Quantum Gravity and Black Hole Dynamics in 1+1 Dimensions

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Abstract

We study the quantum theory of 1+1 dimensional dilaton gravity, which is an interesting toy model of the black hole dynamics. The functional measures are explicitly evaluated and the physical state conditions corresponding to the Hamiltonian and the momentum constraints are derived. It is pointed out that the constraints form the Virasoro algebra without central charge. In ADM formalism the measures are very ambiguous, but in our formalism they are explicitly defined. Then the new features which are not seen in ADM formalism come out. A singularity appears at $\varphi^2 = \kappa(>0)$, where $\kappa = (N - 51/2)/12$ and $N$ is the number of matter fields. Behind the singularity the quantum mechanical region $\kappa > \varphi^2 > 0$ extends, where the sign of the kinetic term in the Hamiltonian constraint changes. If $\kappa < 0$, the singularity disappears. We discuss the quantum dynamics of black hole and then give a suggestion for the resolution of the information loss paradox. We also argue the quantization of the spherically symmetric gravitational system in 3+1 dimensions. In appendix the differences between the other quantum dilaton gravities and ours are clarified and our status is stressed.

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1. Introduction

The quantum dynamics of the black hole is an important issue relating to fundamental laws of physics both in cosmology and field theories. Since the discovery of Hawking,\cite{Hawking1974} many authors have investigated whether the usual rules of quantum mechanics can be applied to quantum black holes or not.\cite{Page1976} Do black holes really evaporate and, if it is true, are informations indeed lost? No definite argument has not been yet. To resolve these problems the gravity also should be quantized.

The black hole evaporation is caused by non-perturbative quantum effects. Davies, Fulling and Unruh\cite{Davies1977} discussed the black hole dynamics in two dimensional equivalent of the Schwarchild black hole and showed that the conformal anomaly induces the emission of thermal radiation. This indicates that when we argue the black hole dynamics we must carefully evaluate divergence properties of quantum fields.

As a quantization method of gravitation, Arnowitt-Deser-Misner (ADM) formalism or Wheeler-DeWitt approach is well-known. There are, however, some serious problems in ADM formalism, which are the issues of measures and orderings. These are the most important points when we discuss quantum field theories. As far as ignoring these effects we cannot say anymore beyond WKB approximation. Anomalies cannot be derived from WKB approximation. Namely, it is necessary to quantize the gravitation exactly when we discuss the dynamics of black hole.

Recently Callan, Giddings, Harvey and Strominger\cite{Callan1992} proposed an interesting toy model of gravity in 1+1 dimensions. It is called the dilaton gravity. The model has interesting features similar to the spherically symmetric gravitational system in 3+1 dimensions. The essence of the black hole dynamics appears to be included enough. Really in the semi-classical approximation the dynamics can be discussed in completely parallel with the case of the spherically symmetric black hole. Furthermore they advanced the arguments so that the gravitational back-reaction effects were included systematically by introducing the large number of matter fields.\cite{Callan1992,Callan1993,Callan1994}
In this paper we develop the argument to the quantum gravity. In Sect. 2 we first define the quantum theory of the dilaton gravity and clarify the differences from the other definitions (see also appendix). Then our status is stressed. We explicitly evaluate the contributions of measures of gravity part and fix the diffeomorphism invariance completely in conformal gauge by using the techniques developed in two dimensional quantum gravity. In Sect. 3 and 4 we derive the physical state conditions that correspond to the Hamiltonian and the momentum constraints and discuss the algebraic structure of them. Then the new features which are not seen in ADM formalism come out. A singularity appears at $\varphi_2 = \kappa (> 0)$, where $\kappa = (N - 51/2)/12$ and $N$ is the number of matter fields. Behind the singularity the quantum mechanical region $\kappa > \varphi_2 > 0$ extends, where the sign of the kinetic term in the Hamiltonian constraint changes. If $\kappa < 0$, the singularity disappears. The existence of the quantum mechanical region gives a new insight when we discuss the dynamics of black holes in Sect. 5. We argue a possibility of gravitational tunneling and give a suggestion for the resolution of the information loss paradox. In Sect. 6 we attempt to quantize the spherically symmetric gravitational system in 3+1 dimensions. In this case some problems appear.

2. Quantum dilaton gravity

The theory of 1+1 dimensional dilaton gravity is defined by the following action $^\dagger$

\[
I(g, \varphi, f) = I_D(g, \varphi) + I_M(g, f),
\]

\[
I_D(g, \varphi) = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left( R_g \varphi^2 + 4g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + 4\lambda^2 \varphi^2 \right),
\]

\[
I_M(g, f) = -\frac{1}{4\pi} \sum_{j=1}^N \int d^2x \sqrt{-g} \left( g^{\alpha\beta} \partial_\alpha f_j \partial_\beta f_j \right),
\]

where $\varphi = e^{-\phi}$ is the dilaton field and $f_j$’s are $N$ matter fields. $\lambda^2$ is the cosmo-

$^\dagger$ Here we do not discuss the model coupled with gauge fields, which is discussed in ref.15.
logical constant. \( R_g \) is the curvature of the metrics \( g \). The classical equations of motion can be solved exactly and one obtains, for instance, the black hole geometry

\[
\varphi^2 = e^{-2\rho} = \frac{M}{\lambda} - \lambda^2 x^+ x^- , \quad f_j = 0 ,
\]

where \( g_{\alpha\beta} = e^{2\rho} \eta_{\alpha\beta} \), \( \eta_{\alpha\beta} = (-1, 1) \) and \( x^\pm = x^0 \pm x^1 \). \( M \) is the mass of the black hole. More interesting geometry is the gravitational collapse.\(^{[16]} \) It is given by

\[
\varphi^2 = e^{-2\rho} = -\frac{M}{\lambda x_0^+} (x^+ - x_0^+) \vartheta(x^+ - x_0^+) - \lambda^2 x^+ x^- ,
\]

where \( \vartheta \) is the step function. The infalling matter flux is given by the shock wave along the line \( x^+ = x_0^+ \)

\[
\frac{1}{2} \sum_{j=1}^{N} \partial_+ f_j \partial_+ f_j = \frac{M}{\lambda x_0^+} \delta(x^+ - x_0^+) .
\]

