Standard Model Derivation from a 4-d Pseudo-Conformal Field Theory

C. N. Ragiadakos
IEP, Ministry of Education
email: ragiadak@gmail.com

ABSTRACT

Pseudo-conformal field theory (PCFT) is a 4-d action, which depends on the lorentzian Cauchy-Riemann (LCR) structure, determined by a tetrad satisfying precise integrability conditions. This LCR-tetrad defines a class of Einstein metrics and an electroweak U(2) connection. A static massive and a massless LCR-manifolds are found. The massive soliton is compatible with the Kerr-Newman manifold. Its two conjugate LCR-structures have $g=2$ gyromagnetic ratio and opposite charges, suggesting their identification with the electron and positron particles with a naked ring essential singularity. Their background CP(3) formulation bypasses the Hawking-Penrose singularity theorems. The massless LCR-manifold does not have a charge, suggesting its identification with the neutrino. The LCR-structure formalism provides the particles separated into left and right handed chiral parts, the left and right columns of the homogeneous coordinates of the grassmannian G(4,2). The electron LCR-tetrad explicitly provides its gravitational and electroweak potentials (dressings). Their distributional nature permit us to use the Bogoliubov causal perturbative approach (improved by Epstein-Glaser and Scharf et. al. techniques) as a pure mathematical harmonic expansion in the Gelfand rigged Hilbert-Fock space of tempered distributions of the Poincare representations (corresponding free fields). This S-matrix computational procedure in the proper Gelfand triplet, provides the standard model lagrangian for the electromagnetic, weak and Higgs interactions. The interacting terms and the relation between the masses and the coupling constants are implied by the Scharf et. al. operational algorithm on the free fields. In PCFT the computed gluon potential (static quark dressing) cannot be treated with the Bogoliubov procedure. Possible solutions of the dark matter and neutrino mixing problems are discussed.
1 INTRODUCTION

It is well known that 4-dimensional generally covariant lagrangian models, based on riemannian geometry, are not renormalizable. Even if they are endowed with the Weyl symmetry, they turn out not to be compatible with quantum mechanics, because of the emergence of the product of two Weyl tensors. It is well understood that lagrangians with second order derivatives generate negative norm states in the Hilbert space. Hence we have to look for metric independent lagrangians, which are not topological.

The original idea[23][29] to study (Cauchy-Riemann) CR-structure dependent field theories emerged from the observation that the Polyakov string action does not essentially depend on the metric $\gamma_{\alpha\beta}$ of the 2-dimensional surface, because in the light-cone coordinates $(\xi_-, \xi_+)$ it takes the metric independent form

$$I_S = \int d^2z \partial_- X^\nu \partial_+ X^\nu \eta_{\mu\nu}$$

which is not a topological lagrangian. This metric independence is based on the fundamental property of the 2-dimensional riemannian manifolds to admit a coordinate system $(\xi_-, \xi_+)$ such that $ds^2 = 2\gamma d\xi_+ d\xi_-$. This metric independence of the action, without being topological, is the crucial property of the Polyakov action, which should be transferred to four dimensions and not the simple Weyl invariance. That is, the four dimensional analogous symmetry has to be a form of pseudo-conformal symmetry (Cauchy-Riemann structure) and not the conventional Weyl symmetry.

Four dimensional spacetimes metrics cannot generally take the form (1.2). Only metrics which admit two geodetic and shear free null congruences $\ell^\mu \partial_\mu$, $n^\mu \partial_\mu$ can take[11][12] the analogous form

$$ds^2 = 2g_{a\beta} dz^\alpha dz^{\bar{\beta}} , \quad \alpha, \bar{\beta} = 0, 1$$

where $z^b = (z^\alpha(x), z^{\bar{\beta}}(x))$ are generally complex coordinates. In this case we can write down the following metric independent Yang-Mills-like action

$$I_G = \int d^4z \sqrt{-g} g^{\alpha\bar{\beta}} F_{\alpha\beta} F_{\bar{\alpha}\bar{\beta}} + c.c. = \int d^4z F_{01} F_{\bar{0}\bar{1}} + c.c. \quad (1.4)$$

which depends on the CR-structure coordinates, but it does not depend on the metric.

Notice the similarity of this 4-dimensional action with the 2-dimensional Polyakov action (1.2). In the place of the "field" $X^\nu$, which is interpreted as the background 26-dimensional Minkowski spacetime in string theory, we now have a gauge field $A_{\mu}$, which we have to interpret as the gluon, because the field equations generate a linear potential instead of the Coulomb-like $(\frac{1}{r})$ potential of ordinary Yang-Mills action.
The present action is based on the lorentzian CR-structure[31], which is determined by two real and one complex independent 1-forms \((\ell, n, m, \overline{m})\), which satisfy the relations

\[
\begin{align*}
d\ell &= Z_1 \wedge \ell + i \Phi_1 m \wedge \overline{m} \\
dn &= Z_2 \wedge n + i \Phi_2 m \wedge \overline{m} \\
dm &= Z_3 \wedge m + \Phi_3 \ell \wedge n
\end{align*}
\]

(1.5)

where the vector fields \(Z_1^\mu, Z_2^\mu\) are real, the vector field \(Z_3^\mu\) is complex, the scalar fields \(\Phi_1\), \(\Phi_2\) are real and the scalar field \(\Phi_3\) is complex. This structure essentially replaces the riemannian structure of the spacetime in the Einstein general relativity. The form (1.5) is completely integrable via the (holomorphic) Frobenious theorem, which implies that the lorentzian CR-manifold (LCR-manifold) is defined[1] as a 4-dimensional totally real submanifold of \(\mathbb{C}^4\) determined by four special (real) functions,

\[
\begin{align*}
\rho_{11}(z^\alpha, \overline{z}^\alpha) &= 0 \\
\rho_{12}(z^\alpha, \overline{z}^\tilde{\alpha}) &= 0 \\
\rho_{22}(\overline{z}^\tilde{\alpha}, z^\tilde{\alpha}) &= 0
\end{align*}
\]

(1.6)

where \(\rho_{11}, \rho_{22}\) are real and \(\rho_{12}\) is a complex function and \(z^b = (z^\alpha, \overline{z}^\tilde{\alpha})\), \(\alpha = 0, 1\) are the local structure coordinates in \(\mathbb{C}^4\). Notice the special dependence of the defining functions on the structure coordinates. They are not general functions of \(z^b\). The separation of chiralities in the standard model is caused to this property. The LCR-structure is more general than the riemannian structure of general relativity and permits the invariance of the set of solutions to the pseudo-conformal transformations (in the E. Cartan and Tanaka terminology)[4].

The action (1.4) takes the following generally covariant form

\[
I_G = \int d^4x \sqrt{-g} \left\{ (\ell^\mu m^\nu F_{j\mu\nu}) (n^\rho \overline{m}^\sigma F_{j\rho\sigma}) + (\ell^\mu m^\nu F_{j\mu\nu}) (n^\sigma \overline{m}^\rho F_{j\rho\sigma}) \right\}
\]

(1.7)

\[
F_{j\mu\nu} = \partial_\mu A_{j\nu} - \partial_\nu A_{j\mu} - \gamma f_{jik} A_{j\mu} A_{k\nu}
\]

where we have to consider the additional action term with the integrability conditions on the tetrad

\[
I_C = -\int d^4x \left\{ \phi_0(\ell^\mu m^\nu - \ell^\nu m^\mu)(\partial_\mu \ell_\nu) + \phi_1(\ell^\mu m^\nu - \ell^\nu m^\mu)(\partial_\mu m_\nu) + \phi_2(n^\rho \overline{m}^\sigma - n^\sigma \overline{m}^\rho)(\partial_\mu n_\nu) + \phi_3(\overline{m}^\rho - \overline{m}^\rho)(\partial_\mu \overline{m}_\nu) + \text{c.conj.} \right\}
\]

(1.8)

These Lagrange multipliers introduce the integrability conditions of the tetrad and make the complete action \(I = I_G + I_C\) self-consistent and the usual quantization techniques may be used[22]. The action is formally renormalizable[28], because it is dimensionless and metric independent. Its path-integral quantization is also formulated[32] as functional summation of open and closed 4-dimensional lorentzian CR-manifolds in complete analogy to the summation of
2-dimensional surfaces in string theory\textsuperscript{22}. These transition amplitudes of a quantum theory of LCR-manifolds provides the self-consistent algorithms for the computation of the physical quantities. But unfortunately, I have not yet found a method to compute these functional integrals, therefore I will use a "solitonic" technique\textsuperscript{25},\textsuperscript{27}, which appears in the linearized Einstein gravity approximation. The present paper should be considered as a continuation of the last one\textsuperscript{32}, which will be called [paper I]. In this [paper I] the reader may find a review of the (lorentzian) LCR-structure\textsuperscript{31}, which is the fundamental mathematical structure of the present pseudo-conformal field theory (PCFT). The properties of this structure will be used in the present work without proof, in order to facilitate the understanding of the general framework of the procedure.

The LCR-structure defining relations are invariant under the following tetrad-Weyl transformations

\begin{align}
\ell'_\mu &= \Lambda \ell_\mu, \quad n'_\mu = N n_\mu, \quad m'_\mu = M m_\mu \\
n'^\mu &= \frac{1}{N} n^\mu, \quad m'^\mu = \frac{1}{M} m^\mu
\end{align}

(1.9)

with non-vanishing $\Lambda$, $N$, $M$. I point out that we have not yet introduced a metric. The tetrad with upper and lower indices is simply a basis of tangent and cotangent spaces. But the tetrad does define a class $[g_{\mu\nu}]$ of symmetric tensors

\begin{align}
g_{\mu\nu} &= \ell_\mu n_\nu + \ell_\nu n_\mu - m_\mu \overline{m}_\nu - m_\nu \overline{m}_\mu
\end{align}

(1.10)

Every such tensor may be used as a metric to build up the riemannian geometry of general relativity, because its local signature is $(1, -1, -1, -1)$. But this form always admits two geodetic and shear free null congruences and hence it does not cover all the metrics of general relativity. I think that this restriction will not cause any phenomenological problem to the model, because all the known gravitational objects do admit such congruences. Besides, notice that the tetrad-Weyl symmetry (1.9) is larger than the well known metric-Weyl symmetry of the quadratic Weyl tensor lagrangian. Because of these symmetries the PCFT is renormalizable\textsuperscript{28}.

The conventional solitons\textsuperscript{10} are defined as classical solutions with finite mass determined via the energy-momentum conserved current, which besides, are "protected" to deform to the vacuum configuration by topological invariants. In the present context the soliton is a LCR-manifold which in the linearized Einstein $(g_{\mu\nu})$ gravity approximation has finite mass computed from the conserved gravitational source. The LCR-structure is "protected" by topological invariants and/or its relative invariants defined\textsuperscript{31},\textsuperscript{30} from the non-vanishing $\Phi_i$.

The standard model gauge field will be explicitly defined from the LCR-tetrad. The electron solitonic LCR-surface and its electromagnetic, weak interactions and Higgs potentials will be also determined. The construction of the effective "quantum field model" will be performed using the Bogoliubov-Medvedev-Polivanov (BMP)\textsuperscript{2} axiomatic framework. The S-matrix is function of the free effective fields and it is constructed order by order. Starting from the classical interaction (correspondence principle) the effective QFT will
be built up introducing the necessary additional terms and conditions such that the final action to be well defined. The incorporation of the Epstein-Glaser ([9]) regularization procedure and the Scharf $Q$-charge technique [36] [37] provides the standard model action.

In order to make things as simple as possible I will proceed step by step. In section II, the linearized Einstein gravity approximation is described and the graviton and its source current is defined. In the subsection we prove how the LCR-structure solves the puzzle of the naked singularity of Kerr-Newman manifold with the electron mass, charge and spin parameters. It makes clear that LCR-structure may bypass the Hawking-Penrose singularity theorem in riemannian geometry, which did not permit to relate the electron with the Kerr-Newman manifold. In section III, the degenerate LCR-structure will be studied, which is assumed to be the vacuum [32] of the effective QFT. I repeat this analysis, already done in [paper I], in order to make clear not only the vacuum conservation of the Poincaré group, but also the deep LCR-structure origin of the chirality, which is fundamental in the standard model. The left and right separation of the infinite group of pseudo-conformal transformations (in the E. Cartan and Tanaka terminology) is a fundamental property of the LCR-structure. We will show that outside of the distributional singularity of the solitonic potentials (dressings) the symmetry group is the Poincaré group which is essential to consider the Bogoliubov perturbative expansion as a Poincaré harmonic expansion. In section IV, the static LCR-manifold is explicitly derived, which is identified with the electron. Its complex conjugate is identified with the positron, because they are found to have opposite charges. The electromagnetic potentials are defined by a self-dual 2-form, which happens to be locally integrable. The photon interacts with the electron and positron currents with opposite charges. The corresponding metric is the Kerr-Newman manifold, where the electromagnetic potential is found to be proportional to the geodetic and shear-free null vector $\ell_{\mu}$. It is not a computational accident, because the defined (in section V) electroweak connection extends this relation to all the electroweak $U(2)$ gauge fields. A subsection is devoted to describe the BMP procedure, which suggests that the order-by-order implied formal electromagnetic potential of the electron should be identified to the corresponding expansion of $A_{\mu}$ relative to the Kerr-Newman parameter $a$. The left and right chiralities emerge from the homogeneous coordinates of grassmannian manifold $G_{4,2}$ of the lines of $CP^3$. In order to have a physical intuition through all the mathematical steps, I will use the generally complex Newman trajectories [17] [18] to determine the LCR-structure. They are essentially characteristic properties of the ruled surfaces $G_{4,2}$ of $CP^3$

In section V, the massless stationary LCR-structure, determined from a reducible quadratic Kerr polynomial, is computed. It has a clear asymmetry between the left and right handed parts of the $G_{4,2}$ homogeneous coordinates. This LCR-structure is identified with the neutrino, because only its left-handed part is not degenerate (trivial), while the right part is that of the trivial vacuum. Besides, its integrable self-dual 2-form does not define any charge. The general LCR-tetrad ($\ell, m; n, \overline{m}$) can be adapted into the [2.78 of paper I] $U(2)$ group and
the electroweak potentials are computed in the electron static LCR-structure. I
think that this discovery fulfills the Einstein’s goal towards a geometric unified
theory.

In section VI, the gauge field compatible with the symmetries of LCR-
structure is identified with the gluon field and a confining gauge potential is
computed in the static “quark” LCR-structure. But the gluon propagator does
not coincide with that of conventional quantum chromodynamics, because it
does not permit an expansion in the spin parameter $a$. In section VII the dark
matter and the neutrino mixing problems proposing possible solutions.

