Naked Singularities as Possible Candidates for Gamma-ray Bursters

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Abstract

Naked singularities appear naturally in dynamically evolving solutions of Einstein equations involving gravitational collapse of radiation, dust and perfect fluids, provided the rate of accretion is less than a critical value. We propose that the gamma-ray bursters (GRBs) are examples of these naked singularity solutions. For illustration, we show that according to solutions involving spherically symmetric collapse of pure radiation field, the energy $E_\gamma$ and the observed duration $\Delta t_o$ of a GRB should satisfy, $\frac{E_\gamma}{\Delta t_o} \leq 4.5 \times 10^{58} \ f_\gamma \ \text{erg sec}^{-1}$, $f_\gamma$ being the fraction ($10^{-2}$ to $10^{-3}$) of energy released as gamma rays. All the presently observed GRBs satisfy this condition; those satisfying the condition close to equality must necessarily be of cosmological origin with the red-shift factor $z$ not exceeding $\sim 2 - 10$ depending on exact observed flux.
Gamma-ray bursts (GRBs), both weak and strong, are believed to be isotropically distributed as revealed by PVO and BATSE experiments\textsuperscript{1,2}. These observations strongly argue for their cosmological origin\textsuperscript{3-5}. The bursts have typical flux of $10^{-5}$ to $2 \times 10^{-4}$ erg cm\textsuperscript{-2} with the rise time as low as $10^{-4}$s and the duration of burst from $10^{-2}$s to $10^{3}$s. The origin of GRBs is not yet known. Speculations involve merger of binary neutron stars\textsuperscript{5-6} and capture of neutron stars by black holes\textsuperscript{7}.

In the present letter, we propose that the GRBs could be the naked singularity solutions of Einstein’s equations describing collapse of radiation, dust or matter shells. Recently, a large number of such solutions have been proposed\textsuperscript{8-18}. The nature of such a naked singularity has been analyzed in detail and it has been shown\textsuperscript{8-13} that the space-time curvatures and gravitational tidal forces grow very strongly in the vicinity of these singularities which turn out to be strong curvature singularities in a very powerful sense. Hence, it appears that for several reasonable equations of state satisfying the positivity of energy, a strong curvature naked singularity may be formed in the space-time as a result of the gravitational collapse. During the collapse, such naked singularities could emit powerful bursts of radiation visible to an external observer situated far away from the sight of collapse.

To focus on a particular solution of such kind, we consider the case of gravitational collapse of a spherical shell of radiation at the center of the symmetry. The total mass of the singularity grows from zero to a finite total of $M$ when the final collapsing shell has arrived at the singularity\textsuperscript{14}. A naked singularity forms at the origin $t = 0, r = 0$, which is a two-sphere in a space-time, or a null surface when represented in a Penrose diagram. The gravitational potentials (in the units $c = 1, G = 1$) within the radiation zone are described by a Vaidya metric and are given by,

$$g_{uu} = - \left( 1 - \frac{2m(u)}{r} \right), \quad g_{ur} = 1, \quad g_{\theta\theta} = r^2, \quad g_{\phi\phi} = r^2 \sin^2 \theta \quad (1)$$

where $u$ is the advanced time given by $u = t + r$ and all other metric components vanish. After the completion of the collapse, the solution settles to an external Schwarzschild
geometry with a singularity of mass $M$ at the center. It has been shown\textsuperscript{8,9,16} that the
naked singularity at the center forms independently of the details of exact functional
form of the mass function. For the purpose of illustration, it is convenient to choose a
linear form for the mass function, say, $m(u) = \lambda u$. Here, $\lambda$ is a constant defining the
accretion rate of the collapse at $r = 0$, which in CGS unit is given by,

$$\frac{dm}{dt}|_{r=0} = \frac{\lambda c^3}{G} \sim \Lambda \times 10^{38} \text{gm sec}^{-1}. \quad (2)$$

