A Microscopic Liouville Arrow of Time

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Abstract

We discuss the treatment of quantum-gravitational fluctuations in the space-time background as an ‘environment’, using the formalism for open quantum-mechanical systems, which leads to a microscopic arrow of time. After reviewing briefly the open-system formalism, and the motivations for treating quantum gravity as an ‘environment’, we present an example from general relativity and a general framework based on non-critical strings, with a Liouville field that we identify with time. We illustrate this approach with calculations in the contexts of two-dimensional models and D branes. Finally, some prospects for observational tests of these ideas are mentioned.

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1 A New Time for the New Millennium?

The nature of time remains one of the great mysteries of physics. During the nineteenth century, the monotonic increase of entropy incarnated in the laws of thermodynamics made manifest a macroscopic arrow of time, but did not explain it. During the twentieth century, time was first reinterpreted as a coordinate of the relativistic space-time continuum, and later understood to be distorted by gravitational fields. However, time still appeared in both special and classical general relativity as a reversible coordinate, although an arrow reappeared at the macroscopic level via the expansion of the universe. Microscopically, neither did a microscopic arrow appear in the quantum treatment of time in Schrödinger’s equation and its relativistic generalizations. In quantum field theory, time reversal (T) is not necessarily an exact symmetry, and it is known experimentally to be violated in the neutral kaon system. Nevertheless, the combination of T with charge conjugation (C) and parity (P) is known to be an exact symmetry of quantum field theory, thanks to the CPT theorem\footnote{that follows from Lorentz invariance, locality and causality. Thus time has no apparent arrow at the microscopic level.}

There have been many suggestions that the only arrow of time is cosmological, and that the increase in thermodynamic entropy is due to entanglement with the macroscopic environment. There is, however, a minority view that there might be a microscopic arrow of time, engendered by quantum gravity. This suggestion originates from the formulation by Hawking\footnote{and Bekenstein\footnote{of black-hole thermodynamics, which was first derived at the semi-classical level. It was subsequently suggested that quantum-gravitational entropy growth might also appear at the microscopic level\footnote{entailing abandonment of the S-matrix description of particle scattering in conventional quantum field theory, and its replacement by a non-factorizing superscattering operator between asymptotic in- and out-states. As has been pointed out by Wald\footnote{any such formalism would entail abandoning the strong form of the CPT theorem valid in conventional quantum field theory. If this is the case, the conventional microscopic quantum-mechanical time evolution of elementary particles would need to be modified. It has been suggested that they may need to be described as open quantum-mechanical systems interacting with an ‘environment’ furnished by quantum-gravitational background fluctuations\footnote{that introduces a microscopic arrow of time.}.

Any such suggestion must be measured by the standards of the emerging quantum theory of gravity provided by what used to be called string theory\footnote{This framework is apparently powerful enough to provide a consistent quantum description of microscopic gravitational fluctuations in the space-time vacuum background. We have developed\footnote{one approach to this problem, using the tools of non-critical string theory with non-trivial conformal dynamics parametrized by a Liouville field evolving according to the renormalization group. In this approach, time is identified as a renormalization-scale parameter provided by the zero mode of the Liouville field, and has an arrow associated with the orientation of the renormalization group flow.}}. This framework is apparently powerful enough to provide a consistent quantum description of microscopic gravitational fluctuations in the space-time vacuum background. We have developed\footnote{one approach to this problem, using the tools of non-critical string theory with non-trivial conformal dynamics parametrized by a Liouville field evolving according to the renormalization group. In this approach, time is identified as a renormalization-scale parameter provided by the zero mode of the Liouville field, and has an arrow associated with the orientation of the renormalization group flow.} one approach to this problem, using the tools of non-critical string theory with non-trivial conformal dynamics parametrized by a Liouville field evolving according to the renormalization group. In this approach, time is identified as a renormalization-scale parameter provided by the zero mode of the Liouville field, and has an arrow associated with the orientation of the renormalization group flow.}
The purpose of this article is to review (in Section 2) the basic ideas of this formalism, which leads to a derivation of a modified quantum Liouville equation for the density matrix, and to outline (in Section 3) some model calculations that exemplify this general approach. We conclude (in Section 4) by discussing the outlook for this approach.

2 A Brief History of Liouville Time

2.1 Quantum Mechanics of Open Systems: Density Matrix Formalism and Induced Decoherence

A quantum-mechanical (sub)system interacting with some environment does not satisfy a standard Schrödinger equation. The interaction with the environment implies that the proper description of the system is via a density matrix \( \rho \equiv \langle \Psi | M | \Psi \rangle \), where \( | \Psi \rangle \) is a state vector for the subsystem, and \( Tr_M \) denotes an average over unobserved states of the external environment. The temporal evolution of \( \rho \) is therefore given by:

\[
\partial_t \rho = i[\rho, H] + \delta H(\rho)
\]

where the correction \( \delta H(\rho) \) is \textit{a priori} a non-linear function of the density matrix, describing the interaction of the subsystem with the environment. Linearizations of this interaction have been considered in the literature \cite{9, 10}, in particular in the context of quantum-gravitational environments \cite{6, 4}, which we pursue here. If this is so, \( \delta H \) is an operator independent of \( \rho \), which however depends on the nature of the environment \cite{6}:

\[
\partial_t \rho = i[\rho, H] + \delta H \rho
\]

We give below a more explicit form for such linear environmental entanglement.

Consider, for concreteness, an open system described by a Hamiltonian \( H \), in interaction with a quantum-mechanical environment, described generically by annihilation and creation operators \( B_m, B_m^\dagger \). For simplicity, we assume that the environmental entanglement conserves the total probability and energy on the average. Such an environmental entanglement is described by the so-called Lindblad formalism \cite{9} for the evolution equation of the density matrix of the system:

\[
\dot{\rho} \equiv \partial_t \rho = i[\rho, H] - \sum_m \{B_m^\dagger B_m, \rho\}_+ + 2 \sum_m B_m \rho B_m^\dagger
\]

This type of linear evolution equation for \( \rho \) is used generally to describe Markov processes in open quantum-mechanical systems \cite{10}. Recalling that the density matrix may be expressed in terms of the state vector \( | \Psi \rangle \) via

\[
\rho = Tr_M | \Psi \rangle \langle \Psi |
\]

with the trace being taken over an ensemble of theories \( \mathcal{M} \), it is possible to derive a time-evolution equation for the state vector \( | \Psi \rangle \), which is of stochastic Ito form \cite{11}:

\[
|d \Psi > = -\frac{i}{\hbar} H | \Psi \rangle + \sum_m \langle B_m^\dagger | \Psi \rangle B_m - \frac{1}{2} B_m^\dagger B_m -
\]
\[
- \frac{1}{2} < B_m^\dagger >_\Psi < B_m >_\Psi \rangle |\Psi > dt + \sum_m (B_m - < B_m >_\Psi) |\Psi > d\xi_m 
\]

where \(< \ldots >_\Psi >\) denote averages with respect to the state vector \(|\Psi >\), and \(d\xi_m\) are complex differential random matrices, associated with white-noise Wiener or Brownian processes.