The quantum theory of the dilaton gravity is defined by

\[
Z = \int \frac{Dg(Dg(\varphi)Dg(f))}{\text{Vol(Diff.)}} e^{iI(g,\varphi,f)} ,
\]

where \( \text{Vol(Diff.)} \) is the gauge volume. The functional measures are defined from the following norms

\[
< \delta g, \delta g >_g = \int d^2 x \sqrt{-g} g^{\alpha\beta} g^{\gamma\delta} (\delta g_{\alpha\gamma} \delta g_{\beta\delta} + u \delta g_{\alpha\beta} \delta g_{\gamma\delta}) ,
\]

\[
< \delta \varphi, \delta \varphi >_g = \int d^2 x \sqrt{-g} \delta \varphi \delta \varphi ,
\]

\[
< \delta f_j, \delta f_j >_g = \int d^2 x \sqrt{-g} \delta f_j \delta f_j \quad (j = 1, \cdots N) ,
\]

where \( u > -1/2. \) The integration range of \( \varphi \) is the whole real values. Physically we should restrict the values of \( \varphi \) within the non-negative values. However, since the action (2.1) is invariant under the change \( \varphi \to -\varphi \), it seems that our definition is meaningful enough when we discuss the quantum dynamics of black holes.

\(^b \) Then the measure (2.13) becomes positive definite.
The several authors discuss the other type of quantum theory.\cite{8,9,10,11,17} If we carry out the field transformation
\[
\chi = \varphi^2, \quad h = e^{2\omega} g, \quad \omega = \frac{1}{2} \log \chi - \frac{1}{2} \chi, \quad (2.7)
\]
the classical dilaton action becomes\cite{9,10,17}
\[
I_D(h, \chi) = \frac{1}{2\pi} \int d^2 x \sqrt{-h} \left( R_h \chi + h^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi + 4 \lambda^2 e^\chi \right). \quad (2.8)
\]
The matter action does not change under this transformation. Then the measures are defined for the fields \( \chi \) and \( h \) instead of \( \varphi \) and \( g \). This is a definition of quantum gravity, but this definition has a demerit. The theory does not have the \( Z_2 \) symmetry under the change \( \chi \to -\chi \) so that the restriction to \( \chi \geq 0 \) seems to be crucial. Thus it is not suited for discussing the quantum dynamics of black holes. If one ignores the restriction, the quantum theory becomes very simple. It reduces to a free-like field theory, which means that the short distance behavior becomes that of the usual free field in two dimensions. The quantization of this theory is discussed in appendix. The quantum theories of ref.8 are very similar to this one. On the other hand the quantum theory \( (2.5) \) has quite different features as discussed below. Really it does not become a free-like theory.

Let us first discuss the measure of the metrics. We decompose the metrics into a conformal factor \( \rho \) and a background metric \( \hat{g} \) as \( g = e^{2\rho} \hat{g} \). This is the conformal gauge-fixing condition adopted here. The change in the metric is given by the change in the conformal factor \( \delta \rho \) and the change under a diffeomorphism \( \delta \xi \) as
\[
\delta g_{\alpha\beta} = 2\delta \rho g_{\alpha\beta} + \nabla_\alpha \delta \xi_\beta + \nabla_\beta \delta \xi_\alpha = 2\delta \rho' g_{\alpha\beta} + (P_1 \delta \xi)_{\alpha\beta}, \quad (2.9)
\]
where
\[
\delta \rho' = \delta \rho + \frac{1}{2} \nabla^\gamma \delta \xi_\gamma, \quad (P_1 \delta \xi)_{\alpha\beta} = \nabla_\alpha \delta \xi_\beta + \nabla_\beta \delta \xi_\alpha - g_{\alpha\beta} \nabla^\gamma \delta \xi_\gamma. \quad (2.10)
\]
The variations \( \delta \rho' g_{\alpha\beta} \) and \( (P_1 \delta \xi)_{\alpha\beta} \) are orthogonal in the functional space defined
by the norms (2.6). Therefore the measure over metrics can be decomposed as

\[ D_g(g) = D_g(\rho')D_g(P_1\xi) = D_g(\rho)D_g(\xi_\alpha)\det_g P_1. \]  

(2.11)

The functional integration over \( \xi_\alpha \) cancels out the gauge volume. The Jacobian \( \det_g P_1 \) can be represented by the functional integral over the ghosts \( b, c \). Thus the partition function (2.5) becomes

\[ Z = \int D_g(\rho)D_g(\varphi)D_g(f)D_g(b)D_g(c) \exp \left[ i I_D(g, \varphi) + i I_M(g, f) + i I_{gh}(g, b, c) \right], \]

(2.12)

where \( I_{gh} \) is the well-known ghost action (see for example ref.13). The measure \( D_g(\rho) \) is defined from the norm (2.6) by

\[ < \delta \rho, \delta \rho >_g = \int d^2x \sqrt{-g(\delta \rho)^2} = \int d^2x \sqrt{-\hat{g} e^{2\rho}(\delta \rho)^2}. \]  

(2.13)

This is not the end of the story. The expression (2.12) has serious problems. The measure (2.13) is not invariant under the local shift \( \rho \to \rho + \epsilon \) and also the measures of the fields \( \varphi, f, b \) and \( c \) explicitly depend on the dynamical variable \( g = e^{2\rho}\hat{g} \). This is quite inconvenient because we must pick up contributions from the measures when the conformal factor \( \rho \) is integrated. So we will rewrite the measures on \( g \) into more convenient ones defined on the background metric \( \hat{g} \).

