Research Article

Study on Quality Decisions in Supply Chain considering the Lagged Time and Retailers Competition

Yuan Peng1,2 and Liang Ning1

1School of Business Administration, Jiangxi University of Finance and Economics, No. 169, East Shuanggang Avenue, Changbei, Nanchang, Jiangxi 330013, China
2College of Computer and Information Engineering, Jiangxi Agricultural University, No. 1101, Zhiming Avenue, Changbei, Nanchang, Jiangxi 330013, China

Correspondence should be addressed to Yuan Peng; 1178177307@qq.com

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1. Introduction

With increasingly fierce market competition, the competition among enterprises has transformed into a competition of supply chain. Quality is the lifeline of an enterprise, and brand goodwill is the driving force of its development. For the final consumer market, brand goodwill is not only reflected in the design and manufacturing quality of products but also includes the service quality of retailers. Many consumers in the market often experience product quality differences through the sales services of retailers. In the first half of 2019, the case of “Protection of Mercedes Benz Consumer Rights” in Xi’an City of China reflected to a certain extent the importance of dealer service quality and vehicle manufacturer product quality to supply chain operation. This case was triggered by the rights protection caused by the service quality problems of car dealers when consumers encounter the occasional quality problems of automobile products and seek after-sales service, which has seriously negative effect on the brand of Benz automobile. Therefore, the manufacturers and retailers in a supply chain can obtain greater competitive advantages through quality cooperation. In reality, manufacturers may motivate retailers to improve their service quality by bearing part of the service costs of retailers.

The impact of product quality improvement on goodwill will not bring prompt outcomes. In fact, it takes time to exert an influence on goodwill because consumers cannot evaluate products until they have purchased them and experienced them. Then, they would circulate their experiences about...
these products through the word-of-mouth effect. Therefore, the goodwill effect of quality improvement often lags behind for a period of time, which is the lagged time of quality on goodwill.

It is not clear what is the impact that the lagged time will have on supply chain decision-making. In addition, when the retailers have competition, what is the relationship between the lagged time of product quality to goodwill and the profit of supply chain members and what is the effect of competition coefficient on the profit of supply chain?

Therefore, for the secondary supply chain, this paper considers the lagged time of product quality level, when discussing the influence of lagged effect and competitive factors on product quality and service quality decision-making of supply chain, and how to choose the path of supply chain decision.

2. Literature Review

Many scholars have studied the decision-making of supply chain from different perspectives. This paper has a threefold research topic, namely, the product quality of supply chain, the service quality of supply chain, and the lagged time; we sort out the relevant literature combined with the research content of the paper.

A lot of literature on the decision-making of supply chain product quality mainly discusses how to coordinate the quality of supply chain. When there is cooperation in the supply chain, most of the researches focus on the contract design of the supply chain, which mainly considers the core issues such as the overall income distribution and cost allocation of the supply chain.

Zhu et al. mainly studied the supply chain of OEM. In the supply chain, the buyers and sellers share the goodwill cost related to quality, the after-sales service cost, and the market share loss cost. The authors explored the decision-making problem of improving product quality [1]. Gurnani and Erkoc constructed the demand function of product quality and marketing effort and compared three different supply chain contract forms [2]. Dan et al. constructed a quality-based demand function for the secondary supply chain and realized the quality coordination of the supply chain in different situations by designing cost sharing contract and revenue sharing contract [3]. Ma et al. studied how to design the manufacturer’s product quality contract in the two-stage supply chain [4]. Karray analyzed how to design the product quality coordination contract in two distribution channels under a horizontal strategy and a vertical strategy in supply chain and provided the corresponding coordination strategies [5]. Seyyed-Mahdi et al. studied green product quality strategy under different decision-making scenarios and designed the coordination and cooperation mechanisms of supply chain [6].

In the case of noncooperation, most previous studies focus on how to choose quality coordination strategies. They mainly reviewed the quality balance strategies under different competition situations and how to formulate the quality control and inspection strategies of all parties in the supply chain. Boyaci and Gallego reviewed two competitive supply chains, in which each supply chain consisted of a supplier and a retailer. They studied three competitive situations of these supply chains and obtained the quality equilibrium strategies in each circumstance [7]. Xiao et al. studied the quality and price equilibrium strategy under four decision-making scenarios (centralized decision-making, supplier cooperation, supplier noncooperation, and mixed scenarios) as well as the influence of quality competition on the supply chain strategy solution under different scenarios [8]. Zhu et al. discussed the quality decision-making of manufacturers under two incentive mechanisms [9]; then they further studied the strategy selection of suppliers to control product quality under different distribution channel modes [10]. Cai et al. studied the quality control and quality risk prevention of fresh products in a supply chain. Based on their analysis, they proposed quality risk prevention measures and constructed a supply chain optimization control model to solve the problem [11]. Liu and Wang constructed a product quality control model of service supply chain and considered the impact of different risk attitudes on product quality decision-making [12]. Sarkar and Saren demonstrated that when there were defects in quality inspection, the profit could be increased by formulating a product quality inspection strategy based on the cost of paying a guarantee fee [13]. He et al. [14] and Chenavaz [15] studied the dynamic quality decision-making of enterprises with reference to the quality behavior of consumers. Hong and Huang divided quality management activities into quality control and quality improvement. They used differential game theory to study the quality strategy of supply chain under four different game situations. They found that the best way to cooperate is to make quality decisions by taking into account the supply chain as a whole [16].

