Effect of magnetic field on over-doped HTc superconductors: Conflicting predictions of various HTc theories

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Following the recent NMR experiments by Gorny et al., we discuss the effect of a magnetic field on the superconducting $T_c$ and the spin pseudo-gap $T^*$. As a testable prediction, we argue that a spin pseudo-gap should also be observed in over-doped samples in a magnetic field (while normally there is no pseudo-gap above $T_c$). We find that different theoretical approaches have marked differences in their predictions for over-doped HTc cuprates.

I. INTRODUCTION

In a high accuracy NMR experiment on near optimal doped YBCO, Gorny et al. found that a magnetic field of 14.8 Tesla shift $T_c$ down by as much as $8K$, while the spin pseudo-gap remains unaffected (as measured by $(T_1T)^{-1}$. They concluded that it is an evidence that "...hence the pseudo-gap is unrelated to superconducting fluctuations".

Here, we make a testable prediction: That, over-doped HTc samples (which normally do not show a spin pseudo-gap state) when subjected to a moderate magnetic field will unprecedentedly reveal a spin pseudogap above $T_c (B)$ starting from approximately the original $T_c (B = 0)$ of zero magnetic field. Our prediction is based both on a new phenomenological interpretation of the experiments (in contrast with the original interpretation of Gorny et al.), and further on a microscopic stripes theoretical approach.

As elaborated below, the above prediction is not shared by several other current HTc theories. The general difference in the prediction of various theoretical approaches for the effect of a magnetic field on over-doped HTc cuprates can be understood from the difference in the position of the spin pseudo-gap line, $T^*$, in the two theoretical phase diagrams depicted in Figure-1. Therefore, repeating the experiments of Gorny et al. on over-doped HTc samples constitute a crucial experiment to determine the proper form of the HTc phase diagram in the over-doped region, and hence to provide further theoretical constraints.

II. TWO PHASE DIAGRAMS: CONFLICTING PREDICTIONS

The spin pseudo-gap is often referred to as a peculiarity of underdoped and near optimal doped HTc superconductors, where the gap evolves smoothly as the temperature is increased through $T_c$ and remains significant up to a cross-over temperature $T^* > T_c$. This is in sharp contrast to the behavior of over-doped cuprates (doping $x > 0.2$) and conventional superconductors where the gap closes at $T ≥ T_c$. Hence, it is common to find references to the over-doped cuprates as more conventional, (even by theorists who otherwise advocate non-conventional mechanisms).

At the under-doped and optimal regions of doping ($0.05 < x < 0.2$), there is a growing agreement that two cross-over temperatures can be identified: A doping dependent cross-over temperature $T_0(x)$ is experimentally marked by a broad maxima in the spin susceptibility $\chi_0(T)$. In addition, below $T_0(x)$, $T_1T$ decreases linearly in temperature and there is a downward deviation of the in-plane resistivity $\rho_{ab}(T)$. At a lower temperature $T^*(x)$, a second cross-over occurs, when a pseudo-gap feature appears in NMR, ARPES, neutron scattering, and specific heat measurements. Below $T^*(x)$, $\chi_0(T)$ continues to decrease even more rapidly down to $T_c$, but $T_1T$ exhibit a minimum followed an increase as temperature is lowered further, which is suggestive of a spin gap formation. (sometime both cross-overs are referred to as "pseudo-gaps", which remains a cause for confusion).

In the over-doped region, the continuation of the pseudo-gap lines below $T_c$ in Figure-1 should be under-
stood as "what would be if there was no superconducting phase", which is exactly what a magnetic field does [3]. The difference between phase diagrams (A) and (B) in Figure-1 is in the over-doped region, and the proposed NMR experiment will reveal the correct one [4] (and thus pose a challenge to the other theories).

(a) Figure-1A is the one most commonly found in the literature [4]. Near an over-doping point $x_{over} \approx 0.2$, where $T^* \approx T_0 = T_c$, the pseudo-gap lines end sharply (i.e., cross the $T_c$ line). Explicit examples of such phase diagrams are currently drawn by Pines and collaborators [5] (which advocate a spin fluctuation exchange mechanism), and by the Rome group of Castellani, DiCastro, Grilli and collaborators [6] (which advocate a quantum critical point fluctuations mechanism). In addition, if the pseudo-gap below $T^*$ is not related to pairing then there is no reason for it to be correlated with $T_c$ in the over-doped region. The choice of the $x_{over} \approx 0.2$ point is not arbitrary. There are various experimental indicators for a significant qualitative change in the cuprates beyond this point; A prime example is the experiment of Boebinger [4] which implies a metal-insulator quantum phase transition. The physical significance of each cross-over line (and the $x_{over} \approx 0.2$ point) is, of course, varying between theories.

