Nucleon Decay
in the Minimal Supersymmetric
$SU(5)$ Grand Unification

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Abstract

We make a detailed analysis on the nucleon decay in the minimal supersymmetric $SU(5)$ grand unified model. We find that a requirement of the unification of three gauge coupling constants leads to a constraint on a mass $M_{HC}$ of color-triplet Higgs multiplet as $2 \times 10^{13} \text{ GeV} \leq M_{HC} \leq 2 \times 10^{17} \text{ GeV}$, taking both weak- and GUT-scale threshold effects into account. Contrary to the results in the previous analyses, the present experimental limits on the nucleon decay turn out to be consistent with the SUSY particles lighter than 1 TeV even without a cancellation between matrix elements contributed from different generations, if one adopts a relatively large value of $M_{HC}$ ($\geq 2 \times 10^{16} \text{ GeV}$). We also show that the Yukawa coupling constant of color-triplet Higgs multiplet does not necessarily blow up below the gravitational scale ($2.4 \times 10^{18} \text{ GeV}$) even with the largest possible value of $M_{HC}$. We point out that the no-scale model is still viable, though it is strongly constrained.
1 Introduction

The hierarchy problem has been the most serious problem in the grand unified theory (GUT) [1]. At present, the only feasible solution to this problem is to introduce the supersymmetry [2]. Furthermore, the supersymmetric (SUSY) SU(5) model [3] is now strongly supported phenomenologically by the $\sin^2 \theta_W$ measurement [4] made at the LEP experiments [5]. Once we regard the SUSY-GUT as a serious candidate of the physics beyond the standard model, a natural question is how we can test the model. The most striking consequence of the grand unification is the instability of nucleons. However, the nucleon decay via exchanges of $X$ and $Y$ gauge bosons is strongly suppressed as

$$\tau_{\text{n,p}}^{-1} \propto M_{\text{GUT}}^{-4}$$

in the SUSY-GUT because of the large unification scale

$$M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV}. \quad (1.1)$$

On the other hand, the nucleon decay via exchanges of color-triplet Higgs multiplet [6], which is suppressed only by $M_{\text{GUT}}^{-2}$, may still allow us to verify the model in the near future.

The main purpose of this paper is to study the implication of the present experimental limits on the nucleon decay in the minimal SUSY SU(5) GUT (MSGUT). Similar analyses have been carried out by Ellis, Nanopoulos, and Rudaz [7], and later by Arnowitt, Chamseddine, and Nath [8, 9] rather thoroughly. However, there has been no criterion given on how heavy the color-triplet Higgs multiplet can be. In this paper, we examine the experimental limits in the most conservative way, making the color-triplet Higgs multiplet as heavy as we can, allowed from the renormalization group (RG) analysis of the gauge coupling constant unification [10]. As a consequence, we find weaker constraints than those given in the previous analyses. The authors of Refs. [9] have claimed that the data of nucleon-decay experiments at that time are already stringent enough so that the SUSY particles below 1 TeV are excluded unless there is a delicate cancellation between the proton decay matrix elements from second- and third-generation contributions. On the contrary, we find that the present limits from the nucleon-decay experiments are still consistent with the SUSY particles below 1 TeV even without such
a cancellation. We also study a possible reach of the superKAMIOKANDE experiment. It will be shown that superKAMIOKANDE, together with LEP-II, is capable of covering most of the region with SUSY particles below 1 TeV, and hence it is highly expected to observe the nucleon decay at superKAMIOKANDE. It will be also stressed that more precise measurements on the gauge coupling constants, especially on that of QCD, will give a strong impact on the determination of the color-triplet Higgs mass $M_{H_C}$.

The paper is organized as follows. A brief review on the MSGUT is presented in Sect. 2, to summarize our conventions. We critically re-examine the analysis of Refs. [8, 9] in Sect. 3. We find that the coefficients of the dimension-five operators are larger than theirs by a factor of 2. The decay rates for various modes are presented. As pointed out in Ref. [9], there may occur a cancellation between second- and third-generation contributions. We present the partial lifetimes in terms of unknown parameters $y^{tK}$ or $y^{t\pi}$ which represent the ratios of the third- to the second-generation contributions. In Sect. 4, we give an upper bound on the mass of color-triplet Higgs multiplet, requiring that the gauge coupling constants are unified. Then the present experimental limits are examined in Sect. 5. There it is explicitly shown that the present data still allow for SUSY particles below 1 TeV, even without the cancellation between matrix elements mentioned above. The reach of the superKAMIOKANDE and the LEP-II experiments is discussed in Sect. 6. Sect. 7 is devoted to conclusions and discussions.

An analysis on the dimension-six operators is presented in Appendix A. Appendix B summarizes discussions on the renormalization effects on the dimension-five operators. We improve the analysis given in Ref. [1], but the difference turns out to be small. The chiral Lagrangian technique adopted to calculate the nucleon-decay matrix elements is described in Appendix C.

## 2 Minimal SUSY $SU(5)$ GUT

In this section, we review the minimal SUSY $SU(5)$ GUT (MSGUT) [11], summarizing our conventions. We also clarify the origin of new CP-violating phases in Yukawa
coupling constants of color-triplet Higgs multiplet to matter multiplets.

There are quite a few multiplets in the MSGUT. An adjoint Higgs multiplet \( \Sigma(24) \) breaks the \( SU(5) \) GUT group down to \( SU(3)_C \times SU(2)_L \times U(1)_Y \), and a pair of quintets \( H(5) \) and \( \overline{H}(5^*) \) contain doublet Higgs multiplets \( H_f, \overline{H}_f \) in the minimal SUSY standard model as well as their color-triplet partners \( H_C, \overline{H}_C \). The superpotential of this model is

\[
W = \frac{f}{3} \text{Tr} \Sigma^3 + \frac{1}{2} f V \text{Tr} \Sigma^2 + \lambda \overline{H}_C (\Sigma^\alpha_\beta + 3 V^\alpha_\beta) H^\beta \\
+ \frac{h^{ij}}{4} \varepsilon_{\alpha\beta\gamma\delta} \psi_i^\alpha \psi_j^\beta H^\gamma + \sqrt{2} f \psi_i^\alpha \phi_j^\alpha \overline{H}_C ,
\]

where the Latin indices \( i, j = 1, 2, 3 \) refer to families, and the Greek ones \( \alpha, \beta, \gamma \cdots \) represent the \( SU(5) \) indices. The chiral superfields \( \psi(10), \phi(5^*) \) are matter multiplets. Contents of the Higgs multiplets are

\[
\Sigma = \Sigma^a T^a \\
= \left( \begin{array}{cc} \Sigma_8 & \Sigma_{(3,2)} \\
\Sigma_{(3^*,2)} & \Sigma_3 \end{array} \right) + \frac{1}{2\sqrt{15}} \left( \begin{array}{cc} 2 & 0 \\
0 & -3 \end{array} \right) \Sigma_{24},
\]

\[
t_H = (H_C, H_C, H_C, H^+_f, H^0_f),
\]

\[
t_{\overline{H}} = (\overline{H}_C, \overline{H}_C, \overline{H}_C, \overline{H}^+_f, -\overline{H}^0_f),
\]

and those of the matter multiplets are

\[
\psi = \frac{1}{\sqrt{2}} \left( \begin{array}{cccccc} 0 & u^c & -u^c & u & d \\
-u^c & 0 & u^c & u & d \\
u^c & -u^c & 0 & u & d \\
-u & -u & -u & 0 & e^c \\
d & -d & -d & -e^c & 0 \end{array} \right),
\]

\[
t_\phi = (d^c, d^c, d^c, e, -\nu).
\]

where all the matter multiplets are written in terms of the chiral (left-handed) superfields. The chiral superfields \( u \) and \( d \) contain left-handed up-type and down-type quarks, \( u^c \) and \( d^c \) the charge conjugations of right-handed up-type and down-type quarks, \( e \) and
ν left-handed charged leptons and neutrinos, and \( e^c \) the charge conjugations of right
handed charged-leptons. In the following, \( ^tQ \equiv (u,d) \) and \( ^tL \equiv (\nu,e) \) will denote chiral
superfields of weak-doublet quarks and leptons, respectively.

The \( SU(5) \) GUT symmetry is broken by a vacuum expectation value of the \( \Sigma \) field,

\[
\langle \Sigma \rangle = V \begin{pmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & -3
\end{pmatrix},
\]

(2.6)
giving masses to \( X \) and \( Y \) gauge bosons

\[
M_V \equiv M_X = M_Y = 5\sqrt{2}g_5 V,
\]

(2.7)

where \( g_5 \) is the unified \( SU(5) \) gauge coupling constant. The invariant mass parameter
of \( H \) and \( \overline{H} \) is fine-tuned to realize masslessness of \( H_f \) and \( \overline{H}_f \), while it keeps their
color-triplet partners, \( H_C \) and \( \overline{H}_C \), superheavy as

\[
M_{H_C} = M_{\overline{H}_C} = 5\lambda V.
\]

(2.8)
The components \( \Sigma_8 \) and \( \Sigma_3 \) acquire the same mass

\[
M_\Sigma \equiv M_{\Sigma_8} = M_{\Sigma_3} = \frac{5}{2} f V,
\]

(2.9)
while the (physical) components \( \Sigma_{(3^*2)} \) and \( \Sigma_{(3,2)} \) form superheavy vector multiplets of
mass \( M_V \) together with the gauge multiplets. The mass of the singlet component \( \Sigma_{24} \) is
\( (1/2)fV \).

To analyze the dimension-five operators, we have to examine the Yukawa couplings
of \( H \) and \( \overline{H} \) to matter multiplets. An important question is how many independent
parameters we have in the Yukawa couplings \([12]\). The Yukawa coupling constants \( h^{ij} \)
and \( f^{ij} \) in Eq. (2.1) form a parameter space \( C^6 \times C^9 \), since \( h^{ij} \) is a symmetric matrix.
The freedom of field re-definition is \( U(3) \times U(3) \), corresponding to the choice of the basis
of $\psi_i$ and $\phi_i$. Thus the physical degrees of freedom of the Yukawa coupling constants is $(6 + 9) \times 2 - 9 \times 2 = 12 = 3 + 3 + 4 + 2$. First two 3’s stand for the eigenvalues for up- and down-type mass matrices, 4 for the Kobayashi-Maskawa matrix elements, and 2 for the additional phase degrees of freedom. We will parameterize the coupling matrices $h^{ij}$ and $f^{ij}$ as

$$h^{ij} = h^i e^{i\varphi_i} \delta^{ij}, \quad (2.10)$$

$$f^{ij} = V_{ij}^* f^j, \quad (2.11)$$

with $V_{ij}$ being the Kobayashi-Maskawa matrix. Only two of the phases $e^{i\varphi_i}$ are independent, and we can take

$$\varphi_u + \varphi_c + \varphi_t = 0. \quad (2.12)$$

In this parameterization, the corresponding bases of the matter multiplets are

$$\psi_i \ni tQ_i \equiv (u_i, d'_i) = (u_i, V_{ij} d_j), \quad (2.13)$$

$$\psi_i \ni e^{-i\varphi_i} u^c_i, \quad (2.14)$$

$$\psi_i \ni V_{ij} e^c_j, \quad (2.15)$$

$$\phi_i \ni d^c_i, \quad (2.16)$$

$$\phi_i \ni tL_i \equiv (\nu_i, e_i), \quad (2.17)$$

in terms of the mass eigenstates $u_i$, $d_i$, $u^c_i$, $d^c_i$, $\nu_i$, $e_i$, $e^c_i$. Then the Yukawa couplings of Higgs to matter multiplets are given by

$$W_Y = h^i Q_i u^c_i H_f + V_{ij}^* f^j Q_i d^c_j \overline{H}_f + f^i e^c_i L_i \overline{H}_f + \overline{H}_C \quad (2.18)$$

$$+ \frac{1}{2} h^i e^{i\varphi_i} Q_i Q_i H_C + V_{ij}^* f^j Q_i L_j \overline{H}_C$$

$$+ h^i V_{ij} u^c_i e^c_j H_C + e^{-i\varphi_i} V_{ij}^* f^j u^c_i d^c_j \overline{H}_C.$$\n
It should be clear from the above expression that the phases $e^{i\varphi_i}$ would be completely irrelevant if $H_C$ and $\overline{H}_C$ were absent. The phases appearing in the Yukawa couplings of $H_C$ and $\overline{H}_C$ cannot be absorbed by the field re-definition without affecting the couplings of $H_f$ and $\overline{H}_f$. As we will see later, these phases are important in the nucleon-decay
amplitudes induced by the $H_C$ and $\overline{H}_C$ exchanges. However, they are perfectly irrelevant to the nucleon decay caused by the $X$ and $Y$ gauge-boson exchanges.

3 Dimension-Five Operators and Decay Rates

In this section, we re-examine the previous analyses by Ellis, Nanopoulos, and Rudaz \cite{7}, and by Arnowitt, Chamseddine, and Nath \cite{8, 9}. Several corrections to the formula and numerical factors are made, and as a consequence nucleon-decay amplitude turns out to be smaller than their result by a factor of 2.

