MULTI-OBJECTIVE CHANCE-CONSTRAINED BLENDING OPTIMIZATION OF ZINC SMELTER UNDER STOCHASTIC UNCERTAINTY

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(Communicated by Changjun Yu)

ABSTRACT. Considering the uncertainty of zinc concentrates and the shortage of high-quality ore inventory, a multi-objective chance-constrained programming (MOCCP) is established for blending optimization. Firstly, the distribution characteristics of zinc concentrates are obtained by statistical methods and the normal distribution is truncated according to the actual industrial situation. Secondly, by minimizing the pessimistic value and maximizing the optimistic value of object function, a MOCCP is decomposed into a MiniMin and MaxiMax chance-constrained programming, which is easy to handle. Thirdly, a hybrid intelligent algorithm is presented to obtain the Pareto front. Then, the furnace condition of roasting process is established based on analytic hierarchy process, and a satisfactory solution is selected from Pareto solution according to expert rules. Finally, taking the production data as an example, the effectiveness and feasibility of this method are verified. Compared to traditional blending optimization, recommended model both can ensure that each component meets the needs of production probability, and adjust the confident level of each component. Compared with the distribution without truncation, the optimization results of this method are more in line with the actual situation.

1. Introduction. Blending is an important process of the zinc hydrometallurgy processes. The result of blending process can affect all subsequent processes. Effective optimization can reduce the fluctuation of ore grade, enhance the stability of quality of mixed raw materials, and cut production costs.

There have been many studies on blending optimization, including deterministic model and uncertain model. In view of different industrial background, many
researchers have established deterministic blending optimization models \([1,5,14]\). However, due to various sources and great difference of raw materials, each component has uncertainty in real industrial process. The advantage of chance-constrained programming (CCP) is that it permits the decision to not satisfy the constraints to some extent, but the probability of constraints is not less than a certain confidence level. Therefore, CCP is widely used in industry, such as oil, zinc smelting, iron making and other blending optimization to solve the uncertainty \([2,11–13,18]\).

To a certain extent, these methods solve blending optimization problems under uncertainty. However, most methods for the blending problem are based on chance-constrained programming with normal distribution \([2,11–13]\), and they assume that the inventory of raw materials is sufficient. This will lead to two results. One is that the chance-constrained programming has no feasible solution when some raw materials are not enough. Another is that the tail of normal distribution is serious, which leads to the serious tail of distribution of mixed zinc concentrate, even the situation is not in line with the actual situation.

In practice, there are three main challenges. The first challenge is how to extract accurate information from uncertain data, as some parts of the fitted distribution may not be realistic. The second one is how to effectively solve the multi-objective, uncertainty, and nonlinear optimization problems that cannot be addressed directly due to the coexistence of multiple distributions. The last one is how to determine the working conditions according to multi-time scale data and choose the satisfactory solution of the Pareto front.

In this study, a multi-objective blending optimization method under uncertainty is proposed to solve the problem that the general chance constrained programming can not be solved in the case of shortage of high-quality mineral resources and the normal distribution leads to serious tailing of optimization results. The method solves the problem of unrealistic fitting and uncertain multi-objective optimization. Firstly, the parameters of the distribution function are obtained according to actual production data. The truncated normal distribution is used to solve the problem that the tail of the normal distribution causes some data to be inconsistent with actual practices. Then, a multi-objective blending model is proposed, in addition, a hybrid intelligent algorithm is presented to obtain Pareto front. Finally, the evaluation mechanism based on AHP and expert experience is established to identify the working conditions and obtain the satisfactory solution from the Pareto front.

The rest of this thesis is organized as follows. The key problems of blending are introduced in Section 2. The Multi-objective optimization model of blending and its solution are introduced in detail in Section 3. The simulation using field data is discussed in Section 4. We make a summary in Section 5.
2. Blending problem of zinc hydrometallurgy.

2.1. Description of the blending process. Blending process is the first process of zinc smelter and previous process of roasting. The blending process is as follows. Firstly, we set the ratio of each ore. Then, the Zinc concentrates are uniformly mixed by grab crane and disc feeder. Lastly, they are sent to drying kiln by a belt conveyor, and sent to roasting workshop, as shown in Fig. 1. Raw materials are stored in semi-underground warehouse, as shown in Fig. 2. According to the content of raw materials, they are divided into five categories. The classification criteria are shown in Table 1.

2.2. Key issues in blending optimization. There are two main problems in blending optimization: uncertainty of Zinc concentrates components and shortage of high-quality ore. One of the key difficulties of blending is the uncertainty of the Zinc concentrates components [2]. The main reasons are as follows: (1) The company has more than 100 suppliers, and the quality of raw material varies greatly. The main composition range and the composition requirements of mixed Zinc concentrates in one month are shown in Table 2. (2) The same batch of zinc concentrate from the same supplier has different components even on the same day, as shown in Table
3. (3) A small number of ore bins leads to rough classification, which increases uncertainty.

