Cognitive Diagnosis with Explicit Student Vector Estimation and Unsupervised Question Matrix Learning

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Abstract
Cognitive diagnosis is an essential task in many educational applications. Many solutions have been designed in the literature. The deterministic input, noisy “and” gate (DINA) model is a classical cognitive diagnosis model and can provide interpretable cognitive parameters, e.g., student vectors. However, the assumption of the probabilistic part of DINA is too strong, because it assumes that the slip and guess rates of questions are student-independent. Besides, the question matrix (i.e., Q-matrix) recording the skill distribution of the questions in the cognitive diagnosis domain often requires precise labels given by domain experts. Thus, we propose an explicit student vector estimation (ESVE) method to estimate the student vectors of DINA with a local self-consistent test, which does not rely on any assumptions for the probabilistic part of DINA. Then, based on the estimated student vectors, the probabilistic part of DINA can be modified to a student-dependent model that the slip and guess rates are related to student vectors. Furthermore, we propose an unsupervised method called heuristic bidirectional calibration algorithm (HBCA) to label the Q-matrix automatically, which connects the question difficulty relation and the answer results for initialization and uses the fault tolerance of ESVE-DINA for calibration. The experimental results on two real-world datasets show that ESVE-DINA outperforms the DINA model on accuracy and that the Q-matrix labeled automatically by HBCA can achieve performance comparable to that obtained with the manually labeled Q-matrix when using the same model structure.

Keywords: Cognitive Diagnosis, DINA, Q-matrix, Unsupervised Labelling

1 Introduction
Recently, many studies have been devoted to computer-aided applications, e.g., computer-adaptive tests [5, 9], teaching plan improvements [2] and personalized recommendation [15]. Among these applications, cognitive diagnosis used to diagnose a student’s degree of mastery of knowledge [3, 19] is an essential task. Moreover, it is important to note that the effectiveness of cognitive analysis is usually validated by the predicting examinee performance (PEP) task, which utilizes trained cognitive parameters from the previously obtained examinee responses to predict unseen scores.

Many cognitive diagnosis models (CDMs) have been developed to define cognitive parameters and progress, such as MIRT [1], DINA [8, 4] and FuzzyCDF [19]. CDMs assume that examinees can be characterized by the proficiency on specific skills, where a Q-matrix [13] denotes the skill distribution of all questions, and the skill distribution of a question composes one binary question vector. The Q-matrix is a key feature of the question database. However, the annotation of the Q-matrix is always arduous [6, 7], because experts need to extract specific knowledge of each question, which requires professional abstractions. For another, the deterministic input, noisy “and” gate model (DINA) model is a well-known CDM baseline due to its interpretable student vector parameters [19, 17], and it is composed of a cognitive part and a probabilistic part. The cognitive part assumes that a student can answer a question correctly in theory if he or she masters all the skills tested by the question. Here, the student’s mastery degrees of all skills comprise a binary student vector. Moreover, its probabilistic part represents the students’ scores with slip and guess rates. Both of these two parts of DINA are explanatory. However, DINA utilizes a strong assumption that the slip and guess rates of each question are student-independent, which reduces the complexity of modeling but goes against common sense. For example, elementary students and college students can have different guess rates on the same college questions.

In this paper, we propose an explicit student vector estimation (ESVE) method to estimate student vectors of DINA locally without any assumptions for its probabilistic part. Specifically, we filter slipped or guessed questions by testing the self-consistency of the question vectors with answer labels, which only requires the cognitive part of DINA. Next, ESVE-DINA estimates student vectors with their bounds from the remaining questions that are not guessed and slipped. Then, based on the estimated student vectors, the probabilistic part of DINA can be modified to a student-dependent model
that the slip and guess rates are related to student vectors. Furthermore, we propose a heuristic bidirectional calibration algorithm (HBCA) to label the $Q$-matrix automatically with an initialization method and a bidirectional calibration process. First, the $Q$-matrix is initialized using a heuristic assumption that the relatively easier questions examine fewer skills. Then, we obtain these relative difficulty from the answer results and label questions by taking relatively easier questions as bases. Additionally, we find that the self-consistency test of ESVE-DINA can circumvent the errors of $Q$-matrix. Then, a dual algorithm (DA) of ESVE-DINA is designed to estimate the $Q$-matrix from the estimated student vectors, which is a dual task of student vector estimation from the $Q$-matrix. Thus, the $Q$-matrix can be bidirectionally calibrated by conducting the fault tolerance of both ESVE-DINA and DA. The main contributions of this paper are summarized as follows:

- We propose an ESVE algorithm to estimate the student vectors of DINA, which requires no assumption for its probabilistic part. Moreover, the student-independent probabilistic part of DINA can be modified to a student-vector-related model based on the estimated student vectors.
- We also propose an unsupervised method HBCA to label the $Q$-matrix automatically, which connects the question difficulty relation and the answer results for initialization and uses the fault tolerance of ESVE-DINA for calibration.
- Experiments on two real-world datasets Fraction and ASSISTments2015 show that ESVE-DINA outperforms DINA model on accuracy, and the $Q$-matrix labeled automatically by HBCA can achieve performance comparable to that obtained with manual $Q$-matrices using the same model structure.

