Dynamical charge and spin density wave scattering in cuprate superconductors

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Abstract. We show that a variety of spectral features in high-$T_c$ cuprates can be understood from the coupling of charge carriers to some kind of dynamical order which we exemplify in terms of fluctuating charge and spin density waves. Two theoretical models are investigated which capture different aspects of such dynamical scattering. The first approach leaves the ground state in the disordered phase but couples the electrons to bosonic degrees of freedom, corresponding to the quasi singular scattering associated with the closeness to an ordered phase. The second, more phenomenological approach starts from the construction of a frequency dependent order parameter which vanishes for small energies. Both theories capture scanning tunneling microscopy and angle-resolved photoemission experiments which suggest the protection of quasiparticles close to the Fermi energy but the manifestation of long-range order at higher frequencies.
1. Introduction

Two major lines of thought are presently debated for the understanding of the high-temperature superconducting cuprates. On the one hand \cite{1, 2, 3} these systems are seen as doped Mott insulators, where strong electron-electron correlations only play the major role. In this case at low doping (the underdoped region), short-range antiferromagnetic correlations spoil the metallic Fermi liquid (FL) phase producing singlet pairs, which give rise to a pseudogap below a temperature $T^*$ and eventually condensing into a superconducting state below $T_c < T^*$. Upon increasing doping, correlations progressively lose strength and $T^*$ merges into the $T_c$ line decreasing in the overdoped region. The second point of view \cite{4, 5, 6, 7} is that a more or less hidden electronic order is present in underdoped cuprates. For sure correlations favor the occurrence of this order, but normal-state anomalies and high-temperature superconductivity essentially stem from the presence of this order: The proximity to an instability (particularly if it is a second-order “critical line” ending at zero temperature into a quantum critical point) marking the onset of order naturally brings along abundant fluctuations and leads to strongly temperature- and doping-dependent features, which accounts for the non-FL properties and for a strong pairing interaction. These two distinct points of view particularly face themselves in the so-called “glue issue” \cite{8, 9}: what is the source of strong scattering/pairing between the charge carriers leading to high-$T_c$? In the case of a doped Mott insulator, one naturally expects a nearly instantaneous magnetic coupling $J$ to play the role of glue in forming the singlets, while for the “quantum critical” scenario the order-parameter fluctuations provide a glue mechanism, which is inherently retarded due to the slow dynamics of bosonic critical fluctuations. In this situation, we find that the two debated issues in the cuprates, namely to identify the source of electronic scattering and to identify the phase (if any) competing or coexisting with pairing in the underdoped region, are the two faces of the same medal.

From the above discussion it should be clear that to detect and to characterize any boson-like excitation coupled to the quasiparticles (QPs) is a major issue. This is precisely the scope of our paper, where we focus on the spectroscopic effects of order-parameter fluctuations. We will show that their dynamical character can make them rather elusive at low energy, while they leave clear signatures at higher energies. Moreover, the specific wavevector dependence of these excitations, as inferred from experiments, indicates that these retarded bosonic excitations are due to dynamical charge and spin ordering fluctuations. This implies that a locally ordered dynamical state is the natural candidate as the competing phase of underdoped cuprates. Far from being alternative to charge ordering fluctuations, we also believe \cite{10} that spin fluctuations are also relevant since they are sustained and supported up to large dopings by the occurrence of fluctuating charge-depleted regions. In the next Section we will briefly overview previous results within this fluctuating order scenario. In Sect. 3 we will present the general theoretical framework, which will find two distinct phenomenological
realizations in the subsequent Sects.4 and 5. Our concluding remarks are reported in Sect. 6.

2. Charge Ordering: a brief overview

Since the discovery of high-temperature superconductors by Bednorz and Müller [11], numerous experiments have evidenced the existence of electronic inhomogeneities in these compounds (cf. e.g. Ref. [12]). While early on these inhomogeneities where believed to be predominantly due to material imperfections like disorder induced by the dopant ions, it was subsequently realized that the strongly correlated character of the cuprate superconductors and thus the electronic subsystem itself favors the formation of the inhomogeneities. According to the theoretical analysis in Ref. [4] the reduction of the kinetic energy of the doped charge carriers caused by the strong correlations together with a short range attractive force, provided e.g. by electron-lattice interactions, gives rise to a phase separation instability. On the other hand, the long-range repulsive Coulomb interaction will spoil the associated zero-momentum instability in the charge sector and instead shift the wave-vector of the ordering transition to finite values which thus corresponds to an incommensurate charge ordering (CO). This is the so-called frustrated phase separation [13].

Alternatively, Hartree-Fock investigations of Hubbard (and tJ)-type hamiltonians [14, 15, 16, 17] have suggested early on that these models favor solutions with a combined charge- and spin-density wave. These solutions, which have been confirmed later by more sophisticated numerical methods (cf. e.g. [18, 19, 20]), are characterized by one-dimensional hole-enriched domain walls where the antiferromagnetic (AF) order changes sign.

The existence of such textures in lanthanum cuprates, codoped with Nd, was confirmed by elastic neutron scattering experiments [21, 22, 23]. These so called ‘stripes’ have also been found in other cuprate materials, codoped with Ba [24] or Eu [25] and the associated CO ordering has been explicitly established by soft resonant x-ray scattering [26, 27].

How generic are these charge- and spin density waves in the family of high-Tc materials? First, it is interesting to observe that the non-codoped lanthanum cuprates show strong similarities in the spin channel to their codoped counterparts. This includes not only the doping dependence of the low energy incommensurability [28], but also the spectrum of high energy magnon excitations which shows the same ‘hour glass’ shape in La$_{2-x}$Ba$_x$CuO$_4$ (LBCO) [29] and La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) [30]. These features can be well described on the basis of striped ground states [31, 32, 33] which also can account for the doping dependence of mid-infrared excitation in LSCO [34]. The ‘hour glass’ magnetic spectrum is now also well established in YBCO superconductors [35] where in the strongly underdoped regime even a static incommensurate spin response has been observed [36].

While neutron scattering provides a clear picture in the spin channel a similar tool
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for the charge channel does not exist. Electron energy loss spectroscopy or resonant inelastic x-ray scattering, although potentially being such tool, do not have yet the resolution to investigate the dynamical charge response with a similar accuracy as the one we have for spin fluctuations. Thus we are rather “blind” with respect to charge fluctuations.

One can hope to see evidence of charge and spin order in the one particle spectral properties accessible through angle resolved photoemission spectroscopy (ARPES) and scanning tunneling spectroscopy. However, direct evidence for charge and spin order is quite elusive at low energies and in this paper we argue that the reason for this 'invisibility' may be due to the dynamical nature of the scattering. We will discuss two different flavors of dynamical charge or spin ordering. Both of them have “protected” low-energy QPs and thus an untouched Fermi surface (FS). However, in the first version there is no long range order, while in the second version long range order is present. Within the former scenario the system is characterized by long (but finite) range spatial correlations which dynamics is associated with a characteristic time scale $\tau \sim 1/\omega_0$. Thus electrons with energy (measured with respect to the Fermi energy) $\omega < \omega_0$ will average over the fluctuations of the order parameter and therefore keep the QP properties of an ordinary homogeneous FL. On the other hand, a snapshot on time scales $t < \tau$ (i.e. frequencies $\omega > \omega_0$) will detect an almost ordered system and we thus expect the features of a conventional ordered system to become apparent in the spectral function at large enough energies. The idea of a low energy sector with protected quasiparticles and a high-energy sector displaying some form of order is also closely related to a recent model of FeAs based superconductors[39].

