Reliability-based evaluation of scouring around dual bridge piers

The causes of uncertainty involved in temporal variation of the depth of cavern formation due to clear water scouring at dual piers with tandem arrangement are identified in this study. In an example, the probability of failure induced by scouring around dual piers is estimated for various pier spacing using probabilistic analysis. The effects of changes in the coefficient of variation of probabilistic scouring variables and probability distributions on the probability of the loss of stability, are also investigated. It was established that the effects of variations in the coefficient of variation on the probability of the loss of stability are more significant compared to that on the probability distributions of the scour depth.

Key words: bridge, pier, probability of the loss of stability, scour, tandem arrangement, uncertainty

Prethodno priopćenje

Ocjena podlokavanja oko dvojnih stupova mosta primjenom analize pouzdanosti

U ovom se istraživanju određuju uzroci nepouzdanosti uslijed vremenske varijacije dubine kaverne nastale podlokavanjem u uvjetima bistre vode na mjestu dvojnih stupova tandemskog razmještaja. Na primjeru se probabilističkom analizom ocjenjuje vjerojatnost gubitka stabilnosti uslijed podlokavanja oko dvojnih stupova, za razne razmake stupova. Istraženi su i utjecaji promjene koeficijenta varijacije varijabli podlokavanja, te funkcije razdiobe vjerojatnosti, na vjerojatnost gubitka stabilnosti. Utvrđeno je da su utjecaji promjene koeficijenta varijacije na vjerojatnost gubitka stabilnosti značajniji od razdiobe vjerojatnosti dubine podlokavanja.

Ključne riječi: most, stup, vjerojatnost gubitka stabilnosti, podlokavanje, tandemski razmještaj, nepouzdanost

Beurteilung der Unterspülung um die Doppelpfeiler einer Brücke anhand einer Zuverlässigkeitsanalyse

In dieser Studie werden die Ursachen für die Unzuverlässigkeit durch die zeitliche Variation der Tiefe der Kaverne bestimmt, die durch Unterspülung mit klaren Wasserbedingungen am Standort von Doppelpfeilern in Tandemanordnung erzeugt wird. Am Beispiel wird mittels der probabilistischen Analyse die Wahrscheinlichkeit eines Stabilitätsverlustes aufgrund der Unterspülung von Doppelpfeilern mit unterschiedlichen Säulenabständen bewertet. Die Auswirkungen der Änderung des Variationskoefizienten probabilistischer Variablen der Unterspülung sowie der Wahrscheinlichkeitsverteilungsfunktion auf die Wahrscheinlichkeit eines Stabilitätsverlustes wurden ebenfalls untersucht. Es wurde festgestellt, dass die Auswirkungen der Änderung des Variationskoefizienten auf die Wahrscheinlichkeit eines Stabilitätsverlustes signifikant waren als die Wahrscheinlichkeitsverteilung der Unterspülungstiefe.

Schlüsselwörter: Brücke, Säule, Wahrscheinlichkeit eines Stabilitätsverlustes, Unterspülung, Tandemanordnung, Unzuverlässigkeit
1. Introduction

Excessive scouring around bridge piers and abutments causes loss of soil support around these elements, which may eventually lead to collapse. Safe design of bridges crossing wide rivers with loose boundaries is based on reliable determination of the local scour depth at bridge infrastructural elements. Deterministic scour prediction equations consider constant values for variables involved in the scouring phenomenon. In fact, most of the scour influencing variables are of probabilistic nature. Therefore, uncertainty-based scour computation is essential. Depending on the magnitude of the uncertainty, and hence the probability of failure, a bridge may be severely damaged or may even collapse during its designed lifetime. That is why computation of the uncertainty-based probability of failure is superior to the conventional evaluation of safety using a deterministic safety factor [1, 2].

