Off-shell $\omega$ production in proton-proton collisions near threshold

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The $\omega$ production in nucleon-nucleon collisions is described through decays of intermediate nucleon resonances. The near-threshold $pp \rightarrow pp\omega$ cross section is found to be dominated by the off-shell production of $\omega$ mesons with masses far below the physical $\omega$ mass. The $N^*(1535)$ resonance plays thereby a crucial role. Due to a strong $N\omega$ coupling, this resonance leads to an off-shell contribution which is about one order of magnitude larger than the experimentally measured contribution from the $\omega$ peak. After a subtraction of the theoretical “background” from the off-shell $\omega$ production, the available data are accurately reproduced over the entire energy range from 5 MeV up to several GeV above the threshold. The scenario of a weaker $N^*(1535) \rightarrow N\omega$ decay mode which is still consistent with electro- and photoproduction data is discussed as well. In the latter case, the off-shell contribution to the $pp \rightarrow pp\omega$ cross section is substantially reduced but the description of the experimental cross section is poor above threshold.

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I. INTRODUCTION

The studies of vector meson production in nucleon-nucleon collisions are motivated by several facts. First, the short range part of the nucleon-nucleon ($NN$) interaction is dominated by the isoscalar $\omega$ meson exchange [1] and thus information from inelastic $NN$ collisions can contribute to better understanding of the nuclear forces. In a dense hadron environment which exists in cores of massive neutron stars or is transiently created in energetic heavy-ion reactions one can expect significant changes of the quark condensates, which manifest themselves in mass shifts of the vector mesons [2], suggested also by QCD sum rules [3]. In a hadronic picture the coupling to many-body correlations modifies the in-medium masses and the spectral properties of the mesons [4,5]. To search for modifications of the in-medium properties of vector mesons is the major issue of dilepton spectroscopy in energetic heavy-ion collisions [6].

However, for the study of medium effects in heavy-ion reactions the theoretical understanding of vector meson production in elementary processes is a prerequisite. In the vicinity of the threshold only the $\omega$ production has been studied experimentally. This is due to the small $\omega$ width of 8.4 MeV which allows an identification of the narrow $\omega$ peak in missing mass spectra already close to threshold. Data for the $\omega$ production in proton-proton ($pp \rightarrow pp\omega$) collisions at small excess energies were recently taken at SATURNE [7] and with the COSY-TOF spectrometer [8]. The excess energy $\epsilon$ is thereby defined as $\epsilon = \sqrt{s} - (2m_p + m_\omega)$ with $\sqrt{s}$ being the proton-proton center-of-mass energy, $m_p$ the proton mass and $m_\omega = 782$ MeV the physical $\omega$ mass at its pole value. The SATURNE experiment was performed very close to threshold ($\epsilon = 4 \div 30$ MeV) whereas the COSY-TOF Collaboration measured the $\omega$ production at $\epsilon = 92$ and 173 MeV. Both experiments served also to examine the validity of the Okubo-Zweig-Iizuka (OZI) selection rule [9]. As compared to the $\omega$ cross section, the $\phi$ meson which couples to the strangeness content of the nucleon should be strongly suppressed by the OZI rule which forbids disconnected quark line diagrams. The assumption of only a small OZI violation leads to a $\phi$ over $\omega$ ratio of $\sim 4 \cdot 10^{-3}$ in $pp$ reactions at comparable excess energies [10] while experimentally this ratio was found to be about one order of magnitude larger [7,8,11].

Microscopic calculations for the $\omega$ production in nucleon-nucleon collisions are, however, rare. In ref. [12] the on-shell $\omega$ production was described through the coupling to nucleon currents and to meson exchange currents including thereby the $NN$ final-state interaction (FSI). The $\omega$-meson was treated as an elementary field and thus off-shell effects in the $\omega$ production were missing from the beginning. Adjusting the relevant form factors, the experimental data [7] in the vicinity of the threshold were well reproduced.

In the present approach the vector meson production is described through a two-step mechanism via the excitation of nuclear resonances. This picture is motivated by existence of a variety of nucleon resonances which have large branchings to the $N\rho$ decay modes, such as $N^*(1520)^{\pm}, N^*(1535)^{\pm}$, and $N^*(1720)^{\mp}$. The couplings to the $\omega$-mesons have been observed in multichannel $\pi N$ partial wave analyses [13,14] and were predicted by quark models [15]. A strong coupling of the $N\omega$ system to nuclear resonances has also been reported in [16] using an EFT coupled channel approach. It is further a well established fact that within a hadronic picture the coupling to nuclear resonances is a major source for the modification of vector meson properties in a dense nuclear environment. E.g. in the medium the $\rho$ acquires its major modifications by the coupling to $N^*$-hole excitations, especially the $N^*(1520)N^{-1}$ [4,5].

The resonance model provides a unified description of a large variety of phenomena such as nucleon electro- and photoproduction, mesonic decays of nuclear resonances...
II. RESONANCE MODEL

The vector meson production is described through a two-step mechanism via the excitation of nuclear resonances, i.e. \( NN \to NR, R \to NV \), with \( V = \rho, \omega \). The nucleon-nucleon cross sections for the resonance production \( NN \to NR \) have been determined by fitting the available data on pion, double-pion, and \( NNPP \) (\( P = \pi, \eta, \ldots \)) are described by the two-step process \( NN \to NR \) with the subsequent decays of the resonances \( R \)'s \([19]\). The influence of the \( NN \) FSI which is generally expected to be important near the thresholds is thereby effectively included in the phenomenological matrix elements \( M_R \)'s for the resonance production. On the other hand, the present approach accounts also effectively for the FSI between one nucleon and the produced vector meson. To take the FSI between the nucleon and the meson in specific partial waves into account is equivalent to include those nucleon resonances into the reaction dynamics which have the same quantum numbers as the corresponding partial waves (final-state interaction theorem \([20]\)). The nucleon resonance propagators, in particular, reoccur due to the division of the bare amplitudes by Jost functions evaluated for \( NV \) resonating phase shifts.

