Collective stimulated Brillouin scatter

Alexander O. Korotkevich1, Pavel M. Lushnikov1 and Harvey A. Rose2, 3

1 Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA
2 New Mexico Consortium, Los Alamos, New Mexico 87545, USA
3 Theoretical Division, Los Alamos National Laboratory, MS-B213, Los Alamos, New Mexico, 87545

(Dated: January 20, 2013)

We develop a statistical theory of stimulated Brillouin backscatter (BSBS) of a spatially and
temporally partially incoherent laser beam for laser fusion relevant plasma. We find a new collective
regime of BSBS which has a much larger threshold than the classical threshold of a coherent beam in
long-scale-length laser fusion plasma. We identify two contributions to BSBS convective instability
increment. The first is collective with intensity threshold independent of the laser correlation time
and controlled by diffraction. The second is independent of diffraction, it grows with increase of
the correlation time and does not have an intensity threshold. The instability threshold is inside
the typical parameter region of National Ignition Facility (NIF). We also find that the bandwidth
of KrF-laser-based fusion systems would be large enough to allow additional suppression of BSBS.

PACS numbers: 52.38.-r 52.38.Bv

Inertial confinement fusion (ICF) experiments require propagation of intense laser light through underdense
plasma subject to laser-plasma instabilities which can be deleterious for achievement of thermonuclear target
ignition because they can cause the loss of target symmetry, energy and hot electron production [1]. Among
laser-plasma instabilities, backward stimulated Brillouin
metry, energy and hot electron production [1]. Among
laser-plasma instabilities, backward stimulated Brillouin
backscatter (BSBS) has long been considered a serious
danger because the damping threshold of BSBS of coher-
ent laser-plasma instabilities, backward stimulated Brillouin
backscatter (BSBS) has long been considered a serious
danger because the damping threshold of BSBS of coher-
ent laser beams is typically several order of magnitude
less than the required laser intensity \( \sim 10^{15} \text{ W/cm}^2 \) for
ICF. BSBS may result in laser energy retracing its path
to the laser optical system, possibly damaging laser com-
ponents [1, 2].

Theory of laser-plasma interaction (LPI) instabilities
is well developed for coherent laser beam [3]. However,
ICF laser beams are not coherent because temporal and
spatial beam smoothing techniques are currently used
to produce laser beams with short enough correlation
time, \( T_c \), and lengths to suppress speckle self-focusing.
The laser intensity forms a speckle field - a random in
space distribution of intensity with transverse correla-
tion length \( L_s \approx F \lambda_0 \) and longitudinal correlation length
(speckle length) \( L_{\text{speckle}} \approx 7F^2 \lambda_0 \), where \( F \) is the optic
f/# and \( \lambda_0 = 2\pi/k_0 \) is the wavelength (see e.g. [1, 4]).

There is a long history of study of amplification in ran-
don media (see e.g. [1, 4] and references there in). For
small laser beam correlation time \( T_c \), the spatial insta-
bility increment is given by a Random Phase Approxima-
tion (RPA). Beam smoothing for ICF typically has \( T_c \)
much larger than the for the regime of RPA applicability.
There are few examples in which the implications of laser
beam spatial and temporal incoherence have been ana-
lyzed for such larger \( T_c \). One example is forward stim-
ulated Brillouin scattering (FSBS). Although FSBS for
a strictly coherent laser beam is a classic linear theory,
we have obtained [8, 9] its dispersion relation for laser
beam correlation time \( T_c \) too large for RPA relevance,
pling of $E$ and $B$ to plasma density fluctuations gives

$$R_{EB}^{-1}E = \left[ i\left( c^{-1}\partial_t + \partial_z \right) + \frac{1}{2k_0} \nabla^2 \right] E = \frac{k_0}{4} n_e \sigma B, \quad (2)$$

$$R_{BB}^{-1}B = \left[ i\left( c^{-1}\partial_t - \partial_z \right) + \frac{1}{2k_0} \nabla^2 \right] B = \frac{k_0}{4} n_e \sigma^* E, \quad (3)$$

$$\nabla = (\partial_x, \partial_y), \text{ and } \sigma \text{ is described by the acoustic wave equation coupled to the ponderomotive force } \propto E^2 \text{ which results in the envelope equation}$$

$$R_{\sigma\sigma}^{-1}\sigma^* = \left[ i\left( c^{-1}\partial_t + 2\nu_ia_k0 + \partial_z \right) - (4k_0)^{-1}\nabla^2 \right] \sigma^* = -2k_0 E^* B. \quad (4)$$

The response of the slowly varying part of $\delta n_e$ to the slowly varying part of the ponderomotive force, proportional to $|E|^2 + |B|^2$, responsible for self-focusing, is neglected. $\nu_ia = \nu_i/2k_0c_s$ is the scaled acoustic Landau damping coefficient. $E$ and $B$ are in thermal units (see e.g. [8]).

