Fisher information analysis and preconditioning in electrical impedance tomography

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Abstract. In this contribution, it is described how the Fisher information can be computed by using adjoint field techniques, and integrated with the gradient calculations used for optimization in electrical impedance tomography. In particular, the Fisher information can be used as a preconditioner to obtain improved convergence properties and a regularization for quasi-Newton optimization algorithms in electrical impedance tomography. Experimental data have been used to study the possibility of combining a good reconstruction quality with a low system complexity, which is achieved by using a four-electrode measurement technique to avoid the need of high-precision electrode modeling and allowing a very coarse FEM grid. Here, the Fisher information based preconditioning implies a regularization and an algorithm that works well on a coarse FEM grid with very small electrodes modeled as point sources.

1. Introduction

Gradient based least squares optimization and adjoint field techniques have great advantages of generality and simplicity, and can be used to solve a variety of inverse problems with applications in electromagnetics, acoustics, geoscience, etc., as long as there are fast direct solvers available, see e.g., [4, 5, 11]. In Electrical Impedance Tomography (EIT), the gradient expressions and related computations of the Jacobian are often based directly on the degrees of freedom that are present in the actual FEM formulation, cf., e.g., the so called standard method [2, 12, 13]. In this contribution, it is described how the Fisher information can be computed by using adjoint field techniques, and integrated with the gradient calculations used for optimization in electrical impedance tomography. The formulation is general and independent of the actual numerical method. As a numerical example, the Fisher information analysis is employed here in a preconditioning/regularization scheme for quasi-Newton optimization algorithms [8, 9], which has been formulated here in the context of a four-electrode measurement technique for EIT [6].

Accurate modeling of the electrodes is an important issue in EIT, particularly in the medical applications, see e.g., [1, 12]. Hence, the complete electrode model was developed due to the
need of an improved precision in the forward modeling, see e.g., [12] for a survey on this topic. In this contribution, general gradient expressions are given that are based on the adjoint field approach, and which can be shown to be consistent with the commonly used shunt model and the complete electrode model [10, 12]. It should be noted that even though accurate electrode modeling is an important issue in EIT, it is also well known that the application of four-electrode measurement techniques [1, 6] is an efficient way of reducing the errors due to electrode impedances (both modeling and instrumentation errors), and hence to reduce the complexity of the numerical solver as well as of the measurement equipment. In this contribution, the Fisher information based preconditioner [8] is used to devise an algorithm that works well on a coarse FEM grid with very small electrodes.

2. Fisher information analysis
Consider the following frequency dependent parameter model for the complex conductivity [10]

\[ \sigma = \sigma_R + i \omega_0 \epsilon_r = \gamma_0 (\beta + i \nu \epsilon_r) \] (2.1)

where \( \omega_0 \) is the center frequency, \( \nu = \omega / \omega_0 \), \( \gamma_0 = \omega_0 \epsilon_0 \), \( \epsilon_0 \) the permittivity of free space and where \( \beta = \sigma_R / \omega_0 \epsilon_0 \) and \( \epsilon_r \) are the dimensionless real conductivity and permittivity parameters, respectively. Let \( \theta \) denote the parameter vector \( \theta = [\beta \ \epsilon_r]^T \). By employing the definition of the Fisher information integral operator according to [9], together with a Gaussian statistical measurement model as in \([p \ q]\) the following expression for the Fisher information integral kernel is obtained

\[ I(r', r'') = 2 \text{Re} \sum_n \sum_m \sum_q \frac{|I_0^{(mnq)}|^2}{R_{mnq}} \delta_{\theta(r')} Z_{mnq} \delta_{\theta(r'')} Z_{mnq} \] (2.2)

where \( Z_{mnq} \) denotes the impedance parameter corresponding to a particular four-electrode current excitation configuration (n), voltage measurement configuration (m) and frequency (q). Here \( r', r'' \) denotes the spatial variables, \( \delta_{\theta(r)} = [\delta_{\beta(r)} \ \delta_{\epsilon_r(r)}]^T \) a vector gradient operator [9], \( I_0^{(mnq)} \) the excitation current and \( R_{mnq} \) the variance of the measurement noise.

Using the adjoint-field techniques [4, 5, 11], it can be shown that the gradients of \( Z_{mnq} \) with respect to \( \beta \) and \( \epsilon_r \) can be calculated as

\[ \begin{cases} \delta_{\beta(r)} Z_{mnq} = -\gamma_0 E^{(n)} \cdot E^{(m)} \\ \delta_{\epsilon_r(r)} Z_{mnq} = -i \nu \gamma_0 E^{(n)} \cdot E^{(m)}, \end{cases} \] (2.3)

where \( E^{(n)} \) and \( E^{(m)} \) are the calculated electric fields with (unit) current excitation at the sensor electrode pairs corresponding to the current excitation (n) and the voltage measurement (m), respectively.

