Formation and transportation of sand-heap in an inclined and vertically vibrated container

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Abstract

We report the experimental findings of formation and motion of heap in granular materials in an inclined and vertically vibrated container. We show experimentally how the transport velocity of heap up container is related to the driving acceleration as well as the driving frequency of exciter. An analogous experiment was performed with a heap-shaped Plexiglas block. We propose that cohesion force resulted from pressure gradient in ambient gas plays a crucial role in enhancing and maintaining a heap, and ratchet effect causes the movement of the heap. An equation which governs the transport velocity of the heap is presented.

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It is well known that under vibration many processes, e.g. segregation, convection, heap-\textcolor{red}{ing}, density wave, and anomalous sound propagation, which govern the physics of granular \textcolor{red}{materials}, are quite unusual \cite{1}, so that the properties of such materials are not well understood. For example, heaping is one of long-standing problem since Faraday \cite{2,3,4,5}, and several physical mechanisms have been identified as possible causes of it: friction between the walls and particles \cite{6}, analog of acoustic streaming if the shaking is nonuniform \cite{7}, gas pressure effect \cite{8}, and auto-amplification \cite{9}. Recently we have observed experimentally the formation of a heap and the motion of it from lower to higher end of an inclined and vertically vibrated container. The experiment was conducted in a Plexiglas rectangular container \[370 \text{mm (length)} \times 25 \text{mm (width)} \times 80 \text{mm (height)}\]. We investigated the behavior of two types of quartz sands: spheres of diameter 0.15-0.20 \text{mm} and grains of irregular shape or coarse surface with diameter 0.3-0.5 \text{mm}. The container was inclined with an inclination \(\alpha\) from 0.04 radian to 0.25 radian by putting a pad underneath it. The vibration exciter (Brüel & Kjær 4805) was driven by a sinusoidal signal, and controlled by a vibration exciter control (Brièl & Kjær 1050). Driving frequency \(f\) and dimensionless acceleration amplitude \(\Gamma = \frac{4\pi^2f^2A}{g}\) (where \(A\) is driving amplitude and \(g\) the gravitational acceleration) were used as two control parameters.

The experiment shows that even if in horizontal container the horizontal acceleration can cause movement of a heap. The horizontal acceleration of our exciter is about 2.5\% of the vertical acceleration. To get rid of the influence of the horizontal component of the driving acceleration on the movement of the heap along the longer direction of the container, we adjusted the longer direction of the container orthogonal to horizontal acceleration of the exciter. The ranges of \(\Gamma\) and \(f\) we used were from 1.4 to 2.8 and from 11 Hz to 20 Hz respectively, which are good ranges for heap formation. At first, about 80 ml of sands was uniformly put into the lower part of the vessel. Then as \(\Gamma\) increased to and beyond

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{heap_images.png}
\caption{The center-high heap (a) and the wall-high heap (b) of coarse sands. \(\alpha = 0.087\) radian, \(f = 15\) Hz, \(\Gamma = 2\).}
\end{figure}
FIG. 2: The transport velocity $V$ of heap formed by coarse sands. (a) $V$ vs $\Gamma$ for different frequencies. The solid lines are fits by $V = A(\Gamma - \Gamma_c)p\alpha q + B$ with $\alpha = 0.045$ radian. (b) $(V - B)/(A\alpha q)$ vs $\Gamma - \Gamma_c$ for the data in (a). The solid line is $(\Gamma - \Gamma_c)p$. (c) $V$ vs $\alpha$ for different frequencies. The solid lines are fits by $V = A(\Gamma - \Gamma_c)p\alpha q + B$ with $\Gamma = 2$. (d) $(V - B)/[A(\Gamma - \Gamma_c)p]$ vs $\alpha$ for the data in (c). The solid line is $\alpha q$. (e) $(V - B)/A$ vs $(\Gamma - \Gamma_c)p\alpha q$ for the data both in (a) and (c). The solid line is $(\Gamma - \Gamma_c)p\alpha q$.

