The mixed quark-gluon condensate from an effective quark-quark interaction

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Abstract

We exhibit the method for obtaining non perturbative quark and gluonic vacuum condensates from a model truncation of QCD. The truncation allows for a phenomenological description of the quark-quark interaction in a framework which maintains all global symmetries of QCD and allows an \( \frac{1}{N_C} \) expansion. Within this approach the functional integration over the gluon fields can be performed and therefore any gluonic vacuum observable can be expressed in terms of a quark operator and the gluon propagator. As a special case we calculate the mixed quark gluon condensate \( g_s \langle \bar{q} G_{\mu\nu} \sigma^{\mu\nu} q \rangle \). We investigate how the value depends on the form of the model quark-quark interaction. A comparison with the results of quenched lattice QCD, the instanton liquid model and QCD sum rules is drawn.

Key words: non perturbative methods in QCD, global color model, Dyson-Schwinger equation, \( \frac{1}{N_C} \) expansion.

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The existence of finite vacuum condensates such as the quark condensate \( \langle \bar{q}q \rangle \), the mixed quark gluon condensate \( g_s \langle \bar{q} G_{\mu\nu} \sigma^{\mu\nu} q \rangle \), the gluon condensate \( \alpha_s \langle G_{\mu\nu} G^{\mu\nu} \rangle \) or the four quark condensate \( \langle \bar{q} \Gamma qq \Gamma q \rangle \) reflects in a direct way the non perturbative structure of the QCD vacuum. Their determination within a certain approach provides therefore important information about its ability to describe sufficiently the physics of strong interaction at low and intermediate energies. One major field of application of these condensates are QCD sum rules [1–3], which are based on the operator product expansion (OPE). Hereby the vacuum condensates serve as the basic input parameters in which all the non perturbative effects are incorporated whereas all the short distance effects are treated within the standard perturbative diagram technique. The knowledge of the vacuum values at least of the lower dimensional quark and

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gluonic operators is therefore essential for the applicability of the QCD sum rule method.

In this paper we want to focus on the mixed condensate $g_s \langle \bar{q} G_{\mu\nu} \sigma^{\mu\nu} q \rangle$ of dimension five, which is among the least well determined ones. Previous studies of this condensate include QCD sum rules themselves where it was treated as fit parameter in the analysis of heavy-light quark system spectra [3,4], quenched lattice QCD [5] and, very recently, the instanton liquid model [6].

It is the aim of this letter to consider vacuum condensates in general and in particular the mixed condensate $g_s \langle \bar{q} G_{\mu\nu} \sigma^{\mu\nu} q \rangle$ in the framework of a truncation of QCD which is based on an effective quark-quark interaction. For this let us consider the QCD partition function for massless quarks in Euclidean space

$$Z_{\text{QCD}} \equiv \int Dq D\bar{q} D\bar{A} e^{-S_{\text{QCD}}[q, \bar{q}, A]}$$

(1)

with the Euclidean action

$$S_{\text{QCD}}[q, \bar{q}, A] = \int dx \left\{ \bar{q}(\not{\partial} - ig_s A) q + \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a \right\}$$

(2)

where

$$A_\mu = A_\mu^a \lambda^a, \quad G_{\mu\nu} = G_{\mu\nu}^a \lambda^a, \quad G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c. \quad (3)$$

Eq.(1) can be rewritten as

$$Z_{\text{QCD}} \equiv \int Dq D\bar{q} e^{-\bar{q} \not{\partial} q} e^{W[i g_s \bar{q} \gamma_\mu \frac{\lambda^a}{2} q]}$$

(4)

where

$$e^{W[j]} \equiv \int DA e^{\int \left( -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + j_\mu^a A_\mu^a \right)}.$$  

(5)

