Can Flavor-Independent Supersymmetric Soft Phases Be the Source of All CP Violation?

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Abstract

Recently it has been demonstrated that large phases in softly broken supersymmetric theories are consistent with electric dipole moment constraints, and are motivated in some (Type I) string models. Here we consider whether large flavor-independent soft phases may be the dominant (or only) source of all CP violation. In this framework $\epsilon$ and $\epsilon'/\epsilon$ can be accommodated, and the SUSY contribution to the B system mixing can be large and dominant. An unconventional flavor structure of the squark mass matrices (with enhanced super-CKM mixing) is required for consistency with B and K system observables.

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Although the first experimental evidence of CP violation was discovered over thirty years ago in the K system [1], the origin of CP violation remains an open question. In the Standard Model (SM), all CP violation arises due to a single phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [2]. While the SM framework of CP violation provides a natural explanation for the small value of $\epsilon$ in the K system and is supported by the recent CDF measurement of $\sin 2\beta$ through the decay $B \to \psi K_S$ [3], it is not clear whether the SM prediction is in agreement with the observed value of $\epsilon'/\epsilon$ recently measured by [4] (confirming the earlier results of [5]) due to theoretical uncertainties [6]. However, the SM cannot account for the baryon asymmetry [7], and hence new physics is necessarily required to describe all observed CP violation.

In this paper, we investigate the possibility of a unified picture of CP violation by adopting the hypothesis that all observed CP violation can be attributed to the phases which arise in the low energy minimal supersymmetric standard model (MSSM), as first suggested by Frère et al. [8]. The issue of CP violation in supersymmetric theories is not a new question [8–12]. However, much of our analysis is motivated from embedding the MSSM into a particular string-motivated D-brane model at high energies [13], which departs significantly from the standard results for CP violation in SUSY models (which we summarize for the sake of comparison). The CP-violating phases of the MSSM can be classified into two categories: (i) the flavor-independent phases (in the gaugino masses, $\mu$, etc.), and (ii) the flavor-dependent phases (in the off-diagonal elements of the scalar mass-squares and trilinear couplings). We focus here on the flavor-independent phases; these phases have traditionally been assumed small ($< \sim 10^{-2}$) if the sparticle masses are $\mathcal{O}(\text{TeV})$ as the phases are individually highly constrained by the experimental upper bounds on the electric dipole moments (EDMs) of the electron and neutron [14–16]. However, a reinvestigation of this issue [17,18] has demonstrated that cancellations between different contributions to the EDMs can allow for viable regions of parameter space with phases of $\mathcal{O}(1)$ and light sparticle masses.

In recent work [13], we found a (Type I) string-motivated model of the soft breaking terms based on embedding the SM on five-branes in which large flavor-independent phases can be accommodated. The large relative phases between the gaugino mass parameters in this model play a crucial role in providing the cancellations in the EDM’s, yielding regions of parameter space in which the electron and neutron EDM bounds are satisfied simultaneously. In this model, the CP-violating phases in the soft breaking terms are due to the (assumed) presence of complex F-component VEV’s of moduli fields. Complex scalar moduli VEV’s can in principle also lead to phases in the superpotential Yukawa couplings; however, for simplicity we assume here that the phase of the CKM matrix is numerically close to zero [19]. The crucial feature of our scenario compared to previous work is that all flavor-independent phases in the soft SUSY breaking sector can be large, with the EDM constraints satisfied by cancellations motivated by the underlying theory. We will show that SUSY can account for all observed CP violation with large flavor-independent phases (including the relative phases of the gaugino masses, which are zero in many SUSY models) and a particular flavor structure of the squark mass matrices. We focus on the low $\tan \beta$ regime, distinguishing our results from other recent work [12]. The baryon asymmetry can be explainable in SUSY [20]; see [21] for a study of baryogenesis within this approach.

The CP-violating and FCNC processes that we consider are presented in Table I (we do not list the electron EDM ([18,13]), but only consider parameter sets which satisfy
the electron and neutron EDM constraints). First note that generically the matrices which diagonalize the quark mass matrices and those which diagonalize the squark mass matrices are not equivalent due to SUSY breaking effects. The sfermion mass matrices are expressed in the super-CKM basis, in which the squarks and quarks are rotated simultaneously. In this basis the sfermion mass matrices are non-diagonal, and the amplitudes depend on the insertion 

\[ \Gamma_{\text{SKM}}^{\text{LR}} = \begin{pmatrix} 1 & \lambda' + \lambda & \lambda' c_\theta & 0 & 0 & -\lambda' s_\theta e^{i\varphi_i} \\ -\lambda' & 1 & \lambda' c_\theta & 0 & 0 & -\lambda' s_\theta e^{i\varphi_i} \\ -\lambda' & -\lambda' & c_\theta & 0 & 0 & -s_\theta e^{i\varphi_i} \end{pmatrix} ; \]

where \( \lambda' \lesssim \lambda \equiv \sin \theta_c, \theta, \varphi_i \) denote the stop mixing parameter and its phase, and entries of \( O(\lambda^2) \) are neglected. Note that the mixing in the LL sector is enhanced as compared to that of the SM, while in the RR sector it is negligible (this is easily seen by setting \( \theta = 0 \)).

