Rethinking the $P_c(4457)^+$ as the $P_{cN}^\Lambda(4457)$ isoquartet $\bar D^*\Sigma_c$ molecule

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The nature of the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ pentaquarks is a fascinating theoretical question. Within the molecular picture their more usual interpretation is that of $I = \frac{1}{2} \bar D\Sigma_c$ and $\bar D^*\Sigma_c^*$ bound states. Here we argue in favor of interpreting the $P_c(4457)$ pentaquark as a $I = \frac{3}{2} \bar D^*\Sigma_c^*$ bound state (with spin $J = \frac{3}{2}$) instead. Owing to isospin symmetry breaking effects, with this identification the partial decay width of the $P_c(4457)^+$ into $J/\psi p$ will be of the same order of magnitude as the $P_c(4312)^+$ and $P_c(4440)^+$, in contrast with the considerably larger partial decay width in the $I = \frac{1}{2}$ scenario. This leads to a different hidden-charm molecular pentaquark spectrum, in which there are only four or five $P_{cN}^\Lambda$ bound states instead of the usual seven, which might explain why the predicted $J = \frac{3}{2}$ and $\frac{5}{2}$ $\bar D^*\Sigma_c^*$ molecular partners of the $P_c(4312)$ and $P_c(4440)$ have not been observed.

Four years ago the LHCb collaboration announced the discovery of three hidden-charmed pentaquarks [1] in the $J/\psi p$ invariant mass distribution — the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ — with masses and widths (in units of MeV)

\[
\begin{align*}
M &= 4311.9 \pm 0.7^{+6.8}_{-0.6}, \quad \Gamma = 9.8 \pm 2.7^{+3.7}_{-4.5}, \quad (1) \\
M &= 4440.3 \pm 1.3^{+4.1}_{-4.7}, \quad \Gamma = 20.6 \pm 4.9^{+8.7}_{-10.1}, \quad (2) \\
M &= 4457.3 \pm 0.6^{+4.1}_{-1.7}, \quad \Gamma = 6.4 \pm 2.0^{+5.7}_{-1.9}, \quad (3)
\end{align*}
\]

which we will refer to as $P_{c1}$, $P_{c2}$ and $P_{c3}$. Their closeness to the $D\Sigma_c$ and $\bar D^*\Sigma_c^*$ threshold, together with the existence of previous predictions [2, 3], has prompted their explanation as meson-baryon bound states with $I = \frac{1}{2}$ [4, 12], though their nature is still far from determined and there are alternative explanations too [13, 16].

Here we will consider the description of these three pentaquarks in the molecular picture. With the recent proposal of a new naming convention [17], the $P_{c1}$, $P_{c2}$ and $P_{c3}$ would be referred to as the $P_{cN}^\Lambda(4312)$, $P_{cN}^\Lambda(4440)$ and $P_{cN}^\Lambda(4457)$ within most of the molecular interpretations available, where the superscript $N$ indicates that in principle these pentaquarks are suspected to have the same quantum numbers as a nucleon. We will revisit this assumption for the case of the $P_c(4457)$, which we argue is better explained if its quantum numbers are those of the $\Delta$ isobar instead of the nucleon. Thus it might be better referred to as the $P_{cN}^\Lambda(4457)$.

The usual molecular interpretation of the three LHCb pentaquarks as $I = \frac{1}{2}$ (i.e. the octet representation of SU(3)-flavor) $D\Sigma_c$ and $\bar D^*\Sigma_c^*$ states is not entirely free of problems. As pointed out previously [3, 11], this interpretation usually implies the existence of a heavy-quark spin symmetry (HQSS) multiplet of seven molecular pentaquarks where all possible octet $D^{(*)}\Sigma_c^{(*)}$ configurations bind. However, three of the pentaquarks in this multiplet will have markedly larger partial decay widths into $J/\psi p$ than the others: the $J = \frac{3}{2}$ $\bar D^*\Sigma_c^*$ and the $J = \frac{1}{2}$ $\bar D^*\Sigma_c^*$ states [18]. Naively this implies that these last two $\bar D^*\Sigma_c^*$ states should appear as prominent peaks in the $J/\psi p$ invariant mass distribution, yet they don’t, though this could be explained if their production rates were to be smaller than those of the other pentaquarks.

