Thermodynamics of absolute stiff matter

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Abstract

The pressure, particle number density and heat capacity of 'absolute stiff' matter \((P = E)\) at finite temperature are obtained for fermions and bosons. The behavior of 'absolute stiff' medium depends on the characteristic temperature \(T_c = \frac{6\pi^2 n}{\gamma m^2}\), and low temperature expansion \((T \ll T_c)\) is considered. We also find that the equation of state \(P = wE\) at arbitrary constant \(w\) can be modeled by an ideal gas of quasi-particles with the energy spectrum \(\varepsilon_p = a p^w q \) in \(q\)-dimensional space.

1 Introduction

The 'absolute stiff' matter \((P = E)\) can be a Fermi or Bose gas of particles with the energy spectrum \(\varepsilon_p \sim p^q\) in \(q\)-dimensional space. We obtain its pressure, particle number density and heat capacity at finite temperature. The behavior of 'absolute stiff' medium is determined by characteristic temperature \(T_c \sim n\). At low temperature the heat capacity obeys the linear law \(C_V \sim T\) for both fermionic and bosonic matter, the pressure of 'absolute stiff' fermions \(P \sim n^2 + O(T^2)\), while the 'absolute stiff' Bose gas never reveals Bose-Einstein condensation.

The equation of state (EOS) is a fundamental characteristic of material. It is a functional link \(P(E)\) between the pressure \(P\) and the energy density \(E\). Its knowledge allows to predict the behavior of matter and calculate
parameters of astrophysical object that contain this matter. The EOS can be
given in the form
\[ P = wE \]  
where \( w \) is dimensionless parameter that, in general, depends on \( E \). Particularly, the EOS of ideal gas of non-relativistic particles has constant \( w = \frac{2}{3} \). The EOS with \( w = \frac{1}{3} \) describes radiation, the EOS of dust has \( w = 0 \), while tachyon matter admits \( P > E \) \[1, 2\]. One of the most exotic examples
\[ P = E \]  
ocorresponds to the so-called 'absolute stiff' matter, that may appear in various problems of astrophysics. Particularly, it is often considered in the interiors of neutron stars \[3, 5\] and problems of cosmology \[6, 7\]. However, it is still uncertain what physical particles can form this material. It is clear
that free particles with the energy spectrum
\[ \varepsilon_p = \sqrt{p^2 + m^2} \]  
cannot constitute 'absolute stiff' matter, and only interaction between nucleons results in the EOS very close to \[2\].

The estimation of interaction is the main difficulty because it requires solution of the quantum many-body problem. It is often performed in the frames or Hartree-Fock approximation. An alternative method is formulated in the density functional theory \[8\], where a system of interacting particles is modeled by a system of free hypothetical quasi-particles moving in some external field. Although the system of free non-interacting quasi-particles is considered as an ideal gas, their energy spectrum differs from the energy spectrum of free particles \[3\]. The role of interaction can be reflected in the effective mass and effective chemical potential as it is done in the Walecka nuclear model \[9\], but in general the energy spectrum of quasi-particles may differ sufficiently from \[3\]. Of course, it is no more than a model because such hypothetical quasi-particles do not exist in nature, but this model is helpful for calculations of the EOS.

In the present paper we use this model of free quasi-particles for description of exotic forms of matter that appear in astrophysical problems. We consider an ideal gas with EOS \( P = wE \) \[11\] at constant \( w \) and establish the energy spectrum of quasi-particles that can constitute this matter. We also study thermodynamical properties of Fermi and Bose gases with 'absolute stiff' EOS \[2\] and outline constraints applicable to neutron stars.

The standard relativistic units \( c = \hbar = k_B = 1 \) are used in the paper.
2 Thermodynamical functions

Consider an ideal gas of free particles with the single-particle energy spectrum $\varepsilon_p$ at finite temperature $T$ and in a $q$-dimensional space. Let $\mu$ be the chemical potential of this system. The pressure $P$, energy density $E$, entropy density $S$, and the particle number density $n$ are determined by the standard formulas

\begin{align}
P &= -T \ln Z \\
E &= \frac{\gamma}{(2\pi)^q} \int f_p \varepsilon_p d^q p \\
n &= \frac{\gamma}{(2\pi)^q} \int f_p d^q p
\end{align}

where

$$
\ln Z = \pm \frac{\gamma}{(2\pi)^q} \int \ln \{1 \pm \exp[(\varepsilon_p - \mu)/T]\} d^q p
$$