This idea of (incoherent) high-energy states carrying a specific momentum structure might seem strange, but another example can clarify the concept of an incoherent part with a strong momentum dependence which carries physical information on the short range physics. Lets consider the more standard issue of large and small FSs in heavy fermions represented in Fig. 1. In heavy-fermions strongly correlated electrons in a narrow half-filled $f$ level hybridize with electrons in a conduction band and give rise to a Kondo resonance at the Fermi level formed by coherent QP states. The width (and weight) of this QP band is usually quite small and sets the scale of the coherence energy in these systems. Now consider the momentum distribution function defined by

$$n_k \equiv \int d\omega A(k,\omega)f(\omega).$$

Here $f(\omega)$ is the Fermi function and the spectral density

$$A(k,\omega) \equiv \frac{1}{\pi} ImG(k,\omega) = \frac{1}{\pi} \frac{\Sigma''(k,\omega)}{(\omega - \Sigma'(k,\omega))^2 + \Sigma''^2(k,\omega)}$$

is proportional to the imaginary part of the electron Green’s function with real (imaginary) part of the self-energy $\Sigma'$ ($\Sigma''$). One can see that $n_k$ involves all the excitation energies and its features might be dominated by the incoherent part of the spectrum if the QPs have a minor weight. Indeed strictly speaking the true FS at zero temperature is given by the small jump in the Fermi distribution function determining
Figure 1. Schematic view of a heavy-fermion system with a Kondo-like resonance arising at the Fermi energy $E_F$ from the mixing of a deep narrow $f$ level (not shown) and the conduction band (dashed line). Two QP bands, $\varepsilon^{1,2}(k)$, arise. The corresponding momentum distribution function $n(k)$ is shown below with a true Fermi momentum $k_{F}^{QP}$ and a “fictitious” FS at $k_{c}^{F}$, where the conduction band (dashed line) crosses the Fermi level (thin red line).

the Fermi momenta of the QPs at $k_{F}^{QP}$ (see Fig. 1). This FS is large and satisfies the Luttinger theorem with a number of carriers including the electrons in the $f$ level. This FS would naturally be determined by following the QP dispersion $[\varepsilon^{1}(k)]$ in the upper panel of Fig. 1. On the other hand the shape of $n_{k}$ is substantially determined by the (incoherent) part of the spectral function, which has strong weight at energies corresponding to both the $f$ level and the conduction band. This latter gives rise to a rather sharp decrease of $n_{k}$ at a “fictitious” Fermi momentum $k_{F}^{c}$, corresponding to the FS that the electrons in the conduction band would have in the absence of mixing with the $f$-level. If the hybridization between the $f$ level and the conduction band is turned off so that the QP weight $z$ is driven to zero one reaches a situation in which the decrease at $k_{F}^{c}$ becomes a discontinuity and the small jump at $k_{F}^{QP}$ disappears. It is clear that the sharp decrease of spectral weight at $k_{F}^{c}$ for finite hybridization has strong physical content for an observer who ignores the underlying model. Thus in this example we see the recurrent picture of low energy quasiparticles appearing due to coherence effects and high energy incoherent excitations with a relevant momentum structure which carry information on the short range physics.
In a series of papers [38, 43, 44] we have worked out the scenario of Fermionic QP coupled to dynamical CO fluctuations with regard to several experimental observations. One first important remark is that all the conclusions drawn in the case of QP coupled to spin excitations [37] might also be obtained for the case of CO fluctuations (with, of course, suitable changes in the energy scales and in the momentum dependencies). As an example we report in Fig. 2 the electronic dispersion along the BZ diagonal obtained when the QPs are coupled to a charge fluctuations with flat distribution of momenta and a Lorentzian distribution of frequencies (see below, in Sect. IV). The typical frequency of the CO fluctuations is $\omega_0 = 75$ meV, and the width of the distribution is $\gamma = 40$ meV. The bare electronic dispersion (with Fermi velocity $v_F = 2/\pi$ eV (in units of lattice spacing) is dressed by the real-part of the self-energy $\Sigma'$ reported in the inset together with the experimental data reported in Ref. [40]. Ref. [40] also reported a remarkable isotopic shift of the electronic dispersions, which had opposite sign in different regions of the BZ. Although this effect is controversial [41, 42], we were able to show that an isotopic dependence of the coherence length of CO dynamical fluctuations can reproduce this effect and account for its strong momentum and doping dependence [43].

Dynamical spin fluctuations, instead, would produce a similar effect, but with a wrong momentum dependence. Another non trivial effect of dynamical CO fluctuation is related to the dichotomy in the Fermi surface of high-$T_c$ cuprates [45, 46]. Especially these latter ARPES studies on underdoped LSCO support our picture of dynamical order in the cuprates. On the one hand the momentum dependence of the low-energy part of the energy distribution curves was followed, thereby reconstructing the low-
energy QP dispersion. In this way a large FS was found corresponding to the FL LDA band-structure and fulfilling the Luttinger requirement that the volume of the FS encircled the whole number of fermionic carriers $n = 1 - x$. On the other hand the FS was determined from the momentum distribution $n_k$, obtained by integrating the spectral function over a broad energy window ($\sim 300$ meV). Then the locus of momentum-space points where $n_k$ displays a sharp decrease, marked a FS formed by two nearly parallel (weakly modulated) lines along the $k_x$ direction and crossing two similar lines along the $k_y$ direction. This crossed FS would naturally arise in a system with one-dimensional stripes along the $y$ and $x$ directions. Thus the coexistence of stripe-like spectral features at large energies with a ‘protected’ FS at low energies due to coherence effects exactly corresponds to the scenario of fluctuating order sketched above.

Further even more direct evidence for dynamical CO in cuprate superconductors is provided by scanning tunneling microscopy (STM) experiments. STM investigations performed on bismuthate and oxychloride superconductors see a complex modulation of the local density of states (LDOS) both in the superconducting (SC) state and above $T_c$. In both cases one observes peaks in the Fourier transform of the real space LDOS at wave-vectors $Q = 2\pi/(4a_0)\ldots2\pi/(5a_0)$ suggestive of checkerboard or stripe charge order. However, the debate is about the question whether these peaks are non-dispersive in energy (and thus signature of ‘real’ charge order) or follow a bias-dependent dispersion due to QP interference. In the latter case the spatial LDOS variations can be understood from the so-called octet model which attributes the modulations to the elastic scattering between the high density regions of the Bogoljubov ‘bananas’ in the superconducting state. Recent STM investigations may resolve this apparent conflict since they suggest that both, dispersive and non-dispersive scattering originates from different regions in momentum and energy space. The states in the nodal region which are well defined in k-space and undergo a transition to a $d$-wave SC state below $T_c$ are then responsible for the low energy QP interference structure of the LDOS, whereas the ill-defined k-space ‘quasiparticle’ states in the antinodal regions are responsible for the non-dispersive CO above some energy scale $\omega_0$. A phenomenological model of dynamical CO (but in the presence of long-range order also) was recently adopted to describe this dichotomic behavior observed in STM experiments. In particular, assuming a specific frequency-dependent CO order parameter producing a marginal-FL type self-energy we were able to reconcile the simultaneous existence of low energy Bogoljubov quasiparticles and high energy electronic order. Moreover the theory also accounts for the CO specific contrast reversal in the STM spectra between positive and negative bias where the energy scale for the modulation of the LDOS is essentially determined by the pairing gap. We also notice in passing that recent microscopic calculations based on QPs coupled to diffusive/propagating CO collective modes demonstrate that despite the strong momentum dependence of CO fluctuations, they can also give rise to a marginal-FL scattering in integrated quantities (like in LDOS).

