Design of Petri Net Supervisor with 1-monitor place for a Class of Behavioral Constraints

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ABSTRACT
This paper studies the design of supervisory controllers with a minimum number of monitor places for Manufacturing System modeled as safe Petri Nets. The proposed approach considers a class of safety specifications known as Behavioral Constraints with a restricted syntax. The set of Behavioral Constraints are represented as predicate logic formulas in normal conjunctive form. Then, each Behavioral Constraint induces a set of algebraic linear inequalities. The approach establishes an equivalence in order to minimize the number of monitor places. Thus, each Behavioral Constraint induces a single linear inequality, giving rise to a 1-monitor place Petri Net supervisor. The approach is illustrated with the design and implementation of 1-monitor place modular supervisor for an automated manufacturing prototype.

1 Introduction

The operation of manufacturing systems is increasingly challenging because of the execution of more complex tasks. In order to reduce periods for manufacturing procedures, but complying with regulatory standards to guarantee a proper operation and product quality, plenty of manufacturing features have been improved in recent years ([1]). The reconfigurability allows to change the entire procedure of an Automated Manufacturing System (AMS), but it also must minimize the use of time and resources ([2]). The safety of the operation, with all the automatic processes occurring in the AMS is a critic feature, leading to the existence of entities with the propose of guarantee safety operation, such as Supervisory Controllers (SCs). For AMS modeled as discrete event systems, Supervisory Control Theory (SCT) proposed by Wonham in [3] is a well-accepted paradigm frequently employed for designing logic controllers at the coordination and basic layers of control systems. Petri Nets provides a formal logic platform for modeling and synthesis of logic controllers as well as analysis widely used in AMS (e.g. [4] [5]). The synthesized Supervisory Controller (SC) is a Petri Net (PN) with a finite number of places, which are called monitor places. Some of the advantage of PN, more compact representations of the supervisor than their automata counterparts are usually achieved and accepts concurrency in the execution of transitions. Among several design methods considering safety specifications, the Invariant Based Control Design method [6] has been successfully employed to deal with forbidden states [7] and Behavioral Constraints [8]. However, the resulting PN may not be a minimal realization of the SC. Synthesis strategies for PN supervisor with a reduced number of monitor places have been proposed for forbidden state avoidance [9] only, not for Behavioral Constraints. This paper studies the synthesis of 1-monitor place supervisory controllers for safe PN. The proposed design approach employs the Invariant Based Control Design (IBCD) method and a class of safety specifications [10] that can be modeled as Behavioral Constraints [8]. Section 2 introduces the fundamentals of PN and SCT and the representation of Behavioral Constraints (BCs) as a set of linear inequalities. Section 3 shows the proposed technique to transform the set of Behavioral Constraint (BC) into a smaller set of linear inequalities, leading to a PN supervisor with a reduced number of monitor places using the IBCD method. Section 3 also establishes the conditions for a Supervisory Controller based on Be-

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havioral Constraints (SCBC) to be proper. Section 4 shows the case study used in this work, an AMS, presenting its description and modeling. Then, Section 5 presents a set of BCs to be imposed in the AMS, the representation as linear inequalities and the resulting SC designed using the IBCD method, as well as its implementation as a ladder diagram.

2 Fundamentals

In this Section the basic definitions of Petri Nets and Supervisory Control Theory are introduced.

2.1 Petri Nets fundamentals

For modeling techniques, as well as structural and dynamic properties of PN the reader is refereed to [11].

Definition 1 (Petri Net) A Petri Net is defined as the triplet \((S, T, F)\) with \(S\) as the set of places, \(T\) as the set of transitions and \(F\) : \(\{S \rightarrow T, T \rightarrow S\}\) a total relation.

Definition 2 (Marking vector) Let \(N\) be a PN and \(S = \{s_1, s_2, ..., s_n\}\) its set of places. The marking vector is a mapping \(M : S \rightarrow N \cup \{0\}\) represented by \([M(s_1) M(s_2) ... M(s_n)]\).

Definition 3 (Enabled transition) Let \(N\) be a PN and \(t \in T\) a transition of \(N\). The transition \(t\) is said to be enabled if the marking of all input places is grater or equal than 1.

Definition 4 (Initial marking vector) Let \(N\) be a PN and \([M(s_1) M(s_2) ... M(s_n)]\) its marking vector. The initial marking vector is defined as the marking vector when no transition has been fired.

Definition 5 (PN System) Let \(N\) be a PN and \(M_0\) its initial marking vector. A PN system is defined as a the pair \((N, M_0)\).

Definition 6 (Boundedness) Let \((N, M_0)\) be a PN system. The system is called bounded if for each place \(s\) exists a natural number \(b\) such that \(M(s) \leq b\) for all reachable markings from \(M_0\).

If \(M(s) \leq b\) holds for any place \(s\), then the system is called \(b\)-bounded.

Definition 7 (Liveness) Let \((N, M_0)\) be a PN system. The PN system is called live if, for any reachable marking \(M\) and any transition \(t\), there is a marking \(M\) which enables \(t\).

Definition 8 (Safe Petri Net System) Let \((N, M_0)\) be a PN system. The system is called Safe if the system is 1-bounded and live.

Even though the term Safe is defined for systems, if a PN structurally generates a Safe system is usually called Safe PN.

2.2 Supervisory Control Theory (SCT)

The automata version of SCT is developed in [3]. In this subsection, the fundamentals SCT for discrete event system modeled as PN are introduced, as seen in [6]. Moreover, the basic concepts and definitions of BC are discussed in [12] and presented in the current section.

Definition 9 (Control pattern) Let \(N\) be a PN and \(T\) be its set of transitions.

