Inflation from Multiple Pseudo-Scalar Fields:
PBH Dark Matter and Gravitational Waves

Alireza Talebian,† Seyed Ali Hosseini Mansoori,‡ and Hassan Firouzjahi

1School of Astronomy, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran, P.O. Box 19395-5531
2Faculty of Physics, Shahrood University of Technology, P.O. Box 3619995161 Shahrood, Iran

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We study a model of inflation with multiple pseudo-scalar fields coupled to a $U(1)$ gauge field through Chern-Simons interactions. Because of parity violating interactions, one polarization of the gauge field is amplified yielding to enhanced curvature perturbation power spectrum. Inflation proceeds in multiple stages as each pseudo-scalar field rolls towards its minimum yielding to distinct multiple peaks in the curvature perturbations power spectrum at various scales during inflation. The localized peaks in power spectrum generate Primordial Black Holes (PBHs) which can furnish a large fraction of Dark Matter (DM) abundance. In addition, gravitational waves (GWs) with non-trivial spectra are generated which are in sensitivity range of various forthcoming GW observatories.

**Introduction:** Inflation is the leading paradigm for early universe cosmology and the mechanism behind the generation of large scale structures. Among the basic predictions of models of inflations are that the primordial perturbations are nearly scale invariant, adiabatic and Gaussian which are well consistent with cosmological observations [1]. While the simplest models of inflation are based on a single scalar field, having inflation driven by multiple scalar fields along with other types of fundamental fields in the spectrum is well-motivated in models of high energy physics [2–4]. In particular, there have been growing interests in the axion models [5–13] to amplify the primordial power spectrum for PBHs formation and to generate detectable GWs signals.

PBHs are distinct from their astrophysical counterparts in several ways. Among all, PBHs could form in the early universe from the collapse upon horizon re-entry of perturbations generated during inflation which may comprise a large fraction of DM energy density [14–18]. However, PBHs can form through different channels in the early universe as well [19, 20]. Remarkably, unlike astrophysical black holes, PBHs can include a vast range of masses. Therefore, the recent observations of GWs from merging binary systems with about 30 times solar mass [21] together with the lack of observational signals of particle DM have renewed the interests in PBHs from inflation [22–24]. To produce PBHs from inflation one requires that the amplitude of the primordial curvature perturbation is large enough, at least $10^7$, requires that the amplitude of the primordial curvature perturbation is large enough, at least $10^7$, at least $10^7$.

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In this model, the collective contributions of N axion fields yield a long enough period of inflation to solve the flatness and horizon problems. In this picture, inflation is divided to N slow-roll phases where each phase is driven by one axion while others are nearly frozen. Inspired by N-flation model, in this work, we study an inflationary model with multiple pseudo-scalar fields coupled to a $U(1)$ gauge field through Chern-Simons types of interaction. We examine the enhancement of the curvature power spectrum to form PBHs at small scales. We show that PBHs can be formed abundantly (in the allowed window where PBHs could provide a substantial part of the DM, if not all) without introducing specific features on the inflationary potentials. In addition, tensor perturbations with non-trivial spectrum are generated which may be detected in upcoming GWs experiments.

**The Model and Background Dynamics:** We consider N pseudo-scalar fields $\Phi_a$ ($a = 1, 2, \cdots, N$) driving inflation in N stages. While our starting discussions are general but for specific examples studied below, we consider the cases $N = 2, 3$ specifically. In each stage, only one pseudo-scalar field can slow-roll and then decay, while others remain frozen. The next inflationary stage is driven by the second field before it decays and so on. For this picture to be realized, we need a working hierarchy on the masses of $\Phi_a$, such that the most massive field starts rolling first, then the second most massive field and so on [28]. For example if the ratio of the mass of $\Phi_1$ to $\Phi_2$ is at the order 10 or so, then we can safely assume that the first period of inflation is driven by $\Phi_1$. As in single field axion model we demand that all pseudo-scalar fields couple to a $U(1)$ gauge field $A_\mu$ through the Chern-Simons interactions [29, 30] in the following action