Some critical acceleration $\Gamma_c(>1)$, center-high heaps formed, meanwhile they moved up the container. As center-high heaps reached higher end wall of the container, they moved forward continuously until they became wall-high heaps. Fig. 1 shows the photos of heaps formed by coarse sands. Fig. 1(a) is center-high heap, and (b) is wall-high heap. For center-high heap, the back (or right shown as in the figure) surface is longer than the frontal (or left) one, but the dynamical angles of repose (slightly smaller than the maximum angle of repose of the static heap) of both frontal and back surfaces relative to horizontal are the same. The difference between the heaps formed by two types of sands is: the dynamical angle of repose of the heap formed by coarse sands is larger than that formed by spherical sands. The heaps moved up the container with nearly uniform velocity. We measured the velocities $V$ of the heaps. Fig. 2 gives the results for coarse sands. Fig. 2(a) shows $V$ vs $\Gamma$ for different $f$. One can see that $V$ increases with $\Gamma$ for all of frequencies, but decreases as $f$ increases for all values of $\Gamma$. The velocity of heap formed by coarse sands is greater than that of heap formed by spherical sands for all sets of values of $\Gamma$ and $f$. Fig. 2(c) shows the
velocity of heap as a function of inclination of the container. It is shown that $V$ increases as $\alpha$, and behaves in the same way as in Fig. 2(a) as $f$ changes. The heap formed by coarse sands moved faster than the heap formed by spherical sands did for the same inclination and the same driving parameters.

Obviously, the transport of the heap is a cooperative behavior of the granular materials. To examine this idea we put a Plexiglas block the same in shape and dimension as the sand heap on the same vibrated inclined container, a similar transport of the block up the container was observed. So we consider that the transport of the heap is similar to that of a solid block. But why does it move, or what is the mechanism of the transport? We used a high speed camera (Redlake MASD MotionScope PCI 2000 SC) to record the movement of the block as it moved up the container with record rate of 250 fps (frames per second), then played back slowly (25 fps). In this way, we can see the detail of the movement of the block. We marked the center of mass of the block with a black point $C$ (shown as in Fig. 4). Fig. 3(a) is the orbit of the center of mass of the block measured experimentally in laboratory reference frame. Fig. 3(b) is a schematic diagram of Fig. 3(a), with which we can discuss the movement of the block conveniently. To examine the effect of ambient gas, we performed two things. First, to get rid of the pressure difference between the gap and the above of the block, we drilled a number of holes vertically and parallel each other through the block, and observed no transport of the block. And second, we evacuated the air from the container and also observed no transport of the block (either with or without holes drilled through the block). These imply that air pressure difference plays a critical role for the movement of the block up the container. Therefore we propose a mechanism for the

\[
\begin{align*}
V & = V_0 \sin \alpha \\
\alpha & = 0.06 \text{ radian, } f = 15 \text{ Hz, } \Gamma = 2.3.
\end{align*}
\]

FIG. 3: (a) The orbit of the center of mass of the block measured experimentally. $\alpha = 0.06$ radian, $f = 15$ Hz, $\Gamma = 2.3$. (b) The schematic diagram of (a).
movement of the block up the container as follows. In each cycle, when $\Gamma \cos 2\pi ft < -g$, block separates with container, and a gap forms between block bottom and the container floor. The pressure $p_1$ in gap is less than the atmospheric pressure $p_0$ above the block on an average (ref. [10] shows that the mean pressure in a gap between the bottom of granular bed and the container floor is below atmospheric pressure. And as Faraday Said: in this gap, “it forms a partial vacuum”. We consider this also suits to the case of block). This pressure difference causes a force exerting on the block. Fig. 4 is a schematic diagram for the forces acting on the block (or heap) during free flight. The forces acting on the frontal (I) and back (II) parts (with dashed line as intersection) are represented by vectors $\mathbf{F}_1$ and $\mathbf{F}_2$ at the centers of mass of two parts, $c_1$ and $c_2$, respectively. They are perpendicular to frontal and back surface (i. e. the frontal and back sides in the figure), respectively. The magnitudes of them are proportional to lengths of the frontal and back sides, respectively. They are decomposed into two components: those parallel (represented by $\mathbf{F}_{1c}$ and $\mathbf{F}_{2c}$) and those perpendicular (represented by $\mathbf{F}_{1d}$ and $\mathbf{F}_{2d}$) to the bottom of the block, respectively. The parallel components $\mathbf{F}_{1c}$ and $\mathbf{F}_{2c}$ equal in magnitude but opposite in direction. So the total force $\mathbf{F}$ is only the sum of perpendicular components $\mathbf{F}_{1d}$ and $\mathbf{F}_{2d}$, acts at the center of mass of the block, $C$, is perpendicular to the bottom of block, and makes an angle $\alpha$ (i. e. the inclination of the container) with gravitational force of the block. This force, together with gravitation force $Mg$ ($M$ is mass of the block), force the block to move along a ballistic trajectory until collides with the container [$A \rightarrow B$ in Fig. 3(b)]. Upon colliding with the container, the block slides down the container [$B \rightarrow C$ in Fig. 3(b)] due to gravity, meanwhile moves together with the container in vertical direction until next separation with container [$C \rightarrow D$ in Fig. 3(b)]. Then a new cycle begins. But due to the friction between the block and the container floor, the distance up the container during free flight is much