The control pattern \(\Gamma\) is defined as the set of transitions enabled in a marking \(M\) of \((N, M)\).

Definition 10 (Transition sequence) Let \((N, M)\) be a PN system and \(T\) be its set of transitions.

\(\sigma = t_1 t_2 ... t_n\) is a transition sequence of transitions such that

- \(M_{i-1} \xrightarrow{t_i} M_i\)
- \(t_i\) is enabled in \(M_{i-1}\)
- with \(t_i \in T\), \(\forall i = 1, 2, ..., n\).

Definition 11 (Petri Net Supervisor) Let \(L \cdot M \leq b\) a constraint for the marking vector of a PN system \((N, M)\) with incidence matrix \(D\). \(S : M \rightarrow \Gamma\) is a supervisor for PN system \((N, M)\). Let \(C\) be a PN with marking \(M_0\) and set of transition \(T\). \(C\) is the implementation of \(S\) as a PN such that

- Marking vector \(M_t = b - L \cdot M_0\).
- Incidence matrix \(Dc = -L \cdot D\)
- \(\Gamma\) is the control pattern for \((C, M_c)\).

Definition 12 (Open loop system) Let \((N, M)\) be a PN system. \((N, M)\) is also called an Open loop system.

Definition 13 (Closed loop system) Let \((N, M)\) a PN system and \((C, M_c)\) a PN implementing \(S\), with \(S\) a supervisor. The closed loop system is defined as the synchronization of \(N\) and \(C\).

Definition 14 (Controllability) Let \((N, M)\) be a PN system and \(T\) be its set of transitions. Let \(\Sigma \subset T^*\) be the set of all transitions sequences \(\sigma\). \(\Sigma\) is called controllable if the prefix closure of \(\Sigma\) is invariant under the occurrence of uncontrollable transitions in \(N\).

Definition 15 (Admissible marking) Let \(M_a\) be reachable from initial marking \(M_0\) of a system \((N, M_0)\) with uncontrollable transitions. \(M_a\) is an admissible marking for the constraint \(L \cdot M \leq b\) if the following conditions hold

- \(L \cdot M_a \leq b\)
- For all reachable markings \(M_r\) from \(M_a\) through the occurrence of uncontrollable transitions in \((N, M)\) \(L \cdot M_r \leq b\).

Definition 16 (Admissible constraint) Let \((N, M)\) a PN system with initial marking \(M_0\). An admissible constraint satisfies

- \(L \cdot M_0 \leq b\)
2.3 Predicate representation of Behavioral Constraints

Let \( N \) be a safe PN with firing vector \( Q = [q_1, q_2, \ldots, q_l] \) and let \((N, M)\) be a system with marking vector \( M = [m_1, m_2, \ldots, m_l] \).

**Definition 17** Predicate variable \( A : Q \rightarrow \{True, False\} \) associated to a firing transition \( T_i \) is defined with the rule

\[
A(q_i) = \begin{cases} 
True & \text{if } q_i = 1 \\
False & \text{if } q_i = 0 
\end{cases}
\]

**Definition 18** Predicate variable \( \Theta : M \rightarrow \{True, False\} \) associated to a marking place \( m_i \) is defined with the rule

\[
\Theta(m_i) = \begin{cases} 
True & \text{if } m_i = 1 \\
False & \text{if } m_i = 0 
\end{cases}
\]

**Definition 19 (Behavioral Constraint (BC))** A BC is defined with the following predicate logic syntax

\[
A(q_a) \rightarrow \Phi
\]

with \( A \) being a predicate variable associated to firing transition \( T_a \) and \( \Phi \) a formula in conjunctive normal form, composed by predicate variables associated to marking places, that is

\[
\Phi = \phi_1 \land \phi_2 \land \ldots \land \phi_n
\]

with \( \phi_i(z_r) = \Theta(m_{r_1}) \lor \Theta(m_{r_2}) \lor \ldots \lor \Theta(m_{r_n}) \)

(3)

with \( r_j \) as the place index in \( N, j = 1, 2, \ldots, l \), with \( l \) the number of places associated in Eq. 3 and

\[
\phi(z) = \begin{cases} 
True & \text{if } z \geq 1 \\
False & \text{if } z = 0 
\end{cases}
\]

Eqs. 1 and 2 are equivalent to

\[
(A(q_a) \rightarrow \phi_1) \land (A(q_a) \rightarrow \phi_2) \land \ldots \land (A(q_a) \rightarrow \phi_n)
\]

(5)

**Definition 20** (\( P(S \leq 0) \)). Let \( S \) be an algebraic expression formed by elements in \( Q \) and in \( M \). The associated predicate \( P : Q \times M \rightarrow \{True, False\} \) is defined with the rule \( S \leq 0 \).

**Proposition 21** Let \( A(q_a), \Theta(m_k) \) be predicate variables and \( P(q_a - m_k) \leq 0 \) be their associated predicate. The following expressions are equivalent

\[
A(q_a) \rightarrow \Theta(m_k)
\]

(6)

\[
P(q_a - m_k) \leq 0
\]

(7)

**Proof.** Let \( N \) be a safe net, thus \( N \) is a 1-bounded net. Hence the marking vector takes only 0 and 1 values. Therefore Table [21] holds.

| \( q_a \) | \( m_k \) | \( A \rightarrow \Theta \) | \( P(q_a - m_k) \leq 0 \) |
|-------|-------|----------------|----------------|
| 0     | 0     | \( T \)         | \( T \)         |
| 0     | 1     | \( T \)         | \( T \)         |
| 1     | 0     | \( F \)         | \( F \)         |
| 1     | 1     | \( T \)         | \( T \)         |

Table [21] Truth table of Proposition [21]

Using Proposition [21], BC presented in Eq. 5 can be written in an equivalent form, as shown in Lemma [22].

