QCD Corrections to $b \to se^+e^-$ Decay

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Abstract

We give a more complete calculation of $b \to se^+e^-$ decay including leading log QCD corrections from $m_{top}$ to $M_W$ in addition to corrections from $M_W$ to $m_b$. The differential decay rate is found to be slightly suppressed for a large invariant mass of the $e^+e^-$ pair; while the integrated width is slightly enhanced comparing with the results without the QCD running from $m_{top}$ to $M_W$. 
1 Introduction

The rare decay $b \rightarrow se^+e^-$ is one of the very useful channels for the study of models beyond standard model. This deeply depends on more precise calculations of this decay rate. Although some calculations show that there exist large $\alpha\tau$ resonance contributions, which interfere with short distance contributions \[1\], there is still a window in the invariant mass spectrum of the $e^+e^-$ pair for short distance contributions to be dominant \[4\]. Furthermore, window also exists for short distance to be dominant in the one lepton energy spectrum \[3\]. The QCD corrected coefficients of effective operators from $b \rightarrow se^+e^-$ are also important for the exclusive processes such as $B \rightarrow K(K^*)e^+e^-$. The decay of $b \rightarrow se^+e^-$ and its large leading log QCD corrections have already been calculated in many papers \[4, 5, 6, 7, 8\]. And also some efforts are made to give a next to leading log calculations \[3, 10\] which is estimated within 20% contribution. All these efforts make it easy for experiments to detect this channel. However, all these papers do not include the QCD running from $m_{\text{top}}$ to $M_W$. Since the top quark is found to be 2-times heavier than W gauge boson \[11\], it needs a detail calculation for the effect of the QCD running from $m_{\text{top}}$ to $M_W$. Furthermore, unlike the $b \rightarrow s\gamma$ case, here the electromagnetic penguin is numerically less important than the box diagrams and Z boson penguin, although it is much enhanced by leading-log QCD corrections. Meanwhile, the important box diagrams and Z boson penguins have no leading-log QCD corrections as a running from $M_W$ to $m_b$ \[12\]. The only leading-log QCD corrections may come from running from $m_{\text{top}}$ to $M_W$.

In the present paper, by using effective field theory formalism in standard model, we recalculate the $b \rightarrow se^+e^-$ decay including QCD running from $m_{\text{top}}$ to $M_W$, in addition to corrections from $M_W$ to $m_b$, so as to give a complete leading log results. First in the next section, we integrate out the top quark, generating a five-quark effective hamiltonian. By using the renormalization group equation, we run the effective field theory down to the W-scale, so as to give out the QCD corrections from $m_{\text{top}}$ to $M_W$. In section 3, the weak gauge bosons are integrated out at $M_W$ scale. Then we continue running the effective hamiltonian down to b-quark scale to include QCD corrections from $M_W$ to $m_b$. In section 4, the differential branching ratio is given as a function
of the invariant mass of the $e^+e^-$ pair. Our results will be useful for experiments to distinguish backgrounds like $c\bar{c}$ resonance. Section 5 is a short summary.

2 QCD Corrections from $\mu = m_{\text{top}}$ to $\mu = M_W$ Scale

At first, in the standard model Lagrangian, we integrate out the top quark, generating an effective theory, introducing dimension-5 and dimension-6 effective operators as to include effects of the absent top quark. Higher dimension operators are suppressed by a factor of $p^2/m_t^2$, where $p^2$ characterize the interesting external momentum of b quark $p^2 \sim m_b^2$. For leading order of $m_b^2/m_t^2$, dimension-6 operators are good enough to make a complete basis of operators:

\[
O_{LR}^1 = -\frac{1}{16\pi^2} m_b \bar{s}_L D^2 b_R,
\]
\[
O_{LR}^2 = \mu \epsilon/2 \frac{g_3}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} X^\alpha b_R G^\mu_{\alpha\nu},
\]
\[
O_{LR}^3 = \mu \epsilon/2 \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} b_R F_{\mu\nu},
\]
\[
Q_{LR} = \mu g_3^2 m_b \phi_+ \phi_0 \bar{s}_L b_R,
\]
\[
P_{L}^{1,A} = -\frac{i}{16\pi^2} \bar{s}_L T^A_{\mu\nu\sigma} D^\mu D^\nu D^\sigma b_L,
\]
\[
P_{L}^2 = \mu \epsilon/2 \frac{e Q_b}{16\pi^2} \bar{s}_L \gamma^\mu b_L \partial^\nu F_{\mu\nu},
\]
\[
P_{L}^4 = i \mu \epsilon/2 \frac{e Q_b}{16\pi^2} \bar{s}_L \gamma^\mu \gamma^5 D^\nu b_L,
\]
\[
R_{L}^1 = i \mu \epsilon g_3^2 (D^\sigma \phi_+) \phi_0 \bar{s}_L \partial b_L,
\]
\[
R_{L}^2 = i \mu \epsilon g_3^2 (D^\sigma \phi_+) \phi_0 \bar{s}_L \partial b_L,
\]
\[
W_{LR} = -i \mu \epsilon g_3^2 m_b W^\nu \bar{W}_- \bar{s}_L \sigma_{\mu\nu} b_R,
\]
\[
W_{L}^1 = i \mu \epsilon g_3^2 W^\nu \bar{W}_- \bar{s}_L \sigma_{\mu\nu} b_L,
\]
\[
W_{L}^2 = i \mu \epsilon g_3^2 (D^\sigma W^\nu) \bar{s}_L \gamma^\mu \gamma_\nu \gamma_\sigma b_L,
\]
\[
W_{L}^3 = i \mu \epsilon g_3^2 W^\nu \bar{s}_L \gamma^\mu \gamma_\nu \gamma_\sigma b_L,
\]
\[
W_{L}^4 = i \mu \epsilon g_3^2 W^\nu \bar{W}_- \bar{s}_L (\bar{D}^\mu \gamma_\nu + \gamma_\mu \bar{D}^\nu) b_L.
\]
\[
Z_{L}^1 = i \mu \epsilon g_3^2 \frac{e}{\cos \theta_w \sin \theta_w} Z^\mu \bar{s}_L \gamma_\mu b_L \phi^+ \phi^-,
\]
\[
Z_{L}^2 = i \mu \epsilon g_3^2 \frac{e}{\cos \theta_w \sin \theta_w} Z^\nu \bar{s}_L \gamma_\mu \gamma_\nu \gamma_\sigma b_L W^\mu W^\sigma_+,
\]
\[ Z_L^3 = \frac{i\mu^e}{16\pi^2 \cos \theta_w \sin \theta_w} M_W^2 Z_\nu \bar{\psi}_L \gamma^\nu b_L, \]
\[ O_9 = (e^2/16\pi^2)(\bar{\psi}_L \gamma^\mu b_L) \bar{\psi}_\gamma e, \]
\[ O_{10} = (e^2/16\pi^2)(\bar{\psi}_L \gamma^\mu b_L) \bar{\psi}_\gamma \gamma_5 e. \]

(1)

Here \( \bar{\psi}_L \gamma_\nu b_L \) stands for \( (\bar{\psi}_L D_\mu \gamma_\nu b_L + (D_\mu \bar{\psi}_L) \gamma_\nu b_L) \) and the covariant derivative is defined as
\[ D_\mu = \partial_\mu - i\mu^e/2g_3 X^a g_\mu^a - i\mu^e/2eQA_\mu, \]
with \( g_3 \) denoting the QCD coupling constant. The tensor \( T_{\mu\nu\sigma}^A \) appearing in \( P_{L,A}^{1,4} \) assumes the following Lorentz structure, the index \( A \) ranging from 1 to 4:
\[ T_{\mu\nu\sigma}^1 = g_{\mu\nu} \gamma^\sigma, \quad T_{\mu\nu\sigma}^2 = g_{\mu\sigma} \gamma^\nu, \]
\[ T_{\mu\nu\sigma}^3 = g_{\nu\sigma} \gamma^\mu, \quad T_{\mu\nu\sigma}^4 = -i\epsilon_{\mu\nu\sigma} \gamma^\tau \gamma_5. \]

The subscript \( L \) and \( R \) in the above formula denote left-handed and right-handed quarks, respectively. Here operators involving \( bsz\phi W \) are not included because their coefficients are suppressed by \( m_b/m_t \) and they do not mix with other operators.

Then we can write down our intermediate effective hamiltonian:
\[ \mathcal{H}_{eff} = 2\sqrt{2} G_F V_{tb} V_{ts}^* \sum_i C_i(\mu) O_i(\mu), \]

(2)

where \( V_{ij} \) represents the \( 3 \times 3 \) unitary Kobayashi-Maskawa matrix elements.

