REVISITING COINCIDENCE RATE BETWEEN GRAVITATIONAL WAVE DETECTION AND SHORT GAMMA-RAY BURST FOR THE ADVANCED AND THIRD GENERATION

T. Regimbau, K. Siellez, D. Meacher, B. Gendre, and M. Böer
UMR ARTEMIS, CNRS, University of Nice Sophia-Antipolis, Observatoire de la Côte d’Azur, CS 34229 F-06304 NICE, France; regimbau@oca.eu
Received 2014 July 30; accepted 2014 November 4; published 2015 January 15

ABSTRACT

We use realistic Monte Carlo simulations including both gravitational-wave (GW) and short gamma-ray burst (sGRB) selection effects to revisit the coincident rate of binary systems composed of two neutron stars or a neutron star and a black hole. We show that the fraction of GW triggers that can be observed in coincidence with sGRBs is proportional to the beaming factor at \( z = 0 \), but increases with the distance until it reaches 100% at the GW detector horizon distance. When this is taken into account the rate is improved by a factor of three compared to the simple beaming factor correction. We provide an estimate of the performance future GRB detectors should achieve in order to fully exploit the potentiality of the planned third-generation GW antenna Einstein Telescope, and we propose a simple method to constrain the beaming angle of sGRBs.

Key words: gamma-ray burst: general – gravitational waves – stars: black holes – stars: neutron

1. INTRODUCTION

The coalescence of compact binary systems, either neutron star–neutron star (BNS) or neutron star–black hole (NS–BH) are among the most promising sources (Abadie et al. 2010b; Abbott et al. 2008) for one of the first direct detections of gravitational waves (GWs) with the new generation of interferometers: the two Advanced-LIGO (aLIGO; Harry et al. 2010) in Hanford and Livingston and Advanced-Virgo (AdV; Acherese et al. 2006) in Cascina. These facilities will be able to detect the late stage of the coalescence, the merger, and the ring down of the binary systems located within a few hundred Mpc. The GW signal during the adiabatic inspiral phase up to near the last stable orbit and the final damped ring down of the final black hole are accurately described by post-Newtonian expansion and BH perturbation theories, while the progress of numerical relativity over the last decade has provided deep understanding of the merger (Buonanno et al. 2009), giving a good level of confidence for a detection by the network of aLIGO and AdV (in the following we name the combined network consisting of the three interferometers aLIGO and AdV as ALV). With the planned third-generation GW detector, Einstein Telescope (ET; Puntero et al. 2010), envisioned to consist of three independent V-shaped Michelson interferometers with 60° opening angles and arm lengths of 10 km, arranged in a triangle configuration and placed underground to reduce the influence of seismic noise, the maximum detection distance is expected to increase significantly over that of ALV, reaching cosmological distances (\( z \simeq 4 \)) for BNS.

The coalescence of BNS systems is also believed to be at the origin of the short, hard gamma-ray bursts (sGRBs; Eichler et al. 1989). In this scenario, the merger of the system produces a transient accretion disk, the gamma-ray emission being produced by the synchrotron and/or inverse Compton scattering from shocks in an ultra-relativistic jet (see Berger 2014, for a review). Short GRBs might also produce a so-called kilonova through the synchrotron and inverse Compton processes (Li & Paczynski 1998; Rosswog 1999; Tanvir et al. 2005; Berger et al. 2013). The standard hypothesis is that the BNS coalescence results in a black hole, though it is possible that a magnetar, or a transient magnetar, is produced as the result of the merger (Usov 1992; Zhang et al. 2006; Corsi & Mészáros 2009; Zhang et al. 2009).

In this context the coincident detection of both GWs and electromagnetic radiation (EM) would be of paramount importance.

1. The coincident detection of a GW and EM event would greatly improve the detection confidence with ALV during the early operations of the facilities.
2. By using an EM detection of a sGRB as a trigger, one can perform a targeted GW search. This would allow for the detection of fainter signals, resulting in a larger horizon distance.
3. On the other hand, GW alerts sent early to GRB satellites and EM telescopes could increase the chance of an EM detection, if the error on sky localization is smaller than the area covered by the satellite (Cannon et al. 2012).
4. Coincident detections could help solve the enigma of GRB progenitors (for instance BNS or NS–BH) but also of the central engine (the GW signature depends on the fate of the system, with the formation of either a black hole or magnetar) and give increased insight in the physics and dynamics of the system.
5. GRB with measured redshift, observed also in GW, could be used as standard sirens to constrain the Hubble constant and the dark energy equation of state (Schutz 1986; Dalal et al. 2006; Nissanke et al. 2010; Sathyaprakash et al. 2010; Zhao et al. 2011) or to recover the intrinsic mass distribution by breaking the observed mass—redshift degeneracy.

While long GRBs (lGRBs), assumed to originate from massive core collapse supernova, have been detected out to a redshift \( z = 8.3 \) (Zhang et al. 2009; Salvaterra et al. 2009), the maximum observed distance for sGRBs is considerably closer. From a comprehensive search for sGRBs in the rest frame, Siellez et al. (2014) found a maximum distance of \( z = 2.74 \). However, Swift is not optimal for the detection of sGRBs whose spectrum is on average harder than that of IGRBs. BATSE and Fermi/GBM may have been able to detect sGRBs out to larger distance, but the larger sky localization errors associated with these experiments prevented any firm association with a given counterpart and thus redshift measurement. Hopefully the situation is improving thanks to the intermediate Palomar Transient Factory (iPTF; Singer et al. 2013) and other proposed wide-field experiments such as the French ORMES project.
The EM emission of sGRBs (as well as that of lGRBs) is emitted in a narrow beam, though several estimates of the aperture of the jet have been reported (Aasi et al. 2014). This drastically reduces the chances to have a detection.

