New clean fission with hadronic molecular states

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Nuclear fission and related technology release tremendous amounts of energy and greatly affect modern science and society. However, it is difficult to dispose of the long-lived radioactive wastes associated with nuclear fission, and these wastes will threaten our environment for a long time. We propose using hadronic molecular states in a new clean fission that does not produce long-lived radioactive wastes, and we investigate the fission of $X(3872)$ in detail. Fission of $X(3872)$ can release approximately 142 MeV of energy, approximately 4% of its mass, which is much larger than that of nuclear fission. The cross section of fission of $X(3872)$ is comparable with and even larger than that of nuclear fission. The charmed meson breeds in the induced fission of $X(3872)$, which makes the chain reaction possible in the $X(3872)$ matter. This special phenomenon in the fission of hadronic molecules will help us better understand the mechanism of fission in the future.

Fission is the disintegration of a nucleus into two or more lighter nuclei accompanied by energy release [1]. It may occur spontaneously or result from a neutron striking a nucleus. The total mass of the initial states is larger than that of the final states in the fission process, and such a mass defect produces a very large amount of nuclear energy in a nuclear reactor. Currently, nuclear power generation accounts for 10% of the world’s total power generation [2].

Even though nuclear power is widely used, it is difficult to dispose of the resulting wastes. Many isotopes associated with nuclear fission are radioactive, and their half lives vary from fractions of a second to millions of years [3]. For example, iodine-131 may cause thyroid cancer, and its half-life is approximately 8 days. Moreover, some transuranic elements produced in fission plants have half-lives of millions of years, which means they can remain radioactive for very long periods of time.

These long-lived fission wastes may be utilized to release energy further in advanced reactor designs and be transformed into short-lived radioisotopes in the future, but currently, disposal of nuclear waste is still one of the greatest challenges for nuclear power plants. In this letter, we propose a new clean fission using hadronic molecular states and show that it can be more energy efficient than ordinary fission and produces only short-lived products.

The components of a nucleus, neutrons, are one kind of hadron that includes various baryons and mesons. It is natural to expect a bound state that contains other hadrons, e.g., a ground state neutral charmed or anticharm meson ($D^0$ or $\bar{D}^0$) associated with an excited state ($D^{*0}$ or $\bar{D}^{*0}$). Such a bound state is called a hadronic molecular state [4, 5]. Currently, studies of hadronic molecular states focus on those composed of two hadrons. In experiment, many hadronic molecule candidates have been observed, such as $X(3872)$, $T_{cc}$, and hidden-charmed $P_{c}$ states [6–9].

$X(3872)$ is the first observed hadronic molecule candidate with a mass very close to the total mass of a ground state and an excited charm meson [7]. Hence, it is suggested to be a hadronic molecular state composed of two (anti)charm mesons [10]. Its wave function can be written as [11],

$$|X⟩ = \frac{1}{\sqrt{2}}(|\bar{D}^{*0}D^0⟩ - |D^{*0}\bar{D}^0⟩),$$

which means that $X(3872)$ can be either the $\bar{D}^{*0}D^0$ molecule or the $D^{*0}\bar{D}^0$ molecule with equal probability. That is, it is the superposition of the two molecular states that can be understood and treated analogously, and thereafter, we take the first term $\bar{D}^{*0}D^0$ as representative. $X(3872)$ carries the spin parity $J^P = 1^+$, which cannot decay into a $D^0\bar{D}^0$ pair due to the conservation of spin parity. In fact, if we do not consider the small decay probabilities of the $D^{*0}$ and $D^0$ mesons, this hadronic molecular state should be stable, similar to a deuteron. Similar to neutron-induced fission of the nucleus, the fission of $X(3872)$ can be induced by an additional $D^0$ meson. Hence, $X(3872)$ is an appropriate example to show the effects of induced fission with hadronic molecular states. The classical fission reactions of neutron-induced uranium and plutonium fission are as follows [3],

$$n + ^{235}\text{U} \rightarrow 2^{236}\text{U} \rightarrow 141\text{Ba} + 92\text{Kr} + 3n, \quad \frac{\Delta M}{M_{^{235}\text{U}}} = 0.06%,$$

$$n + ^{238}\text{U} \rightarrow 2^{239}\text{U} \rightarrow 96\text{Sr} + 140\text{Xe} + 3n, \quad \frac{\Delta M}{M_{^{238}\text{U}}} = 0.08%,$$

$$n + ^{239}\text{Pu} \rightarrow 2^{240}\text{Pu} \rightarrow 96\text{Sr} + 140\text{Ba} + 4n, \quad \frac{\Delta M}{M_{^{239}\text{Pu}}} = 0.08%. \quad (2)$$

Here, $\Delta M$ is the mass defect of the fission process. As shown above, the mass defect of nuclear fission is approximately 0.1% of the nuclear mass. The explicit scheme of the induced fission with $X(3872)$ is shown on the upper left of Fig. 1.

