\Theta^+ hypernuclei in relativistic mean field model

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Abstract

We have investigated the properties of \( \Theta^+ \) in nuclei within the framework of relativistic mean field. The coupling constants are educed with quark meson coupling model. There is strong attractive interaction for \( \Theta^+ \)-nucleus and \( \Theta^+ \) can be bind in nuclei. The depth of optical potential for \( \Theta^+ \) in nuclear matter is estimated.

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1 Introduction

Since the extremely narrow positive strangeness pentaquark state \( \Theta^+ (uudds) \) of mass 1.54 GeV and \( J^P = 1/2^+ \) is predicted[1] and with several experimental searches being successfully undertaken[2, 3, 4, 5, 6, 7, 8], pentaquark becomes the hot topic of investigation. The discovery of the exotic baryon \( \Theta^+ \) with positive strangeness opens new possibilities of forming exotic \( \Theta^+ \) hypernuclei which, like in the case of negative strangeness \( \Lambda, \Sigma, \Xi \) hypernuclei which have obtained steady progress at the experimental and theoretical levels[10, 11, 12, 13, 14], can provide information unreachable or complementary to that obtained in elementary reactions.

Suggestions that \( \Theta^+ \) could be bound in nuclei have already been made [15, 16] and the selfenergy of the \( \Theta^+ \) pentaquark in nuclei is calculated by D. Cabrera et al.[16], they show that the in-medium renormalization of the pion in the two meson cloud of the \( \Theta^+ \) leads to a sizable attraction, enough to produce a large number of bound and narrow \( \Theta^+ \) states in nuclei. However, the spin, isospin and parity of \( \Theta^+ \) are still not very determinate, there are many discussions about these aspects[1, 15, 17, 18, 19, 20]. In this work, we assume that \( J^P = 1/2^+ \) and \( I = 0 \) for \( \Theta^+ \) as some suggestions in Refs.[1, 15], thus we can study \( \Theta^+ \) hypernuclei in the framework of relativistic mean field (RMF), which has been used to investigate \( \Lambda, \Sigma, \Xi \) hypernuclei[21, 22, 23, 24].

In the present work, our main aim is to determine the coupling constants \( \sigma - \Theta^+ , \omega - \Theta^+ \) for the RMF calculation. Then we attempt to investigate the \( \Theta^+ \) hypernuclei in the framework of relativistic mean field, from which we want to know whether the \( \Theta^+ \) baryon could be bound in nuclei and the how much the depth of the \( \Theta^+ \)-nucleus potential.

This paper is organized as follows. The framework of hypernuclei in RMF is introduced in Sec.2. The coupling constants are educed and determined in Sec.3. The calculations and analysis is given in Sec.4. The last Sec. is the summary.
2 Hypernuclei in RMF model

In the RMF theory, the effective Lagrangian density which includes the nucleon and Θ⁺-hyperon can be written as

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_{\Theta^+},$$

with

$$\mathcal{L}_N = \bar{\Psi}_N (i\gamma^\mu \partial_\mu - M_N) \Psi_N - g^N_\sigma \bar{\Psi}_N \sigma \Psi_N - g^N_\omega \bar{\Psi}_N \gamma^\mu \omega_\mu \Psi_N - g^N_\rho \bar{\Psi}_N \rho^a_\mu \tau^a \Psi_N + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{3}{2} g_2^2 \sigma^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{2} \frac{1}{4} \frac{1}{2} m_\omega^2 \omega_\mu \omega_\mu - \frac{1}{4} R_{\mu\nu}^a R^a_{\mu\nu}$$

$$- \frac{1}{4} g^2_3 \sigma^4 - \frac{1}{4} \Omega_{\mu\nu}^a \Omega_{\mu\nu}^a + \frac{1}{2} R_{\mu\nu}^a R^a_{\mu\nu}$$

$$+ \frac{1}{4} \rho_{\mu\nu}^a \rho_{\mu\nu}^a - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - e \bar{\Psi}_N \gamma^\mu A^\mu_1 \frac{1}{2} (1 - \tau^3) \gamma^\mu \Psi_N.$$

$$\mathcal{L}_{\Theta^+} = \bar{\Psi}_{\Theta^+} (i\gamma^\mu \partial_\mu - M_{\Theta^+}) \Psi_{\Theta^+} - g^\Theta_\sigma \bar{\Psi}_{\Theta^+} \sigma \Psi_{\Theta^+} - g^\Theta_\omega \bar{\Psi}_{\Theta^+} \gamma^\mu \omega_\mu \Psi_{\Theta^+} - g^\Theta_\rho \bar{\Psi}_{\Theta^+} \rho^a_\mu \tau^a \Psi_{\Theta^+} + \frac{1}{2} \bar{\Psi}_{\Theta^+} \gamma^\mu A^\mu_1 \frac{1}{2} (1 - \tau^3) \gamma^\mu \Psi_{\Theta^+}$$

