Sensitivity analysis of impact ionization coefficients in an electronic device

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Abstract. Terminal current in a device increases when energetic carriers create additional carriers by impact ionization. Okuto and Crowell suggested an empirical model for describe this phenomenon. In this paper, Monte Carlo techniques were used to observe the effect of variability in the impact ionization coefficients on the results obtained from a computational model for electrons and holes transport. The model was implemented in FEM simulation tool, in order to study avalanche current in a MOSFET including uncertainty of the impact ionization coefficients of material.

1. Introduction
The technology of semiconductors devices is in continuous growth. In this way, the semiconductor devices modeling and computational simulation techniques play a very important role. In general, in the fields of the electronic industry and research new methodologies for the study of complex devices such as the MOSFET are developed. It is necessary to consider all the aspects concerning the mode of static and dynamic operation of the MOSFET. In the particular case of the excess current that is produced by the impact ionization, several models have been proposed. The Okuto-Crowell Impact Ionization model is one of the most used since it coincides quite well with the experimental results.

2. Drift-Diffusion model
To obtain the current in a device it is necessary to use the Poisson equation coupled with the continuity equation
\[ \nabla \cdot \mathbf{D} = e(p - n + N_D - N_A), \]
\[ -e \frac{\partial n}{\partial t} = -\nabla \mathbf{J}_n + e(R - G), \]
\[ e \frac{\partial p}{\partial t} = -\nabla \mathbf{J}_p - e(R - G), \]
\[ \mathbf{D} = \varepsilon \mathbf{E} \quad (4) \]
\[ \mathbf{E} = -\nabla \varphi, \quad (5) \]
\[ \mathbf{J}_n = e\mu_n \mathbf{E} + eD_n \nabla n, \quad (6) \]
\[ \mathbf{J}_p = e\mu_p \mathbf{E} + eD_p \nabla p, \quad (7) \]

where \( \mathbf{D} \) is the electric displacement, \( \mathbf{E} \) the electric field, \( \mathbf{J}_n \) the electron current density, \( \mathbf{J}_p \) the hole current density, while \( \varepsilon \) is the dielectric constant, \( \varphi \) the electrostatic potential, \( p \) and \( n \) the hole and electron concentrations, \( N_D \) and \( N_A \) the doping concentrations, \( e \) the elementary charge, \( \mu_n \) and \( \mu_p \) the electron and hole mobilities, \( D_n \) and \( D_p \) the electron and hole diffusivities, the magnitudes \( R \) and \( G \) are the recombination and generation rates.

A MOSFET is a device of high geometrical and physical complexity. The figure 1 (left) shows a simple schematic 2D diagram of a MOSFET. A MOSFET model was created in the COMSOL finite element program, by means of which the current in the drain was obtained for three different gate voltage values as shown in figure 1 (right).

![Schematic 2D diagram of a MOSFET](image)

**Figure 1:** Schematic 2D diagram of a MOSFET (left). Terminal MOSFET current with \( V_{GS}=2.5 \) V, 3.5 V and 4.5 V (right).

### 3. Okuto–Crowell Impact Ionization model

In the presence of high electric fields, the energy of the electrons and holes reaches values greater than the gap energy. In these circumstances, the generation of electron hole pairs increases in a process of chained impacts called impact ionization, which is responsible for the phenomenon of avalanche breakdown in semiconductors.

The carrier generation rate due to impact ionization is given by

\[ G_n = G_p = -\frac{\alpha_n}{e} \mathbf{J}_n - \frac{\alpha_p}{e} \mathbf{J}_p, \quad (8) \]

Okuto and Crowell created an empirical model for impact ionization, giving the following values to the coefficients \( \alpha_n \) and \( \alpha_p \):

\[ \alpha_n = a_n (1 + c_n (T - T_{ref})) \mathbf{E} \exp\left(-\frac{b_n (1 + d_n (T - T_{ref}))}{\mathbf{E}}\right)^2, \quad (9) \]
\[
\alpha_p = a_p(1 + c_p(T - T_{ref}))E \exp(-\frac{b_p(1 + d_p(T - T_{ref})^2)}{E}),
\]

where \(E\) is the component of the electric field parallel to the electron and hole currents. \(T_{ref}\), \(a_n\), \(a_p\), \(b_n\), \(b_p\), \(d_n\) and \(d_p\) are material coefficients. Table 1 shows the impact ionization coefficients for silicon which are used to obtain the impact ionization current in figure 2 (left). Figure 2 (right) depicts the logarithm of the impact ionization source. Observe that the generation is localized at the drain junction, where the electrical field is more intense, especially where the curvature of the junction is more pronounced.

