Wave processes in an extended underground pipeline interacting with soil according to the model of an "ideal elastoplastic body"

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Abstract. A piecewise-linear problem of the effect of a wave propagating in soil on an extended underground pipeline interacting with soil according to the model of an "ideal elastoplastic body" is solved by the finite difference method. The results of calculating the velocities, strains and lateral shear stress for three waveforms: step-like, impulse and harmonic ones are presented. The analysis of the results obtained is performed and the effect of reverse outward forcing of the pipeline under the action of a displacement impulse propagating in soil in the form of sine squared is shown. An approach to the stationary mode under sinusoidal impact is considered.

1. Introduction
The study of underground pipeline behavior as a life support system during earthquakes is an important problem. A review of publications related to this field is given in [1–3]. Experimental studies [1, 2, 4] made it possible to substantiate simplified models of viscoelastic and elastoplastic interaction of a pipeline with soil. In these articles, the main role is played, under certain conditions, by the soil properties; this is confirmed by the experiments conducted to determine the shear modulus of fine-grained soil at different loading rates [5].

In linear problems of seismodynamics of underground structures, the system of equations of motion includes terms without derivatives of displacements and rotation angles [1, 6]. The construction of finite-difference schemes for such equations without parasitic oscillations is given in [7–9]. Spatial problems for complex systems of underground pipelines are considered in [10–11].

In nonlinear problems of seismodynamics of underground structures, various models of the pipeline interaction with soil are used. In [12], the effect of a wave propagating in soil on an infinite rectilinear pipeline under dry friction is considered. A solution to the stationary problem is constructed and its solution is described. Non-stationary problems for a rod with external dry friction were solved by the method of characteristics in [4, 13, 14]. In [15], a finite-difference approximation of the equation of motion of a rod with external dry friction was constructed and an algorithm for its solution was constructed; then, this algorithm was used in [16–18]. The stationary problem of seismodynamics of an extended rectilinear pipeline with nonlinear interaction models using the plasticity function was considered in [19].
In this article, the problem of seismodynamics of an extended rectilinear underground pipeline with an “ideal elastoplastic body” interaction model is solved by the finite-difference method using a logical algorithm for determining transitions to the limiting state and back.

2. Material and methods
Let a plane longitudinal wave \( v_s(t - x / c_s) \) propagate in soil at velocity \( c_s \); the normal to its front is parallel to the axis of the pipeline of length \( l \). The origin of the O\( x \) coordinate axis is located at the left end of the pipeline. Suppose that the soil movement is set and is not distorted due to the presence of a pipeline, modeled by an elastic bar. The interaction of the pipeline with surrounding soil is taken into account according to the model of an “ideal elastoplastic body”; the constants of the model are determined experimentally. The dynamics equations for an extended underground pipeline are presented in the following form:

\[
\begin{align*}
\frac{\partial v}{\partial t} &= c^2 \frac{\partial e}{\partial x} + \frac{\pi D}{F \rho} \tau, \\
\frac{\partial e}{\partial t} &= \frac{\partial v}{\partial x}, \\
\tau &= \begin{cases} 
\tau_s + k_s \left( u_s - u - U_s \right), \text{ at } |\tau| < \tau_p; \\
\text{sign}(v_s - v) \cdot \tau_p, \text{ when the limit value is reached; }
\end{cases}
\end{align*}
\]

with initial conditions \( e|_{x=0} = 0 \) and \( v|_{x=0} = 0 \), and stress-free boundary conditions. Here \( c = \sqrt{E/\rho} \) is the velocity of the wave propagation in the pipeline; \( E, \rho \) are the modulus of elasticity and the density of the pipeline material; \( e, v, u \) are the strain, velocity and displacement of particles along the axis of the pipeline; \( v_s, u_s \) are the velocity and displacement of soil particles along the axis of the pipeline; \( D, F \) are the diameter and cross-sectional area of the pipeline; \( k_s \) is the coefficient of elastic interaction of the pipeline surface with soil; \( \tau_p \) is the absolute value of the limiting shear stress; \( \tau_s, U_s \) are the values of the limiting lateral shear stress and the difference between the displacements of the corresponding points of soil and the pipeline at the time of the s-th transition from the limiting state to the state of elastic interaction ( \( \tau_0 = 0; U_0 = 0 \) ). Experimental data \([4, 5]\) show that the values of \( k_s \) and \( \tau_p \) depend on static and dynamic pressure, and on the loading rate. For the convenience of the analysis of the solutions obtained the values of \( k_s \) and \( \tau_p \) are assumed as constants. It is necessary to pay attention to two circumstances: the velocities of wave propagation in soil and in the pipeline differ by several times, and the pipeline-soil interaction is described by a piecewise linear model.

