Zero-Field Fiske Resonance Coupled with Spin-waves in Ferromagnetic Josephson Junctions

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AC Josephson current in a Josephson junction with voltage bias is spatially modulated by an external magnetic field, and induces an electromagnetic (EM) field inside the junction. The current-voltage \((I-V)\) curve exhibits peaks due to the resonance between the EM field and the spatially modulated AC Josephson current. This is called \textit{Fiske resonance}. Such a spatially modulated Josephson current can be also induced by a non-uniform insulating barrier and the Fiske resonance appears without external magnetic field. This is called zero-field Fiske resonance (ZFFR). In this paper, we theoretically study the ZFFR coupled with spin-waves in a superconductor/ferromagnetic insulator/superconductor junction (ferromagnetic Josephson junction) with a non-uniform ferromagnetic insulating barrier. The resonant mode coupled with spin-waves can be induced without external magnetic field. We find that the \(I-V\) curve shows resonant peaks associated with composite excitations of spin-waves and the EM field in the junction. The voltage at the resonance is obtained as a function of the normal modes of EM field. We show several current-density dependences of the ZFFRs coupled with spin-waves.

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I. INTRODUCTION

The DC Josephson effect is characterized by the DC current flowing without a voltage-drop between two superconductors separated by a thin insulating barrier. When a DC voltage \((V)\) is applied to the junction, the AC Josephson current with frequency \((2\epsilon/h)\) flows in the junction driven by the difference of phases in two superconducting order parameters, i.e., \textit{Josephson-phase} \((\theta)\). If both the DC voltage and a magnetic field are applied to the junction, whose width \(L\) is smaller than the Josephson penetration depth \(\lambda_j\), the AC Josephson current is spatially modulated and generates the electromagnetic (EM) field inside the junction. In this case, the current-voltage \((I-V)\) curve exhibits peaks due to the resonance between the AC Josephson current and the EM field. This is called \textit{Fiske resonance}.

Josephson junctions composed of ferromagnetic metal (FM) and superconductors (Ss) are extensively studied for the last decade. The S/FM/S junctions exhibits fascinating phenomena which are not observed in the conventional Josephson junctions \cite{7,11}. The interaction between Cooper pairs and spin waves in the FM is of importance in the transport properties in the S/FM/S and S/I/FM/S junctions. The dynamics of \(\theta\) coupled with spin-waves in the FM has been investigated theoretically \cite{12,20} and experimentally \cite{21,23}. However, the Fiske resonance coupled with spin-waves is not yet observed experimentally.

Another type of Josephson junction with ferromagnetic insulator (FI) instead of the FM is also examined. It is reported that the dissipation effect in the S/FI/S junction is smaller than that in the S/FM/S junction \cite{22,26}. The damping of spin-waves is also very small in the FI compared to the case in the FM \cite{27,29}. Therefore, the coupling between \(\theta\) and spin-waves can be observed more clearly in the S/FI/S junction. In fact, in the S/FI/S junction, it is expected that the Fiske resonance has clear multiple structures associated with spin-wave excitations \cite{30}.

Here, we note that the Fiske resonance in the conventional Josephson junction is also induced by the non-uniform insulating barrier in the junction, since AC Josephson current driven by a DC voltage is spatially modulated and then the EM field is generated inside the junction. In this case, the Fiske resonance occurs without external magnetic field. It is called zero-field Fiske resonance (ZFFR), which originates from the resonance between the EM field and the spatially modulated AC Josephson current due to the non-uniform insulating barrier. This phenomenon has been widely studied experimentally and theoretically in the Josephson junction \cite{31,34}.

In this paper, we theoretically study the ZFFR coupled with spin-waves in a S/FI/S junction with a non-uniform FI. The merit of such a non-uniform geometry of junction is that the Fiske resonance occurs without external magnetic field due to the spatially modulated AC Josephson current. By solving the equation of motion of \(\theta\) coupled with spin-waves, it will be found that the \(I-V\) curve shows resonant peaks. The \(I-V\) curve is obtained as a function of the normal modes of EM field, which indicates composite excitations of the EM field and spin-waves in the S/FI/S junction. Dependence of those resonances on distributions of the current density is presented.