with $\Lambda = 4\lambda$. It was shown\textsuperscript{11-12} that the occurrence of the naked singularity depends
on the rate of accretion $\Lambda$. For $\Lambda \leq \frac{1}{2}$, the collapse produces naked singularity, but for
$\Lambda > \frac{1}{2}$, black hole solutions are formed. We now assume that gamma ray luminosities
from GRBs should not exceed the rate of collapse of the radiant energy. From the linear
dependence of $m(u)$ on $u$, it then follows that for a naked singularity solution,

$$\frac{E_\gamma}{\Delta t_o} \leq 4.5 \times 10^{58} f_\gamma \text{erg sec}^{-1}. \quad (3)$$

Here, $f_\gamma$ denotes the conversion fraction of the collapsing energy which is received as
the gamma radiation and $\Delta t_o$ is the duration of observation of the burst. From eqn. (3)
one readily observes that a collapse could correspond to conversion of a $20M_\odot$ object in
about a millisecond as measured on earth! On the other hand, from the observational
data\textsuperscript{1}, we require that,

$$\frac{E_\gamma|_{obs}}{4\pi d_L^2} = 10^{-5} \text{ to } 2 \times 10^{-4} \text{ erg cm}^{-2}$$

where, $d_L$ is the luminosity distance of the source, $d_L = 2R_0(1 + z - \sqrt{1 + z})$, with
$R_0 = c/H_0 = 3 \times 10^3 h_{100}^{-1}$ Mpc. Here $h_{100}$ is the Hubble’s constant ($H_0$) in units of 100
km sec$^{-1}$ Mpc$^{-1}$, and $z$ denotes the redshift of the object due to the expansion of the
universe. Thus, $E_\gamma|_{obs} = 4.1 \times 10^{52}$ to $8.1 \times 10^{53}(1 + z - \sqrt{1 + z})^2 h_{100}^{-2}$ erg. If we assume
$f_\gamma = 10^{-3}$ and $\Delta t_o = 10^{-2}$ sec, it is easy to verify that the condition (3) is readily satisfied
for all the observed GRBs unless they are very distant ($z \geq 1.25 - 4.6$). Instead, if we
had chosen $f_\gamma = 10^{-2}$, eqn. (3) would have been satisfied unless $z \geq 3.5 - 13.5$. In
this context, we would like to recall that in the models of GRBs involving merger of
neutron stars it is customary to choose\textsuperscript{1} $f_\gamma = 10^{-2}$ to $10^{-3}$.
In this letter we have demonstrated that the collapse of spheres filled with radiation fluid could produce bursts of energetic radiation. The energy is expected to be in the band \( \sim \alpha m_p c^2 \text{erg} \sim 100 \text{Mev} \), where \( m_p \) is the mass of the proton, and \( \alpha \sim 0.1 \) depends on the red-shift factor and the efficiency of conversion of accretion energy into radiation. The naked singularities could thus be possible basis for the gamma-ray bursters. The existence of a cut-off in the accretion rate \( \lambda \) enabled us to separate the naked singularity solutions from those which produce black holes at cosmological distances. We show that depending upon the exact observed flux and the efficiency, GRBs should not be located beyond \( z \sim 2 - 10 \). This could be a signature of the proposed mechanism. In more realistic collapse of dust\(^{13,15}\), aspherical collisionless gas\(^{18}\), and perfect fluids\(^{10,17}\), one also has similar parameters which produce naked singularity solutions in a certain range of their values. An interesting property of some of these solutions is that through a single collapse, separate singularities might be developed at different times. These solutions could be relevant to explain the repeated bursts\(^1\) which are observed. Some repeaters could also be due to quasi-periodic oscillations induced during the collapse process. In passing, we may remark that the time-variabilities of emitted radiation from such astrophysical systems should not be limited by the light-crossing time of the Schwarzschild radius, as is currently assumed in describing Active Galaxies and Quasars. Detailed behavior of these solutions in the context of observed astrophysical processes, such as line emissions from GRBs, etc. will be dealt with elsewhere.

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