The functional $W[j]$ can be formally expanded in the current $j_\mu^a$ which leads to an expansion in terms of gluon $n$-point functions,

$$W[j] \equiv \frac{1}{2} \int dx_1 dx_2 j_{\mu_1}^{a_1}(x_1) D_{\mu_1 \mu_2}^{a_1 a_2}(x_1, x_2) j_{\mu_2}^{a_2}(x_2) + \frac{1}{3!} \int D_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3} j_{\mu_1}^{a_1} j_{\mu_2}^{a_2} j_{\mu_3}^{a_3} + \ldots . \quad (6)$$
The first nontrivial contribution arises from the gluon 2-point function

\[ D^{ab}_{\mu\nu}(x, y) = D^{ab}_{\mu\nu}(x - y) = \int \mathcal{D} A A^a_\mu(x) A^b_\nu(y) e^{-S_{\text{QCD}}[0, 0, A]} . \]  

(7)

It should be noted that in this definition of \( D_{\mu\nu} \) does not include quark loops.

Our model truncation consists in truncating the series (6) after \( n = 2 \), i.e. we include only the gluon 2-point function. This defines an effective model based on the bilocal quark-quark interaction kernel \( D^{ab}_{\mu\nu}(x - y) \). This model truncation respects all global symmetries of QCD. It has been used extensively in the literature and is known as the global color model (GCM) [7–11]. The main features of QCD, which are lost by this truncation are local SU(3) gauge invariance and renormalizability.

The partition function of this truncation is given by

\[ Z_{\text{GCM}} = \int \mathcal{D} q \mathcal{D} \bar{q} e^{-\frac{1}{2} \int \bar{q} [\gamma_\mu \gamma_\nu \frac{\delta}{\delta A^a_\mu} q(x)] D^{ab}_{\mu\nu}(x - y) [\bar{q}(y) \gamma_\nu \frac{\delta}{\delta A^b_\nu} q(y)]} \]  

(8)

or

\[ Z_{\text{GCM}} = \int \mathcal{D} q \mathcal{D} \bar{q} \mathcal{D} A e^{-\int \bar{q} [\gamma_\mu i g_\mu A^a_\mu] q + \int \frac{1}{2} A D^{-1} A} \]  

(9)

Eqs. (8) and (9) are connected by the functional integration rule

\[ e^{W[j]} = e^{\frac{1}{2} j D j} = \int \mathcal{D} A e^{-\frac{1}{2} A D^{-1} A + j A} \]  

(10)

where an obvious short hand notation for the integrations has been used.

The gluon 2-point function \( D^{ab}_{\mu\nu}(x - y) \) is treated as the model input parameter, which, as we will discuss later, is chosen to reproduce certain aspects of low energy hadronic physics. For convenience we will use the Feynman like gauge \( D_{\mu\nu}(x - y) = \delta_{\mu\nu} \delta^{ab} D(x - y) \) from now on.

The next step is to apply the standard bosonization procedure, which consists in rewriting the partition function in terms of bilocal meson-like integration variables and expanding about the the classical vacuum, i.e. the saddle point of the action [7,8,12].

The resulting expression for the partition function in terms of the bilocal field
integration is \( \mathcal{Z}_{\text{GCM}} = \int \mathcal{D}\mathcal{B} e^{-S[\mathcal{B}]} \) where the action is given by

\[
S[\mathcal{B}] = -\text{TrLn} \left[ S^{-1} \right] + \int dx dy \frac{\mathcal{B}^g(x, y) \mathcal{B}^g(y, x)}{2g_s^2 D(x - y)}
\]

(11)

and the quark inverse Green’s function, \( S^{-1} \), is defined as

\[
S^{-1}(x, y) \equiv \phi_x \delta(x - y) + \Lambda^{\theta} \mathcal{B}^g(x, y)
\]

(12)

The quantity \( \Lambda^{\theta} \) arises from Fierz reordering the current-current interaction in (8)

\[
\Lambda^{\theta}_{ji} \Lambda^{\theta}_{lk} = \left( \gamma_\mu \frac{\lambda^a}{2} \right)_{jk} \left( \gamma_\mu \frac{\lambda^a}{2} \right)_{li}
\]

(13)