In summary, we found a wealth of evidences that dynamical order fluctuations...
can be present to provide a boson-like retarded scattering mechanism. Of course the spectral distribution of these excitations has a strong influence on the resulting effective interaction. The last example of STM spectra shows that this distribution should be rather flat and featureless (i.e. $\gamma \gg \omega_0$) so that some marginal FL self-energy should result. On the other hand the presence of a marked kink feature in ARPES spectra requires that the bosonic energy scale, at least in this momentum and energy range, is rather well defined. To emphasize more clearly the role of dynamics in the ordering fluctuations, in the rest of this paper we will rather focus on the effects of narrow boson spectral distributions.

3. General formalism and self-energies

In translationally invariant systems the Greens function in real space $G_{ij}(\omega)$, corresponding to an annihilation of a quasiparticle at site $R_i$ and subsequent creation at site $R_j$, only depends on the difference between these two sites, i.e. $G_{ij}(\omega) = G(R_i - R_j, \omega)$. The same holds for the self-energy $\Sigma_{ij}(\omega) = \Sigma(R_i - R_j, \omega)$ so that the Dyson equation in real and momentum space reads as

\begin{equation}
G_{ij}(\omega) = G_{ij}^0(\omega) + \sum_{n,m} G_{i n}^0(\omega) \Sigma_{n m}(\omega) G_{m j}(\omega)
\end{equation}

\begin{equation}
G_k(\omega) = G_k^0(\omega) + G_k^0(\omega) \Sigma_k(\omega) G_k(\omega).
\end{equation}

This (standard) route is followed in Sec. 4 where we consider a self-energy $\Sigma_k(\omega)$, derived from the coupling of quasiparticles to a set of singular bosons. This allows us to study the effect of a dynamical protection of the Fermi surface in a homogeneous system close to an ordering instability. We will show that the quasiparticles close to $E_F$ are not affected by the proximity to the instability whereas at high energies the system looks ordered.

On the other hand, a description of an electronically inhomogeneous state, as observed in STM above some energy scale, necessarily requires the generalization of Eq. (3) to the case where both, $G_{ij}(\omega)$ and $\Sigma_{ij}(\omega)$, separately depend on sites $R_i$ and $R_j$. For simplicity we will consider periodic modulations of the spin and charge density. We assume that the system consists of $N_c$ non-equivalent sites which form a supercell. These supercells repeat periodically thus generating a Bravais lattice. The latter has a reciprocal lattice with exactly $N_c$ non-equivalent reciprocal lattice vectors $Q_n$. For example, if the electronic order is characterized by some density modulation with periodicity $\lambda = N_c a$ in the $x$-direction, then the reciprocal lattice vectors are $Q_n = n \frac{2\pi}{\lambda}(1,0)$ with $n = 1\ldots N_c$. In this case the Dyson equation can be written in momentum space as

\begin{equation}
G_{k+Q_m, k+Q_m} = G_{k+Q_m, k}^0 \delta_{m,n} + G_{k+Q_m, k}^0 \sum_s \Sigma_{k+Q_m, k+Q_m} G_{k+Q_m, k+Q_m}.
\end{equation}

The self-energy now becomes a $N_c \times N_c$ matrix where only the diagonal elements $\Sigma_{k+Q_m, k+Q_m}$ (in case of retarded GF’s) need to obey the condition $Im \Sigma \geq 0$. While the off-diagonal elements still obey Kramers-Kronig relation, the corresponding imaginary
part can have sign changes as a function of $\omega$. This may lead to different low frequency behavior for the off-diagonal $\text{Re}\Sigma$ as for the 'usual' diagonal part.

4. Kampf-Schrieffer approach: systems without long range charge ordering

Here we follow an approach introduced by Kampf and Schrieffer in connection with pseudogap physics due to strong AF fluctuations [74]. The quasiparticles are coupled to boson excitations which are strongly peaked in frequency and momentum space. In our following calculations the uncoupled ground state is a d-wave superconductor and for simplicity the boson correlator is factorized in a frequency and $\mathbf{q}$-dependent part.

The self-energy is then obtained from

$$\Sigma_{\mathbf{k}}(i\omega) = -\frac{1}{2\beta} \sum_{\mathbf{q},ip} J(\mathbf{q}) D(ip) \tau_z G_{\mathbf{k}-\mathbf{q}}(i\omega-ip) \tau_z \quad (6)$$

where we have used Nambu-Gorkov notation so that the unperturbed Green’s function is represented by

$$G^0_{11}(k, i\omega) = \frac{u_k^2}{i\omega - E_k} + \frac{v_k^2}{i\omega + E_k} \quad (7)$$
$$G^0_{22}(k, i\omega) = \frac{v_k^2}{i\omega - E_k} + \frac{u_k^2}{i\omega + E_k} \quad (8)$$
$$G^0_{12}(k, i\omega) = G^0_{21}(k, i\omega) = -u_k v_k \left[ \frac{1}{i\omega - E_k} - \frac{1}{i\omega + E_k} \right] \quad (9)$$

The BCS coherence factors are defined as $u_k^2 = \frac{1}{2} (1 + \frac{\epsilon_k - \mu}{E_k})$ and $v_k^2 = \frac{1}{2} (1 - \frac{\epsilon_k - \mu}{E_k})$ respectively, and the propagator

$$D^{\text{tot}}(i\omega) = -\int d\nu W(\nu) \frac{2\nu}{(\omega^2 + \nu^2)} \quad (10)$$
describes the distribution of dispersionless propagating bosons.

The momentum dependent coupling is contained in the function

$$J(\mathbf{q}) = g^2 N \sum_{\pm q_x^c, \pm q_y^c} \frac{\Gamma}{\Gamma^2 + 2 - \cos(q_x - q_x^c) - \cos(q_y - q_y^c)} \quad (11)$$

which is enhanced at the four equivalent critical wave vectors ($\pm q_x^c, \pm q_y^c$). $N$ is a suitable normalization factor introduced to keep the total scattering strength constant while varying $\Gamma$.