2 RELATED WORK

We briefly summarize our related work for cognitive diagnosis from two aspects: cognitive diagnosis methods and the question information annotation domain.

2.1 Cognitive Diagnosis Methods In educational psychology, a fundamental CDM is the deterministic inputs, noisy “and” gate (DINA) model [8 7 3]. It assumes that a student can answer a question correctly when he or she masters all the test skills. Next, he or she may slip or guess this question after the ideal process. Though these parts are both reasonable, DINA utilizes a strong assumption that the slip and guess rates are student-independent, decoupling the student vectors and the slip and guess rates to achieve an acceptable complexity of modeling. However, ESVE method can estimate student vectors with only the cognitive hypothesis of DINA, which avoids this restriction.

2.2 Question Information Annotation The problem of question information annotation is often polarized and is generally either finely carried out by experts in a costly manner or coarsely labeled with fuzzy annotations. For one, as shown in [6 17], labeling of a detailed $Q$-matrix such as the dataset Fraction [14] requires domain experts with abstractions and is quite costly. For another, many question datasets have sparse skill labels, such as the family of ASSISTments datasets [4] Its questions usually have 1 to 3 sparse and nonspecific skill labels, notably increasing the burden of modeling. In this paper, we show that the initialization of the $Q$-matrix can be related to the answer results, and the fault tolerance of the solving algorithm can calibrate the initialized $Q$-matrix.

3 Problem Definition

Here, we will introduce the formal definition of cognitive diagnosis and the DINA model.

We study cognitive diagnosis for cognitive parameters $V$ of $S$ students on $M$ questions with question parameters $B$. Given the existing answer results $X$, skill distribution of questions $Q$-matrix and score model $CDM = f(V, B, Q)$, we need to solve student parameters $V$ and question parameters $B$, which can also be evaluated by the PEP task of predicting unseen $X$ to show the rationality of a CDM and the estimated $V$.

Let us first review the DINA model [8 3 4]?

\begin{equation}
(3.1) \quad \xi_{ij} = \prod_{k=1}^{N} a_{ik}^{Q_{jk}}.
\end{equation}

\begin{equation}
(3.2) \quad s_j = P(X_{ij} = 0|\xi_{ij} = 1).
\end{equation}

\begin{equation}
(3.3) \quad g_j = P(X_{ij} = 1|\xi_{ij} = 0).
\end{equation}

\begin{equation}
(3.4) \quad P(X_{ij} = 1|\xi_{ij}) = (1 - s_j)\xi_{ij} g_j^{1-\xi_{ij}}.
\end{equation}

DINA uses a binary latent variable $\xi_{ij}$ to denote the cognitive part. Ideally, $i$-th student can answer $j$-th question correctly when he or she masters all the test skills. If there exists a $k$-th skill that is tested ($Q_{jk} = 1$) but is not mastered ($\alpha_{ik} = 0$), $\xi_{ij}$ is false. $Q_j$ stands for the binary question vector of the $j$-th question, and $\alpha_i$ is the binary student vector of the $i$-th student. The probabilistic part of DINA uses $s_j$ to denote the slip rate of $j$-th question, i.e., the probability that the ideal result of the $j$-th question is correct but is answered incorrectly. Then, the probability of $X_{ij}$ being true is $1 - s_j$. $g_j$ denotes the guess rate in a similar manner. However, every $s_j$ and $g_j$ are student-independent for their independence on the student index $i$. 

\footnote{https://sites.google.com/site/assistmentsdata/home}
4 DINA with Explicit Student Vector Estimation

In this section, we will introduce our ESVE method, which filters slipped and guessed questions by a local self-consistency test of the Q-matrix with answer labels. This indicates that we will derive each student vector $\alpha_i$ with only Eq. (3.1) but no probabilistic assumption for $s_j$ and $y_j$ of DINA, while traditional methods [4] optimize the joint probability Eq. (3.4) of all students.

ESVE is a two-step method, where in the first stage, the observed case is converted to an ideal case in which there are no slipped or guessed questions, and in the second stage, the feasible student vector is estimated from the ideal case. Then, primarily, we infer some ideal intermediate relations when there are no slipped or guessed questions; i.e., for all $j$, $s_j = g_j = 0$.

First, we define some intermediate variables.

