QUCARK SPIN IN THE PROTON

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The proton spin puzzle has challenged our understanding of QCD for the last 20 years. We survey new developments in theory and experiment. The proton spin puzzle seems to be telling us about the interplay of valence quarks with chiral dynamics and the complex vacuum structure of QCD.

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1. Introduction

Protons behave like spinning tops. Unlike classical tops, however, the spin of these particles is an intrinsic quantum mechanical phenomenon. This spin is responsible for many fundamental properties of matter, including the proton’s magnetic moment, the different phases of matter in low-temperature physics, the properties of neutron stars, and the stability of the known universe. How is the proton’s spin built up from the spin and orbital angular momentum of the quarks and gluons inside?

Polarized deep inelastic scattering experiments have revealed a small value for the nucleon’s flavour-singlet axial-charge $g_{A(0)}^{(1)}_{\text{pDIS}} \sim 0.3$ suggesting that the quarks’ intrinsic spin contributes little of the proton’s spin. The challenge to understand the spin structure of the proton has inspired a vast programme of theoretical activity and new experiments. Why is the quark spin content $g_{A(0)}^{(0)}_{\text{pDIS}}$ so small?

We start by recalling the $g_1$ spin sum-rules, which are derived starting from the dispersion relation for polarized photon-nucleon scattering and, for deep inelastic scattering, the light-cone operator product expansion. One finds that the first moment of the $g_1$ spin structure function is related to the scale-invariant axial-charges of the target nucleon by

$$
\int_0^1 dx \, g_1^p(x, Q^2) = \left( \frac{1}{12} g_A^{(3)} + \frac{1}{36} g_A^{(8)} \right) \left\{ 1 + \sum_{\ell \geq 1} c_s \alpha_s^{(\ell)}(Q) \right\}
+ \frac{1}{9} g_{A(0)}^{(0)}_{\text{inv}} \left\{ 1 + \sum_{\ell \geq 1} c_s \alpha_s^{(\ell)}(Q) \right\} + O\left( \frac{1}{Q^2} \right) + \beta_\infty.
$$

(1)
Here \( g_A^{(3)} \), \( g_A^{(8)} \) and \( g_A^{(0)}_{\text{inv}} \) are the isovector, SU(3) octet and scale-invariant flavour-singlet axial-charges respectively. The flavour non-singlet \( c_{\text{NSf}} \) and singlet \( c_{\text{Sf}} \) Wilson coefficients are calculable in \( \ell \)-loop perturbative QCD. The term \( \beta_\infty \) represents a possible leading-twist subtraction constant from the circle at infinity when one closes the contour in the complex plane in the dispersion relation. If finite, the subtraction constant affects just the first moment. The first moment of \( g_1 \) plus the subtraction constant, if finite, is equal to the axial-charge contribution.

In terms of the flavour dependent axial-charges

\[
2 M s_\mu \Delta q = \langle p, s | \bar{q} \gamma_\mu \gamma_5 q | p, s \rangle
\]  

the isovector, octet and singlet axial-charges are:

\[
g_A^{(3)} = \Delta u - \Delta d
\]

\[
g_A^{(8)} = \Delta u + \Delta d - 2 \Delta s
\]

\[
g_A^{(0)}_{\text{inv}} / E(\alpha_s) = g_A^{(0)} = \Delta u + \Delta d + \Delta s.
\]  

Here \( E(\alpha_s) = \exp \int_0^{\alpha_s} d\alpha_s \gamma(\alpha_s)/\beta(\alpha_s) \) is a renormalization group factor which corrects for the (two loop) non-zero anomalous dimension \( \gamma(\alpha_s) \) of the single axial-vector current. \( J_{\alpha5} = \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s \), and which goes to one in the limit \( Q^2 \to \infty \); \( \beta(\alpha_s) \) is the QCD beta function. The singlet axial-charge, \( g_A^{(0)}_{\text{inv}} \), is independent of the renormalization scale \( \mu \) and corresponds to \( g_A^{(0)}(Q^2) \) evaluated in the limit \( Q^2 \to \infty \). The axial-charges \( g_A^{(3)} \) and \( g_A^{(8)} \) are renormalization group invariants.

