Data selection for multi-task learning under dynamic constraints

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Abstract—Learning-based techniques are increasingly effective at controlling complex systems using data-driven models. However, most work done so far has focused on learning individual tasks or control laws. Hence, it is still a largely unaddressed research question how multiple tasks can be learned efficiently and simultaneously on the same system. In particular, no efficient state space exploration schemes have been designed for multi-task control settings. Using this research gap as our main motivation, we present an algorithm that approximates the smallest data set that needs to be collected in order to achieve high control performance for multiple learning-based control laws. We describe system uncertainty using a probabilistic Gaussian process model, which allows us to quantify the impact of potentially collected data on each learning-based controller. We then determine the optimal measurement locations by solving a stochastic optimization problem approximately. We show that, under reasonable assumptions, the approximate solution converges towards that of the exact problem. Additionally, we provide a numerical illustration of the proposed algorithm.

I. INTRODUCTION

The success of data-driven techniques in control crucially depends on the quality of the available training data set [1]–[3]. In reinforcement learning, this difficulty is tackled through task-oriented exploration, i.e., by collecting data that is particularly useful for the given task [3]. However, if the task changes, e.g., the system is required to follow a different reference trajectory, then the available data might be unsuited to train the corresponding control law, and a new exploration phase is necessary. This type of scenario is addressed by multi-task reinforcement learning approaches, where policies are sequentially trained for different tasks in order to achieve good overall performance [4]. However, multi-task reinforcement learning approaches often do not consider constraint requirements [5]–[7]. Furthermore, if all task-related exploration requirements are amalgamated into a single exploration phase, then the number of system interaction requirements to obtain good control performance across all tasks is potentially reduced. This is generally desirable, as system interactions are often considered costly [8].

Most techniques for system exploration aim to steer the state to regions that correspond to high system uncertainty [9], [10], i.e., they aim to achieve a globally accurate model. However, this is intractable for large state spaces, as it implies prohibitively long exploration periods. Moreover, some regions of the state space do not need to be explored in order to obtain good control performance. Hence, these approaches are not suited to efficiently collect data for multi-task reinforcement learning.

Efficiently exploring the state space of a system to gather data for multiple different tasks poses a twofold challenge. Firstly, the optimal set of hypothetical system measurements needs to be determined. Secondly, an efficient exploration trajectory needs to be determined. In this work, we address this dilemma by proposing an algorithm that approximates the minimal number of hypothetical measurement points required to achieve good control performance in several different tasks. This is the main contribution of our paper. We employ a probabilistic Gaussian process model to quantify model uncertainty, and measure control performance by computing the probability of constraint violation given dynamic constraints. Our algorithm employs a random sampling-based approximation, which we show to be exact as the number of samples tend to infinity.

This paper is structured as follows: After a formal problem definition in Sec. II, the considered Bayesian model is introduced, in Sec. III. Section IV presents the algorithm for approximating the optimal measurement locations, which is the main contribution of our paper. A numerical illustration, in Sec. V, is followed by a conclusion, in Sec. VI.

II. PROBLEM STATEMENT

We consider a stochastic nonlinear system of the form

\[ x_{t+1} = f(x_t, u_t) + g(x_t, u_t) + w_t \]

where \( x_t \in \mathbb{X} \subseteq \mathbb{R}^{d_x} \), \( u_t \in \mathbb{U} \subseteq \mathbb{R}^{d_u} \) are the system’s states, control inputs at time step \( t \in \mathbb{N} \), respectively. The system is perturbed by normally distributed process noise \( w_t \sim \mathcal{N}(0, I_{d_w}) \). The vector \( x_t := (x_t, u_t) \in \mathbb{X} \), where \( \mathbb{X} := \mathbb{X} \times \mathbb{U} \), concatenates the state \( x_t \) and the control inputs \( u_t \), and is introduced for the sake of brevity. The function \( f : \mathbb{X} \mapsto \mathbb{X} \) is known a priori, whereas \( g : \mathbb{X} \mapsto \mathbb{X} \) is an unknown function, for which we assume to have a probabilistic model, as discussed in Section III.