is the statistical sum,

$$
f_p = \frac{1}{\exp[(\varepsilon_p - \mu)/T] \pm 1}
$$

is the distribution function, and the sign "+" or "−" corresponds to fermions and bosons. The volume of $q$-dimensional hypersphere is defined as

$$
d^q p = \frac{q \pi^{q/2}}{\Gamma \left(\frac{q}{2} + 1\right)} p^{q-1} dp
$$

Partial integration of (7) and its substitution in (4) yields

$$
P = \frac{\gamma}{(2\pi)^q} \frac{\pi^{q/2}}{\Gamma \left(\frac{q}{2} + 1\right)} \int f_p \frac{\partial \varepsilon_p}{\partial p} p^q dp
$$

For example, in 3 dimensions

$$
d^3 p = 4\pi p^2 dp
$$

and

$$
P = \frac{\gamma}{6\pi^2} \int f_p \frac{\partial \varepsilon_p}{\partial p} p^3 dp
$$
Let us imagine that the medium with EOS $P = wE$ is an ideal gas of free quasi-particles with the energy spectrum $\varepsilon_p$. From (1), (5) and (10) we get equation:

$$P - \xi E = \frac{\gamma}{(2\pi)^q \Gamma \left(\frac{q}{2} + 1\right)} \int f_p \left( \frac{p \, d\varepsilon_p}{q \, dp} - w\varepsilon_p \right) p^{q-1} dp = 0$$

(13)

whose solution is

$$\varepsilon_p = ap^{wq}$$

(14)

where $a$ is an arbitrary constant.

It differs from the standard single-particle energy spectrum of free particles (3) and the objects with the energy spectrum (14) should be referred as quasi-particles or excitations. Particularly, the energy spectrum in the form

$$\varepsilon_p = ap^q$$

(15)

belongs to the 'absolute stiff' matter (2), while the exotic matter with $P = -E$ is composed of quasi-particles with the energy spectrum

$$\varepsilon_p = \frac{a}{p^q}$$

(16)

The dust matter with $P = 0$ corresponds to $\varepsilon_p = a = \text{const}$, and massless particles with the energy spectrum

$$\varepsilon_p = cp$$

(17)

always have the same EOS

$$P = \frac{E}{q}$$

(18)

The energy spectrum of 'absolute stiff' matter (15) in 3-dimensional space has the form

$$\varepsilon_p = \frac{p^3}{m^2}$$

(19)

while in 2D space it is

$$\varepsilon_p = \frac{p^2}{2m}$$

(20)

and in 1D space it is

$$\varepsilon_p = cp$$

(21)
where constant $m$ is the mass parameter. Formula (20) implies that the ordinary particles in a thin film do form an ’absolute stiff’ matter (2), the fact already known in statistical mechanics of non-relativistic gases in two dimensions [11, 12]. Phonon-like excitations (21) in a thin channel form an "absolute stiff" matter.

Substituting the energy spectrum of 'absolute stiff' material [15] in (6) and (5), we have

$$n = \Sigma_q \frac{T}{a} \int_0^\infty \frac{dx}{\exp(x - \mu/T) + \sigma} \quad (22)$$

$$E = P = \Sigma_q \frac{T^2}{a} \int_0^\infty \frac{x dx}{\exp(x - \mu/T) + \sigma} \quad (23)$$

where

$$\Sigma (q) = \frac{\gamma }{(2\pi)^q \Gamma (\frac{q}{2} + 1)}$$

$$x = \frac{\varepsilon_p}{T} \quad (24)$$

and $\sigma = \pm 1$ (upper sign for fermions, lower sign for bosons). Hence, we get universal formulas for the particle number density

$$n = \Sigma_q \frac{T}{a \sigma} \ln \left[1 + \sigma \exp \left(\frac{\mu}{T}\right)\right] \quad (26)$$

and the pressure

$$E = P = \Sigma_q \frac{1}{a \sigma} \left\{ \frac{\mu^2}{2} + \left(\frac{\ln^2 \sigma}{2} + \frac{\pi^2}{6}\right) T^2 + T^2 \text{dilog} \left[1 + \frac{1}{\sigma} \exp \left(-\frac{\mu}{T}\right)\right] + \mu T \ln \sigma \right\} \quad (27)$$

where

$$\text{dilog} (z) = \text{Li}_2 (1 - z) = \sum_{k=1}^\infty \frac{(1 - z)^k}{k^2} \quad (28)$$

is a dilogarithm function [13].

