A method for bubble volume calculating in vertical two-phase flow

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Abstract. The movement of bubble is a basic subject in gas-liquid two-phase flow research. A method for calculating bubble volume which is one of the most important characters in bubble motion research was proposed. A suit of visualized experimental device was designed and set up. Single bubble rising in stagnant liquid in a rectangular tank was studied using the high-speed video system. Bubbles generated by four orifice with different diameter (1mm, 2mm, 3mm, 4mm) were recorded respectively. Sequences of recorded high-speed images were processed by digital image processing method, such as image noise remove, binary image transform, bubble filling, and so on. then, Several parameters could be obtained from the processed image. Bubble area, equivalent diameter, bubble velocity, bubble acceleration are all indispensable in bubble volume calculating. In order to get the force balance equation, forces that work on bubble along vertical direction, including drag force, virtual mass force, buoyancy, gravity and liquid thrust, were analyzed. Finally, the bubble volume formula could be derived from the force balance equation and bubble parameters. Examples were given to shown the computing process and results. Comparison of the bubble volume calculated by geometric method and the present method have shown the superiority of the proposed method in this paper.

1. INTRODUCTION

In recent years, gas-liquid two-phase flow has been widely used in biological fermentation, polymer polymerization, industrial wastewater treatment, environmental protection and so on. The movement of bubble is a basic subject in two-phase flow research as bubble in water always plays an important role in solving problems of experiments and projects. The state and motion of bubble are closed with the operating conditions, the nature of liquid and the form of ventilation. If bubble state could be got by visualized method, that will inspirit the development of chemical industry and its relevant filed.

High-speed visual is a non contact measurement which dose not interfere with the movement of bubble in flow. Many researchers have done to learning the characters of bubble using visualized method. Maurus et al.(2002) used digital imaging and analysing techniques to study the subcooled flow boiling and got the bubble characters and local void fraction. Rodriguez-Rodriguez et al.(2003) applied the particle tracking velocimetry approach of high-speed video images to a novel tracking and bubble break-up detection. Zaruba and Krepper et al.(2005) have used high-speed video observation to make study on bubble motion in a rectangular column and an individual bubble in the gas swarm could be tracked.

In addition, theoretic analysis of the force work on bubble has been studied in recent years. Marco et al.(2003) compared the theories of force balance along the vertical with the measured bubble rising
velocities of gas bubble in a still liquid to explain the phenomena of sensible steady oscillation of velocity. A dynamic model of bubble based on pressure balance, force balance and mass transfer considerations has been developed to investigate the dynamics and mass transfer of a single bubble of carbon dioxide rising in a strong alkaline solution hydroxide by Madhavi et al.(2006). The total drag coefficient of single bubble rising freely with high viscosity was calculated from the accelerating motion to the steady motion with the added mass (virtual mass) and history force included by Zhang et al.(2008).

Bubble volume is important in studying bubble motion for it is closed related to void fraction. However, volume of bubble is difficult to get directly, especially for distorted bubble. A method for bubble volume calculation is proposed in this article. First of all, a suit of visualized device is designed and set up to make experiments. Signal bubble rising in stagnant water in a rectangular tank. Secondly, continues image of bubble rising in water was recorded by high-speed video camera. Thirdly, these images need to be processed by digital image processing algorithm. Several useful parameters could be acquired from the processed images. Such as, bubble area, equivalent diameter, bubble velocity and acceleration, and so on. Finally, analysis the force that bubble endured along vertical direction. The calculation formulas of every force have been summarized for the volume calculation. The computing formula of bubble volume was derived from force balance equation and bubble parameters. Examples have shown the analytic steps and calculated results run though the paper to verify the feasibility of this method for bubble volume calculation.

2. EXPERIMENTAL METHOD

High-speed video measurement is one of the important measuring technique of the movement of bubble in air-water two-phase flow. It's very important to get sharp, undistorted, continuous images to observe the flow character of two-phase flow. Therefore, the property of gas and liquid, the refractive index of fluid and channel and the gas speed must be considered during the system design. The experimental facility contains two parts: two-phase flow filed simulating and high-speed image recording.

2.1 Two-phase Flow Simulating

A schematic view of the visualized experimental test facility is present in Fig. 1. The vertical rectangular tank that made of Plexiglas has a cross-section of 20cm×20cm and is 125cm in height. As the tank is rectangular and the refractive index of Plexiglas is matched with that of liquid.

