Direct-Effect Risk Minimization for Domain Generalization

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Abstract

We study the problem of out-of-distribution (o.o.d.) generalization where spurious correlations of attributes vary across training and test domains. This is known as the problem of correlation shift and has posed concerns on the reliability of machine learning. In this work, we introduce the concepts of direct and indirect effects from causal inference to the domain generalization problem. We argue that models that learn direct effects minimize the worst-case risk across correlation-shifted domains. To eliminate the indirect effects, our algorithm consists of two stages: in the first stage, we learn an indirect-effect representation by minimizing the prediction error of domain labels using the representation and the class labels; in the second stage, we remove the indirect effects learned in the first stage by matching each data with another data of similar indirect-effect representation but of different class labels in the training and validation phase. Our approach is shown to be compatible with existing methods and improve the generalization performance of them on correlation-shifted datasets. Experiments on 5 correlation-shifted datasets and the DomainBed benchmark verify the effectiveness of our approach. Our code is available at https://github.com/Liyuhui-12/DRMforDG.

1. Introduction

Machine learning has achieved huge success in many fields, yet they mostly rely on the independent and identically distributed (i.i.d.) assumption. When it comes to an out-of-distribution (o.o.d.) test domain, machine learning models usually suffer from a sharp performance drop [8][10][51]. The o.o.d. data typically come in the form of correlation shift, where spurious correlations of attributes vary between training and test domains, or diversity shift, where the shifted test distribution keeps the semantic content of the data unchanged while altering the data style [22]. The focus of this work is on the former setting known as correlation shift. That is, given stable causality and spurious correlations between attributes, how to disentangle the stable causality and the spurious correlations from the training data. Figure 1 shows the performance gain of our method on the correlation shift datasets.

Much effort has been devoted to learning representations that are invariant across training environments, where many works have introduced the tools from causality to address the o.o.d. generalization problems. When the data are of high dimension and multiple attributes are entangled, it is...
challenging to identify invariant causality across domains. Many methods have been designed to resolve the issue. Representative methods include incorporating invariance constraints by designing new loss functions \[ \text{[6][1][3][2]} \], learning latent semantic features in causal graphs by VAE \[ \text{[4][4][5]} \], and eliminating selection bias by matching \[ \text{[4][7][8]} \]. However, these methods, despite their theoretical guarantees, fail to show empirical improvement over Empirical Risk Minimization (ERM) as verified by the DomainBed benchmark \[ \text{[2][8][0]} \].

This paper is the first attempt to use the tool of direct and indirect effects from causal inference to analyze the correlation shift problem. We argue that under certain conditions, models that learn direct effects minimize the worst-case risk across domain-shifted domains. To learn the direct effects, we propose a two-stage approach: in the first stage, we use an extractor to infer the indirect-effect representation \(Z\) from the data \(X\) such that \(Z\) can predict the domain label \(E\) through a discriminator head (see the blue box in Figure 2). In the second stage, we construct a balanced batch by augmenting the original training batch and validation set with data of the same indirect-effect representation \(Z\) but of a different class label \(Y\). We demonstrate that our validation balancing approach can overcome the inconsistency of the validation distribution with the test distribution, which was shown to be an important reason for the performance degradation of many existing approaches under the DomainBed protocol. We test our approach on the DomainBed benchmark. On the correlation shift dataset Colored MNIST, our model obtains an average accuracy of 71.2% over three domain generalization problems. While the information-theoretic best accuracy on the Colored MNIST dataset is 75%, our method achieves an accuracy as high as 69.7% in the most difficult \(\sim 90%\) environment. Moreover, we provide evidence that the foundation models can alleviate the diversity shift problem but cannot solve the correlation shift problem well, demonstrating that our approach closes the gap between foundation models and domain generalization to some extent. Our method can be combined with existing domain generalization methods to significantly improve their performance on correlation shift datasets. Our main contributions are as follows:

- We present a framework to analyze the correlation shift problem based on direct/indirect causal effects.
- We propose a new approach to improve o.o.d. generalization. We recover the indirect-effect representation and eliminate the indirect effect during training and validation. We also show that our model selection method can largely overcome the model selection problem caused by the inconsistency between the validation and the test distribution. Our approach can be easily compatible with other algorithms and substantially improve their performances.
- Our method outperforms baselines by a large margin on the correlation-shifted datasets. For example, our approach achieves up to 15% absolute improvement on the Colored MNIST dataset and up to 11% absolute improvement on the CelebA datasets over the state-of-the-art in terms of average accuracy over three domains.

2. Preliminaries

Notations. In this paper, we will use capital letters such as \(X, Y, Z\) to represent random variables, lower-case letters such as \(x, y, z\) to represent realization of random variables, and letters with hat such as \(\hat{Z}\) to represent inferred variables by the model. We use the calligraphic capital letter \(\mathcal{E}\) to represent the set of environments and by lower-case letter \(e\) the domain label. \(X \perp Y\) means that random variables \(X\) and \(Y\) are independent. We use \(\mathcal{D}\) and \(\mathcal{P}\) to denote the distribution and the corresponding probability density function (PDF) in environment \(e\), respectively. We add the superscript \(e\) to a variable such as \(x^e\) to indicate
that the variable is sampled from the distribution of the environment \( e \), and \((x_e^i, y_e^i, e)\) refers to an instance sampled from \( \mathcal{D}^e \). We denote by \( \mathcal{H} \) the hypothesis class of models and by \( h : \mathcal{X} \to \mathcal{Y} \) the predictor. \( R_e^e(h) \) refers to the risk of predictor \( h \) in the environment \( e \). Environment and domain are of the same concept, and we use them interchangeably throughout the paper.

### 2.1. Correlation shift

We consider the correlation shift problem in this paper. In a supervised learning setting, the goal is to learn the labeling function \( f : \mathcal{X} \to \mathcal{Y} \), which is consistent in all environments. However, there often exists a variable set \( Z \) such that there are spurious correlations between \( Z \in \mathcal{Z} \) and the label \( Y \). When the spurious correlation changes with the environment, the model that utilizes the spurious correlation may face a performance breakdown in the new test environment. Spurious correlations may originate from the data generation process or selection bias, which is very common in reality.

Consider a binary classification problem of cows and camels (see Figure 3). We assume that the animal category and background are the two attributes that contribute to the generation of an image. Our goal is to predict the animal category from image generation. Spurious correlation shifts and the DAG are general in nature and consistent with other recent works like [85].

**Definition 1 (Correlation shift)** Assume that we have a training environment \( e_S \in \mathcal{E}_{\text{train}} \) and a test environment \( e_T \in \mathcal{E}_{\text{test}} \), whose probability density functions are \( \mathbb{P}^{e_S} \) and \( \mathbb{P}^{e_T} \), respectively. Assume that \( \mathbb{P}^{e_S}(y) = \mathbb{P}^{e_T}(y) \) for every \( y \in \mathcal{Y} \), and that \( e_S \) and \( e_T \) share the same labeling function. Then there exists correlation shift between \( e_S \) and \( e_T \) if there exists a set \( Z \) such that

\[
\sum_{y \in \mathcal{Y}} \int_Z |\mathbb{P}^{e_S}(z|y) - \mathbb{P}^{e_T}(z|y)|dz \neq 0,
\]

where \( \mathbb{P}^{e_S}(z) \times \mathbb{P}^{e_T}(z) \neq 0 \).

By definition, we consider a direct acyclic graph (DAG) describing the data generation process (see Figure 3), where the pathways from \( Y \) to \( X \) are composed of two parts: \( Y \to X \) and \( Y \to Z \to X \). In causal inference, the former is referred to as the direct effect of \( Y \) on \( X \), while the latter is referred to as the indirect effect. \( Z \) is a child of the environment \( E \), which leads to the indirect effects varying with the environment. In a supervised learning setting, models are trained to learn the reversed process of the data generation process, i.e., \( X \to Y \) or \( X \to Z \to Y \). To obtain models that can generalize across different domains under correlation shift, we need to cut off the indirect effect pathway (\( Y \to Z \to X \)) and force the model to learn the reversed mapping of the robust direct effects (\( Y \to X \)).

It is worth noting that in fact there is often only a correlation between the label \( Y \) and the mediator \( Z \). For example, the presence of a cow does not lead to the presence of grass. For the model, however, it is reasonable to interpret this correlation as an indirect causal effect in the absence of human knowledge. Under this interpretation, all confounders between \( Y \) and \( X \) can be included by \( Z \), thus there are no unblocked backdoor pathways [57] between \( Y \) and \( X \). Our description of the data generation process under correlation shifts and the DAG are general in nature and consistent with other recent works like [85].

### 2.2. Problem Setting

We consider a standard domain generalization setting, where the data come from different environments \( e \in \mathcal{E}_{\text{all}} \). Assume that we have the training data collected from a finite subset of training environments \( \mathcal{E}_{\text{train}} \subseteq \mathcal{E}_{\text{all}} \). For every environment \( e \in \mathcal{E}_{\text{train}} \), the training dataset \( \{(x_{e,i}^T, y_{e,i}^T, e)\}_{i=1}^{N_e} \) is sampled from the distribution \( \mathbb{D}^e \). The PDF of the distribution is \( \mathbb{P}^e(X^e, Y^e) = \mathbb{P}(X,Y | E = e) \), where \( X \) is the instance (e.g., an image), \( Y \) is the class label, \( E \) is the domain label, and \( N_e \) is the number of training data in environment \( e \). The goal of domain generalization is to train a model with data from training environments \( \mathcal{E}_{\text{train}} \) that generalizes well to all environments \( e \in \mathcal{E}_{\text{all}} \). Our goal is to find a predictor \( h^* : \mathcal{X} \to \mathcal{Y} \) in the hypothesis class \( \mathcal{H} \) such that the worst-case risk is minimized:

\[
h^* = \arg\min_{h \in \mathcal{H}} \max_{e \in \mathcal{E}_{\text{all}}} R_e^e(h),
\]
where $R^e(h)$ is the risk of predictor $h$ in environment $e$.
We argue that the model learning the stable direct effects is robust when the environment changes, which satisfies equation[1]. To enable the model to learn the direct effects in the data, it is desirable to cut off the pathway between $Z$ and $Y$ so that they are independent. To this end, we designed a novel framework with improved training process and model selection.

