Large $B$–Fields and Noncommutative Solitons

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Abstract. The purpose of this talk is to review a few issues concerning noncommutativity arising from String Theory. In particular, it is shown how in Type IIB Theory, the annihilation of a $D3 - \overline{D3}$ brane pair yields a $D1$–string. This object, in the presence of a large $B$–field and fermions, happens to be a complex noncommutative soliton endowed with superconductivity.

INTRODUCTION

Type IIB Superstring Theory allows two interesting ingredients: stable-BPS RR-charged $Dp$-branes ($p$ odd) and a massless antisymmetric $B_{\mu\nu}$ field. Although a $D3 - \overline{D3}$ brane system is stable; a $D3 - \overline{D3}$ case is not, due to the presence of a tachyon in its spectrum [1]. The job of the $B$-field in the low energy limit is to introduce noncommutativity [2]. In the $D3 - \overline{D3}$ brane configuration, we may turn on a large $B$-field along two worldvolume spatial coordinates. The effect of this is that the complex tachyon allows a GMS soliton [3]. This object appears to be superconducting in the presence of fermions arising from the open string sector [4].

$B$-FIELDS AND NONCOMMUTATIVITY

Consider an open string attached to a $D$-brane. The OPE has the form

$$e^{ik_1 \cdot X} e^{ik_2 \cdot X} \sim (\tau - \tau')^{2\alpha' g_{\mu\nu} k_{1\mu} k_{2\nu}} \times e^{i(k_1 + k_2) \cdot X} + \ldots . \quad (1)$$

However, the introduction of a large $B$-field alters this to

$$e^{ik_1 \cdot X} e^{ik_2 \cdot X} \sim (\tau - \tau')^{2\alpha' G^{\mu\nu} k_{1\mu} k_{2\nu}} \times \left[ e^{-\frac{i}{2} \Theta^{\mu\nu} k_{1\mu} k_{2\nu}} \right] \times e^{i(k_1 + k_2) \cdot X} + \ldots . \quad (2)$$

1) This work is dedicated to my parents Juan Moreno and Martha Soto.
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where $G^{\mu\nu} = \left(\frac{1}{g+2\pi\alpha'} B \frac{1}{g-2\pi\alpha'} \right)^{\mu\nu}$ is the effective metric seen by the open string modes, and $\Theta^{\mu\nu} = - (2\pi \alpha')^2 \left(\frac{1}{g+2\pi\alpha'} B \frac{1}{g-2\pi\alpha'} \right)^{\mu\nu}$ is the noncommutativity parameter matrix [3]. Likewise, the new term in brackets is known -in configuration space- as the Moyal $*$ product. Thus, in the low energy effective theory, fields get $*$-multiplied.

**NONCOMMUTATIVE SOLITONS**

The idea of GMS solitons was cleverly used in [5] to construct real solutions to the tachyon in bosonic $D$-branes. In this work, complex tachyons in RR $D$-brane pairs are considered instead. For a $D3-\bar{D}3$ brane pair in the presence of a large $B$-field along the $x-y$ plane, the action of the tachyon is

$$S^{(\Sigma_{3+1})} = \int_{\Sigma_{3+1}} dxdydzdt \left[ \mathcal{D}^\mu T \ast D_\mu T - V_*(T, \bar{T}) \right],$$

(3)

where $[x, y] = i\theta$, and $z$ and $t$ are commutative coordinates. Also, $D_\mu T = \partial_\mu T - iA_\mu \ast T + i\widetilde{A}_\mu \ast T$, where $A_\mu$ and $\widetilde{A}_\mu$ are the respective gauge fields in each of the $D$-brane’s Chan-Paton $U(1)$ symmetry groups.

We make three assumptions:

- The potential is a polynomial:

$$V_*(T, \bar{T}) = \sum_{k=1}^{n} a_k (T \ast T)^k.$$  

(4)

- The gauge field $R_\mu \equiv A_\mu - \widetilde{A}_\mu$ has the form:

$$R_\mu = R_\mu (z, t).$$

(5)

- We’ll focus on solutions of the form:

$$T = T(x, y), \quad \bar{T} = \bar{T}(x, y).$$

(6)

Eventually, it is shown that the solution is

$$T = t_* T_0, \quad \bar{T} = \overline{t_*} \overline{T_0},$$

(7)

where $t_*$ and $\overline{t_*}$ solve the equations of motion in the commutative case, while $T_0, \overline{T_0}$ satisfy

$$(\overline{T_0} \ast T_0)^k = \overline{T_0} \ast T_0.$$
Since \([x, y] = i\theta\) is analogous to \([\hat{q}, \hat{p}] = i\) in quantum mechanics, we identify 
\(T_0 \leftrightarrow i|0\rangle \langle 0|\) and \(\overline{T}_0 \leftrightarrow -i|0\rangle \langle 0|\) in the Simple Harmonic Oscillator basis.

