Electroweak Baryogenesis with a Second Order Phase Transition

ROBERT H. BRANDENBERGER

and

ANNE-CHRISTINE DAVIS

1) Physics Department, Brown University
   Providence, RI 02912, USA

2) Department of Applied Mathematics
   and Theoretical Physics & Kings College
   University of Cambridge, Cambridge, CB39EW, U.K.

ABSTRACT

If stable electroweak strings are copiously produced during the electroweak phase transition, they may contribute significantly to the presently observed baryon to entropy ratio of the universe. This analysis establishes the feasibility of implementing an electroweak baryogenesis scenario without a first order phase transition.

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1. **Introduction**

The mechanisms suggested so far\(^1\text{−}^4\) for electroweak baryogenesis all rely on having a first order phase transition. The resulting bubble walls were required in order to obtain a region of unsuppressed baryon number violation occurring out of thermal equilibrium. Our work\(^5\) is based on the observation that topological defects forming in a second order phase transition may play a similar role to the bubble walls. We propose a specific mechanism in which electroweak strings are responsible for baryogenesis.

Electroweak strings\(^6\) are nontopological solitons which arise in the standard electroweak theory (and extensions thereof). They are essentially Nielsen-Olesen strings of \(U(1)_Z\) embedded in the \(SU(2) \times U(1)\) theory (\(U(1)_Z\) is the Abelian subgroup which is broken during the electroweak phase transition). For certain ranges of the parameters of the standard model, electroweak strings are energetically stable\(^7\) (They are not topologically stable).

If, however, we are in a region of parameter space in which electroweak strings are stable, a network of such strings will form during the electroweak phase transition, even if it is second order. Inside the strings, anomalous baryon number violating processes are unsuppressed. If the strings move, the out of thermal equilibrium condition will be satisfied. Finally, the standard model contains \(CP\) violation. Hence all of Sakharov’s criteria are satisfied. As we shall demonstrate, it is in fact possible to generate a substantial \(n_B/s\) using electroweak strings.

In order to obtain a sufficiently large baryon to entropy ratio, the standard electroweak model must be extended by adding new terms in the Lagrangian which contain explicit \(CP\) violation. An often used prototype theory is the two Higgs model\(^2\text{−}^4\)

The construction of nontopological vortex solutions in theories which do not satisfy the topological criterion for strings is not specific to the minimal standard model. Thus, we expect electroweak strings to exist also in extensions of the
Weinberg-Salam model (This has recently been demonstrated in the two Higgs model\textsuperscript{8}). It is possible that these strings could be stable even for experimentally allowed values for the model parameters. In the following we shall assume that electroweak strings exist and are stable.

In models admitting stable electroweak strings, a network of such strings will form during the electroweak phase transition. If we consider a theory with Higgs potential

\[ V(\phi) = \lambda(\phi^+ \phi - \eta^2/2)^2, \]  

then the initial correlation length (mean separation of strings) will be\textsuperscript{9})

\[ \xi(t_G) \simeq \lambda^{-1} \eta^{-1}, \]  

where \( t_G \) is the time corresponding to the Ginsburg temperature of the phase transition.

The initial network of electroweak strings will be quite different from that of cosmic strings, the reason being that electroweak strings can end on local monopole and antimonopole configurations. From thermodynamic considerations\textsuperscript{10}), we expect most of the strings to be short, \( i.e., \) of length \( l \simeq \xi(t_G) \), since this maximizes the entropy of the network for fixed energy.

After the phase transition, the vortices will contract along their axes and decay after a time interval

\[ \Delta t_s \simeq \frac{1}{v}(\lambda\eta)^{-1} \]  

where \( v \) is the velocity of contraction (expected to be \( \approx 1 \)). In the following, we shall demonstrate that the string contractions will produce a net baryon symmetry. We are using units in which \( c = \hbar = k_B = 1 \).
2. The Baryogenesis Mechanism

We shall consider an extension of the standard electroweak theory in which there is additional CP violation in the Higgs sector. An example is the two Higgs model used in Refs. 2-4. We assume that electroweak strings can be embedded in this model\(^8\), and we choose the values of the parameters in the Lagrangian for which these strings are stable. Furthermore, the phase transition is taken to be second order.

A key issue is the formation probability of electroweak strings. In the following, we make the rather optimistic assumption that both the mean length and average separation of electroweak strings at \(t_G\) will equal the correlation length \(\xi(t_G)\). For topological defects, this result follows from the Kibble mechanism\(^9\). When applied to electroweak strings, the Kibble mechanism implies that the vortex fields \(\phi\) and \(Z\) have the correlation length \(\xi(t_G)\). However, to form an electroweak string, the other fields must be sufficiently small such that the configuration relaxes to the exact electroweak string configuration. Obviously, the restriction this imposes (and the consequent increase in the mean separation of electroweak strings) is parameter dependent - the more stable the strings, the smaller the increase in the mean separation. Pieces of string are bounded by monopole-antimonopole pairs. Energetic arguments tell us that the string will shrink. We now argue that the moving string ends will have the same effects on baryogenesis as the expanding bubble walls in Refs. 2&3.

