Effective, Efficient and Robust Neural Architecture Search

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Abstract

Recent advances in adversarial attacks show the vulnerability of deep neural networks searched by Neural Architecture Search (NAS). Although NAS methods can find network architectures with the state-of-the-art performance, the adversarial robustness and resource constraint are often ignored in NAS. To solve this problem, we propose an Effective, Efficient, and Robust Neural Architecture Search (E2RNAS) method to search a neural network architecture by taking the performance, robustness, and resource constraint into consideration. The objective function of the proposed E2RNAS method is formulated as a bi-level multi-objective optimization problem with the upper-level problem as a multi-objective optimization problem, which is different from existing NAS methods. To solve the proposed objective function, we integrate the multiple-gradient descent algorithm, a widely studied gradient-based multi-objective optimization algorithm, with the bi-level optimization. Experiments on benchmark datasets show that the proposed E2RNAS method can find adversarially robust architectures with optimized model size and comparable classification accuracy.

1. Introduction

Deep learning has achieved great success in many areas, such as computer vision, nature language processing, speech, gaming and so on. The design of the neural network architecture is important for such success. However, such design relies heavily on the knowledge and experience of experts and even experienced experts cannot design the optimal architecture. Therefore, Neural Architecture Search (NAS), which aims to design the architecture of neural networks in an automated way, has attracted great attentions in recent years. NAS has demonstrated the capability to find neural network architectures with state-of-the-art performance in various tasks [9, 20, 29, 32]. Search strategies in NAS are based on several techniques, including reinforcement learning [23, 37], evolutionary algorithms [19, 25], Bayesian optimization, and gradient descent [20, 33, 3]. As a representative of gradient-descent-based NAS methods, the Differentiable ARchiTecture Search (DARTS) method [20] becomes popular because of its good performance and low search cost.

However, those NAS methods are typically only de-
signed for optimizing the accuracy during the architecture searching process while neglecting other significant objectives, which results in very limited application scenarios. For example, a deep neural network with high computational and storage demands is difficult to deploy to embedded devices (e.g. mobile phone and IoT device), where the resource is limited. Besides, the robustness of deep neural networks is also important. It is well known that the trained neural networks are easily misled by adversarial examples [11, 17, 30], which makes them risky to deploy in real-world applications. For example, a spammer can easily bypass the anti-spam email filter system by adding some special characters as perturbations, and a self-driving car cannot recognize the guideboard correctly after sticking some adversarial patches.

Therefore, multi-objective NAS has drawn great attention recently because we need to consider more than performance when NAS meets real-world applications [4, 9, 29]. In [14, 9, 4], the model size and computational cost are considered to satisfy some resource constraint. Besides, some works [8, 12] search for differentiable architectures that can defend adversarial attacks. However, to the best of our knowledge, there is no work to simultaneously optimize the three objectives, i.e. the performance, the robustness, and the resource constraint.

To fill this gap, this paper proposes an Effective, Efficient, and Robust Neural Architecture Search method (E2RNAS) to balance the trade-off among multiple objectives. Built on DARTS, the proposed E2RNAS method formulates the entire objective function as a bi-level multi-objective optimization problem where the upper-level problem is a multi-objective optimization problem, which can be viewed as an extension of the objective function proposed in DRATS. To the best of our knowledge, there is little work to solve such bi-level multi-objective optimization problem based on gradient descent techniques. To solve such problem, we propose an optimization algorithm by combining the multiple gradient descent algorithm (MGDA) [7] and the bi-level optimization algorithm [5].

Specifically, the contributions of this paper are threefold:

• We propose the E2RNAS method for searching effective, efficient and robust network architectures, leading to a practical DARTS-based framework for multi-objective NAS.

• We formulate the objective function of the E2RNAS method as a novel bi-level multi-objective optimization problem and propose an efficient algorithm to solve it.

• Experiments on benchmark datasets show that the proposed E2RNAS method can find adversarially robust architectures with optimized model size and comparable classification accuracy.

2. Related Works

2.1. Adversarial Attack and Defence

Deep neural networks are not robust while facing adversarial attacks [28]. Most adversarial attacks are white-box attacks that assume attack algorithms can access to all configurations of the trained neural network, including the architecture and model weights.

Fast Gradient Sign Method  Goodfellow et al. [11] propose Fast Gradient Sign Method (FGSM) for generating adversarial examples. It directly uses the sign of the gradient of the loss function with respect to weights as the direction of the adversarial perturbation as

$$\delta = \epsilon \cdot \text{sign}(\nabla_{x} \ell(\theta, x, y)),$$

where x is the original input, \(\epsilon\) is a small scalar to represent the strength of the perturbation, \(\theta\) denotes parameters of the victim model, y is the original ground-truth label for input x, \(\text{sign}(\cdot)\) denotes the elementwise sign function, \(\ell(\cdot, \cdot, \cdot)\) denotes the loss function used for training the victim model, and \(\nabla_{x} \ell(\theta, x, y)\) denotes its gradient with respect to x.

Projected Gradient Descent (PGD) Instead of generating one-step perturbations as in FGSM, Kurakin et al. [17] propose the PGD method by applying a small number of iterative steps. To ensure the perturbation is in \(\epsilon\)-neighborhood of the original image, the PGD method clips the intermediate results after each iteration as

$$\delta_{t+1} = \text{clip}_{\epsilon}(\delta_{t} + \xi \cdot \text{sign}(\nabla_{x} \ell(\theta, x + \delta_{t}, y))),$$

where \(\delta_{t}\) is the perturbation generated in the \(t\)-th steps, \(\xi\) is the attack step size. \(\text{clip}_{\epsilon}(\cdot)\) means to elementwisely clip the input to lie within an interval \([-\epsilon, \epsilon]\), i.e. \(||\delta_{t+1}||_{\infty} \leq \epsilon\).

