no coordinate shrinkage of the dS region. We return to this in the conclusions}. We conclude that the initial region must be a complicated ”cosmic emulsion” made of many connected spheres. Let $\epsilon$ be the minimal ratio of normal volume to dense black hole fluid volume, consistent with survival of the normal region. We find that after a cosmic time $T \sim \frac{1}{\epsilon^2}$, the volume of the universe (or of the largest coordinate sphere surrounding the cosmic emulsion) is dominated by a normal radiation gas. The dilute black hole gas, originally occupying the normal region, quickly decays to radiation by the Hawking process.

The regions of dense black hole fluid in the interstices of the cosmic emulsion, now behave like black holes of average size $T$, embedded in a radiation dominated universe. This black hole fluid is on the edge of the transition between the two phases. The
physical volumes covered by black holes and by radiation gas are approximately equal. If we wait one ten-folding of the universe the dilute phase is established. The large black holes now dominate the energy density of the universe but they behave as a dilute gas. It is only at this point in time that conventional notions of field theory in a background space-time begin to make sense. Fluctuations in the energy density which were generated during the $p = \rho$ era are now encoded in fluctuations in the number density and masses of black holes. For consistency of the picture, the fluctuations must be small. Order one fluctuations would lead to larger black holes, via collision processes, and the universe would quickly evolve back into the dense black hole fluid phase. We do not yet have an estimate of how small the fluctuations must be in order for a normal region to survive.

The energy density (we always use Planck units) at the point when normal field theoretic cosmology becomes valid is \( \frac{1}{aT^2} \), where we have estimated \( a \sim 10 \). Our model assumes that there is a field\(^2\) whose potential energy density has the form

\[ \mu^4 v\left(\frac{\phi}{m_P}\right) \]

, where \( m_P \) is the reduced Planck mass. We further assume that \( \mu^4 \leq (10)^{-3}T^{-2} \), so that as the scale factor \( a \) increases, the black hole energy density becomes smaller than the energy density in this field. Note that since the holographic initial conditions have already led to a highly homogeneous universe, it is entirely plausible that there are regions where the field \( \phi \) is spatially homogeneous over scales of order \( \frac{m_P}{\mu^2} \). Furthermore, since the field evolution begins at energy densities \( \gg \mu^4 \), its initial motion is certainly friction dominated. Thus, the initial conditions are set up for slow roll inflation.

The fluctuations in black hole density, which were generated during the $p = \rho$ era, are imprinted on the scalar field, because the criterion for the beginning of inflation in some region of space is that the black hole energy density in that region drop below \( \mu^4 \). These fluctuations will thus appear in the density of radiation produced by the decay of the inflaton at the end of inflation. In addition, there may be fluctuations produced by the quantum mechanics of the scalar field during inflation. Note however, that the inflationary fluctuations might be quite small if \( \mu \) is very small.

2. Generation of fluctuations in the $p = \rho$ era

The mathematical model of [2] is very far from quantum field theory in a background geometry. The geometry is generated dynamically, in such a way that the system’s

\(^2\)The single field could be an effective description of a single trajectory in a multi-dimensional field space.
entropy automatically scales like the area. Thus, we cannot study fluctuations in this system by using the conventional perturbation theory of Einstein’s equations. A rigorous study of perturbations would require us to define another solution of the consistency conditions of [2] corresponding to a small perturbation of the $p = \rho$ solution. This is hard, and we have not yet done it.

However, the form of the two point fluctuation is completely determined by the symmetries in the model. The quantum system constructed in [2] led naturally to the introduction of a homogeneous isotropic FRW space-time manifold. Imagine that we have succeeded in modifying the construction, so that we can talk about a normal region embedded in the $p = \rho$ background, and let $P(x, t)$ be the density of normal regions in the vicinity of the space-time point $(x, t)$. The model had exact quantum symmetries corresponding to rotation and translation of the $x$ coordinates, as well as to the conformal transformation $t \to bt, \quad x \to b^{-\frac{2}{3}}x$. The measure $d^3x P(x, t)$ should be invariant under these symmetries. As a consequence, the two point function for fluctuations of $P(x, t)$ must have the form:

$$< P(x, t) P(y, s) > = \int d^3xe^{i k(x-y)} G(t | k|^3/2, s | k|^3/2).$$

As noted above, the energy density fluctuations in the normal region are fluctuations in the density of interstitial black holes in the cosmic emulsion. These are related to $P(x, t)$ by convolution with a transfer function