In refs. \([5,17,18]\), the Vector Meson Dominance (VMD) model was applied to determine the \( RN_\omega \) coupling strengths from radiative and mesonic decay data. In its monopole form, i.e., only including the ground-state \( \rho \) and \( \omega \), the naive VMD is known to underestimate systematically the mesonic \( R \to N\rho \) decays when a normalization to radioactive \( R \to N\gamma \) decays is performed \([21]\). The inclusion of higher radial \( \rho \) meson excitations in the VMD helps to resolve this problem \([18,21]\) and provides the correct asymptotes of the \( R \to N\gamma \) transition form factors given by the quark counting rules \([22]\).

The extended \((e)\)VMD model assumes that radial excitations \( (\rho(1450), \rho(1700), \ldots) \) interfere with the ground-state \( \rho \)-mesons in radiative processes. Already in the case of the nucleon form factors, radially excited vector mesons should be added in order to provide a dipole behavior for the Sachs form factors and describe the experimental data \([23]\). Details of the calculations of the magnetic, electric, and Coulomb \( R \to N\gamma \) transition form factors and the branching ratios of the nucleon resonances can be found in refs. \([18,24]\). The model parameters are fixed from photo- and electro-production data and using results from \( \pi N \) scattering multichannel partial-wave analyses. Where experimental data are not available, predictions from non-relativistic quark models are used. In \([25]\) the \( e\)VMD model was successfully applied to a systematic study of meson decays to dilepton pairs and in \([21]\) to the description of dilepton production in \( pp \) reactions \([26]\).

The \( pp \to pp\omega \) cross section is given as follows

\[
\frac{d\sigma(s,M)_{pp\to pp\omega}}{dM^2} = \sum_{R} \int_{(m_p+M)^2}^{(\sqrt{s}-m_p)^2} \frac{d\sigma(s,\mu)_{pp\to pR}}{d\mu^2} \frac{dB(\mu,M)_{R\to p\omega}}{dM^2} \, . \tag{1}
\]

The cross sections for the resonance production are given by

\[
d\sigma(s,\mu)_{pp\to pR} = \frac{|M_R|^2}{16p_i\sqrt{s}\pi^2} \Phi_2(\sqrt{s},\mu, m_p) dW_R(\mu) \tag{2}
\]

with \( \Phi_2(\sqrt{s},\mu, m_p) = \pi p^*(\sqrt{s},\mu, m_p) / \sqrt{s} \) being the two-body phase space, \( p^* (\sqrt{s},\mu, m_p) \) the final c.m. momentum, \( p_i \) the initial c.m. momentum, and \( \mu \) and \( m_R \) the running and pole masses of the resonances, respectively. \( m_p \) is the proton mass. The mass distribution \( dW_R(\mu) \) of the resonances is described by the standard Breit-Wigner formula:

\[
dW_R(\mu) = \frac{1}{\pi} \frac{\mu \Gamma_R(\mu) d\mu^2}{(\mu^2 - m_R^2)^2 + (\mu \Gamma_R(\mu))^2} \, . \tag{3}
\]

The sum in (1) runs over the nucleon resonances given in Table 1. This includes all well established \((4*)\) resonances.
The branching to the ω decay mode is given by

$$dB(μ, M)^{R→πω} = \frac{Γ_R^{πω}(μ, M)}{Γ_R(μ)} dW_ω(M).$$

The ω mass distribution $dW_ω(M)$ is also described by a Breit-Wigner distribution, i.e., substituting $R → ω$ and $μ → M$ in Eq. (3). The energy dependence of the ω width $Γ_ω(M)$ can be calculated according to the two-step process $ω → ρπ → 3π$, as proposed by Gell-Mann, Sharp, and Wagner [28]. The effective vertices describing the $ω → ρπ$ and $ρ → 2π$ decays have the form $L_{ωρπ} = f_{ωρπ}τ_αμντ_αωτ_αρ_αρ_βρ_β$ and $L_{ρππ} = -\frac{1}{2} f_{ρππ}ε_{αβγ}ρ_α^μπ^βδ_απ^γ$, with $f_{ωρπ} = 16.3$ GeV$^{-1}$ and $f_{ρππ} = 6.03$ (see e.g., [17]). The decay width can be found to be

$$Γ_ω(M) = \frac{1}{48π^5 M f_{ωρπ}^2 f_{ρππ}^2} \int_{4m_π^2}^{(M-m_π)^2} dM^2 \int_{-1}^{+1} \frac{dz}{2}$$

$$\times |D(k^+ + k^-) + D(k^+ - k^-) + D(k^- + k^-)|^2$$

$$\times (m_π^2 p^2 q^2 - p^2 (kq)^2 - q^2 (kp)^2)$$

$$\times Φ_2(M, M', m_π) Φ_2(M', m_π, m_π)$$

where $m_π$ is the pion mass, $k^+, k^-$, and $k$ are the pion $π^+$, $π^-$, and $π^0$ momenta, $p = k^+ + k^-$, $q = (k^+ - k^-)/2$, $p^2 = M^2$, $k^2 = (M^2 - M^2 - m_π^2)/2$, $q^2 = -(p^2 (M', m_π, m_π))$, $k q = -p^* (M', M', m_π) p^* (M', m_π, m_π) M z/M'$. The value of $z$ is the cosine between the vectors $q$ and $k$ in the c.m. frame of the charged pions. $D(μ)$ is the $ρ$-meson propagator with the total width of 150 MeV. The width $ω$ can be parameterized by

$$Γ_ω(M) = Γ_ω(m_ω) \left( \frac{p^*(M, m_π, 2m_π)}{p^*(M, m_π, 2m_π)} \right)^a$$

with the coefficient $a = 8$ at $M < m_ω$ and $a = 10$ at $M > m_ω$. Expression (6) is compared to Eq. (5) in Fig. 1. The width $ω$ goes to zero at the $3m_π$ threshold which is also the physical threshold for the $pp → ppω$ reaction.