In the SUSY-GUT there are several baryon-number violating operators since it has many scalar bosons with color quantum numbers. Dimension-six operators induced by the $X$ and $Y$ gauge-boson exchanges are suppressed by $1/M_{GUT}^2$. These operators cause unacceptably large nucleon-decay rates in the minimal non-SUSY $SU(5)$ GUT \cite{14}, but there is no problem in the MSGUT since $M_{GUT}$ is much larger than in the non-SUSY case. In fact, we have analyzed the nucleon decays caused by the dimension-six operators, and found that they are always suppressed compared to those caused by the dimension-five operators. A brief discussion on the dimension-six operators is presented in Appendix A. The dimension-five operators are much more dangerous. These operators are generated by $H_C$ and $\overline{H}_C$ exchanges in the MSGUT, and is suppressed only by $1/M_{GUT}$ \cite{3}. The nucleon-decay amplitudes are obtained by dressing these operators by SUSY particle exchanges to convert scalar bosons to light fermions. Therefore, the nucleon-decay rates are sensitive to SUSY particle masses as well as the mass of $H_C$ and $\overline{H}_C$. It has been also noted that there may be baryon-number violating dimension-four operators \cite{13}. However, they can be forbidden by imposing the $R$-parity invariance, and we will not consider them in this paper.

Let us now discuss the dimension-five operators that cause the nucleon decay. A supergraph is presented in Fig. 1. The operators can be written explicitly as

$$W_5 = \frac{1}{2M_{H_C}} h^i e^{i\varphi_i} V_{kl}^* f^l (Q_i Q_i)(Q_k L_l) + \frac{1}{M_{H_C}} h^i V_{ij} e^{-i\varphi_j} V_{kl}^* f^l (u_i^c e_j^c)(u_k^cd_l^c), \quad (3.1)$$
where the contraction of the indices are understood as

\[(Q_i Q_i)(Q_k L_l) = \varepsilon_{\alpha \beta \gamma} (u^\alpha_i d^\beta_i - d^\alpha_i u^\beta_i) (u^\gamma_k e_l - d^\gamma_k u^\nu_l), \quad (3.2)\]

\[(u^c_i e^c_j)(u^c_k d^c_l) = \varepsilon_{\alpha \beta \gamma} u^c_{i \alpha e^c_j} u^c_{k \beta d^c_l}. \quad (3.3)\]

with $\alpha, \beta, \gamma$ being color indices. Note that the total anti-symmetry in the color index requires that the operators are flavor non-diagonal ($i \neq k$). Therefore dominant decay modes in the MSGUT generally involve strangeness, like $n, p \rightarrow K \bar{\nu}$ [15, 7].

The dimension-five operators will be converted to four-fermi operators at the SUSY breaking scale, by exchanges of gauginos or doublet Higgsino. However, the important contributions come only from the charged-wino dressing of the $Q_i Q_i (Q_k L_l)$ operators, and we will concentrate to this case.

The exchanges of gluino, neutral gaugino and neutral Higgsino are small in general [1], since they are flavor diagonal and hence suppressed by the Yukawa coupling constants of first and second generations appearing in the dimension-five operators. Though the gluino exchanges have stronger gauge coupling $\alpha_3$ than the wino exchanges, it will vanish completely in the limit where all the squark masses are degenerate [15]. Since the high degeneracy is required to suppress the unwanted flavor changing neutral current*, the gluino exchanges turn out to be always small. The charged-Higgsino exchanges are also suppressed due to their small Yukawa coupling constants to the first or second generations.† Thus the charged-wino exchanges give dominant contributions. Note that charged-wino dressing is impossible for the second operators $(u^c_i e^c_j)(u^c_k d^c_l)$, since all the fields involved are right-handed fields. Though the left-right mixing due to the $A$-terms and superpotential $|\partial W/\partial H|^2$ induces the wino dressing to the operators $(u^c_i e^c_j)(u^c_k d^c_l)$, their contribution is suppressed similarly to the charged-Higgsino exchanges since the

\[\begin{align*}
* & \text{A high degeneracy is required to suppress flavor-changing neutral currents, especially between first and second generations. See Ref. [16].} \\
† & \text{There are contributions from the third generation to the charged-Higgsino exchange amplitudes. However, the bottom-quark Yukawa coupling constant $f_b$ is smaller than the SU}(2) \text{ gauge coupling constant } g_2 \text{ unless } \tan \beta_H \text{ in Eq. [3.5] is extremely large. Therefore, the charged-Higgsino exchanges are most likely smaller than the charged-wino exchanges.}
\end{align*}\]
mixing is proportional to the Yukawa coupling constants. In the following we refer to the charged-wino simply as wino.

By dressing the dimension-five operators with the wino exchanges, we will get four-fermi operators. The results depend on the masses of charginos, squarks, and sleptons in the loops. There is a mixing between wino and charged Higgsino, with the mass matrix

$$M_{\text{chargino}} = \begin{pmatrix} m_{\tilde{w}} & \sqrt{2} m_W \cos \beta_H \\ \sqrt{2} m_W \sin \beta_H & \mu \end{pmatrix}. \tag{3.4}$$

Here, $m_{\tilde{w}}$ is a pure wino mass, $\mu$ a pure Higgsino mass, and $\beta_H$ a vacuum angle of doublet Higgs scalars, defined by

$$\tan \beta_H = \frac{\langle H^0_f \rangle}{\langle H^0_f \rangle}. \tag{3.5}$$

The interaction of squarks and sleptons with wino is fixed by

$$\mathcal{L} = g_2 (\tilde{u}_L^* \tilde{w}^+ d_L + \tilde{d}_L^* \tilde{w}^- u_L + \tilde{\nu}_L^* \tilde{w}^+ e_L + \tilde{e}_L^* \tilde{w}^- \nu_L) + \text{h.c.}, \tag{3.6}$$

giving the triangle diagram factor \[f\]

$$\frac{\alpha_2}{2\pi} f(u, d) \equiv g_2^2 \int \frac{d^4 k}{i(2\pi)^4} \left( \frac{1}{m_u^2 - k^2} \right) \left( \frac{1}{m_d^2 - k^2} \right) \left( \frac{1}{M_{\text{chargino}} - k} \right). \tag{3.7}$$

We have taken only the $(1, 1)$ component of the chargino propagator, since the nucleon-decay amplitudes are dominated by the pure wino component in the chargino states. Though the integral Eq. (3.7) depends on the mass eigenvalues and the mixing angles, we have found that it is well approximated by the pure wino exchange, and then it can be given by

$$\frac{\alpha_2}{2\pi} f(u, d) \equiv \frac{\alpha_2}{2\pi} m_{\tilde{w}} \frac{1}{m_u^2 - m_d^2} \left( \frac{m_u^2}{m_u^2 - m_{\tilde{w}}^2} \ln \frac{m_u^2}{m_{\tilde{w}}^2} - \frac{m_d^2}{m_d^2 - m_{\tilde{w}}^2} \ln \frac{m_d^2}{m_{\tilde{w}}^2} \right). \tag{3.8}$$

This approximation can be easily justified if $m_{\tilde{w}} \gg m_W$, since then the off-diagonal elements in the mass matrix Eq. (3.4) can be neglected. On the other hand, if $m_{\tilde{w}} \sim m_W$, the off-diagonal elements cannot be neglected in general. However, if $m_{\tilde{Q}}, m_{\tilde{L}} \gg m_{\tilde{w}}$, the triangle diagram factor $f$ in Eq. (3.7) is simply given by

$$f \sim \frac{(M_{\text{chargino}})_{11}}{m_{\tilde{Q}}^2} = \frac{m_{\tilde{w}}}{m_{\tilde{Q}}}, \tag{3.9}$$
and hence the above approximation is again justified. The region \( m_{\tilde{\omega}} \sim m_{\tilde{Q}} \sim m_{\tilde{L}} \sim \mu \) \( \sim m_W \) requires an exact treatment of the mixing, but this region turns out to be already excluded, and is irrelevant to our analyses.

Notice that the nucleon decay rates depend on the SUSY particle masses only through the function \( f \). It is useful to see the dependence of the function \( f \) on \( m_{\tilde{\omega}} \), \( m_{\tilde{Q}} \), and \( m_{\tilde{L}} \). In the limit \( m_{\tilde{\omega}} \ll m_{\tilde{Q}} \sim m_{\tilde{L}} \), \( f \) behaves as in Eq. (3.9). In the other limit \( m_{\tilde{\omega}} \gg m_{\tilde{Q}} \sim m_{\tilde{L}} \), it behaves as

\[
 f \simeq \frac{1}{m_{\tilde{\omega}}} \ln \frac{m_{\tilde{\omega}}^2}{m_{\tilde{Q}}^2}.
\]  

(3.10)

These behaviors will be used to put bounds on these masses in section 5.

The resulting four-fermi operators can be written down explicitly as

\[
 \mathcal{L} = \frac{1}{M_{\text{HC}}} \frac{\alpha_2}{2\pi} h^i e^{i\phi_i} V_{j k} f^k \varepsilon_{\alpha\beta\gamma} 
 \times \left[ (u^i \bar{d}^\alpha)(d^j \nu_k)(f(u_j, e_k) + f(u_i, d^i_j)) + (d^\alpha_i u^i) (u^i_j e_k)(f(u_i, d^i_j) + f(d^i_j, \nu_k)) 
 + (d^\alpha_i \nu_k)(d^\beta_j u^i_k)(f(u_i, e_k) + f(u_i, d^i_j)) + (u^i \bar{d}^\beta)(u^i_k e_k)(f(d^i_j, u_j) + f(d^i_j, \nu_k)) \right],
\]  

(3.11)

where the contraction of spinor indices are taken in each brackets \( () \). Here we have assumed that the mixing between squarks is negligible. This is true in most of the supergravity models (for example, see Ref. [17]) which ensure the absence of flavor-changing neutral current. Notice that Eq. (3.11) is larger than that given in Refs. [8, 9] by a factor of 2. We suspect that this difference arises from an inconsistency of the normalization of their Yukawa coupling constants (See Eqs. (1.5), (1.6) in Ref. [8]).

We have to include three kinds of renormalization effects to perform quantitative analyses. First, the Yukawa coupling constants appearing in the Eq. (3.11) are those evaluated at the GUT-scale, and we have to calculate their magnitudes using the low-energy quark masses. Second, the dimension-five operators receive anomalous dimensions due to the wave-function renormalizations of the external lines. Third, the four-fermi operators obtained after the gaugino-dressing will be further renormalized from the SUSY breaking scale down to 1 GeV. These three effects are first discussed by Ellis, Nanopou-
los, and Rudaz [7]. However, they dropped the $SU(2)$ gauge interactions in estimating the first renormalization effects, which led to an overestimation by 50%. They also neglected the contributions from the top-quark Yukawa coupling. Since the top-quark mass is heavier than 90 GeV, these contributions, which appear in the wave-function renormalization of Higgs doublets, enhance the dimension-five operators by $\sim 30\%$. However, corrections on these two points give roughly the same result as theirs. The details are explained in the Appendix B.

After taking the renormalization effects into account, the nucleon-decay operators at 1 GeV are given by

$$
\mathcal{L} = \frac{2\alpha_2^2}{M_{HC}} \frac{m_u m_d}{m_W^2 \sin 2\beta} \frac{e^{i\phi_c} V^*_{jk}}{A_S(i, j, k) A_L} \times \
$$

$$\varepsilon_{\alpha\beta\gamma} \left[ (u^c_i d^c_l)(d^c_j \nu_k)(f(u_j, e_k) + f(u_i, d'_i)) + (d^c_l u^c_i)(u^c_j e_k)(f(u_i, d_i) + f(d'_j, \nu_k)) + (d'^c_l u^c_i)(d'^c_j u^c_k)(f(u_i, e_k) + f(u_i, d'_j)) + (u^c_i d'^c_j)(u^c_k e_k)(f(d'_i, u_j) + f(d'_i, \nu_k)) \right],
$$

(3.12)

where $A_S(i, j, k)$ represents the short-range renormalization effect between GUT- and SUSY breaking scales depending on the flavor $i, j, k$, and $A_L$ the long-range renormalization effect between SUSY scale and 1 GeV. Here, the quark masses $m_u$ and $m_d$ are defined at 1 GeV in the $\overline{MS}$ scheme [4].

We first show the prediction of nucleon-decay rates for the dominant modes, $n, p \rightarrow K\bar{\nu}_\mu$. The main contributions in the four-fermi operators Eq. (3.12) come from the terms $(i = c, j = u, k = s)$ (proportional to $m_c m_s$), and $(i = t, j = u, k = s)$ (proportional to $m_t m_s$). The relevant terms are given by

$$
\mathcal{L} = \frac{2\alpha_2^2}{M_{HC}} \frac{m_s V^*_{us}}{m_W^2 \sin 2\beta} A_L \times \
$$

$$\varepsilon_{\alpha\beta\gamma} \left[ (d^c u^3)(s^c \nu_\mu) + (s^c u^3)(d^c \nu_\mu) \right] \times 
$$

$$A_S(c, u, s) m_c e^{i\phi_c} V_{cs} V_{cd}(f(c, \mu) + f(c, d')) \times 
$$

$$+ A_S(t, u, s) m_t e^{i\phi_t} V_{ts} V_{td}(f(t, \mu) + f(t, d')) \right].
$$

(3.13)

We have neglected the terms proportional to $m_u$. Though the terms coming from the $c$-exchange can be computed precisely, the contribution of $t$-exchange is ambiguous due
to the unknown Kobayashi-Maskawa matrix elements for top quark \cite{18}. In fact, the ratio of the \tilde{t}\-contribution relative to the \tilde{c}\-one \cite{8},

\[
y^{tK} \equiv \frac{\overline{m}_t e^{i\phi_t} V_{td} V_{td} A_S(t, u, s) (f(t, \mu) + f(t, d'))}{\overline{m}_c e^{i\phi_c} V_{cs} V_{cd} A_S(c, u, s) (f(c, \mu) + f(c, d'))}
\]

ranges between

\[
0.096 < |y^{tK}| < 1.3,
\]

if we take the triangle diagram factors \( f \) to be common, and the top-quark mass \( m_t \) to be 100 GeV.\footnote{Note that \( m_t \) in Eq. (3.14) is defined at the renormalization point 1 GeV. For example, \( m_t = 270 \) GeV for \( m_t = 100 \) GeV.}

Note that \( e^{i\phi_t} \) and \( e^{i\phi_c} \) in Eq. (3.14) are independent of each other. We cannot measure these phases from the present-day experiments, since they are irrelevant to any observables as far as \( H_C \) and \( \overline{H}_C \) are decoupled. Thus we are completely ignorant whether \( y^{tK} \) is constructive or destructive to the \( \tilde{c}\)-exchange amplitude. We should regard this complex parameter free in the present analyses.

