Constraining the physics of the r-mode instability in neutron stars with X-ray and UV observations

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ABSTRACT

Rapidly rotating Neutron Stars in Low Mass X-ray Binaries (LMXBs) may be an interesting source of Gravitational Waves (GWs). In particular, several modes of stellar oscillation may be driven unstable by GW emission, and this can lead to a detectable signal. Here we illustrate how current X-ray and ultra-violet (UV) observations can constrain the physics of the r-mode instability. We show that the core temperatures inferred from the data would place many systems well inside the unstable region predicted by standard physical models. However, this is at odds with theoretical expectations. We discuss different mechanisms that could be at work in the stellar interior, and we show how they can modify the instability window and make it consistent with the inferred temperatures.

Key words: stars: neutron — X-rays: binaries — gravitational waves

1 INTRODUCTION

Low Mass X-ray Binaries (LMXBs) were suggested as interesting sources of Gravitational Waves (GWs) more than thirty years ago (Papaloizou & Pringle 1978; Wagoner 1984). In these systems, a compact object, which in the case of interest is a neutron star (NS), accretes mass from a less evolved low mass companion. The mass donor fills its Roche lobe, and matter is stripped from the outer layers and forms an accretion disc. The disc matter gradually loses angular momentum and spirals in, until it is eventually accreted by the NS. This process leads to angular momentum being transferred to the NS which can then be spun up to millisecond periods in what is known as the “recycling” scenario (Alpar et al. 1982; Radhakrishnan & Srinivasan 1982).

The main reason for invoking GW emission from these systems is the fact that the distribution of spin rates of both LMXBs and millisecond radio pulsars (MSRP) appears to have a cutoff at around 730 Hz (Chakrabarty et al. 2003), which is well below the centrifugal break up limit (Cook et al. 1994; Haensel et al. 1999). This observation still holds true today, even as more systems have been added to the sample (Patruno 2010). Thus it is natural to seek a physical mechanism that can prevent NSs from spinning up further. The most obvious candidate is the accretion process itself, as the interaction between the accretion disc and the star can lead to spin equilibrium if the system approaches a propeller phase and further accretion is centrifugally inhibited. This mechanism dictates a correlation between the magnetic field strength and accretion rate (White & Zhang 1997), a problem which led to the proposal of several GW emission mechanisms that could generate a strong enough torque to set the spin equilibrium of LMXBs (Bildsten 1998; Andersson, Kokkotas & Stergioulas 1999; Ushomirsky, Cutler & Bildsten 2000; Cutler 2002). Although several authors have reassessed this problem (Andersson et al. 2005; Ho, Maccarone & Andersson 2011; Patruno, Haskell & D’Angelo 2011), the question remains unresolved and current GW searches are not sensitive enough to give strong constraints (Abbott et al. 2010).

The main GW emission mechanisms that could be at work in accreting systems are ”mountains”, either on the crust (Bildsten 1998; Ushomirsky, Cutler & Bildsten 2000; Haskell, Jones & Andersson 2003) or in the core (Owen 1994; Andersson, Haskell & Comer 2010), deformations due to the magnetic field of the star (Cutler 2002; Haskell et al. 2008; Melatos & Payne 2005), and modes of oscillation of the star being driven unstable and growing to large amplitudes (Andersson, Kokkotas & Stergioulas 1999).

We shall focus on the last, specifically the r-mode instability. An r-mode is a toroidal mode of oscillation for which the restoring force is the Coriolis force. It is particularly interesting because it is not only generically unstable to GW emission (Andersson 1998; Friedman & Morsink 1998) and can thus potentially grow to amplitudes large enough to explain the spin equilibrium of LMXBs, but its modelling requires a detailed understanding of the physics of NS interiors. The r-mode can grow unstable if GW emission drives it faster than viscosity damps it. This will
only happen in a range of temperatures and spin frequencies which depends strongly on the details of the damping mechanisms. In the standard picture, the main damping agent at low temperatures (below \(10^{10}\) K) is the viscous boundary layer at the crust-core interface (Bildsten & Ushomirsky 2004; Levin & Ushomirsky 2004), while bulk viscosity is the strongest source of damping at high temperatures (Andersson, & Kokkotas 2001). The nature of the damping mechanisms is very sensitive to the interior microphysics and presence of exotica, such as hyperons and deconfined quarks, on large scale superfluid and/or superconducting components (Andersson, Jones & Kokkotas 2002; Navarro & Owen 2003; Haskell & Andersson 2013; Mannarelli, Manuel & Sa'd 2008; Andersson, Haskell & Comer 2010; Lindblom & Mendell 2000; Haskell, Andersson & Passamonti 2009; Alford, Mahmoodifar & Schwenzer 2010). Furthermore, r-mode oscillations distort the stellar magnetic field, lead to energy dissipation, and possibly prevent the mode from being driven unstable (Rezzolla, Lamb & Shapiro 2006).

