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Dynamic modeling of ultra-precision fly cutting machine tool and the effect of ambient vibration on its tool tip response

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Abstract
The dynamic performances of an ultra-precision fly cutting machine tool (UFCMT) has a dramatic impact on the quality of ultra-precision machining. In this study, the dynamic model of an UFCMT was established based on the transfer matrix method for multibody systems. In particular, the large-span scale flow field mesh model was created; and the variation in linear and angular stiffness of journal and thrust bearings with respect to film thickness was investigated by adopting the dynamic mesh technique. The dynamic model was proven to be valid by comparing the dynamic characteristics of the machine tool obtained by numerical simulation with the experimental results. In addition, the power spectrum density estimation method was adopted to simulate the statistical ambient vibration excitation by processing the ambient vibration signal measured over a long period of time. Applying it to the dynamic model, the dynamic response of the tool tip under ambient vibration was investigated. The results elucidated that the tool tip response was significantly affected by ambient vibration, and the isolation foundation had a good effect on vibration isolation.

Keywords: ultra-precision fly cutting machine tool, transfer matrix method for multibody systems, dynamic response of tool tip, power spectrum density estimation method, ambient vibration

1. Introduction
To meet the precision and close tolerance requirements of industries such as aerospace, precision instrumentation, and national defense, the quality of manufacturing equipment is continuously being improved. An ultra-precision fly cutting machine tool (UFCMT) is one of the key manufacturing machines used to produce ultraprecise components in those industries. The surface morphology of UFCMT-machined components can be influenced by a variety of factors, including the machining parameters and the structural dynamics of the machine. However, the tool tip response is mainly responsible for the good or bad surface morphology of a finished workpiece. Thus, in order to improve processing precision, research on the dynamic performances of machine tools is becoming more and more important.

Scholars have been studying the dynamic modeling of machine tools for several decades. Gurney and Tobias [1] found that a machine tool can be equated to a damped mass-spring system. Hijink and van der Wolf [2] transformed a milling machine tool into a beam model and calculated the natural frequencies with corresponding mode shapes. The finite element method (FEM) has risen to become one of the
most commonly used methods of analyzing the performances of machine tools. Zatarain et al. [3] conducted modal analysis of a milling machine by using FEM and showed that the accuracy of the calculation increases as the finite element nodes increase. Jiang et al. [4] applied FEM and structural topology optimization to improve the connection of a simple mechanical structure. In recent years, with the development of computer technology, commercial software is widely used in the dynamic analysis of machine tools, which makes it possible for the holistic modeling of complex and large UFCMTs. Liang et al. [5] combined FEM and MATLAB Simulink® to predict the processing surface of a workpiece, and harmonic response analysis was conducted on the whole machine tool [6]. Similarly, Chen et al. [7] determined the relationship between the tool tip displacement response and the cutting time using FEM simulation. Based on the criterion of having the same dominant structural frequency as the machine tool, they also presented a novel simulation method where the machine tool was replaced by an equivalent cylinder; and then the cutting process was simulated in Abaqus FEA software [8]. Thus, the surface profile of a workpiece can be analyzed during the manufacturing process. Gao et al. [9] established a dynamic model of an UFCMT by using ANSYS FEA software, and the topography of a machined surface was simulated by combining with a mathematic model. However, it is too time-consuming when using FEM to analyze the performances of machine tools; and ill-conditioning may occur due to the high order of the system matrix and complex rigid-flexible coupling problems, although the accuracy is relatively high. Thus, the damped mass-spring system is used by some researchers to build dynamic model of machine tools. In the research of Wang et al. [10], the beams and columns of the milling machine tool were considered as lumped mass; and the joints were modeled by spring elements. Sato et al. [11] modeled a machine tool based on a damped mass-spring dynamic model, and the relationship between machine tool behavior and cutting force in time domain was analyzed. Nevertheless, when applying the damped mass-spring system, the degrees of freedom are often not enough, which leads to low accuracy. In addition, the overall dynamic equations of the system are always established, which complicates the modeling process.

In this study, the transfer matrix method for multibody systems (MSTMM) was used in the dynamic modeling of an UFCMT. Rui [12–14] presented the transfer matrix method for multibody systems, also called ‘Rui method’ in academia, to overcome the shortcomings of the traditional modeling methods described above. It is a highly efficient dynamic calculation method that can decrease the order of a system matrix, avoid ill-conditioning, and significantly improve calculation speed. Today, MSTMM has been widely applied in various fields of scientific research and practical engineering [15–17].

In some of the literature mentioned above [5–9], aerostatic bearings are used because they meet the high precision requirements of ultra-precision machine tools. The gas film was regarded as a series of springs when analyzing the dynamic performances of the spindle, which hints at the importance of accurate stiffness calculation of these springs. Moreover, it was already established by the previous research that the dynamic response of the machine tool is greatly influenced by aerostatic bearings. At present, the FEM [18, 19] and the finite difference method (FDM) [20, 21] are widely used to solve the Reynolds equation to obtain the gas pressure distribution. However, the results calculated by FEM or FDM largely depend on the discharge coefficient [22], which mainly depends on the orifice parameters and operational conditions which are difficult to accurately determine. In order to overcome this problem and make the results more accurate, computational fluid dynamics (CFD) was adopted to directly solve the Navier–Stokes (N–S) equations [23, 24]. Nevertheless, it takes substantial time when calculating the gas film stiffness under different geometric parameters due to the mesh generation. The dynamic mesh technique (DMT) provides an efficient approach to analyze the performances of aerostatic bearings through mesh deformation, which greatly reduces the time required for repeatedly meshing. Chen et al. [25] and Zhuang et al. [26] applied DMT to investigate the dynamic performances of gas thrust bearings. The grids were repeatedly deformed in the direction of bearing gravity to obtain the relationship between bearing displacement and gas pressure. In addition to linear stiffness, more and more attention is being paid to the tilt effect of bearings [19, 27]. However, DMT is rarely used for the analysis of bearing angular stiffness, especially for journal bearings. This is mainly because of the greater ratio of the diameter and length of a bearing to the gas film thickness, which results in increased difficulty in generating mesh. Moreover, the negative grid is more likely to appear during mesh deformation due to the curvature of gas mesh in the circumferential direction.

