Reduced order model-based observer design for online temperature distribution estimation in lithium-ion batteries

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Abstract Time/space separation-based modeling methods have been widely researched for estimating lithium-ion battery (LIB) thermal dynamics. However, these methods have been developed in an offline environment and may not perform well in real-time application since the battery systems in electric vehicles (EVs) are usually subject to external disturbances. Furthermore, the onboard measurements of temperature are often corrupted by significant error. To address these problems, we present a reduced model-based observer design for online temperature distribution estimation in LIBs. First, an extreme learning machine (ELM)-based offline spatiotemporal model is constructed to approximate the thermal dynamics of LIB. Second, an adaptive reduced order observer is designed based on the offline model developed in the previous step. The offline model is then updated with the estimation results of the observer. As the performance of the estimator is highly related to the placement of sensors, a genetic algorithm (GA)-based integrated optimization strategy is also developed to determine the optimal sensor location for online estimation. Finally, the whole temperature distribution is estimated in real time using the observer, the measured voltage, current and the limited available temperature data. Two experiments on different batteries with different input currents verify the effectiveness of this developed model.

Keywords Spatiotemporal model · Lithium-ion batteries · State observer · Optimal sensor placement · Extreme learning machine

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1 Introduction

With oil prices rocketing and climate change concerns growing, energy and environmental problems due to oil consumption have become a major challenge of our time [1]. To solve this problem, new energy is being developed rapidly [2–4]. Lithium-ion batteries (LIBs) have been widely used as the power sources for electric vehicles (EVs) and hybrid electric vehicles (HEVs) because of their high energy density, high specific energy, and low environmental pollution [5, 6]. Since the life, efficiency, and safety of the battery all depend on its thermal behavior, an accurate battery thermal management system (BTMS) is indispensable to keep the battery operating within an optimal temperature range [7, 8].

The thermal process inside batteries belongs to a nonlinear distributed parameter system (DPS), which is described in nonlinear partial differential equations (PDEs) [9–12]. The input, output, even the state parameters of DPS can vary in both time and space directions [13, 14]. However, real-time measurement of temperature distribution is very challenging because only a limited number of sensors can be placed in batteries. Therefore, a detailed mathematical model that can be used and updated online is required for online monitoring of the temperature distribution. Plenty of researchers have studied the accurate thermal model of LIBs, which can be classified into two parts: lumped thermal models and distributed thermal models.

The lumped thermal models assume that the temperature distribution in LIBs is uniform. That is to say, the temperature only varies in the time direction [15, 16]. Due to the simplicity of such models, they are widely used to characterize the thermal behavior of batteries. There are many studies on lumped models combined with the electrochemical model [17, 18] or equivalent-circuit electrical model [19, 20], which have been successfully applied to battery cells or cylindrical batteries. These models are simple enough for temperature control and BTMS design. However, they do not consider the temperature difference in battery cell spaces [21]. To monitor and predict the temperature more accurately, distributed thermal models that can explain the temperature in time and space are needed. Various distributed models to characterize the thermal behavior of LIBs have been studied in recent years [22–25]. As with lumped models, most of the distributed thermal models were combined with an electrochemical model to explain the heat generation. These models can provide very accurate information for battery design. However, due to the large amount of calculation, they are not suitable for online temperature monitoring and control related applications [26]. Recently, some studies have been done in the construction of reduced order model oriented for BTMS application. Cai and White [27] proposed a reduced electrochemical-thermal model for a LIB cell based on proper orthogonal decomposition (POD), which is also called the Kurhunen-Loève (KL) method. Muratori et al. [28] proposed a reduced order thermal model of prismatic and cylindrical LIB based on Laplace transformation. However, this method is only applicable to line models, and it cannot be used to model nonlinear battery thermodynamics effectively. Liu et al. [29] proposed a novel KL-based time/space separation model for a two-dimensional battery thermal process. An extreme learning machine (ELM) was applied to approximate the heat generation term and produced satisfactory model performance. The Isometric Mapping (ISOMAP)-based spatiotemporal model is proposed for the estimation of the temperature distribution of the LIB under the working conditions simulated [30]. The representative local nonlinear dimension reduction method, locally linear embedding (LLE)-based method has been proposed and performs better than traditional KL-based spatiotemporal model [31].

