On Braneworld Inflation Models in Light of WMAP7 data

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Abstract

We are interested on studying various inflationary spectrum perturbation parameters in the context of the Randall-Sandrum type 2 Braneworld model. We consider in particular three types of potentials. We apply the slow-roll approximation in the high energy limit to constraint the parameter potentials by confronting our results to recent WMAP7 observations. We show that, for some values of the e-folding number $N$, the monomial potential provides the best fit results to observations data.

Keywords: RS Braneworld, Perturbation Spectrum, WMAP7.

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1 Introduction

In the last few years, braneworld model[1] has become a fundamental paradigm of modern cosmology. One of the most used scenario is the Randall-Syndrum type-2 model[2]. In this theory, our four-dimensional Universe is considered as a 3-brane embedded in five-dimensional anti-de Sitter space-time (AdS5), while gravity can be propagated in the bulk. The main characteristic feature of this model is that the Friedmann equation is modified by an additional term proportional to quadratic energy density[3]. Braneworld inflation, was mainly shown to play a fundamental role in divers cosmological investigations of early universe and has been proposed to solve some problems of actual cosmological observations like as dark energy[4], tachyonic inflation[5] or black holes systems[6]. Recently, some generalized potentials and supersymmetric versions were studied in the framework of Randall-Syndrum model and a best fit to recent observations was given[7, 8].

In the present work, we consider various types of potential models, in particular the exponential, monomial and inverse power law. We have analyzed in some details divers perturbation parameter spectrum and show that the monomial potential provides the best fit results to more recent WMAP7 data.

In Sec.2, we begin by recalling the foundation of the braneworld inflation. We have adopted here the slow-roll approximation and have considered the high energy limit to derive various inflation observables. In Sec.3, we present our work concerning a different models of potential in the framework of the Randall-Syndrum type 2 model and give a comparative study of a exponential, monomial and inverse power law potentials. In the last section, a conclusion and perspectives are given.

2 Slow-roll approximation in braneworld inflation

In this section, we start by recalling briefly some fundamentals of Randall-Sundrum type-2 model. One of the most relevant consequences of this model is the modification of the Friedmann equation for energy density of the order of the brane tension, and also the appearance of an additional term, usually considered as dark radiation term. In the case where the dark radiation term is neglected, the gravitational Einstein equations, leads to the modified Friedmann equation on the brane as[3]

\[ H^2 = \frac{8\pi}{3m^2_{pl}} \rho \left[ 1 + \frac{\rho}{2\lambda} \right], \]

(1)

where \( H \) is the Hubble parameter, \( \rho \) is the energy density, \( \lambda \) is the brane tension and \( m_{pl} \) is the Planck mass. Note that the crucial correction to standard inflation is given by the density quadratic term \( \rho^2 \).

Note also that in the limit \( \lambda \to \infty \), we recover standard four-dimensional general relativistic results. Moreover, in the high energy limit i.e. \( \rho \gg \lambda \), the dynamic of the universe in 5 dimensions will be governed by the simplest equation given by

\[ H^2 = \frac{4\pi}{3m^2_{pl}} \frac{\rho^2}{\lambda}. \]

(2)

On the other hand, the matter in 3-brane is dominated by a scalar field with energy density of the form \( \rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \), where \( V(\phi) \) is the scalar field potential responsible of inflation.

Along with these equations, one also has a second inflation Klein–Gordon equation governing the dynamic of the scalar field as

\[ \ddot{\phi} + 3H\dot{\phi} + V' = 0, \]

