Moduli fields and brane tensions: generalizing the junction conditions

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2 August 2000; LATEX-ed October 22, 2018

Abstract

Taking the Randall–Sundrum models as background scenario, we derive generalized Israel–Lanczos–Sen thin-shell junction conditions for systems in which several bulk scalar fields are non-minimally coupled to gravity. We demonstrate that the form of the junction conditions (though arguably not the physics) depends on the choice of frame. We show that generally (in any frame except the Einstein frame) the presence of a thin shell induces discontinuities in the normal derivative of the scalar field, even in the absence of any direct interaction between the thin shell and the scalar field. For some exceptional scalar field configurations the discontinuities in the derivatives of the metric and the scalar fields can feed back into each other and so persist even in the absence of any thin shell of stress-energy.

PACS: 04.60.Ds, 04.62.+v, 98.80 Hw

Keywords:
Junction conditions, thin shells, moduli, brane world, dilaton, Randall–Sundrum.

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1 Introduction

The gravitational interaction is by many orders of magnitude the weakest of all known interactions. The Planck length $l_P$, related to Newton’s constant through $l_P = \sqrt{\frac{G\hbar}{c^3}}$, is $10^{12}$ smaller than the smallest length scale explored with standard-model physics, and is more than $10^{32}$ smaller than the length scales probed by direct gravitational experiments. It is commonly believed that the classical geometrical picture with which we interpret large-scale physics must at some stage be abandoned as we approach smaller and smaller scales, but when and how this exactly happens is a matter of much debate. In the usual scenario we might think of the Planck length as a fundamental cutoff scale and carry the 4-dimensional geometric picture down to (say) some hundreds of Planck lengths, at which stage we would presumably enter the domain of pure quantum gravity.

A radically different possibility which has recently attracted much attention is that there might already be observable deviations from this usual scenario at sub-millimetre distances. In the “large extra dimensions” scenario the geometry of spacetime is the product of our usual 4-dimensional spacetime times some relatively large but mathematically compact extra dimensions [1], giving rise to a rich phenomenology at low energies. (At even smaller scales this geometrical multi-dimensional continuum would melt into a more fundamental quantum gravity theory, such as for instance string theory.) In this scenario the Planck length would be an effective scale derived from the more fundamental electroweak scale, and the hierarchy problem (between these energy scales) would be translated into a hierarchy between the fundamental and the compactification scales. In this approach, the standard-model fields in our 4-dimensional hypersurface would not be part of a standard Kaluza–Klein reduction [1]. (See also [2] for a survey of other older non-standard Kaluza–Klein proposals.)

Another possibility with similar phenomenological richness which, moreover, avoids the new hierarchy problem left over by the previous approach, is the “brane world” scenario. Randall and Sundrum recently proposed that the extra dimensions could be non-compact provided that they were sufficiently “warped” [3]. (For similar suggestions, see also [4] and [5]. Considerably older precursors to the idea of non-compact extra dimensions can be found in [6] and [7].) Inspired by string theory, Randall and Sundrum proposed than we live on a 3-brane [(3+1)-dimensional hypersurface] evolving in a (4+1)-dimensional anti-de Sitter bulk. Everything is confined to live on the brane, except for gravity itself. To “weakly trap” gravity the specific form of the spacetime geometry provides a normalizable Kaluza–Klein zero mode, corresponding to a 4-dimensional graviton (trapped near but not on the brane), plus a continuum of massive Kaluza–Klein graviton modes modifying the gravitational interaction at small distance scales (scales comparable with the warp length).

Within this brane world scenario much work has been done analyzing the effective gravity induced on the brane [3, 4, 10]. From a cosmological perspective, it has been demonstrated that we can recover a Friedmann-like evolution for the late brane world if there is a large negative cosmological constant in the bulk and a large positive intrinsic
tension on the brane. These are carefully arranged to counteract each other, and so leave a small (possibly zero) effective cosmological constant \[8, 9\]. In the early brane world, when the energy density of matter fields reaches values of the order of the brane tension, the cosmological evolution will be modified giving place to a polynomial inflation \[8\]. From a more general point of view, the circumstances under which it is possible to recover standard general relativity on the brane have been studied, and possible modifications to the standard picture have been analyzed \[10\].

It is important to realize that apart from gravity itself there could be other fields not constrained to live on the brane. By considering the low-energy field theory limit of string theory one can deduce the existence of a plethora of bulk fields: primarily the dilaton field and the torsion field, but generically there will also be many moduli fields (corresponding to “soggy” deformations of the six-dimensional Calabi–Yau manifold that is also part of these string-induced scenarios). Also, adopting a modification of more conventional Kaluza–Klein ideas, it is possible that our 4-dimensional (light) standard-model fields might be the reduction to 4 dimensions of some 5-dimensional fields with Planck scale masses, with the effective 4-dimensional mass being greatly suppressed compared to the more fundamental 5-dimensional mass parameter \[11\]. Thus it is important to understand the role that additional bulk fields might play in the brane world scenario.

For instance, the existence of scalar fields in the bulk could provide an answer to the cosmological constant problem \[12\] (for early ideas on this subject, see \[13\]). For arbitrary values of the brane tension (an intrinsic vacuum energy density) there exist solutions of gravity plus a dilaton scalar field which lead to zero bulk cosmological constant, and are compatible with a 4-dimensional Poincare invariant brane world \[12\]. (This implies a vanishing effective cosmological constant for the brane evolution.) In these solutions the warp factor becomes zero at a finite proper distance from the brane in the extra dimension and so it is “effectively compactified” leaving standard 4-dimensional gravity for large scales.

In the models considered to date the bulk scalar fields have no direct coupling to the bulk scalar curvature. In the case of the dilaton field this is accomplished by working in the Einstein frame and not in the (perhaps more natural) string frame. There are many interesting scalar fields theories that cannot easily be dealt with in the current approach, such as the Brans–Dicke theory (and more generically, scalar-tensor theories), non-minimally coupled scalars, or the dilaton field in the string frame. In this paper we will work out a formalism to deal with a large class of scalar field theories in arbitrary frames, by allowing arbitrary couplings to the bulk curvature scalar.

In section 2, we will review the Israel–Lanczos–Sen thin-shell formalism \[14\], obtaining the junction conditions in an easy to generalize manner by beginning directly from the gravitational action. This method will be then be applied the case in which several very general scalar fields interact with gravity. We will obtain generalized junction conditions for these systems. In sections 3 and 4 we will discuss these new junction conditions. We will see that these junction conditions are now much more complicated than the usual ones, and that many intuitive arguments involving brane tensions no longer work. The
point is that the insertion of a “kink” in the geometry, achieved by inserting a thin shell of stress-energy, now gives a contribution to the formation of a “kink” in the scalar field configuration. *Vice-versa*, the presence of a “kink” in the scalar field now makes a contribution to the formation of a “kink” in the geometry. Finally, in section 5 we summarize our results and try to put things in perspective.

2 Action principle and junction conditions

We are interested in geometries in $n$ spacetime dimensions that have an internal hypersurface of co-dimension one on which there is no well defined notion of tangent plane. The derivatives of the metric, with respect to a proper normal coordinate across the hypersurface, undergo a jump on passing through it. We can interpret the presence of this jump in the normal derivative of the metric as produced by an infinitely thin shell of stress-energy located on the hypersurface (one can speak equivalently of a thin shell of stress-energy, an idealized domain wall, or an evolving $n-2$ brane). The junction conditions give a relation between the intrinsic energy-momentum tensor of the brane and the form and strength of the jump in derivatives of the metric (encoded in the jump in the extrinsic curvature of the hypersurface as seen from its two faces).