Though the above methods are suitable for online applications, they cannot perform in different working conditions, and they are very sensitive to external disturbances. Furthermore, the aging of electrochemical related parameters also leads to the time-varying thermal dynamics feature of LIB [6]. Therefore, the thermal model of LIB must be updated online to adapt to the external disturbances and variation of working conditions with few measured temperatures [32]. Some work has been reported on online thermal model development of batteries. Taking the advantage of the thermal sensor on the surface or inside the battery, the online thermal estimation became possible. However, the current research does not involve much about the number and location of thermal sensors. An internal temperature estimation model is proposed to estimate the core temperature of battery, and an embedded thermal sensor is set up to carry out the experiment [33]. The battery with embedded
Reduced order model-based observer design for online temperature distribution estimation

2 Problem description and methodology

2.1 Problem description

The thermal dynamics of the LIB under investigation is a typical nonlinear DPS. Because the heat flux in the z direction is negligible compared to those in the x and y directions of Fig. 1, a two-dimensional thermal model is studied here. According to heat transfer law, the fundamental heat transfer equation of a LIB can be described as follows [11, 23, 28]:

\[ \rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + Q \]  

(1)

The boundary conditions and the initial condition are described as follows:

\[ -k_x \frac{\partial T}{\partial x} \bigg|_{x=x_0} = h(T - T_{air}), T_0 = T(x, y, 0) \]  

(2)

Here, \( T(x, y, 0) \) is temperature distribution of the LIB; \( \rho (\text{kg/m}^3) \) and \( c (\text{J/kg}^\circ \text{C}) \) are density and specific heat of the LIB, respectively; \( k_x (\text{W/m}^\circ \text{C}) \) and \( k_y (\text{W/m}^\circ \text{C}) \) are thermal conductivities along the x and y direction, respectively; the heat generation term \( Q \) is an unknown nonlinear function of input current \( I \) and measured voltage \( V \) of the LIB; \( h (\text{W/m}^2\circ \text{C}) \) is the convective heat transfer coefficient on the surfaces of the LIB, and \( T_{air} \) is the environment’s temperature.

For this type of thermal process, it is difficult to accurately model the temperature distribution due to the following issues:

1. Time-varying process: due to the unknown environmental disturbance and battery aging, the thermal process is time-varying, which is hard to model.

Fig. 1 Structural sketch of the lithium-ion batteries
(2) Limited number of sensors: a reduced number of sensors is available for online model adaptation.

2.2 Methodology

To solve the problems mentioned, the reduced order, model-based observer is designed for estimating the online temperature distribution in LIBs. As shown in Fig. 2, this method contains the following main ideas:

(1) Offline spatiotemporal model identification: First, the KL method is applied to calculate the optimal spatial basis functions \( \{ \varphi_i(x,y) \}_{i=1}^{\infty} \) for time/space separation. Second, with the obtained basis functions, mathematical expression of the reduced order model is derived by Galerkin’s method, and the unknown structure and parameters are then determined using the ELM algorithm. Finally, the offline spatiotemporal model is reproduced using time/space reconstruction.

(2) Online observer design: First, an adaptive reduced order observer is designed based on the obtained offline spatiotemporal model, and the stability is assured by Lyapunov stability theory. Second, the GA-based integrated optimization method is developed for optimal sensor placement. Finally, the real-time temperature distribution is predicted using the observer, input current, measured voltage, and the limited available temperature data.