The general result is that the gravitational, electromagnetic, weak and Higgs
interactions are exactly derived, but the gluon propagator of QCD must be
replaced with the present confining gauge field of PCFT. The interested reader
may find more details in my frequently updated Research eBook [34].

2 GENERAL RELATIVITY AND GRAVITON

The geometric dynamical variables of the present model are the two real and
the one complex vectors $(\ell, n, m, \overline{m})$, which define the lorentzian CR-structure.
They determine the symmetric tensor (1.10), which will be identified with the
Einstein metric. But these metrics are not invariant under the tetrad-Weyl
symmetry (1.9) of the LCR-structure. Therefore a LCR-structure defines a
class of metrics $[g_{\mu\nu}]$. Two metrics related by a tetrad-Weyl symmetry belong
to the same class.

On the other hand the local $SO(1, 3)$ symmetry[6] of this symmetric tensor
does not preserve the geodetic and shear free conditions ($\kappa = \sigma = \lambda = \nu = 0$)[12]
of two null vectors, which are equivalent to the definition (1.6) of the LCR-
structure. Therefore it is not a symmetry of the present fundamental (geometric)
LCR-structure.

The tetrad $(\ell, n, m, \overline{m})$ may be used to write down other symmetric tensors,
but it is the form (1.10) that makes the lagrangian (1.4) of the model metric
independent. Besides, this precise metric form permits us to define the flat
spacetimes and asymptotically flat spacetimes at null infinity using directly the
LCR-structure solutions.

If $[g_{\mu\nu}]$ contains the Minkowski metric, the LCR-structure is determined [31] by
an element of the $G_{4,2}$ grassmannian projective space with homogeneous co-
ordinates $X^m$, which belong to the Kerr surface $K(X^i), i = 1, 2$ of $CP^3$, and
such that

$$X^i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} X = 0 \quad (2.1)$$

Notice that the inverse is also true. These LCR-structures always define a
Minkowski metric on the "real axis" of the Siegel domain, up to the appearance
of singularities. This is the characteristic (Shilov) boundary of the $SU(2, 2)$
symmetric classical domain. This manifold generally admits an infinite number
of LCR-structures locally determined by an irreducible or reducible Kerr homo-
geneous polynomials. The characteristic property of these "flat" LCR-structures
is that $X$ admits the form

$$X = \begin{pmatrix} \lambda \\ -i\xi\lambda \end{pmatrix}$$  \hspace{1cm} (2.2)$$

where $x$ is a hermitian, $\lambda$ is a complex $2 \times 2$ matrix, and the two columns are determined by the Kerr homogeneous polynomials. That is, their "left" and "right" parts decuple. The LCR-structures, which cannottake the form (2.1) will be called "curved", and vice-versa their approximations restricted to these terms will be called the "flatprints" of a generic LCR-structure.

In the linearized Einstein gravity approximation \cite{16}, we find the following linearized gravity relations in the limit

$$g_{\mu\nu} = \eta_{\mu\nu} + kh_{\mu\nu} + O(k^2)$$

for the curvature tensor. The second Bianchi identity takes the form

$$\partial_\mu \hat{R}_{\nu\rho\sigma\tau} = 0$$ \hspace{1cm} (2.4)$$

where the covariant derivative becomes minkowiskian and $\ldots$ denotes antisymmetrization. They imply the conservation condition of the Einstein tensor

$$\partial_\mu \hat{E}_\mu^\nu = \partial_\mu [\hat{R}_\mu^\nu - \frac{1}{2} \delta_\mu^\nu \hat{R}] = 0$$ \hspace{1cm} (2.5)$$

This means that the Einstein tensor is conserved in the linearized Einstein gravity limit. Besides, in the empty space it becomes the free wave equation of a massless spin-2 particle.

Recall that in relativistic quantum field theory the field, which satisfies a spin-s free wave equation, describes\cite{36} a representation of the Poincaré group i.e. a spin-s particle, and vice-versa, a spin-s particle is described by a field representation of the Poincaré group, which satisfies the free wave equation. Hence, in the present model, Einstein’s general relativity naturally emerges. The existence of a graviton is simply implied in the weak gravity limit.

In the Penrose spinorial formalism\cite{20}, the linearized Bianchi identity takes the form

$$\partial^A_B \hat{\Psi}_{ABCD} = \partial^A_B [\hat{\Psi}_{ABCD} \lambda^A \lambda^B \lambda^C \lambda^D]$$ \hspace{1cm} (2.6)$$

where $\ldots$ denotes symmetrization. The left-hand side contains the Weyl tensor and the right-hand side of the relation contains the Ricci tensor, which describes the sources. It is considered as the graviton wave equation.

I have already pointed out that PCFT defines only metrics which admit geodetic and shear-free congruences. In this case the flags $\lambda^A$ of the LCR-structure tetrad must satisfy the condition

$$\Psi_{ABCD} \lambda^A \lambda^B \lambda^C \lambda^D \simeq k \tilde{\Psi}_{ABCD} \lambda^A \lambda^B \lambda^C \lambda^D + O(k^2) = 0$$ \hspace{1cm} (2.7)$$
where the linearized gravity approximation has been also considered. The Weyl tensor of the Minkowski spacetime vanishes. Therefore the gravitational content remains in $\hat{\Psi}_{ABCD}$ and $\hat{\lambda}^A(x)$ are the flags of the flatprint of the LCR-structure tetrad.

Notice that the gravitational singularities are locally determined by the zeroes and infinities of the metric and its inverse, which cannot be removed by a coordinate change. These metric-singularities essentially coincide with the LCR-structure singularities, because $\det(g_{\mu\nu}) = -C(\det[\ell_{\mu}, m_{\mu}, m_{\mu}])^2$. Hence, the singularity sources of the gravitational radiation coincide with the singularities of the LCR-structure, which defines the corresponding metric. But in the linearized Einstein gravity limit, the singular region of the form (2.2) is determined by the Kerr functions. These are the regions where two roots of the homogeneous functions $K_i(X^i), i = 1, 2$ coincide. More details will be given in the next sections, where the same method will be used to define the electromagnetic potential.

Newman has found\[17\] that the Kerr function condition (for a null congruence to be geodetic and shear-free) may be replaced with a (generally complex) trajectory $\xi^a(\tau)$. In the present case of the LCR-structure formalism, this is done by assuming that the $G_{4,2}$ two homogeneous coordinates $i = 1, 2$ must have the form

$$X^i = \begin{pmatrix} \lambda^i \\ -i\xi_{(i)}(\tau)i\lambda^i \end{pmatrix}$$

(2.8)

$$\xi_{(i)}(\tau) = \eta_{ab} \sigma^b$$

where $\sigma^b$ and $\eta_{ab}$ are the Pauli matrices and the Minkowski metric respectively. Here, I have to point out that the consideration of two generally different complex Kerr homogeneous functions is somehow misleading. In conventional algebraic geometry the notion of reducible polynomial is used. The irreducible Kerr polynomial

$$K(Z) = Z^1Z^2 - Z^0Z^3 + 2aZ^0Z^1$$

(2.9)

of the electron LCR-structure is equivalent with the complex trajectory $\xi^a = (\tau, 0, 0, ia)$. The complex trajectory is a characteristic property of the ruled surfaces\[34\] of $CP^3$.

The flatprint LCR-structure coordinates are determined by the condition

$$(x - \xi_{(i)}(\tau_i))\lambda^i = 0$$

(2.10)

that admits one non-vanishing solution for every column $i = 1, 2$ of the homogeneous coordinates of $G_{4,2}$. This is possible if

$$\det(x - \xi_{(i)}(\tau_i)) = \eta_{ab}(x^a - \xi^a_{(i)}(\tau_i))(x^b - \xi^b_{(i)}(\tau_i)) = 0$$

(2.11)

which gives the two solutions $z^0 = \tau_1(x)$ and $z^\tilde{0} = \tau_2(x)$. The other structure coordinates are $z^1 = \frac{\lambda^1}{\lambda^2}$ and $z^\tilde{1} = -\frac{\lambda^2}{\lambda^1}$ where

$$\lambda^{A\bar{J}} = \begin{pmatrix} (x^1 - i\tau^2) - (\xi^1_{(j)}(\tau_j) - i\xi^1_{(j)}(\tau_j)) \\ (x^0 - x^3) - (\xi^0_{(j)}(\tau_j) - \xi^0_{(j)}(\tau_j)) \end{pmatrix}$$

(2.12)
Notice that the trajectory technique for computation of the structure coordinates incorporates the notion of the classical causality, which is apparently respected by (2.11).

The singularity of the flatprint LCR-structure occurs at \( \det[\lambda^{A1}(x), \lambda^{A2}(x)] = 0 \). Recall that the left and right columns of the homogeneous coordinates of \( G_{4,2} \) may be determined ("move") with different trajectories, if the corresponding homogeneous Kerr polynomial is reducible. In the simple case when both move with the same trajectory \( \xi^a(\tau) = (\tau, \xi^1(\tau), \xi^2(\tau), \xi^3(\tau)) \), the singularity occurs at \( \tau_1(x) = \tau_2(x) \), which is

\[
(x^i - \xi^i(t))(x^j - \xi^j(t))\delta_{ij} = 0
\]

(2.13)

If \( \xi^i_R \) and \( \xi^i_I \) are the real and imaginary parts of the trajectory, we find that the locus of the solitonic LCR-structure is

\[
(x^i - \xi^i_R(t))(x^j - \xi^j_R(t))\delta_{ij} - (x^i - \xi^i_I(t))(x^j - \xi^j_I(t))\delta_{ij} = 0
\]

(2.14)

Note that if \( \xi^i_I(t) \) is bounded, the LCR-structure may be interpreted as a soliton with trajectory \( \xi^i_R(t) \) and a locus at the perimeter of the circle of radius \( (\xi^i_R(t))^2 \) around its trajectory. This locus (a two dimensional surface) is a singularity of the gravitational potential and a source of the corresponding gravitational radiation, but it is not a singularity of the LCR-structure viewed as a surface of the \( G_{4,2} \) grassmannian, because the matrix \( X^{mi} \) has not rank two at this surface.

### 2.1 Solving the electron naked singularity "problem"

We saw that the LCR-structure implies Einstein’s general relativity based on the metric geometric structure. But there is an essential difference. The LCR-structure bypasses the naked singularity problem of the Kerr-Newman metric. This metric admits two geodetic and shear free null congruences, which are related with the Kerr polynomial \( (2.9) \). It also admits two commuting Killing vectors, which are identified with the time-translation and \( z \)-rotation generators of the Poincaré group. Carter’s[5] discovery that the gyromagnetic ratio of the Kerr-Newman manifold is fermionic (that of the electron \( g = 2) \)[19] shocked the community of general relativists. Many tried to identify the Kerr-Newman spacetime with the electron without success, because the electron constants imply the existence of a naked singularity in the Kerr-Newman spacetime.

The electron mass \( M_e \), charge \( e^2 \) and spin parameter \( a \) have the values

\[
M = \frac{M}{M_p} = 4.18 \times 10^{-23}
\]

\[
e^2 = \frac{q^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137}
\]

\[
a = \frac{\hbar}{2M_e} = 2.09 \times 10^{23}
\]

(2.15)

\[
a^2 \gg e^2 \gg M^2
\]
Hence \( a^2 + e^2 - M^2 > 0 \), and the electron metric has an essential naked singularity, which is not permitted in riemannian geometry. This is a problem for general relativity, because its fundamental quantity, the metric, does not "see" the algebraic structure. It is known (and well described in many books of general relativity) that its analytic extension has two sheets \( x^b \) and \( x'^b \) which are determined by the two roots

\[
\begin{align*}
r &= \pm \left\{ \frac{(x^1)^2 + (x^2)^2 + (x^3)^2 - a^2}{2} + \sqrt{\left[\frac{(x^1)^2 + (x^2)^2 + (x^3)^2 - a^2}{2}\right]^2 + a^2(x^3)^2} \right\}^{\frac{1}{2}} \tag{2.16}
\end{align*}
\]

These two surfaces constitute the boundary \( U(2) \) of the bounded realization of the \( SU(2, 2) \) classical domain and their correspondence is the well known Cayley transformation. The spinorial electron naked singularity in \( U(2) \) universe can be properly incorporated in PCFT, while it is rejected as "unphysical" by the riemannian formalism. In the context of the unbounded realization this may be studied using the following LCR-ray tracing.

We have already found that in the flatprint electron LCR-structure the structure coordinates are

\[
\begin{align*}
z^0 &= t - r + ia \cos \theta, \quad z^1 = e^{i\varphi} \tan \frac{\theta}{2}, \\
z^0 &= t + r - ia \cos \theta, \quad z^1 = \frac{r + ia}{r - ia} e^{-i\varphi} \tan \frac{\theta}{2}, \tag{2.17}
\end{align*}
\]

and the cartesian coordinates are

\[
\begin{align*}
x^0 &= t \\
x^1 + ix^2 &= (r - ia) \sin \theta e^{i\varphi} \\
x^3 &= r \cos \theta \quad \tag{2.18}
\end{align*}
\]

Then \( L^\mu \partial_\mu z^\alpha = 0 \) implies that the outgoing \( L^\mu \) integral curves (rays) are determined by the surfaces

\[
s_1 := t - r, \quad s_2 := \theta, \quad s_3 := \varphi \tag{2.19}
\]

Assuming the caustic coordinates \((r, s_1, s_2, s_3)\), which have the property \((0, s_1, \frac{\pi}{2}, s_3)\) to be on the caustic. In this caustic coordinate system the LCR-rays are traced by the relation

\[
\begin{align*}
x^0_L(r) &= s_1 + r \\
x^1_L(r) &= (r \cos \varphi + a \sin \varphi) \sin \theta \\
x^2_L(r) &= (r \sin \varphi - a \cos \varphi) \sin \theta \\
x^3_L(r) &= r \cos \theta \quad \tag{2.20}
\end{align*}
\]

\[
\text{Jacobian} = |r^2 + a^2 \cos^2 \theta| \sin \theta
\]

The source of the LCR-rays are at \( r = 0 \), i.e.

\[
\begin{align*}
x^0_L(0) &= s_1 \\
x^1_L(0) &= a \sin \varphi \sin \theta \\
x^2_L(0) &= -a \cos \varphi \sin \theta \\
x^3_L(0) &= 0 \tag{2.21}
\end{align*}
\]
the disk found above. Notice that the rays with \( s_2 := \theta \neq \frac{\pi}{2} \) pass through the disk.