Remark I: Assuming that \( f(\cdot) \) is known does not constitute a restrictive requirement, as it encompasses the scenario

\[ \mathbb{P}(\cdot) \] denotes the power set operator. We employ bold notation to denote vectors and matrices and \( \leq \) to denote component-wise inequality. Given matrices \( A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times l} \), \( m, n, l \in \mathbb{N} \), we employ brackets accompanied by subscripts \( \left[ A \right]_{ij} \) to denote the entry in the \( i \)-th row and \( j \)-th column of \( A \), and brackets without subscripts \( \left[ AB \right] \) to denote the matrix concatenation of \( A \) and \( B \). \( \lceil \cdot \rceil \) denotes the ceiling operator, and \( I_n, n \in \mathbb{N} \) denotes the \( n \)-dimensional identity matrix.

Let \( \mathbb{N} \) denote the positive integers, \( \mathbb{N}_0 := \mathbb{N} \cup \{ 0 \} \) the non-negative integers, \( \mathbb{R} \) the real numbers, and \( \mathbb{R}_- \) the negative real numbers. A matrix \( A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times l} \), \( m, n, l \in \mathbb{N} \), we employ brackets accompanied by subscripts \( \left[ A \right]_{ij} \) to denote the entry in the \( i \)-th row and \( j \)-th column of \( A \), and brackets without subscripts \( \left[ AB \right] \) to denote the matrix concatenation of \( A \) and \( B \). \( \lceil \cdot \rceil \) denotes the ceiling operator, and \( I_n, n \in \mathbb{N} \) denotes the \( n \)-dimensional identity matrix.
without any precise prior system knowledge, i.e., \( f(\hat{x}_t) = x_t \).

We assume that we are given \( x \) without any precise prior system knowledge, i.e., \( f \) is commonly used control laws, e.g., PID-controllers and neural networks. This applies for many cases of energy or input saturation constraints, or polynomial, smooth with respect to the state. This applies for many time-varying reference trajectories. This type of control law is frequently employed in learning-based settings [10], [11].

For the sake of notational simplicity, we henceforth use time-varying reference trajectories. This type of control law can be achieved:

\[
\mathcal{D}_N := \{ (\hat{x}^{(i)}_t, f(\hat{x}^{(i)}_t) + g(\hat{x}^{(i)}_t) + w^{(i)}_t) \}_{i \in \mathbb{N} \leq N},
\]

which is to be collected, e.g., via system exploration. The third argument of \( w^{(\cdot, \cdot, \cdot)} \) is the time step \( t \), which accounts for any time-dependent component of the control laws, e.g., time-varying reference trajectories. This type of control law is frequently employed in learning-based settings [10], [11].

For the sake of notational simplicity, we henceforth use the locations of \( N \) system measurements. Furthermore, make the following assumption.

**Assumption 1:** The control laws \( w^{(\cdot, \cdot, \cdot)} \) are real analytic with respect to the first argument.

In particular, this implies that the control laws \( w^{(\cdot, \cdot, \cdot)} \) are smooth with respect to the state. This applies for many commonly used control laws, e.g., PID-controllers and neural networks with smooth activation functions.

Each control law \( w^{(\cdot, \cdot, \cdot)} \) is required to fulfill a different task, which is expressed as a series of constraints

\[
h_j^t(\hat{x}^t_i) \leq 0, \quad \forall \ t \in \mathbb{N} \leq H, \ j \in \mathbb{N} \leq L
\]

over a finite time horizon of \( H \) steps. Here \( h_j^t : \mathbb{R}^S \to \mathbb{R} \) are nonlinear constraint functions, \( S \in \mathbb{N} \) denotes the number of constraints corresponding to the \( j \)-th control law, \( \hat{x}^t_i := (x_n, w^t(x_t, D)) \). Such constraints are often linear, e.g., in the case of energy or input saturation constraints, or polynomial, e.g., in the case of tracking error performance requirements.

