Equation of Motion of an Electric Charge

Amos Harpaz\textsuperscript{1,2} and Noam Soker\textsuperscript{2}

1. Institute of Theoretical Physics, Technion, Haifa, ISRAEL
2. Department of Physics, Oranim, Tivon 36006, ISRAEL

phr89ah@tx.technion.ac.il
soker@physics.technion.ac.il

Abstract

The appearance of the time derivative of the acceleration in the equation of motion (EOM) of an electric charge is studied. It is shown that when an electric charge is accelerated, a stress force exists in the curved electric field of the accelerated charge, and this force is proportional to the acceleration. This stress force acts as a reaction force which is responsible for the creation of the radiation (instead of the “radiation reaction force” that actually does not exist at low velocities). Thus the initial acceleration should be supplied as an initial condition for the solution of the EOM of an electric charge. It is also shown that in certain cases, like periodic motions of an electric charge, the term that includes the time derivative of the acceleration, represents the stress reaction force.
1. Introduction

The equation of motion (EOM) for a charged particle was calculated by Schott[1] in a three dimensional formalism, and by Dirac[2] in a four dimensional formalism:

\[ ma^\mu = F^\mu_{\text{ext}} + \Gamma^\mu = F^\mu_{\text{ext}} + \frac{2e^2}{3c^3}(\dot{a}^\mu - \frac{1}{c^2}v^\mu a^\alpha a^\alpha) \].

The first term in \( \Gamma \) (known as Schott term) includes the time derivative of the acceleration (a third time derivative of the position), and this raised the question as “what is the role of the third time derivative in the EOM”, as the presence of a third derivative in the EOM demands the definition of three initial conditions for the solution of such an equation, where in classical mechanics we usually need only two initial conditions for the solution of a regular EOM. Is the appearance of the third time derivative in the EOM just a sad accident that somehow should be bypassed, or is it a legitimate requirement that characterizes this kind of motion? We also notice that the second term in \( \Gamma \) equals the power carried by the radiation (Larmor formula). It is clear that Schott term should include the physical factors that create this power. In certain physical situations, like a motion of an electric charge in a cyclotron, the third derivative of the position plays an important role. It appears as a part of the “radiation force”, \( F_{\text{rad}} \). We return to this point in more details in section 3.

One way of bypassing the question of the third derivative is described in [3]. According to this suggestion, Schott term is isolated somehow from \( \Gamma \) and is adapted to the acceleration term in the left side of the EOM (eq. 1). To solve this equation in its new form, a multiplication by an integration factor \( e^{-\frac{\tau}{\tau_0}} \) is needed. This term, which includes the proper time in an exponential form, leads later to divergent solutions that must be discarded. It also causes an acceleration that begins some time before the force that creates the acceleration acts, and this “preacceleration” should be discarded by causality considerations. We shall discuss in detail the question of the third derivative in section 2.

Landau and Lifshitz[4, p. 203] also obtain the third derivative using a different approach. They show that when a charge is accelerated by an external electric field, in addition to the force acting according to Lorentz formula, it is acted upon by a force \( f = \frac{2e^2a^2}{3c^3} \), and by multiplying this force by the velocity, and taking the average over time, they find the power, \( P \), created by this force: \( P = \frac{2e^2a^2}{3c^3} \). They call this force “radiation damping”, and argue that it represents the reaction of the radiation on the charge. However, as was shown in [5], at low velocities the radiation is emitted in a plane perpendicular to the direction of motion, and no counter momentum is imparted to the charge by the radiation, and no radiation reaction force (or a radiation damping force) exists. The reaction force to the motion of the accelerated charge should be found elsewhere. Later we show that the reaction force is actually the stress force that exists in the curved electric field of the accelerated charge. The work performed in overcoming this reaction force is the source of energy carried by the radiation.

Until recently, \( \Gamma \) term (also known as Abraham four vector) was considered as representing the radiation reaction force[6,7], and the fact that in a hyperbolic motion \( \Gamma \) vanishes raised many questions, known as the “energy balance paradox”[8]. The paradox is, that the
vanishing of $\Gamma$, leaves no radiation reaction force and no source for a work that may create
the energy carried by the radiation.