First we rewrite the measures of the dilaton, the matter and the ghost fields into the convenient ones. For the measures of the matter and the ghost fields it is realized by using the well-known transformation property (see for example ref.14)

\[ D_{e^{2\rho}\hat{g}}(f)D_{e^{2\rho}\hat{g}}(b)D_{e^{2\rho}\hat{g}}(c) = \exp \left[ i \frac{N - 26}{12\pi} S_L(\rho, \hat{g}) \right] D_{\hat{g}}(f)D_{\hat{g}}(b)D_{\hat{g}}(c), \]  

(2.14)

where \( S_L(\rho, \hat{g}) \) is what is called the Liouville action defined by

\[ S_L(\rho, \hat{g}) = \frac{1}{2} \int d^2x \sqrt{-\hat{g}} (\hat{g}^{\alpha\beta} \partial_\alpha \rho \partial_\beta \rho + \hat{R} \rho). \]  

(2.15)

Note that the actions of the matter and the ghost fields are invariant under the Weyl rescalings, or \( I_M(g, f) = I_M(\hat{g}, f) \) and \( I_{gh}(g, b, c) = I_{gh}(\hat{g}, b, c) \).
For the measure of the dilaton field the following relation is realized,
\[
\int D_{e^{2\rho\hat{g}}}(\varphi)e^{iI_D(e^{2\rho\hat{g}},\varphi)} = \exp \left[ i\frac{c_\varphi}{12\pi}S_L(\rho,\hat{g}) \right] \int D_{\hat{g}}(\varphi)e^{iI_D(e^{2\rho\hat{g}},\varphi)} \tag{2.16}
\]
with \(c_\varphi = -1/2\). A notable point is that the dilaton action \(I_D\) is not invariant under the Weyl rescalings. Pay attention to the \(\rho\)-dependence of each side of (2.16). This expression is proved by comparing the \(\rho\)-dependence of the functional integrations of each side. The l.h.s. gives the determinant
\[
\int D_{\hat{g}}(\varphi)e^{iI_D(\hat{g},\varphi)} = L[\det_{\hat{g}}\hat{D}]^{-1/2}, \quad \hat{g} = e^{2\rho\hat{g}}, \tag{2.17}
\]
where the operator \(D\) is defined by
\[
D = \Delta_g + \frac{1}{4}R_g + \lambda^2 = e^{-2\rho}\hat{\Delta} + \frac{1}{4}e^{-2\rho}(\hat{R} + 2\hat{\Delta}\rho) + \lambda^2 \tag{2.18}
\]
and \(L\) is a constant factor and \(\Delta\) is the Laplacian defined by \(-\nabla^\alpha\nabla_\alpha\). The functional integration of r.h.s. gives the determinant
\[
\int D_{\hat{g}}(\varphi)e^{iI_D(\hat{g},\varphi)} = L[\det_{\hat{g}}\hat{D}]^{-1/2}, \tag{2.19}
\]
where \(\hat{D}\) is defined by
\[
\hat{D} \equiv e^{2\rho}D = \hat{\Delta} + \frac{1}{4}(\hat{R} + 2\hat{\Delta}\rho) + \lambda^2 e^{2\rho}. \tag{2.20}
\]
The determinants (2.17) and (2.19) can be evaluated by using the heat-kernel method. Here we want to know only the difference between them. Paying attention to the \(\rho\)-dependence, we get the simple relation
\[
\delta_\rho \log \det_g D - \delta_\rho \log \det_{\hat{g}}\hat{D} = -2Tr(\delta_\rho e^{-i\varepsilon D}) = \delta_\rho \left[-i\frac{c_\varphi}{12\pi} \int d^2x \sqrt{-g}(\rho\hat{\Delta}\rho + \hat{R}\rho) + \Lambda \int d^2x \sqrt{-g}e^{2\rho} \right], \tag{2.21}
\]
where \(\varepsilon\) is a infinitesimal parameter to regularize divergences. \(\Lambda\) is the divergent constant \(\frac{1}{4\pi}(-\frac{1}{\varepsilon} + i\lambda^2)\), which is renormalized to zero by introducing a bare term.
\[ \mu_0 \int d^2 x \sqrt{-g} \] and adjusting the bare constant \( \mu_0 \) properly. The details of the calculation appear in ref.7. From eq.(2.21) we obtain the expression (2.16).

From the expression (2.14) and (2.16) we get

\[
Z = \int D_{e^{2\rho}g}(\rho)D_{\hat{g}}(\varphi)D_{\hat{g}}(f)D_{\hat{g}}(b)D_{\hat{g}}(c) \exp \left[ i \frac{c_\varphi + N - 26}{12\pi} S_L(\rho, \hat{g}) + i I_D(e^{2\rho} \hat{g}, \varphi) + i I_M(\hat{g}, f) + i I_{gh}(\hat{g}, b, c) \right].
\] (2.22)

Next we rewrite the measure of \( \rho \). According to the procedure of David-Distler-Kawai (DDK),\textsuperscript{[12]} we assume the following relation

\[
D_{e^{2\rho}g}(\rho) = D_{\hat{g}}(\rho) \exp \left[ i \frac{A}{12\pi} S_L(\rho, \hat{g}) \right].
\] (2.23)

Note that the measure \( D_{\hat{g}}(\rho) \) is invariant under the local shift of \( \rho \). The parameter \( A \) is determined by the consistency. Since the original theory depends only on the metrics \( g = e^{2\rho} \hat{g} \), the theory should be invariant under the simultaneous shifts

\[
\rho \to \rho - \sigma, \quad \hat{g} \to e^{2\sigma} \hat{g}.
\] (2.24)

This requirement leads to \( A = 1 \). The exact proof is given in ref.7. Finally we get the expression

\[
Z = \int D_{\hat{g}}(\Phi) e^{i \hat{I}(\hat{g}, \Phi)},
\] (2.25)

where \( \Phi \) denotes the fields \( \rho, \varphi, f, b \) and \( c \). \( \hat{I} \) is the gauge-fixed action

\[
\hat{I} = \frac{1}{2\pi} \int d^2 x \sqrt{-\hat{g}} \left[ 4 \hat{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + 4 \hat{g}^{\alpha\beta} \varphi \partial_\alpha \varphi \partial_\beta \rho + \hat{R} \varphi^2 + 4\lambda^2 \varphi^2 e^{2\rho}
\]
\[
+ \kappa (\hat{g}^{\alpha\beta} \partial_\alpha \rho \partial_\beta \rho + \hat{R} \rho) - \frac{1}{2} \sum_{j=1}^N \hat{g}^{\alpha\beta} \partial_\alpha f_j \partial_\beta f_j \right] + I_{gh}(\hat{g}, b, c)
\] (2.26)

with

\[
\kappa = \frac{1}{12} (1 + c_\varphi + N - 26) = \frac{N - 51/2}{12}.
\] (2.27)

