A Statistical Treatment of the Gamma-Ray Burst “No Host Object” Issue

David L. Band$^1$ and Dieter H. Hartmann$^2$

$^1$CASS, UC San Diego, La Jolla, CA 92093
$^2$Dept. of Physics & Astronomy, Clemson University, Clemson, SC 29634

Abstract. Various burst origin scenarios require a host object in the burst error box or near a well-located position. For example, a host galaxy should be present in the standard cosmological models. We present a methodology which evaluates whether the observed detections and nondetections of potential host objects in burst error boxes are consistent with the presence of the host, or whether all the detections can be attributed to background objects (e.g., unrelated background galaxies). The host object’s flux distribution must be modeled. Preliminary results are presented for the “minimal” cosmological model.

INTRODUCTION

In many gamma-ray burst scenarios a host object should be detected when an error box is observed to sufficiently faint fluxes. However, once an error box has been observed, how do we know whether the host object has been detected? Most cosmological models predict that bursts occur in or near galaxies. Since the study of X-ray and optical transients indicate that some and probably all bursts are cosmological, here we will focus on galaxies as the host objects, although the concepts and methodology can be applied to other host object types.

The study of burst error boxes consists of three interrelated aspects. First is the observations of the error boxes, which we assume result in a list of galaxies which are brighter than a limiting flux. These observations can be in any wavelength band in which imaging is possible, although usually optical or infrared images are used. Second is the model for the host object, which guides both the observations and the analysis. For example, the assumption that bursts are cosmological leads the observer to ignore the stars in the error box, although the observer (hopefully) notices any unusual objects in the field. Third is the analysis of the observations in terms of the model. Beyond deciding whether the observations support the model, the analysis methodology also guides the observer as to which error boxes should be searched and to what detection limit. Here we present a new methodology for
analyzing burst error box observations, and present preliminary results. A more complete presentation is in press [1].

We emphasize that the analysis of burst error boxes must be made in the context of a model of the expected host. It is nonsensical to ask merely whether galaxies are present in an error box because if one searches deep enough one will find a multitude of faint galaxies. Here we test the “minimal” cosmological model used in most studies of burst ensembles, particularly those analyzing burst error boxes. Bursts are assumed to be standard candles in this model: a basic burst property such as total emitted energy or peak photon luminosity is constant for all bursts, and does not evolve with redshift. Therefore, there is a one-to-one mapping between a burst’s redshift and the observed intensity corresponding to the standard candle (e.g., energy fluence for a constant total emitted energy); the redshift-intensity relationship is derived from the intensity distribution under the assumption that the comoving density of burst sources does not evolve. Of course, in this model bursts occur in galaxies. Since a neutron star-neutron star merger is a possible origin of a burst’s energy, and the number of compact binary systems is presumably proportional to a galaxy’s mass, the burst rate per galaxy is assumed to be proportional to the galaxy’s luminosity (for a constant mass-to-light ratio) [2]. Undoubtedly bursts are characterized by a luminosity function, and cosmological density and luminosity evolution is likely, but this “minimal” model has been a reasonable working assumption in the absence of additional data.

B. Schaefer [3] first reported that the galaxies in 8 burst error boxes were fainter than expected. Specifically, Schaefer calculated a large burst energy (up to $2 \times 10^{53}$ ergs) if the brightest galaxy in an error box was as bright as M31. Fenimore et al. [2] introduced the statistic

$$S = \int_0^{f_{\text{det}}} df \psi(f)$$

where the brightest galaxy in an error box has a flux of $f_{\text{det}}$ and $\psi(f)$ is the flux distribution of the expected host galaxies; $S$ is the fraction of the distribution which is fainter than $f_{\text{det}}$. If $f_{\text{det}}$ is indeed the flux of the host galaxy, and $\psi(f)$ is the correct distribution, then $S$ should be distributed uniformly between 0 and 1, with an average of $1/2 \pm (12N)^{-1/2}$ for $N$ error boxes (this test is similar to the $V/V_{\text{max}}$ test). Based on the minimal cosmological model, Fenimore et al. found $\langle S \rangle = 0.44 \pm 0.10$ for Schaefer’s data. While this $\langle S \rangle$ is consistent with the minimal model, the value of $S$ for a given error box is only an upper limit since the brightest galaxy may be a background galaxy instead of the host galaxy. Similarly, although S. Larson [4–6] reported an overabundance of bright galaxies in his K-band observations of nine IPN$^3$ boxes, he recognized that many of these galaxies are unrelated background galaxies.

We therefore derived an analysis methodology which includes the unrelated background galaxies in evaluating whether a host galaxy is present. An additional guiding principle was the use of all available information. Thus the method uses all the detected galaxies in the observations. We describe the error box by a probability
density ρ(Ω), where Ω represents the spatial coordinates. Typically it is assumed that ρ(Ω) = 1/Ω₀ within the 99% contour (a region of size Ω₀), and ρ(Ω) = 0 outside, but more sophisticated treatments are possible. The detection threshold may vary across the error box, e.g., as a result of mosaicing the box with multiple observations of differing quality. Currently we do not include the clustering of background galaxies, which should be a small effect.

**METHODOLOGY**

Our method is a Bayesian comparison of two hypotheses: H₀—a host galaxy is present in addition to unrelated background galaxies; and H₁—only background galaxies are present. Assume the observations of an error box reveal n_d galaxies with fluxes f_i above a detection limit f_{lim}(Ω); these results we represent by the statement D. We set up an odds ratio

\[ O(H_0, H_1) = \frac{p(H_0 | D)}{p(H_1 | D)} = \frac{p(H_0) p(D | H_0)}{p(H_1) p(D | H_1)} \]  

where: p(H_x | D) is the probability that hypothesis H_x is true given the observations D; p(H_x) is the “prior,” our assessment of the validity of H_x before obtaining the new data D; and p(D | H_x) is the likelihood of H_x, the probability of obtaining D if the hypothesis H_x is correct. For simplicity we set the two priors equal to each other, p(H_0) = p(H_1). Therefore, the odds ratio is the ratio of the likelihoods, \( O(H_0, H_1) = p(D | H_0)/p(D | H_1) \).