Following ref. [9] we adopt the approach given in a recently published work of Rohrlich[10]. The equation of motion given in [10] is:

$$m_0 \dot{v}^\mu = F_{\text{ext}}^\mu + F_{\text{self}}^\mu ,$$

where $m_0$ is the physical rest mass involved in the process ($m_0 = m + \delta m$, [10]), and $F_{\text{self}}^\mu$ actually equals Abraham 4-vector. $\delta m$ represents the electromagnetic mass. Rohrlich concludes that the fourth component of Schott term includes the power created by the reaction force, and the vanishing of $\Gamma$ in a hyperbolic motion shows that Schott term is the source of the power carried by the radiation. However, the spatial part of Schott term (the force that creates this power) is not found, as the radiation reaction force does not exist. In section 3 we show that there is a reaction force, but it is not a radiation reaction force, but it is the stress force that exists in the curved electric field of the accelerated charge. This reaction force should be included in the spatial part of Schott term, and the work done in overcoming this force creates the energy carried by the radiation.

2. Initial conditions

The answer to the question about the role of the derivative of the acceleration demands analysis of all the factors involved in the motion of a charged particle. It comes out that an important factor is usually overlooked - this factor is the stress force that exists in the curved electric field of an accelerated charged particle[5]. The interaction of the curved electric field with the accelerated charge that induced the field appears as a reaction force that acts on the accelerated charge, and the work performed by the external force ($F_{\text{ext}}$) to overcome this reaction force is the source of the energy carried by the radiation[5]. The stress force is inversely proportional to the radius of curvature, $R_c$, of the electric field (see eq. 9 further on). For the simple case of a hyperbolic motion, $R_c$ is given by:

$$R_c = \frac{c^2}{a \sin \theta} ,$$

where $a$ is the acceleration of the charged particle, and $\theta$ is the angle between the direction of motion and the direction of the field line at the location of the charge. We find that the reaction force is proportional to the acceleration and this shows that the value of the acceleration at any moment has an important role in the determination of the motion parameters, as it determines part of the forces that affect the motion. Hence the initial value of the acceleration is needed to define the parameters of the motion, and the appearance of the derivative of the acceleration in the EOM is not a sad mishap, but a legitimate requirement, needed for a complete solution of the physical situation. Later it is found[9], that when this stress force is calculated, its value should be inserted into the spatial part of Schott term, and then, the power invested in overcoming this force, balances the radiation power that appears in the second part of $\Gamma^\mu$ that appears in eq. 1. Thus $\Gamma^\mu$ is fully balanced as required (see [10]), and the “energy balance paradox” is resolved. Dirac treats the first term in eq.
1 \((ma^\mu)\) as the “source of the kinetic energy”, while Schott term is treated by him as the “source of the acceleration energy”. This may be justified by noticing that this term includes the power created in overcoming the stress force (the reaction force) which is proportional to (and created by) the acceleration.

The situation described here resembles the situation in which a charge is supported statically in a gravitational field. It is found\cite{5} that this situation, which seems static is actually not static, but it is a steady state situation - the electric field of the charge is detached from the charge\cite{11}, and it falls in a free fall in the gravitational field. The electric field becomes curved, and a stress force exists in this field. When the physical parameters are calculated for such a situation, for the most simple case (an electric charge located in a homogenous gravitational field\cite{12}), it is found that in the close vicinity of the charge, the radius of curvature of the electric field, \(R_c\), equals (for the first approximation):

\[
R_c \simeq \frac{c^2}{g \sin \theta},
\]

where \(g\) is the gravitational acceleration, and \(\theta\) is the angle between the direction of \(g\) and the electric field line at the charge location. This equation is similar to eq. 3, and this fact tempts us to deduce that there indeed exists an equivalence between an acceleration of a charge in a free space, and a static location of a charge in a gravitational field, and to the conclusion that a charge supported at rest in a gravitational field does radiate.

3. The Stress Force

Let us calculate in detail the power created by the stress force for the case of an hyperbolic motion. The equations for the electric field as calculated by Fulton and Rohrlich\cite{6} using the retarded potentials method are given in cylindrical coordinates \((z, \rho, \phi)\), by:

\[
E_\rho = \frac{8e\alpha^2\rho z}{\xi^3}
\]

\[
E_z = -\frac{4e\alpha^2}{\xi^3}[\alpha^2 + (ct)^2 + \rho^2 - z^2]
\]

\[
B_\phi = \frac{8e\alpha^2\rho ct}{\xi^3}
\]

where

\[
\xi^2 = [\alpha^2 + (ct)^2 - \rho^2 - z^2]^2 + (2\alpha\rho)^2,
\]

and all other field components vanish. \(z = \alpha = c^2/a\) is the location of the charge at time \(t = 0\), and it is also the characteristic radius of curvature of the electric field. The equation for the field lines was calculated by Singal\cite{13}, and they are drawn in Fig. 1.