**Lemma 22** Let \( A(q_a) \) and \( \Theta(m_{k_1}), \Theta(m_{k_2}) \ldots \Theta(m_{k_l}) \) be predicate variables. The BC [5] is equivalent to predicate system [9]

\[
P(q_a - m_{r_1} + m_{r_2} + \ldots + m_{r_j}) \leq 0
\]

(8)

with \( i \) as the number of disjunction variables in each formula \( \phi_i \).

**Proof.** It follows from applying Proposition [21] to BC [5].

3 Supervisory Controllers design using an Equivalent representation of a set of Behavioral Constraints

Using the \( n \) inequalities induced by predicate system [8] with the IBCD method [18], a PN supervisor is obtained with \( n \) monitor places, each one with a bidirectional arc to transition \( t_a \). It is presented below a procedure to design a PN SC, based on a BC as in Eq. [1] with a single monitor place.

**Theorem 23** Let \( A(q_a), \Theta(m_{k_1}), \Theta(m_{k_2}) \ldots \Theta(m_{k_l}) \) be variables as in definitions [17] and [18]. Let a BC for restricting the system behavior be

\[
A(q_a) \rightarrow (\Theta(m_{k_1}) \land \Theta(m_{k_2}) \land \ldots \land \Theta(m_{k_l}))[(\Theta(m_{j_1}) \lor \Theta(m_{j_2}) \lor \ldots \lor \Theta(m_{j_m}))]
\]

(9)

A 1-monitor place PN supervisor can be synthesized (i.e. its incidence matrix can be calculated) with the IBCD method using linear inequality

\[
m[q_a - m_K] + [q_a - m_f] \leq 0
\]

(10)

with \( m_K = m_{k_1} + m_{k_2} + \ldots + m_{k_l} \) and \( m_f = m_{j_1} + m_{j_2} + \ldots + m_{j_m} \) and \( m > 0 \)

**Proof.** See Appendix A.
Corollary 24 Let equation \( A(q_a) \rightarrow \Theta(m_{k_1}) \wedge \Theta(m_{k_2}) \wedge \cdots \wedge \Theta(m_{k_n}) \) be a BC for restricting the system behavior. A 1-monitor place PN supervisor can be synthesized (i.e., its incidence matrix can be calculated) with the IBCD method using linear inequality

\[
[m_{q_a} - m_K] \leq 0
\]

with \( m_K = m_{k_1} + m_{k_2} + \cdots + m_{k_n} \).

3.1 Properness of a Supervisory Controller based on Behavioral Constraints

The conditions for a SCBC to be non-blocking and controllable are studied in this subsection.

Definition 25 (System Under Supervision) Let \( N \) be a safe net and \( M \) its marking vector. Let \( C \) be the PN that implements a supervisor for \( N \) and \( M \), the marking vector of \( C \).

A System Under Supervision (SUS) is defined as

\[
(N || C, [MM_e])
\]

where \( N || C \) represents the synchronization of nets \( N \) and \( C \) with marking vector \([MM_e]\).

This definition complements definition adding the marking vector. In the rest of the document, closed loop system will be refereed as SUS.

A supervisor is proper if the SUS in non-blocking and controllable [3].

3.1.1 Liveness analysis

A necessary condition for non-blocking is liveness. For safe PN modeling AMS, the condition of liveness is required, as shown in this subsection. An AMS is composed by sub systems, each modeled as a live and bounded PN circuit.

Definition 26 (Partial blocking) A system \((N, M)\) is called partially blocking if there is a sub system \((N_1, M_1)\) of \((N, M)\) which is blocking.

Lemma 27 Let \( N \) be a safe PN. System \((N, M)\) is live if and only if it is not partially blocking.

Proof. As necessary condition, if a system is not partially blocking, then there is the system is live. For the sufficiency, it is enough to prove that in a partially blocking system there is a transition not enabled in every reachable marking of \( M \). Assuming a blocking system \((N_1, M_1)\) with \( N_1 \) a sub net of \( N \). Let \( t \) be an output transition to a place \( s \) of \( N_1 \) and \( t \) is not enabled in marking \( M_1 \), \( s \) has no tokens in \( M_1 \). The system is partially blocking \( M_1 \), hence the reachable markings from \( M \) contains element such that \( s \) has no tokens. If \( s \) has no tokens, transition \( t \) is not enabled. Therefore \((N, M)\) is not live.

Therefore, for safe PN, non-partial blocking is required in order to ensure a full functionality in the AMS. Hence by Lemma 27 liveness is required.

Now, the condition for a SCBC to be live is established. Using definition 28 of Proposition 29 and Lemma 31 are proved. Proposition 29 establishes conditions to guarantee reachability of a marking vector. Lemma 31 demonstrates if an associated marking vector is reachable, then SUS is live. Finally, Theorem 32 follows from Proposition 29 and Lemma 31 establishing a condition for a SUS to be live.

Definition 28 (Marking vector associated to constraints)

Let \( A(q_a) \rightarrow \Theta(m_{k_1}) \wedge \Theta(m_{k_2}) \wedge \cdots \wedge \Theta(m_{k_n}) \) be a BC. The marking vector associated to the above constraint is defined as

\[
m_{k_j} = \begin{cases} m_k & \text{if place } k_j \text{ is not in } BC \\ 1 & \text{if place } k_j \text{ is in } BC \\
\end{cases}
\]

Proposition 29 Let a BC of the form \( A(q_a) \rightarrow \Theta(m_{k_1}) \wedge \Theta(m_{k_2}) \wedge \cdots \wedge \Theta(m_{k_n}) \) be a BC. The marking vector associated to the above constraint is defined as

\[
R^T = \begin{bmatrix} m_1 & m_2 & \cdots & 1 & 1 & \cdots & 1 & m_{2+k_n} & \cdots & m_l \end{bmatrix}
\]

with \( l \) as the number of places, is reachable.