The \( N^\mu \partial_\mu \tilde{z} = 0 \) implies that its incoming \( N^\mu \) rays are determined by the surfaces

\[
s'_1 := t + r, \quad s'_2 := \theta, \quad s'_3 := \varphi + \arctan \frac{2ar}{a^2 - r^2} \tag{2.22}
\]

Then we find the congruence

\[
\begin{align*}
x^0_N(r) &= s'_1 - r \\
x^1_N(r) &= [r \cos s'_3 - a \sin s'_3] \sin \theta \\
x^2_N(r) &= [r \sin s'_3 + a \cos s'_3] \sin \theta \\
x^3_N(r) &= r \cos \theta \tag{2.23}
\end{align*}
\]

\( \text{Jacobian} = [r^2 + a^2 \cos^2 \theta] \sin \theta \)

As expected the velocities \( x^i_L(t) \) and \( x^i_N(t) \) have asymptotically opposite radial directions.

We will now show that the origin of the essential singularity of the Kerr manifold is the intersection of the two sheets of the static electron regular quadric (in the unbounded Siegel realization) \( CP^3 \). In the flatprint case we have

\[
\begin{align*}
X^0 &= 1, \quad X^1 = \lambda, \quad X^2 = -i[(x^0 - x^3) - (x^1 - ix^2)\lambda] \\
X^3 &= -i[-(x^1 + ix^2) + (x^0 + x^3)\lambda] \tag{2.24}
\end{align*}
\]

and the Kerr polynomial and its two solutions are

\[
(1 + ix^2)\lambda^2 + 2(x^3 - ia)\lambda - (x^1 + ix^2) = 0
\]

\[
\lambda_{1,2} = \frac{-(x^3 - ia) \pm \sqrt{\Delta}}{x - iy}, \quad \Delta = (x^1)^2 + (x^2)^2 + (x^3)^2 - a^2 - 2iax^3 \tag{2.25}
\]

where \( \lambda_{1,2} \) are the two values of \( \lambda \) on the two sheets of the quadric. The intersection curve of these two sheets is

\[
\Delta = (x^1)^2 + (x^2)^2 + (x^3)^2 - a^2 - 2iax^3 = 0
\]

\[
x^3 = 0, \quad (x^1)^2 + (x^2)^2 = a^2 \tag{2.26}
\]

which, after the LCR projection to \( \mathbb{R}^4 \), becomes the singularity ring of the electron (Kerr-Newman) manifold. Notice that the quadratic surface is regular and the intersection of the two branches is implied by the projection. The points of the algebraic intersection curve (the branch curve) of the (regular) quadric of \( CP^3 \) are regular points like any other point of the quadric.

In paper I we saw that the bounded realization of a flat LCR-manifold is \( U(2) \), which is covered by two \( \mathbb{R}^4 \) sheets through the Cayley \( 2 \to 1 \) transformation

\[
\begin{align*}
\text{For } s := R_0 \frac{\sin \rho}{\cos \tau + \cos \rho} > 0 \\
x^0 &= T_0 \frac{\sin \rho}{\sin \tau} \\
x^1 + ix^2 &= R_0 \frac{\sin \rho}{\cos \tau + \cos \rho} \sin \sigma e^{i\chi} \\
x^3 &= R_0 \frac{\sin \rho}{\cos \tau + \cos \rho} \cos \sigma \tag{2.27}
\end{align*}
\]
and the second $\mathbb{R}^4$ is identified with $s < 0$,

\[
\begin{align*}
F o r \ s := R_0 \frac{\sin \rho}{\cos \tau + \cos \rho} < 0 \\
x'^0 &= T_0 \frac{\sin \tau}{\cos \tau + \cos \rho} \\
x'^1 + ix'^2 &= -R_0 \frac{\sin \rho}{\cos \tau + \cos \rho} \sin \sigma e^{ix} \\
x'^3 &= -R_0 \frac{\sin \rho}{\cos \tau + \cos \rho} \cos \sigma
\end{align*}
\]  
\tag{2.28}

The constants $T_0$ and $R_0$ are related to the time and space sizes. Notice that this is the Penrose artificial compactification of the Minkowski spacetime, but in the context of PCFT, this is implied by the formalism itself. In the case of the Penrose artificial compactification these two sheets $s \geq 0$ communicate through the scri+ and scri- infinities. In the case of the electron flatprint LCR-structure, these two sheets communicate through the glued two discs $(x^1)^2 + (x^2)^2 < a^2$ too, because we may assume

\[
\begin{align*}
& r = + \left( \frac{s^2-a^2}{2} + \sqrt{\left(\frac{s^2-a^2}{2}\right)^2 + a^2(x^3)^2} \right)^{\frac{1}{2}} \text{ for } s > 0 \\
& r = - \left( \frac{s^2-a^2}{2} + \sqrt{\left(\frac{s^2-a^2}{2}\right)^2 + a^2(x^3)^2} \right)^{\frac{1}{2}} \text{ for } s < 0
\end{align*}
\]  
\tag{2.29}

Notice that in the identified region (the disc for both sheets) $r = 0$ in both sheets. That is, $r = 0$ occurs at $x^3 = 0$ and $s^2 \leq a^2$ for both sheets $s \geq 0$.

The two LCR-congruence $\ell^\mu = \frac{dx^\mu}{dr}$ and $n^\mu = \frac{dz^\mu}{dr}$ of the flatprint electron LCR-manifold can be easily implied from the calculations of the previous section. The starting idea is that the structure coordinates $z^\alpha(x)$ provide the three invariants $(s_1, s_2, s_3)$ along the ray, which label the $\ell$-ray $x^\mu_\ell(r)$, and the structure coordinates $z^\alpha(x)$ provide the invariants $(s_1', s_2', s_3')$, which label the $n$-ray $x^\mu_n(r)$. Hence we simply have the same forms, but we let $r \in (-\infty, +\infty)$ and at $r = 0$ we pass to the second $x^\mu_\ell(r), x^\mu_n(r) \in \mathbb{R}^4$ sheet.

A complete visualization of the rays $w_{L,N}(r; s_1, s_2, s_3) \in U(2)$ taking $r \in (-\infty, +\infty)$ can be done in the bounded realization of the flatprint electron (as the $U(2)$ boundary of the $SU(2, 2)$ classical domain). From the relation

\[
\begin{align*}
Y^0 = \frac{1}{\sqrt{2}}(X^0 + X^2) \quad& Y^1 = \frac{1}{\sqrt{2}}(X^1 + X^3) \\
Y^2 = \frac{1}{\sqrt{2}}(X^0 - X^2) \quad& Y^3 = \frac{1}{\sqrt{2}}(X^1 - X^3)
\end{align*}
\]  
\tag{2.30}

between the bounded $Y^{ni}$ and unbounded $X^{ni}$ homogeneous coordinates and

\[
X^{ni} = \begin{pmatrix}
1 & -\bar{z}^1 \\
\bar{z}^1 & \bar{z}^\dagger \\
-i(z^0 - ia) & i(z^0 + ia)\bar{z}^\dagger \\
-i(z^0 + ia)z^1 & -i(z^0 + ia)
\end{pmatrix}
\]  
\tag{2.31}
we find
\[ Y^{mi} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 - i(z^0 - ia) & (-1 + i(z^0 - ia)z^1) \\
1 - i(z^0 + ia) & 1 - i(z^0 + ia) \\
1 + i(z^0 - ia) & -(1 + i(z^0 - ia)z^1) \\
1 + i(z^0 + ia) & 1 + i(z^0 + ia)
\end{pmatrix} \] (2.32)

Like previously, we use the relations (2.17) to find the labels of \( L^\mu \) rays
\[ w_{11} = \frac{Y^{21}Y^{12} - Y^{11}Y^{22}}{Y^{01}Y^{12} - Y^{11}Y^{02}} \, , \, w_{12} = \frac{Y^{01}Y^{22} - Y^{21}Y^{02}}{Y^{01}Y^{12} - Y^{11}Y^{02}} \] (2.33)

between the bounded projective \( w \in U(2) \) and homogeneous \( Y^{mi} \) coordinates, we finally find the rays \( w_L(r; s_1, s_2, s_3) \in U(2) \) in the complete bounded universe \( U(2) \).

The intersection (touching) of the two \( \mathbb{R}^4 \) sheets in \( U(2) \) coordinates can be computed by simply making the Cayley transformation of the cartesian form of the ring singularity. Then we find that in \((\tau, \rho, \sigma, \chi)\) coordinates the ring singularity (the caustic of the congruence) and its "tube" connecting the two sheets is
\[ \sigma = \frac{\pi}{2} \, , \, R_0^2 \frac{\sin^2 \rho}{(\cos^2 \tau + \cos^2 \rho)^2} \leq a^2 \]
\[-\pi < \rho < \pi \, , \, -\pi < \tau < \pi \] (2.34)

which apparently contains both rings of the two \( \mathbb{R}^4 \) copies.

In principle we can compute the explicit form of the \( L^\mu \) ray tracing in \( U(2) \), but it is too complicated. From the cartesian coordinates we have
\[ x^0 = \frac{T_0 \sin \tau}{\cos \tau + \cos \rho} \, , \, x^1 + ix^2 = \frac{R_0 \sin \rho}{\cos \tau + \cos \rho} \sin \sigma e^{i\chi} = \sqrt{r^2 + a^2}e^{-i\arctan \frac{r}{\sqrt{r^2 + a^2}}} \sin \theta e^{i\varphi} \]
\[ x^3 = \frac{R_0 \sin \rho}{\cos \tau + \cos \rho} \cos \sigma = r \cos \theta \]
\[ s := \frac{T_0 \sin \tau}{\cos \tau + \cos \rho} \]
which imply the following relations of the curve determining variables \((s_1, s_2, s_3)\)
\[ s_1 := \frac{R_0 \sin \tau}{\cos \tau + \cos \rho} - r \]
\[ s_2 := \tan \theta = \frac{r}{\sqrt{r^2 + a^2}} \tan \varphi \]
\[ s_3 := \varphi = \chi + \arctan \frac{a}{r} \] (2.36)

and the convenient affine parameter of the congruence is \( r \). The bounded \((s_1, s_2, s_3)\) curve in \( SU(2) \) for \( \tau = 0 \) can be easily seen.
3 THE VACUUM

The permitted (restricted) holomorphic transformations \( z'^a = f^a(z^\beta) \), \( z'^{\tilde{a}} = f^{\tilde{a}}(z^{\tilde{\beta}}) \) may be used to find coordinates (called regular coordinates) such that (1.6) take the forms

\[
\begin{align*}
\rho_{11}(\overline{z^\alpha}, z^\tilde{0}) &= \Im z^\tilde{0} - \phi_{11}(z^1, \Re z^0) \\
\rho_{12}(\overline{z^\alpha}, z^\tilde{0}) &= z^{\tilde{1}} - z^1 - \phi_{12}(z^\tilde{1}, z^0) \\
\rho_{22}(\overline{z^{\tilde{\alpha}}}, z^{\tilde{0}}) &= \Im z^{\tilde{0}} - \phi_{22}(z^1, z^0) \\
\phi_{ij}(0) &= 0, \quad d\phi_{ij}(0) = 0
\end{align*}
\]

(3.1)

where \( z^1, z^{\tilde{1}} \), are the complex coordinates of \( \mathbb{C}P^1 \), because this regular form of the LCR-structure continues to permit the following \( SL(2, \mathbb{C}) \) transformation

\[
\begin{align*}
z'^1 &= \frac{e + dz^1}{a + bz^1}, \quad z'^{\tilde{1}} = \frac{-\overline{e} + \overline{d}z^{\tilde{1}}}{\overline{a} + \overline{b}z^{\tilde{1}}} \\
ad - bc &= 1
\end{align*}
\]

(3.2)

That is, the corresponding spinors transform relative to the conjugate representations of \( SL(2, \mathbb{C}) \)

\[
\left( \begin{array}{c}
\lambda' \\
\lambda' z'^1 \\
\end{array} \right) = \left( \begin{array}{cc}
a & b \\
c & d \\
\end{array} \right) \left( \begin{array}{c}
\lambda \\
\lambda z^1 \\
\end{array} \right)
\]

\[
\left( \begin{array}{c}
-\overline{\lambda' z'^1} \\
-\overline{\lambda} \\
\end{array} \right) = \left( \begin{array}{cc}
a & b \\
c & d \\
\end{array} \right)^{-1} \left( \begin{array}{c}
-\overline{\lambda z^1} \\
-\overline{\lambda} \\
\end{array} \right)
\]

(3.3)

\[
ad - bc &= 1
\]

The action is generally covariant without a precise metric. Therefore the observed in nature Poincaré symmetry must be found in the set of solutions and it must preserve the physical vacuum. The LCR-structure has been extensively studied in the context of general relativity under the name of spacetimes with two geodetic and shear free null congruences. In this context we see that a quite general class of LCR-manifolds take the form of real surfaces of the Grassmannian manifold \( G_{4,2} \). The charts of its typical non-homogeneous coordinates are determined by the invertible pairs of rows. If the first two rows constitute an invertible matrix, the chart is determined by \( \det \lambda \neq 0 \) and the corresponding affine space coordinates \( r \) are defined by

\[
X = \begin{pmatrix}
X^{01} & X^{02} \\
X^{11} & X^{12} \\
X^{21} & X^{22} \\
X^{31} & X^{32}
\end{pmatrix}
= \left( \begin{array}{c}
\lambda^A \\
-i r A^A \lambda^A
\end{array} \right)
\]

(3.4)

\[
r_A^A = \eta_{ab} r^a \sigma^b_{A'A}
\]
Then the LCR-structure defining relations take the form
\[
\begin{align*}
\rho_{11}(X^{m1}, X^{n1}) &= 0 = \rho_{22}(X^{m2}, X^{n2}) \\
\rho_{12}(X^{m1}, X^{n2}) &= 0 \\
K(X^{mj}) &= 0
\end{align*}
\]  
(3.5)
where all the functions are homogeneous relative to \(X^{n1}\) and \(X^{n2}\) independently, which must be roots of the homogeneous holomorphic (generally reducible) Kerr polynomial \(K(Z^m)\). In this context, we see that the LCR-structures determined by the relations
\[
X^{mi}E_{mn}X^{nj} = 0 \quad , \quad K(X^{mj}) = 0
\]  
(3.6)
are flat, i.e. they generate a minkowskian class of metrics \([\eta_{\mu\nu}]\). Besides, the very fruitful notion of asymptotically flat spacetimes at null infinity\(^{20}\) may be transferred to the asymptotically flat LCR-structures, which satisfy the conditions
\[
\begin{align*}
X^{m1}E_{mn}X^{n1} &= 0 = X^{m2}E_{mn}X^{n2} \quad , \quad \rho_{12}(X^{m1}, X^{n2}) = 0 \\
K(X^{mj}) &= 0
\end{align*}
\]  
(3.7)
Notice that \(SU(2, 2)\) is the symmetry group of these solutions. The consideration of open LCR-manifolds implies the removal of a point (infinity) of the Shilov boundary of the bounded \(SU(2, 2)\) classical domain\(^{21}\), which restricts the group down to its Poincaré group up to an additional dilation group, which will be finally broken by the mass of the electron. This Poincaré symmetry group is identified with the observed Poincaré symmetry in nature.