Some scholars have explored the quality decision-making of supply chain from an irrational perspective. For example, considering downstream retailers’ loss aversion preference, Liu and Fan discussed the quality decision-making of supply chain under centralized and decentralized decision-making scenarios [17]. Xiao et al. studied how supply chain members’ overconfidence in market demand affects supplier’s quality investment strategy [18]. In the literature above, the authors mainly considered the influence of upstream manufacturers or supply chain suppliers on customer demand by designing product quality, but they did not consider the influence of the retailer on sales through improvements in the service quality level.

Studies on the service quality level have also attracted the attention of many scholars. For example, Tsay and Agrawal studied a dynamic model of price and service competition with capacity constraints [19]. In the online and offline channels of supply chain, both Ren et al. [20] and Dan et al. [21] considered service competition. Zhang et al. [22] and Fan and Liu [23] discussed service competition strategy in a two-channel supply chain model. Qin et al. explored service quality coordination between online retailers and the supply chain of third-party logistics enterprises [24]. Yao and Chen choose two retailers who provide competitive services as their background and investigated the decision-making problem of after-sales service capability [25]. Zhang et al. explored the
service decision-making problem of retailers with both online and offline shops under perturbation [26]. In addition, some scholars have discussed the service quality level in supply chain, such as Shi et al. [27] and Ali et al. [28].

Reviewing the previous literature, we found that although the product quality strategy and service quality strategy of supply chain have attracted the attention of many scholars, there are few studies combining product quality strategy and service quality strategy from the supply chain perspective. He et al. discussed the decision-making and coordination of the product quality and service quality of supply chain under the condition of supplier risk aversion [29]. Focusing on a supply chain with one retailer and two manufacturers, Huang et al. studied the coordinating operation strategy of supply chain members when considering product price, product quality, and service competition [30]. However, the literature mentioned above mainly constructs supply chain optimization models from a static perspective. The optimal solution obtained under a static situation is the optimal strategy of enterprises only for a short period of time.

In the existing literatures, the research on lagged effect mainly focuses on the lagged time of advertising investment on goodwill; Nerlove and Arrow believed that advertising will not directly affect consumer demand [31] but indirectly affect consumer purchase behavior through brand favorability. Advertising will make consumers have higher expectations for product quality. If the quality of the product does not meet the expectations of consumer, it will reduce consumer brand favorability. Therefore, product quality is an important factor affecting consumer brand favorability. 

In comparison with the previous literature, there are two original points in this paper: first, in consideration of the lagged time of product quality level in the secondary supply chain, this paper discusses the influence of lagged effect and competition factors on the decision-making of product quality and service quality in the retailer market competition environment; secondly, we establish a time-lagged differential equation concluding influence of quality on goodwill and construct a Hamilton function with one manufacturer and two competing retailers using the maximum principle lagged differential equation. Then, the optimal product quality and the optimal service quality are attained under different decision-making scenarios, as well as the optimal service quality cost participation rate of manufacturer under decentralized decision-making and how to choose the decision path of supply chain.

Finally, on the basis of a numerical analysis, this paper analyzes the impact of the lagged time on supply chain profits, product goodwill, product quality, and other aspects as well as the impact of competition on supply chain profits to verify the effectiveness of supply chain optimal decision-making.

3. Basic Hypotheses and Symbol Descriptions

The object of this paper is a two-stage supply chain, which is assumed to be composed of one manufacturer and two competing retailers. The manufacturer is the core enterprise in the supply chain, and it improves product quality through research and development, the application of new technologies, the transformation of manufacturing processes, and so forth. The retailers improve the service quality of products by providing good presale service consultation, an experiential shopping environment, and after-sales return and exchange services. To motivate the downstream retailers to improve their service quality, the manufacturer is willing to offer the subsidy rate for the retailers’ quality improvement cost.

**Assumption 1.** The product quality level of the manufacturer has a positive impact on product goodwill. Based on the Nerlove–Arrow [31] model, a lagged differential equation with the influence of improving quality on product goodwill is constructed:

\[
\dot{G}(t) = \gamma Z(t - d) - \delta G(t),
\]

\[
G(0) = G_0,
\]

where \(G(t)\) is the product’s accumulated goodwill over time \(d\) and \(G_0 \geq 0\) is the initial goodwill at time 0. Parameter \(d\) denotes the lagged time between the improvements of product quality at time \(t - d\) to accumulated goodwill at time \(t\). The lagged time is mainly affected by product quality, business model, and so on. The lagged time reflects the fact that goodwill is accumulated for period of time in which consumers have carried out word-of-mouth publicity after purchasing and experiencing products. Parameter \(\gamma\) is quality’s efficiency to goodwill accumulation under the lagged time.