We divide the pipeline of length \( l \) into segments of \( \Delta x \) into \( m \) parts \( l = m \Delta x \) . With the variable \( t \), we define the time step \( \Delta t = \Delta x / c_s \), which is the limiting condition of Courant stability for an explicit finite-difference scheme. Let us introduce the following notation \( \chi = \frac{\pi D}{F \rho} \).

Discrete values of strain are taken at the ends of segments \( \Delta x \), and the particle velocities are taken in the middle of segments \( \Delta x \). Discrete values of strain in time are taken in the middle of the step and the velocity of particles is taken at each step in time.

Let us represent equation (1) by their finite-difference approximation with the first order of accuracy in \( \Delta x \) and \( \Delta t \).
\[ \frac{v_{i+1/2}^{j+1} - v_i^{j+1/2}}{\Delta t} = c^2 \frac{E_{i+1/2}^{j+1} - E_i^{j+1}}{\Delta x} + \chi \frac{\tau_{i+1/2}^{j+1} + \tau_i^{j+1/2}}{2}; \]

\[ \tau_{i+1/2}^{j+1} = \begin{cases} \tau_i + k_s \left( u_{g,i+1/2}^{j+1} - u_i^{j+1/2} - \Delta t v_i^{j+1/2} - U_i \right), & \text{at } |v_i^{j+1/2}| < \tau_p; \\ \text{sign}(v_{g,i+1/2}^{j+1} - v_i^{j+1/2}) \cdot \tau_p, & \text{when the limit value is reached}; \end{cases} \]

From equations (2) we determine consecutively \( E_{i+1/2}^{j+1}, v_{i+1/2}^{j+1}, u_{g,i+1/2}^{j+1} \) and \( u_{i+1/2}^{j+1} \). The value of \( \tau \) is checked at all points, at each time step. At the points where \(|\tau| \geq \tau_p\), the particle velocity with the average value of \( \tau \) is recalculated from the previous time step and \( \text{sign}(v_{g,i+1/2}^{j+1} - v_i^{j+1/2}) \cdot \tau_p \). Next, we iteratively improve the solution using the Newton-Raphson method

\[ \Delta v^{(k)} = -\left( \frac{v_i^{j+1/2} - v_i^{j+1}}{\Delta t} + c \left( E_i^{j+1/2} - E_i^{j+1} \right) + \text{sign}(v_{g,i+1/2}^{j+1} - v_i^{j+1/2}) \cdot \chi \cdot \Delta t \cdot \tau_p \right), \]

where \( k \) is the iteration number. The iterative process continues until the required computation accuracy is reached. We store information about the transition to the limiting state, at this point, with the direction of the friction force. At subsequent time steps at these points, the last term of the first equation of system (2) must be changed to the limiting value of the friction force. Then, a check of the transition to the state of elastic unloading is made by changing the sign of the relative velocity \( v_x - v \). At the time of unloading, the values of \( \tau_x, U_x \) are stored. The first step of unloading is accompanied by the velocity calculation with the averaged value of \( \tau \) at this time step and \( \text{sign}(v_{g,i+1/2}^{j+1} - v_i^{j+1/2}) \cdot \tau_p \). Here we also make iterative improvement of the solution according to the following algorithm

\[ \Delta v^{(k)} = -\left( \frac{v_{i+1/2}^{j+1} - v_i^{j+1}}{\Delta t} + c \left( E_i^{j+1/2} - E_i^{j+1} \right) + \text{sign}(v_{g,i+1/2}^{j+1} - v_i^{j+1/2}) \cdot \chi \cdot \Delta t \cdot \tau_p + \right. \]

\[ \left. + \chi \cdot k_s \left( u_{g,i+1/2}^{j+1} - u_i^{j+1/2} - \Delta t \left( v_i^{j+1/2} + v_i^{j+1} \right) \right) / \left( 1 - \chi \cdot \Delta t \cdot k_s \right) \right), \]

\[ \left( v_{i+1/2}^{j+1} \right)^{(k+1)} = \left( v_{i+1/2}^{j+1} \right)^{(k)} + \Delta v^{(k)}. \]

