Nonlinear Optical Properties of a Quantum Well With Short-Range Bottomless Exponential Potential

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Abstract. In the present study, the effect of the depth of the confinement potential on the nonlinear optical of a GaAs quantum well with short-range bottomless exponential potential is studied in detail. The energy eigenvalues and eigenfunctions of this structure are calculated within the framework of effective mass and envelope function approximations. Analytic formulas for the linear, third-order nonlinear and total absorption coefficients and relative refractive index changes are obtained using the compact-density matrix approach (CDMA) and iterative method. Based on this model, our obtained numerical results are reported as a function of incident photon energy for several values of the depth of the confinement potential. The results show that the linear, third order nonlinear, and total absorption coefficients and relative refractive index changes are strongly affected by the depth of the confinement potential.

Keywords: Quantum well, nonlinear optical properties.

1. INTRODUCTION

Low dimensional semiconductor heterostructures, such as quantum wells (QWs), quantum well wires (QWWs), quantum rings (QRs), and quantum dots (QDs) have been a subject of great interest in the recent years due to their fundamental properties and their potential device applications. Among these heterostructures, QWs having any desired potential shape, such as square, parabolic, semi-parabolic, inverse parabolic, graded, and V-shaped have received more attention from researchers. In particular, recent advances in the material growth techniques, such as molecular-beam epitaxy (MBE) and metal organic chemical vapor deposition (MOCVD), combined with the use of the modulation doping technique,
have made it possible to fabricate high quality low-dimensional semiconductor QWs. These QWs have a different shaped potential well which provides an additional quantum confinement to free carriers. This potential well can provide high density two-dimensional electron gases (2DEGs) in these structures. Furthermore, the quantum confinement of charge carriers in the QWs, which leads to the formation of discrete energy levels called subbands, results in a drastic change in the nonlinear optical properties related to intersubband transitions (ISBTs) between the subbands within the conduction band of the QWs. For these reasons, recent studies have been focused on the nonlinear optical properties of QWs since these structures have various potential applications for optoelectronic devices based on ISBT, such as infrared lasers [1], ultra-fast infrared detectors [2], high-speed electro-optical modulators[3], and all optical switches[4].

The nonlinear optical properties of QWs, in particular optical absorption coefficients (OACs), the change of index changes (RICs), second (SHG) and third harmonic generations (THG) or nonlinear optical rectification (NOR), have attracted a lot of attention in theoretical studies in recent years[5-20]. For example, Aytekin et. al [15] calculated the NOR, SHG and THG for a Pöschl-Teller QW under electric and magnetic fields. Their numerical results show that the NOR, SHG and THG are strongly influenced by the quantum confinement, electric and magnetic fields. Panda and Panda[16] discussed the effect of intense laser field (ILF) on the nonlinear optical susceptibilities in an asymmetric single QW. Campione et. al[17] theoretically analyzed the SHG capacity of two-dimensional periodic meta-materials comprising sub-wavelength resonators strongly coupled to ISBTs in QWs at mid-infrared frequencies. The effect of non-resonant ILF on the linear and nonlinear ISB optical properties in a strained InGaN/GaN QW has been investigated by Karimi and Vafaei[18]. Orozco et al.[19] reported the computation of the linear and third order corrections, due to ILF effects, of the ACs and RICs, for an asymmetric double δ-doped GaAs MIGFET-like potential profile. Li et al.[20] intensively studied the effect of position-dependent effective mass on the nonlinear optical properties in a GaAs/AlAs semiconductor QW. They found that the position-dependent effective mass has a significant impact on the ACs, RICs and THG in a QW. Recently, we have studied the ISBT optical ACs and RICs in the δ-doped GaAs QWs under ILF[21]. We have also studied the ILF effect on the electronic states and optical properties of n-type double δ-doped GaAs QWs [22]. Although many works have been done on the linear and nonlinear optical properties of low-dimensional semiconductor heterostructures, but the OACs and RICs of GaAs quantum well with short-range bottomless exponential potential has not studied so far. For this purpose, we intend to study this problem in detail.

