Quantization and Bit Allocation for Channel State Feedback in Relay-Assisted Wireless Networks

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Abstract

This paper investigates quantization of channel state information (CSI) and bit allocation across wireless links in a multi-source, single-relay cooperative cellular network. Our goal is to minimize the loss in performance, measured as the achievable sum rate, due to limited-rate quantization of CSI. We develop both a channel quantization scheme and allocation of limited feedback bits to the various wireless links. We assume that the quantized CSI is reported to a central node responsible for optimal resource allocation. We first derive tight lower and upper bounds on the difference in rates between the perfect CSI and quantized CSI scenarios. These bounds are then used to derive an effective quantizer for arbitrary channel distributions. Next, we use these bounds to optimize the allocation of bits across the links subject to a budget on total available quantization bits. In particular, we show that the optimal bit allocation algorithm allocates more bits to those links in the network that contribute the most to the sum-rate. Finally, the paper investigates the choice of the central node; we show that this choice plays a significant role in CSI bits required to achieve a target performance level.

I. INTRODUCTION

It is well established that using relays can significantly improve the communication capacity and reliability of wireless networks [1]. Based on approaches suggested in [2], the work in [3] and [4] analyzes different relaying strategies such as the decode-and-forward (DF) and amplify-and-forward (AF) relaying techniques. Our focus is on DF, wherein the relay must decode and then re-encode the source data. The potential gains associated with the relay systems, and

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cooperative diversity in general, has attracted a great deal of research into the optimization of the relay-assisted network performance. For wireless networks, the optimization is mainly in terms of resource allocation, specifically power and/or bandwidth allocation, relay routing, and selection of relaying strategy [5]–[7].

Although the literature on relay network optimization shows that there are significant performance improvements to be had, most of the analysis is based on the crucial assumption that some central node has exact knowledge of the network-wide channel state information (CSI). This assumption, however, is impossible to satisfy in a practical system implementations due to the limited resources available for CSI training and feedback. As a result, in practice, network-wide CSI is not known perfectly; only a quantized version of the information may be available via feedback. Since the performance of resource allocation algorithms in cooperative relay networks depends heavily on the availability of CSI, it becomes essential to investigate the performance of resource allocation schemes under the assumption of limited-rate quantization of CSI.

This paper takes a step in this direction by investigating the uplink in a relay-assisted wireless cellular network (much of our analysis can also be applied to the downlink as well). Our system model considers multiple sources communicating to a base-station (BS) with the help of a single relay. We assume that while the receiver end in any link has perfect CSI of that link, a quantized version of all channels is available at a central node responsible for resource allocation. The main goal here is to (i) optimize the quantization of CSI and, (ii) given a constraint on the number of bits for CSI feedback, allocate those bits across all links in the network.

The performance of cooperative networks with limited CSI has been addressed in the available literature for several communication scenarios. As a general result, it is shown that providing even a few bits of quantized CSI significantly improves the performance of cooperative systems [8]–[13]. The authors of [8], in particular, investigate optimal temporal resource allocation between the source and the relay under quantized CSI using a DF relaying strategy and show that even with a single bit of feedback provides significant gains in bit error rate. The work in [9] shows similar results for diversity gains for both AF and DF relaying strategies.

The authors of [13] investigate the optimum throughput in a cooperative network using DF. This work maximizes an upper bound on overall throughput, thereby deriving a suboptimal resource allocation scheme. The authors of [10] investigate optimal relay selection in multi-relay AF cooperative networks. This work investigates network performance with quantized and
statistical CSI; again a few CSI bits are shown to provide significant performance gains.

The works mentioned so far do not specifically investigate how to quantize CSI or allocate bits. The authors of [11] consider a cooperative communication system in a cellular network with inter-cell interference. The paper adopts zero-forcing beamforming and finds an approximate expression for the received signal-to-interference-plus-noise ratio (SINR), based on which bit allocation is optimized across the network links. The authors also determine the minimum number of CSI bits that are required to outperform a non-cooperative network. The authors of [12] design quantization codebooks for the transmit power vectors in a single-relay network with DF. For the design, however, they adopt the Lloyd algorithm with Euclidian-distance as the design metric.

This paper takes a different tack by analyzing the loss due to quantization in a cooperative wireless network. We start by introducing the performance loss as the loss in the maximum achievable sum rate due to CSI quantization. To the best of our knowledge, an analysis of this communication scenario under quantized CSI has not been presented before. Our analysis includes proposing bounds on the performance loss and, then, using these bounds to formulate and optimize quantization schemes for the network-wide CSI. Our main contributions are:

- derivation of a tight upper bound on the performance loss due to quantized CSI for the sum-rate maximization problem in cellular networks;
- using the upper bound to formulate the optimal CSI quantizer design problem. By using the proposed quantizer, the bound on the performance loss is shown to grow extremely slowly with respect to the average link signal-to-noise ratio (SNR) and, as is more common, decreases exponentially with the number of quantization bits;
- investigating the optimal allocation of CSI quantization bits across the wireless channels to minimize the performance loss. It is shown that most of the quantization bits should be used for the links that contribute the most to the sum-rate;
- a discussion of the choice of the central node to show that this choice can have a significant effect on the CSI required to achieve a given performance target.

The remainder of this paper is organized as follows. Section II presents the system and CSI quantization models. Section III derives an upper bound on the performance loss due to quantization. Section IV then formulates the optimization problem for quantizer design and presents the corresponding performance analysis. This is followed by Section V which investigates optimal bit allocation to minimize the upper bound on the performance loss due to quantized CSI and
discusses the selection of the central node. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

The network model comprises $N_S$ source nodes ($S_1 \ldots S_{N_S}$) communicating with a single destination $D$ through a single relay node $R$. To avoid multiuser interference, each source is allotted an orthogonal channel. This model most closely represents the uplink of a relay-assisted cellular network, where due to the unavailability of direct source-destination ($S_i$-$D$) links, a relay node is deployed between the source nodes (mobile users) and the destination (base station) to facilitate communication.

Transmission occurs in two consecutive time slots: the first time slot is dedicated to source-relay transmission, while in the second time slot, the relay, using DF, forwards the source messages to the destination. We further assume that the receiver, in any specific link, knows the CSI of that specific link exactly, e.g., via adequate training at the start of each the transmission phase. Such channel estimation is generally necessary to demodulate and is not an additional requirement imposed by the resource allocation process.

We assume there exists a central node that collects the quantized network-wide CSI. This node is responsible for optimal power allocation at the relay by using the available quantized CSI. Since the relay uses DF, only the channel magnitudes are required. Specifically, to calculate the optimal power allocation required at $R$, the central node needs the magnitudes of the $R$-$D$ and all the $S_i$-$R$ channels.

We assume that the long-term average channel powers of all links are known a priori at the central node. These average powers are functions of the large scale fading parameters of the links that vary slowly as compared to the instantaneous channel values. The channel for a link between a transmitter $X$ and receiver $Y$ is denoted by $g_{XY}$ and the corresponding normalized channel power is defined as $h_{XY} = |g_{XY}|^2/E[|g_{XY}|^2]$. Here, $E[\cdot]$ denotes expectation. Since the average power, $E[|g_{XY}|^2]$, is known at the central node, we focus on quantizing the normalized channel power, $h_{XY}$. The probability density functions (pdf) of all the normalized channels are assumed identical for all links. For the random normalized channel power $h$, $f_H(h)$ and $F_H(h)$ denote, respectively, the pdf and cumulative distribution function (cdf). Finally, we assume $f_H(h)$ is bounded and has a bounded derivative almost everywhere.

The central node is to be given some knowledge of the channel powers of all the links in the
network. In the case of quantized CSI for the link X-Y with log_2 N bits for quantization, the quantization rule q[h_{XY}] is implemented as follows:

- the range [0, ∞) is divided into N + 1 disjoint quantization intervals defined by their boundaries \( \{q_n\}_{n=-1}^N \) where \( q_{-1} = 0 \) and \( q_N = \inf \{h \geq 0 : F_{H_{XY}}(h) = 1\} \), i.e., the maximum possible value of h (for many pdfs, \( q_N = \infty \)). Note that N - and so the boundaries (and associated intervals) - may be different for different channels.

- The receiver node, Y, observes its instantaneous normalized channel power \( h_{XY} \) and when this value falls within the \( n \)-th interval, i.e., \( h_{XY} \in [q_{n-1}, q_n) \), the index \( n \) is fed back to the central node.

- The central node then assumes the quantized channel power as \( q[h_{XY}] = q_{n-1} \), i.e., the most conservative value is chosen so the resulting sum rate obtained can be guaranteed.

On receiving the network-wide CSI, the central node calculates the power allocation (or equivalently the rate allocation) at the relay node for all the sources. The relay has a power constraint of \( P_R \). The resource allocation problem for sum-rate maximization is:

\[
\max_{\mathbf{P}} \sum_{i=1}^{N_s} R_i \quad \text{(1)}
\]

subject to: \( 1^T \mathbf{P} \leq P_{RD} \), \( \text{(2)} \)

where \( R_i \) is the rate achieved by source \( S_i \) and \( 1 \) is a length-\( N_s \) vector of ones. In (2), \( P_{RD} = |g_{RD}|^2(P_R/\sigma^2) \) is the SNR at the destination (\( \sigma^2 \) denotes the noise variance and \( P_R \) the power at the relay). Accounting for the R-D channel gain within the power constraint simplifies the notation in the upcoming analysis. The optimization is over the vector \( \mathbf{P} = [P_1, \ldots, P_{N_s}]^T \) which also includes the R-D channel gain. \( P_i \) then denotes the receive SNR (at the destination) that the relay node provides to the source node \( S_i \). This SNR is the actual power allocated by the relay to source \( S_i \) multiplied by the factor of \( |g_{RD}|^2/\sigma^2 \).