Proof. First the following implication is proved using its contra-positive. If the associated marking vector is reachable, then there is not more that 1 place in the BC belonging to the same minimal S-invariant. Consider \( S \) a minimal S-invariant containing 2 or more places included in the BC, and vector \( S^1 = [1 1 \cdots 1] \) of length \( m \), with \( m \) as the number of places in \( S \). The next equation is the invariance condition and guarantees that the number of tokens in an S-invariant is conservative.

\[
M_{\text{in}} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix} \cdot M_{\text{in}} = 1
\]

\( M_{\text{in}} \) is the initial marking of the places in \( S \), and for the conservativeness of the S-invariant, this value holds for any reachable marking. Let \( R' \) be a projection containing the values of \( R \) corresponding to the places in \( S \).

Multiplying \( S^1 \) by \( R' \)

\[ S^1 \cdot R' \geq 2 \]

The above expression violates conservativeness, hence the marking is not reachable.

For the converse implication, consider that there is not more than 1 place in the BC belonging to the same minimal S-invariant \( S \). Therefore, all places of the BC belongs to different and disjoints minimal S-invariant, this is concluded from the fact that the net \( N \) is 1-bounded and system \((N, M)\) is live. The last claim implies that every minimal S-invariant is marking in \( M \), because \( N \) is a free-choice PN (see [11] Commoner Theorem). Thus, every S-invariant has a token in the initial marking, the system is live and by Lemma 27 it is not partially blocking. Hence, there is a reachable marking of the system \((N, M)\) such that every place in the BC has one token simultaneously (invariants are disjoints) and the associated marking vector is reachable.

Proposition 30 Let a BC of the form \( A(q_a) \rightarrow \Theta(m_{k_1}) \wedge \Theta(m_{k_2}) \wedge \cdots \wedge \Theta(m_{k_n}) \) imposed to a system \((N, M)\) and \( R \) its associated marking vector.
The formula $\Phi$ is true if and only if the system $(N,M)$ reaches marking $R$.

**Proof.** By definition, vector $R$ changes its values only in the places appearing in formula $\Phi$. The sufficiency condition, if formula $\Phi$ is true the marking of the system is $R$. Assuming $\Phi$ true in some marking $M_i$, all $\Theta$ variables are true and, by definition, all places in the formula have one token in marking $M_i$. Hence, $R=M_i$.

For the necessary condition consider that system $(N,M)$ has reached marking $R$ after some firing sequence. Using a similar argument, all places appearing in $\Phi$ have one token in $R$, all $\Theta$ variables are true in $R$, hence $\Phi$ is true in $R$. ■

Using previous results two useful conditions for liveness under supervision are developed.

**Lemma 31** Let a BC of the form $A(q_a) \rightarrow \Phi$ and $C$ a PN representing a supervisor for $N$. If the associated marking vector of $\Phi$ is not reachable, then the SUS of $C$ is not live.

**Proof.** If associated marking vector is not reachable, it means that the formula $\Phi$ of the BC never is true, thus transition $t_a$ is never enabled. The system is not live. ■

**Theorem 32** Let $A(q_a) \rightarrow \Phi$ and $C$ a PN representing a supervisor for $N$.

SUS of $C$ is live if and only if there is a reachable marking $M_i$ such that formula $\Phi$ is true and $t_a$ is enabled. By $[30]$ associated marking vector of the BC is not reachable, hence by $[31]$ SUS is not live, leading to a contradiction.

Now for the sufficiency condition, assume that marking $M_i$ is reachable and formula $\Phi$ is true and $t_a$ is enabled. By $[30]$ associated marking vector of the BC is not reachable, hence by $[31]$ SUS is not live, leading to a contradiction.

**3.1.2 Non-conflict analysis**

If a set of BC is non-conflicting then the resulting SC is non-blocking $[4]$. As before, liveness is required for manufacturing systems. Hence, a set of BC is called non-conflicting if the SUS is live.

**Theorem 33** Let $A(q_1) \rightarrow \Phi_1$, $A(q_2) \rightarrow \Phi_2$, \ldots $A(q_n) \rightarrow \Phi_n$ be BCs that satisfy conditions of Lemma $[32]$. Let $C$ be the net representing the supervising of all the constraints.

The set of BC is non-conflicting if and only if, every subnet of $PN$ generates a live subsystem.

**Proof.** Necessary condition. A set of constraints is non-conflicting if the SUS is live. Assume a SUS such that there is a subnet $C_1$ of $C$ generating a non-live system $(C_1,M_1)$. Since is not live, there is a transition $t_1$ disabled in all reachable markings from some marking $M_i$, $t_1$ is a transition of the SUS also, therefore the SUS is not live, leading to a contradiction.

For the sufficiency, assume that a SUS is not live. Therefore, at least a transition of $N$ is not enabled for all reachable markings. In the first case, $t$ is connected to $C$. Then, there is a place $c$ input to $t$ in $C$ with no tokens for all reachable markings. There is a transition $T_1$ input to $c$ not enabled and following the same idea that $t$, assuming $T_1$ is connected to $C$ there is $c_1$ input to $T_1$ in $C$. Recursively until place $c_n$ is place $c$ (there is a finite number of places in $C$), there is a subnet of $C$ with a disabled transition, hence the subnet is not live. If transition $t$ is not connected to $C$, there is a transition $t’$ in the same minimal $S$-invariant of $t$ connected to $C$, and the above procedure can be followed for $t’$.

