Measurements of $Z^0$ to Heavy-quark couplings at SLD

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Abstract

We present measurements of $Z^0$ to heavy-quark coupling electroweak parameters, $R_b$, $R_c$, and parity-violation parameter $A_c$, from SLD. The measurements are based on approximately 550k hadronic $Z^0$ events collected in 1993-98. Obtained preliminary results of $R_b$ and $R_c$ measurements are $R_b = 0.2159 \pm 0.0014 \pm 0.0014$ and $R_c = 0.1685 \pm 0.0047 \pm 0.0043$. In the $A_c$ measurement, we use four methods to determine the initial-quark charge: combined Kaon charge and Vertex charge, lepton, exclusively reconstructed $D^*$, $D$-mesons, and a new method using inclusive soft-pion from $D^*$. The preliminary results of these four methods were combined to give $A_c = 0.634 \pm 0.027$.

Presented at the American Physical Society (APS) Meeting of the Division of Particles and Fields (DPF 99), 5-9 January 1999, University of California, Los Angeles

*Work supported by Department of Energy contract DE–AC03–76SF00515 (SLAC).
1 Introduction

In the Standard Model, the electroweak interaction has both vector ($v$) and axial-vector ($a$) couplings. Measurements of two independent parameters, the ratio of widths, $R_f$, and the parity-violation parameter, $A_f$, at the $Z^0$ resonance probe combinations of these two couplings of the $Z^0$ to fermions,

$$R_f = \frac{\Gamma(Z^0 \rightarrow f \bar{f})}{\Gamma(Z^0 \rightarrow \text{Hadrons})} = \frac{v_f^2 + a_f^2}{\sum_{i}^{udscb} (v_i^2 + a_i^2)}$$

$$A_f = \frac{2v_f a_f}{v_f^2 - a_f^2}.$$ 

The parameter $R_f$ measures $Zf\bar{f}$-coupling strength compared to other quark flavors, while $A_f$ expresses the extent of parity violation at the $Zf\bar{f}$ vertex. These measurements provide sensitive tests of the Standard Model.

The measurements described here are based on a 550k $Z^0$-decay data sample taken in 1993-98 at the Stanford Linear Collider (SLC), with the SLC Large Detector (SLD). A general description of the SLD can be found elsewhere[1]. Polarized electron beams, a small and stable SLC interaction region, and the excellent CCD vertex detector[2] provide precision electroweak measurements, especially in the heavy-quark sector.

2 Flavor Tagging

Topologically reconstructed mass of the secondary vertex[3] is used by many analyses at the SLD for heavy-quark tagging. To reconstruct the secondary vertices, the space points where track density functions overlap are searched in the 3-dimensional space. Only the vertices that are significantly displaced from the primary vertex (PV) are considered to be possible B- or D-hadron decay vertices. The mass of the secondary vertex is calculated using the tracks that are associated with the vertex. Since the heavy-hadron decays are frequently accompanied by neutral particles, the reconstructed mass is corrected to account for this fact. By using kinematic information from the vertex flight path and the momentum sum of the tracks associated with the secondary vertex, we calculate the $P_T$-corrected mass $M_{P_T}$ by adding a minimum amount of missing momentum to the invariant mass. This is done by assuming the true momentum of heavy hadron is in a direction which minimizes the amount of transverse momentum added to the momentum sum of the tracks associated with the secondary vertex, and given by

$$M_{P_T} = \sqrt{M^2_{VTX} + P_T^2 + |P_T|},$$

where $M_{VTX}$ is the momentum sum for the tracks associated with the reconstructed secondary vertex. In this correction, vertexing resolution as well as the PV resolution are crucial. Due to the small and stable interaction point at the SLC and the excellent vertexing resolution from the SLD CCD Vertex detector, this technique has so far only been
Figure 1: a) Distributions of the $P_T$ corrected vertex mass for data (points) and Monte Carlo prediction of b, c and uds. b) Scatter plots of vertex momentum and mass for c (left) and b (right) events.

