Scalable Distributed Non-Convex ADMM-based Active Distribution System Service Restoration

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Abstract—Distributed restoration can harness distributed energy resources (DER) to enhance the resilience of active distribution networks. However, the large number of decision variables, especially the binary decision variables of reconfiguration, bring challenges on developing effective distributed distribution service restoration (DDSR) strategies. This paper proposes a scalable distributed optimization method based on the alternating direction method of multipliers (ADMM) for non-convex mixed-integer optimization problems, and applies to develop the DDSR framework. The non-convex ADMM method consists of relax-drive-polish phases, 1) relaxing binary variables and applying the convex ADMM as a warm start; 2) driving the solutions toward Boolean values through a proximal operator; 3) fixing the obtained binary variables to polish continuous variables for a high-quality solution. Then, an autonomous clustering strategy together with consensus ADMM is developed to realize the distributed cluster-based framework of restoration. The non-convex ADMM-based DDSR can determine DER scheduling and switch status for reconfiguration and load pickup in a distributed manner, energizing the out-of-service area from local faults or total blackouts in large-scale distribution networks. The effectiveness and scalability of the proposed DDSR framework are demonstrated through testing on the IEEE 123-node and IEEE 8500-node test feeders.

Index Terms—Alternating direction method of multipliers (ADMM), autonomous clustering, distributed service restoration, distributed energy resources, non-convex, reconfiguration

I. INTRODUCTION

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mart grid technologies have been applied to enhance the resilience of distribution networks (DN); however, faults and outages are still inevitable due to challenges from natural disasters or man-made attacks [1]. It is critical to effectively respond to extreme events and optimally restore the electric service [2]. For example, after an outage, microgrids can be utilized to supply critical loads [3] or maximize restored loads [4]; distributed energy resources (DER) can be harnessed to energize islanded microgrids [5]; and advanced communication and control devices, such as remote-controlled switches, provide great potentials for advanced restoration strategies [6].

Distribution service restoration (DSR) aims to restore maximum out-of-service loads through finding available paths by switching operation and picking up loads after a blackout [7]. Large-scale DNs and the increasing penetration of DERs make DSR one of the most complicated and challenging problems of active distribution networks (ADN) [8]. The prevailing centralized restoration scheme requires a powerful centralized controller to communicate with components, collect data, carry out large-scale sophisticated computations, and send out commands [9]. However, centralized infrastructures are costly, might suffer from single-point failures, and are limited by information privacy of entities [7].

Distributed optimization and control is a promising solution to address the aforementioned issues, but most of the methods require convexity to guarantee the convergence, which is difficult to meet due to the mixed-integer characteristic of many power system problems including DSR [7]. For example, the multi-agent based distributed service restoration scheme has been developed to address the restoration problem [9–11]. However, these methods require agents to access all information against data privacy, and usually neglect the scalable problem formulation for large-scale DNs. Authors in [12] proposed a distributed restoration framework based on the alternating direction method of multipliers (ADMM). However, simply projected ADMM to deal with binary variables usually obtains infeasible solutions [13], and the proposed clustering structure can not include switches within clusters.

The ADMM method was originally developed for convex optimization problems, but it turns out to be a powerful heuristic method even for non-convex (NC) problems [14]. Recently, it has gained a lot of attention to find the approximate solution for NP-hard problems [15]. Although there is no guarantee for ADMM-based methods converge to a global solution for NC problems, it is an attempt to find a high-quality local solution, considering the difference between global and local optimal solutions in practice may not be significant [15]. Accordingly, [13] and [15] proposed a heuristic projection method based on ADMM for general NC problems, which usually yields infeasible or low-quality local solutions. Therefore, in this paper, a distributed optimization method based on ADMM is proposed to achieve a high-quality solution for mixed-integer optimization problems. Different from [16] that locks binary variables during convergence, the proposed method tries to drive them simultaneously toward Boolean values. The developed NC-ADMM method, together with an autonomous clustering scheme, is applied to the DSR problem.

The main contributions of this paper are threefold: (1) Developed a cluster-based distributed distribution service restoration (DDSR) framework including reconfiguration and load pickup for unbalanced large-scale distribution networks, based on the proposed heuristic NC-ADMM algorithm. The DDSR framework establishes clusters and solves the DSR problem through solving small subproblems for each smart local agent (SLA) and exchanging limited information between adjacent clusters. (2) Developed a heuristic distributed approach based on ADMM through relax-drive-polish phases for NC mixed-
integer problems, and applied to the DDSR problem for a high-quality suboptimal solution. (3) Developed an autonomous two-stage clustering strategy to enhance the scalability of NC-ADMM-based DDSR for large-scale distribution networks.

