On Generation, Motions, and Collisions of Dowsons

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Dowsons are ±2π point singularities of the unitary complex order parameter $e^{i\phi}$ characterizing the so-called dowser texture in a thin nematic layer with homeotropic boundary conditions. Dowsons are therefore similar to disclinations in freely-standing smectic C films or to vortices in two-dimensional superfluids or superconductors. Using especially tailored setups called dowsons’ colliders, pairs of dowsons of opposite signs are generated and set into motion on counter-rotating trajectories leading to collisions. In a first approximation, the velocity of dowsons is orthogonal and proportional to the local phase gradient $\nabla \phi$. The outcome of collisions, i.e., either annihilation or bypass, depends on the distance of trajectories $\Delta \phi$ in terms of the phase: for $\Delta \phi < \pi$ a collision of a pair of dowsons leads to their annihilation, while for $\Delta \phi > \pi$ the dowsons are passing by. This rule is valid only for quasi-static stationary wound up textures and can be easily broken by application of a Poiseuille flow in an appropriate direction.

Keywords: nematic, topological defects, dowser texture, complex order parameter, collider, annihilation

1. DOWSONS: DEFECTS OF THE DOWSER TEXTURE

1.1. The Dowser Texture

As stressed by de Gennes in his pioneering paper on classification of topological defects [1], superfluids (or superconductors) (Figure 1A) and smectics A (Figure 1B) are characterized by complex order parameters $\Psi$ and $e^{i\phi}$. Later, smectics C (Figure 1C) have been added to this list. Beside phases in the thermodynamic sense, the complex order parameter characterizes also textures of a homeotropic nematic layer above the Freedericks transition (Figure 1D) as well as the so called dowser texture in a nematic layer with homeotropic boundary conditions (Figure 1E).

The dowser texture, known as the quasi-planar texture for decades [3], was believed unduly to be unstable, with respect to the homogeneous homeotropic texture, so that it has been scarcely studied in past. Recent work [4] proved that in practice the quasi-planar texture is only metastable and can be preserved indefinitely in certain conditions. Experiments with this persistent version of the quasi-planar texture have unveiled its remarkable qualities such as its sensitivity to magnetic [5], mechanical [6], or electric [7] perturbations. For this reason, as well because of the resemblance with the wooden dowser tool, the persistent version of the quasi-planar texture was dubbed “the dowser texture.”

1.2. Dowsons $d_+$ and $d_-: the +2\pi$ and $-2\pi$ Singularities of the Phase Field $\phi(x, y, t)$ of the Dowser Texture

The dowser texture is fully characterized by the azimuthal angle $\phi$ of the unitary two-dimensional dowser field $d = (\cos \phi, \sin \phi)$ (Figure 1E) which is equivalent to the phase $\phi$ of the complex order parameter $e^{i\phi}$.
Thanks to the birefringence of nematics, the phase \( \phi(x,y,t) \) of the dowser field \( \mathbf{d}(x,y,t) \) is directly observable in polarized light so that its \(+2\pi\) and \(-2\pi\) topological singularities are easily identifiable [5]. Let us note that when considered in three dimensions of the nematic layer, these singularities of the 2D dowser field \( \mathbf{d}(x,y) \) appear as nematic monopoles [8], that is to say, point singularities of the 3D director field \( \mathbf{n}(x,y,z,t) \).

In the present work, devoted to motions and collisions of topological singularities of the dowser field we will call them particularly tailored setups called “dowsons’ colliders” (see section 2.1 and Figure 4).

Let us stress that in contradistinction with the dowser field, the isophasic trajectories can be alternatively called isoform. This structure (radial, circular, or spiral) remains the same. Therefore, when the dowson \( d_+ \) is imbedded in a wound up dowser texture, one can still define isophasic lines by equation

\[
\psi(x_i, y_i, t) = \text{const}
\]  

When the dowson \( d_+ \) is moving on such isophasic trajectories, its structure (radial, circular, or spiral) remains the same. Therefore, the isophasic trajectories can be alternatively called isoform. This second denomination is more convenient in practice: when the orientation of the cross-shaped isogyres of a dowson remains the same, its trajectory is isophasic.

\[ 1.3. \text{Trajectories and Collisions of Dowsons} \]

Previous experiments with dowsons [4, 5] have shown that pairs of dowsons “\( d_+ \)” and “\( d_- \)” can be easily generated, set into motion and brought into collisions. In certain conditions collision of pairs of dowsons \( (d_+, d_-) \) can result in their annihilation. Here, we will explore these processes by means of especially tailored setups called “dowsons’ colliders” (see section 2.1 and Figure 4).

The principal role of dowsons’ colliders consists in driving motions of dowsons which is achieved by a controlled winding of the phase of the dowser field. Indeed, like vortices in superconductors which are set in motion by phase gradients (the Lorentz force is exerted on a flux quantum by a transport current proportional to the phase gradient), the motion of dowsons is also driven by phase gradients.

\[ 1.4. \text{Single Dowsons Inserted in a Wound Up Dowser Field} \]

This is explained on the first example shown in Figure 2 where one dowson \( d_+ \) is imbedded in a wound up dowser texture. Before considering forces involved in the motion of this dowson \( d_+ \), let us emphasize that its structure depends on the phase \( \phi_i = \phi(x_i, y_i) \) at the insertion point \((x_i, y_i)\). Figures 2A–C show that for \( \phi_i = 0 \) the structure of the dowson \( d_+ \) is radial with the field \( \mathbf{d} \) directed outward. For \( \phi_i = \pi/2 \) the structure becomes circular anticlockwise (see Figures 2D–F) and for \( \phi_i = \pi \) it is radial directed inward (see Figures 2G–I).

