The Renormalized Tensor Interaction in a Nucleus

S.J.Q. Robinson\textsuperscript{1} and L. Zamick\textsuperscript{2}

\textsuperscript{1}Department of Physics, University of Southern Indiana, Evansville, Indiana 47712
\textsuperscript{2}Department of Physics and Astronomy, Rutgers University, New Brunswick, New Jersey 08903, USA

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Abstract

We show several examples were the tensor interaction of the lowest order G matrix in a nucleus is too strong. The examples include the quadrupole moment of $^6\text{Li}$, the isosplitting of the lowest $0^-$ states in $^{16}\text{O}$, the near vanishing Gamow-Teller matrix element in the weak decay of the J=0 T=1 state of $^{14}\text{O}$ to the J=1 T=0 ground state of $^{14}\text{N}$, and the magnitude of the deformation of $^{12}\text{C}$. It would appear that we could get better results by decreasing the tensor interaction strength by about a factor of two. We then examine the simple estimates of Gerry Brown concerning second order tensor effects. We note that for the triplet even channel the combination of first and second order tensor does indeed yield an effective weaker tensor interaction and helps to get better agreement with experiment.

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I. INTRODUCTION

In order to study the effects of the tensor and spin orbit interactions in nuclei we use a simple interaction

\[ V = V_c + x V_{s.o.} + y V_t \]

where \( c \equiv \text{central}, s.o. \equiv \text{spin-orbit} \) and \( t \equiv \text{tensor} \). For \( x=1, y=1 \) we select \( V \) so as to be close to a realistic G matrix like Bonn A. We can turn the spin orbit interaction off (on) by setting \( x=0 \) (\( x=1 \)). Likewise we can turn the tensor interaction off (on) by setting \( y=0 \) (\( y=1 \)). This allows us to study behaviors as a function of \( x \) and/or of \( y \).

This interaction [1] is a modification and correction of a previous interaction [2]. This change does not affect calculations purely in the p shell but there are some changes when core excitations are included. To avoid confusion we call the current interaction \( V(2005) \) and the previous one \( V(1991) \), i.e. after the year of publication. The details and reasons for the changes are given in ref. [1].

Our main thesis will be that the tensor interaction given by a bare G matrix is too strong in the isospin \( T=0 \) channel. By simply making it weaker we can correlate a lot of data and be rid of a lot of anomalies.

We conclude by presenting simple arguments by Gerry Brown to justify using a weaker tensor interaction in the valence space. Care must be taken in that alternate explanations could give the same result as a weaker tensor interaction e.g. a stronger spin-orbit interaction.

The topics we discuss are:

1. The quadrupole moment of the \( J=1^+ \) state of \( ^6\text{Li} \).
2. The \( T=1 \rightarrow T=0 \) energy splitting of \( 0^- \) states in \( ^{16}\text{O} \).
3. The near vanishing of the Gamow-Teller Matrix Element for \( A=14 \) \(^{14}\text{O}(J=0 \ T=1) \rightarrow ^{14}\text{N}(J=1 \ T=0) \).
4. The effect of the tensor interaction on Single Particle Energies in Open Shell Nuclei - \(^{12}\text{C} \) and \(^{14}\text{N} \).

Some of these results have been discussed in previously [3] - [7].
TABLE I: Static quadrupole moments e fm\(^2\) for various model spaces and tensor interaction strengths (y) using the bare electric charges \(e_p = 1.0\) and \(e_n = 0.0\). All calculations are done with the full spin-orbit strength \(x=1\).

| Model Space | y=0   | y=0.5 | y=1.0  |
|-------------|-------|-------|--------|
| 0 ℏω        | +0.1204 | -0.0853 | -0.281 |
| 0.2 ℏω      | +0.1011 | -0.095 | -0.279 |
| 0.2,4 ℏω    | +0.0773 | -0.222 | -0.485 |
| 0.2,4,6 ℏω  | +0.06893 | -0.2788 | -0.5815 |
| 0.2,4,6,8 ℏω | +0.06568 | -0.3052 | -0.6301 |

II. THE QUADRUPOLE MOMENT OF \(^6\text{Li}\) 1\(^+\) STATE

The nucleus \(^6\text{Li}\) is often described in cluster models as a deuteron plus an alpha particle. However, the quadrupole moment of the deuteron is positive \(Q = 0.288\) e fm\(^2\) whereas that of \(^6\text{Li}\) is negative, \(Q = -0.083\) e fm\(^2\).