The remainder of the paper is as follows. Sections II and III introduce the distributed scheme for DDSR problem and DSR problem formulation, respectively. Section IV presents the proposed autonomous clustering strategy for large-scale distribution networks. Section V introduces the proposed NC-ADMM method with the application on the DDSR problem. Finally, section VI demonstrates numerical results and analysis, and section VII concludes the paper.

II. DDSR Framework

DDSR can be categorized into emergency service restoration and blackout service restoration [7]. The emergency service restoration aims to isolate faulted areas by opening sectionalizing switches and to energize unaffected out-of-service areas by closing tie-switches. For blackout service restoration, the entire DN is out of service, and the restoration can be accomplished through multiple time steps, depending on the bulk power system restoration procedure. In this paper, the proposed DDSR can handle both top-down and bottom-up restoration strategies, aiming to promptly restore as much load as possible in areas where electric service is disrupted.

The cluster-based DDSR framework is shown in Fig. 1. The distribution network is divided into multiple agents, and these agents can be realized by any node, DER, or smart switch. Rather than controlling each node independently as in our previous work [7], which requires many intelligent entities, a cluster of nodes is controlled by assigning an SLA. Each SLA monitors, controls, and dispatches all the loads, DERs, switches, and capacitor banks (CB) within its cluster, and also exchanges data with neighboring clusters through a two-way communication network. Neighboring clusters are those connected through power delivery elements such as distribution lines. The cluster-based structure improves the scalability and convergence speed of the proposed distributed algorithm and also enables the practical implementation of distributed communication and control. It is worth mentioning that each SLA is responsible for its own territory and the boundaries of clusters are artificial. Moreover, the boundary elements could be controlled by all related SLAs when the proposed algorithm reaches consensus on the operation.

The proposed heuristic consensus NC-ADMM method for DDSR can be realized through solving a small subproblem by each SLA and exchanging limited information among neighboring clusters as shown in Fig. 1. Each subproblem includes various constraints, such as power flow and security constraints, to guarantee the solution feasibility for the restoration procedure. When faults occur, the protection system detects and isolates faults through sectionalizing switches. Then, depending on the type of event, the restoration procedure starts and might last from one to several time steps, each of which has enough duration for operation and stabilization of all involved components. In the case of a bulk system outage, the steps are imposed by the generation capability curve in transmission system, i.e., available power at each distribution substation. Furthermore, the restoration might include the energization of unaffected out-of-service areas through closing tie-switches. Due to the protection system limitation, the tree topology of the entire DN must be preserved during restoration, which is a challenging constraint for DDSR [17].

Accordingly, the optimal restoration plan of DDSR can be achieved in the following iterative steps:

1) **DSR subproblems step:** The local DSR subproblems are solved by each SLA for the entire restoration span. A decomposable Lagrange function is developed for the whole restoration problem and then decomposed into subproblems for each agent. These subproblems can be solved independently by each SLA while consensus variables are being fixed in this step. Then, each agent determines its load pickup, switching, generation dispatch, CB operation, voltage, and power flow for all loads and power delivery elements including inner and boundary ones. It is worth mentioning that in this step all binary variables are relaxed and confined into a convex hull; therefore, all subproblems are convex optimization problems. Also, the Boolean value for these variables is achieved through the relax-drive-polish phases of the NC-ADMM method.

2) **Consensus step:** All neighboring agents exchange information related to the voltage, power flow, lines energization, and radiality constraints, and update the consensus variables.

3) **Lagrange multipliers update step:** The Lagrange multipliers related to the boundary and binary variables are updated by each SLA based on results from the previous two steps and exchanged data. Updating these multipliers enforces general constraints for the entire distribution network, including supplying critical loads in other clusters, radial topology, and total power balance.