with

$$\Omega_{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu,$$

$$R_{\mu\nu}^a = \partial^\mu \rho_{\nu}^a - \partial^\nu \rho_{\mu}^a,$$

$$F_{\mu\nu}^a = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

where the meson fields are denoted by $\sigma, \omega_\mu, \rho_\mu$, and their masses by $m_\sigma, m_\omega, m_\rho$, respectively; $\Psi_N$ and $\Psi_{\Theta^+}$ are the nucleon and $\Theta^+$ baryon fields with corresponding masses $M_N$ and $M_{\Theta^+}$, respectively; $A_\mu$ is the electromagnetic field. $g^N_\sigma (g^\Theta_\sigma), g^N_\omega (g^\Theta_\omega)$ and $g^N_\rho (g^\Theta_\rho)$ are the $\sigma - N(\sigma - \Theta), \omega - N(\omega - \Theta)$ and $\rho - N(\rho - \Theta)$ coupling constants, respectively. The isospin Pauli matrices are written as $\tau^a$, $\tau^3$ being the third component of $\tau^a$. For $\Theta^+$ baryon, we assume isospin $I = 0, \tau^a = 0$, there is no coupling to $\rho$ meson.

3 How to determine the coupling constants

According the quark meson coupling model developed in Ref.[25], the baryon coupling constants can be determined. The equation of motion for meson field operators are

$$\partial_\mu \partial^\mu \bar{\sigma} + m_\sigma^2 \bar{\sigma} = g^\Theta_\sigma \bar{q} q,$$

$$\partial_\mu \partial^\mu \bar{\omega}^\nu + m_\omega^2 \bar{\omega}^\nu = g^\Theta_\omega \bar{q} \gamma^\nu q,$$

$$\partial_\mu \partial^\mu \bar{\rho}_{\mu\alpha} + m_\rho^2 \bar{\rho}_{\mu\alpha} = g^\Theta_\rho \bar{q} \gamma^\nu \tau^a_\nu \frac{1}{2} q,$$

where $g^\Theta_\sigma, g^\Theta_\omega$ and $g^\Theta_\rho$ are the quark-meson coupling constants for $\sigma, \omega$ and $\rho$, respectively.

The mean fields are defined as the expectation values in the ground state of the nucleus, $|A\rangle$:

$$\langle A | \bar{\sigma} (t, r) | A \rangle = \sigma (r),$$

$$\langle A | \bar{\omega}^\nu (t, r) | A \rangle = \delta (\nu, 0) \omega (r),$$

$$\langle A | \bar{\rho}_{\mu\alpha} (t, r) | A \rangle = \delta (\nu, 0) \delta (\alpha, 3) \rho (r),$$

$$\langle A | \bar{\psi} (t, r) | A \rangle = \delta (\nu, 0) \delta (\alpha, 3) \rho (r),$$

$$\langle A | \bar{\psi} (t, r) | A \rangle = \delta (\nu, 0) \delta (\alpha, 3) \rho (r),$$

$$\langle A | \bar{\psi} (t, r) | A \rangle = \delta (\nu, 0) \delta (\alpha, 3) \rho (r).$$
where \((t,r)\) are the coordinates in the rest frame of the nucleus. In the mean field approximation the sources are the sum of the sources created by each nucleon - the latter interacting with the meson fields. Thus we write

\[
\bar{q}q(t, r) = \sum_{i=1, A} \langle \bar{q}q(t, r) \rangle_i,
\]

\[
\bar{q}\gamma^\nu q(t, r) = \sum_{i=1, A} \langle \bar{q}\gamma^\nu q(t, r) \rangle_i,
\]

where \(\langle \rangle_i\) denotes the matrix element in the nucleon \(i\) located at \(R_i\) at time \(t\). According to the Born-Oppenheimer approximation, the baryon structure is described by \(n\) quarks in the lowest mode. Therefore, in the instantaneous rest frame (IRF) of the nucleon \(i\), we have

\[
\langle \bar{q}' q'(t', r') \rangle_i = n \sum_m \bar{\phi}_{i}^{0,m}(u') \phi_{i}^{0,m}(u') = n s_i(u'),
\]

\[
\langle \bar{q}' \gamma^\nu q'(t', r') \rangle_i = n \delta(\nu, 0) \sum_m \bar{\phi}_{i}^{0,m}(u') \phi_{i}^{0,m}(u') = n w_i(u'),
\]

where \((t', r')\) are the the coordinates and \(q'\) is the quark field in the IRF. \(\phi_{i}^{0,m}\) is the complete and orthogonal set of eigenfunctions which are defined by Eq.(23) in Ref.[25].

