Vacuum energy for Yang-Mills fields in $R^d \times S^1$: One-loop, two-loop, and beyond

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Abstract
The vacuum energy is calculated for Yang-Mills (YM) system defined in $D$ dimensional space-time of $S^1 \times R^d$ ($D = d + 1$), where the possibility of the YM fields to acquire the vacuum expectation values on $S^1$ is taken into account. The vacuum energy has already been obtained to the order of one-loop in many people. Here we calculate the vacuum energy in $D$ dimensions to two-loop order. With an intention to reach higher loops, an approximation method is proposed, which is especially effective in higher dimensions. By this method, we can treat the higher-loop contributions of YM interactions as easily as we treat one-loop effect. As a check, we show reproduction of the two-loop contribution ($D$-dependence of the coefficient as well as the functional form) when the coupling constant is small. This approximation method is useful not only for the Kaluza-Klein theories but also for the finite temperature-density system (as a quark-gluon plasma).

1 Introduction
The quantum field theories in the space-time of non-trivial topology have been concerned with various physical situations. For instance, the Casimir effect [1] is a well-known example of them. This effect is shown experimentally by measuring the force between the conducting plates. Thus it is important to investigate the quantum effect to understand the forces working in the topologically non-trivial space-time and their integrals, that is, the “vacuum energy” (see, however, [2]).

The vacuum energy and symmetry breaking by scalar fields have been studied in the compact spaces including a torus and a sphere [3]. The quantum effects in compact spaces being as internal ones have been considered in relation to Kaluza-Klein theories [4, 5]. It has been pointed out that the quantum effect in a topologically non-trivial space plays an important role in symmetry breaking in string theory [6] and in Kaluza-Klein theories [7].

Now we take up as an example the quantum effect in Kaluza-Klein theories [5, 7] in order to see how the calculation of the effect has been performed. In
generic Kaluza-Klein theories the stability of the extra space has been considered, where the estimation of the strength of Casimir-like force plays the crucial role. The contribution of gravitons and matter fields to the vacuum energy has been also computed [5].

On the other hand, the model with symmetry breaking by gauge fields defined on the extra space has been investigated. The vacuum energy in the presence of the background gauge field has been calculated and is used to determine the true vacuum among vacua which belongs to various gauge symmetry [7].

The approaches so far are to calculate the vacuum energies to one-loop order. They are expressed symbolically as

\[(\text{one-loop vac. energy}) = \log \text{Det} (-\Delta),\] (1)

where \(A\) stands for the generalized Laplacian associated with the field under consideration. That is the inverse of propagator.

The reasons why the one-loop approximation is often examined are: i) the technique of calculation and regularization methods (for instance, the \(\text{if-function}\) regularization and the dimensional regularization) are well-defined and can be generalized to an arbitrary dimensional case, and ii) moreover we can carry out one-loop calculation only by the knowledge of the free propagator and thus we need not to worry about the renormalization scheme (of interactions) in the way of calculation.

However we know how important the interaction of fields is in the quantum effect. It is true that the interaction has an essential effect on the quantum effect in the gauge theory including QCD in four dimensions (see [8] in the context of Kaluza-Klein theory).

In the next section we perform the two-loop calculation of the vacuum energy for YM fields on the \(R^d \times S^1\) background topology. This is a generalization of the two-loop calculation of the free energy for YM fields at finite temperature which has been calculated before. For later use, the background gauge field is taken into account in our calculation. In the present paper, we discuss only the vacuum energy (the free energy) for simplicity.

The two-loop calculations of the vacuum energy including Dirac fermions are also shown in Sect. 2. These have not been sufficiently surveyed even in the four-dimensional finite-temperature system. We consider the fermions in fundamental and adjoint representations, possessing a general boundary condition, an arbitrary dimension in the presence of a background gauge field on \(S^1\).

In Sect. 3, we try to include the effect YM interactions beyond two-loops. The approximation scheme proposed there is valid for a large number of dimensions, \(D\).

The last section is devoted to the summary and consideration of future problems.