If the modes \( n, p \rightarrow K\bar{\nu}_\mu \) have a cancellation between \( \tilde{c}\)- and \( \tilde{t}\)-exchange amplitudes, we have to study the other possible decay modes. Next-leading modes are \( n, p \rightarrow \pi \bar{\nu}_\mu \), suppressed by the Cabbibo angle \( \sin^2 \theta_C \) compared to the \( K\bar{\nu}_\mu \) modes. There are, similarly to the \( K\bar{\nu}_\mu \) modes, contributions from \( \tilde{c}\)- and \( \tilde{t}\)-exchange to these modes, and their ratio

\[
y^{t\pi} \equiv \frac{\overline{m}_t e^{i\phi_t} V^2_{td} A_S(t, u, s) (f(t, \mu) + f(t, d'))}{\overline{m}_c e^{i\phi_c} V^2_{cd} A_S(c, u, s) (f(c, \mu) + f(c, d'))},
\]

ranges as

\[
0.041 < |y^{t\pi}| < 1.7,
\]

with common \( f \) and \( m_t = 100 \) GeV. Though \( y^{tK} \) and \( y^{t\pi} \) are correlated, we also regard \( y^{t\pi} \) as an independent parameter because of too large uncertainties in the Kobayashi-Maskawa matrix elements

\[
\frac{|y^{t\pi}|}{|y^{tK}|} = \frac{|V_{td} V_{cs}|}{|V_{ts} V_{cd}|} = 0.22 - 2.94.
\]
We find that a perfect “double” cancellation in $K\bar{\nu}$ and $\pi\bar{\nu}$ modes is not possible, since $y_{tK}$ and $y_{t\pi}$ have a non-vanishing relative phase coming from the CP-violating one in the Kobayashi-Maskawa matrix. Thus, we will not consider the “double” cancellation further in this paper, and take $|1 + y_{t\pi}| = 1$ throughout. A “double” cancellation is logically possible only if the CP-violation is dominated by a non-KM mechanism. Even in the presence of such a “double” cancellation, there are decay modes which do not have such ambiguities (i.e., $p \rightarrow K^0 \mu^+, \eta^0 \mu^+, \pi^0 \mu^+$ and $n \rightarrow \pi^- \mu^+$). The operators containing charged leptons have only a contribution from the up-quark Yukawa coupling constant. Therefore, the decay rates do not suffer from the ambiguity of a possible cancellation. However, the decay rates are very small since the up-quark Yukawa coupling constant is tiny.

To obtain matrix elements at the hadronic level from the operators written in terms of the quarks fields, we adopt a chiral Lagrangian technique [19, 20]. Details on this method is shown in appendix C. We present the results on the partial lifetimes of nucleons in Tables 1–3, in terms of the parameters $eta, M_{HC}, A_S, \beta_H, y_{tK}, y_{t\pi}$, and the triangle factors $f$’s. A parameter $\beta$ is the hadron matrix element parameter used in the chiral Lagrangian technique [21, 22]

$$\beta u_L(k) \equiv \epsilon_{\alpha\beta\gamma}(0)(d_L^\alpha u_L^\beta u_L^\gamma |p, k), \quad (3.19)$$

which ranges as

$$\beta = (0.003 - 0.03) \text{GeV}^3, \quad (3.20)$$

depending on the methods of the theoretical estimation. Due to the uncertainty of an order of magnitude, the predictions of nucleon partial lifetimes receive an ambiguity of two orders of magnitude. A more precise determination of $\beta$ is strongly desired.

Table 1 summarizes predictions on the nucleon partial lifetimes for the dominant modes $n, p \rightarrow K\bar{\nu}$. One sees that the partial lifetimes of neutron are a little shorter than ones of proton. This is because the former has a larger chiral Lagrangian factor.
Table 2 presents next-dominant modes $n, p \to \pi \bar{\nu}$. These modes are suppressed by the Cabbibo-angle $\sin^2 \theta_C$. Ratio of the decay rates into $\bar{\nu}_\mu$ and $\bar{\nu}_e$ is simply the squared ratio of strange- and down-quark masses. The decay rates into charged lepton $\mu^+$ are listed in Table 3. These decay rates do not suffer from the ambiguity of a possible cancellation. However, the partial lifetimes are very long because of the small Yukawa coupling constant of up quark.

The dimension-five operators given by Eq. (3.12) are larger than the expressions given in Refs. [8, 9]. However, our final results shown in Tables 1–3 are smaller than the conclusion in Ref. [4] by a factor of 2 which may originate in an inconsistency between their analytic formula Eq. (2.3) and its numerical evaluations Eq. (2.7) in Ref. [8].

So far we have not discussed the decay rates of the modes containing $\bar{\nu}_\tau$. This is because we cannot make definite predictions for these decay modes since $V_{ub}^*$ has a large ambiguity. While the decay rates of $\bar{\nu}_\tau$ modes can be as large as ones of $\bar{\nu}_\mu$ mode if we take the largest possible $V_{ub}^*$ value, they can be also negligible with the smallest possible $V_{ub}^*$ value. In any case the decay rates can be only comparable to those into the $\bar{\nu}_\mu$, and hence the total decay rate into $\bar{\nu}$ is raised at most by a factor of two. This gives only a factor of $\sqrt{2}$ stronger constraint on the squark masses, and we will not include the $\bar{\nu}_\tau$ modes, hereafter. Once we know $V_{ub}^*$ more precisely, it is easy to incorporate the $\bar{\nu}_\tau$ modes into the present analyses.

Finally, we note that dimension-five operators depend sensitively on quark masses. We use the central values of current quark masses at 1 GeV which are estimated from the chiral perturbation theory and QCD sum rule in Ref. [23]. Quark masses of the first and second generations have large ambiguities. Especially, the strange-quark mass has a large error-bar (i.e., $m_s(1 \text{ GeV}) = 175 \pm 55 \text{ MeV}$, given in the $\overline{MS}$ scheme). In our analyses we use $m_s(1 \text{ GeV}) = 175 \text{ MeV}$. 

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4 Constraints on the GUT-scale Mass Spectrum

Since the nucleon-decay rates due to the dimension-five operators are proportional to the inverse square of the color-triplet Higgs multiplet mass $M_{H_C}^{-2}$, it is very important to determine it by some means. In the previous analyses [7, 8, 9], the authors have chosen $M_{H_C} = (1 - 2) \times 10^{16}$ GeV ad hoc. However, we have shown recently [10], that one can obtain limits on the GUT-scale mass spectrum in the MSGUT, just by requiring the unification of three gauge coupling constants. In particular, we have derived the upper bound on $M_{H_C}$ without any theoretical prejudice. A theoretical requirement that the Yukawa couplings remain perturbative below the gravitational scale $M_p/\sqrt{8\pi}$ (2.4 × $10^{18}$ GeV), poses further constraint on the GUT-scale mass spectrum.

We first discuss the renormalization-group (RG) evolution of three gauge coupling constants. It was shown in Refs. [24, 25] that the simple step-function approximation is accurate for supersymmetric theories, justified in the “supersymmetric regularization” $\overline{\text{DR}}$ scheme [26]. To illustrate how the GUT-scale spectrum receives constraints, we first discuss the one-loop RG equations. After that we include the two-loop corrections.

The running of three gauge coupling constants in the MSGUT can be obtained easily at the one-loop level as

$$a_3^{-1}(m_Z) = a_5^{-1}(\Lambda) + \frac{1}{2\pi} \left\{ \left( -2 - \frac{2}{3} N_g \right) \ln \frac{m_{SUSY}}{m_Z} + \left( -9 + 2N_g \right) \ln \frac{\Lambda}{m_Z} - 4 \ln \frac{\Lambda}{M_V} + 3 \ln \frac{\Lambda}{M_\Sigma} + \ln \frac{\Lambda}{M_{H_C}} \right\}. \quad (4.1)$$

$$a_2^{-1}(m_Z) = a_5^{-1}(\Lambda) + \frac{1}{2\pi} \left\{ \left( -\frac{4}{3} - \frac{2}{3} N_g - \frac{5}{6} \right) \ln \frac{m_{SUSY}}{m_Z} + \left( -6 + 2N_g + 1 \right) \ln \frac{\Lambda}{m_Z} - 6 \ln \frac{\Lambda}{M_V} + 2 \ln \frac{\Lambda}{M_\Sigma} \right\}. \quad (4.2)$$

$$a_1^{-1}(m_Z) = a_5^{-1}(\Lambda) + \frac{1}{2\pi} \left\{ \left( \frac{2}{3} N_g - \frac{1}{2} \right) \ln \frac{m_{SUSY}}{m_Z} + \left( 2N_g + \frac{3}{5} \right) \ln \frac{\Lambda}{m_Z} - 10 \ln \frac{\Lambda}{M_V} + \frac{2}{5} \ln \frac{\Lambda}{M_{H_C}} \right\}. \quad (4.3)$$

Here, the scale $\Lambda$ is supposed to be larger than any of the GUT-scale masses. The number of generations $N_g$ is three, and we have assumed a common mass $m_{SUSY}$ for all
the SUSY particles and for the scalar component of one of the Higgs doublets. A mass of the other doublet Higgs boson is taken at $m_Z$. By eliminating $\alpha_5^{-1}$ from the above equations, we obtain simple relations:

\[
(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_Z) = \frac{1}{2\pi} \left\{ \frac{12}{5} \ln \frac{M_{HC}}{m_Z} - 2 \ln \frac{m_{SUSY}}{m_Z} \right\}, \quad (4.4)
\]

\[
(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z) = \frac{1}{2\pi} \left\{ 12 \ln \frac{M_V^2 M_{\Sigma}}{m_Z^3} + 8 \ln \frac{m_{SUSY}}{m_Z} \right\}. \quad (4.5)
\]

The Eqs. (4.4), (4.5) imply that we can probe the GUT-scale mass spectrum from the weak-scale parameters (i.e., gauge coupling constants and mass spectrum of the SUSY particles). Especially, $M_{HC}$ is determined independently of $M_V$ and $M_{\Sigma}$. Eq. (4.5) determines a combination of the vector and adjoint-Higgs masses $(M_V^2 M_{\Sigma})^{1/3}$, and we will call it as “GUT-scale” $M_{GUT} = (M_V^2 M_{\Sigma})^{1/3}$, hereafter.

So far we have assumed a common mass $m_{SUSY}$ for the SUSY particles, but the mass splitting among the SUSY particles is also important to determine the GUT-scale mass spectrum. To avoid unnecessary complications, we restrict ourselves to the minimal supergravity model [17], where the SUSY-breaking mass parameters at the weak-scale can be determined from a small number of parameters at the Planck scale, by using the RG equations [28]. Therefore, the squark and the slepton masses are given by

\[
\begin{align*}
m_{\tilde{u}}^2 &= m^2 + 6.28M^2 + 0.35m_Z^2 \cos 2\beta_H, \\
m_{\tilde{d}}^2 &= m^2 + 6.28M^2 - 0.42m_Z^2 \cos 2\beta_H, \\
m_{\tilde{e}}^2 &= m^2 + 5.87M^2 + 0.16m_Z^2 \cos 2\beta_H, \\
m_{\tilde{e}^c}^2 &= m^2 + 5.82M^2 - 0.08m_Z^2 \cos 2\beta_H, \\
m_{\tilde{e}}^2 &= m^2 + 0.52M^2 - 0.27m_Z^2 \cos 2\beta_H,
\end{align*}
\]

\[\text{§}\]

It was claimed in Ref. [27] that the threshold corrections at the GUT-scale is so large that one cannot predict the SUSY-breaking scale even if measurements on $\alpha_3$ become much more precise. This high sensitivity on GUT-scale mass spectrum implies that one can probe it through precision measurements on the weak-scale parameters. Therefore, our result is consistent with their claim.

\[\text{¶}\]

This “GUT-scale” $M_{GUT}$ does not necessarily correspond to the scale where all three gauge coupling constants meet.
\[ m_\nu^2 = m^2 + 0.52M^2 + 0.50m_Z^2 \cos 2\beta_H, \]
\[ m_{\tilde{e}c}^2 = m^2 + 0.15M^2 - 0.23m_Z^2 \cos 2\beta_H, \]
in terms of the universal scalar mass \( m \) and the gaugino mass \( M \) at the GUT-scale. We have neglected the contributions from the Yukawa couplings to the renormalization of the particle masses. Also, the gaugino masses at the weak-scale are given by

\[ m_{\tilde{B}} = \frac{\alpha_1(m_Z)}{\alpha_5(M_{GUT})} M, \]
\[ m_{\tilde{\omega}} = \frac{\alpha_2(m_Z)}{\alpha_5(M_{GUT})} M, \]
\[ m_{\tilde{g}} = \frac{\alpha_3(m_Z)}{\alpha_5(M_{GUT})} M, \]

where \( m_{\tilde{B}} \) and \( m_{\tilde{g}} \) represent masses of bino and gluino, respectively.