We now estimate the SUSY contributions to the observables in Table I. We will be working in the framework similar to the one laid out in [13], except we will also assume significant flavor mixing in the trilinear soft terms already at the GUT scale. In particular, we assume the \( A \)-terms to be of the form \( e^{i\theta_0}BY^{u,d}B' \) where \( B, B' \) are real matrices with considerable off-diagonal elements. Further, we assume that the squarks (except for the lightest stop) are degenerate in mass, and retain only the lightest stop except in the case of \( \epsilon \) and \( \epsilon' \) (for which the first two generations give the leading contribution), and neglect all but top quark masses unless the other fermion masses give leading contributions. For the purpose of presentation, we separate the stop left-right mixing from the family mixing. The family mixing matrices \( \tilde{K}_{ij}^{L,R} \) are defined as 

\[ \tilde{K}_{ij}^{L} = (\Gamma_{UL}^{SKM})_{ij}|_{\vartheta = 0}, \tilde{K}_{ij}^{R} = (\Gamma_{UR}^{SKM})_{ij+3}|_{\vartheta = 0} \]

with \( i, j = 1, 3 \). In accordance with the chosen form of the \( \Gamma \)'s, we assume \( \tilde{K}_{ij}^{L} \sim \lambda/3 \) and \( \tilde{K}_{ij}^{R} \sim 0 \) for \( i \neq j \). These matrices are real, as the only source of CP-violating phases in the \( \Gamma \)'s is the stop mixing. We assume maximal chargino and stop mixings, and the following parameter values: \( m_{\tilde{t}} \sim 140 \text{ GeV}, m_{\tilde{\chi}} \sim 100 \text{ GeV}, m_{\tilde{q}} \sim m_{\tilde{g}} \sim 350 \text{ GeV}, \) and \( A \sim 250 \text{ GeV} \). Our estimates agree within better than an order of magnitude with the numerical results to be presented in [22].

Let us first turn to the discussion of \( \epsilon \) and \( \epsilon' \). Here we utilize the mass insertion approximation and the associated \( (\delta_{ij})_{AB} \) parameters (see e.g. [4]). Since we study the impact of flavor-independent phases at high energies, the LL and RR insertions are essentially real (their phases are produced effectively at the two-loop level; see RGE’s in [10]). The LR insertion always occurs in combination with the gluino phase \( \varphi_3 \) due to reparameterization invariance; the physical combination of phases is \( (\delta_{12})_{LR}e^{i\phi_3} \) (the gluino phase has generally
been neglected in earlier work). Our numerical studies show that the observed values of $\epsilon$ and $\epsilon'$ can be reproduced for $|\langle\delta^d_{12}\rangle_{LR}| \approx 3 \times 10^{-3}$ and $Arg((\langle\delta^d_{12}\rangle_{LR})e^{i\alpha}) \approx 10^{-2}$, in agreement with [3,11]. This value of $|\langle\delta^d_{12}\rangle_{LR}|$ can be obtained in models with a large flavor violation in the $A$-terms. Note also that this value of $\langle\delta^d_{12}\rangle_{LR}$ leads to a significant gluino contribution to $\Delta m_K$.

The leading chargino contribution to $(M_K)^{ij}_{12}$ is CP-conserving, as can be seen from

$$\langle M_K \rangle^{ij}_{12} \sim \frac{g^4}{384\pi^2} \frac{m_K f^K_2}{m_t^2} \left( \tilde{K}^{L,R}_{ls} \right)^2 |V_{11} T_{11}|^4 , \tag{3}$$

(recall $\tilde{K}^{L,R}$ are real). $V$ and $T$ denote the chargino and stop mixing matrices; to simplify this expression we employed the approximation $m_t^2 \gg m_\frac{1}{2}^2$. This contribution gives $\Delta m_K \sim 10^{-16}$ GeV, well below the experimental value. Therefore $\Delta m_K$ is dominated by the Standard Model and gluino contributions (as in [9,11]).