Before closing this section, we mention some comments. In the background configuration considered in this paper, both the curvature and the field strength of gauge field vanish. Thus we naively expect vanishing expectation value for
local counter terms. That is why there is no difficulty in obtaining the vacuum energy in the general dimensions. Though we can proceed to a neat discussion on the regularization making use of the operator regularization method [9], that is beyond the scope of this paper.

2 The two-loop contributions to vacuum energy of Yang-Mills system with fermions in $R^d \times S^1$ space

In finite-temperature systems, or systems in Euclidean space-time ($R^3 \times S^1$), vacuum energy of YM fields to two-loop order has already been calculated by many authors [10, 11]. We compute the vacuum energy in an arbitrary space-time dimensions $D(= d + 1)$ in this section. In the present paper, we restrict ourselves to the case for $SU(2)$ gauge group throughout this paper.

We assume $R^d \times S^1$ as the background geometry. The circumference of $S^1$ is set to $L$. We take gauge field condensation on $S^1$ into consideration. It is parametrized as the following (matrix-)form.

$$\langle A_I \rangle = \frac{\phi}{2gL} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (2)$$

where $g$ is the YM coupling constant. Because of the presence of the background field, we use the so-called covariant background gauge method in the calculations, in which quantum and classical fields are neatly separated (see, for example, [12]). Furthermore, we take the Feynman gauge ($\xi = 1$) unless we indicate the gauge explicitly.

Figure 1: The two-loop contributions of YM bosons to the vacuum energy

There are three graphs for two-loop vacuum diagrams of YM fields, of the order of $g^2$ (see Fig. 1). Each contribution to the vacuum energy in $R^d \times S^1$ space is

$$(a) : \quad + \frac{1}{2} g^2 D(D - 1)I(\phi)\{I(\phi) + 2I(0)\}, \quad (3)$$
(b) : \[-\frac{3}{2}g^2(D - 1)I(\phi)\{I(\phi) + 2I(0)\}, \tag{4}\]
(c) : \[\frac{1}{2}g^2I(\phi)\{I(\phi) + 2I(0)\}, \tag{5}\]

where the function \(I\) is defined as
\[
I(x) = \frac{\Gamma\left(\frac{D-2}{2}\right)}{2\phi^{D/2}L^{D-2}} \sum_{k=1}^{\infty} \frac{\cos kx}{k^{D-2}}. \tag{6}\]

\(I(x)\) comes from the loop integration obtained after performing a suitable regularization which is chosen as in the four-dimensional case.

Summing over the three contributions, we obtain the two-loop contribution to vacuum energy for pure YM system:
\[
(2 - \log \text{vac. energy for YM}) = +\frac{1}{2}g^2(D - 2)^2I(\phi)\{I(\phi) + 2I(0)\}. \tag{7}\]

This calculation is a generalization of the result of [11]. Here, we must note: i) the functional form of each contribution of diagram (a-c) is identical, ii) however, the dependence of coefficient on the dimensionality \(D\) is different from each other, iii) if \(D = 2\), the total contribution vanishes (as expected).

Next we study the fermion loops. The one-loop contribution of the fermion to vacuum energy can be easily obtained. We show here the result for the two-loop contribution which is made of fermion and YM boson in a general dimension \(D\). This is also of the order of \(g^2\).

In a finite-temperature system, the calculation for fermion contribution is important particularly in the study of the so-called quark-gluon plasma. For future applications, we consider a generic boundary condition of fermions on \(S^1\) as well as a background gauge field on \(S^1\). We assume that the fermion fields has the following dependence on the translation in the coordinate of \(S^1\), which we denote \(y\):
\[
\Psi(y + L) = e^{i\delta} \Psi(y). \tag{8}\]

The only graph to calculate is given in Fig. 2.