In subsequent time steps, calculations were performed in accordance with the state of the interaction process at each point. It should be noted that the iterative improvement of the solution did not substantially change the result in specific calculations, the number of iterations was one when calculating the velocity with an accuracy of \( 10^{-4} \text{ m/s} \) and the iterative increment was of the order of \( 10^{-5} \text{ m/s} \).
3. Results and discussion

The calculations were performed with the following initial data: \( l = 1000 \text{ m} \); \( D = 0.61 \text{ m} \); \( F = 0.019 \text{ m}^2/\text{s} \); \( c_g = 500 \text{ m/s} \); \( c = 5000 \text{ m/s} \); \( k_i = 10^3 \text{ N/m}^3 \); \( \tau_p = 10 \text{ kPa} \); \( \Delta t = 0.0001 \text{ s} \).

Let us calculate the effect of the step-like wave of an amplitude \( v_{gm} = 0.19 \text{ m/s} \). Figure 1 shows the dimensionless velocities of the pipeline and soil particles at different points in time along the length of the pipeline. An excitation propagates ahead of the wave front in soil along the pipeline; its amplitude decreases with propagation. If the left end of the pipeline would move in the same way as soil, then the wave front in the pipeline would be discontinuous and would propagate without attenuation [6]. As seen from Figures 1 (b, c), the process of wave propagation in the pipeline nearly reaches a stationary mode [12, 13], the influence of low-amplitude waves reflected from the ends of the pipeline is noticeable.

![Figure 1. Dimensionless velocities of soil \( v_{gm} = v_g / v_{gm} \) and pipeline \( v_n = v / v_{gm} \) particles at points in time: \( t = 0.05 \text{ s} \) (a), \( t = 0.3 \text{ s} \) (b), \( t = 0.9 \text{ s} \) (c).](image)

Figure 2 shows the normalized graphs of soil strain \( \varepsilon_{gm} = \varepsilon_g / \varepsilon_{gm} \) (here \( \varepsilon_{gm} = 0.00038 \)) and the pipeline \( \varepsilon_n = \varepsilon / \varepsilon_{gm} \), and the lateral shear stress \( \tau_n = \tau / \tau_p \) at different points in time. Here one can see the discontinuity of the strain at the wave front in soil and the areas with the limiting state according to the graph of the lateral shear stress. It should be noted that the lateral shear stress reaches the limiting state before the discontinuous wave front in soil due to the wave propagating in the pipeline.

![Figure 2. Normalized strains in soil \( \varepsilon_{gm} = \varepsilon_g / \varepsilon_{gm} \), the pipeline \( \varepsilon_n = \varepsilon / \varepsilon_{gm} \), and the lateral shear stress at points in time: \( t = 0.05 \text{ s} \) (a), \( t = 0.3 \text{ s} \) (b), \( t = 0.9 \text{ s} \) (c).](image)

Figures 3 and 4 show the results of calculations when a wave is represented as an impulse \( v_g = 2v_{gm} \sin[\pi(t - x / c_g)/t_0] \cos[\pi(t - x / c_g)/t_0][H(t - x / c_g) - H(t - t_0 - x / c_g)] \), where \( H(t) \) is the Heaviside function, \( t_0 = 0.165 \text{ s} \). Figure 3 shows the process of wave formation in the pipeline. Due to the presence of the sliding motion areas with friction between the pipeline and soil, the velocity waveform takes the form shown in figure 3 and then propagates unchanged. Since, in this case, there is no discontinuous wave front in soil, the limiting state of the lateral shear stress occurs, for a short time, before the wave front in soil at a short distance from the left end of the pipeline and then disappears. Behind the wave front in soil, there are two sections with the ultimate shear stress, and the length of the first section is twice the length of the second section (figure 4). In a shorter section at the left end of the
pipeline, the lateral shear stress can also reach the limiting state due to the wave reflection from the stress-free end. The front part of the strain wave in soil is compressive and a transition to the limiting state occurs in that section; the energy transferred to the pipeline propagates through it at a higher velocity in the form of a compressive wave and is then spent to overcome the force of the spring. The rear part of the wave in soil is tensile, so the energy transferred to the pipeline generates a tensile wave, which propagates at a higher velocity, reaches the zone with the limiting state, reducing the value of compressive strains and thereby increasing the length of this zone. As a result, the effect of reverse outward forcing of the pipeline occurs due to the presence of sliding areas with friction. As shown in figure 4 (c), \( u, u_g \) are the displacements of points of the pipeline and soil, \( m \), respectively. Such a pattern is not observed in the case of the elastic interaction model.

