Suggestions on Photons and on Fermions

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In this paper we suggest a configuration of photons consistent with a spin \( h \), and a configuration of the fermions coherent with a spin \( h/2 \). These suggested configurations open the way to further analyses which lead to the following conclusions:

- There cannot exist elementary particles of spin \( h/2 \) with a mass inferior to \( 1m_e \) or with a radius greater than \( 1\lambda_e \).
- The electrostatic force derives from the centrifugal forces inherent to the spin and are propagated by photons.
- The derivation of the electrostatic force explains the existence of positive and negative charges and Coulomb’s law.
- The enormous differences between the centrifugal forces and the centripetal forces at the surface of the protons give rise to quantic fluctuations of space which generate the energy flows necessary for equilibrium. These energy flows can explain gravitation and the strong force.
- The mass of the proton, \( m_p \), and the mass of the neutron, \( m_n \), must each have a concrete value required for the cohesion of the atomic nuclei. The suggested hypotheses show that the relation \( m_n/m_p \) must be very slightly greater than 1.00, and that, by a first approximation, both \( m_n \) and \( m_p \) must be slightly less than 1851 \( m_e \).
- There are no “gravitons” and no “gluons”; the “messenger particles” are always the very real photons.

I. PREAMBLE

Goudsmit and Uhlenbeck discovered electron spin in 1925, while trying to find out what causes very close parallel lines to appear in the optical spectrum of hydrogen. They were able to show that these lines could be due to the fact that the electron possesses a natural angular impulse and bipolar magnetic momentum, whose component must be equal to \( h/2 \) or \( -h/2 \).

When describing the circumstances of the discovery, Uhlenbeck commented that both he and Goudsmit imagined that the electron would be configured as a sphere in permanent rotation, but that they realised that, given the mass of the electron a spin momentum of \( h/2 \) would require the tangential velocity at its equator to be many times the speed of light. Two years later, the experiments of Phipps and Taylor showed clearly that the electron does in fact possess a natural angular impulse, \( s \), whose value is \( h/2 \), and soon after, Dirac used Schrödinger’s equation while substituting the classic equation \( E = (p^2/2m_0) + V \) by its relativistic form \( E = (c^2p^2 + m_0c^4)^{1/2} + V \), to show that the electron must have a spin of \( s = h/2 \). Robert Eisberg hailed Dirac’s achievement as “a great victory for the theory of relativity, since it established electron spin on very sound theoretical foundations, and showed that it is intimately connected to relativity”. [1] pp. 326-27.

In 1916 Nernst suggested that the quantic fluctuations of space must cause an electromagnetic radiation which would therefore be inherent to space and, consequently, have a relativistically invariant spectrum.

In 1958 Sparnaay found this radiation when he was measuring the Casimir effect at temperatures close to absolute zero. He detected some radiation, which was independent of temperature and whose spectrum was such that the intensities of its flows are inversely proportional to the cubes of the wavelengths, which is a necessary condition for the radiation to be relativistically invariant. In 1997, S.K. Lamoreaux carried out new measurements of the intensity of the energy flow of zero-point radiation, using a different method, and reached the same measurements as Sparnaay’s.

A function of spectral distribution which is inversely proportional to the cubes of the wavelengths implies a distribution of energies which is inversely proportional to the 4th power of the wavelengths, because the energies of the photons are inversely proportional to their wavelengths. In 1969, Timothy H. Boyer showed that the spectral density function of zero-point radiation is:

\[
 f_\varphi(\lambda) = \frac{1}{2\pi^2 \left(\lambda_\ast\right)^3},
\]

where \( \lambda_\ast \) is the number giving the measurement of the wavelength \( \lambda \).

This function produces the next for the corresponding energies.

\[
 E_\varphi(\lambda) = \frac{1}{2\pi^2} \frac{hc}{\lambda_\ast} \left(\frac{1}{\lambda_\ast}\right)^3
\]

For \( \lambda \to 0 \) \( E_\varphi(\lambda) \to \infty \). There must be, therefore, a threshold for \( \lambda_\ast \), which hereafter will be designated by the symbol \( q_\lambda \).

* * *

This paper is closely related to “Interactions between zero-point radiation and electrons” ([2] in REFERENCES) and we include here the following extracts from it, to facilitate the subsequent argument.

Zero-point radiation proceeds equally from all directions of space, and its interactions with electrons could play the role of the “Poincaré tensions”, preventing the electrons from shattering as a result of the repulsion of...
their charge against itself. For this to be the case, there must operate the equation:

\[ x^3 = \frac{4\pi^3}{3\alpha}(k\lambda)^4(r_x)^4[B]_m; \quad (1) \]

equation (17) in [2], where:

\[ x = \text{measurement of the wavelength of the photons with the greatest energy in zero-point radiation, expressed in } q\lambda \text{ (quanta of wavelength).} \]

\[ k\lambda = \text{measurement of the unit of length, } l_e, \text{ in the } (e, m_e, c) \text{ system, expressed in } q\lambda. \]

\[ r_x = \text{measurement of the radius of the electron, expressed in } l_e. \]

\[ [B]_m = \frac{7}{48}B - \frac{11}{50}B^2 + \cdots + T_mB^m; \]

where \( B = \frac{2\pi}{\alpha} \left( \frac{k\lambda}{\alpha} \right) \) and

\[ T_m = (-1)^{m-1} \left[ \frac{1}{m+1} + \frac{2}{m+2} - \frac{3}{m+3} - 1 - \frac{m(m-1)}{6} \right] \]

p.9 in [2].

The hypothesis that zero-point radiation is also the effective cause of gravitational attraction between two electrons leads to the equation:

\[ x^3 = \frac{\pi^3}{3\alpha}(k\lambda)^2(r_x)^2[G_e]_m; \quad (2) \]

In [2] it was also deduced that

\[ r_x = 1l_e \]

\[ k\lambda = \left( \frac{1}{2\pi G_e} \right)^{1/2}, \quad G_e = \frac{1}{2\pi(k\lambda)^2} \quad (3) \]

where \( G_e \) is the numerical coefficient of the gravitational constant in the \((e, m_e, c)\) system \( q\lambda = \alpha^{1/2}L_P \), where \( L_P \) is the Planck length.

\[ k\lambda = 8.143375 \times 10^{20} \]

\[ x = 5.257601 \times 10^{27} \] \quad (4)

**II. CONJECTURES ON PHOTONS**

We know that photons have the following properties:

a) They are “energy packets” which possess the quantity of action \( h = \frac{2\pi m_e c^2}{\alpha} \), and can behave as particles.

b) They move at the speed of light along rectilinear trajectories. These are geodesic lines, in a space without curvature. Within the space defined by the 3-dimensional spherical surface \( w^2 + x^2 + y^2 + z^2 = R^2 \), the geodesic lines are great circles, but in the case of our Universe, the immense size of \( R \) makes it very difficult not to confound them with straight lines.

c) Their wave-motion follows planes perpendicular to their trajectories, producing a spin of \( s = h = \frac{m_e c}{\alpha} \).

d) They only differ from each other in the quantity of energy, \( E \), which is contained in the packets in question. This quantity is given by the equations

\[ E = h\nu = \frac{hc}{\lambda} \]

where \( \nu \) is the frequency (number of photons per time-unit) and \( \lambda \) is the wavelength. Since \( \nu \) and \( \lambda \) are linked by the relation \( \lambda\nu = c \), we need to consider only one of these variables.