**Corollary 34** Let $A(q_1) \rightarrow \Phi_1$, $A(q_2) \rightarrow \Phi_2$, \ldots $A(q_n) \rightarrow \Phi_n$ be BCs that satisfy conditions of Lemma $[32]$. Let $C$ be the net representing the supervisor of all the constraints.

If all the supervisory nets generated by the set of BC are disjoint then the SUS is live (i.e. the set of BCs is non-conflicting)

**Proof.** If the nets are disjoint and the conditions of Lemma $[32]$ are satisfied all the nets generate live systems, hence by Theorem $[33]$ the set of BCs is non-conflicting. ■

**Definition 35 (Controlled Siphon)** Let $R$ be a siphon in a net $N$ with $M_R$ as its marking vector. $R$ is a controlled siphon if for all marking $M_R$ reachable from $M_R$, $|M_R| \geq 1$. Otherwise, it’s an uncontrolled siphon.

That is, a controlled siphon is a siphon that never becomes unmarked.

**Corollary 36** Let $A(q_1) \rightarrow \Phi_1$, $A(q_2) \rightarrow \Phi_2$, \ldots $A(q_n) \rightarrow \Phi_n$ be BCs that satisfy conditions of Lemma $[32]$. Let $C$ be the net representing the supervisor of all the constraints.

Consider that every BC has only one variable in each respective formula $\Phi_i$.

The set of BCs is non-conflicting if and only if there is not an uncontrolled siphon

**Proof.** The same argument used in $[43]$ is applied. Since all the BCs have only one variable, their corresponding nets have only one place each. The condition of a not live subnet becomes then in a subnet with no tokens, hence an uncontrolled siphon. ■

**3.1.3 Controllability analysis**

This subsection shows that a SUS synthesized using the IBCD method with BC is, in fact, controllable.

**Lemma 37** Let $A(q_a) \rightarrow \Phi$ be a BC imposed to the PN $N$. Let $C$ be the net representing the corresponding supervisor and let $t_b \neq t_a$.

If a transition $t_b$ is enabled in a marking $M$ of $N$, then it is enabled in marking $M_c$ of $(N\|C)$.

**Proof.** Consider $s_b$ an input place to $t_b$. Assume $s_b$ as part of the constraint. The net $N$ is safe, hence it is composed by $S$-invariants (i.e. state machines), thus every transition has one only input place. Hence if $T_b$ is enabled, then $s_b$ has a token. Obtaining the marking of
the monitor place using the formula \( m_c = -L \cdot M \) [6] the
monitor place has at least B tokens (B being the coefficient
responding the place \( s_b \) in vector L). Calculating
the incidence matrix \( D_e \) of the monitor place using
\( D_e = -L \cdot D_p \), for the place \( s_b \), the value corresponding to
transition \( t_b \) is coefficient \(-B\). Therefore transition \( t_b \) is enabled because the
monitor place has at least B tokens, necessary to fire transition B.

**Theorem 38** Let \( A(q_\chi) \rightarrow \Phi \) be a BC imposed to net N.
Constraint is admissible (RW-controllable) if and only if transition \( T_{\chi'} \) is controllable.

**Proof.** By Lemma [37] no other transition of N will be
disabled by the supervisor. The supervisor can disable
only transition \( T_{\chi'} \). Hence the supervisor only disables
controllable transitions. Therefore, any sequence of the SUS is invariant to the occurrence of uncontrollable transitions.

4 AMS case study

4.1 System description and open loop model description

The AMS employed as a case study is a pneumatic
punching center whose topology is illustrated in Fig. [1].
The manufacturing procedure begins when a piece
arrives to the storage unit (SU), then valve B (VB)
opens, activating the input piston (IP). IP pushes the
piece into the slot 1 (S1) of the rotatory table, while
valve A (VA) retracts the IP. The motor (MR) is turned
on, generating a rotation of 90 degrees clockwise in
the rotor, and the piece advances to slot 2 (S2). The
piece is processed by the punching machine (PM) at
slot 2, using valve E (VE) to activate the PM. Then,
the motor turns 90 degrees clockwise again, placing
piece into slot 3 (S3). The piece at slot 3 is pushed by
the output piston (OP), activated by valve D (VD), to
a conveyor belt, and finally, valve C (VC) retracts the
OP.

Each elementary component of the AMS is modeled as a two-places PN block. A place is added to the block associated to each discrete value. The set of transitions are defined as the events to change the discrete value of a component. A transition is added to the model for each event. For the initial marking, a token is added to the associated place of the initial discrete value of each component. The rest of the places remain with no tokens. Table [3] enlists the elementary components with the associated semantics of each place and transition. Fig. [2] shows the PN blocks of the AMS.

The following causal relationships complete in the open loop behavior of the AMS. Bidirectional arcs are added to the model to include the relationships in the behavior, as shown in Fig. [2].

- A piece can arrive to slot 1 only if input piston is out and there is a piece in storage (bidirectional arcs from P2 and P4 to T5).
- Input piston can be activate only if valve A is open, and it can be retract only if valve B is open (bidirectional arcs from P18 to T4 and from P20 to T3).
- Punching machine can be activate only if valve E is on (a bidirectional arc from P26 to T11).
- Output piston can be activate only if valve C is open, and it can be retract only if valve D is open (bidirectional arcs from P22 to T14 and from P24 to T13).

