Anapole moment of the lightest neutralino in the cMSSM

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Abstract

We study the anapole moment of the lightest neutralino in the constrained Minimal Supersymmetric Standard Model (cMSSM). The electromagnetic anapole is the only allowed electromagnetic form factor for Majorana fermions, such as the neutralino. Since the neutralino is the LSP in many versions of the MSSM and therefore a candidate for dark matter, its characterization through its electromagnetic properties is important both for particle physics and for cosmology. We perform a scan in the parameter space of the cMSSM and find that the anapole moment is different from zero, albeit very small ($\lesssim 10^{-3}$ GeV$^{-2}$), well below experimental bounds. Despite its small value, it is still possible to distinguish between different regions of parameter space through the anapole moment, adding it thus to the list of experimental constraints when looking for extensions of the Standard Model.

1 Introduction

One of the best motivated extensions of the Standard Model (SM) is the Minimal Supersymmetric Standard Model (MSSM), since, besides giving a solution to the hierarchy problem, it provides us with a good candidate for cold dark matter (CDM), namely, the lightest neutralino.
There are currently several experiments under way, and more planned for the future for direct and indirect detection of dark matter (DM) (for recent reviews on dark matter detection see [1,2]). If detected, it will be necessary to discriminate between different candidates. To this end, it will be important to characterize as much as possible the different candidates. Therefore, the neutralino electroweak properties can give us some insight into its nature.

In the last years, there has been intense work on the electroweak properties of dark matter since they might be relevant in the calculation of DM decays and annihilations [3–15], which have consequences in astrophysical processes [16,17] and are thus important in indirect astrophysical searches for DM, as in the calculation of the annihilation cross section of the DM itself.

Motivated by the above mentioned problems, we have been studying the toroidal dipole moment of Majorana particles, which is related to the anapole moment, one of the least studied electromagnetic properties of a particle [18–21]. The anapole moment corresponds to a $T$ invariant interaction, which is $C$ and $P$ non-invariant [22]. It does not have a simple classical analogue, so the toroidal dipole moment (TDM) was introduced as a more convenient description of $T$ even, $C$ and $P$ odd interactions [23–26]. The electromagnetic vertex of a particle can thus be expressed in a multipole parametrization, including the toroidal moments (see for instance [27]), which provides a one to one correspondence between the form factors and the multipole moments.

Pospelov and ter Veldhuis have obtained an upper limit for the anapole moment of WIMPS [28], using results from the DAMA and CDMS experiments [29,30]. In case the neutralino is the main component of dark matter, its anapole moment should comply with this limit.

Recently, Ho and Scherrer have proposed that dark matter interacts with ordinary matter exclusively through the anapole moment [31]. They calculate the anapole moment needed to obtain the right amount of DM relic abundance, and the anapole DM signatures that could be observed in the LHC [32]. Haish and Kahlhoefer have shown the importance of loop contributions to the scattering cross section of dark matter, in particular those induced by the anapole interaction [33]. More recently, del Nobile et al made a halo-independent analysis of direct DM detection data considering that it has only anapole and magnetic moment dipole interactions [34].

In this paper we calculate the anapole moment of the neutralino at one-loop level within the constrained Minimal Supersymmetric Standard Model. In the MSSM the anapole contributions of the neutralino arise exclusively
by radiative corrections to the vertex $\chi \bar{\chi} \gamma$. We do a scan in the five parameter space of the cMSSM, assuming that dark matter is composed exclusively by neutralinos, and compare the results for the anapole moment with the above mentioned experimental limit. In our analysis we take into account also other experimental constraints, namely the Higgs boson mass and the decays $b \to s \gamma$ and $B \to \mu^+ \mu^-$ in order to find the viable regions of the parameter space. We find that although the anapole moment is very small throughout the regions studied, it is possible to distinguish between the different regions of parameter space through it, which makes it indeed an important property when characterising dark matter. Our results agree qualitatively with those found by Ho and Scherrer [31].

The article is organised as follows: in section II we present a very brief summary of some aspects of the constrained MSSM (cMSSM) relevant to our calculation. In section III we review the general form for the electromagnetic vertex of a particle, and in particular for a Majorana particle. We introduce the anapole moment and its relation to the toroidal dipole moment. In section IV we explain the methodology used to calculate the anapole moment of the neutralino in the cMSSM and evaluate it for different values of the parameters. Section V presents the results obtained and our conclusions.

