A Key Experiment of Quantum Optics: The Transfer of Spin Angular Momentum from Photons to a Birefringent Particle

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Abstract. Rotating small birefringent particles with the spin angular momentum of light is a key experiment of quantum optics. We derive the equation of motion of small retarders in vis cose liquids, demonstrate their sometimes irregular rotation in polarized light, and discuss possible technical applications.

1. Introduction
The momentum and the angular momentum of the photon behave in a classical way when transferred from light to a macroscopic object. From sunlight to the dust particles of a comet tail momentum transfer is an obvious natural phenomenon, while angular momentum transfer from a laser beam to an inch size quartz plate was first shown by Beth [1] in 1935 in a difficult experiment. In 1998 Friese et al. [2] repeated this key experiment of optics with microscopic calcite platelets and demonstrated the fast rotation of these smaller objects in the polarized laser beam of optical tweezers. In this work we want to expound a number of techniques how to rotate small particles with light and present some experimental consequences and applications.

Small reflecting propellers [3] change the momentum of a light beam just like the pinwheels of children the momentum of the wind and due to radiation pressure they start to rotate. Since the twist of the blades of the propeller is fixed, the direction of the beam has to be reversed, if the direction of rotation has to be changed. Instead of the particle also the light beam can be twisted: Laguerre-Gaussian modes [4] consist of intertwined helices, which carry an orbital angular momentum \( \hbar l \). Any small object, which absorbs this light, will start to rotate and the direction of rotation can be changed by changing the handedness of the twist by optical means. But the twist of the light beam can also just be the circular polarization of a normal Gaussian laser beam. A circularly polarized photon has intrinsic or spin angular momentum \( \pm \hbar \) which can be absorbed by a particle, but can also be changed when a photon passes a birefringent platelet [1,2]. In both cases angular momentum is transferred, but in the latter case the photon is not destroyed and stays in the beam. This is the case we want to investigate in the following.
2. Theory
Feynman made the transfer of angular momentum from circularly polarized light to a charge plausible. For left-handed circularly polarized light the electric field rotates anticlockwise if viewed by an observer looking into the source. In a dielectric medium this field will impart positive angular momentum to the electrons [5,6]. Thus, left-handed circularly polarized light has a positive angular momentum parallel to the wave vector. Correspondingly, right-handed circularly polarized light – its electric field has the shape of a normal screw from the hardware store (traditional optical terminology) - has a negative angular momentum. The spin angular momentum does not depend on the frequency of the light or the refractive index of the dielectric [6] – very much in contrast to the momentum of light.

![Figure 1. Polarized states on the Poincaré-sphere. Left and right circular polarization \( \vec{L}, \vec{R} \) lie on the poles and linear polarization on the equator.](image)

Now we have to connect the polarization of the photon and spin angular momentum in a more quantitative way. This is done by introducing the Poincaré-sphere, which maps the states of polarization of light onto the surface of a sphere [7]. The two angles \( \lambda \) and \( \omega \) determine the orientation \( \lambda \) of the large axis \( a \) of an elliptically polarized state \( \vec{P} \) in space and its ellipticity \( \tan \omega = \pm b/a \) respectively. They fully characterize a polarized state on the sphere, and determine its Stokes-vector \( \vec{P}(\varphi = 2\lambda, \delta = 90^\circ - 2\omega) \), figure1. The Poincaré-sphere allows a descriptive representation of the change of polarization in a birefringent plate, which transforms a polarization \( \vec{P}_0 \) into an other one \( \vec{P}_1 \): \( \vec{P}_1 \) lies on the end of the small circle arch \( \Delta \), which is traced out by rotating \( \vec{P}_0 \) anticlockwise around the Stokes-vector, that represents the fast axis of a plate with retardation \( \Delta \). For a linearly birefringent plate, which we use for our experiments, the fast axis lies on the equator of the Poincaré-sphere. The spin angular momentum \( S(\varphi, \delta) \) of a photon with polarization \( \vec{P}(\varphi, \delta) \) is given by \( S = h \sin \delta \). Thus it does not depend on the orientation of the ellipse, but only on its ellipticity. The spin angular momentum of a photon, which is transferred to the plate per photon is the difference of the SAM of the incident photon and the SAM of the photon when it leaves the plate and the plate is driven by the torque of this difference. With spherical trigonometry we then find for the equation of motion of a linearly birefringent plate in an incident light beam of polarization \( \vec{P}(\lambda_0, \omega_0) \):