In this paper, we examine these mechanisms and compare them to observational constraints on NS spins and temperatures. We use available data on NS surface temperatures from X-ray observations of LMXBs in quiescence and UV observations of millisecond pulsars. We also present new analysis of five systems, which leads to new upper limits on their surface temperatures. We conclude that the minimal NS model, i.e., that of a star composed of neutrons, protons and electrons (possibly muons) and whose r-mode damping at low temperatures is due to Ekman pumping at the crust-core interface, is not consistent with observations and that additional damping mechanisms are required, unless the r-mode saturates at a very small amplitude. In this case GW emission would not affect the evolution of the system.

We also discuss additional damping mechanisms that are likely to be at work in NS interiors and may be consistent with observations.

2 R-MODE INSTABILITY WINDOW

An r-mode is a fluid mode of oscillation of a NS for which the restoring force is the Coriolis force. To leading order in a slow rotation analysis, it is purely toroidal and has the form

\[
\delta \mathbf{v} = \alpha \left( \frac{\Omega}{P} \right)^3 R \mathbf{Y}_{\ell m} \exp i \omega t, \tag{1}
\]

where \(\delta \mathbf{v}\) is the Eulerian perturbation of the total fluid velocity, \(\mathbf{Y}_{\ell m}^B\) is the magnetic-type vector spherical harmonic, \(R\) is the stellar radius and \(\alpha\) is the (dimensionless) mode amplitude (Owen et al. 1998). The fluid displacement gives rise to a current quadrupole moment and to the emission of GWs, which can drive the mode unstable via the Chandrasekhar-Friedman-Schutz mechanism (Chandrasekhar 1970; Friedman & Schutz 1978; Andersson 1995; Friedman & Morsink 1998). If GW emission drives the mode growth, then eventually the mode will saturate when energy is transferred to higher order modes due to non-linear couplings. Given the complexity of the full non-linear problem, this process is highly uncertain. Nevertheless, most recent estimates indicate a saturation amplitude \(\alpha \approx 10^{-6} - 10^{-5}\) (Bondarescu, Teukolsky & Wasserman 2007). This can be compared to an upper limit of \(\alpha < 10^{-4}\) from GW searches conducted with LIGO (Owen 2010). Note that, as we are dealing with a superfluid star, the super-fluid neutrons can also flow independently from the charged component (protons and electrons), leading to relative motion. In fact, to second order in the slow rotation analysis, the r-mode will acquire poloidal components along the relative velocity \(\delta w_{nm}\) (Passamonti, Andersson & Haskell 2009; Haskell, Andersson & Passamonti 2009).

An r-mode can be driven unstable as long as GW emission drives the oscillation faster than viscosity damps it. This is usually studied in terms of the critical frequency at which the driving and damping timescales are equal. Solving for the roots of

\[
\frac{1}{\tau_{GW}} = \frac{1}{\tau_{V}} \tag{2}
\]

yields an instability curve that depends on frequency and temperature. \(\tau_{GW}\) is the GW driving timescale which (for an \(l = m = 2\) r-mode and an \(n = 1\) polytrope) is given by (Andersson & Kokkotas 2001)

\[
\tau_{gw} = -47 M_{1.4}^{-1} R_{10}^{-1} P_{ms}^{6} s, \tag{3}
\]

with \(M_{1.4}\) is the NS mass in units of 1.4 \(\odot\), \(R_{10}\) is the NS radius in units of 10 Km and \(P_{ms}\) is the NS rotation period in milliseconds. The viscous damping timescale \(\tau_{V}\) is given by

\[
\frac{1}{\tau_{V}} = \sum_i \frac{1}{\tau_{i}}, \tag{4}
\]

where the summation is over the various dissipative channels, labelled with ‘i’. At high temperature (above \(10^{10}\) K) the main contribution is bulk viscosity due to the modified Urca reaction, with a timescale given by (Andersson & Kokkotas 2001)

\[
\tau_{BV} = 2.7 \times 10^{31} M_{1.4} R_{10}^{-1} P_{ms}^{2} T_9^{-6} s, \tag{5}
\]

where \(T_9\) is the NS core temperature in units of 10^9 K. Note that this form for the bulk viscosity is only appropriate for small perturbations, such that perturbations of the chemical potentials is much smaller than the thermal energy \(kT\). For much larger perturbations, the effect of bulk viscosity is significantly stronger, effectively blocking the growth of the r-mode (Alford, Mahmoodifar & Schwenzer 2011). However, the amplitudes that are necessary for such a scenario are significantly larger than the saturation amplitudes we consider here, so such a possibility will not be discussed further.