Assume that laser beam was made partially incoherent through induced spacial incoherence beam smoothing [12], which defines stochastic boundary conditions at $z = 0$ for the spacial Fourier transform (over $r$) components $E(k)$, of laser beam amplitude $[8]$:

$$\hat{E}(k, z = 0, t) = |E_k| \exp[i\phi_k(t)], \quad \langle \exp[i\phi_k(t) - \phi_k(t')] \rangle = \delta_{kk'} \exp(-|t-t'|/\Delta t), \quad |E_k| = \text{const}, \quad k < k_m; \quad E_k = 0, \quad k > k_m. \quad (5)$$

chosen as the idealized "top hat" model of NIF optics [10]. Here $k_m \approx k_0/(2F)$ and the average intensity, $\bar{I} \equiv \langle |E|^2 \rangle \approx \bar{I}$ determines the constant.

In linear approximation, assuming $|B| \ll |E|$ so that only the laser beam is BSBS unstable, we can neglect right hand side (r.h.s.) of Eq. (2). The resulting linear equation with top hat boundary condition (5) has the exact solution as decomposition of $E$ into Fourier series, $E(r, z, t) = \sum_j E_{kj}$, with $E_{kj} \propto \exp[i(\phi_{kj}(t - z/c) + k_j \cdot r - k_j^2 z/2k_0)]$.

Figures show the increment $\kappa$, of the spatial growth of backscattered light intensity $\langle |B|^2 \rangle \propto e^{-2k_z z}$ as a function of the rescaled correlation time $\tilde{T} \equiv T_0 k_0 c_s / 4 F^2$ (note that definition is different by a factor $1/2F$ from the definition used for FSBS [3, 4]) obtained from the numerical solution of the linearized equations (2)-(4) using operator splitting method along the characteristics of $E$ and $B$. Here and below we use dimensionless units with $k_0/k_m$ as the unit in $z$ direction, $k_0/k_m c_s$ is the time unit and $\mu \equiv 2\nu_ia k_0^2/k_m^2$. Also $\langle \ldots \rangle$ means averaging over the statistics of laser beam fluctuations [5] and $\bar{I}$ is the scaled dimensionless laser intensity defined as $\bar{I} = \frac{4k_0^2}{\nu_ia} I$. Figure corresponds to the 3 + 1D simulations (three spatial coordinates and $t$) with the boundary and initial conditions (5) in the limit $c \rightarrow \infty$ (i.e., setting $c^{-1}$ terms in (2)-(4) to be zero. Figure shows the result of $2 + 1D$ simulations (only 1 transverse spatial variable is taken into account) for the modified boundary condition compare with the last line in (5) as $|E_k| = k^{1/2} \text{const}, \quad k < k_m; \quad E_k = 0, \quad k > k_m$ which is chosen to mimic the extra factor $k$ in the integral over transverse direction of the full $3 + 1D$ problem. In that case $c/c_s \approx 500$. E.g. for $\bar{T}_c = 0.1$ we typically use 256 transverse Fourier modes and a discrete steps $\Delta z = 0.15$ in dimensionless units with the total length of the system $L_z = 100$ and a time step $\Delta t = \Delta z / c$. For each simulation we typically have to wait $\sim 10^6$ time steps to achieve a statistical steady state and then average over next $\sim 10^6$ time steps to find $\kappa_i$.

We now relate $\kappa_i$ to the instability increments for $\langle B \rangle$ and $\langle \sigma^* \rangle$ (we designate them $\kappa_B$ and $\kappa_{\sigma}$, respectively).
In general, growth rates of mean amplitudes only give a lower bound to \( \kappa_s \). First we look for \( \kappa_s \). Eq. (4) is linear in \( B \) and \( E \) which implies that \( B \) can be decomposed into \( B = \sum_j B_{k_j} \). We approximate r.h.s. of (4) as \( E^*B \simeq \sum_j E_{k_j}B_{k_j} \), so that

\[
R_{\sigma}^{-1}\sigma^* = -2k_0 \sum_j E_{k_j}^*B_{k_j}, \tag{6}
\]

which means that we neglect off-diagonal terms \( E_{k_j}^*B_{k_j} \), \( j \neq j' \). Since speckles of laser field arise from interference of different Fourier modes, \( j \neq j' \), we associate the off-diagonal terms with speckle contribution to BSBS [4, 12, 14]. The neglect of off-diagonal terms requires that during time \( T_e \) light travels much further than a speckle length, \( L_{\text{speckle}} \ll cT_e \) and that \( T_e \ll t_{\text{sat}} \), where \( t_{\text{sat}} \) is the characteristic time scale at which BSBS convective gain saturates at each speckle [13].