Within the stochastic framework (5), it can be shown, under assumptions on the Hamiltonian operator that are not too restrictive, that \(|\Psi >\) will always become localized in some state-space channel \(k\), as a result of environmental entanglement. To prove this formally, one may construct a quantity that serves as a measure of the delocalization of the state vector, and examine its temporal evolution. An example is the quantum dispersion entropy [11]:

\[
K \equiv - \sum_k < P_k >_\Psi \log < P_k >_\Psi
\]

where \(P_k\) is a projection operator onto the state \(k\) of the system. This entropy has been shown to decrease in situations where (5) applies, under some assumptions about the commutativity of the Hamiltonian of the system with \(P_k\), which implies that \(H\) can always be written in a block-diagonal form.

Analyses in the above framework have yielded important results concerning the passage from the quantum to the classical world [6, 12, 11, 8]. In particular, in [12] it was suggested that classical states may appear as a result of quantum decoherence, depending on the nature of the environment [13]. Such states are minimum entropy/uncertainty states whose shape is retained during evolution. Such ‘almost classical’ states are termed ‘pointer states’ by Zurek [12]. Such pointer states emerge, as a result of decoherence, when the localization process of the state vector has stopped at a stage, not where it is complete, but so that the resulting minimum-entropy state is least susceptible to the effects of the environment.

We close this section by recalling a recent experiment [14] in which a mesoscopic ‘Schrödinger’s cat’ was constructed, and the associated quantum decoherence was observed, for the first time in experimental physics. The experiment uses electromagnetic cavities of the type familiar from non-demolition experiments in quantum optics [15, 16]. The ‘cat’ is constructed in two stages: first it involves an interaction of the two-state atom with the cavity field, which results in a coherent state of the combined ‘atom + meter’ system, and then dissipation is induced by coupling the cavity (interpreted as a measuring apparatus) to the environment, thereby inducing decoherence in the ‘atom + meter’ system. The important point is that the more macroscopic is the cavity mode, i.e., the higher the number of oscillator quanta, the shorter is the decoherence time. This is exactly what was to be expected from the general theory [12, 6, 17, 8].
2.2 Quantum Gravity as an Environment: A Microscopic Arrow of Time?

We now examine whether the above approach may be applicable to quantum gravity. Here the chief problem is the present lack of a consistent mathematical formalism for quantum fluctuations of the gravitational field. The appearance of singular space-time configurations, such as black holes, is one of the problems that prevent us from constructing a satisfactory local quantum field theory for the gravitational field. Indeed, the presence of microscopic event horizons, as around black holes of Planckian size, is incompatible with the conventional unitary temporal evolution of quantum mechanics.

To understand this, consider the possibility of an asymptotic initial pure quantum state of matter at $t \to -\infty$, characterized by two sets of quantum numbers $\{A\}$ and $\{B\}$. Suppose that a measurement is made in the asymptotic future, $t \to \infty$, by an observer located at spatial infinity from a black hole which is formed, either by collapsing matter or by quantum fluctuations, at the time $t = 0$. Assume further that the quantum numbers $\{B\}$ are absorbed by the black hole. Due to quantum fluctuations the black hole will emit Hawking radiation [4] and eventually evaporate, apparently without any memory of the quantum numbers $\{B\}$. Hence an asymptotic observer at future spatial infinity, who sees around him/herself a macroscopically-flat space time, has to average over this set of quantum numbers, corresponding to an average over the unobservable states of the black hole. This implies that he/she is forced to use a density matrix $\rho_{\text{out}}$ to describe the asymptotic state, which is no longer a pure state, but rather a statistical mixture. This in turn requires non-unitary evolution, since pure states evolve into mixed ones.

This incompatibility with conventional quantum mechanics suggests that the quantum-mechanical system becomes open in the presence of singular quantum-gravitational configurations or fluctuations. Quantum gravity acts as an environment, whose averaging prevents factorization of the scattering matrix. Hawking [4] has argued that the relevant well-defined concept is that of the superscattering matrix $\tilde{S}$ introduced to generalize the scattering matrix in the case of statistical mixtures of asymptotic states:

$$\rho_{\text{out}} = \tilde{S} \rho_{ij} , \quad \tilde{S} \neq SS^\dagger$$

(7)

where $S \sim e^{iHt}$ is a conventional unitary Heisenberg scattering matrix.

Non-factorizability of the superscattering would imply generically some breaking of the strong form of CPT symmetry, as shown by Wald [5]. Moreover, there is entropy production, in agreement with the open nature of the observable subsystem. Passing from the asymptotic description (7) to finite-time evolution, one expects a modified Liouville equation for the density matrix of the form (2), (3), where the ‘environment’ operators now represent quantum-gravitational coherent states [6, 17].

An explicit toy model for the interaction of low-energy propagating matter with a dissipative quantum-gravity environment consisting of virtual wormholes [18] was studied in [17] from a phenomenological viewpoint. Coleman argued that the worm-
hole state was likely to be coherent, and used this argument to support the the vanishing of the cosmological constant. However, this coherence assumption was questioned later, and our subsequent studies of the nature of the space-time foam in quantum gravity and string theory suggest that this is not the case. However, one can still construct a model interaction Hamiltonian between operators describing the low-energy probe $O_P$ and the wormhole state $|a>$:

$$H_I \propto O_P(a^\dagger + a)$$  (8)

where $a^\dagger, a$ are creation and anihilation operators for wormhole states. In the example of [17], $O_P$ was taken to be a four-fermion effective interaction

$$O_P \propto \mathcal{O}(\frac{1}{m_P^2})\bar{\psi}_1\gamma^\mu\psi_1\bar{\psi}_2\gamma^\mu\psi_2$$  (9)

A low-energy observer has to average out the unobservable wormhole effects, with the result that the low-energy probe $P$ becomes an open system. The simple case of a Gaussian distribution for the wormhole configurations was assumed in [17], and the time scale of the induced decoherence of the low-energy probe $P$ was estimated, using the phenomenological equation for the density matrix suggested in [6], which was characterized by probability and energy conservation of the probe. In the framework of the previous section, this coupling may be considered as providing only phase damping for the atom. The situation resembles that of the atom + cavity system of [14], with the essential difference that the effect is not due to quantum electrodynamics, but to quantum-gravitational interactions. The rôle of the cavity is played by the microscopic space-time foam [4, 6].

As was shown in [17], the wormhole-probe coupling is enhanced for large numbers of atoms. Consequently, the decoherence of off-diagonal elements of the density matrix $\rho(x,x')$, in a ‘pointer’ basis $|x>$, where $x$ is the center-of-mass location in space time of a system of $N$ particles, is of the form [17]:

$$\rho(X', X, t) \sim \rho_0(X', X, t)\exp[-ND(X' - X)^2t]$$  (10)

where $D$ represents the coupling of a single particle with a single coherent mode of the wormhole state, and is estimated to be of order $D \sim m^6/M_P^3$ for a particle of mass $m$ in a four-dimensional space time, where $M_P$ is the Planck mass. A uniform density of wormholes of the order of one per Planck volume in space time was assumed in making the estimate (10), and all other interactions of the microscopic particles among themselves have been ignored. One can readily see from (10) the characteristic feature that the decoherence rate is proportional to the square of the distance between the pointer states [12], which is a generic feature of Markov-type decoherence [14].