At low temperatures, the main source of damping is the viscous boundary layer at the crust-core interface, which leads to a damping timescale

\[
\tau_{EK} = 3 \times 10^{7} P_{ms}^{12/5} T_9 s, \tag{6}
\]

where we use the estimate of Glampedakis & Andersson (2006) with a “slippage” factor \(S = 0.05\). The slippage factor accounts for the fact that the crust will not be completely rigid, but will also participate in the oscillation. It is essentially the ratio between the crust/core velocity difference and the mode velocity, so that \(S = 1\) corresponds to a completely rigid crust, while smaller values indicate that the mode can penetrate the crust to some extent. Shear viscosity will also play a role at low temperatures, but its effect will be weaker than that of the crust-core interface (Andersson, & Kokkotas 2001); thus we neglect it here. In
For small saturation amplitudes, the time spent in the unstable region increases, but the spin and temperature variations are modest \((\alpha \approx 10^{-5})\). For large amplitudes, the system undergoes a thermal runaway and heats up significantly but spends much less than 1% of the time in the unstable region. The excursion in temperature was constrained by fitting the UV spectrum. As is obvious from the discussion in Section 2, if we wish to construct an instability curve in the frequency versus temperature plane, it is necessary to estimate the temperature of the NS core, on which the damping timescales will depend. This is clearly not a straightforward task, as what is measured is the surface emission as detected by a distant observer. In order to estimate the core temperature, we shall use X-ray observations of LMXBs in quiescence (when most of the thermal emission is thought to come directly from the NS surface; see, e.g., [Brown, Bildsten & Rutledge 1998] and the few available millisecond radio pulsar thermal spectra observed in UV. This is in contrast to the estimates made by [Ho, Andersson & Haskell 2011], which made use of X-ray observations of LMXBs during bursts.

Several LMXBs have surface temperatures obtained from blackbody fits to their observed X-ray spectrum. For others, the spectrum is completely non-thermal, and only upper limits on the temperature can be obtained. In table 1, we list LMXBs that have a measured temperature (or upper limit) and spin rate. The spin rates are either measured directly for those NSs that display coherent X-ray pulsations (indicated as “accretion powered”) or inferred from the frequency of oscillations seen during thermonuclear type-I X-ray bursts (labelled as “nuclear powered”). The spin rates are taken from the overview given by [Patruno 2011]. For the temperatures, we use the overview compiled by [Heinke al. 2007, 2009] and include 10 additional sources reported in the literature or analyzed in this work (see Section 3.1). We also include three millisecond radio pulsars for which the temperature was constrained by fitting the UV spectrum.

Figure 1. R-mode instability window for the “minimal” NS model described in the text, for which the main damping mechanism at low temperature is the Ekman layer at the base of the crust. We schematically illustrate the trajectory a system would follow for high saturation amplitudes \((\alpha \approx 1)\) and low saturation amplitudes \((\alpha \approx 10^{-5})\). For large amplitudes, the system undergoes a thermal runaway and heats up significantly but spends much less than 1% of the time in the unstable region. For small saturation amplitudes, the time spent in the unstable region increases, but the spin and temperature variations are modest \((\approx 10\%)\). [Bondarescu, Teukolsky & Wasserman 2007].

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The minimal cooling model described above is modified if there are hyperons in the NS core. The bulk viscosity can then be much stronger at lower temperatures. The effect of hyperon bulk viscosity on the r-mode instability window has been studied in detail for superfluid NSs by [Haskell & Andersson 2010], whose results we shall use below.

Furthermore if, as is generally believed, the core of the NS contains large scale superfluid components, these will rotate by forming an array of quantised vortices. The interaction of vortices with the charged components gives rise to a dissipative force known as mutual friction. For temperatures well-below the superfluid transition temperature, the damping timescale for mutual friction is roughly constant, but the timescale can vary considerably when the temperature is near the transition temperature. Here we shall use the detailed results of [Haskell, Andersson & Passamonti 2009]. The main microphysical input that is needed to calculate the mutual friction damping timescale is the value of the (dimensionless) drag parameter \(\mathcal{R}\). It has been shown by [Lindblom & Mendell 2000] and [Haskell, Andersson & Passamonti 2009] that the standard drag parameter (describing electron scattering off vortex cores; \(\mathcal{R} \approx 10^{-4}\)) does not significantly affect the instability window. However, the situation may be considerably different if the core of the NS is in a type II superconducting state. In this case, the magnetic field is arranged in flux tubes, and their interaction with neutron vortices could lead to strong dissipation, with drag parameters possibly of the order of \(\mathcal{R} \approx 10^{-2}\) [Jones 1992, Link 2003; Haskell, Pizzochero & Sidery 2011].