In this paper, the dynamic model of the UFCMT fitted with a gas bearing is established by applying MSTMM. The large-span scale mesh model of a radial-thrust aerostatic bearing was built; and the bearing stiffness was calculated by adopting DMT based on CFD. The dynamic characteristics of the machine tool were found to be in good accordance with the experimental results. Moreover, the field ambient vibration was tested, and then the power spectrum of the ground vibration was obtained according to the data measured. Based on the verified dynamic model, the dynamic response of a tool tip under measured ambient vibration was investigated. The results showed that the ambient vibration cannot be ignored in order to obtain a more accurate dynamic response of any system, and the isolated foundation was proven to be effective.

2. Dynamic modeling of the UFCMT

2.1. Dynamic model of the UFCMT multibody system

As shown in figure 1, the UFCMT is one kind of two-axis machine tool with a hydrostatic guide rail and aerostatic spindle. During the machining process, the aerostatic spindle
rotates at a constant speed around the z axis; then the cutterhead and tool holder are driven. The workpiece is fixed on the sliding table and fed slowly along the hydrostatic guide rail in the y direction.

The components of UFCMT can be regarded as rigid bodies, elastic beams, lumped masses, and elastic hinges, according to the natural properties. The dynamic model of UFCMT is composed of 17 bodies and 39 joints. The topology figure is shown in figure 2. As prescribed by the principle of numbering in the MSTMM:

- The ground boundary is numbered 0.
- The foundation, lathe bed, sliding table, left and right columns, and cross-member are numbered 1, 2, 3, 4, 5, and 10.
- The left and right guide rails are numbered 6, 7, 8, and 9.
- In the spindle system, the torque motor, aerostatic spindle, cutterhead, and two tool holders are numbered 11, 12, 14, 15, 16, and 17.
- The aerostatic spindle is considered to be two beams with single input and single output ends connected by a massless dummy body and is numbered 13.

All of the bodies are connected by spatial elastic hinges whose numbers are clearly marked in figure 2.

For the topology figure of the closed-loop multibody system, after ‘cutting’ at the junction of elements 3/27, 3/28, 10/30, 10/37, and 10/39, the original closed-loop system becomes a tree system, as shown in figure 3.

The interactions among the body elements are reflected in the vibration characteristics and their effect on the dynamic response of the system. The solution procedures for vibration analysis of the UFCMT system are shown in figure 4.

2.2. State vector, transfer equation, and transfer matrix

The state vector is defined as a column matrix denoting the mechanics state of a point. The state variables include the
displacements and the internal forces at that point. For the UFCM system, the state vectors of each connection point can be expressed as

$$Z_{i,j} = [X, Y, Z, \Theta_x, \Theta_y, \Theta_z, M_x, M_y, M_z, Q_x, Q_y, Q_z, U_{i,j}^T]$$  \hspace{1cm} (1)$$

in which the variables are the modal coordinates of the displacement, angular displacement, internal momentum, and internal force along the x, y, and z directions, respectively.

After combining the main transfer equation and geometric equations of all relevant elements, the overall transfer equation of the UFCM system can be obtained:

$$U_{all}Z_{all} = 0,$$ \hspace{1cm} (2)$$

where

$$Z_{all} = [Z_{18,0}^T, Z_{16,0}^T, Z_{17,0}^T, Z_{28,0}^T, Z_{30,0}^T, Z_{37,0}^T, Z_{39,0}^T]^T$$ \hspace{1cm} (3)$$

The equation describing the relations among the state vectors of each boundary end can be easily deduced, as shown in equation (5).

$$Z_{18,0} = T_{16-18}Z_{16,0} + T_{17-18}Z_{17,0} + (T_{27-18} + T_{28-18} + T_{29-18})Z_{28,0} + (T_{30-18} + T_{31-18} + T_{32-18})Z_{30,0} + (T_{37-18} + T_{38-18} + T_{39-18})\hspace{1cm} (5)$$

where

$$
\begin{align*}
T_{16-18} &= \begin{bmatrix}
-1 & T_{16-18} \\
0 & G_{16-15} \\
0 & 0 & G_{12-15} \\
0 & G_{16-13} & 0 & G_{16-12} \\
0 & G_{16-10} & 0 & G_{16-10} \\
0 & 0 & 0 & G_{16-10} \\
0 & G_{16-2} & 0 & G_{16-2} \\
0 & G_{16-2} & 0 & G_{16-2} \\
0 & G_{16-2} & 0 & G_{16-2} \\
0 & G_{16-2} & 0 & G_{16-2} \\
1 & 0 & 0 & 0
\end{bmatrix} \\
T_{17-18} &= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix} \\
T_{27-18} &= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0
\end{bmatrix} \\
T_{28-18} &= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0
\end{bmatrix} \\
T_{30-18} &= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0
\end{bmatrix} \\
T_{37-18} &= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0
\end{bmatrix} \\
T_{39-18} &= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0
\end{bmatrix}
\end{align*}$$

$$
\begin{align*}
T_{5-18} &= \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix} \\
T_{10-18} &= \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix} \\
T_{10-18} &= \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\end{align*}$$

$$
\begin{align*}
T_{10-18} &= \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\end{align*}$$
Taking the example of body element 2, the geometric equations can be deduced

\[
\begin{align*}
G_{16-2} &= -H_{2,1} U_{21} U_{29} U_{10,4} U_{38} U_{13,4} U_{33} U_{14} U_{34} U_{15,4} U_{18} U_{16} \\
G_{17-2} &= -H_{2,1} U_{21} U_{29} U_{10,4} U_{38} U_{13,4} U_{33} U_{14} U_{34} U_{15,4} U_{17} \\
G_{37-2} &= -H_{2,1} U_{21} U_{29} U_{10,4} U_{38} U_{13,4} U_{33} U_{12} U_{35} U_{11} U_{37} \\
G_{39-2} &= -H_{2,1} U_{21} U_{29} U_{10,4} U_{38} U_{13,4} U_{33} U_{14} U_{34} U_{15,4} U_{39} \\
G_{10-2,1} &= -H_{2,1} U_{21} U_{49} U_{10,4} \\
G_{10-2,1} &= -H_{2,1} U_{21} U_{49} U_{10,4} \\
G_{27-2} &= H_{2,4} U_{23} U_{6} U_{25} U_{8} U_{27} \\
G_{28-2} &= H_{2,4} U_{24} U_{7} U_{26} U_{9} U_{28} \\
G_{30-2,4} &= H_{2,4} U_{22} U_{5} U_{30} \\
G_{3-2,1} &= H_{2,4} U_{20} U_{1,4} \\
G_{3-2,1} &= H_{2,4} U_{20} U_{1,4} \\
\end{align*}
\]

