A comprehensive study of vector leptoquark with $U(1)_{B_3-L_2}$ on the $B$-meson and Muon g-2 anomalies

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Recently reported anomalies in various $B$ meson decays and also in the anomalous magnetic moment of muon $(g-2)_\mu$ motivate us to consider a particular extension of the standard model incorporating new interactions in lepton and quark sectors simultaneously. Our minimal choice would be leptoquark. In particular, we take vector leptoquark ($U_1$) and comprehensively study all related observables including $(g-2)_\mu$, $R_{K^{(*)}}$, $R_{D^{(*)}}$, $B \to (K)\ell\ell'$ where $\ell\ell'$ are various combinations of $\mu$ and $\tau$, and also lepton flavor violation in the $\tau$ decays. We find that a hybrid scenario with additional $U(1)_{B_3-L_2}$ gauge boson provides a common explanation of all these anomalies.

I. INTRODUCTION

Physics is a data-driven science, and we are keen to modify the standard model (SM) when experimental results deviate from the theoretical predictions. Over the past several years, the $B$-physics experiments BaBar, Belle, and LHCb have reported several anomalous results in the $b \to s\ell\ell$ and $b \to c\ell\nu$ processes, which are not properly explained within the SM thus call for new physics. In particular, the lepton flavor universality (LFU), which is one of the approximate symmetries in the SM, seems to be broken beyond the expected range according to the observables of $R_D$, $R_{D^*}$, $R_K$, and $R_{K^*}$, which measured the ratios of different lepton flavors

$$R_{D^{(*)}} = \frac{B(B \to D^{(*)}\tau\nu)}{B(B \to D^{(*)}\ell\nu)}, \quad R_{K^{(*)}} = \frac{B(B \to K^{(*)}\mu^+\mu^-)}{B(B \to K^{(*)}\ell^+\ell^-)}.$$ (1)

The precise measurement of those quantities would test the basic structure of the SM since LFU is only violated by the lepton masses in the SM.

The world averaged experimental values [1] based on measurements from BaBar [2], Belle [3–5], and LHCb [6, 7] are

$$R_D = 0.340 \pm 0.027 \pm 0.013 \quad R_{D^*} = 0.295 \pm 0.011 \pm 0.008,$$ (2)

and the combined discrepancy to SM prediction is at the 3.1$\sigma$ level [1, 8].

The most precise measurement to date of the $R_K$ has been performed by LHCb [9]

$$R_K = 0.846^{+0.044}_{-0.041}, \quad q^2 \subseteq [1.1, 6.0] \text{ GeV}^2,$$ (3)

which has 3.1$\sigma$ deviation from the SM expectation. For the $R_{K^*}$, LHCb Run-1 provides [10]

$$R_{K^*} = \begin{cases} 0.66^{+0.11}_{-0.07} \pm 0.03, & q^2 \subseteq [0.045, 1.1] \text{ GeV}^2, \\ 0.69^{+0.11}_{-0.07} \pm 0.05, & q^2 \subseteq [1.1, 6.0] \text{ GeV}^2. \end{cases}$$ (4)
Combining both $q^2$ bins, it has 2.5σ tension with the SM. On the other side, the $R_{K^*}$ and $R_K$ measurements from Belle [11][12]

$$R_{K^*} = \begin{cases} 
0.90^{+0.27}_{-0.21} \pm 0.10, & q^2 \subseteq [0.1, 8.0] \text{ GeV}^2, \\
1.18^{+0.52}_{-0.32} \pm 0.10, & q^2 \subseteq [15, 19] \text{ GeV}^2, 
\end{cases}$$

$$R_K = \begin{cases} 
0.98^{+0.27}_{-0.23} \pm 0.06, & q^2 \subseteq [1.0, 6.0] \text{ GeV}^2, \\
1.11^{+0.29}_{-0.26} \pm 0.07, & 14.18 \text{ GeV} < q^2, 
\end{cases}$$

are still compatible with SM predictions within their large uncertainties. In the near future, Belle II is expected to significantly improve the uncertainties [13].

The other long-standing problem is the anomalous magnetic moment of muon. Recently, the Muon $(g-2)$ experiment at Fermilab reported the value, $a_{\mu}^{\text{FNAL}} = (116592040 \pm 54) \times 10^{-11}$ [14], or, the discrepancy from the SM

$$\Delta a_{\mu}^{\text{FNAL}} = a_{\mu}^{\text{FNAL}} - a_{\mu}^{\text{SM}} = (230 \pm 69) \times 10^{-11},$$

which is a 3.3σ deviation. Since the value is compatible with the earlier value from BNL [15][16], the significance is now strengthened to 4.2σ level:

$$\Delta a_{\mu}^{\text{BNL+FNAL}} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (251 \pm 59) \times 10^{-11}.$$ 

Even though it is not completely settled down from the lattice calculations [17], certainly it is worth considering new physics as its solution.

Finding a common origin of the $B$-meson and $(g-2)_\mu$ anomalies is non-trivial, but it is appealing from the theoretical point of view [1]. In early attempts [16][31][30], the $U_1 = (3, 1)_{2/3}$ singlet vector Leptoquark, in general couples to both left-handed (LH) and right-handed (RH) SM fermions, as single-mediator accounts for all the low-energy data. Its simultaneous explanations of the $R_{K^*}$ and $R_{D^{(*)}}$ anomalies only require the LH couplings between second and third-generation quarks and leptons. However, the LH couplings cannot produce large enough muon magnetic moment for $(g-2)_\mu$ anomaly. In this work, we further extend non-zero RH couplings of $U_1$ such that it substantially enhances the contribution to $(g-2)_\mu$. In addition, we will also consider $U(1)_{B_3-L_2}$ gauge boson $(X)$ [41][43] to improve our fit to the experimental data. We explored the plausible parameter space and search the common solution for both $B$-meson and $(g-2)_\mu$ anomalies.

II. MODEL

In this section, we set our theoretical model to explain $B$-anomalies and $(g-2)_\mu$.