**Definition 4.1.** If a student did a question set, then the question answered correctly by guessing or incorrectly by slipping is called an unreliable question, and the corresponding question vector is an unreliable question vector. Otherwise, the question is called a reliable question and corresponds to a reliable question vector.

Meanwhile, the question vector set are divided into two subsets by the answer results $X_i$ of each student $i$, the correct question set and the incorrect question set; their corresponding question vector sets are annotated as $Q^T$ and $Q^F$ respectively.

Then, we rewrite Eq. (3.1) as

\[ X_{ij} = \xi_{ij} = \begin{cases} 1, & \text{if for all } k, \alpha_{ik} \geq Q_{jk}, \\ 0, & \text{otherwise}. \end{cases} \]

Here, $X_{ij} = \xi_{ij}$ is inferred by Eq. (3.4) with the ideal case that for all $j$, $s_j = g_j = 0$. The second equation means that the $i$-th student can answer the $j$-th question correctly once he or she masters all the tested skills. Eq. (4.5) has the same output as Eq. (3.4) for identical input.

Thus, based on Eq. (4.5), ideally, $X_{ij} = 0$ means that there exists at least one $k$, $\alpha_{ik} < Q_{jk}$. Similarly, $X_{ij} = 1$ means that for all $z$, $\alpha_{iz} \geq Q_{iz}$. When considering all the question vectors that are divided into $Q^T$ ($X_{ij} = 1$) and $Q^F$ ($X_{ij} = 0$), we obtain that

\[ \forall q_k \in Q^T, \forall z, \alpha_{iz} \geq Q_{pz}, \]

\[ \forall q_k \in Q^F, \exists k, \text{ s.t. } \alpha_{ik} < Q_{qk}. \]

In the following two subsections, we first infer the conflict degrees of the question vectors with observed answer labels to filter unreliable questions, converting the observed answer results to only ideal reliable questions. Second, we induce a feasible student vector satisfying all bounds from the remaining reliable questions.

4.1 Filtering Unreliable Question Vectors

In this subsection, we describe a method called **conflict detection** to obtain conflict degrees, which represent the unreliable degrees of question vectors with observed result labels $(Q^T, Q^F)$. Then, we filter all unreliable questions based on conflict degrees.

Here, we will analyze the relationship between the question vector pairs with observed right and wrong labels $(Q^T_p$ and $Q^F_q)$. First, a globally reliable condition between $Q^T_p$ and $Q^F_q$ can be inferred by the ideal conditions in Eq. (4.6) and Eq. (4.7). From these two equations, by setting $z$ as a specific $k$, we obtain that

\[ \forall q_k, \forall q_k, \exists k, \text{ s.t. } Q^T_{pk} \leq \alpha_{ik} < Q^F_{qk}. \]

Then, we can find a locally self-consistent condition between each $Q^T_p$ and $Q^F_q$ pair as

\[ \exists k, \text{ s.t. } Q^T_{pk} < Q^F_{qk}. \]

Here, the local self-consistent condition between $Q^T_p$ and $Q^F_q$ means that both of them are more reliable locally, because these two question vectors satisfy a part of the whole reliable conditions in Eq. (4.8). Thus, an approach to identify the unreliable questions is to count the question vectors that break the globally self-consistent condition in Eq. (4.8) the most. The reverse condition of Eq. (4.9) becomes

\[ \forall k, Q^T_{pk} \geq Q^F_{qk}. \]

The condition in Eq. (4.10) for detecting each right and wrong question pair is called the **conflict condition**. Intuitively, we should treat right and wrong questions equally, which means that if a $Q^F_q$ and $Q^T_p$ pair satisfy Eq. (4.10), the conflict degrees of question $q$ and $p$ should be increased by one together. Then, we can consider the conflict degrees to be the representation of unreliable degrees, and the questions with the maximum conflict degrees will be filtered because they break the globally reliable condition in Eq. (4.8) the most.

Thus, we can count the conflict degrees of all questions by traversal detection of every right and wrong question pairs using Eq. (4.10). We call this **conflict detection**. Moreover, we can convert observed questions to reliable questions by conflict detection and filtering questions with maximum conflict number until there is no conflict. Here, we can observe that the filtering progress uses no probabilistic assumption outside of the cognitive part of DINA, and then, it avoids the strong probabilistic assumption of DINA.

4.2 Estimating Student Vector from Reliable Question Vectors

In this subsection, we estimate
every student vector from reliable question vectors 
\((Q^T_{rel}, Q^F_{rel})\) by estimating the upper and lower bounds of its component, which means that this is an ideal case shown in §4. We still use \(Q^T, Q^F\) for simplicity.