![Block diagram of multi-objective chance-constrained blending optimization](image)

**Figure 3.** Block diagram of multi-objective chance-constrained blending optimization

**Table 1.** Classification methods

|     | #5 High-silicon ore | #4 High-lead ore | #3 Low-purity ore | #2 High-purity ore | #1 High-quality ore |
|-----|------------------|------------------|-------------------|--------------------|-------------------|
| Zn% | <44              | 44<Zn<47         | >47               |                    |                   |
| Pb% | >1.8             | <1.8             | <1.8              | <1.8               |                   |
| SiO2%| >3               | <3               | <3                | <3                 |                   |
Table 2. Common composition range of zinc concentrates

|        | Zn(%) | Fe(%) | SiO2(%) | Pb(%) | Sb(%) | Ge(%) | Co(%) |
|--------|-------|-------|---------|-------|-------|-------|-------|
| Min    | 41.46 | 2.93  | 1.25    | 6.71  | 0.013 | 0.0027| 0.00125|
| Max    | 55.37 | 17.2  | 7.65    | 0.72  | 1.21  | 0.0025| 0.006 |
| Requirement | 47> | <12  | <3 | <1.8 | <0.1 | <0.006 | <0.004 |

Table 3. Test valve of zinc concentrate from one supplier

| Date       | Suppliers | Material       | Zn%  | Pb%  | SiO2% |
|------------|-----------|----------------|------|------|-------|
| 2020/9/7   | Company of A | Zinc concentrates | 49.32 | 1.79 | 2.47  |
| 2020/9/7   | Company of A | Zinc concentrates | 49.50 | 1.64 | 2.69  |
| 2020/9/7   | Company of A | Zinc concentrates | 49.33 | 1.78 | 2.51  |
| 2020/9/7   | Company of A | Zinc concentrates | 49.51 | 1.71 | 2.69  |
| 2020/9/7   | Company of A | Zinc concentrates | 49.27 | 1.30 | 3.64  |
| 2020/9/7   | Company of A | Zinc concentrates | 44.59 | 1.35 | 3.71  |

Another key difficulty in blending is shortage of high-quality ore. To improve the utilization of funds and the competitiveness of enterprises, inventory is kept at a low level at some enterprises [6, 15]. Due to the influences of supply chain, it is impossible to ensure that every stock has enough raw materials every day. Therefore, there may be a shortage of high-quality ore, which leads to challenges in obtaining the ratio that fully meets the production requirements.

3. Multi-objective chance-constrained blending optimization in zinc smelter.

Due to the uncertainty of Zinc concentrates composition, the mixed Zinc concentrates after blending may not meet the requirements. Owing to supply chain constraints, there is a shortage of high-quality ore inventory, which leads to no feasible solution for the general chance-constrained programming model [2]. To solve it, a multi-objective chance-constrained optimization method based on the actual working conditions is proposed, as shown in Fig. 3. It includes three parts. For the first part, the probability distributions of the components are fitted by the maximum likelihood method and MOCCP is built for blending problem. For the second part, to solve the MOCCP, a hybrid intelligent algorithm is presented to obtain Pareto front. For the third part, an evaluation mechanism based on AHP and expert experience is established, and the optimization result is obtained from the Pareto front.

3.1. Formulation of the blending problem.

3.1.1. Description of the uncertain parameters. Many studies have found that the parameters of uncertainty distributions either obey normal distribution or lognormal distribution [2, 10, 12]. According to the historical data statistics, it is found that the distribution parameters of each component of zinc concentrate also obey these two distributions. In this paper, it is assumed that the content of each element in the raw material is independent of each other. The dry weight of each car of Zinc concentrates is used to express the frequency of each car of Zinc concentrates. According to the maximum likelihood method, the components of Zinc concentrates...
can be fitted by the normal distribution and lognormal distribution from historical data. Taking high-purity ore as an example, most of the zinc content is between 47% and 53%, and only a small proportion of zinc is between 53% and 56%. The abscissa is the zinc content of each vehicle minus 47%, and the ordinate is the ratio of the dry weight to the total dry weight. The normal distribution $X = N(\mu, \delta^2)$ can be obtained by fitting, as shown in Fig. 4b. For the high-quality ore, the abscissa is the zinc content of each vehicle minus 47%, which is symmetrically passed by $x = 0$. Ordinates are the same as above. Then, Fig. 4c was obtained by fitting the lognormal distribution. The mean and variance of lognormal distribution can be obtained by Eq. (1) and Eq. (2):

$$E(x) = e^{\mu+\sigma^2/2}$$
$$D(x) = (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$$