We restrict to the leading order one-loop contribution of the self-energy Eq. (6), i.e. we replace the full by the non-interacting Green’s function on the r.h.s. In order to illustrate the basic features of the present approach, we show in Figs. 3-5 the spectral function for a one-dimensional system of electrons (dispersion $\epsilon_k = -\cos(k)$) exposed to dynamical CO scattering with $Q = \pi$. We start by considering the case of a single bosonic mode oscillating at a fixed frequency $\omega_0$. Formally this corresponds to $W(\nu) = \delta(\omega_0 - \nu)$, which was extensively discussed in Ref. [44]. In Fig. 3(a) we report the case of such a single dynamical mode with a small but finite coupling to a system of one-dimensional electrons. To clarify the effect of dynamics, in panel (b) we also consider
the case of a vanishingly small coupling $g = 0$. The spectral function carries information on the excitations of the system with one added or removed particle. For $g = 0$ the lowest energy excitations consist of an added or removed free fermion which produce the dispersion relation indicated by the full line in Fig. 3(b). However there are also excitations in which the fermion is added/removed with the addition of a boson which carries momentum $Q = \pi$ and energy $\omega_0$. The dispersion relation of these excitations is depicted by the dashed line. For $g = 0$ all the weight is in the main band labeled $\xi_k$.

The effect of a finite coupling $g$ is to give some spectral weight to the shadow band at $\xi_{k-Q} \pm \omega_0$ and to introduce some level repulsion when the bands cross. The important point is that in contrast to a really statically ordered system the shadow band never touches the Fermi level, but it is separated from it by the energy to create the bosonic excitation. We associate the main band with the QP band and the shadow band with the incoherent spectral weight arising from the scattering with CO fluctuations. Clearly close enough to the Fermi level only the QP band exists but the high energy spectral function resembles that of an ordered system. Furthermore, the momentum distribution function is practically identical to that of an ordered system, so that a naive analysis of a direct measurement of $n_k$ by Compton scattering or by integrating the photoemission spectral function will lead to a different conclusion on the ordering of the system than a low energy photoemission experiment.

The above example illustrate in a simple way the main idea of a protected Fermi surface in an almost ordered system. On the other hand, the assumption $\Gamma \to 0$ is
too singular from the point of view of Fermi liquid theory and yields unphysical results if the interaction is increased beyond a certain value or in the case of nesting. For larger coupling the backbending of the main band in Fig. 3 will induce an additional FS crossing and thus the generation of spurious ‘shadow features’ at the Fermi level. In Ref. [44] we have circumvented this complication by the introduction of phenomenological vertex corrections aimed to suppress the coupling to the boson at the Fermi energy. Here we show that the problem can also be avoided by employing a more physical coupling which is less singular. Fig. 4a reports the spectral function for the same parameters as Fig. 3 but with a finite correlation length (i.e. finite $\Gamma = 0.1$ in Eq. (11)). Clearly the backbending of the main dispersion is significantly reduced thus derogating the tendency towards shadow feature formation at the Fermi level. On the other hand the high energy states still resemble the structure of a $Q = \pi$ ordered state with of course somewhat smaller weight as compared to Fig. 3. Furthermore the momentum distribution function shown in the lower panel, still resembles that of an ordered system, with a momentum broadening of the shadow features similar to the momentum broadening of the small Fermi surface of the heavy-fermion system of Fig. 1.

We now move to the case of a distribution of bosons which can be tuned to be more or less broad. Our results on cuprates in the second part of this section are obtained for a linear distribution of bosonic modes up to some cutoff energy $\omega_{\text{max}}$, i.e. $W(\nu)$ in Eq. (10) is given by $W(\nu) = (2\nu/\omega_{\text{max}}^2) \Theta(\omega_{\text{max}} - \nu)$. Note however, that the basic features of the spectra do not depend significantly on this choice, but similar results would be obtained by a lorentzian distribution of bosonic modes.

In order to exemplify the influence of a frequency broadening on the spectra, Fig. 4b reports the case of $\omega_{\text{max}} = 0.3$ for a doped one-dimensional system which can be compared with the single frequency case reported in Fig. 3. Also in this case, for the doped system the spectra at higher energy resemble those of an ordered system due to the appearance of the shadow bands, but again these shadow bands do not cross the Fermi level and are gapped on a scale of the bosonic excitation energy $\omega_{\text{max}}$. Therefore close to the Fermi level the system is protected by the scattering which only becomes apparent on an energy scale larger than that of the bosons. As can be seen from the lower panels of Fig. 4 both in the case of momentum broadening as well as frequency broadening the protected Fermi-liquid like quasiparticles coexist with a momentum distribution function resembling that of a system with long-range order. This thus mimicks the results obtained by Zhou et al. in cuprates using ARPES. [45, 46]
We proceed by applying the Kampf-Schrieffer-type approach to the investigation of quasiparticle spectra in the superconducting state of high-\(T_c\) superconductors. In this context we ask the question how this scattering affects the effective spectral gap of the system and in which way it influences the quasiparticle weight. Our investigations are based on a parametrization of the dispersion of Bi2201 from Ref. [63]

\[
\varepsilon_k = -2t[\cos(k_x) + \cos(k_y)] - 4t' \cos(k_x) \cos(k_y) \\
- 2t''[\cos(2k_x) + \cos(2k_y)] - u
\]  

(12)
with $t = 217.5 \text{meV}$, $t' = -60 \text{meV}$ and $t'' = 20 \text{meV}$. The chemical potential is
$u = -0.23 \text{eV}$ corresponding to a particle number of $n \approx 0.81$. The CO scattering vectors are restricted to $Q_m = (\pm 0.851/a, 0), (0, \pm 0.851/a)$ which connect the FS segments at
the antinodal $(0, \pi)$ and $(\pi, 0)$ points, respectively. For simplicity, the results below are derived for the case of infinite correlation length (i.e. $\Gamma \to 0$). However, the model can be extended to CO scattering with finite correlation length which from a technical point of view requires some more numerical effort. To circumvent the $\Gamma \to 0$ complications associated with nesting in the two-dimensional case, we will concentrate on the superconducting ($d$-wave) state so that the FS is gapped in those
regions of momentum space where the scattering could fulfill the nesting condition. Please remember that these complications are avoided by a finite $\Gamma$. The SC gap is approximated by a simple harmonic $d$-wave structure $\Delta(k) = \Delta_0[\cos(k_x) - \cos(k_y)]$ with $\Delta_0 = 30 \text{meV}$ and the linear distribution of boson modes is cut off at $\omega_{\text{max}} = 0.1 \text{eV}$.

Figure 6. Minimum gap deduced from the 'minimum gap' locus for various couplings of the dynamical CO scattering.

The evolution of the SC gap along the underlying FS is shown by the square
symbols in Fig. 6 (plotted as a function of the angle $\phi$ defined in the inset). It follows approximately the relation $\Delta(\phi) = \tilde{\Delta}_0 \cos(2\phi)$ with $\tilde{\Delta}_0 = 28.5 \text{meV} < \Delta_0$ due to the displacement of the Fermi segments from the M-points. Switching on the CO scattering alters the underlying FS and we have to find the effective minimum gap by scanning over the whole Brillouin zone. Fig. 6 shows that the scattering induces a deviation from the simple harmonic $d$-wave structure and increases the effective gap upon approaching the M-point. We can fit the resulting angular gap dependence by including higher harmonics which for the coupling $g^2 = 0.004(\text{eV})^2$ (circles in Fig. 6) leads to $\Delta(\phi) = \tilde{\Delta}_0[0.9 \cos(2\phi) + 0.1 \cos(6\phi)]$ with $\tilde{\Delta}_0 = 38 \text{meV}$. This behavior is close to the observed midpoint shift of the EDC leading edge in recent ARPES experiments [61] where an even larger anharmonicity was reported. Whereas we could easily reproduce larger effective gaps around the M-points this would put in jeopardy our weak coupling approach. We, however, note that the deviation from the harmonic gap structure in our calculations occurs at essentially the same angle ($\approx 0.15\pi$) as in the experiment.
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reported in Ref. [61]. The anharmonicity of the gap function is also supported by STM experiments on underdoped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ [64] where the fitting of the local density of states (LDOS) requires an angular dependence of the superconducting gap which is composed of a harmonic $d$-wave and an additional anharmonic contribution which contributes in the antinodal regions.

