Detectability of the Supernova Relic Neutrinos and Neutrino Oscillation

S. Ando\textsuperscript{a}, K. Sato\textsuperscript{a,b}, and T. Totani\textsuperscript{c,d}

\textsuperscript{a}Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
\textsuperscript{b}Research Center for the Early Universe, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
\textsuperscript{c}Princeton University Observatory, Peyton Hall, Princeton, NJ 08544, USA
\textsuperscript{d}Theory Division, National Astronomical Observatory, Mitaka, Tokyo, 181-8588, Japan

Abstract

We investigate the flux and the event rate of the supernova relic neutrino background (SRN) at the SuperKamiokande detector for various neutrino oscillation models with parameters inferred from recent experimental results. A realistic model of neutrino emission from supernova explosions and several models of the cosmic star formation history are adopted in the calculation. The number flux over entire energy range is found to be $11 - 15 \text{ cm}^{-2}\text{s}^{-1}$. We discuss the detection possibility of SRN at SuperKamiokande, comparing this SRN flux with other background neutrinos in more detail than previous studies. Even though there is no energy window in which SRN is dominant, we might detect it as the distortion of the other background event. We found in the energy range $17 - 25 \text{ MeV}$ the expected event rate at SuperKamiokande $0.4 - 0.8 \text{ yr}^{-1}$. In this range, ten-year observation might enable us to detect SRN signal (at one sigma level) in the case of LMA solar neutrino solution. We also investigate event rate at SNO and KamLAND. Although we can find energy window, the expected event rate is rather small ($0.03 \text{ yr}^{-1}$ for SNO, $0.1 \text{ yr}^{-1}$ for KamLAND).

Key words: diffuse background, supernovae, neutrino oscillation

PACS: 98.70.Vc, 14.60.Pq, 95.85.Ry

\textit{Email address: ando@utap.phys.s.u-tokyo.ac.jp} (S. Ando).
1 Introduction

A core-collapse supernova explosion produces a number of neutrinos and 99% of the gravitational energy is transformed to neutrinos. It is generally believed that the core-collapse supernova explosions have traced the star formation history in the universe and have emitted a great number of neutrinos, which should make a diffuse background. This supernova relic neutrino (SRN) background is one of the targets of the currently working large neutrino detectors, SuperKamiokande (SK) and Sudbury Neutrino Observatory (SNO). Comparing the predicted SRN spectrum with the observations by these detectors provides us potentially valuable information on the nature of neutrinos as well as the star formation history in the universe. This SRN background has been discussed in a number of previous papers [1]-[11]. The work after Totani et al. [8] takes into account the realistic star formation history inferred from various observations and theoretical modeling of galaxy formation, to calculate the SRN flux. Totani et al. [8] calculated the energy flux of SRN at SK detector and compared it with neutrinos emitted by other sources (solar, atmospheric, and reactor neutrinos and so on). Then they concluded that the visible event rate of SRN at SK is $1.2 \, \text{yr}^{-1}$ in the energy range from 15 to 40 MeV. On the other hand, Kaplinghat et al. [11] calculated the upper limit of SRN and discussed the possibility of the detection at SK detector comparing to the other backgrounds. Their result is that at the energy range from 15 to 40 MeV, where no significant background had been considered to exist in previous studies, there is a huge background of the invisible muon decay and the detection of SRN is difficult. They also discussed the effects of neutrino oscillation and briefly mentioned that if the SRN flux is in the vicinity of their upper bound and all three flavors are maximally mixed, it may be detectable as a distortion of the expected muon background.

In this paper we calculate the SRN flux and the event rate at SK, and discuss about the detectability of SRN, with the following new aspects compared with previous studies: 1) realistic neutrino oscillation parameters are incorporated based on the recent solar and atmospheric neutrino experiments, 2) a realistic neutrino spectrum from one supernova explosion is used, which is obtained from a numerical simulation by the Lawrence Livermore group, and 3) we have examined other contaminating background events against the detection of SRN, in more detail than previous studies.

Recent experiments of SK and SNO support neutrino oscillation by solar [12]-[14] and atmospheric neutrino [15] data. Although some previous studies (e.g. [11]) mentioned the possibility of neutrino oscillation, no quantitative calculation of SRN has been made incorporating the realistic oscillation parameters. Here we consider four neutrino oscillation models, which satisfy the solar and atmospheric neutrino data, and investigate the dependence of the positron
spectra at SK on the oscillation models. If neutrino oscillation occurs, $\bar{\nu}_{\mu,\tau}$'s are converted into $\bar{\nu}_e$'s which are mainly detected at SK detector. Because $\bar{\nu}_{\mu,\tau}$'s interact with matter only through the neutral-current reactions in supernovae, they are weakly coupled with matter compared to $\bar{\nu}_e$. Thus the neutrino sphere of $\bar{\nu}_{\mu,\tau}$'s is deeper in the core than $\bar{\nu}_e$'s and their temperatures are higher than $\bar{\nu}_e$'s. Therefore neutrino oscillation enhances the mean $\bar{\nu}_e$'s energy and enhances event rate at SK detector.

In all of the past studies, neutrinos emitted from a supernova are assumed to obey the Fermi-Dirac distribution with zero chemical potential. However, since neutrinos are not in the thermal equilibrium states in supernovae, the real spectrum should be different from the pure Fermi-Dirac distribution. Thus we use in this paper a realistic supernova model established by the Lawrence Livermore group [16] which in fact shows clear difference from the Fermi-Dirac distribution [17].

