A SUPERSPACE FORMULATION FOR THE BATALIN VILKOVISKY FORMALISM WITH EXTENDED BRST INVARIANCE

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Abstract

A superspace formulation for the Batalin Vilkovisky formalism (also called field-antifield quantization) with extended BRST invariance (BRST and anti-BRST invariance) for gauge theories with closed algebra is presented. In contrast to a recent formulation, where only BRST invariance holds off shell, two collective sets of fields are introduced and an off shell realization of the extended algebra in a superspace with two Grassmann coordinates is obtained. The example of the Yang Mills theory is also considered.

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1 Introduction

The Lagrangian BRST quantization procedure of Batalin and Vilkovisky\cite{1,2} (BV) is a very powerful framework for the quantization of gauge field theories \cite{3,4}. One of it’s important features is that when applied to the quantization of gauge theories with generators that form an open algebra it furnishes a systematic way of building up the ghost structure of the theory, even for infinitely irreducible theories, where an infinite chain of ghosts is naturally introduced\cite{2}.

Very interesting results have also recently shown up in the application of the BV procedure to the quantization of anomalous gauge theories. Once one is able to regularize a theory in order to give a well defined meaning to the quantum Master Equation\cite{5}, the anomalies can be calculated and, by extending the field anti-field space\cite{4,6}, the Wess Zumino terms can be naturally introduced in such a way that the theory is BRST invariant at the quantum level.

The standard BV quantization of refs.\cite{1,2} is based on the requirement of BRST invariance\cite{7} of the total action. A Lagrangian quantization procedure, with a larger anti-field structure, based on the requirement of invariance of the total action under both BRST and anti-BRST\cite{8} transformations was presented in \cite{9,10}. We will call it extended BV formalism from now on.

A superspace formulation for the BV action at classical level (zero order in $\hbar$) has recently been given in ref\cite{11}. In this article, a BRST invariant formulation was found, by implementing in a superspace with one grassmann coordinate, an alternative derivation of the Batalin Vilkovisky action, presented in ref.\cite{12}. In this derivation, the complete set of fields (fields of the classical action, ghosts, antighosts and auxiliary fields associated to the original gauge symmetry ) in the theory is doubled by adding the so called collective fields, increasing in a trivial way the total symmetry of the action. The extra symmetries (shift symmetries) are then fixed in such a way that the BV action is found, with each of the antighosts of the shift symmetries playing the role of the associated antifield.

In Ref.\cite{11} the possibility of implementing a superspace formulation with extended BRST (BRST and anti-BRST) invariance for the BV action was also investigated, but using only one set of collective fields, just to see how far can one go within that framework. It was found that the extended BRST invariance would work just on shell. This result was in fact expected, taking into account the particular features of the extended BV procedure presented in \cite{11}. In this extended formalism there are three different antifields associated to each field, one generating BRST transformations, one generating anti-BRST transformations and one generating mixed transformations. A derivation of this extended BV action by means of adding trivial symmetries and then fixing them (collective field approach) was presented in \cite{13,14}. In this references it is shown that two collective sets of fields are necessary in order to recover the complete antifield structure of the extended BV formalism.

The aim of the present article is to present a superspace version for the extended BV formalism for gauge theories with closed algebra at one loop order. As it will become clear, this will not be a trivial generalization of the results of \cite{11} since the gauge fixing structure in this case is considerably different. In section (2) we
will briefly review a derivation of the BV action with extended BRST invariance by means of the introduction of two sets of collective fields. In section (3) we present our superspace formulation for general gauge theories with closed algebra. The case of the Yang Mills theory will be presented, as an example, in section (4). Section (5) is devoted to some concluding remarks. Some technical details of the superspace structure for general theories and also the complete expressions for the superfields that show up in the Yang Mills example are presented in the appendix.

