LIMITS ON THE MASS OF THE LIGHTEST SUSY HIGGS

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We study the limits on the mass of the lightest Higgs in supersymmetric models extended with a gauge singlet when perturbative unification is required. We find that when maximum intermediate matter is added, the different evolution of the gauge couplings raises the mass bound from 135 GeV to 155 GeV. In these models perturbative unification of the gauge couplings is achieved in a natural way.

The minimal supersymmetric extension of the standard model (MSSM) successfully accommodates the standard model (SM) at low energies, and is arguably the best candidate to describe physics at energies beyond 1 TeV. The MSSM also predicts the unification of the gauge couplings at a scale of \( M_X \sim 2 \times 10^{16} \) GeV, a striking result that spans over many orders of magnitude. This unification can be regarded as a high energy prediction obtained from low energy measurements \( \text{(1)} \).

Supersymmetric models have been up to now flexible enough to respect all experimental constraints. However, this flexibility does not translate into a complete lack of predictivity. The most compelling low energy outcome is probably the existence of a light Higgs. In the MSSM the lightest state is a CP even neutral Higgs. Its mass is bounded at tree level by

\[
m_h^2 \leq M_Z^2 \cos^2 2\beta ,
\]

where \( \tan \beta \) is the ratio between the vacuum expectation values \( v \) and \( \bar{v} \) of the Higgs fields \( H \) and \( \bar{H} \) that give masses to the up and down quarks respectively \( \text{(2)} \). Eq. 1 is a very stringent bound as the only supersymmetric parameter that appears is \( \tan \beta \). It tells us that at tree level the lightest Higgs should have a mass lighter than \( M_Z \). Radiative corrections to Eq. 1 can be calculated and are known to relax the bound considerably \( \text{(3)} \).

\[^1\text{Talk presented at the XXXIIIrd Rencontres de Moriond, Electroweak and Unified Theories}\]
When the MSSM is enlarged with gauge singlets, trilinear terms appear in the superpotential

$$W \supset \lambda SH\bar{H}$$

and the tree-level bound becomes

$$m_h^2 \leq M_Z^2 \cos^2 2\beta + \lambda^2 \nu^2 \sin^2 2\beta,$$

with $\nu = \sqrt{v^2 + \bar{v}^2} = 174$ GeV. For an unrestricted value of $\lambda$ no bound can be extracted from (3). However, since the celebrated unification of the gauge couplings is obtained perturbatively, it is physically sound to impose the perturbative unification criterion. That is, all coupling constants should remain perturbative up to the unification scale $[4]$. This condition translates into a bound on $\lambda$. Indeed, consider the beta-function governing the evolution of $\lambda$ at one loop

$$\beta_\lambda = \frac{\lambda}{16\pi^2}(4\lambda^2 + 3h_t^2 + 3h_b^2 - g_1^2 - 3g_2^2),$$

where $h_t$ and $h_b$ are the top and bottom Yukawa couplings, and $g_1$ and $g_2$ are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, respectively. The positive coefficients of $\lambda$ and $h_t$ (the other terms have a smaller impact) imply that the running value of $\lambda$ increases with energy. The starting low energy value of $\lambda$ has to be small enough to avoid blowing up before reaching the unification scale. For a top quark of 180 GeV this argument implies a bound on $\lambda$ of $\sim 0.7$ $[5]$.

A possible way to increase the allowed low energy value of $\lambda$ is to introduce matter fields at intermediate scales. The effect of these fields would be indirect in the sense that they will modify the evolution of the gauge couplings. This argument was outlined in $[6]$. There, Higgs doublets were considered because they increase the evolution rate of $g_1$ and $g_2$ couplings that enter with negative sign in Eq. (4). However, it was found that the effect was always small, because the addition of matter is also limited by the perturbative criterion for the gauge couplings. Furthermore, the presence of such doublets spoils their unification.
Figure 2: Limits on the value of $\lambda$ at the weak scale. We plot the singlet model in the cases with a maximal matter content at intermediate scales (upper) and without extra matter (lower).

