Fracture of quasi-brittle geomaterials with a circular hole under compression

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Abstract. The new nonlocal fracture criteria are proposed, which are the extension of the average stress criterion and the point stress criterion, and which contain a complex parameter that characterizes the size of the fracture process zone and accounts not only for the material microstructure, but also ductile properties of the material, geometry of the specimen, and its loading conditions. The formulas are obtained for the critical pressure in the problem of tensile fracture initiation in the specimens of geomaterials containing a circular hole and subjected to uniformly or nonuniformly distributed compression. The results of the calculations are in good agreement with the experimental data on failure of gypsum plates with a hole.

1. Introduction
Structurally nonuniform geomaterials (concrete, gypsum, rocks) demonstrate dependence of their strength on the loaded volume (the size effect). This effect is most pronounced under conditions of stress concentration when the effective loaded volume is governed by the stress concentration zone which is smaller than the typical size of a deformable body. In such cases, the critical load is calculated using nonlocal and gradient fracture criteria [1, 2]. The nonlocal fracture criteria use the notion of a pre-fracture zone such stress redistribution takes place there while the bulk material deforms elastically to complete failure. The common property of the nonlocal criteria is the introduction of an intrinsic material length $d_0$, which characterizes the structure of a material. This makes it possible to describe the size effect under stress concentration and, thereby, to expand the range of application of the nonlocal fracture criteria as compared with the conventional criteria. At the same time, the application range of the nonlocal criteria is mostly brittle fracture [13]. The nonlocal criteria are applicable in case of quasi-brittle fracture with formation of the pre-fracture zone $d$ if it size is not far different from $d_0$, i.e. in case that $d \approx d_0 = \text{const}$. This paper discusses expandability of the application range of the nonlocal fracture criteria for the cases of quasi-brittle fracture with developed pre-fracture zone in terms of the problem on tensile fracture initiation in geomaterials with a circular hole under nonuniformly distributed compression.

2. Quasi-brittle fracture criteria
Hereinafter, we understand the quasi-brittle fracture as unexpected propagation of an unstable fracture with formation of a large pre-fracture zone. The pre-fracture zone size $d$ is correlated not with the fracture size as in fracture mechanics but with the typical structural size $d_0$ of a material. In case that
\(d = d_0\), fracture is brittle, when \(d > d_0\), fracture is quasi-brittle and changes into viscous fracture at the limit when \(d \gg d_0\). Stress re-distribution within \(d_0\) is unrelated with plastic (at a microscopic scale) deformation of a material. Plasticity appears when \(d > d_0\) and intensifies with higher ratio of \(d\) to \(d_0\). In this regard, we present \(d\) as:

\[
d = d_0 + \beta L_e, \tag{1}
\]

where \(L_e\) is the size of stress concentration zone; \(\beta\) is the dimensionless characteristic of plasticity of a material. For brittle materials, \(\beta = 0\), for plastic materials, \(\beta \gg 1\). When \(\beta \approx 1\) the material is moderately plastic. The first member in (1) characterizes the structure of the material, the second member is representative of inelastic deformations. Using the proposed approach, some new (modified) nonlocal criteria of fracture are developed.

Out of the nonlocal criteria, the most commonly used criterion is the average stress criterion (ASC): \(\langle \sigma_e \rangle_d < \sigma_0\), where \(\langle \sigma_e \rangle_d\) is the averaged value of equivalent stress at the distance \(d\) along a hazardous cross-section; \(\sigma_0\) is the tensile strength of a material. For brittle materials, \(d = d_0 = \text{const}\). The equivalent stress is determined from the theory of the maximum tension stresses.

Let us consider biaxial loading of a flat specimen with a circular hole having radius \(a\) by the tensile force \(\alpha \sigma\) along the horizontal axis and by the compressive forces \(\sigma\) along the vertical axis of the specimen. The critical stress to initiate tensile fractures at the hole boundary is given by [14]:

\[
\sigma_c = 2\sigma_0\left[1 + \gamma^{-3} + \alpha\left(1 + \gamma^{-1}\right)\left(2 + \gamma^{-2}\right)\right]^{-1}, \tag{2}
\]

where \(\gamma = 1 + d / a\). When \(\gamma = 1\) formula (2) yields the critical stress in accordance with the conventional fracture criterion.