Fig. 2 Reduced order model-based observer design scheme
3 Offline spatiotemporal model identification

According to the theorem of separation of variables, the spatiotemporal variable can be approximately separated into the following time/space decoupled form:

\[ T_n(x, y, t) = \sum_{i=1}^{n} \phi_i(x, y)a_i(t) \]  

(3)

where \( \phi_i(x, y) \) is the spatial basis function; \( a_i(t) \) is the corresponding low order temporal model, and \( n \) is the model order. Since \( \phi_i(x, y) \) can be estimated using KL decomposition [27], the next stage is to estimate the expression of the temporal model \( a_i(t) \). Substituting (3) into the physics-based model (1), the equation residual can be described as follows:

\[
R = \frac{\partial T_n}{\partial t} - (k_0 \frac{\partial^2 T_n}{\partial x^2} + k_1 \frac{\partial^2 T_n}{\partial y^2} + k_2 F(T_n) \\
+ k_3 Q_n(x, y, t))
\]

(4)

With Galerkin’s method [36], a discrete form of can be written in the following form:

\[ a_1(k + 1) = \sum_{j=1}^{n} k_{ij} a_j(k) + \bar{k}_i \phi_i(k) \]  

(5)

where \( k_{ij} \) and \( \bar{k}_i \) are constants. Then (5) can be described in a component form as:

\[ a(k) = K_1 a(k - 1) + K_2 q(k - 1) \]  

(6)

\[ T(x, y, k) = Ca(k) \]  

(7)

where: \( a(k) = [a_1(k), \ldots, a_n(k)]^T \), \( K_1 = \{k_{ij}\}_{n \times n} \),

\[ K_2 = diag(\bar{k}_1, \ldots, \bar{k}_n) q(k) = [\phi_1(k), \ldots, \phi_n(k)]^T C \]

The unknown nonlinear function \( q(k) \) in (6) can be approximated using a single, hidden layer feedforward neural (SLFN) network. Then in (6) can be expressed as follows:

\[ a(k) = K_1 a(k - 1) + K_2 \sum_{p=1}^{N} \beta_p G(\omega_p \cdot z(k - 1) + \eta_p) \]  

(8)

where \( N \) is the number of hidden neurons of the SLFN; \( \beta_p \) is the output weights connecting the corresponding hidden neurons and the output neuron of the networks; \( \omega_p \) is the input weights connecting the corresponding hidden neurons and the input neurons; \( \eta_p \) is the threshold of corresponding hidden neurons, and \( G(\cdot) \) is the activation function of the hidden layer, \( z(k) = [I(k), V(k)]^T \).

The value of corresponding temporal coefficients \( a(k) \) corresponding to the spatiotemporal measurements \( \{T(x, y, k)\} \) can be obtained from the following equation [28]:

\[ a(k) = \langle \phi_i(x, y), T(x, y, k) \rangle \]  

(9)

Then the model parameters in Eq. (6) can be identified with a set of data \( \{z(k), a(k)\}_{t=1}^{n} \).

Model (8) can be expressed in linear regression form as:

\[ a(k) = H(k - 1) \theta \]  

(10)

where: \( H(k - 1) = [a(k - 1), G(\omega_1 \cdot z(k - 1) + \eta_1), \ldots, G(\omega_N \cdot z(k - 1) + \eta_N)]^T \), \( \theta = [K_1, K_2 \beta_1, \ldots, K_2 \beta_N]^T \) is the parameters’ vector to be estimated.

Here, ELM algorithms [37] will be introduced to identify the unknown parameters that exist in (10). With ELM, \( \omega_p \) and \( \eta_p \) are randomly generated independent of the training data and each other. Once they are estimated, the value is fixed in the following learning procedure.

Equation (8) can be written in matrix form as:

\[ a = H \theta \]  

(11)

where \( a = [a(2), a(3), \ldots, a(n)]^T \), and \( H = [H(1), H(2), \ldots, H(n-1)]^T \). Then the parameters \( \theta \) can be calculated analytically as:

\[ \hat{\theta} = H^+ a \]  

(12)

where \( H^+ \) is the Moore–Penrose generalized inverse of matrix \( H \).

While the reduced order model (8) is trained very well, the offline spatiotemporal model of the battery thermal process can be reconstructed using time/space synthesis form as in Eq. (3).