At zero temperature $T = 0$ the distribution function [8] of a Fermi gas is replaced by the Heaviside step function

$$f_p = \Theta (p - p_F) = \Theta (\varepsilon - \varepsilon_F) \quad (29)$$
where $p_F$ is the Fermi momentum and $\mu|_{T=0} = \varepsilon_F = a p_F^2$ is the Fermi energy. Then, according to (26) and , we find the particle number density
\[
n = \Sigma_q \frac{\varepsilon_F}{a} = \Sigma_q p_F^2
\]
and the energy density
\[
E = P = \Sigma_q \frac{\varepsilon_F^2}{2a} = \frac{\Sigma_q a p_F^2}{2}
\]
that implies
\[
P = E = \frac{1}{2\Sigma_q} an^2
\]
Particularly, in 3 dimensions it is
\[
P = E = \frac{3\pi^2}{\gamma} an^2
\]
while in 2 dimensions it is
\[
P = E = \frac{2\pi}{\gamma} an^2
\]
The 'absolute stiff'' matter at finite temperature and in 3D space is a substance considered in astrophysical problems \[3, 4, 5, 6, 7\]. Let us consider it in more detail.

3 Fermionic absolute stiff matter

Let us consider an 'absolute stiff' matter \[2\] in 3 dimensional space. As we already have seen, this matter can be modeled by an ideal gas of free quasi-particles with the energy spectrum \[19\]. It can be either a Fermi gas or a Bose gas, and its thermodynamical functions are proportional to thermodynamical functions of nonrelativistic Fermi or Bose gas in two dimensions because the latter has the same 'absolute stiff' EOS $P = E$ \[11, 12\]. However, the energy spectrum of 'absolute stiff' quasi-particles \[19\] differs from the energy spectrum of ordinary nonrelativistic particles \[20\], and we need to obtain all formulas in detail and find exact coefficients.
Choosing the sign ”+” in (8) for fermions, defining

\[ x = \frac{m^4}{\bar{p}^3T} \]  

(35)

and substituting (19) in (6) and (10), we find the particle number density

\[ n = \frac{\gamma m^2 T}{6\pi^2} \int_0^\infty \frac{dx}{\exp(x - \mu/T) + 1} \]  

(36)

and the pressure and energy density

\[ E = P = \frac{\gamma m^2 T^2}{6\pi^2} \int_0^\infty \frac{x dx}{\exp(x - \mu/T) + 1} \]  

(37)

All integrals are calculated in the explicit form or through special functions, and we immediately find the particle number density

\[ n = \frac{\gamma m^2}{6\pi^2} T \ln \left[ 1 + \exp \left( \frac{\mu}{T} \right) \right] \]  

(38)

and the pressure

\[ P = \frac{\gamma m^2}{6\pi^2} \left\{ \frac{1}{2} \mu^2 + \frac{\pi^2}{6} T^2 + T^2 \text{dilog} \left[ 1 + \exp \left( -\frac{\mu}{T} \right) \right] \right\} \]  

(39)

Formula (38) also implies

\[ \mu = T \ln \left[ \exp \left( \frac{6\pi^2 n}{\gamma m^2} \right) - 1 \right] = T \ln \left[ \exp \left( \frac{\varepsilon_F}{T} \right) - 1 \right] \]  

(40)

where

\[ \varepsilon_F = \frac{6\pi^2 n}{\gamma m^2} \]  

(41)

is the Fermi energy.

At low temperature \( T \ll \varepsilon_F \) the chemical potential (40) tends to a constant limit

\[ \mu \simeq \varepsilon_F \]  

(42)
and the pressure (39) is approximated as

\[ P \simeq \frac{\gamma m^2}{6\pi^2} \left( \frac{1}{2} \varepsilon_F^2 + \frac{\pi^2}{6} T^2 \right) \]  

(43)
giving

\[ P = \frac{\gamma m^2}{12\pi^2} \varepsilon_F = \frac{\gamma}{12\pi^2} \frac{p_F^6}{m^2} \]  

(44)
at zero temperature, where

\[ p_F = \left( \frac{6\pi^2 n}{\gamma} \right)^{1/3} \]  

(45)
is the Fermi momentum. Hence, we find an important relation

\[ P = \frac{3\pi^2}{\gamma m^2} n^2 \]  

(46)
that coincides with (33) if \( a = 1/m^2 \).

