Restrictions on dilatonic brane-world models

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Abstract

We consider dilatonic brane world models with a non-minimal coupling between a dilaton and usual matter on a brane. We demonstrate that variation of the fundamental constants on the brane due to such interaction leads to strong restrictions on parameters of models. In particular, the experimental bounds on variation of the fine structure constant rule out non-minimal dilatonic models with a Liouville-type coupling potential $f(\varphi) = \exp(b\varphi)$ where $b \sim O(1)$.

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Brane-world models have been the subject of intensive investigation for the last few years. They offer an interesting alternative (with respect to the Kaluza-Klein model) to the standard multidimensional gravity and cosmology. The main feature of this approach consists in a proposal where the standard matter (SM) fields are localized on the brane (4-dimensional hypersurface which corresponds to our Universe) whereas the gravitational field can propagate in the full multidimensional space-time. It sheds a new light on the problem of the large hierarchy and leads to new designing properties and phenomena for multidimensional models. Thus, it is important to predict observable effects which can confirm such brane-world approach.

Obviously, SM particles may escape from the brane into a bulk resulting in the violation of the energy-momentum and charge conservation laws in the brane \[\Box\]. Such effect can take place, for example, if SM particles interact with bulk fields. A lot of papers were

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devoted to the problem of the interaction between radions and SM fields (see [2, 3] and references therein). Radions usually describe relative motion of the branes. For realistic models, it is usually supposed that there is a mechanism for the brane stabilization with respect to each other. Let $b_0$ be the scale of stabilization of the inter-brane distance and $\psi(x)$ the small fluctuations (radions) around it. Then, an induced 4-D metric on the brane located in an additional dimension at $y = y_0$ reads: $h_{\mu\nu}(x, y_0) = A_0 \exp(c_0 \psi(x)) \tilde{h}_{\mu\nu}(x)$, where $A_0$ is a dimensionless warp factor corresponding to the scale of stabilization $b_0$ and $c_0 \sim 1/M_{EW}$ (in the ADD (Arkani-Hamed, Dimopoulos, Dvali) brane approach $c_0 \sim 1/M_{Pl}$ [3, 4]). Let $\Phi(x)$ represents a matter field (SM) on the $D_0$-dimensional brane with a Lagrangian $L = L(\Phi(x), h(x, y_0))$ and following action

$$S = \int d^{D_0}x \sqrt{|h|} L(\Phi(x), h(x, y_0)).$$  

(1)

The corresponding Lagrangian density of the interaction between radions and field $\Phi$ is

$$\mathcal{L}_{\text{int}} = \psi \frac{\delta \mathcal{L}}{\delta \psi} \bigg|_{\psi=0} = \psi \left( \frac{\delta \mathcal{L}}{\delta h_{\mu\nu}} \frac{\delta h_{\mu\nu}}{\delta \psi} \right) \bigg|_{\psi=0} = -(c_0/2) \psi \left( \sqrt{|h|} T_{\mu\nu} \right)_{\psi=0},$$

(2)

where $T_{\mu\nu}$ is a trace of the energy-momentum tensor for the Lagrangian $L$ with respect to the metric $h_{\mu\nu}$:

$$T_{\mu\nu} = -2 \frac{\delta L}{\delta h_{\mu\nu}} + h_{\mu\nu} L \Leftrightarrow \sqrt{|h|} T_{\mu\nu} = 2 \frac{\delta L}{\delta h_{\mu\nu}}, \quad \mathcal{L} = \sqrt{|h|} L.$$

(3)

Thus, the interaction between radions and SM field is absent for fields with vanishing $T_{\mu\nu}$, e.g. for massless fermions and massless gauge bosons which are the quanta present in high-energy experiments. By this reason graviscalars were neglected in colliding experiments for studying the brane-world physics.

