The Analysis of BDS Measurements Quality and Stochastic Characters

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Abstract. Carrier phase measurements are always used in high precision situations. Though in short baseline case, double difference technic can eliminate most of the systematic errors, some errors still remain. Therefore, the stochastic characters and measurement quality of carrier phase is of vital importance. In this paper, the residuals of zero baseline and short baseline carrier phase are evaluated. Then the observation qualities of carrier phase from different frequencies are analyzed. Besides, the multipath of pseudorange is also investigated. The result showed that in zero baseline case the accuracy of B3 carrier phase is always worse than that of the B1. But, in general, the accuracy of all satellites at the B1 and B3 frequencies is high which ranges from 0.15mm and 0.35mm. In short baseline case, the accuracy of carrier phase is related to satellite elevations. The carrier phase accuracy is worse than that of the zero baseline case which is about 2mm but is better than 1/100 wavelength. The multipath of the three frequencies has a relationship which is B1 > B2 > B3. This is related to different wavelength and code length.

1. Introduction

BeiDou navigation system (BDS) is a global navigation system owned by China. At present, the construction of BDS-2 has completed. The BDS-3 is now under construction. Positioning and orientating using satellite navigation system has many advantages comparing to inertial navigation system.

The double differencing function model are often used in BDS orientating. In short baseline case, double difference technic can eliminate most of the systematic errors which make the solution unbiased [1-2]. However, besides systematic errors, the stochastic characters of observations also have influences on the solution optimality. The stochastic characters are related to lots of factors, such as hardware design, measurement snr ration and satellite elevation [3-6]. If the measurement quality representing factors can be chosen properly, the appropriate stochastic models can be established. Then the solution accuracy and stability can be improved [7].

Therefore, the stochastic characters and measurement quality of carrier phase is of vital importance. In this paper, the residuals of zero baseline and short baseline carrier phase are evaluated. Then the observation qualities of carrier phase from different frequencies are analyzed. Besides, the multipath of pseudorange is also investigated.
2. Observation accuracy estimation

Single differenced carrier phase always used to modeling stochastic model. Its equation can be written as

\[ \lambda \Delta \phi_i = \Delta \rho_i + \Delta \delta_i + \lambda_i \Delta N'_{i} + \Delta \epsilon_i \]  \hspace{1cm} (1)

Where \( \Delta \) represents single difference (SD) operator, \( \Delta \delta_i \) is hardware delay. Reparameterizing Eq. 6 we can obtain

\[ \Delta L_i = \Delta \delta_i + \lambda_i \Delta N_{i} + \Delta \epsilon_i \]  \hspace{1cm} (2)

Where \( \Delta L = \lambda_i \Delta \phi_i - \Delta \rho_i + \lambda_i \Delta V N'_{i} \) and after the double differenced ambiguities are resolved, there is no unknown parameters in \( \Delta L \), \( \Delta V N'_{i} = \Delta N'_{i} - \Delta N'_{i} \) is double differenced ambiguities, \( \Delta N'_{i} \) is the single differenced ambiguities of reference satellite. It is obvious that in Eq. 7, \( \Delta \delta_i + \lambda_i \Delta N_{i} \) will be a constant if there is no cycle slip. Therefore the residual estimation of satellite \( k \) is

\[ \Delta \epsilon_k = \frac{\sum_{i=1}^{m} \Delta L_i}{m} \]  \hspace{1cm} (3)

Where \( m \) is the number of satellite observed. Similarly, the residuals of rest satellites can also be calculated. The relationship between SD residual estimation accuracy and SD residual is

\[ D(\Delta \epsilon_k) = D(\Delta L_i) - \frac{1}{m} \sum_{i=1}^{m} D(\Delta L_i) \]  \hspace{1cm} (4)

For \( D(\Delta L_i) = D(\Delta \epsilon_i) \), then Eq. 9 can be further written as

\[ \frac{D(\Delta \epsilon_k)}{D(\Delta \epsilon_i)} = \left( \frac{m-1}{m} \right) \sum_{i=1}^{m} \frac{D(\Delta L_i)}{D(\Delta L_i)} \]  \hspace{1cm} (5)

