Gauge Symmetry Breaking in Matrix Models

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Abstract

We argue that some features of the standard model, in particular the fermion assignment and symmetry breaking, can be obtained in matrix model which describes noncommutative gauge theory as well as gravity in an emergent way. The mechanism is based on the presence of some extra (matrix) dimensions. These extra dimensions are different from the usual ones which give to a noncommutative geometry of the Grönewold-Moyal type, and are reminiscent of the Connes-Lott model, although the action is very different.

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1 Introduction

Matrix models such as the ones introduced in [1, 2, 3] and noncommutative geometry [4, 5, 6, 7] may be the appropriate tools to describe physics in the quantum gravity regime, where the ordinary concepts of manifold may no longer be valid. The matrix models are known to describe noncommutative gauge theory [8, 9], and contain gravity as an emergent phenomenon a la Sakharov [11, 12]. On the other side noncommutative geometry can describe the Higgs mechanism of symmetry breaking [13, 15, 16] and more in general the standard model coupled with gravity [4, 13, 14] in a natural way by extending the space with the addition of some extra (in general noncommuting) coordinates, while the rest of the coordinates remain the same.

We will describe a construction presented in [17] (see also [18]) which is able to reproduce some of the features of the standard model, mainly in relation to symmetry breaking. The symmetry breaking happens through VEV’s in the extra coordinates analogous to the ideas in [19], and all fermions of the standard model are accommodated. The model presented here is not yet fully realistic, and at present it has no phenomenological valence, it nevertheless shows that matrix models in the context of noncommutative geometry has the potentiality to describe models which in the future may have an actual predictive power.

2 The matrix Model

The starting point is the action:

$$S_{YM} = -(2\pi)^2 \frac{\Lambda_{NC}^4}{g^2} \text{Tr} \left( [X^a, X^{b'}][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} + \bar{\Psi}\Gamma_a[X^a, \Psi] \right)$$

where $X^a$ are infinite-dimensional hermitian matrices, $\eta_{aa'}$ is the flat Minkowski (or Euclidean) metric and $\Psi$ is a corresponding (Grassmann-valued) spinor which is also an infinite matrix, $g$ is a couling constant and $\Lambda$ is a an energy scale which plays the role of noncommutativity scale. We will not specify further which type of matrices we are dealing with, since we are describing a crude approximation of a more refined (and yet unknown) mathematical structure.
Consider first the bosonic part of the model, the equations of motion for the $X$’s are

$$[X^a, [X^b, X^{a'}]]\eta_{aa'} = 0 \quad (2.2)$$

and the model is invariant under the symmetry

$$X^a \rightarrow UX^aU^{-1}, \quad U \in U(\mathcal{H}). \quad (2.3)$$

Apart from the null solution or the case in which all matrices commutes, there is an important solution to the equations of motion:

$$[X^a_0, X^b_0] = i\theta^{ab} \quad (2.4)$$

with $\theta^{ab}$ constant. We call this solution the “scalar Moyal-Weyl” vacuum since in this case the $X$’s are the generators of an algebra which is isomorphic (under appropriate regularity conditions) to the algebra of functions multiplied with the Grönelwol-Moyal $\star$-product. This matrix model describes a noncommutative space with a constant commutator. To a first approximation this is true for sufficiently short distance scale. We will later consider the matrix model obtained by letting this background fluctuate. There is a gauge invariance which at first sight appears to be the unitary group of matrices functions of $X$, but as shown in [10] the $U(1)$ part of the group contributes to the gravitational degree of freedom, we refer to the original paper for details.

The Moyal vacuum is of course not the only minimum of the action, for example

$$\bar{X}^a = X^a_0 \otimes 1_n \quad (2.5)$$

is another solution of (2.2), which correspond a noncommutative $U(n)$ gauge symmetry because in the semiclassical limit it corresponds to a nonabelian gauge theory. Again the $U(1)$ degree of freedom is absorbed by gravity and therefore the theory corresponds to a $SU(n)$ gauge theory.

We can also consider the possibility that some of the dimension are of a different kind altogether, so to have a four dimensional spacetime which in some commutative limit goes to the usual Minkowskian space, plus an internal space described by the tensor product of finite dimensional matrices times the identity. This is actually a noncommutative version of the programme that Connes and collaborators have been developing for some time to describe the standard model. The origins of the model lie in the aim to describe the
Higgs mechanism by some noncommutative internal coordinates, which in some cases are taken to be fermionic \[15, 16\]. In the original Connes-Lott model \[13\] model the internal coordinates were two by two diagonal matrices, but then in the evolutions of the model \[20, 21, 14\] the internal space is described by the matrix algebra \(M_3(\mathbb{C}) \times \mathbb{H} \times \mathbb{C}\), with \(M_3(\mathbb{C})\) the algebra of three by three complex valued matrices, and \(\mathbb{H}\) the algebra of quaternions. The unimodular part of this algebra corresponds to the standard model group. The action is composed of two parts, the fermionic action is the usual one, while the bosonic action is a regularized version of the trac of the covariant Dirac operator. It is actually possible to derive the bosonic action from the fermionic one imposing scale invariance and demanding cancellation of the anomalies \[22, 23\]. In our case the action is different, but the idea is similar, i.e. we consider the case for which the dimensions come in two kinds,