In Figure 2 the dowser field is wound up in the \( y \) direction \( \nabla \psi / \phi \) so that the phase \( \phi_i = \psi(x_i, y_i) \) does not depend on the coordinate \( x_i \) of the insertion point. Therefore, lines defined by \( y_i = \text{const} \) are isophasic and can be considered as isophasic trajectories of the dowson. In the general case of an arbitrarily wound up dowser field, one can still define isophasic lines by equation

\[
\psi(x_i, y_i, t) = \text{const}
\]
FIGURE 2 | (A–I) Dowson +2\pi imbedded in a wound up dowser texture. (A,D,G) Radial, orthoradial, and antiradial configurations of the dowson d+ alone. The orthoradial configuration has the lowest elastic energy [5]. (B,E,H) Phase field of the dowser texture wound up in y direction: \( \phi = 2\pi y/\lambda \). (C,F,I) The wound up phase field with the dowson d+ imbedded respectively at y = 0 (C), y = \lambda/4 (F), and y = \lambda/2 (I). (J–O) Dowson -2\pi imbedded in a wound up dowser texture. (J,M) Configurations of the dowson d− depend on the phase \( \phi_i = \phi(x_i, y_i) \) at the insertion point \( (x_i, y_i) \). They result from rotation by the angle \( \phi_i \). (K,N) Phase field of the dowser texture wound up in y direction: \( \phi = 2\pi y/\lambda \). (L,O) The wound up phase field with the dowson d− imbedded respectively at y = 0 (L) and y = \lambda/4 (O).

Similar consideration on the insertion of one dowson d− into a wound up dowser field (illustrated by Figures 2J–O) leads to the conclusion that the "hyperbolic" structure of the dowson d− rotates as a whole when \( \phi_i \) varies. Such a transformation of the dowson d− does not change its elastic energy so that trajectories of the dowson d− are not submitted to elastic constraints.

On the contrary, as stated above, the structure of the dowson d+ varies with \( \phi_i \). Therefore, due to the elastic anisotropy, the elastic energy of the dowson d+ depends on \( \phi_i \) so that its trajectories are submitted to an elastic constraint. As we will point out below, dowsons d+ tend to follow isophasic trajectories.

In Figure 2C, the dowson d+ is located at the left extremity of a 2\pi wall. The elastic energy stored in this wall is relaxed when
the dowson \( d_+ \) moves to the right because the wall is shortened by this means.

Qualitatively, a wall of width \( \lambda \) exerts on the dowson \( d_+ \) the force which is of the order of the elastic energy per unit length stored in it:

\[
\tau_{el} = \frac{1}{2} K_{eff} h \int_0^\lambda \left( \frac{\partial \varphi}{\partial \xi} \right)^2 d\xi \approx K_{eff} \frac{2\pi^2 h}{\lambda}
\]  

(2)

During the motion of the dowson with velocity \( v \), the driving force \( \tau_{el} \) is opposed by another one \( \tau_{visc} \) resulting from the viscous dissipation:

\[
\tau_{visc} \approx \pi \gamma \eta h
\]  

(3)

Therefore, the velocity of the dowson is given by:

\[
v \approx \frac{2\pi}{\gamma_1} \frac{K_{eff}}{\lambda}
\]  

(4)

In conclusion, the velocity of the dowson should be independent of the local thickness but should decrease as \( 1/\lambda \) with the local width \( \lambda \) of the wall.

When instead of the dowson \( d_+ \), the dowson \( d_- \) is imbedded in the same wound un dowser field (see Figure 2J), it is positioned at a right extremity of the 2\( \pi \) wall and therefore will move to the left.

1.5. Pair of Dowsons (\( d_+,d_- \)) Inserted in a Wound Up Dowser Field

Figure 3 represents the case of a pair of dowsons \( d_+ \) and \( d_- \) inserted in the same wound up dowser field. Analytically, the phase field of the wound up dowser texture with the pair of \( d_+ \) and \( d_- \) dowsons inserted respectively at points \((x_\pm,y_\pm)\) can be expressed as

\[
\varphi(x,y) = \frac{2\pi}{\lambda} y + \arctan\left( \frac{y - y_+}{x - x_+} \right) + \arctan\left( \frac{y - y_-}{x - x_-} \right)
\]  

(5)

When the two dowsons are far enough, i.e., when \(|x_+ - x_-| > \lambda\), they move on trajectories defined by \( y(t) = y_+ \) and \( y(t) = y_- \). We can thus define the distance of trajectories as

\[
\delta = y_+ - y_-
\]  

(6)

in terms of the length or as

\[
\Delta \varphi = 2\pi \frac{y_+ - y_-}{\lambda}
\]  

(7)

in terms of the phase. The set of seven pictures in Figures 3A–G illustrates graphically this concept of the distance of trajectories leading to collisions.