In this work, we are concerned with a theoretical study of the effect of the depth of the confinement potential on the nonlinear optical of a GaAs quantum well with short-range bottomless exponential potential. In this regard, we calculate the eigenfunctions and the corresponding energy eigenvalues by solving the Schrödinger equation. After the energies and related wave functions are obtained, the nonlinear optical properties, such as OACs and RICs are obtained from CDMA and iterative procedure. The organization of this paper is as follows: In Section 2, the description of the theoretical model is presented. The obtained numerical results are given and discussed in Section 3. Finally, the main conclusions of our work are given in Section 4.
2. THEORY

Let us consider a GaAs QWs with short-range bottomless exponential potential grown in the $z$-axis, where electrons act freely in the $(x, y)$ plane. In the effective-mass approximation, the Hamiltonian of one electron in this structure is given by

$$
H = -\frac{\hbar^2}{2m^*} \nabla^2 + V_{SRTEP}(z),
$$

(1)

In this expression, $m^*$ is the conduction band effective mass, $\hbar$ is the reduced Planck constant, and $V_{SRTEP}(z)$ represents the confined potential of the well potential which is given by the following expression [23]:

$$
V_{SRTEP}(z) = \frac{V_0}{\sqrt{1-e^{-z}}} - V_0
$$

(2)

where $V_0$ is the depth of the well potential. According to the effective-mass and envelope wave function approach, the Schrödinger equation for one electron in the GaAs QWs with short-range bottomless exponential potential is given by

$$
\left[ -\frac{\hbar^2}{2m^*} \frac{d^2}{dz^2} + \frac{V_0}{\sqrt{1-e^{-z}}} - V_0 \right] \psi(z) = E\psi(z)
$$

(3)

where $E$ and $\psi(z)$ are eigenvalues and eigenfunctions, respectively. The eigenvalues and eigenfunctions of the GaAs QWs with short-range bottomless exponential potential were calculated numerically using diagonalization method [24]. After finding the electronic states, by using the CDMA via the iterative method, the expressions for the linear, third-order nonlinear, and total OACs and RICs are given as [21, 22, 24]

$$
\beta^{(1)}(\omega) = \omega \left[ \frac{\mu}{\varepsilon_0} \frac{|M_{12}|^2 \gamma_0 \hbar \Gamma_{10}}{(\Delta E - \hbar \omega)^2 + (\hbar \Gamma_{10})^2} \right],
$$

(4)

$$
\Delta n^{(1)}(\omega) = \frac{\sigma_0 M_{12}^2}{2 \pi^2 n_r^2} \left[ \frac{(\Delta E - \hbar \omega)^2}{(\Delta E - \hbar \omega)^2 + (\hbar \Gamma_{10})^2} \right],
$$

(5)

$$
\beta^{(3)}(\omega, I) = -2\omega \left[ \frac{\mu}{\varepsilon_0} \frac{|M_{12}|^2 \gamma_0 \hbar \Gamma_{10}}{(\Delta E - \hbar \omega)^2 + (\hbar \Gamma_{10})^2} \right] \left[ 1 - \frac{|M_{22}-M_{11}|^2}{|M_{12}|^2} \frac{(\Delta E - \hbar \omega)^2}{(\Delta E - \hbar \omega)^2 + (\hbar \Gamma_{10})^2} \right]
$$

(6)

$$
\Delta n^{(3)}(\omega, I) = \frac{\mu |M_{12}|^2}{4 \pi^2 n_r^2} \left[ 4(\Delta E - \hbar \omega)^2 - (M_{22}-M_{11})^2 \left[ (\Delta E - \hbar \omega)^2 + (h \Gamma_{10})^2 \right] \right].
$$

(7)

Here, $n_r$ is the refractive index, $\mu$ is the magnetic permeability, $\varepsilon_0$ is the dielectric permittivity of the vacuum, $\gamma_0$ is the electron density, $\omega$ is the angular frequency of the incident photon, $I$ is the intensity of electromagnetic field, $\Delta E = (E_1 - E_0)$, $M_{ij} = \langle \psi_j | z | \psi_i \rangle$, $(i, j = 0, 1)$ is the product of matrix elements and also means geometric factor, $\Gamma_{10} = 1/\tau_{10}$ indicates the relaxation rate of the final and initial states.