The rest of this paper is organized as follows. In Sec. II, we formulate the DC Josephson current induced by
the ZFFR. In Sec. III, the ZFFR with spin-waves in the $I$-$V$ curve is shown. Summary is given in Sec. IV.

II. FORMULATION OF JOSEPHSON CURRENT IN A FERROMAGNETIC JOSEPHSON JUNCTION WITH MAGNETIC INSULATOR

The system considered is a Josephson junction with the FI sandwiched by two superconductors with $s$-wave symmetry as shown in Fig.1. The geometry of the junction is assumed to be trapezoid to impose non-uniform Josephson current density without external magnetic field. The magnetization in the FI is parallel to the $z$-direction. Here, we adopt a simple model of non-uniform Josephson current density given by,

$$J_j(y,t) = J_c(y) \sin[\omega_j t + \theta(y,t)],$$  \hspace{1cm} (1)

$$J_c(y) = J_c p_L(y) \left[ \frac{1 - \zeta}{\cosh[\kappa(1-2y/L)]} + \frac{\zeta \sinh[\kappa(1-2y/L)]}{\sinh(\kappa)} \right],$$ \hspace{1cm} (2)

where $J_c$ and $\omega_j = (2e/h)V$ are the Josephson critical current density and Josephson frequency with bias voltage $V$, respectively. The Josephson phase $\theta(y,t)$ depends on space and time. The inhomogeneity of the junction is given by $\kappa$ and $\zeta$, where we assume that $0 \leq \zeta < 1$. In the S/FI/S junction, the EM field inside the FI induced by the AC Josephson current can excite spin-waves. In this situation, the equation of motion for $\theta(y,t)$ coupled with spin waves is described by 

$$\frac{\partial^2 \theta(y,t)}{\partial y^2} = \frac{1}{c^2} \left[ \frac{\partial^2 \theta(y,t)}{\partial t^2} + \frac{1}{\mu_0} \int_{-\infty}^{\infty} dy' dt' \chi(y-y',t-t') \frac{\partial^2 \theta(y',t')}{\partial t'^2} \right]$$

$$+ \frac{\Gamma \partial \theta(y,t)}{\partial t} + \frac{1}{\mu_0} \int_{-\infty}^{\infty} dy' dt' \chi(y-y',t-t') \frac{\partial \theta(y',t')}{\partial t'}$$

$$+ \frac{1}{\lambda_J^2 (J_c(y))} J_j(y,t) + \frac{1}{\lambda_J^2 (J_c(y))} \frac{1}{\mu_0} \int_{-\infty}^{\infty} dy' dt' \chi(y-y',t-t') J_j(y',t'),$$ \hspace{1cm} (4)

$$\langle J_c(y) \rangle = \frac{1}{L} \int_0^L dy J_c(y),$$ \hspace{1cm} (5)

with the effective velocity of light in the FI, $c = \sqrt{d/(d+2\lambda_L^2 \mu_0)}$, the Josephson penetration depth, $\lambda_J = \sqrt{\hbar/[2e\mu_0(d+2\lambda_L^2)]}$. The London penetration depth is denoted by $\lambda_L$ and $\Gamma \equiv (\epsilon R)^{-1}$ means the damping factor caused by quasi-particle resistivity ($R$) in the FI. The magnetic susceptibility of the FI in the linearized Landau-Lifshitz-Gilbert equation is given by 

$$\chi(q,\omega_j) = \gamma J_z \frac{\Omega_S + i\alpha_\omega_j}{\Omega_S^2 - (1 + \alpha^2)\omega_j^2 + i2\alpha\Omega_S \omega_j},$$ \hspace{1cm} (6)

