Coherent control via interplay between driving field and two-body interaction in a double well

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We investigate interplay between external field and interatomic interaction and its applications to coherent control of quantum tunneling in a periodically driven double well. A full solution of the system is generated analytically as a coherent non-Floquet state by using the Floquet states as a set of complete bases. It is demonstrated that the photon resonance of interaction leads to translation of the Floquet level-crossing points, and the non-resonant interaction causes avoided crossing of partial levels. In the non-Floquet states, the bosons beyond the crossing points slowly vary their populations, and the resonant (non-resonant) interactions enhance (decrease) the tunneling rate of the paired particles. Three different kinds of the coherent destructions of tunneling (CDT) at the crossing, avoided-crossing and uncrossing points, and the corresponding stationary-like states, are illustrated. The analytical results are numerically confirmed and perfect agreements are found. Based on the results, an useful scheme of quantum tunneling switch between stationary-like states is presented.

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I. INTRODUCTION

Coherent control of quantum tunneling in a double well via periodical driving has been researched extensively from both theoretical and experimental sides [1-4]. Some interesting phenomena, such as the coherent destruction or construction of tunneling (CDT or CCT) [4-10], chaos enhancing tunneling [11, 12], and photon-assisted tunneling [13-14] have been found. Many works focus on single- or many-particle systems. Few-particle systems are of a intermediate class between the both systems, which deserve further study for comprehensively understanding tunneling dynamics. However, investigations on quantum control to two particles in a periodically driven double well are extremely rare, except for the cases of two particles in a one-dimensional lattice which can be reduced to a two-site trap [15-16] or two particles in a driven double-well train [17-20]. Recently, some relevant researches have been completed for a non-driven few-particle system [21-23]. Several interesting phenomena of quantum tunneling were shown for two repulsive bosons in a non-driven double well, which include the Rabi oscillations and correlated pair tunneling [24-26]. The interatomic interaction adjusted by the Feshbach- resonance technique [27] plays an important role in tunneling dynamics of the non-driven two-particle system. Here we are interested in the combined effects of the driving and interaction on tunneling dynamics of double-well coupled two bosons.

In this paper, we study coherent control of quantum tunneling via the competition and cooperation between atomic interaction and driving field for a pair of repulsive bosons confined in a periodically driven double well. In the high-frequency regime, we obtain a set of Floquet quasienergies and Floquet states of invariant population, which contain the quasi-NOON state, an interesting entanglement state [28-29]. The general non-Floquet state of slowly varying population is generated as a full solution which is a coherent superposition of the Floquet states. The quasienergies as function of the driving parameters are plotted for several different values of interaction strength. The quasienergy spectra exhibit that comparing with the non-interacting case, the photon resonance [15] leads to translation of the level-crossing points, and the non-resonant interactions cause avoided crossing of partial levels. Subsequently, exploiting the coherent non-Floquet solutions, we investigate time evolution of the particle population and demonstrate that beyond the crossing points, the bosons slowly vary their populations compared to the high-frequency driving. The resonant or non-resonant interactions can enhance or decrease the tunneling rate of the paired particles. Three different kinds of CDT, respectively at the level-crossing points for the superposition of three states, at the avoided-crossing points for the superposition of two states and at the uncrossing points for the single Floquet states, are illustrated, which result in different stationary-like states of invariant populations. The analytical results are confirmed by direct numerical simulations and good agreements are shown. As an application of the above results, an interesting scheme of quantum tunneling switch between stationary-like states is presented by using CDT or CCT to close or open the quantum tunneling, which could be useful for the quantum control of two bosons in a double well.