There are several background events which hinder the detection of SRN. These includes atmospheric and solar neutrinos, anti-neutrinos from nuclear reactors, and decay electrons from invisible muons. We should find the energy region which is not contaminated by these background events and then calculate the detectable event rate of SRN. By careful examination of these events, we found that there is a narrow energy window at which the detection of the SRN event might be possible.

In addition we use three models of the cosmic star formation history, i.e., the evolution of star formation rate (SFR) density of high redshift galaxies (e.g., [18]), which is inferred from several star formation indicators such as rest-frame UV luminosity, H$\alpha$ lines, or dust emission in far-infrared or submillimeter wavebands. It should be noted that SFR models contain various uncertainties, especially at high redshift regions. (e.g., the faint ends of luminosity functions have not been well established at high redshifts and the uncertainties in dust extinction are large at all redshifts [19].) This is the reason we applied several SFR evolution models described below.

Throughout this paper, we consider only electron antineutrinos ($\bar{\nu}_e$'s) from collapse-driven supernovae, because $\bar{\nu}_e$'s are most easily detected in a water Čerenkov detector like SK.

This paper is organized as follows: In Section 2, we illustrate the neutrino oscillation models, the supernova model, and the supernova rate models considered in this paper and discuss about these models. Formulations for calculation of flux and event rate at SK are given in Section 3. In Section 4, we show the calculated flux and event rate. Detailed discussions especially on other background events are presented in Section 5.
2 Models of Neutrinos and Supernova Rate

2.1 Supernova Model

We use a realistic model of a collapse-driven supernova calculated by the Lawrence Livermore group [16]. We show in Fig. 1 the time-integrated spectrum for $\bar{\nu}_e$'s (e.g., see Ref. [17] for detail). We also show the Fermi-Dirac (FD) distribution with zero chemical potential for comparison. Comparing with the FD model used in the other studies, the deficit of both low- and high-energy neutrinos can be seen.

In this paper, we assume that this numerical model represents all of the past supernovae, although in this model the progenitor of the supernova has been assumed to have the mass of $\sim 20M_\odot$. It is clearly an oversimplification, but we note that the mean mass of progenitor stars of type II supernovae (above $8M_\odot$) weighted by number is about $\sim 15M_\odot$ when a typical initial mass function is applied [3].

Another assumption made in the simulation is the isotropic radiation of neutrinos. All supernova progenitors are rotating, and it may have significant effect on the degree of isotropy of neutrino emission [20], although the effect is very difficult to estimate quantitatively. However, since our interest is in the SRN background, which is the sum of all past supernova neutrinos, the rotation effect is expected to be small.

2.2 Neutrino Oscillation Models

We adopt the four parameter sets for the neutrino mixing (see Table 1) and the normal mass hierarchy. These parameter sets are introduced to explain the observations of the solar and the atmospheric neutrinos [12]-[15]. The neutrino spectra emitted by each supernova are calculated numerically by Takahashi et al. [21] for the same parameter sets assuming the normal mass hierarchy. (Supernova model they have used is the same one discussed in the previous subsection.) We use their calculated $\bar{\nu}_e$ spectrum as that from any supernova.

In Table 1, “LMA” and “SMA” indicate MSW solution of the solar neutrino problem (see Ref. [22], for the review of MSW effect). Recent SK and SNO observations [12,14] show that LMA solution are more favorable but SMA solution cannot be rejected at more than $3\sigma$ level (see Ref. [23] and references therein). Therefore we try the SMA solution as well. The suffixes “-L” and “-S” indicate whether $\theta_{13}$ is large or small. A large (small) $\theta_{13}$ means “higher resonance” is adiabatic (nonadiabatic). (There are two resonance points where
neutrino mass eigenstates may flip in the supernova matter, and we call them lower and higher resonance points. The lower resonance point is at lower density, and the higher resonance point at higher density.) The adiabatic higher resonance enhances the energy of the electron neutrinos and enhances the event rate of $\nu_e$ scattering, but does not affect the electron anti-neutrinos [24]. In this paper, since we deal with only the reaction of $\bar{\nu}_e$’s, this “-L” and “-S” hardly influence the result.

Now, we semi-qualitatively present a simple illustration of the expected spectral shape of $\bar{\nu}_e$’s for LMA and SMA models. We can naively deal with anti-neutrino oscillation effect as vacuum oscillation, since $\bar{\nu}_e$’s are not affected by the resonance. Further we take two generation formalism, for simplicity (this approximation is justified when $\theta_{13}$ is sufficiently small). The conversion probability from $\bar{\nu}_\mu$ to $\bar{\nu}_e$ (and its inverse) averaged over distance is

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = P(\bar{\nu}_e \to \bar{\nu}_\mu) = \frac{1}{2} \sin^2 2\theta_{12}. \tag{1}$$