A. Vector Leptoquark $U_1 = (3, 1)_{2/3}$

Leptoquark is a natural candidate of new physics linking quark sector and lepton sector [16]. In particular, we focus on the $U_1 = (3, 1)_{2/3}$ weak singlet vector leptoquark because it could provide simultaneous explanations for $R_{K^*}$ and $R_{D^{(*)}}$ anomalies with its coupling with LH fermions [30][33]. However, the general Lagrangian includes the $U_1$ couplings to both LH and RH fermion under the SM gauge symmetry. Including the most relevant interactions with the LH couplings to the 2nd and 3rd generations of leptons and quarks and also the RH couplings, we consider the model Lagrangian:

$$\mathcal{L} \supset U_{1\mu} \sum_{i,j=1,2,3} \left[ x_{ij}^L (\bar{d}_L^i \gamma^\mu v_L^j) + (V_{CKM} x_{ij}^L)_{ij} (\bar{u}_L^i \gamma^\mu \nu_L^j) + x_{ij}^R (\bar{d}_R^i \gamma^\mu e_R^j) \right] + h.c. $$

1 See [18][29] for some of the earlier attempts to account $(g-2)$ and also various anomalies and possible experimental probes.
2 When we are finishing our paper, similar idea has been considered in [44]. Unfortunately, however, they have missed some relevant constraints from low-energy experiments. In particular, the experimental $B_s \rightarrow \mu\mu$ data conflicts with their preferred parameter region.
We adopt the real parts of CKM and PMNS matrices which is conveniently written as \([16]\)

\[
V_{\text{CKM}}x_L U_{\text{PMNS}}^* = \begin{pmatrix}
0.974 & 0.225 & 0.001 \\
-0.224 & 0.974 & 0.042 \\
0.009 & -0.041 & 0.999
\end{pmatrix}
\begin{pmatrix}
x_{L1} & x_{L2} & x_{L3} \\
x_{L2} & x_{L3} & x_{L1} \\
x_{L3} & x_{L1} & x_{L2}
\end{pmatrix}
\begin{pmatrix}
0.821 & 0.551 & -0.150 \\
-0.307 & 0.600 & 0.739 \\
0.481 & -0.580 & 0.657
\end{pmatrix}
\]

above expression omits the imaginary parts in \(V_{\text{CKM}}\), but we adopt the full CKM parameterization from Ref \([15]\), in our numerical computation. The couplings \(x_{Lj}\) to the first generation leptons and quarks are strongly constrained from \(\mu - e\) conversion on nuclei, and atomic parity violation on \(\mathcal{B}(K \rightarrow \pi \nu \bar{\nu})\), therefore we simply set them zero.

The most relevant Wilson coefficients of the effective Lagrangian in \(R_K(\ast)\), \(R_D(\ast)\), and \(\mathcal{B}(B_s \rightarrow \mu \mu)\) are \(C_{9}^{\mu \mu} = -C_{10}^{\mu \mu}\) \([30]\), and they are induced from the LH couplings

\[
C_{9}^{\mu \mu} = -C_{10}^{\mu \mu} = -\frac{\pi v^2}{V_{tb}V_{ts}^*}\frac{x_{L}^{22} (x_{L}^{32})^*}{m_{U_1}^2},
\]

where \(v = 246\) GeV is the vacuum expectation value of the Higgs, and \(\alpha_{\text{EM}}\) is the fine structure constant. From the fit to \(R_K\), \(R_K\)- and \(\mathcal{B}(B_s \rightarrow \mu \mu)\) data, we find the parameter window for couplings and the mass of \(U_1\)

\[
C_{9}^{\mu \mu} = -C_{10}^{\mu \mu} \subseteq [-0.85, -0.50] \Rightarrow -\frac{x_{L}^{22} (x_{L}^{32})^*}{m_{U_1}^2} \subseteq [0.83, 1.41] \times 10^{-3} \text{ TeV}^{-2}.
\]

The interactions from Eq.(8) also give rise to the effective coefficient \([30]\)

\[
g_{\nu_L} = \frac{v^2}{2m_{U_1}^2} \left( x_{L}^{22} \right)^* \left[ x_{L}^{22} + \frac{V_{cs}}{V_{cb}} x_{L}^{32} + \frac{V_{cd}}{V_{cb}} x_{L}^{32} \right],
\]

and contribute to \(b \rightarrow c\ell\bar{\nu}\). It becomes one solution for the \(R_D\) and \(R_{D\ast}\) anomalies, and the 1\(\sigma\) region for \(b \rightarrow c\tau\bar{\tau}\) requires

\[
g_{\nu_L} \subseteq [0.09, 0.13] \Rightarrow \frac{(V_{cs} x_{L}^{23} + V_{cb} x_{L}^{33}) (x_{L}^{33})^*}{m_{U_1}^2} \subseteq [0.12, 0.18] \text{ TeV}^{-2}.
\]

The RH couplings \(x_R\), combining with \(x_{L}^{22}\) and \(x_{L}^{32}\), contribute to the Wilson coefficients \((C_S)' = (C_P)'\), thus their magnitudes are bounded by the \(B \rightarrow K\tau\bar{\mu}\) and \(B_s \rightarrow \mu \mu\) data, which hampers from generating large enough muon magnetic dipole for \((g - 2)_\mu\) anomaly. Therefore, we are motivated to further extend our model.

### B. Vector Leptoquark with \(U(1)_{B_3 - L_2}\) X boson

In the following comprehensive analysis, with the preferred parameter region for \(R_K(\ast)\) and \(R_D(\ast)\) anomalies, the \(U_1\) leptoquark itself may still not be able to provide large enough contribution for \((g - 2)_\mu\) anomaly. Therefore, we invoke the additional particle \(X\), \(U(1)_{B_3 - L_2}\) gauge boson to enhance the contribution, which is alternatively plausible candidate for \(R_K\) anomaly and significantly change the preferred parameter space. We take the most relevant interactions with \(X\) boson in the effective Lagrangian

\[
\mathcal{L}_{\text{eff}}^X \supseteq -g_X \bar{\nu}_\mu \gamma^\alpha \mu X_\alpha - g_X \bar{\nu}_\mu \gamma^\alpha P_L \nu_\mu X_\alpha
+ \frac{g_X}{3} u_L \gamma^\alpha \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} u_L X_\alpha + \frac{g_X}{3} d_L \gamma^\alpha \begin{pmatrix} |V_{td}|^2 & V_{ts} V_{td}^* & V_{tb} V_{td}^* \\ V_{td} V_{ts}^* & |V_{ts}|^2 & V_{tb} V_{ts}^* \\ V_{td} V_{tb}^* & V_{ts} V_{tb}^* & |V_{tb}|^2 \end{pmatrix} d_L X_\alpha
+ \frac{1}{2} m_X^2 X_\mu X_\mu
\]

\[= V_{\text{CKM}} V_{\theta_3} V_{\text{CKM}}\]
where $u_L^T \equiv (u_L, c_L, t_L)$ and $d_L^T \equiv (d_L, s_L, b_L)$. Here, we chose the configuration that the left-handed down-quark mixing contributes entire CKM matrix to avoid the constraint from $D - \bar{D}$ mixing. We assume that the $m_X$ is induced by a new Higgs mechanism but the additional contribution from the new Higgs can be neglected. The flavor-changing neutral current (FCNC) in down quarks from the $X$ boson contributes to $B \rightarrow K\mu\mu$ and $B_s \rightarrow \mu\mu$. At the same time, the muon coupling modifies the $(g - 2)_\mu$. In the following, we assume the $U(1)_{B_3 - L_2}$ are broken under energy scale $\mathcal{O}(100)$ GeV, which justifies the non-zero low-energy effective couplings at $\mathcal{O}(m_b)$ scale in Eq. (8).