Let us now consider the LCR-structures determined by a generally complex Newman trajectory\(^{18}\) \(\xi^b(\tau)\) via the relations
\[
\begin{align*}
X &= \left( \begin{array}{c} \lambda^{Aj} \\ -i\sigma_{\lambda}^{\lambda A}A^{Aj} \end{array} \right) = \left( \begin{array}{c} \lambda^{Aj} \\ -i\xi_{\lambda}^{\lambda A}A^{Aj} \end{array} \right) \\
\det(r_{A'\Lambda} - \xi_{A'\Lambda}(\tau)) &= \eta_{ab}(r^a - \xi^a(\tau))(r^b - \xi^b(\tau)) = 0 \\
(r_{A'\Lambda} - \xi_{A'\Lambda}(\tau))\lambda^{Aj}A^j = 0
\end{align*}
\]  
(3.8)
The coordinate system of an observer is determined by a word line \(\xi^a(\tau) = (\tau, 0, 0, 0)\), which defines a LCR-structure compatible with the Minkowski metric via the relations
\[
\begin{align*}
\left( \begin{array}{c} \lambda^i \\ -i\xi_a(\tau)\sigma^a\lambda^i \end{array} \right) &= \left( \begin{array}{c} \lambda^i \\ -ix_a\sigma^a\lambda^i \end{array} \right)
\end{align*}
\]  
(3.9)
\( \tau_i \) are the two solutions and \( \lambda^i \) are the spinors of the corresponding (future pointing) null vectors, i.e.

\[
\det[(x_a - \xi_a(\tau))\sigma^a] = 0 \quad , \quad \tau_{1,2} = x^0 \mp \sqrt{(x^i)^2} \tag{3.10}
\]

and the corresponding null vectors are

\[
\begin{pmatrix}
\sqrt{(x^i)^2} \\
x^1 \\
x^2 \\
x^3
\end{pmatrix} = \lambda^1 \sigma^a \lambda^1, \quad \begin{pmatrix}
\sqrt{(x^i)^2} \\
x^1 \\
x^2 \\
x^3
\end{pmatrix} = \lambda^2 \sigma^a \lambda^2 \tag{3.11}
\]

This shows that the two spinors \( \lambda^{A1} \) and \( \lambda^{A2} \) define spatially inverted null vectors. Hence, these two spinors must belong to the conjugate chiral representations of the \( SL(2,\mathbb{C}) \) group. This means that the spinors defined by the left and right columns of the homogeneous coordinates of the vacuum (degenerate) LCR-structure must have\(^{13}\) opposite chiralities, because parity is an external automorphism of the orthochronous proper Lorentz group. It corresponds a spinor of the fundamental representation to a spinor of its conjugate representation.

Note that this trajectory satisfies the Poincaré invariant normalization condition \( \eta_{ab} \frac{dx^a}{d\tau} \frac{dx^b}{d\tau} = 1 \). This is the vacuum of the precise observer. Any other Poincaré transformed observer \( \xi^a(\tau) = (v^0, v^i, c^i) \) has a pseudo-conformally equivalent vacuum (LCR-structure). In [paper I], I have already shown that this vacuum is invariant under the Poincaré transformations determined with infinity fixed with the projective chart condition \( \det \lambda = 0 \).

In the same chart we may define a different LCR-manifold, which apparently belongs to a different representation of the Poincaré group, because it is determined by the two real trajectories \( \xi^a(\tau) = (\tau, 0, 0, \mp \tau) \), which satisfy the Poincaré invariant normalization condition \( \eta_{ab} \frac{dx^a}{d\tau} \frac{dx^b}{d\tau} = 0 \). The homogeneous coordinates of this LCR-structure and the appropriate structure coordinates \( z^\alpha, z^\beta \) are

\[
X = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-i(x^0 - x^3) & i(x^1 - i x^2) & 1 & 0 \\
i(x^1 + ix^2) & -i(x^0 + x^3) & -i z^0 & -i z^1
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & 1 \\
-i z^0 & -i z^1 \\
-i z^1 & -i z^0
\end{pmatrix} \tag{3.12}
\]

In complete analogy to the preceding vacuum [paper I], the degenerate relations

\[
z^0 - \overline{z^0} = 0 \quad , \quad z^\alpha - \overline{z^\alpha} = 0 \quad , \quad z^\beta - \overline{z^\beta} = 0 \tag{3.13}
\]
remain formally invariant under the Lorentz subgroup

\[
\begin{pmatrix}
X''_1 \\
X''_2
\end{pmatrix} = \begin{pmatrix} B_{11} & 0 \\
0 & B_{22} \end{pmatrix} \begin{pmatrix} X'_1 \\
X'_2
\end{pmatrix}
\]

\[
B_{11} = \begin{pmatrix} a & b \\
c & d \end{pmatrix}, \quad B_{22} = \begin{pmatrix} \frac{1}{b} & -\frac{1}{a} \\
-\frac{1}{b} & \frac{1}{a} \end{pmatrix}
\]

(3.14)

and the real translation subgroup

\[
\begin{pmatrix}
X'_1 \\
X'_2
\end{pmatrix} = \begin{pmatrix} I & 0 \\
B_{21} & I \end{pmatrix} \begin{pmatrix} X_1 \\
X_2
\end{pmatrix}
\]

\[
B_{21} + B_{21}^\dagger = 0
\]

(3.15)

of the \(SU(2, 2)\) group.

In the following sections, quantum field theory of the standard model will be viewed as a harmonic analysis of distribution valued free fields rigged Hilbert-Fock space of the Poincaré group representations. This mathematical picture is based on our consideration that the universe is described by a LCR-structure condition (3.1) \(\rho_{ij} = 0\). The singular regions of the Schwartz distributions determine the ”location” of the particles and their potentials (gravitational, electroweak and Higgs) are their corresponding representative functions. Recall that these representatives have locally integrable singularities at the location of the particle and are regular outside. This framework is characterized by the geometric Kaehler structure

\[
ds^2 = 2\frac{\partial^2 \det(\rho_{ij})}{\partial z^a \partial z^b} dz^a d\bar{z}^b, \quad \omega = 2i \frac{\partial^2 \det(\rho_{ij})}{\partial z^a \partial z^b} dz^a \wedge d\bar{z}^b
\]

(3.16)

This means that the universe LCR-submanifold \(\rho_{ij} = 0\) is real analytic in the ”empty” space. Besides, it is a totally real lagrangian submanifold of \(\mathbb{C}^4\) with the induced metric identified with the Einstein metric of the universe. The reader must realize that here we are not talking about the phenomenological astrophysical metrics, but for everything from the very ”small” to the ”largest” possible. We know [1] that any totally real submanifold admits an analytic transformation \(z^b = f^b_p(r^a)\) in the neighborhoods of their analytic points \(p\), which trivializes the real submanifold

\[
\rho_{ij} = \frac{\tilde{r} - \tilde{r}^\dagger}{2i} = 0
\]

\[
\tilde{r} := \begin{pmatrix} r^0 - r^3 \\
-(r^1 + ir^2) \\
-(r^1 - ir^2) \\
r^0 + r^3 \end{pmatrix}
\]

\[
r^a := x^a + iy^a
\]

(3.17)

This analytic transformation does not preserve the LCR-structure, but it constitutes a legitimate transformation in the Kaehler manifold and the real vari-
ables are the well-known Darboux real coordinates. The metric and the symplectic 2-form in $\mathbb{C}^4$ are

$$ds^2 = \frac{1}{2} \sum_{a,b} \frac{\partial^2 (-(\hat{r}_c - \hat{r}_c')^2)}{\partial r_a \partial r_b} dr^a dr^b = \eta_{ab}(dx^a dx^b + dy^a dy^b)$$

$$\omega = \frac{1}{2} \sum_{a,b} \frac{\partial^2 (-(\hat{r}_c - \hat{r}_c')^2)}{\partial r_a \partial r_b} dr^a \wedge dr^b = i\eta_{ab}dr^a \wedge dr^b = 2\eta_{ab}dx^a \wedge dy^b \tag{3.18}$$

Apparently this locally analytic transformation cannot be extended to the singular regions of the universe. Therefore our approach has to be properly adapted. We will keep the Minkowski coordinates and we will extend the set of functions to the set of Schwartz distributions (generalized functions).

Notice that the 2-form (3.18) is not the ordinary form of the symplectic phase space. The emergence of the Minkowski metric implies that locally the phase space is the $SU(2, 2)$ symmetric Siegel domain $\mathfrak{F}_{\mathfrak{r}} > 0$ with the corresponding projective form $X^1 E_{\mathfrak{U}} X > 0$. The (singular) Schwartz distributions are characterized by their characteristic locally integrable functions, which we will call potentials. Their derivatives are not locally integrable, but they are proper distributions because of the space of the appropriate test functions. We will use the Bogoliubov perturbative approach based on the tempered distributions and the corresponding rigged Hilbert space of the Poincaré representations (free fields).

In order to clarify the subsequent mathematical approach let me mention the well known mathematical problem that Pythagoras affronted and the final solution that Cauchy gave about 2000 years later. Integers and their ratios (the rational numbers) were the only numbers that Pythagoras new. Pythagarian theorem gave him the possibility to compute/measure square roots of numbers (like $\sqrt{2}$). Near the end of his life, he realized that $\sqrt{2}$ is not a rational number. We now know that this is a real number, and we need an infinite series of rational numbers to approximate it. It was Gelfand who through his triple (rigged Hilbert space) permitted the harmonic expansion of the generalized functions to representations of the Poincaré group. Hence the discovery of the electron and neutrino as free stable distributional solitons, the computation of its generalized state, gravitational and electroweak potentials turn out to be a mathematical and not a physical problem. For the computation of the hypotenuse we simply need the proper completion of the rational numbers to the complete set (Hilbert space) of real numbers.

4 BOGOLIUBOV’S PERTURBATIVE QFT

The Bogoliubov-Medvedev-Polivanov method approaches the axiomatic formulation of a quantum field theory starting from the S-matrix and the introduction of a ”switching on and off” function $c(x) \in [0, 1]$ and assuming the following expansion of the S-matrix

$$S = 1 + \sum_{n \geq 1} \frac{1}{n} \int S_n(x_1, x_2...x_n)c(x_1)c(x_2)...c(x_n) [dx] \tag{4.1}$$
where $S_n(x_1, x_2, \ldots x_n)$ depends on the complete free field functions (the local Poincaré representations of the particles) and not its separate "positive" and "negative" frequency parts. That is, the S-matrix is an operator in the Fock space of free relativistic particles. It satisfies the following axioms

**Poincaré covariance:**  
$U_P S_n(x_1, x_2, \ldots x_n) U_P^\dagger = S_n(Px_1, Px_2, \ldots Px_n)$

**Unitarity:**  
$SS^\dagger = S^\dagger S = 1$

**Microcausality:**  
$\frac{\delta}{\delta c(x)} \left[ \frac{\delta S(c)}{\delta c(x)} \right] = 0$ for $x \preceq y$

**Correspondance principle:**  
$S_1(x) = iL_{int}[\phi(x)]$

where $\phi(x)$ denotes the free particle fields and $x \preceq y$ means $x^0 < y^0$ or $(x - y)^2 < 0$. A general solution of these conditions is the Dyson form of the time evolution unitary matrix (S-matrix)

$S = T[\exp(iL[\phi(x); c(x)])]$

$L[\phi(x); c(x)] = L_{int}[\phi(x)]c(x) + \sum_{n \geq 1} \frac{1}{n} \int A_{n+1}(x, x_1, \ldots x_n)c(x)c(x_1)\ldots c(x_n)[dx]$

where $A_n(x, x_1, \ldots x_n)$ are quasilocal quantities, which permit the renormalization process. This order by order construction of a finite S-matrix (with possibly infinite hamiltonian and lagrangian) provides a well established algorithm to distinguish renormalizable with non-renormalizable interaction lagrangians\[3\].

The advantage of the BMP procedure is that it can be used in the opposite sense. Knowing the Poincaré representations, they are identified with "free particles" with precise mass and spin. Then they are described with the corresponding free fields, which are used to write down an effective interaction lagrangian, suggested by the fundamental dynamics. In the present case, the fundamental dynamics is the PCFT and the particles are the solitonic solutions and their corresponding potentials which satisfy the wave equations. The suggested interaction takes the place of the "correspondence principle" in the BMP procedure. The order by order computation introduces counterterms to the action (with up to first order derivatives). If the number of the forms of the counterterms is finite, the action is normalizable and the model is considered compatible with quantum mechanics.

Epstein-Glaser\[9\] noticed that the renormalization procedure is an artifact of the improper replacement of the Dyson time-ordering with the step function (distribution). The multiplication of this distribution with the Wightman distributions is not mathematically permitted. They developed a proper regularization procedure using the scale function. This is very well described in the first book of Scharf\[36\]. In his second book\[37\], Scharf used the operational Krein structure to annihilate order by order the non-physical modes of the free fields. These two ameliorations made the Bogoliubov procedure an algorithmic Poincaré group harmonic expansion of the S-matrix as an operator distribution valued in the Gelfand rigged Hilbert-Fock space\[2\]

$S(\mathbb{R}^n) \subset L^2(\mathbb{R}^n) \subset S'(\mathbb{R}^n)$

$$20$$
of the tempered distributions, where the space $S(\mathbb{R}^n)$ is dense in $L^2(\mathbb{R}^n)$ relative to the supremum definition topology. Hence we only need now is to find the asymptotic fields (particles). The existence of the electron LCR-structure with its gravitational and electromagnetic potentials "triggers" the algorithmic process, providing the entire S-matrix of quantum electrodynamics.

The Kerr-Newman electrified spacetime is one of the physically interesting solutions in general relativity. It admits two geodetic and shear free null congruences, which are related with the Kerr polynomial (2.9). It also admits two commuting Killing vectors, which are identified with the time-translation and $z$-rotation generators of the Poincaré group. Carter's [5] discovery that the gyromagnetic ratio of the Kerr-Newman manifold is fermionic (that of the electron $g = 2$) [19] shocked the community of general relativists. Many tried to identify the Kerr-Newman spacetime with the electron without success.

After the identification of the phenomenological Poincaré symmetry with the $SU(2, 2)$ subgroup, which preserves infinity, it is straight-forward to compute the asymptotically flat LCR-structure, which admits the time-translation and $z$-rotation Killing vectors. It coincides with the LCR-structure found applying the Kerr-Schild ansatz procedure [27]. Recall that the electron is the unique charged stable leptonic particle of current phenomenology.