In order to estimate the coincident rate, one has to account for all the selection effects present in both GW and GRB observations. This is not trivial as there may be overlaps between them. For instance sGRB jets should be directed toward the observer to have a chance to be detected with GRB satellites, but source orientation also affect the strength of the GW signal. In this work, we use Monte Carlo simulations that take into account the selection effects of both GW and sGRB observations to provide realistic estimates on the rate of coincident GW/sGRB detections with both ALV and ET.

The paper is organized as follows: in Section 2 we present our simulations, in Section 3 we derive the coincident efficiency, in Section 4 we estimate the coincident rate, in Section 5 we propose a simple method to measure the average value of the sGRB beaming angle, and finally, in Section 6 we summarize our main conclusions.

2. MONTE CARLO SIMULATIONS

In order to investigate the expected rates of coincident detections of both GWs and sGRBs we perform Monte Carlo simulations using distributions in the expected parameter values. Figure 1 shows a detailed flow chart of all the source parameters that are used in this work and how they relate to each other.

2.1. Simulation of a Population of BNS or NS–BH

Short GRBs are thought to be associated in majority with the coalescence of two neutron stars but it has been suggested that the merger of a neutron star and a black hole could also produce

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1 However, since they have longer evolution time and thus are more numerous at small redshift than primordial binaries, this population may represent the majority of currently observed sGRBs (Wanderman & Piran 2014).
a beam of gamma-ray emission. Since this scenario cannot be excluded, we consider in this paper separately the two possible sources as the progenitors of sGRBs, binary neutron stars, and neutron star–black holes. For each population, we first begin by drawing the source parameters, following a procedure similar to that described in Regimbau et al. (2012, 2014).

1. The redshift is drawn from a probability distribution $p(z)$ (see in Figure 2) constructed by normalizing (in the interval 0–10) the coalescence rate $(dR/dz)(z)$, as detailed in Section 4.

2. Each event is given a sky position in equatorial coordinates (declination and right ascension) that is drawn from an isotropic distribution. The polarization angle, $\psi$, is selected from a uniform distribution from [0, 2$\pi$]. The cosine of the inclination angle, $\cos \iota$, is also drawn from a uniform distribution in the range of [−1, 1].

3. The time interval between two successive events is given by the probability distribution $P(t) = \exp(-t/\lambda)$, assuming coalescences in the observer frame is a Poisson process. The average waiting time $\bar{\lambda}$ is computed from the inverse of the merger rate integrated over all redshifts. Equivalently, we can consider that the coalescence time in the observer frame $t_c$ is a uniform distribution in the interval [0, 2$\pi$] so as to represent one revolution of the Earth about its axis.

4. For the initial set of simulations we consider a delta function for the distribution of the masses with $m_{\text{NS}} = 1.4 M_\odot$ and $m_{\text{BH}} = 10 M_\odot$. We will later consider the case of more realistic mass distributions.

5. In order to model the properties of sGRBs, we also set the beaming angle (taken here as the half opening angle of the jet). We first investigate fixed angles covering a large range of values in the interval [5°–30°]. As for the masses, we will consider use of a distribution of angles at a later stage.

6. The intrinsic peak luminosity $L_p$ (in erg s$^{-1}$) is drawn from the standard broken power law distribution first proposed by Guetta et al. (2005):

$$
\Phi(L_p) \propto \begin{cases} 
(L_p/L_\ast)^\alpha & \text{if } L_\ast/\Delta_1 < L_p < L_\ast, \\
(L_p/L_\ast)^\beta & \text{if } L_\ast < L_p < \Delta_2 L_\ast,
\end{cases}
$$

with $\alpha = -0.6$, $\beta = -2$. We adopt the value of $L_\ast = 10^{51}$ erg s$^{-1}$, corresponding to model ii of Guetta & Piran (2005); Hopman et al. (2006); Guetta & Stella (2009) for primordial binaries, to which we have applied a factor of $\sim 1/2$ correction to convert $L_\ast$ in the band 50–300 keV to the band 15–150 keV used in this paper (see Section 2.3). This is in agreement with the recent work of Wanderman & Piran (2014).\(^2\) We also consider a conservative value of $L_\ast = 5 \times 10^{50}$ erg s$^{-1}$ (the lower bound of model ii of Guetta & Piran 2005), which accounts for a possible extra bias arising if, among the observed sample of sGRBs, those with redshift measurement are the most luminous. We choose $\Delta_1 = 100$ and $\Delta_2 = 10$, in order to cover more than 99% of the luminosities. Notice that taking $\Delta_1 = 30$ as suggested by Guetta & Piran (2005) has a very small effect and does not affect the final results. We neglect in this work any possible evolution of the luminosity with redshift (Butler et al. 2010; Howell et al. 2014).

7. The log of the intrinsic duration of the burst, $\log T_\iota$, is drawn from a Gaussian distribution of mean $\mu_{\log T_\iota} = -0.458$ and standard deviation $\sigma_{\log T_\iota} = 0.502$, derived by fitting the sample of Zhang et al. (2012). We neglect here any possible correlation between the peak luminosity and the duration.

2.2. GW Selection Effects

For each coalescence we must first determine if its resultant GW emission is detectable. For this purpose we calculate the event’s coherent signal-to-noise ratio ($S/N$), for the detector network, in the ideal case of Gaussian noise (see Ghosh & Bose 2013 for a more sophisticated scenario including the possibility of false alarms).

The $S/N$ detected by matched filtering with an optimum filter, in a detector labeled $A$, is

$$
\rho^2_A = 4 \int_0^{\infty} \left| \tilde{h}_+ F_{+,A} + \tilde{h}_x F_{x,A} \right|^2 \frac{S_{n,A}(f)}{\rho_T^2} \, df, 
$$

where $f$ is the frequency of the GW in the observer frame, $\tilde{h}_+$ and $\tilde{h}_x$ the Fourier transforms of the GW strain amplitudes of polarizations + and $\times$, $F_{+,A}$ and $F_{x,A}$ the antenna response functions to the GW polarizations, and $S_{n,A}(f)$ the one-sided noise power spectral density (PSD) of detector $A$ (see Figure 3).

