Effect of curved rigid surface on the collapsing cavitating bubble in cryogenic environment

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Abstract. The high impact erosive pressure resulting from collapsing cavitating bubble has advantages in stone fragmentation and shock wave lithotripsy. They carry damage potential also and can erode the curved hydrofoil of any turbo-machinery, and, still somewhat obscure in cryogenic liquids. Once the material loses its surface smoothness, the flow-field surrounding the collapsing bubbles is affected by the newly formed irregular surface geometry. In this way, the bubble collapse is significantly influenced by the curvature of the rigid boundary. In this work, a collapsing cavitating bubble near a curved rigid surface dipped in cryogenic fluid has been investigated numerically to illustrate the effect of different surface configurations (i.e. Convex i.e. \( \xi < 1 \) and Flat \( \xi = 1 \) surface) using volume-of-fluid (VOF) method in a compressible framework for different standoff distance (\( \gamma_s \)). TAIT equation for the surrounding liquid is used to model shock perturbations during the collapse, whereas ideal gas equation is used for the modeling of compressibility of vapor (or gas) bubble. Here, different dynamical features such as jet formation, bubble-shape evolution (necking and splitting) and vortex ring motion (free and wall vortex) is shown for collapsing cavitating bubble in cryogenic environment and compared with the room-temperature fluid combination i.e. water-air for flat and convex wall configurations for different stand-off distance.

1. Introduction
In reality, bubbles can only be generated either by rupturing the liquid (cavitation) realized by high energy deposit (laser experiments) or tearing (tension) liquid aside. It frequently occurs in a wide variety of cryogenic systems since they operate closer to the critical temperature of the working fluid [1]. The violent collapse of a cavitating bubble driven by anisotropic pressure field created by the nearby rigid surfaces can trigger different dynamic properties such as high-speed jets [2, 3], shock waves during early growth and rebound stage [4, 5, 6], and even light [7, 8]. Such high intensity pressure waves or the impact of the high-speed liquid jets can be attributed to the high local stresses caused by collapsing cavitating bubbles which can even erode stainless steel [9].

These erosive effects of collapsing bubbles can be detrimental for the fluid-handling equipments [10, 11]. Nevertheless, these can be beneficial during ultrasonic cleaning [12], stone fragmentation in shock wave lithotripsy [13, 14], sonoporation [15] etc. However, such dynamic of collapsing bubbles and the mechanism behind their erosive effect is not yet explored in cryogenic
fluids. Non-flat boundaries are often encountered in bubble applications. Cavitating bubbles and their erosive potential are strongly affected by the curvature of nearby surface. It is necessary to investigate the effect of curved surfaces on the collapsing bubble and emanating high-speed microjets. Here, an attempt has been made to elucidate the effect of curved rigid surface on the collapsing cavitating bubble dynamics and results are compared with the flat rigid surface configuration.

2. Problem description and objective
The collapse of the cavitating bubble is a very complex sequence of dynamically changing interfacial structures. Moreover, it is challenging to unravel the physical aspect collapsing cavitating bubble in the cryogenic environment by experimental techniques due to difficulties concomitant with storage and safety. Study of individual collapsing bubbles is still a cornerstone to understanding the erosive damage process.

Therefore, in this work a strongly collapsing inertial bubble near different type of rigid surface configurations is chosen to study the dynamics of collapsing bubble in uniform quiescent two-dimensional liquid domain as depicted in Fig. 1a and Fig. 1 b using high fidelity direct numerical simulations for water and nitrogen (non-cryogenic and cryogenic, respectively) as working fluids. An attempt has been made to understand the re-entrant liquid jets, quantify their jet and impact velocities and discuss energy concentration based on the magnitude of the pressure waves e.g. implosion shock during the collapse. Here, subsequent pressure loadings at the bubble centre and at the wall are also determined. For cryogenic configuration, \( R_{\text{Max}} \) is kept same to ensure comparison. Flow dynamics are compared with respect to microjet velocity, microjet impact pressure and vortex ring dynamics near different flat and convex wall configurations for \( 0.17 < \gamma_s \leq 1.6 \).
3. Numerical methodology