Our aim, to investigate possible configurations of photons, implies that it is preferable to characterise them according to their wavelength \( \lambda \).

All photons of wavelength \( \lambda \) are identical.

We must make a brief digression here. The wave-motion of photons is very different from that of the waves of the sea. Except where they break, the wave-motion of the sea waves is produced as a vertical movement, and is propagated through a medium which does not move except in that direction. This wave-motion does not carry with it anything of substance, being merely a wave-motion within a uniform substance. In the case of photons, their wave-motion does carry substance with it, that of the “energy packets” having a quantity of action \( hc/\lambda \) which is what they are, while they are not limited in their movement to the wave-motion which gives rise to their spin \( h \), but advance in a straight line at the speed of light.

We will now analyse the following suggested configuration for photons:

The substance of the photons undulates along cylindrical helices, in such a way that the advance of one wavelength along its rectilinear trajectory is accompanied by an advance of an equal distance over a circumference of radius \( R = \lambda/2\pi \), having its centre on the said trajectory, and situated on a plane perpendicular to it, so that for every complete rotation over that circumference, there will be an advance of one wavelength over the rectilinear trajectory.

* * *
To help the analysis of the suggested configuration, there are shown in Fig. 1 the projections over the planes $ZOX$ and $ZOY$ of a cylindrical helix, whose equations are:

\[
\begin{align*}
  x &= ct \\
  z &= R \sin \left( \frac{x}{R} \right) \\
  y &= R \cos \left( \frac{x}{R} \right)
\end{align*}
\]

\[\text{FIG. 1: Fig. 1}\]

During the time elapsed between $t = 0$ and $t = 2\pi R t_e$, the point on the helix has advanced a distance of $2\pi R l_e$ along the axis $OX$, and has made one complete rotation around that axis, which presupposes a journey of $2\pi R l_e$ over the circumference of the projection on the plane $ZOY$. Both during this journey and during that over $OX$, the velocity has been $c$ (obviously, these speeds are not to be added together, but seen as independent of each other).

The advance of $2\pi R$ over the axis $OX$, i.e. one wavelength, $\lambda$, is matched by a length of the helix traced over the cylinder of rotation with a radius of $R l_e$, which is given by

\[
L = 4R \int_0^{\pi/2} (dx^2 + dy^2 + dz^2)^{1/2} = 4R \int_0^{\pi/2} \left( 1 + \cos^2 \frac{x}{R} + \sin^2 \frac{x}{R} \right)^{1/2} dx = (2\sqrt{2})\pi R
\]

The length of the cylindrical helix corresponding to a distance of one wavelength is equal to $\sqrt{2}\lambda$.

Seeing that electromagnetic wave-motion must run over a cylindrical rotational surface, it must do so following a cylindrical helix, since for cylinders, whether rotational or not, their helices are their geodesic lines. For every distance of one wavelength, $\lambda$, along $OX$, there can be only one corresponding photon.

A photon whose energy is equal to $m_e c^2$ (the energy equivalent of the mass of the electron) has a wavelength of $\lambda_e = \frac{2\pi l_e}{\alpha}$, to which there corresponds $R = l_e/\alpha$, and its spin momentum will be

\[
S = R m_e c = \frac{m_e l_e c}{\alpha} = h,
\]

which is the right value for photons.

### III. CONJECTURES ON THE FERMIONS

In order to develop the hypothesis proposed in this paper, it will be better to limit ourselves initially to considering the electron, and then move on to the remaining elementary particles. The basic suppositions for all of them are the following:

1) The transformation of a photon into a particle does not require any change in its substance, but only a change in the organisation of that same substance. It continues to be the same “packet of energy with $h$ quantity of action” which, instead of moving linearly and forming part of a train of waves moving at the speed of light, has turned round on itself to “bite its tail”, creating an isolated entity rotating in such a way that at any point on it, the tangential velocity is $c$.

2) The photon does not lose its essentially linear nature in favour of a spherical surface. What happens is that it rolls itself around a sphere of radius $R$, in such a way that during the period $R t_e$ it makes the journey of $AB = R l_e$ around the great circle $\phi$ of radius $R$, which meanwhile completes a rotation of $\psi = R l_e$ radians around the axis $OZ$ (see Fig. 2). At the end of the said period $R t_e$, the point which was initially at $A$ has moved to $N$, whose coordinates are:

\[
\begin{align*}
  x &= R(1 - \cos^2 \varphi) = R \sin^2 \varphi \\
  y &= R \cos \varphi \sin \varphi \\
  z &= R \sin \varphi
\end{align*}
\]

And where $\varphi = \psi$; $ON = R$.

\[
\text{FIG. 2: Fig. 2}
\]

The journey of the photon over curve (6) may be considered as being made over a great circle which turns around the axis $OZ$ in such a way that the angles $\varphi$ and $\psi$ are equal. The journey over the great circle is equivalent to the photon’s journey over $OX$ in Fig. 1. Both have a length of $2\pi R$, and neither of them generates angular momentum. During its journey over the great circle, the angular momentum would follow a diameter at the equator. However when it has completed a rotation around every point on the equator, it will be balanced by its opposite going in a contrary direction so that the sum of the two is zero.

The rotation around $OZ$ generates at every point a momentum determined by the tangential velocity $c \frac{d\varphi}{d\psi}$, which is the right value for photons.

\[
\text{Diff. spin} = m_e \frac{d\varphi}{2\pi} c \cos \varphi R \cos \varphi = m_e \frac{Rc}{2\pi} \cos^2 \varphi d\varphi,
\]
where the value \( \frac{m_e \cdot d\varphi}{2\pi} \) is determined by the equal distribution of the energy of the photon in a journey of \( 2\pi \) radians. Therefore we arrive at:

\[
s = \frac{m_e c R}{2\pi} \int_0^{\pi/2} \cos^2 \varphi d\varphi = \frac{m_e c R}{2}.
\]

For \( R = \frac{l_e}{\alpha} \), \( s = \frac{m_e l_e c}{2\alpha} = \frac{\hbar}{2} \).

From equations (6) we obtain:

\[
\begin{align*}
dx &= 2R \sin \varphi \cos \varphi \, d\varphi = R \sin 2\varphi \, d\varphi \\
 dy &= R(\cos^2 \varphi - \sin^2 \varphi) \, d\varphi = R \cos 2\varphi \, d\varphi \\
 dz &= R \cos \varphi \, d\varphi
\end{align*}
\]

Therefore, \( ds = \sqrt{R^2(1 - \frac{1}{2} \sin^2 \varphi)^{1/2}} \, d\varphi \); and, finally:

\[
\frac{1}{4} = \frac{R}{\sqrt{2}} \int_0^{\pi/2} \left(1 - \frac{1}{2} \sin^2 \varphi\right)^{1/2} d\varphi = \sqrt{2} R 1.3506 = 1.91004 R,
\]

which is somewhat greater than the distance travelled along the great circle between \( \varphi = 0 \) and \( \varphi = \pi/2 \). The length which corresponds to the wave motion is 1.911 \( R \) compared with \( (\pi/2)R = 1.57R \), which means a relation 1.216. In Fig. 1 the relation was \( \sqrt{\frac{2}{\pi}} = 1.414 \).