Nucleon resonances with spin $J > 1/2$ and arbitrary parity have three independent transition amplitudes, while spin-1/2 resonances have only two independent amplitudes. In terms of the magnetic, electric, and Coulomb couplings $g_{M}^{±}$, $g_{E}^{±}$, and $g_{C}^{±}$, the differential decay widths of nucleon resonances with spin $J → l+1/2$ into an $ω$-meson with arbitrary mass $M$ has the form [18]

$$dΓ_{Nω}^R(μ, M) = \frac{9}{64που(2l+1)!}$$

$$\times \left[ \frac{m_π^2 (m_π^2 - M^2)^{l+1/2}(m_π^2 - M^2)^{-l-1/2}}{M_π^{2l+1}m_π^2} \right] \frac{(l+1)}{l} \frac{g_ω^{(+)}(M)}{g_ω^{(±)}(M)}$$

$$+ (l+1)(l+2) \left( \frac{g_ω^{(±)}(M)}{M_π^2} \frac{g_ω^{(±)}(M)}{g_ω^{(±)}(M)} \right) dW_ω(M),$$

where $M_π = μ ± m_π$. The signs ± refer to the natural parity ($1/2^-, 3/2^-, 5/2^-, ...$) and abnormal parity ($1/2^2, 3/2^2, 5/2^2, ...$). $g_ω^{(±)}(M)$ means $g_ω^+ + g_ω^-$. The above equation is valid for $l > 0$. For $l = 0$ ($J = 1/2$) one obtains

$$dΓ_{Nω}^R(μ, M) = \frac{1}{32πμ} \left[ m_π^2 - M^2 \right]^{3/2} \left[ m_π^2 - M^2 \right]^{1/2}$$

$$\times \left( \frac{2 g_ω^{(±)}(M)}{M_π^2} \right)^2 \left( \frac{g_ω^{(±)}(M)}{M_π^2} \right)^2 \right) dW_ω(M).$$

Due to the subthreshold character of the $ω$ production in decays of on-shell nucleon resonances, the $M$-dependence of the coupling constants $g_{M}^{±}$, $g_{E}^{±}$, and $g_{C}^{±}$ can be important. At the $ω$ pole mass $m_ω$ these couplings are proportional to residues of the magnetic, electric, and Coulomb transition form factors. The corresponding values at $M^2 = m_ω^2$ can be found in Table 1. We assume that the coupling constants which enter into the covariant representation of the form factors are not mass dependent. The $M$-dependence of $g_{M}^{±}$, $g_{E}^{±}$, and $g_{C}^{±}$ arises then exclusively from the $M$-dependent transformation from the covariant basis to the multi-pole basis according to

$$g_{T}^{(±)}(M^2) = \sum_{kT} M_{kT}(M^2) M_{kT}^{-1}(m_ω^2) g_{T}^{(±)}(m_ω^2),$$

with $T$ and $T'$ being $M, E, C$. The matrices $M_{kT}(M^2)$ which transform the multi-pole form factors to the covariant form factors can be found in ref. [18].

According to our analysis of ref. [18] the $N^+(1535)$ is the only resonance with a large branching to the $Nω$ channel among those resonances which lie far below the $ω$ peak. Since it turns out that the $ω$ production reacts very sensitive to this fact, in particular close to threshold, we consider also an alternative scenario with a weaker $N^+(1535)$-$Nω$ coupling. In the eVMD model the $Nρ$ and $Nω$ decays are fixed simultaneously. However, in the case of the $N^+(1535)$ there were neither experimental data nor quark model predictions for the $Nω$ channel available [18]. The available data in the $Nρ$ channel and the fact

![FIG. 1. The $ω$-meson decay width $Γ_ω(M)$ versus the off-shell $ω$-meson mass $M$. The solid line shows the calculation (5) according to Gell-Mann, Sharp, and Wagner [28]. The dotted line is the parametrisation of Eq. (6).](image-url)
that quark models provide here partially contradictory results leave some freedom in the determination of the $N^*$(1535)$\omega$ coupling strength. In Fig. 11 shown in the Appendix, we discuss two different fits to the electro- and photoproduction data, the $\pi N$ multichannel scattering analyses and the quark model predictions in more detail. The essential distinction between these two procedures lies in the different normalizations to the $\rho$-meson decay amplitudes. Konik [15], Manley and Saleski [14] and the PDG [30] give similar predictions for the $s_{1/2}$ wave and predict the same sign for the $d_{3/2}$ wave. The second set of amplitudes stems from quark model calculations by Capstick and Roberts [29]. Their values are noticeable smaller than those proposed in [15,14,30]. Both sets do not provide $\omega$-meson amplitudes. To investigate the stability of our results, we introduced one point for the $\omega$-meson $s_{1/2}$ wave around zero. The original solution [18], which uses the results of refs. [15,14,30] turns out to be rather stable and does not allow for a significant reduction of the $s_{1/2}$ amplitude. The second set allows the reduction of the amplitude by a factor of 6 to 8, however, at the expense of a moderately higher $\chi^2$. A further going reduction of the amplitude is hardly possible. In the second fit the deviation from the experimental $p^*(1535)$ Coulomb amplitude is larger and the reproduction of the transversal $p^*(1535)$ amplitude is also worse but the $\rho$-meson amplitudes are better reproduced than with the first set of input parameters. Notice that the $d_{3/2}$ amplitude of the $\omega$-meson was set equal to zero to ensure an unique eVMD solution for the fitted data. The value of $d_{3/2}$ does not significantly affect other observables.

