Quasi-Degenerate Neutrino Mass Spectrum, $\mu \rightarrow e + \gamma$ Decay and Leptogenesis

S. Pascoli $^a$, S. T. Petcov $^{b,c}$ \footnote{Also at: Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria} and C. E. Yaguna $^b$

$^a$Department of Physics, University of California, Los Angeles CA 90095-1547, USA
$^b$Scuola Internazionale Superiore di Studi Avanzati, I-34014 Trieste, Italy
$^c$Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, I-34014 Trieste, Italy

Abstract

In a large class of SUSY GUT models with see-saw mechanism of neutrino mass generation, lepton flavor violating (LFV) decays $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc., are predicted with rates that are within the reach of present and planned experiments. A crucial element in these predictions is the matrix of neutrino Yukawa couplings $Y_{\nu}$ which can be expressed in terms of the light and RH heavy neutrino masses, the neutrino mixing PMNS matrix $U$, and an orthogonal matrix $R$. Leptogenesis can take place only if $R$ is complex. Considering the case of quasi-degenerate neutrinos and assuming that $R$ is complex, we derive simple analytical expressions for the $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$ and $\tau \rightarrow e + \gamma$ decay rates. Taking into account the leptogenesis constraints on the relevant parameters we show that the predicted rates of the LFV decays $\mu \rightarrow e + \gamma$, and $\tau \rightarrow e + \gamma$ are generically enhanced by a factor of $\sim 10^3$ to $\sim 10^6$ with respect to the rates calculated for real $R$, while the $\tau \rightarrow \mu + \gamma$ decay rate is enhanced approximately by two orders of magnitude.
1 Introduction

The solar neutrino experiments \[1, 2, 3, 4\], the data on atmospheric neutrinos obtained by the Super-Kamiokande collaboration \[5\], and the results from the KamLAND reactor antineutrino experiment \[6\], provide very strong evidences for mixing and oscillations \[7, 8, 9\] of flavour neutrinos. The evidences for solar $\nu_e$ oscillations into active neutrinos $\nu_\mu,\tau$, in particular, were spectacularly reinforced by the combined Super-Kamiokande and first SNO \[3\] data, by the more recent SNO data \[4\], and by the just published first results of the KamLAND \[6\] experiment.

The interpretation of the solar and atmospheric neutrino, and of the KamLAND data in terms of neutrino oscillations requires the existence of 3-neutrino mixing in the weak charged lepton current (see, e.g., \[10, 11\]):

$$\nu_{lL} = \sum_{j=1}^{3} U_{lj} \nu_{jL}.$$  \hspace{1cm} (1)

Here $\nu_{lL}, l = e, \mu, \tau$, are the three left-handed flavor neutrino fields, $\nu_{jL}$ is the left-handed field of the neutrino $\nu_j$ having a mass $m_j$ and $U$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix \[7\]. It follows from the results of the $^3\text{H}\beta$-decay experiments \[12\] that $m_j < 2.2$ eV. The existence of the flavour neutrino mixing, eq. \[11\], implies that the individual lepton charges, $L_e, L_\mu$ and $L_\tau$ are not conserved (see, e.g., \[13\]). Therefore, lepton flavour violating (LFV) processes like $\mu \rightarrow e + \gamma$, $\mu^- \rightarrow e^- + e^+ + e^-$, $\tau \rightarrow \mu + \gamma$, $\mu^- + (A, Z) \rightarrow e^- + (A, Z)$, are allowed. However, if the neutrino (lepton) mixing in the weak charged lepton current is the only source of $L_e, L_\mu$ and $L_\tau$ non-conservation, as in the minimally extended Standard Theory with massive neutrinos, the rates and cross-sections of the LFV processes are suppressed by the factor \[14\] $(m_j/M_W)^4 < 5.6 \times 10^{-43}$, $M_W$ being the $W^\pm$ mass, which renders them unobservable.

The experimentally suggested smallness of the neutrino masses can naturally be explained by the see-saw mechanism of neutrino mass generation \[15\]. The see-saw mechanism requires the existence of heavy right-handed (RH) Majorana neutrinos. Right-handed neutrinos \[8, 16\] are completely neutral under the Standard Theory gauge symmetry group. Consequently, they can acquire Majorana masses $M_R$ that are not related to the electroweak symmetry breaking mechanism, and can, in principle, be much heavier than any of the known particles. The heavy RH Majorana neutrinos can generate through their CP-violating decays the observed baryon asymmetry of the Universe \[17\]. In grand unified theories (GUT) their masses are typically by a few to several orders of magnitude smaller than the scale of unification of the electroweak and strong interactions, $M_X \sim 2 \times 10^{16}$ GeV. However, their presence in a theory can lead to a severe hierarchy problem associated with the existence of two very different mass (energy) scales: the electroweak symmetry breaking and the RH Majorana mass scale. In supersymmetric (SUSY) GUT theories the hierarchy between these two mass scales is stabilized. Hence, the SUSY GUT theories incorporating the see-saw mechanism of neutrino mass generation provide a consistent and appealing framework to account for neutrino masses and for the baryon asymmetry in the Universe.

SUSY theories have additional sources of lepton charge non-conservation. In spite of the possible flavor-blindness of SUSY breaking, the supersymmetrization of the see-saw mechanism, for instance, can induce new LFV effects \[18\]. If SUSY is broken above the RH
Majorana mass scale, as, e.g. in gravity-mediated breaking scenarios, there are renormalization group effects that generate new lepton charge non-conserving couplings at low energy even if such couplings are absent at the GUT scale. In contrast to the non-supersymmetric case, these couplings give contributions to the amplitudes of the LFV decays and reactions which are not suppressed by the small values of neutrino masses and the LFV processes can proceed with rates and cross-sections which are within the sensitivity of presently operating and proposed experiments (see, e.g., [19]).

The solar and atmospheric neutrino data and the data from the reactor $\bar{\nu}_e$ experiments KamLAND, CHOOZ and Palo Verde, were used successfully for determining the pattern of the $3 - \nu$ mixing and the values of the two independent neutrino mass-squared differences, $\Delta m^2_\odot$ and $\Delta m^2_\ast$, which drive the solar and atmospheric neutrino oscillations. Under the rather plausible assumption of CPT-invariance, for instance, the recent KamLAND results practically establish [6] the large mixing angle (LMA) MSW solution as unique solution of the solar neutrino problem, with $\Delta m^2_\odot \sim 7 \times 10^{-5} \text{ eV}^2$ and $\tan^2 \theta_\odot \sim 0.40$ favored by the data, $\theta_\odot$ being the mixing angle which controls the solar $\nu_e$ oscillations. The analyses of the atmospheric neutrino data show that $\Delta m^2_\ast$ and the mixing parameter $\sin^2 2\theta_\Lambda$, responsible for the dominant atmospheric $\nu_\mu$ ($\bar{\nu}_\mu$) oscillations into $\nu_\tau$ ($\bar{\nu}_\tau$), have values $\Delta m^2_\Lambda \sim 3 \times 10^{-3} \text{ eV}^2 \gg \Delta m^2_\odot$ , and $\sin^2 2\theta_\Lambda \sim (0.9 - 1.0)$. The existing data, however, does not allow to determine the sign of $\Delta m^2_\Lambda$. Furthermore, the neutrino oscillations are not sensitive to the absolute values of neutrino masses. Correspondingly, there are three different types of 3-neutrino mass spectra which are compatible with the existing neutrino oscillation data [20] (see also, e.g., [21]): normal hierarchical (NH), $m_1 \ll m_2 \ll m_3$, inverted hierarchical (IH), $m_1 \ll m_2 \cong m_3$, and quasi-degenerate (QD), $m_1 \simeq m_2 \simeq m_3$, $m^2_{1,2,3} \gg \Delta m^2_\Lambda$.

