Domain walls, $Z(N)$ charge and $A_0$ condensate: 
a canonical ensemble study

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Abstract

The deconfinement phase transition is studied in the ensemble canonical with respect to triality. Since this ensemble implies a projection to the zero triality sector of the theory we introduce a quantity which is insensitive to $Z(N_c)$ symmetry but can reveal a critical behaviour in the theory with dynamical quarks. Further, we argue that in the canonical ensemble description of full QCD there exist domains of different $Z(N_c)$ phases which are degenerate and possess normal physical properties. This contradicts the predictions of the grand canonical ensemble. We propose a new order parameter to test the realization of the discrete $Z(N_c)$ symmetry at finite temperature and calculate it for the case of $Z(2)$ gauge fields coupled to fundamental fermions.

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1 Introduction

Ever since the end of the seventies there has been increasing interest towards discrete global and local gauge symmetries. In 1978 G. ’t Hooft [1] and G. Mack [2] conjectured that the discrete center $Z(N_c)$ of an underlying gauge group $G$ can be of crucial importance for quark confinement. Two years later the $Z(N_c)$ mechanism of confinement by means of vortex condensation was presented [3]. The next important step in understanding the role of discrete symmetries was the awareness of the deep connection between the spontaneous breaking of the global $Z(N_c)$ symmetry and the deconfinement phase transition in pure gauge models [4]. Recently, different aspects of global and local discrete symmetries have been an object of numerous investigations both on the lattice and in continuum. This article contributes to previous studies from the point of view of the canonical ensemble (CE) description of hot gauge theories with fundamental quarks.

We propose a new order parameter to test a realization of the discrete $Z(N_c)$ symmetry at finite temperature and calculate it for the case of $Z(2)$ gauge fields coupled to fundamental fermions. It is argued that domains of different N-ality in full QCD may exist, are degenerate and possess normal physical properties. What we would like to stress here is that in some important aspects the two possible descriptions produce different results and, consequently it would be safer to use the CE with respect to triality.

We begin over viewing the standard $Z(N_c)$ picture in QCD at finite temperature [4]. Gluon fields are strictly periodic in time, while quarks are anti periodic with a period being the inverse temperature. Pure gauge theory has an exact $Z(N_c)$ global symmetry. The gauge invariant operator, the Polyakov loop (PL), $L_{\vec{x}} = \frac{1}{N_c} Tr \prod_{t=1}^{N_t} U_0(\vec{x}, t)$ transforms under $Z(N_c)$ global transformations as

$$L_{\vec{x}} \rightarrow Z L_{\vec{x}}, \quad Z = \exp\left[\frac{2\pi i}{N_c} n\right], \quad n = 0, \ldots, N_c - 1.$$  \hspace{1cm} (1)

The PL can be used as an order parameter to test $Z(N_c)$ symmetry in pure gauge theory. The expectation value of the PL is interpreted as the free energy of a probe quark $F_q$ immersed in a pure gluonic bath

$$< L_{\vec{x}} > = \exp\left(-\frac{1}{T} F_q\right).$$ \hspace{1cm} (2)

Unbroken $Z(N_c)$ symmetry implies $< L > = 0$ and $F_q = \infty$. When the global $Z(N_c)$ symmetry is spontaneously broken, $< L >$ differs from zero and $F_q$ has a finite value, i.e. it costs only finite energy to create a single quark in the gluonic bath. To our knowledge, the first objection against this lore was discussed in [3]
where it was pointed out that in a system with a finite UV cutoff the free energy
should not diverge. Usually, MC and analytical calculations are performed also with
periodic boundary conditions in space directions. It has been shown, however, that
Gauss’ law and this p.b.c. in space are inconsistent unless the sum of quark and
gluon colour charges vanishes, which is impossible since they have different triality.
It was concluded that for space p.b.c. the expectation value of the PL is not the
free energy of a heavy quark. The second trouble appears if we realize that in the
spontaneously broken phase \(< L >\) may pick up \(N_c\) different values corresponding to
\(N_c\) equivalent minima of the free energy. Thus, \(L\) can be negative or even complex.
Eq.(2) tells us that the free energy could be a complex number. This gives rise to
doubts that Eq.(2) has the proper physical interpretation. When dynamical quarks
are included the picture becomes more complicated and new troubles appear. The
fermion determinant generates loops going around the lattice a number of times
which is not a multiple of \(N_c\). Such loops present a propagation of single quarks
and transform non trivially under \(Z(N_c)\). This means that dynamical quarks break
\(Z(N_c)\) symmetry explicitly and screen sources of heavy quarks at any temperature.
The expectation value of the PL prefers the phase with \(\text{arg} L = 0\) which provides the
minimum of the free energy. Other \(Z(N_c)\) phases with \(\text{arg} L = \frac{2\pi k}{N_c}, k = 1, \ldots, N_c - 1\)
become metastable. They possess, however, such unphysical properties as a complex
free energy or entropy \([6]\). Recently, it has been discovered that chiral symmetry
is not restored in \(Z(N_c)\) phases \([7]\). This led to a reexamination of degenerate
\(Z(N_c)\) phases and interfaces between them in pure gauge theory \([8, 9]\). Although
not common but the most popular opinion is that all \(Z(N_c)\) phases correspond to
the same physical state and the interface is unphysical.