In Table 1 the corresponding values for the $N\omega$ decay widths are given at the resonance pole masses. The branching of the $N^*(1440)$ at the resonance pole is small since it lies deeply below the $N\omega$ threshold. However, at higher masses where the $N^*(1440)$ is off-shell it receives a sizable $\omega$ decay width. In Table 1, the contributions from different partial waves, including relative signs, are also shown. In the case of the $N^*(1535)$ the values for the two different parameter sets resulting in a strong (s), respectively a weaker (w) coupling to the $N\omega$ channel are given. The corresponding matrix elements $M_R$ for the resonance production in $pp$ collisions, Eq. (2), are taken from ref. [19].

The energy dependence of the total resonance decay widths $\Gamma_R^0(\mu)$ is scaled according to the $\pi N$ phase space and the Blatt-Weisskopf suppression factor. In a consistent treatment, the $N\rho$ and $N\omega$ decay channels have to be taken into account in the total width

$$\Gamma_R(\mu) = \Gamma_R^0(\mu) + \Gamma_{N\rho}^R(\mu) + \Gamma_{N\omega}^R(\mu) + \delta\Gamma_R$$

where $\delta\Gamma_R = -\Gamma_{N\rho}^R(m_R) - \Gamma_{N\omega}^R(m_R)$ ensures the normalization of the total width at the resonance pole mass $m_R$. Since the total width appears in the numerator and squared in the denominator of the corresponding Breit-Wigner distribution (3) the partial widths are not simply added in a perturbative way but the coupled channel problem is more complicated. The appearance of new channels shifts generally strength from the pole to the tails of the distribution. However, the shape of the spectral function around the $\omega$ pole depends crucially on the magnitude of the partial decay widths. In the case of a strong $N\omega$ coupling, the rapidly increasing partial width leads to a strong reduction at the $\omega$ threshold, an effective enhancement below the $\omega$ peak, and shifts strength further out. A weak coupling, on the other hand, leads to a visible $\omega$-peak in the resonance spectral function. The consequences of these two different scenarios will become more clear when the cross sections are considered in the next section.

In Fig. 2 we show the energy dependence of the $N^* \rightarrow N\omega$ widths versus the off-shell masses $\mu$ of the resonances. Results were obtained with the $\omega$ spectral function of ref. [18]. We show two solutions for the $N^*(1535)$ resonance: The dashed line refers to the original set of input parameters with strong coupling (s) while the solid line refers to the new set with weaker coupling (w).

When the off-shell mass of a $N^*$ resonance exceeds the $\omega$-meson production threshold given by the $\omega$ pole mass $(m_N + m_\omega)$, the decay width sharply increases since the narrow $\omega$ peak acts similar as a $\delta$-function. The width of a resonance which lies below the $\omega$ pole and has already there a noticeable branching to this decay channel acquires a large width at the $\omega$ pole. Such a step-like behavior is not unusual. If there is a wide potential barrier of height $U_0$ $(= m_R + m_\omega)$ the life time of a resonant state vanishes at an energy $E > U_0$ $(\mu > m_R + m_\omega)$. Moreover, the life time can vanish sharply if the potential
barrier is flat (a square wall). Deeply below $U_0$, the barrier penetration factor $D(E) = \exp(-2\sqrt{2m(U_0 - E)/a})$ is almost constant (with $a$ being the barrier width), just below $U_0$ it increases exponentially, and just above $U_0$ $D(E) = 1$

The $N^*(1535)$ off-shell partial width is especially large for the strong coupling amplitude (s), being 16 GeV above the $\omega$ threshold. We interpret this large $N^*(1535)$ width as an indication for a dissolving resonant state $N^*(1535)$ at running masses $\mu$ above the $\omega$ production threshold. Fig. 3 shows the spectral function of the $N^*(1535)$. When large values of the decay width appear (dashed curve), also the resonance spectral function drops sharply above the $\omega$ production threshold which demonstrates from another side the disappearance of the resonant states with increasing running mass.

![Figure 3](image)

**FIG. 3.** Spectral distribution of the $N^*(1535)$ resonance using an energy dependent total width, and adding the partial $N\rho$ and $N\omega$ widths. The two cases of a strong (s) and weak (w) coupling to the $N\omega$ channel are distinguished.

The implications of a full dissolution of the $N^*(1535)$ are quite straightforward: Integrals over the running mass $\mu$ should be cut at $\mu > m_R + m_{\omega}$. In our model such an effect arises automatically through the appearance of the large total resonance width in the denominator of the Breit-Wigner distribution. The large $N^*(1535)$ width demonstrates that we are somewhere above a potential barrier where the resonance does not exist any more. However, in this case the resonance model allows to treat all resonances on the same footing, i.e. accounting in the calculations of the cross section for the full energy range $m_p + m_\pi < \mu < \sqrt{s} - m_\pi$. For the $N^*(1535)$ the interval $m_p + m_\omega < \mu < \sqrt{s} - m_p$ does not significantly influence observables due to its suppression through the large width. However, the scaling by the Blatt-Weisskopf factor leads to a suppression of $\Gamma_{N\omega}^R$ at $\mu \geq 2$ GeV which results in a small bump in the $N^*(1535)$ spectral function at high values of $\mu$. This suppression gives some additional strength to the cross section at high energies. This fact has, however, no influence on the threshold behavior of the cross section on which the present work focuses. Relatively large $N\omega$ widths are not too surprising. According to the $SU(3)$ symmetry the $\omega$ coupling to nucleons is 3 times greater than the $\rho$ coupling. One can therefore expect that at equal kinematical conditions the off-shell $N\omega$ widths above the $\omega$ production threshold will typically be an order of magnitude greater than the $N\rho$ widths. The off-shell partial widths of the weakly coupled $N^*(1535)$ (solid curve) and of the other resonances are below 1 GeV.

