Dynamical determination of the quadrupole mass moment of a white dwarf

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Abstract

In this paper we dynamically determine the quadrupole mass moment $Q$ of the magnetic white dwarf WD 0137-349 by looking for deviations from the third Kepler law induced by $Q$ in the orbital period of the recently discovered brown dwarf moving around it in a close 2-hr orbit. It turns out that a purely Newtonian model for the orbit of WD 0137-349B, assumed circular and equatorial, is adequate, given the present-day accuracy in knowing the orbital parameters of such a binary system. Our result is $Q = (-1.5 \pm 0.9) \times 10^{47}$ kg m$^2$ for $i = 35$ deg. It is able to accommodate the 3-sigma significant discrepancy of $(1.0 \pm 0.3) \times 10^{-8}$ s$^{-2}$ between the inverse square of the phenomenologically determined orbital period and the inverse square of the calculated Keplerian one. The impact of $i$, for which an interval $\Delta i$ of possible values close to 35 deg is considered, is investigated as well.

Key words: binaries: close: stars: individual: BPS CS 29504-0036: stars: brown dwarfs

1 Introduction

The recently discovered binary system WD 0137-349 [17, 4, 14], composed by a brown dwarf of 0.053 solar masses orbiting a white dwarf of 0.39 solar masses along a $\approx$ 2-hr circular orbit, offers us a nice opportunity to dynamically determine the quadrupole mass moment $Q$ of the white dwarf by suitably analyzing the orbital period of their relative motion.

Theoretical calculation of quantities related in various ways to such a bulk property of white dwarfs, whose knowledge is important for the equation of state of matter at very high densities as those present in such compact objects, can be found, e.g., in [6, 7, 16, 1, 11, 8].

The approach followed here has been recently adopted to determine $Q$ in the double pulsar PSR J0737-3039A/B system [9] and to put upper limits on it in binary systems hosting millisecond pulsars [10].
Table 1: Relevant orbital parameters of the WD 0137-349 system [14]. The mass of the white dwarf is $m_1$, the mass of the brown dwarf is $m_2$ and the baricentric semimajor axis is denoted with $a_{bc}$. The inclination $i$ is the angle between the plane of the sky, perpendicular to the line-of-sight, and the orbital plane.

| $P_b$ (days) | $a_{bc} \sin i$ (R$_\odot$) | $m_1$ (M$_\odot$) | $m_2$ (M$_\odot$) |
|-------------|--------------------------|-----------------|-----------------|
| 0.0803 ± 0.0002 | 0.375 ± 0.014 | 0.39 ± 0.035 | 0.053 ± 0.006 |

2 Model of the orbital period of WD 0137-349B

Given the distance and mass scales involved in our problem (see Table 1 for the relevant orbital parameters), the first post-Newtonian approximation is quite adequate to describe the motion of an object like WD 0137-349B around its parent white dwarf. The equation of motion, in standard post-Newtonian coordinates, can be written as [23]

$$\frac{dv}{dt} = -\nabla U + \frac{1}{c^2} \left[ -(\beta + \gamma)\nabla U^2 + 2(\gamma + 1)v(v \cdot \nabla)U - \gamma v^2 \nabla U \right].$$

(1)

We will assume that WD 0137-349 rigidly rotates and is endowed with both axial symmetry about the equator, taken to be the reference $\{xy\}$ plane\(^1\). Thus, the gravitational potential $U$ can be written as

$$U \equiv U_0 + U_Q,$$

(2)

with [18, 12, 22]

$$\begin{align*}
U_0 &= -\frac{GM}{r}, \\
U_Q &= -\frac{GQ}{r^3} \left( \frac{3 \cos^2 \theta - 1}{2} \right).
\end{align*}$$

(3)

In eq. (3) $M = m_1 + m_2$ and $\theta$ is the co-latitude angle ($\theta = \pi/2$ for points in the equatorial plane). The relative acceleration due to the gravitational potential of eq. (3) is, in spherical coordinates

$$\begin{align*}
A_r &= -\frac{GM}{r^2} - \frac{3GQ}{2r^3}(3 \cos^2 \theta - 1), \\
A_\theta &= -6\frac{GQ}{r^3} \sin 2\theta, \\
A_\varphi &= 0.
\end{align*}$$

(4)

\(^1\)Accounting for deviations from such approximations is not needed, given the present-day modest accuracy in determining $Q$. 

2
We will now make the simplifying assumptions that the motion of WD 0137-349B occurs in a circular orbit of radius \( r_0 \) in the equatorial plane of the white dwarf. This hypothesis is quite reasonable because tidal forces are very strong in short period binaries, and act to quickly circularize the orbit. In this case, \( A_\theta = A_\varphi = 0 \) and only the equation for the radial acceleration survives in eq. (4) as

\[
 r_0^2 \dot{\varphi}^2 = \frac{GM}{r_0^2} - \frac{3GQ}{2r_0^4}.
\] (5)

By posing \( r_0 \equiv a \), a simple integration of eq. (5) yields

\[
 Q = \frac{2}{3} Ma^2 - \frac{8}{3} \pi^2 \frac{a^5}{GP_b^2}.
\] (6)

Note that eq. (6) is an exact result.