Figure 2: The two-loop contributions of fermion and YM boson to the vacuum energy

We consider Dirac fermions which belong to fundamental and adjoint representations. After a small handwork, we obtain:

(2-loop vac. energy for fund. fermion)
\[
\text{(2-loop vac. energy for adj. fermion)}
\]
\[
= -\frac{1}{2}g^2\frac{D-2}{2} \cdot \frac{1}{2} \left\{ I\left(\delta + \frac{\phi}{2}\right) + I\left(\delta - \frac{\phi}{2}\right) \right\} \left\{ I(0) + 2I(\phi) \right\} \\
- I\left(\delta + \frac{\phi}{2}\right)I\left(\delta - \frac{\phi}{2}\right) \\
- \frac{1}{4} \left\{ I\left(\delta + \frac{\phi}{2}\right)^2 + I\left(\delta - \frac{\phi}{2}\right)^2 \right\} \\
\]
(9)

These results for general conditions are obtained for the first time even in four dimensions. For application to a finite-temperature system, we set \( D = 4 \) and replace \( L \) with \( \beta = T^{-1} \) (where \( T \) is the temperature of the system). A peculiarity is known that \( I(x) \) is linear in \( x \) for small \( x \) when \( D = 4 \). Because of this behaviour of \( I(x) \), the free energy (density) as a function of \( r \) has a minimum located around \( \phi \approx \frac{g^2}{4\pi} \). This suggests that a symmetry breaking occurs in the YM system at high temperature [11]. Using the result here, it is revealed that the inclusion of arbitrary numbers of fermions belonging to fundamental or adjoint representations of SU(2) does not modify the vacuum expectation value of \( r \) to the order of \( g^2 \) [13].

After completion of the first draft of the present paper, we find a very recent paper of [18]. In the paper, Belyaev showed that the vacuum condensation \( \phi \approx \frac{g^2}{4\pi} \) is not a true order parameter and the study of Wilson loop exhibits that there is no symmetry breaking in finite-temperature pure YM systems. Invariance of the minimum at \( \phi \approx \frac{g^2}{4\pi} \) when fermions are added must be crucial in the analysis of Wilson loop in the system including fermions. We will treat the subject elsewhere.

3 Beyond the two loop

As in the previous section, one can calculate the vacuum energy to higher-order in \( g \) by computing the higher-loop diagrams (whereas, for instance in four dimensions, we should be careful to sum over loops which yields the contribution of the order \( g^3 \) [10]). If the coupling \( g \) takes a large value, however, such a perturbative approach loses its reliability. When we consider QCD as a non-Abelian gauge theory, validity of perturbation is confirmed in certain occasions; on the other hand it is necessary to examine non-perturbative effects quantitatively as well as qualitatively.
As an example, we take the gauge field condensation mentioned in the previous section. The minimum of free energy has been obtained for small $g$. For large $g$, this perturbative analysis is no longer reliable. However, we are interested in another possibility of gauge field condensation in the non-perturbative region.

In QCD, it is significant to study the coupling dependence of various physical quantities in order to investigate not only cosmology of the early universe but also experiments of heavy nuclei.

It is also pointed out that transition of a certain model of unified theory cries out for consideration of non-perturbative effect in gauge theory [14]. Effort in treating the non-perturbative phenomena analytically is of importance.

From the point of view of the computational technique, it will be convenient to treat all-order effect by an extension of one-loop treatment. In this paper we try to take out much information of non-perturbative physics from the representation of vacuum energy.

In Sect. 2, we have calculated the two-loop vacuum diagrams. The $D$-dependence of the coefficient of each diagram is as follows (see (3, 4, 5) and Fig. 1): for (a) $\approx D^2$, for (b) $\approx D^4$, and for (c) $\approx D^0$ for large $D$. Namely, the graph including YM four-point interaction dominates for large $D$. Here $D$ comes from the trace of the metric at a closed loop. The graph (b) is sub-dominant because three-point interaction has a derivative coupling. The contribution of the graph (c) does not have the trace of the metric.

Therefore it is conceivable that for large $D$ only four-point interaction is important and this simplifies the treatment of YM interactions of higher order. In the rest of this section we consider an approximation method based on this observation.

The $D$-dependence appears also in $I(x)$, that is, in the momentum integrations. Thus the mass insertian reduces the order of $D$. As we can see later, this effect is naturally involved by our approximation.

