How big is the source that produces quark gluon plasma in heavy ion collisions?

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We study, for the first time, the spatial extension of the "source" that produces quark gluon plasma (QGP) in ultra relativistic heavy ion collisions (URHIC). The longitudinal dimension is studied as a function of time as the system evolves. The source size is found to exhibit a novel non-classical feature.

I. INTRODUCTION

The ultimate aim of studying ultra relativistic heavy ion collisions (URHIC) is not merely to establish the production of Quark Gluon Plasma (QGP) - by looking at various signals - but to get a complete picture of the space-time evolution of QGP. In short, one hopes to describe the evolution of this unique deconfined state of hadronic matter in all its stages - production, equilibration and hadronisation. Ambitious that this might seem, it has indeed proved impossible to separate the study of signals such as J/Ψ suppression or strangeness enhancement without having an at least approximate idea of the production and evolution of QGP.

We have recently proposed a mechanism [1] for producing QGP in URHIC - the soft and the semisoft quarks and gluons that constitute the bulk of the plasma. The mechanism manifests as the source term in the transport equation for the distribution functions for the partons. This source term, which is the rate at which the partons are produced in the extended phase space, has the following features: (i) it is defined in the two particle phase space, (ii) the production rate is non-Markovian in time [3], (iii) the vacuum has a dynamical role to play [2] - it acts both as a source and a sink, (iv) the partons are quasiparticles with a finite life time [4], (v) the rate evaluation takes into account the time scales involved in the production vis-a-vis the time intervals over which the rate is determined, (vi) the phase space dependence does not violate the uncertainty principle, and finally, (vii) the dynamical nature of the colour charge is manifest. Given all these attributes, it is natural to ask whether the source term can throw light on the spatial dimensions of the QGP as it evolves in time. Although a complete answer to this question has to be kept pending until the transport equation is solved in a self consistent manner, the purport of this paper is to show that the source term by itself does have interesting information. We shall illustrate this by looking at the longitudinal dimensions in a generic example.

One promising observational tool for studying dynamically the spatial extension of the fireball is the measurement of HBT correlations [5–7] in two and three pions (and kaons) that are produced in URHIC. In particle physics, this idea was first proposed by Goldhaber et. al. [8] for pp collisions. In the context of QGP, a major impetus for such a study is the hope that a knowledge of the system size would also settle whether the source producing the pions/kaons is nuclear, or its deconfined state. A useful tool that it is, measurements in HBT are still beset with many uncertainties - of extracting and interpreting data. Much of it is due to the fact that the source here is not static; it evolves rapidly, over time scales of several fermis and over length scales of the same order. The reconstruction of the source history from HBT observations is, therefore, not straightforward: it has been pointed out [9,10] that what gets determined is not the true "size" of the system, but only the so called regions of "space time homogeneity". The other complications are the finite life time of the source, inhomogeneous temperature profiles and a strong collective dynamical expansion [9]. At a more basic level, HBT studies have the best utility if the pions are produced from a chaotic source; there is no clinching justification (theoretically) to assume that in the case of URHIC; studies based on the Lund model [11,12] indicate that the sources do possess non-chaoticity. An empirical test to check the chaoticity is to measure the so called λ parameter in two pion correlations. However, λ itself depends on other parameters which are either incompletely or inaccurately known [13].

Apart from these caveats, the extraction of radii involves additional assumptions: the interpretation and parametrisation of HBT data assume a Gaussian profile for the source. Reasonable and natural that it seems, it still needs justification. Next, the (spurious) contributions from the Coulomb interaction have to be subtracted. More importantly, one needs a criterion for "emission instant" and the "duration of emission of the particles" - both of which are ill defined in quantum mechanics [1,10]. The phase space coordinates should also be sufficiently coarsened so as not to violate uncertainty principle [1]. These problems are nontrivial to handle both in experiments and theoretical simulations [10]. An illustrative instance of the present situation is the so called RHIC puzzle [14] which is still not understood properly.

In short, the utility of the HBT analyses gets enhanced if they can be supplemented with an independent theoretical investigation of the space time dynamics of QGP. As mentioned at the beginning, the transport studies pro-
vide the required frame work. We shall show that the
the source term proposed in [1] does indeed give valuable
information on the spatial extension of QGP.

The next section introduces the source term derived
in [1] and the subsequent section discusses the spatial
extension of the source. For the sake of simplicity, we
consider only the longitudinal dimension.