In the case of QD spectrum, neutrino masses can be measured directly in the $^3\text{H} \beta$-decay experiments which are sensitive to the $\bar{\nu}_e$ mass, $m_{\bar{\nu}_e} \cong m_{1,2,3}$. The present bound obtained in these experiments reads [12], $m_{1,2,3} \cong m_{\bar{\nu}_e} < 2.2 \text{ eV}$. Sensitivity to values of $m_{1,2,3} \simeq 0.35 \text{ eV}$ are planned to be reached in the KATRIN experiment [22]. If the massive neutrinos $\nu_j$ are Majorana particles, as is predicted by the see-saw mechanism, neutrinoless double-beta decay experiments can also provide information on the type of the neutrino mass spectrum and on the absolute neutrino mass scale (see, e.g., [23] [21] and the references quoted therein). They measure a combination of masses and mixing parameters known as the effective Majorana mass parameter, $|< m>|$ (see, e.g., [13] [21]). The most stringent constraints on $|< m>|$ were obtained in the $^{76}\text{Ge}$ experiments: $|< m>| < 0.35 \text{ eV}$ [24] (90% C.L.), and $|< m>| < (0.33 - 1.35) \text{ eV}$ [25] (90% C.L.). Higher, or considerably higher, sensitivities to $|< m>|$ are planned to be achieved in several $(\beta \beta)_{0v}$-decay experiments of the next generation (for a review see, e.g., [26]). If neutrinos have a QD mass spectrum, they can be relevant cosmologically through their contribution to the hot dark matter component of the Universe. The sum of neutrino masses ($m_1 + m_2 + m_3$) can be determined with a precision of $\sim (0.04 - 0.10) \text{ eV}$ from cosmological and astrophysical data [27].

The Universe seems to be made only of matter; cosmologically significant amounts of antimatter have never been observed. This asymmetry between matter and antimatter can be understood as the result of the dynamical evolution of an initially symmetric Universe in which baryon number is not conserved. $C$- and $CP$- symmetries are violated and a deviation from thermal equilibrium exists [28]. If these conditions are fulfilled, baryogenesis, the process which generates an excess of baryons over antibaryons, can take place. At present,
one of the most favored scenarios for baryogenesis is the leptogenesis scenario \[17\] in which the heavy RH neutrinos play a fundamental role. Their CP-violating and out-of-equilibrium decays produce a lepton asymmetry that is partially converted into a baryon asymmetry through anomalous electroweak processes. Leptogenesis has the attractive feature of providing a link between neutrino masses and the baryon asymmetry.

In a large class of SUSY GUT models with see-saw mechanism of neutrino mass generation and flavour-universal soft SUSY breaking at the GUT scale (see, e.g., \[29\, 30\]), the LFV processes and leptogenesis are related: they both depend (although in different ways) on the matrix of neutrino Yukawa couplings $Y_\nu$. The latter is one of the basic ingredients of the see-saw mechanism. The matrix $Y_\nu$ can be expressed in terms of the light neutrino and heavy RH neutrino masses, the neutrino mixing PMNS matrix $U$, and an orthogonal matrix $R$. Leptogenesis can take place only if $R$ is complex. Working in the framework of the indicated class of theories and taking $R$ to be complex, we derive in the present article simple analytical expressions for the $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$ and $\tau \rightarrow e + \gamma$ decay rates in the case of quasi-degenerate neutrino mass spectrum. We use the model of leptogenesis of \[31\], in which the heavy RH neutrinos are produced non-thermally in inflation, to obtain constraints on the parameters which determine the leading contribution in the LFV decay rates. Taking into account the leptogenesis constraints, we show that the rates of the LFV decays $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$ and $\tau \rightarrow e + \gamma$, obtained for complex $R$, are generically strongly enhanced with respect to those calculated for real $R$. We present quantitative results for the enhancement factors for the indicated three LFV decays.

Detailed predictions for the rates of the LFV processes in the class of SUSY GUT models with see-saw mechanism considered in our work were obtained, e.g., in refs. \[29\, 30\, 32\, 33\, 34\]. The case of quasi-degenerate neutrinos we analyze was discussed, in particular, in \[29\, 32\, 34\]. However, the results in these articles were obtained for real matrix $R$. In \[33\] the case of hierarchical neutrino mass spectrum was considered. The articles quoted in ref. \[30\] contain rather comprehensive study of the LFV processes, including the case of complex $R$ and the leptogenesis constraints, but for light neutrino mass spectra with normal and with inverted hierarchy.

2 The neutrino Yukawa coupling

The superpotential of the lepton sector in the MSSM with RH neutrinos is given by:

$$W_{\text{lepton}} = \hat{l}_L^c T Y_e \hat{L} \cdot \hat{H}_d + \hat{N}_L^c e^T Y_\nu \hat{L} \cdot \hat{H}_u - \frac{1}{2} \hat{N}_L^c T M_R \hat{N}_L^c,$$  \(2\)

where the family indices were suppressed. Here $\hat{L}_j$, $j = e, \mu, \tau \equiv 1, 2, 3$, represent the chiral super-multiplets of the $SU(2)_L$ doublet lepton fields, $\hat{l}_j^c$, $j = e, \mu, \tau \equiv 1, 2, 3$, is the super-multiplet of the $SU(2)_L$ singlet lepton field $l_j^c \equiv C \hat{l}_j^c$, where $C$ is the charge conjugation matrix and $l_{jR}$ is the right-handed charged lepton field, $\hat{N}_L^c$ is the super-multiplet of the $SU(2)_L$ singlet neutrino field $N_{jL}^c \equiv C \hat{N}_{jL}^T$, where $N_{jR}$ is the RH neutrino field, and $\hat{H}_u$ and $\hat{H}_d$ are the super-multiplets of the two Higgs doublet fields $H_u$ and $H_d$ carrying weak hypercharges $-\frac{1}{2}$ and $\frac{1}{2}$, respectively. In eq. \(2\), $Y_\nu$ is the $3 \times 3$ matrix of neutrino Yukawa couplings, $Y_e$ is the $3 \times 3$ matrix of the Yukawa couplings of the charged leptons, and $M_R$ is the Majorana mass matrix of the RH neutrinos $N_{jR}$. We can always choose a basis in
which both $Y_e$ and $M_R$ are diagonal. We will work in that basis and will denote by $D_M$ the corresponding diagonal RH neutrino mass matrix, $D_M = \text{diag}(M_1, M_2, M_3)$.

The see-saw mechanism generates a Majorana mass matrix for the left-handed neutrinos of the form:

$$m_\nu = (Y_\nu v_u)^T D^{-1}_M (Y_\nu v_u),$$  \hspace{1cm} (3)

where $v_u$ is the vacuum expectation value of $H_u$. The neutrino mass matrix $m_\nu$ is diagonalized by a single unitary matrix $U$ according to

$$D_m = U^T m_\nu U \equiv \text{diag}(m_1, m_2, m_3),$$  \hspace{1cm} (4)

where $U$ is the PMNS matrix in the weak charged lepton current, eq. \text{(1)}.

