On the Particle Data Group evaluation of $\chi_c$ and $\psi'$ branching ratios

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I propose a new evaluation of $\psi'$ and $\chi_c$ branching ratios which avoids the correlations affecting the current Particle Data Group evaluation.

These correlations explain the apparent technique-dependent discrepancies between the available determinations of the $B(\chi_c \to p\bar{p})$ and $\Gamma(\chi_c \to \gamma\gamma)$ under the hypothesis that the current values of the $\psi' \to \chi_c\gamma$ branching ratios are overestimated.

In the process I also noticed that the Particle Data Group has not restated many of the older measurements, when necessary, for the new value of $B(J/\psi \to l^+l^-)$, which significantly affects the evaluation of some relevant $\psi'$ and $\chi_c$ exclusive branching ratios.

I. INTRODUCTION

All of the older measurements of $\chi_c$ branching ratios where performed in $e^+e^-$ experiments, where the $\chi_c$ were studied in the $\psi'$ radiative decays.

In recent years new measurements appeared for $B(\chi_c \to p\bar{p})$ and $\Gamma(\chi_c \to \gamma\gamma)$ obtained in $e^+e^-$, $pp$ and $e^+e^- \to e^+e^-\gamma\gamma$ experiments.

Although the accuracy of these measurements is limited, it’s becoming evident that the values of $B(\chi_c \to p\bar{p})$ derived by measurements performed in $p\bar{p}$ annihilation experiments tend to be systematically higher than the values obtained in $e^+e^- \to \psi'$ experiments (Tab. I), while for the $\Gamma(\chi_c \to \gamma\gamma)$ the values derived by measurements in $p\bar{p}$ experiments are significantly lower than the values obtained in $e^+e^- \to e^+e^-\gamma\gamma$ experiments (Tab. I).

I reviewed the procedure and the data used by the Particle Data Group (PDG [1]) to evaluate these quantities and I realized that this peculiar pattern of technique-dependent discrepancies could be explained by a systematic error in $B(\psi' \to \chi_c\gamma)$.

The problem is however of a more general kind, since it depends on the fact that in many cases the PDG uses in its evaluation quantities that are derived from the measurement. At least for some of the $\psi'$ and $\chi_c$ branching ratios, this procedure introduces a correlation among the values obtained by different experiments which has not been properly taken into account.

I also noticed that in most cases the older data used by PDG to evaluate the $\psi'$ branching ratios to final states including a $J/\psi$ and the $\chi_c$ radiative decay branching ratios had not been restated for the current world average of $B(J/\psi \to l^+l^-)$, whose current central value is now about 15% lower than the value currently accepted at the time most of the measurements were performed.

While this only involves a restatement of a number of experimental results, it affects the value of exclusive $\psi'$ and $\chi_c$ branching ratios which are relevant for a large number of other experiments.

The outline of the paper is as follows: in section II the problems found in the evaluation of $\psi'$ and $\chi_c$ branching ratios are discussed; in section III a new evaluation is presented. The results are discussed in section IV and compared with some theoretical expectations for the partial widths of the radiative transitions.

II. RATIONALE FOR A NEW EVALUATION OF $\psi'$, $\chi_c$ BRANCHING RATIOS

There is often an ambiguity in presenting the experimental result between the measurement and the determination of what the experiment considers the relevant branching ratio, extracted from its measurement based on other, better known quantities, to which the result is clearly correlated.

While this serves the legitimate purpose of showing the impact of a new measurement, it might contribute to an improper treatment of data by PDG if the correlation to other measurements is not given enough evidence.

As an example, the PDG compilation lists as independent measurement of $B(\chi_{c0} \to p\bar{p})$, the values quoted by BES [2] and E835 [3].
E835 explicitly mentions in the abstract that the value they quote for $\mathcal{B}(\chi_{c0} \to p\bar{p})$ is derived from a direct measurement of
\[ \Gamma(\chi_{c0} \to p\bar{p})\mathcal{B}(\chi_{c0} \to J/\psi\gamma)\mathcal{B}(J/\psi \to e^+e^-) = 2.89^{+0.67}_{-0.53} \pm 0.14 \text{ eV}, \] (1)
while it is not immediate to recognize that the value of $\mathcal{B}(\chi_{c0} \to p\bar{p})$ quoted by BES is obtained by a measurement of
\[ \frac{\mathcal{B}(\psi' \to \chi_{c0}\gamma)\mathcal{B}(\chi_{c0} \to p\bar{p})}{\mathcal{B}(\psi' \to J/\psi\pi^+\pi^-)} = (4.6 \pm 1.9) \times 10^{-5} \] (2)
because this value is not explicitly quoted in [2] and the value in eq. (2) must be calculated from the number of $\psi' \to \chi_{c0}\gamma \to p\bar{p}\gamma$ decays observed, the corresponding efficiency, and the total number of $\psi'$, which in turn, as specified in one of their references [3], is obtained from the observed number of $\psi' \to J/\psi\pi^+\pi^-$ using for this decay mode the value of branching ratio taken from the 1996 compilation of PDG (PDG96) [4].

The value of $\mathcal{B}(\chi_{c0} \to p\bar{p})$ derived from eq. (2) is strongly anticorrelated to that obtained from eq. (1), since in the first case the measurement is multiplied by $\Gamma(\chi_{c0} \to J/\psi\gamma)^{-1}$ which is derived as
\[ \Gamma(\chi_{c0} \to J/\psi\gamma)^{-1} = \frac{\mathcal{B}(\psi' \to \chi_{c0}\gamma)}{\mathcal{B}(\psi' \to \chi_{c0}\gamma \to J/\psi\gamma\gamma)} \frac{1}{\Gamma(\chi_{c0})}, \] (3)
while in the second case it is multiplied by
\[ \frac{1}{\mathcal{B}(\psi' \to \chi_{c0}\gamma)}. \] (4)

The multiplicative factor is directly proportional to $\mathcal{B}(\psi' \to \chi_{c}\gamma)$ for $p\bar{p}$ annihilation experiments and inversely proportional to it for $e^+e^- \to \psi'$ experiments. (The same applies of course to the various determinations of $\mathcal{B}(\chi_{c1,2} \to p\bar{p})$).

Even if it is not immediately evident, the same kind of technique-dependent anticorrelation is found among the different $\Gamma(\chi_{c1,2} \to \gamma\gamma)$ determinations, as discussed in more detail in the appendix. The multiplicative factor in this case is directly proportional to $\mathcal{B}(\psi' \to \chi_{c}\gamma)$ for $e^+e^- \gamma\gamma$ experiments and inversely proportional to it for $p\bar{p}$ annihilation experiments.

The most interesting consequence of this correlation is that a systematic error in $\mathcal{B}(\psi' \to \chi_{c}\gamma)$ would induce a peculiar pattern of apparent technique-dependent discrepancies in both $\mathcal{B}(\chi_{c} \to p\bar{p})$ and $\Gamma(\chi_{c} \to \gamma\gamma)$ obtained by this procedure.

The observed pattern shown in tabs. [1] and [1] suggests the hypothesis that the current values of $\mathcal{B}(\psi' \to \chi_{c}\gamma)$ are overestimated.

This is not the only problem in the evaluation of $\psi$ and $\chi_{c}$ branching ratios. An inconsistency was already noticed by Gu and Li [4] in the 1998 edition of the PDG compilation (PDG98) [3] concerning the evaluation of some $\psi'$ branching ratios which included the measurements performed by E760 [3].

PDG98, based on the results quoted in the abstract of the paper, listed the E760 result as independent measurements of $\mathcal{B}(\psi' \to J/\psi\pi^+\pi^-)$, $\mathcal{B}(\psi' \to J/\psi\pi^0\pi^0)$, and $\mathcal{B}(\psi' \to J/\psi\eta)$, while the experiment, as specified in the paper, indeed measured the ratio of these branching ratios to $\mathcal{B}(\psi' \to J/\psi X)$, whose values are not explicitly quoted in the paper and must be calculated from efficiency and event ratios.

The inconsistency in this case was due to a circular dependence, since the multiplicative factor used in PDG98 to derive the above branching ratios from the E760 measurement, $\mathcal{B}(\psi' \to J/\psi X)$, was itself correlated to the E760 measurement, since it was obtained as
\[ \mathcal{B}(\psi' \to J/\psi X) = \mathcal{B}(\psi' \to J/\psi\pi^+\pi^-) + \mathcal{B}(\psi' \to J/\psi\pi^0\pi^0) + \mathcal{B}(\psi' \to J/\psi\eta) + \mathcal{B}(\psi' \to \chi_{c1}\gamma)\mathcal{B}(\chi_{c1} \to J/\psi\gamma) + \mathcal{B}(\psi' \to \chi_{c2}\gamma)\mathcal{B}(\chi_{c2} \to J/\psi\gamma) \] (5)
based on values for the $\mathcal{B}(\psi' \to J/\psi\pi^+\pi^-)$, $\mathcal{B}(\psi' \to J/\psi\pi^0\pi^0)$, and $\mathcal{B}(\psi' \to J/\psi\eta)$ which included the E760 result in the average.

The problem here is rather subtle: the values of branching ratios quoted in the abstract by E760 are correlated to previous measurements, but there is no circular dependence since the value of $\mathcal{B}(\psi' \to J/\psi X)$, taken from PDG96 [3], is not correlated to the E760 measurement. The same procedure is also used in a recent E835 paper [3] reporting new, more precise measurements of $\frac{\mathcal{B}(\psi' \to e^+e^-)}{\mathcal{B}(\psi' \to J/\psi X)}$, $\frac{\mathcal{B}(\psi' \to J/\psi\pi^+\pi^-)}{\mathcal{B}(\psi' \to J/\psi X)}$, and $\frac{\mathcal{B}(\psi' \to J/\psi\eta)}{\mathcal{B}(\psi' \to J/\psi X)}$, the quantities directly measured are clearly specified, but the abstract quotes only the non-independent estimate of branching ratios derived using the value of $\mathcal{B}(\psi' \to J/\psi X)$ from PDG96.

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Following Gu and Li, the 2000 edition of the PDG compilation uses in its evaluation the E760 measurement of \( \frac{B(\psi' \to \psi\pi\pi)}{B(\psi' \to J/\psi X)} \) and the value of \( \frac{B(\psi' \to J/\psi\pi^-\pi^0)}{B(\psi' \to J/\psi \pi^+\pi^-)} \) derived from the E760 measurement, because they claim this makes the E760 input data independent. While this approach avoids the inconsistency described above, it doesn’t make proper use of measurements that constrain which fractions of \( B(\psi' \to J/\psi X) \) are due to \( B(\psi' \to J/\psi\pi^0\pi^0) \) and \( B(\psi' \to J/\psi\pi^+\pi^-) \) separately, not only their ratio.

An analogous problem of circular dependence is found in the 2000 edition of PDG in the \( B(\psi' \to J/\psi\eta) \) and \( B(\chi_{c1,2} \to J/\psi\gamma) \) evaluations.

The MARK-II results on these branching ratios [10] depend on the value of \( B(\psi' \to J/\psi\pi^+\pi^-) \) (this decay mode is used to determine the total number of \( \psi' \) as specified in their ref. [10]), whose average value is no longer independent (because of the E760 measurement) from the average value of \( B(\psi' \to J/\psi X) \), which in turn depends on the average value \( B(\psi' \to J/\psi\eta) \) and \( B(\chi_{c1,2} \to J/\psi\gamma) \), that include the MARK-II result.

Notice that in the last two cases the circular dependence arises from the choice of using the value \( B(\psi' \to J/\psi X) \) determined by eq. [3] rather than the average of direct measurements.

The correlation between the determined values of \( B(\psi' \to J/\psi\pi^+\pi^-) \), \( B(\psi' \to J/\psi\pi^0\pi^0) \), \( B(\psi' \to J/\psi\eta) \), and \( B(\chi_{c1,2} \to J/\psi\gamma) \) is unavoidable given that they are derived from measurements of their ratios to \( B(\psi' \to J/\psi X) \) and/or \( B(\psi' \to J/\psi\pi^+\pi^-) \).

On the other side, the PDG has excluded from its averages the value of \( B(\psi' \to e^+e^-) \) derived from the E760 measurement of \( \frac{B(\psi' \to e^+e^-)}{B(\psi' \to J/\psi X)} \) while in this case the determined value, although dependent on \( B(\psi' \to J/\psi X) \), is not correlated to the (less precise) value of \( B(\psi' \to e^+e^-) \) derived from the line shape analysis.

For all of these reasons, and also because the PDG missed to restate quite a few older measurements for the new products of branching ratios directly measured by the experiments.

Unfortunately this implies that it is no longer possible to evaluate the \( \chi_c \) branching ratios independently from the \( \psi' \) branching ratios.

III. A NEW EVALUATION OF \( \psi' \), \( \chi_c \) BRANCHING RATIOS

A fit to the \( \psi' \) and \( \chi_c \) branching ratios simultaneously based on all the measurements listed in Tabs. IV–V and VII is performed.

In these tables I've listed the quantities directly measured by each experiment, even if the results are usually presented using the procedure discussed above.

When the quantity directly measured is not explicitly quoted in the paper, I calculated it either from the number of events and efficiencies or by rescaling the final result by the value of the branching ratios used in its derivation, as specified in the tables’ footnotes.

The values have been corrected when appropriate for the current world average of \( B(J/\psi \to l^+l^-) \).

The fit is done by a standard \( \chi^2 \) minimization using the MINUIT program [12]. The \( \chi^2 \) is defined as:

\[
\chi^2 = \sum_i \frac{(M_i - F_i(\vec{\theta}))^2}{\sigma_i^2}
\]

(6)

where \( M_i \) are the quantities directly measured by each experiment, and \( F_i \) are the corresponding expected values given the set of parameters \( \vec{\theta} \)

\[
\vec{\theta} = \left( B(\psi' \to \psi\pi\pi), B(\psi' \to \psi\eta), B(\psi' \to \chi_c\gamma), B(\chi_c \to J/\psi\gamma), B(\chi_c \to pp), B(\chi_c \to \gamma\gamma), \Gamma(\chi_c) \right)
\]

In all cases systematic and statistical errors have been added in quadrature. No attempt has been made to take into account the correlation between systematic errors on measurements performed by the same experiment.

The fit uses 63 measurements to fix 17 free parameters and the results are listed in Tab. VII. It has a \( \chi^2=56.0 \) for 46 degrees of freedom which corresponds to a probability of 14.8% under the assumption of gaussian errors.

1The new E835 measurements on \( \psi' \) [4] and \( \chi_c \to \gamma\gamma \) [11] are included. The last table includes also some measurements already listed in Tabs. IV–VI.
Given the large number of free parameters, I verified the stability of the results by fixing some of them to their current PDG averages.

I’ve chosen the variables to fix as those for which an average based on direct independent measurements is available. Fixing the values of the total widths $\Gamma(\chi_c)$ the number of free parameters is reduced to 14.

I’ve also fixed the values of the $B(\psi' \rightarrow X_c\gamma)$’s to their current PDG averages.

The value of $B(\psi' \rightarrow J/\psi\pi^+\pi^-)$ is weakly correlated to $B(\psi' \rightarrow \chi_{c1,2}\gamma \rightarrow J/\psi\gamma\gamma)$ through the $B(\psi' \rightarrow J/\psi X)$ which is calculated by eq. [3].

Ignoring this weak correlation and fixing $\langle B(\psi' \rightarrow J/\psi\pi^+\pi^-) \rangle$ to the central value of the only direct measurement, performed by MARK-I back in 1975 [13], the complexity of the fit is greatly reduced.

For this test a separate fit to each one of the $\chi_c$ states can be performed using only the measurements listed in Tabs. III-IV. Each fit has now at most 5 free parameters:

$$
\vec{\theta} = \left( B(\psi' \rightarrow \chi_c\gamma), B(\chi_c \rightarrow J/\psi\gamma), B(\chi_c \rightarrow p\bar{p}), B(\chi_c \rightarrow \gamma\gamma), \Gamma(\chi_c) \right).$

In all cases the results are stable, with the only exception of the fit in which the $\psi'$ radiative decay branching ratios are fixed to their current world average. The $\chi^2$ is also worse in this case, as would be expected in case of a systematic error on these quantities.

IV. DISCUSSION OF THE RESULTS

As a result of the fit I obtain values for the branching ratios of the radiative $\psi'$ and $\chi_c$ decays that are significantly different from the current PDG average.

The difference between the $\chi_c$ radiative branching ratios $(1.2^{+0.3}_{-0.2})\% (\chi_{c0})$, $(31.8^{+3.6}_{-3.1})\% (\chi_{c1})$, and $(18.7^{+2.8}_{-2.3})\% (\chi_{c2})$ and the corresponding current PDG averages $(0.66 \pm 0.18 \pm 0.07)\%$, $(27.3 \pm 1.6 \pm 2.5)\%$, and $(13.5 \pm 1.1 \pm 1.4)\%$ can only in part be ascribed to the new value of $B(J/\psi \rightarrow l^+l^-)$, which only accounts for an increase by 15%.

The differences between the new determination of the $\psi'$ radiative branching ratios, $(7.1 \pm 1.2)\%$, $(8.4 \pm 0.8)\%$ and $(6.8 \pm 0.8)\%$ for $\chi_{c0}$, $\chi_{c1}$ and $\chi_{c2}$ respectively, and the corresponding PDG averages of $(9.3 \pm 0.9)\%$, $(8.7 \pm 0.8)\%$ and $(7.8 \pm 0.8)\%$ can only be explained in term of a systematic error in the data, since in this case the PDG performs an average of genuinely independent, and direct measurements of these quantities.

It is interesting to notice that for all of the above branching ratios the precision on the fit result is worse than that quoted by PDG, with the only exception of the $\chi_{c0} \rightarrow J/\psi\gamma$. This could be a symptom of discrepancies in the data.

The above results for the $\psi'$ branching ratios can also be interpreted as a confirmation that for $\chi_{c0}$ and $\chi_{c2}$ the technique-dependent discrepancies between the $\Gamma(\chi_{c2} \rightarrow \gamma\gamma)$ and $B(\chi_{c0} \rightarrow p\bar{p})$ determinations are significant enough to support the hypothesis that the current values of $B(\psi' \rightarrow \chi_{c0,2}\gamma)$ (whose average is dominated by the Crystall Ball measurement [14]) are overestimated by $\sim 10-20\%$.

The different measurements involving the $B(\chi_{c1} \rightarrow p\bar{p})$, given the present accuracy, are not sensible to an hypothetical systematic error in $B(\psi' \rightarrow \chi_{c1}\gamma)$.

The hypothesis of a systematic error on the $B(\psi' \rightarrow \chi_c\gamma)$ would also explain why the error on the $B(\chi_c \rightarrow p\bar{p})$ and $B(\chi_c \rightarrow \gamma\gamma)$ is sometimes drastically reduced.

For the $p\bar{p}$ branching ratios I obtain for $\chi_{c0}$, $\chi_{c1}$, and $\chi_{c2}$ $(2.4 \pm 0.6) \times 10^{-4}$, $(7.1^{+1.5}_{-1.2}) \times 10^{-5}$ and $(7.4^{+1.4}_{-1.2}) \times 10^{-5}$ respectively.

These values must be compared with the corresponding current PDG evaluation: $(2.2 \pm 1.3 \pm 0.2) \times 10^{-4}$, $(8.2 \pm 1.3 \pm 0.8) \times 10^{-5}$ and $(9.8 \pm 1.0 \pm 1.0) \times 10^{-5}$. For the $\chi_{c0}$ the error is reduced from $\sim 60\%$ to $\sim 25\%$.

For the $B(\chi_c \rightarrow \gamma\gamma)$ the increased precision in the new determination, $(2.0^{+1.0}_{-0.9}) \times 10^{-4}$ and $(2.2 \pm 0.4) \times 10^{-4}$ (for $\chi_{c0}$ and $\chi_{c2}$ respectively), with respect to the corresponding PDG evaluation $(2.7 \pm 1.9 \pm 0.3) \times 10^{-4}$ and $(1.6 \pm 0.5 \pm 0.2) \times 10^{-4}$ is only in part due to the new E835 measurements [11].

The reasonable agreement between predicted and measured partial widths, in particular for the radiative decay modes, has always been regarded as one of the most remarkable features of the quarkonium model.

The values obtained here for the $\psi'$ and $\chi_c$ radiative decays are in general reasonably compatible with theoretical predictions of the radiative widths for these states as shown in Tab. VIII, at least to the same level as the current PDG values.
V. CONCLUSIONS

I have presented a new evaluation of some $\psi'$ and $\chi_c$ branching ratios which is, I believe, more adequate to evaluate them based on a set of independent measurements which involve different combinations of $\psi'$ and $\chi_c$ branching ratios simultaneously.

The new values for the $\mathcal{B}(\psi' \to \chi_{c0,2})$’s and $\mathcal{B}(\chi_{c0,2} \to J/\psi\gamma)$’s obtained here are significantly different from the current PDG averages.

It must be noticed that the current world average of the $\psi'$ and $\chi_c$ radiative decays are all dominated by the Crystall Ball measurement [12] [13], and that if there is indeed a systematic error, it is not unreasonable to expect that it be common to all $\psi' \to \chi_c \gamma$ and $\chi_c \to J/\psi\gamma$ measurements.

New, more precise measurements of $\mathcal{B}(\psi' \to \chi_{c1}\gamma \to p\bar{p}\gamma)$ could help to clarify the problem. Also new measurements in $p\bar{p}$ or $\gamma\gamma$ experiments of selected $\chi_c$ branching ratios to decay modes other than $J/\psi\gamma$ or $\gamma\gamma$ could provide valuable information.

As a last remark, I must mention that it has not always been easy to extract the values directly measured from the original papers, since they are sometimes not explicitly quoted, and in a few cases relevant information had to be gathered from references.

As shown by the case presented here, the ambiguity between measurement and values derived from it may hide the correlations, which must be understood to properly analyze the data from different experiments.

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APPENDIX: THE PDG EVALUATION OF $\mathcal{B}(\chi_c \to p\bar{p})$ AND $\Gamma(\chi_c \to \gamma\gamma)$

The evaluation of $\langle \mathcal{B}(\chi_c \to p\bar{p}) \rangle$ and $\langle \Gamma(\chi_c \to \gamma\gamma) \rangle$ is based on measurement performed with a variety of techniques.

The values of $\mathcal{B}(\chi_{c0,1,2} \to p\bar{p})$ quoted in the original papers are extracted from the measurements according to

$$\mathcal{B}(\chi_c \to p\bar{p}) = \begin{cases} \mathcal{B}(\psi' \to \chi_c \gamma)\mathcal{B}(\chi_c \to p\bar{p}) \frac{1}{\mathcal{B}(\psi' \to \chi_c \gamma)} & (e^+e^-) \\ \Gamma(\chi_c \to p\bar{p})\mathcal{B}(\chi_c \to J/\psi\gamma) \frac{1}{\Gamma(\chi_c)\mathcal{B}(\chi_c \to J/\psi\gamma)} & (p\bar{p}) \end{cases}$$

(A1)

where $\langle \rangle$ indicates the world average for the given quantity.

Analogously the values quoted for $\Gamma(\chi_{c0,2} \to \gamma\gamma)$ are obtained as

$$\Gamma(\chi_c \to \gamma\gamma) = \begin{cases} \mathcal{B}(\psi' \to \chi_c \gamma)\mathcal{B}(\chi_c \to \gamma\gamma) \frac{\langle \Gamma(\chi_c) \rangle}{\mathcal{B}(\psi' \to \chi_c \gamma)} & (e^+e^-) \\ \Gamma(\chi_c \to \gamma\gamma)\mathcal{B}(\chi_c \to J/\psi\gamma) \frac{1}{\mathcal{B}(\chi_c \to J/\psi\gamma)} & (\gamma\gamma) \\ \mathcal{B}(\chi_c \to p\bar{p})\mathcal{B}(\chi_c \to \gamma\gamma) \frac{\langle \Gamma(\chi_c) \rangle}{\mathcal{B}(\chi_c \to p\bar{p})} & (p\bar{p}) \end{cases}$$

(A2)

The PDG evaluates the $\langle \mathcal{B}(\chi_{c0,1,2} \to p\bar{p}) \rangle$ and $\langle \Gamma(\chi_{c0,2} \to \gamma\gamma) \rangle$ averaging the values derived by the different experiments according to eqs. (A1) and (A2).

\footnote{The last expression in Eq. (A2) is used in a recent E835 paper [1] to present a new determination of $\Gamma(\chi_{c0,2} \to \gamma\gamma)$. It is not yet included in PDG averages.}

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There is more than one reason to object to this procedure. First of all the error attributed to the $\langle B(\chi_c \to J/\psi\gamma) \rangle$, calculated by

$$B(\chi_c \to J/\psi\gamma) = \frac{\mathcal{B}(\psi' \to \chi_c\gamma \to \chi_c \to J/\psi\gamma\gamma)}{\mathcal{B}(\psi' \to \chi_c\gamma)}$$

(A3)

doesn’t even include the $\sim 10\%$ uncertainty in $\langle B(\psi' \to \chi_c\gamma) \rangle$ which is the dominant source of error on the $\langle B(\chi_c \to J/\psi\gamma) \rangle$.

A more serious objection regards the fact that the different values of $\mathcal{B}(\chi_c \to p\bar{p})$ and $\Gamma(\chi_c \to \gamma\gamma)$ obtained in this way are not independent.

If we substitute Eq. (A3) in Eqs. (A1) and (A2), and consider that the $\langle B(\chi_c \to p\bar{p}) \rangle$ is dominated by the measurements performed in $p\bar{p}$ experiments, we obtain

$$\mathcal{B}(\chi_c \to p\bar{p}) = \frac{\mathcal{B}(\psi' \to \chi_c\gamma)\mathcal{B}(\chi_c \to p\bar{p})}{\langle B(\psi' \to \chi_c\gamma) \rangle} \frac{1}{\langle B(\psi' \to \chi_c\gamma) \rangle}$$

(e$^+e^-$)

(A4)

$$\Gamma(\chi_c \to \gamma\gamma) = \frac{\mathcal{B}(\chi_c \to \gamma\gamma)\mathcal{B}(\chi_c \to J/\psi\gamma)}{\langle B(\psi' \to \chi_c\gamma \to J/\psi\gamma\gamma) \rangle}$$

(A5)

Not only have we a correlation between $\mathcal{B}(\chi_c \to p\bar{p})$ and $\Gamma(\chi_c \to \gamma\gamma)$, but all of the values are correlated to $\langle B(\psi' \to \chi_c\gamma) \rangle$, directly or through the $\langle B(\chi_c \to J/\psi\gamma) \rangle$.

It must be noticed that this correlation has been ignored already in the original literature, where the values of branching ratios extracted from the measurements according to eqs. (A1) and (A2) are sometimes compared to each other without mentioning it.

From eqs. (A4) and (A5) it is also clear that any systematic error in the $\langle B(\psi' \to \chi_c\gamma) \rangle$ would induce an apparent and technique-correlated disagreement between the determinations of $\mathcal{B}(\chi_c \to p\bar{p})$ and $\Gamma(\chi_c \to \gamma\gamma)$ simultaneously.

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TABLE I. Comparison of $B(\chi_c \to p\bar{p})$ as obtained by different experimental techniques

| Experimental technique | $\psi' \to \chi_c \gamma \to p\bar{p}\gamma \ (\times 10^4)$ | $p\bar{p} \to \chi_c \to J/\psi\gamma$ |
|------------------------|------------------------------------------------|----------------------------------|
| $B(\chi_{c0} \to p\bar{p})$ | $1.59 \pm 0.43 \pm 0.53$ | $4.8 \pm 0.8 \pm 0.1$ |
| $B(\chi_{c1} \to p\bar{p})$ | $0.42 \pm 0.22 \pm 0.28$ | $0.86 \pm 0.10 \pm 0.1$ |
| $B(\chi_{c2} \to p\bar{p})$ | $0.58 \pm 0.31 \pm 0.32$ | $0.97 \pm 0.44 \pm 0.08$ |

*Calculated as $\Gamma(p\bar{p})/\Gamma$ from PDG [1]

TABLE II. Comparison of $\Gamma(\chi_c \to \gamma\gamma)$ as obtained by different experimental techniques

| Experimental technique | $\psi' \to \chi_c \gamma \to \gamma\gamma \gamma\gamma \gamma\gamma$ | $p\bar{p} \to \chi_c \gamma \to \gamma\gamma$ |
|------------------------|----------------------------------------------------------------|----------------------------------|
| $\Gamma(\chi_{c0} \to \gamma\gamma)$ (KeV) | $4.0 \pm 2.8$ | $1.45 \pm 0.75 \pm 0.42$ |
| $\Gamma(\chi_{c2} \to \gamma\gamma)$ (KeV) | $0.58 \pm 0.31 \pm 0.32$ | $0.270 \pm 0.049 \pm 0.033$ |

*Calculated using $\Gamma(\chi_{c0}) = 14.9^{+2.6}_{-2.4}$ MeV from PDG [1]
| Quantity measured | Value | Ref. |
|-------------------|-------|-----|
| $\frac{B(\psi' \rightarrow \chi_{c0}\gamma)B(\chi_{c0} \rightarrow p\bar{p})}{B(\psi' \rightarrow J/\psi\pi^+\pi^-)} \times 10^5$ | $4.6\pm1.9^{ab}$ | BES [2] |
| $\frac{B(\chi_{c0} \rightarrow \gamma\gamma)}{B(\chi_{c0} \rightarrow J/\psi\gamma)}$ (%) | $1.45\pm0.74^b$ | E835 [1] |
| $\Gamma(\chi_{c0} \rightarrow p\bar{p})B(\chi_{c0} \rightarrow J/\psi\gamma)$ (eV) | $48.7\pm11.5^b$ | E835 [3] |
| $B(\psi' \rightarrow \chi_{c0}\gamma)B(\chi_{c0} \rightarrow J/\psi\gamma)$ (%) | $3.3\pm1.7$ | SPEAR [2] |
| | $0.16\pm0.11^b$ | DORIS [24] |
| | $0.4\pm0.3^{bd}$ | DASP [27] |
| | $0.069\pm0.018^{bc}$ | C.Ball [13] |
| | $0.073\pm0.024$ | Average ($s=1.3$) |
| $B(\psi' \rightarrow \chi_{c0}\gamma)B(\chi_{c0} \rightarrow \gamma\gamma) \times 10^5$ | $3.7\pm1.8\pm1.0$ | C.Ball [18] |
| $B(\psi' \rightarrow \chi_{c0}\gamma)$ (%) | $7.5\pm2.6$ | MARK I [28] |
| | $7.2\pm2.3$ | SPEAR [2] |
| | $9.9\pm0.5\pm0.8$ | C.Ball [14] |
| | $9.3\pm0.8$ | Average |
| $\Gamma(\chi_{c0})$ (MeV) | $13.5\pm3.3\pm4.2$ | C.Ball [14] |
| | $14.3\pm2.0\pm3.0$ | BES [2] |
| | $16.6^{+5.2}_{-3.7}\pm0.1$ | E835 [3] |
| | $14.9^{+2.6}_{-2.3}$ | Average |

TABLE III. Available data used in the evaluation of $\chi_{c0}$ B's.

*aUses $B(\psi' \rightarrow J/\psi\pi^+\pi^-)=0.324\pm0.026$ (see [4]) and $B(\psi' \rightarrow \chi_{c0}\gamma)=0.093\pm0.008$.

*bUsing $B(J/\psi \rightarrow e^+e^-)=0.0593\pm0.0010$ and/or $B(J/\psi \rightarrow \mu^+\mu^-)=0.0588\pm0.0010$.

*cSystematic error rescaled for the quoted $B(J/\psi \rightarrow l^+l^-)$ contribution and added in quadrature to the statistical error.

*dUses $B(J/\psi \rightarrow \mu^+\mu^-)=0.076\pm0.011$. 
| Quantity measured | Value | Ref. |
|-------------------|-------|------|
| $\frac{B(\psi' \to \chi_{c1}\gamma)B(\chi_{c1} \to p\bar{p})}{B(\psi' \to J/\psi\pi^+\pi^-)} \times 10^5$ | $1.1 \pm 0.6^b$ | BES [2] |
| $\Gamma(\chi_{c1} \to p\bar{p})B(\chi_{c1} \to J/\psi\gamma)$ (eV) | $19.9 \pm 4.4^b$ | R704 [17], E760 [16] |
| | $21.7 \pm 2.7^b$ | Average |
| | $21.2 \pm 2.3$ | |
| $B(\psi' \to \chi_{c1}\gamma)B(\chi_{c1} \to J/\psi\gamma)$ (%) | $2.8 \pm 0.9^b$ | MARK I [28], [29] |
| | $5.0 \pm 1.5^e$ | SPEAR [25] |
| | $2.9 \pm 0.5^b$ | DORIS [26] |
| | $2.2 \pm 0.5^f$ | DASP [4] |
| | $2.78 \pm 0.30^b$ | C.Ball [24] |
| | $2.56 \pm 0.12 \pm 0.20$ | C.Ball [14] |
| | $2.65 \pm 0.16$ | Average |
| $\frac{B(\psi' \to \chi_{c1}\gamma)B(\chi_{c1} \to J/\psi\gamma)}{B(\psi' \to J/\psi\pi^+\pi^-)} \times 10^2$ | $8.5 \pm 2.1^{b,d}$ | MARK II [10] |
| $B(\psi' \to \chi_{c1}\gamma)$ (%) | $7.1 \pm 1.9^e$ | SPEAR [25] |
| | $9.0 \pm 0.5 \pm 0.7$ | C.Ball [14] |
| | $8.7 \pm 0.8$ | Average |
| $\Gamma(\chi_{c1})$ (MeV) | $0.88 \pm 0.11 \pm 0.08$ | E760 [14] |

**TABLE IV.** Available data used in the evaluation of $\chi_{c1}$ $B$‘s.

- $^a$Uses $B(\psi' \to J/\psi\pi^+\pi^-)=0.324 \pm 0.026$ (see [4]) and $B(\psi' \to \chi_{c1}\gamma)=0.087 \pm 0.008$
- $^b$Using $B(J/\psi \to e^+e^-)=0.0593 \pm 0.0010$ and/or $B(J/\psi \to \mu^+\mu^-)=0.0588 \pm 0.0010$
- $^c$Systematic error rescaled for the quoted $B(J/\psi \to l^+l^-)$ contribution and added in quadrature to the statistical error
- $^d$Uses $B(\psi' \to J/\psi\pi^+\pi^-)=0.33 \pm 0.03$ (see their ref. [10])
- $^e$Assumes isotropic $\gamma$ distribution
- $^f$Uses $B(J/\psi \to \mu^+\mu^-)=0.076 \pm 0.011$
| Quantity measured | Value | Ref. |
|--------------------|-------|------|
| $\frac{B(\psi' \to \chi_{c2} \gamma)B(\chi_{c2} \to pp)}{B(\psi' \to J/\psi \pi^+ \pi^-)} \times 10^5$ | 1.4±1.1$^{ab}$ | BES [2] |
| $\frac{B(\chi_{c2} \to \gamma \gamma)}{B(\chi_{c2} \to J/\psi \gamma)} \times 10^3$ | 0.99±0.18$^b$ | E835 [1] |
| $B(\chi_{c2} \to pp)B(\chi_{c2} \to \gamma \gamma) \times 10^8$ | 9.9±4.5 | R704 [22] |
| | 1.60±0.42 | E760 [20] |
| | **1.67 ± 0.77** | Average ($s=1.8$) |
| $\Gamma(\chi_{c2} \to pp)B(\chi_{c2} \to J/\psi \gamma)$ (eV) | 36±8$^b$ | R704 [17] |
| | 28.2±2.6$^b$ | E760 [16] |
| | **28.9±2.5** | Average ($s=1.8$) |
| $B(\psi' \to \chi_{c2} \gamma)B(\chi_{c2} \to J/\psi \gamma)$ (%) | 1.2±0.7$^b$ | MARK I [28] |
| | 2.2±1.2$^c$ | SPEAR [25] |
| | 1.2±0.2$^b$ | DORIS [20] |
| | 1.8±0.5$^b$ | DASP [27] |
| | 1.47±0.17$^{bc}$ | C.Ball [18] |
| | 0.99±0.10±0.08 | C.Ball [14] |
| | **1.20±0.11** | Average ($s=1.2$) |
| $\Gamma(\chi_{c2} \to \gamma \gamma)B(\chi_{c2} \to J/\psi \gamma)$ (eV) | 139±55±23$^{bg}$ | L3 [13] |
| | 240±64±55$^{bg}$ | OPAL [21] |
| | 150±42±36$^{bg}$ | CLEO2 [23] |
| | 470±240±120$^{bg}$ | TPC/2$\gamma$ [24] |
| | **168±36** | Average |
| $\frac{B(\psi' \to \chi_{c2} \gamma)B(\chi_{c2} \to J/\psi \gamma)}{B(\psi' \to J/\psi \pi^+ \pi^-)} \times 10^2$ | 3.9±1.2$^{bd}$ | MARK II [10] |
| $B(\psi' \to \chi_{c2} \gamma)B(\chi_{c2} \to \gamma \gamma) \times 10^5$ | 7.0±2.1±2.0 | C.Ball [18] |
| $B(\psi' \to \chi_{c2} \gamma)$ (%) | 7.0±2.0$^e$ | SPEAR [25] |
| | 8.0±0.5±0.7 | C.Ball [14] |
| | **7.8±0.8** | Average |
| $\Gamma(\chi_{c2})$ (MeV) | 1.98±0.17±0.07 | E760 [16] |
| | 2.6$^{+1.4}_{-1.1}$ | R704 [17] |
| | 2.4$^{+1.0}_{-0.9}$ | C.Ball [14] |
| | **2.0±0.18** | Average |

TABLE V. Available data used in the evaluation of $\chi_{c2}$ $B$’s.

$^a$Uses $B(\psi' \to J/\psi \pi^+ \pi^-)=0.324±0.026$ (see [1]) and $B(\psi' \to \chi_{c2} \gamma)=0.078±0.008$

$^b$Using $B(J/\psi \to e^+ e^-)=0.0593±0.0010$ and/or $B(J/\psi \to \mu^+ \mu^-)=0.0588±0.0010$

$^c$Systematic error rescaled for the quoted $B(J/\psi \to l^+ l^-)$ contribution and added in quadrature to the statistical error

$^d$Uses $B(\psi' \to J/\psi \pi^+ \pi^-)=0.33±0.03$ (see their ref. [10])

$^e$Assumes isotropic $\gamma$ distribution

$^f$Uses $B(J/\psi \to \mu^+ \mu^-)=0.076±0.011$

$^g$Uses $B(\chi_{c2} \to J/\psi \gamma)=0.135±0.011$
| Quantity measured | Value       | Ref.          |
|-------------------|-------------|---------------|
| \(B(\psi' \to J/\psi X)\) | 0.51±0.12   | DASP [24]     |
|                   | 0.57±0.08   | MARK I [13]   |
|                   | **0.55±0.07** | Average     |
| \(B(\psi' \to J/\psi \pi^+ \pi^-)\) | 0.32±0.04   | MARK I [13]   |
| \(B(\psi' \to J/\psi \eta)\) | 0.042±0.006^a | DORIS [28]     |
|                   | 0.045±0.012^ab | DASP [29]     |
|                   | 0.0255±0.0029^c | C.Ball [31]   |
|                   | **0.0294±0.0051** | Average (s=2.0) |
| \(B(\psi' \to \chi_{c1}\gamma \to J/\psi \gamma \gamma)\) | 0.028±0.009^a | MARK I [28]     |
|                   | 0.050±0.015^d | SPEAR [29]     |
|                   | 0.020±0.005^* | DORIS [28]     |
|                   | 0.022±0.005^ab | DASP [27]     |
|                   | 0.0278±0.0030^c | C.Ball [31]   |
|                   | **0.0256±0.0012±0.0020** | Average |
| \(B(\psi' \to \chi_{c2}\gamma \to J/\psi \gamma \gamma)\) | 0.012±0.007^a | MARK I [28]     |
|                   | 0.022±0.012^d | SPEAR [29]     |
|                   | 0.012±0.002^* | DORIS [28]     |
|                   | 0.018±0.005^ab | DASP [27]     |
|                   | 0.0147±0.0017^c | C.Ball [31]   |
|                   | **0.0099±0.0010±0.0008** | Average (s=1.2) |
| \(B(\psi' \to J/\psi \pi^+ \pi^-)\) | 0.496±0.037f | E-760 [6]     |
| \(B(\psi' \to J/\psi X)\) | 0.73±0.09 | MARK I [32]   |
| \(B(\psi' \to J/\psi \pi^0 \pi^-)\) | 0.323±0.033f | E-760 [6]     |
| \(B(\psi' \to J/\psi \pi^+ \pi^-)\) | 0.328±0.015  | E-835 [4]     |
|                   | **0.327±0.014** | Average     |
| \(B(\psi' \to J/\psi \eta)\) | 0.091±0.021^ac | MARK II [10]  |
| \(B(\psi' \to J/\psi \pi^+ \pi^-)\) | 0.069±0.008 | Average     |
| \(B(\psi' \to \chi_{c1}\gamma \to J/\psi \gamma \gamma)\) | 0.085±0.021^ac | MARK II [10]  |
| \(B(\psi' \to \chi_{c2}\gamma \to J/\psi \gamma \gamma)\) | 0.039±0.012^ac | MARK II [10]  |

**TABLE VI.** Available data used in the evaluation of \(\psi'\) \(B\)'s

- \(^a\)Using \(B(J/\psi \to e^+e^-) = 0.0593±0.0010\) and/or \(B(J/\psi \to \mu^+\mu^-) = 0.0588±0.0010\)
- \(^b\)Uses \(B(J/\psi \to \mu^+\mu^-) = 0.076±0.011\)
- \(^c\)Systematic error rescaled for the quoted \(B(J/\psi \to \ell^+\ell^-)\) contribution and added in quadrature to the statistical error.
- \(^d\)Assumes isotropic \(\gamma\) distribution
- \(^e\)Uses \(B(\psi' \to J/\psi \pi^+ \pi^-) = 0.33±0.03\) (see their ref. [10])
TABLE VII. \( \psi' \) and \( \chi_c \) B's as determined from the fits described in the text. The second column gives the results which would be obtained by ignoring the correlation to \( \mathcal{B}(\psi' \rightarrow J/\psi \pi^+ \pi^-) \) and performing a separate fit for each one of the \( \chi_c \) states

\( ^a \) Calculated as \( \mathcal{B}(\psi' \rightarrow J/\psi \pi^+ \pi^-) + \mathcal{B}(\psi' \rightarrow J/\psi \pi^0 \pi^0) + \mathcal{B}(\psi' \rightarrow J/\psi \eta) + \mathcal{B}(\psi' \rightarrow \chi_{c1} \gamma) + \mathcal{B}(\psi' \rightarrow \chi_{c1} \gamma) + \mathcal{B}(\psi' \rightarrow \chi_{c2} \gamma) \)

\( ^b \) The second error refers to the uncertainty in the \( \mathcal{B}(\psi' \rightarrow \chi_{c1} \gamma) \)

| Parameter | \( \mathcal{B}(\psi' \rightarrow \chi) \) | \( \mathcal{B}(\psi' \rightarrow J/\psi \pi^+ \pi^-) \) | \( \Gamma \) fixed | New fit | PDG |
|-----------|----------------------------------|----------------------------------|----------------|----------|-----|
| \( \Gamma(\chi'_{c0}) \) (MeV) | 16.3±2.3 | 1.0±0.2±0.1 | 1.0±0.2±0.1 | 1.0±0.2±0.1 | 1.0±0.2±0.1 | 1.0±0.2±0.1 |
| \( \Gamma(\chi'_{c1}) \) (MeV) | 9.2±0.13 | 9.2±0.13 | 9.2±0.13 | 9.2±0.13 | 9.2±0.13 | 9.2±0.13 |
| \( \Gamma(\chi'_{c2}) \) (MeV) | 2.07±0.17 | 2.07±0.17 | 2.07±0.17 | 2.07±0.17 | 2.07±0.17 | 2.07±0.17 |
| \( \mathcal{B}(\psi' \rightarrow J/\psi X \%) \) | 56±4 | 56±4 | 56±4 | 56±4 | 56±4 | 56±4 |

| \( \mathcal{B}(\psi' \rightarrow J/\psi \gamma \gamma \%) \) | 1.0±0.2±0.1 | 1.0±0.2±0.1 | 1.0±0.2±0.1 | 1.0±0.2±0.1 | 1.0±0.2±0.1 | 1.0±0.2±0.1 |
| \( \mathcal{B}(\psi' \rightarrow \chi'_{c0} \gamma \%) \) | 9.2±0.13 | 9.2±0.13 | 9.2±0.13 | 9.2±0.13 | 9.2±0.13 | 9.2±0.13 |
| \( \mathcal{B}(\psi' \rightarrow \chi'_{c0} \gamma \%) \) | 2.07±0.17 | 2.07±0.17 | 2.07±0.17 | 2.07±0.17 | 2.07±0.17 | 2.07±0.17 |
| \( \mathcal{B}(\psi' \rightarrow J/\psi \gamma \gamma \%) \) | 56±4 | 56±4 | 56±4 | 56±4 | 56±4 | 56±4 |

\( \chi^2/N_{DOF} \) | 58.6/42 | 9.1/9 (\( \chi_{c0} \)) | 55.9/42 | 56/46 |

\( \chi^2/N_{DOF} \) | 5.7/9 (\( \chi_{c1} \)) | 26.5/18 (\( \chi_{c2} \)) |
| Transition         | PDG   | This fit | Predicted |
|-------------------|-------|----------|-----------|
| \(\psi' \to \chi_{c1} \gamma\) | 24±3  | 23±4     | 20±3      |
| \(\psi' \to \chi_{c2} \gamma\) | 21±3  | 19±3     | 14±3      |
| \(\chi_{c1} \to J/\psi \gamma\) | 240±44 | 282±52   | 273±3     |
| \(\chi_{c2} \to J/\psi \gamma\) | 270±43 | 374±62   | 347±3     |
| \(\chi_{c1} \to \gamma \gamma\) | 1.9±1.0b | 3.0±1.5 | 3.72±0.11 |
| \(\chi_{c2} \to \gamma \gamma\) | 0.32±0.09b | 0.44±0.09 | 0.49±0.15 |

**TABLE VIII.** Comparison of partial widths with some theoretical expectations. All values are in KeV.

- \(^{a}\)Calculated using the \(\chi_c\) branching ratios which would be obtained following the PDG procedure with the new value of \(B(J/\psi \to l^+l^-)\).
- \(^{b}\)Using also E835 [11] result in the average