**Assumption 2.** Market demand is affected by factors such as a retailer’s service quality level, goodwill, and competitor service quality level. The potential sales volume of retailer \(i\) before an improvement in product quality and service quality is \(a_i\). Assume that retailer 1 and retailer 2 have the same potential market sales; namely, \(a_1 = a_2 = a\):

\[
D_i(t) = a + bG(t) + \mu S_i(t) + \beta(S_i(t) - S_{i-1}(t)).
\]

**Assumption 3.** The investment cost of product quality and service quality is an increasing function of the product
quality level and service quality level, and the second-order reciprocal is greater than 0; that is, $C_M'(Z) > 0$ and $C_R''(S_i) > 0$. The quality investment cost functions of the manufacturer and retailer I are as follows:

$$C_M = \frac{1}{2} k_M Z^2(t),$$  \hspace{1cm} (3)

$$C_Ri = \frac{1}{2} kRI S_i^2(t).$$

The relevant symbols and descriptions are shown in Table 1.

This paper also assumes that the members of the supply chain make decisions based on the optimal profits in the infinite time zone. To conclude, we obtain the net discount profit functions of the manufacturer, the retailers, and the whole supply chain:

$$J_M = \int_0^\infty \left\{ e^{-\lambda t} \sum_{i=1}^2 \left[ p_M D_i - \frac{1}{2} \phi_i kRI S_i^2(t) \right] - \frac{1}{2} k_M Z^2(t) \right\} \, dt,$$

$$J_Ri = \int_0^\infty \left\{ e^{-\lambda t} \left[ p_Ri D_i - \frac{1}{2} (1 - \phi_i) kRI S_i^2(t) \right] \right\} \, dt,$$

$$J_MR = \int_0^\infty \left\{ e^{-\lambda t} \sum_{i=1}^2 \left[ (p_Ri + p_M) D_i - \frac{1}{2} kRI S_i^2(t) \right] - \frac{1}{2} k_M Z^2(t) \right\} \, dt.$$  \hspace{1cm} (4)

4. Decentralized Decision-Making

As independent entities, the manufacturer and retailer $i$ make decisions on product quality and service quality, respectively, based on the principle of optimizing their own interests. To expand the market scale, the manufacturer encourages the retailers to improve their service level and promises to share a certain proportion of their own quality investment costs. Accordingly, the manufacturer and retailers formulate product quality and service quality strategies, respectively.

In this scenario, the decision sequences of the members are described as follows: (i) First, the manufacturer offers the subsidy rate $\phi$ for the retailers’ quality improvement cost. (ii) Next, after observing the manufacturer’s subsidy action, the manufacturer and the retailers determine the product quality level and service quality level, respectively, along time $t$ and simultaneously. (iii) Finally, the manufacturer determines the optimal subsidy rate $\phi$ according to the optimal product quality level and the optimal service quality level.

Proposition 1. Under decentralized decision-making, the manufacturer’s optimal product quality level and the retailers’ optimal service quality level, respectively, are as follows:

$$Z^* = \frac{2 p_M b y e^{\lambda t}}{k_M (\lambda + \delta)^2},$$

$$S_i^* = \frac{p_Ri (\mu + \beta)}{(1 - \phi) kRI}.$$  \hspace{1cm} (7)

The optimal goodwill of a product is as follows:

$$G^* = \frac{2 p_M b y e^{\lambda t}}{\delta k_M (\lambda + \delta)} (1 - e^{-\delta t}) + G_0 e^{-\delta t}.$$  \hspace{1cm} (8)

The optimal share proportion of service quality investment of manufacturers is as follows:

$$\phi_i = \begin{cases} 2 p_M - p_Ri (\mu + \beta) \frac{p_M}{p_Ri} > \frac{\mu + \beta}{2}\mu, \\ 2 p_M + p_Ri (\mu + \beta) \frac{p_M}{p_Ri} \leq \frac{\mu + \beta}{2}\mu. \end{cases}$$  \hspace{1cm} (9)

Proof. First, we assume that the proportion of the manufacturer’s service quality investment sharing is a fixed value $\phi_i$. Accordingly, we obtain the optimal quality levels of the manufacturer and the retailers.

Combined with equation (1), the optimal control of the manufacturer’s profit is $\max_{Z(t)} J_M$.

Using the maximum principle, we obtain the following Hamilton function:

$$H_M = e^{-\lambda t} \left\{ \sum_{i=1}^2 \left[ p_M (a + b G + \mu S_i + \beta (S_i - S_{i-1}) \right) + \frac{1}{2} k_M Z^2 \right\} + K_m [\gamma Z(t - d) - \delta G].$$  \hspace{1cm} (10)

We obtain

$$\frac{dH_M}{dZ} = -e^{-\lambda t} k_M Z + \frac{dZ(t - d)}{dZ} = 0,$$

$$K_m = -\frac{dH_M}{dG} = K_m \delta - 2 p_M be^{-\lambda t}.$$  \hspace{1cm} (11)

At the same time, we assume that $(dZ(t - d)/dZ(t)) = M(t)$. Thus, the manufacturer’s optimal quality level strategy satisfies

$$Z(t) = \frac{K_m \gamma M(t)}{k_M} e^{\lambda t}. $$  \hspace{1cm} (13)