In this work, we consider the following, more general case:

**Assumption 2:** The entries \( h_j^t(\cdot) \) of the functions \( h_j^t(\cdot) \) are non-constant and real analytic [12].

**Remark 2:** The proposed method extends straightforwardly to the more general case where both the horizon \( H \) and number of constraints \( S \) are different for each control law. However, we do not consider this case, as it would incur cumbersome notation.

We aim to obtain the smallest possible set of measurement locations \( \tilde{X}^* := \{ (\tilde{x}^{(i)}_t, s) \}_{i = 1, \cdots, N} \), such that the corresponding data set \( \mathcal{D}^* \), if collected and used to design the control laws \( w^{(\cdot, \cdot, \cdot)} \), yields system trajectories that satisfy (3) with high probability, i.e.,

\[
\hat{X}^* = \arg \min_{X_N \in \mathcal{P}(\tilde{X})} \mathbb{E}_{N \in \mathcal{P}(\tilde{X})} \text{ s.t. } \mathcal{D}_N = \{ (\hat{x}^{(i)}_t, f(\hat{x}^{(i)}_t) + g(\hat{x}^{(i)}_t) + w^{(i)}_t) \}_{i \in \mathbb{N} \leq N}.
\]

where \( 0 < \delta < 1 \) is a predetermined scalar that specifies the desired probability of constraint violation. The probability operator \( \mathbb{P}(\cdot) \) describes the probability of an event given process noise \( w_t \) and the a priori distribution that we assume for the unknown function \( g(\cdot) \), as discussed in Section III.

**Remark 3:** Since the system dynamics are unknown, the measurements in an arbitrary data set \( \mathcal{D}_N \) are hypothetical. However, by assuming a priori distribution over \( g(\cdot) \), we are able to determine the impact of measurement locations \( \tilde{X}_N \) on control performance.

**Remark 4:** In order to guarantee convergence of the method proposed in this paper, we require a solution \( \hat{X}^* \) of (3) to satisfy the chance constraints strictly. However, this is not a severe restriction, as \( \delta \) is a design choice.

Finding an optimal set \( \hat{X}^* \) under uncertainty is generally impossible without considering further assumptions. Hence, we restrict ourselves to the case where the controllers are specified in a way that the desired closed-loop behavior is achievable:

**Assumption 3:** The optimization problem (3) is feasible for a finite \( \hat{X}^* \), i.e., \( |\hat{X}^*| =: N^* < \infty \).

Furthermore, we assume that the optimal data set is contained within a known compact subset of \( \tilde{X} \):

**Assumption 4:** There exists a known compact subset \( \hat{X}^* \subset \tilde{X} \), such that \( \tilde{x}^{(i),*} \in \hat{X}^* \) for all \( i \in \{1, \cdots, N^*\} \). This does not constitute a very restrictive assumption, since control tasks are typically restricted to a compact subset of the state space, which in turn implies that only system information within a compact subset is required to achieve good control performance.

In order to streamline notation, we henceforth subsume measurement data and system trajectories of (1) as

\[
\mathcal{Z}_N := \{ (\tilde{z}_n, f(\tilde{z}_n) + g(\tilde{z}_n) + w_n)_{n \in \mathbb{N} \leq N} \}
\]

where \( N := N + L(H + 1) \),

\[
\tilde{z}_n := \begin{cases}
\tilde{x}^{(d)}_n, & n = 0 \cdots, N - 1 \\
\tilde{x}^{(n)}_n, & n = N, \cdots, N^* 
\end{cases}
\]

\[
d_n := n + 1, j_n := [(n - N)/(H + 1)], t_n := n - N - (j(n) - 1)(H + 1).
\]

