Study on the superposition of multi-harmonic sources in electrified railway traction substations

Xiaohong Geng\textsuperscript{1,3}, Jun Wen\textsuperscript{1}, Sichao Wang\textsuperscript{1}, Dongshan He\textsuperscript{1} and Yeniu Qian\textsuperscript{2}

\textsuperscript{1}School of Electrical & Electronic Engineering, North China Electric Power University, Beijing 102206, China
\textsuperscript{2}State Grid Beijing Electric Power Company, Beijing 102206, China
\textsuperscript{3}E-mail: 1390215919@qq.com

Abstract. The harmonic current generated by electrified railway traction substations has stochastic volatility. Aiming at the superposition of multi-harmonic sources caused by electrified railway traction substations in a certain area, this paper establishes the harmonic sources model of electrified railway traction substations, and calculates to superpose harmonic sources with random amplitude and phase angle by the national standard method and Monte Carlo method in MATLAB. Then built the power grid model of the area to verify the above methods based on ETAP simulation software. The results show that Monte Carlo method has less calculation error in calculating random harmonic sources and is more suitable for evaluating the superposition of multi-harmonic sources in electrified railway traction substations.

1. Introduction
In recent years, a large number of electrified railway traction substations (hereinafter referred to as ‘traction substations’) have been accessed intensively \textsuperscript{C}, the traction substations obtain power from the AC system and simultaneously inject plenty of harmonics into the power grid as a harmonic source. For the point of common coupling \textsuperscript{(PCC)} bus of the power grid with multiple traction substations connected, evaluating the harmonic level becomes the research emphasis of the regional power quality. Traction substation is a typical dynamic harmonic source. As a special power electronic equipment, the harmonics generated by traction station not only cause the power quality degradation, but also may cause the resonance of AC system.

At present, domestic and foreign scholars have done some research on the superposition of multi-harmonic sources \textsuperscript{[1-11]}. Documents \textsuperscript{[1-4]} obtain the statistical characteristics of harmonic current by actual measurement and statistics. However, it is impossible to install harmonic detection devices in each node of power system, and the investment is high. In reference \textsuperscript{[5]}, a random phase probability analysis method is proposed, which converts the sum of random vectors with \([0,2\pi]\) distribution into the sum of random variables. At present, it is limited to convolution method. The International Electrotechnical Commission (IEC) and Chinese harmonic standard for power quality public power grid (GB/T 14549-93) have proposed an approximate method to investigate the superposition of multi-harmonic sources \textsuperscript{[6-7]}. The literature \textsuperscript{[8]} uses Laguerre polynomial to calculate the superposition of multi-harmonic sources with different phase angle distributions in the power system, but Laguerre polynomial will bring the truncation error. Monte Carlo (MC) method is proposed in \textsuperscript{[9-10]} which calculate the superposition current of multi-harmonic source with unknown phase angle. This method
avoids the cumbersome convolution integral. However, the stochastic volatility of the amplitude of harmonic current is not considered in the calculation process.

To solve the superposition of multi-harmonic sources caused by intensive harmonic source access of massive electrified railway traction substations in an area of China, firstly, this paper studies the model of harmonic source of traction substations, and deduces the theoretical formula of the superposition of multi-harmonic sources. Secondly, Monte Carlo method, which can realize random sampling, is introduced into the superposition of multi-harmonic sources with random phase and amplitude of harmonic current. Subsequently, the specific harmonic superposition process and its error estimation are investigated, and the superposition calculation of multi-harmonic source with random amplitude and phase angle is realized by programming in MATLAB. Finally, the area power grid is built using the power system analysis software ETAP to verify the accuracy of the proposed method in the calculation for the superposition of multi-harmonic sources.

2. Harmonic source model of traction substation

As a single-phase high-power rectifier load, traction load generates plenty of harmonics which flow along the traction network to the traction substations. Therefore, for the power system, each traction substation is a harmonic current source.

2.1. Single harmonic source

When only one traction substation is accessed to the power grid, the Norton equivalent circuit of the $h$th harmonic current source generated by the traction substation is shown in Figure 1. In the figure, $u_{sh}$ and $Z_{sh}$ are the equivalent voltage source and impedance on the system side, $i_{sh}$ is the $h$th harmonic current flowing into the power system, and $i_{1h}$ and $Z_{1h}$ are the Norton equivalent current and impedance of the harmonic current source in the traction substation respectively.

