Instabilities in the Early Solar System Due to a Self-gravitating Disk

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Abstract

Modern studies of the early solar system routinely invoke the possibility of an orbital instability among the giant planets triggered by gravitational interactions between the planets and a massive exterior disk of planetesimals. Previous works have suggested that this instability can be substantially delayed (about hundreds of megayears) after the formation of the giant planets. Bodies in the disk are typically treated in a semi-active manner, wherein their gravitational force on the planets is included, but interactions between the planetesimals are ignored. We perform N-body numerical simulations using GENOA, which makes use of GPUs to allow for the inclusion of all gravitational interactions between bodies. Although our simulated Kuiper Belt particles are more massive than the probable masses of real primordial Kuiper Belt objects, our simulations indicate that the self-stirring of the primordial Kuiper Belt is very important to the dynamics of the giant planet instability. We find that interactions between planetesimals dynamically heat the disk and typically prevent the outer solar system’s instability from being delayed by more than a few tens of megayears after giant planet formation. Longer delays occur in a small fraction of systems that have at least 3.5 au gaps between the planets and planetesimal disk. Our final planetary configurations match the solar system at a rate consistent with other previous works in most regards. Pre-instability heating of the disk typically yields final Jovian eccentricities comparable to the modern solar system’s value, which has been a difficult constraint to match in past works.

Key words: planets and satellites: dynamical evolution and stability – planets and satellites: formation

1. Introduction

Over the last few decades, it has become clear that the solar system’s outer planets have evolved substantially from their initial orbital configuration (Fernandez & Ip 1984; Malhotra 1993). While evidence of this process mostly comes from the observed features of bodies orbiting beyond Neptune, the effects of giant planet migration would have been felt throughout the solar system. One particularly successful realization of giant planet migration known as the “Nice model” assumes that the gas giants were able to form in a different configuration before the dissipation of the primeval gaseous disk (Gomes et al. 2005; Tsiganis et al. 2005). However, the resulting system of giant planets was only metastable, where an instability caused these planets to scatter into the orbital configuration we see today (Gomes et al. 2005; Tsiganis et al. 2005; Morbidelli et al. 2007, 2009).

The Nice model has been used to explain different dynamical phenomena in the solar system, such as Jupiter’s Trojans (Morbidelli et al. 2005; Robutel & Ghezzi 2006), the capture of irregular satellites (Cuk & Gladman 2005; Jewitt & Haghighipour 2007; Nesvorny et al. 2007), the dynamical structure of the asteroid belt (Morbidelli et al. 2009; Minton & Malhotra 2011; Roig & Nesvorny 2015), and a possible dynamical trigger for the Late Heavy Bombardment (LHB; Bottke et al. 2007, 2012; Morbidelli 2007; Thommes et al. 2008). The initiation of the instability and the scattering of small bodies that followed has been linked to an epoch of bombardment on the Moon that has been recorded within its craters (see Bottke & Normand 2017 for a review). This late period of bombardment is thought to set the timescale for the giant planets’ scattering at ~500–700 Myr after the condensation of calcium–aluminum-rich inclusions in the early protoplanetary disk (Gomes et al. 2005; Tsiganis et al. 2005), although recent studies (Toliou et al. 2016; Deienno et al. 2017; Zellner 2017; Clement et al. 2018; Morbidelli et al. 2018) have begun to call into question this assumption.

Due to the broad range of masses (i.e., asteroids, comets, embryos, and planets), numerical studies have often used approximations in order to make the computations more tractable. For instance, the outer debris disk is usually described as swarms of real disk particles that either interact only with the gas giants, ignoring the gravitational interactions between swarms, or allow interactions between swarms but only within a few mutual Hill spheres to approximate the effects of viscous self-stirring (Gomes et al. 2005; Tsiganis et al. 2005; Levison et al. 2011). The latter approach was used by Levison et al. (2011), who assumed that most of the self-heating in the outer disk should have come from encounters between small particles in the disk that represented the bulk of its mass and ~1000 Pluto-sized particles, which represented the “heavy” part of the size distribution. That method proved to be computationally efficient, and self-stirring was shown to be small but not negligible. For a large enough spacing between the outer gas giant and the inner edge of the disk, there were practically no encounters between particles and planets for a very long time due to negligible disk spreading in the semimajor axis. Self-stirring was found to secularly transfer energy to the giant planets, which eventually became unstable and constituted a regime of solutions, all necessarily late.

Within modeling efforts of these types, there has also been work on the number of giant planets present within this era of solar system evolution. Such works have explored how the existence of four, five, or six giant planets would affect the final orbital architectures, the migration rate, and the instability timescale necessary to match observational constraints in the asteroid belt and the craters of the inner solar system (Tera et al. 1974; Michel & Morbidelli 2007; Bottke et al. 2012; Morbidelli et al. 2017). In particular, recent studies have shown that a giant planet instability very often destabilizes the
terrestrial planets, again calling into question the timing of the instability (Brasser et al. 2009, 2013; Nesvorný 2011; Agnor & Lin 2012; Nesvorný & Morbidelli 2012; Kaib & Chambers 2016). A less delayed instability occurring before the completion of terrestrial planet formation may be more compatible with the present-day solar system (Clement et al. 2018; Nesvorny et al. 2018).

Fan & Batygin (2017) used a self-gravitating disk model and found consistent final giant planet architectures but did not evaluate the full timescales associated with the Nice model. In this work, we employ models with a self-interacting disk to revisit the timing and outcomes of an outer solar system instability on 100 Myr timescales. Through these models we estimate the magnitude of excitation within the disk from three sources: external forcing by the giant planets, self-stirring due to particle–particle interactions, and artificial enhancement of the self-stirring due to the mass resolution of particles. Our initial setup and methodology are summarized in Section 2. We describe our results for special cases in Sections 3.1 and 3.2, whereas our broader results for four and five giant planet systems are presented in Sections 3.3 and 3.4, respectively. We discuss the dependence of our results on the number of particles assumed in Sections 3.5 and 3.6. The conclusions of our study are summarized in Section 4.

2. Methodology

2.1. Numerical Setup

Our numerical study makes use of a relatively new code based on the popular integration package mercury (Chambers 1999) that has been adapted for GPUs called GENGA (Grimm & Stadel 2014). This code has been well tested and shown to yield results consistent with the mercury package (Grimm & Stadel 2014; Hoffmann et al. 2017). In our study, we use NVIDIA Tesla K20m cards with a compute capability of 3.5 and version 8.0 of the CUDA drivers. In practice, the code allows for three modes of operation—fully active, semi-active, and test particles—similar to the standard version of mercury. The test particle mode allows large bodies (fully active particles) to influence the motions of massless bodies (test particles), ignoring interactions between the test particles and reaction forces on the massive bodies. The semi-active mode upgrades the test particles to small bodies that have mass, and allows reaction forces on large bodies but still ignores interactions between small bodies. We primarily use the fully active mode, where all gravitational interactions are included between all bodies and use the other modes for studies of special cases.