### III. LOW-ENERGY OBSERVABLES

In this section, we summarize the low-energy observables with the $U_1$ vector leptoquark contributions.

#### A. $(g - 2)_\mu$

The previous result of Muon $(g - 2)$ experiment with the BNL E821 was $3.7\sigma$ from the SM. After the FNAL result, the difference between experiment and SM has become $14, 46$, which justifies the non-zero low-energy effective couplings at $\mathcal{O}(m_b)$ scale in Eq. (8).

\[ \Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}, \]

which is a deviation of $4.2\sigma$ significance from the SM prediction and this enhances the motivation for SM extensions for new couplings with leptons. In this paper, we present the single leptoquark which is described in Sec. II A to explain not only $(g - 2)_\mu$ anomaly but also $B$-meson anomalies.

For the large mass of $U_1$ leptoquark, it contributes to the $(g - 2)_\mu$ anomaly as

\[ \Delta a_\mu = \frac{N_c}{16\pi^2} \sum_i \left[ 2Q_U \tilde{\kappa}_Y \text{Im}(x_L^2(x_R^2)^*) \frac{m_d^2 m_\mu}{m_{U_1}^2} \left( \ln \left( \frac{\Lambda_{UV}^2}{M_{U_1}^2} \right) + \frac{5}{2} \right) + 2\text{Re}(x_L^2(x_R^2)^*) \frac{m_d m_\mu}{m_{U_1}^2} (2Q_d + Q_{U_1}) \left( (1 - \kappa_Y) \text{ln} \left( \frac{\Lambda_{UV}^2}{M_{U_1}^2} \right) + \frac{1 - 5\kappa_Y}{2} \right) \right] - \left( |x_L^2|^2 + |x_R^2|^2 \right) \frac{m_\mu^2}{m_{U_1}^2} \left( \frac{4}{3} Q_d + Q_{U_1} \left( (1 - \kappa_Y) \text{ln} \left( \frac{\Lambda_{UV}^2}{M_{U_1}^2} \right) - \frac{1 + 9\kappa_Y}{6} \right) \right) \right], \]

If $\kappa_Y \neq 1$ and $\tilde{\kappa}_Y \neq 0$, the dipole moment exhibits logarithmic dependence on the cut-off scale $\Lambda_{UV}$ not far above the leptoquark mass. So, the leptoquark contribution to $(g - 2)_\mu$ anomaly becomes

\[ \Delta a_\mu = \frac{N_c}{16\pi^2} \sum_i \left[ 2\text{Re}(x_L^2(x_R^2)^*) \frac{m_d m_\mu}{m_{U_1}^2} (2Q_d + Q_{U_1}) \left( \frac{1 - 5\kappa_Y}{2} \right) \right] - \left( |x_L^2|^2 + |x_R^2|^2 \right) \frac{m_\mu^2}{m_{U_1}^2} \left( \frac{4}{3} Q_d + Q_{U_1} \left( - \frac{1 + 9\kappa_Y}{6} \right) \right), \]

where we use $\kappa_Y = 1$, $\tilde{\kappa}_Y = 0$, $N_C = 3$, $Q_b = -1/3$, and $Q_{U_1} = 2/3$. We handle the renomalization group running from leptoquark scale down to muon mass by evaluating the quark masses at $\mathcal{O}(\text{TeV})$ scale in Eq. [16], e.g. $m_b(\text{TeV}) \simeq 2.4$ GeV [8].

The $X$ boson also contributes to $(g - 2)_\mu$ as

\[ \Delta a_\mu^X = \frac{g_X^2}{8\pi^2} \int_0^1 dz \frac{2s(1 - z)^2}{(1 - z)^2 + z(m_X / m_\mu)^2} \approx (3g_X^2 / 4\pi)(m_\mu^2 / m_X^2) \text{ when } m_X \gg m_\mu \]

\[ \approx 2.7 \times 10^{-11} \times \left( \frac{9 \text{ GeV}}{0.01} \right)^2 \left( \frac{100 \text{ GeV}}{m_X} \right)^2, \]

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3 We use the same $\kappa$ and $\tilde{\kappa}$ parameters defined in Ref. [5], which are the tri-gauge boson couplings between $U_1$ leptoquarks and $B_\mu$ gauge boson from $U(1)_Y$. In Eq. [16], we set $\kappa = 1$ and $\tilde{\kappa} = 0$, in such a way, the dipole moment becomes independent on the logarithmic term and UV theory.
thus it is negligible compared to the experimental value for $m_X \simeq 100$ GeV and $g_X \simeq 0.01$. However, with $X$ boson contributions, the preferred parameter space for $B$-meson anomalies would move and our fit to the data can be significantly improved as we will describe below.

B. $R_K^\ast, R_D^\ast$

To explain the experimental result of $R_K^\ast$, it requires the Wilson coefficients as

$$\Delta C_9^{\mu\mu}|_{\text{exp}} = -0.40 \pm 0.12, \quad \Delta C_9^{U}|_{\text{exp}} = -0.50 \pm 0.38,$$

with correlation $-0.5\ ^{32, 33, 47}$ between them. And the ratio of the SM predictions and experimental observation is

$$\frac{R_D^{\text{exp}}}{R_D^{\text{SM}}} = 1.14 \pm 0.10, \quad \frac{R_D^{\text{exp}}}{R_D^{\text{SM}}} = 1.14 \pm 0.05,$$

with correlation $-0.37\ ^{33}$. For the $U_1$ leptoquark, the contribution to Wilson coefficients is $^{32, 33}$

$$\Delta C_9^{\mu\mu} = -\Delta C_9^{10} = -\frac{4\pi^2}{c^2} \frac{v^2}{m_{U_1}^2} x_L^L (x_R^L)^* \frac{V_{tb}^*}{V_{tb}},$$

$$\Delta C_9^{U} \simeq -\frac{1}{V_{tb} V_{ts}^*} \frac{2}{3} \frac{v^2}{m_{U_1}^2} x_L^L (x_R^L)^* \ln \left(\frac{m_b^2}{m_{U_1}^2}\right),$$

where $\Delta C_9^{U}$ is the lepton-universal contribution to $b \to s\ell\ell$ and originates from the $x_L^L$ contributing through a log-enhanced photon penguin diagram $^{33, 10}$. Similarly, the $U_1$ contribution to $R_D$ and $R_D^*$ are $^{33}$$

$$\frac{R_D}{R_D^{\text{SM}}} \simeq \left[ 1 + \frac{v^2}{m_{U_1}^2} \text{Re} \left\{ (x_L^L - 1.5\eta_S (x_R^L)^*) \frac{(V_{tb} V_{ts}^*)^2}{V_{cb}} \frac{(V_{cb} x_L^L + V_{cs} x_{R_1}^L + V_{cd} x_{R_3}^L)}{V_{cb}} \right\} \right],$$

$$\frac{R_D^*}{R_D^{\text{SM}}} \simeq \left[ 1 + \frac{v^2}{m_{U_1}^2} \text{Re} \left\{ (x_L^L - 0.14\eta_S (x_R^L)^*) \frac{(V_{tb} V_{ts}^*)^2}{V_{cb}} \frac{(V_{cb} x_L^L + V_{cs} x_{R_1}^L + V_{cd} x_{R_3}^L)}{V_{cb}} \right\} \right],$$

where $\eta_S \simeq 1.8$ accounts for the running of the scalar operator from $m_{U_1} = 4$ TeV to $m_b$.