It is the direct product of Dirac, flavor \( SU(3) \) and color matrices

\[
\Lambda^{\theta} = \frac{1}{2} \left( 1_D, i\gamma_5, \frac{i}{\sqrt{2}} \gamma_\mu, \frac{i}{\sqrt{2}} \gamma_\mu \gamma_5 \right)
\]

\[
\otimes \left( \frac{1}{\sqrt{3}} 1_F, \frac{1}{\sqrt{2}} \lambda^a_F \right)
\]

\[
\otimes \left( \frac{4}{3} 1_C, \frac{i}{\sqrt{3}} \lambda^a_C \right)
\]

(14)

The saddle-point of the action is defined as \( \frac{\delta S}{\delta \mathcal{B}} |_{\mathcal{B}_0} \) and is given by

\[
\mathcal{B}^g_0(x, y) = g_s^2 D(x - y) \text{tr} \left[ \Lambda^{\theta} S_0(x - y) \right]
\]

(15)

where \( S_0 \) stands for \( S[\mathcal{B}^g_0] \). These configurations provide self-energy dressing of the quarks with “rainbow” gluons through the definition \( \Sigma(p) \equiv \Lambda^{\theta} \mathcal{B}^g(p) = i\mathcal{P} [A(p^2) - 1] + B(p^2) \). The self energy functions \( A \) and \( B \) are determined by the “rainbow” Dyson-Schwinger equation

\[
\left[ A(p^2) - 1 \right] p^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} g_s^2 D(p - q) \frac{A(q^2)}{q^2 A^2(q^2) + B^2(q^2)}
\]

\[
B(p^2) = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} g_s^2 D(p - q) \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)}
\]

(16)

This dressing comprises the notion of “constituent” quarks by providing a mass \( M(p^2) = B(p^2)/A(p^2) \), reflecting a vacuum configuration with dynamically broken chiral symmetry.
We will calculate the vacuum condensates from the above saddle-point expansion, that is, we will work at the mean field level. This is consistent with the lowest order $1/N_C$ in the quark fields for a given model gluon 2-point function. It should be noted, however, that the gluon 2-point function $D$ itself contains all powers of $1/N_C$. 

The mesonic modes can be obtained within in the standard RPA technique by considering small fluctuations around the saddle point configuration and solving for the eigenmodes of the arising quadratic kernels. This leads then automatically to the ladder Bethe Salpeter equations for the $\bar{q}q$ bound states. Mesonic properties have been extensively studied in refs.[7,8]. In ref.[12] a detailed investigation of the low energy sector was performed by deriving the general form of the effective chiral action for the SU(3) Goldstone bosons and determining $f_\pi$ and most of the chiral low energy coefficients (Gasser-Leutwyler coefficients) $L_i$, which, in turn, determines the physics of the $\pi$, $K$ and $\eta$ mesons at low energies [13]. This was done for various forms of the model gluon 2-point function $D(q^2)$ all of them reproducing $f_\pi$. In the present approach we are following this spirit by choosing gluon 2-point functions which have this feature and moreover reproduce values for the chiral coefficient $L_i$, $i = 1, 2, 3, 5, 8$ which are compatible with the phenomenological ones [13].

It is now rather straightforward to calculate the vacuum expectation value of any quark operator of the form

$$Q_n \equiv \left(\bar{q}_{j_1} \Lambda_{j_1i_1}^{(1)} q_{i_1}\right) \left(\bar{q}_{j_2} \Lambda_{j_2i_2}^{(2)} q_{i_2}\right) \cdots \left(\bar{q}_{j_n} \Lambda_{j_ni_n}^{(n)} q_{i_n}\right)$$

(17)

in the mean field vacuum. Here the $\Lambda^{(i)}$ stands for an operator in Dirac, flavor or color space. This can be done by defining the functional

$$G[\eta, \bar{\eta}] \equiv \frac{\int \mathcal{D}q \mathcal{D}\bar{q} \, e^{-\sum_{ij} \bar{q}_i (S_0^{-1})_{ij} q_j + \sum_i \bar{\eta}_i q_i + \sum_i q_i \eta_i}}{\int \mathcal{D}q \mathcal{D}\bar{q} \, e^{-\sum_{ij} \bar{q}_i (S_0^{-1})_{ij} q_j}} \equiv \sum_{ij} \bar{\eta}_i (S_0)_{ij} \eta_j$$