It is convenient to choose $m_j > 0$, to number the massive neutrinos in such a way that $m_1 < m_2 < m_3$, and to work with Majorana neutrino fields $\nu_j$ which satisfy the Majorana condition: $C(\bar{\nu}_j)^T = \nu_j$, $j = 1, 2, 3$. In this case the PMNS matrix $U$ can be written as

$$U = V \cdot \text{diag}(1, e^{i\alpha}, e^{i\beta}),$$  \hspace{1cm} (5)

where $\alpha$ and $\beta$ are two Majorana $CP$-violating phases \cite{35}. For $V$ one can use the standard parametrization

$$V = \begin{pmatrix}
    c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\
    s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},$$  \hspace{1cm} (6)

with the usual notations, $s_{ij} \equiv \sin\theta_{ij}$, etc. If, for instance, $\Delta m^2_\odot = \Delta m^2_{21}$ (neutrino mass spectrum with normal hierarchy) and $\Delta m^2_\odot = \Delta m^2_{23}$, one can identify $\theta_{12} = \theta_\odot$, $\theta_{23} = \theta_\Lambda$, while $\theta_{13}$ is limited by the data from the CHOOZ and Palo Verde experiments \cite{36, 37}, $\sin^2\theta_{13} < 0.05$.

The matrix $D_m$ can be expressed as

$$D_m = U^T Y_\nu^T v_u D^{-1}_M Y_\nu v_u U = U^T Y_\nu^T v_u D^{-1/2}_M D^{-1/2}_M Y_\nu v_u U.$$  \hspace{1cm} (7)

Following ref. \cite{29}, we define the complex matrix $R$:

$$R \equiv D^{-1/2}_M Y_\nu^T v_u U D^{-1/2}_M.$$  \hspace{1cm} (8)

Given $D^{1/2}_M$, $D^{-1/2}_M$ and $U$, the most general neutrino Yukawa coupling matrix reads

$$Y_\nu = \frac{1}{v_u} D^{1/2}_M R D^{1/2}_M U^\dagger.$$  \hspace{1cm} (9)

It follows from eq. \text{(7)} that $R$ is an orthogonal matrix, $RR^T = 1$. In order for the leptogenesis scenario of baryon asymmetry generation to work, $R$ must be complex and we will keep $R$ complex throughout this study. As we will see, apart from being a necessary condition for leptogenesis, this leads also to drastically different predictions for the rates of the LFV processes like $\mu \rightarrow e + \gamma$, $\tau \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$.

The see-saw model contains 18 physical parameters - 6 phases and 12 moduli. These include, in the basis we work, the 3 (real) masses of the heavy RH Majorana neutrinos,
and 9 moduli and 6 phases of $Y_\nu$ (3 of the 9 phases in $Y_\nu$ can be eliminated through a rephasing of the LH charged lepton fields). At low energies it is convenient to parametrize the model by the 3 angles and 3 phases of the PMNS mixing matrix $U$, eqs. (10) and (11), the 3 light neutrino masses, $m_{1,2,3}$, and the 6 parameters - 3 moduli and 3 phases, of the complex orthogonal matrix $R$. The 3 additional real parameters of the model are the 3 heavy RH Majorana neutrino masses, contained in $D_M$.

The 3 lepton mixing angles in the PMNS matrix $U$ and the two neutrino mass squared differences, $\Delta m^2_\odot$ and $\Delta m^2_\Lambda$, can be measured with a relatively high precision in neutrino oscillation experiments. These experiments could also provide information on the Dirac CP-violating phase $\delta$, whereas information on the two Majorana CP-violating phases, $\alpha$ and $\beta$, can be obtained, in principle, in processes in which the Majorana nature of neutrinos manifests itself, such as $(\beta\bar{\beta})_{0\nu}$-decay, $K^- \rightarrow \pi^+ + \mu^- + \nu$ decay, etc. (see, e.g., [38, 39]). The measurement of the neutrino mixing parameters would be complete with the determination of the type of the neutrino mass spectrum and of the absolute neutrino mass scale.

The probabilities of the LFV processes and the baryon asymmetry in leptogenesis depend on the see-saw parameters respectively via the quantities

$$Y_\nu Y_\nu = \frac{1}{v^2_u} UD^{1/2}_m R \hat{D}_m R^{1/2} U^\dagger, \quad (10)$$

and

$$\text{Im} [ (Y_\nu Y_\nu^\dagger)_{ij}]^2 = \frac{1}{v^2_u} \text{Im} [ (D^{1/2}_m R \hat{D}_m R^{1/2})_{ij}]^2, \quad i \neq j. \quad (11)$$

Thus, the matrix $R$ enters into both the expressions for the rates of the LFV processes and for the baryon asymmetry.

We will consider in what follows the case of quasi-degenerate neutrino mass spectrum, $m_1 \simeq m_2 \simeq m_3, m^2_{1,2,3} >> \Delta m^2_\odot, \Delta m^2_\Lambda$. We can then write

$$m_1 \equiv m_\nu, \quad m_2 = m_\nu + \frac{1}{2m_\nu} \Delta m^2_\odot, \quad m_3 = m_\nu + \frac{1}{2m_\nu} \Delta m^2_\Lambda, \quad (12)$$

where $m_\nu$ is the neutrino mass determining the absolute neutrino mass scale which is not known, $m_\nu < 2.2$ eV [12]. It is natural to assume that also $D_M$ has quasi-degenerate eigenvalues, $M_{1,2,3} \simeq M_R, \quad D_M \simeq M_R 1$; otherwise, an exceptional fine-tuning between $Y_\nu$ and $D_M$ would be needed in order to obtain a QD spectrum for the light neutrinos.

In the case of QD neutrino mass spectrum one has

$$Y_\nu \simeq \frac{1}{v_\nu} M^{1/2}_R m^{1/2}_\nu R \text{diag}(1,1+\Delta m^2_\odot/(4m^2_\nu),1+\Delta m^2_\Lambda/(4m^2_\nu)) U^\dagger \equiv \frac{1}{v_\nu} M^{1/2}_R m^{1/2}_\nu R U^\dagger. \quad (13)$$

Hereafter corrections $O(\Delta m^2_\odot/(2m^2_\nu))$ and $O(\Delta m^2_\Lambda/(2m^2_\nu))$ will be neglected.

The matrix $R$ can be parametrized as

$$R = e^{i A} O, \quad (14)$$

where $A$ and $O$ are real matrices. The orthogonality of $R$ implies that $O$ is orthogonal and $A$ is antisymmetric. A different parametrization of $R$ has been used in previous works (see, e.g., [29]), but this one is particularly useful if the neutrino mass spectrum is of the QD type.
Up to corrections of the order of $\Delta m^2_\Lambda / (2m^2_\nu)$ and $\Delta m^2_{\odot} / (2m^2_\nu)$, i.e., in the approximation of exact degeneracy of the three Majorana neutrinos $\nu_{1,2,3}$, the matrix $O$ can be absorbed in the PMNS matrix $U$ - the latter is defined up to a real orthogonal matrix and $U$ and $UO$ lead to the same physics [40]. Thus, up to relatively small corrections, $O$ can effectively be taken to be the unit matrix, $O \approx 1$. This simplification is due to an additional $O(3)$ symmetry present in the lepton sector when the neutrino mass spectrum is exactly degenerate [40].