Adversarial Training  Adversarial training is an effective method for defending adversarial attacks [11, 17, 21, 30]. Goodfellow et al. [11] leverage the FGSM as a regularizer to train deep neural networks and make the model more resistant to adversarial examples. Wong et al. [30] use FGSM adversarial training with random initialization for the perturbation. The proposed method can speed up the adversarial training process and it is as effective as the PGD-based adversarial training.

2.2. Multi-Objective Optimization

Multi-objective optimization aims to optimize more than one objective function simultaneously. Among different techniques to solve multi-objective problems, we are interested in gradient-based multi-objective optimization algorithms [7, 10, 27], which leverage the Karush-Kuhn-Tucker (KKT) conditions [16] to find a common descent direction for all objectives.

In this paper, we utilize one such method, i.e. MGDA [7]. With \(n\) objective functions \(\{L_{i}(\theta)\}_{i=1}^{n}\) to be minimized, MGDA is an iterative method by first solving the
3.1. Preliminary: DARTS

In this section, we present the proposed E2RNAS method. We first give an overview of the DARTS method and then introduce how to achieve the robustness and formulate the objective to constrain the number of parameters in the search architecture. Finally, we present the bi-level multi-objective problem of the proposed E2RNAS method as well as its optimization.

3.1. Preliminary: DARTS

DARTS [20] aims to learn a Directed Acyclic Graph (DAG) called cell, which can be stacked to form a neural network architecture. Each cell consists of $N$ nodes $\{x_i\}_{i=0}^{N-1}$, each of which denotes a hidden representation. $O$ denotes a discrete operation space. The edge $(x_i, x_j)$ of the DAG represents an operation function $o(\cdot)$ (e.g., skip connection or $3 \times 3$ pooling) from $O$ with a probability $\alpha_{o(i,j)}$ to perform at the node $x_i$. Therefore, we can formulate each edge $(x_i, x_j)$ as a weighted sum function to combine all the operations in $O$ as $f_{i,j}(x_i) = \sum_{o \in O} \frac{\exp(\alpha_{o(i,j)})}{\sum_{o' \in O} \exp(\alpha_{o'(i,j)}/o(x_i))$. An intermediate node $x_j$ is the sum of its predecessors, i.e., $x_j = \sum_{i<j} f_{i,j}(x_i)$. The output of the cell, i.e., node $x_{N-1}$, is the concatenation of all the output nodes excluding the two input nodes $x_0$ and $x_1$. Therefore, $\alpha = \{\alpha_{o(i,j)}\}_{(i,j) \in E, o \in O}$ can parameterize the searched architecture, where $E$ denotes the set of all the edges from all the cells.

Let $X_{tr}$ denote the training dataset and $Y_{tr}$ denote the corresponding set of labels. Similarly, the validation dataset and labels are denoted by $X_{val}$ and $Y_{val}$. We use $\theta$ to denote all the weights of the neural network and $\ell(\theta, x, y)$ to denote the loss function. DARTS is to solve a bi-level optimization problem as

$$\min_{\alpha} \quad L_{val}(\theta^*(\alpha), \alpha)$$

$$\text{s.t.} \quad \theta^*(\alpha) = \arg \min_{\theta} L_{tr}(\theta, \alpha),$$

where $L_{tr}(\theta, \alpha) = \frac{1}{|X_{tr}|} \sum_{(x,y) \in (X_{tr}, Y_{tr})} \ell(\theta, x, y)$ and $L_{val}(\theta, \alpha) = \frac{1}{|X_{val}|} \sum_{(x,y) \in (X_{val}, Y_{val})} \ell(\theta, x, y)$ represent the training and validation losses, respectively. Here $\min_{\alpha} L_{val}(\theta^*(\alpha), \alpha)$ is called the upper-level problem and $\min_{\theta} L_{tr}(\theta, \alpha)$ is called the lower-level problem.

When the search procedure finishes, the final architecture can be determined by the operation with the largest probability in each cell, i.e., $o(i,j) = \arg \max_{o \in O} \alpha_{o(i,j)}$.

3.2. Adversarial Training for Robustness

In E2RNAS, we expect the searched architecture to be robust, which means that for the trained model with the searched architecture, its performance is stable when adding some perturbation to the dataset. To improve the robustness of the searched architecture, we leverage the adversarial training method in [30] to train a robust model.

Following [30], for each sample $x$ and its corresponding label $y$, we can generate a perturbation for $x$ using one single step as

$$\delta'(x, y) = \text{clip}_\epsilon(\delta + \xi \cdot \text{sign}(\nabla_x \ell(\theta, x + \delta, y)),$$

where $\epsilon$ is the perturbation size, $\delta$ is randomly initialized with an uniform distribution on the interval $[-\epsilon, \epsilon]$, and $\xi$ is the attack step size. Therefore, we generate the adversarial instance as $x' = x + \delta'(x, y)$. Obviously, the FGSM is a special case of this method when $\delta$ is initialized with zero.
and $\xi = \epsilon$. This FGSM-based adversarial training method with random initialization for $\delta$ [30] can effectively defend the PGD adversarial attack [17], while not adding much computational cost in the architecture search procedure.

We use these perturbed data to learn the network parameters $\theta$ so that the trained model can defense adversarial attacks. Therefore, we aim to minimize the training loss of the perturbed data as

$$L_{tr}^{adv} (\theta, \alpha) = \frac{1}{|X_{tr}|} \sum_{(x,y) \in (X_{tr}, Y_{tr})} \ell (\theta, x', y).$$  \hspace{1cm} (3)

Note that this adversarial training method trains the model only on adversarial examples, which is different from the FGSM-based adversarial training method [11] that uses them as a regularization term for training.