(3)

where \( \dot{\phi} = \frac{\partial \phi}{\partial t} \), \( \ddot{\phi} = \frac{\partial^2 \phi}{\partial t^2} \) and \( V' = \frac{dV(\phi)}{d\phi} \).
This is a second-order evolution equation which follows from conservation condition of energy–momentum tensor $T_{\mu\nu}$. To calculate some physical quantities as scale factor or perturbation spectrum, one has to solve Eqs.(1,3) for some specific potentials $V(\phi)$. To do so, the slow-roll approximation was introduced and applied by many authors to derive perturbation spectrum of inflation\textsuperscript{[9]}. In this work, we apply slow-roll approximation ($\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \ll H\dot{\phi}$ ) and we use the well known slow-roll parameters\textsuperscript{[10]}, to calculate perturbation spectrum.

$$\epsilon = \frac{m^2_{\text{pl}} V''}{4\pi V^3}, \quad \eta = \frac{m^2_{\text{pl}} V'''}{4\pi V^2},$$

(4)

where $V'' = \frac{d^2V}{d\phi^2}$. We signal that, during inflation we have the following conditions

$$\epsilon \ll 1, \quad |\eta| \ll 1.$$  
(5)

The small quantum fluctuations in the scalar field lead to fluctuations in the energy density which was studied in a perturbative theory\textsuperscript{[11]}. As discussed in\textsuperscript{[12]} quantum fluctuations effect of the inflaton are generally negligibles, since the coupling of the scalar field to bulk gravitational fluctuations only modifies the usual 4D predictions at the next order in the slow-roll expansion. So, one can define the power spectrum of the curvature perturbations as

$$P_R (k) = \frac{H^2}{2\pi^2} \left( \frac{\epsilon}{V''} \right)^2,$$

(6)

where $k$ is the wave number.

In relation to $P_R (k)$, the scalar spectral index is defined as\textsuperscript{[11]}

$$n_s - 1 = \frac{d\ln P_R (k)}{d\ln k}, \quad (7)$$

$$= -6\epsilon + 2\eta.$$  
(8)

On the other hand, the quantum fluctuations in the scalar field lead also to fluctuations in the metric. In this way, one can define the amplitude of tensor perturbations as\textsuperscript{[13]}

$$P_g (k) = \frac{64\pi}{m^2_{\text{pl}}} \left( \frac{H}{2\pi} \right)^2 F^2 (x),$$

(9)

where $x = H m_{\text{pl}} \sqrt{\frac{2}{4\pi \lambda}}$ and $F^2 (x) = \left( \sqrt{1 + x^2} - x^2 \sinh^{-1} \left( \frac{1}{x} \right) \right)^{-1}$. Note that in the high-energy limit ($V \gg \lambda$), $F^2 (x) \approx \frac{3}{2} x = \frac{3}{2} \sqrt{V} \lambda$.

These results lead to the ratio of tensor to scalar perturbations $r$

$$r = \frac{P_g (k)}{P_R (k)}.$$  
(10)

As function of $\epsilon$, the inflation parameter $r$ become

$$r = 24\epsilon.$$  
(11)

Other perturbation quantity is the running of the scalar index $\frac{dn_s}{d\ln k}$, which is given in terms of $V (\phi)$ as

$$\frac{dn_s}{d\ln k} = \frac{m^2_{\text{pl}} V' \lambda}{2\pi V^2} \left( 3\frac{\partial \epsilon}{\partial \phi} - \frac{\partial \eta}{\partial \phi} \right),$$

$$= -\frac{m^4_{\text{pl}} \lambda^2}{8\pi^2} \left( \frac{V'^4}{V^6} - \frac{3V'' V'^2}{V^5} + \frac{V'''}{V^4} \right).$$

(12)
where \( V''' = \frac{d^3V}{d\phi^3} \).

Another important characteristic inflationary parameter is the number of e-folding \( N \) defined by

\[
N = -\frac{4\pi}{\lambda m_p^2} \int_{\phi_s}^{\phi_{\text{end}}} \frac{V'}{V} d\phi,
\]

where \( \phi_s \) and \( \phi_{\text{end}} \) are the values of the scalar field at the epoch when the cosmological scales exit the horizon and at the end of inflation, respectively.

In the next section, we give our results concerning a three potential models and present the evolution of various inflationary perturbation spectrum according to different potential parameters.