That the deuteron has a quadrupole moment leads to it having a \(J=1^+\) ground state and hence the isospin must be \(T=0\). Without a tensor interaction the quadrupole moment of the deuteron would be zero.

Expanding on the work in \[^3\] we calculate the value of \(Q\) in various model spaces and for various strengths of the tensor interaction i.e. y as seen in Table I.

We see first of all that we do need a tensor interaction for \(Q\) to be negative. However for the full strength of the tensor interaction the \(Q\) value is too negative, a situation that is not aided by the expansion of the model space. Indeed, as the model space grows, the value of \(Q\) becomes increasingly negative, requiring further quenching of the tensor interaction.

Note that the results with our current V(2005) interaction are qualitatively different than with the previous V(1991). With V(2005) we get the quadrupole moment of the 1\(^+\) state of \(^6\text{Li}\) becoming more negative with increasing \(ℏω\), and getting further away from experiment. With V(1991) the opposite happened. At the zero \(ℏω\) level the values of \(Q\) in ref \[^4\] for 0,0+2 and 0+2+4 \(ℏω\) were respectively -0.360,-0.251 and -0.0085 e fm\(^2\).
TABLE II: The Isosplitting of the $0^-$ states in $^{16}$O for various spin-orbit and tensor interaction strengths.

| Interaction | 1p-1h | 1p-1h+3p-3h | 1p1h+3p-3h+2p2h |
|-------------|-------|-------------|-----------------|
| x=0 y=0    | 0.011 | -0.035      | 0.019           |
| x=0 y=1    | 3.144 | 3.044       | 2.238           |
| x=1 y=0    | 0.046 | -0.036      | 0.156           |
| x=1 y=1    | 2.932 | 2.900       | 1.919           |
| EXPT.       | 1.845 MeV |           |                 |

III. THE $0^-_{T=1}$ TO $0^-_{T=0}$ SPLITTING IN $^{16}$O

In $^{16}$O the lowest $0^-_{T=0}$ state is at an excitation energy of 10.952 MeV while the lowest $0^-_{T=1}$ state is at 12.797 MeV i.e. an energy splitting $\Delta E = 1.845$ MeV [4, 5]. In the next table we present the values of $\Delta E$ in various model spaces.

We see that in the absence of a tensor interaction the splitting is negligibly small as discussed in the work of B. R.Barrett [8]. In the smallest space (1p-1h) the splitting is too large with the full interaction 2.932 MeV. Adding 3p-3h makes very little difference (2.900 MeV). Only when 2p-2h is added to 1p-1h + 3p-3h do we get reasonably close to 1.919 MeV calculated vs. 1.845 MeV. This is a case where the spin-orbit interaction plays a relatively minor role.

Again, if we insist on working in the 1p-1h space we need a weaker tensor interaction to explain the data. We can also give results for the Bonn interactions. In a relativistic formulation of matter in 1p-1h Bonn A with an effective mass $m^* = 930$ gives $\Delta E = 3.08$ MeV in the 1p-1h space but yields 1.87 MeV in the full (1+3) $h\omega$ space (1p-1h + 3p-3h + 2p-2h).