| Parameter | Magnitude | Unit |
|-----------|-----------|------|
| \(n\) - electrons | \(a_n\) | 0.426 | \(V^{-1}\) |
| | \(a_p\) | 0.243 | \(V^{-1}\) |
| \(p\) - holes | \(b_n\) | \(4.81 \times 10^5\) | \(V/cm\) |
| | \(b_p\) | \(6.53 \times 10^5\) | \(V/cm\) |
| | \(c_n\) | \(3.05 \times 10^{-5}\) | \(K^{-1}\) |
| | \(c_p\) | \(5.35 \times 10^{-5}\) | \(K^{-1}\) |
| | \(d_n\) | \(6.86 \times 10^{-4}\) | \(K^{-1}\) |
| | \(d_p\) | \(5.67 \times 10^{-4}\) | \(K^{-1}\) |

Figure 2: Terminal current with impact ionization for \(V_g = 3.5\) V (left). Logarithm of the impact ionization source (right).

4. Sensitivity analysis of MOSFET impact \(a_n\) and \(b_n\) coefficients
In the first instance, the \(a_n\) and \(b_n\) coefficients are assumed to vary from 99 to 101 percent of the original value. This is approximated using a normal distribution with a mean value equal to the original coefficient and a standard deviation of \(0.033a_n\) (so that most of the values lie within the assumed range of 99–101\%). See figure 3 (left). Figures 4 and 5
(left) show the MOSFET impact ionization current for some random values of the $a_n$ and $b_n$ coefficients within an input variability of 1%. Figure 3 (right) shows the $a_n$ coefficient with a variability of 10% and figure 4 (right) corresponds to impact ionization current for this case. Finally, figure 5 (right) illustrates the impact ionization current when the $b_n$ coefficient varies in a range of 10%.

Figure 3: Implementation of the coefficient $a_n$ within a random variability of 1% (left) and 10% (right).

Figure 4: Impact ionization current for some random values of the coefficient $a_n$ within an input variability of 1% (left) and 10% (right).

Figure 5: Impact ionization current for some random values of the coefficient $b_n$ within an input variability of 1% (left) and 10% (right).
5. Conclusions
When the $a_n$ and $b_n$ silicon impact ionization coefficients vary in a range of 1% the terminal current does not change appreciably. In the development of this article only the coefficients corresponding to the electrons were taken because the MOSFET channel is $n^+$ type, this means that this device conducts electrons preferentially. The coefficients $c_n$ and $d_n$ were not studied since they are linked to the operating temperature of the device.

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References
[1] Okuto Y and Crowell C R 1975 Threshold Energy Effect on Avalanche Breakdown in Semiconductor Junctions Solid-State Electronics Vol 18 pp 161-168
[2] Chau H and Pavlidis D 1992 A physics-based fitting and extrapolation method for measured impact ionization coefficients in II-VI semiconductors J. Appl. Phys. 72 (2) 15
[3] Cao L et al 2018 Experimental characterization of impact ionization coefficients for electrons and holes in GaN grown on bulk GaN substrates Appl. Phys. Lett. 112 262103
[4] Mauro A et al 2015 3D finite element modeling and simulation of industrial semiconductor devices including impact ionization Journal of Mathematics in Industry 5:1
[5] Zhou Q et al 2011 GaN/SiC avalanche photodiodes Appl. Phys. Lett. 99 131110
[6] Van Overstraeten R and De Man H 1970 Measurement of the ionization rates in diffused silicon p-n junctions Solid-State Electronics Vol 13 pp 583-608
[7] Datta A and Rakesh V 2010 An Introduction to Modeling of Transport Processes Cambridge University Press
[8] Chang Y-C et al 1983 Monte Carlo simulation of impact ionization in GaAs including quantum effects Appl. Phys. Lett. 42(1) 1
[9] Jung H K et al 1996 Impact ionization model for full band Monte Carlo simulation in GaAs Journal of Applied Physics 79 2473
[10] Marsland J S 2011 Comparison of different models of non-local impact ionization for low noise avalanche photodiode Physica Status Solidi C Vol 8-9