Seiberg-Witten theory for a non-trivial compactification from five to four dimensions

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The prepotential and spectral curve are described for a smooth interpolation between an enlarged $N = 4$ SUSY and ordinary $N = 2$ SUSY Yang-Mills theory in four dimensions, obtained by compactification from five dimensions with non-trivial (periodic and antiperiodic) boundary conditions. This system provides a new solution to the generalized WDVV equations. We show that this exhausts all possible solutions of a given functional form.

1. In their pioneering paper [1], N. Seiberg and E. Witten suggested that a hidden (duality) symmetry of the Yang-Mills dynamics could be exploited to provide an ansatz for exact low-energy effective actions for various $N = 2$ supersymmetric YM models in four dimensions. This work uncovered three fundamental objects: a Riemann surface, the moduli space of the surface and a given meromorphic one-form on the surface, the Seiberg-Witten differential $dS$. Such data had also appeared previously in the study of completely integrable systems and the Seiberg-Witten (SW) ansatz was interpreted in [2] in terms of Whitham dynamics associated with a particular finite-dimensional integrable system describing the effective dynamics along the minima in the space of the scalar fields. The Riemann surface is the spectral curve of the integrable system and the moduli of the curve are the values taken by the various commuting Hamiltonians. In the SW context the moduli are the vacuum expectation values of the various scalar fields and these have also been interpreted as the vacuum expectation values fields dynamically develop when a 5-brane is wrapped around a bare spectral curve [3]. Further, the periods $a_i$, $a_i^D$ of the Seiberg-Witten differential are the adiabatic invariants (the action variables $\oint \vec{p} d\vec{q}$) of integrable system and these satisfy the Picard-Fuchs equations on the moduli space of spectral curves. These periods define a prepotential $\mathcal{F}$ via $a_i^D = \oint B_i dS \equiv \frac{\partial F}{\partial a_i}$ and this prepotential is central to the theory.

Numerous examples of this correspondence between SW-theory and integrable systems have now been worked out for four and five-dimensional $N = 2$ supersymmetric YM models with various gauge groups and matter hypermultiplets [4–11]. Unfortunately this correspondence at present remains phenomenological and its full explanation is an important outstanding problem.
in the area. The purpose of this Letter is to add new and very important model to the
list of known examples. Indeed our example further highlights an additional feature of the
correspondence to which we now turn.

The (perturbative) prepotentials $\mathcal{F}(a_i)$ of SW-theories have been observed to satisfy the
generalized WDVV equations on the moduli space \cite{12, 13, 14}:

$$\mathcal{F}_i \mathcal{F}_k^{-1} \mathcal{F}_j = \mathcal{F}_j \mathcal{F}_k^{-1} \mathcal{F}_i, \quad \forall i,j,k; \quad (\mathcal{F}_i)_{jk} \equiv \frac{\partial^2 \mathcal{F}}{\partial a_i \partial a_j \partial a_k}. \quad (1)$$

These equations differ from the standard WDVV equations (of say two-dimensional topological
field theory) which may be cast $\mathcal{F}_i \mathcal{F}_1^{-1} \mathcal{F}_j = \mathcal{F}_j \mathcal{F}_1^{-1} \mathcal{F}_i$ (for all $i,j$). Here a particular direction
(the 1) has been singled out and one further imposes that $\mathcal{F}_1$ is a constant. Physically this
was motivated by the special status of the vacuum state and two-point correlation functions.
Actually as long as $\mathcal{F}_k$ is invertible then (1) follows from $\mathcal{F}_i \mathcal{F}_1^{-1} \mathcal{F}_j = \mathcal{F}_j \mathcal{F}_1^{-1} \mathcal{F}_i$ and so (1) may
be viewed as a projectivised form of the usual equations, putting all of the coordinates on a
similar footing. The significant part of the generalization of \cite{12, 13, 14} lies in not requiring any
of the $\mathcal{F}_k$’s to necessarily be constant. Indeed the solutions given by the SW-theories are not.
Again a fundamental understanding of the appearance of (1) is still lacking. From the integrable
system side of the correspondence these equations may be understood in terms of Whitham
dynamics \cite{15} which enables one to construct a $\tau$-function. Presumably this corresponds to a
generating functional for the correlation functions of the light fields of the theory \cite{16}. Although
we still await a deeper understanding of (1) we may enquire of its solutions. By doing this one
discovers the models of this letter.

