A model of a reflective surface under the Shklovsky-de Genet topology

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Abstract. A rough surface is studied. Its reflective features are related to the geometry of a random allocation of a solid opaque medium, the surface of which separates it from a translucent medium. A model of a real rough surface is suggested. It is based on the Shklovsky-de Genet topology, which uses the concept of a connecting cluster, which is divided into a skeleton (core) and dead ends (drop points). A step parameter is chosen as the roughness value in the model, as it is reliably detected using a profilometer. It is demonstrated that data concerning the step parameter allows to analytically evaluate the spectral reflection coefficient of surfaces with a known reflection indicatrix in a wide range of wavelengths.

1. Introduction

It has been reported that the surfaces of real bodies are rough (with the exception of crystal cleavage surfaces). While solving the problem of assessing the contribution of reflected light to the normalized illumination and quality of illumination of agricultural production areas, there is a necessity to design a mathematical reflectance model from rough surfaces [1]. The simulation is aimed at solving a direct problem - to assess the spectral reflectance \( \rho(\lambda) \) by known roughness parameters [2]. The reflective surfaces of the room can be regarded as additional light sources reducing the energy bills. Thus, for a material with unknown behavior \( \rho(\lambda) \), the challenge is urgent.

Its resolution is possible based on a rough surface model accounting for irregularities at the micro- and nanoscale. Therefore, the basis of the mathematical reflectance model is a rough surface model. Its adequacy depends on the accord of the theory with the experiment.

In most of the papers still based on the Torrens-Sparrow [3] and Cook-Torrens [4] models, Gaussian statistics of irregularities are assumed. Therefore, the rough surface is presented as a set of randomly oriented mirror-reflecting microplats (facets) or steps [5]. Nevertheless, real surfaces can differ considerably from Gaussian ones. This results in an incorrect assessment of the roughness dispersion, and the mirror reflection of facets generates challenges in explaining the absorption capacity of a rough surface.

The main disadvantage, which is identified in the analysis of papers devoted to solving a direct task, consists in a considerable discrepancy between theory and experiment in a wide range of wavelengths. This fact is the ground for the search and development of new models of the real surface.

Our article proposes a model of a real rough surface based on the Shklovsky-de Genet topology [6,7]. It uses the concept of a connecting cluster, which is subdivided into the so-called core and dead ends.

The objective is to: 1) to substantiate a model of a real rough surface based on the Shklovsky-de Genet topology and choose an experimental parameter characterizing the real surface roughness; 2) to...
obtain an analytical assessment of the spectral reflection coefficient in a wide range of wavelengths using the selected parameter for a known reflection indicatrix.

2. Materials and methods

2.1. An observed rough surface geometry
According to GOST 2789-73, the Russian standard for normalizing surface roughness has identified six geometric roughness parameters for quantifying the surface, given in Figure 1: altitudinal \( (R_a, R_z, R_{\max}) \), step \( (S_m, S) \) and the parameter of the relative reference length of the profile \( (t_p) \).

\[\begin{align*}
R_a & \text{ parameter is the arithmetic mean of the absolute values of the profile deviations within the base length } l; \\
R_z & \text{ – altitude of profile irregularities at ten points, equal to the sum of the average absolute values of altitudes } Y_{pmi}; \\
R_{\max} & \text{ – the distance between the lines of peaks and dents within the base length}; \\
S_m & \text{ – average step of irregularities within the base length}; \\
S & \text{ – the average value of the step of local peaks within the base length}; \\
t_p & \text{ – the relative reference length of the profile is the ratio of the reference length of the profile to the base length.}
\end{align*}\]

To a large number of dimensions \( Y(x) \), the surface roughness is defined by dispersion averaging over a series of multiple measurements \( \sigma^2 = \overline{Y(x)^2} \). The reference point is chosen by \( \overline{Y(x)} = 0 \). It follows from [9] that if the measurements \( Y \) are performed at the maximum length \( \ell \), then the dispersion will be defined as \( \sigma^2 = kl \), where \( k \) – a proportional constant. An increase in the dispersion with a growth in length \( \ell \) indicates Gaussian statistics of the altitude distribution.