The coefficients \( C_i(\mu = m_{top}) \) can be derived through matching Green functions calculated from the standard model with that from the intermediate effective theory \([13]\). Keeping only leading order of \( p^2/m_t^2 \), the coefficients relevant to \( b \to s\gamma \) decay are already given in ref.\([13]\), here we only give the new ones:

\[ C_{Z_L^3} = \frac{1}{g_3^2} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta \right), \]
\[ C_{Z_L^2} = \frac{2}{g_3^2} 3 \sin^2 \theta_w \delta, \]
\[ C_{Z_L^4} = \frac{1 - 6\delta}{2\delta (1 - \delta)} - 2\delta + \frac{1 + 2\delta - 6\delta^2 + 12\delta^3}{2(1 - \delta)^2} \log \delta - 2 \log \delta, \]
\[ + \sin^2 \theta_w \left( \frac{1}{3} + 2\delta - \frac{1}{3} \log \delta - \frac{14}{3} \delta \log \delta \right), \]
\[ C_{O_9} = -\frac{1}{2\sin^2 \theta_w} \left( \frac{1 + \delta + \frac{1}{3} \delta^2}{1 - \delta} + \frac{\delta - \delta^2 + \frac{1}{2} \delta^3}{(1 - \delta)^2} \log \delta \right), \]
\[ C_{O_{10}} = \frac{1}{2\sin^2 \theta_w} \left( \frac{\delta + \frac{1}{3} \delta^2}{1 - \delta} + \frac{\delta - \delta^2 + \frac{1}{2} \delta^3}{(1 - \delta)^2} \log \delta \right), \]
with $\delta = M_W^2/m_t^2$. The renormalization group equation satisfied by the coefficient functions $C_i(\mu)$ is

$$\mu \frac{d}{d\mu} C_i(\mu) = \sum_j (\gamma^T)_{ij} C_j(\mu). \quad (8)$$

The anomalous dimension matrix $\gamma_{ij}$ is calculated in practice by requiring renormalization group equations for Green functions with insertions of composite operators to be satisfied order by order in perturbation theory.

Only the last five operators in equation (1) are different from that of $b \to s\gamma$ case [13]. After evaluating the loop diagrams, we get the leading order anomalous dimensions for each of the operators in our basis. Since there are so many operators, it is a very large matrix of anomalous dimensions [13]. Here we only list the part new from that of $b \to s\gamma$ case:

$$\gamma = \begin{pmatrix}
R_L^2 & W_L^2 & Z_L^1 & Z_L^2 & Z_L^3 & O_9 & O_{10} \\
R_L^2 & 0 & 0 & 0 & -\frac{1}{2} + \sin^2 \theta_w & 0 & 0 \\
W_L^2 & 0 & \frac{23}{48\pi^2} & 0 & 0 & \frac{1}{4 \sin^2 \theta_w} & \frac{-1}{4 \sin^2 \theta_w} \\
Z_L^1 & 0 & 0 & \frac{23}{48\pi^2} & 0 & 1 & 0 \\
Z_L^2 & 0 & 0 & 0 & \frac{23}{48\pi^2} & 2 & 0 \\
Z_L^3 & 0 & 0 & 0 & 0 & 0 & 0 \\
O_9 & 0 & 0 & 0 & 0 & 0 & 0 \\
O_{10} & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \quad 2g_3^2. \quad (9)$$

The solution to renormalization group equation (8) appears in obvious matrix notation as

$$C(\mu_2) = \left[ \exp \int_{\mu_1}^{\mu_2} \frac{d\mu}{\beta(\mu)} \frac{\gamma^T(\mu)}{\beta(\mu)} \right] C(\mu_1). \quad (10)$$

After inserting anomalous dimension matrix (8), we can have the coefficients of operators at $\mu = M_W^+$, where the W boson has not been integrated out.