\[
X^a = (X^\mu, \chi^i), \quad \mu = 0, \ldots, 3, \ i = 1, \ldots, n
\]  

with the \(X^\mu\)'s (quantized) coordinate functions of the form \(2.5\), which generate the Moyal-Weyl plane, and \(n\) extra generators \(\chi^i\). These coordinate are a solution of \(2.2\) because

\[
[\bar{X}^\mu, \bar{X}^\nu] = i\theta^{\mu\nu} \otimes 1_N, \quad [\bar{X}^\mu, \bar{\chi}^i] = 0
\]  

with \(\theta\) constant. The symmetry is still \(SU(N)\).

The fluctuations around this solution can be expressed in terms of two fields, \(A\) and \(\Phi\) and the noncommutativity scale \(\Lambda_{NC}\) as

\[
X^\mu = \bar{X}^\mu + A^\mu, \quad \Phi^i = \Lambda_{NC}^2 \chi^i
\]  

As mentioned the trace-\(U(1)\) give rise to emergent gravity and will be ignored in the rest of these proceedings. The remaining \(SU(N)\)-valued fluctuations

\[
A^\mu = -\theta^{\mu\nu} A_\nu^\alpha(x) \otimes \lambda_\alpha
\]  

correspond to \(SU(N)\)-valued gauge fields, while the fluctuations in the internal degrees of freedom

\[
\Phi^i = \Phi^i_{\alpha\alpha}(x) \otimes \lambda_\alpha
\]  

correspond to scalar fields in the adjoint. The matrix model action \(2.1\) therefore describes \(SU(N)\) gauge theory on \(\mathbb{R}^4_{\theta}\) coupled to \(n\) scalar fields. Hereafter we drop the \(\otimes\) sign.
Noncommutative gauge theory is obtained from the matrix model using

$$[\bar{X}^\mu + A^\mu, f] = i\theta^{\mu\nu}(\partial_{\bar{x}^\nu} + i[A_{\nu}, .])f \equiv i\theta^{\mu\nu}D_{\nu}f.$$  \hspace{1cm} (2.11)

The matrix model action (2.1) can then be written as

$$S_{YM} = \frac{1}{g^2} \int d^4\bar{x} \text{tr} \left( G^\mu^\nu G^{\mu'\nu'} F_{\mu\nu} F_{\mu'\nu'} \right.$$  
$$+ 2G^\mu_{\nu} D_\mu \Phi^i D_{\nu'} \Phi^j \delta_{ij} - [\Phi^i, \Phi^j][\Phi^{i'}, \Phi^{j'}] \delta_{ij} \delta_{i'j'}$$  
$$+ \bar{\Psi} D\Psi + \bar{\Psi} \Gamma_{\nu}[\Phi^i, \Psi] \right).$$  \hspace{1cm} (2.12)

This is the action of a $SU(N)$ gauge theory on $\mathbb{R}^4_\theta$, with effective metric given by

$$G^\mu_{\nu} = \rho\theta^\mu_{\nu'} \theta^{\nu'\nu}, \quad \rho = (\det \theta^\mu_{\nu'})^{-1/2} = \Lambda_{NC}^4,$$  \hspace{1cm} (2.13)

which satisfies $\sqrt{|G|} = 1$. Here $F^\mu_{\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$ is the field strength on $\mathbb{R}^4_\theta$ and $D_\mu \equiv \partial_\mu + i[A_\mu,.]$ is the covariant derivative for fields in the adjoint, and $\text{tr}()$ denotes the trace over the $SU(N)$ components. The effective Dirac operator is given by

$$D\Psi = \Gamma_{\mu} [X^\mu, \Psi] \sim i\gamma^\mu D_\mu \Psi$$  \hspace{1cm} (2.14)

where \cite{24}

$$\gamma^\mu = \sqrt{\rho} \Gamma_{\nu} \theta^\nu_{\mu}, \quad \{\gamma^\mu, \gamma^\nu\} = 2G^\mu_{\nu}.$$  \hspace{1cm} (2.15)

The fermions have been rescaled appropriately, and a constant shift as well as total derivatives in the action are dropped. Note that $g$ is now identified as the coupling constant for the nonabelian gauge fields on $\mathbb{R}^4_\theta$.

