Generalizing the effect of Indian populace with the help of mathematical modeling

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Abstract. In this paper, a mathematical model of Indian populace is investigated. For this, a secondary data of the Indian populace has been collected from IDB (International Data Base) of the years from 1960 to 2018 (inclusive). Also, we will deduce the expected populace of India up to the year 2060 by the using the least square method. Then, we will represent this populace in form of graphs using MATLAB. Also, we will find out the carrying capacity of future Indian populace. Finally we will find out the effect of this populace on India.

Keywords: Malthus and logistic growth model, MATLAB, carrying capacity, vital coefficient, IDB (International database), populace.

1. Introduction
In this paper, a mathematical model of Indian populace is investigated. For this purpose, a secondary data of the Indian populace has been collected from IDB (International Data Base) of the years from 1960 to 2018 (inclusive). Also, we will deduce the expected populace of India up to the year 2060 by the using the least square method. Then we will represent this populace in the form of graphs using MATLAB. Also we will find out the carrying capacity of future Indian populace. Finally we will find out the effect of this populace on India. Increasing or decreasing of the populace of a country is directly effects on poverty, employment, economy, culture, education, natural resources and environment of that country. Mainly populace effects water, food, space and natural resources. There are two different graph which is show that the effects populace on water, food, space and natural resources.
In these figures we see the effect i.e., if populace decreases then basic need of humans that is water, space, food and natural resources increases and also if the populace increases then the basic need of human that is water, space, food and natural resources decreases. Populace problem has become one of the most important problems in front of a country. Government always requires precise design of populace. Here we predict future populace of India [5].

2. Methodology
In this paper, we are collecting the data of Indian populace for the years 1960 to 2018 (conceding the populace of 2018 also) from IDB (International Data Base). Also we compute the predicted populace and then represent it in the form of graphs by using MATLAB software and comparison will be done between actual populace and predicted by using least square method we will find populace growth, rate caring capacity and also determine the years in which India population will become approximately half of the value of its carrying capacity.
3. Development of the model

Let \( B(t) \) denotes the populace clan at time \( t \) and ‘a’ is the inequality between its birth rate and death rate. If this populace is isolated from other, then by [2, 3], \( \frac{d}{dt} B(t) \), i.e., the rate of change of populace equal to ‘a \( B(t) \)’, where ‘a’ is a constant by [8]. In this case, differential equation of populace growth is

\[
\frac{d}{dt} B(t) = aB(t) \tag{3.1}
\]

where ‘a’ is the Malthusian factor that determines the growth rate [1]. Equation (3.1) is a non-homogeneous linear first order differential equation called Malthusian law of populace growth. ‘\( B(t) \)’ is discontinuous function of ‘t’ and takes further integral appraisals [2,3]. Although, it may be approximated by differential and continuous function as soon as the no. of individual is large enough [2, 3, 8]. On solving equation (3.1), we get

\[
B(t) = B_0 e^{at} \tag{3.2}
\]

So, any clan satisfying the Malthusian law of populace growth extend accumulate with time [2, 3]. It can be relate as approximate physical law as it is generally acknowledged that not a single thing can extend at a constant rate indefinitely when the size of populace is increases in size [8]. A Belgian mathematician Verhust [4] define that the populace size not lone build upon populace size but further on how faraway this size is from its maximal limit [2, 3, 8]. He mutated Malthus model. [2, 3]

\[
\frac{a-bB(t)}{a} \tag{3.3}
\]

where ‘a’ and ‘b’ are vital coefficient of populace. Equation (3.3) reflects how faraway the populace is from its maximal limit [2, 3]. Although, while the populace appraisal increases and gets closer to \( \frac{a}{b} \), this is very small term and approach to zero [2, 3],

\[
\frac{d}{dt} B(t) = \frac{aB(t)(a-bB(t))}{a} \tag{3.4}
\]

Here the populace ‘\( B(t) \)’ on the right of equation (3.4) is being multiplied by itself. This equation is called logistic law of populace growth. Put \( B = B_0 \) for \( t = 0 \), where \( B_0 \) is the populace at same particular time, \( t = 0 \) so, equation (3.4) becomes [2, 3]

\[
\frac{d}{dt} B = aB - bB^2 \tag{3.5}
\]

Separating and integrating equation (3.5), we get

\[
\int \frac{1}{a \left( \frac{1}{B} + \frac{b}{a-bB} \right)} dB = t + c' \]

\[
\frac{1}{a} \left( \log B - \log (a - bB) \right) = t + c' \tag{3.6}
\]

Use \( t = 0 \) and \( B = B_0 \), So,

\[
c' = \frac{1}{a} \left( \log B_0 - \log (a - bB_0) \right)
\]

\[
\frac{1}{a} \left( \log B - \log (a - bB) \right) = t + \frac{1}{a} \left( \log B_0 - \log (a - bB_0) \right)
\]
Solving for B, we have [2] \[ B = \frac{a}{b} \left( 1 + \frac{a}{b_0 - 1} \right) e^{-at} \] \quad (3.7)

Taking the limit \( t \to \infty \) in equation (3.7), we get (since \( a > 0 \))

\[ B_{\text{max}} = \lim_{t \to \infty} B = \frac{a}{b} \] \quad (3.8)

Now we have to find the appraisal of a, b and \( B_{\text{max}} \) by using least square method [8].