4 Online observer design

For the model developed in the previous section to be updated in real time to adapt to the variation of
working environments and external disturbances, we propose an adaptive observer based online temperature estimation scheme for online battery temperature distribution estimation with the help of a few sensors. As shown in Fig. 2b, based on the offline spatiotemporal model obtained in Section III, the online estimation scheme includes two steps. The first step is the reduced observer design, and the second step is the sensor location design.

With the feedback of the error between the estimated and online measured temperature, the observer can be designed based on the identified offline spatiotemporal model, as:

$$\hat{a}(k + 1) = f(\hat{a}(k), I(k), V(k)) + L(T_m - \hat{T}_m) \quad (13)$$

$$\hat{T}_m(k) = C_m \hat{a}(k) \quad (14)$$

where $T_m$ is the real time temperature data; $m$ is the number of online sensors and $m \ll M$; $M = n_x \times n_y$ is the number of offline sensors that collect temperature distribution data for offline spatiotemporal model identification; $\hat{T}_m$ is the reconstructed temperature field with the estimated $\hat{a}(k)$. $L$ is the gain matrix of the observer determines the rate at which $\hat{a}(k) \rightarrow a(k)$.

$C_m$ is of the following form:

$$C_m = \begin{bmatrix} \varphi_1(X_1) & \cdots & \varphi_n(X_1) \\ \vdots & \ddots & \vdots \\ \varphi_1(X_m) & \cdots & \varphi_n(X_m) \end{bmatrix} \quad (15)$$

The performance of the observer is highly related to both the gain matrix $L$ and the placement of the sensors $m$. In this paper, the choice of the sensor placement $m$ is integrated with the design of the observer for online temperature distribution estimation. The goal is to find the appropriate choice of $m$ and value of $L$ simultaneously so that the following objective function $J(m, L, \hat{T})$ can be minimized:

$$\min J(m, L, \hat{T}) = \frac{1}{2} \| \hat{T}(X, k) - T_m(X, k) \|_Q \quad (16)$$

Subject to:

$$\begin{cases} \hat{a}(k + 1) = f(\hat{a}(k), I(k), V(k)) + L(T_m - \hat{T}_m) \\ \hat{T}_m(k) = C_m \hat{a}(k) \end{cases}$$

where $Q$ is a positive definite weighting matrix.

The integration cost function is a nonlinear constrained optimization problem. It is very hard and complex to optimize the sensor placement parameter $m$ and the observer gain matrix $L$ together using analytical optimization methods. To solve this problem, a hierarchical optimization framework [38] is adopted here to decompose the integration optimization problem into two nested steps: embedded observer design (inner loop) and master sensor placement optimization (outer loop). In the inner loop, an adaptive observer based on the offline spatiotemporal model is designed. In the outer design loop, a GA-based algorithm is used to optimize the sensor placement problem. A detailed description can be found in the following subsection.

4.1 Adaptive observer design

Given the designed sensor number and location, the observer is designed to guarantee stability and convergence. With the ELM-based low-order model developed in Section 3, the observer can be designed as:

$$\hat{a}(k) = K_1 \hat{a}(k - 1) + g(I(k - 1), E(k - 1)) + L(T_m - \hat{T}_m) \quad (17)$$

$$\hat{T}_m(x, y, k) = C_m \hat{a}(k) \quad (18)$$

where: $g(k) = [g_1(k), g_2(k), \ldots, g_n(k)]^T$

The gain $L$ is chosen for $K_1 - LC_m$ in order to be stable. The proof is present as follows.

With the estimation error $e_k = a(k) - \hat{a}(k)$ and the measurement error $e_f = T_m(k) - C_m \hat{a}(k) = C_m e_k$, the following can be obtained:

$$e_k = a(k) - \hat{a}(k) = a(k) - (K_1 \hat{a}(k - 1) + g(I(k - 1), E(k - 1)) + L(T_m - \hat{T}_m)) = (K_1 - LC_m)e_{k-1} = \omega e_{k-1}. \quad (19)$$

Consider the following Lyapunov function:

$$V_k = e_k^T e_k \quad (20)$$

Then:
According to Eq. (21), $\Delta V_k < 0$, when $||\omega|| < 1$; therefore, the output error $e_T$ will converge to zero.

