Anomaly-free $U(1)$ gauge symmetries in neutrino seesaw models

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Adding right-handed neutrino singlets and/or fermion triplets to the particle content of the Standard Model allows for the implementation of the seesaw mechanism to give mass to neutrinos and, simultaneously, for the construction of anomaly-free gauge group extensions of the theory. We consider Abelian extensions based on an extra $U(1)_X$ gauge symmetry, where $X$ is an arbitrary linear combination of the baryon number $B$ and the individual lepton numbers $L_{e,\mu,\tau}$. By requiring cancelation of gauge anomalies, we perform a detailed analysis in order to identify the charge assignments under the new gauge symmetry that lead to neutrino phenomenology compatible with current experiments. In particular, we study how the new symmetry can constrain the flavor structure of the Majorana neutrino mass matrix, leading to two-zero textures with a minimal extra fermion and scalar content. The possibility of distinguishing different gauge symmetries and seesaw realizations at colliders is also briefly discussed.

I. INTRODUCTION

Neutrino oscillation experiments have firmly established the existence of neutrino masses and lepton mixing, implying that new physics beyond the Standard Model (SM) is required to account for these observations (for reviews, see e.g. Refs. [1, 2]). One of the most appealing theoretical frameworks to understand the smallness of neutrino masses is the seesaw mechanism. In this context, the tree-level exchanges of new heavy states generate an effective dimension-five Weinberg operator, which then leads to an effective neutrino mass matrix at low energies. A simple possibility consists of the addition of singlet right-handed neutrinos (type I seesaw). Alternatively, color-singlet $SU(2)$-triplet scalars (type II) or $SU(2)$-triplet fermions (type III) can be introduced.

As is well known, theories that contain fermions with chiral couplings to the gauge fields suffer from anomalies, i.e. the breaking of gauge symmetries of the classical theory at one-loop level. To make such theories consistent, anomalies should be exactly canceled. In the SM, this cancellation occurs between quarks and leptons within each generation [3, 5]. Adding new chiral fermions requires us to arrange the chiral sectors of the new theory so as to cancel the arising gauge anomalies.

One attractive possibility is to realize the anomaly cancellation through the modification of the gauge symmetry. Extra $U(1)$ symmetries naturally arise in a wide variety of grand unified and string theories. One of the interesting features of such theories is their richer phenomenology, when compared with the SM (for reviews, see e.g. Refs. [6, 7]). In particular, the spontaneous breaking of additional gauge symmetries leads to new massive neutral gauge bosons which, if kinematically accessible, could be detectable at the Large Hadron Collider (LHC). Clearly, the experimental signatures of these theories crucially depend on whether or not the SM particles have nontrivial $U(1)_X$ charges. Assuming that the SM fermions are charged under the new gauge group, and that the new gauge boson $Z'$ has a mass around the TeV scale, one expects some effects on the LHC phenomenology.

In the context of neutrino seesaw models, the implications of anomaly-free constraints based on the gauge structure $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ have been widely studied in the literature [8–12]. In particular, assuming family universal charges, it was shown in Ref. [12] that type I and type III seesaw mechanisms cannot be simultaneously realized, unless the $U(1)_X$ symmetry is a replica of the standard hypercharge or new fermionic fields are added to the theory. On the other hand, when combined type I/II or type III/II seesaw models are considered, it is always possible to assign nontrivial anomaly-free charges to the fields. Models based on gauge symmetries that are linear combinations of the baryon number $B$ and the individual lepton flavor numbers $L_{\alpha}$ ($\alpha = e, \mu, \tau$) have also been extensively discussed [13–20]. From the phenomenological viewpoint many aspects of the latter symmetries are similar to those of the $B-L$ symmetry, with $L = \sum_\alpha L_{\alpha}$ being the lepton number. Nevertheless, the flavor information encoded in the new gauge symmetry can lead to definite predictions on the neutrino mass spectrum and mixing.

In this paper, we consider Abelian extensions of the SM based on an extra $U(1)_X$ gauge symmetry, with $X = aB - \sum_\alpha b_{\alpha}L_{\alpha}$ being an arbitrary linear combination of the baryon number $B$ and the individual lepton numbers $L_{\alpha}$. Our aim is to perform a systematic study, thus complementing previous works in several aspects. In particular, we classify all the anomaly-free $U(1)_X$ gauge...
symmetries that lead to predictive two-zero textures in the effective neutrino mass matrix, obtained via type I, type III or mixed type I/III seesaw mechanisms, with a minimal extra matter content. Some of these symmetries have been recently identified in Ref. 20 in a type I seesaw framework. We extend the analysis and obtain new solutions not previously found in the literature. Furthermore, we study in detail the phenomenological implications of such symmetries on neutrino oscillation analysis through matter effects. Finally, we also discuss the possibility of discriminating at collider experiments the flavor structure of the neutrino mass matrix and its corresponding gauge symmetry, by studying the decays of the $Z'$ boson into leptons and third-generation quarks.

The paper is organized as follows. In Sec. II by requiring cancellation of gauge anomalies, we study the allowed charge assignments under the new gauge symmetry, when two or three right-handed neutrino singlets or fermion triplets are added to the SM particle content. In Sec. III we then discuss the phenomenological constraints on these theories, requiring consistency with current neutrino oscillation data. In particular, by extending the SM with a minimal extra fermion and scalar content, we study how the new gauge symmetry can constrain the flavor structure of the effective neutrino mass matrix, obtained through a type I or type III seesaw mechanism. The possibility of distinguishing different charge assignments (gauge symmetries) and seesaw realizations at collider experiments is also briefly addressed. Our conclusions are summarized in Sec. IV.

II. ANOMALY CONSTRAINTS

We consider a renormalizable theory containing the SM particles plus a minimal extra fermionic and scalar content, so that light neutrinos acquire seesaw masses. We include singlet right-handed neutrinos $\nu_R$ and color-singlet $SU(2)$-triplet fermions $\Sigma$ to implement type-I and type-III seesaw mechanisms, respectively. Besides the SM Higgs doublet $H$ that gives masses to quarks and leptons, a complex scalar singlet field $S$ is introduced in order to give Majorana masses to $\nu_R$ and $\Sigma$.