II. ANALYTICAL SOLUTIONS IN HIGH-FREQUENCY REGIME

We consider two ultracold bosons confined in a periodically driven double well with the governing Hamiltonian [30-31]

\[
H(t) = \frac{g(t)}{2}(b_1^{\dagger}b_1 - b_2^{\dagger}b_2) + \gamma(b_1^{\dagger}b_2 + b_2^{\dagger}b_1) + \frac{U}{2}[b_1^{\dagger}b_1(b_1^{\dagger}b_1 - 1) + b_2^{\dagger}b_2(b_2^{\dagger}b_2 - 1)],
\]

where \(b_i(b_i^{\dagger})\) for \(i = 1, 2\) are the annihilation (creation) operators in the \(i\)th localized state which may be a line superposition of the lowest doublet of single-particle energy eigenstates

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[32]. Their commutation relations read $[b_j, b^*_l] = \delta_{jl}$. The parameters $\gamma$ and $U$ are the tunneling coefficient between two wells and the interaction strength between two bosons. The function $e(t) = \tilde{e}_0 \cos(\omega t)$ describes the external ac field in which $\omega$ and $\tilde{e}_0$ are the driving frequency and amplitude, respectively. For simplicity, we have taken $\hbar = 1$ and taken the reference frequency $\omega_0 = 10^5 \text{ s}^{-1}$ [33] so that $\tilde{e}_0$, $\gamma$, $\omega$ and $U$ are in units of $\omega_0$, and time $t$ is normalized in units of $\omega_0^{-1}$.

Quantum state $|\psi(t)\rangle$ of system (1) can be expanded in Fock bases $\{|0,2\rangle, |1,1\rangle, |2,0\rangle\}$ as [15]

$$|\psi(t)\rangle = a_0(t)|0,2\rangle + a_1(t)|1,1\rangle + a_2(t)|2,0\rangle,$$

(2)

where $|j\rangle = |j,2-j\rangle (j=0,1,2)$ denote $j$ bosons reside in left well, $2-j$ bosons reside in right well; $a_j(t)$ represents probability amplitude of the system in $j$-th Fock state $|j\rangle$, which obey the normalization condition $|a_0(t)|^2 + |a_1(t)|^2 + |a_2(t)|^2 = 1$. Inserting Eqs. (1) and (2) into the time-dependent Schrödinger Equation $i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = H(t)|\psi(t)\rangle$ results in the coupling equations

$$iu_0(t) = \sqrt{2\gamma}a_1(t),
$$
$$iu_1(t) = \sqrt{2\gamma}a_2(t),
$$
$$iu_2(t) = \sqrt{2\gamma}a_0(t) + \sqrt{2\gamma}a_1(t).
$$

(3)

It is difficult to get exact analytical solutions of Eq. (3) with periodic driving $e(t) = \tilde{e}_0 \cos(\omega t)$. However, we can construct the approximate analytical solution in the high-frequency limit, $\omega \gg 1$ and $u, \gamma \ll \omega$. To do this, we adopt the idea of reduced interaction strength [15] to rewrite the interaction strength as $U = n\omega + u$ for $0 \leq u < \omega$, $n = 0,1,2,\ldots$, with $u$ being the reduced interaction strength, and make the function transformation $a_0(t) = b_0(t)e^{i(\omega t - nu)t}$, $a_1(t) = b_1(t)$, $a_2(t) = b_2(t)e^{-i(\omega t + nu)t}$, where $b_j(t)$ ($j = 0,1,2$) are slowly-varying functions of time, then Eq. (3) becomes new equations in terms of $b_j(t)$. Because of $|b_j(t)| = |a_j(t)|$, the high-frequency limit implies that the occupied probabilities of Fock states slowly vary in time. Exploiting Fourier expansion $\exp\{i(\pm \epsilon t - in\omega t)\} = \exp\{i(\pm \epsilon t) \sin(\omega t) - in\omega t\} = \sum_{n=-\infty}^{\infty} \mathcal{F}_{\epsilon,\omega}(\pm n) \exp(\pm i \omega t)$, we easily obtain the time-average of the rapidly oscillating function as $\mathcal{F}_{\epsilon,\omega}(\pm n)$ which are the $n$-order bessel function. Under the high-frequency approximation, the rapidly oscillating function can be replaced by its time-averaging value [30] such that the equations of $b_j(t)$ are transformed into

$$ib_0(t) = ub_0(t) + J_0 \tilde{e}_0 \omega b_1(t),
$$
$$ib_1(t) = J_0 \tilde{e}_0 \omega b_0(t) + (-1)^n J_n \tilde{e}_0 \omega b_2(t),
$$
$$ib_2(t) = ub_2(t) + (-1)^n J_n \tilde{e}_0 \omega b_1(t).
$$

(4)

In the calculations, we have employed the formula $\mathcal{F}_{\epsilon,\omega}(\pm n) = (-1)^n \mathcal{F}_{\epsilon,\omega}(\pm n)$ for positive integer $n$, and the renormalized coupling coefficient $J_n \tilde{e}_0 \omega = \sqrt{2\gamma} \mathcal{F}_{\epsilon,\omega}(\pm n)$. Starting from Eq. (4), we obtain the interesting analytical solutions as follows.