(7)

where

\[
\begin{align*}
G_{16-2} &= \frac{Z_{16,0} + G_{17-2} Z_{17,0} + (G_{10-2,1} + G_{10-2,1}) Z_{18,0} + (G_{17-2} + G_{10-2,1}) Z_{17,0} + (G_{19-2} + G_{10-2,1}) Z_{19,0} = 0} \\
G_{16-2} &= \frac{Z_{16,0} + G_{17-2} Z_{17,0} + G_{10-2,1} Z_{18,0} + (G_{17-2} + G_{10-2,1}) Z_{17,0} + (G_{19-2} + G_{10-2,1}) Z_{19,0} = 0} \\
G_{16-2} &= \frac{Z_{16,0} + G_{17-2} Z_{17,0} + G_{10-2,1} Z_{18,0} + (G_{17-2} + G_{10-2,1}) Z_{17,0} + (G_{19-2} + G_{10-2,1}) Z_{19,0} = 0} \\
G_{16-2} &= \frac{Z_{16,0} + G_{17-2} Z_{17,0} + G_{28-2} Z_{28,0} + G_{10-2,1} Z_{18,0} + (G_{17-2} + G_{10-2,1}) Z_{17,0} + (G_{19-2} + G_{10-2,1}) Z_{19,0} = 0} \\
G_{16-2} &= \frac{Z_{16,0} + G_{17-2} Z_{17,0} + G_{28-2} Z_{28,0} + G_{10-2,1} Z_{18,0} + (G_{17-2} + G_{10-2,1}) Z_{17,0} + (G_{19-2} + G_{10-2,1}) Z_{19,0} = 0} \\
G_{16-2} &= \frac{Z_{16,0} + G_{17-2} Z_{17,0} + G_{28-2} Z_{28,0} + G_{10-2,1} Z_{18,0} + (G_{17-2} + G_{10-2,1}) Z_{17,0} + (G_{19-2} + G_{10-2,1}) Z_{19,0} = 0} \\
\end{align*}
\]

The transfer matrices of rigid bodies, beams, and elastic hinges and the detailed process of deduction can be seen in [28].

The state vectors are defined in the form of equation (1). Specifically, \( Z_{18,0} \) is a fixed boundary; and its displacement and angle are zero. The displacement and angle of boundary ends \( Z_{16,0} \) and \( Z_{17,0} \) are unknown, while the force and moment are zero.

After cutting the closed-loops, new boundaries with equivalent state vectors are generated, i.e. \( Z_{27,0\varepsilon} \), \( Z_{28,0\varepsilon} \), \( Z_{30,0\varepsilon} \), \( Z_{37,0\varepsilon} \), and \( Z_{39,0\varepsilon} \).

\[
\begin{align*}
Z_{18,0} &= [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \quad (i = 16, 17) \\
Z_{1,0} &= [X, Y, Z, \Theta_x, \Theta_y, \Theta_z, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \quad (i = 27, 28, 30, 37, 39) \\
\end{align*}
\]

Substituting equation (9) into (2), the system’s characteristics equation can be obtained:

\[
\text{det} \mathbf{U}_{\text{all}} = 0. 
\]

(10)

Solving equation (10), natural frequencies \( \omega_k (k = 1, 2, \ldots) \) can be obtained.

2.3. Bending of the revolving cutterhead

Due to the height difference between the centroids of the cutterhead and tool holder in the direction of the \( z \) axis, as the UFCMT starts rotating, the centrifugal force causes the tool holder to produce a moment on the cutterhead. Thus, the cutterhead bends upward during the machining process; and the higher the spindle speed, the more the bending. Although Ding et al. [29] and Lu et al. [30] analyzed the dynamic performances of UFCMT using MSTMM, they did not consider the influence of cutterhead bending.

The FDM is applied to calculate the elastic deformation of the cutterhead. The cutterhead is regarded as a multistep beam composed of \( n \) sections with uniform cross-section and equal length, as shown in figure 5.

For the section \( i,j = 0, 1, 2, 3 \), the displacement of the output end in the \( z \) axis is described as \( z_{i,j} \). The displacements of the adjacent sections satisfy equation (11)

\[
z_{i,j+1} - 2z_{i,j} + z_{i,j-1} = d^2 \frac{M_j}{E J},
\]

where \( M_j \) and \( E J \) are moment and flexural rigidity on the \( j \)th section, and \( d \) is the length of each section.
Assuming the slope of the deflection at the section $i_0$ equals zero, then

$$\left( \frac{dz}{dy} \right)_0 \approx \frac{z_{i_0} - z_{i_0}}{2d} = 0. \quad (12)$$

By iterative computation, the displacements of each section can be obtained.

2.4. System response

The UFCMT multibody system is composed of body elements, such as rigid bodies, beams, and others, connected by various hinges. Meanwhile, as the damping forces are included in the external forces, the dynamic equation of any body

Figure 4. Vibration characteristics and dynamic response procedure.

Figure 5. Partition of the cutterhead.
element \( i \) can be written in the matrix form

\[
M_i v_i + K_i v_i = f_i,
\]

where \( M_i \) and \( K_i \) are parameter matrices of mass and stiffness, \( v_i \) is a column matrix composed of displacements and angular displacements indicating the motion state of the body element, and \( f_i \) is the column matrix of external forces and external torques acting on the body element. The subscript \( t \) denotes differentiation with respect to time.

