What is the discrete gauge symmetry of the \( R \)-parity violating MSSM?

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Abstract

The lack of experimental evidence for supersymmetry motivates \( R \)-parity violating realizations of the minimal supersymmetric standard model. Dropping \( R \)-parity, alternative symmetries have to be imposed in order to stabilize the proton. We determine the possible discrete \( R \) and non-\( R \) symmetries, which allow for renormalizable \( R \)-parity violating terms in the superpotential and which, at the effective level, are consistent with the constraints from nucleon decay. Assuming a gauge origin, we require the symmetry to be discrete gauge anomaly-free, allowing also for cancellation via the Green Schwarz mechanism. Furthermore, we demand lepton-number violating neutrino mass terms either at the renormalizable or nonrenormalizable level. In order to solve the \( \mu \) problem, the discrete \( Z_N \) or \( Z_N^R \) symmetries have to forbid any bilinear superpotential operator at tree level. In the case of renormalizable baryon number violation the smallest possible symmetry satisfying all conditions is a unique hexality \( Z_6^R \). In the case of renormalizable lepton-number violation the smallest symmetries are two hexalities, one \( Z_6 \) and one \( Z_6^R \).

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1 Introduction

The Standard Model (SM) provides a remarkably successful description of particle physics as observed in past and current experiments. Yet, it is considered to be incomplete due to the occurrence of quadratic divergences which directly entail the so-called technical hierarchy problem \[1, 2\]. Various extensions of the SM have been proposed to cure this problem, the most popular one being low-scale supersymmetry \[3, 4\]. However, so far no signs of supersymmetry nor, with the exception of massive neutrinos, any physics beyond the SM\[1\] have been seen at the LHC, which severely challenges the minimal supersymmetric extension of the SM (MSSM) \[9–12\]. The lack of experimental evidence for supersymmetry motivates the study of nonminimal incarnations of supersymmetry, in particular $R$-parity violating scenarios, see e.g. \[13–19\].

Originally, $R$-parity \[20\], or equivalently matter parity, was introduced in order to ensure a stable proton \[21\]. This is achieved by forbidding both baryon ($B$) and lepton ($L$) number violation at the renormalizable level. Without imposing $R$-parity, the most general renormalizable superpotential including the SM particles is given by \[22\]

\[
W = y_u Q H_u U^c + y_d Q H_d D^c + y_e L H_d E^c + \mu H_d H_u + \kappa L H_u + \lambda L L E^c + \lambda' L Q D + \lambda'' U^c D^c D^c .
\] (1.1)

Here, $Q$ and $L$ denote the left-handed quark and lepton doublets, while $U^c$, $D^c$ and $E^c$ correspond to the right-handed fields; $H_u$ and $H_d$ are the up- and the down-type Higgs fields. Note that we have suppressed all color, weak and family indices. The terms in the second line of Eq. (1.1) violate either baryon or lepton number. Forbidding these by imposing $R$-parity clearly stabilizes the proton in a renormalizable theory. However, as proton decay requires baryon- as well as lepton-number violation, it is equally possible to allow for $L$ ($B$) violation provided the $B$ ($L$) violating terms are forbidden. This can be achieved by virtue of $R$-parity violating discrete symmetries such as e.g. baryon triality \[23, 24\].

In general, alternatives to $R$-parity can be classified according to

- the allowed renormalizable and nonrenormalizable operators
  (i.e. baryon- and lepton-number violating dimension three, four, five terms; $\mu$ term),
- discrete anomaly considerations, cancellation with or without the Green-Schwarz (GS) mechanism \[25\],
- their compatibility with grand unified theories (GUTs).

The idea of constraining possible discrete symmetries using anomaly considerations was first brought forward by Ibáñez and Ross \[23, 26\]. As global discrete symmetries are violated by quantum gravity effects, it is desirable to obtain them as remnants of a spontaneously broken (continuous) gauge symmetry \[27–31\]. In order to be mathematically consistent,
the underlying gauge theory must be anomaly-free; the corresponding anomaly conditions can then be translated to (weaker) discrete anomaly conditions which constrain the low-energy discrete symmetry. Assuming family-independent charges as well as no exotic light particles, Ibáñez and Ross studied the anomaly-free $Z_2$ and $Z_3$ symmetries [23,26], identifying only two interesting candidates, matter parity $M_p$ and baryon triality $B_3$. A subsequent extension of their work to $Z_N$ symmetries with arbitrary values of $N$ revealed another attractive symmetry, namely proton hexality $P_6$ [32]. As shown in Ref. [33], adding right-handed neutrinos to the particle content gives rise to an infinite set of new discrete symmetries if one assumes neutrinos to be Dirac particles. All these discrete symmetries allow for the bilinear term in the first line of Eq. (1.1). As such they do not provide a natural solution to the $\mu$ problem [34] because the dimensionful $\mu$ parameter is expected to take a value at a scale much higher than the phenomenologically required electroweak scale.

A straightforward way to alleviate this state of affairs consists in forbidding the $\mu$ term with the discrete symmetry. A weak scale $\mu$ term must then be generated dynamically [35,36]. Adopting the idea of eliminating the $\mu$ term through a discrete symmetry, it was shown in Refs. [37,38] that $SU(5)$ GUT-compatible discrete charge assignments are inconsistent with the discrete anomaly conditions unless discrete symmetries ($Z_N$) are extended to discrete $R$ symmetries ($Z_R^N$). Requiring $SO(10)$ compatibility, a unique $Z_4^R$ symmetry was identified which forbids the $\mu$ term as well as dimension three, four and five baryon- and lepton-number violating operators. See also Ref. [39]. When this symmetry is broken to standard matter parity, the $\mu$ term is generated at the electroweak scale. For a similar discussion in an $SU(5) \times U(1)$ setting, see Ref. [40]. Often the GS mechanism is imposed in order for the symmetries to be consistent with discrete anomaly considerations. For earlier work on $Z_R^N$ symmetries which forbid the $\mu$ term but do not invoke the GS anomaly cancellation mechanism, see e.g. Refs. [41,42], where $R$-parity violating operators are suppressed (and absent at the renormalizable level).

It is the purpose of this paper to systematically investigate the $R$-parity violating discrete family-independent $Z_N$ and $Z_R^N$ symmetries which forbid the bilinear terms in Eq. (1.1). We consider discrete symmetries with either (i) renormalizable $B$ violation or (ii) renormalizable $L$ violation, but not both simultaneously. These symmetries are further constrained by requiring the absence of $B$- and/or $L$-violating dimension-five operators which, if present, would mediate rapid proton decay [21,44,45]. In order to stay as general as possible, we first impose the discrete GS anomaly cancellation condition only [46]. The so-obtained infinite list of discrete symmetries can be significantly reduced by either demanding anomaly freedom without the GS mechanism or, alternatively, consistency with the type I seesaw mechanism [5–8].

The paper is organized as follows. In Sec. 2 we present the discrete anomaly coefficients for $Z_N$ and $Z_R^N$ symmetries and discuss the resulting anomaly condition invoking the GS mechanism. The phenomenological constraints on the dimension-five baryon and lepton number violating operators in the presence of renormalizable $R$-parity violation are listed in Sec. 3. Combining these constraints with the GS anomaly condition in Sec. 4 we obtain all

\footnote{$B$ and $L$ violation in models with discrete $R$ symmetries which (in the symmetry limit) forbid the $\mu$ term as well as all renormalizable $B$- and $L$-violating operators was studied e.g. in Ref. [43].}
possible allowed $Z_N^{[R]}$ symmetries. This set of viable $R$-parity violating discrete symmetries is thinned out by adding further constraints in Sec. 5 and Appendix A. In Sec. 6 we discuss the implications of dynamically generating the $\mu$ term. We conclude in Sec. 7.

2 Discrete anomaly coefficients

The discrete anomaly coefficients are derived from the anomaly coefficients of the underlying gauge theory. We assume this to be the SM gauge group $SU(3)_C \times SU(2)_W \times U(1)_Y$ augmented by the $U(1)_{[R]}$ gauge symmetry which gives rise to the discrete $Z_N^{[R]}$ symmetry. Disregarding the anomaly coefficients involving only SM factors, we encounter three linear anomaly coefficients

$$ A_{SU(3)_C - SU(3)_C - U(1)_{[R]}}, \quad A_{SU(2)_W - SU(2)_W - U(1)_{[R]}}, \quad A_{\text{grav} - \text{grav} - U(1)_{[R]}}, \quad (2.1) $$

where “grav” stands for gravity, as well as three purely Abelian anomalies

$$ A_{U(1)_Y - U(1)_Y - U(1)_{[R]}}, \quad A_{U(1)_Y - U(1)_{[R]} - U(1)_{[R]}}, \quad A_{U(1)_{[R]} - U(1)_{[R]} - U(1)_{[R]}}, \quad (2.2) $$

We shall not be concerned with the Abelian anomalies in Eq. (2.2) as they are less general [30, 31]. For instance, the cubic anomaly $A_{U(1)_{[R]} - U(1)_{[R]} - U(1)_{[R]}}$ is derived from the $U(1)_{[R]}$ charges of all fields, including the massive ones. The possibility of having fractionally charged heavy particles then allows for solutions to the cubic anomaly with any $U(1)_{[R]}$ charge assignments for the light states, see e.g. [31]. Similarly, the other Abelian anomalies in Eq. (2.2) provide only marginal constraints related to heavy fractionally (hyper-)charged particles. On the other hand, the linear anomalies of Eq. (2.1), lead to severe constraints on the allowed $U(1)_{[R]}$ charge assignments, and thus on the set of possible discrete symmetries.

