Chiral extrapolation and physical insights

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It has recently been established that finite-range regularisation in chiral effective field theory enables the accurate extrapolation of modern lattice QCD results to the chiral regime. We review some of the highlights of extrapolations of quenched lattice QCD results, including spectroscopy and magnetic moments. The Δ resonance displays peculiar chiral features in the quenched theory which can be exploited to demonstrate the presence of significant chiral corrections.

1. INTRODUCTION

Over the past few years there has been a significant effort into the study of the chiral extrapolation problem for lattice QCD. Extrapolation of lattice QCD simulation results to the light quark mass regime is nontrivial due to nonanalytic variation of hadron properties with quark mass. Such nonanalytic behaviour arises as a consequence of spontaneously broken chiral symmetry in the QCD ground state.

Early research in extrapolations of nucleon properties [1,2,3] discovered that one could obtain reliable extrapolations by incorporating the model-independent constraints of chiral symmetry near the chiral limit. Importantly, these studies highlighted the fact that the rapid, nonanalytic variation with quark mass must be suppressed at some intermediate energy scale. Above pion masses $\sim 500$ MeV, nucleon properties become smoothly varying functions of the quark mass, $m_q \propto m_\pi^2$. This observation has a simple physical explanation. Once the Compton wavelength of the pion becomes small ($\lambda_C \sim m_\pi^{-1}$) relative to the finite extent of the nucleon, chiral loop effects become suppressed [4].

Recent work has established that these features are all naturally built into chiral perturbation theory (χPT) when evaluated with a finite-range regulator (FRR) [5]. The formulation of χPT with chiral loop integrals cut-off in momentum space at a finite energy scale has been established by Donoghue et al. [6]. Finite-range cutoff effects in chiral effective field theory have also been studied in the context of lattice regularised χPT [7].

A case study of the nucleon mass expansion [5] has demonstrated that the use of FRR improves the convergence properties of the χPT expansion. The expansion has been demonstrated to be applicable up to pion masses of at least $m_\pi^2 \sim 0.8$ GeV$^2$. This allows a reliable connection with modern lattice QCD simulations. A range of functional forms for the momentum space cutoff have been investigated and the systematic errors induced by the cutoff have been found to be below the 1% level.

Following this study it has been demonstrated that one-loop FRR-χPT is able to extrapolate modern lattice QCD simulations of the nucleon mass with minimal systematic uncertainty [8]. Figure 1 shows extrapolations of two-flavour dynamical QCD lattice data [9] based upon four different functional forms for the regulator. The residual dependence on the regulator is beyond the resolution of this figure. With an increasing number of dynamical baryon simulations in lattice QCD [9,10,11,12] it is essential to incorporate a correct treatment of chiral physics. Only then can an interesting comparison of theory and experiment be carried out.

In this paper we highlight some of the features learned about the chiral structure of baryons. In particular we look at the nucleon and the Δ resonance in both dynamical and quenched simulations of QCD. The majority of the effects of dy-
Figure 1. Extrapolation of nucleon mass lattice data using four different finite-range regulators [8]. The physical nucleon mass is not a constraint in these fits.

2. BARYON MASSES

As outlined in the Introduction the use of a FRR in \( \chi \)PT is essential for the chiral expansion to be applicable at the quark masses simulated in state of the art dynamical lattice simulations. The application of quenched \( \chi \)PT [16] to quenched simulations has demonstrated similar success with the use of a FRR [13]. The remarkable discovery of this work was that the primary difference between quenched and dynamical spectroscopy can be explained by the differences in the chiral loop contributions.

Using FRR we express the chiral expansion of the nucleon mass as

\[
m_B = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \Sigma_B(m_\pi, \Lambda),
\]

(1)

where \( \Sigma_B \) is the total contribution to the nucleon mass from chiral-meson loop diagrams and \( \Lambda \) is a regulator parameter governing the range. For the nucleon mass the corresponding contributions are shown in Fig. 2 with analogous diagrams for the \( \Delta \). These diagrams give rise to the leading and next-to-leading nonanalytic behaviour in the chiral expansion. It is important to note that although \( \Sigma_B \) is a function of \( \Lambda \), the dependence on \( \Lambda \) is removed through renormalisation of the chiral expansion [5].