We assume that each fermion field $f$ has a charge $x_f$ under the new $U(1)_X$ gauge symmetry. We work in a basis of left- and right-handed fermions: $q_L \equiv (u_L, d_L)^T, \ell_L \equiv (\nu_L, e_L)^T, u_R, d_R, e_R, \nu_R$. For quarks, a family universal charge assignment is assumed, while leptons are allowed to have nonuniversal $X$ charges. To render the theory free of the $U(1)_X$ anomalies, the following set of constraints must then be satisfied:

$$
\begin{align*}
A_1 &= n_G (2x_q - x_u - x_d) = 0 , \\
A_2 &= \frac{3n_G}{2} x_q + \frac{1}{2} \sum_{i=1}^{n_G} x_{\ell_i} - 2 \sum_{i=1}^{n_G} x_{\sigma_i} = 0 , \\
A_3 &= n_G \left( \frac{x_q}{6} - \frac{4x_u}{3} - \frac{x_d}{3} \right) + \sum_{i=1}^{n_G} \left( \frac{x_{\ell_i}}{2} - x_{\sigma_i} \right) = 0 , \\
A_4 &= n_G \left( x_q^2 - 2x_u^2 + x_d^2 \right) + \sum_{i=1}^{n_G} \left( -x_{\ell_i}^2 + x_{\sigma_i}^2 \right) = 0 , \\
A_5 &= n_G \left( 6x_q^3 - 3x_u^3 - 3x_d^3 \right) + \sum_{i=1}^{n_G} \left( 2x_{\ell_i}^3 - x_{\sigma_i}^3 \right) - \sum_{i=1}^{n_R} x_{\nu_i}^3 - 3 \sum_{i=1}^{n_G} x_{\sigma_i}^3 = 0 , \\
A_6 &= n_G \left( 6x_q - 3x_u - 3x_d \right) + \sum_{i=1}^{n_G} \left( 2x_{\ell_i} - x_{\sigma_i} \right) - \sum_{i=1}^{n_R} x_{\nu_i} - 3 \sum_{i=1}^{n_G} x_{\sigma_i} = 0 ,
\end{align*}
$$

where $n_G = 3$ is the number of generations, $n_R$ is the number of right-handed neutrinos and $n_\Sigma$ is the number of $SU(2)$-triplet fermions. The equations for $A_1, \ldots, A_5$ arise from the requirement of the cancellation of the axial-vector anomalies [21], while the equation for $A_6$ results from the cancellation of the mixed gravitational-gauge anomaly [22].

Since the anomaly equations are nonlinear and contain many free parameters, some assumptions are usually made to obtain simple analytic solutions. For in-

\footnote{For a type I (type III) seesaw mechanism alone, consistency with neutrino oscillation data requires $n_R \geq 2 \ (n_\Sigma \geq 2)$. Aside from this constraint, the number of right-handed neutrinos (fermion triplets) is arbitrary. If both seesaws are simultaneously allowed, cases with $n_R \geq 2$ and $n_\Sigma \geq 1$, or $n_R \geq 1$ and $n_\Sigma \geq 2$, are viable as well.
stance, in family universal models, universal charges are assigned so that anomaly cancellation is satisfied within each family. Family universality is nevertheless not necessarily required and nonuniversal solutions can be equally found [23]. Assuming, for instance, a nonuniversal purely leptonic gauge symmetry with \( x_{\ell i} = x_{\nu i} \) and \( n_R = n_\Sigma = n_G = 3 \), the anomaly equations (1) lead to the following charge constraints:

\[
\begin{align*}
\sum_{x_{\ell 1} + x_{\ell 2} + x_{\ell 3} &= 0,} \\
x_{\ell 1} + x_{\ell 2} + x_{\ell 3} &= 0, \\
x_{\sigma 1} + x_{\sigma 2} + x_{\sigma 3} &= 0, \\
x_{\ell 1}x_{\ell 2}x_{\ell 3} - x_{\ell 1}x_{\nu 2}x_{\nu 3} - 3x_{\sigma 1}x_{\sigma 2}x_{\sigma 3} &= 0.
\end{align*}
\]  

This system of equations has an infinite number of integer solutions. For example, with the charge assignment \( (x_{\ell 1}, x_{\ell 2}, x_{\ell 3}) = (x_{\ell 1}, x_{\ell 2}, x_{\ell 3}) = (1, 2, -3) \), one can have the solutions \( (x_{\nu 1}, x_{\nu 2}, x_{\nu 3}) = (-1, -3, 4) \) and \( (x_{\sigma 1}, x_{\sigma 2}, x_{\sigma 3}) = (1, 2, -3) \), or \( (x_{\nu 1}, x_{\nu 2}, x_{\nu 3}) = (1, 3, -4) \) and \( (x_{\sigma 1}, x_{\sigma 2}, x_{\sigma 3}) = (-1, 1, 2) \), among many others.

In this work, we shall consider models where \( X \equiv aB - \sum_{i=1}^{n_G} b_iL_i \) is an arbitrary linear combination of the baryon number \( B \) and individual lepton numbers \( L_i \), simultaneously allowing for the existence of right-handed neutrinos and fermion triplets that participate in the seesaw mechanism to generate Majorana neutrino masses. Under the gauge group \( U(1)_X \), the charge for the quarks \( q_L, u_R, d_R \), is universal, while the charged leptons \( e_R \) and the family nonuniversal charge assignment \( x_{\ell i} = x_{\nu i} = -b_i \), with all \( b_i \) different. The latter condition guarantees that the charged lepton mass matrix is always diagonal (i.e., it is defined in the charged lepton flavor basis), assuming that the SM Higgs is neutral under the new gauge symmetry. The right-handed neutrinos \( \nu_R \) and/or the triplets \( \Sigma \) are allowed to have any charge assignment \( -b_k \), where \( k = 1 \ldots n_G \).