The bridge pier scouring reliability can be computed using a scour-prediction equation derived according to a wide range of governing independent variables provided that the probability distributions and coefficients of variation of the governing variables involved in this equation are known. To be on the safe side, a sensitivity analysis needs to be conducted for various scenarios that are likely to occur during the design life of a bridge. For example, possible alterations in watershed use may influence the flow and sedimentologic regime of the river. Therefore, some variables, such as the approach flow velocity, flow depth, bed material characteristics, etc., may change from the original values considered in the design phase. This may not only lead to changes in coefficients of variation but also to the variation in probability distribution of governing variables involved in the scouring mechanism. Such possible adverse effects can be simulated by assigning different coefficient of variations and probability density functions to those variables [2]. Since the probabilistic approach based on incorporation of uncertainties intends to quantify the probability of failures in a more realistic manner, the corresponding design of scour countermeasures would be conservative. Thus, in a conservative design, where the scour countermeasures are designed and placed considering all adverse hydraulic conditions, the annual costs of scour repair and maintenance will decrease substantially. That is why there is a growing tendency to apply the reliability theory to structural systems in the recent years [3]. A pioneering study on the reliability based analysis of bridge pier scouring was conducted by [1] who developed a relationship between the probability of failure and safety factor for a particular combination of variables.

Depending on the structural/geotechnical requirements as well as loading conditions, bridges may have a group of piers. In case of dual bridge piers, as discussed in the next section, the maximum instantaneous clear water scour depth at dual piers, where the approach flow does not contain sediment, is higher compared to single pier case due to additional scouring mechanisms [4]. For the same depth of footing, a single pier case will be subject to less scour compared to the case of dual piers, and so the probability of failure is much smaller compared to the dual pier case. The use of a single pier equation for dual pier case will underestimate failure probability and weaken the safety of the bridge. Therefore, conducting separate reliability analyses for both cases is of importance. Earlier studies dealing with the probability of failure [1, 2] considered mainly single pier to set up a methodology. After studying scouring patterns at dual piers [4], it may be appropriate to quantify probability of failure for dual piers, perhaps as a pioneering study.

The motivating aim of this study is to close a gap in the literature considering the lack of reliability-based evaluation of temporal variation of scour depth around dual bridge piers. This study deals with the computation of probability of failure induced by clear water scouring action at dual piers with tandem arrangement, based on the use of a scour depth equation obtained in a different study [5]. This equation presents the explicit time-dependent scour depth for dual piers. Uncertainties involved in governing variables are interpreted and defined by the coefficient of variation and probability density functions characterizing the probabilistic variation of the variable concerned, i.e. the approach flow depth, $d_0$, the mean approach flow velocity, $u$, and the median sediment size, $D_{50}$. Monte Carlo simulations are employed to generate random scour influencing variables to enable computation of the probability of failure. The methodology is demonstrated with an application in which a relationship is obtained between the safety factor and the probability of failure. A sensitivity analysis is also conducted to assess the effects of possible variations of statistical data on the probability of failure. The final decision on the depth of a combined pier footing is given according to the worst possible scenario which could lead to the highest probability of failure at a particular bridge site.

2. Clear water scouring at dual piers

2.1. Scour mechanisms in dual piers

In the case of dual bridge piers with tandem arrangement, the clear water scour holes around the front and rear pier may overlap and result in a combined scour hole. When the pier spacing ratio, $b/d > 0.09$ the scour holes around the front and rear piers overlap (in this ratio $b$ is the pier diameter and $d$ is the centre-to-centre spacing between piers). When $b/d < 0.09$, these scour holes act as separate holes [6]. The primary mechanisms of local scour at a single bridge pier are downflow, horse-shoe vortices, and wake vortices.
In the case of dual piers with tandem arrangement, where the front and rear pier scour holes are overlapping, additional mechanisms take place, namely the reinforcing, sheltering, and vortex shedding [6]. Among these three additional mechanisms, reinforcing is responsible for the increased maximum instantaneous scour depth at dual piers compared to that of a single pier. Reinforcing mechanism affects the scour depth at the front pier. Due to the interference of the scour holes around the front and rear piers, the scour depth at the upstream face of the front pier increases according to this mechanism. When the scour holes overlap, the slope of the intermediary dune, which is the accretion zone between the piers, decreases and the bed level at this zone lowers as shown in Figure 1. Therefore, transportation of bed material around the front pier is facilitated and the scour depth, \( d_s \), at the upstream of the front pier increases compared to the single pier case. The intensity of this mechanism decreases with an increase in pier spacing [7].

