GLUONS IN THE LATTICE SU(2) CLASSICAL GAUGE FIELD

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The SU(2) gluonic correlation functions, glueball effective masses in the $J^P = 0^+, 2^+$ and $0^-$ channels were calculated from the lattice classical gauge configurations which were obtained by smoothing the thermal gauge configurations through the improved cooling method. The instanton-induced attractive force in the $0^+$ channel and the repulsive force in the $0^-$ channel are confirmed in the Monte Carlo simulation. There is evidence that the instanton vacuum contribution to the $0^+$ glueball mass is significant.

1 Introduction

QCD predicts there exist glueballs. The structure and formation mechanism of glueballs are so extricated that it must be explored nonperturbatively. Extensive Monte Carlo simulations have been performed and a few statements seem to be firmly established in the glueball sector: i) the scalar glueball is the lightest with the mass in the 1.6-1.8 GeV range; ii) the tensor glueball and the pseudoscalar one are significantly heavier than the scalar one with the mass ratio $m_{2^+}/m_{0^+} \simeq 1.4$ and $m_{0_-}/m_{0^+} \simeq 1.5 - 1.8$, respectively; iii) The scalar is much smaller than other glueballs($r_{0^+} \simeq 0.2 fm$, $r_{2^+} \simeq 0.8 fm$). From these statements one can find that glueballs are much heavier than typical quark-model hadrons and the size difference between the scalar and the tensor glueball is much larger than that of the similar mesons( a similar measurement for the $\pi$ and $\rho$ mesons gives $0.32 fm$ and $0.45 fm$). This indicates that the spin-dependent forces between gluons are stronger than that between quarks.

The information about the gluonic interactions can be obtained by studying the correlation functions of gluonic operators with the relevant quantum numbers. Schäfer and Shuryak calculated the correlation functions and the Bethe Salpeter amplitudes in an instanton-based model of the QCD vacuum. Their results show that instantons lead to a strong attractive force in the $J^{PC} = 0^{++}$ channel, thus the scalar glueball is much smaller than other glueballs. In the $0^{-+}$ channel the corresponding force is repulsive, and in the $2^{++}$ case it is absent.

Recently the cooling method, a smoothing algorithm for the gauge configurations, has been developed to extract classical contents( such as instantons)
of QCD vacuum on the lattice. This motivates us to study the relationship of instantons and glueballs by the lattice simulation. The goal of this work is to study the instanton effects in the glueball masses and gluonic correlation functions in the lattice SU(2) gauge theory. The tadpole improved Symanzik’s action is used on an anisotropic lattice to perform the Monte Carlo simulation as accurate as possible on coarse lattices. The smearing and fuzzy scheme are applied to the glueball operators to increase the signal-to-noise ratio. These schemes have been verified to be successful in Morningstar and Peardon’s calculation and in our work. We employ the improved cooling to extract the instanton configurations. The gluonic correlators are measured during the cooling procedure and the glueball masses are extracted by the exponential fall-off of these correlators with respect to the Euclidean time. To investigate the spin dependence of the instanton effects, the behaviors of the gluonic vacuum correlation functions are also explored in this work.

The simulation details, the results and the discussion are given in section II. Section III is the conclusion and summary.

2 Lattice Calculation

If the lattice action is properly improved, the costs of the Monte Carlo simulation will be greatly reduced and the numerical study can be performed on much coarser lattice system. In this work, the gluonic correlation functions and the masses of the lowest-lying glueballs in the SU(2) gauge theory are calculated from uncooled and cooled configurations. The simulations are performed on an $8^3 \times 24$ anisotropic lattice by using tadpole-improved Symanzik’s action. The lattice spacing in the temporal direction is smaller than that in the spatial direction with the anisotropy ratio $\xi = a_s/a_t = 3$, so that the physically useful signals can be obtained before they would be undermined by the statistical fluctuations. The form of the lattice gauge action we use is expressed as

$$S_1 = \beta \sum_{x,s > s'} \left[ \frac{5}{3} P_{ss'} \xi u_s^4 - \frac{1}{12} R_{ss'} \xi u_s^6 - \frac{1}{12} R_{st's} \xi u_s^6 \right]$$

where $\beta = \frac{4}{g^2}$, $g$ is the gauge coupling, $P_{\mu\nu}$ is the plaquette operator and $R_{\mu\nu}$ 2 $\times$ 1 rectangular operator, $u_s$ and $u_t$ are tadpole improvement parameters. $u_s$ is defined by $u_s = \langle (1/2) Tr W_{sp} \rangle^{1/4}$, while $u_t$ is set to be 1 because of $u_t = 1 - O(\xi^{-2})$ ($\xi$ is much bigger than one).
In the continuum theory, the gluonic operators with quantum numbers $0^+$, $0^−, 2^+$ are defined by the field strength squared, the topological charge density, and the energy density, respectively,

$$O_S = (gG_{\mu\nu}^a)^2, \quad O_P = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} g^2 G_{\mu\nu} G_{\rho\sigma},$$

$$O_T = \frac{1}{4} (gG_{\mu\nu}^a)^2 - g^2 G_{0\alpha}^a G_{0\alpha}^a. \quad (2)$$