In our approach the SM tree diagrams for B decays are real, and there is negligible interference with the superpenguin diagrams. Therefore the B system is essentially superweak, with all CP violation due to mixing. In contrast to the case of $K-\bar{K}$ mixing, $B-\bar{B}$ mixing is dominated by the chargino contribution:

$$\langle M_B \rangle^{ij}_{12} \sim \frac{g^4}{384\pi^2} \frac{m_B f^K_2}{m_t^2} \left( \tilde{K}^{L,R}_{ls} \right)^2 |V_{11} T_{11}|^4 \left( 1 - \frac{h_4 V_{12} T_{12} \tilde{K}^{R*}_{tb}}{g V_{11} T_{11} \tilde{K}^{L*}_{tb}} \right) . \tag{4}$$

The corresponding $\Delta m_B$ is of order $10^{-13}$ GeV, which is roughly the observed value. The SM contribution to $\Delta m_B$ is significantly smaller since the CKM orthogonality condition forces $V_{td}$ to take its smallest allowed value. The CP-violating gluino contribution requires two LR mass insertions and, as a result, is suppressed by $(m_b/m_t)^2$. Similar considerations hold for $B_s - \bar{B}_s$ mixing although the mixing phase is generally smaller than that in $B_d - \bar{B}_d$ due to a significant CP-conserving SM contribution.

Although the CP asymmetries and CKM entries are not related, $\sin 2\beta$ and $\sin 2\alpha$ can be defined in terms of the above asymmetries ($\sin 2\gamma$ can be defined via the CP asymmetry in $B_s \rightarrow \rho K_s$). The angles of the “unitarity triangle” given in this way need not sum to $180^\circ$ as in the SM. Our results demonstrate that the chargino contribution alone is sufficient to account for the observed value of $\sin 2\beta$ reported in the CDF preliminary results [3]. This can be seen from (4) since the mixing phase can be as large as $\pi/2$ if $O(1)$ phases are present in $V$ and $T$. In Fig. 1 we show contour plots of both $\sin 2\beta$ and $\Delta m_B$ in the $\varphi_1-\varphi_2$ plane.

The CP-asymmetries in $B \rightarrow \psi K_s$ and $B \rightarrow \pi^+\pi^-$ are related: $\sin 2\beta = -\sin 2\alpha$. This relation is characteristic of superweak models with a real CKM matrix ( [23,24]), and is not consistent with the SM, as seen using the “sin” relation: $\sin \beta / \sin \alpha = |V_{ub}|/|V_{cb}| \sin \theta_c|$. The LHS implies $|V_{ub}|/|V_{cb}| \sin \theta_c = 1$, while the experimental upper bound on the RHS is 0.45, verifying the nonclosure of the unitarity triangle.

We now turn to the $b \rightarrow s\gamma$ CP asymmetry $A_{CP}(b \rightarrow s\gamma)$. The dominant contribution is due to mixing between the magnetic penguin operator Wilson coefficients $C_7$ and $C_8$ [20]

$$A_{CP}(b \rightarrow s\gamma) \sim -\frac{4\alpha_s(m_b)}{9|C_7|^2} \Im(C_7 C_8^*) . \tag{5}$$
FIG. 1. Contours of $\sin 2\beta$ and $\Delta m_B$ for $\lambda' = 0.07$, $\theta = \pi/5$, and the lightest stop mass $m_{\tilde{t}} \sim 140$ GeV. The absolute value of $\sin 2\beta$ can be as large as 0.78 for this choice of parameters. $\Delta m_B^{(\text{exp})} \sim 3.1 \times 10^{-13}$ GeV and $\sin 2\beta^{(\text{exp})} = 0.79 \pm 0.44$.

Both $C_7$ and $C_8$ receive real SM contributions, and hence the SUSY contribution from the chargino-stop loop has to be competitive while at the same time respect the experimental limits on $\text{BR}(b \to s\gamma)$. As a result, larger values of $A_{CP}(b \to s\gamma)$ usually imply branching ratios further away from the experimental central value. Typical results still predict asymmetries larger than in the SM (of order several percent).

We checked that the enhanced super-CKM mixing does not lead to an overproduction of $D - \bar{D}$ mixing. Since the chargino contribution is subject to strong GIM cancellations, the leading contribution is given by the gluino-stop loop:

$$ (M_D)^{i\bar{g}}_{12} \sim -\frac{\alpha_s^2 m_D f_D^2}{27 m_{\tilde{g}}^2} \ln \frac{m_{\tilde{g}}^2}{m_{\tilde{t}}^2} \left( \tilde{L}_{tu} \tilde{L}_{tc}^* \right)^2 |T_{11}|^4, $$

where $\tilde{L}$ is a real matrix which has roughly the same form as $\tilde{K}$. $\Delta m_D$ is of order $10^{-14}$ GeV which corresponds to $x = (\Delta m/T)_{Ds}$ between $10^{-3}$ and $10^{-2}$, which is in the range of the SM prediction and is consistent with recent CLEO measurements [27].