4.2 GA-based optimal sensor location decision

As shown in Fig. 3, the optimal sensor locations are decided by solving the cost function, where the observer gains have been decided in the previous part. This optimization problem is difficult to solve by analytical methods since it may be non-convex and not differential with respect to the design variable. Here, a genetic algorithm (GA)-based method is proposed to solve the optimization problem.

GA is derived from the mechanisms of natural selection and nature genetics. By searching for the fitness through an iterative procedure, the optimal (or sub-optimal) solution can be found without calculating the derivatives of the function to be optimized. The GA manipulation is based on a population of individuals, and each individual represents a possible solution of the optimization problem. In the GA-based optimal sensor placement design, it is assumed the number of online measurements $m$ is fixed. The same data used for offline model identification can be used for sensor location design. Each population consists of $M$ elements where $m$ elements are given 1 and others are given 0. The procedures for implementing the GA algorithm are summarized as below:

**Step 1:** Population initialization: generate a random population of $n$ individuals.

**Step 2:** Fitness calculation: evaluate the fitness of each individual by the cost function $J(m, L, \hat{T})$.

**Step 3:** New population generation: create a new population by repeating the following process:

1. Selection: Select the parent individuals for crossover according to their fitness value: The individual with the best fitness is preserved and used to replace the individual with the worst individual; other individuals are selected using roulette wheel selection.
2. Crossover: Two randomly selected individuals from the parents will be crossed over with a certain crossover probability to generate the next generation.
3. Mutation: To maintain the number of sensors, $n_d$ randomly chosen elements are mutated, with $n_d$ denoting the difference between the number of sensors in the new generation (indicated as 1 in individuals) and the number of sensors $m$ predesigned for online estimation.

**Step 4:** Repeat steps 2 and 3 until a stopping criterion is met. Then, the best solution in the current population is returned.

5 Experimental verification

To evaluate the effectiveness of the proposed online temperature estimation strategy, real-time experiments and comparisons on two batteries are conducted. For easy comparison of the model performance of each method, the following evaluation criterions are defined:

- Spatiotemporal prediction error
  $$e(S, t) = T(S, t) - \hat{T}_N(S, t)$$
Root of mean square error (RMSE)

\[ RMSE = \sqrt{\frac{1}{n_sn_t} \sum_{k=1}^{n_S} \sum_{t=1}^{n_T} (T(S_k, t) - \tilde{T}_N(S_k, t))^2} \]

Temporal normalized absolute error (TNAE)

\[ TNAE = \frac{1}{n_t} \sum_{t=1}^{n_t} |e(S, t)| \]

Spatial normalized absolute error (SNAE)

\[ SNAE = \frac{1}{n_S} \sum_{k=1}^{n_S} |e(S_k, t)| \]

Absolute relative errors (ARE)

\[ ARE = \frac{1}{n_S} \sum_{k=1}^{n_S} \frac{|e(S_k, t)|}{|S_k|} \]

Fig. 4  Dimension and sensors arrangement of the LIB

Fig. 5  LIB cycle charging and discharging experiment platform
5.1 Case 1: experimental setup

In the experiment, a 60Ah LiFePO4/graphite LIB is used as showed in Fig. 4. It is a conventional lithium-ion battery with high power density, long life, low self-discharge, low maintenance cost and low environmental impact. The LIB can generate a lot of heat while working time increases, and the temperature is higher near the electrode and lower at locations far from the electrodes. Obviously, the temperature of the LIB has a spatiotemporal property. Since the thickness of the cell stacks are much shorter than the other dimensions of the electrodes, and the temperature variations across the cell stacks are neglected. Therefore, the thermal process of the LIB can be simplified as a two-dimensional thermal model. The LIB is cycled with Battery Thermal System (BTS) Integrated Battery Tester, as shown in Fig. 5. Totally 20 thermal sensors are uniformly arranged along the directions of x and y for temperature data collection. As shown in Fig. 1, the sensors marked in “cross” are used for model estimation. The applied input current and working voltage of the LIB is measured using the Integrated Battery Tester.