The correlation functions $C(x)$ for the Euclidean separation $x$ are defined as

$$C_T(x) = \langle 0| O_T(x) O_T(0) |0 \rangle. \quad (3)$$

The glueball spectra are obtained by the spatial integral of two points functions of operators with definite $J^PC$ quantum numbers, such as

$$C_T(t) = \int d\vec{x} \langle 0| O_T(x) O_T(0) |0 \rangle$$

$$= \sum_{n_T} \frac{1}{2m_{n_T}} |\langle 0| O_T| n_T, \vec{p} = 0 \rangle|^2 e^{-m_{n_T} |t|}$$

$$\propto |\langle 0| O_T| n_T, \vec{p} = 0 \rangle|^2 e^{-m_{n_T} |t|} (t \to \infty).$$

The lattice simulation of masses of lowest-lying hadrons is well established. The lattice version of $O_T$ are defined as:

$$O_{0^+}(t) = \sum_{\vec{x}} [P_{12}(\vec{x}, t) + P_{23}(\vec{x}, t) + P_{31}(\vec{x}, t)], \quad (5)$$

$$O_{2^+}(t) = \sum_{\vec{x}} [P_{12}(\vec{x}, t) - P_{13}(\vec{x}, t)]. \quad (6)$$

The corresponding glueball masses are extracted from the exponential falloff of the lattice correlation functions.

$$m_T(t) = \frac{1}{a_t} \ln \left( \frac{C(t)}{C(t - a_t)} \right) \quad (7)$$

We first calculate the lowest-lying glueball masses on the uncooled configurations at $\beta = 1.0, 1.1, 1.2$ through the standard procedure, where the fuzzy and smearing methods are used to increase the signal-to-noise ratio. The configurations are thermalized through 5000 Monte Carlo sweeps (each sweep is composed of four heatbath sweeps and one micro-canonical iteration) and the
measurements are performed on 10000 configurations. We analyze the statistical error by dividing the 10000 configuration into 20 bins. The physical scale is set by calculating the heavy quark potential. With the value of the string tension $\sqrt{\sigma} = 440 MeV$, $a_s$ takes the value 0.277 $fm$, 0.233 $fm$, and 0.183 $fm$ respectively for $\beta = 1.0, 1.1, 1.2$. The glueball masses at $\beta = 1.0$ are extracted to be

$$
\begin{align*}
m_{0^+} &= 1634 \pm 60 MeV \\
m_{2^+} &= 2305 \pm 72 MeV \\
m_{0^-} &= 2574 \pm 224 MeV
\end{align*}
$$

Our results are in good agreement with that of other work. To explore the role of instantons in the gluonic correlation functions and glueball masses, the cooling method is used to get the instanton configurations. The link-updating of the cooling iteration is defined as below

$$
U_\mu \rightarrow U'_\mu = c\Sigma^+\mu,
$$

where $c$ is the normalization factor so that $U'_\mu$ is an element of SU(2) group. $\Sigma_\mu$ is defined by the local action

$$
S(U_\mu) = 1 - \frac{1}{2}TrU_\mu \sum_{\nu \neq \mu} (staples) \equiv 1 - \frac{1}{2}TrU_\mu \Sigma_\mu,
$$

This transformation justifies that the local action will be minimized. We take the measurement on a sample of 500 configurations generated and cooled by the lattice action $S_1$. The gluonic correlators are measured every five cooling sweeps and the spectra are extracted by the same approach as that was used in the uncooled case. In order to avoid the auto-correlation, every two adjacent configuration are separated by 20 Monte Carlo sweeps. Each configuration is cooled by 100 steps. The cooling procedure is monitored by calculating topological charges at each cooling step. The lattice charge density operator ($Q_{cont.}(x) = \frac{1}{16\pi^2}TrF_{\mu\nu}(x)F_{\mu\nu}(x)$ in continuum theory) used in this work is the improved version,

$$
Q_L(x) = -\frac{1}{512\pi^2} \left( \frac{5}{3} Q_P(x) - \frac{1}{6} Q_S(x) \right) \equiv a^4(Q_{cont.}(x) + O(a^4))
$$

where

$$
Q_P(x) = \sum_{\mu\nu\rho\sigma=\pm1} \tilde{e}_{\mu\nu\rho\sigma} Tr(P_{\mu\nu}(x)P_{\rho\sigma}(x))
$$

4
Figure 1: The correlation function $C(t)$ in $0^+$ channel at various cooling steps are illustrated. The dots, the rectangulars, the triangles and the upside-down triangles correspond to the cooling steps 10, 25, 50 and 100, respectively.