It has also been noticed that within the molecular picture the interpretation of the $P_c(4457)$ as an octet $\bar D^*\Sigma_c^*$ molecule is potentially problematic. In [19] it is argued that the amplitude analysis of the $\Lambda_b \to J/\psi pK^-$ decays suggests the interpretation of $P_{c3}$ as a cusp rather than as a bound state. Ref. [20] considers the experimental constrains from $\Lambda_b$ decays and photoproduction, which suggests that while the $P_{c1}$ and $P_{c2}$ are easily explainable as $I = \frac{3}{2} \bar D\Sigma_c$ and $J = \frac{3}{2} \bar D^*\Sigma_c^*$ molecules, this is not the case for the $P_{c3}$. More recently Ref. [21] argues from a fit to the $J/\psi p$ invariant mass spectrum for the interpretation of the $P_{c3}$ either as a $D^*\Sigma_c$ cusp, a $D\Lambda_c(2595)$ triangular singularity or a $\bar D\Lambda_c(2595)$ bound state. This last interpretation has previously appeared in works about the spectroscopy of molecular pentaquarks [22, 23].

Along the present manuscript we will follow [22, 23] and consider the $P_{c1}$ and $P_{c2}$ to be octet $J = \frac{1}{2} \bar D\Sigma_c$ and $J = \frac{3}{2} \bar D^*\Sigma_c^*$ bound states. The opposite identification, namely $P_{c3}$ as a $J = \frac{1}{2} \bar D\Sigma_c$ bound state, is more difficult to reconcile with the known experimental information about this resonance and it will thus not be considered in this work. Our argument exploits isospin breaking effects to reduce the problematically large $J/\psi p$ partial decay width of the $J = \frac{1}{2} \bar D\Sigma_c$ configuration, where this mechanism only works if we are dealing with a state that is close to threshold. Previously isospin breaking effects have been discussed in the context of the possible $P_{c3} \to J/\psi \Delta$ decays of the $P_c(4457)$ as an octet pentaquark [12].

Here we will explore a molecular explanation in which the $P_{c3}(4457)$ or $P_{c3}$ is a decuplet $I = \frac{3}{2}$ $J = \frac{1}{2} \bar D^*\Sigma_c^*$ molecule. To illustrate the potential problems of the octet or $I = \frac{1}{2}$ molecular description of the $P_c(4457)$ or $P_{c3}$ pentaquark, we will begin by reviewing the decays of an $I = \frac{3}{2} \bar D^*\Sigma_c^*$ meson-baryon pair into $J/\psi N$, which can be derived from the light- and heavy-quark spin decomposition of the meson-baryon
respectively, where it can be appreciated that the $B$ discussed in [18, 20].

The r-space wave function at the origin takes the form

$$\langle \psi \rangle_{J} \propto P_{c}(0)$$

with $g$ an unknown coupling constant. From the previous it is apparent that the $J = \frac{1}{2} D^* \Sigma_c$ configuration has a particularly large relative coupling with $J/\psi N$. The partial decay width of a bound meson-baryon pair into $J/\psi N$ will be given by

$$\Gamma(P_c \rightarrow J/\psi N) = \frac{\mathcal{P}_{J/\psi N}}{\pi} \frac{\omega_{J/\psi} \omega_{N}}{m_{P_c}} g^2_{\rho_{P_c}} \langle |\psi_j(0)|^2 \rangle,$$  

where $\mathcal{P}_{J/\psi N}$ is the center-of-mass momentum of the final $J/\psi N$ state, $\omega_{J/\psi}$ and $\omega_{N}$ the energies of the final $J/\psi$ and $N$, $m_{P_c}$ the pentaquark mass, $g_{\rho_{P_c}}$ the coupling times the numerical factor from the light- and heavy-quark spin decomposition in Eqs. [18, 20]. The $c$ represents the r-space wave function of the pentaquark at the origin ($\vec{r} = 0$). If we assume that the pentaquarks can be described in a contact-range theory, the r-space wave function at the origin takes the form

$$\Psi_{P_c}(0) = N_{P_c} \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{f(q/A)}{2\mu_{P_c} B_{P_c} + q^2},$$

