ABSTRACT

We consider the problem of scheduling a group of heterogeneous, distributed processes, with different relative priorities and service preferences, to a group of heterogeneous virtual machines. Assuming linearly elastic IT resource needs, we extend prior results on proportional fair and max-min fair scheduling to a constrained multiresouce case for a family of fairness criteria (including our recently proposed Per-Server Dominant-Share Fairness). Performance comparison is made by illustrative numerical example. We conclude with a discussion of scheduling problems for a public cloud with heterogeneous instances and servers.

1. INTRODUCTION

Recently, there have been a number of proposed scheduling (resource provisioning) problems involving plural heterogeneous processes\(^1\) and plural heterogeneous workers\(^2\) capable of serving them. In addition, the processes may have different service priorities and service constraints (as in e.g., service-quality constraints \(^3\) or cache-affinity constraints). In prior work, to show fairness properties of a proposed scheduler, a resource-congested regime is typically considered for competing processes, with elastic resource demand that is linearly proportionate with workload intensity. Let \(x_{n,i}\) be the workload intensity of process \(n\) and worker \(i\), let \(d_{n,r}\) be the demand for resource \(r\) per unit-work load of process \(n\), and assume processes have sufficient demand to occupy all the resources of the workers. Thus, the capacity constraints are

\[
\sum_n x_{n,i} d_{n,r} \leq c_{i,r} \tag{1}
\]

where \(c_{i,r}\) is the amount of resource \(r\) in worker \(i\). As an alternative to “Global” Dominant Resource Fairness (DRF, cf. (17)) and its variants, e.g., [6, 5, 4, 15], we recently proposed the Per-Server Dominant-Share Fairness (PS-DSF) criterion ([9], cf. (12)) to more efficiently schedule heterogeneous processes to heterogeneous workers. In the following, we formulate optimization problems as [3, 10, 8] that employ such fairness criteria and at whose solutions corresponding max-min and proportional fairness properties are achieved\(^3\). Greedy iterative approaches to these static optimization problems (e.g., projected gradient, “progressive filling” [3, 6]) can be heuristically adapted to more dynamic settings with process and worker churn.

2. MAX-MIN FAIRNESS

To generalize Theorem 1 of [8] to multiple resource types, consider the following general-purpose fairness criterion,

\[
U_n = \phi_n^{-1} \sum_i u_{n,i} x_{n,i}, \tag{2}
\]

for scalars \(u_{n,i} > 0\). In addition, consider the service-preference sets

\[
N_i = \{n \mid \delta_{n,i} = 1\} \quad \text{where} \quad x_{n,i} > 0 \Rightarrow \delta_{n,i} > 0. \tag{3}
\]

Assume the demand model (1) and strictly concave and increasing \(g\) with \(g(0) = 0\). Define the optimization problem

\[
\max \sum_n \phi_n g(U_n) \tag{4}
\]

such that (here restating (1))

\[
\forall i, r, \sum_{n \in N_i} x_{n,i} B_{n,i,r} \leq 1 \quad \text{and} \quad \forall n, i, x_{n,i} \geq 0, \quad \text{where} \quad B_{n,i,r} := d_{n,r}/c_{i,r}. \tag{5}
\]

Note that the objective is continuous and strictly concave and the domain given by (5) (equivalently (1)) is compact. So, simply by Weierstrass’s Extreme Value Theorem, there exists a unique maximum.

Regarding fully booked resources in worker \(i\) under allocations \(x = \{x_{n,i}\}\), also let

\[
R_i := \{ (x,r) \mid \sum_{n \in N_i} x_{n,i} B_{n,i,r} = 1 \}.
\]

For the following definition, assume that \(\forall n, i, r, B_{n,i,r} > 0\).

\textbf{Definition 1.} A feasible allocation \(\{x_{n,i}\}\) satisfying (5) is said to be U-Max-Min Fair (MMF) if:

\[
U_f/\phi_f > U_m/\phi_m, \quad x_{m,i} > 0, \quad \& \exists r \text{ s.t. } \sum_{n \in N_i} x_{n,i} B_{n,i,r} = 1
\]

implies that \(x_{f,i} = 0\).

\(^{\dagger}\)Other desirable properties identified for DRF [6] are studied for PS-DSF in [9].
Note that if instead \( x_{\ell,i} > 0 \) in this definition, then \( x_{\ell,i} \) can be reduced and \( x_{\ell,m} \) increased to reduce \( U_m/\phi \ell - U_m/\phi_m \). Also, if \( \{x_{n,i}\} \) is \( U \)-MMF and \( x_{n,i}, x_{\ell,i} > 0 \) for some worker \( i \) then \( U_m = U_\ell \). Quantization issues associated with workload resource demands are considered in [5].

