A phenomenological intra-laminar plasticity model for FRP composite materials

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Abstract. The nonlinearity of fibre-reinforced polymer (FRP) composites have significant effects on the analysis of composite structures. This article proposes a phenomenological intra-laminar plasticity model to represent the nonlinearity of FRP composite materials. Based on the model presented by Ladeveze et al., the plastic potential and hardening functions are improved to give a more rational description of phenomenological nonlinearity behavior. A four-parameter hardening model is built to capture important features of the hardening curve and consequently gives the good matching of the experiments. Within the frame of plasticity theory, the detailed constitutive model, the numerical algorithm and the derivation of the tangent stiffness matrix are presented in this study to improve model robustness. This phenomenological model achieved excellent agreement between the experimental and simulation results in element scale respectively for glass fibre-reinforced polymer (GFRP) and carbon fibre-reinforced polymer (CFRP). Moreover, the model is capable of simulating the nonlinear phenomenon of laminates, and good agreement is achieved in nearly all cases.

1. Introduction
FRP composite materials are widely used in aircrafts and many other industries, due to their high strength, stiffness and good fatigue resistance. Generally, composite materials exhibit noticeable nonlinear behavior before failure. Experimental investigations reported in public literature shows that many FRP laminates with multiple-angle layups exhibit obvious nonlinear or irreversible deformations before destructive failure. Another well-known feature is that most FRP composite materials show significant nonlinearity in shear. O’Higgins [1] studied the damage initiation and growth in CFRP HTA/6376 and GFRP S2FM94 composite materials. In his research, nonlinear performances were observed, relating to not only the stress-strain response in shear, but also to the tensile responses of laminates with either [±22.5°]2, [±35°]2, [±55°]2, angle-ply layups or [45°]6 off-ply layup. H. Koerber [2] found similar phenomena in the off-ply laminates of carbon-epoxy GFRP IM7-8552. T. J. Vogler [3] also observed nonlinear behavior in the shear and transverse compression of unidirectional AS4/PEEK and investigated their interaction both experimentally and numerically.

Plasticity is not the only effect concerning nonlinearity. Additionally, material continuum degradation due to matrix micro-cracks or fibre/matrix debonding is another important feature inducing a soft decline; it is phenomenologically the same as nonlinearity. Consequently, many inelastic models have been proposed. Ladeveze [4] presented a widely used two-dimensional (2D)

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inelastic model including the combination of plasticity and the continuum damage mechanics (CDM) model. This model performed very well in simulations of composite structures, such as jointed fibre metal laminate structures [5] and composite bolted joints. More complexly, Comanho et al. [6] proposed a new three-dimensional (3D) transversely isotropic elastic-plastic constitutive model for unidirectional FRPs under the framework of the plasticity theory.

Unlike previous inelastic models, an attempt is made here to simplify the model describing the nonlinearity of composites. Although FRP composite materials generally have obvious non-linear behavior due to many physical reasons, such as damage accumulation resulting from fibre or matrix micro-cracks, fibre/matrix interface debonding and the nonlinearity of each individual constituent [7], disregarding the physical origins of the nonlinearity, a plasticity model is proposed to describe the nonlinear phenomenon. In addition, the hardening function is improved to make the plasticity model more rational.

2. Intra-laminar plastic model

2.1. The constitutive model

Nonlinearity is plastic deformation that occurs under mechanical loading. The model proposed here contains plasticity, so it predicts stress-strain behavior using plastic strain. The constitutive relations are given by:

\[ \sigma = C^e : \varepsilon^e \]  

(1)

where \( C^e \) is the fourth order tensors of unidirectional composite laminates, and superscript \( e \) represents elasticity. \( \varepsilon^e \) is the elastic strain tensor. \( \sigma \) is the stress tensor, and the symbol (:) denotes a double dot product of two tensors.

For the elastic-plastic constitutive model presented in this article, the total strain, \( \varepsilon \), is a sum of the elastic strain \( \varepsilon^e \) and plastic strain \( \varepsilon^p \):

\[ \varepsilon = \varepsilon^e + \varepsilon^p \]  

(2)

The yield domain, \( F \), is then defined as a function of the effective stress:

\[ F = F^p(\sigma) - R_0 - R(\tilde{\varepsilon}_p) \]  

(3)

where \( F^p(\sigma) = \sqrt{\sigma_{11}^2 + a^2 \sigma_{22}^2 + b \text{sign}(\sigma_{22})} \), and is called the plastic potential. \( R_0 \) is the initial threshold, defined as \( R(\tilde{\varepsilon}_p = 0) \), and it is evaluated experimentally. \( R(\tilde{\varepsilon}_p) \) is the hardening function, and \( a^2 \) is the shear-transverse plasticity coupling property. Empirically, the compensate parameter for transverse compressive stresses, \( b \) is given as:

\[ b = \frac{\sigma_{21,p}^o}{\sigma_{21,p}^o + \sigma_{22,p}^o} \]  

(4)

where \( \sigma_{21,p}^o \) and \( \sigma_{22,p}^o \) are the initial yielding stress for transverse tension and transverse compression, respectively.