Most of our simulations begin with a specified giant planet resonant configuration and an outer disk of 1500 smaller bodies. The outer disk is composed of equal-mass bodies distributed following a surface density profile $\Sigma \propto a^{-1}$, which is common among previous investigations (see Levison et al. 2011). We vary the inner edge of the disk $a_o$, or disk gap ($\Delta = a_i - a_o^{cp}$), following previous works within $2.3-6.3$ au of the outermost giant planet, $a_0^{cp}$, in increments of 0.125 au at our highest resolution. The outer boundary of the disk is initially kept fixed at 30 au. However, we expect spreading due to interactions within the disk that push bodies beyond this boundary. We use 100 au as a radial boundary to consider bodies to be ejected, and bodies that extend beyond this boundary are removed from a given simulation. We perform a subset of runs using this basic setup, where we primarily vary the number and/or mass of the small bodies to measure the extent of self-stirring using the rms eccentricity of disk particles on a 1 Myr timescale.

A large fraction of our simulations are terminated when 50 Myr of simulation time has elapsed, while a subset of runs are extended beyond 50 Myr. These runs are chosen because they contain a significant portion of their disk at 50 Myr and have at least four giant planets, which are necessary for a delayed instability scenario to remain plausible. We use a timestep of 180 days for our runs, which is typical in previous works (Levison et al. 2011; Nesvorný & Morbidelli 2012). The initial eccentricities and inclinations of the disk particles are chosen randomly from a Rayleigh distribution with a scale parameter $\sigma = 0.001$. The other orbital parameters (argument of periastron, ascending node, and mean anomaly) are chosen randomly from a continuous uniform distribution between 0° and 360°. Figure 1 shows the initial state of our simulations from a top–down perspective for both the four giant planet (left) and five giant planet (right) configuration.

2.2. Criteria for Success

Following Nesvorný & Morbidelli (2012), we prescribe four criteria to measure the overall success of our simulations. These criteria are outlined as follows:

(A) The final planetary system must have four giant planets.
(B) The final orbits of the giant planets must resemble the current solar system.
(C) The $e_{55}$ amplitude must be greater than half of its current value (0.044).
(D) The period ratio between Jupiter and Saturn changes from $<2.1$ to $>2.3$ within 1 Myr.

Our criteria are virtually identical to those of Nesvorný & Morbidelli (2012) so that we can fairly compare our results to those of previous investigations (i.e., Levison et al. 2011; Nesvorný & Morbidelli 2012). Similar to Nesvorný & Morbidelli (2012), we expect that only a few percent of systems will satisfy all four criteria. The justification for Criterion A is self-evident when comparing to the actual solar system. Criterion B is defined further to include semimajor axes within 20% of the current values of the giant planets, mean final eccentricities of each planet $<0.11$, and mean final inclinations of each planet $<2°$. The mean final eccentricities and inclinations of each planet are determined from an additional integration after the instability occurs in isolation (without disk particles) for 10 Myr. Numerical simulations of this type are susceptible to the inherent chaos within dynamical systems resulting in mainly statistical comparisons, and from this we justify a fairly wide range in Criterion B.

Criterion C was previously justified because the eccentricity modes of the giant planets were relatively hard to excite given that the self-gravity of the disk was largely ignored. We keep this criterion in our analysis but find the $e_{55}$ amplitude to be excited relatively easily. In order to determine the value of $e_{55}$, we perform an integration for 10 Myr using the final giant planet configurations and discard any remaining disk bodies. This secondary integration is analyzed using the frequency-modified Fourier transform1 by Šidlichovský & Nesvorný (1996). The secular mode of Jupiter has been used to broadly

1. https://www.boulder.swri.edu/~davidn/fmft/fmft.html.
describe the long-term evolution of the solar system as it affects the observed structure found in populations of small bodies. Criterion D institutes a quick transition through resonances that would sweep through the terrestrial region of the solar system (Brasser et al. 2009; Kaib & Chambers 2016). We keep Criterion D in our analysis for completeness but do not strictly adhere to it when describing simulations as successful or not because its importance is diminished if the instability occurs before the epoch of terrestrial planet formation is complete (Clement et al. 2018).

2.3. Giant Planet Architectures

Previous studies have used giant planet architectures, where the period ratios \((T_{\text{out}}:T_{\text{in}})\) between successive planets are integer ratios \(a:b\) that correspond to the mean motion resonances (MMRs) between the giant planets expanding radially outward (e.g., Morbidelli 2007; Batygin & Brown 2010). A wide range of initial configurations have been proposed, where several authors start Jupiter and Saturn near the 2:1 MMR as it is the strongest resonance (Tsiganis et al. 2005; Morbidelli 2007; Zhang & Zhou 2010; Pierens et al. 2014; Izidoro et al. 2016). However, Nesvorný & Morbidelli (2012) were unable to sufficiently excite \(e_{55}\) with this starting condition, so we restrict our initial conditions to resonant chains where Jupiter and Saturn occupy a 3:2 MMR.

Our simulations use architectures of four and five giant planets, which have been shown to be successful in prior investigations (Levison et al. 2011; Nesvorný & Morbidelli 2012). The system of four giant planets considers Jupiter and Saturn as fully grown planets and two ice giants (15.75 \(M_\oplus\) each) in a 3:2, 3:2, 4:3 resonant configuration. The system of five giant planets follows a 3:2, 3:2, 2:1, 3:2 resonant configuration, which has been shown to be particularly good for a delayed instability (Nesvorný & Morbidelli 2012; Deienno et al. 2017). Using GENGA, we find that both of these resonant configurations are stable up to 600 Myr without including any disk particles.

3. Results and Discussion

3.1. Short-term Evolution of an Isolated Disk

In order to make fair comparisons with previous works, we perform a series of simulations for 1 Myr using initial conditions similar to those in Levison et al. (2011) for four giant planet and five giant planet configurations from Nesvorný & Morbidelli (2012). The particles used in most of our runs are more massive than the largest known Kuiper Belt objects (KBOs), and thus it is important to understand how our assumed mass resolution of particles and disk surface density will affect the outcomes in our simulations. In particular, we want to estimate how much disk stirring is enhanced by using super-Pluto-mass particles. To do this, we can compare the behavior of short fully interacting simulations with that of similar simulations done with Pluto-mass bodies in past work (Levison et al. 2011). From these simulations, we measure the viscous stirring of disk particles through the rms eccentricity, \(\epsilon_{\text{rms}}\).

First, we evaluate the short-term evolution of an outer disk using our four and five giant planet configurations that begin in the resonant configuration in Figures 2 and 3, respectively, where we systematically explore four different scenarios. In each scenario, we evaluate the prescribed conditions with and without the giant planets so that we can more easily separate the interactions of the disk bodies from either the self-stirring of the disk or secular forcing from the inner giant planets.

The first scenario in Figure 2(a) starts with an outer disk composed of 1000 Pluto-mass bodies, where the initial surface density \((\sigma_{\text{DP}})\) of the disk varies between simulations. We keep the total mass constant and change the inner edge of the disk \((a_i)\) while keeping the outer edge of the disk fixed at 30 au. Beginning the inner edge of the disk at \(\sim23.4\) or 26.9 au results in doubling or quadrupling, respectively, the initial surface density.
density, σ_D, compared to setting the inner edge at 14 au. As one would expect, the viscous self-stirring increases in proportion to increases in σ_D for isolated disks (e.g., Stewart & Ida 2000; Levison et al. 2011), where our simulations (dashed lines) indicate self-stirring between small bodies contributes a significant fraction to the total viscous stirring of the disk. In particular, we find good agreement with previous works (Levison et al. 2011, their Figure 2), which have found an \( e_{\text{rms}} \sim 0.02 \) after 1 Myr when the inner edge of the disk is at 14 au.