We construct our models by first solving the generalized WDVV equations (1) for a general
class of perturbative prepotentials $\mathcal{F}_{\text{pert}}$ assuming the functional form

$$\mathcal{F} = \sum_{\alpha \in \Phi} f(\alpha \cdot a), \quad (2)$$

where the sum is over the root system $\Phi$ of a Lie algebra. This functional form is motivated
by the several existing calculations that may be found in the literature. Imposing this ansatz
reduces the WDVV equations to a single functional equation. Such functional equations arise in
many guises in the context of integrable systems \cite{17} and we are able to give a general solution,
so yielding all solutions of (1) of this given form. Our solution may be interpreted in terms of the
compactification of a five-dimensional SUSY theory to four dimensions where supersymmetry
breaking is achieved by imposing appropriate boundary conditions. The correspondence
between SUSY Yang-Mills theory and an integrable system is then most straightforwardly
provided by identifying effective charges (couplings) with period matrices,

$$T_{ij} = \frac{\partial^2 \mathcal{F}}{\partial a_i \partial a_j}. \quad (3)$$

An outline of the Letter is as follows. In section 2 we begin by describing a class of models
that may arise by compactification of a five-dimensional SUSY theory to four dimensions, giving
their expected perturbative prepotential based on known SW solutions. In section 3 we solve
the functional equations described above. For those less comfortable with functional equations
section 4 gives a physical interpretation of our equation in terms of a boson/fermion equivalence.
Section 5 develops the connection with the spectral curve and identification of the associated
integrable system. We finish with a brief conclusion. We shall restrict ourselves in this Letter
to the case of $SU(N)$ gauge group.
2. Four dimensional \( N = 2 \) supersymmetric YM theories can be obtained by softly breaking \( N = 4 \) supersymmetric YM in two different ways. The most commonly discussed way is to directly add a mass term for the adjoint hypermultiplet in four dimensions and to take the double-scaling limit \( m \to \infty, \tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} \to i\infty \), while keeping \( \Lambda^N = m^N e^{2\pi i \tau} = \text{finite} \). On the integrable system side this is described in terms of the elliptic Calogero-Moser model: here \( m \) plays the role of the coupling constant and this double-scaling limit was shown by Inosemtsev to turn the Calogero-Moser system into that of the Toda chain. (Various details of this construction may be found in [6].) An alternative construction of a four dimensional theory is to begin with a five dimensional \( N = 2 \) supersymmetric YM theory and compactify onto a circle with periodic boundary conditions. One can then break \( N = 4 \) SUSY down to \( N = 2 \) by imposing non-trivial (antiperiodic) boundary conditions on half of the fields. A smooth interpolation between the four dimensional \( N = 4 \) and \( N = 2 \) models is then provided by the change of compactification radius from \( R = \infty \) to \( R = 0 \).

The perturbative prepotential for these theories may be calculated. As explained in detail in [13], the perturbative (1-loop) contribution to the prepotential \( \mathcal{F}_{\text{pert}} \) may be constructed from the mass spectrum and the Coleman-Weinberg type formula:

\[
\mathcal{F}_{\text{pert}}(a) = \frac{1}{2} \sum_M \sum_{i<j} \pm (a_{ij} + M)^2 \log(a_{ij} + M), \quad a_{ij} \equiv a_i - a_j, \quad \sum a_i = 0. \tag{4}
\]

The choice of sign \( \pm \) depends on the nature of the supermultiplet. In the case of a massive adjoint hypermultiplet (Calogero-Moser model) we have

\[
\mathcal{F}^{\text{Cal}}_{\text{pert}}(a) = \frac{1}{2} \sum_{i<j} (a_{ij})^2 \log(a_{ij}) - \frac{1}{2} \sum_{i<j} (a_{ij} + m)^2 \log(a_{ij} + m). \tag{5}
\]

Suppose now a four dimensional model is obtained by compactification from five dimensions. Now one will encounter a whole tower of massive hypermultiplets with \( M = 2\pi k/R \) and \( M = 2\pi (k - 1/2)/R \), and

\[
\mathcal{F}^{\text{comp}}_{\text{pert}}(a) = \frac{1}{2} \left(\frac{2\pi}{R}\right)^2 \sum_{i<j} \sum_{k = -\infty}^\infty \left\{ \left( \frac{Ra_{ij}}{2\pi} + k \right)^2 \log \left( \frac{Ra_{ij}}{2\pi} + k \right) - \left( \frac{Ra_{ij}}{2\pi} + k - \frac{1}{2} \right)^2 \log \left( \frac{Ra_{ij}}{2\pi} + k - \frac{1}{2} \right) + \text{regulator} \right\}. \tag{6}
\]

This will be the form of the prepotential we obtain by seeking solutions to (4).