As can be expected, the field lines are curved and the radius of curvature, \(R_c\), is inversely proportional to the acceleration and is given in eq. 3. These equations were calculated earlier (in a three dimensional formalism) by Schott\cite{1}. They were also calculated by Gupta
Figure 1: The field lines of a charge moving with a constant acceleration

and Padmanabhan[14] who calculated first the electric field of the accelerated charge in its own system of reference, and then performed transformations to the free space system of reference. They also show that by using the retarded variables (x_{ret}, t_{ret}), they recover the equations usually given in the textbooks[15, 16].

The stress force density, \( f_s \), is given (see [17] for more details) by:

\[
f_s = \frac{E^2}{4\pi R_c} \tag{9},
\]

where \( E \) is the electric field. When we substitute in eq. 9 for \( R_c \) and for \( E = \frac{e^2}{r^2} \) (which is correct for the close vicinity of the charge) we get for the stress force density:

\[
f_s = \frac{e^2 a \sin \theta}{4\pi r^4 c^2} \tag{10}.
\]

The component of this force in the direction of the acceleration is \( -f_s \sin \delta \), where \( \delta \) is the angle between the field line and the direction of motion. In the close vicinity of the charge, where the direction of the field lines did not change much, \( \delta \simeq \theta \), and we find for the parallel component of the force density:

\[
-f_s \sin \delta \simeq -f_s \sin \theta = \frac{-e^2 a \sin^2 \theta}{4\pi c^2 r^4} \tag{11},
\]

and we have to integrate this expression over a sphere around the charge. To avoid divergence at the center \( (r \rightarrow 0) \), we take for the lower limit of the integration a small distance from the center, \( c\Delta t \), where \( \Delta t \) is infinitesimal. As the upper limit of the integration we take some large distance, \( r_{up} \), where \( c\Delta t \ll r_{up} \ll c^2/a \). The integration is carried in the free space system of reference, \( S \), which momentarily coincides with the system of reference of the charge at time \( t = 0 \). This integration yields the stress force \( F_s \), by which the curved electric field acts on the accelerated charge. It acts against the acceleration, and the external
force that accelerates the charge, should overcome this stress force, in addition to the work it does in creating the kinetic energy of the charged particle. This work is the source of the energy carried by the radiation.

\[ F_s = \int -f_s \sin \theta dV = -\frac{e^2 a}{4\pi c^2} \int_0^{2\pi} d\phi \int_0^\pi \sin^3 \theta d\theta \int_{c\Delta t}^{r_{wp}} \frac{dr}{r^2} = \left[ -\frac{2e^2 a}{3c^2r} \right]_{c\Delta t}^{r_{wp}} \left[ \frac{2e^2 a}{3c^3} \Delta t \right] \left[ 1 - \frac{c\Delta t}{r_{wp}} \right]. \] (12)

Certainly, the second term in the square parenthesis can be ignored. In order to find the power, \( P_s \), created by this force, we multiply \(-F_s\) by the velocity of the charge at the time \( t = \Delta t \), \( v = a\Delta t \), and find:

\[ P_s = -F_s a\Delta t = \frac{2e^2 a^2}{3c^3} \] (13)

which is the power carried by the radiation (Larmor formula). This power should be inserted in the fourth component of Schott term, where it balances the power carried by the radiation.

4. A charge in a periodic motion

We present here two examples in which we find that \( F_s \), which is responsible for the creation of the radiation, is proportional to \( \dot{a} \).