2.1 Israel–Lanczos–Sen junction conditions: Einstein Frame

Let us first review (and extend) the derivation of the standard Israel–Lanczos–Sen junction conditions. We will use a method easily generalizable to more complicated systems involving non-minimal couplings between gravity and additional fields. (The paper by Chamblin and Reall \[15\] contains a similar derivation of the Einstein frame junction conditions.) We start with the Einstein–Hilbert action, supplemented by the Gibbons–Hawking boundary term \[16\], and add both bulk fields and an intrinsic action for the brane:

$$S = \frac{1}{2} \int_{\text{int}(\mathcal{M})} \sqrt{-g} \, d^n x \, (R - 2\Lambda) - \int_{\partial \mathcal{M}} \sqrt{-\tilde{q}} \, d^{n-1} x \, K + \int_{\text{int}(\mathcal{M})} \sqrt{-g} \, d^n x \, \mathcal{L}_{\text{bulk}} + \int_{\text{brane}} \sqrt{-\tilde{q}} \, d^{n-1} x \, \mathcal{L}_{\text{brane}}. \tag{2.1}$$

Here $q_{AB}$ represents the induced metric, either on the $(n-1)$-dimensional hypersurface forming the boundary, or on that defined by the brane. (We adopt the conventions of Misner, Thorne, and Wheeler \[17\].) This expression makes perfectly good sense because the scalar of curvature is well defined (in the distributional sense) even at points on the brane itself.

For definiteness we shall split the bulk matter Lagrangian $\mathcal{L}_{\text{bulk}}$ into contributions from the scalar moduli fields $\phi^i$, which for generality will be described by a non-linear
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\[ \mathcal{L}_{\text{bulk}}(g_{AB}, \phi, \psi) = -\frac{1}{2} H_{ij}(\phi) \left[ g^{AB} \partial_A \phi^i \partial_B \phi^j \right] - V(\phi, \psi) + \mathcal{L}_{\text{bulk}}(g_{AB}, \psi). \]  

(2.2)

As usual, we assume that \( \mathcal{L}_{\text{bulk}}(g_{AB}, \psi) \) contains no second derivatives of \( \psi \), and for convenience assume it contains not even first derivatives of the metric. (This last condition is not essential, but relaxing it would involve additional unnecessary algebraic complexity.)

In the spirit of the brane world models, let us now deal with spacetimes that are decomposable as \( \Sigma \times \mathbb{R} \), where \( \mathbb{R} \) represents a spatial dimension parametrized by \( \eta \) and \( \Sigma \) is a \( n-1 \) dimensional spacetime (locally, this is always possible). We consider a formal evolution in the \( \eta \) parameter between two fixed hypersurfaces located at \( \eta_i \) and \( \eta_f \) (they are assumed to comprise the boundary \( \partial M \)), leaving the brane located at \( \eta = 0 \). (In most of the literature, the fifth dimension \( \eta \) is taken to be spacelike, and we also take this option; it is for this reason that we speak of \( \eta \) as “formal” evolution parameter, for timelike fifth dimension a few minus signs will change.) The total action (2.1) can be re-written as

\[ S = \lim_{0^- \eta \to 0^+} \left\{ \frac{1}{2} \int_{\eta_i}^{\eta_f} \sqrt{-g} \, d\eta \, d^{n-1}x \left[ (n-1)R - K^{AB}K_{AB} + K^2 - 2\Lambda \right] 
+ \frac{1}{2} \int_{0^+}^{\eta_f} \sqrt{-g} \, d\eta \, d^{n-1}x \left[ (n-1)R - K^{AB}K_{AB} + K^2 - 2\Lambda \right] 
+ \frac{1}{2} \int_{0^-}^{\eta_i} \sqrt{-g} \, d\eta \, d^{n-1}x \left[ (n-1)R - K^{AB}K_{AB} + K^2 - 2\Lambda \right] 
- \int_{\partial M} \sqrt{-q} \, d^{n-1}x \, K \left|_{0^+}^{0^-} \right. \right\} 
+ \int_{\text{int}(M)} \sqrt{-g} \, d^{n}x \, \mathcal{L}_{\text{bulk}} 
+ \int_{\text{brane}} \sqrt{-q} \, d^{n-1}x \, \mathcal{L}_{\text{brane}}. \]  

(2.3)

In obtaining this expression we have used the fact that one can unambiguously separate \( R \) into a term that contains no second derivatives of the metric, and a second term (a total divergence) isolating all the second derivatives of the metric:

\[ R = (n-1)R - K^{AB}K_{AB} + K^2 + 2\nabla_A(n^A K), \]  

(2.4)

with \( n_A \) denoting the normal to the foliation. To verify this, see for example, MTW (21.84), (21.86), and (21.88), supplemented with exercise (21.10), and equation (21.82), being careful to keep to a timelike hypersurface (spacelike normal) throughout \[17\]. Also note that in MTW conventions

\[ K_{AB} = -\frac{1}{2} \frac{\partial q_{AB}}{\partial \eta} \]  

(2.5)

for both spacelike and timelike hypersurfaces.

For the class of geometries we are considering the scalar of curvature is everywhere finite except for a possible delta-function contribution at \( \eta = 0 \). If \( K \) undergoes a finite
jump at $\eta = 0$, then the term with second derivatives of the metric (first derivatives of $K$) in equation (2.4) will give rise to this delta-function contribution. Then, it is easy to see that
\[
\lim_{0^+ - 0^- \rightarrow 0^+} \left[ \frac{1}{2} \int_{0^-}^{0^+} \sqrt{-g} \, d\eta \, d^{n-1}x \right] R - \int_{0^-}^{0^+} \sqrt{-q} \, d^{n-1}x \, K \bigg|_{0^-}^{0^+} = 0,
\]
(2.6)

independently of the existence or not of any delta-function contribution.

As a consequence, the total action (though not the bulk Einstein–Hilbert Lagrangian) is finally formed by two standard bulk pieces [the first two terms in (2.3)] plus the contribution of the brane [last term in (2.3)]. We explicitly see that the action (2.1) contains no second derivatives of the metric:
\[
S = \frac{1}{2} \int_{\eta_i}^{\eta_f} \sqrt{-g} \, d\eta \, d^{n-1}x \left[ (n-1) R - K^{AB} K_{AB} + K^2 \right]
+ \frac{1}{2} \int_{\eta_i}^{\eta_f} \sqrt{-g} \, d\eta \, d^{n-1}x \left[ (n-1) R - \kappa^{AB} K_{AB} + K^2 \right]
+ \int_{\text{int}(\mathcal{M})} \sqrt{-g} \, d^n x \, \mathcal{L}_{\text{bulk}}
+ \int_{\text{brane}} \sqrt{-q} \, d^{n-1}x \, \mathcal{L}_{\text{brane}},
\]
(2.7)

This action can now be re-expressed in a Hamiltonian form
\[
S = \int_{\eta_i}^{\eta_f} \sqrt{-g} \, d\eta \, d^{n-1}x \left[ \pi^{AB} \frac{dq_{AB}}{d\eta} - \mathcal{H}(q_{AB}, \pi^{AB}) \right]
+ \int_{\eta_i}^{\eta_f} \sqrt{-g} \, d\eta \, d^{n-1}x \left[ \pi^{AB} \frac{dq_{AB}}{d\eta} - \mathcal{H}(q_{AB}, \pi^{AB}) \right]
+ \int_{\text{brane}} \sqrt{-q} \, d^{n-1}x \, \mathcal{L}_{\text{brane}},
\]
(2.8)

where we have temporarily suppressed the bulk fields $\phi^j$ and $\psi$. Here $\mathcal{H}$ is the appropriate Hamiltonian density (its exact form is not needed for the following argument), and we use the following definition for the canonical momentum
\[
\pi^{AB} = \frac{1}{2} \sqrt{-g} \left( K^{AB} - q^{AB} K \right).
\]
(2.9)

The canonical Hamiltonian will also depend on the lapse and shift function of the standard [(n − 1) + 1] ADM decomposition, but we have omitted it for simplicity (as it does not contribute in the following argument). We have partially fixed the gauge by placing the brane at $\eta = 0$. Therefore, we are going to consider only diffeomorphisms that do not affect the boundaries or the location of the brane.