Eqs. (3) and (5) result in the closed expression

\[
R_{\sigma}^{-1}(\sigma^*) = -(k_0/2)\langle n_i/n_e\rangle \langle E^*R_{BB}B^*E \rangle \text{ which has the same form as the Bouret approximation [3].}
\]

We look for the solution of that expression in exponential form \( B_j, \sigma^* \propto e^{i(\kappa z+k r-w t)} \), then the exponential time dependence in (5) allows to carry integrations in that expression explicitly to arrive at the following relation in dimensionless units

\[
-i \omega + \mu + i \kappa - (i/4)k^2
= 8iF_i \frac{n_e}{n_i} \sum_j \frac{\omega \mu c}{c^2} + \kappa - k^2 + \frac{k^2}{2} - \frac{k^2}{2} c \kappa - \kappa - 2i \frac{c^2}{c^2} \frac{c}{T_e}. \tag{7}
\]

where \( 1/k_m \) is the transverse unit of length and vectors \( k_j \) span the entire top hat (5), i.e. \( I = \sum_j |E_{k_j}|^2 \).

In the continuous limit \( N \to \infty \), sum in (7) is replaced by integral which gives for the most unstable mode \( k = 0 \):

\[
-i \omega + \mu + i \kappa + i \frac{\mu}{4} \ln \frac{1 - \kappa - \omega \mu c}{c^2} - 2i \frac{c^2}{c^2} \frac{c}{T_c} = 0. \tag{8}
\]

The relation (8) supports the convective instability with the increment \( \kappa_\sigma \equiv Im(\kappa) > 0 \) only for \( I > I_{\text{convthresh}} \), where \( I_{\text{convthresh}} \) is the convective CBSBS threshold given by

\[
I_{\text{conveshrsh}} = \frac{4F^2 n_e}{\nu_{ia}} I_{\text{convresh}} = 4/\pi. \tag{9}
\]

In the limit \( c/c_e \to \infty \), the increment \( \kappa_\sigma \) is independent of \( T_e \) which suggests that we refer to it as the collective instability branch. For finite but small \( c_e/c \ll 1 \) and \( I > I_{\text{convthresh}} \) there is sharp transition of \( \kappa_\sigma \) as a function of \( T_e \) from 0 for \( T_e \to 0 \) to \( T_e \)-independent value of \( \kappa_\sigma \).

That value can be obtained analytically from (8) for \( I \) just above the threshold as follows: \( \kappa_i = i(\mu/4)(I - I_{\text{convresh}})/(\mu I - 1) \).

The increment \( \kappa_B \) is obtained in a similar way by statistical averaging of equation (3) for \( \langle B \rangle \) with \( \sigma^* \) from equation (1) which gives

\[
-i \omega c + i \kappa + i \frac{\mu}{4} \frac{1}{\kappa - \omega - i \mu - i \frac{c^2}{c^2}} = 0. \tag{10}
\]

Here we neglected the contribution to \( \kappa_B \equiv Im(\kappa) \) from diffraction which gives negligible correction. Equation (10) does not have a convective threshold (provided we neglect here light wave damping) while \( \kappa_B \) has near-linear dependence on \( T_e : \kappa_B \propto \mu TT_e/4 \) for \( T_e < 1/\mu \) which is typical for RPA results. It suggests that we refer \( \kappa_B \) as the RPA-like branch of instability.

We choose \( \omega = 0.5 \) in (5) and \( \omega = 0 \) in (10) to maximize \( \kappa_\sigma \) and \( \kappa_B \), respectively. Equation (5) also predicts absolute instability for \( I > \mu + 3 \mu^{-1} + O(\mu^{-3}) + O(T_e^{-1} \epsilon_c/c) \), which is slightly above the coherent absolute threshold \( I = \mu \) but here we emphasize the convective regime. Figures 1 and 1b show that the analytical expression \( \kappa_B + \kappa_\sigma \) is a reasonable good approximation for numerical value of \( \kappa_\sigma \) above the convective threshold (9) for \( T_e \lesssim 0.1 \) which is the main result of this Letter. Below this threshold the analytical and numerical results are in qualitative agreement at best but in that case we replace \( \kappa_B + \kappa_\sigma \) by \( \kappa_B \) because \( \kappa_\sigma < 0 \) in that case.

