Dissipation induced Tonks-Girardeau gas of photons

M. Kiffner$^1$ and M. J. Hartmann$^1$

$^1$Technische Universität München, Physik-Department I, James-Franck-Straße, 85748 Garching, Germany

A scheme for the generation of a Tonks-Girardeau (TG) gas of photons with purely dissipative interaction is described. We put forward a master equation approach for the description of stationary light in atomic four-level media and show that, under suitable conditions, two particle decays are the dominant photon loss mechanism. These dissipative two-photon losses increase the interaction strength by at least one order of magnitude as compared to dispersive two-photon processes and can drive the photons into the TG regime. Our scheme allows for measurements of various characteristic correlations of the TG gas via standard quantum optical techniques, including quantities that distinguish it from free fermions.

Quantum mechanics categorizes particles into fermions or bosons. In three dimensions only these two categories are possible, whereas more exotic anyons can exist in two dimensions [1]. In one dimension, the particle statistics cannot be considered without taking inter-particle interactions into account [2]. A prominent example are bosons that interact via strong repulsive forces in a one-dimensional setting and can enter a Tonks-Girardeau (TG) gas regime [3], where they behave with respect to many observables as if they were fermions. A TG gas can be described as the strong interaction limit of the Lieb-Liniger model [4].

Strong correlations in many-particle systems, such as in the TG gas, give rise to interesting and partly not yet well understood physics. A substantial amount of research is thus currently devoted to these systems and progress in cooling and trapping of atoms and ions has opened up possibilities to study strongly interacting many-body systems experimentally with unprecedented precision. Eventually, this progress enabled the observation of a TG gas of atoms in an optical lattice [5].

Later, an experiment [6] with cold molecules showed that not only elastic interactions, but even two-particle losses alone are able to create a TG gas where two molecules never occupy the same position, thereby avoiding dissipation of particles. This counterintuitive result can be regarded as a manifestation of the quantum Zeno effect [6].

In contrast to atoms, photons are massless particles that do not interact at all. Nonetheless, effective many-body systems of photons and polaritons can be generated by employing light matter interactions. This concept has been introduced recently [8, 9, 10, 11] and is currently receiving increasing attention [12, 13, 14, 15]. To enter the strongly correlated regime and access its rich physics, sufficiently strong effective interactions are needed. Suitable experimental setups, like arrays of coupled microcavities doped with emitters [16] or optical fibers that couple to atoms [12, 17], thus need to combine strong photon-emitter coupling and low-loss photon propagation. In all these setups, the main challenge for realizing strong correlations is to make the polariton-polariton interactions much stronger than photon losses which are inevitably present in every experiment.

Here we present an effective many-body system of polaritons where the ubiquitous but usually undesired dissipative processes become the essential ingredient for the creation of strong many-particle correlations. This paradigm shift allows us to relax some conditions on the model parameters such that the achievable nonlinearities in our approach are at least an order of magnitude larger than their conservative counterparts [8, 12, 14]. In particular, we show that the dissipative nonlinearities in our system give rise to a TG gas of photons. For this regime, fermionic (e.g. Friedel oscillations) as well as non-local (e.g. the single particle density matrix) correlations of the TG gas can be measured via standard quantum optical techniques.

We consider photons guided in an optical fiber that interact with nearby atoms [12, 17], where stationary light [18] is created via Electromagnetically Induced Transparency (EIT) [19]. As compared to coupled microparticles, the fiber approach is appealing due to the much stronger than photon losses.

PACS numbers: 42.50.Ct, 42.50.Ex, 67.10.Fj, 42.50.Gy

![Figure 1](https://example.com/fig1.png)

**FIG. 1:** (Color online) (a) Considered setup of $N$ atoms confined to an interaction volume of length $L$ and transverse area $A$. $\Omega_{\pm}$ are the Rabi frequencies of the classical control fields, and $E_{\pm}$ are the quantum probe fields. (b) Atomic level scheme. $\gamma_{ij}$ is the full decay rate on the $|i\rangle \leftrightarrow |j\rangle$ transition, $\delta$ and $\Delta$ label the detuning of the probe fields with states $|3\rangle$ and $|4\rangle$, respectively, and $\varepsilon$ is the two-photon detuning.
low photon loss of the fiber and since the longitudinal trapping of light is done optically, thus avoiding the need to build many mutually resonant cavities. We thus focus on this setup here. However, our mechanism for building up correlations works equally well in cavity arrays, and the dissipative nonlinearities we discuss here are always stronger than their conservative counterparts independent of the geometry of the experimental device [20].

