One-dimensional analysis of unsteady flows due to supercritical heat addition in high speed condensing steam

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Abstract. Unsteadiness in supersonic flow in nozzles can be generated by the release of heat due to spontaneous condensation. The heat released is termed “supercritical” and may be responsible for turbine blades failure in turbine cascade as it causes a supersonic flow to decelerate. When the Mach number is reduced to unity, the flow can no longer sustain the additional heat and becomes unstable. This paper aims to numerically investigate the unsteadiness caused by supercritical heat addition in one-dimensional condensing flows. The governing equations for mass, momentum and energy, coupled with the equations describing the wetness fraction and droplet growth are integrated and solved iteratively to reveal the final solution. Comparison is made with well-established experimental and numerical solution done by previous researchers that shows similar phenomena.

1. Introduction
The aim of every steam turbine design is an optimum efficiency operation. However, overall efficiency of a steam turbine power plant strongly depends on the turbine’s performance. Design of the nozzle and the blades is important as it affects the performance of the turbine. Thus any slight improvement, can increase power availability, decrease costs, and generate savings in operation and development cost. While the High Pressure (HP) turbine and Intermediate Pressure (IP) turbine operate with superheated steam, Low Pressure (LP) turbine has to work with wet steam. This is due to the occurrence of supersaturation and spontaneous condensation in expanding the steam flow at this stage.

To understand the effect of spontaneous condensation on LP turbines, the fundamental process of nucleation must first be understood. When steam expands through a nozzle, droplets do not appear instantly as it crosses the saturation line, but it first becomes metastable or usually referred to as supersaturated or subcooled. Supersaturation is a necessary but not a sufficient precondition for spontaneous condensation to occur. Before it is initiated by either homogeneous or heterogenous nucleation, it must exceed the limiting condition, at which random kinetic motions of molecules create sufficient stable microclusters for equilibrium to be recovered through condensation. In contrast to heterogenous nucleation, homogeneous condensation occurs when there are no foreign nuclei, dust particles, ions, etc and when a rapid formation and growth of clusters from metastable to stable or equilibrium size starts. Condensation in a nozzle usually occurs in the divergent section just downstream of the throat with the consequence of the considerable release of latent heat, $Q$, to the flow. Apart from that, it also has the effect of moving the flow Mach number towards unity as the additional heat causes the flow to decelerate. The effect produced when Mach number equals to unity was described as “thermal choking” by Pouring [1]. This phenomenon results in flow instability and induces oscillations. Previously, flow oscillations induced by the supercritical heat addition during condensation of water vapour near the throat of converging-diverging nozzles have been observed in details by Pouring [1], Wegener [2], Barschdorff [3] and Skillings [4].
In the current study, a one-dimensional (1-D) time-marching compressible Euler solver has been developed to simulate the phenomenon in steam nozzle flows. It uses the second order cell-vertex finite volume spatial discretization and the standard fourth order Runge-Kutta temporal integration has been developed. As the flow is unsteady, the time accuracy of the scheme is retained. Inclusion of higher virial coefficient was made in order to increase the accuracy for the calculation of real gas properties. This model has been applied to the case of unsteady supercritical heat addition in Skilling’s nozzle [4]. The shape of the nozzle is shown in Fig. 1. In this case, the inlet total temperature and inlet total pressure were set to be 347.9 K and 0.3514 bar respectively.

2. The governing equations
The governing equations describing the conservation of mass, momentum and energy are written in the form of partial differential equations, as shown in Equations (1), (2) and (3) respectively:

Conservation of mass: \( \frac{\partial (\rho A)}{\partial t} + \frac{\partial (\rho AV)}{\partial x} = 0 \)  

Conservation of momentum: \( \frac{\partial}{\partial t} (\rho AV) + \frac{\partial}{\partial x} (\rho AV^2) = -A \frac{\partial (\rho p)}{\partial x} \)  

Conservation of energy: \( \frac{\partial}{\partial t} \left[ f \left( e + \frac{V^2}{2} \right) A \right] + \frac{\partial}{\partial x} \left[ f \left( e + \frac{V^2}{2} \right) AV \right] = -\frac{\partial}{\partial t} (\rho AV) \)  

The equation of state used for the calculation of real gas is truncated at second virial coefficient based on Keenan et al. [5]. It is shown in equation (4):

\[ p = \rho RT_c (1 + B \rho) \]  

Where, \( R = 0.4615 \text{ kJ/(kg.K)} \) is a gas constant and \( B \) is the second virial coefficient. The coupling between the vapour, \( G \) and droplet, \( L \) phase is by means of wetness fraction, which is denoted by \( \omega \):

\[ \omega = \frac{m_L}{m_G + m_L} \]  

The wetness fraction is introduced into the expression for mixture enthalpy \( h \) and density \( \rho \).