In the quenched case we first note the inclusion of two additional diagrams which are absent in the case of QCD. The flavour-singlet \( \eta' \) is also a light degree of freedom in the quenched theory [16]. Details of the role of the \( \eta' \) in the context of baryon mass extrapolation have been discussed extensively in Ref. [13].

The contributions from the standard \( \pi \)-meson loops are also modified in the quenched theory. In QCD meson-baryon loops are attractive and hence the binding energy has the effect of lowering the effective mass of the state in question. Of particular interest in the quenched theory is the \( N-\pi \) loop contribution to the \( \Delta \) mass. The sign of this diagram is reversed and the loop acts repulsively, raising the mass of the resonance in the quenched theory.

This phenomenology can easily be understood by considering a pictorial, quark-flow description of the processes contributing to the \( N-\pi \) loop in full QCD. Figure 2 shows all different topological contributions to the \( \Delta^{++} \to N\pi \) loop diagram. Diagrams (b) and (c) both indicate the propagator.

Figure 2. Leading chiral loop corrections to the nucleon mass in QCD and quenched QCD. Solid, double and dashed curves correspond to nucleon, \( \Delta \) and meson propagators. The cross denotes a hairpin insertion in the \( \eta' \) propagator.
Figure 3. Pictorial quark-flow view of the $N-\pi$ meson-cloud contributions to the $\Delta$ baryon in full QCD.

In Fig. 3 we show the fits to preliminary results of the CSSM Lattice Collaboration using FLIC fermions [17]. The lattice scale has been set via the Sommer scale, $r_0 = 0.5$ fm. This gives a lattice spacing of 0.128 fm on a $20^3 \times 40$ volume. The flattening of the $\Delta$ in the chiral regime is clearly displayed by these new simulation results. This effect is primarily caused by the reversal in sign of the $\Delta \to N\pi$ contribution as discussed above. The dashed curves in Fig. 4 represent our estimate of unquenching effects where $\Sigma^{(Q)}_B$ has been replaced by $\Sigma_B$. The chiral contributions here have been evaluated using a 0.8 GeV dipole regulator. This value was the optimal $\Lambda$ found in Ref. [5] which gave best agreement between dynamical lattice QCD and experiment. These results are preliminary and finite volume effects are yet to be taken into account, yet we already see remarkable agreement with the experimentally measured masses.

We see from Fig. 4 that the $\Delta$ suffers from larger quenching effects due to the repulsive $N\pi$ loop contribution. It has also been recognised that other excited states seem to lie systematically high in the quenched approximation. Using chirally improved quark actions the BGR Collaboration have noted that there seems to be a correlation between the quenched discrepancies and the physically observed widths [18]. As chiral extrapolation techniques for excited states are de-
Figure 4. Solid curves show quenched fits to preliminary FLIC fermion lattice data. Dashed curves display estimates of the physical theory obtained by unquenching the chiral loops. Stars indicate experimental values.

-developed it will be interesting to see if more quenched spectroscopy can be similarly understood in terms of chiral interactions.

3. MAGNETIC MOMENTS

Chiral loop effects are known to give large contributions to the electromagnetic structure of baryons. In fact the leading nonanalytic contribution to the neutron’s magnetic moment is nearly one third the experimental value. Chiral symmetry effects in nucleon electromagnetic structure describe dynamical, nonperturbative structure beyond the simple quark model of QCD. Such is the magnitude of the pion loop effects that one should expect dramatic results in lattice calculations as the light quark mass regime is probed.

Once again we find that the repulsive interaction of the $N\pi$ loop means that the magnetic moment of the $\Delta$ baryon is especially interesting in QCD.