### 3 Perturbation spectrum for various potentials

#### 3.1 Exponential model

The exponential potential was studied in various occasions, for example the authors in ref.\[14\] have shown that inflation becomes possible in Braneworld model for a class of potentials ordinarily too steep to sustain accelerated expansion. They have also shown that this potential allows a particularly natural implementation of reheating via gravitational particle production.

Here we consider an exponential potential of following type

\[
V = V_0 \exp \left( -\frac{\alpha}{m_p} \phi \right),
\]

where \( V_0 \) and \( \alpha \) are constants.

For this type of potential, the scalar spectral index \( n_s \) and the ratio of tensor to scalar perturbations \( r \) take respectively the following expressions

\[
n_s - 1 = -\frac{\alpha^2 \lambda}{\pi V},
\]

\[
r = \frac{6\alpha^2 \lambda}{\pi V}.
\]

The running of the scalar index is presented by

\[
\frac{dn_s}{d \ln k} = -\frac{\alpha^4 \lambda^2}{4 \pi^2 V^2}
\]

Inflation ends when \( \epsilon = 1 \), the equation \( \epsilon = \frac{\alpha^2 \lambda}{4 \pi^2 V^2} \) leads to

\[
V_{\text{end}} = \frac{\alpha^2 \lambda}{4 \pi}.
\]

Taking into account the equation (13), the value of the potential when the cosmological scales exit the horizon is

\[
V_s = \frac{\alpha^2 \lambda}{4 \pi} (N + 1).
\]

In terms of \( N \) the inflation parameters become

\[
n_s - 1 = -\frac{4}{N + 1},
\]

\[
r = \frac{24}{(N + 1)^2},
\]

\[
\frac{dn_s}{d \ln k} = -\frac{4}{(N + 1)^2}.
\]


By using the equations (20) and (21) and WMAP7 observations data[15],

\[
0.963 \leq n_s \leq 1.002 \quad (95\% CL),
\]

\[
r < 0.36 \quad (95\% CL),
\]

we can show that \( N > 107 \). Thus, we can deduce an upper limit for the e-folding number without running.

The power spectrum of the curvature perturbations is given by

\[
P_R = \frac{\lambda \alpha^6 (N+1)^4}{48\pi^2 m_{pl}^4}.
\]

The observed value for \( P_R \) from WMAP7 is

\[
P_R = (2.28 \pm 0.15) \times 10^{-9} \quad (95\% CL).
\]

According to Eq.(26), the condition \( N > 107 \) implies that

\[
\lambda \lesssim 2.65 \times 10^{-14} \alpha^6 m_{pl}^4,
\]

which constitutes the necessary and sufficient condition on the brane tension with respect to parameter \( \alpha \) so that the exponential potential can describe the early inflation of the universe.

### 3.2 Monomial model

The exponential potential was recently introduced to describe tachyonic inflation, but in standard inflation this potential cannot reproduce many phenomena such as dissipation. The chaotic potential and its generalisation[16], can occur for field values, when the cosmological scales exit the horizon \( \phi_\ast \), below the four-dimensional Planck scale i.e. \( \phi_\ast < m_{pl} \). In ref.[7], we have studied a more generalized version of this potential in Braneworld model, and shown that the observation bound are satisfied. On the other hand, liddle et al.[9] have shown that observational constraints can be respected for an monomial potential, in particular for \( n = 2 \). Here, we are interested on studying the variation of various inflationary parameters, in Braneworld scenario, as function of \( n \) for diverse e-folding number values.

In this work we consider a potential of the form

\[
V = M\phi^n,
\]

where \( n \) is constant and \( M \) is a parameter of dimension \( [E]^{4-n} \).