For the components with normal distribution, the parameters of zinc content could be obtained by the direct fitting method in high-silica ore. However, there was a serious tailing phenomenon in the normal distribution, so the truncated normal distribution was adopted, as shown in Fig. 4a. The mean and variance of the truncated normal distribution $TN(\mu, \delta^2, a, b)$ can be obtained by Eq. (3) and (4):

$$E(x) = \mu + \left[ \frac{\phi(\frac{b-u}{\sigma}) - \phi(\frac{b-u}{\sigma})}{\Phi(\frac{b-u}{\sigma}) - \Phi(\frac{b-u}{\sigma})} \right] \sigma$$
$$VAR(x) = \sigma^2 \left[ 1 + \left( \frac{a-u}{\sigma} \right) \phi\left( \frac{a-u}{\sigma} \right) - \phi\left( \frac{b-u}{\sigma} \right) \phi\left( \frac{b-u}{\sigma} \right) \right] - \sigma^2 \left[ \frac{\phi\left( \frac{a-u}{\sigma} \right) - \phi\left( \frac{b-u}{\sigma} \right)}{\Phi\left( \frac{b-u}{\sigma} \right) - \Phi\left( \frac{a-u}{\sigma} \right)} \right]^2$$

$\Phi$ is the standard normal cumulative distribution function, and $\phi$ is the standard normal probability density function.

3.1.2. Multi-objective chance-constrained blending optimization. A blending optimization model was established with the objective of minimizing the cost of raw materials per ton and maximizing the zinc content of mixed zinc concentrate. There are five types of raw materials and eight components.

The mathematical model is as follows:

![Figure 4. Statistical diagram and distribution fitting diagram of each component (a: Truncated normal distribution for high-purity ore; b: Lognormal distribution for high-purity ore; c: Lognormal distribution for high-quality ore).]
Table 4. Model parameters description

| Model Parameters | explanatory notes |
|------------------|-------------------|
| $i$              | $i = 1, 2, 3, 4, 5$; |
| $P_i$            | price per ton of zinc concentrate $i$; |
| $T_y$            | Maximum allowable of $y$, $y = \{SiO_2, Pb, Fe, S, Ni, Co, Sb, Ge\}$ |
| $m$              | amount of blending ($t$); |
| $\bar{m}$       | allowance of mixed zinc concentrates; |
| $\underline{m}$| minimum demand for mixed zinc concentrates; |
| $X_{\text{max}}$| allowance of raw material $i$, and |
| $X_{\text{min}}$| minimum demand for raw material $i$. |

Random variables

- $\bar{W}_i$ zinc content percentage of raw material $i$;
- $\bar{T}_{yi}$ $y$ content percentage of raw material $i$.

Decision variables

- $X_i$ amount of zinc concentrates $i$.

\[
\begin{align*}
\text{min} J &= \frac{\sum_{i=1}^{5} (X_i \times P_i)}{\sum_{i=1}^{5} X_i \times \bar{W}_i} \\
\text{max} f &= \frac{\sum_{i=1}^{5} (X_i \times \bar{T}_{yi})}{\sum_{i=1}^{5} X_i} \leq \frac{\sum_{i=1}^{5} X_i}{T_y} \\
\text{s.t.} & \\
\sum_{i=1}^{5} (X_i \times T_{yi}) & \leq \bar{m} \\
X_{\text{min}} & \leq X_i \leq X_{\text{max}} \\
y & = \{SiO_2, Pb, Fe, S, Sb, Ni, Ge, Co\}
\end{align*}
\]

The first objective function is to minimize the amount of zinc per ton. The second objective function is to maximize the zinc content. The first constraint indicates that the impurity composition requirements should be met. Since only the main components are considered, the sum of the components isn’t 100%. The second constraint is to limit the total amount of mixed zinc concentrate. The third constraint represents the upper and lower limits of each bin usage.

Figure 5. Optimization idea
As the multi-objective optimization model cannot be directly optimized, the optimization idea is proposed as shown in Fig. 5. First, the original problem is decomposed into two single-objective optimization problems, so Eq. (6) and Eq. (7) are obtained.

\[
\begin{align*}
\text{min } J &= \frac{\sum_{i=1}^{5} (X_i \times P_i)}{\sum_{i=1}^{5} X_i \times \tilde{W}_i} \\
\text{s.t.} & \quad \sum_{i=1}^{5} (X_i \times T_{yi}) / \sum_{i=1}^{5} X_i \leq T_y \quad (6) \\
& \quad m \leq \sum_{i=1}^{5} X_i = m \leq \bar{m} \\
& \quad X_{\text{min}} \leq X_i \leq X_{\text{max}} \\
& \quad y = \{\text{SiO}_2, \text{Pb, Fe, S, Sb, Ni, Ge, Co}\} \\
\text{max } f &= \frac{\sum_{i=1}^{5} (X_i \times \tilde{W}_i)}{\sum_{i=1}^{5} X_i} \\
\text{s.t.} & \quad \sum_{i=1}^{5} (X_i \times T_{yi}) / \sum_{i=1}^{5} X_i \leq T_y \quad (7) \\
& \quad m \leq \sum_{i=1}^{5} X_i = m \leq \bar{m} \\
& \quad X_{\text{min}} \leq X_i \leq X_{\text{max}} \\
& \quad y = \{\text{SiO}_2, \text{Pb, Fe, S, Sb, Ni, Ge, Co}\}
\end{align*}
\]