For low mass systems such as BNS or NS–BH, the $S/N$ is dominated by the inspiral part of the signal and can reduce to

$$
\rho^2_A = \frac{5}{6} \left( GM(1+z)^{5/3} F^2_A \right) \int_{f_{\text{min}}}^{f_{\text{ISO}(z)}} f^{-7/3} S_{n,A}(f) \, df.
$$

Here $M$ is the intrinsic chirpmass, a combination of the two component masses, $d_L(z)$ is the luminosity distance, $G$ is the gravitational constant, $c$ is the speed of light, $f_{\text{min}}$ is the low frequency limit of the detector and $f_{\text{ISO}(z)} = f_{\text{LSO}}(1+z)$ is the observed (redshifted) GW frequency of the last stable orbit. The factor

$$
F^2_A = \frac{(1 + \cos^2 \iota)^2}{4} F^2_{+,A} + \cos^2 \iota F^2_{x,A},
$$

characterizes the detector response. Assuming uncorrelated noise, the combined $S/N$ for the network of detector is simply the quadrature sum $\rho^2 = \sum \rho^2_A$ of individual $S/N$s. If $\rho$ is larger than a set $S/N$ threshold level ($\rho \geq \rho_T$) then we say that the event is detectable.\(^2\) Actually there is a factor of $\sim 20$ difference in the results of Wanderman & Piran (2014) since they used a larger energy band (1–10,000 keV).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Probability distribution of the redshift of BNS (blue, smaller peak) and NS–BH (red, higher peak), assuming the star formation rate of Hopkins & Beacom (2006), a distribution of the delay of the form $P(t_d) \propto 1/t_d$ with a minimum delay of 20 Myr for BNS and 100 Myr for NS–BH.}
\end{figure}
2.3. EM Selection Effects

2.3.1. The Beaming Angle

The first selection effect affecting the detection of sGRBs in EM is the strong focusing of the ultra-relativistic jetted emission, i.e., the beaming angle $\theta_B$. Only the fraction $\Theta_B = (1 - \cos \theta_B)$ of sources with inclination angle

$$\left| \cos \iota \right| \leq \cos \theta_B \quad (0 \leq \iota \leq \theta_B \text{ or } \pi \leq \iota \leq \pi + \theta_B),$$

(5)
can be observed on Earth.

Because of the small number of detections of afterglow associated with sGRBs, this angle is constrained to $5^\circ$–$10^\circ$ for only a handful of cases (see eg Soderberg et al. 2006; Fong et al. 2012; Nicuesa Guelbenzu et al. 2011). The non-detection of a jet break also provides lower limits on the jet opening angle between $5^\circ$ and $25^\circ$ (Coward et al. 2012; Berger 2014; Fox & Mészáros 2006; Grupe et al. 2006). Based on a sample of 79 IGRBs and 13 sGRBs (Fong et al. 2012) propose a median of about $10^\circ$. Because of this uncertainty we use a wide range of values for the jet opening angle between $5^\circ$–$30^\circ$.

2.3.2. Instrumental Effects

The other obvious selection effects are related to the instrument: the burst can be undetected due to its faintness, to the fact that it is not located in the field of view (FOV), or because it occurs while the instrument cannot record it. This last effect is dominated by passes through the Southern Atlantic Anomaly (SAA), which reduce the duty cycle (DC) to $\sim$80% of the total time. In the following, we take the effects of the FOV and the DC as independent factors reducing the probability of detection, independently of the source distance and spectrum. Table 1 gives a summary of the characteristics of the detectors we have considered in this work.

An important point to take into account is the complicated trigger mechanism of GRB detectors. Currently, two main instruments can be used to detect a large sample of bursts: 

- **Swift**:
- **Fermi**. While the later is more suited to detect high energy photons, characteristic of sGRBs, only the former provides a statistical sample of sGRBs with a measured redshift. As a matter of consequence, we will use **Swift** in the following as our reference instrument.

Whether a sGRB at redshift $z$ is bright enough to be detected is given by

$$L_p \geq L_{\lim}(z) \text{ with } L_{\lim}(z) = F_{\lim}4\pi d_L(z)^2k(z),$$

(6)

where $L_p$ is the peak luminosity of a source at redshift $z$, $F_{\lim}$ is the limiting flux for a sGRB detection, and $k(z)$ is the k-correction due to the finite observation band at a given redshift. For a generic sGRB defined by its peak energy $E_p = 440$ keV and low and high energy spectrum power indices $\alpha = -0.5$, and $\beta = -3.2$, the Band law (Band et al. 1993) gives $F_{\lim} \sim 0.4$ photons s$^{-1}$ cm$^{-2}$ for on-axis triggers in the 1–1000 keV observation band. A moderate variation of $E_p$ (up to a factor of 2–3) does not change significantly this result. For the **Swift** detection band 15–150 keV, one obtains a threshold of $F_{\lim} \sim 0.56$ photons s$^{-1}$ cm$^{-2}$, which translates to $F_{\lim} \sim 1.5$ photons s$^{-1}$ cm$^{-2}$ for off-axis sources. In addition to these optimal and averaged values, we also consider a pessimistic value of 2.5 photons s$^{-1}$ cm$^{-2}$, corresponding to sGRBs with redshift measurement. In practice, the trigger mechanism can be more complex than Equation (6) and very difficult to model in detail (Lien et al. 2014), but this equation provides a first-order estimate of the trigger condition, and the threshold is derived using flight data (Band 2006) in order to reduce the uncertainties.

