Finite and infinite time horizon for BSDE with Poisson jumps.

Ahmadou Bamba Sow\(^{(*)}\)

May 3, 2014

\(^{(*)}\) LERSTAD, UFR de Sciences Appliquées et de Technologie, Université Gaston Berger, BP 234, Saint-Louis, SENEGAL. email : ahmadou-bamba.sow@ugb.edu.sn

Abstract

This paper is devoted to solving a real valued backward stochastic differential equation with jumps where the time horizon may be finite or infinite. Under linear growth generator, we prove existence of a minimal solution. Using a comparison theorem we show existence and uniqueness of solution to such equations when the generator is uniformly continuous and satisfies a weakly monotonic condition.

Keywords : Backward stochastic differential equation, random Poisson measure, Doléans Dade exponential.

AMS Subject Classification: 60H05, 60G44.

1 Introduction

After the pioneer work of Pardoux and Peng [10] on linear Backward stochastic differential equation (BSDE in short) with Lipschitz generator, the interest in such stochastic equations has increased thanks to the many domains of applications including stochastic representation of solutions of partial differential equations (PDEs in short). For example, Pardoux and Peng [11] and Peng [13] proved that BSDEs provide a probabilistic formula for solutions of quasilinear parabolic PDEs.

BSDEs with Poisson Process (BSDEP in short) were first discussed by Tang and Li [15] and Wu [17]. Studying such equations, Barles et al [2] generalized the result in
[11], and obtained a probabilistic interpretation of a solution of a parabolic integral-partial differential equation (PIDE). This was done by means of a real-valued BSDEP with Lipschitzian generator. Since then many efforts have been done in relaxing the Lipschitz assumption of the generator of the BSDEs (see [1, 7, 8, 9] among others) and the BDSEP (see [12, 14, 16, 21]). In [12], the author solved a multidimensional BSDEP and showed an existence result under monotonicity in the second variable of the drift and Lipschitz condition in the other ones. Royer [14] focused in weakening the Lipschitz condition required on the last variable of the generator and improved upon the results given in [2]. The key point is a strict comparison theorem and a representation of solution of the one dimensional BSDEP in terms of non-linear expectation. But all these results are established with a fixed time horizon $T$. A natural question is under which condition on the coefficients the stochastic equation still has a solution given a square integrable terminal value $\xi$? In fact this problem has been investigated by Peng [13] and Darling and Pardoux [4] and others researchers when the terminal value $\xi$ is null or satisfies the integrability condition $E(e^{\lambda T}\xi^2) < \infty$, for some $\lambda > 0$ and random terminal time $T$. Chen and Wang [3] established the first existence and uniqueness of solution to BSDE with infinite time horizon when the generator satisfies a Lipschitz type condition. Recently Fan et al [6] weakened assumptions required in [3] and prove an existence and uniqueness result under mild conditions of the generator with finite or infinite time horizon.

The aim of this paper is to extend the result established in [6] to the case of BSDEP. Our motivation comes from the recent work of Yao [19]. The author proves an existence and uniqueness result of BSDEP with infinite time interval and some monotonicity condition stronger than those in [6]. In this work we show that the results obtained in [6] can be extended to BSDEP. The paper is organized as follows. We first prove existence of a minimal solution in Section 2 and a comparison theorem in Section 3. Thanks to these statements we deal with the solvability of finite or infinite BSDEP in Section 4.

2 BSDE with Poisson Jumps

2.1 Definitions and preliminary results

Let $\Omega$ be a non-empty set, $\mathcal{F}$ a $\sigma$—algebra of sets of $\Omega$ and $P$ a probability measure defined on $\mathcal{F}$. The triplet $(\Omega, \mathcal{F}, P)$ defines a probability space, which is assumed to be complete. We are given two mutually independent processes:

- a $d$—dimensional Brownian motion $(W_t)_{t \geq 0}$,
• a random Poisson measure $\mu$ on $E \times \mathbb{R}_+$ with compensator $\nu(dt, de) = \lambda(de)dt$

where the space $E = \mathbb{R} - \{0\}$ is equipped with its Borel field $\mathcal{E}$ such that $\{\tilde{\mu}([0, t] \times A) = (\mu - \nu)[0, t] \times A\}$ is a martingale for any $A \in \mathcal{E}$ satisfying $\lambda(A) < \infty$. $\lambda$ is a $\sigma$-finite measure on $\mathcal{E}$ and satisfies

$$\int_E (1 \wedge |e|^2) \lambda(de) < \infty.$$ 

We consider the filtration $(\mathcal{F}_t)_{t \geq 0}$ given by $\mathcal{F}_t = \mathcal{F}^W_t \vee \mathcal{F}^\mu_t$, where for any process $\{\eta_t\}_{t \geq 0}$, $\mathcal{F}^\eta_{s,t} = \sigma\{\eta_r - \eta_s, s \leq r \leq t\} \vee \mathcal{N}$, $\mathcal{F}^\eta_t = \mathcal{F}^\eta_{0,t}$. $\mathcal{N}$ denotes the class of $\mathcal{P}$-null sets of $\mathcal{F}$.

For $Q \in \mathbb{N}^*$, $| \cdot |$ stands for the euclidian norm in $\mathbb{R}^Q$.

We consider the following sets (where $E$ denotes the mathematical expectation with respect to the probability measure $P$), and a non-random horizon time $0 < T \leq +\infty$:

• $S^2(\mathbb{R}^Q)$ the space of $\mathcal{F}_t$-adapted càdlàg processes

$$\Psi : [0, T] \times \Omega \rightarrow \mathbb{R}^Q, \|\Psi\|^2_{S^2(\mathbb{R}^Q)} = \mathbb{E}\left(\sup_{0 \leq t \leq T} |\Psi_t|^2\right) < \infty.$$ 

• $H^2(\mathbb{R}^Q)$ the space of $\mathcal{F}_t$-progressively measurable processes

$$\Psi : [0, T] \times \Omega \rightarrow \mathbb{R}^Q, \|\Psi\|^2_{H^2(\mathbb{R}^Q)} = \mathbb{E}\int_0^T |\Psi_t|^2 dt < \infty.$$ 

• $L^2(\tilde{\mu}, \mathbb{R}^Q)$ the space of mappings $U : \Omega \times [0, T] \times E \rightarrow \mathbb{R}^Q$ which are $\mathcal{P} \otimes \mathcal{E}$-measurable s.t.

$$\|U\|^2_{L^2(\mathbb{R}^Q)} = \mathbb{E}\int_0^T \|U_t\|^2_{L^2(E, \mathcal{F}, \lambda, \mathbb{R})} dt < \infty,$$

where $\mathcal{P}$ denotes the $\sigma$-algebra of $\mathcal{F}_t$-predictable sets of $\Omega \times [0, T]$ and

$$\|U_t\|^2_{L^2(E, \mathcal{F}, \lambda, \mathbb{R})} = \int_E |U_t(e)|^2 \lambda(de).$$

We may often write $| \cdot |$ instead of $\| \cdot \|_{L^2(E, \mathcal{F}, \lambda)}$ for a sake of simplicity.