In this work, a single fluid homogeneous mixture model based VOF interface capturing method is used in a compressible framework of Navier-Stokes equation as discussed in section 3.1. The heat transfer and phase change over the bubble interface are neglected due to its rapid growth. We have used TAIT equation for the surrounding liquid to model shock perturbations during the early growth and collapse stage. Gas contained inside the bubble obeys adiabatic compression and expansion based on ideal gas law. The pressure in the interior domain is assumed initially to be homogeneous in space. Outer boundaries except bottom solid boundary of the computational domain are approximated as non-reflecting and has been pushed sufficiently far away ($z_{max} \approx 80R_{max}$) to avoid the influence of reflected pressure wave on the dynamics of collapsing bubble (see Fig. 1a and 1b). Gravity can be neglected for the small bubbles. Viscosity ($\mu$) and surface tension ($\sigma$) effects of the both fluids is included as it is important for intermediate sized bubbles [16, 10].

3.1. Governing equations

Following are the governing equations:

a) Continuity equation: The overall continuity in homogeneous mixture modeling approach is obtained by summing the phase continuity for each phase:

$$\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho u) = 0,$$  \hspace{1cm} \text{(1)}

b) Momentum equation:

$$\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho u \otimes u) = -\nabla p + \nabla \cdot \tau + f_{ST},$$  \hspace{1cm} \text{(2)}

where $\rho$ is mixture density i.e. $\rho = \sum_{i=l,g} \rho_i \alpha_i$, $\mu$ is mixture viscosity i.e. $\mu = \sum_{i=l,g} \mu_i \alpha_i$ and $\tau$ is stress tensor i.e. $\tau = \mu \left[ \nabla u + (\nabla u)^T - \frac{2}{3} (\nabla \cdot u) I \right]$. Here surface tension forces $f_{ST} = \int_S \sigma \kappa n \delta (x - x') dS'$ are modeled using the continuous-surface-force method [17]. In order to obtain the thermodynamic closure of the equations of motion, following equations of state used for the surrounding liquid and gas bubble:

c) Tait equation:

$$p(\rho) = (p_{\infty} + B) \left( \frac{\rho}{\rho_{\infty}} \right)^{\eta_T} - B$$  \hspace{1cm} \text{(3)}

d) Ideal gas equation

$$p(\rho) = p_{ref} \left( \frac{\rho}{\rho_{ref}} \right)^{\kappa}$$  \hspace{1cm} \text{(4)}

where $\rho_{ref} = 998.56$ km$^{-3}$ (water), $\rho_{ref}=808.1$ km$^{-3}$ (LN2) $\rho_{ref}=1.2$ km$^{-3}$ (gas) $p_{\infty} = 101325$ Pa(water) and $p_{\infty} = 296000$ Pa(LN2) and different thermophysical properties of fluids (B, $\eta_T$ and $\kappa$) are defined in Table 2.

The implicit Euler scheme has been employed for discretizing transient terms. To avoid the interface distortion, maximum Courant number is set as 0.2, and to minimize the effect of the numerical diffusion, second order upwind schemes has been selected for the discretization of the convective terms [18]. An adjustable time step based on the Courant number has been selected to yield an accurate temporal discretization.
3.2. Mesh statistics
A non-uniform grid spacing with fine resolution near the bubble vicinity is chosen in fluid domain that completely covers the bubble and extends down to the wall as shown in Fig. 2a and Fig. 2b. Typical meshing parameters are enlisted in Table 1.

| Parameters                  | Flat wall | Convex wall |
|-----------------------------|-----------|-------------|
| Nodes                       | 1577600   | 560000      |
| Skewness (MAX)              | 6.62e-12  | 0.355       |
| Aspect ratio (MAX)          | 25        | 11.68       |

Figure 2. Grid schematic: Non-uniform grid spacing with fine resolution in the vicinity of collapsing bubble for all the a) Flat wall and b) Convex wall cases, detailed mesh statistics are given below in Table 1.