The effect of the mass splitting can be taken into account by replacing \( \ln(m_{\text{SUSY}}/m_Z) \) in Eqs. (4.4), (4.5) as

\[-2 \ln \frac{m_{\text{SUSY}}}{m_Z} \rightarrow 4 \ln \frac{m_{\tilde{g}}}{m_{\tilde{\omega}}} + \frac{N_g}{5} \ln \frac{m_\tilde{\nu}^2}{m_Q^2 m_L^2} - \frac{8}{5} \ln \frac{m_{\tilde{h}}}{m_Z} - \frac{2}{5} \ln \frac{m_H}{m_Z} \]

in Eq. (4.4), and

\[ 8 \ln \frac{m_{\text{SUSY}}}{m_Z} \rightarrow 4 \ln \frac{m_{\tilde{g}}}{m_Z} + 4 \ln \frac{m_{\tilde{\omega}}}{m_Z} + N_g \ln \frac{m_Q^2}{m_\tilde{\nu}^2 m_{\tilde{e}c}} \]

in Eq. (4.3). Two doublet Higgs bosons are assumed to have masses at \( m_H \) and \( m_Z \), respectively. The symbol \( m_{\tilde{h}} \) represents a mass of doublet Higgsino. We have neglected the mixings among gauginos and doublet Higgsino.

For the time being we will restrict ourselves to the case where the universal scalar mass dominates the SUSY breaking (i.e., \( m \gg M \)). The terms \( \ln(m_\tilde{\nu}^2/m_Q^2 m_L^2) \) in Eq. (4.5) and \( \ln(m_Q^2/m_\tilde{\nu}^2 m_{\tilde{e}c}) \) in Eq. (4.3) are negligibly small. The term \( \ln(m_{\tilde{g}}/m_{\tilde{\omega}}) \) stays constant, since \( m_{\tilde{g}}/m_{\tilde{\omega}} = \alpha_3/\alpha_2 \approx 3.5 \). The dependence on \( m_H \) in Eq. (4.8) is weak due to its small coefficient, and we set \( m_H = 1 \) TeV. Therefore, we find that \( M_{Hc} \) depends mainly on the Higgsino mass \( m_{\tilde{h}} \), and \( M_{GUT} \) on the product of gaugino masses
We have also examined the constraint on $M_{HC}$ in the no-scale model \cite{29} (i.e., $m = 0$), and found that the difference is negligible.

Now we are at the stage to derive the GUT-scale mass spectrum from the above RG analysis. In our numerical calculation, we use the one-loop RG equations for the weak- and the GUT-scale thresholds, and the two-loop ones between these two distant scales. The two-loop RG equations in the minimal SUSY standard model are \cite{24}

\[
\mu \frac{\partial g_i}{\partial \mu} = \frac{1}{16\pi^2} b_i g_i^3 + \frac{1}{(16\pi^2)^2} \sum_{j=1}^{3} b_{ij} g_j^2 g_i^3 \tag{4.10}
\]

where $i, j = 1, 2, 3$, and

\[
b_i = \begin{pmatrix}
0 \\
-6 \\
-9
\end{pmatrix} + \begin{pmatrix}
2 \\
2 \\
0
\end{pmatrix} N_g + \begin{pmatrix}
\frac{3}{10} \\
\frac{1}{2} \\
0
\end{pmatrix} N_{H_f}, \tag{4.11}
\]

\[
b_{ij} = \begin{pmatrix}
0 & 0 & 0 \\
0 & -24 & 0 \\
0 & 0 & -54
\end{pmatrix} + \begin{pmatrix}
\frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\
\frac{2}{5} & 14 & 8 \\
\frac{11}{15} & 3 & \frac{68}{3}
\end{pmatrix} N_g + \begin{pmatrix}
\frac{7}{10} & \frac{9}{10} & 0 \\
\frac{2}{10} & \frac{7}{2} & 0 \\
0 & 0 & 0
\end{pmatrix} N_{H_f}. \tag{4.12}
\]

Here, $N_{H_f}$ is the number of doublet Higgs multiplets ($N_{H_f} = 2$). The threshold corrections at the two-loop level are expected to be small, since their mass splittings only within the same order of magnitude do not produce large logarithms. As the input parameters, we use the $\overline{MS}$ gauge coupling constants at the $Z$-pole given in Ref. \cite{30}, $\alpha = 127.9 \pm 0.2$, $\sin^2 \theta_W = 0.2326 \pm 0.0008$, and $\alpha_3 = 0.118 \pm 0.007$. However, the use of the simple step-function approximation is only justified in the $\overline{DR}$-scheme. Since we employ the simple step-function approximation, we have to convert these coupling constants at the $Z$-pole into the $\overline{DR}$-scheme by

\[
\frac{1}{\alpha_i^{DR}} = \frac{1}{\alpha_i^{MS}} - \frac{C_i}{12\pi}, \tag{4.13}
\]

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Combining all the above discussions, we find that $M_{H^C}$ is constrained to the range

$$2.2 \times 10^{13} \text{ GeV} \leq M_{H^C} \leq 2.3 \times 10^{17} \text{ GeV},$$

(4.14)

and the “GUT-scale” is tightly constrained as

$$0.95 \times 10^{16} \text{ GeV} \leq (M_1^2 M_2)_{1/3} \leq 3.3 \times 10^{16} \text{ GeV},$$

(4.15)

for $100 \text{ GeV} \leq m_{\tilde{g}} \leq 1 \text{ TeV}$. The allowed range of $M_{H^C}$ is much less constrained, because of the small gauge-group representation for the Higgs multiplets. The large ambiguity of $M_{H^C}$ comes mainly from the uncertainty in the strong coupling constant $\alpha_3$, and the prediction on the nucleon decay will be drastically improved if the uncertainty diminishes. In Fig. 2 we present the GUT-scale spectrum derived from the present gauge coupling constants, and also that expected if the error-bar of $\alpha_3$ is reduced by a factor of 2 with the same central value. The importance of more precise measurements on $\alpha_3$ should be clear from the figure.

When one uses these constraints, Eq. (4.14) and Eq. (4.15), one needs to pay two attentions. First, we have taken only one standard deviation for gauge coupling constants. If we allow two standard deviations, allowed region of $M_{H^C}$ spreads to both ends by two orders of magnitude, losing any practical limits. On the other hand, the “GUT-scale” is still constrained tightly. Therefore, we restrict our analyses to only one standard deviation as in Eqs. (4.14), (4.15). Second, we have used the RG equations at the two-loop level. One may be concerned for whether three-loop corrections are important. The difference in $M_{H^C}$ between the one-loop and the two-loop results is a factor of 30, which is a very small factor compared to the large ratio $M_{GUT}/m_Z$ appearing in the

---

The bound on $M_{GUT}$ quoted in Ref. [3], $0.90 \times 10^{16} \text{ GeV} \leq M_{GUT} \leq 3.1 \times 10^{16} \text{ GeV}$, includes a minor mistake, and should be replaced by Eq. (4.15).

**In other words, it is still possible to raise $M_{H^C}$ up to near the gravitational scale if we allow two standard deviations. However, we tentatively take this one-sigma bound seriously to perform further analyses. It should be noted that nucleon decay can be generated at observable rates for reasonable range of parameters, even with the color-triplet Higgs of mass at the gravitational scale.**
solutions of the RG equations. Thus we expect that the three-loop corrections are much less than $O(1)$.

Although we have concentrated to the RG analysis on the gauge coupling constants to determine the GUT-scale mass spectrum, we will obtain further constraint on the mass spectrum from the RG analysis on Yukawa coupling constants. As shown in Eq. (2.8), $M_{H_C}$ is given by a Yukawa coupling constant $\lambda$ between $H_C$, $H_C$ and $\Sigma$, which is not known. On the other hand, the mass $M_V$ is determined by the $SU(5)$ gauge coupling constant $g_5$, whose strength is known by the RG analysis. A large mass splitting $M_V \ll M_{H_C}$ requires that the $\lambda$ is very large compared to $g_5$. Thus the applicability of the perturbation theory restricts the mass splitting to be not large. The same argument applies to the mass $M_\Sigma$, which originates in a self-coupling of the adjoint-Higgs as seen in Eq. (2.9).

A constraint arises by requiring that those Yukawa coupling constants do not blow-up below the gravitational scale, $M_P/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV. The running of the Yukawa coupling constants in Eq. (2.1) are described by the RG equations,

$$\mu \frac{\partial \lambda}{\partial \mu} = \frac{1}{(4\pi)^2} \left(-\frac{98}{5} g_5^2 + \frac{53}{10} \lambda^2 + \frac{21}{40} f^2 + 3 (h^t)^2 \right) \lambda, \quad (4.16)$$

$$\mu \frac{\partial f}{\partial \mu} = \frac{1}{(4\pi)^2} \left(-30 g_5^2 + \frac{3}{2} \lambda^2 + \frac{63}{40} f^2 \right) f, \quad (4.17)$$

$$\mu \frac{\partial h^t}{\partial \mu} = \frac{1}{(4\pi)^2} \left(-\frac{96}{5} g_5^2 + \frac{12}{5} \lambda^2 + 6 (h^t)^2 \right) h^t, \quad (4.18)$$

$$\mu \frac{\partial g_5}{\partial \mu} = -\frac{3}{(4\pi)^2} g_5^3, \quad (4.19)$$

where $h^t$ is the Yukawa coupling constant to top quark. The conservative limit on $\lambda$ can be obtained in the case $f = h^t = 0$. A numerical study shows that the mass $M_{H_C}$ is limited from above,

$$M_{H_C} = \frac{\lambda}{\sqrt{2g_5}} M_V < 2.0 M_V. \quad (4.20)$$

A similar limit on $M_\Sigma$ can be obtained with $\lambda = h^t = 0$,

$$M_\Sigma = \frac{f}{2 \sqrt{2g_5}} M_V < 1.8 M_V. \quad (4.21)$$
One may feel uneasy about the assumption that there is no new physics between the GUT-scale and the gravitational scale. We have examined the above analysis again requiring that the Yukawa coupling constants do not blow-up below $10^{17}$ GeV. This requirement relaxes the constraint Eqs. (4.20), (4.21) at most by a factor of 2. Therefore, in the following calculation we use Eqs. (4.20), (4.21).

We obtain constraints on $M_V$ and $M_\Sigma$ separately, combining the discussions above. The limits Eq. (4.15) and Eq. (4.21) give

$$M_V > 0.78 \times 10^{16} \text{ GeV},$$

$$M_\Sigma < 4.9 \times 10^{16} \text{ GeV}. $$

Eq. (4.22) will be used to put limits on the dimension-six operators in the following sections.

5 Present Limits on Dimension-Five Operators

In this section we combine the predictions obtained in the previous sections with the results of the nucleon-decay experiments to see the present status of the MSGUT. We find that the present lower bounds on the nucleon partial lifetimes are still consistent with the SUSY particles below 1 TeV if one adopts a relatively large value of $M_{H_C} (\geq 2 \times 10^{16} \text{ GeV})$. In Table 4, we have listed the experimental lower limits on the partial lifetimes of nucleon [18].

The most dominant decay mode by dimension-five operators is $n \rightarrow K^0 \bar{\nu}_\mu$, as shown in Table 1. This mode dominates slightly over the similar decay mode, $p \rightarrow K^+ \bar{\nu}_\mu$, because of the chiral Lagrangian factor. These decay modes, however, have an ambiguity in the parameter $y^{tK}$, which is the relative ratio of the $\tilde{t}$-exchange to the $\tilde{c}$-exchange contributions. Furthermore, the parameter $y^{tK}$ contains an unknown phase factor $e^{i(\phi_c - \phi_t)}$, which cannot be determined from any low-energy experiments. In fact, Arnowitt, Chamseddine, and Nath [8] have shown a possible cancellation between the second- and third-generation contributions. However, if the combination $|1 + y^{tK}|$ de-
creases, the modes \( n, p \to K\bar{\nu}_\mu \) cease to be dominant. Then the experimental limit on the other modes \( n, p \to \pi\bar{\nu}_\mu \) become important. This interchange occurs at

\[
|1 + y^{tK}| = 0.40, \quad (5.1)
\]

when \( |1 + y^{t\pi}| = 1 \). The parameters \( y^{tK} \) and \( y^{t\pi} \) are correlated as clear from their definitions (Eqs. (3.14), (3.16)). However, we find it impossible that \( |1 + y^{tK}| \) and \( |1 + y^{t\pi}| \) are canceled out simultaneously, as explained in section 4. We take \( |1 + y^{t\pi}| = 1 \) throughout.