### 3. Method

#### 3.1. Recovering Indirect Effects

Since the variable $Z$ on the indirect-effect pathway is often not observable, we design an extractor to recover the representation $Z$ of the indirect effect from $X$ by learning a discriminator head in the first stage. From Figure [3], we observe that the indirect-effect representation $Z$ and the class label $Y$ form a Markov blanket for the domain label $E$, which means that $E$ is independent of other variables given $Y$ and $Z$. Hence the discriminator head needs $Z$ and $Y$ to predict $E$. If we take the output of the extractor and $Y$ as the input of the discriminator head, the discriminator head will force the extractor to recover $Z$ from $X$. Specifically, assume that the dataset is sampled from $N_s$ training domains. We set up an extractor $G(\cdot; \Theta_G) : \mathcal{X} \rightarrow \mathcal{Z}$ and a discriminator head $D(\cdot, \cdot; \Theta_D) : \mathcal{Z} \times \mathcal{Y} \rightarrow [0, 1]^{N_s}$ that outputs the probability that a sample belongs to each training domain, and update the parameters of both models by minimizing the prediction error of domain label $e$:

$$\Theta_G^*, \Theta_D^* := \arg\min_{\Theta_G, \Theta_D} \mathbb{E}_{x, y, e} \mathbb{C} \left( D(G(x; \Theta_G), y; \Theta_D), e \right),$$

where $\mathbb{C}$ is the Cross Entropy loss, $\Theta_G$ and $\Theta_D$ stand for the parameters of the extractor $G$ and the discriminator head $D$, respectively, and $(x, y, e)$ is a training sample. We use the learned extractor to obtain the representation $\hat{z}_i = G(x_i^e; \Theta_G^*)$ for every instance $(x_i^e, y_i^e, e)$.

Many methods learned domain discriminators by a minimax problem [4][24][37]. These methods extracted features that could maximize the domain discriminator error. In our approach, on the other hand, the representation vector $Z$ is obtained by minimizing the domain discrimination error. This makes our model easier to optimize and more stable than a minimax game.

#### 3.2. Eliminating Indirect Effects in Training (TB)

In the model training stage, we remove the indirect effects from the data by creating balanced batches based on the representation $\hat{Z}$, which is referred to as TB in the following. We start by defining the balanced batch.

**Definition 2 (Balanced Batch)** For any sample in a balanced batch, denoted by $(x_i^e, y_i^e, e, \hat{z}_i)$, there exists a corresponding sample $(x_j^e, y_j^e, e, \hat{z}_j)$ with probability $P$, such that $\hat{z}_i = \hat{z}_j$, $y_i^e \neq y_j^e$, and $P_{\text{Batch}}(Y) = P_{\text{D}}(Y)$, where $P_{\text{Batch}}(Y)$ and $P_{\text{D}}(Y)$ are marginal probability density functions of $Y$ in the batch and the training set, respectively.

Ideally, for each sample $x_i$, we can find a corresponding sample $x_j$ with the same indirect-effect representation $\hat{z}_i = \hat{z}_j$. However, we cannot always find exactly equal $\hat{z}$ as in the ideal case. To resolve this problem, for each sample $(x_i^e, y_i^e, e, \hat{z}_i)$, we search for another sample $(x_j^e, y_j^e, e, \hat{z}_j)$ such that $\hat{z}_j$ is the nearest neighbor of $\hat{z}_i$. To ensure that the marginal distribution of label $Y$ does not change, we include the matched sample into the batch with a probability that depends on the proportion of each class of samples in the training set.

Taking the cow and camel classification problem in Figure 3 as an example, we search for a camel image with the same background for each cow image in the batch, e.g., a camel standing on grass for a cow standing on grass, as well as a cow image with the same background for each camel image. Thus our training batches consist of pairs of images. We train the model on balanced batches constructed as described above.

#### 3.3. Model Selection (VB)

According to DomainBed [26], whether to follow the protocol of random hyperparameter search can drastically affect the performance of a method, especially for correlation-shifted datasets. DomainBed recommended that researchers should disclaim any oracle-selection results as such and specify policies to limit access to the test domain. Unlike the i.i.d. task, the distributions of the training and testing domains are significantly different in a domain generalization problem, and there is a large performance gap between selecting a model on the test domain distribution and the training domain distribution. For example, as shown in Figure [4], while using the method of training-domain validation, the validation and test accuracy rates are often inconsistent. Checkpoints with high accuracy on the validation set do not perform well on the test set. DomainBed searches for hyperparameters randomly to ensure no access to the test domain, thus the results of many existing methods reported by DomainBed may be lower. We believe that one of the very significant reasons for this decline in performance is that the spurious correlation in the validation set misleads the model selection. Models that utilize spurious correlation are able to perform well on validation sets where spurious correlation exists, while they cannot generalize to unseen test domains with reversed correlation.

Considering this inconsistency, our DRM framework also includes a novel approach to model selection based on
Algorithm 1 Direct-Effect Risk Minimization (DRM)

Input: Training set $D$; validation set $V$; initial predictor $f_{\theta_{e}}$; training steps $T$; checkpoint frequency $C$; learning rate $\epsilon$;

Output: Predictor $f_{\theta_{e}}$;

1: Update $\Theta_{G}, \Theta_{D}$ by the equation (2) and get $\Theta_{G}^{*}, \Theta_{D}^{*}$;
2: $V_{b} \leftarrow \emptyset$;
3: for $(x^{e}, y^{e})$ in $V$ do
4: \[ \hat{z}^{e} \leftarrow G(x^{e}; \Theta_{G}^{*}); \]
5: Search $V$ for $(x_{b}^{e}, y_{b}^{e})$ with the closest $\hat{z}_{b}^{e}$ to $\hat{z}^{e}$, and $y^{e} \neq y_{b}^{e}$;
6: Add $(x^{e}, y^{e})$, $(x_{b}^{e}, y_{b}^{e})$ to $V_{b}$;
7: end for
8: $t \leftarrow 0$;
9: while $t \leq T$ do
10: Sample a batch $B = \{(x_{i}^{e}, y_{i}^{e})\}_{i=1}^{\text{batchsize}}$ from $D$;
11: for $(x^{e}, y^{e})$ in batch do
12: \[ \hat{z}^{e} \leftarrow G(x^{e}; \Theta_{G}^{*}); \]
13: Search $D$ for $(x_{b}^{e}, y_{b}^{e})$ with the closest $\hat{z}_{b}^{e}$ to $\hat{z}^{e}$, and $y^{e} \neq y_{b}^{e}$;
14: Add $(x_{b}^{e}, y_{b}^{e})$ to $B$;
15: end for
16: Run ERM or other algorithms on $B$ and update $f_{\theta_{e}}$;
17: if $C|t$ then
18: Evaluate model on balanced validation set $V_{b}$;
19: end if
20: $t \leftarrow t + 1$;
21: end while

Figure 4. The inconsistency of validation set accuracy and test set accuracy during the training process. (a) is for IRM, (b) is for VREx, and (c) is for CORAL.

the above-mentioned balancing approach, which is referred to as VB in the following sections. Specifically, we create balanced validation sets in the same way as in Section 3.1 and Section 3.2 on which we evaluate the models. The validation data are all divided from the training domains, which is consistent with the training-domain validation protocol in DomainBed and ensures that the model has no access to the test set.

For the main results of our paper (Table 1), we run ERM on balanced batches and evaluate the models on balanced validation sets (ERM+VB+TB). However, it is worth mentioning that our VB and TB methods can also be combined with many existing methods and substantially improve their performances. We show these results in Table 2. All of our experiments followed the DomainBed protocol, and to be fair, we only compared DRM with methods that follow the same protocol and have no access to test domains.

The pseudo-code description of the whole DRM framework is shown in Algorithm 1.

4. Experiments

We compare DRM with 14 baseline methods, including: ERM [79], IRM [8], GroupDRO [70], Mixup [86, 88, 89, 90], MLDG [35], CORAL [76], MMD [38], DANN [24], CDANN [40], MTL [13], SagNet [52], ARM [98], VREx [33] and RSC [29], which appeared in the DomainBed benchmark [25]. We evaluate the performance of our approach on datasets classified as correlation-shifted datasets by [87, 92], in which there are spurious correlations between the class label and the features such as the color or the background of the images. Learning the test environment is difficult since there might be a spurious-correlation flip between the training and test environments. We show that the performance of i.i.d. algorithms such as ERM will significantly drop in this case, while our approach achieves improved performance in these difficult environments. We strictly follow the protocol of DomainBed by conducting random searches for all hyperparameters in all stages. For comparison, we use the codes provided by DomainBed to run the above-mentioned 14 methods. We defer the details of the experiment setting and results to Appendix B.

4.1. Datasets

We evaluate our approach on MNIST dataset, 3DShapes dataset, DSprites dataset, and CelebA dataset, which are common correlation shift datasets used to evaluate domain generalization methods [70, 87, 92].

Colored MNIST [8, 34] creates a spurious correlation between colors and digits by artificially coloring the digits red or green. The correlations between colors and labels in three environments are +90%, +80%, and −90%, respectively. For example, in the “+90%” environment, 90% images with label 1 are dyed red, while 90% images with label 0 are dyed green. In addition, the dataset randomly flips 25% of the class labels, which results in 75% correlation between shapes and digital labels, lower than that between colors and labels. Thus, an i.i.d. learning approach like ERM prefers to learn correlations between colors and labels. To test the validity of our method in a broader and more realistic context, we also introduced four correlation shift datasets, which are 3DShapes [15], DSprites [43], CelebA-HB, and CelebA-NS [44]. The stable and spurious features for these datasets are “floor hue” and “orientation”, “Position X” and “Position Y”, “No Beard” and “Wearing
Hat”, “Smiling” and “Wearing Necktie”, respectively. We defer the specific details about the dataset to Appendix B.2.

While focusing on the correlation shift problem, we also conduct experiments on the diversity-shifted dataset contained in DomainBed such as PACS and VLCS to show that DRM will not hurt the model performance on such datasets. We defer the results to the appendix. Diversity shift is considered as a different domain generalization problem with correlation shift in our paper, which is consistent with [92]. An example of diversity shift is that the images in the training domain are art paintings and cartoons, while they are photos in the test domain. We show in Section 4.3 that ERM suffers from more severe performance degradation on the correlation-shifted datasets than on the diversity-shifted datasets in the o.o.d. case, and they may require different solutions. It is to the former that our paper proposes an effective solution.

We would also like to emphasize that the type of domain shift is independent of whether a dataset is real-world or not. Both CelebA-NS and CelebA-HB datasets in our paper are real-world datasets with correlation shifts. The correlation shift is more likely to arise from the selection bias in real-world scenarios, as simulated by the CelebA datasets. However, since the datasets used in research are often shuffled, researchers tend to ignore the presence of selection bias (correlation shift), even though this is an important factor.

