Proton Spin Based On Chiral Dynamics

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Abstract

Chiral spin fraction models agree with the proton spin data only when the chiral quark-Goldstone boson couplings are pure spinflip. For axialvector coupling from soft-pion physics this is true for massless quarks, and at high momentum for light quarks. Axialvector quark-Goldstone boson couplings with constituent quarks are found to be inconsistent with the proton spin data.

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I. INTRODUCTION

The nonrelativistic quark model (NQM) explains qualitatively many of the strong, electromagnetic and weak interaction properties of the nucleon and other octet and decuplet baryons in terms of three valence quarks whose dynamics is motivated by quantum chromodynamics (QCD), the gauge field theory of the strong interaction. Due to the spontaneous chiral symmetry breakdown ($\chi$SB) of QCD, the effective degrees of freedom at the scale $\Lambda_{QCD}$ are expected to be quarks and Goldstone bosons.

Here we point out that a naive use of a constituent quark mass for all observables, and in axialvector quark-Goldstone boson couplings in particular, leads to disagreement with the proton spin fractions, because the non-spinflip contributions dominate over spinflip at low momentum.

Understanding the internal proton structure is one of the goals of particle physics and, over the past dozen years or more, has led to extensive studies of the spin and flavor contents of the nucleon in terms of measured deep inelastic structure functions (DIS) [1,2]. Since spin fractions ultimately must derive from and be consistent with polarized DIS proton structure functions that are integrated over Bjorken $x$, we have used such a general DIS formalism [3] to analyze recent chiral models that have succeeded in reproducing the spin fractions of the proton.

The effects of chiral dynamics on the spin fractions are discussed in Sects. II and III in the framework of chiral field theory applied to deep inelastic scattering.
II. QUARK SPIN FRACTIONS FROM CHIRAL DYNAMICS

Chiral field theory involves the effective strong interactions commonly used in chiral perturbation theory (\(\chi PT\)) and applies at scales from \(\Lambda_{QCD}\) up to the chiral symmetry restoration scale \(\sim \Lambda_{\chi} \approx 4\pi f_\pi \approx 1.17\ \text{GeV}\), where \(f_\pi = 0.093\ \text{GeV}\) is the pion decay constant.

If the chiral symmetry breakdown is based on \(SU(3)_L \times SU(3)_R\), then the effective interaction between quarks and the octet of Goldstone boson (GB) fields \(\Phi_i\) involves the axial vector coupling

\[
\mathcal{L}_{\text{int}} = -\frac{g_A}{2f_\pi} \sum_{i=1}^{8} \bar{q}\gamma^\mu \gamma^5 \lambda_i \Phi_i q
\]

that is well known from soft-pion physics. In Eq. (1), \(g_A\) is the dimensionless axial vector-quark coupling constant that is taken to be 1 here. As a consequence, the polarization of quarks flips in chiral fluctuations, \(q_{1,\uparrow} \rightarrow q_{i,\uparrow} + GB\), into pseudoscalar mesons of the \(SU(3)\) flavor octet of Goldstone bosons, but for massive quarks the non-spinflip transitions from \(\gamma^\pm \gamma^5 k^\pm\) that depend on the quark masses are not negligible. Let us also emphasize that, despite the non-perturbative nature of the chiral symmetry breakdown, the interaction between quarks and Goldstone bosons is small enough for a perturbative expansion to apply.

Chiral field theory dissolves a dynamical or constituent quark into a current quark and a cloud of virtual Goldstone bosons. In this context, it was first shown in [5] that chiral dynamics leads to a reduction of the proton spin fractions carried by the valence quarks and also to a reduction of the axial vector coupling constant \(g_A^{(3)}\), based on one overall chiral strength parameter, \(a\), that contains the scale (cf. Table 1). It is well known that relativistic effects reduce the axial charge further, and this causes problems for the spin fractions [4]. In addition, the violation of the Gottfried sum rule [3], which signals an isospin asymmetric quark sea in the proton, i.e. \(\bar{u} < \bar{d}\), became plausible. \(SU(3)\) symmetric chiral spin fraction models explain spin and sea quark observables of the proton, except for the ratio of axial charges \(\Delta_3/\Delta_8 \approx 5/3\) and the weak axial vector coupling constant of the nucleon, \(g_A^{(3)} = \mathcal{F} + \mathcal{D}\). In [3], the effects of \(SU(3)\) breaking (adding \((1 + \lambda_8 \epsilon)\) in \(\mathcal{L}_{\text{int}}\)) were more systematically built into these chiral models and shown to lead to a remarkable further improvement of the spin and quark sea fractions in comparison with the data.