Substituting the \( U(1)_X \) charge values into the anomaly equations (1), we obtain the constraints

\[
\begin{align*}
\sum_{k \leq n_G} b_k &= 0, \\
\sum_{i=1}^{n_G} b_i &= \sum_{j \leq n_R} b_j = n_G, \ a, \\
\sum_{i=1}^{n_G} b_i^3 - \sum_{j \leq n_R} b_j^3 - 3 \sum_{k \leq n_G} b_k^3 &= 0.
\end{align*}
\]  

The solutions of this system of equations and the corresponding symmetry generators \( X \) are presented in Table I for minimal type I and type III seesaw realizations with \( n_R + n_\Sigma \leq 4 \). We note that in the absence of right-handed neutrinos only purely leptonic (\( a = 0 \)) gauge symmetry extensions are allowed. This is a direct consequence of the second constraint in Eq. (3). Given the charge assignments, one can identify the maximal gauge group corresponding to each solution. For instance, when \( n_R = 3 \) and \( n_\Sigma = 0 \), the maximal anomaly-free Abelian gauge group extension is \( U(1)_{B-L} \times U(1)_{L_-} \), as recently pointed out in Ref. [20].

In the next section, we shall study the phenomenological implications of different anomaly-free \( U(1)_X \) gauge symmetries on the flavor structure of the effective neutrino mass matrix, assuming that the new symmetries lead to texture zeros that are consistent with the present neutrino oscillation data (see Appendix A). In particular, several neutrino matrix patterns, with a maximum of two independent zeros [24, 25], turn out to be compatible with all data at 1σ level [27].

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\( n_R \) & \( n_\Sigma \) & Anomaly constraints & Symmetry generator \( X \) \\
\hline
2 & 0 & \( b_i + b_j = 3a, b_k = 0 \) & \( B - 3L_j - b'_i(L_i - L_j) \) \\
0 & 2 & \( b_i + b_j = 0, b_k = 0 \) & \( L_i - L_j \) \\
2 & 1 & \( b_i + b_j = 3a, b_k = 0 \) & \( B - 3L_j - b'_i(L_i - L_j) \) \\
1 & 2 & \( b_i + b_j = 0, b_k = 0 \) & \( L_i - L_j \) \\
3 & 0 & \( b_i + b_j + b_k = 3a \) & \( B - L_j + (1 - b'_i)(L_i - L_j) + (1 - b''_i(L_i - L_k)) \) \\
0 & 3 & \( b_i + b_j = 0, b_k = 0 \) & \( -b'_i(L_i - L_k) - b''_j(L_j - L_k) \) \\
3 & 1 & \( b_i + b_j = 3a, b_k = 0 \) & \( B - 3L_j - b'_i(L_i - L_j) \) \\
1 & 3 & \( b_i + b_j = 0, b_k = 0 \) & \( L_i - L_j \) \\
2 & 2 & \( b_i + b_j = 0, b_k = 0 \) & \( L_i - L_j \) \\
\hline
\end{tabular}
\caption{Anomaly-free solutions for minimal type I and/or type III seesaw realizations and their symmetry generators. In all cases, \( i \neq j \neq k \) and \( b'_i \equiv b_i/a \). Cases with \( a = 0 \) correspond to a purely leptonic symmetry.}
\end{table}
III. PHENOMENOLOGICAL CONSTRAINTS

A. Neutrino mass matrix and texture zeros from the gauge symmetry

Realistic gauge theories should include mechanisms to explain fermion masses. This is achieved in the SM by introducing gauge-invariant Yukawa interactions, which give rise to nonvanishing quark and charged lepton masses once the Higgs field acquires a vacuum expectation value (VEV). Neutrinos, on the other hand, are massless in the SM, so new physics is required to account for their nonzero masses. An attractive and economical framework is the seesaw mechanism.

The Lagrangian terms that generate the SM fermion masses and are compatible with (minimal) type I and type III seesaw models for Majorana neutrinos are given by

\[
\begin{align*}
Y_a \overline{q_L} u_R \tilde{H} + Y_d \overline{q_L} d_R H + Y_e \overline{\ell_L} e_R H + Y_\nu \overline{\nu_L} \nu_R \tilde{H} \\
+ \frac{1}{2} m_{1R} \overline{v_R} C\nu_R + Y_1 \overline{v_R} C\nu_R S + Y_2 \overline{v_R} C\nu_R S^* \\
+ \frac{1}{2} m_{\Sigma} \text{Tr}\left(\Sigma^T C\Sigma\right) + Y_T \overline{\ell_L} i\tau_2 \Sigma H \\
+ Y_3 \text{Tr}\left(\Sigma^T C\Sigma\right) S + Y_4 \text{Tr}\left(\Sigma^T C\Sigma\right) S^* + \text{H.c.,}
\end{align*}
\]

where \( H = (H^I H^I)^T \) is the SM Higgs doublet, assumed neutral under the \( U(1)_X \) gauge symmetry, \( \tilde{H} = i\tau_2 H^I \), and \( S \) is a complex singlet scalar field with a \( U(1)_X \) charge equal to \( x_s \). Here \( Y_{a,d,e} \) and \( Y_T \) are \( n_G \times n_G \), \( n_G \times n_R \), and \( n_G \times n_\Sigma \) Yukawa complex matrices, respectively; \( m_R \) and \( Y_{1,2} \) are \( n_R \times n_R \) symmetric matrices, while \( m_\Sigma \) and \( Y_{3,4} \) are \( n_\Sigma \times n\Sigma \) symmetric matrices.

Notice that, in general, the \( U(1)_X \) symmetry does not forbid bare Majorana mass terms for the right-handed neutrinos and fermion triplets. For matrix entries with \( X = 0 \), such terms are allowed. In turn, entries with \( X \neq 0 \) are permitted in the presence of the singlet scalar \( S \), charged under \( U(1)_X \). The latter gives an additional contribution to the Majorana mass terms once it acquires a VEV.