4.2. Results.

Table 1 shows the performance of our approach under correlation shift. Under the DomainBed protocol, both ERM and the domain generalization algorithms officially reported by DomainBed do not perform well for correlation shift because they all suffer from a sharp performance drop when the test environment has reversed correlation with the training environment. Their accuracy rates are all very close, which is consistent with the results of CMNIST reported in DomainBed [26]. For Colored MNIST, on which the information-theoretic best accuracy is 75% due to the 25% noise, the accuracy of ERM and other domain generalization algorithms are no more than 10.5% in the most difficult “−90%” environment, which is far lower than random guess. In contrast, our DRM approach achieves 69.7% accuracy, almost 60% higher than the other algorithms. At the same time, our approach does not hurt performance on the “+90%” and “+80%” domains. On average, our approach outperforms ERM by 20% and outperforms the best previous approach by 15%. For the other datasets, the results show the same pattern. Our approach is substantially ahead of the other methods by about 50% on the most difficult domain and brings significant performance improvement of more than 15% for the correlation shift problems.

Results interpretation. We attribute the significant performance improvement of DRM to its ability to reduce the spurious correlations in the training and validation set to a large extent, even on more realistic datasets, as shown in Figure 5. We show eight examples of balanced CMNIST image pairs in Figure 6. The label-1 image with a red background is matched with the label-0 image with a red background, and the label-0 image with a green background is matched with the label-1 image with a green background, thus the correlation between the background color and the label is reduced. Models trained with balanced data are less susceptible to misleading correlations and so have better o.o.d. generalization capabilities.

![Figure 5. The spurious correlation before and after balancing.](image)

![Figure 6. Eight examples of balanced CMNIST image pairs.](image)

Ablation study. Our approach is based only on an improvement of the sampling phase, so both training set balancing (TB) and validation set balancing (VB) can be easily combined with other algorithms. In this section, we analyze the contribution of VB and TB in our approach, respectively. Results are shown in Table 2. Both training set balancing and validation set balancing can improve the o.o.d. performance of the original model significantly, showing that both of them are effective and important components of our approach and can be used as a general framework to mitigate the correlation shift problem. VB demonstrates the ability to exploit the potential of existing methods such as IRM, as they can significantly outperform ERM simply by adding VB. When both VB and TB are used, the performances of existing methods achieve further improvements. ERM performs well in this case because VB and TB have largely eliminated spurious correlations.

Accuracy curves analysis. Many mainstream domain generalization methods design new loss functions by incorporating invariant constraints, such as IRM and VREx, which leads to a very unstable training process. In contrast, as shown in Figure 7, the fluctuation of the accuracy curve...
and Avg are the minimum value and the average accuracy for all test environments, respectively.

| Algorithm | CMNIST | DSprites | CelebA-HB | CelebA-NS |
|-----------|--------|----------|-----------|-----------|
| Min       | Avg    | Min      | Avg       | Min       | Avg       | Min       | Avg       |
| ERM       | 10.0 ± 0.1 | 51.5 ± 0.1 | 10.1 ± 0.1 | 53.3 ± 0.1 | 13.8 ± 0.5 | 54.0 ± 0.1 | 16.8 ± 1.2 | 52.0 ± 0.5 | 21.1 ± 0.4 | 52.7 ± 0.5 |
| I RM      | 10.2 ± 0.3 | 52.0 ± 0.1 | 10.0 ± 0.0 | 53.2 ± 0.1 | 14.5 ± 0.3 | 54.0 ± 0.1 | 20.4 ± 2.1 | 52.1 ± 0.7 | 21.5 ± 0.9 | 53.2 ± 0.4 |
| GroupDRO  | 10.0 ± 0.2 | 52.1 ± 0.0 | 10.5 ± 0.4 | 53.4 ± 0.1 | 15.0 ± 0.4 | 54.4 ± 0.2 | 18.3 ± 1.5 | 52.8 ± 0.9 | 21.2 ± 0.2 | 53.3 ± 0.2 |
| Mixup     | 10.1 ± 0.1 | 52.1 ± 0.2 | 10.2 ± 0.1 | 53.4 ± 0.2 | 14.0 ± 0.3 | 53.9 ± 0.0 | 17.9 ± 3.4 | 52.4 ± 1.1 | 22.2 ± 1.5 | 53.7 ± 0.7 |
| MLDG      | 9.8 ± 0.1  | 51.5 ± 0.1 | 10.1 ± 0.1 | 53.5 ± 0.1 | 14.3 ± 0.3 | 54.2 ± 0.1 | 20.0 ± 2.1 | 53.0 ± 0.7 | 22.7 ± 1.7 | 53.7 ± 0.6 |
| CORAL     | 9.9 ± 0.1  | 51.5 ± 0.1 | 10.0 ± 0.0 | 53.3 ± 0.1 | 13.8 ± 0.2 | 53.9 ± 0.0 | 17.7 ± 1.6 | 52.4 ± 0.6 | 22.1 ± 1.1 | 53.4 ± 0.4 |
| MMD       | 9.9 ± 0.3  | 51.5 ± 0.1 | 10.0 ± 0.1 | 53.2 ± 0.1 | 14.4 ± 0.0 | 51.4 ± 2.1 | 17.4 ± 1.8 | 50.7 ± 0.5 | 22.6 ± 0.6 | 53.3 ± 0.1 |
| DANN      | 10.0 ± 0.0 | 51.5 ± 0.3 | 10.0 ± 0.0 | 53.3 ± 0.0 | 14.7 ± 0.3 | 54.1 ± 0.3 | 16.9 ± 1.7 | 51.7 ± 0.3 | 21.8 ± 1.5 | 53.7 ± 0.8 |
| CDANN     | 10.2 ± 0.1 | 51.7 ± 0.1 | 10.0 ± 0.0 | 53.3 ± 0.1 | 14.4 ± 0.2 | 54.0 ± 0.1 | 18.6 ± 2.6 | 52.5 ± 0.6 | 22.5 ± 1.2 | 53.9 ± 0.4 |
| MTL       | 10.5 ± 0.1 | 51.4 ± 0.1 | 10.1 ± 0.0 | 53.4 ± 0.1 | 14.8 ± 0.5 | 54.3 ± 0.1 | 23.5 ± 1.4 | 53.7 ± 0.6 | 27.6 ± 1.2 | 54.9 ± 0.3 |
| SagNet    | 10.3 ± 0.1 | 51.7 ± 0.0 | 10.1 ± 0.1 | 53.4 ± 0.1 | 13.6 ± 0.1 | 54.0 ± 0.0 | 14.9 ± 0.9 | 50.4 ± 0.3 | 22.0 ± 0.6 | 53.1 ± 0.2 |
| ARM       | 10.2 ± 0.0 | 52.0 ± 0.0 | 10.0 ± 0.0 | 55.2 ± 0.3 | 14.5 ± 0.6 | 59.7 ± 0.4 | 22.8 ± 2.3 | 54.1 ± 0.6 | 21.1 ± 1.4 | 53.0 ± 0.5 |
| VREx      | 10.2 ± 0.3 | 52.0 ± 0.1 | 10.8 ± 0.3 | 53.5 ± 0.1 | 13.8 ± 0.3 | 53.9 ± 0.1 | 19.2 ± 1.9 | 52.5 ± 0.7 | 20.3 ± 0.4 | 53.2 ± 0.3 |
| RSC       | 10.0 ± 0.2 | 51.7 ± 0.2 | 10.1 ± 0.0 | 53.2 ± 0.1 | 13.3 ± 0.2 | 53.8 ± 0.1 | 18.9 ± 1.1 | 52.5 ± 0.5 | 23.7 ± 0.8 | 54.3 ± 0.5 |
| DR M (ours) | 69.7 ± 1.5 | 71.2 ± 0.6 | 74.5 ± 0.2 | 74.8 ± 0.1 | 73.3 ± 0.5 | 73.8 ± 0.2 | 61.0 ± 4.9 | 66.1 ± 0.6 | 59.9 ± 2.6 | 65.4 ± 1.2 |

Table 2. Ablation study for CMNIST, DSprites, and CelebA-HB. VB stands for balancing during validation, while TB stands for balancing during training.

Table 1. Experimental results on the correlation-shifted datasets, where the experiments are run by following the DomainBed setting. Min and Avg are the minimum value and the average accuracy for all test environments, respectively.

Table 3. Experimental results on the correlation-shifted datasets, where the experiments are run by following the DomainBed setting. Min and Avg are the minimum value and the average accuracy for all test environments, respectively.
respective. For CelebA-NS, the indirect effect is the pathway between “Wearing Necktie” and “Smiling”.

Analysis of indirect effect representation. Our DRM approach recovers the indirect effect in the first stage. Thus the quality of indirect effect representation $\hat{Z}$ is important. We use t-SNE [78] to reduce the dimension of $\hat{Z}$ extracted in the first stage to 2 and show the results in Figure 8. We can observe that data points with the same spurious feature show a clustering effect, which means that $\hat{Z}$ is an appropriate representation of the spurious feature.

Attention map. In Figure 9 we present the attention maps of the last convolution layer for ERM (the first row) and DRM (the second row). The model trained by ERM focuses on the spurious feature “Wearing Hat” and “Wearing Necktie”, while the model trained by DRM focuses on the stable feature “No Beard” and “Smiling”.

5. Related Works

Domain generalization with causality. A large body of works has introduced tools from causality inference to the domain generalization problem. Causality has been shown to be robust across domains [59], and some works discussed which causal factors can be extracted [71][72] and the connection between causality and generalization [19][56] gave a definition of natural direct effects. Although the concept has been introduced in early works [28][61][97], no work analyzed domain generalization using this framework.

Matching based methods. Matching is a common approach that aims to eliminate selection bias in causal inference by matching comparable instances [67][47] proposed an unsupervised matching algorithm and [85] introduced the propensity score matching method to balance the mini-batch. Our method also uses a mini-batch balancing approach in the second stage. However, we extract the indirect-effect representation in the first stage by learning a domain discriminator and propose a balancing-based model selection method, which are different from above approaches and help our approach to achieve better performance.

6. Conclusion

In this paper, we introduce the concept of direct and indirect effects from causal inference to the domain generalization. We propose a domain generalization method to extract the indirect-effect representation and remove the indirect effects during training. We also propose a new approach to do model selection in the o.o.d. setting. Both our domain generalization approach and model selection approach can be combined with other existing algorithms and improve their
performance significantly. Experimental results show that our approach achieves the state-of-the-art performance.

References

[1] Kartik Ahuja, Ethan Caballero, Dinghuai Zhang, Jean-Christophe Gagnon-Audet, Yoshua Bengio, Ioannis Mitliagkas, and Irina Rish. Invariance principle meets information bottleneck for out-of-distribution generalization. Advances in Neural Information Processing Systems, 34:3438–3450, 2021.

[2] Kartik Ahuja, Karthikeyan Shanmugam, Kush Varshney, and Amit Dhurandhar. Invariant risk minimization games. In International Conference on Machine Learning, pages 145–155. PMLR, 2020.