### 3 Symmetry breaking

In \cite{17} we presented two mechanisms for the breaking of the symmetries, one based on constant matrices and the other on seeing the internal space as fuzzy spheres. For reasons of space in these proceedings we will present only the former, which can be also seen as an effective version of the latter.

Consider a model with $X^\mu$ as in \cite{25} with $n = 7$ and one extra coordinate:

$$\langle \mathcal{X}^\Phi \rangle = \begin{pmatrix} \alpha_1 \mathbb{1}_2 \\ \alpha_2 \mathbb{1}_2 \\ \alpha_3 \mathbb{1}_3 \end{pmatrix}.$$  \hspace{1cm} (3.1)
The $\alpha$’s are constant quantities with the dimensions of a length, all different among themselves. These new coordinates are still solutions of the equations of the motion because, but the $SU(7)$ symmetry is broken down because of $X^\Phi$. The residual unbroken group is $SU(3) \times SU(2) \times U(1) \times U(1) \times U(1)$.

In the bosonic action as the spacetime ($\mu \nu$) part of action remains unchanged, while for the $\mu \Phi$ components we obtain, in the Moyal-Weyl background,

$$[\bar{X}^\mu + A^\mu, X^\Phi] = i \theta^{\mu \nu} D_\nu X^\Phi = i \theta^{\mu \nu} (\partial_\nu + i A_\nu) X^\Phi,$$

$-(2\pi)^2 \text{Tr} [X^\mu, X^\Phi][X^\nu, X^\Phi]\eta_{\mu \nu} = \int d^4 x G^{\mu \nu} \left( \partial_\mu X^\Phi \partial_\nu X^\Phi - [A_\mu, X^\Phi][A_\nu, X^\Phi] \right).$

Note that the mixed terms $\int \partial^\mu X^\Phi [A_\mu, X^\Phi] = -\frac{1}{2} \int X^\Phi [\partial^\mu A_\mu, X^\Phi] = 0$ vanish, assuming the Lorentz gauge $\partial^\mu A_\mu = 0$.

Now consider the vacuum (3.1). Since $X^\mu$ and $\langle X^\Phi \rangle$ commute, this means $\langle X^\Phi \rangle = \text{const}$ and the first term in the integral above vanish. We can therefore separate the fluctuations of this extra dimension which are a field, the (high energy) Higgs field. In the action the first term is nothing but the derivative of it. The second term instead is

$$[A^\mu, \langle X^\Phi \rangle] = \begin{pmatrix} 0 & (\alpha_2 - \alpha_1) A_{12}^\mu & (\alpha_3 - \alpha_1) A_{13}^\mu \\ (\alpha_1 - \alpha_2) A_{21}^\mu & 0 & (\alpha_3 - \alpha_2) A_{23}^\mu \\ (\alpha_1 - \alpha_3) A_{31}^\mu & (\alpha_2 - \alpha_3) A_{32}^\mu & 0 \end{pmatrix},$$

(3.3)

where we consider the block form of $A^\mu$

$$A^\mu = \begin{pmatrix} A_{11}^\mu & A_{12}^\mu & A_{13}^\mu \\ A_{21}^\mu & A_{22}^\mu & A_{23}^\mu \\ A_{31}^\mu & A_{32}^\mu & A_{33}^\mu \end{pmatrix}.$$ (3.4)

Therefore (3.2) leads to the mass terms for the off-diagonal gauge fields,

$$-(2\pi)^2 \text{Tr} [X^\mu, \langle X^\Phi \rangle][X^\nu, \langle X^\Phi \rangle]\eta_{\mu \nu} = \int d^4 x G^{\mu \nu} \left( \sum (\alpha_i - \alpha_j)^2 A_{\mu,ij} A_{\nu,ji} \right).$$ (3.5)

which is nothing but the usual Higgs effect. If the differences $\alpha_i - \alpha_j$ are large (which we assume) the non diagonal blocks of $A^\mu$ acquire large masses $m_{ij}^2 \sim (\alpha_i - \alpha_j)^2$, and effectively disappear from the spectrum.
4 Particles and symmetries

We now show how the fermions in the standard model can be naturally accommodated in the framework of matrix models. This is nontrivial because the fermions in the matrix model are necessarily in the adjoint of some basic $SU(N)$ gauge group. In [17] we have also shown how the electroweak symmetry can be broken through a somewhat modified Higgs sector, and the Yukawa couplings which are obtained.