![Figure 1. Schematic representation of reinforcing and sheltering effect (Redrawn from [8])](image)

2.2. Evolution of scouring at dual piers

An experimental study was conducted by [5, 9] in the Hydromechanics Laboratory of the Civil Engineering Department, Orta Dogu Teknik Universitesi, Ankara, Turkey. A rectangular flume with the constant bed slope, filled with uniform bed material, was used in the experiments. Two identical PVC pier models with tandem arrangement were tested with three different pier diameters, \( b \). The piers were placed with various pier spacing ratios, \( b/d \), ranging from 0.917 to 0.946, were used in the experiments, where \( u \) is the mean approach flow velocity and \( u_r \) is the mean threshold velocity of the approach flow initiating the motion of sediment particles at the bed surface. Therefore, combined scour holes were obtained for all values of \( b/d \) under clear water conditions. Further information regarding the experimental setup, scope, procedure, and results is available in [5, 9]. A photo of the bed topography obtained by one of the experiments where \( b = 7.5 \text{ cm} \), \( b/d = 0.25 \), and \( t = 360 \text{ min} \), is given in Figure 2 showing the combined scour hole around dual piers.

![Figure 2. A photo of bed topography](image)

For dual piers, a combined footing may be designed, whose depth is decided according to the maximum depth of scour in close vicinity to piers [4]. At any time, the maximum instantaneous clear water scour depth in a combined scour hole occurs at the upstream face of the front pier [4]. Therefore, in the experiments, scour depth, \( d_s \), at the upstream face of the front pier versus time, \( t \), is recorded at frequent intervals. The continuous scour depth is measured by means of the SeaTek 5 MHz Ultrasonic Ranging System [10]. Using the measured \( d_s \) versus \( t \) values, a multiple linear regression equation, having a determination coefficient, \( R^2 = 0.84 \), is obtained for the scour depth at the upstream of the front pier [5]:

\[
\frac{d_s}{b} = 0.80\left(\frac{d_0}{b}\right)^{0.155}F_{d}^{2.357}(T_s)^{0.123}\left(\frac{1}{b}\right)^{0.102}
\]

(1)

where \( d_s \) is the scour depth, \( b \) is the pier diameter, \( d_0 \) is the approach flow depth, \( F_d \) is the densimetric Froude number, \( T_s = tD_{50}(\Delta gD_{50})^{-0.5/b} \) is the dimensionless time, \( t \) is the time, \( D_{50} \) is the median sediment size, \( \Delta \) is the relative submerged density of sediment particles, \( g \) is the gravitational acceleration, and \( d \) is the centre-to-centre spacing between piers. To the authors’ knowledge, this equation presents - for the first time in literature - the explicit time-dependent scour depth variation for dual piers. The results of the experimental study show that the maximum instantaneous scour depth increases with an increase in the piers spacing ratio, \( b/d \), in the studied range.
3. Interpretation of sources of scouring uncertainties

The accuracy of computing the probability of failure is closely linked to the level at which uncertainties induced by different sources are incorporated into the simulations. The model uncertainty arising from eqn. (1) is ignored and only the parameter uncertainty is incorporated in the analyses. Most of the governing scouring variables are uncertain in nature, such as the approach flow velocity and depth, bed material characteristics including particle size distribution and type of bed material, whereas the pier size, spacing, and flood duration are accepted as deterministic variables. Progressive alteration of bed resistance in rivers subject to frequent flash floods may affect the velocity distributions. Therefore, even under a constant stage, the flow velocity and flow depth in movable beds may exhibit time-wise variations. Uncertainty in flow velocity arises from the lack of precise prototype data because of difficulty of using measuring instruments in the field composed of movable bed, their low precision, location of the measurement, and human-induced errors. In nature, median sediment size cannot attain a constant value and therefore be represented by a probability density function for the movable bed conditions. Particle size distribution may alter even from flood to flood. Therefore, it is of uncertain nature [3, 11, 12]. On the other hand, for a group of piers, uncertainties in flow velocity and flow depth may further increase, depending on the degree of interference of the flow fields with vortices among the bridge piers. Use of vortex flow meters in a flow field under the combined time-dependent effects of wake vortices of the front pier and horse-shoe vortices of the rear pier may not provide precise results. The flow field may also be simulated via some software using a 3D approach based on the detailed turbulence modelling. However, this approach would also be subject to a calibration problem since velocity measurements are associated with uncertainties, unless they have been performed using large precise datasets. Therefore, as a preliminary approach, the aforementioned uncertainties can be amplified arbitrarily with reference to the case of a single pier, and hence such possible effects can be incorporated by sensitivity analyses.