The iterative procedure of DDSR continues until satisfying convergence criteria or time limits. Then, the algorithm provides a sequence of control actions for each cluster to restore its loads based on their priority, and operate switches and various components in the network within each time step of the total restoration span.
III. DSR Problem Formulation

A. Model Representation

This section provides DSR formulation for an unbalanced DN consisting of \( N \) nodes denoted by \( N := \{1, \ldots, N\} \). Also, \( L, E, \) and \( G \) show the set of loads, lines, and DERs, respectively. Superscripts \( S \) and \( F \) represent subsets of switchable lines and faulted lines, as \( E^S \subseteq E \); and superscripts \( d \) and \( nd \) stand for subsets of dispatchable and non-dispatchable loads as \( L^d \cup L^{nd} = L \). Furthermore, \( T \) denotes the set of the restoration time steps as \( t \in T \), and \( \phi \) stands for three phases of \( \{a, b, c\} \). The inner product of two vectors \( \vec{x}, \vec{y} \) is denoted by \( \langle \vec{x}, \vec{y} \rangle \), and the element-wise product and division are denoted by \( \odot \) and \( \oslash \), respectively.

Three-phase restored load \( i \) at time \( t \) is represented by \( \vec{P}_{i,t} \) and \( \vec{Q}_{i,t} \) in \( \mathbb{R}^{3 \times 1} \). Let \( \vec{P}_{i,t} + j \vec{Q}_{i,t} \) denote three-phase power generation of DERs, and \( \vec{P}_{i,t} + j \vec{Q}_{i,t} \) in \( \mathbb{R}^{3 \times 1} \) and \( r_{ij} + jx_{ij} = z_{ij} \in \mathbb{C}^{3 \times 3} \) denote the power flow and impedance of distribution line \( ij \) between nodes \( i \) and \( j \). In order to capture the energization status of load \( i \) and bus \( i \), and the connection status of line \( ij \) at time \( t \), a set of binary variables are defined as \( x_{i,t}^L, x_{i,t}^B, \) and \( \alpha_{ij,t} \). Nevertheless, to simplify the DSR problem formulation and solution algorithm, binary variables \( \beta_{ij,t} \) are only assigned for switchable lines.

B. Objective Function

The DSR problem is modeled as a mixed-integer convex optimization problem. A multi-objective function is developed to maximize total restored loads at all time steps considering load priorities, as well as the optimal operation of switches. Binary variables of switch status are prioritized so that the normally-closed switches have a higher priority compared to the normally-open ones. The motivation is to keep network topology close to the one under the normal operation. These two terms are regularized through \( c_1 \) and \( c_2 \) as below:

\[
\max \sum_{t \in T} \left( c_1 \sum_{i \in L} \left( w_{i,t}^L + P_{i,t}^L \right) + c_2 \sum_{(i,j) \in L} w_{ij} \alpha_{ij,t} \right) \tag{1}
\]

where \( w_{i,t}^L \) and \( w_{ij} \) denote priority of load \( i \) and switchable line \( ij \).

C. Constraints

1) Security constraints: Security constraints guarantees the operational limits of various elements in DN. Constraints (2) and (3) represent the load pickup capacity of non-dispatchable and dispatchable loads. Nodal voltage and branch power flow should be within the limit for energized nodes and lines, as in (4) and (5) where \( U_{i,t} \) is the square of voltage magnitude at node \( i \). Constraint (6) demonstrates the restorative capacity of transmission network \( \vec{P}_{i,t}^B + j \vec{Q}_{i,t}^B \), as \( P_{ij,t}^B + jQ_{ij,t}^B \) is the total three-phase power flow of the connected line to the substation. The output of energized CB is limited by its maximum capacity in \( \mathbb{C} \). A convex quadratic constraint is used to limit the output power of inverter-based photovoltaic (PV) generators in (8) which are the only type of DERs modeled in this paper. Finally, Voltage regulators (VR) are assumed to be wye-connected type B, which the voltage between primary and secondary sides are represented by (9).

This equation can be linearized based on \( \mathbb{C} \); however, for simplicity, VRs are assumed to be constant during restoration.

\[
\beta_{ij,t} \geq \alpha_{ij,t}, \forall i,j \in L \tag{2}
\]

\[
0 \leq \beta_{ij,t} \leq \beta_{ij,t}^{max}, \forall i,j \in L \tag{3}
\]

\[
\vec{P}_{i,t}^B + \beta_{ij,t} \vec{P}_{ij,t}^B, \quad \vec{Q}_{i,t}^B + \beta_{ij,t} \vec{Q}_{ij,t}^B, \quad \forall i \in L \tag{4}
\]

\[
0 \leq \vec{P}_{ij,t}^\phi \leq \vec{P}_{ij,t}^{sub}, \quad 0 \leq \vec{Q}_{ij,t}^\phi \leq \vec{Q}_{ij,t}^{sub} \tag{5}
\]