In Fig. 3, we show the lower bound on \( M_{H_C} \) derived from the present nucleon-decay experiments by varying \( |1 + y^{tK}| \). We choose other parameters such that the nucleon lifetimes become as long as possible (i.e., \( m_{\tilde{Q}} = m_{\tilde{L}} = 1 \) TeV, \( m_{\tilde{w}} = 45 \) GeV, and \( \tan \beta_H = 1 \)). In this figure, the upper horizontal line corresponds to the maximum value \( (M_{H_C} = 2.3 \times 10^{17} \) GeV) in Eq. (4.14). There are two curves representing the lower limit on \( M_{H_C} \) obtained from the experimental limits on the nucleon lifetimes. The upper curve corresponds to the case of the hadron matrix element \( \beta = 0.03 \) GeV\(^3\), and the lower curve to the case of \( \beta = 0.003 \) GeV\(^3\). The smaller hadron matrix element \( \beta \) gives weaker constraint as expected. Thus, the conservative lower bound on \( M_{H_C} \) from the nucleon-decay experiments is

\[
M_{H_C} \geq 5.3 \times 10^{15} \text{ GeV}. \quad (5.2)
\]

We illustrate how the lower bounds on \( M_{H_C} \) depend on the SUSY breaking parameters, the wino mass \( m_{\tilde{w}} \) and the squark mass \( m_{\tilde{Q}} \) in Fig. 4, assuming \( m_{\tilde{L}} \simeq m_{\tilde{Q}} \). In this figure the dashed line shows the dependence on \( m_{\tilde{w}} \) when we choose the most conservative set of parameters, \( m_{\tilde{Q}} = m_{\tilde{L}} = 1 \) TeV, \( \tan \beta_H = 1 \), \( |1 + y^{tK}| < 0.4, A_S = 0.67 \), and \( \beta = 0.003 \) GeV\(^3\). The lower bound on \( M_{H_C} \) rises linearly on \( m_{\tilde{w}} \) in the region \( m_{\tilde{w}} < 1 \) TeV. However, the lower bound decreases as \( m_{\tilde{w}}^{-1} \) beyond 1 TeV, and we do not obtain an upper bound on \( m_{\tilde{w}} \) with this conservative choice of parameters. The

††The situation does not change even if we allow mass splittings between sleptons and squarks (say, \( m_L \ll m_{\tilde{Q}} \)), since the denominator in Eq. (3.9) is dominated by the heavier mass, \( m_{\tilde{Q}} \).
dash-dotted line shows the dependence on $m_{\tilde Q}$ again for the most conservative case, $m_{\tilde w} = 45$ GeV, $\tan \beta_H = 1$, $|1 + y^{tK}| < 0.4$, $A_S = 0.67$, and $\beta = 0.003$ GeV$^3$. The lower bound on $M_{H_C}$ goes down as $1/m_{\tilde Q}^2$, leading to the lower bound on $m_{\tilde Q}$.

$$m_{\tilde Q} \geq 150 \text{ GeV.} \quad (5.3)$$

We also show the dependence on $\tan \beta_H$ in Fig. 5. We have fixed $m_{\tilde Q} = m_{\tilde L} = 1$ TeV, $m_{\tilde w} = 45$ GeV, $\beta = 0.003$ GeV$^3$, and $|1 + y^{tK}| = 1.0$ or $|1 + y^{tK}| < 0.4$. The lower bound on $M_{H_C}$ is proportional to

$$\frac{1}{\sin 2\beta_H} = \frac{1}{2} \left( \frac{1}{\tan \beta_H} + \tan \beta_H \right) . \quad (5.4)$$

When $\tan \beta_H \gg 1$, the dependence is almost linear. We find a constraint on $\tan \beta_H$,

$$\tan \beta_H \leq 85, \quad (5.5)$$

which is, however, much weaker than $\tan \beta_H < 40$ obtained from the requirement that the Yukawa coupling constant $f^b$ for bottom quark remains in the perturbative regime below the “GUT-scale”.‡‡

Taking $M_{H_C}$ as heavy as possible given in Eq. (4.14), we obtain limits on the masses $m_{\tilde w}$ and $m_{\tilde Q}$. The allowed region is shown in Fig. 6. Here we have taken $|1 + y^{tK}| = 1$ and $m_{\tilde L} \sim m_{\tilde Q}$. The present experimental limits on the wino and the squark masses from direct-search experiments at LEP [32] and CDF [33] are shown for comparison. Since the decay rate behaves like $(m_{\tilde w}/m_{\tilde Q}^2)^2$ in the region $m_{\tilde w} \ll m_{\tilde Q}$, the lower bound on $m_{\tilde Q}$ behaves like $m_{\tilde w}^{1/2}$. In the other extreme, $m_{\tilde w} \gg m_{\tilde Q}$, the decay rate goes like $1/m_{\tilde w}^2$, and around $m_{\tilde w} \sim 10^5$ GeV the constraint on $m_{\tilde Q}$ from the nucleon-decay experiments becomes weaker than that from the CDF experiments. We see that the “natural” mass region $\lesssim 1$ TeV for the SUSY particles still survives the nucleon-decay experiments.

Though the authors of Ref. [1] claimed that the present limits on the nucleon decay are stringent enough to exclude the SUSY particles lighter than 1 TeV in the absence

‡‡The $\tan \beta_H$ has to be larger than 0.5, since otherwise the top-quark Yukawa coupling constant will blow up below the “GUT-scale” with $m_t \geq 90$ GeV.
of a delicate cancellation between matrix elements of the dimension-five operators (i.e., \( \beta \simeq 0.003 \text{ GeV}^3 \), \(|1 + y^{tK}| \simeq 0.2\)), we see now that there is a wide allowed range. This is mainly because we use the maximum value of \( M_{H_C} (2.3 \times 10^{17} \text{ GeV}) \) given from the RG analysis while they chose \( M_{H_C} \simeq M_{GUT} \) just by hand (i.e., \( 2.0 \times 10^{16} \text{ GeV} \)).

We show similar limits from the \( \pi \bar{\nu}_\mu \) mode in Fig. 7. As discussed above, these decay modes become dominant if \(|1+y^{tK}| < 0.4\). The limit is weaker compared to the previous case without the cancellation as shown in Fig. 6.

We have taken the largest value of \( M_{H_C} \) in Eq. (4.14) in Figs. 6,7. This value requires \( M_V \) larger than \( 1 \times 10^{17} \text{ GeV} \) in order to satisfy the requirement Eq. (4.20), which leads to \( M_\Sigma \) smaller than \( 3 \times 10^{15} \text{ GeV} \). Though this case needs a mass splitting among the heavy particles, it is still within two orders of magnitude, and it is completely acceptable phenomenologically.

In the minimal supergravity model, the SUSY particle masses are determined mainly by the universal scalar mass and the gaugino mass. In models where the universal scalar mass dominates the SUSY breaking parameters, squarks and sleptons are almost degenerate, that are larger than wino. Therefore, models of this type are preferred. In the no-scale model the SUSY breaking is dominated by the gaugino mass \([29]\), which results in a definite prediction of the SUSY particle masses \( (m_{\tilde{Q}} \sim m_{\tilde{g}} \simeq 3m_{\tilde{\omega}} \sim 3m_{\tilde{L}}) \) \([35]\). Since all the mass parameters in the triangle factor \( f \) defined in Eq. (3.8) are proportional to \( m_{\tilde{\omega}} \), \( f \) behaves as \( m_{\tilde{\omega}}^{-1} \). This enables us to derive the lower bound on \( m_{\tilde{\omega}} \) from the nucleon-decay experiments. We have found that lower limit on \( m_{\tilde{\omega}} \) is 70 GeV (equivalently, \( m_{\tilde{Q}} > 210 \text{ GeV} \)), in the most conservative case (i.e., \( \beta = 0.003 \text{ GeV}^3 \), \( M_{H_C} = 2.3 \times 10^{17} \text{ GeV} \), \( \tan \beta_H = 1 \), and \(|1+y^{tK}| = 0.4\)). Thus, we conclude that the no-scale model is still surviving.\(^*\)

Finally, we briefly discuss the nucleon decay caused by the dimension-six operators, which come from the \( X \) and \( Y \) gauge-boson exchanges. We show the decay rates by the

\(^*\)In the absence of the cancellation (i.e., \(|1+y^{tK}| = 1\)), the corresponding limit is \( m_{\tilde{\omega}} > 180 \text{ GeV} \).

\(^\dagger\)The wino mass in this region is consistent with the limit \( m_{\tilde{\omega}} < 300 \text{ GeV} \) \([35]\) in the radiative breaking scenario of the electroweak symmetry in the no-scale model.
dimension-six operators in Appendix A. We find that dimension five operators always dominate dimension-six operators, in agreement with the naive expectations. The ratio \( (R_\tau) \) of the partial lifetime of the decay \( n \to K^0\bar{\nu}_\mu \) via the dimension-five operators to that of the decay \( p \to \pi^0e^+ \) via the dimension-six operators is

\[
R_\tau \equiv \frac{\tau(n \to K^0\bar{\nu}_\mu)}{\tau(p \to \pi^0e^+)} \leq 2.8 \times 10^{-4} \times \left( \frac{M_{H_C}}{M_V} \right)^4 \left( \frac{10^{16} \text{ GeV}}{M_{H_C}} \right)^2,
\]

where all parameters besides \( M_{H_C} \) were chosen so that \( R_\tau \) becomes as large as possible. The ratio of \( M_{H_C} \) to \( M_V \) is smaller than 2.0 from Eq. (4.20), and \( R_\tau \) is always smaller than \( \frac{1}{60} \). The MSGUT predicts that the dimension-five operators should be observed earlier than dimension-six operators.

### 6 Future Tests on the Minimal SUSY \( SU(5) \) GUT

Now we examine how stringent constraint on MSGUT we can obtain from the nucleon-decay experiments in the near future. The superKAMIOOKANDE experiment will push up the lower bound of the nucleon lifetime by a factor of 30. Meanwhile, the LEP-II experiment will be able to find wino below \( m_Z \). Since the larger \( m_{\tilde{\ell}} \) means faster nucleon decay as we discussed in previous sections, the combination of these two experiments can put more stringent limits on the dimension-five operators.

In Figs. 8 and 9, we show the expected limits on the dimension-five operators if superKAMIOOKANDE does not observe the \( K\bar{\nu}_\mu \) and \( \pi\bar{\nu}_\mu \) decay modes, and if LEP-II does not discover wino below \( m_Z \). For the \( K\bar{\nu}_\mu \) decay modes, superKAMIOOKANDE and LEP-II will be able to exclude most of the region with the SUSY particles below 1 TeV, even with the smallest hadron matrix element, \( \beta = 0.003 \text{ GeV}^3 \). In the case where the \( \pi\bar{\nu}_\mu \) mode is dominant, the region below 1 TeV is almost closed leaving a little window. Thus one can see that the LEP-II and the superKAMIOOKANDE experiments are extremely important for testing the MSGUT.
One may be concerned for a little region below 1 TeV which may be left by these experiments. This window is open because of the large maximum value of $M_{H_C}$ obtained from the gauge coupling unification. If the error-bar of $\alpha_3$ is reduced by a factor of 2 with the same central value in the future, the maximum value of $M_{H_C}$ becomes $6.1 \times 10^{16}$ GeV, and one will be able to close the window completely. In Fig. 10 we demonstrate the case where the error-bar of $\alpha_3$ is reduced by a factor of 2. If one wishes to test the MSGUT completely, it is quite important to reduce the error-bar of $\alpha_3$.

At the end of this section, we see whether the nucleon decay via the dimension-six operators can be observed in the near future. From Eq. (4.22) and Eq. (A.2), the theoretical lower limit of the partial lifetime of the decay $p \to \pi^0 e^+$ is obtained as

$$\tau(p \to \pi^0 e^+) \geq 4.1 \times 10^{33} \left( \frac{0.03 \text{ GeV}^3}{\alpha} \right)^2 \text{ years},$$

in terms of the hadron matrix element $\alpha$ ($=0.003$–0.03 GeV$^3$, see Appendix A). Since superKAMIOKANDE is expected to reach up to $\tau(p \to \pi^0 e^+) \simeq 10^{34}$ years [36], there is a possibility to observe the nucleon decay via the dimension-six operators in the MSGUT.

7 Conclusions and Discussions

We have analyzed the nucleon decay in the minimal SUSY $SU(5)$ GUT (MSGUT) in details. First, we have studied the GUT-scale particle spectrum using the RG analysis, and found a maximum value of $M_{H_C}$ to be $2.3 \times 10^{17}$ GeV. Then, we have studied the nucleon partial lifetimes with the largest possible $M_{H_C}$. We have found that the present nucleon-decay experiments are still consistent with the MSGUT, even without a cancellation between the matrix elements from exchanges of squarks in different generations. We have emphasized the important role of precise measurement of the gauge coupling constants, especially the QCD coupling constant $\alpha_3$ to determine the mass $M_{H_C}$. We have also stressed that the combined information of the lower bound on the chargino mass (LEP-II) and on the nucleon lifetimes (superKAMIOKANDE), will give
a stringent constraint on the MSGUT.

It deserves to mention that the nucleon decay prefers supergravity models where the SUSY breaking parameters are dominated by scalar masses, rather than by gaugino masses. For example, the no-scale model, where the SUSY breaking parameters come only from the gaugino masses, is strongly constrained from the nucleon-decay experiments, though it is still viable. Increasing the limits on the gaugino masses will have strong impact on the MSGUT phenomenology.