It was first shown in ref. [4], and subsequently confirmed [10,11] that the \(\eta'\) meson, proposed in [12] mainly to decrease the antiquark fraction \(\bar{u}/\bar{d}\) from the \(SU(3)\) symmetric value \(3/4\) to \(\sim 0.5\), gives an almost negligible contribution to the spin fractions of the nucleon, not only because of its large mass but also due
to the small singlet chiral coupling constant. It is therefore often ignored, and this is consistent with the understanding that, due to the axial anomaly, the $\eta'$ meson is not a genuine Goldstone boson. Pions and kaons are well established Goldstone bosons. For a discussion of the controversial role of the $\eta$ meson as the hypercharge or octet Goldstone boson, see [13].

Chiral fluctuations occur with probability densities $f(u^\uparrow \to \pi^+ + d^\downarrow),...$ which, from Eq. 1, may be written as coefficients in the following chiral reactions:

\begin{align*}
  u^\uparrow &\to f_{u^\to\pi^+d}(x_\pi, \vec{k}_\perp)(\pi^+ + d^\downarrow) + f_{u^\to\eta u} \frac{1}{6}(\eta + u^\downarrow) + f_{u^\to\pi^0 u} \frac{1}{2}(\pi^0 + u^\downarrow) \\
  &\quad + f_{u^\to K^+ s}(K^+ + s^\downarrow), \\
  d^\uparrow &\to f_{d^\to\pi^- u}(\pi^- + u^\downarrow) + f_{d^\to\eta d} \frac{1}{6}(\eta + d^\downarrow) + f_{d^\to\pi^0 d} \frac{1}{2}(\pi^0 + d^\downarrow) \\
  &\quad + f_{d^\to K^0 s}(K^0 + s^\downarrow), \\
  s^\uparrow &\to f_{s^\to\eta u}(\eta + u^\downarrow) + f_{s^\to\pi^- d}(\pi^- + d^\downarrow) + f_{s^\to\bar{K}^0 d}(\bar{K}^0 + d^\downarrow),
\end{align*}

(2)

and corresponding ones for the other initial quark helicity. The factors $1/2, 1/6, 1, ...$ in Eq. 2 for $u^\to\pi^0 u, u^\to\eta u, u^\to\pi^+ d, ...$, respectively, originate from the flavor dependence in Eq. 1 and are denoted as $p_m$ for the Goldstone boson $m$ for brevity. After integrating over transverse momentum in the infinite momentum frame, the coefficients in Eq. 2 become the polarized (− sign) and unpolarized (+ sign) chiral splitting functions,

\begin{equation}
P^{+}_{GB/q}(x) = \int d^2 k_\perp f_{q^\to q'GB}(x, \vec{k}_\perp),
\end{equation}

(3)

The unpolarized splitting function $P^+$ determines the (spinflip plus non-spinflip) probability for finding a Goldstone boson of mass $m_{GB}$ carrying the longitudinal momentum fraction $x_{GB}$ of the parent quark’s momentum and a recoil quark $q'$ with momentum fraction $1 - x_{GB}$ for each fluctuation in Eq. 2.

In (non-renormalizable) chiral field theory with cutoff $\Lambda_{\chi}$ of ref. [5], the unpolarized chiral splitting function takes the form

\begin{equation}
P^{+}_{q^\to q'+GB}(x_{GB}) = \frac{g_A^2}{f_\pi} \frac{x_{GB}}{32\pi^2} (m_q + m_{q'})^2 \int_{-\Lambda^2}^{t_{max}} dt \frac{(m_q - m_{q'})^2 - t}{(t - m_{GB}^2)^2},
\end{equation}

(4)

where $t = k^2 = -(k_\perp)^2 + x_{GB}(m_{q'})^2 - (1 - x_{GB})(m_q)^2)/(1 - x_{GB})$ is the square of the Goldstone boson four-momentum. The polarized splitting function is obtained using that it contains the difference of non-flip and helicity-flip probabilities. Since quarks are on their mass shell in the light front dynamics.
used here, the axialvector quark-Goldstone boson interaction is equivalent to the simpler $\gamma_5$ coupling. Except for an overall factor, the relevant unpolarized chiral transition probability is proportional to

$$-\frac{1}{2} tr[(\gamma \cdot p + m_q)\gamma_5(\gamma \cdot p + m_q)\gamma_5] = 2(pp' - m_q m'_q) = (m_q - m'_q)^2 - k^2, \quad (5)$$

where $2pp' = m_q^2 + m_q^2 - k^2$. Eq. 5 is the numerator in Eq. 4 which can also be written as