Let \( P_{S_iR} = h_{S_iR}(P_S/\sigma^2) \), where \( P_S \) is the source transmit power. Then \( R_i \) is given by (3):

\[
R_i = \min(C(P_{S_iR}), C(P_i)), \quad \text{(3)}
\]

with \( C(p) = \ln(1 + p) \), i.e., rate is measured in nats.

To further simplify the notation, we express \( P_{RD} \) and \( P_{S_iR} \) in terms of the normalized channel powers, by writing \( P_{RD} = \gamma_{RD} h_{RD} \) and \( P_{S_iR} = \gamma_{S_iR} h_{S_iR} \), where \( \gamma_{RD} = (P_R/\sigma^2)E[|g_{RD}|^2] \) and \( \gamma_{S_iR} = (P_S/\sigma^2)E[|g_{S_iR}|^2] \) are the average SNR for the R-D and \( S_i-R \) links, respectively.
Let $R_i^*$ denote the optimal transmission rate of source $S_i$ obtained by solving (I) assuming perfect CSI. Similarly, let $R_i^{q*}$ denote the solution to the same problem using quantized CSI, i.e., the solution to (I) when one replaces $P_{S_i R}$ and $P_{R D}$ by $q[P_{S_i R}] = \gamma_{S_i R} q[h_{S_i R}]$ and $q[P_{R D}] = \gamma_{R D} q[h_{R D}]$. Our main goal is to investigate the performance loss due to quantization, i.e. the difference between the sum-rate found by solving (I) with perfect and quantized CSI. We address this problem in the next section by deriving tight bounds on the performance loss.

### III. Upper Bound on Performance Loss

Throughout this paper, the term performance loss or simply loss refers to the difference between the optimal sum-rate for the perfect and quantized CSI scenarios. In this section, we provide an upper bound on this loss in terms of the quantization levels and CSI statistics. This bound is then used in Section IV to optimize the quantizer and eventually derive the optimal bit allocation across the links in Section V. The performance loss is defined as

$$\Delta = \sum_{i=1}^{N_S} \Delta_i = \sum_{i=1}^{N_S} (R_i^* - R_{i}^{q*}),$$

where $\Delta_i$ represents the rate loss seen by source $S_i$. We are interested in the expectation of this loss, i.e., the expected value of (4) over the channel variables. For each node $i$ define

$$E[\Delta_i] = E[R_i^* - R_{i}^{q*}] = E[\min(C(P_{S_i R}), C(P_i^*)) - \min(C(q[P_{S_i R}]), C(P_{i}^{q*}))].$$

In (5), $P_i^*$ and $P_i^{q*}$ are, respectively, the optimal power (including the channel gain) allocated by the relay to source $S_i$ in the perfect CSI and quantized CSI cases.

Due to the function $\min(\cdot, \cdot)$ in (5) the integration region is divided into four distinct sets. In order to distinguish these sets, for the source $S_i$, define $A_i = \{h : P_{S_i R} \leq P_i^*\}$ and similarly, $B_i = \{h : q[P_{S_i R}] \leq P_{i}^{q*}\}$. Here, $h = [h_{S_1 R}, h_{S_2 R} \ldots h_{S_{N_S} R}, h_{R D}]^T$ is the vector of variables to be quantized. The sets $A_i$ and $B_i$ are, respectively, the regions where the source-relay channel capacity is the bottleneck for the perfect and quantized CSI scenarios. By definition, the capacity function $C(\cdot)$ is increasing and (5) can be expressed as

$$E[\Delta_i] = \int_{h \in A_i \cap B_i} (C(P_{S_i R}) - C(q[P_{S_i R}]))) f_H(h) dh + \int_{h \in A_i' \cap B_i} (C(P_i^*) - C(q[P_{S_i R}])) f_H(h) dh$$
$$+ \int_{h \in A_i \cap B_i'} (C(P_{S_i R}) - C(P_{i}^{q*}))) f_H(h) dh + \int_{h \in A_i' \cap B_i'} (C(P_i^*) - C(P_{i}^{q*}))) f_H(h) dh,$$
where $A_i^c$ and $B_i^c$ represent the complements of $A_i$ and $B_i$.

From the definitions of $A_i$ and $B_i$,

$$C(P_i^*') \leq C(P_{S,R}) \forall h \in A_i^c \cap B_i,$$  \hspace{1cm} (7)  

$$C(P_{S,R}) \leq C(P_i^*) \forall h \in A_i \cap B_i^c.$$  \hspace{1cm} (8)

Now from (6), (7), and (8) we have the following upper bound on the performance loss

$$E[\Delta_i] \leq \Delta_{S,R} + \Delta_{RD,i},$$  \hspace{1cm} (9)  

where

$$
\Delta_{S,R} = \int_{h \in B_i} (C(P_{S,R}) - C(q[P_{S,R}])) f_H(h) dh,
$$  \hspace{1cm} (10)  

and

$$
\Delta_{RD,i} = \int_{h \in B_i^c} (C(P_i^*) - C(P_i'^*)) f_H(h) dh.
$$  \hspace{1cm} (11)

Equation (10) is an upper bound on the average performance loss due to quantization of the link $S_i-R$ and is found by merging the first two terms of (6) using (7); similarly, (11) defines an upper bound the performance loss due to the power allocated to source $S_i$ based on quantization of the link $R-D$ derived from the third and fourth terms of (6) using (8).

A similar analysis can be proposed to derive a lower bound on (6) with an expression resembling (9) where the integration region is replaced by the set $A_i$ and $A_i^c$. Since we are focusing on the achievable rate regions for the proposed system model, we will continue with the upper bound on performance loss which ultimately leads to a lower bound on the achievable rates. In the next section, we further bound the terms in (9).

**A. Loss due to the Quantization of $S_i-R$ Links**

In this section we focus on analyzing (10). From our assumption that all channels are mutually independent, the joint pdf of $h$, $f_H(h)$, has a product form. However, in (10), the region of integration is coupled across the channel variables making the integration complicated. To overcome this problem we define a larger region, $B_i^l$, which includes $B_i$, and results in a product form for the integral region in (10). The set $B_i^l$ is defined as

$$B_i^l = \{ h : q[P_{S,R}] \leq P_{RD} \} = \{ h : \gamma_{S,R}q[h_{S,R}] \leq \gamma_{RD}h_{RD} \}.$$  \hspace{1cm} (12)

To see $B_i \subseteq B_i^l$ note that $\forall i$ and $h \in B_i$ we have $q[P_{S,R}] \leq P_i'^* \leq q[P_{RD}] \leq P_{RD} \Rightarrow h \in B_i^l$. 

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Since \( B_i \subseteq B_i^t \), we achieve an upper bound on the term \( \Delta_{S_i,R} \). From (10) and (12):

\[
\Delta_{S_i,R} \leq \int_{h \in B_i^t} (C(P_{S_i,R}) - C(q[P_{S_i,R}]就来看看\( f_H(h)dh \)
\]

\[
= \int_0^\infty \int_{h' \geq \gamma_{S_i,R}h} (C(\gamma_{S_i,R}h) - C(\gamma_{S_i,R}q[h])) f_{HRD}(h')f_{HS_i,R}(h)dh'dh
\]

\[
= E\left[ \ln \left( \frac{h_{S_i,R} + \gamma_{S_i,R}^{-1}}{q[h_{S_i,R}] + \gamma_{S_i,R}^{-1}} \right) (1 - F_{HRD}(\alpha_i[q[h_{S_i,R}]])) \right],
\]

with \( \alpha_i = \gamma_{S_i,R}^{-1}\gamma_{RD} \) and using \( C(p) = \ln(1 + p) \). The expectation is over \( h_{S_i,R} \).

The term \( 1 - F_{HRD}(\alpha_i[q[h_{S_i,R}]] \) in (13) shows that, in general, the quantization of one \( S_i-R \) link depends on the distribution of the \( R-D \) link channel power. This renders the quantization optimization intractable. We therefore upper-bound (13) by dropping the term \( 1 - F_{HRD}(\alpha_i[q[h_{S_i,R}]]\):

\[
\Delta_{S_i,R} \leq E_h \left[ \ln \left( \frac{h + \gamma_{S_i,R}^{-1}}{q[h] + \gamma_{S_i,R}^{-1}} \right) \right].
\]

(14)

The upper-bound in (14) can be minimized with respect to the quantization rule \( q[\cdot] \). This ultimately leads to the optimal quantization levels for \( h_{S_i,R} \), the normalized \( S_i \)-\( R \) channel power. Crucially, by using (14) as the objective function to optimize the quantization, the quantization levels found for quantization of \( S_i-R \) link depend on the statistics the \( S_i-R \) channel power only. As is shown in the following sections, adopting the upper bound in (14) also leads to separable problems for the optimal quantization design and bit allocation.

After finding the optimal quantization levels based on (14) in Section [V], we will return to (13) in Section [V-A] to discuss the optimal bit allocation across the wireless channels.