$$C_{P_L^2}(M_W^+) = C_{P_L^2}(m_t) + \frac{81}{226}(\zeta_{113/138}^1 - 1) \left[ C_{P_L^{1,2}}(m_t) + C_{P_L^{1,4}}(m_t) \right]$$

$$- \frac{1}{2} g_3^2(m_t) \left[ C_{R_L^2}(m_t) + 2C_{W_L^2}(m_t) \right] \log \delta,$$

$$C_{P_L^4}(M_W^+) = C_{P_L^4}(m_t) + 12g_3^2(m_t)C_{W_L^1}(m_t) \log \delta,$$

$$C_{Z_i}(M_W^+) = \zeta C_{Z_i}(m_t), \quad i = 1, 2.$$
\[ C_{Z \Delta}(M_W^+) = C_{Z \Delta}(m_t) + g_3^2(m_t)C_{Z \Delta}(m_t) \log \delta + 2g_3^2(m_t)C_{Z \Delta}(m_t) \log \delta - \left( \frac{1}{2} - \sin^2 \theta_w \right)g_3^2(m_t)C_{R_L^\mu}(m_t) \log \delta - 6(1 - \sin^2 \theta_w)g_3^2(m_t)C_{W_L^\mu}(m_t) \log \delta, \]

\[ C_{O_9}(M_W^+) = C_{O_9}(m_t) + \frac{1}{4 \sin^2 \theta_w}g_3^2(m_t)C_{W_L^\mu}(m_t) \log \delta, \]

\[ C_{O_{10}}(M_W^+) = C_{O_{10}}(m_t) - \frac{1}{4 \sin^2 \theta_w}g_3^2(m_t)C_{W_L^\mu}(m_t) \log \delta, \]

(11)

with \( \zeta = \alpha_s(m_t)/\alpha_s(M_W) \). The coefficients of other operators at \( \mu = M_W^+ \) like \( P_L^{1,2}, P_L^{1,4}, W_L^2, R_L^2 \), are given at the appendix of ref.[14].

3 QCD Corrections from \( \mu = M_W \) to \( \mu = m_b \) Scale

In order to continue running the operator coefficients down to lower scales, one must integrate out the weak gauge bosons W, Z and would-be Goldstone bosons \( \phi \) at \( \mu = M_W \) scale. The matching conditions involving four-quark operators and photon, gluon penguin diagrams are the same as \( b \to s \gamma \) case[13]. We here only display the following relations between coefficient functions just below(-) and above(+) \( \mu = M_W \) scale, which is relevant to \( b \to s e^+e^- \):

\[ C_{P_L^{1,2}}(M_W^-) = C_{P_L^{1,2}}(M_W^+) - 7/9, \]

\[ C_{P_L^{1,4}}(M_W^+) = C_{P_L^{1,4}}(M_W^+) + 1, \]

\[ C_{P_L^2}(M_W^-) = C_{P_L^2}(M_W^+) - g_3^2(M_W)C_{W_L^2}(M_W^+) - 3/2, \]

\[ C_{P_L^2}(M_W^+) = C_{P_L^2}(M_W^+) + 9, \]

\[ C_{O_9}(M_W^-) = C_{O_9}(M_W^+) - \frac{g_3^2}{8 \sin^2 \theta_w}C_{W_L^2}(M_W^+) - \frac{1}{4 \sin^2 \theta_w} + \frac{4 \sin^2 \theta_w - 1}{4 \sin^2 \theta_w}C(M_W) - \frac{1}{3}D(M_W) - \frac{4}{9}, \]

\[ C_{O_{10}}(M_W^-) = C_{O_{10}}(M_W^+) + \frac{g_3^2}{8 \sin^2 \theta_w}C_{W_L^2}(M_W^+) + \frac{1}{4 \sin^2 \theta_w} + \frac{1}{4 \sin^2 \theta_w}C(M_W). \]

(12)

The first two terms in the right side of \( C_{O_9}(M_W^-) \) and \( C_{O_{10}}(M_W^-) \) arise from box diagrams with W boson couples to top quark; the third terms arise from box diagrams with W boson coupling to charm quark. The function \( C(M_W) \) arise from graphs with a Z gauge boson coupling to the \( e^+e^- \) pair, while the \( D(M_W) \) function arises from graphs with a photon couples to the \( e^+e^- \) pair. They are defined as,

\[ C(M_W) = C_{Z_L^2}(M_W^+) - g_3^2C_{Z_L^2}(M_W^+) + \left( \frac{1}{2} - \sin^2 \theta_w \right)g_3^2C_{R_L^2}(M_W^+) + 2 \cos^2 \theta_w g_3^2C_{W_L^2}(M_W^+), \]
\[ D(M_W) = C_{P_L^2}(M_W) - \frac{1}{2} C_{P_L^1 2}(M_W^-) - \frac{1}{2} C_{P_L^1 4}(M_W^-) + \frac{1}{2} C_{P_L^2}(M_W^-). \]  