In the linearized Einstein-gravity approximation, the Kerr-Schild ansatz coincides with the approximation itself. This fact facilitates our calculation and interpretation. Hence, the $G_{4,2}$ point of the flatprint of the electron LCR-structure is determined from the static trajectory $\xi^b = (\tau, 0, 0, ia)$. The corresponding two spinors $\lambda^A$, which appear in its representation in the homogeneous coordinates have the form

$$\lambda^A = \begin{pmatrix} x^1 - i x^2 \\ x^0 - x^3 - \tau_1 - ia \\ x^0 - x^3 - \tau_2 - ia \end{pmatrix}$$

and the flat null tetrad is

$$L^a = \frac{1}{\sqrt{2}} \xi^A \lambda^{A1} \sigma^{a}_{A'B} \quad , \quad N^a = \frac{1}{\sqrt{2}} \xi^A \lambda^{A1} \sigma^{a}_{A'B}$$

$$M^a = \frac{1}{\sqrt{2}} \xi^A \lambda^{A1} \lambda^{B1} \sigma_{AB}$$

$$\epsilon_{AB} \lambda^{A1} \lambda^{B2} = 1$$

(4.6)

where the spinors have been properly normalized. In the Lindquist coordinates it takes the form

$$L^\mu \partial_\mu = \partial_t + \partial_r$$

$$N^\mu \partial_\mu = \frac{1}{2 \sqrt{r^2 + a^2 \cos^2 \theta}} \left( \partial_t - \partial_r + \frac{2a}{r^2 + a^2} \partial_\phi \right)$$

$$M^\mu \partial_\mu = \frac{1}{\sqrt{2} \sqrt{r^2 + ia \cos \theta}} \left( ia \sin \theta \partial_r + \partial_\theta + \frac{i}{\sin \theta} \partial_\phi \right)$$

(4.7)
Its covariant form is

\[
L_\mu dx^\mu = dt - dr - a \sin^2 \theta \ d\phi \\
N_\mu dx^\mu = \frac{r^2 + a^2}{2(r^2 + a^2 \cos^2 \theta)} [dt + \frac{r^2 + 2a^2 \cos^2 \theta - a^2}{r^2 + a^2} dr - a \sin^2 \theta \ d\phi] \\
M_\mu dx^\mu = \frac{-1}{\sqrt{2(r^2 + a^2 \cos^2 \theta)}} \left[ -i \sin \theta (dt - dr) + (r^2 + a^2 \cos^2 \theta) d\theta + i \sin \theta (r^2 + a^2) d\phi \right]
\]

(4.8)

The Kerr-Schild ansatz gives [24] [27] the general form of the curved LCR-manifold

\[
\ell_\mu = L_\mu \ , \ m_\mu = M_\mu \ , \ n_\mu = N_\mu + \frac{h(r)}{2(r^2 + a^2 \cos^2 \theta)} L_\mu
\]

(4.9)

where \( h(r) \) is an arbitrary function. Notice that for \( h(r) = -2mr + e^2 \) the Kerr-Newman space-time is found with electromagnetic potential

\[
A = \frac{q}{4\pi (r^2 + a^2 \cos^2 \theta)} (dt - dr - a \sin^2 \theta d\phi)
\]

(4.10)

\[
r^4 - [(x^1)^2 + (x^2)^2 + (x^3)^2 - a^2]r^2 - a^2(x^3)^2 = 0
\]

The fermionic parameter \( a \) appears in the electromagnetic and gravitational potentials (dressings) of the electron. Besides the electromagnetic potential is proportional to the null tetrad covector \( \ell_\mu \). The explicit computation of the electroweak connection (in section V) will make clear that it is not a computational accident.

From the deriving relations

\[
(x_a - \xi_a(\tau_j)) \sigma_\alpha^{A} \lambda^A_j = 0
\]

\[
x^a - \xi^a(\tau_j) = \left( \begin{array}{c}
\pm \sqrt{(x^1)^2 + (x^2)^2 + (x^3 - ia)^2} \\
x^1 \\
x^2 \\
x^3 - ia
\end{array} \right)
\]

(4.11)

we see that \( \lambda^{A2} \) satisfies a relation implied after a temporal reflection of the corresponding relation that \( \lambda^{A1} \) satisfies, because \((x^0 - \tau_2) = -(x^0 - \tau_1)\). Hence, these two spinors must belong to the conjugate chiral representations of the \( SL(2, \mathbb{C}) \) group. This means that the spinors defined by the left and right columns of the homogeneous coordinates of the electron LCR-structure must have opposite chiralities, because temporal reflection (like parity) is an external automorphism of the orthochronous proper Lorentz group. I want to point out that this relation is generalized only in the case of LCR-structures determined by one trajectory. In the general case of left and right columns of \( X^{ui} \) implied by different trajectories (reduced Kerr polynomials), they are not related with such a discrete symmetry. One has to go back to the regular LCR-structure coordinates \( \{x_{1,2}\} \) to reveal opposite chirality between left and right columns of the homogeneous coordinates of \( G_{4,2} \).
Because of the importance of the chirality emergence, I will now explicitly show that the temporal (and spatial) reflection, applied directly to the geodetic and shear free condition on $\lambda^A \partial x^\sigma$:

$$\lambda^A \lambda^B \sigma^b_{A' A} \frac{\partial}{\partial x^\sigma} \lambda^B = 0$$

where $\lambda^A = \lambda^0 \left( \frac{1}{\lambda} \right)$

implies the change of the $SL(2, \mathbb{C})$ representation.

Using my notation

$$x_{A'} = x_\mu \sigma^\mu_{A'} = \left( \begin{array}{c} x^0 - x^3 \\ -x^1 + ix^2 \\ x^0 + x^3 \end{array} \right)$$

I make the temporal reflection

$$x' = \left( \begin{array}{c} -x^0 - x^3 \\ -(x^1 - ix^2) \\ -x^0 + x^3 \end{array} \right) = -\epsilon \epsilon^{-1}$$

which implies

$$\frac{\partial \lambda'}{\partial x^\sigma} + \lambda' \frac{\partial \lambda'}{\partial x^\sigma_{A'}} = 0$$

where

$$\lambda' = -\frac{1}{\lambda}$$

As expected, the representation of the spinor changes to its conjugate one.

We saw that the chirality distinction is fundamental in the pseudo-conformal field theory (PCFT). The massive Poincaré representation of the flat LCR-structure is determined with the complex linear trajectory

$$\xi^b(s) = v^b s + c^b + i a^b$$

where $v^b$, $c^b$, $a^b$ are the real constants, which represent the constant velocity, the initial position and the spin of the classical configuration of the electron. Note that the present normalization of the parametrization is properly changed in order to assure the massive character of the representation. In the next section we will argue that the complex linear trajectories with $(\xi^b)^2 = 0$ is related to the
neutrino. That is the neutrino is the "boosted" electron in the projective LCR-structure formalism. Considering the accelerating electron as a ruled surface of \( CP^3 \) with a general trajectory \( \xi^b(\tau) \) the electron corresponds to the non-vanishing gaussian curvature and its neutrino to the corresponding tangential developable surface, which has vanishing gaussian curvature. Hence my conclusion is that the electron and its massless neutrino have to be treated as free Dirac fields in the Bogoliubov expansion. Besides recall that the computed\[5\] gyromagnetic ratio of the electron LCR-manifold is fermionic \( g = 2 \).

4.1 Derivation of quantum electrodynamics

In addition to the symmetric tensor \( g_{\mu\nu} \), the LCR-structure tetrad also defines a class of three self-dual and three antiself-dual 2-forms (relative to the defined metric (1.10))

\[
V^0 = \ell \wedge m \quad , \quad V^\tilde{0} = n \wedge \overline{m} \quad , \quad V = 2\ell \wedge n - 2m \wedge \overline{m} \quad (4.17)
\]

which satisfy the relations

\[
dV^0 = [(2\varepsilon - \rho)n + (\tau - 2\beta)\overline{m}] \wedge V^0 \\
dV^\tilde{0} = [(\mu - 2\gamma)\ell + (2\alpha - \pi)m] \wedge V^\tilde{0} \\
dV = [2\mu\ell - 2\rho m - 2\pi m + 2\tau \overline{m}] \wedge V \quad (4.18)
\]

where the small greek letters are the connection parameters of the spin-coefficient formalism[20]. In fact any non conformally flat metric, which admits geodetic and shear free null directions, define a finite number of triplets of such self-dual 2-forms. This number is related to the Petrov type of the metric, and we will discuss it below.

If the LCR-structure is realizable[1], there are always functions such that

\[
0 = d(dz^0 \wedge dz^1) = d[(f_{00} f_{11} - f_{01} f_{10}) \ell \wedge m] \\
0 = d(dz^\tilde{0} \wedge dz^{\tilde{1}}) = d[(f_{\tilde{0}\tilde{0}} f_{\tilde{1}\tilde{1}} - f_{\tilde{0}\tilde{1}} f_{\tilde{1}\tilde{0}}) n \wedge \overline{m}] \quad (4.19)
\]

But for the third self-dual 2-form \( V \), there is not always a function, which makes it closed i.e. such that \( d(fV) = 0 \). This happens if

\[
d[2\mu\ell - 2\rho m - 2\pi m + 2\tau \overline{m}] = 0 \quad (4.20)
\]

In fact, if there is a member of the tetrad-Weyl equivalent class of 2-forms, which implies \[1,20\], this member may be assumed as the physical representative, because it defines a conserved "charge". That is, the existence of a 2-form which defines the conserved quantity "charge" breaks the tetrad-Weyl symmetry down to the ordinary Weyl symmetry. The remaining Weyl symmetry will be finally
restricted to one tetrad, from the definition of the mass from the Einstein gravity source.

The electron LCR-structure \([4.8]\) satisfies this condition, because

\[
2\mu\ell - 2\rho n - 2\pi m + 2\tau m = d[\ln(r - ia\cos\theta)^2] \quad (4.21)
\]

Hence, the self-dual 2-form

\[
F^+ = \frac{1}{(r - ia\cos\theta)^2} (2\ell \wedge n - 2m \wedge m) = F - i * F
\]

\[
A = \frac{aq^3}{4\pi(r^4 + a^4(x^4))^2} (dx^0 - \frac{r^{x^1 - ax^2}}{r^2 + a^2} dx^1 - \frac{r^{x^2 + ax^1}}{r^2 + a^2} dx^2 - \frac{x^3}{r} dx^3)
\]

is closed. It defines a real 2-form \(F\), which is identified with the electromagnetic field, as we see from its electromagnetic potential, written in cartesian coordinates in order to compare it with the first order electron dressing computed below from the Bogoliubov procedure.

The solitonic feature of the electron LCR-structure is protected by the non-vanishing of all the three relative invariants \(\Phi_i\) of the LCR-structure. Recall that the trivial vacuum has all the three relative invariants equal to zero. In the physical interpretation, this LCR-manifold has to be identified with the electron and its complex conjugate structure with the positron, because it has opposite charge.

Notice that the electromagnetic field is essentially determined by the flat-print of the electron LCR-structure. The second term of the tetrad form of the Kerr-Schild ansatz \([4.9]\) does not contribute to the definition of \(F\). Therefore we should consider that the electromagnetic field is a particle (Poincaré representation determined by the singularity of the LCR-structure). Hence the correct field equation is

\[
d * F = j \quad , \quad dj = 0
\]

\[
\partial_{\mu} F^{\mu\nu} = j^{\nu} \quad , \quad \partial_{\nu} j^{\nu} = 0
\]

(4.23)

where the singularity gives a conserved current. The implied conserved quantity is the electron charge.

The positron is identified with the conjugate electron LCR-structure, which is found by simply interchanging \((m \leftrightarrow \overline{m})\). Then the metric remains the same, which implies that electron and positron have the same masses. But the 2-forms change, implying that electron and positron have opposite charges.

The energy-momentum are the conserved quantities determined from the source \(T^\mu_\nu(p)\) of the (linearized) Einstein equation. This satisfies the Bianchi identities, which must be valid even at the "singularities". Recall that this point was essentially used by Einstein and coworkers\(^7\) to derive the equations of motion. On the other hand the EM-equations happens to be satisfied by the static soliton. They are not satisfied for any LCR-structure.
I want to point out that the definition of the LCR-structure (1.5) implies the tetrad-Weyl invariants $F_i = dZ_i$ and the relative invariants $\Phi_i$. In the case of the Kerr-Schild ansatz (4.9) the invariants of the LCR-structure

\[
F_1 = dZ_1 = \frac{4ra^2 \sin \theta \cos \theta}{(r^2 + a^2 \cos \theta)^2} dr \wedge d\theta
\]

\[
F_2 = dZ_2 = \frac{4ra^2 \sin \theta \cos \theta}{(r^2 + a^2 \cos \theta)^2} dr \wedge d\theta
\]

\[
F_3 = dZ_3 = -\frac{4ra^2 \sin \theta \cos \theta}{(r^2 + a^2 \cos \theta)^2} dr \wedge d\theta
\]

(4.24)

do not depend on $h(r)$, and the relative invariants

\[
\rho - \overrightarrow{\rho} = \frac{-2ia \cos \theta}{(r + ia \cos \theta)(r - ia \cos \theta)} = i \Phi_1
\]

\[
\mu - \overrightarrow{\mu} = \frac{ia(r^2 + a^2 \cos \theta)}{(r + ia \cos \theta)^2(r - ia \cos \theta)^2} = -i \Phi_2
\]

\[
\tau + \overrightarrow{\tau} = \frac{i\sqrt{2}ar \sin \theta}{(r + ia \cos \theta)^2(r - ia \cos \theta)^2} = \Phi_3
\]

(4.25)
do not vanish. Hence, $h(r)$ is not obstructed from taking the self-consistent form.