For this type of potential, the scalar spectral index \( n_s \) and the ratio \( r \) are respectively given by

\[
n_s - 1 = \frac{m_{pl}^2 \lambda}{2\pi M} \frac{n}{\phi^{n+2}} (2n+1),
\]

\[
r = \frac{6m_{pl}^2 \lambda}{\pi M} \frac{n^2}{\phi^{n+2}}.
\]

The running of the scalar index is presented by

\[
\frac{dn_s}{d\ln k} = -\frac{m_{pl}^4 \lambda^2 n^2 (n+2) (2n+1)}{8\pi^2 M^2 \phi^{2n+4}}.
\]

To express the previous parameters of inflation only in terms of the e-folding number \( N \) and the exponent \( n \), it is convenient to calculate the values of \( \phi_{\text{end}} \) and \( \phi_\ast \) using Eq.(13)

\[
\phi_{\text{end}}^{n+2} = \frac{m_{pl}^2 \lambda}{4\pi M} n^2,
\]

\[
\phi_\ast^{n+2} = \frac{m_{pl}^2 \lambda}{4\pi M} n (N+1)+2N.
\]
The power spectrum of the curvature perturbations is given by

\[ P_R = \frac{16\pi \frac{n+4}{4\pi} M \frac{n+2}{n+2} m_{pl}^{2n+2}}{3n^2} \times \left( \frac{n(n+1)+2N}{4\pi} \right)^{\frac{n+2}{n+2}}. \]  

(34)

Eqs.(26, 34) imply that

\[ \lambda^{\frac{n-4}{n+2}} \leq \frac{1.45 \times 10^{-10} n^2 m_{pl}^{2n+2}}{M^{\frac{n+4}{4\pi} \left( \frac{n(n+1)+2N}{4\pi} \right)^{\frac{n+2}{n+2}}}} \]  

(35)

This is the condition for the brane tension according only to power spectrum of the curvature perturbations observational value for a relevant \( M \) energy scale of the potential. For a complete study, it’s necessary to check the other inflationary parameters. According to Eq.(33), the inflationary observables become

\[ n_s - 1 = -\frac{4n+2}{n(n+1)+2N}, \]  

(36)

\[ r = \frac{24n}{n(n+1)+2N}, \]  

(37)

\[ \frac{dn_s}{d\ln k} = -\frac{2(n+2)(2n+1)}{(n(n+1)+2N)^2}. \]  

(38)

Note that when \( n \to \infty \), the inflation parameters \( n_s, r \) and \( \frac{dn_s}{d\ln k} \) reduce to expressions of exponential potential case(Eqs. 20, 21, 22). In the following, we plot these observables as functions of \( n \).

Fig.1: \( n_s \) vs \( n \) for \( N = 50, 55, 60 \) for monomial potential \( V = M\phi^n \)

The fig.1 shows that there exist a significant region of variation of \( n_s \) according to \( n \), where the results are consistent with observations, in particular for large values of \( N \) and small values of \( n \). For large values of \( n \), the scalar spectral index becomes in disagreement with WMAP7 data except for very large \( N \). In that case, we recover the results obtained in the case of the exponential potential (Eq.14).

In particular, for \( n = 2 \) which corresponds to chaotic case, the observational constraints for \( n_s \) require that \( N \geq 68 \), Eq.(35) reduces then to

\[ \lambda \gtrsim \frac{M^3}{2 \times 10^{-27} m_{pl}^2} \]  

(39)
Fig.2: $r$ vs $n$ for $N = 50, 55, 60$ for monomial potential $V = M\phi^n$

In this case, the fig.2 shows that we can recover the observational results for large range of variation of exponent $n$ provided that $N$ be large.

Fig.3: $\frac{dn_s}{d\ln k}$ vs $n$ for $N = 50, 55, 60$ for monomial potential $V = M\phi^n$

In fig.3, we have plotted the variation of $\frac{dn_s}{d\ln k}$ according to $n$. We see that the values of the e-folding number $N$ compatibles with WMAP7 data, the running of the scalar index is small and negative. Thus, we can conclude that the spectral scalar index $n_s$ is scale invariant. This phenomena was already predicts by inflation theory (Harrison-Zeldovich invariant)[17].