5.2 Case 1: model estimation and verification

With the known physical model of a battery, appropriate input signals must be designed to fully excite the state of charge and thermal dynamics in the entire working range, and then informative output temperature samples can be collected for model identification. Here, a multi-step input current (Fig. 6a) same to that used in [39] is applied here. The entire thermal process takes 5400 s. The corresponding output voltage is measured using the integrated battery tester, as illustrated in Fig. 6b, and will be used as the input signal along with the input current for model estimate. As the spatiotemporal output, the sensors capture a total of 5400 temperature samples, with the first 3600 samples serving as training data and the last 1800 serving as testing data.

With the collected temperature data, the KL method is first applied to calculate the dominant spatial basis

\[ ARE = \frac{|e(S, t)|}{T(S, t)} \]
functions for time/space separation, where the order of basis functions is selected as 5. Then the low-order temporal model described in (8) is determined. Finally, the spatiotemporal dynamic of LIB can be reconstructed based on the obtained spatial basis functions and determined low order temporal model. When the ELM-based spatiotemporal model is trained properly, a state observer as described in section IV can be designed. Four sensors were used for online temperature data collection. The GA-based optimal sensor placement strategy described in section IV. B is used to design the sensor location for online estimation. The optimization result is shown in Fig. 7, where the “circle” points denote the obtained sensor location for online estimation through optimization, and two sensors are selected randomly from the rest cross points to be used for model validation.

The 1800 testing samples are employed to assess the models’ performance, and the prediction error distributions at \( t = 4200 \) s and \( t = 5400 \) s are presented. To compare with other existing methods, LLE (locally linear embedding) based model and ISOMAP (Isometric Mapping)-based model are also studied in the same environmental condition. ISOMAP and LLE

Fig. 8 Measured temperature distribution using the testing input signal

Fig. 9 Model prediction errors using the proposed method
are two successful methods for modeling battery thermal process in the laboratory environment without observer. The instantaneous temperature distribution at 4200 and 5400 s is shown in Fig. 8a and b, respectively. From Fig. 8a and b, it is obvious that the internal temperature is increased due to the electrochemical reaction. The corresponding spatiotemporal model output and prediction errors at corresponding time are shown in Figs. 9, 10 and 11. The experimental results indicate that the maximum error values of the proposed method, the LLE based model and ISOMAP-based model are 0.1, 0.55 and 0.5, respectively. That means the percentage error is within 1.67%, if the lowest temperature is 33 °C. Therefore, all these three spatiotemporal models have good performance on the testing data. Especially, the proposed method has the lowest maximum error value compared with the other two models. Obviously, the proposed spatiotemporal model can approximate the thermal behavior of the original system very well.
Meanwhile, it is found that the model errors at 5400 s are larger than at 4200 s. This is due to the effects of cumulative error and time-varying dynamic. The proposed spatiotemporal model has the optimization of sensor position by GA and observer design, which can adjust the model output online with current data. This makes the proposed method more suitable for different working states of batteries.

For further comparisons, the results of the three spatiotemporal models in terms of SNAE, TNAE, and ARE are shown in Figs. 12, 13 and 14. The error index RMSE of the three models over the testing process is shown in Table 1. From the aforementioned figures and table, it is obvious that the proposed method has superior model performance than the other two spatiotemporal models in the modeling of LIB thermal process.

### Table 1 Model performance comparison in terms of RMSE

| Method            | Testing RMSE |
|-------------------|--------------|
| The proposed method | 0.0619       |
| LLE based method  | 0.1325       |
| ISOMPA based method | 0.1209      |

Meanwhile, it is found that the model errors at 5400 s are larger than at 4200 s. This is due to the effects of cumulative error and time-varying dynamic. The proposed spatiotemporal model has the optimization of sensor position by GA and observer design, which can adjust the model output online with current data. This makes the proposed method more suitable for different working states of batteries.