These properties reflect the GCE description of QCD with respect to triality. In
our previous papers we introduced an ensemble canonical with respect to triality \([10]\)
(see also \([11]\)) and studied the QCD phase structure within this ensemble \([12, 13]\).
This ensemble reveals the following important properties of QCD with dynamical
quarks:

1) In the low temperature phase every state has triality zero. In the deconfinement
phase the system as a whole has zero triality. Since all \(Z(N_c)\) noninvariant
variables are projected out of the theory, the PL itself has only little meaning in
the CE. A single quark does not appear in the spectrum. Since the Debye mass is
nonzero, chromoelectric fields of quarks are screened. To distinguish the low temper-
ature phase from a non-confining phase one can use the phase of the \(SU(N_c)/Z(N_c)\)
PL. Since this phase is insensitive to \(Z(N_c)\) transformations it can be used also in
the theory with dynamical quarks \([13]\). The deconfinement phase transition can
be seen in the CE similarly to pure gauge theory \([13]\).

2) Metastable minima with unphysical properties are absent, all \(Z(N_c)\) phases are
degenerate just as in a pure gauge model. It follows, $Z(N_c)$ phases and corresponding domain walls may exist in full QCD [10, 12, 13].

3) What is not so obvious is the behaviour of the quark condensate in different $Z(N_c)$ phases. It was shown, for example, that chiral symmetry is always broken in the metastable minimum of $SU(2)$ theory and the quark condensate differs from zero in this minimum at arbitrary high temperature [7]. The canonical partition function demonstrates opposite behaviour: chiral symmetry is restored in all $Z(N_c)$ phases and at the same temperature. We have shown this in our recent paper [13] by calculating the quark condensate in the CE in strong coupling $SU(2)$ lattice gauge theory.

It should be stressed that the canonical ensemble with respect to the total fermion number [14] also respects these features since it does not violate explicitly the $Z(N_c)$ global symmetry.

One may ask at this stage how to treat pure gauge theory where dynamical quarks are absent. The resolution is trivial if we remember that pure gauge theory is a limit of the full theory when the quark mass goes to infinity. This limit should be taken after the thermodynamical limit $N \to \infty$, i.e. the pure gauge partition function is defined as follows

$$Z_{PG} = \lim_{m_q \to \infty} \lim_{N_c \to \infty} \frac{1}{N_c^2} \sum_{k=1}^{N_c} \int \prod_{l} dU_l \prod_{x,i} d\bar{\Psi}_i^x d\Psi_i^x e^{-S_W(Z_k U_0) - S_F}. \quad (3)$$

Thus, by definition a single quark does not appear in the spectrum and nonzero triality contributions are excluded by construction. Hence, the problem is only a careful mathematical treatment of the last equation. The puzzle mentioned in [3] and discussed in [4] does not appear. All the information can be extracted from correlators of PLs. In this respect the observation that the CE destroys the clusterization property has no sense as long as the PL itself has no meaning in the CE. If one wants to have this property and relation between the correlator in the limit $R \to \infty$ and the expectation value of the PL, the projection to another triality sector has to be performed. This later sector, however, does not present the ground state of QCD. $< L >$ is a thermodynamical mixture of a heavy-light meson, a heavy-light baryon and so on (see for details, [12]). The correlator therefore gives the free energy of these states placed at distance $R$.

