Threshold $\pi^0$ photo- and electro-production in a meson-exchange model

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We show that, within a meson-exchange dynamical model describing well most of the existing pion electromagnetic production data up to the second resonance region, one is also able to obtain a good agreement with the $\pi^0$ photo- and electroproduction data near threshold. In the case of $\pi^0$ production, the effects of final state interaction in the threshold region are nearly saturated by single charge exchange rescattering. This indicates that in ChPT, it might be sufficient to carry out the calculation just up to one-loop diagrams for threshold neutral pion production.

Photo- and electroproduction of $\pi^0$ near threshold have been a subject of many experimental and theoretical studies in the last decade. It was prompted by the discrepancy between the "old" low-energy-theorem (LET) prediction of $E_{\pi^0} = -2.4 \times 10^{-3}/m_{\pi}$ and the "new" experimental measurements [1, 2], which yielded $E_{\pi^0} \sim -1.3 \times 10^{-3}/m_{\pi}$. The discrepancy between LET and the experimental data was eventually resolved by the chiral perturbation theory (ChPT) calculation [3] which showed that the loop corrections gave rise to nonanalytical terms in $m_{\pi}$. Since then precise measurements on the $\pi^0$ electromagnetic (EM) production near threshold have been performed [4] and the ChPT calculations to one loop $O(p^3)$ ($O(p^4)$ in the case of photoproduction) have been carried out in the heavy baryon formulation [5]. Nice agreement between theory and experiment was reached not only for the $S$-wave multipoles but also for the $P$-wave amplitudes.

Meson-exchange models (MEM’s), as in ChPT, also start from an effective chiral Lagrangian. However, they differ from ChPT in the approach to calculate the scattering amplitudes. In ChPT, crossing symmetry is maintained in the perturbative field-theoretic calculation, and the agreement between its predictions and the data is expected as long as the series converges. In MEM’s, the effective Lagrangian is used in the construction of potential for use in the scattering equation. The solutions of the scattering equation include rescattering effects to all orders and hence unitarity is ensured, while crossing symmetry is violated. Such models [6, 7, 8, 9, 10, 11, 12] have been able to provide a good description of $\pi N$ scattering lengths and phase shifts in $S$-, $P$-, and $D$-waves up to 600 MeV pion laboratory kinetic energy.

MEM’s have been constructed for pion EM production as well [10, 13] and good
agreement with the data has also been achieved up to 1.3 GeV total πN c.m. energy. However, the predictive power of the MEM for EM pion production near threshold has not been fully explored even though the importance of final state interaction (FSI) for threshold π⁰ photoproduction had been demonstrated in several dynamical model studies [7, 14] prior to the 1-loop calculations of ChPT [3].

In this talk we present the predictions of the Dubna-Mainz-Taipei (DMT) dynamical model, based on meson-exchange picture, which we recently developed in Ref. [15] for the threshold EM pion production and compare them with the recent experimental data [2, 16, 17, 18, 19] for the S- and P-wave multipoles and cross sections, and with the results of ChPT. In our DMT model, contributions which are related to the excitation of resonances are considered phenomenologically using standard Breit-Wigner forms. Such an approach gives an good description of EM pion production up to the second resonance region [20].

In the dynamical model for EM pion production [21], the t-matrix is given as

\[ t_{\gamma\pi}(E) = v_{\gamma\pi} + v_{\gamma\pi}g_0(E)t_{\pi N}(E), \]

where \( v_{\gamma\pi} \) is the \( \gamma\pi \) transition potential, \( g_0 \) and \( t_{\pi N} \) are the \( \pi N \) free propagator and \( t \)-matrix, respectively, and \( E \) is the total energy in the c.m. frame. In the present study, \( t_{\pi N} \) is obtained in a meson-exchange \( \pi N \) model [8] constructed in the Bethe-Salpeter formalism and solved within Cooper-Jennings reduction scheme [22]. Both \( v_{\pi N} \) and \( v_{\gamma\pi} \) are derived from an effective Lagrangian containing Born terms as well as \( \rho \)- and \( \omega \)-exchange in the \( t \)-channel [23]. For pion electroproduction we restore gauge invariance by the substitution, \( J_\mu \rightarrow J_\mu - k_\mu (k \cdot J/k^2) \), where \( J_\mu \) is the electromagnetic current corresponding to the background contribution of \( v_{\gamma\pi} \).