Fig. 7 reports the spectral function for selected points along the 'minimum gap' locus. Since the scattering momentum connects the antinodal segments of the underlying FS the corresponding spectrum (panel a) displays the weak incoherent shadow features both below and above $E_F$, which are additionally shifted in energy by the mixing to the boson excitations. Upon moving away from the antinodes (panel b) the scattering predominantly affects states below $E_F$ so that the associated high energy spectrum acquires large incoherent characteristics. On the other hand the nodal regions (panels c,d) are hardly affected by the scattering and the corresponding dispersion resembles that of a 'clean' $d$-wave superconductor.

We proceed by calculating the spectral weight of the SC coherence peaks in different parts of the Brillouin zone. The corresponding experimental ARPES study has revealed a strong angular dependence of the weight upon scanning from the antinodes towards the nodal region [62]. In this work the (symmetrized) energy distribution curves for $\mathbf{k} = \mathbf{k}_F$ (≡ Fermi momentum) at temperatures slightly above $T_c$, $\rho^>_{\mathbf{k}F}(\omega)$, have been substracted from those obtained below $T_c$, $\rho^<_{\mathbf{k}F}(\omega)$. The coherent spectral weight $W_{CP}$ is then defined as the integral over the positive area of this difference, i.e.

$$ W_{CP} = \int_{-\infty}^{\omega_c} d\omega \left( \rho^<_{\mathbf{k}F}(\omega) - \rho^>_{\mathbf{k}F}(\omega) \right) $$

and $\omega_c$ is defined as the binding energy where the integrand becomes negative close to $E_F$. Analysis of the ARPES data [62] revealed a characteristic angular dependence of $W_{CP}$ (Fig. 2h in Ref. [62]) which vanishes at the node and is additionally suppressed around the antinodes. The suppression increases with underdoping so that $W_{CP}$ develops a (doping-dependent) maximum at intermediate angles between nodal and antinodal regions.

In order to understand these results let us first start with the case of an 'unperturbed' ideal superconductor. Here the difference between SC and normal state spectral function on the Fermi surface $\delta \text{Im}G_{\mathbf{k}}(\omega) = \text{Im}G_{\mathbf{k}}^{SC}(\omega) - \text{Im}G_{\mathbf{k}}^{NS}(\omega)$, consists of two delta-peaks at $\omega = \pm \Delta(\mathbf{k})$ with weight one-half and a negative delta-peak at $\omega = 0$ with weight one, respectively. Thus the coherent spectral weight $W_{CP}$ as the integral over the $(\omega \leq 0)$ coherence peak yields $W_{CP} = 1/2$ for a finite SC gap and $W_{CP} = 0$ for $\Delta = 0$. In case of disorder the delta-peaks are broadened into lorentzians (width $\delta$) for which an analogous consideration yields

$$ W_{CP}(\mathbf{k}) = \frac{1}{\pi} \text{arctan} \left( \sqrt{1 + \alpha_k^2} \right) - \frac{1}{2\pi} \text{arctan} \left( \frac{\sqrt{1 + \alpha_k^2 + 2\alpha_k}}{2\pi} \right) 
- \frac{1}{\pi} \text{arctan} \left( \sqrt{1 + \alpha_k^2 - 2\alpha_k} \right) $$

with $\alpha_k = \Delta_k/\delta$. The function Eq. (14) is shown in the inset of Fig. 9 and suggests that the slope of the observed angular dependence of $W_{CP}$ around the nodes is probably
Figure 7. Spectral function along selected radial cuts of the Brillouin zone. (cf. inset to Fig. 6) as a function of the $k_x$ projection. From a) to d) $\phi/\pi = 0, 0.1, 0.2, 0.25$. The intensity scale corresponds to $\frac{1}{\pi} Im G^{k_1}_{k} (\omega) \delta \omega$. Parameters: $g^2 = 0.002 (eV)^2$, $\Delta_0 = 30 meV$.

determined by the amount of disorder in the sample. On the other hand we can attribute the suppression of $W_{CP}$ around the antinodes to the influence of the fluctuating CO order which predominantly affects these regions in momentum space. Fig. 8 shows the difference between normal state and SC EDC curves (both under the presence of dynamical CO scattering) for selected momentum points along the minimum gap locus. The suppression of coherent spectral weight around the antinodes due to the scattering is clearly apparent. On the other hand, close to the nodes the coherence peaks shift to small energies and due to the broadening the associated weight is partially absorbed by the negatively weighted normal state peak causing the vanishing of $W_{CP}$ for $\phi \rightarrow \pi/4$.

We are now in the position to theoretically model the angular dependence of $W_{CP}$ as shown in Fig. 2h of the ARPES study of Ref. [62]. We select parameters in such a way that the overdoped sample is described by an unperturbed $d$-wave superconductor, but with significant broadening of the spectral function due to the dopant induced disorder. Optimally doped and underdoped samples are modelled by a smaller amount of disorder but with increasing coupling to the CO fluctuations. The resulting angular dependencies of $W_{CP}$ are shown in Fig. 9 and qualitatively reproduce the experimental ones of Ref. [62]. It should be noted that the experimental data of Ref. [62] report a complete suppression $W_{CP}$ around the antinodes for the underdoped sample. Within the present approach this would require much larger couplings beyond our weak coupling computation. However, it should also be mentioned that the suppression may not be as strong as the analysis of Ref. [62] suggests. In fact, as discussed above, their method of extracting $W_{CP}$ is based on the positive area difference between EDC’s below and above $T_c$ which (since spectral weight is conserved) should be the same as the negative area difference which has not been analyzed. Since the latter is still substantial in both optimally doped and underdoped samples around the antinodes (cf. Figs. 2d,f in Ref.
Figure 8. Difference between EDC’s in the SC and normal state for selected momenta along the ‘minimum gap’ locus. From top to bottom \( \phi/\pi = 0.05, 0.1, 0.2 \). Parameters: \( g^2 = 0.004(eV)^2 \), \( \Delta_0 = 30\text{meV} \).

This may point towards a less complete suppression than what is suggested by Fig. 2h of Ref. [62].

Note that the dip feature around \( \phi \approx 0.05 \) in the curve for optimal doping (square symbols) results from the scattering of near-antinodal points between adjacent Brillouin zones. As can be seen this dip is smeared out for larger coupling and is obviously a result of our single momentum CO scattering approximation. We expect that the implementation of finite range correlations by e.g. a lorentzian distribution of scattering vectors would wash out the features associated with nesting, however, the mean magnitude should still be comparable with the separation of the antinodal FS segments so that the scattering still induces the suppression of \( W_{CP} \) in that area of the Brillouin zone.