Closing this section there are some remarks. We showed that the theory (which
includes the measures) is invariant under the simultaneous shifts (2.24). Furthermore the measure $D_{\hat{g}}(\rho)$ is invariant under the local shift of $\rho$. So the theory is invariant under conformal changes of the background metric $\hat{g}$: $\hat{g} \rightarrow e^{2\sigma} \hat{g}$. More explicitly the Liouville-dilaton part is transformed as

$$\int D_{e^{2\sigma} \hat{g}}(\rho) D_{e^{2\sigma} \hat{g}}(\varphi) \exp \left[ \frac{i\kappa}{\pi} S_L(\rho, e^{2\sigma} \hat{g}) + iI_D(e^{2\rho} e^{2\sigma} \hat{g}, \varphi) \right]$$

$$= \int D_{e^{2\sigma} \hat{g}}(\rho) D_{e^{2\sigma} \hat{g}}(\varphi) \exp \left[ i\frac{\kappa}{\pi} S_L(\rho - \sigma, e^{2\sigma} \hat{g}) + iI_D(e^{2\rho} \hat{g}, \varphi) \right]$$

$$= \exp \left[ i\frac{1}{12\pi} \frac{c_{\rho}}{2} S_L(\sigma, \hat{g}) \right] \int D_{\hat{g}}(\rho) D_{\hat{g}}(\varphi) \exp \left[ i\frac{\kappa}{\pi} S_L(\rho - \sigma, e^{2\sigma} \hat{g}) + iI_D(e^{2\rho} \hat{g}, \varphi) \right]$$

$$= \exp \left[ -i\frac{N - 26}{12\pi} S_L(\sigma, \hat{g}) \right] \int D_{\hat{g}}(\rho) D_{\hat{g}}(\varphi) \exp \left[ i\frac{\kappa}{\pi} S_L(\rho, \hat{g}) + iI_D(e^{2\rho} \hat{g}, \varphi) \right],$$

where in the last equality we use the relation for the Liouville action

$$S_L(\rho - \sigma, e^{2\sigma} \hat{g}) = S_L(\rho, \hat{g}) - S_L(\sigma, \hat{g}).$$

(2.29)

The extra Liouville action $-i\frac{N - 26}{12\pi} S_L(\sigma, \hat{g})$ cancels out with that induced from the measures of the matter and ghost fields (see eq.(2.14)) so that the partition function is invariant under the conformal change of $\hat{g}$. This invariance is quite reasonable because the background metric $\hat{g}$ is very artificial. The theory should be independent of how to choose the background metric.

Here there is a question whether the theory (2.25) is regarded as a kind of conformal field theory (CFT) on $\hat{g}$ or not. The usual definition of CFT is that the action is invariant under the conformal transformation. According to this definition the Liouville theory is not CFT. However, the Liouville action satisfies the relation (2.29), which means that the Poisson brackets of the classical energy-momentum tensor satisfy the Virasoro algebra with central extension $-12\kappa$. Furthermore, as shown in ref.18, the quantum Liouville theory realizes the Virasoro algebra with central charge $c_\rho = 1 - 12\kappa$ (which is easily derived by ignoring the dilaton field in eq.(2.28)). Thus the Liouville theory is considered as a kind of CFT. In the
theory (2.25), we must treat the fields \( \rho \) and \( \varphi \) in pairs because the theory has the derivative coupling of the “third order” of fields. The equation (2.28) says that the Liouville-dilaton part of the quantum energy-momentum tensor satisfies the Virasoro algebra with central extension \( c_{\rho \varphi} = 1 + c_{\varphi} - 12\kappa = 26 - N \). In general CFT is described by a set of free fields, while the theory (2.25) has the non-trivial coupling and is not free-like so that it is quite different from usual CFT. The Virasoro structure of this theory is realized in the non-trivial way, which is discussed in Sect. 4.

The second remark is that the partition function is a scalar. This is manifest in the definition (2.5). After rewriting the partition function into the expression (2.25), however, this invariance is hidden. It is instructive to show that the partition function is really scalar. The Liouville field \( \rho \) is transformed as

\[
\rho'(x') = \rho(x) - \gamma(x), \quad \gamma(x) = \frac{1}{2} \log \left| \frac{\partial x'}{\partial x} \right|^2,
\]

where we only consider the conformal coordinate transformation \( x^{\pm'} = x^{\pm}(x^{\pm}) \) to preserve the conformal gauge and use the notation \( |x|^2 = x^+x^- \). On the other hand the background metric is not transformed: \( \hat{g}'(x') = \hat{g}(x) \). It is natural because the background metric is not dynamical. Therefore the gauge-fixed action is transformed as

\[
\hat{I}' = \hat{I} - \frac{\kappa}{\pi} S_L(\gamma, \hat{g}),
\]

where note that \( R_g \) is a scalar, but \( \hat{R} \) is transformed as \( \hat{R}' = \left| \frac{\partial x'}{\partial x} \right|^2(\hat{R} + 2\hat{\Delta} \gamma) \). The measures defined on \( \hat{g} \) are also non-invariant under the coordinate transformation. The extra Liouville term \( S_L(\gamma, \hat{g}) \) cancels out with that coming from the measures so that the partition function is invariant. By replacing \( \gamma \) with the conformal change \( \sigma \), it is seen that the invariance under the conformal change of \( \hat{g} \) after all guarantees the invariance under the coordinate transformation.
3. Physical state conditions

Now we carry out the canonical quantization of the gauge-fixed 1+1 dimensional dilaton gravity. As mentioned in Sect. 2 the theory should be independent of how to choose the background metric \( \hat{g} \). Thus the variation of the partition function with respect to \( \hat{g} \) vanishes

\[
0 = \frac{\delta Z}{\delta \hat{g}^{\alpha\beta}} = \int D\hat{g}(\Phi) \frac{\delta \hat{I}}{\delta \hat{g}^{\alpha\beta}} e^{i(\hat{g}, \Phi)} + \int \frac{\delta D\hat{g}(\Phi)}{\delta \hat{g}^{\alpha\beta}} e^{i(\hat{g}, \Phi)}.
\] (3.1)

The first term of r.h.s. is nothing but \( \langle i \frac{\delta \hat{I}}{\delta \hat{g}^{\alpha\beta}} \rangle_{\hat{g}} \). The second term picks up an anomalous contribution. But if we choose the Minkowski background \( \hat{g} = \eta \), this contribution vanishes. So it is convenient to choose the Minkowski background metric. Then the physical state conditions are

\[
\langle \frac{\delta \hat{I}}{\delta \hat{g}^{\alpha\beta}} \rangle_{\hat{g} = \eta} = 0 \quad (3.2)
\]

or

\[
\langle \hat{T}_{00} \rangle_{\hat{g} = \eta} = \langle \hat{T}_{01} \rangle_{\hat{g} = \eta} = 0, \quad (3.3)
\]

where the energy-momentum tensor \( \hat{T}_{\alpha\beta} \) is defined by \( \hat{T}_{\alpha\beta} = -\frac{2}{\sqrt{-g}} \frac{\delta \hat{I}}{\delta \hat{g}^{\alpha\beta}} \). The condition for \( \hat{T}_{11} \) reduces to the one for \( \hat{T}_{00} \) by using the \( \rho \)-equation of motion. Furthermore we restrict the physical state to the one which satisfies the condition \( \langle \hat{T}_{\alpha\beta} \rangle_{\hat{g} = \eta} = 0 \) because the ghost flux should vanish in the flat space-time.