2 The MSSM and the neutralino as candidate for dark matter

The cold dark matter density is known to be [35]

$$\Omega_{_{DM}}h^2 \sim 0.1109,$$  \hspace{0.5cm} (1)

where $h$ is the Hubble constant in units of 100 km sec$^{-1}$ Mpc$^{-1}$. The thermally averaged effective cross section times the relative speed of the dark matter particle, needed to get this relic density is [36-38]

$$<\sigma v> \propto \frac{g_{weak}^4}{16\pi^2} m_x^2$$  \hspace{0.5cm} (2)

consistent with the assumption of a weakly interacting dark matter particle (WIMP) with a mass between 10 GeV - (few) TeV.

The minimal supersymmetric extension of the Standard Model (MSSM) provides us with one of the best WIMP candidates for dark matter: The lightest neutralino (for reviews on SUSY see for instance [39,40]). The MSSM
requires two complex Higgs electroweak doublets to give mass to the up and down type quarks in order to avoid chiral anomalies. After electroweak symmetry breaking five physical Higgs states remain: two neutral CP invariant ($h^0, H^0$), two charged CP invariant ($H^+, H^-$), and one neutral CP-odd ($A^0$).

The MSSM has a new discrete symmetry, R parity, defined as $R = (-1)^{3B+2S+L}$, where $B$ and $L$ are the baryonic and leptonic numbers respectively. This symmetry assigns a charge $+1$ to the SM particles and $-1$ to the supersymmetric partners, thus making the lightest supersymmetric particle (LSP) stable.

Supersymmetry has to be broken, or it would have already been observed. To break supersymmetry explicitly, without the reappearance of quadratic divergencies, a set of super-renormalizable terms are added to the Lagrangian, the so-called soft breaking terms. The Lagrangian for the soft breaking terms is given by

$$\mathcal{L}_{soft} = -\frac{1}{2} M_a \lambda^a \lambda^a - \frac{1}{6} A^{ijk} \phi_i \Phi_j \phi_k - \frac{1}{2} B^{ij} \phi_i \phi_j + c.c. - (m^2)^i_j \phi^j \phi_i , \quad (3)$$

where $M_a$ are the gaugino masses, $A^{ijk}$ and $B^{ij}$ are trilinear and bilinear couplings respectively, and $(m^2)^i_j$ are scalar squared-mass terms. It is assumed that supersymmetry breaking happens in a hidden sector, which communicates to the observable one only through gravitational interactions, and that the gauge interactions unify. This means that at the GUT scale the soft breaking terms are “universal”, i.e., the gauginos $M_a$ have a common mass, as well as the scalars $(m^2)^i_j$ and the trilinear couplings, $A^{ijk}$. Requiring electroweak symmetry breaking fixes the value of $B^{ij}$ and the absolute value of the Higgsino mixing parameter $|\mu|$. This is known as the constrained MSSM (cMSSM) which is described by five parameters: the unified gaugino mass $m_{1/2}$, the universal scalar mass $m_0$, the value of the universal trilinear coupling $A_0$, the sign of Higgsino mass parameter $\mu$, and the ratio of the vacuum expectation values of the two Higgses, $\tan \beta$.

After the electroweak symmetry breaking the neutral and charged states in the MSSM can mix. In the case of the neutral ones they give rise to a set of four mass eigenstates, the neutralinos. It is the lightest one of these that is the LSP and a good candidate to dark matter in many SUSY models. The lightest neutralino, in the gauge eigenstate basis, is thus a function of the neutral Higgsinos and the neutral gauginos (Wino and Bino)

$$\psi_0 = (\tilde{B}, \tilde{W}^0, \tilde{H}^0_u, \tilde{H}^0_d) . \quad (4)$$
The properties of the neutralinos will depend on the mixing, which in turn depends on the soft breaking parameters. Thus, the lightest neutralino can range from almost pure Bino to almost pure Higgsino.

Before WMAP, the cMSSM was compatible with the limit $\Omega_{DM,0}h^2 \sim 0.1 - 0.3$ and other direct and indirect low energy and collider data in a huge parameter space region called the “bulk”. However, after the constraint by WMAP to $\Omega_{DM,0}h^2$, and with the recent limits to the sparticles masses from the LHC, which excludes light masses, the bulk region in $m_0 - m_{1/2}$ is no longer viable. Moreover, in the cMSSM the LSP neutralino is, practically for all cases, an almost pure bino state which annihilates itself more efficiently into leptons through right hand sleptons due to their higher hypercharge. However, with the newest data from WMAP, this mechanism is not sufficiently efficient. There are still three favoured scenarios that require some very specific accidental relations between some parameters at the electroweak scale.