### 3.3. Objective Function of Resource Constraints

Architectures with a small number of parameters have more application scenarios even in resource-constrained mobile devices. Therefore, we regard resource constraints as one of the desired objectives.

By following DARTS [20], we determine the operation of each cell in the final architecture as the one with the largest probabilities. So the number of parameters in an architecture can be computed as

$$N(\alpha) = \sum_{(i,j) \in E} n_{\arg \max_{o \in O} o^{(i,j)}},$$  \hspace{1cm} (4)

where $n_o$ denotes the number of parameters corresponding to the operation $o$.

Note that $\arg \max_{o \in O} o^{(i,j)}$ in Eq. (4) is a non-differentiable operation, making the computation of the gradient of $N(\alpha)$ with respect to $\alpha$ infeasible. To make such operation differentiable, we use the softmax trick to approximate the arg max operation and then formulate the approximation $\hat{N}(\alpha)$ as

$$\hat{N}(\alpha) = \sum_{(i,j) \in E} \sum_{o \in O} \frac{\exp(o^{(i,j)} \mid o)}{\sum_{o' \in O} \exp(o'^{(i,j)} \mid o')} n_o.$$  \hspace{1cm} (5)

Furthermore, to prevent the model to search oversimplified architectures (i.e. the one containing too many parameter-free operations) that leads to unsatisfactory performance, we add a lower bound $L$ to the parameter size in Eq. (5), i.e. $\hat{N}(\alpha) \geq L$. Therefore, the objective function of the resource constraint can be formulated as

$$\Psi(\alpha) = \max(\hat{N}(\alpha), L).$$  \hspace{1cm} (6)

Different from RC-DARTS [14] that directly adds the resource constraint into the original DARTS objective function (2) as a constraint and formulates the objective function as a constrained optimization problem, here we take it as an objective function.

### 3.4. Bi-level Multi-Objective Formulation

E2RNAS aims to search the architecture parameter $\alpha$ to minimize the validation loss for the effectiveness and the number of parameters for the efficiency, while achieving the robustness via the adversarial training. Thus, we combine Eqs. (3) and (6) as well as the adversarial training to formulate the entire objective function as

$$\min_{\alpha} \left( L_{val}(\theta^*(\alpha), \alpha), \Psi(\alpha) \right)$$  \hspace{1cm} s.t. $\theta^*(\alpha) = \arg \min_{\theta} L_{tr}^{adv}(\theta, \alpha).$  \hspace{1cm} (7)

Problem (7) is similar to the bi-level optimization problem (2) in the DARTS, where the lower-level problem (i.e. $\min_{\theta} L_{tr}^{adv}(\theta, \alpha))$ is similar, but there exists significant differences in that the upper-level problem (i.e. $\min_{\alpha}(L_{val}(\theta^*(\alpha), \alpha), \Psi(\alpha)))$ contains two objectives. So problem (7) is a bi-level multi-objective optimization problem which is a generalization of problem (2) in the DARTS. There are few works on bi-level multi-objective optimization [2, 6, 36, 26] and to the best of our knowledge, the proposed optimization algorithm as introduced in the following is the first gradient-based algorithm to solve general bi-level multi-objective optimization problems.

Problem (7) can be understood as a two-stage optimization. Firstly, when given an architecture parameter $\alpha$, we can learn a robust model with optimal model weights $\theta^*$ via the empirical risk minimization on adversarial examples. Secondly, given $\theta^*$, the architecture parameter $\alpha$ is updated on the validation dataset by making a trade-off between its performance and model size. Therefore, we can solve problem (7) in two stages, which are described as follows.

**Updating $\theta$** Given the architecture parameter $\alpha_t$, $\theta$ can be simply updated as

$$\theta_{t+1} = \theta_t - \eta_{\theta} \nabla_{\theta} L_{tr}^{adv}(\theta_t, \alpha_t),$$  \hspace{1cm} (8)

where $t$ denotes the index of the iteration and $\eta_{\theta}$ denotes the learning rate.

**Updating $\alpha$** After obtaining $\theta_{t+1}$, we can optimize the upper-level problem to update the architecture parameter $\alpha$. As the upper-level problem is a multi-objective optimization problem, we adopt the MGDA to solve it. In MGDA, we first need to solve problem (1), which requires the computation of the gradients of the two objectives with respect to $\alpha$. The gradient of $\Psi(\alpha_t)$ with respect to $\alpha$ is easy to compute, while the gradient of $L_{val}(\theta^*(\alpha_t), \alpha_t)$ with respect to $\alpha$ is a bit complicated as $\theta^*(\alpha_t)$ is also a function of $\alpha$ and it is too expensive to obtain $\theta^*(\alpha_t)$. Therefore, we use a second-order approximation as

$$\nabla_{\alpha} L_{val}(\theta^*(\alpha_t), \alpha_t) \approx \nabla_{\alpha} L_{val}(\theta_{t+1} - \eta_{\theta} \nabla_{\theta} L_{tr}^{adv}(\theta_{t+1}, \alpha_t), \alpha_t),$$  \hspace{1cm} (9)
Obviously when $\eta_0 = 0$, $\theta_{t+1}$ becomes an approximation of $\theta^*(\alpha_t)$ and Eq. (9) degenerates to the first-order approximation, which can speed up the gradient computation and reduce the memory cost but lead to worse performance [20]. So we use the second-order approximation in Eq. (9). Then due to the two objectives in the upper-level problem of problem (7), we can simplify problem (1) as a one-dimensional quadratic function of $\gamma$ as

$$\min_{0 \leq \gamma \leq 1} \|u_1 + (1 - \gamma)u_2\|^2, \quad (10)$$

where $u_1 = \nabla_{\alpha_t} L_{\text{val}}(\theta^*(\alpha_t), \alpha_t)$ and $u_2 = \nabla_{\alpha_t} \Psi(\alpha_t)$ denote the gradients of two objectives, respectively. Here $\gamma$ can be viewed the weight for the first objective and $1 - \gamma$ is for the second objective. It is easy to show that problem (10) has an analytical solution as

$$\hat{\gamma} = \max\left(\min\left(\frac{(u_2 - u_1)^T u_2}{\|u_1 - u_2\|^2}, 1\right), 0\right). \quad (11)$$