where $N_{P_c}$ is the normalization of the wave function, $f(x)$ a regulator function, $\rho_{P_c}$ the reduced mass of the meson-baryon system and $B_{P_c}$ the binding energy. If we use a Gaussian regulator $f(x) = e^{-x^2}$ and a cutoff $\Lambda = 0.75$ GeV (of the order of the $\rho$ meson mass), the ratio of the partial decay widths for $P_{c1}, P_{c2}$ and $P_{c3}$ will be

$$1 : 1.8 : 11.5,$$  

respectively, where it can be appreciated that the $J = \frac{1}{2} P_{c3}$ pentaquark has a $J/\psi N$ partial decay width one order of magnitude larger than the other two pentaquarks, as previously discussed in [18, 20].

Experimentally what we know are the production fractions of each of the pentaquarks, defined as

$$\mathcal{F}(P_c) = \frac{\mathcal{B}(\Lambda_{b}^{0} \rightarrow K^{-}P_{c}^{+}) \mathcal{B}(P_{c}^{+} \rightarrow J/\psi p)}{\mathcal{B}(\Lambda_{b}^{0} \rightarrow K^{-}J/\psi p)}.$$  

where $\mathcal{B}$ denotes the branching ratio of a particular decay, with $\mathcal{F}_i = 0.30^{+0.35}_{-0.11} \times 1.11^{+0.40}_{-0.34} \times 0.53^{+0.22}_{-0.21}$ for $i = 1, 2, 3$ [11]. The ratios of the production fractions are then

$$\frac{\mathcal{F}_1}{\mathcal{F}_1^{\text{exp}}} = 1 : 3.7^{+2.5}_{-2.3} : (1.8 \pm 1.2),$$

which though not directly comparable with the $J/\psi p$ partial decay width ratios of Eq. [9] should still be of the same order of magnitude. Actually, only the ratios of $\mathcal{B}(\Lambda_{b}^{0} \rightarrow K^{-}P_{c}^+) \mathcal{B}(P_{c}^{+} \rightarrow J/\psi p)$ can be obtained from the experimental decay widths and the relative partial decay widths in Eq. [5]. If we define $\mathcal{R}_i = B(\Lambda_{b}^{0} \rightarrow K^{-}P_{c}^+)/B(\Lambda_{b}^{0} \rightarrow K^{-}P_{c}^+(4312))$ with $i = 2, 3$, we can express the theoretical prediction for the production fractions as

$$\frac{\mathcal{F}_1}{\mathcal{F}_1^{\text{th}}} = 1 : (0.86^{+0.10}_{-0.05}) R_2 : (18^{+16}_{-13}) R_3,$$

which also includes the uncertainties coming from the experimental decay widths (summed in quadrature). These ratios indicate that unless the relative production rate $R_i$ for the $P_{c}(4457)$ pentaquark is considerably smaller than for the other two, we will have an inconsistency with the experimental data.

This changes if we consider the $P_{c3}$ isospin wave function

$$|P_{c}(4457)^{+}\rangle = \cos \theta_{l} |\bar{D}^{0} \Sigma_{c}^{+}\rangle + \sin \theta_{l} |D^{*} \Sigma_{c}^{++}\rangle,$$

with the isospin angle $\theta_{l} = -54.7^{\circ}$ and $35.3^{\circ}$ for a pure $I = \frac{1}{2}$ and $\frac{3}{2}$ state, respectively. If we additionally assume a $J = \frac{1}{2}$ $P_{c3}$, the decay amplitude reads

$$\langle D^{*} \Sigma_{c}(J = \frac{1}{2})|H|J/\psi p\rangle = \frac{5}{6} g \left(\frac{1}{\sqrt{3}} \cos \theta_{l} - \frac{2}{\sqrt{3}} \sin \theta_{l}\right).$$