4. Results and discussions
4.1. Bubble motion near boundaries and formation of a liquid jet
It has been noticed that the bubble period is shorter near convex surfaces ($\xi < 1$) as compared to collapse near flat surface as shown in Fig. 3. It may be interesting to explore the prolongation coefficient ($\mu_p$) for $\mu_p > 1$ as it has been observed that the simulation points deviates from the relation $\gamma_p = 1 + 0.4065\mu_p$ for higher value of $\mu_p$, as no experimental data is available.

| Properties                  | Air       | Water    | GN2      | LN2      |
|-----------------------------|-----------|----------|----------|----------|
| Adiabatic index, $\kappa$   | 1.4       | 1.4      |
| Surface tension, $\sigma$ [Nm$^{-1}$] | 0.0725 | 6.5582e-3 |
| B [MPa]                     | -         | 304.6    | -        | 37       |
| $\eta_T$                    | -         | 7.15     | -        | 18.9     |
0.003 0.01 0.1 1
0.5
1.0
1.5
2.0
2.5
3.0
Non-dimensionalized collapse time, \( \gamma_P \)

Prolongation coefficient, \( \mu_p \)

\[
\mu_p = 1 + 0.4065 \mu_p \quad \text{(Tomita, 2002)}
\]

- Flat wall (Tomita, 2002)
- Convex wall (Tomita, 2002)
- Sphere (\( a^* = 1.1364 \), Tomita, 2002)
- Flat wall (Theo.)
- Flat wall (Numerical-Water)
- Flat wall (Numerical-Cryogenic)
- Convex wall (Theo. with \( a^* = 2.63 \))
- Convex wall (Numerical-Water)
- Convex wall (Numerical-Cryogenic)

**Figure 3.** Effect of different surface configuration on the collapsing bubble

\[
\mu_p = \frac{a^*}{\gamma_s(2a^* + \gamma_s)} - \frac{1}{a^*} \ln \left\{ \frac{(a^* + \gamma_s)^2}{\gamma_s(2a^* + \gamma_s)} \right\} = \frac{\xi^2}{\gamma_s(1 + \xi)}
\]  

(5)

here \( \mu_p \) is prolongation coefficient affected only by the surface geometry, \( \xi \) is curvature coefficient, \( a^* \) is dimensionless convex surface radius, \( \gamma_s \) dimensionless stand-off distance from the surface to bubble centre. Note that the value of the surface parameter \( \xi \) varies with the value \( \gamma_s \) even when we employ the same boundary [19].

### 4.2. Effect of stand-off distance

**Figure 4.** a) Effect of stand-off distance on the microjet speed in vertical direction at the bubble centre and b) “Bubble necking and splitting” at \( \gamma_s = 0.17 \) and \( \xi = 0.78 \)

Effect of stand-off distance i.e. normalized distance between the bubble centre to the surface is shown in Fig 4.a. These results resembles the trends similar to as reported by Tomita et
al. [20]. Discrepancy in the magnitude might be subject to the nature of virtual observations followed in experiments, initial bubble energy etc. It has been observed that for same $\gamma_s$ value the maximum jet velocity at the bubble centre is less ($1/2 \sim 1/3$ of water-air) for the LN2-GN2 as compared to water-air combination. It has been also noticed in Fig. 4.b that for $\gamma_s = 0.17$, the lateral parts of the bubble collapse faster than the far end. This results in necking and splitting of the bubble similar to the experimental observation of [21]. One part repels from the convex surface during its subsequent expansion, whereas the other part develops a fast re-entrant jet from where the split occurred, which is directed towards the convex surface.