Thus, up to corrections of the order of $\Delta m^2_\Lambda / (2m^2_\nu)$ and $\Delta m^2_{\odot} / (2m^2_\nu)$, the matrix $R$ in the expression for $Y_\nu$, eq. (13), is effectively given by $e^{iA}$. The matrix $e^{iA}$ can be explicitly calculated in terms of the three non-zero elements of $A$. If we write

$$A = \begin{pmatrix}
0 & a & b \\
-a & 0 & c \\
-b & -c & 0
\end{pmatrix},$$

then

$$e^{iA} = 1 - \frac{\cosh r - 1}{r^2} A^2 + i \frac{\sinh r}{r} A,$$

where $r = \sqrt{a^2 + b^2 + c^2}$.

Our final expression for $Y_\nu$ in the case of QD neutrino mass spectrum is

$$Y_\nu \approx \frac{1}{v_u} M^{1/2}_R m^{1/2}_\nu e^{iA} U^\dagger,$$

with $e^{iA}$ given by (16).

If we take the mixing angles $\theta_{12} = \theta_{\odot}$ and $\theta_{23} = \theta_{A}$ as known and neglect $\sin \theta_{13}$ in $U$, both $Y_\nu^\dagger Y_\nu$ and $Y_\nu Y_\nu^\dagger$ depend on 5 real parameters: $M_R$, $m_\nu$, $a$, $b$, $c$; $Y_\nu$ depends in addition on the phases $\alpha$ and $\beta$.

Yukawa couplings are expected to have moduli less than one, $|Y_\nu| \leq 1$. Taking $a = b = c \equiv k$ we get from the diagonal elements of $Y_\nu$ the condition

$$\cosh 3k \geq \left| \frac{261\text{GeV}}{\sqrt{M_R m_\nu}} - \frac{1}{2} \right|,$$

so that $k \in \{1.4, 0.9, 0.3\}$ for $m_\nu = 0.2$ eV and $M_R = \{10^{10}, 10^{12}, 10^{14}\}$ GeV, respectively.

3 The processes $\ell_i \to \ell_j + \gamma$

The existence of two Yukawa couplings in the lepton sector generally causes lepton flavour violation in a way analogous [14] to its quark sector counterpart in the Standard Theory. In the minimally extended Standard Theory with massive neutrinos and in the non-supersymmetric versions of the see-saw model, the decay rates and cross sections of the LFV processes are extremely suppressed: one has, for example, for the branching ratio of the $\mu \to e + \gamma$ decay, $BR(\mu \to e + \gamma) < 10^{-47}$ [41, 14]. Such small branching ratios are unobservable. The present experimental limit is [12]

$$BR(\mu \to e + \gamma) < 1.2 \times 10^{-11}.$$
This bound is expected to be improved at least by a few orders of magnitude in the future. In an experiment under preparation at PSI [43], for instance, it is planned to reach a sensitivity to

$$BR(\mu \to e + \gamma) \sim 10^{-14}. \quad (20)$$

As we have seen, the rates of the LVF processes in the minimally extended Standard Theory with massive neutrinos are so strongly suppressed that these processes are not observable in practice. In a SUSY theory the situation is very different because there is a new source of lepton flavor violation: the soft SUSY breaking Lagrangian, $\mathcal{L}_{\text{soft}}$. The breaking of SUSY will, generally, cause lepton flavor violation. Indeed, off-diagonal elements in the neutrino Yukawa coupling can give rise to off-diagonal elements in the slepton mass matrix at low energies through renormalization group effects.

The slepton sector of the soft supersymmetry breaking Lagrangian has the form

$$-\mathcal{L}_{\text{soft}} = (m^2_{\tilde{L}})_{ij}\tilde{L}_i^\dagger \tilde{L}_j + (m^2_{\tilde{e}})_{ij}\tilde{e}_R^\dagger \tilde{e}_R + (m^2_{\tilde{\nu}})_{ij}\tilde{\nu}_R^\dagger \tilde{\nu}_R + \left(A^e_{ij}H_d\tilde{e}_R^\dagger \tilde{L}_j + A^\nu_{ij}H_u\tilde{\nu}_R^\dagger \tilde{L}_j + h.c.\right). \quad (21)$$

Lepton flavor violation can be generated by off-diagonal elements of the soft SUSY breaking parameters. The most conservative starting point for $\mathcal{L}_{\text{soft}}$ is the assumption of universality at the GUT scale $M_X$:

$$\begin{align*}
(m^2_{\tilde{L}})_{ij} &= (m^2_{\tilde{e}})_{ij} = (m^2_{\tilde{\nu}})_{ij} = \delta_{ij}m_0^2, \\
\tilde{m}^2_{H_d} &= \tilde{m}^2_{H_u} = m_0^2, \\
A^\nu &= Y_\nu a_0 m_0, \quad A^e = Y_e a_0 m_0.
\end{align*} \quad (22)$$

Thus, at the unification scale, flavor is exactly conserved by $\mathcal{L}_{\text{soft}}$. Nevertheless, soft SUSY breaking terms suffer from renormalization via Yukawa and gauge interactions. In this way, LFV in the Yukawa couplings will induce LFV in the slepton mass matrices at low energy even if the slepton masses are flavour-universal at high energy.

The RGE for the left-handed slepton mass matrix is given by (see, e.g., [29, 30])

$$\mu \frac{d}{d\mu} (m^2_L)_{ij} = \mu \frac{d}{d\mu} (m^2_L)_{ij}\bigg|_{\text{MSSM}} + \frac{1}{16\pi^2} \left[ (m^2_L Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m^2_L)_{ij} \\
+ 2(Y_\nu^\dagger m^2_{Y_\nu} + \tilde{m}^2_{H_u} Y_\nu^\dagger Y_\nu + A^\nu_{ij} A^\nu_{ij}) \right],$$

where the first term is the standard MSSM term which has no LFV, while the second one is the source of LFV. In the leading-log approximation and with universal boundary conditions, the off-diagonal elements of the left-handed slepton mass matrix at low energy are given by

$$(m^2_L)_{ij} \approx -\frac{1}{16\pi^2} (6 + 2a_0^2)m_0^2(Y_\nu^\dagger Y_\nu)_{ij} \log \frac{M_X}{M_R}. \quad (23)$$

This equation shows the connection between LFV in neutrino Yukawa couplings and LFV in slepton mass terms.