2 The BV formalism with extended BRST symmetry

In this section we will briefly review a derivation of the extended version of the BV quantization method, by means of the introduction of collective fields [13, 14]. We will use the same notation and conventions as in these references. Let us consider a Classical Action $S_{\text{class}}(\phi)$ of some gauge theory. We introduce ghosts, anti-ghosts and auxiliary fields associated to the gauge symmetries of $S_{\text{class}}$, as usual, denoting this enlarged set of fields as $\phi^A = (\phi^i, c^\alpha, \pi^\alpha, G^\alpha)$. The extended transformations for these fields are represented in the compact notation:

$$\delta_a^\phi \phi^A = R^A_a(\phi)$$

where the index $a = 1, 2$ corresponds respectively to the BRST and anti-BRST transformations.

Now we introduce two new sets of collective fields: $\varphi^{A1}$ and $\varphi^{A2}$ and substitute the fields $\phi^A$ by $(\phi^A - \varphi^{A1} - \varphi^{A2})$. This enlarges in a trivial way the symmetry content of the theory, adding to each value of the index $A$ two independent shift symmetries. In order to build up a representation for the enlarged BRST algebra corresponding to the original gauge symmetries plus the new ones, one associates to the shift symmetries two sets of ghosts: $\pi^{A1}$ and $\phi^{*A2}$, two sets of antighosts $\phi^{*A1}$ and $\pi^{A2}$ and two sets of auxiliary fields: $B^A$ and $\lambda^A$. There is a large freedom in choosing the transformation of the individual fields, since only $\phi^A - \varphi^{A1} - \varphi^{A2}$ is the relevant quantity. One possible way of writing out the extended BRST algebra for this enlarged system is:

$$\delta_a^\phi \phi^A = \pi^A_a$$
$$\delta_a \varphi^A_b = \delta_{ab}[\pi^A_a - \epsilon_{ac} \phi^{*A}_{c} - R^A_a(\phi^{*A} - \varphi^{*A})] + (1 - \delta_{ab}) \epsilon_{ac} \phi^A_{c}$$
$$\delta_a \pi^A_b = \epsilon_{ab} B^A$$
$$\delta_a B^A = 0$$
$$\delta_a \phi^{*A}_b = -\delta_{ab}[(1)^a \lambda^A + \frac{1}{2}(B^A + \delta_{cB} \frac{\delta R_1^A(\phi^{*A} - \varphi^{*A})}{\delta \phi^B} R_2^B(\phi^A - \varphi^{A1} - \varphi^{A2})]$$
$$\delta_a \lambda^A_b = 0$$

where the left arrow indicates right derivatives [4].
The gauge fixing of the shift symmetries is implemented by means of the gauge fixing action:

\[ S_{\text{col.}} = -\frac{1}{4} \epsilon^{ab} \delta_a \delta_b \left[ \varphi^A M^{AB} \varphi^B_1 - \varphi^A M^{AB} \varphi^B_2 \right] \]  

(3)

where \( M^{AB} \) is a c-number matrix defined in such a way that \( \phi^A M^{AB} \phi^B \) has ghost number zero and even Grassmann parity.

The gauge fixing of the original symmetry is implemented by introducing a trivial BRST anti-BRST invariant:

\[ S_F = \frac{1}{2} \epsilon^{ab} \delta_a \delta_b F(\phi^A) \]  

(4)

where \( F \) depends only on the original fields \( \phi^A \).

The total action corresponds to

\[ S = S_{\text{class.}}[\phi^i] + S_{\text{col.}}[\varphi^A_1, \varphi^A_2] + S_F[\phi^A] \]  

(5)

As discussed in detail in [13, 14], action (5) leads to the extended BV formalism, with the (redefined) fields:

\[ \phi^{*Aa'} = (-1)^{Aa'} M_{BA} (-1)^{a+1} \]  

(6)

playing the role of the three sets of antifields of ref. [9].