Finally it should be noted that, in magnetised stars, fluid motion distorts magnetic field lines, possibly leading to energy being drawn from the mode faster than GW emission can drive it [Rezzolla, Lamb & Shapiro 2000].

3 NEUTRON STAR TEMPERATURES AND SPIN RATES

As is obvious from the discussion in Section 2, if we wish to construct an instability curve in the frequency versus temperature plane, it is necessary to estimate the temperature of the NS core, on which the damping timescales will depend. This is clearly not a straightforward task, as what is measured is the surface emission as detected by a distant observer. In order to estimate the core temperature, we shall use X-ray observations of LMXBs in quiescence (when most of the thermal emission is thought to come directly from the NS surface; see, e.g., [Brown, Bildsten & Rutledge 1998] and the few available millisecond radio pulsar thermal spectra observed in UV. This is in contrast to the estimates made by [Ho, Andersson & Haskell 2011], which made use of X-ray observations of LMXBs during bursts.

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3.1 Observations

In order to obtain constraints on the surface temperature for some sources, we first simulate a fiducial X-ray spectrum with the software package XSpec [v 12.6; Arnaud 1996]. This is done using the NS atmosphere model NSATMOS [Heinke et al. 2006], where we take $M = 1.4 M_\odot$ and $R = 10$ km. We assume the entire NS surface is emitting (i.e., model normalization is fixed to 1) and use source distances reported in the literature (see below). After constructing such a model for each source, we determine the NS temperature that produces the observed (quiescent) thermal flux limit. This value is then considered to be the upper limit on the NS surface temperature $T_s$.

We use flux upper limits reported in the literature to infer constraints on the surface temperature for three sources: IGR J17191-2812 ($D = 11$ kpc; Altamirano et al. 2010a), NGC X-2 ($D = 8.5$ kpc; Heinke et al. 2010) and Swift J1756-2508 ($D = 8$ kpc; Patruno et al. 2010). In the case of IGR J17511-3057, nothing is reported in the literature about its quiescent properties. However, we found two observations obtained with the X-ray Telescope (XRT) onboard Swift, which did not reveal the source during its quiescent state. We obtain an upper limit on the 0.5–10 keV unabated flux of $\sim 7.5 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$. We infer a NS surface temperature of $T_s < 1.44 \times 10^{6}$ K for the fiducial model parameters mentioned above and assuming $D = 6.9$ kpc [Altamirano et al. 2010a]. We analysed a recent XMM-Newton observation of Swift J1749.4-2807 during quiescence (Degenaar et al. in preparation). The source is clearly detected during the observation, but its X-ray spectrum is completely non-thermal. For a distance of $D = 6.7$ kpc [Altamirano et al. 2011], we obtain an upper limit on the NS surface temperature of $T_s < 1.66 \times 10^{6}$ K. Finally, HETE J1900.1-2455 has been continuously active since its discovery in 2005, but the source intensity dropped dramatically during a short $\sim 20$-day interval in 2007 (Degenaar et al. 2007a). At a certain point the source could not be detected with Swift/XRT; this resulted in an upper limit on the quiescent X-ray flux [Degenaar et al. 2007b]. We re-analysed the data of this non-detection to estimate the upper limit on the NS surface temperature, where we assumed a source distance of $D = 3.6$ kpc [Galloway et al. 2008].