IV. THE NEAR VANISHING OF THE GAMOW-TELLER MATRIX ELEMENT $^{14}$O ($J=0$, $T=1$) → $^{14}$N ($J=1$, $T=0$)

The transition $^{14}$O($J=0$, $T=1$) → $^{14}$N($J=1$, $T=0$) should be an allowed Gamow-Teller decay. But the matrix element for this A(GT) is close to zero. It was shown by Inglis [9]...
TABLE III: The Gamow-Teller matrix elements for the A=14 decay as a function of tensor strength 
y

| y  | A(GT) |
|----|-------|
| 0  | -1.19 |
| 0.25 | -0.95 |
| 0.49 | 0.00  |
| 1.00 | 1.38  |

TABLE IV: Wavefunction of the 1+ state in LS coupling as a function of tensor strength

| y  | C^s_f | C^p_f | C^d_f |
|----|-------|-------|-------|
| 0  | -0.47 | 0.30  | 0.83  |
| 0.25 | -0.36 | 0.30  | 0.88  |
| 0.49 | 0.09  | 0.22  | 0.97  |
| 1  | 0.68  | 0.03  | 0.74  |

that this was not possible for a 2 hole configuration p−2 if only a central and spin-orbit interaction were present. Jancovici and Talmi [10] showed early-on that this near vanishing could be explained by the presence of a two-body tensor interaction. The decay of 14O as well as its mirror 14C was extensively studied experimentally by Sherr et.al. [11].

We give the values of A(GT) for x=1 as a function of y in III

We see that we get a vanishing of A(GT) for y = 0.49, about half of the tensor strength needed to fit the bare G matrix like Bonn A. This fits in with what occurs for other cases like Jπ = 0− isosplitting in 16O.

The 1+ wave function changes drastically as we increase y as can be seen in Table IV
We see that as we go from y=0 to y=0.49 the 3S amplitude drops noticeably and the 3D amplitude increases. Of course a pure 3D state cannot decay to 3S or 1P, which are the only two components of the J=0 ground state of 14C in the p−2 model space.

In fairness, there is another way to get a vanishing GT. For y=1, we can make the spin-orbit interaction stronger. For y=1 x=1.44 we get a vanishing of A(GT). Hence if we limit ourselves to this example alone we cannot conclude that we need a weaker tensor interaction.
to fit the data. However, in other cases e.g. $^6$Li quadruple moment and the $0^-$ splitting in $^{16}$O, one needs an effective weaker tensor interaction and simply strengthening the spin-orbit does not resolve the issues in those nuclei.

In using Kuo’s realistic matrix elements but with a 30 percent increase in the one body spin orbit splitting, Zamick managed to get a vanishing B(GT) \[12\]. Recently a Stony Brook - Idaho collaboration focused on the spin-orbit interaction for the near vanishing of B(GT) for A=14 \[13\] using arguments of Brown-Rho scaling \[14\].

Since in the A=14 problem we get entangled in two different physics mechanisms, namely the weakening of the tensor interaction and the strengthening of the spin-orbit interaction, it is vital to look at other examples where only one of the two effects is important, as an example the $0^-$ isospin splitting where only the tensor interaction is important.

There are theories in which the spin-orbit interaction becomes larger e.g. in Dirac Phenomenology with Dirac Effective Mass Ratio $m^*/m$ less than one. The spin-orbit splitting is proportional to $m/m^*$.

V. THE EFFECT OF THE TENSOR INTERACTION ON SINGLE PARTICLE ENERGIES IN OPEN SHELL NUCLEI - $^{12}$C

We note that for a closed major shell like $^4$He or $^{16}$O the tensor interaction does not contribute to the single particle splitting in lowest order. When ground state correlations are allowed as in ref \[1\] there are some contributions but they are small. However, for an open shell nucleus like $^{12}$C in lowest order we will see otherwise.

Consider the splitting $\epsilon_{p_{1/2}} - \epsilon_{p_{3/2}}$ for various strengths of the spin-orbit interaction (x) and the tensor interaction (y).

\begin{tabular}{|c|c|}
\hline
$^4$He & $\epsilon_{p_{1/2}} - \epsilon_{p_{3/2}}$ \\
\hline
xy & \\
00 & 0.000 \\
10 & 3.38 \\
01 & 0 \\
11 & 3.38 \\
\hline
\end{tabular}
Note that for $^4\text{He}$ and $^{16}\text{O}$ we get positive splitting, 3.38 MeV and 5.00 MeV respectively but for $^{12}\text{C}$ the value is negative -0.25 MeV. As seen in the table the tensor interaction gives no contribution to the splitting in $^4\text{He}$ or $^{16}\text{O}$ but it gives a large negative contribution for $^{12}\text{C}$. In this nucleus the spin-orbit and tensor interactions act in opposite ways. That the tensor interaction acts like an opposite sign spin-orbit force was suggested first by Wong and Scheerbaum.