The uncertainty of a variable can be expressed with the coefficient of variation, $COV = \sigma / \mu$, in which $\sigma$ is the standard deviation and $\mu$ is the mean of the variable. The deviation of a variable from its mean can be determined by measurements and observations.

4. Application problem

Computation of the probability of failure is demonstrated using the following application. At a bridge site, the following data are available: diameter of identical cylindrical piers with tandem arrangement is 1.8 m, the mean values of the approach flow depth and the mean approach flow velocity are 0.8 m and 0.85 m/s, respectively, mean value of the median sediment size is 6 mm, the centre-to-centre distance between the piers is 4.55 m, and the flood duration, $t_f$ is 6 hours. In the analyses, the mean approach flow velocity, approach flow depth, and median sediment size, are accepted to be probabilistic variables, whereas the pier sizes, flood duration, and the spacing between piers, are taken to be deterministic variables. Based on the structural and geotechnical surveys, the depth of the combined footing for the dual piers, $d_f$ is taken as approximately 2.0 m. To observe the effect of pier footing depth on the probability of failure, various $d_f$ values are tested in the range 1.7 m and 2.3 m. With the given data, mean threshold velocity of the approach flow, $u_c$ can be computed as 1.0 m/s from [13]:

$$\frac{u_c^2}{\Delta g D_{50}} = 2.0 \left( \frac{D_{50}}{d_0} \right)^{1/3}$$

(2)

where, $u_c$ is the mean threshold velocity of the approach flow, $\Delta$ is the relative submersed density of the sediment particles, $g$ is the gravitational acceleration, $D_{50}$ is the median sediment size, and $d_0$ is the approach flow depth. Here $\Delta$ can be taken as 1.65 for quartz sand [13]. Therefore, clear water conditions prevail at the riverbed during which the scour depth rapidly changes at the beginning of the flood. Then, the rate of scouring decreases and the scour depth reaches an equilibrium depth asymptotically [13]. When $d_f$ is expressed from eqn. (1) by taking $\Delta = 1.65$ and $g = 9.81$ m/s², the following equation is obtained:

$$d_s = 0.00351 b^{0.701} d_0^{0.155} u^{0.357} D_{50}^{-0.994} t_f^{0.123} d_f^{0.102}$$

(3)

where $d_s$ is the scour depth, $b$ is the pier diameter, $d_0$ is the approach flow depth, $u$ is the mean approach flow velocity, $D_{50}$ is the median sediment size, $t_f$ is the time, and $d_f$ is the centre-to-centre spacing between piers.

4.1. Computation of probability of failure

Conventional deterministic design approaches require attaining a safety factor, which is greater than unity. However, when uncertainties associated with the governing variables are accounted for, it may be possible to obtain a probability of failure, $P_f$ value (risk), which cannot be accepted with confidence. The acceptability of the risk depends on the importance level and bridge location. For example, in the USA, for the bridges crossing small rivers, the limit $P_f$ value can be taken as $10^{-3}$, whereas for larger bridges that are exposed to high traffic intensity, this value can be considered as $10^{-5}$ [14]. It is clear that structures may be exposed to such loads that cause failure during their lifetime. However, the probability of failure can be decreased by regular inspection and maintenance. For instance, the application of riprap or partially grouted riprap...
around the piers of bridges can increase the resistance level. The vulnerability of a bridge to scouring can be assessed in the scope of regular inspections [15, 16]. Thus the resistance of the structure can be assessed and precautions can be taken (additional repairs) when the safety level decreases. The probability of failure, $P_f$, of a bridge due to excessive scour around its piers is defined by [1] as the probability of the safety margin, $SM$, being less than zero. The $SM$ and $P_f$ are defined as $SM = d_f - d_s$ and $P_f = P_{SM < 0}$, respectively, where $d_f$ is the depth of the combined footing for the dual piers, $d_s$ is the scour depth, and $P$ is the probability.