In order to formulate the anomaly coefficients for both $U(1)$ and $U(1)_R$ symmetries simultaneously, we follow Ref. [37] and introduce the variable $R$. For $U(1)_R$ symmetries we set $R = 1$, while a regular $U(1)$ symmetry has $R = 0$. This is necessary since, in supersymmetry, a $U(1)$ symmetry assigns equal charge to the scalar and the spin one-half components of a chiral superfield, and the components of a vector superfield remain neutral. The situation is quite different for a $U(1)_R$ symmetry which (depending on the convention) assigns a charge of +1 to the superspace variable $\theta$. Denoting the $U(1)_R$ charge of a chiral superfield

$$ \Phi = \varphi + \theta \psi + \theta^2 F, \quad (2.3) $$

by $x$, the spin one-half component $\psi$, which is the particle contributing to the anomaly, has a charge of $x - 1$. Hence, in a unified notation, a chiral superfield with charge $x$ enters

3We adopt the notation $Z_N^{[R]}$ to refer to two cases of discrete $R$ symmetries ($Z_N^R$) and discrete non-$R$ symmetries ($Z_N$); analogously for the continuous symmetries $U(1)_{[R]}$.

4For gauged $R$ symmetries and their anomalies see Refs. [47, 49].

5In models based on an underlying GUT with a simple Lie algebra, the hypercharge of all the fields is quantized relative to each other as they necessarily originate from some GUT multiplet. Such a GUT framework would render the anomaly coefficient $A_{U(1)_Y - U(1)_Y - U(1)_{[R]}}$ more significant.
the anomaly coefficients with a factor of

\[ x - R , \quad \begin{cases} R = 0 , & \text{for } U(1) , \\ R = 1 , & \text{for } U(1)_R . \end{cases} \]  

(2.4)

Concerning \( U(1)_{[R]} \) neutral vector superfields,

\[ V = \theta \sigma^\mu \bar{\theta} A_\mu + \theta^2 \bar{\theta} \bar{\lambda} + \bar{\theta}^2 \theta \lambda + \theta^2 \bar{\theta}^2 D , \]

(2.5)

which correspond to the gauge fields of the theory, it is clear that the fermionic components, the gauginos \( \lambda \), carry the same \( U(1)_{[R]} \) charge as \( \theta \), i.e. the charge \( R \).

The linear anomaly coefficients are calculated as the weighted sums of the \( U(1)_{[R]} \) charges of all fermions. For the color anomaly we get

\[ A_{SU(3) - SU(3) - U(1)_{[R]}} = \left( \sum_{i=\text{colored}} \ell(r_i) (x_i - R) \right) + \ell(8) R , \]

(2.6)

where \( \ell(r_i) \) denotes the Dynkin index of the \( SU(3) \) representation \( r_i \) and the sum is over colored chiral states only. These colored states could be fundamental triplets or higher-dimensional representations like sextets, octets, etc. The corresponding Dynkin indices are defined up to an overall normalization. Here, we adopt the standard normalization with \( \ell(\text{fund}) = \frac{1}{2} \). Then the Dynkin index of an octet becomes \( \ell(8) = 3 \) meaning that the gluinos contribute to the anomaly with the term \( 3R \).

In order to derive the discrete anomaly coefficient from Eq. (2.6), we need to relate the \( U(1)_{[R]} \) charges \( x_i \) to the \( Z_N^{[R]} \) charges \( q_i \). Assuming all \( x_i \) to be integers\(^7\), we can readily express this relation, after \( U(1)_{[R]} \rightarrow Z_N^{[R]} \) breaking, as

\[ x_i = q_i + m_i N , \]

(2.7)

with \( m_i \in \mathbb{Z} \). Inserting this into Eq. (2.6) we find

\[ A_{SU(3) - SU(3) - U(1)} = \left( \sum_{i=\text{colored}} \ell(r_i) (q_i - R) \right) + 3R + \frac{1}{2} k \cdot N , \]

(2.8)

where the integer \( k = \sum_i 2 \ell(r_i)m_i \) is unspecified in the low-energy theory. The factor of \( \frac{1}{2} \) arises due to the standard normalization of the Dynkin indices. At this level, the sum is over light and heavy fermions alike. Heavy particles, that is particles which decouple from the low-energy theory and should therefore not occur in any useful discrete anomaly condition, have a \( Z_N^{[R]} \) invariant mass term. This allows us to remove their contribution from the explicit sum and absorb it into the third term of Eq. (2.8), proportional to \( k \), as we show now. Assuming the heavy particle to be Dirac entails two independent chiral superfields; their discrete charges \( q_{D1} \) and \( q_{D2} \) have to add up to \( 2R \mod N \) in order to be

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\(^6\)Note that a different convention is adopted in Ref. [48].

\(^7\)If they are fractional (but still quantized), it is possible to rescale the charges by a common factor of \( f \) such that they become integers. However, this will entail the \( U(1)_R \) charge of \( \theta \) to be \( f \) rather than 1, potentially leading to more general sets of discrete \( Z_N^{[R]} \) symmetries, as pointed out recently in Ref. [50].
compatible with a bilinear mass term in the superpotential. Therefore, their contribution to the discrete anomaly coefficient is given by

$$\ell(r_D) (q_{D_1} + q_{D_2} - 2R) = \ell(r_D) k' N ,$$

(2.9)

where $k' \in \mathbb{Z}$. With $\ell(r_D)$ necessarily being a multiple of $\frac{1}{2}$ it is clear that such a contribution simply amounts to redefining the unspecified parameter $k$ in Eq. (2.8). A heavy Majorana particle, on the other hand, contributes to the discrete anomaly coefficient with the charge $q_M$ of only one chiral superfield. The existence of the mass term requires $2q_M = 2R \mod N$. Its effect on the anomaly coefficient reads

$$\ell(r_M) (q_M - R) = \ell(r_M) \frac{1}{2} k'' N ,$$

(2.10)

with $k'' \in \mathbb{Z}$. The Dynkin index of a Majorana particle is constrained, as the representation must be real. In $SU(3)$ these are the $1, 8, 27, 64, ...$. It is now possible to show that the Dynkin indices of these and all other real representations are even multiples of $\ell(\text{fund.}) = \frac{1}{2}$ and thus integers $\frac{1}{2} + n$. Similar to the case of a heavy Dirac particle, the contribution of a heavy Majorana particle to the discrete anomaly coefficient can therefore be absorbed into the third term of Eq. (2.8).

In summary, the structure of the discrete anomaly coefficient of Eq. (2.8) is unchanged once the heavy particles are removed from the first term. Then the explicit sum is over the light particles of the model. Assuming the MSSM particles to be the only light fields as well as family-independent discrete charges, we obtain

$$A_{SU(3)_C - SU(3)_C - U(1)_{[R]}} = 3 \cdot \left(\frac{1}{2} (2q_Q + q_{U^C} + q_{D^C} - 4R) + 3R + \frac{1}{2} k \cdot N \right).$$

(2.11)

The factor of 3 accounts for the number of families. Similarly one can work out the discrete anomaly coefficient of the weak anomaly

$$A_{SU(2)_W - SU(2)_W - U(1)_{[R]}} = 3 \cdot \left(\frac{1}{2} (3q_Q + q_L - 4R) + \frac{1}{2} (q_{H_u} + q_{H_d} - 2R) + 2R + \frac{1}{2} \tilde{k} \cdot N \right),$$

(2.12)

where we have assumed one pair of Higgs doublets and $\tilde{k} \in \mathbb{Z}$.

Turning to the gravitational anomaly $A_{\text{grav} - \text{grav} - U(1)_{[R]}}$, we first remark that it does not involve any Dynkin indices. We simply need to add the $U(1)_{[R]}$ charges of all the fermions in the theory, i.e. the quarks and leptons, the Higgsinos, the gauginos, the gravitino as well as any additional SM neutral fermions. The MSSM gauginos (gluino, wino, bino) enter with their multiplicities, $8R + 3R + R$, and the $R$-gaugino, see Ref. [18], adds the term $R$.

