Cavity QED implementation of the multi-qubit refined Deutsch–Jozsa algorithm

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Received 29 October 2009, in final form 20 December 2009
Published 11 February 2010
Online at stacks.iop.org/JPhysB/43/055501

Abstract

We theoretically study the realization of a multi-qubit refined Deutsch–Jozsa (DJ) algorithm using resonant interaction of many Rydberg atoms with a single-mode microwave cavity, in which the multi-qubit controlled phase gates could be accomplished efficiently. We show how to achieve a multi-qubit refined DJ algorithm in high fidelity, even in the case of a weak cavity decay and some imperfection. We argue that the required operations in our scheme are almost within the present experimental possibilities.

(Some figures in this article are in colour only in the electronic version)

Quantum algorithms have displayed unusual power in improving the computational speed over their classical counterparts due to computational parallelism or interference effects. Among the most frequently mentioned algorithms [1–3], the Deutsch–Jozsa (DJ) algorithm is the simplest, but demonstrates the power of quantum mechanics by distinguishing the constant functions from the balanced functions using only one-step logical computation regardless of the input size. Experimentally, the original DJ algorithm [3] and/or its modified version (i.e. refined DJ algorithm) [4] have been implemented in the nuclear magnetic resonance (NMR) system [5, 6], quantum dot [7], linear optical system [8] and trapped ions [9]. Additionally, there have been some theoretical proposals for achieving the DJ algorithm using trapped electrons [10], polyatomic molecules [11], atomic ensembles [12], Josephson charge qubits [13] and cavity quantum electrodynamics (QED) [14–16].

In this paper, we will focus on an implementation of the multi-qubit refined DJ algorithm with Rydberg atoms simultaneously passing through a single-mode microwave cavity system. Relevant experiments have been carried out for the resonant interaction of two or three atoms with the cavity mode [17], the entanglement between two atoms in a microwave cavity [18] and the two-qubit phase gate [19]. Based on the available techniques, our scheme could have the following favourable characters: (i) by a smart encoding, we may accomplish the multi-qubit gating by one step, which helps to achieve a straightforward and fast implementation of the refined DJ algorithm. This makes the necessary operations more efficient with respect to the previous ideas [15, 16] using two-qubit conditional gates; (ii) as the DJ algorithm realized in our scheme is the refined version [4], the auxiliary qubits are not necessary, which might greatly simplify the experimental requirement particularly in the case of scalability; (iii) although the resonant interaction is unavoidably affected by the cavity decay, the fast implementation could effectively suppress the detrimental effect in the case of the weak cavity decay.

Let us first briefly review the main idea of the original DJ algorithm, which can distinguish constant functions \( f_C(x) \) from balanced functions \( f_B(x) \) by a single query of \( y = \{0, 1\}^N \) \( \mapsto \{0, 1\} \) [3]. In the original DJ algorithm [3], the function is characterized by the unitary operation \( U(x)|y\rangle = |x\rangle|y \oplus f(x)\rangle \), where \( x \) is an \( N \)-qubit input but the auxiliary qubit \( y \) must be prepared in the superposed state \( (|0\rangle - |1\rangle)/\sqrt{2} \).
which results in the final transformation

\[ U(x)|y\rangle = U(x)(|0\rangle - |1\rangle)/\sqrt{2} \]

\[ \rightarrow (-1)^{f(x)}|0\rangle - |1\rangle)/\sqrt{2}. \]  

(1)

|y\rangle seems superfluous because it remains unchanged during the operation process, although it actually plays a crucial role in the above DJ algorithm. This redundancy of the auxiliary qubit y is fully removed in the refined DJ algorithm [4], where the action of the f-controlled gate \( \hat{U}^{(N)}_\text{fc} \) can be denoted by \( \hat{U}^{(N)}_\text{fc}|x\rangle = (-1)^{f(x)}|x\rangle \) with \( N \) being the qubit number. The basic quantum circuit to perform this refined DJ algorithm is sketched in figure 1.