We can phrase our argument either in terms of the language of Ref. 2 or of Ref. 3. The phase of the extra \(CP\) violation is nonvanishing in the region in which the Higgs fields \(\phi\) are changing in magnitude, \(i.e.,\) at the edge of the string. Since \(|\phi|\) increases in magnitude, \(CP\) violation has a definite sign. Hence, in the language of Ref. 3, a chemical potential with definite sign for baryon number is induced at the tips of the string (where \(|\phi|\) is increasing). This chemical potential induces a nonvanishing baryon number.

In the language of Ref. 2, the \(CP\) violation with definite sign at the tips of the
string leads to preferential decay of local texture configurations with a definite net change in Chern-Simons (i.e., baryon) number.

Let us now estimate the magnitude of this effect. The rate of baryon number violating events inside the string (in the unbroken phase) is

$$\Gamma_B \sim \alpha_w^4 T^4. \quad (4)$$

The volume in which $CP$ violation is effective changes at a rate ($g$ is the gauge coupling constant)

$$\frac{dV}{dt} = g^2 w^2 V, \quad (5)$$

where $w \simeq \lambda^{-1/2} \eta^{-1}$ is the width of the string and $v$ is its contraction velocity. The factor $g$ comes from the observation that baryon number violating processes are unsuppressed only if $|\phi| < g\eta$.11) The rate of baryon number generation per string is

$$\frac{dN_B}{dt} \sim w^2 v \Gamma_B \epsilon \Delta t_c, \quad (6)$$

where $\epsilon$ is a dimensionless constant measuring the strength of $CP$ violation and

$$\Delta t_c = \frac{gw}{v} \frac{1}{\gamma(v)} \quad (7)$$

is the time a fixed point in space is in the transition region. Here, $\gamma(v)$ is the usual relativistic $\gamma$ factor. Since there is one string per correlation volume $\xi(t_G)^3$, the resulting rate of increase in the baryon number density $n_B$ is

$$\frac{dn_B}{dt} \sim \lambda^{-3/2} \eta^{-3} g^3 \alpha_w^4 T^4 \frac{1}{\gamma(v)} \xi(t_G)^{-3}. \quad (8)$$

The net baryon number density is obtained by integrating (8) from $t_G$, the time
corresponding to the Ginsburg temperature, and $t_G + \Delta t_S$ (see (3)). The result is

$$n_B \sim \frac{\lambda}{v\gamma(v)} g^3 \alpha_w^4 T_G^3 \epsilon.$$  \hspace{1cm} (9)

Our result (9) must be compared to the entropy density at $t_G$:

$$s(t_G) = \frac{\pi^2}{45} g^* T_G^3,$$  \hspace{1cm} (10)

where $g^*$ is the number of relativistic spin degrees of freedom. From (9) and (10) we obtain

$$\frac{n_B}{s} \sim \frac{45}{\pi^2 g^* \gamma(v) v} \frac{\lambda}{\epsilon g^3 \alpha_w^4}.$$  \hspace{1cm} (11)

For $\lambda \sim v \sim 1$ and $\epsilon \sim 1$, the ratio obtained is only slightly smaller than the observational value.

In order for our mechanism to work, the core radius of the string ($|\phi| < g\eta$) must be large enough to contain the nonperturbative configurations which mediate baryon number violating processes. This leads to the condition $\lambda < g^4$, i.e. small Higgs mass. In addition, the sphaleron must be sufficiently heavy such that sphaleron transitions in the broken symmetry phase are suppressed for $T = T_G$. For small values of $\lambda$, this condition will automatically be satisfied. Finally, the model parameters must be such that the phase transition is of second order. In the standard electroweak theory, this condition is incompatible with $\lambda \ll g^4$. In any extended electroweak theory, the consistency of the above conditions must be satisfied in order for our baryogenesis mechanism to be effective.
3. Discussion

We have presented a counterexample to the “folk theorem” stating that electroweak baryogenesis requires a first order electroweak phase transition. We propose a mechanism in which finite length electroweak strings during their contraction generate a nonvanishing net baryon number. The strings play a similar role to the expanding bubble walls in a first order phase transition: they provide out of equilibrium processes, and also a region where \( CP \) violation occurs.

The mechanism presented here requires stable electroweak strings and an extra source of \( CP \) violation (which is present in the two Higgs models used in Refs. 2-4). Based on the stability analysis of electroweak strings in the standard model\(^7\), it is unlikely that these strings will be stable for experimentally allowed values of the parameters in the Lagrangian.

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