If one assumes no twist-two subtraction constant (\( \beta_\infty = O(1/Q^2) \)) the axial-charge contributions saturate the first moment at leading twist. The isovector axial-charge is measured independently in neutron \( \beta \)-decays (\( g_A^{(3)} = 1.270 \pm 0.003 \)) and the octet axial-charge is commonly taken to be the value extracted from hyperon \( \beta \)-decays assuming a 2-parameter SU(3) fit (\( g_A^{(8)} = 0.58 \pm 0.03 \)). Using the sum-rule for the first moment of \( g_1 \), given in Eq. (1), polarized deep inelastic scattering experiments have been interpreted in terms of a small value for the flavour-singlet axial-charge. If we take \( g_A^{(8)} = 0.58 \pm 0.03 \), then inclusive \( g_1 \) data with \( Q^2 > 1 \text{ GeV}^2 \) gives

\[
g_A^{(0)}_{|_{\text{pDIS}, Q^2 \to \infty}} = 0.33 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.})
\]

– considerably smaller than the value of \( g_A^{(8)} \) quoted above.

In the naive parton model \( g_A^{(0)}_{|_{\text{pDIS}}} \) is interpreted as the fraction of the proton’s spin which is carried by the intrinsic spin of its quark and antiquark constituents. When combined with \( g_A^{(8)} = 0.58 \pm 0.03 \) the value of \( g_A^{(0)}_{|_{\text{pDIS}}} \) in Eq.(4) corresponds to a negative strange-quark polarization

\[
\Delta s_{Q^2 \to \infty} = \frac{1}{3} (g_A^{(0)}_{|_{\text{pDIS}, Q^2 \to \infty}} - g_A^{(8)}) = -0.08 \pm 0.01(\text{stat.}) \pm 0.02(\text{syst.})
\]

– that is, polarized in the opposite direction to the spin of the proton.

What physics separates the values of the octet and singlet axial-charges?
2. Spin and the singlet axial-charge $g_A^{(0)}$

There are two key issues: the physics interpretation of the flavour-singlet axial-charge $g_A^{(0)}$ and possible SU(3) breaking in the extraction of $g_A^{(8)}$ from hyperon $\beta$-decays.

First consider $g_A^{(0)}$. Gluonic information feeds into $g_A^{(0)}$ through the QCD axial anomaly. QCD theoretical analysis leads to the formula\cite{1,8–11}

$$g_A^{(0)} = \left( \sum_q \Delta q - \frac{3}{2\pi} \frac{\Delta g}{\alpha_s} \right)_{\text{partons}} + C_{\infty}. \quad (6)$$

Here $\Delta q_{\text{partons}}$ is the amount of spin carried by polarized gluons in the polarized proton ($\alpha_s \Delta q \sim \text{constant as } Q^2 \to \infty$\cite{8,19}) and $\Delta q_{\text{partons}}$ measures the spin carried by quarks and antiquarks carrying “soft” transverse momentum $k^2_t \sim P^2, m^2$ where $P$ is a typical gluon virtuality and $m$ is the light quark mass. The polarized gluon term is associated with events in polarized deep inelastic scattering where the hard photon strikes a quark or antiquark generated from photon-gluon fusion and carrying $k^2_t \sim Q^2$\cite{10,11}. $C_{\infty}$ denotes a potential non-perturbative gluon topological contribution which is associated with the possible subtraction constant in the dispersion relation for $g_1$ and Bjorken $x = 0$\cite{11} $g_A^{(0)}|_{\text{pDIS}} = g_A^{(0)} - C_{\infty}$.

The subtraction constant, if finite, is a non-perturbative effect and vanishes in perturbative QCD. It is sensitive to the mechanism of axial U(1) symmetry breaking and the realization of axial U(1) symmetry breaking by instantons: spontaneous U(1) symmetry breaking by instantons naturally generates a subtraction constant whereas explicit symmetry breaking does not\cite{12}. The QCD vacuum is a Bloch superposition of states characterised by non-vanishing topological winding number and non-trivial chiral properties. When we put a valence quark into this vacuum it can act as a source which polarizes the QCD vacuum with net result that the spin “dissolves” and some fraction of the spin of the constituent quark is associated with non-local gluon topology with support only at Bjorken $x = 0$.

Possible explanations for the small value of $g_A^{(0)}|_{\text{pDIS}}$ extracted from the polarized deep inelastic experiments include screening from positive gluon polarization, negative strangeness polarization in the nucleon, a subtraction at infinity in the dispersion relation for $g_1$ associated with non-perturbative gluon topology and connections to axial U(1) dynamics\cite{13,14} as well as possible SU(3) breaking in $g_A^{(8)}$ – possibly as large as 20%\cite{14,15}. The QCD axial anomaly decouples from the non-singlets $g_A^{(3)}$ and $g_A^{(8)}$.