Figure 4 shows $L_{\lim}(z)$ for these three limits, along with a sample of 17 observed sGRBs with redshift measurement (Zhang et al. 2012), and 1000 simulated data with the conservative peak luminosity distribution ($L_p = 5 \times 10^{51}$), which best fit the observed sample (using $L_p = 10^{51}$ produces too many detected sGRBs at large redshift).

3. DETECTION EFFICIENCY

In the following sections, we present the various selection effects described above in terms of efficiency, i.e., the fraction of sources, relative to the total, which can be detected at a given redshift.

3.1. GW Efficiency

Several factors affect the maximum distance to which a GW detector will be able to detect a source (the horizon).

1. The relative position of the source with respect to the detector plane at the time of coalescence: a detector is most sensitive to a GW signal that propagates orthogonally to the plane of the detector, meaning that the signal will be affected by both the position in the sky and the time of arrival.

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**Table 1**

| Mission     | DC   | FoV  | Energy Band (keV) |
|-------------|------|------|-------------------|
| Swift       | 80   | 1.4  | 15–150            |
| Fermi-GBM   | 80   | 9.5  | 8–30000           |
| SVOM        | 80   | 2    | 4–5000            |
| LOFT        | 80   | $\pi$| 2–80              |

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*Figure 3. Projected sensitivity for second-generation (advanced) detectors (here the aLIGO high-power zero detuning sensitivity (Harry et al. 2010) and Adv Virgo BNS optimized (Acernese et al. 2006)) and for the initial configuration of ET, ET-B, considered in the Design Study, and the most evolved configuration ET-D (Punturo et al. 2010). The sensitivity of first generation detectors LIGO and Virgo is also shown for comparison.*
at low redshift. It would not explain why they would be sub-luminous though. Of 3 Gyr (Wanderman & Piran 2014), may explain why they are more numerous netars or dynamical BNS or NS–BH whose long evolution times, of the order to reproduce with the simulations and could be either a population of magnitude population, indicated by a red circle in the bottom left of the plot, is difficult to explain by the simulations and could be either a population of magnetars or dynamical BNS or NS–BH whose long evolution times, of the order of 3 Gyr (Wanderman & Piran 2014), may explain why they are more numerous at low redshift. It would not explain why they would be sub-luminous though.

2. The second factor is the inclination angle, i.e., the angle between the normal of the source orbital plane and the observer’s line of sight.

Figure 5 displays the efficiency of ALV and ET for the two cases BNS and NS–BH. The S/N threshold is set to $\rho_T = 12$ for ALV (Kelley et al. 2013), corresponding roughly to an S/N of 8 on at least two detectors. The observation of an EM counterpart, by increasing detection confidence, may allow to reduce the S/N threshold, thus we also use a less conservative value of $\rho_T = 8$, corresponding to an S/N of about 6 on two detectors (P. Sutton 2014, private communication). For the planned ET, we use $\rho_T = 8$ as suggested by mock data challenges (Regimbau et al. 2012). At a redshift $z \sim 0$, all the sources are detected, but as the distance increases, only the best located and oriented sources reach the required threshold. The horizon of the detector corresponds to the maximum distance of detection for the optimally oriented (face-on) and positioned sources, and increases as the S/N threshold decreases. The ALV horizon is about 460 Mpc for BNS and 1 Gpc for NS–BH, assuming $\rho_T = 12$, or 720 Mpc for BNS and 1.6 Gpc Mpc for NS–BH, assuming $\rho_T = 8$. The ET horizon is $z = 4$ for BNS and $z = 13.5$ for NS–BH.

3.2. Coincidence Efficiency: the Case of a Perfect GRB Detector

In order to model the EM selection effects, we first consider the case of a perfect GRB detector with FOV of $4\pi$ sr, duty cycle of 100%, and infinite sensitivity, so that the only selection effect is the beaming fraction. Compared to the previous case, we also require that the inclination $i$ is equal or smaller than the beaming angle, i.e., one of the two opposite jets is directed toward the Earth. The resulting efficiency presented in Figure 6, $\varepsilon_{\text{cap}}(z)$, is equal to the beaming factor at $z = 0$ and shows a plateau until a redshift $z_e$. Short GRBs closer than $z_e$ can all be detected in GWs, then the efficiency decreases until the GW detector horizon. The end of the plateau correspond to the redshift that gives an S/N $\rho = \rho_T$ when the inclination is equal to the beaming angle and the position in the sky and the polarization are such that $F$ is minimum.

Figure 7 shows the efficiency ratio between $\varepsilon_{\text{GW}}(z)$ and $\varepsilon_{\text{cap}}(z)$. The fraction of GW/EM coincident detections corresponds to the beaming angle efficiency at $z = 0$, then increases with the distance to reach 1 close to the GW horizon, where the only sources that can be detected are the best oriented and satisfy Equation (5).

Our assumption of the same averaged value of the beaming angle for the whole population of sGRBs is certainly not

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3. Our horizon distance is a bit higher than the one obtained in Abadie et al. (2010a), as we have taken into account the redshift (see Equation (3)). For a single aLIGO detector, we would obtain a horizon of 485 Mpc instead of 445 Mpc for BNS, and 1090 Mpc instead of 927 Mpc for NS–BH.
Figure 6. Left: GW/GRB coincident detection efficiency of BNS (top) and NS–BH (bottom), for ALV, assuming infinite sensitivity, an FOV of $4\pi$ and a duty cycle of 100% for the GRB satellite, and signal-to-noise ratio threshold of 12 (continuous lines) and 8 (dashed lines). The curves that extend to larger distances are for a threshold of 8. Right: GW/GRB coincident detection efficiency as a function of redshift of BNS (top) and NS–BH (bottom), for ET, infinite sensitivity, an FOV of $4\pi$ and a duty cycle of 100% for the GRB satellite, and signal-to-noise ratio threshold of 8. The different lines indicate different values of the beaming angle. From top to bottom: 30°, 20°, 15°, 10°, and 5°.