For the deterministic variables, the mean value of the parameter is used in risk assessment computations. The deterministic variables are provided in Table 1. In this table, Case 2 presents the original case defined in the application problem, whereas Cases 1 and 3 have different pier spacing and diameters and Cases 4 and 5 have different pier spacing with the same diameter. Considering structural requirement, the pier size increased in 20 cm intervals with increasing pier spacing. Case 2, in fact, represents a typical application for a two-lane bridge. The main idea behind the selection of the cases outlined in Table 2 is to limit the deflection of the bridge deck, i.e. smaller pier sizes require smaller spacing among the piers. On the contrary, a greater spacing can be used for a thicker pier. The variability of the uncertain parameters, i.e. $d_s$ and $u$, increases when the spacing gets smaller because of the interference of flow field around the piers; whereas the COVs in Case 3 are decreased considering the increase in the spacing between the piers. For the probabilistic variables, random numbers are generated using the probability density function, PDF, the mean and the COV of the variable. To this end, PDFs and COVs of $d_s$, $u$, and $D_{50}$ are obtained from previous studies [2, 3, 11, 12, 17, 18] and the recommended PDFs and COV values are used as presented in Combination (As) of Table 2.

Therefore, the bed regime and scour type is only influenced by the local flow characteristics. The ratio of the pier footing depth to the maximum scour depth at the end of the flood duration is defined as the factor of safety, $FS$. Furthermore, additional sensitivity analyses are conducted to investigate the possible effects of the distribution type and COV values of variables on the probability of failure. The resources in the literature define different PDFs and COV values for the same variables, i.e. $d_s$, $u$, and $D_{50}$[2, 3]. Therefore, within the scope of this study, the distribution types and COV values are altered considering these and presented in Combination (Bs) and (Cs) in Table 2. In Combination A, which presents statistical properties of the parameters defined in recent literature, the coefficient of variation values are relatively smaller and the distribution types are Gaussian (normal distribution). From Combination A to Combination C, COV values are slightly increased and the PDFs of the mean approach flow velocity and median sediment size are varied from normal distribution to triangle and uniform distributions except for Cases 4 and 5 since they are considered to investigate the influence of bridge pier spacing on the probability of failure and scour depth statistics.

### Table 1. Deterministic variables and their values for cases considered

| Case | $d$ [m] | $b$ [m] | $t_f$ [hr] |
|------|---------|---------|-----------|
| 1    | 3.60    | 1.6     | 6         |
| 2    | 4.55    | 1.8     | 6         |
| 3    | 7.20    | 2.0     | 6         |
| 4    | 3.60    | 1.8     | 6         |
| 5    | 5.45    | 1.8     | 6         |

For the probability of failure computation using $P_f = P_{SM < 0}$, Monte Carlo simulation is utilized to generate random numbers, as also used in previous studies for single piers [1, 2]. Since it influences the accuracy of $P_f$, it is important to determine the probability of failure and scour depth statistics.