In Figure 3(a) we perform the same experiment on our five-planet configuration. Here we see that the self-stirring in isolated disks (dashed lines) is comparable to that in Figure 2(a). However, when we introduce giant planets (solid lines) into the system, the disk is driven to much higher eccentricities compared to those in the corresponding four-planet simulations with a similar disk gap. This appears to be driven by planet–disk interactions. Previous work (Levison et al. 2011) has shown that planet–disk interactions are strongest for portions of the disk within or closer to the 3:2 resonance with the outer giant. In our four-planet setup, this region of the disk hosts only \( \sim 4\% \) of our particles (assuming a \( \sim 3\) au gap between the disk and planets). Meanwhile, 51% of disk particles are found in this region if we assume the same planet–disk gap in our five-planet

**Figure 2.** Measures of viscous stirring in terms of \( e_{\text{rms}} \) considering a Nice model configuration with four giant planets where the outer disk has (a) 1000 Pluto-mass bodies with three values for the inner edge of the disk (\( a_i \)), (b) a constant surface area but a different number of Pluto-mass bodies, (c) a constant total mass and area using three particle masses, and (d) a constant surface area and number of bodies using three particle masses. The colors distinguish different values (\( a_i \), particle number, and particle mass), where we include simulations with (solid) and without (dashed) the giant planets.
configuration! Thus, a much larger fraction of the disk is strongly influenced by the giant planets. One can also see this in the red and blue solid curves in Figure 3(a), where we move the inner edge of the disk to 26.6 and 28.4 au, respectively, just beyond the location of the 3:2 MMR. In these disks, the growth in eccentricity is greatly diminished compared to that in a disk with the inner edge at 22.7 au.

In the second scenario (Figures 2(b) and 3(b)), we keep the surface area of the disk fixed and vary the number of Pluto-mass bodies (1000–4000) present to simulate more realistic conditions (i.e., Nesvorný & Vokrouhlický 2016). We find that the viscous self-stirring increases with the number of Pluto-mass bodies included, which is likely due to the relative strength and frequency of close encounters between particles. Levison et al. (2011) performed simulations that employed 1000 Pluto-mass bodies, but more recent studies of Kuiper Belt formation indicate that the primordial belt may have contained as many as 4000 Pluto-mass bodies (Nesvorný & Vokrouhlický 2016). When we introduce the planets to our simulations in our four-planet systems (Figure 2(b)), the rms eccentricity increases by ∼0.005–0.01, where the increase is smaller (∼0.005) for a greater number of particles (N = 4000) and vice versa. However, our five-planet simulations (Figure 3(b)) show the effects of particle number variations are dwarfed by the eccentricity increases driven by the planets. In these cases, the rms eccentricity increases by ∼0.1.
In our third set of experiments (Figures 2(c) and 3(c)), we keep the total surface density of our disks fixed and vary the mass resolution of individual particles (0.5–5.0 \(M_{\text{Pluto}}\)). This allows us to isolate how the level of disk self-stirring will be impacted by how well we can resolve the mass of the primordial disk. As a result of varying particle mass over an order of magnitude, we find the viscous stirring variations (as measured by \(e_{\text{rms}}\)) to be similar to those seen when varying the number of Pluto-mass bodies between 1000 and 4000, which spans the actual range of uncertainty of this number. In fact, the difference in viscous stirring in our isolated disks of 5.0 \(M_{\text{Pluto}}\) is less than a factor of two larger than if 0.5 \(M_{\text{Pluto}}\) bodies were used. The larger effect arises when we use four (Figure 2(c)) or, much more so, five (Figure 3(c)) giant planets. The effect in the five giant planet scenario is important because this scenario has been shown to better reproduce the current solar system architecture (i.e., Nesvorný & Morbidelli 2012).

The final scenario (Figures 2(d) and 3(d)) investigates how the viscous stirring changes with the mass resolution of particles, but the number of particles (2000) and surface area of the outer disk remain fixed. Increasing particle mass here lowers the mass resolution and increases the disk surface density, both of which lead to increased disk stirring. As expected, increasing the particle mass drives increases in the disk \(e_{\text{rms}}\), but as we see in our other sets of experiments with our five-planet configuration, the particle/disk-dependent eccentricities are smaller than the increase in eccentricity that occurs when we introduce the giant planets to the system.

In each of our short-term experiments we see that the eccentricity stirring is affected by particle number, particle mass, disk surface density, and perturbations of the giant planets. In particular, we find that the largest increases in \(e_{\text{rms}}\) occur when we embed our five-planet configuration interior to the primordial disk. This effect dominates all of the variations in our disk properties. Much of this is due to the fact that a much larger fraction of the disk resides closer to the Sun than to the 3:2 MMR with the outer giant. In addition, the eccentricity evolution of the four and five giant planets in isolation for 1 Myr shows that the eccentricities of the outer ice giants are much larger (up to 2\(\times\)) in the five giant planet system. These higher planetary eccentricities widen resonances and increase secular forcing, which add to the larger eccentricity growth seen in disks surrounding our five-planet configuration. This is important because our longer-term simulations use particle masses that are typically a factor of a few greater than Pluto’s, and these shorter simulations indicate that our computationally driven choice of larger particle masses should not dramatically alter the evolution of our disks because their stirring is primarily enhanced by the inclusion of the five planets. It is also possible that some of the enhanced eccentricity growth of the five-planet system is due to the inner edge of the disk being very close to the planets, where particles at the inner edge of the disk get scattered by the planets to high eccentricities very fast (or gain eccentricity in the resonances with the outer giant). Hence \(e_{\text{rms}}\) increases faster than it would in a disk with a significant source of dynamical friction (e.g., dust), creating some artificial enhancement to the growth of the disk \(e_{\text{rms}}\).

3.2. Longer-term Evolution of an Isolated Disk
We perform a number of simulations to evaluate the extent of self-stirring within isolated disks (35 and 20 \(M_{\odot}\)) using...
equal-mass planetesimals on longer timescales (10–100 Myr). Longer simulation timescales also translate into longer computational times, where we limit the scope of our numerical tests in response.

First, we show the evolution of a 35 $M_⊕$ disk ($a_i = 14$ au) composed of 1500 bodies (∼11 Pluto masses) in terms of the rms eccentricity and rms inclination in Figures 4(a) and (b), respectively. The $e_{\text{rms}}$ and $i_{\text{rms}}$ of the disk particles increase exponentially in simulation time, where doubling occurs roughly with an order of magnitude longer simulations. The magnitude of the increase is likely related to our mass resolution, the infrequent nature of collision events, and the propensity of bodies to scatter off one another. The 10th percentile of the disk particles in their periastron distances ($q_{10}^{10}$, blue) and the 90th percentile in their apastron distances ($Q_{90}^{10}$, red) are also provided in Figure 4(c) to demonstrate the extent of the disk spreading through the difference in their initial values (dashed horizontal lines). The spreading in these distances indicates that the self-stirring of the disk particles will interact with the giant planets eventually and induce the migration of the outermost giant planet through scattering events.