$$S = \int d^4x \sqrt{-g} \left[ \frac{M^2_{\text{Pl}}}{2} R - \frac{1}{2} \delta^{ab} g^{\mu\nu} \partial_\mu \Phi_a \partial_\nu \Phi_b - V(\Phi_a) ight] - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \sum_{a=1}^{N} \delta_a \left( \frac{\Phi_a}{M_{\text{Pl}}} \right) F_{\mu\nu} \tilde{F}^{\mu\nu} , \hspace{1cm} (1)$$

in which $M_{\text{Pl}}$ is the reduced Planck mass, $R$ is the Ricci...
so for \( \tilde{\phi} \) mentioned before, we assume that all \( \tilde{\phi} \) turbations via \( A \) only of small strength tensor \( F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \). Finally, \( \tilde{\alpha}_a \) is a dimensionless parameter controlling the coupling of the \( a \)-the pseudo-scalar field to the electromagnetic field [29].

To simplify the analysis below, we assume that all \( \tilde{\alpha}_a \) have the same sign. This is a technical tuning which simplifies the analysis significantly but it can be relaxed in a more general consideration.

In the presence of coupling \( \tilde{\alpha}_a \), the gauge field quanta exhibit tachyonic instability sourced by the rolling pseudo-scalar fields. More precisely, during the \( a \)-th stage of inflation, only the field \( \Phi_a \) rolls slowly. The rolling of this \( \Phi_a \) amplifies one polarization (e.g. the negative-helicity) of the gauge field, leading to [31]

\[
A_k^{(-)} \approx \frac{e^{\pi \xi - \sqrt{8 \xi k/(aH)}}}{\sqrt{2k}} \left( \frac{k}{2 aH} \right)^{1/2} \xi \equiv \sum_{a=1}^{N} \frac{|\tilde{\alpha}_a \phi_a|}{2H},
\]

where \( k \) is the comoving Fourier mode of the gauge field, \( a \) and \( H \equiv \dot{a}/a \) are the scale factor and the Hubble expansion rate during inflation and \( \phi_a \) is the homogeneous part of the pseudo-scalar field \( \Phi_a \). The dot denotes the derivative with respect to the cosmic time. The above solution well describes the growth of the mode functions in the interval \( (8 \xi)^{-1} \leq k/(aH) \leq 2 \xi \) [8]. Note also that the other polarization state (here the positive-helicity) is not amplified and can therefore be ignored. The so-called instability parameter \( \xi \) can be considered nearly constant, as its time variation is subleading in a slow-roll expansion. It is worth mentioning that the gauge quanta (2) not only affect the background dynamics of \( \phi_a \) and the scale factor but also source scalar perturbations via inverse decay [7, 8, 32]. We assume that \( \dot{\phi}_a < 0 \) during inflation as in the large field models like (5) so for \( \tilde{\alpha}_a > 0 \) the negative-helicity is amplified. As mentioned before, we assume that all \( \tilde{\alpha}_a \) are positive so only \( A_k^{(-)} \) is amplified at each stage of inflation.

Since the gauge field has no background value one can calculate their effects on the background dynamics via mean field approximation method [11, 13], yielding to

\[
3M_{\text{Pl}}^2 H^2 - V - \frac{1}{2} \delta^{ab} \phi_a \phi_b \simeq \frac{\Gamma(7)H^4}{21a^2 \pi^2} e^{2\pi \xi},
\]

\[
\phi_a + 3H \dot{\phi}_a + \frac{\partial V}{\partial \phi_a} \simeq -\tilde{\alpha}_a \Gamma(8)H^4 \frac{2a^2 \pi^2}{\xi^5} e^{2\pi \xi}.
\]

