Ion-acoustic waves in non-isothermal electron distribution using particle-in-cell method

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Abstract. We consider the ion-acoustic waves mode in plasma where electrons are consisted of free and trapped by ion-acoustic waves. The electron distribution for free and trapped electrons are the Maxwellian distribution function with the different temperatures. The system is considered as a one dimensional and simulates a collisionless, unmagnetized plasma. The Particle-In-Cell (PIC) method is applied to show the ion-acoustic oscillation as well as two-stream instability of this system.

1. Introduction

Plasmas is the many body system where positive charged particles (ions) are together with negative charged particles (electrons) [1]. The oscillations of these particles due to the electric and magnetic fields are known as the plasma oscillation modes. One of these modes that we are considering is the ion-acoustic waves. We will concentrate on one-dimensional ion-acoustic waves in plasma without the external magnetic field can be described by the governed fluid equations [1],

\[
\frac{d n}{dt} + n \frac{du_x}{dx} = 0, \tag{1}
\]

presents the continuity equation of ions, where \(n\) denotes the ion density, \(u\) is the ion velocity. The subscripts denote derivatives.

\[
u_t + u u_x = -\phi_x, \tag{2}
\]

is known as the equation of motion, in which \(\phi\) is the electric potential. The last equation is known as the Poisson equation,

\[
\frac{d^2 \phi}{dx^2} = n_e - n, \tag{3}
\]

where \(n\) is normalized by the ion number density at equilibrium, and the spatial and temporal coordinates are normalized by Debye length and ion plasma frequency, respectively. In this work, the Particle-in-Cell method [2] (PIC) will be applied to solve these equations. The idea behind this method can be shown in figure 1. We start with the initial values of particle speeds and the electric field and using the Maxwell’s equations to update the new fields as well as the Newton’s second law to update the new particle’s position. The particle velocity and electrostatic potential are normalized by \(c_s\) and \(k_B T/e\), respectively. In this work, we consider
the interaction between ion-acoustic wave and electron, where some electrons will be trapped in the waves while some are not.

2. Ion-acoustic waves with trapped electrons
Schamel [3, 4] proposed the effect of ion-acoustic waves and electrons interaction where some electrons can be trapped in the ion-acoustic waves while another still move freely in the space. This implies that the plasma system consists of two possible types of electrons. However, Schamel [3, 4] consider the Maxwellian Distribution for both free and trapped electrons.

The electron density can be

\[ n_e = e^\phi \text{erfc}(\phi^{1/2}) + |\beta|^{-1/2} \left\{ \frac{e^{3\phi} \text{erf}(\beta \phi^{1/2})}{(2/\pi^{1/2})W((\beta \phi)^{1/2})} \right\} \beta \geq 0, \]

\[ \beta < 0, \]

(4)

where \( W \) is the Dawson integral,

\[ W(x) = e^{-x^2} \int_0^x e^{t^2} dt. \]

3. Particle-In-Cell method
The main idea of this method is the solve particle trajectories of ions and electrons in plasmas [2,5–7]. We will follow the steps inside this book. There are 3 main steps in the method, the first one is to calculate the electric field from the particle density or solving the Maxwell equations,

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}. \]

where \( \rho \) is the ion density surrounded by free and trapped electrons and \( \epsilon_0 \) is the electric permittivity in free space. The next step is to interpolate of these updated fields and space grid. The last step is to calculate the new positions of particles due to the updated fields or solving the Newton’s law with the Lorentz force,

\[ \frac{d\mathbf{v}}{dt} = \frac{q \mathbf{E}}{m}, \]

where \( q \) is the ion charge and \( m \) denotes the ion mass. This can be shown in figure 1.

**Figure 1.** Figure shows the schematic for PIC method.

In this work we applied the central discretizing for the temporal space [2],

\[ \frac{df}{dt} = \frac{f_{n+1/2} - f_{n-1/2}}{\Delta t}. \]
The solution of Poisson equation, we applied the central finite difference to calculate the electric potential,

\[ \frac{dE}{dx} = \frac{\rho(x)}{\epsilon_0}. \]

Consider the relation between the electric field and the electric potential,

\[ E = -\frac{d\phi}{dx}, \]

The Poisson equation can be written as

\[ \frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon_0}. \]

Therefore,

\[ \frac{\phi_{i-1} - 2\phi_i + \phi_{i+1}}{(\Delta x)^2} = \frac{\rho(x_i)}{\epsilon_0}, \]

in which \( \rho(x_i) \) is updated, \( E_i \) running from 0 to \( L/\Delta x \) where \( L \) is the length of the system of \( N \) points, where the periodic boundary conditions are applied to the system. There are reports about the PIC method for studying the ion-acoustic waves in some plasma systems [8–10] but the trapped electron in ion-acoustic waves hasn’t reported with the PIC method yet.

4. Results and discussion

We first determine the ion-acoustic wave. We used the followed parameters to obtain all results, total number of particle is 10000, time step (\( dt \)) is 0.01, \( \beta = 1 \). and the grid space is 0.01. The wave can be found via the energies of ions as shown in figure 2, where the initial positions of ion is chosen as

\[ x_i(t = 0) = x_{i0} + 0.5 \cos(x_{i0}). \]

![Figure 2](image-url)

**Figure 2.** Figure shows energies oscillation of the ion.

This figure combines the total, kinetic as well as potential energies of the ion and can be implied that the particle is oscillating with some period. The next behaviour we will consider the two-stream instability. This phenomenon occurs when there are two counter-streaming plasma
flow in the velocity space. We consider two types of ion with the Maxwellian distribution function,

\[ f(x) = f_0 e^{-(v-v_s)^2/2}, \]

where \( v_s \) is the speed of ion acoustic wave, which we used \( v_s = \pm 5 \) for each particle beam, and \( f_0 \) is 0.5. The phase space for the two-stream instabilities are shown in figure 3.

![Phase space of two-stream instability](image)

**Figure 3.** Figures show phase-space of two-stream instability for \( t = 69 \) (left) and \( t = 168 \) (right).

At \( t = 69 \), particles in each stream are interacted to each other completely (the red particles are almost mixed in the blue particles and vice versa). This situation is known as the two-stream instability. At \( t = 168 \) we observed the characteristic vorticity which is the result of the two-stream instability.

5. Conclusion

We applied the PIC method to investigate the oscillation mode of the ion-acoustic waves with the non-isothermal electrons, where some of electrons are moving in the plasma freely and others are trapped in the ion-acoustic waves. We also observed the two-stream instability when we injected two counter streams of ions into the plasma.

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