3. Examination of the known perturbative SW prepotentials reveals the structure (2) as a minimal component. Here we pose the following general question: for what functions \( f \) does the prepotential

\[
\mathcal{F} = \sum_{i<j} f(a_{ij}), \quad a_{ij} \equiv a_i - a_j, \quad \sum a_i = 0 \tag{7}
\]

satisfy the generalized WDVV equations? Our first result is that (4) are satisfied for the prepotential of the generic form (7) if and only if \( g(a) = \left[ \frac{\partial f}{\partial a^*} \right]^{-1} \) satisfies the functional equation

\[
g(a_{12})g(a_{34}) - g(a_{13})g(a_{24}) + g(a_{14})g(a_{23}) = 0. \tag{8}
\]

\footnote{The case of compactified five dimensional \( (N = 1 \text{ SUSY}) \rightarrow (4d \ N = 2 \text{ SUSY}) \) discussed in [8, 9] is not of the type (4), because \( \mathcal{F}_{\text{pert}}(a) \) contains cubic terms depending not only on the differences \( a_{ij} \).}
To show this we extend the analysis of [12]. Two types of entry appear when evaluating $F_i^{-1}F_jF_k^{-1} - F_j^{-1}F_kF_i^{-1}$: the first corresponds exactly with (8) establishing the necessity, while a second type corresponds to several combinations of (8). Consequently the vanishing of (8) is also a sufficient condition.

Up to the invariance $g(x) \to \lambda g(\gamma x)$ the general solution of (8) is $g(x) = x, \sin(x), \sinh(x)$. This may be argued as follows. Upon setting $a_1 = -a_4 = x$ and $a_2 = a_3 = 0$ in (8) we deduce that $g(0) = 0$. (A priori we do not know $g(0)$ to be finite.) Similarly upon setting $a_1 = a_3 = 0$, $a_2 = -a_4 = x$ and using our first result we find $g(x)$ to be an odd function. Now apply $\partial_3(\partial_1 + \partial_2)$ to (8) and set $a_1 = a_3 = a$ and $a_2 = a_4 = 0$. This yields the differential equation

$$g''(a)g(a) - g'(a)g'(a) + g'(0)g'(0) = 0.$$  

(9)

With $\phi(a) \equiv \log g(a)$ this may be reexpressed as the Liouville type equation

$$\phi''(a) = g'(0)^2e^{-2\phi(a)}$$  

(10)

which has the first integral $\phi'(a)^2 + g'(0)^2e^{-2\phi(a)} = \text{const} \equiv R^2$, and yields

$$g(a) = \frac{g'(0)}{R} \sin R(a - a_0).$$  

(11)

Now using the oddness of $g(a)$ and the invariance noted above one finds that $a_0 = 0$ and

$$g(a) = \frac{1}{R} \sin Ra.$$  

(12)

Corresponding to this solution one finds that

$$f(a) = \frac{\text{Li}_3(e^{-Ra}) - \text{Li}_3(-e^{-Ra})}{2R^2},$$  

(13)

where (for $|x| < 1$) the trilogarithm is defined by

$$\text{Li}_3(x) \equiv \sum_{k=1}^{\infty} \frac{x^k}{k^3},$$  

(14)

and this may be extended by analytic continuation [18]. Indeed, the correspondence with (8) is most readily done from the form

$$g(a) = \frac{1}{R} \sin Ra, \quad f''(a) = \frac{R}{\sin Ra}, \quad f''(a) = \log \left( \alpha \tan \left( \frac{Ra}{2} \right) \right),$$  