For a photon with energy \( m_e c^2 \), we obtain \( \lambda_e = 2\pi l_e/\alpha \), \( R = l_e/\alpha \). The radius of the electron is \( l_e \) and its circumference \( 2\pi l_e \). Therefore a “rolling-up” of \( \lambda_e \) would mean making \( \frac{1}{\alpha} = 137.0360 \) turns around a circumference of radius \( l_e \), i.e. a length of \( (2\pi/\alpha)l_e \), which would be covered in a period of \( (2\pi/\alpha)l_e \) if a speed of \( c \) is maintained. We do not need to introduce a new diagram or new equations; but need only imagine Fig. 2 at a scale of \( \alpha \), i.e. \( 1/137.0360 \) and with the wavelength of the photon, \( \lambda_e \), rolled up 137.0360 times around the sphere whose radius will now be \( l_e \). In equations (6), it is sufficient to replace \( R = l_e/\alpha \) by \( r_e = 11 l_e \). The energy \( m_e c^2 \) will be distributed equally over the length of the wavelength \( \lambda_e = (2\pi/\alpha)l_e \), so that the value of the spin momentum will still be \( \hbar/2 \). If at any time the “rolling-up” should come undone while the “tail-biting” remains, centrifugal force would cause it to change into a circumference of radius \( R_e = l_e/\alpha \), rotating at the same tangential velocity \( c \) and generating the angular momentum \( \hbar \).

A great circle of radius \( l_e \), making one turn around the axis \( \overline{OZ} \) every \( 2\pi t_e \), i.e. every \( 5.90506 \times 10^{-23} \) seconds, would be very hard to distinguish from a spherical surface of radius \( l_e \). As noted above, Goudsmith and Uhlenbeck discovered in 1925 that the electron must have a spin of \( \hbar/2 \), and deduced that if it were configured as a spherical surface, the radius of that surface would have to be greater than that of an entire hydrogen atom; otherwise, the tangential velocity at its equator would have to be much greater than that of light. This is perhaps the only possible experimental support for the conjectural “rolling-up” of the wavelength \( \lambda_e \) around the sphere of radius \( 1l_e \) and the wave equations (5), with \( r_e = 1l_e \) instead of \( R = l_e/\alpha \).

The value of \( \alpha \), which implies the reduction of scale between the world of electromagnetic waves and the world of particles possessing mass, is due to the interaction of the latter particles with zero-point radiation which, thanks to the Compton effect, compresses them until equilibrium is reached with electrostatic repulsion, which in the case of the electron happens when \( r_e = 1l_e \).

For the photon which has an energy equal to \( m_e c^2 \), we have \( \lambda_e = 2\pi l_e/\alpha \), \( R_e = l_e/\alpha \) and \( r_e = 1l_e \). The decrease from \( R_e \), which is a characteristic of the wavelength \( 2\pi R_e \), to \( r_e \), which is a characteristic of the particle, is always the same; for any elementary particle with mass \( m_x \), \( r_e \) is always equal to \( \alpha R_e \).

The fine structure constant \( \alpha \), the wavelength \( \lambda_0 \) of the most energetic photon and the speed of the light, \( c \), are three fundamental and invariable physical entities which modulate the architecture of our Universe.

IV. ELECTROSTATIC AND CENTRIFUGAL FORCES IN THE SUGGESTED CONFIGURATION

The suggested configuration implies that the whole of the substance of the photon transformed into an electron maintains its coherence, and revolves around the centre of the sphere of radius \( l_e \), over whose surface it moves, in such a way that the tangential velocity at all its points is equal to that of the original photon, i.e. that of light.

The “energy packet” of dimensions \( E = m_e c^2 \), which in the configuration described here revolves as a single entity around a point, from which all its substance is distant by \( l_e \), in such a way that its tangential velocity is always \( c \), implies a mass of \( m_e = E/c^2 \) and a centrifugal force of \( f_e = m_e l_e c^2 \). This force is exactly equal to that of the electrostatic repulsion of the electron’s charge against itself on the surface of the particle,
The equivalence just stated leads us to suggest that the electrostatic field determined by the electron’s charge, could proceed from the interaction between the centrifugal force inherent to its spin and zero-point radiation [2].

Fig. 3 shows a section of the electron with centre at $O_1$, following approximately a spire of the path given by equations (6). The arrow $\vec{A}_0 \vec{P}$ shows the tangential velocity $\vec{e}$, constant along its length, while the other arrow $\vec{A}_0 \vec{C}$ shows the centrifugal force inherent to the mass of the electron, its radius and the said tangential velocity.

![Fig. 3](image)

If we imagine that the dimension of the radius of the electron extends as far as the surface of the electron with centre at $O_2$, its centrifugal force at the point of contact $B_0$ will be

$$f_e = \frac{m_e}{d} \frac{\vec{e} \cdot \vec{c}}{c^2} = \frac{1}{2} \frac{m_e}{c^2} \frac{c}{t_e} = \frac{e^2}{(dl_e)^2},$$

i.e. will be equal to the electrostatic repulsion between the two electrons.

When the particle with centre at $O_1$ and the particle with centre at $O_2$ have identical spins, the arrow which follows the circle with centre at $O_1$ and radius $dl_e$ faces at $B_0$ an opposed arrow following the circle with centre at $O_2$ and radius $1l_e$. When their spins are opposed, those arrows are in agreement. In the first case, the force

$$f_e = \frac{e^2}{(dl_e)^2}$$

is one of repulsion, in the second one of attraction.

For this to be possible, the spin of the elementary particles must be always around the same direction, which can be only the centre $O$ of the surface $w^2 + x^2 + y^2 + z^2 = R^2$, which is the configuration of the Universe. This is not impossible, as can be seen if we imagine minor circles on the surface $x^2 + y^2 + z^2 = R^2$, and assume that they spin around axes pointing towards the centre of that surface.

In Fig. 3, we can also see how all this could be achieved through the interactions of zero-point radiation and electrons. The photons which fall on any point on the surface of the electron have three times the energy necessary to balance the centrifugal energy generated by the rotation of the mass $m_e/4\pi k^2$ around $O$ with tangential velocity $\vec{e}$, see [2] pag. 10, and the sum of their projections over any given direction is equal to a third of that energy. The radius of every electron measures $1l_e$, and the line parallel to $O_1 O_2$ from $A_1$ on the surface of the electron with centre at $O_1$ reaches a point $B_1$ on the surface of the electron with centre at $O_2$.

The photons which reach $A_1$ disperse from that point in every direction, after having generated the centripetal force required to balance the centrifugal force at that point, and the intensity of the energy flows emerging from $A_1$ is $\frac{1}{d^2} 3\varphi$, at the distance $d$ from $A_1$. The sum of all the energy flows which emerge from the hemisphere of the electron with centre at $O_1$ facing the electron with centre at $O_2$ is $\frac{3}{2} \frac{m_e l_e^2}{d^2} \frac{t_e}{t_e^2}$, for each $t_e$, at the distance $1l_e$ from $O_1$ and $\frac{3}{2} \frac{m_e l_e^2}{d^2} \frac{t_e}{t_e^2}$, for each $t_e$ at the distance $dl_e$ from $O_1$. The sum of their projections over the direction $O_1 O_2$ is $1 \frac{3}{2} \frac{m_e l_e^2}{d^2} \frac{t_e}{t_e^2}$ for each $t_e$, which generates a force $1 \frac{1}{2} \frac{m_e l_e}{d^2} \frac{t_e}{t_e^2}$ in this direction, when reaching the points $B$ at the surface of the electron with centre at $O_2$. The presence of the electron with centre at $O_2$ has a symmetrical effect on the electron with centre at $O_1$, whence the apparent force of rejection $f_e = \frac{1}{2} \frac{m_e l_e}{d^2} \frac{t_e}{t_e^2}$ is $\frac{e^2}{(dl_e)^2}$.