### A. Quasienergies and Floquet states

Because the time-dependent Hamiltonian (1) has the period $T = \frac{2\pi}{\omega}$, we can make use of Floquet theory [34] to get its solution in the form $|\psi(t)\rangle = e^{-iE_1 t}|\varphi(t)\rangle$, where $|\varphi(t)\rangle$ is the Floquet state with the same period $\frac{2\pi}{\omega}$, $E$ is called the Floquet quasienergy which has been normalized in units of $\hbar \omega_0$. Noting the same period of the transformation function $e^{i\left(\pm \epsilon t - in\omega t\right)}$ between $a_i(t)$ and $b_j(t)$, to generate the Floquet states, we seek the stationary solutions [35] of Eq. (4) $b_0 = Ae^{-iE_1 t}$, $b_1 = Be^{-iE_1 t}$, $b_2 = Ce^{-iE_1 t}$ with constants $A, B, C$ obeying the normalization condition $|A|^2 + |B|^2 + |C|^2 = 1$. Inserting these into Eq. (4), we establish the equations of the stationary solutions as

$$\left( E - u \right) A - J_0 \left( \frac{\tilde{e}_0}{\omega} \right) B = 0,
$$
$$-J_0 \left( \frac{\tilde{e}_0}{\omega} \right) A + E B - \left( -1 \right)^n J_n \left( \frac{\tilde{e}_0}{\omega} \right) C = 0,
$$
$$-\left( -1 \right)^n J_n \left( \frac{\tilde{e}_0}{\omega} \right) B + \left( E - u \right) C = 0.
$$

(5)

By solving Eq. (5), we obtain three Floquet quasienergies $E_l$ and three sets of constants $A_l, B_l, C_l$ for $l = 0,1,2$.

$$E_0 = u, A_0 = \frac{1}{\sqrt{2}}, B_0 = 0, C_0 = (-1)^{n+1} \frac{1}{\sqrt{2}}.
$$

(6)

$$E_1 = \frac{1}{2}(u - k_n), A_1 = \frac{2J_n}{\sqrt{8J_n^2 + (u + k_n)^2}},
$$
$$B_1 = \frac{-u + k_n}{\sqrt{8J_n^2 + (u + k_n)^2}}, C_1 = (-1)^n \frac{2J_n}{\sqrt{8J_n^2 + (u + k_n)^2}}.
$$

(7)

$$E_2 = \frac{1}{2}(u + k_n), A_2 = \frac{2J_n}{\sqrt{8J_n^2 + (k_n - u)^2}},
$$
$$B_2 = \frac{-k_n - u}{\sqrt{8J_n^2 + (k_n - u)^2}}, C_2 = (-1)^n \frac{2J_n}{\sqrt{8J_n^2 + (k_n - u)^2}}.
$$

(8)

Here, the simplified parameter $k_n = \sqrt{8J_n^2 + u^2}$ has been adopted, so that the Floquet quasienergies are functions of driving parameters and interaction strength. Obviously, for $u, \gamma \ll \omega$ we have $E_l \ll \omega$. Thus as the functions of $\exp\{-iE_1 t\}$ the stationary solutions $b_j(t)$ are slowly varying indeed. Given $b_j(t)$, the rapidly oscillating function $a_i(t)$ are obtained immediately. Substituting such $a_i(t)$ into Eq. (2), Floquet states $|\psi(t)\rangle$ are expressed as

$$|\psi(t)\rangle = e^{-iE_1 t}(A_1 e^{\frac{\sqrt{2}}{\omega} \sin(\omega t) - in\omega t})|0,2\rangle + B_1 |1,1\rangle
$$
$$+ C_1 e^{\frac{\sqrt{2}}{\omega} \sin(\omega t) - in\omega t} |2,0\rangle
$$