II. THE SOURCE TERM

We refer to [1] for the motivation and details of de-
ivering the source term. To summarise in a nut shell,
the production of QGP takes place via the decay of a
mean chromoelectric field (CEF) that is produced in be-
tween the two nuclei soon after they have collided and
start receding from each other. By energy momentum
conservation, the CEF acquires a space - time dependence,
thanks to which the instability of the QCD vac-
uum may be studied perturbatively. It may be empha-
sised that the existence of CEF is itself a consequence of
the non-perturbative aspects of QCD, Indeed, the chro-
meoelectric field may be considered to be a manifestation
of CEF. The state at any time is then projected on to the
non-perturbative aspects of QCD. Indeed, the chro-
moelectric field may be considered to be a manifestation
of the strings in the colour flux tube model [15], which is
known to provide a natural setting for discussing quark
confinement [16]. The main difference between this work
and the conventional approaches is that the latter do not
take into cognisance the space time dependence of the
CEF in invoking the production mechanism; they employ
Schwinger mechanism [17] which is valid only if the field
is uniform and constant. We make no such assumption
here (see also [18] in this context).

Consider the gluons. The lagrangian for pair produc-
tion may be set up by expanding the gauge potential
$A_\mu^a$ as a sum of the background classical potential
$C_\mu^a$ and its fluctuation $\phi_\mu^a$, which is operator valued. Keeping terms
quadratic in the fluctuations in the Yang - Mills action, we get

$$L_{2g} = -\frac{g}{2} f^{abc} [ (\partial_\mu C_\nu^a - \partial_\nu C_\mu^a) \phi^{ab}_\eta \phi^{bc}_\eta + \cdots ] + O(g^2).$$  

(1)

Taking into account the Wu Yang ambiguity [19], and
the studies of Brown and Weissberger [20], one can ar-
ge that $C_\mu^a$ should have an abelian form, at least for
non vanishing leading order contributions. By a suit-
able gauge choice, we may write $C_\mu^a = \delta^a_\mu,0 \sum_i C_i(t, r) \delta_{a,i}$
where the summation runs only over the diagonal gener-
ators of the gauge group. Note that $C_\mu^a$ generates only
an electric component.

In determining the production rate, as a function of
time, the crucial step is not to evaluate the S-Matrix. In-
stead, we study the time dependent evolution of the state
$|\Psi > (t)\rangle$ in the Fock space using the standard Schwinger-
Dyson expansion for the Unitary operator $U(t, 0)$, with
the boundary condition $|\Psi > (t = 0)\rangle = |vac >$. The in-
stant $t = 0$ is singled out as the moment of the creation
of CEF. The state at any time is then projected on to the
two gluon sector. The mass shell condition is imposed as
a constraint on the physical states.

Denote the two gluon state as $|gg\rangle \equiv |p_1, p_2; s_1, s_2; c_1, c_2 >$, labelled by the momentum, spin and colour quantum numbers respec-
tively. In the leading order, the production amplitude
may be written as

$$< gg | T(t) | 0 \rangle = \frac{g}{(2\pi)^3} \frac{(E_2 - E_1)}{2\sqrt{E_1 E_2}} \varepsilon^{\nu}(p_1) \cdot \varepsilon^\nu(p_2) f^{a_1 a_2}$$

$$C^{0,a}(E_1 + E_2; p_1 + p_2; t)$$

(2)

where $T(t) \equiv U(t, 0) - 1$. Further,

$$C_0^a = \int_0^t dt e^{-i(E_1 + E_2)t} \int d^3 r \xi \exp(i(p_1 + p_2) \cdot r) C^{0,a}(t_1, r)$$

is the incomplete Fourier transform of the gauge field
$E_i$ and the energies carried by the gluons. The cor-
responding expression for the $q\bar{q}$ production is given by

$$< q\bar{q} | T(t) | 0 \rangle = \frac{g}{(2\pi)^3} \frac{m}{\sqrt{E_1 E_2}} C_0^a T_{c_1 c_2 s_1}^{a} (p_1) v_{s_2} (-p_2),$$  

(3)

with $|q\bar{q}\rangle \equiv |p_1, p_2; s_1, s_2; c_1, c_2 >$. $T^a$ are the generators
of the gauge group in the fundamental representation,
while u, v are the usual Dirac spinors.

The probability that a pair is produced any time
during the interval $(0, t)$ is given by $| <q\bar{q}|T(t)|vac>|^2$, $\xi$
standing for either $gg$ or $q\bar{q}$. In [1], rates were extracted by taking the derivative of the probability with respect
to time. Here, we study the probabilities directly.

In order to extract the size parameters, we deviate from
the evaluation performed in [1]. We label the two parton
states by their position coordinates $\vec{r}_1, \vec{r}_2$, which entails
the standard Fourier transform of Eqns. 2 and 3 in the
variables $p_1, p_2$. The magnitude squared of the ampli-
dudes now has the significance that it is the probability
that a pair gets created in the interval $(0, t)$, with the
particles found at $\vec{r}_1, \vec{r}_2$ at the instant $t$. For our pur-
poses we may sum over the spin, colour and one of the
position coordinates. The resulting quantity $P(\vec{r}, t)$ gives the (unnormalised) probability density for the parton
as a function of time. Please note that it is incorrect to
interpret $t$ as the instant at which the parton is created.