Fig. 1 Schematic view of the experimental setup

There are 4 orifices with different diameter (1mm, 2mm, 3mm, and 4mm) at the bottom centre of the channel. They could generate bubbles in liquid. Under each orifice, there is a simplex gas check to ensure that the liquid can not leak from the orifice. An air control valve which connected with the
simplex gas check is the master switch. It controls the rising rate, frequency, size and stability of bubble by regulating the flow of air. A air pump connected to the air control value supplies air to the tank. At the bottom of the whole device, a square corrosion resistant plate connected with four column stainless steel tube was used to support the tank to stabilize the two-phase flow system.

2.2 High-speed Image Recording

A high-speed video camera consists of an advanced CMOS sensor was used to record images of the rising bubble. Its frame frequency is up to 32000fps and its highest resolution is $1280 \times 1024$. However, higher spatial resolutions result in longer recording times and fewer video frames per unit time. A compromise between recording speed and image resolution has to be found. Experimentally, $640 \times 480$ special resolution and 500fps frequency were enough to meet the requirement in the experimental process and was adopted in all experiments records in this study. Auxiliary backlighting with 5400K color temperature was also used to outline the bubbles. As gas, water and tank are all transparent, back light was adopted in the experiment. The whole experiment platform was shown in fig. 2.

![Fig. 2 High-speed image recording system](image)

2.3 Experimental Steps

First, connect the camera to computer and turn on the camera. Second, adjust focus of high-speed camera until the image is clear. Third, turn on the light and pump, turn the air control value to make bubbles rising from the orifice. Fourth, record continuous images of bubble rising. Each orifice should be measured independently. It’s also necessary to change bubble rising rate and move the camera and the light to record bubbles rising images at different working condition. The following section select the single bubble images of the part from 14cm to 25.6cm from the tank bottom as the research object.

3. IMAGE PROCESSING AND PARAMETER CALCULATING

3.1 Image Processing

In order to abstract the character information from high-speed images the original images should be processed by digital image processing. First of all, transform RGB image into gray image. As digital images have a variety of noises, wavelet filter was adopted to emphasize some features or remove noises (Lv et al. 2007). Second, a brightness threshold $I$ is applied to transform gray image into binary image as follows:
if \( k(i, j) < t \), then \( k(i, j) = 0 \)
if \( k(i, j) \geq t \), then \( k(i, j) = 255 \)

For the reverberation of bubble, the center of bubble has the same gray value as the background in binary image. It’s necessary to fill the center of bubble to calculate the area or other parameters of bubble in two-phase flow. Infected algorithm which used to fill bubble of binary image is adopted in this paper. First, set a original infective point which is labeled value \( a \). Second, scan the image matrix began at the original infective point. If the point in binary image is effected, then the points in its 8-neighborhood which is 0 will be labeled to number \( p \). The formulation of infected algorithm is:

\[
\begin{cases}
    h(i, j) = p, & \left[ \bigcup_{n=1}^{i+1} \bigcup_{m=1}^{j+1} h(m, n) \right] \cap \overline{k(i, j)} = 1 \\
    h(i, j) = 0, & \text{else}
\end{cases}
\]  \hspace{1cm} (1)

Finally, creat a zero matrix as large as the processed binary image matrix. Migrate all the infected points into the zero matrix to get the filled image.

Fig.3 has shown four typical images cut from a sequence of image when the diameter of the orifice is 4mm. and the processed results of them were also shown out.

![Original Image](image1)

(a) Original image

![Gray Image](image2)

(b) Gray image

![Binary Image](image3)

(c) Binary image

![Filled Image](image4)

(d) Filled image

Fig. 3 Example of high-speed image and the processing results

3.2 Parameters Calculating
Parameters that abstracted from processed image play a very important role in bubble rising study. Area, equivalent diameter, rising velocity and acceleration of bubble are all necessary in bubble volume calculation.