5. Phenomenological approach with long range charge ordering

While the Kampf-Schrieffer limit captures in principle the dichotomy between low energy protected quasiparticles and high energy CO scattering, it preserves the homogeneity of the ground state. However, STM experiments show that the real space spectra taken at
different sites are not equivalent often showing a glassy character. Even more, as outlined in the introduction, the observation of almost dispersionless LDOS modulations at higher energy [58] provides additional evidence for real translational symmetry breaking. In this case a translational invariant treatment is clearly inappropriate.

For simplicity we will restrict to systems in which the symmetry breaking is periodic but the same physics applies to disordered systems. As discussed in Sec. 3, symmetry breaking is described by the off-diagonal GF and self-energy contributions in Eq. (5) which will be phenomenologically constructed in the present section.

For this purpose we consider particles on a lattice which gain kinetic energy from hopping processes ($\sim t_{ij}$) but are additionally scattered by an inhomogeneous, local, and frequency dependent self-energy. Thus in Eq. (5) we set $\Sigma_{ij}(\omega) \rightarrow \Sigma_i(\omega)\delta_{ij}$ and Dyson equation in real space reads,

$$ (\omega - \Sigma_i(\omega)) G_{\sigma}^{\sigma}_{ij} = \delta_{ij} + \sum_p t_{ip} G_{\sigma}^{\sigma}_{pj}. $$(15)

Our objective in the following is to construct a phenomenological self-energy $\Sigma_i(\omega)$ which leaves the states around $E_F$ protected from the scattering but is consistent with some kind of electronic order at higher binding energies. Note that the local charge density at site $m$ is determined by the sum over all frequencies of the off-diagonal GF

$$ \langle n_m \rangle = \frac{1}{\beta N} \sum_{k,s,i,p} \exp(iQ_s R_m) G_{k,k+Q_s}(ip) $$

where the $Q_s$ are reciprocal lattice vectors introduced in Sec. 3. One is thus able to construct a $\Sigma_i(\omega)$ which vanishes for $\omega \rightarrow 0$ (and thus also the off-diagonal GF) but causes a inhomogeneity in $\langle n_m \rangle$ via its high frequency part.

In order to accomplish this task it is necessary to maintain the correct analytical properties of the self-energy and GF. Without loss of generality we can introduce a pole
expansion for the self-energy,
\[ \Sigma_i(\omega) = v_i^2 \sum_n \frac{1}{\omega - \epsilon_{n,\sigma}} + \Delta_i \equiv v_i^2 f_i(\omega) + \Delta_i. \]  
with the constant \( \Delta_i \) controlling the high frequency limit. The expansion in Eq. (17) coincides with the self energy of the following Fano-Anderson Hamiltonian,
\[ H_{aux} = H^0 + \sum_{i,n,\sigma} v_{i,n} \left[ c_{i,\sigma}^{\dagger} f_{i,n,\sigma} + h.c. \right] + \sum_{i,n,\sigma} \epsilon_{i,n} f_{i,n,\sigma}^{\dagger} f_{i,n,\sigma} \]  
and which is illustrated in Fig. 10 for a one-dimensional system. Besides the hopping between nearest neighbor sites (\( \sim t \)), itinerant (c) electrons (representing the quasiparticles) on each site can be transferred (\( \sim v_{i,n} \)) to a bath of auxiliary f-states ('impurities') representing the collective fluctuating charge or spin environment which scatters the itinerant carriers. It is straightforward to show that the GF for the (c) quasiparticles obeys Eq. (15) and the local self-energy is determined by the coupling to the f-level distribution. The mapping to this auxiliary hamiltonian problem ensures that the phenomenological self-energy has all the correct analytical properties.

The function \( f_i(\omega) \) describes the dynamics of the CO and depends on the site and frequency. \( \Delta_i \) describes the frequency independent modulation of onsite levels.

Since the self-energy is local, upon Fourier transforming it no longer depends on \( \mathbf{k} \) but only on the transferred momentum, i.e. \( \Sigma_{\mathbf{k}+\mathbf{Q}_n,\mathbf{k}+\mathbf{Q}_m} \rightarrow \Sigma_{\mathbf{Q}_m-\mathbf{Q}_n} \). Thus the diagonal elements of the self-energy matrix are equal and given by \( \Sigma_0 \) while other values of the transferred momentum correspond to off-diagonal elements.

To gain some insight into the properties of this approach consider a half-filled one-dimensional model (dispersion \( \epsilon_k = -\cos(k) \)) with period doubling (\( N_c = 2 \)). In this case the Fourier transformed \( \Sigma_Q(\omega) = 1/N \sum \exp(iQR_j) \Sigma_j(\omega) \) has only two non-vanishing components, \( \Sigma_{Q=0}(\omega) \) (diagonal) and \( \Sigma_{Q=\pi}(\omega) \) (off-diagonal).

With regard to the auxiliary hamiltonian Eq. (18) there are essentially two possibilities to realize the dynamical modulation: An alternating coupling to a site independent frequency spectrum (in the following referred to as 'variant 1') or a constant coupling to a frequency spectrum which alternates from site to site ('variant 2').

Figure 10. Schematic structure a 1D model hamiltonian implementing dynamical CO order. The itinerant 'c'-states represent the non-interacting quasiparticles of the system which are coupled to a bath of f-states representing collective charge or spin fluctuations. Static CO corresponds to local shift of the 'c'-state onsite levels by \( \Delta_i \).
Consider first variant 1. The (site-independent) frequency spectrum is assumed to be peaked at $\omega = \pm \omega_0$ and reads as

$$f(\omega) = \frac{1}{\omega - \omega_0} + \frac{1}{\omega + \omega_0}. \quad (19)$$

Coupling only odd sites ($v_{2i}^2 = 0$, $v_{2i+1}^2 = 2v^2$) to the spectrum yields the self-energies

$$\Sigma_{Q=0}(\omega) = -\Sigma_{Q=\pi}(\omega) = v^2 f(\omega). \quad (20)$$

In the weak coupling limit ($\lambda \equiv v/\omega_0 \ll 1$) we can expand the resulting Green’s function in the limits $\omega > \omega_0$ and $\omega < \omega_0$ which yields

$$G_k(\omega) = \frac{Z}{\omega - Z \varepsilon_k} \quad \text{for } \omega < \omega_0 \quad (21)$$

$$G_k(\omega) = \frac{\omega + \varepsilon_k - \frac{2g^2}{\omega}}{(\omega - E_k)(\omega + E_k)} \quad \text{for } \omega > \omega_0$$

with $Z = 1/(1 + \lambda)$ and $E_k = \sqrt{\varepsilon_k^2 + 4v^2}$. Fig. 11(a) displays the resulting spectral function for variant 1. The spectrum is composed of the low energy part ($\omega < \omega_0$) with renormalized dispersion and quasiparticle weight ($\sim Z$). Also apparent is the weak shadow band due to the low energy scattering induced by $\Sigma_{Q=\pi} = \lambda^2 \omega$. At high energy ($\omega > \omega_0$) the electronic structure changes to that of a CDW with poles at $\pm E_k$, i.e. a CDW which is given by $\Delta = 2v$.