The additional correction from $X$ boson to the $\Delta C_9^{\mu\mu}$ is given as $^{43}$

$$\Delta C_9^X = \left( \frac{g_X^2}{3} \frac{V_{ts} V_{tb}^*}{m_X} \right) \left( \frac{36 \text{TeV}}{m_X} \right)^2 \approx -0.18 \times \left( \frac{g_X}{0.01} \right)^2 \left( \frac{100 \text{ GeV}}{m_X} \right)^2$$

It is important to notice that the negative value from $V_{ts}$ makes it trend toward the experimental value for $B$-meson anomalies but does not contribute to $(g - 2)_\mu$.

C. $B^- \to \tau^- \nu_\tau, B^+ \to \tau^+ \nu_\tau$

The bound from observation $^{48, 49}$ and SM prediction for $B_c \to \tau \nu$ are $^8\ ^{50}$

$$B(B_c^- \to \tau^- \nu_\tau)_{\text{exp}} \leq 0.60$$

$$B(B_c^- \to \tau^- \nu_\tau)_{\text{SM}} = (2.21 \pm 0.09) \times 10^{-2}$$

(26)

The proportion of experimental value and SM prediction for $B^+ \to \tau^+ \nu_\tau$ is $^8$

$$\frac{B(B^+ \to \tau^+ \nu_\tau)_{\text{exp}}}{B(B^+ \to \tau^+ \nu_\tau)_{\text{SM}}} = 1.30 \pm 0.29.$$
And the $U_1$ leptoquark contributions to each of observables are \cite{8}

\begin{equation}
\frac{B(B_c^+ \to \tau^- \bar{\nu}_\tau)}{B(B_c^+ \to \tau^- \nu_\tau)_{\text{SM}}} = 1 - \frac{(V_{cdx_L}^{13} + V_{csx_L}^{23} + V_{cbx_R}^{33})}{V_{cb}} \frac{v^2}{m_{U_1}^2} \left( \frac{(x_L^{33})^*}{2} + \frac{(x_R^{33})^* m_{B_s}^2}{m_\tau (m_b + m_c)} \right)^2,
\end{equation}

\begin{equation}
\frac{B(B_c^0 \to \tau^+ \nu_\tau)}{B(B_c^0 \to \tau^+ \bar{\nu}_\tau)_{\text{SM}}} = 1 - \frac{(V_{udx_L}^{13} + V_{usx_L}^{23} + V_{ubx_R}^{33})}{V_{ub}} \frac{v^2}{m_{U_1}^2} \left( \frac{(x_L^{33})^*}{2} + \frac{(x_R^{33})^* m_{B_s}^2}{m_\tau (m_b + m_c)} \right)^2.
\end{equation}

D. $B_s^0 \to \tau^+ \tau^-$, $B_s^0 \to \mu^+ \mu^-$, $B_s^0 \to \tau^+ \mu^+$ and $B^+ \to K^+ \tau^+ \tau^-$

The experimental value from LHCb is \cite{51} and SM prediction is \cite{52}

\begin{equation}
B(B_s^0 \to \tau^+ \tau^-)_{\text{exp}} < 6.8 \times 10^{-3} \text{ at } 95\% \text{ C.L.},
\end{equation}

\begin{equation}
B(B_s^0 \to \tau^+ \tau^-)_{\text{SM}} = (7.73 \pm 0.49) \times 10^{-7}.
\end{equation}

And the related contribution for $U_1$ leptoquark is \cite{8}

\begin{equation}
\frac{B(B_s^0 \to \tau^+ \tau^-)}{B(B_s^0 \to \tau^+ \bar{\nu}_\tau)_{\text{SM}}} = \frac{16\pi^4}{e^2(C_{10}^{SM})^2 m_{U_1}^2 m_W^2} \left( \frac{(x_L^{33})^* x_R^{23} - (x_R^{33})^* x_L^{23}}{V_t V_b} \right)^2 \left( 1 - \frac{4m_{B_s}^2}{m_{B_s}^2} \right)
+ \frac{4\pi^2}{e^2 C_{10}^{SM}} \frac{v^2}{m_{U_1}^2} \left( \frac{(x_L^{33})^* x_R^{23} + (x_R^{33})^* x_L^{23}}{V_t V_b} - \frac{m_{B_s}^2 (x_R^{33})^* x_R^{23} + (x_R^{33})^* x_L^{23}}{m_\tau (m_b + m_c)} \right)^2,
\end{equation}

where $C_{10}^{SM} \approx -4.1$ which we use for a normalization such that the SM value for the Wilson coefficient \cite{53}.

The ratio between the SM prediction and experimental value is \cite{8, 52}

\begin{equation}
\frac{B(B_s^0 \to \mu^+ \mu^-)_{\text{exp}}}{B(B_s^0 \to \mu^+ \mu^-)_{\text{SM}}} = 0.73^{+0.13}_{-0.10}.
\end{equation}

And the following $U_1$ vector leptoquark and $X$-boson total contribution is written by \cite{8, 51}

\begin{equation}
\frac{B(B_s^0 \to \mu^+ \mu^-)}{B(B_s^0 \to \mu^+ \bar{\nu}_\tau)_{\text{SM}}} = \frac{16\pi^4}{e^2(C_{10}^{SM})^2 m_{U_1}^2 m_W^2} \left( \frac{(x_L^{33})^* x_R^{23} - (x_R^{33})^* x_L^{23}}{V_t V_b} \right)^2
+ \frac{4\pi^2}{e^2 C_{10}^{SM}} \frac{v^2}{m_{U_1}^2} \left( \frac{(x_L^{33})^* x_R^{23} + (x_R^{33})^* x_L^{23}}{V_t V_b} - \frac{m_{B_s}^2 (x_R^{33})^* x_R^{23} + (x_R^{33})^* x_L^{23}}{m_\tau (m_b + m_c)} \right)
- \frac{\alpha}{2\pi \sin^2 \theta_W} Y \left( \frac{m_{B_s}^2}{m_W^2} \right)^{-1} \left( \frac{2g_3^2 m_Z^2}{3g_2^2 m_X^2} \right)^2,
\end{equation}

where the last term in the second absolute bracket comes from the $X$-boson. Here we take $Y(m_B^2/m_W^2) = 1.05$, and $g \approx 0.652$ the $SU(2)_L$ coupling constant.