Hardening is assumed to be isotropic, so \( R(\tilde{\varepsilon}_p) \) is obtained using the power-law relation as:

\[ R(\tilde{\varepsilon}_p) = R_o (1 - \eta e^{\beta \tilde{\varepsilon}_p})(1 - e^{\beta \tilde{\varepsilon}_p}) \]  

(5)

where \( \beta \), \( \eta \) and \( \mu \) are determined to fit the experimental hardening curve. Note that \( \tilde{\varepsilon}_p \) is the equivalent effective plastic strain, while \( \varepsilon^p \) is the effective plastic strain tensor. The effective plastic strain rate \( d\tilde{\varepsilon}_p \) is given as:
Where $\lambda$ is the plastic multiplier. The principle of plastic work equivalence is defined as [8]:

$$F^p(\sigma)d\varepsilon^p = \sigma : d\varepsilon^p$$

(7)

Substituting equations (3) and (6) into equation (7), the following relation is derived:

$$d\varepsilon^p = d\lambda$$

(8)

2.2. Numerical integration algorithm for plasticity

For a given load step, an elastic predictor stress at the beginning of an increment is used to evaluate whether the current load step is in the elastic or plastic domain. The yield function, $F$, will be less or equal to zero if no plasticity is occurring during the current increment. If $F$ is greater than zero, the correct plastic strains have to be determined to update stress. The back Euler implicit integration algorithm is employed to compute the plastic strains [7]. The integration scheme is implicit in terms of plasticity parameter $\Delta\lambda_{n+1}$.

$$\varepsilon_{n+1} = \varepsilon_n + \Delta\varepsilon_{n+1}$$

(9-a)

$$\varepsilon^p_{n+1} = \varepsilon^p_n + \Delta\varepsilon^p_{n+1} = \varepsilon^p_n + \Delta\lambda_{n+1} \frac{\partial F_{n+1}}{\partial \sigma_{n+1}}$$

(9-b)

$$\varepsilon^p_{n+1} = \varepsilon^p_n + \Delta\lambda_{n+1}$$

(9-c)

$$\bar{\sigma}_{n+1} = \sigma^{\text{yield}} - \Delta\sigma = C : (\varepsilon_{n+1} - \varepsilon^p_{n+1})$$

(9-d)

$$F_{n+1} = F(\bar{\sigma}_{n+1}, \varepsilon^p_{n+1})$$

(9-e)

The trial stress and plastic corrector are both in terms of the undamaged material stiffness tensor. The equation system equation (9) is solved iteratively using the Newton-Raphson algorithm, equations (10) and (11). For the numerical implementation in UMAT, it is assumed that the total strain, $\varepsilon_{n+1}$, remains constant during the iteration, so that the initial conditions include $\Delta\varepsilon_{n+1}$, $\varepsilon^p_n$, $\varepsilon^p_n$, $\Delta\lambda_n = 0$. Then, the aim of the Newton-Raphson iteration is to achieve proper $\Delta\lambda_{n+1}$; subsequently, $\varepsilon^p_{n+1}$ and $\varepsilon^p_{n+1}$ make the yield domain, $F(\bar{\sigma}_{n+1}, \varepsilon^p_{n+1}) \leq TOL$, where $TOL \leq 1 \times 10^{-5}$ is the error tolerance.

$$F^{(n)} + \left( \frac{dF}{d\Delta\lambda} \right)^{\text{(a)}} \delta\lambda^{(n)} = 0$$

(10)

Where

$$\delta\lambda^{(n)} = \Delta\lambda_{n+1} - \Delta\lambda_n$$

(11)

2.3. The tangent stiffness matrix

The consistency condition for the yield domain is written as:

$$\frac{\partial F}{\partial \sigma} : d\sigma + \frac{\partial F}{\partial \varepsilon^p} : d\varepsilon^p = 0$$

(12)

The elastic strain increment is given by:
\[
d e^\varepsilon = d \varepsilon - d \varepsilon_p
\]  
(13)

Subsequently, the stress increment is:

\[
d \sigma = D : d e^\varepsilon
\]  
(14)

Substituting equations (6)-(8) into equation (12), the increment of the plastic multiplier is given as:

\[
d \lambda = \frac{\partial F}{\partial \sigma} : D : d e^\varepsilon - \frac{\partial F}{\partial \sigma} : \frac{\partial F}{\partial \sigma} : \frac{\partial \varepsilon}{\partial \varepsilon_p}
\]  
(15)

The tangent stiffness matrix \( D^{eq} \) is given by dividing the stress increment and the total strain increment:

\[
D^{eq} = \frac{d \sigma}{d \varepsilon}
\]  
(16)

After substituting equations (7), (8), (13), (14) and (15) into equation (16), the tangent stiffness matrix is:

\[
D^{eq} = D - D : \frac{\partial F}{\partial \sigma} \otimes \frac{\partial F}{\partial \sigma} : D
\]  
(17)

Where \( \otimes \) is the dyadic product.