Note that the contributions in right hand sides above come respectively from \( (E_2 + B^2) \) and \( (E \cdot B) \) where the electric and magnetic fields, in the Coulomb-radiation gauge \( A_0 = 0 = \partial_\theta A_\theta \), are defined as \( E^i = -a^{-1} \dot{A}^i \) and \( B^i = a^{-2} e^{i/k} \partial_\theta A_k \) respectively. The exponential enhancement reflects significant non-perturbative gauge particle production in the regime \( \xi \simeq 1 \) [6]. To ensure that the tachyonic growth of gauge field fluctuations does not spoil the inflationary dynamics, we demand \( H^2/|\phi_a|^2 \ll O(10^3) \xi^{3/2} e^{-2\pi \xi} \) at each \( a \)-th stage [7, 8, 13]. From Eq. (4), one finds that the growth of \( \xi \) comes to a halt when the back-reaction term becomes large enough. Note that the system experiences a nonlinear phase for large coupling, e.g. \( \tilde{\alpha} \gtrsim 20 \). This regime is known as the strong back-reaction regime [33]. Furthermore, \( \xi \) does not experience the oscillatory epoch discussed in [33–36], before entering this phase at the end of previous stage, the next rolling field dictates the evolution of \( \xi \).

We work in the regime of negligible back-reaction such that the system never enter this phase and the evolution of \( \xi \) can not destroy the inflationary dynamics driven by the pseudo-scalar \( \phi_a \).

A simple choice for the inflation potential is the chaotic-type potentials \( V = \sum_{a=1}^{N} m_a^2 \phi_a^2 \) [27–29] as in N-flation such that during the \( a \)-th stage only the field \( \phi_a \) rolls down towards it potential minimum for some e-folds, oscillating rapidly at the bottom of its potential till its amplitude is effectively died out and the next field starts its rolling. However, the chaotic potentials is rule out by current Planck constraints [1] even in the multi-field configuration [37, 38] due to the large tensor-to-scalar ratio value, \( r_t \). For example, for the two-field and three-field \( (N = 2, 3) \) axion models with the pure chaotic potentials, our numerical results indicate that \( r_t \gtrsim 0.1 \) which is in conflict with large scale CMB observations [1]. One possible way to avoid this issue is to consider the following simple potential form [39–43]

\[
V(\Phi^a) = V_0 \frac{\phi_a^2}{\Phi_1^2 + m_1^2} + \sum_{a=2}^{N} \frac{1}{2} m_a^2 \phi_a^2,
\]

where \( V_0, m_1, \) and \( m_a \) are constant parameters. In two-field case, this potential represents the well-known dilaton-axion inflation [40, 44]. We arrange that on CMB scales the first pseudo-scalar field (the dilaton field) drive inflation while the remaining pseudo-scalar fields \( (N \geq 2) \) with the standard chaotic type potential (now specifically called axionic fields) drive the rest of inflation. Recently, in [41–43] the authors have shown that PBHs and GWs might be generated by considering a non-flat field space with the negative curvature in the absence of Chern-Simons coupling. In comparison, here we work with the flat field space while the instabilities induced from the Chern-Simons coupling are responsible for amplification of power spectra. Also note that instead of potential (5) one may consider different examples as well. We only need to assume the first stage of inflation is driven with a potential different than simple chaotic potential such that on CMB scales the value of \( r_t \) is small enough.

After presenting the general setup, in the following we consider two specific models: dilaton-axion (model I) and dilaton-axion-axion (model II). Table I presents the initial conditions and model parameters. The parameters are fixed to produce the correct COBE normalized at the CMB pivot scale \( k_{\text{pivot}} = 0.05 \text{Mpc}^{-1} \) [1]. As shown in Figs. 1 and 2, the background experiences several inflationary phases in both models. The first inflationary phase is driven by dilaton \( \Phi_1 \) while the other fields remain frozen. After \( \Phi_1 \) has reached to its minimum and
Its energy is died out after a few rapid oscillations the axion fields drive the next inflationary phase each in turn. The evolution of the Hubble parameter is also presented in Figs. 1 and 2 in accord with multiple inflationary phases. The numerical values of $\{N_{\text{end}}, r_n, n_s\}$ for the total number of $e$-folding, the tensor-to-scalar ratio, and the spectral index on CMB scales are $\{61.1, 0.014, 0.961\}$ for model I and $\{62.5, 0.032, 0.935\}$ for model II which are in close agreement with analytic results [40].