[3] Kartik Ahuja, Jun Wang, Amit Dhurandhar, Karthikeyan Shanmugam, and Kush R Varshney. Empirical or invariant risk minimization? a sample complexity perspective. In International Conference on Learning Representations, 2020.

[4] Isabela Albuquerque, João Monteiro, Mohammad Darvishi, Tiago H Falk, and Ioannis Mitliagkas. Generalizing to unseen domains via distribution matching. arXiv preprint arXiv:1911.00804, 2019.

[5] Alvin Alpher. Frobnication. Journal of Foo, 12(1):234–778, 2002.

[6] Alvin Alpher and Ferris P. N. Fotheringham-Smythe. Frobnication revisited. Journal of Foo, 13(1):234–778, 2003.

[7] Alvin Alpher, Ferris P. N. Fotheringham-Smythe, and Gavin Gomaw. Can a machine frobnicate? Journal of Foo, 14(1):234–778, 2004.

[8] Martin Arjovsky, João Monteiro, Mohammad Darvishi, Tiago H Falk, and Ioannis Mitliagkas. Generalizing to unseen domains via distribution matching. arXiv preprint arXiv:1911.00804, 2019.

[9] Yogesh Balaji, Swami Sankaranarayanan, and Rama Chellappa. Metareg: Towards domain generalization using meta-regularization. Advances in neural information processing systems, 31, 2018.

[10] Sara Beery, Grant Van Horn, and Pietro Perona. Recognition in terra incognita. In European Conference on Computer Vision, pages 472–489. Springer, 2018.

[11] Alexi Bellot and Mihaela van der Schaar. Accounting for unobserved confounding in domain generalization. arXiv preprint arXiv:2007.10653, 2020.

[12] Shai Ben-David, John Blitzer, Koby Crammer, and Fernando Pereira. Analysis of representations for domain adaptation. Advances in neural information processing systems, 19, 2006.

[13] Gilles Blanchard, Gyemin Lee, and Clayton Scott. Generalizing from several related classification tasks to a new unlabeled sample. Advances in neural information processing systems, 24, 2011.

[14] Manh-Ha Bui, Toan Tran, Anh Tran, and Dinh Phung. Exploiting domain-specific features to enhance domain generalization. Advances in Neural Information Processing Systems, 34:21189–21201, 2021.

[15] Chris Burgess and Hyunjik Kim. 3d shapes dataset. https://github.com/deepmind/3dshapes-dataset/, 2018.

[16] Fabio M Carlucci, Antonio D’Innocente, Silvia Bucci, Barbara Caputo, and Tatiana Tommasi. Domain generalization by solving jigsaw puzzles. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 2229–2238, 2019.

[17] Junbum Cha, Sanghyuk Chun, Kyungjae Lee, Han-Cheol Cho, Seunghyun Park, Yunsung Lee, and Sungae Park. Swad: Domain generalization by seeking flat minima. Advances in Neural Information Processing Systems, 34:22405–22418, 2021.

[18] Mathieu Chevalley, Charlotte Bunne, Andreas Krause, and Stefan Bauer. Invariant causal mechanisms through distribution matching. arXiv preprint arXiv:2206.11646, 2022.

[19] Rune Christiansen, Niklas Pfister, Martin Emil Jakobsen, Nicola Gnecco, and Jonas Peters. A causal framework for distribution generalization, 2020.

[20] Alexey Dosovitskiy, Lucas Beyer, Alexander Kolesnikov, Dirk Weissenborn, Xiaohua Zhai, Thomas Unterthiner, Mostafa Dehghani, Matthias Minderer, Georg Heigold, Sylvain Gelly, et al. An image is worth 16x16 words: Transformers for image recognition at scale. arXiv preprint arXiv:2010.11929, 2020.

[21] Qi Dou, Daniel Coelho de Castro, Konstantinos Kamitkas, and Ben Glocker. Domain generalization via model-agnostic learning of semantic features. Advances in Neural Information Processing Systems, 32, 2019.

[22] Chen Fang, Ye Xu, and Daniel N Rockmore. Unbiased metric learning: On the utilization of multiple datasets and web images for softening bias. In Proceedings of the IEEE International Conference on Computer Vision, pages 1657–1664, 2013.

[23] Yuxin Fang, Wen Wang, Binhui Xie, Quan Sun, Ledell Wu, Xinggang Wang, Tiejun Huang, Xinlong Wang, and Yue Cao. Eva: Exploring the limits of masked visual representation learning at scale. arXiv preprint arXiv:2211.07636, 2022.

[24] Yaroslav Ganin, Evgeniya Ustinova, Hana Ajakan, Pascal Germain, Hugo Larochelle, François Laviolette, Mario Marchand, and Victor Lempitsky. Domain-adversarial training of neural networks. The journal of machine learning research, 17(1):2096–2030, 2016.

[25] Muhammad Ghifary, W Bastiaan Kleijn, Mengjie Zhang, and Amit Dhurandhar. Invariant risk minimization games. arXiv preprint arXiv:2211.07636, 2022.

[26] Mohammad Ghifary, Evgeniya Ustinova, Hana Ajakan, Pascal Germain, Hugo Larochelle, François Laviolette, Mario Marchand, and Victor Lempitsky. Domain-adversarial training of neural networks. The journal of machine learning research, 17(1):2096–2030, 2016.

[27] Muhammad Ghifary, W Bastiaan Kleijn, Mengjie Zhang, and Amit Dhurandhar. Invariant risk minimization games. arXiv preprint arXiv:2211.07636, 2022.

[28] Tom Heskes, Evi Sijben, Ioan Gabriel Bucur, and Tom Claassen. Causal shapley values: Exploiting causal knowledge to explain individual predictions of complex models. Advances in neural information processing systems, 33:4778–4789, 2020.
[29] Zeyi Huang, Haohan Wang, Eric P Xing, and Dong Huang. Self-challenging improves cross-domain generalization. In European Conference on Computer Vision, pages 124–140. Springer, 2020.

[30] Yusuke Iwasawa and Yutaka Matsuo. Test-time classifier adjustment module for model-agnostic domain generalization. Advances in Neural Information Processing Systems, 34:2427–2440, 2021.

[31] Pritish Kamath, Akilesh Tangella, Danica Sutherland, and Nathan Srebro. Does invariant risk minimization capture invariance? In International Conference on Artificial Intelligence and Statistics, pages 4069–4077. PMLR, 2021.

[32] Dahee Kim, Youngjun Yoo, Seunghyun Park, Jinkyu Kim, and Jaekoo Lee. Selfreg: Self-supervised contrastive regularization for domain generalization. In Proceedings of the IEEE/CVF International Conference on Computer Vision, pages 9619–9628, 2021.

[33] David Krueger, Ethan Caballero, Joern-Henrik Jacobsen, Amy Zhang, Jonathan Binns, Dinhui Zhang, Remi Le Priol, and Aaron Courville. Out-of-distribution generalization via risk extrapolation (rex). In International Conference on Machine Learning, pages 5815–5826. PMLR, 2021.

[34] Yann LeCun. The mnist database of handwritten digits. http://yann.lecun.com/exdb/mnist/, 1998.

[35] Da Li, Yongxin Yang, Yi-Zhe Song, and Timothy Hospedales. Learning to generalize: Meta-learning for domain generalization. In Proceedings of the AAAI conference on artificial intelligence, volume 32, 2018.

[36] Da Li, Yongxin Yang, Yi-Zhe Song, and Timothy M Hospedales. Deeper, broader and artier domain generalization. In Proceedings of the IEEE international conference on computer vision, pages 5542–5550, 2017.

[37] Haoliang Li, Sinno Jialin Pan, Shiqi Wang, and Alex C Kot. Domain generalization with adversarial feature learning. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 5400–5409, 2018.

[38] Haoliang Li, Sinno Jialin Pan, Shiqi Wang, and Alex C Kot. Domain generalization with adversarial feature learning. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 5400–5409, 2018.

[39] Haoliang Li, YuFei Wang, Renjie Wan, Shiqi Wang, Tie-Qiang Li, and Alex Kot. Domain generalization for medical imaging classification with linear-dependency regularization. Advances in Neural Information Processing Systems, 33:3118–3129, 2020.

[40] Ya Li, Xinmei Tian, Mingming Gong, Yajing Liu, Tongliang Liu, Kun Zhang, and Dacheng Tao. Deep domain generalization via conditional invariant adversarial networks. In Proceedings of the European Conference on Computer Vision (ECCV), pages 624–639, 2018.

[41] Yiying Li, Yongxin Yang, Wei Zhou, and Timothy Hospedales. Feature-critic networks for heterogeneous domain generalization. In International Conference on Machine Learning, pages 3915–3924. PMLR, 2019.

[42] Yong Lin, Hanze Dong, Hao Wang, and Tong Zhang. Bayesian invariant risk minimization. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 16021–16030, 2022.

[43] Chang Liu, Xinwei Sun, Jindong Wang, Haoyue Tang, Tao Li, Tao Qin, Wei Chen, and Tie-Yan Liu. Learning causal semantic representation for out-of-distribution prediction. Advances in Neural Information Processing Systems, 34:6155–6170, 2021.

[44] Ziwei Liu, Ping Luo, Xiaogang Wang, and Xiaoou Tang. Deep learning face attributes in the wild. In Proceedings of International Conference on Computer Vision (ICCV), December 2015.

[45] Chaochao Lu, Yuhuai Wu, Jos´e Miguel Hern´andez-Lobato, and Bernhard Sch¨olkopf. Invariant causal representation learning for out-of-distribution generalization. In International Conference on Learning Representations, 2021.

[46] Fangrui Lv, Jian Liang, Shuang Li, Bin Zang, Chi Harold Liu, Ziteng Wang, and Di Liu. Causality inspired representation learning for domain generalization. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 8046–8056, 2022.

[47] Divyat Mahajan, Shruti Tople, and Amit Sharma. Domain generalization using causal matching. In International Conference on Machine Learning, pages 7313–7324. PMLR, 2021.

[48] Loic Matthey, Irina Higgins, Demis Hassabis, and Alexander Lerchner. dsprites: Disentanglement testing sprites dataset. https://github.com/deepmind/dsprites-dataset/, 2017.

[49] Saeid Motiian, Marco Piccirilli, Donald A Adjeroh, and Giambattista Parascandolo, Alexander Neitz, Antonio Orvieto, Luigi Gresele, and Bernhard Schölkopf. Understanding the failure modes of out-of-distribution generalization. In Proceedings of the IEEE international conference on computer vision, pages 5715–5725, 2017.