$$\frac{1}{1 - x_{GB}}[(k_\perp)^2 + [m'_q - (1 - x_{GB})m_q]^2], \quad (6)$$

and has the following physical interpretation. Recall that the axialvector quark-Goldstone boson coupling $\gamma_\mu \gamma_5 k^\mu$ in Eq. 1 involves the spin raising and lowering operators $\sigma_1 \pm i\sigma_2$ in a scalar product with the transverse momentum $\vec{k}_\perp$ of the recoil quark, which suggests that the $k_{\perp}^2$ term in $P^+$ of Eq. 6 represents the helicity-flip probability of the chiral fluctuation, while the longitudinal and time components, $\gamma_\pm \gamma_5 k^\pm$, induce the non-spinflip probability, which depends on the quark masses. This can be seen from the helicity non-flip probability

$$|\bar{u'}_\uparrow \gamma_5 u_\uparrow|^2 = |\bar{u'}_\downarrow \gamma_5 u_\downarrow|^2 \sim (m_q' - x'm_q)^2, \quad x' = 1 - x_{GB}, \quad (7)$$

using light cone spinors and suppressing the spinor normalizations. Thus Eq. 6 identifies the mass term in Eq. 6 as the helicity non-flip chiral transition. Similarly, the helicity-flip probability is obtained from

$$|\bar{u'}_\downarrow \gamma_5 u_\uparrow|^2 \sim (p'_\perp)^2 + x'^2(p_{\perp})^2 - x'p'_\perp \cdot p_{\perp} \quad (8)$$

which, in frames where $p_{\perp} = 0$, reduces to $(k_{\perp})^2$, i.e. the net helicity flip probability generated by the chiral splitting process. In the nonrelativistic limit, where $|p'_\perp| \ll m_q', |\vec{p}_\perp| \ll m_q$, clearly non-spinflip dominates over spinflip, while spinflip dominates at high momentum (it is not clear how ref. reached the opposite conclusion which, obviously, flies in the face of an extensive low-energy nuclear physics lore).

The polarized splitting function $P^-$ therefore has the same quark mass dependence as $P^+$, but involves the helicity flip probability with the opposite sign, i.e. has

$$\frac{1}{1 - x_{GB}}[-(k_{\perp})^2 + (m_q' - (1 - x_{GB})m_q)^2] \quad (9)$$

in its numerator, which agrees with ref. Only for massless quarks there are no non-spinflip chiral transitions, so that $P^- = -P^+$ holds which is characteristic of pure spinflip chiral transitions.
The splitting of quarks into a Goldstone boson and a recoil quark corresponds to a factorization of DIS structure functions that leads to a convolution of quark distributions with splitting functions. Thus, chiral fluctuations in lowest order of perturbation theory contribute convolution integrals

\[
\sum_{q,m} p_m P^{+}_{u\downarrow \rightarrow q\downarrow m} \otimes u^0_{\uparrow} + \sum_{q,m} p_m P^{+}_{d\uparrow \rightarrow q\downarrow m} \otimes d^0_{\uparrow},
\]

\[
\sum_{q,m} p_m P^{+}_{u\downarrow \rightarrow +q\uparrow m} \otimes u^0_{\downarrow} + \sum_{q,m} p_m P^{+}_{d\downarrow \rightarrow q\uparrow m} \otimes d^0_{\downarrow}
\]

(10)

to the quark distributions \(q\downarrow, \uparrow\), respectively. Chiral fluctuations also cause a reduction of the valence quark probabilities

\[
(1 - P_q)q^0_{\uparrow, \downarrow},
\]

(11)

where \(P_q\) are the total fluctuation probabilities.

III. SPIN DISTRIBUTION RESULTS

In chiral dynamics, antiquarks originate only from the Goldstone bosons via their standard quark-antiquark composition. Therefore, antiquarks are unpolarized. Small antiquark polarizations are consistent with the most recent SMC data [1], so that we expect only small corrections if we use \(\bar{u}\uparrow = \bar{u}\downarrow\) in the spin fractions \(\Delta u = u\uparrow - u\downarrow + \bar{u}\uparrow - \bar{u}\downarrow\), etc., i.e. \(\Delta s = \Delta s_{sea}, \Delta \bar{u} = \Delta \bar{d} = \Delta \bar{s} = 0\).

Let us now return to the polarized quark distributions and their lowest moments, the spin fractions.

Upon generalizing the chiral spin fraction formalism of [9,10,12] to the polarized quark distributions, the probabilities displayed in Eq. 2,10,11 yield

\[
q_\uparrow(x) = (1 - P_q)q^0_\uparrow(x) + \sum_{m,q'} p_m P^{+}_{q\downarrow \rightarrow q\downarrow m} \otimes q^0_{\downarrow} + ...
\]

(12)

which obviously are based on pure spinflip chiral transitions. The corresponding result holds for the other quark helicity. The ellipses in Eq. 12 denote double convolution terms with \(q^0_{\downarrow}\) from a Goldstone boson \(m\) that cancel in \(q\uparrow - q\downarrow\). The opposite quark helicity on the rhs of Eq. 12 implies the negative sign of all chiral contributions to the spin distributions

\[
\Delta q(x) = (1 - P_q)\Delta q^0(x) - \sum_{m,q'} p_m P^{+}_{q\downarrow \rightarrow q\downarrow m} \otimes \Delta q^0_{\downarrow}.
\]