(1) Area of Bubble
Area of bubble is a useful parameter in obtaining void fraction. For simply, the area value of the bubble is defined as the total pixel number of a consecutive area in the bubble filled image. It is calculated according to:

\[ A_i = \sum_j n_{ij} \]  

(2) Equivalent Diameter
In two phase flow, the diameter of bubble or particle is a important parameter. As bubble in two phase flow is not normally spherical, the diameter of sphere that has the same volume with bubble is defined as equivalent diameter of the bubble (Darton, 1977). The measurement of high-speed image is based on 2-D image, so equivalent diameter is the diameter of circle which has the same area with the bubble shadow. The formula of the equivalent diameter is:

\[ d = \frac{4A_i}{\pi} \]  

(3) The Center of the Bubble
Under the press of water, bubble is distorting during the rising movement. So the position of bubble is difficult to be found. However, get the position of bubble in image is a pre-requisited work to do in analysing the movement of bubble (Zhang et al. 2006). Gray-scale centroid method is applied to calculate the coordinate of bubble center based on gray image. The computing formulation is shown as follows:

\[ x_c = \sum_{i,j \in \Omega} \frac{k(i,j)j}{M}, \quad y_c = \sum_{i,j \in \Omega} \frac{k(i,j)i}{M} \]  

where \( M = \sum_{i,j \in \Omega} k(i,j) \) is the grand total of gray value of image. It can be seen that weighted average of the gray value in the image is the center of bubble.

(4) Bubble Velocity and acceleration
Bubble velocity is a key parameter in two-phase flow measurement, which infect other parameters directly. The velocity was calculated by measuring the distance of a bubble in two successive images for the time interval \( \Delta t \) between two relative image is known. Compute out the coordinate of bubble center \( (x_c, y_c) \) in every image. Get the displacement of bubble by subtracting bubble center of two successive images. Suppose the bubble centers are \( (x_{c1}, y_{c1}) \) and \( (x_{c2}, y_{c2}) \). Bubble velocity \( v_x, v_y \) is a calculated by the formula as follows:

\[ v_x = \frac{x_{c2} - x_{c1}}{\Delta t}, \quad v_y = \frac{y_{c2} - y_{c1}}{\Delta t} \]  

As the frequency of high-speed is very high, the displacement of bubble and time interval between two images are so small that the velocity can be regarded as instantaneous speed. And then, acceleration \( a \) of bubble can be computed out by definition as follows:
\[ a_x = \left( v_{x2} - v_{x1} \right) / \Delta t, \quad a_y = \left( v_{y2} - v_{y1} \right) / \Delta t \]  

\( a_x \) and \( a_y \) is the acceleration of horizontal direction and vertical direction respectively. \( a_y \) is a useful parameter in bubble volume calculating.

Table 1 gives the calculated results of four kinds parameter of which bubbles generate from orifices with different diameter. 20 images have been chosen to be processed in each situation. However, table 1 only shown the parameters of 6 images which is cut from the 20 images for your information only.

| Diameter of orifice | Parameter | Parameter value of 6 images |
|---------------------|-----------|-----------------------------|
|                     | \( A(\text{mm}^2) \) | 9.05 8.52 8.23 8.41 8.35 7.88 |
|                     | \( d(\text{mm}) \)  | 3.39 3.29 3.24 3.27 3.26 3.17 |
|                     | \( v_y(\text{m/s}) \) | 0.152 0.311 0.132 0.248 0.228 0.152 |
|                     | \( a_y(\text{m/s}^2) \) | 7.935 -8.939 5.770 -0.959 -3.832 6.234 |
| 1mm                 | \( A(\text{mm}^2) \) | 20.62 15.89 12.32 13.84 18.22 14.25 |
|                     | \( d(\text{mm}) \)  | 5.12 4.50 3.96 4.20 4.82 4.26 |
|                     | \( v_y(\text{m/s}) \) | 0.188 0.2448 0.204 0.225 0.171 0.188 |
|                     | \( a_y(\text{m/s}^2) \) | 2.813 -2.016 1.069 -2.713 0.846 2.318 |
| 2mm                 | \( A(\text{mm}^2) \) | 15.07 12.97 15.54 23.07 22.19 19.86 |
|                     | \( d(\text{mm}) \)  | 4.38 4.06 4.45 5.42 5.32 5.03 |
|                     | \( v_y(\text{m/s}) \) | 0.265 0.177 0.216 0.192 0.215 0.199 |
|                     | \( a_y(\text{m/s}^2) \) | -4.431 1.960 -1.219 1.183 -0.818 0.911 |
| 3mm                 | \( A(\text{mm}^2) \) | 26.63 25.29 29.08 27.62 20.38 19.62 |
|                     | \( d(\text{mm}) \)  | 5.82 5.67 6.08 5.93 5.094 5.00 |
|                     | \( v_y(\text{m/s}) \) | 0.216 0.221 0.243 0.247 0.195 0.206 |
|                     | \( a_y(\text{m/s}^2) \) | 0.274 1.094 0.204 -2.616 0.536 -0.301 |
| 4mm                 |                     |                      |