### III. PROBABILISTIC MODEL

In order to quantify the uncertainty corresponding to the unknown component \( g(\cdot) \), we assume a GP distribution over \( g(\cdot) \). Formally, a GP is a collection of random variables, of which any finite subset is jointly normally distributed [13]. In order to assess how data collected in the future will potentially affect control performance, we need to quantify how model uncertainty decreases as new data points are added. To this end, we consider hypothetical data sets \( \mathcal{S}_N := \{ (\tilde{x}_n, f(\tilde{x}_n) + g^*(\tilde{x}_n) + w_n)_{n \in \mathbb{N} \leq N} \} \), which are sampled from the GP distribution. Here we employ the superscript \( s \) to emphasize that \( g^*(\cdot) \) is a sample function evaluation, as opposed to an evaluation of the true function \( g(\cdot) \). This is explained in detail in the sequel.

**Remark 5:** A GP model can be trained using measurement data from the true system (1). For the sake of notational simplicity, we analyze the setting where no prior measurement
data from the true system is available, and show exclusively how to draw samples from a GP in a recursive fashion. However, this does not constitute a loss of generality, a posteriori GP distribution after training satisfies the requirements used in this paper [13].

We begin by introducing GPs for the case where \( d_x = 1 \), and then describe how they can be generalized to a multivariate setting. A GP is fully specified by a mean function, which we set to zero without loss of generality [13], and a positive definite kernel \( k: \mathbb{R}^{d_x} \times \mathbb{R}^{d_x} \rightarrow \mathbb{R} \). Given a sample data set \( S_n \), a subsequent sample evaluation at an arbitrary augmented state \( \tilde{x} \) is normally distributed, i.e.,

\[
g^*(\tilde{x}) \sim \mathcal{N}(\mu_{n+1}(\tilde{x}), \sigma^2_{n+1}(\tilde{x}))
\]

Here \( p(\zeta_n) = \mathcal{N}(0, I_{2d_x}) \). Note that we require the random variables \( \zeta_i \) to have dimension \( 2d_x \) in order for the GP samples

\[
g^*(\tilde{s}_n) = \mu(\tilde{s}_n) + \sigma_i(\tilde{s}_n)[\zeta_n]_{1:d_x},
\]

where \( \zeta_n[1:d_x] \) denotes the first \( d_x \) entries of \( \zeta_i \), to be uniquely defined [13].

Since our goal is to find the smallest possible set of measurement points \( \tilde{X}^* \), it is reasonable to assume that \( \tilde{X}^* \) does not contain any measurement locations that provide identical information. In terms of a GP distribution, this is expressed as follows:

Assumption 6: Let \( \tilde{X}^* \) be the minimizer of (6). Then \( \sigma_n(\tilde{x}^{(n+1),*}) \neq 0 \) holds for \( n \in \mathbb{N}_{N-1} \).

For many commonly used kernels, e.g., squared exponential kernels, Assumption 6 implies that \( \tilde{X}^* \) does not contain identical measurement locations.

### IV. Two Stage Optimization

We now describe the optimization scheme used to approximate the optimal solution of (4), and provide a corresponding theoretical analysis.

Since each control law \( \mathbf{u}'(\cdot, \cdot, \cdot) \) is fully specified by the training data \( D_N \), the probability distribution of a trajectory obtained using any two different control laws \( \mathbf{u}'(\cdot, \cdot, \cdot) \), \( \mathbf{u}''(\cdot, \cdot, \cdot) \), \( i \neq j \), are conditionally independent given \( D_N \), i.e.,

\[
p(\tilde{x}_t^i | D_N) = p(\tilde{x}_t^i | D_N)p(\tilde{x}_t^j | D_N) \quad \text{for all } t, \tau \in \{1, \cdots, H\}.
\]