![Figure 1. Norton equivalent model of power system with single current harmonic source.](image)

2.2. Multiple harmonic source

When $m$ traction substations are accessed to PCC points at the same time, the Norton equivalent circuit of the multi-harmonic source system with harmonic order $h$ is shown in Figure 2. In the figure, $i_{kh}$ and $Z_{kh}$ ($k=1,2,\ldots,m$) are the Norton equivalent current and equivalent impedance of the $k$th harmonic current source respectively, A is the PCC bus of the system, $i_{sh}$ is the $h$th harmonic current flowing into bus A.

Assuming that $I_{kh}$ is the $h$th harmonic current when the $k$th harmonic source acts alone, its amplitude and phase angle is $I_{kh}$, $\phi_{kh}$ respectively, then

$$I_{kh} = I_{kh} \angle \phi_{kh}$$  \hspace{1cm} (1)

According to Kirchoff's Law of Current (KCL), the $h$th harmonic current flowing into bus B can be expressed as
\[ i_{sh} = i_{1h} + i_{2h} + \cdots + i_{mh} \]
\[ = \sum_{k=1}^{m} I_{kh} \cos \varphi_{kh} + j \sum_{k=1}^{m} I_{kh} \sin \varphi_{kh} \]  

(2)

Figure 2. Norton equivalent model of power system with multiple current harmonic sources.

3. Calculation method of the superposition of multi-harmonic sources

The conventional method of harmonic source analysis considers that the injected harmonic current generated by the non-linear load is fixed, and the harmonic superposition result can be obtained by substituting the fixed amplitude and phase angle of the harmonic current source into Equation (2). However, in the actual power system, the amplitude and phase angle of harmonic current generated by non-linear load have stochastic volatility, which makes it difficult to evaluate the harmonic level of PCC bus. At present, the commonly used methods to analyse the superposition of multi-harmonic sources are the Chinese standard method and the Monte Carlo method.

3.1. Chinese standard method

GB/T 14549-1993 points out that when the phase angle \( \theta_{h} \) between the same order harmonic currents \( A_{hi} \) and \( A_{hj} \) generated by two harmonic sources in the power grid is determined, the superposition formula of the harmonics is as follows [6].

\[ M_{h} = \sqrt{A_{hi}^{2} + A_{hj}^{2} + 2A_{hi}A_{hj} \cos \theta_{h}} \]  

(3)

When the phase angles of two harmonic sources are unknown, the superposition formula can be expressed as

\[ M_{h} = \sqrt{A_{hi}^{2} + A_{hj}^{2} + K_{h}A_{hi}A_{hj}} \]  

(4)

The coefficient \( K_{h} \) is shown in Table 1.

| \( h \text{th} \) | 3 | 5 | 7 | 11 | 13 | 9 | >13 | even |
|---------------|---|---|---|----|----|---|-----|-----|
| \( K_{h} \)    | 1.62 | 1.28 | 0.72 | 0.18 | 0.08 | 0 | 0 | 0 |

When multiple harmonic current sources are superimposed, firstly add two of the same order harmonic currents, and then superimpose the superposition result with the third harmonic current, and so on.
In some power systems, the harmonic superimposed current calculated by Equation (4) has a certain deviation from the actual value. In order to obtain a more reasonable result of the superposition of multi-harmonic sources, the Monte Carlo method is introduced to explore the superposition of multi-harmonic sources.

3.2. Monte Carlo method
Monte Carlo method belongs to a branch of experimental mathematics. It is based on sampling theory and uses random numbers to obtain statistical eigenvalues through statistics of random variables, and takes them as numerical solutions to be solved [7, 10, 12].

As the mathematical basis of Monte Carlo method, the law of large numbers and the central limit theorem of probability theory represent the mathematical properties of Monte Carlo method, theoretically guarantee the correctness of Monte Carlo method, the stability and convergence are verified by the law of large numbers, and the calculation error and convergence rate are analysed by the central limit theorem.