4. BUBBLE VOLUME CALCULATION

Bubble volume is a very important parameter in two phase flow, which is difficult to measure. In this paper, we proposed a method based on 2-D high-speed image analysis to estimate volume of bubble. First, analysis the forces which bubble subjected in vertical direction. Then, get the force balance formulation based on Newton’s second law.

4.1 Force Analysis in Vertical Direction

Usually, there are 5 kinds of force in vertical direction. They are drag force, virtual mass force, buoyancy, gravity and liquid thrust (Sherwood et al., 1999, Zhang et al., 2001).

4.1.1 Drag Force

Drag is the resistance of bubble moving in the water(Krepper et al., 2007). It can be expressed as:

\[
F_D = -\frac{1}{2} \rho \, C_D \, |u_1 - u_s| \, (u_g - u_1) \, s \quad (7)
\]

\( s \) is the front face area of bubble. For bubble keep rolling during the experiment, \( s \) was approximated
to the average area $A$ of the side projection area of bubble. $C_D$ is the Drag coefficient, which is related to Reynolds number.

By the definition, Reynolds number is the ratio of inertial force and viscous force. It is expressed as follow:

$$\text{Re} = \frac{F_g}{F_m} = \frac{\rho_g Su_g^2}{\frac{S}{d} \eta g} = \frac{u_g d}{\nu}$$

(8)

where $u$ is average velocity of bubble. $d$ is the equivalent diameter of bubble. $v, \eta$ are the coefficient of kinematics viscosity and dynamic viscosity of the flow in motion state. When the temperature is 20 degrees Celsius, the dynamic viscosity of air is $18.09 \times 10^{-6}$ Pa·s, the kinematics viscosity of air is $14.8 \times 10^{-6}$ m2/s. Combining the working condition and formula (8), the Reynolds number could computed out.

In 1979, Ishii proposed Ishii formula. When $20 < \text{Re} < 260$, $C_D$ could be calculated by:

$$C_D = \frac{24}{\text{Re}} \left( 1 + 0.1 \text{Re}^{0.75} \right)$$

(9)

Table 2 have computed the Reynolds number and drag coefficient of the images refer to table 1. It’s obviously that the smaller the diameter of the orifice is, the bigger the two parameters of the same bubble varying.

| Diameter of orifice | Coefficient value of 6 images |
|---------------------|--------------------------------|
| $d=1$ mm            | 34.94 69.24 28.93 54.75 50.33 32.49 |
| $d=2$ mm            | 65.02 74.17 54.54 63.86 55.62 54.05 |
| $d=3$ mm            | 78.51 48.51 64.88 70.13 77.28 67.55 |
| $d=4$ mm            | 84.98 84.91 100.05 99.14 67.15 69.51 |

| Name of coefficient | Diameter of orifice | Coefficient value of 6 images |
|---------------------|---------------------|--------------------------------|
| Reynolds number (Re) | $d=1$ mm            | 1.085 0.606 1.26 0.744 0.800 1.151 |
|                     | $d=2$ mm            | 0.641 0.570 0.747 0.651 0.734 0.752 |
|                     | $d=3$ mm            | 0.542 0.825 0.642 0.599 0.550 0.620 |
|                     | $d=4$ mm            | 0.505 0.505 0.436 0.439 0.623 0.604 |

4.1.2 Virtual Mass Force

Liquid that surrounding the bubble will accelerate with the relative acceleration of bubble rising. An additional mass called virtual mass which caused by bubble acceleration will the give an additional force called virtual mass force. Virtual mass will increase the effective inertial mass (Chuag M. et al., 2001).