Moreover, since the control laws \( \mathbf{u}' \) are deterministic given \( D_N \), we have \( p(\tilde{x}_t^i | D_N) = p(\tilde{x}_t^i | D_N) \). Hence, similarly to (10), computing the probability of constraint satisfaction for a set of measurement points \( \tilde{X}_N \) amounts to evaluating the integral

\[
C_N \left( \tilde{X}_N \right) := \mathbb{P} \left( \mathbf{h}_t^i(\tilde{x}_t^i) \leq 0, \quad \forall \ t \in T_{N} \leq H, \ j \in \mathbb{N}_{N} \leq L \right)
\]

\[
= \prod_{n=1}^{N} \int_{X_{n}^N} \mathbf{1}_{R_{d_x}^N}(\tilde{s}_n,\zeta_n) \ p (\tilde{s}_n, \zeta_n) \ d\zeta_n
\]

(12)

\[
\mathbf{1}_{R_{d_x}^N}(\cdot) \text{ is the indicator function of } \mathbb{R}_{d_x}^N.
\]

Generally, computing (12) is intractable, which renders a direct approach to solving (4) infeasible. In this work, we employ a two-stage optimization approach, which yields an approximation of the optimal solution \( \tilde{X}^* \) with probability 1. This is achieved by repeatedly defining a fixed number of data points \( N \) and maximizing the Monte Carlo approximation \( C_N^M(\tilde{X}_N) \) of (12). If the maximal approximate probability of constraint satisfaction is lower than the desired bound \( 1 - \delta \), the number of data points \( N \) is increased and the procedure is repeated. This is detailed in Algorithm 1.

#### A. Theoretical Analysis

We now derive formal guarantees for the approximate solution \( \tilde{X}_N^M \) obtained with Algorithm 1. To this end, we prove some preliminary results.
This enables us to show that the state is on a set of measure $M$, the state lies within an arbitrary set of measure zero at time $\sigma \in [0, 1]$, and $\sigma$ is also real analytic. These are well-known properties of real-analytic functions. As $\sigma$ is a real-analytic function of $x$, and we can apply the same argument as in the case $t = 0$ and obtain the desired result for an arbitrary $t$ and $j = 1$.

Due to Assumption 5, we can assume that $\sigma_{N}(\cdot)$ is invertible for data sets of size $N \neq 0$, which enables us to extend the proof to arbitrary $N$ using the same argument.

This directly yields the following result:

**Lemma 3:** Let Assumptions 2, 5, and 6 be satisfied. Then $P(h_{1}^{j}(\tilde{x}) = 0) = 0$ holds for all $t \in [0, T], j \in N_{\mathbb{N}}$.

Proof: Since $[h_{1}^{j}(\tilde{x})]_{i}$ are real-analytic, $[h_{1}^{j}(\tilde{x})]_{i} \neq 0$ holds for all $i \in N_{\mathbb{N}}$ and all $\tilde{x} \in \mathbb{X}$ up to a set of measure zero. By employing Lemma 2 and the union bound, we obtain

$$
P\left(h_{1}^{j}(\tilde{x}) = 0\right) \leq \bigcup_{i \in N_{\mathbb{N}}} P\left(h_{1}^{j}(\tilde{x})_{i} = 0\right) = 0. \quad (14)$$

We now show that the sample average approximations used in Algorithm 5 converge to the true probabilities of constraint satisfaction (12).

**Lemma 4:** Let Assumptions 1, 3, 5, and 6 hold, and let $\tilde{X}^{*}$ be given as in Assumption 4. Then, for an arbitrary $N \in \mathbb{N}$, the expected value of $C_{N}(\cdot)$ is finite valued and continuously differentiable on $(\tilde{X}^{*})^{N}$, and $C_{N}(\cdot)$ converges to $C_{N}(\cdot)$ with probability 1 uniformly in $(\tilde{X}^{*})^{N}$ as $M \to \infty$.

