ROTATIONAL BROWNIAN MOTION OF A MASSIVE BINARY

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ABSTRACT

The orientation of a massive binary undergoes a random walk due to gravitational encounters with field stars. The rotational diffusion coefficient for a circular-orbit binary is derived via scattering experiments. The binary is shown to reorient itself by an angle of the order of \((m/M)^{1/2}\) during the time that its semimajor axis shrinks appreciably, where \(M\) is the binary mass and \(m\) the perturber mass. Implications for the orientations of rotating black holes are discussed.

Subject heading: stellar dynamics

1. INTRODUCTION

Two sorts of Brownian motion are defined by statistical physicists. Translational Brownian motion is the irregular motion exhibited by a massive particle as it collides with molecules in a fluid. Collisions also affect the spin and orientation of particles; theories of rotational Brownian motion are concerned with the evolution of these quantities due to encounters. Rotational Brownian motion was first discussed by Debye (1913, 1929) in the context of dielectric theory. The polarization of a dielectric material is a competition between torques due to the imposed field, which tend to align polar molecules, and collisions, which tend to destroy the alignment.

The likely existence of black holes with masses much greater than those of stars has led to a renewed interest in Brownian motion among astrophysicists (Merritt 2001, hereafter Paper I; Milosavljevic & Merritt 2001; Dorband, Hemsendorf, & Merritt 2001). Translational Brownian motion of black holes in galactic nuclei may be directly observable (e.g., Reid et al. 1999; Backer & Sramek 1999), allowing constraints to be placed on black hole masses. Translational Brownian motion may also affect the rate of tidal disruption of stars by a nuclear black hole (e.g., Young 1977) or the efficiency with which a binary black hole can interact with stars as it wanders through a nucleus (Quinlan & Hernquist 1997; Milosavljevic & Merritt 2001). In Paper I, the translational Brownian motion of a massive binary was discussed; it was shown that the amplitude of the wandering can be increased by a modest factor compared to that of a single point mass due to exchange of energy between field stars and the binary.

Here we consider the second kind of Brownian motion experienced by a binary, the irregular variation of the orientation of the binary’s spin axis due to encounters. A single field star that passes within a distance \(\sim a\) of the binary, with \(a\) the binary’s semimajor axis, will exchange orbital angular momentum with the binary, leading to both a change in the binary’s orbital eccentricity as well as a change in the orientation of the binary’s spin axis. Repeated encounters will cause the binary’s orbital eccentricity to evolve (usually in the direction of increasing eccentricity) and will also cause the orientation of the binary to undergo a random walk. The first process has been discussed extensively (e.g., Mikkola & Valtonen 1992; Quinlan 1996); the second, discussed here for the first time, is called rotational Brownian motion, by analogy with the similar process that occurs in a fluid of polar molecules (e.g., McConnell 1980).

The orbital orientation of a black hole binary influences the direction of the spin axis of the single black hole that forms after coalescence of the binary due to emission of gravitational radiation (e.g., Flanagan & Hughes 1998). The spin axis, in turn, is thought to determine the plane of the inner accretion disk that forms around the black hole (Bardeen & Petterson 1975), the direction of jets launched from the accretion disk (Rees 1978), etc. Thus, changes in the orientation of a black hole binary as it coalesces may have observable consequences.

In §2 the conservation equations for the interaction of a field star with a massive binary are presented; most of these have been seen before and were used in earlier studies to derive the rates of change of the binary’s separation and eccentricity. The rotational diffusion equation is presented in §3 and the order of magnitude of the rotational diffusion coefficient \((\Delta\Omega)^2\) is derived. Section 4 describes the numerical derivation of \((\Delta\Omega)^2\) from scattering experiments. Section 5 presents solutions of the diffusion equation and derives a relation between the degree of hardening of a binary and the change in its orientation.