An incoming $D^0$ meson is used to bombard the molecular state $X(3872)$, which produces an intermediate state as $D^0\bar{D}^{*0}D^0$ that can decay into three ground mesons as

$$D^0 + X(3872) \rightarrow D^0\bar{D}^{*0}D^0 \rightarrow 2D^0 + D^0, \quad \frac{\Delta E}{M_{X}} = 3.67%. \quad (3)$$
FIG. 1: Schematic depiction of the induced fission and chain reaction of \( X(3872) \). (a) The induced fission of \( X(3872) \) with an incoming \( D^0 \) meson. The blue and green balls denote \( D^0 \) and \( \bar{D}^0 \) mesons, respectively. The collision happens between the incoming \( D^0 \) meson and \( \bar{D}^0 \) meson in the \( X(3872) \) which is denoted by a small oval. (b) The Feynman diagram corresponding to the induced fission in (a) with \( \rho / \omega \) exchange. (c) The chain reaction of \( X(3872) \) matter. The large oval means an induced fission in (a). The overlap of the ovals and the meson therein means that the final meson of previous fission becomes the incoming meson of next fission.

Here, the masses are cited from PDG [6]. In the reaction, the incoming \( D^0 \) meson is analogous to the neutron in nuclear fission. Such a reaction is analogous to neutron-induced nuclear fission shown in Eq. (2). We can define such a reaction as the induced fission of a molecular state.

The chain reaction is important to make nuclear fission proceed continuously and is the key mechanism of nuclear bombs and nuclear power plants. If a fission reaction occurs in \( X(3872) \) as shown in the right part of Fig. 1, the produced \( D^0 \) mesons will induce the fission of more molecular states. Thus, the chain reaction of induced fission is also possible for hadronic molecular states.

Because the hadronic molecular state is much lighter than the heavy nucleus, the relative mass defect is much larger than that of nuclear fission, as shown in Eqs. (2) and (3). That is, \( X(3872) \) can release more (approximately 40 times) more energy than an equal mass of uranium!

The probability of induced fission can be estimated by the cross section \( \sigma \). In the induced fission of \( X(3872) \), the incoming \( D^0 \) meson strikes on the \( \bar{D}^0 \) meson in \( X(3872) \) while the interior \( D^0 \) meson is a spectator, and then one \( \bar{D}^0 \) and two \( D^0 \) mesons are produced. This can be described by a Feynman diagram within field theory, as shown on the bottom left of Fig. 1. From the diagram, the \( \bar{D}^0 \) meson in \( X(3872) \) exchanges a \( \rho / \omega \) meson with the incoming \( D^0 \) meson and transforms into a \( D^0 \) meson. For meson-induced fission of a molecular state it is logical to use the rest frame of the molecular state, that is, the so-called laboratory frame. In this reference frame, the cross section for the reaction \( D^0 + X \rightarrow 2D^0 + \bar{D}^0 \) is expressed as,

\[
d\sigma = \frac{1}{4(p_1 \cdot p_2)^2 - m_{1,2}^2 m_{1,2}^2} \frac{1}{2} \sum |M_i|^2 d\Phi_3 \frac{1}{2},
\]

where the \( p_{1,2} \) and \( m_{1,2} \) are the momentum and mass of incoming \( D^0 \) meson or \( X(3872) \). The phase space \( d\Phi_3 \) is produced with the help of GENEV code inFAW as \( R_3 = (2\pi)^3 d\Phi_3 = \prod \frac{d^k \varepsilon_{\lambda} \lambda \sum n_i k_i - P} \) where the \( k_i \) and \( E_i \) are the momentum and energy of final particle \( i \). The mechanism of the fission reaction can be described by an amplitude \( M_i \) with \( \lambda \) being the helicity of \( X(3872) \). For the first term of the wave function in Eq. (1), the amplitude can be written as

\[
M_i = \frac{A_{\text{Fist}}(X \rightarrow \bar{D}^0 D^0) A_{\text{Fist}}(D^0 D^0 \rightarrow \bar{D}^0 D^0)}{p^2 - m_{1,2}^2},
\]

where the different helicities for intermediate \( \bar{D}^0 \) meson \( \lambda_{D^0} \) should be summed up and the helicities will be omitted if not necessary.

