LARGE CONNECTED STRONGLY REGULAR GRAPHS ARE HAMILTONIAN

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ABSTRACT. We prove that every connected strongly regular graph on sufficiently many vertices contains a Hamiltonian cycle. We prove this by showing that, apart from three families, connected strongly regular graphs are (highly) pseudo-random. Our results suggest a number of new questions and conjectures.

We also show that dense graphs with primitive automorphism groups are pseudo-random unless their automorphism groups are almost abelian.

0. INTRODUCTION

A graph is strongly regular with parameters \((v, k, \lambda, \mu)\) if it is a \(k\)-regular graph on \(v\) vertices such that

(i) each edge is in \(\lambda\) triangles,

(ii) any two non-adjacent vertices have \(\mu\) common neighbours.

(We exclude the complete and empty graphs).

For example, the Petersen graph is strongly regular with parameters \((10,3,0,1)\).

It is conjectured by Brouwer and Haemers [BH2] that the Petersen graph is the only connected non-Hamiltonian strongly regular graph.

They proved the conjecture for graphs on at most 98 vertices and remarked that it can be shown to hold for graphs with “most admissible parameter sets for strongly regular graphs” by means of a sufficient condition for hamiltonicity given in [BH2]. It is certainly not hard to show that the members of several infinite families of strongly regular graphs are Hamiltonian. These include complete multipartite graphs of type \(K_{m,m,...,m}\), line graphs of Steiner systems and Latin square graphs (these families will be discussed later).

We prove the following.

**Theorem 1.** There is a number \(N\) such that if \(G\) is a connected strongly regular graph on \(v > N\) vertices, then \(G\) contains a Hamiltonian cycle.

Connected strongly regular graphs are exactly distance regular graphs of diameter 2. Motivated by Theorem 1 and some additional evidence (discussed in Section 3) we make the following (somewhat provocative) conjecture.

**Conjecture 1.** Connected distance regular graphs are Hamiltonian with finitely many exceptions.
If $G$ is a $k$-regular graph then the eigenvalues of its adjacency matrix are real numbers $k = \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_v$. Set $\Lambda = \Lambda(G) = \max\{|\lambda_i(G)| \mid i = 2, 3, \ldots, v\}$. The parameter $\Lambda$ is usually called the second eigenvalue of $G$.

Our proof rests on the following hamiltonicity criterion of Krivelevich and Sudakov [KS2].

**Theorem 2** (Krivelevich, Sudakov). Let $G$ be a $k$-regular graph on $v$ vertices. If $v$ is large enough and $k/\Lambda > 1000 \log v (\log \log v) / (\log \log v)^2$, then $G$ is Hamiltonian.

Let us call a connected strongly regular graph *exceptional* if it is not a complete multipartite graph or a Steiner graph or a Latin square graph. It seems to be impossible to classify exceptional strongly regular graphs. The central result of this note is the following.

**Theorem 3.** Let $G$ be an exceptional strongly regular graph. Then $k/\Lambda > v^{1/10}/2$.

By the above discussion Theorem 1 follows. One can give an astronomical but explicit estimate for the value of $N$ in Theorem 1 by glancing at the proof of Theorem 2. To prove that connected strongly regular graphs on say, at least 1000 vertices are Hamiltonian seems to be a challenging task.

Theorem 3 shows that exceptional strongly regular graphs are highly pseudo-random. Pseudo-random graphs, that is graphs that have edge distribution similar to that of a truly random graph were first systematically investigated by Thomason [Th1], [Th2]. Thomason already noted that many families of strongly regular graphs have certain pseudo-random properties. This observation is made more explicit in the recent survey [KS1].

As explained e.g. in [KS1] if $G$ is a $k$-regular graph and $k/\Lambda$ is large then $G$ has almost uniform edge distribution i.e. it is pseudo-random (we do not give a formal definition, for various possibilities see [AS], [KS1]). In [BL] it is shown that on the other hand if a $k$-regular graph has almost uniform edge-distribution then $k/\Lambda$ is large.