Suppose there are \( m \) harmonic current sources of traction substations in the power system, let \( \Phi_{kh} \) \((k=1,2,\ldots,m)\) denote the random phase of the \( k \)th \( h \)th harmonic current, then the \( i \)th test harmonic current amplitude and phase sample value can be defined as

\[
\begin{bmatrix}
I_{ih} \\
\theta_{ih}
\end{bmatrix} = \begin{bmatrix}
I_{1ih}, I_{2ih}, \ldots, I_{mih}
\end{bmatrix},
\]

where \( I_{khh} \) is the random amplitude of the \( h \)th harmonic current generated when the \( k \)th harmonic source acts alone in the \( i \)th experiment, \( I_{ih} \) is a row vector which contains the random amplitudes of \( m \) harmonic current sources, \( \Phi_{khh} \) \((k=1, 2, \ldots, m)\) is the random phase of the \( k \)th harmonic current in the \( i \)th test, \( \theta_{ih} \) is a column vector which embodies the random phases of \( m \) harmonic current sources.

If \( \hat{I}(I_{ih}, \theta_{ih}) \) represents the harmonic current phasor superimposed by \( m \)th harmonic currents in the \( i \)th test, then \( \hat{I}(I_{ih}, \theta_{ih}) \) can be expressed as

\[
\hat{I}(I_{ih}, \theta_{ih}) = I_{ih} \cos \theta_{ih} + j I_{ih} \sin \theta_{ih}
\]

Assuming that the number of sampling tests is \( N \), the sample space of the harmonic current’s amplitude is \( \{I_{1ih}, I_{2ih}, \ldots, I_{Nih}\} \), the phase sample space is \( \{\theta_{1ih}, \theta_{2ih}, \ldots, \theta_{Nih}\} \), and the average value of the harmonic currents in \( N \) sampling tests is given as

\[
\bar{I}_{Nh} = \frac{1}{N} \sum_{i=1}^{N} \hat{I}(I_{ih}, \theta_{ih})
\]

If \( f(I_{ih}, \theta_{ih}) \) represents the probability density distribution of \( (I_{ih}, \theta_{ih}) \), then the mathematical expectation of \( \hat{I}(I_{ih}, \theta_{ih}) \) can be found from

\[
\mathbb{E} \hat{I} = \int_0^\infty \int_0^{2\pi} \hat{I}(I_{ih}, \theta_{ih}) f(I_{ih}, \theta_{ih}) dI_{ih} d\theta_{ih}
\]

When \( N \) is relatively large, \( \bar{I}_{Nh} \) is considered to be an estimate of \( \mathbb{E} \hat{I} \), which approximates the actual harmonic current.

According to Wiener-khinchin law of large numbers, if the random variable sequence \( \{\hat{I}(I_{ih}, \theta_{ih}), i = 1, 2, \ldots, N\} \) is independently and identically distributed, and its mathematical expectation exists, then
\[
P\left( \lim_{N \to \infty} I_{Nh} = EI \right) = 1
\]  

That is to say, when \( N \to \infty \), the estimated value \( \bar{I}_{Nh} \) of the statistic \( I(I_{Nh}, \theta_{Nh}) \) converges to the expected \( EI \) with probability 1.

According to Lindburg-Levy central limit theorem in probability theory, when \( N \to \infty \), it can be deduced that \( \lim_{N \to \infty} P\left( \frac{|I_{Nh} - EI|}{\sqrt{N}} < \frac{X_{\alpha} \sigma}{\sqrt{N}} \right) = 1 - \alpha \) is established with probability 1-\( \alpha \).

\[
\lim_{N \to \infty} P\left( \frac{|I_{Nh} - EI|}{\sqrt{N}} < \frac{X_{\alpha} \sigma}{\sqrt{N}} \right) = 1 - \alpha
\]

where \( \sigma \) is the standard deviation, \( \alpha \) is the confidence degree, 1-\( \alpha \) is called the confidence level, \( \alpha \) corresponds to \( X_{\alpha} \), and \( X_{\alpha} \) is the normal difference. Their values are shown in Table 2.

Table 2. The relation between confidence degree, confidence level and normal difference.

| \( \alpha \) | 1-\( \alpha \) | \( X_{\alpha} \)   |
|-------------|-------------|------------------|
| 0.01        | 0.99        | 2.2728           |
| 0.05        | 0.95        | 1.96             |
| 0.1         | 0.9         | 1.64485          |

If \( \sigma \neq 0 \), it is known from the inequality in Equation (10) that for \( h \)th harmonic, the relative error \( \varepsilon_i \) of the statistical estimate \( \bar{I}_{Nh} \) is

\[
\varepsilon_i = \frac{X_{\alpha} \sigma}{\sqrt{NI_{Nh}}} 
\]

It can be seen from Equation (11) that the calculation error of the Monte Carlo method is the probability error at a certain confidence level, which is inversely proportional to the square root of the number of samples \( N \). When the number of samples \( N \) is sufficiently large, it is guaranteed to meet the required accuracy requirements.