### Table 2. Statistical properties of scour parameters considered

| Variable | Mean value [μ] | Comb. | Case 1 COV | Case 2 COV | Case 3 COV | Case 4 COV | Case 5 COV | PDF       |
|----------|---------------|-------|------------|------------|------------|------------|------------|-----------|
| $d_s$    | 0.8 m         | A     | 0.10 (0.10)| 0.10 (0.10)| 0.10 (0.10)| 0.10 (0.10)| 0.10 (0.10)| Normal    |
|          |               | B     | 0.20 (0.15)| 0.15 (0.12)| NA (NA)    | NA (NA)    | NA (NA)    | Normal    |
|          |               | C     | 0.20 (0.15)| 0.15 (0.12)| NA (NA)    | NA (NA)    | NA (NA)    | Normal    |
| $u$      | 0.85 m/s      | A     | 0.010 (0.010)| 0.010 (0.010)| 0.010 (0.010)| 0.010 (0.010)| 0.010 (0.010)| Normal    |
|          |               | B     | 0.020 (0.015)| 0.012 (NA)  | NA (NA)    | NA (NA)    | NA (NA)    | Normal    |
|          |               | C     | 0.015 (0.010)| 0.008 (NA)  | NA (NA)    | NA (NA)    | NA (NA)    | Triangle  |
| $D_{50}$ | 6 mm          | A     | 0.050 (0.050)| 0.050 (0.050)| 0.050 (0.050)| 0.050 (0.050)| 0.050 (0.050)| Normal    |
|          |               | B     | 0.075 (0.075)| 0.075 (NA)  | NA (NA)    | NA (NA)    | NA (NA)    | Normal    |
|          |               | C     | 0.075 (0.075)| 0.075 (NA)  | NA (NA)    | NA (NA)    | NA (NA)    | Uniform   |
4.2. Results and discussion

The mean and coefficient of variation of the pier scour depths, factor of safety values, and the probability of failure of all cases and combinations, are computed and presented in Table 3. For Cases 1 to 3, it can be seen that the mean scour depths, $\mu(d_s)$, obtained from the simulations, do not vary significantly from one combination to the other, while the coefficient of variation of the scour depth, $d_s$, notably changes. The $COV$ of $d_s$ may vary noticeably (up to 64%) through combinations with the change of $COVs$ and $PDFs$ of the approach flow depth, velocity and the sediment size. Similarly, the failure probability varies remarkably as the combinations change. The changes in $COV(d_s)$ and $P_f$ are more pronounced in Combination (B) when compared to that of Combination (C).s. Therefore, it can be stated that $COV(d_s)$ and the probability of failure are more sensitive to the changes in the $COVs$ of the input variables than their $PDFs$. Along with these, comparisons can be made between Case 2-Combination (A) and Cases 4 and 5, where these cases have the same pier diameter but different spacing. It can be seen that, when the spacing between tandem piers increases, the mean pier scour depth decreases, the safety factor increases, and the failure probability decreases. This is an expected result since the scour depth is inversely related to the spacing between piers, see eq (3).

For all cases, Combination A ended up with the minimum probabilities of failure for a certain safety factor since this combination has the smallest $COV$ values. For Combinations B and C, in which the $COV$ values are higher, $P_f$ values are found to be up to $10^4$ times greater than those of Combination A. Difference between the calculated probability of failure of the bridge for Combinations B and C is smaller than between any of them and Combination A. This showed that the changes in $PDFs$ of $u$ and $D_{50}$ from normal to triangle and uniform distributions do not significantly affect the probabilistic behaviour of the bridge considered in this study. Therefore, it can be said that the probability of failure of the bridge is less sensitive to the variation of the $PDF$ type of the variables. However, even slight changes in the $COV$ values affect the safety of the structure. In the design process of river bridges, which are not located in a densely populated urban area, a $P_f$ value of less than $10^{-3}$ ($P_f \leq 0.001$) may be considered reasonable for the safety of the structure [1]. For the bridges having tandem piers, such a $P_f$ value is observed when the factor of safety is greater than 1.30 for all scenarios considered. The final decision on the depth of combined pier footings is to be given according to the worst possible scenario that is most likely to occur at

| Case | Comb. | $\mu(d_s)$ [m] | $COV(d_s)$ | $P_f$ |
|------|-------|----------------|------------|-------|
| 1    | A     | 1.604          | 0.058      | 0.007$x10^{-3}$ |
|      | B     | 1.608          | 0.095      | 5.000$x10^{-3}$ |
|      | C     | 1.607          | 0.089      | 3.080$x10^{-3}$ |
| 2    | A     | 1.701          | 0.058      | 1.080$x10^{-3}$ |
|      | B     | 1.705          | 0.087      | 24.100$x10^{-3}$ |
|      | C     | 1.704          | 0.083      | 16.900$x10^{-3}$ |
| 3    | A     | 1.748          | 0.058      | 5.750$x10^{-3}$ |
|      | B     | 1.753          | 0.083      | 43.200$x10^{-3}$ |
|      | C     | 1.754          | 0.079      | 37.400$x10^{-3}$ |
| 4    | A     | 1.742          | 0.058      | 4.840$x10^{-3}$ |
| 5    | A     | 1.669          | 0.058      | 0.272$x10^{-3}$ |