The best lattice calculations of the electromagnetic structure of baryons have so far been restricted to the quenched model of QCD. Because of the absence of $q\bar{q}$ pair-creation, pion loop effects are typically suppressed in the quenched approximation. However, the inclusion of the flavour singlet $\eta'$ does offer the opportunity for detection of enhanced chiral behaviour in quenched simulations. As a result of double-hairpin loops the leading nonanalytic contribution is $\log m_\pi$ — i.e. magnetic moments diverge in the chiral limit.

In the case of the $\Delta$ it is the next-to-leading order corrections which cause the most significant chiral effects. The flavour symmetry of the $\Delta$ interpolating fields ensure that in the quenched approximation, assuming isospin symmetry, the magnetic moment is proportional to the charge. Just considering the $\Delta^{++}$ is therefore sufficient to determine the chiral behaviour of all charge states in the quenched theory.

We apply the diagrammatic method of Leinweber to determine the quenched chiral corrections to the $\Delta$ baryon magnetic moment. Preliminary results have been documented in Refs. As for the nucleon, the vertex correction from the double hairpin $\eta'$ dressing gives rise to a leading logarithmic divergence. The magnetic current induced by a pion loop is zero in the quenched theory. In the case of the $\Delta^{++}$ this is a trivial observation, as meson fields can only be generated by $u\bar{u}$-pairs and must therefore be neutral.

It is the vertex corrections to the magnetic moment which give rise to the most interesting physics in the quenched approximation. In particular, it is once again the contribution from the intermediate $uuu$-baryon which gives rise to a clear signal of quenched chiral physics. The contribution from the electromagnetic current coupling to the baryon in Fig. 3(c) causes a reversal of the chiral effect relative to physical QCD. As for the case of the $\Delta$ mass as described above, the repulsive interaction of this loop enhances the effect of quenching in the chiral regime.

We show preliminary results of FLIC fermion simulations of baryon magnetic moments. In particular, we plot together the $p$ and $\Delta^+$ magnetic moments in Fig. 5. We observe that at moderate quark masses the expected heavy-quark theory result is observed, with the $\Delta^+$ lying slightly above the proton. However, in the light-quark...
regime chiral physics becomes apparent with the clear separation of the two signals. The downward curvature in the $\Delta^+$ is dominated by the coupling to an intermediate octet baryon. This demonstrates a clear picture of nonperturbative effects resulting from dynamical chiral symmetry breaking in quenched lattice QCD.

Finite volume effects cannot play a significant role in the conclusions drawn from this study. The opposite sign on the $\Delta \rightarrow N\pi$ loop means that such a vertex correction to the $\Delta$ baryon magnetic moment will also have a sign which is opposite to that of QCD. The pion loop contributions to the magnetic moments are $p$-wave and hence the main effect of the finite volume is to create a gap in momentum space between 0 and $2\pi/L$. Because the loop integrals are positive definite for both the proton and $\Delta$ when $m_\pi + m_N > m_\Delta$, the finite volume can only suppress the magnitude of loop effects but cannot change their sign. Studying the nucleon and $\Delta$ magnetic moments together therefore provides a clear signal of quenched chiral physics independent of volume effects.

Finite volume effects in the octet baryon sector have been studied in more detail in Ref. [27]. It is found that the finite volume discrepancies in the meson-loop contributions are negligible at the pion masses of interest.

4. CONCLUSIONS

The study of both masses and magnetic moments in the quenched approximation helps to provide a deeper understanding of chiral extrapolation. This has also helped in developing a deeper understanding of the internal structure of baryons. In particular, we have identified that the chiral structures evaluated in FRR $\chi$PT are able to describe quenched discrepancies in baryon mass calculations.

We identify that the $\Delta$ resonance offers unique features to study the differing chiral properties of quenched and physical QCD. The flattening of the $\Delta$ mass observed in FLIC fermion simulations demonstrates a clear signal of quenching effects. In the case of the magnetic moments the effect is much more dramatic. We see the turnover in the $\Delta$ magnetic moment in the chiral regime signifies clear nonperturbative effects, showing strong deviation from the constituent-quark picture of QCD.

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