| Source              | $\nu$ (Hz) | $T_\infty/10^6$ K | $T_{\text{core}}/10^6$ K | Type     | Reference           |
|---------------------|------------|-------------------|--------------------------|----------|---------------------|
| Aql X-1            | 550        | 1.09              | 1.08                     | AP       | Heinke et al. (2007) |
| 4U 1608-52          | 620        | 1.97              | 4.55                     | NP       | Heinke et al. (2007) |
| KS 1731-260         | 526        | 0.73              | 0.42                     | NP       | Cackett et al. (2010) |
| MXB 1659-298        | 556        | 0.63              | 0.31                     | NP       | Cackett et al. (2008) |
| SAX J1748.9-2021    | 442        | 1.01              | 0.89                     | AP       | Heinke et al. (2007) |
| IGR 00291+5934      | 599        | 0.82              | 0.54                     | AP       | Heinke et al. (2009) |
| SAX J1808.4-3658    | 401        | <0.35             | <0.11                    | AP       | Heinke et al. (2009) |
| XTE J1751-305       | 435        | <0.82             | <0.54                    | AP       | Heinke et al. (2009) |
| XTE J0929-314       | 185        | <0.58             | <0.26                    | AP       | Heinke et al. (2009) |
| XTE J1807-294       | 190        | <0.59             | <0.27                    | AP       | Heinke et al. (2009) |
| XTE J1811-338       | 314        | <0.80             | <0.51                    | AP       | Heinke et al. (2009) |
| EXO 0748-676        | 552        | 1.26              | 1.58                     | NP       | Degenaar et al. (2011) |
| HETE J1900.1-2455   | 377        | <0.65             | <0.33                    | AP       | This work           |
| IGR J17191-2812     | 294        | <0.86             | <0.60                    | NP       | This work           |
| IGR J17511-3057     | 245        | <1.10             | <1.10                    | AP       | This work           |
| SAX J1750.8-2900    | 601        | 1.72              | 3.38                     | NP       | Lowell et al. (in preparation) |
| NGC 6440 X-2        | 205        | <0.37             | <0.12                    | AP       | This work           |
| SWIFT J1756-2508    | 182        | <0.96             | <0.78                    | AP       | This work           |
| SWIFT J1749.4-2807  | 518        | <1.27             | <1.61                    | AP       | Degenaar et al. (in preparation) |
| J0437-4715          | 174        | 0.12              | 0.018                    | RP       | Kargaltsev, Pavlov & Romani (2004) |
| J2124-3358          | 203        | <0.46             | <0.17                    | RP       | Kargaltsev, Pavlov & Romani (2004) |
| J0030+0451          | 205        | <0.92             | <0.70                    | RP       | Kargaltsev, Pavlov & Romani (2004) |

3.2 Neutron star core temperatures

Having determined NS surface temperatures, we now estimate the core temperatures. We assume that the core and crust are nearly isothermal (which is very nearly the case since the thermal conductivity of the crust is high, as indicated by recent cooling observations of X-ray transients; Brown & Cumming 2009) and that the core temperature is simply the temperature at the base of the heat blanketing envelope. As we are considering accreting systems and radio pulsars that are thought to have been recycled through accretion, we use the relation between surface temperature and envelope base temperature for a partially accreted crust given by Potekhin, Chabrier & Yakovlev (1997), where we follow Brown & Cumming (2009) by considering a layer...
of light elements down to a column depth of $P/g = 10^9$ g/cm$^2$.

We assume that millisecond radio pulsars have been recycled to rapid rotation by accretion and that they have similar crustal compositions as the LMXBs; thus we again use the relation of [Potekhin, Chabrier & Yakovlev 1997] to estimate their core temperature. This is of course a crude assumption. However by using the iron envelope relation of [Gudmundsson, Pethick & Epstein 1983], the estimated temperatures change by a factor of approximately 2. As we shall see, the MSRPs fall in a region of the instability window for which such a correction has no effect on our conclusions; this justifies our use of the relation from [Potekhin, Chabrier & Yakovlev 1997]. Let us remark that the estimates of core temperature have large uncertainties, as not only is the composition of the envelope uncertain, but some systems may still be thermally relaxing to a steady state after an outburst and may have sizable temperature gradients in the crust. These effects lead to an uncertainty of a factor of a few in the inferred core temperatures (Brown & Cumming 2009). This has no qualitative impact on our conclusions, but this uncertainty should be kept in mind and we shall attempt to quantify it in the following sections.

### 3.3 Strange stars

It is possible that the most stable form of matter at the high densities that characterise a NS interior may be that of a conglomerate of deconfined quarks (Hoff70; Witter84). In fact, it has been suggested that all NSs may be strange stars (Alcock, Farhi & Olinto 1986). Although the ground state of matter at asymptotically high densities and low temperatures is known to be given by paired quarks in the so-called “Colour Flavour Locked” (CFL) phase (Alford et al. 2008), the properties of matter at realistic NS densities are still uncertain. The effect of a CFL core on the instability window was calculated by [Andersson, Haskell & Comer 2010] and found to be quite weak, so we shall not consider this possibility any further. We shall thus consider the effect of a conglomerate of unpaired quarks in the NS interior on the r-mode instability window, as this can be quite significant. In this scenario the shear viscosity dissipation timescale for a strange star (again assuming an $n = 1$ polytrope as the background model) is found to be (Andersson, Jones & Kokkotas 2002)

$$t_s \approx 7.4 \times 10^7 \left( \frac{\alpha s}{0.1} \right)^{5/3} M_4^{5/9} R_{10}^{1/3} T_9^{5/3} \text{s},$$  