**Proposition 1.** A solution \( x = \{x_{n,i}\} \) of the optimization (4) s.t. (5) has at least one resource \( r \) fully booked in each worker \( i \). In addition, there is a unique \( U \)-MMF solution if also
\[
\delta_{m,i} = 1 = \delta_{i,j} \& (x, r) \in R_i \Rightarrow d_{m,r} = d_{i,r}. 
\] (6)

Remarks: Regarding (6) - a strong assumption that processes that can share workers have homogeneous demand for fully utilized resources - note that \( d_{m,r} = d_{i,r} \Rightarrow B_{m,i,r} = B_{i,i,r} \).

Proof. Define the Lagrangian to be maximized over \( x \) and over Lagrange multipliers \( \lambda, \nu \geq 0 \):
\[
L = \sum_{n} \phi_n g(U_n) + \sum_{n,i} \lambda_{n,i} (1 - \sum_{n \in N_i} x_{n,i} B_{n,i,r}) 
+ \sum_{i, n \in N_i} \nu_{n,i} x_{n,i}. \]
The first-order optimality condition,
\[
\forall i, n \in N_i, \delta_{n,i} = 1, 
0 = \frac{\partial L}{\partial x_{n,i}} = u_{n,i} g'(U_n) - \sum_r \lambda_{n,i} B_{n,i,r} + \nu_{n,i}. \] (7)
and \( g \) strictly increasing imply
\[
\forall i, n \in N_i, \sum_r \lambda_{n,i} B_{n,i,r} > \nu_{n,i} \geq 0. \] (8)
So, \( \forall i, \exists r \text{ s.t. } \lambda_{n,i} > 0. \) Thus, complementary slackness is
\[
\forall i, r, \lambda_{n,i} (1 - \sum_{n \in N_i} x_{n,i} B_{n,i,r}) = 0 \] (9)
\[
\Rightarrow \forall i, \exists r \text{ s.t. } \sum_{n \in N_i} x_{n,i} B_{n,i,r} = 1, \] (10)
\text{i.e., in every worker } \( i \), one resource \( r \) (which may depend on \( i \)) is fully booked. So, the set of fully booked resources in worker \( i \) under allocations \( x = \{x_{n,i}\} \) can be characterized by \( \{r \mid \lambda_{n,i} > 0\} \). Now by (7) and assumed strict concavity of \( g \), uniquely
\[
\forall i, n \in N_i, U_n = (g')^{-1} \left( \sum_r \lambda_{n,i} B_{n,i,r} \frac{u_{n,i}}{u_{n,i}} \right) = \left( g' \right)^{-1} \left( \sum_{r, \lambda_{n,i} > 0} \lambda_{n,i} B_{n,i,r} - \nu_{n,i} \right). 
\]
Now consider two processes \( m \) and \( \ell \) and worker \( i \) such that \( x_{m,i} > 0 \) and \( \delta_{m,i} = 1 = \delta_{i,j} \). So, complementary slackness
\[
\forall i, n \in N_i, \nu_{n,i} x_{n,i} = 0. 
\] (11)
implies \( \nu_{m,i} = 0. \) Thus, because \( (g')^{-1} \) is strictly decreasing (g strictly concave),
\[
U_m = (g')^{-1} \left( \sum_{r, \lambda_{m,i} > 0} \lambda_{m,i} B_{m,i,r} \frac{u_{m,i}}{u_{m,i}} \right) \leq (g')^{-1} \left( \sum_{r, \lambda_{i,r} > 0} \lambda_{i,r} B_{i,r} \left( \frac{u_{i,r}}{u_{i,r}} \right) - \frac{\nu_{i,r}}{u_{i,r}} \right) = U_\ell, 
\] where we have used assumption (6) for the inequality. Because of this and (10), a solution \( x = \{x_{n,i}\} \) of the optimization (4) s.t. (5) is \( U \)-MMF. \( \square \)

