Determination of the mixing angle between new charmonium states

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Abstract

Using the pictures for $X(3872)$ as a mixture of charmonium and molecular $D^{*0}D^0$ states; $Y(3940)$ as a mixture of $\chi_{c0}$ and $D^*D^{*0}$ states, and $Y(4260)$ as a mixture of the tetra-quark state with charmonium states, the corresponding mixing angles are estimated within the QCD sum rules. We obtain that our predictions for the mixing angles of the $X(3872)$, $Y(4260)$ and $Y(3940)$ states are considerably smaller compared to works in which mixing angles are estimated from the condition in reproducing the mass of these states. Our conclusion is that the considered pictures for the $X(3872)$, $Y(4260)$ and $Y(3940)$ states are not successful in describing these states.

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1 Introduction

The analysis of the spectroscopy and decays of the heavy flavored mesons is an essential source for obtaining useful information about the dynamics of QCD at “low” energies. Remarkable progress in this direction has been made on the experimental side. Starting from the observation of $X(3872)$ [1] up to present time 23 new charmonium line states have been discovered (see the recent review [2]). All observed charmonium states might have more complex structures compared to those predicted by the simple quark model. These new states (referred as $XYZ$ states in the text) can be potential candidates of exotic states, and for this reason theorists pay great effort for understanding the dynamics of these states (for a review, see [3]). There are two attractive pictures in interpretation of all observed states: tetra-quark, and bound states of two mesons (meson molecules).

The theoretical approaches which have been employed in investigation of these states are QCD sum rules, lattice QCD, effective Lagrangian method, chiral perturbation theory, quark model, etc. Among all approaches the QCD sum rules occupies a special place [4], which is based on the fundamental QCD Lagrangian. The mass and some of the strong coupling constants of $XYZ$ mesons with light mesons are widely discussed in framework of the QCD sum rules method [3]. It is assumed in [5] that $X(3872)$ is a mixture of charmonium and molecular states, whose interpolating current is taken as:

$$j_\mu = \cos \theta j_{\mu}^{(2)} + \sin \theta j_{\mu}^{(4)},$$  \hspace{1cm} (1)

where

$$j_{\mu}^{(2)} = \frac{1}{6\sqrt{2}}(\bar{q}q)c\gamma_\mu\gamma_5c,$$  \hspace{1cm} (2)

$$j_{\mu}^{(4)} = \frac{1}{\sqrt{2}}\left[(\bar{u}\gamma_5c)(\bar{c}\gamma_\mu u) - (\bar{u}\gamma_\mu c)(\bar{c}\gamma_5 u)\right],$$  \hspace{1cm} (3)

and whose analysis in QCD sum rules method predicted that if the mixing angle $\theta$ lies in the range $(9 \pm 4)^0$, it can provide good agreement with the experimental value of the mass and decay width. Using the QCD sum rules, the mass and the decay width of the channel $J/\Psi\omega$ for the $Y(3940)$ state is studied in [6], assuming that it is described by the mixed scalar $\chi_{c0}$ and $D^*\bar{D}^*$ states, i.e.,

$$j = -\frac{\langle \bar{q}q \rangle}{\sqrt{2}} \cos \theta j_{\chi_{c0}} + \sin \theta j_{D^*\bar{D}^*},$$  \hspace{1cm} (4)

where $j_{\chi_{c0}} = \bar{c}c$ and $j_{D^*\bar{D}^*} = (\bar{q}\gamma_\mu c)(\bar{c}\gamma_\mu q)$. As a result of this study it is found that, one can reproduce the mass and decay width of $Y(3940)$ in very good agreement with the experimental result if the mixing angle is chosen to be $\theta = (76 \pm 5)^0$.