Let us turn now to the lepton-flavor violating processes of the type $\ell_i \to \ell_j + \gamma$. The amplitude for this process has the general form

$$T = \epsilon^\alpha \bar{\ell}_j m_{\ell_\alpha} i\sigma_{\alpha\beta} q^\beta (A_L P_L + A_R P_R)\ell_i, \quad (24)$$
where $q$ is the momentum of the photon, $P_{R(L)} = (1 + (-)\gamma_5)/2$ and $A_L (A_R)$ is the coefficient of the amplitude when the decaying lepton is left-handed (right-handed). The corresponding branching ratio is

$$BR(\ell_i \to \ell_j + \gamma) = \frac{12\pi^2}{G_F^2} |A_L|^2 + |A_R|^2.$$  \hspace{1cm} (25)

The terms $|A_{L,R}|$ contain the contributions of the neutralino and the chargino loops (see Fig. 1). Explicit expressions for $A_L$ and $A_R$ can be found in the literature. In the mass insertion approximation, the diagrams contributing to $\ell_i \to \ell_j + \gamma$ have the generic form shown in Fig. 1 and the branching can be estimated using the expression

$$BR(\ell_i \to \ell_j + \gamma) \approx \frac{12\pi^2}{G_F^2} |A_R|^2 \approx \frac{\alpha^3}{G_F^2} \frac{|(m_L^2)_{ij}|^2}{m_S^2} \tan^2 \beta,$$  \hspace{1cm} (26)

where $m_S$ represents a scalar lepton mass. In the leading-log approximation, using the expression (25), one finds

$$BR(\ell_i \to \ell_j + \gamma) \approx \frac{\alpha^3}{m_S^2 G_F^2} \left[ \frac{3 + a^2}{8\pi^2} m_0^2 \log \frac{M_X}{M_R} \right] \frac{2}{2} |(Y_\nu^\dagger Y_\nu)_{ij}|^2 \tan^2 \beta. \hspace{1cm} (27)$$

Therefore, the off-diagonal elements of $Y_\nu^\dagger Y_\nu$ are the crucial quantities needed to estimate the branching ratios.

Using the expression for $Y_\nu$ in eq. (17) we find that in the case of QD neutrino mass spectrum and in the approximation of negligible splitting between the neutrino masses,

$$(Y_\nu^\dagger Y_\nu)_{ij} \approx \frac{1}{v^2_m} M_R m_{\nu}(Ue^{i2A}U^\dagger)_{ij}.$$  \hspace{1cm} (28)

For small values of $a$, $b$, and $c$, and negligible $s_{13}$ we obtain

$$(Ue^{i2A}U^\dagger)_{21} \approx 2i \left[ -a(c_{12}^2 e^{i\alpha} + s_{12}^2 e^{-i\alpha}) c_{23} - e^{i\beta} s_{23}(bc_{12} + cs_{12} e^{-i\alpha}) \right]$$  \hspace{1cm} (29)

$$(Ue^{i2A}U^\dagger)_{31} \approx 2i \left[ a(c_{12}^2 e^{i\alpha} + s_{12}^2 e^{-i\alpha}) s_{23} - e^{i\beta} c_{23}(bc_{12} + cs_{12} e^{-i\alpha}) \right]$$  \hspace{1cm} (30)

$$(Ue^{i2A}U^\dagger)_{32} \approx 2i \left[ -2ias_{12} c_{12} c_{23} s_{23} \sin \alpha + (bs_{12} - cc_{12} e^{i\alpha})(s_{23}^2 e^{i\beta} + c_{23}^2 e^{-i\beta}) \right]$$  \hspace{1cm} (31)

These elements control the $\mu \to e + \gamma$, $\tau \to e + \gamma$ and $\tau \to \mu + \gamma$ decay rates, respectively.
This result means that the branching ratio of the $\alpha$ is larger than (34). We have, for instance, where we used eq. (28) with $\delta = 0$, $\Delta \rightarrow m_{\nu}$ one has to the rates calculated using eq. (33), because in the case of QD neutrino mass spectrum $m_{\nu}$ enhancement depends on the values of the parameters $a$, $b$ and $c$, contained in $A$.

Expressions (29) - (31) and (33) differ significantly in one more aspect: in contrast to $Y_{\nu}^\dagger Y_{\nu}|_R$, eq. (33), the quantity $Y_{\nu}^\dagger Y_{\nu}$ calculated for complex $R$ depends on the Majorana CP-violating phases $\alpha$ and $\beta$ in the PMNS matrix $U$. Thus, the observation of the LFV processes $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. could allow one to get information about these phases. Let us recall that determining or even constraining the Majorana CP-violating phases in the neutrino matrix $U$ is a formidable problem (see, e.g., 21, 38).

In the numerical estimates which follow we take $s_{12} \equiv \sin \theta_{\odot} = 0.6$, $s_{23} \equiv \sin \theta_{A} = 1/\sqrt{2}$, $\delta = 0$, $\Delta m_{A}^{2} = 3 \times 10^{-3}$ eV$^2$, $\Delta m_{\odot}^{2} = 7 \times 10^{-5}$ eV$^2$, $m_{\nu} = 0.3$ eV and consider two different values for $s_{13}$: 0; 0.2. We have

$$
\left| (Y_{\nu}^\dagger Y_{\nu}|_R)_{31} \right|^2 \sim \left| (Y_{\nu}^\dagger Y_{\nu}|_R)_{21} \right|^2 \simeq \frac{M_{R} m_{\nu}^2}{v_{u}^4} \times \left\{ \begin{array}{ll}
6 \times 10^{-6} & \text{if } s_{13} = 0.2, \\
1.4 \times 10^{-8} & \text{if } s_{13} = 0.0.
\end{array} \right. \quad (34)
$$

We will compare these results with the results we get for a complex matrix $R$. We set $\alpha = \pi/2$ and $\beta = 0$. If we choose $|a|, |b|, |c| \simeq O(10^{-1})$, we always get a result that is much larger than (33). We have, for instance,

$$
\left| (Y_{\nu}^\dagger Y_{\nu})_{21} \right|^2 \simeq \frac{M_{R}^{2} m_{\nu}^2}{v_{u}^4} \times \left\{ \begin{array}{ll}
0.34, & \text{for } (a, b, c) = (0.2, -0.4, 0.5), \\
0.81, & \text{for } (a, b, c) = (0.4, 0.3, 0.2)
\end{array} \right. \quad , \quad (35)
$$

where we used eq. (28) with $s_{13} = 0$ (the results for $s_{13} = 0.2$ are only slightly different). We see that the coefficient in the right-hand side of the above equation is always $O(0.1 - 1.0)$. This result means that the branching ratio of the $\mu \rightarrow e + \gamma$ decay will be enhanced with respect to the prediction based on eq. (33) approximately by a factor of $10^5$ to $10^8$.

---

This is valid also in the cases of neutrino mass spectra with normal and inverted hierarchy 30.

That if $R$ is complex, $BR(\mu \rightarrow e + \gamma)$ could be enhanced with respect to the branching ratio predicted for real $R$, was noticed in 29.
depending on the value of \( s_{13} \). Even if we take \(|a|, |b|, |c| \simeq \mathcal{O}(10^{-2})\), there is still an enhancement of about three to six orders of magnitude. The same results are valid for the \( \tau \to e + \gamma \) decay rate.

The \( \tau \to \mu + \gamma \) decay rate is also enhanced, but the magnitude of the enhancement is smaller than in the case of the \( \mu \to e + \gamma \) decay rate: by a factor of \( \sim 10^4 \) for \(|a|, |b|, |c| \simeq \mathcal{O}(10^{-1})\), and of \( \sim 10^2 \) for \(|a|, |b|, |c| \simeq \mathcal{O}(10^{-2})\). This is due to the fact that the leading term in \((Y^\nu_3 Y^\nu_\nu^\dagger)_\overline{32}\) is not suppressed by \( s_{13} \).