We start with a more detailed description of our one-dimensional model shown in Fig. 1. Each of the \( N \) atoms interacts with control and probe fields denoted by \( \Omega_{\pm} \) and \( \hat{E}_{\pm} \), respectively. The control fields of frequency \( \omega_c \) are treated classically and \( \Omega_+ (\Omega_-) \) labels the Rabi frequency of the control field propagating in the positive (negative) \( z \) direction. In addition, we assume that the control fields are spatially homogeneous but may depend on time. The probe fields \( \hat{E}_+ \) and \( \hat{E}_- \) are quantum fields that propagate in the positive and negative \( z \) direction, respectively. They are defined as \( \hat{E}_+ (z) = \sum_{k} a_{\pm K} e^{iKz} \), where \( a_{\pm K} \) are photon annihilation operators. The wave numbers \( K \) are positive and of the order of the wave number \( k_c \) of the control field.

We model the time evolution of the atoms and the quantized probe fields by a master equation [21] for their density operator \( \hat{\rho} \), \( \dot{\hat{\rho}} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] + \mathcal{L} \hat{\rho} \), where \( \mathcal{L} \hat{\rho} \) describes spontaneous emission from states \( |3\rangle \) and \( |4\rangle \), and the full decay rate on the transition \( |i\rangle \leftrightarrow |j\rangle \) is denoted by \( \gamma_{ij} \) (see Fig. 1). In a rotating frame that removes the time-dependence of the classical laser fields, the system Hamiltonian \( \hat{H} \) reads

\[
\hat{H} = \frac{\hbar}{2} \sum_{K} \left( \omega_p - \omega_K \right) \left( a_K^\dagger a_K + a_K a_K^\dagger \right)
- \hbar \sum_{\mu=1}^{N} \left[ \varepsilon A_{22}^{(\mu)} + \delta A_{43}^{(\mu)} + (\Delta + \varepsilon) A_{43}^{(\mu)} \right],
\]

where we neglected corrections of order \( 1/k_c \) of the probe fields are empty and all atoms are in state \( |1\rangle \). The operators \( \psi_k \) are defined as [2]

\[
\psi_k = A_k \cos \theta - X_{k2}^4 \sin \theta,
\]

the operators \( \psi_k \) obey bosonic commutation relations in \( \mathcal{H}_{FE} \), \( \langle \psi | \sum_{\mu=1}^{N} A_{\mu}^{(\mu)} | \psi \rangle \approx N \) for all \( |\psi\rangle \in \mathcal{H}_{FE} \). This follows that the operators \( \psi_k \) obey bosonic commutation relations in \( \mathcal{H}_{FE} \).

The dark-state polaritons are eigenstates of \( \hat{H} \), but the remaining parts \( \hat{H}_0 \) and \( \hat{H}_{NL} \) of the system Hamiltonian give rise to a non-trivial time evolution of \( \hat{\rho}_0 \). Fortunately, this dynamics can be studied entirely in terms of bosonic quasi-particle excitations if the system dynamics is restricted to the subspace \( \mathcal{H}_{FE} \). In particular, the free time evolution \( \hat{H}_0 \) introduces a coupling of dark-state polaritons to bright polaritons, \( \phi_k = A_k \sin \theta + X_{k2}^4 \cos \theta \), and photons, \( D_k = (a_{k,+} + a_{k,-})/\sqrt{2} \). Excitations in the states \( |3\rangle \) and \( |4\rangle \) are in turn coupled to the excited state \( |3\rangle \). Furthermore, \( \hat{H}_{NL} \) introduces a direct coupling of dark-state polaritons to the excited state \( |4\rangle \) via a two-particle process [23]. Excitations in the states \( |3\rangle \) and \( |4\rangle \) are created by \( P_{k,+} \) and \( U_{k,-} \), respectively, where

\[
P_{k,\pm} = \sum_{\mu=1}^{N} \left[ S_{31}^{(\mu)} e^{i(k_{\pm}+k)z} \pm S_{31}^{(\mu)} e^{-i(k_{\pm}-k)z} \right]/\sqrt{2N},
\]