Next consider the CP violating decay $K_L \to \pi^0 \nu \bar{\nu}$, which in the SM provides an alternate way to determine $\sin \beta$ [28]. It proceeds through a CP-violating $Zds$ effective vertex, for which the dominant SUSY contribution is the chargino-stop loop [29]:

$$ Z_{ds}^i \sim \frac{1}{4} m_{\tilde{t}}^2 \ln \frac{m_{\tilde{g}}^2}{m_{\tilde{t}}^2} |V_{11} T_{11}|^2 \tilde{K}_{td} \tilde{K}_{ts}. $$

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This contribution conserves CP, and thus we expect the branching ratio $K_L \to \pi^0 \nu \bar{\nu}/K^+ \to \pi^+ \nu \bar{\nu}$ to be $\mathcal{O}(\epsilon)$. This clearly violates the SM relation between the CP asymmetry in $B \to \psi K_s$ and the branching ratio of $K_L \to \pi^0 \nu \bar{\nu}$. However, the CP-conserving (charged) mode of this decay is dominated by the SM and chargino contributions. Typically we expect $Z^i_{ds}$ to be of order $10^{-4}$ which translates into the branching ratio for $K^+ \to \pi^+ \nu \bar{\nu}$ of the order of $10^{-10}$. In certain regions of the parameter space, this branching ratio can be significantly enhanced (up to an order of magnitude) over the SM prediction.

To summarize: our approach provides a unified view of all CP violation (including the baryon asymmetry [21]) which is testable at future colliders [30] and at the B factories, tying its origin to fundamental CP-violating parameters within a (Type I) string-motivated context. CP violation in the K system is mainly due to the gluino-squark diagrams, with phases from the gluino mass $M_3$ and the trilinear coupling $A$. As the CKM matrix is by assumption (approximately) real, the B system is superweak: CP violation occurs mainly due to mixing. Therefore the unitarity triangle does not close, and we expect $\sin 2\beta/\sin 2\alpha \simeq -1$. $\Delta m_K$ is dominated by the SM and gluino contributions, while $\Delta m_B$ is dominated by the chargino-stop contribution. $K^+ \to \pi^+ \nu \bar{\nu}$ can be enhanced while $K_L \to \pi \nu \bar{\nu}$ is suppressed compared to the SM predictions. $D - \bar{D}$ mixing is expected to occur at a level somewhat below the current limit. The CP asymmetry in $b \to s \gamma$ can be considerably enhanced over its SM value. The electric dipole moments of the electron and neutron are suppressed by cancellations and should have values near the current limits. Our approach dictates an unconventional and interesting flavor structure for the squark mass matrices at low energies which is required for consistency with the preliminary experimental value of $\sin 2\beta$. An investigation of the connection of these matrices to the flavor structure of a basic theory at high energies is underway [22].

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Table I: We list the CP-violating observables and our dominant one-loop contributions (we work within the decoupling limit and hence neglect the charged Higgs). The third column schematically shows the flavor physics. Basically the $\delta$’s are elements of the squark mass matrices normalized to some common squark mass, and the $\tilde{K}$’s are related to the $\Gamma^{U}$ matrices defined in the text (with the stop mixing factored out, so they represent the family mixing only). Subscripts label flavor or chirality. The table is designed to demonstrate symbolically which observables are related (or not) to others. (More technically, in the down-squark sector, we utilize the $(\delta_{ij})_{AB}$ parameters of the mass insertion approximation. $\tilde{K}_{ij}$ labels the flavor factors which enter in diagrams involving up-type squarks. The flavor factors which enter the $b \to s\gamma$ and the nEDM amplitudes are different from the $\tilde{K}$ matrices but the flavor structure is similar (analogous statements apply for $D - \bar{D}$ mixing).
REFERENCES