\[
0 \leq \beta_{ij,t}^{cap} \leq \beta_{ij,t}^{cap, max} \tag{6}
\]

\[
\vec{P}_{ij,t}^G \odot (\vec{Q}_{ij,t}^G) \leq \vec{P}_{ij,t}^{sub}, \quad \vec{Q}_{ij,t}^G \odot (\vec{Q}_{ij,t}^{sub}) \leq \vec{Q}_{ij,t}^{cap} \tag{7}
\]

\[
\vec{U}_{i,t} \leq \vec{U}_{i,t}^{max} \quad \forall i \in L \tag{8}
\]

2) Power flow constraints: Power flow equations are based on the branch flow model for three phase unbalanced DN in \( \mathbb{C} \). For simplicity, these equations are linearized by removing the quadratic loss term \( \mathbb{C} \). The power flow equations include the linearized voltage drop for node \( i \) in (10a), and active and reactive power balance in (10b) and (10c), where \( \vec{r}_{ij} \) and \( \vec{x}_{ij} \) are unbalanced line impedance matrices, as referred in \( \mathbb{C} \), and \( j \) and \( \delta(i) \) are parent node and set of children nodes of node \( i \). For switchable lines, the big M method can be applied on equation (10a) to enable it based on the connectivity.

\[
\vec{U}_i = \vec{U}_j - 2 \left( \sum_{m \in \delta(i)} \vec{P}_{im,t} - \vec{Q}_{im,t} \right) \tag{10a}
\]

\[
\vec{P}_{ij,t} = \vec{P}_{ij,t}^L + \sum_{m \in \delta(i)} \vec{P}_{im,t} - \vec{Q}_{im,t}^L \tag{10b}
\]

\[
\vec{Q}_{ij,t} = \vec{Q}_{ij,t}^L + \sum_{m \in \delta(i)} \vec{Q}_{im,t} - \vec{Q}_{ij,t}^L \tag{10c}
\]

3) Topological, connectivity and sequencing constraints: These constraints guarantees radial operation, isolation of fault, and sequential restoration of DN. Load shedding is prohibited by (11) for energized loads during restoration. If a fault happens, the protection system can locate and isolate the faulted area through sectionalizing switches. Then, by forcing the related binary variables of faulted lines to be zero as (12), the faulted area remains isolated during the DSR procedure. The radial configuration of DN is guaranteed through spanning tree constraints (13) \( \mathbb{C} \). Two auxiliary binary variables \( \beta_{ij,t} \) and \( \beta_{ij,t}^* \) are associated with each line \( ij \), denoting the direction of flow if any. Then, equations (13a) and (13b) indicate that for each line \( ij \), either node \( j \) is the parent of node \( i \) (\( \beta_{ij,t} = 1 \)), or node \( i \) is the parent of node \( j \) (\( \beta_{ij,t}^* = 1 \)). Also, equation (13c) requires every node other than substation has exactly one parent node, while substation does not have any parent.
IV. AUTONOMOUS CLUSTERING ALGORITHM

A two-level autonomous clustering strategy is developed to maximize the convergence speed and minimize the information exchange. The first level is to identify the optimal number and size of clusters, and the second level is to detect and generate clusters within a DN. This self-organizing strategy is applicable for various communication typologies, different types of information sharing, and a large number of nodes.

Proposition 1. The optimal number of clusters in a DN to maximize the convergence speed and minimize the information exchange is achieved by square root of nodes.

Proof. Considering a DN with $N$ nodes divided into $k$ clusters each with $m$ nodes, an optimization problem is formulated with the objective function of minimizing $\lambda_1 k + \lambda_2 m$ subject to $km = N$, where $\lambda_1$ and $\lambda_2$ are weighting factors for a trade-off between communication requirement and computational burden of each cluster for the convergence speed. Substituting the constraint as $m = N/k$ and assuming $\lambda_1 = \lambda_2 = 1$, yields the problem as minimizing $k + N/k$. Then, by taking the derivative, the objective value obtains as $k = \sqrt{N}$ and subsequently $m = \sqrt{N}$.

In second level, a bottom-up travers method is developed for DNs with tree structure. This method starts from the leaves of the tree and ends by the root node, as provided in Algorithm 1. First, the algorithm assigns a weight to each node demonstrating its subtree size. Next, the algorithm finds the nodes with the weight close to the ideal number and detaches them as a cluster. This procedure is repeated until all nodes are removed from the tree structure and been divided into clusters.

