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Apparent Resistivity Calculation Method of the Wide-area, Multi-component Transient Electromagnetic Field

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Abstract

From the transient electromagnetic vector field theory, this paper analyzed the mathematics characteristic of the transient electromagnetic vector field nuclear function. By analyzing, we know that the function forms of vertical and horizontal components of electromagnetic field at non-central points are similar to the function form of the central vertical component. So using curve fitting method, the polynomial mathematical expressions of vertical and horizontal components of electromagnetic field at non-central points can be received. Then, through the analysis of the transient electromagnetic early and late conditions, the paper deduced the apparent resistivity expression of vertical and horizontal components of electromagnetic field at non-central points. Through the array field observation method, vertical component and horizontal component can be synthesized to total vector, which enriched the transient electromagnetic detection observation technology and explain theory. A lot of the theoretical model calculation verifies that the method is correct and effective.

Key words: transient electromagnetic field; apparent resistivity definition; multi-component; vector synthesize

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1. Introduction

The transient electromagnetic method is a kind of time domain method of electromagnetic exploration. Due to its high efficiency and it can directly measuring the second field and other advantages, the TEM method applied in geophysical exploration more and more widely [1-4]. Today, most people only acquisition vertical component and analyze the response character. On one hand, due to the limitation of the instrument, on another hand, because easily affected by noise and other characteristics, research scholar of studying horizontal component is few. So, the research is still not deep enough. Because the horizontal component can provide some only through the vertical component can't get about geologic information, studying the horizontal component response characteristics is necessary, and with the vertical component of comprehensive analysis will be more meaningful [5].

2. Basic theory

2.1 The expansion of the wide area, multi-component field

From the basic theory of electromagnetic field, transient electromagnetic each component of the frequency domain response, excited by a large loop in homogenous half space, under cylindrical coordinate system can be deduced as [6-8]:

\[
H_z = I_0 R \int_0^\infty \frac{\lambda^2}{\lambda + u_i} J_1(\lambda R) J_0(\lambda r) d\lambda
\]

(1)

\[
H_r = I_0 R \int_0^\infty \frac{\lambda u_i}{\lambda + u_i} J_1(\lambda R) J_1(\lambda r) d\lambda
\]

(2)

Where, \(H_z\) presents the vertical component of TEM, \(H_r\) presents the horizontal component of TEM; \(I_0\) presents the transmitter current; \(R\) is the side length of transmitter coil; \(u_i = \sqrt{\lambda^2 + k_i^2}\); \(k_i^2 = -i\omega\sigma\mu_0\); \(r\) presents the offset distance.

For the central point, \(r = 0\), according to the particularity of Bessel function, \(J_0(0) = 1\) and \(J_1(0) = 0\), so horizontal component is zero. Through two special integral (Sommerfeld integral and Lipschitz integral), the vertical component can be simplified as:

\[
H_z|_{r=0} = \frac{I_0}{k_i^2 a^3} \left[ 3 - \left( 3k_i a + k_i^2 a^2 \right) e^{-k_i a} \right]
\]

(3)

The integral kernel function of vertical component at non-central points is:

\[
f(\lambda) = \frac{\lambda^2}{\lambda + u_i} J_1(\lambda R) J_0(\lambda r)
\]

(4)
where $\frac{\lambda^2}{\lambda + u_1}$ is complex variable function, $J_1(\lambda R)J_0(\lambda r)$ is real function, so $f(\lambda)$ also is complex variable function.

Fig.1 (a) is the integral kernel function real part function diagram of the vertical component at non-central points. Fig.1 (b) is the integral kernel function imaginary part function diagram of the vertical component at non-central points.

From fig.1, the real part function diagram of $f(\lambda)$ has the characteristics such as high oscillation frequency, almost no energy attenuation. For this kind of function, it is difficult to integral calculation in $[0, \infty)$. The imaginary part function diagram of $f(\lambda)$ has the characteristics of single-peak and quickly attenuation. Therefore it is easy to integral calculation for the imaginary part function.