Nevertheless, massless SM particles on the brane can interact at tree level with other bulk fields, e.g. with a non-minimal dilaton field. Moreover, as this scalar field lives in full 5-D space-time (in the bulk), the coupling constant is $c_0 \sim 1/M$ (if 5-D gravitational constant $\kappa_5^2 = M^{-3}$). Such interaction may play an important role in the brane-world physics. Thus, it is of interest to predict observable effects following from this type of interaction and to obtain experimental restrictions on parameters of the models. There is an extensive list of papers devoted to the investigation of the dilatonic brane-world models with a slightly different form of the action (e.g. [4, 5, 8, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21]). They naturally follows from a low-energy limit of string theories and have a dilatonic bulk potential and a dilatonic coupling potential of the form of

\footnote{We should emphasize that in the standard Kaluza-Klein model the interaction (with $c_0 \sim 1/M_{Pl}$) between graviscalars (gravexcitons [3]) and massless particles is possible at tree level [3].}
the Liouville potential \[^{[3, 8, 14, 19]}\]. For example, the action describing non-minimal coupling between dilaton and a brane matter (in the model with one brane) can be written in the form

\[
S = \frac{1}{2\kappa_5^2} \int_{M_5} d^5x\sqrt{|g|}e^{-2\varphi}\left\{ R[g] + 4\bar{g}^{ab}\partial_a\varphi\partial_b\varphi - 2\kappa_5^2 V \right\} + \\
+ \int_{M_4} d^4x\sqrt{|\bar{h}|}e^{-2\varphi}\{-T + L_m[\bar{h}]\},
\]

where \(M_5\) is the 5-D manifold with metric \(\bar{g}_{ab}\) \((a, b = 0, 1, 2, 3, 5)\) and the 4-D hypersurface \(M_4\) is the brane with induced metric \(\bar{h}_{\mu\nu}\) \((\mu, \nu = 0, 1, 2, 3)\). \(T\) is a tension of the brane. The Lagrangian \(L_m[\bar{h}]\), constructed with the help of the metric \(\bar{h}\), corresponds to SM fields on the brane. 5-D gravitational constant \(\kappa_5^2\) is connected with 5-D fundamental mass as follows: \(\kappa_5^2 = M^{-3}\) and we usually suppose \[^{[4]}\] \(M = M_{EW} \sim 1\text{TeV}\). The dilatonic field \(\varphi\) is dimensionless. Its dimensions are restored with the help of the 5-D fundamental mass \(M\): \(\varphi = M^{-3/2}\phi = M^{-1}\phi\) where \(\phi\) and \(\phi\) have dimensions of \(\mathcal{O}(m^{3/2})\) and \(\mathcal{O}(m)\) \((m\) is a unit of mass\), respectively. A scalar field \(\phi\) has usual dimensions for scalar fields in 4-D space-time \((\text{cf. \cite{24}})\). The bulk potential \(V\) \((\text{which we consider as the bulk cosmological constant})\) has dimensions \(\mathcal{O}(m^5)\). The tension \(T\) and the matter Lagrangian \(L_m\) have dimensions \(\mathcal{O}(m^4)\).

Action \(^{(4)}\) is written in the string frame, where we suppose that 5–D original metric \(\bar{g}_{ab}\) and its induced metric \(\bar{h}_{\mu\nu}\) do not depend on the dilaton field \(\varphi\). Conformal transformation to the Einstein frame

\[
g_{ab} = \Omega_1^2(\varphi)\bar{g}_{ab} \equiv e^{-(4/3)\varphi}\bar{g}_{ab}
\]

yields

\[
S = \frac{1}{2\kappa_5^2} \int_{M_5} d^5x\sqrt{|g|}\left\{ R[g] - \frac{4}{3}\bar{g}^{ab}\partial_a\varphi\partial_b\varphi - 2\kappa_5^2 e^{(4/3)\varphi} V \right\} + \\
+ \int_{M_4} d^4x\sqrt{|\bar{h}|}\left\{-e^{(2/3)\varphi}T + e^{(2/3)\varphi}L_m[\varphi, h]\right\},
\]

where \(L_m[\bar{h}] = L_m[\Omega^{-2}\bar{h}] \equiv L_m[\varphi, h]\).

\(^2\)For simplicity, we consider the case of one brane located at the additional coordinate \(y = y_0\). Let \(n_a\) be a unite space-like vector normal to the brane. Then, the induced metric on the brane is \(\bar{h}_{ab} = \bar{g}_{ab} - n_a n_b\). We also suppose that all space-time can be covered by the normal Gauss coordinates where \(n_a = n^a = (0, 0, 0, 0, 1)\).

In this case \(\bar{h}_{a5} = \bar{h}_a^5 = \bar{h}_5^a = 0\) and \(\bar{h}_a^a = \delta_a^a\). These simplifications do not affect the results of our paper.