Since a universal \( U(1)_X \) charge is assigned to quarks, the new gauge symmetry does not impose any constraint on the quark mass matrices. However, a choice of a nonuniversal charge assignment, with \( x_{fi} = x_{ei} = -b_i \) and all \( b_i \) different, forces the charged lepton mass matrix to be diagonal, so leptonic mixing depends exclusively on the way that neutrinos mix. The effective neutrino mass matrix \( m_\nu \) is obtained after integrating out the heavy right-handed neutrinos and fermion triplets. In the presence of both (type I and type III) seesaw mechanisms it reads as

\[
m_\nu \simeq -m_D M_R^{-1} m_D^T - m_T M_\Sigma^{-1} m_T^T,
\]

where

\[
\begin{align*}
m_D &= Y_\nu(H), \quad M_R = m_R + 2Y_1 \langle S \rangle + 2Y_2 \langle S^* \rangle, \\
m_T &= Y_T(H), \quad M_\Sigma = m_\Sigma + 2Y_3 \langle S \rangle + 2Y_4 \langle S^* \rangle.
\end{align*}
\]

In what follows we restrict our analysis to minimal seesaw scenarios with \( n_R + n_\Sigma \leq 4 \). The requirement that charged leptons are diagonal \( (b_1 \neq b_2 \neq b_3) \) imposes strong constraints on the matrix textures of \( m_D \) and \( m_T \). Indeed, considering either a type I or a type III seesaw framework, only those matrices with a single nonzero element per column are allowed. Furthermore, matrices with a null row or column are excluded since they lead to a neutrino mass matrix with determinant equal to zero, not belonging to any pattern with only two independent zeros.\(^2\)

We look for anomaly-free \( U(1) \) gauge symmetries that lead to phenomenologically viable two-zero textures of the neutrino mass matrix \( m_\nu \) (i.e., to patterns \( A_{1,2}, B_{1,2,3,4}, C \)) given in Appendix A. Solutions were found only within a type I seesaw framework with three right-handed neutrinos, or in a mixed type I/III seesaw framework with three right-handed neutrinos and one fermion triplet. Table II shows the allowed solutions, for the cases when the Dirac-neutrino mass matrix \( m_D \) is diagonal, which implies the charge assignment \( x_{ei} = -b_i \). All the solutions belong to the permutation set \( P_1 \) [see Eq. (A2)]. We remark that, for each pattern of \( m_\nu \), there are another 20 solutions corresponding to matrices \( m_D \) with 6 zeros (i.e., permutations of the diagonal matrix) and their respective charge assignments. Thus, all together there exist 96 viable solutions. No other anomaly-free solutions are obtained in our minimal setup.\(^3\) For a mixed type I/III seesaw with \( n_R = 3 \) and \( n_\Sigma = 1 \), only the set of solutions with \( |x_s| = 3 \) in Table II are allowed, since the anomaly equations imply that the \( b_i \) coefficient associated to the fermion triplet charge is always zero.

Notice also that, starting from any pattern given in Table II other patterns in the table can be obtained by permutations of the charged leptons. For instance, starting from the symmetry generators that lead to the \( A_1 \) pattern, those corresponding to \( A_2 \) and \( B_1 \) are obtained by \( \mu \leftrightarrow \tau \) and \( e \leftrightarrow \mu \) exchange, respectively. Similarly, the \( B_3 \) texture can be obtained from \( A_2 \) through the \( e \leftrightarrow \tau \) exchange.

B. Scalar masses and mixing

We assume a minimal scalar content: one SM Higgs doublet \( H \), neutral under \( U(1)_X \), and a complex singlet scalar \( S \) with the charge assignment \( x_s \) under \( U(1)_X \). The VEV of the scalar \( S \) breaks this symmetry spontaneously, giving a contribution to the masses of the right-handed neutrinos and fermion triplets. The scalar poten-

\(^2\) Mixed type I/III seesaw mechanisms can relax this constraint.
\(^3\) Solutions leading to \( m_D = D_1, D_2, B_3, B_4 \) have been recently considered in Ref. [20].
The mass of the new $Z'$ gauge boson is given by $m_{Z'} = |x_s| g_X v_S$, where $g_X$ is the $U(1)_X$ gauge coupling.

An indirect constraint on $m_{Z'}$ comes from analyses of LEP2 precision electroweak data \cite{28}:

$$m_{Z'} = |x_s| g_X v_S > 13.5 \text{ TeV}. \quad (16)$$

Thus, depending on the charge $x_s$, different lower bounds on the breaking scale of the $U(1)_X$ gauge symmetry are obtained. For the anomaly-free scalar charges given in Table \ref{tab:anomaly_free_susy_charges} namely $|x_s| = 2, 3, 6$, one obtains the bounds $v_S > 6.75$ TeV, 4.5 TeV, and 2.25 TeV, respectively. To put limits on the $Z'$ mass, the gauge coupling strength must be known. Assuming, for definiteness, $g_X \sim 0.1,$
the bound in Eq. (16) implies $m_{Z'} \gtrsim 1.4$ TeV. Such masses could be probed through the search of dilepton $Z'$ resonances at the final stage of the LHC, with a center-of-mass energy $\sqrt{s} = 14$ TeV and integrated luminosity $L \simeq 100$ fb$^{-1}$ [29, 30]. Recent searches for narrow high-mass dilepton resonances at the LHC ATLAS [31] and CMS [32] experiments have already put stringent lower limits on extra neutral gauge bosons. In particular, from the analysis of $pp$ collisions at $\sqrt{s} = 8$ TeV, corresponding to an integrated luminosity of about 20 fb$^{-1}$, these experiments have excluded at 95% C.L. a sequential SM $Z'$ (i.e. a gauge boson with the same couplings to fermions as the SM Z boson) lighter than 3 TeV.