Summarizing, I have already shown that the static LCR-manifold is a soliton protected by its mass and its relative invariants, and it belongs to the massive spinorial representation. Hence, it is represented with the Dirac free field $\psi(x)$, which satisfies the massive Dirac equation. This Dirac field represents the "left" and "right" columns $X^{nj}$ of the homogeneous grassmannian coordinates of the moving electron-soliton. It has an EM-potential (dressing), which satisfies the massless wave equation, hence it is a spin-1 particle represented with a vector field $A_\mu(x)$. The interaction of these two formal "quantum" free fields is apparently the well known electromagnetic interaction

\[
L_{EM} = e \gamma_\mu \psi_e^* \psi_e A_\mu
\]

(4.26)

The Bogoliubov procedure permit us to find the 1st order electromagnetic potential

\[
A_{1\mu}(x; 1) \simeq -i \frac{e}{2} \Phi_1 \delta_\mu^\nu \Phi_1 \Big|_{\nu = 0}
\]

\[
\hat{S}_2(J) = \int T((L_1(x_1) + A_\nu(x_1)J^\nu(x_1))(L_1(x_2) + A_\nu(x_2)J^\nu(x_2))[dx]
\]

(4.27)
in the Bogoliubov book notation, which becomes

\[
A_1^\mu(x) \simeq -e \int D\delta(x - y) \overleftrightarrow{\psi_e}(y) \gamma^\mu \overleftrightarrow{\psi_e}(y) \Phi_1 d^4y
\]

\[
\Phi_1 p = (2\pi)^2 \delta_{a_p} (\overrightarrow{p}) \Phi_0
\]

(4.28)

But (4.22) is singular at the ring with radius $a$, while the above perturbative term is singular at the point $\overrightarrow{a} = 0$, which emerge after an expansion of (4.22)
and the definition of $r$ in powers of $a = \frac{\hbar}{2m}$. The emergence of the Plank constant $\hbar$ strongly indicates that the electromagnetic field includes the contributions of loop diagrams. This observation makes clear that if PCFT is the background dynamics of the Bogoliubov harmonic expansion, there must be a shelf-consistency condition between the "classical" solutions (like the Kerr-Newman manifold) and the summation of the "quantum" series. Besides, the Bogoliubov free-field expansion should be the positive energy representations of the conformal group with finite component fields.

5 ELECTROWEAK GAUGE FIELD

We saw that the symplectic 2-form is defined in the unbounded $SU(2, 2)$ symmetric Siegel domain $X^1E \cdot X > 0$. Its boundary is $\mathbb{R}^4$ with a Minkowski induced metric. The corresponding bounded realization $Y^1E \cdot Y > 0$ has is $U(2)$. The "natural U(2)" LCR-structure is

$$e = -iw^{-1}dw = \left(\ell \atop m \atop n\right), \quad de - ie \wedge e = 0$$

$$d\ell = im \wedge \overline{m}, \quad dn = -im \wedge \overline{m}, \quad dm = i(\ell - n) \wedge m$$

This form strongly suggests to osculate the LCR-structure with the $U(2)$ group. The first step of that is to cast a LCR-tetrad into the hermitian matrix

$$e' := \left(\ell' \atop m' \atop n'\right) = i(\partial - \overline{\partial})\left(\rho_{11} \atop \rho_{12} \atop \rho_{22}\right)$$

where $\rho_{ij}$ is the hermitian LCR embedding relations. Following the Maurer-Cartan procedure we consider the hermitian matrix $e'$ an element of $u(2)$ Lie algebra, i.e. a $U(2)$ connection with non-vanishing curvature. The connection and the corresponding curvature are

$$B = B_{\mu}dx^\mu t_1 = \left(\ell' \atop m' \atop n'\right), \quad [t_i, t_j] = iC_{iJK}t_K$$

$$F = dB - iB \wedge B \rightarrow DF := dF + iB \wedge F - iF \wedge B = 0$$

where $t_j$ are generators of $U(2)$. Apparently a gauge transformation breaks the tetrad-Weyl symmetry. That is, the $U(2)$ transformation is expected to transform LCR-structures to other LCR-structures, like the weak $U(2)$ transforms electron to its neutrino and vice-versa. The $SO(1, 3)$ transformation of the null tetrads also breaks the LCR-structure. Therefore we chose the LCR-tetrad $e'$ to be such that $\Phi_1 = 1 = -\Phi_2$. That is, we partly fix the tetrad-Weyl symmetry for non-trivial LCR-structures with $\Phi_1 \neq 0 \neq \Phi_2$. Recall the general tetrad-Weyl
transformation is
\[ \ell' = \Lambda \ell , \quad n' = N n , \quad m' = M m \]

\[ Z'_1 = Z_1 + d(\ln \Lambda) , \quad \Phi'_1 = \frac{\Lambda}{M \Lambda} \Phi_1 \]
\[ Z'_2 = Z_2 + d(\ln N) , \quad \Phi'_2 = \frac{\Lambda}{M N} \Phi_2 \]
\[ Z'_3 = Z_3 + d(\ln M) , \quad \Phi'_3 = \frac{\Lambda}{MN} \Phi_3 \]  

(5.4)

In the case of the following generators
\[ B_{0\mu} + \frac{1}{2} B_{3\mu} = \ell'_\mu , \quad B_{0\mu} - \frac{1}{2} B_{3\mu} = n'_\mu , \quad \frac{1}{2}(B_{1\mu} + i B_{2\mu}) = m'_\mu \]
\[ F_{0\mu\nu} = \partial_\mu B_{0\nu} - \partial_\nu B_{0\mu} - \epsilon_{ijk} B_{j\mu} B_{k\nu} \]

(5.6)

The standard model relations between the \( U(1) \) gauge potential \( B_{0\mu} \) and the \( SU(2) \) gauge potentials \( B_{ij\mu} \) suggest us to identify the electromagnetic potential \( A_\mu \) with \( \ell'_\mu \), the neutral potential \( Z_\mu \) with \( n'_\mu \) and the charged potential \( W_\mu \) with \( m'_\mu \). Besides, the relative invariants are apparently related to the Higgs field.

In the case of the electron LCR-tetrad \([4.19]\) the three relative invariants are \([4.20]\)

\[ \Phi_1 = -2a \cos \theta \]
\[ \Phi_2 = \frac{a^2 \cos^2 \theta + h}{(r^2 + a^2 + h) a \cos \theta} \]
\[ \Phi_3 = \sqrt{2(r + h \cos \theta)^2 (r - h \cos \theta)} \]

(5.7)

We first make the tetrad-Weyl transformation to reach the conditions \( \Phi'_1 = 1 = -\Phi'_2 \). We find

\[ N = -\frac{r^2 + a^2 \cos^2 \theta}{r^2 + a^2 \cos^2 \theta + h} \Lambda \]
\[ M M' = -\frac{2a \cos \theta}{r^2 + a^2 \cos^2 \theta + h} \Lambda \]

(5.8)

The electromagnetic dressing \([4.10]\) is found with \( \Lambda = \frac{qr}{r^2 + a^2 \cos^2 \theta + h} \). Then the electroweak connection \( B \) \([5.6]\) is found with

\[ \Lambda = \frac{qr}{r^2 + a^2 \cos^2 \theta + h} \]
\[ N = -\frac{2a \cos \theta}{4\pi (r^2 + a^2 + h)} \]
\[ M M' = -\frac{qr a \cos \theta}{2\pi (r^2 + a^2 \cos^2 \theta + h)} \]

(5.9)

up to an \( M \) phase tetrad-Weyl transformation. That is, we find the following electroweak potentials (dressings) of the electron

\[ A = \frac{qr}{4\pi (r^2 + a^2 \cos^2 \theta)} (dt - dr - a \sin^2 \theta d\varphi) \]
\[ Z = \frac{-qr}{8\pi (r^2 + a^2 \cos^2 \theta)} (dt + \frac{r^2 + 2a^2 \cos^2 \theta - a^2 h - 2a \sin^2 \theta \cos \theta}{r^2 + a^2 + h} dr - a \sin^2 \theta d\varphi) \]
\[ W = \frac{-M}{\sqrt{2(r + i a \cos \theta)}} [-i a \sin \theta (dt - dr) + (r^2 + a^2 \cos^2 \theta) d\theta + i \sin \theta (r^2 + a^2) d\varphi] \]

(5.10)
where the tetrad-Weyl factor $M$ will be computed below through the Higgs dressing.

The third (complex) relative invariant $\Phi_3'$ (5.4) is not completely fixed. Its phase is absorbed by $W$ and the remaining scalar real field $\Phi_3'$ will be finally related with the electron Higgs potential

$$
M = \frac{i}{r - ia \cos \theta} \left[ \frac{g r a \cos \theta}{\pi (r^2 + a^2 \cos^2 \theta)^2} \right]^{\frac{1}{2}}
$$

$$
\Phi_3' = \frac{2 \sin \theta (r^2 + a^2 \cos \theta)}{r^2 + a^2 \cos \theta} \left[ \frac{\pi a}{q r} \cos \theta \right]^{\frac{1}{2}}
$$

We see that Einstein’s $SO(1,3)$ connection and the $U(2)$ gauge field are directly related to the LCR-tetrad and the Higgs field is related with the $\Phi_i$ relative invariants of the LCR-structure. Hence all the known fields of the electroweak interactions exist in the LCR-structure. They are introduced in the algorithm of Bogoliubov, Epstein-Glaser, Scharf and collaborators as asymptotic free fields. The recursive relations provide all the relations between the masses and the coupling constants.

The idea of Scharf and collaborators was to exploit the fact that the $S$-matrix is an operator valued distribution which is expanded to the operator valued distributions of precise free fields (Poincaré representations) in the rigged Hilbert-Fock space of tempered distributions. In this framework we have to consider the operator form of the gauge transformations

$$
A^\mu = e^{-i\lambda Q} A^\mu e^{-i\lambda Q}
$$

$$
\delta A^\mu = -i\lambda [Q, A^\mu]
$$

(5.12)

where $Q$ is a nilpotent operator. For every gauge field $A^\mu$ we introduce the anticommuting ghost fields $u(x), \tilde{u}(x)$ and the implied gauge field triplet acquire the transformations

$$
d_Q A^\mu = [Q, A^\mu] = i \partial^\mu u
$$

$$
d_Q u = [Q, u] = 0
$$

$$
d_Q \tilde{u} = [Q, \tilde{u}] = -i \partial_\mu A^\mu
$$

(5.13)

The invariance of the $S$-matrix essentially relative to this deformation essentially protect the generation of non-physical modes. That is, the physical Fock space is the $\ker(Q^\dagger Q + QQ^\dagger)$.

Applying this algorithm to our case of a pair of asymptotic (free) massive fermions $\psi_e, \psi_\nu$, and a $U(2)$ gauge field with one massless and three massive, we start from general interaction lagrangian. The leptonic and gauge field variations are

$$
d_Q \psi_e = 0 , \quad d_Q \psi_\nu = 0
$$

$$
d_Q A^\mu = i \partial^\mu u_0 , \quad d_Q W_{1,2}^\mu = i \partial^\mu u_{1,2} , \quad d_Q Z^\mu = i \partial^\mu u_3
$$

(5.14)

The gauge variations for the Higgs $\phi$ and the unphysical scalar fields $\Phi_{1,2,3}$ are

$$
d_Q \phi = 0 , \quad d_Q \Phi_{1,2} = im W_{1,2} u_1 , \quad d_Q \Phi_3 = im Z u_3
$$

(5.15)
The gauge variations for the fermionic ghosts are

\[ dQ u_b = 0, \quad b = 0, 1, 2, 3 \]
\[ dQ \tilde{u}_0 = i\partial_\mu A_\mu, \quad dQ \tilde{u}_{1,2} = -i\partial_\mu(W_{1,2}^\mu + m_1 \Phi_{1,2}) \]
\[ dQ \tilde{u}_{1,2} = -i\partial_\mu(Z_\mu + m_2 \Phi_3) \] 

(5.16)

After very long calculations Scharf and collaborators succeeded to reproduce the standard model lagrangian with the proper relations between coupling constants and masses! Besides the application of this algorithm to the gravitational interactions, they essentially found that the protection from the unphysical modes restricts the action to the Einstein-Hilbert form, because simply it is the only scalar form without second order derivatives, which have negative norm states.

5.1 On the origin of the leptonic generations

The electron and its neutrino LCR-structures derived from the linear trajectory \( \xi^a = v^a \tau + c^a \) or equivalently the quadratic Kerr polynomial

\[ K(Z) = iZ^0 Z^0 [(v^0 - v^3)(c^1 + ic^2) - (v^1 + iv^2)(c^0 - c^3)] + \]
\[ + iZ^0 Z^1 [(v^0 + v^3)(c^0 - c^3) - (v^1 - iv^2)(c^1 + ic^2)] - Z^0 Z^2 (v^1 + iv^2) - \]
\[ - Z^0 Z^3 (v^0 - v^3) + iZ^1 Z^1 [(v^1 - iv^2)(c^0 - c^3) - (v^0 + v^3)(c^1 - ic^2)] + \]
\[ + Z^1 Z^2 (v^0 + v^3) + Z^1 Z^3 (v^1 - iv^2) \] 

(5.17)

with \( c^a \) generally complex. This is the most general quadratic Kerr polynomial which incorporates all the parameters of the Poincaré representation. The singular points of this quadratic surface satisfy the relations

\[ \partial_a K(Z) = 0, \quad Z^n \neq 0 \] 

(5.18)

We finally find that there are the following two cases

1st: \( \text{If } v^a v^b \eta_{ab} \neq 0 \text{ the surface is irreducible} \) 

2nd: \( \text{If } v^a v^b \eta_{ab} = 0 \text{ the surface is reducible} \) 

(5.19)

The first case gives the electron and positron LCR-solitons and the second reducible surface gives the left-handed chiral part of the neutrino. The electron LCR-structure is determined with an irreducible quadratic polynomial and the neutrino LCR-structure is determined with the corresponding reducible quadratic polynomial. Their gravitational metrics \( g_{\mu\nu} \) admit only two geodetic and shear free null congruences and are type-D spacetimes in the Petrov classification. They constitute the electronic generation of the observed standard model. In order to look for the other two generations (the muon and tau families) we have to describe the general framework of the solutions of the pseudo-conformal manifolds and their topological obstructions.

The grassmannian manifold \( G_{4,2} \) is the set of lines of \( CP^3 \). A point of \( G_{4,2} \) is a line of \( CP^3 \), which is determined by the two columns \( X^{n1} \) and \( X^{n2} \) viewed
as two points of $CP^3$. The linear transformation $SL(4, C)$, which applies from the left side, preserves the LCR-structure. The $SL(2, C)$ linear transformation, which applies from the right side, preserves the line of $CP^3$, but it does not preserve the LCR-structure. Hence, in the present formalism the points of the Minkowski spacetime are lines of $CP^3$.