For over-doping $x > x_{over} \approx 0.2$ in Figure-1A, it means that $T^* \approx T_0 < T_c$. Therefore, if Figure-1A is the correct phase diagram then for an over-doped HTc sample under a magnetic field $B$ of about $8-14$ Tesla, the following predictions are implied:

1. Though $T_c$ will be suppressed by a few degrees, there will remain no signature of a spin pseudo-gap behavior above $T_c$ (B).
2. In particular, $\chi_0 (T)$ will continue to increase with decreasing temperature down to $T_c$, in sharp contrast with a pseudo-gap behavior where below $T^*$ it decreases rapidly down to $T_c$.

(b) In contrast, we now introduce arguments in favor of the phase diagram depicted in Figure-1B, where the $T^*$ cross-over line merges continuously with $T_c$, but it is still "there" (as a pairing mechanism) and can be revealed in the appropriate NMR experiment.

To explain the pseudo-gap phenomenon below $T^*$, a general argument base on superconducting phase fluctuations was introduced by Emery&Kivelson [7] and elaborated by Millis and collaborators [8]. As depicted in figure-2, the superconducting transition temperature $T_c$ is determined by the lowest of two parameters; the pairing temperature $T_{pair} \sim \Delta (0)/2$, and the phase ordering temperature $T_\theta$. The establishment of a significant pairing amplitude is determined by the pairing energy scale which is given by the spin gap $\Delta (T)$. The classical phase ordering temperature $T_\theta$ is obtained by considering the disordering effects of only the classical phase fluctuations as $T_\theta \sim \chi_\theta$, where $\chi_\theta = \frac{k^2 n_s(0) x}{4 m_e}$ is the zero-temperature value of the “phase stiffness” (which sets the energy scale for the spatial variation of the superconducting phase). In conventional weak coupling BCS superconductors $T_\theta \gg T_{pair}$ and hence $T_c = T_{pair}$.

Note that the phase stiffness can be reduced either by increasing the effective quasiparticle mass (i.e., when $m^* \gg m_e$), or by low superfluid density $n_s(0)$. It is argued that in the HTc cuprates $n_s(0)$ is indeed low enough that phase fluctuations become important (see further discussion below). The density of mobile charge carriers, and hence the superfluid density $n_s(0)$, naturally increases with increased doping $x$.

FIG. 2. The upper figure depicts the usual plot, in zero magnetic field, showing that $T_c$ is limited by $T_\theta$ in the case of low superfluid density $n_s(0)$, and by $T_{pair}$ at higher $n_s(0)$. A pseudo-gap correspond to the region where $T_\theta < T_{pair}$. The dashed line at the putative $x = 0.2$ doping serves as a guide to the eye. The lower figure depicts our suggested effect of a magnetic field for the case of HTc superconductors. As explained in the text, the $T_\theta$ line is shifted down while $T_{pair}$ remains nearly unsuppressed. As a result, the pseudo-gap region now extends to higher dopings.

Figure-2A depicts the resulting theoretical phase diagram in the absence of a magnetic field [3] (and neglecting competition with other order parameters such as AFM). In the under-doped and optimal-doped regions, where $T_{pair} > T_\theta$, $T_c$ is determined by the phase ordering temperature $T_\theta$. In particular, there is a temperature range $T_c < T < T_{pair}$ where there is significant pairing amplitude without global phase coherence. Therefore, within this framework we make the identification of the spin pseudo-gap temperature $T^* = T_{pair}$. In contrast, in the over-doped region, $T_c$ is determined by $T_{pair}$, i.e., by the pairing energy scale as in conventional weak coupling BCS.

Focussing here on the over-doped region, one would be first lead to the conclusion that the above picture entails that the effect of a magnetic field on an over-
doped sample would be similar to the case of conventional superconductors.