(9)

for $l = 0,1,2$. Here $n$ is a positive integer adjusted by the interaction strength. If the system is prepared in the Floquet states initially, probabilities of the system in different Fock states are the constants $|A|^2$, $|B|^2$ and $|C|^2$, respectively, so that tunneling of atoms between two wells is suppressed completely, i.e., CDT occurs.
IV. EFFECTS OF DRIVING AND INTERACTION ON QUASIENERGY

We have already obtained the Floquet quasienergies, Floquet states and non-Floquet states in high frequency regime. The general non-Floquet state of Eq. (10) is a coherent superposition of two or three Floquet states with two or three nonzero superposition constants $c_l$ determined by the initial setup. It is worth noting that in Eqs. (11-13) the three quasienergies appear in the time-dependent phases of the probability amplitudes and directly affect the tunneling probabilities through the phase coherence. When the quasienergies of the Floquet states are different each other, the tunneling probabilities are periodic or quasiperiodic functions of time, which describe coherent population oscillation of the system. If all the three quasienergies are the same, Eqs. (11-13) mean invariant populations with constant $|b_j(t)|$, namely the level-crossing leads to CDT and stationary-like states. Similarly for the case $E_0 = E_2 \neq E_1$, the superposition state of two Floquet states $|\psi(0)|$ and $|\psi(2)|$ describes the second kind of CDT, where the energy $E_1$ avoids the level-crossing. Therefore, the analytical results render the relation between level-crossing and CDT more transparent and the dependence of tunneling dynamics on the values of Floquet quasienergies more evident. From the analytical results of Eqs. (6-8) we know that the quasienergies are the functions of interaction strength and driving parameters. In this section, we study interplay between the external driving and interatomic interaction, through the quasienergy spectra.

For a high frequency the expression $U = n\omega + u$ with $u \ll \omega$ implies that $n$ denotes noninteracting or weakly interacting case, $n \geq 1$ correspond to strongly interacting case. Here, we will consider only the cases $n = 0$ and $n = 1$, due to the interaction $U = \omega + u$ strong enough. From Eqs. (6-8) the Floquet quasienergies as functions of the driving parameters for several different values of interacting strength $U$ and the same coupling coefficients $\gamma$ are shown as the circles in Fig. 1. We also calculate the quasienergy spectra numerically from Eq. (3) for the same parameters as those of the analytical results. The numerical results are plotted as the solid curves of Fig. 1 and both the results consistently illustrate the following interesting phenomena.

Photon resonance leads to translation of the level-crossing points. In Fig. 1(a) and Fig. 1(c) we show that for the noninteracting and resonant interaction cases with $u = 0$ and $n = 0, 1$, quasienergy spectra exhibit exact level-crossings. Three quasienergies are same at the crossing points, $E_0 = E_1 = E_2$, such that the probabilities $|b_j(t)|^2$ $(j = 0, 1, 2)$ of Eqs. (11)-(13) do not change in time, and the CDT of the first kind occurs. In the absence of interaction ($U = 0$), the inset of Fig. 1(a) shows the first level-crossing location $x_1 = \epsilon_0/\omega = 2.405$ which is the first root of equation $J_0(x) = 0$. While for the strong interaction ($U = 50$) the inset of Fig. 1(c) indicates the first level-crossing location $y_1 = \epsilon_0/\omega = 3.832$ which is the first root of $J_1(y) = 0$. The results mean that photon resonance with $U = \omega$ results in translation of the location of level-crossing point from $(\epsilon_0/\omega, U) = (x_1, 0)$ to $(y_1, \omega)$. This implies that in the strong interaction case the onset of
CDT requires a greater $\varepsilon_0/\omega$ value, because of $y_1 > x_1$.