Algorithm 1 Bottom-up travers clustering algorithm

Input: $T$: The network tree, $N$: number of nodes, $k$: desired number of clusters, $r$: relaxation factor;
Output: $T'$: Clustered network tree.

1: Calculate the number of nodes in each cluster ($m$)
2: Traverse the tree in the bottom-up postorder manner
3: for each node $v$ Assign its subtree size
4: Find all nodes $v'$ with subtree size $= m \pm r$
5: Cluster each node $v'$ with its subtree
6: Remove all clustered nodes
7: Repeat Algorithm 1 until all nodes been clustered

Remark 1. Finding $\sqrt{N}$ clusters with $\sqrt{N}$ nodes is a hard problem and might be impossible due to the topology of the network. Accordingly, the proposed algorithm provides a high quality solution even for large-scale DNs, in a swift manner.

V. DISTRIBUTED SOLUTION METHODOLOGY

A. Proposed Distributed Algorithm of Non-convex ADMM

This paper developed a heuristic approach, including the relax-drive-polish procedure, to enhance the performance of heuristic ADMM for non-convex problems. It solves the relaxed convex problems in each iteration of the ADMM, and assigns auxiliary variables to force relaxed binary variables toward Boolean values during convergence. It also combines with the consensus ADMM [13], which provides a parallel computational framework for all agents to reach consensus on binding variables through limited information exchange.

Definition 1. Binding variables are those involved in binding constraints among agents (SLA) in a decomposed problem.

The optimization problems of DSR with decomposable objective function for $K$ clusters can be generalized as following:

$$
\min_{\tilde{x}, \tilde{z}} \sum_{i \in K} f_i(\tilde{x}, \tilde{z}_i) \quad \text{s.t.} \quad \tilde{x}, \tilde{z} \in C; \tilde{z} \in S
$$

where $\tilde{x} \in \mathbb{R}^n$ and $\tilde{z} \in \mathbb{R}^q$ are decision variables with a convex objective function, and inequality and equality constraints define a convex set $C$ for $\tilde{x}$ and $\tilde{z}$. Respectively, decision variables can be classified for each cluster $i$ as $[\tilde{x}_i, \tilde{z}_i]$. Based on Definition 1, these variables can be categorized into binding and interior variables as $[\tilde{x}^B_i, \tilde{x}^N_i, \tilde{z}^B_i, \tilde{z}^N_i]$. Furthermore, $S$ represents the non-convex set, which for DSR problem is Boolean set as $S = \{0, 1\}^q$. The relaxation of Boolean constraints is achieved through introducing auxiliary variables of $\tilde{y}$ within a convex hull and making them equal to $\tilde{z}$ as a consensus constraint $\tilde{y} = \tilde{z}$, $0 \leq \tilde{y} \leq 1$.

Through introducing augmented Lagrangian over consensus constraint in (14), the ADMM solution procedure can be formed as (15) where $\tilde{u}$ is the Lagrange multiplier for ADMM in scaled form. However, the convergence procedure is unsmooth and the results are usually infeasible. Therefore, a non-convex ADMM method is developed next.

$$(\tilde{x}, \tilde{y})(k+1) := \min_{\tilde{x}, \tilde{y} \in C, 0 \leq 1} \left( \sum_{i \in K} f_i(\tilde{x}, \tilde{y}_i) + \frac{\mu}{2} \|\tilde{y} - \tilde{z}(k) + \tilde{u}(k)\|^2_2 \right)$$

$$(\tilde{z}(k+1) := \Pi_S(\tilde{y}(k+1) + \tilde{u}(k))$$

$$(\tilde{u}(k+1) := \tilde{u}(k) + \tilde{y}(k+1) - \tilde{z}(k+1)$$

Remark 2. The $\Pi_S$ stands for the projection set $S = \{0, 1\}^q$ which is given by rounding the entries to 0 or 1.