It is important to understand that the microscopic pairing mechanism and the phase coherence mechanism are distinct. The effect of a magnetic field is sensitive to microscopic details which are not part of the macroscopic phase fluctuation theory. In weak coupling BCS theory, a significant part of the pairing energy is an outcome to the overlap between the pairs (due to the large pair coherence length), which leads to coherent scattering among many pairs within the single pair coherence length. In this sense, phase coherence (on a local scale) between pairs is affecting also the pairing scale (i.e., the length). In this sense, phase coherence (on a local scale) is affecting also the pairing scale (i.e., the length). In this sense, phase coherence (on a local scale) is affecting also the pairing scale (i.e., the length). In this sense, phase coherence (on a local scale) is affecting also the pairing scale (i.e., the length).

The dependence of $T_c(H)$ on the magnetic field will also be quite peculiar, and different from what is observed in under-doped samples. Initially, when still $T_B(H) > T_{\text{pair}}(H) = T_c(H)$, there will be very little suppression of $T_c(H)$ due to the relatively small suppression of the pairing mechanism. Yet, below a critical magnetic field $T_B(H_{cr}) = T_{\text{pair}}(H_{cr}) = T_c(H_{cr})$, there will be a more rapid suppression of $T_c(H)$ with increased magnetic field (since now $T_c(H) > T_B(H) < T_{\text{pair}}(H)$).

We add a remark that the effect of a magnetic field on phase coherence and pair fluctuation effects may depend on microscopic details. In a weak coupling BCS model of a d-wave superconductor, Eschig et al. [10] concluded that the magnetic field reduces also the fluctuation corrections to $(T/T_c)^{-1}$, i.e., lead to also lower $T^*$. Similarly, Pines (3, page:14) state that moderate magnetic fields will have a dephasing effect on the pairing channel (via AFM spin fluctuation exchange) and thus significantly suppress $T^*$ (in contrast with our phenomenological assumption above, and with the stripe model described below).

Emery-Kivelson and Collaborators [1] elaborated a theoretical approach to HTc which is based on coupled fluctuating spin and charge stripes in real space. The stripes are local phase separated electronic structures made out of quasi one-dimensional hole rich conducting electronic filaments (referred to as "hole-lines") and confined in between them are narrow ladder-like half-filled regions (which thus have substantial AFM correlations).

The systematics of phase fluctuations [6], mentioned above, suggests that pairing on a high energy scale does not require interaction between metallic charge stripes. Instead, pairing is established first on single stripes, independently, at temperature $T^*$ (the single stripe is modelled as a 1D electron gas coupled to the various low-energy states of an insulating ladder-like environment [11]). Below $T^*$, each charge stripe can be regarded as a spin gaped one dimensional extended "grain" with enhanced pairing. In turn, $T_c$ is controlled by the Josephson coupling required to establish phase coherence for an array of stripes [12].

Another way of looking at the situation is to compare the superfluid density $n_s(0)$ with the number of particles $n_p$ involved in pairing. In BCS theory, at $T = 0$, $n_p$ is of order $\Delta_0/E_F$ (where $E_F$ is the Fermi energy) and $n_s(0)$ is given by all the particles in the Fermi sea; i.e., $n_p \ll n_s(0)$. For Bose condensation $n_p = n_s(0)$. In the stripes model of high temperature superconductors, $n_p \gg n_s(0)$; most of the electrons in the Fermi sea participate in the spin gap below $T^*$ (since both the electronic ladder environment and the hole-lines develop the spin gap) but the superfluid density of the doped insulator is small, since the mobile charge density, proportional to $x$, includes only the charges in the hole lines.

Though the stripes are extended objects, their effective one-dimensionality entails that a magnetic field does not significantly alter the electron pairing dynamics on individual stripes, and thus $T^*(H)$ is predicted to remains almost constant. Similarly, the lack of large fluctuation diamagnetism between $T^*$ and $T_c$ is readily understood, since an applied magnetic field does not drive any significant orbital motion until coherence develops in two (and ultimately three) dimensional patches, close to $T_c$ [11]. Below $T^*$, Josephson coupling between stripes leads to the establishment of global phase coherence. Hence, as in conventional granular superconductors, a magnetic field suppresses the phase coherence between stripes.

In conclusion, the effect of a magnetic field on the microscopic dynamics of the stripes model leads to the same predictions which were deduced above following

\[ T_c(H) = T_c(0) - \frac{\Delta_0^2}{E_F} H \]

\[ T^*(H) = T^*(0) - \frac{\Delta_0^2}{E_F} H \]

\[ \Delta_0^2(H) = \Delta_0^2(0) - \frac{\Delta_0^2}{E_F} H \]

\[ \lambda_c(H) = \lambda_c(0) - \frac{\lambda_c}{E_F} H \]

\[ \lambda_p(H) = \lambda_p(0) - \frac{\lambda_p}{E_F} H \]

\[ \lambda_{\text{pair}}(H) = \lambda_{\text{pair}}(0) - \frac{\lambda_{\text{pair}}}{E_F} H \]
a phenomenological re-interpretation of the experiments of Gorny et al due to the separation of pairing an phase coherence scales.