Mass resolution undoubtedly plays a role in the excitation of the disk particles and can contribute to an enhancement in the actual self-stirring within a disk, where more realistic conditions (Nesvorný & Vokrouhlický 2016) would offset this enhancement, but then a large number of particles would be required. To investigate the potential effects on our results, we simulate isolated 20 $M_⊕$ disks, varying the number (750–24,000) of equal-mass planetesimals with the inner edge of the disk beginning at 22.7 au for 10 Myr. In Figure 5, we show the excitation of the eccentricity and the drift of the inner portion of the disk through the 10th percentile of the semimajor axis distances. The symbols in Figure 5 denote the respective values at two different epochs, where the color code denotes simulations using either the full extent of the disk (blue) or only the inner half (by area) of the disk with an equivalent mass resolution (red). Probing mass resolution in this way is
important because it can play a role in influencing the outer giant through semimajor axis diffusion of the inner disk and the outer half of the disk contributes on much longer timescales due to the increase in orbital period.

Even for 10 Myr integration times, simulating disks with several thousand particles results in very long run times. Thus, for the tests with the highest number of particles (12,000 and 24,000), we instead only simulate the inner half of the disk, which just consists of 6000 and 12,000 particles, respectively. Our justification for this approach is that any given particle in an isolated disk is primarily stirred via interactions with other nearby particles, so the influence of the outer half of the disk on the stirring of the inner half is likely small on short (\(\lesssim 10\) Myr) timescales. To verify this, we simulate full disks (up to 6000 particles) for 10 Myr and then repeat the simulation using only the inner half of the disk resolved with half as many particles. Examining Figure 5(a), we see that the rms values of disk eccentricity are nearly identical for both systems consisting of more than 1500 particles after 10 Myr. In addition, we see that the inner disk edge (as measured by the 10th percentile of the semimajor axis distribution) has diffused by nearly the same amount when the number of particles is larger than 3000. With these results, we can effectively study the evolution of a 12,000- or 24,000-particle disk by modeling just the inner half with 6000 or 12,000 particles, respectively.

Figure 5(a) shows the \(e_{\text{rms}}\) after 10 Myr of simulation time for disks composed of relatively large equal-mass bodies (\(\sim 11\) Pluto masses) down to a much smaller mass resolution (\(\sim 0.4\) Pluto masses), where there is a factor of \(\sim 2-3\) (see red dots in Figure 5(a)) in self-stirring between these mass scales. KBO surveys indicate that the large KBO size distribution follows a \(q \sim -5\) size distribution (Shankman et al. 2013; Fraser et al. 2014; Lawler et al. 2018). If we assume there were originally 4000 primordial objects more massive than Pluto and extend such a size distribution to lower masses, we would estimate that there were \(\sim 6000\) primordial belt objects with masses above 0.75 Pluto masses. This is within a factor of two of our 12,000-particle simulation. As we see in Section 3.1, changing particle number by a factor of two results in modest changes, especially compared to the effects of planet–disk interactions. Moreover, the average particle mass in our highest-resolution simulation (\(\sim 0.4\) Pluto masses) is less than the average mass of all bodies larger than 0.75 Pluto masses under the \(q \sim -5\) distribution, and our underestimate of mass will somewhat offset our overestimate of objects. Thus, the behavior of this simulated disk, which is not radically different from our coarser isolated disks, should resemble the behavior of the high-mass objects in the actual primordial belt. We note that smoothly extending such a distribution down to \(\sim 100\) km bodies implies an unrealistically massive primordial belt. We just employ this example to illustrate that our simulation resolution approaches physical values, given the uncertainty in the primordial belt’s properties.

While eccentricity stirring can cause disk particles to strongly interact with planets and trigger an instability, the giant planets can also be destabilized through energy diffusion in the disk, which causes the inner disk edge to bleed inward toward the planets. Thus, we also study the evolution of the 10th percentile of the semimajor axis distribution (as a proxy for its inner edge) in Figure 5(b). The diffusion in the 10th percentile semimajor axis distribution for disks composed of a
smaller number of more massive particles ($\Delta a_{10} \sim 1.5$ au) is greater than that for disks composed of a larger number of less massive particles ($\Delta a_{10} \sim 0.5$ au). Our simulations with giant planets mitigate this enhanced diffusion, due to the mass resolution, by starting with a gap ($\sim 2.4$ au) between the disk and the outer giant. With such a gap, diffusion due to self-stirring should take $\sim 100$ Myr to begin crossing orbits with the outermost giant planet considering particle masses $\lesssim 6$ times that of Pluto. Secular forcing due to the giant planets will likely shorten this timescale, where larger gap sizes will counteract the effect.

There is a substantial computational cost to considering disks with a larger number of less massive particles, where compute times become intractable using Pluto-mass to sub-Pluto-mass resolutions for our current study. As a result, the focus of our work is limited to 4500 particles, or $\sim 2$ Pluto masses. With a more realistic mass resolution (24,000 particles), the $e_{\text{rms}}$ value is 45% smaller than with 4500 particles, but secular forcing of eccentricity due to the five giant planets will likely overshadow this difference (see Figure 3(c)). In the case of four giant planets (see Figure 2(c)) secular forcing is not as large as for five giant planets, but such forcing will likely dominate over the enhanced self-stirring due to the mass resolution because the surface density $\sigma_D$ in our simulations with four giant planets is about half as much as that in the runs with five giant planets.

3.3. Four Giant Planets

The classical Nice model posits that the four giant planets evolved into an MMR before the dispersion of the gaseous disk and later transitioned to the current configuration (Gomes et al. 2005; Morbidelli et al. 2007). Although most recent research has focused on a five-planet initial configuration, we first explore the effects of a self-gravitating disk on four-planet setups for completeness and comparison with past work. Our simulations explore how one of the best four giant planet cases (3:2, 3:2, 4:3) fares when interactions between all disk particles are included. Previous investigations (e.g., Levison et al. 2011) have sought to approximate the excitation of the outer disk through different algorithms to mimic viscous self-stirring. Levison et al. justified their approximation because fully active models are limited to artificially large bodies that result in enhanced numerical heating of the planetesimal disk (e.g., Stewart & Ida 2000). While Levison et al. (2011) included stirring due to close encounters between 1000 Pluto-mass bodies, their technique ignored long-range interactions between the bodies, which also significantly contribute to self-stirring (Stewart & Ida 2000). Moreover, these Pluto-mass bodies constituted $< 10\%$ of the total disk mass, and the remaining disk mass was not able to self-stir.

Within numerical software like GENGA and mercury, we have the option to ignore the interactions between disk particles, which we call the semi-active mode of the software,
and perform tests to examine differences in the excitation of the disk. In Figure 6, we examine the eccentricity state as a function of the semimajor axis at two epochs (1 and 10 Myr) to better understand the influence of the giant planets and the extent of mixing due to disk excitation (using the color scale). Figures 6(a) and (d) show the full extent of the viscous self-stirring within an isolated disk (no giant planets) at these two different epochs. Within 10 Myr, the maximum eccentricity grows to \( \sim 0.2 \), allowing significant portions of the outer disk to expand. The stirring mixes the middle (\( \sim 20-25 \) au) portion of the disk and transports some bodies that originate near the inner edge to the outer edge.