Detailed predictions for the rate of the \( \mu \to e + \gamma \) decay, obtained for real \( R \) and for QD neutrino mass spectrum, can be found in [29, 32, 34]. They can be used, together with eqs. (29) and (33), to estimate \( B(R(\mu \to e + \gamma)) \) for complex \( R \) we have considered. The substantial enhancement we have found certainly makes the importance of the searches for this decay even more significant.

Within the see-saw model, the neutrino Yukawa couplings, \( Y^\nu \), plays a major role in the generation of neutrino masses and in determining the rates of the LFV processes such as \( \mu \to e + \gamma \), \( \tau \to \mu + \gamma \) and \( \tau \to e + \gamma \) decays. It plays a fundamental role also in leptogenesis.

4 The Leptogenesis Constraints

The convenient dimensionless number which characterizes the magnitude of the baryon asymmetry of the Universe is the ratio of the baryonic charge density, \( n_B - n_{\bar{B}} \), to the entropy density, \( s \). The presently observed baryon asymmetry is

\[
Y_B = \frac{n_B - n_{\bar{B}}}{s} = (0.1 - 1) \times 10^{-10}. \tag{36}
\]

The aim of baryogenesis is to explain this number in terms of processes and fundamental parameters of particle physics. In leptogenesis, the out of equilibrium decays of heavy RH neutrinos produce a lepton asymmetry which is reprocessed by sphaleron processes into a baryon asymmetry. If the light neutrinos are quasi-degenerate, \( m_\nu \gtrsim 0.1 \text{ eV} \), the out-of-equilibrium condition cannot be satisfied and the amount of produced lepton asymmetry is strongly suppressed (see, e.g., [45, 46]), unless the RH neutrinos are produced non-thermally. We shall consider leptogenesis via decays of RH neutrinos \( N_i \) which are produced through inflaton decays [31].

At tree level the decay width of a heavy neutrino \( N_i \) is,

\[
\Gamma_{D_i} = \Gamma(N_i \to H_u + l) + \Gamma(N_i \to H^c_u + l^c) = \frac{1}{8\pi}(Y^\nu Y^\nu_\nu^\dagger)_{ii} M_i. \tag{37}
\]

If \( CP \) is not conserved by the neutrino Yukawa couplings, the interference between the tree and the one-loop diagram contributions to the \( N_i \) decay amplitudes results in a lepton number production. The lepton number asymmetry per decay of a RH neutrino is

\[
\epsilon_i \equiv \frac{\Gamma(N_i \to H_u + l) - \Gamma(N_i \to H^c_u + l^c)}{\Gamma(N_i \to H_u + l) + \Gamma(N_i \to H^c_u + l^c)} \approx -\frac{1}{8\pi} \frac{1}{(Y^\nu Y^\nu_\nu^\dagger)_{ii}} \sum_{j \neq i} \text{Im} \left[ \left( (Y^\nu Y^\nu_\nu^\dagger)_{ij} \right)^2 \left[ f(M^2_j/M^2_i) + g(M^2_j/M^2_i) \right] \right]. \tag{38}
\]
Here $H_u, l$, and $N_i$ denote scalar or fermionic components of the corresponding supermultiplets, $f$ is the contribution from the one-loop vertex correction,

$$f(x) = \sqrt{x} \left[ \log \left( \frac{1 + x}{x} \right) \right], \quad (39)$$

and $g$ is the contribution from the one-loop self energy diagrams, which can be reliably calculated in perturbation theory if the condition

$$|M_i - M_j| \gg |\Gamma_i - \Gamma_j| \quad (40)$$

holds. One finds

$$g(x) = \frac{2\sqrt{x}}{x - 1}. \quad (41)$$

For quasi-degenerate RH neutrinos, $x \simeq 1$ and $g \gg f$.

The ratio of the lepton number density $n_L$ to the entropy density $s$ produced by the inflaton decay is given by

$$\frac{n_L}{s} = \frac{3}{2} \sum_i \epsilon_i BR(\phi \rightarrow N_i N_i) \frac{T_R}{m_\phi}, \quad (42)$$

where $\phi$ denotes the inflaton field, $BR(\phi \rightarrow N_i N_i) \equiv Br^{(i)}$ is the $\phi \rightarrow N_i N_i$ decay branching ratio, $m_\phi$ is the mass of the inflaton and $T_R$ is the reheating temperature after the inflation. We have assumed that $M_R \geq T_R$ in order to prevent lepton-number violating processes from washing out the lepton asymmetry after the $N$'s have decayed. Part of the lepton asymmetry is immediately converted into baryon asymmetry via the sphaleron effect,

$$\frac{n_B}{s} = C \frac{n_L}{s}, \quad (43)$$

with $C \simeq -0.35$ in the MSSM.

Using eq. (9) we obtain

$$Y_\nu Y_\nu^\dagger = \frac{1}{v_u^2} D_M^{1/2} R D_m R^\dagger D_M^{1/2}. \quad (44)$$

Hence, in general, leptogenesis is independent of the mixing angles and phases contained in the PMNS matrix $U$. If $R$ is real, $\text{Im}(Y_\nu Y_\nu^\dagger) = 0$ and leptogenesis cannot work. This is a model independent statement and it is the main reason we have to assume that $R$ is complex.

For quasi-degenerate neutrinos, eq. (44) can be further simplified,

$$Y_\nu Y_\nu^\dagger \simeq \frac{M_R m_\nu}{v_u^2} e^{-2A}. \quad (45)$$

It is well-known that if the RH neutrinos are completely degenerate, the generated lepton asymmetry is zero [49]. Thus, one has to break the exact degeneracy in the heavy RH neutrino masses. We write

$$M_2 = M_1(1 - \varepsilon_2), \quad M_3 = M_1(1 - \varepsilon_3), \quad |\varepsilon_3| \gg |\varepsilon_2|. \quad (46)$$
The maximal values of $|\varepsilon_2|$ and $|\varepsilon_3|$ which are naturally consistent with a low-energy quasi-degenerate neutrino mass spectrum are $|\varepsilon_2| \approx \Delta m^2_{10}/2m_{\nu}^2$ and $|\varepsilon_3| \approx \Delta m^2_{13}/2m_{\nu}^2$.

Conditions \((40)\) for small \(r\) translate into

\[
|\varepsilon_2| \gg \frac{1}{4\pi} |c^2 - b^2| \frac{M_R}{10^{14} \text{GeV}},
\]

\[
|\varepsilon_3| \gg \frac{1}{4\pi} |c^2 - a^2| \frac{M_R}{10^{14} \text{GeV}},
\]

\[
|\varepsilon_3| \gg \frac{1}{4\pi} |b^2 - a^2| \frac{M_R}{10^{14} \text{GeV}},
\]

where we have used $m_{\nu} = 0.3 \text{eV}$ and $v_u = 174 \text{GeV}$. Since $|\varepsilon_2| \approx 10^{-4}$ and $|\varepsilon_3| \approx 10^{-2}$, for the extreme value $M_R \approx 10^{14} \text{GeV}$ these conditions are satisfied as long as $|b|, |c| \lesssim 10^{-2}$ and $|a| \lesssim 10^{-1}$. For $M_R \approx 10^{10} \text{GeV}$, eqs. \((47)\) - \((49)\) lead to the constraints $|a|, |b|, |c| \lesssim 1$.