For \( N = 2 \), the operation \( \hat{U}^{(N)}_\text{fc} \) can be specifically expressed as

\[ \hat{U}^{(2)}_\text{fc} = \text{diag}((-1)^{f(00)}, (-1)^{f(01)}, (-1)^{f(10)}, (-1)^{f(11)}) \]

in the state space spanned by \{00\}, \{01\}, \{10\}, \{11\}). It has been shown that \( \hat{U}^{(N)}_\text{fc} \) can be reduced to a direct product of single-qubit operations if \( N \leq 2 \). This implies that the realization of the DJ algorithm could be considerably simple. However, for \( N \geq 3 \), the situation will become much more intricate. In what follows, we will mainly work on a three-qubit implementation of the refined DJ algorithm. Our scheme is directly extendable to many-qubit cases.

In the three-qubit case, along with two f-controlled gates \( \hat{U}^{(3)}_\text{fc} = \pm \text{diag}[1, 1, 1, 1, 1, 1, 1, 1, 1] \) corresponding to the constant functions \( f_\text{fc}(x) \), the number of \( \hat{U}^{(3)}_\text{fc} \) corresponding to the balanced functions \( f_\text{b}(x) \) is \( C_2^3 = 70 \). Actually, there is only one \( \hat{U}^{(3)}_\text{fc} \) and 35 nontrivial and distinct \( \hat{U}^{(3)}_\text{fc} \) if we take the symmetry into account and neglect the overall phase factors. So our task here is to implement 36 unitary transformations.

As the goal of the DJ algorithm is to differentiate the constant functions from the balanced functions, instead of finding how \( \hat{U}^{(N)}_\text{fc} \) works specifically, we may simply consider the case below with one balanced function and the corresponding f-controlled operation:

\[ \hat{U}^{(3)}_\text{fc} = \text{diag}[1, -1, 1, -1, 1, 1, 1, 1, 1, 1], \]  

(2)

where the state space is spanned by \{000\}, \{001\}, \{010\}, \{011\}, \{100\}, \{101\}, \{110\}, \{111\}. For clarity, we first consider an ideal situation without cavity decay, where three identical three-level atoms are input with identical velocities and simultaneously interact with the single-mode vacuum cavity field. The atomic internal states are denoted by \( |ij\rangle \), \( |gj\rangle \) and \( |ej\rangle \), with \( |ij\rangle \) decoupled from other two states throughout our scheme due to large detuning, as shown in figure 2(a). In units of \( \hbar = 1 \), the effective Hamiltonian in the interaction picture reads

\[ H_I = \sum_{j=1}^{3} \Omega_j (a^\dagger S_j^+ + a S_j^-), \]

where \( \Omega_j \) is the coupling constant of the \( j \)-th atom to the cavity mode, \( S_j^+ = |e_j\rangle\langle g_j| \) and \( S_j^- = |g_j\rangle\langle e_j| \) are the atomic spin operators for raising and lowering, respectively, and \( a^\dagger (a) \) is the creation (annihilation) operator for the cavity mode.

The effective Hamiltonian \( H_I \) is similar to the quantum computing model of trapped ions in linear trap [20], where the multi-qubit controlled phase flip (CPF) gate \( J_p = I - 2|\rho\rangle\langle \rho| \) with \( I \) the identity matrix and \( |\rho\rangle = |000\rangle, |001\rangle, \ldots, \) and \( |111\rangle \), could be achieved with a high success probability and high fidelity. In the present scheme, we will try to move such an idea to the cavity QED system for designing the CPF gate, which is essential to the f-controlled operation \( \hat{U}^{(N)}_\text{fc} \). Different from the time-varying coupling strength \( \Omega_j(t) \) in [20], however, the coupling strength in our scheme is time-independent, i.e.,

\[ \Omega_j = \Omega_0 \cos(2\pi z/\lambda_0) \exp(-r^2/w^2) \sim \Omega_0 \cos(2\pi z/\lambda_0) \]

\[ \sim \Omega_0 \cos(2\pi z/\lambda_0) \]

