Radiative capture of protons by deuterons

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The differential cross section for radiative capture of protons by deuterons is calculated using different realistic NN interactions. We compare our results with the available experimental data below \( E_x = 20 \) MeV. Excellent agreement is found when taking into account meson exchange currents, dipole and quadrupole contributions, and the full initial state interaction. There is only a small difference between the magnitudes of the cross sections for the different potentials considered.

The angular distributions, however, are practically potential independent.

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The radiative capture of protons by deuterons and the inverse reaction, the photodisintegration of \(^3\)He, have been investigated experimentally and theoretically over the last decades with quite some interest. Despite the various corresponding investigations, the theory is only in rough agreement with experiment, and there are inconsistencies between the data up to 30% in the magnitudes of the cross sections. The experimental results by Belt et al. \(^1\) and King et al. \(^2,3\) are in good agreement. Those by Matthews et al. \(^4\) and Skopik et al. \(^5\) agree in the angular distributions, but disagree in the magnitudes of the cross sections. This indicates a calibration problem of the measurements.

From the theoretical side several attempts have been made to describe the cross sections in this energy region. In the early calculations by Barbour et al. \(^6\) phenomenological interactions were used. It was shown that the final state interaction is quite important, and that the E2 contributions in the electromagnetic interaction are needed in the differential cross section. In the calculations by Gibson and Lehman \(^7\) a more realistic Yamaguchi interaction, but only the E1 components were employed. King et al. \(^8\) performed an effective two-body, direct capture calculation with the initial state being treated as a plane wave, or as a scattering state generated from an optical potential. Fonseca and Lehman \(^9\) calculated the polarization observables \( A_{yy} \) and \( T_{20} \) at the excitation energy \( E_x = 14.75 \) MeV including only the E1 interaction. A calculation at \( E_x = 15 \) MeV based on realistic interactions and both, the E1 and E2 contributions has been done by Ishikawa and Sasakawa \(^10\). Another calculation of \( A_{yy} \) in this energy region is by Jourdan et al. \(^11\). It was found in all these investigations that \( T_{20} \) is independent of the deuteron and the helium D-state probability, whereas \( A_{yy} \) shows a weak dependence on these quantities.

Very-low-energy radiative capture processes are of considerable astrophysical relevance. The \( p-d \) radiative capture, which at such energies is almost entirely a magnetic dipole (M1) transition, was studied in plane wave (Born) approximation by Friar et al. \(^12\). In these investigations the authors employed their configuration-space Faddeev calculations of the helium wave function, with inclusion of three-body forces and pion exchange currents. Various trends, e.g., the correlation between cross sections and helium binding energies, and their potential dependence were pointed out. More recently a rather detailed investigation of such processes has been performed by Viveni et al. \(^13\). Their calculations employed the quite accurate three-nucleon bound- and continuum states obtained in the variational pair-correlated hyperspherical method, developed, tested and applied over years by this group.

In Refs. \(^13,14\) we have treated the \(^3\)He photodisintegration and the inverse radiative capture process within the integral equation approach discussed below. These calculations were based on the Paris, Bonn A, and Bonn B potentials in Ernst-Shakin-Thaler (EST) representation: PEST, BAEST, BBEST \(^15,16\). We have demonstrated in particular the role of E2 contributions, meson exchange currents, and higher partial waves at \( E_x = 12 \) MeV and \( E_x = 15 \) MeV. The sensitivity against the underlying potentials, moreover, was pointed out. In the present paper we extend these investigations and compare our calculations with all sufficiently accurate data below \( E_x = 20 \) MeV.

The Alt-Grassberger-Sandhas (AGS) equations are well known to go over into effective two-body Lippmann-Schwinger equations \(^18\) when representing the input two-body \( T \)-operators in separable form. The proton-deuteron scattering amplitude, thus, is determined by

\[
T(q, q'') = \mathcal{V}(q, q'') + \int d^3q' \mathcal{V}(q, q') G_0(q') T(q', q'').
\]

(1)

Applying the same technique to the \(^3\)He photodisintegra-
expression by using the principle of detailed balance \[19\].

