Residual-Based Adaptive Coefficient and Noise-Immunity ZNN for Perturbed Time-Dependent Quadratic Minimization

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Abstract—The time-dependent quadratic minimization (TDQM) problem appears in many applications and research projects. It has been reported that the zeroing neural network (ZNN) models can effectively solve the TDQM problem. However, the convergent and robust performance of the existing ZNN models are restricted for lack of a joint-action mechanism of adaptive coefficient and integration enhanced term. Consequently, the residual-based adaption coefficient zeroing neural network (RACZNN) model with integration term is proposed in this paper for solving the TDQM problem. The adaptive coefficient is proposed to improve the performance of convergence and the integration term is embedded to ensure the RACZNN model can maintain reliable robustness while perturbed by variant measurement noises. Compared with the state-of-the-art models, the proposed RACZNN model owns faster convergence and more reliable robustness. Then, theorems are provided to prove the convergence of the RACZNN model. Finally, corresponding quantitative numerical experiments are designed and performed in this paper to verify the performance of the proposed RACZNN model.

Index Terms—Quadratic minimization, time-dependent, zeroing neural network (ZNN), adaptive coefficient.

I. INTRODUCTION

The quadratic minimization (QM) as a branch of optimization theory has been extensively studied in controller design [1], communication engineering [2], energy system design [3], image processing [4], robot kinematics [5], and is still a very active research area [6], [7]. In summary, when facing general QM problems, the conventional scheme is to solve them with the help of some numerical or iterative algorithms [8]. Nikolova and Chan present an image restoration algorithm with an implicit QM problem via the gradient linearization iteration method [4]. A message-passing scheme for solving QM problems is presented in [9] by Ruozzi and Tatikonda. In addition, Zhang et al. present a QM-based dual-arm CMG manipulator control scheme and analyze its properties from the perspective of cybernetics [5]. It is worth noting that although considerable research has been devoted to solving conventional QM problems, studies aimed explicitly at the time-dependent quadratic minimization (TDQM) problem are insufficient. Traditional solutions have serious lag errors when facing large-scale time-dependent issues, resulting in inadequate solution accuracy and even the collapse of the solution system [10], [11].

To break through the dilemma that traditional algorithms cannot effectively deal with time-dependent problems, Zhang et al. design the original zeroing neural network (OZNN) model [12]. The OZNN model can make full use of the derivative information of the time-dependent problem to predict its evolution direction and continuously adjust the solution strategy of the solution system through a named evolution function [13]–[15]. Therefore, with the help of the OZNN model, it can cope with various time-dependent problems with extremely high accuracy [16], [17]. The OZNN model has been successfully applied to the fields of signal processing and automatic control due to its high solution accuracy and real-time solution advantages [18]. However, the biggest drawback of the OZNN model is its sensitivity to measurement noise. Its solution accuracy will be reduced distinctly in the presence of noise interference [19], [20]. Besides, the scale parameter of the OZNN model needs to be manually set and adjusted. Hence tedious and repeated adjustments are required when facing actual engineering application problems [21]. In recent years, it has been reported that much research aims to improve the performance of the zeroing neural network models. Xiao et al. present another new type of finite-time zeroing neural network termed the FTZNN model to solve the TDQM problem [6], which overcomes the shortcomings of the OZNN model that it takes time to converge when it approaches infinity, and accelerates the model to globally converge within a finite time [22]. On this basis, a strictly predefined-time convergent zeroing neural network model (PTCZNN) is presented by Li et al. Theoretical analysis is performed with the convergence of the models, including the predefined-time convergence of the PTCZNN model rigorously proved [23]. Noteworthily, both the FTZNN model and the PTCZNN model implement the convergence properties of the control solution model by constructing a special activation function and not synthesizing the scale parameter in models [24]. Consequently, Zhang et al. design a varying-parameter convergent-differential neural network (termed VP-CDNN) model [25]. Different from these fixed-valued neural-dynamic models, the VP-CDNN model is based on monotonically increasing time-varying design parameters. The VP-CDNN model converges exponentially and maintains better robustness under perturbation conditions than the OZNN model. Although the VP-CDNN model has...
VI. Besides, the following parts summarize the main contributions of this paper:

The abovementioned advantages in theory, it still has several inevitable defects. Specifically, in the process of model implementation and engineering applications, as the time continues to increase, the monotonically increasing time-varying design parameters may be too large to be achieved or violate the objective limit, which result in the solving failure [26]. When implementing the zeroing-type models, it is unavoidable that models may be interfered by various measurement noises, leading to the reduction of system solution accuracy and even the collapse of the solution system. In order to eliminate the influence of measurement noise on the solution of zeroing-type models and improve the robustness of the system, Jin et al. present the modified zeroing neural network (MZNN) model in [27], which introduces the integral information into the solution evolution formula for the first time. However, the parameters of the MZNN model still require tedious manual adjustments, which leads to a lot of additional computational resources and redundant adjustment processes [28]. He et al. present a residual learning framework to simplify the training process of deep neural networks, which explicitly reformulate the layers as learning residual functions concerning the layer inputs [29]. Inspired by the residual learning framework and combined with the advantages of the abovementioned zeroing-type models, this paper proposes a residual-based adaptive coefficient zeroing neural network model design framework embedded with adaptive scale coefficient and adaptive feedback coefficient, and for the first time this model is applied to solve the measurement noises perturbed TDQM problem.

The rest of this paper is arranged as the following five sections. The problem preliminaries and benchmark scheme are presented in Section II. The adaptive scale coefficient and adaptive feedback coefficient design framework and the evolution function of the proposed residual-based adaption coefficient zeroing neural network (RACZNN) models are formulated in Section III. Section V contains the corresponding quantitative simulation experiment and results investigation. Finally, the conclusion of this paper is arranged in Section VI. Besides, the following parts summarize the main contributions of this paper:

- This paper presents a novel design framework for constructing adaptive scale coefficient and adaptive feedback coefficient for the first time, which can accelerate the global convergence of the model and enhance the robustness of the solution system.
- Based on the design framework of adaptive scale coefficient and adaptive feedback coefficient proposed in this paper, a RACZNN model for solving the TDQM problem with perturbed measurement noise is proposed. Subsequently, the global convergence of the RACZNN model is analyzed from the perspective of from the perspective of Lyapunov stability theory.
- Corresponding quantitative numerical experiments are given to verify the performance of the RACZNN model in solving the TDQM problem with various measurement noise pollution.
- A dynamic localization scheme is proposed based on the RACZNN model with adaptive coefficients, which has higher robustness and solution accuracy than existing methods.

II. Preliminaries and Related Scheme Formulation

In general, the unified form of the time-dependent quadratic minimization (TDQM) problem can be written as

\[
\min \frac{1}{2} z^T(t) M(t) z(t) + b^T(t) z(t),
\]

where the parameters \( M(t) \in \mathbb{R}^{n \times n} \) and \( b(t) \in \mathbb{R}^n \) denote the smoothly time-dependent Hessian matrix and vector, respectively. The parameter \( z(t) \in \mathbb{R}^n \) represents the unknown vector that should be solved in real time. The superscript \( ^T \) denotes the transpose of a vector. For further investigation and solving the time-dependent quadratic minimization problem [1], a function \( F(z(t), t) \) is defined as \( F(z(t), t) = \frac{1}{2} z^T(t) M(t) z(t) + b^T(t) z(t) \). Consequently, the gradient of the function \( F(z(t), t) \) can be described as follows:

\[
\nabla F(z(t), t) = \frac{\partial F(z(t), t)}{\partial z(t)} = M(t) z(t) + b(t).
\]

Note worthily, by zeroing \( \nabla F(z(t), t) \) in each time instant \( t \in [0, +\infty] \), the theoretical solution of the TDQM problem [1]
can be obtained in real time. Hence, the following equation is formulated to zeroing the equation (3):

\[ M(t)z(t) + b(t) = 0. \]  
(3)

The following error function is arranged to monitor and revise the evolution direction of the solving system:

\[ e(t) = M(t)z(t) + b(t). \]  
(4)

According to the OZNN model construction framework, the evolution direction of the error function should be satisfied that \( \hat{e}(t) = -\eta \Omega(e(t)) \), where the \( \eta \) represents the scale coefficient and \( \Omega(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n \) denotes the activation function. Therefore, the OZNN model for solving the TDQM problem can be obtained in real time. Hence, the following equation is rearranged to construct the adaptive scale coefficient and adaptive feedback coefficient, respectively. The following two methods can be employed to construct the adaptive scale coefficient \( \xi(\cdot) \):

- Power adaptive scale coefficient:
  \[ \xi(e(t)) = \| e(t) \|^2 + a, \]  
  where the parameter \( a > 0 \).
- Exponential adaptive scale coefficient:
  \[ \xi(e(t)) = \rho \| e(t) \|^2 + \| e(t) \|_2 + a, \]  
  where the parameter \( \rho > 1 \).