After that, we can update $\alpha_t$ by minimizing

$$U(\theta^*(\alpha_t), \alpha_t) = \hat{\gamma} L_{\text{val}}(\theta^*(\alpha_t), \alpha_t) + (1 - \hat{\gamma}) \Psi(\alpha_t)$$

$$\alpha_{t+1} = \alpha_t - \eta_\alpha \nabla_{\alpha_t} U(\theta^*(\alpha_t), \alpha_t), \quad (12)$$

where $\eta_\alpha$ denotes the learning rate for $\alpha$.

**Algorithm 1 E2RNAS**

**Input:** Dataset $\{X_{tr}, Y_{tr}, X_{val}, Y_{val}\}$, batch size $B$, perturbation size $\epsilon$, minimum constraint $L$, learning rates $\eta_\alpha$ and $\eta_\theta$

**Output:** Learned architecture parameter $\alpha$

1. Randomly initialized $\alpha_0$ and $\theta_0$;
2. $t := 0$;
3. while not converged do
   4. Sample a mini-batch of size $B$;
   5. Compute $L_{\text{val}}(\theta_t, \alpha_t)$ according to Eq. (3);
   6. Update $\theta_{t+1}$ according to Eq. (8);
   7. Compute two objective functions $L_{\text{val}}(\theta^*(\alpha_t), \alpha_t)$, $\Psi(\alpha_t)$ and the corresponding gradients;
   8. Compute $\hat{\gamma}$ according to Eq. (11);
   9. Update $\alpha_t$ according to Eq. (12);
10. $t := t + 1$;
11. end while

**Comparison between E2RNAS and DARTS** Though the proposed E2RNAS method is based on the DARTS, there are two key differences between them, which are shown in Figure 1. Firstly, E2RNAS adopts the adversarial training to improve the robustness of the corresponding neural network. Secondly, E2RNAS evaluates model with two objectives: minimizing the validation loss for the effectiveness and the number of parameters for the efficiency. Therefore, E2RNAS can search an effective, efficient, and robust architecture. The whole algorithm is summarized in Algorithm 1.

**4. Experiments**

In this section, we empirically evaluate the proposed E2RNAS method on three image datasets, including CIFAR-10 [15], CIFAR-100 [15], and SVHN [22]. Details about these datasets are presented in the Appendix.

**4.1. Implementation Details**

**Search Space** The search space adopts the same setting as DARTS [20]. There are two types of cells, i.e., the reduction cell and the normal cell. The reduction cell is located at the 1/3 and 2/3 of the total depth of the network and other cells belong to the normal cell. For both reduction and normal cells, there are 7 nodes in each cell, including four intermediate nodes, two input nodes, and one output node. In both normal and reduction cell, the set of operations $O$ contains eight operations, including $3 \times 3$ separable convolutions, $5 \times 5$ separable convolutions, $3 \times 3$ dilated separable convolutions, $5 \times 5$ dilated separable convolutions, $3 \times 3$ max pooling, $3 \times 3$ average pooling, identity, zero. For the convolution operator, the ReLU-Conv-BN order is used.

**Training Settings** By following DARTS [20], a half of the standard training set is used for training a model and the other half for validation. A small network of 8 cells is trained via the FGSM-based adversarial training method [30] in Eq. (3) with the batch size as 64 and initial channels as 16 for 50 epochs. Following the setting of [30], the perturbation of the FGSM adversary is randomly initialized from the uniform distribution in $[-\epsilon, \epsilon]$, where $\epsilon = 2/255$. The attack step size $\xi$ is set to 1.25$\epsilon$. The SGD optimizer with the momentum 0.9 and the weight decay $3 \times 10^{-4}$ is used. The proposed method is implemented in PyTorch 0.3.1 and all the experiments are conducted in Tesla V100S GPUs with 32G CUDA memory.

**Evaluation Settings** A large network of 20 cells is trained on the full training set for 600 epochs, with the batch size as 96, the initial number of channels 36, a cutout of length 16, the dropout probability 0.2, and auxiliary towers of weight 0.4. To make the model size comparable, we adjust the initial channels of each cell for both DARTS and the proposed E2RNAS method, which is denoted by “\{model-C#channels\}”. The accuracy is tested on the full testing set. Adversarial examples are generated using the PGD attack [17] with the perturbation size $\epsilon = 2/255$ on the testing set. The PGD attack takes 10 iterative steps with the step size of 2.5$\epsilon$ as suggested in [21].

