Mapping the QCD Phase Transition with Accreting Compact Stars

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Abstract. We discuss an idea for how accreting millisecond pulsars could contribute to the understanding of the QCD phase transition in the high-density nuclear matter equation of state (EoS). It is based on two ingredients, the first one being a “phase diagram” of rapidly rotating compact star configurations in the plane of spin frequency and mass, determined with state-of-the-art hybrid equations of state, allowing for a transition to color superconducting quark matter. The second is the study of spin-up and accretion evolution in this phase diagram. We show that the quark matter phase transition leads to a characteristic line in the $\Omega - M$ plane, the phase border between neutron stars and hybrid stars with a quark matter core. Along this line a change in the pulsar’s moment of inertia entails a waiting point phenomenon in the accreting millisecond X-ray pulsar (AMXP) evolution: most of these objects should therefore be found along the phase border in the $\Omega - M$ plane, which may be viewed as the AMXP analog of the main sequence in the Hertzsprung-Russell diagram for normal stars. In order to prove the existence of a high-density phase transition in the cores of compact stars we need population statistics for AMXP’s with sufficiently accurate determination of their masses and spin frequencies.

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INTRODUCTION

Accreting compact stars in low-mass binary systems undergo a stage with disc accretion leading to both spin-up and mass increase. Initial indications for spin frequency clustering in low-mass X-ray binary (LMXB) systems, reported by measurements with the Rossi-XTE, have lead to the interpretation of such a correlation as a waiting-point phenomenon where stellar configurations cross the border between pure neutron stars and hybrid stars in the spin frequency-mass plane \cite{1}. A systematic analysis of the critical line for a deconfinement phase transition in the “phase diagram” for accreting compact stars \cite{2} has revealed that the suggested population clustering due to the phase transition will instead lead to mass clustering. For homogeneous interiors like in the case of strange stars, however, such an effect shall be absent \cite{3}. For generic polytropic forms of the EoS of quark and hadronic matter, the relationship between softness or hardness of the EoS and the structure of this phase diagram has been demonstrated in Ref. \cite{4}.

In the present contribution we give an updated view on these ideas, based on a recently developed hybrid EoS \cite{5} which fulfills constraints from compact star observations and
heavy-ion collision (HIC) experiments. This example demonstrates the compatibility of an onset of deconfinement in stars with typical masses $\sim 1.4 M_\odot$ with a high maximum mass $\sim 2.0 M_\odot$. We speculate that the class of objects for which mass clustering due to a phase transition in their interior applies might include also most of the first-born neutron stars in low-kick, non-excentric double neutron star as well as in pulsar-white dwarf systems which have undergone a mass accretion stage [7]. Upon further elaboration and proper selection of the pulsar population, the mass clustering phenomenon could prove to be a direct observation of a phase transition in compressed nuclear matter, such as the QCD chiral symmetry restoration transition.

**HYBRID EOS: MASS-RADIUS VS. FLOW CONSTRAINTS**

One of the most challenging tasks in fundamental nuclear and particle physics is the delineation of the border between hadronic matter and quark-gluon matter in the temperature-density plane: the QCD phase diagram. At high temperatures and low baryon densities, numerical simulations of QCD as a lattice gauge theory indicate that both, chiral symmetry restoration and deconfinement are crossover transitions and their critical temperatures coincide at a value of $T_\chi = T_d = 196$ MeV, obtained by the Bielefeld-Brookhaven-Columbia-Riken collaboration [8]. This value is consistent with a statistical model analysis of the freeze-out temperature $T_f = 160$ MeV from hadron production in nuclear collisions at RHIC for the highest presently available c.m.s. energy of $\sqrt{s} = 200$ GeV [10, 11].

At finite densities and $T \approx 0$, the situation is quite different. Both lattice QCD simulations and heavy-ion collision experiments cannot access this region and many questions are yet unanswered, like:

(i) Is (are) the transition(s) of first order so that there must be a critical endpoint in the QCD phase diagram [12]?

(ii) Is quark matter at low $T$ a color superconductor and how does such a property manifest itself [13]?

Other points of discussion which we will not discuss in detail here are:

(iii) Do all quark flavors appear simultaneously or rather sequentially [14, 15, 16, 17]?

(iv) Do chiral and deconfinement transition happen at the same critical chemical potential $\mu_\chi = \mu_d$ or is there a so-called *quarkyonic phase* [18]?