Since the functional measures are defined on the Minkowski background metric, we can set up the canonical commutation relations as usual. The conjugate momentums for \( \rho, \varphi \) and \( f_j \) are given by

\[
\Pi_{\rho} = -\frac{\kappa}{\pi} \dot{\rho} - \frac{2}{\pi} \dot{\varphi} \dot{\phi},
\]

\[
\Pi_{\varphi} = -\frac{4}{\pi} \dot{\phi} - \frac{2}{\pi} \dot{\varphi} \dot{\rho},
\]

\[
\Pi_{f_j} = \frac{1}{2\pi} \dot{f}_j,
\]

where the dot stands for the derivative with respect to the time coordinate. Then
the physical state conditions (3.3) can be expressed as

\[
\left[ \frac{\pi/2}{\varphi^2 - \kappa} \left( \Pi^2 - \varphi \Pi \rho + \frac{\kappa}{4} \Pi^2 + \frac{2}{\pi} (\varphi \varphi'' - \varphi \rho' - \lambda^2 \varphi^2 e^{2\rho}) \right) - \frac{\kappa}{2\pi} (\rho'^2 - 2\rho''') + \sum_{j=1}^{N} \left( \frac{\pi \Pi f_j + \frac{1}{4\pi} f_j^2}{j} \right) \right] \Psi = 0
\]

(3.5)

and

\[
(\rho' \Pi + \Pi' \varphi + \sum_{j=1}^{N} \Pi f_j f_j') \Psi = 0 ,
\]

(3.6)

where \( \kappa \) is defined by eq. (2.27). \( \Psi \) is a physical state. The prime stands for the derivative with respect to the space coordinate.

Here we have two remarks. The first is that the fields \( \rho \) and \( \varphi \) are dynamical variables so that it is significant to consider the equations of motion of \( \rho \) and \( \varphi \). But \( \hat{g} \) is not dynamical. So we should not regard the physical state conditions as the equations of motion of \( \hat{g} \). The conditions come from the symmetry of the theory. In this point of view the conditions indeed correspond to the constraints. Therefore we call eqs. (3.5) and (3.6) the Hamiltonian and the momentum constraints respectively. These are the modified versions of the Wheeler-DeWitt equations.\(^\dagger\)

The second remark is that the energy-momentum tensor \( \hat{T}_{\alpha\beta} \) is transformed as non-tensor because the Liouville field \( \rho \) is transformed as (2.30) for the conformal coordinate transformation \( x^{\pm'} = x^{\pm}(x^\pm) \). In the light-cone coordinate we get

\[
\hat{T}_{\pm\pm}(x') = \left( \frac{\partial x^{\pm}}{\partial x^{\pm'}} \right)^2 \left( \hat{T}_{\pm\pm}(x) + \frac{\kappa}{\pi} t_{\pm}(x) \right) ,
\]

\[
\hat{T}_{+-}(x') = \left. \frac{\partial x^{\pm}}{\partial x' \partial x'} \right| \hat{T}_{+-}(x) ,
\]

(3.7)

\(^\dagger\) The usual Wheeler-DeWitt equations are derived, for example, in ref.19, where the spherically symmetric gravitational system in 3+1 dimensions is discussed. Application to the 1+1 dimensional dilaton gravity is straightforward.
where \( t_\pm(x) \) is the Schwarzian derivative

\[
t_\pm(x) = \frac{\partial \gamma(x)}{\partial x^\pm} \frac{\partial \gamma(x)}{\partial x^\pm} - \frac{\partial^2 \gamma(x)}{\partial x^{\pm 2}} , \quad \gamma(x) = \frac{1}{2} \log \left| \frac{\partial x'}{\partial x} \right|^2 . \tag{3.8}
\]

Therefore the physical state conditions (3.5-6) correspond to the case of \( t_\pm = 0 \). To determine what coordinate system corresponds to this case is a physical requirement. It is natural that the coordinate system which is joined to the Minkowski space time (asymptotically) is considered as the coordinate system with \( t_\pm = 0 \).

If \( \kappa > 0 \), there is a singularity at finite \( \varphi^2 = \kappa \). The region \( \varphi^2 > \kappa \) is the classically allowed region, whereas the region \( \kappa > \varphi^2 > 0 \) is called the Liouville region, where the sign of the kinetic term of the Hamiltonian constraint changes. This is the classically forbidden region. The existence of the Liouville region is interesting. There may be some possibility of gravitational tunneling through this region. If \( \kappa < 0 \), the situation drastically changes. In this case the singularity disappears.

4. On Virasoro algebra in quantum dilaton gravity

The constraints should form the closed algebra without central extension. We first discuss the Poisson brackets between the constraints. The Poisson brackets are defined by

\[
\{ \rho(x), \Pi_\rho(y) \}_{P.B.} = \delta(x - y) , \quad \{ \varphi(x), \Pi_{\varphi}(y) \}_{P.B.} = \delta(x - y) . \tag{4.1}
\]

Here we concentrate on the Liouville-dilaton part. Then the Poisson brackets

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\( ^\ddagger \) Here \( \hat{I} \) is considered as a classical action
become

\[ [H^{\rho\varphi}(x), H^{\rho\varphi}(y)]_{P.B.} = 2P^{\rho\varphi}(x)\delta'(x-y) + P^{\rho\varphi'}(x)\delta(x-y), \]

\( [P^{\rho\varphi}(x), P^{\rho\varphi}(y)]_{P.B.} = 2P^{\rho\varphi}(x)\delta'(x-y) + P^{\rho\varphi'}(x)\delta(x-y), \tag{4.2} \)

and

\[ [H^{\rho\varphi}(x), P^{\rho\varphi}(y)]_{P.B.} = [P^{\rho\varphi}(x), H^{\rho\varphi}(y)]_{P.B.} \]

\[ = 2H^{\rho\varphi}(x)\delta'(x-y) + H^{\rho\varphi'}(x)\delta(x-y) + \frac{\kappa}{\pi}\delta'''(x-y), \tag{4.3} \]

where

\[ H^{\rho\varphi}(x) = \hat{T}^{\rho\varphi}_{00}(x), \quad P^{\rho\varphi}(x) = \hat{T}^{\rho\varphi}_{01}(x). \tag{4.4} \]

The Poisson brackets of the matter and the ghost parts are the same as the expression above without the central extension.