(15)

where $\alpha$ is a constant. Now upon using the infinite product expansion

$$f''(a) = \log \left( \alpha \tan \left( \frac{Ra}{2} \right) \right) = \log \left( \alpha \frac{Ra}{2} \frac{\prod_{k=1}^{\infty} (1 - \frac{(Ra)^2}{(2k-1)^2})}{\prod_{k=1}^{\infty} (1 - \frac{Ra^2}{(2k-1)^2})} \right)$$

$$= \log \left( \alpha \frac{Ra}{2} \prod_{k=1}^{\infty} \frac{(\frac{Ra}{2\pi} + k)(\frac{Ra}{2\pi} - k)(k - 1/2)^2}{(\frac{Ra}{2\pi} + k - 1/2)(\frac{Ra}{2\pi} - k + 1/2)k^2} \right),$$  

(16)

integration yields

$$f(a) = \frac{1}{2} a^2 \log \left( \frac{Ra}{2\pi} \right) + \left( \frac{2\pi}{R} \right)^2 \frac{1}{2} \sum_{k=1}^{\infty} \left\{ \left( \frac{Ra}{2\pi} + k \right)^2 \log \left( \frac{Ra}{2\pi} + k \right) \right.$$  

$$+ \left( \frac{Ra}{2\pi} - k \right)^2 \log \left( \frac{Ra}{2\pi} - k \right) - \left( \frac{Ra}{2\pi} + k - 1/2 \right)^2 \log \left( \frac{Ra}{2\pi} + k - 1/2 \right)$$

$$- \left( \frac{Ra}{2\pi} - k + 1/2 \right)^2 \log \left( \frac{Ra}{2\pi} - k + 1/2 \right) + \alpha_k \left( \frac{Ra}{2\pi} \right)^2 + \beta_k \left( \frac{Ra}{2\pi} \right) + \gamma_k \}.$$  

(17)
Here $\alpha_k$, $\beta_k$ and $\gamma_k$ may be chosen to make the sum convergent. This quadratic term corresponds to the regulator of (9) and by appropriate regrouping we obtain our earlier prepotential (8).

We remark that the Calogero-Moser prepotential is known not to satisfy the WDVV equations (presumably) because it has an extra modulus $\tau$. However, in the double-scaling limit (when $g(a) \to a$) it does yield a solution to the WDVV equations.

4. In this paragraph we wish to view our functional equation (8) from a rather different perspective. This functional equation may be interpreted as the consistency condition for realising a four-point correlation function in terms of both bosonic and fermionic operators. To see this, first observe that (8) is equivalent to

$$H_{1234} \equiv h(a_{12})h(a_{23})h(a_{34})h(a_{41}) + h(a_{13})h(a_{32})h(a_{24})h(a_{41}) + h(a_{14})h(a_{42})h(a_{23})h(a_{31}) = 0.$$  

(18)

Here $h(a) \equiv \frac{\partial^3 f}{\partial a^3} = g(a)^{-1}$ and we have used the oddness of this function. This is schematically depicted in Fig. 1.

![Fig. 1](image)

The solution $h(a) = 1/a$ to this equation may be viewed as expressing Wick’s theorem for the (two-dimensional) bosonic current $J(a) = \partial \phi(a)$ in the four-point correlation function $\langle J(a_1)J(a_2)J(a_3)J(a_4) \rangle \equiv C_{1234}$. Wick’s theorem here states that $C_{1234} = C_{12}C_{34} - C_{13}C_{24} + C_{14}C_{23}$, where $C_{ij}$ denotes a pair correlator. Now in a fermionic representation of the current, $J(a) = \bar{\psi}(a)\psi(a)$, Wick’s theorem gives us that $C_{1234} = C_{12}C_{34} - C_{13}C_{24} + C_{14}C_{23} + H_{1234}$, where the first three terms are depicted in Fig. 2 and the diagrams for the fourth term are just those in Fig. 1, where $h(a_{ij})$ plays the role of the fermionic propagator.
Consistency between the bosonic and fermionic realisations implies eq. (18), i.e. $H_{1234} = 0$. This consistency certainly holds on a plane, where $h_{\text{plane}}(a_{ij}) = \sqrt{da_i da_j}/a_{ij}$. It also holds on a cylinder, where $h_{\text{cyl}}(a_{ij}) = R \sqrt{da_i da_j}/\sinh Ra_{ij}$. Clearly we may obtain $h_{\text{plane}}$ from $h_{\text{cyl}}$ by taking the limit $R \to 0$. Indeed, we may also obtain $h_{\text{cyl}}$ from $h_{\text{plane}}$ (for any $R$) by a conformal transformation of the plane to the cylinder, since

$$
\frac{\sqrt{da_i da_j}}{2 \sinh a_{ij}} = \frac{\sqrt{da_i da_j}}{e^{a_{ij}} - e^{-a_{ij}}} = \frac{\sqrt{de^{2a_i} de^{2a_j}}}{e^{2a_i} - e^{2a_j}}.
$$