For the case of quasi-brittle fracture, the averaging size is determined from formula (1) with the stress concentration zone size \(L_e = a + \frac{1 + 3\alpha}{5 + 7\alpha}\). Accordingly, the expression of \(\gamma\) takes on form:

\[
\gamma = 1 + \frac{d_0}{a} + \beta\frac{1 + 3\alpha}{5 + 7\alpha}. \tag{3}
\]

Alongside with ASC, another criterion is also widely used—the point stress criterion (PSC). In this criterion, integration is replaced by calculation of an equivalent stress \(\sigma_e\) at a certain point at the distance \(d\) from the point of maximum. The strength criterion is \(\sigma_e(x_0 + d) < \sigma_0\), where \(x_0\) is the coordinate of the maximum equivalent stress point. The parameter \(d\) is assumed as a constant of the material and, broadly speaking, disagreeable with the analogous parameter in the criterion ASC. The critical stress for a specimen with a circular hole under biaxial loading is given by [14]:

\[
\sigma_c = 2\sigma_0\left[-\gamma^{-2} + 3\gamma^{-4}\right] + \alpha\left(2 + \gamma^{-2} + 3\gamma^{-4}\right)\left[\gamma^{-2} + 3\gamma^{-4}\right]^{-1}. \tag{4}
\]

For the quasi-brittle materials, the parameter \(\gamma\) is determined from formula (3).

3. Experimental validation

Applicability of the quasi-brittle fracture criteria is validated using the experimental data on fracture of flat specimens with a hole under uniformly distributed (uniaxial) and nonuniformly distributed compression. In the latter case, compression was applied to a specimen via rigid gaskets inserted between the specimen and the pressing plates. In this case, in the center of the specimen (beyond the
influence zone of the hole), the uniform biaxial stress state is materialized: tension along the horizontal axis and compression along the vertical axis of the specimen. The experimental procedure is described in [14].

3.1 Nonuniform compression

The test specimens 200×200 mm in size and 36 mm thick were made of a gypsum material with hemihydrate plaster content of 80–84% (gypsum 1). Before testing, circular holes with a diameter from 1 to 20 mm were drilled in the specimens. The load \( p \) was applied to the specimens via gaskets 120 mm in size. The brittle fracture mechanism was implemented due to sudden initiation of tensile fractures at the hole boundary and their rapid growth along the axis of compression.

The values of \( \sigma \) and \( \alpha \) were calculated using the finite element method in the center of specimens without holes subjected to loading via preset size gaskets. In case of the gaskets used in the tests, \( \sigma = 0.764 p \) and \( \alpha = 0.187 \). In accordance with formula (2) and with regard to the evaluated \( \sigma \) and \( \alpha \), we write down the expression for critical pressure in a specimen with a circular hole by the criterion ASC:

\[
p_c = 2\chi C_0\left[0.764(1 + \gamma)^{-3} + 0.143\left(1 + \gamma^{-1}\right)^2\gamma^{-2}\right]^{-1},
\]

where \( \chi = \sigma_0 / C_0 \); \( C_0 \) is the ultimate compression strength of the material. The parameter \( \gamma \) is found from formula (3), where \( \alpha = 0.187 \). For quasi-brittle materials with moderate plastic properties, the asymptotic (at \( a \to \infty \)) value of the critical stress \( T_s \approx T_0(1 + \beta / 2) \), where \( T_0 = 0.838\chi C_0 \).