4.3. Dynamics of collapsing bubble

![Dynamics of collapsing bubble](image)

Figure 5. Dynamics of collapsing bubble for a) Flat wall i.e. $\gamma_s = 0.94$ $\xi = 1$ and b) Convex wall i.e. $\gamma_s = 0.65$ $\xi = 0.54$ for cryogenic fluid combination

Bubble shape evolution for flat and convex wall configurations is shown in Fig. 5a and Fig. 5b for the LN2-GN2 combination, respectively. In both the cases bubble initially grows spherically, and reaches its maximum size ($R_{max} = 3.1 mm$) at $\approx 300 \mu s$ simultaneously. During the collapse bubble acquires a elongated cone shape owing the presence of anisotropy in the flow field, followed by a high speed jet ($34.67 ms^{-1}$) for the convex surface as compared to flat wall case ($27.5 ms^{-1}$) as shown in Fig. 6c). These high speed jets in cryogenic environment can be useful for the fragmentation and generation of nanoparticle synthesis etc. Splashes after the jet impingement are also observed as reported by Cui et al. [21]. It can be also noticed from the figure rebound after the jet impingement is much faster and prominent in case of convex surface. An annular liquid nano jet in the torus bubble has been also noticed at 560 $\mu s$ (see Fig. 5a) near flat wall similar to Lechner et al [22].

Fig. 6 represents the comparative time history of normalized pressure and velocity for $\gamma_s = 0.94$ $\xi = 1$ and $\gamma_s = 0.65$ $\xi = 0.54$. It has been observed in Fig. 6a and Fig. 6b that the first collapse is accelerated near convex surface. Fig. 6b shows the pressure pulses during the early growth and at the first collapse. It has been also observed that though the velocity of jet is significant at the bubble centre (see Fig. 6c) but the velocity of jet impact (see
Figure 6. Time history of the normalized pressure variation a) at the centre i.e. \( \frac{p_{\text{centre}}}{p_\infty} \) b) at the wall i.e. \( \frac{p_{\text{wall}}}{p_\infty} \) and normalized maximum velocity of c) jet at the centre i.e. \( v_{\text{jet,centre}} \left( \frac{p_\infty - p}{p_\infty} \right)^{0.5} \) and d) impact at the wall i.e. \( v_{\text{jet,wall}} \left( \frac{p_\infty - p}{p_\infty} \right)^{0.5} \), where \( t/t_c \) is the normalized time. Fig. 6d) is very low for the higher \( \gamma_s \) due to the viscous dissipation of kinetic energy of the jet while crossing the fluid layers between bubble and the surface.

4.4. Vortex ring motion

Figure 7. Vortex ring motion of a collapsing cavitating bubble. Note Vorticity and Average velocity color legends are adjusted for the comparison.

Generally the vorticity is introduced by the collapsing bubble into the liquid. These vortex
motion associated with the formation of a vortex ring prevails longer than the bubble itself. In Fig. 7 vortex ring motion is analysed for different values of $\gamma_s$ for flat and convex surfaces. It has been elucidated that no vortex ring seems to be created as of no directed motion of the bubble fragments is noticed for $\gamma_s = 0$. For both flat and curved rigid walls for the cryogenic fluids, as reported by [23] for water experiments. Free vortex ring occurs at the centre and stagnates the axial jet for low $\gamma_s = 0.48$ and $\gamma_s = 0.65$ for the flat and convex ($\xi = 0.54$) walls, respectively. A wall vortex traverses towards the wall with opposite sign of circulation compared to the free vortex. These wall vortices have been observed for higher $\gamma_s$, where the jet pierces the bubble and impacts on the wall.

5. Conclusions
This work is an attempt to determine the primary reason of erosive strength of collapsing bubble in cryogenic environment for different surface configurations. It is observed that these cryogenic jets are faster near the convex surfaces as compared to the flat surface and can be used for particle fragmentation and able to break coal effectively as observed by [24]. Vortex ring motion is also studied. It is observed that for $\gamma_s > 0.94$ for the convex surfaces gives wall vortex with $\omega_z = \pm 1 e + 5 s^{-1}$ which can results in mechanical shearing of the nearby rigid surfaces.

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