This probability depends only on $\theta_{12}$. Then, $P(\bar{\nu}_\mu \to \bar{\nu}_e) = P(\bar{\nu}_e \to \bar{\nu}_\mu) = 0.44$(LMA), $2.5 \times 10^{-3}$(SMA). Using these conversion probabilities we can present $\bar{\nu}_e$ flux as

$$F_{\bar{\nu}_e} = [1 - P(\bar{\nu}_e \to \bar{\nu}_\mu)]F_{\bar{\nu}_e}^0 + P(\bar{\nu}_\mu \to \bar{\nu}_e)F_{\bar{\nu}_\mu}^0 \tag{2}$$

\[
\simeq \begin{cases} 
0.57F_{\bar{\nu}_e}^0 + 0.44F_{\bar{\nu}_\mu}^0 & \text{(LMA)} \\
F_{\bar{\nu}_e}^0 & \text{(SMA)}
\end{cases}
\tag{3}
\]

where $F_{\bar{\nu}_e}^0$ means neutrino flux for no oscillation model. This equation shows that in LMA models the $\bar{\nu}_e$ flux at higher energy region is enhanced, on the other hand that at lower energy region is suppressed, because the original mean $\bar{\nu}_\mu$’s energy is higher than that of $\bar{\nu}_e$’s as mentioned in Section 1.

However, in the case of the inverted hierarchy, this situation changes dramatically although we do not give calculation for this case. In this case, the higher resonance point is in the anti-neutrino sector [24], and hence resonant conversion of $\bar{\nu}_e$’s and $\bar{\nu}_{\mu,\tau}$’s is possible. The conversion probability would be dependent on the neutrino energy. Another possibility of such resonant conversion effect is the spin-flip oscillation of $\bar{\nu}_e$’s by a flavor-changing neutrino magnetic moment [25].
2.3 Supernova Rate

A number of studies have modeled the expected evolution of the cosmic SFR with redshift. The cosmic SFR can now be traced to \( z \approx 4 \) observationally, although some details remain controversial. We use three SFRs per unit comoving volume which are also used in Ref. [26], where all of them are derived following the same method employed by Madau et al. [18] first. They use the intergalactic absorption to identify high redshift galaxies in broadband multicolor survey, and convert the UV luminosity to SFR and the metal ejection rate assuming a typical initial mass function (IMF) of stars. The supernova rate is expected to be proportional to SFR since the lifetime of progenitors of core-collapse supernovae is much shorter than the cosmological time scale. We show in Fig. 2 the supernova rates corresponding to these three SFR models that are explained below.

In the Einstein-de Sitter (EdS) universe \((\Omega_m = 1.0, \Omega_\lambda = 0.0)\), the first (hereafter SF1) is taken from Ref. [27]:

\[
R_{SF1}(z) = 0.3h_{65} \frac{\exp(3.4z)}{\exp(3.8z) + 45} M_{\odot} \text{ yr}^{-1} \text{Mpc}^{-3},
\]  

(4)

where \( h_{65} = H_0/65 \text{ km s}^{-1} \text{ Mpc}^{-1} \). This SFR increases rapidly between \( z = 0 \) and \( z = 1 \), peaks between \( z = 1 \) and \( z = 2 \), and gently declines at higher redshifts. This includes an upward correction for dust reddening of \( A_{1500} = 1.2 \) mag. The original SFR (before correction) should be interpreted as lower limits to the real values, since a significant fraction of UV light from young stars could be absorbed and re-emitted in far-infrared band. In the second model, the SFR remains instead roughly constant at \( z \gtrsim 2 \) (SF2) [19], as:

\[
R_{SF2}(z) = 0.15h_{65} \frac{\exp(3.4z)}{\exp(3.4z) + 22} M_{\odot} \text{ yr}^{-1} \text{Mpc}^{-3}.
\]  

(5)

We should consider SF2 in addition to SF1, because of the uncertainties associated with the incompleteness of the data sets and the amount of dust extinction at early epochs. The third SFR (SF3) [26],

\[
R_{SF3}(z) = 0.2h_{65} \frac{\exp(3.05z - 0.4)}{\exp(2.93z) + 15} M_{\odot} \text{ yr}^{-1} \text{Mpc}^{-3}
\]  

(6)

represents even more star formation at early epochs. This SFR is based on the studies which suggest that the evolution of the SFR up to \( z \approx 1 \) may have been overestimated [28], while the rates at high-z may have been severely underestimated due to large amounts of dust extinction [29].
These SFR evolutions should change when a different cosmological model is assumed, since they are based on the observed data of high-z galaxies. The correction to other cosmological models can be written as (e.g., [30]):

$$R_{SF}(z; \Omega_m, \Omega_\Lambda, h_{65}) = h_{65}\sqrt{(1 + \Omega_m z)(1 + z)^2 - \Omega_\Lambda(2z + z^2)}$$

$$\times R_{SF}(z; 1, 0, 1). \quad (7)$$

To obtain the supernova rate ($R_{SN}$), we multiply the SFRs by the coefficient

$$\int_{125}^{8} dm \phi(m) \int_{125}^{0.01} dmm \phi(m) = 0.0122 M_{\odot}^{-1}, \quad (8)$$

where $\phi(m)$ is the Salpeter IMF ($\phi(m) \propto m^{-2.35}$) and $m$ is the stellar mass in solar units [26]. (Here, we assume that all stars whose mass is greater than $8M_{\odot}$ explode as core-collapse supernovae.) This resulting rates agree with the local observed value. We label these models “SN1”, “SN2”, and “SN3”, respectively.