Non-resonant interaction causes avoided crossing of partial levels. In Fig. 1(b) and Fig. 1(d) we show that for $u \neq 0$ and $n = 0$, the crossing of three levels is replaced by that of two levels at the locations $(\varepsilon_0/\omega, U) = (2.405, 2)$ and $(3.832, 52)$, respectively, namely the avoided crossing of one level appears. Such avoided level-crossing points of partial levels correspond to driving parameters obeying $J_{\pm}(\varepsilon_0/\omega) = 0$, which are the points of closest approach of energies in quasi-energy spectra. At such points the coherence between phases $E_{0f}$ and $E_{1f}$ may exist in the non-Floquet state of Eq. (10) for the nonzero $c_1$, $I = 0, 1, 2$. However, for the constants $c_1 = 0$ and $c_2 \neq 0$, the non-Floquet state becomes the superposition of two Floquet states with the same quasienergy and time-dependent phase, so the probabilities $|b'_j(t)|^2 (j = 0, 1, 2)$ of Eqs. (11-13) don’t vary in time and the second kind CDT occurs.

\begin{align*}
E_1 &= 0.6, \quad E_2 = 1.5, \quad E_3 = 2.5 \\
E_1 &= 0.3, \quad E_2 = 1.5, \quad E_3 = 2.5 \\
E_1 &= 0.3, \quad E_2 = 1.5, \quad E_3 = 2.5
\end{align*}

\begin{align*}
\varepsilon_0 &= 0.5, \quad \omega = 50, \quad u = 0, \quad (a) \ U = 0 \\
\varepsilon_0 &= 0.5, \quad \omega = 50, \quad u = 10, \quad (b) \ U = 2 \\
\varepsilon_0 &= 0.5, \quad \omega = 50, \quad u = 50, \quad (c) \ U = 50
\end{align*}

\begin{align*}
\varepsilon_0 &= 0.5, \quad \omega = 50, \quad u = 50, \quad (d) \ U = 50 + u = 52.
\end{align*}

Hereafter any quantity plotted in the figures is dimensionless, circles or circular points denote the analytical results and solid curves the numerical correspondences respectively, unless it is specially indicated.

**IV. CONTROL AND SWITCH OF QUANTUM TUNNELING**

We have known that at the points without level-crossing the bosonic population oscillates periodically or quasi-periodically, and at the level-crossing points the initial population can be kept. Now we investigate the coherent control of quantum tunneling by applying the interplay between the external field and interatomic interaction. For the level-uncrossing case we study how to coherently manipulate the tunneling rates by setting and adjusting the values of the system parameters. For the level-crossing and avoided crossing cases we perform control to the stationary-like states via CDT of different kinds. Then we apply these results to present a scheme for designing the quantum tunneling switch from a given state to different stationary-like states under CDT.

Hereafter, all the analytical results are based on the general non-Floquet state of Eq. (10), and the initial conditions $b'_0(0) = 1, b'_1(0) = b'_2(0) = 0$ correspond to Eqs. (14-16) and other initial setups are associated with Eqs. (11-13).

### A. Manipulating tunneling rates

At first, we consider the case in which the system parameters beyond level-crossing points and the bosonic population oscillates periodically. We study how to control the tunneling rates by setting and adjusting the values of interaction strength and driving parameters.

Letting $P_j(t) = |b'_j(t)|^2$ be probability of the system in the $j$-th state, from Eqs. (14-16) we plot time evolutions of the probabilities for $j = 0, 1, 2$, $\omega = 50$, $\varepsilon_0 = 2\omega = 100$ and $\gamma = 0.5$, as shown in Fig. 2. In this figure we show that all the probabilities are slowly varying functions of time comparing with the high-frequency driving. Defining tunneling time as the needed time of the paired bosons tunneling from state $|0, 2\rangle$ to $|2, 0\rangle$, a rough estimate from curves 1, 2, 3 and 4 in Fig. 2(a) and the inset of Fig. 2(b) indicates non-resonant tunneling time of the paired particles for the weak interaction $U = 2$ to be about 125 and for the strong interaction $U = 52$ to be about 20, compared to 6 with resonant interaction $U = 50$ and 14 without interaction. The results mean that photon resonance ($U = \omega$) induces coherent construction of tunneling (CCT) which leads to increase of the tunneling rate of paired particles comparing with the non-resonance case. Especially, for the weak interaction ($U = 2$), $P_1$ oscillates around 0 nearly and its values are negligible. Thus the two bosons tunnel as pair between wells such that at any time, quantum state of the system is a superposition of states $|0, 2\rangle$ and $|2, 0\rangle$, which arrives at the NOON state with probabilities $P_0 = P_2 = 1/2$ periodically.