Notice that the space $\mathcal{B}^2(\mathbb{R}^Q) = S^2(\mathbb{R}^Q) \times H^2(\mathbb{R}^Q) \times L^2(\tilde{\mu}, \mathbb{R}^Q)$ endowed with the norm

$$\|(Y, Z, U)\|_{\mathcal{B}^2(\mathbb{R}^Q)} = \|Y\|^2_{S^2(\mathbb{R}^Q)} + \|Z\|^2_{H^2(\mathbb{R}^Q)} + \|U\|^2_{L^2(\mathbb{R}^Q)}$$

is a Banach space.
Finally let $S$ be the set of all non-decreasing continuous function $\varphi(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying $\varphi(0) = 0$ and $\varphi(s) > 0$ for $s > 0$.

Let $f : \Omega \times [0, T] \times \mathbb{R} \times \mathbb{R}^d \times L^2(E, \mathcal{E}, \lambda, \mathbb{R}) \to \mathbb{R}$ be jointly measurable. Given $\xi$ a $\mathcal{F}$–measurable $\mathbb{R}$–valued random variable, we are interested in the BSDEP with parameters $(\xi, f, T)$:

$$Y_t = \xi + \int_t^T f(r, \Theta_r) \, dr - \int_t^T Z_r dW_r - \int_t^T \int_E U_r(e) \tilde{\mu}(dr, de), \quad 0 \leq t \leq T,$$

where $\Theta_r$ stands for the triple $(Y_r, Z_r, U_r)$.

For instance let us precise the notion of solution to (2.1).

**Definition 2.1.** A triplet of processes $(Y_t, Z_t, U_t)_{0 \leq t \leq T}$ is called a solution to eq. (2.1), if $(Y_t, Z_t, U_t) \in \mathcal{B}^2(\mathbb{R})$ and satisfies eq. (2.1).

First we state some results in the case of Lipschitz type conditions of the generator. Suppose that assumption (A) holds (where $0 < T \leq \infty$):

(A1) : For all $(y, z, u) \in \mathbb{R} \times \mathbb{R}^d \times \mathbb{R}$, $f(\cdot , y, z, u)$ is a progressively measurable process and satisfies $E \left[ \left( \int_0^T |f(r, 0, 0, 0)| \, dr \right)^2 \right] < \infty$.

(A2) : There exist two non-random functions $\gamma(\cdot), \rho(\cdot), : [0, T] \to \mathbb{R}^+$ such that for $0 \leq t \leq T$ and $(y, y') \in \mathbb{R}^2$, $(z, z') \in (\mathbb{R}^d)^2$ and $u \in L^2(E, \mathcal{E}, \lambda, \mathbb{R})$,

$$|f(t, y, z, u) - f(t, y', z', u)| \leq \gamma(t)|y - y'| + \rho(t)|z - z'|.$$

(A3) : There exists $-1 < c \leq 0$ and $C > 0$, a deterministic function $\sigma(\cdot) : [0, T] \to \mathbb{R}_+$ and $\beta : \Omega \times [0, T] \times \mathcal{E} \to \mathbb{R}$, $\mathcal{P} \otimes \mathcal{E}$–measurable satisfying $c(1 \wedge |e|) \leq \beta(t) \leq C(1 \wedge |e|)$ such that for all $y \in \mathbb{R}$, $z \in \mathbb{R}^d$ and $u, u' \in (L^2(E, \mathcal{E}, \lambda, \mathbb{R}))^2$,

$$f(t, y, z, u) - f(t, y, z, u') \leq \sigma(t) \int_E (u(e) - u'(e)) \beta_t(e) \lambda(de).$$

(A4) : The integrability condition holds : $\int_0^T (\gamma(s) + \rho^2(s) + \sigma^2(s))ds < \infty$.

**Remark 2.2.** Let us mention that (A3) implies that $f$ is $\sigma(t)$–Lipschitz in $u$ since we have (where $\tilde{c}$ is a universal positive constant)

$$|f(t, y, z, u) - f(t, y, z, u')| \leq \tilde{c} \sigma(t) \int_E |u(e) - u'(e)| (1 \wedge |e|) \lambda(de) \leq \tilde{c} \sigma(t) \left( \int_E |u(e) - u'(e)|^2 \lambda(de) \right)^{1/2} \leq \tilde{c} \sigma(t) \|(u - u')\|_{L^2(E, \mathcal{E}, \lambda, \mathbb{R})}$$
We have the following result which is a consequence of Lemma 2.2 in [21].

**Lemma 2.3.** Let $\xi \in L^2(\Omega, \mathcal{F}, \mathbb{P})$ and $0 < T \leq \infty$. If (A) holds then eq. (2.1) with parameters $(\xi, f, T)$ has a unique solution $(Y_t, Z_t, U_t)_{0 \leq t \leq T}$.

The proof of our main result need a comparison theorem in infinite time horizon. Given two parameters $(\xi^1, f^1, T)$ and $(\xi^2, f^2, T)$, we consider the BSDEPs, $i = 1, 2$,

$$Y^i_t = \xi^i + \int_t^T f^i(r, \Theta^i_r) \, dr - \int_t^T Z^i_r dW_r - \int_t^T \int_E U^i_r(e) \tilde{\mu}(dr, de), \quad 0 \leq t \leq T,$$

where for $i = 1, 2$, $\Theta^i$ stands for the triple $(Y^i, Z^i, U^i)$.

Assume in addition that

(A5) : $\xi^1 \leq \xi^2$ and $\forall (\omega, t, y, z, u)$, $f^1(\omega, t, y, z, u) \leq f^2(\omega, t, y, z, u)$.

We have the following result which is proved in [14] in the case $T < +\infty$ (see Theorem 2.5). The proof when $T = +\infty$ is given in Section 5.

**Theorem 2.4.** Suppose that $f^1$ and $f^2$ satisfy (A1)-(A5) and $0 < T \leq +\infty$. If $(Y^i_r, Z^i_r, U^i_r), \ i = 1, 2$ are solutions to (2.3), then we have

$$Y^1_t \leq Y^2_t, \quad \mathbb{P} - a.s.$$

**Remark 2.5.** Theorem 2.4 established a comparison theorem in the case of Lipschitz coefficients for either $T < \infty$ or $T = +\infty$. Basically it improves the well known result in the finite time horizon.