Transition operators of atom \( \mu \) at position \( z_\mu \) are defined as \( S_{ij}^{(\mu)} = |i\rangle \langle j| \).
\[ U_{k,\pm}^{(4)} = \sum_{\nu=1}^{N} |S_{\nu}^{(4)} e^{-i(k_n-k_m)z_{\nu}} \pm S_{\nu}^{(4)H} e^{-i(k_n-k_m)z_{\nu}}| \sqrt{2N}. \]

Finally, we note that spontaneous emission from states |3⟩ and |4⟩ results in the decay of excitations created by \( P_{k,\pm}^{(1)} \) and \( U_{k,\pm}^{(1)} \).

We employ projection operator techniques \[21\] to derive a master equation for \( \hat{\rho}_D \) which is obtained from \( \hat{\rho} \) by a partial trace over all excitations except for the dark state polaritons \( \psi_k \). We restrict our analysis to the so-called slow-light regime where \( \sin^2 \theta \approx 1 \) and \( \cos^2 \theta \ll 1 \). In this case, the coupling of dark-state polaritons to excitations in states |3⟩ and |4⟩ is much slower than the decay of the relevant correlation functions \( \langle \hat{\phi}_k \hat{\phi}_p \rangle \) and \( \langle \hat{D}_k \hat{D}_p \rangle \) and \( \langle \hat{U}_{k,\pm} \hat{U}_{p,\pm} \rangle \), which happens on a timescale given by the lifetimes of the exited states [3] and [4]. This existence of two different time scales allows us to derive a master equation in Born-Markov approximation if \( 4g_2^2 \cos^2 \theta \Delta n_{ph} \ll \gamma_{d2} \), \( \cos^2 \theta \kappa^2_{max} / \Omega_0^2 \ll 1 \), \( \cos^2 \theta \Delta \omega^2 / \Omega_0^2 \ll 1 \), and \( \Omega_0 \gg \gamma_G / 6 \). Here \( c \) is the speed of light, \( \Delta \omega = \omega_p - \omega_\hat{c} \) is the frequency difference between the probe and control fields and \( N_{ph} \) is the number of photons in the pulse. We describe the polariton pulse by the field operator \( \hat{\psi}(z) = (1/\sqrt{\mathcal{L}}) \sum_k e^{i k z} \hat{\psi}_k \) which obeys the commutation relations \( [\hat{\psi}(z), \hat{\psi}^\dagger(z')] = \delta(z - z') \). The maximal wave number contributing to \( \hat{\psi} \) is \( k_{max} \).

Furthermore, our derivation assumes that fast oscillating spin coherences with wave number \( \pm 2k \) are washed out due to atomic motion \[18\]. For a small two-photon detuning \( \epsilon = -\cos^2 \theta \Delta \omega \), we obtain

\[ \dot{\hat{\rho}}_D = -i \hat{H}_{\text{eff}} \hat{\rho}_D + i \dot{\hat{\rho}}_D H_{\text{eff}} + \mathcal{L}_D \hat{\rho}_D + \mathcal{L}_2 \hat{\rho}_D, \]

where \( \hat{H}_{\text{eff}} \) is a non-hermitian Hamiltonian,

\[ H_{\text{eff}} = \frac{\hbar^2}{2m_{ph}} \int_0^L dz \partial_x \hat{\psi}_\dagger \partial_x \hat{\psi} + \frac{\hat{\theta}}{2} \int_0^L dz \partial_x \hat{\psi}^\dagger \partial_x \hat{\psi}^\dagger, \]

\[ m_{eff} = -\hbar g_2^2 / (24\epsilon_2^2 \cos^2 \theta) \]

is the effective mass of the polaritons, \( \hat{\theta} = 2tL \sigma_{z} \cos^2 \theta / (\Delta - \cos^2 \theta \Delta \omega + i \gamma_{d2}/2) \) is the complex coupling constant, and

\[ \mathcal{L}_D \dot{\rho}_D = -\text{Im}(\hat{\theta}) \int_0^L dz \partial_x \hat{\psi}_\dagger \partial_x \hat{\psi}^\dagger, \]

\[ \mathcal{L}_2 \dot{\rho}_D = -\frac{\hbar \Gamma \Delta \omega^2 D[\hat{\psi}]^\dagger}{2\Omega_0^2 / \cos^2 \theta}, \]