1) Step 1-Relax: Inspired by the Douglas-Rachford splitting method [19], instead of projection in (15), the proximal operator, as defined in (16), is applied to drive $\tilde{z}$ toward Boolean values as (17), where $t$ is the regularization factor. In order to drive $\tilde{z}$ toward the Boolean values, it is proposed to establish $I(\tilde{z}) := \frac{1}{2} \| \tilde{z} - \Pi_S(\tilde{u}) \|^2_2$.

$$\text{prox}(\tilde{w}) := \argmin_{\tilde{z}} \frac{1}{2t} \| \tilde{z} - \tilde{w} \|^2_2 + I(\tilde{z})$$

$$\tilde{z}^{(k+1)} := \text{prox}_{t,I}(\tilde{y}^{(k+1)} + \tilde{u}^{(k)}) := \text{prox}(\tilde{w})$$

Then, the optimization problem of (16) is a compromise to choose $\tilde{z}$ between the Boolean value and the consensus value of $\tilde{w}$. The closed-form solution of (16) is obtained as (18).

$$\tilde{z}^{(k+1)} = (1 + \tilde{p}^{(k)})^{-1} \left( \tilde{w} + \tilde{p}^{(k)} \Pi_S(\tilde{w}) \right)$$
2) Step 2-Drive: The relaxation converts (14) into a convex problem by setting \( \tilde{t} = 0 \), and the solutions are used as a warm start for the drive phase. It is proposed to adjust \( \tilde{t} \) as (19), based on the primal and dual residuals defined in (20) [13].

\[
\tilde{t}^{(k)} := \tilde{t}^{(k-1)} + c \left( 1/r_p^{(k-1)} + 1/r_d^{(k-1)} \right) \quad (19)
\]

\[
r_p^{(k)} = \|\tilde{y}^{(k)} - \hat{z}^{(k)}\|_2^2, \quad r_d^{(k)} = \rho \|\tilde{z}^{(k)} - \hat{x}^{(k-1)}\|_2^2 \quad (20)
\]

This ensures that the rate of changing \( \tilde{t} \) corresponds to the stabilized convergence during the procedure. If residuals are small, the algorithm is stabilized and \( \tilde{t} \) can be increased to further push binary variables toward Boolean values; otherwise, if residuals are large, more iterations are required and \( \tilde{t} \) should not be boosted. After reaching specific iterations or residuals, the procedure might be completed by \( \tilde{t} \to \infty \) or the projection of remained variables onto final Boolean values.

3) Step 3-Polish: In order to verify the results, the polishing phase fixes the values of binary variables based on the results obtained from the previous phase, and solves the convex problem using the convex ADMM for the remaining variables.

To realize distributed framework and enhance the scalability, the proposed NC-ADMM method integrates the relax-drive-polish procedure with the consensus ADMM, as shown in (21) for cluster \( i \). The idea is to introduce and reparametrize consensus continuous variables as \( \tilde{z} \) and consensus binary variables among each \( B \) neighbor clusters. Then, \( \tilde{z} \) reappears as a consensus binary variable, being integrated with all binding and inner ones, which \( B = 1 \) for inner variables.

\[
(\bar{x}_i, \bar{y}_i)^{(k+1)} := \arg\min_{x'_i, y'_i} f_i(x'_i, y'_i) + \frac{\rho}{2} \|x_i^{B_i} - x_i^{(k)} + u_1^{(k)}\|_2^2 + \frac{\rho}{2} \|\bar{y}_i - \bar{z}_i + u_2^{(k)}\|_2^2 \quad (21a)
\]

\[
\bar{x}_i^{(k+1)} := \frac{1}{B} \sum_{i=1}^{B} (\bar{x}_i^{B_i^{(k+1)}} + u_1^{(k)}) \quad (21b)
\]

\[
\bar{z}_i^{(k+1)} := \text{prox}_{\ell_t(z)} \left( \frac{1}{B} \sum_{i=1}^{B} (\bar{y}_i^{(k+1)} + u_2^{(k)}) \right) \quad (21c)
\]

In order to calculate the residuals in the NC-ADMM method, (20) should incorporate the consensus continuous variables, as shown in (22). The detailed procedure of the proposed NC-ADMM method is provided in Algorithm 2.

\[
r_p^{(k)} = \|\tilde{y}^{(k)} - \hat{z}^{(k)}\|_2^2 + \|\tilde{x}^{(k)} - \hat{x}^{(k-1)}\|_2^2 \quad (22a)
\]

\[
r_d^{(k)} = \rho \|\tilde{z}^{(k)} - \hat{x}^{(k-1)}\|_2^2 + \rho \|\tilde{x}^{(k)} - \hat{x}^{(k-1)}\|_2^2 \quad (22b)
\]

B. Application of NC-ADMM to DDSR

The DN is clustered based on the clustering strategy in Algorithm 1, and the DDSR is solved using the NC-ADMM method in Algorithm 2. For each cluster \( i \), decision variables of problem (17) can be decomposed as \( \bar{x}_i = [\bar{x}_{i,t}], \bar{y}_i = [\bar{y}_{i,t}], \tilde{z}_i = [\tilde{z}_{i,t}], z_i = [z_{i,t}], \forall i \in \mathcal{C}, t \in T \), as defined in (23).