In the cMSSM at low $m_0$, there is a region with almost degenerate $\tilde{\tau} - \tilde{\chi}^0_1$. In this case the populations of these two particles are almost the same, making the NLSP $\tilde{\tau}$ thermally accesible. The mass difference between the scalar tau and the highest neutralino, $\Delta M = m_\tilde{\tau} - m_\chi^0_1$, controls the population ratio of these two species through the Boltzmann factor $\exp(-\Delta M/T_f)$. Therefore, it is a very sensitive parameter which enters into the calculation of the relic density. Whenever the coannihilation takes place, through the participation of $\tilde{\tau}$ in processes like $\tilde{\tau}_1\chi^0_1 \rightarrow \tau\gamma$ or even $\tilde{\tau}_1\tilde{\tau}_1 \rightarrow \tau\tilde{\tau}$, the relic density can be reduced in comparison to the case of the bulk scenario. In this region, the LSP neutralino is mainly bino with a mass essentially stablished by $M_1$ up to corrections of order $M_2/\mu$ ($\mu$ is high). The approximate formulae for the mass of the neutralino and the mass of $\tilde{\chi}^0_1$ suggest that degeneration happens for $m_0 \sim 0.145 m_{1/2}$.

A sudden increase in the usual mechanism of coannihilation to reduce the relic density can occur if $m_{\chi^0_1}$ is close to a pole. Faster and more efficient annihilation can take place through Higgs resonance. Given the Majorana nature of the neutralino, the resonant enhancement is obtained only via the pseudo-scalar Higgs boson. The colliders constrains to the LSP in the cMSSM allow the heavy “Higgs funnel”, where $\chi^0_1\chi^0_1 \rightarrow A \rightarrow b\bar{b}/\tau\tilde{\tau}$, which happens for high $\tan \beta$. Therefore, the quantities that establish this scenario are the quantity $2m_{\chi^0_1} - m_A$ and the amplitude of the pseudo-scalar, since they define the resonance profile of $A$.

In most of the cMSSM, $\mu$ is very high. However, you can exceptionally
have that $\mu \sim M_1$, which allows much more efficient coannihilation through reactions as $\tilde{\chi}_0^0 \tilde{\chi}_1^0 \rightarrow WW/ZZ/Zh/t\bar{t}$. This happens in the so-called focus point region where $m_0$ is very high. The focus point region corresponds to high values of $m_0$ close to the border of viable electroweak symmetry breaking, where the value of $\mu$ decreases rapidly. When $\mu \sim M_1, M_2$, the LSP has a significant fraction of higgsino, and the next lightest sparticles ($\tilde{\chi}_2^0$ or $\tilde{\chi}_1^\pm$) have also a significant component of higgsino and are not much heavier than the LSP. Thus, the coannihilation channels are favoured. However, coannihilation cannot be very efficient, otherwise the relic density would be less than what is actually measured. In this scenario, all the sfermions are very heavy (more than 4 TeV) to be accessible to one of the proposed colliders. The LSP mass goes from close to 150 to 350 GeV, with higgsino-type neutralinos being 100 to 50 GeV heavier. From the perspective of a linear collider, an energy of more than 800 GeV is needed to reveal some of the properties of this scenario. The pseudo-scalar has a mass higher than 1 TeV and very likely would not be found directly in the LHC.

3 Anapole Moment

For 1/2-spin particles the most general expression for the electromagnetic vertex function, which characterizes the interaction between the particle and the electromagnetic field, is:

$$\Gamma_\mu(q) = f_Q(q^2)\gamma_\mu + f_\mu(q^2)i\sigma_{\mu\nu}q^\nu\gamma_5 - f_E(q^2)\sigma_{\mu\nu}q^\nu + f_A(q^2)(q^2\gamma_\mu - qq_\mu)\gamma_5,$$

(5)

where $f_Q(q^2)$, $f_\mu(q^2)$, $f_E(q^2)$ and $f_A(q^2)$ are the so-called charge, magnetic dipole, electric dipole and anapole form factors, respectively; in here $q_\mu = p_\mu - p_\mu$ is the transferred 4-momentum and $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$ [25,41]. These form factors are physical observables when $q^2 \to 0$, and their combinations define the well-known electric charge (Q), magnetic dipole ($\mu$), electric dipole ($d$) and anapole ($a$) moments.