Furthermore, there is the contribution of the gravitino which adds $-21R$ to the anomaly coefficient $[16, 19, 52, 53]$. The gravitational discrete anomaly coefficient is then given by

$$A_{\text{grav} - \text{grav} - U(1)_{[R]}} = 3 \cdot \left(6q_Q + 3q_{U^C} + 3q_{D^C} + 2q_L + q_{E^C} - 15R \right) + \left(2q_{H_u} + 2q_{H_d} - 4R \right) - 21R + 8R + 3R + 1R + 1R$$

$$+ \tilde{k} \cdot N + \sum_{i=\text{SM neutral}} q_i ,$$

(2.13)

\footnote{This relies on the observation that $\ell(r_1 \times \overline{r}_1) = d(r_1)d(\overline{r}_1) + \ell(r_1)d(\overline{r}_1) = 2d(r_1)\ell(r_1)$, where $d(r_1)$ is the dimension of the representation.}

\footnote{The latter is not included in Ref. [37] as the authors envisage a scenario without a local $U(1)_R$ symmetry.
where the integer $\hat{k}$ originates from the difference between the $U(1)_{[R]}$ and the $Z_{N}^{[R]}$ charges, see Eq. (2.7). The sum over SM neutral fermions includes both heavy and light degrees of freedom. Since the existence of light hidden fermions is not excluded, this sum can yield an arbitrary contribution which is not necessarily a half-integer multiple of $N$. In the following, we will therefore not make use of the gravitational anomaly to constrain the set of allowed $Z_{N}^{[R]}$ symmetries.

Invoking the GS mechanism, the discrete anomaly coefficients of Eqs. (2.11) and (2.12) have to satisfy the universality condition

$$A_{SU(3)_{C} - SU(3)_{C}} - U(1)_{[R]}_{C} = \frac{A_{SU(2)_{W} - SU(2)_{W} - U(1)_{[R]}}}{k_{W}} = \delta_{GS},$$

(2.14)

where $\delta_{GS} \in \mathbb{R}$ is a constant. Setting this constant to zero, is tantamount to satisfying the discrete anomaly conditions without an underlying GS mechanism. $k_{C}$ and $k_{W}$ label the Kac-Moody levels of the corresponding gauge algebra; they are integers for non-Abelian factors, and furthermore identical in superstring theories [54]. In fact, in most string models, the Kac-Moody levels of the non-Abelian gauge groups are just one. We will therefore assume $k_{C} = k_{W}$ (see Ref. [39] for alternative choices) so that the relevant discrete anomaly condition reduces to

$$3 \cdot (q_{Q} + q_{L} - q_{U^{c}} - q_{D^{c}}) + (q_{H_{u}} + q_{H_{d}}) - 4R = 0 \mod N .$$

(2.15)

### 3 Phenomenological constraints

Besides the anomaly condition in Eq. (2.15), the set of allowed $Z_{N}^{[R]}$ symmetries is constrained by various requirements. First, the discrete symmetry must allow for the up- and the down-type quark as well as the charged lepton Yukawa terms in Eq. (1.1). Second, as pointed out above, it must forbid the bilinear superpotential terms in order to avoid the $\mu$ (and $\kappa$) problem. Third, the symmetry should guarantee a sufficiently stable proton.

Being interested in discrete symmetries which violate $R$-parity at the renormalizable level, we have to distinguish two (exclusive) cases:

- (i) demand renormalizable $B$ violation, i.e. the term $U^{c}D^{c}D^{c}$,
- (ii) demand renormalizable $L$ violation, i.e. the term $LLE^{c}$.

Requiring the operator $LLE^{c}$ in the latter case entails the existence of the other trilinear $L$-violating term of Eq. (1.1), i.e. $LQD^{c}$, because the Yukawa operators $LH_{d}E^{c}$ and $QH_{d}D^{c}$ are both present. In order to prevent rapid proton decay we need to forbid renormalizable $L$ violation in case (i), and renormalizable $B$ violation in case (ii). Furthermore, it might be necessary to prohibit some of the dimension-five $B$- and/or $L$-violating superpotential operators [23, 32, 55],

$$\begin{align*}
\mathcal{O}_{1} &= [QQQL]_{F}, & \mathcal{O}_{2} &= [U^{c}U^{c}D^{c}E^{c}]_{F}, \\
\mathcal{O}_{3} &= [QQQH_{d}]_{F}, & \mathcal{O}_{4} &= [QU^{c}E^{c}H_{d}]_{F}, \\
\mathcal{O}_{5} &= [LH_{u}LH_{u}]_{F}, & \mathcal{O}_{6} &= [LH_{u}H_{d}H_{u}]_{F}, \\
\mathcal{O}_{7} &= [U^{c}D^{c}E^{c}]_{D}, & \mathcal{O}_{8} &= [H_{u}^{c}H_{d}E^{c}]_{D}, \\
\mathcal{O}_{9} &= [QU^{c}L^{c}]_{D}, & \mathcal{O}_{10} &= [QQD^{c}]_{D},
\end{align*}$$

(3.1)
where the subscripts $F$ and $D$ denote the $F$- and $D$-term of the corresponding product of superfields. Before discussing their role in destabilizing the proton, it is worthwhile to emphasize that several of these terms are allowed or forbidden simultaneously. This is due to the fact that the quark and charged lepton Yukawa terms, the first line of Eq. (1.1), are necessarily allowed by the $Z_N^{[R]}$ symmetry. To give an example, let us combine the second and the complex conjugate of the third term of Eq. (1.1) and multiply it by $U^cU^{c*}$. The resulting product is neutral under $Z_N^{[R]}$. Regrouping it as

$$QH_dD^c\left(LH_uE^c\right)^*U^cU^{c*} = QU^cL^*\left(U^cD^cE^c\right)^*H_dH_d^*,$$

(3.2)

shows that $O_7$ and $O_9$ have identical $Z_N^{[R]}$ charges. Similarly it is possible to show that forbidding the bilinear superpotential term $LH_u$ automatically removes the operators $O_4$, $O_7$, $O_8$, $O_9$. Furthermore, one can easily check that the operators $O_3$ and $O_{10}$ are also simultaneously allowed or forbidden. With these observations, we are well-equipped to discuss the constraints on the $Z_N^{[R]}$ symmetries originating from the dimension-five operators of Eq. (3.1).

The operators $O_1$ and $O_2$ violate $B$ and $L$ simultaneously; hence, they can mediate proton decay without any extra source of $B$ or $L$ violation. The operator $QQQL$ needs to be forbidden unless unnaturally small coefficients are assumed. On the other hand, the contribution of the operator $U^cU^cD^cE^c$ to proton decay is suppressed by small Yukawa couplings $\lambda_{22}^2, \lambda_{112}^2, \lambda_{33}^2$. In the following we will therefore demand the $Z_N^{[R]}$ symmetry to forbid $O_1$ but not necessarily $O_2$. In our list of possible discrete symmetries we will, however, explicitly mark those cases which allow for the operator $O_2$.

The remaining operators of Eq. (3.1) violate either $B$ or $L$, but not both. In order for them to contribute significantly to proton decay, they need to be combined with a renormalizable $R$-parity violating operator. Note, however, that the Weinberg operator $O_5$ [57] violates $L$ by two units; it therefore does not yield proton decay even when combined with the $B$-violating term $U^cD^cD^c$, so it need not be forbidden by the discrete symmetry.

The $B$-violating operators $O_3$ and $O_{10}$ can mediate proton decay only in case $(ii)$, i.e. the case with renormalizable $L$ violation. The simultaneous presence of the terms $QH_dD^c$ and $LQD^c$ requires $H_d$ and $L$ to have identical $Z_N^{[R]}$ charges. As a consequence, forbidding $QQQL$ ($O_1$) automatically also removes $QQQH_d$ ($O_3$), and with it $O_{10}$. Therefore it is not necessary to separately forbid the operator $O_3$.

The last dimension-five operator to discuss is $O_6$. Violating lepton number, it has to be combined with the renormalizable term $U^cD^cD^c$ to mediate proton decay. A possible such diagram is sketched in Fig. 1. Integrating out the neutralino $\chi^0$, the effective couplings on the left- and right-hand side of the squark propagator multiply to give $y_6\frac{\nu_6\nu_{6*}}{M_{Pl}}\xi_{22}Y^d_{22}\lambda_{112}^n$. Here we assume the nonrenormalizable operator to be suppressed by the reduced Planck mass $M_{Pl} \sim 10^{19}$ GeV and denote the corresponding dimensionless coupling constant by $y_6$; in addition, the $(2,2)$ element of the down-type quark Yukawa matrix $Y^d$ enters at the vertex on the right. Comparing this to the better-known bound $\lambda_{22}^n\lambda_{112}^n \lesssim 10^{-27}\left(\frac{\bar{m}_s}{100\text{GeV}}\right)^2$ [58], obtained from the diagram involving the renormalizable $B$- and $L$-violating operators of Eq. (1.1), yields $y_6\lambda_{112}^n \lesssim 10^{-8}\left(\frac{\bar{m}_s}{100\text{GeV}}\right)^2$ where we have used $\frac{\nu_{6*}Y^d_{22}}{M_{Pl}} \sim \frac{m_s}{M_{Pl}} \sim 10^{-19}$ as well as the assumption $\frac{\nu_{6*}}{\bar{m}_s} \sim 1$. Even though this bound suggests that one should forbid the
Figure 1: Contribution to proton decay obtained from combining $LH_u H_d H_u$ with $U^c D^c D^c$. Shown here is one possible diagram relevant for the decay channel $p \rightarrow K^+ \bar{\nu}$.

operator $O_6$ in case (i), this need not necessarily be the case since the coupling $\chi''_{112}$ is already experimentally bounded to be smaller than $10^{-4}$, but could even be much smaller depending on a hadronic scale parameter [59,60]. We therefore do not impose the condition that $O_6$ vanish in case (i), but rather state if a given $Z_N^{[R]}$ symmetry allows for this nonrenormalizable operator or not.