Figure 7. Ratio of total GW events to those that can be observed as sGRBs, assuming infinite sensitivity, an FOV of $4\pi$ and a duty cycle of 100% for the GRB. Left: ALV with signal-to-noise ratio threshold of 12. Right: ET with signal-to-noise ratio threshold of 8. The behavior is similar for NS–BH. The different lines indicate different values of the beaming angle. From top to bottom: 30°, 20°, 15°, 10°, and 5°.
realistic. We considered also a set of simple distributions (uniform and Gaussian for $\theta_B$ and $\cos \theta_B$ respectively) as well as the log-Gaussian distribution of Goldstein et al. (2011), derived from observations, with average value $\mu_{\log \theta_B} = 2.0794$ and standard deviation $\sigma_{\log \theta_B} = 0.69 (\theta_B$ in degrees). We found that the efficiency is essentially sensitive to the average value of the beaming factor $(\Theta_B) = (1 - \langle \cos \theta_B \rangle)$ and that the width and the shape of the distribution has very little impact. The efficiency is equal to $(\Theta_B)$ during the plateau and the only difference is visible at the very end, where the distribution of the beaming angle results on a smoother transition between the plateau and the sharp decrease of the efficiency curve.

On the other hand, NS are expected to have a very narrow mass distribution centered around 1.4 $M_o$, but the BH mass distribution is more uncertain. Changing the mass of the system affects the redshift $z_e$ at which the plateau ends as well as the horizon distance. For a delta function, the S/N is shifted toward lower values when the mass, as well as $z_e$ and the horizon, decreases. For a broader distribution (for example, a Gaussian or a uniform distribution), there is a smooth transition between the plateau and the sharp decrease of the efficiency curve, starting at the critical redshift $z_e(M_{\text{min}})$ corresponding to the minimum value of the chirp mass $M_{\text{min}}$ and ending at $z_e(M_{\text{max}})$ corresponding to the maximum value of $M_{\text{max}}$. The horizon is the maximum distance observed for $M_{\text{max}}$.

### 3.3. Case of a Realistic GRB Detector

We now consider the case of a realistic GRB detector with finite sensitivity, and reduced FOV and duty cycle. The FOV, the duty cycle, and the flux limit of the GRB satellites being independent of the GW detection, we write the final efficiency as the product

$$\epsilon_{\text{cd}}(z) = \epsilon_{\text{FOV}} \times \epsilon_{\text{DC}} \times \epsilon_{\text{cdp}}(z) \times \epsilon_{\text{sat}}(z),$$  

(7)

where $\epsilon_{\text{FOV}}$ is the FOV divided by $4\pi$ sr, $\epsilon_{\text{DC}}$ is the duty cycle, $\epsilon_{\text{cdp}}$ is the efficiency for a perfect detector found in Section 3.2, and $\epsilon_{\text{sat}}(z)$ is the fraction of sGRBs whose flux is larger than the limiting flux $F_{\text{lim}}$ (see Section 2.3).

The efficiency of Swift derived from our Monte Carlo procedure is presented in Figure 8 for a flux threshold of 1.5 photons s$^{-1}$ cm$^{-2}$, a pessimistic value of 2.5 photons s$^{-1}$ cm$^{-2}$, corresponding to sGRBs with redshift measurement, and an optimal value of 0.56 photons s$^{-1}$ cm$^{-2}$, corresponding to on-axis sources. The continuous lines correspond to a peak luminosity probability distribution with $L_\star = 10^{50}$ erg s$^{-1}$ and $\Delta_1 = 100$, and the dashed line to $L_\star = 5 \times 10^{50}$ and $\Delta_1 = 100$. For comparison, we have also indicated the efficiency for a larger value of the low luminosity bound ($L_\star = 5 \times 10^{50}$ and $\Delta_1 = 30$) in dash-dotted blue. The efficiency is calculated for an FOV of $4\pi$ and a duty cycle of 100%, in order to have an efficiency of 1 at $z = 0$.

A value of 0.2 photons s$^{-1}$ cm$^{-2}$ would give 80% of the sources above the threshold at $z = 1$ and 1 photons s$^{-1}$ cm$^{-2}$ 50%.

### 4. RATE

In this paper, we assume the coalescence occurs after two massive stars in a binary system have burned all their nuclear fuel, have evolved into red giants, and the cores have collapsed, possibly after supernova explosions, forming a bound system of two compact objects (neutron stars or black holes) inspiralling each other due to the emission of GWs. Another scenario suggests that NS or BH binaries could form through dynamical captures in dense stellar environment. However, most simulations indicate that the chance for this to occur is small, due to the presence of massive black holes at the center that substitute into binaries during dynamical interactions, so that this population may not represent a significant fraction of the total coalescence rate (Abadie et al. 2010a).  

The coalescence rate per interval of redshift,

$$\frac{dR}{dz}(z) = \rho(z) \frac{dV}{dz},$$

(8)

is obtained by multiplying the element of comoving volume $(dV/dz)$ and the coalescence rate per unit of volume,

$$\dot{\rho}(z) \propto \int \frac{\dot{\rho}_s(z_f)}{1 + z_f} P(t_d) dt_d \quad \text{with} \quad \dot{\rho}(0) = \rho_0.$$  

(9)

In this equation, $\dot{\rho}_s$ is the star formation rate (SFR), $\rho_0$ the local coalescence rate in Mpc$^{-3}$ Myr$^{-1}$, $z_f$ the redshift at

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4 Notice that the dynamical origin is favored in the recent study of Wandelman & Piran (2014) as sGRB progenitors, which may suggest that they are more numerous that what is predicted by simulations or that dynamical binaries are not affected by the same selection effects as primordial binaries (Grindley et al. 2006).
Figure 9. GW/GRB detection efficiency of BNS (left) and NSBH (right), for ALV (top) and ET (bottom), and the Swift satellite sensitivity with a flux limit of 1.5 photons s$^{-1}$ cm$^{-2}$ (continuous blue line) a pessimistic value of 2.5 photons s$^{-1}$ cm$^{-2}$ (dashed red line), corresponding to sGRBs with redshift measurement, and an optimal value of 0.56 photons s$^{-1}$ cm$^{-2}$ (dash-dotted green line), corresponding to on-axis sources. The black curve corresponds to the efficiency for an infinite sensitivity satellite and is shown for comparison. The efficiency is calculated for an FOV of 4$\pi$ and a duty cycle of 100%.