Next we derive the essential results of this letter from the master equation \[5\] that describes a one-dimensional system of interacting bosons. The first contribution to \( H_{\text{eff}} \) in Eq. \[4\], \( (\hbar^2 / 2m_{eff}) \int_0^L dz \partial_x \hat{\psi}_\dagger \partial_x \hat{\psi} \), represents a kinetic energy term with quadratic dispersion relation for the polaritons. The term proportional to \( \hat{\theta} \) in Eq. \[3\] and \( \mathcal{L}_D \hat{\rho}_D \) in Eq. \[4\] account for elastic and inelastic two-particle interactions that originate from the coupling of dark-state polaritons to the excited state |4⟩. More precisely, the real part of \( \hat{\theta} \) gives rise to a hermitian contribution to \( H_{\text{eff}} \) that accounts for elastic two-particle collisions. On the other hand, the imaginary part of \( \hat{\theta} \) together with \( \mathcal{L}_D \hat{\rho}_D \) gives rise to a two-particle loss term that can be written in Lindblad form as \( -\text{Im}(\hat{\theta}) D[\hat{\psi}_\dagger]^2 \).

The contributions \( \mathcal{L}_1 \hat{\rho}_D \) and \( \mathcal{L}_2 \hat{\rho}_D \) describe single-particle losses that can be omitted under the following conditions. Since \( \mathcal{L}_1 \hat{\rho}_D \) is proportional to \( \Delta \omega^2 \), single-particle losses are minimized by minimizing \( |\Delta \omega| \). Note that this fact has not been pointed out so far. From now on we assume that \( |\Delta \omega| \) is small enough such that \( \mathcal{L}_2 \hat{\rho}_D \) represents the dominant single particle loss. This is reasonable if \( |\Delta \omega| \) is at most of the order of GHz and implies \( |\epsilon| \ll |\gamma_{d2}| \). The term \( \mathcal{L}_2 \hat{\rho}_D \) is negligible if two conditions are met. First, the dynamics induced by the kinetic energy term proportional to \( m_{eff} \) in Eq. \[5\] must be fast as compared to the inverse decay rate of polaritons introduced by \( \mathcal{L}_2 \hat{\rho}_D \). This can be achieved if we set \( |\delta| \gg \Gamma \). Second, losses due to \( \mathcal{L}_2 \hat{\rho}_D \) must be negligible which imposes a limit on the maximal evolution time \( t_{\text{max}} \ll 2\Omega_0 / (\Gamma c^2 k_{max} \cos^2 \theta) \). This implies that \( t_{\text{max}} \) can be of the order of \( 1 / (\cos^2 \theta \Gamma) \gg 1 / \Gamma \).

Under these conditions, the master equation \[5\] reduces to \( \dot{\hat{\rho}}_D = -i \hat{H}_{\text{eff}} \hat{\rho}_D + i \dot{\hat{\rho}}_D H_{\text{eff}} + \mathcal{L}_D \hat{\rho}_D \) and can be identified with the generalized Lieb-Liniger model \[3\] for a one-dimensional system of bosons with mass \( m_{eff} \) and complex interaction parameter \( \hat{\theta} \). All features of the Lieb-Liniger model \[4\] are characterized by a single, dimensionless parameter \( G = m_{eff} \hat{\theta} / (\hbar^2 N_{ph} / L) \), where \( N_{ph} \) is the number of photons in the pulse. The absolute value of \( G \) is

\[ |G| = \frac{g_1^2 g_2^2 L^2 N}{c^2 |\delta| \sqrt{\Delta^2 + 2\Delta N_{ph}}} = \frac{(1/16) \Gamma \gamma_{d2} OD^2}{|\delta| \sqrt{\Delta^2 + 2\Delta N_{ph}}}, \]

