Neutron unpolarized structure function $F_2^n(x)$ from deep inelastic scattering off $^3\text{He}$ and $^3\text{H}$

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Abstract

The possibility to safely extract the neutron deep inelastic structure function $F_2^n(x)$ in the range $0 \leq x \leq 0.9$ from joint measurements of deep inelastic structure functions of $^3\text{He}$ and $^3\text{H}$ is demonstrated. While the nuclear structure effects are relevant, the model dependence in this extraction linked to the $N - N$ interaction is shown to be weak.
The knowledge of both the proton and neutron deep inelastic structure functions (DISF) at large \( x \) could give access to the valence \( u \) and \( d \) quark distributions in the nucleon [1]. Usually deuteron data have been employed to gain information on the neutron unpolarized DISF, \( F_2^n(x) \), but uncertainties remain. In particular, it has been recently argued [2] that the standard treatment of deuteron data [3], leading to the value 1/4 for the ratio \( r(x) = F_2^n(x)/F_2^p(x) \) when \( x \rightarrow 1 \), could be unfair. An improved analysis could move such a value towards that of 3/7, suggested by pQCD arguments [4], and closer to that of 2/3, expected within SU(6) symmetry.

Recently, the possible use of an unpolarized \(^3H\) target has been discussed [5]. In particular, an experiment has been proposed [6], aimed to the measurement of \( F_2^n(x) \), using the ratio of unpolarized structure functions of \(^3He, F_2^H(x)\), and \(^3H, F_2^T(x)\), to reduce systematic errors in the measurements, as well as theoretical model dependences.

In this work a safe procedure is proposed to extract \( F_2^n(x) \) from this ratio [4], although it will be shown that, at high values of Bjorken variable \( x = Q^2/(2M\nu) \), nuclear structure effects are relevant and cannot be overlooked. Nucleon structure in \(^3He\) and \(^3H\) is assumed to be the same as for free nucleons, while nucleon motion and nucleon binding in the three-nucleon systems are accurately taken care of and the Coulomb interaction (CI) is explicitly considered in the evaluation of the \(^3He\) wave function. For an easy presentation only the case of infinite momentum transfer in the Bjorken limit will be considered, but it is straightforward to generalize our approach to finite momentum transfer values [7].

The DISF for \(^3He\) and for \(^3H\) in impulse approximation (IA) can be written as follows

\[
F_2^H(x) = 2 \int_x^A F_2^p(x/z) f_p^H(z) dz + 2 \int_x^A F_2^n(x/z) f_n^H(z) dz \tag{1}
\]

\[
F_2^T(x) = 2 \int_x^A F_2^p(x/z) f_p^T(z) dz + 2 \int_x^A F_2^n(x/z) f_n^T(z) dz \tag{2}
\]

where the distribution \( f_p^{H(T)} (z) \) describes the structure of the trinucleon system [8]

\[
f_{p(n)}^{H(T)} (z) = \int dE \int d\vec{p} \, F_{p(n)}^{H(T)} (|\vec{p}|, E) \, \delta \left( z - \frac{pq}{M\nu} \right) z C . \tag{3}
\]

In Eq. (3) the functions \( P_{p(n)}^{H(T)} (|\vec{p}|, E) \) are the proton and neutron spectral functions (SF) in \(^3He\) \((^3H)\), respectively, \( \vec{p} \) and \( E \) the nucleon momentum and removal energy, \( M \) is the nucleon mass, and \( C \) a normalization factor.

Let us define by \( E(x) \) the experimental ratio between the \(^3He\) and \(^3H\) unpolarized DISF : \( E(x) = F_2^H(x)/F_2^T(x) \), and by \( R(x) \) the super-ratio (S-R)

\[
R(x) = \frac{F_2^H(x)/(2F_2^p(x) + F_2^n(x))}{F_2^T(x)/(2F_2^p(x) + F_2^n(x))} = E(x) \frac{2r(x) + 1}{2 + r(x)} \tag{4}
\]

In IA the S-R is a functional of \( r(x) \) ( \( R(x) = R[r(x)] \) ). Indeed from Eqs. (1,2,3) one has

\[
R[r(x)] = \frac{2 \int_x^A p(x/z) f_p^H(z) dz + 2 \int_x^A p(x/z) r(x/z) f_n^H(z) dz}{2 \int_x^A p(x/z) f_p^T(z) dz + 2 \int_x^A r(x/z) p(x/z) f_n^T(z) dz} \frac{2r(x) + 1}{2 + r(x)} \tag{5}
\]
Figure 1. (a) The super-ratio $R(x)$ (Eq. (4)) with $F_2^{n(p)}(x)$ from Ref. [10] for different $N-N$ interactions. Solid, dashed and long-dashed lines correspond to the RSC and Av14 interactions plus CI and to Av14 without CI for $^3$He, respectively. The dotted line is obtained as the solid one, but using the nucleon momentum distributions, instead of the nucleon SF. (b) The super-ratio $R(x)$ for the RSC interaction plus CI. Solid and dotted lines correspond to the models of Ref. [10] and Ref. [11] for $F_2^{n(p)}(x)$, respectively; dashed line: as the solid one with $F_2^{n(p)}(x)$ multiplied by a factor $(1 + 0.5x^2)$; long-dashed line: $R(x)$ obtained with the arbitrarily modified $F_2^{n(p)}(x)$, but evaluating the corresponding variations of $R(x)$ only at the first order in $\delta r^{(0)}(x) = 0.5x^2 F_2^{n(p)}(x)/F_2^{p(p)}(x)$ (see text).

