Mathematical connection ability: teacher's perception and experience in learning

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Abstract. In learning mathematics mathematical connections ability is one of the mathematical ability that must be possessed by students. The strong connection between concepts in mathematics implies that mathematical connections ability also contains other mathematical aspects or vice versa. Viewing mathematics as a whole is very important in learning and thinking about connections between topics in mathematics. This study uses qualitative research methods with phenomenology research design types. The participants in this study were 8 middle school mathematics teachers who taught at different schools. This study uses a short interview form which consists of five open questions to determine the teacher's understanding of mathematical connection ability. It was found that middle school math teachers had sufficient comprehensive and adequate knowledge of their views on mathematical connection ability, moreover there were also respondents who could not provide examples of mathematical problems that contained the characteristics of mathematical connection ability even though they were able to define what mathematical connection is. Likewise, the development of teachers in mathematical connection ability in learning is still limited to the scope of the relationship of mathematics with mathematics itself and mathematics with everyday life.

1. Introduction

In essence, mathematics as a structured and systematic science means that concepts and principles in mathematics are interrelated with one another. As an implication, learning mathematics to achieve meaningful understanding of students must have adequate mathematical connection ability. Mathematical connection ability is the ability to associate mathematical concepts both between concepts in mathematics itself and linking mathematical concepts to other concepts in another fields [1]. The mathematics education literature supports the belief that mathematical understanding requires students to make connections between mathematical ideas, facts, procedures, and relationships [2, 3, 4, 5, 6]; thus, building, reviewing and understanding relationships is fundamental in carrying out the work of teaching mathematics.

The strong connection between mathematical concepts implies that aspects of mathematical connections also contain other mathematical aspects or vice versa. Viewing mathematics as a whole is very important in learning and thinking about connections between topics in mathematics. So that in conveying a concept B for example, a teacher must introduce or pay attention to the concept of A first. But the fact is that current learning support such as teaching materials have not been able to facilitate students in linking or connecting between one concept and another concept.

Mathematical connection ability is an important part that must be mastered by students at every level of education. Because mathematical connections help students see the interrelationships and benefits of mathematics itself [7]. By making connections, mathematical concepts that have been learned are not
left as a separate part, but are used as basic knowledge to understand new concepts. Through the teaching process that emphasizes the relationship between mathematical ideas, students will not only learn about mathematics, but about the usefulness of mathematics.

Mathematical connection is an ability that must be built and studied, because with good mathematical connection ability will help students to be able to know the relationship of various concepts in mathematics and apply mathematics in everyday life. With mathematical connection ability students will feel the benefits of learning mathematics, and the students' understanding of the concepts they learn will last longer. In mathematics curriculum at the school mathematical connection is one of the basic ability of mathematics that must be mastered by high school students.

Ministry of National Education [8] states that mathematics learning objectives include that students have the ability: 1) understand mathematical concepts, explain the interrelationships between concepts and apply concepts or algorithms, flexibly, accurately, efficiently, and precisely in problem solving, 2) using reasoning on patterns and traits, 3) solving problems, 4) communicating ideas with symbols, tables, diagrams, or other media, and 5) having an attitude of appreciating the usefulness of mathematics in life. While the National Council of Teachers of Mathematics [7], sets the standards of mathematical ability such as problem solving, reasoning and proof, communication, connection, and representation, should be owned by students.

Wherefore mathematical connection ability is an ability that underlie mathematical thinking. Coxford [9], explains mathematical connection ability is the ability to connect conceptual and procedural knowledge, use mathematics on other topics, use mathematics in life activities, know connections between topics in mathematics. Mathematical connection ability is one aspect of important mathematical ability that must be achieved through mathematics learning activities. The reason this ability important is because by knowing mathematical relationships, students will better understand mathematics and also give them greater mathematical power. Krulik states that according to Bruner there is no concept or operation that is not connected with other concepts or operations in a system, it is a fact that the essence of mathematics is one topic related to another topics [10]. Thus, in order for students to succeed in learning mathematics, students should be given more opportunities to see the links.