We remark that the eigenvalues of the non-exceptional strongly regular graphs are well-known. Hence Theorem 3 largely clarifies the connection between pseudo-randomness and strongly regular graphs discussed e.g. in [Th2], [KS1] and [Ni].

Using related ideas we exhibit other large, natural classes of pseudo-random graphs. For example we prove the following.

**Theorem 4.** Let $G_n$ be sequence of $k_n$-regular graphs of order $v_n$ with $k_n > \epsilon v_n$ (for some $\epsilon > 0$). Assume that the groups $X_n = \text{Aut}(G_n)$ are primitive permutation groups and that the index of the largest abelian normal subgroup of $X_n$ goes to infinity. Then $\Lambda(G_n) = o(k_n)$, hence $G_n$ is pseudo-random.

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1. **General bounds**

We call a strongly regular graph $G$ *primitive* if both $G$ and its complement are connected (i.e. if $G$ is not a complete multipartite graph of type $K_{m,m,\ldots,m}$.)
or the complement of such a graph). In this section we will derive bounds for the eigenvalue ratios of primitive strongly regular graphs using well-known results. Apart from the eigenvalue $k$ a connected strongly regular graph $G$ has two distinct eigenvalues which satisfy $r > 0$ and $-1 > s$ (hence $\Lambda = \max \{r, -s\}$). Denote by $f$ and $g$ their respective multiplicities.

We need the following basic restrictions on the parameters.

**Proposition 1.1.** Let $G$ be a strongly regular graph. Then we have

(i) $k^2 + fr^2 + gs^2 = kv$,

(ii) $k(k - 1 - \lambda) = \mu(v - k - 1)$,

(iii) $rs = \mu - k$,

(iv) $r + s = \lambda - \mu$,

(v) $r$ and $s$ are integers, except perhaps when $G$ is a conference graph, that is, $(v, k, \lambda, \mu) = (4t + 1, 2t, t - 1, t)$ for some integer $t, r = (\sqrt{4t + 1} - 1)/2$ and $s = (-\sqrt{4t + 1} - 1)/2$.

The first equation follows from $k^2 + fr^2 + gs^2 = \text{Tr}(A^2) = kv$, where $A$ denotes the adjacency matrix of $G$. For the rest see e.g. [BH1].

Seidel’s absolute bound [Se] will provide us with a crucial starting point.

**Theorem 1.2** (Seidel). Let $G$ be a primitive strongly regular graph. Then we have

(i) $v \leq f(f + 3)/2$,

(ii) $v \leq g(g + 3)/2$.

**Lemma 1.3.** Let $G$ be a primitive strongly regular graph. Then we have $k/\Lambda > \sqrt{k/\sqrt{v}}$

**Proof.** Our statement is equivalent to $\Lambda^4 < vk^2$. We prove $r^4 < vk^2$ (a similar argument shows that $s^4 < vk^2$). Theorem 1.2 implies that $v - 1 \leq f^2$. Proposition 1.1 (i) implies that $fr^2 \leq k(v - k)$. Hence $r^4 < vk^2(v - k)/f^2 \leq vk^2$ as required.

Lemma 1.3 combined with Theorem ?? already shows that large enough connected strongly regular graphs with $k > \sqrt{v}(1000 \log v)^2$ are Hamiltonian. Note that we have $k > \sqrt{v - 1}$ for any connected strongly regular graph.

Another consequence of Lemma 1.3 is the following.

**Corollary 1.4.** Let $G$ be a primitive strongly regular graph. Then we have $|\lambda - \mu| < v^{3/4}$.

**Proof.** Using Proposition 1.1 (iv) we obtain that $|\lambda - \mu| \leq \Lambda < v^{1/4}\sqrt{K} \leq v^{3/4}$ as required.

Earlier it was shown by Nikiforov [Ni] using Szemerédi’s Regularity lemma that $|\lambda - \mu| = o(v)$, hence strongly regular graphs of degree $\geq \varepsilon v$ are pseudo-random for any fixed $\varepsilon > 0$. This was a useful hint.