In this context it should be also mentioned that expression (5) for the energy dependent $\omega$ width $\Gamma_\omega(M)$ contains still uncertainties away from the $\omega$ peak: First, the coupling constants $f_{\omega\rho}$ and $f_{\pi\pi\omega}$ in eq. (5) can be $M$-dependent. Furthermore, the Blatt-Weisskopf factor which is, however, not included here would lead to a suppression of the off-shell widths above and enhancement below the resonance masses. This factor is usually applied to two-body decays. However, the $\omega$ decay has a three-body final state and thus an appropriate modification of the spectral function is not straightforward. Notice that the $\omega \to 3\pi$ decay cannot simply be treated as a quasi two-body decay either, due to the coherent superposition of the $\rho$-meson propagators entering the integrand of Eq. (5). In the case of the strong $N^*(1535)$ coupling an enhancement of $\Gamma_\omega$ at $M < m_\omega$ would result in a further increase of the off-shell part of the cross section as discussed below, but not in an increase of its measurable peak contribution. For the weak coupling, where the background is not very pronounced the cross section is practically not affected by this uncertainty.

III. THE PP → PPω CROSS SECTION

In this section we turn to the calculation of the $\omega$ production in pp reactions. We distinguish between the two scenarios of a strong and a weak $N^*(1535)N\omega$ coupling. The decay modes of the other resonances are kept fixed as determined in [18] and given in Table 1.

A. Strong $N^*(1535) – N\omega$ coupling

First we consider the differential cross section $d\sigma/dM$. Fig. 4 shows the differential cross section $d\sigma/dM$ for typical excess energies of the SATURNE ($\epsilon = 3.8, 19.6, 30.1$ MeV) and the COSY-TOF ($\epsilon = 173$ MeV) experiments. At small excess energies the cross section is dominated by off-shell omega production with masses far below the $\omega$ pole mass of 782 MeV. At the lowest considered energy, i.e. 3.8 MeV, there is even no clear $\omega$ peak structure visible in the spectrum whereas at larger values of $\epsilon$ the
The 

peak is well developed. With increasing energy the off-shell production with masses far below the quasi-particle pole becomes less important. E.g. at 173 MeV the off-shell strength is already moderate. Here the cross section originates to most extent from the peak but the off-shell part leads still to a non-vanishing renormalization of the cross section.

To explore the origin of the large off-shell component in more detail Fig. 5 shows separately the contributions of the various resonances at an excess energy of \( \epsilon = 19.6 \text{ MeV} \). As can be seen from there the decay of the \( N^*(1535) \) is responsible for the large off-shell \( \omega \) production with masses far below the pole mass whereas the off-shell contributions of the other resonances are small. However, at the \( \omega \) pole the situation is opposite, i.e. the contribution from the \( N^*(1535) \) is strongly suppressed. The other resonances show a clear peak structure.

The reason for this peculiar behavior of the \( N^*(1535) \) lies in its large coupling to the \( N\omega \) channel already far below the threshold (given by the physical \( \omega \) mass). Since the decay width is large at the resonance pole, i.e. 167 MeV below the \( \omega \) pole, this resonance provides significant strength to the \( \omega \) production in a kinematical regime where the \( \omega \) meson is far off-shell. When the \( \omega \) mass approaches the on-shell value the partial decay width to the \( N\omega \) channel increases strongly. However, this strong increase suppresses the on-shell production of the meson. The suppression is already reflected in the spectral distribution of the \( N^*(1535) \) resonance, shown in Fig. 3. This is a general feature for the decay of a broad resonance to another particle which arises under particular kinematical conditions. As discussed by Knoll for the case of the \( \rho \) decay [31] the opening of new decay channels suppresses generally the corresponding contributions at the on-shell values. The reason lies in the non-perturbative way by which the new channel enters into the resonance spectral function where it adds to the total resonance width [31]. In the case of the \( N^*(1535) \) this effect is particularly pronounced due to the large \( N\omega \) decay width and the corresponding kinematical conditions. At \( \epsilon = 19.6 \text{ MeV} \), the background generated by the \( N^*(1535) \) stays below the experimental background [7]. It is also smooth enough that it does not generate a visible structure in the data [7]. Hence, the possible existence of such an off-shell background is not in contradiction to current experimental facts.

The mechanism which is responsible for the occurrence of the strong off-shell \( \omega \) contribution is illustrated in Fig. 6. We show the kinematical limits for the integrations of the cross section, Eq. (1). The integration region is the two-dimensional parameter space of the off-shell \( \omega \)-meson mass \( M \) and the off-shell \( N^* \) mass \( \mu \). Along the horizontal axis we plotted schematically the \( N^* \to N\omega \) width as a function of \( \mu \). On the vertical axis, we show schematically the Breit-Wigner function for the \( \omega \)-meson, which enters into Eq. (1). The sharp increase of the \( N^* \to N\omega \) width can result in a strong suppression of the integrand in the shaded area due to Breit-Wigner function (3). The \( \omega \) peak appears just in the region where such a suppression is possible. The occurrence of the "theoretical background" has therefore a simple kinematical origin. Whether it occurs depends on the off-shell \( N^* \to N\omega \) width in the vicinity of the \( \omega \)-meson production threshold. The integration boundaries for this example are determined for an excess energy of \( \epsilon = 30 \text{ MeV} \). Notice
that the suppression of the $\omega$ peak in the strong coupling regime is still present at increasing $\sqrt{s}$.

![Graph](image)

FIG. 6. The integration region for the $\omega$ production cross section is displayed. The integration is performed in the two-dimensional parameter space of the off-shell $\omega$-meson mass $M$ and the off-shell $N^*$ mass $\mu$. Along the horizontal axis the behavior of the $N^* \rightarrow N\omega$ width as a function of $\mu$ is indicated schematically, on the vertical axis the $\omega$-meson spectral distribution is indicated. The sharp increase of the $N^* \rightarrow N\omega$ width can result in a strong suppression of the integrand in the dashed region where the $\omega$ peak appears.

![Graph](image)

FIG. 7. Differential $pp \rightarrow pp\omega$ cross section at an excess energy of $\epsilon = 19.6$ MeV. The theoretical background due to off-shell $\omega$ production (dashed line) is subtracted from the full cross section (solid line). The hatched area shows the remaining contribution from the $\omega$ peak.