### 3 Superspace Formulation

We will now develop a superspace formulation for the extended BV formalism. As discussed already in [14], one of the most important aspects of this kind of formulation is the way of handling the antifields. If we simply try to insert them in some superfields we would not get the usual BV conditions of relating the antifields to functional derivatives of some gauge fixing functional. In the collective field approach to the extended BV formalism reviewed in section (2) there is, in principle, just a standard gauge theory with no antifields (but with a trivial shift symmetry structure). Only after a judicious choice of how to gauge fix the extra shift symmetries is that one recovers the BV formalism, interpreting some fields as playing the role of antifields. That is why we are going to build up a superspace version of section (2).

As explained in the appendix, we consider a superspace \((x, \theta^a)\) (with \( a = 1, 2 \)) and build up superfields in such a way that the extended BRST transformations will be realized as translations in the \( \theta^a \) components. Considering a gauge field theory involving \( \varphi^A(x) \), as in section (2), the set of fields \( \phi^A(x) \) enlarged by the addition of the collective fields \( \varphi^A_1 \) and \( \varphi^A_2 \), we introduce the associated superfields, represented here and in the rest of the article as underlined quantities: \( \underline{\phi}^A(x, \theta^a), \underline{\varphi}^A_1(x, \theta^a) \) and \( \underline{\varphi}^A_2(x, \theta^a) \) that realize the extended algebra of eq.(2):
\[ \phi^A(x, \theta^a) = \phi^A(x) + \pi^A(x) \theta^a + B^A(x) \theta^2 \theta^1 \]

\[ \varphi^{A1}(x, \theta^a) = \varphi^{A1}(x) + \left[ \pi^{A1} - \phi^* A^2 - R^A_1 (\phi - \phi^1 - \phi^2) \right] \theta^1 - \phi^* A^1 \theta^2 \]
\[ + \left[ -\lambda^A + \frac{1}{2} (B^A + \frac{1}{\delta \phi^C} R^A_1 (\phi - \phi^1 - \phi^2)) \right] \theta^2 \theta^1 \]

\[ \varphi^{A2}(x, \theta^a) = \varphi^{A2}(x) + \phi^* A^2 \theta^1 + \left[ \pi^{A2} + \phi^* A^1 - R^A_2 (\phi - \phi^1 - \phi^2) \right] \theta^2 \]
\[ + \left[ \lambda^A + \frac{1}{2} (B^A + \frac{1}{\delta \phi^C} R^A_1 (\phi - \phi^1 - \phi^2)) \right] \theta^2 \theta^1 \]

The gauge invariance of the Classical action implies also it’s BRST and anti-BRST invariance. Therefore, in this superspace, it corresponds just to a superfield with only the first component:

\[ S_{\text{class.}}[\phi^i - \phi^i^1 - \phi^i^2] = S_{\text{class.}}[\phi^i - \phi^i^2 - \phi^i^2] \]  

The gauge fixing action for the shift symmetries can be written as

\[ S_{\text{col.}} = \int dx \frac{1}{2 \theta^2} \frac{\partial}{\partial \theta^1} \left[ \varphi^{A1} M^{AB} \varphi^{B1} - \varphi^{A2} M^{AB} \varphi^{B2} \right] \]  

where \( M^{AB} \) is a c-number matrix with the same properties as in the previous chapter. Since \( M^{AB} \) is field independent, it is trivially BRST and anti-BRST invariant, corresponding thus, in our superspace to a quantity with no \( \theta \) components. (\( M^{AB} = M^{AB} \)).

The gauge fixing of the original symmetry is obtained using a bosonic (super) functional \( F[\phi^A(x, \theta^a)] \) of only the superfields \( \phi^A(x, \theta^a) \). To arrive at a gauge fixing action that corresponds to a product of a BRST and an anti-BRST transformations acting on a gauge fixing bosonic quantity that depends only on the original fields \( \phi^A(x) \), we must impose a restriction on the functional \( F \). The first component of this functional must involve just the fields \( \phi^A \), that correspond to the first components of the superfields \( \phi^A \), otherwise, as we can see from the component expansion for this superfield (eq. (7)) it would involve not only the original fields, but also ghosts, antighosts and auxiliary fields of the shift symmetry, and would thus not lead to a gauge fixing in the standard way. This condition corresponds to the requirement that \( F \) does not involve derivatives with respect to \( \theta^a \). Imposing this restriction, the general expansion for the gauge fixing functional gets:

\[ F[\phi^A(x, \theta^a)] = F[\phi^A] + \delta_a F \theta^a + \delta_1 \delta_2 F \theta^2 \theta^1 \]
\[ = F[\phi^A] + \frac{\delta}{\delta \phi^A} \pi^A \theta^a \]
Using this superfunctional we write the gauge fixing of the original gauge symmetries as

\[
S_F = \int d^2\theta \int dx \mathcal{F}[\hat{\phi}^A(x, \theta^a)]
\]  

The total action gets thus

\[
S[\hat{\phi}^A, \varphi^A_1, \varphi^A_2] = S_{\text{class.}}[\hat{\phi}^i - \varphi^1 - \varphi^2] + S_{\text{col.}}[\varphi^A_1, \varphi^A_2] + S_F[\hat{\phi}^A] \\
= \int d^2\theta \left\{ S_{\text{class.}}[\hat{\phi}^i - \varphi^1 - \varphi^2] \theta^2 \theta^1 \right\} + \frac{1}{2} \int dx \left[ \varphi^A_1 M^A B B_1 - \varphi^A_2 M^A B B_2 \right] + \int dx \mathcal{F}[\hat{\phi}^A] \right\} 
\]  

In order to show that this superspace expression would lead to the BV quantization with extended BRST invariance, as in ref. [9], we can insert the expansions (7) for the superfields in (12). The result corresponds to equation (10)

\[
\delta R^A B^A + \frac{1}{2} \epsilon^{ab} \pi^B B_1 \left[ \delta \mathcal{F} \right] \right\]  

The vacuum functional is build up functionally integrating over all the fields involved in (13), represented here as \(\mu_\alpha\)

\[
Z = \int \prod_\alpha [D\mu_\alpha] \exp \left\{ \frac{i}{\hbar} S[\hat{\phi}^A, \varphi^A_1, \varphi^A_2] \right\} 
\]  

Integrating over \(\lambda_A, \varphi^A_2, B^B, \varphi^A_1, \phi^A_1, \phi^A_2, \pi^B_1, \pi^B_2\), we get the same result as in reference [9]:

\[
Z = \int [D\phi^A] \exp \left\{ \frac{i}{\hbar} S_{\text{class.}} + \int dx \left[ \frac{\left< \frac{\delta}{\delta \phi^A} \right> R^A_1}{\left< \frac{\delta}{\delta \phi^B} \right> R^B_2} + \frac{1}{2} R^B_1 \left[ \frac{\delta}{\delta \phi^A} \right] \frac{\delta}{\delta \phi^B} R^A_1 \right] \right\} 
\]
If $F$ depends just on the classical fields $\phi^i$, this expression will correspond just to the Faddeev Popov result.

The interpretation of all the steps considered in the present section for general gauge theories with closed algebra will be illustrated in the following section by the example of the Yang Mills theory.

4 Example: The Yang Mills Theory

Let us work out the Yang Mills theory as an illustrative example of our superspace formulation. The classical Action in this case is

$$S_{\text{class.}} [A_{\mu}] = -\frac{1}{4} \int dx \, Tr \left( F^{\mu\nu} F_{\mu\nu} \right)$$

(16)

The set of fields that realize the algebra of extended BRST transformations corresponds, in this model, to the set of original gauge fields plus ghost, antighost and auxiliary field: $\phi^A = (A_{\mu}, c, \bar{c}, G)$. The standard extended transformations (without yet the introduction of the collective fields) are:

$$\delta_1^o A_{\mu} \equiv R_1^{[A_{\mu}]} = D_{\mu} c$$
$$\delta_1^o c \equiv R_1^{[c]} = \frac{1}{2} [c, c]_+$$
$$\delta_1^o \bar{c} \equiv R_1^{[\bar{c}]} = G$$
$$\delta_1^o G \equiv R_1^{[G]} = 0$$

