Tree Level Mediation of Supersymmetry Breaking

Marco Nardecchia
SISSA/ISAS and INFN, I–34013 Trieste, Italy
E-mail: marco.nardecchia@sissa.it

Abstract. We propose a new scheme in which supersymmetry breaking is communicated to the MSSM sfermions by GUT gauge interactions at the tree level. The (positive) contribution of MSSM fields to Str($\mathcal{M}_2^2$) is automatically compensated by a (negative) contribution from heavy fields. Sfermion masses are flavour universal, thus solving the supersymmetric flavour problem. In the simplest SO(10) embedding, the ratio of different sfermion masses is predicted and differs from mSugra and other schemes, thus making this framework testable at the LHC. Gaugino masses are generated at the loop level but enhanced by model dependent factors.

We finally entered the LHC era. Hopefully, the ongoing experiments at CERN will clarify the nature of the electroweak symmetry breaking. The simplest and most economical solution to obtain the \( SU(2)_L \otimes U(1)_Y \) breaking is through the Higgs mechanism, and a light SM Higgs scalar is also preferred by the EWPT.

The presence of an elementary scalar in the theory poses new questions and an extension of the SM is expected at the TeV scale. In particular, if supersymmetry exists close to the TeV energy scale, it allows for a solution of the naturalness problem of the SM.

In the MSSM, supersymmetry is explicitly broken by the presence of the soft terms. Fully generic flavor-breaking structures in the soft terms are ruled out by experimental constraints. However, these constraints can be used to identify the restricted class of allowed soft terms, providing useful guidelines to understand the mechanism of the supersymmetry breaking and its mediation.

Recently [1] we considered a new option in which spontaneous supersymmetry breaking is communicated to the observable sector at the tree level through GUT gauge interactions.

Tree level supersymmetry breaking is sometimes considered not to be viable because the supertrace formula [2]. This clearly represents a problem if the only fermions in chiral superfields are the SM ones, as the experimental constraints rather require a significantly larger sfermion total squared mass.

In our scheme, the supertrace does vanish (in the full theory at the GUT scale), but the positive contribution from the MSSM matter fields is automatically compensated by a negative contribution from heavier chiral superfields. In order for this to work, a SM-neutral gauge U(1) in addition to the SM hypercharge is needed to avoid a stronger implication of the supertrace formula, which requires the lightest squark in either the up or down sector not to be heavier than the corresponding lightest quark [3]. The main features of our model arise from requiring that such an extra U(1) be part of a unified group.

Before presenting the model, let us motivate its gauge structure and field content. Our aim is to identify the supersymmetry breaking messengers with heavy vector superfields corresponding...
Figure 1: Tree level gauge mediation supergraph inducing a soft mass for the sfermion $\tilde{Q}$.

to broken generators, $X$, of a simple grand unified group, as illustrated in Fig. 1. There, $N'$ is a SM singlet superfield whose $F$-term breaks supersymmetry, $\langle N' \rangle = F \theta^2$ (the prime is there just for consistency with the notations used below). As $N'$ has to couple to the heavy vector $V$ associated to the broken generator $X$, $N'$ must belong to a non-trivial multiplet of the unified group. $Q$ represents a generic MSSM superfield. In the effective theory below $M_{GUT}$, the diagram in Fig. 1 induces a non-renormalizable contribution

$$-2g^2X_N X_Q (Q^\dagger Q N'^\dagger N')/M_V^2$$

to the Kähler potential, analogous to the ones of effective supergravity, but flavour universal ($X_{N,Q}$ are the $X$-charges of $N', Q$, $M_V$ is the vector mass). A sfermion mass $\tilde{m}_{\tilde{Q}}^2 = 2g^2X_NX_Q(F/M_V)^2$ is then generated. In the full theory at $M_{GUT}$, on the other hand, everything takes place at the renormalizable level. In fact, the sfermion masses arise because $N'$ couples to the broken generator $X$. As a consequence, its $F$-term generates a non-vanishing vev for the corresponding $D$-term

$$\langle DX \rangle = -2gX_N \left( \frac{F}{M_V} \right)^2,$$

which in turn induces the soft mass

$$\tilde{m}_{\tilde{Q}}^2 = -gX_Q\langle DX \rangle = 2g^2X_NX_Q \left( \frac{F}{M_V} \right)^2$$

for the sfermion $\tilde{Q}$.