Although the integration region displayed in Fig. 6 increases the effective integration region is restricted by the $\omega$ pole mass. Distinct from other resonances, the contribution of the $N^*(1535)$ to the cross section does therefore not increase with energy. This explains why the theoretical "background" becomes less important at high energy.

At small excess energies the cross section receives its major strength from off-shell $\omega$ production. However, experimentally only the $\omega$ peak can be identified in missing mass spectra and the off-shell part of the cross section is attributed to the general experimental background. Hence, for a meaningful comparison to data the same procedure has to be applied to theory which means to take only the contribution form the $\omega$ peak into account. The subtraction of this theoretical "background" from off-shell meson production is demonstrated in Fig. 7 for $\epsilon = 19.6$ MeV. Like in the experimental analysis [8] the background is smoothly interpolated by splines. The remaining peak contribution resembles a Breit-Wigner distribution around the $\omega$ pole mass which, due to the energy dependence of $\Gamma_\omega$ and the phase space factor $\Phi_3(\sqrt{s},m_p,m_p,M)$ entering into the cross section, is asymmetric. As soon as a clear peak structure appears the decomposition of the cross section is straightforward and rather unique as indicated by the error bars shown in Fig. 8.

![Graph](image)

FIG. 8. Exclusive $pp \rightarrow pp\omega$ cross section obtained in the resonance model. Data are taken from [7,8] and [32,33]. The solid curve shows the full cross section including the contributions from off-shell $\omega$ production. The dashed curve corresponds to the renormalized cross section where only the experimentally detectable contribution from the $\omega$ peak is taken into account.

The correspondingly renormalized total cross section is shown in Fig. 8 as a function of the excess energy $\epsilon$. The theoretical calculation provides now a very accurate description of the exclusive experimental $\omega$ cross section.
section from close to threshold [7,8] up to excess energies of several GeV. In particular, at low energies the slope of the cross section is well reproduced. The full theoretical cross section, i.e. including off-shell $\omega$ production, is also shown in Fig. 8. The total cross section is systematically larger than the renormalized and the experimental cross sections. Above excess energies of the DISTO point [32] (440 MeV) the off-shell production starts to become negligible and both curves almost coincide. Due to the increasing phase space of the $\omega$ width at large excess energies there arise new off-shell contributions from the high momentum tail of the $\omega$ spectral function. Since the present work concentrates on the threshold behavior, we accounted only partially for high momentum corrections to the cross section which are small anyhow. However, close to threshold the contribution from the off-shell “background” to the total cross section is about one order of magnitude larger than the measurable pole part of the cross section.

B. Weak $N^*(1535) - N\omega$ coupling

If one assumes a coupling of the $N^*(1535)$ to the $N\omega$ channel which is 6 to 8 times weaker than in the previous case the $pp \rightarrow pp\omega$ cross section has a completely different behavior close to threshold. This scenario is based on the alternatively selected data set for this resonance (see the Appendix). We show the differential cross section again for $\epsilon = 19.6$ MeV. First of all, the large contribution form off-shell $\omega$ production vanishes. There remains still strength at masses far below the $\omega$ pole but compared to the previous case the off-shell production is almost negligible. On the other hand, in contrast to the case of a strong $N^*(1535) - N\omega$ coupling discussed in the previous section, the on-shell $\omega$ production from the $N^*(1535)$ is not suppressed but contributes maximally to the on-shell cross section. Since the contributions from the other resonances remain unchanged the magnitude of the cross section at the $\omega$ pole mass is about 30% larger as in the previous case. The deviation increases with decreasing $\sqrt{s}$.

Now the “theoretical background” due to off-shell production is much smaller than in the previous case and thus the distinction between the on-shell and the moderate off-shell part of the cross section is not straightforward. Nevertheless, very close to threshold the background is present and the cross section does not vanish when the excess energy $\epsilon$ goes to zero. When the $\omega$ meson is treated as an elementary field [12] the cross section, according to phase space, vanishes by construction at the threshold. However, here the physical threshold for the $\omega$ production is given by the $3m_\pi$ threshold instead of the $\omega$ pole mass. The threshold behavior is qualitatively similar to a recent calculation from the Jülich group [34] where the finite mass distribution of the $\omega$ has been taken into account. However, close to threshold our result is significantly larger and overestimates the SATURNE [7] and the COSY [8] data. A subtraction of the small and not so well defined off-shell background is now a delicate procedure which depends strongly on the particular experimental treatment of the background. We estimated the magnitude of this effect and found that it might bring the theoretical result into slightly better agreement with experiment at the lowest measured excess energies [7]. But already at $\epsilon = 30$ MeV and in particular for the COSY points [8] such corrections are practically zero and thus the experimental results are still overestimated. In the COSY regime the tendency is similar as in the calculations from [12,34] which show also a too steep increase with energy.

At large excess energies above 1 GeV the cross section is somewhat reduced compared to the case with large $N^*(1535) - N\omega$ coupling and also compared to experiment. The reason is missing strength from this resonance at high energies. This behavior is already reflected in the spectral function, Fig. 3. The large $N^*(1535) - N\omega$ coupling depletes the spectral function at the $\omega$ threshold and shifts strength to the high energy tail which is missing in the second case. There the strength is concentrated around and a few hundred MeV above the $\omega$ threshold which results in the enhanced cross section in the COSY regime.