However, the objective functions are nonlinear and cannot be transformed into an equivalent deterministic optimization model by using Taha’s method [16]. To solve the functions, Eq. (6) is transformed into a MiniMin model by minimizing the pessimistic value of the objective function and Eq. (7) is transformed into a MaxiMax model by maximizing the optimistic objective function. Then, Eqs. (8) and (9) can be obtained by introducing two new variables \(J\) and \(\tilde{f}\).

\[
\begin{align*}
\text{min } \min_{X} J & \quad \text{s.t.} \\
& \quad \sum_{i=1}^{5} (X_i \times P_i) \leq J \\
& \quad \sum_{i=1}^{5} (X_i \times T_{yi}) / \sum_{i=1}^{5} X_i \leq T_y \\
& \quad m \leq \sum_{i=1}^{5} X_i = m \leq \bar{m} \\
& \quad X_{\text{min}} \leq X_i \leq X_{\text{max}} \\
& \quad y = \{\text{SiO}_2, \text{Pb, Fe, S, Sb, Ni, Ge, Co}\} \\
\text{max } \max_{X} \tilde{f} & \quad \text{s.t.} \\
& \quad \sum_{i=1}^{5} (X_i \times \tilde{W}_i) / \sum_{i=1}^{5} X_i \geq \tilde{f} \\
& \quad \sum_{i=1}^{5} (X_i \times T_{yi}) / \sum_{i=1}^{5} X_i \leq T_y \quad (9)
\end{align*}
\]
\[ \begin{align*} 
m &\leq \sum_{i=1}^{5} X_i = m \leq \bar{m} \\
X_{\text{min}} &\leq X_i \leq X_{\text{max}} \\
y &\{\text{SiO}_2, \text{Pb}, \text{Fe}, \text{S}, \text{Sb}, \text{Ni}, \text{Ge}, \text{Co}\} \\
\end{align*} \]

Then, Eq. (8) and (9) are transformed into probability problems, and Eq. (10) and (11) for the chance-constrained programming are obtained.

\[
\begin{align*}
\min \ min \ J \\
\text{s.t.} \\
Pr \left\{ \frac{\sum_{i=1}^{5} (X_i \times P_i)}{\sum_{i=1}^{5} X_i \times \tilde{W}_i} \leq J \right\} \geq \vartheta_1 \\
Pr \left\{ \frac{\sum_{i=1}^{5} (X_i \times T_y)}{\sum_{i=1}^{5} X_i \leq T_y} \geq \beta_j \right\} \\
\min \ J \\
\text{s.t.} \\
Pr \left\{ \frac{\sum_{i=1}^{5} (X_i \times \tilde{W}_i)}{\sum_{i=1}^{5} X_i \times \tilde{W}_i} \leq J \right\} \geq \vartheta_2 \\
Pr \left\{ \frac{\sum_{i=1}^{5} (X_i \times T_y)}{\sum_{i=1}^{5} X_i \leq T_y} \geq \beta_j \right\} \\
\end{align*}
\]

Where \( \vartheta_1 \) and \( \vartheta_2 \) are the confidence levels. The confidence levels \( \beta_j \) are determined by the actual demand. Usually, they can be set according to historical statistics and results. \( \min J \) is the \( \vartheta_1 \)-pessimistic value of the objective function \( J \). \( \max f \) is the \( \vartheta_2 \)-optimistic value of the objective function \( f \).

3.2. **Hybrid intelligent optimization algorithm.** A multi-objective hybrid intelligent algorithm is proposed to solve MOCCP, as shown in Fig. 6. Firstly, Monte Carlo method is introduced to create stochastic numbers for uncertain parameters. Then, by approximating the uncertain function with neural network, the MiniMin and MaxiMax MOCCP can be converted to a deterministic equivalent model. Finally, the Pareto front is obtained by using NSGA-II of the elite strategy.

According to Liu’s method [9], Eq. (10) and (11) can be expressed as Eq. (12).

\[
\begin{align*}
U_a &= \min \left\{ \left( \sum_{i=1}^{4} (X_i \times P_i) \right) \text{Pr} \left\{ \frac{\sum_{i=1}^{5} X_i \times \tilde{W}_i}{\sum_{i=1}^{5} X_i \times \tilde{W}_i} \leq J \right\} \geq \vartheta_1 \right\} \\
\end{align*}
\]
Using the Monte Carlo simulation method for Eq. (12)
Obtain a set of independent $x$ and corresponding $U$
Train the neural network to approximate Eq. (12) by using this set of $x$ and $U$
Use NSGA-II to solve multi-objective optimization problems
Obtain the Pareto frontier