1.6. Aims of Experiments With Dowsons’ Colliders

One of aims of our experiments performed with dowsons’ colliders is to find conditions which determine the outcome of collisions [9, 10]. When \( \Delta \varphi = 0 \) (see Figure 3A), the two dowsons are located at extremities of the same 2\( \pi \) wall. It seems therefore that annihilation of the \((d_+,d_-)\) pair must occur. Inversely, when \( \Delta \varphi > \pi \) (see Figure 3G), the two dowsons are located at extremities of two distinct 2\( \pi \) walls so that the annihilation of such a pair will be avoided. We will thus generate experimentally numerous pair collisions with the aim to find the annihilation cross section of dowsons.

Before that, we will focus on the primary aim of our experiments which consists in observing motions of dowsons and measuring their velocities. Knowing that the elastic force driving their motion is inversely proportional to the wave length \( \lambda \) of the wound up texture, we have to wind up the dowser texture more or less expecting that the velocity of dowsons should increase when the phase gradient grows.

2. DOWSONS’ COLLIDERS

2.1. Experimental Setups

2.1.1. The Double Dowsons’ Collider

The first setup shown in Figure 4A, called here “Double Dowson Collider” or DDC, was developed during the study of the rheotropism of the dowser texture [6]. It consists mainly of a convex lens (50 mm in diameter) and of a glass slide (25 × 75 × 1 mm) supported at one end by a translation stage as shown in Figure 4A. The radius of curvature of the convex lens is 140 mm. A droplet of a nematic (5CB) is held by capillarity in the gap between the lens and the slide. Typically the diameter of the squeezed droplet is 10 mm and its thickness in the center (regulated by means of the translation stage) is of the order of a few \( \mu \)m. The glass slide is set into vibrations by the force exerted on small magnets by the magnetic field of the coil. Due to the mirror symmetry [with respect to the \((x,z)\) plane] of this device, only the flexural modes of vibration \( \zeta = \zeta(x,t) \) are excited in it.

As explained in Pieranski et al. [6], vibration of the slide (in its flexural modes) results in two harmonic motions at the drop center: 1—modulation of the gap thickness and 2—rotation around the \( y \) axis. By this means, two Poiseuille flows, radial and dipolar, shifted in phase by \( \pi/2 \), are driven simultaneously. The resulting effective flows are elliptical: clockwise and anticlockwise in the two halves of the droplet symmetrical with respect to the mirror plane \((x,z)\).

The rheotropic (weathercock-like) behavior of the dowser field results in rotation of the dowser field \( \text{d}\) with the angular velocity \( \omega(x,y,t) = d\varphi/dt \) depending on the \((x,y)\) position in the droplet. In the DDC, the torque \( \Gamma(x,y,t) \) exerted by the elliptical Poiseuille flow on the dowser field can be approximated, heuristically, by the function \( f_{DDC}(r) \cos(\theta) \), with \( f_{DDC}(r) = re^{-r^2} \), \( r = \sqrt{x^2 + y^2} \) and \( \theta = \arctan(y/x) \), plotted in Figure 4C.

A typical pattern of a wound up dowser texture observed in experiments between crossed polarisers is shown in Figure 4E. It is symmetrical with respect to the \((x,z)\) plane.

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2.1.2. The Circular Dowsons’ Collider

For the purpose of the present study of collisions of dowsons, we developed a second setup (see Figure 4B) tailored for production of a circularly wound up pattern. As we will see below, trajectories of dowsons in this Circular Dowsons’ Collider are circular and respectively clockwise and anticlockwise for the $d_+$ and $d_-$ defects.

In this second setup, the mirror symmetry is broken by its structure. The magnet is now located at one extremity of an additional glass slide ($10 \times 75 \times 1\text{mm}$) which is attached at its second extremity to the principal glass slide. The force $f_{\text{exc}}$ exerted by the coil on the magnet produces now also a torque $f_{\text{exc}}\Delta y$ which drives the torsional mode of the principal glass slide. As the flexural and torsional modes have different eigenfrequencies, the $\pi/2$ phase shift between them can be obtained by an appropriate choice of the excitation frequency, which typically is of the order of 360–440 Hz. In such a case, the motion of the principal glass slide at the center of the drop is conical: the normal to it precesses on a cone centered on the z axis. The Poiseuille flow of the droplet is now circular (orthonormal) with the amplitude $\nu$ varying between 0 to $3\pi$ from (A–G). The color code for the phase is given in (H).

![Figure 4D](attachment:Figure_4D.png)

**FIGURE 4D** | Dowson’s Collider. (A) The isogyres’ pattern of the dowser texture is shown on a white background. The color code for the phase is given in (B). (C) Dowson(s) imbedded in a wound up texture. The distance of their trajectories in terms of the phase difference $\Delta\psi$ is plotted in (D). (E) Dowson(s) imbedded in a wound up texture. The distance of their trajectories in terms of the phase difference $\Delta\psi$ is plotted in (F). (G) Dowson(s) imbedded in a wound up texture. The distance of their trajectories in terms of the phase difference $\Delta\psi$ is plotted in (H).