The electromagnetic properties of Majorana fermions (like the neutralino) are described by a unique form factor, the anapole, $f_A(q^2)$. This is a consequence of CPT-invariance and the C, P, T properties of $\Gamma_\mu(q^2)$ and the interaction Hamiltonian. Thus, the electromagnetic vertex function of a neutralino can be written just as

$$\Gamma_\mu(q^2) = f_A(q^2)(q^2\gamma_\mu - qq_\mu)\gamma_5.$$

(6)
The anapole moment was introduced by Zel’dovich to describe a T-invariant interaction that does not conserve P and C parity \[22\]. In contrast to the electric and magnetic dipole moments, the anapole moment interacts only with external electromagnetic currents \( J_\mu = \partial_\nu F_{\mu\nu} \). In the non-relativistic limit, the interaction energy with an external electromagnetic field takes the form

\[
H_{\text{int}} \propto -\mu (\sigma \cdot B) - d (\sigma \cdot E) - a (\sigma \cdot \nabla \times B),
\]

where \( B \) and \( E \) are the strength of the magnetic and electric fields, and \( \sigma \) are the Pauli spin matrices.

The anapole moment does not have a simple classical analogue, since \( f_A(q^2) \) does not correspond to a multipolar distribution. A more convenient quantity to describe this interaction was proposed by V. M. Dubovik and A. A. Cheshkov \[42\]: The toroidal dipole moment (TDM), \( \tau(q^2) \). The TDM and the anapole moment coincide in the case of \( m_i = m_f \), i.e. the incoming and outgoing particle are the same. This type of static multipole moments does not produce any external fields in vacuum but generate a free-field (gauge invariant) potential \[25\], which is responsible for topological effects like the Aharonov-Bohm one.

The simplest TDM model (anapole) was given by Zel’dovich as a conventional solenoid rolled up in a torus and with only one poloidal current, see fig. [1]. For such stationary solenoid, without azimuthal components for the current or the electric field, there is only one magnetic azimuthal field.
different from zero inside the torus.

4 One-loop calculation

A neutralino is a Majorana particle, and necessarily electrically neutral. This fact does not allow for a tree level electromagnetic coupling. Therefore the electromagnetic properties of the Majorana particle –the anapole– arise only via loop contributions. The anapole moment of the neutralino may be defined in the one-loop approximation in the MSSM by the Feynman diagrams shown in figs. 2 and 3 where \( f \) represents the charged fermions of the SM. Taking each fermionic family separately we obtain 96 Feynman diagrams in total, corresponding to self-energies and vertex corrections.

We use \textit{FeynCalc} [43] to calculate the amplitude of these diagrams. Since we are only interested in the terms that contribute to the anapole form factor, we isolate the ones that have the Lorentz structure \( \gamma_\mu \gamma_5 \). One of the first results we obtain is that the self-energies \( \gamma H^0, \gamma h^0, \gamma A^0 \) and \( \gamma G^0 \) do not contribute to the calculation at all. If we call \( \Xi_i \) the coefficient that multiplies \( \gamma_\mu \gamma_5 \) for the \( i \)th diagram, then we have that

\[
\sum_i \Xi_i = f_A(q^2)q^2. \tag{8}
\]

To obtain the anapole moment \( a = f_A(0) \) we use the l’Hopital rule and get

\[
a = f_A(0) = \lim_{q^2 \to 0} \frac{\sum_i \Xi_i}{q^2} = \frac{\partial}{\partial q^2} \left|_{q^2 \to 0} \sum_i \Xi_i \right. \tag{9}
\]

Two- and three-point Passarino-Veltman scalar functions arise in the calculation of each diagram. The two-point PV scalar function is defined as

\[
B_0(q^2; m_1^2, m_2^2) \equiv \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^Dk}{[k^2 - m_1^2][(k + q)^2 - m_2^2]}, \tag{10}
\]

and the three point PV scalar function is defined as

\[
C_0(A, B, C; m_1^2, m_2^2, m_3^2) \equiv \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^Dk}{[k^2 - m_1^2][(k + p_1)^2 - m_2^2][p_2^2 - m_3^2]}. \tag{11}
\]
where $A = p_1^2, B = (p_1 - p_2)^2$ and $C = p_2^2$. The self-energies contain two point Passarino-Veltman scalar functions of the type $B_0(q^2, x^2, x^2)$ and $B_0(0, x^2, x^2)$. Likewise, the contributions to the vertex corrections have two and three point scalar functions of the type $B_0(q^2, x^2, x^2), B_0\left(M_{\tilde{\chi}_1^0}^2, y^2, x^2\right)$ and $C_0\left(q^2, M_{\tilde{\chi}_1^0}^2, M_{\tilde{\chi}_1^0}^2, x^2, x^2, y^2\right)$. In both cases $x$ and $y$ represent the masses of the particles in the loop.