Similar analysis is carried out in [7] for the $Y(4260)$ state by assuming that it can be described by the mixture of the tetra-quark and charmonium currents, i.e.,

$$j_\mu = \cos \theta j_{\mu}^{(2)} + \sin \theta j_{\mu}^{(4)},$$  \hspace{1cm} (5)

where

$$j_{\mu}^{(4)} = \frac{1}{\sqrt{2}}\delta_{abc}\varepsilon_{dec}\left[(q_a^T C\gamma_5 c_b)(\bar{q}_d\gamma_\mu\gamma_5 C\bar{c}^T) + (q_a^T C\gamma_5\gamma_\mu c_b)(\bar{q}_d\gamma_5 C\bar{c}^T)\right],$$  \hspace{1cm} (6)

$$j_{\mu}^{(2)} = \frac{1}{\sqrt{2}}\langle \bar{q}q \rangle\bar{c}\gamma_\mu c,$$  \hspace{1cm} (7)
and it is found that the experimental result can be reproduced by setting the mixing angle to $\theta = (53 \pm 5)^0$.

Note that, the mixing studied in [8] between the two- and four-quark states is suggested for the light quark sector. This mixing is analyzed with the use of currents, and can be extended to the charm sector. The origin of this mixing can be explained as follows: The $\bar{c}c$ state can emit a gluon, which subsequently splits into a light quark-antiquark living like a molecular state during some time interval. In the present note we will calculate the mixing angle directly from QCD sum rules method following the works [8, 9]. Let us briefly remind the main steps of this in calculation of the mixing angle. The two physical hadronic states are both considered to be the mixing of the two states

$$
\eta_{H_1} = \cos \theta \eta_{H_1}^0 + \sin \theta \eta_{H_2}^0 , \\
\eta_{H_2} = - \sin \theta \eta_{H_1}^0 + \cos \theta \eta_{H_2}^0 ,
$$

(8)

and then considering the correlation function

$$
\Pi = i \int d^4xe^{ipx} \langle 0 \left| T \{ \eta_{H_1}(x)\bar{\eta}_{H_2}(0) \} \right| 0 \rangle .
$$

(9)

According to the general strategy of the QCD sum rules method, this correlation function is calculated in terms of hadrons on the one side; and in terms of quarks and gluons on the other side. Using the duality ansatz these two representations are then equated to obtain the QCD sum rules. The correlation function from the hadronic side is calculated by saturating it with the corresponding hadrons carrying the same quantum numbers as the interpolating current. Obviously, the hadronic part of the correlation function should be equal to zero after this procedure, since the hadronic states given by Eq. (8) are orthogonal. Using Eq. (8), the mixing angle can be calculated from Eq. (9) whose expression can be written as

$$
\tan 2\theta = \frac{2\Pi^{(0)12}}{\Pi^{(0)11} - \Pi^{(0)22}} ,
$$

(10)

where $\Pi^{(0)ij}$ are the correlation function corresponding to the unmixed case, i.e.,

$$
\Pi^{(0)ij} = i \int d^4xe^{ipx} \langle 0 \left| T \{ \eta^i(x)\bar{\eta}^j(0) \} \right| 0 \rangle ,
$$

where $i = 1$ or 2, and $j = 1$ or 2. In the case of scalar current, the correlation function contains only one invariant function which we shall denote by $\Pi^{(0)ij}$.

In the case of vector (axial-vector) current the two-point correlation function can be written in terms of two independent invariant functions as follows:

$$
\Pi^{(0)ij}(p^2) = \Pi^{(1)ij}(p^2) \left( g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \Pi^{(2)ij}(p^2)\frac{p_\mu p_\nu}{p^2} .
$$

(11)

The invariant functions $\Pi^{(1)ij}(p^2)$ and $\Pi^{(2)ij}(p^2)$ in Eq. (11) are associated with the spin-1 and spin-0 mesons, respectively. Since $X(3872)$ and $Y(4260)$ mesons both have $J = 1$ quantum number, in further discussions we shall consider only the structure $(g_{\mu\nu} - p_\mu p_\nu/p^2)$, i.e., we shall analyse the invariant function $\Pi^{(1)ij}(p^2)$ only.
Using the currents given in Eqs. (1), (4), (7) and their orthogonal combinations, the corresponding correlation functions are calculated in terms of quarks and gluons in the deep euclidean region $p^2 \ll 0$ using the operator product expansion. The results for each current are presented in Appendix.