Let us now deal with our problem.

### 2.2 Existence of a minimal solution

In this section, we will prove existence of a minimal solution for BSDEPs when their generators are continuous and have a linear growth (see Theorem 2.8 below). First let us give the

**Definition 2.6.** A solution $(Y_t, Z_t, U_t)_{0 \leq t \leq T}$ of eq. (2.1) is called a minimal solution if for any other solution $(\tilde{Y}_t, \tilde{Z}_t, \tilde{U}_t)_{0 \leq t \leq T}$ to (2.1) we have for each $0 \leq t \leq T$, $\ Y_t \leq \tilde{Y}_t$.

We introduce the following list of conditions weaker than those required in [2, 14, 19, 21].

---

5
We assume that $0 \leq T \leq +\infty$ and the generator $f$ satisfies assumptions \((H1)\):

\begin{enumerate}[\textbf{(H1.1)}]
\item There exist three functions $\gamma(\cdot)$, $\rho(\cdot)$, $\sigma(\cdot) : [0, T] \to \mathbb{R}^+$ satisfying \((A4)\).
\item There exists a $\mathcal{F}_t-$progressively measurable nonnegative process $(f_t)_{0 \leq t \leq T}$ s.t.
\end{enumerate}

\[ E \left[ \left( \int_0^T f_t dt \right)^2 \right] < \infty \]

and for $(t, y, z, u) \in [0, T] \times \mathbb{R} \times \mathbb{R}^d \times L^2(E, \mathcal{E}, \lambda, \mathbb{R})$,

\[ |f(t, \omega, y, z, u)| \leq f_t(\omega) + \gamma(t)|y| + \rho(t)|z| + \sigma(t)|u|. \]

\begin{enumerate}[\textbf{(H1.3)}]
\item $f(\omega, t, \cdot, \cdot, \cdot) : \mathbb{R} \times \mathbb{R}^d \times L^2(E, \mathcal{E}, \lambda, \mathbb{R}) \to \mathbb{R}$ is continuous.
\end{enumerate}

As in [8], we are led to consider the sequence $f_n : \Omega \times \mathbb{R} \times \mathbb{R}^d \times L^2(E, \mathcal{E}, \lambda, \mathbb{R}) \to \mathbb{R}$ associated to $f$ defined by $\forall (t, \omega, y, z, u) \in \Omega \times \mathbb{R} \times \mathbb{R}^d \times L^2(E, \mathcal{E}, \lambda, \mathbb{R})$

\[ f_n(t, \omega, y, z, u) = \inf_{(y', z', u') \in \mathbb{R}^d \times L^2(E, \mathcal{E}, \lambda, \mathbb{R})} [f(t, \omega, y', z', u') + n(|y - y'| + |z - z'| + |u - u'|)]. \]

Using similar computations as in proof of Lemma 1 in [8], one can obtain the following proposition. We omit its proof.

\textbf{Proposition 2.7.} Assume that $f$ satisfies \((H1)\). Then the sequence of functions $f_n$ is well defined for each $n \geq 1$, and it satisfies, $d\mathbb{P} \times dt$-a.s.

\begin{enumerate}[\textbf{(i)}]
\item Linear growth: $\forall n \geq 1, \forall y, z, u$, $|f_n(\omega, t, y, z, u)| \leq f_t(\omega) + \gamma(t)|y| + \rho(t)|z| + \sigma(t)|u|$. \\
\item Monotonicity in $n$: $\forall y, z, u$, $f_n(\omega, t, y, z, u)$ increases in $n$. \\
\item Convergence: $\forall (\omega, t, y, z, u) : \Omega \times [0, T] \times \mathbb{R} \times \mathbb{R}^d \times L^2(E, \mathcal{E}, \lambda, \mathbb{R})$,
\end{enumerate}

\[ f_n(\omega, t, y, z, u) \xrightarrow{n \to +\infty} f(\omega, t, y, z, u). \] 

\begin{enumerate}[\textbf{(iv)}]
\item Lipschitz condition: $\forall n \geq 1, \forall y, y', z, z', u, u'$, we have
\end{enumerate}

\[ |f_n(\omega, t, y, z, u) - f_n(\omega, t, y', z', u')| \leq n\gamma(t)|y - y'| + n\rho(t)|z - z'| + n\sigma(t)|u - u'|. \]

Thus by Lemma 2.3, the BSDEP with parameters $(\xi, f_n, T)$:

\[ Y^n_t = \xi + \int_t^T f_n(r, \Theta^n_r) dr - \int_t^T Z^n_r dW_r - \int_t^T \int_E U^n_r(e) \tilde{\mu}(dr, de), 0 \leq t \leq T, \] 

has a unique solution $(\Theta^n_t)_{0 \leq t \leq T} = (Y^n_t, Z^n_t, U^n_t)_{0 \leq t \leq T}$.

The Main result in this section is the following
Theorem 2.8. Let $\xi \in L^2(\Omega, \mathcal{F}, P)$ and $0 < T \leq \infty$. Under assumption (H1), the BSDEP (2.1) has a minimal solution $(Y_t, Z_t, U_t)_{0 \leq t \leq T}$.

Proof. We follow the proof of Theorem 1 in [6]. Consider $F : \Omega \times [0, T] \times \mathbb{R} \times \mathbb{R}^d \times L^2(E, \mathcal{E}, \lambda, \mathbb{R}) \to \mathbb{R}$ given by

$$
\forall (\omega, t, y, z, u), \quad F(\omega, t, y, z, u) = f_t(\omega) + \gamma(t)|y| + \rho(t)|z| + \sigma(t)|u|.
$$

Applying H"older’s inequality in the two last integrals, we obtain

$$
L \leq \mathbb{E}[\sup_{0 \leq s \leq T} |Y_s(\omega)|^2] \leq \mathbb{E}(G^2) < \infty.
$$

Itô’s formula applied to eq. (2.5), yields $(0 \leq t \leq T)$

$$
\mathbb{E}|Y_t^n|^2 + \mathbb{E} \int_0^T |Z_r^n|^2 dr + \mathbb{E} \int_0^T \int_E |U_r^n(e)|^2 \lambda(de) dr = \mathbb{E}|\xi|^2 + 2 \mathbb{E} \int_0^T Y_r^n f_n(r, \Theta^n_r) dr
$$

$$
\leq \mathbb{E}|\xi|^2 + 2 \mathbb{E} \int_0^T |Y_r^n| (f_r + \gamma(r)|Y_r^n| + \rho(r)|Z_r^n| + \sigma(r)|U_r^n|) dr
$$