**B. Loss due to the Quantization of R-D Link**

The analysis for the \( R-D \) link follows the same approach as that of Section [III-A]. Following (11) we define the \( R-D \) loss component \( \Delta_{RD} \) as

\[
\Delta_{RD} = \sum_{i=1}^{N_S} \Delta_{RD,i} = \sum_{i=1}^{N_S} \int_{B_i^c} (C(P^*_i) - C(P_{q^*})) f_H(h)dh.
\]

(15)

The problem of evaluating (15) for a general distribution function is intractable. Therefore, similar to the analysis in the previous section, we extend the region of integration in (15), resulting in an upper bound on \( \Delta_{RD} \). To this end, define \( B = \bigcup_{i=1}^{N_S} B_i^c \) which readily yields \( B_i^c \subseteq B \). Since
the integrand in (15) is positive, we have

\[ \Delta_{RD,i} = \int_{B} (C(P_i^*) - C(P_i^{q*})) f_H(h)dh \leq \int_{B} (C(P_i^*) - C(P_i^{q*})) f_H(h)dh \]  

(16)

\[ \Rightarrow \Delta_{RD} \leq \int_{B} \sum_{i=1}^{NS} (C(P_i^*) - C(P_i^{q*})) f_H(h)dh. \]  

(17)

The set of power variables \( \{P_i^{q*}\}_{i=1}^{NS} \) represent the optimal power allocation maximizing the sum-rate under the quantized CSI; therefore

\[ \sum_{i=1}^{NS} C(P_i^{q*}) \geq \sum_{i=1}^{NS} C \left( \frac{q[P_{RD}]}{P_{RD}} P_i^* \right), \]  

(18)

which is true since \( \{\frac{q[P_{RD}]}{P_{RD}} P_i^*\}_{i=1}^{NS} \) is a valid, and likely suboptimal, solution to the max sum-rate problem satisfying the power constraint. From (17) and (18) it follows that

\[ \Delta_{RD} \leq \int_{h \in B} \sum_{i=1}^{NS} \left[ C(P_i^*) - C \left( \frac{q[P_{RD}]}{P_{RD}} P_i^* \right) \right] f_H(h)dh \]

\[ \leq \int_{h \in B} \sum_{i=1}^{NS} \ln \left( \frac{1 + P_i^*}{1 + \frac{q[P_{RD}]}{P_{RD}} P_i^*} \right) f_H(h)dh. \]  

(19)

For any channel power \( h \in B \) we have \( \sum_{i=1}^{NS} P_i^* \leq P_{RD} \). On the other hand, the integrand in (19) is a concave function of \( P_i^* \) for \( P_i \geq 0 \) and all \( i \). Using Jensen’s inequality, we have

\[ \frac{1}{NS} \sum_{i=1}^{NS} \ln \left( \frac{1 + P_i^*}{1 + \frac{q[P_{RD}]}{P_{RD}} P_i^*} \right) \leq \ln \left( \frac{1 + P_{RD}}{1 + \frac{q[P_{RD}]}{NS} P_{RD}} \right). \]  

(20)

Using (19) and (20), we have

\[ \Delta_{RD} \leq NS \int_{h \in B} \ln \left( \frac{1 + \frac{P_{RD}}{NS}}{1 + \frac{q[P_{RD}]}{NS}} \right) f_H(h)dh = NS \int_{h \in B} \ln \left( \frac{\gamma_{RD} + h_{RD}}{\gamma_{RD} + q[h_{RD}]} \right) f_H(h)dh. \]  

(21)

In order to proceed with the evaluation of (21), we need to simplify the definition of set \( B \). This can be achieved by applying the following lemma.

**Lemma 1:** If the solution to the sum rate maximization problem in (1) for some channel vector \( h \) leads to \( P_i^* \leq P_{S_i,R} \) for some source \( S_i \), then the following inequality is valid

\[ \sum_{i=1}^{NS} P_{S_i,R} \geq \sum_{i=1}^{N_S} P_i^*. \]  

(22)

**Proof:** See Appendix A. \( \blacksquare \)
According to the definition of set $B$, we have $P_{i}^{q*} < q[P_{S,R}]$ for at least one $i$. Then by Lemma 1, $\sum_{i=1}^{NS} q[P_{S,R}] \geq q[P_{RD}]$. This leads to an alternative representation of the set $B$ as

$$B = \bigcup_{i=1}^{NS} B_i^* = \{h : \exists i \text{ such that } q[P_{S,R}] \geq P_{i}^{q*}\} = \left\{h : \sum_{i=1}^{NS} q[P_{S,R}] \geq q[P_{RD}]\right\}. \quad (23)$$

According to (23), the integration region in (21) is defined in terms of the quantized values of channel powers. Due to the complexity of working with quantized random variables, we will introduce a slightly larger set $B'$ described by the true $S_i$-$R$ channel powers. Define $B'$ as

$$B' = \left\{h : \sum_{i=1}^{NS} P_{S,R} \geq q[P_{RD}]\right\} \supset B. \quad (24)$$

By defining $Y = \sum_{i=1}^{NS} P_{S,R} = \sum_{i=1}^{NS} \gamma_{S_i} h_{S_i} R_i$, it follows for (21) that

$$\Delta_{RD} \leq NS \int_{h \in B} \ln \left(\frac{NS \gamma_{RD} + h_{RD}}{NS \gamma_{RD} + q[h_{RD}]}\right) f_{H}(h) dh$$

$$\leq NS \int_{h \in B'} \ln \left(\frac{NS \gamma_{RD} + h_{RD}}{NS \gamma_{RD} + q[h_{RD}]}\right) f_{H}(h) dh$$

$$= NS \int_{0}^{\infty} \int_{y \geq \gamma_{RD} q[h]} \ln \left(\frac{h + NS \gamma_{RD}^{-1}}{q[h] + NS \gamma_{RD}^{-1}}\right) f_{H_{RD}}(h) f_{Y}(y) dy dh$$

$$= NS \int_{0}^{\infty} \ln \left(\frac{h + NS \gamma_{RD}^{-1}}{q[h] + NS \gamma_{RD}^{-1}}\right) (1 - F_{Y}(\gamma_{RD} q[h])) f_{H_{RD}}(h) dh,$$

$$\leq NS \int_{0}^{\infty} \ln \left(\frac{h + NS \gamma_{RD}^{-1}}{q[h] + NS \gamma_{RD}^{-1}}\right) f_{H_{RD}}(h) dh, \quad (25)$$

where the first inequality uses $B \subset B'$ and the next separates out the integral into an integral over $Y$ and $h_{RD}$. The next step completes the integration over $Y$. The final inequality drops the $(1 - F_{Y}(\gamma_{RD} h))$ term as in the previous section.

This series of steps leaves us with the same objective function as that of (13) used for the optimal quantization problem. Therefore, based on our analysis, the same quantization structure is optimal for the upper bounds derived for all links across the network. After investigating the optimal quantizer in the following section, we return to (14) and (25) in Section V-B for the analysis of the optimal bit allocation.

**IV. DERIVATION OF THE OPTIMAL QUANTIZER**

The general structure of a quantizer requires quantization intervals followed by a choice of quantization levels. As described in Section III, our approach requires the quantization level to
be the lowest value of the chosen quantization interval. The quantizer is, therefore, completely
calibrated by the quantization vector \( \mathbf{q} = [q_0, q_1, ..., q_{N-1}] \), i.e., a vector comprising the \( N \)
non-zero quantization levels (note that by definition, \( q_{-1} = 0 \)). Then, according to the results
from the previous section, for both the \( S_i-R \) and \( R-D \) channels, the optimal quantizer is the one
which minimizes \( \delta(\mathbf{q}) \) where

\[
\delta(\mathbf{q}) = E \left[ \ln \left( \frac{h + \gamma^{-1}}{q[h] + \gamma^{-1}} \right) \right] = \int_0^\infty \ln \left( \frac{h + \gamma^{-1}}{q[h] + \gamma^{-1}} \right) f_H(h) dh. \tag{26}
\]

In \( \gamma \) is the average SNR and equals \( \gamma_{S_iR} \) for the case of the \( S_i-R \) channel and \( \gamma_{RD} \) for the
\( R-D \) channel. Moreover, the expectation in \( \delta(\mathbf{q}) \) is with respect to \( h \), the instantaneous channel
power of the corresponding link, whose distribution is that of \( h_{S_iR} \) or \( h_{RD} \) for the \( S_i-R \) and
\( R-D \) links, respectively. For the function \( \delta(\mathbf{q}) \) in \( \delta(\mathbf{q}) \) we have

\[
\delta(\mathbf{q}) = \sum_{n=-1}^{n=N-1} I_n = \sum_{n=-1}^{n=N-1} \int_{q_n}^{q_{n+1}} \ln \left( \frac{h + \gamma^{-1}}{q_n + \gamma^{-1}} \right) f_H(h) dh, \tag{27}
\]

i.e., \( I_n \) is as the component of the expectation integral over the interval \([q_n, q_{n+1}]\). Note that, as
defined earlier, we set two fixed quantization levels \( q_{-1} = 0 \) and \( q_N = +\infty \) (or \( q_N = \inf \{h \geq 0 : F_H(h) = 1\} \) if the pdf has finite support).

In our model we only consider channels distributions with finite average power. More specifically, we assume \( E[h] = 1 \). This assures \( \delta(\mathbf{q}) < \infty \) and consequently, from the continuity of
\( f_H(\cdot) \), the function \( \delta(\cdot) \) is differentiable with respect to the quantization levels \( q_n \), \( 0 \leq n \leq N-1 \).
Therefore, the optimal quantization level \( q_n \) satisfies the following

\[
\frac{\partial}{\partial q_n} \delta(\mathbf{q}) = \frac{\partial}{\partial q_n} (I_{n-1} + I_n) = 0. \tag{28}
\]

The following theorem presents the fundamental iterative relation between the optimal quanti-
tzation levels and is a key contribution of this paper.