When including the effects coming from the different masses of the $D^{*} \Sigma_{c}^{+}$ and $D^{*} \Sigma_{c}^{++}$ thresholds, we find the partial decay width ratios

$$1 : 1.8 : 1.0 \text{ or } 1 : 1.8 : 0.035,$$

for $\theta_{l} = 20.1^{\circ}$ and $35.3^{\circ}$, respectively. Alternatively, if we consider instead the ratios of the production fractions we find

$$\frac{\mathcal{F}_1}{\mathcal{F}_1^{\text{th}}} = 1 : (0.86^{+0.10}_{-0.05}) R_2 : (1.5^{+14}_{-11}) R_3 \quad \text{or}$$
$$1 : (0.86^{+0.10}_{-0.05}) R_2 : (0.054^{+0.049}_{-0.042}) R_3,$$

for $\theta_{l} = 20.1^{\circ}$ and $35.3^{\circ}$, with $R_i$ the ratios of $B(\Lambda_{b}^{0} \rightarrow K^{-}P_{c}^+)$. In Fig. [1] we illustrate the dependence of $\mathcal{F}_3/\mathcal{F}_1$ on the isospin angle $\theta_{l}$ and compare it with the experimental $\mathcal{F}_3/\mathcal{F}_1^{\text{exp}} = 1.8 \pm 1.2$ under the assumption that the production rates are identical ($R_{3} = 1$). From Fig. [1] it can be appreciated that even for identical production rates there is a wide band of values of $\theta_{l}$ that are potentially compatible with the experimental production fractions, though with a preference for values of $\theta_{l}$ closer to $I = 3/2$ than to $I = 1/2$ (unless $R_{3} \ll 1$).

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1. This value of the cutoff maximizes the momenta for which the contact-range description is valid ($k < \Lambda$, with $k$ the center-of-mass momentum of the meson-baryon system), while not being as hard as to resolve the short-range details of the meson-baryon potential ($\Lambda > m_{\rho}$ with $m_{\rho}$ the $\rho$ mass, if we assume that their short-range potential is described by vector meson exchange).

2. The quantity that we name here as production fraction is actually closely related to the fit fraction — loosely speaking, the fraction of the $X \rightarrow ABC$ decay that has a resonance $R$ as an intermediate state, i.e. $X \rightarrow A(R) \rightarrow ABC$ — though they are not equivalent. For a detailed discussion on their relation, we recommend Ref. [28].
A second argument in favor of the $P_c^0 (4457)$ assignment comes from the full decay widths of the pentaquarks, which are thought to be dominated by the meson-baryon decays, i.e. the $\bar{D}\Lambda_c$ and $\bar{D}^*\Lambda_c$ channels. The reason is (a) the very small decay branching ratios of the pentaquarks into $J/\psi p$, which in the GlueX experiment [24] have been found to be $\mathcal{B}(P_c \to J/\psi p) < 4.6, 2.3, 3.8 \%$ for $i = 1, 2, 3$, respectively, (b) the relatively small partial decay width into $\bar{D}^{(*)}\Lambda_c\pi$ of the order of 2 MeV [20] (i.e. about the width of a free $\Sigma_c$ [25]). The decay amplitudes into $\bar{D}^{(*)}\Lambda_c$ are given by [21, 26, 27]

$$\langle \bar{D}\Sigma_c | J = \frac{1}{2}, I = \frac{1}{2} | H | \bar{D}^*\Lambda_c \rangle = \sqrt{3} \, E_b, \quad (17)$$

$$\langle \bar{D}\Sigma_c | J = \frac{1}{2}, I = \frac{1}{2} | H | \bar{D}\Lambda_c \rangle = \sqrt{3} \, E_b, \quad (18)$$

$$\langle \bar{D}^\star\Sigma_c | J = \frac{1}{2}, I = \frac{1}{2} | H | \bar{D}^*\Lambda_c \rangle = -2 \, E_b, \quad (19)$$

$$\langle \bar{D}^\star\Sigma_c | J = \frac{1}{2}, I = \frac{1}{2} | H | \bar{D}\Lambda_c \rangle = \quad E_b, \quad (20)$$

which, if we determine the coupling $E_b$ from the $P_c (4312)$ decay width, will lead to

$$\Gamma_{P_c} \approx 9.8^{+4.6}_{-5.2} \text{ MeV} \quad \text{and} \quad \Gamma_{P_{c^3}} \approx 81^{+38}_{-43} \text{ MeV}. \quad (21)$$

In the case of the $P_{c^3}$ the calculated width is clearly inconsistent with the experimental one. In this case, the assumption of a predominantly $I = \frac{1}{2} \frac{3}{2}$ $P_{c^3}$ plus isospin breaking effects will lead instead to

$$\Gamma_{P_{c^3}} \approx 7.0^{+3.3}_{-3.7} \text{ MeV} \quad \text{or} \quad 0.2 \pm 0.1 \text{ MeV}, \quad (22)$$

for $\theta_1 = 20.1^\circ$ and $35.3^\circ$, respectively. This comparison is however considerably less reliable than the $J/\psi N$ one for the following reasons:

(i) First, if the pentaquarks are molecular, being as close as they are to threshold, the Breit-Wigner resonance profile might not be ideal, meaning that their actual widths might be different from the experimental ones [28]. This is in contrast with the $J/\psi N$ partial decay widths, which are an important factor in how visible the pentaquarks are in the $J/\psi N$ invariant mass distribution.

(ii) Second, the extraction of the branching ratios for the $J/\psi p$ decays from GlueX [24] depends on the JPAC model for $J/\psi$ photoproduction [29]. In view of the recent GlueX photoproduction data [30], the JPAC collaboration itself has updated its priors regarding one of the assumptions within their model — vector meson dominance in the heavy sector — which is no longer considered to be reliable [31]. As a consequence the actual $J/\psi p$ branching ratios might be very different from current estimations. In particular, if the $J/\psi p$ branching ratios happen to be much above the single digit percentage level estimated by GlueX [24], then the possible inconsistencies in the $\bar{D}\Lambda_c$ and $\bar{D}^*\Lambda_c$ decays would be a secondary concern (though in this case the $J/\psi p$ partial decay widths will represent a larger contribution of the pentaquark widths, and the problems related with them will be less dependent on the unknown production rates and thus more pressing). Yet, the fit of Ref. [21] to the $J/\psi p$ invariant spectrum, which does not rely on vector meson dominance, suggests even smaller $J/\psi p$ branching ratios (of the order of $10^{-3}$) than those of GlueX. Be it as it may, future experimental and theoretical results will be necessary to better determine these ratios.

(iii) Third, very probably the final meson-baryon states are strongly interacting, which might in turn change considerably the previous predictions for the decay widths. This is particularly true for the $P_{c (4312)} \to \bar{D}^*\Lambda_c$ decay, in which the final center-of-mass momentum of the meson-baryon system is merely 190 MeV, i.e. not that far from threshold, suggesting the possibility of a considerably larger partial decay width in the $\bar{D}^*\Lambda_c$ interaction is attractive.

(iv) Fourth, the center-of-mass momentum is very different for the three pentaquarks, from which we do not only expect a different role of the final state interaction but also of the relative importance of finite hadron size effects (e.g. form factors and regulators).

(v) Fifth, we have assumed that these decays are S-wave. Yet, it could happen that D-wave decays are as important or more than the S-wave ones.
For these reasons, even though the comparison of the $\tilde{D}^{*}\Lambda_c$ decays is interesting, they should be considered as less compelling than the $J/\psi N$ ones.

Next we will consider the $P_c(4457)$ spectroscopy. We will begin with the description of the molecular pentaquarks within the lowest order (LO) of a contact-range EFT, as has been discussed in the literature \[ [32] \]. Within this type of EFT the momentum space potentials for the $I = \frac{1}{2}$ or octet configurations are

\[
V(\tilde{D}_s\Sigma_c, J = \frac{1}{2}) = C^O_a, \\
V(\tilde{D}^*\Sigma_c, J = \frac{1}{2}, I = \frac{1}{2}) = C^O_a - \frac{1}{3}C^O_b, \\
V(\tilde{D}^*\Sigma_c, J = \frac{1}{2}, I = \frac{1}{2}) = C^O_a + \frac{2}{3}C^O_b,
\]

with $C^O_a$ and $C^O_b$ the two octet couplings. These potentials are singular (they correspond to a Dirac-delta in coordinate space) and require regularization, for which we choose here a separable regulator of the type

\[
\langle p' | V_c | p \rangle = C(\Lambda) \frac{f(L)}{\Lambda} \frac{f(L')}{\Lambda},
\]

where $C(\Lambda)$ is the coupling (which runs with the cutoff) and $f(x)$ a regulator function, for which we use a Gaussian: $f(x) = e^{-x^2}$. Then, to find the poles, we plug this potential into the Lippmann-Schwinger equation

\[
1 + 2\mu C(\Lambda) \int \frac{d^3 \tilde{q}}{(2\pi)^3} f^2(\tilde{q}) M_{th} + \frac{M^2}{\tilde{q}^2} - M_P = 0,
\]

with $\mu$ the reduced mass, $M_{th}$ the threshold mass and $M_P$ the mass of the molecular pentaquark. The coupling $C(\Lambda)$ is renormalized from the condition of reproducing $M_P$ for a given molecular pentaquark candidate.