![FIG. 4: A schematic diagram for the forces acting on block (heap) during free flight.](image-url)
larger than the distance down the container during bed-floor collision, i.e. the block moves one step up the container in each cycle. The analysis above shows that the transport of block up the container is a ratchet effect caused by the pressure difference (between the gap and above of the block) and the friction force (between block and the container floor).

For the granular bed, if for any reason, some small (or flatter) initial heap has formed. In the period of free flight, the pressure difference between the gap and above of the heap leads to a pressure gradient (or force) in the interior of the heap. Also cf. Fig. 4, and we still use $F_1$ and $F_2$ as representatives of the total force acting on frontal and back parts of the heap, respectively. These two forces are perpendicular to the frontal and back surface, respectively. Here we call the parallel components $F_{1c}$ and $F_{2c}$ as a couple of cohesion force, i.e., they make granular materials cohesive and enhance the heap. Under the action of the cohesion force, heap is compressed in direction parallel to heap bottom while elongated in direction perpendicular to heap bottom. And this makes the heap bottom convex, as we have observed experimentally in both inclined and horizontal containers. Fig. 5(a) is a schematic diagram of the velocity (or mass flow) field in the interior of the heap formed by coarse sands during free flight. The total force $\mathbf{F}$ acting on the center of mass of heap, together with gravitational force $\mathbf{Mg}$, force the heap as a whole to move along a ballistic trajectory and up the container. Upon colliding with the container, the center part of the heap bottom touches floor first, then the other parts, from center to outer, touch floor consecutively. This results in a further enhancement of the heap. In this way the slope of the heap is getting larger and larger. Once the slope angle reaches and exceeds the dynamic angle of repose of the heap, and upon colliding with container, the avalanche occurs on the surface of the heap. Then the process repeats periodically: during the free flight, the compression due to cohesion force makes the slope of heap exceeding the dynamic angle of repose, and during the bed-floor collision, avalanche occurs. So when the heap reaches a

![Fig. 5: The schematic diagram of the velocity (or mass flow) field in a heap formed by coarse sands during the period of free flight (a) and heap-floor collision(b), respectively.](image)
steady state, in the laboratory reference frame, one can see a steady convection flow: in the interior of the heap, grains move upward, while at the surface grains move rapidly downward [schematically shown as in Fig. 5(b)]. This analysis is also suitable to wall-high heap, in which the cohesion force points higher end wall. Similar to the block, the transport of the heap up the container is also a ratchet effect, which is caused by the pressure gradient in the heap and the friction force between the heap and the container floor. If we pump the air out of the container, as pressure is reduced, heap reaches a flat state gradually. This shows that ambient gas plays a important role in enhancing and maintaining a heap.