For a rigid body \( i \) vibrating in space with \( N \) input and \( L \) output ends, the motion can be described by linear displacements \( r_i \) and angular displacements \( \theta_i \) of the first input end \( I_1 \). The motion equation of the rigid body is

\[
m_i \ddot{r}_i - m_i \dot{I}_{hC} \dot{\theta}_i - \sum_{n=1}^{N} q_{k,i} - \sum_{l=1}^{L} q_{O,i} = F_i.
\]

The rotation equation of the rigid body rotating about \( I_1 \) is

\[
m_i \dot{I}_{hC} \dot{F}_i + J_{hC} \dot{\theta}_i + \sum_{n=1}^{N} m_{k,i} - \sum_{l=1}^{L} m_{O,l} - \sum_{n=2}^{N} \dot{I}_{k,n} \dot{q}_{k,i} - \sum_{l=1}^{L} \dot{I}_{O,l} \dot{q}_{O,i} = m_i \ddot{I}_{hD} F_i,
\]

where \( m_i \) and \( F_i \) are external principal moment and force acting on the rigid body. The internal force and moment of the input end and output end are \( q_{k,i} \), \( q_{O,i} \), \( m_{k,i} \), and \( m_{O,l} \). The coordinate matrices of the center of mass, output point, input points, expect the first input point, and the force acting point relative to the first input point \( I_1 \) are \( I_{hC,i} \), \( I_{O,i} \), \( I_{k,i} \), \( I_{l,i} \), and \( I_{D,i} \), respectively. The moment of inertia matrix of the rigid body with respect to point \( I_1 \) is \( J_{h,i} \). The simplified center of external force is \( D \).

For a beam undergoing transversal, longitudinal, and torsional vibrations, the dynamic equations are

\[
\begin{align*}
(EI)_i \frac{\partial^4 x}{\partial z^4} + m_i \frac{\partial^2 x}{\partial t^2} &= f_x(z_1,t) - \frac{\partial}{\partial z_1} m_x(z_1,t) & (0 \leq z_1 \leq l) \\
(EI)_i \frac{\partial^4 y}{\partial z^4} + m_i \frac{\partial^2 y}{\partial t^2} &= f_y(z_1,t) - \frac{\partial}{\partial z_1} m_y(z_1,t) & (0 \leq z_1 \leq l) \\
\dot{m}_i \frac{\partial^2 z}{\partial t^2} - (EA)_i \frac{\partial^2 z}{\partial z_1^2} &= f_z(z_1,t) & (0 \leq z_1 \leq l) \\
\rho_i \frac{\partial^2 \theta}{\partial z^2} - (GJ)_i \frac{\partial^2 \theta}{\partial z_1^2} &= m_z(z_1,t) & (0 \leq z_1 \leq l)
\end{align*}
\]

where \( x(z_1,t) \), \( y(z_1,t) \), and \( z(z_1,t) \) denote the transversal and longitudinal displacements of any point on the beam relative to its equilibrium position, respectively. The torsion angle of an arbitrary cross-section relative to its equilibrium position is...
\( \theta_\xi (z_1, t) \). The external distributional force and moment in the \( x, y, \) and \( z \) directions are denoted by \( f_x (z_1, t), f_y (z_1, t), f_z (z_1, t), m_x (z_1, t), m_y (z_1, t), \) and \( m_z (z_1, t) \), respectively.

Arranging the body dynamic equations of each element, the body dynamic equation of the system can be constituted

\[
Mv + K\ddot{v} = f,
\]

where

\[
M = \begin{bmatrix}
M_{B_1} & M_{B_2} & \cdots & M_{B_n}
\end{bmatrix},
\]

\[
K = \begin{bmatrix}
K_{B_1} & K_{B_2} & \cdots & K_{B_n}
\end{bmatrix},
\]

\[
v = \begin{bmatrix}
v_{B_1} \\
v_{B_2} \\
\vdots \\
v_{B_n}
\end{bmatrix},
\]

\[
f = \begin{bmatrix}
f_{B_1} \\
f_{B_2} \\
\vdots \\
f_{B_n}
\end{bmatrix}.
\]

The subscripts \( B_1, B_2, \ldots \) and \( B_n \) are the sequence numbers of the body elements.

Applying modal technology, assume that

\[
v = \sum V^k q^k(t),
\]

where \( V^k = [V_{B_1}^k, V_{B_2}^k, \ldots, V_{B_n}^k]^T \) \( (k = 1, 2, \ldots) \).
Table 1. Natural frequencies and mode shapes.

| Order | Modal test (Hz) | MSTMM (Hz) | Relative error (%) | Mode shape |
|-------|-----------------|------------|--------------------|------------|
| 1     | 7.45            | 7.4        | −0.67              | The rotation of the UFCMT and foundation around the z axis in the same direction |
| 2     | 9.99            | 10.00      | 0.10               | The movement of the UFCMT and foundation along the z axis in the same direction |
| 3     | 42.72           | 42.83      | 0.26               | The wiggle of the UFCMT and foundation around the x axis in the opposite direction |
| 4     | 53.56           | 51.60      | −3.66              | The wiggle of the UFCMT and foundation around the y axis in the opposite direction |
| 5     | 70.98           | 71.98      | 1.41               | The movement of the UFCMT and foundation along the z axis in the opposite direction |
| 6     | 100.09          | 101.07     | 0.98               | The rotation of the cutterhead and lathe bed in the same direction |
| 7     | 106.75          | 110.03     | 3.07               | The rotation of the UFCMT around the x axis |
| 8     | 180.66          | 176.36     | −0.03              | The torsion of the cutterhead and cross-member around the x axis in the same direction |
| 9     | 222.48          | 224.1      | 0.73               | The torsion of the cross-member to lathe bed around the z axis in the opposite direction |
| 10    | 261.92          | 253.36     | −1.46              | The movement of the cutterhead along the z axis |
| 11    | 366.74          | 370.12     | 0.92               | The vibration of the cutterhead and machine tool along the z axis in the opposite direction |
| 12    | 442.49          | 449.81     | 1.65               | The vibration of the cutterhead and cross-member along the z axis in the opposite direction |
| 13    | 582.37          | 582.66     | 0.05               | The transformation of the cutterhead |
| 14    | 690.48          | 687.79     | −0.39              | The transformation of the sliding table, the torsion of lathe bed |
| 15    | 808.58          | 808.09     | −0.06              | The vibration of the tool holder and sliding table along the z axis in the same direction |
| 16    | 961.27          | 946.69     | −1.52              | The vibration of the tool holder along the z axis |
The augmented eigenvectors $V^k$ include the modal coordinates of displacements and angular displacements of each discrete element and the mode shape of each continuous element corresponding to the $k$th eigenfrequency $\omega_k$ of the machine tool system.