Solving differential equations (12) and (14) can be obtained as follows:

where

$$K_m = ce^{\lambda t} + \frac{2 p_M b e^{-\lambda t}}{\lambda + \delta}, \quad c \in R.$$  \hspace{1cm} (14)

According to the literature [31], \( M(t) = \Phi(t - d, t) = e^{\delta t} \); therefore,
$Z^* (t) = \frac{\gamma c}{k_M} e^{\delta t + \delta d} + \frac{2p_M b_y}{k_M} (\lambda + \delta) e^{\delta d}$. (15)

When $\rightarrow \infty$, the product quality level of the manufacturer is thus finite; therefore,

$$\lim_{t \to \infty} Z(t) < \infty. \quad (16)$$

From the equation above, we judge that $c = 0$. After rearrangement, we obtain the optimal quality level of the manufacturer:

$$Z^* (t) = \frac{2p_M b_y}{k_M} (\lambda + \delta) e^{\delta d}. \quad (17)$$

Combined with constraint equation (1), the retailers’ optimal decision problem is expressed as $\max_{\eta > 0} J_M$.

Using the maximum principle, we construct the following Hamilton function:

$$H_{Ri} = e^{-\delta t} \left[ p_{Ri} [a + bG + \mu S_i + \beta (S_i - S_{i-1})] - \frac{1}{2} S_i^2 \right] + k_{Ri} [\gamma Z (t-d_e) - \delta G].$$

Similarly, using the maximum principle, we obtain the optimal service quality level of the retailers:

$$S_i^* = \frac{p_{Ri}(\mu + \beta)}{(1 - \phi_i)k_{Ri}}. \quad (19)$$

Substituting equations (17) and (19) into (1), we obtain the solution of the product goodwill lagged differential equation (1):

$$G = \frac{2 \gamma p_M b_y}{\delta k_M (\lambda + \delta)} (1 - e^{-\delta t}) e^{\delta d} + G_0 e^{-\delta t}. \quad (20)$$

Second, the manufacturer designs the optimal service quality investment sharing ratio. We substitute equations (17), (19), and (20) into (4) and simplify it. Thus, we can obtain the following equation:

$$J_M = \frac{1}{\lambda} \left[ \sum_{i=1}^{2} \left( \frac{p_{Ri} (\mu + \beta)}{k_{Ri} (1 - \phi_i)} \right)^2 - \frac{2p_M}{\delta} \left( \frac{\beta c}{2} - \frac{\mu S_i}{\delta} + \beta S_{i-1} \right) \right]$$

$$+ \frac{1}{2} k_M Z^2 - \frac{2p_M}{\delta} \left( \frac{\beta c}{2} - \frac{\mu S_i}{\delta} + \beta S_{i-1} \right). \quad (21)$$

Maximizing $J_M$ to $\phi_i$, we obtain the optimal service quality investment sharing ratio of the manufacturer:

$$\frac{dJ_M}{d\phi_i} = \frac{1}{2} \left[ \frac{p_{Ri} (\mu + \beta)}{k_{Ri} (1 - \phi_i)} \right]^2 - \frac{p_M p_{Ri} (\mu + \beta)}{2k_{Ri}} \left( \frac{\beta c}{2} - \frac{\mu S_i}{\delta} + \beta S_{i-1} \right). \quad (22)$$

Obtain $dJ_M/d\phi_i = 0$, and we obtain the following:

$$\phi_i = \begin{cases} \frac{p_M (\mu + \beta)}{p_{Ri}} & \text{if } p_M (\mu + \beta) > 2 \mu \\
0 & \text{if } p_M (\mu + \beta) \leq 2 \mu \end{cases} \quad (23)$$

From Proposition 1, we know that when there is service cooperation, the optimal profits of the manufacturer and the retailers are as follows:

$$J_M = \frac{2b p_M}{\lambda} \left( G_0 - \gamma Z^* \right) + \frac{1}{\lambda} \left[ \sum_{i=1}^{2} S_i \left( \mu - \frac{1}{2} k_{Ri} S_i^* \right) \right]$$

$$- \frac{1}{2} k_M Z^2 + \frac{2b p_M (\gamma + \mu)}{\delta} Z^* + 2p_M a,$$

$$J_{Ri} = \frac{b p_i}{\lambda} \left[ G_0 - \gamma Z^* \right] e^{-\delta t}$$

$$+ \frac{1}{\lambda} \left[ \frac{b p_i (\gamma + \mu)}{\delta} Z^* + p_i a + S_i^* \left( \mu - \frac{1}{2} (1 - \phi_i) k_{Ri} S_i^* \right) \right]. \quad (24)$$
Corollary 1. Under decentralized decision-making, (i) the greater the marginal profits \( p_R \) and \( p_M \), the higher the optimal product quality level provided by the manufacturer and the optimal service quality level provided by the retailers and the higher the quality investment cost. (ii) When members’ marginal profit satisfies the restraining condition \( p_M / p_R > (\mu + \beta) / 2\mu \), the manufacturer is willing to offer the subsidy rate \( \phi \) for the retailers’ service quality improvement cost and subsidy rate \( \phi \) is positively correlated with service quality level. (iii) The manufacturer’s marginal profit is positively related to subsidy rate \( \phi \), while the retailers’ marginal profit is negatively correlated with the subsidy rate. (iv) The degree of competition and the retailers’ marginal profit are positively correlated with the retailers’ service quality level.