In ref. [11], the experimental results were compared to a cross section which is assumed to be proportional to the $NN\omega$ phase space corrected by the proton-proton $s$-wave FSI and the finite $\omega$ width. It was pointed out that the energy of the protons is sufficiently high to excite higher partial waves where the proton-proton correlations
are not important. A smooth interpolation to lower $\sqrt{s}$ should then be possible if there exists no strong coupling of nucleon resonances to the $N\omega$ channel. Apparently, our model is based on the opposite assumption, namely that the dynamics is governed by nucleon resonances. With respect to the FSI, our model is complementary in the sense that we account for the $N\omega$ FSI through the inclusion of resonances whereas the proton-proton FSI is missing. Near the threshold the proton-proton FSI generally enhances the amplitude which is relevant for the SATURNE data [7]. For the weak $N^*(1535)$ coupling the amplitude is in our model already overestimated already due to the inclusion of the $N\omega$ FSI. The reproduction of the data with a $NN\omega$ contact term for strongly correlated $s$-wave nucleons would require an amplitude of opposite sign and a delicate cancellation between the proton-proton and the $N\omega$ FSI at threshold. The resonance model proposed by Teis et al. [19] contains such a contact term for a direct $NN\pi$ coupling which could be attributed to the $NN$ FSI and/or a non-resonant background. Its contribution is, however, not so important at threshold for the $pp\pi$ final states. This fact can be interpreted to mean that the proton-proton FSI and/or a background are effectively already included into the phenomenological matrix elements for the resonance production.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig10.png}
\caption{Exclusive $pp \rightarrow pp\omega$ cross section obtained in the resonance model. Data are taken from [7,8] and [32,33].}
\end{figure}

In summary, the comparison to the data favors the calculation based on the large $N^*(1535)N\omega$ branching ratio. An additional fact which supports this picture is the strongly anisotropic $pp\omega$ angular distribution observed in [8] at $\epsilon = 173$ MeV. As can be seen from Table 1 only the $N^*(1535)$ has a large $s$-wave component in the $N\omega$ decay. The contributions from the other resonances involve higher partial waves which lead to anisotropic angular distributions. As we have seen, the strong $N^*(1535)N\omega$ coupling suppresses the contribution from this resonance at the $\omega$ pole which leads to the following decomposition of the on-shell cross section at $M = m_\omega$: $N^*(1535)$ (7%), $N^*(1650)$ (26%), $N^*(1520)$ (17%), $N^*(1440)$ (22%) and $N^*(1720)$ (26%). Hence the on-shell cross section contains only a small $s$-wave component. In the alternative case the $s$-wave contribution from the $N^*(1535)$ is much larger, the cross section decomposes according to: $N^*(1535)$ (35%), $N^*(1650)$ (18%), $N^*(1520)$ (12%), $N^*(1440)$ (15%) and $N^*(1720)$ (18%). An highly anisotropic angular distribution indicates furthermore that contribution from meson exchange currents which are missing in our treatment are small [12].

IV. SUMMARY

The $\omega$ production in nucleon-nucleon collisions has been described via the excitation of nucleon resonances. The approach is based on the extended VMD (eVMD) model which successfully describes the mesonic $R \rightarrow NV$ and radiative $R \rightarrow N\gamma$ decays of nucleon resonances. Among the considered resonances the $N^*(1535)$ turns out to play a special role for the $\omega$ production. The reason is a large decay mode to the $N\omega$ channel in a kinematical regime where the $\omega$ is far off-shell. A strong $N^*(1535)N\omega$ coupling is implied by the available electro- and photo-production data. As a consequence large off-shell contributions in the $\omega$ production cross section appear. In particular close to threshold the off-shell production is dominant. On the other hand, the on-shell production of $\omega$ mesons at their physical masses is suppressed, since above the $\omega$ production threshold the $N^*(1535)$ acquires a large width and dissolves. This feature appears generally when a broad resonance has a large branching to a narrow state already far below the pole mass of the produced particle. In quantum mechanics, a similar behavior is experienced by resonance states at energies just above a potential barrier.

We found that near threshold, the off-shell production of the $\omega$‘s becomes dominant. This part of the cross section can, however, be hardly identified experimentally and is attributed currently to the background. To compare to data we applied the same procedure as experimentalists: The theoretical ”background” from the off-shell production was subtracted and only the measurable pole part of the cross section was taken into account. Doing so, the available data are accurately reproduced starting from energies very close to threshold up to energies significantly above threshold without adjusting any new parameters. At small excess energies the full cross section is about one order of magnitude larger than the measurable pole part.

Since the observed results depend crucially on the role of the $N^*(1535)$ we considered also an alternative sce-
nario which is possible within experimental uncertainties. Since the $N\omega$ decay of this resonance has not been measured directly, the existing $N\rho$ data leave some freedom to fix the eVMD model parameters. A different normalization to the $N\rho$ channel, making thereby use of an alternative set of quark model predictions, allows to reduce the $N\omega$ decay mode by maximally a factor of 6 to 8, however, at the expense of a slightly worse reproduction of the existing data set. With the reduced $N\omega$ coupling the $N^*(1535)$ shows no extraordinary behavior and the off-shell contributions are substantially reduced. However, this resonance contributes now fully to the peak part of the cross section which leads to a significant overestimation of the experimental data around and several 100 MeV above threshold. The discrepancy is quite strong and can apparently not be attributed solely to the $NN$ FSI and/or a non-resonant background.

We conclude that, consistent with the analysis of electro- and photoproduction data, the measured $\omega$ production in $pp$ reactions favors the scenario of a large coupling of the $N^*(1535)$ to the $N\omega$ channel. The consequences are large off-shell contributions in the $\omega$ production cross section around threshold. Experimentally this part of the cross section is hardly accessible since it is hidden in the general experimental background. However, if the off-shell $\omega$ production is large, the number of dileptons produced in $pp$ collisions should be significantly greater than one would expect from estimates based on the on-shell production of $\omega$'s. This effect should manifest itself in the description of the dilepton production in $pp$ collisions at kinetic beam energies which correspond to the $\omega$-meson threshold. It can have consequences for the dilepton production in heavy-ion collisions.

The hypothesis of a large off-shell $\omega$ production is also appealing from another point of view. The $\phi$ meson decays dominantly to the $K\bar{K}$ mode with a threshold very close to the physical $\phi$ mass. Hence one does not expect sizable off-shell effects in this case. When off-shell $\omega$'s are counted, the $\phi/\omega$ ratio could come into agreement with the OZI rule, i.e. the experimentally observed deviations from the OZI predictions for the $\phi/\omega$ ratio might be explained by the neglect of the off-shell $\omega$-meson production.