**4.2. Analysis on Experimental Results**

**Search Architecture on CIFAR-10** The normal and reduction cells searched by the E2RNAS method on the CIFAR-10 dataset are presented in Figures 2 and 3, respectively. Different from DARTS [20], the reduction cell
| Architecture                | Test Err. (%) ↓ | Params (MB) ↓ | PGD Acc. (%) ↑ | Search Cost (GPU days) ↓ | Search Method          |
|-----------------------------|-----------------|---------------|----------------|--------------------------|------------------------|
| DenseNet-BC† [13]           | 3.46            | 25.6          | -              | -                        | manual                 |
| NASNet-A [37]               | 2.65            | 3.3           | -              | -                        | RL                     |
| AmoebaNet-B [25]            | 2.55±0.05       | 2.8           | -              | -                        | evolution              |
| Hierarchical Evolution† [19]| 3.75±0.12       | 15.7          | -              | -                        | evolution              |
| PNAS† [18]                  | 3.41±0.09       | 3.2           | -              | -                        | SMBO                   |
| ENAS [23]                   | 2.89            | 4.6           | -              | 0.5                      | RL                     |
| DARTS‡ [20]                 | 2.59            | 3.349         | 6.57           | 0.595*                   | gradient-based         |
| DARTS-C28‡                  | 2.68            | 2.061         | 5.42           | 0.595*                   | gradient-based         |
| DARTS-C20‡                  | 3.15            | 1.083         | 3.90           | 0.595*                   | gradient-based         |
| DARTS-C12‡                  | 3.09            | 0.416         | 3.08           | 0.595*                   | gradient-based         |
| P-DARTS‡ [3]                | 2.59            | 3.434         | 8.35           | 0.247*                   | gradient-based         |
| PC-DARTS‡ [33]              | 2.65            | 3.635         | 9.53           | 0.426*                   | gradient-based         |
| E2RNAS-C46                  | 3.64            | 3.383         | 10.21          | 0.836*                   | gradient-based         |
| E2RNAS-C36                  | 4.19            | 2.102         | 9.61           | 0.836*                   | gradient-based         |
| E2RNAS-C25                  | 4.86            | 1.042         | 7.76           | 0.836*                   | gradient-based         |
| E2RNAS-C16                  | 6.03            | 0.449         | 6.76           | 0.836*                   | gradient-based         |

Table 1. Comparison with state-of-the-art NAS methods on the CIFAR-10 dataset. † represents training without the cutout augmentation. ‡ indicates the use of the code released by original authors. ↑ indicates a larger value is better, while ↓ indicates a lower value is better. “{model-C#channels}” means the architecture searched by “model” is evaluated with the initial number of channels as “channels”. * means the search cost is recorded on a single Tesla V100S GPU.

in E2RNAS contains many convolution operations and the normal cell only includes one operation with parameters (i.e., the 5×5 separable convolution). Thus, the parameter size of the architecture searched by E2RNAS is lower than that of DARTS because E2RNAS searched an architecture with fewer reduction cells.

**Figure 2.** The normal cell in E2RNAS learned on CIFAR-10.

**Figure 3.** The reduction cell in E2RNAS learned on CIFAR-10.

**Architecture Evaluation on CIFAR-10** The comparison of the proposed E2RNAS method with state-of-the-art NAS methods on the CIFAR-10 dataset is shown in Table 1. Notably, E2RNAS outperforms these NAS methods in [37, 25, 19, 18] by searching for a more lightweight architecture with lower search costs of three to four orders of magnitude and a slightly higher test error rate. Moreover, although ENAS [23] slightly outperforms E2RNAS in the test accuracy and search time, it finds a deeper architecture with about doubled model size (i.e., 4.6MB for “ENAS” vs. 2.102MB for “E2RNAS-C36”).

Compared to the original DARTS in [20], “E2RNAS-C36” significantly improves the robustness with lower model size and comparable search cost while the classification error increases slightly. Some studies [24, 34] show that the increased robustness is usually accompanied by decreased test accuracy. Therefore, the increased test error of E2RNAS is because of the improved robustness and the decreased parameter size, which indicates E2RNAS can make a better trade-off among these three goals than DARTS.

Besides, both P-DARTS [3] and PC-DARTS [33] search for a deeper architecture with less search cost than “E2RNAS-C36”, so they slightly outperform in the test error rate with competitive PGD accuracy. We can apply the E2RNAS method to P-DARTS and PC-DARTS to make a trade-off among multiple objectives (i.e., the accuracy, the robustness and the number of parameters) in future work.

To further compare the performance of E2RNAS and DARTS, we change the initial number of channels in the architecture evaluation for both methods to keep a roughly similar model size. According to the results shown in Ta-
CIFAR-100 and SVHN datasets.

Table 2. Comparison with state-of-the-art NAS methods on the CIFAR-100 and SVHN datasets.

| Dataset | Architecture | Test Err. (↓)(%) | Params (MB) ↓ | PGD Acc. (↑)(%) |
|--------|--------------|------------------|-------------|----------------|
| CIFAR-100 | DARTS$^1$ [20] | 17.17 | 3.401 | 2.06 |
| | DARTS-C34$^1$ | 17.70 | 3.047 | 1.67 |
| | DARTS-C27$^1$ | 17.78 | 1.960 | 1.70 |
| | DARTS-C19$^1$ | 19.15 | 1.010 | 1.34 |
| | P-DARTS$^1$ [3] | 15.67 | 3.485 | 4.58 |
| | PC-DARTS$^1$ [33] | 16.66 | 3.687 | 4.29 |
| | E2RNAS-C38 | 19.30 | 3.459 | 4.90 |
| | E2RNAS-C36 | 19.19 | 3.120 | 4.00 |
| | E2RNAS-C29 | 19.80 | 2.075 | 3.78 |
| | E2RNAS-C20 | 22.97 | 1.041 | 3.44 |
| SVHN | DARTS$^1$ [20] | 2.16 | 3.449 | 46.78 |
| | DARTS-C34$^1$ | 2.18 | 2.998 | 41.32 |
| | DARTS-C28$^1$ | 2.13 | 2.061 | 35.35 |
| | DARTS-C20$^1$ | 2.16 | 1.083 | 40.38 |
| | P-DARTS$^1$ [3] | 2.12 | 3.433 | 49.11 |
| | PC-DARTS$^1$ [33] | 2.20 | 3.635 | 54.81 |
| | E2RNAS-C39 | 2.21 | 3.421 | 44.15 |
| | E2RNAS-C36 | 2.14 | 2.935 | 52.38 |
| | E2RNAS-C30 | 2.13 | 2.075 | 53.82 |
| | E2RNAS-C21 | 2.21 | 1.062 | 54.96 |

In Table 1, we can see that E2RNAS remarkably improves the robustness with comparable classification accuracy. For example, compared to "E2RNAS-C46" with "DARTS", the PGD accuracy increases about 1.6 times, while the test error increases by only around 0.9%.

