Comment on ”Collective excitations of a degenerate gas at the BEC-BCS crossover”

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Very recent experiments have studied for the first time collective excitations of an ultracold $^6$Li gas [1,2], covering in particular the BEC-BCS crossover domain [2]. We wish to point out that the results for the axial mode, through hydrodynamics, give direct access to the (3D) equation of state of the strongly interacting gas, mostly near the unitarity limit. On the other hand the surprising results found for the radial mode are actually not necessarily in contradiction with the expectations from superfluid hydrodynamics.

Indeed the radial mode frequencies $\Omega_r$ are near [1] $2\pi 2400$ Hz and [2] $2\pi 1200$ Hz. The frequency $\omega_F = E_F/\hbar$ corresponding to the Fermi energy is [2] $1.6 \times 10^5$ Hz. If on the BCS side we estimate the gap by $\hbar \Omega_r/\Delta \sim 0.13$ at the trap center, this ratio increasing when we go away from the center. This is large enough to make questionable hydrodynamics, which assumes $\hbar \Omega_r/\Delta \ll 1$ in order to be accurate, and may explain the 10% discrepancy between theory and experiment at unitarity. Similarly one can estimate that the ratio $\xi/l_c$ between the Cooper pair size and the transverse size of the trap is at best given by this same figure $\sim 0.1$, which makes doubtful the accuracy of the local density approximation (a necessary ingredient in the hydrodynamic result). The situation in Ref. [1] is even worse. This is also consistent with a natural superfluid interpretation of the strong attenuation [2] at 910 G as a pair-breaking peak corresponding to $\hbar \Omega_r = 2\Delta(T, B)$, occurring because $T_c$ and $\Delta$ decrease with increasing field $B$. Clearly in this case $\hbar \Omega_r/\Delta$ is no longer small. Taking for example $\Delta(T, B) \approx T_c$ (the gap depends on $T/T_c$, which is not known) gives $T_c/E_F \sim 0.02$, coherent with the estimated temperature $T$ in this experiment.

On the other hand the axial mode is a very good case for making use of the hydrodynamic limit. Indeed its frequency is very low $\hbar \Omega_a/\Delta \sim 5 \times 10^{-3}$, which is the appropriate range for this approximation. Moreover the experimental temperature is certainly quite low, as confirmed by the very low damping found in most of the magnetic field range. This makes it possible to neglect the effect of dissipation on the frequency of the modes. Finally for the very elongated traps used in experiments, one has to deal with a simple effective one-dimensional problem. In this case, when the chemical potential $\mu(n)$ of the (3D) gas is a power law of the total particle density $n^{1/3}$, the axial mode frequency is given by an exact analytical result [3,4] $\omega^2/\omega_a^2 = 2 + 1/(p + 1)$. In the present situation this case is found in the BEC limit (small positive scattering length $a$), with $p = 1$, leading to $\omega^2/\omega_a^2 = 5/2$, in the BCS limit (small negative scattering length $a$), with $p = 3/2$ and $\omega^2/\omega_a^2 = 12/5$. Moreover quite remarkably [5] this same value $p = 3/2$ applies also in the unitarity limit, where $a$ is very large. These values are in fair agreement with experiment [2] for the BEC and the unitarity cases.

We have shown recently [6] how it is possible to invert such experimental data to obtain the equation of state of the gas. This requires only that $\mu(n)$ is known in some limiting case, from which one then go away by an iterative procedure, making use of the experimental knowledge of the mode frequency as a function of density. The basic ingredient of this method has been shown recently [7] to have an accuracy of order $10^{-3}$. In the present case we have in principle the choice between the three limiting cases mentioned above, the most convenient one being the unitarity limit. However there are not enough data to carry out the above program with a sensible precision. Nevertheless we can analyze the region in the vicinity of the unitarity limit, where a fairly linear behaviour is obtained experimentally. In this case we have the simpler problem of performing a perturbative calculation [7].

![FIG. 1. Reduced axial mode frequency as a function of the inverse scattering length $a^{-1}$ for the model in the text. Heavy line: linear approximation near unitarity. Red squares: experimental data of Ref. 2.](chart.png)
\((256/875\pi)(S/\xi)(1/k_{F_{\text{max}}})\) where \(k_{F_{\text{max}}}\) is the (3D) Fermi wavevector at the center of the trap. Taking \([2]\) \(E_F = 1.2 \mu K\) and \(\delta(1/a) = 5.7 \times 10^4 m^{-1} G^{-1}\) in the vicinity of the resonance, this gives \(\delta(\omega^2/\omega_0^2) \simeq 10^{-3} S/\xi G^{-1}\). Comparing with the experimental result \(\simeq 1.1 \times 10^{-3} G^{-1}\) we obtain \(S/\xi \simeq 1.1\). If we take for \(\xi = 1 + \beta\) a value \([5]\) \(\xi = 0.45\) which is most likely both experimentally \([9]\) and theoretically \([10]\), we find the experimental result \(S \simeq 0.5\).

Finally a quite simple model in reasonable agreement with known constraints is \(f(y) = 1/2 - (1/\pi) \arctan(\pi y/2)\). Its linearization for \(y \sim 0\) leads to take \(\xi = S = 0.5\). It gives the proper limit for the weakly interacting Fermi gas and in the BEC limit it yields \(a_m = 6a/\pi\) for the molecular scattering length, which is not too different from the result \(a_m = 0.6a\) of Petrov et al \([11]\). Although it is very simple this model gives already a quite good agreement \([12]\) with experiment \([2]\) in the resonance region as can be seen in Fig.1.

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