We employ the same CEF that was introduced in [1]
study the production rate in phase space. It is given by

$$E_i^a = \delta_{i, z} E_0 (\delta_{a,3} \delta_{a,8}) \exp\{(|z| - t)/t_0\} \theta(t) \theta(t^2 - z^2)$$

This choice is close to the boost invariant configuration
required in the Bjorken scenario [21]. The gauge group
is $SU(3)$, and the field is dependent only on $z, t$. The
probability along the transverse directions is, therefore,
uniform, \(i.e.,\) a gaussian distribution with a width given by the diameter of the nucleus. This dimension remains constant in time. However, the longitudinal extension has to be determined explicitly.

### III. THE LONGITUDINAL EXTENSION

We obtained the production probability density for gluons and quarks in the one-particle configuration space by performing a multi-dimensional fast fourier transform on the momentum space amplitude, and integrating over the unwanted degrees of freedom. The densities are shown in Figs. 1 and 2 as functions of \(z\) for different time instants namely \(t = 1, 2, 5, 10, 20\) in units of \(t_0\). The probability densities are displayed only on the positive \(z\) axis. The negative half is symmetric about \(z = 0\). The range of the probabilities (unnormalised!) is truncated in order to improve the visibility of the important features of the curves. The probability densities at \(z = 0\) are large but finite. These curves shed light on an interesting aspect of the source.

Recall that the CEF has a support in the interval \(z \in (-t, +t)\) at any time \(t\), with the two nuclei at \(|z| = t\) moving with unit speed in opposite directions. We may expect, na"ively, that the particle production should also be restricted within the interval \((-t, t)\). The graphs clearly bely these classical expectations. The required features are most pronounced in Fig. 1, where we show the results for the gluons. Strikingly, the probability density does not terminate at \(z = t\). Instead, it extends beyond, as a broad plateau all the way up to \(z = 2t\), where it falls abruptly, and almost discontinuously. This feature is universal for all the curves, indicating that the point \(z = 2t\) is naturally chosen as the boundary by the system dynamics. In that sense, we do not have to extract any length scale by curve fittings.

The quark results are shown in Fig. 2, again for times ranging from \(t = 1\) to \(t = 20\), in units of \(t_0\). The results are not that vivid at earlier times. However, the features get more pronounced as the system evolves so that the plateau structure is clearly delineated at \(t = 10 \text{fm}\).

The remarkable aspect of these conclusions is that the size of the system is quintessentially non-classical. Indeed, the particles are found in regions where the field identically vanishes. Since the boundary of the CEF located at \(z = \pm t\) is moving with the speed of light, causality forbids the particles produced any time in the interval \((0, t)\) within the range \(z \in (-t, t)\) from reaching the regions outside the interval. One concludes that the particles are indeed produced in the region where CEF vanishes identically! We suspect that this could be a generic property of the class of CEF that we have employed. It is important to note that the plateau position and its extension has a topological nature: the magnitudes change smoothly as the system evolves with the time, but the location of the shoulder is pegged at the value \(|z| = t\). A straightforward conclusion that one draws is that longitudinally, the QGP source is twice as large as the space between the two nuclei. The fireball extends beyond the two nuclei.

![FIG. 1. Unnormalized longitudinal probability density for gluons for times \(t = 1, 5, 10\) and 20.](image1.png)

![FIG. 2. Unnormalized longitudinal probability density for quarks as at \(t = 1, 5, 10\) and 20.](image2.png)

A more quantitative understanding can be obtained by looking at the relative probability for finding the particles in the regions \(|z| < t\) and \(t < |z| < 2t\). The ratio for the gluons varies from a maximum of about 38\% at \(t = 1.0\), to 15\% at \(t = 20\). For the quarks, on the other hand, it varies from 19\% at \(t = 1.0\) to 7\% at \(t = 20\). Clearly, the non-classical features are the most pronounced when the field is varying most rapidly, and gets less important as the field evolves in time. Also, should these results seem surprising, we may recall that the probabilities in the regions where the field is vanishing are, in fact smaller than the probability that an oscillator in its ground state is found in the classically forbidden region. With this perspective, we may conclude that the production in the regions where the field vanishes identically is a quantum
effect; the occurrence of the step at $|z| = 2t$ is perhaps a feature of the class of fields which have a boost invariant nature, or are close to it.

A final question remains as to why the spread in the quarks is less than that of gluons as indicated by the numbers above. The reason for this may be attributed, in part, to the fact that the two gluon amplitude is antisymmetric in the colour and the momentum indices of the two gluons (see Eqn. 2. By Bose symmetry, it is symmetric in the spins). Consequently, the amplitude will be antisymmetric in the position variables $\vec{r}_1, \vec{r}_2$ as well. The quark case on the other hand, has the charge conjugation symmetry which, when employed similarly, leads to a symmetric behaviour in the spatial part which inhibits the spread. In fact, the spread in the momentum probabilities is larger for quarks than for the gluons, as found in [1]. By the uncertainty principle, one may expect the quark size to be smaller than that of the gluons.

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