[50] Krikamol Muandet, David Balduzzi, and Bernhard Schölkopf. Domain generalization via invariant feature representation. In International Conference on Machine Learning, pages 10–18. PMLR, 2013.

[51] Vaishnavh Nagarajan, Anders Andreassen, and Behnam Neyshabur. Understanding the failure modes of out-of-distribution generalization. In International Conference on Learning Representations, 2020.

[52] Hyeonseob Nam, HyunJae Lee, Jongchan Park, Wonjun Yoon, and Donggeun Yoo. Reducing domain gap via style-agnostic networks. arXiv preprint arXiv:1910.11645, 2(7):8, 2019.

[53] Full Author Name. The frobnicable foo filter, 2014. Face and Gesture submission ID 324. Supplied as additional material fg324.pdf.

[54] Full Author Name. Frobnication tutorial, 2014. Supplied as additional material tr.pdf.

[55] Giambattista Parascandolo, Alexander Neitz, Antonio Orvieto, Luigi Gresele, and Bernhard Schölkopf. Learning explanations that are hard to vary. arXiv preprint arXiv:2009.00329, 2020.

[56] Judea Pearl. Direct and indirect effects. Probabilistic and Causal Inference: The Works of Judea Pearl, page 373, 2001.
[57] Judea Pearl. *Causality*. Cambridge university press, 2009.
[58] Xingchao Peng, Qinxun Bai, Xide Xia, Zijun Huang, Kate Saenko, and Bo Wang. Moment matching for multi-source domain adaptation. In *Proceedings of the IEEE/CVF international conference on computer vision*, pages 1406–1415, 2019.
[59] Jonas Peters, Peter Bühlmann, and Nicolai Meinshausen. Causal inference by using invariant prediction: identifica-
tion and confidence intervals. *Journal of the Royal Statistical
Society: Series B (Statistical Methodology)*, 78(5):947–1012, 2016.
[60] Vihari Piratla, Praneeth Netrapalli, and Sunita Sarawagi. Efficient domain generalization via common-specific low-
rank decomposition. In *International Conference on Ma-
chine Learning*, pages 7728–7738. PMLR, 2020.
[61] Jiaxin Qi, Yulei Niu, Jianqiang Huang, and Hanwang Zhang. Two causal principles for improving visual dialog. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pages 10860–10869, 2020.
[62] Fengchun Qiao, Long Zhao, and Xi Peng. Learning to
learn single domain generalization. In *Proceedings of the
IEEE/CVF Conference on Computer Vision and Pattern
Recognition*, pages 12556–12565, 2020.
[63] Alec Radford, Jong Wook Kim, Chris Hallacy, Aditya Ramesh, Gabriel Goh, Sandhini Agarwal, Girish Sastry, Amanda Askell, Pamela Mishkin, Jack Clark, et al. Learning transferable visual models from natural language supervision. In *International conference on machine learning*, pages 8748–8763. PMLR, 2021.
[64] Alec Radford, Jong Wook Kim, Chris Hallacy, Aditya Ramesh, Gabriel Goh, Sandhini Agarwal, Girish Sastry, Amanda Askell, Pamela Mishkin, Jack Clark, Gretchen Krueger, and Ilya Sutskever. Learning transferable visual models from natural language supervision. *CoRR*, abs/2103.00020, 2021.
[65] Alexandre Rame, Corentin Dancette, and Matthieu Cord. Fishr: Invariant gradient variances for out-of-distribution generalization. In *International Conference on Machine Learning*, pages 18347–18377. PMLR, 2022.
[66] Alexander Robey, George J Pappas, and Hamed Hassani. Model-based domain generalization. *Advances in Neural Information Processing Systems*, 34:20210–202229, 2021.
[67] Paul R Rosenbaum and Donald B Rubin. The central role of the propensity score in observational studies for causal effects. *Biometrika*, 70(1):41–55, 1983.
[68] Elan Rosenfeld, Pradeep Kumar Ravikumar, and Andrej Risteski. The risks of invariant risk minimization. In *International Conference on Learning Representations*, 2020.
[69] Jongbin Ryu, Gitaek Kwon, Ming-Hsuan Yang, and Jong-woo Lim. Generalized convolutional forest networks for domain generalization and visual recognition. In *International conference on learning representations*, 2019.
[70] Shiori Sagawa, Pang Wei Koh, Tatsunori B Hashimoto, and Percy Liang. Distributionally robust neural networks for group shifts: On the importance of regularization for worst-case generalization. *arXiv preprint arXiv:1911.08731*, 2019.
[71] Bernhard Schölkopf, Dominik Janzing, Jonas Peters, Eleni Sgouritsa, Kun Zhang, and Joris Mooij. On causal and anti-
causal learning. *arXiv preprint arXiv:1206.6471*, 2012.
[72] Bernhard Schölkopf, Francesco Locatello, Stefan Bauer, Nan Rosemary Ke, Nal Kalchbrenner, Anirudh Goyal, and Yoshua Bengio. Toward causal representation learning. *Proceedings of the IEEE*, 109(5):612–634, 2021.
[73] Soroosh Shahtalebi, Jean-Christophe Gagnon-Audet, Touraj Laleh, Mojtaba Faramarzi, Kartik Ahuja, and Irina Rish. Sand-mask: An enhanced gradient masking strategy for the discovery of invariances in domain generalization, 2021.
[74] Shiv Shankar, Vihari Piratla, Soumen Chakrabarti, Siddhartha Chaudhuri, Preethi Jyothi, and Sunita Sarawagi. Generalizing across domains via cross-gradient training. In *International Conference on Learning Representations*, 2018.
[75] Yuge Shi, Jeffrey Seely, Philip HS Torr, N Siddharth, Awni Hannun, Nicolas Usunier, and Gabriel Synnaeve. Grad-
ient matching for domain generalization. *arXiv preprint
arXiv:2104.09937*, 2021.
[76] Baochen Sun and Kate Saenko. Deep coral: Correlation alignment for deep domain adaptation. In *European conference on computer vision*, pages 443–450. Springer, 2016.
[77] Josh Tobin, Rachel Fong, Alex Ray, Jonas Schneider, Wojciech Zaremba, and Pieter Abbeel. Domain randomization for transferring deep neural networks from simulation to the real world. In *2017 IEEE/RSJ international conference on intelligent robots and systems (IROS)*, pages 23–30. IEEE, 2017.
[78] Laurens Van der Maaten and Geoffrey Hinton. Visualizing data using t-sne. *Journal of machine learning research*, 9(11), 2008.
[79] Vladimir Vapnik. *Statistical learning theory* wiley. New
York, 1(624):2, 1998.
[80] Ramakrishna Vedantam, David Lopez-Paz, and David J Schwab. An empirical investigation of domain generalization with empirical risk minimizers. *Advances in Neural Information Processing Systems*, 34:28131–28143, 2021.
[81] Hemanth Venkateswara, Jose Eusebio, Shayok Chakraborty, and Sethuraman Panchanathan. Deep hashing network for unsupervised domain adaptation. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 5018–5027, 2017.
[82] Riccardo Volpi, Hongseok Namkoong, Ozan Sener, John C Duchi, Vittorio Murino, and Silvio Savarese. Generalizing to unseen domains via adversarial data augmentation. *Advances in neural information processing systems*, 31, 2018.
[83] Yoav Wald, Amir Feder, Daniel Greenfeld, and Uri Shalit. On calibration and out-of-domain generalization. *Advances in neural information processing systems*, 34:2215–2227, 2021.
[84] Ruoyu Wang, Mingyang Yi, Zhitang Chen, and Shengyu Zhu. Out-of-distribution generalization with causal invariant transformations. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 375–385, 2022.
[85] Xinyi Wang, Michael Saxon, Jiachen Li, Hongyang Zhang, Kun Zhang, and William Yang Wang. Causal balancing for domain generalization. *arXiv preprint arXiv:2206.05263*, 2022.

[86] Yufei Wang, Haoliang Li, and Alex C Kot. Heterogeneous domain generalization via domain mixup. In *ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 3622–3626. IEEE, 2020.

[87] Olivia Wiles, Sven Gowal, Florian Stimberg, Sylvestre Alvise-Rebuffi, Ira Ktena, Taylan Cemgil, et al. A fine-grained analysis on distribution shift. *arXiv preprint arXiv:2110.11328*, 2021.

[88] Minghao Xu, Jian Zhang, Bingbing Ni, Teng Li, Chengjie Wang, Qi Tian, and Wenjun Zhang. Adversarial domain adaptation with domain mixup. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 6502–6509, 2020.

[89] Shen Yan, Huan Song, Nanxiang Li, Lincan Zou, and Liu Ren. Improve unsupervised domain adaptation with mixup training. *arXiv preprint arXiv:2001.00677*, 2020.

[90] Fu-En Yang, Yuan-Chia Cheng, Zu-Yun Shiau, and Yu-Chiang Frank Wang. Adversarial teacher-student representation learning for domain generalization. *Advances in Neural Information Processing Systems*, 34:19448–19460, 2021.

[91] Xiangyu Yue, Yang Zhang, Sicheng Zhao, Alberto Sangiovanni-Vincentelli, Kurt Keutzer, and Boqing Gong. Domain randomization and pyramid consistency: Simulation-to-real generalization without accessing target domain data. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 2100–2110, 2019.

[92] Junzhe Zhang and Elias Bareinboim. Fairness in decision-making—the causal explanation formula. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 32, 2018.

[93] Shanshan Zhao, Mingming Gong, Tongliang Liu, Huan Fu, and Dacheng Tao. Domain generalization via entropy regularization. *Advances in Neural Information Processing Systems*, 33:16096–16107, 2020.

[94] Kaiyang Zhou, Yongxin Yang, Timothy Hospedales, and Tao Xiang. Deep domain-adversarial image generation for domain generalisation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 13025–13032, 2020.

[95] Jun-Yan Zhu, Taesung Park, Phillip Isola, and Alexei A Efros. Unpaired image-to-image translation using cycle-consistent adversarial networks. In *Proceedings of the IEEE international conference on computer vision*, pages 2223–2232, 2017.
A. Other Related Works

**Learning invariant features.** To enable classifiers to generalize across domains, a very intuitive idea is to force the model to learn a representation with cross-domain invariance [49,33], which is usually implemented by adding a regularization term to the loss [39,66,84,99,100]. Representative methods include using maximum mean discrepancy as a divergence measure [50], seeking a data representation such that the predictors using the representation are invariant (IRM) [8], encouraging the training risks in different domains to be similar [33], or using a correlation matrix to construct a new loss function [46]. Some works studied the conditions under which invariance can guarantee domain generalization from a theoretical point of view [91], while other works quantified transferability of feature embeddings learned by domain generalization models [95]. [3] analyzed different finite sample and asymptotic behavior of ERM and IRM, and [2] expanded IRM from a game theory perspective. However, existing works have demonstrated that it is difficult to achieve good generalization performance by relying only on the constraint of cross-domain invariance [14,47], and some other works claimed that many of these approaches still fail to capture the invariance [31,68]. Empirical work has also questioned the effectiveness of these methods [26]. On average, ERM outperforms the other methods [80].