![Figure 3](image1)

**Figure 3.** Dimensionless velocities of soil \( v_{gn} = v_g / v_{gm} \) and pipeline \( v_n = v / v_{gn} \) particles at points in time: \( t=0.05 \text{ s} \) (a), \( t=0.3 \text{ s} \) (b), \( t=0.9 \text{ s} \) (c).

![Figure 4](image2)

**Figure 4.** Normalized strains in soil \( \varepsilon_{gn} = \varepsilon_g / \varepsilon_{gm} \), the pipeline \( \varepsilon_n = \varepsilon / \varepsilon_{gm} \), and lateral shear stress at points of time: \( t=0.05 \text{ s} \) (a), \( t=0.9 \text{ s} \) (b), and displacements (in meters) at \( t=0.9 \text{ s} \) (c).

Figure 5 shows the results of calculations when a wave is represented as an impulse \( v_g = v_{gm} \cos \left[ \pi(t - x / c_g) / t_0 \right] \left[ H(t - x / c_g) - H(t - t_0 - x / c_g) \right] \), \( t_0 = 0.165 \text{ s} \). Figure 5 shows graphs similar to the ones given in figure 4. In this case, the front of the strain wave in soil has a discontinuity, so, the sliding motion with friction occurs before this front; the same pattern is observed behind the rear front of the wave. The displacement graph along the pipeline length differs from the displacement graph given in figure 4 (c). This means that the residual effects after the wave propagation depend on the waveform.

Figure 6 shows the results of calculations when a wave is represented as an impulse \( v_g = v_{gm} \left[ H(t - x / c_g) - H(t - t_0 / 2 - x / c_g) - v_{gm} \left[ H(t - t_0 / 2 - x / c_g) - H(t - t_0 - x / c_g) \right] \right] \), \( t_0 = 0.165 \text{ s} \). Graphs in figure 6 also confirm that the residual effects after the wave propagation depend on the waveform.

Figures 7 and 8 show the results of calculations when a wave is represented in the form \( v_g = v_{gm} \cos \left[ \pi(t - x / c_g) / t_0 \right] H(t - x / c_g), t_0 = 0.165 \text{ s} \). These figures show the process of reaching the stationary mode after the first half-period \( t_0 \) of the wave propagating in soil. Near the discontinuous front of a wave propagating in soil, at the left end, the velocity of the pipeline particles begins to increase,
then, with distance, it decreases, and then, at some distance from the left end, it goes into a stationary mode.

Figure 5. Normalized strains in soil $\varepsilon_g = \varepsilon / \varepsilon_m$, the pipeline $\varepsilon_n = \varepsilon / \varepsilon_m$, and lateral shear stress $\tau_n = \tau / \tau_p$ at points of time: $t=0.05 \text{ s} \ (a)$, $t=0.9 \text{ s} \ (b)$, and displacements (in meters) at $t=0.9 \text{ s} \ (c)$.

Figure 6. Normalized strains in soil $\varepsilon_g = \varepsilon / \varepsilon_m$, the pipeline $\varepsilon_n = \varepsilon / \varepsilon_m$, and lateral shear stress $\tau_n = \tau / \tau_p$ at points of time: $t=0.05 \text{ s} \ (a)$, $t=0.9 \text{ s} \ (b)$, and displacements (in meters) at $t=0.9 \text{ s} \ (c)$.

Figure 7. Dimensionless velocities of soil $v_g = v / v_m$ and pipeline $v_n = v / v_m$ particles at points of time: $t=0.05 \text{ s} \ (a)$, $t=0.3 \text{ s} \ (b)$, $t=0.9 \text{ s} \ (c)$.

Figure 8. Normalized strains in soil $\varepsilon_g = \varepsilon / \varepsilon_m$ and pipeline $\varepsilon_n = \varepsilon / \varepsilon_m$, and lateral shear stress $\tau_n = \tau / \tau_p$ at points of time: $t=0.05 \text{ s} \ (a)$, $t=0.3 \text{ s} \ (b)$, $t=0.9 \text{ s} \ (c)$.

4. Conclusion
The non-stationary problem of the effect of a plane longitudinal wave propagating in soil on a finite length underground pipeline interacting with soil according to the model of an “ideal elastoplastic body”
is solved by an explicit finite difference method. Numerical values of velocities, strains, and lateral shear stresses are obtained for three waveforms: step-like, impulse, and harmonic ones. The process of wave formation and propagation in the pipeline is shown. The residual effects after the wave propagation depend on the waveform. The effect of the outward forcing of the pipeline under the action of a displacement impulse propagating in soil in the form of sine squared and an isosceles triangle is obtained. An approach to the stationary mode of wave propagation in the pipeline is shown for a sinusoidal wave in soil.