The qualitative explanation why \( \kappa_B + \kappa_\sigma \) is a surprisingly good approximation to \( \kappa_i \) is based on the following argument. First imagine that \( B \) propagates linearly and not coupled to the fluctuations of \( \sigma^* \), so its source is \( \sigma^*E \to \langle \sigma^* \rangle E \) in r.h.s of (3). If \( \langle \sigma^* \rangle \propto e^{i \sigma z} \) grows slowly with \( z \) (i.e. if \( \langle \sigma^* \rangle \) changes a little over the speckle length \( L_{\text{speckle}} \) and time \( T_e \)), then so will \( \langle |B|^2 \rangle \) at the rate \( 2 \kappa_\sigma \). But if the total linear response \( R_{BB}^t (R_{BB}^o) \) is the renormalization of bare response \( R_{BB} \) due to the coupling in r.h.s of (3) is unstable then its growth rate gets added to \( \kappa_\sigma \) in the determination of \( \langle |B|^2 \rangle \) since in all theories which allow factorization of 4-point function into product of 2-point functions, \( \langle B(1) B^*(2) \rangle = R_{BB}^t(B(1),1')S(1,1')R_{BB}^o B^*(2,2) \) Here \( S(1,2) \) \( \equiv \langle \sigma^*(1) \sigma(2) \rangle \langle E(1) E^*(2) \rangle \sim \langle \sigma^*(1) \sigma(2) \rangle \langle E(1) E^*(2) \rangle \) and ”1”, ”2” etc. mean of all spatial and temporal arguments.

The applicability conditions of the Bouret approximation used in derivation of (8) and (10) in the dimensionless units are

\[
\Delta \omega_B \Delta \omega_\sigma \gg 1/2. \tag{11}
\]

and \( \Delta \omega_B \gg (c/c_e)|\kappa_B| \) as well as \( \Delta \omega_\sigma \gg \mu \). Here \( \gamma_0 \) is the temporal growth rate of the spatially homogenous solution which is given by \( \gamma_0^2 = (1/4)(c/c_e) \mu I \). Also \( \Delta \omega_\sigma = 1/T_e \) is the bandwidth for \( \sigma \) and \( \Delta \omega_B \) is the effective bandwidth for \( B \). \( \Delta \omega_B \) is dominated by the diffraction in (3) which gives in the dimensionless units \( \Delta \omega_B = c/c_e \). Then (11) reduces to \( T_e \ll 4/(\mu I) \) and \( |\kappa_B| \ll 1 \). Together with the condition \( T_e \gg L_{\text{speckle}}/c \) used in
the derivation of \([8]\) and assuming that \(\tilde{I} \approx \tilde{I}_{\text{convthresh}}\), it gives a double inequality \((7\pi/2)(c_s/c) \ll \tilde{T}_c \ll \pi/\mu\) which can be well satisfied for \(\mu \approx 5\), i.e. for \(\nu_a \approx 0.01\) as in gold NIF plasma but not for \(\mu \approx 50\) as in low ionization number \(Z\) NIF plasma. Also \(|\kappa_B| < 1\) implies that \(\tilde{I} > \tilde{I}_{\text{convthresh}}\) because otherwise, below that threshold, \(\kappa_B \approx -\mu\) which would contradict \(|\kappa_B| < 1\). All these conditions are satisfied for \(\tilde{T}_c \ll 1/4\) for the parameters of Figure \(\ref{fig:1}\) with \(I = 2\) or \(I = 3\) (solid lines in Figure \(\ref{fig:1}\)) but not for \(I = 1\) (dashed lines in Figure \(\ref{fig:1}\)). Additionally, an estimate for \(T_c \ll t_{\text{sat}}\) from the linear part of the theory of Ref. \([13]\) results in the condition \(\tilde{T}_c \ll 8\tilde{I}/\mu\) which is much less restrictive than the previous condition. These estimates are consistent with the observed agreement between \(\kappa_i = \kappa_a + \kappa_B\) and \(\kappa_i\) from simulations (filled circles in Figure \(\ref{fig:1}\)) for \(\tilde{I}\) above the threshold \([9]\). We conclude from Figure \(\ref{fig:1}\) that the applicability condition for the Bourret approximation is close to the domain of \(\tilde{T}_c\) values for which \(\kappa_i = \kappa_a + \kappa_B\).