Figures 6(b) and (e) show that MMRs with the giant planets work to excite the inner portions (\(< 20 \) au) of the disk when using the semi-active mode, while the rest of the disk remains dynamically “cold.” Also, the state of the four giant planets does not appreciably change on this timescale, and the amount of disk spreading is minimal. This is in contrast to Figures 6(a) and (d), where the disk is much more excited despite the lack of giant planets in the simulations. Also, our tests using the semi-active mode show that an isolated disk undergoes a negligible amount of excitation or spreading on these timescales.

Figures 6(c) and (f) show a considerably different scenario when the interactions between planetesimals are included. In 1 Myr, disk bodies begin to cross orbits with the outer ice giant and induce its outward migration. For this particular case, a resonance crossing between the giant planets ensues within 10 Myr, which is too quick to connect with the delayed instability in the classical Nice model (Gomes et al. 2005). The interactions with the outer planetesimal disk are significant enough to trigger substantial migration of the outer ice giant and can lead to an overall instability of the giant planets on a 10 Myr timescale. This may be a special case that depends on the disk gap \( \Delta \) between the outer ice giant and the planetesimal disk, where a larger gap could delay the instability substantially (Gomes et al. 2005).

Thus, we perform a range of simulations, given in Figure 7, which systematically show results as a function of the disk gap \( \Delta \) (e.g., Levison et al. 2011) in order to uncover whether a delayed instability is possible or likely when considering our four giant planet resonant configuration. The vertical placement of symbols in Figure 7 denotes when the first giant planet is ejected within a given simulation, and the symbols themselves mark how many giant planets remain in the system up to the termination of the run. There are two possibilities that can allow for Criterion A: a smooth migration of the outer ice giants that gradually depletes the outer disk or a scattering event that disrupts most of the outer disk but manages to retain all the giant planets.

We find that both scenarios (smooth migration and scattering) are represented in our simulations and 4 out of 32 runs (\( \sim 13\% \)) satisfy Criterion A. However, this typically includes a scattering event within 50 Myr and a smooth migration over the remaining 500 Myr. In contrast to Levison et al. (2011), our broader results show this to be a relatively uncommon occurrence, where \( \sim 60\% \) of our simulations lose both ice giants and 20% lose a single ice giant. Nesvorný &

![Figure 8. Final giant planet architectures of a Nice model configuration with four giant planets along with a 35 \( M_\oplus \) outer disk. The filled points enclosed in a box represent the initial giant planet configuration of each simulation, whereas the open points denote the final architectures. Horizontal lines are given at the semimajor axis values of the current solar system, and the gray bars represent the range of values within 20%. The check marks at the bottom identify those simulations that end with four giant planets.](image-url)
Morbidelli (2012) also found that similar resonant configurations with low disk masses ($M_D < 50 M_\oplus$) typically lead to violent instabilities and planet ejection. The mass of our disk particles may also play a role in allowing for early instabilities, but finding conditions that prevent planet ejection and replicate the outer solar system has been difficult for four giant planet configurations. We have included points (in red) that mark when 50% of the outer disk remains, and error bars that represent when either 16% (lower bound) or 84% (upper bound) of the disk is lost for cases where all four giant planets survive to 550 Myr. Figure 7 shows ∼16% of our runs could be consistent with a late (∼550 Myr) instability; thus we continue the runs that survive for 50 Myr for an order of magnitude longer simulation time to 550 Myr. Only one of these simulations undergoes an instability on this longer timescale; the other four simulations have smooth migrations of the outer giants. The erosion of the disk is substantial, and only ∼3%–4% of the disk remains after the simulation ends at 550 Myr.

Figure 8 illustrates the giant planet architectures in terms of their configuration. We have included points in red, allowing it to scatter off Jupiter and escape from the system. There is a secondary interaction a few megayears later between the outer two ice giants, but this one is much milder and allows the outermost ice giant to migrate outward and closer to the present-day semimajor axis of Neptune. There are four giant planets at the end of the simulation in Figure 9, which satisfies Criteria A. Criterion B requires that the giant planets reside near their present-day semimajor axes and maintain average eccentricities <0.11 and average inclinations <2°, which are both evident in Figure 9.

Figure 10 shows the evolution of the disk in a way similar to that in Figure 5 (isolated disk) but with five giant planets included and approximately double the surface density. The different mass resolutions (1500, 3000, and 4500 particles) are color-coded (black, blue, and red, respectively). From Figure 3(c) we may expect higher eccentricities in Figure 10(a) at 1 Myr, but the results in Figure 3(c) use a smaller disk gap resulting in a more significant initial perturbation from the outer giant planet and Figure 3(a) (red line) illustrates that the disk eccentricities are comparable to those in 10(a) at 2–4 Myr. Figure 10(b) shows the diffusion of the disk through the 10th percentile in the periapsis distribution. The spreading of the disk causes orbit crossings with the outermost giant planet, where the timescale for these interactions depends on the mass resolution but only up to a factor of a few (i.e., less than an order of magnitude).

Our results in Figure 11 indicate that 8 out of 32 runs (25%) are able to scatter an ice giant out of the system within 50 Myr, while 3 runs (10%) take a longer timescale to satisfy Criterion A. To achieve this, it appears that the disk gap ($\Delta = 3.5$ au) needs to be larger than was assumed in prior studies, where $\Delta \sim 1$ au (Gomes et al. 2005; Nesvorný & Morbidelli 2012). Levison et al. (2011) varied the disk gap for four-planet systems and found that stable (1 Gyr) systems occurred once

### Table 1

| $a_i$ (au) | $\Delta$ (au) | $t$ (Myr) | A | B | C | D |
|-----------|--------------|----------|---|---|---|---|
| 14.000    | 2.329        | 9.462    | ✓ | x | ✓ | x |
| 14.125    | 2.454        | 550.00   | ✓ | x | ✓ | x |
| 14.250    | 2.579        | 13.947   | ✓ | x | ✓ | x |
| 14.375    | 2.704        | 45.733   | ✓ | x | ✓ | x |
| 14.500    | 2.829        | 21.832   | ✓ | x | ✓ | x |
| 14.625    | 2.954        | 5.125    | ✓ | x | ✓ | x |
| 14.750    | 3.079        | 550.00   | ✓ | ✗ | ✓ | x |
| 14.875    | 3.204        | 5.273    | ✓ | ✓ | ✓ | x |
| 15.000    | 3.329        | 13.010   | ✓ | x | ✓ | x |
| 15.125    | 3.454        | 4.879    | ✓ | x | ✓ | x |
| 15.250    | 3.579        | 30.702   | ✓ | x | ✓ | x |
| 15.375    | 3.704        | 6.259    | ✓ | x | ✓ | x |
| 15.500    | 3.829        | 21.388   | ✓ | x | ✓ | x |
| 15.625    | 3.954        | 39.918   | ✓ | x | ✓ | x |
| 15.750    | 4.079        | 119.90   | ✓ | x | ✓ | x |
| 15.875    | 4.204        | 550.00   | ✓ | ✗ | ✓ | x |
| 16.000    | 4.329        | 6.949    | ✓ | x | ✓ | ✗ |
| 16.125    | 4.454        | 25.626   | ✓ | x | ✓ | ✗ |
| 16.250    | 4.579        | 12.912   | ✓ | x | ✓ | ✗ |
| 16.375    | 4.704        | 10.349   | ✓ | x | ✓ | ✗ |
| 16.500    | 4.829        | 11.828   | ✓ | x | ✓ | ✗ |
| 16.625    | 4.954        | 8.279    | ✓ | x | ✓ | ✓ |
| 16.750    | 5.079        | 10.497   | ✓ | x | ✓ | ✓ |
| 16.875    | 5.204        | 9.610    | ✓ | x | ✓ | ✓ |
| 17.000    | 5.329        | 10.300   | ✓ | x | ✓ | ✓ |
| 17.125    | 5.454        | 22.078   | ✓ | x | ✓ | ✓ |
| 17.250    | 5.579        | 35.828   | ✓ | x | ✓ | ✓ |
| 17.375    | 5.704        | 550.00   | ✓ | x | ✓ | ✓ |
| 17.500    | 5.829        | 20.747   | ✓ | x | ✓ | ✓ |
| 17.625    | 5.954        | 11.926   | ✓ | x | ✓ | ✓ |
| 17.750    | 6.079        | 14.242   | ✓ | x | ✓ | ✓ |
| 17.875    | 6.204        | 19.121   | ✓ | x | ✓ | ✓ |