In other words, \( D(\Delta \epsilon_k) = r \cdot D(\Delta \epsilon_i) \), \( r = (m-1)/m \). Assuming the measure accuracy of the two receivers are same. The the relationship between carrier phase accuracy and SD residual accuracy is \( D(\lambda_i \Delta \phi_i) = D(\Delta \epsilon_i)/2 \). Therefore carrier phase accuracy can be obtained by

\[ D(\Delta \epsilon_i) = \frac{D(\Delta \epsilon_k)}{2r} \]  \hspace{1cm} (6)

In short period, the standard deviation of \( \Delta \epsilon_k \) can be assumed as a constant. So its deviation can be obtained from multi-epoch data.
Where \( n \) is the number of satellites. Substituting Eq. 12 to Eq. 11, the final equation to calculate carrier phase accuracy is obtained

\[
D(\Delta \hat{e}_k^i) = \sum_{i=1}^{n} (\Delta \hat{e}_{i,j}^k)^2
\]

(8)

3. Multipath Estimation

Based on the observation equations of carrier phase and pseudorange, the following equation can be obtained

\[
R_i^s - \lambda_i \phi_i^s = (\rho^s - c(\Delta t_r - \delta t_r) + \delta_{\text{ion}}(f_i) + \delta_{\text{trop}} + \delta_{\text{tide}} + \delta_{\text{mulR}} + \delta_{\text{rel}} + \varepsilon_R)
\]

\[-(\rho^s + \lambda_i N_i + c(\Delta t_r - \delta t_r) - \delta_{\text{ion}}(f_i) + \delta_{\text{trop}} + \delta_{\text{tide}} + \delta_{\text{mulP}} + \delta_{\text{rel}} + \varepsilon_p)
\]

\[= (\delta_{\text{mulR}} - \delta_{\text{mulP}}) - \lambda_i N_i + 2\delta_{\text{ion}}(f_i) + \varepsilon_i
\]

(9)

Where \( \delta_{\text{mulR}} \) and \( \delta_{\text{mulP}} \) are multipath errors from phase and pseudorange, \( \lambda \) is wavelength, \( N \) is ambiguity, \( \delta_{\text{ion}} \) is ionosphere delay, \( \varepsilon \) is residual, \( i \) represents the \( i \)th frequency. In this equation, errors such as clock errors and troposphere delay have been eliminated. But the multipath errors, ambiguity and ionosphere delay remains. For the pseudorange multipath are much more significant than carrier phase. Therefore, \( \delta_{\text{mulP}} \) can be ignored. The model of ionosphere delay is given by

\[
\delta_{\text{ion}}(f_i) = \frac{A}{f_i^2}
\]

(10)

Thus, the relationship between two different frequencies can be derived

\[
f_i^2 \delta_{\text{ion}}(f_i) = f_j^2 \delta_{\text{ion}}(f_j)
\]

(11)

Then the estimator of ionosphere delay can be given by

\[
\delta_{\text{ion}}(f_i) = \frac{f_j^2}{f_i^2 - f_j^2} (\lambda_j \phi_j - \lambda_i \phi_i) + \frac{f_j^2}{f_i^2 - f_j^2} (N_j - N_i)
\]

(12)

Substitute the estimator to Eq.9, the equation to calculate pseudorange multipath is obtained

\[
MP_i = R_i^s - \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} \lambda_i \phi_i + \frac{2f_i^2}{f_i^2 - f_j^2} \lambda_j \phi_j + C_i + \varepsilon_{MP}
\]

(13)

Where \( \varepsilon_{MP} \) is residual,

\[
C_i = -\frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} \lambda_i N_i + \frac{2f_i^2}{f_i^2 - f_j^2} \lambda_j N_j
\]

(14)
When there is no cycle slip, \( C_j \) will be a constant which can be regarded as a systematic bias. For in Eq.14 ambiguities are difficult to resolve, the values of \( MP_i \) cannot be obtained directly. Thus, we analyze the sequence after \( MP_i \) removes its mean value in a short period. Though this sequence is not the actual multipath value, it can reflect the multipath variations.