From the Lorentz transformation properties of the fields we get

\[
\langle \bar{q}q(t, r) \rangle_i = \frac{n}{(2\pi)^3}((\cosh \xi_i))^{-1} \int dke^{i k \cdot (r - R_i)} S(k, R_i),
\]

\[
\langle \bar{q}\gamma^\nu q(t, r) \rangle_i = \frac{n}{(2\pi)^3} \int dke^{i k \cdot (r - R_i)} W(k, R_i),
\]

\[
\langle \bar{q}\gamma q(t, r) \rangle_i = \frac{n}{(2\pi)^3} v_i \int dke^{i k \cdot (r - R_i)} W(k, R_i),
\]

with

\[
S(k, R_i) = \int du e^{-i (k_{\perp} \cdot u_{\perp} + k_L u_L / \cosh \xi_i)} s_i(u),
\]

\[
W(k, R_i) = \int du e^{-i (k_{\perp} \cdot u_{\perp} + k_L u_L / \cosh \xi_i)} w_i(u).
\]

The corresponding physical quantities such as \(\xi_i\), \(u_{\perp}\), \(u_L\) etc. are defined in Ref.[25]. Finally, the mean field expressions for the meson properties take the form

\[
\langle A \mid \bar{q}q(t, r) \mid A \rangle = \frac{n}{(2\pi)^3} \int dke^{i k \cdot r} \langle A \mid \sum_i ((\cosh \xi_i))^{-1} e^{-i k \cdot R_i} S(k, R_i) \mid A \rangle,
\]

\[
\langle A \mid \bar{q}\gamma^\nu q(t, r) \mid A \rangle = \frac{n}{(2\pi)^3} \int dke^{i k \cdot r} \langle A \mid \sum_i e^{-i k \cdot R_i} W(k, R_i) \mid A \rangle,
\]

\[
\langle A \mid \bar{q}\gamma q(t, r) \mid A \rangle = 0.
\]

To simplify further, we remark that a matrix element of the form

\[
\langle A \mid \sum_i e^{-i k \cdot R_i} \ldots \mid A \rangle
\]

is negligible unless \(k\) is less than, or of the order of, the reciprocal of the nuclear radius.
We now define the scalar, baryon and isospin densities of the nucleons in the nucleus by

\[
\rho_s = \langle A | \sum_i \frac{M^*_B(R_i)}{E_i - V(R_i)} \delta(r - R_i) | A \rangle, \tag{24}
\]

\[
\rho_B = \langle A | \sum_i \delta(r - R_i) | A \rangle, \tag{25}
\]

\[
\rho_3 = \langle A | \sum_i \frac{\tau^B_3}{2} \delta(r - R_i) | A \rangle, \tag{26}
\]

where we have used \( E_i = M^*_B \cosh \xi_i + V(R_i) \) (Eq.(58) of Ref.[25]) to eliminate the factor \((\cosh \xi)^{-1}\). The meson sources take the simple form

\[
\langle A | \bar{q}q(t, r) | A \rangle = nS(r)\rho_s, \tag{27}
\]

\[
\langle A | \bar{q}g^\rho q(t, r) | A \rangle = n\rho_B, \tag{28}
\]

\[
\langle A | \bar{q}g^\nu \frac{\tau^\alpha}{2} q(t, r) | A \rangle = \delta(\nu, 0)\delta(\alpha, 3)\rho(3). \tag{29}
\]

We have used the notation

\[
S(r) = S(0, r) = \int du u_r(u) = \frac{\Omega_0/2 + m^*_q R_B(\Omega_0 - 1)}{\Omega_0(\Omega_0 - 1) + m^*_q R_B/2},
\]

\[
\Omega_0 = \sqrt{\chi^2 + (R_B m^*_q)^2},
\]

\[
m^*_q = m_q - g^2_\sigma \sigma(r) \tag{30}
\]

Here, \( \chi \) and \( m_q \) are the parameters for lowest eigenvalues for the quarks and the corresponding current quark masses. \( R_B \) is the radius of the bag.