**Data augmentation.** Data augmentation is a kind of valid methods to improve the generalization ability of models. Domain Randomization bridge the simulated environment and the real world by generating rich enough data, which can benefit the o.o.d. generalization [77]. Some works performed domain adversarial training to generate data across environments [74,32]. Generative models and transformation models such as CycleGAN [103] are also used to perform data augmentation [62,93,101].

**Other approaches.** In addition to the above approaches, many works have been done to enhance the performance of o.o.d. generalization in a variety of different ways. A great deal of work has been done to improve generalization performance by analyzing and designing new neural network structures [36,69,94]. Meta-learning is another helpful direction, and many approaches based on it have emerged [9,21,35,41]. In addition, [90] proposed an approach of Adversarial Teacher-Student Representation Learning to derive generalizable representations; [102] proposed an approach based on feature statistics mixing across source domains; [60] joint learned common components and domain-specific components by modifying the last classification layer; [16] combined supervised with self-supervised learning to improve the generalization performance of the model by solving the Jigsaw puzzles.

B. Experimental Details

B.1. Hyperparameter search

We search hyperparameters with the same distribution as DomainBed. On DomainBed benchmark, some distribution of hyperparameters are related to image size. To avoid human intervention, we resize images of CelebA-HB and CelebA-NS to $224 \times 224$, which is the standard size on DomainBed benchmark. To save computing resource and time, we reduce the number of search to eight for all diversity shift datasets and small-image (smaller than $224 \times 224$) correlation shift datasets. DRM will augment the batch, so we reduce the batchsize for big-image datasets to avoid GPU memory overflow. The two stages of DRM use the same model: Resnet-50 [27] for big-image datasets and MNIST-CNN for small-image datasets, which is consistent with DomainBed. And hyperparameters distribution of two stages are the same, which is also consistent with DomainBed. In stage 1, we divide the data from train environments into the training set and the validation set in a ratio of 8:2. We choose the model which perform best on the validation set.

B.2. Datasets

**3DShapes [15].** To demonstrate that our approach can eliminate spurious correlations between attributes, we run our algorithm on the 3DShapes dataset. The 3DShapes is a dataset with six attributes, among which we choose the “floor hue” and “orientation” to form the spurious correlation. Specifically, we divide the orientation of the graph into two categories. Our goal is to predict which category the orientation belongs to. We use the same construction as Colored MNIST to build three environments and add label noise.

**DSprites [48].** We also evaluate our DRM algorithm on the DSprites dataset, which has six attributes. In this paper, “Position X” and “Position Y” are chosen to form spurious correlations in the DSprites. We argue that these two attributes are similar, and thus it is challenging to identify the invariant features across all environments.

**CelebA-HB and CelebA-NS [44].** We introduce the CelebA dataset to test the performance of our approach. CelebA is a large-scale face attribute dataset with 40 attribute annotations, e.g., eyeglasses, wearing hat and bangs. Any two attributes can form a correlation shift dataset, one of which acts as the label to be predicted and the other is used to create spurious correlations. In this paper, for CelebA-HB, “No Beard” is the label and there exists spurious correlation between the attribute “No Beard” and the attribute “Wearing Hat”. For CelebA-NS, “Smiling” is the label and the correlation between “Wearing Necktie” and “Smiling” is unstable. Unlike above mentioned datasets, correlation shift on CelebA comes from non-random sampling, which is called selection bias in causal inference.
B.3. Full results

We conducted experiments on different test domains. Below are the experimental results for each domain on different datasets. We use accuracy as metrics.

For the diversity shift datasets, the support sets of data on different environments have no overlap, and the test distribution keeps the semantic content of the data unchanged while altering the data style. For instance, in the PACS dataset, the training environment and the test environment can be photos and cartoons, respectively. We test our approach on the diversity shift datasets. Following DomainBed, we present results on RotatedMNIST [25], VLCS [22], PACS [36], Office-Home [81], Terra Incognita [10] and DomainNet [58]. In addition to the 14 methods we mentioned above, we also report the results of other recent methods, including: Fish [75], Fishr [65], AND-mask [55], SAND-mask [73], SelfReg [52], CauseRL [18], mDSDI [14], SWAD [17], and T3A [30]. We report the experimental results in Table 5. It shows that DRM does not hurt model performance for diversity shift. We observe that performance of ERM and other domain generalization algorithms are similar. The performance of our DRM approach is comparable with others on the diversity shift datasets, and improves by $\sim 2\%$ on average concerning the DomainBed benchmark.

### Table 4. The result for CMNIST

| Algorithm   | +90% | +80% | -90% | Avg  |
|-------------|------|------|------|------|
| ERM         | 71.7 | 72.9 | 10.0 | 51.5 |
| IRM         | 72.5 | 73.3 | 10.2 | 52.0 |
| GroupDRO    | 73.1 | 73.2 | 10.0 | 52.1 |
| Mixup       | 72.7 | 73.4 | 10.1 | 52.1 |
| MLDG        | 71.5 | 73.1 | 9.8  | 51.5 |
| CORAL       | 71.6 | 73.1 | 9.9  | 51.5 |
| MMD         | 71.4 | 73.1 | 9.9  | 51.5 |
| DANN        | 71.4 | 73.1 | 10.0 | 51.5 |
| CDANN       | 72.0 | 73.0 | 10.2 | 51.7 |
| MTL         | 70.9 | 72.8 | 10.5 | 51.4 |
| SagNet      | 71.8 | 73.0 | 10.3 | 51.7 |
| ARM         | 82.0 | 76.5 | 10.2 | 56.2 |
| VREx        | 72.4 | 72.9 | 10.2 | 51.8 |
| RSC         | 71.9 | 73.1 | 10.0 | 52.1 |
| DRM (ours)  | 71.4 | 72.4 | 69.7 | 71.2 |

### Table 5. The results for 3DShapes

| Algorithm   | +90% | +80% | -90% | Avg  |
|-------------|------|------|------|------|
| ERM         | 73.4 | 75.5 | 10.1 | 53.3 |
| IRM         | 74.2 | 75.4 | 10.0 | 53.2 |
| GroupDRO    | 74.6 | 75.1 | 10.5 | 53.4 |
| Mixup       | 74.6 | 75.4 | 10.2 | 53.4 |
| MLDG        | 75.0 | 75.4 | 10.1 | 53.1 |
| CORAL       | 74.6 | 75.2 | 10.0 | 53.3 |
| MMD         | 74.6 | 75.2 | 10.0 | 53.2 |
| DANN        | 74.6 | 75.2 | 10.0 | 53.3 |
| CDANN       | 74.7 | 75.4 | 10.1 | 53.4 |
| MTL         | 74.7 | 75.4 | 10.0 | 53.4 |
| SagNet      | 74.9 | 75.1 | 10.1 | 53.4 |
| ARM         | 81.8 | 73.9 | 10.0 | 55.2 |
| VREx        | 74.6 | 75.2 | 10.8 | 53.5 |
| RSC         | 74.4 | 75.1 | 10.1 | 53.2 |
| DRM (ours)  | 74.5 | 75.1 | 74.8 | 74.8 |

### Table 6. The results for DSprites

| Algorithm   | +90% | +80% | -90% | Avg  |
|-------------|------|------|------|------|
| ERM         | 73.5 | 74.8 | 13.8 | 54.0 |
| IRM         | 73.5 | 74.2 | 14.5 | 54.0 |
| GroupDRO    | 73.8 | 74.4 | 15.0 | 54.4 |
| Mixup       | 73.4 | 74.3 | 14.0 | 53.9 |
| MLDG        | 73.9 | 74.4 | 14.3 | 53.4 |
| CORAL       | 73.4 | 74.5 | 13.8 | 53.9 |
| MTL         | 65.8 | 74.2 | 14.0 | 53.4 |
| SagNet      | 73.7 | 73.9 | 14.7 | 53.1 |
| ARM         | 86.9 | 77.6 | 14.5 | 59.7 |
| VREx        | 73.4 | 74.6 | 13.8 | 53.9 |
| RSC         | 73.7 | 74.5 | 13.3 | 53.8 |
| DRM (ours)  | 73.7 | 74.4 | 73.3 | 73.8 |

### Table 7. The results for CelebA-HB

| Algorithm   | +90% | +80% | -90% | Avg  |
|-------------|------|------|------|------|
| ERM         | 67.3 | 71.8 | 16.8 | 52.0 |
| IRM         | 65.9 | 70.0 | 20.4 | 52.1 |
| GroupDRO    | 68.3 | 71.9 | 18.3 | 52.8 |
| Mixup       | 68.4 | 70.8 | 17.9 | 52.4 |
| MLDG        | 68.3 | 70.6 | 20.0 | 53.0 |
| CORAL       | 68.3 | 71.2 | 17.7 | 52.4 |
| MMD         | 64.5 | 70.1 | 17.4 | 50.7 |
| DANN        | 67.0 | 71.2 | 16.9 | 51.7 |
| CDANN       | 66.8 | 72.1 | 18.6 | 52.5 |
| MTL         | 65.9 | 71.6 | 23.5 | 53.7 |
| SagNet      | 64.6 | 71.6 | 14.9 | 50.4 |
| ARM         | 66.7 | 72.8 | 22.8 | 54.1 |
| VREx        | 66.9 | 71.4 | 19.2 | 52.5 |
| RSC         | 67.4 | 71.1 | 18.9 | 52.5 |
| DRM (ours)  | 68.1 | 69.3 | 61.0 | 66.1 |
Table 8. The results for CelebA-NS

| Algorithm | +90% ± | +80% ± | -90% ± | Avg  |
|-----------|--------|--------|--------|------|
| ERM       | 67.8 ± 1.0 | 69.1 ± 0.6 | 21.1 ± 0.4 | 52.7 |
| IRM       | 68.2 ± 0.3 | 69.8 ± 0.1 | 21.5 ± 0.9 | 53.2 |
| GroupDRO  | 68.0 ± 0.4 | 70.6 ± 0.3 | 21.2 ± 0.2 | 53.3 |
| Mixup     | 67.7 ± 0.6 | 70.2 ± 0.7 | 22.2 ± 1.5 | 53.7 |
| MLDG      | 67.7 ± 0.3 | 70.8 ± 0.1 | 22.7 ± 1.7 | 53.7 |
| CORAL     | 68.2 ± 0.5 | 70.8 ± 0.1 | 22.7 ± 1.7 | 53.4 |
| MMD       | 67.6 ± 0.4 | 69.4 ± 0.4 | 22.5 ± 0.6 | 53.3 |
| DANN      | 69.3 ± 0.6 | 70.9 ± 0.6 | 21.8 ± 1.5 | 53.7 |
| CDANN     | 68.0 ± 0.5 | 71.1 ± 0.4 | 22.5 ± 1.2 | 53.9 |