The aforementioned refers to the rough surface geometry seen with the help of profilometers. Nevertheless, the profilometer sensor can move in the cavity space only along the macro normal to the surface (in Figure 1 only along \( Y \) axis). This restriction does not allow detecting possible roughness in the direction perpendicular to the \( Y \) axis. The presence of unobservable roughness of peaks and dents with the help of profilometers is proved in the Shklovsky-de Genet topology. According to it, a model of a real rough surface, briefly called the dead ends model, is proposed.

\[\begin{align*}
\text{Figure 1. Rough surface profile [8]. } Y \text{ axis sets the direction of the macro normal to the surface.}
\end{align*}\]
2.2. The justification of the model of dead ends based on the Shklovsky-de Genet topology

It is recognized that the sizes of characteristic profile irregularities of hard rough surfaces, measured using probe methods (for example, using a scanning probe atomic-force microscope – AFM), lie in the range from hundredths of a nanometer to several micrometers. Nevertheless, the movement of the AFM sensor needle also has the above limitation, which does not allow detecting possible roughness of the surfaces of dents and peaks. We have formulated this opportunity in the form of an assumption: the lateral surfaces of dents and peaks have peaks and dents-branches of the first order. In return, these are also rough and have peaks and dents-branches of the second order ... and so on up to atomic scales. Meanwhile, the peaks and dents (Figure 1), observed using probe methods, have zero order.

Our assumption (at least about first-order branches) is proved by computer modeling of systems with a random distribution of two phases in percolation [10] by labeling clusters, for example, according to the Hoshen-Kopelman algorithm [11].

To resolve the direct task, a translucent medium and a substance with a rough surface, the relief of which was formed under the influence of random mutually independent factors, can be regarded as a random two-phase system. Its reflecting features are associated with the random distribution geometry of an opaque media (a solid is the first phase), whose surface separates it from a translucent medium (usually air is the second phase). A cluster linking a line of depressions with a line of peaks by means of a translucent medium (Figure 1), we will call a connecting cluster.

In the Shklovsky-de Genet model [6,7], the connecting cluster is subdivided into the so-called skeleton (core) and dead ends (dead-end branches) oriented randomly (Figure 2). Meanwhile, there are isolated or finite clusters with their own dead-end branches. The connecting cluster without dead-end branches is a core that is detected using profilometers. In Figure 1, the connecting cluster (literally connecting a line of dents and a line of peaks at a distance $R_{\text{max}}$) has no lateral dead-end branches.

Dead-end branches and isolated end clusters can be discovered in practice in the polishing process, which can be performed using a material (usually fabric, felt or leather). Its hardness is tens or hundreds of times lower than the hardness of the processed material. Such a contradiction can be attributed to the reality of the existence of isolated finite clusters and dead-end branches (both connecting clusters and isolated finite clusters), which serve as a kind

![Figure 2](image-url)
destruction, contributing to a relatively easy cleavage of the processed material. By polishing, the surface is rough without flat pads. Nevertheless, the space of isolated finite clusters with their dead-end branches is separated from the incident ray. In this regard, we exclude them from further consideration.

Therefore, computer modeling and polishing practice confirm the existence of dead-end branches. Let us limit ourselves to considering the primary branches, each of which has depth at any scale $L_i$ and the input transverse dimension $S_i$ (Figure 2); meanwhile, from $S_i$ the following parameter depends on the area of the micro-hole - the entrance to the branch. Then the rough surface can be regarded as a set of randomly oriented micro-holes of primary branches. This model of a real surface will be referred to for brevity as the “dead ends model”, retaining the terminology of the Shklovsky-de Genet topology. Since the primary branches may have endings at a depth below the line of dents (determined by the profilometer), the experimental assessment of the variance $\sigma^2$ based on altitude parameters will be incorrect. Hence, in the dead-end model, another parameter is required as the roughness option. From the step dimensional parameters, the $S_m$ parameter is safely identified with the help of a profilometer, applied for the estimation of the spectral coefficient of back reflection $\rho(\lambda)$.