Substituting equation (19) into (17) yields

$$
\sum_{k=1}^{\infty} MV^k \ddot{q}^k(t) + \sum_{k=1}^{\infty} KV^k q^k(t) = f.
$$

Taking the inner product for both sides of equation (20) using augmented eigenvectors $V^p$ ($p = 1, 2, \ldots, n$) and the orthogonality of the augmented eigenvectors,

$$
\langle MV^k, V^p \rangle = \delta_{k,p} M_p, \quad \langle KV^k, V^p \rangle = \delta_{k,p} K_p.
$$

The generalized coordinate equations of the system can be obtained:

$$
\ddot{\bar{q}}^p(t) + \omega_p^2 \bar{q}^p(t) = \frac{\langle f, V^p \rangle}{M_p} \quad (p = 1, 2, \ldots, n).
$$

Finally, according to the initial condition $\bar{v}(x,0)$ and $\bar{v}_t(x,0)$, the solution’s generalized coordinates $\bar{q}^p(t)$ are easily determined, and the dynamics response $\bar{v}$ can be computed.

### 3. Stiffness analysis of aerostatic bearings

#### 3.1. Geometric model of aerostatic bearings

Figure 6 illustrates the configuration of a vertical aerostatic bearings system in the UFCMT, which consists of a journal bearing and a thrust bearing. Externally pressurized gas supplied at gas inlets, flows through the gas channel and cavity and then forms the bearings’ gas film of journal and thrust, which maintains the rotational and translational motion of bearings.

#### 3.2. Governing equations of fluid field

For compressible Newtonian fluid, the pressure distribution in the fluid domain is governed by N–S equations, assuming that the fluid domain is isothermal during the calculation process. The mass and momentum conversation equations in the $x$, $y$, and $z$ orientation are depicted as follows:

$$
\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{u}) = 0
$$

Figure 12. Power spectrum density functions.

Figure 13. Welch method procedure.
where
\[
\begin{align*}
\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) &= -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + F_x, \\
\frac{\partial (\rho \mathbf{u} \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{v} \mathbf{v}) &= -\frac{\partial p}{\partial y} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z} + F_y, \\
\frac{\partial (\rho \mathbf{u} \mathbf{w})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{w} \mathbf{w}) &= -\frac{\partial p}{\partial z} + \frac{\partial \tau_{zz}}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + F_z.
\end{align*}
\]

(24)}

where $t$ is time; $\mathbf{u}$ is the velocity vector; $u$, $v$, and $w$ are the velocity components in the three coordinate orientations; $p$ is pressure; $F_x$, $F_y$, and $F_z$ are the body force in the three coordinate orientations; $\tau$ is viscous stress; and $\lambda = -2/3$.

### 3.3 Dynamic mesh technique

In this paper, the ratio of length to thickness of the gas film is more than $1 \times 10^4$, which leads to grid distortion and even the generation of negative grids during the meshing and calculating process. In order to improve the quality of the mesh, the multiblock method has been applied to generate the flow field topology structure in the ANSYS® ICEM CFD® software, hence sophisticated grid division of a large-span scale flow field is achieved. The pressure distribution at the end of the inlet is complicated, the pressure drop is large, and O-block is an effective method of encrypting the mesh of a cylindrical flow field. Therefore, grid encryption has been performed around the inlet hole, and the O-block has been created at the gas inlet. Figure 7 illustrates the mesh model and boundary conditions of aerostatic bearings in the numerical calculation process.

The DMT has been used to calculate the radial and axial stiffness of journal and thrust bearing in this paper. By utilizing the user defined function microinstruction DEFINE_GRID_MOTION in ANSYS® Fluent® to define the translational motion in $x$ and $z$ orientations of the rotational wall, the load capacity and stiffness of bearings are obtained, and the calculation efficiency has also been improved. The diffusion smooth method has been used in DMT to update the mesh deformation, which is expressed as the following equation:

\[
\nabla \cdot (\gamma \nabla \mathbf{u}_m) = 0,
\]

(26)

where
\[
\gamma = \frac{1}{\rho_0},
\]

(27)

where $\mathbf{u}_m$ is the mesh deformation velocity, $\gamma$ is the diffusion coefficient which controls the influence of boundary mesh motion on internal mesh motion, and $l$ is the orthogonal distance from the boundary. In this paper, the value of $\alpha$ is set as zero so that the boundary mesh motion guarantees uniform diffusion.

The position of mesh nodes is updated as follows:

\[
\mathbf{x}^{i+1} = \mathbf{x}^i + \mathbf{u}_m \Delta t,
\]

(28)

where $\mathbf{x}^{i+1}$ is the position of updated mesh nodes, $\mathbf{x}^i$ is the position of mesh nodes before update, and $\Delta t$ is the time step.

Newton’s law is solved, and the resultant force in the radial and axial directions is obtained, as well as the moment of journal and thrust bearings. According to equations (29) and (30), the linear and angular stiffness of bearings can be
\[ K_h = \frac{\Delta W}{\Delta h} \]  \hspace{1cm} (29)

\[ K_\theta = \frac{\Delta M}{\Delta \theta} \]  \hspace{1cm} (30)

where \( \Delta W \) and \( \Delta h \) are the variation of load capacity and thickness of the gas film, respectively; and \( \Delta M \) and \( \Delta \theta \) are the variation of deflection moment and deflection angle, respectively.

3.4. Variation of bearings stiffness

In order to calculate the vibration response of the UFCMT, the stiffness of the journal and thrust bearings must be acquired first. Since the cutting force of vertical bearings is only a few Newtons, the amount of radial eccentricity is extremely small, so the spindle and journal restrictor are almost concentric during the calculation. When the gas film thickness of the journal bearings is 15 \( \mu \text{m} \), the sum of the upper and lower thrust film thickness is 30 \( \mu \text{m} \), the inlet pressure is 0.74 MPa, the rotational speed of the bearings (for the purposes of this paper) is 280 rpm, and the total weight of
Figure 19. Acceleration response of tool tip: (a) without vibration isolation and (b) with vibration isolation.