In summary, experiment results in Table 1 show that E2RNAS can search significantly robust architectures with a lower model size and comparable classification accuracy, compared with state-of-the-art NAS methods.

Architecture Evaluation on CIFAR-100 and SVHN

The comparison of E2RNAS with DARTS on the CIFAR-100 and SVHN datasets is presented in Table 2. The performance of E2RNAS on the CIFAR-100 dataset is similar to that on the CIFAR-10 dataset in that E2RNAS can search a robust architecture with a lower model size and a slightly decreased test accuracy. For example, compared to DARTS, "E2RNAS-C36" reduces the number of parameters by 0.3MB and improves the PGD accuracy by nearly twice times, though the test error is slightly increased (about 2%). In addition, E2RNAS shows excellent results on the SVHN dataset. It not only significantly improves the robustness but also achieves competitive test accuracy with a lower parameter size. For instance, compared to DARTS, "E2RNAS-C36" reduces the model size by about 15% and increases the PGD accuracy, while keeping competitive performance. Therefore, those quantitative experiments indicate that E2RNAS can search robust architectures with a lower model size and comparable performance.

4.3. Ablation Study

In this section, we study how each design in E2RNAS influences its performance on different objectives. The corresponding results are presented in Table 3. The adversarial training (abbreviated as adv) in the lower-level problem of problem (7) transforms training data to adversarial examples and hopes to learn a robust model when given an architecture. The resource constraint (abbreviated as nop) in the upper-level problem of problem (7) expects to constrain the parameter size of the searched architecture. The multiple-gradient descent algorithm (abbreviated as MGDA) is applied to solve the upper-level problem of problem (7), which is a multi-objective problem to minimize both the validation accuracy and the model size. If without MGDA, it means that we solve the upper-level problem by minimizing an equally weighted sum of two objectives (i.e., $\gamma = 0.5$ in Eq. (12)). The lower bound (abbreviated as L) of the number of parameters expects to prevent the model to search over-simplified architectures.

Impact of Adversarial Training

The adversarial training, which trains a neural network on adversarial examples, is an effective method for improving the robustness of a neural network. Thus, we apply it in the lower-level problem of problem (7) and hope the searched architecture can defense adversarial attacks. Here we discuss two impacts of the adversarial training in details.

Firstly, using the adversarial training tends to reduce the number of parameters, which may leads to worse accuracy. We notice that the parameter size of the architecture searched by DARTS with adversarial training (i.e., "E2RNAS w/o nop") in Table 3 is only 1.37MB, which means that the searched architecture contains many parameter-free operations. Therefore, it has a larger test error because of its simplistic architecture, although its PGD accuracy is larger than DARTS with a comparable model size (i.e., "DARTS-C20" in Table 1). Besides, compared with E2RNAS, the model size of "E2RNAS w/o adv" increases by 1.631MB, which indicates that the adversarial training significantly decreases the number of parameters. However, it can be alleviated by constraining the parameter size with a lower bound L.

Secondly, using the adversarial training can help E2RNAS make a trade-off between the robustness and accuracy. We notice adversarial training can significantly influences the model size. Therefore, to make a fair comparison, we set the number of initial channels of "E2RNAS w/o adv" (i.e., "E2RNAS w/o adv (C27)") in the architecture evaluation to keep its model size roughly similar to E2RNAS. The result in Table 3 shows "E2RNAS" has a
Table 3. Ablation study on the CIFAR-10 dataset. adv means using adversarial training in the low-level problem of problem (7); nop indicates adding the resource constraint into the upper-level problem of problem (7); MGDA denotes using MGDA to make a trade-off between the accuracy and model size and if without MGDA, it means equal weights of the two objectives in problem (7) are used (i.e. $\hat{\gamma} \equiv 0.5$ in Eq. (12)). $L$ is the lower bound of the number of parameters. ↑ indicates larger value is better, while ↓ indicates lower value is better. “C27” means the initial number of channels in the architecture evaluation is changed to 27, instead of the default number of 36.

| Method       | adv | nop | MGDA | $L$ (MB) | Test Err. (%) ↓ | Params (MB) ↓ | PGD Acc. (%) ↑ |
|--------------|-----|-----|------|----------|-----------------|---------------|---------------|
| E2RNAS       | √   | √   | √    | 1        | 4.19           | 2.102         | 9.61          |
| w/o adv      | √   | √   |      | 1        | 2.75           | 3.733         | 10.35         |
| w/o adv (C27)| √   | √   |      | 1        | 2.84           | 2.148         | 8.91          |
| w/o adv ($L = 0$) | √ |      |      | 0        | 7.95           | 1.370         | 4.00          |
| w/o nop      |      |     | √    |          | 8.29           | 1.370         | 5.21          |
| w/o MGDA     | √   | √   |      | 1        | 5.48           | 2.105         | 8.11          |
| w/o $L$      | √   | √   |      |          | 8.30           | 1.370         | 4.39          |

Figure 4. Architecture evaluation of E2RNAS on the CIFAR-10 dataset using different minimum constraint $L$ in Eq. (6).