Table 9. Experimental results on the DomainBed benchmark. We use accuracy as metrics.

| DomainBed | Algorithm | CMNIST | RMNIST | VLCS | PACS | OfficeHome | TerraInc | DomainNet |
|-----------|----------|--------|--------|------|------|------------|----------|-----------|
| ERM       | 51.5 ± 0.1 | 98.0 ± 0.0 | 77.5 ± 0.4 | 85.5 ± 0.2 | 66.5 ± 0.3 | 46.1 ± 1.8 | 40.9 ± 0.1 | 66.6 |
| IRM       | 52.0 ± 0.1 | 97.7 ± 0.1 | 78.5 ± 0.5 | 83.5 ± 0.8 | 64.3 ± 2.2 | 47.6 ± 0.8 | 33.9 ± 2.8 | 65.4 |
| GroupDRO  | 52.1 ± 0.0 | 98.0 ± 0.0 | 76.7 ± 0.6 | 84.4 ± 0.8 | 66.0 ± 0.7 | 43.2 ± 1.1 | 33.3 ± 0.2 | 64.8 |
| Mixup     | 52.1 ± 0.2 | 98.0 ± 0.1 | 77.4 ± 0.6 | 84.6 ± 0.6 | 68.1 ± 0.3 | 47.9 ± 0.8 | 39.2 ± 0.1 | 66.7 |
| MLDG      | 51.5 ± 0.1 | 97.9 ± 0.0 | 77.2 ± 0.4 | 84.9 ± 1.0 | 66.8 ± 0.6 | 47.7 ± 0.9 | 41.2 ± 0.1 | 66.7 |
| CORAL     | 51.5 ± 0.1 | 98.0 ± 0.1 | 78.8 ± 0.6 | 86.2 ± 0.3 | 68.7 ± 0.3 | 47.6 ± 1.0 | 41.5 ± 0.1 | 67.5 |
| MMD       | 51.5 ± 0.2 | 97.9 ± 0.0 | 77.5 ± 0.9 | 84.6 ± 0.5 | 66.3 ± 0.1 | 42.2 ± 1.6 | 23.4 ± 9.5 | 63.3 |
| DANN      | 51.5 ± 0.3 | 97.8 ± 0.1 | 78.6 ± 0.4 | 83.6 ± 0.4 | 65.9 ± 0.6 | 46.7 ± 0.5 | 38.3 ± 0.1 | 66.1 |
| CDANN     | 51.7 ± 0.1 | 97.9 ± 0.1 | 77.5 ± 0.1 | 82.6 ± 0.9 | 65.8 ± 1.3 | 45.8 ± 1.6 | 38.3 ± 0.3 | 65.6 |
| MTL       | 51.4 ± 0.1 | 97.9 ± 0.0 | 77.2 ± 0.4 | 84.6 ± 0.5 | 66.4 ± 0.5 | 45.6 ± 1.2 | 40.6 ± 0.1 | 66.2 |
| SagNet    | 51.7 ± 0.0 | 98.0 ± 0.0 | 77.8 ± 0.5 | 86.3 ± 0.2 | 68.1 ± 0.1 | 48.6 ± 1.0 | 40.3 ± 0.1 | 67.2 |
| ARM       | 56.2 ± 0.2 | 98.2 ± 0.1 | 77.6 ± 0.3 | 85.1 ± 0.4 | 64.8 ± 0.3 | 45.5 ± 0.3 | 35.5 ± 0.2 | 66.1 |
| VREx      | 51.8 ± 0.1 | 97.9 ± 0.1 | 78.3 ± 0.2 | 84.9 ± 0.6 | 66.4 ± 0.6 | 46.4 ± 0.6 | 33.6 ± 2.9 | 65.6 |
| RSC       | 51.7 ± 0.2 | 97.6 ± 0.1 | 77.1 ± 0.5 | 85.2 ± 0.9 | 65.5 ± 0.9 | 46.6 ± 1.0 | 38.9 ± 0.5 | 66.1 |

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| DomainBed | Algorithm | CMNIST | RMNIST | VLCS | PACS | OfficeHome | TerraInc | DomainNet |
|-----------|----------|--------|--------|------|------|------------|----------|-----------|
| Fish      | 51.6 ± 0.1 | 98.0 ± 0.0 | 77.8 ± 0.3 | 85.5 ± 0.3 | 68.6 ± 0.4 | 45.1 ± 1.3 | 42.7 ± 0.2 | 67.1 |
| Fishr     | 52.0 ± 0.2 | 97.8 ± 0.0 | 77.8 ± 0.1 | 85.5 ± 0.4 | 67.8 ± 0.1 | 47.4 ± 1.6 | 41.7 ± 0.0 | 67.1 |
| ANDmask   | 51.3 ± 0.2 | 97.6 ± 0.1 | 78.1 ± 0.9 | 84.4 ± 0.9 | 65.6 ± 0.4 | 44.6 ± 0.3 | 37.2 ± 0.6 | 65.5 |
| SANDmask  | 51.8 ± 0.2 | 97.4 ± 0.1 | 77.4 ± 0.2 | 84.6 ± 0.9 | 65.8 ± 0.4 | 42.9 ± 1.7 | 32.1 ± 0.6 | 64.6 |
| SelfReg   | 52.1 ± 0.2 | 98.0 ± 0.1 | 77.8 ± 0.9 | 85.6 ± 0.4 | 67.9 ± 0.7 | 47.0 ± 0.3 | 42.8 ± 0.0 | 67.3 |
| CausIRL_CORAL | 51.7 ± 0.1 | 97.9 ± 0.1 | 77.5 ± 0.6 | 85.8 ± 0.1 | 68.6 ± 0.3 | 47.3 ± 0.8 | 41.9 ± 0.1 | 67.3 |
| CausIRL_MMD | 51.6 ± 0.1 | 97.9 ± 0.0 | 77.6 ± 0.4 | 84.0 ± 0.8 | 65.7 ± 0.6 | 46.3 ± 0.9 | 40.3 ± 0.2 | 66.2 |

Codes by authors

| DomainBed | Algorithm | CMNIST | RMNIST | VLCS | PACS | OfficeHome | TerraInc | DomainNet |
|-----------|----------|--------|--------|------|------|------------|----------|-----------|
| mDSI      | 52.2 ± 0.2 | 98.0 ± 0.1 | 79.0 ± 0.3 | 86.2 ± 0.2 | 69.2 ± 0.4 | 48.1 ± 1.4 | 42.8 ± 0.1 | 67.9 |
| SWAD      | -        | -      | 79.1 ± 0.1 | 88.1 ± 0.1 | 70.6 ± 0.2 | 50.0 ± 0.3 | 46.5 ± 0.1 | - |
| T3A       | -        | -      | 80.0 ± 0.2 | 85.3 ± 0.6 | 68.3 ± 0.1 | 47.0 ± 0.6 | -          | - |

Reported by authors

| DomainBed | Algorithm | CMNIST | RMNIST | VLCS | PACS | OfficeHome | TerraInc | DomainNet |
|-----------|----------|--------|--------|------|------|------------|----------|-----------|
| DRM (ours) | 71.2 ± 0.6 | 97.6 ± 0.1 | 77.9 ± 0.5 | 84.8 ± 0.5 | 65.7 ± 0.6 | 48.2 ± 0.2 | 41.0 ± 0.2 | 69.5 |
### Table 10. The result for RMNIST

| Algorithm | 0     | 15    | 30    | 45    | 60    | 75    | Avg  |
|-----------|-------|-------|-------|-------|-------|-------|------|
| ERM       | 95.9 ± 0.1 | 98.9 ± 0.0 | 98.8 ± 0.0 | 98.9 ± 0.0 | 98.9 ± 0.0 | 96.4 ± 0.0 | 98.0 |
| IRM       | 95.5 ± 0.1 | 98.8 ± 0.2 | 98.7 ± 0.1 | 98.6 ± 0.1 | 98.7 ± 0.0 | 95.9 ± 0.2 | 97.7 |
| GroupDRO  | 95.6 ± 0.1 | 98.9 ± 0.1 | 98.9 ± 0.1 | 99.0 ± 0.0 | 98.9 ± 0.0 | 96.5 ± 0.2 | 98.0 |
| Mixup     | 95.8 ± 0.3 | 98.9 ± 0.0 | 98.9 ± 0.0 | 98.8 ± 0.1 | 98.8 ± 0.1 | 96.5 ± 0.3 | 98.0 |
| MLDG      | 95.8 ± 0.1 | 98.9 ± 0.1 | 99.0 ± 0.0 | 98.9 ± 0.1 | 99.0 ± 0.0 | 95.8 ± 0.3 | 97.9 |
| CORAL     | 95.8 ± 0.3 | 98.8 ± 0.0 | 98.9 ± 0.0 | 99.0 ± 0.0 | 98.9 ± 0.1 | 96.4 ± 0.2 | 98.0 |
| MMD       | 95.6 ± 0.1 | 98.9 ± 0.1 | 99.0 ± 0.0 | 99.0 ± 0.1 | 98.9 ± 0.0 | 96.0 ± 0.2 | 97.9 |
| DANN      | 95.0 ± 0.5 | 98.9 ± 0.1 | 99.0 ± 0.0 | 90.0 ± 0.1 | 98.9 ± 0.0 | 96.3 ± 0.2 | 97.8 |
| CDANN     | 95.7 ± 0.2 | 98.8 ± 0.0 | 98.9 ± 0.1 | 98.9 ± 0.1 | 99.0 ± 0.1 | 95.8 ± 0.2 | 97.9 |
| SagNet    | 95.9 ± 0.3 | 98.9 ± 0.1 | 99.0 ± 0.1 | 99.0 ± 0.1 | 99.0 ± 0.1 | 96.3 ± 0.1 | 98.0 |
| ARM       | 96.7 ± 0.2 | 99.1 ± 0.0 | 99.0 ± 0.0 | 99.0 ± 0.1 | 99.1 ± 0.1 | 96.5 ± 0.4 | 98.2 |
| VREx      | 95.9 ± 0.2 | 99.0 ± 0.1 | 98.9 ± 0.1 | 98.9 ± 0.1 | 98.7 ± 0.1 | 96.2 ± 0.2 | 97.9 |
| RSC       | 94.8 ± 0.5 | 98.7 ± 0.1 | 98.8 ± 0.1 | 98.8 ± 0.0 | 98.9 ± 0.1 | 95.9 ± 0.2 | 97.6 |