\(^3\)This assumption is valid for 5–D brane-world models of the Randall–Sundrum–type \cite{22}, where gravity is effectively 4–dimensional below a length scale of the order \(M_{Pl}^2/M_{EW}^3\). In the case of 5–D ADD approach, gravity becomes 5–dimensional below \(M_{Pl}^2/M_5^3\). For such ADD models, to be in accordance with gravitational experiments \((\text{see e.g. \cite{23}})\), mass scale \(M\) should satisfy the following restriction: \(M \gtrsim 10^8\text{GeV}\).
It is clear that a non-minimal interaction of the dilatonic field with SM fields can result in violation of the matter conservation on the brane (see footnote 5 below). To be in accordance with observations, $\varphi$ should be stabilized on the brane near some value $\varphi_0$ or slightly vary during the Universe evolution (at least from the time of nucleosynthesis). To estimate a possible restrictions on the rate of such variations, it is necessary to investigate a Lagrangian of interaction between dilaton and SM fields. Let $\varphi_0$ is the present value of $\varphi$ and $\eta = M^{-1}\psi$ are small fluctuations around it. Additionally, we slightly generalize our model to an arbitrary dilaton coupling potential: $e^{-2\varphi} \rightarrow f(\varphi)$ supposing only that function $f(\varphi)$ (as well as $\Omega_1(\varphi)$) is the Liouville-type potential. It is of interest to compare the Lagrangians of interaction in different frames.

1. String frame.

Here, the action of the SM fields on the brane and the Lagrangian density of interaction read, correspondingly:

$$S_m = \int_{M_4} d^4x \sqrt{|h|} f(\varphi) L_m[h],$$

$$L_{int} = \eta \left. \frac{\delta \left( \sqrt{|h|} f(\varphi) L_m[h] \right)}{\delta \varphi} \right|_{\varphi=\varphi_0} = \eta \left. \frac{df}{d\varphi} \sqrt{|h|} L_m[h] \right|_{\varphi=\varphi_0}. \quad (7)$$

2. Einstein frame.

In this frame, the SM action can be written as follows:

$$S_m = \int_{M_4} d^4x \sqrt{|h|} \Omega_1^{-4}(\varphi) f(\varphi) L_m[\varphi, h] \equiv \int_{M_4} d^4x \sqrt{|h|} F(\varphi) L_m[h],$$

where $F(\varphi) \equiv \Omega_1^{-4}(\varphi) f(\varphi) f_1(\varphi)$ and we imply $L_m[\varphi, h] \equiv L_m[\Omega_1^{-2} h] = f_1(\varphi) L_m[h]$. The latter equality (resulting in factorization) usually takes place for massless particles. Precisely this kind of SM fields we shall consider below. The exact expression for $f_1(\varphi)$ is defined by the form of the Lagrangian $L_m$. Obviously, if $\Omega_1$ is a Liouville-type function, $f_1$ also belongs to this class of functions. Corresponding Lagrangian density of interaction reads

$$L_{int} = \eta \left. \frac{\delta \left( \sqrt{|h|} F(\varphi) L_m[h] \right)}{\delta \varphi} \right|_{\varphi=\varphi_0} = \eta \left\{ \frac{df}{d\varphi} \sqrt{|h|} L_m[h] + F \frac{\partial h^\mu}{\partial \varphi} \frac{\delta \left( \sqrt{|h|} L_m[h] \right)}{\delta h^\mu} \right\}_{\varphi=\varphi_0} \left( \varphi_0 \right), \quad (10)$$
where $T^\mu_\mu[h]$ is a trace of the energy-momentum tensor for the matter Lagrangian $L_m[h]$ and $\Omega_1(\varphi)$ is a conformal factor connecting metrics in the string and Einstein frames (see Eq. (5)). It is worthy of note that Lagrangian density $\sqrt{h}L_{em}[h]$ is invariant under conformal transformation of metric for 4-D electromagnetic fields: $\sqrt{h}L_{em}[\bar{h}] = \sqrt{h}L_{em}[h]$ and $F(\varphi) = f(\varphi)$. Thus, because a trace $T_{em} = 0$, the Lagrangians of interactions (8) and (10) formally coincide with each other.