For the physical multipoles in channel \( \alpha = \{l, j, I\} \), Eq. (1) gives [21]

\[ t_\alpha(q_E, k) = \exp(i\delta_\alpha) \cos\delta_\alpha \left[ v_\alpha(q_E, k) + P \int_0 dq' \frac{R_\alpha(q_E, q')}{E(q_E) - E(q')} v_\alpha(q', k) \right], \]

where \( \delta_\alpha \) and \( R_\alpha \) are the \( \pi N \) phase shift and reaction matrix, in channel \( \alpha \), respectively, \( q_E \) is the pion on-shell momentum and \( k = |k| \) the photon momentum. In order to ensure the convergence of the principal value integral, we introduce a dipole-like off-shell form factor characterizing the finite range aspect of the potential with \( \Lambda = 440 \text{ MeV} \).

For \( \pi^0 \) photoproduction, we calculate the multipole \( E_{0^+} \) near threshold by solving the coupled channels equation within a basis with physical pion and nucleon masses. Results for \( Re E_{0^+} \) are shown in Fig. 1. One sees that our results (solid curve) agree well with the experimental data and ChPT calculations (dash-dotted-dotted curve) [5]. The FSI contributions from the elastic (\( \pi^0 p \)) and charge exchange (\( \pi^+ n \)) channels, are shown by the short-dashed and dash-dotted curves, respectively, while the dotted curve corresponds to the LET results, i.e., without the inclusion of FSI. Our results clearly indicate that practically all of the FSI effects originate from the \( \pi^+ n \) channel. Note that the main contribution stems from the principal value integral of Eq. (2).

In the approach considered above, \( t_{\pi N} \) contains the effect of \( \pi N \) rescattering to all orders. However, we have found that only the first order rescattering contribution, i.e. the 1-loop diagram, is important. This result is obtained by replacing \( t_{\pi N} \) in Eq. (1) by the \( v_{\pi N} \). As can be seen in Fig. 1, the thus obtained results given by the long-dashed curve,
differ from the full calculation by 5% only. This indicates that the 1-loop calculation in ChPT could be a reliable approximation for \( \pi^0 \) production in the threshold region.

Similar results are also obtained for the \( \pi^0 \) photoproduction on neutron where 1-loop contribution with \( \pi^-p \) intermediate states is found to be large. In Table 1, the results obtained up to tree, 1-loop, and 2-loop approximations for all four possible pion photoproduction channels are listed and compared to the experiments and ChPT results. We see that for \( \pi^0 \) production from both proton and neutron, it is necessary to include one-loop contribution while tree approximation is sufficient for the charged pion productions.

In Fig. 2, we compare the predictions of our model for the differential cross section with recent photoproduction data from Mainz [16,19]. The dotted and solid curves are obtained without and with FSI effects, respectively. It is seen that both off-shell pion rescattering and cusp effects substantially improve the agreement with the data. This indicates that our model gives reliable predictions also for the threshold behaviour of the \( P \)-waves without any additional arbitrary parameters. A detailed comparison [24] showed that our predictions for \( P \)-waves are in good overall agreement with the ChPT predictions [5] and the experimental values extracted from recent TAPS polarization measurements [19]. However, there is a 15% − 20% difference in \( P_3 = 2M_{1^+} + M_{1^-} \) which leads to an underestimation of our result for the photon asymmetry. Note that, in contrast to our model, \( P_3 \) is essentially determined by a low energy constant in ChPT.