By varying this action with respect $q_{AB}$ and $\pi^{AB}$ we obtain
\[
\delta S = \int_{\eta_i}^{\eta_f} \sqrt{-g} \, d\eta \, d^{n-1}x \left[ \frac{dq_{AB}}{d\eta} - \frac{\partial \mathcal{H}}{\partial \pi^{AB}} \right] \delta \pi^{AB}
\]
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\[- \int_0^{\eta_f} \sqrt{-q} \, d\eta \, d^{n-1}x \left[ \frac{d\pi^{AB}}{d\eta} + \frac{\partial H}{\partial q_{AB}} \right] \delta q_{AB} + \int \sqrt{-q} \, d^{n-1}x \, \pi^{AB} \delta q_{AB}\bigg|_{\eta_f}^{\eta_i} + \int \sqrt{-q} \, d^{n-1}x \left( \pi^{AB} - \pi^{+AB} \right) \delta q_{AB}\bigg|_{\eta=0} + \frac{1}{2} \int \sqrt{-q} \, d^{n-1}x \, S^{AB} \delta q_{AB}, \tag{2.10}\]

where $S^{AB}$ is the energy-momentum tensor of the brane.

This action has an extremum among all the geometries that connect the two fixed hypersurfaces at $\eta_i$ and $\eta_f$ when: (i) it satisfies the classical equation of motion in the two bulk regions (the full diffeomorphism invariance in each bulk region will result in the additional classical constraint equations not of direct interest), and, (ii) when the junction condition

\[\pi^{+}_{AB} - \pi^{-}_{AB} = \frac{1}{2} \sqrt{-q} \, S_{AB} \tag{2.11}\]

holds.

Now define

\[\mathcal{K}_{AB} \equiv K^+_{AB} - K^-_{AB}, \tag{2.12}\]

and substitute the explicit form of the momentum (2.9) into (2.11). We obtain the usual Israel–Lanczos–Sen junction conditions

\[\mathcal{K}_{AB} - \mathcal{K} \, q_{AB} = S_{AB}. \tag{2.13}\]

For future generalization, we find it convenient to split this result into trace-free part and a trace, so that

\[\mathcal{K}_{AB} = \frac{1}{n-1} \mathcal{K} \, q_{AB} = S_{AB} - \frac{1}{n-1} \, S \, q_{AB}, \tag{2.14}\]

and

\[\mathcal{K} = - \frac{S}{n-2}. \tag{2.15}\]

This can be reassembled to yield the equivalent form

\[\mathcal{K}_{AB} = S_{AB} - \frac{1}{n-2} \, S \, q_{AB}. \tag{2.16}\]

These junction conditions have a tremendous number of applications, well beyond the confines of the brane world scenario, and are a basic tool of general applicability \[\text{[18]}\].

Now reinstate the non-gravitational bulk fields $\phi^i$ and $\psi$, and repeat the argument: there will be several additional conjugate momenta

\[\pi^{i}_{\phi^i} = -\sqrt{-q} \, H_{ij} (\phi) \left( \frac{\partial \phi^j}{\partial \eta} \right), \tag{2.17}\]
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and

\[ \pi_\psi = \sqrt{-g} \left( \frac{\partial \mathcal{L}_{\text{bulk}}(g_{AB}, \psi)}{\partial (\partial \psi / \partial \eta)} \right). \]  

(2.18)

There will also be additional junction conditions on the bulk fields \( \phi^i \) and \( \psi \):

\[ (\pi_{\phi^i})^+ - (\pi_{\phi^i})^- = \sqrt{-q} \frac{\partial \mathcal{L}_{\text{brane}}}{\partial \phi^i}, \]  

(2.19)

and

\[ (\pi_\psi)^+ - (\pi_\psi)^- = \sqrt{-q} \frac{\partial \mathcal{L}_{\text{brane}}}{\partial \psi}. \]  

(2.20)

Because we have decided on an explicit form for the \( \phi^i \) Lagrangian, we can simplify that junction condition considerably. First introduce the notation

\[ J^i \equiv n^A (\partial_A \phi^i)^+ - n^A (\partial_A \phi^i)^- \equiv \left( \frac{\partial \phi^i}{\partial \eta} \right)^+ - \left( \frac{\partial \phi^i}{\partial \eta} \right)^-, \]  

(2.21)

Furthermore, let \( H^{ij}(\phi) \) denote the inverse matrix to \( H_{ij}(\phi) \). Then

\[ J^i = -H^{ij}(\phi) \frac{\partial \mathcal{L}_{\text{brane}}}{\partial \phi^j}. \]  

(2.22)

On the other hand, because we have made no specific commitment as to the form of \( \mathcal{L}_{\text{bulk}}(g_{AB}, \psi) \) the best we can say for those fields is that

\[ \left[ \frac{\partial \mathcal{L}_{\text{bulk}}(g_{AB}, \psi)}{\partial (\partial \psi / \partial \eta)} \right]^+ - \left[ \frac{\partial \mathcal{L}_{\text{bulk}}(g_{AB}, \psi)}{\partial (\partial \psi / \partial \eta)} \right]^-= \frac{\partial \mathcal{L}_{\text{brane}}(g_{AB}, \psi)}{\partial \psi}. \]  

(2.23)

Though abstract, this bulk-field junction condition is perhaps the clearest general statement one can make regarding the coupling of non-gravitational bulk fields to the brane. In the next subsection we will see how this method of obtaining the junction conditions is easily generalizable to even more complicated systems, and in particular how to deal with generic frame dependence and non-minimal curvature coupling.

2.2 Moduli fields — Generic frames

We are now going to analyze a system in which there is not only gravity in the bulk, but also additional bulk scalar fields with a non-minimal coupling to the curvature scalar. Explicitly, we take an action of the form

\[ S = \frac{1}{2} \int_{\text{int}(M)} \sqrt{-g} \, d^n x \, F(\phi) \left[ R - 2\Lambda \right] - \int_{\partial M} \sqrt{-q} \, d^{n-1} x \, F(\phi) \, K \]

\[ + \int_{\text{int}(M)} \sqrt{-g} \, d^n x \left\{ -\frac{1}{2} H_{ij}(\phi) \left[ g^{AB} \partial_A \phi^i \partial_B \phi^j \right] - V(\phi, \psi) + \mathcal{L}_{\text{bulk}}(g_{AB}, \psi) \right\} \]

\[ + \int_{\text{brane}} \sqrt{-q} \, d^{n-1} x \, \mathcal{L}_{\text{brane}}(g_{AB}, \phi, \psi). \]  

(2.24)
The Gibbons–Hawking boundary term now takes the form

$$-\int_{\partial M} \sqrt{-q} \, d^{n-1} x \, F(\phi) \, K. \quad (2.25)$$

This is specifically chosen to cancel the second derivatives of the metric in the Einstein–Hilbert action. The two (in principle) arbitrary functions, $F(\phi)$ and $H_{ij}(\phi)$, allow us to take into account a large class of possible moduli fields.

In this notation the dilaton field in the string frame corresponds to

$$F(\phi) = e^{-2\phi}; \quad H(\phi) = -4 \, e^{-2\phi}. \quad (2.26)$$

In the Einstein frame the dilaton corresponds to

$$F(\phi) = 1; \quad H(\phi) = \frac{4}{n-2}. \quad (2.27)$$

Standard non-minimally coupled scalars correspond to

$$F(\phi) = 1 - \xi_{ij} \, \phi^i \, \phi^j; \quad H_{ij}(\phi) = \delta_{ij}. \quad (2.28)$$

This is typically (though perhaps misleadingly) referred to as the Jordan frame, in this context meaning the frame in which these particular scalars have canonical kinetic energy. Note that non-minimally coupled scalars (e.g., conformally coupled) are from a quantum field theory point of view generic \cite{19}; however from a gravitational point of view they are potentially problematic, leading to classical violations of the energy conditions and worse \cite{19, 20}. Note that you can always make non-minimal coupling go away by performing a conformal transformation and shifting frame to the Einstein frame — there is however a conservation of difficulty and the price paid is to modify the scalar kinetic energies. It is often convenient to keep the scalar kinetic energies simple and instead confront non-minimal coupling head-on. Finally, we can describe a Brans–Dicke theory by means of

$$F(\phi) = \phi; \quad H(\phi) = \frac{\omega(\phi)}{\phi}. \quad (2.29)$$

Confusingly enough, this is also typically referred to as the Jordan frame, in this context meaning the frame in which the kinetic energies of all other non-Brans–Dicke bulk fields are canonical.