Two questions are proper here. The first one concerns a suitable quantity which could characterize the critical behaviour of the system if one wants to use the CE. The second one is whether there is no contradiction between the screening of quarks and zero triality of the whole system. Indeed, it would naively seem that in the Debye screening phase there are no long-range forces. If we have a single quark at position $\vec{r}$ how can this quark feel what is going on in any other point far away
from it and, thus, how may the system have zero triality? It turns out that this later question is closely connected to the problem of detecting $Z(N_c)$ charges at long distances. All these problems are the object of the next two sections.

2 Order parameter for $Z(N_c)$ charge and domain walls

As an ensemble canonical with respect to $N$-ality does not break $Z(N_c)$-symmetry explicitly there appears the interesting question if $Z(N_c)$-symmetry can be broken spontaneously in this ensemble and which quantity could be used to detect nontrivial $Z(N_c)$ charges at long distances. An operator which detects a nontrivial phase of some state in a region enclosed by a surface $\Sigma$ is

$$A(\Sigma, C) = \frac{< F(\Sigma) W(C) >}{< F(\Sigma) > < W(C) >},$$

(4)

and the limits $\Sigma, C \to \infty$ have to be taken. $W(C)$ is a space-time Wilson loop nontrivial on the $Z(N_c)$ subgroup. $\Sigma$ may be thought as a set of plaquettes from one time-slice frustrated by $Z(N_c)$ singular transformations in such a way that the dual links form a closed two-dimensional surface for this time slice. $\Sigma$ has to be chosen to enclose a line of the Wilson loop $C$ going in time direction. The action of this order parameter is based on the Aharonov-Bohm effect: despite the absence of chromoelectric fields a singular potential influences particles at arbitrary long distances giving them a nontrivial phase during winding around the solenoid. In our case a $Z(N_c)$ charge may be viewed as a kind of such singular solenoid placed at some space position. To get this configuration we should take the Wilson loop $C$ and consider the limit $C \to \infty$. Since the Wilson loop is a source of $Z(N_c)$ charge we have the nontrivial charge and probe it via the “device” $F(\Sigma)$. A nontrivial $Z(N_c)$ charges may be detected only in the case when the fermionic screening is suppressed relatively to the Debye screening

$$\lim_{\Sigma, C \to \infty} A(\Sigma, C) = \exp \left( \frac{2\pi i K(\Sigma, C)}{N_c} \right),$$

(5)

where $K(\Sigma, C)$ is the linking number of the surface $\Sigma$ and loop $C$. If the triality charge is totally screened one gets $\lim_{\Sigma, C \to \infty} A(\Sigma, C) = 1$.

Since the fundamental PL is a source of $Z(N)$ charge at finite temperature we can insert the correlation function of PLs into (4) instead of the Wilson loop

$$A(\Sigma, R) = \frac{< F(\Sigma) L_0 L_R >}{< F(\Sigma) > < L_0 L_R >},$$

(6)
Another interpretation for the influence of $F(\Sigma)$ is the following. $F(\Sigma)$ tries to implement a phase change at the spatial surface $\Sigma$. If $Z(N_c)$ symmetry is spontaneously broken $F(\Sigma)$ produces a stable interface between the volume enclosed by $\Sigma$ and the surrounding vacuum. The different phases can be detected by a Wilson loop and the order parameter $A(\Sigma, C)$. If the symmetry is unbroken $F(\Sigma)$ cannot produce a stable interface and the value of the Wilson loop is not influenced by $F(\Sigma)$ which is far apart. In the case of the grand canonical ensemble $Z(N_c)$ symmetry is explicitly broken, there exists a preferred phase (zero) and therefore a phase change is annihilated after a short distance from $\Sigma$. At high and low temperatures $\lim_{\Sigma, C \to \infty} A(\Sigma, C) = 1$ and cannot be used as an order parameter. $Z(N_c)$ charges are completely screened by the nonzero $N_c$-ality contributions to the partition function which are present in the grand canonical ensemble even at high temperature and in thermodynamical limit. As described above we expect however a nontrivial behaviour of this order parameter in the ensemble canonical with respect to $N_c$-ality.