Electroweak precision data severely constrain any mixing with the ordinary Z boson [7]. The $Z - Z'$ mixing may appear either due to the presence of Higgs bosons which transforms nontrivially under the SM gauge group and the new $U(1)_X$ Abelian gauge symmetry or via kinetic mixing in the Lagrangian [33]. The mass mixing is not induced in our case because the SM Higgs doublet is neutral under $U(1)_X$, while kinetic mixing may be avoided (up to one loop), if $U(1)_Y$ and $U(1)_X$ are orthogonal [34]. Although a detailed analysis of the $Z - Z'$ mixing is beyond the scope of our work, it is worth noting that, in general, it imposes additional restrictions on these models. For simplicity, hereafter we assume that mixing is negligible and restrict ourselves to the case with no mixing.

C. Nonstandard neutrino interactions

Neutrino oscillation experiments are sensitive to new degrees of freedom that mediate neutral-current interactions. The effects of new physics are usually parametrized by the so-called nonstandard interactions (NSI), which affect neutrino production and detection processes as well as neutrino propagation in matter. In particular, NSI are generated in seesaw models once the heavy (singlet or triplet) fermions are integrated out. In the canonical basis of the kinetic terms, the induced $d = 6$ operators lead to modified couplings of the leptons to gauge bosons and, consequently, to deviations from unitarity of the leptonic mixing matrix (see, e.g., Ref. [2] and references therein).

Nonstandard neutrino interactions are also induced in the presence of new heavy gauge bosons with nonuniversal lepton couplings. These can be parametrized in terms of the effective four-fermion operators

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2} G_F \varepsilon^{fP}_{\alpha\beta} (\bar{\nu}_\alpha \gamma_\mu L \nu_\beta)(\bar{f} \gamma^\mu P f), \quad (17)$$

where $G_F$ is the Fermi constant, $P = L$ or $R$ stands for the chiral projection operator, $f$ denotes a SM fermion, $\alpha, \beta = e, \mu, \tau$, and $\varepsilon^{fP}_{\alpha\beta}$ are dimensionless couplings that encode deviations from standard interactions. A simple estimate allows us to relate the magnitude of the $\varepsilon$ parameters to the new physics scale. Assuming that NSI are mediated by some intermediate particles with masses of the order of $m_{\text{NSI}}$, one expects $|\varepsilon| \sim m_F^2/m_{\text{NSI}}^2$, leading to $|\varepsilon| \sim 10^{-2} (10^{-4})$ for $m_{\text{NSI}} \sim 1 (10)$ TeV.

In phenomenological studies of neutrino propagation in matter, it is customary to define the effective NSI parameters

$$\varepsilon_{\alpha\beta} = \sum_f \frac{n_f}{n_e} \varepsilon^{fP}_{\alpha\beta}, \quad (18)$$

where $n_f$ is the number density of the fermion species $f = e, u, d$ in matter. For neutral Earth-like matter the relation

$$\varepsilon_{\alpha\beta} \simeq \sum_P \left( \varepsilon^{P}_{\alpha\beta} + 3\varepsilon^{uP}_{\alpha\beta} + 3\varepsilon^{dP}_{\alpha\beta} \right) \quad (19)$$

holds, while for neutral solar-like matter

$$\varepsilon_{\alpha\beta} \simeq \sum_P \left( \varepsilon^{P}_{\alpha\beta} + 2\varepsilon^{uP}_{\alpha\beta} + \varepsilon^{dP}_{\alpha\beta} \right). \quad (20)$$

The above quantities can be approximately bounded by

$$\varepsilon^{\oplus}_{\alpha\beta} \simeq \sqrt{\sum_P \left( \varepsilon^{P}_{\alpha\beta} \right)^2 + (3\varepsilon^{uP}_{\alpha\beta})^2 + (3\varepsilon^{dP}_{\alpha\beta})^2} \quad (21)$$

and

$$\varepsilon^{\ominus}_{\alpha\beta} \simeq \sqrt{\sum_P \left( \varepsilon^{P}_{\alpha\beta} \right)^2 + (2\varepsilon^{uP}_{\alpha\beta})^2 + (\varepsilon^{dP}_{\alpha\beta})^2}, \quad (22)$$

for neutral Earth-like ($\oplus$) and solar-like ($\ominus$) matter, respectively. The following model-independent bounds are then obtained [33]:

$$|\varepsilon^{\oplus}_{ee}| < 4.2, \quad |\varepsilon^{\oplus}_{\mu\mu}| < 0.068, \quad |\varepsilon^{\oplus}_{e\tau}| < 21.0; \quad (23)$$

$$|\varepsilon^{\ominus}_{ee}| < 2.5, \quad |\varepsilon^{\ominus}_{\mu\mu}| < 0.046, \quad |\varepsilon^{\ominus}_{e\tau}| < 9.0.$$  

For the anomaly-free models discussed in Sec. III A and summarized in Table III, the deviations from standard interactions are given by

$$\varepsilon^{fP}_{\alpha\beta} = \frac{x^2}{2} \frac{x_f x_{\nu_\alpha}}{x^2} \delta_{\alpha\beta}, \quad (24)$$

Equations (21) and (22) then imply

$$\varepsilon^{m}_{\alpha\alpha} \simeq \frac{v^2}{v_S^2} |x_{\nu_\alpha}| C_{m}, \quad (25)$$

with

$$C_{\oplus} = \frac{1}{x^2} \sqrt{\frac{x^2}{2} + 1}, \quad C_{\ominus} = \frac{1}{x^2} \sqrt{\frac{x^2}{2} + 5/18}. \quad (26)$$

The limits given in Eq. (23) can be translated into lower bounds on the $U(1)_X$ gauge symmetry breaking scale $v_S$. For each anomaly-free solution of Table III the corresponding bounds (obtained from NSI of neutrinos in Earth-like and solar-like matter) are given in Table III. As can be seen from the table, these bounds are less restrictive than those obtained from electroweak precision tests.
D. Gauge sector and flavor model discrimination

For the effects due to the new gauge symmetry to be observable, the seesaw scale should be low enough. One expects a phenomenology similar to the case with a minimal $B - L$ scalar sector $^{30}$. Nevertheless, by studying the $Z'$ resonance and its decay products, one could in principle distinguish the generalized $U(1)_X$ models from the minimal $B - L$ model.