Let us study an oscillatory motion of a charge in a linear antenna whose length is \( 2D \) in the \( x \) direction. The equations of motion for this case are:

\[ x = D \sin \omega t \quad ; \quad v = \omega D \cos \omega t \] (14.a)
\[ a = -\omega^2 D \sin \omega t = -\omega^2 x \quad ; \quad \dot{a} = -\omega^3 D \cos \omega t = -\omega^2 v. \] (14.b)

At time \( t = \Delta t \), (where \( \Delta t \) is infinitesimal), we use the approximations (ignoring second and higher powers of \( \Delta t \) and assuming \( D\omega \ll c \)): \( \sin \omega t \approx \omega \Delta t \), and \( \cos \omega t \approx 1 \), and find:

\[ x = D\omega \Delta t \quad ; \quad v = D\omega \] (15.a)
\[ a = -D\omega^3 \Delta t \quad ; \quad \dot{a} = -D\omega^3. \] (15.b)

We find: \( a = \dot{a}\Delta t \). (Note that in section 3 we had \( a \to \text{const.}, v \to 0 \), while here, \( a \to 0, v \to \text{const.} \). The motion is linear, and we recall from eq. 12 that the reaction force \( F_{reac} \) created in a linear acceleration motion at low velocity after short time \( \Delta t \) is: \( F_s = \frac{2e^2 a}{3c^3} \Delta t \). Using the relation we found above between \( a \) and \( \dot{a} \) we find \( F_{reac} = F_s = \frac{2e^2 a}{3c^3} \dot{a} \), which is the expression used by Jackson [15, 17.8] for the radiation force \( F_{rad} \), which is responsible for the creation of the radiation. However, it was already shown that no radiation reaction force exists in a linear acceleration at low velocities, and \( F_{reac} \), which should replace Jackson’s \( F_{rad} \), is the reaction force that exists as a stress force in the curved electric field of the accelerated charge. The force \(-F_s\), should be inserted as the spatial part (three vector component) of Schott
term, and a term \(-F_s\), should be added to the accelerating force \(F_{ext}\), which is needed to overcome the stress force.

As a second example, consider the case in which a charge is moving in a circular motion like in a cyclotron. When a charge moving with a uniform velocity in a free space enters into a homogenous magnetic field, \(B\), which is aligned in a right angle to its direction of motion, it will be acted upon by a force \(F_B\), which is perpendicular both to its direction of motion and to the direction of \(B\). Since the force is always perpendicular to the charge’s velocity, no work is performed by \(F_B\), and the charge will perform a circular motion due to the radial acceleration imparted to it. If in such a motion no loss of energy takes place, this motion may continue forever. However, due to the acceleration imparted to the charge, it radiates, and loses energy through this radiation. The central force cannot perform the work that creates the radiation, because it is always perpendicular to the direction of motion. In the equipment (cyclotron) in which this motion takes place, a force \(F_{ext}\), that acts along the direction of motion (along the circle) should act, where the work performed by \(F_{ext}\), creates the energy carried away by the radiation. Usually, this force is implemented by an oscillatory electric field, whose frequency equals the frequency of the cyclotron. For simplicity, we assume that this force acts continuously along the trajectory of the charge. It is clear that some force exists in the system which “tries” to slow down the charge in its motion (it acts in an opposite direction to the velocity), and this force acts as a reaction force, \(F_{reac}\), to the motion. \(F_{ext}\) should overcome \(F_{reac}\), and the question is: what is the source of \(F_{reac}\).

The usually given answer to this question is that \(F_{reac}\) is a radiation reaction force, and it is called by Jackson[15, eq. 17.8], \(F_{rad}\). However, close inspection of the equation that gives the angular distribution of the radiation emitted from a charge moving in a circular motion [15, eq. 14.44], shows that for low velocities of the charge in its circular motion \((\beta \ll 1)\), the radiation is emitted in a plane perpendicular to the direction of the acceleration, and due to the symmetry of the radiation in this plane, no counter momentum is imparted by the radiation to the charge, and no radiation reaction force exists.

It was shown in [5], that when a charge is accelerated, its electric field becomes curved,
and a stress force, $F_s$, exists in this curved field. $F_s$ actually acts as a reaction force, which $F_{ext}$ has to overcome, and doing this, $F_{ext}$ performs the work that creates the energy carried away by the radiation. We are interested here in the component of $F_s$ in the counter direction of the velocity. As shown in [5], the calculations are carried during an arbitrary infinitesimally time interval, $\Delta t$. In Fig. 2 we present the calculations: The figure is drawn in the $x, z$ plane, as shown in the figure. In this figure, the center of the circular orbit (with radius $R$), is the point $C$. At time $t = 0$, the charge is located at the point $P$. The velocity of the charge is $V$, and we analyze the situation in the system of reference $B$, which momentarily coincides with the system of reference of the charge at time: $t = 0$. In this system the linear velocity of the charge vanishes, and a central velocity $V_z$ is created due to the central acceleration $a$. At the end of the time interval $\Delta t$ a velocity $V_z = a\Delta t$ is created. We recall from [5] (and from eq. 12 above) that the component of the stress force parallel to the acceleration is: $F_s = \frac{2e^2a}{3c^3} \Delta t$. We are interested in the component $F_{track}$ of $F_s$, which is parallel (or anti-parallel) to the circular velocity $V$, at time: $t = \Delta t$. From trigonometric relations we find that at the time $t = \Delta t$:

$$\frac{F_{track}}{F_s} = \frac{V_z}{V}.$$  \hspace{1cm} (16)