Here, \(k\) is the relative momentum state of the proton, \(H_{\text{em}}\) denotes the electromagnetic operator. In other words, with this replacement any working program for \(p\)-\(d\) scattering, based on separable representations or expansions of the two-body potential, can immediately be applied to calculating the full \(^3\)He photodisintegration amplitude with inclusion of the final-state interaction. The cross section for the \(p\)-\(d\) capture process is obtained from the corresponding photodisintegration expression by using the principle of detailed balance \[19\].

\[
B(q) = \langle \psi_d | H_{\text{em}} | \psi_{^3\text{He}} \rangle. \tag{3}
\]

Here, \(|\psi_{^3\text{He}}\rangle\) and \(|\psi_d\rangle\) are the \(^3\)He and deuteron states, \(|q\rangle\) is the relative momentum state of the proton, \(H_{\text{em}}\) denotes the electromagnetic operator. In other words, with this replacement any working program for \(p\)-\(d\) scattering, based on separable representations or expansions of the two-body potential, can immediately be applied to calculating the full \(^3\)He photodisintegration amplitude with inclusion of the final-state interaction.

The cross section for the \(p\)-\(d\) capture process is obtained from the corresponding photodisintegration expression by using the principle of detailed balance \[19\].

\[
\frac{d\sigma}{d\Omega} = \frac{3}{2} \frac{k^2}{Q^2} \frac{d\sigma_{\text{cap}}}{d\Omega}. \tag{4}
\]

Here, \(k\) and \(Q\) are the momenta of the proton and the photon, respectively. In the present treatment no Coulomb forces have been taken into account. The matrix element \[3\] for \(p\)-\(d\) capture differs from the corresponding \(n\)-\(d\) expression only in its isospin content.

The results presented in this paper are obtained by employing the PEST, BAEST and BBEST potentials as input \[17,21,22\], however, with an improved parameterization by Haidenbauer \[20\]. The high quality of this input has been demonstrated in bound-state and scattering calculations \[17,21,22\].
FIG. 4. Differential cross section for $p$-$d$ capture for energies $E_x$ from near threshold up to 16 MeV. The data are from [1-4]. The data set by Matthews et al. [4] has been renormalized with the $A_0$ from King et al. [2,3].

Details concerning their high quality are given in [25]. For the initial state all partial waves with $j \leq 2$ have been included in order to get a converged calculation of the cross section [13,14].

Usually the differential cross section is expanded in terms of Legendre polynomials

$$\sigma(\theta) = A_0 \left( 1 + \sum_{k=1}^{4} a_k P_k(\cos \theta) \right).$$

The total cross section is obtained by integrating over the angle $\theta$ between the incoming photon and the outgoing proton

$$\sigma = 4\pi A_0.$$
energy, or the $D$-state probability of the $^3\text{He}$ wave function.

Figure 3 shows the angular distribution coefficient $a_2$ of the expansion (8) compared to the coefficients extracted from experiment (9). In accordance with Figure 2 there is almost no potential dependence, i.e. no dependence on the three-body binding energy and the $D$-state probability, although this probability varies for the three potentials considered between 6 to 8% [25].

Figure 4 shows the differential cross sections obtained for these potentials at various energies compared to the experimental data. Due to the slight potential dependence of the total cross section and, thus, of $A_0$, the magnitudes of the curves differ correspondingly. In all cases there is good agreement between theory and experiment. As pointed out in (10) this agreement can only be achieved by taking into account $E_1$ and $E_2$ contributions of the electromagnetic interaction, meson exchange currents, and higher partial waves in the potential and in the three-body wave function. It should be mentioned that for increasing energies the peak is slightly shifted to the right-hand side, because of a smaller $E_1$ and a somewhat higher $E_2$ contribution. Note that, due to the missing $E_1$-$E_2$ interference term, the quadrupole contribution is practically negligible in the total cross section.

In (11,12) we have shown that for different potentials the low-energy peak heights of the $^3\text{He}$ photodisintegration cross sections are strictly correlated with the corresponding $^3\text{He}$ binding energies, and with the number of partial waves included. The magnitude of the present radiative capture process, i.e., the constant $A_0$, appears to be similarly fixed by the three-body binding energy. In other words, at the energies discussed, the radiative capture process, i.e., the constant $A_0$, appears to be similarly fixed by the three-body binding energy.

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