For future reference and convenience, we summarize the constraints on the $Z_N^{[R]}$ symmetries discussed in this section for both cases,

(i) with renormalizable $B$ violation:

- demand existence of $U^c D^c D^c$,
- forbid $LLE^c$ (thus automatically $LQD^c$),
- forbid $H_d H_u$,
- forbid $LH_u$ (thus automatically $O_4$, $O_7$, $O_8$, $O_9$),
- forbid $O_1 = QQQL$;

(ii) with renormalizable $L$ violation:

- demand existence of $LLE^c$ (thus automatically $LQD^c$),
- forbid $U^c D^c D^c$,
- forbid $H_d H_u$ (thus automatically $LH_u$, $O_4$, $O_7$, $O_8$, $O_9$),
- forbid $O_1 = QQQL$ (thus automatically $O_3$ and $O_{10}$).

4 Possible $Z_N^{[R]}$ symmetries

In this section we combine the phenomenological constraints of the previous section with the discrete anomaly condition of Eq. (2.15). Including the right-handed neutrino $N^c$ [10] we

\footnote{Being SM gauge singlets, the right-handed neutrinos $N^c$ do not alter the anomaly coefficients $A_{SU(3)c-SU(3)c-U(1)_R}$ and $A_{SU(2)_W-SU(2)_W-U(1)_R}$. Hence, the results of this section remain valid in scenarios where $N^c$ is absent.}
need to fix the generation-independent discrete $Z_N^{[R]}$ charges of eight superfields. The first set of constraints on the charge assignments arises from requiring the Yukawa couplings
\[ QH_u U^c, \quad QH_d D^c, \quad LH_d E^c, \quad LH_u N^c. \] (4.1)

A further simplification is achieved by shifting all discrete charges by an amount which is proportional to the hypercharge of the respective superfield. In other words, starting with any given set of discrete charges we can obtain a physically equivalent set using such hypercharge shifts. With the quark doublet having the smallest (absolute value of) hypercharge, it is possible to set $q_Q = 0$ without loss of generality. The resulting discrete charges take integer values, so that we can parameterize the most general $Z_N^{[R]}$ symmetry by three integers $p, n, m$ as follows,
\[ q_Q = 0, \quad q_{U^c} = -m, \quad q_{D^c} = m - n, \]
\[ q_L = -n - p, \quad q_{E^c} = m + p, \quad q_{N^c} = -m + n + p, \]
\[ q_{H_u} = m + 2R, \quad q_{H_d} = -m + n + 2R. \] (4.2)

We remark that these discrete charges as well as the parameters $p, n, m$ are only defined modulo $N$. For notational simplicity we do not require values between 0 and $N - 1$. However, equivalent choices must not be counted separately. The parameter $R$ is introduced in the Higgs charges in order to take into account the possibility of an $R$ symmetry; $R = 0$ for a $Z_N$ symmetry, while $R = 1$ in the case of a $Z_N^{[R]}$ symmetry. Concerning the latter case, we emphasize that the choice of introducing the parameter $R$ in the Higgs charges, rather than anywhere else, is completely general, since the other parameters $p, n, m$ have not been fixed at this stage.

### 4.1 Imposing Green-Schwarz anomaly cancellation

Anomaly considerations further constrain the allowed set of charges in Eq. (4.2). Let us first assume a setup in which the GS anomaly-cancellation mechanism is at work. Then the anomaly coefficients need not vanish identically. Instead we only have to satisfy Eq. (2.15). Inserting the charges given in Eq. (4.2) yields
\[ n = 3p \mod N, \] (4.3)
independent of $R$. In other words, the parameter $n$ is uniquely determined by the value of $p$. As we are only interested in symmetries that allow for renormalizable $B$ or $L$ violation, we can determine the third parameter $m$ from demanding either (i) the $B$-violating operator $U^c D^c D^c$ or (ii) the $L$-violating operator $LLE^c$. Table 1 shows the resulting parameterization of $Z_N^{[R]}$ symmetries in the two cases of interest. Varying $p$ between 0 and $N - 1$ shows that, for any given $N$, there are at most $N$ different $B$- / $L$-violating $Z_N^{[R]}$

---

\(^{11}\)Comparing the number of parameters and constraints we find: eight discrete charge parameters minus the four constraints of Eq. (4.1) minus another degree of freedom related to the hypercharge shift. This leaves us with three undetermined parameters.
symmetries which allow the superpotential terms of Eq. (4.1) and are consistent with the GS anomaly-cancellation condition of Eq. (2.15).

Several of these symmetries additionally allow for superpotential terms which, if present, would lead to rapid proton decay. As discussed in Sec. 3, we need to forbid the operators \( LLE^c, H_u H_d, H_u L \) and \( QQQL \) in the \( B \)-violating case, leading to the inequalities

\[
\begin{align*}
(\text{i}) & \quad p \neq 0 \text{ mod } N \quad (LLE^c), \\
& \quad 3p + 2R \neq 0 \text{ mod } N \quad (H_u H_d), \\
& \quad 2p + 2R \neq 0 \text{ mod } N \quad (H_u L), \\
& \quad 4p + 2R \neq 0 \text{ mod } N \quad (QQQL).
\end{align*}
\]

Likewise, the operators \( U^c D^c D^c, H_u H_d \) and \( QQQL \) have to be forbidden in the \( L \)-violating case. Thus, the corresponding conditions reduce to only three inequalities

\[
\begin{align*}
(\text{ii}) & \quad p \neq 0 \text{ mod } N \quad (U^c D^c D^c), \\
& \quad 3p + 2R \neq 0 \text{ mod } N \quad (H_u H_d), \\
& \quad 4p + 2R \neq 0 \text{ mod } N \quad (QQQL).
\end{align*}
\]

Notice that the constraints on \( p \) in the \( L \)-violating case, Eq. (4.5), must be satisfied in the \( B \)-violating case as well. The \( B \)-violating \( Z_N^{[R]} \) symmetries are, however, additionally constrained by the third inequality of Eq. (4.4).

Taking these restrictions into account it is straightforward to determine the allowed \( Z_N^{[R]} \) symmetries which rely on GS anomaly-cancellation. We have listed the smallest ones in Table 2 for the \( B \)-violating and in Table 3 for the \( L \)-violating case. For \( R = 0 \) the symmetries defined by \((N, p, n, m), (N, -p, -n, -m) = (N, N - p, N - n, N - m)\) and \((\frac{N}{d}, \frac{p}{d}, \frac{n}{d}, \frac{m}{d})\), where \( d \) is the greatest common divisor, are equivalent and therefore only one of them is shown. A similar overcounting of symmetries does not occur in the case of \( R = 1 \), with, however, one exception: for \( N = 4 \) the symmetries defined by \((p, n, m)\) and \((4 - p, 4 - n, 4 - m)\) are indeed equivalent. This becomes clear by noticing that \( 2 = \pm 2 \text{ mod } 4 \). Going from the former symmetry to the latter changes all charges of Eq. (4.2), including the charge of the Higgses. Any superpotential operator which is allowed by one symmetry hence satisfies the condition

\[
\sum_i q_i = 2 \text{ mod } 4 = -2 \text{ mod } 4 = \sum_i (-q_i),
\]

and, so, is automatically allowed by the other symmetry as well. For each of the symmetries listed in Tables 2 and 3 we have marked whether or not they are discrete anomaly-free
Table 2: The list of all $B$-violating $Z^{|R|}_N$ symmetries with $N \leq 10$ which fulfill the Green-Schwarz anomaly cancellation condition and the conditions listed in Eq. (4.4). None of these symmetries allows for the superpotential term $U^c U^c D^c E^c$. Furthermore it is shown which of the symmetries allow for the Weinberg operator as well as the potentially critical superpotential operators $LH_u H_d H_u$ (only in the $B$-violating case) and $U^c U^c D^c E^c$. We point out that the latter carries a $Z^{|R|}_N$ charge of $-2p$ and is thus forbidden by all $B$-violating symmetries that satisfy the third inequality of Eq. (4.4).