At high temperature \( T \gg \varepsilon_F \) formula (40) yields

\[ \mu \to T \ln \left( \frac{\varepsilon_F}{T} \right) \ll -T \]  

(47)
implying that the chemical potential is definitely negative. Hence, formula (38) also implies

\[ n \simeq \frac{\gamma m^2}{6\pi^2} T \exp \left( \frac{\mu}{T} \right) \]  

(48)
while the pressure (39) tends to asymptotic

\[ P \to \frac{\gamma m^2}{6\pi^2} T^2 \exp \left( \frac{\mu}{T} \right) \]  

(49)
that corresponds to the standard EOS of classical Maxwell-Boltzmann gas

\[ P = nT \]  

(50)
whose particle number density

\[ n = \frac{\gamma m^2}{6\pi^2} T \exp \left( \frac{\mu}{T} \right) \]  

(51)
and pressure
\[ E = P = \frac{\gamma m^2 T^2}{6\pi^2} T^2 \exp \left( \frac{\mu}{T} \right) \]  
(52)
are obtained by formulas (6) and (10) with the distribution function
\[ f_p = \exp \left( \frac{\mu - \varepsilon_p}{T} \right) \]  
(53)

If we consider 'absolute stiff' thermal excitations whose number is not conserved, it is necessary to put the chemical potential equal to zero \( \mu = 0 \). According to (38) and (39), their particle number density and pressure are
\[ n_{\text{exc}} = \frac{\gamma \ln 2}{6\pi^2} m^2 T \]  
(54)
and
\[ P_{\text{exc}} = \frac{\gamma m^2}{72} T^2 \]  
(55)

Now
\[ P_{\text{exc}} = \frac{\pi^4}{2 \ln^2 2} \frac{n_{\text{exc}}^2}{\gamma m^2} \]  
(56)
that differs from the relevant identity for the cold Fermi gas (46), however, proportionality \( P \sim n^2/m^2 \) is the same in both cases.

4 Bosonic absolute stiff matter

Choosing the sign "−" in (8) we obtain the thermodynamical functions of bosonic 'absolute stiff' matter
\[ n = \frac{\gamma m^2 T}{6\pi^2} \int_0^\infty \frac{dx}{\exp (x - \mu/T) - 1} \]  
(57)
\[ E = P = \frac{\gamma m^2 T^2}{6\pi^2} \int_0^\infty \frac{x dx}{\exp (x - \mu/T) - 1} \]  
(58)

All integrals are obtained in the explicit form or through special functions, and we immediately find the particle number density
\[ n = -\frac{\gamma m^2}{6\pi^2} T \ln \left( 1 - \exp \frac{\mu}{T} \right) \]  
(59)
and the pressure
\[ P = \frac{\gamma m^2}{6\pi^2} \left\{ \frac{1}{2} \mu^2 + \frac{\pi^2}{6} T^2 + T^2 \text{dilog} \left[ \exp \left( -\frac{\mu}{T} \right) \right] - \mu T \ln \left[ 1 - \exp \left( \frac{\mu}{T} \right) \right] \right\} \]

where function \( \text{dilog} \left( x \right) \) is defined by (28). Formula (59) allows to determine the chemical potential of the Bose gas
\[ \mu = T \ln \left[ 1 - \exp \left( -\frac{6\pi^2 n}{\gamma m^2 T} \right) \right] = T \ln \left[ 1 - \exp \left( -\frac{T}{T_c} \right) \right] \]

where the characteristic temperature is
\[ T_c = \frac{6\pi^2 n}{\gamma m^2} \]

The chemical potential \(|\mu|\) is growing with the growth of temperature, it is always negative and attains zero \( \mu \rightarrow 0 \) only in the limit \( T \rightarrow 0 \). So, there is no Bose-Einstein condensation of the 'absolute stiff' bosons.

At low temperature \( T \ll T_c \) the chemical potential (61) tends to zero
\[ \mu \simeq -T \exp \left( -\frac{T}{T_c} \right) \rightarrow 0 \]

while the pressure (60) is approximated by formula
\[ P \simeq \frac{\gamma m^2}{36} T^2 \]

It should be noted that the number of particles (59) is divergent
\[ n = \frac{\gamma m^2}{6\pi^2} T \ln \left( \frac{T}{-\mu} \right) \]

and there is no Bose-Einstein condensation because the chemical potential (63) never attains zero level.