Such a scheme requires specific gauge structures and field contents. First of all, the heavy vector field $V$ in Fig. 1 must be a SM singlet, as $N'$ is. Then, SU(5) does not provide viable candidates for the gauge messenger $V$ and the minimal option is identifying the broken generator with the SU(5) singlet generator $X$ of SO(10). As for the SM singlet $N'$ whose $F$-term breaks supersymmetry, it must belong to a non-trivial SO(10) multiplet such that $N'$ has a non-vanishing charge under $X$. Limiting ourselves to representations with dimension $d < 126$, the only possibility is that $N'$ be the singlet component of a spinorial representation, 16 or $\overline{16}$. We also need a $16 + \overline{16}$ participating to SO(10) breaking at the GUT scale. At least two $16 + \overline{16}$ are then required, one getting a vev along the scalar component and the other along the $F$-term component. Finally, the standard embedding of a whole MSSM family into a 16 of SO(10) would not work, as it would lead to negative sfermion masses for some of the sfermions. That is why we distribute the matter fields in three 16 and three 10 of SO(10).

Having motivated some of its features, we now illustrate a minimal model satisfying the above requirements. The gauge group is SO(10). The matter fields (negative $R$-parity) are three $16_i = (\overline{5}_i^{16}, 10_i^{16}, 1_i^{16})$ and three $10_i = (\overline{5}_i^{10}, 5_i^{10})$, $i = 1, 2, 3$, where the SU(5) decomposition is also indicated. Supersymmetry and SO(10) breaking to SU(5) are provided by $16 = (\overline{5}^{16}, 10^{16}, N)$,
\[ \mathbf{16} = (5^{16}, \overline{10}^{16}, \overline{N}), \mathbf{16}' = (5^{16}, 10^{16}, N'), \overline{\mathbf{10}} = (5^{16}, \overline{10}^{16}, \overline{N'}) \] (positive $R$-parity), with
\[ \langle N' \rangle = F \theta^2 \quad \langle \overline{N} \rangle = 0 \quad \langle N \rangle = M \quad \langle \overline{N} \rangle = M, \] (3)
\[ \sqrt{F} \ll M \sim M_{\text{GUT}}. \] The D-term condition forces $|\langle N \rangle| = |\langle \overline{N} \rangle|$ and the phases of all the vevs can be taken positive without loss of generality. The MSSM up Higgs $h_u$ is embedded in a $10 = (5^{10}, \overline{5}^{10})$ of SO(10), while the down Higgs $h_d$ is a mixture of the doublets in the 10 and the 16,
\[ 10 = h_u + c_d h_d + \text{heavy}, \quad 16 = s_d h_d + \text{heavy}, \] (4)
where $c_d = \cos \theta_d$, $s_d = \sin \theta_d$ and $0 < \theta_d < \pi/2$ parametrizes the mixing in the down Higgs sector.

At this point we are in the condition of calculating the sfermion masses induced by integrating out the heavy vector fields:
\[ m_Q^2 = \frac{X_Q}{2X_N} m^2, \quad m = \frac{F}{M}. \] (5)

In the normalization we use for $X, X_N = 5$. In order to determine the $X$ charge of the SM fermions we need to specify their embedding in the matter fields $16_i, +10_i$. We do that by first writing the most general $R$-parity conserving superpotential, except a possible mass term for the $10_i$, as
\[ W = \frac{y_{ij}}{2} 16_i 16_j 10 + h_{ij} 16_i 10 + h'_{ij} 16_i 10 + h_{\text{vev}} 16_i 10 + W_{\text{NR}}, \] (6)
where $W_{\text{vev}} = W_{\text{vev}}(16, \overline{16}, \mathbf{10}, \ldots)$ does not involve the matter fields and takes care of the vevs, the doublet triplet splitting, and the Higgs mixing, and $W_{\text{NR}}$ contains non-renormalizable contributions to the superpotential needed in order to account for the measured ratios of down quark and charged lepton masses (we will ignore such issue here).