2.2. Experiments With the Double Dowsons’ Collider

2.2.1. Velocity of Single Dowsons on Straight Trajectories

As shown in Figure 5E the isogyres’ pattern of the dowser field wound up in the double dowsons’ collider is (almost) symmetrical with respect to the mirror plane $(x,z)$, Therefore, when a single dowson is imbedded in the wound up dowser field in the vicinity of this plane, the $2\pi$ wall to which it is attached is parallel to the x axis as discussed in the Introduction (see Figure 2). This is the case in the series of five pictures in Figures 5A–E showing the motion of a single dowson $d_+$ “pulled” by a $2\pi$ wall along the x axis.

These pictures are extracted from a video containing 55 pictures recorded at intervals of 20 s. Using all of them, we measured the velocity $v$ of the dowson and the width $\lambda$ of the $2\pi$ wall to which it is attached. The result, $v(\lambda)$, is plotted in Figure 5F. Arrows labeled from a to e indicate measurement points corresponding to the five picture above.

From the Equation (4) in the section 1.4 we expect that the velocity of the dowson should grow as $v \sim \lambda^{-1}$ with the local wave length $\lambda$ of the wound up phase field. The dashed line in the diagram of Figure 5F represents the best fit to this law. Clearly, the slope of the measured variation $v(\lambda)$ is slightly steeper. We have therefore tempted to fit experimental results with a more general power law $v \sim \lambda^{\alpha}$. The continuous line represents the fit with $\alpha = -1.24$ which clearly is better than the one with $\alpha = -1$.

2.2.2. Dowsons’ Sprint

In the search for reasons of this discrepancy, we performed another experiment which could be called “the dowsons sprint.” It starts with a simultaneous generation of a row of $(d_+,d_-)$ pairs in a wound up dowser texture by means of a shear flow applied in the y direction (see Figures 6A,B). (We postpone the detailed discussion of this issue to another paper.) At $t = 0$ s, the dowsons $d_+$ “in statu nascendi” are aligned on a slightly curved line AB while the dowsons $d_-$ are aligned on another line CD parallel to AB. As expected, all dowsons $d_+$ start to move to the left while the dowsons $d_-$ move to the right.

For the purpose of the further discussion we will label seven neighboring dowsons $d_+$ on the start line AB with an integer index $i = 1,2,3,\ldots$ (see Figure 6C).

At the very beginning of this race, the motion of dowsons is driven exclusively by shortening of the $2\pi$ walls connected to them, as discussed in the section 1.4. Therefore, they have therefore the same velocity $v_i = \text{const}$ and conserve their alignment on the curved line which is moving to the left as a whole.
However, soon after the departure, an instability occurs: the set of all dowsons is split into two subsets defined by the parity of the index \( i \) and, for example, dowsons with \( i \) odd (see Figure 6D) begin to move more slowly than those with \( i \) even. This retardation of odd dowsons (with \( i = 2n+1 \)) is easy to understand: the width \( \lambda_{2n+1} \) of the \( 2\pi \) walls to which they are attached is twice larger than that of the even dowsons \( \lambda_{2n} \).

If the force \( f_i \) pulling dowsons was determined only by the width \( \lambda_i \) of the \( 2\pi \) walls to which they are attached, the ratio of velocities \( v_{2n}/v_{2n+1} \) should be 2. However, measurements of the dowsons’ velocity have shown that \( v_{2n}/v_{2n+1} \approx 3 \).

Explanation of this apparent discrepancy involves a more detailed evaluation of the elastic energy released during the motion of dowsons. If the “lanes” left behind faster dowsons stayed free of distortion, the force acting on them would remain constant during the race. However, as shown in Figure 6D, these lanes are filled by enlargement of the lanes of slower dowsons. The corresponding amount of the released elastic energy per unit length is equal to the force \( f_{2n+1} \) pulling slower dowsons. In conclusion, the elastic force \( f_{2n} \) acting on faster dowsons is not two but three times larger than \( f_{2n+1} \). A more detailed
FIGURE 5 | Motion of the dowson $d_+$ in the phase gradient generated in the dowson collider DDC. The series of five pictures shows successive positions of the dowson $d_+$ at: (A) $t = 0$ s, (B) $t = 380$ s, (C) $t = 520$ s, (D) $t = 620$ s, and (E) $t = 700$ s. For a better visibility, small areas in vicinity of the dowson have been enlarged in pictures (B–E). (F) Plot of the velocity of the dowson $d_+$ vs. the local wave length of the wound up dowser texture. The continuous red line represents the fit to the power law $v = A\lambda^\alpha$ with $\alpha = -1.24$. The dashed blue line corresponds to $\alpha = -1$. 
FIGURE 6 | Dowsons race. (A) Central portion of the dowser texture wound up in the DDC. It can be seen as a stack of $2\pi$ walls. (B) Collective generation of $(d_+, d_-)$ pairs by application of a transient shear flow in $y$ direction which “breaks” simultaneously all $2\pi$ walls. (C) At the beginning of the race, dowsons $d_+$ (or $d_-$) are moving with the same velocity. (D) Odd-even instability: odd dowsons ($i = 2n+1$) stay behind even dowsons ($i = 2n$) because they become about three times slower.

(Collaboration with Elise Hadjefstatiou and Lisa-Marie Montagnat).

discussion of the dowsons’ race is postponed to another article.