Table 4. RMS and maximum amplitudes.

|                         | RMS (m s^{-2}) | Maximum (m s^{-2}) |
|-------------------------|----------------|--------------------|
| Without vibration isolation | 0.0112         | 0.0515             |
| With vibration isolation  | 0.0066         | 0.0350             |

the bearings and cutterhead is 220 kg. The pressure contour of the aerostatic bearings system can be obtained, as shown in figure 8.

The maximum pressure is at the gas inlet, and the gas outlet is approximately equal to atmospheric pressure. Due to the eccentricity of the journal and thrust bearings, the thickness of the journal gas film is uneven, so the pressure is not a periodic distribution. However, eccentricity of thrust only reduces the thickness of the lower gas film, so the pressure of the lower gas film is greater than that of the upper gas film. The variations of linear and angular stiffness with respect to the film thickness of the journal and thrust bearings are calculated as shown in figures 9 and 10, respectively.

It can be observed from figure 9 that the linear stiffness of the journal and thrust bearings can be affected by the gas film thickness. The linear stiffness increases rapidly to peak and then decreases slowly with the growth of the film thickness. The journal bearing radial stiffness is maintained at a relatively high level when the film thickness is between 9 and 13 μm, and the maximum radial stiffness of 748.36 N μm^{-1} can be obtained at a journal bearing film thickness of 10 μm. For thrust bearing, a relatively high axial stiffness exists at the sum of the upper and lower thrust film thickness of 10–22 μm. The maximum axial stiffness of 2336.04 N μm^{-1} can be obtained at the sum of the upper and lower thrust film thickness of 14 μm.

Figure 10 presents the variation of angular stiffness of the journal and thrust bearings with respect to gas film thickness. With the same results as shown in figure 9, the angular stiffness of the journal and thrust bearings also strongly depends on the film thickness. The angular stiffness rises to a peak and then drops with the growth of film thickness. The angular stiffness decreases sharply near the peak value. The journal and thrust bearings can obtain relatively high angular stiffness at a journal film thickness of 8–11 μm and a sum of the upper and lower thrust film thickness of 10–22 μm, respectively. The maximum angular stiffness of 1.312 × 10^6 N m rad^{-1} and 9.865 × 10^6 N m rad^{-1} of the journal and thrust bearings can be acquired at a journal film thickness of 9 μm and a thrust film thickness of 14 μm, respectively.

4. Dynamic characteristics analysis of the UFCMT

To validate the dynamic model of the UFCMT, a modal test was conducted. By exerting the excitation at the test points with an impact hammer, as shown in figure 11, the vibration response of each point was measured in sequence, which was one column of the frequency response function matrix. The modal parameters of the UFCMT were obtained by using the curve fitting method. In this test, 364 test points were arranged to clearly describe the overall shape of the UFCMT, the analyzing frequency f_n was from 0 to 2048 Hz, the sample length ΔT was 2 s, and the frequency resolution Δf was 0.5 Hz.

Considering the lumped average curve of amplitude-frequency in three directions, the natural frequency and mode shape were selected and are shown in table 1. Meanwhile, the natural frequencies ω_k (k = 1, 2, ... ) obtained by MSTMM were compared with the experimental data. The maximum relative error was 3.66%. Moreover, the frequencies and mode shapes were all in good agreement, which indicates the dynamic model of the UFCMT is appropriate.

5. Analysis and discussion

Due to their distinct traits, namely high precision, high resolution, and high sensitivity, precision machine tools, such as the UFCMT, are able to machine components that have surface roughness in nonmetric levels. Therefore, even the interference of ambient vibration affects dynamic response of the machine tool. The vibration-sensitive frequency of ultra-precision machining equipment ranges from 1 to 150 Hz, and the acceleration is generally below 2 × 10^{-2} m s^{-2}. To provide optimum performance, foundation isolation is critical, even in the toughest applications and the most demanding circumstances. Considering the extreme requirements for machining precision, technical isolation foundation was
designed to avoid disturbance from the surrounding environment.

The fundamental requirement for designing a functioning foundation isolation system is to establish the proper relationship between the forcing frequency and the natural frequency of the isolation system, and the objective is to ensure that the natural frequency is lower than the forcing frequency. The existing isolation foundation contains several procedures for achieving foundation isolation. An inertia block isolation system is the simplest, most practical approach with a natural frequency distributed in 6–15 Hz. It consists of a concrete inertia block sized to provide adequate support for the UFCMT being installed. It also supplies mass to give needed damping to the UFCMT system being supported by a series of foundation isolation pad materials. Meanwhile, Regufoam® isolation materials with a natural frequency range of 6–15 Hz were applied as vibration isolating structural elements to ensure maximum performance. These installations provide a cost-effective, excellent approach to preventing the transmission of structure-born noise and vibration.

5.1. Power spectrum of ambient vibration

Due to the diversity of ambient vibration sources, the dynamic excitation characteristics of the actual system present complex randomness; and it is difficult to study those using accurate analytical methods. Therefore, statistical analysis becomes a more reasonable option, specifically the power spectrum density estimation method. Common methods of obtaining the power spectrum density function include the periodogram method, autocorrelation function method, Burg method, and Welch periodic average method. The experimental ambient vibration signal is processed by these four methods and is shown in figure 12.

After comparing the power spectrum density function curves in figure 12 using the Welch method, the peak values are clearest, and the burr is minimal. The Welch method is applicable to processing a smooth signal, and the ambient vibration signal is precisely the smooth one. Therefore, the Welch method was selected, and the solution procedure is shown in figure 13.

The Welch method divides the signal sequences \( x(n) \) with \( N \) samples into overlapping data sections, and each section has \( M \) samples. The section of data can be expressed as

\[
u(n) = x(n + iD), \quad 0 \leq n \leq M - 1, \quad 0 \leq i \leq K - 1,
\]

where \( K \) is the number of sections, and \( iD \) is the origin of the \( i \)th section. The overlap degree between data sections is defined as \( D/M \), e.g., as \( D = M/2 \), the overlap degree is 1/2.