where $p(x, z) = F_2^{p}(x/z)/F_2^{p}(x)$. From Eq. (4) one can immediately obtain the following equation for the ratio $r(x)$:

$$r(x) = \frac{E(x) - 2R[r(x)]}{R[r(x)] - 2E(x)}$$  (6)

The above equation is a self-consistent equation, which allows one to determine $r(x)$. If the distribution $f_{p(n)}^{H(T)}(z)$ is similar to a Dirac $\delta$ function, $f_{p(n)}^{H(T)}(z) \sim \delta(z-1)$, then $R(x) = 1$ and Eq. (6) becomes trivial. Actually this hypothesis is only an approximate one, so that $R(x) \neq 1$ at high $x$, as shown in Fig. 1. Furthermore, sensible differences are obtained for $x \geq 0.6$, if the nucleon SF are replaced by the nucleon momentum distributions. On the contrary, the model dependence of $R(x)$, due to the $N-N$ interaction, is weak for any $x$ (see Fig. 1(a)). In particular, the introduction of a three-body force yields negligible effects in $R(x)$ at $x \leq 0.90$.

It is interesting to note that, if in Eq. (6) $r(x)$ is replaced by $r^{(0)}(x) = r(x) + \delta r^{(0)}(x)$, then the variations of $R(x)$ can be accurately estimated using the expression obtained at the first order in $\delta r^{(0)}(x)$ (see Fig. 1(b)). Thus, if in the right hand side of Eq. (6) the exact $r(x)$ is replaced by some approximation $r^{(0)}(x)$, the expression for the super-ratio $R(x)$ at the first order in $\delta r^{(0)}(x)$ can be safely used. From this expression one easily realizes that the
values of $r(x)$ calculated from Eq. (3) are closer to the exact ones than the approximation $r^{(0)}(x)$. This observation allows one to solve Eq. (3) by recurrence

$$r^{(n+1)}(x) = \frac{E(x) - 2 R[r^{(n)}(x)]}{R[r^{(n)}(x)] - 2 E(x)}$$

starting from a reasonable zero order approximation for $F^n_2(x)$.

To study the convergence of this recurrence relation, we evaluated both $E(x)$ and $R(x)$ using the nucleon SF obtained from the RSC plus CI interaction in Ref. [10]. The nucleon structure functions of Ref. [10] were used in $E(x)$, while for $R(x)$ the neutron one was arbitrarily modified by the factor $(1 + 0.5x^2)$ to generate the zero-order approximation $r^{(0)}(x)$. As shown in Fig. 2(a), a sequence which rapidly converges to $r(x)$ of Ref. [10] is obtained in the range $0 \leq x \leq 0.9$. In particular we find that up to $x = 0.9$ an accuracy better than 0.1% is obtained with only ten iterations. A convergence of a similar quality to the same function is also obtained using for $r^{(0)}(x)$ the DISF of Ref. [11]. On the contrary, if the momentum distribution is used for the evaluation of $R(x)$, instead of the SF, the iterative procedure converges to a function $r(x)$, which differs from the correct one more than 10% for $x \geq 0.8$. 

Figure 2. (a) The ratio $r(x)$ obtained from the recurrence relation (7), using the RSC interaction plus CI both for $E(x)$ and $R(x)$. Solid and dot-dashed lines correspond to the nucleon DISF of Ref. [10] and [11], respectively, while dotted line corresponds to the model of Ref. [10], multiplied by $(1 + 0.5x^2)$ used as our $r^{(0)}(x)$ (see text). Long-dashed, dashed, and short-dashed lines are $r^{(n)}(x)$ for $n = 3, 10, 20$, iterations, respectively. (b) The same as in (a), but dashed, short-dashed, and long-dashed lines are $r^{(20)}(x)$, obtained using for $R(x)$ the nucleon SF obtained from the RSC interaction without CI for $^3$He, the Av14 interaction and the Av14 interaction without CI for $^3$He, respectively [9]. The dot-dashed line is the result of the iterative procedure obtained using the nucleon momentum distribution for the evaluation of $R(x)$. 


In order to check the model dependence of our approach, we repeated the whole procedure, using for $R(x)$ SF corresponding to different interactions than the $RSC$ one plus $CI$ used for $E(x)$. In the range $0 \leq x \leq 0.85$ the ratio $r(x)$ extracted by the recurrence relation after twenty iterations differs from the one used for $E(x)$ less than 2.5%, for any of the considered $N-N$ interactions (see Fig. 2 (b)). We have also successfully applied our recurrence procedure to the extraction of $r(x)$ from In conclusion the proposed procedure, which has been successfully applied also to the $^2H-\text{proton}$ system and to the $^3He-^2H$ system, is able to yield reliable information on $F_n^2(x)$ in the $x$ range accessible at the upgraded TJNAF, whenever nucleon binding in nuclei is correctly taken into account.

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