The above description by no means constitutes a microscopic description of decoherence in quantum gravity. Because the underlying theory is still unknown, one cannot describe the pertinent ‘environment’ in a detailed way. However, in subsequent sections of this review we present a formalism which may provide a way to
formulate the task. We preface our discussion with a suggestive example from general relativity, and then move to an analysis in the context of a non-critical Liouville string approach.

2.3 Time as a Renormalization-Group Scale:
A Suggestive Example from General Relativity

Aspects of the above intuitive arguments are illustrated in a study\[20\] of a scalar field of mass $\mu$ coupled to an Einstein-Yang-Mills (EYM) black hole in four dimensions. This study was purely in the context of quantum gravity, with no string degrees of freedom, but features gauge hair and entanglement entropy. The renormalizability of the EYM field theory enables the partition function and entropy of the four-dimensional black hole to be calculated with the inclusion of quantum corrections \[20\].

Some of the quantum corrections to the entropy can be absorbed into the semiclassical Hawking-Bekenstein entropy $S = A/(4G_N)$, where $A$ is the area of the horizon, via a renormalization of Newton’s constant $G_N : G_{N_0} \rightarrow G_{N_{ren}}$. However, not all the quantum corrections can be absorbed into a bare parameter in this way, pointing to a new effect beyond the reach of the conventional renormalization programme. Formally, it may be absorbed into a quantum ‘horizon area’, which depends logarithmically on an ultraviolet cutoff $\epsilon$, and induces similar dependences in the partition function $Z$ and the entropy $S$:

$$ F \equiv -\log Z, \quad S \sim \log\left(\frac{\epsilon}{r_h}\right) $$

(11)

where $r_h$ is the semi-classical horizon radius. Since Hawking radiation causes $r_h$ to vary in time, we \[21\] identify $\log\left(\epsilon/r_h\right)$ with the target time $t$, fixing the sign by consistency with Hawking’s quantum evaporation of the black hole \[4\].

As shown in more detail in \[21\], this induced time-evolution of the density matrix of the scalar field in the EYM black hole background cannot be described by a conventional Hamiltonian $H$. It requires an open quantum-mechanical evolution equation of the form (2), with

$$ \delta H = \beta \left( -\frac{\partial H}{\partial t} + \frac{\partial F}{\partial t} \right) $$

(12)

$$ = O\left(\frac{\mu^2}{M_P}\right) + O\left(\log\left(\frac{\epsilon}{r_h}\right)^{-1}\right) + \ldots, $$

(13)

and the entropy of the scalar field $\phi$ increases with time:

$$ \frac{\partial S}{\partial t} = O\left(\frac{\mu^2}{M_P}\right) + O\left(\frac{r_h}{\epsilon}\right) + \ldots $$

(14)

The fact that the scalar field $\phi$ state becomes more mixed as a result of the $\delta H$ term (13) in the quantum Liouville equation (2) is due to a change in the entanglement
of the external $\phi$ field with the unmeasurable modes interior to the black hole. The amount of this entanglement depends on the quantum numbers of the EYM background. In particular, the more gauge hair it has, the smaller the $\epsilon$ dependence in the partition function [21], and hence the slower the rate of information loss in (14).

The quadratic dependence of $\delta H$ on the scalar mass $\mu$, divided in order of magnitude by just one power of $M_P$, is the largest that could be expected for any such modification of the quantum Liouville equation: \textit{a priori}, it could have been suppressed by one or more additional powers of $\mu/M_P$, or an exponential, or even absent all together. We are not in a position to estimate the coefficient of this $\mu^2/M_P$ term. Nor, indeed, can we be sure that such a parametric dependence would persist in a complete quantum theory of gravity. However, in Section 3 we review some explicit calculations within the Liouville string approach to quantum gravity, where we show that such a quadratic dependence indeed appears as a possibility [22, 23].

2.4 Liouville-String Approach to Quantum Gravity: Time as a World-Sheet Renormalization-Group Scale

This is not the place to review the great promise that string theory [7], now in its non-perturbative incarnation as $M$ theory, holds as a possible consistent quantum theory of gravity. Instead, we focus on the heterodox aspects of our Liouville string approach [24, 25]. Most discussions of string theory concentrate on classical backgrounds, which are described by conformal field theories on the string world sheet. In this case, one has a decoupling of the Liouville mode $\phi$ that scales the world-sheet metric:

\begin{equation}
\gamma_{\alpha\beta} = e^{2\hat{\gamma}_{\alpha\beta}} \equiv e^{\phi/Q}\hat{\gamma}_{\alpha\beta}
\end{equation}

where $\hat{\gamma}$ is some suitable fiducial metric on the world sheet, and $Q$ is defined below, and the Liouville dynamics is trivial as a result of conformal symmetry. Instead, we take the point of view that in order to discuss quantum fluctuations in the string background or transitions between generic different string vacua, one needs a broader framework than conformal field theories on the world sheet.

We therefore consider a generic conformal field theory action $S[g^*]$ perturbed by non-conformal deformations $\int d^2z g^i V_i$, whose couplings have non-trivial world-sheet renormalization-group $\beta$-functions:

\begin{equation}
\beta_i = (h_i - 2)(g^i - g^{*i}) + c^i_{jk}(g^j - g^{*j})(g^k - g^{*k}) + \ldots,
\end{equation}

where the $c^i_{jk}$ are operator product expansion (OPE) coefficients defined in the normal way. Coupling this theory to two-dimensional quantum gravity restores conformal invariance at the quantum level, with the Liouville field acting as a dynamical local scale on the world-sheet, that makes the gravitationally-dressed operators $[V_i]_\phi$ exactly marginal. The corresponding gravitationally-dressed conformal theory is:

\begin{equation}
S_{L-m} = S[g^*] + \frac{1}{4\pi\alpha} \int d^2z \{ \partial_\alpha \phi \partial^\alpha \phi - Q R^{(2)} + \lambda^i(\phi)V_i \}
\end{equation}
where \[24, 26\]:

\[
\lambda_i(\phi) = g_i e^{\alpha_i \phi} + \frac{\pi}{Q \pm 2\alpha_i} c_{ijk} g_j^i g_k^j \phi e^{\alpha_i \phi} + \ldots ; \quad \alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} - (h_i - 2)} \quad (18)
\]

Within this general framework, we focus on the operators \(V_i\) that are \((1, 1)\) but not exactly marginal, i.e., have \(h_i = 2\) but \(c_{ijk} \neq 0\). Their couplings obey:

\[
\frac{d}{d\tau} \lambda_i(\phi) = \beta_i ,
\]

where \(\tau = -\frac{1}{g_s} \log A : \quad A \equiv \int d^2z \sqrt{\gamma} e^{\alpha \phi(z, \bar{z})}\) is the world-sheet area, and \(\alpha = -\frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 8}\) with:

\[
Q = \sqrt{\frac{|25 - C[g, \phi]|}{3g_s^2}} + \frac{1}{2} \beta^j G_{ij} \beta^i \quad (19)
\]

Here \(C[g, \phi]\) is the Zamolodchikov C function \[27\], which is given according to the \(C\) theorem \[27\] by:

\[
C[g, \phi] = c^* - g_s^\chi \int_{\phi_*}^{\phi} d\phi' \beta^j G_{ij} \beta^i \quad (20)
\]

where * denotes a fixed point of the world-sheet flow, \(g_s\) is the string coupling and \(\chi\) is the Euler characteristic of the world-sheet manifold. The other part of \(Q\) is due to the local character of the renormalization-group scale \[8\], with \(G_{ij}\) related to divergences of \(< V_i V_j >\) and hence the Zamolodchikov metric \[27\] in the space of possible theories.