**Theorem 1:** The quantization levels of the optimal quantizer minimizing \( \delta(\mathbf{q}) \) in \( \delta(\mathbf{q}) \) satisfy

\[
(q_n + \gamma^{-1}) \ln \left( \frac{q_n + \gamma^{-1}}{q_{n-1} + \gamma^{-1}} \right) = \frac{F_H(q_{n+1}) - F_H(q_n)}{f_H(q_n)}, \quad 0 \leq n \leq N - 1. \tag{29}
\]

**Proof:** Refer to Appendix B.

The quantization levels proposed in Theorem 1 are optimal for a variety of distributions,
including the uniform distribution. We first investigate the structure of the optimal quantizer for
the uniform distribution and then extend the results to more general distributions of channel
power.
A. Optimal Quantization for the Uniform Distribution

In this section we focus on the uniform distribution for the channel power and present the optimal quantization vector \( q \) which minimizes \( \delta(q) \) in (26). For the uniform distribution and from the assumption \( E[h] = 1 \) we have \( f_H(h) = \frac{1}{2} \) for \( 0 \leq h \leq 2 \) and \( f_H(h) = 0 \) for \( h > 2 \). Since the pdf is a constant, for any \( 0 \leq q_n < q_{n+1} \leq 2 \), we have

\[
F_H(q_{n+1}) - F_H(q_n) = f_H(q_n)(q_{n+1} - q_n).
\]

(30)

This will simplify the optimality condition proposed in Theorem 1. Essentially it follows from (29) and (30) that

\[
(q_n + \gamma^{-1}) \ln \left( \frac{q_n + \gamma^{-1}}{q_{n-1} + \gamma^{-1}} \right) = q_{n+1} - q_n, \quad 0 \leq n \leq N - 1,
\]

(31)

where \( q_N = \inf\{h \geq 0 : F_{H_X}(h) = 1\} = 2 \). Setting \( n = N - 1 \) and adding \( (q_{N-1} + \gamma^{-1}) \) to both sides leads to

\[
(q_{N-1} + \gamma^{-1}) \left( 1 + \ln \left( \frac{q_{N-1} + \gamma^{-1}}{q_{N-2} + \gamma^{-1}} \right) \right) = 2 + \gamma^{-1}.
\]

(32)

Theorem 2 now specifies all \( N \) non-zero quantization levels for the uniform distribution.

**Theorem 2:** The \( n \)-th quantization level of the optimal quantizer for the uniform distribution is given by

\[
q_n = \prod_{i=0}^{n} r_i^\gamma - \gamma^{-1}, \quad 0 \leq n \leq N
\]

(33)

where \( r_i \) is an iterated logarithmic sequence defined as

\[
r_i = 1 + \ln r_{i-1}, \quad 1 \leq i \leq N.
\]

(34)

Finally, the optimal value of \( r_0 \) satisfies

\[
\prod_{n=0}^{N} r_n = 2\gamma + 1.
\]

(35)

**Proof:** Refer to Appendix C.

Note that in the high-SNR regime, we can ignore the \( \gamma^{-1} \) term and we have \( r_i = q_i/q_{i-1}, 0 \leq i \leq N \), i.e., the ratio of consecutive quantization levels.

While Theorem 2 clearly defines the optimal quantizer for the uniform distribution, this distribution is impractical. Therefore, in the next section, we extend this quantizer to a general distribution function. This is an intractable problem for arbitrary \( N \) and we focus on the case of asymptotically large \( N \).
B. Asymptotically Optimal Quantization for General Distributions

The iterated logarithm in (34) is a direct result of (30) which holds exactly for the uniform distribution. For a general distribution function \( f_H(h) \), (30) is the result of a first-order Taylor-series approximation of \( F_H(q_{n+1}) \) at \( h = q_n \). This approximation becomes accurate for large \( N \), i.e., \( F_H(q_{n+1}) - F_H(q_n) \to f_H(q_n)(q_{n+1} - q_n) \) as \( q_n \to q_{n+1} \). As \( N \to \infty \), we have \( q_n \to q_{n+1} \). Then the optimality condition for the quantization levels \( q_0, q_1, \ldots, q_{N-2} \) presented in (29) is consistent with that of the uniform quantizer in (31). As a result, the quantizer structure presented through Theorem 2 is asymptotically optimal.

However, this statement is not true for all \( n \), specifically, \( n = N - 1 \). For a general distribution, \( q_N = \infty \) and the Taylor series approximation cannot be applied to the last interval, \([q_{N-1}, \infty)\). Therefore, for a general distribution, \( q_{N-1} \) remains unspecified. The value of \( q_{N-1} \) should be chosen such that \( I_{N-1} \) in (27) is small, in turn making \( \delta(q) \to 0 \) with \( N \to \infty \). We call a quantizer consistent if \( \delta(q) \to 0 \) as \( N \to \infty \). Unfortunately, using (33)-(35) results in a bounded value for \( q_{N-1} \) even if \( N \to \infty \), in turn making \( I_{N-1} \) always non-zero. This would, therefore, lead to an inconsistent quantizer. In this respect, (35) needs to be modified.

The key is to realize that by using (33) and (34) finding the proper value of \( q_{N-1} \) is equivalent to choosing an appropriate \( r_0 \). This choice is based on the behavior of \( f_H(h) \) at large \( h \); specifically, the value of \( r_0 \) needs to increase with \( N \) to guarantee that all \( I_n \) approach zero for large \( N \). This can be achieved by replacing (35) with the following

\[
\prod_{n=0}^{N} r_n = \kappa_N \gamma + 1, \tag{36}
\]

where the constant 2 in (35) is replaced with \( \kappa_N \). Here \( \kappa_N \) increases with \( N \) to ensure that the quantizer is consistent - for large \( N \), \( h \) is quantized such that \( I_n \) becomes sufficiently small \( \forall n \). The following theorem develops an appropriate choice of \( \kappa_N \) for a general distribution function.

**Theorem 3:** Consider the proposed quantizer with \( r_0 \) found from (36). For a general distribution function \( f_H(h) \) with cdf \( F_H(h) \), the quantization loss, defined in (26), is bounded by

\[
\delta(q) \leq \mathcal{O} \left( \frac{\ln(\kappa_N^*)}{N} \right), \tag{37}
\]

where \( \kappa_N^* = F_H^{-1}(1 - N^{-1}) \).

**Proof:** Refer to Appendix [D]
Note that, using (33) and (36), the suggested choice of $\kappa_N^*$ ensures that the probability of the channel falling in the final interval, i.e., $Pr\{H > q_{N-1}\}$, is almost $1/N$.

**Corollary 1:** For any channel with $E[h] = 1$, $\kappa_N^* < N$ and therefore the quantizer is consistent.

**Corollary 2:** Under Rayleigh fading, i.e., $H \sim e^{-h}$, and $N \to \infty$, we have $\kappa_N^* = \ln N$ and

$$\delta(q) \leq O\left(\frac{\ln N}{N}\right).$$

**Corollary 3:** For channel power uniformly distributed in $[0,2)$, for any $N > 1$, $\kappa^* = 2$ and

$$\delta(q) \leq \frac{c}{N}$$

for some fixed constant $c > 0$.

**C. Performance loss for High Average SNR**

In the previous section we considered the asymptotic $N \to \infty$ case, but for finite SNR. In this section we consider the reverse and investigate how the quantizer levels (equivalently ratios) must change as a function of SNR; specifically we investigate the high-SNR regime. However, we do not assume that $N \to \infty$. We are, therefore, interested in the limiting behavior of $\delta(q)$ defined in (26) for the optimal quantizer vector $q$ designed using Theorems 2 and 3 when $\gamma \to \infty$.

To illustrate the importance of this analysis we first consider the performance of a fixed quantizer, denoted by $q'$, for Rayleigh fading. The quantization levels in $q'$ do not change with SNR, specifically $q'_0$ is constant with $\gamma$. Then using $q'_{-1} = 0$ and the concavity of $\ln(\cdot)$ we have the following lower bound on $I_{-1}$ (defined in (27))

$$I_{-1} = \int_0^{q'_0} \ln(\gamma h + 1)f_H(h)dh \geq \frac{1}{2} q'_0 e^{-q'_0} \ln (\gamma q'_0 + 1) \approx \frac{q'_0}{2} \ln \gamma.$$  \hfill (40)

From (40) we see that $\delta(q') \geq O(\ln \gamma)$ as $\gamma \to \infty$. The loss in sum-rate would, therefore, be at least $O(\ln \gamma)$. On the other hand, as $\gamma \to \infty$, the overall sum rate is also $O(\ln \gamma)$. Therefore for a fixed quantizer, at least a fixed percentage of the transmission rate is lost due to CSI quantization. For small values of $N$ this loss becomes quite significant.

The key contribution of this section is to show that, in the limit as $\gamma \to \infty$, our proposed quantizer results in a $\delta(q)$ that grows at a pace much slower than $\ln \gamma$. As a consequence, the relative rate loss due to CSI quantization tends to zero as the SNR grows. This is true for a wide class of channel distributions and even for small values of $N$. The next theorem summarizes the main results on the high SNR behavior of $\delta(q)$ for the proposed quantizer.
Theorem 4: In the high SNR regime, where $\gamma \to \infty$, the quantizer described by Theorem 2 leads to a quantization loss $\delta(q)$ (defined in (26)) which scales as the $N$-th order iterated logarithm of the average SNR, $\ln^{(N+1)} \gamma$, where $\ln^{(0)} x = x$ and $\ln^{(n)} x$ is defined as

$$\ln^{(n)} x = 1 + \ln \left( \ln^{(n-1)} x \right), \quad n > 0, x \geq 1. \quad (41)$$

Proof: Refer to Appendix E.