Pion electroproduction provides us with information on the \( Q^2 \) dependence of the transverse \( E_{0^+} \) and longitudinal \( L_{0^+} \) multipoles in the threshold region. It is known that at threshold, the \( Q^2 \) dependence is given mainly by the Born plus vector meson contributions

| Channel          | Tree | 1-loop | 2-loop | Full  | ChPT   | Exp   |
|------------------|------|--------|--------|-------|--------|-------|
| \( \pi^0p \)     | 2.26 | 1.06   | -1.01  | -1.00 | -1.1   | 1.33  | 0.11  |
| \( \pi^+n \)     | 27.72| 28.62  | 28.82  | 28.85 | 28.2 ± 0.6 | 28.3 ± 0.3 |
| \( \pi^0n \)     | 0.46 | 2.09   | 2.15   | 2.18  | 2.13   |       |       |
| \( \pi^-p \)     | -31.65| -32.98 | -33.27 | -33.31| -32.7 ± 0.6 | -31.8 ± 1.9 |

Figure 1. \( ReE_{0^+} \) for \( \gamma p \rightarrow \pi^0p \). Notations are given in the text. Data points are from (△) [16], (●) [2], and (○) [19].
Figure 2. Differential cross sections for $\gamma p \rightarrow \pi^0p$. For notations, see the text. Data points are from (●) [16] and (○) [19].

Figure 3. $\text{Re}E_{0^+}$ and $L_{0^+}$ at $Q^2=0.1 (\text{GeV/c})^2$. Notations same as in Fig. 3. Data points are from (○) [17] and (△) [18].

In $\nu\gamma\pi$, as described in Ref. [23]. In Fig. 3 we show our results for the cusp and FSI effects in the $E_{0^+}$ and $L_{0^+}$ multipoles for $\pi^0$ electroproduction at $Q^2 = 0.1 (\text{GeV/c})^2$, along with the results of the multipole analysis from NIKHEF [17] and Mainz [18]. Note that results of both groups were obtained using the $P$-wave predictions given by ChPT. However, there exist substantial differences between the $P$-wave predictions of ChPT and DMT model at finite $Q^2$. To understand the consequence of these differences, we have made a new analysis of the Mainz data [18] for the differential cross sections, using DMT prediction for the $P$-wave multipoles instead. The $S$-wave multipoles extracted this way are also shown in Fig. 3 by solid circles. We see that the results of such a new analysis give a $E_{0^+}$ multipole closer to the NIKHEF data and in better agreement with our dynamical model prediction. However, the results of our new analysis for the longitudinal $L_{0^+}$ multipole stay practically unchanged from the values found in the previous analyses. Note that the dynamical model prediction for $L_{0^+}$ again agrees much better with the NIKHEF data.

In Fig. 4, DMT model predictions (dashed curves) are compared with the Mainz experimental data [18] for the unpolarized cross sections $d\sigma/d\Omega$, and for the longitudinal-transverse cross section $d\sigma_{TL}/d\Omega$. Overall, the agreement is good. The solid curves are the results of our best fit at fixed energies (local fit) obtained by varying only the $E_{0^+}$ and $L_{0^+}$ multipoles. We have found that the differences between the solid and dashed curves in Fig. 4 are mostly due to the difference in the $L_{0^+}$ multipole (see also Fig. 3).

In summary, we have shown that within a meson-exchange dynamical model [15], one is able to describe pion photo- and electroproduction in the threshold region in good
agreement with the data. The model has been demonstrated to give a good description of most of the existing pion electromagnetic production data up to the second resonance region [20]. The success of such a model at intermediate energies is perhaps not surprising since unitarity plays an important role there. However, it is not a priori clear that our model should also work well near threshold, even though we do start from an effective chiral Lagrangian. In principle, crossing symmetry is violated and the well-defined power counting scheme in ChPT is lost by rescatterings. On the other hand, MEM’s [11, 12] have also been shown to give a good description of low energy $\pi N$ data, in addition to an excellent agreement with the data at higher energies. It is therefore assuring that similar success can also be achieved for the pion EM production.

Finally, we found that the effects of FSI in the threshold region and in the case of $\pi^0$ production, are nearly saturated by the single rescattering term. Therefore, the existing one-loop calculations in ChPT could be a good approximation to threshold $\pi^0$ production.

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