In the momentum equation of dispersion flows, virtual mass force is used to abrupt accelerate the continuous phase surround the dispersed phase. If the density of the dispersed phase is far outweighs that of the continuous phase, then the influence of virtual mass can be neglected. However, the density of dispersed phase air in this paper is much smaller than that of continuous phase water. Therefore, virtual mass force could not be neglected in this study. The expression of virtual mass force is:
\[ F_v = \frac{1}{2} V_g \rho_l \frac{d}{dt} (u_i - u_g) \]  

(10)

\subsection*{4.1.3 Buoyancy}
Object in the liquid will suffer an upward force named buoyancy produced by the surrounding liquid or gas in which it is fully or partially immersed, due to the pressure difference of the fluid between the top and bottom of the object. According to Archimedes principle, the upward buoyancy force \( F_B \) is equal to the magnitude of the weight of fluid displaced by the body. The computing formula is:

\[ F_B = G_i = \rho_g V g \]  

(11)

Where \( g \) is the acceleration of gravity and \( g=9.8\text{m/s}^2 \).

\subsection*{4.1.4 Gravity}
Gravity is proportional to the mass of the body. So it was expressed as follow:

\[ G = -m g \]  

(12)

\subsection*{4.1.5 Liquid Thrust}
If liquid is flowing and the flowing direction of liquid is the same as bubble moving to, then bubble will endure the thrust of liquid. The thrust is:

\[ F_p = \rho V g \frac{du_i}{dt} \]  

(13)

For the liquid is static in this subject, the flow rate of water \( u_j \) is 0. Thus, the liquid thrust dose not work on bubble moving.

\subsection*{4.2 Bubble Volume Calculating}

\subsection*{4.2.1 Formula for volume}
According to Newton's Second Law, the acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object. In other words, the net force is equated to the product of the mass times the acceleration \( a \). It can be expressed in equation form as follows:

\[ F_c = ma \]  

(14)

where \( F_c \) is the net force. The direction of the net force is in the same direction as the acceleration.

Considering the above analysis of force, the net force of the rising bubble in static water is:

\[ F_c = F_B + F_D + F_v + G \]  

(15)

Combine formula (7) to (15), the force balance equation can be expressed as follows:

\[ m g a_y = -\frac{1}{2} \rho_l C_m u_g^2 A - \frac{1}{2} \rho g V g a_y + (\rho_i - \rho_g) g V g \]  

(16)
The finishing formula for volume is:

\[ V_g = \frac{\rho_l C_D u^2 A}{2(\rho_l - \rho_g)g - (\rho_l + 2\rho_g) a_y} \] (17)

Table 3 have calculated the volume of bubble refered to table 1 and table 2. And the average volume of 20 images were also shown in table 3.

| Diameter of orifice | Volume value of 6 images | Average value |
|---------------------|--------------------------|---------------|
| d=1mm               | 19.60 17.52 13.19 18.68 14.88 15.68 | 17.03         |
| d=2mm               | 27.81 25.00 20.64 20.49 20.86 21.92 | 22.28         |
| d=3mm               | 23.94 18.97 22.35 27.58 27.70 26.06 | 24.54         |
| d=4mm               | 32.51 33.92 38.72 33.46 25.38 25.27 | 31.81         |

It is worth while to note that the drag coefficient \( C_D \) in formula (17) is related to the equivalent diameter \( d \) which supposed bubble to be a normal sphere. If we supposed the bubble is a normal sphere, bubble volume can be computed by \( V_d = 4/3\pi (d/2)^3 \). But bubbles deform during the rising course, the volume that only depends on equivalent diameter \( d \) will result in large error. However, formula (17) considered the forces endured on bubble in hydronamics, which combined many factors to analyse bubble motion and weakened the effect of the assumption that bubble is normal sphere in drag coefficient \( C_D \) computation. So, force analysis method for bubble volume calculation could be considered as an improved method which is much more accurate than direct calculation.

4.2.2 Comparison of Bubble Volume

Bubble volume also can be calculated by geometric methold from direct image measurement. In order to vertify the validity of the calculated results for the method metioned above, geometric method was introduced. And the standard deviation of bubble volume computed by this two method were compared with each other.

Under the press of water, bubble is distorting during the rising movement. However, the horizontal forces that worked on the bubble in the same high level are equal to offset each other. So, the cross section is quasi-circular. Therefore, the whole bubble could be described as an ellipsoid.