2.3. Analytical assessment of the spectral coefficient of back reflection $p(\lambda)$ using a step parameter with a known reflection indicatrix

To evaluate $p(\lambda)$, we take the assumption that the reflection indicatrix does not depend on the wavelength of the incident ray.

Let $E$ ray of unpolarized light with a wavelength $\lambda$, nm, normally falls on a small area $S$, $m^2$, rough surface (Figure 3). What is the probability $p(\lambda)$ that the ray will be reflected back? In [1], the probability $p(\lambda)$ and the back reflection coefficient are regarded as identical quantities equal to the fraction of the reversed flux. For a rough surface, the reflected ray $n$ will have a random direction in the hemisphere where the source is located. Let us put a radius vector $R_n$ with a random module along a random direction $R_n$. In a spherical coordinate system $R_n$ may be determined by 3 numbers: $R_n$ and two angles $\theta$ and $\phi$ (Figure 3).

![Figure 3. Incident and reflected rays in Cartesian and spherical reference systems.](image1)

![Figure 4. Angles $\gamma_1$ and $\gamma_2$, limiting reflection indicatrix.](image2)

The possible values of the angles $\theta$ and $\phi$ are in the range from $\gamma_1$ to $\gamma_2$ (in Figure 4, the interval is shaded). Moreover, $\gamma_1$ and $\gamma_2$ are determined by the reflection indicatrix and can have values ranging from $0$ to $\pi/2$. In [1] it is shown that by averaging the projections of $R_n$ on the Cartesian axes, the concerned probability $p(\lambda)$ can be represented as
\[ \rho(\lambda) = C(\gamma_1, \gamma_2) P(\lambda), \]  

where \( C(\gamma_1, \gamma_2) \) – the probability that the incident ray was reflected into the hemisphere in the form of a ray \( n \) and in general has the form:

\[ C(\gamma_1, \gamma_2) = \left[ (\cos \gamma_1 - \cos \gamma_2) / (\gamma_2 - \gamma_1) \right]^2, \]  

and the function \( P(\lambda) \) is the conditional probability that the reflected ray \( n \) is directed back to the incident, provided that it is reflected into the hemisphere in the form of a ray \( n \):

\[ P(\lambda) = \frac{1}{1 + \exp[-a(\lambda - \lambda_0)]}, \]  

where \( \lambda_0 \) matches the conditional probability \( P(\lambda_0) = 0.5 \).

According to the presumption of the reflection indicatrix does not depend on the wavelength. Thus, \( C(\gamma_1, \gamma_2) \) for a given surface texture is a constant value.

Let us contrast the exponent with the unit in the denominator (3). For this, consider the normalization condition

\[ \int_0^\infty dP(\lambda) = 1, \]  

which can be performed if \( \exp(a\lambda_0) >> \exp(-a\lambda_0) \), then

\[ \exp(-2a\lambda_0) << 1 \text{ or } \exp(2a\lambda_0) >> 1, \]  

consequently, \( a > 0 \). Let us write (1) in (3):

\[ \rho(\lambda) = \frac{1}{1 + \exp[-a(\lambda - \lambda_0)]} C(\gamma_1, \gamma_2). \]  

The positive value of \( a \) indicates that \( p(\lambda) \) is an increasing function of the wavelength. If \( \lambda = \lambda_0 \), the function \( p(\lambda) \) grows most rapidly, since it has the greatest slope equal to \( aC(\gamma_1, \gamma_2)/4 \). This suggests the proportionality of the wavelength \( \lambda_0 \) of the incident light with some characteristic size of the connecting cluster \( d_0 < S \) (Figure 2); thus \( \lambda_0 = d_0 \) can be accepted.