For typical NIF parameters \([1, 9]\), \(F = 8\), \(n_c/n_e = 0.1, \lambda_0 = 351\text{nm}\) and \(c_s = 6 \times 10^7\text{cm s}^{-1}\) and the electron plasma temperature \(T_e \approx 5\text{keV}\), we obtain from \([9]\) that \(\tilde{I}_{\text{convthresh}} \approx 2.2 \times 10^{14}\text{W/cm}^2\) for gold plasma with \(\nu_a \approx 0.01\), in the range of NIF single polarization intensities. So we conclude that for gold NIF plasma \(I > \tilde{I}_{\text{convthresh}}\) while for low \(Z\) plasma with \(\nu_a \approx 0.1\) \(I\) is well below \(\tilde{I}_{\text{convthresh}}\). Fig. \(\ref{fig:2}\) shows \(\kappa_i\) in the limit \(c_s/c = 0\), \(\tilde{T}_c \rightarrow 0\) from simulations, analytical result \(\kappa_a\) (solid curve) and coherent theory \(\kappa_{\text{coherent}}\) (dotted curve). Upper grid corresponds to laser intensity in dimensional units for NIF parameters and gold plasma \(T_e \approx 5\text{keV}\), \(F = 8\), \(n_c/n_e = 0.1, \nu_a = 0.01, \lambda_0 = 351\text{nm}\).

In conclusion, we identified a collective threshold for BSBS instability of partially incoherent laser beam for ICF relevant plasma. Above that threshold the BSBS increment \(\kappa_i\) is well approximated by the sum of the collective like increment \(\kappa_a\) and RPA-like increment \(\kappa_B\). We found that \(\kappa_i\) is significantly below the BSBS increment \(\kappa_{\text{coherent}}\) of the coherent laser beam.

We acknowledge helpful discussions with R. Berger and N. Meezan. P.L. and H.R. were supported by the New Mexico Consortium and Department of Energy Award No. DE-SC0002238.

- Electronic address: har@lanl.gov

[1] J. D. Lindl, et al., Phys. Plasmas 11, 339 (2004).
[2] N. B. Meezan, et al., Phys. Plasmas, 17, 056304 (2010).
[3] W. L. Krueer, The physics of laser plasma interactions, Addison-Wesley, New York (1990).
[4] H. A. Rose, Phys. Plasmas 2, 2216 (1995).
[5] J. Garnier, Phys. Plasmas 6 1601 (1999).
[6] A.A. Vedenov, and I.I. Rudakov, Sov. Phys. Doklady, 9, 1073 (1965); A.M. Rubenchik, Radiophysics and Quant. Electron., 17 1249 (1976); V.E. Zakharov, S.L. Mushet, and A.M. Rubenchik, Phys. Rep., 129 285 (1985).
[7] D. Pesme, et. al., Natl. Tech. Inform. Document No. PB92-100312 (1987); arXiv:0710.2195.
[8] P. M. Lushnikov and H. A. Rose, Phys. Rev. Lett., 92, 255003 (2004).
[9] P.M. Lushnikov and H.A. Rose, Plasma Physics and Controlled Fusion, 48, 1501 (2006).
[10] H. A. Rose and D. F. DuBois, Phys. Rev. Lett. 72, 2883 (1994).
[11] A. J. Schmitt and B. B. Afeyan, Phys. Plasmas 5, 503 (1998); D. Pesme et al., Phys. Rev. Lett. 84, 278 (2000); A. V. Maximov et al., Phys. Plasmas 8, 1319 (2001); P.
Loiseau, et al., Phys. Rev. Lett. 97, 205001 (2006).

[12] H. A. Rose and Ph. Mounaix, Phys. Plasmas 18, 042109 (2011).

[13] Ph. Mounaix, et al., Phys. Rev. Lett., 85 4526 (2000).

[14] D. F. DuBois, B. Bezzerides, and H. A. Rose, Phys. of Fluids B: Plasma Physics 4, 241 (1992).

[15] R. H. Lehmberg and S. P. Obenschain, Opt. Commun.

[16] Subsequent analysis can be easily generalized to include polarization smoothing [1].

[17] H.A. Rose and D.F. DuBois, Phys. of Fluids B5, 3337 (1993).