Theorem 4 shows that the loss, $\delta(q)$, approaches infinity extremely slowly - at a rate of $O(\ln^{(N+1)} \gamma)$. This result is valid for any finite valued channel distribution function and as long as $N > 1$, the relative performance loss vanishes with $\gamma \to \infty$.

D. Numerical Validation

To validate our analysis we investigate the loss defined in (26) through computer simulations. Fig. 1 plots the objective function $\delta(q)$ in bits (as opposed to nats) for two quantizers: the first adapts to the average SNR by setting the quantization levels according to Theorem 2. The second quantizer is similar but the levels are optimized for an average SNR of $10$ dB and then kept fixed. Note that $\delta(q)$ is, for any specific link, the upper bound on the loss in the rate of that link - see (26). This figure is obtained by numerically generating channel powers drawn from the uniform distribution - we use Theorem 2 to obtain the quantizer - and then averaging the resulting $\delta(q)$ over many channel realizations. As Fig. 1 shows, even for $N = 3$ there is a significant difference between the performance of adaptive and fixed quantizers. Importantly, as suggested in Section IV-C, with a fixed quantizer the loss is linear in average SNR (measured in dB), while, for the optimal quantizer the loss grows, but very slowly.

In a second test scenario, we simulate a network comprising two source nodes and a relay node. We assume Rayleigh fading with the same average SNR for all links. Also all channels deploy the same $k_{avg}$-bit quantizer ($N + 1 = 2^{k_{avg}}$). We compare the optimal quantizer and the max-entropy quantizer [14], i.e., the quantizer which maximizes the entropy of CSI messages by creating equi-probable quantization intervals. Fig. 2 illustrates the performance loss for the two quantizers. The curves in Fig. 2 show the percentage of the perfect CSI rate which is lost to quantization as a function of average SNR. As predicted, this fraction goes to zero for the optimal quantizer while, for the max-entropy quantizer, it increases as a function of average SNR and converges (from below) to a constant.
V. Optimal Bit Allocation

In Section IV we derived the optimal quantizer based on the upper bound on performance loss developed in Section III. We then provided appropriate choices of parameters that ensured a consistent quantizer. A remaining question is, given a budget on the overall number of bits available for CSI, how to allocate these CSI bits to different channels in the network. The key question considered in this section is how to determine the relative importance of each link in the network. Based on previous results and assuming the quantizer structure of Section IV, we analyze the performance loss to find the optimal bit allocation algorithm.

To proceed, we continue from the results of (9), (13), and (25). It follows that

\[ \Delta \leq \sum_{i=1}^{N_S} E \left[ \ln \left( \frac{h_{S_iR} + \gamma_{S_iR}^{-1}}{q_i[h_{S_iR}] + \gamma_{S_iR}^{-1}} \right) (1 - F_H(\alpha_i q_i[h_{S_iR}]]) \right] 
+ \sum_{i=1}^{N_S} E \left[ \ln \left( \frac{h_{RD} + N_S \gamma_{RD}^{-1}}{q_R[h_{RD}] + N_S \gamma_{RD}^{-1}} \right) (1 - F_Y(\gamma_{RD} q_R[h_{RD}]]) \right] , \quad (42) \]

where we have added indices for the quantizer functions to emphasize each link is quantized according to different quantization levels.

According to Theorem 3, the optimal quantizer is consistent with \( N \), i.e., the performance loss \( \delta(q) \) is \( \mathcal{O}(\ln(\kappa^*_N)/N) \). Furthermore, the corollaries showed that, differing distributions of power, \( f_H(h) \), result in a wide variety of appropriate choices of \( \kappa^*_N \). However, it appears that in all cases, the numerator is a relatively slowly increasing function of \( N \), while the denominator is consistently \( N \). To allow for a general - and tractable - analysis, we approximate the loss as \( \delta(q) \propto \eta/(N+1) \). This property is tested in Fig. 3 for the Rayleigh fading channel distribution \( H \propto e^{-h} \) where the channel is quantized according to the proposed quantizer in Section IV. The figure shows that the rate of the decrease in the performance loss is almost \( N^{-1} \).

Following this assumption, (42) can be written as

\[ \Delta \leq \sum_{i=1}^{N_S} \frac{\eta_i}{2^{k_i}} + \frac{\eta_{RD}}{2^{k_{RD}}} , \quad (43) \]

where \( k_i = \log_2(N_i + 1) \) and \( k_{RD} = \log_2(N_{RD} + 1) \) are the number of CSI quantization bits allocated to the quantization of links \( S_i-R \) and \( R-D \), respectively. This is equivalent to assigning \( N_i \) and \( N_{RD} \) quantization levels to the corresponding normalized channel powers. The coefficients \( \eta_i \) and \( \eta_{RD} \) are the key parameters in defining the bit allocation algorithm across wireless links and are referred to as the loss coefficients for the \( S_i-R \) and \( R-D \) links, respectively. In the next
sections we will extract these loss coefficients based on the results at the end of Sections [III-A] and [III-B] and assuming the optimal quantizer proposed in Section [IV].

A. Evaluating the loss coefficient for the link $S_i$-$R$: $\eta_i$

Let $N_i$ be the number of quantization levels used for the quantization of this link. From (13)

\[
\Delta_{S_i,R} \leq \sum_{n=-1}^{N_i-1} (1 - F_{H_{RD}}(\alpha_i q_n)) I_n = I_{-1} + \sum_{n=0}^{N_i-1} (1 - F_{H_{RD}}(\alpha_i q_n)) I_n,
\]

(44)

For large $N_i$ we can eliminate consideration of the edge terms, setting $I_{-1}, I_{N_i-1} \simeq 0$. Also for the optimal quantizer and for all $q_0 < h < q_{N_i-1}$, we have $q_n + \gamma_{S_i,R}^{-1} \leq h + \gamma_{S_i,R}^{-1} < r_{n+1}(q_n + \gamma_{S_i,R}^{-1})$, which means $\frac{h + \gamma_{S_i,R}^{-1}}{r_{n+1}} - \gamma_{S_i,R}^{-1} < q_n$. Therefore, we have

\[
(1 - F_{H_{RD}}(\alpha_i q_n)) I_n \leq \int_{q_n}^{q_{n+1}} \ln \left( \frac{h + \gamma_{S_i,R}^{-1}}{q_n + \gamma_{S_i,R}^{-1}} \right) \left( 1 - F_{H_{RD}} \left( \frac{h}{r_{n+1}} \right) \right) f_H(h) dh
\]

\[
\leq \int_{q_n}^{q_{n+1}} \ln \left( \frac{h + \gamma_{S_i,R}^{-1}}{q_n + \gamma_{S_i,R}^{-1}} \right) \left( 1 - F_{H_{RD}} \left( \frac{h}{r_1} \right) \right) f_H(h) dh, 0 \leq n \leq N_i - 2,
\]

(45)

where the last inequality is true since, from (34), $r_1 \geq r_n$ for $n > 1$. Note that the distribution $f_H(h)$ in this equation corresponds to the $S_i$-$R$ channel under consideration. It follows that

\[
\sum_{n=0}^{N_i-2} (1 - F_{H_{RD}}(\alpha_i q_n)) I_n \leq \int_{q_0}^{q_{N_i-1}} \ln \left( \frac{h + \gamma_{S_i,R}^{-1}}{q_i[h] + \gamma_{S_i,R}^{-1}} \right) \left( 1 - F_{H_{RD}} \left( \frac{h}{r_{1}} \right) \right) f_H(h) dh,
\]

(46)

and from the Cauchy-Schwarz inequality we have

\[
\sum_{n=0}^{N_i-2} (1 - F_{H_{RD}}(\alpha_i q_n)) I_n \leq \int_{q_0}^{q_{N_i-1}} \ln \left( \frac{h + \gamma_{S_i,R}^{-1}}{q_i[h] + \gamma_{S_i,R}^{-1}} \right) f_H(h) dh
\]

\[
	imes \int_{q_0}^{q_{N_i-1}} \left( 1 - F_{H_{RD}} \left( \frac{h}{r_{1}} \right) \right) dh
\]

\[
\leq \frac{r_1}{\alpha_i} \int_{q_0}^{q_{N_i-1}} \ln \left( \frac{h + \gamma_{S_i,R}^{-1}}{q_i[h] + \gamma_{S_i,R}^{-1}} \right) f_H(h) dh.
\]

(47)

Since $F_{H_{RD}}(\alpha_i q_n) \leq 1$, the coefficient $r_1/\alpha_i$ is replaced by $\min(1, \frac{r_1}{\alpha_i})$ to ensure that (13) does not exceed (14). Finally, from (44) and (46) we have

\[
\sum_{n=-1}^{N_i-1} (1 - F_{H_{RD}}(\alpha_i q_n)) I_n \leq I_{-1} + \min \left( 1, \frac{r_1}{\alpha_i} \right) \sum_{n=0}^{N_i-2} I_n + (1 - F_{H_{RD}}(\alpha_i q_{N_i-1})) I_{N_i-1}
\]

\[
\simeq \min(1, \frac{r_1}{\alpha_i}) \sum_{n=-1}^{N_i-1} I_n = \min \left( 1, \frac{r_1}{\alpha_i} \right) E \left[ \ln \left( \frac{h + \gamma_{S_i,R}^{-1}}{q_i[h] + \gamma_{S_i,R}^{-1}} \right) \right].
\]

(48)
Based on Theorem 3, we know that the expectation above is inversely proportion to \( N \). Therefore,
\[
\Delta_{S,R} \leq \min \left( 1, \frac{r_i}{\alpha_i} \right) \frac{c_q}{N_i + 1} = \min \left( 1, \frac{r_i}{\alpha_i} \right) \frac{c_q}{2^k_i}.
\] (49)

From (49) we obtain \( \eta_i = \min \left( 1, \frac{r_i}{\alpha_i} \right) c_q \) where \( c_q \) is a constant independent of \( N_i \).