Since their sources are time independent and since they do not propagate, the mean fields are also time independent. So, we get the desired equations for \( \sigma(r) \), \( \omega(r) \) and \( \rho(r) \):

\[
\{ - \Delta + m^2_\sigma \} \sigma(r) = g_\sigma C(r) \rho_s(r), \tag{31}
\]

\[
\{ - \Delta + m^2_\omega \} \omega_0(r) = g_\omega \rho_B(r), \tag{32}
\]

\[
\{ - \Delta + m^2_\rho \} \rho_0(r) = g_\rho \rho_3(r). \tag{33}
\]

where the nucleon coupling constants and \( C(r) \) are defined by

\[
g_\sigma = ng^2_\sigma S(\sigma = 0), g_\omega = ng^2_\omega, g_\rho = g^2_\rho, C(r) = S(r)/S(\sigma = 0). \tag{34}
\]

In an approximation where the \( \sigma(r) \), \( \omega(r) \) and \( \rho(r) \) fields couple only to the \( u \) and \( d \) quarks[26, 27], the coupling constants in the strange baryons are obtained as \( g^S_\omega = (n_q/n)g_\omega \) and \( g^S_\rho \equiv g_\rho = g^2_\rho \), with \( n_q \) being the total number of valence \( u \) and \( d \) quarks in the baryon \( S \). For the strange baryon \( S \), the coupling constant \( \sigma - S \) can be written as

\[
g^S_\sigma = n_q g^2_\sigma S_S(\sigma = 0), \tag{35}
\]

where

\[
S_S(r) = \frac{\Omega_0/2 + m^*_S R_S(\Omega_0 - 1)}{\Omega_0(\Omega_0 - 1) + m^*_S R_S/2}. \tag{36}
\]
where $R_S$ is the bag radius for the strange baryon. By combining the Eq.(24) and Eq.(25) we get

$$g^S_\sigma = \frac{n_q}{n} g_\sigma S_S(\sigma = 0)/S(\sigma = 0) = \frac{n_q}{n} g_\sigma \Gamma_{S/B}. \quad (37)$$

According Eq.(24)

$$g_\sigma = \frac{n_q}{3} g^N_\sigma, g_\omega = \frac{n_q}{3} g^N_\omega, g_\rho = g^N_\rho, \quad (38)$$

where $g^N_\sigma$, $g^N_\omega$, $g^N_\rho$ are the coupling constants for nucleon. So, we can easily get the relations

$$g^S_\sigma = \frac{n_q}{3} g^N_\sigma \Gamma_{S/B}, g^S_\omega = \frac{n_q}{3} g^N_\omega, g^S_\rho = g^N_\rho. \quad (39)$$

### 4 Calculations and Analysis

In this section we will show the our numerical results using relativistic mean field theory model. In the mean-field and the no-sea approximations[28] the contributions of anti-(quasi) particles and quantum fluctuations of mesons fields can be thus neglected. The contribution of the tensor coupling is neglected for simple, although it is considered in some Refs.[22, 23]. The coupling constants of nucleon to mesons employed are NL-SH[30], which are list in table 1. The coupling constants of $\Theta^+$ to mesons are obtained from the relations Eq.(39). For $\Theta^+$, $n_q=4$, isospin $I=0$ means no coupling with $\rho$ meson. Thus, we have the relations

$$g_\sigma^{\Theta} = \frac{4}{3} g^N_\sigma \Gamma_{\Theta/B}, g_\omega^{\Theta} = \frac{4}{3} g^N_\omega. \quad (40)$$

In practice the value for $\Gamma_{S/B} \approx 1$ for all hyperons even though $R_B \neq R_S$[29], thus in our calculation, we use $\Gamma_{\Theta/B} = 1$.

#### 4.1 The results for $\Theta^+$ hypernuclei in RMF

In figure 1, we show the single-particle energy spectrum such as $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$... of $\Theta^+$ in $^6$Li, $^{12}$C, $^{16}$O, $^{40}$Ca and $^{208}$Pb, which is denoted by 1, 2a, 3, 4a, 5 respectively. 2b is the energy spectrum for $^{12}$C when the coupling constant change a little to $g^3_\sigma = \frac{4}{3} g^N_\sigma \times 0.95$. 2c, 2d and 4b, 4c are the results for $^{13}$C and $^{21}$Ca at the depth of potential 60 MeV and 120 MeV respectively of ref[16].