$$\frac{\delta \rho}{\rho}(x, t) = \int_1^t ds d^3y F(|x-y|, t, s) P(y, s).$$

The transfer function relates the density of normal regions in the emulsion to the density of interstices which are filled with $p = \rho$ matter. It depends on the local geometry of the emulsion, but it should fall off at large distances at all times (its Fourier transform should approach a constant for $|k|$ smaller than a few inverse Planck lengths.). The fluctuation spectrum at the instant, $T$, after the transition to the dilute black hole gas phase is thus:

$$< \frac{\delta \rho}{\rho}(k, T) \frac{\delta \rho}{\rho}(-k, T) > = |F|^2 \int_1^T dt \int_1^T ds G(tk^{3/2}, sk^{3/2}),$$

where we have factored out the constant value of the Fourier transform of $F$. Rescaling the integration variables, we get

$$< \frac{\delta \rho}{\rho}(k, T) \frac{\delta \rho}{\rho}(-k, T) > = \frac{|F|^2}{k^3} \int_1^{Tk^{3/2}} dt \int_1^{Tk^{3/2}} ds G(t, s).$$

These are scale invariant fluctuations over a range of $k$ whose physical size at the time of the transition runs from the inverse horizon size at that time, down to a few Planck lengths.
lengths. The fluctuations must be small, but we do not have a quantitative estimate of how small. We also do not have an argument about higher moments of the distribution.

3. The scale of fluctuations today

After the end of the dense black hole era, the universe is matter dominated while the scale factor increases to \( a = (T^2\mu^4)^{-\frac{1}{2}} \). This is followed by \( N \) e-folds of inflation, after which the universe reheats to a temperature \( T_{RH} = \mu^3 \). The correlated fluctuations, which were of physical size \( T \) at the end of the dense black hole era, have size:

\[
R_{RH} = T(T^2\mu^4)^{-1/3}e^N\mu^{-4},
\]

at the end of reheating. Their current size is \( R_{RH}T_{NOW} \). This should be compared to the current horizon size, which satisfies \( R_{NOW}T_{NOW} \sim 10^{29} \). If we wish to assert that the fluctuations we now observe in the CMB, originated in the \( p = \rho \) era, we must require that the correlation length of fluctuations is at least as large as \( R_{NOW} \). This leads to the inequality

\[
T^{1/3}\mu^{-7/3}e^N \geq 10^{29}
\]

We must also require that the fluctuations extend over 3 decades in wavelength (five decades if we want scale invariant fluctuations at galaxy scales). This means \( T_2^2 \geq 10^3(10^5) \). Note that something like the latter inequality is probably required for self-consistency of our picture, independent of the observations. The description of the microscopic dynamics of [2] by a \( p = \rho \) fluid is valid only at scales much larger than the Planck scale. Thus, \( T \), the horizon size at the end of the \( p = \rho \) era, must certainly be much larger than the Planck scale.

Another observational bound comes from requiring that the reheat temperature be larger than 6 MeV, so that nucleosynthesis proceeds according to the standard theory. This implies \( \mu > 10^{-7} \).

In principle, one should eventually be able to calculate \( T, \mu \) and \( N \) from a microscopic theory. At this phenomenological stage we can only discuss scenarios. The above bounds apply to the scenario in which the fluctuations generated during the \( p = \rho \) era are the ones we observe on the sky. It is also possible that the inflationary period generates the observed fluctuations. Note however that the \textit{a priori} lower bounds on \( T \) suggest that \( \mu \) may be quite small. The inflationary fluctuations of a simple slow roll model would then be unobservable, and we would have to turn to a more complicated hybrid model.
The most exciting scenario is the one we have outlined above. It predicts an exactly scale invariant adiabatic fluctuation spectrum over a range of scales which extends from some $R_{\text{max}}$ down to $T^{-\frac{2}{3}}R_{\text{max}}$. The UV and IR cut-offs on this spectrum are rather sharp. If, for some reason, $R_{\text{max}}$ were of order the current horizon scale, this could explain the low angular momentum anomalies in the CMB. Although the fluctuations are definitely small, we have not yet been able to quantify their size, nor to determine whether they are Gaussian. These questions depend on the microscopic dynamics of the cosmic emulsion.