At present, the description of cold, dense matter phases within QCD is out of reach. Therefore, effective models for quark matter like those of the Nambu–Jona-Lasinio (NJL) type [19] may help to obtain quantitative estimates for the dense matter EoS and phase transitions, once the free parameters can be fixed. In this situation, astrophysical observations of compact stars may provide constraints for the behavior of matter under

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1. We want to remark that a considerably lower value of $T_\chi = 151$ MeV has been reported by Fodor et al. [9]. The discrepancy to Cheng et al. [8] is not yet resolved.
high compression and isospin asymmetry, complementary to data from lattice QCD and heavy-ion collisions.

![Graph](image)

**FIGURE 1.** Left: Hybrid EoS with DBHF nuclear matter and color superconducting NJL quark matter fitted to obey the flow constraint \[22\]. Right: Mass-Radius constraints from thermal radiation of the isolated neutron star RX J1856.5-3754 (grey hatched region) and from QPOs in the LMXB’s 4U 0614+09 (green hatched area, the “wedge”) and 4U 1636-536 (orange hatched region) which shall be regarded as separate conditions to the EoS, see Ref. \[5\]. The controversial interpretation that the 1122 Hz pulsations for XTE J1739-285 correspond to its spin frequency \[23\] entails a stringent mass-radius constraint for this compact star \[24\]. The theoretical $M - R$ relations correspond to hybrid star configurations with a DBHF hadronic shell and a color superconducting NJL quark matter core. The underlying hybrid EoS is the isospin-asymmetric generalization of that in the left panel, under $\beta$-equilibrium with electrons and muons. Shown are also lines of constant surface redshift $z$. In the spirit of two-phase models for the dense matter EoS, the nuclear matter EoS can be described by the ab-initio Dirac-Brueckner-Hartree-Fock (DBHF) approach \[20, 21\] using the Bonn-A nucleon-nucleon potential and the quark matter EoS is given by a color superconducting three-flavor NJL model \[15\] augmented by a vector meanfield contribution \[5\] which stiffens the deconfined phase. In the left panel of Fig. 1 we illustrate how the coupling strengths in the scalar diquark ($\eta_D$) and in the vector meson ($\eta_V$) channels as free parameters of the NJL model description can be fixed by the requirement that the isospin-symmetric hybrid EoS resulting from a Maxwell construction with the DBHF EoS is as stiff as possible but still compatible with the constraint region from HIC flow data at AGS and SIS energies \[22\]. Resulting preferable parameter values are $\eta_V = 0.5$ and $\eta_D$ in the range $\eta_D = 1.02...1.03$. The onset of the hadron-to-quark matter transition in symmetric nuclear matter is at a density of $n_d = 0.55 \text{ fm}^{-3} = 3.5 \ n_0$, where $n_0 = 0.16 \text{ fm}^{-3}$ denotes the nuclear matter saturation density. The generalization of this hybrid EoS to the isospin-asymmetric case under $\beta$-equilibrium with electrons and muons can be used to predict the critical mass of a neutron star for the onset of deconfinement in its interior in the range of $M_{\text{crit}} \sim 1.35...1.0 \ M_\odot$, see also \[5\]. The corresponding critical density for the hadron-to-quark matter transition in compact stars is at about $n_{\text{crit}} = 0.4 \text{ fm}^{-3} = 2.5 \ n_0$. In the right panel of Fig. 1 we show the sequences of hadronic compact stars (red line) and the hybrid star sequences with color superconducting quark matter cores (black and blue lines). We would like to stress two results: (i) we predict that compact stars in the typical mass range of $1.35 \pm 0.1 \ M_\odot$ may have quark matter interiors, and (ii) the maximum mass for compact stars with quark matter
interiors exceeds $2 \, M_\odot$. The latter result disproves the claim \cite{25} that the occurrence of deconfined quark matter in compact stars would necessarily lead to a softening of the EoS which excludes the possibility of hybrid star configurations with large masses and radii, see also Ref. \cite{26}.

However, this demonstrates only that the possibility of compact stars having quark matter interiors cannot be excluded by the measurement of a large mass. What does it take to discover a high-density phase transition by astrophysical observations? With the newly developed hybrid star EoS at hand we want to revisit the suggestion that a clustering of frequencies \cite{1} and/or masses \cite{2} in the population of compact stars in LMXB’s may be seen as a signal for deconfinement and add a new twist to this hypothesis by suggesting to include double neutron stars into these considerations!