Note. The columns correspond to the heliocentric inner edge of the disk $a_i$, the distance between the inner disk edge and the outer ice giant $\Delta$, the time of the giant planet instability $t$, and whether the given conditions meet (✓) or fail (✗) each of our criteria for success, A–D.
Δ > 3.8 au. Deienno et al. (2017) performed some tests varying Δ and found that Δ > 2 au could be consistent with a late instability. However, they induced the migration of the outer ice giant with dust rather than with encounters with planetesimals in the outer disk. Our study finds that the timing of the instability typically occurs within 10–40 Myr without a strong trend in the disk gap Δ. Systems where all five giant planets are retained occur at a lower level (∼3%) than those we would deem to be a success, while most of the remaining simulations result in only two giant planets surviving the instability (∼44%). We include red points with error bars to show that even though all five giant planets survive, a large portion (86%) of the outer disk is lost within ∼50 Myr. Over the longer timescale (550 Myr), the outer disk continues to erode until 97% of the disk is lost. The open symbols in Figure 11 denote runs that are unstable on timescales greater than 50 Myr but less than 550 Myr. These runs show that instabilities can occur fairly late, even with a small amount of disk material; Table 2 shows the instability times and whether each run meets our success criteria. However, the minimal amount of disk material left also suggests that such late instabilities may be unable to generate the intense bombardment associated with the LHB, the original motivation for a late instability.

We show the architectures in the same way as in Section 3.3 in Figure 12 but mark the cases with four and five planets surviving with a check mark (√) and an “X,” respectively. From this view, we find that the run with five giant planets remaining does not allow for a large migration of Saturn or the inner ice giant, where the outer ice giant is able to substantially migrate outward and begin depleting the outer disk. The eight cases that produce systems with four giant planets typically allow for them to arrive near the present-day semimajor axes. There are two exceptions, where Saturn is either transported into the inner solar system or ejected entirely. Figure 12 also shows that the next most common outcome (two planets) typically leaves the system with only Jupiter and Saturn.

Table 2 summarizes our results with respect to our success criteria. Eleven of our simulations satisfy Criterion A, but only four of those also satisfy Criterion B. This is because the final mean eccentricity for these cases is larger than 0.11, indicating that the system becomes too dynamically “hot” to resemble the current solar system. Similar to our results in Section 3.3, Criterion C is satisfied ∼68% of the time. One-third of our runs that satisfy Criterion A do not satisfy Criterion D. However, satisfying Criterion D is less of a concern because it is instituted to mitigate the strength of sweeping resonances from exciting the terrestrial region at a later epoch (∼550 Myr). Only one simulation satisfies all four criteria, but this may change when we vary the total number of particles (see Section 3.5). Overall, our results imply that a giant planet instability can occur within 50 Myr after the dispersal of the primordial gaseous disk, which overlaps with the late stages of accretion for the terrestrial planets, and largely resemble the current architecture of the giant planets. Although we do not rule out the possibility of a delayed instability, this is not common in our simulation set and generally occurs after the bulk of the disk mass has been dynamically depleted.
3.5. Dependence on the Number of Particles with Giant Planets

Nesvorný & Morbidelli (2012), Reyes-Ruiz et al. (2015), and Deienno et al. (2017) evaluated whether the timing of the giant planet instability varies as a function of the number of bodies within the planetesimal disk. We are motivated to do the same for this work and perform simulations based on a subset of configurations detailed in Sections 3.3 and 3.4. In Section 3.3, we perform eight runs beginning with \( a_i = 14 \) au and incrementing by 0.5 au. At each disk gap, the number of particles is changed by a factor of 0.5, 2, and 3 while keeping the initial disk mass constant (35 \( M_\oplus \)). As a result, the mass of our disk particles ranges from several times the mass of the Moon down to a few times the mass of Pluto.

The results of these simulations are given in Figure 13, where the points are color-coded relative to the initial number of particles within the outer planetesimal disk. None of these cases produce a scenario consistent with a delayed instability. The median instability epoch is 10.5, 14.1, 17.3, and 18.9 Myr after our simulations begin considering 750, 1500, 3000, and 4500 disk particles, respectively. The scattering process is a chaotic one, and our results show a large spread of actual outcomes relative to the number of giant planets remaining (e.g., Kaib & Chambers 2016). Due to the small number of trials, we cannot rule out a delayed instability and can only infer that an early instability occurs more often irrespective of the number of disk particles.

We perform another test using our 8 five-planet runs from Section 3.4 that undergo a giant planet instability within 50 Myr, allowing one giant planet to escape and leaving four giant planets behind (i.e., the filled stars in Figure 11). These simulations are not uniform relative to the disk gap. These simulations vary the number of particles by a factor as before and keep the initial disk mass constant (20 \( M_\oplus \)), where the mass of the disk particles is 1.75 times smaller than that in our four giant planet runs. The results of these simulations are given in Figure 14, where the median instability epoch is 19.6, 26.2, 53.7, and 66.4 Myr after our simulations begin considering 750, 1500, 3000, and 4500 disk particles, respectively.

The median instability epoch increases for these simulations with increasing particle number, but the rate of increase is not dramatic. In terms of mean object mass, our 12,000-particle disk is comparable to the mean mass expected for the 12,000 most massive bodies in the primordial belt (Shankman et al. 2013; Nesvorný & Vokrouhlický 2016). From Figure 5, we find that an isolated 4500-particle disk has an \( e \)-heating rate within 15% of an isolated 12,000-particle disk and an \( a \)-spreading rate within 30%—40% of an isolated disk without any planetary stirring. If we include the giant planets, the mass resolution effects should be diminished further. Finally, our higher-resolution simulations only consider large disk gaps (>3.5 au). A larger initial disk gap would lower the immediate effects on the orbital evolution of the outermost giant planet due to the potential enhancement in \( e_{\text{rms}} \) and \( a \)-spreading from the assumed mass resolution on 100 Myr timescales. Based on this, we do not expect a still more realistic mass resolution will increase the typical disk stability times by 1–2 orders of magnitude. Moreover, numerical experiments probing such mass resolutions, beyond what we present here, exceed our current computing capabilities.
A majority of these additional runs result in giant planet instabilities on a timescale of $10^{3}–10^{4}$ Myr as before, where a subset ($33\%$) of these runs last for longer timescales. In six of our runs (open symbols in Figure 14; two points overlap when $a_{i} = 24.322$ au), five giant planets remain with a depleted disk after 100 Myr of simulation time. We continue these runs to 550 Myr, a timescale consistent with a delayed instability. Two simulations (750 and 4500 particles) where $\Delta \sim 4$ au retain all five giant planets on this timescale, and $97\%$ of the disk mass is ejected. The other four runs undergo a giant planet instability on a timescale of $\sim 150–400$ Myr and lose more than one giant planet.