Using the inequality $2ab \leq a^2/\varepsilon + (b^2/\varepsilon)$ for every $a \geq 0, b \geq 0$ and $\varepsilon > 0$, we deduce that

$$
\mathbb{E} \int_0^T |Z_r^n|^2 dr + \mathbb{E} \int_0^T \int_E |U_r^n(e)|^2 \lambda(de) dr \leq \mathbb{E}|\xi|^2 + (1 + \delta + \delta') \mathbb{E}(G^2)
$$

$$
+ \mathbb{E} \left[ \left( \int_0^T f_r dr \right)^2 \right] + 2 \mathbb{E}(G^2) \cdot \int_0^T \gamma(r) dr
$$

$$
+ \frac{1}{\delta} \mathbb{E} \left[ \left( \int_0^T \rho(r)|Z_r^n| dr \right)^2 \right] + \frac{1}{\delta'} \mathbb{E} \left[ \left( \int_0^T \sigma(r)|U_r^n(e)| \lambda(de) dr \right)^2 \right].
$$

Applying Hölder’s inequality in the two last integrals, we obtain

$$
\mathbb{E} \left[ \int_0^T |Z_r^n|^2 dr + \int_0^T \int_E |U_r^n(e)|^2 \lambda(de) dr \right] \leq M + \frac{1}{2} \mathbb{E} \left[ \int_0^T |Z_r^n|^2 dr + \int_0^T \int_E |U_r^n(e)|^2 \lambda(de) dr \right]
$$

$$
+ \frac{1}{\delta} \mathbb{E} \left[ \left( \int_0^T \rho(r)|Z_r^n| dr \right)^2 \right] + \frac{1}{\delta'} \mathbb{E} \left[ \left( \int_0^T \sigma(r)|U_r^n(e)| \lambda(de) dr \right)^2 \right].
$$

7
where
\[ M = E|\xi|^2 + (1 + \delta + \delta')E(G^2) + E \left[ \left( \int_0^T f_r dr \right)^2 \right] + 2E(G^2) \cdot \int_0^T \gamma(r) dr > 0 \]
and depend only on the parameters \( f, \xi \) and \( T \). Consequently we have
\[ \sup_{n \in \mathbb{N}} E \int_0^T |Z_n|^2 dr \leq 2M \quad \text{and} \quad \sup_{n \in \mathbb{N}} E \int_0^T \int_E |U_n^m(e)|^2 \lambda(de) dr \leq 2M. \]

Let us define for \( W \in \{ Y, Z, U \} \) and integers \( n, m \geq 1 \), \( W^{n,m} = W^n - W^m \).

Applying again Itô’s formula, we deduce from (2.5),
\[ E|Y_{t}^{n,m}|^2 + E \int_t^T |Z_{r}^{n,m}|^2 dr + E \int_t^T \int_E |U_{r}^{n,m}(e)|^2 \lambda(de) dr \]
\[ = 2E \int_t^T Y_{r}^{n,m} (f_n(r, \Theta^n_r) - f_m(r, \Theta^m_r)) dr, \quad 0 \leq t \leq T. \]

Using once again Hölder’s inequality and assumption (H1) we obtain
\[ E|Y_0^{n,m}|^2 + E \int_0^T |Z_{r}^{n,m}|^2 dr + E \int_0^T \int_E |U_{r}^{n,m}(e)|^2 \lambda(de) dr \]
\[ \leq 4E \int_0^T |Y_{r}^{n,m}| f_r dr + 4 \left( E(G^2) \right)^{1/2} \cdot \left( E \left[ \left( \int_0^T |Y_{r}^{n,m}| \gamma(r) dr \right)^2 \right] \right)^{1/2} \]
\[ + 2\sqrt{8M} \cdot \left( E \left[ \int_0^T |Y_{r}^{n,m}|^2 \rho^2(r) dr \right] \right)^{1/2} + 2\sqrt{8M} \cdot \left( E \left[ \int_0^T |Y_{r}^{n,m}|^2 \sigma^2(r) dr \right] \right)^{1/2} \]

In particular Lebesgue’s dominated convergence theorem implies that \( \{ Z^n \} \) (respectively \( \{ U^n \} \)) is a Cauchy sequence in \( H^2(\mathbb{R}^d) \) (respectively \( L^2(\tilde{\mu}, \mathbb{R}) \)). Hence there exists \( (Z, U) \in H^2(\mathbb{R}^d) \times L^2(\tilde{\mu}, \mathbb{R}) \) such that
\[ \| Z^n - Z \|^2_{H^2(\mathbb{R}^d)} \rightarrow 0 \quad \text{and} \quad \| U^n - U \|^2_{L^2(\mathbb{R})} \rightarrow 0, \quad \text{as} \quad n \rightarrow \infty, \]
which implies along a subsequence if necessary
\[ Z^n \xrightarrow{H^2(\mathbb{R}^d)} Z \quad \text{and} \quad U^n \xrightarrow{L^2(\tilde{\mu}, \mathbb{R})} U, \quad \text{as} \quad n \rightarrow \infty. \]
Further by virtue of (2.4), we have $f_n(s, Y^n_s, Z^n_s, U^n_s) \xrightarrow{n \to \infty} f(s, Y_s, Z_s, U_s)$, $0 \leq s \leq T$ and arguing as in [6, Theorem 1], we deduce that
\[
\lim_{n \to \infty} \mathbb{E} \left( \left( \int_0^T |f_n(r, \Theta^n_r) - f(r, \Theta_r)|dr \right)^2 \right) = 0 \quad \text{and} \quad \lim_{n \to \infty} \mathbb{E} \left( \sup_{0 \leq t \leq T} |Y^n_t - Y_t|^2 \right) = 0.
\]

This is enough to deduce that $Y \in S^2(\mathbb{R})$. Letting $n \to +\infty$ in (2.5), we prove that $(Y_s, Z_s, U_s)_{0 \leq s \leq T}$ is solution to (2.1).

Let $(Y', Z', U') \in B^2(\mathbb{R})$ be a solution of eq. (2.1). Thanks to Theorem 2.4, we have
\[
\forall n \geq 1, \quad Y^n \leq Y'.
\]
Letting $n \to \infty$, we get $Y \leq Y'$. This implies that $Y$ is the minimal solution to (2.1). \qed

### 3 Comparison theorem

We intend to prove a comparison theorem under mild conditions on the drift of the BSDEP. This result is useful for the proof of existence and uniqueness of solution.

Let us introduce the following assumptions $(\text{H2})$ on the generator $f$ where $0 < T \leq +\infty$.