To illustrate these ideas we studied a simple theory of $Z(2)$ gauge spins coupled to massless naive fermions described in the CE by the partition function

$$Z = \frac{1}{2} \sum_{k=\pm} \sum_{s_l=\pm} \int d\bar{\Psi} x d\Psi x e^{S_W + S_F}. \quad (7)$$

Sum over $k$ is the sum over two $N_c$-ality sectors. We have denoted

$$S_W = \lambda \sum_p S_p, \quad (8)$$

where $S_p$ is a product of $Z(2)$ gauge link variables $s_l$ around plaquette and

$$S_F = \alpha \frac{1}{2} \sum_{x,n} s_n(x)[\bar{\Psi}_x \Psi_{x+n} - \bar{\Psi}_{x+n} \Psi_x] + \alpha \frac{1}{2} \sum_x s_0(x)k[\bar{\Psi}_x \Psi_{x+0} - \bar{\Psi}_{x+0} \Psi_x].$$

We fix a static gauge where all gauge spins in time direction $s_0(x)$ are set to 1 except for one time-slice which includes the frustrated plaquettes $\Sigma$. We consider the region $\lambda \gg 1$ and $\alpha \ll 1$ which corresponds to the weak coupling high temperature phase. The fermionic screening is suppressed as $\alpha^{2N_f}$. The Debye screening comes from the pure gluonic action and produces the formula for the correlation of PLs

$$\ln < L_0 L_R > \propto \exp[-M_D R], \quad (9)$$

where $M_D$ is the Debye mass. It is straightforward to calculate $< F(\Sigma) >$ in leading order in $\alpha$

$$< F(\Sigma) >= \exp[-\delta S_\Sigma + O(\alpha^8)], \quad (10)$$
where \( \delta \approx \frac{2n^4 \tanh \lambda}{1 + \alpha^2} \). The main contribution comes from configurations of the gauge fields \( s_0(x) \) flipped in a volume enclosed by \( \Sigma \) relatively to \( s_0(x) \) outside of this volume. Since the PL \( L_0 \) in the origin penetrates this volume one time \( L_0 \) changes its sign. The corresponding expansions in large \( \lambda \) and in small \( \alpha \) are converging. Therefore, in the limit of infinite \( \Sigma \) and \( R \) all the corrections go to zero and we find

\[
A(\Sigma, R) = -I. \quad (11)
\]

In the region \( \alpha \gg 1 \) the fermionic screening can dominate \( <L_0L_R> \) and we find \( A(\Sigma, R) = I \). It follows that \( Z(2) \) may be broken in this region. We have thus shown that \( A(\Sigma, R) \) can be used in the CE as an order parameter to distinguish the phase of broken \( Z(N_c) \) symmetry from the phase where the triality charge can be detected at long range. The operator \( <F(\Sigma)> \) introduces a \( Z(N_c) \) string which becomes the boundary of a domain wall when the \( Z(N_c) \) symmetry is spontaneously broken. This domain wall is stable in pure gauge theory but appears to be unstable in full QCD if we treat it in the GCE. In the CE the domain wall is stable just as in pure gauge theory. Thus, the question of the realization of the domain walls in QCD is only the question whether the local \( Z(N_c) \) symmetry can be spontaneously broken in the CE in the sense described here. Application of these results to \( SU(N_c) \) theory is conceptually straightforward. Calculations however are much more complicated and could be advanced using Monte-Carlo simulations.

### 3 \( A_0 \) condensate in the CE

We consider in this section another quantity which is a good candidate for characterizing the deconfinement phase, the so-called \( A_0 \) condensate. It carries other useful features than the order parameter proposed above. Let us perform the following decomposition of the \( SU(N_c) \) PL \( V_x \) before tracing \( (L_x = TrV_x) \)

\[
V_x = Z_x \bar{V}_x, \quad (12)
\]

where \( \bar{V}_x \in SU(N_c)/\mathbb{Z}(N_c) \) and \( Z_x \in \mathbb{Z}(N_c) \). For the invariant measure we have in this case

\[
\int D\mu(V) = \frac{1}{N_c} \sum_Z \int D\mu(\bar{V}), \quad (13)
\]

where \( D\mu(\bar{V}) \) is the invariant measure on \( SU(N_c)/\mathbb{Z}(N_c) \) group. Let us recall, that the invariant measure on the \( SU(N_c)/\mathbb{Z}(N_c) \) group coincides with the \( SU(N_c) \) measure up to the restriction

\[
-\frac{2\pi}{N} \leq arg[TrV] \leq \frac{2\pi}{N}. \quad (14)
\]
We chose the static diagonal gauge for $V_x$ gauge field matrices. Then, $\bar{V}_x$ has the diagonal representation with angles $\phi_k$ obeying Eq. (14). The corresponding continuum potential can be written in the form $\beta g(A_0 + a_0(x))$ with a constant part $A_0$ taking values as in (14) and a quantum fluctuating part $a_0(x)$. This constant value may be interpreted as the $A_0$ condensate, see [17] for a review on this question. Our definition is, however, slightly different just because of the restriction (14). We have found that this definition is more proper both for the continuum theory and for the lattice theory. $Z$ configurations are responsible for the disorder in the confinement phase and $A_0$ takes values on the edge of the integration region (e.g., $\beta g A_0 = \pm \pi$ for $SU(2)$). In the high temperature phase, it is possible to show that an actual value of $A_0$ is shifted from the edge of the integration region forming a non-trivial saddle point in the $SU(N_c)/Z(N_c)$ subgroup [16] (truly, we did not prove that this saddle point survives a transition to the continuum)\(^2\).