**POPULATION CLUSTERING AS A SIGNAL FOR DECONFINEMENT**

Our investigation is based on the classification of rotating compact star configurations in the plane of angular velocity $\Omega$ and baryon number $N$ (or gravitational mass $M$), the so called phase diagram for compact stars \cite{27}. It is defined by at least three lines: the maximum frequency $\Omega_{\text{max}}(N)$ for which stable rotation without mass shedding can be sustained, the maximum baryon number $N_{\text{max}}(\Omega)$ the star can carry without undergoing gravitational collapse and the critical line $N_{\text{crit}}(\Omega)$, which separates the region of quark core configurations from the hadronic ones, see Fig. 2. It has been shown that the latter line is correlated with the local maxima of the moment of inertia with respect to changes of the baryon number at given $\Omega$ due to the change of the internal structure of the compact object at the deconfinement phase transition. Therefore, we expect that the rotational behavior of these objects changes in a characteristic way when this line is crossed. Early suggestions of a deconfinement signal following from this characteristic behavior have considered the spin-down of isolated radio pulsars without mass accretion for which a characteristic deviation of the braking index from the value $n = 3$ for dipole emission shall signal the transition \cite{28, 29}. It has also been noted that the star has to spend about $10^8$ yr for crossing the configuration border in the phase diagram, the typical time it takes the star to loose by dipole emission the amount of angular momentum $\Delta J = \Omega \Delta I$ which corresponds to the change in the moment of inertia $\Delta I$ due to the phase transition in the star’s interior. The consequence will be an increase of the population of stars at this critical line which could be observed. Provided that a sufficiently large number of accretors will be discovered and their masses and spin frequencies \cite{2} could be determined. Then the phase transition would reveal itself by a population clustering along the line $N_{\text{crit}}(\Omega)$ in the phase diagram, which according to the above results for nonrotating stars should be well separated from the black hole limit $N_{\text{max}}(\Omega)$.

The main problem with this signal is the shape of the critical line in the phase diagram which for typical hybrid EoS \cite{4} would suggest a mass clustering \cite{2} rather than a frequency clustering \cite{1}. In order to measure the star mass, however, one would need a companion star which for isolated pulsars is absent!

Therefore, the suitable population for which this statistical phase transition test shall be applicable are compact stars in binary systems with mass transfer via Roche-lobe
overflow. There are two such systems which we want to focus on: (i) LMXB’s and (ii) double neutron stars (DNS’s), see [7] for a recent discussion.

The spin evolution of a compact star under mass accretion from a low-mass companion star can be regarded as a sequence of stationary states of configurations (points) in the phase diagram. It is governed by the change in angular momentum of the star

\[
\frac{d}{dt}(I(N,\Omega)) = K_{\text{ext}}, \quad K_{\text{ext}} = \sqrt{GMM^2r_0} - N_{\text{out}},
\]

where \(K_{\text{ext}}\) denotes the external torque due to both the specific angular momentum transferred by the accreting plasma and the magnetic plus viscous stress given by \(N_{\text{out}} = \kappa \mu^2 r_c^{-3}\), \(\kappa = 1/3\) [31]. For a star with radius \(R\) and magnetic field strength \(B\), the magnetic moment is given by \(\mu = R^3 B\) and \(r_c = (GM/\Omega^2)^{1/3}\) is the co-rotating radius, see [1, 2] and references therein for details. From Eq. (1) follows the evolution equation for the angular velocity

\[
\frac{d\Omega}{dt} = \frac{K_{\text{ext}}(N,\Omega) - K_{\text{int}}(N,\Omega)}{I(N,\Omega) + \Omega \partial I(N,\Omega)/\partial N}, \quad K_{\text{int}}(N,\Omega) = \Omega N \left(\frac{\partial I(N,\Omega)}{\partial N}\right). \tag{2}
\]

Solutions of (2) are trajectories in the \(\Omega - N\) plane describing the spin evolution of accreting compact stars. Since \(I(N,\Omega)\) exhibits characteristic functional dependences at the deconfinement phase transition line \(N_{\text{crit}}(\Omega)\) we expect observable consequences in the \(P - P\) plane when this line is crossed.

