8D Parameters Estimation for Bistatic EMVS-MIMO Radar via the nested PARAFAC

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Abstract—In this letter, a novel nested PARAFAC algorithm was proposed to improve the 8D parameters estimation performance for the bistatic EMVS-MIMO radar. Firstly, the outer part PARAFAC algorithm was carried out to estimate the receive spatial response matrix and its first way factor matrix. For the estimated first way factor matrix, a theory is given to rearrange its data into an new matrix, which is the mode-

1 unfolding matrix of a three-way tensor. Then, the inner part PARAFAC algorithm was used to estimate the transmit steering vector matrix, the transmit spatial response matrix and the receive steering vector matrix. Thus, the transmit 4D parameters and receive 4D parameters can be accurately located via the abovementioned process. Compared with the original PARAFAC algorithm, the proposed nested PARAFAC algorithm can avoid additional reconstruction process when estimating the transmit/receive spatial response matrix. Moreover, the proposed algorithm can offer a highly-accurate 8D parameters estimation than that of the original PARAFAC algorithm. Simulated results verify the effectiveness of the proposed algorithm.

Index Terms—Bistatic EMVS-MIMO radar, nested PARAFAC algorithm, tensor model, parameters estimation

I. INTRODUCTION

RECENTLY, the research about the electromagnetic vector sensors (EMVS) has attracted extensive attention due to its excellent measurement capabilities of angle parameters and polarization parameters [1]-[5]. In order to estimate the polarization parameters in bistatic EMVS-MIMO radar, the ESPRIT algorithm was firstly proposed in [6]. The significance of the ESPRIT algorithm is to derive a method to reconstruct the transmit/receive spatial response matrix, but, its computational complexity is large. In order to reduce the computational complexity, the PM algorithm was proposed in [7], which can estimate the signal subspace matrix by avoiding the singular value decomposition process of the covariance matrix. To make full use of the space-time multidimensional structure of the array received data, the HOSVD algorithm based on the fourth-order tensor was raised in [8]. However, the algorithms reported in [6]-[8] need the pair matching process for the 2D-DOD and 2D-DOA estimation. Therefore, a modified PM algorithm in [9] and PARAFAC algorithm in [10] were proposed to realize the 2D-DOD and 2D-DOA automatically paired. In [11], the bistatic coprime EMVS-MIMO radar was designed to improve the 2D-DOD and 2D-DOA estimation performance.

Through analysis, it can be found that the above mentioned algorithms can be used to estimate the 8D parameters in bistatic EMVS-MIMO radar. Amongst, the HOSVD algorithm can offer the best estimation performance, but its shortcoming is the high computational load. The modified PM algorithm and the PARAFAC algorithm can realize the 2D-DOD and 2D-DOA automatically paired. But, the estimation performance of them are relatively poor. According to [12]-[13], the nested PARAFAC algorithm can improve the estimation performance of the angle parameter, but, it cannot be directly used in the bistatic EMVS-MIMO radar due to the special structure of the receive data. Motivated by the strategy in [12]-[13], we proposed a new nested PARAFAC algorithm. Firstly, the outer part PARAFAC was conducted to estimate the receive spatial response and the first way factor matrix. Then, the inner part three way tensor can be constructed by rearranging the estimated first way factor matrix, which can be used to estimate transmit steering vector matrix, transmit spatial response matrix and the receive steering vector matrix. Compared with the HOSVD algorithm, the raised new nested PARAFAC algorithm has lower computational complexity. And this method presents a higher estimation accuracy than the original PARAFAC algorithm as well as the modified PM algorithm.

II. DATA MODEL

As shown in Fig. 1 for a bistatic EMVS-MIMO radar system consists of $M$ transmit EMVS and $N$ receive EMVS, the transmit steering vector and receive steering vector can be expressed as

$$c_{tk} = a_{tk} \otimes q_{tk} (\theta_{tk}, \phi_{tk}, \gamma_{tk}, \eta_{tk})$$

Fig. 1. Bistatic EMVS-MIMO radar system
\[ c_{rk} = a_{rk} \otimes q_{rk} (\theta_{rk}, \phi_{rk}, \gamma_{rk}, \eta_{rk}) \]  

where \( k = 1, 2, \ldots, K \) denote the number of targets, \( \theta_{rk}, \phi_{rk} \in [0, \pi), \phi_{rk}, \gamma_{rk} \in [0, 2\pi), \gamma_{rk}, \eta_{rk} \in [0, \pi/2) \) and \( \eta_{rk}, \gamma_{rk} \in [-\pi, \pi) \) denote the transmit/receive elevation angle, transmit/receive azimuth angle, transmit/receive polarization angle and transmit/receive polarization phase difference, respectively. \( q_{rk} = (\theta_{rk}, \phi_{rk}, \gamma_{rk}, \eta_{rk}) \) represent the spatial response of the transmit EMVS and the receive EMVS of the \( k \)-th target. \( a_{rk} = [e^{-j\pi \sin(\theta_{rk})}, e^{-j2\pi \sin(\theta_{rk})}, \ldots, e^{-jM \pi \sin(\theta_{rk})}]^T \), \( a_{rk} = [e^{-j\pi \sin(\theta_{rk})}, e^{-j2\pi \sin(\theta_{rk})}, \ldots, e^{-jN \pi \sin(\theta_{rk})}]^T \). According to (1), the spatial response \( q \) of an EMVS can be expressed as

\[
q(\theta, \phi, \gamma, \eta) = 
\begin{bmatrix}
\cos(\phi) \cos(\theta) & -\sin(\phi) \\
\sin(\phi) \cos(\theta) & \cos(\phi) \\
-\sin(\theta) & 0 \\
-\cos(\phi) \cos(\theta) & \cos(\phi) \\
0 & \sin(\theta)
\end{bmatrix}
\begin{bmatrix}
\sin(\gamma) e^{j\eta} \\
\cos(\gamma)
\end{bmatrix} 
\]

where \( F(\theta, \phi) \in \mathbb{C}^{6 \times 2} \) denotes the spatial angular location matrix, \( g(\gamma, \eta) \in \mathbb{C}^{2 \times 1} \) denotes the polarization states vector.

The data model at the receive front can be expressed as

\[
y(t) = [(A_t \otimes Q_t) \odot (A_r \otimes Q_r)] s(t) + n(t) 
\]

where \( A_t = [a_{t1}, a_{t2}, \ldots, a_{tK}] \), \( A_r = [a_{r1}, a_{r2}, \ldots, a_{rK}] \), \( Q_t = [q_{t1}, q_{t2}, \ldots, q_{tK}] \) and \( Q_r = [q_{r1}, q_{r2}, \ldots, q_{rK}] \) denote the transmit steering matrix, the receive steering matrix, the transmit spatial response matrix and the receive spatial response matrix, respectively. For the total \( L \) snapshots, the sample data can be further expressed written as

\[
Y = [(A_t \otimes Q_t) \odot (A_r \otimes Q_r)] S + N
\]

III. THE PROPOSED STRATEGY

A. The nested PARAFAC algorithm

In order to reserve the multiple space-time structure in \( Y \), a nested-tensor signal model are designed as follows

\[
Y = [(A_t \otimes Q_t) \odot (A_r \otimes Q_r)] S + N
\]

The different slices of the third-order tensor \( Y \) can be further expressed as

\[
Y_i^{[i,i]} = Q_t^D_i (S^T) A_{tqr}^T, \quad i = 1, 2, \ldots, L \\
Y_j^{[j,j]} = S^T D_j (A_{tqr}) Q_r^T, \quad j = 1, 2, \ldots, 6MN
\]

where \( D_i, D_j, D_k \) represent the diagonal matrix operation, and the elements on the diagonal matrices are the \( k \)-rows of the factor matrices \( Q_r, A_{tqr}, \) and \( S \), respectively.

In order to realize the estimation of the factor matrices \( Q_r, A_{tqr}, \) and \( S \), the trilinear alternating least squares algorithm is used

\[
\min_{\hat{A}_{tqr}} ||Y_i^{[i,i]} - (Q_r \odot S^T) A_{tqr}^T||^2_F \\
\min_{\hat{Q}_r} ||Y_j^{[j,j]} - (S^T \odot A_{tqr}) Q_r^T||^2_F \\
\min_{\hat{S}} ||Y_j^{[j,j]} - (A_{tqr} \odot Q_r) S||^2_F
\]

Then, let \( \hat{S}, \hat{Q}_r, \) and \( \hat{A}_{tqr} \) denote the estimated factor matrices, respectively

\[
\hat{A}_{tqr}^T = (Q_r \otimes S^T)^T \hat{Y}_i^{[i,i]} \\
\hat{Q}_r^T = (S^T \otimes A_{tqr})^T \hat{Y}_j^{[j,j]} \\
\hat{S} = (A_{tqr} \otimes \hat{Q}_r)^T \hat{Y}_j^{[j,j]}
\]

It can be found that the factor matrices \( \hat{Q}_r \) and \( \hat{A}_{tqr} \) can be obtained. And, the receive elevation angle, receive azimuth angle, receive polarization angle and receive polarization phase difference can be estimated by performing the vector-cross-product operation on \( \hat{Q}_r \).

Then, in order to estimate the transmit 4D parameters, the inter part in \( \hat{A}_{tqr} \) needs to be considered. It is invalid to direct to perform inner-part PARAFAC on estimated \( \hat{A}_{tqr} \). Under this circumstance, a new three way tensor should be constructed as follows

\[
Y_1 = \sum_{k=1}^{K} A_r \otimes q_r \otimes A_t
\]

However, the factor matrix \( \hat{A}_{tqr} \neq Y_1^{[i]} \) is not true. Thus, the conversion of the factor matrix \( \hat{A}_{tqr} \) into the mode-i, \( i = 1, 2, 3 \) unfolding of the tensor \( Y_1 \) is necessary. By analysis, the relationship \( \hat{A}_{tqr} \) and \( Y_1^{[i]} \) is considered. The detailed form of the mode-1 unfolding of the tensor \( Y_1 \) is

\[
\hat{A}_{tqr}^T = [Y_1^{[1,i]}] = [Q_t \otimes A_t] A_r^T \in \mathbb{C}^{6M \times N}
\]

Here, we give an theory to show how to rearrange the factor matrix \( \hat{A}_{tqr} \) into the matrix \( \widehat{A}_{tqr} \).

**Theorem 1** The matrix \( \widehat{A}_{tqr} \in \mathbb{C}^{6M \times N} \) is equal to the inverse matrix vectorization by row of the vector \( \sum_{k=1}^{K} A_{tqr}(:,k) \in \mathbb{C}^{6M \times 1} \), namely

\[
\widehat{A}_{tqr} = vec \sum_{k=1}^{K} A_{tqr}(:,k)
\]

where vec denotes the inverse matrix vectorization by row.
Proof: The conclusion is a direct result of the following equalities
\[
\sum_{k=1}^{K} A_{tqr}(;k) \iff \sum_{k=1}^{K} (A_t(;k) \otimes Q_t(;k) \otimes A_r(;k)) \\
\iff \sum_{k=1}^{K} (a_{tk} \otimes q_{tk} \otimes a_{rk}) \\
\iff \sum_{k=1}^{K} vec(a_{rk}(a_{tk} \otimes q_{tk})^T) \\
\iff vec(A_t (A_t \otimes Q_t)^T) \\
\iff vec((Q_t \otimes A_t)^T) \\
\iff vec(A_{tqr})^T
\]
where the \( \iff \) is satisfied by using the relationship between\( a \otimes b = a \odot b = vec(ba^T) \). Let \( B = Q_t \otimes A_t = [b_1, b_2, \ldots, b_K] \), the \( \iff \) is equivalent by the following analysis
\[
vec(A_tB^T) \iff vec\left([a_{t1}, a_{t2}, \ldots, a_{tK}] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix}\right) \\
\iff vec(a_{t1}b_1 + a_{t2}b_2 + \ldots + a_{tK}b_K) \\
\iff \sum_{k=1}^{K} vec(a_{tk}b_k) \\
\iff \sum_{k=1}^{K} vec(a_{rk}(a_{tk} \otimes q_{tk})^T)
\]
Consequently, the matrix \( \tilde{A}_{tqr} \) can be obtained by using the above theory. Subsequently, the inter part three-way tensor \( Y_1 \) can be constructed as
\[
Y_1 = reshape(\tilde{A}_{tqr}, [N, 6, M]) = \sum_{k=1}^{K} A_t \circ q_t \circ A_t
\]
Therefore, the PARAFAC algorithm can be used again to estimate the factor matrices \( \tilde{Q}_t, \tilde{A}_t \) and \( \tilde{A}_r \). The derivation process is similar to \( [8][10] \). Thus, through the utilization of the outer part PARAFAC and inner part PARAFAC, the factor matrices \( \tilde{Q}_t, \tilde{Q}_t, \tilde{A}_t \) and \( \tilde{A}_r \) can be obtained.

B. Estimation of the angle and polarization parameters

For the estimated \( \tilde{Q}_t, \tilde{Q}_t, \tilde{A}_t \) and \( \tilde{A}_r \), the corresponding transmit 4-D parameters \( (\theta_r, \varphi_r, \gamma_r, \eta_r) \) and the receive 4-D parameters \( (\theta_r, \varphi_r, \gamma_r, \eta_r) \) can be attained. As the derivation is similar, only the derivation for \( (\theta_r, \varphi_r, \gamma_r, \eta_r) \) is provided here. For estimated \( \tilde{A}_r \) via the inner part PARAFAC algorithm, the following selection matrices can be constructed
\[
\begin{cases}
J_{r1} = [I_{N-1} \ 0_{(N-1)\times1}] \\
J_{r2} = [0_{(N-1)\times1} \ I_{N-1}]
\end{cases}
\]
Then, the following rotation-invariant relationship for \( \tilde{A}_r \) can be obtained as
\[
J_{r1} \tilde{A}_r \Phi(\theta_r) = J_{r2} \tilde{A}_r
\]
where
\[
\Phi(\theta_r) = \begin{bmatrix}
e^{j \pi \sin(\theta_{r1})} \\
\vdots \\
e^{j \pi \sin(\theta_{rK})}
\end{bmatrix}
\]
Resultantly, the estimation of \( \Phi(\theta_r) \) can be achieved by employing the least squares
\[
\hat{\Phi}(\theta_r) = (J_1 \tilde{A}_r) ^T J_2 \tilde{A}_r
\]
Furthermore, by performing the singular value decomposition on \( \tilde{\Phi}(\theta_r) \), the corresponding eigenvalues \( \lambda_1, \lambda_2, \ldots, \lambda_K \) can be attained. So, the receive elevation angle \( \theta_{rk}, k = 1, 2, \ldots, K \) can be estimated as
\[
\hat{\theta}_{rk} = \arcsin(\frac{\lambda_k}{\pi}) \quad k = 1, 2, \ldots, K
\]
Let \( \bar{\theta} = [\hat{\theta}_{r1}, \hat{\theta}_{r2}, \ldots, \hat{\theta}_{rK}] \). For the estimated \( \tilde{Q}_t \), through by using the outer part PARAFAC algorithm, the detailed derivation process for \( (\hat{\theta}_r, \hat{\phi}_r, \hat{\gamma}_r, \hat{\eta}_r) \) can be conducted through the following process.

The normalized Poynting vector of \( \tilde{Q}_t \) is
\[
\begin{bmatrix}
\bar{u}_{rk} \\
\bar{v}_{rk} \\
\bar{w}_{rk}
\end{bmatrix} \triangleq \frac{e_{rk}}{|e_{rk}|} \times \frac{h_{rk}^*}{|h_{rk}|} = \begin{bmatrix}
sin(\theta_{rk}) \cos(\phi_{rk}) \\
\sin(\theta_{rk}) \sin(\phi_{rk}) \\
\cos(\theta_{rk})
\end{bmatrix}
\]
Therefore, the estimated transmit elevation angle and azimuth angle \( (\hat{\theta}_{rk}, \hat{\phi}_{rk}), k = 1, 2, \ldots, K \) can be expressed as
\[
\begin{bmatrix}
\hat{\theta}_{rk} = \arctan(\frac{\bar{v}_{rk}}{\bar{u}_{rk}}) \\
\hat{\phi}_{rk} = \arcsin(\sqrt{(\bar{u}_{rk})^2 + (\bar{v}_{rk})^2})
\end{bmatrix}
\]
After obtaining \( (\hat{\theta}_{rk}, \hat{\phi}_{rk}) \), the corresponding receive polarization state vector \( g_{rk}(\gamma_{rk}, \eta_{rk}) \), \( k = 1, 2, \ldots, K \) can be expressed as
\[
g_{rk}(\gamma_{rk}, \eta_{rk}) = [g_{1rk} \ g_{2rk}] = [F(\hat{\theta}_{rk}, \hat{\phi}_{rk})]^T \tilde{Q}_t
\]
Moreover, the polarization parameters \( (\hat{\gamma}_{rk}, \hat{\eta}_{rk}), k = 1, 2, \ldots, K \) of the receive EMVS can be expressed as
\[
\begin{bmatrix}
\hat{\gamma}_{rk} = \arctan(\frac{g_{1rk}}{g_{2rk}}) \\
\hat{\eta}_{rk} = \angle g_{rk}
\end{bmatrix}, k = 1, 2, \ldots, K
\]
Finally, the receive 4-D parameters \( (\theta_r, \phi_r, \gamma_r, \eta_r) \) and the transmit 4-D parameters \( (\theta_t, \phi_t, \gamma_t, \eta_t) \) can be attained by the abovementioned process.
C. Some Remarks

Remark 1: The transmit 4-D parameters \((\theta_t, \phi_t, \gamma_t, \eta_t)\) are automatically paired, since the estimated \(\hat{Q}_t\) and \(\hat{A}_t\) are corresponding to each other via the employment of the inner PARAFAC algorithm. But, the estimated \(\hat{Q}_r\) by outer PARAFAC algorithm and \(\hat{A}_r\) by inner PARAFAC algorithm are not one to one correspondence. Thus, the pairing process for \(\hat{\theta}\) estimated by \(\hat{A}_r\) and \((\hat{\theta}_r, \hat{\phi}_r, \hat{\gamma}_r, \hat{\eta}_r)\) estimated by \(\hat{Q}_r\) is inevitable. The detailed pairing process can be carried out according to Algorithm 1.

Algorithm 1 pseudocode for pairing process

Input: \(\hat{\theta}\) estimated via \(\hat{A}_r\), \(\hat{\theta}\) estimated via \(\hat{Q}_r\)
Output: pair matching \((\hat{\theta}_t, \hat{\phi}_r, \hat{\gamma}_r, \hat{\eta}_r)\)
1: \(\text{Index} = \text{zeros}(1, K)\)
2: for \(i = 1 : K\)
3: for \(j = 1 : K\)
4: \(\text{Index}(i) = \text{argmin}_j |\hat{\theta}(i) - \hat{\theta}(j)|\)
5: \(\hat{\theta}_1 = \hat{\theta}[\text{Index}]\)
6: return \((\hat{\theta}_1, \hat{\phi}_r, \hat{\gamma}_r, \hat{\eta}_r)\)

Afterwards, the pair matching receive 4-D parameters \((\hat{\theta}_1, \hat{\phi}_r, \hat{\gamma}_r, \hat{\eta}_r)\) can be achieved, in which case, the transmit 4-D parameters and the receive 4-D parameters are also paired.

Remark 2: Based on the Kruskal’s condition in [14]-[17], the uniqueness of the outer and inner PARAFAC decomposition can be guaranteed by

\[
\begin{align*}
2K + 2 &\leq \kappa_{A_{tqr}} + \kappa_{Q_r} + \kappa_S \\
2K + 2 &\leq \kappa_{A_t} + \kappa_{Q_t} + \kappa_{A_r}
\end{align*}
\]

And, according to (18), \(K\) also relies on the rotation invariant relationship when estimating the transmit/receive elevation angle, namely

\[
\begin{align*}
K &\leq M - 1 \\
K &\leq N - 1
\end{align*}
\]

Thus, the maximum resolvable targets \(K = \min[M - 1, N - 1]\).

Remark 3: The computational complexity of the proposed nested-PARAFAC algorithm is mainly determined by the outer alternating ALS and inter alternating ALS. Thus, the total computational complexity is \(O(\kappa [(6MN + M + N + L + 12) K^2])\).

IV. SIMULATION RESULTS

Here, the RMSEs performance of different algorithms versus SNR are compared. The ESPRIT algorithm in [6], the PM algorithm in [7], the Tensor subspace-based algorithm in [8] and the PARAFAC algorithm in [10] are considered for comparison. The average root mean square error is calculated as \(RMSE = \sqrt{\frac{1}{MNP} \sum_{m=1}^{M} \sum_{p=1}^{P} ||\alpha - \alpha^*||^2}\), where \(\alpha\) and \(\alpha^*\) denote the true angle parameters and estimated angle parameters, respectively. The number of transmit MEVS and receive EMVS are set to 9 and 10, respectively. Assume that there are \(K = 3\) far-field targets with \(\theta_t = [40^\circ, 20^\circ, 30^\circ]\), \(\phi_t = [15^\circ, 25^\circ, 35^\circ]\), \(\gamma_t = [10^\circ, 22^\circ, 35^\circ]\), \(\eta_t = [38^\circ, 48^\circ, 56^\circ]\), \(\theta_r = [24^\circ, 38^\circ, 16^\circ]\), \(\phi_r = [21^\circ, 32^\circ, 55^\circ]\), \(\gamma_r = [42^\circ, 33^\circ, 60^\circ]\), \(\eta_r = [17^\circ, 27^\circ, 39^\circ]\), respectively. The SNR increases from 0dB to 20dB and the snapshot is set to 200 for different SNRs. The suffix ‘-d’ and ‘-p’ in the legend refer to angle parameter and polarization parameter, respectively. Fig. 2 shows that the proposed algorithm exhibits an excellent 8D parameters estimation performance compared with the state-of-art algorithms on the conditions of different SNRs.

V. CONCLUSION

In this letter, a high accuracy estimation for the transmit 4D parameters and the receive 4D parameters in the bistatic EMVS-MIMO radar has been investigated. By utilizing the tensor structure, the array receive data are rearranged into an outer part three way tensor models and an inner part three way tensor models, which are resolved by the proposed nested PARAFAC algorithm. The proposed nested PARAFAC algorithm can effectively solve the transmit/receive elevation angle, transmit/receive azimuth angle, transmit/ receive polarization angle and transmit/ receive polarization phase difference. More significant is that the proposed algorithm provides a new strategy to deal with the parameters estimation problem for other signal models similar to the one in this letter. In the upcoming studies, we will consider how to apply the proposed nested PARAFAC algorithm to the bistatic EMVS-MIMO radar with sparse transmit EMVS and sparse receive EMVS.
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