3. Minimal frame.

It can be easily seen, that there is additionally a specific frame with a minimal coupling between dilaton and SM fields on the brane. It corresponds to the conformal transformation

$$g_{ab} = \Omega_2^4(\varphi)\bar{g}_{ab},$$

which yields

$$S_m = \int_{M_4} d^4x \sqrt{|\bar{h}|L_m[\bar{h}]}.$$

To achieve it, the conformal factor $\Omega_2^4(\varphi)$ should satisfy the following condition

$$\Omega_2^{-4}(\varphi)f(\varphi)f_2(\varphi) = 1,$$

where, by the full analogy with function $f_1(\varphi)$, the exact expression for $f_2(\varphi)$ is defined by the form of the Lagrangian $L_m$. Obviously, for 4-D electromagnetic field $f_2 = \Omega_2^4$ and equation (13) leads to $f(\varphi) \equiv 1$, i.e. dilaton should be minimally coupled with the brane electromagnetic field from the very beginning. Thus, in the case of non-minimal coupling, the transition to the minimal frame is impossible for electromagnetic field.

In the minimal frame, the Lagrangian density of the interaction is

$$L_{int} = \eta \frac{\delta \left( \sqrt{h}L_m[h] \right)}{\delta \varphi} \bigg|_{\varphi = \varphi_0} = \eta \frac{\partial \bar{h}^{\mu \nu}}{\partial \varphi} \left( \sqrt{h}L_m[h] \right) \bigg|_{\varphi = \varphi_0} = -\eta \frac{d\Omega_2}{\Omega_2} \frac{d\varphi}{d\varphi} \sqrt{h}T^\mu_\mu[\bar{h}] \bigg|_{\varphi = \varphi_0},$$

and has the form (2) of interaction between radion and SM fields. Therefore, in the minimal frame traceless fields do not interact with dilaton at tree level. However, as we stressed above, the minimal frame is absent for some of SM fields (if original theory is non-minimal), e.g. for 4-D electromagnetic field.

Now, we shall concentrate on the experimental consequences of the non-minimal coupling between dilaton and 4-D electromagnetic field. In this case, transition to the minimal frame is absent and the Lagrangian density of interaction has the same form in the
string as well as Einstein frames. In the following, to be more concrete, we shall use the Einstein frame. In spite of the traceless character of the electromagnetic field energy-momentum tensor, Eqs. (8) and (10) show that dilatonic fields can interact on the brane with electromagnetic fields at the tree level. It is the main difference with graviscalars considered in Eq. (2). Eq. (10) can be rewritten as follows:

$$L_{int} = \frac{\beta}{M} L_m [h] = \beta \left. \frac{\psi}{M} F_{\mu \nu} F^{\mu \nu} \right|_{h}$$

(15)

with the coefficient $\beta := \frac{df}{d\varphi}|_{\varphi_0}$. As we wrote above, $\varphi_0$ is the present value of $\varphi$ and $\eta = M^{-1}\psi$ are small fluctuations around it. Interaction (15) is suppressed by the electroweak mass $M = M_{\text{EW}} \sim 1\text{TeV}$ in contrast to the interaction with WIMP’s (Weakly-Interacting Massive particles) which are suppressed by 4-D Planck mass $M_{\text{Pl}} \sim 10^{16}\text{TeV}$. Thus, the interaction SM fields with dilatons in brane worlds can be much more effective than with WIMP’s in the standard Kaluza-Klein approach.

Obviously, interactions between dilatons and massless SM particles, e.g. photons, are of great interest in high-energy colliding experiments. If the dilaton field $\varphi$ is stabilized on the brane at $\varphi_0$ corresponding to a minimum of an effective potential and small fluctuations near this position constitute quanta $\psi$ with a mass $m$, then a decay rate (due to the interaction (15)) of these quanta into 2 photons are

$$\Gamma \sim \beta^2 \frac{m^3}{M^2}$$

(16)

with a life-time $t \sim 1/\Gamma \sim \beta^{-2}(M^2/m^3)(\hbar/c^2)$. Thus, the dilatons with masses

$$m \lesssim \beta^{-2/3} \left[ M_{\text{Pl}}^4 \frac{T_{\text{Pl}}}{t_{\text{univ}}} M^2 M_{\text{Pl}} \right]^{1/3} \sim \beta^{-2/3} 10^{-4}\text{eV}$$

(17)

have life-time $t \geq 10^{10}\text{sec} > t_{\text{univ}} \sim 10^{18}\text{sec}$ greater than the age of the Universe. They are rather light particles. For heavier dilatons the decay plays important role during the Universe evolution.