Now differentiating equation (3.7), twice with respect to \( t \), we get [2, 3, and 8]

\[ \frac{d^2B}{dt^2} = \frac{Ra^3e^{at}(R-e^{at})}{b(R+e^{at})^3} \] \quad (3.9)

Where \( R = \frac{a}{b_0} - 1 \)

At point of inflection B must be equal to zero, then

\[ R = e^{at} \] \quad (3.10)

This equation solving for \( t \), we get

\[ t = \log \frac{R}{a} \] \quad (3.11)

This is the time when the populace is a half of the appraisal of its carrying capacity, when the point of inflection is occurs [8]. Let time when the point of inflection occurs be \( t = t_k \) then \( R = e^{at} \) becomes \( R = e^{at_k} \) using latest appraisal of R and renewal \( \frac{a}{b} \) aside P equation (3.7) develop into. [2, 3, 8]

\[ B = \frac{P}{1 + e^{-a(t-t_k)}} \] \quad (3.12)

Let us assume the coordinates of actual populace appraisals is \((t, t')\) and the coordinates of the predicted populace appraisals with same abscissa on the fitted curve be \((t, B)\), then in this case error be \((B - t')\). Since some of the actual populace data point lie below the curve of predicted appraisals as other lie above it, we square \((B - t')\) to ensure that the error is positive. Thus, the total squared error, \( e \), in fitting the curve is given by [2, 3, and 8]

\[ e' = \sum_{k=1}^{n} (B_k - t'_k)^2 \] \quad (3.13)

Equation (3.13) have three restrictions P, a, and \( t_k \) for ejecting P, we assume

\[ B = Ph' \] \quad (3.14)

Where \( h' = \frac{1}{1 + e^{-a(t-t_k)}} \) \quad (3.15)

Using the appraisal of B from equation (3.14) and algebraic product of equation (3.13), we have [2, 3, and 8]
\[ e' = \sum_{k=1}^{n} (B_k - t_k')^2 \]
\[ = (B_1 - t_1')^2 + \ldots + (B_n - t_n')^2 \]
\[ = (P h_i' - t_i')^2 + \ldots + (P h_n' - t_n')^2 \]
\[ = |(P h_1', \ldots, P h_n')(t_1', \ldots, t_n')|^2 \]
\[ = |P H' - V|^2 \]
\[ = P^2(H', H') - 2P \langle H', V \rangle \]

Where \( H' = (h_1', \ldots, h_n') \) and \( V = (t_1', \ldots, t_n') \) Thus,

\[ e' = P^2(H', H') - 2P \langle H', V \rangle \]  \hspace{1cm} (3.16)

Partially differentiate of \( e' \) with respect to \( P \) and taking equal to zero, we get

\[ 2P \langle H', H' \rangle - 2 \langle H', V \rangle = 0 \] So, \([2, 3, 8]\)

\[ P = \frac{\langle H', V \rangle}{\langle H', H' \rangle} \]  \hspace{1cm} (3.17)

Putting the appraisal of \( P \) in equation (3.16), we obtain

\[ e = \langle V, V \rangle - \frac{\langle H', V \rangle^2}{\langle H', H' \rangle} \]  \hspace{1cm} (3.18)

This equation is changed into an error function, that contains just two restrictions ‘a’ and ‘\( t_k' \)’. There appraisals were found, and used in equation (3.17) to find the appraisal of \( P \) \([8]\).

4. Results

In 2015 populace of India was 1.31 billion. If the populace growth rate like this, then in 2022 India will become world’s most populated country and this is not a good thing because if populace increases, then many problems will have to face the country. The basic need of human will be difficult to complete like water, space, food and natural resources. These will effects the Indian culture also. Now following are different solution of overpopulation.

4.1 One child policy of China

In front of this question, many people thinks about one child policy of china \([7]\) but further certain man’s know that it was a fail liar due to this China faces Gender imbalance and female filicide increases because China is also traditionally gender bias society like India in China people are prefer to that they have at least one boy child. That’s why after 30 years, China withdraw this policy in 2015. So, on the basis of all these cases in India one child policy of China is not a solution.