The central extension of the Poisson bracket (4.3) reflects that the gauge-fixed action \( \hat{I} \) is not invariant under the coordinate transformation and transformed as eq.(2.31). The extra Liouville action of eq.(2.31) indeed corresponds to the central extension of the Poisson bracket.

The results in the path integral show that the conformal invariance is recovered by the quantum corrections which come from the measures defined on \( \hat{g} \). In terms of the operator formalism it means that, if we replace the Poisson brackets with the commutators and define the normal ordering properly, the central term of the Poisson bracket is canceled out completely.

What is the proper normal ordering consistent to the path integral results? For the matter and the ghost fields we can adopt the free field normal ordering, but for the Liouville and the dilaton fields we cannot adopt the free-like one. At present we do not find the proper normal ordering yet. Here we only give a suggestion. To cancel the prefactor \( (\varphi^2 - \kappa)^{-1} \) of the Hamiltonian constraint, the most singular part of the operator product between the two \( \Pi_{\rho} \)'s should behave like \( \Pi_{\rho}(x)\Pi_{\rho}(y) \sim (\varphi^2 - \kappa)/(x-y)^2 + \cdots \). The similar structure should be realized for \( \Pi_\varphi \). The field dependence of the most singular term indicates that the theory has
the non-trivial coupling. This structure is very different from the other quantum gravity models in two dimensions.

After properly normal ordered, the commutation relations of the constraints ought to satisfy the closed algebra without the central charge. Combining the Hamiltonian and the momentum constraints as \( \hat{T}_{\pm \pm} = \frac{1}{2}(H \pm P) \), we get

\[
\begin{align*}
[\hat{T}_{\pm \pm}(x), \hat{T}_{\pm \pm}(y)] &= \pm 2i\hat{T}_{\pm \pm}(x)\delta'(x - y) \pm i\hat{T}'_{\pm \pm}(x - y), \\
[\hat{T}_{++}(x), \hat{T}_{--}(y)] &= 0 .
\end{align*}
\]  

This commutation relations generate the well-known Virasoro algebra without central charge. This algebra guarantees the general covariance of the theory.

5. Black hole dynamics

Until now the arguments are completely non-perturbative. If we can solve the physical state conditions exactly, the solution should include the complete dynamics of black hole. Unfortunately it is a very difficult problem so that we take an approximation. The original action (2.1) is order of \( 1/\hbar \), but the Liouville part of \( \hat{I} \) is zeroth order of \( \hbar \). However, if \( |\kappa| \) is large enough, then it is meaningful to consider the “classical” dynamics of \( \hat{I} \). This is nothing but the semi-classical approximation, which is valid only in the case of \( M \gg 1 \) and \( N \gg 1 \). In the other cases the quantum effect of gravitation becomes important. The classical dynamics of \( \hat{I} \) is ruled by the equations \( \hat{T}_{\alpha \beta} = 0 \) and the dilaton equation of motion

\[
\begin{align*}
-2\partial_+ \varphi \partial_+ \varphi + 2\varphi \partial_+^2 \varphi - 4\varphi \partial_+ \varphi \partial_+ \rho + \frac{1}{2} \sum_{j=1}^{N} \partial_+ f_j \partial_+ f_j \\
-\kappa(\partial_+ \rho \partial_+ \rho - \partial_+^2 \rho + t_+) = 0 , \\
-2\partial_- \varphi \partial_- \varphi + 2\varphi \partial_-^2 \varphi - 4\varphi \partial_- \varphi \partial_- \rho + \frac{1}{2} \sum_{j=1}^{N} \partial_- f_j \partial_- f_j \\
-\kappa(\partial_- \rho \partial_- \rho - \partial_-^2 \rho + t_-) = 0 , \\
-2\partial_+ \varphi \partial_- \varphi - 2\varphi \partial_+ \partial_- \varphi - \lambda^2 \varphi^2 e^{2\rho} - \kappa \partial_+ \partial_- \rho = 0 .
\end{align*}
\]  

(5.1)
and

\[4\partial_+\partial_-\varphi + 2\varphi\partial_+\partial_-\rho + \lambda^2\varphi e^{2\rho} = 0. \quad (5.2)\]

These are nothing but the CGHS equations\(^4\) with the coefficient \(\kappa\) instead of \(N/12\) in front of the Liouville part. Many authors have solved these equations for \(\kappa > 0\) and derived the dynamics of evaporating black hole.\(^5,6\) Giving the expression (2.4) as the infalling matter flux, we can get the exact solution of the equations along the line of \(x^+ = x^+_0\)

\[\partial_+\varphi(x^+_0, x^-) = \frac{\lambda}{2} \sqrt{-\frac{x^-}{x^+_0}} - \frac{M}{2\lambda x^+_0} \sqrt{-\lambda^2 x^+_0 x^- - \kappa}. \quad (5.3)\]

The (apparent) horizon, which is defined by the equation \(\partial_+\varphi(x) = 0\),\(^{19}\) locates at

\[x^- = -\sqrt{\left(\frac{M}{\lambda^3 x^+_0}\right)^2 + \left(\frac{\kappa}{2\lambda^2 x^+_0}\right)^2 - \frac{\kappa}{2\lambda^2 x^+_0}}, \quad x^+ = x^+_0. \quad (5.4)\]

Initially the location of the horizon shifts to the outside of the classical horizon defined through the solution (2.3) by quantum effects (almost matter’s effects). Then the black hole evaporates and the horizon approaches to the singularity asymptotically. The location of the singularity is determined by the equation \(\varphi^2 = \kappa\), which is easily proved by combining the equations (5.1) and (5.2) properly (at \(x^+ = x^+_0\), it is \(x^- = \frac{\kappa}{\lambda^2 x^+_0}\)). It coincides with that determined from the Hamiltonian constraint. Note that at the singularity the curvature is singular, but the metric is regular. As far as the gauge-fixed action is treated classically, it seems that the horizon does not cross the singularity. As mentioned before the quantum mechanical region \(\kappa > \varphi^2 > 0\) extends behind the singularity, where the quantum gravitational effects become important.