The ratio of $V_z$ to $V$ is:

$$\frac{V_z}{V} = \frac{a}{V} \Delta t = \frac{V}{R} \Delta t,$$  \hspace{1cm} (17)

and we find for $F_{track}$:

$$F_{track} = F_s \frac{V_z}{V} = -F_s \frac{V}{R} \Delta t = \frac{2e^2}{3c^3} \frac{a}{R} V = \frac{-2e^2 V^3}{3c^3 R^2},$$  \hspace{1cm} (18)

where the last expression is obtained by substituting $a = -\omega^2 R = \frac{-V^2}{R}$. Using the same value for the acceleration, we have: $-\omega^2 V = \frac{V^3}{R^2} = \dot{a}$, and we find that:

$$F_{track} = \frac{2e^2}{3c^3} \dot{a},$$  \hspace{1cm} (19)

which equals the expression given by Jackson for $F_{rad}$, the force that causes the creation of the radiation. What we have found is that this force is not a radiation reaction, but it is a reaction force created by the stress force that exists in the curved electric field of the accelerated charge. Multiplying $-F_{track}$ as given in eq. 16 by $V$, yield the power created by this force:

$$-F_{track} \cdot V = \frac{2e^2 V^4}{3c^3 R^2} = \frac{2e^2 a^2}{3c^3},$$  \hspace{1cm} (20)

which is the power carried by the radiation. These calculations approve that also in the case of a circular motion (at low velocities), the stress force does the main role of the reaction force which is responsible for the creation of the radiation.

5. A Charge Supported in a Gravitational Field

The situation of a relative acceleration between a charge and its electric field also exists for a charge supported statically in a gravitational field. The electric field, which is not
supported with the charge, falls in a free fall (see [11]), and there exists a relative acceleration between the charge and its field. Prima facie, the situation seems static, but it is not. It is a steady state situation. It is correct that in the lab system (the system of the supported charge), the Poynting vector vanishes, but a Poynting vector is not an invariant, and cannot be taken as a covariant measure for the existence of radiation. We demand that a covariant measure for the existence of radiation, is the non-vanishing of the Poynting vector in the system of reference defined by the geodesics. Such a system is the one that falls freely parallel to the charge, and in this system, the supported charge is accelerated upward. A magnetic field exists in this system, and the Poynting vector does not vanish (see Rohrlich[12]). In this system, the electric field is curved, a stress force exists, and calculations similar to those carried for the accelerated charge, yield the power carried by the radiation. The power can be calculated according to Larmor formula, where we put $g$ for the acceleration, namely:

$$P = \frac{2 g^2 e^2}{3 c^3}.$$  

(13a)

As a general measure for the radiation, we take the four acceleration vector $A^\mu$. It is calculated by:

$$A^\mu = D_4 U^\mu = \frac{d}{dx^4} U^\mu + \{4^\mu_\nu\} U^\nu,$$  

(21)

where $D$ represents a covariant drivative and $\{\gamma^\mu_\nu\}$ is the Christoffel symbol. For a static charge $U^\mu = \left( \frac{0}{c} \right)$. The regular derivative in eq. 14 vanishes, and we are left with:

$$A^\mu = \{4^\mu_4\} U^4$$  

(22)

and for Larmor formula we should use: $a^2 = g_{\mu\nu} A^\mu A^\nu$, where $a$ is the proper acceleration. Let us examine two cases:

1. A homogenous static gravitational field.
2. A Schwarzschild field.

1. Consider a homogenous gravitational field directed downward in the $z(= x^3)$ direction, that generates an acceleration $g$. This field is characterized by $u(z)$ (a function of $z$):

$$\frac{1}{u} = \cosh \sqrt{(1 - g z)^2 - 1}, \quad u' = du/dz, \text{ and } g_{33} = (u'/g)^2 \text{ (Rohrlich[12]).}$$

$$A^3 = \{4^3_4\} U^4 = g^2 \frac{u}{u'}$$  

(23)

and

$$g_{33} A^3 A^3 = g^2 u^2,$$  

(24)

which at the charge location ($z \to 0$), yields $g^2$ to be introduced in Larmor formula.