Due to their low background and neat identification, leptonic final states give the cleanest channels for the discovery of a new neutral gauge boson. In the limit that the fermion masses are small compared with the $Z'$ mass, the $Z'$ decay width into fermions is approximately given by

$$\Gamma(Z' \to jj) \approx \frac{g^2}{24\pi} m_{Z'} (x_{fL}^2 + x_{fR}^2),$$  \hfill (27)$$

where $x_{fL}$ and $x_{fR}$ are the $U(1)_X$ charges for the left and right chiral fermions, respectively.

It has been shown $^{37,38}$ that the decays of $Z'$ into third-generation quarks, $pp \to Z' \to b\bar{b}$ and $pp \to Z' \to t\bar{t}$ can be used to discriminate between different models, having the advantage of reducing the theoretical uncertainties. In particular, the branching ratios $R_{b/\mu}$ and $R_{t/\mu}$ of quarks to $\mu^+\mu^-$ production,

$$R_{b/\mu} = \frac{\sigma(pp \to Z' \to b\bar{b})}{\sigma(pp \to Z' \to \mu^+\mu^-)} \approx 3K_b \frac{x_q^2 + x^2}{x_{t2}^2 + x_{c2}^2},$$

$$R_{t/\mu} = \frac{\sigma(pp \to Z' \to t\bar{t})}{\sigma(pp \to Z' \to \mu^+\mu^-)} \approx 3K_t \frac{x_q^2 + x^2}{x_{t2}^2 + x_{c2}^2},$$  \hfill (28)$$

could serve as discriminators. The $K_{b/t} \sim O(1)$ factors incorporate the QCD and QED next-to-leading-order correction factors. Substituting the quark and charged-lepton $U(1)_X$ charges, i.e. $x_q = x_u = x_d = a/3$ and $x_{t2} = x_{c2} = -b_2$, we obtain

$$R_{b/\mu} \approx \frac{K_b a^2}{3b_2^2}, \quad R_{t/\mu} \approx \frac{K_t a^2}{3b_2^2},$$  \hfill (29)$$
yielding $R_{b/\mu} \approx R_{t/\mu}$. Figure 1 shows the $R_{t/\mu} - R_{b/\mu}$ branching ratio plane for the anomaly-free solutions given in Table III which lead to the viable neutrino mass matrix patterns $\mathbf{A}_{1,2}$ and $\mathbf{B}_{3,4}$, with two independent zeros. As can be seen from the figure, the solutions split into five different points in the plane, which correspond to the allowed values of the $b_2$ coefficient, $|b_2| = 1, 3, 5, 6, 9$, assuming $a = 1$. The allowed $m_\nu$ patterns are shown at each point.

The ratio $R_{\tau/\mu}$ of the branching fraction of $\tau^+\tau^-$ to $\mu^+\mu^-$ has also proven to be useful for understanding models with preferential couplings to $Z'$ $^{38}$. It is approximately given in our case by

$$R_{\tau/\mu} = \frac{\sigma(pp \to Z' \to \tau^+\tau^-)}{\sigma(pp \to Z' \to \mu^+\mu^-)} \approx K_{\tau} \frac{x_{t3}^2 + x_{c3}^2}{x_{t2}^2 + x_{c2}^2} \approx K_t \frac{b_3^2}{b_2^2},$$  \hfill (30)$$

where in the last expression we have used the charge relation $x_{t3} = x_{c3} = -b_3$. Clearly, this ratio can be used to distinguish models with generation universality ($R_{\tau/\mu} \sim 1$) from models with nonuniversal couplings, as those given in Table III. The $R_{t/\mu} - R_{\tau/\mu}$ branching ratio plane is depicted in Fig. 2. In this case, the neutrino mass matrix patterns exhibit a clear discrimination in the plane, having overlap of two solutions in just three points.
patterns of type free solutions of Table II, leading to neutrino mass matrix patterns with two independent zeros, obtained via Abelian gauge symmetries that are linear combinations of the ratios of third generation final states (τ, b, t) to μ decays of the new gauge boson Z′ could be useful in distinguishing between different gauge symmetry realizations. This provides a complementary way of testing flavor symmetries and their implications for low-energy neutrino physics.

FIG. 2. \( R_{\ell/\mu} - R_{\nu/\mu} \) branching ratio plane for the anomaly-free solutions of Table II leading to neutrino mass matrix patterns of type \( A_{1,2} \) and \( B_{3,4} \).

IV. CONCLUSIONS

We have considered extensions of the SM based on Abelian gauge symmetries that are linear combinations of the baryon number \( B \) and the individual lepton numbers \( L_{\tau, \mu, \tau} \). In the presence of a type I and/or type III seesaw mechanisms for neutrino masses, we then looked for possible charge assignments under the new gauge symmetry that lead to cancellation of gauge anomalies and, simultaneously, to a predictive flavor structure of the effective Majorana neutrino mass matrix, consistent with present neutrino oscillation data. Our analysis was performed in the physical basis where the charged leptons are diagonal. This implies that the neutrino mass matrix patterns with two independent zeros, obtained via the seesaw mechanism, are directly linked to low-energy parameters. We recall that, besides three charged lepton masses, there are nine low-energy leptonic parameters (three neutrino masses, three mixing angles, and three CP violating phases). Two-zero patterns in the neutrino mass matrix imply four constraints on these parameters. Would we consider charge assignments that lead to non-diagonal charged leptons, then the predictability of our approach would be lost, since rotating the charged leptons to the diagonal basis would destroy, in most cases, the zero textures in the neutrino mass matrix.

Working in the charged lepton flavor basis, we have found that only a limited set of solutions are viable (cf. Table II), leading to two-zero textures of the neutrino mass matrix with a minimal extra fermion and scalar content. All allowed patterns were obtained in the framework of the type I seesaw mechanism with three right-handed neutrinos (or in a mixed type I/III seesaw framework with three right-handed neutrinos and one fermion triplet), extending the SM scalar sector with a complex scalar singlet field.