The continuous consensus variables of (23c) are derived from the power flow equations, which each cluster also considers the voltage of boundary nodes and the power flow of joint lines from neighboring clusters.

\[
\tilde{x}_{i,t} := [\bar{x}_{j,t}^{B_i}, \bar{Q}_{j,t}^{L}, \bar{P}_{j,t}^{L}, \bar{P}_{j,t}^{G}, \bar{Q}_{j,t}^{G}, \bar{Q}_{j,t}^{cap,i}] \quad (23a)
\]

\[
\bar{y}_{i,t} := [x_{j,t}^{L}, x_{j,t}^{B_i}, o_{j,t}, \bar{P}_{j,t}^{L}, \bar{P}_{j,t}^{G}] \quad (23b)
\]

\[
\tilde{z}_{i,t} := [\bar{z}_{j,t}, \bar{P}_{j,t}^{L}, \bar{Q}_{j,t}, Q_{j,t}] \quad (23c)
\]

Each SLA solves a subproblem of (21a), in which the related convex set of \( \mathcal{C}_i \) is defined by (21b) and (21c). The exchanged data include 1) all binding continuous variables and related Lagrange multipliers; and 2) consensus binary variables, such as the status of switchable joint lines, their related auxiliary binary variables, and their associated Lagrange multipliers. In this process, all SLAs need to comply with the proposed NC-ADMM algorithm in each of the relax-drive-polish phases during convergence.
Sub. picked up during the first time step of DDSR. Clearly, all high priority loads have been open switches remain open to prevent any loop during the network operation. As there is not any faulted line in the network, all normally-open switches are operated to pick up more loads. Fig. 3 shows the status of restoration. All energized and de-energized loads are shown by green and red dots, respectively. Blue downside and yellow upside arrows show closed and open switch during restoration. The ADMM parameter $\rho$ restoration. The ADMM parameter $\rho$ is set as 1.

1) Scenario 1: It shows how the proposed method deals with a network with outages after a blackout. The provided power through substation gradually increases, and PV generators are operated to pick up more loads. Fig. 4 shows the status of switching operation and load pickup for the first time step of restoration. All energized and de-energized loads are shown by green and red dots, respectively. Blue downside and yellow upside arrows show closed and open switch during restoration. As there is not any faulted line in the network, all normally-open switches remain open to prevent any loop during the network operation. Clearly, all high priority loads have been picked up during the first time step of DDSR.

2) Scenario 2: Considering previous scenario, there is also a faulted line between nodes 67 and 76, which is isolated through opening switches 4, 5, and 7 as shown in Fig. 5. The convergence of the NC-ADMM-based DDSR, in terms of total restored loads for each cluster in scenario 1, is shown in Fig. 6. PV generators are dispatched to provide more power for the load pickup. Due to space limitations, it only shows the first time step of DDSR in which all high priority loads have been picked up. Furthermore, the normally-open switch 6 is closed to provide power to the unfaulted out-of-service area without any loop in operation.

B. Performance of NC-ADMM Method

The performance and scalability of NC-ADMM-based DDSR are demonstrated though testing on IEEE123-node and IEEE 8500-node test feeders. Each time step is assumed to be 15 minutes (1 p.u.) for operation and stabilization. Simulations are implemented on Python, using Gurobi as solver and OpenDSS through COM interface as data provider.

A. NC-ADMM-based DDSR for Unbalanced Network

IEEE 123-node DN is modified by connecting two inverter-based PV units at nodes 66 and 105, each with the maximum capacity of 300kW. Loads at nodes 48 and 65 have higher priority, and loads at nodes 47 and 76 are dispatchable. There are total 3 time steps following transmission restoration, which reflect the gradually increasing generation capabilities of the bulk system as [400kW, 1400kW, 3500kW] [20]. It is assumed that all loads have a constant power factor of 0.9, and the nodal voltage is limited within 0.95 pu and 1.05 pu.

Based on the proposed clustering strategy, the DN is divided into 11 clusters, as shown in Fig. 3. Two outage scenarios are considered in which scenario 1 represents a restoration after blackout and scenario 2 combines scenario 1 with emergency restoration.