When evaluating (6), derivatives of the Passarino-Veltman functions ap-
Figure 3: One-loop corrections to the self-energy for the process $\gamma \rightarrow \chi^0_1\chi^0_1$. 
To evaluate the $B_0$'s, as well as their derivatives, we use *LoopTools* \[44\].

To evaluate the $C_0$'s and their derivatives we expand them in a power series around $q^2 = 0$. In this way it is possible to find an analytic approximation which coincides with the full expression in the limit $q^2 = 0$, simplifying enormously the calculation (see appendix).

In all regions of parameter space (except for $m_{1/2} \gg m_0$, which is ruled out by cosmological constrains since the LSP is charged) it was observed that the two triangle diagrams involving $\tilde{\tau}$ in the loop are almost completely dominant, see fig. 4. The expressions for the contributions of these two diagrams are

\[
\Xi_1 = \frac{a_1 B_0(q^2, m_\tau^2, m_{\tilde{\tau}}^2) + a_2 B_0(M_{\tilde{\chi}_1^0}^2, m_\tau^2, m_{\tilde{\tau}}^2) + a_3 C_0(q^2, M_{\tilde{\chi}_1^0}^2, M_{\tilde{\tau}}^2, m_\tau^2, m_{\tilde{\tau}}^2)}{32\pi^2 \cos^2 \theta_W \sin^2 \theta_W M_{\tilde{\chi}_1^0}^2 (q^2 - 4M_{\tilde{\chi}_1^0}^2)}
\]

(12)

and

\[
\Xi_2 = \frac{a'_1 B_0(q^2, m_\tau^2, m_{\tilde{\tau}}^2) + a'_2 B_0(M_{\tilde{\chi}_1^0}^2, m_\tau^2, m_{\tilde{\tau}}^2) + a'_3 C_0(q^2, M_{\tilde{\chi}_1^0}^2, M_{\tilde{\tau}}^2, m_\tau^2, m_{\tilde{\tau}}^2)}{32\pi^2 \cos^2 \theta_W \sin^2 \theta_W M_{\tilde{\chi}_1^0}^2 (q^2 - 4M_{\tilde{\chi}_1^0}^2)}
\]

(13)

respectively, where $a_i$ and $a'_i$ are coefficients that depend on $M_{\tilde{\chi}_1^0}$, $m_\tau$, $m_{\tilde{\tau}}$, $\cos \beta$ and the elements of the mixing matrix for the staus. This is explained by the fact that in most parts of the parameter space for this particular model the $\tilde{\tau}$ is the NLSP, if not the LSP itself. Given that the PV functions involved have the term $M_{\tilde{\chi}_1^0}^2 - m_\tau^2$ in the denominator, these two diagrams tend to contribute more than the others.

Figure 4: Dominant Feynman diagrams for calculation of neutralino’s TDM.
The expression obtained for the anapole moment depends on various parameters of the MSSM, including the supersymmetric particles masses as well as the mass mixing matrix elements, the value of $\tan \beta$, and the values of the soft breaking terms. We evaluate the anapole moment within the cMSSM using Suspect [45], by fixing the value of $A_0$, $\tan \beta$ and sign$\mu$, and scanning over the other two parameters, $m_0$ and $m_{1/2}$. We then vary $A_0$, $\tan \beta$ and sign$\mu$ and repeat the procedure. Comparing the different results, no dependence on $A_0$ or sign$\mu$ is shown. Although sign$\mu > 0$ may solve the problem of the discrepancy between the measured value of $g - 2$ of the muon and the one predicted by the SM, this does not mean negative sign$\mu$ is ruled out since this problem might be solved through other mechanisms, therefore sign$\mu < 0$ should be also taken into consideration. However, the TDM does depend slightly on $\tan \beta$ as can be seen in figure.