(13)

In addition to these, there are new four-quark operators from integrating out the W boson\[^6, 9\]:

\[ O_1 = (\bar{c}_L \gamma^\mu b_{L\alpha})(\bar{s}_L \gamma_\mu c_{L\beta}), \]
\[ O_2 = (\bar{c}_L \gamma^\mu b_{L\alpha})(\bar{c}_{L\beta} \gamma_\mu c_{L\beta}), \]
\[ O_3 = (\bar{s}_L \gamma^\mu b_{L\alpha})(\bar{u}_{L\beta} \gamma_\mu u_{L\alpha} + \cdots + \bar{b}_{L\beta} \gamma_\mu b_{L\beta}), \]
\[ O_4 = (\bar{s}_L \gamma^\mu b_{L\beta})(\bar{u}_{L\beta} \gamma_\mu u_{L\alpha} + \cdots + \bar{b}_{L\beta} \gamma_\mu b_{L\alpha}), \]
\[ O_5 = (\bar{s}_L \gamma^\mu b_{L\alpha})(\bar{u}_{R\beta} \gamma_\mu u_{R\alpha} + \cdots + \bar{b}_{R\beta} \gamma_\mu b_{R\alpha}), \]
\[ O_6 = (\bar{s}_L \gamma^\mu b_{L\beta})(\bar{u}_{R\beta} \gamma_\mu u_{R\alpha} + \cdots + \bar{b}_{R\beta} \gamma_\mu b_{R\alpha}), \]

(14)

with coefficients

\[ C_i(M_W) = 0, \quad i = 1, 3, 4, 5, 6, \quad C_2(M_W) = -1. \]

There should also be two magnetic moment operators relevant to \( b \to s \gamma \),

\[ O_7 = \frac{e}{16\pi^2} m_b s_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \]
\[ O_8 = \frac{g_3}{16\pi^2} m_b s_L \sigma^{\mu\nu} X^a b_R C^a_{\mu\nu}, \]

(15, 16)

their coefficients at \( \mu = M_W^- \) are given in ref.\[^{[13]}\].

The operator basis now consists of 10 operators \( O_1 - O_{10} \). The effective hamiltonian appears just below the W-scale as

\[ \mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(M_W^-) O_i(M_W^-). \]

(17)

If the QCD corrections from \( m_{\text{top}} \) to \( M_W \) are ignored [by setting \( \alpha_s(m_t) = \alpha_s(M_W) \) in eqn.\(^{(12)}\)], the above results\[^{(12)}\] would reduce to the previous results \[^{[6, 9]}\] exactly, where the top quark and W bosons are integrated out together. This is a necessary consistent check.

The running of the coefficients of operators from \( \mu = M_W \) to \( \mu = m_b \) was well described in previous papers\[^{[6, 8]}\]. After running we have the coefficients of operators at \( \mu = m_b \) scale \[^{16}\].

\[ C_7(m_b) = \eta^{16/23} C_7(M_W) + \frac{8}{3}(\eta^{14/23} - \eta^{16/23}) C_8(M_W) + C_2(M_W) \sum_{i=1}^{8} h_i \eta^a_i, \]

(18)
\[ C_9(m_b) = C_9(M_W) + \frac{C_2(M_W)}{\alpha_s(m_b)} \left( -0.0938/\eta + \sum_{i=1}^{8} p_i \eta^{a_i} \right), \quad (19) \]

\[ C_{10}(m_b) = C_{10}(M_W), \quad (20) \]

with \( \eta = \alpha_s(M_W)/\alpha_s(m_b) \), \( h_i, a_i \), and \( p_i \) defined in ref.[16].