We will shortly see that the presence of a function $F(\phi) \neq 1$ can yield to very interesting effects. To also take into account possible interactions between the brane and the bulk fields, in our analysis we will allow the brane Lagrangian to depend arbitrarily on the metric and on the bulk scalar fields, (this is in addition to its dependence on the fields trapped on the brane). We will allow also a bulk potential term for the scalar fields, and a cosmological constant in the bulk.
Now, let us follow the same steps as in the previous subsection. The whole action (2.24) can be separated into two bulk contributions without any second derivative of the metric plus the brane action. From the term
\[ \frac{1}{2} \int \sqrt{-g} \, d^n x \, F(\phi) \, R \] (2.30)
we can see that the bulk actions include a term of the form
\[ \frac{1}{2} \int \sqrt{-g} \, d\eta \, d^{n-1} x \, F(\phi) \left[ (n-1)R - K^{AB} K_{AB} + K^2 - 2 \frac{\partial \ln F(\phi)}{\partial \phi^i} \left( \frac{\partial \phi^i}{\partial \eta} \right) K \right], \] (2.31)
with the prime denoting ordinary derivative with respect to \( \phi \). Indeed the total action is
\[ S = \frac{1}{2} \int_0^\eta \sqrt{-g} \, d\eta \, d^{n-1} x \]
\[ F(\phi) \left[ (n-1)R - K^{AB} K_{AB} + K^2 - 2 \frac{\partial \ln F(\phi)}{\partial \phi^i} \left( \frac{\partial \phi^i}{\partial \eta} \right) K - 2\Lambda \right] \]
\[ + \frac{1}{2} \int_0^{\eta'} \sqrt{-g} \, d\eta \, d^{n-1} x \]
\[ F(\phi) \left[ (n-1)R - K^{AB} K_{AB} + K^2 - 2 \frac{\partial \ln F(\phi)}{\partial \phi^i} \left( \frac{\partial \phi^i}{\partial \eta} \right) K - 2\Lambda \right] \]
\[ + \int_\eta^0 \sqrt{-g} \, d\eta \, d^{n-1} x \left\{ -\frac{1}{2} H_{ij}(\phi) \left[ g^{AB} \partial_A \phi^i \partial_B \phi^j \right] - V(\phi, \psi) + \mathcal{L}_{\text{bulk}}(g_{AB}, \psi) \right\} \]
\[ + \int_0^{\eta'} \sqrt{-g} \, d\eta \, d^{n-1} x \left\{ -\frac{1}{2} H_{ij}(\phi) \left[ g^{AB} \partial_A \phi^i \partial_B \phi^j \right] - V(\phi, \psi) + \mathcal{L}_{\text{bulk}}(g_{AB}, \psi) \right\} \]
\[ + \int_{\text{brane}} \sqrt{-q} \, d^{n-1} x \, \mathcal{L}_{\text{brane}}. \] (2.32)

When re-expressed in a Hamiltonian form we get
\[ S = \int_\eta^0 \sqrt{-g} \, d\eta \, d^{n-1} x \left[ \pi^{AB} \frac{dq_{AB}}{d\eta} + \pi_{\phi^i} \frac{d\phi^i}{d\eta} + \pi_\psi \frac{d\psi}{d\eta} - \mathcal{H}(q_{AB}, \phi, \psi, \pi^{AB}, \pi_{\phi^i}, \pi_\psi) \right] \]
\[ + \int_0^{\eta'} \sqrt{-g} \, d\eta \, d^{n-1} x \left[ \pi^{AB} \frac{dq_{AB}}{d\eta} + \pi_{\phi^i} \frac{d\phi^i}{d\eta} + \pi_\psi \frac{d\psi}{d\eta} - \mathcal{H}(q_{AB}, \phi, \psi, \pi^{AB}, \pi_{\phi^i}, \pi_\psi) \right] \]
\[ + \int_{\text{brane}} \sqrt{-q} \, d^{n-1} x \, \mathcal{L}_{\text{brane}}, \] (2.33)
where again we do not need to know the detailed form of the Hamiltonian. Following the same reasoning as before, we conclude that the modified junction conditions read
\[ \pi^{+}_{AB} - \pi^{-}_{AB} = \frac{1}{2} \sqrt{-q} \, S_{AB}, \] (2.34)
\[ \pi^{\phi^i}_+ - \pi^{\phi^i}_- = \sqrt{-q} \, \frac{\partial \mathcal{L}_{\text{brane}}}{\partial \phi^i}, \] (2.35)
\[ \pi_\psi^+ - \pi_\psi^- = \sqrt{-q} \, \frac{\partial \mathcal{L}_{\text{brane}}}{\partial \psi}, \] (2.36)
but now with a different definition for the gravitational momentum
\[ \pi_{AB} = \frac{1}{2} \sqrt{-q} F(\phi) (K_{AB} - q_{AB} K) + \frac{1}{2} \sqrt{-q} F'_i(\phi) \left( \frac{\partial \phi^i}{\partial \eta} \right) q_{AB}. \] (2.37)

Here for convenience we have defined
\[ F'_i(\phi) \equiv \partial_i F(\phi) = \frac{\partial F(\phi)}{\partial \phi^i}. \] (2.38)

The momentum canonically conjugate to \( \phi \) is also altered and now reads
\[ \pi_{\phi^i} = -\sqrt{-q} H_{ij}(\phi) \left( \frac{\partial \phi^j}{\partial \eta} \right) - \sqrt{-q} F'_i(\phi) K. \] (2.39)

Obviously, the generic expression (2.18) for the momentum canonically conjugated to \( \psi \) remains unchanged and so its associated junction condition (2.23). For simplicity, from here on we will not consider explicitly these additional fields. Rearranging the previous expressions [(2.34) to (2.39)] and using the previous definition of \( J^i \) we arrive at
\[ F(\phi) (K_{AB} - q_{AB} K) + F'_i(\phi) J^i q_{AB} = S_{AB}. \] (2.40)

and
\[ -H_{ij}(\phi) J^j - F'_i(\phi) K = \frac{\partial \mathcal{L}_\text{brane}}{\partial \phi^i}. \] (2.41)

This shows how discontinuities in the extrinsic curvature mix with discontinuities in the normal derivatives of the scalar field. (The junction condition for \( \psi \) is unaltered because of our technical assumption that \( \mathcal{L}_\text{bulk}(g_{AB}, \phi) \) contains no metric derivatives.) In the Einstein frame \( [F(\phi) = 1] \) this mixing switches off and we recover the ordinary Israel–Lanczos–Sen junction condition, supplemented by the bulk-field junction conditions of the preceding subsection. If we move away from the Einstein frame (and it is often convenient to do so) the price paid is that the junction conditions become inextricably intertwined.

For these generalized junction conditions it is extremely useful to separate the equation (2.40) into a trace-free portion and a trace. That is
\[ K_{AB} - \frac{1}{n-1} \mathcal{K} q_{AB} = \frac{1}{F(\phi)} \left( S_{AB} - \frac{1}{n-1} S q_{AB} \right), \] (2.42)

and
\[ (n-2) F(\phi) \mathcal{K} - (n-1) F'_i(\phi) J^i = -S. \] (2.43)

Inverting the general equations (2.41) and (2.43) yields our final form of the generalized junction conditions. For the trace of extrinsic curvature
\[ \mathcal{K} = -\frac{S + (n-1) H^{ij}(\phi) F'_i(\phi) (\mathcal{L}_\text{brane})'_j}{(n-2) F(\phi) + (n-1) H^{ij}(\phi) F'_k(\phi) F'_l(\phi)}. \] (2.44)
For the normal discontinuity in the scalar derivative

\[ J^i = H^{ij}(\phi) \left[ F'_j \frac{S + (n-1)H^{pq}(\phi) F'_p(\phi)}{(n-2)F(\phi) + (n-1)H^{kl}(\phi) F'_k(\phi) F'_l(\phi)} - (\mathcal{L}_{\text{brane}})'_j \right]. \]  

(2.45)

By looking at the expressions (2.42), (2.44), and (2.45) the first thing that we can notice is that for \( F(\phi) = 1 \) neither does \( K_{AB} \) depend on \( L'_b \) nor does \( J \) depend on \( S_{AB} \). As we have already seen in the previous subsection, in the Einstein frame the coupling of the brane degrees of freedom to the scalar field and to the metric separately induce uncorrelated kinks in the geometry and in the scalar field. When \( F(\phi) \neq 1 \), however, both effects become interconnected: a kink in the scalar field configuration can be produced even though there is no direct coupling between the bulk scalar field and the fields on the brane, and vice-versa.