We have also studied how the nonuniversal \( U(1)_X \) charge assignments to neutrinos affect neutrino oscillation analysis through the matter effects. We found that current model-independent limits on the effective NSI parameters lead to lower bounds on the \( U(1)_X \) symmetry breaking scale that are less restrictive than those obtained from electroweak precision tests and, in particular, from recent LHC experiments. Finally, we briefly addressed the possibility of discriminating the different charge assignments (gauge symmetries) and seesaw realizations at the LHC. We have shown that the measurements of the ratios of third generation final states (τ, b, t) to μ decays of the new gauge boson \( Z′ \) could be useful in distinguishing between different gauge symmetry realizations. This provides a complementary way of testing flavor symmetries and their implications for low-energy neutrino physics.

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Appendix A: Two-zero textures for the neutrino mass matrix and their seesaw realization

The neutrino mass matrix \( m_\nu \) is a symmetric matrix with six independent complex entries. There are \( 6!/[n!(6 - n)!] \) different textures, each containing \( n \) independent texture zeros. One can show that any pattern of \( m_\nu \), with more than two independent zeros \( (n > 2) \) is not compatible with current neutrino oscillation data. For \( n = 2 \), there are fifteen two-zero textures of \( m_\nu \), which can be classified into six categories (A, B, C, D, E, F):

\[
\begin{align*}
A_1 & : \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, & A_2 & : \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ * & 0 & * \end{pmatrix}; \\
B_1 & : \begin{pmatrix} 0 & 0 & * \\ * & 0 & * \\ 0 & * & * \end{pmatrix}, & B_2 & : \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}; \\
B_3 & : \begin{pmatrix} 0 & 0 & * \\ * & * & * \\ * & * & 0 \end{pmatrix}, & B_4 & : \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & * & 0 \end{pmatrix};
\end{align*}
\]
TABLE IV. Viable type I (type III) seesaw realizations of two-zero textures of the effective neutrino mass matrix \( m_\nu \) when \( n_R = 3 \) \((n_\Sigma = 3)\) and the Dirac-neutrino Yukawa mass matrix \( m_D \) \((m_T)\) is diagonal, i.e. \( m_{D,T} = \text{diag} \,(\ast, \ast, \ast) \). All cases belong to the permutation set \( P_1 \).

| \( m_{D,T} \) | \( M_{R,\Sigma} \) | \( m_\nu \) |
|------------|----------------|-----------|
| \( \ast \ast \ast \) | \( \ast \ast \ast \) | \( \ast \ast \ast \) |
| \( \ast \ast \ast \) | \( \ast \ast \ast \) | \( \ast \ast \ast \) |
| \( \ast \ast \ast \) | \( \ast \ast \ast \) | \( \ast \ast \ast \) |

TABLE V. Viable type I (type III) seesaw realizations of two-zero textures of \( m_\nu \) when \( n_R = 3 \) \((n_\Sigma = 3)\) and assuming that \( m_D \) \((m_T)\) belongs to a permutation set \( P_i \) \((i = 1, 2, 3, 4)\). Only matrices \( m_D \) \((m_T)\) and \( m_\nu \) contained in \( P_1 \) are allowed, sharing always the same pattern.

\[
C : \begin{pmatrix} \ast \ast \ast \\ \ast \ast \ast \end{pmatrix}; \quad D_1 : \begin{pmatrix} \ast \ast \ast \\ \ast \ast \ast \\ \ast \ast \ast \end{pmatrix}; \quad D_2 : \begin{pmatrix} \ast \ast \ast \\ \ast \ast \ast \end{pmatrix}; \quad E_1 : \begin{pmatrix} 0 \ast \ast \\ 0 \ast \ast \\ 0 \ast \ast \end{pmatrix}; \quad E_2 : \begin{pmatrix} \ast \ast \ast \\ \ast \ast \ast \\ \ast \ast \ast \end{pmatrix}; \quad E_3 : \begin{pmatrix} \ast \ast \ast \\ \ast \ast \ast \\ \ast \ast \ast \end{pmatrix}; \quad F_1 : \begin{pmatrix} 0 \ast \ast \\ \ast \ast \ast \\ 0 \ast \ast \end{pmatrix}; \quad F_2 : \begin{pmatrix} 0 \ast \ast \\ \ast \ast \ast \\ \ast \ast \ast \end{pmatrix}; \quad F_3 : \begin{pmatrix} \ast \ast \ast \\ \ast \ast \ast \\ 0 \ast \ast \end{pmatrix};
\]

\( \ast \) denotes a nonzero matrix element. In the flavor basis, where the charged-lepton mass matrix \( m_l \) is diagonal and \( m_l = \text{diag} \,(m_e, m_\mu, m_\tau) \), only seven patterns, to wit \( A_1, B_{1,2,3,4} \) and \( C \) \([14]\), are compatible with the present neutrino oscillation data \([27]\).

We remark that, since the \( U(1)_X \) gauge symmetry does not constrain the values of the Yukawa couplings, any ordering of the charged leptons in the flavor basis is allowed. Therefore, any permutation transformation acting on the above patterns is permitted, provided that it leaves \( m_l \) diagonal. In particular, we find the following permutation sets:

\( P_1 \equiv (A_1, A_2, B_3, B_4, D_1, D_2), \)
\( P_2 \equiv (B_1, B_2, E_3), \)
\( P_3 \equiv (C, E_1, E_2), \)
\( P_4 \equiv (F_1, F_2, F_3). \)
TABLE VI. Viable type I (type III) seesaw realizations that lead to the two-zero pattern $C$ in $m_\nu$. The cases with $n_R = 2$ ($n_\Sigma = 2$) and $n_R = 3$ ($n_\Sigma = 3$) are displayed.

| $m_{D,T}$ | $M_{R,\Sigma}$ | $m_\nu$ |
|-----------|----------------|----------|
| $\begin{pmatrix} * & * & 0 \\ * & 0 & 0 \\ 0 & 0 & * \end{pmatrix}$, $\begin{pmatrix} * & * & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}$ | $\begin{pmatrix} 0 & * \\ * & 0 \\ 0 & 0 \end{pmatrix}$ | $C$ with $\det C = 0$ |
| $\begin{pmatrix} * & * & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}$, $\begin{pmatrix} * & * & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ | $C$ |
| $\begin{pmatrix} * & * & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}$, $\begin{pmatrix} * & * & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ | |
| $\begin{pmatrix} * & * & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}$, $\begin{pmatrix} * & * & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ | |

Starting from any pattern belonging to a particular set, one can obtain any other pattern in the same set by permutations.