In regard to the post-Newtonian term \( P^{(\text{PN})} \) coming from the \( c^{-2} \) part of eq. (1), in general relativity (\( \beta = \gamma = 1 \)) it is

\[
P^{(\text{PN})} \equiv P^{(1/c^2)} + P^{(Q/c^2)},
\] (7)

where

\[
\begin{align*}
P^{(1/c^2)} &= \frac{3\pi}{c^2} \sqrt{GMa}, \\
P^{(Q/c^2)} &= -\frac{13\pi Q}{2c^2} \sqrt{\frac{G}{Ma^*}}.
\end{align*}
\] (8)

\( P^{(1/c^2)} \) was calculated by Soffel in [20] and Mashhoon et al. in [13]; \( P^{(Q/c^2)} \) can be worked out from (6a) of [19] in the case of equatorial and circular orbits.

Let us now check if the precision with which the orbital period of WD 0137-349B is known requires to account for the post-Newtonian terms as well. From Table 1 and by using the following relation for the relative semimajor axis \( a \)

\[
a = \left(1 + \frac{m_2}{m_1}\right) a_{bc} + \mathcal{O}(c^{-2}),
\] (9)

it turns out

\[
\begin{align*}
P^{(1/c^2)} &= 2 \times 10^{-7} \text{ d}, \\
P^{(Q/c^2)} &= -1.9 \times 10^{-54} Q \text{ d}.
\end{align*}
\] (10)

Since the uncertainty in the brown dwarf’s orbital period amounts to \( 2 \times 10^{-4} \) d, it is clear that the Newtonian model of eq. (5) is quite adequate for our purposes.
3 Determination of $Q$

In order to calculate $Q$ and assess in a realistic and conservative way its uncertainty, let us make the following considerations. We are looking for a deviation from the ‘pure’ third Kepler law induced by $Q$, so that the values of the system’s parameters to be used with eq. (6) should not come from the third Kepler law itself. This is just the case, apart from the inclination angle $i$. Indeed, the mass of the white dwarf comes from an analysis of its optical spectrum, independently of any dynamical effect involving the orbital motion of its companion [2, 5, 14]; the same also holds for the brown dwarf’s mass which is determined from the ratio of the masses measured by means of accurate spectroscopy of the H$\alpha$ emission and absorption lines of WD 0137-349 [14]. Moreover, the dynamical observables at our disposal are the orbital period and two semi-amplitude velocities from which the projected semimajor axis is phenomenologically determined. The inclination $i$, instead, can only be measured from the expression of the mass function $F$ obtained with the third Kepler law, i.e.

$$F = \frac{m_1^3 \sin^3 i}{(m_1 + m_2)^2}, \quad (11)$$

according to [14], $i \approx 35$ deg. Thus, we will not treat $i$ as an estimated parameter with an associated experimental error; instead, we will keep it fixed to given values close to 35 deg, and for them we will determine $Q$, so to have a realistic idea of what could be the impact of $i$ on $Q$. We will assume $\Delta i/i = 28\%$, i.e. $i = 35 \pm 5$ deg, and $\Delta i/i = 11\%$, i.e. $i = 35 \pm 2$ deg.

Let us start with $i = 35$ deg. The values of Table 1 and eq. (6) yield

$$Q = -1.4615 \times 10^{47} \text{ kg m}^2. \quad (12)$$

According to Table 1 and eq. (6), the error can be conservatively evaluated as

$$\delta Q \leq \delta Q|_a + \delta Q|_M + \delta Q|_P + \delta Q|_G = 0.9004 \times 10^{47} \text{ kg m}^2, \quad (13)$$
with

\[
\begin{align*}
\delta Q|_a & \leq \left| \frac{4}{3}Ma - \frac{40}{3}\pi^2 \frac{a^4}{GP_b^2} \right| \delta a = 7.401 \times 10^{46} \text{ kg m}^2, \\
\delta Q|M & \leq \left| \frac{2}{3}a^2 \right| \delta M = 1.449 \times 10^{46} \text{ kg m}^2, \\
\delta Q|P_b & \leq \left| \frac{16}{3}\pi^2 \frac{a^5}{GP_b^2} \right| \delta P_b = 1.49 \times 10^{45} \text{ kg m}^2, \\
\delta Q|G & \leq \left| \frac{8}{3}\pi^2 \frac{a^5}{GP_b^2} \right| \delta G = 4 \times 10^{43} \text{ kg m}^2,
\end{align*}
\]