At high temperature \( T \gg T_c \) the chemical potential (61) is approximated so
\[ \mu \rightarrow -T \ln \left( \frac{T}{T_c} \right) \ll -T \]
while formula (59) also implies

\[ n \simeq \frac{\gamma m^2}{6\pi^2} T \exp \left( \frac{\mu}{T} \right) \]  

(67)

and the pressure (60) tends to

\[ P \rightarrow \frac{\gamma m^2}{6\pi^2} T^2 \exp \left( \frac{\mu}{T} \right) \]  

(68)

that again gives the EOS of Maxwell-Boltzmann gas \( P = nT \).

5 Anyonic matter and heat capacity

For the general anyon distribution function

\[ f_p = \frac{1}{\exp \left[ (\varepsilon_p - \mu)/T \right] + \sigma} \]  

(69)

with arbitrary \( \sigma \) and ‘absolute stiff’ energy spectrum (19) formulas (6) and (10) yield the particle number density

\[ n = \frac{\gamma m^2}{6\pi^2} \frac{T}{\sigma} \ln \left[ 1 + \sigma \exp \left( \frac{\mu}{T} \right) \right] \]  

(70)

and pressure

\[ E = P = \frac{\gamma m^2}{6\pi^2} \left\{ \frac{\mu^2}{2} + \left( \frac{\ln^2 \sigma}{2} + \frac{\pi^2}{6} \right) T^2 + T^2 \text{dilog} \left[ 1 + \frac{1}{\sigma} \exp \left( -\frac{\mu}{T} \right) \right] + \mu T \ln \sigma \right\} \]  

(71)

that at \( \sigma = \pm 1 \) are reduced to (38)-(39) and (59)-(60), respectively.

The the entropy density \( S \) and the heat capacity \( C_V \) are defined by formulas

\[ S = -\frac{\partial (T \ln Z)}{\partial T} = \frac{\partial P}{\partial T} \quad C_V = T \frac{\partial S}{\partial T} \]  

(72)

Substituting (71) in (72), we get

\[ S = \frac{\gamma m^2}{6\pi^2 \sigma} \left\{ T \left( \ln^2 \sigma + \frac{\pi^2}{3} \right) + 2T \text{dilog} \left[ 1 + \frac{1}{\sigma} \exp \left( -\frac{\mu}{T} \right) \right] - \mu \ln \left[ \frac{1}{\sigma} + \frac{1}{\sigma^2} \exp \left( -\frac{\mu}{T} \right) \right] \right\} \]  

(73)
and
\[ C_V = \frac{\gamma m^2}{6\pi^2\sigma} \left\{ \frac{\pi^2}{3} + 2T \text{dilog} \left[ 1 + \frac{\exp\left(-\frac{\mu}{T}\right)}{1 + \exp\left(\frac{\mu}{T}\right)} \right] - \frac{\mu^2/T}{1 + \exp\left(\frac{\mu}{T}\right)} - 2\mu \ln \left[ 1 + \frac{1}{\sigma} \exp\left(-\frac{\mu}{T}\right) \right] \right\} \] (74)

Particularly, at \( \sigma = 1 \) we have the entropy density of 'absolute stiff' Fermi gas
\[ S_F = \frac{\gamma m^2}{6\pi^2} \left\{ \frac{\pi^2}{3} + 2T \text{dilog} \left[ 1 + \exp\left(-\frac{\mu}{T}\right) \right] - \mu \ln \left[ 1 + \exp\left(-\frac{\mu}{T}\right) \right] \right\} \] (75)

and at \( \sigma = -1 \) we have the entropy density of 'absolute stiff' Bose gas
\[ S_B = \frac{\gamma m^2}{6\pi^2} \left\{ \frac{2\pi^2}{3} T + 2T \text{dilog} \left[ 1 - \exp\left(-\frac{\mu}{T}\right) \right] + \mu \ln \left[ \exp\left(-\frac{\mu}{T}\right) - 1 \right] \right\} \] (76)

Substituting (75) in (72) we find the heat capacity of 'absolute stiff' Fermi gas
\[ C_F = \frac{\gamma m^2}{6\pi^2} \left\{ \frac{\pi^2}{3} + 2T \text{dilog} \left[ 1 + \exp\left(-\frac{\mu}{T}\right) \right] - \mu \ln \left[ 1 + \exp\left(-\frac{\mu}{T}\right) \right] \right\} \] (77)