Corollary 1 shows that marginal profit has a great impact on the quality investment decision-making of the enterprises. If the marginal profit is larger, the enterprises will be more enthusiastic about improving their quality level. The greater the manufacturer’s marginal profit is, the more active it will be in supporting the retailers to improve their service quality level. In contrast, if the marginal profit is low, the manufacturer will be less enthusiastic about supporting the retailers to improve their service quality level. The manufacturer undertakes part of the investment cost of service quality, which to a certain extent reduces the economic burden of the downstream retailers and encourages them to improve their service quality. The higher the competition of the retailers, the greater the number of consumers they must attract by improving their service quality. At the same time, the higher the marginal profit is, the more active the retailers will be in improving their service quality.

Corollary 2. Under decentralized decision-making, if the lagged time is longer, the optional product quality level of the manufacturer will be higher; the optional profit of manufacturers decreases with the increase in the lagged time of the product quality improvement influencing goodwill, while the retailers’ optimal profit increases with the increase in the lagged time.

Corollary 2 shows that the manufacturer will invest more in product development and technology introduction when the lagged time of the product quality improvement affecting goodwill is longer. This higher investment will increase the cost incurred by the enterprise. Despite the improvement in market demand, the profit obtained will decrease. At the same time, the product quality level has positive effects on the market demand. The increase of the retailers’ sales revenue will make more profits for them.

5. Centralized Decision-Making

Under centralized decision-making, the manufacturer and two retailers are regarded as three departments within an enterprise. Designing the optimal service quality \( S(t) \) and the optimal product quality \( z(t) \), the profit of the whole supply chain can be maximized.

Proposition 2. Under centralized decision-making, the optimal product quality and service quality of the manufacturer and the retailers, respectively, are as follows:

\[
Z^{**} = \frac{\beta}{k_M} \left( \frac{\sum_{i=1}^{n} (p_R_i + p_M_i)}{\lambda + \delta} \right) e^{\lambda t},
\]

\[
S_{i}^{**} = \frac{(p_R_i + p_M_i)(\mu + \beta)}{k_{Ri}}.
\]

The optimal goodwill on the product is as follows:

\[
G^* \left( t \right) = \sum_{i=1}^{n} \left( p_R_i + p_M_i \right) y \left( 1 - e^{-\delta t} \right) e^{\delta t} + G_0 e^{-\delta t}.
\]

Proof. First, the optimal decision-making problem of the supply chain is characterized as the optimal control problem max\( \sum_{Z>0,S>0} J_{MR} \).

Using the maximum principle, we construct the following Hamilton function:

\[
H_{MR} = e^{-\lambda t} \left\{ \sum_{i=1}^{n} \left( p_R_i + p_M_i \right) D_i - \frac{1}{2} k_{Ri} S_i^2 \right\} - \frac{1}{2} e^{-\lambda t} k_M Z^2 + K_c \left[ \gamma Z(t - d) - \delta G \right].
\]

We obtain the following:

\[
\frac{dH_{MR}}{dZ} = -e^{-\lambda t} k_M Z + \frac{K_c \gamma y Z(t - d)}{dZ} = 0,
\]

\[
\frac{dH_{MR}}{dS_i} = e^{-\lambda t} \left[ (p_R_i + p_M_i)(\mu + \beta) - k_{Ri} S_i \right] = 0.
\]

The Hessian matrix of \( H_{MR} \) with respect to \( Z(t) \) and \( S(t) \) is as follows:

\[
H \left( H_{MR} \right) = \begin{bmatrix}
-e^{-\lambda t} k_M & 0 & 0 \\
0 & -e^{-\lambda t} k_{R1} & 0 \\
0 & 0 & -e^{-\lambda t} k_{R2}
\end{bmatrix}.
\]

Because the first-order principal subequation \( <0 \), the second-order principal subequation \( >0 \), and the third-order principal subequation \( <0 \). Therefore, the matrix of \( H \left( H_{MR} \right) \) is negative definite. Thus, the unique optimal solution of \( J_{MR} \) is \( Z^{**}(t) \) and \( S^{**}(t) \), which are the best quality levels of the manufacturer and the retailers, respectively. Similar to the solution of Proposition 1, we obtain the following:

\[
Z^{**} = \frac{\beta}{k_M} \left( \frac{\sum_{i=1}^{n} (p_R_i + p_M_i)}{\lambda + \delta} \right) e^{\lambda t},
\]

\[
S_{i}^{**} = \frac{(p_R_i + p_M_i)(\mu + \beta)}{k_{Ri}}.
\]

Substituting equations (30) and (31) into (1), we obtain the solution of the product goodwill lagged differential equation (1):
\[ G^{**}(t) = \left(1 - e^{-\delta t}\right) by^2 \sum_{i=1}^{2} \left( P_{Ri} + P_{M} \right) e^{\delta t} + G_{0} e^{-\delta t}. \]  
(32)