We interpret such non-critical strings \[28, 8\] as effective string models that describe the dynamics of modes observable in local scattering experiments. It is known that generic string backgrounds, such as black holes, also have unobservable delocalized modes which combine with local modes to provide exactly marginal deformations in the presence of singular space-time fluctuations. The corresponding local modes are not exactly marginal, and should be described by the formalism of the previous paragraph. The couplings of such local modes to delocalized modes lead to quantum entanglement and the possibility of information loss and entropy growth.

The zero mode of the Liouville field in such a string theory may be identified \[28, 8, 27\] with a target-time variable, since its kinetic term has negative metric, as seen in (17) and as contrasted with the kinetic terms for flat space coordinates in the fixed-point action:

\[
S[g^*] = -\frac{1}{4\pi \alpha^'} \int d^2z \partial_\alpha X^i \partial^\alpha X^i \quad (21)
\]

The opposite sign between (17) and (21) arises because the effective non-critical string theory is supercritical, i.e., its matter central charge is greater than the critical (conformal) value, which is 25 for the bosonic strings discussed here \[28, 24, 25\]. This identification of time with the Liouville field entails a modification of the conventional Hamiltonian time evolution, reflecting the above-mentioned quantum entanglement, which results in an irreversible temporal evolution in target space, with decoherence and associated entropy production, as we now review.
The effective density matrix for the local modes is:

\[ \tilde{\rho}(\text{local}, t) = \int d(\text{delocal}) \rho(\text{local}, \text{delocal}) \quad (22) \]

where \( \rho \) denotes the full density matrix, and the \textit{delocal} states play a role analogous to those of the unseen states inside the black-hole horizon in the arguments of [4] and the previous subsections. The integration over \textit{delocal} in (22) ensures that the reduced density matrix \( \tilde{\rho} \) is mixed in general, even if the full \( \rho(\text{local}, \text{delocal}) \) is pure. We have shown that \( \tilde{\rho} \) obeys a modified quantum Liouville equation of the form (2) [8, 29] with a Hamiltonian \( H \) and

\[ \delta H = -i \Sigma_{i,j} \beta^i G_{ij} \quad (23) \]

where the indices \((i, j)\) label all possible microscopically-distinct string background states with coordinate parameters \( g^i \), and \( G_{ij} \) is a metric in the space of such possible backgrounds [27]. Since these are not conformally invariant once one integrates out the \textit{delocal} degrees of freedom, the corresponding renormalization functions \( \beta^i \) are non-trivial, reflecting the back reaction of the light particles on the background metric. We note further that the background fields \( g^i \) must be quantized, as a result of the summation over world-sheet topologies in the Liouville string [8]. In the toy context of two-dimensional black-hole space times, the \textit{delocal} degrees of freedom are topological higher-level discrete string modes, which represent infinite-dimensional stringy gauge symmetries described mathematically by \( W_\infty \) algebras [30, 8, 31].

There are general properties of the Liouville system that follow from the renormalizability of the world-sheet \( \sigma \)-model theory [8]. These include energy conservation on the average, and probability conservation. The nature of the renormalization group on the two-dimensional world-sheet [27] entails monotonic entropy increase, \( \partial_t S \propto \beta^i G_{ij} \beta^j \geq 0 \), and hence a microscopic arrow of time. The maximum magnitude of effect that we can imagine is

\[ \delta H \simeq H^2/M_P \quad (24) \]

which would be around \( 10^{-19} \ldots 10^{-20} \) GeV for a typical low-energy probe. A contribution to the evolution rate equation (23) of this order of magnitude would arise if there were some Planck-scale interaction contributing an amplitude \( A \simeq 1/M_P^2 \) and hence a rate \( R \simeq 1/M_P^4 \), to be multiplied by a density of singular microscopic backgrounds \( n \simeq L_P^{-3} \simeq M_P^3 \), yielding the overall factor of \( \simeq 1/M_P \) shown in (24) [17]. As we have seen above, such an estimate was found in a pilot study of a scalar field in a four-dimensional black-hole background [21], and such an estimate is also found in a Liouville-string representation of Dirichlet membranes (D branes) [32], as we shall discuss in section 3.

Similar conclusions are reached in a study of asymptotic scattering, since it is impossible to define factorizable S-matrix elements in Liouville string theory, once we interpret the Liouville field as time. To see this, we first note that correlation functions in Liouville theory may be written in the form

\[ A_N \equiv \langle V_{i_1} \ldots V_{i_N} \rangle_{\mu} = \Gamma(-s)\mu^s < \int d^2z \sqrt{\gamma} e^{i\phi} \tilde{V}_{i_1} \ldots \tilde{V}_{i_N} >_{\mu=0} \quad (25) \]
where the $\tilde{V}_i$ have the Liouville zero mode removed, $\mu$ is a scale related to the worldsheet cosmological constant, and $s$ is the sum of the anomalous dimensions of the $V_i$: $s = -\sum_{i=1}^{N} \frac{Q}{2} - \frac{Q}{2}$. As it stands, (23) is ill defined for $s = n^+ \in Z^+$, because of the $\Gamma(-s)$ factor [33]. To regularize this factor, we use the integral representation [34, 8]

$$\Gamma(-s) = \int dA e^{-A A^{-s-1}},$$

where $A$ is the covariant area of the world sheet, and analytically continue to the contour shown in Fig. 1. Interpreting the Liouville field $\phi$ as time [8]: $t \propto \log A$, we interpret the contour of Fig. 1 as representing evolution in both directions of time between fixed points of the renormalization group: Infrared fixed point $\rightarrow$ Ultraviolet fixed point $\rightarrow$ Infrared fixed point.

Within this approach, it is not difficult to see that conventional $S$-matrix elements are in general ill-defined in Liouville-string theory, and that scattering must be described by a non-factorizable $S$-matrix. Decomposing the Liouville field in an orthonormal mode sum: $\phi(z, \bar{z}) = \sum_n c_n \phi_n = c_0 \phi_0 + \sum_{n \neq 0} \phi_n$, where $\nabla^2 \phi_n = -\epsilon_n \phi_n : n = 0, 1, 2, \ldots$, we separate the zero mode $\phi_0 \propto A^{-\frac{1}{2}}$ with $\epsilon_0 = 0$. One may now consider $\tilde{A}_N$, the correlation function with $\phi_0$ subtracted, and its behaviour under an infinitesimal Weyl transformation $\gamma(x, y) \rightarrow \gamma(x, y)(1 - \sigma(x, y))$. We have found that the correlator $\tilde{A}_N$ transforms as follows [32]:

$$\delta \tilde{A}_N \propto \sum_i h_i \sigma(z_i) + \frac{Q^2}{16\pi} \int d^2x \sqrt{\hat{g}} \hat{R} \sigma(x) + \mathcal{O}\left(\frac{s}{A}\right)$$

(26)

where the hat notation denotes transformed quantities. We see explicitly that (26) contains non-covariant terms $\propto A^{-1}$ if the sum of the anomalous dimensions $s \neq 0$. Thus the generic correlation function $A_N$ does not have a well-defined limit as $A \rightarrow 0$.