4. Numerical experiment and observation analysis

4.1. Zero baseline accuracy

Zero baseline single differenced (SD) observations can eliminate nearly all systematic errors. Thus, they can be used to evaluate the receiver accuracy. The experiment is carried out using two dual frequency receivers with a sample rate of 1s. The receiver is capable of receiving frequency B1 and B3. Signals of the same antenna are respectively introduced into two receivers through a splitter. Recorded data was processed by self-developed GNSS Multiple Frequency ToolBox. The results are showed in Fig.1 and Table 1. Fig.1 illustrates the SD carrier phase residuals with satellite elevation where red lines represent frequency B1 and blue lines represent frequency B3. Table 1 lists the carrier phase mean observation accuracy on B1 frequency and B3 frequency.

As shown in Fig.1, the carrier phase residuals are related to the satellite elevation. The higher the elevation, the smaller the residuals. Though the residuals of GEO satellites are relatively stable, it still can be found out that GEO satellites that with lower elevation preserve larger residuals. Table 1 lists the mean observation accuracy of all satellites. Comparing the accuracy of each satellite in this table, the accuracy of B3 observations is always worse than that of the B1 observations. The mean STD of all satellites at B1 is about 0.2056mm, while it is 0.2724mm for B3. But, in general, the STD of all satellites at the B1 and B3 frequencies is relatively stable which ranges from 0.15mm and 0.35mm.

\[
\text{(a) GEO satellites C1, C3 and C4} \quad \text{(b) IGSO satellite C6} \quad \text{(c) MEO satellite C14}
\]

**Figure 1.** Zero baseline carrier phase residuals
Table 1. The mean standard derivation (STD) of zero baseline carrier phase (mm)

| No.  | C1          | C2          | C3          | C4          | C5          | C6          | C7          |
|------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| STD  | 0.2422      | 0.2505      | 0.1949      | 0.1844      | 0.1726      | 0.2005      | 0.1807      |
| No.  | C8          | C9          | C10         | C11         | C12         | C13         | C14         |
| STD  | 0.2338      | 0.1984      | 0.1865      | 0.1852      | 0.2535      | 0.1906      | 0.2052      |
| B3   | No.         | C1          | C2          | C3          | C4          | C5          | C6          | C7          |
| STD  | 0.3453      | 0.3394      | 0.2615      | 0.2524      | 0.2280      | 0.2688      | 0.2510      |
| No.  | C8          | C9          | C10         | C11         | C12         | C13         | C14         |
| STD  | 0.3027      | 0.2734      | 0.2681      | 0.2193      | 0.2842      | 0.2817      | 0.2374      |

4.2. Short baseline accuracy

Short baseline single differenced (SD) observations can eliminate the effects of troposphere delay and ionosphere delay. Though some errors like multipath still remains, the observations can be used to evaluate the observation accuracy.

The experiment is carried out using two dual frequency receivers with a sample rate of 1s. The receivers connects to two antennas which are about 10 meters away from each other. The receiver is capable of receiving frequency B1 and B3. The process results are shown in Fig.3.

Figure 2. Short baseline carrier phase residuals and accuracy
From Fig.2 (a), the variation of the GEO satellite residuals are relatively flat because of their stable geometric position. The residuals of IGSO satellites and MEO satellites in Fig.2 (b, c) have significant changes, especially when the satellite is about to be observed or just entering the observation range. Because the elevation angle is at this time is relatively low and the multipath effect is serious. In addition, the residuals of the B1 and B3 have no significant difference making it difficult to distinguish.