From Eq. (4.6), we obtain that all \(Q^T_{pq}\) is not larger than \(\alpha_{iq}\), and since the relation in Eq. (4.6) is for all components \(z\), we can set \(z\) equal to \(k\) for the unity of derivation, and we call the union of all components of \(Q^T_{pk}\) the lower component of \(\alpha_{ik}\). Thus, we obtain that

\[
\alpha_{ik} \geq \alpha^\text{lower}_{ik} = \bigcup_{p=1}^{u} Q^T_{pk}.
\]

Here, the OR operation means that student \(i\) should master skill \(k\) if the right question set tests skill \(k\), and \(\alpha^\text{lower}_{ik}\) can be thought as the maximum value of all \(Q^T_{pk}\). \(u\) denotes the size of the right question set.

As there are no unreliable questions, the whole Q-matrix and student vector \(\alpha_i\) must be self-consistent. Thus, from Eq. (4.11) and Eq. (4.7), we obtain that

\[
\forall Q^F \in Q^F, \exists k, \text{s.t. } Q^F_{qk} > \alpha_{ik} \geq \alpha^\text{lower}_{ik}.
\]

As \(\alpha_{ik}\) is unknown, Eq. (4.12) can be written as

\[
\forall Q^F \in Q^F, \exists k, \text{s.t. } Q^F_{qk} > \alpha^\text{lower}_{ik}.
\]

Likewise, we can consider the upper component of \(\alpha_{ik}\), which is defined as a component that is larger than \(\alpha_{ik}\), which comes from \(Q^F_{qk}\) in Eq. (4.12). As \(\alpha_{ik}\) is unknown, we use Eq. (4.13) to infer the upper component of \(\alpha_{ik}\), which implies that

\[
\alpha^\text{upper}_{ik} = \bigcap_{q=1}^{v} Q^F_{qk}, \text{if } Q^F_{qk} > \alpha^\text{lower}_{ik}.
\]

Here, the AND operation means taking the minimum values by the definition of the upper components. However, this operation is not unique. For example, consider the case of only two reliable questions, a wrong question vector \([1, 1, 1]\) and a right question vector \([0, 0, 1]\). We can see that the first and the second components can both satisfy Eq. (4.14), so they are all upper components. But there may be three reasons why the student answered the question \(q\) incorrectly; namely, he or she lacks skill 1 or skill 2 or both. Nevertheless, to punish wrong questions, we choose the third-worst solution, i.e., choosing the worst combination of the observed lacked skill set.

Next, a feasible \(\alpha_{ik}\) satisfying the upper and lower components of \(\alpha_{ik}\) will be selected. The upper and lower bounds of each \(\alpha_{ik}\) are inferred by Eq. (4.11) and Eq. (4.14) from reliable question vectors. Then, we just choose a feasible \(\alpha_{ik}\) that meets the condition

\[
\alpha^\text{lower}_{ik} \leq \alpha^\text{upper}_{ik}.
\]

As \(\alpha_{ik}\) is either 0 or 1, \(\alpha_{ik}\) is set as follows:

\[
\alpha_{ik} = \begin{cases} 
0, & \text{if } \alpha^\text{upper}_{ik} = 1; \\
1, & \text{if } \alpha^\text{lower}_{ik} = 1; \\
\text{random}(0, 1), & \text{others.}
\end{cases}
\]

Therefore, every student vector \(\alpha_i\) are obtained by the same operation on each \(\alpha_{ik}\).

4.3 DINA with Explicit Student Vector Estimation

Combining §4.1 and §4.2, the student vectors of DINA are solved by ESVE. First, the Q-matrix are divided into \(Q^T, Q^F\) by the observed answer results. Then, the unreliable questions are filtered by conflict detection using Eq. (4.10). Next, the upper and lower bounds of each \(\alpha_{ik}\) are estimated by the remaining reliable question vectors using Eq. (4.11) and Eq. (4.14). Finally, every feasible \(\alpha_{ik}\) are obtained by Eq. (4.15). The flowchart of ESVE-DINA is shown in Fig. 1.

Here, we can observe that ESVE-DINA estimates student vectors with no probabilistic assumption outside of the cognitive part of DINA. Then, the prediction method based on ESVE-DINA can be quite flexible.

A trivial method is the student-independent (SI) prediction method of DINA, where \(s_j, g_j\) are unrelated to the student vector \(\alpha_i\) or other possible parameters. \(s_j, g_j\) are obtained by their definitions. For example, \(s_j = N(\text{question } j \text{ filtered from wrong question set})/N(\text{examinee of question } j)\). Here, \(N(X)\) means the number of \(X\), and a question filtered from wrong question set means that it should be right but in fact is wrong, meaning it is a slipped question. \(g_j\) counts the number filtered from the right question set similarly.