Regarding to this, the teacher plays an important role to improve mathematical connection ability through classroom activities. They need class discussions that allow students to create alternative solutions, to share their solutions and ideas, and to communicate with each other so that they can improve their mathematical connection ability in solving the problem. Considering the opportunity to discuss their ideas with colleagues and to develop their mathematical understanding through conversation, students have a greater opportunity to develop mathematical connection ability. Therefore, it is very important that the teacher has knowledge of mathematical connections and able to create a rich environment to support the development of mathematical connection ability among students.

Reflect to student performance in the mathematical ability described above, it can be concluded that teachers need to use classroom activities to be more effective in developing students' mathematical understanding and thinking ability. In this context, this research focuses on mathematical connections as an important way to grow students' mathematical thinking. This study was conducted on experienced teachers who work in secondary schools, so it is important to understand their views and experiences about mathematical connections as a tool for developing mathematical thinking ability. Understanding the views and experiences of the teacher will provide a basis for discussing the use of mathematical connections to support students' thinking processes and mathematical understanding.

2. Method
This study used qualitative research methods with the type of phenomenology research design, this choice is based on the desire of researchers to produce a clear description of the perception of mathematics teachers and their experience in using mathematical connection ability in mathematics learning to foster students' mathematical thinking. The phenomenon that will be examined is the perception of the mathematics teacher and their experience with mathematical connection ability. Lincoln & Guba [11] have outlined the criteria standards in qualitative researchers to establish trust in qualitative data. These criteria include credibility, dependence, adjustment, and transfer.
The participants in this study were 8 middle school math teachers who taught at different schools. Participants were selected through random sampling and voluntarily participated in the study. They vary in teaching experience, which ranges from 2 to 12 years, and represents different grade levels. To ensure that the research code of ethics of the participants is informed by a name code because of research ethics, they are identified by the number specified as R1, R2, ..., R8.

This study uses a short interview form which consists of five open questions to determine the teacher's understanding of mathematical connection skills. The following table presents questions in the interview form.

**Table 1. List of Interview Form Questions**

| Question                                                                 | Respondent |
|--------------------------------------------------------------------------|------------|
| How do you define mathematical connections?                              | R1         |
| What do you think about the impact of mathematical connections on students’ mathematical thinking skills? | R2         |
| Can you give an example of the learning process you did about mathematical connections? | R3         |
| What approach do you use to support students’ mathematical connection skills? | R4         |
| Below is an example of some problem solutions obtained from students’. What can you conclude from this solution regarding students’ mathematical connections? | R5         |

The first and last questions are designed to determine the teacher's theoretical mathematical connection knowledge while the second, third and fourth questions are intended to explore the teacher's thinking about mathematical connections in practice based on the students' examples and answers given. Individual interview forms are given to teachers for 1 week. Teachers complete the form without assistance or intervention by the researcher in any way.

Reviewing the qualitative data, this study uses the content analysis method for data analysis. Content analysis is carried out to uncover facts that may be contained in the data obtained and to figure out and describe this data in a way that the reader can understand by bringing similar data together in the context of certain concepts and themes [12]. In the first stage of data analysis, each question is analyzed separately; researchers examine data simultaneously and independently from each other and produce code tables with categories and subcategories. In the second stage, the researcher encodes all data. Next, the researchers compared the data together to eliminate differences in coding. In the third stage, the researcher encodes all data. Next, the researchers compared the data together to eliminate differences in coding. In the third stage, the researcher encodes all data. Next, the researchers compared the data together to eliminate differences in coding. In the third stage, the researcher encodes all data. Next, the researchers compared the data together to eliminate differences in coding. In the third stage, the researcher encodes all data. Next, the researchers compared the data together to eliminate differences in coding. In the third stage, the researcher encodes all data. Next, the researchers compared the data together to eliminate differences in coding. In the third stage, the researcher encodes all data. Next, the researchers compared the data together to eliminate differences in coding.