Consider now the variant 2 with a constant coupling to an alternating frequency spectrum. For simplicity we assume that the latter is determined by a pole at $\omega = (-1)^R \omega_0$ so that the self-energies read as

$$\Sigma_{Q=0} = v^2 \left[ \frac{1}{\omega - \omega_0} + \frac{1}{\omega + \omega_0} \right] \quad (22)$$

$$\Sigma_{Q=\pi} = v^2 \left[ \frac{1}{\omega - \omega_0} - \frac{1}{\omega + \omega_0} + \frac{2}{\omega_0} \right]. \quad (23)$$

Note that a static component $\Delta = \lambda^2 \omega_0 = \frac{\omega^2}{\omega_0}$ has been added to the finite momentum self-energy in order to keep the limit $\Sigma_{Q=\pi}(\omega = 0) = 0$.

The difference to the previous case is first in the low energy behavior of $\Sigma_{Q=\pi}$ which now is of the order $O(\omega^2)$ [56]. This further reduces the intensity of the low energy shadow bands which is apparent from the spectral function shown in Fig. 11(b). Second, the high-energy part of the off-diagonal self-energy Eq. 23 approaches $\Sigma_{Q=\pi}(\omega \to \infty) = 2v^2/\omega_0$ and thus corresponds to the self-energy of the static solution. Thus in contrast to variant 1, which for the present choice of $f(\omega)$ (cf. Eq. 19) leads to a vanishing CO scattering in the limit $\omega \to \infty$, variant 2 leads to a finite off-diagonal Green’s function $G_{k,k+Q}$ for all energies $\omega \gg \omega_0$. It should be noted that in case of variant 2 the weak coupling expansion of the diagonal Green’s function $G_{k,k}$ at low and high energies leads to the same results as before [cf. Eq. 21].

Being phenomenological in nature we do not have a microscopic electron-electron or electron-boson model from which this self-energy can be derived (contrary to the Kampf-Schrieffer self-energy which can be derived from a fermion-boson model). However, we
can motivate the above self-energy variants from a more microscopic point of view by considering the coupling of a local charge at site \( \mathbf{R}_i \) (operator \( c_i^{(\dagger)} \), site energy \( \varepsilon_i \), coupling constant \( g_i \)) to a bosonic excitation (operator \( b_i^{(\dagger)} \), frequency \( \omega_0 \)). If the site is empty, the lowest order addition states are obtained from \( c_i^{\dagger}|0\rangle \) and \( c_i^{\dagger}b_i^{\dagger}|0\rangle \) with addition energies \( \varepsilon_i \) and \( \varepsilon_i + \omega_0 \), respectively. The addition self-energy thus becomes

\[
\Sigma^+(\omega) = g_i^2(1 - \langle n_i \rangle)/(\omega - \varepsilon_i - \omega_0).
\]

Analogously the lowest order removal states are obtained from \( c_i|i\rangle \) and \( c_i b_i^{\dagger}|i\rangle \) with removal energies \( \varepsilon_i \) and \( \varepsilon_i - \omega_0 \), respectively. Here \( |i\rangle \) denotes the occupied state at site \( \mathbf{R}_i \) and the removal self-energy is given by

\[
\Sigma^-(\omega) = g_i^2\langle n_i \rangle/(\omega - \varepsilon_i + \omega_0).
\]

The above variant '2' thus has an asymmetry which mimics a strong charge modulated state in the sense that on sites with large (small) charge density the self-energy is peaked at \( \omega = \pm \omega_0 \) (\( \varepsilon_i = 0 \)).

On the other hand the variant '1' can be motivated from the weakly charge modulated limit where the intensity of the bosonic satellites at \( \pm \omega_0 \) is approximately the same at each site.
In cuprates evidence for inhomogeneity in the frequencies and coupling constants of vibrational modes has been found by STM experiments in Ref. [57]. According to our previous discussion a situation where the charge carriers are inhomogeneously and weakly coupled to these modes, would correspond to variant 1. On the other hand, a stronger inhomogeneous coupling, which would also result in a stronger electronic inhomogeneity and thus a stronger variation of the local phonon addition and removal spectra, could be formally captured by variant 2.

In any case this phenomenological theory reproduces our initial scenario of low energy protected quasiparticles but emerging spectral properties of CO scattering at large energies. In contrast to the Kampf-Schrieffer approach which is based on homogeneous ground states the present theory explicitely describes systems with broken symmetry.

In Ref. [68] we have used the approach corresponding to 'variant 1' in order to investigate the bias dependence of the LDOS in connection with the STM experiments in Ref. [58]. Besides the fact that the theory can reconcile the simultaneous existence of low energy Bogoljubov quasiparticles and high energy electronic order, it can further account for the CO specific contrast reversal in the STM spectra between positive and negative bias [70], where the energy scale for the modulation of the local density of states is essentially determined by the pairing gap. Furthermore, from Eq. (17) it turns out that the scattering rate (i.e. the \( q = 0 \) contribution to the self-energy) is determined by the sum of amplitudes of the electronic inhomogeneity \( \sim v_0^2 \). Such an intrinsic relation between electronic inhomogeneity and inelastic scattering rate has been recently revealed by STM experiments on Bi2212 materials [59] where it has been shown that the LDOS spectra can be parametrized based on a model with SC \( d \)-wave order supplemented by an energy dependent scattering rate \( \Gamma^{LDOS}_\omega = \alpha \omega \). The parameter \( \alpha \) varies spatially and in the regions with pronounced charge order acquires values up to \( \alpha \approx 0.4 \).
Figure 13. Constant energy scans of the spectral function for dynamic (a-f) and static (g,h) stripes at (a,g) 0 meV, (b) 5 meV, (c) 80 meV, (d) 100 meV, (e) 150 meV, (f) 250 meV, and (h) 200 meV below $E_F$. Window of integration: $\delta \omega = \pm 1$ meV. The energy scale in the two-pole Ansatz for variants 1,2 is $\omega_0 = 50$ meV. Figs. (a) and (b) look the same for both variants 1 and 2. The panels (c) and (d) are obtained implementing variant 1, while panels (e) and (f) are obtained within variant 2.

Ref. [68] we have used this experimental finding to approximate the function $f(\omega)$ by a marginal-Fermi liquid type self-energy [69].

Here we supplement these investigations by an alternative choice for $f(\omega)$ which is motivated from the observation of kink structures at some energy scale in the LDOS of STM experiments on cuprate superconductors that separate homogeneous from inhomogeneous electronic states [59]. Such kinks arise via a peak in the imaginary part of the self-energy which suggests the implementation of Eq. (19) for the frequency dependent spectrum $f(\omega)$. For the real space modulation we take an array of bond-centered stripes separated by four lattice constants (cf. Fig. 12). Thus the present model corresponds to combined dynamical charge and spin order. Our aim here is to establish a connection with the ARPES experiments of Refs. [45, 46]. In fact, motivated by our one-dimensional example at the beginning of this section we investigate the problem whether our present approach can capture the experimentally observed dichotomy in the FS structure and reproduce the characteristic features of straight FS segments at higher binding energies. It should be noted that ARPES experiments suggest that the true self-energy should be a combination of both parts, i.e. some bosonic peaky feature [65, 40, 42] and a MFL part [66, 67] (cf. also Fig. 2). Here our aim is to implement these frequency structures in a scheme which allows for symmetry broken solutions. Since the MFL part has already been investigated in Ref. [68] in the following we focus on the
two pole Ansatz Eq. (19) and Eqs. (22, 23), respectively.