From Proposition 2, we further obtain the optimal profit of the whole supply chain:

\[ J_{MR} = \frac{b}{\lambda + \delta} \sum_{i=1}^{2} \left( P_{M} + p_{i} \right) \]
\[ + \frac{1}{\lambda} \left\{ \sum_{i=1}^{2} S_{i}^{*} \left( -k_{R} S_{i}^{*} \right) - \frac{1}{2} k_{M} Z^{**2} + b y \sum_{i=1}^{2} \left( P_{M} + p_{i} \right) Z^{**} + a \sum_{i=1}^{2} \left( P_{M} + p_{i} \right) \right\}. \]  
(33)

**Proposition 3.** Under centralized decision-making, the optimal product quality level of the manufacturer, the optimal service quality level of the retailers, the product market sales volume, and goodwill are all greater than those under decentralized decision-making. That is, \( Z^{**}(t) > Z^{*}(t), S^{**}(t) > S^{*}(t), G^{**}(t) > G^{*}(t), \) and \( D^{**}(t) > D^{*}(t). \)

Proof.

\[ Z^{**}(t) - Z^{*}(t) = \sum_{i=1}^{2} \left( P_{Ri} + P_{M} \right) \frac{by^2}{\lambda + \delta} e^{\delta t} - \frac{2P_{M} by}{\lambda + \delta} e^{\delta t} \]
\[ = \frac{by^2}{\lambda + \delta} \sum_{i=1}^{2} \left( P_{Ri} + P_{M} \right) e^{\delta t} > 0. \]  
(34)

Therefore, we obtain \( Z^{**}(t) > Z^{*}(t). \)

\[ S_{i}^{**}(t) - S_{i}^{*}(t) = \frac{\left( P_{Ri} + P_{M} \right) \left( \mu + \beta \right)}{k_{R}} - \frac{P_{Ri} \left( \mu + \beta \right)}{1 - \phi_{i} k_{Ri}} \]
\[ = \frac{P_{Ri} \left( \mu + \beta \right) + 2P_{M} \beta}{2k_{Ri}}. \]  
(35)

Therefore, we obtain \( S^{**}(t) > S^{*}(t). \)

\[ G^{**}(t) - G^{*}(t) = \left(1 - e^{-\delta t}\right) by^2 \sum_{i=1}^{2} \left( P_{Ri} + P_{M} \right) e^{\delta t} \]
\[ - \left(1 - e^{-\delta t}\right) \frac{2P_{M} by}{\lambda + \delta} e^{\delta t} \]
\[ = \left(1 - e^{-\delta t}\right) by^2 \sum_{i=1}^{2} \left( P_{Ri} + P_{M} \right) e^{\delta t} > 0. \]  
(36)

From equations (35) and (36), we obtain \( G^{**}(t) > G^{*}(t). \)

According to equations (34)–(36), we obtain the following:

\[ \sum_{i=1}^{2} D_{i}^{**} - \sum_{i=1}^{2} D_{i}^{*} = \frac{2b(1 - e^{-\delta t}) y^2 by}{\delta k_{M} \lambda} \sum_{i=1}^{2} P_{Ri} e^{\delta t} \]
\[ + 2(\mu + \beta) \sum_{i=1}^{2} P_{Ri} (\mu + \beta) \]  
(37)

Equation (37) shows that the market demand under centralized decision-making is greater than that under decentralized decision-making:

\[ \sum_{i=1}^{2} D_{i}^{**} > \sum_{i=1}^{2} D_{i}^{*}. \]  
(38)

**Proposition 4.**

(1) When the lagged time satisfies the inequality \( d \geq 1/\delta \ln((A_{2} + \sqrt{A_{2}^{2}4A_{2}A_{1}})/2A_{1}), \) we obtain \( J_{MR} \leq J_{M} + J_{R}. \) That is, the profit of a supply chain under centralized decision-making will not be greater than that under decentralized decision-making.

(2) When \( 0 < d < 1/\delta \ln((A_{2} + \sqrt{A_{2}^{2}4A_{2}A_{1}})/2A_{1}), \) we obtain \( J_{MR} > J_{M} + J_{R}. \) That is, the profit of the supply chain under decentralized decision-making is greater than that under decentralized decision-making.

Proof.

\[ J_{MR} - J_{M} - J_{R} = \frac{1}{2} \left\{ \sum_{i=1}^{2} \left( P_{Ri} (\mu + \beta) + \mu P_{M} - \beta P_{R3(\beta)} \right) \right\} \]
\[ + \frac{b^2 y \sum_{i=1}^{2} P_{Ri} v}{\lambda k_{M} \lambda} \left( P_{R1} + P_{R2} + 2P_{M} \right) e^{\delta t} \]
\[ - \frac{by \sum_{i=1}^{2} (P_{Ri} + 2P_{M}) \sum_{i=1}^{2} P_{Ri} e^{\delta t}}{2\lambda (\lambda + \delta) k_{M}} > 0. \]  
(39)