### 3 Formulation of Flux Calculations

In section 2, we calculated the number of emitted $\bar{\nu}_e$'s from a supernova per unit energy $q$, $dN_{\bar{\nu}}/dq$, and supernova rate, $R_{SN}$. We remark that $R_{SN}(z)$ is supernova rate per comoving volume, and hence we should multiply the factor $(1 + z)^3$ to obtain the rate per physical volume at that time. The present number density of $\bar{\nu}_e$'s, whose energy is in the interval of the $q \sim q + dq$, emitted in the interval of the redshift $z \sim z + dz$ is given by

$$dn_{\bar{\nu}}(q) = R_{SN}(z)(1 + z)^3 \frac{dt}{dz} dN_{\bar{\nu}}(1 + z)q(1 + z)dq(1 + z)^{-3}$$

$$= R_{SN}(z) \frac{dt}{dz} dN_{\bar{\nu}}(1 + z)q(1 + z)dq, \quad (9)$$

$$= R_{SN}(z) \frac{dt}{dz} dN_{\bar{\nu}}(1 + z)q(1 + z)dq, \quad (10)$$

where the factor $(1 + z)^{-3}$ comes from the expansion of the universe. The Friedmann equation gives the relation between $t$ and $z$ as follows:

$$\frac{dz}{dt} = -H_0(1 + z)\sqrt{(1 + \Omega_m z)(1 + z)^2 - \Omega_\Lambda(2z + z^2)}. \quad (11)$$
We now obtain the differential number flux of SRN, $dF_\nu(q)/dq$, using the relation $dF_\nu(q)/dq = c[d\nu(q)/dq]$:

$$\frac{dF_\nu}{dq} = \frac{c}{H_0} \int_0^{z_{\text{max}}} R_{\text{SN}}(z) \frac{dN_\nu((1+z)q)}{dq} \times \frac{dz}{\sqrt{(1 + \Omega_m z)(1 + z)^2 - \Omega_\Lambda(2z + z^2)}} \quad (12)$$

where we assume that gravitational collapses began at the redshift parameter $z_{\text{max}} = 5$.

It can be seen that, from equations (7) and (12), the flux does not depend on the cosmological parameters such as $\Omega_m$, $\Omega_\Lambda$, and $H_0$, while in Ref. [8], the flux depend on these parameters. The illustration of this difference is as follows: Totani et al. used supernova rate evolution derived from their theoretical model of galaxy evolution which reproduces various properties of present-day galaxies. On the other hand, our supernova rate is based on the observational estimate of luminosity densities of high-$z$ galaxies, and the dependence of the cosmological volume element on the parameters such as $\Omega_m$ and $\Omega_\Lambda$ is cancelled out. Therefore, our supernova rate depends on the cosmological parameters while the neutrino flux does not depend on them.

4 Results

4.1 Calculation of SRN Flux

We calculate the flux of the SRN using the formula eq. (12) for various models. In Fig. 3, the flux for the three supernova rate models are shown assuming no oscillation model. The fluxes of these models are almost the same above $\sim 8$ MeV and the models with more SFR at early epochs have a higher peak at lower neutrino energy. These properties come from the following effects: The energy of neutrinos which were emitted at redshift $z$ is reduced by a factor $(1+z)^{-1}$ when we observe. Then at observation, the high energy tail ($\gtrsim 8$ MeV) is mainly contributed by low redshift supernovae. Since at low redshift three supernova rate models are almost the same, the fluxes at $E_{\nu_e} \gtrsim 8$ MeV are not much different. Similarly, because high redshift supernova neutrinos contribute more at low energy region when we observe, we can see model dependence at low energy clearly.

In Fig. 4, we show the flux of various neutrino oscillation models assuming SN1 model. In SMA and no oscillation models we can see higher peak around
\[ \sim 5 \text{ MeV} \text{ than in LMA models, because in LMA models the low energy } \bar{\nu}_e \text{'s are deficit due to the conversion into } \bar{\nu}_{\mu,\tau} \text{'s. But above } \sim 10 \text{ MeV, in LMA models we can see more flux, because } \bar{\nu}_{\mu,\tau} \text{'s which have higher mean energy at production have more changed into } \bar{\nu}_e \text{'s than in the case of SMA or no oscillation. (See also qualitative explanation in Section 2.2.)} \]

Integrated flux over the entire neutrino energy range is shown in Table 2.

### 4.2 Event Rate at SuperKamiokande Detector

The SK detector is a water Čerenkov detector whose fiducial mass is 22,500 ton. The detector efficiency is 100% for electrons (positrons) whose energy is above 5 MeV, and 50% at 4.2 MeV. We only consider the reaction \( \bar{\nu}_e + p \rightarrow e^+ + n \), because the cross section \( [9.52 \times 10^{-44} (E_e/\text{MeV}) (p_e/\text{MeV}) \text{cm}^2] [31] \) is much larger than the other reactions.

In Fig. 5, the event rate at SK of the three supernova rate models are shown assuming no oscillation model. Because of the detector threshold, we see only the positrons whose energies are above \( \sim 5 \text{ MeV} \), so that the differences between models are small as shown also in Fig. 3 where the model dependence is also small for neutrinos above \( \sim 7 \text{ MeV} \). Then, although our three supernova rate models have very different properties at high redshift regions, our results are not influenced by the behavior of SFRs at those high \( z \) regions. (These results are hardly changed when we assume the other LMA or SMA models, instead of the no oscillation model.)

In Fig. 6, we show the event rate of various neutrino oscillation models assuming SN1 model. We can see clear difference of the LMA model from SMA or no oscillation models, especially at the high energy tail. This property results from the flux dependence on oscillation models (see also Fig. 4).

