MMV-based OMP for DOA estimation with 1-bit measurement

Dan Chen¹*, Longwei Tian¹ and Changqing Xu¹

¹Shanghai Key Laboratory of Beidou Navigation and Location Services, School of Electronic Information and Electrical Engineering, Shanghai Jiao Tong University, Shanghai, Shanghai, 200000, China
*Corresponding author’s e-mail: achendan@sjtu.edu.cn

Abstract. This paper shows a novel scheme for direction-of-arrival (DOA) estimation of narrow-band signals. In the scheme, the 1-bit measurement is used to reduce the system cost. A recovery compressed sensing (CS)-based algorithm with multiple measurement vector (MMV) is proposed to make up the information loss. The purposed MMV-OMP algorithm provides a new idea which make the compress sensing framework applicable directly to an MMV model. The effectiveness of the proposed approach is assessed through numerical simulations under different environment configurations. Comparisons with other methods are reported as well.

1. Introduction

Direction of arrival (DOA) estimation methods are used extensively in quality enhancement of wireless communication such as beamforming and beam alignment. Many high-resolution estimation methods have been studied, among which subspace algorithm represented by MUSIC and ESPRIT are the most prominent ones[1][2]. Spectrum estimation techniques based on compressed sensing (CS) have also been developed because of the sparsity of signals in the spatial domain.

Some work has been done on how to mold the DOA estimation problem in compress sensing framework. Single snapshot DOA estimation can be formulated as a problem of finding a sparse representation[3]. On the other hand, some technics have been proposed in the case of multiple snapshots which is of greater practical importance.

The technics can be divided into two categories when multiple snapshots are presented: treating each time sample independently and combining the time sample prior to solving the inverse problem. The main drawback of treating each time sample independently is no operation between these subproblems, where small perturbations in data can lead to different result. We can expect to correct these perturbations in combining solutions. When the source has a strong non-zero temporal mean, data can be combined by taking temporal average, which often shows up in localization of moving vehicles. If the signal is zero-mean and correlated, the approach to combine the data is to merge the subproblems for different time samples into a single larger inverse problem[4][5][6].

Based on this framework, many CS-based algorithms have been proposed for multiple measurement vector (MMV) DOA estimation. In particular, the ℓ₁-SVD is a powerful sparse recovery algorithm which incorporates the singular value decomposition (SVD) with ℓ₁-norm penalty[7]. Joint ℓ₂,₀ approximation DOA (JLZA DOA) algorithm proposed in[8] show that MMV-based DOA estimation can be solved by minimizing a mixed ℓ₂,₀ norm approximation. L₁-ACCV algorithm solves the estimation problem through second-order cone (SOC) framework after it transforms the MMV model...
to single snapshot model through an array cross-correlation vector (ACCV) technic[9].

Although high-resolution quantization is preferred by theoretical research and simulation, these methods are not attractive in applications due to the high hardware cost caused by an analog to digital (ADC) devices[10]. As a result, designing high-speed system by reducing the number of bits of ADC precision has been widely studied[11]. Many works have been done to verify the effectiveness of CS reconstruction under quantized or even 1-bit measurement[12][13].

In this context, we focus on the problem of 1-bit DOA estimation using compressed sensing methods. A model of multiple measurement vector (MMV) for DOA estimation is applied to make up the information loss cause by 1-bit quantization.

The notation used in this paper are defined as follows: the operator $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose; the operator $\|\cdot\|_2$ is the two-norm of the vector; $\phi$ and $\cup$ represent the empty set and union set.

2. Signal Model

Assume that $K$ narrowband far-field uncorrelated sources impinge on the $M$-element uniform linear array (ULA) from different directions $\{\theta_1, \theta_2, \cdots, \theta_K\}$. Figure 1 is the sketch of the DOA estimation configuration.

At the time instant $t$, the signal acquired by the antenna element $m \in \{1,2,\cdots,M\}$ is given by

$$x_m(t) = \sum_{k=1}^{K} s_k(t) e^{-j2\pi\frac{d}{\lambda}(m-1)\sin \theta} + n_m(t) \quad (1)$$

when being extended into multiple $N$ snapshots, the output matrix $X = [x_1(t), x_2(t), \cdots, x_M(t)] \in \mathbb{C}^{M \times N}$ can be expressed as

$$X = AS + N \quad (2)$$

where $A = [a(\theta_1), a(\theta_2), \cdots, a(\theta_K)] \in \mathbb{C}^{M \times K}$ represent the steering matrix of the array. $a = [1, e^{-j2\pi\frac{d}{\lambda}\sin \theta}, \cdots, e^{-j2\pi\frac{d}{\lambda}(M-1)\sin \theta}]^T \in \mathbb{C}^{M \times 1}$ is the steering vector with array interspace $d$ which is usually set as half of the wave length to avoid phase ambiguity. $S$ is the signal vector given as $[s_1(t), \cdots, s_K(t)] \in \mathbb{C}^{K \times N}$ and $N$ is the additive Gaussian noise matrix with the dimension $M \times N$.