The octet molecular pentaquark potential contains only two parameters but there are three pentaquarks, which means that we can check the consistency of the molecular hypothesis by using two pentaquarks as input and predicting the third one. If we assume that the $P_{c2}$ and $P_{c3}$ are spin $\frac{3}{2}$ and $\frac{1}{2}$, respectively (in agreement with the arguments of \[ [33] \] and previous explorations in the molecular model \[ [10, 11, 34, 35] \]), there are three possible ways in which to calibrate the $C^O_a$ and $C^O_b$ couplings: (i) from the $P_{c1}$ and $P_{c2}$ pentaquarks, (ii) from the $P_{c2}$ and $P_{c3}$ and (iii) from the $P_{c1}$ and $P_{c3}$. We will refer to them as set (i), (ii) and (iii).

For the uncertainties, we will consider the following two error sources and sum them in quadrature. The first is varying the cutoff around the $\Lambda = 0.75$ GeV central value, for which we choose the $(0.5 - 1.0)$ GeV window \[ [1] \]. The second is the truncation error of the couplings: $C^O_a$ and $C^O_b$ are leading order couplings, yet the couplings that we know ($C(P_{c1})$ with $i = 1, 2, 3$) contain all subleading order corrections, which are undetermined by the available experimental data. Thus the LO couplings might differ from the full ones by a relative error of $\gamma/m_P$ (the EFT truncation error), with $\gamma$ the wave number of the pentaquark from which the coupling is obtained and $m_P$ the rho meson mass (we remind that $\gamma = \sqrt{2}\mu(M_{th} - M_P)$, with $\mu$, $M_{th}$ and $M_P$ defined below Eq. \[ (27) \]). For instance, if $C^O$ is the LO coupling and $C(P_{c1})$ the coupling reproducing the $P_{c1}$ pentaquark, the uncertainty will be determined from the condition $C^O_a (1 + O(\gamma/m_P)) = C(P_{c1})$. With this for each set we get the couplings (i) $C^O_a = -1.19^{+0.17}_{-0.26}$ $(-2.16 - 0.80)$ fm$^2$ and $C^O_b = -0.38^{+0.08}_{-0.15}$ $(-1.07 - 0.18)$ fm$^2$; (ii) $C^O_a = -1.30^{+0.10}_{-0.19}$ $(-2.52 - 0.85)$ fm$^2$ and $C^O_b = -0.207^{+0.048}_{-0.082}$ $(-0.543 - 0.107)$ fm$^2$; (iii) $C^O_a = -1.19^{+0.17}_{-0.26}$ $(-2.16 - 0.80)$ fm$^2$ and $C^O_b = -1.120^{+0.014}_{-0.018}$ $(-0.279 - 0.068)$ fm$^2$. The first value is the $\Lambda = 0.75$ GeV coupling and its expected EFT truncation error, while the values in parentheses represent their determination for $\Lambda = 0.5 - 1.0$ GeV. These uncertainties can then be propagated into the predicted mass of the third pentaquark.

We show the results in Table \[ I \] where it can be appreciated that in general this works well: the hypothesis that the three pentaquarks are molecular is self-consistent. However, if the input states are the $P_{c3}$ and $P_{c2}$, the $P_c(4457)$ predicted as a near threshold virtual (instead of bound) state. This by itself is not a serious issue, as a virtual state close to threshold could still be detected in experiments. But it will be interesting to explore the possibility that the $P_c(4457)$ is an $\frac{1}{2}$ state.