We obtain the following:

\[ A_{1} = \frac{(\mu + \beta) \sum_{i=1}^{2} P_{Ri}}{2\lambda k_{R} \lambda} \left\{ \sum_{i=1}^{2} \left[ P_{Ri} (\mu + \beta) + \mu P_{M} - \beta P_{R3(\beta)} \right] \right\}, \]
\[ - \frac{(\mu + \beta)}{4} \sum_{i=1}^{2} 3P_{Ri} + 4P_{M} \],
\[ A_{2} = \frac{b^2 y \sum_{i=1}^{2} P_{Ri} v}{\lambda k_{M} \lambda} \left( P_{R1} + P_{R2} + 2P_{M} \right), \]
\[ A_{3} = \frac{by \sum_{i=1}^{2} (P_{Ri} + 2P_{M}) \sum_{i=1}^{2} P_{Ri} e^{\delta t}}{2\lambda (\lambda + \delta) k_{M}}. \]  
(40)
Arranging the above equation, we obtain the following:

\[ J(d) = J_{MR} - J_M - J_R = A_1 + A_2 e^{\delta d} - A_3 e^{2\delta d}. \]  

(41)

(1) When

\[ d \geq \frac{1}{\delta} \ln \left( \frac{A_2 + \sqrt{A_2^2 + 4A_3A_1}}{2A_3} \right), \]

we obtain \( J(d) \leq 0, \) \( J_{MR}^* \leq J_M^* + J_R^* \).

(2) When

\[ \frac{1}{\delta} \ln \left( \frac{A_2 + \sqrt{A_2^2 + 4A_3A_1}}{2A_3} \right) < d < \frac{1}{\delta} \ln \left( \frac{A_2 + \sqrt{A_2^2 + 4A_3A_1}}{2A_3} \right), \]

we obtain \( J(d) \leq 0, \) \( J_{MR}^* \leq J_M^* + J_R^* \).

Proposition 4 shows how the supply chain members choose reasonably the decision-making mode depending on lagged time. Within various lagged time intervals, there are different relationships between the overall profits of the supply chain under the two decision-making scenarios. If supply chain members adopt centralized decision-making mode and can sign agreements to achieve a reasonable distribution of profits, then the optimal profits of manufacturers and retailers will obtain Pareto improvement.

6. Numerical Analysis

By numerically analyzing the differences in the profit of the supply chain, the goodwill of the product, and the quality level under decentralized decision-making and centralized decision-making, we can verify the above theoretical results and explain them from the perspective of supply chain decision-making. Assume that the values of each parameter in the model are as follows: the discount rate \( \lambda = 0.1 \); the attenuation \( \delta = 0.01 \); the marginal profit \( P_M = 5 \), \( P_{R1} = 4 \), and \( P_{R2} = 3 \); the influencing factors \( b = 1.2 \), \( \theta = 3 \), \( \mu = 2 \), \( \gamma = 2 \), \( \beta = 1 \), \( k_m = 1 \), and \( k_r = 1 \); the initial value of brand credit \( G(0) = 1 \); and the potential market sales \( a = 2 \).

First, according to the benchmark parameters, the optimal trajectories of product goodwill under centralized and decentralized decision-making are plotted with \( d = 1 \) and \( d = 5 \), as shown in Figure 1.

Figure 1 shows that, over time, the product goodwill under centralized decision-making is higher than that under decentralized decision-making. The difference between the values of product goodwill under centralized decision-making and decentralized decision-making is plotted with \( d = 1 \) and \( d = 5 \), as shown in Figure 1.

Second, according to the benchmark parameters, the influence of the lagged time on the equilibrium strategies under decentralized and centralized decision-making is drawn, as shown in Figures 2 and 3.

From Figures 2 and 3, under centralized decision-making, the manufacturer’s product quality level and the retailers’ service quality level are higher than those under decentralized decision-making. The longer the lagged time of the influence of the manufacturer’s product quality on
goodwill under the two decision-making scenarios is, the higher the manufacturer’s investment will be. This indicates that the lagged time can motivate the manufacturer to invest more in product quality and to improve product quality. The longer the lagged time is when the manufacturer’s product quality improvement affects goodwill under the two decision-making scenarios, the higher the manufacturer’s investment will be. This indicates that the lagged time can motivate the manufacturer to invest more in product quality and to improve product quality. The retailers’ service quality will not change with changes in the lagged time.

However, under decentralized decision-making, the profit of manufacturer and retailer is plotted with lagged time when the manufacturer’s product quality improvement affects goodwill, as shown in Figure 4.

Figure 4 shows that, under decentralized decision-making, the manufacturer’s profit decreases with the increase in the lagged time when the manufacturer’s product quality improvement affects goodwill, and the retailers’ profit increases with the increase in the lagged time of product quality improvement. Therefore, the lagged time of product quality is favorable to retailers and unfavorable to manufacturers. Additionally, retailers with higher marginal profit will gain more profits. Next, we plot the change in the overall profit of the supply chain with the lagged time of product quality improvement under the two decision-making scenarios, as shown in Figure 5.

