Treatment of the $\Delta$ current in electromagnetic two-nucleon knockout reactions

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Abstract

The contribution of the $\Delta(1232)$ isobar to the electromagnetic current of the two-nucleon system and its role in ($\gamma$,NN) processes is investigated. The difference between the genuine $\Delta$-excitation current and that part of the current connected to the deexcitation of a preformed $\Delta$ in the target nucleus is stressed. The latter cannot lead to a resonant behaviour of matrix elements for energies in the $\Delta$ region. The reaction $^{16}\text{O}(\gamma,pp)^{14}\text{C}$, where the $\Delta$ contribution is dominant at intermediate energies, is considered. The large variations found in the cross sections for different treatments stress the need for a proper treatment of the $\Delta$ current for a clear understanding of the reaction mechanism of two-nucleon emission processes.

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I. INTRODUCTION

The electromagnetically induced two-nucleon knockout serves as a tool to study short-range correlations between two nucleons in a nucleus. Thereby, one assumes that the photon interacts with the correlated pair through a one-nucleon current. However, nucleon pairs can also be ejected by two-nucleon currents which effectively take into account the influence of subnuclear degrees of freedom like mesons and isobars. Therefore, in order to estimate this competing effect quantitatively, a reliable treatment of meson exchange as well as isobar currents is necessary before one can draw definite conclusions on the role of short-range correlations. Experimental data of two-nucleon photoemission in the \( \Delta (1232) \) region were taken in different laboratories \([1–4]\), while only one exploratory experiment on the \((e, e' pp)\) reaction \([5]\) was completed. However, many new results will become available in the near future, exploiting continuous-wave accelerators and tagged-photon facilities.

In this note we would like to point out a pitfall which one may encounter when considering contributions from intermediate \( \Delta \) isobars to electromagnetic processes via the effective operator approach. To this end we remind the reader at the two first-order contributions of the \( \Delta \) to the effective electromagnetic two-body current of a two-nucleon subsystem shown diagramatically in Fig. 1. The first one (I) describes the \( \Delta \) excitation with subsequent deexcitation by pion exchange while the second (II) describes the time interchange of the two steps, i.e., first excitation of a virtual \( \Delta \) by pion exchange in a NN collision and subsequent deexcitation by photon absorption. In other words parts I and II correspond to N\( \Delta \) admixtures in the final and the initial NN subsystem, respectively. They would be treated as explicit components of the wave function in the approach of nuclear isobar configurations (IC) \([6,7]\). Consequently, it is important to note that in diagram II the \( \Delta \) is always far off-shell being part of the initial state irrespective of the energy transferred to the system by the real (or virtual) photon, whereas in diagram I the \( \Delta \) can become on-shell for a sufficiently high energy transfer. This then gives rise to a pronounced resonant behaviour of the matrix elements of diagram I when varying the energy transfer in the region between, say, 200 and

\( \ldots \).
400 MeV. These features appear automatically in the IC approach, however, in the effective operator approach only if one keeps the full Δ propagator in the intermediate state, making the effective two-body operator nonlocal.

For (γ,NN) calculations on complex nuclei, it is necessary to avoid this nonlocality in order to keep the numerical effort within reasonable limits. Therefore, often the Δ propagator is taken in the simplest static approximation keeping only the baryon mass difference $M_\Delta - M_N$. In this case the two contributions of Fig. I can be combined into one effective local two-nucleon operator. This approximation appears reasonable at low energies but it certainly fails when the transferred energy allows the excitation of an on-shell Δ. In the latter case one might be tempted to replace $(M_\Delta - M_N)^{-1}$ by a resonant energy-dependent Δ propagator. Indeed this procedure has been followed in the past. However, it leads to a wrong effective operator, as is outlined in detail in the next section, and results in a strong overestimation of the contribution of diagram II. This is shown in section [III], where different treatments of the Δ current are compared by means of a calculation for the $^{16}\text{O}(\gamma, pp)^{14}\text{C}$ process within the framework of Refs. [8,9].