- **(H2.1)**: $f$ is weakly monotonic in $y$ i.e. there exists $\gamma(\cdot) : [0, T] \to \mathbb{R}_+$ satisfying $\int_0^T \gamma(t)dt < \infty$ and a function $\varrho \in \mathcal{S}$ s.t. $\int_0^+ \frac{1}{\varrho(r)}dr = +\infty$ and for any $(y, y') \in \mathbb{R}^2$, $z \in \mathbb{R}^d$, $u \in L^2(E, \mathcal{E}, \lambda, \mathbb{R})$,
\[
(y - y')(f(t, y, z, u) - f(t, y', z, u)) \leq |y - y'| \theta(t) \varrho(|y - y'|)
\]

and we assume that $\varrho(x) \leq k(x + 1)$ where $k$ denotes the linear growth constant of $\varrho$.

- **(H2.2)**: $f$ is uniformly continuous in $z$ and there exists $\rho(\cdot) : [0, T] \to \mathbb{R}_+$ satisfying $\int_0^T \rho^2(t)dt < \infty$ and $\phi \in \mathcal{S}$ s.t.
\[
|f(t, y, z, u) - f(t, y, z', u)| \leq \rho(t) \phi(|z - z'|)
\]

and we assume that $\phi(x) \leq ax + b$, $a > 0, b > 0$.

- **(H2.3)**: There exists $-1 < c \leq 0$ and $C > 0$, a deterministic function $\sigma(\cdot) : [0, T] \to \mathbb{R}_+$ satisfying $\int_0^T \sigma^2(s)ds < \infty$ and $\beta : \Omega \times [0, T] \times E \to \mathbb{R}$, $\mathcal{P} \otimes \mathcal{E}$-measurable satisfying $c(1 \wedge |e|) \leq \beta_t \leq C(1 \wedge |e|)$ such that for all $y \in \mathbb{R}$, $z \in \mathbb{R}^d$ and $u, u' \in (L^2(E, \mathcal{E}, \lambda, \mathbb{R}))^2$,
\[
f(t, y, z, u) - f(t, y, z, u') \leq \sigma(t) \int_E (u(e) - u'(e)) \beta_t(e) \lambda(de).
\]
Given two parameters \((\xi^1, f^1)\) and \((\xi^2, f^2)\), we are interested in two one-dimensional BSDEPs (with \(0 \leq t \leq T\))

\[
Y^1_t = \xi^1 + \int_t^T f^1(r, \Theta^1_r) \, dr - \int_t^T Z^1_r \, dW_r - \int_t^T \int_E U^1_r(e) \tilde{\mu}(dr, de), \tag{3.3}
\]

\[
Y^2_t = \xi^2 + \int_t^T f^2(r, \Theta^2_r) \, dr - \int_t^T Z^2_r \, dW_r - \int_t^T \int_E U^2_r(e) \tilde{\mu}(dr, de), \tag{3.4}
\]

and we assume in addition that

\((H2.4): \forall (t, y, z, u), f^1(t, y, z, u) \leq f^2(t, y, z, u)\) and \(\xi^1 \leq \xi^2\).

We state the following result (see [6, Lemma 3]) which will be useful in the sequel

**Lemma 3.1.** Let \(\Psi(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+\) be a nondecreasing function with linear growth which means

\[\exists K > 0 \text{ s.t. for all } x \in \mathbb{R}^+, \quad \Psi(x) \leq K(x + 1).\]

Then for each \(n \geq 2K\) we have, \(\Psi(x) \leq nx + \Psi \left( \frac{2K}{n} \right),\) \(x \geq 0\).

Before proving the main statement of this section, let us recall the Girsanov theorem for discontinuous processes. If \(\mathcal{M}^2\) denotes the set of square integrable martingales, we can define thanks to the martingale representation (see [15, Lemma 2.3]) a mapping \(\Phi : \mathcal{M}^2 \to H^2(\mathbb{R}^d) \times L^2(\tilde{\mu}, \mathbb{R})\)

\[M \mapsto (\theta, \upsilon) \quad \text{such that} \quad M_t = \int_0^t \theta_s \, dW_s + \int_0^t \int_E \upsilon_r(e) \tilde{\mu}(de, dr).\]

Let \(\mathcal{M} = \left\{ (M_t)_{t \geq 0} \in \mathcal{M}^2 |||\theta||| \leq C, \upsilon_s(x) > -1, |\upsilon_s(x)| \leq C(1 \wedge |x|), \text{a.s. with } \Phi(M) = (\theta, \upsilon) \right\}\). For \(M \in \mathcal{M}\), the Doléans-Dade exponential of \(M\) is defined by

\[\mathcal{E}(M)_T = e^{M_T - \frac{1}{2} \langle M, M \rangle_T} \prod_{0 < s \leq T} (1 + \Delta M_s) e^{-\Delta M_s}.\]

We have

**Theorem 3.2** (Girsanov Theorem). Let \((\overline{Z}, \overline{U}) \in H^2(\mathbb{R}^d) \times L^2(\tilde{\mu}, \mathbb{R})\) and \(K_t = \int_0^t \overline{Z}_s \, dW_s + \int_0^t \int_E \overline{U}_r(e) \tilde{\mu}(de, dr)\). If \(M \in \mathcal{M}\) then the process \(\tilde{K} = K - \langle K, M \rangle\) is a martingale under the probability measure \(\tilde{P}\) s.t \(d\tilde{P} / dP = \mathcal{E}(M)_T\).
Here is the main result of this section.

**Theorem 3.3.** Let $0 < T \leq +\infty$. Assume given $f^1$, $f^2$ and $(\xi^1, \xi^2) \in (L^2(\Omega, \mathcal{F}_T, P))^2$ such that (H2) holds. If $(Y_t^1, Z_t^1, U_t^1)_{0 \leq t \leq T}$ and $(Y_t^2, Z_t^2, U_t^2)_{0 \leq t \leq T}$ are solutions of eq. (3.3) and eq. (3.4) respectively, then we have

$$
\forall 0 \leq t \leq T, \quad Y_t^1 \leq Y_t^2, \quad P - a.s.
$$

**Proof.** We assume $d = 1$. Putting

$$
\hat{\Theta}_t = (\hat{Y}_t, \hat{Z}_t, \hat{U}_t) = (Y_t^1 - Y_t^2, Z_t^1 - Z_t^2, U_t^1 - U_t^2), \quad \hat{\xi} = \xi^1 - \xi^2,
$$

then $(\hat{\Theta}_t)_{0 \leq t \leq T}$ satisfies the BSDEP $(0 \leq t \leq T)$

$$
\hat{Y}_t = \hat{\xi} + \int_t^T [f^1 (r, \Theta_r^1) - f^2 (r, \Theta_r^2)] \, dr - \int_t^T \hat{Z}_r \, dW_r - \int_t^T \int_E \hat{U}_r (e) \hat{\mu} (dr, de).
$$