[1] J. Christenson, J. Cronin, V. Fitch, and R. Turlay, Phys. Rev. Lett. 13 (1964) 138.
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
[3] CDF Collaboration, CDF/PUB/BOTTOM/CDF/4855 (1999).
[4] A. Alavi-Harati et al., KTeV Collaboration, Phys. Rev. Lett. 83 (1999) 22.
[5] G. D. Barr et al., NA31 Collaboration, Phys. Lett. B317 (1993) 233.
[6] See e.g. A. Buras, hep-ph/9905437, hep-ph/9908393 and references therein.
[7] M. Gavela et al., Mod. Phys. Lett. A9 (1994) 795, Nucl. Phys. B430 (1994) 382; P. Huet and E. Slather, Phys. Rev. D51 (1995) 379; G. R. Farrar and M. E. Shaposhnikov, Phys. Rev. D50 (1994) 382.
[8] J. M. Frere and M. Gavela, Phys. Lett. B132 (1983) 107; S. Abel and J. M. Frere, Phys. Rev. D55 (1997) 1623.
[9] F. Gabbiani et al., Nucl. Phys. B477 (1996) 321.
[10] S. Bertolini et al., Nucl. Phys. B353 (1991) 591.
[11] M. Brhlik and H. Murayama, Phys. Rev. Lett. 83 (1999) 9107.
[12] K. Babu and S. Barr, Phys. Rev. Lett. 72 (1994) 2831; K. Babu, et al., hep-ph/9905464; S. Baek et al., hep-ph/9907572, and references therein; S. Baek, P. Ko, hep-ph/9904283; S. Khalil, T. Kobayashi, and A. Masiero, Phys. Rev. D60 (1999) 075003; R. Barbieri, et al., hep-ph/9908235; A. Buras et al., hep-ph/9908371; G. Eyal et al., hep-ph/9908382; D. Demir, A. Masiero, and O. Vives, hep-ph/9909323; hep-ph/9911337; A. Kagan and M. Neubert, hep-ph/9908404, Y. Grossman et al., hep-ph/9909297.
[13] M. Brhlik et al., Phys. Rev. Lett. 83 (1999) 2124; hep-ph/9908326.
[14] J. Ellis, S. Ferrara, and D.V. Nanopoulos, Phys. Lett. B114 (1982) 231; W. Buchmüller and D. Wyler, Phys. Lett. B121 (1983) 321; J. Polchinski and M. Wise, Phys. Lett. B125 (1983) 393; F. del Aguila et al., Phys. Lett. B126 (1983) 71.
[15] M. Dugan, B. Grinstein, and L. Hall, Nucl. Phys. B255 (1985) 413.
[16] R. Garisto, Nucl. Phys. B419 (1994) 279.
[17] T. Ibrahim and P. Nath, Phys. Rev. D58 (1998) 111301; Phys. Rev. D57 (1998) 478, Phys. Rev. D58 (1998) 019901; Phys. Lett. B418 (1998) 98; T. Falk and K. Olive, Phys. Lett. B439 (1998) 71; S. Pokorski, J. Rosiek, and C. Savoy, hep-ph/9906206.
[18] M. Brhlik, G. Good, and G. L. Kane, Phys. Rev. D59 (1999) 115004.
[19] Within the Type I string models, the matter fields arise from open strings which start and end on D-branes; in the heterotic language, they all have effective “modular weights” of $-1$, like untwisted sector fields. Therefore the trilinear superpotential couplings are $O(1)$, and other mechanisms (e.g. higher-dimensional operators) will be required to obtain realistic fermion textures (in contrast to perturbative heterotic orbifold models, in which the moduli dependence of the twisted sector Yukawa couplings can allow for the generation of realistic fermion mass matrices at the trilinear order). The question of flavor physics within Type I models is currently under investigation.
[20] See eg. M. Carena et al., Nucl. Phys. B503 (1997) 387; J. Cline et al., Phys. Lett. B417 (1998) 79; A. Riotto Phys. Rev. D58 (1998) 095009 and references therein.
[21] M. Brhlik, G. Good, and G. L. Kane, hep-ph/9911243.
[22] M. Brhlik et al., in progress.
[23] O. Lebedev, Phys. Lett. B452 (1999) 294.
[24] G. C. Branco, F. Cagarrinho and F. Krüger, *Phys. Lett.* **B459** (1999) 224.
[25] S. Adler et. al., *Phys. Rev. Lett.* **79** (1997) 2204.
[26] A. Kagan and M. Neubert, *Phys. Rev.* **D58** (1998) 094012.
[27] CLEO collaboration, hep-ex/9908040.
[28] See e.g. A. Buras, A. Romanino, and L. Silvestrini, *Nucl. Phys.* **B520** (1998) 3.
[29] G. Colangelo and G. Isidori, *JHEP* 9809:009 (1998).
[30] G. L. Kane, S. Mrenna, and L. Wang, hep-ph/9910477.