In light nuclei like $^6$Li (1), $^{12}$C(2a) and $^{16}$O(3) we find several bound states. The separation of the two deepest bound states (1s1/2 and 1p3/2) is about 24 MeV for $^6$Li, about 60 MeV for $^{12}$C and around 30 MeV for $^{16}$O. For medium and heavy nuclei, as in $^{40}$Ca, $^{208}$Pb shown in the figure, we find more bound states and the energy separation between 1s1/2 and 1p3/2 is about 18 MeV for $^{40}$Ca and about 4 MeV for $^{208}$Pb. There is a trend that the separation between 1s1/2 and 1p3/2 becomes smaller and smaller and the bound state becomes more and more with the increasing of the nucleon number. It is interesting that the width of $\Theta^+$ is very narrow with upper limit as small as 15 MeV predicted by the chiral soliton model in Ref. [1], and 9 MeV[37].Studies based on $K^+ N$ scattering suggest that the width should be smaller than 5 MeV[34, 35, 36]. Cahn and trilling have extracted $\Gamma(\Theta) = 0.9 \pm 0.3$ MeV from an analysis of Xenon bubble chamber[38]. The narrow upper limit width of $\Theta^+$ is often smaller than the separation of the two deepest bound states (1s1/2 and 1p3/2) for light and medium nuclei, which would make a clear case for the experimental observation of these states.
The splitting of spin-orbit for Θ+ hypernuclei can be get from our calculation with RMF. Such as, from figure 1, we can find that the splitting between 1p3/2 and 1p1/2 is about 5 MeV for 6Li (1) and 16O, 10 MeV for 12C, The splitting is smaller for medium nucleus 40Ca (about 2 MeV) and there is no obvious splitting for heavy nucleus 208Pb. If the prediction based on K+N scattering suggest that the width should be smaller than 5 MeV[34, 35, 36] is true, the the splitting between 1p3/2 and 1p1/2 is also bigger than the narrow upper limit width of Θ+, which means that we may observe splitting of spin-orbit for light Θ+ hypernuclei. There is a very interesting phenomenon that both separation between 1s1/2 and 1p3/2 and splitting of spin-orbit between 1p3/2 and 1p1/2 are much larger than the others.

For Comparison, we also yield the the single-particle energy spectrum of Θ+ from Ref[16] by solving Schrödinger equation with two potentials. It is a very simple model and can not give the splitting of spin-orbit. It estimates that the binding energy (1s) of 12C is about 116 MeV and 81 MeV for 40Ca, and the energy separation between two bound states is about 18 MeV for 40Ca and 60 MeV for 12C. Our results are very close to the the results estimated by solving Schrödinger equation in ref[16].

In table 1 we list the calculated binding energy per baryon, -E/A, r.m.s. charge radius, r_{ch}, and r.m.s. radii of the Θ+ baryon and the neutron and proton distributions (r_Θ, r_n and r_p). For comparison, we also give these quantities without Θ+. The differences in values for finite nuclei and hypernuclei listed in table 1 reflects the effects of the Θ+. The appearance of Θ+ in nuclei leads to larger binding energy per baryon, -E/A. From table 1, it is found that the values of r_n, r_p and r_{ch} for 9Be, 12C are obvious less than those for the corresponding normal nuclei. This is the shrinkage effect which has been found in lighter Λ hypernuclei[24, 31]. In our calculation for 9Be, 12C, we also find that there is shrinkage effect and gluelike role for Θ+. But this phenomenon is not found for 7Li and it is not obvious for 17O, 41Ca and 209Pb.

The key point for our calculation is how to determine the coupling constants of Θ+ to mesons as accurate as possible. In the calculation, we find that the binding energy is very sensitive to the coupling constants. In Refs[21], they determine the the coupling constants of hyperon to mesons by adjusting g_ΘY/g_ΘN around 2/3 (according Eq.(39), g_ΘY/g_ΘN ≈ 2/3 ) to fit the experimental data because of the effect of medium in nuclei. Where Y stands the hyperon. To consider the effect of medium to the coupling constant g_ΘY, we list the result of 12C in figure 1, denoted with 2b, when the coupling constant g_ΘY changes to g_ΘY×0.95. From the figure, we see the binding energy of 1s1/2 have a large change from 116 to 60 MeV, but there is still several bound states and the energy separation energy between 1s1/2 and 1p3/2 is around 32 MeV.

We also the list results of the ground state for 12C when the coupling constant g_ΘY changes from 13.925 to 11.725 (g_ΘY×0.84) in table 2. From it, we can see that the binding energy of 1s1/2 is sensitive to the coupling constant, if g_ΘY < 11.725(13.925 × 0.84), there are no bound states for Θ+ in 12C. In Ref.[21], g_ΘY/g_ΘN is 0.621 (0.667 × 0.93) and 0.619 (0.667 × 0.928) for Λ−σ and Σ−σ, respectively. Thus, from the calculation of before[21], we can conclude that the effect of medium is not very large, the considered medium effective coupling constant is about 0.93 × g_ΘY for hyperon. If we let g_ΘY = 0.93 × 13.925=12.95, from table 1, we can see that the binding energy of 1s1/2 for 12C is between 52 and 40 MeV, which is agree with the estimation 34 ~ 87 MeV in the Ref[16].