For this last scenario — $P_c(4312)$ and $P_c(4457)$ as octets and $P_c(4457)$ as a decuplet — we will explicitly include isospin breaking effects for the $P_c(4457)$, in which case the potential in the $D^{*}\Sigma^{-}_c - D^{*}S^{-}_c$ basis will read

\[
V(\tilde{D}^*\Sigma_c, J = \frac{1}{2}) = \begin{pmatrix}
\frac{1}{2} C^O + \frac{3}{2} C^D \\
\frac{1}{2} C^O - \frac{3}{2} C^D
\end{pmatrix} + \begin{pmatrix}
\frac{1}{2} C^O - C^D \\
\frac{1}{2} C^O + C^D
\end{pmatrix},
\]

where $C^O$ and $C^D$ are the octet and decuplet potential for this configuration, i.e. $C^O = C^O_a - \frac{1}{2} C^O_b$ and $C^D = C^D_a - \frac{1}{2} C^D_b$. If we calibrate $C^D$ as to reproduce the location of the $P_c(4457)$, we obtain $C^D = -0.97^{+0.11}_{-0.14}$ $(-1.65 - 0.70)$ fm$^2$ and $\theta_1 = (26.6^{+7.6}_{-8.4})^{\circ}$ $((30.7 - 23.3)^{\circ})$, which is relatively close to a pure $I = \frac{1}{2}$ state and where the errors come from propagating the uncertainties of $C^O_a$ and $C^D_a$ into $C^D$ and $\theta_1$. From this angle,
TABLE II. Predictions for the octet \( (I = \frac{1}{2}) \) and decuplet \( (I = \frac{1}{2}) \) molecular pentaquark spectrum from the conditions of (a) reproducing the \( P_c(4312) \) and \( P_c(4440) \) as \( I = \frac{1}{2} D^* \Sigma \) and \( J = \frac{1}{2} D^* \Sigma \) bound states, (b) the \( P_c(4457) \) as a \( I = \frac{1}{2}, J = \frac{1}{2} D^* \Sigma \) molecule and (c) the phenomenologically inspired relation \( C_b^D = \pm 2C_b^D \). “Molecule” shows the meson-baryon configuration (when two configurations are shown it indicates that we include explicit isospin breaking effects, which we only do for pentaquarks predicted really close to threshold), \( I(J^P) \) is the isospin, spin and parity of the state and \( M \) is the mass (in MeV). For molecules in which we include isospin breaking the consideration as it is \( I = \frac{1}{2} \) and \( I = \frac{1}{2} \) is that the central value of the isospin angle is \( \theta_1 < 0 \) and \( \theta_1 > 0 \), respectively (see Eq. (13)). However, owing to the large uncertainties in \( \theta_1 \), the opposite isospin identification cannot be completely discarded for the \( J = \frac{1}{2} D^* \Sigma \) and \( J = \frac{1}{2} D^* \Sigma \) molecules.

The relative partial decay widths into \( J/\psi p \) of the three pentaquarks will be

\[
1 : 1.83 (1.84 - 1.82) : 0.44_{-0.41}^{+0.64} (0.26 - 0.66), \quad (29)
\]

to be compared with Eq. (9) for the octet hypothesis. The previous will translate into a production fraction ratio of

\[
\frac{\mathcal{T}_I}{\mathcal{T}_{I_{th}}} = 1 : 0.87^{+1.10}_{-0.53} (0.88 - 0.87) \mathcal{R}_2
\]
\[
: 0.67^{+5.69}_{-6.67} (0.40 - 1.01) \mathcal{R}_3, \quad (30)
\]

which also include the uncertainties in the experimental decay widths. These values are potentially more compatible with the experimental production fractions of Eq. (11) without requiring unnaturally large or small \( \mathcal{R}_2 \) and \( \mathcal{R}_3 \). Meanwhile the decay width into \( D^{*+}\Lambda_c \) is estimated to be \( 3.1^{+2.8}_{-1.1} (1.2 - 6.1) \) MeV, which is of the correct order of magnitude.