Although only four runs have instabilities later than 150 Myr, a common feature among them is that their primordial Kuiper Belts are all heavily depleted by the time of the instability. Indeed, when we look at all of our five-planet simulations, we see that the instability time is strongly correlated with the amount of dynamical erosion of the disk prior to the onset of the instability. Moreover, this correlation appears to be largely independent of simulation resolution. Figure 15 shows the disk mass ($M_{\oplus}$) of our 8 five-planet simulations just prior to the instability time with respect to the initial number of particles. We find that every simulation with an instability after 100 Myr has a disk mass of $2M_{\oplus}$ or less just prior to the instability. This could have implications for the amount of material available for an LHB or for whether a low disk mass could sufficiently damp the remaining giant planets’ orbits after the instability, leading to another instability at a later time. While the timing of the early instabilities tends to vary with the initial number of particles (within a factor of 2–3), later instabilities seem to always be associated with a heavily depleted disk, regardless of particle number.

Having only four late instabilities makes it very difficult to assess the likelihood of the various orbital outcomes of late instabilities. We can further build our statistics by restarting these simulations right before they become unstable with slightly different conditions. The instability process is so chaotic that these slight shifts in conditions yield totally different final orbital configurations. We probe how the outcome can vary with small perturbations on the innermost ice giant and how this affects our conclusions with respect to our success criteria.

Small sets of simulations are performed using a state a few hundred thousands of years before the giant planets leave a compact configuration using the disk gaps and particle numbers shown in Figure 14 (open symbols). Perturbations are made by modifying the $x$-coordinate of the innermost ice giant randomly by 1 km and continuing the simulation for 10 Myr. Table 3 shows the percentage of runs that satisfy our Criteria A–D individually and simultaneously for short- and long-lived timescales. We estimate our uncertainty in satisfying all four criteria by assuming binomial noise for $\sigma_{A–D}$. These simulations suggest that the minority of systems that do experience a late instability meet our success criteria at a lower rate ($6\%$) than the systems which undergo early ($t < 100$ Myr) instabilities ($12\%$).
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3.6. Comparison with a Still Higher Mass Resolution

Until now, each of our simulations employs only one mass for every primordial belt object, and that mass is at least 2 Pluto masses in most of our simulations. Meanwhile, KBO observations and formation models suggest that the primordial belt possessed 1000–4000 Pluto-mass objects and a much larger population of smaller objects (Shankman et al. 2013; Nesvorný & Vokrouhlický 2016). The number of particles necessary to accurately model the lower bounds of this constraint is ~80,000 assuming a bimodal distribution consisting of 2000 Pluto-mass particles and the remainder in bodies that are 0.1 \( M_{\oplus} \) Pluto for a 20 \( M_{\oplus} \) disk. This is currently beyond our capability to simulate efficiently.

However, we can integrate a more realistic primordial Kuiper Belt mass distribution for a modest time span and then compare this to our highest mass resolution, fully interacting runs to better gauge their dynamical accuracy. To do this, we utilize the semi-active mode of GENGA for the large population of smaller bodies. We construct a 20 \( M_{\oplus} \) disk that contains 40% of its mass in 2000 fully active 2 \( M_{\oplus} \) Pluto bodies.

The other 60% of the disk mass is comprised of 30,000 bodies (each of 0.2 \( M_{\oplus} \) Pluto) that operate in the semi-active mode. While this disk’s fully active bodies are still too massive by a factor of two, the majority of the mass is in the semi-active form, which cannot self-stir, so these two effects will offset each other to some degree. Interior to this highest-resolution disk, we embed our five giant planet configuration, where the inner edge of the disk \( (a_i) \) begins at 25 au so that we can compare this with our run with \( a_i = 24.947 \) au and 4500 particles. This higher-resolution simulation is stopped after 3 Myr so that our comparison primarily captures the relative magnitude of viscous stirring within the disk. Figure 16 illustrates the state of our higher-resolution simulation (Figures 16(c) and (d)) with a comparable simulation with 4500 equal-mass particles (Figures 16(a) and (b)). The black points represent those particles that are fully active, whereas the color-coded points denote semi-active particles. The eccentricity distributions appear strikingly similar on this timescale.

To distinguish between particle types, we provide the time evolution of the mass-weighted mean eccentricity and inclination of the disk particles in Figure 17. We find that the smaller semi-active particles (dashed blue) closely track the evolution of the equal-mass particles (solid black) in both the mean eccentricity and inclination. The fully active particles within the high-resolution run (dashed red) lag behind likely due to dynamical friction from the semi-active particles. The total mass-weighted mean eccentricity of our most realistic disk will lie slightly closer to the blue line than to the red, because 60% of the disk mass is in the form of semi-active bodies. After 3 Myr of evolution, the mean eccentricities of the two disks are not radically different (~0.07 versus ~0.08). Thus, we conclude that the primordial Kuiper Belt’s real dynamics and self-excitation should not be radically different from the highest mass resolution of this work on the timescales considered (around hundreds of megayears).

### Table 2

Summary of Results Based on a Nice Model Configuration (3-2, 3-2, 2-1, 3-2) with Five Giant Planets (\( a_i^{\text{d}} \approx 20.3 \text{ au} \)) along with a 20 \( M_{\oplus} \) Outer Disk

| \( a_i \) (au) | \( \Delta \) (au) | \( t \) (Myr) | A | B | C | D |
|----------------|-----------------|---------------|---|---|---|---|
| 22.697         | 2.329           | 16.361        | * | * | * | * |
| 22.822         | 2.454           | 17.002        | * | * | * | * |
| 22.947         | 2.579           | 19.220        | * | * | * | * |
| 23.072         | 2.704           | 25.947        | 2.454 | 16.361 | * | * |
| 23.197         | 2.829           | 23.507        | * | * | * | * |
| 23.322         | 2.954           | 21.635        | * | * | * | * |
| 23.447         | 3.079           | 13.257        | * | * | * | * |
| 23.572         | 3.204           | 15.277        | * | * | * | * |
| 23.697         | 3.329           | 14.390        | * | * | * | * |
| 23.822         | 3.454           | 41.840        | * | * | * | * |
| 23.947         | 3.579           | 288.83        | * | * | * | * |
| 24.072         | 3.704           | 16.016        | * | * | * | * |
| 24.197         | 3.829           | 17.495        | * | * | * | * |
| 24.322         | 3.954           | 15.622        | * | * | * | * |
| 24.447         | 4.079           | 378.33        | * | * | * | * |
| 24.572         | 4.204           | 28.287        | * | * | * | * |
| 24.697         | 4.329           | 133.16        | * | * | * | * |
| 24.822         | 4.454           | 40.509        | * | * | * | * |
| 24.947         | 4.579           | 16.706        | * | * | * | * |
| 25.072         | 4.704           | 24.409        | * | * | * | * |
| 25.197         | 4.829           | 24.049        | * | * | * | * |
| 25.322         | 4.954           | 13.799        | * | * | * | * |
| 25.447         | 5.079           | 550.00        | * | * | * | * |
| 25.572         | 5.204           | 39.869        | * | * | * | * |
| 25.697         | 5.329           | 70.423        | * | * | * | * |
| 25.822         | 5.454           | 13.405        | * | * | * | * |
| 25.947         | 5.579           | 35.039        | * | * | * | * |
| 26.072         | 5.704           | 31.688        | * | * | * | * |
| 26.197         | 5.829           | 30.062        | * | * | * | * |
| 26.322         | 5.954           | 39.380        | * | * | * | * |
| 26.447         | 6.079           | 33.067        | * | * | * | * |
| 26.572         | 6.204           | 89.988        | * | * | * | * |