Tanaka-Meyer’s formula yields (where $x^+ = \max(x, 0)$)

$$
\begin{align*}
\hat{Y}_t^+ & \leq \hat{\xi} + \int_t^T \mathbf{1}_{\{\hat{Y}_r^+ > 0\}} [f^1 (r, \Theta_r^1) - f^2 (r, \Theta_r^2)] \, dr - \int_t^T \mathbf{1}_{\{\hat{Y}_r^+ > 0\}} \hat{Z}_r \, dW_r \\
& \quad - \int_t^T \int_E \mathbf{1}_{\{\hat{Y}_r^+ > 0\}} \hat{U}_r (e) \hat{\mu} (de, dr), \quad 0 \leq t \leq T.
\end{align*}
$$

Further we have

$$
f^1 (r, \Theta_r^1) - f^2 (r, \Theta_r^2) = [f^1 (r, \Theta_r^1) - f^1 (r, \Theta_r^2)] + [f^1 (r, \Theta_r^2) - f^2 (r, \Theta_r^2)]
$$

and assumption (H2.4) implies that the right-hand side is less than

$$
[f^1 (r, \Theta_r^1) - f^1 (r, Y_r^2, Z_r^1, U_r^1)] + [f^1 (r, Y_r^2, Z_r^1, U_r^1) - f^1 (r, Y_r^2, Z_r^2, U_r^2)] \\
+ [f^1 (r, Y_r^2, Z_r^2, U_r^1) - f^1 (r, \Theta_r^2)].
$$

Hence applying (H2.1) and (H2.3) we deduce that

$$
\mathbf{1}_{\{\hat{Y}_r^+ > 0\}} [f^1 (r, \Theta_r^1) - f^2 (r, \Theta_r^2)] \leq \gamma (r) \varrho (\hat{Y}_r^+) + \mathbf{1}_{\{\hat{Y}_r^+ > 0\}} \rho (r) \phi (|\hat{Z}_r|)
$$

$$
+ \int_E \mathbf{1}_{\{\hat{Y}_r^+ > 0\}} \hat{U}_r (e) \hat{\beta}_r (e) \lambda (de).
$$

By Lemma 3.1 we have (with $\Psi (\cdot) = \phi (\cdot); K = c = a + b$)

$$
\mathbf{1}_{\{\hat{Y}_r^+ > 0\}} \rho (r) \phi (|\hat{Z}_r|) \leq \mathbf{1}_{\{\hat{Y}_r^+ > 0\}} n \rho (r) |\hat{Z}_r| + \mathbf{1}_{\{\hat{Y}_r^+ > 0\}} \rho (r) \phi \left( \frac{2c}{n} \right), \quad n \geq 2c.
$$
Putting pieces together, we derive from (3.7)

\[
\hat{Y}_t^+ \leq a_n + \int_t^T \gamma(r) g(\hat{Y}_r^+) dr + \tilde{K}_t
\]

(3.8)

where

\[
\tilde{K}_t = \int_t^T \left[ 1_{\{\hat{Y}_r^+ > 0\}} \hat{Z}_r \left( \frac{n \rho(r) \hat{Z}_r}{|\hat{Z}_r|} 1_{\{\hat{Z}_r \neq 0\}} \right) + \int_E 1_{\{\hat{Y}_r^+ > 0\}} \hat{U}_r(e) \beta_r(e) \lambda(de) \right] dr
\]

\[- \int_t^T 1_{\{\hat{Y}_r^+ > 0\}} \hat{Z}_r dW_r - \int_t^T \int_E 1_{\{\hat{Y}_r^+ > 0\}} \hat{U}_r(e) \tilde{\mu}(de, dr)\]

and (where \(b\) is given in (H2.2))

\[
a_n = 1_{b \neq 0} \phi \left( \frac{2c}{n} \right) \cdot \int_0^T \gamma(r) dr \xrightarrow{n \to \infty} 0.
\]

Define

\[
M_t = \int_0^t \left( \frac{n \rho(r) \hat{Z}_r}{|\hat{Z}_r|} 1_{\{\hat{Z}_r \neq 0\}} \right) dW_r + \int_0^t \int_E \beta_r(e) \tilde{\mu}(de, dr), \quad 0 \leq t \leq T,
\]

\[
K_t = \int_0^t 1_{\{\hat{Y}_r^+ > 0\}} \hat{Z}_r dW_r + \int_0^t \int_E 1_{\{\hat{Y}_r^+ > 0\}} \hat{U}_r(e) \tilde{\mu}(de, dr), \quad 0 \leq t \leq T.
\]

By Theorem 3.2, it follows that \(\tilde{K}_t\) is a martingale under the probability measure \(\tilde{\mathcal{P}} = \mathcal{E}(M)_T \cdot \mathcal{P}\). Hence taking \(\tilde{E}(\cdot | \mathcal{F}_t)\) the conditional expectation given \(\mathcal{F}_t\) under the probability measure \(\tilde{\mathcal{P}}\), and taking in account \(\varrho\) is concave, we deduce that

\[
\tilde{E} \left( \hat{Y}_s^+ | \mathcal{F}_t \right) \leq a_n + \int_t^s \gamma(r) \varrho \left( \tilde{E} \left[ \hat{Y}_r^+ | \mathcal{F}_t \right] \right) dr, \quad t \leq s \leq T.
\]

Thus Lemma 5 in [6] implies that \(\hat{Y}_t^+ = 0\) which is true if and only if \(Y_t^1 \leq Y_t^2\).

The following corollary is immediate.

**Corollary 3.4.** Let \(0 < T \leq +\infty\). If \(\xi \in L^2(\Omega, \mathcal{F}, \mathcal{P})\) and \(f\) satisfies (H2), then the BSDEP (2.1) with parameters \((\xi, f, T)\) has at most one solution.
4 Existence and uniqueness of solution

Thanks to the results establish in the previous section, we investigate in this section the solvability of our equations under weaker conditions on the generator.

Assume that \( f : \Omega \times [0,T] \times R \times R^d \times L^2(E,E,\lambda,R) \rightarrow R \) is uniformly continuous with respect to its variables and satisfies \((H3)\):

\[
|f(t,y,z,u) - f(t,y',z',u)| \leq \gamma(t)\phi(|y-y'|) + \rho(t)\phi(|z-z'|),
\]

\[
f(t,y,z,u) - f(t,y,z,u') \leq \sigma(t) \int_E (u(e) - u'(e)) \beta(e) \lambda(de)
\]

where \( \gamma, \rho, \sigma, \phi \) and \( \beta \) are as in \((H2)\).

We claim

**Theorem 4.1.** Let \( 0 < T \leq +\infty \) and \( \xi \in L^2(\Omega,F,P) \). If \( f \) satisfies \((H3)\) and \((A1)\) then equation \((2.1)\) admits a unique solution.