2. ENCOUNTER KINEMATICS

Consider an encounter of a single particle (the “field star”) with a binary. Assume that the binary remains bound during the encounter, appropriate if the mass of the field star is much less than the mass of each of the components of the binary or if the binary is sufficiently hard. At early times, the field star has velocity \(\mathbf{v}_0\) with respect to the center of mass of the field star–binary system, and its impact parameter is \(p\). A long time after the encounter, the velocity \(\mathbf{v}\) of the field star attains a constant value. Conservation of linear momentum implies that the change \(\Delta V\) in the velocity of the binary’s center of mass is given by

\[
\Delta V = -\frac{m_f}{M_{12}} \Delta \mathbf{v},
\]

where \(m_f\) is the mass of the field star and \(M_{12} = M_1 + M_2\) is the mass of the binary. This relation was used in Paper I in conjunction with scattering experiments to investigate the translational Brownian motion of a binary due to encounters.
The energy of the field star–binary system, expressed in terms of pre-encounter quantities, is

\[ E_0 = \frac{1}{2} m_f v_0^2 + \frac{1}{2} M_{12} V_0^2 - \frac{GM_1 M_2}{2a_0} \quad (2a) \]

\[ = \frac{1}{2} m_f \left( 1 + \frac{M_f}{M_{12}} \right) v_0^2 - \frac{GM_1 M_2}{2a_0} , \quad (2b) \]

with \( V_0 = -(m_f/M_{12}) v_0 \) the initial velocity of the binary’s center of mass and \( a_0 \) the binary’s initial semimajor axis. After the encounter, we have

\[ E = \frac{1}{2} m_f \left( 1 + \frac{m_f}{M_{12}} \right) v^2 - \frac{GM_1 M_2}{2a} \quad (3) \]

and \( E = E_0 \), so that

\[ \delta \left( \frac{1}{a} \right) = \frac{m_f (v^2 - v_0^2)}{GM_1 M_2} \left( 1 + \frac{m_f}{M_{12}} \right) \approx \frac{m_f (v^2 - v_0^2)}{GM_1 M_2} , \quad (4b) \]

where the latter relation assumes \( M_{12} \gg m_f \). This result was used by Hills (1983, 1992), Mikkola & Valtonen (1992), and Quinlan (1996) to compute the hardening rate \( (d/dt)(1/a) \) of a massive binary due to encounters.

The angular momentum of the field star–binary system about its center of mass, expressed in terms of pre-encounter quantities, is

\[ \mathbf{L}_0 = m_f \left( 1 + \frac{m_f}{M_{12}} \right) \mathbf{l}_0 + \mu_{12} \mathbf{l}_{10} , \quad (5) \]

where \( \mu_{12} \equiv M_1 M_2 / M_{12} \), \( \mathbf{l}_0 \equiv \mathbf{p} v_0 \), and \( \mathbf{l}_{10} \equiv \mathbf{L}_{10}/\mu_{12} \), with \( \mathbf{L}_{10} \) the binary’s spin angular momentum. Conservation of angular momentum during the encounter gives

\[ \delta \mathbf{l}_b = -\frac{m_f}{\mu_{12}} \left( 1 + \frac{m_f}{M_{12}} \right) \delta \mathbf{l} \approx -\frac{m_f}{\mu_{12}} \delta \mathbf{l} . \quad (6b) \]

Changes in \( |\mathbf{b}| \) correspond to changes in the binary’s orbital eccentricity \( e \) via the relation \( e^2 = 1 - l_b^2 / GM_{12} \) (Mikkola & Valtonen 1992; Quinlan 1996). Changes in the direction of \( \mathbf{b} \) correspond to changes in the orientation of the binary, leading to rotational diffusion.