The likelihoods are calculated by breaking into little bins the three-dimensional space formed by the two spatial dimensions and the flux, and calculating the probability that a host or background galaxy is present or absent in each bin. If the galaxy redshifts are also available, then the redshift can be added as a fourth dimension. Poisson statistics characterize the probability that a background galaxy is found in a given bin. The likelihood for H₀, \( p(D | H_0) \), is the sum of every possibility for the presence of a host galaxy: either the host is fainter than the limit \( f_{lim} \) or it is one of the detected galaxies. Consequently [1]

\[ \frac{p(D | H_0)}{p(D | H_1)} = \int dΩ \int_{f_{lim}(Ω)} df \frac{Ψ(f)ρ(Ω)}{φ(f, z)} + \sum_{i=1}^{n_d} \frac{Ψ(f_i, z_i)ρ(Ω_i)}{φ(f_i, z_i)} \]  

where Ψ(f, z) is model-dependent host galaxy distribution, ρ(Ω) is the burst location’s probability density across the error box, φ(f, z) is distribution of background galaxies, and n_d is the number of detected galaxies. If the redshifts of the detected galaxies are unknown, then the \( z \)-dependence of Ψ and φ should be dropped. In this equation, the first term on the right is the probability that the host galaxy can be hidden below the detection limit, while the sum compares for each detected galaxy the probability that it is the host galaxy to the probability that it is an unrelated background galaxy. Clustering of the background galaxies can be included.
by multiplying $\phi(f_i, z_i)$ by a function of the distance to the other detected galaxies. This additional factor will usually be of order unity, and will not affect our results qualitatively.

For a database with a number of error boxes the likelihood ratio for the ensemble is the product of the ratios for each box. If the resulting odds ratio is much larger than one, then the presence of host galaxies has been demonstrated. If the ratio is much less than one, then the host galaxy model is incorrect. Finally, if the ratio is of order unity, then the data are insufficient to distinguish between the hypotheses.

By evaluating the likelihood ratio for the expected host and background galaxies, we can determine the method’s sensitivity for a given error box. For reasonable assumptions about the distributions, we find that an observation can distinguish between hypotheses if the error box is small enough so that the expected host galaxy is much brighter than average background galaxy. Only then is it clear that a galaxy is the host and not a background galaxy. As currently formulated, this methodology tests a given hypothesis. However, it can easily be modified to fit model parameters.

This methodology was developed for finite size error boxes, such as has been available from the various IPNs. However, the methodology can be readily adapted for other circumstances. Afterglows localize the burst with very small uncertainties (e.g., a fraction of an arcsecond). However, unless the model being tested places the burst in a galactic nucleus, a galaxy within a certain region around the burst would be acceptable as the host; this region can be treated as the error box. Similarly, the burst source might have been ejected from the host galaxy. The distance the source might have traveled before bursting can be used to define the error box around an afterglow; finite size error boxes should be expanded by this distance.

**APPLICATIONS**

As examples, we apply this methodology to published datasets to test the “minimal” cosmological model described above. In the future we plan to test variants of the cosmological model using a more extensive dataset.

Larson and McLean [6] presented K-band observations of 9 IPN$^3$ error boxes with an average size of 8 arcmin$^2$. They listed only the flux of the brightest galaxy in each box, and therefore we use this galaxy as the single galaxy detection and its flux as the detection limit. For all 9 error boxes we find

$$\prod_{j=1}^{9} O_j = 0.25$$

(4)

which indicates that based on this data we cannot determine whether or not a host galaxy is present. The reason the analysis of these data is inconclusive is that the fluxes of the average expected host galaxy, the detection limit, and the average brightest background galaxy are all comparable; therefore even if the host galaxy is present, the odds ratio will be of order unity.
Schaefer et al. [7] observed 4 small (1/4-2 arcmin$^2$) burst error boxes with the Hubble Space Telescope. The detection of objects exhibiting bizarre behavior (e.g., proper motion) was the primary purpose of these observations, but our methodology can be applied to the data, nonetheless. Galaxies were detected in 2 of these error boxes. The odds ratio for the four boxes together is

$$\prod_{j=1}^{4} O_j = 2 \times 10^{-6} .$$

This is a clear statement that host galaxies expected by the minimal model are not present.

ACKNOWLEDGEMENTS

We thank B. Schaefer, C. Luginbuhl and F. Vrba for stimulating discussions. This research is supported by the CGRO guest investigator program (DLB and DHH) and NASA contract NAS8-36081 (DLB).

REFERENCES

1. Band, D., and Hartmann, H., Ap. J. 493, in press (1998).
2. Fenimore, E. E., et al., Nature 366, 40 (1993).
3. Schaefer, B. E., in Gamma-Ray Bursts: Observations, Analyses and Theories, eds. C. Ho, R. I. Epstein, and E. E. Fenimore (Cambridge: Cambridge Univ. Press), 107 (1992).
4. Larson, S. B., McLean, I. S., and Becklin, E. E., Ap. J. 460, L95 (1996).
5. Larson, S. B., Ap. J. 491, in press (1997).
6. Larson, S. B. and McLean, I. S., Ap. J. 491, in press (1997).
7. Schaefer, B. E., Cline, T. L., Hurley, K. C., and Laros, J. G., Ap. J., in press (1997).