A more reasonable method is that \(s_j, g_j\) are student-dependent (SD) and they can be related to the student vector \(\alpha_i\). We assume that \(s_j\) is related to the mastery skill number (i.e., level) of \(\alpha_i\), meaning that we think that students with the same skill number have identical slip rates on each question. Furthermore, we assume that \(g_j\) is related to the lacked skill number (i.e., deficiency) of some student on question \(j\). SD \(s, g\) are computed in a similar manner as above SI \(s, g\), both the numerator and denominator plus a condition that the level or deficiency is equal to a specific number.
5 Heuristic Bidirectional Calibration Algorithm

In this section, we introduce our HBCA method. HBCA is a bidirectional calibration (BC) process with an initialization procedure. The initialization of the Q-matrix spans a question tree, and it heuristically assumes that relatively easier questions test fewer skills, which is called the question spanning tree (QST) algorithm. Besides, the BC process repeatedly utilizes ESVE-DINA and its dual algorithm (DA). The fault tolerance of ESVE-DINA and DA are used for calibration of the Q-matrix. The following subsections will introduce QST, DA and HBCA.

5.1 Question Spanning Tree Algorithm In this subsection, we show how to initialize the Q-matrix automatically. We assume that the relatively easier questions examine fewer skills, and an example of QST with three questions is shown in Fig. 2. There are two steps. The first is to find the relations from the student answer results, and the second is to initialize a Q-matrix by spanning a question tree with these relations.

First, we discuss the relations between questions. Supposing that there are two questions, we can know that a covering relation such as \( Q_3 \) and \( Q_1 \) in Fig. 2 can be useful, which means that every component of one question vector is equal to or greater than another.

The covering relation can efficiently restrict the solution space of the Q-matrix. If question \( w \) covers a question set with question vector set \([Q_1, Q_2, \ldots, Q_z]\), then we can obtain the following inequality:

\[
\forall k, Q_{wk} \geq \bigcup_{p=1}^{z} Q_{pk}.
\]

Here, the OR operation takes the maximum value.

5.1.1 Covering Relation Construction In this subsection, we demonstrate a method for obtaining covering relations. First, the conditional probability of the question’s accuracy is defined and can be computed by counting statistics.

\[
\beta_{wz} = P(X_{iz} = 1 | X_iw = 1).
\]

Based on our assumption that relatively easier questions test fewer skills, if \( \beta_{wz} \) is very large; i.e., \( \beta_{wz} \geq \eta \) and \( \eta \) is large, then question \( z \) is relatively easier than question \( w \) and we think question \( w \) covers question \( z \).

Moreover, from the standpoint of entropy [?], a large \( \eta \) means that the answer of question \( z \) severely tracks question \( w \) and it is a compact piece of information. Meanwhile, the \( \eta \) value can be guided by the generated Q-matrix. Once it is too small, the whole covering relation will so redundant that the generated Q-matrix will have many full binary question vectors.

5.1.2 Q-matrix Initialization In this subsection, we describe the initialization of the Q-matrix with covering relations. First, the question pair satisfying covering relations are called a parent and child pair. As parent and child are the terms of tree structure [10], we consider that our method spans the question tree and call it the question spanning tree algorithm.

Next, we show how to initialize the Q-matrix with the parent and child relation. From Eq. (5.16), we know that if the children of question \( w \) is question \( z \), \( Q_{wk} \) can be set as its lower bound \( \bigcup_{p=1}^{z} Q_{pk} \), such as \( Q_3 \) and \( Q_1, Q_2 \) in Fig. 2. Meanwhile, to make some randomness of generated \( Q_w \), we can set some probability to roll over its zero component. For example, \( Q_3 \) in Fig. 2 may become [1, 1, 1] instead of [0, 1, 1]. Furthermore, we can randomly initialize the leaf nodes without the children. Moreover, we can span the question tree in the descending order of the parent number, which means that we label the questions from easy to difficult. Here, more parents means an easier question. Thus, the Q-matrix is initialized with our relative relation assumption and base questions, i.e., children questions.

5.2 Dual Algorithm In this subsection, we introduce the fault tolerance of ESVE-DINA and estimate the Q-matrix from student information with DA.

First, we analyze the process of ESVE-DINA. ESVE-DINA filters unreliable questions by the self-consistency test of the question vectors with answer labels. If a question vector has more wrong labels with fixed result labels (result labels are unrelated to \( Q \)), it may fail the self-consistency test easily and will not influence the estimation of student vectors. Hence, ESVE-DINA may eliminate some mistakes in the Q-matrix. Thus, if we can avoid some mistakes of estimated student vectors, we can calibrate the Q-matrix by these bidirectional fault tolerances. Fortunately, we can design a dual algorithm of ESVE-DINA to achieve this goal by the duality between the student vector and the
question vector. The duality is as follows:

(1) The answers from student \( i \) depend on all question vectors and one unknown student vector \( \alpha_i \).