In addition, the variation of probability of failure with respect to the factor of safety is presented in Figure 3. For a constant depth of footing, the safety factor decreases for combinations with an increase in scour depth. Besides, when the $COV$ of the parameters increases, the probabilistic analyses are conducted with parameter values away from the median of the parameter concerned. This results in smaller depths of scour if the parameter values are sampled from the left-hand side of its probability distribution. Conversely, greater scour depths are obtained if the parameter values are sampled from the right-hand side of the distribution. The values are randomly sampled from both the left and right-hand sides of the median of the distribution and they are not equally sampled in number. Therefore, the most observed $P_f$ value in 15000 Monte Carlo simulations is considered as the target $P_f$ value. As a result of these, the probability of having a larger scour depth increases when the $COV$ of the variables increases (see Figure 3).
the site of the bridge concerned. The worst possible scenario can be obtained under joint consideration of variation of time-dependent hydrologic, hydraulic, and bed material characteristics throughout the physical life of the bridge. Possible changes in the watershed use, climatic factors, local conditions in close vicinity of the bridge, etc., may lead to the entirely different conditions contrary to the design data. Therefore, one should consider the expected changes in the aforementioned characteristics according to the location of the bridge in the watershed, and COV values may be assigned accordingly.

In this study, the equation for the temporal variation of scour depth for dual piers is not compared with an equation giving similar tendency for single piers. Due to reinforcing effect, the maximum instantaneous scour depth at dual bridge piers is higher compared to single pier case. For the same depth of footing, a single pier case will be subject to less scour compared to the case of dual piers, and will end up with much smaller $P_f$ values than dual pier case. Therefore, using a single pier equation for dual pier case will underestimate the failure probability and weaken the safety of the bridge. This highlights the importance of conducting separate reliability analyses for both cases. This study focuses on the reliability-based analysis of pier scour in dual pier case, to close a gap in the literature considering the lack of reliability-based evaluation of temporal variation of scour depth around dual bridge piers.

The goal of this study is to determine the representative PDF for dual pier scour. Therefore, statistical properties of the scour depth around dual bridge piers need to be assessed in detail. To this end, the scour depth data obtained from Monte Carlo simulations are statistically analysed by obtaining the descriptive statistics, the frequency histogram, and the box-plots of the data. The results of the descriptive statistics analyses are treated as an intermediate tool in interpreting data and are not provided as the findings of this study. However, the frequency histograms, box-plots of the data, and the goodness of fit tests results, are of great importance and are presented in detail. The frequency histograms, fitted probability density functions, and the box-plots of the scour depth data of all cases and combinations are presented in figures 4 to 7. The box-plots enable presentation of the minimum, maximum, median, and the first and third quartiles of the data. Also, the spread and symmetry of the distributions can be drawn from these graphs. According to the results, relatively symmetrical distributions are obtained in Combination A; Combination B distributions are commonly skewed to the left; whereas Combination C distributions are relatively symmetrical. It should be noted that, although Combination C distributions are symmetrical, they are flattened, and this made these distributions difficult to define with a common probability density function. It can be deduced from Figure 7 that the distribution shapes of pier scour depths of Cases 2, 4 and 5 are very similar in terms of the skewness and flatness; however, their mean and median values are different. Compared to other cases, Case 4 has the largest mean and median value.