LHCb search on $B_s^0 \to \tau^+ \mu^+$ provides an upper limit

\begin{equation}
B(B_s^0 \to \tau^+ \mu^+)_{\text{exp}} < 2.1 \times 10^{-5},
\end{equation}

at 95 \% confidence level \cite{55}. SM prediction of this branching fraction is extremely small as $O(10^{-54})$ \cite{56}. The expression of $U_1$ contribution to $B_s^0 \to \tau^+ \mu^+$ is \cite{33}

\begin{equation}
B(B_s^0 \to \tau^+ \bar{\nu}_\tau) = \frac{1}{\Gamma_{B_s}} \frac{m_{B_s} f_{B_s} G_F^2}{8\pi} \frac{\alpha}{m_{B_s}^2} \left( 1 - \frac{m_{B_s}^2}{m_{B_s}^2} \right)^2
\times \frac{v^4}{4m_{U_1}^2} \left| x_L^{22} (x_R^{33})^* - \frac{2\eta_5 m_{B_s}^2}{m_\tau (m_b + m_c)} x_L^{22} (x_R^{33})^* \right|^2.
\end{equation}
where \( G_F = 1.166 \times 10^{-5} \) GeV\(^{-2}\) is the Fermi constant, \( f_{B_s} = 0.225 \) GeV \(^[67]\) is the lepton decay constant of \( B_s^0 \), and \( \Gamma_{B_s} = 4.34 \times 10^{-13} \) GeV is the total width of \( B_s^0 \).

BaBar experiment measured the branching fraction \(^{[58]}\)

\[
B(B^+ \to K^+\tau^+\tau^-)_{\text{exp}} = (1.31 \pm 0.71) \times 10^{-3} .
\]

with an upper limit of \( \text{Br}(B^+ \to K^+\tau^+\tau^-) < 2.25 \times 10^{-3} \) at the 90% confidence level. The expression of \( U_1 \) leptoquark contribution to this process is given by \(^{[33]}\)

\[
B(B^+ \to K^+\tau^+\tau^-) \approx 1.5 \times 10^{-7} + 10^{-3} \frac{\nu^2}{2\mu_{U_1}^2} \left[ 1.4 \text{Re} \left( x_L^{23} (x_L^{33})^* \right) - 3.3 \text{Re} \left( x_L^{23} (x_R^{33})^* \right) \right] + \frac{\nu^4}{4m_{U_1}^4} \left| x_L^{33} \right|^2 + 16.4 \text{Re} \left( x_R^{33} (x_L^{33})^* \right) + 95.0 \left| x_R^{33} \right|^2 .
\]

where \( \nu = 246 \) GeV is the electroweak vacuum expectation value.

**E. \( B^+ \to K^+\tau^+\mu^- \), \( B^+ \to K^+\tau^-\mu^+ \)**

From BaBar experiment, we obtain upper limits \(^{[59]}\)

\[
\begin{align*}
B(B^+ \to K^+\tau^+\mu^-) &< 2.8 \times 10^{-5} , \\
B(B^+ \to K^+\tau^-\mu^+) &< 4.5 \times 10^{-5} . 
\end{align*}
\]

at 90% confidence level. The leptoquark contribution is given by \(^{[33]} \)[60]\]

\[
\begin{align*}
B(B^+ \to K^+\tau^+\mu^-) &\approx \frac{\nu^4}{4m_{U_1}^4} \left| x_L^{23} \right|^2 \left[ 8.3 \left| x_L^{33} \right|^2 + 155.2 \left| x_R^{33} \right|^2 - 42.3 \text{Re} \left( (x_L^{33})^* x_R^{33} \right) \right] , \\
B(B^+ \to K^+\tau^-\mu^+) &\approx \frac{\nu^4}{4m_{U_1}^4} \left| x_L^{23} \right|^2 \left( x_R^{33} (x_L^{33})^* \right)^2 .
\end{align*}
\]

**F. \( \tau \to \mu\gamma \), \( \mu\phi \) and LFU in \( \tau \) decays**

Due to its sizeable couplings to muon and tau leptons, \( U_1 \) leptoquark can significantly affect the Lepton-flavor-violation in \( \tau \) decays. The experimental upper limits are \(^{[61] \,[62]}\)

\[
\begin{align*}
B(\tau \to \mu\gamma) &< 3.0 \times 10^{-8} , \\
B(\tau \to \mu\phi) &< 5.1 \times 10^{-8} .
\end{align*}
\]

at 90% confidence level. In addition, the LFU in the decay of charged leptons can give stringent bounds on the leptoquark couplings. The experimentally measured values are \(^{[61] \,[63]}\)

\[
\begin{align*}
(g_{\tau}/g_{\mu})_{\text{exp}} &= 1.0000 \pm 0.0014 , \\
(g_{\tau}/g_e)_{\text{exp}} &= 1.0010 \pm 0.0015 , \\
(g_{\tau}/g_{\pi})_{\text{exp}} &= 0.9961 \pm 0.0027 , \\
(g_{\tau}/g_{\kappa})_{\text{exp}} &= 0.9860 \pm 0.0070 .
\end{align*}
\]

For the \( U_1 \) leptoquark contribution to \( B(\tau \to \mu\gamma) \), we adopt complete formula of the decay width \(^{[64]}\)

\[
\Gamma(\tau \to \mu\gamma) = \frac{\alpha_{\text{EM}} \left( m_{\tau}^2 - m_{\mu}^2 \right)^3}{4m_{\tau}^2} \left( |x_L^{32}|^2 + |x_R^{32}|^2 \right) ,
\]

\[\text{where } \alpha_{\text{EM}} = \frac{e^2}{4\pi} .\]
where

$$\sigma_{L}^{32} = -\frac{iN_e}{16\pi^2m_{U_1}^2} \sum_{k=1,2,3} \left\{ \frac{2}{3} \left[ \left( x_{R}^2 x_{R}^3 m_{\tau} + x_{L}^2 x_{L}^3 m_{\mu} \right) g(t_k) + x_{R}^2 x_{L}^3 m_{d_k} j(t_k) \right] \right. $$

$$- \frac{1}{3} \left[ \left( x_{R}^2 x_{R}^3 m_{\tau} + x_{L}^2 x_{L}^3 m_{\mu} \right) f(t_k) + x_{R}^2 x_{L}^3 m_{d_k} h(t_k) \right] \right\} $$

$$\sigma_{R}^{32} = -\frac{iN_e}{16\pi^2m_{U_1}^2} \sum_{k=1,2,3} \left\{ \frac{2}{3} \left[ \left( x_{L}^2 x_{R}^3 m_{\tau} + x_{L}^2 x_{R}^3 m_{\mu} \right) g(t_k) + x_{L}^2 x_{R}^3 m_{d_k} j(t_k) \right] \right. $$

$$- \frac{1}{3} \left[ \left( x_{L}^2 x_{R}^3 m_{\tau} + x_{L}^2 x_{R}^3 m_{\mu} \right) f(t_k) + x_{L}^2 x_{R}^3 m_{d_k} h(t_k) \right] \right\} $$

(49)

with \( t_k \equiv m_{d_k}^2/m_{U_1}^2 \) and \( f, g, h, j \) are loop functions \[64\].