In table 2, the binding energy per baryon, -E/A (in MeV), r.m.s. charge radius r_{ch} (in fm), r.m.s. radii of Θ+, neutron and proton, r_Θ, r_n and r_p (in fm) are also shown respectively. It is seen in table 2 that the radii is increasing and the binding energy per baryon is decreasing with the decreasing of the coupling constant. When g_ΘY < 12.325, r_Θ < r_{ch}, which means that
the Θ⁺ is out of the nucleus $^{12}$C. It can only form Θ⁺ atom.

Table 1: The parametrization of the nucleonic sector (adopted from Ref.[30]). The masses are given in MeV and the coupling $g_2$ in fm$^{-1}$.

| $M_N$  | $m_\sigma$ | $m_\omega$ | $m_\rho$     | $g_\sigma^N$ | $g_\omega^N$ | $g_\rho^N$ | $g_2$ | $-15.8337$ |
|--------|------------|------------|---------------|---------------|---------------|------------|-------|-------------|
| 939.0  | 526.059    | 783.0      | 763.0         | 10.444        | 12.945        | 4.383      | -6.9099 |             |

Table 2: Binding energy per baryon, $-E/A$ (in MeV), binding energy of 1s1/2 for Θ⁺ in nuclei, $-B$ (in MeV), r.m.s. charge radius $r_{ch}$ (in fm), r.m.s. radii of hyperon, neutron and proton, $r_\Theta$, $r_n$ and $r_p$ (in fm), respectively. The configuration of Θ⁺ is 1s1/2 for all hypernuclei.

| $^A_Z$  | $r_p$(fm) | $r_n$(fm) | $r_\Theta$(fm) | $r_{ch}$(fm) | $-E/A$(MeV) | $-B$(MeV)  |
|---------|-----------|-----------|-----------------|--------------|-------------|------------|
| $^6$Li | 2.37      | 2.32      | 2.51            | 2.50         | 5.67        | 52.13      |
| $^7_\Theta$Li | 2.36      | 2.36      | 1.38            | 2.50         | 6.21        | 52.13      |
| $^8$Be | 2.34      | 2.30      | 2.48            | 2.36         | 5.42        | 67.72      |
| $^9_\Theta$Be | 2.21      | 2.19      | 1.35            | 11.43        | 2.36        | 67.72      |
| $^{12}$C | 2.32      | 2.30      | 2.46            | 2.32         | 7.47        | 116.24     |
| $^{13}_\Theta$C | 2.17      | 2.15      | 1.19            | 2.32         | 116.24      | 116.24     |
| $^{16}$O | 2.58      | 2.55      | 2.70            | 2.70         | 8.04        | 83.66      |
| $^{17}_\Theta$O | 2.58      | 2.54      | 1.46            | 2.70         | 8.60        | 83.66      |
| $^{40}$Ca | 3.36      | 3.31      | 3.46            | 3.45         | 8.52        | 81.35      |
| $^{41}_\Theta$Ca | 3.35      | 3.30      | 1.86            | 10.18        | 81.35       | 81.35      |
| $^{208}$Pb | 5.45      | 5.71      | 5.51            | 8.19         | 69.54       | 69.54      |
| $^{209}_\Theta$Pb | 5.44      | 7.70      | 3.87            | 2.49         | 69.54       | 69.54      |

4.2 Θ⁺ potential depth in nuclear matter

In nuclear matter the Θ⁺ potential depth is written as[24]

$$U = g_\sigma^\Theta \sigma^{eq} + g_\omega^\Theta \omega_0^{eq}$$

(41)

where $\sigma^{eq}$ and $\omega_0^{eq}$ are the values of $\sigma$ and $\omega$ fields at the saturation nuclear density, respectively. Use the relations $\sigma^{eq} = (M_N^* - M_N)/g_\sigma^N$ and $\omega_0^{eq} = g_\omega^N g_0/m_\omega^2$ at the saturation nuclear density, we can get[32]

$$U = M_N (M_N^* - 1) \frac{g_\sigma^\Theta}{g_\sigma^N} + \left( \frac{g_\omega^N}{m_\omega} \right) \frac{g_\omega^\Theta}{g_\omega^N} g_0$$