**Proof.** Uniqueness follows from Corollary 3.4 since \((H3)\) implies \((H2)\). Moreover from \((H3)\) one can derive that

\[
|f(\omega,t,y,z,u)| \leq \gamma(t)\phi(|y|) + \rho(t)\phi(|z|) + \tilde{c} \sigma(t) \left( \int_E |u(e)|^2 \lambda(de) \right)^{1/2} + |f(\omega,t,0,0,0)|
\]

\[
\leq f_t + k\gamma(t)|y| + a\rho(t)|z| + \tilde{c} \sigma(t)|u|
\]

where \( f_t = k\gamma(t) + b\rho(t) + |f(\omega,t,0,0,0)| \). Hence Theorem 2.8 ensures existence of a minimal solution. This completes the proof. \( \square \)

5 Proof of Theorem 2.4

This section is devoted to establishing the comparison theorem under assumptions \((A1)-(A5)\) and a horizon time \( T \) satisfying \( 0 < T \leq +\infty \). We consider the case \( T = +\infty \) since the result for \( T < \infty \) is well known. The key point is to expressed the difference of two solutions as a conditional expectation in a suitable probability space. To do this we need to apply Girsanov theorem. This is the guiding line of the following computations. To begin with, let us establish the following result.

**Proposition 5.1.** Let \( (a_t)_{t \geq 0}, (b_t)_{t \geq 0} \) be adapted processes satisfying a.s. \( |a_t| \leq \gamma(t); |b_t| \leq \rho(t) \). Assume that there exist a constant \( C_{5,1} > 0 \) and a process \( (\alpha_t)_{t \geq 0} \) satisfying

\[
\]
\( \alpha_t(e) > -1 \) and \( |\alpha_t(e)| \leq C_{5,1}(1 \wedge |e|) \) a.s. and an adapted process \( \{\varphi_t\}_{t \geq 0} \) satisfying

\[
\mathbb{E} \left[ \left( \int_0^\infty |\varphi_t| dt \right)^2 \right] < \infty.
\]

If \( (Y_t, Z_t, U_t) \) is solution to the BSDEP

\[
Y_t = \xi + \int_t^\infty \left( \varphi_s + a_s Y_s + b_s Z_s + \int_E \alpha_s(e) U_s(e) \lambda(de) \right) ds
\]
\[- \int_t^\infty Z_s dW_s - \int_t^\infty \int_E \alpha_s(e) \tilde{\mu}(ds, de), \quad t \geq 0.
\]

then there exists a probability measure \( \tilde{\mathbb{P}} \) such that

\[
Y_t = \tilde{\mathbb{E}} \left[ \exp \left( \int_t^\infty a_s ds \right) + \int_t^\infty \varphi_s \exp \left( \int_t^s a_r dr \right) ds \bigg| \mathcal{F}_t \right], \quad t \geq 0,
\]

where \( \tilde{\mathbb{E}} \) stands for the expectation under \( \tilde{\mathbb{P}} \).

**Proof.** Thanks to assumptions on \( b \) and \( \alpha \), it is easily seen that the stochastic process \( M = (M_t)_{0 \leq t \leq T} \) given by

\[
M_t = \int_0^t b_s dW_s + \int_0^t \int_E \alpha_s(e) \tilde{\mu}(ds, de), \quad 0 \leq t \leq T
\]

belongs in \( \mathcal{M}^2 \). So let \( \mathcal{E}(M)_t \) be the Doléans-Dade exponential of \( M \). By Theorem 3.2 there exists a probability measure \( \tilde{\mathbb{P}} \) such that

\[
\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}} |_{\mathcal{F}_t} = \mathcal{E}(M)_t
\]

\[
= \exp \left( \int_0^t b_s dW_s - \frac{1}{2} \int_0^t b_s^2 ds \right) \times \prod_{0 \leq s \leq t} \left( 1 + \int_E \alpha_r(e) \mu(\{s\}, dr) \right) \exp \left( - \int_0^t \int_E \alpha_r(e) \lambda(de) dr \right).
\]

Moreover the process \( W^*_t = W_t - \int_0^t b_r dr \) is a Brownian Motion under \( \tilde{\mathbb{P}} \) and \( \mu^*(dr, de) = \tilde{\mu}(dr, de) - \alpha_r(e) \lambda(de) dr \) is \( \tilde{\mathbb{P}} \)- martingale. Let \( 0 < T < \infty \) be fix. One can see that \( Y_t \) can be rewritten as

\[
Y_t = Y_T + \int_t^T (\varphi_r + a_r Y_r) dr - \int_t^T Z_r dW^*_r - \int_t^T \int_E \varphi_r(e) \mu^*(dr, de), \quad 0 \leq t \leq T < \infty.
\]
Define $\Gamma_t(\omega) = e^{\int_0^t a_r(\omega)dr}$, $\omega \in \Omega$, $t \geq 0$. It follows from Itô’s formula

$$
\Gamma_t Y_t = \Gamma_T Y_T - \int_0^T \Gamma_r dY_r - \int_0^T Y_r d\Gamma_r - \int_0^T d[\Gamma_r, Y_r] - \int_t^T \int_0^T \Gamma_r \varphi_r dr - \int_t^T \int_0^T \Gamma_r Z_r dW_r - \int_t^T \int_E \Gamma_r U_r(e) \mu^*(dr, de)
$$

Taking conditional expectation $\mathbf{E}(\cdot | \mathcal{F}_t)$, we deduce that for any $0 < t \leq T < \infty$,

$$
\Gamma_t Y_t = \mathbf{E} \left[ \Gamma_T Y_T + \int_t^T \varphi_r \Gamma_r dr \bigg| \mathcal{F}_t \right]
$$

which implies

$$
Y_t = \mathbf{E} \left[ Y_T \exp \left( \int_0^T a_s ds \right) + \int_t^T \varphi_s \exp \left( \int_t^s a_r dr \right) ds \bigg| \mathcal{F}_t \right].
$$

Letting $T \to \infty$, we deduce that

$$
Y_t = \mathbf{E} \left[ \xi \exp \left( \int_0^\infty a_s ds \right) + \int_t^\infty \varphi_s \exp \left( \int_t^s a_r dr \right) ds \bigg| \mathcal{F}_t \right].
$$

**Lemma 5.2.** Assume given $f^1, f^2$ and $(\xi^1, \xi^2) \in (L^2(\Omega, \mathcal{F}_T, \mathbf{P}))^2$ such that (A1)-(A4) hold. If $(\Theta^1, \Theta^2) = (Y^1, Z^1, U^1)$ is the corresponding solution, then there exists a probability $\mathbf{P}$ such that

$$
\tilde{Y}_t = \mathbf{E} \left[ \tilde{\xi} \exp \left( \int_t^\infty a_s ds \right) + \int_t^\infty \left[ f^1(s, \Theta^2_s) - f^2(s, \Theta^2_s) \right] \exp \left( \int_t^s a_r dr \right) ds \bigg| \mathcal{F}_t \right]
$$

where $\tilde{Y}_t$ and $\tilde{\xi}$ are given by (3.5).