These complete sets of generalized junction conditions [either (2.40) and (2.41), or (2.42), (2.44), and (2.45)] are very powerful. They cover most of the scalar fields one can encounter in the literature, and have wide applicability beyond the confines of the brane world scenario. The most important differences between these junction conditions and the standard ones show up when \( F(\phi) \neq 1 \). In the next section we will discuss the peculiarities of these conditions when \( F(\phi) \neq 1 \), that is, when there is a scalar field that is not minimally coupled to the geometry.

Finally for completeness we mention that it is still possible (but perhaps algebraically not so useful) to reassemble the trace and trace-free parts of the extrinsic curvature to yield

\[ K_{AB} = \frac{S_{AB}}{F(\phi)} - \left\{ \frac{[F(\phi) + H^{ij}(\phi) F'_i(\phi) F'_j(\phi)] S + F(\phi) H^{ij}(\phi) F'_i(\phi) (\mathcal{L}_{\text{brane}})'_j}{(n-2)F(\phi) + (n-1)H^{ij}(\phi) F'_i(\phi) F'_j(\phi)} \right\} q_{AB} \frac{F'(\phi)}{F(\phi)}. \]  

(2.46)

### 3 The exceptional case

An exceptional case occurs when the denominator in the generalized junction conditions vanishes, that is, when

\[ F(\phi) = -\frac{(n-1)}{(n-2)} H^{ij}(\phi) F'_i(\phi) F'_j(\phi). \]  

(3.47)

This can only occur for \( F(\phi) H^{ij}(\phi) \) not positive definite as we are assuming \( n \geq 4 \) to avoid too simple a gravity law. Also note that a dilaton field never satisfies this exceptionality condition, either in the Einstein or the string frames.

In this case the expressions (2.44), (2.45), and (2.46) are not well defined. However, what this condition really means is that the matrix inversion used to go from (2.41) and (2.43) to (2.44) and (2.45) is singular, though the trace-free relationship (2.42) remains
perfectly valid. The attempt to invert the singular matrix now instead results in a compatibility condition between $S$ and $\mathcal{L}_{\text{brane}}'$

$$(\mathcal{L}_{\text{bulk}})'_i = \frac{1}{n-2} \frac{F'_i(\phi)}{F(\phi)} S,$$  \hfill (3.48)

and a reduced set of linear constraints between $K$ and $J^i$ [say equations (2.41)], so that we could view $K$ (for instance) as freely specifiable.

As a particular example, for these exceptional systems one can have “phantom branes” or “Cheshire cat” configurations with both a kink in the geometry and in the scalar field without the need to add any external thin shell of energy. From (2.40) and (2.41), taking $L_{\text{brane}}(g_{AB}, \phi, \psi) \equiv 0$ so that both $S_{AB} = 0$ and $(\mathcal{L}_{\text{brane}})'_i = 0$, and imposing the exceptionality condition (3.47) one arrives at

$$K_{AB} = \frac{1}{n-2} \frac{F'_i(\phi)}{F(\phi)} J^i q_{AB}, \quad \text{and} \quad J^i = -H^{ij}(\phi) F'_j(\phi) K,$$  \hfill (3.49)

that is, a simple relation between the jumps in the extrinsic curvature and in the normal derivative of the scalar field. (And note in particular that there is a unique direction in field space for which there is a discontinuity; effectively, in these “Cheshire cat” configurations only one of the scalar fields is allowed to have a discontinuity.)

To better understand what this system means we can perform a conformal transformation to the Einstein frame. The above condition between $F(\phi)$ and $H(\phi)$, equation (3.47), is nothing more than the condition to (at the point in question) eliminate the kinetic term for one of the scalar fields in the Einstein frame. (And so locally turn that scalar field equation, in the Einstein frame, into an algebraic constraint rather than a differential relation.) To see this, note that transforming from the generic $F(\phi) \neq 1$ frame to the Einstein frame implies the conformal redefinition

$$g_{AB} = \Omega^2(\phi) [g_E]_{AB},$$  \hfill (3.50)

with

$$\Omega(\phi)^{n-2} = F(\phi)^{-1},$$  \hfill (3.51)

and the concomitant redefinition

$$H_{ij}(\phi) \rightarrow [H_E]_{ij}(\phi) = H_{ij}(\phi) + \frac{(n-1)}{(n-2)} \frac{F'_i(\phi)}{F(\phi)} F'_j(\phi).$$  \hfill (3.52)

Then

$$\det[H_E] = \det(H) \left\{ 1 + \frac{(n-1)}{(n-2)} \frac{H^{ij}(\phi) F'_i(\phi) F'_j(\phi)}{F(\phi)} \right\}. \hfill (3.53)$$

So $\det[H_E] = 0$ is equivalent to the exceptionality condition (3.47).

If we insist that the exceptionality condition hold as an identity throughout field space rather than “accidentally” at some specific value of the field, then we can think of this
kind of scalar field as nothing more than the conformal part of the metric in a pure-gravity theory (with its corresponding “negative” kinetic energy). That is, if we start with the ordinary Einstein–Hilbert action and split the metric into a conformal factor times some fiducial metric, then the conformal factor (when viewed as a scalar field propagating in the geometry of the fiducial metric), identically satisfies the exceptionality condition above.

Indeed if $g_{AB}$ and $\phi$ are the only fields present in the bulk then these “Cheshire cat” branes can always be smoothed away in this manner, leaving absolutely no trace of their existence, and so they should then be viewed as completely unphysical mathematical artifacts. If there are several scalar fields, then this argument still goes through (modulo potential problems from globally diagonalizing the Einstein frame sigma model metric $[H_{E}]_{ij}$).

On the other hand, consider the very intriguing possibility that the exceptionality condition holds not as an identity in field space, but rather as an “accident” at a point in field space corresponding to the value of the scalar fields on the brane. Then “Cheshire cat” branes of this type cannot be globally transformed away in the above manner. You can eliminate the kink in the geometry, and the kink in the scalar fields, but there will now be at least one scalar field with a very peculiar kinetic energy term: a kinetic energy that vanishes at some (but not all) points in field space. There is a conservation of difficulty and one has a tradeoff between kinks in the geometry and scalar fields versus exceedingly peculiar kinetic energies.

4 Moduli fields and brane tensions

Let us now restrict the discussion to “bare” brane Lagrangians of the form

$$\mathcal{L}_{\text{brane}} = -T f(\phi),$$

that is, the brane only provides a vacuum energy modulated by the value of the scalar field:

$$S_{AB} = -T f(\phi) q_{AB}.$$  \hfill (4.55)

An immediate consequence is that the trace-free part of $\mathcal{K}_{AB}$ is zero,

$$\mathcal{K}_{AB} = \frac{1}{n-1} K q_{AB},$$ \hfill (4.56)

and that all of the interesting physics is hiding in the trace $\mathcal{K}$ and in $J$.

4.1 Dilaton field in the Einstein frame

Consider $F(\phi) = 1$ and $H(\phi) = 4/(n - 2)$. This is the common case analyzed in the literature — a dilaton field in the Einstein frame \cite{12}. Then a positive brane tension, $T f(\phi) > 0$ yields a warp factor that decreases when going away from the brane. (Strictly speaking this is true only when one imposes a $Z_2$ symmetry on the solutions, or adopts
the “one-sided” view discussed in [9, 21]. More generally we have to say that is the sum
of the variation of the warp factor when departing from the brane in the two possible
directions that decreases away from the brane.)