We, thus, adjust the following definitions for the $A_0$ condensate:

1. On the lattice we define it as a saddle point configuration for the invariant integrals over the zero component gauge field matrix. Any such saddle point, if it exists, is invariant by the lattice construction. A physical meaning of this definition follows from the Hamiltonian formulation of lattice QCD: a constant saddle point is an (imaginary) chemical potential for the global colour charge [18]. The existence of this constant saddle point configuration was demonstrated in [16] for the Lagrangian formulation and in [19] for the Hamiltonian formulation.

2. In the continuum the $A_0$ condensate is defined as the position of the minimum of the effective potential obtained by perturbative integration over gauge potentials over the vacuum $A_i = 0$, $A_0 =$constant with fixed gauge. The proof of an independence of the condensation phenomenon of the gauge fixing as well as the independence of the physical quantities of the gauge in the theory with nonzero condensate was done in Refs. [20]. Besides the same physical interpretation as before, the condensate plays the role of an infrared regulator of the theory. It was shown, for example, that there are no infrared divergences in the two-loop polarization tensor in a theory with such a condensate [21].

One important advantage of our definition of $A_0$ is that it is insensitive to the $Z(N_c)$ symmetry because $\bar{V}_x$ is invariant under $Z(N_c)$ transformation. Hence, it may be used both in a pure gauge theory and in a theory with dynamical fermions in both

\(^2\)It should be mentioned that there is no a common opinion in the literature concerning the $A_0$ condensate. Very different points of view have been stated and advocated during last five years, see [17] [16] and references therein. We are not going to discuss here all the issues on this subject conjecturing that a generation of $A_0$ is possible both on the lattice and in the continuum spacetime. We claim, however, that the value of $A_0$ should be calculated after a summation over $Z(N)$ configurations contained into the PL [14].
ensembles to reveal some features of the high-temperature phase. We think that $A_0$ could be used as a sort of “order parameter” to distinguish the confining from the non-confining phase in the CE. Presumably, it is not an order parameter in the exact meaning, rather it shows preferred values of the $SU(N_c)/Z(N_c)$ configurations of the PL in different phases. In fact, this is very close to what the adjoint PL demonstrates in the $SO(3)$ LGT at finite temperature [22]. Even though there is no global $Z(2)$ symmetry in the $SO(3)$ model, the phases of the adjoint PL behave differently in the confinement and in the deconfinement phases as described above. The $SO(3)$ model was considered in this respect also in [17]. We have proved for the simple example of a pure $SO(3)$ model in the strong coupling region that the phases of the adjoint PL can indeed be used to test the deconfinement phase. In the continuum theory in the CE the $A_0$ condensate was calculated in [23] up to second loop order. It shows the expected behaviour at high temperatures. The loop expansion, however does not allow to observe a critical behaviour of the system. This is a question we address here. We are able now to demonstrate this expected behaviour in the strong coupling $SU(2)$ LGT with Kogut-Susskind fermions. We work in the Hamiltonian formulation. The Hamiltonian of the lattice gluodynamics in the strong coupling approximation includes only the chromoelectric part

$$H = \frac{g^2}{2a} \sum_{\text{links}} E^2(l),$$

where $E(l) = i\partial/\partial(A_l)$ are the chromoelectric field operators. In this approach the chromomagnetic term can be treated perturbatively at $g^2 \to \infty$. The determination of the partition function

$$Z = \tilde{Sp} \exp(-\beta H)$$

is connected with the summation over local gauge-invariant states. This is reflected by the symbol $\tilde{Sp}$ in [16], $\beta$ is the inverse temperature. The corresponding physical Hilbert space is determined by Gauss’ law. After the conventional procedure one gets the partition function of the form (see, for instance, [17])

$$Z = \int \prod_x d\mu(\phi_x) \prod_{x,n} \left[ \sum_{l=0,\frac{1}{2},...} e^{-\gamma C_l} \Omega_l(\phi_x) \Omega_l(\phi_{x+n}) \right].$$