where $J_z$, $\alpha$, and $\gamma$ are the $z$-component of the magnetization, Gilbert damping factor, and the gyromagnetic ratio, respectively. Magnetic susceptibility and spin-wave energy $\hbar \Omega_S$ in a magnetic material are generally modified by geometry and thickness. On the other hand, Eq. (6) is obtained assuming an uniform FI. This is justified, since the magnetic susceptibility and $\hbar \Omega_S$ are insensitive to the thickness of FI, provided that the conformation of the ferromagnetic materials changes on a scale of nanometers. Therefore, we adopt Eq. (6) and $\hbar \Omega_S$ obtained in the uniform FI [57] for the trapezoidal FI as an approximation, since we consider the thickness change of FI to be in a range of a few nanometers. In the uniform FI, the dispersion relation of spin-waves with the frequency
$\Omega_S$ is given by
\[ \Omega_S = \Omega_B + \frac{n^2 \eta^2}{h}, \tag{7} \]
where $\Omega_B = \gamma(H_K - M_z / \mu_0)$. The anisotropic field and the stiffness of spin-waves in the FI are denoted by $H_K$ and $\eta$, respectively. The spin-wave having a finite wave number $q$ is neglected in the Fiske resonance because of the following reason: In Eq. (7), the first term $\Omega_S$ is caused by the anisotropic and demagnetizing fields, and the wave number $q$ is given by $n \pi / L$. In a conventional FI, $\hbar \Omega_B$ is about tens of $\mu eV$ \cite{27}. On the other hand, $\eta q^2$ is of the order $peV$ due to the small stiffness of spin-waves \cite{28} when $L$ is a few mm. Below, we only consider $q = 0$ mode for spin-waves with the constant frequency $\Omega_B$.

In order to obtain the solution of Eq. (4), we expand $\theta(y, t)$ in terms of the normal modes of the EM field generated by the AC Josephson current as follows,
\[ \theta(y, t) = \text{Im} \left[ \sum_{n=0}^{\infty} g_n e^{i \omega_n t} \cos(k_n y) \right], \tag{8} \]
where $g_n$ is a complex number and $k_n = n \pi / L$. This equation of $\theta(y, t)$ satisfies $\partial \theta / \partial y |_{y=0} = [\partial \theta / \partial y] y=L = 0$, which is Kulik’s boundary condition \cite{29, 30}. We consider $\theta(y, t)$ to be a small perturbation and solve Eq. (8) by taking $J_\ell(y, t)$ to be $J_c(y) \sin(\omega_d t)$. Substituting Eq. (8) into Eq. (4), $g_n$ is determined as,
\[ g_n = -\frac{e^2 J_c}{\lambda_\ell^2} \left( \frac{1 + \chi(-\omega_d)}{\mu_0} \right) \times \frac{\omega_n^2 [1 + \chi(-\omega_d)] / \mu_0 - \omega_n^2 + i \Gamma \omega_d [1 + \chi(-\omega_d)] / \mu_0}{(1 - \zeta) B_n + \zeta C_n}, \tag{9} \]
where $\omega_d = (c / \pi L n)$.

Next, we calculate the DC Josephson current density $J_{DC}$ coupled with spin waves as a function of $V$. The function, $\sin(\omega_d t + \theta(y, t))$, is expanded in terms of $\theta(y, t)$ and $J_{DC}$ is given by
\[ J_{DC} \approx \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \frac{1}{L} \int_0^L dy J_c(y) \cos(\omega_d t) \theta(y, t). \tag{10} \]
Introducing Eqs. (3) and (9) into Eq. (10), the analytic formula of $J_{DC}$ without external magnetic field is obtained as,
\[ J_{DC} \equiv \sum_{n=0}^{\infty} J_{n}^{DC} \approx \frac{e^2 \kappa J_c}{\lambda_\ell^2 (1 - \zeta) \tan(\kappa)} \sum_{n=0}^{\infty} \Psi_n(\omega_1) \left[ (1 - \zeta) \frac{\kappa \cos^2 \left( \frac{n \pi}{2} \right) \tan(\kappa)}{\kappa^2 + (n \pi / 2)^2} + \frac{\kappa \sin^2 \left( \frac{n \pi}{2} \right) \tan^{-1}(\kappa)}{\kappa^2 + (n \pi / 2)^2} \right], \tag{11} \]
\[ \Psi_n(\omega_1) = \frac{\Gamma \omega_1^2 [1 + 2 \chi(\omega_1)] / \mu_0 + \omega_n^2 \chi_2(\omega_1) / \mu_0 + \Gamma \omega_1 \chi_1(\omega_1) + \chi_2(\omega_1) / \mu_0^2}{\left[ \omega_n^2 [1 + \chi(\omega_1)] / \mu_0 \right] - \omega_n^2 + \Gamma \omega_1 \chi_2(\omega_1) / \mu_0^2} + \frac{\Gamma \omega_1 [1 + \chi(\omega_1)] / \mu_0 + \omega_n^2 \chi_2(\omega_1) / \mu_0^2}, \tag{12} \]
where $\chi_1(\omega_1) = \text{Re}[\chi(\omega_1)]$ and $\chi_2(\omega_1) = \text{Im}[\chi(\omega_1)]$. $J_n^{DC}$ is DC Josephson current density for a mode number $n$ of EM field. Equation (11) clearly demonstrates that zero-field resonant modes depend on the parameter $\kappa$ and $\zeta$ which determines the distribution of the Josephson current density inside the FI. Hence, one can easily find that three cases are possible for the zero-field resonance. When $\zeta = 0$ ($\zeta = 1$), the zero-field resonance only appears at even (odd) numbers of $n$. On the other hand, when $\zeta \neq 0, 1$, the zero-field resonance appears at all integers $n$. These are also clearly demonstrated by numerical calculations later on.