II. FORMALISM

The effective current operator shown in Fig. I is given by

$$j_\Delta = j_\Delta^{(I)} + j_\Delta^{(II)} + (1 \leftrightarrow 2).$$

(1)

In the following we restrict ourselves to the dominant magnetic dipole N\leftrightarrow Δ transition. For simplicity the hadronic NΔ \leftrightarrow NN transition is described by static π-exchange. The inclusion of the ρ-exchange is straightforward but not essential for our purpose. Under these assumptions, the excitation current reads

$$j_\Delta^{(I)}(\vec{q}) = \gamma \vec{\tau}_N \cdot \vec{\tau}_N \sigma^{(1)}_N \sigma^{(2)}_N \frac{\vec{k} \sigma^{(2)}_N \cdot \vec{k}}{\vec{k}^2 + m_\pi^2} G_{\Delta}(\sqrt{s_f}) \vec{\tau}_{\Delta N,3} i\vec{\sigma}_N \times \vec{q},$$

(2)

and the deexcitation part
\[ f^{(II)}_{\Delta}(q) = \gamma \bar{\tau}^{(1)}_{N\Delta} \cdot i\sigma^{(1)}_{N\Delta} \times \bar{q} \cdot G_{\Delta}(\sqrt{s_{II}}) \bar{\tau}^{(1)}_{NN} \cdot \bar{\tau}^{(2)}_{NN} \cdot \sigma^{(1)}_{N\Delta} \cdot \vec{k} \cdot \sigma^{(2)}_{NN} \cdot \vec{k} \cdot k^2 + m^2 \pi, \]  

where \( \vec{q} \) is the photon momentum, and \( \vec{k} \) is the momentum of the exchanged pion. The factor \( \gamma \) collects various coupling constants, \( \gamma = f_{N\Delta}f_{\pi NN}f_{\pi N\Delta}/m^2 \pi \). The propagator of the resonance is denoted by \( G_{\Delta} \). It depends on the invariant energy \( \sqrt{s} \) of the \( \Delta \). Neglecting the kinetic energy of the relative motion of the intermediate \( N\Delta \) state, one obtains a local approximation to \( G_{\Delta} \). It reads

\[ G_{\Delta}(\sqrt{s}) = \frac{1}{M_{\Delta} - \sqrt{s} - \frac{i}{2} \Gamma_{\Delta}(\sqrt{s})}, \]  

where \( \Gamma_{\Delta} \) is the energy-dependent decay width of the \( \Delta \) and \( M_{\Delta} = 1232 \text{ MeV} \) its mass.

Clearly, \( \sqrt{s} \) can be very different in diagram I and II. In diagram II, it does not depend on the photon energy \( E_{\gamma} \), and it is reasonable to approximate it by the nucleon mass

\[ \sqrt{s_{II}} = M_N. \]  

On the other hand, \( \sqrt{s_I} \) depends on \( E_{\gamma} \). It grows with \( E_{\gamma} \) and for \( \sqrt{s_I} \approx M_{\Delta} \) the \( \Delta \) is essentially on-shell. Following the recent suggestion made in [10] for the choice of \( \sqrt{s_I} \), the calculations presented below use

\[ \sqrt{s_I} = \sqrt{s_{NN}} - M_N, \]  

where \( \sqrt{s_{NN}} \) is the experimentally measured invariant energy of the two fast outgoing nucleons in an \( A(\gamma, NN)A - 2 \) reaction. The energy dependence of the two parts of the \( \Delta \) current can also be considered from a different point of view. For forward propagating exchanged pions, both parts are related to the process of electromagnetic pion production on one nucleon followed by its reabsorption on the second nucleon. Part I is connected to the \( s \)-channel contribution of the \( \Delta \) which leads to the well-known resonant \( M^{3/2}_{1+} \) pion production multipole, whereas part II is connected to the \( u \)-channel contribution which has only a smooth energy dependence.