(42)

where $M_N^*$ is the effective mass of nucleon and $g_0$ is the saturation nuclear density. From Eq.(42), we can see that the depth of potential $U$ is determined by $\frac{M_N^*}{M_N}$, $\frac{g_\sigma^\Theta}{g_\sigma^N}$ and $g_0$, if we let $\frac{g_\omega^\Theta}{g_\omega^N} = 4/3$
seen that as “experimental input”. In the RMF calculation, potential depth at saturation nuclear density easily. From figure 2 (a), we can see as in Eq.(40). Usually, the effective mass $\frac{M^*_{NN}}{M_N}$ and saturation nuclear density $\sigma_0$ are considered as “experimental input”. In the RMF calculation, $\frac{M^*_{NN}}{M_N} \sim 0.60$, $\sigma_0 \sim 0.15\text{fm}^{-3}$[33]. Here, we let them have some uncertain and can change in a little region. According the calculation and analysis of section 3, when $g^\Theta_\sigma < 11.725(13.925 \times 0.84)$, there are no bound states for $\Theta^+$ in $^{12}\text{C}$, thus we let it as a approximation that there is no bound states for $\Theta^+$ in nuclear matter when $g^\Theta_\sigma < 11.725(13.925 \times 0.84)$, namely the depth of potential $U \geq 0$.

In figure 2, we show the relation of the potential depth changing with the effective mass $\frac{M^*_{NN}}{M_N}$ for $\sigma_0 = 0.15, \sigma_0 = 0.16$ and $\sigma_0 = 0.17\text{fm}^{-3}$ in a, b and c respectively. From figure 2(a), it is easily seen that $U=0$ corresponds to effective mass $\frac{M^*_{NN}}{M_N} = 0.60$ when $\sigma_0 = 0.15\text{fm}^{-3}$. With the same method we can determine $\frac{M^*_{NN}}{M_N} = 0.575$ for $\sigma_0 = 0.16\text{fm}^{-3}$ and $\frac{M^*_{NN}}{M_N} = 0.546$ for $\sigma_0 = 0.17\text{fm}^{-3}$ from b and c. The results of effective mass $\frac{M^*_{NN}}{M_N}$ is reasonable, because they are around 0.6, which agrees with the fit data in RMF[33].

Since the effective mass $\frac{M^*_{NN}}{M_N}$ and saturation nuclear density $\sigma_0$ are determined, we can get the potential depth at saturation nuclear density easily. From figure 2 (a), we can see $U=-81$ MeV at $g^\Theta_\sigma = 13.925$ and $U=-46$ MeV at $g^\Theta_\sigma = 13.925 \times 0.93$ when $\frac{M^*_{NN}}{M_N} = 0.60, \sigma_0 = 0.15\text{fm}^{-3}$. Considered the effect of medium, if the $\sigma - \Theta^+$ coupling constant is in the region $13.925 \leq g^\Theta_\sigma \leq 13.925 \times 0.93$, the potential depth is in the region $-81 \leq U \leq -46$ MeV. From figure 2(b) and (c), we also get $-84 \leq U \leq -47$ MeV for $\sigma_0 = 0.16\text{fm}^{-3}$ and $-92 \leq U \leq -52$ MeV for $\sigma_0 = 0.17\text{fm}^{-3}$ when $13.925 \leq g^\Theta_\sigma \leq 13.925 \times 0.93$ in the same way. From the calculation, it is found that the uncertain of the saturation nuclear density can bring some uncertain for the potential depth in several MeV. Because there is still no experimental data for $\Theta^+$ in nuclear matter, we can only estimate the potential depth in theory, the uncertain for the potential depth in several MeV is allowed. In fact, the potential depth estimated by us agrees well with that of in Ref.[16] $-120 \leq U \leq -60$ MeV.