The integral kernel function of horizontal component at non-central points is:

$$g(\lambda) = \frac{\lambda u_1}{\lambda + u_1} J_1(\lambda R)J_1(\lambda r)$$

Fig.2 (a) is the integral kernel function real part function diagram of the horizontal component at non-central points. Fig.2 (b) is the integral kernel function imaginary part function diagram of the horizontal component at non-central points.

The function attribute of $g(\lambda)$ and $f(\lambda)$ is similar. $\frac{\lambda u_1}{\lambda + u_1}$ is complex variable function, $J_1(\lambda R)J_0(\lambda r)$ is real function, so $g(\lambda)$ is complex variable function.

Fig.2 (a) is the integral kernel function real part function diagram of the horizontal component at non-central points. Fig.2 (b) is the integral kernel function imaginary part function diagram of the horizontal component at non-central points.
It can be seen from Fig.2 (a) that the real part function of \( g(\lambda) \) has the characteristics such as high oscillation frequency, almost no energy attenuation and it is hard to calculate the integral in interval \([0, \infty)\), which is the same with the real part function of \( f(\lambda) \). It also can be got from Fig.2 (b) that the imaginary part function of the horizontal component \( g(\lambda) \) attenuates to zero quickly, and it is easy to calculate the integral of this kind of slightly oscillation.

According to the analysis of nature of the Bessel function and magnetic field component kernel function, when the integrand is Double Bessel function, firstly it should be to find out the zero points of integrand. Then it can do integral calculation in each zero point interval using adaptive Simpson integral method. It can avoid the oscillation of the integrand, and ensure the calculation precision [9,10].

The frequency response of vertical component and horizontal component can be got from numerical integral. It can be found that frequency response of each component in different offset and the vertical component of central point has similar function form and nature. So each component of non-central point can be expressed as polynomial form similar with central point, which is shown as below:

\[
\hat{H}_z = \frac{I_0}{k_1 R} \left[ C_{z0} + \left( C_{z1} + C_{z2} k_1 R + C_{z3} k_1^2 R^2 \right) e^{-k_1 R} \right] \\
\hat{H}_r = \frac{I_0}{k_1^2 R^2} \left[ C_{r0} + \left( C_{r1} + C_{r2} k_1 R + C_{r3} k_1^2 R^2 \right) e^{-k_1 R} \right]
\]

where \( \hat{H}_z \) is fitting function of vertical component, \( \hat{H}_r \) is fitting function of horizontal component, \( C_0, C_1, C_2 \) and \( C_3 \) are fitting polynomial coefficients of horizontal component.

The polynomial coefficients of each component in different offset can be calculated using least square method. Basing on this method, the polynomial coefficients of vertical component and horizontal component in different offset can be obtained and shown below:

| \( r \) (m) | \( c_{z0} \)     | \( c_{z1} \)   | \( c_{z2} \) | \( c_{z3} \) |
|----------|----------------|--------------|-------------|-------------|
| 1        | 2.9986764     | -2.99867648 | -2.9986766 | -0.9994712 |
| 3        | 3.0074970     | -3.00749558 | -3.00749558 | -1.0036697 |
| 5        | 3.02358198    | -3.02355    | -3.02357757 | -1.01139504 |
| 7        | 3.047408484   | -3.047408522 | -3.047399635 | -1.022861738 |
| 9        | 3.079015809   | -3.079015873 | -3.079001037 | -1.038097974 |
| 45       | 3.862334112   | -3.862335357 | -3.862049473 | -1.509118498 |

| \( r \) (m) | \( c_{r0} \)   | \( c_{r1} \)   | \( c_{r2} \) | \( c_{r3} \) |
|----------|----------------|--------------|-------------|-------------|
| 5        | -7.9725952 E-02 | 7.9725877 E-02 | 7.9743062 E-02 | 1.3398733 E-02 |
Then, according to Fourier transformation, time domain response value of vertical component and horizontal component can be calculated:

\[
\frac{\partial B_z(t)}{\partial t} = \frac{I_0 \rho_z}{R^3} \left[ C_{z0} + C_{z1}(1 - \phi(u)) + C_{z2} \frac{2}{\pi} \int_0^t e^{-u} e^{-\tau^2/2} d\tau + C_{z3} \frac{2}{\pi} \int_0^t \tau e^{-u} e^{-\tau^2/2} d\tau \right]
\]

\[
\frac{\partial B_r(t)}{\partial t} = \frac{I_0 \rho_r}{R^3} \left[ C_{r0} + C_{r1}(1 - \phi(u)) + C_{r2} \frac{2}{\pi} \int_0^t e^{-u} e^{-\tau^2/2} d\tau + C_{r3} \frac{2}{\pi} \int_0^t \tau e^{-u} e^{-\tau^2/2} d\tau \right]
\]

2.2. Apparent resistivity definition of wide-area, multi-component field

Through the Fourier transform, the time domain expression of each component of TEM has been obtained. Then it is easy to establish the function relationship of the time domain component and the apparent resistivity. The apparent resistivity definition formula about each component of TEM sounding can be deduced finally [11-13].

Let \( u >> 1 \), \( \frac{2\pi R}{\tau} >> 1 \), \( t \) must be very small at this time. It means in the early stage of transient field. Now \( \phi(u) \) tends to 1, then \( f(u) \to Cz0 \), formula (8) can be approximated as:

\[
\frac{\partial B_z(t)}{\partial t} = \frac{I_0 \rho_z Cz0}{R^3}
\]

Therefore the apparent resistivity expression in early stage of vertical component at non-central points in large loop can be obtained:

\[
\rho_t \left( \frac{\partial B_z(t)}{\partial t} \right) = \frac{R^3}{I_0 Cz0} \frac{\partial B_z(t)}{\partial t}
\]

The apparent resistivity expression in early stage of horizontal component at non-central points in large loop can also be obtained in this way:

\[
\rho_t \left( \frac{\partial B_r(t)}{\partial t} \right) = \frac{R^3}{I_0 Cz0} \frac{\partial B_r(t)}{\partial t}
\]

Let \( u >> 1 \), \( \frac{2\pi R}{\tau} << 1 \), it is in the late stage of transient field. The apparent resistivity expression in late stage of vertical component at non-central points in large loop can be deduced by expanding the
probability integral $\varphi(u)$, function $e^{-u^2/2}$ with Taylor formula and substituting the expansion equation into the time domain response expression.

\[
\rho_1 \left( \frac{\partial B_z(t)}{\partial t} \right) = \frac{\mu_0}{m} \left[ \frac{C_r \pi R^2 I_0 \mu_0}{4t} \frac{\partial B_z(t)}{\partial t} \right]^{\frac{3}{2}}
\]

(13)

where $C_r = \frac{C_{r2}}{8} - \frac{C_{r1}}{40} - \frac{C_{r3}}{2}$.

The apparent resistivity expression in late stage of horizontal component at non-central points in large loop can also be obtained in this way:

\[
\rho_2 \left( \frac{\partial B_z(t)}{\partial t} \right) = \frac{\mu_0}{m} \left[ \frac{C_r \pi R^2 I_0 \mu_0}{4t} \frac{\partial B_z(t)}{\partial t} \right]^{\frac{3}{2}}
\]

(14)

where $C_r = \frac{C_{r2}}{8} - \frac{C_{r1}}{40} - \frac{C_{r3}}{2}$.

To verify the algorithm above, the forward modeling calculation of different type of theoretical model will be done below.