Only for low energy transfers, say below 100 MeV, it may be justified to approximate also \( \sqrt{s_I} \approx M_N \). Then the \( \Delta \) propagators in the excitation and deexcitation parts are equal
and the spin and isospin structure of their sum simplifies due to the cancellation of terms. To see this, one first has to rewrite Eqs. (2) and (3) using the following identity for the $N \leftrightarrow \Delta$ transition spin (isospin) operators

$$\vec{\sigma}_{N\Delta} \cdot \vec{a} \vec{\sigma}_{N\Delta} \cdot \vec{b} = \frac{2}{3} \vec{a} \cdot \vec{b} - \frac{i}{3} \vec{\sigma}_{NN} \cdot \vec{a} \times \vec{b},$$  

(7)

where $\vec{a}$ and $\vec{b}$ are two arbitrary vectors. One finds

$$J^{(I)}_\Delta(q) = \frac{1}{9} \gamma \left[ 2\vec{T}_{NN,3}^{(2)} - i \left( \vec{T}_{NN,3}^{(1)} \times \vec{T}_{NN,3}^{(2)} \right) \right] \left( 2i\vec{k} - \vec{k} \times \vec{\sigma}_{NN}^{(1)} \right) \times \vec{q} G_\Delta(\sqrt{s_I}) \frac{\vec{\sigma}_{NN}^{(2)} \cdot \vec{k}}{\vec{k}^2 + m^2_\pi},$$  

(8)

and

$$J^{(II)}_\Delta(q) = \frac{1}{9} \gamma \left[ 2\vec{T}_{NN,3}^{(2)} + i \left( \vec{T}_{NN,3}^{(1)} \times \vec{T}_{NN,3}^{(2)} \right) \right] \left( 2i\vec{k} + \vec{k} \times \vec{\sigma}_{NN}^{(1)} \right) \times \vec{q} G_\Delta(\sqrt{s_{II}}) \frac{\vec{\sigma}_{NN}^{(2)} \cdot \vec{k}}{\vec{k}^2 + m^2_\pi},$$  

(9)

Then, in the low-energy (le) approximation, i.e. using $G_\Delta(\sqrt{s_I}) = G_\Delta(\sqrt{s_{II}}) = (M_\Delta - M_N)^{-1}$, one obtains for the total current

$$J^{(le)}_\Delta(q) = \frac{2}{9} \gamma i \left[ 4\vec{T}_{NN,3}^{(2)} \vec{k} + \left( \vec{T}_{NN,3}^{(1)} \times \vec{T}_{NN,3}^{(2)} \right) \vec{k} \times \vec{\sigma}_{NN}^{(1)} \right] \times \vec{q} \frac{1}{M_\Delta - M_N} \frac{\vec{\sigma}_{NN}^{(2)} \cdot \vec{k}}{\vec{k}^2 + m^2_\pi}. $$  

(10)

This form is usually quoted in the literature (see e.g. [11,12]). It serves as starting point for model calculations in heavier nuclei [12,14] as well as in few-nucleon reactions [15,17].

The simple replacement of the low energy propagator $(M_\Delta - M_N)^{-1}$ in Eq. (10) by the resonant $G_\Delta$ of Eq. (4) in order to obtain an operator which is more appropriate for studying photon absorption at higher energies is, however, in no way justified. This procedure implies a strong overestimation of the deexcitation part at higher energies since it introduces a resonant behaviour also there. Nevertheless, it has been used in the past (see e.g. [18,19]). To make this point clearer, it is useful to consider the isospin matrix structure of the excitation and deexcitation current. Table 1 summarizes the relevant isospin matrix elements. Note
that transitions between \( np \) pairs with isospin \( T = 1 \) are always forbidden for an isovector current. According to Table I, the above replacement leads to wrong conclusions, in particular when the absorption on \( pp \) pairs is studied, or in cases where the absorption on \( np \) pairs with isospin \( T = 1 \) is expected to be relevant. In \((\gamma, np)\) reactions, only those contributions which proceed via absorption on \( np \) pairs with \( T = 0 \) (like the quasi deuteron mechanism for example) remain unaffected with respect to the correct prescription, as the contribution of the operator (8) vanishes.