Figure 2. (a) Schematic setup for implementing the three-qubit refined DJ algorithm in the case that the f-controlled gate is \( \hat{U}^{(3)}_\text{fc} = \text{diag}[1, -1, 1, -1, 1, 1, 1, 1, 1, 1, 1] \), where the inset shows the atomic level structure, and \( D_1, D_2, D_3 \) are state-selective field-ionization detectors. (b) The detailed configuration of the unitary operations \( J_{Ri(R'j)}, J_{Ri(1j)}, J_{Ri(2j)} \) and \( J_{Ri(12)} \). The cavity is a microwave cavity sustaining a single mode with a standing-wave pattern along the z-axis. The atoms 1, 2 and 3 prepared in high-lying circular Rydberg states are sent through the cavity with a proper speed, resonantly interacting with the cavity mode. \( R_i (R'_i) \) with \( i = 1, 2, 3 \) denote Ramsey zones for performing single-qubit rotations as explained in the text.
where $r$ is the distance of the atom away from the cavity centre, and $\lambda_0$ and $w$ are the wavelength and the waist of the cavity mode, respectively. We use the same qubit definitions as in [20]; that is, the logic state $|0\rangle$ (|1\rangle) of qubit 1 is denoted by $|g_1\rangle$ ($|e_1\rangle$) of atom 1, $|g_2\rangle$ and $|i_2\rangle$ of atom 2 encode the logic state $|0\rangle$ (|1\rangle) of qubit 2, the logic state $|0\rangle$ (|1\rangle) of qubit 3 is represented by $|g_3\rangle$ ($|i_3\rangle$) of atom 3. If we assume the different atoms with coupling constants $\Omega_1 : \Omega_2 : \Omega_3 = 1 : 10 : 10$ and the gating time $T_0 = \pi/\Omega_1$, we can obtain an approximate three-qubit CPF gate $J_{i_{1,2,3}} = J_{i_{1,2,3},i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, depicted in figure 2(a), where $\Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$ in the computational subspace spanned by $|g_1\rangle|g_2\rangle|g_3\rangle$, $|g_1\rangle|i_2\rangle|g_3\rangle$, $|g_1\rangle|i_2\rangle|i_3\rangle$, $|e_1\rangle|g_2\rangle|g_3\rangle$, $|e_1\rangle|g_2\rangle|i_3\rangle$, $|e_1\rangle|i_2\rangle|g_3\rangle$, $|e_1\rangle|i_2\rangle|i_3\rangle$, $|e_1\rangle|i_2\rangle|i_3\rangle$, where $\alpha_i = \left[\Omega_1^2 \cos(\Theta_i) + \Theta_i^2 - \Omega_2^2\right]/\Theta_i^2 \approx 1$ with $i = 1, 2, 3$, and $\Theta_i = \sqrt{\Omega_1^2 + \Omega_2^2 + \Omega_3^2}$. Based on the gate $J_{i_{1,2,3}}$, we construct the $f$-controlled gate $U^{(3)}_{f_{m}}$ by a straightforward way, i.e.

$$U^{(3)}_{f_{m}} = J_{i_{1,2,3}} E_{R_{m}} E_{L_{m}} R_{m},$$

where $J_{i_{1,2,3}} = J_{i_{1,2,3},i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$. The other four indispensable CPF gates could be created as $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$, $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}/\Gamma_{i_{1,2,3}}$. Using these eight CPF gates, we could construct other 35 $f$-controlled gates $U^{(3)}_{f_{m}}$ ($m = 1, 2, \ldots, 35$), with each $U^{(3)}_{f_{m}}$ involving four different CPF gates.