It is important to see whether predictions on the dimension-five operators become drastically altered by modifying the MSGUT. In the MSGUT, there is a prediction of mass relation, \( m_b = m_\tau, m_s = m_\mu, \) and \( m_d = m_e, \) at the GUT-scale. It is known that, though \( m_b = m_\tau \) is consistent with observations if top quark is not too heavy [37], \( m_s = m_\mu \) and \( m_d = m_e \) are not. Therefore, the modification of the Yukawa coupling structure is needed. Georgi and Jarlskog proposed that \( m_b = m_\tau, 3m_s = m_\mu, \) and \( m_d = \frac{1}{3}m_e \) at the GUT-scale, introducing a 45 dimensional Higgs scalar [38]. Also, Kim and Özer proposed to make an “effective” 45 dimensional Higgs scalar by using higher dimension operators [39]. These modifications may produce more uncertainties in the Yukawa couplings of \( \mathcal{H}_C \) and \( \mathcal{H}_C \) to matter multiplets. However, we have checked that these models receive at most a few times stronger constraint, and hence the main conclusion in the present analyses does not change qualitatively.

It has been argued that the introduction of a Peccei-Quinn symmetry may be able to eliminate the dimension-five operators [3]. However, the requirement of the coupling constant unification does not allow us to introduce new multiplets of arbitrary masses. We have shown in a separate paper [40] that a suppression of the dimension-five operators by introducing the Peccei-Quinn symmetry cannot be stronger than in the MSGUT.

Another interesting result of our RG analysis on the GUT-scale spectrum is that one can raise the grand unification scale up to the gravitational scale, by lowering the \( M_\Sigma \) down to the intermediate scale \( (10^{11-12} \text{ GeV}) \). In this case the heavy vector multiplet as well as \( H_C \) and \( \overline{H}_C \) lie at the gravitational scale, if one allow for two standard deviations in Eq. (4.14). These light \( \Sigma_8 \) and \( \Sigma_3 \) require an extremely small Yukawa coupling \( f \Sigma^3 \),
leading to a very flat potential of $\Sigma$. The possibility that the three gauge coupling constants meet at the gravitational scale has come out with a seemingly accidental parameter tuning. However, this may suggest a completely different underlying physics (non-GUT) like the superstring theory [41].

**Acknowledgement**

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**Note added**

After completing this work, we have received preprints by R. Arnowitt and P. Nath [42]. In these works, they also derive a constraint on $M_{HC}$, requiring the Yukawa coupling constants to remain in the perturbative regime. The difference between their constraint $M_{HC} < 3M_V$ and our Eq. (4.20) is due to different requirements. Namely, they impose that the Yukawa coupling constant $\lambda$ should not blow up below $2M_{HC}$, while we impose that below the gravitational scale. However, they are not aware of the possibility to raise $M_V$ by lowering $M_C$, and hence obtain a smaller upper bound on $M_{HC}$. This leads to the opposite conclusion on the no-scale model, where they claim it to be excluded while we have found it still viable.

**Appendix A  Decay Rates via Dimension-six Operators**

In this appendix we analyze the nucleon-decay rates via dimension-six operators. The dimension-six operators are caused by exchanges of $X$ and $Y$ gauge-bosons or color-triplet Higgs scalars. The color-triplet Higgs scalars interact with matter only by small Yukawa coupling constants, and also its mass should be larger than $5 \times 10^{15}$ GeV (see
Eq. (5.2)). Thus the dimension-six operators induced by the color-triplet Higgs scalar exchanges are negligible.

The dimension-six operators via the $X$ and $Y$ gauge-boson exchanges are dominated by generation-diagonal decay modes, and the decay modes containing strangeness are suppressed by the Cabibbo angle $\sin \theta_C$. We discuss here only the dominant decay mode $p \rightarrow \pi^0 e^+$. The amplitude by the $X$ and $Y$ gauge-boson exchanges is

$$\mathcal{L}_{X,Y} = A_R e^{i\phi_u} \frac{g_5^2}{M_V^2} \epsilon_{\alpha\beta\gamma} \left( (d_{R}^{\alpha} u_{R}^{\beta})(u_{L}^{\gamma} e_{L}) + (1 + |V_{ud}|^2)(d_{L}^{\alpha} u_{L}^{\beta})(u_{R}^{\gamma} e_{R}) \right).$$  \hspace{0.5cm} (A.1)

The renormalization factor $A_R$ calculated by the authors in Ref. [43] is $A_R = 3.6$. Since the decay rate of this mode caused by the dimension-five operator is extremely suppressed, the observation of this decay mode would suggest the presence of the $X$ and $Y$ gauge-bosons exchanges. We calculate the decay rate using the chiral Lagrangian technique, as

$$\tau(p \rightarrow \pi^0 e^+) = 1.1 \times 10^{36} \times \left( \frac{M_V}{10^{16} \text{ GeV}} \right)^4 \left( \frac{0.003 \text{ GeV}^3}{\alpha} \right)^2 \text{ years.}$$  \hspace{0.5cm} (A.2)

Here, $\alpha$ is the hadron matrix element

$$\alpha u_L(\vec{k}) \equiv \epsilon_{\alpha\beta\gamma} \langle 0 | (d_{R}^{\alpha} u_{R}^{\beta})(u_{L}^{\gamma} e_{L}) | p, \vec{k} \rangle,$$  \hspace{0.5cm} (A.3)

whose absolute value is the same as $\beta$ (i.e., $|\alpha| = |\beta|$) [21]. It is straightforward to compare it with the nucleon-decay rates via the dimension-five operators. The result is given in Eq. (5.6).

**Appendix B  Renormalization Factors**

In this appendix, we present several formulae necessary to compute the renormalization factors which have appeared in the section 3. There are three kinds of renormalization effects. First, we have to derive the Yukawa coupling constants of $H_C$ and $\bar{H}_C$ from the observed quark masses. Second, the dimension-five operators derived at the GUT-scale receive anomalous dimensions from the wave-function renormalizations of the external
fields. Third, the four-fermi operators dressed at the SUSY breaking scale should be
renormalized down to 1 GeV. In the text the short-range renormalization factor between
the SUSY breaking and the GUT-scales is denoted as $A_S$, and the long-range renormal-
ization factor between 1 GeV and the SUSY breaking scale as $A_L$. Though $A_S$ and $A_L$
are estimated by the authors in Ref. [4], we need minor corrections to their calculation
of $A_S$. We demonstrate the derivation of $A_S$, and also comment on $A_L$.

Since the Yukawa coupling constants in Eq. (3.4) are those given at the GUT-scale,
we have to calculate the values of the Yukawa coupling constants by solving the RG
equations from the SUSY breaking scale up to the GUT-scale. This was done in the
Ref. [4]. Since the Yukawa couplings $H_f$ and $\overline{H}_f$ are $F$-terms, what we have to compute
is only the wave-function renormalizations of each chiral superfields thanks to the non-
renormalization theorem of the $F$-terms [34]. The authors of Ref. [4] have given the
following general formula for the running of arbitrary $F$-terms. Any $F$-terms can be re-written in the following form,

$$W = \frac{1}{3!} h^{klm} \phi_k \phi_l \phi_m,$$  (B.1)

where $\phi$’s are chiral superfields, and the coupling constant $h^{klm}$ are supposed to be
completely symmetric under the interchange of the indices $k, l, m$. If the fields $\phi_k$’s
belong to the representation $R^k$ of certain gauge group $i$, the coupling constants $h^{klm}$
follow the RG equation

$$\frac{dh^{klm}}{d \ln \mu} = \frac{1}{(4\pi)^2} \left[ \theta_{k'}^{k} h^{k'l'm} + \theta_{l'}^{l} h^{kl'm} + \theta_{m'}^{m} h^{klm'} \right],$$  (B.2)

where the constants $\theta$’s are given by

$$\theta_{k'}^{k} = -2 C_2^i (R^k) g_i \delta_{k'}^{k} + \frac{1}{2!} h^{kpq} h_{k'pq}.$$  (B.3)

The gauge coupling constant is denoted by $g_i$, and the second Casimir of the representation $R^k$ by $C_2^i (R^k)$. If the coupling constant $h^{klm}$ can be neglected in the expression
of $\theta$, the RG equation can be easily integrated to the form

$$h^{klm}(\mu) = h^{klm}(\mu_0) \prod_{r=k,l,m} \prod_{i=1,2,3} \left( \frac{\alpha_i(\mu_0)}{\alpha_i(\mu)} \right)^{C_2^i(R^r)/b_i}.$$  (B.4)
Here, the coefficient of the $\beta$-function $b_i$ is defined as

$$\frac{d\alpha_i^{-1}}{d \ln \mu} = -\frac{b_i}{2\pi}. \quad (B.5)$$

The explicit forms of $b_i$'s in the minimal SUSY standard model are given in Eq. (4.11). Thus, in our case, the Yukawa coupling constants of the lower generations will be renormalized from the SUSY breaking to the GUT-scales by

\[
\begin{align*}
    h^u(M_{GUT}) &= h^u(m_Z) \left( \frac{\alpha_1(m_Z)}{\alpha_5(M_{GUT})} \right)^{13/198} \left( \frac{\alpha_2(m_Z)}{\alpha_5(M_{GUT})} \right)^{3/2} \left( \frac{\alpha_3(m_Z)}{\alpha_5(M_{GUT})} \right)^{-8/9}, \\
    f^d(M_{GUT}) &= f^d(m_Z) \left( \frac{\alpha_1(m_Z)}{\alpha_5(M_{GUT})} \right)^{7/198} \left( \frac{\alpha_2(m_Z)}{\alpha_5(M_{GUT})} \right)^{3/2} \left( \frac{\alpha_3(m_Z)}{\alpha_5(M_{GUT})} \right)^{-8/9}, \quad (B.6) \\
    f^e(M_{GUT}) &= f^e(m_Z) \left( \frac{\alpha_1(m_Z)}{\alpha_5(M_{GUT})} \right)^{3/22} \left( \frac{\alpha_2(m_Z)}{\alpha_5(M_{GUT})} \right)^{3/2}.
\end{align*}
\]

We have assumed the SUSY breaking scale to be $m_Z$. Here, $h^u$, $f^d$, $f^e$ are the Yukawa coupling constants of $H_f$ and $\overline{H}_f$ to up-, down-type quark and charged-lepton multiplets. Note that the expressions differ from those given in Ref. [7]. The most important difference from the previous analysis is that the formula in Ref. [7] does not have the factors of $\alpha_2$, and hence they overestimated the Yukawa coupling constants at the GUT-scale.

Another ingredient which has not been included in Ref. [7] is the large top-quark Yukawa coupling constant. Its effect appears in two points. First, the large Yukawa coupling constant contributes to the wave-function renormalization of the Higgs multiplet. If top quark becomes heavy, its large Yukawa coupling constant cannot be neglected in the RG equation of the Yukawa coupling constants of first and second generations. Its effect is to enhance the Yukawa coupling constants at the GUT-scale, and hence enhancing the dimension-five operators. Second, the top-quark Yukawa coupling constant

\(^\dagger\)The authors in Ref. [7] seem to have considered the renormalization of the mass operators rather than the Yukawa coupling constants. Since what determines the coefficient of the dimension-five operators is the Yukawa coupling constants themselves rather than the mass parameters induced by the $SU(2) \times U(1)$ breaking, it is clear that one should consider the running of the Yukawa coupling constants up to the GUT-scale.
itself become quite large at the GUT-scale, due to the wave-function renormalization of the top-quark multiplet. This further enhances the dimension-five operators for the third-generation contribution. Since the RG equation cannot be solved analytically if one includes the top-quark Yukawa coupling constant, we can only present the numerical results.

From the Yukawa coupling constants at the GUT-scale, we obtain the coefficient of the dimension-five operators in Eq. (B.1). The dimension-five operators receive the anomalous dimensions, which we have to estimate to know their coefficients at the SUSY breaking scale. Here again, since the dimension-five operators are $F$-terms, all the renormalization effects come from the wave-function renormalizations of the external lines. We obtain the same result as that given in Ref. [7]. The $QQQL$ operators including only first- and second-generation fields in Eq. (B.1) receive an enhancement factor

$$\left(\frac{\alpha_1(m_Z)}{\alpha_5(M_{GUT})}\right)^{-1/33} \left(\frac{\alpha_2(m_Z)}{\alpha_5(M_{GUT})}\right)^{-3} \left(\frac{\alpha_3(m_Z)}{\alpha_5(M_{GUT})}\right)^{4/3}. \quad (B.7)$$

The enhancement factor of the operators including the third-generation fields is a little reduced by the effect of the top-quark Yukawa coupling.