Let us now describe the ratchet effect little more concretely but qualitatively [also cf. Fig. 3(b) and Fig. 4]. We denote $\beta$ as the ratio of free-flight time to the excitation period $T$ in each cycle. In each cycle, the up-container distance of free flight by heap is $s_1 \sim a_1 \beta^2 T^2 / \cos \alpha$, where $\alpha$ is the inclination of the container, and $a_1$ is the mean acceleration of the center of mass of the heap due to the horizontal component $F_x$ of the force $F$. Then $a_1 \sim F \sin \alpha (F = |F|)$, and $s_1 \sim F \beta^2 T^2 \tan \alpha$. In period of bed-floor collision, heap slides down the container. The distance of sliding is $s_2 \sim a_2 (1 - \beta) T^2$, where $a_2$ is the mean acceleration down the container, which is determined by the friction coefficient $\mu$ between the granular bed and the container floor, the inclination $\alpha$ of the container, and gravitational acceleration $g$ through $a_2 = g \sin \alpha - \mu g \cos \alpha$. The total (or net) displacement of heap up the container in each cycle is $s = s_1 - s_2$, timing driving frequency $f$ gives the velocity $V$ of heap up the container. The ratio $\beta$ increases with driving amplitude $A = \Gamma g/4\pi^2 f^2$, i.e. $V$ increases as $\Gamma$, but decreases as $f$ increases. The larger the $\mu$, the smaller the $s_2$, and then the larger the $V$. Because the friction between coarse sands and the container floor is larger than that between spherical sands and the container floor, the transport velocity of heap formed by coarse sands is larger than that of heap formed by spherical sands. In view of $s_2 \ll s_1$ and $\alpha \ll 1$ in our experiment, then $V$ increases as inclination $\alpha$ of the container. The pressure gradient or force in the interior of heap depends on driving acceleration and frequency, inclination of the container, and properties of grains in a specific and complex way, and changes with time and space. The investigation on this problem is under way.

Any way, from the description above we have already had a qualitative picture on how the transport velocity of the heap up the container depends on driving acceleration and frequency, inclination of the container, properties of the grains, and friction between heap and container floor. Based on this and on the analysis of experimental results, the experimental
data in Fig. 2(a) and (c) are well fitted by the equation

\[ V = A(\Gamma - \Gamma_c)^p\alpha^q + B, \tag{1} \]

with \( p = 0.6, q = 0.8, \) and \( \alpha = 0.045 \) radian for Fig 2(a) and \( \Gamma = 2 \) for Fig 2(c), respectively. The solid lines in the figure are nonlinear least-square fits by Eq. (1). The parameters \( A, B \) and \( \Gamma_c \) are depend on the driving frequency \( f \), the dynamic angle of repose of heap which depends on properties (e.g., size and shape etc.) of grains, and the friction coefficient \( \mu \) between granular bed and container floor. If the data of Fig. 2(a) are plotted as \((V - B)/(A\alpha^q)\) vs \( \Gamma - \Gamma_c \), they collapse onto a single curve, Fig. 2(b); and if the data of Fig. 2(c) are plotted as \((V - B)/(A(\Gamma - \Gamma_c)^p)\) vs \( \alpha \), they also collapse onto a single curve, Fig. 2(d), within the experimental resolution. Moreover, if the data of both Fig. 2(a) and Fig. 2(c) are plotted as \((V - B)/A\) vs \((\Gamma - \Gamma_c)^p\alpha^q\), they all collapse onto a single curve, Fig. 2(e).

This model is also suitable for the heap (either the center-high or the wall-high heap) formed in a horizontal container, where the cohesion force (as above) plays the same role (enhancing and maintaining the heap) as in an inclined container. The total force \( F \) due to the pressure gradient is perpendicular to heap bottom, i.e. is parallel to gravitational force \( Mg \), and no force forces heap to move in horizontal direction.

Our conclusion is: pressure gradient in ambient gas, which makes cohesionless granular materials cohesive, plays a crucial role in enhancing and maintaining a heap in vibrating granular materials; pressure gradient in ambient gas and friction force between the granular bed and the container floor, which lead to a ratchet effect, are unique cause for the transport of a heap up an inclined container. The transport velocity of the heap is well described by the equation (1). Our mechanism for enhancing and maintaining heap also shows that one reason for lacking of heaping in molecular dynamics (MD) simulation may be that those models did not take into account the cohesion force due to ambient gas effect.

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