We can now see that the vev of the 16 gives rise to the mass term $h_{ij} M_{16}^{16} 5_{10}^{10}$, which makes the $\overline{5}_{16}^i$ and $5_{10}^{10}$ heavy. Only the MSSM superfield content survives at the electroweak scale (assuming the three singlets in the 16_i get mass e.g. from non-renormalizable interactions with the \[ \mathbf{16}. \] Moreover, the three MSSM families turn out to be embedded in the three \[ 10_i^{16}, \] with $X = 1$ and in the three \[ \overline{5}_{10}^{10}, \] with $X = 2$. We can then go back to eq. (5) and obtain
\[ m_{q}^2 = m_{c}^2 = m_{e}^2 = m_{\overline{10}}^2 = \frac{1}{10} m^2, \quad m_{l}^2 = m_{\overline{5}^2} = m_{\overline{5}}^2 = \frac{1}{5} m^2 \] (7)
\[ m_{h_u}^2 = -\frac{1}{5} m^2, \quad m_{h_d}^2 = \frac{2c_d^2 - 3s_d^2}{10} m^2 \] (8)
at the GUT scale. The result in eq. (7) is quite general, as it only depends on the choice of the gauge group and on the embedding of the three MSSM families in the \[ 10_i^{16}, \overline{5}_{10}^{10}. \] We note a few interesting features of this result.

- All the sfermion masses turn out to be positive. This is because the negative $X$ charges (which must be there as $X$ is traceless) happen to be associated to the fields that get an heavy supersymmetric mass.
- The sfermions masses are flavour universal, thus solving the supersymmetric flavour problem.
- The sfermions masses belonging to the 10 and 5 of SU(5) are related by
\[ m_{q,u,c,e}^2 = \frac{1}{2} m_{d,e}^2 \] (9)
at the GUT scale, a peculiar prediction that allows to distinguish this model from mSugra, gauge mediation, and other models of supersymmetry breaking.
Let us now consider gaugino masses. While the tree-level prediction for the sfermion masses, eq. (7), only depends on the choice of the unified gauge group and the MSSM embedding, gaugino masses arise at one loop, as in standard gauge mediation, and depend on the superpotential parameters. The chiral multiplets $\tilde{\psi}_i^{16}$ and $\tilde{\psi}_j^{10}$ get an heavy supersymmetric mass $h_{ij} M$ and their scalar components get a supersymmetry breaking mass $h_{ij} F$. They play the role of three pairs of chiral messengers in standard gauge mediation and give rise to one loop gaugino masses. The contribution of each messenger arises at a different scale. In the one loop approximation for the RGE running, the total gaugino masses at lower scales can be calculated by running effective GUT-scale gaugino masses given by

$$M_a = \frac{\alpha}{4\pi} \text{tr}(h'h^{-1}) m \equiv M_{1/2}, \quad a = 1, 2, 3,$$

(10)

where $\alpha$ is the unified coupling.

Let us compare gaugino and sfermion masses. Particularly interesting is the ratio $\tilde{m}_t/M_2$. In fact, the $W$-ino mass $M_2$ is at present bounded to be heavier than about 100 GeV, while $\tilde{m}_t$ enters the radiative corrections to the Higgs mass. Therefore, the ratio $\tilde{m}_t/M_2$ should not be too large in order not to increase the fine-tuning and not to push the stops and the other sfermions out of the LHC reach. From

$$\frac{M_2}{\tilde{m}_t}|_{\text{M}_{\text{GUT}}} = \frac{3\sqrt{10}}{(4\pi)^2} \lambda, \quad \lambda = \frac{g^2 \text{tr}(h'h^{-1})}{3}$$

(11)

we see first of all that the loop factor separating $\tilde{m}_t$ and $M_2$ is partially compensated by a combination of numerical factors: $(4\pi)^2 \sim 100$ (leading to $\tilde{m}_t \gtrsim 10$ TeV for $\lambda = 1$) becomes $(4\pi)^2/3\sqrt{10} \sim 10$ (leading to $\tilde{m}_t \gtrsim 1$ TeV for $\lambda = 1$). Note that the factor $\sqrt{10}$ is related to the ratio of $X$ charges in eq. (5) and the factor 3 corresponds to the number of families ($\text{tr}(h'h^{-1}) = 3$ for $h = h'$).