We also obtain numerical results from Eq. (3) with the same parameters and initial conditions $a_0(0) = b'_0(0) = 1, a_1(0) = a_2(0) = b'_1(0) = b'_2(0) = 0$ as those of the analytical case, which are shown as solid lines of Fig. 2. The analytical results are in good agreement with the numerical results except the slight deviation for the case $U = 52$.

### B. Preparing stationary-like states via CDT of different kinds

When the tunneling rate is controlled to zero, CDT occurs and the stationary-like state of invariant population is prepared. Such stationary-like state may be a single Floquet state or the superposition of three or two Floquet states, which correspond to CDT of three different kinds, respectively.

**CDT at the level-crossing points.** At level-crossing point $(\varepsilon_0/\omega, U) = (2.405, 0)$ and for the high-frequency $\omega = 50$, time evolution of $P_0(t) = |b'_0(t)|^2$ of Eq. (14) is shown as curve 1 in Fig. 3(a). It is observed from this curve that $P_0(t)$ maintains the initial value $P_0(0) = 1$ so that tunneling between wells is suppressed completely and CDT of the first kind occurs. Under the same driving parameters, for the level-uncrossing points with strong interaction $U = 50$ and 52, $P_0(t)$ still oscillates between 0 and 1, as indicated by the
Such a time-dependent state possesses the same constant population of Eq. (9), non-Floquet state of Eq. (10) actually equates the first Floquet solution with the same initial conditions and system parameters. The analytical results are confirmed numerically from Eq. (3) for the same initial conditions and system parameters as those of the analytical calculations.

**CDT at the avoided level-crossing points.** For the avoided level-crossing points of partial levels, \((\epsilon_0/\omega, U) = (2.405, 2)\) and \((3.832, 52)\), from Eq. (14) the time evolutions of \(P_0(t)\) are plotted as the coincided curve 1 and curve 4 of Fig. 3 respectively. The invariant probability \(P_0(t) = 1\) means CDT of the second kind, where Eq. (10) is the superposition state of \(|\psi_0\rangle\) and \(|\psi_2\rangle\) with the same energy \(E_0 = E_2\), due to \(c_1 = \lim_{J_n \to 0} (k_n - u) \sqrt{8J_n^2 + (k_n + u)^2}/(8J_nk_n) = 0\) for the given parameters. The analytical results are confirmed numerically from Eq. (3) for the same initial conditions and system parameters as those of the analytical calculations.

**CDT for arbitrary values of the system parameters.** If the system is prepared initially in a single Floquet state of Eq. (9), probability \(P_j(t)\) of the system in any \(j\)-th Fock state \(|j, 2-j\rangle\) is a constant for arbitrary values of the system parameters in high-frequency regime, including those at the points with level-crossing and without level-crossing. Such an invariant population means CDT of the third kind. Based on such CDT, we can prepared different quasi-stationary states. As an example, we here are interested in the initial NOON state \(|\psi_0(0)\rangle = \frac{1}{\sqrt{2}}[|0, 2\rangle + (-1)^n|2, 0\rangle]\) of Eqs. (9) and (6).

Under the corresponding initial conditions \(b_0'(0) = -b_2'(0) = 1/\sqrt{2}, b_1'(0) = 0\), starting form Eqs. (11-13), time evolutions of \(P_j(t)\) are shown in Fig. 3(c) for \(\omega = 50, \epsilon_0/\omega = 2\), and \(U = 2\), where the values of \(P_0 = |A_0|^2\) and \(P_2 = |C_0|^2\) are always 1/2 and the value of \(P_1 = |B_0|^2\) is always 0. Thus the non-Floquet state of Eq. (10) actually equates the first Floquet state of Eq. (9), \(|\psi(t)\rangle = |\psi_0(t)\rangle\), due to \(c_0 = 1, c_1 = c_2 = 0\). Such a time-dependent state possesses the same constant population with a NOON state and is called the quasi-NOON state thereby. For the same parameters and the initial conditions \(a_0(0) = 1/\sqrt{2}, a_1(0) = 0, a_2(0) = -1/\sqrt{2}\), numerical results based on Eq. (3) are shown by the solid lines of Fig. 3(c), which coincide with the obtained analytical results. The results consistently demonstrate that the entangled quasi-NOON state can be prepared by setting the initial NOON state and applying the high-frequency driving.