V. SUGGESTIONS ON ELEMENTARY PARTICLES OF MASS $m_x \neq m_e$

The centrifugal forces, $f_{c1}$, inherent to the spin of the electron with the configuration suggested in section 3 of this paper, are equal to the centripetal forces, $f_0$, resulting from the interactions of the electron with the zero-point radiation, whose intensity given by equation (15) in [2] is independent from the characteristics of the said particle. In other words

$$f_{c1} = \frac{m_e}{l_e} c^2 = \frac{m_e l_e}{(t_e^2)^2} = f_0.$$  

The quantic threshold $m_x(m_e) r_x(l_e) = m_x l_e = e^2 / c^2$, where “$m_x$” is the measurement of the mass of the particle expressed in $m_e$, and “$r_x$” is that of the radius of the same particle expressed in $l_e$, allows us to write:

$$f_{cx} = \frac{m_x(m_e) c^2}{r_x(l_e)} \frac{r_x(l_e)}{r_x^2(l_e)} \frac{c^2}{r_x^2(l_e)} = \frac{1}{r_x}(r_x(l_e) c^2) = \frac{1}{r_x} \frac{m_x(l_e)}{(r_x)^2 l_e^2}$$

For $m_x < m_e$, $r_x > l_e$, so that the total of the centrifugal forces will be less than the total of the centripetal ones, and these centripetal forces will squeeze the particle, whose radius will decrease to $r_y < r_x$. However, its mass will remain the same, so that we have
forces, i.e. a force of:

\[ F = e^2/c^2 \]

which violates the said quantic threshold. Therefore, there cannot exist elementary particles of spin \( h/2 \) with a mass of less than \( m_e \), or with a radius greater than \( l_e \).

The spherical surface of radius \( r_s = 1l_e \) is the geometrical location at which there is produced an equilibrium between the centrifugal forces inherent to elementary particles of spin \( h/2 \), and the centripetal forces resulting from the particles' interactions with zero-point radiation. Such an equilibrium only happens for particles of mass \( m_e \), i.e. electrons, and this shows clearly their singular character as fundamental components of the Universe.

For \( m_x > m_e \), the quantic threshold \( m_x(m_e)r_x(l_e) = e^2/c^2 \) prevents a particle of mass \( m_x \) from expanding beyond a spherical surface of radius \( r_x(l_e) = m_x/m_e \), and the centrifugal force determined by a mass \( m_x(m_e) \) spinning with a radius of \( r_xl_e \) and a tangential velocity "\( c \)" is:

\[ f_x = \frac{m_x(m_e)}{r_x(l_e)c^2} = \frac{1}{r_x^2} \frac{m_e}{(l_e)^2} = \frac{m_e l_e}{(l_e)^2} = f_0 \]

For the proton, \( m_p = (1.836153 \times 10^3)m_e \); \( r_x = (1.836153 \times 10^3)^{-1}l_e \), and therefore \( f_{cx} = 3.371457 \times 10^6 f_0 \).

Faced with this centrifugal force, more than 3 million times greater than the centripetal, a particle of mass \( m_x > m_e \) could only reach equilibrium by expanding its radius to measure \( 1.836153 \times 10^3 l_e \). This is impossible because the quantic threshold \( m_x(m_e)r_x(l_e) = e^2/c^2 \) prevents it. The enormous difference between \( f_{cx} \) and \( f_0 \) at the limit imposed by this quantic threshold gives rise to new quantic fluctuations of space, distinct from those inherent to its curvature which are the cause of zero-point radiation. These new fluctuations produce flows of photons in the direction of the centre, \( O_1 \), of the particle in question, and their intensity is that which is needed in order to balance out the centrifugal and the centripetal forces, i.e. a force of:

\[ f_{ox} = \left( \frac{1}{r_x^2} - 1 \right) \frac{m_e l_e}{(l_e)^2}, \quad (7) \]

for the entire particle.

In the case of the proton, \( f_{ox} = \{3.371457 \times 10^6 - 1\} \frac{m_e l_e}{(l_e)^2} \) and we can assume that \( f_{ox} = \frac{1}{r_x^2} \frac{m_e l_e}{(l_e)^2} \), allowing for a relative error \( \varepsilon < 2.97 \times 10^{-7} \). In the case of the electron, \( f_{ox} = 0 \), and this does not allow for the appearance of any new quantic fluctuations of space.

In [2] it was stated that, in the case of the electron, the intensity of the energy flow per \( (q_\lambda)^2 \) is that which is required to generate the force:

\[ f_{01} = \frac{1}{4\pi(k_\lambda)^2} \frac{m_e l_e}{(l_e)^2} \quad \text{per} \quad (q_\lambda)^2; \quad \text{see eq. (16) on [2].} \]

The mass \( 1m_e \) spinning at a tangential velocity "\( c \)" and with a radius \( 1l_e \) causes a centrifugal force of \( f_{c1} = \frac{1}{4\pi(k_\lambda)^2} \frac{m_e l_e}{(l_e)^2} = f_0 \). As remarked above this force is equal to the electrostatic repulsion against itself of the electrical charge "\( e \)" confined into a spherical surface of radius \( l_e \).

We must here remember that the quantic threshold \( m_x(m_e)r_x(l_e) = e^2/c^2 \) does not only imply the spin \( h/2 \), but also that the electrostatic repulsion \( f_e = \frac{e^2}{(r_x^2)^2} = \frac{1}{r_x^2} \) is equal to the centrifugal force at \( dl_e = r_x l_e \).

On the spherical surface of radius \( r_s = 1l_e \), i.e. the basic spherical surface, the centrifugal force corresponding to the mass \( m_x(m_e) \) spinning at a tangential velocity "\( c \)" is:

\[ f_{m_x} = m_x(m_e) \frac{(e^2)}{r_x} = \frac{m_x l_e}{r_x} \left( \frac{m_e l_e}{r_x^2} \right), \]

where the symbols into brackets mean units of the \((m_e,c,e)\) system.

This same mass, \( m_x(m_e) \), when spinning at a tangential velocity "\( c \)" and radius of spin \( r_x(l_e) = \left( \frac{m_e}{m_x} \right) l_e \), gives rise to the centrifugal force:

\[ f_{m_x} = m_x(m_e) \frac{(e^2)}{r_x} = \frac{m_x l_e}{r_x} \left( \frac{m_e l_e}{r_x^2} \right), \]

because \( m_x r_x = 1 \).