Successfully applied at the SLD. FIG. 1-a) shows the $P_T$-corrected mass distributions for the data and Monte-Carlo predictions. To select the $Z^0 \rightarrow b\bar{b}$ events, we apply the cut of $M_{P_T} > 2 \text{GeV}/c^2$, which provides 98% purity.

Charm tag relies on the intermediate mass region ($0.55 \text{GeV}/c^2 < M_{P_T} < 2 \text{GeV}/c^2$). Additional separation is provided by the 2-dimensional cut in the momentum-mass plane for the secondary vertex, as shown in FIG. 1-b) The cuts of $P_{vtx} > 5 \text{GeV}/c$ and $15M_{vtx} - P_{vtx} < 10$ provides 70% purity and 16% efficiency for $Z^0 \rightarrow c\bar{c}$ events.

3 Measurements of $R_b$ and $R_c$

The SLD $R_b$ measurement is based on the double-tag technique. Events are divided into two hemispheres by the plane perpendicular to the thrust axis of the event, and a $b$-tag algorithm is applied to each hemisphere in turn. The fraction of hemispheres tagged as originating from $b$-quarks (single-tag) is given by

$$F_s = R_b \epsilon_b + R_c \epsilon_c + (1 - R_c - R_b) \epsilon_{uds},$$

and the fraction of events with both hemispheres tagged as originating from $b$-quarks (double-tag) is given by

$$F_d = R_b (\epsilon_b^2 + \lambda_b (\epsilon_b - \epsilon_b^2)) + R_c (\epsilon_c^2 + \lambda_c (\epsilon_c - \epsilon_c^2)) + (1 - R_c - R_b) \epsilon_{uds}^2.$$

The above two equations are solved for both $R_b$ and the $b$-tag efficiency $\epsilon_b$. The background tagging efficiencies for uds- and $c$-hemispheres, $\epsilon_{uds}$ and $\epsilon_c$, as well as the $b$-tag hemisphere
correlation \( \lambda_b = (\epsilon_b^{\text{double}} - \epsilon_b^2)/(\epsilon_b - \epsilon_b^2) \) are estimated from the Monte-Carlo. \( R_c \) is assumed to be a Standard Model value.

For the \( R_c \) measurement, the double-tag technique is extended to include both charm and bottom tags. Using the similar equations as above, we add the fraction of hemispheres tagged as originating from c-quarks

\[
G_s = R_b \eta_b + R_c \eta_c + (1 - R_c - R_b) \eta_{uds},
\]

and the fraction of events with both hemispheres tagged as originating from c-quarks

\[
G_d = R_b (\eta_b^2 + \lambda_b (\eta_b - \eta_b^2)) + R_c (\eta_c^2 + \lambda_c (\eta_c - \eta_c^2)) + (1 - R_c - R_b) \eta_{uds}^2.
\]

In the \( R_c \) measurement, we have one more fraction of events where one hemisphere is tagged as \( b \) and another hemisphere is tagged as c (mixed-tag)

\[
M = 2 [R_b \epsilon_b \eta_b + R_c \epsilon_c \eta_c + (1 - R_c - R_b) \epsilon_{uds} \eta_{uds}] .
\]

The last three equations are solved for \( R_c \), c-tag efficiency \( \eta_c \), and \( b \)-tag efficiency \( \eta_b \). Where the \( uds \) efficiency \( \eta_{uds} \) and the correlations are taken from the Monte Carlo. \( R_b \) and \( \epsilon_b \) are known from the first two equations. In general, a high purity tag is needed for a double-tag measurement. However, the residual background in the c-tag sample are mainly \( b \)’s and the mixed-tag equation allows us to solve \( \eta_b \) from the data, using the high purity \( b \)-tag in the opposite hemisphere.