The behaviour can be easily read from (2.15) and (2.22) [and also from the more
general (2.44) and (2.45) when particularized to this case]:

\[
\mathcal{K} = -\frac{1}{n-2} S = \frac{n-1}{n-2} T f(\phi). \quad (4.57)
\]

The behaviour of the scalar field away from the brane depends upon the derivative of
\(f(\phi)\),

\[
J = -\frac{n-2}{4} T f'(\phi). \quad (4.58)
\]

For positive tension branes \((T f(\phi) > 0)\), the condition \([\ln f(\phi)]' > 0\) yields a scalar
field that decreases away from the brane; with \([\ln f(\phi)]' < 0\) the scalar field increases.
Generically it is found that positive tension branes give rise to bulk configurations in
which at some finite distance from the brane the spacetime “terminates” in the form of a
naked singularity [12].

When \(F(\phi) \neq 1\) this is no longer true. The existence of a coupling between the scalar
field and the curvature makes the junction conditions more involved. The thin shell of
energy contributes not only to the kink in the geometry but also to that in the scalar
field. To see this with more detail, let us consider the case of a dilaton field in the string
frame in 5 dimensions.

### 4.2 Dilaton field in the string frame

Consider \(F(\phi) = \exp(-2\phi)\) and \(H(\phi) = -4 \exp(-2\phi)\). Write the junction conditions
(2.44) and (2.45) as

\[
\mathcal{K} = -(n-1) T e^{2\phi} \left[ f(\phi) + \frac{1}{2} f'(\phi) \right], \quad (4.59)
\]

\[
J = T e^{2\phi} \left[ \frac{n-1}{2} f(\phi) + \frac{n-2}{4} f'(\phi) \right]. \quad (4.60)
\]

Imagine that we take an ansatz for the scalar field-brane coupling function \(f(\phi) = e^{-\alpha \phi}\).
Then, for \(2(n-1)/(n-2) < \alpha < +\infty\) the warp factor and the dilaton field decrease away
from the brane. For \(\alpha = 2(n-1)/(n-2)\) the warp factor still decreases away from the
brane but the dilaton configuration has no kink. For \(2 < \alpha < 2(n-1)/(n-2)\), the warp
factor decreases away from the brane but the dilaton field increases. For \(\alpha = 2\), which
is the case usually chosen as the appropriate coupling, there is no kink in the geometry
and the dilaton increases away from the brane. For \(\alpha < 2\) (the case \(f(\phi) = 1\) would
coincide to \(\alpha = 0\) the warp factor and the dilaton increase away from the brane.

In view of the previous discussion some observations are in order. Let us take a
coupling function \(f(\phi) = e^{-2\phi}\), and restrict to the case \(n = 5\). This is used commonly as
representing the coupling of the dilaton field to the brane [12]; (in the Einstein frame used
in those works the corresponding coupling function is $f_E(\phi) = e^{2\phi/3}$. For this case we see that we can have 3-branes with arbitrarily large tensions but, nonetheless, with a smooth string-frame geometry (that is, without any hypersurface on which there is a jump in the extrinsic curvature). We would notice the existence of such a 3-brane only through its effect on the scalar field configuration. However, in transforming this metric-scalar configuration to the Einstein frame we will recover a kink in the Einstein frame geometry at the location of the 3-brane. This is due to the fact that the Einstein frame metric is a function of both the string frame metric and the scalar field. From the point of view of the Einstein frame is the existence of a kink in the geometry that is trapping 4-dimensional gravitons near the brane world. From the string frame point of view, this trapping will be due to a non-differentiable variation of the effective Newton constant when moving across the brane. From the Einstein frame point of view a finite 4-dimensional Newton constant would show up in the brane if the extra dimension has a finite volume per unit area. (By this we mean that $\int a(\eta) \, d\eta = < \infty$ where $a(\eta)$ is the scale factor of the geometry.) Taking the string frame point of view, the relevant quantity is the volume of the extra dimension weighted by the scale factor derived from the effective Newton constant.

5 Summary and discussion

We have generalized the Israel–Lanczos–Sen thin-shell junction conditions to the case in which the bulk spacetime is filled, apart from gravity itself, by a set of general scalar fields with non-trivial curvature coupling. First, we have derived the standard Israel–Lanczos–Sen junction conditions beginning directly from the gravitational action. The method we have followed turns out to be straightforwardly generalizable in the presence of other fields in the bulk. We have found these new junction conditions for gravity plus a very general set of moduli fields with arbitrary couplings to the scalar curvature. Our formalism is capable of dealing with the dilaton field, in the string and/or Einstein frames, metric-scalar theories à la Brans–Dicke, and non-minimally coupled scalars.

The existence of couplings between the moduli fields and the curvature scalar makes the generalized junction conditions interconnected. One can have a “kink” in the scalar field configuration even though there is no direct coupling between the scalar fields and the thin shell of stress-energy. Conversely, these couplings between the thin-shell and the scalar fields make contributions to the form and strength of the “kink” in the geometry.

An exceptional case appears for some moduli field configurations. These exceptional field configurations are characterized by the vanishing of the kinetic term of some of the scalar fields when transforming to the Einstein frame. In this exceptional case we showed that one can find “kink” metric-scalar configurations in the original frame even in the absence of any thin-shell of stress-energy. The “kink” in the geometry and that in the scalar fields feed back into each other. The possibility that the exceptional condition could be satisfied for some particular values of the moduli fields could give place to very interesting “Cheshire cat” configurations.
Partializing to the dilaton field, we showed that the geometric picture that emerges when working in the string or Einstein frames are very different. It is even possible that a geometry with a “kink” in the Einstein frame has a counterpart which in the string frame is perfectly smooth. The Einstein frame geometric “kink” is absorbed into the conformal factor in such a way that in the string frame only the dilaton field undergoes a discontinuity in its normal derivative as one crosses the brane. These apparently different configurations are however equivalent. Up to this point in the discussion the choice of frame is a matter of convenience.

However this might not always be the case. First, the complete equivalence between frames is only guaranteed if the conformal transformation relating them is not singular. When this is not true one can find configurations in one frame that have no complete analogue in the other frame. As an specific example, for a non-minimally coupled scalar field the present authors have found some traversable wormhole solutions [19] for which the term \( \kappa - \xi \phi^2 \) becomes zero in some specific points of the geometry (on the other hand, the “Jordan” frame geometry is perfectly regular). The relation between de Einstein and “Jordan” frames can be written as \([g_E]_{AB} = (\kappa - \xi \phi^2)[g_J]_{AB}\). Therefore, the whole wormhole configuration in the “Jordan” frame has no non-singular counterpart in the Einstein frame.

Second, complete equivalence between choice of conformal frames is guaranteed only if you simultaneously change both frame and the equation of motion for test particles. If physical test particles are observed to follow geodesics in one conformal frame then they will not follow geodesics in any other conformal frame. If free fall is truly universal (as the Eötvös experiments indicate), then one conformal frame is more equal than the others. (Brans–Dicke type theories are set up in such a manner that this special frame is the Jordan frame, and for this reason the Jordan frame is often called the “physical” frame.)

Thus, by performing experiments on bulk matter one should be able to distinguish between different frames [22], and the discussion about which frame is the “most physical” will come into play [23, 24]. We emphasise the well-known, albeit commonly overlooked fact that string theory does not satisfy the weak equivalence principle, (except approximately), let alone the Einstein equivalence principle [22, 23, 24]. If one uses experiment to decide which is the “most physical” frame, then to work in another frame would be nothing more than a mathematical trick (although sometimes potentially useful).

However, taking the Randall-Sundrum models as background scenario it is far from clear if the differences in bulk physics will affect experiments performed in the brane itself. Since bulk matter is now trapped on the brane, direct free-fall experiments could at best suggest the choice of a particular frame on the brane, but would leave the off-brane conformal frame a free variable whose choice would be driven by esthetics and mathematical convenience rather than by physics.

At a more basic level, when proposing the fundamental Lagrangian, for each new field that one adds it is necessary to decide in which of the previous possible frames it will couple minimally. For example, if we want to begin with a non-zero cosmological constant for the bulk, defined as a proper tendency of the bulk spacetime to be curved, in
which frame should the expression $G_{AB} = -\Lambda g_{AB}$ hold? Side-stepping the answer to this question for now, it is imperative to analyze (as in this paper) systems with very general scalar potentials, to obtain results that can be applied in any frame.