5. Conclusions

In this paper, we propose the E2RNAS method that optimizes multiple objectives simultaneously to search an effective, efficient and robust architecture. The proposed objective function is formulated as a bi-level multi-objective problem and we design an algorithm to integrate the MGDA with the bi-level optimization. Experiments demonstrate that E2RNAS can find adversarial robust architecture with optimized model size and comparable classification accuracy on various datasets. In our future study, we are interested in extending the proposed E2RNAS method to search for multiple Pareto-optimal architectures at one time.

better robustness but lower accuracy than “E2RNAS w/o adv (C27)”.

Therefore, using adversarial training can help E2RNAS to make a trade-off among multiple objectives and search a robust architecture with a lower model size.

Effectiveness of MGDA MGDA is used to solve the upper-level problem of problem (7). We quantitatively compare the performance of E2RNAS with and without MGDA (i.e. “E2RNAS” vs. “E2RNAS w/o MGDA” in Table 3) and find that solving with MGDA achieves much better results on the test accuracy, parameter size, and PGD accuracy. So instead of using equal weights, using MGDA can find a good solution of weights and make a trade-off among multiple objectives.

Necessity of $L$ We find that training E2RNAS without the minimum constraint $L$ (i.e. “E2RNAS w/o $L$” in Table 3) searches an architecture with many parameter-free operations (i.e. its parameter size is only 1.370MB). There are three reasons for this phenomenon. Firstly, the instability of DARTS sometimes makes it converge to extreme architectures (e.g. full of skip-connects) \cite{35, 3}. Secondly, as discussed above, using the adversarial training in the lower-level problem tends to reduce the number of parameters. Finally, only optimizing the number of parameters in the upper-level problem (i.e. “E2RNAS w/o adv ($L = 0$)” in Table 3) also results in searched architectures with many parameter-free operations. Therefore, it is necessary to constrain the number of parameters with a lower bound $L$ to prevent E2RNAS to search over-simplified architectures.

Figure 4 shows the architecture evaluation results of E2RNAS on the CIFAR-10 dataset using different $L$ in Eq. (6). Hence, we set this hyperparameter $L$ to 1 in our work because E2RNAS achieves the best performance (i.e. lowest test error rate in Figure 4(a), acceptable model size in Figure 4(b), highest PGD accuracy in Figure 4(c)) when $L = 1$. 
References

[1] Kaifeng Bi, Lingxi Xie, Xin Chen, Longhui Wei, and Qi Tian. Gold-nas: Gradual, one-level, differentiable. arXiv preprint arXiv:2007.03331, 2020. 3

[2] Herminia I Calvete and Carmen Galé. Linear bilevel programs with multiple objectives at the upper level. Journal of Computational and Applied Mathematics, 234(4):950–959, 2010. 4

[3] Xin Chen, Lingxi Xie, Jun Wu, and Qi Tian. Progressive differentiable architecture search: Bridging the depth gap between search and evaluation. In Proceedings of the IEEE International Conference on Computer Vision, pages 1294–1303, 2019. 1, 6, 7, 8

[4] Zewei Chen, Fengwei Zhou, George Trimponias, and Zhenguo Li. Multi-objective neural architecture search via non-stationary policy gradient. CoRR, abs/2001.08437, 2020. 2, 3

[5] Benoît Colson, Patrice Marcotte, and Gilles Savard. An overview of bilevel optimization. Annals of operations research, 153(1):235–256, 2007. 2

[6] Kalyanmoy Deb and Ankur Sinha. Solving bilevel multi-objective optimization problems using evolutionary algorithms. In International conference on evolutionary multi-criterion optimization, pages 110–124. Springer, 2009. 4

[7] Jean-Antoine Désidéri. Multiple-gradient descent algorithm (MGDA) for multiobjective optimization. Comptes Rendus Mathematique, 350(5):313–318, 2012. 2

[8] Minjing Dong, Yanxi Li, Yunhe Wang, and Chang Xu. Adversarially robust neural architectures. CoRR, abs/2009.00902, 2020. 2

[9] Thomas Elsken, Jan Hendrik Metzen, and Frank Hutter. Efficient multi-objective neural architecture search via lamarckian evolution. In Proceedings of the 7th International Conference on Learning Representations, 2019. 1, 2, 3

[10] Jörg Fliege and Benar Fux Svaiter. Steepest descent methods for multicriteria optimization. Mathematical Methods of Operations Research, 51(3):479–494, 2000. 2

[11] Ian J. Goodfellow, Jonathon Shlens, and Christian Szegedy. Explaining and harnessing adversarial examples. In Yoshua Bengio and Yann LeCun, editors, 3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings, 2015. 2, 4

[12] Minghao Guo, Yuzhe Yang, Rui Xu, Ziwei Liu, and Dahua Lin. When NAS meets robustness: In search of robust architectures against adversarial attacks. In 2020 IEEE/CVF Conference on Computer Vision and Pattern Recognition, CVPR 2020, Seattle, WA, USA, June 13-19, 2020, pages 628–637. IEEE, 2020. 2

[13] Gao Huang, Zhuang Liu, Laurens Van Der Maaten, and Kilian Q Weinberger. Densely connected convolutional networks. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 4700–4708, 2017. 6

[14] Xiaojie Jin, Jiang Wang, Joshua Slocum, Ming-Hsuan Yang, Shengyang Dai, Shuicheng Yan, and Jiashi Feng. Re-darts: Resource constrained differentiable architecture search. arXiv preprint arXiv:1912.12814, 2019. 2, 3, 4

[15] Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images. 2009. 5

[16] Harold W Kuhn and Albert W Tucker. Nonlinear programming. In Traces and emergence of nonlinear programming, pages 247–258. Springer, 2014. 2