The total ACs and RICs can be written as follows:

$$
\beta(\omega, I) = \beta^{(1)}(\omega) + \beta^{(3)}(\omega, I)
$$

(8)

$$
\Delta n(\omega, I) = \frac{\Delta n^{(1)}(\omega)}{n_r} + \frac{\Delta n^{(3)}(\omega, I)}{n_r}.
$$

(9)

3. RESULTS AND DISCUSSIONS

For numerical calculations, we take $m^* = 0.067 m_0$ where $m_0$ is the free electron mass, $n_r = 3.2, I = 0.05 mW/cm^2, \gamma_0 = 3.0 \times 10^{22} m^{-3}, \mu = 4\pi \times 10^{-7} Hm^{-1}$, and $\tau_{10} = 0.14 ps$. 
To make the analysis of the depth of the confinement potential effect on the OACs and RICs in the GaAs QWs with short-range bottomless exponential potential, in Figure 1(a) and 1(b), we show the squared wave functions of the first two quantum confined energy levels together with the form of the modified GaAs QWs with short-range bottomless exponential potential for the system under study. The modified potential profile and the absolute square of the wave functions have been presented as a function of the z-axis. As seen from these figures, the width of the well where the electrons are surrounded begins to increase with the increase of the $V_0$. The subband energy levels move away from each other as a result of this effect. Furthermore, the electron wave-functions to be more localized at the center of the well potential since the effective width of the potential increases with $V_0$. Thus, we observe from these figures that the confinement potential profile and the subband energy states change drastically by the increase of $V_0$.

Figure 1. The variations of the confinement potential profile, energies of the first two subband energy levels and the squared wave functions corresponding to these energy levels for two different the depth of the confinement potential.

In figure 2, we depict the OACs as a function of the photon energy for two different values of $V_0$. This figure obviously demonstrates that, when increase the $V_0$, the OACs coefficient moves toward the higher energies (blue-shift). This is clarified by the increase of the subband energy difference $E_{10}$ when the $V_0$ increases. Furthermore, it is clearly seen that the maximum value of the OACs coefficient increases by increasing the $V_0$. This is clarified by the increase of the dipole matrix element and the enhance of the effective width of the confinement potential, which means that the overlap of the wave function of the first two energy levels are increased when the $V_0$ enhances.

Figure 2. The variation of the OACs coefficient with the incident photon energy for two different $V_0$ values.

The RICs as a function of the photon energy for two different values of $V_0$ are plotted in Figure 3. From this figure, we can see that the magnitude of the RICs decreases with the augment of $V_0$. The different behavior of RICs according to the OACs owing to the
additional terms in the matrix elements that appear in equations (5) and (7) and compensate the decrement in the difference between energy levels. Furthermore, we can see another significant feature in this figure that the resonant peak position of the RICs shift toward the higher energies with the increment of $V_0$. The physical reason for this trait is that as $V_0$ enhances, the quantum confinement becomes strong, which results in the increment of the energy differences $E_{10}$. Therefore, the peak positions of the RICs move to the higher energies.

![Figure 3](image_url)

**Figure 3.** The variation of the RICs coefficients as a function of the incident photon energy for two different $V_0$ values.

4. CONCLUSION

In this study, we first numerically solved the eigenvalues and their corresponding eigenvectors of a GaAs QWs with short-range bottomless exponential potential. Then, we have calculated the effect of the depth of the confinement potential on the OACs and RICs of the system. The obtained results demonstrate that the OACs and RICs are significantly influenced by the depth of the confinement potential. Therefore, the sensitivity of the OACs and RICs their distributions to the depth of the confinement potential is very beneficial for novel devices based on depend GaAs QWs with short-range bottomless exponential potential.

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