This PN is live and 1-bounded, i.e. is a safe PN. The incidence matrix \( d \) of each PN module is of the form of Eq. [14]. Hence, the incidence matrix \( D_e \) of the entire system is a 28x28 block matrix in Eq. [15]. The initial marking vector \( m \) of each module is shown in Eq. [16]. Hence, the initial marking vector \( M_e \) of the AMS is shown as a block vector in Eq. [17].

\[
d = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad (14)
\]

\[
D_e = \text{blockdiag}[d] \quad (15)
\]

\[
m = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (16)
\]

\[
M_e^T = \begin{bmatrix} m & m & m & m & m & m & m & m & m & m \end{bmatrix} \quad (17)
\]

4.2 Closed loop specification modeling

The specifications to be imposed upon the AMS are described in this subsection. Four safety specifications are defined to ensure the AMS safe operation. Matching definition [19] each specification have a corresponding BC.

1. If turning on motor (T27) is enabled, then both piston (P3, P13) and punching machine (P11) are in the withdrawn position and there is a manufacturing piece in slot 1 (P6) or in slot 2 (P8). The corresponding BC is shown in Eq. [18].

2. If opening valve B to activate input piston (T19) is enabled, then there is a piece in storage (P2) and table is in load position (P15). The corresponding BC is shown in Eq. [19].

3. If opening valve D to activate output piston is enabled (T23), then there is a piece in slot 3 (P10). The corresponding BC is shown in Eq. [20].

4. If opening valve E to activate punching machine (T25) is enabled, then there is a piece in slot 2 (P8). The corresponding BC is shown in Eq. [21].

\[
A(q_{27}) \rightarrow \Theta(m_3) \land \Theta(m_{13}) \land \Theta(m_{11}) \land [\Theta(m_6) \lor \Theta(m_8)] \quad (18)
\]

\[
A(q_{19}) \rightarrow \Theta(m_2) \land \Theta(m_{15}) \quad (19)
\]
Figure 1: AMS Topology

Using Lemma 22, the induced system for the BCs from Eqs. 18-21 is presented in Eq. system 22 consisting of a linear system of 8 inequalities. Employing the method proposed in section 3 (Theorem 23 and corollary 24) Eqs. 18-21 are transformed into a set of 4 linear inequalities shown in Eq. system 23.

\[
\begin{align*}
A(q_{23}) & \rightarrow \Theta(m_{10}) \\
A(q_{25}) & \rightarrow \Theta(m_{8}) \\
q_{27} - m_{3} & \leq 0 \\
q_{27} - m_{13} & \leq 0 \\
q_{27} - m_{11} & \leq 0 \\
q_{27} - m_{6} - m_{8} & \leq 0 \\
q_{19} - m_{2} & \leq 0 \\
q_{19} - m_{15} & \leq 0 \\
q_{23} - m_{10} & \leq 0 \\
q_{25} - m_{8} & \leq 0 \\
7q_{27} - 2m_{3} - 2m_{13} - 2m_{11} - m_{6} - m_{8} & \leq 0 \\
2q_{19} - m_{2} - m_{15} & \leq 0 \\
q_{23} - m_{10} & \leq 0 \\
q_{25} - m_{8} & \leq 0
\end{align*}
\]

Using Eq. system 23 with the IBCD method, a PN supervisor is designed. The matrix $L$ is defined in Eq. system 24. Using the equation $D_c = -L \cdot D_p$ incidence matrix $D_c$ of the supervisor is calculated, and it is shown in Eq. system 25. Four self-looped arcs are added, one for each BC, connecting the monitor place with the corresponding controllable transition. The weight of each arc is the corresponding coefficient for the transition in the set of induced inequalities shown in Eq. system 23. The equation $M_{oc} = -L \cdot M_p$ is used for calculating the initial marking vector $M_{oc}$ of PN supervisor, shown in Eq. 26. Each monitor place is connected only to some transitions in the open loop model. Thus, each monitor place can be represented as a modular supervisor, using only the PN blocks connected to the monitor place. The resulting 4 modular PN supervisors are shown in Fig. 4.

4.3 Properness analysis

This subsection presents the analysis to show that the designed SCBC is in fact proper, i.e. the SUS is live, non-conflicting and controllable. For each BC, there are not 2 places belonging to the same PN block. Each PN block is a minimal S-invariant (see [11]). Therefore, there are not 2 places belonging to the same minimal S-invariant. Hence, by Proposition 29 the associated marking vectors for all the BCs are reachable. Now, by Proposition 30 in those markings the respective formulas $\Phi$ are true. Since all transition of the BC are enabled in its respective associated reachable markings by Theorem 32 the SUS for every BC is live.

Now, by Theorem 33 the PN supervisor must not have any not live subnet in order to prove that the set of constraints is non-conflicting. However, the only not disjoint subnet of PN supervisor is concerned to transitions $T_7$ and $T_8$. From a quick analysis it is clear that this particular subnet is live. Hence, by 34 and Theorem 33 the SUS is live, i.e. the set of BCs is non-conflicting.

The set of constraints must be proven admissible. By Theorem 38 the set of constraints is proven admissi-
possible since transitions $T_{27}, T_{17}, T_{21}$ and $T_{25}$ are controllable.

4.4 Ladder diagram implementation of supervisory controller

A PN can be translated into a ladder diagram for its implementation in a control device (e.g., a PLC). The general procedure for the translation of PN into ladder diagram is explained in [14]. Every place has a corresponding register in the ladder diagram. Every transition has a corresponding contact and its execution generates the change of the contact state.