(17)

$$\delta_2^o A_{\mu} \equiv R_2^{[A_{\mu}]} = D_{\mu} \bar{c}$$
$$\delta_2^o c \equiv R_2^{[c]} = -(G - [c, \bar{c}])_+$$
$$\delta_2^o \bar{c} \equiv R_2^{[\bar{c}]} = \frac{1}{2} [\bar{c}, \bar{c}]_+$$
$$\delta_2^o G \equiv R_2^{[G]} = [G, \bar{c}]$$

(18)

Following the procedure of the previous chapter we enlarge the field content of the theory, by introducing two collective sets of fields that will be denoted for simplicity as: $\varphi^{A_1} = (\tilde{A}_{\mu}, \tilde{c}, \tilde{\bar{c}}, \tilde{G})$ and $\varphi^{A_2} = (\tilde{\tilde{A}}_{\mu}, \tilde{\tilde{c}}, \tilde{\tilde{\bar{c}}}, \tilde{\tilde{G}})$. The fields $\phi^A$ are replaced by the corresponding $\varphi^A - \varphi^{A_1} - \varphi^{A_2}$ enlarging the symmetry content of the theory. In order to represent the enlarged extended BRST algebra, corresponding to the Yang Mills version of eq.(3) we will need the extra ghosts, antighosts and auxiliary fields that will be denoted respectively as:

$$\pi^{[A_{\mu}]}_1, \pi^{[c]}_1, \pi^{[\bar{c}]}_1, \pi^{[G]}_1; \phi^*[A_{\mu}]_2, \phi^*[c]_2, \phi^*[\bar{c}]_2, \phi^*[G]_2;$$

(19)

$$\phi^*[A_{\mu}]_1, \phi^*[c]_1, \phi^*[\bar{c}]_1, \phi^*[F]_1; \pi^{[A_{\mu}]}_2, \pi^{[c]}_2, \pi^{[\bar{c}]}_2, \pi^{[G]}_2;$$

(20)
In order to build up the superfields corresponding to the Yang Mills version of eq. (7), we must calculate the crossed terms that contribute to the last components of the superfields $\varphi^{Aa}$, that means:

$$
\frac{\delta}{\delta \phi^{A\mu}} R_2^C = D_\mu (G - \tilde{G} - \tilde{\tilde{G}}) - [D_\mu (c - \tilde{c} - \tilde{\tilde{c}}), \tilde{c} - \tilde{\tilde{c}} - \tilde{\tilde{c}}]^+ \\
\frac{\delta}{\delta \phi^{c}} R_2^C = [G - \tilde{G} - \tilde{\tilde{G}}, c - \tilde{c} - \tilde{\tilde{c}}] + [\tilde{\tau} - \tilde{\tilde{\tau}}, (c - \tilde{c} - \tilde{\tilde{c}})^2]^+ \\
\frac{\delta}{\delta \phi^{G}} R_2^C = [G - \tilde{G} - \tilde{\tilde{G}}, \tau - \tilde{\tau} - \tilde{\tilde{\tau}}] \\
\frac{\delta}{\delta \phi^{A\mu}} R_2^C = 0
$$

(22)

Inserting these results in (7) we find the expressions for the twelve superfields, that will be represented as: $\tilde{A}_\mu$, c, $\tilde{c}$, G; $\tilde{\tilde{A}}_\mu$, $\tilde{\tilde{c}}$, $\tilde{\tilde{\tau}}$, $\tilde{\tilde{\tilde{G}}}$; $\tilde{\tilde{\tilde{A}}}_\mu$, $\tilde{\tilde{c}}$, $\tilde{\tilde{\tau}}$, $\tilde{\tilde{\tilde{G}}}$, $\tilde{\tilde{G}}$. The complete expressions for these superfields are given in the appendix.