| $g^\Theta_\sigma$ | $\langle r_p \rangle$ (fm) | $\langle r_n \rangle$ (fm) | $\langle r_\Theta \rangle$ (fm) | $\langle r_{ch} \rangle$ (fm) | $-E/A$ (MeV) | $-B$ (MeV) |
|------------------|-----------------|-----------------|-----------------|-----------------|-------------|-------------|
| 13.925           | 2.17            | 2.15            | 1.19            | 2.32            | 14.24       | 116.24      |
| 13.725           | 2.18            | 2.16            | 1.22            | 2.33            | 13.27       | 100.00      |
| 13.525           | 2.19            | 2.17            | 1.28            | 2.34            | 12.33       | 83.70       |
| 13.325           | 2.21            | 2.19            | 1.35            | 2.36            | 11.43       | 67.72       |
| 13.125           | 2.24            | 2.21            | 1.46            | 2.38            | 10.60       | 52.92       |
| 12.925           | 2.26            | 2.24            | 1.59            | 2.41            | 9.85        | 40.17       |
| 12.725           | 2.29            | 2.27            | 1.76            | 2.44            | 9.19        | 29.67       |
| 12.525           | 2.32            | 2.29            | 1.95            | 2.46            | 8.60        | 21.20       |
| 12.325           | 2.34            | 2.31            | 2.20            | 2.48            | 8.10        | 14.43       |
| 12.125           | 2.35            | 2.32            | 2.50            | 2.49            | 7.68        | 9.08        |
| 11.925           | 2.35            | 2.32            | 2.90            | 2.49            | 7.34        | 4.98        |
| 11.725           | 2.35            | 2.32            | 3.43            | 2.49            | 7.10        | 2.01        |
5 Summary

We have educed the coupling constants for strange many quarks system with quark meson coupling model. We get the relation \( g_\Theta^\sigma = \frac{4}{3} g_\sigma^N \Gamma_{\Theta/B} \), \( g_\Theta^\omega = \frac{4}{3} g_\omega^N \) for \( \Theta^+ \) coupling with \( \sigma \) and \( \omega \) meson. With the coupling constants for \( \sigma - \Theta^+ \) and \( \omega - \Theta^+ \) educed by us, the calculations for \( \Theta^+ \) in nuclei are carried out in the framework of RMF. From the single-particle levels of \( \Theta^+ \) in nuclei, it is found that there are several bound states for light nuclei, such as \( ^7_\Lambda Li \), and more bound states for heavier nuclei. The binding energies of \( 1s1/2 \) are usually as high as 50 MeV to 120 MeV, which means that there are strong attractive interaction for \( \Theta^+ \)-nucleus. This prediction of RMF is agree with the prediction of Refs.[15, 16]. The strong attractive interaction for \( \Theta^+ \)-nucleus induces stronger per nucleon binding energy than normal nuclei, which can be seen obviously in table 2. Because of the strong attractive interaction for \( \Theta^+ \)-nucleus, the shrinking effect is found for light nuclei \( ^6_\Lambda Be \) and \( ^{13}_\Lambda C \), in medium or heavy nuclei there is no obvious shrinking effect. The shrinking effect is first found in the light \( \Lambda \)-hypernucleus for lithium hypernuclei[30], but there is no shrinking effect for \( ^7_\Lambda Li \) in RMF prediction.

The depth of optical potential for \( \Theta^+ \) in nuclear matter is investigated, the region is given. If we assume the saturation nuclear density \( \rho_0 = 0.15 \) 0.16 and 0.17 fm\(^{-3}\) with the coupling constant \( 13.925 \leq g_\sigma^\Theta \leq 13.925 \times 0.93 \), the depth of potential \(-81 \leq U \leq -46\) MeV, \(-84 \leq U \leq -47\) MeV and \(-92 \leq U \leq -52\) MeV, respectively.

In this work we assume that \( J^P = 1/2^+ \) and \( I = 0 \) for \( \Theta^+ \), and when we induce the coupling constants in section 3 with QMC model, we do an approximation where the \( \sigma(r) \) \( \omega(r) \) and \( \rho(r) \) fields couple only to the \( u \) and \( d \) quarks. These assume and approximation may be affect our results.

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Figure 1: Single-particle levels for $\Theta^+$ in $^6$Li, $^{12}$C, $^{16}$O, $^{40}$Ca and $^{208}$Pb denoted with 1,2a,2b,2c,2d,3,4a,4b,4c and 5 respectively. 1, 2a, 3, 4a, 5 are our results with RMF at $g^\Theta_{\sigma} = 13.925$ and 2b is our result at $g^\Theta_{\sigma} = 13.925 \times 0.95$. 2c,2d and 4b,4c are the results from Ref.[16] in $^{12}$C and $^{40}$Ca at the depth of potential 60 MeV and 120 MeV respectively.
Figure 2: The depth of potential for $\Theta^+$ in nuclear matter versus effective mass $M^*/M$ for fixed values of $U$ at the saturation nuclear density. a, b and c is for $\rho_0 = 0.15, 0.16, 0.17$fm$^{-3}$ respectively. The dash dot line is for $g_\Theta = 13.925 \times 0.84$, dash line is for $g_\Theta = 13.925 \times 0.93$, and solid line is for $g_\Theta = 13.925$. 