We used \( \delta G = 0.0010 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \) \([15]\) and

\[
\delta a \leq \delta a|_{abc} + \delta a|_{m_1} + \delta a|_{m_2} = 0.045R\odot, \tag{15}
\]

with

\[
\begin{align*}
\delta a|_{abc} & \leq \left| 1 + \frac{m_2}{m_1} \right| \delta a_{bc} = 0.027R\odot, \\
\delta a|_{m_2} & \leq \left| \frac{m_{bc}}{m_1} \right| \delta m_2 = 0.010R\odot, \\
\delta a|_{m_1} & \leq \left| \frac{m_{bc}}{m_1^2} \right| \delta m_1 = 0.008R\odot.
\end{align*}
\]

Now, let us repeat the same process for \( i = 30 \) deg; we obtain

\[
Q = (-3.9541 \pm 1.8210) \times 10^{47} \text{ kg m}^2. \tag{17}
\]

For \( i = 40 \) deg we get

\[
Q = (-4.657 \pm 4.983) \times 10^{46} \text{ kg m}^2. \tag{18}
\]

Let us now see what happens for values closer to \( i = 35 \) deg. A departure of 2 deg yields

\[
Q = (-2.1852 \pm 1.1754) \times 10^{47} \text{ kg m}^2 \tag{19}
\]

for \( i = 33 \) deg and

\[
Q = (-9.582 \pm 7.022) \times 10^{46} \text{ kg m}^2 \tag{20}
\]

for \( i = 37 \) deg. Thus, for \( \Delta i/i = 28\% \) we have \( \Delta Q/Q \approx 200\% \), while a narrower variation \( \Delta i/i = 11\% \) yields \( \Delta Q/Q \approx 83\% \). Such results have not to be considered as experimental errors.
4 Discussion

In regard to the adopted method, let us note the following remarks.

- The orbital period $P_b$ was determined in a purely phenomenological way from spectroscopical measurements, independently of any gravitational theory, so that it fully accounts for all the dynamical features of WD 0137-349B’s motion, within the errors.

- There is a significant discrepancy

$$\Delta = (1.0 \pm 0.3) \times 10^{-8} \, \text{s}^{-2} \, (i = 35 \, \text{deg})$$

between the inverse square of the phenomenologically determined orbital period $1/P^2_b$ and the inverse square of the purely Keplerian period $1/P^2(0)$ which is not compatible with 0 at 3-sigma level; now, eq. (12) and eq. (13) yield from eq. (6)

$$\frac{-3GQ}{8\pi^2a^3} = (1.0 \pm 0.3) \times 10^{-8} \, \text{s}^{-2} \, (i = 35 \, \text{deg}).$$

The same holds also for $i = 30 \, \text{deg}$ and $i = 40 \, \text{deg}$.

- According to eq. (12), $3Q/2Ma^2 = -1.2 \, (i = 35 \, \text{deg})$, so that the use of the exact, non-approximated expression of eq. (6) is fully justified.

- Let us note that with the result of eq. (12) the second post-Newtonian term of eq. (8) becomes $P^{(Q/c^2)} = 3 \times 10^{-7} \, \text{d} \, (i = 35 \, \text{deg})$: thus we can well justify, a posteriori, our choice of neglecting it in our analysis.

In conclusion, we have determined the quadrupole mass moment of the system WD 0137-349 to be $Q = (-1.5 \pm 0.9) \times 10^{17} \, \text{kg m}^2$ for $i = 35 \, \text{deg}$.

Our measured $Q$ should, in fact, be regarded as an effective quadrupole mass moment which may also include, in principle, contributions from the other multipole mass moments of higher degrees, from the white dwarf’s magnetic field [11, 8] and from the oblateness of the brown dwarf itself.

Another possible approach to the problem tackled here would be to re-analyze the raw data of the WD 0137-349 system by fitting them with a new orbital model including a quadrupole mass term as well, but it is beyond the scope of the present work.

Finally, in order to make easier a comparison with our results, in Table 2 we quote the numerical values used for the relevant constants entering the calculation.
Table 2: Values used for the defining, primary and derived constants
(http://ssd.jpl.nasa.gov/?constants#ref).

| constant     | numerical value          | units          | reference |
|--------------|--------------------------|----------------|-----------|
| c            | 299792458                | m s$^{-1}$     | [15]      |
| GM$_\odot$   | $1.32712440018 \times 10^{20}$ | m$^3$ s$^{-2}$ | [21]      |
| G            | $(6.6742 \pm 0.0010) \times 10^{-11}$ | kg$^{-1}$ m$^3$ s$^{-2}$ | [15]      |
| R$_\odot$    | $6.95508 \times 10^{8}$ | m             | [3]       |
| 1 mean sidereal day | 86164.09054         | s             | [21]      |

Acknowledgements

I gratefully thank P. Maxted for useful information about the orbital geometry of the considered system and J. Katz for important discussion about the inclination and the third Kepler’s law.

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