It is instructive to see explicitly why there are no $N = 2, 3, 4$ solutions in the $B$-violating case and no $N = 2$ solutions in the $L$ violating case. $N = 2$ is excluded by the inequality of Eqs. (4.4,4.5) related to forbidding the term $QQQL$ as the left-hand side is always even. In the $B$-violating case, non-$R$ symmetries with $N = 3$ and $N = 4$ are forbidden by the second and the fourth inequality of Eq. (4.4), respectively. Furthermore, the $B$-violating $Z^{|R|}_3$ symmetries with $p = 1$ and $p = 2$ are excluded by the fourth and the third inequality of
| $N$ | $R$ | $p$ | $n$ | $m$ | anomaly-free | $LH_uLH_u$ | $U^cU^cD^cE^c$ |
|-----|-----|-----|-----|-----|-------------|-----------|---------------|
| 3   | 1   | 2   | 0   | 1   | ✓           | ✓         | ✓             |
| 4   | 1   | 1   | 3   | 1   |             | ✓         | ✓             |
| 5   | 0   | 1   | 3   | 2   |             | ✓         | ✓             |
| 5   | 0   | 2   | 1   | 4   |             | ✓         | ✓             |
| 5   | 1   | 3   | 4   | 3   |             | ✓         | ✓             |
| 5   | 1   | 4   | 2   | 0   |             | ✓         | ✓             |
| 6   | 0   | 1   | 3   | 1   | ✓           | ✓         | ✓             |
| 6   | 1   | 2   | 0   | 4   | ✓           | ✓         | ✓             |
| 6   | 1   | 3   | 3   | 5   | ✓           | ✓         | ✓             |
| 6   | 1   | 5   | 3   | 1   | ✓           | ✓         | ✓             |
| 7   | 0   | 1   | 3   | 0   |             |           |               |
| 7   | 0   | 2   | 6   | 0   |             |           |               |
| 7   | 0   | 3   | 2   | 0   |             |           |               |
| 7   | 1   | 1   | 3   | 2   |             |           |               |
| 7   | 1   | 2   | 6   | 2   |             |           |               |
| 7   | 1   | 5   | 1   | 2   |             |           |               |
| 7   | 1   | 6   | 4   | 2   |             | ✓         | ✓             |
| 8   | 0   | 1   | 3   | 7   |             |           |               |
| 8   | 0   | 3   | 1   | 5   |             |           |               |
| 8   | 1   | 1   | 3   | 1   |             |           |               |
| 8   | 1   | 3   | 1   | 7   |             | ✓         | ✓             |
| 8   | 1   | 4   | 4   | 6   |             |           |               |
| 8   | 1   | 5   | 7   | 5   |             |           |               |
| 8   | 1   | 6   | 2   | 4   |             |           |               |
| 8   | 1   | 7   | 5   | 3   |             | ✓         | ✓             |
| 9   | 0   | 1   | 3   | 7   | ✓           |           |               |
| 9   | 0   | 2   | 6   | 5   | ✓           |           |               |
| 9   | 0   | 4   | 3   | 1   | ✓           |           |               |
| 9   | 1   | 1   | 3   | 0   |             |           |               |
| 9   | 1   | 2   | 6   | 7   |             | ✓         |               |
| 9   | 1   | 3   | 0   | 5   |             |           |               |
| 9   | 1   | 5   | 6   | 1   |             | ✓         |               |
| 9   | 1   | 6   | 0   | 8   |             |           |               |
| 9   | 1   | 7   | 3   | 6   |             |           |               |
| 9   | 1   | 8   | 6   | 4   |             | ✓         | ✓             |
| 10  | 0   | 1   | 3   | 7   |             |           |               |
| 10  | 0   | 3   | 9   | 1   |             |           |               |
| 10  | 1   | 1   | 3   | 9   |             |           |               |
| 10  | 1   | 3   | 9   | 3   |             |           |               |
| 10  | 1   | 4   | 2   | 0   |             | ✓         | ✓             |
| 10  | 1   | 5   | 5   | 7   |             |           |               |
| 10  | 1   | 8   | 4   | 8   |             |           |               |
| 10  | 1   | 9   | 7   | 5   |             | ✓         | ✓             |

Table 3: The list of all $L$-violating $Z_N^{[R]}$ symmetries with $N \leq 10$ which fulfill the Green-Schwarz anomaly cancellation condition and the conditions listed in Eq. (4.5). As will be discussed in Sec. 6, the symmetries allowing for $U^cU^cD^cE^c$ are disfavored once the generation of an effective $\mu$ term is imposed.
Eq. (4.4), respectively. Finally, the \( B \)-violating \( Z^R_4 \) symmetries with \( p = 1 \) and \( p = 2 \) are excluded by the third and the second inequality of Eq. (4.4), respectively.

5 Further constraints on the list of \( Z^R_N \) symmetries

We have seen in the previous section that there exists an infinite number of both \( B \) and \( L \)-violating symmetries which satisfy the GS anomaly cancellation condition of Eq. (2.15). Several of these are anomaly free even without invoking the GS mechanism. In this section we first identify the complete set of anomaly-free \( Z^R_N \) symmetries parameterized by an integer \( \ell \). Another strategy to thin out the list of possible \( Z^R_N \) symmetries given in Sec. 4 consists of looking for symmetries which are consistent with the type I seesaw mechanism. Again the obtained solutions can be parameterized by an integer \( \ell \).

It is worth emphasizing that compatibility with standard grand unified theories cannot be achieved for \( R \)-parity violating discrete symmetries which stabilize the proton. For instance, in \( SU(5) \), the renormalizable superpotential operators \( LQD^c \) and \( U^cD^cD^c \) both originate in the same \( SU(5) \) term, namely \( 5\bar{5} 10 \); allowing for one of the two operators is accompanied by having the other as well. Therefore, an \( SU(5) \) compatible \( Z^R_N \) symmetry would violate both \( B \) and \( L \) at the renormalizable level, and the proton would decay rapidly. Similar arguments hold for \( SO(10) \) as well as \( SU(5) \times U(1) \). In contrast, a setup based on Pati-Salam ‘unification’ \[61\] might be consistent with \( R \)-parity violating \( Z^R_N \) symmetries, however, in the following we shall not adopt Pati-Salam compatibility as a constraint on the list of discrete symmetries.

5.1 Imposing anomaly freedom

To further reduce the number of possible discrete symmetries, we can search for solutions which do not necessarily have to rely on the GS anomaly-cancellation mechanism. That is, in this subsection we are interested in discrete anomaly free \( Z^R_N \) symmetries. This adds one more condition to the discussion of Sec. 4 to wit

\[
A_{SU(3)C-SU(3)C-U(1)_{[R]}} = 0 \quad \Rightarrow \quad 3p = \left( \frac{k_1}{3} N - 2R \right) \text{mod} N ,
\]

(5.1)

where we have replaced \( n \) by \( 3p \) using the condition of Eq. (4.3). Due to the modulo \( N \) ambiguity it is sufficient to vary \( k_1 \) between 0 and 2. With \( k_1 = 0 \) we get \( 3p+2R = 0 \text{mod} N \) which is in contradiction to the requirement of forbidding the \( \mu \) term, see Eqs. (4.4,4.5). The remaining two cases \( k_1 = 1, 2 \) yield fractional values for \( 3p \) unless \( N \) is a multiple of 3. We must therefore assume that 3 divides \( N \), i.e. \( 3|N \), which allows us to define an integer \( N' \) such that

\[
N = 3N' .
\]

(5.2)

\footnotetext[12]{Another problem which would have to be addressed in GUT extensions without a \( \mu \) term is the existence of massless colored Higgs states.}

\footnotetext[13]{The condition in Eq. (4.3) ensures that the discrete anomaly coefficients \( A_{SU(3)C-SU(3)C-U(1)_{[R]}} \) and \( A_{SU(2)W-SU(2)W-U(1)_{[R]}} \) are identical. Demanding the former (latter) to vanish, automatically also sets the latter (former) to zero.
Using this and expressing the mod \( N \) explicitly by \( k_2N \), we can rewrite Eq. (5.1) as

\[ p = \frac{1}{3}(k_1N' - 2R) + k_2N' . \]  

(5.3)

Since values of \( p \) which are identical modulo \( N \) are equivalent, we can restrict ourselves to the cases where \( k_2 = 0, 1, 2 \). Moreover, the fact that \( p \) must be integer requires the factor \((k_1N' - 2R)\) to be a multiple of 3, which in turn entails several restrictions and case distinctions:

- (a) \( R = 0 \rightarrow (3|N') \rightarrow N' = 3\ell \),
- (b) \( R = 1 \land k_1 = 2 \rightarrow N' = 1 + 3\ell \),
- (c) \( R = 1 \land k_1 = 1 \rightarrow N' = 2 + 3\ell \),

with \( \ell \in \mathbb{N} \). This shows that the possible discrete symmetries can be classified according to the value of \( N' \). The allowed values of \( k_1 \) and \( k_2 \) then give rise to a small set of different \( Z_N^{[R]} \) symmetries. For (b) and (c), there are only three different choices, corresponding to \( k_2 = 0, 1, 2 \). In the case of (a) one can additionally vary \( k_1 \), but it is easy to show that the choices with \( k_1 = 2 \) are related to those with \( k_1 = 1 \) as we discuss now. Inserting \( R = 0 \) and \( N' = 3\ell \) into Eq. (5.3) gives

\[ p = (k_1 + 3k_2)\ell \equiv c_{k_1,k_2}\ell , \]  

(5.4)

where the coefficient \( c_{k_1,k_2} \in \{1, 2, 4, 5, 7, 8\} \) takes six different values (recall \( k_1 \neq 0 \)). Note, however, that its value can always be shifted by a multiple of 9 without affecting the obtained discrete symmetry. This is because in case (a) \( N = 3N' = 9\ell \). Shifting \( c_{k_1,k_2} \) by 9 corresponds to shifting \( p \) by \( 9\ell \) which is just \( N \), and all values are defined mod \( N \). For \( k_1 = 1 \) we find \( c_{k_1,k_2} \in \{1, 4, 7\} \), while for \( k_1 = 2 \) we get \( \{2, 5, 8\} = \{-7, -4, -1\} \mod 9 \). The overall minus sign between the latter and the former solution is fed through from the parameter \( p \) to the parameter \( n \), Eq. (4.3), and eventually – due to \( R = 0 \) – also to \( m \) and the discrete charges \( q_i \) of Eq. (1.2). Hence, the solutions with \( k_1 = 2 \) are physically identical to the solutions with \( k_1 = 1 \), in case (a).