3. ROTATIONAL DIFFUSION EQUATION

Let \( F(\theta, \phi, \mathbf{d} \Omega) \) be the probability that the spin axis of the binary is oriented within the solid angle \( d\Omega \) at time \( t \). We seek an equation describing how \( F \) evolves with time because of encounters of field stars with the binary, if each encounter is assumed to produce a negligibly small change in \( \theta \) and \( \phi \). If we imagine that \( \mathbf{L}_b \) is initially oriented parallel to the \( \mathbf{d} \) axis and that the encounters leading to changes in the orientation of the binary are isotropic in velocity and direction, then \( F \) will evolve in such a way as to remain a function of \( \theta \) alone, \( F = F(\theta, t) \). These are the same assumptions made by Debye (1929) in his theory of the rotational Brownian motion of spherical molecules. Debye showed that the evolution equation for \( F \) in the absence of an external torque is

\[ \frac{\partial F}{\partial t} = \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \langle \Delta \theta^2 \rangle}{\partial \theta} \right) , \quad (7) \]

where \( \langle \Delta \theta^2 \rangle \) is the rotational diffusion coefficient,

\[ \langle \Delta \theta^2 \rangle = \int \Psi(d\Omega, \ d\Omega') d\theta' d\Omega' , \quad (8) \]

\( \Psi(d\Omega, d\Omega') \) being the probability that during a unit interval of time a binary whose angular momentum \( \mathbf{L}_b \) is directed toward \( d\Omega \) will reorient itself such that \( \mathbf{L}_b \) lies within \( d\Omega' \), and \( \theta \) is the angular separation between \( d\Omega \) and \( d\Omega' \).

From its definition, \( \langle \Delta \theta^2 \rangle \) is the sum over a unit interval of time of \( (\delta \theta)^2 \) due to encounters with field stars. It can be expressed in terms of mean changes in the components of the binary’s angular momentum as follows. Let the binary be initially oriented with its spin vector along the z-axis, \( \mathbf{l}_{10} = l_{10} \mathbf{e}_z \). After an encounter, the angle \( \theta \) between initial and final spins is

\[ \cos \theta = \frac{l_{b,z}}{l_b} = \frac{l_{b,z}}{\sqrt{l_{b,x}^2 + l_{b,y}^2 + l_{b,z}^2}} . \quad (9) \]

Expanding equation (9) to second order in \( \theta \) and taking averages, we find

\[ \langle \Delta \theta^2 \rangle = \frac{\langle \Delta l_{b,z}^2 \rangle + \langle \Delta l_{b,y}^2 \rangle}{l_{b}^2} , \quad (10) \]

which can further be expressed in terms of changes in the angular momentum components of the field star via equation (6b) as

\[ \langle \Delta \theta^2 \rangle = \frac{m_f^2}{\mu_{12}^2} \left( \langle \Delta l_{b,z}^2 \rangle \langle \Delta l_{b,y}^2 \rangle \right) \approx \frac{m_f^2}{GM_1 M_2 \mu_{12} a} \left( \langle \Delta l_{b,z}^2 \rangle \langle \Delta l_{b,y}^2 \rangle \right) . \quad (11b) \]

The last relations assume \( M_{12} \gg m_f \). Specializing to the case of an equal-mass circular-orbit binary, equation (11b) becomes

\[ \langle \Delta \theta^2 \rangle = 16 \left( \frac{m_f}{M_{12}} \right)^2 \frac{\langle \Delta l_{b,z}^2 \rangle \langle \Delta l_{b,y}^2 \rangle}{GM_1 M_2 a} . \quad (12) \]

Equation (12) is a prescription for computing the rotational diffusion coefficient from scattering experiments. This prescription is applied in the next section.

We can estimate the order of magnitude of the rotational diffusion coefficient by noting that \( \langle GM_{12} l_b^2 \rangle = V_{bin} \), the relative velocity of \( M_1 \) and \( M_2 \), and that a single close encounter produces a change in \( l \) of the field star of the order of \( V_{bin} a \). The rate of close encounters is \( \sim 2\pi GM_{12} n_f a / \sigma_f \), with \( n_f \) and \( \sigma_f \) the number density and one-dimensional velocity dispersion of the field stars (Paper I). Thus,

\[ \langle \Delta l_{b,z}^2 \rangle = \langle \Delta l_{b,y}^2 \rangle \approx \frac{m_f^2}{GM_{12} n_f} \frac{2\pi GM_1 M_2 a}{\sigma_f} . \quad (13) \]

where \( \rho_f \equiv m_f n_f \).