Every two intersection points of the line and the surface $K(Z^m) = 0$ determine a LCR-structure. In every affine space of $G_{4,2}$, the line is projectively represented with a $2 \times 2$ matrix $r = r_a \sigma^a_{A, A}$, which, after the application of the implicit function theorem for the solution of the four (real) relations of the LCR-structure, takes the form

$$r^a = x^a + iy^a(x^b)$$ (5.20)

The LCR-structure with vanishing $y^a(x^b) = 0$ are compatible with the Minkowski metric. Therefore we may consider this imaginary part as the gravitational content of the LCR-structure. Hence, the linearized Einstein gravity approximation projects the LCR-structure down to its "flatprint". The well known to general relativists $SO(1, 3)$ local transformations of the null tetrad, which preserve the metric, coincide with the line preserving $SL(2, C)$ transformation. In fact the quartic polynomial (2.7) is the maximum degree Kerr polynomial permitted by a regular Einstein metric, which admits geodetic and shear free null congruences $\ell^\mu \partial_\mu$ and $n^\mu \partial_\mu$. Taking into consideration that the degree of the Kerr polynomial is a topological invariant of the corresponding surface of $CP^3$, we expect the existence of two more chiral currents to be permitted in addition to the above studied quadratic ones. Those determined by cubic Kerr polynomials, which we identify with the muon generation, and those determined by quartic Kerr polynomials, which we identify with the tau generation.

The preceding analysis indicates to correlate the lepton numbers with the degrees of the Kerr polynomial. Then the limitation of the number of generations is imposed by the Einstein gravity. At each spacetime point, which is determined by a line of $CP^3$, there are at most four intersections between the Kerr surface and the line. The 1st generation $(e, \nu_e)$ corresponds to quadratic surfaces, which we have extensively studied before.

Notice that the Kerr polynomial of the curved LCR-manifold is the same with its corresponding flattened. Therefore we will make the computation of the Hopf in this simplified case. Every column $i$ of the $G_{4,2}$ homogeneous coordinates $X^{ni}$ determine a complex function $\lambda^A_i(x)$ in $S^2$. That is, for any LCR-structure we have two functions

$$\lambda^A_i(x) : S^1 \times S^3 \rightarrow S^2$$ (5.21)

understood to be in the bounded $U(2)$ realization of spacetime. It is known that the homotopy group $\pi_1(S^2)$ is trivial but $\pi_3(S^2) = Z$. The Hopf invariant is determined using the sphere volume 2-form

$$\omega = \frac{i}{2\pi} d\lambda \wedge d\lambda \frac{d\lambda}{(1 + \lambda^2)^2}$$ (5.22)
which is closed. This implies that in $S^3$ there is an exact 1-form $\omega_1$ such that
$\omega = d\omega_1$. Then the Hopf invariant of $\lambda(x)$ is

$$H(\lambda) = \int \lambda^*(\omega) \wedge \omega_1$$

(5.23)

The Hopf invariants (linking numbers) of the electron and neutrino LCR-structures have been computed[29] and found to be $\pm \frac{a}{|a|}$. The higher Hopf invariants may be viewed as a composition of the above simple ones and the higher $S^2 \to S^2$.

In the flatprint electron LCR-structure, the relation between the cartesian coordinates and the structure coordinates are

$$x^0 = t$$
$$x^1 + ix^2 = (r - ia) \sin \theta e^{i\varphi}$$
$$x^3 = r \cos \theta$$

(5.24)

For constant time, the (left) causal ray $\ell^a(r)$ is

$$x^0 = t = 0$$
$$x^1 = (r \cos \varphi + a \sin \varphi) \sin \theta$$
$$x^2 = (r \sin \varphi - a \cos \varphi) \sin \theta$$
$$x^3 = r \cos \theta$$

(5.25)

Recall that the entire $SU(2)$ space is covered by considering $r \in (-\infty, +\infty)$. This 3-dimensional space may be considered as the initial data of the electron LCR-manifold.

Let us now consider the following initial data

$$x^0 = t = 0$$
$$x^1 = (r \cos k \varphi + a \sin k \varphi) \sin \theta$$
$$x^2 = (r \sin k \varphi - a \cos k \varphi) \sin \theta$$
$$x^3 = r \cos \theta$$

(5.26)

and compute the linking number of the circles

$$\vec{x}' = (0, 0, r) \ , \ \theta = 0, \ \varphi = 0$$
$$\vec{x} = (a \sin k \varphi, -a \cos k \varphi, 0) \ , \ \ r = 0, \ \theta = \frac{\pi}{2}$$

(5.27)

$$l = \frac{1}{4\pi} \int \frac{\epsilon_{ijk}(x'^i - x^i)dx^j \wedge dx^k}{|\sum_{l}^{(x'^i - x^i)|^2}|^{\frac{3}{2}}}$$

$$\vec{x}' = (0, 0, r) \ , \ \theta = 0, \ \varphi = 0$$
$$\vec{x} = (a \sin k \varphi, -a \cos k \varphi, 0) \ , \ \ r = 0, \ \theta = \frac{\pi}{2}$$

(5.28)

$$l = \frac{ka^2}{4\pi} \int \frac{dx \varphi}{(a^2 + r^2)^{\frac{3}{2}}} = \frac{ka}{2|a|} \int_{-\infty}^{\infty} \frac{dx'}{(1+r'^2)^{\frac{3}{2}}} = \frac{ka}{|a|}$$

32
It apparently counts how many times the circle of the ring singularity winds around \( x'(r) \), the \( \rho \)-closed curve of \( SU(2) \).

This theoretical reasoning, based on \( \pi_2(S^3) = \mathbb{Z} \), needs to be accompanied with the reason why the observed leptonic generations are three and not infinite as it is suggested. The above computations are done in zero gravity limit. Hence the restriction of the number of generations should be caused by gravity through the following reasoning.

The gravitational dressing of the elementary particles satisfies Einstein’s equations through the metric \( g_{\mu\nu} \), which admits geodetic and shear-free null congruences. These congruences are principal null directions of the Weyl tensor which is formally written

\[
\ell^\mu = \mathcal{R}^A_{\ C} \sigma^b \, A^A \, \kappa^\mu \, \epsilon^b_c , \quad \Psi_{ABCDA_\, B_\, \kappa^\mu \kappa^C D} = 0 \tag{5.29}
\]

in the Newman-Penrose formalism. Hence the number of gravitational principal directions cannot exceed four and subsequently the degree of the Kerr polynomial cannot be higher than four.

Let us now pose the question "how many static massive and (stationary) massless representations of polynomial multiplets of a given degree exist?". The knowledge of the Poincaré group permit us to answer this question by simply noticing that such a multiplet must contain a polynomial, which admits the infinitesimal z-rotations and time-translation as automorphisms.

I have worked it out\(^{29}\) and found that only the irreducible quadratic surface\(^{23}\) determines a polynomial multiplet. If our hypothesis that the lepton numbers are the topological degrees of the Kerr polynomial, then we have to conclude that the muon and tau are unstable solitonic surfaces, which is also observed. Therefore it would be very interesting to find solitonic third and fourth degree surfaces.

6 HADRONIC SECTOR AND CONFINEMENT

The following tetrad-Weyl invariant gauge field equations

\[
\frac{1}{\sqrt{-g}} (D_\mu)_{ij} \left\{ \sqrt{-g} \left( \Gamma^{\mu\nu\rho\sigma} - \Gamma^{\nu\rho\sigma}_{\mu} \right) F_{j\rho\sigma} \right\} = 0
\]

\[
\Gamma^{\mu\nu\rho\sigma} = \frac{1}{2} \left( \eta^{\mu\nu} \eta^{\rho\sigma} - \eta^{\mu\rho} \eta^{\nu\sigma} - \eta^{\mu\sigma} \eta^{\nu\rho} \right) + \eta^{\mu\nu} \eta^{\rho\sigma} \eta^{\alpha\beta} \eta_{\alpha\beta} \left( \ell^{\alpha} \sigma^{\beta} - \ell^{\beta} \sigma^{\alpha} \right) \left( \ell^{\sigma} \sigma^{\rho} - \ell^{\rho} \sigma^{\sigma} \right)
\]

\[
(D_\mu)_{ij} = \delta_{ij} \partial_\mu - \gamma f_{ikj} A_{k\mu}
\]

(6.1)

may be written down. The do not have sources. Therefore I find convenient to give them PDEs the following forms

\[
(-) \quad \rightarrow \quad \frac{1}{\sqrt{-g}} (D_\mu)_{ij} \left\{ \sqrt{-g} \left( \eta^{\mu\nu} \eta^{\rho\sigma} F_{j\rho\sigma} \right) + \eta^{\mu\nu} \eta^{\rho\sigma} \eta_{\alpha\beta} \eta^{\alpha\beta} \left( \ell^{\alpha} \sigma^{\beta} - \ell^{\beta} \sigma^{\alpha} \right) \left( \ell^{\rho} \sigma^{\sigma} - \ell^{\sigma} \sigma^{\rho} \right) \right\} = -k_i^\nu
\]

\[
(+) \quad \rightarrow \quad \frac{1}{\sqrt{-g}} (D_\mu)_{ij} \left\{ \sqrt{-g} \left( \eta^{\mu\nu} \eta^{\rho\sigma} F_{j\rho\sigma} \right) + \eta^{\mu\nu} \eta^{\rho\sigma} \eta_{\alpha\beta} \eta^{\alpha\beta} \left( \ell^{\alpha} \sigma^{\beta} - \ell^{\beta} \sigma^{\alpha} \right) \left( \ell^{\rho} \sigma^{\sigma} - \ell^{\sigma} \sigma^{\rho} \right) \right\} = -ik_i^\nu
\]

(6.2)
where \( k'_{\nu}(x) \) is a real vector field which we will consider as distributional (localized) sources. That is, taking into account that the sum of the two terms is self-dual

\[
\Gamma^{\mu\nu\rho\sigma} F_{j\rho\sigma} =: G_j^{\mu\nu} - i * G_j^{\mu\nu}
\]

we may find a real gauge field with electric sources in the first case (-), and magnetic sources in the second case (+). The framework of distributional solitons seems to be studied the problem of hadronic sector in PCFT.

We will here study the properties of such non-null abelian solitons found in the case of the flatprint static LCR-structure. It is more convenient to make calculations using the differential forms of (6.2)

\[
d\{ (n^\rho m^\sigma F_{\rho\sigma}) \ell \wedge m + (\ell^\rho m^\sigma F_{\rho\sigma}) n \wedge \overline{m} \} = i * k
\]

Using the Stoke's theorem procedure, we find the non-vanishing closed 2-forms (with real sources) in the case of flatprint massive static LCR-tetrad

\[
(L^\mu M^\sigma F_{\rho\sigma}) = \frac{C''(r+ia \cos \theta)}{(r^2+a^2)^{\frac{1}{2}}} L \wedge M - \frac{C'(r+ia \cos \theta)}{(r^2+a^2)^{\frac{1}{2}}} N \wedge \overline{M} - \frac{C''(r+ia \cos \theta)}{(r+ia \cos \theta)^{\frac{1}{2}}} \epsilon_{\mu}\rho \sigma_{\nu}\lambda F_{\rho\sigma}
\]

where \( C' \) and \( C'' \) are arbitrary complex constants, which are fixed using Stokes' theorem and the reality conditions for gluonic sources. We also assume that

\[
[(L^\mu N^\nu - M^\mu \overline{M}^\nu)F_{j\mu\nu}] = 0
\]

because it cannot be determined by the sources. That is

\[
F_{\rho\sigma} = - \frac{C'}{\sin\theta(r-ia \cos \theta)} (L_\rho M_\sigma - L_\sigma M_\rho) - \frac{C''(r+ia \cos \theta)}{(r^2+a^2)^{\frac{1}{2}}} (N_\rho \overline{M}_\sigma - N_\sigma \overline{M}_\rho) + c.c.
\]

For the static flatprint LCR-tetrad the solutions have the explicit forms

\[
F = i * F := - \frac{2C'}{\sin\theta(r-ia \cos \theta)} L \wedge M - \frac{2C''(r+ia \cos \theta)}{(r^2+a^2)^{\frac{1}{2}}} N \wedge \overline{M} = \frac{2C' + C''}{\sqrt{2}} \left[ \frac{ia}{(r^2+a^2)^{\frac{1}{2}}} dt \wedge dr + \frac{1}{\sin \theta} dt \wedge d\theta + a \sin \theta d\theta \wedge d\phi \right] + \frac{2C' - C''}{\sqrt{2}} \left[ \frac{ia}{(r^2+a^2)^{\frac{1}{2}}} dt \wedge dr + i dt \wedge d\phi - \frac{r^2+a^2 \cos^2 \theta}{(r^2+a^2)^{\frac{1}{2}}} dr \wedge d\theta \right]
\]

After a straightforward calculation I find

\[
\int_{r,t=const} \left[ \frac{2C'}{\sin\theta(r-ia \cos \theta)} L \wedge M + \frac{2C''(r+ia \cos \theta)}{(r^2+a^2)^{\frac{1}{2}}} N \wedge \overline{M} \right] = \frac{(2C' + C'')4\pi a}{\sqrt{2}} =: -i \gamma
\]

where \( \gamma \) is the real hadronic constant. Assuming \( 2C' - C'' = 0 \) (because it is not determined by the gluonic charge), the arbitrary constants are completely fixed and the solutions are

34
\[ F = \frac{\gamma}{4\pi} \left[ \frac{1}{\alpha} \frac{d\alpha}{\tan^{-1} \frac{\alpha}{\tan \theta}} dt \wedge dr + dr \wedge d\varphi \right] = \frac{d}{\tan^{-1} \frac{\alpha}{\tan \theta}} \left( \tan^{-1} \frac{\alpha}{\tan \theta} + r d\varphi \right) \]

\[ *F = \frac{\gamma}{4\pi} \left[ \frac{1}{\alpha \sin \theta} dt \wedge d\theta + \frac{\alpha \sin \theta}{r^2 + a^2} dr \wedge d\theta + \sin \theta d\theta \wedge d\varphi \right] \]

where the corresponding gluonic potential (quark dressing) been apparent. In cartesian coordinates the potential and the gluonic field strength take the form

\[ A^g = \frac{\gamma}{4\pi a} \left( \tan^{-1} \frac{\alpha}{\tan \theta} + r d\varphi \right) = \frac{\gamma}{4\pi a} \left( \tan^{-1} \frac{\alpha}{\tan \theta} - \frac{ax_1 + ax_2}{(x_1^2 + (x_2)^2)} dx_1 + \frac{ax_1 - ax_2}{(x_1^2 + (x_2)^2)} dx_2 \right) \]

\[ F^g = \frac{\gamma}{4\pi a} \left( \frac{ax_1 + ax_2}{(x_1^2 + (x_2)^2)} dx_1 \wedge dx_2 + \frac{ax_1 - ax_2}{(x_1^2 + (x_2)^2)} dx_1 \wedge dx_2 + \frac{r x_1 - r x_2}{r^2 + a^2} dx_1 \wedge dx_2 + \frac{r x_1 - r x_2}{r^2 + a^2} dx_2 \wedge dx_3 \right) \]

\[ F^e = \frac{\gamma}{4\pi a r} \left[ (x_1^2 + (x_2)^2) dx_1 \wedge dx_2 + \frac{r x_1 - r x_2}{r^2 + a^2} dx_1 \wedge dx_2 + \frac{r x_1 - r x_2}{r^2 + a^2} dx_2 \wedge dx_3 \right] \]

while the form of the electric potential is

\[ A^e = \frac{\gamma}{4\pi r^2 + a^2} \left( dx_0 - \frac{r x_1 - r x_2}{r^2 + a^2} dx_1 - \frac{r x_1 - r x_2}{r^2 + a^2} dx_2 - \frac{r x_1 - r x_2}{r^2 + a^2} dx_3 \right) \]

It has a line singularity along the z-axis like the Dirac magnetic monopole, but its asymptotic behavior is different. The "electric" and "magnetic" parts are

\[ A^g = \frac{\gamma}{4\pi a} \left( \tan^{-1} \frac{\alpha}{\tan \theta} \right) - \frac{d}{\tan^{-1} \frac{\alpha}{\tan \theta}} \left[ \frac{1}{\alpha} \frac{d\alpha}{\tan \theta} \right] \]

\[ F^g = \frac{\gamma}{4\pi a} \left[ \frac{1}{\alpha} \frac{d\alpha}{\tan \theta} \right] \left( \frac{1}{r^2 + a^2} \right) \left( x_1^2 + x_2^2 \right) \]

\[ r = \pm \left\{ \frac{(x_1^2 + x_2^2 - a^2)}{2} + \sqrt{\frac{(x_1^2 + x_2^2 - a^2)^2}{2} + a^2 (x_3)^2} \right\}^{\frac{1}{2}} \]

where the last term of \( \widetilde{A}^g \) is a singular gauge. The "magnetic" part of the potential is linear indicating confinement.