Monte Carlo method can be summarized as the following four calculation steps:

1. Establish a probability model based on actual problems or actual systems;
2. Sample values are generated by random sampling according to the probability distribution of random variables;
3. Determine the functional relationship between the statistic and the random variable, and select the statistic;
4. Estimates of statistics are obtained from the arithmetic mean of statistics as an approximate estimate to solve the problem.

3.3. Comparison of the two methods

In the example, the confidence degree \( \alpha \) is 0.05, the confidence level 1-\( \alpha \) is 0.95, the normal difference \( X_{\alpha} \) is 1.96, the given error EPS is 0.01, the sampling number is 20. Models of harmonic source’s amplitude for traction substations can be referred in Table 3. In MATLAB, the harmonic current data in Table 3 is input, and use random numbers to generate random amplitudes. When the harmonic order is less than 7, the phase angle of harmonic current produced by each traction substation fluctuates randomly in the first quadrant. When the harmonic order is greater than 7, the phase angle of harmonic current produced by each traction substation fluctuates randomly in four quadrants [10]. Monte Carlo method and Chinese standard method are used to calculate the superposition of multi-harmonic sources. The results are shown in Figure 3.
Figure 3. Calculation results of 2 methods for multi-harmonic sources superposition.

As can be seen from Figure 3, the two curves above are the superposition currents of each harmonic when five harmonic sources are superimposed, and the two curves below are the superposition currents of each harmonic when two harmonic sources are superimposed. With the increase of the number of harmonic sources, the Monte Carlo method and the Chinese standard method have a consistent trend in the calculation of harmonic superimposed currents, and both show an increasing trend, but there is a big gap between the two calculation results. In order to select a more reasonable method to explore the superposition of multi-harmonic sources, the power system analysis software ETAP is used to verify the simulation.

4. Case verification

4.1. Introduction of the case

The electrified railway of 220 kV power grid in an area of China is accessed intensively. The power grid in this area includes four substations and five traction substations, as shown in Figure 4. In the figure, the substation and the traction station are marked with their apparent power, unit MVA; the line type and their length are marked on each transmission line. In order to ensure the reliability of traction substation power supply, each traction substation uses double circuit line to supply power. When one circuit breaks down, the other circuit can guarantee normal power supply. In this paper, the harmonic current of SD–H transmission line is taken as the research focus to evaluate the impact of harmonic source intensively access of multiple traction substations on the harmonic level of H substation.

Figure 4. Power grid structure in an area of China.

The measured harmonic data are from 5 traction substations in an area of China. The data are obtained by Fluke power quality analyser. The harmonic currents of 2-25 times are measured by 24
hours continuous monitoring method. The measured harmonic data of traction substation show that even harmonic current is very small, so only odd harmonic current with larger value is selected in this paper, as shown in Table 3.

| nth | HT     | XHT    | LS     | HS     | LD     |
|-----|--------|--------|--------|--------|--------|
| 3   | 0.473  | 2.181  | 1.617  | 2.439  | 3.829  |
| 5   | 0.635  | 1.127  | 1.530  | 1.548  | 1.688  |
| 7   | 0.473  | 1.000  | 0.849  | 1.054  | 1.631  |
| 11  | 0.307  | 0.793  | 0.536  | 0.634  | 1.046  |
| 13  | 0.105  | 0.634  | 0.262  | 0.356  | 0.628  |
| 17  | 0.375  | 0.493  | 0.825  | 0.862  | 1.003  |
| 19  | 0.292  | 0.390  | 1.262  | 1.188  | 1.606  |
| 23  | 0.266  | 0.264  | 0.363  | 0.401  | 0.791  |
| 25  | 0.215  | 0.144  | 0.183  | 0.551  | 1.060  |

4.2. Simulation calculation

Based on the ETAP simulation model (ETAP 16.2.0 Chinese, and the basic data package and harmonic analysis module are used in this paper), the relative errors of the fundamental current and the measured values of the above five traction substations are 1.07%, 0.49%, 1.15%, 1.28% and 0.06% respectively, and their values are less than 2%. The results show that the simulation model in ETAP can be used to evaluate harmonic levels in the region.