How does all this manifest itself? One can look at the excitation energies of $1^+ \ T=0$ and $1^+ \ T=1$ states in $^{12}\text{C}$. We will show that energy considerations alone are misleading. In a 1p-1h calculation relative to a closed $p_{3/2}$ shell it is not surprising that the excitation energies of these states is proportional to in the spin-orbit strength. Note however that for $x=0$ the two states above are at negative energies (i.e. below the ground state) -4 MeV for the $T=0$ state and -1 MeV for the $T=1$ state. This is due to the tensor interaction which effectively acts as a “negative” spin-orbit interaction. But in full p shell calculations the energies of these states is relatively flat as a function of the spin-orbit strength – not at all linear – from $x=0$ to about $x=2$.

This means that energy considerations are of no value. However the transition rates from $J=0$ to $J=1^+$ are dependent on the spin-orbit and tensor interactions. The $J=1^+ \ T=0$ rate is severely altered by weak isospin admixtures.

Note that for $x=0$, $y=0$ we have the SU(4) limit for which the spin part of the M1 amplitude vanishes. We see that indeed $A(M1)$ increases as $x$ increases. However $A(M1)$ decreases as $y$ increases supporting the fact that the tensor interaction acts like an “anti-spin
TABLE V: The M1 matrix element for the excitation of a $J=1^+$ $T=1$ state in $^{12}$C for various $x$ and $y$’s

| $y=1$ | $x$ | $A(M1)$ | $x=1$ | $y$ | $A(M1)$ |
|-------|-----|---------|-------|-----|---------|
| 0.0   | 0.03| 0       | 0.5   | 0.57| 0.5     |
| 1.0   | 0.94| 1.0     | 1.5   | 1.37| 1.5     |
| 2.0   | 1.86| 2.0     | 0.0   | 0.03| 0       |
| 0.5   | 0.57| 0.5     | 1.0   | 0.94| 0.94    |
| 1.0   | 0.94| 1.0     | 1.5   | 0.81| 0.81    |
| 2.0   | 0.79| 2.0     | 0.0   | 0.03| 0       |

orbit force”.

In the last few years increasing interest has developed in the topic of the effects of the tensor interaction in open shell nuclei by Otsuka et. al. [17, 18, 19]. In [19] they also note that a weaker tensor interaction is needed in the $T=0$ channel, but not $T=1$. See also the work of Brink and Stancu [20].

VI. SECOND ORDER TENSOR CONTRIBUTIONS

Following arguments of Gerry Brown [21] we make simple evaluations of second order tensor interaction contributions to the effective central and tensor interactions in a nucleus.

We use the Hamada-Johnson interaction [22] which is here given:

$$V_C = 0.08 \frac{1}{3} \mu (\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2)Y(x)(1 + a_C Y(x) + b_C Y^2(x))$$ (1)

$$V_T = 0.08 \frac{1}{3} \mu (\tau_1 \cdot \tau_2)Z(x)[1 + a_T Y(x) + b_T Y^2(x)]$$ (2)

Here the unitless parameter $x$ is $x = \mu r$ with $\mu^{-1} = 1.415$fm and in terms of energy $\mu=139.4$ MeV

Where

$$Y(x) = \frac{e^{-x}}{x}$$ (3)

and

$$Z(x) = (1 + \frac{3}{x} \frac{3}{x^2})Y(x)$$ (4)

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As we are interested only in the $S=1\ T=0$ case, $\tau \cdot \tau$ will be -3 and $\sigma \cdot \sigma$ will be 1. The coefficients are given in Table VI.