References

[1] Rashidov T 1973 Dynamic theory of seismic resistance of complex systems of underground structures (Tashkent: Fan) p 180 URL: https://ru.b-ok.as/book/2975567/59d466 (in Russian)
[2] Rashidov T and Khozhmetov G X 1985 Seismic resistance of underground pipelines (Tashkent: Fan) p 152 URL: https://www.twirpx.com/file/1141472 (in Russian)
[3] Israilov M Sh 2017 Seismodynamics of extended underground structures: the limits of applicability of engineering approaches and the illegality of the analogy with aboveground structures Earthquake-resistant construction. Safety of structures № 1 pp 55–60 (in Russian)
[4] Sultanov K S 2016 Wave theory of seismic resistance of underground structures (Tashkent: Fan) p 392 https://www.twirpx.com/file/2097102 (in Russian)
[5] Massarsh K R 2005 Deformation properties of fine-grained soils based on seismic test indicators Urban reconstruction and geotechnical construction № 9 pp 203–220 (in Russian)
[6] Rashidov T R, Kuznetsov S V, Mardonov B M and Mirzaev I 2019 Applied problems of seismodynamics of structures. Book 1. The effect of seismic waves on an underground pipeline and foundations of structures interacting with the soil environment (Tashkent: "Navruz") p 268 URL: http://mechmath.ipmnet.ru/lib/? S = solid & book = 33827 (in Russian)
[7] Mirzaev I M and Nikiforovskii V S 1973 Plane wave propagation and fracture in elastic and imperfectly elastic jointed structures Soviet Mining Science 9 pp 161–165 doi:10.1007/BF02506181
[8] Nikifirovsksky V S and Shemyakin E I 1979 Dynamic Fracture of Solids (Novosibirsk: Nauka) p 272 URL: https://www.twirpx.com/file/232741 (in Russian)
[9] Virginia Corrado, Berardino D’Acunto, Nicola Fontana and Maurizio Giugni 2012 Inertial Effects on Finite Length Pipe Seismic Response Mathematical Problems in Engineering Hindawi Publishing Corporation 2012 Article ID 824578 doi:10.1155/2012/824578
[10] Bekmirzaev D A and Mirzaev I 2020 Dynamic Processes in Underground Pipelines of Complex Orthogonal Configuration at Different Incidence Angles of Seismic Effect International journal of scientific & technology research 9 issue 04 pp 2449–2453
[11] Bekmirzaev D, Mirzaev I, Mansurova N, Kosimov E and Juraev D 2020 Numerical methods in the study of seismic dynamics of underground pipelines. IOP Conf. Series: Materials Science and Engineering 869 052035 doi:10.1088/1757-899X/869/5/052035
[12] Ilyushin A A and Rashidov T R 1971 On the effect of a seismic wave on an underground pipeline Izv. AN RUs. Ser. Tech. Science № 1 pp 3–11 (in Russian)
[13] Nikitin L V 1998 Statics and dynamics of rigid bodies with external dry friction (Moscow: Moscow Lyceum) p 261 URL: https://www.twirpx.com/file/1003867 (in Russian)
[14] Mogilevsky R I, Ormonbekov T O and Nikitin L V 1993 Dynamics of rods with interfacial dry friction J. Mech. Behav. Mater. 5 issue 1 pp 85–93
[15] Mirzaev IM 1985 Dynamics of a prestressed bar under the action of a shock load Dynamics of a continuous medium 71 pp 65–74 (in Russian)
[16] Isakov A L and Shmelev V V 1998 Wave processes when driving metal pipes into the ground using shock-pulse generators Journal of Mining Science 34 pp 139–147
[17] Smirnov A L 1989 Computation of the process of impact submersion of a pile in the ground Soviet Mining Science 25 pp 359–365
[18] Aleksandrova N I 2012 Numerical-analytical investigation into impact pipe driving in soil with dry friction Part I: Nondeformable external medium *Journal of Mining Science* 48 pp 856–869

[19] Kolmakova E 1985 The effect of a stationary seismic wave on a long pipeline taking into account the plastic properties of interaction with soil *Proc. Int. Scientific Conf. on Friction, wear and lubricants* 5 (Tashkent: Fan) pp 139–140 (in Russian)