In the literature [12], the \( X(3872) \) splitting into \( \bar{D}^0 D^0 \) can be related to the scattering of \( D^0 \bar{D}^0 \). The coupling of the molecular state to its constituents can be related to binding energy [13]. Hence, the amplitude for \( X(3872) \) splitting into \( \bar{D}^0 D^0 \) is determined by the scattering length \( a \) as [12],

\[
\mathcal{A}(X \rightarrow \bar{D}^0 D^0) = \sqrt{\frac{15\pi m_X m_D m_D}{\mu^2 a}} \lambda \cdot \epsilon,
\]

where \( m_{X,D} \) is the mass of \( X(3872) \), the constituent \( \bar{D}^0 \), or \( D^0 \). The \( \epsilon \) and \( \epsilon \) are the polarized vectors for \( X(3872) \) and \( \bar{D}^0 \). Scattering length \( a = 1/2\mu E_B \) with the reduced mass \( \mu = m_D m_D/(m_D + m_D) \) and the \( E_B \) being the binding energy.

The propagator of \( \bar{D}^0 \) in laboratory frame, where the \( X(3872) \) is static, can be written as

\[
\mathcal{A}(X \rightarrow \bar{D}^0 D^0) = \sqrt{\frac{15\pi m_X m_D m_D}{\mu^2 a}} \lambda \cdot \epsilon,
\]

Before being struck, the \( X(3872) \) is static and the binding energy is very small, which suggest that the momenta of the constituent mesons is small. The energy of two constituent mesons can be safely approximated as \( E_3 = \frac{m_1^2 + m_2^2}{2m_D} \) and \( E_3 = \frac{m_1^2 + m_2^2}{2m_D} \). As in Ref. [14], the amplitudes for \( X(3872) \) splitting with the propagator of \( D^0 \) meson can be expressed with wave function of \( X(3872) \) as

\[
\mathcal{A}(X \rightarrow \bar{D}^0 D^0) = \sqrt{\frac{15\pi m_X m_D m_D}{\mu^2 a}} \lambda \cdot \epsilon,
\]

where wave function is

\[
\psi(k) = \frac{8\pi}{\alpha} \frac{1}{k^2 + 1/\alpha^2},
\]

with normalization \( \int d^4k/(2\pi)^4 |\psi(k)|^2 = 1 \). Such wave function is consistent with the wave function adopted by Voloshin in coordinate space [15].

The energy-releasing transition of the \( \bar{D}^0 \) meson to the \( D^0 \) meson is induced by the incoming \( D^0 \) meson, which involves the inelastic scattering \( \bar{D}^0 D^0 \rightarrow D^0 D^0 \) through vector exchange, as shown in Fig. 1. To depict the scattering, the following Lagrangians under the heavy quark and chiral symmetries are adopted [16],

\[
\mathcal{L}_{\rho \gamma} = -i\lambda_{\rho \gamma} \varepsilon_{\rho \mu \alpha \beta} \left( \partial_\mu D^\alpha \partial_\nu D^\beta + D^\alpha \partial_\nu D^{\alpha \beta} \right) V^\beta
\]
\[ \mathcal{L}_{PPV} = -i g_V \sum_{i \neq j} \langle \bar{D}^0 \gamma^\mu \bar{p} \bar{D}^0 + \bar{D}^0 \gamma^\mu D^0 \rangle \bar{p}^\rho \gamma^\rho, \]

where \( V = \rho^0 \) or \( \omega \), and the parameters involved here were determined in the literature as \( \beta = 0.9, \lambda = 0.56 \text{GeV}^{-1} \), and \( g_V = 5.9 \) [16, 17].

Applying standard Feynman rules, the amplitude for the inelastic scattering \( \bar{D}^0 D^0 \to \bar{D}^0 D^0 \) can be written as

\[
\mathcal{A}_{\bar{D}^0 D^0} = 2 \sqrt{2 \pi} g^2 V_{D_{SD}} P(k^2) \left( 1/(q^2 - m_1^2)(q^2 - \Lambda^2)/(q^2 - \Lambda^2) \right) \text{with a standard cutoff} \Lambda = 1 \text{GeV}.
\]

The above amplitudes for splitting of \( X(3872) \) and inelastic scattering \( \bar{D}^0 D^0 \to \bar{D}^0 D^0 \), the amplitudes for total reaction \( \bar{D}^0 + X \to 2D^0 + D^0 \) can be written as,

\[
\mathcal{M} = -8 \pi g^2 \sqrt{m_X m_D m_D} P(q^2) \frac{[\psi(k_3) \bar{D}_{SD} p^\mu_1 k_3^\mu p^\mu_2 \bar{D}_{SD}]}{m_X - m_D - m_D} (2 \leftrightarrow 3).
\]

The \( (2 \leftrightarrow 3) \) term is an exchange of the momenta for final particle 2 and 3, which is for the second term \( D^0 D^0 \) in the wave function in Eq. (2).