We are now in a position to formulate the most general anomaly-free \( Z_N^{[R]} \) symmetries. We summarize our findings in Table 4, where we have made use of the mod \( N \) ambiguity to simplify the expressions. Notice that for \( R = 0 \), i.e. the case (a), the value of \( N = 9 \) is uniquely specified as a possible overall factor of \( \ell \) would only rescale the charges without changing the physics. For each of the three cases, (a), (b) and (c), we have three subcases corresponding to the three possible choices of \( k_2 = 0, 1, 2 \) in Eq. (5.3).

Having found the anomaly-free baryon- and lepton-number violating \( Z_N^{[R]} \) symmetries of Table 4, it is necessary to identify the subset which is compatible with the restrictions of Eqs. (4.4, 4.5). The discussion of Appendix A shows that all symmetries of Table 4 with \( \ell \geq 1 \) are allowed. With \( \ell = 0 \), only two \( L \)-violating symmetries, \( (R, N, p, n, m) = (1, 3, 2, 0, 1) \) and \( (1, 6, 2, 0, 4) \), are consistent with Eqs. (4.4, 4.5), while all other symmetries with \( \ell = 0 \) are phenomenologically forbidden.
Table 4: The list of all anomaly-free discrete (i) baryon and (ii) lepton number violating $Z_N^{[R]}$ symmetries. Here $N = 3N'$ and $\ell \in \mathbb{N}$ is a free parameter. The parameters $p, n, m$ can be translated to the discrete charges $q_i$ using Eq. (4.2). As discussed in Appendix A, for $\ell \geq 1$ all symmetries are consistent with the restrictions of Eqs. (4.4, 4.5). For $\ell = 0$, only two $L$ violating symmetries defined by $p = 2$ together with $N' = 1, 2$ are phenomenologically viable; however, we show in Sec. 6 that these are disfavored once the generation of an effective $\mu$ term is imposed.

The superpotential operators $LH_uH_dH_u$ and $U^cU^cD^cE^c$, which have the potential to mediate proton decay, can be shown to exist only for a finite number of anomaly-free $Z_N^{[R]}$ symmetries. Imposing the constraints of Sec. 4, $LH_uH_dH_u$, which has to be considered only in the $\mathbb{B}$ case, carries a $Z_N^{[R]}$ charge of $5p + 8R$ [see Eq. (4.2) and Table 4], so it is only allowed if

$$5p + 6R = 0 \mod N.$$  \hspace{1cm} (5.5)

Comparing this condition with the results of Table 4 shows that $LH_uH_dH_u$ is only present for the following two $\mathbb{B}$ anomaly-free discrete $R$ symmetries $(R, N, p, n, m) = (1, 12, 6, 6, 2)$ and $(1, 24, 18, 6, 14)$. Again using solely the constraints of Sec. 4, the discrete charge of the second operator, $U^cU^cD^cE^c$, is given by $-2p$, regardless of $Z_N^{[R]}$ violating $B$ or $L$ at the renormalizable level. Hence, this operator is allowed if

$$2p + 2R = 0 \mod N.$$  \hspace{1cm} (5.6)

As this is inconsistent with the third inequality of Eq. (4.4), all $B$-violating symmetries forbid $U^cU^cD^cE^c$. In the $L$-violating case, one can easily show that the only anomaly-free discrete $R$ symmetries satisfying Eq. (5.6) are the two symmetries allowed with $\ell = 0$ in Table 4 i.e. $(R, N, p, n, m) = (1, 3, 2, 0, 1)$ and $(1, 6, 2, 0, 4)$. Anticipating the results of Sec. 6, we point out that these $L$-violating symmetries are disfavored as the mechanism which generates an effective $\mu$ term can be adopted to generate an effective $U^cD^cD^c$ term at a dangerous level.

| class | $R$ | $N'$ | $p$ | $n$ | $m$ | (i) $B$ case | (ii) $L$ case |
|-------|-----|------|-----|-----|-----|--------------|--------------|
| (a)   | 0   | 3    | 1   | 3   | 6   | 7            |              |
|       |     |      |     |     |     |              |              |
|       |     |      | 4   | 3   | 6   | 1            |              |
|       |     |      | 7   | 3   | 6   | 4            |              |
| (b)   | 1   | $1 + 3\ell$ | $2\ell$ | $6\ell$ | $-1 + 3\ell$ | $-1 + 5\ell$ | $8\ell$ |
|       |     |      | $1 + 5\ell$ | $6\ell$ | $-1 + 3\ell$ | $-2 + 2\ell$ | $1 + 8\ell$ |
|       |     |      | $2 + 8\ell$ | $6\ell$ | $-1 + 3\ell$ | $-2 + 2\ell$ | $2 + 2\ell$ |
|       |     |      | $4 + 7\ell$ | $3\ell$ | $2 + 6\ell$ | $2 + 7\ell$ | $2 + 8\ell$ |
| (c)   | 1   | $2 + 3\ell$ | $\ell$ | $3\ell$ | $2 + 6\ell$ | $2 + 7\ell$ | $4 + 7\ell$ |
|       |     |      | $2 + 4\ell$ | $3\ell$ | $2 + 6\ell$ | $2 + \ell$ | $2 + 8\ell$ |
|       |     |      | $4 + 7\ell$ | $3\ell$ | $2 + 6\ell$ | $4 + \ell$ | $2 + 8\ell$ |
5.2 Requiring the seesaw mechanism

Tables 2 and 3 show that many of the phenomenologically viable $Z_N^{[R]}$ symmetries which satisfy the discrete GS anomaly-cancellation condition forbid the Weinberg operator $LH_u L H_u$. If allowed, such an operator can naturally be obtained in the framework of the attractive type I seesaw mechanism; alternatively it can be generated as an effective nonrenormalizable operator from Planck scale physics. From the symmetry point of view both options are identical since we have defined the charge of the right-handed neutrinos $N^c$ such that the Dirac Yukawa term $L H_u N^c$ is allowed. As a consequence, demanding the existence of the right-handed Majorana mass term $N^c N^c$, is equivalent to demanding the Weinberg operator $L H_u L H_u$. In this subsection we will constrain the set of $Z_N^{[R]}$ symmetries found in Sec. 4 by requiring the presence of the Weinberg operator. Thus we identify those symmetries which are consistent with the type I seesaw mechanism. We first discuss the $B$-violating case and turn to the $L$-violating case thereafter.

The absence of $L H_u L H_u$ in the $B$-violating case entails that neutrinos are Dirac particles, a scenario which would be ruled out if neutrinoless double beta decay was observed. This motivates the extraction of only those $Z_N^{[R]}$ symmetries of Table 2 and its extension to arbitrary $N$ which are consistent with Majorana neutrinos, i.e. which allow for $L H_u L H_u$. All possible $B$-violating $Z_N^{[R]}$ symmetries which satisfy Eq. (2.15) are parameterized in terms of three integers $(R, N, p)$, cf. Table 1. Demanding the Weinberg operator yields the condition

$$2(m - n - p + 2R) - 2R = 4p + 6R = 0 \mod N. \tag{5.7}$$

It is clear that $R = 0$ conflicts with the absence of the term $QQQL$, see Eq. (4.4). We are therefore left with $R = 1$, leading to the condition

$$2p = \left(-3 + \frac{k_3}{2}N\right) \mod N, \tag{5.8}$$

with $k_3 = 0, 1$. As $p$ is defined to be an integer, the two cases allow only particular values of $N$,

(a) $k_3 = 0 \rightarrow N = 3 + 2\ell$

(b) $k_3 = 1 \rightarrow N = 2(1 + 2\ell)$

where $\ell \in \mathbb{N}$. Inserting this into Eq. (5.8) determines $p$ as a function of $\ell$. In case (a), we find $p = \ell$, leading to the set of symmetries defined by

(a) $\rightarrow (R, N, p, n, m) = (1, 3 + 2\ell, \ell, -3 + \ell, -4 + 2\ell). \tag{5.9}$

It is straightforward to show that consistency with the restrictions of Eq. (4.4) requires $\ell \geq 2$. The choice $\ell = 0$ would allow $LLE^c$, while $\ell = 1$ would allow $H_uH_d$. Turning to case (b) we obtain two possible solutions, $p = -1 + \ell$ as well as $p = 3\ell$, leading to

(b) $\rightarrow (R, N, p, n, m) = \begin{cases} (1, 2 + 4\ell, -1 + \ell, -3 + 3\ell, -6 + 2\ell), \\
(1, 2 + 4\ell, 3\ell, -4 + \ell, -6 + 2\ell). \end{cases} \tag{5.10}$
Again it is possible to verify that all such symmetries with \( \ell \geq 3 \) are consistent with the restrictions of Eq. (4.4); moreover, \( \ell = 2 \) is allowed in the first subcase (with \( p = -1 + \ell \)) and \( \ell = 1 \) is possible in the second (with \( p = 3\ell \)). The choice \( \ell = 0 \) would allow \( H_uL \) in both subcases, while \( \ell = 1 \) would allow \( LLE^c \) in the first subcase, and choosing \( \ell = 2 \) in the second would yield the \( \mu \) term \( H_uH_d \). We summarize our findings of the allowed GS anomaly-free \( B \)-violating \( Z_N^{[R]} \) symmetries which are consistent with the type I seesaw mechanism, and thus light Majorana neutrinos, in Table 5. One can quickly confirm that these solutions do not satisfy Eq. (5.5), so that \( LH_uH_dH_u \) is forbidden. \( U^cU^cD^cE^c \) is forbidden even without demanding the Weinberg operator due to the incompatibility of Eq. (5.6) with the third inequality of Eq. (4.4).