Model

Constructing the layered theoretical model for the forward modeling, using the apparent resistivity definition formula above, the apparent resistivity curves of horizontal component at different offsets can be got. The work device is large loop. Transmitter length is 100m, offset is 10m, dot pitch is 10m. The survey line is the center line of the loop. Model parameters are:

- **D**: $\rho_1 = 100\,\Omega\cdot m$, $\rho_2 = 10\,\Omega\cdot m$, $h_1=80m$;
- **G**: $\rho_1 = 10\,\Omega\cdot m$, $\rho_2 = 100\,\Omega\cdot m$, $h_1=80m$;
- **H**: $\rho_1 = 100\,\Omega\cdot m$, $\rho_2 = 10\,\Omega\cdot m$, $\rho_3 = 100\,\Omega\cdot m$, $h_1=80m$, $h_2=50m$.

![Fig3](image-url) The apparent resistivity curve of the horizontal component about D, G and H
3. Model calculation

3.1. Single abnormal body model

Firstly the 3D cube model will be built in homogenous half space. The resistivity of the homogenous half space is $\rho = 75\Omega\cdot m$. The abnormal body is a cube with dimensions $50m \times 50m \times 50m$, its resistivity is $\rho = 1\Omega\cdot m$. The top of the cube is $30m$ below the ground. The model scheme shows Fig.4, Overhead view is shown in Fig.5.

![Model scheme](image)

![Overhead view](image)

The observation profile in Fig.5 is the principal section crossing the center of the abnormal body. “×” presents the position of the observation point. Dot pitch is $5m$. There are 21 observation points in a line. The work device is large loop with the dimension $100m \times 100m$.

Through forward modeling calculation, the attenuation voltage of the magnetic vertical component and horizontal component of each observation point can be obtained in large loop. Multichannel profile of the principal section can be got firstly, as below:

![Attenuation voltage multichannel profile](image)

The attenuation voltage attenuates gradually from central field to both edges which can be seen from Fig.6. The peak value presents at the central point of the principal section which indicates that a geologic body with relatively low resistance exists under the central ground of the observation section.

According to the apparent resistivity definition, the apparent resistivity profiles of the vertical component and horizontal component can be obtained.
The horizontal axis of Fig.7 indicates the position of the 21 observation points. The vertical axis indicates the 20 time channel. From Fig.7 which can be seen that the background value of the apparent resistivity around the sectional region is about 75Ω.m. It is the same with the value set above. A geologic body with relatively low resistance exists in the upper central region of the section profile. It can be seen that the apparent resistivity of early channels is relatively low, and the corresponding part is the 3D cube model with low resistivity, which reflects the existence of the low resistance.

The distribution of the abnormal body with low apparent resistivity is much the same in Fig.7 and Fig.8. The difference is that the abnormal body with low apparent resistivity in Fig.8 has stronger reaction within the scope of about 50m in the central section, and the apparent resistivity is lower (9Ω.m~17Ω.m). It can be conclude that the horizontal resolution ability of horizontal component is better than the vertical component.

The horizontal and vertical resolution ability can be improved by combining and comparing the apparent resistivity profiles of the horizontal component and vertical component.

In addition, four observation lines are set on the both sides of the principal section for forward modeling calculation. The abnormal body reflected by each observation line is the same with the principal section. The 3D chart can be composed by the data of 9 observation lines which have 189 points totally.

From the above two figures, the distribution of single abnormal body can be clearly obtained, therefore the space location and the size of the abnormal body can be precisely determined.

### 3.2 Two abnormal bodies model
The array field distribution will be adopted for two abnormal bodies. There are two 3D cube model in homogenous half space. The resistivity of the homogenous half space is $\rho=75\,\Omega\cdot m$. The two abnormal bodies are cube with dimension of $50m \times 50m \times 50m$, its resistivity is $\rho = 1\,\Omega\cdot m$. The top of the cube is $30m$ below the ground, and the distance between the two cubes is $40m$. The model scheme shows below:

![Model scheme of two 3D cube](image1)

![The apparent resistivity profiles of the vertical component of two 3D model array fields](image2)