The superspace version for the classical action is just

$$
S_{\text{class.}}[A_\mu - \tilde{A}_\mu - \tilde{\tilde{A}}_\mu] = -\frac{1}{4} \int dx Tr (F_{\mu\nu} (A_\mu - \tilde{A}_\mu - \tilde{\tilde{A}}_\mu) F^{\mu\nu} (A_\mu - \tilde{A}_\mu - \tilde{\tilde{A}}_\mu))
$$

(23)

Now let us build the gauge fixing action $S_{\text{col.}}$ for the shift symmetries. We will choose the only non vanishing terms of $M^{AB}$ to be:

$$
M[A_\mu A_\mu] = 1 ; \; M[c^\tau] = 1 ; \; M[c \tau] = -1 ; \; M[G^G] = 1 ;
$$

(24)

It is easy to see that this matrix satisfies the requirements of chapter (2). With this choice, we get:

$$
S_{\text{col.}} = \int dx \frac{\delta}{\delta \theta^2} \frac{\delta}{\delta \theta^1} \left[ A_\mu \tilde{A}_\mu + A_\mu \tilde{\tilde{A}}_\mu + 2 \tilde{c} \tilde{\tau} + 2 \tilde{\tilde{c}} \tilde{\tilde{\tau}} + G \tilde{G} + \tilde{\tilde{G}} \tilde{\tilde{G}} \right]
$$

(25)

The gauge fixing action for the original gauge symmetry is of the same form as eq. (10). In order to recover the usual Faddeev Poppov result, we will assume that the gauge fixing functional $F$ depends only on the superfields associated to the original classical fields $A_\mu$:

$$
S_F = \int d^2 \theta \int dx F[A_\mu]
$$

(26)
and also, as discussed in section (3), that it’s first component involves just $A_\mu$. If we then consider the vacuum functional for this theory integrated over all the ghosts, antighosts and auxiliary fields associated to the shift symmetry and also over the collective fields, we get the Yang Mills version of equation (13):

$$Z = \int [dA_\mu] [dc] [d\tau] [dG] \exp \frac{i}{\hbar} \left\{ S_{\text{class.}} + \int dx \left[ \frac{\delta F}{\delta A_\mu} \frac{\delta}{\delta A_\nu} R[A_\mu] - R[A_\nu] \right] \right\}$$

(27)

Let us now consider the gauge fixing functional to be:

$$F = -\frac{1}{2} Tr A_\mu A^\mu$$

(28)

with this particular choice, eq. (27) gets

$$Z = \int [dA_\mu] [dc] [d\tau] [dG] \exp \frac{i}{\hbar} \left\{ S_{\text{class.}} + \int dx \left[ D_\mu GA^\mu \alpha + \partial_\mu \tau(D^\mu c) \right] \right\}$$

(29)

Corresponding to the usual gauge fixed Faddeev Popov action for the Yang Mills field in the Lorentz gauge.

5 Conclusion

In the Batalin Vilkovisky Lagrangian quantization procedure one extends the configuration space by introducing anti-fields corresponding to all the original fields present in the theory. In the case of the extended BV formalism, with extended BRST invariance, there are three antifields associated to each of the original fields. In this space, one can define antibrackets that generate the extended BRST transformations with the classical BV action as the generator. The complete quantum action is obtained as a proper solution of the master equation in a power series in $\hbar$ which coincides with the classical BV action (up to renormalization) when there are no anomalies present. In this paper we have shown that this Extended BV quantization, at order zero in loops can be formulated in a superspace with two Grassmann coordinates. The Yang Mills theory provides a nice example of how the gauge fixing structure works in this superspace formulation of the extended BV. One realizes, comparing with reference [11] the more intricate structure that one needs in the present approach, with extended BRST invariance.