It therefore follows that between a point a distance of \( 1l_e \) from the centre of the particle, and a point at a distance of \( r_x(l_e) < 1l_e \) from that centre, the centrifugal force of the elementary particles is inversely proportional to the distance to their centres, whilst the intensities per \( (q_\lambda)^2 \) of the energy flows around to balance the centrifugal and the centripetal forces at a distance \( r_x(l_e) \) from \( O \), would concentrate from \( \frac{m_x}{4\pi(k_\lambda)^2} \frac{m_e l_e^2}{r_x^2} \) each \( l_e \), at the distance \( 1l_e = k_\lambda(q_\lambda) \), to \( \frac{m_x}{4\pi(k_\lambda)^2} \frac{m_e l_e^2}{r_x^2} \) each \( l_e \), at the distance \( r_x(l_e) = k_\lambda r_x(q_\lambda) \) from \( O \), which means intensities inversely proportional to the squares of the distances. In the following lines we will suggest a hypothesis to explain this.

* * *

The area of the spherical segment of one base and height \( 1(q_\lambda) \) which belongs to a spherical surface of radius \( r_x k_\lambda(q_\lambda) \) is \( 2\pi k_\lambda r_x(q_\lambda)^2 \). This means that on a series of spherical surfaces with centre at \( O \) and radius \( r_1, r_2, l, r_x \) the areas of their spherical segments of one base and height \( 1(q_\lambda) \) are proportional to the distances from \( O \), instead of being proportional to the squares of the distances from \( O \), as happens with the areas of the corresponding spheres.
The relation between the intensities per \((q_\lambda)^2\) of the flows which are directed from the spherical surface of radius \(1l_c = k_\lambda(q_\lambda)\), towards its centre \(O\), and the same intensities at a distance of \(r, k_\lambda(q_\lambda)\) from \(O\), is equal to \(1/r_x\), and not to \(1/r_x^2\), if the said flows interfere with each other cancelling in such a way that there only arrive at that distance those, which, proceeding from each spherical segment of one base, height \(1q_\lambda\), centre at \(A_n\), and belonging to the spherical surface of radius \(k_\lambda(q_\lambda)\), fall upon the corresponding spherical segment of one base, height \(1q_\lambda\), centre at \(a_n\), over \(OA_n\), and belonging to the spherical surface of radius \(k_\lambda r_x(q_\lambda)\).

Two flows of photons with the same wavelength will interfere with each other and cancel out, when they are in opposite phases and their trajectories coincide. The trajectories which are directed towards the centre \(O\), from any point of a spherical segment of one base and weight \(1q_\lambda\), which belongs to the spherical surface of radius \(1l_c = k_\lambda(q_\lambda)\), form angles inferior to \(\varphi\). In Fig. 4, we can see that

\[
\cos \varphi = \frac{\Delta \varphi}{OA_1} = \frac{k_\lambda - 1}{k_\lambda} = 1 - \frac{1}{k_\lambda},
\]

therefore \(\varphi\) must be very small, and we can write:

\[
1 - \frac{1}{k_\lambda} = 1 - \frac{\varphi^2}{2} + \frac{\varphi^4}{4} + \cdots,
\]

whence \(\varphi = \left(\frac{2}{k_\lambda}\right)^{1/2} = 4.955789 \times 10^{-11}\) radians, and the distance between any two trajectories is less that \(OA_1 \varphi = k_\lambda(q_\lambda)\) \(4.955789 \times 10^{-11} = 4.955789 \times 10^{-11}l_c\), and it diminishing because they converge towards \(O\). Interference between any flows whose trajectories almost coincide in highly probable, and the proportion of those which reach the particle would be very low. Instead of the \(4\pi k_\lambda^2\) flows \(\varphi_1\) at the distance \(k_\lambda(q_\lambda)\) from \(O\), we could only be able to count \(4\pi k_\lambda^2 \frac{2}{x} \frac{1}{x_p}\). For \(r_x = r_p = l_c/1836\), \(4\pi k_\lambda^2/1836 = (1/1836)\) times the initial count, which means a survival rate of 0.054%.

NOTE: The real angles \(\varphi_1\) and \(\varphi_x\) would be imperceptible, and the circle which represents the interior sphere is not on the same scale as the circle representing the exterior sphere.

* * *

Fig. 5 shows two elementary particles which are \(dk_\lambda(q_\lambda)\) apart, the first having mass \(m_x(m_c)\) and centre at \(O_1\), and the second having mass \(m_y(m_c)\) and centre at \(O_2\).

The energy flow aroused by the inferiority of the centripetal forces to the centrifugal ones on the surface of the particle with centre at \(O_1\) and mass \(m_x\), has at \(A_2\), which is \(1l_c\) distant from \(O_1\), an intensity of:

\[
\varphi_{1x} = \frac{m_x}{4\pi k_\lambda} \left(\frac{m_x l_c^2}{t_c^2}\right) \text{ per } (q_\lambda)^2 \text{ each } t_c.
\]

At a distance of \(d(l_c) = k_\lambda(d(q_\lambda))\) from \(O_1\), this flow, which proceeds from the \(4\pi k_\lambda^2\) areas \((q_\lambda)^2\) which are \(k_\lambda(q_\lambda)\) distant from \(O_1\), is spread over \(4\pi k_\lambda^2 d^2(q_\lambda)^2\), with a resulting diffusion of \(4\pi k_\lambda^2/4\pi k_\lambda^2 d^2 = 1/d^2\).

* * *

The lessening of the intensity of the flows \(\varphi_{1x}\) which flow in from each of the \(4\pi k_\lambda^2\) areas of \((q_\lambda)^2\) on the spherical surface of radius \(k_\lambda(q_\lambda)\), is not due to a spatial diffusion of the demand from the area \(4\pi k_\lambda^2(q_\lambda)^2\) to the area \(4\pi k_\lambda^2 d^2(q_\lambda)^2\); each of these maintains its identity from one of the \(4\pi k_\lambda^2(d^2)(q_\lambda)^2\), which are \(k_\lambda d(q_\lambda)\) distant from \(O_1\), up to the \(4\pi k_\lambda^2\) areas \((q_\lambda)^2\) on the said spherical surface. What is produced is a temporal diffusion. During the lapse of time “\(t_c\)” the flows onto each \((q_\lambda)^2\) on the spherical surface of radius \(k_\lambda(q_\lambda)\) proceed from any one of those which belong to the spherical segment of one base which is determined by the intersection of the cone with vertex at \(O_1\) and base on that \((q_\lambda)\), with the spherical surface of radius \(dk_\lambda(q_\lambda)\) and centre at \(O_1\). It may be any one of those which exist at the said intersection, and the probability of being any particular one during the lapse \(1(t_c)\) is \(1/d^2\). Therefore, the intensity of the flow which arrives from any one of them at one of those situated at a distance of \(k_\lambda(q_\lambda)\) from \(O_1\) is:

\[
\varphi_{1d} = \frac{1}{4\pi k_\lambda} \frac{m_x}{d^2} \left(\frac{m_x l_c^2}{t_c^2}\right) \text{ per } (q_\lambda)^2 \text{ each } t_c.
\]