The SLD preliminary results of

\[
R_b = 0.2159 \pm 0.0014(\text{stat.}) \pm 0.0014(\text{syst.}),
\]

\[
R_c = 0.1685 \pm 0.0047(\text{stat.}) \pm 0.0043(\text{syst.}),
\]

are obtained from the 1993-98 winter SLD run (400k \( Z^0 \)) and the 1993-98 whole run (550k \( Z^0 \)), respectively. Both results are in good agreement with the Standard-Model predictions. The largest uncertainties in \( R_b \) and \( R_c \) measurements are detector systematics, and Monte-Carlo statistics of the uds background, respectively. FIG. 2. shows the comparison of the preliminary results of \( R_b \) and \( R_c \) measurements from the SLD and LEP experiments.

4 \( A_c \) measurements

\( A_f \) can be extracted by forming the forward-backward asymmetry

\[
A_{FB}^f(z) = \frac{\sigma^f(z) - \sigma^f(-z)}{\sigma^f(z) + \sigma^f(-z)} = A_e A_f \frac{2z}{1 + z^2},
\]

where \( z = \cos \theta \) is the direction of the outgoing fermion relative to the incident electron. \( A_{FB} \) for quarks depends on both the initial state \( A_e \) and the final state \( A_f \). At the SLC, the ability to manipulate the longitudinal polarization of the electron beam allows the isolation of the
Figure 2: Comparison of world $R_b$ (left) and $R_c$ (right) measurements. The inner and outer error bars present statistical and total errors, respectively.

Parameter $A_f$ independently of the $A_e$, through formation of the left-right forward-backward double asymmetry:

$$\tilde{A}_{FB}^f(z) = \frac{[\sigma^f_L(z) - \sigma^f_R(-z)] - [\sigma^f_R(z) - \sigma^f_L(-z)]}{[\sigma^f_L(z) + \sigma^f_R(-z)] + [\sigma^f_R(z) + \sigma^f_L(-z)]} = |P_e|A_f\frac{2z}{1 + z^2},$$

where $P_e$ is the longitudinal polarization of the electron beam. The high polarization of $\sim 77\%$ at the SLC also provides a large statistical advantage of $(P_e/A_e)^2 \sim 25$ compared to the $A_{FB}^f$ on the sensitivity to $A_f$.

In the actual analyses, we use an unbinned maximum likelihood fit based on the Born-level cross section for fermion production in $Z^0$-boson decay, to extract the $A_e$, instead of using the double asymmetry. The likelihood function used in the analyses is

$$\ln L = \sum_{i=1}^{n} \ln \left\{ f_c \cdot [(1 - P_c A_e)(1 + z_i^2) + 2(A_e - P_e)z_i \cdot A_c \cdot (1 - \Delta_{QCD}^c(z_i))] \right\}$$

$$+ f_b \cdot [(1 - P_b A_e)(1 + z_i^2) + 2(A_e - P_e)z_i \cdot A_b \cdot (1 - 2\bar{\chi}) \cdot (1 - \Delta_{QCD}^b(z_i))$$

$$+ f_{BG} \cdot [(1 + z_i^2) + 2A_{BG}z_i] \right\}$$

where $n$ is the total number of candidates, $f_c$, $f_b$, and $f_{BG}$ indicates the probabilities that a candidate is a signal from $c\bar{c}$, $b\bar{b}$, or background, respectively. $\bar{\chi}$ is the $B^0\bar{B}^0$ mixing parameter, and $\Delta_{QCD}^f(y)$ is the $O(\alpha_s)$ QCD correction to the asymmetry.
At the SLD, four different techniques are used to measure the $A_c$: inclusive charm-asymmetry measurement with Kaon charge and Vertex charge, lepton, exclusively reconstructed $D^*$ and $D$-mesons, and a new method using inclusive soft-pion from $D^*$. An inclusive charm tag using intermediate vertex mass is used to select charm events in a similar manner as the SLD $R_c$ analysis\cite{6}. A $b$ veto is also applied to reject any event with high vertex mass in either hemisphere. For the hemispheres with a secondary vertex, a secondary track identified as $K^\pm$ from the CRID, or a non-zero vertex charge, is used to sign the charm quark direction. The background is mostly $b$ events and its fraction is constrained by the double-tag calibration as for the $R_c$ measurement. The preliminary result from the 1993-98 data sample is $A_c = 0.603 \pm 0.028\text{(stat.)} \pm 0.023\text{(syst.)}$. This analysis has significantly high statistical power and the systematic errors are still very much under control.