The following rules are an adaptation of the translation procedure developed in [14]. Let $T_a$ be a transition in the supervisor PN. Let $P_a$ be an output place of $T_a$, connected by an arc with weight $na$. Let $P_b$ be an input place of $T_a$, connected by an arc with weight $nb$.

- Each transition $T_a$ is represented as a contact in a ladder segment.
- If $P_a$ is 1–bounded, then it is represented by a coil with a set function. If $P_a$ is not 1–bounded, then it is represented by an add block, adding $na$ tokens to $P_a$.
- If $P_b$ is 1–bounded, then it is represented by a coil with a reset function. Also, a normally open contact is associated to $P_b$ in the segment.
- If $P_a$ is not 1–bounded, then it is represented by a subtract block, subtracting $nb$ tokens to $P_b$. Also, a comparison contact is associated to $P_b$, with the rule, greater or equal than $nb$.
- If $P_a = P_b$ (self-loop), then the number of tokens holds. Thus, there are not output blocks associated to $P_a$ in the segment.

The resulting ladder diagram for the SCBC is composed by 28 segments, one for each transition of the AMS model. A part of this ladder diagram is shown in Fig. 4.4. Each segment contains the conditions to enable the corresponding transition. For example, monitor place C1 must have at least 7 tokens for enabling transition $T_{27}$. The number 7 is the coefficient corresponding to transition $T_{27}$ in the Eq. system 23. Moreover, in the Fig. 4, the weight of the bidirectional arc from monitor place C1 to transition $T_{27}$ is 7. Monitor place C4 must have a token for enabling transition $T_{25}$. 

Figure 2: AMS model
Figure 3: Elementary components, discrete values and events of AMS with the corresponding places and transitions assignment (uc, uncontrollable; c, controllable)

### 5 Conclusions

The approach presented in this work reduces the number of monitor places needed to impose a set of constraints in a AMS. In the case study, the safety specification were successfully imposed in the system behavior using 4 monitor places, showing the exact same results that using the classical approach with 8 monitor places.

The incidence matrix of a discrete event system modeled as a PN usually has a lot of zero entries. The proposed approach reduces the dimension of Matrix L of the IBCD method, avoiding unnecessary by-zero multiplications giving a computational numerical advantage.

In the context of discrete event system the state expansion leads to complicated and unreadable graphs representations, such as Finite State Machines. The use of PN gives a more compact representation of the system, but it is still possible to find very complex graphs representations when a SC is design.

It has been proposed a synthesis method for a class of BC with a restricted syntax. Giving rise to a minimal PN SC. This increases the variety that can be considered in the synthesis (i.e. forbidden states) using a solid and mathematically established procedure.

The safety specifications ensure a behavior that forbids to any unwanted situation occurs in the system. The implementation was made using techniques previously proposed. The resulting implementation is compact and is a more usable approach for manufacturing systems.

**Appendix A: Proof of Theorem 23**

**Proposition 39** Let \( x, y_1, y_2 \) be integer variables with
\[ L_1 = \begin{bmatrix} 0 & 0 & -2 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & -2 & 0 & -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ L = \begin{bmatrix} L_1 & 0 \end{bmatrix} \]

\[ D_{c_1} = \begin{bmatrix} 0 & 0 & -2 & 2 & 1 & -1 & 1 & -1 & 0 & 0 & -2 & 2 & 2 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ D_c = \begin{bmatrix} D_{c_1} & 0 \end{bmatrix} \]

\[ M_{tc}^T = \begin{bmatrix} 6 & 1 & 0 & 0 \end{bmatrix} \]

![Figure 4: a) Modular supervisor for monitor place C1. b) Modular supervisor for monitor place C2. c) Modular supervisor for monitor place C3. d) Modular supervisor for monitor place C4](image)

**domain** \([0, 1]\). The solution set for inequality

\[(x - y_1) + (x - y_2) \leq 0 \quad (27)\]

is the same solution set for the system

\[
\begin{align*}
  x - y_1 & \leq 0 \\
  x - y_2 & \leq 0
\end{align*} \quad (28)
\]

**Proof.** The solution set of an inequalities system agrees to the intersection of each inequality solution set. Let predicate \(29\) be associated to system \(28\) and predicate \(30\) be associated to ineq. \(27\)

\[ P[(x - y_1 \leq 0) \wedge (x - y_2 \leq 0)] \quad (29) \]

\[ P[(x - y_1) + (x - y_2) \leq 0] \quad (30) \]

Table \(39\) shows that both expressions are equivalent.

![Table tab:T1 Truth table for Proposition 39](image)

| \(x\) | \(y_1\) | \(y_2\) | Eq. \(29\) | Eq. \(30\) |
|------|------|------|--------|--------|
| 0    | 0    | 0    | \(T\)   | \(T\)   |
| 0    | 1    | 0    | \(T\)   | \(T\)   |
| 0    | 1    | 1    | \(T\)   | \(T\)   |
| 1    | 0    | 0    | \(F\)   | \(F\)   |
| 1    | 0    | 1    | \(F\)   | \(F\)   |
| 1    | 1    | 0    | \(F\)   | \(F\)   |
| 1    | 1    | 1    | \(T\)   | \(T\)   |

**Lemma 40** Let \(x, y_1, \ldots, y_n\) be integer variables with domain \([0, 1]\) and \(n \geq 2\). \(Y = y_1 + y_2 \cdots y_n\). Let \(R = \{(x, Y) \mid x = [0, 1], Y = [0, 1, \ldots, n]\}\) be the constrained domain. Let \(\Sigma \subset R\) the solution set for the inequalities sys-
Theorem
$$x - y_1 \leq 0$$
$$x - y_2 \leq 0$$
$$\vdots$$
$$x - y_n \leq 0$$
Then $\Sigma$ is the solution set for the inequality
$$(x - y_1) + (x - y_2) + \cdots + (x - y_n) \leq 0$$
or in a compact form
$$nx - Y \leq 0$$
Proof. (By mathematical induction) Let the base case be $\text{Proposition 39}$. The induction hypothesis of the inductive step is the Lemma statement. Therefore, it must be proved that the solution set of ineq. 34 and system 35 is the same.