3. Result and Discussion

This section presents the findings from the analysis of answers provided by the teachers to the questions in the interview form. The researcher checks each question through analyzing content and identifying different themes. Respondents' responses in this study are categorized in five broad thematic topics: (1) Concepts related to mathematical connections, (2) Strategies to improve mathematical connections, (3) Objectives and benefits of mathematical connections, (4) Impact of mathematical connections on mathematical thinking ability, (5) Application of mathematical connections in class. After the interview form is returned by the teacher, the researcher analyzes each of the answers to the questions in the interview form. The following is the result of analyzing the participants' answers to the first question, which is about how respondents define mathematical connections.

**Table 2. Themes that Appear from First Question Results**

| Themes                                      | Respondent |
|---------------------------------------------|------------|
| Regarding the relationship of mathematics   | R1         |
| Using all thought strategies needed to answer mathematics problems | R2         |
| Use mathematical knowledge to develop alternative solutions | R3         |
| Connecting mathematics                      | R4         |
| Link mathematical knowledge with previous knowledge | R5         |
In the first question, the respondents (teachers) were asked to define mathematical connections. Based on the table above, it can be concluded that there are three different themes about teacher responses to the definition of mathematical connections. These themes show that most teachers define mathematical connections as mathematical relationships. However, there is something interesting when the teacher also understands mathematical connections as a form of the ability to understand mathematical problems and can also solve them.

According to the teachers, this emphasizes the importance of understanding the problem in relation to mathematical connections. The teacher with initials R6 states that when students can understand the problem and solve it, students will actually be able to see the relationship contained in the material being taught. Furthermore, there are some teachers who explain that mathematical connections are mathematical knowledge related to prior knowledge and associate mathematical knowledge with real life situations. One of them, connecting mathematics by producing new models through mathematical knowledge and reinterpreting that knowledge, explains mathematical connections as an attempt to connect numbers, algebraic expressions, mathematical thinking to prior knowledge to produce new models and to reinterpret (R5).

On the second question, the teachers were asked about their thoughts about the impact of mathematical connections on students' mathematical thinking skills. The teachers believe that mathematical connections improve mathematical thinking skills. This is generally illustrated in their statement that mathematical connections are a tool to develop their students' mathematical thinking skills, this can be seen from the definitions conveyed earlier and strengthened by the hierarchical nature of mathematics so that it requires mathematical connection skills to foster students' mathematical thinking skills. Teachers generally emphasize the impact of mathematical connections on understanding concepts, problem solving, reasoning and critical thinking skills. In line with the participants' connection association with their understanding, they also think that mathematical connections can also develop their students' conceptual understanding. One teacher (R3) stated that "when students understand the relationships contained in mathematics they will try to understand concepts and apply knowledge in their lives". Teacher with initial R2 emphasizes, "with mathematical connections, students will be encouraged to investigate and develop connections". While the teacher (R8) states "with mathematical connections, thinking ability, the ability to see different points of view, critical thinking and problem solving skills are easier to develop". The teachers also believe that with mathematical connections students' critical thinking skills develop. For example, one teacher states that, "when students start thinking about the relationship of a concept, their critical thinking skills develop and they the "why" questions often appear in other subjects".

Third, the teachers were asked to give examples of the learning process carried out about mathematical connections. The following themes outline that can be concluded from the results of the teacher's interview answers.

| Themes | Respondent |
|--------|------------|
| Examples for cases that involve associations of mathematical knowledge that are different from each other | R3, R5 |
| Produce alternative solutions | R1 |
| Generalization | R7 |
| Relationship between Concepts A and B | R4, R8 |
| Irrelevant | R2, R6 |

Some examples given by the teacher related to the relationship of concept A and concept B are as follows: "Mr. Burhan has a circular plot of land, he wants to build a swimming pool 8 m long and 6 m
wide: a) How wide is the swimming pool Mr. Burhan has in units of cm?; b) If you want to make a rocky road around Mr. Burhan’s swimming pool, what is the cost for Mr. Burhan if the cost per meter is Rp. 25,000.00?”