3.3 Inverse power law potential

The inverse power law potential ($V = \frac{\mu}{\phi^m}$), has been studied in various context, specially, for modelisation of quintessence matter on the Brane world inflation[13] where it was shown that inflation occurs only for $m > 2$. This potential was also studied in tachyonic inflation model[19]. It have been shown that if the tachyon dominates the background dynamics, then it will either go into a dust-dominated phase ($m > 2$), power-law expansion for $m = 2$, or quasi de-Sitter accelerated expansion for $m << 2$. 
We consider the following inverse power law potential

\[ V = \frac{\mu}{\phi^m}, \]  

(40)

\(\mu\) is a parameter of dimension \([E]^{4+m}\).

For this type of potential, the scalar spectral index and the ratio of tensor to scalar perturbations take respectively the following expressions

\[ n_s - 1 = (-2m + 1) \frac{\lambda m_{pl}^2 m}{2\pi \mu} \phi^{(m-2)}, \]  

(41)

\[ r = 6\frac{\lambda m_{pl}^2 m^2}{\pi \mu} \phi^{(m-2)}. \]  

(42)

The running of the scalar spectral index is presented by

\[ \frac{dn_s}{d\ln k} = -m_{pl}^2 \frac{\lambda^2 m^2}{8\pi^2 \mu^2} \phi^{2m-4}. \]  

(43)

As above, we evaluate either the values of \(\phi_{\text{end}}\) and \(\phi_*\) using Eq.(13)

\[ \phi_{\text{end}}^{(m-2)} = \frac{4\pi \mu}{\lambda m_{pl}^2 m^2}, \]  

(44)

\[ \phi_*^{(m-2)} = \frac{4\pi \mu}{\lambda m_{pl}^2 m (m (N + 1) - 2N)}. \]  

(45)

The power spectrum of the curvature perturbations is given by

\[ P_R = \frac{16\pi \lambda \frac{2^{\frac{m+8}{m^2}} m_{pl}^{\frac{2m+8}{m^2}}}{3m^2 \mu^{\frac{6}{m+2}}}}{m (m (N + 1) - 2N)} \left( \frac{4\pi}{m (m (N + 1) - 2N)} \right)^{2 - \frac{4m}{m^2}} \]  

(46)

Eqs.(26, 46) imply that

\[ \lambda \geq \left( 1.27 \times 10^{-10} m^2 \right) \frac{2^{\frac{m+8}{m^2}}}{m_{pl}^{\frac{2m+8}{m^2}}} \mu^{\frac{6}{m+2}} \frac{1}{m (m (N + 1) - 2N)^{\frac{2m+8}{m^2}}}. \]  

(47)

As for the previous models, Eq.(47) shows the condition for the brane tension according to power spectrum of the curvature perturbations observational value for a relevant \(\mu\) energy scale of the potential.

To complete our study, we analyse the variations of all other inflationary parameters with respect to \(m\).

So, in terms of \(N\) and \(m\), the inflationary parameters become

\[ n_s - 1 = \frac{2 - 4m}{m (N + 1) - 2N}, \]  

(48)

\[ r = \frac{24m}{m (N + 1) - 2N}, \]  

(49)

\[ \frac{dn_s}{d\ln k} = -2 \frac{(m - 2) (2m - 1)}{(m (N + 1) - 2N)^2}. \]  

(50)

Note that when \(m \rightarrow \infty\), the inflation parameters \(n_s, r\) and \(\frac{dn_s}{d\ln k}\) reduce to expressions of exponential potential case(Eqs. 20, 21, 22). In the following, we plot these observables as functions of \(n\).
Fig.4: $n_s$ vs $m$ for $N = 50, 55, 60$ for inverse power law potential $V = \frac{\mu}{\phi^m}$

Fig.4 shows the variation of $n_s$ as function of $m$. We observe that, in order to confront $n_s$ with WMAP7 data we must have very large value of $N$. This result is compatible with the exponential potential case.