[17] Alexey Kurakin, Ian J. Goodfellow, and Samy Bengio. Adversarial examples in the physical world. In 5th International Conference on Learning Representations, ICLR 2017, Toulon, France, April 24-26, 2017, Workshop Track Proceedings. OpenReview.net, 2017. 2, 4, 5

[18] Chenxi Liu, Barret Zoph, Maxim Neumann, Jonathon Shlens, Wei Hua, Li-Jia Li, Li Fei-Fei, Alan Yuille, Jonathan Huang, and Kevin Murphy. Progressive neural architecture search. In Proceedings of the European Conference on Computer Vision (ECCV), pages 19–34, 2018. 6

[19] Hanxiao Liu, Karen Simonyan, Oriol Vinyals, Chrisantha Fernando, and Koray Kavukcuoglu. Hierarchical representations for efficient architecture search. arXiv preprint arXiv:1711.00436, 2017. 1, 6

[20] Hanxiao Liu, Karen Simonyan, and Yiming Yang. DARTS: differentiable architecture search. In Proceedings of the 7th International Conference on Learning Representations, 2019. 1, 3, 4, 5, 6, 7

[21] Aleksander Madry, Aleksandar Makelov, Ludwig Schmidt, Dimitris Tsipras, and Adrian Vladu. Towards deep learning models resistant to adversarial attacks. In 6th International Conference on Learning Representations, ICLR 2018, Vancouver, BC, Canada, April 30 - May 3, 2018, Conference Track Proceedings. OpenReview.net, 2018. 2, 5

[22] Yuval Netzer, Tao Wang, Adam Coates, Alessandro Bissacco, Bo Wu, and Andrew Y Ng. Reading digits in natural images with unsupervised feature learning. 2011. 5
[23] Hieu Pham, Melody Y Guan, Barret Zoph, Quoc V Le, and Jeff Dean. Efficient neural architecture search via parameter sharing. *arXiv preprint arXiv:1802.03268*, 2018. 1, 6

[24] Aditi Raghunathan, Sang Michael Xie, Fanny Yang, John C Duchi, and Percy Liang. Adversarial training can hurt generalization. *arXiv preprint arXiv:1906.06032*, 2019. 6

[25] Esteban Real, Alok Aggarwal, Yanping Huang, and Quoc V Le. Regularized evolution for image classifier architecture search. In *Proceedings of the aaai conference on artificial intelligence*, volume 33, pages 4780–4789, 2019. 1, 6

[26] Sauli Ruuska and Kaisa Miettinen. Constructing evolutionary algorithms for bilevel multiobjective optimization. In *2012 IEEE Congress on Evolutionary Computation*, pages 1–7. IEEE, 2012. 4

[27] Stefan Schäffler, Reinhart Schultz, and Klaus Weinzierl. Stochastic method for the solution of unconstrained vector optimization problems. *Journal of Optimization Theory and Applications*, 114(1):209–222, 2002. 2

[28] Christian Szegedy, Wojciech Zaremba, Ilya Sutskever, Joan Bruna, Dumitru Erhan, Ian J. Goodfellow, and Rob Fergus. Intriguing properties of neural networks. In Yoshua Bengio and Yann LeCun, editors, *2nd International Conference on Learning Representations, ICLR 2014, Banff, AB, Canada, April 14-16, 2014, Conference Track Proceedings*, 2014. 2

[29] Mingxing Tan, Bo Chen, Ruoming Pang, Vijay Vasudevan, Mark Sandler, Andrew Howard, and Quoc V. Le. MnasNet: Platform-aware neural architecture search for mobile. In *Proceedings of IEEE Conference on Computer Vision and Pattern Recognition*, pages 2820–2828, 2019. 1, 2, 3

[30] Eric Wong, Leslie Rice, and J. Zico Kolter. Fast is better than free: Revisiting adversarial training. In *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net, 2020. 2, 3, 4, 5

[31] Bichen Wu, Xiaoliang Dai, Peizhao Zhang, Yanghan Wang, Fei Sun, Yiming Wu, Yuandong Tian, Peter Vajda, Yangqing Jia, and Kurt Keutzer. Fbnet: Hardware-aware efficient convnet design via differentiable neural architecture search. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 10734–10742, 2019. 3

[32] Sirui Xie, Hehui Zheng, Chuxiaoch Liu, and Liang Lin. SNAS: stochastic neural architecture search. In *Proceedings of the 7th International Conference on Learning Representations*, 2019. 1

[33] Yuhui Xu, Lingxi Xie, Xiaopeng Zhang, Xin Chen, Guo-Jun Qi, Qi Tian, and Hongkai Xiong. Pc-darts: Partial channel connections for memory-efficient architecture search. In *International Conference on Learning Representations*, 2019. 1, 6, 7

[34] Yao-Yuan Yang, Cyrus Rashtchian, Hongyang Zhang, Ruslan Salakhutdinov, and Kamalika Chaudhuri. A closer look at accuracy vs. robustness. *Advances in Neural Information Processing Systems*, 33, 2020. 6

[35] Arber Zela, Thomas Elsken, Tonmoy Saikia, Yassine Marrakchi, Thomas Brox, and Frank Hutter. Understanding and robustifying differentiable architecture search. *arXiv preprint arXiv:1909.09656*, 2019. 8

[36] Tao Zhang, Tiesong Hu, Yue Zheng, and Xuning Guo. An improved particle swarm optimization for solving bilevel multiobjective programming problem. *Journal of Applied Mathematics*, 2012, 2012. 4

[37] Barret Zoph and Quoc V Le. Neural architecture search with reinforcement learning. *arXiv preprint arXiv:1611.01578*, 2016. 1, 6