However, there are also examples given by the teacher for irrelevant mathematical connections or failing to provide examples, which are also findings that imply that the teacher has not promoted or facilitated mathematical ability in their learning, such as the following example: “Andi goes to school on a bicycle every day covering a distance of 628 m. Andi’s bicycle wheel spin around 500 times to travel the distance. Calculate the circumference and length of the radius of the Andi’s bicycle wheel”.

Fourth, teachers were asked to explain about what approach they use to support students’ mathematical connection ability. In this case the teachers emphasized several things, namely (a) challenges and (b) suggestions for developing mathematical connections in the classroom. Their responses are analyzed in these two sub-themes.

**Table 4. Themes that Appear from Fourth Question Results**

| Themes                                      | Respondent |
|---------------------------------------------|------------|
| Challenge:                                  |            |
| 1) Lack of time                             | R1         |
| 2) Demands in completing the curriculum     | R4         |
| Suggestion:                                 |            |
| 1) Develop teaching materials               | R2, R5     |
| 2) Student centered approach                | R1, R2, R6 |
| 3) Using non-routine problem situations     | R3, R7     |
| 4) Arrange questions that are intended to link previous knowledge with new ones | R5, R6 |
| 5) Directing students to produce problems   | R2         |
| 6) Problems of situations that require students to relate to real life situations | R7, R8 |
| 7) Create a class discussion                | R1, R2, R3, R4, R5, R6, R7, R8 |
| 8) Directing students toward group work     | R1, R2, R3, R4, R5, R6, R7, R8 |
| 9) Create an appropriate class environment  | R1, R2, R3, R4, R5, R6, R7, R8 |

Most respondents complained about the intensity of the curriculum as a challenge to apply mathematical connections. For example, one of them said, “I have no time in compiling teaching materials that can facilitate mathematical connections.” Another teacher point out, “To apply a mathematical connection, I use a student centered approach.” One teacher answer using learning method that usual used, because they must complete the curriculum at the end of the year. Other teachers explain “how the curriculum should be organized to provide effective space to facilitate students’ mathematical connection skills”.

Based on the opinions of teachers, another way to support students’ mathematical connection ability is to support them to involve ways of dealing with problems. With activities like this of course students will be motivated to get more concepts related to this problem so that students will realize the connection in mathematics. As addition, there are some teachers who give different suggestions and argue that this can encourage students to relate their knowledge to real life, to justify, to solve problems in their own words, to make problems, to make class discussions.

The fifth question from the interview guide that must be answered by the teacher is about examples of several solutions to the problem of student answers and what can be concluded from the solution regarding the students’ mathematical connections. The following picture shows the answer solution given by students from the question: “A satellite crosses an orbit at a distance of 1,000 km above the surface of the earth. If the length of the earth’s radius is 6,400 km and to cross the orbit it takes 30 hours. Calculate satellite path length and satellite speed in km / hr! And explain what relationships are found in this problem”. 
Based on the results of teacher's evaluation of the student's answers above, the teacher's answers can be categorized in the following themes:

| Themes                                             | Respondent          |
|----------------------------------------------------|---------------------|
| Perform calculations correctly                     | R1, R2, R3, R4, R5, R6, R7, R8 |
| Cannot explain the operation that is being carried out | R1, R6, R7, R8      |
| Does not describe the relationship in question     | R2, R3, R5, R6, R7, R8 |

Referring to the table above it can be concluded that the respondents in this case the teachers stated that the results of calculations made by students were correct. The following are some of the statements expressed by the teachers:

"Students can do calculations correctly. However, students do not clearly explain the calculation process and have not explained the follow-up questions related to the link in the problem" (R1).

"Here, students reach the solution correctly even though it is not perfect" (R6).

"The connection found in the problem is the relationship between mathematical concepts and mathematics and also with other disciplines" (R7).