(18)

taking the appropriate number of derivatives with respect to external sources $\eta_i$ and $\bar{\eta}_j$ and putting $\eta_i = 0$ and $\bar{\eta}_j = 0$ [14]. This gives

$$\langle :Q_n: \rangle = (-)^n \sum_{\Pi} (-)^\Pi \left\{ \Lambda_{j_1i_1}^{(1)} \cdots \Lambda_{j_ni_n}^{(n)} (S_0)_{i_1j_1(x(1))} \cdots (S_0)_{i_nj_n(x(n))} \right\}$$

(19)

where $\Pi$ stands for a permutation of the $n$ indices. In particular we obtain for
the quark condensate $\langle \bar{q}q \rangle$

$$\langle \bar{q}q \rangle_\mu = (-)tr_C \{ S_0(x, x) \}_{x=0} = (-) \left( \frac{N_C}{16\pi^2} \right) \left\{ 4 \int_0^\mu ds \frac{B(s)}{X(s)} \right\}$$  \hspace{1cm} (20)

where $X(s) \equiv sA(s)^2 + B(s)^2$. $\mu$ is the renormalization scale which we chose to be $1\text{GeV}^2$. As indicated the trace in eq.(20) is to be taken in Dirac and color space, whereas the flavor trace has been separated out.

Another important consequence from (19) is the fact that the four quark operators factorize in the mean field vacuum. For instance in case of $\Gamma^{(1)} = \Gamma^{(2)} = \gamma_\mu \frac{\lambda^a}{2}$, one finds from (19) and after reapplying the Fierz transformation (13)

$$\left\langle : \left( \bar{q} \gamma_\mu \frac{\lambda^a}{2} q \right) \left( \bar{q} \gamma_\mu \frac{\lambda^a}{2} q \right) : \right\rangle = (-)\frac{1}{4} \frac{16}{9} \langle \bar{q}q \rangle^2$$  \hspace{1cm} (21)

i.e. our approach is consistent with the vacuum saturation assumption of ref.[1]. The fact, that the four quark condensate factorizes due to eq.(21) is independent of the model gluon 2-point function which is used. That means that there are no four quark correlations at the mean field level, which is of course expected in the large $N_C$ limit.

Let us now turn to gluonic observables. Because the functional integration over the $A$ field in (9) is quadratic for a given quark-quark interaction $D$ we can perform the integration over any number of gluon fields analytically. We obtain as a generalization of (10) using the same short hand notation

$$\int \mathcal{D}A e^{-\frac{1}{2} \mathcal{A}^{-1} A + jA} \equiv e^{\frac{1}{2}jDj}$$

$$\int \mathcal{D}AA \ e^{-\frac{1}{2} \mathcal{A}^{-1} A + jA} \equiv (jD)(y_1) \ e^{\frac{1}{2}jDj}$$

$$\int \mathcal{D}AA^2 \ e^{-\frac{1}{2} \mathcal{A}^{-1} A + jA} \equiv \left[ D(y_1, y_2) + (jD)^2 \right] \ e^{\frac{1}{2}jDj}$$

$$\ldots$$  \hspace{1cm} (22)

This means that the gluon vacuum average renders effectively a quark color current $\bar{q} \gamma_\mu \frac{\lambda^a}{2} q$ together with the gluon 2-point function $D$. The integration over the quark operators is then performed in the mean field vacuum as described above. The remaining expression contains the gluon 2-point function $D$ as well as the quark propagator $S_0$, which, in turn is given by the self energy functions $A$ and $B$ through the Dyson-Schwinger equation (17). The integrations are most conveniently performed in momentum space. This way we can
in principle obtain the vacuum expectation value for any gluonic and combined quark-gluonic operator. It should be stated, however, that the number of terms produced by (22) will increase rapidly with the number of gluon fields. For instance for the gluon condensate $\langle G^2 \rangle$, which contains an $A^4$ integration the calculation gets already rather involved.