4 Results

In a spectator model, the inclusive decay \( \overline{B} \rightarrow X_se^{+}e^{-} \) is mainly contributed from the b quark decay \( b \rightarrow se^{+}e^{-} \). To leading order, the differential decay rate \( d\Gamma(\overline{B} \rightarrow X_se^{+}e^{-})/d\hat{s} \), where \( \hat{s} = (p_+^e + p_-^e)^2/m_b^2 \), is given by:

\[
\frac{1}{\Gamma(\overline{B} \rightarrow X_c\ell\nu)} \frac{\text{d}}{\text{d}\hat{s}} \Gamma(\overline{B} \rightarrow X_se^{+}e^{-}) = \frac{\alpha^2_{QED}}{4\pi^2 f(m_c/m_b)} \left( 1 - \hat{s} \right)^2 \left[ (1 + 2\hat{s}) \left( |C_{eff}^9|^2 + C_{10}^2 \right) 
+ 4 \left( 1 + \frac{2}{\hat{s}} \right) |C_7|^2 + 12C_7\text{Re}(C_{eff}^9) \right].
\]  

\[ (21) \]

Here

\[ C_{eff}^9 = C_9(m_b) + g(m_c/m_b, \hat{s})(3C_1 + 2C_2 + 3C_3 + C_4 + 3C_5 + C_6) \]

\[ - \frac{1}{2} g(1, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) - \frac{1}{2} g(0, \hat{s})(C_3 + 3C_4), \quad (22) \]

where \( g(m_c/m_b, \hat{s}) \) arises from the one-loop matrix element of the four-quark operators, which can be written as [3]

\[ g(z, \hat{s}) = -\frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} \beta - \frac{2}{9} (2 + \beta) \sqrt{|1 - \beta|} \left\{ \ln \left( \frac{\sqrt{1-\beta+1}}{\sqrt{1-\beta-1}} \right) + i\pi, \quad \beta < 1, \right. 
\]

\[ \left. 2 \arctan(1/\sqrt{\beta - 1}), \quad \beta > 1, \right. \quad (24) \]

with \( \beta = 4z^2/\hat{s} \). The factor of \( f(m_c/m_b) \) arises because of the dependence of the semileptonic decay rate on the nonnegligible ratio \( m_c/m_b \), and is given by

\[ f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x. \]

In eqn.(24) the masses of the electron and the strange quark were neglected.

The semileptonic decay \( \overline{B} \rightarrow X_c\ell\nu \) [17] is used to eliminate large uncertainties of \( m_b^5 \) in the decay width formula. The dependence on the weak mixing angles also cancels out. If we take experimental result \( Br(\overline{B} \rightarrow X_c\ell\nu) = 10.8\% \) [17], the branching ratios of \( \overline{B} \rightarrow X_se^{+}e^{-} \) is found.
Taking values as $M_W = 80.22\text{GeV}$, $m_t = 175\text{GeV}$, $m_b = 4.8\text{GeV}$ and the QCD coupling constant $\alpha_s(m_Z) = 0.117$ [12], the differential branching ratios are depicted in Fig.1 as a function of $\hat{s}$. It is shown that the QCD corrections from $m_t$ to $M_W$ slightly suppress the $b \to se^+e^-$ differential decay rate for a large invariant mass of the $e^+e^-$ pair; while enhance it for a very small invariant mass of the $e^+e^-$ pair. Since the enhancement is larger than the suppression in the invariant mass spectrum of the $e^+e^-$ pair, the total branching ratio is found to be slightly enhanced about $4\%$ comparing with the one without QCD correction from $m_{top}$ to $M_W$.

5 Conclusion

As a conclusion, we have given the full leading log QCD corrections (including QCD running from $m_{top}$ to $M_W$) to $b \to se^+e^-$ decay in the standard model.

The QCD correction from $m_t$ to $M_W$ slightly suppresses the $b \to se^+e^-$ differential decay rate for a large invariant mass of the $e^+e^-$ pair. While the integrated width is slightly enhanced comparing with that without the QCD running from $m_{top}$ to $M_W$.

Although this result is not quite different from the previous calculations, our improvement lies in reducing some theoretical uncertainties.

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Figure Captions

Fig.1 Differential branching ratios of $b \to s e^+ e^-$, as a function of $s = (p^+ + p^-)^2 / m_b^2$. The solid line denotes the result with full QCD corrections, while the dashed one corresponds to result without QCD corrections from $m_t$ to $M_W$. 
Fig. 1

\[ s = \frac{(p + p^*)^2}{m_b^2} \]

\[ (g - 01) \frac{s_p}{(\theta^+ \theta^- s^+ q) \theta_p} \]