We also measure the charm asymmetry with traditional technique using electrons and muons which not only tag the $c$ events but also determine the $c$-quark direction from the lepton\cite{7}. We get the preliminary result $A_c = 0.567 \pm 0.051\text{(stat.)} \pm 0.064\text{(syst.)}$ from the 1993-98 (muon) and 1993-97 (electron) SLD data.

The exclusive reconstruction of charmed mesons provide the cleanest technique for the charm-asymmetry measurements\cite{8}. We use four decay modes to identify $D^{*+}$: the decay $D^{*+} \rightarrow \pi^+_s D^0$ followed by $D^0 \rightarrow K^-\pi^+$, $D^0 \rightarrow K^-\pi^+\pi^0$ (Satellite resonance), $D^0 \rightarrow K^-\pi^+\pi^-\pi^+$, or $D^0 \rightarrow K^- l^+\nu_l$ ($l = e$ or $\mu$). We also identify $D^+$ and $D^0$ mesons via the decay of $D^+ \rightarrow K^-\pi^+\pi^+$ and $D^0 \rightarrow K^-\pi^+$ (not from $D^{*+}$). In this analysis, we reject $Z^0 \rightarrow b\bar{b}$ events using $P_T$-corrected mass of the reconstructed vertices. We required that reconstructed vertices had a mass of less than 2.0 GeV/$c^2$. This cut rejected 57\% of $b\bar{b}$ events with 99\% of the remaining being $c\bar{c}$ events. The random-combinatoric background can be estimated from the mass sidebands. The SLD preliminary result from this analysis using 550k of data from 1993-98 runs is $A_c = 0.690 \pm 0.042\text{(stat.)} \pm 0.022\text{(syst.)}$.

A new analysis using inclusive soft-pion from $D^*$ has been introduced by SLD in Winter-99. Since the decay $D^{*+} \rightarrow D^0\pi_s$ has small Q value of $m_{D^*} - m_{D^0} - m_{\pi} = 6\text{ MeV}/c^2$, the maximum transverse momentum of $\pi_s$ with respect to the $D^*$ flight direction is only 40 MeV. To determine the $D^*$ direction, charged tracks and neutral clusters are clustered into jets, using an invariant-mass algorithm. We also reject the $b\bar{b}$ background using $P_T$-corrected-mass information of reconstructed vertices. The background shape was determined by the function of $F_{BG}(P_T^2) = a/(1 + bP_T^2 + c(P_T^2)^2)$. FIG. 3. shows the $P_T^2$ distribution for the soft-pion tracks. The region of $P_T^2 < 0.01\text{ (GeV}/c^2)$ is regarded as a signal region, where a signal-to-background ratio of 1:2 is observed. From the 1993-98 SLD data, we get the preliminary result of $A_c = 0.683 \pm 0.052\text{(stat.)} \pm 0.050\text{(syst.)}$. The largest systematic uncertainty is the choice of background shape.
FIG. 3. $P_T^2$ distribution for the soft-pion tracks. Background shape is obtained by the function described in the text.

FIG. 4. shows the preliminary results from the SLD and LEP measurements, where the LEP measurements are derived from $A_c = 4A_{FB}^{0.1b}/(3A_c)$ using $A_c = 0.1491 \pm 0.0018$ (the combined SLD $A_{LR}$ and LEP $A_{lepton}$). The combined preliminary SLD result for $A_c$ is obtained as

$$A_c = 0.634 \pm 0.027.$$

5 CONCLUSION

SLD produces world class electroweak-parameter measurements in the heavy-quark sector. The SLD measurements of $R_{cb}$ and $A_c$ are now the most precise single measurements in the world. The measured $R_{cb}$, $R_s$ and $A_c$ results are consistent with the Standard Model, and some analyses will improve when the full set of 1993-98 SLD data is included.

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