Next we establish half of Theorem 2.

**Lemma 1.5.** Let $G$ be a primitive strongly regular graph. Then we have

$$
\frac{k}{s} > \frac{v^{1/6}}{2}.
$$
Lemma 2.2. Let \( s \) be an integer. Then we have
\[
\frac{k}{r} > \frac{v^4}{2}
\]
Proof. Denote \(|s| = -s\) by \( m \). If \( k > \sqrt{4} \) then \( k/m > v^{1/4}/2 \) follows from Lemma 1.3.

By Proposition 1.1 (iv) we have \( m = -s = r + \mu - \lambda \leq r + \mu \).

If \( r \geq \mu \) then this implies \( m \leq 2r \). By Proposition 1.1 (iii) we have \( rm = -rs = k - \mu \leq k \). It follows that \( m \leq \sqrt{2k} \). Using \( k^2 + 1 \geq v \) we obtain that \( k/m \geq \sqrt{k^2/2} \geq (v - 1)^{1/4}/\sqrt{2} \geq 1/2 \).

Assume finally that \( 4k \leq v \) and \( \mu > r \). Then using Proposition 1.1 (ii) we see that \( m \leq r + \mu - 2\mu = 2k(k - 1 - \lambda)/(v - k - 1) < 2k^2/(v - k) \) and hence \( k/m > (v - k)/2k \). On the other hand Lemma 1.3 implies that \( m^2 < v/2 \) and hence \((k/m)^2 > k^2/v \). Multiplying we obtain that \((k/m)^6 > (v - k)^2/4v \). Using \( v \geq 4k \) we see that \((k/m)^6 > v^2/24 \) as required.

\[\square\]

2. Exceptional vs non-exceptional graphs

We now discuss the three classes of connected non-exceptional graphs (see [Go2] for more details)

(i) Complete multipartite graphs of type \( K_{m,m,...,m} \) with \( n \) parts of size \( m \). The eigenvalues of these graphs are \( m(n - 1), 0, -m \).

(ii) Latin square graphs: given \( m - 2 \) mutually orthogonal Latin squares of order \( n \), the vertices are the \( n^2 \) cells, two vertices are adjacent if they lie in the same row or column or have the same entry in one of the squares. We allow \( m = 2 \) (in which case the graph is isomorphic to the line graph of \( K_{m,m} \)). The eigenvalues of these graphs are \( m(n - 1), n - m, -m \).

(iii) Steiner graphs: the vertices are the blocks of a Steiner system \( S(2,m,n) \) (that is, they are \( m \)-subsets of an \( n \)-set with the property that any two elements of the set lie in a unique block): two vertices are adjacent if and only if the corresponding blocks intersect. We allow \( m = 2 \). The eigenvalues of these graphs are \( m((n - 1)/(m - 1) - 1), (n - 1/m - 1) - m - 1, -m \).

It is an easy exercise to show that graphs in the first two classes are Hamiltonian. For the third class see [HR]. It is clear from the above that for certain types of strongly regular graphs such as line graphs of Steiner triple systems \( k/\lambda \) can be quite small and hence these graphs are far from pseudo-random.

To complete the proof of Theorem 2 we will need a well-known deep result of Neumaier [Ne].

Theorem 2.1 (Neumaier). Let \( G \) be an exceptional strongly regular graph with \( s \) an integer. Then we have \( r \leq s(s + 1)(\mu + 1)/2 - 1 \).

Lemma 2.2. Let \( G \) be an exceptional strongly regular graph. Then we have \( k/r > v^{1/10} \).

Proof. If \( G \) is a conference graph then our statement follows by direct computation. Hence we can assume that \( s \) is an integer and \( s \leq -2 \). Set \( m = -s \).

If \( k > v^{7/10} \) then our statement follows from Lemma 1.3.