(17)

$C_l$ is here the quadratic Casimir operator, $\gamma = \frac{g^2}{2a}$. $\Omega_l$ is the character of $l$-th irreducible representation of the $SU(2)$ group

$$\Omega_l(\phi) = \frac{\sin(2l+1)\phi}{\sin \phi}.$$

(18)
Notice, that the invariant measure $d\mu(\phi_x)$ appeared after the representation of the Gauss’ law delta-function on the $SU(2)$ group. Following formulae (12)-(14) we represent $SU(2)$ characters as

$$\Omega_l(\phi) = s^{2l} \bar{\Omega}_l(\phi).$$

Here, we may choose $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$ which follows from (14). We would like to calculate an effective potential for the phase $\phi$ which we set constant further. In this way, using a minimization procedure, we find a saddle point for integrals over the $SU(2)/Z(2)$ subgroup. Substituting (18) and (19) into (17) and summing up over all representations of the $SU(2)$ group, one arrives to the effective potential of the form

$$V_{eff}(\phi) = 2d \ln F_0(\phi) - (2d - 1) \ln \sin^2 \phi + \frac{1}{N_s} \ln Z_{IM}(\phi),$$

where $N_s$ is a number of lattice sites and we have taken into account the measure contribution $\ln \sin^2 \phi$ which is invariant under $Z(2)$. $Z_{IM}$ is the partition function of the Ising model

$$Z_{IM} = \sum_{s_x=\pm 1} \prod_l [1 + \tanh Y s_x s_{x+n}] \prod_x [1 + M \cos \phi s_x],$$

where

$$\tanh Y = \frac{F_1(\phi)}{F_0(\phi)}$$

and

$$F_0(\phi) = \Theta_2(e^{-\gamma}, 0) - \Theta_2(e^{-\gamma}, 2\phi),$$

$$F_1(\phi) = \Theta_3(e^{-\gamma}, 0) - \Theta_3(e^{-\gamma}, 2\phi).$$

$\Theta_i$ is the Jacobi theta-function. The quark contribution

$$Z_q = \prod_x [1 + M \cos \phi s_x]$$

was taken as in [24] with $M = \exp(-\beta m)$. The first two terms from (20) coincide exactly with the effective potential for pure the $SO(3)$ model in the same approximation [17]. The existence of a nontrivial solution for $\phi$ can easily be established in this case for $\gamma > \gamma_c$. For $\gamma < \gamma_c$ one finds $\phi = \pm \frac{\pi}{2}$. Thus, for this case our conclusion coincides with the one of Ref.[22]. The third term in (21) establishes a difference from the $SO(3)$ theory. It is clear, however, that this term cannot influence drastically the qualitative picture in the deconfinement phase. We performed
numerical calculations to find maxima of the potential for the CE, where for the quark contribution we should take

$$Z_{q}^{CE} = \sum_{k=0}^{1} \prod_{x} \left[ 1 + M \cos(\phi + k\pi) s_{x} \right],$$  \hspace{1cm} (26)

whereas for the $SO(3)$ model this contribution is

$$Z_{q}^{SO(3)} = \prod_{x} \left[ 1 + M \cos^{2} \phi \right].$$  \hspace{1cm} (27)

Detailed calculations with a chromomagnetic part of the Hamiltonian are at the moment in progress and will be presented elsewhere. The result for the effective potential (20) including the quark contribution is, however, qualitatively the same both for the $SO(3)$ model and for the $SU(2)$ model in the CE, namely $A_{0}$ forms a nontrivial saddle point above some critical value. Below this value $\phi$ is $\pm \frac{\pi}{2}$. Therefore, we established that $A_{0}$ defined in such a manner, indeed can serve as an order parameter to probe phase transitions in a gauge theory with fundamental matter fields.

As a last point we would like to mention an application of our result. We have already stressed that the $A_{0}$ condensate has the meaning of an imaginary chemical potential for the global colour charge. Thus, if the condensate appears above some critical point, a spontaneous generation of nonzero colour charge might happen in the quark sector of the theory. This charge should have the same absolute value in all $Z(N_{c})$ phases. Since it must be zero in the confinement region this charge could be an exact order parameter for full QCD. If this phenomenon really happens it could lead to significant improvement of our knowledge of the quark-gluon plasma phase.

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