The same encounters that induce rotational diffusion will also cause the binary to harden, according to equation (4b),
The hardening rate can be written
\[ \frac{d}{dt} \left( \frac{1}{a} \right) = \frac{G \dot{M} a}{\sigma_f}, \]
with \( H \approx 15 \) for a hard binary (e.g., Quinlan 1996). Thus,
\[ \langle \Delta a^2 \rangle \approx \frac{32 \pi m_f}{H M_{12}} \int \frac{d}{dt} \left( \frac{1}{a^2} \right) \approx \frac{32 \pi m_f}{H M_{12} \Delta a^2}, \]
with \( \Delta a^2 \approx a(d/dt)(1/a) \). Equation (15) suggests that rotational diffusion occurs on a timescale that is of the order of \( M_{12}/m_f \) times the hardening time, or that the binary will rotate by an angle of the order of \( (m_f/M_{12})^{1/2} \) in the time it takes to harden significantly. This result is assessed more quantitatively in §5.

4. NUMERICAL COMPUTATION OF THE DIFFUSION COEFFICIENT

The same set of scattering experiments described in Paper I were used here to evaluate \( \langle \Delta \ell \rangle \) and \( \langle \Delta \ell^2 \rangle \). As described more fully in that paper, field stars were treated as massless particles moving in the potential of the binary whose center of mass and orbital parameters remained fixed. Orbits were integrated using the routine DOP853 of Hairer, Nørsett, & Wanner (1991), an 8th (6th) -order embedded Runge-Kutta integrator; each field star was assumed to begin at a position \((x, y, z) = (-\infty, p, 0)\) and was advanced from \( r = \infty \) to \( 50a \) along a Keplerian orbit about a point mass \( M_{12} \). The integrations were terminated when the star had moved a distance from the binary that was at least 100 times its initial distance with positive energy. The binary’s orbit was assumed to be circular; the orientation of the binary’s orbital plane with respect to the initial velocity vector of the field star and the initial phase of the binary were chosen randomly for each integration. Unless otherwise indicated, distances and velocities are given below in program units of \( a \) and \( (GM_{12}/a)^{1/2} \), respectively. The orbit integrations and the expressions given above relating changes in the field star’s angular momentum to the orientation of the binary were checked by carrying out a limited set of experiments using a fully general, three-body integrator and various values for \( \eta/\eta_{12} \). The expressions derived above for \( \delta \theta \), etc. were found to be accurately reproduced in the limit of small \( \eta/\eta_{12} \).

Consider first the rotational diffusion coefficient corresponding to interactions with stars of a single velocity \( V \) at infinity. Multiplying \( \langle \delta \theta^2 \rangle \) in a single encounter by \( 2\pi \eta \dot{m}_V \), with \( \eta \) the parameter and \( \dot{m}_V \) the density of field stars, and integrating over \( p \) gives
\[ \langle \delta \theta^2 \rangle (V, \eta_{\text{max}}) = 2\pi \eta \dot{m}_V \int_0^{\eta_{\text{max}}} dp \dot{p} \delta \theta^2 \]
\[ = 2\pi \eta \dot{m}_V \int_0^{\eta_{\text{max}}} dp \frac{\delta \ell^2}{\ell^2} \]
\[ = \frac{32 \pi \eta \dot{m}_V}{GM_{12} a} \left( \frac{m_f}{M_{12}} \right)^2 \int_0^{\eta_{\text{max}}} dp \frac{\delta \ell^2}{\ell^2} \]
(c.f. eq. [12]), where \( \overline{\delta \ell^2}, \overline{\delta \ell^2} \) are defined as mean square changes at fixed \( V, v_{\text{max}} \), averaged over many encounters in which the orientation of \( V \) with respect to the binary’s spin axis is taken to be random. Figure 1 shows the distribution of field-star velocity changes as a function of \( p \) for \( V = 0.5V_{\text{bin}} \).