In this paper, we use a standard flat cold dark matter ($\Lambda$CDM) model for the universe, with $\Omega_m = 0.3$ and $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$.

In order to account for the uncertainty in the star formation history, we consider seven different SFRs described in detail in Regimbau & Hughes (2009); Regimbau (2011) and plotted in Figure 10. As a reference, we use the SFR of Hopkins & Beacom (2006), which is derived from measurements of the galaxy luminosity function in the ultraviolet (UV) and far infrared (FIR) wavelengths, and is normalized by the Super Kamiokande limit on the electron anti-neutrino flux from past core-collapse supernovae. This model is expected to be quite accurate up to $z \sim 2$, with very tight constraints at redshifts $z < 1$ (to within 30%-50%). Fardal et al. (2007) used a different set of measurements and a different dust extinction correction. The SFR found in Fardal et al. (2007) is the same as that of Hopkins & Beacom (2006) up to $z \sim 1$, but decreases slightly at higher redshifts. We also consider the model described by Wilkins et al. (2008), which is derived from measurements of the stellar mass density. The SFR is equivalent to that in Hopkins & Beacom (2006) and Fardal et al. (2007) for $z \lesssim 0.7$, but again is lower at higher redshifts. Note that at present there

the time of formation of the massive binary system, and $P(t_d)$ the probability distribution of the delay between the formation and the coalescence. The number of massive systems that remain bounded after two supernovae (or prompt core-collapses) is uncertain, as well as the time to coalescence (the delay), which depends on complicated evolution scenario involving common envelope and mass transfer. We assume a distribution of the form $P(t_d) \propto 1/t_d$ with a minimum delay of 20 Myr for the population of BNS and 100 Myr for BH-NS, as suggested by the population synthesis software StarTrack (Dominik et al. 2012), and we leave $\dot{\rho}_0$, which is given between 0.001–10 Mpc$^{-3}$ Myr$^{-1}$ by Abadie et al. (2010a) as a free parameter. The co-moving volume element is given by

$$\frac{dV}{dz}(z) = 4\pi \frac{c}{H_0} \frac{r(z)^2}{E(\Omega, z)},$$  \hspace{1cm} (10)

where

$$r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(\Omega, z')}.$$  \hspace{1cm} (11)

and

$$E(\Omega, z) = \sqrt{\Omega_\Lambda + \Omega_m (1 + z)^3}.$$  \hspace{1cm} (12)
is a discrepancy between the “instantaneous” SFR, measured from the emission of young stars in star forming regions, and the SFR as determined from extragalactic background light. This could have an important impact on the contribution of at $z > 2$. Finally, we consider the analytical SFR of Springel & Hernquist (2003) derived from cosmological smoothed particle hydrodynamics numerical simulations, the model of Nagamine et al. (2006) derived from the fossil record of star formation in nearby galaxies, which probably underestimate the SFR at small redshifts but may be more accurate that and is constant at high redshifts due to the contribution of elliptical galaxies, and the SFR of Tornatore et al. (2007), which combined observations and simulations at higher redshift. For completeness, we also considered a previous model derived from the UV continuum and Hα, up to $z \sim 4$, where the main uncertainty comes from dust extinction, which spreads the UV luminosity into the FIR (Madau et al. 1998).

4.1. Detection Rates

The GW detection rate is obtained by integrating over redshift the product of the coalescence rate given in Equation (9) and the GW efficiency,

$$R_{gw} = \int_{z_0}^{z_{max}} \epsilon_{gw}(z) \frac{dR}{dz}(z) dz,$$

where $z_{max}$ corresponds to the beginning of stellar activity. From our models, $z_{max} \sim 10$–20.

This equation is different than the approximated expression $R_{gw} = (4/3) \pi (d_{max}/2.26)^3$ (Fin & Chernoff 1993), where $d_{max}$ is the horizon distance and the factor 2.26 is a correction needed to average over sky location and orientation. These two equations are similar when facing with the probe of a small volume, like in the current era of advanced detectors, but strongly diverge for a larger horizon, which is the key signature of ET. In such a case, including the efficiency, the star formation history and the cosmology becomes crucial.

In the same way, we calculate the GW/sGRB detection rate as

$$R_{cd} = \int_{z_0}^{z_{max}} \epsilon_{cd}(z) \frac{dR}{dz}(z) dz.$$

Compared to the simple beaming factor correction $R_{cd} = \Omega B R_{gw}$, this equation results in an improvement of the final coincident rate by a factor of ~3 in the case of a perfect sGRB detector, only limited by the beaming selection effect.

The GW coincident rates for ALV and the Swift detector are presented in Table 2, for different beaming angles. We used as a reference, the local rates of $\rho_{SFR}^{\alpha} = 0.06 \text{ Mpc}^{-3} \text{ Myr}^{-1}$ for BNS and 0.003 Mpc$^{-3}$ Myr$^{-1}$ for NS–BH, obtained recently by Dominik et al. (2014) for the StarTrack standard model. For other local rates, one should multiply the values of this table by $\rho_{SFR}^{\alpha}/0.06$ for BNS or $\rho_{SFR}^{\alpha}/0.003$ for NS–BH. The values in the last column indicates the GW detection rate. The rates for the other satellites can be obtained by multiplying these results by FOV/1.4 sr, with the FOV given in Table 1.