4.2 India’s force sterilization

Many people are thinking that force sterilization is a solution to overcome the populace. India tried this in 1976 in emergency. In this program govt. sterilized 10 million peoples forcefully. But the bigger
thing is that after this forcefully sterilization there is no change on Indian populace growth. Infect there was no noticeable change there. And also this is against human rights. So on the basis of all this in India forcefully sterilization is not a solution.

4.3 Sifted on different planet

Some people are thinking like that people sifted on different planet but we don’t know about any planet which have life. Also it is very expansive. A govt. or a common person can’t afford this program. So, this is also not a good solution for over populace. Now we are plotting some graphs on the basis of different year’s populace which is secondary data from IDB (international Data Base) and also some year’s data which will be predicted by the use of least square method. These are given below:

Table I: Actual appraisals of populace

| Sr.No. | Years | populace       |
|--------|-------|----------------|
| 1      | 1960  | 449480608      |
| 2      | 1965  | 497702365      |
| 3      | 1970  | 553578513      |
| 4      | 1975  | 621301720      |
| 5      | 1980  | 696783517      |
| 6      | 1985  | 781666671      |
| 7      | 1990  | 870133480      |
| 8      | 1995  | 960482795      |
| 9      | 2000  | 1015974042     |
| 10     | 2005  | 1144118674     |
| 11     | 2010  | 12030980691    |

Graph of the Indian population from year 1950 to 2010
In table-I we have a secondary data of Indian populace from the year 1960 to 2010 this data is collected from IDB (International Data Base). From this data we plot the graph of actual populace of India from the year 1960 to 2010 by the using of MATLAB.

**Table 2.** Actual appraisals of populace.

| Sr.No. | Years   | populace       |
|-------|---------|----------------|
| 1     | 2011    | 1247236029     |
| 2     | 2012    | 1263065852     |
| 3     | 2013    | 1278562207     |
| 4     | 2014    | 1263859294     |
| 5     | 2015    | 13101524032016 |
| 6     |         | 1324517249     |
| 7     |         | 20171338676785 |
| 8     |         | 2018135264228020191369587667 |
| 9     |         |                |

Figure 3. Graph for table-I.

Figure 4. Graph for table-2.
In table-2 we have a secondary data of Indian populace from the year 2011 to 2019 this data is collected from IDB (International Data Base). From this data we plot the graph of actual populace of India from the year 2011 to 2019 by the using of MATLAB.

### Table 3. Predicted appraisals of populace.

| Sr.No. | Years | populace     |
|--------|-------|--------------|
| 1      | 2020  | 1384845360   |
| 2      | 2025  | 1461133827   |
| 3      | 2030  | 1537422296   |
| 4      | 2035  | 1613710764   |
| 5      | 2040  | 1689999231   |
| 6      | 2045  | 1766287700   |
| 7      | 2050  | 1842576168   |
| 8      | 2055  | 1918864635   |
| 9      | 2060  | 1998864635   |

![Graph of the Indian population of the year from 2020 to 2060](image)

**Figure5.** Graph for table-3.

In table-3 we have a predicted data of Indian populace from the year 2020 to 2060 this data is predict by using the least square method. From this data we plot the graph of predicted populace of India from the year 2020 to 2060 by the using of MATLAB.
Also we are plot a multiple graph of the populace from 1960 to 2010, 2011 to 2019 and 2020 to 2060 from the combination of Table-I, Table-II and Table-III. In all these graph of Indian populace we noticed that the Indian populace is increased every time, this is not a good think. Also this is a very big problem in front of India. Some solutions are given above but these are not work in front of Indian populace. So the question is that “what is the realistic solution of Indian populace”

4.4 Realistic solution over populace

So what is the realistic solution of Overpopulation the answer of that question is Education and Health care a very important point is there, that is fertility rate means on an average a woman of a count How many children born in over life. If the replacement fertility rate is 2.2 of a country then the correlation between the fertility rate and literacy rate becomes stable. So a country that is achieve the replacement fertility rate 2.2 Then the future populace of that country is stable. In India some state have fertility rate below then 2.2 but some state have fertility rate above then 2.2 Mainly Rajasthan, Madhya Pradesh, Utter Pradesh Bihar and Haryana. Due to these states, India becomes over populated.

5. Conclusion

If the replacement fertility rate is 2.2 of a country then the correlation between the fertility Rate and literacy rate becomes stable. So a country that is achieved the replacement fertility rate 2.2, then the future populace of that country is stable. In India some state have fertility rate below then 2.2 But some states have fertility rate above then 2.2 mainly Rajasthan, Madhya Pradesh, Utter Pradesh, Bihar and Haryana. Due to these states India becomes over populated. If we are successful to decrease the fertility rate of this state then we can stop to increase to populace. It is possible further when we are work on education and health care.

6. References

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