By Proposition 1.1 (iii) we have \( k = rm + \mu \). Hence \( k/r \geq m \) and if \( m > v^{1/10} \) our statement follows.

Furthermore if \( m \geq \mu \) then using Theorem 2.1 we obtain that \( \sqrt{v - 1} \leq k = rm + \mu \leq \frac{1}{2}(m^2(m - 1)(\mu + 1)) - m + \mu < m^2/2 \) and therefore \( m > v^{1/8} \).
Assume now that $m < \mu, k \leq \frac{v^7}{10}$ and $m < \frac{v^4}{10}$. This implies in particular that $v > 2^{10}$ and $v > 8k$.

As above we have $k = rm + \mu \leq 1/2(m^2(m-1)(\mu+1)) - m + \mu$ hence $k \leq (m^3)\mu/2 + \mu = (m^3 + 2)\mu/2$.

By Proposition 1.1 (ii) we have $\mu \leq \frac{k(k-1)}{(v-k-1)} < k^2/(v-k)$. This implies $k \leq (m^3 + 2)k^2/2(v-k)$ and hence $2(v-k) \leq (m^3 + 2)k$. Using our assumptions we obtain that $2(7v/8) \leq ((m^3 + 2)/m^3)(km^3) < (10/8)v$, a contradiction. 

The above lemma was inspired by a result of Spielman [Sp, Corollary 9], which says that if $G$ is an exceptional strongly regular graph with $k = o(v)$ then $r = o(k)$.

Spielman used his result as a starting point for an algorithm testing isomorphism of strongly regular graphs in time $v^{o(\mu/3 \log v)}$.

Lemma 2.2 completes the proof of Theorem 2.

As noted above connected non-exceptional strongly regular graphs are Hamiltonian. Combining Theorem 2.1 and Theorem 2 we obtain that large enough exceptional strongly regular graphs are also Hamiltonian. This completes the proof of Theorem 1.

In fact one can see that if $G$ is an exceptional strongly regular graph of degree $k$ with $v$ sufficiently large then deleting roughly $k/2$ edges from each vertex in an arbitrary way we still obtain a Hamiltonian graph. This follows immediately from Theorem 2 and a qualitative strengthening of Theorem 2.1 due to Sudakov and Vu [SV].

Theorem 2.3 (Sudakov, Vu). For any fixed $\varepsilon > 0$ and $v$ sufficiently large the following holds. Let $G$ be a $k$-regular graph on $v$ vertices such that $k/\Lambda > (\log n)^2$. If $H$ is a subgraph of $G$ with maximum degree $\Delta(H) \leq (1/2 - \varepsilon)k$ then $G' = G - H$ contains a Hamiltonian cycle.

One can also see that if $G$ is an exceptional strongly regular graph then the number of Hamiltonian cycles in $G$ is very large.

This follows from Theorem 2 and the following result of Krivelevich. [Kr]

Theorem 2.4 (Krivelevich). Let $G$ be a $k$-regular graph on $v$ vertices such that

(i) $\frac{k}{\log v} \geq (\log v)^{1+\varepsilon}$ for some $\varepsilon > 0$
(ii) $\log k \cdot \log \frac{k}{\log v} \gg \log v$

Then the number of Hamiltonian cycles in $G$ is $v!(\frac{k}{\log v})^v (1 + o(1))^v$.

The toughness of a finite connected graph $G$ with vertex set $V$ is defined as the minimum of the quotient $|X|/c(V \setminus X)$ over all subsets $X$ of $V$ such that $c(V \setminus X) > 1$ where $c(Y)$ denotes the number of connected components of the graph induced on $Y$ by $G$.

Improving an earlier result of Alon, Brouwer [Br] proved that if $G$ is a connected noncomplete $k$-regular graph then its toughness $t$ satisfies $t > k/\Lambda - 2$. Hence Theorem 2 implies that exceptional strongly regular graphs are very tough.

