Ground-Truth Adversarial Examples

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Abstract

The ability to deploy neural networks in real-world, safety-critical systems is severely limited by the presence of adversarial examples: slightly perturbed inputs that are misclassified by the network. In recent years, several techniques have been proposed for training networks that are robust to such examples; and each time stronger attacks have been devised, demonstrating the shortcomings of existing defenses. This highlights a key difficulty in designing an effective defense: the inability to assess a network’s robustness against future attacks. We propose to address this difficulty through formal verification techniques. We construct ground truths: adversarial examples with provably minimal perturbation. We demonstrate how ground truths can serve to assess the effectiveness of attack techniques, by comparing the adversarial examples produced to the ground truths; and also of defense techniques, by measuring the increase in distortion to ground truths in the hardened network versus the original. We use this technique to assess recently suggested attack and defense techniques.

1 Introduction

While machine learning, and in particular neural networks, have seen significant success, recent work [19] has shown that an adversary can cause unintended behavior by performing slight modifications to the input at test-time. In neural networks used as classifiers, these adversarial examples are produced by taking some normal instance that is classified correctly, and applying a slight perturbation to cause it to be misclassified as any target desired by the adversary. This phenomenon, which has been shown to affect most state-of-the-art networks, poses a significant hindrance to deploying neural networks in safety-critical settings.

Many effective techniques have been proposed for generating adversarial examples [2, 5, 16, 19]: and, conversely, several techniques have been proposed for training networks that are more robust to these examples [6, 8, 15, 20]. Unfortunately, it has proven difficult to accurately assess the robustness of any given defense by evaluating it against existing techniques for generating adversarial examples. In several cases,
a defensive technique that was at first thought to produce robust networks was later shown to be susceptible to new kinds of attacks [3]. This ongoing cycle thus cast a doubt in any newly-proposed defensive technique.

In recent years, new techniques have been proposed for the formal verification of neural networks [4, 9, 12, 17, 18]. These techniques take a network and a desired property, and formally prove that the network satisfies the property — or provide an input for which the property is violated, if such an input exists. Verification techniques can be used to find adversarial examples for a given input point and some allowed amount of distortion, but they tend to be significantly slower than gradient-based techniques [11, 12, 18].

In this paper we propose a method for using formal verification to assess the effectiveness of techniques for producing adversarial examples or defending against them. The key idea is to examine networks and apply verification to identify ground-truth adversarial examples. Given a neural network $F$, a distance metric $d$, and an input $x$, we say that another input $x'$ is a ground-truth adversarial example for $x$ if it is the nearest point (with respect to the metric $d$) such that $F$ assigns different labels to $x$ and $x'$. It follows that all points whose distance to $x$ is smaller than the distance between $x$ and $x'$ are assigned the same label as $x$. The distance to the ground truth is thus an indication of how robust the network is to adversarial attacks at point $x$. Ground truths can serve multiple purposes: (i) if ground-truth adversarial examples are known for a set of points drawn from some sort of meaningful distribution thought to represent real-world inputs, they can serve to estimate the robustness of a network as a whole, against any possible attack; (ii) they can be used for assessing attack techniques, by measuring the proximity of the adversarial examples that these attacks produce to the ground truths; and (iii) they can be used in assessing the effectiveness of defense techniques, by measuring the distance to the ground truths in the hardened network and comparing it to the original network.

Existing verification techniques are designed to answer a yes/no question — does a certain property hold for a given network, or is it violated? However, by iteratively invoking a verification tool and repeatedly having it check the property “there does not exist an adversarial example within distance $\delta$ of point $x'$”, we can perform a binary search and find a ground-truth adversarial example up to a desired level of precision. When a property fails to hold, the verification tool returns a counter-example, which constitutes the ground-truth example (up to the specified precision) that we are after.

Our contributions can thus be summarized as follows:

- We define ground-truth adversarial examples, the provably closest adversarial with respect to some distance metric, as a tool for studying attacks and defenses.
- We find that first-order attack algorithms often produce near-optimal results, i.e. results that are close to a ground-truth adversarial examples.
- We study adversarial training and find that it does increase robustness and does not overfit to a specific attack, as long as the attack is strong.

The rest of this paper is organized as follows. In Section 2 we provide some necessary background. We then describe the experiments that we conducted in Section 3 and analyze their results in Section 4. Finally, we conclude with Section 5.
2 Background and Notation

Neural network notation. We regard a neural network as a function $F(\cdot)$ consisting of multiple layers $F = F_n \circ F_{n-1} \circ \cdots \circ F_1 \circ F_0$. In this paper we exclusively study feed-forward neural networks used for classification, and so the final layer $F_n$ is the softmax activation function. We refer to the output of the second-to-last-layer of the network (the input to $F_n$) as the logits and denote this as $Z = F_{n-1} \circ \cdots \circ F_1 \circ F_0$. We define $\ell_F(x, y)$ to be the cross-entropy loss of the network $F$ on instance $x$ with true label $y$.

We focus here on networks for classifying greyscale MNIST images. Input images with width $W$ and height $H$ are represented as points in the space $[0, 1]^{W \cdot H}$.

Adversarial examples. Given an input $x$, classified originally as target $t = F(x)$, and a new desired target $t' \neq t$, we call $x'$ a targeted adversarial example if $F(x') = t'$ and $x'$ is close to $x$ under some given distance metric. In this paper we focus on the $L_\infty$ and $L_1$ distance metrics.

Generating adversarial examples. We make use of three popular methods for constructing adversarial examples:

1. The Fast Gradient Method (FGM) [5] is a one-step algorithm that takes a single step in the direction of the gradient.

$$x' = \text{FGM}(x) = \text{clip}_{[0, 1]}(x + \epsilon \text{sign}(\nabla \ell_F(x, y)))$$

where $\epsilon$ controls the step size taken, and clip ensures that the adversarial example resides in the valid image space from 0 to 1.

2. The Basic Iterative Method (BIM) [14] (sometimes also called Projected Gradient Descent [15]) can be regarded as an iterative application of the fast gradient method. Initially it lets $x'_0 = x$ and then uses the update rule

$$x'_{i+1} = \text{clip}_{[x-x, x+x]}(\text{FGM}(x'_i))$$

Intuitively, in each iteration this attack takes a step of size $\epsilon$ as per the FGM method, but it iterates this process while keeping each $x'_i$ within the $\alpha$-sized ball of $x$.

3. The Carlini and Wagner (CW) [2] method is an iterative attack that constructs adversarial examples by approximately solving the minimization problem

$$\min d(x, x') \text{ such that } F(x') = t'$$

where $d(\cdot)$ is an appropriate distance metric. Since the constrained optimization is difficult, instead they choose to solve

$$\min d(x, x') + c \cdot g(x')$$

where $g(x')$ is a loss function that encodes how close $x'$ is to being adversarial. Specifically, they set

$$g(x') = \max \{\max \{Z(x')_i : i \neq t\} - Z(x')_t, 0\}.$$
Neural network verification. The intended use of deep neural networks as controllers in safety-critical systems \cite{1,10} has sparked an interest in developing techniques for verifying that they satisfy various properties \cite{4,9,12,17,18}. Here we focus on the recently-proposed Reluplex algorithm \cite{12}: a simplex-based approach that can effectively tackle networks with piecewise-linear activation functions, such as rectified linear units (ReLUs) or max-pooling layers. Reluplex is known to be sound and complete, and so it is suitable for establishing ground truths.

In \cite{12} it is shown that Reluplex can be used to determine whether there exists an adversarial example within distance $\delta$ of some input point $x$. This is performed by encoding the neural network itself and the constraints regarding $\delta$ as a set of linear equations and ReLU constraints, and then having Reluplex attempt to prove the property that “there does not exist an input point within distance $\delta$ of $x$ that is assigned a different label than $x$”. Reluplex either responds that the property holds, in which case there is no adversarial example within distance $\delta$ of $x$, or it returns a counter-example which constitutes the sought-after adversarial input. By invoking Reluplex iteratively and applying binary search, one can approximate the optimal $\delta$ (i.e., the largest $\delta$ for which no adversarial example exists) up to a desired precision \cite{12}.

The proof-of-concept implementation of Reluplex described in \cite{12} supported only networks with the ReLU activation function, and could only handle the $L_{\infty}$ norm as a distance metric. Here we use a simple encoding that allows us to use it for the $L_1$ norm as well.

Adversarial training. Adversarial training is perhaps the first proposed defense against adversarial examples, and is a conceptually straightforward approach. The defender trains a classifier, generates adversarial examples for that classifier, retrains the classifier using the adversarial examples, and repeats.

Recent work has shown \cite{15} that for networks with sufficient capacity, adversarial training can be an effective defense even against the most powerful attacks today. However, it has been shown that performing adversarial training with a weak attack, such as the fast gradient sign method, does not increase robustness substantially against stronger attacks. It is an open question whether adversarial training using stronger attacks would actually increase robustness to adversarial examples, or will such training be effective against known attacks but be vulnerable against future, stronger attacks.

3 Model Setup

The problem of neural network verification that we consider here is an NP-complete problem \cite{12}, and despite recent progress only networks with a few hundred nodes can be soundly verified. Thus, in order to evaluate our approach we trained a small network over the MNIST data set. This network is a fully-connected, 3-layer network that achieves a 97% accuracy despite having only 20k weights and consisting of fewer than 100 hidden neurons (24 in each layer). As verification of neural networks becomes more scalable in the future, our approach could become applicable to larger networks and additional data sets.
For verification, we use the proof-of-concept implementation of Reluplex available online [13]. The only non-linear operator that this implementation was originally designed to support is the ReLU function, but we observe that it can support also max operators using the following encoding:

$$\max(x, y) = \text{ReLU}(x - y) + y$$

This fact allows the encoding of max operators using ReLUs, and consequently to encode max-pooling layers into Reluplex (although we did not experiment with such layers in this paper). Thus, it allows us to extend the results from [12] and measure distances with the $L_1$ norm as well as the $L_\infty$ norm, by encoding absolute values using ReLUs:

$$|x| = \max(x, -x) = \text{ReLU}(2x) - x$$

Because the $L_1$ distance between two points is defined as a sum of absolute values, this encoding allowed us to encode $L_1$ distances into Reluplex without modifying its code. We point out, however, that an increase in the number of ReLU constraints in the input adversely affects Reluplex’s performance. For example, in the case of the MNIST dataset, encoding $L_1$ distance entails adding a ReLU constraint for each input of the 784 input coordinates. It is thus not surprising that experiments using $L_1$ typically took longer to finish than those using $L_\infty$.

Each individual experiment that we conducted included a network $F$, a distance metric $d \in \{L_1, L_\infty\}$, an input point $x$, a target label $\ell' \neq F(x)$, and an initial adversarial input $x'_{\text{init}}$ for which $F(x'_{\text{init}}) = \ell'$. The goal of the experiment was then to find a ground-truth example $x_{\ell'}$, such that $F(x_{\ell'}) = \ell'$ and $d(x, x_{\ell'})$ is minimal. As explained in Section 2, this is performed by iteratively invoking Reluplex and performing a binary search. This procedure is given as Algorithm 1.

**Algorithm 1 Find Ground Truth $(F, d, x, \ell', x'_{\text{init}})$**

1: $x'_{\text{best}} := x'_{\text{init}}$
2: $\delta_{\text{min}} := 0$
3: $\delta_{\text{max}} := \|x - x'_{\text{init}}\|_d$
4: while $\delta_{\text{max}} - \delta_{\text{min}} > 10^{-3}$ do
5: \hspace{1em} $d := (\delta_{\text{max}} + \delta_{\text{min}})/2$
6: \hspace{1em} Invoke Reluplex to test whether $\exists x'. \|x - x'\|_d \leq \delta \land F(x') = \ell'$
7: \hspace{1em} if $x'$ exists then
8: \hspace{2em} $\delta_{\text{max}} := \delta$
9: \hspace{2em} $x'_{\text{best}} := x'$
10: \hspace{1em} else
11: \hspace{2em} $\delta_{\text{min}} := \delta$
12: return $\delta_{\text{max}}, x'_{\text{best}}$

Intuitively, $\delta_{\text{max}}$ indicates the distance to the closest adversarial input currently known, and the ground-truth input is known to be in the range between $\delta_{\text{min}}$ and $\delta_{\text{max}}$. Thus, $\delta_{\text{max}}$ is initialized using the distance of the initial adversarial input provided, and $\delta_{\text{min}}$ is initialized to 0. The search procedure iteratively shrinks the range $\delta_{\text{max}} - \delta_{\text{min}}$ until it is below a certain threshold (we used $10^{-3}$ for our experiments). It then returns
\( \delta_{\text{max}} \) as the distance to the ground truth, and this is guaranteed to be accurate up to the specified precision. The ground-truth input itself is also returned.

For the initial \( x'_{\text{init}} \) in our experiments we used an adversarial input found using the CW attack. We note that Reluplex invocations are computationally expensive, and so it is better to start with \( x'_{\text{init}} \) as close as possible to \( x \), in order to reduce the number of required iterations until \( \delta_{\text{max}} - \delta_{\text{min}} \) is sufficiently small. For the same reason, experiments using the \( L_1 \) distance metric were slower than those using \( L_{\infty} \): the initial distances were typically much larger, which required additional iterations.

4 Evaluation

For evaluation purposes we arbitrarily selected 10 source images with known labels from the MNIST test set. We considered two neural networks — the one described in Section 3, denoted \( N \), and also a version of \( N \) that has been trained with adversarial training as described in [15], denoted \( \bar{N} \). We also considered two distance metrics, \( L_1 \) and \( L_{\infty} \). For every combination of neural network, distance metric and labeled source image \( x \), we considered each of the 9 other possible labels for \( x \). For each of these we used the CW attack to produce an initial targeted adversarial example, and then used Algorithm 1 to search for a ground-truth example. The results are given in Table 1, with graphical representations attached as appendices at the end of the paper.

Each major row of the table corresponds to specific neural network and distance metric (as indicated in the first column), and describes 90 individual experiments (10 inputs, times 9 target labels for each input). The first sub-row within each row considers just those experiments for which Algorithm 1 terminated successfully, whereas the second sub-row considers all 90 examples, including those where Algorithm 1 timed out. Whenever a timeout occurred, we considered the last (smallest) \( \delta_{\text{max}} \) that was discovered by the search before it timed out as the algorithm’s output. The other columns of the table indicate the average distance to the adversarial examples found by the CW attack, the average distance to the ground-truth adversarial examples found by our technique, and the average improvement rate of our technique over the CW attack.

Below we analyze the results in order to draw conclusions regarding the CW attack and the defense of [15]. While these results naturally hold only for the networks we used and the inputs we tested, we believe they provide some intuition as to how well the tested attack and defense techniques perform. We intend to make our data publicly available, and we encourage others to (i) evaluate new attack techniques using the ground-truth examples we have already discovered, and on additional ones; and (ii) to use this approach for evaluating new defensive techniques.

4.1 Evaluating Attacks

Iterative attacks produce near-optimal adversarial examples. As is shown by Table 1, the adversarial examples produced by the CW attack are on average within 11.6% of the ground truth when using the \( L_{\infty} \) norm, and within 6.2% of the ground truth when
using \( L_1 \) (we consider here just the terminated experiments, and ignore the \( N, L_1 \) category where too few experiments terminated to draw a meaningful conclusion). In particular, iterative attacks perform substantially better than single-step methods, such as the fast gradient method. This is an expected result and is not surprising: the fast gradient method was designed to show the linearity of neural networks, not to produce high-quality adversarial examples.

There is still room for improving iterative attacks. Even on this very small and simple neural network, we observed that in many instances the ground-truth adversarial example has a 30% or 40% lower distortion rate than the best iterative adversarial example. The cause for this is simple: gradient descent only finds a local minimum, not a global minimum. We have found that if we take a small step from the original image in the direction of the ground truth, then gradient descent will converge to the ground truth much more often.

Suboptimal results are correlated. We have found that when the iterative attack performs suboptimally compared to the ground truths for one target label, it will often perform poorly for many other target labels as well. These instances are not always of larger absolute distortion, but a larger relative gap on one instance often indicates that the relative gap will be larger for other targets. For instance, on the adversarially trained network attacked under \( L_\infty \) distance, the ground-truth adversarial examples for the digit 9 were from 21\% to 47\% better than the iterative attack results.

When we examined the most extreme cases in which this phenomenon was observed, we found that, similarly to the case described above, the large gap was caused by gradient descent initially leading away from the ground truth for most targets, resulting in the discovery of an inferior, local minimum.

### 4.2 Evaluating Defenses

For the purpose of evaluating the defensive technique of \([15]\), we compared the \( N, L_\infty \) and \( \bar{N}, L_\infty \) experiments (the \( L_1 \) experiments were disregarded because of the small
number of experiments that terminated for the $N, L_1$ case). Specifically, we compared the $N, L_\infty$ and $\bar{N}, L_\infty$ experiments on the subset of 35 instances that terminated for both experiments. The results appear in Table 2.

|        | #Points | CW   | Ground Truth | % Improvement |
|--------|---------|------|--------------|---------------|
| $N, L_\infty$ | 35/35   | 0.042 | 0.039        | 12.319        |
| $\bar{N}, L_\infty$ | 35/35   | 0.18  | 0.165        | 11.153        |

The defense of Madry et al. [15] is effective. Our evaluation suggests that the Madry defense is indeed effective: it improves the distance to the ground truth by an average of 423% (from an average of 0.039 to an average of 0.165) on our small network.

Another interesting observation is that while the Madry defense improves the overall situation, we found several points in which it actually made things worse — i.e., the ground truth for the hardened network was smaller than that of the original network. This behavior was observed for 7 out of the 35 aforementioned experiments, with the average percent of degradation being 12.8%. This seems to highlight the necessity of evaluating the effectiveness of a defensive technique, and the robustness of a network in general, over a large dataset of points. The question of how to pick a “good” set of points that would adequately represent the behavior of the network remains open.

Training on iterative attacks does not overfit. Overfitting is an issue that is often encountered when performing adversarial training, meaning that a defense may overfit to the type of attack used during training. When this occurs, the hardened network will have high accuracy against the one attack used during training, but low accuracy on other attacks. We have found no evidence of overfitting when performing the adversarial training of [15]: the ground truths improve on the CW attack by 12% on both the hardened and untrained networks.

The defense of Madry et al. is easier to formally analyze. For both the $L_\infty$ and $L_1$ distance metrics, it seems significantly easier to analyze the robustness of the adversarially trained network: when using $L_\infty$, Algorithm terminated on 81 of the 90 instances on the adversarially trained network, versus 38 on the standard network; and for $L_1$, the termination rate was 64 for the hardened network compared to just 6 on the standard network. We are still looking into the reason for this behavior. Naively, one might assume that because the initial adversarial examples $x_{\text{init}}$ provided to Algorithm have larger distance for the hardened network, that these experiments will take longer to converge — but we were seeing an opposite behavior.

One possible explanation could be that the adversarially trained network makes less use of the nonlinear ReLU units, and is therefore more amenable to analysis with Reluplex. We empirically verify that this is not the case. For a given instance, we track, for each ReLU unit in the network, whether it is in the saturated zero region, or the linear $x = y$ region. We then compute the nonlinearity of the network as the number
of units that change from the saturated region to the linear region, or vice versa, when
going from the given input to the discovered adversarial example. We find that there is
no statistically significant difference between the nonlinearity of the two networks.

5 Conclusion

Neural networks hold great potential to be used as controllers in safety-critical systems,
but their susceptibility to adversarial examples poses a significant hindrance. The de-
velopment of defensive techniques is difficult when they are measured only against
existing attacks. The burgeoning field of neural network verification can mitigate this
problem, by allowing us to obtain an absolute measurement of the usefulness of a de-
fense, regardless of the attack to be used against it.

In this paper, we introduce ground-truth adversarial examples and show how to
construct them with formal verification approaches. We evaluate one recent attack [2]
and find it often produces adversarial examples whose distance is within 6.6% to 13%
of optimal, and one defense [15], and find that it increases distortion to the nearest
adversarial example by an average of 423% on the MNIST dataset for our tested net-
works. To the best of our knowledge, this is the first proof of robustness increase for a
defense.

Currently available verification tools afford limited scalability, which means ex-
periments can only be conducted on small networks. However, as better verification
techniques are developed, this limitation is expected to be lifted. Orthogonally, when
preparing to use a neural network in a safety-critical setting, users may choose to de-
sign their networks as to make them particularly amenable to verification techniques
— e.g., by using specific activation functions or network topologies — so that strong
guarantees about their correctness and robustness may be obtained.

Acknowledgements. This work was partially supported by grants from the FAA, Intel,
the AFOSR under MURI award FA9550-12-1-0040, the Hewlett Foundation through
the Center for Long-Term Cybersecurity, and Qualcomm.

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