(7)

where $\alpha s$ is the strong coupling constant. For the temperature range of interest (below $\approx 10^9$ K), the bulk viscosity damping timescale is (Madsen 1992; Andersson, Jones & Kokkotas 2002)

$$t_{Bv} \approx 7.9 \left( \frac{m_8}{100 \text{MeV}} \right)^{-4} M_4^{2} R_{10}^{-4} T_9^{-2} P_{ms}^2 \text{s},$$  

(8)

where $m_8$ is the mass of the strange quark. A strange star can also support a thin crust of normal nuclear matter up to the neutron drip density, after which free neutrons will drip into the strange core. Such a crust would obviously be much thinner and much less massive than that of a NS, with a maximum mass of approximately $10^{-5} M_\odot$ (Glendenning & Weber 1992). This crust would be much less rigid than a standard NS crust and not contribute significantly to the damping (Andersson, Jones & Kokkotas 2002). We shall thus not consider damping due to the crust-core interface in the discussion on strange stars, although its inclusion would not qualitatively change our conclusions.

The presence of the crust is, however, very significant for estimating the temperature of the core, as it provides a “heat blanket” for the strange core, allowing the outgoing radiation to thermalise (which would not be the case for a bare strange star, for which the spectrum would be considerably harder; [Page & Usov 2002]). Furthermore, many of the systems we consider show not only coherent pulsations but also thermonuclear bursts, which would be challenging to explain if there is no crust of normal matter. We shall thus use the same prescription as in the NS case to estimate the core temperature (see also the discussion in [Pizzochero 1991]).

### 4 OBSERVATIONAL CONSTRAINTS ON THE INSTABILITY WINDOW

Let us now examine how the observational evidence compares with theoretical calculations of the r-mode instability window. First of all, we begin by comparing the measured temperatures/spins with the minimal model instability window of Section 2. In Figure 2 we show the inferred core temperatures and in Figure 3 we estimate the uncertainty due to the modelling of the outer layers of the star. The error bars on the core temperatures inferred from observations have been obtained by considering two extreme compositions for the stellar envelope, the properties of which (composition, thermal conductivity etc...) control the heat flow from the core to the exterior. We have thus calculated a “minimum” temperature by assuming a completely accreted crust of light elements (Potekhin, Chabrier & Yakovlev 1997) and a “maximum” temperature by assuming an iron envelope (Gudmundsson, Pethick & Epstein 1984). As we can see this produces an uncertainty of a factor of a few, which will dominate over the observational uncertainty but does not affect our conclusions. It is obvious from Figures 2 and 3 that, even accounting for theoretical and observational uncertainties associated with temperature measurements, several systems are well inside the unstable region. As already mentioned, this would be possible if the saturation amplitude is very large and one is lucky enough to catch the system while it is still in the GW emitting part of its duty cycle. However, given that for large amplitudes ($\alpha > 10^{-3}$) one would expect the system to spend less than 1% of the time in the unstable region, it is highly unlikely that we are observing so many systems in this phase. Furthermore, such a system would be spinning down rapidly due to the emission of GWs, but one of the systems, IGR J00291+5934, has a measured spin-down rate in quiescence of $\dot{\nu} \approx 3 \times 10^{-15}$ Hz s$^{-1}$ (Patruncie 2010; Hartman, Galloway & Chakraborty 2011; Papitto et al. 2011), which is consistent with purely electromagnetic spin-down due to a $B \approx 10^8$ G magnetic field (although one cannot rule out a much weaker magnetic field and low level GW emission, see [Haskell & Patruncie 2011] for a discussion of why this is unlikely to be the case in two other sources, SAX J1808.4-3658 and...
Figure 2. R-mode instability window of LMXBs and MSRP\$s that have estimates of both the spin frequency and surface temperature (arrows indicate upper limits). The right panel is the same as the left panel but focused on the low temperature region in which the observed systems are located. It is obvious that a significant number of systems is well inside the “minimal” instability window, where one would not expect to find so many systems. In fact, for realistic values of the saturation amplitude, a star could not heat up enough to be significantly inside the unstable region, while for high values of the saturation amplitude a system would spend only a very small fraction of the time (less than 1\%) above the instability curve, making it very unlikely to catch systems in this region. The only possibilities are thus that either the instability curve is significantly different from our minimal model curve due to additional damping mechanisms or the saturation amplitude is small enough not to affect the evolution of the systems.