For \( B(\tau \rightarrow \mu \phi) \), it is given by \[33\] \[65\]

$$B(\tau \rightarrow \mu \phi) = \frac{1}{\Gamma_\tau} f_\phi^2 G_F^2 \int m^3 \left( 1 - \frac{m^2}{m^2} \right)^2 \left( 1 + \frac{2m^2}{m^2} \right) \frac{\nu^4}{4m_{U_1}^4} \left| x_{L}^{23}(x_{L}^{22})^* \right|^2, $$

(50)

where \( \phi \) is the \( s\bar{s} \) vector meson with \( m_\phi = 0.225 \text{ GeV} \), \( m_\phi = 1.019 \text{ GeV} \) \[65\]. For LFU in lepton decays, we use the expression \[33\] \[63\]

$$\left( \frac{g_\tau}{g_\mu} \right)_{\ell, \pi, K} \simeq 1 - 0.08 \times \frac{(x_{L}^{33})^2 \nu^2}{4m_{U_1}^2}. $$

(51)

More specifically, they can be written in terms of the effective Lagrangian for leptonic decay \[63\]

$$\mathcal{L}_{\ell \rightarrow \ell' \nu \bar{\nu}} = -\frac{4G_F}{\sqrt{2}} \left[ \left( C^\nu_{LL}^{V} \right)_{\rho_{\alpha \beta}} \left( \rho_{\bar{\nu} \nu} \rho_{\alpha \beta} \right) \left( \bar{\ell}^\nu_{L} \ell'^{\nu}_{L} \right) + \left[ C^\nu_{LR}^{V} \right]_{\rho_{\alpha \beta}} \left( \rho_{\bar{\nu} \nu} \rho_{\alpha \beta} \right) \left( \bar{\ell}^\nu_{L} \ell'^{\nu}_{L} \right) \right], $$

(52)

and for hadronic decay \[63\]

$$\mathcal{L}_{\tau \rightarrow h

result in the following expressions for couplings with \( \tau \) and \( \mu \)

$$\left( \frac{g_\tau}{g_\mu} \right)_\ell = \left[ \sum_{\rho} \left( \delta_{\rho 3} \delta_{\tau 1} + \left[ C^\nu_{LL}^{V} \right]_{\rho_{\alpha \beta}} \left[ C^\nu_{LR}^{V} \right]_{\rho_{\alpha \beta}} \right) \right]^{1/2} \left[ \sum_{\rho} \left( \delta_{\rho 2} \delta_{\tau 1} + \left[ C^\nu_{LL}^{V} \right]_{\rho_{\alpha \beta}} \left[ C^\nu_{LR}^{V} \right]_{\rho_{\alpha \beta}} \right) \right]^{1/2} \right. $$

$$\left( \frac{g_\tau}{g_{\mu_\pi}} \right) = \left[ \sum_{\rho} \left( \delta_{\rho 3} \delta_{\tau 1} + \left[ C^\nu_{LL}^{V} \right]_{\rho_{\alpha \beta}} \left[ C^\nu_{LR}^{V} \right]_{\rho_{\alpha \beta}} \right) \right]^{1/2} \left[ \sum_{\rho} \left( \delta_{\rho 2} \delta_{\tau 1} + \left[ C^\nu_{LL}^{V} \right]_{\rho_{\alpha \beta}} \left[ C^\nu_{LR}^{V} \right]_{\rho_{\alpha \beta}} \right) \right]^{1/2} \right. $$

$$\left( \frac{g_\tau}{g_{\mu_\pi}} \right)_K = \left[ \sum_{\rho} \left( \delta_{\rho 3} \delta_{\tau 1} + \left[ C^\nu_{LL}^{V} \right]_{\rho_{\alpha \beta}} \left[ C^\nu_{LR}^{V} \right]_{\rho_{\alpha \beta}} \right) \right]^{1/2} \left[ \sum_{\rho} \left( \delta_{\rho 2} \delta_{\tau 1} + \left[ C^\nu_{LL}^{V} \right]_{\rho_{\alpha \beta}} \left[ C^\nu_{LR}^{V} \right]_{\rho_{\alpha \beta}} \right) \right]^{1/2} \right. $$

(54)

For \( U_1 \) leptoquark, by using the Fierz transformation in Eq. (5),

$$[\bar{u}_{1L} \gamma^\mu u_{2L}][\bar{u}_{3L} \gamma^\mu u_{4L}] = -[\bar{u}_{1L} \gamma^\mu u_{4L}][\bar{u}_{3L} \gamma^\mu u_{2L}], $$
TABLE I. The list of observables for the $\chi^2$ scanning with the measured values and predictions from SM. The equations of the $U_1(+X)$ model are also referenced.