This analysis can nevertheless be improved [7]. First, it is important to note that $g_3$ and $h_t$, have the highest numerical value and therefore have a significant impact in the evolution equations. If the evolution rate of $g_3$ is increased (by the addition of coloured matter), then the evolution rate of $h_t$ is slowed because $g_3$ enters with a large negative coefficient in its beta–function (see [7] for the complete renormalization group equations up to two loops). Lower values of $h_t$ in Eq. (4) imply higher allowed starting values for $\lambda$. This effect is shown in Fig. 1, where the evolution of $g_3$, $h_t$ and $\lambda$ is depicted with and without intermediate matter. One sees that when matter is present $h_t$ even decreases with energy, mainly due to the higher evolution rate of $g_3$. Second, there is a way to introduce extra matter fields that respect perturbative unification of the gauge couplings. Complete representations of a simple group [$SU(5)$, $SO(10)$, $E_6$, ...] that contains $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a subgroup modify the (one-loop) running of the three gauge couplings in such way that they still meet at the same unification scale $M_X$, but with a higher final value. This scenario allows larger intermediate values of $g_1$ and $g_2$ together with smaller values of $h_t$ and defines the setting for the absolute perturbative bound on $\lambda$. The presence of extra matter can be motivated by models with gauge mediated supersymmetry breaking (GMSB) [8]. Minimal scenarios of GMSB could in fact be closer to the singlet model than to the MSSM.

We can now obtain the bound on $\lambda$ with vector–like matter at intermediate scales. We consider the full two loop renormalization group equations for the evolution of the couplings. The first and second coefficients of the beta–functions are of opposite sign. The couplings would then evolve up to the point where the beta–functions cancel. We shall consider that any coupling $g$ is non-perturbative if at a scale below $M_X$ the running value is $\frac{g}{\pi} > 0.3$. The change from the perturbative to the non-perturbative regime (when the constants reach their limiting value) is quite abrupt, and therefore the results do not depend appreciably on the actual cutting value chosen.
The results for the bounds on $\lambda$ are depicted in Fig. 2. The lower curve corresponds to the case of the singlet model without extra matter. On the left and right hand side of the plot the first couplings to become non–perturbative are the top and bottom Yukawas respectively. In the middle zone, the $\lambda$ coupling itself imposes the dominant condition. The allowed range for $\tan\beta$ is $1.88 \leq \tan\beta \leq 51.2$. Beyond these values, even for $\lambda = 0$, the perturbative criterion is not satisfied. The absolute limit is $\lambda = 0.69$, which occurs at $\tan\beta = 10$.

In the upper curve a maximal content of extra matter is added before the gauge couplings become non–perturbative. There are several ways to reach this maximal content. We have plotted the case of four $5+\bar{5}$ representations of SU(5), which is the lowest dimensional vector representation of a single group containing the SM symmetries, introduced at 250 GeV. The same three zones appear, but now the allowed range of $\tan\beta$ is enlarged, $\tan\beta$ is $1.19 \leq \tan\beta \leq 74.4$. This a relevant outcome, as in these models the maximum value of the mass bound is obtained at lower $\tan\beta$. The absolute maximum on $\lambda$ is 0.82 at $\tan\beta = 8$, a 19% higher than the previous case.

The bounds on $\lambda$ can now be translated into mass bounds, once the radiative corrections of the top-quark loops are added [1], which amount a shift of around 30 GeV for squark masses of 1 TeV and no mixing. Our results are shown in Fig. 3. As is clear from the plot, if the presence of a singlet takes the MSSM bound from $m_h \leq 128$ GeV to $m_h \leq 135$ GeV, the presence of matter at intermediate scales pushes this bound further up, to $m_h \leq 155$ GeV. The actual numerical values of the bounds depend on the precise input parameters (top mass, strong coupling constant, mixings, etc.), however it is a generic feature that the addition of vector–like matter has a significant impact in raising the value of the bound without spoiling unification of the gauge couplings.
Acknowledgments

This work was supported by CICYT under contract AEN96-1672 and by the Junta de Andalucía under contract FQM-101.

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