Acknowledgments

The research of CB was supported by the Spanish Ministry of Education and Culture (MEC). MV was supported by the US Department of Energy.
References

[1] I. Antoniadis, “A Possible New Dimension At A Few TeV,” *Phys. Lett.* B246 (1990) 377.
N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “The Hierarchy Problem and New Dimensions at a Millimeter”, *Phys. Lett.* B429 (1998) 263. [hep-ph/9803317].
“New dimensions at a millimeter to a Fermi and superstrings at a TeV,” I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* B436 (1998) 257. [hep-ph/9804398].
N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “Phenomenology, Astrophysics and Cosmology of Theories with Sub-Millimeter Dimensions and TeV Scale Quantum Gravity” *Phys. Rev.* D59 (1999) 086004. [hep-ph/9807344].
N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, “Stabilization of Sub-Millimeter Dimensions: The New Guise of the Hierarchy Problem”, [hep-th/9809124].

[2] J. M. Overduin and P. S. Wesson, “Kaluza-Klein gravity,” *Phys. Rept.* 283 (1997) 303. [gr-qc/9805018].

[3] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” *Phys. Rev. Lett.* 83 (1999) 3370. [hep-ph/9905221].
L. Randall and R. Sundrum, “An alternative to compactification” *Phys. Rev. Lett.* 83 (1999) 4690. [hep-th/9906064].

[4] M. Gogberashvili, “Four dimensionality in non-compact Kaluza-Klein model”, *Mod. Phys. Lett.* A14 (1999) 2025. [hep-ph/9904383].
M. Gogberashvili, “Our world as an expanding shell”, *Europhys. Lett.* 49 (2000) 396. [hep-ph/9812367].
M. Gogberashvili, “Hierarchy problem in the shell-universe model”, [hep-ph/9812290].
M. Gogberashvili, “Gravitational Trapping for Extended Extra Dimension”, [hep-ph/9908347].

[5] N. Arkani-Hamed, S. Dimopoulos and G. Dvali and N. Kaloper “Infinitely Large New Dimensions” *Phys. Rev. Lett.* 84 (2000) 586. [hep-th/9907209].

[6] K. Akama, “An early proposal of ‘brane world’ ”, [hep-th/0001113].
This is an electronic reprint of: “Pregeometry” Lect. Notes Phys. 176 (1982) 267–271.

V. A. Rubakov and M. E. Shaposhnikov, “Do We Live Inside A Domain Wall?”, *Phys. Lett.* B125 (1983) 136.
M. Visser, “An Exotic Class Of Kaluza-Klein Models”, *Phys. Lett.* B159 (1985) 22. [hep-th/9910093].
E. J. Squires, “Dimensional Reduction Caused By A Cosmological Constant,” *Phys. Lett.* B167 (1986) 286.
P. Laguna–Castillo and R. A. Matzner, “Surfaces of discontinuity in five dimensional Kaluza–Klein models”, Nucl. Phys. B282 (1987) 542.

G. W. Gibbons and D. L. Wiltshire, “Space-Time As A Membrane In Higher Dimensions,” Nucl. Phys. B287 (1987) 717.

[7] M. Gell-Mann and B. Zwiebach, “Dimensional Reduction Of Space-Time Induced By Nonlinear Scalar Dynamics And Noncompact Extra Dimensions,” Nucl. Phys. B260 (1985) 569.

[8] N. Kaloper, “Bent domain walls as braneworlds”, Phys. Rev. D60 (1999) 123506. [hep-th/9905210].

T. Nihei, “Inflation in the five-dimensional universe with an orbifold extra dimension”, Phys. Lett. B465 (1999) 81. [hep-ph/9905487].

C. Csáki, M. Graesser, C. Kolda, and J. Terning, “Cosmology of one extra dimension with localized gravity”, Phys. Lett. B462 (1999) 34. [hep-ph/9906513].

J. M. Cline, C. Grojean, and G. Servant, “Cosmological expansion in the presence of extra dimensions”, Phys. Rev. Lett. 83 (1999) 4245. [hep-ph/9906523].

P. Kanti, I. I. Kogan, K. A. Olive, and M. Pospelov, “Cosmological 3-brane solutions”, Phys. Lett. B468 (1999) 31. [hep-ph/9909481].

P. Kraus, “Dynamics of anti-de Sitter domain walls”, JHEP 9912 (1999) 011. [hep-th/9910149].
radion stabilization”, [hep-ph/9911406].
T. Li, “Classification of 5-dimensional space-time with parallel 3-branes”, [hep-th/9912182].
I. I. Kogan, S. Mouslopoulos, A. Papazoglou, G. G. Ross, and J. Santiago, “A three three-brane universe: New phenomenology for the new millennium?”, [hep-ph/9912552].
J. M. Cline, “Cosmological expansion in the Randall-Sundrum warped compactification”, [hep-ph/0001285].
C. Csáki, J. Erlich, T. J. Hollowood, and J. Terning, “Holographic RG and cosmology in theories with quasi-localized gravity”, [hep-th/0003076].
H. Collins and B. Holdom, “Brane cosmologies without orbifolds”, [hep-ph/0003173].
R. N. Mohapatra, A. Pérez-Lorenzana, and C. A. de Sousa Pires, “Cosmology of brane-bulk models in five dimensions”, [hep-ph/0003328].
N. Doreelle and T. Dolezel, “Brane versus shell cosmologies in Einstein and Einstein-Gauss-Bonnet theories”, [gr-qc/0004021].
R. Maartens, “Cosmological dynamics on the brane”, [hep-th/0004166].
C. van de Bruck, M. Dorca, R. Brandenberger and A. Lukas, “Cosmological Perturbations in Brane-World Theories: Formalism”, [hep-th/0005032].
P. Kanti, K.A. Olive and M. Pospelov, “Solving the Hierarchy Problem in Two-Brane Cosmological Models” [hep-ph/0005146].
J.M. Cline, H. Firouzjahi “Brane-World Cosmology of Modulus Stabilization with a Bulk Scalar Field”, [hep-ph/0005237].
K. Koyama and J. Soda, “Evolution of Cosmological Perturbations in the Brane World”, [hep-th/0005239].
D. Langlois, “Brane cosmological perturbations”, [hep-th/0005023].
B. Grinstein, D.R. Nolte and W. Skiba, “Adding Matter to Poincare Invariant Branes”, [hep-th/0005001].
A. Chamblin, A. Karch and A. Nayeri, “Thermal Equilibration of Brane-Worl ds”, [hep-th/0007060].
L. Anchordoqui, C. Nuñez, K. Olsen, “Quantum Cosmology and AdS/CFT” [hep-th/0007064].

[9] C. Barceló and M. Visser, “Living on the edge: Cosmology on the boundary of anti-de Sitter space,” Phys. Lett. B482 (2000) 183. [hep-th/0004056].

[10] A. Chamblin and G.W. Gibbons, “Supergravity on the Brane,” Phys. Rev. Lett. 84 (2000) 1090. [hep-th/9909130].
A. Chamblin, S. W. Hawking and H. S. Reall, “Brane-world black holes”, Phys. Rev. D61 (2000) 065007. [hep-th/9909203]
C. Grojean, J. Cline, and G. Servant, “Supergravity inspired warped compactifications and effective cosmological constants”, Nucl. Phys. B578 (2000) 259. [hep-th/9910081].
R. Emparan, G. T. Horowitz and R. C. Myers, “Exact description of black holes on
branes,” *JHEP* **0001** (2000) 007. [hep-th/9911043].

D. Youm, “Solitons in brane worlds,” *Nucl. Phys.* **B576** (2000) 106. [hep-th/9911218];

M. Sasaki, T. Shiromizu, and K. Maeda, “Gravity, stability and energy conservation on the Randall-Sundrum brane-world”, *Phys. Rev.* **D62** (2000) 024008. [hep-th/9912233].

S. Mukohyama, T. Shiromizu, and K. Maeda, “Global structure of exact cosmological solutions in the brane world”, *Phys. Rev. D62* (2000) 024028. [hep-th/9912287].