Next, we comment on the $\mu$ problem. Relating the $\mu$-term to supersymmetry breaking is a highly model-dependent issue, but in our scenario we have a simple possibility in which both the $F$-term, $(N') = F \theta^2$ and $\mu$ originate from the same parameter $m \sim$ TeV in the superpotential: $W \supset m N N'$. Once $N$ is forced to get its vev $(N) = M \sim M_{\text{GUT}}$, $N'$ acquires an $F$-term $F = m M$ (so that $m$ is indeed the parameter introduced in eq. (5)). In our setup, $N'$ and $\overline{N}$ are part of the SO(10) multiplets $16'$ and $\overline{16}$ respectively. A $\mu$ term related to the supersymmetry breaking scale $\mu \sim m$ is then therefore generated if $h_u$ has a component in $\overline{16}$ and $h_d$ has a component in $16'$. Such a situation can be achieved with an appropriate superpotential. Contrary to standard gauge mediation, there is no $\mu$-$B\mu$ problem here, as $B\mu/\mu$ is not enhanced by an inverse loop factor. $B\mu$ can be generated at the tree level, for example as in [4], or it can be generated by the RGE evolution.

We now illustrate an example of low energy spectra that can be obtained in our framework. We neglect the (small, for our purposes) effect of the intermediate scale $\tilde{\psi}_i^{16}$ and $\tilde{\psi}_j^{10}$ and use the MSSM RGE equations, with boundary conditions at high energy as in eqs. (7,8), the $A$-terms set to zero. We assume the messenger mass to coincide with the GUT scale, $M = M_{\text{GUT}}$. The overall normalization of the unified gaugino masses $M_{1/2}$ can be considered as a free parameter due to the presence of the factor tr($h'h^{-1}$) in eq. (10), or equivalently of the factor $\lambda$ in eq. (11). As the size of the parameters $\mu$ and $B\mu$ is model dependent, we consider them as free parameters as well and recover them as usual in terms of $M_2$ and $\tan \beta$.

Table 2 shows the low-energy spectrum corresponding to $\theta_d = \pi/6$, $\tan \beta = 30$ and $\text{sign}(\mu) = +$. The common gaugino mass is $M_{1/2} = 150$ GeV, near the minimal value allowed at
| Concept       | Mass (GeV) |
|--------------|------------|
| Higgs        |            |
| $m_{h^0}$    | 114        |
| $m_{H^0}$    | 1543       |
| $m_{A^0}$    | 1543       |
| $m_{H^\pm}$ | 1545       |
| Gluinos      |            |
| $\tilde{M}_g$| 448        |
| Neutralinos  |            |
| $m_{\tilde{\chi}_1^0}$| 62    |
| $m_{\tilde{\chi}_2^0}$| 124   |
| $m_{\tilde{\chi}_3^0}$| 1414  |
| $m_{\tilde{\chi}_4^0}$| 1415  |
| Charginos    |            |
| $m_{\tilde{\chi}_1^\pm}$| 124   |
| $m_{\tilde{\chi}_2^\pm}$| 1416  |
| Squarks      |            |
| $m_{\tilde{u}_L}$ | 1092  |
| $m_{\tilde{u}_R}$ | 1027  |
| $m_{\tilde{d}_L}$ | 1095  |
| $m_{\tilde{d}_R}$ | 1494  |
| $m_{\tilde{t}_1}$ | 1007  |
| $m_{\tilde{t}_2}$ | 1038  |
| $m_{\tilde{b}_1}$ | 1069  |
| $m_{\tilde{b}_2}$ | 1435  |
| Sleptons     |            |
| $m_{\tilde{e}_L}$ | 1420  |
| $m_{\tilde{e}_R}$ | 1091  |
| $m_{\tilde{\tau}_1}$ | 992   |
| $m_{\tilde{\tau}_2}$ | 1387  |
| $m_{\tilde{\nu}_e}$ | 1418  |
| $m_{\tilde{\nu}_\tau}$ | 1382  |

Figure 2: An example of spectrum, corresponding to $m = 3.2$ TeV, $M_{1/2} = 150$ GeV, $\theta_d = \pi/6$, $\tan \beta = 30$ and $\text{sign}(\mu) = +$, $A = 0$, $\eta = 1$. All the masses are in GeV, the first two families have an approximately equal mass.

present by chargino direct searches. The value of $m$ is near the minimal value allowed by the bound $m_h > 114$ GeV. This spectrum corresponds to $\lambda = 2.5$. Given the (moderate) hierarchy between $M_{1/2}$ and the sfermion masses, the sfermion RGEs are not significantly affected by the gaugino masses and the sfermion mass relations characterizing the model, eq. (7), survive, to some extent, at low energy.