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V. APPENDIX

The eVMD model parameters for the description of $N\omega$ decays of nuclear resonances are determined form experimental data.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig11.png}
\caption{Electric and Coulomb transition form factors $G_E$ and $G_C$ (in GeV$^{-1}$), helicity amplitudes $A_{1/2}$ and $S_{1/2}$ (in GeV$^{-1/2}$), and partial-wave amplitudes (in GeV$^{-1}$) are shown for the $N^*(1535)$ decay to the $\rho$- and $\omega$-meson channels. The value $G_D(t) = 1/(1 - t/0.71)^2$ is the dipole function. The experimental photo- and electro-production data $A_{1/2}$ and $S_{1/2}$ for the $p^*(1535)$ are taken from [30]. The $\rho$ meson decay amplitudes (filled symbols) are from [15,14,30], the opaque circles are the quark model predictions from [29]. The dot-dashed lines correspond to fit to the strong coupling set [15,14,30] while the solid lines correspond to the weak coupling set [29].}
\end{figure}
in the normalizations to the $\rho$-meson decay amplitudes. The original fit of [18] (dot-dashed lines) is based on the the results of refs. [15,14,30]. The quark model of Koniuk [15], the $\pi N$ analysis of Manley and Saleski [14], and the PDG. [30] give close predictions for the $s_{1/2}$ wave and predict the same sign for the $d_{3/2}$ wave. Using this set of data the solution is rather stable and yields a large $N\omega$ $s_{1/2}$ wave amplitude.

The second set fit (solid lines) is based on the $s_{1/2}$ and $d_{3/2}$ $N\rho$ amplitudes taken from quark model calculations by Capstick and Roberts [29]. Their values are significantly smaller than those proposed by [15,14,30]. This allows a reduction of the $N\omega$ $s_{1/2}$ amplitude by a factor of 6 to 8, however, by the price of a moderately higher $\chi^2$. A further going reduction of the amplitude is hardly possible. For the second fit the deviation from the experimental $p^*(1535)$ Coulomb amplitude is larger and the reproduction of the transversal $p^*(1535)$ amplitude is also worse but the $\rho$-meson amplitudes are better reproduced than with the first set of input parameters. Notice that the $d_{3/2}$ amplitude of the $\omega$-meson was set equal to zero to ensure an unique eVMD solution for the fitting procedure.

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TABLE I. Resonances ($R$) included into the $NN \to NN\omega$ cross section through the two-step mechanism $NN \to NR, R \to N\omega$. The second column shows the total widths of the resonances. The third column shows the partial widths for $N\omega$ decays ($\sqrt{\Gamma_{N\omega}}$ in MeV$^{1/2}$). The next three columns show the partial-wave decomposition of the $N\omega$ widths, including signs of the amplitudes. The coupling constants $g_M$, $g_E$, and $g_C$ at the $\omega$ pole are given in the last three columns in units GeV$^{-1}$ for spin $J = 1/2$ and GeV$^{-(J+1)}$ for $J = l + 1/2$. The $N\omega$ decay modes are determined by fitting the photo- and electro-production data, results of the multichannel $\pi N$ partial-wave analyses and quark model predictions. In the case of the $N^*(1535)$, the results from the two different fits with a strong (s), respectively, and a weak (w) $N\omega$ branching are given.

| Resonance       | $\Gamma_0$ [MeV] | $\sqrt{\Gamma_{N\omega}}$ [MeV$^{1/2}$] | $N\omega$ | $N\omega$ | $N\omega$ | $g_M$ | $g_E$ | $g_C$ |
|-----------------|------------------|------------------------------------------|----------|----------|----------|------|------|------|
| $N^*(1535)_{1/2}^+$ | 150              |                                          | $s_{1/2}$ | $d_{3/2}$ | -28.03   | -42.67 |
|                 |                  |                                          | $w$: 1.43 | 0.05     | -4.22    | -6.34  |
| $N^*(1650)_{1/2}^+$ | 150              |                                          | $d_{1/2}$ | $d_{3/2}$ | $s_{3/2}$ |      |      |      |
|                 |                  |                                          | -0.97    | -0.02    | 2.01     | 4.14   |
| $N^*(1520)_{3/2}^+$ | 120              |                                          | $d_{1/2}$ | $d_{3/2}$ | $s_{3/2}$ |      |      |      |
|                 |                  |                                          | -0.02    | 0.03     | -0.02    | 2.07   | 4.22 | 6.34 |
| $N^*(1675)_{5/2}^+$ | 150              |                                          | $p_{1/2}$ | $p_{3/2}$ | $f_{3/2}$ |      |      |      |
|                 |                  |                                          | < 0.01   | < 0.01   | < 0.01   | 2.01   |      |      |
| $N^*(1440)_{1/2}^+$ | 350              |                                          | $p_{1/2}$ | $p_{3/2}$ | $f_{3/2}$ |      |      |      |
|                 |                  |                                          | < 0.01   | < 0.01   | < 0.01   | 2.07   |      |      |
| $N^*(1720)_{5/2}^+$ | 150              |                                          | $f_{1/2}$ | $f_{3/2}$ | $p_{3/2}$ |      |      |      |
|                 |                  |                                          | 5.69     | 5.29     | -2.09    | 0.14   | 0.14 | 0.14 |
| $N^*(1680)_{3/2}^+$ | 130              |                                          | $f_{1/2}$ | $f_{3/2}$ | $p_{3/2}$ |      |      |      |
|                 |                  |                                          | 0.71     | 0.09     | 0.58     | 0.40   | 1.34 | 1.75 |
$\sigma$ [mb] vs $\epsilon$ [GeV]

- **Saturne**
- **COSY-TOF**

- **full**
- **on-shell**
- **on-shell 5 bphs**