In order to implement the stripe modulation in our approach we calculate the corresponding local chemical potential variation $\lambda_i(> 0)$ (sketched in Fig. 12) within an unrestricted Gutzwiller approximation [72]. The $\lambda_i$ can be used to construct the self-energies Eq. (17) for dynamical stripes in terms of variants 1,2 in the following way. For variant 1 we make the correspondence $\lambda_i = \alpha v_i^2$, $\Delta_i = 0$ with some scaling parameter $\alpha$. For the static solution the correspondence simply reads $v_i^2 = 0$, $\Delta_i = \lambda_i$.

In case of variant 2 the two-pole Ansatz is extended as follows

$$f_i(\omega) = \left[ \frac{\alpha_i}{\omega - \omega_0} + \frac{\beta_i}{\omega + \omega_0} \right]$$

$$\Delta_i = \frac{\alpha_i - \beta_i}{\omega_0}$$

with $\alpha_i + \beta_i = 1$ and $v_i^2 \equiv v^2$ (cf. Eq. (17)). Thus at each site an asymmetry is introduced in the spectral distribution which is taken to be proportional to the charge modulation. Denoting by $\bar{\lambda} \equiv \lambda_{q=0}$ the average local chemical potential the weights of the poles are implemented as $\alpha_i = 1/2[1 - tanh(1 - \lambda_i/\bar{\lambda})]$. As in case of the commensurate example, the static component Eq. (25) is necessary to guarantee the vanishing of the scattering at low energy ($f_i(\omega = 0) = 0$). In addition it leads to a non-vanishing off-diagonal Green’s function at large frequencies.

Notice that for the sake of simplicity, for both variants 1,2, our phenomenological form of $f(\omega)$ has the same frequency structure for both dynamical charge- and spin scattering.

Finally, the underlying bare dispersion is

$$\varepsilon_k = -2t [\cos(k_x) + \cos(k_y)] - 4t' \cos(k_x) \cos(k_y) - \mu$$

with $t = 250 meV$ and $t'/t = -0.2$ as appropriate for lanthanum cuprates [71]. The chemical potential is adjusted to yield a doping $n = 0.12$ for all following results.

Panels (g,h) display cuts of the spectral function for the static stripe solution at $E_F$ (panel g) and 200 meV below $E_F$ (panel h). Naturally the one-dimensional nature of the ground state leads to almost straight segments in the ‘FS’ where the residual ’wiggly’ structure is due to the overlap of the wave-functions between adjacent stripes. For the chosen doping $n = 0.12$ and stripe separation (4 lattice constants), stripes are almost half-filled so that the main segments appear at momentum $k_y = \pm \pi/4$ and weaker higher harmonic structures at $k_y = \pm 3\pi/4$. At higher binding energies the cuts of the static solution reveal the AF order associated with the stripe solution. Panel (h) shows the corresponding pockets around the nodal points at a binding energy of $\omega = 200 meV$. Let us now turn to the resulting cuts for the dynamical stripe solutions calculated within variant 1 (panels a,b,c,d) and variant 2 (panels a,b,e,f). For both dynamical stripe variants the scattering on the FS vanishes by construction and correspondingly panel (a) in Fig. 13 just displays the bare electronic structure at $\omega = E_F$. Below the binding energy scale $\omega_0 = 50 meV$ the dispersion for both variants is gradually renormalized and for the present parameters changes to a large electron like FS already for 5 meV.
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below \( E_F \) (cf. panel (b) which is the same for variants 1,2). Can we reproduce the formation of 'stripe segments' within the dynamical stripe variants at binding energies beyond the energy scale \( \omega_0 \)? Panels (c,d) show the high-energy cuts of the spectral function for variant 1. As discussed before, the scattering induced by the corresponding off-diagonal self-energy \( \Sigma_{Q \neq 0} \) vanishes for \( \omega \gg \omega_0 \) and thus can lead to a reconstruction of electronic states only in a restricted frequency range above \( \omega_0 \). One can in fact observe at \( \omega = 80\,\text{meV} \) (panel (c)) the appearance of a small 'segment' feature at \( k_y = \pi/4 \) but the scattering is not sufficient to 'gap' completely the electronic structure at \( k_y = \pi \).

Nevertheless at slightly larger binding energies (\( \omega = 100\,\text{meV} \), panel d) the spectral distribution shows the characteristic pocket feature reminiscent of the associated AF order. Panels (e,f) show the analogous cuts for the implementation of dynamical stripes within variant 2. Due to the fact that in this case the high energy scattering completely resembles that of the static solution one now indeed observes in panel (e) the same segment structure as for the FS of static stripes (panel g), but now at large binding energies (\( \omega = 150\,\text{meV} \)). Moreover the spectral function changes to the characteristic AF pocket structure at even higher energies (\( \omega = 250\,\text{meV} \), panel (f)), again similar to the static solution reported in panel (h). Therefore, quite remarkably, assuming a site-dependent fluctuation spectrum, the spectral function has the desired characteristics: A uniform FL at low energy and spectral features typical of stripes at high energy.

6. Conclusions

In the present paper we have investigated the consequences of dynamical charge (and spin) order on the spectral properties of cuprate superconductors. We have introduced two phenomenological schemes which can account for the dichotomy between low energy 'unperturbed' quasiparticles and high energy charge (and spin) order as indicated by STM [58] and ARPES [45, 46] experiments. Our first approach is a generalization of the Kampf-Schrieffer method [74] originally introduced to deal with fluctuating AF order in the vicinity of a magnetic quantum phase transition. At large binding energies one obtains a diagonal Green's function which has the same analytic structure as that of the broken symmetry state. However, since the off-diagonal GF vanishes the homogeneity of the ground state is preserved. By applying the Kampf-Schrieffer model to dynamical incommensurate CO fluctuations in a \( d \)-wave superconductor we have shown that the associated scattering can reduce the weight of the Bogoliubov coherence peaks in the vicinity of the antinodal points in agreement with ARPES data on Bi2201 compounds [62].

We also considered another phenomenological approach, which is based on the mapping to an auxiliary hamiltonian, appropriate for systems with spontaneous breaking of symmetry. This allows for the construction of wave-functions which are homogeneous for energies close to \( E_F \), but manifest the symmetry-broken nature above some energy scale. We have seen that the most promising way for such a construction is via a coupling of the charge carriers to a spatially varying fluctuation spectrum which has
been taken to have the symmetry of charge- and spin stripes. In this way we obtain the
quite non trivial result for the spectral intensity reported in panels (a),(b),(e),(f) of Fig.
where, upon increasing energy, a large LDA FS gradually evolves into a “segment-
like” energy profile typical of well-formed stripes and, at even larger energies, into hole
pockets typical of a antiferromagnetically ordered state. This result clearly captures the
physical idea of slowly fluctuating charge collective modes, and more fastly fluctuating
spin degrees of freedom. The simple analytic (peak-like) form of the phenomenological
self-energy also provides clear hints and constraints for the outcomes of forthcoming
microscopic theories.

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