**Curvature Perturbations and PBH Formation:**

Now we study the evolution of fluctuations in our setup. Because of the coupling between the scalar fields and the gauge field, there will be source terms in the equation of motion of the scalar field fluctuations. The scalar field $\Psi^h$ can be decomposed into the background part $\phi^h$ and its canonical perturbation $\hat{Q}^h$ as $\Psi^h = \phi^h + a \hat{Q}^h$. In the spatially flat gauge, the equation of motion for the modes $\hat{Q}^h$ in momentum space reads [11, 45, 46]

$$\left( \partial^2 + k^2 - \frac{a''}{a} \right) \hat{Q}^h_k(\tau) + a^2 \left( \frac{\partial^2 V}{\partial \phi^a \partial \phi^b} \right) \hat{Q}^h_k(\tau) = \frac{\hat{a}^a a^3}{f} \int \frac{d^3k}{(2\pi)^{3/2}} e^{-ic \cdot \vec{E}} \vec{B} \cdot \vec{E},$$  

(6)

where $\tau$ is the conformal time, $d\tau \equiv a(t)dt$. The solution for $\hat{Q}^h$ can be separated into two uncorrelated parts $\hat{Q}^h = \hat{Q}^{(v)}_h + \hat{Q}^{(s)}_h$ where $\hat{Q}^{(v)}_h$ represents the solution to the homogeneous part of Eq. (6) which reduces to Bunch-Davies vacuum on small scales, whereas $\hat{Q}^{(s)}_h$ is the particular solution obtained by the Green function [7]. Also, since there is no interaction between the fields, the equations for $\hat{Q}_h$ are decoupled. Finally, the power spectrum of curvature perturbations, which is defined as $P_R = \sum_a (H/|a|) \hat{Q}_h^a$ at horizon crossing time, $N_k$, becomes [7, 8]

$$P_R(k) \simeq \frac{H^2}{8\pi^2 M^2_P \epsilon_H} \left( 1 + \frac{H^2}{8\pi^2 M^2_P \epsilon_H} f_2(\xi) e^{4\pi\xi} \right),$$  

(7)

where $\epsilon_H$ is the first slow-roll parameter and the dimensionless function $f_2(\xi)$ can be estimated for large $\xi$ as $10^{-5}/\xi^6$ [7]. The first term in Eq. (7) stands for the standard vacuum contribution to the power spectrum [4, 28].

The curvature perturbations power spectrum for the models in Table I are illustrated in Fig. 1 and 2. As can be seen, a rise in $\xi$ amplifies the scalar power spectrum for the mode that leaves the Hubble radius at the transition time between the two stages. The location of $a$-th peak, $N_a$ (number of $e$-fold since the start of inflation),

| Model | $(\tilde{a}_1, \tilde{\phi}_1)$ | $(\tilde{a}_2, \tilde{\phi}_2, m_1)$ | $(\tilde{a}_3, \tilde{\phi}_3, m_2)$ |
|-------|-----------------|---------------------------------|-----------------|
| I     | (10.74, 5.8)    | (5.10, 5.125)                   | -               |
| II    | (9.22, 5)       | (13.81, 0.2)                    | (5.10, 0.02)    |