3. Model verification and simulation results

3.1. Hardening curve

The parameter comparisons of the four-parameter plasticity model proposed in this paper are shown in figure 1. \( R_\infty = 73.38 \), \( \eta = 0.6347 \), \( \beta = -61.51 \) and \( \mu = -871.2 \) were obtained from data fitting, which matches the HTA/6376 hardening curve very well. For comparison, two additional alternative values for each parameter were given. The hardening stress here corresponds to \( R(\varepsilon_p) \) in equation (3), not including the initial hardening stress \( (R_\eta) \). As we can see, each parameter plays its own irreplaceable role in the shape of the hardening curve. Physically, \( R_\infty \) is the ultimate hardening stress, which mainly controls the amplitude of the hardening curve. \( \eta \) is more related to the slope of the initial stage, playing the stretching-like role, while \( \beta \) influences the quantity of the hardening curve after the initial stage. Additionally, \( \mu \) slightly adjusts the incline extent of the first half of the hardening curve.

It can be seen that the four-parameter model is capable of giving a more rational description of the hardening stress curve. Each parameter has its own role, which increases not only the understanding of the hardening curve, but also of the data fitting.
3.2. Stress-strain response

To validate the proposed plasticity model, comparisons were carried out at the element scale. Stress-strain curves from two advanced FRP composite material systems for glass, GFRP E-glass/MY750/HY917/DY063 [9] and CFRP HTA/6376 [1], were compared with the experiment's method.

Figure 1. Parameter comparison for hardening model: (a) $R_c$ (b) $\eta$ (c) $\beta$ (d) $\mu$.

Figure 2. Stress-strain validation for E-glass/MY750/HY917/DY063: (a) compression (b) in-plane.
The numerical and experimental results of E-glass/MY750/HY917/DY063 (shown in Figure 2) exhibit significantly nonlinear behavior under the transverse compression and in-plane shear. Figure 3 shows the plasticity hardening process in the shears of HTA/6376. The above simulation results in elementary scale match the experiment remarkably. This indicates that the plasticity model, including the four-parameter hardening model and the improved plastic potential, has the capability to simulate nonlinearity correctly.

3.3. Intra-laminar nonlinear response of laminates

3.3.1. Model details. Four types of selected laminate lay-ups were analyzed under uniaxial tensile load, which were respectively $[\pm 35^\circ]_2$, $[45^\circ]_8$, $[\pm 55^\circ]_2$, and $[\pm 67.5^\circ]_2$ with a nominal ply thickness of 0.13 mm. Specially, $[45^\circ]_8$ is an off-axis layup. The typical stress-strain comparisons were made between experiments and simulations for these four laminates.

The axial stress in each of laminate specimen was calculated according to:

$$\sigma_x = \frac{P}{wt}$$  \hspace{1cm} (18)

where $P$ is the applied load, $w$ is the measured specimen width, and $t$ is the measured specimen thickness.

The axial strain in each of the laminates was calculated in the experiment as:
where $\delta$ is the recorded extensometer displacement in the experiments, $l_g$ is the extensometer gauge length, and $l_{off}$ is the gauge length offset value recorded when the applied load was zero. In simulations, the axial strain is given as:

$$\varepsilon_x = \frac{u_x}{l}$$

(20)

where $u_x$ is the applied displacement, and $l$ is the geometric length of the finite element model in the loading direction.

As shown in figure 4, because of the asymmetry of off-ply laminates, the constraints for the finite element model of off-ply laminates is different from those of the angle-ply laminates. The constraint end of the angle-ply laminates is held stationary, while for off-ply laminates, the constraint in the vertical direction of the loading is freed at the holding end.

3.3.2. Simulation results.

All simulations were implemented using implicit finite element model formulation. Abaqus (TM) solid elements C3D8R were chosen for the discretization. The minimum size of the mesh is 0.5 mm x 0.5 mm x 0.13 mm, a relatively fine mesh that is able to minimize mesh dependence. Since this study focuses on the plasticity of composites, laminate failure is not considered. However, the failure point is marked, as shown in figure 5.

**Figure 5.** Stress-strain curves of laminates: (a) $[\pm35^\circ]_2$, (b) $[45^\circ]_8$, (c) $[\pm55^\circ]_2$, (d) $[\pm67.5^\circ]_2$.

All four laminates exhibit obvious nonlinearity, as seen in figure 5. The $[\pm67.5^\circ]_2$ laminate does not show as much plasticity as the other three laminates, due to the relatively higher shear stresses of
the other three laminates. This indicates that shear nonlinearity plays a more important role in the CFRP HTA/6376 material system. There was very good agreement between experimental data and simulations in nearly all cases (figure 5). In only the 45° off axis tensile test, hardening behavior was slightly underestimated.

4. Conclusion
The proposed plasticity model is capable of simulating the nonlinearity of FRP composite materials under the frame of plasticity theory. Hardening behavior was successfully represented by the four-parameter hardening function, of which each parameter makes significant contributions. With the robustness added by the numerical algorithm and the detailed derivation of the tangent stiffness, the plasticity model in simulations achieved very good agreement with experiments.

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