One would like to understand the dynamics which appears to suppress the singlet axial-charge extracted from polarized deep inelastic scattering relative to the OZI prediction $g_A^{(0)} = g_A^{(8)}$ and also the sum-rule for the longitudinal spin structure of the nucleon

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \Delta g + L_q + L_g \quad (7)$$

where $L_q$ and $L_g$ denote the orbital angular momentum contributions. There is
presently a vigorous programme to disentangle the different contributions. Key experiments include semi-inclusive polarized deep inelastic scattering (COMPASS and HERMES) and polarized proton-proton collisions (PHENIX and STAR at RHIC), as well as deeply virtual Compton scattering to learn about total angular momentum.

3. The shape of $g_1$

To understand the proton spin puzzle, it is interesting to look at the $x$ dependence of the measured $g_1$ spin structure function. Deep inelastic measurements of $g_1$ have been performed in experiments at CERN, DESY, JLab and SLAC. There is a general consistency among all data sets. COMPASS are yielding precise new data at small $x$, down to $x \sim 0.004$. JLab are focussed on the large $x$ region.

Precise measurements of the deuteron spin structure function $g_1^d$ show the remarkable feature that $g_1^d$ is consistent with zero in the small $x$ region between 0.004 and 0.02. In contrast, the isovector part of $g_1$ is observed to rise at small $x$ as $\sim x^{-0.22\pm0.07}$ and is much bigger than the isoscalar part of $g_1$. This is in sharp contrast to the situation in the unpolarized structure function $F_2$ where the small $x$ region is dominated by isoscalar pomeron exchange. The $g_1^{p-n}$ data are consistent with quark model and perturbative QCD predictions in the valence region $x > 0.2$.

The size of $g_A^{(3)}$ forces us to accept a large contribution from small $x$ and the observed rise in $g_1^{p-n}$ is in excellent agreement with the prediction $g_1^{p-n} \sim x^{-0.22}$ of hard Regge exchange - in particular a possible $a_1$ hard-pomeron cut involving the hard-pomeron which seems to play an important role in unpolarized deep inelastic scattering.

The “missing spin” is associated with a “collapse” in the isosinglet part of $g_1$ to something close to zero instead of a valence-like rise for $x$ less than about 0.02. This isosinglet part is the sum of SU(3)-flavour singlet and octet contributions. If there were a large positive polarized gluon contribution to the proton’s spin, this would act to drive the small $x$ part of the singlet part of $g_1$ negative - that is, acting in the opposite direction to any valence-like rise at small $x$. However, gluon polarization measurements at COMPASS, HERMES and RHIC constrain this spin contribution to be small in measured kinematics meaning that the sum of valence and sea quark contributions is suppressed at small $x$. (Soft Regge theory predicts that the singlet term should behave as $\sim N \ln x$ in the small $x$ limit, with the coefficient $N$ to be determined from experiment.)

There is presently a vigorous programme to disentangle the different contributions involving experiments in semi-inclusive polarized deep inelastic scattering and polarized proton-proton collisions. These direct measurements show no evidence for negative polarized strangeness in the region $x > 0.006$ (in apparent contrast to the extraction of negative strangeness polarization extracted from inclusive measurements of $g_1$). For gluon polarization, present measurements suggest $| -3\frac{\alpha_s}{\pi} \Delta g | < 0.06$ corresponding to $|\Delta g | < 0.4$ with $\alpha_s \sim 0.3$. That is, they
are not able to account for the difference \((g_A^{(0)}|_{\text{pDIS}} - g_A^{(8)}) \sim -0.25\) obtained via Eq.(4). An independent measurement of the strange-quark axial-charge could be made through neutrino-proton elastic scattering.\(^{10}\) The axial-charge measured in \(\nu p\) elastic scattering is independent of any assumptions about the presence or absence of a subtraction at infinity in the dispersion relation for \(g_1\) and the \(x \sim 0\) behaviour of \(g_1\). Further measurements to push the small \(x\) frontier in polarized deep inelastic scattering would be possible with a polarized \(ep\) collider.\(^{33}\)