XTE 1814-338). A final possibility is that systems inside the window have undergone a thermal runaway and have reached an equilibrium between heating and cooling [Bondarescu, Teukolsky & Wasserman 2007]; they are now either at spin equilibrium (i.e., with the GW spin-down torque balancing the accretion spin-up torque) or approaching spin equilibrium (as could be the case for IGR J00291+5934 which exhibits long-term spin-up). We discuss this possibility further in the following section. It is, however, clear that the minimal model is not consistent with observations.

We now discuss the possible mechanisms that may be at work in a realistic NS and that could be consistent with observations. We first examine effects due to properties of the crust. One is that the crust may be more rigid than is commonly assumed. This would lead to stronger dissipation at the crust-core interface. In Figure 4, we show the effect of increasing the “slippage” factor $S$ from a standard value of $S = 0.05$ (Glampedakis & Andersson 2006) to $S = 1$ (a completely rigid crust). It is obvious that a more rigid crust could allow all the systems to be stable (see also Wen et al. 2011). However such a rigid ($S = 1$) crust is not realistic. Alternatively, given the frequency range of r-modes, the mode may couple effectively to torsional oscillations of the crust. This would produce strong dissipation at the resonance frequency [Levin & Ushomirsky 2001; Glampedakis & Andersson 2006; Ho, Andersson & Haskell 2011].

Another possibility is that core bulk viscosity may be much stronger at low temperatures. For example, if hyperons are present in the core, then a significantly restricted unstable region is created, as illustrated in Figure 5. The situation for strange stars is somewhat similar, with bulk viscosity playing a much stronger role at low temperatures and leading to a reduced unstable region, as shown in Figure 6 (see Alford, Mahmoodifar & Schwenzer 2011) for more detailed discussion of the r-mode instability window in strange stars.

Figure 3. The same r-mode instability window as in the right panel of Figure 2 where we have also estimated the error bars due to the uncertainty in modelling the outer layers of the NS, as described in the text. We can see that although there is a significant uncertainty on the inferred core temperatures it is not large enough to modify the conclusion that many of the systems appear to be well inside the unstable region.
Constraining the physics of the r-mode instability

Figure 4. R-mode instability window for different values of the “slip” parameter $S$ (Glampedakis & Andersson 2006; see text). A large slip parameter, corresponding to a nearly completely rigid crust, appears to be necessary to explain the observations.

Figure 5. R-mode instability window for a $M = 1.4M_\odot$ star with hyperons in the core. We use the results of Haskell & Andersson (2010) for different NS radii and values of the coupling parameter $\chi$, which parametrises in-medium effects (see text). A very interesting mechanism is one that involves strong vortex-mediated mutual friction. If the core of the NS contains a type I superconductor, then mutual friction will not be strong enough to significantly affect the instability window (Sedrakian 2005; although see Jones 2006, for a discussion of strong drag in type I superconductors). However, if the core contains a type II superconductor, then the interaction of vortices with flux tubes will lead to strong mutual friction if a large fraction of vortices can “creep” through the flux tubes. Examples of this are shown in Figure 5.

Finally a promising scenario involves magnetic damping of the r-mode (Rezzolla, Lamb & Shapiro 2000). Given the high electrical conductivity of the NS interior, the magnetic field lines are frozen in with the fluid and can thus be distorted and wound up by the oscillatory motion of the r-mode. Even for relatively weak magnetic fields, this could lead to rapid damping and could close the instability window (Rezzolla et al. 2001a,b; Cuofano & Drago 2010).

5 SPIN EQUILIBRIUM

We now examine the possibility that GW emission due to an unstable r-mode may be setting the spin equilibrium for LMXBs. This could be the case if the critical frequency increases with temperature at around $10^7$ K (e.g., for hyperon and quark bulk viscosity or for strong mutual friction). As a result, thermal runaway is halted, and the system reaches an equilibrium state, such that viscous heating due to the r-mode is balanced by neutrino emission and the GW torque balances the accretion torque at the observed spin pe-
Taking the heat from r-mode dissipation to be lost by neutrino emission [i.e., \( L_{\text{heat}} \approx L_{\nu}(T) \)], the core temperature can be inferred. In order to determine the rate at which neutrino emission cools the system, it is important to account for superfluidity, as this will lead not only to a reduction in the emission rates for the modified Urca emission processes but also to additional neutrino emission from the formation of Cooper pairs. We use the latest constraints on superfluid transition temperatures \( \text{Page et al. (2011)} \) and Shternin et al. (2011) obtained from the observed rapid cooling of the neutron triplet transition temperatures from \( \text{Page et al. (2011)} \) and \( \text{Shternin et al. (2011)} \) is \( \lesssim \) fifteen percent; see \( \text{Ho, Andersson & Haskell (2011)} \) for derived core temperatures assuming only modified Urca neutrino emission.