Applying an appropriate window function \( w(n) \) to ensure the non-negativity of the spectral estimation, the spectral estimation of each section is given by

\[
\hat{S}_w(w) = \frac{1}{MU} \left| \sum_{n=0}^{M-1} u_i(n)w(n)e^{-j\omega n} \right|^2,
\]

where \( U = \frac{1}{M} \sum_{n=0}^{M-1} w^2(n) \) is the normalization factor to guarantee that the final power spectrum estimation is unbiased. Finally, Welch spectral estimation can be obtained by averaging the spectral estimation of each section

\[
\hat{S}_w(w) = \frac{1}{KU} \sum_{i=1}^{K} \hat{S}_w(w).
\]

The original acceleration signal of ambient vibration is shown in figure 14(a). The simulated ambient vibration acceleration signal can be obtained by the trigonometric series method, as in equation (34) and shown in figure 14(b).

\[
x(t) = \sum_{k=1}^{N} a_k \cos(\omega_k t + \phi_k),
\]

where \( \phi_k \) is the independent random phase angle evenly distributed in \([0, 2\pi]\), and \( \omega_k \) is the frequency obtained by equation (35)

\[
\omega_k = \omega_i + \left( k - \frac{1}{2} \right) \Delta \omega, \quad k = 1, \ldots, N,
\]

where \( \Delta \omega = (\omega_{i+1} - \omega_i)/N \) is the interval of frequency, and \( [\omega_i, \omega_{i+1}] \) is the frequency range in which the power spectral density function \( S_k(\omega) \) has salient value. The amplitude \( a_k \) can be obtained by equation (36)

\[
a_k = \sqrt{2S_k(\omega_k) \Delta \omega}, \quad k = 1, \ldots, N.
\]

5.2. Vibration test of tool tip and analysis

The ambient vibration acceleration signal was applied to the foundation, i.e. Body 1 in the dynamic model, as an external excitation. The dynamic response was calculated by solving the overall transfer equation and dynamics equation deduced in section 2. The vibration test to measure the vibration acceleration response of the tool tip was set up as shown in figure 15. The acceleration signal was detected by acceleration sensor PCB 355B04, which was glued near the tool tip. The data was collected and stored by the micro dynamic data acquisition system DH 5916, which was fixed at the bottom of the cutterhead.

The acceleration signal of ambient vibration, measured under start-up without rotating, was added into the dynamic model. Then the acceleration response of the tool tip could be simulated. The simulated results were compared with the experimental vibration response of the tool tip both in the time domain and frequency domain, as shown in figures 16 and 17.

As shown in figure 16, the simulated acceleration of the tool tip is in good agreement with the experimental acceleration. The root mean square (RMS) and maximum amplitudes are shown in table 2. The simulation values and experimental results have negligible difference.

Fourier transform was applied to the vibration acceleration signal, and the corresponding frequency spectrums are shown in figure 17. The main frequencies of the experimental
acceleration and simulated acceleration were in good agreement. Furthermore, these frequencies also matched well with natural frequencies from the modal test, as shown in table 3.

5.3. Effect of ambient vibration on the UFCMT

In figure 18, the ambient vibration monitoring point that lies outside the isolation foundation is marked as P1, and the monitoring point on the foundation is P2. Using these two sensors, the ambient vibration signals with and without the vibration isolation were measured.

Using the power spectrum density estimation method, the measured vibration acceleration signals of these two points can be simulated into statistical signals. As the external excitation, the vibration acceleration signals were added into the dynamic model established in section 2. Then the acceleration response of the tool tip was simulated (see figure 19).

From figure 19, it is apparent that the acceleration response of the tool tip decreased through the vibration isolation. The RMS and maximum amplitudes of acceleration are shown in table 4. The RMS decreased by 41.1%, and the maximum declined by 32.0%.

As the rotating speed of the spindle was 280 rpm and the feed rate of the sliding table was 15 mm min$^{-1}$, the machining process on the workpiece with a diameter of 50 mm was simulated. Figure 20(a) shows the whole machining process after obtaining the displacement response of the tool tip relative to the workpiece, and the total machining time was 200 s. The time for one revolution of the spindle was 0.214 s, and figure 20(b) shows the displacement in one revolution. Figure 20(c) represents the cutting process in which the tool tip came into contact with the workpiece.

With the vibration isolation foundation, the displacement response of the tool tip was smaller compared to the response without vibration isolation (see figure 20). Selecting the cutting process, the maximum displacements were 140.47 and 123.52 nm. Due to the role of isolation foundation, the displacement of the tool tip relative to the workpiece was reduced by 12.1%.

After obtaining the displacement of the tool tip, the simulated machined surface was acquired by using the surface topography model and considering the interference phenomenon of the cutting profile and geometry of the tool tip, as shown in figure 21.

The corresponding peak-to-valley (PV) values of the simulated machined surfaces were 302.68 and 289.51 nm, and the PV value decreased by 4.4%. Therefore, ambient vibration had great influence on the response of the tool tip, which would be the crucial factor in machining quality.
6. Conclusion

(1) The dynamic model of the UFCMT multi-rigid-flexible system was established based on MSTMM. The bending of the cutterhead due to rotation was considered and calculated by FDM. The characteristic equation of the system was deduced theoretically, and the simulated vibration characteristics were consistent with the modal test results. The effective simulation of the dynamic response was realized by solving the dynamic equations, and the simulation results were validated by the vibration test.

(2) The mesh model of the large-span scale flow field was established, and the DMT was proposed to calculate the linear and angular stiffness of the journal and thrust bearings at different film thicknesses. Both linear and angular stiffness increased to the peak, and then decreased with the growth of the film thickness. The most suitable radial and angular stiffness of the journal bearing was obtained at a film thickness of 9–11 μm. When the sum of the upper and lower thrust film thickness was 12–18 μm, the optimum axial and angular stiffness of the thrust bearing was also acquired. The angular stiffness of the thrust bearing was close to ten times that of the angular stiffness of the spindle bearing, so the main anti-deflection capacity came from thrust bearing.

(3) Due to the diversity of ambient vibration sources, the short-time measured signal did not reflect the dynamic excitation characteristics with complex randomness. Thus, the power spectrum density estimation method was adopted for statistical analysis. After comparing the four methods of applying the power spectrum density function calculation, the Welch method was chosen to simulate the equivalent ambient vibration excitation by processing the long-time measured ambient vibration signal.