2.2.3. Are Trajectories of Dowsons Isophasic?

In experiments with dowsons’ colliders, the $2\pi$ walls can be defined as bundles of four adjacent isogyres; when one crosses one of such bundles, the phase varies by $2\pi$.

In the vicinity of the mirror symmetry plane $(x,z)$ of the Double Dowson Collider, the $2\pi$ walls are parallel to the $x$ axis so that the dowsons $d_+$ and $d_-$ are moving on straight isophasic trajectories. However, as we know already from section 2.1 (see Figure 4C), the whole dowser texture, wound up in the Double Dowsons’ Collider, can be seen as made of $2\pi$ walls forming closed loops in the absence of defects. Let us suppose that a pair $(d_+, d_-)$ of dowsons has been generated by breaking one of these $2\pi$ walls. Pulled in opposite directions by the broken $2\pi$ wall these dowsons will move apart. Will their trajectories remain isophasic? If it was the case, they would remain connected to the same $2\pi$ wall which would became shorter and shorter so that, finally, the two dowsons would meet and annihilate. Such a behavior was indeed observed in first experiments with the dowser texture wound up by a rotating magnetic field [5].

As we will see below, experiments with dowsons’ colliders have shown that trajectories of dowsons are not necessarily isophasic so that they do not remain connected to the same $2\pi$ wall. Therefore, when after a half turn of the wound up dowser texture the two dowsons of the pair meet again, the distance of their trajectories $\Delta \varphi$ in terms of the phase is not necessarily zero so that their annihilation is not granted.

2.2.4. The First Evidence for Non-isophasic Trajectories of Dowsons

The issue of non isophasic trajectories of dowsons was raised for the first time in experiments with the DDC. Let us consider a typical experiment illustrated in the Figures 7A,B by a view of one of the two target patterns of the wound up dowser texture. We identify here four dowsons $d_+$ and three dowsons $d_-$. On this background we represented by rows of circular markers successive positions, recorded at intervals of 30 s, of dowsons $d_+$ (Figure 7A) and $d_-$ (Figure 7B).

Several conclusions can be drawn from this figure:

1. Dowsons $d_+$ and $d_-$, pulled by $2\pi$ walls, circulate in opposite directions, as expected.
2. The velocity of dowsons is correlated to the local width $\lambda$ of $2\pi$ walls, as expected.
3. The trajectory of the dowson $d_+$ is parallel to isogyres while the one of the dowson $d_-$ is crossing isogyres. In other words, the trajectory of the dowson $d_+$ seems to be isophasic while that of the dowson $d_-$ is not isophasic.
4. The non isophasic behavior of dowsons $d_-$ is even more obvious when one considers the one labeled with a dashed circle in Figure 7. It is located in the center of the target pattern and this central position is dynamically stable during the phase winding. Now, as during the phase winding, the
angular velocity $\omega = d\phi/dt$ is the largest here, this central position is obviously not isophasic. Consequently, the maltese cross (formed by four isogyres) of this dowson is rotating as a whole with the angular velocity $\omega$.

Knowing that the circular markers in Figure 7 indicate successive positions of dowsons at time intervals of 30 s, the velocity $v$ of dowsons has been determined. Simultaneously the local width $\lambda$ of the $2\pi$ walls pulling on dowsons has been measured in this...
Figure 7A. Results obtained with the dowson $d_+$ are plotted with blue crosses in Figure 7D. The best fit to the power law $v \sim \lambda^\alpha$ plotted with the blue plain line was obtained with $\alpha = -1.14$. On the same diagram of Figure 7D we have plotted once again (with red crosses and a red line) results shown previously in Figure 5.

### 2.3. Experiments With the Circular Dowsons’ Collider

The most recent experiments performed with the Circular Dowsons’ Collider confirmed these conclusions but also unveiled other remarkable properties of the dowsons dynamics. In particular, we have found that the result of the phase winding process in the Circular Dowsons’ Collider depends on the initial state of the dowser field as well as on the amplitude of the excitation. In general, for topological reasons (homeotropic boundary conditions at the nematic/air interface of the meniscus), the dowser field can contain only an odd number $2n+1$ of dowsons $d_+$ and an even number $2n$ of dowsons $d_-$. We will show below that two different dynamically stable states C-B1 or C-B2 can be reached when, respectively, $n = 0$ and $n > 0$.

#### 2.3.1. Cladis-Brand Stationary State C-B1: One Dowson $d_+$ Orbiting Around the Target Pattern

In the simplest case of $n = 0$, one dowson $d_+$ is located initially at the center $O$ of the drop and the dowser field has the radial configuration imposed by the cuneitropisme [4] of the dowser texture. This radial configuration also satisfies the homeotropic boundary conditions at the nematic/air interface on the edge of the droplet (see Figure 8A).

![FIGURE 8](https://www.frontiersin.org/article/10.3389/fphy.2020.00023/figs/8.png)
When the rheotropic driving torque due to the circular Poiseuille flow is applied to the dowser field, it starts to rotate with the angular velocity $\omega(r, t)$, varying with the distance $r$ from the center as shown in Figure 4D. Rotation of the dowser field is thus clockwise in the center at $r=0$, then the angular velocity $\omega(r, t)$ decreases and changes its sign at $r = r_c$ (dashed circle in Figure 8B). As a result, the maltese cross formed by four isogyres shown in Figure 8A is deformed: its four arms become spiral as shown in Figure 8B. Later, the dowson $d_+$ leaves the center O and a target pattern of loop-like isogyres starts its growth from the center O and their radii are growing (see Figure 8).