B. Evaluating the \( R-D \) loss coefficient: \( \eta_{RD} \)

For the \( R-D \) link we follow the result in (25). Evaluating (25) for a general distribution function \( f_H(h) \) is intractable. Therefore, we resort to the Rayleigh fading channel model leading to the negative exponential distribution on the channel power.

To find (25) we need the cdf function \( F_Y(\cdot) \) where \( Y \) is a weighted sum of negative exponential random variables as defined in Section III-B. We can approximate this random variable with an Erlang-2 random variable. More specifically, we define the following:
\[
Z = 2 \sum_{i=1}^{N_S} H_{S,R} \simeq \frac{2Y}{\sum_{i=1}^{N_S} \gamma_{S,R}}.
\] (50)

The random variable \( Z \) is a standard Erlang-2 random variable with cdf \( F_Z(z) = \left[ 1 - \sum_{n=0}^{2N_S-1} e^{-\frac{z}{n!}} \right] \). From (50) we can write (25) as
\[
\Delta_{RD} \leq N_S \int_0^\infty \ln \left( \frac{h + N_S \gamma_{RD}^{-1}}{q[h] + N_S \gamma_{RD}^{-1}} \right) (1 - F_Z(\beta q[h])) f_H(h) dh,
\] (51)

where \( \beta = 2\gamma_{RD} / \sum_{i=1}^{N_S} \gamma_{S,R} \). From the cdf of the Erlang-2 distribution, (51) implies
\[
\Delta_{RD} \leq N_S \sum_{k=0}^{2N_S-1} J_k,
\]

with \( J_k \) defined as
\[
J_k = \int_0^\infty \ln \left( \frac{h + N_S \gamma_{RD}^{-1}}{q[h] + N_S \gamma_{RD}^{-1}} \right) \frac{(\beta q[h])^k}{k!} e^{-\beta q[h]} f_H(h) dh = \frac{(-\beta)^k}{k!} \frac{\partial^k J_0}{\partial \beta^k}.
\] (52)

\(^1\)Note that this approximation results in a tight upper bound on (25). The intuition is that the random variable \( Z \) (with \( E[Z] = E[Y_{norm}] = N_S \)) with \( Y_{norm} = Y/E[Y] \) has a much smaller variance than \( Y_{norm} \). Therefore, one could imagine the distribution of \( Z \) being more concentrated around its mean, \( N_S \), whereas the distribution of \( Y_{norm} \) has a wider spread around its mean. This makes \( 1 - F_Z(h) \geq 1 - F_{Y_{norm}}(h) \) for \( h \leq N_S \). In the region \( h > N_S \), we already have \( f_H(h) = e^{-N_S} \ll 1 \) which diminishes the effect of the approximation error in the overall value of the integral.
Here we assume high resolution quantization such that $\beta q[h] + h \simeq (\beta + 1)h$ and also $(1 + \beta)q[\frac{h}{1+\beta}] \simeq h$. Now by defining the auxiliary variable $u = (\beta + 1)h$, $J_0$ reduces to the following

$$J_0 \simeq \frac{1}{\beta + 1} \int_{0}^{\infty} \ln \left( \frac{u + N_S(\beta + 1)\gamma_{RD}^{-1}}{q[u] + N_S(\beta + 1)\gamma_{RD}^{-1}} \right) e^{-u} du$$

$$= N_S \frac{1}{\beta + 1} E \left[ \ln \left( \frac{h + \gamma_{RD}^{-1}}{q[h] + \gamma_{RD}^{-1}} \right) \right],$$

with $\gamma_{RD}^{-1} = N_S \left( (2 \sum_{i=1}^{N_S} \gamma_{S_i R})^{-1} + \gamma_{RD}^{-1} \right)$. On the other hand, from (52) we have:

$$J_k \simeq \frac{\beta^k}{(\beta + 1)^{k+1}} E \left[ \ln \left( \frac{h + \gamma_{RD}^{-1}}{q[h] + \gamma_{RD}^{-1}} \right) \right],$$

resulting in

$$\Delta_{RD} \leq \sum_{k=0}^{2N_S-1} J_k = N_S \left( 1 - \left( \frac{\beta}{\beta + 1} \right)^{2N_S} \right) E \left[ \ln \left( \frac{h + \gamma_{RD}^{-1}}{q[h] + \gamma_{RD}^{-1}} \right) \right].$$

(53)

Similar to the discussion at the end of section V-A we have

$$\Delta_{RD} \leq \sum_{k=0}^{2N_S-1} J_k = N_S \left( 1 - \left( \frac{\beta}{\beta + 1} \right)^{2N_S} \right) \frac{c_q}{N_{RD}} = N_S \left( 1 - \left( \frac{\beta}{\beta + 1} \right)^{2N_S} \right) \frac{c_q}{2^{k_{RD}}},$$

(54)

The result in (54) suggests that $\eta_{RD} = N_S \left( 1 - \left( \frac{\beta}{\beta + 1} \right)^{2N_S} \right) c_q$. Note that since $c_q$ is common to both the $S_i$-R and R-D channels, it is irrelevant.

C. Bit Allocation

As explained in the beginning of Section V, the upper bound in (43) can be used to formulate the bit allocation problem. In particular, we look at the problem of bit allocation in a scenario where the system imposes a cap on the overall number of CSI bits in each transmission phase. We assume this number is $k_{max} > N_S + 1$. Based on this model and from (43), the optimal bit allocation problem is formulated as

$$\min_k \sum_{i=1}^{N_S} \frac{\eta_i}{2^{k_i}} + \frac{\eta_{RD}}{2^{k_{RD}}}$$

subject to:

$$\sum_{i=1}^{N_S} k_i + k_{RD} \leq k_{max},$$

$$k_i \geq 1, \forall i,$$

(55)
where $k_i$ denotes the number of CSI bits allocated to the $S_i$-$R$ link and $k_{RD}$ the number allocated to the $R$-$D$ link. Finally, $k = [k_1, \ldots, k_{NS}, k_{RD}]$. The last constraint ensures that the transmission rate for all nodes is non-zero (if CSI bits are not allocated to a link, the corresponding channel power is quantized to zero; in turn the achievable rate for that link is zero).

The solution to the bit allocation problem follows a simple iterative algorithm. Initialize the allocation vector as $k^1$ as the all-ones vector. Assume $k^j = [k^j_1, k^j_2, \ldots, k^j_{NS}, k^j_{RD}]$ denotes the pattern of bit allocation at iteration $j$. At the iteration $j + 1$ we look for the link with the largest effect on the performance loss. Link $n$ has the largest contribution to the upper bound on loss if $n = \arg \max_m 2^{-k^j_m} \eta_m$. Then, the next bit is allocated to link $n$, i.e., $k^j_{n+1} = k^j_n + 1$. This procedure is repeated until all $k_{max}$ bits are allocated to the source nodes (in $k_{max} - N_S - 1$ iterations).

To illustrate the performance gain through optimal bit allocation, we simulate a two-source network where the sources are randomly located in front of a relay which is at a fixed distance to the destination. We assume a Rayleigh fading model for all links and $E[\gamma_{S_i,R}] = 25$dB and $\gamma_{RD}$ fixed at 20dB, i.e., the scenario where the relay is located closer to the source nodes rather than the destination. We compare optimal bit allocation and uniform bit allocation, (i.e., $N_{S_i,R} = N_{RD}$). Also we include the performance of both the max-entropy quantizer and the quantizer proposed in Section IV in order to illustrate the performance gains through optimal quantization. Fig. 4 illustrates the percentage of the perfect CSI sum rate achieved for each case under quantized CSI. It is seen that through optimal bit allocation we can achieve considerable performance gain (as opposed to uniform bit allocation) and this difference is particularly interesting when the $k_{max}$ is small. Furthermore, Fig. 4 shows that our proposed quantizer always outperforms the max-entropy quantizer. For the given network parameters and at the 80% target level, it is observed that the proposed quantizer saves almost one bit per link compared to the max-entropy quantizer while this saving grows to more than 1.5 bits per link after bit allocation. One important reason for this difference is the adaptability of quantization to SNR.

D. Central Node

In the discussion so far, we assumed that the quantized CSI is reported to a node that is not part of the network. A more detailed look at the terms in (43) reveals that choice of the central node will drastically affect the value of performance loss. Before the uplink data transmission
phase starts, the CSI for links $S_i-R$ are already available at the relay and the CSI for $R-D$ link is available at the destination via training. Selecting the relay or the destination as the central node is equivalent to assuming the perfect CSI of the $S_i-R$ links or the $R-D$ link is available at the central node. This is equivalent to letting $k_i$ for all $i$ or $k_{RD}$ grow to infinity where $k_i$ and $k_{RD}$ are defined in (42). In short, the selection between the destination and the relay reduces to comparing the first and second terms in (42).