The geometric method is computed based on edge detected images. Suppose the background of edge detected image is white and the bubble edge is black. The geometric method of bubble volume computing is described as follows:

1. Scan the edge detected image from the top down, left-to-right. To find the coordination of point with value 0.
2. Compute the distance between the edge point \((i, j)\) and the Center Coordinate. The distance \(r\) is calculating by:

\[ r = \sqrt{(i - x_c)^2 + (j - y_c)^2} \] (18)

3. Determine the size of \(r\) and \(d/2\). The mean value of the bigger is regarded as the major semi-axis \(a\); but that of the smaller is regarded as the minor semi-axis \(b\).
(4) Construct a ellipsoid with semi-axis: a, b, and c. where c=b. Compute the volume of the ellipsoid with volume formula:

\[ V = \frac{4}{3} \pi abc \quad (19) \]

Table 4 has shown the comparison of the standard deviation for the volume calculated by geometric method and the method proposed in this paper. It’s obverse that the result of bubble volume calculated by the method proposed in this paper is much more accuracy than that of geometric method from table 4.

The geometric method for geometric volume computing of bubble is only based on 2-D high-speed image processing. For the shape changing during bubble rolling rising, the geometric volume has a larger fluctuation and compute error than that of the volume computed by the proposed method. However, the force analysis method based on a powerful theory produce stabilized results and small error. It is superior to geometric method in bubble volume calculating.

| Diameter of orifice | Standard deviation of geometric volume | Standard deviation of presented method |
|--------------------|----------------------------------------|---------------------------------------|
| d=1mm              | 1.3742                                 | 0.4133                                |
| d=2mm              | 1.5685                                 | 0.4562                                |
| d=3mm              | 1.3507                                 | 0.4129                                |
| d=4mm              | 1.4590                                 | 0.6126                                |

5. CONCLUSIONS
An experimental apparatus was set up and operated to study gas bubble motion. In order to get high-quality image of rising bubble by high-speed camera, two-phase flow filed was established by injecting gas bubbles into a rectangular tank through orifices with different diameter. Sequential images which acquired by the visualized system needed to be processed by digital image processing algorithm. Affective algorithm was applied to fill bubble in binary image. The area, equivalent diameter, center coordinate, velocity and acceleration of bubble were calculated by the processed images.

Additionally, when the contributions of drag force, virtual mass force, buoyancy and gravity acting on a single bubble were combined, the volume could be derived using Newton’s second law combines with the experimental measurements of parameters of rising single bubbles released from orifices.

Experimental examples of bubble parameters calculation were shown in the paper. As bubble rolling in water, the same parameter of the same bubble in different image is different. Average value of volume could be viewed as the final rough estimated results. However, the accuracy of the results calculated method is much better than that got from direct image measurement after the standard deviation comparing.

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NOMENCLATURE
\[ A \quad \text{area value of bubble} \quad [\text{mm}^2] \]
\(a\)  the acceleration of bubble \([\text{m/s}^2]\)

\(C\)  drag coefficient

\(d\)  the equivalent diameter of bubble \([\text{mm}]\)

\(F\)  force \([\text{N}]\)

\(G\)  gravity \([\text{N}]\)

\(g\)  acceleration of gravity \([\text{m/s}^2]\)

\(h\)  gray value of image

\((i, j)\) coordinate of pixel point

\(K\)  gray value of image

\(m\)  mass \([\text{kg}]\)

\(n\)  the number of pixels

\(M\)  grand total of gray value of image

\(\text{Re}\)  Reynolds number

\(s\)  front face area of bubble \([\text{mm}^2]\)

\(t\)  brightness threshold

\(u\)  flow velocity \([\text{m/s}]\)

\(V\)  volume \([\text{mm}^3]\)

\(v\)  bubble velocity \([\text{m/s}]\)

\(x\)  coordinate of x-axis

\(y\)  coordinate of y-axis

**Greek Letters**

\(\Omega\)  the gather of the pixels of the same bubble

\(\rho\)  density \([\text{kg/m}^3]\)

\(\nu\)  kinematics viscosity coefficient \([\text{m}^2/\text{s}]\)

\(\eta\)  dynamic viscosity coefficient \([\text{Pa}\cdot\text{s}]\)

\(\Delta t\)  time interval between image \([\text{ms}]\)

**Subscripts**

\(B\)  buoyancy

\(C\)  centre of bubble

\(D\)  drag force

\(I\)  the i-th bubble

\(j\)  the j-th pixel value

\(l\)  liquid

\(g\)  gas

\(V\)  virtual mass

\(x\)  parameter of horizontal direction

\(y\)  parameter of vertical direction

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