Along with the above-mentioned CPF gates, three-qubit Hadamard gates $H^{(3)} = \prod_{i=1}^{3} H_i$ should be performed to encode the input and decode the output, respectively, as depicted in figure 2(a), where $H_i = [(|0\rangle_0 + |1\rangle_0)/\sqrt{2}]_0 + [(|0\rangle_1 + |1\rangle_1)/\sqrt{2}]_1$ is the Hadamard gate acting on the $i$th atom. These gates could be achieved using external microwave pulses. As a result, a full three-qubit refined DJ algorithm is available. Taking the $f$-controlled gate $U^{(3)}_{f_{m}}$ as an example, we have designed a three-qubit refined DJ algorithm setup in figure 2(b), where three Rydberg atoms prepared in the state $|\Psi_0\rangle = |g_1g_2g_3\rangle$ are initially encoded by the three-qubit Hadamard gate $H^{(3)}$, and then sent through the cavity with the identical velocities. The implementation could be simply described as

$$|\Psi_0\rangle \xrightarrow{H^{(3)}U^{(3)}_{f_{m}}H^{(3)}} |\Psi_f\rangle = \pm |000\rangle,$$

$$|\Psi_0\rangle \xrightarrow{H^{(3)}U^{(3)}_{f_{m}}H^{(3)}} |\Psi_f\rangle = \pm |000\rangle.$$

For simplicity, we assume here that we could individually address the atoms for implementing Hadamard gates. In real experiments, however, due to the long wavelength of the microwave, we have to employ inhomogeneous electric fields for single-qubit operations [22].

where $\ell$ is a normalized coefficient with $A_m, B_m, \ldots, G_m$ being 0 or $\pm 1$. It implies that, if $f(x)$ is constant, the state of the atoms becomes $|000\rangle$; but if $f(x)$ is balanced, the state of the atoms becomes a superposition state $|\Psi_f\rangle_{m}$, excluding the component $|000\rangle$. So we can efficiently determine whether the function is constant or balanced by a collective measurement on the output state of the three atoms.

Under the assumption of a weak cavity decay that no photon actually leaks out of the microwave cavity during our implementation, we may reconsider our scheme using the quantum trajectory method [21],

$$H_D = \sum_{j=1}^{3} \Omega_j (a^\dagger S_j + a S_j^\dagger) - \frac{\kappa}{2}a^\dagger a,$$

where $\kappa$ is the cavity decay rate. When we choose the atom–cavity interaction time $T_D = \pi/\Lambda_{1k}$ with $\Lambda_{1k} = \sqrt{\Omega_1^2 - \kappa^2/16}$, and meet the condition $\Omega_1 : \Omega_2 : \Omega_3 = 1 : 10 : 10$, the approximate three-qubit CPF gate in the decay case becomes $J_{i_{1,2,3}} = \Gamma_{i_{1,2,3}}$ with $\Gamma_{i_{1,2,3}} = \pi/\sqrt{\Omega_1^2 + \Omega_2^2 + \Omega_3^2}$, and the gating time becomes $T_D = \pi/\sqrt{\Omega_1^2 - \kappa^2/16}$.

For an initial state $|\Psi_0\rangle = |000\rangle$, after the operations in figure 2, the output state of the three atoms in the decay case is given by $|\Psi_f\rangle_{m} = R_{m}|000\rangle + A_m|001\rangle + B_m|010\rangle + C_m|011\rangle + D_m|100\rangle + E_m|101\rangle + F_m|110\rangle + G_m|111\rangle$, where $R_m \approx 0$ and $A_m, B_m, \ldots, G_m$ are slightly deviated from $A_m, B_m, \ldots, G_m$. In figure 3(a), we demonstrate the fidelity according to the following relation: $F_m = \langle \Psi_f\rangle_{m} |\Psi_f\rangle_{m} |\Psi_f\rangle_{m}$ [23], in which we have also considered the influence from the deviation $\Delta$ due to slightly different atomic velocities.