$$Q_S(x) = \sum_{\mu\nu\rho\sigma = \pm 1}^4 \tilde{\epsilon}_{\mu\nu\rho\sigma} Tr \left[ \left( S_{1\mu}^2(x) + S_{2\mu}^1(x) \right) \left( S_{1\rho\sigma}^2(x) + S_{2\rho\sigma}^1(x) \right) \right].$$

Here $\tilde{\epsilon}_{\mu\nu\rho\sigma}$ is the standard Levi-Civita tensor for positive directions while for negative ones the relation $\tilde{\epsilon}_{\mu\nu\rho\sigma} = -\tilde{\epsilon}_{-\mu\nu\rho\sigma}$ holds. $P_{\mu\nu}$ is the plaquette operator in the $\mu - \nu$ plane, while $S_{ij}^{\mu\nu}$ is $i \times j$ rectangular operator. We find that after several cooling sweeps the charges will reach meta-stable plateaus with approximate integer values.

Fig. 1, 2, 3 show the correlation functions of the three channels. In each channel, the correlation function is plotted at 10, 25, 50 and 100 cooling steps. The glueball “masses” are extracted in the same way as in the uncooled case and are plotted in Fig. 4, 5, 6.

Shuryak and Schäfer has calculated the gluonic correlation functions in the single-instanton background field. Taking the free gluon propagator into consideration, the correlators are expressed as:

$$C_S(x) = \frac{384g^4}{\pi^4x^8} + n\rho^4C_{inst}(x),$$  \hspace{1cm} (10)

$$C_P(x) = -\frac{384g^4}{\pi^4x^8} + n\rho^4C_{inst}(x),$$  \hspace{1cm} (11)

$$C_T(x) = \frac{24g^4}{\pi^4x^8},$$  \hspace{1cm} (12)
where $g$ is the running coupling constant and $C_{\text{inst}}(x)$ is the instanton contribution. $C_{\text{inst}}(x)$ is definitely positive and its detailed expression is abbreviated here. The signs in Eqn.(10) and Eqn.(11) are impressive: the two contributions reinforce in the scalar channel (attractive) but cancel to some extent in the pseudoscalar channel (repulsive). There is no effect in the tensor channel.

Our results shown in the figures are qualitatively in agreement with these comments. At small $t$, the amplitudes of the correlation functions decrease rapidly with the increase of the cooling steps, so do the extracted masses. In the intermediate and large $t$ range, the correlation functions are less sensitive to the cooling. The "masses" take out short plateaus in the 0$^+$ channel and the 2$^+$ channel in the intermediate $t$. These are the results of that the ultraviolet components of the gauge fields are removed by the cooling but their topological structure is kept more stable. Small instantons are smoothed out by the cooling iterations (we monitor the cooling procedure and find that only the instantons with sizes greater than $2a_s$ survive), so the correlation functions and the "masses" at small $t$ may be underestimated. The fact that in the 2$^+$ channel the correlation function tends to be flat (Fig. 2) and the mass-plateau decreases rapidly (Fig. 5) imply that there is no instanton effect in this channel. In contrast to this, the fall-off the correlation functions with $t$ in the 0$^+$ and 0$^-$ channels are obvious even after 100 cooling sweeps. The height of the mass plateau in the 0$^+$ channel is less sensitive to the cooling steps in the intermediate $t$ range (Fig. 4). This means the contribution of instantons to the glueball mass in the 0$^+$ channel is significant. As illustrated in Fig. 3, the
correlation function in the $0^-$ channel even changes sign at $t = 5a_t - 8a_t$ after 50 cooling steps. This is an evidence of that the gluonic interaction induced by instantons is repulsive in the pseudoscalar channel. The mass information can not be clearly extracted in this channel (Fig. 6).

3 Conclusion

We simulated the SU(2) glueball masses in the cooled and cooled gauge configurations. The results in the uncooled case are in agreement with the existing work. In the cooled case there is evidence of the instanton-induced attractive and repulsive interactions in the $0^+$ channel and the $0^-$ channel, respectively, which are predicted by Shuryak and Schäfer based on the instanton liquid models. The contribution of instantons to the $0^+$ glueball mass is significant.

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Figure 4: The $0^+$ channel 'glueball masses' are extracted from the exponential decay of the correlators in cooled case. The masses vs. Euclidean time at various cooling steps are plotted.

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Figure 5: The $2^+$ channel 'glueball masses' are extracted from the exponential decay of the correlators in cooled case. The masses vs. Euclidean time at various cooling steps are plotted.

Figure 6: The $0^-$ channel 'glueball masses' are extracted from the exponential decay of the correlators in cooled case. The masses vs. Euclidean time at various cooling steps are plotted. No plateaus appear. The mass information can not be obtained in this channel.