Having all the necessary formulas, we can now proceed for the numerical analysis of the mixing angles. It follows from the expressions of the mixing angles that the main input parameters involved in the numerical calculations are the quark and gluon condensates and mass of the quarks whose values are given as: $\langle \bar{q}q \rangle (1 \text{ GeV}) = - (246^{+28}_{-19} \text{ MeV})^3 \ [10]$, $\langle g^2 G^2 \rangle = 0.47 \text{ GeV}^4$, $m_0^2 = 0.8 \text{ GeV}^2 \ [11]$. For the mass of the $c$ quark we have used its $\overline{\text{MS}}$ value $\bar{m}_c(\bar{m}_c) = 1.28 \pm 0.03 \text{ GeV} \ [12]$.

Beside these input parameters, the sum rules do also contain two auxiliary parameters, namely, Borel mass parameter $M^2$ and the continuum threshold $s_0$. The continuum threshold is correlated with the energy of the first excited state. It is usually chosen as $\sqrt{s_0} = (m_{\text{ground}} + 0.5) \text{ GeV}$ and we look for the domain of $s_0$ which satisfies this restriction. In this work the continuum thresholds for $X(3872)$, $Y(3940)$ and $Y(4260)$ are chosen as $\sqrt{s_0} = (4.2 \pm 0.1) \text{ GeV} \ [5]$, $\sqrt{s_0} = (4.4 \pm 0.1) \text{ GeV} \ [6]$, $\sqrt{s_0} = (4.7 \pm 0.1) \text{ GeV} \ [7]$, respectively, which are obtained from the analysis of two-point function. The working region of the Borel mass parameter $M^2$ can be obtained using the procedure. The upper bound of $M^2$ is determined from the condition that the continuum and higher state contributions exceed the nonperturbative ones. From these conditions we get the following working regions for $M^2$: $2 \leq M^2 \leq 4 \text{ GeV}^2$ for $X(3872)$, $Y(3940)$; and $2 \leq M^2 \leq 5 \text{ GeV}^2$ for the $g_{\mu \nu} - p_\mu p_\nu/p^2$ structure.

In Fig. (1) we present the dependence of the mixing angle $\theta$ on Borel mass parameter $M^2$ at $\sqrt{s_0} = (4.4 \pm 0.1) \text{ GeV}$ for the $g_{\mu \nu} - p_\mu p_\nu/p^2$ structure for the $X(3872)$ state. We observe from this figure that this structure predicts for the mixing angle $\theta \simeq (2.4 \pm 0.6)^0$, which is considerably small compared to the value of $\theta$ obtained in [5] in order to reproduce the mass of $X(3872)$.

In Fig. (2) we present the dependence of the mixing angle $\theta$ on $M^2$ at fixed value of $\sqrt{s_0} = (4.4 \pm 0.1) \text{ GeV}$ for the $Y(3940)$ state. The mixing angle we obtain from this figure is $\theta = (20 \pm 2)^0$, which is about 3.5 times smaller compared to that predicted in [6].

In Fig. (3) we present the dependence of the mixing angle $\theta$ on Borel mass parameter $M^2$ at $\sqrt{s_0} = (4.6 \pm 0.1) \text{ GeV}$ for the $g_{\mu \nu} - p_\mu p_\nu/p^2$ structure for the $Y(4260)$ state. We see from this figure that the mixing angle has the value $\theta = (20 \pm 3)^0$, which is approximately 2.5 times smaller than the one predicted in [7].
In summary, in this work, based on the pictures that the $X(3872)$ is a mixture of charmonium and $D^*\bar{D}^0$ states, $Y(3940)$ is a mixture of scalar $\bar{c}c$ and $D^*D^*$ molecule, and $Y(4260)$ is a mixture of tetra-quark and charmonium states, we estimate the respective mixing angles within the QCD sum rules method. It is obtained that, the mixing angles calculated in the present work for all considered pictures are considerably smaller than the ones predicted in [5–7]. Therefore, in our view the considered pictures for $X(3872)$, $Y(3940)$ and $Y(4260)$ are not successful ones in describing these states.
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Appendix A

In this appendix we present the spectral densities for the $Y(4260)$, $X(3872)$ and for the $Y(3940)$ states. Note that the spectral density is the imaginary part of the correlation function, i.e., $\pi \rho(s) = \text{Im}\Pi(q^2 = s)$.