Further, the following methods can be adopted to implement the adaptive feedback coefficient \( \kappa(\cdot) \):

- Magnification adaptive feedback coefficient:
  \[ \kappa(e(t)) = \lambda \int_0^t |e(\delta)|^2 + c, \]  
  where the parameters \( \lambda > 1 \) and \( c > 0 \).

- Power adaptive feedback coefficient:
  \[ \kappa(e(t)) = \| e(t) \|^2 + b + c, \]  
  where the parameter \( b > 0 \) and \( c > 0 \).

Consequently, the proposed RACZNN model with adaptive scale coefficient for solving the TDQM problem can be written as follows:

\[ M(t)z(t) = -\hat{M}(t)z(t) - \hat{b}(t) \]
\[ -\xi(e(t))(\hat{M}(t)z(t) + \hat{b}(t)) \]
\[ -\kappa(e(t)) \int_0^t (M(\epsilon)z(\epsilon) + b(\epsilon)) d\epsilon. \]  
(11)

Besides, the RACZNN model inevitively is perturbed by various measurement noises in the practical application. Therefore, the RACZNN model for solving the TDQM problem perturbed by noise is described as

\[ M(t)z(t) = -\hat{M}(t)z(t) - \hat{b}(t) \]
\[ -\xi(e(t))(\hat{M}(t)z(t) + \hat{b}(t)) \]
\[ -\kappa(e(t)) \int_0^t (M(\epsilon)z(\epsilon) + b(\epsilon)) d\epsilon + \vartheta(t), \]  
(12)

where the noise perturbation item \( \vartheta(t) \in \mathbb{R}^n \). In order to ensure the completeness and conciseness of the paper, the noise perturbation item \( \vartheta(t) \) will be omitted before the robustness analysis is finished.

Taking into account that convergence is a key criterion for the RACZNN model, we propose the following theorem and corresponding proof process to analyze the global convergence of the RACZNN model.

**Theorem 1:** Given any solvable TDQM problem (1), the computational solution vector \( z(t) \) of the proposed RACZNN model globally converges to the theoretical solution of the TDQM problem (1) from any random initial state.

**Proof:** The \( i \)th subsystem of the RACZNN model evolution formula \( \hat{e}(t) = -\xi(e(t))e(t) - \kappa(e(t)) \int_0^t e(\delta)d\delta \) can be depicted as

\[ \hat{e}_i(t) = -\xi(e(t))e_i(t) - \kappa(e(t)) \int_0^t e_i(\delta)d\delta. \]  
(13)

The following Lyapunov candidate function is presented for investigating the global convergence of the system:

\[ y_i(t) = e_i^2(t) + \kappa(e(t)) \int_0^t e_i(\delta)d\delta \]  
(14)

which indicates the Lyapunov function candidate \( y_i(t) > 0 \) when \( e_i(t) \neq 0 \) or \( \int_0^t e_i(\delta)d\delta \neq 0 \). If and only if \( e_i(t) = \int_0^t e_i(\delta)d\delta = 0 \), \( y_i(t) = 0 \). Thus the Lyapunov function candidate \( y_i(t) \) is positive semi-definite. Considering that the adaptive feedback coefficient \( \kappa(e(t)) \) is a constant \( \kappa \) in each time interval and taking the time derivative of function leads:

\[ \frac{dy_i(t)}{dt} = e_i(t)\dot{e}_i(t) + \kappa(e(t))e_i(t) \int_0^t e_i(\delta)d\delta \]
\[ = e_i(t)(\dot{e}_i(t) + \kappa(e(t)) \int_0^t e_i(\delta)d\delta) \]
\[ = -\xi(e(t))e_i^2(t) \leq 0. \]
That is to say, the Lyapunov function candidate \( y_i(t) \) is negative semi-definite. Thus, according to the definition of the Lyapunov theory, the function \( \epsilon_i(t) \) will ultimately converge to zero. It can be generalized and concluded that \( \epsilon_i(t) \) globally converges to zero for each \( i \in \{1, 2, \ldots, n\} \). In summary, the error function \( \epsilon(t) \) globally converges to zero over time. In other words, the proposed RACZNN model (11) globally converges to the theoretical solution of the TDQM problem (1). The proof is thus completed.