We do not see a mechanism in this scenario, for generating gravitational waves of wavelength shorter than $T$ prior to the beginning of inflation. Thus, observable gravitational wave backgrounds could only be generated by inflation. However, since $\mu < 10^{15}$ GeV, the inflationary gravitational wave amplitude is too small to be observed in the foreseeable future. Note that this part of the conclusion is valid, even if the observable scalar CMB fluctuations come from inflation. We consider the conclusion about gravitational waves generated prior to inflation to be preliminary.

4. Conclusions

Holographic cosmology is a quantum mechanical description of the Big Bang. It sets up the proper initial conditions for inflation. It also solves the homogeneity, isotropy and flatness problems without recourse to inflation. Depending on parameters we are currently unable to calculate, it may produce an observable, non-inflationary contribution to CMB fluctuations. This contribution is adiabatic, and exactly scale invariant, between sharply defined UV and IR cut-offs. Preliminary arguments indicate that the primordial gravitational wave spectrum predicted by this model is below the level one can expect to observe in the near future.

One final note about how stable and generic our construction is, must be addressed. The discussion up to this point referred an infinite $p = \rho$ universe with an infinite cosmic emulsion embedded in it. The initial coordinate density of cosmic emulsion is $\epsilon$, which should be taken as small as possible to obtain the most generic initial conditions compatible with survival of the normal region. A much more probable initial condition would be a finite cosmic emulsion in an infinite $p = \rho$ universe. This is potentially unstable, because the Israel junction condition requires a sphere of normal region to shrink in coordinate size. A way to avoid this conclusion is to insist that the normal region asymptote to a de Sitter space. The cosmological horizon of de Sitter space can be joined to a marginally trapped surface in the $p = \rho$ background. That is, the normal universe is the interior of a $p = \rho$ black hole! One way to understand this is that empty de Sitter space has the same amount of entropy as the ball of dense black hole fluid
which it displaces. Thus, although the normal region of the universe begins as a low entropy region, it eventually evolves to a maximal entropy region. This introduces the cosmological constant as a new parameter. It is clear that in terms of the measure on initial conditions, a universe which will evolve to have a large cosmological constant is initially more entropic than a universe which evolves to small $\Lambda$. That is, in causal diamonds much smaller than the event horizon, the dS vacuum has very small entropy.

Thus, we are led to a picture in which the cosmological constant is a random variable, with an a priori measure which favors large $\Lambda$. Weinberg’s bound[5] now implies that if our definition of a “normal” universe includes the requirement that galaxies can form, then the most probable normal universe in our model will have $\Lambda \sim Q^3 \rho_0$. Here $Q$ is the amplitude of primordial density fluctuations at horizon crossing, and $\rho_0$ is the dark matter density at the beginning of the matter dominated era. In inflationary models of the fluctuations, $Q$ is independent of $\Lambda$ for $\Lambda \ll \mu^4$, and the same is true a fortiori for our $p = \rho$ generated fluctuations, which arise even earlier than the inflationary era. $\rho_0$ is typically determined by microscopic physics and will also be independent of $\Lambda$ for small $\Lambda$. Assuming these numbers take on their real world values (in the eventual Theory of Everything) then these considerations would “explain” the value of $\Lambda$ as corresponding to the most probable initial condition for a universe containing galaxies.

5. Acknowledgments

The research of T.B was supported in part by DOE grant number DE-FG03-92ER40689, the research of W.F. was supported in part by NSF grant- 0071512.
References

[1] T. Banks, W. Fischler, *M-theory observables for cosmological spacetimes*, hep-th/0102077; T. Banks, W. Fischler, *An Holographic Cosmology*, hep-th/0111142; T. Banks, W. Fischler, *Holographic Cosmology 3.0*, hep-th/0310288.

[2] T. Banks, W. Fischler, L. Mannelli, *Microscopic Quantum Mechanics of a $p = \rho$ Universe*, hep-th/0408076.

[3] W. Fischler, L. Susskind, *Holography and Cosmology*, hep-th/9806039; R. Bousso, *Holography in general space-times*, JHEP 9906, 028 (1999) hep-th/9906022

[4] T. Banks, W. Fischler, *An Holographic Cosmology*, hep-th/0111142

[5] S. Weinberg, *Anthropic Bound on the Cosmological Constant*, Phys. Rev. Lett. 59, 2606, (1987).