The PS-DSF criterion can be written as
\[
K_{n,j} = \frac{\sum_{\ell} x_{n,j,\ell} \phi_{\ell,\rho(n,j)}}{\phi_n \phi_{\ell,\rho(n,j)}} = \frac{B_{n,j,\rho(n,j)} x_n}{\phi_n}, \] (12)
where \( \rho \) is such that \( B_{n,j,\rho(n,j)} := \max_r B_{n,j,r} \) when \( \delta_{n,j} = 1. \) Define
\[
K_n = \sum_{i} K_{n,i} \delta_{n,i} = \phi_n^{-1} x_n \sum_{i} B_{n,i,\rho(n,i)} \delta_{n,i}. \] (13)
So \( U_n = K_n \) when, \( \forall \), scalars \( u_{n,j} = \sum_{i} B_{n,i,\rho(n,i)} \delta_{n,i}. \)

**Corollary 1.** A solution \( x = \{x_{n,i}\} \) of the optimization problem
\[
\max_x \sum_n \phi_n g(K_n) \text{ s.t. (5)}, 
\]
is such that at least one resource \( r \) is fully booked in each worker \( i \). Also, there is a unique \( K \)-MMF solution if in addition
\[
\delta_{m,i} = 1 = \delta_{i,j} \& (x, r) \in R_i \Rightarrow \frac{B_{m,i,r}}{\sum_{j} B_{m,j,\rho(n,j)}} = \frac{B_{i,r}}{\sum_{j} B_{i,j,\rho(n,j)}}. \] (14)
Note that if there is a single resource type \( r \) in each worker (as [16, 8]), then (14) trivially holds. See Theorem 1 of [8].

**3. PROPORTIONAL FAIRNESS**

In the final step of the proof of Proposition 1, we see the need to divide by the weight \( \phi \) in the argument of the objective function \( g(\cdot) \). For weighted proportional fairness, we change the objective to
\[
\max_x \sum_n \phi_n g_\alpha(x_n), 
\] (15)
i.e., without dividing by \( \phi_n \) in the argument of \( g_\alpha \) [10]. For parameter \( a > 0 \) specifically take
\[
g_\alpha(X) = \left\{ \begin{array}{ll}
\log(X) & \text{if } a = 1 \\
(1 - a)^{-1} X^{1-a} & \text{else}
\end{array} \right.
\]
i.e., \( g'_\alpha(X) = 1/X^a \), again see [10]. Obviously, in the case of \( a = 1 \) \( (g = \log) \), whether the factor \( \phi \) in the argument of \( g \) is immaterial.
The following generalizes Lemma 2 of [10] on Proportional Fairness.

**Proposition 2.** A solution \( x^* \) of the optimization (15) s.t. (5) is uniquely (weighted) \( (\phi, a) \)-proportional fair, i.e., for any other feasible solution \( x \),
\[
\Phi(x, x^*) := \sum_n \phi_n \frac{x_n - x_n^*}{(x_n^*)^a} \leq 0. 
\]
4. NUMERICAL EXAMPLE

Under global-DRF, processes $n$ are selected using criterion

$$M_n = \max_r \frac{a_{n,i,r} c_{i,r}}{\phi_n \sum_{i,r} c_{i,r}}.$$  

(17)

Obviously, Corollaries 1 and 3 also hold for global-DRF. Illustrative examples showing PS-DSF is more efficient than C-DRF-H [5] and TSP [15] are given in [9]. Here consider the following example without preference constraints (i.e., $\delta_{n,i} \equiv 1$), with two heterogeneous processes ($n = 1, 2$) having resource demands per unit workload $d_{1,1} = 1, d_{2,2} = 5, d_{2,1} = 5, d_{1,2} = 1$, and two heterogeneous workers ($i = 1, 2$) having resource capacities $c_{1,1} = 100, c_{1,2} = 30, c_{2,1} = 30, c_{2,2} = 100$. Suppose the workers are chosen in random order where workers are randomly permuted in each round (as the scheduling framework Mesos may operate in practice). An iterative, incremental greedy approach is used to select the processes $n$. On average under DRF, the allocations are $x_{n,i} \equiv 5$ for all $n, i$, i.e., both processes share both workers. Under PS-DSF, the average allocations are $x_{n,i} = 19.5, x_{1,2} = 0.6, x_{2,1} = 0.35, x_{2,2} = 19.65$, i.e., more efficient use of resources and increased total allocation $x_n = x_{n,1} + x_{n,2} = 20 > 10$. Similar results for PS-DSF ensue if greedy progressive filling is used to choose both processes and workers, or if a centralized optimization is used to select both processes and workers (as in [9] and the above propositions).