(13)

This result is common to all recent successful chiral models of spin fractions. Let us emphasize that the general reduction of the valence quark spin fractions
\( \Delta q^0 \) by chiral fluctuations in lowest order in Eq. [13] is the crucial property responsible for the remarkable success of chiral field theory for the proton spin fractions. Eq. [13] can be compared to the corresponding one from DIS involving the polarized splitting functions

\[
P^{-q \rightarrow q' \text{GB}}(y) = \int d^2 k_\perp f^{-q \rightarrow q' \text{GB}}(y, \vec{k}_\perp),
\]

which has the same form as Eq. [13]

\[
\Delta q(x) = (1 - P_q) \Delta q^0(x) + \sum_{m,q'} p_m p^{-q \rightarrow q'm} \otimes \Delta q^0
\]

except for the replacement of \(-P^+\) by the corresponding polarized splitting function \(P^-\). A comparison with Eqs. [6,9] shows that this approximation, \(P^- \approx -P^+\), corresponds to neglecting the non-spinflip probability, which is valid only if the quark mass term in the numerator (i.e. Eq. [9] of the splitting functions is negligible compared to the transverse momentum scale. This is not the case for constituent quarks [3,14].

Thus, when these \(\Delta q(x)\) of Eq. [15] are integrated over Bjorken \(x\), the lowest moments reproduce precisely the structure of the results for the spin fractions (cf. Table 1).

**TABLE I. Quark Spin Observables of the Proton (from ref. [9]), \(a = \text{chiral strength}, \zeta = \text{relative singlet to octet coupling and } \epsilon = \text{SU}_3 \text{ breaking parameter in } \lambda_8 \text{ direction.}**

| Data          | NQM       | \(a = 0.12\) | \(a = 0.12\) |
|---------------|-----------|--------------|--------------|
| E143 [2]      |           |              |              |
| at 3 GeV\(^2\)| 0.84±0.05 | 0.83         | 0.81         |
| SMC [1]       | 0.82±0.02 |              |              |
| at 5 GeV\(^2\)| -0.43±0.05| -0.39        | -0.39        |
|               | 0.84±0.05 |              |              |
|               | -0.39±0.02| -0.39        | -0.39        |
| \(\Delta u\)  |           |              |              |
| \(\Delta d\)  | -0.08±0.05| -0.07        | -0.07        |
| \(\Delta s\)  | -0.10±0.02| 0.36         | 0.35         |
| \(\Delta \Sigma\)| 0.30±0.06 | 0.36         | 0.35         |
| \(\Delta \Sigma\)| 0.29±0.06 |              |              |
| \(\Delta s/\Delta s\)| 2.09±0.13 | 5/3          | 2.12         |
| \(g_A^{(3)}\)| 1.2573±0.0028 [13] | 5/3          | 1.22         |
| \(f_D^c\)| 0.575±0.016 | 2/3          | 0.58         |
| \(I_G\)| 0.235±0.026 | 1/3          | 0.27         |
|               | 1.21      |              |              |
|               | 0.58      |              |              |
|               | 0.25      |              |              |
IV. CONCLUSIONS

The detailed comparison in Sect. IV shows unambiguously that the success of several recent chiral models [5,6,8–12] for the spin fractions $\Delta q$ can be attributed to pure spinflip quark-Goldstone boson couplings that do not account for quark mass terms from the non-spinflip chiral transitions in the lowest moments of the splitting functions in standard chiral field theory.

Whenever constituent quark masses are used in Eqs. 6, then the positive contributions in $P^-$ corresponding to the non-spinflip probability represented by the quark mass term substantially reduce the chiral subtractions so that no agreement with the proton spin data can be achieved [3,14]. These results were obtained with quite different initial valence quark distributions and both show that the proton spin observable $\Delta\Sigma$ stays above the value 1/2. This disagreement can be interpreted so that the constituent quark model in the framework of standard chiral field theory, which is often called chiral quark model [16], is ruled out by the proton spin data.

The success of the chiral spin fraction models suggests that axialvector quark-Goldstone boson couplings are consistent with current quarks, but not constituent quarks. This usage conforms with the chiral field theory practiced in chiral perturbation theory [4]. It is interesting to note that proton spin data are successfully described by models based on an instanton fluid in the vacuum, where the instanton-quark interaction is also pure spinflip [17].

The proton spin data does not challenge the vast nuclear theory based on pion exchange directly – as the pion exchange potential has been successfully tested through its pion exchange current predictions – but it suggests understanding better its derivation from QCD concepts such as current quark masses and quark condensates, and not merely from constituent quark models.

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