Fig.2 (d, e, f) show the relationship between carrier phase accuracy and elevation angle. It is obvious that the accuracy of the IGSO satellites and the MEO satellites are proportional to the elevation angle. The observation accuracy of the GEO satellites is also related to the elevation angle. The observation accuracy of the C3 satellite which has a high elevation angle is much higher than others. Moreover, the carrier phase accuracy is worse than that of the zero baseline case. The observation accuracy of GEO satellites is between 0.2mm and 1.5mm, the IGSO satellites are between 0.2mm and 1.8mm, and the MEO satellites are between 0.3mm and 1.3mm. The accuracy of the B1 and the B3 is basically the same. There is no significant difference. In addition, the wavelength of the B1 frequency is 0.1920m, and the wavelength of the B3 frequency is 0.236m, so the carrier phase accuracy of all satellites is better than 1/100 wavelength.

4.3. Multipath
The experiment was carried out for about 24 hours, the sampling interval was 2s, and the frequency bands in which the data were collected were B1, B2 and B3. The data processing was carried out by self-developed multi-frequency GNSS data processing toolkit. The results are shown in Fig.3 and Table 2.

Combined with Fig.2 and Table1, the multipath of the GEO satellites is stable in any frequency and it does not change much throughout the day. However, the multipath average of C4 and C5 is higher than the other three, and there is a relationship: C4 > C5 > C2 > C1 > C3. This is mainly caused by the difference in elevation. The elevation angles of C4 and C5 are approximately 23 degrees and 25 degrees, while C1, C2 and C3 are 37 degrees, 44 degrees and 50 degrees.

Compared to GEO satellites, the multipath of IGSO satellites and MEO satellites has significant changes, especially when satellites are about to be observed or when satellites are just emerging. At this time, the elevations of the satellites are low, and the multipath is significantly higher than other times.

In addition, comparing the multipath on different frequencies of the three types of satellites, it can be clearly found that the multipath of the three frequencies has a relationship of B1 > B2 > B3. The multipath of the B3 frequency is much smaller than B1 and B2. B2 is smaller than B1 mainly because the wavelength of B2 is longer. The reason that B3 is less affected by multipath is mainly because the code length of B3 frequency is about 10 times higher than that of B1 and B2.
Figure 3. The multipath STD of pseudorange

Table 2. The multipath STD of pseudorange (m)

| Freq. | C1   | C2   | C3   | C4   | C5   | C6   | C7   |
|-------|------|------|------|------|------|------|------|
|       |      |      |      |      |      |      |      |
| B1    | 0.1603 | 0.185 | 0.1244 | 0.2469 | 0.2284 | 0.1708 | 0.1755 |
| B2    | 0.1124 | 0.1108 | 0.087 | 0.1661 | 0.1547 | 0.1254 | 0.1174 |
| B3    | 0.066  | 0.0594 | 0.0421 | 0.0907 | 0.0812 | 0.0788 | 0.0741 |
|       |      |      |      |      |      |      |      |
| Freq. | C8   | C9   | C10  | C11  | C12  | C13  | C14  |
|       |      |      |      |      |      |      |      |
| B1    | 0.1904 | 0.1531 | 0.1581 | 0.1507 | 0.1523 | 0.1338 | 0.1279 |
| B2    | 0.145  | 0.1132 | 0.1184 | 0.1089 | 0.1059 | 0.1249 | 0.1061 |
| B3    | 0.1137 | 0.0763 | 0.0787 | 0.0811 | 0.0755 | 0.0708 | 0.0626 |

5. Conclusion
In this paper, the residuals of zero baseline and short baseline carrier phase are evaluated. Then the observation qualities of carrier phase from different frequencies are analyzed. Besides, the multipath of pseudorange is also investigated. The results showed that in zero baseline case the accuracy of B3 carrier phase is always worse than that of the B1. But, in general, the accuracy of all satellites at the B1 and B3 frequencies is high which ranges from 0.15mm and 0.35mm. In short baseline case, the accuracy of carrier phase is related to satellite elevations. The carrier phase accuracy is worse than that of the zero baseline case which is about 2mm but is better than 1/100 wavelength. The multipath of the three frequencies has a relationship which is B1 > B2 > B1. This is related to different wavelength and code length.

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