Finally, we comment about cosmology. As in loop gauge mediation, the LSP is the gravitino. In fact, the supersymmetry breaking parameter is given by

$$\sqrt{F} \approx 0.8 \cdot 10^{10} \text{GeV} \left( \frac{\tilde{m}_{10}}{\text{TeV}} \frac{M}{2 \cdot 10^{16} \text{GeV}} \right)^{1/2}$$  \hspace{1cm} (12)

and the gravitino mass by

$$m_{3/2} = \frac{F}{\sqrt{3}M_p} \approx 15 \text{GeV} \left( \frac{\tilde{m}_{10}}{\text{TeV}} \frac{M}{2 \cdot 10^{16} \text{GeV}} \right),$$  \hspace{1cm} (13)
where $\tilde{m}_{10}$ is the tree-level mass of the sfermions in the 10 of SU(5) at the GUT scale. Note that $F$ and the gravitino mass are smaller than in loop gauge mediation, for a given messenger scale $M$, because of the absence of a loop factor in eqs. (12,13). For a stable (on the age of the universe timescale) gravitino with a mass as large as in eq. (13), a dilution mechanism such as inflation is necessary in order for the its energy density not to exceed the dark matter one. The upper bound on the reheating temperature $T_R$ depends on the gravitino and the gaugino masses [5].

The thermal contribution to the gravitino energy density, for a reheating temperature around $10^9$ GeV is given by

$$\Omega_{\tilde{G}}^{TP} h^2 \approx 6 \times 10^{-2} \left( \frac{T_{RH}}{10^9 \text{ GeV}} \right) \left( \frac{15 \text{ GeV}}{m_{3/2}} \right) \left( \frac{M_{1/2}}{150 \text{ GeV}} \right)^2.$$  \hspace{1cm} (14)

For the spectrum in Table 2, the bound $\Omega_{\tilde{G}}^{TP} h^2 \leq \Omega_{DM} h^2 = 0.11$ translates in $T_R < 2 \cdot 10^9$ GeV.

We then have to take care of the decays of the NLSP into the gravitino, which might spoil big bang nucleosynthesis (BBN) unless it is fast enough. The fate of BBN depends on what the NLSP is. In the bulk of the parameter space we expect the NLSP to be the lightest neutralino or a stau. In the example in Table 2, the NLSP is essentially a Bino. For $m_{3/2} \sim 15$ GeV, the decay of a Bino NLSP into its coupling to the Goldstino component of the gravitino is way too slow (one would need $m_{3/2} < 100 \text{MeV}$ in order not to spoil BBN [6]). A Bino NLSP therefore requires a much faster decay channel. The latter can be provided by a tiny amount of $R$-parity violation [7]. Such a possibility is also consistent with thermal leptogenesis and gravitino dark matter. The other possibility is that the NLSP is a stau. In this case, all the BBN constraints can be satisfied if the lifetime of the stau is $\tau_{\tilde{\tau}} \approx 48 \pi m_{3/2}^2 M_F^2 / m_{\tilde{\tau}}^2 \lesssim 6 \cdot 10^3 \text{s}$ [8]. This is a viable possibility, which however requires large $\lambda = \mathcal{O}(100)$ and sizable gaugino masses. For such large values of $\lambda$, radiative contributions to sfermion masses (from RGEs and the standard gauge mediation contribution) dominate over the tree level one, the spectrum approaches the usual one loop gauge mediated one, and the peculiar relation between sfermion masses at the messenger scale gets hidden.

In conclusion, we have shown that is possible to communicate the supersymmetry breaking through a tree level renormalizable exchange of a gauge (GUT) messenger, as in Fig. 1. This scheme solves the supersymmetric FCNC problem and, in its simplest implementation, leads to peculiar relations among sfermion masses that can be tested at the LHC.

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