Chvatal conjectured long ago that $t$-tough graphs are Hamiltonian if $t$ is a large enough constant. Motivated by the above discussion we suggest the following
weaker problem: Prove that log $v$ tough graphs are Hamiltonian if the number of vertices $v$ is sufficiently large!

As we saw above the pseudorandomness of exceptional strongly regular graphs implies that these graphs have some interesting graph-theoretic properties, for many more see [KS2]. It would be interesting to see whether connected non-exceptional strongly regular graphs also possess similar properties e.g. whether an appropriate analogue of the conclusion of Theorem 2.3 and Theorem 2.4 holds for them.

3. More pseudo-random graphs

In the last section we will show that in two other natural classes of $k$-regular graphs $G/k/\Lambda(G)$ is often relatively large.

A graph is called distance regular if, given any two vertices $g$ and $h$, the number of vertices in $G$ at distance $i$ from $g$ and distance $j$ from $h$ only depends on $i$, $j$ and the distance between $g$ and $h$. As noted before a connected strongly regular graph is a distance regular graph of diameter two.

For any graph $G$ with vertex set $V(G)$ let $G_r$ denote the $r$-th distance graph of $G$ i.e. the graph on the same vertex set in which two vertices $u$ and $v$ in $V(G)$ are connected if their distance in $G$ is exactly $r$. We call $M$ a merged graph of $G$ if it is the union of some of the $G_r$.

Denote the adjacency matrix of $G_r$ by $A_r$. It is well-known see (e.g. [Go2]) that if $G$ is a distance regular graph then $A_r$ is a polynomial of degree $r$ in $A = A_1$, say $A_r = p_r(A)$. It follows that if $\theta$ is an eigenvalue of $A$ with eigenspace $V_\theta$, then each vector $v$ in $V_\theta$ is an eigenvector of $A_r$ with eigenvalue $p_r(\theta)$. This implies that if $M$ is a merged graph of $G$ and $\lambda$ is an eigenvalue of $M$ with eigenspace $V_\lambda$, then $V_\lambda$ is the direct sum of some eigenspaces of $G$.

We will use the following theorem of Godsil [Go1].

**Theorem 3.1** (Godsil). Let $G$ be a connected distance regular graph of degree $k$ ($k > 2$). Suppose that $G$ is not a complete multipartite graph and that $\theta$ is an eigenvalue of $G$ with multiplicity $m$. Then if $\theta \neq \pm k$, the diameter of $G$ is at most $3m - 4$ and $k \leq (m - 1)(m - 2)/2$.

In particular we have $|G| < m^{6m}$

We prove the following.

**Corollary 3.2.** Let $G$ be a connected non-bipartite distance-regular graph on $v$ vertices which is not a complete multipartite graph. Let $M$ be a connected merged graph of $G$ of degree $p$. Then $\Lambda(M) < \sqrt{6pv\log \log v/\log v}$.

**Proof.** Let $k$ be the degree of $G$ and $m$ the smallest multiplicity of an eigenvalue $\theta \neq k$ of $G$. By the above discussion the multiplicity $t$ of an eigenvalue $\lambda \neq p$ of $M$ is at least as large as $m$. By Theorem 3.1 we have $t > m \geq \log v/6\log \log v$. Similarly to the proof of Proposition 1.1 (i) we see that $t\lambda^2 < tr(\text{Adj}(M)^2) = pv$, where $\text{Adj}(M)$ is the adjacency matrix of $M$. Hence $\lambda^2 < 6pv\log \log v/\log v$ and our statement follows. \qed
In particular if we have a family of merged graphs $M_n$ as above satisfying $p_n > \varepsilon v_n$ for some $\varepsilon > 0$, then $\Lambda(M_n) = o(p_n)$. This extends the main result of [Ni] to distance regular graphs.

These results yield some partial evidence supporting Conjecture 4 from the introduction. By a result of Bang, Dubickas, Koolen and Moulton [BDKM] there are only finitely many distance regular graphs of fixed valency greater than two. Hence in Conjecture 4 we may assume that the valency is sufficiently large.