4. SU(3) breaking and \(g_A^{(8)}\)

Given that the contributions to \(g_A^{(0)}\) from the measured distribution \(\Delta s\) and from \(-3\alpha_s^2 \pi \Delta g\) are small, it is worthwhile to ask about the value of \(g_A^{(8)}\). The value \(0.58\) is extracted from a 2 parameter fit to hyperon \(\beta\)-decays in terms of the SU(3) constants \(F = 0.46\) and \(D = 0.80\)\(^{10}\) - see Table 1. The fit is good to \(\sim 20\%\) accuracy.\(^{18,33}\) The uncertainty quoted for \(g_A^{(8)}\) has been a matter of some debate. There is considerable evidence that SU(3) symmetry may be badly broken and some have suggested that the error on \(g_A^{(8)}\) should be as large as \(25\%\).\(^{18}\) More sophisticated fits will also include chiral corrections. Calculations of non-singlet axial-charges in relativistic constituent quark models are sensitive to the confinement potential, effective colour-hyperfine interaction,\(^{34-37}\) pion and kaon clouds plus additional wavefunction corrections\(^{38}\) chosen to reproduce the physical value of \(g_A^{(3)}\).

This physics has recently been investigated by Bass and Thomas\(^{17}\) within the Cloudy Bag model (CBM)\(^{38,39}\) which has the attractive feature that when pion cloud and quark mass effects are turned off the model reproduces the SU(3) analysis. One finds that chiral corrections significantly reduce the value of \(g_A^{(8)}\). This, in turn, has the effect of increasing the value of \(g_A^{(0)}|_{\text{pDIS}}\) and consequently reducing the absolute value of the “polarized strangeness” extracted from inclusive polarized deep inelastic scattering.

The Cloudy Bag\(^{10}\) was designed to model confinement and spontaneous chiral symmetry breaking, taking into account pion physics and the manifest breakdown of chiral symmetry at the bag surface in the MIT bag. If we wish to describe proton spin data including matrix elements of \(J_{\mu 5}^3\), \(J_{\mu 5}^8\) and \(J_{\mu 5}\), then we would like to know that the model versions of these currents satisfy the relevant Ward identities. For the scale-invariant non-singlet axial-charges \(g_A^{(3)}\) and \(g_A^{(8)}\), corresponding to the

| Process | measurement | SU(3) combination | Fit value | MIT + OGE |
|---------|-------------|-------------------|-----------|-----------|
| \(n \to p\) | \(1.270 \pm 0.003\) | \(F + D\) | \(1.26\) | \(-\frac{1}{3}B' + G\) |
| \(\Lambda^0 \to p\) | \(0.718 \pm 0.015\) | \(F + \frac{1}{2}D\) | \(0.73\) | \(B'\) |
| \(\Sigma^+ \to n\) | \(-0.340 \pm 0.017\) | \(F - D\) | \(-0.34\) | \(-\frac{1}{3}B' - 2G\) |
| \(\Xi^- \to \Lambda^0\) | \(0.25 \pm 0.05\) | \(F - \frac{1}{2}D\) | \(0.19\) | \(\frac{1}{3}B' - G\) |
| \(\Xi^0 \to \Sigma^+\) | \(1.21 \pm 0.05\) | \(F + D\) | \(1.26\) | \(\frac{1}{3}B' + G\) |
matrix elements of partially conserved currents, the model is well designed to make a solid prediction.

The effective colour-hyperfine interaction has the quantum numbers of one-gluon exchange (OGE). In models of hadron spectroscopy this interaction plays an important role in the nucleon-Δ and Σ − Λ mass differences, as well as the nucleon magnetic moments and the spin and flavor dependence of parton distribution functions. It shifts total angular-momentum between spin and orbital contributions and, therefore, also contributes to model calculations of the octet axial-charges. In Bag model calculations one also needs to include wavefunction corrections associated with the well known issue that, for the MIT and Cloudy Bag models, the nucleon wavefunction is not translationally invariant and the centre of mass is not fixed. To compare the model results with experiment we take the view that, in principle, the model - with corrections - should give the experimental value of $g_A^{(3)}$. We therefore choose the centre-of-mass factor phenomenologically to give the experimental value of $g_A^{(3)}$. This then fixes the parameters of the model and allows us to use it to make a model prediction for $g_A^{(8)}$.