In our model calculations we assume that both the mass accretion and the angular momentum transfer processes are slow enough to justify the assumption of quasistationary rigid rotation without convection. The baryon number for the constant accreting rate \(\dot{N}\) is given by \(N(t) = N(t_0) + (t - t_0)\dot{N}\) and for the magnetic field of the accretors we consider the exponential decay \(B(t) = [B(0) - B_{\infty}] \exp(-t/\tau_B) + B_{\infty}\). We solve the equation for the spin-up evolution (2) of the accreting star for decay times \(\tau_B = 10^9\) yr and initial magnetic fields in the range \(0.2 \leq B(0)[\text{G}] \leq 4.0\). The remnant magnetic field is chosen to be \(B_{\infty} = 10^{-4}\text{G}[32]\), where 1 TG = \(10^{12}\) G.

The question arises whether there is a characteristic feature in the spin evolution when the trajectories traverse the critical phase transition line. In order to perform a more quantitative discussion of possible signals of the deconfinement phase transition we investigate the spin-up evolution for stars with \(N(0) = 1.4N_\odot\) and \(\Omega(0) = 1\) Hz in the initial state. In the case of high accretion rate \((\dot{N} = 10^{-8}N_\odot/\text{yr}, \text{e.g. for Z sources})\) and long-lived magnetic field \((\tau_B = 10^9\text{yr})\) there is a dip in the period derivative when the star evolves into the quark core region of the phase diagram. This feature can be quantified by the distribution of a waiting time \(\tau = |P/\dot{P}| = \Omega/\dot{\Omega}\) in the \(\Omega - N\) plane.

In Fig.2 we show contours of waiting time regions in the phase diagram. The region of longest waiting times is located in a narrow branch around the phase transition border and does not depend on the evolution scenario after the passage of the border, when the depopulation occurs and the probability to find an accreting compact star is reduced.
Another smaller increase of the waiting time and thus a population clustering could occur in a region where the accretor is already a quark core star. For an estimate of the magnetic field influence we show in Fig. 2 also the region of evolutionary tracks when the values of initial magnetic field vary within $0.6 \leq B(0) [\text{TG}] \leq 1.0$.

As a strategy of search for QCSs we suggest to select from the LMXBs exhibiting the QPO phenomenon those accreting close to the Eddington limit [33] and to determine simultaneously the spin frequency and the mass [34] for sufficiently many of these objects. The emerging statistics of accreting compact stars should then exhibit the population clustering shown in Fig. 2 when a deconfinement transition is possible. If a structureless distribution of objects in the $\Omega - N$ plane will be observed, then no firm conclusion about quark core formation in compact stars can be made as, e.g., for strange stars [3]. There is a problem with the simultaneous analysis of masses and spin frequencies for LMXB’s which is still rather dependent on the model employed [35, 36].

Following van den Heuvel [7], we may suggest to apply our approach also to the population of the first-born stars in double neutron star systems with low eccentricity and pulsar-white dwarf double systems, as they should have undergone a mass accretion stage similar to the one described above which is a prerequisite for the applicability of the population clustering signal of deconfinement. For the specific example of the hybrid EoS discussed above with a DBHF hadronic phase and a color superconducting stiff quark matter phase, we show in Fig. 3 the phase diagram of rotating compact stars together with the masses and spin frequencies obtained for double neutron star systems.
We observe an interesting correlation of the distribution of these objects with the critical deconfinement phase transition line for diquark coupling in the range $\eta_D = 1.017...1.02$. There might be other reasons for the mass clustering of spinning pulsars [7], but the suggestion to relate it to a phase transition in the compact star interior cannot be excluded.

**SUMMARY**

The model independent result of our study is that a population clustering in the phase diagram for accreting compact stars shall measure the critical line $N_{\text{crit}}(\Omega)$ which separates neutron stars from hybrid stars where the shape of this curve can discriminate between different models of the nuclear EoS at high densities.

For the new hybrid equation of state discussed in this contribution, we expect the suggested population clustering as a signal of the deconfinement transition in the mass range $M \sim 1.3...1.4 M_\odot$ which might therefore serve as one aspect for the explanation of the well-known mass clustering in double neutron stars in that same mass range. The above rather fresh ideas suggest that future observational programs may contribute to unraveling most actual problems of fundamental physics of dense baryonic matter.
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