We shall compute next the lepton number asymmetries, eq. \((38)\). From \((16)\) we get

\[
\text{Im} \left[ (e^{i2A})_{12} (e^{i2A})_{12} \right] = 2 \frac{abc}{v_u^3} \sinh 2r (\cosh 2r - 1)
\]

\[
= \text{Im} \left[ (e^{i2A})_{23} (e^{i2A})_{23} \right]
\]

\[
= \text{Im} \left[ (e^{i2A})_{31} (e^{i2A})_{31} \right]
\]

and \(\text{Im} \left[ (e^{i2A})_{ij} (e^{i2A})_{ij} \right] = -\text{Im} \left[ (e^{i2A})_{ji} (e^{i2A})_{ji} \right]\). Assuming that \(a\), \(b\) and \(c\) are small, we can expand the hyperbolic functions in \((50)\) to obtain

\[
\epsilon_1 = \epsilon_2 \approx \frac{1}{\pi} \frac{M_R m_{\nu} abc}{v_u^2 \varepsilon_2}
\]

\[
\epsilon_3 \approx -\frac{2}{\pi} \frac{M_R m_{\nu} abc}{v_u^2 \varepsilon_3}
\]

The baryon asymmetry thus generated is

\[
\frac{n_B}{s} \approx 1.4 \times 10^{-8} \left( \frac{2M_R}{m_{\phi}} \right) \left( \frac{T_R}{10^8 \text{GeV}} \right) \left( \frac{m_{\nu}}{\text{0.1eV}} \right) \left( \frac{abc}{\varepsilon_2} \right) (B^{(1)}_r + B^{(2)}_r),
\]

where we have neglected the \(\varepsilon_3\) contribution. Hence, the empirical baryon asymmetry is obtained with a reheating temperature of $T_R \approx 10^8 \text{GeV}$ for $|abc/\varepsilon_2| \approx 10^{-1}$ and a natural choice of the remaining parameters. Since $|\varepsilon_2| \approx 10^{-4}$, we have

\[
|abc| \approx 10^{-5}.
\]

Larger values of $|abc|$ are possible if $2M_R/m_{\phi} \ll 1$. Higher reheating temperatures would be compatible with smaller values of the product $|abc|$, but would also lead to the cosmological gravitino problem \[50\].

The baryon asymmetry is proportional to the product $abc$ and therefore none of the three parameters can be zero. Moreover, they cannot be exceedingly small, otherwise it would be impossible to reproduce the baryon asymmetry. Equation \((53)\) implies that at least one of them - $|a|$, $|b|$, or $|c|$ - must be of order $10^{-2}$ or larger. This number fixes the possible enhancement of $BR(\mu \to e + \gamma)$: about four orders of magnitude for $s_{13} = 0.2$ and six orders of magnitude for $s_{13} = 0$. $BR(\tau \to e + \gamma)$ is enhanced by similar factors, while $BR(\tau \to \mu + \gamma)$ is enhanced approximately by two orders of magnitude.
5 Conclusions

We have considered the $\mu \to e + \gamma$, $\tau \to \mu + \gamma$ and $\tau \to e + \gamma$ decay branching ratios in a class of SUSY GUT models with see-saw mechanism of neutrino mass generation. We have assumed that the orthogonal $R$ matrix which was introduced in \cite{29} and which is related to the neutrino Yukawa coupling matrix $Y_\nu$, is complex. This is required in order for the model to be compatible with the leptogenesis scenario of generation of the baryon asymmetry. In this case $R$ can be represented as $R = e^{iA}O$, where $A$ and $O$ are respectively real antisymmetric and real orthogonal matrices. The matrix $A$ can be parametrized by 3 real parameters, $a$, $b$ and $c$. We have considered the case of quasi-degenerate spectrum of light neutrinos, $m_{1,2,3} \approx m_\nu$, $m_\nu^2 >> \Delta m_A^2$, $\Delta m_\odot^2$, where $\Delta m_A^2$ and $\Delta m_\odot^2$ are the neutrino mass-squared differences which drive the atmospheric and solar neutrino oscillations. Assuming that the heavy right-handed neutrinos are also quasi-degenerate in mass, $M_{1,2,3} \approx M_R$, and that the soft SUSY breaking slepton mass terms are flavour-universal at the GUT scale, we have derived approximate expressions for $\mu \to e + \gamma$, $\tau \to \mu + \gamma$ and $\tau \to e + \gamma$ decay rates. Apart from the standard SUSY soft-breaking parameters $(m_0, a_0, \tan\beta, m_{1/2})$, the decay rates depend on $m_\nu$, $M_R$, on the mixing angles $\theta_{12}$ and $\theta_{23}$ which control the solar and atmospheric neutrino oscillations, on the Majorana CP-violating phases in the PMNS mixing matrix $U$ and on the parameters $a$, $b$ and $c$. We have found that for complex $R$, the branching ratios of the indicated LFV decays are considerably larger than when $R$ is taken to be real: for $a \sim b \sim c \sim 10^{-1}$, for instance, $BR(\mu \to e + \gamma)$ and $BR(\tau \to e + \gamma)$ are enhanced approximately by a factor of $10^5$ to $10^8$ with respect to the case of real $R$, while $BR(\tau \to \mu + \gamma)$ is enhanced by approximately four orders of magnitude. We used the model of leptogenesis with light quasi-degenerate neutrinos, in which the heavy RH Majorana neutrinos are assumed to be produced non-thermally in the inflaton decay, to get constraints on $a$, $b$ and $c$. The baryon asymmetry is proportional to the product $abc$ of the three parameters associated with the complexity of $R$. For values of the RH neutrino mass $M_R$ characteristic for the leptogenesis model, the observed asymmetry can be reproduced for $|abc| \sim 10^{-5}$. If $BR(\mu \to e + \gamma)$ and $BR(\tau \to e + \gamma)$ are evaluated for values of $a$, $b$ and $c$ compatible with the leptogenesis constraint, the enhancement we found is approximately by a factor of $10^3$ and $10^6$ for values of $\sin^2\theta_{13} = 0$ and 0.04. The corresponding enhancement of $BR(\tau \to \mu + \gamma)$ is approximately by two orders of magnitude.

Besides $a$, $b$ and $c$, and the Majorana CP-violating phases in the PMNS matrix $U$, $BR(\mu \to e + \gamma)$ depends also on SUSY soft breaking parameters $(m_0, a_0, \tan\beta, m_{1/2})$ and the RH neutrino mass $M_R$. Given the existing experimental bound on $BR(\mu \to e + \gamma)$, our results can be used, in particular, to further constrain the space of the supersymmetric parameters in the case of quasi-degenerate neutrino mass spectrum. This requires a more detailed numerical analysis which is beyond the scope of the present work.

If neutrinos will be proven experimentally to have a quasi-degenerate mass spectrum and the neutrino masses are generated via the see-saw mechanism within a SUSY GUT theory, the process $\mu \to e + \gamma$ should be observable in the planned experiments of the next generation provided the supersymmetric particles have masses in the range of several hundred GeV. Additional constraints from data on the $\tau \to \mu + \gamma$ and $\tau \to e + \gamma$ decays and from leptogenesis can be used to determine the matrix $A$ and the RH neutrino mass $M_R$. With that information, the neutrino Yukawa couplings $Y_\nu$ could be almost fully reconstructed.
Acknowledgements. We would like to thank W. Rodejohann for useful discussions. S.P. would like to thank SISSA for very kind hospitality during the completion of the present work. This work was supported in part by the EC network HPRN-CT-2000-00152, by the Italian MIUR under the program “Fenomenologia delle Interazioni Fondamentali” (S.T.P.) and by the U. S. Department of Energy (S.P.).