For the sake of this paper we accommodate all known fermions (with the exception of right handed neutrinos) in an upper triangular matrix\[\Psi = \begin{pmatrix} 0_{2 \times 2} & L_L & Q_L \\ 0_{2 \times 2} & 0 & e_R \\ 0_{3 \times 2} & 0_{3 \times 2} & 0_{3 \times 3} \end{pmatrix} \tag{4.1} \]

Here $l_L$ will be the standard (left-handed) leptons, and $e_R$ the right-handed electron. The quark matrix is

$$Q = \begin{pmatrix} Q_L \\ Q_R \end{pmatrix}, \quad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_R = \begin{pmatrix} d_R \\ u_R \end{pmatrix} \tag{4.2}$$

The correct hypercharge, electric charge and baryon number are then reproduced by the following traceless generators

$$Y = \begin{pmatrix} 0_{2 \times 2} & -\sigma_3 & 0 \\ -\sigma_3 & -\frac{1}{3} \mathbb{I}_{3 \times 3} & 0 \\ 0 & 0 & -\frac{1}{3} \mathbb{I}_{3 \times 3} \end{pmatrix} + \frac{1}{7} \mathbb{I} \tag{4.3}$$

$$Q = T_3 + \frac{Y}{2} = \frac{1}{2} \begin{pmatrix} \sigma_3 \\ -\sigma_3 \\ -\frac{1}{3} \mathbb{I}_{3 \times 3} \end{pmatrix} + \frac{1}{14} \mathbb{I} \tag{4.4}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{3} \mathbb{I}_{3 \times 3} \end{pmatrix} + \frac{1}{7} \mathbb{I} \tag{4.5}$$

\[\text{Some of the zero's in the matrix may correspond to other particles, see [17].}\]
which act in the adjoint. Of course at this stage we are still quite far from a complete model. Nevertheless this result points to the possibility to describe the standard model within this noncommutative geometry matrix model.

5 Electroweak breaking

Now we show how electroweak symmetry breaking might be realized in this framework. To explain the idea we will first present a simplified version where the Higgs is realized in terms of a single extra coordinate (resp. scalar) field. This is again not intended as a realistic model, but it shows that suitable Higgs potential can naturally arise within the present framework.

Higgs field connects the left with the right sectors of leptons, and is otherwise colour blind, it is therefore natural to consider another extra coordinate which will have to necessarily be off-diagonal. The following matrix has the correct characteristics:

\[
\mathcal{X}^\phi = L_{NC}^{-2} \begin{pmatrix}
0_{2 \times 2} & \phi & 0_{2 \times 3} \\
\phi^\dagger & 0_{2 \times 2} & 0_{2 \times 2} \\
0_{3 \times 2} & 0_{3 \times 2} & 0_{3 \times 3}
\end{pmatrix}
\]

again we consider the extra variable \( \mathcal{X} \), its vacuum expectation value and the fluctuations which are a physical field. The Higgs \( \phi \) is a \( 2 \times 2 \) matrix which is actually composed of two doublets:

\[
\phi = (\bar{\phi}, \varphi)
\]

The vacuum expectation value of \( \phi \) is an off-diagonal matrix:

\[
\langle \phi \rangle = \begin{pmatrix}
0 & v \\
\bar{v} & 0
\end{pmatrix}
\]

All other components are assumed to be very massive, e.g. due to the commutator with the high-energy breaking discussed before.

Now consider the fermionic part of the action (2.1), which can be written on \( \mathbb{R}^4_0 \) in the form (2.12). The part involving \( X^\mu \) gives the usual Dirac action as in (2.12), and the part involving \( \mathcal{X}^\phi \) yields the Yukawa couplings

\[
S_Y = \text{Tr} \overline{\Psi} \gamma_5 [\mathcal{X}^\phi, \Psi]
\]
giving mass to the fermions. Here we have considered the extra dimension to be a fifth dimension, hence the presence of $\gamma_5$, and $\Psi = \Psi^\dagger \gamma_0$. Then the full Yukawa term is

$$S_Y = \text{Tr} \bar{L}_L \gamma_5 \phi \begin{pmatrix} 0 & e_R \\ e_R & 0 \end{pmatrix} \left( \begin{pmatrix} 0 \\ e_R \end{pmatrix} \gamma_5 \phi^\dagger L + \bar{Q}_L \gamma_5 \phi Q_R + \bar{Q}_R \gamma_5 \phi^\dagger Q_L \right) \quad (5.10)$$

Only the correct couplings appear, albeit all with the same value.

6 Conclusions and Outlook

We have sketched how a matrix model with the capability to describe non-commutative geometry and gravity may also describe some features of the standard model, mostly regarding the issues of symmetry breaking. The model described here is in its simplest form and is not yet phenomenologically viable, it just points the way to further developments. Some further steps were already undertaken in [17, 19] where it was shown that already seeing the extra dimensions as composed of fuzzy spheres gives more liberty in the model, while (near-)realistic models appear possible along the lines of [25]. The hope is that matrix model can not only describe some form of quantum gravity, but also give some input as far gauge theories are concerned.

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