When 1-bit quantizer is employed, the array output is:

$$Y = Q(X) = Q(AS + N) \quad (3)$$

$X$ is complex value, so the quantization procedure composes of two sign function $\text{sign}(\cdot)$, which keeps only the sign of the real and imaginary part,

$$Q(X) = \frac{1}{\sqrt{2}}[\text{sign}(\Re(X)) + j\text{sign}(\Im(X))] \quad (4)$$

In the following algorithm, we shall show that the OMP reconstruction method can be directly employed to the one-bit array output $Y$ without extra pre-processing. In addition, considering both estimation accuracy and algorithm complexity, we combine the previous work and conduct a two-step searching method.
3. Proposed Algorithm

The algorithm we proposed is a two-step searching method in MMV model. It can be divided into a rough sparse signal reconstruction problem and a subtle one-bit DOA estimation.

Angle range is divided by sampling grid \( \{ \varphi_1, \varphi_2, \cdots, \varphi_{N_d} \} \). The steering matrix of the array is extended to a dictionary matrix \( A_\Lambda \), whose \( t \)th column corresponds to the direction \( \varphi_t \) of the sampling grid.

Matrix sparsity can be determined by the dimension contrast between result matrix and the dictionary matrix instead of only by the quantity of non-zero entries.

As we all know, OMP is a greedy reconstruction method which utilize the correlation between the receiving message vector and dictionary matrix \( A_\Lambda = [ a(\varphi_1), a(\varphi_2), \cdots, a(\varphi_{N_d}) ] \), where \( N_d \) is determined by interval \( I \).

In the iterative calculation, drawn dictionary columns from perception matrix \( A_\Lambda \) and make a comparison to each snapshot data of receiving message \( Y \) to get the highest matching degree of the dictionary matrix. As a result, we obtain a vector \( P_t \) with \( N \) by 1 dimensions which each element of the matrix stores the index of the most correlative steering vector to the current snapshot.

\[
P_t = \arg \max_{n \in \{1, 2, \cdots, N\}} \| y_{t-1} A_n \| \tag{5}
\]

Make a full consideration of the multiple snapshot result to get the index which has the highest vote in \( P_t \):

\[
J_t = \arg \max_{i \in \{1, 2, \cdots, N\}} (P_{ti}) \tag{6}
\]

Update the supporting set by importing the result index into current set:

\[
\Phi_t = [\{ \Phi_t, a(q_{J_t}) \}] \tag{7}
\]

Collect the corresponding steering vector to get the provisional solution:

\[
\hat{x}_t = \arg \min_{\hat{x}} \| Y - A_\Lambda \hat{x} \|_2 \tag{8}
\]

The redundant matrix is iteratively updated by the least square (LS) estimation method. The purpose of this step is to eliminate the influence from direction in the supporting set and make the iteration continues.

\[
\gamma_t = Y - A_\Lambda \hat{x}_t \tag{9}
\]

Finally, we can reconstruct the sparse direction by extract the corresponding angle in the supporting set.

The above steps can solve DOA estimation in normal situation. However, there occurs a significant decrease in performance because of the information loss caused by the 1-bit quantization. Therefore, a step two searching is essential.

In this algorithm, a subspace-based searching method is then conducted. We can use the reconstruction result as a prior knowledge to narrow down the searching range and reduce complexity.

Based on the work of Bar-shalom[12], we can reconstruct the unquantized covariance matrix \( R_X = E[X(t)X^H(t)] \) from 1-bit covariance matrix \( R_Y = E[Y(t)Y^H(t)] \) by

\[
R_X = p^{-1} \sin(\frac{\pi}{2} R_Y) \tag{10}
\]

where \( \sin(Z) = \sin(\Re(Z)) + j \sin(\Im(Z)) \), \( p \) is an unimportant scaling parameter which don’t affect DOA estimation.