DRM (ours): 94.5 ± 0.6 | 98.6 ± 0.1 | 98.8 ± 0.1 | 99.1 ± 0.0 | 98.9 ± 0.0 | 96.0 ± 0.3 | 97.6 |

### Table 11. The result for VLCS

| Algorithm | C     | L     | S     | V     | Avg  |
|-----------|-------|-------|-------|-------|------|
| ERM       | 97.7 ± 0.4 | 64.3 ± 0.9 | 73.4 ± 0.5 | 74.6 ± 1.3 | 77.5 |
| IRM       | 98.6 ± 0.1 | 64.9 ± 0.9 | 73.4 ± 0.6 | 77.3 ± 0.9 | 78.5 |
| GroupDRO  | 97.3 ± 0.3 | 63.4 ± 0.9 | 69.5 ± 0.8 | 76.7 ± 0.7 | 76.7 |
| Mixup     | 98.3 ± 0.6 | 64.8 ± 1.0 | 72.1 ± 0.5 | 74.3 ± 0.8 | 77.4 |
| MLDG      | 97.4 ± 0.2 | 65.2 ± 0.7 | 71.0 ± 1.4 | 75.3 ± 1.0 | 77.2 |
| CORAL     | 98.3 ± 0.1 | 66.1 ± 1.2 | 73.4 ± 0.3 | 77.5 ± 1.2 | 78.8 |
| MMD       | 97.7 ± 0.1 | 64.0 ± 1.1 | 72.8 ± 0.2 | 75.3 ± 3.3 | 77.5 |
| DANN      | 99.0 ± 0.3 | 65.1 ± 1.4 | 73.1 ± 0.3 | 77.2 ± 0.6 | 78.6 |
| CDANN     | 97.1 ± 0.3 | 65.1 ± 1.2 | 70.7 ± 0.8 | 77.1 ± 1.5 | 77.5 |
| MTL       | 97.8 ± 0.4 | 64.3 ± 0.3 | 71.5 ± 0.7 | 75.3 ± 1.7 | 77.2 |
| SagNet    | 97.9 ± 0.4 | 64.5 ± 0.5 | 71.4 ± 1.3 | 77.5 ± 0.5 | 77.8 |
| ARM       | 98.7 ± 0.2 | 63.6 ± 0.7 | 71.3 ± 1.2 | 76.7 ± 0.6 | 77.6 |
| VREx      | 98.4 ± 0.3 | 64.4 ± 1.4 | 74.1 ± 0.4 | 76.2 ± 1.3 | 78.3 |
| RSC       | 97.9 ± 0.2 | 64.4 ± 1.4 | 74.1 ± 0.4 | 76.2 ± 1.3 | 77.1 |

DRM (ours): 97.9 ± 0.2 | 65.1 ± 0.7 | 71.5 ± 0.9 | 77.1 ± 1.7 | 77.9 |

### Table 12. The result for PACS

| Algorithm | A     | C     | P     | S     | Avg  |
|-----------|-------|-------|-------|-------|------|
| ERM       | 84.7 ± 0.4 | 80.8 ± 0.6 | 97.2 ± 0.3 | 79.3 ± 1.0 | 85.5 |
| IRM       | 84.8 ± 1.3 | 76.4 ± 1.1 | 96.7 ± 0.6 | 76.1 ± 1.0 | 83.5 |
| GroupDRO  | 83.5 ± 0.9 | 79.1 ± 0.6 | 96.7 ± 0.3 | 78.3 ± 2.0 | 84.4 |
| Mixup     | 86.1 ± 0.5 | 78.9 ± 0.8 | 97.6 ± 0.1 | 75.8 ± 1.8 | 84.6 |
| MLDG      | 85.5 ± 1.4 | 80.1 ± 1.7 | 97.4 ± 0.3 | 76.6 ± 1.1 | 84.9 |
| CORAL     | 88.3 ± 0.2 | 80.5 ± 0.5 | 97.5 ± 0.3 | 78.8 ± 1.3 | 86.2 |
| MMD       | 86.1 ± 1.4 | 79.4 ± 0.9 | 96.6 ± 0.2 | 76.5 ± 0.5 | 84.6 |
| DANN      | 86.4 ± 0.8 | 77.4 ± 0.8 | 97.3 ± 0.4 | 73.5 ± 2.3 | 83.6 |
| CDANN     | 84.6 ± 1.8 | 75.5 ± 0.9 | 96.8 ± 0.3 | 73.5 ± 0.6 | 82.6 |
| MTL       | 87.5 ± 0.8 | 77.1 ± 0.5 | 96.4 ± 0.8 | 77.3 ± 1.8 | 84.6 |
| SagNet    | 87.4 ± 1.0 | 80.7 ± 0.6 | 97.1 ± 0.1 | 80.0 ± 0.4 | 86.3 |
| ARM       | 86.8 ± 0.6 | 76.8 ± 0.5 | 97.4 ± 0.3 | 79.3 ± 1.2 | 85.1 |
| VREx      | 86 ± 1.6 | 79.1 ± 0.6 | 96.9 ± 0.5 | 77.7 ± 1.7 | 84.9 |
| RSC       | 85.4 ± 0.8 | 79.7 ± 1.8 | 97.6 ± 0.3 | 78.2 ± 1.2 | 85.2 |

DRM (ours): 85.0 ± 0.9 | 80.0 ± 0.5 | 96.7 ± 0.6 | 77.5 ± 1.2 | 84.8 |
Table 13. The result for OFFICEHOME

| Algorithm  | A    | C    | P    | R    | Avg |
|------------|------|------|------|------|-----|
| ERM        | 61.3 | 52.4 | 75.8 | 76.6 | 66.5|
| ERM        | 58.9 | 52.2 | 72.1 | 74.0 | 65.3|
| GroupDRO   | 60.4 | 52.7 | 75.0 | 76.0 | 66.0|
| Mixup      | 62.4 | 54.8 | 76.9 | 78.3 | 68.1|
| MLDG       | 61.5 | 53.2 | 75.0 | 77.5 | 66.8|
| CORAL      | 65.3 | 54.4 | 76.5 | 78.4 | 68.7|
| MMD        | 60.4 | 53.3 | 74.3 | 77.4 | 66.3|
| DANN       | 59.9 | 53.0 | 73.6 | 76.9 | 65.9|
| CDANN      | 61.5 | 50.4 | 74.4 | 76.6 | 65.8|
| MTL        | 61.5 | 52.4 | 74.9 | 76.8 | 66.4|
| SagNet     | 63.4 | 54.8 | 75.8 | 78.3 | 68.1|
| ARM        | 58.9 | 51.0 | 74.1 | 75.2 | 64.8|
| VREx       | 60.7 | 53.0 | 75.3 | 76.6 | 66.4|
| RSC        | 60.7 | 51.4 | 74.8 | 75.1 | 65.5|
| DRM (ours) | 60.4 | 52.5 | 74.2 | 75.5 | 65.7|

Table 14. The result for TERRAINCOGNITA

| Algorithm  | L100  | L38   | L43   | L46   | Avg |
|------------|-------|-------|-------|-------|-----|
| ERM        | 49.8  | 42.1  | 56.9  | 35.7  | 46.1|
| IRM        | 54.6  | 39.8  | 56.2  | 39.6  | 47.6|
| GroupDRO   | 41.2  | 38.6  | 56.7  | 36.4  | 43.2|
| Mixup      | 59.6  | 42.2  | 55.9  | 33.9  | 47.9|
| MLDG       | 54.2  | 44.3  | 55.6  | 36.9  | 47.7|
| CORAL      | 51.6  | 42.2  | 57.0  | 39.8  | 47.6|
| MMD        | 41.9  | 34.8  | 57.0  | 35.2  | 42.2|
| DANN       | 51.1  | 40.6  | 57.4  | 37.7  | 46.7|
| CDANN      | 47.0  | 41.3  | 54.9  | 39.8  | 45.8|
| MTL        | 49.3  | 39.6  | 55.6  | 37.8  | 45.6|
| SagNet     | 53.0  | 43.0  | 57.9  | 40.4  | 48.6|
| ARM        | 49.3  | 38.3  | 55.8  | 38.7  | 45.5|
| VREx       | 48.2  | 41.7  | 56.8  | 38.7  | 46.4|
| RSC        | 50.2  | 39.2  | 56.3  | 40.8  | 46.6|
| DRM (ours) | 52.8  | 42.7  | 56.3  | 41.1  | 48.2|

Table 15. The result for DOMAINEET

| Algorithm  | clip  | info  | paint | quick | real  | sketch | Avg  |
|------------|-------|-------|-------|-------|-------|--------|------|
| ERM        | 58.1  | 18.8  | 16.7  | 12.2  | 59.6  | 49.8   | 40.9 |
| IRM        | 48.5  | 15.0  | 38.3  | 10.9  | 48.2  | 42.3   | 33.9 |
| GroupDRO   | 47.2  | 17.5  | 33.8  | 9.3   | 51.6  | 40.1   | 33.3 |
| Mixup      | 55.7  | 18.5  | 44.3  | 12.5  | 55.8  | 48.2   | 39.2 |
| MLDG       | 59.1  | 19.1  | 45.8  | 13.4  | 59.6  | 50.2   | 41.2 |
| CORAL      | 59.2  | 19.7  | 46.6  | 13.4  | 59.8  | 50.1   | 41.5 |
| MMD        | 32.1  | 11.0  | 26.8  | 8.7   | 32.7  | 28.9   | 23.4 |
| DANN       | 53.1  | 18.3  | 44.2  | 11.8  | 55.5  | 46.8   | 38.3 |
| CDANN      | 54.6  | 17.3  | 43.7  | 12.1  | 56.2  | 45.9   | 38.3 |
| MTL        | 57.9  | 18.5  | 46.0  | 12.5  | 59.5  | 49.2   | 40.6 |
| SagNet     | 57.7  | 19.0  | 45.3  | 12.7  | 58.1  | 48.8   | 40.3 |
| ARM        | 49.7  | 16.3  | 40.9  | 9.4   | 53.4  | 43.5   | 35.5 |
| VREx       | 47.3  | 16.0  | 35.8  | 10.9  | 49.6  | 42.0   | 33.6 |
| RSC        | 55.0  | 18.3  | 44.4  | 12.2  | 55.7  | 47.8   | 38.9 |
| DRM (ours) | 58.5  | 19.5  | 45.4  | 13.8  | 59.0  | 49.9   | 41.0 |