First of all, we write down the YM action where three-point interactions are omitted:

$$\frac{1}{4} \text{tr} F^2 \approx \frac{1}{4} \sum_{\mu \nu} \sum_{a=1}^{3} (D^B_{\mu} a^a_{\nu} - D^B_{\nu} a^a_{\mu})^2 + \text{(gauge-fixing term)}$$

$$+ g^2 \{(a^3_\mu a^3_\nu - a^2_\mu a^3_\nu)^2 + (a^3_\mu a^1_\nu - a^3_\nu a^1_\mu)^2 + (a^1_\mu a^2_\nu - a^2_\nu a^1_\mu)^2\} \quad (11)$$

where $a^a_\mu$ is the quantum fluctuation of the gauge field while $D^B_{\mu}$ denotes the covariant derivative involving the background classical fields. If we further neglect the interactions which take the form of $a^1_\mu a^1_\nu a^2_\mu a^2_\nu$, we obtain

$$\sum_{\mu \nu} \left[ \frac{1}{2} \sum_a (D^B_{\mu} a^a_{\nu})^2 + \frac{1}{2} g^2 \{(a^3_\mu a^3_\nu)(a^3_\mu a^3_\nu) \right] \quad (12)$$

The reason why the terms like $a^1_\mu a^3_\nu a^2_\mu a^2_\nu$ can be omitted is that we cannot make the graph like Fig. 3 by only use of those terms, while graphs of the type
of Fig. 4 can be made: although both contributions of Figs. 3 and 4 are of the order of \( g^4 \), the contribution of Fig. 3 is superior to that of Fig. 4 by factor of the order of \( D \) which comes from trace of the metric associated with each loop. Therefore as long as we regard \( D \) as a large number, the YM Lagrangian is well approximated by (12).

Figure 3: The graph of the order of \( g^4 \) which can be led from the effective Lagrangian (12)

Figure 4: The graph of the order of \( g^4 \) constructed by the interaction terms like \( a^1 \mu a^1_\nu a^2_\mu a^3_\nu \) which are not involved in the effective Lagrangian (12)

Owing to this simplification, the calculation of the vacuum energy becomes very transparent. We can utilize auxiliary fields [15] to treat the non-linear interaction in the Lagrangian (12).

Using the knowledge that the matrix

\[
\begin{pmatrix}
  0 & 1 & 1 \\
  1 & 0 & 1 \\
  1 & 1 & 0
\end{pmatrix},
\]

has its inverse matrix

\[
\frac{1}{2} \begin{pmatrix}
  -1 & 1 & 1 \\
  1 & -1 & 1 \\
  1 & 1 & -1
\end{pmatrix},
\]

we rewrite the Lagrangian as

\[
\frac{1}{2} \sum_{\mu \nu} \sum_a (D^\mu_a a^\nu_a)^2 + \frac{1}{2} \sum_{\mu} \sum_a \chi^a a^a_\mu a^a_\mu + \frac{1}{8g^2} \cdot [(\chi^1)^2 + (\chi^2)^2 + (\chi^3)^2 - 2(\chi^2\chi^3 + \chi^3\chi^1 + \chi^1\chi^2)].
\]

The use of the auxiliary fields \( \chi^a \) (\( a = 1, 2, 3 \)) enable us to rewrite the action in the bilinear form of \( a_\mu \)'s.

We calculate the vacuum energy in the presence of the background field (2). The background fields and metric are assumed to be the same as in the previous section. Because of the symmetry, we can set \( \chi^1 = \chi^2 \) and then the
formal expression of the one-loop vacuum energy including the auxiliary fields are given as

\[
\frac{1}{8g^2}((\chi^3)^2 - 4\chi^1\chi^3) \\
+ D^* \int \frac{d^dp}{(2\pi)^d} \sum_{k=-\infty}^{\infty} \ln \left\{ p^2 + \left( \frac{2\pi k + \Phi}{L} \right)^2 + \chi^1 \right\} \\
+ \frac{D^*}{2} \int \frac{d^dp}{(2\pi)^d} \sum_{k=-\infty}^{\infty} \ln \left\{ p^2 + \left( \frac{2\pi k}{L} \right)^2 + \chi^3 \right\}.
\]

(16)

Since we are ignoring the contribution of ghost fields, we denote the coefficient of the integrals using the constant \( D^* \) which is the same order as \( D \). Later we will “adjust” the constant \( D^* \).