If $\omega(0, 0, t)$ is the phase growth rate at the center O, then the rate of nucleation of $2\pi$ walls (each made of four isogyres) is $\omega(0, 0, t)/(2\pi)$.

During this winding process, the dowson $d_+$ is pushed (elastically) by isogyres toward the periphery of the target pattern as shown in Figure 8. By this means, its position inside the evolving phase field $\varphi(x, y, t)$ remains isophasic. This behavior results from the elastic anisotropy of the nematic. Indeed, as shown in Figure 2 the configuration of the dowson $d_+$ depends on the phase $\varphi_i$ at the point of its insertion into the wound up phase field. From Pieranski et al. [5] we know that the elastic energy of the dowson $d_+$ depends on its configuration. As energetically the orthoradial configurations (clockwise or anticlockwise) are the best ones, the dowson tends to conserve its position at $\varphi = \pi/2 \text{ (mod } \pi)$.

Simultaneously, pulled by the $2\pi$ wall to which it is attached, the dowson $d_+$ begins its orbiting motion with velocity $v$ (see Figure 8E) around the target pattern made of concentric $2\pi$ walls. The orbiting dowson $d_+$ can be seen as a “phase sink”: after each whole turn around the target pattern, one $2\pi$ wall is “swollen.” If $T$ is the period of the orbit, then we can define the phase sinking rate as $2\pi/T$.

**FIGURE 9** | Cladis-Brand [11] dynamically stable state C-B1 of the phase winding in the Circular Dowsons’ Collider. (A) Spatio-temporal cross section taken along the dashed line defined in Figure 8F. Four new isogyres are nucleated at the center O during one period $T$ of the orbiting motion of the dowson $d_+$. (B) Blow up of the rectangular domain defined with a dashed line in (A). $\lambda$ is the width of a $2\pi$ wall composed of four oblique trajectories of isogyres. During one period $T = 30$ min, the $2\pi$ wall is shifted by $\lambda$ to the right. (C) Successive positions of the dowson $d_+$ recorded as colored dots at intervals of 10 s during three periods of its orbital motion. The three colors of dots correspond to the three periods $T$ of the orbital motion. (D) Blow up of the rectangular domain defined with dashed line in (C).
During the winding process, the phase growth rate $\omega(r,t)$ decreases because the rheotropic torque is opposed by the growing elastic torque. Simultaneously, the sinking rate increases because the orbiting dowson is moving faster pulled by the narrowing $2\pi$ wall.

The dynamically stable (stationary) phase field $\varphi(x,y,t)$ is achieved when the phase growth rate at the center equals the sinking rate due to the orbiting dowson $d_+: \omega(0,0,t) = 2\pi/T$ (see Figure 9). As Cladis and Brand have formerly discovered in free standing smectic C films the same configuration of a $+2\pi$ defect orbiting around a target pattern [11] we propose to call it “The Cladis-Brand state 1” or shortly C-B1.

2.3.2. C-B2: Second Version of the Cladis-Brand Stationary State

At first sight, upon the action of the rheotropic torque $\Gamma_r(r)$, the dowser field should rotate in the anticlockwise direction for $r > r_c$, $r_c$ being defined in Figure 4D. It seems therefore that new isogyres could nucleate also in the annular area near the second extremum of the torque $\Gamma_r(r)$. In the experiment discussed above and illustrated by the series of six pictures in Figure 8, this is not the case: new isogyres nucleate only at the first extremum of $\Gamma_r(r)$ located in the center $O$ at $r = 0$.

Explanation of this experimental fact is very simple. Beside the rheotropic torque driving the rotation of the dowser field,
there is also the cuneitropic torque $\Gamma_c = (\pi K/h)g \times d$ (see \cite{4]) tending to orient the dowser field $d$ in the direction of the thickness gradient $g$, that is to say in the radial direction $r$ of the sphere/plane geometry of the sample (this is the case in Figure 8A). This cuneitropic torque vanishes at $r=0$ for symmetry reasons but is finite at $r > r_c$. For a given $r$, it reaches its maximum value $\Gamma_{c\text{max}} = (\pi K/h)(g(r))$ when $d$ is orthogonal to $g$. In the experiment of Figure 8, for $r > r_c$ the rheotropic torque is smaller than $\Gamma_{c\text{max}}$ so that rotation of the dowser field is hindered there. In another experiment illustrated by the series of six pictures in Figure 10, the rheotropic torque was much larger so that nucleation of new isogyres occurred also in the secondary extremum of the rheotropic torque.

### 2.3.3. Triplet Stationary State: Two Dowsons $d_+$ Orbiting Around One $d_-$ in the Center

Experiments with the Circular Dowsons’ Collider unveiled a third stationary state (see Figure 11). To reach it, the winding process has to be applied to the dowser field with $n>0$, that is to say containing at least two dowsons $d_-$ and one dowson $d_+$ when $n = 1$.