But the essential difference of the gluonic dressing is its singularity relative to the angular variable \( \alpha \). This does not permit us to make a harmonic expansion into free fields. The Bogoliubov causal perturbative approach and its Scharf version cannot be applied to "derive" quantum effects.

7 BEYOND THE STANDARD MODEL

Standard model may be considered as the great success of modern fundamental physics. In the beginning we thought that extending the \( U(2) \) electroweak group to larger internal groups, like \( SU(5) \), we could incorporate the hadronic sector. The experimental disagreement with the decay rate of the proton blocked these expectations. The next attempt was the consideration of supersymmetric groups, which combine bosons and fermions. The recent (last) experimental
result at CERN, that supersymmetric particles do not exist, blocked this theoretical investigation too. String theory was the last theoretical framework of the beyond the standard attempts to incorporate gravity, but its supersymmetric basis was fatal for this theory too.

Pseudo-conformal field theory (PCFT) is purely geometric. Its fundamental physical principle is the existence of a lorentzian Cauchy-Riemann (LCR) structure defined in the tangent space (bundle) of a manifold (spacetime) like the riemannian metric. The existence of (Schwartz) distributional solutions ”needs” the mathematical extension to the tempered distributions and the harmonic analysis (series) in the corresponding rigged Hilbert space (Gelfand triple). Recall that the Pythagora computation of hypotenuse ”needed” the completion of the rational numbers \( \mathbb{Q} \) to the real numbers \( \mathbb{R} \), computed as infinite summations of rational numbers. In this section I will try to indicate some directions (suggestions) where the explanation of some ”beyond the standard model” may be hidden.

7.1 On the dark matter and energy

The dark energy may be a proper normalization cosmological constant in the Scharf algorithmic graviton field expansion. But my negative results to find other static or stationary solitons drives me to look for possible geometric explanations of cosmological dark matter.

The macroscopic study of the universe should be done through the Kaecker manifold determined by (3.16). ”Dark matter” may be the effect of the second fundamental form of the embedding of the universe LCR-submanifold in \( \mathbb{C}^4 \).

In order to visualize the embedding consequences of the universe as a real submanifold of \( \mathbb{C}^4 \), it is convenient to work in real coordinates. In the Eisenhart book\[8\] notation, the Gauss-Codazzi equations have the form

\[
R_{ijkl} = \sum_{\sigma=0}^8 e_\sigma (\Omega_{\sigma|ik} \Omega_{\sigma|jl} - \Omega_{\sigma|il} \Omega_{\sigma|jk}) + R_{\alpha\beta\gamma\delta} y^{\alpha}_{ij} y^{\beta}_{jk} y^{\gamma}_{kl} y^{\delta}_{il} \\
\Omega_{\sigma|ij,k} - \Omega_{\sigma|ik,j} = \sum_{\tau=0}^8 e_\tau (\mu_{\tau|ij} \Omega_{\tau|ik} - \mu_{\tau|ik} \Omega_{\tau|ij}) + R_{\alpha\beta\gamma\delta} y^{\alpha}_{ij} y^{\beta}_{jk} y^{\gamma}_{kl} e^{\delta}_{\tau} 
\]

(7.1)

where the universe is \( V_4 \) with induced metric \( g_{ij} \), and the enveloping space is \( V_8 \) with metric \( a_{\alpha\beta} \). The latin indices \( i, j, k, ... \) take values up to four and the greek indices \( \alpha, \beta, \gamma, \mu, ... \) take values up to eight. We see that the induced curvature \( R_{ijkl} \) of the surface depends on the second fundamental form \( \Omega_{\sigma|ik} \) and the curvature \( R_{\alpha\beta\gamma\delta} \) of the ambient Kaecker manifold. These relations of the geometric tensors are essentially implied by their precise dependence on the four real embedding functions \( \rho_{ij}(x^1, z^2) \).

One of the most striking effects attributed to ”dark matter” comes from the velocities \( v^j \) of the stars at different distances from the galactic center. This mathematical problem is studied in paragraph 48 of the Eisenhart book\[8\], and
the following relation (48.7 p.165)

\[ \frac{e^1}{\rho_a^2} = \frac{e^2}{\rho_a^2} + \frac{e}{\rho^2} \]  

(7.2)

where we have the curve curvatures: \( \frac{1}{\rho_a} \) relative to the ambient metric \( a_{\alpha\beta} \), \( \frac{1}{\rho_g} \) relative to the spacetime metric \( g_{ij} \), and \( \frac{1}{\rho^2} \) the normal curvature of spacetime and \( e_a, e_g \) and \( e \) are plus or minus one. Apparently the deviation of the observed trajectory from the geodetic one indicates that the observed spacetime is not a totally geodetic submanifold.

### 7.2 Harmonic expansion in the bounded domain

The appearance of \( \eta_{\mu\nu} \) in the "vacuum" metric and symplectic form \(^{(3.18)}\) impose the Poincaré group harmonic expansion. The used Bogoliubov causal approach (with the Scharf et. al. improvements) are heavily based on the unbounded realization of the \( SU(2,2) \) classical domain. But the corresponding bounded realization seems to be more "natural" because the polar decomposition approach covers the entire spacetime (up to a point), but its maximal subgroup changes to \( S(U(2) \times U(2)) \). The Kaehler function is \( \det(I - w^1 w) \) with \( w \) a general \( 2 \times 2 \) matrix which at the \( U(2) \) boundary becomes unitary. The relation between the bounded and unbounded realizations of the conformal group \( SU(2,2) \) and its relation with the Minkowski space distributions has been analyzed by Ruehl[35].

In the bounded realization the maximal subgroup \( S(U(2) \times U(2)) \) is

\[ \begin{pmatrix} Y'_1 \\ Y'_2 \end{pmatrix} = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \]

\[ U_1 , U_2 \in U(2) , \quad \det(U_1 U_2) = 1 \]

(7.3)

where \( U_1 \) and \( U_2 \) are independent \( U(2) \) elements. Its Cartan subalgebra is

\[ H_1 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & 0 \end{pmatrix} , \quad H_2 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^3 \end{pmatrix} \]

\[ H_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \]

(7.4)

The \( z \)-rotation of the Poincaré algebra takes the form \( J_3 = H_1 + H_2 \). in the bounded realization. But the evolution operator in the bounded representation is\(^{(14)}\) \( H_0 = \frac{1}{2}(P^0 + K^0) \), which is essentially the generator of \( U(1) \) factor of \( U_2(2) \) (or its universal covering \( \mathbb{R} \)). It does not coincide with the evolution operator \( P^0 \) of the unbounded realization. Hence the static quadratic Kerr polynomial with \( J_3 \) and \( H_0 \) automorphisms has the form

\[ \begin{cases} 
\text{(bounded)} & K_B = AY^0 Y^3 + BY^1 Y^2 \\
\text{(unbounded)} & K_U = \frac{4 + B}{2}(X^0 X^1 - X^2 X^3) + \frac{4 - B}{2}(X^1 X^2 - X^0 X^3) 
\end{cases} \]

(7.5)
where we have used the transformation
\[
\begin{pmatrix}
Y^0 \\
Y^1 \\
Y^2 \\
Y^3
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
I & I \\
I & -I
\end{pmatrix}
\begin{pmatrix}
X^0 \\
X^1 \\
X^2 \\
X^3
\end{pmatrix}
\] (7.6)

between the bounded coordinates \(Y^\alpha\) and the unbounded ones \(X^\alpha\). Notice the difference between the static LCR-structure (electron) (2.9) written below and the "static" LCR-structure in the bounded realization

\begin{align*}
(bounded) & \quad K_B^{(e)} = \frac{C}{2}(Y^0Y^1 - Y^2Y^3) + \frac{C-2D}{2}(Y^1Y^2 - Y^0Y^3) \\
(unbounded) & \quad K_U^{(e)} = CX^0X^1 + D(X^1X^2 - X^0X^3)
\end{align*}

(7.7)

This difference of the asymptotic bounded and unbounded states may be the origin of the neutrino (and quark) mixing, which appears in their time evolution.
References

[1] M. S. Baouendi, P. Ebenfelt and L. Rothschild, "Real submanifolds in complex space and their mappings", Princeton University Press, Princeton, (1999).

[2] N. N. Bogoliubov, A.A. Logunov and I.T. Todorov, "Introduction to Axiomatic Quantum Field Theory", W.A. Benjamin Publishing Company, Inc. USA (1975).

[3] N. N. Bogoliubov and D. V. Shirkov, "Introduction to the Theory of Quantized Fields", John Wiley and sons, Inc. USA (1980).

[4] E. Cartan, Ann. Math. Pure Appl. (4) 11 (1932), 17.

[5] B. Carter (1968), Phys. Rev. 174, 1559.

[6] S. Chandrasekhar (1983), “The Mathematical Theory of Black Holes”, Clarendon, Oxford.

[7] A. Einstein, L. Infeld and B. Hoffman (1938), Ann. Math. 39, 65.

[8] L. P. Eisenhart, "Riemannian Geometry", Princeton University Press, (1966).

[9] H. Epstein and V. J. Glaser, Annales de l’ Institut Poincaré, A19, 211, (1973).

[10] B. Felsager (1981), “Geometry, Particles and Fields”, Odense Univ. Press.

[11] E. J. Jr Flaherty (1974), Phys. Lett. A46, 391.

[12] E. J. Jr Flaherty (1976), “Hermitian and Kählerian geometry in Relativity”, Lecture Notes in Physics 46, Springer, Berlin.

[13] I. M. Gel’fand, R. A. Minlos and Z. Ya. Shapiro, ”Representations of the rotation and lorentz groups and their applications”, The Macmillan Company, New York, USA (1963).

[14] G. Mack, "All Unitary Ray Representations of the Conformal Group SU(2,2) with Positive Energy", Commun. math. Phys., 55, (1977), 1.

[15] G. Mack and A. Salam, "Finite-Component Field Representations of the Conformal Group", Ann. Phys. 53, (1969), 174.

[16] C. N. Misner, K. S. Thorn and J. A. Wheeler, "GRAVITATION", W. H. Freeman and Co (1973).

[17] E. T. Newman (1973), J. Math. Phys. 14, 102.
[18] E. T. Newman (2004), Class. Q. Grav. 21, 3197 (arXiv:gr-qc/0402056).
[19] E. T. Newman and J. Winicour (1974), J. Math. Phys. 15, 426.
[20] R. Penrose and W. Rindler (1984), “Spinors and space-time”, vol. I and II, Cambridge Univ. Press, Cambridge.
[21] I. I. Piatetsky-Chapiro (1966), “Géométrie de domaines classiques et théorie des fonctions automorphes”, Dunod.
[22] J. Polchinski, "STRING THEORY", vol. I, Cambridge Univ. Press, Cambridge, (2005).
[23] C. N. Ragiadakos, "A Gauge Model Unifying Geometry and Matter” in "LEITE LOPES Festschrift - A pioneer physicist in the third world", Edited by N. Fleury et al., World Scientific, Singapore (1988).
[24] C. N. Ragiadakos (1990), "A Four Dimensional Extended Conformal Model", Phys. Lett. B251, 94.
[25] C. N. Ragiadakos (1991), "Solitons in a Four Dimensional Generally Covariant Conformal Model" Phys. Lett. B269, 325.
[26] C. N. Ragiadakos (1992), "Quantization of a Four Dimensional Generally Covariant Conformal Model", J. Math. Phys. 33, 122.
[27] C. N. Ragiadakos (1999), "Geometrodynamic solitons", Int. J. Math. Phys. A14, 2607.
[28] C. N. Ragiadakos (2008), “Renormalizability of a modified generally covariant Yang-Mills action”, arXiv:hep-th/0802.3966v2.
[29] C. N. Ragiadakos (2008), “A modified Y-M action with three families of fermionic solitons and perturbative confinement”, arXiv:hep-th/0804.3183v1.
[30] C. N. Ragiadakos (2013), "A renormalizable cosmodynamic model", arXiv:hep-th/1302.0512.
[31] C. N. Ragiadakos (2013), "Lorentzian CR structures", arXiv:hep-th/1310.7252.
[32] C. N. Ragiadakos (2017), "Pseudo-conformal Field Theory”, arXiv:hep-th/1704.00321.
[33] C. N. Ragiadakos (2018), "Hadronic Sector in the 4-d Pseudo-conformal Field Theory”, arXiv:hep-th/1811.04428.
[34] C. N. Ragiadakos, Research eBook of "Pseudo-Conformal Field Theory” in my personal page www.pcft.gr.
[35] W. Ruehl, "Distributions on Minkowski Space and Their Connection with Analytic Representations of the Conformal Group", Commun. math. Phys., 27, (1972), 53.

[36] G. Scharf, "Finite Quantum Electrodynamics: The causal approach", Springer-Verlag, Berlin, (1995).

[37] G. Scharf, “Quantum Gauge Theorie: A true ghost story”, John Wiley & Sons, Inc. USA, (2001).

[38] W. Tung, “Group theory in physics”, World Scientific Publishing Co., Singapore, (2003).