In the case of \( L \)-violating discrete \( Z_N^{[R]} \) symmetries, the left-handed neutrinos can acquire a mass radiatively without the assumption of an underlying seesaw mechanism [22,62–64]. Yet, it is conceivable that such loop-induced contributions to the neutrino masses are too small to account for the observed lower mass bound of the heaviest neutrino, and that a seesaw mechanism might still be required. In that context, it is interesting to extract those GS anomaly-free \( L \)-violating \( Z_N^{[R]} \) symmetries which allow for the Weinberg operator. We therefore proceed analogously to the \( B \) violating case. The existence of \( LH_uLH_u \) gives rise to the constraint

\[
2(m - n - p + 2R) - 2R = 6p + 6R = 0 \text{ mod } N .
\]

With \( 0 < p + R \leq N \), there are six possible cases to distinguish,

\[
p = \frac{k_4}{6} N - R ,
\]

with \( k_4 = 1, \ldots, 6 \). For \( k_4 = 1, 5 \), it is obvious that \( N \) has to be a multiple of 6, so we can define an integer \( \ell \in \mathbb{N} \) such that \( N = 6\ell \); inserting this into Eq. (5.12) yields

\[
p = k_4' \ell - R ,
\]

where \( k_4' = k_4 = 1, 5 \). Similarly, for \( k_4 = 2, 4 \), \( N \) must be divisible by 3, so we can define an integer \( \ell \) such that \( N = 3\ell \), leading to Eq. (5.13), but now with \( k_4' = \frac{k_4}{2} = 1, 2 \). Likewise, \( k_4 = 3 \) requires \( N = 2\ell \), giving Eq. (5.13) with \( k_4' = \frac{k_4}{3} \). Finally, \( k_4 = 6 \) corresponds to \( N = \ell \) and results in Eq. (5.13) with \( k_4' = \frac{k_4}{6} = 1 \). These solutions have to be compared

| class | \( R \) | \( N \) | \( p \) | \( n \) | \( m \) | allowed values of \( \ell \) |
|-------|------|------|------|------|------|------------------|
| (a)   | 1    | \( 3 + 2\ell \) | \( -1 + \ell \) | \( -3 + \ell \) | \( -4 + 2\ell \) | \( \ell \geq 2 \) |
| (b)   | 1    | \( 2 + 4\ell \) | \( -1 + \ell \) | \( -3 + 3\ell \) | \( -6 + 2\ell \) | \( \ell \geq 2 \) |
Table 6: The list of all GS anomaly-free $L$-violating $Z_N^{[R]}$ symmetries consistent with the type I seesaw mechanism as well as the restrictions of Eq. (4.5). Here $\ell \in \mathbb{N}$, and the discrete charges $q_i$ of the MSSM superfields are obtained from the parameters $p, n, m$ using Eq. (4.2). The $Z_N^{[R]}$ symmetries with $N = \ell$ and $N = 2\ell$ allow for $U^c U^c D^c E^c$ and are thus disfavored once the generation of an effective $\mu$ term is imposed, see Sec. 6.

| $R$ | $N$ | $p$ | $n$ | $m$ | allowed values of $\ell$ |
|-----|-----|-----|-----|-----|--------------------------|
| 0   | 6   | 1   | 3   | 1   | $\ell \geq 3$           |
| 1   | $\ell$ | $-1 + \ell$ | $-3 + \ell$ | $-5 + \ell$ | $\ell \geq 3$ |
| 1   | $2\ell$ | $-1 + \ell$ | $-3 + \ell$ | $-5 + \ell$ | $\ell \geq 2$ |
| 1   | $3\ell$ | $-1 + \ell$ | $-3 + 3\ell$ | $-5 + \ell$ | $\ell \geq 3$ |
| 1   | $-1 + 2\ell$ | $-3 + 3\ell$ | $-5 + 2\ell$ | $\ell \geq 2$ |
| 1   | $6\ell$ | $-1 + \ell$ | $-3 + 3\ell$ | $-5 + \ell$ | $\ell \geq 2$ |
| 1   | $-1 + 5\ell$ | $-3 + 3\ell$ | $-5 + 5\ell$ | $\ell \geq 2$ |

The list of all GS anomaly-free $L$-violating $Z_N^{[R]}$ symmetries consistent with the type I seesaw mechanism as well as the restrictions of Eq. (4.5). Here $\ell \in \mathbb{N}$, and the discrete charges $q_i$ of the MSSM superfields are obtained from the parameters $p, n, m$ using Eq. (4.2). The $Z_N^{[R]}$ symmetries with $N = \ell$ and $N = 2\ell$ allow for $U^c U^c D^c E^c$ and are thus disfavored once the generation of an effective $\mu$ term is imposed, see Sec. 6.

For $R = 0$ we quickly find that $k_4$ can be either 1 or 5. After rescaling of charges and dropping overall signs, we only find one $L$-violating discrete symmetry with $R = 0$. In the cases where $R = 1$, one can show that all symmetries are consistent with Eq. (4.5) provided that $\ell \geq 3$; some symmetries satisfy the restrictions of Eq. (4.5) also for $\ell = 2$. Our results for the GS anomaly free $L$-violating $Z_N^{[R]}$ which allow for the Weinberg operator are summarized in Table 6 with the last column giving the constraints on $\ell$ arising form the phenomenological restrictions of Eq. (4.5). It is straightforward to prove that all $Z_N^{[R]}$ symmetries obtained from the second ($N = \ell$) and the third ($N = 2\ell$) row of Table 6 satisfy Eq. (5.6), thus allowing for the superpotential operator $U^c U^c D^c E^c$, while the remaining solutions of Table 6 eliminate this term. Note that $L$-violating symmetries which allow for $U^c U^c D^c E^c$ are disfavored, see Sec. 6.

Before concluding this section, we comment on the possibility of combining the compatibility of the $Z_N^{[R]}$ symmetries with the type I seesaw mechanism and the requirement of anomaly freedom, i.e. the results of Sec. 5.1. It is straightforward to verify that there are only two such discrete symmetries in each case,

(i) $B$ case: $(R, N, p, n, m) = (1, 15, 6, 3, 8)$ and $(1, 30, 6, 18, 8)$,

(ii) $U$ case: $(R, N, p, n, m) = (1, 3, 2, 0, 1)$ and $(1, 6, 2, 0, 4)$,

where the $L$-violating $Z_N^{[R]}$ symmetries might lead to rapid proton decay as the same mechanism which generates an effective $\mu$ term can be adopted to generate an effective $U^c D^c D^c$ term, see Sec. 6.

6 Consequences of generating an effective $\mu$ term

From the low-energy perspective, a $\mu$ term at around the electroweak scale is mandatory. Having forbidden this term by the $Z_N^{[R]}$ symmetry, we need to generate it dynamically by
breaking $Z_N^{[R]}$ spontaneously. In this section we will discuss the ensuing phenomenological consequences.

Regardless of the explicit mechanism which is responsible for creating an effective $\mu$ term in the superpotential, it must necessarily break $Z_N^{[R]}$. Let us for concreteness assume the Giudice-Masiero mechanism \[35\] in which the bilinear term $H_d H_u$ is obtained from the nonrenormalizable Kähler potential operator $S \frac{M_{Pl}}{H_d H_u}$. When the $F$-term of $S$ acquires a vacuum expectation value $\langle F_S \rangle \sim \frac{m_3}{2} M_{Pl}$, with $m_3$ denoting the gravitino mass, the term

$$m_{3/2} H_d H_u,$$

(6.1)

is generated in the effective superpotential after integrating out the superspace variable $\bar{\theta}^2$. Using Eq. (4.2), the $Z_N^{[R]}$ charge of this effective term is

$$n + 4 R \neq 2 R \mod N.$$

Before $Z_N^{[R]}$ breaking, the field which gives rise to this effective $\mu$ parameter carries a discrete charge such that $m_{3/2}$ can be regarded as an object with charge $-n - 2R$.