We define a dimensionless, velocity-dependent diffusion coefficient \( D_\Delta \) as
\[ \langle \delta \theta^2 \rangle (V, \eta_{\text{max}}) = \pi \eta^2 \dot{m}_V^2 \left( \frac{m_f}{M_{12}} \right)^2 D_\Delta (V, \eta_{\text{max}}), \]
\[ D_\Delta (V, \eta_{\text{max}}) \equiv 16 \left( \frac{V}{V_{\text{bin}}} \right) \int_0^{\eta_{\text{max}}} d(p/\eta) \frac{\delta \ell^2 + \delta \ell^2}{a^2 V_{\text{bin}}} \]
with \( V_{\text{bin}} = GM_{12} a^{1/2} \). Figure 2 plots \( D_\Delta \) as a function of \( V \) and \( \eta_{\text{max}} \). The most effective encounters are those with small \( V \) and \( \eta_{\text{max}} \); encounters with impact parameters \( p > (2GM_{12}a^{1/2})/V \) have no significant change in the binary’s orientation. Hence, \( D_\Delta \) reaches a maximum value at \( p \approx (2GM_{12}a^{1/2})/V \) before leveling off at larger \( p \).

Multiplying \( \langle \delta \theta^2 \rangle \) by \( f_f (v_f) \), the distribution of field-star velocities, and integrating \( dp \) gives \( \langle \Delta \theta^2 \rangle \). We assume a Maxwellian distribution of field-star velocities,
\[ f_f (v_f) = \frac{1}{(2\pi \sigma_f^2)^{3/2}} e^{-v_f^2/2\sigma_f^2}, \]
so that
\[ \langle \Delta \theta^2 \rangle = \frac{4\pi}{(2\pi \sigma_f^2)^{3/2}} \int_0^{\infty} dV V^2 e^{-V^2/2\sigma_f^2} \langle \delta \theta^2 \rangle. \]

This can be written in terms of a dimensionless coefficient \( L \),
\[ \langle \Delta \theta^2 \rangle = \frac{m_f}{M_{12}} \frac{GM_{12} a}{\sigma_f} L, \]
\[ L \equiv \frac{\sqrt{2}\pi}{2} \int_0^{\infty} dz z^2 e^{-z^2/2} D_\Delta (z V_{\text{bin}}/S, R_p) \]

The term \( L \) is defined in analogy with the dimensionless coefficients \( H, J, \) and \( K \) that describe respectively the rates of change of the binary’s energy, the rate of mass ejection by the binary, and the binary’s eccentricity growth rate (Quinlan 1996). Note that \( L \) is a function of two parameters, \( L = L(R, S) \), where
\[ R \equiv \frac{p_{\text{max}}}{p_f} = \frac{p_{\text{max}} \sigma_f^2}{GM_{12}}, \quad S \equiv \frac{V_{\text{bin}}}{\sigma_f}. \]

The term \( R \) is the maximum impact parameter in units of the radius of gravitational influence of the binary, and \( S \) is the dimensionless hardness of the binary. Figure 3 plots \( L(R, S) \). At a given hardness \( S \), \( L(R, S) \) reaches its limiting value \( L_{\infty} \approx L(\infty, S) \) for \( R \approx 3/S \). For all but the softest binaries, this limiting value is reached before \( R = 1 \), which is roughly the value of \( R \) we expect for a massive binary at the center of a steeply falling density cusp (Paper I). Hence, we ignore the dependence of \( L \) on \( p_{\text{max}} \) in what follows and write
\[ L(S) = L(\infty, S). \]

The quantity \( L(S) \) varies from \( \approx 25 \) for the softest binaries (\( S \approx 1 \)) to \( \approx 60 \) for \( S \approx 10 \).

5. SOLUTIONS OF THE EVOLUTION EQUATION

Equation (7) describes the evolution of \( F(\theta, t) \), the probability distribution for the binary’s orientation, given the
rotational diffusion coefficient \(\Delta \theta^2\). The latter (eq. [20a]) is a function of the binary separation \(a\), directly via the factor \(G\rho_f/a/\sigma_f\), and indirectly via the weak dependence of \(L\) on binary hardness (Fig. 3). Furthermore, the timescale for the binary to reorient itself is shorter than the hardening time \(|(1/a)(da/dt)|^{-1}\) (cf. eq. [15]); hence, the change in \(a\)
cannot be neglected when solving for the evolution of the orientation.