2. In a Schwarzschild field, the only relevant coordinate is $r(= x^1)$. We have:

$$A^1 = \{4^1_4\} U^4 = \frac{m}{r^2} (1 - \frac{2m}{r}).$$  

(25)
The charge has an outward acceleration of \( g(1 - 2m/r) \). For Larmor formula we have to use \( a^2 = g_{11}A^1A^1 = g^2(1 - 2m/r) \), and we find that for a static charge in a weak field, Larmor formula yields a power very close to that radiated by an accelerated charge with an acceleration \( g \).

Let us calculate the electric field of a supported charge in a homogenous gravitational field. Using equations given by Rohrlich[12], we have (in cylindrical coordinates, \( \rho, z, \phi \)):

\[
E_\rho = \frac{8e\alpha^3\rho u^2}{\xi^3} \tag{26}
\]

\[
E_z = \frac{-4e\alpha^3uu'}{\xi^3}(\alpha^2(1 - u^2) + \rho^2) \tag{27}
\]

\[
\xi^2 = (\alpha^2(1 - u^2) - \rho^2)^2 + 4\alpha^2\rho^2, \tag{28}
\]

where \( u(z) \) and \( u' \) are given above, and here \( \alpha = c^2/g \), which is the characteristic radius of curvature of the electric field close to the location of the charge. To calculate the fields in a horizontal plane passing through the location of the charge, we take \( z = 0 \), which gives: \( u \to 1, \quad uu' \to 1/\alpha \). These substitutions yields for the fields:

\[
E_\rho = \frac{e}{\rho^2 [1 + \frac{\rho^2}{4\alpha^2}]^{3/2}} \tag{26a}
\]

\[
E_z = \frac{-e}{2\alpha \rho [1 + \frac{\rho^2}{4\alpha^2}]^{3/2}} = -\frac{eg}{2\rho c^2 [1 + \frac{\rho^2}{4\alpha^2}]^{3/2}}. \tag{27a}
\]

If for the case described in section 3 (a charge moving in a hyperbolic motion) we calculate the electric fields of the charge in the plane \( z = \alpha \), at time \( t = 0 \), we find:

\[
E_\rho = \frac{e}{\rho^2 [1 + \frac{\rho^2}{4\alpha^2}]^{3/2}}, \tag{5a}
\]

\[
E_z = \frac{-e}{2\alpha \rho [1 + \frac{\rho^2}{4\alpha^2}]^{3/2}} = \frac{-ea}{2\rho c^2 [1 + \frac{\rho^2}{4\alpha^2}]^{3/2}}, \tag{6a}
\]

where here \( \alpha = c^2/a \), and we find that these equations are identical to eqs. 26a, 27a, except for the fact that \( g \) is replaced by \( a \).

This means that if a charge accelerating in hyperbolic motion radiates, and the radiation in the plane perpendicular to the direction of motion is described by eqs. 5a, 6a, then the similar equations for the field of the charge supported in a homogenous gravitational field (given by eqs. 26a, 27a), show that this charge also radiates.

Usually, we consider the transverse field \( (E_{\text{trans}} = E_z) \) as describing the radiation field, as it falls in the close vicinity of the charge like \( 1/\text{distance} \). However, the presence of the extra term in the denominator \( ([1 + \frac{\rho^2}{4\alpha^2}]^{3/2}) \) shows that for large distances from the charge, the transverse field does not fall exactly like \( 1/\text{distance} \), as is usually found in the equations given in the textbooks[15,16]. The difference of eqs. 5a, 6a, from the the equations
given in the textbooks confused people who did not notice that equations 5a, 6a describe
the electric field in the free space system of reference, while the equations given in
the textbooks, with the nice neat dependence on 1/distance for the radiation, use the retarded
coordinates \((x_{\text{ret}}, t_{\text{ret}})\). (In ref [14], the authors show how the transformations between these
two sets of coordinates work.) This problem confused Boulware[7], who tried to show that
a coaccelerated observer will not observe radiation from a hyperbolically accelerated charge.
By transforming from the retarded coordinates to the free space coordinates[7, eqs. III.18
- III.20] Boulware found that the nice dependence on 1/distance is lost, and from this fact
he deduced that the coaccelerated observer cannot observe the radiation. This conclusion is
correct for any observer, coaccelerated or not, and hence cannot be accepted for the case of
a coaccelerated observer.