References

[1] B.T. Cleveland et al., Astrophys. J. 496 (1998) 505; Y. Fukuda et al., Phys. Rev. Lett. 77 (1996) 1683; V. Gavrin, Nucl. Phys. Proc. Suppl. 91 (2001) 36; W. Hampel et al., Phys. Lett. B447 (1999) 127; M. Altmann et al., Phys. Lett. B490 (2000) 16.

[2] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 86 (2001) 5651 and 5656.

[3] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 87 (2001) 071301.

[4] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89 (2002) 011302 and 011301.

[5] Super-Kamiokande Collaboration, M. Shiozawa, talk given at the Int. Conf. on Neutrino Physics and Astrophysics “Neutrino’02”, May 25 - 30, 2002, Munich, Germany.

[6] KamLAND Collaboration, K. Eguchi et al., hep-ex/0212021.

[7] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 33 (1957) 549 and 34 (1958) 247; Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[8] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53 (1967) 1717.

[9] V. Gribov and B. Pontecorvo, Phys. Lett. B28 (1969) 493.

[10] S.M. Bilenky, C. Giunti and W. Grimus, Prog. Part. Nucl. Phys. 43 (1999) 1.

[11] S.T. Petcov, hep-ph/9907216.

[12] V. Lobashev et al., Nucl. Phys. Proc. Suppl. 91 (2001) 280; C. Weinheimer et al., hep-ex/0210050.

[13] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59 (1987) 671.

[14] S.T. Petcov, Sov. J. Nucl. Phys. 25 (1977) 340.

[15] T. Yanagida, in Workshop on unified theories, KEK report 79-18 (1979) 95; M. Gell-Mann, P. Ramond, R. Slansky, in Supergravity (eds. P. van Nieuwenhuzen and D. Freedman, North Holland, Amsterdam, 1979); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[16] S.M. Bilenky and B. Pontecorvo, Lett. Nuov. Cim. 17 (1976) 569.

[17] M. Fukugita and T. Yanagida, Phys. Lett. B174 (1986) 45.
[18] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986) 961.
[19] Y. Kuno and Y. Okada, Rev. Mod. Phys. 73 (2001) 151.
[20] S.T. Petcov and A.Yu. Smirnov, Phys. Lett. B322 (1994) 109.
[21] S.M. Bilenky, S. Pascoli and S.T. Petcov, Phys. Rev. D64 (2001) 053010.
[22] A. Osipowicz et al. (KATRIN Project), hep-ex/0109033.
[23] S.M. Bilenky et al., Phys. Lett. B465 (1999) 193; S. Pascoli and S.T. Petcov, Phys. Lett. B544 (2002) 239; S. Pascoli, S.T. Petcov and W. Rodejohann, hep-ph/0212113.
[24] H. V. Klapdor-Kleingrothaus et al., Nucl. Phys. Proc. Suppl. 100 (2001) 309.
[25] C.E. Aalseth, F.T. Avignone III et al., Physics of Atomic Nuclei 63 (2000) 1225.
[26] M. Spiro, Summary talk at the Int. Conf. on Neutrino Physics and Astrophysics “Neutrino’02”, May 25 - 30, 2002, Munich, Germany.
[27] W. Hu and M. Tegmark, Astrophys. J. Lett. 514 (1999) 65; see also, e.g., S. Hannestad, astro-ph/0211106.
[28] A.D. Sakharov, JETP Lett. 5 (1967) 24.
[29] J.A. Casas and A. Ibarra, Nucl. Phys. B618 (2001) 171.
[30] J. Ellis et al., Nucl. Phys. B621 (2002) 208 and hep-ph/0206110; J. Ellis and M. Raidal, Nucl. Phys. B643 (2002) 229; J. Ellis, M. Raidal and T. Yanagida, Phys. Lett. B546 (2002) 228.
[31] G. Lazarides and Q. Shafi, Phys. Lett. B258 (1991) 305; K. Kumekawa, T. Moroi and T. Yanagida, Prog. Theor. Phys. 92 (1994) 437; G.F. Giudice et al., JHEP 9908 (1999) 014; T. Asaka et al., Phys. Lett. B464 (1999) 12, and Phys. Rev D61 (2000) 083512; M. Fujii, K. Hamaguchi and T. Yanagida, Phys. Rev. D65 (2002) 115012; T. Asaka, H. B. Nielsen and Y. Takanishi, Nucl. Phys. B647 (2002) 252.
[32] A. Kageyama et al., Phys. Rev. D65 (2002) 096010 and Phys. Lett. B527 (2002) 206.
[33] S. Lavignac, I. Masina and C.A. Savoy, Phys. Lett. B520 (2001) 269 and Nucl. Phys. B633 (2002) 139.
[34] F. Deppisch et al., hep-ph/0211138.
[35] S.M. Bilenky et al., Phys. Lett. B94 (1980) 495; M. Doi et al., Phys. Lett. B102 (1981) 323.
[36] M. Apollonio et al. (CHOOZ collab.), Phys. Lett. B466 (1999) 415; F. Boehm et al. (Palo Verde collab.), Phys. Rev. Lett. 84 (2000) 3764 and Phys. Rev. D62 (2000) 072002.
[37] G.L. Fogli et al., hep-ph/0212127.

[38] S. Pascoli, S.T. Petcov and L. Wolfenstein, Phys. Lett. B524 (2002) 319; W. Rodejohann, Nucl. Phys. B597 (2001) 110 and hep-ph/0203214; S. Pascoli, S.T. Petcov and W. Rodejohann, Phys. Lett. B524 (2002) 319.

[39] A. de Gouvêa, B. Kayser and R. Mohapatra, hep-ph/0211394.

[40] G.C. Branco, M.N. Rebelo and J.I. Silva-Marcos, Phys. Rev. Lett. 82 (1999) 683.

[41] S.M. Bilenky, S.T. Petcov and B. Pontecorvo, Phys. Lett. B67 (1977) 309; T.P. Cheng and L. Li, Phys. Rev. Lett. 45 (1980) 1908.

[42] M. L. Brooks et al. (MEGA Collaboration), Phys. Rev. Lett. 83 (1999) 1521, hep-ex/9905013.

[43] L. M. Barkov et al., Research Proposal for an experiment at PSI (1999), http://meg.psi.ch.

[44] J. Hisano et al., Phys. Rev. D53 (1996) 2442.

[45] M. Plümmacher, Z. Phys. C74 (1997) 549.

[46] W. Buchmüller and M. Plümmacher, Int. J. Mod. Phys. A15 (2000) 5047.

[47] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B384 (1996) 169.

[48] M. Flanz et al., Phys. Lett. B389 (1996) 693.

[49] J. Liu and G. Segre, Phys. Rev. D49 (1994) 1342; A. Pilaftsis, Int. J. Mod. Phys. A14 (1999) 1811.

[50] M. Y. Khlopov and A. D. Linde, Phys. Lett. B138 (1984) 265; J. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B145 (1984) 181; M. Kawasaki and T. Moroi, Prog. Theor. Phys. 93 (1995) 879; E. Holtmann et al., Phys. Rev. D60 (1999) 023506.