If student \( i \) can answer question \( j \) correctly (T label), then, for all \( k \), \( \alpha_{ik} \geq Q_{jk} \); otherwise (F label), there exists at least one \( k \) such that \( \alpha_{ik} < Q_{jk} \).

**Ideal condition:** \( \forall p \) and \( q \), \( \exists k, Q_{pk}^F > \alpha_{ik} \geq Q_{jk}^T \).

(2) The answers to question \( j \) depend on all student vectors and one unknown question vector \( Q_j \).

If question \( j \) can be answered by student \( i \) correctly (T label), then, for all \( k \), \( Q_{jk} \leq \alpha_{ik} \); otherwise (F label), there exists at least one \( k \) such that \( Q_{jk} > \alpha_{ik} \).

**Ideal condition:** \( \forall p \) and \( q \), \( \exists k, \alpha_{ik} < Q_{jk} \).

From the above comparison, we observe that the ideal condition of the student vector and the question vector only differ in terms of the inequality direction. Thus, we can design a dual algorithm (DA) of ESVE-DINA to estimate the Q-matrix by the following correspondence. Suppose that \( \alpha^T \) has \( r \) elements and that \( \alpha^F \) has \( l \) elements.

(1) Conflict condition of DA: \( \forall k, \alpha_{ik}^T \leq \alpha_{ik}^F \). \iff conflict condition of ESVE-DINA: \( \forall k, Q_{pk}^F \geq Q_{pk}^T \).

(2) upper component of \( Q_{jk} \): \( Q_{jk}^{upper} = \bigcap_{w=1}^{r} \alpha_{wk}^T \) \iff lower component \( \alpha_{ik}^T \): \( \alpha_{ik}^{lower} = \bigcup_{p=1}^{u} Q_{pk}^T \).

(3) lower component of \( Q_{jk} \): \( Q_{jk}^{lower} = \bigcup_{z=1}^{l} \alpha_{zk}^F \), if \( \alpha_{ik}^F < Q_{jk}^{upper} \) \iff upper component of \( \alpha_{ik}^F \): \( \alpha_{ik}^{upper} = \bigcap_{p=1}^{u} Q_{pk}^F \), if \( Q_{jk} < \alpha_{ik}^{lower} \).

Every \( Q_{jk} \) can be set by an equation similar to Eq. (4.15) that satisfies \( Q_{jk}^{lower} < Q_{jk} \leq Q_{jk}^{upper} \).

### 5.3 Heuristic Bidirectional Calibration Algorithm

Based on §5.1 and §5.2, HBCA can label the Q-matrix automatically. First, the Q-matrix are performed by the heuristic initialization of QST, and then, the student vectors (S) and the Q-matrix are bidirectionally calibrated by ESVE-DINA and DA. Furthermore, an optimization strategy is initializing many Q-matrices and update them separately (total T iterations), and if the training does not decrease the prediction error, some new Q-matrices are initialized to replace those with bad property. This method imitates the update of genetic algorithm [18]. In summary, the HBCA framework is shown in Fig. 3, here, the ⊗ means the prediction method of optional specific models.

### 6 Experiments

In this section, first, we compare the performance of ESVE-DINA and HBCA against the baseline approaches (mainly the DINA model) on the PEP task. Next, we utilize some consistency experiments to show that the student-independent s, g assumption of DINA is inappropriate. Finally, we conduct experiments to investigate the hyperparameter sensitivity of HBCA.

#### 6.1 PEP Tasks

##### 6.1.1 Datasets

In our experiments, we adopted Fraction [14, 19, 16] and ASSISTments2015 [16] (ASSIST for short) datasets in our experiments. The Fraction dataset consists of scores of fraction problems and a Q-matrix labeled by experts. The ASSIST dataset is collected by an online education system [12], and its Q-matrix is a sparse identity matrix. We utilized the full Fraction dataset and a part of the ASSIST dataset, because the original ASSIST dataset is sparse and contains duplicate records. We used three steps to select a subset, namely, filtering duplication, selecting questions with a record frequency of more than 20% and selecting students with a response frequency of more than 50%. Table 2 summarizes the data statistics of the selected datasets. Following [19, 17], we then utilize 80% of the dataset chosen randomly for training and the remaining 20% for testing. For additional comparison to [19], we also selected 20%, 40%, 60% and 80% of the dataset for testing on the Fraction dataset.