5.3 Case 2: experimental setup

In the experiment, a 32Ah Li(NiCoMn)O2 ternary lithium battery is used for model verification. Thirty sensors are placed evenly on the surface of battery as shown in Fig. 15. The battery was excited by 0.5C constant discharge current, where the current $I(t)$ and the generated voltage $V(t)$ are presented in Fig. 16. A total of 5400 temperature samples are gathered as spatiotemporal outputs from the sensors, with the first 3600 samples utilized as training data and the last 1800 samples used as test data. The sample interval is 1 s.

5.4 Case 2: model estimation and verification

The modeling procedure is the same as in the Case 1. The optimized sensors using GA in this case are shown in Fig. 17, where the sensors with the “circle” sign are used for online data collected. To verify the performance of the proposed model, a comparative study was conducted with the LLE-based model and KL-based model under the same environmental conditions. The instantaneous temperature distribution of 4200s and 5400s is shown in Fig. 18. Figures 19, 20, 21
Fig. 15  Dimension and sensors arrangement of the LIB

Fig. 16  Current and voltage signals for model estimation

(a) Input current

(b) Measured voltage
shows the model prediction output and errors of the three models at corresponding time points. The experimental results indicate that the maximum error values of the proposed method, the LLE-based model and KL-based model are 0.1, 0.45 and 0.5, respectively. The superior model performance benefits from the design of observer.

To further analyze the advantages of the proposed model, the results of the three spatiotemporal models in terms of SNAE, TNAE, and ARE are shown in Figs. 22, 23 and 24. The error index RMSE of the three models over the testing process is shown in Table 2. From these comparison results, the efficiency and accuracy of our method is further verified in this case. The proposed method performed the best in our experiments. The main reason is the design of reduced-order observer, which can adjust the model output according to the collected online data. This will significantly improve the robustness of the spatiotemporal model and make it more suitable to current working state.

6 Conclusion

In this paper, a systematic online estimation scheme for temperature distribution in LIBs was presented. First, an ELM-based spatiotemporal model was developed to approximate the thermal behavior. Second, an adaptive observer was designed for the determined ELM-based spatiotemporal model. Finally, the spatiotemporal temperature distribution of the battery was estimated online using the designed observer, input current, the measured voltage, and limited temperature values. As the performance of the estimator is highly related to the placement of sensors, a GA-based integrated optimization method was also proposed to decide the optimal sensor location for online estimation. The effects of the sensor location...
and the observer gain matrix were both considered in designing the sensor location. Experiments and comparisons demonstrated the effectiveness of the proposed method. Though this developed model can work very well, still some problems should be addressed in our future works. Currently, we are still in the early stage of theoretical development for analytical modeling. The model is implemented in a laboratory setting that does not consider the sensors number for real application. The model is implemented in a laboratory setting that does not consider the number of sensors for real application, and the real working
operations of EVs (NEDC, WLTP, and EPA). After this work, the attention will be on model implementation for limited sensors under real working conditions of EVs. The main emphasis would be put on

**Figure 21** Model prediction errors using KL based model

**Figure 22** Model performance comparison using SNAE

**Figure 23** Model performance comparison using TNAE

**Figure 24** Model comparison using ARE at the sensor in second row, third column

| Table 2 | Model performance comparison in terms of RMSE |
|---------|---------------------------------------------|
|         | The proposed method | LLE based method | KL-ELM |
| Testing | 0.0569            | 0.1869           | 0.3576 |

operations of EVs (NEDC, WLTP, and EPA). After this work, the attention will be on model implementation for limited sensors under real working conditions of EVs. The main emphasis would be put on
sensor reduction, model simplification, and application in three-dimensional battery pack.

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### Data Availability
All data generated during the study are available from the corresponding author by request.

### Declarations

#### Conflict of interest
The authors declare that they have no conflict of interest.

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