where \( OD = 4N g_2^2 L / (c \Gamma) \) is the optical depth on the probe field transitions. Note that the parameters \( g_1^2 L \) and \( g_2^2 L \) are independent of the length of the system since \( g_1, g_2 \sim 1/\sqrt{AL} \). It follows that the parameter \( G \) and the optical depth depend only on the transverse area \( A \) of the interaction volume, but not on the length \( L \) of the cell. The absolute value of \( G \) characterizes the effective interaction strength between the particles. In the strongly correlated regime \( |G| \gg 1 \), the interaction between the particles creates a Tonks-Girardeau gas where photons behave like impenetrable hard-core particles that
never occupy the same position. Formally, this result can be derived via the pair correlation function 
\[ g^{(2)}(z, z') = \langle \psi^\dagger(z) \psi^\dagger(z') \psi(z) \psi(z') / (\langle \hat{n}(z) \rangle \langle \hat{n}(z') \rangle) \] 
with \( \hat{n}(z) = \psi^\dagger(z) \psi(z) \). For the ground state of the generalized Lieb-Liniger model in the strongly correlated regime, 
\[ g^{(2)}(z, z) = (1 - 1/N_{ph}^2) 4 \pi^2 / (\langle G \rangle^2) \] 
is close to zero and vanishes in the limit \( \langle G \rangle \rightarrow \infty \). Moreover, this ground state is the same \[ 7 \] as in the original model with repulsive interaction for \( \Delta = 0 \). It follows that \( g^{(2)}(z, z') \) for \( z \neq z' \) exhibits Friedel oscillations \[ 20 \] that indicate a crystallization of photons in the fiber.

The parameter \( |G| \) is maximal if the interaction between the polaritons is purely dissipative (\( \Delta = 0 \)). Since the realization of a regime where the two-particle interactions are dominated by elastic processes requires \( \Delta \gg \gamma_{42}/2 \), the conservative nonlinearities are at least an order of magnitude smaller than the dissipative counterparts for \( \Delta = 0 \). It follows that purely dissipative interactions between the polaritons we discuss here are most effective for the generation of correlations.

An analysis of dissipation-induced correlations (\( \Delta = 0 \)) requires at least two photons. Assuming \( |\delta|/\Gamma = 10 \) such that the single-particle loss term \( L_1 g_{12} \) in Eq. \[ 5 \] is negligible and \( N_{ph} = 2 \), Eq. \[ 14 \] shows that \( |G| \) is larger than unity for \( OD^2/N > 160 \). A recent experiment \[ 17 \] with atoms loaded into a hollow fiber reports a value of \( OD^2/N \approx 0.3 \). If we assume for simplicity that the decay rates of the atomic states \[ 3 \] and \[ 4 \] do not depend on the transverse area \( A \), we find \( |G| \propto N\lambda_p^2/A^2 \), where \( \lambda_p \) is the wavelength of the probe field. It follows that \( |G| \) does not depend on the strength of the atomic transition dipole moments and could be increased by a reduction of the area \( A \) or by an increased number of atoms \( N \) inside the fiber. In contrast to cavity QED systems \[ 14 \], we point out that the condition \( g_2 \gg \gamma_{42} \) is not required to obtain large values of \( |G| \).

The observation of the dissipation-induced TG gas regime requires that the system can be prepared in low-energy states. One possibility is the procedure described in \[ 12 \] which relies on an adiabatic state transfer realized by a time-dependent detuning \( \Delta \). A second possibility does not require any tuning of the two-particle losses. The master equation \[ 5 \] implies that losses due to inelastic two-particle interactions are related to the pair-correlation function \( g^{(2)}(z, z) \) via \( \partial_t \langle \hat{n}(z) \rangle = (2/\hbar) \text{Im}(\tilde{g}) g^{(2)}(z, z) \langle \hat{n}(z) \rangle^2 \). It follows that uncorrelated states with \( g^{(2)}(z, z) \approx 1 \) decay much faster than those where \( g^{(2)}(z, z) \approx 0 \). Therefore, a regime where \( g^{(2)}(z, z) < 1 \) should be entered on a time scale \( h / [2 \text{Im}(\tilde{g}) N_{ph} \Omega_p L] \), which is shorter than the maximally allowed evolution time \( t_{\text{max}} = 16g_{22}^2 N_{ph} \Omega_p^2 / (\gamma_{42}\Omega_p^2 k_{\text{max}}^2) \) > 1. Since the ground state of the generalized Lieb-Liniger model decays at the smallest rate \[ 3 \], two-particle losses themselves are then able to drive the system into states close to the ground state. A more rigorous investigation of this point would require a numerical integration of the master equation \[ 5 \] which is beyond the scope of this work.