### Table 2

|          | 5°   | 10°  | 15°  | 20°  | 30°  | GW   |
|----------|------|------|------|------|------|------|
| BNS      |      |      |      |      |      |      |
| $\rho$   | 12   | 0.004–0.005 | 0.01–0.02 | 0.03–0.04 | 0.06–0.07 | 0.11–0.13 | 2.5–3.0 |
| $\rho$   | 8    | 0.01–0.02  | 0.05–0.06 | 0.10–0.13 | 0.18–0.23 | 0.35–0.46 |      |
| NS–BH    |      |      |      |      |      |      |
| $\rho$   | 12   | 0.001–0.002 | 0.006–0.008 | 0.01–0.02 | 0.02–0.03 | 0.04–0.06 | 1.5–2.0 |
| $\rho$   | 8    | 0.004–0.005 | 0.01–0.02 | 0.03–0.04 | 0.05–0.08 | 0.11–0.16 |      |

**Notes.** The range of values reflects the uncertainty on the SFR. The first line correspond to a signal-to-noise ratio thresholds of 12 and the second line to 8. We used the local rates of $\rho_{SFR}^{\alpha} = 0.06 \text{ Mpc}^{-3} \text{ Myr}^{-1}$ for BNS and 0.003 Mpc$^{-3}$ Myr$^{-1}$ for NS–BH, obtained recently by Dominik et al. (2014) for the StarTrack standard model. For other local rates, one should multiply the values of this table by $\rho_{SFR}^{\alpha}/0.06$ for BNS or $\rho_{SFR}^{\alpha}/0.003$ for NS–BH. The values in the last column indicates the GW detection rate. The rates for the other satellites can be obtained by multiplying these results by FOV/1.4 sr, with the FOV given in Table 1.

**Figure 10.** Cosmic star formation rates (in $M_\odot$ Mpc$^{-3}$ yr$^{-1}$) used in this paper: SFR of Hopkins & Beacom (2006, our reference model) in continuous blue, SFR of Fardal et al. (2007) with light blue dots, SFR of Wilkins et al. (2008) in dashed green, SFR of Springel & Hernquist (2003) with red squares, SFR of Nagamine et al. (2006) with orange crosses, SFR of Tornatore et al. (2007) in dot-dashed purple, and the SFR of Madau et al. (1998) in black with plus signs.
Using the efficiency presented in Figure 9, we roughly estimated the beaming angle, should be between 3° and 10°. Between a half jet opening angle of θ_B = 5° and θ_B = 30°, the rate increases by a factor of about 35. Using the efficiency presented in Figure 9, we roughly estimated that for our reference value of the local merger rate, the beaming threshold.\(^5\) The efficiency of coincident detections is then equal to the threshold at a redshift of z = 1, or the lower bound of Coward et al. 2012 (\(~0.1–1\) yr\(^{-1}\) after FOV and DC correction), based on bias correction of Swift data. They are a factor of 10 smaller than the rates found by Pettrillo et al. (2013, 0.2–1 yr\(^{-1}\)). Comparing the different studies is difficult though, as different authors used different assumptions.

As one can see, the coincident rate is very sensitive to the beaming angle θ_B. Between a half jet opening angle of θ_B = 5° and θ_B = 30°, the rate increases by a factor of about ~35. Using the efficiency presented in Figure 9, we roughly estimated that for our reference value of the local merger rate, the beaming angle, should be between 3°–10° in order to reproduce the actual observed rate of eight sGRBs per year with Swift, which is consistent with current models of the sGRB jet. This would favor a coincident rate between ALV and Swift <0.1 yr\(^{-1}\). Increasing the local merger rate would shift the allowed range for the beaming angle toward lower values so that the final coincident rate would be unchanged. However, these estimates of the beaming angle should be considered with precaution due to the uncertainties associated with the intrinsic luminosity distribution derived by population synthesis from the small sample of observed sGRBs.

For ET the GW coincident rates are presented in Table 3 for a detector with a FOV of 4π sr, a duty cycle of 80% and infinite flux sensitivity. This is of course unrealistic but it gives an upper bound of the number of coincidences in 10–20 yr. If the sensitivity of GRB satellites do not improve in the next decade compared to Swift, the best one can expect is the Swift rate of eight detections a year. This is much better that the rates predicted for ALV, but orders of magnitude below the ET potential. As a matter of consequence, we note that the construction of ET should be accompanied by the launch of a new generation of space observatories focused on transient events.

5. CONSTRAINTS ON THE BEAMING ANGLE

Coincident GW/GRB detections can help measuring, or at least putting strong constraints on the beaming angle of sGRBs (Chen & Holz 2013; Diez 2011). We propose here a very simple method to measure the average beaming factor and thus \(\langle \cos \theta_B \rangle\). The number of ALV coincident triggers will be probably too small to do a parameter estimation, thus we consider only the case of ET. We note that it is likely that a further enhancement of ALV, or a new detector before ET provide an intermediate case. For simplicity, we neglect NS–BH sources.

In the redshift interval \(z = 0–0.2\), the GW efficiency of ET is almost 1 (99.5% of the sources can be detected) and we assume the sensitivity of the satellite is good enough so that only a negligible fraction of all the sGRBs is below the flux threshold.\(^5\) The efficiency of coincident detections is then equal to the average beaming factor (times some factor due to the duty cycle and the FOV) and we can construct the estimator of \(\theta_B\):

\[
\hat{\Theta}_B = (\epsilon_{\text{FOV}} \times \epsilon_{\text{DC}})^{-1} \frac{N^0_{\text{GW}}}{N^0_{\text{cd}}},
\]

where \(N^0_{\text{cd}}\) is the number of coincident detections and \(N^0_{\text{GW}}\) the number of GW detections alone, for \(z\) in the bin \([0–0.2]\).