In principle, trilinear terms can be generated by the same mechanism, the only difference being an extra $\frac{1}{M_{Pl}}$ suppression of the operator. In the $B$-violating case (i), the crucial operator to look at is $LQD^c$; if this was generated analogously to the $\mu$ term, we would get

$$\frac{m_{3/2}}{M_{Pl}} LQD^c.$$

(6.2)

Assuming $m_{3/2} \sim 100$ GeV, this corresponds numerically to $\lambda' \sim 10^{-16}$. This, together with the presence of $U^c D^c D^c$ at the renormalizable level, could lead to proton decay at a dangerous rate. Eq. (6.2) does, however, not occur if the charge of $LQD^c$ and the effective charge of $m_{3/2}$ (i.e. $-n - 2R$) add up to something different from $2R \mod N$. Explicitly, we find using Eq. (4.2) and Table 1,

$$m - 2n - p - n - 2R = -4p \neq 2R \mod N.$$

(6.3)

As this condition is identical to the fourth inequality of Eq. (4.4), the $L$ violating renormalizable term $LQD^c$ cannot be obtained in the $B$-violating case (i) by the same mechanism that gives rise to the effective $\mu$ term.

Similar considerations lead to the condition

$$-2p \neq 2R \mod N,$$

(6.4)

on the $L$-violating $Z_N^{[R]}$ symmetries which allow for an effective $\mu$ (and thus also $\kappa$) term and, at the same time, do not generate the $B$ violating term $U^c D^c D^c$. Notice that the condition of Eq. (6.4) is equivalent to forbidding the superpotential term $U^c U^c D^c E^c$, cf. Eq. (5.6). Hence all $L$-violating symmetries which allow for $U^c U^c D^c E^c$, see Table 3 as well as comments below Eqs. (5.6) and (5.13), are disfavored by imposing the generation of an effective $\mu$ term.

We conclude this section by pointing out that the original $Z_N^{[R]}$ symmetry is, in many cases, not completely broken through the mechanism which generates the effective $\mu$ term. There exists a simple criterion for having a residual symmetry: $N$ and $n + 2R$ (i.e. the absolute value of the effective charge of $m_{3/2}$) must have a common divisor. Denoting the greatest common divisor by $M$, a $Z_N^{[R]}$ symmetry is broken to a $Z_M^{[R]}$ symmetry. In the case
of the anomaly-free discrete symmetries listed in Table 4 one can easily show that $M = N'$. As another example we mention the $L$-violating $Z_6$ (non-$R$) symmetry of Table 3, defined by $(p, n, m) = (1, 3, 1)$. With $n + 2R = 3$ we get $M = 3$, so that the residual $Z_3$ symmetry is given by $(p', n', m') = (p, n, m) \mod 3 = (1, 0, 1)$, which is the well-known symmetry baryon triality $B_3$.

7 Discussion and Conclusion

The parameter space of the conventional $R$-parity conserving MSSM is becoming ever more constrained by the ongoing searches for supersymmetry at the LHC. The fact that no signal has yet been found sets quite stringent bounds on the masses of some strongly interacting sparticles. In particular, first generation squarks and gluinos below about 1.5 TeV are excluded if their masses are roughly equal. On the other hand, squark and gluino masses above 1.5 TeV seem already somewhat high, considering that the main motivation for postulating their existence is to stabilize the electroweak hierarchy against radiative corrections. However, these mass limits can be evaded in alternative supersymmetric models such as the $R$-parity violating MSSM, where the lightest supersymmetric particle decays and thus the missing transverse momentum is considerably reduced compared to the $R$-parity conserving MSSM.

In the framework of the $R$-parity violating MSSM, we have identified the (GS and non-GS) anomaly-free discrete gauge ($R$ and non-$R$) symmetries which are consistent with constraints from nucleon decay and which, at the same time, forbid the $\mu$ term. An effective $\mu$ term of electroweak order must then be generated dynamically via a mechanism such as the one proposed by Giudice and Masiero or Kim and Nilles. Furthermore, we consider which symmetries allow for neutrino mass generation via the Weinberg operator $LH_u L H_u$, or equivalently which allow for the type I seesaw mechanism if right-handed neutrinos are added to the particle spectrum.

As the simultaneous presence of renormalizable $B$-violating and $L$-violating terms is disfavored because it would lead to rapid proton decay, we have analyzed the two cases separately. In the case of renormalizable $B$ violation we find exactly two anomaly-free discrete gauge symmetries which allow for the Weinberg operator: a $Z_{15}^R$ and a $Z_{30}^R$, given at the end of Sec. 5. Relaxing the constraints by imposing anomaly-cancellation via the Green-Schwarz mechanism, we also find solutions with smaller values of $N$, the smallest being a unique $B$-violating hexality $Z_6^R$ defined by $(p, n, m) = (3, 3, 2)$, cf. Eq. (4.2). The required dynamical generation of the $\mu$ term entails the breaking of this $Z_6^R$, leaving no residual symmetry at all.

In the $L$-violating case, there are exactly two anomaly-free discrete gauge symmetries which allow for the Weinberg operator: one with $N = 3$ and one with $N = 6$, given at the end of Sec. 5. However these are disfavored due to constraints from proton decay as the mechanism which generates the effective $\mu$ term can be adopted to generate an effective $U^c D^c D^c$ term at a dangerous level (which generally happens for symmetries which do not forbid the term $U^c U^c D^c E^c$). Therefore we again extend the set of symmetries by imposing

\[14\] There is an infinite set of solutions with larger $N$. 

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anomaly-cancellation via the Green-Schwarz mechanism. Then, the smallest solutions are the two $L$-violating hexalities $Z_6$ with $(p, n, m) = (1, 3, 1)$ and $Z_6^R$ with $(p, n, m) = (3, 3, 5)$. The generation of the effective $\mu$ term breaks the $Z_6$ down to baryon triality ($B_3$), while the $Z_6^R$ is broken down to nothing. Alternatively, one can give up the presence of the Weinberg operator, since, in the $L$-violating case, sufficiently large neutrino masses can also be generated radiatively via the dimension-four $LQD$ and $LLE$ operators. In that case, the smallest viable anomaly-free solutions are the three $L$-violating ennealities of Table 3, which all reduce to $B_3$ once the $\mu$ term is generated.

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**Appendix**

A Phenomenologically viable anomaly-free $Z_N^{[R]}$

In Sec. 5.1 we have derived all anomaly-free $B$ or $L$ violating $Z_N^{[R]}$ symmetries regardless of their phenomenological viability. It is the purpose of this appendix to compare the obtained solutions with the constraints of Eqs. (4.4,4.5), and identify those symmetries which are physically relevant. To do so, we tabulate the discrete charges of the offensive operators in Table 7 for each of the nine cases defined by the choice of $(R, N', p)$, cf. Table 4. Clearly, in each case the charges only depend on the integer parameter $\ell$. Using the mod $N$ ambiguity we have shifted all charges such that the coefficient of $\ell$ is always positive and smaller than nine. This way it is straightforward to verify that, for $\ell \geq 1$, the charges of the offensive operators are always non-zero and smaller than $N$. Hence, all symmetries with $\ell \neq 0$ are consistent with Eqs. (4.4,4.5). For $\ell = 0$, there exist several entries in Table 7 which vanish modulo $N$. Hence, with $\ell = 0$ (i.e. $N = 3, 6$) there is no solution which forbids all dangerous operators of the $B$-violating case (i). However, in the $L$-violating case (ii) we do not have to consider the last column of Table 7 as a consequence, the $L$-violating symmetries with $(R, N, p, n, m) = (1, 3, 2, 0, 1)$ and $(1, 6, 2, 0, 4)$ are possible.

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\[15\text{For Greek prefixes see for example }\text{http://en.wikipedia.org/wiki/Number_prefix.}\]
Table 7: The discrete charges of the offensive operators in terms of the parameter $\ell \in \mathbb{N}$. Here we have used the mod $N$ ambiguity to shift the charges such that the coefficient of $\ell$ is positive and smaller than nine. The last column need not be considered in the $L$-violating case $(ii)$.

| $R$ | $N'$ | $p$ | $LLE^c$ | $H_uH_d$ | $QQQL$ | $H_uL$ |
|-----|-----|-----|---------|---------|--------|--------|
| 0   | 3   | 1   | 1       | 3       | 4      | 2      |
|     | 4   | 1   | 1       | 3       | 7      | 8      |
|     | 7   | 1   | 3       | 1       | 5      |        |
| 1   | $1 + 3\ell$ | $2\ell$ | $2\ell$ | $2 + 6\ell$ | $2 + 8\ell$ | $2 + 4\ell$ |
|     | $1 + 5\ell$ | $1 + 5\ell$ | $2 + 6\ell$ | $2\ell$ | $1 + \ell$ |        |
|     | $2 + 8\ell$ | $2 + 8\ell$ | $2 + 6\ell$ | $1 + 5\ell$ | $3 + 7\ell$ |        |
| 1   | $2 + 3\ell$ | $\ell$ | $\ell$ | $2 + 3\ell$ | $2 + 4\ell$ | $2 + 2\ell$ |
|     | $2 + 4\ell$ | $2 + 4\ell$ | $2 + 3\ell$ | $4 + 7\ell$ | $6 + 8\ell$ |        |
|     | $4 + 7\ell$ | $4 + 7\ell$ | $2 + 3\ell$ | $\ell$ | $4 + 5\ell$ |        |

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