**III. RESULTS**

In this section we investigate the dependence of \((\gamma, pp)\) cross sections on the different treatments of the \( \Delta \) current within the theoretical model of Ref. [9]. Although different channels can be considered in the model, we here have chosen the \((\gamma, pp)\) channel since there the \( \Delta \) current is dominant at intermediate energies.

Exclusive cross sections of the \( ^{16}\text{O}(\gamma, pp)^{14}\text{C} \) knockout reaction have been calculated for transitions to the ground state and low-lying discrete excited states of \( ^{14}\text{C} \). In the model the final state \(|J^{\pi}\rangle\) of the residual nucleus is obtained from the removal of a nucleon pair coupled to \( J \). The correlated wave function of the pair is calculated with the single-particle states of Ref. [20] and the Jastrow-type correlation function of Ref. [21]. The final-state interaction is taken into account by means of the optical potential of Ref. [22], describing the interaction of each one of the two protons with the residual nucleus. More details of the model and of the theoretical ingredients of the calculation are given in Ref. [9].

The differential cross sections of the \( ^{16}\text{O}(\gamma, pp)^{14}\text{C}(\text{g.s.}) \) reaction at \( E_{\gamma} = 150 \text{ MeV} \) and \( 300 \text{ MeV} \) are shown in Fig. 2 in a coplanar and symmetrical kinematics as a function of the angle \( \gamma \) between the photon and one of the symmetrically outgoing protons. Three different treatments of the \( \Delta \) current are compared. The solid curves refer to the correct form, i.e., use of the resonant \( \Delta \) propagator in the excitation current only (part I of Fig. 1). Adopting the resonant propagator in both excitation and deexcitation currents (part I and
II) leads to the dotted curves. The dashed curves refer to the low energy approximation of Eq. (10). In the following, these three currents are referred to as \( \vec{j}_\Delta^{(\text{RN})} \), \( \vec{j}_\Delta^{(\text{RR})} \), and \( \vec{j}_\Delta^{(\text{NN})} \), respectively, indicating the use of either the resonant (R) or energy-independent nonresonant (N) propagator in part I and II. Already at 150 MeV, \( \vec{j}_\Delta^{(\text{RN})} \) and \( \vec{j}_\Delta^{(\text{RR})} \) lead to peak cross sections which differ by nearly a factor two. The difference grows with the photon energy and at 300 MeV the peak cross sections differ by more than a factor five. As one would expect, \( \vec{j}_\Delta^{(\text{NN})} \) underestimates and \( \vec{j}_\Delta^{(\text{RR})} \) overestimates the cross section. One notes also a different shape of the angular distributions for \( \vec{j}_\Delta^{(\text{RN})} \) on the one hand and \( \vec{j}_\Delta^{(\text{RR})} \) or \( \vec{j}_\Delta^{(\text{NN})} \) on the other hand. At 300 MeV the minimum is less pronounced for the solid curve. This is a consequence of the fact that \( \vec{j}_\Delta^{(\text{RN})} \) has a different operator structure with respect to its spin and angular momentum dependence, since its parts I and II do not enter with equal weight as in \( \vec{j}_\Delta^{(\text{RR})} \) and \( \vec{j}_\Delta^{(\text{NN})} \). For the same reason one may expect qualitatively different predictions for polarization observables.

In Fig. 3 the differential cross section in coplanar and symmetrical kinematics at zero recoil momentum of the residual nucleus is plotted as a function of the photon energy. This corresponds to the region in Fig. 2 where the cross section is maximal. Fig. 3 clearly shows the strong overestimation of the cross section when \( \vec{j}_\Delta^{(\text{RR})} \) is used. This certainly would affect any analysis of experimental \( (\gamma, pp) \) data in the \( \Delta \) region in view of the role of short-range correlations. The low-energy approximation \( \vec{j}_\Delta^{(\text{NN})} \) can of course not predict a resonance peak. The dash-dotted curve has been calculated with \( \vec{j}_\Delta^{(\text{RR})} \) as the dotted one, but the choice in Eq. (10) has been replaced by the energy assignment \( \sqrt{s_\text{T}} = E_\gamma + M_N \) used, e.g., in Ref. [19]. It shifts the resonance position towards lower energies and thus leads to a further overestimation of the cross section for energies below 260 MeV. The effect of this choice of \( \sqrt{s_\text{T}} \) on the angular distribution has already been discussed in Ref. [9] for the \( (e, e'pp) \) reaction.