For the proposed system model it could be argued that relay is the best choice to serve as the central node. The reason is that each link in the network needs at least one bit for the CSI quantization (otherwise the link is assumed to be dead and the channel is always quantized to zero). This demands a minimum of $N_S$ CSI bits for the quantization of $S_i-R$ links which might lead to large CSI quantization costs for multiple node networks. The selection of the relay as the central node will cancel this requirement. At the same time, providing a few bits to the quantization of the $R-D$ link ensures a considerably small performance loss.

Although setting the relay as the central node is beneficial in terms of CSI demand, we should mention that for the multiple relay networks, it is more reasonable to choose the destination as the central node. This is due to the fact that the destination, as the central node, can resolve the source-relay assignment problem across the network.

VI. CONCLUSIONS

In this paper we developed bounds on the performance loss due to quantization of CSI in a multi-source, single-relay, network. Our system model is most similar to the uplink of a cellular system. Our design metric is the sum-rate achieved over all the nodes; the relay allocates its power among the source nodes in order to maximize the sum rate. Our analysis leads to a tight upper bound on the performance loss which is expanded as the sum of individual terms each representing the loss due to the quantization of a certain wireless link.

We use the upper bound on the performance loss to develop an optimal quantizer. This quantizer is consistent in the sense that the loss approaches zero as the number of quantization levels increases. Moreover, one key result we develop is that the quantizer is strongly robust to the average SNR of the link; the loss is an $N$-th order iterated logarithm of the average SNR. A consequence is that the performance loss stays almost constant over the range of practical values for the average SNR and the relative loss goes to zero as SNR increases.
Using the proposed upper bound and considering the optimal quantizer, the performance loss is further reduced through optimal bit allocation across the wireless links. A key contribution is to quantize the relative importance of each link in the network. Numerical results show that through quantization and bit allocation, considerable savings in the average number of CSI bits per node is obtained. Finally, we argue that for the proposed network model when the number of source nodes is large, it is better to select the relay as the central node.

**Appendix A**

**Proof of Lemma 1**

We prove the lemma by contradiction. The optimal power allocation is the solution to (1) with the constraint in (2). From (3), \( R_i \) is increasing in \( P_i \), but for \( P_i > P_{S_i R} \), \( R_i \) remains constant. Now if there is at least one source node such that \( P_i < P_{S_i R} \), then the optimal power allocation mandates that for all other nodes we must have \( P_j \leq P_{S_j R} \). To see this, assume the opposite, i.e., there is at least one other node \( j \) such that \( P_j > P_{S_j R} \). Then, since \( R_j \) is constant in a neighborhood around \( P_j \), we can simply reduce \( P_j \) by some \( \delta P \leq P_j - P_{S_j R} \) which maintains the value of \( R_j \), and add it to \( P_i \) which increases \( R_i \) by \( C(\min(P_i + \delta P, P_{S_i R}) - C(P_i) \). Therefore, this allocation cannot be optimal.

**Appendix B**

**Proof of Theorem 1**

The theorem sets the iterative relationship between the optimal quantization levels, From the definition in (27) we obtain

\[
I_{n-1} + I_n = \int_{q_{n-1}}^{q_n} \ln \left( \frac{h + \gamma^{-1}}{q_{n-1} + \gamma^{-1}} \right) f_H(h) dh + \int_{q_n}^{q_n+1} \ln \left( \frac{h + \gamma^{-1}}{q_n + \gamma^{-1}} \right) f_H(h) dh \\
= \int_{q_{n-1}}^{q_n+1} \ln(h + \gamma^{-1}) f_H(h) dh \\
- \left( \ln(q_{n-1} + \gamma^{-1}) \int_{q_{n-1}}^{q_n} f_H(h) dh + \ln(q_n + \gamma^{-1}) \int_{q_n}^{q_{n+1}} f_H(h) dh \right). \tag{56}
\]

Now from the optimality condition in (28) we have

\[
\frac{\partial}{\partial q_n} (I_{n-1} + I_n) = \frac{\partial}{\partial q_n} \int_{q_{n-1}}^{q_n+1} \ln(h + \gamma^{-1}) f_H(h) dh - \frac{\partial}{\partial q_n} \left( \ln(q_{n-1} + \gamma^{-1}) \int_{q_{n-1}}^{q_n} f_H(h) dh \right. \\
+ \ln(q_n + \gamma^{-1}) \int_{q_n}^{q_{n+1}} f_H(h) dh) = 0. \tag{57}
\]
The first term of (57) is independent of \( q_n \), hence the derivative is zero. From the second term it follows
\[
\frac{\partial}{\partial q_n} \left( \ln(q_n-1) \int_{q_n-1}^{q_n} f_H(h) dh + \ln(q_n+1) \int_{q_n}^{q_{n+1}} f_H(h) dh \right)
\]
\[
= \ln(q_n-1) f_H(q_n) + \ln(q_n+1) (-f_H(q_n)) + \frac{\int_{q_n}^{q_{n+1}} f_H(h) dh}{q_n+1} = 0.
\]
(58)
Multiplying the sides of (58) by \( \frac{(q_n+\gamma^{-1})}{f_H(q_n)} \) we find (29) and the theorem is proved.

**APPENDIX C**

**PROOF OF THEOREM 2**

Theorem 2 specifies the \( N \) quantization levels for the uniform distribution. The proof follows from (31). We have
\[
(q_n+\gamma^{-1}) \left( 1 + \ln \left( \frac{q_n+\gamma^{-1}}{q_{n-1}+\gamma^{-1}} \right) \right) = q_{n+1} + \gamma^{-1},
\]
which leads to
\[
\frac{q_{n+1}+\gamma^{-1}}{q_n+\gamma^{-1}} = 1 + \ln \left( \frac{q_n+\gamma^{-1}}{q_{n-1}+\gamma^{-1}} \right).
\]
(59)
Define \( r_n = \frac{q_{n+1}+\gamma^{-1}}{q_{n-1}+\gamma^{-1}}. \) Then we have \( q_{n+1} + \gamma^{-1} = r_n (q_n + \gamma^{-1}) \) which directly leads to
\[
q_n = \prod_{i=0}^{n} r_i (q_{i+1} + \gamma^{-1}),
\]
(60)
where from \( q_{-1} = 0 \) in our earlier assumptions, we find (33). Furthermore, by replacing the ratios in (59) with \( r_{n+1} \) and \( r_n \) we find (34). Finally, from (35) and (33) we obtain
\[
(q_{N-1}+\gamma^{-1}) \ln(r_{N-1}) = 2 - q_{N-1}
\]
\[
\Rightarrow (q_{N-1}+\gamma^{-1})(1+\ln(r_{N-1})) = 2 + \gamma^{-1}
\]
\[
\Rightarrow \prod_{n=0}^{N-1} \frac{r_n}{\gamma} r_N = 2 + \gamma^{-1}.
\]
(61)
which leads to (35) and the theorem is proved.
APPENDIX D

PROOF OF THEOREM 3

Theorem 3 sets the value of $\kappa_N$ to ensure a consistent quantizer. To prove the theorem we need the following lemma that sets an upper bound on an iterated logarithmic sequence.

Lemma 2: For large $n$ and some positive constant $c$, the iterated logarithmic sequence defined in (34) is bounded as

$$ r_n \leq 1 + \frac{c}{n}, \quad n > 0. \quad (62) $$

Proof: Here we show that $r_n$ in (34) decreases at least as $O\left(\frac{1}{n}\right)$ for large $n$. For some $n > 1$ choose integer $M$ such that $r_n \leq 1 + \frac{M}{n}$. Then we have:

$$ r_{n+1} = 1 + \ln r_n \leq 1 + \ln \left(1 + \frac{M}{n}\right) = 1 + \ln \left(\frac{M + n}{n}\right). \quad (63) $$

On the other hand, from the Reimann integral of the function $f(x) = 1/x$ over $[n, M + n]$ we have (choosing intervals of length $1$):

$$ \ln \left(\frac{M + n}{n}\right) \leq \sum_{k=n}^{M+n-1} \frac{1}{k} + \sum_{k=n+1}^{n+M-1} \frac{1}{k} \leq \frac{1}{n} + (M - 1) = \frac{M}{n} + 1 - \frac{1}{n + 1} \leq \frac{M}{n + 1} + O\left(\frac{1}{n^2}\right). \quad (64) $$

Then from induction and assuming a large enough $n$ we have, $r_m - 1 = O\left(\frac{1}{m}\right)$ for $m > n$. \(\blacksquare\)