In our approach to string time we identify [8] the target time as $t = \phi_0 = -\log A$, where $\phi_0$ is the world-sheet zero mode of the Liouville field. The normalization follows from a consequence of the canonical form of the kinetic term for the Liouville
field $\phi$ in the Liouville $\sigma$ model \[28,3\]. The opposite flow of the target time, as compared to that of the Liouville mode, is, on the other hand, a consequence of the 'bounce' picture \[34,8\] for Liouville flow of Fig. 1. This identification implies that, as a result of the above-mentioned singular behaviour in the ultraviolet limit $A \to 0$, the correlator $\hat{A}_N$ cannot be interpreted as an $S$-matrix element, whenever there is a departure from criticality $s \neq 0$.

When one integrates over the Saalschultz contour in Fig. 1, the integration around the simple pole at $A = 0$ yields an imaginary part \[34,8\], associated with the instability of the Liouville vacuum. We note, on the other hand, that the integral around the dashed contour shown in Fig. 1, which does not encircle the pole at $A = 0$, is well defined. This can be interpreted as a well-defined $S$-matrix element, which is not, however, factorisable into a product of $S$- and $S^\dagger$-matrix elements, due to the $t$ dependence acquired after the identification $t = -\log A$ \[3\].

### 3 Play Times

In this section we shall present briefly some examples and applications of the above Liouville string formalism. Although the examples considered are simple, we believe these toy cases capture important qualitative feature of realistic string gravity situations.

#### 3.1 Two-Dimensional String Black Holes

As a first illustration of our approach to non-critical string theory, we discuss the two-dimensional black-hole \[36\]. This toy laboratory gives insight into the nature of time in string theory and contributes to the physical effects mentioned in other sections. Its action may be written in the form \[36\]

$$S_0 = \frac{k}{2\pi} \int d^2z [\partial_r \partial_r - \tanh^2 r \partial_t \partial_t] + \frac{1}{8\pi} \int d^2z R^{(2)} \Phi(r)$$

(27)

where $r$ is a space-like coordinate and $t$ is time-like, $R^{(2)}$ is the scalar curvature, and $\Phi$ is the dilaton field. The customary interpretation of (33) is as a string model with $c = 1$ matter, represented by the $t$ field, interacting with a Liouville mode, represented by the $r$ field, which has $c < 1$ and is correspondingly space-like \[28\]. As an illustration of the approach outlined in the previous section, however, we re-interpret \[8,33\] as a fixed point of the renormalization group flow in the local scale variable $t$. In our interpretation, the ‘matter’ sector is defined by the spatial coordinate $r$, and has central charge $c_m = 25$ when $k = 9/4$ \[36\]. Thus the model (27) describes a critical string in a dilaton/graviton background. The fact that this is static, i.e., independent of $t$, reflects the fact that one is at a fixed point of the renormalization group flow \[8\].

\[1\]This formalism is similar to the Closed-Time-Path (CTP) formalism used in non-equilibrium quantum field theories \[8\].
We now outline how one can use the machinery of the renormalization group in curved space, with \( t \) introduced as a local Liouville renormalization scale on the world sheet, to derive the model (33): a detailed technical description is given in \[8\]. There are two contributions to the kinetic term for \( t \) in this approach, one associated with the Jacobian of the path integration over the world-sheet metrics, and the other with fluctuations in the backgroud metric.

To exhibit the former, we first choose the conformal gauge \( \gamma_{\alpha\beta} = e^{\phi} \hat{\gamma}_{\alpha\beta} \) \[24, 25\], where \( \phi \) represents the Liouville mode, as in (15). An explicit computation \[25\] of the Jacobian using heat-kernel regularization yields

\[
-\frac{1}{2\pi} \partial_\alpha \phi \partial^\alpha \phi + \ldots
\]  

(28)

where the dots represent terms that are not essential for this argument, and an appropriate overall normalization has been chosen. This procedure reproduces the critical string results of \[36\] when one identifies the Liouville field \( \phi \) with \( \sqrt{k} t \).

Equation (28) contains a negative (time-like) contribution to the kinetic term for the Liouville (time) field, but this is not the only such contribution. Quantum fluctuations in the background metric yield, after renormalization, a term of the form \[8\]

\[
\frac{k}{8\pi} R \partial_\alpha t \partial^\alpha t
\]  

(29)

where \( R \) is the scalar curvature in target space.

In the case of the Minkowski black-hole model \[30\], the scalar curvature is

\[
R = \frac{4}{\cosh^2 r} = 4 - 4\tanh^2 r,
\]  

(30)

Substituting (30) into equation (29) we obtain the second contribution to the kinetic term for the Liouville field \( \phi \). Combining it with the world-sheet metric Jacobian term in (28) we recover precisely the action (27) for the two-dimensional black hole.

This is a non-trivial check of our identification of time with the Liouville field. The same model can also be used to exemplify the fact that exactly-marginal operators may in general contain combinations of local and delocalized modes. One of the exactly-marginal operators of Euclidean version of the two-dimensional black hole \[36\] takes the form \[31\]

\[
L_0^1 \mathcal{F}_0^1 \propto \mathcal{F}_{\frac{1}{2},0,0}^{c-c} + i(\psi^{++} - \psi^{--}) + \ldots
\]  

(31)

where the \( \psi \) denote higher-level operators, and the first term in (31) is a ‘tachyon’ operator given by

\[
\mathcal{F}_{\frac{1}{2},0,0}^{c-c}(r) = \frac{1}{\cosh r} F\left(\frac{1}{2}, \frac{1}{2}, 1, \tanh^2 r\right)
\]  

(32)

The higher-level string modes cannot be detected in local scattering experiments, because of their delocalized character. An ‘experimentalist’ therefore sees necessarily a truncated matter theory, where the only deformation is the tachyon \( \mathcal{F}_{-\frac{1}{2},0}^{c} \), which
is a (1,1) operator, but is not exactly marginal [8]. When truncated to such local modes, the model is non-critical, and the general analysis of the previous section applies.

3.2 The Arrow of Liouville Time in Two-Dimensional String Universes

As another non-trivial application of the Liouville approach to time, we consider the dilaton-matter black-hole solution found in [37]. In two dimensions, the action of the dilaton-tachyon system coupled to gravity is

\[ S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \{ e^{-2\Phi} [R + 4(\nabla \Phi)^2 - (\nabla T)^2 - V(T) + 4\lambda^2] \} \]  

(33)

where \(4\lambda^2\) is the two-dimensional cosmological constant, which is related to the central charge \(c\) of the corresponding world-sheet \(\sigma\) model via [36]:

\[ \lambda^2 = \frac{k}{3}(c - 26) \]  

(34)

where \(k > 2\) is the level parameter of the Wess-Zumino conformal field theory that has as a conformal solution the target-space two-dimensional theory [33], which is in turn related to the central charge by [36] \(c = (3k/k - 2) - 1\). The quantity \(V(T)\) is the tachyon potential, which is ambiguous in string theory [38]. The only unambiguous term is the quadratic term for the tachyon field, which is also all that we need for our analysis. One solution of the equations of motion derived from this action is the static black hole of [39], which from a string theory point of view is an equilibrium (conformal) fixed point solution in the space of string theories. A non-equilibrium exact solution appears [37] when one considers collapsing tachyonic (massless) matter, \(T = T(x^+)\), where \(x^+\) is a light-cone coordinate. The solution is characterised by an initial singularity at \(x^+ = 0\) and the absence of a white hole.