In the scope of the study, the probabilistic natures of the scour depth around tandem bridge piers are tested to be characterized with a probability density function. To this end, two common goodness of fit tests, Kolmogorov-Smirnov and Chi-square tests, are used at two different significance levels: $\alpha = 5\%$ and $\alpha = 10\%$ [19]. Both goodness of fit tests can be applied to the obtained pier scour depth dataset. The fitted distributions are continuous and so the Kolmogorov-Smirnov test can be used. The dataset is relatively large and can be easily divided into small subsets, i.e. binaries. Therefore, the Chi-squared test can also be applied [19]. The tested PDFs are composed of the most common functions used in water resources engineering, namely normal ($N$), log-normal ($LN$), 3-parameter log-normal ($LN-3P$), Gamma ($G$), log-Pearson type 3 ($LPT3$), and generalized extreme value ($GEV$) distributions. The fitted distribution is accepted if it is accepted by any of the tests. It is said to be rejected when rejected by both tests. The results of the goodness of fit analyses are given in Table 4. The A and R letters in the table stand for “accept” and “reject”, respectively. Besides, the best PDF values tested according to Kolmogorov-Smirnov test are marked in the table with one asterisk, whereas the best PDF with respect to Chi-square test is indicated with double asterisks. The results showed that Combinations A and B of all cases can be represented with almost all the PDFs tested. $LPT3$ and $GEV$ are found to be the best distributions in characterizing the scour depth around tandem bridge piers probabilistically. However, the scour depth cannot be defined with a common PDF type for Combination C of any case in which the PDFs of the mean approach flow velocity and the median sediment size are varied. This is mainly due to the kurtosis of the frequency histograms of the combination. The histograms are seen to have negative excess kurtosis, which means a flattened distribution (see Combination (C)s in Figure 4 to Figure 6).
5. Conclusions

This paper deals with determination of probability of failure induced by scouring action at dual bridge piers having tandem arrangement as a pioneering study. Sources of uncertainties associated with the scouring variables involved in temporal variation of clear water scour at dual bridge piers are interpreted and defined by some probability distributions and coefficients of variation, $COV$. Effects of changes in coefficients of variation and probability distribution of input variables, i.e. the approach flow depth, the mean approach flow velocity, and the median sediment size, are observed through the changes in coefficient of variation of the scour depth and the probability of failure. The results are found to be much more sensitive to the changes in $COV$ values compared to the probability distributions. It can be observed that the probability of failure increases with an increase in the coefficient of

Table 4. Goodness of fit test results for probability density functions

| Case | Comb. | $\alpha$ | PDF type |
|------|-------|----------|----------|
| 1    | A     | 0.10     | R A A A A A** A |
|      |       | 0.05     | A A A A A** A |
|      | B     | 0.10     | R A A R A A** A |
|      |       | 0.05     | R A A R A A** A |
|      | C     | 0.10     | R R R R R R |
|      |       | 0.05     | R R R R R R |
| 2    | A     | 0.10     | R A A* A A* A ** A |
|      |       | 0.05     | R A A A A** A ** A |
|      | B     | 0.10     | R A A R A A* A ** A |
|      |       | 0.05     | R A A A A** A ** A |
|      | C     | 0.10     | R R R R R R |
|      |       | 0.05     | R R R R R R |
| 3    | A     | 0.10     | R A A A A A** A ** A |
|      |       | 0.05     | R A A A A A** A ** A |
|      | B     | 0.10     | R A A R A A* A ** A |
|      |       | 0.05     | R A A A A** A ** A |
|      | C     | 0.10     | R R R R R R |
|      |       | 0.05     | R R R R R R |
| 4    | A     | 0.10     | R A A A A** A A* A A |
|      |       | 0.05     | R A A A A** A A* A A |
| 5    | A     | 0.10     | R A A A A A** A A** A A |
|      |       | 0.05     | R A A A A** A A** A A |

* The best fitted probability density function according to Kolmogorov-Smirnov goodness of fit test.
** The best fitted probability density function according to Chi-Square goodness of fit test.
variation assigned to the variables. The effect of pier spacing was also tested and it was concluded that, as the pier spacing increased, the mean pier scour depth decreased and therefore resulted in a lower probability of failure. As a contribution to the literature, a representative probability density function for dual pier scour is recommended. Results of the goodness of fit tests show log-Pearson type 3 and generalized extreme value distributions to best characterize the dual pier scouring. Finally, the overall study can be implemented to a real case by determining the depth of combined pier footing according to the worst possible scenario, which may lead to the highest probability of failure due to dual pier scouring.

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