(19)

5. It remains to relate our solution of the WDVV equations with a spectral curve and ultimately an associated integrable system. The perturbative prepotential is typically associated with the spectral curve of the form $w = 2P_N(\lambda)$ for pure $SU(N)$ SUSY YM theories, and with $w = 2P_N(\lambda)/\sqrt{Q_{N_f}(\lambda)}$ for the theory that includes $N_f$ matter hypermultiplets (see [13] for details). For the five dimensional case of interest we have in these formulas $P_N(\lambda) = \prod_{i=1}^N (\lambda - e^{2a_i})$ and $Q_{N_f} = \prod_{i=1}^{N_f} (\lambda - e^{2m_\alpha})$, where $m_\alpha$ are the hypermultiplet masses and $\sum \alpha_i = 0$ in the $SU(N)$ case. The relevant Seiberg-Witten differential $dS$ is given by

$$
dS = \frac{1}{2} \log \lambda d\log w
$$

(20)

In order to reproduce the prepotential (7), (13) discussed in the previous sections, we stay with the same $dS$ and the same form of the spectral curve now choosing:

$$
w = (-)^N 2^{N-2} \frac{P_N(\lambda)}{P_N(-\lambda)}, \quad P_N(\lambda) = \prod_{i=1}^N (\lambda - e^{2a_i}), \quad \sum_{i=1}^N a_i = 0
$$

(21)

One can readily check that this leads to the correct result in several ways. Perhaps the simplest way is simply to note that this curve may be obtained from the curve for YM with fundamental matter upon choosing $N_f = 2N$ with masses pairwise coinciding and identification of these masses with $a_i + \frac{i\pi}{2}$. Then the result for the prepotential obtained from (3.37) of [21] coincides exactly with (7), (13).

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To simplify formulas, hereafter we put $R = 2$. 

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Fig.2
A more immediate way to check the curve (21) is to follow the line of reasoning of [13, 21]. We obtain (for $i$ puncture sphere (21), i.e. those having simple poles only at the marked points $\lambda_i = e^{2a_i} \ (\text{cf. with s.3.3 of [21]}). \ We \ obtain \ residues \ straightforwardly \ using \ an \ example \ the \ case \ i,j,k$.

Because $\text{d}\omega_i$ are canonical holomorphic differentials and $dS$ is fixed to be of the form (20). The derivatives of $dS$ with respect to moduli $a_i$ give the set of “holomorphic” differentials on the puncture sphere (21), i.e. those having simple poles only at the marked points $\lambda_i = e^{2a_i} \ (\text{cf. with s.3.3 of [21]}). \ We \ obtain \ (for \ i = 1, \ldots N - 1)\ \ d\omega_i = \frac{\lambda_{iN}d\lambda}{2(\lambda - \lambda_i)(\lambda - \lambda_N)} + \frac{\lambda_{iN}d\lambda}{2(\lambda + \lambda_i)(\lambda + \lambda_N)} = \lambda_{iN} \frac{(\lambda^2 + \lambda_i\lambda_N)}{(\lambda^2 - \lambda_i^2)(\lambda^2 - \lambda_N^2)}d\lambda,$

and we have set $\lambda_{ij} \equiv \lambda_i - \lambda_j.$ Upon writing

$$\text{d}\log w = \sum_{r=1}^N \frac{2\lambda_r d\lambda}{\lambda^2 - \lambda_r^2} = \frac{\sum_{r=1}^N 2\lambda_r \prod_{s \neq r} (\lambda^2 - \lambda_s^2)}{\prod (\lambda^2 - \lambda_r^2)} \equiv \frac{H(\lambda^2)}{\prod_{s=1}^N (\lambda^2 - \lambda_s^2)} \ (24)$$

the residue formula (22) provides

$$\mathcal{F}_{ijk} = \lambda_{iN} \lambda_{jN} \lambda_{kN} \left. \text{res}_{H(\lambda^2) = 0} \frac{\lambda (\lambda^2 + \lambda_i\lambda_N)(\lambda^2 + \lambda_j\lambda_N)(\lambda^2 + \lambda_k\lambda_N) \prod_{s \neq i, j, k}^N (\lambda^2 - \lambda_s^2)}{H(\lambda^2)(\lambda^2 - \lambda_i^2)(\lambda^2 - \lambda_j^2)(\lambda^2 - \lambda_k^2)(\lambda^2 - \lambda_N^2)^2} \right) \ (25)$$