For reasons which so far have been not fully understood, during the winding, the dowson $d_-$ is attracted to the center $O$ [maximum of $\omega(r,t)$] and stays there while the two dowsons $d_+$, on the contrary, are pushed to the periphery of the growing pattern. Let us emphasize that in this new configuration the winding process does not require nucleation of new isogyres. The phase growth in the center is now due to rotation of

![Figure 11](https://example.com/figure11.png)
the dowson \( d_- \) located there. This mechanism is similar to the Frank-Read model of crystal growth in which a spiral step, attached to a dislocation emerging on a crystal facet, is rotating.

When \( n \) is larger than 1, the \((d_+,d_-)\) pairs in excess with respect to \( n = 1 \) are eliminated by annihilation during the winding process as shown in Figures 11B–D.

Like in the Cladis-Brand process, the stationary triplet state is reached when the phase growth rate in the center, due to the rotation of the dowson \( d_- \), is fully compensated by the orbital motion of the two dowsons \( d_+ \) acting as phase sinks. In this stationary state, the two dowsons \( d_+ \) are located on the same orbit (see Figure 12D) and have therefore the same angular velocity \( \omega_{d_+} \). The total phase absorption rate is therefore \( 2\omega_{d_+} \). Therefore, if the \( d_- \) dowson in the center rotates with the angular velocity \( \omega_{d_-} \) then in the stationary state one has:

\[
\frac{d\phi}{dt} = \omega_{d_-} + 2\omega_{d_+} = 0
\]

so that

\[
\omega_{d_-} = -2\omega_{d_+}
\]

This equality is illustrated by in Figures 12A,B.

3. GENERATION, COLLISIONS, AND ANNIHILATION OF DOWSONS’ PAIRS

The dynamically stable states of the Circular Dowsons’ Collider are convenient for studies of generation of dowsons and of their subsequent collisions which can lead to annihilation. Indeed,
like positrons and electrons in a hadron collider, dowsons \(d_+\) and \(d_-\) are moving in the Circular Dowsons’ Collider on respectively clockwise and anticlockwise trajectories so that they can undergo collisions that can result in annihilation of dowsons pairs.

By a collision we mean an event during which the linear distance \(l_{+−}\) between two dowsons, \(d_+\) and \(d_-\), decreases and becomes of the order of the winding wave length \(λ\).

### 3.1. Generation of One \((d_+,d_-)\) Pair

For the purpose of clarity of the forthcoming discussion, let us consider the example represented in Figure 13 which shows in the Figure 13A, a view of the wound up dowser field shortly after generation of just one dowsons pair. The process of generation itself is illustrated by the series of five pictures (Figures 13C–G). It is triggered by a rapid and short forth-and-back motion of the oscillating glass slide applied to the wound up texture visible in Figure 13C. During the motion, the image of the isogyres’ pattern becomes fuzzy (Figure 13D) but shortly after that, at the beginning of the relaxation (Figure 13E), one can distinguish seven \(2π\) walls thinned by the perturbation.

As discussed in Pieranski et al. [6] thinning of \(2π\) walls is due to the rheotropism of the dowser texture, that-is-to-say, to its sensitivity to Poiseuille flows. Anticipating a more detailed discussion in section 3.5 we infer that at the beginning of the relaxation a transitory Poiseuille flow \(2π\) walls occurred.

An excessive thinning of one of the \(2π\) walls leads to its breaking shown in Figure 13E. Subsequently the two dowsons generated by this means are moving in opposite directions on initially isophasic trajectories.

### 3.2. Collision of a \((d_+,d_-)\) Pair

As the isogyres pattern in the wound up Cladis-Brand state is made of concentric rings, one could think that after a half turn of their orbits (see Figure 13B), the freshly generated dowsons should come to a collision on isophasic trajectories. The series of five pictures (Figures 13H–I), shows clearly that this is not the case: there is a \(Δθ \approx 2π\) distance (see Figure 13H), in terms of the phase, between trajectories of the two dowsons coming to their collision. We postpone discussion of this paradox to another paper.

In meantime, let us just say that the two dowsons coming to collision are pulled by two distinct \(2π\) walls so that annihilation is avoided.

### 3.3. Rules for Collisions of \((d_+,d_-)\) Pairs

When more than one pair of dowsons is generated simultaneously, the subsequent collisions occur at variable distances \(Δθ\) of trajectories. From observations of many of such collisions with \(-2π < Δθ < +2π\) we inferred the following rules:

1. **Bypass**: When \(|Δθ| > π\), the annihilation is avoided and the dowsons are passing by (see Figures 14A–I).
2. **Annihilation**: When $|\Delta \phi| < \pi$, the annihilation occurs (see Figures 14J–R).

3. **Critical**: When $|\Delta \phi| = \pi$, the outcome of the collision is random.

### 3.4. Influence of Poiseuille Flows on Collisions of Dowsons Pairs, Experiment

The rules formulated above apply to pairs of dowsons moving inside a very slowly evolving stationary wound up dowser field.

Knowing from former experiments that the dowser texture is very sensitive to Poiseuille flows [6] we used this property, called rheotropism, to influence the outcome of dowsons collisions. As an example we point out in the series of 20 pictures in Figures 15A–T that the annihilation, which should occur in terms of the collisions’ rules applied to the pair of dowsons in Figure 15A, can be avoided by application of a Poiseuille flow in an appropriate direction.