**TABLE I:** Model parameters $(\tilde{a}_a, m_a)$ and initial conditions for pseudo-scalar fields, $\phi^a$ with $V_0 = 500 m_a^2 M_P^2$ and $m_1 = \sqrt{3} M_P$ for both models and define the dimensionless parameters $\tilde{m}_a \equiv 10^3 m_a/M_P$ and $\tilde{\phi}_a \equiv \phi_a/M_P$.
is given by the initial condition \( \phi^0_{\nu} \), while the amplitude of \( \mathcal{P}_\mathcal{R}(k) \) is controlled by \( \tilde{\alpha}_{\nu} \). Interestingly, for large values of \( \tilde{\alpha}_{\nu} \), the enhancement in the power spectrum is large enough to seed PBH formation due to the gravitational collapse of large density fluctuations after horizon re-entry during radiation-dominated era \[18\]. The mass of the formed PBHs is approximately 0.2 of the mass enclosed in the horizon at the time of re-entry of the corresponding mode \[18, 47\]. Assuming an instant reheating at the end of inflation, the mass corresponding to \( \alpha \)-th peak can be estimated as \[12\]

\[
\frac{M_{\alpha}}{M_\odot} \approx 10^{-13} \left( \frac{10^{-6} M_{\odot}}{H_{\text{end}}} \right) e^{2(N_{\text{end}} - N_{A} - 22.25)}, \quad (8)
\]

where \( M_\odot \) is the solar mass and \( H_{\text{end}} \) and \( H_{A} \) are the Hubble rates at \( N_{\text{end}} \) and \( N_{A} \), respectively.

The formed PBHs can contribute to the DM density. The fraction of PBHs against the total DM density at the present is given by \[18\]

\[
f_{\text{PBH}}(M_{\alpha}) \approx 2.7 \times 10^8 \left( \frac{M_{\odot}}{M_{\alpha}} \right)^{1/2} \beta(M_{\alpha}), \quad (9)
\]

where \( \beta \) is the mass fraction of PBHs at the time of formation \[12, 48-50\] which depends on the probability density function (PDF) of curvature perturbation \[51, 52\]. Since \( \hat{Q}^{(s)}_\alpha \), originating from the convolution of two Gaussian gauge fields, generates the peaks in \( \mathcal{P}_\mathcal{R} \), the PDF of curvature perturbation obeys a \( \chi^2 \) statistics \[11\]. In Fig. 3, we have depicted \( f_{\text{PBH}} \) for the models introduced in Table I. As illustrated, the formed PBHs can furnish a large fraction of total DM abundance. In particular, for model I, we obtain \( f_{\text{PBH}} \approx 1 \) corresponding to \( M_{\text{PBH}} \approx 2 \times 10^{-14} M_\odot \), whereas for model II we see two distinct peaks associated with \( M_{\text{PBH}} \approx 10^{-12} M_\odot \) and \( M_{\text{PBH}} \approx 0.8 M_\odot \), which jointly yield to \( f_{\text{PBH}} \approx 1 \).

**Primordial and Induced GWs:** In addition to the quantum vacuum fluctuations of metric during inflation, there are two distinct populations of stochastic GWs in our inflationary scenario. The first contribution is related to the GWs generated form the amplified gauge fields during inflation \[8, 58-61\]. The second contribution is the so-called induced GWs originating from the enhanced second order scalar fluctuations \[62-70\].

Due to the parity-violating nature of the system, the right and left helicities of tensor modes have different amplitudes \[58\]. The equation of motion for two canonical tensor helicity \( \hat{h}_\lambda \) is given by

\[
(\partial^2 + k^2 - \frac{2}{\tau^2})\hat{h}^{(\lambda)}_\tau(\tau) = -\frac{a^3}{M_{\text{Pl}}} \Pi^{(\lambda)}_\omega(k)x_{\lambda}(\tau)
\]

\[
\int \frac{d^3k}{(2\pi)^{3/2}} \frac{e^{-i\mathbf{k} \cdot \mathbf{x}}}{E_i E_j + B_i B_j} \]