In order to test our method, we consider a population of BNS with local rate of \(\dot{\rho} = 0.1 \text{ Mpc}^{-3} \text{ Myr}^{-1}\). Using the SFR of Hopkins & Beacom (2006), we obtain a total of \(N^0_{\text{GW}} \sim 3000\) GW over the operation lifetime of ET we assume to be 10 yr. We build the histogram of \(\hat{\Theta}_B\) from a sample of 10\(^5\) simulations (Figure 11) and find that \(\hat{\Theta}_B\) is a non-biased estimator of \(\Theta_B\) (the average value \(< \hat{\Theta}_B >\) converges to the true value) with an error that depends on the average number of coincident sources in one simulation, and thus on the SFR, the local rate, the beaming angle, the duty cycle and the FOV, and of course the time of observation. We confirm the result found in Section 3.2 that a distribution of the beaming angle does not affect the number of coincident detections, and thus the beaming factor estimator. However, when the distribution of \(\theta_B\) is not known, we can only measure the average value of the cosine.

Table 4 gives the average value of \(\Theta_B\) and the standard deviation for the SFR of Hopkins & Beacom (2006) (\(N^0_{\text{GW}} \sim 3000\)) for \(\epsilon_{\text{FOV}} = \epsilon_{\text{DC}} = 1\) and fixed angles of 5°, 10°, and 15°.

For different combinations of local rates, SFR, duty cycle, and FOV or time of observation \(T\), one should simply rescale the standard deviation as

\[
\sigma' = \sqrt{\frac{3000}{\langle N_s \rangle}} \sigma,
\]

---

\(^{5}\) With \(F_{\text{lim}} = 0.1–0.2 \text{ photons s}^{-1} \text{ cm}^{-2}\), we have 99% of the sources above the threshold at a redshift of \(z = 0.2\) for instance.
Finally we have proposed an original method to estimate the mean jet opening angle of sGRBs. This method can be applied to ALV though with low sensitivity. The accuracy will improve as the sensitivity of the GW detectors enhance.

The coincident rates could slightly increase by considering the population of dynamical binaries that could have formed by captures in dense environment and that could be numerous at low redshift due to the long delay between formation and coalescence (Wanderman & Piran 2014). However we do not expect a big change and our findings emphasize the need of a dedicated, wide field of view, multi-wavelength followup of GW detections with a sensitivity increase by a factor of 5 to 10 compared to current detectors.

We acknowledge the use of public data from the Swift data archive and Eric Howell and David Coward for helpful discussions. D.M. acknowledges the PhD financial support from the Observatoire de la Côte d’Azur and the PACA region.

6. CONCLUSION

In this paper, we have presented Monte Carlo simulations of coincident detections between GW and electromagnetic detectors. We have assumed that sGRBs could be powered by BNS or NS–BH coalescences and we have modeled the different selection effects of both GW and EM detectors. We have calculated a coincident efficiency taking into account the fact that the source inclination affect both the GW and GRB efficiency. Besides the beaming angle and the GRB satellite field of view, which are the most important effects, we have shown that the coincident sensitivity is limited by the GW detector horizon for the network of advanced detectors ALV, while for ET it is the GRB flux threshold that is the most important. For ALV the best GRB satellite will be the one with the largest field of view, independently of the flux sensitivity, but for ET, the most important gain will come from the improvement of the sensitivity. In order to roughly estimate the sensitivity required for such an “optimized” sGRB detector used for the follow-up of ET, we have estimated that reducing the average flux limit to $F_{\text{lim}} \sim 0.1$–0.2 photons s$^{-1}$ cm$^{-2}$ over the next decade, would allow 80% of the sources at a redshift of $z \approx 1$ (corresponding to the end of the plateau of the coincident detection efficiency for BNS) to be above the threshold. The flux limit would have to be reduced to $F_{\text{lim}} \sim 0.02$–0.05 photons s$^{-1}$ cm$^{-2}$ to obtain 80% at $z = 2$, where we have the peak of the distribution of the redshift, and where the difference between dark energy equation of states cosmological models is more visible.

Using a set of star formation rate models, we have calculated the coincident rate for different values of the beaming angle for ALV and ET. Our results predict a small number of coincident detections with ALV (less than one event per year for the Swift field of view of 1.4 sr and a duty cycle of 80%), in agreement with recent studies (see for instance Coward et al. 2012; Petrillo et al. 2013; Siellez et al. 2014). The observation of a sGRB counterpart, by increasing detection confidence, may allow for the reduction of the S/N threshold. We have shown that using a threshold of 8 rather than 12 increases the number of coincident detections by a factor of 3 for ALV. We have found a potential number of coincidences of $\sim 100$–10 000 yr$^{-1}$ for ET assuming GRB satellites can reach the desired flux threshold and a maximum FOV of 4$\pi$, but this number will reduce to a few events per year if the FOV and the sensitivity do not improve compared to Swift.

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{\(N_{\text{gw}}^0\)} & \textbf{\(\langle \dot{\theta}_B \rangle\)} & \textbf{\(\sigma_{\theta_B}\)} & \textbf{Error (\%)} \\
\hline
5 & 0.0039 & 0.0011 & 29.3 \\
10 & 0.015 & 0.0022 & 14.5 \\
20 & 0.060 & 0.00042 & 7.23 \\
\hline
\end{tabular}
\caption{Mean and Standard Deviation of the Beaming Angle Estimator \(\dot{\theta}_B\) for the SFR of Hopkins (\(N_{\text{gw}}^0\) $\sim$ 3000), and Assuming a GRB Satellite with a FOV of 4\(\pi\) sr, Duty Cycle of 100% and Infinite Flux Sensitivity}
\end{table}