Because $H(\lambda^2) \neq 0$ and everything is well-behaved at infinity we may exchange calculating the residues at the zeros of $H(\lambda^2)$ with the residues at the poles $\lambda = \pm \lambda_i$ and so on. Consider for example the case $i, j, k$ distinct. Then

$$\begin{align*}
\mathcal{F}_{ijk} &= \lambda_{iN} \lambda_{jN} \lambda_{kN} \left. \text{res}_{\lambda = \pm \lambda_N} \frac{\lambda (\lambda^2 + \lambda_i\lambda_N)(\lambda^2 + \lambda_j\lambda_N)(\lambda^2 + \lambda_k\lambda_N) \prod_{s \neq i, j, k}^N (\lambda^2 - \lambda_s^2)}{H(\lambda^2)(\lambda^2 - \lambda_i^2)(\lambda^2 - \lambda_j^2)(\lambda^2 - \lambda_k^2)(\lambda^2 - \lambda_N^2)^2} \right) \\
&= -\lambda_{iN} \lambda_{jN} \lambda_{kN} \left. \text{res}_{\lambda = \pm \lambda_N} \frac{\lambda (\lambda^2 + \lambda_i\lambda_N)(\lambda^2 + \lambda_j\lambda_N)(\lambda^2 + \lambda_k\lambda_N) \prod_{s \neq i, j, k}^N (\lambda^2 - \lambda_s^2)}{H(\lambda^2)(\lambda^2 - \lambda_i^2)(\lambda^2 - \lambda_j^2)(\lambda^2 - \lambda_k^2)(\lambda^2 - \lambda_N^2)^2} \right). \\
\end{align*} \ (26)$$

Upon evaluating this at the pole $\lambda = \lambda_N$ we obtain a result in agreement with the form (12). Similar calculations of $F_{iij}$ and $F_{iii}$ are also in agreement, verifying that we have the correct form of the perturbative spectral curve.

The final step is going from the perturbative spectral curve to the full spectral curve. Exactly at this step one identifies the relevant integrable system. As conjectured in [8] the integrable system relevant here is the elliptic Ruijsenaars model. The details of this step will be provided elsewhere.

\[\text{Noting} \frac{\lambda}{(\lambda^2 - \lambda_N^2)^2} = \frac{\lambda}{(\lambda - \lambda_N^2)^2(\lambda + \lambda_N)^2} = \frac{1}{4\lambda_N} \left( \frac{1}{(\lambda - \lambda_N)^2} - \frac{1}{(\lambda + \lambda_N)^2} \right)\]

we obtain residues straightforwardly using

$$\text{res}_{\lambda = \pm \lambda_N} \left. \frac{\lambda}{(\lambda^2 - \lambda_N^2)^2} \right| G(\lambda^2) = \frac{1}{4\lambda_N} \left( \frac{d}{d\lambda} G(\lambda^2)|_{\lambda_N} - \frac{d}{d\lambda} G(\lambda^2)|_{-\lambda_N} \right) = G'(\lambda_N^2).$$
6. The connections between integrable systems, SW theory and the generalized WDVV equations are striking but still await a fuller understanding. This Letter has produced another important example in this circle of ideas. Indeed we have turned the usual line of reasoning around and rather than starting with a SW theory and verifying that its perturbative prepotential satisfies the generalized WDVV equations we have found a new class of solutions for the generalized WDVV equations of a given functional form and associated to these perturbative prepotentials for a SW theory. The theory found may be interpreted in terms of a compactified five-dimensional theory. The resulting $N = 4$ four-dimensional SUSY is broken here by non-trivial boundary conditions on half of the fields and by varying the radius of the theory we may interpolate between $N = 4$ and $N = 2$ theories. A spectral curve has been found for this data and an integrable system associated with the curve. Showing here that we may reverse this circle of ideas serves to strengthen the need for a better understanding of the links between integrability and field theory.

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