Substituting (76) in (72), we find the heat capacity of the 'absolute stiff' Bose gas
\[ C_B = \frac{\gamma m^2}{6\pi^2} \left\{ \frac{2\pi^2}{3} T - 2T \text{dilog} \left[ 1 - \exp\left(-\frac{\mu}{T}\right) \right] + 2\mu \ln \left[ 1 - \exp\left(-\frac{\mu}{T}\right) \right] + \frac{\mu^2/T}{1 - \exp\left(\mu/T\right)} \right\} \] (78)

At low temperature (\( T \ll \varepsilon_F \) and \( T \ll T_c \)) the heat capacity of both fermionic and bosonic 'absolute stiff' matter behaves as
\[ C_F \to C_B \to \frac{\gamma m^2}{18} T \] (79)

At high temperature (\( T \gg \varepsilon_F \) and \( T \gg T_c \)) the heat capacity becomes exponentially small
\[ C_F \to C_B \to \frac{\gamma m^2 \mu^2}{6\pi^2 T} \exp\left(\frac{\mu}{T}\right) \] (80)
According to formulas (55) and (72), the heat capacity of fermionic 'absolute stiff' thermal excitations (that have zero chemical potential $\mu = 0$) is always

$$C_{\text{exc}}^F = \frac{\gamma m^2}{36} T$$  \hspace{1cm} (81)

## 6 Conclusion

The particle number density $n$ and the pressure $P$ of 'absolute stiff' matter with the equation of state $P = E$ [2] at finite temperature are given by formulas (39)-(40) and (60)-(61) for fermions and bosons, respectively. There is a critical temperature

$$T_c = \frac{6\pi^2 n}{\gamma m^2}$$ \hspace{1cm} (82)

that characterizes the behavior of Fermi and Bose gases. At low temperature ($T \ll T_c$) the fermionic pressure is approximated by formula (43), while the bosonic pressure is (64). A mixture of fermions and bosons at low temperature will have the pressure

$$P \approx \frac{3\pi^2}{\gamma_f m_F^2} n^2 + \frac{\gamma_f m_F^2 + \gamma_B m_B^2 T^2}{36\pi^2}$$ \hspace{1cm} (83)

At high temperature ($T \gg T_c$) both fermions and bosons behave as ideal Maxwell-Boltzmann gases with $P = nT$, although the constraint (2) remains valid. The heat capacity of 'absolute stiff' Fermi and Bose gases is given by formulas (77)-(78) and at low temperature it is approximated by the same linear asymptotic (79).

The EOS of 'absolute stiff' Fermi gas at zero temperature is characterized by proportionality (46):

$$P \sim n^2$$ \hspace{1cm} (84)

Although this formula is applied only to a system of fermions at zero temperature, it has can be used in various problems concerned with neutron stars. The central density of neutron $n_c$ exceeds several times the normal nuclear density $n_{nm}$, while the regular nuclear EOS can be accurately estimated when the density is not so high. Some researches appeal to the model of 'absolute stiff' matter when $n$ is larger some fiducial density $n_\perp$ around $4n_{nm}$ [4]. So, we can calculate the pressure $P_\perp$ in the frames of regular nuclear EOS.
at zero temperature. Then, substituting $n_c$ and $n_\perp$ in (84) we immediately determine the pressure in the center of the star

$$P_c = P_\perp \frac{n_c^2}{n_\perp^2}$$

(85)

At finite temperature this constraint should be replaced by

$$P_c = P_\perp \frac{n_c^2 + \alpha T_c^2}{n_\perp^2 + \alpha T_\perp^2} \quad \alpha = \frac{\gamma_F m_F^2 (\gamma_F m_F^2 + \gamma_B m_B^2)}{108\pi^4}$$

(86)

that also includes the central temperature $T_c$ and the temperature $T_\perp$ at the boundary between the envelope and the core with ’absolute stiff’ EOS.

Thus, the EOS $P = wE$ [1] with arbitrary constant $w$ can be modeled by a system of free quasi-particles that have energy spectrum [14], without regard which statistics they obey. Particularly, we can consider fermionic or bosonic stars containing an ideal gas of quasi-particles that constitute exotic matter with $P < -E$ as an alternative to the Chaplygin gas [14]. It is the subject for further research.

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