Fig.5: $r$ vs $m$ for $N = 50, 55, 60$ for inverse power law potential $V = \frac{\mu}{\phi^m}$

This figure which presents $r$ as function of $m$, confirm our above results concerning the very large values of e-folding number $N$ as condition of inflation.

In particular, for $m = 10$, the observational constraints for $n_s$ and $r$ require that $N \geq 128$. Then, for $m = 10$ and $N = 128$, Eq.(47) reduces to

$$\lambda \gtrsim \frac{3.75 \times 10^{-13} \mu^2}{m_{pl}^2}$$

(51)
Fig.6: $\frac{dn_s}{d\ln k}$ vs $m$ for $N = 50, 55, 60$ for inverse power law potential $V = \frac{\mu}{\phi^m}$

It’s clear, as shown in fig.6, that the running of the scalar index $\frac{dn_s}{d\ln k}$ becomes negligible for very large $N$, as predicted by WMAP7 observations.

### 3.4 Recapitulate

We present in this subsection a summary concerning the different inflationary parameters for the three types of potential at $V = V^*$ and $\phi = \phi^*$.

| Potentials | Monomial $V = M\phi^n$ | Inverse power law $V = \frac{\mu}{\phi^m}$ | Exponential $V = V_0 \exp \left( -\frac{\alpha}{m_p} \phi \right)$ |
|------------|------------------------|--------------------------------|-------------------------------------------------|
| $\epsilon$ | $\frac{n}{n(N+1)+2N}$ | $\frac{m}{m(N+1)-2N}$ | $\frac{1}{N+1}$ |
| $\eta$    | $\frac{n+1}{n(N+1)+2N}$ | $\frac{m+1}{m(N+1)-2N}$ | $\frac{1}{N+1}$ |
| $n_s - 1$ | $-\frac{4n+2}{n(N+1)+2N}$ | $\frac{2-4m}{m(N+1)-2N}$ | $-\frac{4}{N+1}$ |
| $r$       | $\frac{2n}{n(N+1)+2N}$ | $\frac{24n}{m(N+1)-2N}$ | $\frac{24}{N+1}$ |
| $\frac{dn_s}{d\ln k}$ | $\frac{-2(m-2)(2m-1)}{m(N+1)-2N}$ | $\frac{-4}{(N+1)^2}$ | $\frac{-4}{(N+1)^2}$ |

This table shows that, for $V = V^*$ and $\phi = \phi^*$, all inflationary parameters depend only on the e-folding number $N$ and exponents of the scalar fields for the two first potentials. Note that for large $n$ and $m$, the monomial and inverse power law potentials lead to similar results in the exponential potential case.

### 4 Conclusion

In this work, we have studied various types of potentials by considering different inflationary perturbation spectrum parameters. We have analyzed in particular the exponential potential: $V = V_0 \exp \left( -\frac{\alpha}{m_p} \phi \right)$, monomial potential: $V = M\phi^n$ and inverse power law model: $V = \frac{\mu}{\phi^m}$ in the framework of the Randall-Sundrum type-2 model. We have shown that, the monomial potential provides the best fit results with recent WMAP7 observational constraints. Indeed, for the exponential potential, the observational constraints require that $N > 107$, so that $n_s$ and $r$ simultaneously are consistent with WMAP7 data assuming a negligible running. However, for the inverse power law potential, the observational constraints on the inflationary parameters $n_s$ and $r$ have lead to consider a very large e-folding number $N$. 
On the other hand, for the monomial potential, the parameters $n_s$ and $r$ are consistent with the observations for small values of $n$ and large $N$ (notably for the scalar spectral index $n_s$). Finally, note that, as $n$ and $m$ increase, the inverse power law potential and monomial present the same behaviors as the exponential one.

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