3. Results and discussions
In order to investigate the unsteady effect, the frozen Mach number within the nozzle at different time intervals during one period of oscillations is plotted in Fig. 2. At time \( t = 0 \), it can be observed from the figure that the nucleation occurs some distance just downstream of the throat with the Mach number at the...
onset ranges from nearly above unity to below unity (1.02 – 0.94). The pattern is similar at time interval \( t = 0.2 \) \( T \) and \( t = 0.4 \) \( T \), but the range of values is slightly lower with not less than 0.9. This is due to the supercritical heat addition that retards the flow and restores the sonic flow at the throat and condensation in the diverging section. As time progresses, the process then repeats itself giving rise to oscillations.

![Figure 2. Frozen Mach number at different time intervals during one period of oscillations](image)

The variations of static pressure at a point located just downstream of the nozzle’s throat is plotted against the number of nucleation and time as in Fig. 3(a) and (b) respectively. As the flow becomes unstable in Fig. 3(a), the pressure decreases significantly and this pressure drops restores the sonic flow at the throat and gives rise to oscillations. In this case, regular oscillations with a frequency of 302 Hz can be clearly seen in Fig. 3(b). The amplitude of oscillations of about 17 mbar was also produced using the current method. The result is good compared to the experimental measurement made by Skillings [4], which is of frequency 380 Hz and amplitude of 18 mbar. He has also attempted to simulate similar one-dimensional model by using MacCormack’s scheme. In the subsequent years, attempt was also been made to model the supercritical heat addition effect to one-dimensional flow using Denton’s method by Guha [6]. Although the unsteady time-marching treatment for condensing flow of steam was adopted by Skillings [4] and Guha [6], however, the frequency was over-predicted, namely 540 Hz to 650 Hz respectively. This may be due to the use of different method in calculating the droplet growth compared to the present method. In recent decades, the study of supercritical heat addition in high speed condensing flow has been carried out by White and Young [7] and Yusoff [8]. The predicted frequency was calculated to be 420 Hz and 393 Hz respectively. Yusoff [8] has also predicted the amplitude of pressure of 20 mbar. The frequency and pressure difference of current results are compared with previous data obtained from other researchers and tabulated in Table 1.

The current model has shown good agreement with the experimental data of Skillings [4]. Reasonable agreement was also achieved when compared to numerical work carried out by other researchers. It is also noted that better agreement was obtained by two-dimensional models due to the significant velocity changes resulted from gradual changes in the nozzles’ cross sectional area.
Figure 3. Static pressure downstream of Skilling’s nozzle’s throat against (a) number of iterations and (b) time.

Table 1. Comparisons of frequency and pressure difference of current work with previous attempts.

| Author          | Case       | Frequency, Hz | Pressure Difference, mbar |
|-----------------|------------|---------------|---------------------------|
| Skillings [4]   | Experiment | 380           | 20                        |
| Skillings [4]   | 1D program | 640           | -                         |
| Guha [6]        | 1D program | 540           | -                         |
| White and Young [7] | 2D program | 420           | -                         |
| Yusoff [8]      | 2D program | 393           | 18                        |
| Current work    | 1D program | 302           | 17                        |

4. Conclusions
This study reveals promising results indicating the capability of the model to calculate unsteady flow due to supercritical heat addition phenomenon which might cause instability in steam turbine blade channel. The heat addition causes the flow to retard and gives rise to oscillations. Comparison with published experiments and numerical simulations is generally good although the computed frequency and amplitude of oscillation is slightly lower. This might be due to fact that some of the nozzle geometries exhibit significant two-dimensional effects, and these are likely to be modified by boundary layer growth in two-dimensional case.

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