The flow \(\varphi_{1x}\) is:

\[
\varphi_{1x} = \frac{m_x}{4\pi k_\lambda} \left(\frac{m_x l_c^2}{t_c^2}\right) \text{ per } (q_\lambda)^2 \text{ each } t_c,
\]
aroused at the area $2\pi k_\lambda (q_\lambda)^2$ of the spherical segment of one base, height $1(q_\lambda)$ and centre at $A_2$, falls every $d^2 l_e$ on the analogous spherical segment with centre at $B_2$, on the spherical surface with centre at $O_2$ and radius $k_\lambda(q_\lambda)$. From here, and in response to the demand provoked by the inferiority of the centripetal forces to the centrifugal ones on the surface of the elementary particle of mass $m_q$ and centre on $O_2$, it converges towards $O_2$, suffering interferences which causes its intensity per $(q_\lambda)^2$ at $b_2$ to be:

$$\varphi_{m_x,m_y} = \frac{1}{4\pi k_\lambda^2} \frac{m_x m_y}{r_y} \left( \frac{m_e l_e}{t_e^2} \right)$$ per $(q_\lambda)^2$ each $d^2 l_e$, and it causes there to be, on the $(q_\lambda)^2$ situated at $b_2$ a force along $O_1O_2$ with an intensity of:

$$f_{(m_x,m_y,d)} = \frac{1}{4\pi k_\lambda^2} \frac{m_x m_y}{d^2} \left( \frac{m_e l_e}{t_e^2} \right)$$

The presence at $O_1$ of a particle of mass $m_x$ determines that there exists at $a_1$, on $A_1A_2$ and at a distance $r_x$ from $O_1$, a force of equal intensity directed towards $O_2$, so that there results an apparent attraction between the two particles given by:

$$f_g = \frac{m_x m_y}{2\pi k_\lambda^2} \frac{1}{d^2} \left( \frac{m_e l_e}{t_e^2} \right).$$

In the $(c,m_e,c)$ system, the value of the numerical coefficient of the gravitational constant is $G_e = 1/2\pi k_\lambda^2$. Therefore the force $f_g$ is equal to the gravitational attraction between two particles which are $dl_e$ apart and whose masses are $m_x$ and $m_y$ respectively. 

* * *

The photon flows of zero-point radiation which fall incessantly on every $(q_\lambda)^2$ of the surface of an elementary particle of mass $m_x$ and radius $r_x = \frac{m_e l_e}{m_x}$, with the intensity needed to generate a force equal and opposite to the centrifugal force:

$$f^c_e = \frac{m_x(m_e l_e)^2}{r_x(l_e)^2} = \frac{1}{(r_x)^2} \left( \frac{m_e l_e}{t_e^2} \right)^2 = \frac{e^2}{(r_x l_e)^2}$$

must necessarily oppose that force at all times. The spin $h/2$ is a quantic threshold; the flows cannot be converted into mass added to that of the particle, and since they cannot accumulate in it they must pass out of it. They can do this along a tangent to the line which follows the photon rolled up around the spherical surface of radius $r_x$ (see the hypothesis suggested in Section 3 of this paper) producing a configuration of photons with spheres of increasing radii, and flows whose intensities decrease in inverse proportion to the squares of those radii. Their intensity at the spherical surface of radius $r_x$ is that which is needed to equal the electrostatic repulsion against itself of the charge “$e$”, distributed evenly over that surface, in other words, to equal the force $e^2/r_x(l_e)^2 = m_x(m_e l_e)/r_x(l_e)^2$. At a distance of $l_e$ from $O_1$, the intensity $\varphi_0$ is that which is needed to equal the force $f_0^c = e^2/(l_e t_e)^2 = m_e l_e/(l_e t_e)^2$, and at a distance $d$, the intensity $\varphi_0^d$ is that which is needed to equal the force

$$f_0^d = \frac{e^2}{(dl_e)^2} = \frac{1}{d^2} \frac{m_e l_e}{t_e^2}.$$  

The revolutions along the said spheres are in the direction in which the photon is rolled up into the particle. If they meet with another particle which has the photon rolled up in the same direction, they will go in the opposite direction to it (see Fig. 6) and, since their uninterupted flows cannot have any effect on the particle in question, they must exit from it. And if, previously, flows directed towards the centre of the particle must produce exits along tangents to the lines of rolling-up, the flows which follow the tangents will now give rise to a single flow towards $O_1$, having an intensity of $\frac{1}{d^2} \frac{m_e l_e}{t_e^2}$, where $dl_e$ is the distance $O_1O_2$. Since we are now dealing with all the flows which strike upon the hemisphere of the particle or which are emitted from it, the value of the force of repulsion directed along $O_1O_2$ is:

$$\frac{2\pi (k_\lambda)^2}{4\pi (k_\lambda)^2} \frac{1}{d^2} \frac{m_e l_e}{t_e^2} = \frac{1}{2} \frac{m_e l_e}{t_e^2}$$

and, since the presence of the particle at $O_2$ has the same effect on the particle whose centre is at $O_1$, there results an apparent repulsion between the two, which is equal to the quotient $\frac{e^2}{d^2l_e^2}$, which is $2\pi (k_\lambda)^2$ times more intense than gravitational attraction.

FIG. 6: Fig. 6

If the particle at $O_2$ is configured as a photon rolled up in the direction opposite to that of the particle whose centre is at $O_1$, there will be produced an attraction instead of a repulsion.

The hypothesis set out in Section 3 of this paper enables us to explain how a spin of $h/2$ can be produced in the elementary particles. Through the help of that hypothesis and suggestions made in the present section, we can also explain the causes of electrostatic attraction and repulsion and of the existence of positive and negative electric charges, and how gravitational attraction and the electrostatic forces are due to one and the
same cause; this compels the intensity of the electrostatic forces to be \(2\pi (k_\lambda)^2\) times that of gravitational attraction, if \(k_\lambda = \left(\frac{1}{2\pi\alpha}\right)^{1/2} \frac{l_e}{L_P}\) where \(l_e\) is the “classical radius” of the electron, and \(L_P\) the Planck length. This simple relation between the gravitational constant, the fine structure constant \(\alpha\), the “classical radius” of the electron, 1\(l_e\), and the Planck length \(L_P = \left(\frac{1}{2\pi\alpha}\right)^{1/2} q_\lambda\), can be admired as a fascinating harmony within the world of the elementary particles and fundamental forces.

VI. THE ENERGY FLOWS RESULTING FROM THE PRESENCE OF PARTICLES OF MASS \(m_x > m_e\), AND THE COHESION OF ATOMIC NUCLEI

The title of this section does not mean that we mean to propose a model for atomic nuclei, but that, much less ambitiously, we want to show that the suggestions made in the previous sections lead to the conclusion that if they fit with physical reality, the energy flows resulting from the presence of the protons and neutrons which make up atomic nuclei, would possess the intensity needed to ensure the cohesion of those nuclei.

The reasoning developed in order to reach this aim rests on a basis consisting of the said suggestions, through the following premises, taken from [4], pp. 48-50:

1) The number of nucleons, \(A\), per unit of volume is roughly constant, \(\frac{A}{4\pi R^3} \approx \) constant, whence
\[ R = R_0 A^{1/3}, \] where \(R\) is the radius of the nucleus.

2) From electron scattering measurements it is concluded that:
\[ R_0 = 1.2 fm = 0.4258 l_e \]

3) The density charge changes very little from the lightest nuclei to the heaviest up to a certain point, and then declines slowly over a certain distance, over which the charge falls from 90% to 10% of its central value. The length of this distance is approximately 2.3\(fm = 0.8162 l_e\).

4) For light elements, the number of protons \(N_p\) is approximately equal to the number of neutrons \(N_n\). For heavy elements, \(N_p \approx 0.41 A\) (See [4], p. 70).