Figure 5 shows that when the lagged time of product quality improvement is relatively short and the manufacturer invests more in product quality under centralized
decision-making, the whole supply chain will gain more profits. At this time, the manufacturer and retailers prefer centralized decision-making. When the lagged time of product quality improvement is long, the lagged time will promote the manufacturer to invest more in product quality and improve the product quality level of the manufacturer. However, it will also lead to an increase in the cost of the manufacturer, and the cost under centralized decision-making will be greater than that under decentralized decision-making. As a result, a long lagged time will reduce the profit of the whole supply chain. Therefore, when the lagged time of product quality improvement influencing on goodwill exceeds a certain range, the profit under decentralized decision-making will be greater than that under centralized decision-making.

Finally, the influence of the competition coefficient beta on the profit of the supply chain is analyzed.

As shown in Figure 6, with the increase in the competition coefficient, the overall trend of the profits of retailer 1 and retailer 2 is decreasing. When competition is low, the influence of competition on the profit of the retailers is not as evident. With the increase in competition, its influence on the retailers is greater and is unfavorable to them. The influence of the competition coefficient on the profit of the manufacturer under decentralized decision-making is shown in Figure 7.
Form manufacturers, as retailers compete more and more fiercely, the manufacturers’ profits will increase slowly. On the one hand, retailers compete fiercely with each other, leading to increased market demand and increased sales revenue for manufacturers. On the other hand, increasing competition makes manufacturers reduce the share of service costs that they pay, and it increases the overall profits of manufacturers. Therefore, increasing competition is beneficial to manufacturers but not to retailers.

Further analysis shows that, with the increase in competition, the profit of the supply chain shows a downward trend, as shown in Figure 7. When the degree of competition is low, the profit of the supply chain under decentralized decision-making is less than that under centralized decision-making. When the degree of competition increases and exceeds a certain threshold, the profit of the supply chain under decentralized decision-making is greater than that under centralized decision-making. This result shows that...
when the competition level is low, the supply chain tends to adopt centralized decision-making. When the competition level is intense, it is appropriate for the supply chain to adopt decentralized decision-making.

At the same time, we can also find that, under centralized decision-making, the profit of a supply chain with a longer lagged time is less than that of a supply chain with a shorter lagged time. However, under decentralized decision-making, the profit of a supply chain with a longer lagged time is larger than that of a supply chain with a shorter lagged time. This result shows that when the lagged time is small, centralized decision-making is preferred. Additionally, decentralized decision-making is preferred when the lagged time is large.

7. Conclusion

Quality is the lifeline of an enterprise. Quality improvement is the driving force for the sustainable development of enterprises. The lagged time is a common phenomenon in the process of quality improvement, which plays an important role in making quality strategy of supply chain. According to previous literatures, this paper establishes a differential game model considering the lagged time and retailers’ competition to study the relationship between the optimal quality level, the product goodwill, and the profit under the two decision scenarios, whether the delay time affects the profit and quality decision, and how competition coefficient affects the relationship of supply chain profit. Some interesting conclusions can be obtained as follows:

(1) The optimal values of product goodwill, product quality, service quality, and demand under centralized decision-making are higher than those under decentralized decision-making. However, in contrast to previous studies, the overall profit of a supply chain under centralized decision-making is not always better than that under decentralized decision-making. The relationship between the overall profits in two scenarios is affected by the lagged time in the process of product quality improvement.

(2) When the lagged time in the process of product quality improvement increases, manufacturer’s optimal profit will decrease. This means that the lagged time of product quality improvement is negatively related to the manufacturer’s optimal profit and positively related to the optimal profit of retailers.

(3) The lagged time of the product quality investment is positively correlated with the product quality improvement. With the increase in the lagged time, the manufacturer will improve its product quality.

(4) Increased competition has negative impact on centralized decision-making in the supply chain. When competition is low, the profit of a supply chain under decentralized decision-making is less than that under centralized decision-making. When the degree of competition increases and exceeds a certain threshold, the profit of a supply chain under decentralized decision-making is larger than that under centralized decision-making. This shows that when the degree of competition is low, supply chain tends to adopt centralized decision-making. When competition is intense, it is appropriate for supply chain to adopt decentralized decision-making.

In accordance with the conclusion above, we could hence be enlightened as follows:

First of all, for the manufacturers in a supply chain, they should try their best to shorten the lagged time of product quality and reduce the lagged time. The longer the lagged time of product quality is, the better it will promote the manufacturers’ investment in product quality and benefit the retailers, but it is not good for the profits of the manufacturers and the overall profits of the supply chain. In order to promote the coordination of the supply chain, the manufacturers should try their best to shorten the lagged time of product quality.

Secondly, manufacturers and retailers should consider how the lagged time affects goodwill of product quality and the threshold of retailer’s competition degree when making centralized decision. When the lagged time and competition degree are within a small threshold range, the overall profit of supply chain under centralized decision-making is greater than that under decentralized decision-making. When both sides make centralized decision, the best effect will be achieved. Otherwise, they should make decentralized decision.

The limitations of our study present direction for future research. We establish a supply chain system that includes manufacturer and two competitive retailers in our model and assume they are homogeneous. The results may be more insightful if we consider the heterogeneous competition.

Data Availability

The simulated input data used to support the findings of this study are included within the article. For further questions, please contact the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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