In case of the mixed condensate $g_s \langle \bar{q} G_{\mu\nu} \sigma^{\mu\nu} q \rangle$ we have to integrate only over powers $A^1$ and $A^2$ and therefore the procedure is still feasible. Applying the method described above we find

$$g_s \langle \bar{q}(x) G_{\mu\nu}(x) \sigma^{\mu\nu} q(x) \rangle = (-2i) N_C \int dz \left[ \partial^{(x)}_\mu g_s^2 D(z - x) \right] \text{tr}_\gamma [S_0(z, x) \sigma_{\mu\nu} S_0(x, z) \gamma_\nu] +$$

$$+ (4i) N_C \int dz_1 dz_2 g_s^2 D(z_1 - x) g_s^2 D(z_2 - x)$$

$$\cdot \text{tr}_\gamma [S_0(z_2, x) \sigma_{\mu\nu} S_0(x, z_1) \gamma_\mu S_0(z_1, z_2) \gamma_\nu] \right) \right). \quad (23)$$

In order to calculate this expression we can make explicit use of the Dyson-Schwinger equation (15) which determines the mean field vacuum configuration and which can be cast into the form

$$\frac{4}{3} g_s^2 D(x - y) S_0(x - y) = \frac{14}{3} \text{tr}_\gamma[\Sigma(x - y)] - \frac{14}{27} \gamma_\mu \text{tr}_\gamma[\Sigma(x - y) \gamma_\mu]. \quad (24)$$

This eliminates the integration over the gluon 2-point function in favor of the quark self energy and therefore strongly simplifies the evaluation of (23). We obtain as final result for the mixed condensate in Minkowski space

$$g_s \langle \bar{q} G_{\mu\nu} \sigma^{\mu\nu} q \rangle = (-) \left( \frac{N_C}{16\pi^2} \right) \left( \frac{27}{4} \int_0^\mu ds d\sigma B \frac{2A(A - 1)s + B^2}{X} \right) +$$

$$\left( - \right) \left( \frac{N_C}{16\pi^2} \right) \left( 12 \int_0^\mu ds d\sigma B \frac{(2 - A)}{X} \right). \quad (25)$$

In tables 1, 2 and 3 we display the result for $\langle \bar{q} q \rangle$ and $g_s \langle \bar{q} \sigma G q \rangle$ for three different model gluon 2-point functions, which are parametrized by a “width” parameter $\Delta$ and a “strength” parameter $\chi$ [12]. As stated above in all cases we fix the pion decay constant in the chiral limit to $f_\pi = 87\text{MeV}$ [13]. The renormalization point is in all cases $\mu = 1\text{GeV}^2$. We also show in all three cases the values obtained for the chiral low energy coefficients $L_i, i = 1, 3, 5, 8$. As we can see the values we obtain for the $L_i$ after fixing $f_\pi$ are compatible with the phenomenological ones [13]:

$L_1 = 0.7 \pm 0.5, L_3 = -3.6 \pm 1.3, L_5 = 1.4 \pm 0.5, L_8 = 0.9 \pm 0.3$. 

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Finally in table 4 we compare with the values which other nonperturbative approaches find for \( \langle \bar{q}q \rangle \) and \( g_s \langle \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q \rangle \): QCD sum rules [3], quenched lattice QCD [5] and the instanton liquid model [6]. As we can see our results for \( g_s \langle \bar{q}G_{\mu\nu}\sigma^{\mu\nu}q \rangle \) are compatible with the range obtained within these methods, whereas the quark condensate \( \langle \bar{q}q \rangle \) itself is systematically smaller than the “standard” value of \(- (220 \text{ MeV})^3\). It should be noted in this context that both the QCD sum rules value from ref.[3] and the quenched lattice values from ref.[5] refer to \( \mu = 1\text{ GeV}^2 \), whereas the instanton liquid model [6] has a renormalization point which is given by the instanton size and therefore somewhat smaller \( \approx (600 \text{ MeV})^2 \).