**Remark 6:** The proofs of Lemma 4 and Theorem 1, which we state in the following, require Theorem 7.48 and Theorem 5.4 from [14], respectively. Due to space limitations, we do not include them here. However, to facilitate interpretation, we enumerate the technical statements in the proofs of Lemma 4 and Theorem 1 such that they correspond to Theorem 7.48 and Theorem 5.4 from [14].
**Proof of Lemma**  We show that the the approximation \( C_N^m(\cdot) \) satisfies all conditions of [14, Theorem 5.4], enumerated in the sequel as i)-iii), which directly yields the desired result.

i) Due to Lemma 3 the functions \( 1_{\mathbb{R}^d} \left( h_t^j(\tilde{s}^m_{n,t,j}) \right) \) are uniquely defined and continuous for an arbitrary \( t, j \in \mathbb{N} \) and almost every sample \( \zeta^m_n \) [14]. Hence, \( C_N^M(X_N) \) is continuously differentiable at any \( X_N \in (\mathbb{R}^*)^N \) for almost every sample \( \zeta^m_n \).

ii) Since \( C_N^M(X_N) \leq 1 \) and \( X_N \in (\mathbb{R}^*)^N \) is compact, the absolute value of \( C_N^M(X_N) \) is upper bounded by an integrable function on \( X_N \in (\mathbb{R}^*)^N \).

iii) The samples \( \zeta_n \) are i.i.d.

**Lemma 5:** Let Assumptions 1-6 hold. Moreover, let \( C_N(\cdot) \) be the probability of constraint satisfaction for a data set of size \( N \), let \( C_N^M(\cdot) \) correspond to its SAA, and let \( \hat{X}^M_N \) denote the output of Algorithm 1. Then, with probability 1, for every \( \varepsilon > 0 \), there exists an \( M_\varepsilon \), such that \( C_N(\hat{X}^M_N) - C_N \leq \varepsilon \) holds for all \( M \geq M_\varepsilon \).

**Proof:** We show that the conditions of [14, Theorem 5.4] are satisfied by \( C_N(\cdot) \) and \( C_N^M(\cdot) \), which yields the desired result. In the following, we employ i)-iv) to enumerate the required conditions, which corresponds to the enumeration in [14, Theorem 5.4].

i) Due to Assumption 4 \((\mathbb{R}^*)^N\) is non-empty and compact.

ii) Due to Lemma 3 \( C_N(\cdot) \) is finite valued and continuously differentiable on \( (\mathbb{R}^*)^N \).

iii) Due to Lemma 4 \( C_M(\cdot) \) converges to \( C_N(\cdot) \) with probability 1 as \( M \to \infty \), uniformly in \((\mathbb{R}^*)^N\).

iv) Since we restrict ourselves to the set \((\mathbb{R}^*)^N, \hat{X}^M_N \in (\mathbb{R}^*)^N \) holds trivially for all \( M \).

**V. Numerical Illustration**

We illustrate the proposed approach with a system of the form given by (1), where \( g(\hat{x}) = (u_1, u_2)^T \),

\[
f(\hat{x}) = \begin{pmatrix} x_1 + (\cos(2\pi x_1) - 1)x_2 \\ 1 + \exp(-5x_1) - \sin(\pi x_2) \end{pmatrix},
\]

and \( \omega_t \sim \mathcal{N}(0, \text{diag}(0.01, 0.01)) \). Due to its highly nonlinear dynamics, it is impossible to extrapolate the system’s behavior from locally collected data. Hence, unless the regions of interest for each control task overlap, each control task requires different measurements to achieve good performance.