Figure 1 depicts the experimental data on the value of the load at the initiation moment of tensile fractures at the hole boundary depending on the hole diameter \( l \) (points) and the critical pressure calculated from (5) at \( \beta = 0 \) (curve 1) and \( \beta = 0.6 \) (curve 2). The size \( d_0 \) was 1.0 mm. The stress \( T_s \) equals \( T_0 \) in the first case (dashed straight line) and \( T_s = 1.3T_0 \) in the second case (solid straight line).

\[\text{Figure 1. Critical pressure–hole diameter curves for gypsum 1. Calculated by ASC criterion.}\]

Figure 1 displays a sensible size effect, i.e. the influence of the hole diameter on the local strength of the material. As the hole diameter is decreased, the critical pressure grows and reaches the ultimate compression strength; when the hole diameter is increased, the critical pressure asymptotically approaches the stress \( T_s \). Such behavior is smoothly described by the modified average stress criterion with the averaging size \( d \) found from formula (1).

In accordance with (4) and with regard to the evaluated \( \sigma \) and \( \alpha \), the expression for the critical pressure in a specimen with a circular hole by the criterion PSC:
For quasi-brittle materials with moderate plastic properties, $T_s \approx T_0(1 + \beta)$. 

Figure 2 shows the experimental data (points) and the calculated pressure from formula (6) at $\beta = 0$ (curve 1) and $\beta = 0.3$ (curve 2). The size $d_0$ was 0.3 mm. The stress $T_s$ is equal to $T_0$ (dashed straight line) in the first case and $T_s = 1.3T_0$ (solid straight line) in the second case. The test results are smoothly described by the modified point stress criterion with the size $d$ found from formula (1).

\[
p_c = 2\chi C_0\left[0.764(-\gamma^{-2} + 3\gamma^{-4}) + 0.143(2 + \gamma^{-2} + 3\gamma^{-4})\right]^{-1}.
\]  

(6)

3.2 Uniaxial compression

The test specimens were made of a gypsum material with hemihydrates plaster content of 60–70% (gypsum 2). The procedures of specimen manufacturing and experimentation are described in [15]. Initiation and propagation of fractures at the boundary of the holes of large diameters (10 mm and more) took place gradually. Such behavior is typical of viscous fracture.

Figure 3 presents the experimental data (points) of the value of the load at the initiation moment of tensile fractures at the hole boundary versus the hole diameter and the calculated critical pressure from formulas (2) and (3) at $\sigma = p$, $\alpha = 0$ by the conventional criterion (curve 1) and modified ASC criterion (curve 2). In the latter case, $\beta = 2.5$. The size $d_0$ was 4.5 mm in the first case and 2.0 mm in the second case. The asymptotic value of the critical pressure is equal to $\sigma_0$ in the first case (dashed straight line) and $2.7\sigma_0$ in the second case (solid straight line). The experimental data are smoothly described by the modified criterion ASC.
The same experimental data (points) and the calculated critical pressure from formulas (4) and (3) at $\sigma = p$, $\alpha = 0$ by the conventional criterion (curve 1) and modified PSC criterion (curve 2) are demonstrated in Figure 4. In the latter case, $\beta = 1$. The size $d_0$ was 2.25 mm in the first case and 0.6 mm in the second case. The asymptotic value of the critical pressure equals $c_0\sigma_0$ in the first case (dashed straight line) and $2.7\sigma_0$ in the second case (solid straight line). The experimental data are smoothly described by the modified criterion PSC.

![Figure 4](image)

**Figure 4.** Critical pressure–hole diameter curves for gypsum 1. Calculated by PSC criterion.

4. Conclusions
The application range of the existing nonlocal fracture criterion with the added constant with length dimension to characterize the structure of a material is reduced to the cases of brittle or quasi-brittle fracture with a small pre-fracture zone. In order to extend this application range to quasi-brittle fracture with the developed pre-fracture zone, it is proposed to abandon the hypothesis on the pre-fracture zone size as the material constant only related with the material structure. For quasi-brittle materials, this parameter is represented by the sum of two components. The first component characterizes the structure of the material, and the second component is reflective of the inelastic deformation and depends on the plastic properties of the material, geometry of the specimen and on the loading conditions (boundary conditions). The proposed approach is used in development of new (modified) criteria of average stresses and point stresses. The applicability of the criteria is validated in terms of tensile fracturing of geomaterial specimens with a circular hole under uniform (uniaxial) and non-uniform compression. The proposed criteria adequately describe the experimental data on fracture of quasi-brittle materials.

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