In addition to the imperfection considered above, there are other noise effects that are necessary to investigate, such as the resonant dipole interaction between two neighbouring Rydberg atoms. Since the typical dipole moment of a Rydberg atom is about several hundreds of $\mu_0 a_0$, where $a_0$ is the Bohr

\[ c_1 = (\Omega^2 |c_{2,4}\rangle + \Omega |c_{2,4}\rangle |\Theta_2\rangle + \Omega |c_2\rangle |\Theta_2\rangle + \Omega |c_2\rangle |\Theta_2\rangle), \]

\[ c_2 = \Gamma_1 e^{\pi/4} + \Omega |c_{2,4}\rangle |\Theta_2\rangle + \Omega |c_2\rangle |\Theta_2\rangle + \Omega |c_2\rangle |\Theta_2\rangle, \]

\[ c_3 = \Gamma_1 e^{\pi/4} + \Omega |c_{2,4}\rangle |\Theta_2\rangle + \Omega |c_2\rangle |\Theta_2\rangle + \Omega |c_2\rangle |\Theta_2\rangle, \]

\[ \Theta_2 = \sqrt{\Theta_2^2 - \kappa^2/16}, \]

\[ A_{1k} = \sqrt{\Theta_2^2 - \kappa^2/16}. \]
radius and $q_i$ is the electron charge, the dipole coupling strength $\delta$ between two neighbouring Rydberg atoms is in the same range as the cavity decay rate $\sim \text{kHz}$. Considering the system Hamiltonian (equation (6)), we take an additional term $TD$ with high fidelity, we have to suppress these above-mentioned consideration above. So to carry out our scheme efficiently and our scheme can still achieve a high fidelity. In a realistic $\delta$ being an even number for simplicity), we have the tracks of the nodes by $\lambda_0/(2\pi) \arccos(1/10)$, but other atoms through the antinodes. Specifically, for the $N$ input atoms (suppose $N$ being an even number for simplicity), we have the tracks of the atomic movement as $z_1 = -(N/2)\lambda_0 + (\lambda_0/2\pi) \arccos(1/10)$, $z_2 = -(N/2 - 1)\lambda_0, \ldots, z_N = (N/2)\lambda_0$. This is of course highly challenging with current experimental technology because we have not yet found any experimental report for $N \geq 3$ atoms simultaneously going through a microwave cavity. However, the two-atom entanglement in a microwave cavity has been achieved using van der Waals collision between the atoms [18] under a non-resonant condition. Moreover, our scheme could also be straightforwardly applied to other quantum information processing candidate systems, e.g. the ion-trap-cavity combinatory setup [25] or cavity-embedded optical lattices confining atoms [26], in which the atoms are localized very well.

Extending to the many-qubit case, we could construct the $N$-qubit CPF gate $\hat{V}_{[\ldots-1]}^{(N)} = \text{diag}[1, 1, 1, \ldots, -1]$ by meeting the condition for coupling constants $\Omega_1 \ll \Omega_2 = \Omega_3 = \ldots = \Omega_N = \Omega$ and by keeping the gating time $T_D = \pi/\sqrt{\Omega_1^2 - \kappa^2}/16$ unchanged. This implies that the CPF gate could be carried out by a constant time irrelevant to the qubit number, which is favourable for a scalable DJ algorithm in the cavity QED system. Since the single-qubit operation takes negligible time in comparison with that for the many-qubit CPF gate, we may roughly omit the single-qubit operation time, which yields the running time of the $N$-qubit refined DJ algorithm to be $T(N) = 2N - 1 \times T_D$. Figure 3(b) demonstrates an example of the four-qubit $f$-controlled gate $\hat{U}_{J_4}^{(4)} = \text{diag}[1, -1, 1, -1, 1, -1, 1, 1, -1, 1, -1, 1, 1, -1, 1]$, constructed by

$$\hat{U}_{J_4}^{(4)} = J_{R_1E_2S_3}J_{R_4E_1S_2}J_{R_1E_2S_3}J_{R_4E_1S_2}J_{R_1E_3S_4}J_{R_4E_1S_2}J_{R_1E_3S_4}J_{R_4E_2S_3}J_{R_1E_3S_4}J_{R_4E_2S_3}J_{R_1E_3S_4}.$$  

(7)