##### 6.1.2 Experimental Setup

Among these experiments, ESVE-DINA has no hyperparameters. In HBCA, for some settings, we used the same values for the two datasets. We initialized 100 Q-matrices for 100 iterations and set the update number of the Q-matrix to 40. Its dual algorithm used 100 random student vectors to estimate the Q-matrix. There are two differences between the two datasets; that is, the threshold \( \eta \) in the Fraction dataset was 0.85, while that in ASSIST was 0.9, and their settings are discussed in §5.1. The HBCA searched the question vector dimension \( \text{dim}_{qv} \) from 5 to 9 in the Fraction dataset, while ASSIST searched \( \text{dim}_{qv} \) from 6 to 10 for more questions. All of these methods
are implemented on a Core i5 2.3GHz machine with a CPU. We built the following models for comparison:

- **ESVE-DINA-SI** This model utilizes the student vectors of ESVE-DINA and the student-independent (SI) assumption of DINA.
- **ESVE-DINA-SD** This model utilizes the student vectors of ESVE-DINA and our student-dependent (SD) s, g assumption that every $s_j$ and $g_j$ is related to the student vectors.
- **HBCA** This model uses the Q-matrix labeled by HBCA instead of the original manual Q-matrix. HBCA will previously choose 20% of the training set as validation to select a best $dim_{vu}$ with the corresponding selecting goal; i.e., $x + HBCA$ means that the selecting goal is MAE of $x$. Since our API of the DINA code has a conflict with HBCA, DINA + HBCA uses Q of HBCA with the selecting goal of ESVE-DINA-SI instead, because they use the same SI s, g assumption.
- **QST** This model uses Q-matrix labeled by QST of the first iteration in HBCA.

### 6.1.3 Baselines

To demonstrate the effectiveness of ESVE-DINA and HBCA, we compare them with some baseline methods, their details are shown as follows:

- **DINA [8, 3]**: A model assumes that the probabilistic parameters of questions are student-independent. Here, we implement DINA with the classic EM algorithm. DINA is a typical baseline [17, 19] for the PEP task.
- **DINA (Wu.)** The results come from a paper [19].
- **FuzzyCDF [19]**: A model that expands each dimension of DINA into an IRT model [11], it has more explainable but complicated cognitive parameters than the DINA model. This is an enhanced comparison for our methods, and the numerical results here come from the original author.

### 6.1.4 Evaluation Metrics

To demonstrate the effectiveness of our models, we conduct experiments with five-time random validation on the PEP task, i.e., predicting response logs. We use the evaluation metrics from both classification aspect [19] and regression aspect [17], including MAE (mean absolute error), RMSE (root mean square error) and AUC (area under the curve).

### 6.1.5 Experimental Results

Table 2 presents results with different test ratios on the Fraction dataset, and Tables 3 and 4 respectively show the results of our methods and other baseline approaches with 20% test ratio on the Fraction and ASSIST datasets. From Tables 3 and 4, we can observe that ESVE-DINA-SD outperforms DINA and ESVE-DINA-SI with experts’ Q on Fraction dataset and Q of HBCA on both datasets, indicating the effectiveness of ESVE-DINA-SD. For HBCA, different models mainly have differences in the selecting goals but have the same initialization and calibration methods, so the results with Q of HBCA are approximately comparable. Second, comparing the same models with different Q-matrices, we can observe that the Q-matrix automatically labeled by HBCA can achieve comparable performance to that of the manual Q-matrices. Specifically, Q of HBCA shows similar results with DINA on both datasets, better results with ESVE-DINA-SI and slightly worse results with ESVE-DINA-SD on the Fraction dataset. Third, according to Table 2, our ESVE-DINA-SD model shows better MAE than FuzzyCDF when the test ratio is more than 20%, indicating that our model has stronger prediction ability with less student information.

One detail is that ESVE-DINA can not solve ASSIST dataset, because its trivial unit Q-matrix cannot be used for the conflict detection of ESVE-DINA.

### 6.2 Consistency Test of the s, g assumption of DINA

To examine the assumption of DINA that the slip and guess rates of each question are student-independent (SI), we compute $s$ on different levels with the golden student results of the test set and solved student vectors of training set as their references:

$$s_{i,j,k}^{ref} = \frac{\sum_{i=0}^{S}(X_{ij} = 0)\&(\xi_{ij} = 1)\&(\text{sum}(\alpha_i) = k)}{\sum_{i=0}^{S}(\xi_{ij} = 1)\&(\text{sum}(\alpha_i) = k)}$$

Here, sum($\alpha_i$) represents the level of student $i$. References of $g$ are similar, the level becomes deficiency, and
Table 3: Experimental results on the predicting examinee performance task of the Fraction dataset.