Calculations for the transition to the excited \( 1^+ \) and to the first excited \( 2^+ \) state of \(^{14}\text{C}\) have also been performed. Qualitatively, the results agree with the former ones. However, the size of the overestimation varies. This again can be traced back to the different operator
structure of $\vec{j}_\Delta$(RN) and $\vec{j}_\Delta$(RR). It becomes visible when the quantum numbers of the active nucleon pair change and cannot simply be simulated even by an energy-dependent rescaling factor.

Similar results are obtained in the $(e, e'pp)$ reaction, where the presence of the longitudinal contribution, due to correlations, reduces the effects of the different treatments of the $\Delta$ current.

**IV. SUMMARY**

In this paper we have discussed the treatment of the $\Delta$ isobar current in electromagnetic two-nucleon knockout reactions based on an effective two-nucleon operator. It has been emphasized that this current consists of a $\Delta$ excitation part and a $\Delta$ deexcitation part corresponding to a $\Delta$ admixture in the initial state. These two parts are related to the $s$- and $u$-channel contributions of the $\Delta$ to electromagnetic pion production on a nucleon. Only the excitation part of the current has a resonant energy dependence which can result in a characteristic energy dependence of observables.

This different energy dependence of excitation and deexcitation parts has to be taken care of when going to higher energy transfers. In particular, it does not allow to replace the propagator $(M_\Delta - M_N)^{-1}$ of the simplest static approximation to the current (which is justified in the low-energy region) by a resonant propagator. The error is two-fold: first, the deexcitation part would get a wrong energy dependence and, secondly, one would lose parts of the excitation current which have been cancelled by terms in the deexcitation current. In view of the fact that this prescription has been used in the literature, we have performed an explicit calculation for the $^{16}\text{O}(\gamma, pp)^{14}\text{C}$ reaction. It shows that the simple replacement may lead to an overestimation of the cross section (in the preferred kinematic region around zero recoil momentum of the residual nucleus) by more than a factor five. Such a variation of the $\Delta$ contribution is definitely too large when one wants to use two-proton knockout as a tool to study short range correlations in the nucleus.
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FIG. 1. The $\Delta$ contribution to the current of the two-nucleon system: $\Delta$-excitation current (I) and $\Delta$-deexcitation current (II).
FIG. 2. The differential cross section of the $^{16}\text{O}(\gamma,pp)^{14}\text{C}(\text{g.s.})$ reaction in coplanar and symmetrical kinematics as a function of $\gamma$ using different treatments of the $\Delta$ current: $\vec{j}_\Delta$(RN) solid, $\vec{j}_\Delta$(RR) dotted, and $\vec{j}_\Delta$(NN) dashed curves.
FIG. 3. The differential cross section of the $^{16}\text{O}(\gamma,pp)^{14}\text{C}(\text{g.s.})$ reaction in coplanar and symmetrical kinematics at zero recoil momentum as a function of the photon energy. Line convention as in Fig. 2. The dash-dotted curve has been calculated with $\vec{j}_\Delta(\text{RR})$, but using the energy assignment $s_I^{1/2} = E_\gamma + M_N$ as in Ref. 19.
TABLE I. Isospin matrix elements (terms in square brackets in Eqs. (8) and (9)) of the excitation (I) and deexcitation part (II) of the $\Delta$ current for various transitions.

| Transition                  | $J_\Delta^{(I)}$ | $J_\Delta^{(II)}$ |
|-----------------------------|------------------|-------------------|
| $np(T = 0) \rightarrow np(T = 1)$ | $-4$             | $0$               |
| $np(T = 1) \rightarrow np(T = 0)$ | $0$              | $-4$              |
| $pp/nn \rightarrow pp/nn$   | $\pm 2$          | $\pm 2$           |