If \(N\) is small, the non-anomalous quantum corrections of gravity part maybe contribute to the dynamics and the approximation becomes bad. Nevertheless we apply the approximation for \(\kappa < 0\) because we hope that some new insights are
obtained from the solution. If $\kappa < 0$, the singularity disappears. The location of the horizon initially shifts to the inside of the classical horizon. If the effective mass of the black hole is defined by $M_{BH} = \lambda \varphi^2|_{\text{horizon}}$, this means that the initial mass of the black hole is less than the infalling matter flux $M$. After the black hole is formed, the positive flux comes in through the horizon and the black hole mass increases. It seems that the horizon approaches to the classical horizon asymptotically and becomes stable. If $\kappa = 0$, the Liouville action disappears and the classical solution (2.3) is dominant.

The problem of the information loss seems to come out in the case of $\kappa > 0$. Then the black hole evaporates and the information seems to be lost. However in this case the Liouville region extends behind the singularity. So it appears that there is a possibility that the informations run away through this region by gravitational tunneling. On the other hand, if $\kappa \leq 0$, the Liouville region disappears. But the black hole seems to be stable. In this case it appears that the problem of the information loss does not exist.

6. Toward the quantization of spherically symmetric gravity

In this section we discuss the quantization of the spherically symmetric gravitational system in 3+1 dimensions. If the 3+1 dimensional metric is restricted as

\[
(ds^{(4)})^2 = g_{ab}^{(4)} dx^a dx^b = g_{\alpha\beta} dx^\alpha dx^\beta + G \varphi^2 d\Omega^2. \tag{6.1}
\]

where $d\Omega^2$ is the volume element of a unit 2-sphere and $G$ is the gravitational constant, the Einstein-Hilbert action becomes\[^{[19]}\]

\[
I_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g^{(4)}} R^{(4)} = \frac{1}{4} \int d^2x \sqrt{-g} \left( R_g \varphi^2 + 2 g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + \frac{2}{G} \right). \tag{6.2}
\]

In the following we set $G = 1$. 

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If the conformal matter defined by the action (2.1) is coupled and the measures are defined by (2.6), the quantization is carried out in the parallel with the case of the dilaton gravity. Then the gauge-fixed action of the spherically symmetric gravity becomes

$$\hat{I}_{SSG} = \frac{\kappa_s \pi}{\kappa} S_L(\rho, \hat{g}) + I_{EH}(e^{2\rho} \hat{g}, \varphi) + I_M(\hat{g}, f) + I_{gh}(\hat{g}, b, c) ,$$

where the coefficient in front of the Liouville action is

$$\kappa_s = \frac{1}{12} (1 - 2 + N - 26) = \frac{N - 27}{12} .$$

The nature of the quantum dynamics becomes the same as that of the dilaton gravity. The differences are only quantitative.

If both the black hole mass $M$ and the parameter $\kappa_s$ are large enough, the classical dynamics of $\hat{I}_{SSG}$ is dominant. This corresponds to taking the semi-classical approximation. As a classical geometry we introduce the shock wave geometry similar to (2.3). It is given by

$$ds^2 = -\left(1 - \frac{2M \vartheta(\bar{v})}{r}\right)\frac{\bar{u}}{\bar{u} + 4M \vartheta(\bar{v})} d\bar{u} d\bar{v} , \quad \varphi = r ,$$

This geometry is derived by sewing the flat space time and the Schwarzshild black hole geometry along the shock wave line. We first define that for $v < 0$ the metric is flat $ds^2 = -dudv$, where $u = v - 2r$, while for $v > 0$ the metric is the Schwarzshild $ds^2 = -(1 - \frac{2M}{r})du^* dv$, where $u^* = v - 2r^*$ and $r^* = r + 2M \log(\frac{r}{2M} - 1)$. Next we relate the coordinate system $(r, v)$ with the coordinate $(\bar{u}, \bar{v})$ describing a gravitational collapse. In the past infinity the geometry is asymptotically flat so that we set $\bar{v} = v$ in the whole space time. Let us take the metric $ds^2 = -d\bar{u}d\bar{v}$ (or $\bar{u} = u$) for $\bar{v} < 0$. Then the metric for $\bar{v} > 0$ is determined by the machining

---

\* This value is given by setting $\xi = 1/2$ in ref.7
condition at $\tilde{v} = 0$. The condition gives the relation $d\tilde{u} = d\tilde{u}^*(\tilde{u} + 4M)/\tilde{u}$ and we get the expression (6.5). This geometry is really a classical solution with the infalling matter flux $T^f_{\tilde{v}\tilde{v}} = M\delta(\tilde{v})$. In $(\tilde{u}, \tilde{v})$ coordinate the location of the horizon is given by $\tilde{u} = -4M$.

By substituting the classical shock wave geometry into the induced energy-momentum tensor $\tilde{T}^\rho_{uu}$ and transforming it into that in the null coordinate $u^*$, we get the Hawking radiation

$$
(\tilde{T}^\rho_{u^*u^*} + \frac{\kappa_s}{\pi} t_{u^*})|_{r=+\infty} = \frac{\kappa_s}{64\pi M^2} \left( 1 - \frac{2M}{r} \right)^2 \left( 1 + \frac{4M}{r} + \frac{12M^2}{r^2} \right).
$$

(6.6)

In the spacial infinity $r \to \infty$, the flux becomes $\kappa_s/64\pi M^2$. This is really the same as the result derived by Hawking if we replace $\kappa_s$ with $N/12$.

The quantum model of spherically symmetric gravity discussed above has some problems. Here we adopt the conformal matter described by the action (2.1). Strictly speaking, however, we should consider the action such as $I_M = -\int d^2x \sqrt{-g} \varphi^2 g^{\alpha\beta} \partial_\alpha f \partial_\beta f$, which is derived by reducing the four dimensional action to the two dimensional one. Ignoring $\varphi^2$-factor corresponds to ignoring the potential which appears when we rewrite the d’Alembertian in terms of the spherical coordinate. The black hole dynamics is determined by the behavior near the horizon so that it seems that this simplification does not change the nature of dynamics.