The binding energy is an important metric of a hadronic molecular state. Though there is a very small suggested value of \( X(3872) \) listed in the PDG [6], we will vary the binding energy from 0.05 to 50 MeV. With such variation, more property of the molecular can be unveiled and it is also helpful to show the behavior of other molecular states with different binding energies. In Fig. 2, for a binding energy of approximately 0.1 MeV, the cross section is smaller. Considering the radius of a \( D \) or \( D^* \) meson is smaller than 1 fm, such binding energy means a large radius, approximately 14 fm, which makes the probability of collision of the incoming \( D^0 \) with \( D^0 \) in \( X(3872) \) very small. With increasing binding energy, the hadronic molecular state becomes more compact, and the cross section increases. If the binding energy is larger than 10 MeV, which corresponds to a radius of approximately 1 fm on the same order as the force range of \( \rho/\omega \) exchange, then the cross section decreases again due to the small size of the molecular state.

In Fig. 3, the cross section with the variation in the momentum of the incoming \( D^0 \) meson \( p_1 \) and the momentum distribution of final particles are presented. The largest cross section is found at a small incoming momentum \( p_1 \). With increasing incoming momentum, the cross section decreases almost linearly and reaches a minimum at approximately 100 MeV. This is reasonable because the faster incoming \( D^0 \) meson has a shorter time to interact, which makes the probability of reaction smaller.

The momentum distributions of the final particles are important in the study of the nuclear fission [3]. It is important to study the momentum distribution of three final mesons. In Fig. 2, the cross section of the \( D^0 + X \to 2D^0 + D^0 \) reaction is shown as a function of the binding energy. The results for three choices of the momentum of the incoming \( D^0 \) meson \( p_1 \) = 1 eV, 1 keV, and 1 MeV, are presented as black, and red, and blue curves. The three subfigures show the radii of the molecular states with binding energies of 0.1, 1, and 10 MeV. The distance 1 fm between collision \( D^0 \) and \( D^0 \) mesons (two upper mesons) represent the interaction range of \( \rho/\omega \) exchange.

As shown above, the behavior of the fission of \( X(3872) \) is quite similar to the nuclear fission. However, different from nuclear fission, the fission of \( X(3872) \) produces only short-lived (anti)charm mesons with half-lives shorter than \( 10^{-12} \) seconds [6]. The (anti)charm mesons soon decay to lighter
particles, most of which also have short half-lives of less than $10^{-5}$ seconds. If we can extract all the kinetic energy of the (anti)charm mesons and the decayed or subsequently decayed products and make them nearly static in the laboratory, then after less than 1 second, the only remaining products are neutrons, electrons, photons, antineutrons, positrons, and neutrinos. Neutrons, electrons, and photons are present everywhere in our world, and neutrinos would depart at the speed of light with almost no effect. Antineutrons and positrons easily trap and annihilate ordinary matter. No long-lived radioactive products remain after the fission of $X(3872)$.

In addition to $D^0$ meson-induced fission, a $\bar{D}^0$ meson can also induce the fission of $X(3872)$ as follows:

$$D^0 + X(3872) \rightarrow \bar{D}^0 D^0 D^0 \rightarrow D^0 + 2 D^0.$$  \hspace{1cm} (13)

Both $D^0$ and $\bar{D}^0$ mesons can play the role of neutrons in nuclear fission and continuously induce chain reactions. Moreover, “cross” chain reactions can also occur; for example, $\bar{D}^0$ meson-induced fission produces one $D^0$ meson, which further induces another kind of fission. The fission of $X(3872)$ may show phenomena other than those of nuclear fission, and these phenomena provide various views and will help us understand the mechanisms of fission more deeply.

The road leading to the experimental realization of fission with hadronic molecules is definitely very long. Less than 20 years has passed since the discovery of $X(3872)$, and currently, only very small amounts of $X(3872)$ can only be produced in the laboratory, and this material and cannot be stored for long periods of time. Nevertheless, its cleanliness and large energy efficiency motivate us to investigate fission with hadronic molecules despite the large challenges.

There can be induced fission with other hadronic molecular states. Here, we list three typical reactions in the strange, charmed, and bottom sectors,

$$K + f_1(1285) \rightarrow K \bar{K} K \rightarrow 2K + \bar{K}, \frac{\Delta E}{M_{X_{c}}} = 18\%,$$
$$D + T_{cc}(3875) \rightarrow DD^* D \rightarrow 3D, \frac{\Delta E}{M_{X_{c}}} = 4.0\%,$$
$$B + Z_{b}(10610) \rightarrow BB^* B \rightarrow 2B + \bar{B}, \frac{\Delta E}{M_{X_{b}}} = 0.4\%.$$  

With an increasing number of new hadrons observed in experiments, various kinds of fission can be proposed; they could be similar to nuclear fission but may also exhibit exotic behavior. A detailed study would further disclose the secrets of fission and help us utilize it better.

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