Cosmological and experimental constrains have highly reduced the allowed regions of the parameter space of the cMSSM. The most recent of these constrains is the one from the CMS and ATLAS collaborations on the mass of the lightest Higgs boson $m_h = 125.8 \pm 0.6$ GeV [46–49]. This constraint is derived from a combination of $5.1$ fb$^{-1}\sqrt{s} = 7$ TeV data and $12.2$ fb$^{-1}\sqrt{s} = 8$ TeV data. In our calculation we take this constraint as $m_h = 126 \pm 3$, where the uncertainty comes from a combination of the experimental and theoretical determinations of the Higgs mass.

There is also a new measurement of $BR(\bar{B}_s \rightarrow \mu^{+}\mu^{-}) = (3.2 \pm 1.5) \times 10^{-9}$ (14) from the LHCb collaboration, derived from $1$ fb$^{-1}$ of data at $\sqrt{s} = 7$ TeV collision energy and $1.1$ fb$^{-1}$ of data at $\sqrt{s} = 8$ TeV collision energy [50]. The excluded region due to this constraint in the cMSSM has already been determined in [51]. The recent measurement of $BR(B \rightarrow X_s \gamma) = (355 \pm 33) \times 10^{-6}$ further constrains the parameter space. We impose also the constraint coming from the branching ratio $b \rightarrow s\gamma$, whose value is given by the Heavy Flavour Averaging Group (HFAG) as [52]

$$BR(b \rightarrow s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}. \quad (15)$$

this result can be determined directly from Suspect. Moreover, if we assume that neutralinos make up all of the dark matter in the universe, the WMAP 7-year dark matter relic abundance value $\Omega_\chi h^2 = 0.1109 \pm 0.0056$ [35] puts even more strict constraints. In general, the “surviving” regions have a very
Figure 5: Anapole moment for three different values of $A_0$ (-1000, 0 and 1000 GeV), with $\tan \beta = 10$ and $\text{sign} \mu > 0$ for a specific region. No dependence on $A_0$ is shown.

small value for the anapole moment of the lightest neutralino $(10^{-6}-10^{-7}$ GeV$^{-2}$), which is consistent with the results of Ho and Scherrer [31].
Figure 6: Anapole moment for sign $\mu > 0$ and sign $\mu < 0$, with $A_0 = 0$ GeV and $\tan \beta = 10$ for a specific region. No dependence on sign $\mu$ is shown.

Figure 7: Anapole moment for two different values of $\tan \beta$ (10 and 50), with $A_0 = 0$ GeV and sign $\mu > 0$ for a specific region. No cosmological or experimental constraint is considered.

In the upper plot of figure 8 we show the anapole moment values for different regions of parameter space in $(m_0, m_{1/2})$ planes in the cMSSM for
Figure 8: Value of the anapole moment in \((m_0, m_{1/2})\) planes in the cMSSM for \(\tan \beta = 10\) (upper plot) and \(\tan \beta = 50\) (lower plot), assuming \(A_0 = 0\) and \(\mu > 0\), see text. The region that complies with all the phenomenological constraints would be to the extreme right of the plots, between the white lines.
\( \tan \beta = 10 \). We have marked in this plot the different phenomenological constraints as follows: the region above the red dashed line is where the Higgs mass is \( m_h = 126 \pm 3 \) GeV, to the left of the pink dot-dashed line the LSP is charged, under the dotted blue line is the region excluded by the value of \( b \rightarrow s\gamma \), whereas the region under the dotted green line is excluded because it does not comply with the requirement of radiative electroweak symmetry breaking. The region where the relic LSP density falls within the range allowed by WMAP is marked with a white line, while a more loose constraint, \( \Omega_\chi h^2 < 0.12 \), assuming the LSP is not the only component of CDM, is delimited by a white line. The lower plot shows the same regions, but with \( \tan \beta = 50 \).

The cMSSM may be too constrained to be realistic, however, using it as a test model, we can see that the anapole moment is indeed different for different regions of the parameter space. Thus, anapole analysis can be used as another criteria to constrain the parameter space of a given model. According to our results, if a non-zero (around \( 10^{-4}-10^{-3} \) GeV\(^{-2} \)) anapole moment could be measured for the neutralino, that would indicate that the favored region of the parameter space of the cMSSM would be high \( m_0 (\geq 800 \text{ GeV}) \) and low \( m_{1/2} (\leq 400 \text{ GeV}) \). Otherwise, other regions are compatible with an anapole moment lower than \( 10^{-5} \) GeV.