VI. Numerical Results

The performance and scalability of NC-ADMM-based DDSR are demonstrated though testing on IEEE123-node and IEEE 8500-node test feeders. Each time step is assumed to be 15 minutes (1 p.u.) for operation and stabilization [7]. Simulations are implemented on Python, using Gurobi as solver and OpenDSS through COM interface as data provider.

Cluster
Children cluster
Parent cluster
$U_{ij,t}$
$i, P_{ij}, t$
$i, Q_{ij,t}$
$i, \alpha_{ij,t}$
$i, \beta_{ij,t}$
Parents

Remark 3. $\alpha_{ij,t}$ and $\beta_{ij,t}$ are the only common consensus binary variables for the joint distribution lines, from power flow equation (10) and spanning tree constraints (13).

Definition 2. Parent cluster is defined as the unique closest neighboring cluster to the substation or the root node, while the others are defined as children clusters.
amount are driven toward Boolean values; 4) at the end of drive phase, by $\tilde{t} \to \infty$, all remaining binary variables are projected to the nearest Boolean values; 5) during the polish phase, binary variables are fixed and continuous variables such as dispatchable loads converge to better optimal values.

Fig. 6 shows the convergence of switching status, which are correctly converged during the relax phase. During the drive phase, despite stimulation of changing and inspecting other values, they are reverted immediately since the initial values are optimal, as all switches remain their original status. It also shows the evolution of $\tilde{t}$ during the convergence, as it equals to zero during the relax phase, and constantly increases to larger values according to (19), by setting $c = 10^{-1}$.

Fig. 7 shows the residuals using the conventional heuristic projection method in (15), and the comparison with the proposed NC-ADMM method is shown in Fig. 8. It is shown that the conventional method of (15) oscillates even after a large number of iterations. If the algorithm halted, the final values of conventional method (15) are usually infeasible, as the second time step in Fig. 8 due to the false switching operation.

C. Scalability of the NC-ADMM-based DDSR

IEEE 8500-node network is integrated with two 1,000 kW PV generators on buses ‘1026706’ and ‘1047592’. The circuit consists of 35 normally-closed switches and 5 tie-switches, and a long switchable tie-line is added between buses ‘L2767341’ and ‘L2955081’. This network consists of 2,522 primary buses while the secondary networks and loads are aggregated into the related secondary transformer buses. The first 100 loads in the related load document are considered as dispatchable, and the rest are binary-valued. There are total 3 restoration steps with
the power capacity of [600kW, 6,000kW, 12,000kW] following transmission restoration.

Using the proposed clustering strategy, the network is divided into 50 clusters as shown in Fig. 9. A blackout restoration is assumed with a faulted line between buses ‘M1125934’ and ‘L2730163’, which is isolated through sectionalizing switches ‘A8645_48332_sw’ and ‘A8611_48332_sw’. Fig. 10 shows the last time step of restoration. The long tie-line switch is closed to provide power for the out-of-service area after the faulted line. It is clear that all other tie-switches are open as shown by a yellow arrow, to prevent any loop in the operation. Furthermore, the convergence of the proposed method, in terms of total restored loads at each time step, is shown in Fig 11. Similarly, SLAs solve the problem in each phase and exchange data among each other, until the related Lagrange multipliers for the power balance affected and the total available power adjusted during the drive phase. During the polish phase, all binary variables are fixed to polish the results toward high-quality solution for the DDSR problem.

VII. CONCLUSION

Service restoration can be formulated as a challenging mixed-integer nonlinear programming problem, which deals with many binary variables representing loads and switching operation. In this paper, a non-convex ADMM-based distributed optimization method is developed and applied to the service restoration problem in large-scale active distribution networks. The developed heuristic ADMM-based algorithm is incorporated with consensus ADMM to provide a fully distributed cluster-based framework. Moreover, an adaptive autonomous clustering strategy is developed for application in large-scale networks, in which each cluster consists of a smart agent to carry out the distributed restoration procedure with limited data exchange with neighbors. Simulation results on large-scale IEEE test networks demonstrate the capability of the distributed restoration to deal with various blackout or emergency restoration problems, and also the superiority of the distributed non-convex method over simple projection methods. In future work, the proposed NC-ADMM method can be further analyzed in terms of the evolution of parameter t during the drive phase to achieve an even better solution.

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