We nevertheless begin by considering an idealized model in which \( \langle \Delta \theta' \rangle \), \( a \), and the parameters \( \langle \rho_f, \sigma_f \rangle \) that describe the background are assumed constant in time. The evolution equation (7) becomes

\[
\frac{\partial F}{\partial t} = \frac{1}{2} \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) \frac{\partial F}{\partial \mu} \right],
\]

where \( \mu = \cos \theta, \tau = t/t_0 \), and

\[
t_0 = \frac{2}{\mu_f} \left( \frac{M_{12}}{m_f} \right) \frac{\sigma_f}{G \rho_f a L} .
\]

Consider the evolution of \( \bar{\mu}(t) = 2\pi \int_1^\infty F(\mu', t) \mu' d\mu' \), the expectation value of \( \mu \). We have

\[
\frac{d\bar{\mu}}{dt} = 2\pi \int_1^\infty \frac{\partial F}{\partial \mu'} \mu' d\mu' = 2\pi \int_1^\infty \frac{\partial F}{\partial \mu'} \mu' \left( 1 - \mu^2 \right) d\mu' = -2\pi \int_1^\infty \frac{\partial F}{\partial \mu'} \mu' d\mu' = -\bar{\mu}
\]

or

\[
\bar{\mu}(t) = \bar{\mu}_0 e^{-\tau} .
\]

Hence, \( t_0 \) is the time constant for relaxation to a uniform distribution of orientations \( \bar{\mu} = 0 \), given a fixed binary separation and unchanging stellar background. It can be expressed in physical units as

\[
t_0 = 6.1 \times 10^9 \text{ yr} \left( \frac{M_{12}}{10^6 m_f} \right) \left( \frac{L}{50} \right)^{-1} \left( \frac{10^6 M_\odot \text{ Mpc}^{-3}}{\rho_f} \right)^{-1} \times \left( \frac{200 \text{ km s}^{-1}}{a} \right) \left( \frac{1 \text{ pc}}{10^9 \text{ yr}} \right)^{-1} .
\]

Next consider the more interesting case in which \( a \), as well as the parameters \( \rho_f \) and \( \sigma_f \) that describe the field stars, may be changing with time. This time dependence is not easily specified, but we can make progress by changing evolution variables from \( t \) to \( x = \log(a/a_0) \). First combining equations (7) and (20a), we find

\[
\frac{\partial F}{\partial t} = \frac{L}{4 M_{12}} \frac{G \rho_f a}{\sigma_f} \frac{\partial}{\partial \mu} \left[ \left( 1 - \mu^2 \right) \frac{\partial F}{\partial \mu} \right] .
\]

Now, changing variables,

\[
\frac{\partial F}{\partial t} = \frac{\partial F}{\partial x} \frac{dx}{dt} = \frac{\partial F}{\partial x} \frac{1}{a} \frac{da}{dt} = \frac{\partial F}{\partial x} \frac{G \rho_f H a}{\sigma_f} (\text{cf. eq. [14]), and so})
\]

\[
\frac{\partial F}{\partial x} = -\frac{1}{4 H M_{12}} \frac{G \rho_f a}{\sigma_f} \frac{\partial}{\partial \mu} \left[ \left( 1 - \mu^2 \right) \frac{\partial F}{\partial \mu} \right] .
\]

This equation gives the evolution of the binary’s orientation in terms of changes in its semimajor axis \( a \), with no explicit dependence on the parameters that describe the stellar background. Ignoring the weak dependence of \( H \) and \( L \) on \( a \), appropriate for a hard binary, we find, as before,

\[
\bar{\mu}(a) = \bar{\mu}_0 \left( \frac{a}{a_0} \right)^{(1/2)/(L/H)(m_f/M_{12})} .
\]

The exponent in this expression is of the order of \( m_f/M_{12} \ll 1 \). Hence, we can write

\[
\bar{\mu}(a) \approx 1 + \frac{1}{2 H M_{12}} \log \left( \frac{a}{a_0} \right) ,
\]

where \( \bar{\mu}_0 \) has been set to unity, corresponding to an initial orientation parallel to the \( \theta = 0 \) axis.