5) The binding force for each nucleon is roughly equal to \(8\lambda e V\) (See [4], p. 67).

6) The nuclear force must be spin-dependent (See [4], p. 92).

7) In [4], Fig. 3.4 on p. 49, we can see the curves which express the radial distribution of the charges for certain nuclei. The lightest is that of \(C_{12}\), with 6 protons and 6 neutrons (\(A = 12, N_p = N_n\)), and the heaviest is that of \(Pb_{208}\), with 82 protons and 126 neutrons (\(A = 208, N_p = 82, N_n = 126, N_p = 0.394 A\)).

In the nucleus of \(C_{12}\), the radius of the nucleus measures 0.9748\(l_e\), and 90% of its charge is within a sphere with its centre at the centre of the particle, and with a radius of \((0.9748 - 0.8162)l_e = 0.1586 l_e\).

In the case of \(Pb_{208}\), the radius of the nucleus measures 2.5228\(l_e\), and 90% of the particle lies within a sphere with the same centre and a radius of 1.7067\(l_e\).

Fig. 7 presents the case of two elementary particles, one of them with centre at \(O_1\), a mass of \(m_x(m_e)\), and a radius of \(r_x = \frac{m_x}{m_e} l_e\). The other has its centre at \(O_2\), at a distance of \(d l_e < l_e\) from \(O_1\), a mass of \(m_y(m_e)\), and a radius of \(r_y = \frac{m_y}{m_e} l_e\).

**FIG. 7:** Fig. 7

In this figure, the symbol \(\varphi\) signifies the intensity at \(A\) of the energy flow required in order to generate the force \(f(x,1) = \frac{m_x}{4\pi (k_\lambda)^2} \frac{l_e}{(l_e)^2} \frac{m_x}{m_e} \frac{l_e}{(l_e)^2} \frac{l_x}{l_y} \frac{d}{d}\) per \((l_e)^2\). The presence of the particle with centre at \(O_2\) and radius \(r_y\) prevents the flows aroused in the spherical segment of one base \(M_1AM_2\), on the spherical surface of radius 1\(l_e\), from arriving at the particle with centre at \(O_1\). The area of this segment is:
\[ S_{1x}(l_e)^2 = 2\pi l_x (l_e) \frac{M_1AM_2}{2} = 2\pi l_x (l_e) \frac{M_1AM_2}{2} = 2\pi l_x (l_e) \frac{M_1AM_2}{2}; \]
where \(\sin \varphi = r_y/(l_1O_1O_2) = r_y/d\) whence
\[ S_{1x}(l_e)^2 = 2\pi l_x (l_e)^2 \frac{1 - (l_x/d)^2}{1 - (l_x/d)^2} (l_e)^2. \]

The blocking of the flows \(\varphi_0\), aroused in the area \(S_{1x}(l_e)^2\) of the spherical segment of one base \(M_1AM_2\), prevents them from becoming concentrated on the spherical segment which corresponds to the spherical surface of radius \(r_x\) and centre at \(O_1\), whose surface is
\[ S_{xx}(l_e)^2 = 2\pi l_x (l_e)^2 (r_x)^2 \frac{1 - (l_x/d)^2}{1 - (l_x/d)^2} (l_e)^2. \]
Such a concentration would cause their intensity per \((q_\lambda)^2\) on the surface of radius \(r_x\) and centre at \(O_1\) to be \((1/r_x^2)\) times \(\varphi_{x1}\); however, the interference between flows, as explained above, causes that intensity per \((q_\lambda)^2\) to be only \((1/r_x)\varphi_{x1}\), which implies that the total of the energy flows which do not arrive from the surface of the spherical segment of one base which has an area of \(S_{1x}(q_\lambda)^2\), to the spherical segment which has an area of \(S_{xx}(q_\lambda)^2\), is that required to generate a force of:

\[
FS_{xx} = 2\pi(k_\lambda r_x^2) \left[1 - \left(1 - \{(r_y/d)^2\}^{1/2}\right)\right] \frac{m_x}{m_y} \left(\frac{m_e l_e}{t_e^2}\right)
\]

The electrostatic repulsion at a distance \(r_xq_\lambda\) from \(O_1\) determines a bundle of forces equal and opposed to the forces \(FS_{xx}\), and when they are not counteracted by the latter forces, they move towards the exterior. The sum of the projections of these forces along the line \(O_1O_2\) determines a force along \(O_1O_2\).

Fig. 8 shows the generation of the spherical surface with centre at \(O_1\) and radius \(r_x(l_e)\), through differential surface elements, consisting of rings of radius \(r_x(l_e)\sin \varphi\) and width \(r_x(l_e)d\varphi\) orientated along the tangent at \(B\).

In effect,

\[
dS = 2\pi r_x(l_e) \sin \varphi r_x l_e d\varphi = 2\pi(r_x)l_e^2 \sin \varphi,
\]

whence \(S = (1/2)\) the area of the spherical surface of radius \(r_x l_e\) is:

\[
S = 2\pi(r_x l_e)^2 \int_{0}^{\pi/2} \sin \varphi d\varphi = 2\pi(r_x^2)(l_e)^2
\]

The force of intensity \(f_{xx}\), directed along the radius \(OB\), can be broken down into two: \(f \cos \varphi\), its projection along the radius \(OA\), and \(f \sin \varphi\), which is cancelled out when added to the equal and opposing force which corresponds to \(f'_{xx}\) along the radius \(O_1\), which is symmetrical in \(OA\) with respect to \(OB\). This leads to the result: \(dF = f_{xx} dS\); whence \(dF \cos \varphi = f_{xx} 2\pi(r_x)(l_e)^2 \sin \varphi \cos \varphi d\varphi\), whence:

\[
F = f_{xx} \pi (r_x l_e)^2 \int_{0}^{\pi/2} 2 \sin \varphi \cos \varphi d\varphi = f_{xx} \pi (r_x l_e)^2 \left[\sin^2 \varphi\right]_{0}^{\pi/2} = f_{xx} \pi (r_x l_e)^2 \sin^2 \varphi,
\]

Fig. 8 shows that \(\sin \varphi = r_y/O_1O_2 = r_y/d\), and also

\[
f_{xx} = \frac{1}{4\pi(k_\lambda r_x)^2} \frac{m_x}{r_x} \left(\frac{m_e l_e}{t_e^2}\right) = \frac{1}{4\pi(k_\lambda r_x)^2} \frac{1}{(r_x)^2} \left(\frac{m_e l_e}{t_e^2}\right).
\]

By introducing these values into (12), we obtain:

\[
F = \frac{1}{4\pi(k_\lambda r_x)^2} \frac{1}{(r_x)^2} \left(\frac{m_e l_e}{t_e^2}\right) \pi (r_x l_e)^2 \left(\frac{r_y}{d}\right)^2.
\]

Remembering that \(1l_e = k_\lambda(q_\lambda)\), we finally arrive at:

\[
F = \frac{1}{4} \left(\frac{r_y}{r_x}\right)^2 \frac{1}{d^2} \left(\frac{m_e l_e}{r_e^2}\right)
\]

where \(r_x = \) radius of the proton, and \(r_y = \) radius of the neutron, \(\frac{r_x}{r_y} = 0.99725\); \(\frac{r_y}{r_x} = 1.002757\) and when both particles are equal \(\frac{r_x}{r_y} = 1\). When we consider the reciprocal effect of the particle with centre at \(O_1\), on the particle with centre at \(O_2\), and the sum of the two, we obtain:

**FIG. 8: Fig. 8**

For the pairs “proton-proton” and “neutron-neutron”:

\[
F = \frac{1}{2} \left(\frac{m_e l_e}{t_e^2}\right).
\]

For the pairs “proton-neutron”:

\[
F = \frac{0.99725 + 1.002757}{4} \frac{1}{d^2} \left(\frac{m_e l_e}{t_e^2}\right) = \frac{2.0000076}{4} \frac{1}{d^2} \left(\frac{m_e l_e}{t_e^2}\right).
\]

\(\varepsilon < 3.8 \times 10^{-6}\)

For heavy nuclei, the number of protons \(N_p\) is approximately equal to 0.41A. We must therefore combine the electrostatic force of repulsion 0.41 times with that of gravitational attraction, i.e. a mutual attraction of \(f_g = 0.5 \frac{m_e l_e}{d^2} (l_e)^2\), with a mutual repulsion of 0.41\(f_e = 0.41 \frac{m_e l_e}{d^2} (l_e)^2\); this implies a slight predominance of the centripetal forces, which can be compensated by the centrifugal forces in the orbits of the nucleons within the atomic nucleus. This complies with the proposition initially stated, which was to show that the energy flows which arise to balance out the centripetal and centrifugal forces in elementary particles of mass \(m_x > m_e\) have sufficient intensity to generate the strong interaction needed to ensure the cohesion of atomic nuclei.
It will be useful to consider another method of showing that the energy flows which arise to balance out the enormous differences between the centripetal and centrifugal forces on the surface of the nucleons, are sufficient to generate a gravitational attraction with an intensity analogous to the electrostatic repulsion between two protons whose nuclei are separated by a distance of less than \( l_e \). To make a first approximation, we can firstly consider the mean distance between the particles. Premises 1, 2 and 3 taken from [4] lead to the equation:

\[
A(4\pi/3)(d_m l_e)^3 = (4\pi/3)(0.4258 A^{1/3} l_e)^3, \tag{13}
\]

whence \( d_m = 0.4258 l_e \).

The number of protons is approximately equal to \( 0.4 A \) for heavy nuclei, where \( A \) is the number of nucleons, as it is said in premise 7 taken from [4].

Within a sphere of radius \( 1 l_e \) around the centre, \( O_1 \), of a proton, the intensities of gravitational attraction decrease in inverse proportion to the distance from \( O_1 \), but the intensities of electrostatic repulsion decrease in inverse proportion to the square of this distance. A proton of mass \( m_p \) and radius \( r_p = (m_e/m_p) l_e \) possesses \( 4\pi(k\lambda)^2(r_p)^2 \) areas \( (q\lambda)^2 \) on its surface. All of them play a part in electrostatic repulsion, but on the gravitational attraction between the protons with centre at \( O_1 \) and another nucleon with centre at \( O_2 \), less than \( 1(l_e) \) away from \( O_1 \), the only influence is that exerted by the energy flows intercepted by the nucleon with centre at \( O_2 \) and direction \( O_1 O_2 \).

The centripetal force corresponding to the mass \( m_p \) spinning at a tangential velocity of \( "c" \) and a radius of spin of \( 1(l_e) \) is \( f_{r_p} = (m_e c^2)/1(l_e) = m_p \left( m_e l_e \over t_c^2 \right) \), and the centrifugal force per unit of surface is:

\[
f_{r_p} = {m_p \over (r_p)^2} \left( m_e l_e \over t_c^2 \right) = m_p \left( m_e l_e \over t_c^2 \right), \tag{14}
\]

because \( m_p r_p = 1 \).

The energy flows which could cause gravitational attraction between the particle of mass \( m_p \), radius \( r_p \) and centre at \( O_1 \) and the similar particle with centre at \( O_2 \), are those which, being directed towards \( O_1 \) fall on the spherical segment of one base, which belongs to the sphere with centre at \( O_1 \) and radius \( 1(l_e) \), whose height is \( h = 1(q\lambda) \) and which has its centre at \( A \). The area of this spherical segment is \( S_1 = 2\pi k\lambda(q\lambda)^2 \), and the projection of this area over the plane perpendicular to \( O_1 A \) at \( O_1 \) does not differ significantly from \( S_2 \). The sum of the flows which fall on the half spherical surface with centre at \( O_1 \) and radius \( r_p l_e \) is proportional to \( 2\pi(k\lambda r_p)^2(q\lambda)^2 \) and the projection of this surface over the said plane perpendicular to \( O_1 A \) at \( O_1 \) has the area \( S_{r_p} = \pi(k\lambda r_p)^2(q\lambda)^2 \).

The quotient \( S_2 \over S_{r_p} = 2 \over k\lambda (r_p)^2 \) is the factor of area, \( \phi_a \), by we must multiply the force per unit of surface at the distance \( r_p l_e \) from \( O_1 \), in order to obtain half the intensity of the energy flow which generates the gravitational force. This flow arrives at a distance \( d l_e \) from \( O_1 \) multiplied by the factor \( 1/d \), because up to distance \( 1(l_e) \) from \( O_1 \) the said flow decrease in intensity in inverse ratio to the distances from \( O_1 \), and not to the square of those distances. Taking all of this together, we obtain \( f_G/2 = \phi_a f_{r_p} l_e / d l_e \). The presence of the particle with center at \( O_1 \) exerts an equal and opposite effect on the particle with center at \( O_2 \), so that we finally obtain:

\[
f_G = \frac{4}{k\lambda r_p^2} \left( \frac{m_p l_e}{t_c^2} \right), \quad \text{i.e.}
\]

\[
f_G = \frac{4}{k\lambda} \left( \frac{m_p}{l_e^2} \right), \quad \text{whence} \tag{15}
\]

\[
m_p = \left( \frac{0.1 k\lambda}{0.4258} \right)^{1/6} = 2400, \quad \text{i.e.} \quad m_p = 2400 m_e
\]

Trying for a better approximation we can write:

\[
\frac{4(m_p)^6}{k\lambda} \left( \frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_n} \right)
\]

\[
= 0.4 \left[ \frac{1}{(d_1)^2} + \frac{1}{(d_2)^2} + \cdots + \frac{1}{(d_n)^2} \right],
\]

in which \( d_1, d_2, l, d_n \), are the distances between a proton and the nucleons which are at a distance of less than \( 1 l_e \) from its centre. Premise 3 in [4] implies a uniform distribution of charges in the heart of the nucleus, and a mean distance of 0.4258l_e determines that in such a case there are within a sphere of radius \( l_e \) around the proton:

\[ n = (0.4258)^{-3} \text{ nucleons, i.e., almost exactly 13 nucleons} \]

For \( n = 13 \), \[ \sum_{1}^{n} n^{-1} = 3.180134; \quad \sum_{1}^{n} n^{-2} = 1.570894 \]

and we obtain

\[
\frac{4(m_p)^6}{k\lambda} \cdot 3.180134 = 0.41570894 m_e,
\]

whence:

\[
m_p = \left( \frac{k\lambda 0.41570894}{12.720583} \right)^{1/6} m_e = 1.851 m_e,
\]

which is a very good approximation to the true value \( m_p = 1836 m_e \).

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