We show the numerical values of the coefficient $A_S(i,j,k)$, the short-range renormalization effect between the GUT- and the SUSY breaking scales. First, if the contribution of the top-quark Yukawa coupling constant is neglected, this value becomes

$$A_S = \left(\frac{\alpha_1(m_Z)}{\alpha_5(M_{GUT})}\right)^{7/99} \left(\frac{\alpha_3(m_Z)}{\alpha_5(M_{GUT})}\right)^{-4/9} = 0.59. \quad (B.8)$$

Thus our short range renormalization factor $A_S$ is smaller than that in Ref. [7] by $\frac{2}{3}$ (their value is 0.91). Next, we show the numerical values of the coefficient $A_S$ in Fig. 11 for varying $m_t/\sqrt{2}\sin \beta_H$. Since $A_S$ depend on the top-quark Yukawa coupling constant rather than $m_t$, only a combination $m_t/\sqrt{2}\sin \beta_H$ is relevant. The solid line represents $A_S$ for the dimension-five operators only with first- and second-generation fields, and the dash-dotted line for the operator $(Q_tQ_t)(Q_cL_\mu)$. The lower horizontal line is $A_S$ with
the top-quark contribution neglected. One can see that the top-quark Yukawa coupling
enhances the dimension-five operators.

The factor $A_L$ in Eq. (3.12) is the long-range renormalization factor due to the QCD
interaction between the SUSY scale and 1 GeV scale, and contain the renormalization
of Yukawa coupling constants and the anomalous dimension of four fermi-operators. It
is given in Ref. [7],

$$A_L = \left( \frac{\alpha_3(1 \text{ GeV})}{\alpha_3(m_c)} \right)^{-2/3}\left( \frac{\alpha_3(m_c)}{\alpha_3(m_b)} \right)^{-18/25}\left( \frac{\alpha_3(m_b)}{\alpha_3(m_Z)} \right)^{-18/23}$$

(B.9)

Combining all the renormalization effects, the four-fermi operators can be written down
as in Eq. (3.12).

**Appendix C  Chiral Lagrangian Technique**

In this appendix we present a chiral Lagrangian technique for translating operators at
the quark level to those at the hadron level. This technique has been developed in the
Refs. [19, 20]

The baryon number violating four-fermi operators derived from the dimension-five
operators are

$$O(duu\nu_i) = \epsilon_{\alpha\beta\gamma}(d_\alpha^\beta u_L^\gamma)(d_i^\gamma \nu_{iL}),$$
$$O(dude_i) = \epsilon_{\alpha\beta\gamma}(d_\alpha^\beta u_L^\gamma)(u_L^\gamma e_{iL}),$$
$$O(sud\nu_i) = \epsilon_{\alpha\beta\gamma}(s_\alpha^\beta u_L^\gamma)(d_i^\gamma \nu_{iL}),$$
$$O(dus\nu_i) = \epsilon_{\alpha\beta\gamma}(d_\alpha^\beta u_L^\gamma)(s_i^\gamma \nu_{iL}),$$
$$O(suu\epsilon_i) = \epsilon_{\alpha\beta\gamma}(s_\alpha^\beta u_L^\gamma)(u_L^\gamma e_{iL}),$$

where quark and lepton fields are in mass eigenstates, and $i$ denotes the generation
indices. There are also baryon number violating four-fermi operators derived from the
dimension-six operators, and we will concentrate only on the operators relevant for the
decay mode \( p \to \pi^0 e^+ \),

\[
\tilde{O}^{(1)} = \epsilon_{\alpha\beta\gamma} (d^\alpha_R u^\beta_R)(u^\gamma_L e_L), \\
\tilde{O}^{(2)} = \epsilon_{\alpha\beta\gamma} (d^\alpha_L u^\beta_L)(u^\gamma_R e_R).
\]  

(C.2)

The effective Lagrangian \( \mathcal{L}^q \) at the quark level for each decay modes can be written as

\[
\mathcal{L}^q(n,p \to \pi(\eta)\bar{\nu}_i) = C(duu\nu_i)O(duu\nu_i), \\
\mathcal{L}^q(n,p \to \pi(\eta)e^+_i) = C(dude_i)O(dude_i), \\
\mathcal{L}^q(n,p \to K\bar{\nu}_i) = C(sud\nu_i)O(sud\nu_i) + C(dus\nu_i)O(dus\nu_i), \\
\mathcal{L}^q(n,p \to Ke^+_i) = C(suue_i)O(suue_i), \\
\mathcal{L}^q(n,p \to \pi e^+_i) = \tilde{C}^{(1)}\tilde{O}^{(1)} + \tilde{C}^{(2)}\tilde{O}^{(2)}. 
\]  

(C.3)

The coefficients \( C \)'s are derived from Eq. (3.12), and \( \tilde{C} \)'s from Eq. (A.1),

\[
C(duu\nu_i) = \frac{4\alpha^2}{M_{HC}} \frac{m_{c\bar{c}}m_{d\bar{d}}e^{i\phi_c} V_{ud}^* V_{cd}^2 A_L A_S(c,u,d_i)}{m_{W}^2 \sin 2\beta_H} (1 + y^\pi)(f(u, d) + f(u, e)), \\
C(dude_i) = \frac{2\alpha^2}{M_{HC}} \frac{m_{u\bar{u}}m_{d\bar{d}}e^{i\phi_u} V_{ud}^* V_{cd}^2 A_L A_S(u,c,d_i)}{m_{W}^2 \sin 2\beta_H} (f(u, d) + f(d, \nu)), \\
C(sud\nu_i) = \frac{2\alpha^2}{M_{HC}} \frac{m_{c\bar{c}}m_{d\bar{d}}e^{i\phi_c} V_{ud} V_{cd} V_{CE}^* A_L A_S(c,u,d_i)}{m_{W}^2 \sin 2\beta_H} (1 + y^K)(f(u, d) + f(u, e)), \\
C(suue_i) = \frac{2\alpha^2}{M_{HC}} \frac{m_{u\bar{u}}m_{d\bar{d}}e^{i\phi_u} V_{ud} V_{cd} V_{CE}^* A_L A_S(u,c,d_i)}{m_{W}^2 \sin 2\beta_H} (f(u, d) + f(d, \nu)), \\
\tilde{C}^{(1)} = \frac{e^{i\phi_u} g^2_R A_R}{M_{\tilde{\nu}^*}}, \\
\tilde{C}^{(2)} = \frac{e^{i\phi_u} g^2_R A_R}{M_{\tilde{\nu}^*}} (1 + |V_{ud}|^2). 
\]  

(C.4)

We need to translate these effective Lagrangians at the quark level \( \mathcal{L}^q \) to the operators written in terms of the baryon and meson fields \( \mathcal{L}^h \).

Let us review the chiral Lagrangian for baryons and mesons. The Nambu-Goldstone bosons \( \Phi \) associated with the spontaneous breaking of chiral \( SU(3)_L \times SU(3)_R \) symmetry can be written by

\[
U = \exp \left( \frac{2i\Phi}{f_\pi} \right) 
\]  

(C.5)

33
where $f_\pi$ is the pion decay constant, and

$$
\Phi = \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & \pi^+ & K^+ \\
\pi^- & -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^0 \\
K^- & -\sqrt{\frac{1}{2}} \pi^0 - \sqrt{\frac{1}{6}} \eta & -\sqrt{\frac{1}{3}} \eta
\end{pmatrix}.
$$

(C.6)

Similarly, the baryon fields can be written in the matrix form,

$$
B = \begin{pmatrix}
\sqrt{\frac{1}{2}} \Sigma^0 + \sqrt{\frac{1}{6}} \Lambda & \Sigma^+ & \rho^+ \\
\Sigma^- & -\sqrt{\frac{1}{2}} \Sigma^0 + \sqrt{\frac{1}{6}} \Lambda & n^0 \\
\Xi^- & \Xi^0 & -\sqrt{\frac{1}{3}} \Lambda
\end{pmatrix}.
$$

(C.7)

Now the most general $SU(3)_L \times SU(3)_R$ invariant Lagrangian for strong interactions of mesons and baryons is

$$
\mathcal{L}_0 = \frac{1}{8} f_\pi^2 \text{Tr}(\partial U)(\partial U^\dagger) + \text{Tr} \bar{B}(i \not\partial - M_{\text{inv}}) B \\
+ \frac{1}{2} i \text{Tr} \bar{B} \gamma^\mu \left[ \zeta (\partial_\mu \zeta^\dagger) + \zeta^\dagger (\partial_\mu \zeta) \right] B \\
+ \frac{1}{2} i \text{Tr} \bar{B} \gamma^\mu B \left[ (\partial_\mu \zeta) \zeta^\dagger + (\partial_\mu \zeta^\dagger) \zeta \right] \\
- \frac{1}{2} i (D - F) \text{Tr} \bar{B} \gamma^\mu \gamma_5 B \left[ (\partial_\mu \zeta) \zeta^\dagger -(\partial_\mu \zeta^\dagger) \zeta \right] \\
+ \frac{1}{2} i (D + F) \text{Tr} \bar{B} \gamma^\mu \gamma_5 \left[ \zeta (\partial_\mu \zeta^\dagger) - \zeta^\dagger (\partial_\mu \zeta) \right] B,
$$

(C.8)

where

$$
\zeta = \exp \left[ \frac{i M}{f_\pi} \right].
$$

(C.9)

Since the (current) quark masses that break the chiral symmetry are small for up, down, and strange quarks, we can use $\mathcal{L}_0$ to estimate the lifetimes of nucleon.

Now we translate the effective Lagrangians containing quark fields $\mathcal{L}^q$ to the ones containing baryons and mesons $\mathcal{L}^h$, by comparing the transformation properties under the $SU(3)_L \times SU(3)_R$ symmetry. The transformation properties of the baryon number violating operators given above are,

$$O(duuv_i), O(dude_i) \quad \text{as} \quad (8, 1),$$

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$$O(sudν_i), O(swue_i) \text{ as } (8, 1),$$
$$O(sudν_i) \text{ as } (8, 1),$$
$$\tilde{O}^{(1)} \text{ as } (3, 3^*),$$
$$\tilde{O}^{(2)} \text{ as } (3^*, 3).$$

Thus, the four-fermi operators translating as $(8, 1)$ can be expressed in terms of the baryon and meson fields with a dimensionful constant $β$,

$$\mathcal{L}^h(n, p \rightarrow π(η)\bar{ν}_i) = βC(duυν_i)ν_{dL} Tr[P_1 ζB_L θ^\dagger] + h.c.,$$
$$\mathcal{L}^h(n, p \rightarrow π(η)e_i^+) = βC(due_ιe_{dL} Tr[P_2 ζB_L θ^\dagger] + h.c. ,$$
$$\mathcal{L}^h(n, p \rightarrow K\bar{ν}_i) = βC(sudν_i)ν_{dL} Tr[P_3 ιB_L θ^\dagger] + βC(dusν_i)ν_{dL} Tr[P_4 ιB_L θ^\dagger] + h.c.,$$
$$\mathcal{L}^h(n, p \rightarrow Ke_i^+) = βC(swue_ι)e_{dL} Tr[P_5 ιB_L θ^\dagger] + h.c.,$$

and the ones translating as $(3, 3^*)$ or $(3^*, 3)$ with a dimensionful constant $α$,

$$\mathcal{L}^h(n, p \rightarrow πe^+) = α\tilde{C}^{(1)} e_{dL} Tr[P_6 ιB_L θ] + α\tilde{C}^{(2)} e_{dR} Tr[P_7 ιB_R θ^\dagger] + h.c.,$$

with the coefficients $C$’s and $\tilde{C}$’s defined in Eq. (C.4). In the above formulae, $P_i$ are projection operators,

$$P_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$
$$P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$P_5 = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$
$$P_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (C.13)$$
The constants $\alpha$ and $\beta$ are the same as those defined in Eqs. (A.3), (3.19) [19]. When one estimates the decay rates of the nucleons, one needs to include the virtual baryon exchanges. For example, to estimate the lifetimes of decay modes $n,p \rightarrow K\bar{\nu}$, one should add the contributions from diagrams with virtual $\Sigma$ and $\Lambda$ exchanges. We show the chiral Lagrangian factors of each decay modes in Table 5. We took $m_B \equiv m_\Sigma = m_\Lambda = 1150$ MeV, $D = 0.81$, $F = 0.44$ in Tables 1–3.

Here we have some comments on the chiral Lagrangian factors in the decay rates of the nucleons. First, the ratio of $K\bar{\nu}$ decay rates of neutron and proton is

$$\frac{\Gamma(n \rightarrow K^0\bar{\nu})}{\Gamma(p \rightarrow K^+\bar{\nu})} = \frac{\left|2 + \frac{2m_n}{m_B}F\right|^2}{\left|1 + \frac{m_p}{m_B}(D + F)\right|^2} = 1.8.$$  \hspace{1cm} (C.14)

This shows that the decay rate of the neutron is larger than that of the proton. The situation is different in $\pi\bar{\nu}$ mode. The decay rate of the mode $p \rightarrow \pi^+\bar{\nu}$ is two times larger than that of $n \rightarrow \pi^0\bar{\nu}$. However, we use $n \rightarrow \pi^0\bar{\nu}$ in section 5 because the experimental lower bound on $\tau(n \rightarrow \pi^0\bar{\nu})$ is longer than that on $\tau(p \rightarrow \pi^+\bar{\nu})$.

Second, the decay rates into $\eta$ of dimension-five operators are as large as those into $\pi^0$. For example, the ratio in $\bar{\nu}$ decay mode is

$$\frac{\Gamma(n \rightarrow \eta\bar{\nu})}{\Gamma(n \rightarrow \pi^0\bar{\nu})} = \frac{(m_n^2 - m_\eta^2)^2}{m_n^4} \frac{3\left|1 - \frac{1}{3}(D - 3F)\right|^2}{\left|1 + D + F\right|^2} = 0.35.$$ \hspace{1cm} (C.15)

Recall that the decay rates into $\eta$ from dimension-six operators is negligible [19]. Thus this mode may be interesting.