IV. SIMULATIONS

Both the visualized and corresponding quantitative experimental are designed and performed in this section. Firstly, the simulation experiment of the proposed RACZNN model (11) for solving the TDQM problem (1) is concluded and visualized. Secondly, we compare the performance of the RACZNN model (11) with other state-of-the-art neural network models, specifically gradient-based RNN (GNN) model, predefined-time convergent zeroing neural network (PTCZNN) model, nonconvex and bound constraint zeroing neural network (NCZNN) model, when solving the TDQM problem (1) with noise-free or different measurement noise. Noting that all of the simulations are conducted via MATLAB R2018a on a computer with Intel Core i5-8300H @2.30 GHz CPU, 16 GB memory, NVIDIA GeForce GTX 1050Ti GPU, and Windows...
A. Time-dependent situation

In this simulation, the time-dependent matrix and vector in the TDQM problem (1) are constructed as follows:

\[
M(t) = \begin{bmatrix} 0.5 \sin(t) + 2 & \cos(t) \\ \cos(t) & 0.5 \cos(t) + 2 \end{bmatrix}, \quad b(t) = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}.
\]

The adaptive scale coefficient \(\xi(\cdot)\) and adaptive feedback coefficient \(\kappa(\cdot)\) of the proposed RACZNN model (11) are set as \(\xi(\epsilon(t)) = ||\epsilon(t)||_2^2 + 5\) and \(\kappa(\epsilon(t)) = 5||\int_0^t \epsilon(\delta)\delta||_2 + 5\), respectively. The corresponding quantitative simulation results of example (IV-A) are arranged in Figs. 1 to 4. Besides, the following models are introduced to solve the TDQM problem (11) as a comparison of the RACZNN model (11).

- **GNN model** presented in [33].

\[
\dot{z}(t) = -\gamma M^T(t) (M(t) z(t) + b(t)).
\] (15)

- **PTCZNN model** presented in [23].

\[
M(t) \dot{z}(t) = -\dot{M}(t) z(t) + \tilde{b}(t) - \gamma \exp(t - 1) (M(t) z(t) - d(t)).
\] (16)

- **NCZNN model** presented in [37].

\[
M(t) \dot{z}(t) = -\dot{M}(t) z(t) + \tilde{b}(t) - \gamma R_T(M(t) z(t) - d(t)).
\] (17)
Where the parameter $R_T(\cdot)$ represents the non-convex and bounded activation function.

Note that the scale parameter $\gamma$ of the GNN model (15), the PTCZNN model (16), and the NCZNN model (17) are arranged as 5.

B. RACZNN model with noise free situation

The quantitative experiment visualization results synthesized by the RACZNN model (11) for solving the TDQM problem example (IV-A) in the noise-free case are arranged in Fig. 1. As demonstrated in Fig. 1(a), beginning with a randomly generated initial vector-formed value, the residual error $||\epsilon(t)||_2$ of the proposed RACZNN model (11) sharply approaches zero, which means the solving system can converge to the theoretical solution. Among the compared five models, the RACZNN model (11) has the second convergence speed. The logarithms of the models’ residual error $||\epsilon(t)||_2$ are shown in Fig. 1(b) which depicts the models’ solution accuracy. As observed in Fig. 1(b), the proposed RACZNN model (11) has significantly higher accuracy when solving the noise-free TDQM problem (IV-A) comparing with the GNN model (15), PTCZNN model (16), and NCZNN model (17). Specifically, the GNN model (15) converges to order $10^{-2}$, PTCZNN model (16) and NCZNN model (17) converge to order $10^{-3}$, and the proposed RACZNN model (11) converges to order $10^{-5}$. Furthermore, the RACZNN model (11) converges to the steady-state residual error at 8.5s. In general, compared with the state-of-the-art models, the proposed RACZNN model (11) has a competitive performance in robustness and convergence speed.

C. RACZNN model with noises situation

The quantitative experiment visualization results synthesized by the RACZNN model (11) for solving the TDQM problem example (IV-A) in the constant noise, linear noise, and bounded random noise cases are arranged in Fig. 2, Fig. 3, and Fig. 4, respectively. Besides, the adaptive scale coefficient and adaptive feedback coefficient of the RACZNN model (11) are $\xi(\epsilon(t)) = ||\epsilon(t)||_2^2 + 5$ and $\kappa(\epsilon(t)) = 5||f_0\int_0^t y(t)dy||_V + 5$, respectively. Based on these three different situations, the following three situations will be analyzed and discussed in detail. Noticeably, the measurement noises perturbed RACZNN model can be expressed as equation (12). Firstly, the amplitude of the constant noise in example (IV-A) is provided as $\vartheta(t) = \vartheta = [5,5]^T$. As shown in Fig. 2(a), beginning with a random-generated initial value, even though the RACZNN model (11) is disturbed by the constant noise, its system residual error $||\epsilon(t)||_2$ can still accurately converge to the theoretical solution. Meanwhile, Fig. 2(b) depicts that the solution accuracy of the GNN model (15), PTCZNN model (16), and NCZNN model (17) remain at a relatively high level. Secondly, the quantitative experimental simulation results of the RACZNN model (11) solving the TDQM problem example (IV-A) under linear noise $\vartheta(t) = \vartheta t \in \mathbb{R}^n$ interference are
Fig. 8. Schematic diagram of AoA dynamic positioning scheme.