The purpose of this study was to investigate the understanding of middle school mathematics teachers about mathematical connections. With this aim, the teacher's definition of mathematical connections is examined first. According to the explanation, it appears that the teachers define mathematical connections as mathematical relationships. However, there is something interesting when the teacher also understands mathematical connections as a form of the ability to understand mathematical problems and can also solve them. This is in accordance with the view of Linto et al (2012) [13] states that mathematical connection ability is the ability to solve mathematical problems related to previously studied material. It can be said that the teacher's definition of mathematical connections is not comprehensive enough. Therefore, even so that teachers who define mathematical connections as relationships or linkages might evaluate or support students' mathematical connections only through this skill. This situation will cause the mathematical connection ability of students to develop incomplete, for example, in defining the connection, the teacher does not explain the relationship between mathematics and other sciences, the teacher is more dominant to see the relationship between mathematics and daily life.

On the second question the teachers were asked about their thoughts about the impact of mathematical connections on students' mathematical thinking skills. The teachers believe that mathematical connections improve mathematical thinking skills. This is generally illustrated in their statement that mathematical connections are a tool to develop their students' mathematical thinking skills, this can be seen from the definitions conveyed earlier and strengthened by the hierarchical mathematical nature so that it requires mathematical connection ability to grow up the mathematical thinking skills. Mathematical connections help students develop understanding and sharpen their thinking about mathematics. Connection refers to the ability to see and make connections between mathematical ideas, between mathematics and other subjects, and between mathematics and everyday life. This helps students understand what they are learning in mathematics [14]. Without mathematical
connection ability, students will experience difficulties in learning and solving mathematical problems. Other research results emphasize that the idea of using real-life connections in mathematics teaching increases students' interest and motivation for mathematics, develops positive attitudes about mathematics, contributes to students for their real-life preparation and develops conceptual understanding [15, 16, 17, 18, 19].

Furthermore, the teachers were asked to explain about what approach they use to support students' mathematical connection ability. In order to support students' mathematical connection ability is to encourage them to engage in various ways to solve problems. With activities like this of course students will be motivated to dig deeper into the concepts related to the problem so that students will realize the connection of mathematics that appear. Chapin et al [20] explain that by facilitating students 'conversations on mathematical problems, concepts and procedures improve students' understanding so that they can make deeper and clearer connections. Bruner [21] also argues that in order for students to learn mathematics more successfully, students should be given more opportunities to see links, both the relation between theorem and theorem, between theory and theory, between topics and topics, or between branch of mathematics (algebra and geometry for example). In a study that focused on informal strategies to support mathematical connections, Cooke & Buchholz [22] described strategies to improve students' mathematical thinking as follows: 1) Provide opportunities for self-expression; 2) Serve as a facilitator; 3) Provide opportunities for students to connect new understanding with previous knowledge; 4) Linking administrative tasks/class routines to mathematics; 5) Asking various questions; and 6) Encourage the use of appropriate mathematical terms.

Based on the description above, it can be concluded that in order to achieve the new goals of mathematics education, creating a learning environment that encourages mathematical connection skills is very important. In addition, teachers have a key role in creating a classroom environment where mathematical connection skills are dimensions that are indispensable in mathematics learning. Understanding teachers' perceptions and experiences in using mathematical connection skills in the classroom is expected to lead to discussions about developing students' high-level thinking skills and mathematical understanding through discussion, sharing, and asking questions in class.

4. Conclusion
Based on the results of this study, it was found that secondary school mathematics teachers have quite comprehensive and adequate knowledge about their views on the ability of mathematical connections. Also found respondents who could not provide examples of mathematical problems that contain characteristics of the strength of mathematical relationships even though they can define what a mathematical connection is. Associated with the development of teachers in the ability to connect mathematics in learning is still limited to the scope of relations between mathematics and mathematics itself and mathematics with everyday life.

Acknowledgments
Overall, it can be understood that secondary school mathematics teachers have sufficient comprehensive and adequate knowledge of their views on mathematical connection skills. However, this ability should not only be limited to defining the definition of mathematical connection ability, but must be transferred into the learning action/process so that it can support students' mathematical ability. Other important things from the results of this study indicate that there are respondents who cannot provide examples of mathematical problems that contain the characteristics of mathematical connection ability even though they are able to define what mathematical connection ability is. Other than that, the development of teachers about mathematical connection ability in learning is still limited to the scope of relations between mathematics and mathematics itself and everyday life.

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