Next, we derive a condition for the ZFFR in the present system by analyzing Eq. (12). When the denominator of $\Psi_n(\omega_1)$ is minimum with respect to $\omega_1$, $\Psi_n(\omega_1)$ takes a maximum, so that the DC Josephson current exhibits the resonant behavior. The DC voltage, at which the resonance occurs, is determined by neglecting the damping term of Eq. (12) as $\alpha = \Gamma = 0$. Setting the denominator of $\Psi_n(\omega_1)$ to be zero, the voltage is given by
\[ V^\pm = \frac{\hbar}{2e} \sqrt{\frac{1}{2} \left[ \omega_n^2 + \Omega_S^2 + \frac{\gamma M_c \Omega_S}{\mu_0} \pm \sqrt{\left( \omega_n^2 + \Omega_S^2 + \frac{\gamma M_c \Omega_S}{\mu_0} \right)^2 - 4 \omega_n^2 \Omega_S^2} \right]^2}, \tag{13} \]
We have two DC voltages, $V^+$ and $V^-$, at which the ZFFR occurs for each $n$. The integer $n$ is determined by the mode of the EM field in the junction. In Eq. (13), it
is found that two dispersion relations in Fig. 4 result from the coupling between the EM field and spin-waves in the FI. The S/FI/S junction generates a composite excitation composed of the EM fields and spin-waves. This result is different from the case of the conventional Josephson junction, in which one can see only one resonance for each mode. Note that the amplitude of ZFFR strongly depends on $\kappa$ and $\zeta$ as we will see in the next section.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, we evaluate numerically Eqs (11) and (2). Parameters are set to be $M_s = 0.1$ T, $\alpha = 1 \times 10^{-4}$ [29], $\Omega_B/\omega_L = 3$, $\Gamma/\omega_L = 0.5$ [32], $\gamma = 2.2 \times 10^5$ m/A-s [40], and $\omega_L = c\pi/L = 30$ GHz. Instead of plotting an $I$-$V$ curve, the DC components of Josephson current density, i.e., $J_{DC}$, which appear at the resonances, will be shown as a function of the voltage $V$. The amplitude of $J_{DC}$ is associated with a height of resonant peak. In the following numerical calculations, we exclude the contribution of $n = 0$ in Eq. (11), since we discuss about the resonance between the spatially modulated AC Josephson current and standing wave of EM field.