We assume to know that \( f(\cdot) \) depends exclusively on \( x \), hence we employ a GP that takes only the state \( x \) as input. Moreover, we employ a squared-exponential kernel \( k(\cdot, \cdot) \) for the GP, which is able to approximate a continuous function arbitrarily accurately on compact sets [15]. We employ GP-based feedback linearizing control laws \( w^j(x, t) = -\mu_{N,t}(x) + x_{ref}^j(t) \) with 3 different reference trajectories

\[
x_{ref}^1(t) = 0
\]

\[
x_{ref}^2(t) = [\sin(2\pi t/50) \cos(2\pi t/50)]^T
\]

\[
x_{ref}^3(t) = [2\sin(2\pi t/25) \cos(2\pi t/100)]^T
\]

The GP used to compute the mean \( \mu_{N,t}(\cdot) \) is identical to the one used to obtain the approximate optimal data set \( \hat{X}^M_N \). Each control law is required to fulfill a single tracking performance requirement \( h_t^j(x) \leq 0, j = 1, 2, 3 \)

\[
h_t^1(x) = \|x - x_{ref,t}\|_2 - \varphi(t), \quad j = 1, 2,
\]

\[
h_t^3(x) = |x_1| - 5/2,
\]

and \( \varphi(t) := \max\{3 \exp(-t/5), 0.1\} \), over a time horizon of \( H = 100 \) steps. We assume that the optimal data set is contained within \( \hat{X}^* = [-3, 3]^2 \), since the control objectives are restricted to this region. Furthermore, we are given 100 prior measurements taken from random samples of the true system, which we use to train the GP. The number of samples used to obtain the approximate optimal data set \( \hat{X}^M_N \) is set to \( M = 100 \), and the desired probability of constraint satisfaction is set to \( 1 - \delta = 0.01 \).

In order to solve the approximate optimization problem, we search for a solution by minimizing the surrogate function

\[
\frac{1}{M} \sum_{m=1}^{M} \prod_{t=1}^{L} \prod_{h=1}^{H} h_t^j(\tilde{s}_{h, m}) \mathbb{1}_{\mathbb{R}^d_+} \left( h_t^j(\tilde{s}^m_{n,t,j}) \right),
\]

which enables us to employ gradient-based methods.

We apply the DS-ML algorithm 10 times using randomly sampled starting points \( x_0 \in \mathcal{U}([-3, 3]^2) \), where \( \mathcal{U}(\cdot) \) denotes a uniform distribution, and obtain an approximate optimal data set \( \hat{X}^M_N \) after \( N \in \{6, \cdots, 9\} \) iterations.
the individual control tasks. However, since we employed
intuitive, since this is the region where the desired trajec-
tories were achievable after $N \in \{6, \ldots, 9\}$ iterations of the DS-ML algorithm.

All approximate optimal sets $\tilde{X}_{N,\star}^{M,*}$ correspond roughly
to points within the circle given by $x_1(t)$. This result is	intuitive, since this is the region where the desired trajectories
specified by eqs. \textcolor{red}{(13)} and \textcolor{red}{(19)} overlap the most. Moreover, as can be seen in Figure \textcolor{red}{1} the approximate optimal solution $\tilde{X}_{N,\star}^{M,*}$ regions that are both unexplored and of interest to
the individual control tasks. However, since we employed
a gradient-based solver, sub-optimal solutions are to be
expected. This is also the case in Figure \textcolor{red}{1} where some data points are close to already available prior data, i.e., a local minimum was found.

After every completion of the DS-ML algorithm, measurements of the true system at the approximate optimal set $\tilde{X}_{N,\star}^{M,*}$ are collected, and we carry out 100 Monte Carlo simulations of the true system. This results in no constraint violation except for task $j = 2$. However, constraint violations are small, as can be seen in Figure \textcolor{red}{3} which indicates that the proposed method yielded a good approximate optimal data set $\tilde{X}_{N,\star}^{M,*}$.

VI. CONCLUSION AND FUTURE WORK

This paper presents an algorithm to approximate the
smallest training set required for successfully completing
multiple tasks in learning-based control. We use a sample-

based approximation that approximates the correct solution
arbitrarily well with probability 1 as the number of samples
increases. In a numerical simulation, the approximate optimal
data sets computed with the proposed method are shown to
yield adequate data sets for multiple tasks.

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