**Figure 6.** Multi-objective hybrid intelligent optimization algorithm

\[
U_b = \max \left\{ \mathcal{T} \left| \mathcal{P} \left\{ \sum_{i=1}^{4} (X_i \times \tilde{W}_i) \geq \sum_{i=1}^{4} X_i \geq \mathcal{T} \right\} \geq \partial_2 \right\} \right. \quad (12b)
\]

\[
U_y = \mathcal{P} \left\{ \sum_{i=1}^{4} (X_i \times \tilde{T}_{yi}) \geq \sum_{i=1}^{4} X_i \leq \mathcal{T}_y \right\} \quad (12c)
\]

\[y = \{SiO_2, Pb, Fe, S, Sb, Ni, Ge, Co\}\]

First, the input and output data of functions (12a) and (12b) are generated by Monte Carlo method and the law of large numbers. Taking Eq. (12a) as an example, its process is shown in Table 5. Similarly, Eq. (12b) can be obtained.

**Table 5.** Eq. (12a) acquisition process

| Algorithm 1. |
|--------------|
| Step 1: Use the uniform distribution to create decision variable $x$; |
| Step 2: Use the Monte Carlo method to produce $L = 1000$ independent random matrices $\xi^1, \xi^2, \cdots, \xi^L$ based on the distribution, and get the sequence $\{J_1, J_2, \cdots, J_L\}$; |
| Step 3: Take $L'$ as the integer part of $\partial_1 L$; |
| Step 4: Select the $L'$th element $J_{L'}$ as an estimate of $U_a = J$ in the sequence $\{J_1, J_2, \cdots, J_L\}$ based on the law of large numbers. |

Then, the Monte Carlo method and Kolmogorov's law of strong numbers are used to generate input-output data for function (12c). The process of Eq. (12c) is shown in Table 6. Let

\[g = \sum_{i=1}^{4} (X_i \times T_{yi}) / \sum_{i=1}^{4} X_i \quad (13)\]
Table 6. Eq. (12c) acquisition process

Algorithm 2.

Step 1: Use the uniform distribution to create decision variable $x$;
Step 2: Use the Monte Carlo method to generate $L = 1000$ independent random matrices $\xi_1, \xi_2, \ldots, \xi_L$ according to the distribution, and obtain the sequence $\{g_1, g_2, g_3, \ldots, g_L\}$ by Eq. (13);
Step 3: Get number $L'$ that satisfies the inequality $g_i \geq W_i$, $i = 1, 2, \ldots, L$ in sequence $\{g_1, g_2, g_3, \ldots, g_L\}$;
Step 4: Estimate the probability $U_{Z_n}$ based on the frequency $L'/L$ according to Kolmogorov’s law of strong numbers.

In addition, the neural network with 5 inputs, 20 Hidden nodes and 4 outputs, is used to approximate the function (12) [7]. The activation function is sigmoid [8]. Thus, the uncertain programming is transformed into the deterministic optimization problem by using neural network and Monte Carlo method.

To solve the optimization problem, a fast and NSGA-II is used [3]. When the constraints are not met, a large penalty is imposed. The parameters setting of NSGA-II are as follows: population size set to 60, crossover probability of 0.8, and mutation probability of 0.3.

3.3. Evaluation mechanism based on AHP and expert rules. The Pareto front is obtained by the hybrid intelligent optimization method. To obtain the results of optimization from the Pareto front, the objective functions $U_a$ and $U_b$ are normalized to obtain $f_1$ and $f_2$ according to Eq. (14).

$$z_k(x) = \frac{f_k(x) - f_k^{min}}{f_k^{max} - f_k^{min}}$$

(14)

In order to convert to the single-objective optimization problem, we need to determine proportion in Eq. (15).

$$\min (\text{proportion} \ast f_1 - (1 - \text{proportion}) \ast f_2)$$

(15)

where the range of proportion is 0 to 1.

Figure 7. Roasting process
In actual production, the proportion in the case of shortage of high-quality ore is mainly determined according to working conditions of roasting furnace. According to experience of workers, the data reflecting the operation of the roasting furnace mainly includes the test value of the Zinc concentrates (mainly reflected in zinc, lead, and silicon), the calcination temperature, and the soluble zinc rate (SZR), as shown in Fig. 7. The main parameters are divided into five levels to deal with the multi-time scale data. Through a qualitative method, we order each index and gave the evaluation of either excellent, good, general, poor, or worst, as shown in Table 7. The content of zinc, lead, and silicon and the SZR are tested twice a day, and the average value is taken as the evaluation basis. The zinc content is greater than 50% in the third level because the zinc content is too high, which would affect the quality of the electrolytic process and subsequent ingredients. The calcination temperature of the boiling furnace have one sampling every minute. The boiling furnace require a standard temperature range of 860°C to 940°C. The ratio of the number (RN) of times that the standard temperature range is exceeded to the total sampling time in one day is used as the evaluation basis. It is necessary to consider the situation that the thrower is blocked for a short time, as shown in the green box in Fig. 8. The temperature is considered to be within the specified range in this situation. In the case of failure, such as shortage of material, ball mill failure, and drying kiln maintenance, all indicators may be affected and may not be resolved through blending. It may need to be handled manually. Thus, the system is not evaluated for working conditions. All data are from the industrial site.