As a result, if we can detect SRN events above \( \sim 10 \text{ MeV} \), we can discriminate the LMA model from the SMA or no oscillation models in any supernova rate models and any sets of cosmological parameters.

### 5 Discussion

#### 5.1 Background Events against the Detection

We discuss in this subsection about neutrinos from other sources which may become an obstacle to the SRN detection. They are atmospheric and solar
neutrinos, anti-neutrinos from nuclear reactors, and decay electrons from invisible muons. We show in Fig. 7 the number flux of SRN and these background events, and we discuss the each background event below.

The flux of the atmospheric neutrinos is usually calculated using Monte Carlo method including various relevant effects (flux of primary cosmic rays, solar modulation, geomagnetic field, interaction of cosmic rays in the air, and so on), and in that simulation one-dimensional approximation is used, i.e., after the interaction of primary cosmic ray particles with air nuclei, all the particles are assumed to be moving along the line of the momentum vector of the primary cosmic ray particles [32]. (Recently preliminary results of three-dimensional flux calculations have been reported [33].) There are many authors who calculated the atmospheric neutrino flux (see, e.g., [34] for a recent result). We use in this paper the flux calculated by Gaisser et al. [35] and Barr et al. [36]. Although their calculations are rather old, the fluxes of low energy neutrinos, in which we are interested, are also calculated, while most of other papers show only higher energy region (> 1 GeV).

Solar neutrino flux is dominant at energy range 10–20 MeV. We use the flux predicted by the standard solar model (SSM) in Fig. 7 [37]. Since the solar neutrinos are not $\bar{\nu}_e$’s but $\nu_e$’s, the cross section for solar neutrinos is about two order smaller than that for $\bar{\nu}_e$’s. Furthermore recoil electrons scattered by solar neutrinos strongly concentrate to the opposite direction of the Sun, in contrast to the isotropic distribution of $\bar{\nu}_e$ events. Therefore the solar neutrino is an avoidable background. In fact, we can demonstrate how we can avoid the solar neutrino background, as follows. The solar neutrino event number is about $5 \times 10^3$ yr$^{-1}(22.5\text{kton})^{-1}$ [12], about 3 or 4 orders of magnitude larger than our expected event rate $\sim 1$ yr$^{-1}(22.5\text{kton})^{-1}$. The recoil electrons scatter obeying the Gaussian of one-sigma error $\sigma \sim 25^\circ$ for 10 MeV electrons [38]. Then, 99.98% of solar neutrino events within $\sim 3.7\sigma$ (corresponding to $\sim 90^\circ$) can be avoided. Therefore, restricting our discussion on the nearby side of the hemisphere from the Sun, we can ignore the solar neutrino events.

The third background which we must consider is anti-neutrinos from nuclear reactors. In each nuclear reactor, almost all the power comes from the fissions of the four isotopes, $^{235}\text{U}$ ($\sim 75\%$), $^{238}\text{U}$ ($\sim 7\%$), $^{239}\text{Pu}$ ($\sim 15\%$), and $^{241}\text{Pu}$ ($\sim 3\%$) [39]. Each isotope produces a unique electron anti-neutrino spectrum through the decay of its fission fragments and their daughters. The $\bar{\nu}_e$ spectrum from $^{235}\text{U}$, $^{239}\text{Pu}$, and $^{241}\text{Pu}$ can be derived using the semi-empirical formula with which we fit data of detected $\beta$-spectrum from fission by thermal neutrons [40]. ($^{238}\text{U}$ undergoes only fast neutron fission and hence electron spectrum from $^{238}\text{U}$ cannot be measured by this kind of experiment.) Above 7 MeV, the number of $\beta$ counts drops dramatically and fitting error becomes large. In addition, with this method, as we determine the maximum $\beta$ energy and derives the energy distribution below that energy, it is difficult to estimate
the errors on the high energy range [41]. While the $\bar{\nu}_e$ spectra in Ref. [40] are given as tables, we use for simplicity somewhat less accurate analytical approximation given in Ref. [42]. As a normalization factor we use energy-integrated $\bar{\nu}_e$ flux at Kamioka, $1.34 \times 10^6$ cm$^{-2}$s$^{-1}$, which are the summation of the flux from various nuclear reactors in Japan and Korea [39]. This fit is not valid above $\sim 8$ MeV. However, there is an estimation that we would get few (10 or less) events per year above 10 MeV from reactors[41].

With three backgrounds we discussed above, we expect the energy window of SRN events opening from 10 MeV to 27 MeV. However, according to Kaplighat et al. [11] electrons or positrons from invisible muons are the largest background in the energy window from 19 to 35 MeV. This invisible muon event is illustrated as follows. The atmospheric neutrinos produce muons by interaction with the nucleons (both free and bound) in the fiducial volume. If these muons are produced with energies below Čerenkov radiation threshold (kinetic energy less than 53 MeV), then they will not be detected (“invisible muons”), but their decay-produced electrons and positrons will be. Since the muon decay signal will mimic the $\bar{\nu}_e p \to n e^+$ process in SK, it is difficult to distinguish SRN from these events. The energy spectrum of this invisible muon events is obtained by the stopped muon decay spectrum

$$\frac{dN}{dE_e} = \frac{G_F^2}{12\pi^3} m_\mu^2 E_e^2 \left( 3 - \frac{4E_e}{m_\mu} \right),$$

(Michel spectrum [43]), where $G_F$ is the Fermi constant, $m_\mu$ the muon mass, and $E_e$ the electron (positron) energy. This equation is valid for $E_e < m_\mu/2$.