It is worth noting that Eqs. (11-13) can fit arbitrary initial conditions of the system. Therefore, based on the different CDTs we can start from any initial state to construct the corresponding stationary-like state. As the considered example, for the initial conditions \(b_0'(0) = 1, b_2'(0) = b_0'(0) = 0\), Eqs. (11-13) are reduced to Eqs. (14-16) which are associated with the initial paired state \(|\psi(0)\rangle = |0, 2\rangle\) and the final stationary-like state \(|\psi(t)\rangle = e^{\frac{i\gamma}{\sqrt{2}}} \sin(\omega t - \epsilon_0/\omega t)|0, 2\rangle\) of Eq. (10). If the initial state is prepared as \(|\psi(0)\rangle = |2, 0\rangle\) or \(|\psi(0)\rangle = (|0, 2\rangle - |2, 0\rangle)/\sqrt{2}\), of course, from Eqs. (11-13) the
final stationary-like state reads \( \psi(t) = e^{-\frac{\omega}{2}\sin(\omega t - i\mu + \omega t)}[2, 0] \) or the quasi-NOON state \( \psi(t) = [e^{-\frac{\omega}{2}\sin(\omega t - i\mu + \omega t)}|0, 2\rangle - e^{-\frac{\omega}{2}\sin(\omega t - i\mu + \omega t)}|2, 0\rangle] / \sqrt{2} \). These results will be used to design the quantum tunneling switch in next subsection.

C. Quantum tunneling switch

From the above two subsections we know that driving field affect tunneling dynamics dramatically, so we can design the quantum tunneling switch through the high-frequency driving field that controls tunneling to open or close, as shown in Fig. 4. Let the two weakly interacting bosons occupy the paired state \(|0, 2\rangle\) initially and fix the parameters \(\omega = 50\) and \(U = 2\). We give the scheme of quantum tunneling switch between different stationary-like states by adjusting the driving strength, namely using CDT or CCT to close or open the tunneling.

Switch of paired bosons tunneling. In Fig. 4(a), one can see that in time interval \(0 \leq t \leq t_1\), we let driving strength keep the value at the avoided level-crossing point \(\epsilon_0 = 2.405\omega\), so the initially occupied state is kept, due to the CDT of the second kind in Fig. 3. At an arbitrarily given time \(t = t_1\), we change the driving strength to \(\epsilon_0 = 2\omega\) and keep this value until \(t = t_3\) at which the system transits completely from state \(|0, 2\rangle\) to state \(|2, 0\rangle\), because of the CCT in this time interval. Then we return the driving strength to the initial value such that the CDT makes the final state the stationary-like state \(e^{-\frac{i\omega}{2}\sin(\omega t - i\mu + \omega t)}[2, 0]\). Thus we transport the two bosons from well 2 to well 1, through the quantum tunneling switch. The switching time is just the tunneling time \(t_3 - t_1 \approx 125(\omega_0^{-1}) = 125/200s = 0.625s\) which is shown in the inset of Fig. 2(b). Note that in the considered case, Fig. 2(b) points out \(P_1(t) \approx 0\).

Switch from the initial state \(|2, 0\rangle\) to the final quasi-NOON state. Before \(t < t_2\), we perform the similar operations with that of Fig. 4(a). At \(t = t_2\) we return the driving strength to the initial value for CDT of the second kind that results in \(P_0(t) \approx P_2(t) \approx 1/2\) for \(t \geq t_2\), as shown in Fig. 4(b). Here we have used \(t_2\) as new initial time and values of \(P_j(t_2)\) for \(j = 0, 1, 2\) as the initial conditions of Eqs. (11-13) to plot \(P_j(t)\) for \(t > t_2\). The normalization implies \(P_j(t_2) \approx 0\), because of \(P_0(t_2) + P_2(t_2) \approx 1\). Therefore we have achieved the transition from the initial state \(|2, 0\rangle\) to the final quasi-NOON state \(\psi(t) = e^{-\frac{\omega}{2}\sin(\omega t - i\mu + \omega t)}[0, 2\rangle - e^{-\frac{\omega}{2}\sin(\omega t - i\mu + \omega t)}|2, 0\rangle] / \sqrt{2}\). The corresponding switching time is the half tunneling time \(t_2 - t_1 \approx 62.5(\omega_0^{-1}) = 0.3125s\).