Table 7. The qualitative assessment method

| Criterion | Excellent | Good | Generally | Bad | Worst |
|-----------|-----------|------|-----------|-----|-------|
| SZR $x_1/%$ | $x_1>96$ | $96>x_1>94$ | $94>x_1>92$ | $92>x_1>90$ | $90>x_1$ |
| RN $x_2$ | $0.5%>x_2$ | $1%>x_2>0.5%$ | $2%>x_2>1%$ | $4%>x_2>2%$ | $x_2>4%$ |
| Zn$% x_3$ | $50>x_3>48$ | $48>x_3>47$ | $x_3>50$ | $47>x_3>46$ | $46>x_3$ |
| Pb$% x_4$ | $1>x_4$ | $1.5>x_4>1$ | $1.8>x_4>1.5$ | $2.0>x_4>1.8$ | $x_4>2.0$ |
| SiO2$% x_5$ | $1.5>x_5$ | $2.0>x_5>1.5$ | $3.0>x_5>2.0$ | $3.5>x_5>3.0$ | $x_5>3.5$ |

$$\min (\text{proportion} \ast f_1 -(1- \text{proportion}) \ast f_2)$$ (16)

where the range of proportion is 0 to 1.

AHP is a multi-criteria decision-making method. It can digitize the thinking process of human beings for complex systems, quantify qualitative analysis that is mainly based on artificial subjective judgment, and quantify the differences among various judgment elements. AHP is widely used in practice [4,17]. Therefore, a multi-time scale evaluation mechanism based on AHP and expert rules is proposed, as shown in Fig. 9. AHP is used to obtain five evaluation indexes and the weights of the five levels of each index. The specific steps are as follows:

**Step 1.** Establish a hierarchical model as shown in the AHP section in Fig. 9. Set the maximum operating condition score as 1.

**Step 2.** A pairwise comparison matrix is established:

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix}$$ (17)
Figure 8. Calciner temperature and feed rate (the upper is the calcination temperature and the lower is the feed amount of the thrower).

Figure 9. Evaluation mechanism of working condition

where \( a_{ij} > 0 \) and \( a_{ij} = \frac{1}{a_{ij}} \).

Table 8. Scale of importance

| Number | Explanation               |
|--------|---------------------------|
| 1      | Equally important         |
| 3      | Slightly important        |
| 5      | Strongly important        |
| 7      | Very strongly important   |
| 9      | Absolutely important      |
| 2, 4, 6, 8 | Intermediate value      |

The importance scale of AHP is shown in Table 8. According to expert experience and Table 8, the importance scales are shown in Table 9. Thus, all pairwise comparison matrices can be obtained. For example, the pairwise comparison matrix of criterion levels is shown in Table 10.
Table 9. The score of the criteria and the five levels

| Target | Criterion | Importance of index | Excellent | Good | General | Poor | Worst |
|--------|-----------|---------------------|-----------|------|---------|------|-------|
| Total  | SZRx1     | 9                   | 9         | 7    | 5       | 3    | 1     |
| score  | RNx2      | 7                   | 9         | 7    | 6       | 4    | 2     |
| u      | Zn% x3    | 4                   | 9         | 8    | 6       | 3    | 1     |
|        | Pb% x4    | 5                   | 9         | 7    | 6       | 2    | 1     |
|        | SiO2% x5  | 3                   | 9         | 8    | 5       | 2    | 1     |

Table 10. Pairwise comparison matrix of criterion levels

|       | SZRx1 | RNx2 | Zn% x3 | Pb% x4 | SiO2% x5 |
|-------|-------|------|--------|--------|----------|
| SZRx1 |       | 1    | 9/7    | 9/4    | 9/5      |
| RNx2  | 7/9   | 1    | 7/4    | 7/5    | 7/3      |
| Zn% x3| 4/9   | 4/7  | 1      | 4/5    | 4/3      |
| Pb% x4| 5/9   | 5/7  | 5/4    | 1      | 5/3      |
| SiO2% x5 | 1/3  | 3/7  | 3/4    | 3/5    | 1        |

Step 3. Hierarchical single sorting and consistency test. The evaluation matrix is generated by experts or decision-makers according to their knowledge and experience. Therefore, it is necessary to judge the consistency of the matrix in order to prove the correctness of the logical relationship between the indicators in the working condition evaluation system. Compute the consistency index (CI) as follows:

\[ CI = \frac{\lambda_{\text{max}} - n}{n - 1} \]  

where \( n \) is the number of items being compared and \( \lambda_{\text{max}} \) is the maximum eigenvalue of evaluation matrix.

Compute the consistency ratio defined as follows:

\[ CR = \frac{CI}{RI} \]  

where \( RI \) is the random consistency index whose values according to \( n \) are shown in Table 11; When \( CR < 0.10 \), the consistency of the judgment matrix is acceptable.