We look for all possible type I and/or Type III seesaw realizations of two-zero textures of the neutrino mass matrix $m_\nu$ compatible with the experimental data, i.e. that lead to a pattern $A_1$, $A_2$, $B_1$, $B_2$, $B_3$, $B_4$ or $C$. We restrict our analysis to scenarios with $n_R + n_\Sigma \leq 4$. Clearly, in our minimal theoretical framework, with only one Higgs doublet and one singlet scalar, not all the solutions found can be implemented in a theory free of anomalies under the gauge group $U(1)_X$, with $X = a B - \sum_{i=1}^{n_\Sigma} b_i L_i$.

In particular, requiring charged leptons to be diagonal ($b_1 \neq b_2 \neq b_3$) imposes strong constraints on the patterns of the Dirac-Yukawa mass matrices. Indeed, only matrices $m_D$ and $m_T$ with one nonzero element per column are permitted. In this appendix, nevertheless, we present the complete set of solutions, regardless of their realization or not as anomaly-free theories, since from the model building viewpoint these solutions may be of interest in the context of extended theories or in the presence of discrete symmetries.

In Table IV we present all viable type I (type III) seesaw realizations of two-zero textures of the effective neutrino mass matrix $m_\nu$ when $n_R = 3$ ($n_\Sigma = 3$) and the Dirac-neutrino Yukawa mass matrix $m_D$ ($m_T$) is diagonal, i.e. $m_{D,T} = \text{diag}(*,*,*)$. All cases belong to the permutation set $P_1$. Table IV shows the viable type I (type III) seesaw realizations of two-zero textures of $m_\nu$ when $n_R = 3$ ($n_\Sigma = 3$) and assuming that $m_D$ ($m_T$) belongs to a permutation set $P_i$ ($i = 1, 2, 3, 4$). Only matrices $m_D$ ($m_T$) and $m_\nu$ contained in $P_1$ are allowed, sharing always the same pattern, i.e. exhibiting “parallel” structures.

One may wonder whether neutrino mass matrices of type $C$, belonging to the permutation set $P_3$, can be obtained. As it turns out, this requires matrices $m_{D,T}$ (and $M_{R,\Sigma}$) with two and four zeros, for $n_R, n_\Sigma = 2$ and $n_R, n_\Sigma = 3$, respectively. In Table VII we present all viable type I (type III) seesaw realizations that lead to the two-zero pattern $C$ in the effective neutrino mass matrix $m_\nu$. The cases with $n_R = 2$ ($n_\Sigma = 2$) and $n_R = 3$ ($n_\Sigma = 3$) are considered. As can be seen from the table, with only two right-handed singlet (fermion triplet) neutrinos, there are only two possible constructions, both leading to a massless neutrino ($\det C = 0$). In fact, these are the only solutions that yield a pattern consistent with neutrino oscillation data; no textures of type $A_i$ or $B_i$ are found. For $n_R = 3$ ($n_\Sigma = 3$), besides the $C$-pattern, there exist several combinations of matrices $m_{D,T}$ and $M_{R,\Sigma}$ (not displayed in the table), that lead to the viable patterns $A_{1,2}$ and $B_{3,4}$. However, no texture of type $B_{1,2}$, belonging to the permutation set $P_2$, can be obtained. The latter can be realized in the context of mixed seesaw schemes. In Table VII several patterns leading to neutrino mass matrices of type $B_{1,2}$ through a mixed seesaw with two right-handed neutrinos and two fermion triplets ($n_R = n_\Sigma = 2$) are shown. The solutions correspond to cases where the Dirac-Yukawa mass matrices $m_{D,T}$ contain the maximum of allowed vanishing matrix elements,
TABLE VII. Examples of type I/III mixed seesaw realizations with two right-handed neutrinos and two fermion triplets \( n_R = n_\Sigma = 2 \) that lead to a neutrino mass matrix of type \( B_{1,2} \). The solutions correspond to cases where the \( 3 \times 2 \) Dirac-Yukawa mass matrices \( m_D \) and \( m_T \) contain the maximum of allowed vanishing elements, i.e. four zeros. We remark that in the mixed cases with \( n_R = 2, n_\Sigma = 1 \) and \( n_R = 1, n_\Sigma = 2 \) there are viable patterns as well, but since they only generate neutrino mass matrices of type \( A_{1,2}, B_{3,4} \) and \( C \), we do not present them in Table VII.

\[
\begin{array}{cccc}
\mathbf{m}_D & \mathbf{M}_R & \mathbf{m}_T, \mathbf{M}_\Sigma & \mathbf{m}_\nu \\
\begin{pmatrix}
0 & 0 \\
0 & * \\
* & 0
\end{pmatrix} & \begin{pmatrix}
0 & * \\
* & 0
\end{pmatrix} & \begin{pmatrix}
0 & * \\
* & 0
\end{pmatrix} & \begin{pmatrix}
0 & * \\
* & 0
\end{pmatrix} \text{ or } \begin{pmatrix}
0 & 0 \\
* & 0
\end{pmatrix} \\
0 & 0 & * & 0 \\
0 & * & 0 & 0
\end{array}
\]

\( B_1 \)

\[
\begin{pmatrix}
0 & 0 & * & 0 \\
0 & 0 & * & 0 \\
0 & 0 & * & 0
\end{pmatrix}
\]

\( B_2 \)

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