From the observation of Kamiokande II detector the estimated event rate from these muon decays is around unity for 0.58 kton yr exposure and this is forming the principal source of background after the various cuts had been implemented [44]. Kaplinghat et al. [11] used this value corrected for SK by multiplying the volume ratio and concluded that it is impossible to detect SRN unless their upper limit realizes and maximum neutrino oscillation occurs. This event rate is now inferred directly from the new data of the SK, which is about 100 per 1,258 days per 22.5 kton fiducial volume above 18 MeV [41,45]. This value corresponds to about 20% of the Kamiokande II data. In Fig. 8 we show SRN event rate compared to the invisible muon events. From this figure we conclude that SRN events can be seen only below about 12 MeV.

In practice, there is another serious background, i.e., spallation products induced by cosmic ray muons. Ultra high energy cosmic ray muons spall oxygens in the detector, and radioactive decay processes of these spalled nuclei occur. The event rate of the spallation background is several hundred per day per 22.5 kton. Although most of them can be rejected by the information of preceding muons, even a small fraction can not be. (Roughly, this spallation products produces about 200,000 events per a year. Because expected SRN event rate is
less than 1 per a year, we should reject all these 200,000 events. For future detectors this problem is also quite difficult to solve.) Then, this makes a serious background at the energy range below the maximum energy of beta spectrum of spallation products, 16 MeV [45,46]. From these discussions presented in this subsection we conclude that there is no energy window of SRN.

5.2 Calculating the detectable event rate at SK

In the previous subsection we have found that there is no energy region where SRN is dominant. However, we can detect the SRN events by subtracting the other background events from total detected events. We consider the energy range $17 < (T_e/\text{MeV}) < 25$, where $T_e$ is positron kinetic energy. This range corresponds to $19 < (E_{\bar{\nu}_e}/\text{MeV}) < 27$ by the simple relation, $E_{\bar{\nu}_e} = T_e + 1.8\text{MeV}$. We find two advantages in using this energy region. First, SRN event rate is rather large, and second, the background (invisible muon) event rate is fairly well known by SK observation. We show in Table 3 the SRN event rate at SK in this energy range, i.e., $0.4 - 0.8 \text{ yr}^{-1}$. In contrast, the event rate of the invisible muon over the same energy range is $3.4 \text{ yr}^{-1}$. When SRN event rate is larger than the statistical error of background event rate, we can conclude that the SRN is detectable as a distortion of the expected invisible muon background event. Unfortunately, only one year observation does not provide any useful information about SRN. However, we can expect that ten-year observation provides several statistically meaningful results. The statistical error of invisible muon events in ten years is $\sqrt{34} = 5.8$, which is smaller than the event rate of LMA models and is larger than that of SMA and no oscillation models. Then we conclude that these neutrino oscillation models can be distinguished by the observation of the event rate of invisible muon events. (If there is a discrepancy from expected event rate, this is due to SRN events and LMA models are favored.)

In future, it is expected that next generation of water Čerenkov detectors have much larger volume than that of SK. For example HyperKamiokande project is now under consideration. HyperKamiokande detector is pland to be a water Čerenkov detector whose mass is about 1,000,000 tons (about 20 times larger than SK), and its location is near SK detector. We expect that the SRN event becomes about 10 per one year for this detector, and statistically sufficient discussion of SRN is possible even using only one year data.

5.3 SNO and KamLAND detectors

In this subsection we discuss SRN detectability with SNO [47] and KamLAND [48]. An advantage of these detectors is that we are able to identify $\bar{\nu}_e$ events
using delayed coincidence signals. For this reason, we can remove other backgrounds from non-$\bar{\nu}_e$ origin (solar neutrinos, invisible muon decay products, and spallation products).

At SNO detector, used reaction for $\bar{\nu}_e$ detection is

$$\bar{\nu}_e + d \longrightarrow e^+ + n + n. \quad (14)$$

Then these neutrons react with surrounding nuclei through

$$n + d \longrightarrow ^3\text{H} + \gamma \ (E_\gamma = 6.3\text{MeV}, \text{efficiency : } 24\%), \quad (15)$$

$$n + ^{35}\text{Cl} \longrightarrow ^{36}\text{Cl} + \gamma \ (E_\gamma = 8.6\text{MeV}, \text{efficiency : } 83\%), \quad (16)$$

where NaCl is added to efficiently capture neutrons. This delayed signal with the preceding Čerenkov radiation from $e^+$ in eq. (14) shows that the detected neutrino is $\bar{\nu}_e$. Using this criterion we can reject backgrounds from non-$\bar{\nu}_e$ origin. These are solar neutrinos, invisible muon decay products, and spallation products, which are great obstacle for SRN detection at SK. Thus, we can find the energy window below $T_e \simeq 23\text{MeV}$. Actual calculation shows that the expected event rate at SNO is $\sim 0.03 \text{ yr}^{-1}$. (We used cross section calculated in Ref. [49].)

We can use a similar criterion at KamLAND. $\bar{\nu}_e$’s are detected through below reactions:

$$\bar{\nu}_e + p \longrightarrow e^+ + n, \quad (17)$$

$$n + p \longrightarrow d + \gamma \ (E_\gamma = 2.2\text{MeV}). \quad (18)$$

The energy window for SRN is from 10 MeV to 25 MeV. (Below 10 MeV reactor $\bar{\nu}_e$’s are large.) In that range the calculated event rate is $\sim 0.1 \text{ yr}^{-1}$.