C. Csaki, J. Erlich, T. J. Hollowood, and Y. Shirman, “Universal aspects of gravity localized on thick branes”, *Nucl. Phys. B581* (2000) 309. [hep-th/0001033].

S. B. Giddings, E. Katz and L. Randall, “Linearized Gravity in Brane Backgrounds,” *JHEP* **0003** (2000) 023. [hep-th/0002091].

J. Garriga and T. Tanaka, “Gravity in the brane-world”, *Phys. Rev. Lett.* **84** (2000) 2778. [hep-th/9911055].

T. Shiromizu, K. Maeda, and M. Sasaki, “The Einstein equations on the 3-brane world”, [gr-qc/9910070].

J. Garriga and M. Sasaki, “Brane-world creation and black holes”, [hep-th/9912118].

A. Chamblin, C. Csáki, J. Erlich and T. J. Hollowood, “Black Diamonds at Brane Junctions,” [hep-th/0002076];

[11] W.D Goldberger and M.B. Wise, “Bulk fields in the Randall-Sundrum compactification scenario” [hep-th/9907218].

[12] P. J. Steinhardt, “General considerations of the cosmological constant and the stabilization of moduli in the brane-world picture”, *Phys. Lett. B462* (1999) 41. [hep-th/9907080].

N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, “A small cosmological constant from a large extra dimension”, *Phys. Lett. B480* (2000) 193. [hep-th/0001197].

C. Grojean, “T Self-Dual Transverse Space and Gravity Trapping”, *Phys. Lett. B479* (2000) 273. [hep-th/0002130].

L. Mersini, “Decaying cosmological constant of the inflating branes in the Randall-Sundrum-Oda model”, [hep-ph/9909494].

O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, “Modeling the fifth dimension with scalars and gravity”, [hep-th/9909134].

S. Kachru, M. Schulz and E. Silverstein, “Self-tuning flat domain walls in 5d gravity and string theory”, [hep-th/0001206].

M. Gremm, “Thick domain walls and singular spaces” [hep-th/0002040].

S. Kachru, M. Schulz and E. Silverstein, “Bounds on curved domain walls in 5d gravity”, [hep-th/0002121].

D. Youm, “Bulk fields in dilatonic and self-tuning flat domain walls”, [hep-th/0002147].

S. Forste, Z. Lalak, S. Lavignac and H.P. Nilles, “A Comment on Self-Tuning and Vanishing Cosmological Constant in the Brane World” *Phys. Lett. B481* (2000) 360.
P. Kanti, K.A. Olive and M. Pospelov, “Static Solutions for Brane Models with a Bulk Scalar Field”, Phys. Lett. B481 (2000) 386. [hep-ph/0002229].

C. Csáki, J. Erlich, C. Grojean and T. Hollowood, “General Properties of the Self-tuning Domain Wall Approach to the Cosmological Constant Problem”, [hep-th/0004133].

G.T. Horowitz, I. Low and A. Zee, “Self-tuning in an Outgoing Brane Wave Model” [hep-th/0004206].

B. Grinstein, D.R. Nolte and W. Skiba, “Adding Matter to Poincare Invariant Branes”, [hep-th/0005001].

N. Alonso-Alberca, B. Janssen and P.J. Silva, “Curved dilatonic brane-worlds and the cosmological constant problem” [hep-th/0005116].

S. Nojiri, O. Obregon and S.D.Odintsov, “(Non)-singular brane-world cosmology induced by quantum effects in d5 dilatonic gravity” [hep-th/0005127].

C. Zhu “A Self-tuning Exact Solution and the Non-existence of Horizons in 5d Gravity-Scalar System”, JHEP 0006 (2000) 034. [hep-th/0005230].

V. Barger, T. Han, T. Li, J.D. Lykken and D. Marfatia, “Cosmology and Hierarchy in Stabilized Warped Brane Models”, [hep-ph/0006279].

P. Binetruy, J.M. Cline and C. Grojean, “Dynamical Instability of Brane Solutions with a Self-Tuning Cosmological Constant”, [hep-th/0007029].

V. A. Rubakov and M. E. Shaposhnikov, “Extra Space-Time Dimensions: Towards A Solution To The Cosmological Constant Problem”, Phys. Lett. B125 (1983) 139.

W. Israel, “Singular hypersurfaces and thin shells in general relativity”, Nuovo Cimento, 44B [Series 10] (1966) 1–14; Errata—ibid 48B [Series 10] (1967) 463–463.

K. Lanczos, “Untersuching über flächenhafte verteilung der materie in der Einstein-schen gravitationstheorie”, (1922), unpublished; Flächenhafte verteilung der materie in der Einsteinschen gravitationstheorie”, Ann. Phys. (Leipzig), 74, (1924) 518–540.

N. Sen, “Über die grenzbedingungen des schwerefeldes an unstetig keitsflächen”, Ann. Phys. (Leipzig), 73 (1924) 365–396.

H.A. Chamblin and H.S. Reall, “Dynamic Dilatonic Domain Walls”, Nucl. Phys. B562 (1999) 133. [hep-th/9903225].

G.W. Gibbons and S.W. Hawking, “Action integrals and partition functions in quantum gravity”, Phys. Rev. D15 (1977) 2752.

S.W. Hawking, “The path integral approach to quantum gravity”, in General Relativity: an Einstein centenary survey, ed. by S.W. Hawking and W. Israel, (Cambridge University Press, Cambridge, England, 1979).

C.W. Misner, K.S. Thorne, and J.A. Wheeler, “Gravitation”, (Freeman, San Francisco, 1973).
[18] For a number of applications of the junction conditions, see for example:
S. K. Blau, E. I. Guendelman and A. H. Guth, “The Dynamics Of False Vacuum Bubbles,” *Phys. Rev.* **D35** (1987) 1747.
S. K. Blau and E. I. Guendelman, “Bubble Dynamics And Dimensional Phase Transitions,” *Phys. Rev.* **D40** (1989) 1909.
M. Visser, “Traversable Wormholes From Surgically Modified Schwarzschild Space-Times,” *Nucl. Phys.* **B328** (1989) 203.
M. Visser, “Traversable Wormholes: Some Simple Examples,” *Phys. Rev.* **D39** (1989) 3182.
P. R. Brady, J. Louko and E. Poisson, “Stability of a shell around a black hole,” *Phys. Rev.* **D44** (1991) 1891.
E. Poisson and M. Visser, “Thin shell wormholes: Linearization stability,” *Phys. Rev.* **D52** (1995) 7318. [gr-qc/9506083].

[19] C. Barceló and M. Visser, “Traversable wormholes from massless conformally coupled scalar fields,” *Phys. Lett.* **B466** (1999) 127. [gr-qc/9908028].
M. Visser and C. Barceló, “Energy conditions and their cosmological implications,” [gr-qc/0001099].
C. Barceló and M. Visser, “Scalar fields, energy conditions, and traversable wormholes,” to appear in Class. Quantum Grav. [gr-qc/0003032].

[20] E. E. Flanagan and R. M. Wald, “Does backreaction enforce the averaged null energy condition in semiclassical gravity?,” *Phys. Rev.* **D54** (1996) 6233. [gr-qc/9602052].

[21] C. Barcelo and M. Visser, “Brane surgery: Energy conditions, traversable wormholes, and voids,” to appear in Nucl. Phys. [hep-th/0004022].

[22] R. Casadio, B. Harms, “Charged Dilatonic Black Holes: String frame versus Einstein frame”, *Mod. Phys. Lett. A* **14** (1999) 1089. [gr-qc/9806032].

[23] V. Faraoni, E. Gunzig and P. Nardone, “Conformal transformations in classical gravitational theories and in cosmology”, [gr-qc/9806032].

[24] G. Magnano and L. M. Sokolowski, “On physical equivalence between nonlinear gravity theories and a general relativistic selfgravitating scalar field,” *Phys. Rev.* **D50** (1994) 5039. [gr-qc/9312008].

[25] J. Polchinski, “String Theory: An introduction to the Bosonic string”, (Cambridge University Press, England, 1998).

[26] C. M. Will, “The confrontation between general relativity and experiment: A 1998 update,” [gr-qc/9811036].