According to the arcsine law, the correlation coefficient between the \( m \)th and \( n \)th sensors follow the expression:

\[
\rho_{y_m y_n} = \frac{2}{\pi} \arcsin(\rho_{x_m x_n}) \tag{11}
\]

So, we can expand the convert equation as follows:

\[
\arcsin(\Re(\rho_{x_m x_n})) = \Re(\rho_{x_m x_n}) + \frac{1}{6} \Re^3(\rho_{x_m x_n}) + \frac{1}{40} \Re^5(\rho_{x_m x_n}) + \cdots \tag{12}
\]
The same expansion can be done with the imaginary part of the element. Therefore, we can still get a relatively satisfying result because the maximal approximation error is \(|\arcsin(1) - 1| = 0.57\).

Based on the conclusion in[13], another important fact is that both real and imaginary part turn to decrease with the decreasing of the signal SNR. Therefore, it is obvious that when the SNR is sufficiently low, the approximation is better satisfied.

Under this circumstance, we have conclusion that the subspace-based method can be directly applied to the one-bit receiving message \(Y\). The specific steps of the algorithm are show in the Table 1 below:

| Algorithm 1: MMV-based OMP |
|-----------------------------|
| **Input:** \(X, K, A, I\)   |
| **Output:** \(\hat{\theta}\) |
| **Initialization:** \(y_0 = Y, A_0 = \phi, t = 1, A_{t}\) |
| 1. While \(t < K\)          |
| 2. \(P_t = \arg \max_{n \in \{1, 2, \ldots, N\}} |\langle y_{t-1}, A_n \rangle|, P_t \in \mathbb{C}^N\) |
| 3. \(J_t = \arg \max_{i \in \{1, 2, \ldots, N\}} (P_t, i)\) |
| 4. \(A_t = A_{t-1} \cup \{J_t\}\) |
| 5. \(\hat{x}_t = A_{A_t}^H (A_{A_t}A_{A_t}^H)^{-1}Y\) |
| 6. Update the redundant matrix \(y_t = Y - A_{A_t}\hat{x}_t, t = t + 1\) |
| 7. **end**                  |
| 8. Calculate the one-bit covariance matrix by \(\hat{R}_Y = \frac{1}{N}YY^H\). |
| 9. Perform singular value decomposition (SVD) and output the estimated noise subspace \(\hat{U}_n\) |
| 10. Spectrum search using \(G_{\theta} = \frac{1}{a^H(\theta)\beta_n\beta_n^H a(\theta)}\) |
| 11. Output the DOA estimations based on the locations of K highest peaks of the spectrum above. |

4. Simulation Results

In this section, several simulations will be carried out to verify our previous analysis and evaluate the performance of the proposed algorithm. The results are obtained in \(R = 100\) Monte Carlo runs. All the sources are uncorrelated.

First, we make a contrast of one-bit OMP in SMV and MMV. For illustration, we assume two signals from 45.135° and 65.896° impinge on a 50-element uniform linear array (ULA) with half-wavelength inter-element spacing. The DOA estimation result is plotted in Figure 2 where the SNR=10 dB, \(M = 50, I = 0.1^\circ\).

The snapshot in these two occasions are set to be 1, 50 and 100. We can notice in Figure 2 that the curve with the snapshots value \(N=100\) has the most obvious fluctuation. MMV can make up the information loss influence by the one-bit quantization.

For a better illustration, we will focus on the DOA estimation error under different parameter settings. The performance is measured in terms of root mean square error (RMSE) between real direction \(\theta_k\) and their estimations \(\hat{\theta}_{k,r}\) in all runs \(r = 1, 2, \cdots R\).

\[
RMSE = \sqrt{\frac{1}{R K} \sum_{r=1}^{R} \sum_{k=1}^{K} |\theta_{k,r} - \theta_k|^2} \quad (13)
\]
Figure 2. DOA estimation under SMV and MMV

Figure 3. The probability of success versus number of array element with source number $K = 2, 3, 4$

Figure 3 plots the probability of success for a different number of array elements. The number of source $K$ changes from 2 to 4. In this part, the DOA estimation is supposed to be a success when the RMSE is less than $5^\circ$.

From the simulation results in Figure 3, the probability of success is increasing with the number of the array. Generally speaking, success probability become acceptable when $M > 20$ with all source numbers. The increase of $K$ put forward a higher requirement of the array elements number. We can detect from the figure that minimum standard of the elements number is 6, 12 and 15 as source number is 2, 3, 4 respectively.

Figure 4. RMSE of DOA estimation versus snapshot number.