Through forward modeling calculation, the attenuation voltage of the magnetic vertical component and horizontal component of each observation point can be obtained. According to the apparent resistivity definition, the apparent resistivity profiles of each point can be obtained and shown in the charts below:

It can be seen from Fig.12 that the location with relatively low resistivity exist at Point 0 and Point 100, which indicates that two geologic bodies with relatively low resistance exist under the corresponding ground. The two abnormal bodies with low apparent resistivity are relatively apparent in the early channels, the apparent resistivity is $43\,\Omega\cdot m \sim 57\,\Omega\cdot m$. While in the late channels the two abnormal bodies have little effect, and the apparent resistivity is basically the background value.

In addition, 20 observation lines are set on the both sides of the principal section for forward modeling calculation. The 3D chart can be composed by the data of 41 observation lines which have 1681 points totally.

![3-D apparent resistivity chart of two abnormal bodies](image3)

![3D apparent resistivity slice chart of two abnormal bodies](image4)
4. Conclusion

According to the feature that the function form of vertical component and horizontal component is similar, the vertical and horizontal response of different offset can be obtained through a series of numerical calculation. At the same time the apparent resistivity expression of each component can be got. Theoretical model calculation proved that the method is correct and effective. The resolution ability can be improved greatly by combining the horizontal component and vertical component.

Array type observation and vector synthesis, also is a new attempt. Theoretical calculation shows that these methods have the very good auxiliary function to improve the precision of the transient electromagnetic explanation, and have certain directive significance to further research of high resolution explain technology.

References

[1] Li Xiu. The theory and application of transient electromagnetic sounding. Xi’An: Shaanxi Science Technology Press, 2002.1-48, 84-97,102-105 (in Chinese).

[2] Piao Huarong. Principles of geoelectromagnetic methods. Beijing : Geological Publishing House, 1990 (in Chinese).

[3] Fang Wenzao, Li Yuguo, Li Xiu. Theory of transient electromagnetic method sounding. Xi’an: Northwest Industry University Press, 1993 (in Chinese).

[4] Niu Zhilian. Time domain transient electromagnetic principle. Changsha: Central South University of Technology Press. 1993, 1-15 (in Chinese).

[5] Xi Zhenzhu; Liu Jian; Long Xia et al. Three-component measurement in transient electromagnetic method. Journal of Central South University (Science and Technology), 2010 2, 41(1): 272-276 (in Chinese).

[6] Kaufman, A. A., G. V. Keller. Frequency and transient sounding. Elsevier Science Publ. Co. Inc. 1983.

[7] Nabighian M. N. Electromagnetic methods in applied geophysics (Volume 1. Theory) Zhao Jingxiang Trans. Beijing: Geology Press, 1992 (in Chinese).

[8] Xue Guoqiang et al. Characters of response of large-loop transient electro-magnetic field [J]. Oil Geophysical Prospecting. 2007, 10, 42(5): 586-590 (in Chinese).

[9] Liu Jintao, Gu Hanming, Hu Xiangyu et al. Analysis of three-component transient electromagnetic [J]. Yangtze River, 2008, 39: 114-116 (in Chinese).

[10] Chen C S, Chiu W H, Lin C R. Three component time-domain electromagnetic surveying: modeling and data analysis [J]. Piers online, 2008, 4(4): 475-480 (in Chinese).

[11] Bai Denghai. Numerical calculation of all-time apparent resistivity for central loop transient electromagnetic method [J]. Chinese Journal of Geophysics. 2003, 46(5): 697-704 (in Chinese).

[12] Li Jiangping, Li Tonglin, Zhao Xuefeng et al. Study on the TEM all-time apparent resistivity of arbitrary shape loop source over the layered medium [J]. Progress in Geophysics. 2007.11, 22(6) (in Chinese).

[13] Ding Lijuan, Cheng Qiyuan. Numerical computing methods . Beijing: Beijing Institute of Technology Press, 2005.8 (in Chinese).