We have recently observed improved efficiency using a “residual” PS-DSF (rPS-DSF) criterion

$$\tilde{K}_{n,j,x_j} = \max_{x} \frac{d_{n,r}}{\phi_n(c_{n,r} - \sum_{x'} x_{n',j} d_{n',r})}.$$  

That is, this criterion makes scheduling decisions using residual capacities based on the current allocations $x$. For the above example under rPS-DSF, $x_{1,1} = 19, x_{1,2} = 2, x_{2,1} = 2, x_{2,2} = 19$. In future work, we will study the fairness properties of rPS-DSF. Also, similar propositions can be explored for other variants of fairness criteria (2) found by instead maximizing or minimizing over workers $i$.

5. DISCUSSION: PRICING CUSTOM INSTANCES

Cost-conscious tenants or their proxies may request custom VMs or containers including specifying

- collocation of different VMs (from Availability Regions to individual racks or servers),
- the amounts of individual IT resources as in a Resource-as-a-Service (RaaS) [2] framework,
- parameters of token bucket mechanisms governing access to IT resources [14], or
- parameters of revocation as in spot or preemptible instances, e.g., [13].

Also, tenants may be able to dynamically resize their VMs, e.g., [1].

So in this setting, a public cloud cannot simply, e.g.,

- plan for regular instance offerings that are equal fractions of their physical servers, and
- consolidate burstable instances onto physical servers from different tenants hoping to exploit statistical multiplexing gains, e.g., [14].

A highly heterogeneous system of instances and servers, together with instance churn, motivates

- efficient dynamic instance consolidation, and
- pricing or auction/spot-pricing system that depends on contention for individual IT commodities [7].
Consider a future, flexible service of the public cloud where tenants $n$ bid $\phi_n$ dollars per unit time per unit VM with resource allocations of their choosing, $d_{n,r} > 0$ for all $r$. So, we have interpreted parameters $\phi$ to be a bid and $d$ to define the desired size of the VM, where the public cloud is not concerned with how the thus allocated VMs are utilized.

Here, to maximize marginal revenue at any point in time, a kind of DRF or PS-DSF criteria can be used by the public cloud to select which bid to accept from among the current tenants $n$ that are bidding (to assign a small resource bundle $d_{n,r} = \{d_{n,r}\}$ from a slave worker $i$ with sufficient spare capacity to do so). Clearly, such a decision will depend on the tenants’ explicitly declared willingness to pay $\phi$ and marginal (incremental) resource demands $d_{n,r}$, as well as the residual capacity of slaves (whether the spare capacity can accommodate the marginal demand of a tenant, in particular). At any point in time, the bid selected implies a kind of spot price for this multicmodity system.

Let $x_{n,i}$ be the current (integer valued) allocations, so that the current residual capacities are $c_{i,r} = c_{i,r} - \sum x_{n,i}d_{n,i}$. A current bid $(n, i)$ is feasible if $c_{i,r} \geq d_{n,r}$ for all $r$. A simple example of a greedy heuristic is to choose the feasible bid $(n, i)$ (accept tenant $n$’s bid and allocate from slave $i$) with smallest

$$\frac{1}{\phi_n} \max_r \frac{c_{i,r}}{c_{i,r} - d_{n,r}}.$$

Alternatively, a criterion can be inspired by the static problem where the public cloud’s objective is to maximize revenue,

$$\sum_n \phi_n \sum_i x_{n,i},$$

subject to the resource capacity constraints of servers $i$ it has allocated for this service, $\sum_n x_{n,i}d_{n,r} \leq c_{i,r} > 0$, and integer allocations $x_{n,i} \geq 0$ for all $n, i, r$. This is a linear program for a static problem that can be progressively solved by the simplex method or an interior point method, e.g., [12, 11].

From [12], we get the following greedy criterion. Based on the current allocations $x$ and residual capacities $\hat{c}$, define

$$\xi = \max_{n, i : \hat{c}_{i,r} > 0 \forall r} x_{n,i} \phi_n.$$

The public cloud chooses from among the current bids $(n, i)$, the feasible bid with smallest

$$\frac{\xi}{x_{n,i} \phi_n}.$$

Other resource scheduling criteria can be gleaned from the progressive approaches to static linear programs corresponding to revenue maximization problems with resource capacity constraints.

In the future, we will further explore how to price instances with parameters specifying:

- individual IT resources [7],
- instance revocation, and
- token bucket parameters [14].

We will also consider consolidation and revocation mechanisms to deal with server and instance heterogeneity, e.g., how burstable instances can be consolidated onto physical servers with statistical multiplexing based overbooking [14]. Finally, we will consider the implications of sub-additive demand $D(x)$ (rather than $xd$) [7].

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