(4) Using the power spectrum density estimation method, the experimental ambient vibration acceleration signals with complex randomness were transformed into statistical signals, which were further applied to the dynamic model. Comparing the dynamic responses of the tool tip with and without vibration isolation, the simulated results indicated that the PV value declined by 4.4% with the vibration isolation, which revealed the great influence of ambient vibration on the dynamics response and machining quality.

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References

[1] Gurney J P and Tobias S A 1961 A graphical method for the determination of the dynamic stability of machine tools Int. J. Mach. Tool Des. Res. 1 148–56
[2] Hijink J A W and van der Wolf A C H V D 1973 Analysis of a milling machine: computed results versus experimental data Proc 14th Int. MTDR Conf. pp 553–8
[3] Zatarain M, Lejardi E and Egana F 1998 Modular synthesis of machine tools CIRP Ann. 47 333–6
[4] Jiang T and Chirehdast M 1997 A systems approach to structural topology optimization: designing optimal connections Trans. ASME, J. Mech. Des. 119 40–7
[5] Liang Y, Chen W, Sun Y, Chen G, Wang T and Sun Y 2012 Dynamic design approach of an ultra-precision machine tool used for optical parts machining Proc. Inst. Mech. Eng. B 226 1930–6
[6] Liang Y, Chen W, Sun Y, Lu X, Lu L and Liu H 2014 A mechanical structure-based design method and its implementation on a fly-cutting machine tool design Int. J. Mach. Tool Manuf. 70 1915–21
[7] Chen W, Liang Y, Luo X, Sun Y and Wang H 2014 Multi-scale surface simulation of the KDP crystal fly cutting machining Int. J. Mach. Tool Manuf. 73 289–97
[8] Chen W, Liu H, Sun Y, Yang K and Zhang J 2016 A novel simulation method for interaction of machining process and machine tool structure Int. J. Mach. Tool Manuf. 88 3467–74

[9] Gao Q, Zhao H, Lu L, Chen W and Zhang F 2020 Investigation on the formation mechanism and controlling method of machined surface topography of ultra-precision flycutting machining Int. J. Adv. Manuf. Technol. 106 3311–20

[10] Wang L, Liu H, Yang L, Zhang J, Zhao W and Lu B 2015 The effect of axis coupling on machine tool dynamics determined by tool deviation Int. J. Mach. Tool Manuf. 88 71–81

[11] Sato R, Noguchi S, Hokazono T, Nishida I and Shirase K 2020 Time domain coupled simulation of machine tool dynamics and cutting forces considering the influences of nonlinear friction characteristics and process damping Precis. Eng. 61 103–9

[12] Rui X, He B, Lu Y, Lu W and Wang G 2005 Discrete time transfer matrix method for multibody system dynamics Multibody Syst. Dyn. 14 317–44

[13] Rui X, Wang G, Lu Y and Yun L 2008 Transfer matrix method for linear multibody system Multibody Syst. Dyn. 19 179–207

[14] Rui X, Bestle D, Zhang J and Zhou Q 2016 A new version of transfer matrix method for multibody systems Multibody Syst. Dyn. 38 137–56

[15] Zhang X, Sørensen R, Iversen M R and Li H 2018 Computationally efficient dynamic modeling of robot manipulators with multiple flexible-links using acceleration-based discrete time transfer matrix method Robot. Comput.-Integr. Manuf. 49 181–93

[16] Abbas L K, Zhou Q, Hendy H and Rui X 2015 Transfer matrix method for determination of the natural vibration characteristics of elastically coupled launch vehicle boosters Acta Mech. Sin. 31 570–80

[17] Wang X, Rui X, Yang F and Zhou Q 2018 Launch dynamics modeling and simulation of vehicular missile system J. Guid. Control Dyn. 41 1–10

[18] Bhat N, Kumar S, Tan W, Narasimhan R and Low T C 2012 Performance of inherently compensated flat pad aerostatic bearings subject to dynamic perturbation forces Precis. Eng. 36 399–407

[19] Cui H, Wang Y, Yang H, Zhou L, Li H, Wang W and Zhao C 2018 Numerical analysis and experimental research on the angular stiffness of aerostatic bearings Tribol. Int. 120 166–78

[20] Fillon M, Dmochowski W and Dadouch A 2007 Numerical study of the sensitivity of tilting pad journal bearing performance characteristics to manufacturing tolerances: steady-state analysis Tribol. Trans. 50 387–400

[21] Lo C Y, Wang C C and Lee Y H 2005 Performance analysis of high-speed spindle aerostatic bearings Tribol. Int. 38 5–14

[22] Belforte G, Raparelli T, Viktorov V and Trivella A 2007 Discharge coefficients of orifice-type restrictor for aerostatic bearings Tribol. Int. 40 512–21

[23] Eleshaky M E 2009 CFD investigation of pressure depressions in aerostatic circular thrust bearings Tribol. Int. 42 1108–17

[24] Li Y, Yin Y, Yang H, Liu X, Mo J and Cui H 2017 Modeling for optimization of circular flat pad aerostatic bearing with a single central orifice-type restrictor based on CFD simulation Tribol. Int. 109 206–16

[25] Chen X, Zhu J and Chen H 2013 Dynamic characteristics of ultra-precision aerostatic bearings Adv. Manuf. 1 82–6

[26] Zhuang H, Ding J, Chen P, Chang Y, Zeng X, Yang H, Liu X and Wei W 2019 Numerical study on static and dynamic performances of a double-pad annular inherently compensated aerostatic thrust bearing J. Tribol.—Trans. ASME 141 (051701)

[27] Dal A and Karaçay T 2017 Effects of angular misalignment on the performance of rotor-bearing systems supported by externally pressurized air bearing Tribol. Int. 111 276–88

[28] Rui X, Yun L, Lu Y and Wang G 2008 Transfer Matrix Method of Multibody System and Its Applications (Beijing: Science Press) (in Chinese)

[29] Ding Y, Rui X, Lu H, Chang Y and Chen Y 2020 Research on the dynamic characteristics of the ultra-precision fly cutting machine tool and its influence on the mid-frequency waviness of the surface Int. J. Adv. Manuf. Technol. 106 441–54

[30] Lu H, Ding Y, Chang Y, Chen G and Rui X 2020 Dynamics modelling and simulating of ultra-precision fly-cutting machine tool Int. J. Precis. Eng. Manuf. 21 189–202