Its stability has been analyzed in [22], whose results we now summarize. The metric element for the \(\lambda = 0\) case can be written in the form

\[ ds^2 = e^{2\tau} \left( -\xi^2 d\tau^2 + d\xi^2 \right) \]  

(35)

where \(\xi \equiv e^{r/2}\) and \(\tau \equiv t/2\). We see from [35] that this expanding space is conformally equivalent to a two-dimensional Rindler space with constant acceleration [39]. An analysis of Bogolubov coefficients has shown that this \(\lambda = 0\) model is thermodynamically stable in the sense of general relativity, with no particle production. This is because of a cancellation of the effects of the spatial expansion visible in [33] and the Rindler-like acceleration [10], as far as particle production is concerned. On the other hand, the \(\lambda \neq 0\) curved space time, formed by collapsing matter \(T(x^+)\), corresponds to time-dependent black-hole solution [37], and was found to exhibit nonthermal particle creation, and to be thermodynamically unstable.
It is tempting to guess that the equilibrium case $\lambda = 0$ constitutes the end-point of the time evolution of the unstable $\lambda^2 \neq 0$ solutions. As we have argued in [22], this point of view is supported by a Liouville-string interpretation of the space-time metric (35), when one identifies $t$ above with the Liouville zero mode. From this point of view, the rate of change of the cosmological constant during the non-equilibrium phase is obtained from the Zamolodchikov $C$ theorem [27], appropriately extended to Liouville strings, once one identifies the Liouville field with a local renormalization scale on the world-sheet [8]:

$$\frac{\partial}{\partial t} \lambda^2 \sim - \sum_{i,j} \beta^i < V_i V_j > \beta^j$$

where the $V_i$ are the vertex operators corresponding to deformations $g^i$, and the summation over $i, j$ also includes spatial integrations $\int dr \int dr'$. The two-point correlators $< V_i V_j >$ constitute a metric in theory space [27], which is positive definite for unitary world-sheet theories, implying an irreversible time flow [8].

Within this approach, it is immediate to deduce from (20) the time dependence of $\lambda(t)$. In the case of small $\lambda$ and weak matter fields $T$, the leading-order effect is associated with the graviton $\beta$ functions. To $O(\alpha')$, the latter are proportional to $RG_{MN}$, where $R$ is the scalar curvature for the generic case $\lambda \neq 0$ of the time-dependent black hole solution of [37], computed in [22]. Using the the initial condition $\lambda^2(0) = \lambda^2_0 \ll 1$, and the fact that, to this order, the Zamolodchikov metric $< V_i V_j >$ is proportional to $\delta_{ij} \delta(r - r')$, one finds:

$$\lambda^2(t) = \frac{\lambda^2_0}{1 + \lambda^2_0 A(e^t - 1)}$$

where

$$A = \left(4C^2_1 a^2 \int_0^\infty dr e^{-r} \right) = 4C^2_1 a^2$$

with $C_1, a$ appropriate constants [22]. Thus we have a quantitative description of the flow in Liouville time towards flat space times: $\lambda^2 \to 0$ as $t \to \infty$. Could such a mechanism also be operational in our four-dimensional world? If so, how rapidly would the cosmological constant be relaxing, and how would its magnitude compare with current experimental limits?

### 3.3 D-Brane Foam and the Decoherent Scattering of Light Particles

As another example, we consider a string picture of space-time foam, i.e., microscopic quantum-gravity fluctuations, described as $D$-branes, which provide a very powerful representation of string solitonic states [41]. A conformal field theory formulation of soliton recoil in string theory, has been developed in [42, 43, 44, 32], exploiting the $D$-brane representation of the horizon of black holes [41] to describe the back reaction of quantum fluctuations of matter on space time.
The basic observation of [42, 43] was that such recoil degrees of freedom can be described by a logarithmic conformal field theory [45]. To see this, we recall that the propagation of a light closed-string particle through this representation of $D$-brane foam involves, at the lowest order, a diagram with a disk topology, internal tachyon vertices, and Dirichlet boundary conditions [41]. In this way one may describe scattering through a real or virtual $D$-brane state, with production and decay amplitudes $A_{TT \, m}$, subject to the energy-conservation condition. The next term in a topological expansion in genus $g = 2 - 2\#_{\text{handles}} - \#_{\text{holes}}$ is an annulus with closed-string operator insertions. As has been discussed elsewhere [46, 47], this has a singularity $A \sim \delta(E_1 + E_2)\sqrt{\frac{1}{\log(\delta)}}$ in the pinched-annulus configuration $\delta \to 0$, which is regularized by introducing recoil operators $C, D$ to describe the back reaction of the struck $D$ brane:

\begin{equation}
V_{\text{rec}} = y_i C + u_i D \quad C \equiv \epsilon \int_{\partial \Sigma} \Theta_\epsilon(X^0) \partial_n X^i \quad D \equiv \int_{\partial \Sigma} X^0 \Theta_\epsilon(X^0) \partial_n X^i \quad (39)
\end{equation}

where $y_i$ and $u_i$ are the position and momentum of the recoiling $D$ brane, and $\lim_{\epsilon \to 0}\Theta_\epsilon(X^0)$ is a suitable integral representation of the step function [43], with $\epsilon$ a suitable infrared regulator parameter.

The quantum treatment of $D$-brane recoil necessitates the introduction of world-sheet annulus diagrams, whose large-size limit is characterized by a size parameter $L$ that, together with a conventional ultraviolet world-sheet regulator parameter $a$, specifies the value of $\epsilon$ [43]:

\begin{equation}
\frac{1}{\epsilon^2} \simeq 2\log|L/a|^2 \quad (40)
\end{equation}

As discussed earlier, we identify the Liouville field $\phi$ with the renormalization scale on the world sheet [8, 29], and its zero mode, $\phi_0$, is further identified with the time variable

\begin{equation}
\phi_0 = t \simeq \log|L/a| \quad (41)
\end{equation}

Note that there is no absolute time in this approach, since physical quantities are described by the renormalization group, which relates different scales $L, L'$ that correspond to time differences $\delta t \simeq \log|L/L'|$. As discussed in [47], we identify $1/\epsilon^2 \sim \log \delta$, and in turn, using the Fischler-Susskind mechanism [15] on the world sheet to relate renormalization-group infinities among different genus surfaces, we identify $t \sim \log \delta$.