| Observable | Experiment | SM predict | $U_1(+X)$ predict |
|------------|------------|------------|-------------------|
| $R_{D^{(*)}}$ | $\frac{R_D^{exp}}{R_D^{SM}} = 1.14 \pm 0.10$, $\frac{R_{D^{(*)}}^{exp}}{R_{D^{(*)}}^{SM}} = 1.14 \pm 0.05$ | | (24) |
| $\Delta c^\mu_{9} = -\Delta c^\mu_{10} (R_{K^{(*)}})$ | $-0.40 \pm 0.12$ | 0 | (22) |
| $\Delta c^\Upsilon_{9} = -0.50 \pm 0.38$ | 0 | | (25) |
| $B^{-}_{c} \rightarrow \tau^{-} \bar{\nu}_{\tau}$ | $\leq 0.60$ | $(2.21 \pm 0.09) \times 10^{-2}$ | (28) |
| $B^{+} \rightarrow \tau^{+} \nu_{\tau}$ | $(1.09 \pm 0.24) \times 10^{-4}$ | $(8.8 \pm 0.6) \times 10^{-5}$ | (29) |
| $B^{0}_{s} \rightarrow \tau^{+} \tau^{-}$ | $< 6.8 \times 10^{-3}$ | $(7.73 \pm 0.49) \times 10^{-7}$ | (32) |
| $B^{0}_{s} \rightarrow \mu^{+} \mu^{-}$ | | $BR(B^{0}_{s} \rightarrow \mu^{+} \mu^{-})^{exp} = 0.73^{+0.13}_{-0.10}$ | (34) |
| $B^{0}_{s} \rightarrow \tau^{+} \mu^{+}$ | $< 2.1 \times 10^{-5}$ | | (36) |
| $B^{+} \rightarrow K^{+} \tau^{+} \tau^{-}$ | $(1.31 \pm 0.71) \times 10^{-3}$ | $(1.20 \pm 0.12) \times 10^{-7}$ | (41) |
| $B^{+} \rightarrow K^{+} \tau^{+} \mu^{-}$ | $\leq 2.8 \times 10^{-5}$ | | (41) |
| $B^{+} \rightarrow K^{+} \tau^{-} \mu^{+}$ | $\leq 4.5 \times 10^{-5}$ | | (41) |
| $\tau \rightarrow \mu \gamma$ | $< 3.0 \times 10^{-8}$ | | (48) |
| $\tau \rightarrow \mu \phi$ | $< 5.1 \times 10^{-8}$ | | (50) |
| LFU in $\tau$ decay | | $(g_{\tau}/g_{\mu})^{exp} = 1.0010 \pm 0.0015$ | |
| | | $(g_{\tau}/g_{\mu})^{SM} = 0.9991 \pm 0.0027$ | (54) |
| | | $(g_{\tau}/g_{\mu})^{SM} = 0.9880 \pm 0.0070$ | |

we replace the Wilson coefficients as

$$
\begin{align*}
[ C^{V,LL}_{\nu e u \nu} ]_{\rho \sigma ij} & \leftrightarrow - \frac{2v^2}{4m^2_{U_1}} (V_{CKM} x_L U^*_{PMNS} )_{j \rho} (x_L)_{i \sigma} , \\
[C^{S,RL}_{\nu e u \nu}] & = [ C^{V,LL}_{\nu e e} ] = [ C^{V,L R}_{\nu e e} ] = 0 .
\end{align*}
$$

(55)
IV. PARAMETER SCANNING

We perform the $\chi^2$ parameter scanning, with the values of all 17 observables constituting the $\chi^2$ listed in Table I, which includes anomalies of $R_{K^{(*)}}$, $R_{D^{(*)}}$, constraints from other $B$-meson decay channels, and constraints from $\tau$ decays. Under the null hypothesis (SM only), we have $\chi^2_{SM} = 26.0$ along with $p_{SM} = 0.074$ where $p_{SM}$ is the $P$-value of the null hypothesis. We compare this result with the following three scenarios:

- **scan-1**: $P_{scan-1} = (x_{22}^L, x_{23}^L, x_{32}^L, x_{33}^L)$, with $m_{U_1} = 2.5$ TeV.
  
  Results are in Fig. 1

- **scan-2**: $P_{scan-2} = P_{scan-1} \oplus x_{32}^R$, with $m_{U_1} = 2$ TeV.
  
  Results are in Fig. 2

- **scan-3**: $P_{scan-3} = P_{scan-2} \oplus g_X$, with $m_{U_1} = 2$ TeV, and $m_X = 100$ GeV.
  
  Results are in Fig. 3

![Scan-1 Diagram](image1)

**FIG. 1.** Scan-1: The region satisfies $\Delta \chi^2 \equiv \chi^2 - \chi^2_{min,1} \leq 2.3$, and $\chi^2_{min,1} = 9.23$.

For scan-1, we choose a set of relevant couplings, $(x_{22}^L, x_{23}^L, x_{32}^L, x_{33}^L)$, that is sufficient to explain the $B$-meson anomalies and satisfies all the low-energy observables. It gives rise to the best fit $\chi^2_{min,1} = 9.23$ and $p_1 = 0.755$ with $(x_{22}^L, x_{23}^L, x_{32}^L, x_{33}^L) = (7.90 \times 10^{-2}, 0.328, -3.83 \times 10^{-2}, 0.862)$ and $m_{U_1} = 2$ TeV. Figure 1 shows the 1σ region for the scan-1. The favored region of the $(x_{23}^L, x_{33}^L)$ plane, shown in the upper-left panel of Fig. 1, is mainly determined by the $R_{K^{(*)}}$, meanwhile the $R_{D^{(*)}}$ dictates the favored region of the $(x_{22}^L, x_{23}^L)$ plane. There is contribution to $\Delta a_\mu$ from the LH couplings, as shown in the upper-right panels of Fig. 1. However, it is not large enough to explain the recent measurement from Fermilab, thus providing the motivation to turn on the RH coupling in the scan-2.

In the scan-2 ($\chi^2_{min,2} = 9.06$ and $p_2 = 0.698$), according to Eq.(16), we found the most efficient way to enhance $\Delta a_\mu$ is to use $x_{32}^R$, due to the fact that the multiplicity of $(x_{23}^L, x_{32}^L)$ has milder mass suppression factor ($m_{b}/m_{U_1}$) than that of pure LH couplings or RH couplings. In the $(x_{32}^R, \Delta a_\mu)$ and $(x_{32}^L, \Delta a_\mu)$ planes of Fig. 2, they show that the value of $\Delta a_\mu$ can be increased by more than an order of magnitude in contrast to the scan-1. However, it still cannot explain the BNL and FNAL results without additional contribution. For example, incorporating a muon-philic vector boson $X$ with coupling $g_X = 0.2$ and mass $m_X = 100$ GeV, which is consistent with current experimental bounds, endows $\Delta a_\mu X = 38 \times 10^{-11}$ shown by the green hatched regions overlapping with $U_1$ 2σ-allowed region in Fig. 2.
\[ \Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}} \leq 2.30 \ (5.99), \text{ and } \chi^2_{\text{min}} = 9.06. \] Here we fix \( m_{U_1} = 2 \text{ TeV}. \)

The green hatched regions show the effect of a muon-philic \( X \) vector boson with coupling \( g_X = 0.2 \) and mass \( m_X = 100 \text{ GeV}, \) which endows \( \Delta a_\mu |_X = 38 \times 10^{-11}. \)