\[
\mathcal{P}_{\mathcal{R}}^{(p)}(k) \approx \frac{H^2}{\pi^2 M_{\text{Pl}}^2} \left( 1 + 2 \frac{H^2}{M_{\text{Pl}}^2} f_{\lambda}(\xi) e^{4\xi} \right), \quad (11)
\]

where the superscript (p) stands for the primordial contribution in which the first term represents the contribution from vacuum fluctuations. Moreover, the dimensionless function \( f_{\lambda}(\xi) \) at large \( \xi \) for the right and left helicities are approximately \( 10^{-6}/\xi^6 \) and \( 10^{-9}/\xi^6 \), respectively \[58\]. Correspondingly, the main contribution to the primordial tensor power spectrum, \( \mathcal{P}_{\mathcal{R}}^{(p)} = \sum_\lambda \mathcal{P}_{\mathcal{R}}^{(p)} \), comes from the right helicity GW modes \[32, 58\].

As stated earlier, the GWs can be induced from the amplified curvature perturbations in Eq. \(7\) \[61\]. Indeed, the large second order scalar fluctuations on small scales induce tensor perturbations after the horizon re-entry during radiation-dominated era. With regard to the population of GWs discussed above, one deals with multiple integrals of the following form \[69\]

\[
\mathcal{P}_{k}^{(\text{ind})} \sim \int dk \int dk' \left[ \int f(k, k', t) dt \right] \mathcal{P}_{\mathcal{R}}(k) \mathcal{P}_{\mathcal{R}}(k') \quad (12)
\]
where \( f(k, k', t) \) is an oscillating function and \( t \) describes the time when the GW is sourced from the scalar modes [69–72]. Finally, the total present-day energy density of GWs is given by [61, 67, 68]

\[
\Omega_{\text{GW}}(k) = \Omega_{\text{GW}}^{(p)} (k) + \Omega_{\text{GW}}^{(\text{ind})} (k),
\]

in which \( \Omega_{\text{GW}}^{(p)} (k) \) and \( \Omega_{\text{GW}}^{(\text{ind})} (k) \) represent the fraction energy density of primordial GWs induced by the tachyonic gauge field mode and the induced GWs from the second order scalar perturbations respectively.

In Fig. 4, we have plotted the quantity \( \Omega_{\text{GW}} h^2 \) against the frequency with \( h^2 = 0.49 \) together with the sensitivity of the various forthcoming GW experiments e.g. the LISA [73], BBO [74–76], SKA [77–79], and PPTA [80, 81]. Clearly, for the models I, \( \Omega_{\text{GW}} h^2 \) falls within the sensitivity of the BBO and peaks well inside the range of detectability of LISA. Remarkably, for the model II, we observe that the double rises in GWs are detectable by LISA and SKA. A similar feature has been observed in [82] as a signal of a non-thermal baryogenesis from evaporating PBHs. Future GW observations can put constrain on the parameters of our model.

**Summary and Discussions:** We have studied a model of inflation with multiple pseudo-scalar fields coupled to a gauge field via the Chern-Simons type interactions. There are multiple stages of inflation driven by each scalar field. To evade the constraint on tensor-to-scalar ratio, we have considered a setup where the first stage is driven by a dilaton field while the remaining stages of inflation below CMB scales are driven with multiple axionic fields with the standard chaotic type potentials. However, our setup can be extended to more complicated potentials, such as \( \alpha \)-attractor model [40]. The enhanced power spectrum from the gauge field instability can generate PBHs with various masses which can furnish a large fraction of total DM while satisfying the bounds on PBHs formation [12]. In addition, GWs can be generated both from second order scalar perturbations as well as from the tachyonic gauge field perturbations with distinct features on the location of the peaks and their oscillatory behaviours. These signals are within the detection range of the future GW observatories. There are a number of directions in which the current investigations can be extended. These include investigating the non-Gaussianity of the perturbations [83] and its effects on the induced GWs [84, 85] and the PBHs formation [86].

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