In order to accurately reflect the stochastic volatility of the amplitude and phase of harmonic current generated by traction substations, according to the distribution characteristics of the actual traction station harmonic current, the random amplitude and phase angle of each harmonic current are simulated in MATLAB. The resulting data is entered into the harmonic current settings in ETAP as the harmonic current for each traction substation. The following two harmonic source injection modes are designed (the calculation results are shown in Figure 5 and Figure 6):

- Mode 1: Inject harmonic current simultaneously at the LS, HS and LD traction substations;
- Mode 2: Harmonic current sources are injected into all 5 traction substations.

![Figure 5. Comparisons of calculation results under harmonic source injection mode 1.](image-url)
Figure 5 to Figure 6 show that when there are multiple traction substation harmonic sources accessed to the region, Monte Carlo method and ETAP calculation results are similar, the relative error is between 1.04% and 9.53%, and the maximum relative error is not more than 10%, the average error is 5.67%; and the relative error of the Chinese standard method and ETAP harmonic current calculation is large, and the average relative error is greater than 10%. When the number of harmonics is less than 13, with the increase of harmonic frequency, the actual harmonic currents’ amplitude of traction substations tend to attenuate (test data in Table 3), so the harmonic superposition current amplitude tends to decrease, while when the number of harmonics is greater than 13, the actual harmonic currents’ amplitude increase(The 17th and 19th harmonic currents in the test data in Table 3 show a quite large value in several traction substations.), so the harmonic superposition current amplitude tends to increase after the 13th harmonic. When the harmonic order is greater than 19, the amplitude of harmonic superposition current decreases.

5. Conclusions
The traction substation injects harmonic current into the power system as a harmonic current source. When multiple traction substations connect to PCC bus, the harmonic currents flowing into PCC bus are superimposed by the harmonic current vectors of each traction substation.

When the number of traction substations connected to the region increases, the amplitude of each harmonic superimposed current increases.

Compared with the calculation results in ETAP, the relative error of Chinese standard method is larger. Monte Carlo method can estimate random variables according to the specific distribution of harmonic current amplitude and phase angle of traction substation, and the relative error is smaller. This paper considers that it is more reasonable to use Monte Carlo method to investigate the superposition of multi-harmonic sources caused by intensively access of electrified railway traction substations.

References
[1] Xie Shaofeng and Li Qunzhan 2005 Study on harmonic distribution characteristic and probability model of the traction load of electrified railway Proceedings of the CSEE (16) 79-83
[2] R E Morrison and A D Clark 1984 Probabilistic representation of harmonic currents in AC traction systems IEE proc. Vol. 131 No. 5
[3] Xiao Y and Yang X 2012 Harmonic summation and assessment based on probability
distribution *IEEE Trans on Power Delivery* 27(2) 1030-1032

[4] Li Xinran, Sun Qian and Zhu Xianyou 2012 A harmonic source model of electrified railway traction load on measured data *Power System Technology* 36(01) 158-162

[5] Sherman W G 1972 Summation of harmonics with random phase angles *Proc IEEE* Vol. 119 1643-1648

[6] IEC 61000-3-6 1996 Assessment of Emission Limits for Distorting Loads in MV and HV Power System *IEC*

[7] State Bureau of Technical Supervision 1993 Electric Energy Quality Communal Electric Fence Harmonic of the national standard (GB/T 14549-93) Beijing: Standards Press of China pp 4

[8] Wang Lei and Yang Honggeng 2005 Summation of random harmonic vectors based on Laguerre polynomials *Automation of Electric Power Systems* 40(04) 40-44

[9] Hua Huichun, Zheng Lu and Wang Li 2016 Calculation method for same-order harmonic superposition of multiple harmonic sources *Automation of Electric Power Systems* 40(19) 107-112

[10] Liu Fan 2003 The research of methods in superposition of multi-harmonics sources in electrical traction system Sichuan University

[11] Hu Wei, Cha Xiaoming and Sun Jianjun 2006 Harmonic sources modeling and harmonic superposition simulation of power system with multi-harmonic sources *Electric Power* (03) 61-65

[12] Zeng Ming, Yang Yongqi and Wang Yuqing 2016 Generation side coordinative power system planning simulation based on monte carlo method *Journal of North China Electric Power University* (Natural Science Edition) 43(05) 94-104