It is well known (see e.g. [24]) that interaction of the form $f(\varphi)F^2$ results in variation of the fine structure constant $\alpha$:

$$\frac{\dot{\alpha}}{\alpha} = \frac{\dot{f}}{\bar{f}},$$

(18)

where the dot denotes differentiation with respect to time. There is an extensive list of papers devoted to the experimental bounds for such variations (e.g. [25, 26, 27] and references therein). Different experiments give different bounds on $|\dot{\alpha}/\alpha|$, from $\lesssim 10^{-12}\text{yr}^{-1}$

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4See also footnote 3 concerning 5-D ADD models. For this approach $M \gtrsim 10^8\text{GeV}$, which is much bigger than 1TeV but is still much less than $M_{\text{Pl}}$. 
for cosmic microwave background \[25\] to \(\lesssim 10^{-17}\) yr\(^{-1}\) for the Oklo experiment \[28\]. Primordial nucleosynthesis gives \(|\Delta \alpha/\alpha| \lesssim 10^{-4}\) at a redshift on the orders \(z = 10^9 - 10^{10}\) \[29\], i.e. \(|\dot{\alpha}/\alpha| \lesssim 10^{-14}\) yr\(^{-1}\). In all these estimates \(\dot{\alpha} = \Delta \alpha/\Delta t\) is the average rate of change of \(\alpha\) for the period \(\Delta t\) (corresponding to a redshift \(z\)). For our calculations we take some averaged estimate \(|\dot{\alpha}/\alpha| \lesssim 10^{-13}\) yr\(^{-1}\). For this bound, from Eq. (18) we obtain:

\[
\left| \frac{\dot{f}}{f} \right| = \left| \frac{\dot{\phi}}{M f} \right| \lesssim 10^{-13}\text{yr}^{-1},
\]

This estimate leads to the following restriction on the parameter \(\beta\) (cf. \[24\]):

\[
|\beta| \approx \Delta t \frac{\dot{\alpha}}{\alpha} \frac{M}{\Delta \phi} \Rightarrow |\beta| \lesssim 10^{-3},
\]

(20)

where we suppose \(\Delta \phi \sim M\) and that the present value of \(f \approx 1\) (that usually is equivalent to the assumption for the dilaton field at the present time: \(\phi_0 \ll M \Rightarrow \varphi_0 \ll 1\)).

As we wrote above, most of the dilatonic models are motivated by string theories which, at a low-energy limit, usually have the Liouville-type potentials (see e.g. Eq. (6)):

\[V(\varphi) = V \exp(a\varphi)\] and \(f(\varphi) = \exp(b\varphi)\) with \(a \sim b \sim \mathcal{O}(1)\) \[8, 9, 14, 19\]. Substitution of the Liouville coupling potential \(f(\varphi) = \exp(b\varphi)\) into estimate (19) leads to the limits on the parameter \(b\):

\[
\left| b \frac{1}{M} \frac{\Delta \phi}{\Delta t} \right| \lesssim 10^{-13}\text{yr}^{-1} \Rightarrow |b| \lesssim 10^{-3}
\]

(21)

for \(\Delta \phi \sim M\) and \(\Delta t \sim 10^{10}\) years. Estimates (20) and (21) coincide with each other because for \(\varphi_0 \ll 1\) : \(\beta = df/d\varphi|_{\varphi_0} = b \exp(b\varphi_0) \sim b\). It is worth pointing out that natural assumption \(\Delta \phi \sim M\) results in independence of estimates (20) and (21) upon the concrete value of \(M\).

It is hardly possible that the Liouville-type potentials for such considered model provide the stabilization of \(\varphi\) on the brane. Thus, the dilatonic models with non-minimal coupling to the SM fields on the brane are ruled out by estimate (21) for theories with \(a \sim b \sim \mathcal{O}(1)\) (e.g. model (4), (6)).