**Note.** The columns correspond to the heliocentric inner edge of the disk \( a_i \), the distance between the inner disk edge and the outer ice giant \( \Delta \), the time of the giant planet instability \( t \) in Myr, and whether the given conditions meet (✓) or fail (✗) each of our criteria for success, A–D.
expect it to evolve similarly to our low-resolution disks, tending toward short instability times. On the other hand, if the protoplanetary disk had a steep size frequency distribution or its mass was segregated into two or more species with large discrepancies in object mass, we would expect the instability times could be long, like those found in Levison et al. (2011).

In our simulations, the instability epoch with four giant planets appears to be independent of the assumed disk gap, or the distance between the inner edge of the disk and the outermost ice giant. While some cases survive for 550 Myr, the amount of disk material is significantly reduced (∼97% $M_D$ lost), thereby leaving the trigger necessary for a delayed instability up to random perturbations between the giant planets and possibly reducing the disk’s ability to damp the eccentricities of the giant planets if an instability does eventually occur. Interestingly, we find the excitation of Jupiter’s eccentricity mode $e_{55}$ proceeds relatively easily compared to that in previous works that used approximations for the interactions within the disk (Levison et al. 2011; Nesvorný & Morbidelli 2012). Previous Nice model studies have generally disfavored Jupiter and Saturn beginning in the 2:1 resonance because this initial configuration fails to excite $e_{55}$ enough, but the enhanced excitation seen in our work may reopen this possibility (Nesvorný & Morbidelli 2012). Future studies of this configuration using a self-stirring disk should address this.

Our five giant planet results show a slight trend with the disk gap, favoring values $>$3.5 au. When we begin our five-planet systems with disks of 1500 equal-mass particles, one-eighth of the post-instability systems reproduce the current orbits of the outer planets quite well (Criteria A, B, and D in Section 2.2). Seven out of 32 of these five-planet systems remain stable for longer than 50 Myr. Upon further integration to 550 Myr, all but one of these systems go unstable. The remaining stable system retains only 3% of its disk mass after 550 Myr. We note that the instabilities could occur at even later times (∼800 Myr), as in the original models proposed by Gomes et al. (2005) and Tsiganis et al. (2005), but the mass flux needed to match an LHB would likely not be available because the disk mass is so dynamically eroded by this point (i.e., all of the mass in our stable system would need to be directed at the inner solar system).

Given that the disk particle masses of our main sets of simulations are several times the mass of Pluto, we also investigate how our systems’ evolution varies with lower particle mass. For our four-planet systems we do not find any significant trends with decreasing particle mass and increasing particle number. However, in our five-planet systems the median instability time increases to ∼66 Myr as the particle mass approaches 1 Pluto mass. All of these additional runs assume a large gap between the disk and planets, and our highest-resolution runs approach mean particle masses and numbers consistent with the primordial belt. Even then, early instabilities within ∼20 Myr occur, and the median instability time is well below 100 Myr. We implement statistical variations to our

![Figure 12. Final giant planet architectures of a Nice model configuration with five giant planets along with a 20 $M_\oplus$ outer disk. The filled points enclosed in a box represent the initial giant planet configuration of each simulation, whereas the open points denote the final architectures. Horizontal lines are given at the semimajor axis values of the current solar system, and the gray bars represent the range of values within 20%. The check marks at the bottom identify those simulations that end with four giant planets, and the “X” marks show the simulations that do not lose any giant planets. Points at the top filled with hatch marks denote which giant planets are lost in a given simulation.](image-url)
Figure 13. Instability times of a Nice model configuration with four giant planets along with a 35 $M_{\oplus}$ outer disk. The symbols indicate how many giant planets remain at the end of the simulation, and are color-coded by the initial number of disk particles.

Figure 14. Instability times of a Nice model configuration with five giant planets along with a 20 $M_{\oplus}$ outer disk. The symbols indicate how many giant planets remain at the end of the simulation, and are color-coded by the initial number of disk particles.
results using a random perturbation to the inner ice giant. We follow these cloned systems through the instability and find that the basic orbital architecture of the outer planets can be reproduced via late or early instabilities.

Our systems with early instabilities successfully replicate the outer planets’ orbits roughly 12% of the time, while the late-instability systems replicate the outer planets at a lower rate of 6%. This difference could be attributed in part to our resolution in disk particle masses. However, comparison of the self-stirring of a more realistic disk model that employs a bimodal set of KBO masses to that in our highest-resolution self-interacting disks suggests that the disk dynamics of simulations will not radically change as we continue to approach more realistic mass resolutions. Nevertheless, such higher mass

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**Figure 15.** Disk masses and instability times of a Nice model configuration with five giant planets along with a 20 $M_\oplus$ outer disk. The symbols indicate how many giant planets remain at the end of the simulation, and are color-coded by the initial number of disk particles.

**Figure 16.** Representations of the system state at 3 Myr considering a Nice model configuration with five giant planets with a 20 $M_\oplus$ outer disk ($a_i \approx 25$ au) using a disk of 4500 equal-mass particles (a and b) and using a bimodal distribution of 32,000 particles (c and d). The color code represents the initial semimajor axis of the semi-active particles, where the size of the points is scaled by the physical radius. The fully active particles (black) are also scaled by the physical radius but are not coded with the initial semimajor axis.

**Table 3**

| $t$ (Myr) | A (%) | B (%) | C (%) | D (%) | A–D (%) | $\sigma_{A–D}$ (%) |
|---|---|---|---|---|---|---|
| <100 | 30% | 20% | 98% | 30% | 12% | 3% |
| >100 | 28% | 11% | 64% | 29% | 6% | 2% |

**Note.** The columns correspond to the temporal range of the giant planet instability $t$ in Myr, the percentage of runs that satisfy Criteria A–D individually, the percentage of runs that satisfy Criteria A–D simultaneously, and the uncertainty in our estimation of satisfying all four criteria simultaneously.
resolution models will be required to confirm our conclusions with higher confidence.

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![Figure 17](image-url)  
Time evolution of the mass-weighted mean eccentricity (top) and inclination (bottom) of the outer disk particles for the simulations given in Figure 16. The simulation with equal masses (solid) is distinguished from the higher-resolution run with a bimodal distribution (dashed), which is delineated further by two mass bins (red and blue) of 2 and 0.2 Pluto-mass bodies.