Figure 5. RMSE of DOA estimation versus SNR

In the third example, we compare the performance of this method with those of $\ell_1$-SVD, JLZA_DOA, MUSIC. RMSE is plotted over the different number of snapshots $N$ in Figure 4 where the SNR=10 dB, $M = 50$, $I = 0.1^\circ$, $K=2$, $\theta = 45.135^\circ, 65.896^\circ$. Our method always outperforms $\ell_1$-SVD. The difference between MMV-OMP and unquantized MUSIC is ignorable. It is worth mentioned that although JLZA_DOA has the best resolution ability, the run time is almost ten times longer.

In Figure 5, the RMSE is plotted with snr set from -10 to 20 for $M = 50$, $N = 100$, $I = 0.1^\circ$. We can detect in the figure that MMV-OMP and Unquantized MUSIC exhibit a quiet similar performance and achieve at most $0.1^\circ$ when the parameter SNR being set at 0 or larger. In addition, although the result turn to be same, the 1-bit MMV-OMP method has the lowest time complexity.
5. Conclusion
In this paper, we show a novel scheme for DOA estimation which uses one-bit measurement to reduce the system cost. A recovery CS-based algorithm with MMV observation is proposed to make up the information loss.

Since OMP was introduced for SMV, we extended it to MMV cases. The proposed algorithm provides a new idea which makes the compress sensing framework applicable directly to an MMV model.

The presented result demonstrates that this method is comparable to the unquantized MUSIC or even better when the SNR and snapshot number are relatively low. Therefore, MMV-OMP is suitable for massive MIMO with many antenna elements. These findings in this work can simplify the implementation of DOA estimation.

Acknowledgments
This research work was supported by the National Natural Science Foundation of China under Grant Nos. 61971278, and the Shanghai Science and Technology Committee under Grant No. 1751106300.

References
[1] Weber, R.J., Huang, Y. (2009) Analysis for Capon and MUSIC DOA estimation algorithms. In: 2009 IEEE Antennas and Propagation Society International Symposium. Charleston. pp.1-4.
[2] Priyadarshini, M.P., Vinutha, R. (2012) Comparative performance analysis of MUSIC and ESPRIT on ULA. In: 2012 International Conference on Radar, Communication and Computing (ICRCC). Tiruvannamalai. pp.120-124.
[3] Fortunati, S., Grasso, R., Gini, F. (2014) Single snapshot DOA estimation using compressed sensing. In: Acoustics, Speech and Signal Processing (ICASSP). Florence. pp. 2297-2301.
[4] Hu, Y., Yu, X. (2017) Research on the application of compressive sensing theory in DOA estimation. In: IEEE International Conference on Signal Processing, Communications and Computing (ICSPCC). Xiamen. pp.1-5.
[5] Nasu, T., Kikuma, N., Sakakibara, K. (2018) Performance Improvement by Two-step Search Method in DOA Estimation Based on Compressed Sensing. In: International Symposium on Antennas and Propagation (ISAP). Busan. pp.1-2.
[6] Zhang, X., Li, Y., Yuan, Y., Jiang, T., Yuan, Y. (2018) Low-Complexity DOA Estimation via OMP and Majorization-Minimization. In: IEEE Asia-Pacific Conference on Antennas and Propagation (APCAP). Auckland. pp.18-19.
[7] Malioutov, D., Cetin, M., Willsky, A.S. (2005) A sparse signal reconstruction perspective for source localization with sensor arrays. IEEE Transactions on Signal Processing, 53(8): 3010-3022.
[8] Hyder, M.M., Mahata, K. (2010) Direction-of-Arrival Estimation Using a Mixed Norm Approximation. IEEE Transactions on Signal Processing, 58(9): 4646-4655.
[9] Xu, D., Hu, N., Ye, Z. (2012) The estimate for DOAs of signals using sparse recovery method. In: IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). Kyoto. pp. 2573-2576.
[10] Walden, R.H. (1999) Analog-to-digital converter survey and analysis. IEEE Journal on Selected Areas in Communications, 17(4): 539-550.
[11] Zymnis, A., Boyd, S., Candes, E. (2010) Compressed Sensing with Quantized Measurements. IEEE Signal Processing Letters. 17(2): 149-152.
[12] Bar-Shalom, O., Weiss, A.J. (2002) DOA estimation using one-bit quantized measurements. IEEE Transactions on Aerospace and Electronic Systems. 38(3):868-884.
[13] Huang, X., Bin, L. (2019) One-Bit MUSIC. IEEE Signal Processing Letters. 26(7):1-1.