**Proof.** W.l.o.g we assume $d = 1$. Define $\varphi_s = f^1(s, \Theta^2_s) - f^2(s, \Theta^2_s)$ and

$$
\begin{align*}
\Delta_y f^1(s) &= \frac{f^1(Y^1_s, Z^1_s, U^1_s) - f^1(Y^2_s, Z^1_s, U^1_s)}{\tilde{Y}_s} 1_{\{\tilde{Y}_s \neq 0\}}, \\
\Delta_z f^1(s) &= \frac{f^1(Y^1_s, Z^1_s, U^1_s) - f^1(Y^1_s, Z^2_s, U^1_s)}{\tilde{Z}_s} 1_{\{\tilde{Z}_s \neq 0\}}, \\
\Delta_u f^1(s, e) &= \frac{f^1(Y^1_s, Z^1_s, U^1_s(e)) - f^1(Y^1_s, Z^1_s, U^2_s(e))}{\tilde{U}_s(e)} 1_{\{\tilde{U}_s(e) \neq 0\}}.
\end{align*}
$$

15
Then \( (\hat{\Theta}_t)_{0 \leq t \leq T} \) is solution to

\[
\hat{Y}_t = \hat{\xi} + \int_t^T \left( \varphi_s + \Delta_y f^1(s) \hat{Y}_s + \Delta_z f^1(s) \hat{Z}_s + \int_E \Delta_u f^1(s,e) \hat{U}_s(e) \lambda(de) \right) ds \\
- \int_t^T \hat{Z}_s dW_s - \int_t^T \int_E \hat{U}_s(e) \tilde{\mu}(ds,de).
\]

(5.2)

By assumptions on the generator \( f^1 \), we have

\[
|\Delta_y f^1(s)| \leq \gamma(s), \quad |\Delta_z f^1(s)| \leq \rho(s), \quad |\Delta_u f^1(s,e)| \leq C(1 \wedge |e|) \quad \text{and} \quad \Delta_u f^1(s,e) > -1
\]

Hence applying Proposition 5.1 with \( a_s = \Delta_y f^1(s) \), \( b_s = \Delta_z f^1(s) \) and \( \alpha_s(e) = \Delta_u f^1(s,e) \) we get the desired result.

\[ \Box \]

**Proof of Theorem 2.4:** Applying the previous Lemma and taking in account assumptions \( (A1) \), we deduce that \( \hat{Y}_t \leq 0 \) since \( \hat{\xi} \leq 0 \) and \( \varphi_s \leq 0 \).

### References

[1] Bahlali, K., Backward stochastic differential equations with locally Lipschitz coefficient, C. R. Acad.Sci. Paris, Ser. I, 333, (2001), 481–486.

[2] Barles, G., Buckdahn, R., Pardoux, É., Backward stochastic differential equations and integral-partial differential equations, Stochastics and Stochastics Reports, 60, (1996), 57–83.

[3] Chen, Z., Wang, B., Infinite time interval BSDEs and the convergence of \( g \)-martingales, J. Austral. Math. Soc. (Series A), 69, (2000) 187–211.

[4] Daling, R., Pardoux, É., BSDE with random terminal time and applications to semilinear elliptic PDE, Ann. Probab. 3, (1997), 1135–1159.

[5] Fan, S., Jiang, L., Finite and infinite time interval BSDEs with non-Lipschitz coefficients, Statistics and Probability Letters, 80, (2010), 962–968.

[6] Fan, S., Jiang, L., Tian, D., One dimensional BSDEs with finite and infinite time horizon, Stochastic Processes and their Applications, 121, (2011), 427–440.

[7] Kobylanski, M., Résultats d’existence et d’unicité pour des équations différentielles stochastiques rétrogrades avec des générateurs à croissance quadratique, C. R. Acad. Sci. Ser. I Math., 324 (1), (1997), 81–86.

[8] Lepeltier, J. P., San Martin, J., Backward stochastic differential equations with continuous coefficients, Statistic. Probab. Letters, 32, (1997), 425–430.

[9] Mao, X., Adapted solution of Backward stochastic differential equations with non-Lipschitz coefficients, Stoch. Proc. Appl, 58, (1997), 281–292.

[10] Pardoux, É., Peng, S., Adapted solutions of backward stochastic differential equations, Systems and Control Letters, 14, (1997), 55–61.
[11] Pardoux, É., Peng, S., Backward stochastic differential equations and quasilinear parabolic PDEs, In: Rozovskii, B.L., Sowers, R.S. (Eds). Stochastic partial differential equations and their applications, Lect. Notes in Control & Info. sci., Springer, Berlin, Heidelberg, New York, 176, (1992), 200–217.

[12] Pardoux, É., Generalized discontinuous backward stochastic differential equations. In Backward stochastic differential equations (Paris, 1995–1996), volume 364 of Pitman Res. Notes Math. Ser., pages 207219. Longman, Harlow, 1997.

[13] Peng, S., Probabilistic interpretation for systems of quasilinear parabolic partial differential equations, Stochastics Stochastics Reports, 37, (1991), 61–74.

[14] Royer, M., Backward stochastic differential equations with jumps and related non-linear expectations, Stochastic Processes and their Applications, 116, (2006), 1358–1376.

[15] Tang, S., Li, X., Necessary conditions for optimal control of stochastic systems with random jumps, SIAM J. Control Optim., 32, (5), (1994), 1447–1475.

[16] Rong, S., On solutions of backward stochastic differential equations with jumps and applications, Stochastic Processes and their Applications, 66, (1997) 209-236.

[17] Wu, Z., FBSDE with Brownian motion and Poisson Process, Acta Mathematica Applicatae Sinica, 15, No 4, (1999), 433-443.

[18] Wang, Y., Huang, Z., Backward stochastic differential equations with non Lipschitz coefficients equations, Statistics and Probability Letters, 79, (2009), 1438–1443.

[19] Yao, S., Lp solutions of Backward differential equation with jumps, Arxiv, 2010.

[20] Yin, J., Mao, X., The adapted solution and comparison theorem for backward stochastic differential equations with Poisson jumps and applications. J. Math. Anal. Appl., 346 (2), (2008), 345–358.

[21] Yin, J., Rong, S., On solutions of forward-backward stochastic differential equations with Poisson jumps. Stochastic Anal. Appl., 21 (6), (2003), 1419–1448.