Another interesting application of these ideas would be to investigate how higher order corrections (in $\hbar$) associated to anomalous gauge theories could also be worked out in this superspace formulation. This is presently under study.
\section*{Appendix}

In this appendix we will work out some details of the structure of the BRST anti-BRST superspace used in this article. More details can be found in ref.\textsuperscript{[15]}. Our superspace consists of the set \((x, \theta^a)\) of spatial coordinates \(x\) plus two Grassmann coordinates \(\theta^a\), with \(a = 1, 2\). The superfields \(\phi(x, \theta^a)\), represented in this article by underlined symbols, are build up with the following Taylor expansion in the Grassmann coordinates:

\[
\phi(x, \theta^a) = \phi(x) + \delta_1 \phi(x) \theta^1 + \delta_2 \phi(x) \theta^2 + \delta_1 \delta_2 \phi(x) \theta^2 \theta^1 \quad (A.1)
\]

where \(\delta_1\) and \(\delta_2\) represent BRST and anti-BRST transformations respectively. In such a way that these transformations are generated as translations in \(\theta^1\) and \(\theta^2\) respectively:

\[
\delta_a \phi(x, \theta^a) = \frac{\partial}{\partial \theta^a} \phi(x, \theta^a) \quad (A.2)
\]

where the left arrow indicates that the derivative is acting from the right.

The transformations \(\delta_1^2\) and \(\delta_2^2\) satisfy the so called extended BRST algebra:

\[
\delta_1^2 = \delta_2^2 = \delta_1 \delta_2 + \delta_2 \delta_1 = 0 \quad (A.3)
\]

this implies that the last component of a superfield is a BRST and anti-BRST invariant quantity.

Now considering the example of the Yang Mills theory in section 4, using eqs. (17) and (22) in the general expression for the superfields (6) we get:

\[
\begin{align*}
A_\mu &= A_\mu + \pi^{[A_\nu]} a \theta^a + B^{[A_\nu]} \theta^2 \theta^1 \\
\tilde{A}_\mu &+ \left( [\pi^{[A_\nu]} - \phi^{* [A_\nu]}] - D_\mu c \right) \theta^1 - \phi^{* [A_\nu]} \theta^2 \\
\tilde{\phi} &+ \left[ \phi^{* [A_\nu]} - 1 \right] B^{[A_\nu]} + D_\mu (G - \tilde{G} - \tilde{G} - [D_\mu (c - \tilde{c} - \tilde{c}), \partial - \tilde{\partial} - \tilde{\partial}]) \theta^2 \theta^1 \\
\lambda &+ \left[ \lambda^{[A_\nu]} + \frac{1}{2} (B^{[A_\nu]} + D_\mu (G - \tilde{G} - \tilde{G} - [D_\mu (c - \tilde{c} - \tilde{c}), \partial - \tilde{\partial} - \tilde{\partial}]) \theta^2 \theta^1 \\
\end{align*}
\]

(A.4)
\[ + \frac{1}{2}(B[c] + [G - \tilde{G}, c - \tilde{c} - \tilde{c}] + [\nabla - \tilde{\nabla}, (c - \tilde{c} - \tilde{c})^2 + \nabla, c - \tilde{c} - \tilde{c}]) \theta^2 \theta^1 \]  
\[ \tilde{c} = c + \pi_a[\nabla] \theta^a + B[\nabla] \theta^2 \theta^1 \]
\[ \tilde{\nabla} = \tilde{c} + [\pi[\nabla] - \phi[\nabla] + G] \theta^1 - \phi[\nabla] \theta^2 \]
\[ \tilde{\nabla} = \tilde{c} + \phi[\nabla] \theta^1 + [\pi[\nabla] + \phi[\nabla] \theta^2 - \frac{1}{2}[\pi, \nabla] \theta^1 \]
\[ \tilde{G} = G + \pi[G] \theta^a + B[G] \theta^2 \theta^1 \]
\[ \tilde{\nabla} = \tilde{G} + [\pi[G] - \phi[G] \theta^1 - \phi[G] \theta^2 \]
\[ \tilde{\nabla} = \tilde{G} + \phi[G] \theta^2 + [\pi[G] + \phi[G] - [G, \nabla]] \theta^2 \]

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