After regularization of (16), we obtain the following expression in the large \( D \) (also \( D^* \)) limit:

\[
\frac{1}{8g^2}((\chi^3)^2 - 4\chi^1\chi^3) \\
- 4D^* \left[ \sum_{k=1}^{\infty} \left( \frac{\sqrt{\chi^1}}{2\pi kL} \right)^{D/2} K_{D/2}(\sqrt{\chi^3}Lk) \cos k\phi \right] \\
- 2D^* \left[ \sum_{k=1}^{\infty} \left( \frac{\sqrt{\chi^3}}{2\pi kL} \right)^{D/2} K_{D/2}(\sqrt{\chi^3}Lk) \right],
\]

(17)

where \( K_n(x) \) is the modified Bessel function. Eliminating the auxiliary fields by applying the equations of motion for \( \chi \), we obtain the vacuum energy including the (four-point) interaction effect. For larger \( D \), more exact result for energy is expected to be given.

As a check, we will show that the result of the one- and two-loop order is obtained when \( g^2 \ll 1 \).

For small \( g \), the equations of motion for \( \chi \) obtained from (17) can be solved approximately as

\[
\chi^1 = g^2D^* \frac{\Gamma \left( \frac{D}{2} - 1 \right)}{2\pi} \left( \frac{1}{\pi L^2} \right)^{D/2-1} \sum_{k=1}^{\infty} \frac{1 + \cos k\phi}{k^{D-2}},
\]

(18)

\[
\chi^3 = g^2D^* \frac{\Gamma \left( \frac{D}{2} - 1 \right)}{2\pi} \left( \frac{1}{\pi L^2} \right)^{D/2-1} \sum_{k=1}^{\infty} \frac{2 \cos k\phi}{k^{D-2}}.
\]

(19)

Thus the vacuum energy for YM system in \( R^d \times S^1 \) for small coupling \( g \) is obtained to the order of \( g^2 \) as:

\[
(\text{vac. energy of YM for small } g^2)
\]
\[
\approx -D^* \frac{\Gamma \left( \frac{D}{2} \right)}{\pi^{D/2} L^D} \sum_{k=1}^{\infty} \frac{1 + \cos k\phi}{k^D} \\
+ \frac{g^2}{2} \left\{ D^* \frac{\Gamma \left( \frac{D}{2} - 1 \right)}{2\pi^{D/2} L^{D-2}} \right\}^2 \left( \sum_{k=1}^{\infty} \frac{\cos k\phi}{k^{D-2}} \right) \left( \sum_{k=1}^{\infty} \frac{2 + \cos k\phi}{k^{D-2}} \right). \tag{20}
\]

Therefore we find that the substitution \( D^* = D - 2 \) leads to the exact result of perturbative calculation of the two-loop diagrams (see (7)).

In physical meaning, \( \chi \) stands for the “mass” of the gauge bosons. In finite-temperature systems, this is finite in general [10]. In four dimensions \((D = 4)\), let us investigate the next-order contribution in \( g^2 \). We rewrite \( T = L^{-1} \) and \( \langle \phi \rangle \) is assumed to vanish. Using asymptotic expansion of \( K_1(x) \), we obtain
\[
\chi \frac{T^2}{2} = \frac{g^2 D^*}{6} - \frac{1}{2\sqrt{6}} (g^2 D^*)^{3/2} + \ldots. \tag{21}
\]

Though the result is not exactly coincident with the known result [10], we obtain the correct order of magnitude of coefficients and the correct fractional dependence on \( g \) in the next-leading term [10]. Our approximation is not so bad even in four dimensions, \( D = 4 \).

Another consequence which can be compared with the perturbative results is the dependence on gauge parameter. So far we take Feynman gauge, \( \xi = 1 \); to take other choice is straightforward because we only need the technique of the one-loop calculation. It is known that there is in fact gauge-dependence of \((\xi - 1)^1\) at the two-loop order, while a naive counting suggests at most \((\xi - 1)^3\) dependence [16]. By our method we find \((\xi - 1)^1\) dependence in the order of \( g^2 \), though we do not give the explicit calculation here.