Table 11. Values of the random consistency index

| \( n \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|--------|---|---|---|---|---|---|---|---|---|----|----|
| \( RI \) | 0 | 0 | 0.58 | 0.9 | 1.12 | 1.24 | 1.32 | 1.41 | 1.46 | 1.49 | 1.52 |

Step 4. Calculate the weight of each layer element. The weights obtained through the above three steps are shown in Table 12.

Based on actual industrial data, the current working condition score of the baking furnace can be obtained by the AHP method. In order to obtain the current condition classification, the expert rules based on workers’ experiences are shown in Table 13. When the system runs well, zinc content and price are treated equally. When the
system runs poorly, we only makes zinc content as high as possible, regardless of the price of raw materials.

Through the evaluation mechanism, the proportion can be obtained by using AHP and expert rules. Therefore, the uncertainty and working condition information can be considered comprehensively, and the optimization results can be easily obtained from the Pareto front.

4. Illustrative examples. The experimental data are from the Zinc Smelting Group of China. In the actual production process, the roasting furnace is mainly affected by zinc, silicon, and lead. Therefore, the method recommended is based on these three main elements and verified by industrial field data. All simulations are implemented on a 2.8 Ghz processor, 8 GB RAM PC with MATLAB.

4.1. Distributed parameters of Zinc concentrates. The main distribution parameters are shown in Table 14. $N$ represents the normal distribution $N(u, \sigma^2)$. $a$ and $b$ indicate the lower and upper limit of truncation parameters, respectively. $LN$ is the lognormal distribution. The $u$ with an asterisk (*) represents the lognormal distribution $ln(x+y) \sim N(u, \sigma^2)$, where zinc, silicon, and lead of $y$ are 47, 3, 1.8, respectively. The box represents the lognormal distribution $ln(47-x) \sim N(u, \sigma^2)$.

### Table 12. Weight of each index

| Target | Criterion | Weights | Excellent | Good | General | Poor | Worst |
|--------|-----------|---------|-----------|------|---------|------|-------|
| Total  | SZR x1    | 0.3103  | 0.36      | 0.28 | 0.2     | 0.12 | 0.04  |
|        | RN x2     | 0.2414  | 0.3214    | 0.25 | 0.2143  | 0.1429 | 0.0714 |
|        | Zn% x3    | 0.1379  | 0.3333    | 0.2963 | 0.2222 | 0.1111 | 0.037 |
|        | Pb% x4    | 0.1724  | 0.36      | 0.28 | 0.24    | 0.08  | 0.04  |
|        | SiO2% x5  | 0.1034  | 0.36      | 0.32 | 0.2     | 0.08  | 0.04  |

### Table 13. Expert rules

| Operation of the system | Range of $u$ | Proportion |
|-------------------------|--------------|------------|
| Excellent               | $u > 0.9$    | 0.5        |
| Good                    | $0.9 > u > 0.8$ | 0.6       |
| General                 | $0.8 > u > 0.7$ | 0.8       |
| Poor                    | $0.7 > u$    | 1          |

### Table 14. Main component parameters of Zinc concentrates

| Ore | Zn (%) | Pb (%) | SiO2 (%) | AP |
|-----|--------|--------|----------|----|
| bin | $u$    | $a$    | $b$      | $Dis$ | $u$    | $a$    | $b$      | $Dis$ | $RMB/t$ |
| #1  | 1.01*  | 0.594  | –        | LN    | 1.021  | 0.38   | 0 1.8    | N      | 0.21   | 0.35   | LN    | 14701 |
| #2  | 0.122□ | 0.351  | –        | LN    | 1.24   | 0.3    | 0 1.8    | N      | 0.19   | 0.38   | LN    | 13236 |
| #3  | 0.103△ | 0.201  | –        | LN    | 1.133  | 0.41   | 0 1.8    | N      | 0.185  | 0.42   | LN    | 12157 |
| #4  | 46.12  | 1.9    | 40       | 52    | N      | 0*     | 0.4      | –      | LN    | 0.22   | 0.4   | LN    | 13368 |
| #5  | 45.31  | 1.62   | 40       | 52    | N      | 1.17   | 0.31     | 0 1.8   | N      | 0.1*   | 0.55  | LN    | 13128 |

$inom{N}{\ell}$ represents the normal distribution $N(u, \sigma^2)$.
The triangle represents the lognormal distribution $\ln(44-x) \sim N(u, \sigma^2)$. $AP$ is the average price per ton of Zinc concentrates. The average values of the parameters in Table 14 are shown in Table 15. The limiting conditions for the percentage of various components and the use amount of Zinc concentrates are shown in Table 16.