The operators $C, D$ constitute a logarithmic pair [43] with $\langle C(z)C(0) \rangle$, $\langle C(z)D(0) \rangle$ non-singular as $\epsilon \to 0^+$, whereas $\langle D(z)D(0) \rangle$ is singular with a world-sheet scale dependence [13] $D \to D + tC$, from which we infer that $u_i \to u_i$, $y_i \to y_i + u_i t$, corresponding to a Galilean time transformation [44] as is appropriate for a heavy $D$ brane with mass $\propto 1/g_s$ [13]. This representation or recoil is a striking confirmation of the utility of our identification of the Liouville field with time.

Continuing the analysis, one finds that the logarithmic operators [39] make divergent contributions to the genus-0 amplitude in the limit where it becomes a pair of Riemann surfaces $\Sigma_1, \Sigma_2$ connected by a degenerate strip [32, 44]:

\begin{equation}
A_{\text{strip}} \sim g_s (\log^2 \delta) \int d^2 z_1 D(z_1) \int d^2 z_2 C(z_2),
\end{equation}
\[
g_s \log \delta \int d^2 z_1 D(z_1) \int d^2 z_2 D(z_2), \quad g_s \log \delta \int d^2 z_1 C(z_1) \int d^2 z_2 C(z_2)\tag{42}
\]

Assuming a dilute gas of monopole defects on the world sheet, the amplitudes \[
(42)
\]
become contributions to the effective action \[32, 44\]. One may then seek to cancel them or else to absorb them into scale-dependent couplings of the world-sheet field theory. If these could all be cancelled, the corresponding theory would be conformally invariant, whereas absorption of these divergent contributions would result in departures from criticality.

The leading double logarithm associated with the CD combination in \[42\] may indeed be cancelled \[44\] by imposing the momentum-conservation condition

\[
u_i = g_s (k_1 + k_2)_i \tag{43}
\]

as expected for a D brane soliton of mass \(1/g_s\). As we now show, this is also consistent with the tree-level energy-conservation condition that is obtained by integration over the Liouville zero mode, supporting its interpretation as time. From the point of view of the Liouville theory on the open world sheet, the tree-level monopole mass term arises from a boundary term \(i \int_{\partial \Sigma} Q X^0 \hat{k}\) in the effective action \[19\], where \(\hat{k}\) is the extrinsic curvature and \(Q\) is given by equation \(19\). When the Liouville integration is performed at the quantum level, \(Q\) is replaced by its value in \(19\), which receives a contribution from \(\beta_y = u\) \[17\], by virtue of the logarithmic operator product expansion of \(C\) and \(D\) \[43\]. Expanding the right-hand side of \(19\) using \(20\), for small \(|u_i u^i| \ll 1\), we find the quantum energy-conservation condition:

\[
E_1 + E_2 = \frac{e}{\sqrt{\beta}} = e (\pi (C - 25)/3 g_s)^{1/2} = \frac{e}{\sqrt{g_s}} (1 + u^2_i/2 + \ldots) \tag{44}
\]

which matches the momentum-conservation condition \[13\] when we set \(e \sqrt{\pi/3} = 1/\sqrt{g_s}\). In this way, we confirm our interpretation of time as the Liouville field \[8, 29\]. The fact that the cancellation of the leading double logarithm in \(12\) enforces energy conservation \(\frac{d}{dt} < E >= 0\) confirms previous arguments \[8\] in the general renormalization-group approach to Liouville dynamics.

The single logarithms associated with the CC and DD contributions in \(12\) are a different story, since they can only be absorbed into quantum coupling parameters \[32, 14\]: \(\hat{y}_i = y_i + \alpha_C \sqrt{\log \delta}, \hat{u}_i = u_i + \alpha_D \sqrt{\log \delta}\). The resulting probability distribution in theory space becomes time dependent \[32, 14\]:

\[
\mathcal{P} \sim \frac{1}{g_s^2 \log \delta} e^{-(q_m - q_m)G^{mn}(q_n - q_n)} \tag{45}
\]

where \(q_m \equiv \{y_i, u_i\}\).

This in turn leads to a breakdown of the usual \(S\)-matrix description of the transition from the initial-state density matrix. To see this, we first use the Liouville dressing of the recoil operators \[13\], and the identification of the Liouville mode with the time \(X^0\) appearing already in the logarithmic pair \[32\], to derive \[32\] the
following form for the singular part of the target-space metric \( G_{MN} \) around the moment of the collision:

\[
G_{00} = -1, \quad G_{ij} = \delta_{ij}, \quad G_{0i} = G_{i0} = \epsilon (\epsilon y_i + u_i t) \Theta_\epsilon(t)
\]  

(46)

It is to be understood that, in addition to (46), there is also a static, spherically-symmetric part of the metric, which is \( \mathcal{O}(M/R) \) at large distances \( R \) from the struck \( D \) brane of mass \( M \). We now consider the scattering of a second low-energy light particle at large impact parameter \( R \), so that we may neglect this spherically-symmetric part in a first approximation, and consider the asymptotic metric as flat to zeroth order in \( M/R \). The physics that interests us is that associated with the \( \epsilon \)-dependent singularity in (46).

There is particle creation associated with the metric (46). To see this, we first note that an on-mass-shell scalar field \( \phi \) of mass \( \mu \) in the background (46) may be expanded in terms of ordinary flat-space Minkowski modes which play the rôle of the “in” modes, since our background space (46) can be mapped, for \( t > 0 \), to a flat space to \( \mathcal{O}(\epsilon^2) \) by means of a simple coordinate transformation:

\[
\tilde{t} = t, \quad \tilde{X}_i = X_i + \frac{1}{2} \epsilon u_i t^2
\]  

(47)

which may be represented by the Penrose diagram shown in Fig. 2.

Each of the four diamonds corresponds in a conventional Rindler space to one of the causally-separated regions [39], and our space-time corresponds to the right-hand diamond. We see that it is flat Minkowski for \( t < 0 \), and that for \( t >> 0 \) the shaded region of the space-time resembles, to \( \mathcal{O}(\epsilon) \), a Rindler wedge, with ‘acceleration’ \( \epsilon u_i \). The event horizons are indicated by the straight lines separating the diamond-shaped regions of the diagram. The dashed line corresponds to the curvature singularity \( \epsilon^2 \delta_\epsilon(t) \), which may be ignored in order \( \mathcal{O}(\epsilon) \). As a consequence of the non-relativistic nature of the heavy \( D \) brane, considered here, this singularity appears to violate causality, lying outside the light cone. This is merely an artifact of taking the limit as the velocity of light \( c \to \infty \), as is appropriate for a non-relativistic (heavy) \( D \) particle. One expects causality to be restored, with a rotation of the singular surface so as to become light-like, in a fully relativistic treatment of the problem, which lies beyond the scope of the present discussion.

Analyzing the non-trivial Bogolubov transformation between the “in” and “out” vacua at \( t < 0 \) and \( t >> 0 \), we find [23] the following number of particles created in the mode labelled by \( E \) and \( K \):

\[
n_{EK} \propto \frac{\epsilon^2(u.K)^2}{E^6}
\]  

(48)

Note that the formula (48) describes particle creation with a non-trivial angular distribution around the direction of the velocity vector \( u_i \). The particle spectrum is not thermal as it would have been in the case of a uniform acceleration [40].
in order of magnitude, given by:

\[ M(50) \text{ agrees with the generic estimates of } [6, 8, 32] \]

Figure 2: Penrose diagram for the space-time environment derived from the quantum recoil of a heavy D brane induced by the scattering of a light closed-string state.