Table 1
\( \langle \bar{q}q \rangle \) and \( g_s \langle \bar{q}\sigma Gq \rangle \) at \( \mu = 1\text{ GeV}^2 \) for \( g_s^2 D(s) = (4\pi^2d) \frac{\chi^2}{s^2 \Delta} \), \( d = \frac{12}{27} \). In all cases \( f_\pi = 87\text{ MeV} \). Also displayed are the chiral low energy coefficients \( L_i, i = 1, 3, 5, 8 \).

| \( \Delta \) | \( \chi \) | \( -\langle \bar{q}q \rangle \) | \( -g_s \langle \bar{q}\sigma Gq \rangle \) | \( L_1 \) | \( L_3 \) | \( L_5 \) | \( L_8 \) |
|-------------|--------|-----------------|-----------------|---------|---------|---------|---------|
| [GeV^4]     | [GeV]  | [MeV^3]         | [MeV^5]         | *10^3   | *10^3   | *10^3   | *10^3   |
| 1 \times 10^{-1} | 1.77   | (183)^3         | (459)^5         | 0.79    | -3.76   | 2.38    | 1.03    |
| 1 \times 10^{-2} | 1.33   | (178)^3         | (457)^5         | 0.80    | -4.01   | 1.88    | 0.93    |
| 1 \times 10^{-4} | 0.95   | (175)^3         | (456)^5         | 0.81    | -4.23   | 1.48    | 0.88    |
| 1 \times 10^{-6} | 0.77   | (171)^3         | (452)^5         | 0.83    | -4.41   | 1.19    | 0.84    |

Table 2
The same as in tab.1 for \( g_s^2 D(s) = 3\pi^2 \frac{\chi^2}{s^2 \Delta} \) e\(^{-\frac{\chi}{s}}\).

| \( \Delta \) | \( \chi \) | \( -\langle \bar{q}q \rangle \) | \( -g_s \langle \bar{q}\sigma Gq \rangle \) | \( L_1 \) | \( L_3 \) | \( L_5 \) | \( L_8 \) |
|-------------|--------|-----------------|-----------------|---------|---------|---------|---------|
| [GeV^2]     | [GeV]  | [MeV^3]         | [MeV^5]         | *10^3   | *10^3   | *10^3   | *10^3   |
| 0.200       | 1.55   | (183)^3         | (448)^5         | 0.81    | -4.07   | 1.56    | 0.80    |
| 0.020       | 1.39   | (167)^3         | (431)^5         | 0.83    | -4.43   | 0.96    | 0.81    |
| 0.002       | 1.23   | (151)^3         | (395)^5         | 0.85    | -4.46   | 0.82    | 0.93    |

To summarize: We have considered a truncation of QCD which gives rise to an effective quark-quark interaction whose kernel is given by the model gluon 2-point function. This gluon 2-point function is chosen to reproduce \( f_\pi \) and it also reproduces the physics of the low energy mesonic sector. We have shown how to obtain vacuum matrix elements for any quark and gluon operator at the mean field level, which is consistent with the large \( N_C \) limit if the quark-quark interaction is fixed. In particular we have found that any gluon operator reduces to an interacting multi-quark operator, whose vacuum expectation value can then be calculated in the mean field vacuum. We have applied this
The same as in tab.1 for $g_2^2 D(s) = 3\pi^2 \frac{x^2}{\lambda} x^{-\lambda} + \frac{4\pi^2 d}{\ln(s/\Lambda^2 + \eta)}$, $d = \frac{12}{27}$, $\Lambda = 200\text{MeV}$.

| $\Delta$ [GeV$^2$] | $\chi$ [GeV] | $-\langle \bar{q}q \rangle$ [MeV$^3$] | $-g_s\langle \bar{q}\sigma G q \rangle$ [MeV$^5$] | $L_1$ | $L_3$ | $L_5$ | $L_8$ |
|---------------------|--------------|---------------------------------|---------------------------------|------|------|------|------|
| 0.200               | 1.65         | (173)$^3$                        | (458)$^5$                        | 0.81 | -4.03| 1.66 | 0.83 |
| 0.020               | 1.55         | (160)$^3$                        | (448)$^5$                        | 0.82 | -4.39| 1.13 | 0.84 |
| 0.002               | 1.45         | (151)$^3$                        | (432)$^5$                        | 0.85 | -4.43| 0.97 | 0.88 |

Technique to calculate the mixed quark-gluon condensate and found a value compatible with the ones of various other nonperturbative approaches.

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