Baryons in the Gross-Neveu model in $1+1$ dimensions at finite number of flavors

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Introduction

- Phase diagram of QCD at intermediate densities is largely unknown.
- Several toy models for QCD have inhomogeneous phases (chiral condensate is a function of space) in the $\mathcal{N}_f \to \infty$ limit / mean-field approximation.
- We focus on the Gross-Neveu (GN) Model in 1+1 dimensions at finite number of flavors.
- The main interests are whether the inhomogeneous phase still occurs, the structure of the phase diagram and the role of baryons.
- This talk is about our two recent papers:
  - [J. Lenz, L. Pannullo, M. Wagner, B. Wellegehausen, A. Wipf, Phys. Rev. D 101, 094512 (2020), arXiv:[2004.00295]]
  - [J. Lenz, L. Pannullo, M. Wagner, B. Wellegehausen, A. Wipf, (2020) arXiv:[2007.08382]]
Gross-Neveu Model
The Gross-Neveu Model is a toy model with crude similarities to QCD:
- The fermions interact by a 4-point fermion interaction.
- A discrete chiral symmetry is realized in the action.
- This symmetry can be spontaneously broken.

Euclidean action of the Gross-Neveu model:

\[
S_E = \int d^2x \left( \bar{\psi}_f (\phi + \gamma_0 \mu) \psi_f - \frac{\lambda}{2N_f} (\bar{\psi}_f \psi_f)^2 \right).
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**Hubbard-Stratonovich transformation:**

\[
Z = \mathcal{N} \int \mathcal{D} \psi_f \mathcal{D} \bar{\psi}_f \mathcal{D} \sigma \exp \left[ - \int d^2 x \left( \bar{\psi}_f (\gamma^0 + \gamma_0 \mu + \sigma) \psi_f + \frac{N_f}{2\lambda} \sigma^2 \right) \right].
\]

\[
\langle \bar{\psi}(x) \psi(x) \rangle = -\frac{N_f}{\lambda} \langle \sigma(x) \rangle \quad \text{→ from now on refer to } \sigma \text{ as chiral condensate}
\]
Rich and interesting phase diagram in the limit of $N_f \rightarrow \infty$
(suppresses all fluctuations of $\sigma$, i.e. only the minimum of the effective action contributes)

Phase diagram of the Gross-Neveu model:

[O. Schnetz, M. Thies and K. Urlichs, Annals Phys. 314, 425 (2004) [hep-th/0402014]]
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Phase diagram of the Gross-Neveu model:

The chiral condensate for $T = 0$ and different $\mu$:

[O. Schnetz, M. Thies and K. Urlichs, Annals Phys. 314, 425 (2004) [hep-th/0402014]]
Baryons in the $N_f \to \infty$ limit

- Baryons align with the condensate in fixed spatial separation.
  - The inhomogeneous phase is interpreted as a crystal of baryons.
- Baryon number matches the number of cycles of the condensate.

The averaged baryon density at $T = 0$: 

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{baryon_density_plot}
\caption{The averaged baryon density at $T = 0$.
\cite{Schnetz:2004kx}}
\end{figure}
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The averaged baryon density at $T = 0$:

\[ n_B(x) \text{ and } \sigma(x) \text{ for } (\mu/\sigma_0, T/\sigma_0) = (0.7, 0): \]

[O. Schnetz, M. Thies and K. Urlichs, Annals Phys. 314, 425 (2004) [hep-th/0402014]]
Lattice simulations at finite $N_f$

- Previously unknown whether inhomogeneous behavior survives at finite $N_f$ in 1+1 dim.
- Not only the minimum of the action contributes.$\rightarrow$ lattice MC simulations
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- Usage of two different derivative discretizations which preserve chirality as a cross-check:
  - Naive discretization
  - SLAC discretization → shown in this talk
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→ Now we turn to the investigation of baryons and their spatial distribution at finite $N_f$. 
Baryons at finite $N_f$
The HMC algorithm is blind to the breaking of translational symmetry.
\[ \langle \sigma(x) \rangle \approx 0 \text{ due to destructive interference} \]

Use correlation observables that preserve the inhomogeneities instead:

\[ C(x) = \left\langle \frac{1}{N_t N_s} \sum_{t,y} \sigma(t, y + x) \sigma(t, y) \right\rangle, \]

\[ C_{n_B \sigma^2}(x) = \left\langle \frac{1}{N_t N_s} \sum_{t,y} n_B(t, y + x) \sigma^2(t, y) \right\rangle. \]
Spatial distribution of the baryon density

\[ C \text{ and } C_{nB}\sigma^2 \text{ for } N_f \rightarrow \infty \text{ at } \]
\[ (\mu/\sigma_0, T/\sigma_0) = (0.700, 0.038): \]

![Graph showing the spatial distribution of the baryon density](image-url)
Spatial distribution of the baryon density

\[ C \text{ and } C_{nB}\sigma^2 \text{ for } N_f \to \infty \text{ at } (\mu/\sigma_0, T/\sigma_0) = (0.700, 0.038): \]

\[ C \text{ and } C_{nB}\sigma^2 \text{ for } N_f = 8 \text{ at } (\mu/\sigma_0, T/\sigma_0) = (0.700, 0.038): \]
Correspondence of condensate cycles and baryon number

- $N_f \to \infty$: Baryon number = number of cycles of the oscillation of the condensate
- The dominating frequency of $C$ is equal to the dominating frequency in $\sigma$.
- Define observable to extract this:

$$k_{\max} = \arg \max_k \tilde{c}(k).$$

- The number of cycles of the oscillation is then

$$\nu_{\max} = \frac{L\langle |k_{\max}| \rangle}{2\pi}.$$
Condensate cycles and baryon number at $T = 0.076$

$B$ and $\nu_{\text{max}}$ at $T/\sigma_0 = 0.076$:

![Graph showing $B$ and $\nu_{\text{max}}$ vs $\mu/\sigma_0$](image-url)
Condensate cycles and baryon number at $T = 0.038$

$B$ and $\nu_{\text{max}}$ at $T/\sigma_0 = 0.038$:
Condensate cycles and baryon number at $T = 0.038$

$B$ and $\nu_{\text{max}}$ at $T/\sigma_0 = 0.038$:

MC history of $B$ and $\nu_{\text{max}}$ at $(\mu/\sigma_0, T/\sigma_0) = (1.100, 0.038)$:
Conclusion

- spatial distribution of the baryons at finite $N_f$ as in $N_f \to \infty$
- $N_f \to \infty$ limit retains important properties of inhomogeneous behavior in finite $N_f$ physics.
- Future steps: apply our techniques to models more relevant to QCD such as the quark-meson model in 3+1 dimensions.