To prove Theorem 3 we start with the definition of $\delta(q)$ in (26). It follows that

$$ \delta(q) = \sum_{n=1}^{N-1} \int_{q_n}^{q_{n+1}} \ln \left(\frac{h + \gamma^{-1}}{q_n + \gamma^{-1}}\right) f_H(h)dh, $$

$$ \leq \sum_{n=1}^{N-2} \ln(r_{n+1})(F_H(q_{n+1}) - F_H(q_n)) + I_{N-1} = \sum_{n=1}^{N-2} \ln(r_{n+1})Q_n + I_{N-1}, \quad (65) $$

where $Q_n = F_H(q_{n+1}) - F_H(q_n)$ and $I_{N-1}$ is defined in (26). We will argue that the sequence $\{Q_n\}_{n=1}^{N-1}$ is increasing with $n$. To see this, first note that the length of the quantization intervals increases with $n$ (we have $q_{n+1} - q_n = q_n(r_{n+1} - 1) \leq q_n(1 - r_n^{-1}) = q_n - q_{n-1}$ where the inequality is a direct result of the fact that for any $r > 1, r^{-1} + \ln r > 1$). Now from (36) we have $q_{N-1} \leq \kappa_N$ and by fixing $\kappa_N = \kappa < N$ and using Lemma 2 it is observed that $q_{n+1} - q_n \leq \kappa/N$. Therefore,
by increasing $N$ the distance between quantization levels approaches zero. This allows for Taylor series approximation of $F_H(h)$ at the quantization levels. Then, by assuming $f_H(h)$ to be almost constant over $[q_{n-1}, q_{n+1}]$, from the definition of $Q_n$ for any $0 \leq n < N - 1$ we have

$$\frac{Q_n}{Q_{n-1}} = \frac{F_H(q_{n+1}) - F_H(q_n)}{F_H(q_n) - F_H(q_{n-1})} \approx \frac{f_H(q_n)(q_{n+1} - q_n)}{f_H(q_n)(q_n - q_{n-1})} = \frac{q_n(r_{n+1} - 1)}{q_n(1 - r_n)} > 1,$$

(66)

Note that for the uniform distribution the result above is valid for all $N > 1$. Considering the fact that $\sum_{n=1}^{N-2} Q_n = F_H(q_{N-1})$, and $r_n$ being a strictly decreasing sequence (follows from the definition in (34)), it simply follows that

$$\sum_{n=1}^{N-2} Q_n \ln(r_{n+1}) \leq \sum_{n=1}^{N-2} \frac{F_H(q_{N-1}) \ln(r_{n+1})}{N-1}.$$

(67)

By taking the natural logarithm of both sides of (36) we have

$$\sum_{n=1}^{N-2} \ln(r_{n+1}) = \ln \left( \frac{\kappa \gamma + 1}{r_N} \right),$$

(68)

which together with (65) and (67) leads to

$$\delta(q) \leq \frac{1}{N-1} \ln \left( \frac{\kappa \gamma + 1}{r_N} \right) + I_{N-1}.$$

(69)

In order to simplify (69), we use Lemma 2 to find $r_N$ for $N \to \infty$. Lemma 2 shows that for $N \to \infty$, $r_N \to 1$ leading to $q_{N-1} \simeq \kappa$. Therefore assuming $\gamma^{-1} \ll \kappa$, we can write (69) in terms of $\kappa$ as in the following

$$\delta(q) \leq \frac{\ln(\kappa \gamma)}{N} + \int_{\kappa}^{q_N} \ln \left( \frac{h}{\kappa} \right) f_H(h) dh.$$

(70)

The upper bound in (70) can be minimized with respect to $\kappa$. This can be achieved by finding the root of the derivative of (70). Through some cumbersome math it is found that the optimal $\kappa$, i.e., $\kappa^*$ satisfies

$$\frac{1}{\kappa^* N} - \frac{1 - F_H(\kappa^*)}{\kappa^*} = 0$$

$$\Rightarrow \kappa^* = \frac{N}{1 - \frac{1}{N}}.$$

(71)
To obtain the smallest bound on $\delta(q)$, we need to evaluate (70) at $\kappa = \kappa^*$. It follows for $I_{N-1}$ that

$$I_{N-1} \simeq \int_{\kappa^*}^{q_N} \ln \left( \frac{h}{\kappa^*} \right) f_H(h) dh$$

$$= (1 - F_H(\kappa^*)) E \left[ \ln \left( \frac{h}{\kappa^*} \right) \mid h > \kappa^* \right]$$

$$\leq \frac{1}{N} \ln \left( \frac{1}{\kappa^*} E[h \mid h > \kappa^*] \right) \leq \frac{c_1}{\kappa^* N}, \tag{72}$$

where $c_1$ is a constant and the latest result is true due to the fact that from (71), $\kappa^* \to \infty$ as $N \to \infty$. Finally, from (72) and (70) we have

$$\delta(q) \leq O \left( \frac{\ln \kappa^*}{N} \right),$$

and the theorem is proved.

**APPENDIX E**

**PROOF OF THEOREM 4**

Theorem 4 states that the loss, is an order- $N$ iterated logarithm of SNR. To prove the theorem, we will require the following lemma that proves some useful properties of the ratios $r_n$:

**Lemma 3:** For all $n \geq 0$, the ratios $r_n$ defined according to (34) have the following properties:

1) $\lim_{\gamma \to \infty} r_n = \infty$

2) $\lim_{\gamma \to \infty} \frac{r_n}{r_n} = \infty$

3) $\lim_{\gamma \to \infty} \frac{r_{n+1}}{r_n} = \lim_{\gamma \to \infty} \frac{1 + \ln r_n}{r_n} = 0$.

**Proof:** The proof follows from (34) and (36). Note that the left hand side of (36) is strictly increasing in $r_0$; hence the first property. For the second property note that from (34) we have $\gamma r_n^{-1} > r_m \to \infty$ for $n \neq m$. Finally, the third property is derived from (34) and the fact that $\ln x/x \to 0$ for large $x$.

To prove Theorem 4 consider a quantizer with the ratios $r_n$ following (34). We proceed by finding an upper bound on the performance loss $\delta(q)$ in (27) in terms of the optimal ratio’s $r_n$ (defined in (34)). The integrand in (27) is increasing in $h$ and from the definition of $r_n$ in Theorem 2 for any $n < N - 2$ we have

$$I_n \leq \ln (r_{n+1}) (F_H(q_{n+1}) - F_H(q_n)), \tag{73}$$
where from (34), \( \ln r_n = r_n - 1 \). Letting \( f_H^{\text{max}} \) be the maximum value of \( f_H(h) \), i.e., \( f_H(H) \leq f_H^{\text{max}} \), \( \forall h > 0 \), from (73) and the definition of quantization levels in Theorem 1 it follows that

\[
I_n \leq (r_{n+2} - 1)(q_{n+1} - q_n)f_H^{\text{max}} \leq \frac{(r_{n+1} - 1)(r_{n+2} - 1)}{\gamma} \prod_{m=0}^{n} r_m f_H^{\text{max}}
\]  

(74)

In order to find the limiting value of \( I_n \) as \( \gamma \to \infty \) we use Lemma 3. From the first property in Lemma 3 and equation (36) we see that the upper bound in (74) approaches zero as \( \gamma \to \infty \). Therefore, it follows for \(-1 \leq n < N - 2 \) that

\[
\lim_{\gamma \to \infty} I_n = 0.
\]  

(75)

For \( I_{N-2} \) we have

\[
I_{N-2} \leq \frac{(r_N - 1)(r_{N-1} - 1)}{\gamma} \prod_{m=0}^{N-2} r_m,
\]  

(76)

where from (36) it readily follows that

\[
\lim_{\gamma \to \infty} I_{N-2} < \kappa_N < \infty.
\]  

(77)

Since in the high SNR regime \( \gamma^{-1} \ll q_{N-1} \), from (27) we have

\[
I_{N-1} \simeq \int_{q_{N-1}}^{\infty} \ln \left( \frac{h}{q_{N-1}} \right) f_H(h)dh,
\]  

where due to concavity of the logarithm together with Jensen’s inequality it follows that

\[
I_{N-1} \leq \ln \left( \frac{\bar{h}_{N-1}}{q_{N-1}} \right) (1 - F_H(q_{N-1})),
\]  

(79)

with \( \bar{h}_{N-1} = E_h[h \mid h > q_{N-1}] \). From (36) we have \( q_{N-1} = \frac{\kappa_N}{r_N} \) and from the first property of Lemma 3 we see that \( q_{N-1} \to 0 \) as \( \gamma \to \infty \). This together with \( E[h] = 1 \) leads to

\[
\bar{h}_{N-1} = \int_{q_{N-1}}^{\infty} h f_H(h) \frac{1}{1 - F_H(q_{N-1})} dh \\
\simeq (1 + F_H(q_{N-1})) \int_{q_{N-1}}^{\infty} h f_H(h)dh \\
\leq (1 + q_{N-1} f_H^{\text{max}})E[h] \\
= 1 + q_{N-1} f_H^{\text{max}}.
\]  

(80)

Finally from (79) and (80),

\[
I_{N-1} \leq \ln(f_H^{\text{max}} + \frac{1}{q_{N-1}})(1 - F_H(q_{N-1})) \\
\leq \ln(f_H^{\text{max}}) + \ln \left( 1 + \frac{r_N}{\kappa_N f_H^{\text{max}}} \right).
\]  

(81)
In conclusion, from (75), (77), and (81) we have

\[ \lim_{\gamma \to \infty} \delta(q) \leq \lim_{\gamma \to \infty} \left( \sum_{n=-1}^{N-3} I_n + I_{N-2} + I_{N-1} \right) \]

\[ \leq 0 + \kappa_N + \ln(f_{max}^H) + \ln \left( 1 + \frac{r_N}{\kappa_N f_{max}^H} \right) \]

\[ \sim \mathcal{O} \left( \ln^{(N+1)} r_0 \right) < \mathcal{O} \left( \ln^{(N+1)} \gamma \right) , \] (82)

which completes the proof.

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Fig. 1: Fixed and optimal quantizers for $N = 3$ (2 bits) and $N = 7$ (3 bits) levels of quantization.

Fig. 2: Percentage of the optimal perfect CSI sum-rate lost to quantization.
Fig. 3: loss vs. number of quantization levels; simulation and analysis for SNR = 10 and 20 dB.

Fig. 4: Percentage of the optimal perfect CSI sum-rate achieved through different methods of quantization and bit allocation for two users.