\[
\Theta \frac{d^2 \lambda}{dt^2} = Nh\{ \sin 2\omega_0(\cos \Delta - 1) + \cos 2\omega_0 \sin \Delta \sin(2\lambda_0 - 2\lambda) \} - \eta \frac{d\lambda}{dt} \tag{1}
\]
We use the parameters $\lambda, \omega, \Delta$ for the orientation of the fast axis of the plate in space and its retardation [8]. $\Theta$ is the moment of inertia of the rotating plate, and the last term describes velocity proportional friction. Figure 2 shows the numerical integration of equation (1) for a $\Delta = 2\pi/4$ retardation and for different ellipticities of the incident polarization $\bar{P}(\lambda_0, \omega_0)$. The rotational behaviour of the plate is characteristic for different polarizations. In the case of a circular polarization of the incident light, the driving force (torque) for the plate is constant in time and the plate behaves like a stone falling with friction in the earth gravitational field: it accelerates until friction compensates the torque. It then has a constant rotational frequency. In the other limiting case, when the incident light is linearly polarized, the plate executes damped oscillations until it comes to a halt with a fixed orientation. In this case both polarizations, the incident and the exiting one, lie on the same altitude on the Poincaré-sphere. Thus no angular momentum is transferred to the plate. If the ellipticity of the incident polarization exceeds a certain value, the plate rotates irregularly. Depending on the orientation angle $\lambda$ the transferred angular momentum to the plate varies and changes its acceleration.

![Figure 2. Rotation angle $\lambda$ of a quarter wave retardation plate under light of different ellipticity.](image)

3. Experiments
A laser with 20 - 100 mW, a high aperture microscope objective, a video camera, some polarization optics [9,10,11], and suitable particles with a high birefringence are needed for the experiments. The focus of the objective forms optical tweezers, which trap the platelet in a drop of water. We use Mercury(I)iodide or PTCDA [8], both dielectrics with a very high birefringence. Since the platelets have to be very small (some microns), we grind them in water and find the proper form and seize by
trying to rotate them. When they rotate in water, they typically reach a speed of a few Hertz. Changing the incident laser beam from left- to right handed circular polarization makes the little platelets change their angular momentum from anticlockwise to clockwise. This is a beautiful experiment for a student’s laboratory.

We filmed the rotating particles and analyzed their orientation with an image processing program. The results are shown in figure 3. The rotation angle versus time shows a step-like behavior due to the periodic acceleration and deceleration of the particle at different orientations. In the case shown, the particle nearly stops between intervals of fast rotation. This indicates that if the light were slightly more elliptical, the particle would come to a complete halt at a fixed orientation.

![Figure 3](image.png)

**Figure 3.** Periodically varying rotational speed of a small platelet of PTCDA in elliptically polarized optical tweezers. \( \omega = -30^\circ \pm 2^\circ \)

4. Conclusions

The scientific importance of the transfer of spin angular momentum to a retardation plate lies in the demonstration that a nonclassical property of a quantum particle - the spin of the photon - has a classical mechanical consequence if transferred to a classical object. By using small birefringent platelets instead of large and massive retardation plates, Beth’s experiment is turned into a simple lecture experiment which can be observed with the naked eye and gives a deep insight into the interlacements of nature. From a didactic viewpoint its importance can be compared with the Millikan-experiment.
The rotation of the retardation plate leads to an energy shift of the exiting light, thus the laser light has sidebands on leaving the rotating plate – or even many, if the rotation is irregular. In our case, the frequency shift is only a few Hertz, but if the plate would rotate in vacuum [12], the rotation frequency can be very high and the frequency shift substantial. The birefringent material has to be high-tensile, so it is not be torn apart by centrifugal forces. The incoherent laser light that is produced this way can for instance be used to reduce disturbing speckle in imaging.

Uniform rotation of a platelet is readily obtained, when the incident light is circularly polarized, or when the retardation Δ is \( \pi \). In addition, the direction of rotation can be easily switched from clockwise to counter clockwise by switching the handedness of the incident light. Applications as micro-motors, micro-pumps, or micro-centrifuges are thinkable. The discussed technique is in competition with orbital angular momentum transfer from light to a particle. The simple absorption of circularly polarized light by a particle also transfers angular momentum, but in cases of absorption the light is lost, in contrast to the angular momentum transfer to a transparent retarder. The momentum transfer to artificially created propeller-shaped particles is an other technique to create micro-rotors. But it lacks the possibility to easily switch the rotation direction. The attractivity of this technique lies in the fact that nature designs bacteria, pollen, and diatoms in the shape of micro-rotors, so that they need not to be artificially produced. For example helical bacteria can be rotated with light [13,14]. Spin angular momentum transfer to biological materials is not yet investigated and there is little information about the birefringence of these materials. Rotating small particles can also be used to measure the micro viscosity of liquids, but also to determine physical parameters of the particles itself, like thickness, shape or birefringence.

The halting of birefringent plates in polarized light of higher ellipticity can be used to place and orient them in space, for instance, micro retardation plates in front of optical fibres. Until now it is to our knowledge unclear, whether Polaroid polarizers can also be turned and oriented by spin angular momentum transfer from light. So we think that many further investigations are necessary to plumb this field in order to find further applications for the angular momentum transfer.

Acknowledgements

This work is funded by Cottrell Research Corporation. E.Frins appreciates the support by the Comisión Sectorial de Investigación Científica (CSIC) of the Universidad de la República (Uruguay) and PEDECIBA.

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