Figure 3 shows $J_{DC}$ induced by ZFFR as a function of $V$ for $\kappa = 0$ and $\zeta = 0.4$. With these parameters, $J_n(y)$ is linearly distributed in the junction (see the inset of Fig. 4), in which interfaces of the junction are assumed to be clean without randomness. The black (solid) and red (dashed) lines are $J_{DC}$ and $J_n^{DC}$ for $n = 1$, respectively. This result clearly demonstrates that the Fiske resonance occurs without external magnetic field, i.e., the ZFFR occurs. The additional resonance peak around $V/(\omega_L h/2e) \approx 3.3$ arises from the presence of spin-wave excitation in the FI. This resonance comes from the inhomogeneity of Josephson critical current density induced by the non-uniform geometry of junction. Moreover, in the present case, Eq. (11) becomes

$$J_{DC} = \frac{\alpha^2 J_e}{\lambda_f(1 - \zeta)} \sum_{n=0}^{\infty} \Psi_n(\omega_1) \zeta^2 \left[ \frac{\sin(n\pi/2)}{(n\pi/2)} \right]^4. \quad (14)$$

In Eq. (14), it is found that the ZFFR only occurs at odd number of $n$. Since resonant peaks of ZFFR with $n > 1$ are much smaller than that with $n = 1$, main contribution to the ZFFR as depicted in Fig. 3 is the mode of $n = 1$.

Figure 4 is the case for $\kappa = 2$ and $\zeta = 0.4$. The black (solid) line is $J_{DC}$. Red (solid), blue (dashed), and green (chain) lines are $J_n^{DC}$ of each $n$. It is found that ZFFR peaks of $J_{DC}$ clearly appear at $n \geq 1$ in Fig. 4 in contrast to Fig. 3. The reason is simply due to the non-linear Josephson critical current density to contain both symmetric and antisymmetric components with respect
Josephson critical current density (see inset of Fig. 5), which is achieved by $\kappa \gg 1$ in the present model. Parameters are set to be $\kappa = 50$ and $\zeta = 0.4$. In this case, Josephson current flows only near edge of the junction as shown in the inset of Fig. 4. In Fig. 5, it is found that many clear resonant peaks appear in common with the case to apply to external magnetic field \[20\]. Therefore, the strong inhomogeneity of the Josephson critical current density induces obvious ZFFR coupled with spin-waves. Second, we consider the limit of symmetric Josephson critical current density (see inset of Fig. 6), which is achieved by $\kappa \gg 1$ in the present model for $\zeta = 0$. Parameter is set to be $\kappa = 50$. Figure 6 shows the numerical result of $J_{DC}$ induced by ZFFR coupled with spin-waves for $\kappa = 50$ and $\zeta = 0$. In this limiting case, Eq. (14) is given by

$$J_{DC}^2 \approx \frac{e^2 J_c}{\chi^2} \sum_{n=0}^\infty \Psi_n(\omega) \left[ \cos \left( \frac{n\pi}{2} \right) \right]^4 \frac{\tanh(\kappa)}{\kappa}. \quad (15)$$

In Eq. (15) and as shown in Fig. 6 we find that the ZFFR only occurs at even number of $n$.

### IV. SUMMARY

We have theoretically studied the zero-field Fiske resonance (ZFFR) in the S/FI/S junction with several patterns of spatial variation in the Josephson critical current density, which is induced by a non-uniform ferromagnetic insulating barrier. Such a non-uniform AC Josephson current density can excite the EM field inside the FI without external magnetic field. It is found that the current-voltage ($I-V$) curve shows two resonant peaks without external magnetic field in the present model for $\kappa = 50$. Parameter is set to be $\kappa = 50$. Figure 6 shows the numerical result of $J_{DC}$ induced by ZFFR coupled with spin-waves for $\kappa = 50$ and $\zeta = 0$. In this limiting case, Eq. (14) is given by

$$J_{DC}^2 \approx \frac{e^2 J_c}{\chi^2} \sum_{n=0}^\infty \Psi_n(\omega) \left[ \cos \left( \frac{n\pi}{2} \right) \right]^4 \frac{\tanh(\kappa)}{\kappa}. \quad (15)$$

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The present study will provide a platform to study the dynamics of Josephson phase and the magnetic excitation. Furthermore, in the non-uniform S/FI/S junction, several applications such as spin-current emitter by utilizing spin-wave excitation in the FI \[20\] may be also possible in analogy with the emission of coherent THz radiation in the high-$T_c$ cuprate \[23\] \[25\]. In fact, the inhomogeneity of the junction was one of essential factors to realize the emission without external magnetic field \[26\] \[27\]. However, novel devices using the S/FI/S junction are beyond the scope of the present paper and will be studied elsewhere.

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