From the previous it is in principle not possible to derive the spectrum of the decuplet pentaquarks, as we do not know the specific values of \( C_a^D \) and \( C_b^D \). However, it is possible to make an educated guess on the basis that the spin-spin contact-range coupling \( C_b \) represents the spin-spin dependence coming from the short range vector meson exchange potential, which is in turn given by \( (11) \)

\[
V_{VY}(r) = \left(1 + \mathbf{r} \cdot \mathbf{T}_2^* \right) f_{V1} f_{V2} \frac{m_v^2}{2M^2} \frac{e^{-m_Y r}}{4\pi r}, \quad (31)
\]

with \( r \) the distance, \( f_{V1} \) and \( f_{V2} \) coupling constants, \( m_Y \) the vector meson mass and \( M \) a scaling mass (e.g. the nucleon mass). The isospin factor in front of the potential takes the values \( (1 + \mathbf{r} \cdot \mathbf{T}_2^* \) = \(-1, 2 \) for \( I = \frac{1}{2} \) and \( \frac{3}{2} \), respectively. That is, for the short-range vector exchange potential from which \( C_b \) is derived \( V_{bY}^D = -2V_{bY}^O \). We might simply assume this to be true for the contact coupling \( C_b \) as well, in which case \( C_b^D = -2C_b^O \pm 30\% \), with \( \pm 30\% \) the expected relative error for this relation, which we will set to be \( 30\% \). With this assumption we end up with \( C_a^O = 0.65_{-0.40}^{+0.52} (1.20 - (0.19)) \) fm\(^2\) and \( C_b^O = 0.76_{-0.28}^{+0.38} (2.14 - 0.37) \) fm\(^2\).

From this we are able to derive the octet and decuplet pentaquark spectra, which we show in Table II. It is interesting to notice that we predict seven pentaquarks bound below their respective thresholds, in agreement with most works implementing HQSS constraints [5, 8, 9, 18, 26, 35, 37]. But the quantum numbers are not the same, as the previous works usually predict seven octet pentaquarks (while here there are five octet and two decuplet bound pentaquarks, plus a few virtual states and resonances). In this regard our octet spectrum is more similar to the one we originally proposed in [32] than to our later predictions [8, 10, 38] (though predictions of a \( J = \frac{1}{2}, I = \frac{1}{2} D^* \Sigma_c \) bound state have appeared in [11, 39]).

The two most interesting configurations are probably the \( I = \frac{1}{2}, J = \frac{3}{2} D^* \Sigma_c \) pentaquarks, for which their isospin quantum numbers are not necessarily clear. For the central values of the couplings the lower (higher) mass state will be the octet (decuplet) one, yet it is within the uncertainties of the theory that the identification might be the opposite one. Indeed, the calculation of the isospin angle for the lower mass state is \( \theta_1 = -48.0^{+9.9}_{-10.0} \) \(^\circ\) \((-52.1 - 47.1)\), which is compatible with \( \theta_1 < 0 \) and the octet identification, though the large uncertainties do not allow to rule out \( \theta_1 > 0 \). The relative \( J/\psi p \) partial decay width relative to the \( P_c(4312) \) is 3.6^{+0.2}_{-0.1} (3.5 - 3.6), but it could be considerably smaller if \( \theta_1 \) turns out to be positive. The non-observation of this state in the \( J/\psi p \) invariant mass suggests \( \theta_1 > 0 \).

To summarize, we have considered the possibility that the \( P_c(4457) \) is a decuplet \( I = \frac{1}{2} D^* \Sigma_c \) bound state, instead of the more usual octet \( I = \frac{1}{2} \) interpretation. There are a few advantages in explaining the \( P_c(4457) \) as a decuplet: first, it generates a partial decay width into \( J/\psi p \) of the same order as the \( P_c(4312) \) and \( P_c(4440) \) (instead of one order of magnitude larger if it is an octet), which might in turn be in line with their experimental production fractions. Second, it implies a smaller \( D^* \Lambda_c \) decay width, which in the octet interpretation
turns out to be too large when compared with the experimental decay width of the $P_c(4457)$. Third, if the $P_c(4457)$ is a decuplet then the predicted spectrum of the molecular pentaquarks will be different: seven bound molecular pentaquarks are predicted in total, but instead of all of them being octets, there will be a mix of octet and decuplet states. This might be more compatible with the non-observation of the $J = \frac{1}{2}^+$, $\frac{3}{2}^+$ $D^*\Sigma^*$ pentaquarks in the $J/\psi\phi$ invariant mass, despite the expectation of them having relatively large $J/\psi\phi$ partial decay widths. The evidence for a decuplet identification of the $P_c(4457)$ is not conclusive though, as there are a few open issues (production rates of the pentaquarks, resonance profiles, branching ratios into $J/\psi\phi$, etc.) that could tip the balance towards a different interpretation.

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