The other problem is in the definitions of measures. As the actions are derived from the four dimensional ones, the two dimensional measures also should be derived from the four dimensional one

$$
<\delta g^{(4)}, \delta g^{(4)}>_{g^{(4)}} = \int d^4x \sqrt{-g^{(4)}} g^{(4)ab} g^{(4)cd} (\delta g^{(4)}_{ac} \delta g^{(4)}_{bd} + u \delta g^{(4)}_{ab} \delta g^{(4)}_{cd}),
$$

(6.7)

\footnote{In $(\tilde{u}, \tilde{v})$ coordinate, $t_{\tilde{u}}$ and $t_{\tilde{v}}$ of (3.8) vanish by the physical requirement, but, in $(u^*, v)$ coordinate, $t_{u^*}$ is non-zero. See the relation (3.7).}
where \( u > 0 \). From this definition we get

\[
< \delta g, \delta g >_g = \int d^2 x \sqrt{-g} \varphi^2 g^{\alpha\beta} g^{\gamma\delta} (\delta g_{\alpha\gamma} \delta g_{\beta\delta} + u \delta g_{\alpha\beta} \delta g_{\gamma\delta}) ,
\]

\[
< \delta \varphi, \delta \varphi >_g = \int d^2 x \sqrt{-g} \delta \varphi \delta \varphi .
\]

And also for the matter fields,

\[
< \delta f_j, \delta f_j >_g = \int d^2 x \sqrt{-g} \varphi^2 \delta f_j \delta f_j \quad (j = 1, \cdots N) .
\]

The difference between (2.6) and (6.8-9) is apparent. The factor \( \varphi^2 \) in the measures of \( g \) and \( f \) prevents us from quantizing the spherically symmetric gravity exactly. We expect that this factor also does not change the nature of quantum dynamics drastically.

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APPENDIX

In this appendix we discuss the nature of the quantum dilaton gravity defined by the action (2.8) and clarify the difference from ours. The quantum theory of (2.8) is defined by

\[
Z_\chi = \int \frac{D_h(h)D_h(\chi)D_h(f)}{\text{Vol}(\text{Diff.})} e^{iI_\chi(h,\chi,f)} ,
\]

where \( I_\chi = I_D(h,\chi) + I_M(h,f) \). The conformal gauge fixing is carried out by separating the metric \( h \) into the conformal factor \( \rho \) and the background metric \( \hat{g} \) as \( h = e^{2\rho} \hat{g} \). The \( \rho \)-dependence of the measure of \( \chi \) is evaluated as follows.
Since the measure of $\chi$ is invariant under the local shift, we can replace $\chi$ into $\chi' = \chi + \frac{1}{2} \Delta_h^{-1} R_h$. Then the dilaton action $I_D(h, \chi)$ becomes

$$
I_D(h, \chi') = \frac{1}{2\pi} \int d^2 x \sqrt{-h} \left[ h^{\alpha\beta} \partial_{\alpha} \partial_\beta \chi' - \frac{1}{4} h R_h \Delta_h^{-1} R_h + 4 \lambda^2 \exp\left( \chi' - \frac{1}{2} \Delta_h^{-1} R_h \right) \right].
$$

(A.2)

When in two dimensions the kinetic term of a field takes the standard quadratic form and there is no derivative coupling, the short distance behavior becomes that of the usual free field in two dimensions since there is no divergence which could modify the singularity of free field theory in perturbation expansion. Therefore the divergence structure of $\chi'$ field is the same as that of a single free boson. This fact leads to the relation

$$
\int D_h(\chi)e^{iI_D(h,\chi)} = \int D_h(\chi')e^{iI_D(h,\chi')}
$$

(A.3)

The relation for the matter and the ghost fields is given by (2.14). According to the procedure of DDK, we finally get

$$
Z_\chi = \int D\hat{g}(\Phi)e^{i\hat{I}_\chi(\hat{g},\Phi)},
$$

(A.4)

where $\Phi$ denotes $\rho, \chi, f, b$ and $c$. The gauge fixed action is

$$
\hat{I}_\chi = \frac{1}{2\pi} \int d^2 x \sqrt{-\hat{g}} \left[ \hat{g}^{\alpha\beta} \partial_\alpha \partial_\beta \chi + 2 \hat{g}^{\alpha\beta} \partial_\alpha \chi \partial_\beta \rho + \cdots \right] + I_{gh}(\hat{g}, b, c)
$$

(A.5)

with

$$
\kappa_\chi = \frac{1}{12} (1 + 1 + N - 26) = \frac{N - 24}{12}.
$$

(A.6)

Note that $\rho - \chi$ coupling including the derivative is the second order, while the $\rho - \varphi$ coupling of the action (2.26) is the third order. This difference is very important
because the former becomes the free-like theory after performing the canonical field transformation as mentioned below, whereas the latter is not so as mentioned in Sect. 4.

By defining the fields $X$ and $Y$ as “linear” combinations of $\rho$ and $\chi$

$$\rho = X - \frac{1}{\kappa\chi}Y, \quad \chi = Y,$$  \hfill (A.7)

we can diagonalize the kinetic term of the gauge-fixed action

$$\hat{I}_x = \frac{1}{2\pi} \int d^2x \sqrt{-\hat{g}} \left[ \kappa\chi \hat{g}^{\alpha\beta} \partial_\alpha X \partial_\beta X + \left(1 - \frac{1}{\kappa\chi}\right) \hat{g}^{\alpha\beta} \partial_\alpha Y \partial_\beta Y + \kappa\chi \hat{R}X ight. \left. + 4\lambda^2 e^{2X+(1-\frac{2}{\kappa\chi})Y} - \frac{1}{2} \sum_{j=1}^{N} \hat{g}^{\alpha\beta} \partial_\alpha f_j \partial_\beta f_j \right] + I_{gh}(\hat{g}, b, c) \hfill (A.8)$$

Since there is no derivative coupling, the short distance behavior of the diagonalized fields $X$ and $Y$ is that of the usual free field in two dimensions. The diagonalized action is nothing but the action derived by Bilal and Callan in ref. 8.

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