Let \( Z_{mnq}^M \) denote the measured impedance parameters. The gradient corresponding to the misfit functional

\[ F(\sigma) = \sum_n \sum_m \sum_q \frac{|I_0^{(mnq)}|^2}{R_{mnq}} |Z_{mnq} - Z_{mnq}^M|^2 \] (2.4)

is then given by

\[ \delta_{\theta(r)} F = 2 \text{Re} \sum_n \sum_m \sum_q \frac{|I_0^{(mnq)}|^2}{R_{mnq}} (Z_{mnq} - Z_{mnq}^M)^* \delta_{\theta(r)} Z_{mnq} \] (2.5)

where \( \delta_{\theta(r)} Z_{mnq} \) is given by (2.3). Note that it is the same impedance gradients \( \delta_{\theta(r)} Z_{mnq} \) given by (2.3), that are used in the calculation of the Fisher information \( I(r', r'') \) in (2.2), as well as in the calculation of the total gradient \( \delta_{\theta(r)} F \) in (2.5).
Let \( \theta_i \) denote either of the real parameter functions \( \beta \) and \( \epsilon_i \) where \( i = 1, 2 \), respectively, and let \( I_{\theta_i}(r', r'') \) denote the corresponding (parameter-diagonal) Fisher information defined by (2.2). To obtain a preconditioner suitable for quasi-Newton optimization algorithms as described in [8], a linear parameter scaling is incorporated where

\[
\theta_i^{sc}(r) = \theta_i(r) \sqrt{I_{\theta_i}(r, r)},
\]

and where the parameter function \( \theta_i^{b} \) corresponds to some known background. Hence, the Fisher information for the scaled parameter function is given by \( I_{\theta_i^{sc}}(r', r'') = 1 / \sqrt{I_{\theta_i^{b}}(r', r') I_{\theta_i^{b}}(r'', r'')} I_{\theta_i}(r', r'') } \). Clearly, if \( \theta_i = \theta_i^{b} \) then the diagonal Fisher information is \( I_{\theta_i^{sc}}(r, r) = 1 \) for \( i = 1, 2 \) and for all \( r \) in the imaging domain. Finally, the gradient corresponding to the misfit functional (2.4) is given by

\[
\delta_{\theta_i^{sc}}(r) F = \frac{1}{\sqrt{I_{\theta_i^{b}}(r, r)}} \delta_{\theta_i}(r) F.
\]

3. Experiments with very small electrodes

The purpose of the experiments described here is to study EIT imaging performance when using very small electrodes. This problem is motivated by the EIT application for non-destructive imaging of living trees [7] where contact electrodes in the form of very thin needles are used to penetrate through the bark layer. Subject to the required constraints of low hardware complexity as well as low computational complexity (fast solvers), a four-electrode measurement system was considered together with a very coarse FEM grid. A FEM numerical solver was employed using 1412 triangles and 753 nodes. The very small electrodes were modeled as single nodes in the FEM mesh.

A two-dimensional imaging domain was considered consisting of a cylindrical container of radius \( a = 0.132 \) m, filled with tap-water to a height of 50 mm. As electrodes were used 16 thin wires (diameter 0.5 mm) placed vertically and evenly distributed on the inner surface of the container. The objects consisted of one or two small cylindrical bottles of glass with diameter \( d = 46 \) mm, placed at different positions inside the imaging domain, see figure 1b. The measurement system consisted of a frequency synthesis generator, a 16-channel analog multiplexing unit, A/D converters, a DSP-based frequency analysis and a PC for control and data acquisition. A four-electrode measurement strategy was chosen based on 15 current excitation configurations and 12 corresponding voltage measurements. All measurements were made at the four frequencies 390, 1562, 7812, and 31250 Hz. Suitable background parameters \( \epsilon_i = 80 \) and \( \sigma_R = 29.9 \) mS/m were found by calibration without objects. A Fisher information analysis revealed that the parameter sensitivity close to the electrodes was about 20 dB in excess of the sensitivity in the general interior of the cylinder, cf., figure 1a.

The following three algorithms were tested. The first algorithm is a quasi-Newton algorithm based on gradient calculations and the BFGS formula to update the inverse of the Hessian [3]. The second algorithm is an improvement of the quasi-Newton algorithm using the Fisher information based preconditioning (2.6). The third algorithm is a linearized one-step Newton algorithm (pseudo-inverse), cf., [10]. In figure 1b is illustrated the four measurement cases together with the reconstructed images of the conductivity \( \sigma_R \). As can be seen in these images, both the first and the third algorithm have a tendency to over-emphasize the sensor positions, something that can be interpreted as a modeling artifact due to the very high sensitivity close to the point sources. However, the Fisher information based preconditioning is able to alleviate this artifact by proper parameter scaling (regularization), and hence to define an algorithm that works well on a coarse FEM grid with very small electrodes.
Figure 1. a) Fisher information $\mathcal{I}_\beta (r, r)$ for the background parameters. Note the excessively high sensitivity at the positions of the small electrodes. b) Four measurement cases and a comparison of using three different reconstruction algorithms for the conductivity parameter $\sigma_R$ (in mS/m). Here, the calibrated background corresponds to $\epsilon_r = 80$ and $\sigma_R = 29.9$ mS/m.

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