5 Conclusions

We calculated the only electromagnetic property of the lightest neutralino: its toroidal dipole moment. Its characterization is extremely valuable for discriminating different models which have the neutralino as dark matter candidate. We performed the calculation in the framework of the cMSSM, however a similar analysis can be performed for other models. We found that the anapole moment of the neutralino is sensitive to \( m_0, m_{1/2} \) and \( \tan \beta \), but non-dependent on \( A_0 \) and sign\( \mu \).

All points in the parameter space we scanned give an anapole moment consistent with the upper limit (\( \sim 10^{-2} \) GeV\(^{-2} \)) obtained by Pospelov and ter Veldhuis \[23\] for WIMPs interacting with heavy nuclei using data from the CDMS and DAMA experiments. However, this data can and will improve in the next few years helping to refine the upper limit, likely ruling out some regions of the parameter space.

The anapole analysis can be used as another criteria to constrain the
parameter space of a given model which has a neutralino as candidate for dark matter. Thus, according to our results, if a non-zero (around $10^{-4}$-$10^{-3}$ GeV$^{-2}$) anapole moment could be measured for the neutralino, that would indicate that the favored region of the parameter space of the cMSSM would be high $m_0$ ($\geq 800$ GeV) and low $m_{1/2}$ ($\leq 400$ GeV). Otherwise, other regions are compatible with an anapole moment lower than $10^{-5}$ GeV.

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Appendix: Scalar Three-point Function

In this appendix, we analyse the Passarino-Veltman scalar three-point function $C_0(q^2, x^2, x^2, z^2, z^2, y^2)$ [53, 54] which appears in the TDM calculation. Here $q^2$ denotes the photon transferred 4-momentum, $x$ is the neutralino mass, and $y$ and $z$ are the masses of the particles running in the loop.

The corresponding plot for this $C_0$ function can be seen in figure 9. The red line shows the numerical solution, the blue line represents the approximate solution, i.e., the Taylor expansion around $q^2 = 0$, which can be written as follows:

$$C_0(q^2, x^2, x^2, z^2, z^2, y^2) = \alpha_0 + \alpha_1 q^2 + O(q^4).$$

The coefficients $\alpha_i$ are functions of the masses:

$$\alpha_0 = \frac{\log \left( \frac{y^2}{z^2} \right)}{2x^2} + a \log \omega,$$

$$\alpha_1 = \frac{x^4 - y^2 x^2 - 2z^2 x^2 + z^4 - y^2 z^2}{6x^2 z^2 (-x + y - z)(x + y - z)(-x + y + z)(x + y + z)} \frac{\log \left( \frac{y^2}{z^2} \right)}{12x^2} + b \log \omega,$$

where
Figure 9: Comparison between numerical (red line) and approximate (blue line) scalar three-point function $C_0(q^2, x^2, x^2, z^2, y^2)$, with $x = 97.7$ GeV, $y = 415.4$ GeV and $z = 80.43$ GeV. The analytical approximation (blue line) is only valid for $q^2 \to 0$.

\[
\omega = \frac{(ix^2 + iy^2 - iz^2 + \sqrt{-y^4 + 2(x^2 + z^2)y^2 - (z^2 - x^2)}) (ix^2 + iy^2 + iz^2 + \sqrt{-y^4 + 2(x^2 + z^2)y^2 - (z^2 - x^2)})}{(-ix^2 + iy^2 - iz^2 + \sqrt{-y^4 + 2(x^2 + z^2)y^2 - (z^2 - x^2)}) (-ix^2 + iy^2 + iz^2 + \sqrt{-y^4 + 2(x^2 + z^2)y^2 - (z^2 - x^2)})},
\]

\[
a = \frac{i(x^2 + y^2 - z^2)}{2x^2 \sqrt{-x^4 + 2y^2x^2 + 2z^2x^2 - y^4 - z^4 + 2y^2z^2}}
\]

and

\[
b = \frac{i(x^2 + y^2 - z^2)(x^4 - 4y^2x^2 - 2z^2x^2 + y^4 + 4 - 2y^2z^2)}{12x^4(-x + y - z)(x + y - z)(-x + y + z)(x + y + z)\sqrt{-x^4 + 2y^2x^2 + 2z^2x^2 - y^4 - z^4 + 2y^2z^2}}
\]

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