It can be shown that the pertinent influence action \( S_{IF} \) for the scale field \([50]\) is, in order of magnitude, given by:

\[ S_{IF} \propto - \hbar \log \mathcal{F}(a, a') \sim \hbar \left| \frac{1}{4} g_s^2 |\hat{u} \cdot K|^2 \right| t + \ldots \]  \hspace{1cm} (49)

where \( \hat{u} \) denotes a unit vector in the direction of \( u \), and the \( \ldots \) indicate terms which are not of interest to us here. Thus the quantum fluctuations of the D-brane recoil velocity, represented by the summation over world-sheet genera \([17, 14]\), induce decoherence for the second light particle that grows linearly in time. If there were only the classical recoil velocity, there would be no \( \log \delta \) dependence and hence no time dependence and no decoherence.

The imaginary coefficient in \([19]\), and its linear dependence on \( t \), imply that the quantum recoil of the D brane, which corresponds in conformal-field-theoretic language to a scenario departure from criticality, induces a non-Hamiltonian contribution \( \delta \mathcal{H} \) \([2]\) into the generic evolution equation of the reduced density matrix for the scalar particle in this background. Since the momentum of the spectator particle, \( K \), may be taken generically to be of the same order of magnitude as its energy \( E \), one obtains from \([19]\) the order-of-magnitude estimate

\[ \delta \mathcal{H} \sim \mathcal{O}(E^2/M_D) \]  \hspace{1cm} (50)

where \( M_D \) is the heavy D-brane mass, which is related to the conventional string scale \( M_s = (\alpha')^{-1/2} \), with \( \alpha' \) the Regge slope, via \( g_s \sim M_s/M_D < 1 \). The formula \((50)\) agrees with the generic estimates of \([3, 8, 2]\), with suppression by only a single power of the heavy mass scale, in this case \( M_D \).
4 What is the Future of Time?

Clearly, the ideas discussed in this review are often heuristic, though based on plausible intuition and some detailed analysis in specific models. Much work is needed to put the results on a firmer theoretical foundation. We are of the firm opinion that string theory [7], in some non-perturbative incarnation such as $M$ theory, provides the most appropriate framework for this necessary work, and the technology of $D$ branes [41] is certainly a very powerful tool to tackle it, as we have attempted in [51]. We do not attempt here to preview future theoretical developments. However, we do believe that one should constantly keep in mind possible experimental probes of such speculative theoretical ideas, and conclude this article by reviewing some tentative suggestions in this direction.

4.1 Uncertain Times

The canonical quantization of string theory space has been discussed in [32, 32]. Motion in this space is characterized as a renormalization-group flow in the space of effective $\sigma$-model couplings $g^i$ on the lowest-genus world sheet, with the flow variable identified in turn with the Liouville field and the target time variable. This flow is classical in the absence of higher-genus effects. However, in their presence, the renormalization-group flow has been shown to obey the relevant Helmholtz conditions which are necessary and sufficient for the canonical quantization of the $\sigma$-model couplings $g^i$. This potentially has interesting consequences for cosmology and provides a possible string scenario for inflation and entropy generation [52].

This summation over genera and the ensuing quantization clearly entail a modification of the conventional conception of space-time coordinates, though the exact nature of the resulting picture is still to be understood. Some aspects are, however, already apparent. It is clear from the identification of target time $t$ with the zero mode of the Liouville field, up to a $g^i$-dependent normalization factor:

$$t = Q[g^i] \varphi_0$$

where $\varphi_0$ is the world-sheet zero mode of the Liouville scale $\varphi$ in (15), and $C[g^i]$ the Zamolodchikov $C$-function, that $t$ also becomes a quantum operator. The identification of the target-space coordinates as zero modes of the $\sigma$-model fields $X$ also implies in non-critical strings an interdependence between these coordinates and the quantum background $\{g^i\}$. Although the precise nature of the (correspondingly) quantized target-space coordinates remains to be fully understood, these observations suggest the appearance of non-trivial space-time commutation and uncertainty relations.

This can be seen somewhat more directly in the dynamical context of $D$ branes treated as non-relativistic heavy objects. The summation over genera in that case can be shown [53] to yield an uncertainty relation between the Liouville time and...
transverse spatial coordinates of the $D$-brane:

$$\delta y \delta t \geq \sqrt{g_R \alpha'},$$

(52)

where $g_R$ is an appropriately renormalized string coupling, and $\alpha'$ is the Regge slope. Uncertainties resembling (52) have also been derived in the context of critical $D$ branes [54], but in that case the right-hand side depends only on the Regge slope $\alpha'$, and not on the string coupling.

These features may lead to uncertainty $\delta t$ in the time delay of light propagation, leading to an energy-dependent dispersion in the apparent velocity of light, which might take the form

$$\delta c \sim (E/M),$$

where $M$ is some gravitational mass scale. The most sensitive probe of such a possibility may be provided by gamma-ray bursters (GRBs), some of which are known to be at cosmological distances, and whose emissions may have short-time structures. We have estimated [55] that a careful observational programme might be sensitive to $M \sim 10^{18}$ GeV, comparable to the possible mass scale of quantum gravity.

4.2 Towards Experimental Tests of Decoherence Induced by Quantum Gravity

Tests of quantum-gravity entanglement may be possible in particle physics, using sensitive probes of quantum mechanics. The most sensitive probe, to date, is the neutral kaon system. Assuming energy and probability conservation, it was shown that one can parametrize the modification term $\delta H$ in (2) as

$$\delta H = \kappa_{\alpha\beta} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -2\alpha & -2\beta \\
0 & 0 & -2\beta & -2\gamma
\end{pmatrix}$$

(53)

where the indices $\alpha, \beta$ label Pauli matrices $\sigma_{\alpha,\beta}$ in the $K_{1,2}$ basis, and we have also assumed that $\delta H$ has $\Delta S = 0$. The three free parameters $\alpha, \beta, \gamma$ must obey the conditions

$$\alpha, \gamma > 0, \quad \alpha\gamma > \beta^2$$

(54)

stemming from the positivity of the matrix $\rho$.

It is easy to see that these parameters induce decoherence and violate CPT [56]. Various observables sensitive to these parameters have been discussed in the literature, including the $2\pi$ decay asymmetry and the double semileptonic decay asymmetry. Measurements of these and other quantities would in principle enable the decohering CPT violation that we propose here to be distinguished from 'conventional' quantum-mechanical CPT violation [56]. The data so far agree perfectly with conventional quantum mechanics, imposing only the following upper limits [57]:

$$\alpha < 4.0 \times 10^{-17}\text{GeV}, \quad \beta < 2.3 \times 10^{-19}\text{GeV}, \quad \gamma < 3.7 \times 10^{-21}\text{GeV}$$

(55)

We cannot help being impressed that these bounds are in the ballpark of $m_K^2/M_P$, which is the maximum magnitude that we could expect any such effect to have.
This and the previous example give us hope that experiments may be able to probe close to the Planck scale, and we would not exclude the possibility of being able to test some of the speculative ideas about quantum gravity reviewed in this article.

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