**Y(4260)**

Spectral densities corresponding to $g_{\mu\nu} - p_{\mu}p_{\nu}/p^2$ structure.

$$\rho_{11}(s) - \rho_{22}(s) = -\frac{1}{3072\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \left\{ 12(1 - \alpha - \beta)\mu_1^2 m_q^2 (\alpha \beta \mu_1 + 6m_Qm_{Q'}) - \mu_1^3 (1 - \alpha - \beta) \left[ 3\alpha \beta (1 + \alpha + \beta) \mu_1 - 2(1 - \alpha - \beta)^2 m_Q m_{Q'} \right] + 16\pi^2 m_q \left[ 12\alpha \beta \mu_1^2 + m_0^2 m_Q m_{Q'} + 6(5 - \alpha - \beta) \mu_1 m_Q m_{Q'} \right] \langle \bar{q}q \rangle \right\}$$

$$+ \frac{\langle \bar{q}q \rangle^2}{192\pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \left\{ m_q (1 - \alpha) \alpha \left[ 6\mu_2^2 - m_0^2 (\mu_2 + 2s) \right] - 6m_0^2 m_q m_Q m_{Q'} - 8\pi^2 \left[ (1 - \alpha) \alpha (m_0^2 - 7\mu_2 + 8s - 2m_q^2) + 10m_Q m_{Q'} \right] \right\},$$

$$\rho_{12}(s) = \rho_{21}(s) = -\frac{\langle \bar{q}q \rangle^2}{8\pi^2} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \left[ (1 - \alpha) \alpha (\mu_2 - s) - m_Q m_{Q'} \right]. \quad (1)$$

**X(3872)**

Spectral densities corresponding to $g_{\mu\nu} - p_{\mu}p_{\nu}/p^2$ structure.

$$\rho_{11}(s) - \rho_{22}(s) = \frac{3\mu_1}{4906\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \left\{ -\alpha \beta \left[ 1 - (\alpha + \beta)^2 \right] \mu_1^3 \right.$$  

$$\left. + 2(1 - \alpha - \beta) \mu_1 m_q \left[ \alpha (3 + \alpha + \beta) \mu_1 m_Q + \beta (3 + \alpha + \beta) \mu_1 m_Q - 12m_q m_Q m_{Q'} \right] \right.$$  

$$\left. - 8\pi^2 \langle \bar{q}q \rangle \left[ m_0^2 - 2(1 + \alpha + \beta) \mu_1 + 2m_q^2 \right] (\alpha m_Q + \beta m_Q) \right.$$  

$$\left. + 32\pi^2 m_q \langle \bar{q}q \rangle \left( \alpha \beta \mu_1 - 4m_Q m_{Q'} \right) \right\}$$

$$+ \frac{\langle \bar{q}q \rangle}{512\pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \left\{ 6m_0^2 \mu_2 \left[ (1 - \alpha) m_Q + \alpha m_{Q'} \right] - 32\pi^2 \left[ m_Q m_{Q'} + m_q^2 \alpha (1 - \alpha) \right] \langle \bar{q}q \rangle \right.$$  

$$\left. + 3m_q^2 (m_0^2 + 4\mu_2) \left[ (1 - \alpha) m_Q + \alpha m_{Q'} \right] \right.$$  

$$\left. - 4m_q \alpha (1 - \alpha) \left[ 3\mu_2^2 + m_0^2 (\mu_2 - s) \right] - 12m_0^2 m_q m_Q m_{Q'} \right.$$  

$$\left. + 24\pi^2 m_q \left[ (1 - \alpha) m_Q + \alpha m_{Q'} \right] \langle \bar{q}q \rangle \right\},$$
\[ \rho_{12}(s) = \rho_{21}(s) = -\frac{\langle \bar{q}q \rangle^2}{96\pi^2} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \left[ \alpha (1 - \alpha) (\mu_2 - s) + m_Q m_{Q'} \right]. \] (2)