In addition, making use of CDT, we can arrive at different final states with the constant probabilities \(P_0(t) = P_0(\tau), P_2(t) = P_2(\tau)\) for \(t \geq \tau\), by returning the driving strength to the avoided level-crossing point at a different fixed time \(\tau \in (t_1, t_2)\). Clearly, if we change the interaction strength from \(U = 2\) to \(U = \omega = 50\) and adjust the driving strength either \(\epsilon_0 = 3.832\omega\) or \(\epsilon_0 = 2\omega\) at appropriate times, the switching time in Fig. 4(a) can be shortened to about \(6(\omega_0^{-1}) = 0.03s\) by CCT from the resonant interaction shown in Fig. 2(a).

FIG. 4: (Color online) Plots showing the schemes of quantum tunneling switch, through the time evolutions of probabilities \(P_0(t)\) and \(P_2(t)\) from Eqs. (11-16) for \(\omega = 50, y = 0.5\). (a) In the time intervals \(0 \leq t < t_1\) and \(t \geq t_1\), the parameter values are taken at the avoided level-crossing point \((\epsilon_0/\omega, U) = (2.405, 2)\), and in the time interval \(t_1 \leq t < t_2\), only the driving strength is changed to \(\epsilon_0 = 2\omega\). (b) In the time intervals \(0 \leq t < t_1\) and \(t \geq t_2\), the parameter values are taken at the same avoided level-crossing point with that of Fig. 4(a), and the driving strength is kept as \(\epsilon_0 = 2\omega\) in \(t_1 \leq t < t_2\). Here \(t_1 = 20, t_2 = 82.5\) and \(t_3 = 145\) are associated with the approximate tunneling time \(t_1 - t_1 = 125(\omega_0^{-1})\) s and its half \(t_2 - t_1 = 125/2 = 62.5(\omega_0^{-1})\) s.

V. CONCLUSION

We have considered two repulsive bosons confined in a periodically driven double well and investigated the interplay between interparticle interaction and external field and its application to coherent control of quantum tunneling dynamics. Under high-frequency limit, we obtained Floquet quasienegies, Floquet states of invariant population. A full solution of the Schrödinger equation is constructed as a general non-Floquet state of slowly variant population, which is coherent supposition of the three Floquet states. The analytical quasienergy spectra exhibit the level-crossing for resonant interaction, the avoided crossings of partial levels for non-resonant interaction, and the translations of level-crossing points caused by the photon resonance. Exploiting the coherent non-Floquet states, we obtain time evolutions of the bosonic population, where beyond the level-crossing points, resonant interaction results in CCT which enhances tunneling rate of the paired bosons. For weak interaction, two repelling bosons tunnel through wells as pair and fall on a NOON state periodically. Three different kinds of CDT are found for the three-level crossing, avoided crossing of partial levels and arbitrary values of the system parameters, respectively. At the level-crossing points, tunneling of two bosons between wells is suppressed completely that means occurrence of the first kind CDT. Such a CDT needs larger driving strength for a stronger interaction. The avoided crossing of partial levels means the two-level crossing, so the superposition state of two Floquet states with the same energy describes CDT of the second kind. If the system is prepared in a single Floquet state initially, CDT of the third kind occurs for arbitrary values of the system parameters in high-frequency regime, including those at the level-crossing and -uncrossing points. The CDTs of different kinds lead to different stationary-like states in which the probability amplitudes are time-dependent and the corresponding probabilities are time-independent.

In order to confirm the analytical results, we make direct numerical simulations and demonstrate perfect agreement between both results. As an application of these results, we present an interesting design scheme of quantum tunneling switch, through the paired-particle tunnelings between differ-
ent stationary-like states which contain the entangled quasi-
NOON state, by using CDT or CCT to close or open the quan-
tum tunneling. Such a scheme could be useful for controlling
quantum tunneling of two bosons in a double well.

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