Indeed, at the beginning of the experiment (see Figures 15A–D) dowsons $d_+$ and $d_-$ coming to collision are almost isophasic and are connected by a $2\pi$ wall which is shortening. The outcome of the forthcoming collision seems unavoidable: an annihilation. However, an application of the Poiseuille flow $\nabla$ in the left direction [parallel to the dowser field in the middle of the wall ($d_+,d_-$)], has a very striking effect well visible in pictures Figures 15E–L: the wall connecting the dowsons pair as well as the whole system of isogyres is split in such a manner that the two dowsons are reconnected to two new, different $2\pi$ walls. These walls, narrowed by the Poiseuille flow, pull strongly on dowsons which move rapidly on distinct trajectories separated now by $2\pi$, in terms of the phase. After cessation of the flow (Figures 15M–T) the system of isogyres relaxes: the trajectories of the two dowsons become almost isophasic again but they diverge now.

### 3.5. Influence of Poiseuille Flows on Collisions of Dowsons Pairs, a Model

Theoretically, this experiment can be modeled as follows. At the beginning of the experiment, the phase field can be expressed...
as before (see Equation 5):

\[
\varphi_o(x,y) = \frac{2\pi}{\lambda} y + \arctan \left( \frac{y - y_+}{x - x_+} \right) + \arctan \left( \frac{y - y_-}{x - x_-} \right)
\]

with \((x_+,y_+) = (-5, -\pi/4)\) and \((x_-,y_-) = (5, \pi/4)\) so that

\[
\Delta \varphi = 2\pi \frac{y_+ - y_-}{\lambda} = \frac{\pi}{2}
\]

(11)

This initial field is depicted in Figure 15U using the color code defined in Figure 2A. Application of the Poiseuille flow of the amplitude \(v_{\text{max}}\) in the \(-x\) direction perturbs the field \(\varphi_o(x,y)\). The rheotropic torque exerted by this Poiseuille flow on the field \(\varphi(x,y)\) can be written as:

\[
\vec{\tau}_{rt} = \frac{2\alpha_2}{\pi} v_{\text{max}} \sin \varphi_o(x,y) \vec{z}
\]

(12)

In the first approximation, the resulting elastic distortion is proportional to this torque so that the perturbed phase field can be written as:

\[
\varphi_{\text{pert}}(x,y) \approx \varphi_o(x,y) + \delta \varphi \sin(\varphi_o(x,y))
\]

(13)

with \(\delta \varphi \sim v_o\). The graphic representation of \(\varphi_{\text{pert}}(x,y)\) in Figure 15V shows an agreement with the experimental picture in Figure 15I.
Let us stress that when the Poiseuille flow is applied in the inverse, +x direction, our model predicts that the 2π wall connecting the dowsons’ pair is no split but narrowed as shown in Figure 15W so that the annihilation is accelerated.

These simulations are in agreement with our experiments: the outcome of the forthcoming collisions can be chosen at will by application of Poiseuille flows in appropriate directions.

4. CONCLUSIONS

The present paper is by no means exhaustive in the matter of generation, motions, and collisions of dowsons. Nevertheless, it raises new issues concerning (1) laws of motion of dowsons driven by phase gradients and (2) laws ruling the outcome of dowsons’ collisions.

In particular, there is a huge difference in the behavior of dowsons d+ and d− during phase winding. The dowsons d+ cannot rotate because of the elastic anisotropy so that they tend to escape from the winding up phase field and are going to areas where the phase growth rate is zero. In the case of a unique dowson d+, this leads to the Cladis-Brand stationary states in which the orbiting dowson d+ absorbs the phase generated by the dowsons’ collider.

The behavior of the dowson d− seems to be a contrary one and much more enigmatic. Indeed, experiments showed that during the winding process the dowsons d− is attracted to the area in which the phase growth rate is maximal. By this means another stationary state, with the dowson d− in the center (acting as a phase source) and two dowsons d+ orbiting around it (acting as phase sinks), can be reached. This gyrophilic behavior of the dowson d− remains to be explained.

The law ruling translational motion of dowsons on their orbits needs also further clarification. Theoretically, in the first approximation, the velocity v of dowsons should be proportional to the local phase gradient \( \nabla \phi = 2\pi/\lambda \cdot \nu \sim \lambda^\alpha \) with \( \alpha = -1 \). Experiments have shown however that in practice the exponent \( \alpha \) is smaller than \(-1\). This discrepancy is probably due to interactions between moving dowsons which certainly play the major role during the dowsons sprint discussed in section 2.2.2.

From observations of dowsons pairs (d+,d−) moving on counter-rotating orbits, a rule for the outcome of their collisions, i.e., either annihilation or bypass, was inferred. The distance of trajectories \( \Delta \phi \) in terms of the phase appeared as the pertinent parameter: for \( \Delta \phi < \pi \) a collision of a pair of dowsons leads to their annihilation, while for \( \Delta \phi > \pi \) the dowsons are passing by. However, this rule is valid only for quasi-static stationary wound up textures and can be easily broken by application of a Poiseuille flow in an appropriate direction.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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**Conflict of Interest:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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