Let us consider the example of the light reflection from polished surfaces close to mirror surfaces, when we can consider that the reflection indicatrix is known and corresponds to the magnitude \( C(\gamma_1, \gamma_2) = 1 \). Then \( p(\lambda) = P(\lambda) \) according to (1). If the reflection is close to the mirror, then \( \lambda > S_m \), which is consistent with the theory of radiation of waveguides and horns [12]; thus \( p(\lambda) = P(\lambda) = 1 \) and if \( \lambda = S_m \).

From (3) at the wavelength \( \lambda = 3\lambda_0 \) we will have the following: \( P(3\lambda_0) = 1 \); if (5), then \( 3\lambda_0 = S_m \). Therefore, the required analytical estimate \( p(\lambda) \) in our case has the final form

\[ p(\lambda) = \frac{1}{1 + \exp[-a(\lambda - S_m/3)]}. \]  

Generally, when \( C(\gamma_1, \gamma_2) < 1 \) a relationship is needed between the characteristic size of the connecting cluster \( d_0 \) (or \( \lambda_0 \) value) with a step parameter \( S_m \).

2.4. Coordination with the experiment

Thus, for well-polished surfaces, appreciable changes in \( p(\lambda) \) are in a wide range of wavelengths \( 0 < \lambda < S_m \). When \( \lambda > S_m \) saturation mode occurs when \( p(\lambda) = P(\lambda) = 1 \), which is consistent with the experimental dependence of the reflection coefficient of the polished surface of the aluminum alloy AMg6 in the UV and visible area (Figure 5) [13].


Figure 5. Experimental and theoretical dependences of the reflection coefficient on the wavelength for the polished surface of the aluminum alloy AMg6.

Generally, when $C(\gamma_1, \gamma_2)<1$, saturation mode corresponding to $P(\lambda)=1$ and $\rho(\lambda) = C(\gamma_1, \gamma_2)$ is proved in the experimental spectral dependences of the reflection coefficient of materials of different physical origin: – plant tissues of spruce, birch and dogwood needles; steel 20 and alloys of copper, nickel, aluminum; silicon and gold [14-16].

3. Results and discussion
1. In the model of dead ends suggested on the basis of the Shklovsky-de Genet topology (hereinafter referred to as the model), the real rough surface is regarded as a set of randomly oriented micro-holes of primary branches at different magnification scales. According to the wavelength of the radiation incident in them and their depth, reflection and absorption are possible. This is the principal difference between the proposed and facet model. The computer modeling and the polishing process confirms the existence of dead ends (dead-end branches) of the connecting cluster of the translucent medium.

2. Naturally, the analogue of the offered surface is the surface of a high-altitude system, pitted with caves having both macroscopic and microscopic dimensions. It should be noted that stalactites and stalagmites can be found inside the caves, with the help of which the internal roughness of the primary branches can be represented.

3. The connection obtained in a particular case between the characteristic size of the connecting cluster $d_0$ (or the magnitude $\lambda_0$) and the step parameter $S_m$ indicates the possibility of spectral reflectance coefficient assessment $p(\lambda)$ using this parameter in a wide range of wavelengths.

4. Conclusions
1. On the basis of the Shklovsky-de Genet topology, a model of dead ends is justified. It regards a real rough surface as a set of randomly oriented micro-holes of branches of the first order distributed on the surface of peaks and dents of the zero order.

2. According to the model, as an experimental parameter characterizing the real surface roughness, the average step of irregularities within the base length is chosen, as reliably determined using a profilometer.

3. In the case of a known reflection indicatrix, an analytical assessment of the spectral reflection coefficient was obtained using the selected parameter, consistent with the experiment in a wide range of wavelengths.

4. The offered rough surface model can be applied in those branches of agriculture where roughness parameters are important operational characteristics. For example: thermal and corrosion resistance, a
number of mechanical features (including wear resistance and sliding and rolling friction coefficients) and many other properties.

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