The second order tensor effects as given by Gerry Brown are

$$\delta(V_C)_{\text{eff}} = -\frac{8V^2(r)}{e_{\text{eff}}}$$

and

$$\delta(V_T)_{\text{eff}} = \frac{2S_{12}V^2(r)}{e_{\text{eff}}}$$

where $e_{\text{eff}}$ is an Energy parameter ranging from 222 to 264 MeV depending on the matrix elements under consideration. Here we choose $e_{\text{eff}} = 250$ MeV for simplicity.

The second order contribution to the central interaction is negative definite and for the tensor it is positive definite. Hence whether there is destructive or constructive interference depends on the sign of the first order term as given above.

We start from a Moszkowski-Scott cutoff radius of 1 fermi corresponding to $x$ of 0.7. They argued that the part of the attraction up to the cut-off radius of 1 fermi cancels out the short-range repulsion whose range is about 0.4 fermi. [23]

The Moszkowski-Scott method plays an important role in the justification of the Vlow k method of the Stony Brook Group. [24].

In figures 1 to 4 we look at the cases of the $V_{\text{central}}$ and $V_{\text{tensor}}$ for both even and odd values of $L$.

The first order central even is negative so adding the second order tensor contributions makes it more negative. This is reasonable from the existence of the $T=0$ even channel bound state for two nucleons, namely the deuteron. The bare central interaction in this channel is not deep enough to support a bound state so the tensor interaction has to contribute. We can see this in Figure 1.

The first order tensor even interaction is negative (when the -3 factor is included) so the combination of first and second order terms must be less negative or weaker. The sign
of the first order tensor interaction is determined phenomenologically by the positive sign of the quadrupole moment of the deuteron. Also the sign is consistent with the one pion exchange potential. This supports all the conclusions of the previous sections where we see repeatedly that the bare G matrix tensor interaction is too strong in the T=0 channel and needs weakening.

For the odd channels we have first in Figure 3 the central odd potential. Here the attractive contribution of the second order term pulls an initially repulsive first order term down sufficiently so that it is slightly attractive. For the tensor odd interaction in 4, the inclusion of the second order term again makes the total tensor portion less attractive.

The topic of the weakening of the tensor interaction is very relevant to the problem of nuclear pairing in the T=0 channel [25] especially in regarding the size of the pairing gap.

We emphasize that the main point of this work is to show that there are clear experimental signatures that require that in the spin triplet channel renormalization relative to a bare G matrix are required. This is especially true for even L states where not only is the effective central part of the interaction made deeper but also the effective tensor interaction is weakened (screening effects). Relative to the use of only a first order tensor interaction, the combined first and second order tensor interaction helps to explain the smaller energy splitting of T=1 and T=0 0− states in 16O and the vanishing of the Gamow-Teller matrix element in the 14C beta decay. Also the anti-spin orbit effects in open shell nuclei like 12C are reduced although they are still substantial. We still have a problem with 6Li. Although we have shown that one needs the tensor interaction to get a negative quadrupole moment we get it to be increasingly negative with increasing model space.

In closing we note that the shell model works very well in the p shell as noted by the many works of Cohen and Kurath [26]. One purpose here is to understand why it works so well. We see that although the explicit configurations involving higher shells are not present in most calculations, their implicit presence is of crucial importance for the success of the model. We have adopted a low-tech approach to illustrate this point. For more trustworthy quantitative results, the high powered ab initio shell model methods or other equivalent methods need to be employed. Nevertheless we feel that the more qualitative methods used here are of considerable value in providing insight into the physics behind
these more complex approaches.

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FIG. 1: The central force for even values of $L$ - the dashed line is without the second order correction, the solid line is with that included.

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FIG. 2: The tensor force for even values of $L$ - the dashed line is without the second order correction, the solid line is with that included.

FIG. 3: The central force for odd values of $L$ - the dashed line is without the second order corrections, the solid line is with it included.
FIG. 4: The tensor force for odd values of L - the dashed line is without the second order correction, the solid line is with that included.