Unfortunately these values (0.03 yr$^{-1}$ for SNO; 0.1 yr$^{-1}$ for KamLAND) are quite small, since the fiducial volume of these detectors (1 kton) is much smaller than that of SK. However, the future same kind of detector of larger volume, if ever built, might detect SRN.

6 Acknowledgments

We would like to thank A. Suzuki, M. Nakahata, and Y. Fukuda for useful discussions and also would like to thank K. Takahashi for preparing the numerical data of the neutrino oscillation and for useful discussions. S. Ando
also would like to thank S. Nagataki for useful discussions. This work was supported in part by Grants-in-Aid for Scientific Research provided by the Ministry of Education, Science and Culture of Japan through Research Grant No.07CE2002.

References

[1] G.S. Bisnovatyi-Kogan, S.F. Seidov, Ann. N.Y. Acad. Sci. 422 (1984) 319.
[2] L.M. Krauss, S.L. Glashow, D.N. Schramm, Nature 310 (1984) 191.
[3] S.E. Woosley, J.R. Wilson, and R. Mayle, Astrophys. J. 302 (1986) 19.
[4] K.S. Hirata (1991) PhD theses, University of Tokyo.
[5] Y. Totsuka, Rep. Prog. Phys. 55 (1992) 377.
[6] H. Suzuki, “Supernova Neutrinos”, in Physics and Astrophysics of Neutrinos edited by M. Fukugita and A. Suzuki, Springer-Verlag (1994) p763.
[7] T. Totani and K. Sato, Astropart. Phys. 3 (1995) 367.
[8] T. Totani, K. Sato, and Y. Yoshii, Astrophys. J. 460 (1996) 303.
[9] R.A. Malavey, Astropart. Phys. 7 (1997) 125.
[10] D.H. Hartmann and S.E. Woosley, Astropart. Phys. 7 (1997) 137.
[11] M. Kaplinghat, G. Steigman and T.P. Walker, Phys. Rev. D 62 (2000) 043001.
[12] S. Fukuda et al., Phys. Rev. Lett. 86 (2001) 5656.
[13] J.N. Bahcall, P.I. Krastev, and A.Yu. Smirnov, Phys. Rev. D 58 (1998) 096016.
[14] Q.R. Ahmad et al., SNO Collaboration, Phys. Rev. Lett. 87 (2001) 071301.
[15] Y. Fukuda et al., Phys. Rev. Lett. 82 (1999) 2644; I. Scholberg, hep-ex/9905016.
[16] J.R. Wilson, R. Mayle, S. Woosley, and T. Weaver, Ann. N.Y. Acad. Sci. 470 (1986) 267.
[17] T. Totani, K. Sato, H.E. Dalhed, and J.R. Wilson, Astrophys. J. 496 (1998) 216.
[18] P. Madau, H.C. Ferguson, M.E. Dickinson, M. Giavalisco, C.C. Steidel, and A. Fruchter, Mon. Not. R. Astron. Soc. 283 (1996) 1388.
[19] C.C. Steidel, K.L. Adelberger, M. Giavalisco, M. Dickinson, and M. Pettini, Astrophys. J. 519 (1999) 1.
[20] T.M. Shimizu, T. Ebisuzaki, K. Sato, and S. Yamada, Astrophys. J. 552 (2001) 756.
[21] K. Takahashi, M. Watanabe, K. Sato, and T. Totani, Phys. Rev. D 64 (2001) 093004.

[22] T.K. Kuo and J. Pantaleone, Rev. Mod. Phys. 61 (1989) 937.

[23] P.I. Krastev and A.Yu. Smirnov, hep-ph/0108177.

[24] A.S. Dighe and A.Yu. Smirnov, Phys. Rev. D 62 (2000) 033007.

[25] T. Totani and K. Sato, Phys. Rev. D 54 (1996) 5975

[26] C. Porciani and P. Madau, Astrophys. J. 548 (2001) 522.

[27] P. Madau and L. Pozzetti, Mon. Not. R. Astron. Soc. 312 (2000) L9.

[28] L.L. Cowie, A. Songaila, and A.J. Barger, Astron. J. 118 (1999) 603.

[29] A.W. Blain, J-P. Kneib, R.J. Ivison, and I. Smail, Astrophys. J. 512 (1999) L87.

[30] T. Totani, Y. Yoshii, and K. Sato, Astrophys. J. 483 (1997) L75

[31] P. Vogel and J.F. Beacom, Phys. Rev. D 60 (1999) 053003.

[32] T. Kajita and Y. Totsuka, Rev. Mod. Phys. 73 (2001) 85.

[33] G. Battistoni, A. Ferrari, P. Lipari, T. Montaruli, P.R. Sala, and T. Rancati, Astropart. Phys. 12 (2000) 315; Y. Tserkovnyak, R. Komar, C. Nally, and C. Waltham, hep-ph/9907450; P. Lipari, Astropart. Phys. 14 (2000) 153.

[34] P. Lipari, T. Stanev, and T.K. Gaisser, Phys. Rev. D 58 (1998) 073003; V. Agrawal, T.K. Gaisser, P. Lipari, and T. Stanev, Phys. Rev. D 53 (1996) 1314; M. Honda, T. Kajita, K. Kasahara, and S. Midorikawa, Phys. Rev. D 52 (1995) 4985.