Without pion cloud corrections the MIT Bag with centre of mass corrections reproduces the SU(3) analysis of the axial-charges extracted from β-decays. This is illustrated in Table 1. Without additional physics input, e.g. pion chiral corrections, there is a simple algebraic relation between the SU(3) parameters $F$ and $D$, the bag parameter $B'$ and the OGE correction $G$: $F = \frac{2}{3}B' - \frac{1}{2}G$ and $D = B' + \frac{3}{2}G$. The numerical agreement is very good.

The pion cloud of the nucleon also renormalizes the nucleon’s axial-charges by shifting intrinsic spin into orbital angular momentum. In the Cloudy Bag Model (CBM) the nucleon wavefunction is written as a Fock expansion in terms of a bare MIT nucleon, $|N\rangle$, and baryon-pion, $|N\pi\rangle$ and $|\Delta\pi\rangle$, Fock states. The probabilities to find the nucleon in each Fock component are determined phenomenologically by fitting to a wealth of nucleon observables. The expansion converges rapidly and we may safely truncate the Fock expansion at the one pion level. When we calculate the pion and kaon cloud chiral corrections to $g_A^{(8)}$ we also have to choose the chiral representation, in particular whether to use the original surface coupling or the later volume coupling version of the Cloudy Bag model.

The extent of the reduction in $g_A^{(8)}$ depends upon the version of the CBM used, lying in the range 0.49 ± 0.02 for the original CBM and 0.42 ± 0.02 for the volume coupling version. These changes alone raise the value of $g_A^{(0)}|_{\text{pDIS},Q^2\to\infty}$ derived from the experimental data from $0.33\pm0.03(\text{stat.})\pm0.05(\text{syst.})$ to $0.35\pm0.03(\text{stat.})\pm0.05(\text{syst.})$ and $0.37\pm0.03(\text{stat.})\pm0.05(\text{syst.})$, respectively. Both of these values have the effect of reducing the level of OZI violation associated with the difference $g_A^{(0)}|_{\text{pDIS}} - g_A^{(8)}$ from $-0.25\pm0.07$ to just $-0.14\pm0.06$ and $-0.05\pm0.06$, respectively. It is this OZI violation which eventually needs to be explained in terms of singlet degrees of freedom: effects associated with polarized glue and/or a topological effect associated with $x = 0$.

The uncertainty in this model calculation lies in the small ambiguity between the
two chiral representations that one can choose. In order to quote an overall value that properly encompasses these possibilities we follow the Particle Data Group procedure for combining data that may not be compatible to estimate the overall error, finding a combined value of \( g_A^{(8)} = 0.46 \pm 0.05 \) (with the corresponding semiclassical singlet axial-charge or spin fraction being \( 0.42 \pm 0.07 \) before inclusion of gluonic effects). Note that the error \( \pm 0.02 \) on \( g_A^{(8)} \) quoted for each version of the model follows from varying over the phenomenological range of possible pion parameters within first order perturbation theory. In terms of analogy to experimental errors, the \( \pm 0.02 \) is like a statistical error and the final \( \pm 0.05 \) error includes systematic effects. With this final value for \( g_A^{(8)} \) the corresponding experimental value of \( g_A^{(0)}|_{pDIS} \) would increase to \( g_A^{(0)}|_{pDIS} = 0.36 \pm 0.03 \pm 0.05 \).

5. Towards possible understanding

Where are we in our understanding of the spin structure of the proton and the small value of \( g_A^{(0)}|_{pDIS} \)? Measurements of valence, gluon and sea polarization suggest that the polarized glue term \( -\frac{3\pi}{2} \Delta g_{\text{partons}} \) and strange quark contribution \( \Delta s_{\text{partons}} \) in Eq.(6) are unable to resolve the small value of \( g_A^{(0)}|_{pDIS} \). Two explanations are suggested within the theoretical and experimental uncertainties depending upon the magnitude of SU(3) breaking in the nucleon and hyperon axial-charges. One is a value of \( g_A^{(8)} \sim 0.5 \) (as suggested by the surface coupling model) plus an axial U(1) topological effect at \( x = 0 \) associated with a finite subtraction constant in the \( g_1 \) dispersion relation. The second is a much larger pion cloud reduction of \( g_A^{(8)} \) to a value \( \sim 0.4 \) (as suggested by the volume coupling model in first order pion cloud perturbation theory). Combining the theoretical error on the pion cloud chiral corrections embraces both possibilities. The proton spin puzzle seems to be telling us about the interplay of valence quarks with chiral dynamics and the complex vacuum structure of QCD.

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