We find that the correct measure for the existence of radiation should be the ratio
\[ \frac{|E_{\text{trans}}|}{|E_{\text{long}}|} > 1, \]
where \(E_{\text{trans}}\) and \(E_{\text{long}}\) are the transverse and longitudinal electric
fields respectively, and \(E_{\text{rad}}\) and \(E_{\text{coul}}\) are the radiation field and the coulomb field respec-
tively. This ratio is found both in eqs. 5a, 6a, for a hyperbolically accelerated charge, and in
eqs. 26a, 27a, for a charge statically supported in a homogenous gravitational field, at large
distances from the charge.

What is the source of the energy carried by the radiation in the gravitational field?
The charge is supported by a solid object, which is static in the GF (Gravitational Field).
This solid object is rigidly connected to the source of the GF. Otherwise, it will fall in the
GF, together with the “supported” charge. Actually, the supporting object is part of the
object that creates the GF.

The charge is static and no work is done on the charge. However, the electric field of
the charge is not static, and it falls in a free fall in the GF. With no interaction between
the electric field and the charge, the field would follow a geodetic line and no work would
be needed to keep it following the geodetic line. But the field is curved, and a stress force is
implied. The interaction between the curved field and the supported charge creates a force
that contradicts the free fall. In order to overcome this force and cause the electric field to
follow the geodetic lines, a work is done on the electric field, and this work is done by the
GF. This work is the source of the energy carried by the radiation. The energy carried away
by the radiation is supplied by the GF, that loses this energy.

5. Conclusions

By analyzing the forces that act on an accelerating charge we found that a stress force that
exists in the curved electric field of the accelerated charge is proportional to the acceleration.
Hence, the initial value of the acceleration is needed as an initial condition for a complete
solution of the charge’s motion, and this property justifies the appearance of the third
derivative of the position \((\ddot{a})\) in the equation of motion of a charged particle. The inclusion
of the stress force as the spatial part of Schott term supplies the source of the energy
carried by the radiation, and thus the “energy balance paradox” is solved. The stress force
is calculated, and the work performed in overcoming it supplies the energy carried by the
radiation. The similarity between the electric field of an accelerated charge, and that of a
charge supported statically in a gravitational field, and the existence of relative acceleration between the charge and its electric field in both cases, leads to the conclusion that a charge supported statically in a gravitational field radiates.

references:

1. G. A. Schott, *Electromagnetic Radiation*, Cambridge University P., 1912.
2. P.A.M. Dirac, *Proc. R. Society*, A167, 148, 1938.
3. F. Rohlich, *Classical Charged Particles*, Addison-Wesley, Reading, MA., 1965.
4. L.D. Landau, E.M. Lifshitz, *Classical Theory of Fields*, Pergamon Press, 3rd Ed., 1971.
5. A. Harpaz, N. Soker, *Found. of Phys.*, 31, 935, 2001.
6. R. Fulton, F. Rohrlich, *Ann. Phys.*, 9, 499, 1960.
7. D.G. Boulware, *Ann. Phys.*, 124, 169, 1980.
8. C. Leibovitz, A. Peres, *Ann. Phys.*, 25, 400, 1963.
9. A. Harpaz, N. Soker, *Proc. Roy. Soc. London A*, submitted (physics/0207038).
10. F. Rohrlich, *Amer. J. of Phys.*, 68, 1109, 2000.
11. A. Harpaz, *Euro. J. of Phys.*, 23, 263, 2002.
12. F. Rohrlich, *Annals of Phys.*, 22, 169, 1963.
13. A.K. Singal, *Gen. Rel. Grav.*, 29, 1371, 1997.
14. A. Gupta, T. Padmanabhan, *Phys. Rev.*, D57, 7241, 1998.
15. J.D., Jackson, *Classical Electrodynamics*, 2nd Ed. (New York, Wiley), 1975.
16. A.K.H. Panofsky, M. Phillips, *Classical Electricity and Magnetism*, 2nd Ed. (Reading, MA., Addison-Wesley), 1964.
17. A. Harpaz, N. Soker, *Gen. Rel. Grav.*, 30, 1217, 1998.