As an additional remark, notice that the under-doped end of the $T^*$ line is drawn as going down sharply towards zero below $x = 0.04$. This is also a testable consequence of the spin-gap-proximity effect mechanism in the stripes approach \cite{1}. Between doping $x = 0.06$ and $x = 0.02$, we may envision two extreme scenarios leading to the same consequence for the spin gap of the effective AFM ladder environment (between each two hole rich lines), which in turn affect the total spin gap: 1) If the hole lines filling fraction remains constant then the AFM ladder triples its width, which implies that the theoretical maximum spin gap decreases by a factor $e^{-3} \approx \frac{1}{20}$. 2) If the width of the ladder environment remains constant then the hole-lines filling "overflows" and dissolves the stripe structure.

III. SUMMARY

The NMR experiment of Gorny et al. indicate that a magnetic field shifts $T_c$ down while the spin pseudo-gap (what ever is its origin) remains relatively unaffected (i.e., $T^* \sim$ constant). We point out that in any model in which superconducting pairing and phase ordering are governed by distinct physics, that distinct dependences on parameters of the pairing scale and the superconducting $T_c$ are to be expected, in conflict with the conclusion of Gorny et al \cite{1}. If the pseudo-gap is associated with local pairing then it implies that a magnetic field is only weakly suppressing the pairing energy scale (unlike weak coupling BCS).

Current theoretical approaches where conceived to agree with the known experimental results in under-doped and optimal-doped cuprates. Yet, their implied characterization of the over-doped region ($x > 0.2$) are distinct. As depicted in Figure-1, the difference is highlighted by the continuation of the $T^*$ line in the over-dope region. In this paper we argued that NMR experiments can reveal those differences.

In particular, we present the following argument and prediction: (1) Let us assume that there is only one and the same mechanism of HTc superconductivity in all the cuprates and over the whole doping range (from under to over doping \cite{12}). (2) Assume that the spin pseudo-gap below $T^*$ is indeed a precursor of the superconducting gap, i.e., an outcome of a developed local pairing amplitude in the absence of global phase coherence. (3) The over-doped cuprates are characterized by $T_c = T^*$ in the absence of an external magnetic field. (4) It follows from the experiment of Gorny et al. \cite{1} that a magnetic field suppresses the superconducting phase coherence temperature $T_c$, while the pairing mechanism remains much less affected (as measured by $(T_c/T^*)^{-1}$). Therefore, we predict that in an over-doped sample ($x > 0.2$) in a moderate magnetic field ($8 - 14$ Tesla) a spin pseudo-gap will be revealed starting from approximately the original $T_c$, at $T^* \approx T_c (H = 0)$, above $T_c (H)$ (while there was no pseudo-gap in the absence of a magnetic field).

The above prediction is a natural consequence of the spin-gap-proximity effect mechanism advanced by Emery-Kivelson-Zachar \cite{11}, but is not shared by several other leading theoretical approaches \cite{13, 14, 15}. Hence, the result of performing the suggested NMR experiment can serve to provide new theoretical constraints; On the one hand, if the pseudo-gap phenomenon will prove to be only a curiosity of underdoping then it does not reflect an essential part of the pairing mechanism. On the other hand, if the above suggested experiment will unprecedentedly reveal a pseudo-gap region also in overdoped samples then it will strengthen the view that the underdoped materials exemplify the essential physics of HTc, which is only getting progressively obscured (due to similar energy scale of otherwise distinct phenomena) in optimal and overdoped samples, and not vice versa.

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[3] The over-doped region of the phase diagram is at many instances not extensively elaborated by the original authors of each theory. Hopefully this article will have a stimulation affect to change that. I made reference only to theories which presented a phase diagram which was elaborated enough to draw some conclusion from. Unfortunately, there aren’t many.
[4] Some reservation needs to be added, that the magnetic field should not be so strong as to significantly suppress also the antiferromagnetic correlations which are related to the origin of the spin pseudo-gap in many theoretical approaches.
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pairing amplitude towards over-doping, some theoretical approaches advocate a qualitative change in the character of the pairing mediated fluctuations.