The results in this appendix depend on the parameters $\alpha$ and $\beta$. However, they are sensitive to hadron dynamics, and they differ for each hadron models. Even among the lattice calculations, the results vary from $0.03 \text{ GeV}^3$ [14] to $0.0056 \text{ GeV}^3$ [22].
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### Table 1

The prediction of the nucleon partial lifetimes for the dominant decay modes, arising from the dimension-five operators \((Q_c Q_c)(Q_u L_\mu)\) and \((Q_t Q_t)(Q_u L_\mu)\). This class of decay modes depends on the parameter \(y^{tK}\). The mass degeneracy \(m_\tilde{c} = m_\tilde{u}\) and \(m_\tilde{\mu} = m_\tilde{e}\) is assumed. The function \(f\) is defined in Eq. (3.8) and depends on the SUSY particle masses. See the text for other variables.

\[
\begin{align*}
\tau(p \rightarrow K^+ \bar{\nu}_\mu) &= 6.9 \times 10^{31} \\
\tau(p \rightarrow K^+ \bar{\nu}_e) &= 1.4 \times 10^{33} \\
\tau(n \rightarrow K^0 \bar{\nu}_\mu) &= 3.9 \times 10^{31} \\
\tau(n \rightarrow K^0 \bar{\nu}_e) &= 7.7 \times 10^{32}
\end{align*}
\]

### Table 2

The prediction of the nucleon partial lifetimes for the next-leading decay modes, arising from the dimension-five operators \((Q_c Q_c)(Q_u L_\mu)\) and \((Q_t Q_t)(Q_u L_\mu)\). This class of modes depends on the parameter \(y^{t\pi}\). The mass degeneracy \(m_\tilde{e} = m_\tilde{u}\) and \(m_\tilde{\mu} = m_\tilde{e}\) is assumed. The function \(f\) is defined in Eq. (3.8) and depends on the SUSY particle masses. See the text for other variables.

\[
\begin{align*}
\tau(p \rightarrow \pi^+ \bar{\nu}_\mu) &= 1.4 \times 10^{32} \\
\tau(p \rightarrow \pi^+ \bar{\nu}_e) &= 2.9 \times 10^{33} \\
\tau(n \rightarrow \pi^0 \bar{\nu}_\mu) &= 2.9 \times 10^{32} \\
\tau(n \rightarrow \pi^0 \bar{\nu}_e) &= 5.7 \times 10^{33} \\
\tau(n \rightarrow \eta^0 \bar{\nu}_\mu) &= 8.2 \times 10^{32} \\
\tau(n \rightarrow \eta^0 \bar{\nu}_e) &= 1.6 \times 10^{34}
\end{align*}
\]
Table 3

The prediction of the nucleon partial lifetimes for the decay modes which depend neither on the parameter $y^{tK}$ nor $y^{t\pi}$. The relevant dimension-five operator is $(Q_uQ_u)(Q_cL_\mu)$ alone. The mass degeneracy $m_\tilde{c} = m_\tilde{u}$ and $m_\tilde{\nu}_\mu = m_\tilde{\nu}_e$ is assumed. The function $f$ is defined in Eq. (3.8) and depends on the SUSY particle masses. See the text for other variables.

$$
\begin{align*}
\tau(p \to K^0\mu^+) &= 1.0 \times 10^{35} \\
\tau(p \to \pi^0\mu^+) &= 2.0 \times 10^{35} \\
\tau(p \to \eta^0\mu^+) &= 5.7 \times 10^{35} \\
\tau(n \to \pi^-\mu^+) &= 9.9 \times 10^{34} \\
\end{align*}
$$

Table 4

The experimental lower bounds on the nucleon partial lifetimes at the 90% C.L. [18].

$$
\begin{align*}
\tau(p \to K^+\bar{\nu}) &> 1.0 \times 10^{32}\text{yrs} \\
\tau(n \to K^0\bar{\nu}) &> 8.6 \times 10^{31}\text{yrs} \\
\tau(p \to \pi^+\bar{\nu}) &> 2.5 \times 10^{31}\text{yrs} \\
\tau(n \to \pi^0\bar{\nu}) &> 1.0 \times 10^{32}\text{yrs} \\
\tau(n \to \eta\bar{\nu}) &> 5.4 \times 10^{31}\text{yrs} \\
\tau(p \to K^0\mu^+) &> 1.2 \times 10^{32}\text{yrs} \\
\tau(p \to \pi^0\mu^+) &> 2.7 \times 10^{32}\text{yrs} \\
\tau(p \to \eta\mu^+) &> 6.9 \times 10^{31}\text{yrs} \\
\tau(n \to \pi^-\mu^+) &> 1.0 \times 10^{32}\text{yrs} \\
\tau(p \to \pi^0e^+) &> 5.5 \times 10^{32}\text{yrs} \\
\end{align*}
$$
Table 5

Chiral Lagrangian factors in the nucleon-decay matrix elements. For notations, see the text.

\[
\Gamma(p \to K^+\bar{\nu}_i) = \frac{(m^2_p - m^2_K)^2}{32\pi m^3_p f^2_{\pi}} \left| C(sud\nu_i) \frac{2m_p}{3m_B} D + C(dus\nu_i) \left[ 1 + \frac{m_p}{3m_B} (D + 3F) \right] \right|^2
\]

\[
\Gamma(n \to K^0\bar{\nu}_i) = \frac{(m^2_p - m^2_K)^2}{32\pi m^3_p f^2_{\pi}} \left| C(sud\nu_i) \left[ 1 - \frac{m_n}{3m_B} (D - 3F) \right] + C(dus\nu_i) \left[ 1 + \frac{m_p}{3m_B} (D + 3F) \right] \right|^2
\]

\[
\Gamma(p \to \pi^+\bar{\nu}_i) = \frac{m_p}{32\pi f^2_{\pi}} \left| C(duu\nu_i) \left[ 1 + D + F \right] \right|^2
\]

\[
\Gamma(n \to \pi^0\bar{\nu}_i) = \frac{m_n}{64\pi f^2_{\pi}} \left| C(duu\nu_i) \left[ 1 + D + F \right] \right|^2
\]

\[
\Gamma(n \to \eta\bar{\nu}_i) = \frac{(m^2_n - m^2_\eta)^2}{64\pi m^3_n f^2_{\pi}} \left| C(duu\nu_i) \left[ 1 - \frac{1}{3} (D - 3F) \right] \right|^2
\]

\[
\Gamma(p \to K^0\pi^+_i) = \frac{(m^2_p - m^2_K)^2}{32\pi m^3_p f^2_{\pi}} \left| C(sue\nu_i) \left[ 1 - \frac{m_p}{m_B} (D - F) \right] \right|^2
\]

\[
\Gamma(p \to \pi^0\pi^+_i) = \frac{m_p}{64\pi f^2_{\pi}} \left| C(duu\nu_i) \left[ 1 + D + F \right] \right|^2
\]

\[
\Gamma(p \to \eta\pi^+_i) = \frac{(m^2_p - m^2_\eta)^2}{64\pi m^3_p f^2_{\pi}} \left| C(duu\nu_i) \left[ 1 - \frac{1}{3} (D - 3F) \right] \right|^2
\]

\[
\Gamma(n \to \pi^0\pi^+_i) = \frac{m_n}{32\pi f^2_{\pi}} \left| C(duu\nu_i) \left[ 1 + D + F \right] \right|^2
\]

\[
\Gamma(p \to \pi^0\pi^+_i) = \frac{m_p}{64\pi f^2_{\pi}} \left( \left| \tilde{C}(1) \right|^2 + \left| \tilde{C}(2) \right|^2 \right) \left[ 1 + D + F \right]^2
\]
Figure Captions

Fig. 1 A supergraph contributing to the dimension-five operators of the nucleon decay.

Fig. 2 Allowed ranges on the color-triplet Higgs mass $M_{H_C}$ and the “GUT-scale” $M_{GUT} \equiv (M_V^2 M_{\Sigma})^{1/3}$ obtained from the renormalization group analysis (thick lines), by varying $m_{\tilde{h}}$ and $m_{\tilde{g}}$ between 100 GeV and 1 TeV. $M_{H_C}$ depends only on $m_{\tilde{h}}$, and $M_{GUT}$ only on $m_{\tilde{g}}$. We use the gauge coupling constants at the weak-scale given in the text. Also shown are the ranges with an improved measurement on the strong coupling constant, $\alpha_3 = 0.118 \pm 0.0035$ (thin lines).

Fig. 3 Lower bound on $M_{H_C}$ derived from the nucleon-decay experiments. The horizontal axis represents $|1 + y^{tK}|$, the sum of the second- and third-generation contributions normalized by the second-generation one. The vertical axis corresponds to $M_{H_C}$. The shaded region is excluded. The upper curve corresponds to the hadron matrix element $\beta = 0.03 \text{ GeV}^3$, the lower one to $\beta = 0.003 \text{ GeV}^3$. The experimental limits come from the mode $n \rightarrow K^0\nu_\mu$ for $|1 + y^{tK}| > 0.4$, and from the mode $n \rightarrow \pi^0\nu_\mu$ for $|1 + y^{tK}| < 0.4$ and $|1 + y^{t\pi}| = 1$. The short-range renormalization factor $A_S$ is taken to be $A_S = 0.67$. The maximum value on $M_{H_C} (= 2.3 \times 10^{17} \text{ GeV})$ from the renormalization-group (RG) analysis requiring gauge coupling unification (see section 4) is also shown.

Fig. 4 The dependence of the lower bound of $M_{H_C}$ on the parameters $m_{\tilde{Q}}$ and $m_{\tilde{w}}$. The dashed line shows the dependence on $m_{\tilde{w}}$ taking $m_{\tilde{Q}} = 1 \text{ TeV}$. The dash-dotted line shows the dependence on $m_{\tilde{Q}}$ when $m_{\tilde{w}} = 45 \text{ GeV}$. In both curves we have taken the most conservative set of parameters, $\tan \beta_H = 1$, $|1 + y^{tK}| < 0.4$, $|1 + y^{t\pi}| = 1$, $A_S = 0.67$, and $\beta = 0.003 \text{ GeV}^3$. We have assumed $m_L \simeq m_{\tilde{Q}}$. The maximum value on $M_{H_C} (= 2.3 \times 10^{17} \text{ GeV})$ from the renormalization-group (RG) analysis requiring gauge coupling unification (see section 4) is also shown.

Fig. 5 The dependence of the lower bound of $M_{H_C}$ on $\tan \beta_H$. The upper curve is obtained with $|1 + y^{tK}| = 1$, and the lower curve with $|1 + y^{tK}| < 0.4$ and $|1 + y^{t\pi}| = 1$. We
have taken $m_{\tilde{Q}} = m_{\tilde{L}} = 1$ TeV, $m_{\tilde{w}} = 45$ GeV, $A_S = 0.67$ and $\beta = 0.003$ GeV$^3$. The maximum value on $M_{H_C}(= 2.3 \times 10^{17}$ GeV) from the renormalization-group (RG) analysis requiring gauge coupling unification (see section 4) is also shown.

**Fig. 6** The limits on $m_{\tilde{w}}$ and $m_{\tilde{Q}}$ from the KAMIOKANDE nucleon-decay experiments, in the absence of the cancellation between second- and third-generation contributions (i.e., $|1 + y^{tK}| = 1$). The most conservative parameters, $M_{H_C} = 2.3 \times 10^{17}$ GeV, $\tan \beta_H = 1$, $\beta = 0.003$ GeV$^3$, and $A_S = 0.67$, are used. We have assumed $m_{\tilde{Q}} \simeq m_{\tilde{L}}$ for simplicity. The shaded region is excluded. Also shown are the limits from the direct search experiments on wino and squarks at LEP and CDF.

**Fig. 7** The same as in Fig. 6, but allowing the cancellation between second- and third-generation contributions (i.e., $|1 + y^{tK}| < 0.4, |1 + y^{t\pi}| = 1$).

**Fig. 8** The same as in Fig. 6, but with an improved constraint by a factor of 30 expected at superKAMIOKANDE. The expected limit on $m_{\tilde{w}}$ from the LEP-II experiment ($m_{\tilde{w}} > 90$ GeV) is also shown.

**Fig. 9** The same as in Fig. 7, but with an improved constraint by a factor of 30 expected at superKAMIOKANDE. The expected limit on $m_{\tilde{w}}$ from the LEP-II experiment ($m_{\tilde{w}} > 90$ GeV) is also shown.

**Fig. 10** The same as in Fig. 9. We have assumed that the error-bar of $\alpha_3$ is reduced by a factor of 2 with the same central value, leading to a stronger upper bound on $M_{H_C}(< 6.1 \times 10^{16}$ GeV) (see Fig. 2).

**Fig. 11** The renormalization factor $A_S$ vs. $m_t/\sqrt{2}\sin \beta_H$. The solid line represents the $A_S$ for the dimension-five operators only with first- and second-generation fields, and dash-dotted line for the operator $(Q_tQ_t)(Q_cL_\mu)$. The upper horizontal line is $A_S$ derived by the authors in Ref. [7]. The lower horizontal line is $A_S$ which does not contain the contribution of the top-quark Yukawa coupling (i.e., $m_t = 0$).