$$(x - y_1) + (x - y_2) + \cdots + (x - y_s) \leq 0$$
(34)

$$x - y_1 \leq 0$$
$$x - y_2 \leq 0$$
$$\vdots$$
$$x - y_s \leq 0$$
$$x - y_{s+1} \leq 0$$
Ineq. 34 holds if and only if
$$x - y_{s+1} \leq 0$$
holds and
$$(x - y_1) + (x - y_2) + \cdots + (x - y_s) \leq 0$$
(37)
also holds. This is derived from the fact that $x$ can only take values 0 and 1. If $\Sigma$ is the solution set for ineqs. 36 and 37, then $\sigma$ is the solution set for 34. By induction hypothesis, if ineq. 37 holds, then system
$$x - y_1 \leq 0$$
$$x - y_2 \leq 0$$
$$\vdots$$
$$x - y_s \leq 0$$
also holds. Therefore, $\Sigma$ is the solution set for system 35 and it is proven that $\Sigma$ is solution for 34 and 35.

Lemma 41 Let $X, y_1, y_2, \cdots, y_n, z_1, z_2, \cdots, z_m$ be integer variables with domain $[0,1]$. Let $Y = y_1 + y_2 + \cdots + y_n$, $Z = z_1 + z_2 + \cdots + z_m$. Let $R = \{(X,Y,Z)|X = [0,1], Y = [0,1,\cdots,n], Z = [0,1,\cdots,m]\}$ be the constrained domain. Let $\Sigma \subset R$ the solution set for the inequality
$$m(nX - Y) + (x - Z) \leq 0$$
(38)
Then $\Sigma$ is also the solution set for the system
$$X - y_1 \leq 0$$
$$X - y_2 \leq 0$$
$$\vdots$$
$$X - y_n \leq 0$$
$$X - Z \leq 0$$
Proof. The proof consists of two steps. First the inequality 38 is derived from a geometrical perspective. Then, it is proven that if $\Sigma$ is solution for eq. 38, then it is also solution for system 39. By Lemma 40, the first $n$ inequalities are equivalent to ineq. 33, therefore system 39 becomes
$$nX - Y \leq 0$$
$$X - Z \leq 0$$
(40)
From a geometric perspective, both inequalities in system 40 have a corresponding plane in a tree dimensional space $(X,Y,Z)$. The solution set for each inequality is constructed with the points contained in domain $R$ and bounded above by the corresponding plane, thus the solution set for system 39 is constructed with the points contained in domain $R$ and bounded above for the intersection of both corresponding planes. Therefore, there is a plane such that contains the intersection of both planes and bounds above all the points contained in domain $R$ and the solution set of system 40. The intersection of these planes is a line containing the points $(0,0,0)$ and $(1,n,1)$. In order to describe a plane equation, an orthogonal vector to the plane is required, and for its calculation a third point is obtained by convenience, $(\frac{m}{mm+1},1,0)$. The orthogonal vector is obtained by calculating the cross product of two vectors in the plane, for simplicity, $v_1 = 1, n, 1 >$ and $v_2 = m, mn + 1, 0 >$.

$$v_3 = \begin{vmatrix} i & j & k \\ 1 & n & 1 \end{vmatrix} = -(mn + 1), m, 1 >$$
(41)
The plane equation is $(mn + 1)X - mY - Z = 0$. Thus, the solution set for $(mn + 1)X - mY - Z \leq 0$ is the same as system 39. Fig. 1 shows the plane and the constrained domain $R$. Now it is proven that solution set $\Sigma$ for ineq. 38 is the same as system 39. System 39 holds for $X = 0$. If $X \neq 0$, because of the domain constraint, then $X = 1$. If $X = 1$, system 39 holds for $y_i \geq 1$ and $Z \geq 1$, then $Y \geq n$. Again because of the domain constraint, $Y = n$. Hence the set $\Sigma$ that holds for expression 41 is the solution set for system 39

$$(x = 0) \lor [(Y = n) \land (Z \geq 1)]$$
(41)
Hence the new system
\[
x = 0 \quad Y = n \quad Z \geq 1 \quad \{x = 0 \lor [(Y = n) \land (Z \geq 1)]
\]

\begin{array}{cccc}
F & F & F & F \\
F & F & T & F \\
F & T & F & F \\
F & T & T & T \\
T & F & F & T \\
T & F & T & T \\
T & T & F & T \\
T & T & T & T \\
\end{array}

Table 41 Truth table of equation 41

The truth table of expression 41 is shown in Table 41. Using definition 20, the truth table for predicate variable for expression 41 and ineq. 38 associated predicate.

By Lemma 40, the first \( n \) inequalities are equivalent to inequality
\[
nq_a - [m_{k_1} + m_{k_2} + \cdots + m_{k_n}] \leq 0
\]

Hence the new system
\[
nq_a - m_{k} \leq 0 \\
q_a - m_{j} \leq 0
\]

Variables \( q_a, m_{k}, m_{j} \) satisfy conditions of Lemma 41. Therefore inequality
\[
m[nq_a - m_{k}] + [q_a - m_{j}] \leq 0
\]
shares the same solution set with system 44 and, henceforth, with system 42.

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