| Methods               | MAE   | RMSE  | AUC  |
|-----------------------|-------|-------|------|
| DINA (Wu.) [19]       | 0.3153| 0.4056| -    |
| DINA                  | 0.3101| 0.3997| 0.8577|
| ESVE-DINA-SI          | 0.2611| 0.4595| 0.7633|
| ESVE-DINA-SD          | 0.2443| 0.3865| 0.8704|
| DINA + HBCA           | 0.3097| 0.3903| 0.8506|
| ESVE-DINA-SI + HBCA   | 0.2556| 0.4242| 0.8032|
| ESVE-DINA-SD + HBCA   | 0.2561| 0.3850| 0.8649|

Table 4: Experimental results on the predicting examinee performance task of the ASSIST dataset.

| Methods               | MAE   | RMSE  | AUC  |
|-----------------------|-------|-------|------|
| DINA                  | 0.5101| 0.5579| 0.6505|
| DINA + HBCA           | 0.4961| 0.5385| 0.6428|
| ESVE-DINA-SI + HBCA   | 0.4366| 0.5813| 0.5859|
| ESVE-DINA-SD + HBCA   | 0.3770| 0.4707| 0.6691|

Figure 4: Results of consistency experiments for examining the SI assumption of DINA.

The values of $X_{ij}, \xi_{ij}$ are reversed. Then, as the golden student results X of test set are unseen, the values of $s^{ref}, g^{ref}$ can be references values of $s, g$. The distribution of $s^{ref}, g^{ref}$ can be validated to determine whether $s, g$ can be SI. Fig. 4 (a) shows the distribution of $s^{ref}$ of DINA with experts’ Q on the Fraction dataset, the results with Q of HBCA on both datasets or the results of $g^{ref}$ are similar. We can observe that the values of most rows have large variances, implying that the slip rates can be related to student levels. Thus, the SI $s, g$ assumption of DINA can be inappropriate.

Next, we compare the distortions between estimated $s, g$ of the training set and their references to demonstrate the rationality comparison between the SI $s, g$ and SD $s, g$ assumption. The distortion of $s$ is defined as follows (distortion of $g$ is similar):

$$s_\delta = \frac{1}{MN} \sum_{j=0}^{M} \sum_{k=0}^{N} |s_{j,k} - s_{j,k}^{ref}|.$$  (6.19)

Here, every $s_{j,k}$ is estimated on the training set, and $s_{j,k}^{ref}$ shown in Eq. (6.18) are references values of the test set. Then, their consistency can partly show the rationality of $s, g$ assumption. Moreover, for ESVE-DINA-SI and DINA, their SI $s_{j,k}$ values of the training set are constant on dimension $k$. Fig. 4 (b) shows the $s_\delta, g_\delta$ comparison with experts’ Q on the Fraction dataset, the results with Q of HBCA on both datasets are similar. We can observe that our SD $s, g$ of ESVE-DINA-SD has smallest $s_\delta, g_\delta$. Thus, our SD $s, g$ assumption is more reasonable than SI $s, g$ assumption.

6.3 Hyperparameter Sensitivity of HBCA

Here, we show the hyperparameter sensitivity of our unsupervised labeling method HBCA. We test two primary factors, namely, the initialization (QST) efficiency and the question vector dimension ($\text{dim}_{qv}$). Fig. 6 shows the MAE comparison of HBCA and QST on the validation and test sets of the Fraction dataset, and similar results are obtained for the ASSIST dataset. From Fig. 5, we can observe that the initialization of HBCA (QST) is not bad, and it ($\text{dim}_{qv} = 9$) is only slightly worse than the results with experts’ Q (0.2443) on the test set. Second, we observe that labeling of HBCA is not sensitive to $\text{dim}_{qv}$, and it has relatively smooth results on the test set. Third, we also observe that the calibration results of HBCA (gap of HBCA and QST) are different on the validation and test sets. This result may be due to the difference between the test pattern and labelling progress, because there is an entire training set to estimate parameters when testing a model, but the labeling progress selects the $Q$-matrix with the best performance of the validation set, which is only a subset of the original training set. In summary, we can observe that the labeling progress of HBCA is efficient due to its good initialization and dimensional robustness.
the initialization of the $Q$-matrix uses the count for the parameter estimation. Second, DINA, because our student-dependent assumption only.

First, a better probabilistic model may improve ESVE-DINA, and it may be further investigated for initial-

ation data: The dina model, classification, latent class

Figure 5: Parameter sensitivity of HBCA.

7 Conclusion and Future Work

In this paper, we propose an ESVE algorithm to estimate student vectors of DINA without any probabilis-

In future work, there are still some further studies. First, a better probabilistic model may improve ESVE-DINA, because our student-dependent assumption only uses the count for the parameter estimation. Second, the initialization of the $Q$-matrix in HBCA is unrelated to DINA, and it may be further investigated for initializing the $Q$-matrix of all CDMs. Third, estimating the $Q$-matrix from the estimated student information may be a novel approach for other CDMs.

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