In Figure 7, we show the temperatures obtained in the spin equilibrium scenario for a “shallow” neutron superfluid transition (see \( \text{Ho, Glampedakis & Andersson (2011)} \) for details) with \( T_{\text{en,max}} \approx 5 \times 10^8 \) K, as in \( \text{Page et al. (2011)} \). The long-term accretion luminosities are taken from \( \text{Watts et al. (2008)} \) and from \( \text{Falanga et al. (2011)} \) for IGR J17511-3057. We can see that many systems appear to be colder than what would be expected in the presence of an unstable r-mode, although for some of the faster systems (which are also the most likely targets for GW searches, given the strong scaling with frequency of the GW torque), GW-driven spin equilibrium may still be possible and cannot be completely ruled out.

Finally we can calculate the maximum amplitude that would be compatible with the inferred core temperatures by assuming that the viscous heating is due to an unstable r-mode with arbitrary amplitude \( \alpha \) (Andersson & Kokkotas 2001)

\[
L_{\text{heat}} = 1.31 \alpha^2 \nu^2 M R^2 \tau_{\nu},
\]

(10)

where \( M \) is the mass of the star, \( R \) its radius, we have assumed that the equation of state is given by an \( n = 1 \) polytrope and \( \tau_{\nu} \) is the shear viscosity damping timescale. If we assume that shear viscosity is mainly due to electron-electron scattering and that modified Urca reactions are the main contribution to the cooling, the maximum amplitude we obtain takes the form

\[
\alpha_m \approx 6.7 \times 10^{-5} \frac{T_b^3}{\nu} \]

(11)

where \( T_b \) is the temperature in units of \( 10^8 \) K and \( \nu \) is the spin frequency of the system. With the notable exception of the two fastest and hottest systems, 4U 1608-52 and SAX...
In this paper, we estimated the core temperature of NSs using data from X-ray observations of LMXBs in quiescence and UV observations of millisecond pulsars, in order to place constraints on the physics of the r-mode instability window. We also presented a new analysis of five systems.

These estimates show that, if one uses a “minimal” NS model, in which shear viscosity is due to dissipation in a boundary layer between the crust and core and bulk viscosity is due to modified Urca processes, the r-mode would be unstable in many of these systems. In particular, many systems are too cold to allow for a spin equilibrium r-mode. However, the more rapidly rotating systems, which are hotter, may be consistent with spin equilibrium.

A promising scenario is that in which damping is due to the crust responding rigidly to the mode displacement in the r-mode frequency range for rapidly rotating system. The phenomenological model of Ho, Andersson & Haskell (2011) shows that this is viable, but more quantitative models are needed. In particular, efforts should be made to better understand the effect on viscous damping timescales of pasta phases at the crust-core interface (Horowitz & Berry 2008). Another scenario that would be consistent with observations is that in which the mode winds up the magnetic field of the star, and energy is extracted from the oscillatory motion more rapidly than GW emission can drive it (Rezzolla, Lamb & Shapiro 2000). Once again this mechanism depends strongly on the internal magnetic field structure and further work is needed in order to assess its relevance, as well as accounting for the presence of superconducting components.

Finally an interesting possibility is that the saturation amplitude of the r-mode is small enough that the GW torque cannot counteract the accretion torque and a system would spin up into the unstable region. In order for this scenario to be consistent with observations (i.e. in order for the heating from the mode to be consistent with the observed temperature), the saturation amplitude should be roughly $\alpha \lesssim 10^{-9} - 10^{-8}$. Such a small amplitude may be consistent with theoretical calculations (Bondarescu, Teukolsky & Wasserman 2007) and would indeed lead to a spin-down torque that is smaller than the...
electromagnetic spin-down torque for a $B \approx 10^8$ G magnetic field, thus not impacting on the evolution of the systems.

We examined the possibility that continuous GW emission from an unstable r-mode may be setting the spin equilibrium period of the LMXBs. This scenario was considered by Brown & Ushomirsky (2000) who found that, if one assumes modified Urca cooling, most systems would be too hot to be consistent with observations. We re-examined this scenario by using the most recent constraints on superfluid transition temperatures obtained from observations of the cooling of the NS in Cassiopeia A. We find that this leads to lower core temperatures (due to stronger neutrino emission) which may be consistent with the more rapidly rotating systems. This is interesting since the GW spin-down torque scales strongly with frequency and is expected to play a stronger role in rapidly rotating systems. Further observational constraints, as may be available from future X-ray observatories such as LOFT and Astrosat, as well as theoretical work on NS composition and viscosity, are crucial to aid in the search for GWs from these systems (Watts et al. 2008).

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