| Ore | Zn (%) | Pb (%) | SiO2 (%) | AP |
|-----|--------|--------|----------|----|
| bin | E | E | E | RMB/t |
| #1  | 50.275 | 1.006 | 1.302 | 14701 |
| #2  | 45.8  | 1.218 | 1.305 | 13236 |
| #3  | 42.87 | 1.09  | 1.314 | 12157 |
| #4  | 46.12 | 2.93  | 1.32  | 13368 |
| #5  | 45.31 | 1.154 | 4.259 | 13128 |

Table 16. Limitation requirements

| Zn% | SiO2% | Pb% | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $m$ |
|-----|-------|-----|-------|-------|-------|-------|-------|-----|
| Min | 47    | 0   | 0     | 0     | 0     | 0     | 0     | 280 |
| Max | 55    | 3   | 1.8   | 20    | 970   | 840   | 350   | 300 |

Table 17. Different probability levels

| Pb  | 0.6 | 0.8 | 0.8 |
| SiO2| 0.6 | 0.8 | 0.95|
| Colour | Blue | Red | Green |

4.2. Comparison of Pareto fronts with the deterministic optimization method. According to Algorithm 1, when two confidence levels $\delta_1$ and $\delta_2$ are manually set to 0.5, the two objective functions are closer to the actual value. The parameters of the set value of each component probability are shown in Table 17. Different Pareto fronts can be obtained by setting different probability levels. In Fig. 10, blue, red, and green are Pareto fronts corresponding to the probability levels $(0.6, 0.6)$, $(0.8, 0.8)$, and $(0.95, 0.8)$, respectively. The higher the general probability level, the higher is the price under the same zinc probability level. However, there is an overlap as shown by the arrow in Fig. 10. There are two main reasons for it. (1) The uncertainty optimization method can only ensure that each component in the optimization result is greater than or equal to the set probability level. (2) NSGA-II cannot guarantee that every solution in the solution set is optimal. The probability level of lead and silicon was set to $(0.6, 0.6)$, and the actual optimization result was $(0.6, 0.8)$. Therefore, when setting the probability level to be $(0.6, 0.8)$, the optimization results will overlap.
In Fig. 9, black is the Pareto front obtained by the corresponding deterministic optimization method. The price of the deterministic optimization method and chance-constrained programming with probability level of (0.95, 0.8) is almost the same at the same zinc content. In the black and green Pareto fronts in Fig. 10, the confidence level of lead and silicon meeting the requirements is shown in Fig. 11. As the deterministic optimization model doesn’t consider the uncertainty, the probability of some components does not meet the set value, such as the probability of lead content in Fig. 11. Because the silicon content of raw materials is generally low, both methods can meet the requirements. However, the confidence level of each component cannot be known by using the deterministic optimization method. It is impossible to adjust the confidence level of each component according to the practical conditions.

4.3. Comparison of optimization results based on non-truncated and truncated normal distribution. According to the actual production data and Table 12, the working condition score of the baking furnace is 0.834, so the working condition is “good”. The proportion is 0.6. After 10 runs of operation, the average solution of the MOCCP is (11.2, 25.5, 192.8, 9.2, 41). Monte Carlo simulation is used to obtain the results of each component distribution using truncated normal distribution and non-truncated normal distribution, as shown in Fig. 12. The abscissa represents the content of each element, and $\text{truc}_x$ represents the distribution of element $x$ under the truncated normal distribution. The distributions of zinc and
silicon are almost the same under the two distributions because zinc only obeys the truncated normal distribution in #4 and #5 and these two warehouses account for a relatively small proportion. All silicon elements are lognormal distribution. However, due to no truncation, the distribution of lead elements is seriously tailed and even negative. This is notably inconsistent with the actual situation. Therefore, the truncated normal distribution is better than the non-truncated normal distribution.

![Graph](image)

**Figure 12.** Distribution of blending results under different distributions

5. **Conclusion.** As companies continue to prefer a low inventory level of raw materials, high-quality ore inventory will be affected by the supply chain, and therefore, it will be difficult to avoid the phenomenon of insufficient inventory. A multi-objective optimization based on actual working conditions is established to solve this issue. Firstly, the distribution of each component of Zinc concentrates is obtained by statistical methods and truncated according to the actual situation. Secondly, a MiniMin and MaxiMax MOCCP is established. Thirdly, a hybrid intelligent algorithm combining Monte Carlo method, neural network algorithm, and NSGA-II is used to obtain the Pareto front. Then, AHP and expert rules are used to identify the working conditions of the roaster, and the most appropriate solution is obtained from the Pareto front. Finally, the model is verified based on actual data of zinc hydrometallurgy. Compared with traditional method, this method can guarantee the probability of each component and adjust the confident level. The situation of serious tailing or even not conforming to the reality is solved by truncating the distribution.

**Declaration of Competing Interest.** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Acknowledgments.** This work is supported by the National Natural Science Foundation of China under (61988101, 61860206014 and 61973321), the National Key R&D Program of China (2019YFB1704703) and Natural Science Foundation of Hunan Province (2019JJ50823).
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Received May 2021; revised July 2021; early access September 2021.

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