Another restrictions on parameter of models can be obtained from experiments on variation of the 4-D gravitational constant. To show it, we represent the brane part of an action (in the Einstein frame) as follows:

\[
S_b = \int_{M_4} d^4x \sqrt{|h|} \left\{ -T(\varphi) + F(\varphi) L_m[h] \right\},
\]

(22)

where \(T(\varphi) \equiv \Omega^{-4}_4(\varphi) f(\varphi) T\). For 4-D electromagnetic field \(F(\varphi) = f(\varphi)\) and we shall use this equality below.
In 4–D projected Einstein equations (see e.g. [16, 17]), linear terms (with respect to the brane matter energy-momentum tensor\(^5\)
\[ T_{\mu\nu}[h] = -2\delta L_m[h]/\delta h^{\mu\nu} + h_{\mu\nu}L_m[h] \]
are responsible for the conventional cosmology on the brane. For the brane-world models with \(S_b\) of the form (22), these linear terms have the form \((\kappa_5^4/6)T(\varphi)f(\varphi)T_{\mu\nu}[h]\). Thus, the quantity
\[ 8\pi G_N \equiv \frac{\kappa_5^4}{6}T(\varphi)f(\varphi) \equiv \frac{\kappa_5^4}{6}\tilde{f}(\varphi) \]
plays the role of a 4-D gravitational constant on the brane. This equation shows that effective 4–D gravitational constant \(G_N\) for models (22) defines by the function \(\tilde{f}(\varphi)\). Thus, variation of \(\tilde{f}(\varphi)\) leads to a variation of \(G_N\). If functions \(\Omega_1(\varphi)\) and \(f(\varphi)\) are of the Liouville-type then \(\tilde{f}(\varphi)\) also belongs to this class of functions and can be written in the form \(\tilde{f}(\varphi) = T\exp(c\varphi)\). Obviously, for models with \(a \sim b \sim \mathcal{O}(1)\) parameter \(c \sim \mathcal{O}(1)\). For example, for model \([4], [3]\) we have: \(T(\varphi) = T\exp((2/3)\varphi)\), \(f(\varphi) = \exp(-2\varphi)\) and \(\tilde{f}(\varphi) = T\exp((-4/3)\varphi)\).

There are a number of observable data for an estimate of a possible time variation of the gravitational constant [27, 30, 31, 32]. They imply \(|\dot{G}_N/G_N| \lesssim 10^{-11}\) yr\(^{-1}\). Thus, we can obtain a limitation of the variation of \(\dot{f}(\varphi)\):
\[ \left| \frac{\dot{f}}{f} \right| \lesssim 10^{-11}\) yr\(^{-1}\),

which for the Liouville potential \(\tilde{f} = T\exp(c\varphi)\) puts on the parameter \(c\) the following restrictions:
\[ |c| \lesssim 10^{-1}, \]

where we suppose \(\Delta \phi/\Delta t \sim M/(10^{10}\) yr\). This estimate is much less severe than (21) and, strictly speaking, not rules out theories with \(c \sim \mathcal{O}(1)\). For example, it is expected [31, 32], that on the Hubble time scale \(|\dot{G}_N/G_N| \lesssim H_0 \sim 10^{-10}\) yr\(^{-1}\). Then, inequality (25) is reduced to the following estimate: \(|c| \lesssim 1\).

In order to avoid the problem of the fundamental constant variation in the non-minimal dilatonic brane-world models, it is natural to suppose that the dilaton is stabilized on the brane (before primordial nucleosynthesis), i.e. \(\varphi \rightarrow \varphi_0 \equiv \text{const}\) where \(\varphi_0\) corresponds to a stable solution of the equation of motion on the brane. It would be very interesting to find explicit models leading to such stabilization. If, in general, such stabilization is

\(^5\)An effective energy-momentum conservation equation for the matter on the brane has the form [17]
\[ (f(\varphi)T^\mu_{\nu}[h])_{,\nu} = \varphi_{,\mu}(df/d\varphi)L_m[h], \]
which shows that the matter is conserved on the brane if the dilaton field is either minimally coupled to the SM \(f \equiv \text{const}\) or stabilized on the brane \((\varphi|_{\text{brane}} \rightarrow \text{const})\).
impossible, then, variations of $\varphi$ with time should be in accordance with experimental bounds on variations of the fundamental constants (see e.g. Eqs. (19) and (24)).

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