Now we consider some examples for applications of our method.

If we consider \( S^1 \) as an extra space, we can calculate vacuum energies for YM fields in the Kaluza-Klein background. The one-loop effects of various fields have been investigated in many authors [5, 7, 8]. The approximation scheme in the present paper is effective in Kaluza-Klein theories because the number of dimensions may be arbitrarily large. Explicit calculations will be shown in [19].

Next we consider finite-temperature systems. We set \( T = L^{-1} \), where \( T \) is the temperature of the system, and \( D = 4 \). At first we assume the background field \( \langle \phi \rangle = 0 \). The free energy of free massless particle system is proportional to \( T^4 \). The effect of YM interaction modifies the relation. To see this, we define an “effective degrees of freedom”, denoted by \(|F|\), as
\[
(\text{effective degrees of freedom}) = \frac{|\text{free energy density}|}{T^4}. \tag{22}
\]

By numerical calculation, we can obtain free energy of YM boson gas. The result is shown in Fig. 5. It is physically natural to see the effective degrees of freedom decreases when \( g^2 \) becomes large. Since its decrease is very moderate, the change on the temperature through the running of the coupling \( g \) is expected
Figure 5: The plot of the effective degrees of freedom vs. $g^2$

to be small. Therefore, for example, the analysis of the finite-volume effect [17] is important on the physical confinement.

We can calculate the vacuum energy even in the presence of the background gauge field. In higher dimensional theory, the symmetry breaking by the field is known as Hosotani mechanism [7]. At the one-loop level, the mechanism is investigated by many authors. We will report the application of our method in higher dimensions elsewhere [19].

Figure 6: The plot of $F(\phi) = (\text{the free energy density}/T^4)$ vs. $\phi$ for $g^2 = 0.01$ and $g^2 = 1$

In four dimensions, the background field at finite temperature is also useful to analyze the finite-volume effect in YM system [17]. In Fig. 6, $F(\phi) = (\text{the free energy})/(\text{temperature})^4$ is plotted against $\langle \phi \rangle$. (Note: $|F| = F(0)$.) The minimum remains located at $\langle \phi \rangle = 0$. Unfortunately, non-perturbative minima have not been found by the present analysis.

The values of $\chi^4/T^2$ in the presence of $\langle \phi \rangle$ are given in Fig. 7.

The curvature of the change in $\phi$ near zero becomes smaller for larger $g$; thus the finite-volume effect, which is obtained by the integration with respect to $\phi$ around $\langle \phi \rangle = 0$ [17], is expected to increase slightly if the coupling $g$ becomes
large.

We hope to study more generic cases, such as with SU(3) YM fields, and thorough analyses on the various systems in the near future.

4 Summary and future problems

We have obtained the vacuum energy for YM system in the D-dimensional space $S^1 \times R^d$ ($D = d + 1$). We have taken the presence of the background gauge field into consideration. In the present paper we have performed the two-loop calculation in D dimensions. We have shown the approximation scheme which is suitable for large $D$ and by the use of this one can investigate the effects of YM interaction. For small $g$, the YM self-coupling, we have shown the reconstruction of free energy to the two-loop order (i.e., the functional form and the dependence on $D$) by our method.

Our computational method is especially well-suited for Kaluza-Klein theories in higher dimensions. Even if the extra space has a complicated structure, we can study the higher-loop effects by our method similarly to the one-loop technique. The study of this is currently in progress.

Similar analyses of higher-loop effects on the vertex and the propagator graphs are possible, though we have treated only the vacuum graph in this paper. This subject is an interesting one which we wish to study.

Another important task is the extension to large symmetry groups. Besides the application to QCD, it is claimed that the behavior of the free energy for the system of gauge bosons which interact themselves strongly is important in the symmetry breaking in the early universe.

We want to develop the method which is more effective in various situations. We hope that various numerical calculations which can be treated by our method will be reported in near future.
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