In the scan-3, we include a specific \( U(1)_{B_s-L_2} \) \( X \) boson in addition to the \( U_1 \) leptoquark framework, and demonstrate that under this framework it can alleviate the \( (g - 2)_\mu \) and \( B \)-physics tensions within 2\( \sigma \). Not only the \( (g - 2)_\mu \), but this \( X \) boson also contributes to both \( \Delta C_9 \) (see Eq. (25)) and \( B_s \to \mu^+\mu^- \) (see Eq. (34)). In particular, we vary the coupling \( 0.01 \leq g_X \leq 0.05 \), and fix \( m_X = 100 \text{ GeV}, \) which is partially allowed under current experimental constraints from neutrino-trident [66], \( B_s^0 - \bar{B}_s^0 \) mixing [67], \( K_L \to \mu^+\mu^- \) [68], ATLAS [69], and CMS [70, 71]. It is worth to mention the reliable range of \( X \) boson parameters consistent with the measurement of i) \( B_s^0 - \bar{B}_s^0 \) mixing (\( \Delta M_s \)), ii) \( \text{Br}(K_L \to \mu^+\mu^-) \) and iii) \( K^0 - \bar{K}^0 \) mixing parameters. The mass difference \( \Delta M_s \) has been precisely measured by CDF2, LHCb and CMS collaborations [72, 73], and its theoretical predictions has been improved by developed sum rules and lattice calculations. We use the weighted average of the latest results given in Ref. [67] as our \( \Delta M_s^{\text{SM}} \). In the presence of \( X \) boson, new physics contribution to \( \Delta M_s \) is approximated by \( \frac{\Delta M_s^{\text{SM}+\text{NP}}}{\Delta M_s^{\text{SM}}} \approx 1 + \left( \frac{3\delta C_9}{-0.53} \right)^2 \left( \frac{m_X / g_X}{1 \text{ TeV}} \right)^2 \) [67]. Putting \( \delta C_9' = -\delta C_{10}' = -0.40 \pm 0.12 \) [82, 83, 47] into the expression and requiring

FIG. 2. Scan-2: The 1\( \sigma \) (2\( \sigma \)) region satisfies \( \Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min},2} \leq 2.30 \ (5.99), \text{ and } \chi^2_{\text{min},2} = 9.06. \) Here we fix \( m_{U_1} = 2 \text{ TeV}. \) The green hatched regions show the effect of a muon-philic \( X \) vector boson with coupling \( g_X = 0.2 \) and mass \( m_X = 100 \text{ GeV}, \) which endows \( \Delta a_\mu |_X = 38 \times 10^{-11}. \)
\[ \Delta M_s = (1.04^{+0.04}_{-0.07}) \Delta M_s^{\text{exp}}, \] the lower limit on the coupling is \( g_X > 1.01 \times 10^{-2} \) within 1\( \sigma \) for \( m_X = 100 \text{ GeV} \). For \( \text{Br}(K_L \rightarrow \mu^+ \mu^-) \), requiring that new physics contribution to dimension-six \( \Delta F = 1 \) operator \((\bar{s}_L \gamma^\mu d_L) (\bar{\mu}_R \gamma_\mu \mu)\) is smaller than SM contribution [68, 75], we impose the upper limit on the coupling \( g_X < 4.61 \times 10^{-2} \) for \( m_X = 100 \text{ GeV} \). Focusing on the upper limit on the short-distance contribution \( \text{Br}(K_L \rightarrow \mu^+ \mu^-)_{\text{SD}} \), a dedicate analysis gives \( g_X |_{m_X=100 \text{ GeV}} < 3.55 \times 10^{-2} \) [76]. For the neutral Kaon mixing, the most stringent bound comes from the measurement of \( \epsilon_K \). Following the approach in Ref. [76], we get the allowed range on the coupling as \(-4.13 \times 10^{-2} < g_X < 4.24 \times 10^{-2} \) for \( m_X = 100 \text{ GeV} \). We assume the left-handed coupling to \( X \) boson is dominant for flavor-violating vertices in the hadronic sector.

Such \( X \) boson has negligible effect on \((g-2)_\mu\), but gives modest contribution to \( \Delta C_9 \). Due to the negative value of \( V_{ts} \approx -0.40 \), the \( X \) boson trends to the observed value \( (C_9^{\mu\mu} = -0.40 \pm 0.12) \). Consequently, the \( U_1 \) leptoquark in

\footnote{Using the global fit \( \delta C_9 = -\delta C_{10}^{\mu\mu} = -0.39 \pm 0.07 \) based on Moriond 2021 result [74], the lower limit on the coupling is \( g_X > 1.15 \times 10^{-2} \) within 1\( \sigma \) for \( m_X = 100 \text{ GeV} \).}
conjunction with \(X\) boson explain the \(R_K(\tau)\) anomaly, but the former solely contributes to the \((g-2)_{\mu}\). Comparing to Scan-2, the \(\chi^2_{\min} = 9.06\) at \((x_{L}^{22}, x_{L}^{23}, x_{L}^{33}, x_{R}^{32}, g_X) = (4.04 \times 10^{-2}, 0.324, -9.30 \times 10^{-3}, 0.810, 3.73 \times 10^{-4}, 1.47 \times 10^{-2})\) is no further reduced by including the \(X\) boson. However, the 2\(\sigma\) chi-square regions (yellow regions in Fig. 3) extend overlapping with \((g-2)_{\mu}\) 2\(\sigma\) region. Two representative points in the overlapped regions are

\[
(x_{L}^{22}, x_{L}^{23}, x_{L}^{33}, x_{R}^{32}, g_X) = \begin{cases} 
(3.38 \times 10^{-4}, 0.167, -0.846, 0.644, +0.520, 1.32 \times 10^{-2}), \\
(-3.08 \times 10^{-4}, 0.227, +0.831, 0.668, -0.554, 1.44 \times 10^{-2}),
\end{cases}
\]

respectively give \((\chi^2, \Delta a_\mu) = (13.9, 193 \times 10^{-11})\) and \((\chi^2, \Delta a_\mu) = (13.7, 201 \times 10^{-11})\). As a result, this hybrid scenario, \(U_1\) leptoquark in conjunction with \(U(1)_{B_3-L_2}\) gauge boson, explains the \(B\)-physics and \((g-2)_{\mu}\) anomalous within 2\(\sigma\).

V. SUMMARY

The recent observational anomalies lead us to consider a vector leptoquark whose couplings with both left and right chiral fermions are essential. It affects various channels of \(B\)-meson decays and generate lepton flavor universality breaking. At the same time, the leptoquark can contribute to \((g-2)_{\mu}\) with significant enhancement. We perform the global analysis to several low-energy observables and encounter both left- and right-handed \(U_1\) leptoquark couplings. Unfortunately, we have not found common parameter region for \(B\) and \((g-2)_{\mu}\) anomalies without additional muon-philic vector boson. Motivated by this, we found the \(U_1\) leptoquark in conjunction with additional \(U(1)_{B_3-L_2}\) gauge boson is able to reconcile the \(B\)-physics and \((g-2)_{\mu}\) anomalies within 2\(\sigma\). We expect the experimental measurements will be much more improved in the future and the leptoquark and the new gauge boson will be better tested accordingly.

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