We briefly address the experimental feasibility of our scheme by considering three high-lying Rydberg atomic levels with principal quantum numbers 49, 50 and 51 to be levels $|g\rangle$, $|i\rangle$ and $|e\rangle$, respectively. Based on the experimental numbers reported in [24], the coupling strength at the cavity centre could be $\Omega_1 = 2\pi \times 51 \text{ KHz}$, and the Rydberg atomic lifetime is $30 \text{ ms}$. Specifically, assuming $\Omega_1 = 2\pi \times 51 \text{ KHz}$, $\Omega_1 = 2\pi \times 5.1 \text{ KHz}$ and $\kappa = 10^{-3}\Omega_1$, a direct calculation shows that the time for a single $N$-qubit CPF gate is $T_D = \pi/\sqrt{\Omega_1^2 - \kappa^2}/16 \approx 98 \mu s$, which is much shorter than either the cavity decay time, i.e. $2\pi/\kappa \approx 0.2 \text{ s}$ or the Rydberg atomic lifetime. Actually, the lifetime of the photon in the superconducting cavity has reached $0.5 \text{ s}$ recently [24]. To make the quantum trajectory method practicable, we require our implementation time to be much shorter than the cavity decay time, which yields the condition $\kappa \ll \Omega_1/2N^2$. Considering the values listed above, we find that $N$ could be 9.

In current microwave cavity experiments [21], the intra-atom interaction occurs in the central region of the cavity with the Rabi frequency $\Omega_j = \Omega_0 \cos(2\pi z/\lambda_0)$ [21]. To meet the condition $\Omega_1 = \Omega_1/10$, the $N$ atoms should be sent through the cavity with the first atom going along the $y$-axis deviating from the nodes by $(\lambda_0/2\pi) \arccos(1/10)$, but other atoms through the antinodes. Specifically, for the $N$ input atoms (suppose $N$ being an even number for simplicity), we have the tracks of the atomic movement as $z_1 = -(N/2)\lambda_0 + (\lambda_0/2\pi) \arccos(1/10)$, $z_2 = -(N/2 - 1)\lambda_0, \ldots, z_N = (N/2)\lambda_0$. This is of course highly challenging with current experimental technology because we have not yet found any experimental report for $N \geq 3$ atoms simultaneously going through a microwave cavity. However, the two-atom entanglement in a microwave cavity has been achieved using van der Waals collision between the atoms [18] under a non-resonant condition. Moreover, our scheme could also be straightforwardly applied to other quantum information processing candidate systems, e.g. the ion-trap-cavity combinatory setup [25] or cavity-embedded optical lattices confining atoms [26], in which the atoms are localized very well.

Alternatively, the superconducting circuit QED [27–29] would also be a suitable candidate for implementing our scheme. In the circuit QED [27, 28], a number of superconducting qubits play the role of artificial atoms and the quantum bus is provided by a 1D superconducting transmission line resonator. The dynamics of this system could be well described by the Jaynes–Cummings Hamiltonian, which is an essential requirement for our scheme. Implementing the multi-qubit refined DJ algorithm in the circuit QED has several merits that are worth mentioning here. First, the experiments performed in the circuit QED setup have achieved the long coherence time and strong coupling [28], which could greatly reduce the gating time and noise effects in our scheme. Secondly, the widely separate superconducting qubits are well localized in the circuit QED, so a fully in situ control over the qubit parameters could be easily made. Thirdly, the strong coupling between the resonator field and the qubits can be used to perform high-efficiency quantum nondemolition (QND) readout of the qubits by a single shot without the need for additional signal ports.

In conclusion, we have proposed a potentially practical scheme for realizing a multi-qubit refined DJ algorithm by resonant interaction of Rydberg atoms in a microwave cavity. We have estimated the influence from the cavity decay on our scheme and shown that our scheme could be achieved efficiently to distinguish the balanced functions from the constant functions with high fidelity. We argue that our present scheme would be helpful for the demonstration of the refined DJ algorithm at a large scale using various devices.
Acknowledgments

This work is supported by NNSF of China under no 10774163 and no 10774042.

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