Consequently, the geometric relationship between the angle of arrival between the observation base station and the target can be described as

$$\tan \theta_i(t) = \frac{y(t) - y_i}{x(t) - x_i}$$

(18)

Noting that parameters $\theta_i(t)$ and $(x(t), y(t))$ represent real-time changing angles and coordinates. Expand and rearrange the equation (18) to get $y_i - x_i \tan \theta_i(t) = -x(t) \tan \theta_i(t) + y(t)$.
Therefore, we can get the following linear equation:

\[
\begin{bmatrix}
-\tan(\theta_1(t)) & 1 \\
-\tan(\theta_2(t)) & 1 \\
\vdots & \vdots \\
-\tan(\theta_n(t)) & 1
\end{bmatrix}
\begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix}
= \begin{bmatrix}
y_1 - x_1 \tan(\theta_1(t)) \\
y_2 - x_2 \tan(\theta_2(t)) \\
\vdots \\
y_n - x_n \tan(\theta_n(t))
\end{bmatrix}, \tag{19}
\]
where the parameter \(n\) denotes the number of observation base stations, and we further express the equation (19) as the following form:

\[
F(t)g(t) = h(t), \tag{20}
\]
where the time-dependent matrix \(F(t) \in \mathbb{R}^{n \times 2}\), time-dependent vector \(g(t) = [x(t), y(t)]^T\) and \(h(t) \in \mathbb{R}^n\). Besides, the error function for equation (20) can be written as \(\epsilon(t) = F(t)g(t) - h(t)\). Subsequently, the RACZNN model (11) for dynamic localization scheme is presented as

\[
F(t)\dot{g}(t) = -\dot{F}(t)g(t) - \dot{h}(t) - \gamma (F(t)g(t) - h(t))
- \kappa(\epsilon(t)) \int_{0}^{t} (F(\delta)g(\delta) + h(\delta))d\delta.
\]

For comparison, the original zeroing neural network (OZNN) for the AoA dynamic positioning scheme are introduced as following:

\[
F(t)\dot{g}(t) = -\dot{F}(t)g(t) - \dot{h}(t) - \gamma (F(t)g(t) - h(t)), \tag{22}
\]
where the parameter \(\gamma\) represents the scale parameter. The corresponding simulation results are provided in the Fig. 6 and Fig. 7. Figure 6 shows the target trajectory and system position error obtained by the dynamic localization scheme based on the OZNN model and the RACZNN model. Obviously, starting from the initial coordinate point \(u_0\), the position error generated by the RACZNN model (21) converges to the \(10^{-5}\) order, which is more accurate than the \(10^{-3}\) order obtained by the OZNN model (22). Besides, as described in Fig. 7(a) and (c), the trajectory obtained by the OZNN model (22) obviously does not converge to the true dynamic target trajectory when disturbed by constant noise \(\theta(t) = [20, 20]^T\). While the trajectory generated by the RATZNN model (21) is well consistent with the dynamic target trajectory. Figure 7(b) and (d) show that in the case of constant noise \(\theta(t)\) interference, the RACZNN model (21) converges to order \(10^{-3}\), while the OZNN model (22) diverges. In general, this part fully demonstrates that the RACZNN model can be effectively applied to the AoA dynamic localization scheme regardless of whether there is noise or not.

VI. CONCLUSIONS

The residual-based adaption coefficient zeroing neural network (RACZNN) model is proposed in this paper for solving the various noises perturbed TDQM problem. Unlike the original zeroing neural network models, the scale coefficient and feedback coefficient of the RACZNN model proposed in this paper is presented from the perspective of adaptive optimization to accelerate the global convergence of the solution system and to improve the robustness of the model. Since the proposed RACZNN model uses a more advanced adaptive scale coefficient and adaptive feedback coefficient, it not only has obvious advantages in robustness but also has a faster convergence speed compared with the commonly used neural network models proposed before. Consequently, this paper presents corresponding theorem and proof procedures from the stability perspective to analyze the global convergence of the RACZNN model. Then, the corresponding numerical experiment is designed and executed, and the numerical results and visualization results of the experiment are given in the form of tables and images, respectively. Finally, the potential of the RACZNN model in practical applications is shown, and the simulation experiment demonstrates the effectiveness and superiority of the dynamic positioning scheme based on the RACZNN model.

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