**Y(3940)**

Spectral densities for the Y(3940) state.

\[ \rho_{11}(s) - \rho_{22}(s) = \frac{3\mu_1}{512\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} d\beta \left\{ -(1 - \alpha - \beta)\mu_1 \left[ \alpha \beta \mu_1^2 + 12m_q^2 m_Q m_{Q'} \right. \\
- \left. 2\mu_1 m_q(a m_{Q'} + \beta m_Q) \right] + 8\pi^2 \left[ \mu_1(a m_{Q'} + \beta m_Q) - 8m_q m_Q m_{Q'} \right] \langle \bar{q}q \rangle \right\} \\
+ \frac{\langle \bar{q}q \rangle}{384\pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \left\{ 6(m_0^2 + 3\mu_2)m_q^2 \left[ (1 - \alpha) m_Q + a m_{Q'} \right] + 9m_0^2 \mu_2 \left[ (1 - \alpha) m_Q + a m_{Q'} \right] \\
- 12m_q^2 (1 - a) \left[ 3\mu_2^2 + m_0^2 (2\mu_2 - s) \right] - 36m_0^2 m_q m_Q m_{Q'} \\
+ 16\pi^2 \left[ -\alpha (1 - \alpha) (14 \mu_2 - 7s + 9m_q^2) + 3(1 - \alpha) m_q m_Q + (3 \alpha m_Q - 13 m_Q m_{Q'}) \langle \bar{q}q \rangle \right] \right\}, \]

\[ \rho_{12}(s) = \rho_{21}(s) = \frac{\langle \bar{q}q \rangle^2}{4\sqrt{2}\pi^2} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \left[ \alpha (1 - \alpha) (2\mu_2 - s) + m_Q m_{Q'} \right]. \] (3)

where,

\[ \mu_1 = \frac{m_Q^2}{\alpha} + \frac{m_{Q'}^2}{\beta} - s, \]

\[ \mu_2 = \mu (\beta \rightarrow 1 - \alpha), \]

\[ \beta_{\text{min}} = \frac{\alpha m_{Q'}^2}{s \alpha - m_Q^2}, \]

\[ \beta_{\text{max}} = 1 - \alpha, \]

\[ \alpha_{\text{min}} = \frac{1}{2s} \left[ s + m_Q^2 - m_{Q'}^2 - \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4m_Q^2 s} \right], \]

\[ \alpha_{\text{max}} = \frac{1}{2s} \left[ s + m_Q^2 - m_{Q'}^2 + \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4m_Q^2 s} \right]. \]
Figure captions

Fig. (1) The dependence of the mixing angle $\theta$ on Borel mass square $M^2$, at the fixed value of the continuum threshold $\sqrt{s_0} = 4.4 \text{ GeV}$, for the structure $g_{\mu\nu} - p_\mu p_\nu/p^2$ for the $X(3872)$ state.

Fig. (2) The dependence of the mixing angle $\theta$ on Borel mass square $M^2$, at the fixed value of the continuum threshold $\sqrt{s_0} = 4.4 \text{ GeV}$, for the $Y(3940)$ state.

Fig. (3) The same as in Fig. (1), but at the fixed value of the continuum threshold $\sqrt{s_0} = 4.6 \text{ GeV}$ for the $Y(4260)$ state.
Figure 1:

\[ \sqrt{s_0} = 4.4 \text{ GeV} \]

\[ X(3872) \]

Figure 2:

\[ \sqrt{s_0} = 4.4 \text{ GeV} \]

\[ Y(3940) \]
$\sqrt{s_0} = 4.6$ GeV

Figure 3: