Neutron skin of $^{48}$Ca consistent with experimental data on skins

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Background: In our previous paper, we predicted neutron skin $r_{\text{skin}}$ and proton, neutron, matter radii, $r_p$, $r_n$, $r_m$ for $^{40–62,64}$Ca after determining the neutron dripline, using the Gogny-D1S Hartree-Fock-Bogoliubov (GHFB) with and without the angular momentum projection (AMP). We found that effects of the AMP are small. Very lately, Tanaka et al. measured interaction cross sections $\sigma_i$ for $^{42–51}$Ca, determined matter radii $r_{\text{m}}(\sigma_i)$ from the $\sigma_i$, and deduced skin $r_{\text{skin}}(\sigma_i)$ and $r_{\text{n}}(\sigma_i)$ from the $r_{\text{m}}(\sigma_i)$ and the $r_p(\text{exp})$ evaluated from the electron scattering. Comparing our results with the data, we find for $^{42–48}$Ca that GHFB and GHFB+AMP reproduce $r_{\text{skin}}(\sigma_i)$, $r_{\text{n}}(\sigma_i)$, $r_{\text{m}}(\sigma_i)$, but not for $r_p(\text{exp})$.

Aim: Our purpose is to determine a value of $r_{\text{skin}}(\sigma_i)$ by using GHFB+AMP and the constrained GHFB (cGHFB) in which the calculated value is fitted to $r_p(\text{exp})$.

Results: For $^{42,44,46,48}$Ca, cGHFB hardly changes $r_{\text{skin}}$, $r_m$, $r_n$ calculated with GHFB+AMP, except for $r_{\text{skin}}$. For $r_{\text{skin}}$, the cGHFB result is $r_{\text{skin}} = 0.190$ fm, while $r_{\text{skin}} = 0.159$ fm for GHFB+AMP. We should take the upper and the lower bound of GHFB+AMP and cGHFB. The result $r_{\text{skin}} = 0.159 – 0.190$ fm consists with the $r_{\text{skin}}^{208}$, $r_{\text{skin}}^{48}$, $r_{\text{skin}}^{208}$, $r_{\text{skin}}^{48}$ results obtained from high-resolution $E1$ polarizability experiment ($E1pE$). Using the $r_{\text{skin}}^{208}$ relation with strong correlation of Ref. [3], we transform the data $r_{\text{skin}}^{208}$ determined by PREX and $E1pE$ to the corresponding values, $r_{\text{skin}}^{208}$ ($t\text{PREX}$) and $r_{\text{skin}}^{208}$ ($tE1pE$), where the symbol ‘t’ stands for the transformed data. Our result is consistent also for $r_{\text{skin}}^{208}$ ($t\text{PREX}$) and $r_{\text{skin}}^{208}$ ($t\text{E1pE}$). Eventually, for $^{42,44,46,48}$Ca, cGHFB reproduces $r_{\text{skin}}(\sigma_i)$, $r_m(\sigma_i)$, $r_n(\sigma_i)$, $r_p(\text{exp})$, while GHFB+AMP does $r_{\text{skin}}(\sigma_i)$, $r_m(\sigma_i)$, $r_n(\sigma_i)$.

I. INTRODUCTION AND CONCLUSION

Background on experiments. Neutron skin thickness $r_{\text{skin}}$ is strongly correlated with the slope parameter $L$ in the symmetric energy of nuclear matter [1,3]. The $r_{\text{skin}}$ is thus important to determine the EoS.

Horowitz, Pollock and Souder proposed a direct measurement for $r_{\text{skin}} = r_n - r_p$ [4], where $r_p$ and $r_n$ are proton and neutron radii, respectively. The measurement consists of parity-violating and elastic electron scattering. The $r_n$ is determined from the former experiment, and the $r_p$ is from the latter. For $r_{\text{skin}}$, in fact, the Lead Radius EXperiment (PREX) [5,7] yields

$$r_{\text{skin}}^{208}\text{(PREX)} = 0.33^{+0.16}_{-0.18} = 0.15 - 0.49\text{ fm}.$$ (1)

The result has a large error. For this reason, the PREX-II and the $^{48}$Ca Lead Radius EXperiment (CREX) are ongoing at Jefferson Lab [5].

As an indirect measurement on $r_{\text{skin}}$, the high-resolution $E1$ polarizability experiment ($E1pE$) was made for $^{208}$Pb [8] and $^{48}$Ca [9] in RCNP. The results are

$$r_{\text{skin}}^{208}(E1pE) = 0.156^{+0.025}_{-0.021} = 0.135 - 0.181\text{ fm},$$ (2)

$$r_{\text{skin}}^{48}(E1pE) = 0.14 - 0.20\text{ fm}.$$ (3)

For $^{208}$Pb, the central value 0.156 fm of the indirect measurement is much smaller than 0.33 fm of the direct measurement. This is a problem to be solved.

Very lately, Tanaka et al. published data on interaction cross sections $\sigma_i$ for $^{42–51}$Ca [10]. The data have high accuracy, since the average error is 1.05%. They determined $r_{\text{m}}(\sigma_i)$ from the $\sigma_i$ using the Glauber model, and deduced $r_{\text{skin}}(\sigma_i)$ from the $r_{\text{m}}(\sigma_i)$ and the $r_p(\text{exp})$ of Ref. [11]. For $^{48}$Ca, a value of $r_{\text{skin}}(\sigma_i)$ is

$$r_{\text{skin}}^{48}(\sigma_i) = 0.086 - 0.206\text{ fm}.$$ (4)

Using the $r_{\text{skin}}^{208}$ relation [3] with high correlation coefficient of $R = 0.99$, $r_{\text{skin}}^{48} = 0.5547 r_{\text{skin}}^{208} + 0.0718$, (5)

we transform $r_{\text{skin}}^{208}(\text{PREX})$ and $r_{\text{skin}}^{208}(E1pE)$ to the corresponding values $r_{\text{skin}}^{48}(t\text{PREX})$ and $r_{\text{skin}}^{48}(tE1pE)$, where the symbol ‘t’ stands for the transformed data. The transformed data are

$$r_{\text{skin}}^{48}(t\text{PREX}) = 0.155 - 0.344\text{ fm}$$ (6)

for PREX and

$$r_{\text{skin}}^{48}(tE1pE) = 0.147 - 0.172\text{ fm}$$ (7)

for $E1pE$.

Background on theories: As an ab initio method for Ca isotopes, we should consider the coupled-cluster method [12,13] with chiral interaction. The coupled-cluster result [12]

$$r_{\text{skin}}^{48}(CC) = 0.12 - 0.15\text{ fm}$$ (8)

is consistent with data $r_{\text{skin}}^{48}(E1pE)$ and $r_{\text{skin}}^{48}(\sigma_i)$.

Among effective interactions, NNLOsat [14] is a chiral interaction constrained by radii and binding energies of selected nuclei up to $A \approx 25$ [12], where $A$ is the mass number. In fact, the ab initio calculations were done for Ca isotopes [12,14,15]. As shown in Fig. 1 Garcia Ruiz et al. evaluated the charge radii $R_{ch}$ for $^{39–44}$Ca [15], using the coupled-cluster method with two low-momentum effective interactions, SRG1 of Ref. [16] and SRG2 of Ref. [17], that are derived from the chiral interaction with the renormalization

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group method. The SRG1 (SRG2) yields the lower (upper) bound of \( r_{\text{ch}} \). Difference of the two results is \( \sim 1.2 \) fm.

\[ r_{\text{skin}} = 0.159 - 0.19 \text{ fm}. \]  

Our result is consistent with \( r_{\text{skin}}^{48} (\text{E1pE}), r_{\text{skin}}^{48} (\sigma_I), r_{\text{skin}}^{48} (\text{tPREX}) \) and \( r_{\text{skin}}^{48} (\text{E1pE}) \), as shown in Fig. 2. The figure also shows that our result is also consistent with the coupled-cluster theory and this work (TW) for \( r_{\text{skin}}^{48} \). for \( r_{\text{skin}}^{48} (\text{tPREX}), r_{\text{skin}}^{48} (\text{E1pE}), r_{\text{skin}}^{48} (\sigma_I), r_{\text{skin}}^{48} (\text{CC}), r_{\text{skin}}^{48} (\text{TW}) \).

**II. MODELS**

We recapitulate GHFB, GFHB+AMP and cGHFB.

**A. GFHB+AMP**

In GHFB+AMP, the total wave function \( |\Psi_{M}^{I}\rangle \) with the AMP is defined by

\[ |\Psi_{M}^{I}\rangle = \sum_{K,K',n} g_{K}^{n} \hat{P}_{M}^{I} |\Phi_{n}\rangle, \]  

where \( \hat{P}_{M}^{I} \) is the angular-momentum-projector and the \( |\Phi_{n}\rangle \) for \( n = 0, 1, \cdots, N \) are mean-field (GHFB) states, where \( N + 1 \) is the number of the states. The coefficients \( g_{K}^{n} \) are obtained by solving the Hill-Wheeler equation

\[ \sum_{K',n'} \mathcal{H}_{K,K',n,n'}^{I} g_{K'^{n'}}^{n} = E_{I} \sum_{K,K',n,n'} N_{K,K',n,n'}^{I} g_{K'^{n'}}^{n}, \]  

with the Hamiltonian and norm kernels defined by

\[ \begin{align*}  
\mathcal{H}_{K,K',n,n'}^{I} &= \langle \Phi_{n} | \hat{H} | \Phi_{n'} \rangle, \ 
N_{K,K',n,n'}^{I} &= \langle \Phi_{n} | \hat{P}_{M}^{I} \hat{P}_{M}^{I} | \Phi_{n'} \rangle. \end{align*} \]

For even nuclei, there is no blocking state, i.e., \( N = 0 \) in the Hill-Wheeler equation. We can thus perform GHFB+AMP. However, we have to find the value of \( \beta \) at which the ground-state energy becomes minimum. In this step, the AMP has to be performed for any \( \beta \), so that the Hill-Wheeler calculation is still heavy. In fact, the AMP is not taken for mean field calculations in many works; see for example Ref. \( [26] \). The reason why we do not take into account \( \gamma \) deformation is that the deformation does not affect \( \sigma_{R} \) \( [27] \). As for GHFB, meanwhile, we do not have to solve the Hill-Wheeler equation.

For odd nuclei, we must put a quasi-particle in a level. The number \( N \) of the blocking states are very large. This makes

**FIG. 1.** A dependence of charge radii \( R_{\text{ch}} \) for \( 40-54 \) Ca taken from Ref. [15].

**FIG. 2.** Comparison among four data and two theoretical results of coupled-cluster theory and this work (TW) for \( r_{\text{skin}}^{48} \); namely, \( r_{\text{skin}}^{48} (\text{tPREX}), r_{\text{skin}}^{48} (\text{E1pE}), r_{\text{skin}}^{48} (\sigma_I), r_{\text{skin}}^{48} (\text{CC}), r_{\text{skin}}^{48} (\text{TW}) \).
it difficult to solve the Hill-Wheeler equation. Furthermore, we have to confirm that the resulting \( |\Psi_M^I| \) converges as \( N \) goes up for any set of two deformations \( \beta \) and \( \gamma \). This procedure is extremely time-consuming. For this reason, we do not consider the AMP for odd nuclei. As for GHFB, meanwhile, we do not have to solve the Hill-Wheeler equation, since we consider the one-quasiparticle state with the lowest energy. However, it is not easy to find the values of \( \beta \) and \( \gamma \) at which the energy becomes minimum in the \( \beta-\gamma \) plane.

As a result of the heavy calculations for even nuclei, we find that \( \beta \) is small for GHFB+AMP; see the table I of Ref. \[18\] for the values of \( \beta \). Meanwhile, the mean-field (GHFB) calculations yield that the energy surface becomes minimum at \( \beta = 0 \). The fact that \( \beta = 0 \) for GHFB and small for GHFB+AMP yields small difference between GHFB results and GHFB+AMP ones. Therefore, we consider GHFB+AMP for even nuclei and GHFB for odd nuclei.

### B. Constrained-GFHB

The difference between \( r_p(GHFB + AMP) \) and \( r_p(exp) \) is largest for \( ^{46}\text{Ca} \) in \(^{40-52}\text{Ca}\). In order to fit \( r_p \) to the central value of \( r_p(exp) \), one use constrained HFB; for example, see Ref. \[28\]. In the framework of GHFB, we modify the Hamiltonian as

\[
\hat{H}_{\text{constraint}} \equiv \hat{H} + \lambda \hat{Q}
\]

with

\[
\hat{Q} = r_p^2 - [r_p(exp)]^2,
\]

and take the expectation value \( \langle \lambda | \hat{Q} | \lambda \rangle \) of \( \hat{Q} \) with the constrained-GHFB (cGHFB) solution \( |\lambda\rangle \). The \( r_p(cGHFB) \) determined by cGHFB agrees with \( r_p(exp) \) under the condition

\[
\frac{d \langle \lambda | \hat{Q} | \lambda \rangle}{d \lambda}|_{\lambda=0} = 0.
\]

In actual calculations, we use the augmented Lagrangian method \[29\].

### III. RESULTS FOR NEIGHBOR NUCLEI OF \(^{48}\text{Ca}\)

As neighbor nuclei of \(^{48}\text{Ca} \), we consider \(^{42-51}\text{Ca} \), since the data are available for \( r_{\text{skin}}, r_m, r_n, r_p \).

Figure 3 shows \( r_p, r_m, r_n, r_{\text{skin}} \) as a function of \( A \). As for \( r_p \), the results of GHFB+AMP and GHFB do not reproduce the data \[11\] for \(^{39-52}\text{Ca} \). For \(^{42,44,46,48,50}\text{Ca} \), we then do cGHFB calculations to fit the theoretical value to the central value of \( r_p(exp) \).

The cGFHB results hardly change the values of \( r_m, r_n, r_{\text{skin}} \) except for \( r_{\text{skin}}^{48} \); see Table 3 for the numerical values of cGFHB and Table 4 for the numerical values of GHFB and GHFB+AMP. The deviation for \( r_p \) is thus not important except for \( r_{\text{skin}}^{48} \). We then take the lower and the upper bound of GHFB+AMP and cGFHB, as shown in our conclusion in Sec. II.

As for \( r_{\text{skin}} \), the results of GHFB, GHFB+AMP, cGFHB reproduce the data \[10\] for \(^{42-48}\text{Ca} \), but underestimate the data for \(^{49-51}\text{Ca} \). The difference between GHFB+AMP and GHFB is small for even Ca isotopes, indicating that effects of AMP are small. Eventually, for \(^{42,44,46,48}\text{Ca} \), cGFHB reproduce \( r_{\text{skin}}(\sigma_1), r_m(\sigma_1), r_n(\sigma_1), r_p(exp) \), while GHFB+AMP does \( r_{\text{skin}}(\sigma_1), r_m(\sigma_1), r_n(\sigma_1) \).

The data on \( r_m \) has a kink at \( A = 48 \). Qualitatively, \( r_m \) may be in inverse proportion to the binding energy per nucleon, \( E_B/A \). We then consider a dimensionless quantity \( \alpha \equiv r_m E_B/(\hbar c) \), where the central values of data \[10,23\] are taken for \( r_m \) and \( E_B/A \). The values of \( \alpha \) are tabulated in Table 3. The average of \( \alpha \) and its error are

\[
\alpha = 0.1535(9)
\]

for \(^{42-51}\text{Ca} \), indicating that \( r_m \) is in inverse proportion to \( E_B/A \). We can say that the kink comes from the shell effect.
FIG. 3. $A$ dependence of $r_p$, $r_n$, $r_m$, $r_{\text{skin}}$. Closed circles denote the GHFB results, while closed squares correspond to the GHFB+AMP results. Open squares show the results of cGHFB for $^{42,44,46,48,50}$Ca. Experimental data are taken from Refs. [10, 11].

TABLE I. Radii of constrained GHFB for $^{42,44,46,48,50}$Ca.

| $A$ | $r_n$ fm | $r_p$ fm | $r_m$ fm | $r_{\text{skin}}$ fm |
|-----|----------|----------|----------|---------------------|
| 42  | 3.417    | 3.411    | 3.414    | 0.006               |
| 44  | 3.477    | 3.424    | 3.453    | 0.053               |
| 46  | 3.530    | 3.401    | 3.475    | 0.129               |
| 48  | 3.575    | 3.385    | 3.497    | 0.190               |
| 50  | 3.658    | 3.429    | 3.568    | 0.229               |

TABLE II. Radii for Ca isotopes. The superscript “AMP” stands for the results of GHFB+AMP, and no superscript corresponds to those of GHFB.

| $A$ | $r_n^{\text{AMP}}$ fm | $r_p^{\text{AMP}}$ fm | $r_m^{\text{AMP}}$ fm | $r_{\text{skin}}^{\text{AMP}}$ fm | $r_n$ fm | $r_p$ fm | $r_m$ fm | $r_{\text{skin}}$ fm |
|-----|------------------|------------------|------------------|------------------|---------|---------|---------|------------------|
| 39  | 3.320            | 3.381            | 3.351            | -0.061           |
| 40  | 3.366            | 3.412            | 3.389            | -0.046           | 3.349   | 3.393   | 3.371   | -0.044           |
| 41  | 3.451            | 3.424            | 3.438            | 0.026            | 3.417   | 3.401   | 3.409   | -0.010           |
| 42  | 3.448            | 3.405            | 3.428            | 0.043            |
| 43  | 3.501            | 3.426            | 3.467            | 0.075            | 3.477   | 3.410   | 3.447   | 0.067            |
| 44  | 3.504            | 3.414            | 3.465            | 0.090            |
| 45  | 3.555            | 3.436            | 3.504            | 0.118            | 3.530   | 3.420   | 3.483   | 0.110            |
| 46  | 3.554            | 3.424            | 3.499            | 0.130            |
| 47  | 3.604            | 3.445            | 3.539            | 0.159            | 3.576   | 3.428   | 3.515   | 0.148            |
| 48  | 3.621            | 3.440            | 3.548            | 0.181            |
| 49  | 3.687            | 3.469            | 3.601            | 0.218            | 3.658   | 3.452   | 3.577   | 0.206            |
| 50  | 3.698            | 3.462            | 3.607            | 0.236            |
| 51  | 3.760            | 3.490            | 3.659            | 0.270            | 3.734   | 3.475   | 3.659   | 0.270            |
| 52  | 3.779            | 3.486            | 3.671            | 0.293            |
| 53  | 3.840            | 3.524            | 3.726            | 0.316            | 3.817   | 3.507   | 3.705   | 0.310            |
| 54  | 3.856            | 3.524            | 3.739            | 0.332            |
| 55  | 3.913            | 3.557            | 3.790            | 0.357            | 3.891   | 3.541   | 3.770   | 0.350            |
| 56  | 3.928            | 3.557            | 3.802            | 0.370            |
| 57  | 3.977            | 3.588            | 3.847            | 0.389            | 3.958   | 3.575   | 3.830   | 0.383            |
| 58  | 3.995            | 3.593            | 3.863            | 0.402            |
| 59  | 4.043            | 3.611            | 3.904            | 0.432            | 4.020   | 3.608   | 3.888   | 0.412            |
| 60  | 4.106            | 3.637            | 3.961            | 0.469            | 4.067   | 3.628   | 3.931   | 0.439            |
| 61  | 4.153            | 3.658            | 4.005            | 0.494            | 4.113   | 3.648   | 3.974   | 0.465            |
TABLE III. Numerical values of $r_m(\sigma_I)$ for $^{42-51}$Ca. The $r_m(\sigma_I)$ are taken from Ref. [10], and the data on $E_B/A$ are from Ref. [23].

| A | $r_m(\sigma_I)$ fm | $E_B/A$ MeV | $\alpha$ |
|---|------------------|-------------|---------|
| 42 | 3.437            | 8.616563    | 0.1501  |
| 43 | 3.453            | 8.600663    | 0.1505  |
| 44 | 3.492            | 8.658175    | 0.1532  |
| 45 | 3.452            | 8.630545    | 0.1510  |
| 46 | 3.487            | 8.668988    | 0.1532  |
| 47 | 3.491            | 8.63935     | 0.1528  |
| 48 | 3.471            | 8.66686     | 0.1524  |
| 49 | 3.565            | 8.594844    | 0.1553  |
| 50 | 3.645            | 8.55016     | 0.1579  |
| 51 | 3.692            | 8.476913    | 0.1586  |

Table IV shows the experimental data [10] on $r_m(\sigma_I)$, $r_n(\sigma_I)$, $r_{\text{skin}}(\sigma_I)$ for $^{42-51}$Ca, together with $r_p(\text{exp})$ [11].

TABLE IV. Numerical values of $r_p(\text{exp})$, $r_m(\sigma_I)$, $r_n(\sigma_I)$, $r_{\text{skin}}(\sigma_I)$ for $^{42-51}$Ca. The numerical values on $r_m(\sigma_I)$, $r_n(\sigma_I)$, $r_{\text{skin}}(\sigma_I)$ are taken from Ref. [10], where the systematic error is included. The $r_p(\text{exp})$ are deduced from the electron scattering [11]. Note that Tanaka et al. provide us the numerical values of $r_m(\sigma_I)$, $r_{\text{skin}}(\sigma_I)$, $r_n(\sigma_I)$.

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[1] B. A. Brown, Phys. Rev. Lett. 111, no. 23, 232502 (2013), [arXiv:1308.3664 [nucl-th]].
[2] X. Roca-Maza, M. Centelles, X. Vinas and M. Warda, Phys. Rev. Lett. 106, 252501 (2011), [arXiv:1103.1762 [nucl-th]].
[3] S. Tagami, N. Yasutake, M. Fukuda and M. Yahiro, [arXiv:2003.06168 [nucl-th]].
[4] C. J. Horowitz, S. J. Pollock, P. A. Souder and R. Michaels, Phys. Rev. C 63, 025501 (2001), [nucl-th/9912038].
[5] R. Michaels et al., Lead Radius Experiment PREX proposal 2005, [http://hallaweb.jlab.org/pparity/prex]
[6] S. Abrahanyan et al., Phys. Rev. Lett. 108, 112502 (2012), [arXiv:1201.2568 [nucl-ex]].
[7] A. Ong, J. C. Berengut and V. V. Flambaum, Phys. Rev. C 82, 014320 (2010), [arXiv:1006.5508 [nucl-th]].
[8] A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011), [arXiv:1104.5431 [nucl-ex]].
[9] J. Birkhan et al., Phys. Rev. Lett. 118, no. 25, 252501 (2017), [arXiv:1611.07072 [nucl-ex]].
[10] M. Tanaka et al., Phys. Rev. Lett. 124, no. 10, 102501 (2020), [arXiv:1911.05262 [nucl-ex]].
[11] I. Angeli and K. P. Marinova, Atom. Data Nucl. Data Tabl. 99, 69 (2013).
[12] G. Hagen et al., Nature Phys. 12, no. 2, 186 (2015), [arXiv:1509.07169 [nucl-th]].
[13] G. Hagen, T. Papenbrock, M. Hjorth-Jensen and D. J. Dean, Rept. Prog. Phys. 77, no. 9, 096302 (2014), [arXiv:1312.7872 [nucl-th]].
[14] A. Ekstrom et al., Phys. Rev. C 91, no. 5, 051301 (2015), [arXiv:1502.04682 [nucl-th]].
[15] R. F. Garcia Ruiz et al., Nature Phys. 12, 594 (2016), [arXiv:1602.07906 [nucl-ex]].

[16] K. Hebeler, S. K. Bogner, R. J. Furnstahl, A. Nogga and A. Schwenk, Phys. Rev. C 83, 031301 (2011), [arXiv:1012.3381 [nucl-th]].

[17] R. J. Furnstahl and K. Hebeler, Rept. Prog. Phys. 76, 126301 (2013), [arXiv:1305.3800 [nucl-th]].

[18] S. Tagami, M. Tanaka, M. Takechi, M. Fukuda and M. Yahiro, Phys. Rev. C 101, no. 1, 014620 (2020), [arXiv:1911.05417 [nucl-th]].

[19] S. Tagami, Y. R. Shimizu, and J. Dudek, J. Phys. G 42 (2015), 015106.

[20] M. Toyokawa, K. Minomo, M. Kohno and M. Yahiro, J. Phys. G 42, no. 2, 025104 (2015), Erratum: [J. Phys. G 44, no. 7, 079502 (2017)] [arXiv:1404.6895 [nucl-th]].

[21] M. Toyokawa, M. Yahiro, T. Matsumoto, K. Minomo, K. Ogata and M. Kohno, Phys. Rev. C 92, no. 2, 024618 (2015), Erratum: [Phys. Rev. C 96, no. 5, 059905 (2017)], [arXiv:1507.02807 [nucl-th]].

[22] M. Toyokawa, M. Yahiro, T. Matsumoto and M. Kohno, PTEP 2018, 023D03 (2018), [arXiv:1712.07033 [nucl-th]]. See http://www.nt.phys.kyushu-u.ac.jp/english/gmatrix.html for Kyushu g-matrix.

[23] the National Nuclear Data Center, NuDat 2.7; https://nucleus.iaea.org/Pages/nu-dat-2.aspx.

[24] O. B. Tarasov et al., Phys. Rev. Lett. 121, 022501 (2018).

[25] S. Michimasa et al., Phys. Rev. Lett. 121, 022506 (2018).

[26] S. Hilaire and M. Girod, Hartree-Fock-Bogoliubov results based on the Gogny force; http://www-phynu.cea.fr/science-en-ligne/carte-potentiels-microscopiques/carte-potentiel-nucleaire-eng.htm

[27] T. Sumi, K. Minomo, S. Tagami, M. Kimura, T. Matsumoto, K. Ogata, Y. R. Shimizu and M. Yahiro, Phys. Rev. C 85, 064613 (2012), [arXiv:1201.2497 [nucl-th]].

[28] E. Khan, J. Margueron, G. Colo, K. Hagino and H. Sagawa, Phys. Rev. C 82, 024322 (2010) doi:10.1103/PhysRevC.82.024322 [arXiv:1005.1741 [nucl-th]].

[29] A. Staszcuk, M. Stoitsov, A. Baran and W. Nazarewicz, Eur. Phys. J. A 46, 85 (2010), [arXiv:1006.4137 [nucl-th]].