[35] T.K. Gaisser, T. Stanev, and G. Barr, Phys. Rev. D 38 (1988) 85.

[36] G. Barr, T.K. Gaisser, and T. Stanev, Phys. Rev. D 39 (1989) 3532.

[37] J.N. Bahcall, Neutrino Astrophysics, Cambridge Univ. Press. (1989).

[38] M. Nakahata et al., Nucl. Instrum. Methods A421 (1999) 113.

[39] C. Bemporad, G. Gratta, and P. Vogel, hep-ph/0107277.

[40] K. Schreckenbach et al., Phys. Lett. B160 (1985) 325; A.A. Hahn et al., Phys. Lett. B218 (1989) 365.

[41] A. Suzuki, private communication.

[42] P. Vogel and J. Engel, Phys. Rev. D 39 (1989) 3378.

[43] L. Michel, Rev. Mod. Phys. 29 (1957) 223.

[44] W. Zhang et al., Phys. Rev. Lett. 61 (1988) 385.

[45] M. Nakahata, private communication.
[46] Y. Fukuda, private communication.

[47] SNO Homepage, http://www.sno.phy.queensu.ca/

[48] KamLAND Homepage, http://www.awa.tohoku.ac.jp/KamLAND/ Proposal for US Participation in KamLAND (1999), http://kamland.lbl.gov/KamLAND.US.Proposal.pdf

[49] S. Ying, W.C. Haxton, and E.M. Henley, Phys. Rev. D 40 (1989) 3211.
Fig. 1. Energy spectrum of $\bar{\nu}_e$ of the numerical supernova model used in this paper. The dashed line is the Fermi-Dirac fits which have the same luminosity with the numerical model. The chemical potential is set to zero for the FD distribution.
Fig. 2. Supernova rate evolution on the cosmological time scale. These lines are for a Λ-dominated cosmology ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$). The Hubble constant is taken to be 70 km s$^{-1}$ Mpc$^{-1}$. 
Fig. 3. Number flux of $\bar{\nu}_e$’s for the three supernova rate models, assuming “no oscillation” case.
Fig. 4. Number flux of $\bar{\nu}_e$'s for the neutrino oscillation models. In this figure “SN1” model is used for supernova rate evolution.
Fig. 5. Event rate of $\bar{\nu}_e$'s at SK for the three supernova rate models, assuming "no oscillation" case.
Fig. 6. Event rate of $\bar{\nu}_e$'s at SK for the neutrino oscillation models. In this figure "SN1" model is used for supernova rate evolution.
Fig. 7. Number flux of SRN compared to other background neutrinos. No oscillation and SN1 model are assumed for SRN flux.
Fig. 8. Event rate at SK detector of SRN and invisible $\mu$ decay products. Two oscillation models are shown (no oscillation and LMA-L) assuming SN1 model for supernova rate evolution.
Table 1
Sets of mixing parameter for calculation

| model   | $\sin^2 2\theta_{12}$ | $\sin^2 2\theta_{23}$ | $\sin^2 2\theta_{13}$ | $\Delta m_{12}^2$ (eV$^2$) | $\Delta m_{13}^2$ (eV$^2$) | $\nu_\odot$ problem |
|---------|------------------------|------------------------|------------------------|-----------------------------|-----------------------------|---------------------|
| LMA-L   | 0.87                   | 1.0                    | 0.043                  | $7.0 \times 10^{-5}$         | $3.2 \times 10^{-3}$         | LMA                 |
| LMA-S   | 0.87                   | 1.0                    | $1.0 \times 10^{-6}$   | $7.0 \times 10^{-5}$         | $3.2 \times 10^{-3}$         | LMA                 |
| SMA-L   | $5.0 \times 10^{-3}$   | 1.0                    | 0.043                  | $6.0 \times 10^{-6}$         | $3.2 \times 10^{-3}$         | SMA                 |
| SMA-S   | $5.0 \times 10^{-3}$   | 1.0                    | $1.0 \times 10^{-6}$   | $6.0 \times 10^{-6}$         | $3.2 \times 10^{-3}$         | SMA                 |

Table 2
Number flux of SRN. Each entry is the number flux in unit cm$^{-1}$ s$^{-1}$ for each supernova rate and oscillation models. These fluxes are integrated over whole energy range.

| models | LMA-L | LMA-S | SMA-L | SMA-S | no oscillation |
|--------|-------|-------|-------|-------|----------------|
| SN1    | 11.2  | 11.3  | 12.3  | 12.2  | 11.9           |
| SN2    | 12.2  | 12.2  | 13.4  | 13.2  | 12.9           |
| SN3    | 13.8  | 13.8  | 15.1  | 14.9  | 14.6           |

Table 3
SRN event rate at SK detector. Each entry is the event rate in unit yr$^{-1}$ for each supernova rate and oscillation models. Integrated energy range is from 17 to 25 MeV.

| models | LMA-L | LMA-S | SMA-L | SMA-S | no oscillation |
|--------|-------|-------|-------|-------|----------------|
| SN1    | 0.73  | 0.72  | 0.46  | 0.45  | 0.44           |
| SN2    | 0.69  | 0.68  | 0.44  | 0.43  | 0.42           |
| SN3    | 0.76  | 0.75  | 0.50  | 0.49  | 0.48           |