Applying the Weyl-Wigner-Moyal correspondence yields the following lowest energy solitonic solution:

\[
T(x, y) = 2it^*e^{-r^2}, \quad \overline{T}(x, y) = -2it^*e^{-r^2},
\]

where \(r^2 = x^2 + y^2\) [4].

**THE NONCOMMUTATIVE SUPERCONDUCTING STRING**

Given a \((3 + 1)\) Dirac spinor \(\Psi\), with two-component entries \(\psi_R\) and \(\psi_L\) obeying \(\vec{\sigma} \cdot \hat{p}\psi_R = \psi_R\) and \(\vec{\sigma} \cdot \hat{p}\psi_L = -\psi_L\), the action for the spinor coupled to the complex soliton has the form:

\[
S^{(\Sigma_{3+1})} = \int_{\Sigma_{3+1}} dt dz dx dy \left[ f \left( T \right) \ast \overline{\Psi} \ast g \left( T \right) \partial \Psi \right],
\]

where \(f\) and \(g\) are polynomials, and \(\partial \Psi = \gamma^\mu (\partial_\mu \Psi - iR_\mu \ast \Psi)\).

In terms of operators, we may express our spinors as [6]:

\[
\hat{\psi}_{L,R}(x^\mu) = \sum_{m,n \geq 0} \psi_{mn}^{L,R}(z,t) \langle m | n \rangle.
\]

In order to find the effective theory along the string (the \(z,t\) coordinates,) we make two assumptions:

- \(\theta \to \infty\), which means that the noncommutative kinetic part is negligible.
- \(\psi \to \psi^L\) (as in Witten’s superconducting string [7]).

Therefore, the action for the Noncommutative D-string is

\[
S^{(\Sigma_{1+1})} = -2\pi i\theta f \left( T_s \right) g \left( t_s \right) \int_{\Sigma_{1+1}} \left[ i\overline{\psi}^L \sigma^a D_a \psi^L - \overline{\psi}^L m\psi^L \right],
\]

where \(\psi^L\) denotes \(\psi^L_{00}\) in (11), \(m\) is a “mass” matrix \((m(z,t) = \sigma^\alpha R_\alpha)^3\).

**SUPERCONDUCTIVITY**

In the massless case (after rescaling the action and getting rid of the unnecessary \(L\) subscript)

\[a = z, t\) and \(\alpha = x, y.\]
\[ S^{(\Sigma_{3+1})} = \int_{\Sigma_{3+1}} dzdt \left( i\overline{\psi} \sigma^a D_a \psi \right). \] (13)

According to the bosonization technique [7], in two dimensions we can equivalently describe the theory by either bosons or fermions. This is done by introducing a scalar field \( \zeta (z, t) \) such that

\[ \overline{\psi} \sigma^a \psi = \frac{1}{\sqrt{\pi}} \varepsilon^{ab} \partial_b \zeta. \] (14)

It can be shown that the kinetic term corresponds to

\[ i\overline{\psi} \sigma^a D_a \psi = \frac{1}{2} (\partial_a \zeta) (\partial^a \zeta) - \frac{1}{\sqrt{\pi}} R_a \varepsilon^{ab} \partial_b \zeta, \] (15)

which is associated to a conserved current

\[ J^a = \partial^a \zeta + \frac{1}{\sqrt{\pi}} \varepsilon^{ab} R_b. \] (16)

However, this current may be expressed in terms of another scalar:

\[ J^a = \varepsilon^{ab} \partial_b \varphi. \] (17)

Thus, \( \partial_b \varphi = -\varepsilon_{ba} J^a \) and

\[ \partial^b \partial_b \varphi = -\partial^b \varepsilon_{ba} \left( \partial^a \zeta + \frac{1}{\sqrt{\pi}} \varepsilon^{ab} R_b \right) = -\frac{1}{\sqrt{\pi}} \partial^b R_b. \] (18)

This means that from \( \partial_a \varphi = -\varepsilon_{ac} J^c \), we get that \( -\varepsilon_{ac} \partial^c J^c = -\frac{1}{\sqrt{\pi}} \partial^a R_a. \) In other words, for \( J^3 \) (the current along the string):

\[ \frac{dJ^3 (z, t)}{dt} = \frac{1}{\sqrt{\pi}} \frac{dR_0 (z, t)}{dz}, \] (19)

which has the following solution:

\[ J^3 (z, t) = \frac{1}{\sqrt{\pi}} [R_0 (z, \tau) - R_0 (z, \tau_i)]. \] (20)

This means that the current is nondecaying as long as the gauge fields has different values when it’s turn on \((t = \tau_i)\) and turn off \((t = \tau)\).

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