The following weaker form of Conjecture 4 seems to be already very hard

**Conjecture 3.1**

Connected distance regular graphs of diameter $d$ are Hamiltonian with finitely many exceptions.

Note that van Dam and Koolen [DK] constructed a family of non-vertex transitive distance regular graphs with unbounded diameter. However such families seem to be hard to find. It may be a reasonable first step to establish the validity of Conjecture 3.1 for distance transitive graphs.

Finally we will prove Theorem 4 in a somewhat more general form.

Let $P$ be a permutation group of degree $n$. If $V$ is an $n$ dimensional vector-space over the complex numbers then $P$ acts on $V$ in a natural way by permuting the elements of an orthonormal basis. Hence we have a complex representation $\pi$ of the group $P$ by permutation matrices. If the group $G$ is transitive then it is well-known (see [Is, Corollary 5.15]) that the trivial representation $1_P$ occurs with multiplicity 1 in a decomposition of $\pi$ into irreducible constituents.

The following result is motivated by an argument of Gowers [Gow].

**Proposition 3.3.** Let $G$ be a $k$-regular connected graph on $v$ vertices and $P$ a group of automorphisms which acts transitively on $G$. Assume that the minimal degree of a non-trivial irreducible constituent of the natural representation $\pi$ of $P$ by permutation matrices is at least $t$. Then $\Lambda(G) < \sqrt{vk/t}$.

**Proof.** It is well-known that the permutation matrices $\pi(p)$ ($p \in P$) commute with the $\text{Adj}(G)$. This implies that the eigenspaces of $\text{Adj}(G)$ are invariant under $\pi(P)$ (see [Ba 1.5]). Since $G$ is connected the eigenspace corresponding to $k$ is the one-dimensional subspace $V_k = \{< x, x \ldots x >\}$. $V_k$ affords the trivial representation $1_P$. By our condition the dimensions of all other eigenspaces are larger than $t$. It follows that the multiplicity of an eigenvalue $\lambda$ different from $k$ is at least $t$. As above we have $t\lambda^2 < \text{Tr}(\text{Adj}(G)^2) = vk$. Our statement follows. \hfill \Box

Recall that a permutation group $P$ acting on a set $\Omega$ is primitive if there is no non-trivial $P$ invariant partition of $\Omega$ or equivalently if all the graphs with vertex set $\Omega$ invariant under $P$ are connected [Cam].

**Corollary 3.4.** Let $G$ be a $k$-regular graph on $v$ vertices such that its automorphism group $\Delta$ is a primitive permutation group. If $S$ is the largest abelian normal subgroup of $\Delta$ then $\Lambda(G) < c_1(vk/\sqrt{\log(\Delta/S)})^{1/2}$ for some constant $c_1 > 0$.

**Proof.** It is well-known [see [Cam] Exercise 2.20] that if $P$ is a primitive group then a non-trivial irreducible constituent of the natural representation $\pi$ by permutation matrices is faithful.
By a classical theorem of Jordan a finite subgroup of $GL(n,C)$ has an abelian normal subgroup of index $J(n)$, where $J(n) < c^2 n^2$ for some constant $c$ (see [Is, Theorem 14.12]).

Hence if $t$ is the smallest degree of a non-trivial irreducible constituent of $\pi$ then we have $t > c_0 \sqrt{\log |A/S|}$. Using Proposition 3.3 we obtain our statement. □

It would be interesting to see whether Corollary 3.4 has some kind of extension to merged graphs of primitive coherent configurations (see [Ba]). Theorem 4 is an immediate consequence of Corollary 3.4.

We remark that if $A$ is a primitive permutation group of degree $v$ then it has a unique largest abelian normal subgroup $S$ which is either trivial or elementary abelian of order $v$. In the first case we have $\Lambda(G) < c_1 k \sqrt{v/k} \log v$. In the second case if $A/S$ is small then a graph $G$ invariant under $A$ is "close" to being a Cayley graph of an elementary abelian group.

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