For the case of binary supermassive black holes, if we define \( a_0 \) as the separation when the binary first forms a bound pair, we expect gravitational radiation coalescence to occur when \( a/a_0 \approx 10^{-2} \) (Merritt 2000). Since \( L/2H \approx 2 \), for a hard binary, we find for the expectation value of \( \mu \) at coalescence:

\[
\bar{\mu} \approx 1 - 10 \left( \frac{m_f}{M_{12}} \right) ,
\]

Writing \( \delta \theta \equiv [2(1-\bar{\mu})]^{1/2} \), the rms change in the angle defined by the binary’s spin axis, this becomes

\[
\delta \theta \approx \sqrt{20 m_f / M_{12}} , \frac{m_f}{M_{12}} \ll 1 .
\]

For \( m_f/M_{12} = 10^{-6} \), e.g., a binary of mass \( 10^6 M_\odot \), surrounded by \( 1 M_\odot \) stars, this predicts \( \delta \theta \approx 0.25 \). For a binary of so-called intermediate-mass black holes with \( M_{12} \approx 10^3 M_\odot \), \( \delta \theta \approx 8^\circ \).

Milosavljevic & Merritt (2001) give information about the rotational Brownian motion of a massive binary in a set of \( N \)-body simulations. They show (in their Fig. 11) the evolution of the orientation of an equal-mass circular-orbit binary in three simulations with \( m_f/M_{12} = \{ 164, 328, 655 \} \). The simulations all follow the evolution of the binary until \( a/a_0 \approx 10^{-4} \); hence, equation (32) predicts \( \delta \theta \approx \{ 14^\circ, 10^\circ, 7^\circ \} \). These values are in excellent agreement with the angular deflections shown in Figure 11 of Milosavljevic & Merritt (2001).

Under what circumstances could the reorientation angle of a supermassive black hole binary be much greater than the modest values predicted here? Since \( \delta \theta \propto m_f/M_{12} \), one possibility is for the perturbing objects to be much more massive than \( 1 M_\odot \); for instance, they could be supergiant stars \( (m_f \approx 10^2 M_\odot) \), star clusters \( (m_f \approx 10^3 M_\odot) \), or giant molecular clouds \( (m_f \approx 10^4 M_\odot) \). Such physically large objects would presumably produce most of their perturbing torques when the binary separation was greater than or comparable to their own size.

An even more extreme possibility is that the perturbing objects are themselves supermassive black holes. Consider for example a binary supermassive black hole at the center of a giant elliptical galaxy. The stellar densities at the centers of such galaxies are low, and one might expect the coalescence of the binary to stall because of a shortage of interacting stars (e.g., Milosavljevic & Merritt 2001). In this case, further decay of the binary might await the infall of a third and subsequent black holes from later galactic mergers (e.g., Valtonen 1996). Mass ratios \( m_f/M_{12} \) would then be of the order of \( 10^{-2} - 10^0 \) and significant reorientation of the binary could occur before coalescence.
Orientations of radio jets in Seyfert galaxies are known to be almost random with respect to the plane of the stellar disk (e.g., Kinney et al. 2000). If jets are launched parallel to the spin axes of rotating black holes, this implies that the spins of nuclear black holes bear almost no relation to disk spins. While the modest reorientation rates derived here are probably insufficient to explain this phenomenon, a related mechanism may suffice. The spin orientation of a supermassive black hole that forms via repeated mergers with smaller black holes would undergo a random walk due to the random orbital inclinations of the infalling black holes. If nuclear black holes in Seyfert galaxies grew through multiple mergers before the formation of the stellar disks, one might expect their spin axes to be uncorrelated with those of the stars and gas.

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