For measurements, we note that the polariton pulse can be released from the fiber without distortion if one control field is adiabatically switched off \[ 12 \] \[ 18 \]. It follows that spatial correlations \( \langle \psi^\dagger(z) \psi(z') \rangle \) and \( \langle \psi^\dagger(z) \psi^\dagger(z') \psi(z) \psi(z') \rangle \) of the trapped pulse are mapped into first and second order correlations in time of the output light, respectively. Since the latter can be detected via standard quantum optical techniques, Friedel oscillations of \( g^{(2)}(z, z') \), the correlations \( \langle \psi^\dagger(z) \psi(z') \rangle \) and the characteristic momentum distribution of the TG gas, can be measured with high precision.

The authors thank W. Zwerger for discussions. This work is part of the Emmy Noether project HA 5593/1-1 funded by the German Research Foundation (DFG).

[1] F. Wilczek, Fractional Statistics and Anyon Superconductivity (World Scientific, 1990).
[2] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008).
[3] M. Girardeau, J. Math. Phys. 1, 516 (1960).
[4] E. H. Lieb and W. Liniger, Phys. Rev. 130, 1605 (1963).
[5] E. R. Paredes et al., Nature 429, 277 (2004); T. Kinoshita, T. Wenger, and D. S. Weiss, Science 305, 1125 (2004).
[6] N. Syassen et al., Science 320, 1329 (2008).
[7] S. Dürr et al., Phys. Rev. A 79, 023614 (2009).
[8] M. J. Hartmann, F. G. S. L. Brandão, and M. B. Plenio, Nat. Phys. 2, 849 (2006).
[9] M. J. Hartmann, F. G. S. L. Brandão, and M. B. Plenio, Laser & Photon. Rev. 2, 527 (2008).
[10] D. G. Angelakis, M. F. Santos, and S. Bose, Phys. Rev. A 76, 031805(R) (2007).
[11] A. Greentree et al., Nat. Phys. 2, 856 (2006).
[12] D. E. Chang et al., Nat. Phys. 4, 884 (2008).
[13] D. Gerace et al., Nat. Phys. 5, 281 (2009); I. Carusotto et al., Phys. Rev. Lett. 103, 033601 (2009).
[14] M. J. Hartmann, and M. B. Plenio, Phys. Rev. Lett. 99, 103601 (2007); M. J. Hartmann, F. G. S. L. Brandão, and M. B. Plenio, Phys. Rev. Lett. 99, 160501 (2007).
[15] D. Rossini and R. Fazio, Phys. Rev. Lett. 99, 186401 (2007).
[16] T. Aoki et al., Nature 443, 671 (2006); K. Hennessy et al., Nature 445, 896 (2007); M. Trupke et al., Phys. Rev. Lett. 99, 063601 (2007); A. Wallraff et al., Nature 431, 162 (2004).
[17] M. Bajcsy et al., Phys. Rev. Lett. 102, 203902 (2009).
[18] M. Bajcsy, A. S. Zibrov, and M. D. Lukin, Nature 426, 638 (2003).
[19] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. 77, 633 (2005).
[20] Here the role of dissipation differs fundamentally from H. Vinck-Posada et al., Phys. Rev. Lett. 98, 167405 (2007).
[21] H.-P. Breuer and F. Petruccione, The Theory of Open Quantum Systems (Oxford University Press, Oxford, 2006).
[22] A. Imamoglu et al., Phys. Rev. Lett. 79, 1467 (1997).
[23] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. 84,
[24] F. E. Zimmer et al., Phys. Rev. A \textbf{77}, 063823 (2008).
[25] S. E. Harris and Y. Yamamoto, Phys. Rev. Lett. \textbf{81}, 3611 (1998).
[26] J. Friedel, Nuovo Cimento \textbf{7}, 287 (1958).