A predictive governing control method of the pumped-storage unit based on lumped-parameter model equivalence

Yang Zheng1*, Qijuan Chen1, Wanying Liu1, Donglin Yan1

1. School of Power and Mechanical Engineering, Wuhan University, Wuhan 430072, P.R. China

Corresponding author: zhengyang@whu.edu.cn

Abstract. The speed governor is known as one of the most important control parts in the pumped-storage unit (PSU). When operating in transient processes, the stability of the PSU’s hydraulic and mechanical states maintains by the precise regulation of the speed governor. For the sake of improving the governing control performances, the lumped-parameter equivalent circuit model is first introduced for precise modelling of hydraulic-mechanical characteristics of PSU. Subsequently, a multi-variable generalized predictive control system based on receding horizon technique is applied at every sampling instant to optimize the control trajectory within certain finite time horizon. To validate the effectiveness of the proposed predictive control method, the simulation experiment of governing movements during turbine start-up process is conducted under a nonlinear lumped-parameter numerical simulation platform of PSU by comparative case studies, the overall control performance of the proposed control scheme is evaluated and compared with that of the traditional PID algorithm. Simulation results indicate that the proposed governing control method can achieve stronger frequency fluctuation alleviation and hydraulic oscillation damping performances than traditional PID.

Key words: pumped-storage unit, governing control, generalized predictive control, lumped-parameter model

1. Introduction

Pumped-storage unit (PSU) plays the paramount peak and frequency modulation role in modern power system with the ever-increasing renewable energy sources online [1]. Compared with traditional hydropower units, PSU possesses the unique characteristics including a wide range of working head and frequent operating condition transitions. The pipeline system of the PSU often has a complicated hydraulic structure, and the stability of the reversible pump-turbine is facing challenges because of the well-known “S” characteristic in turbine mode and the “hump” area in pumping mode. Seeing as the structural complexity of PSU and its connected pipeline system, many important hydraulic-mechanical coupling characteristics of PSU still need further investigation. Under the circumstances of the rapid development of the intelligent power grids, research on the dynamic behaviours of the PSU and its control optimization has become a hotspot in recent years.

The speed governor is one of the major controllers of PSU. Its mathematical modelling and advanced control have attracted many researchers’ attention. The governing system of PSU is composed of
pipeline system, pump-turbine, generator-motor and governor. In General, the conventional modelling methods for governing system of PSU can be divided into two approaches: one is based on state-space model and the other is based on characteristic line model. The state-space model follows a lumped-parameter framework and is convenient for advanced control theory design, while the characteristic line model follows a distributed-parameter framework and performs well in precise transient behaviour description of the whole system. Although characteristic line model inherits many advantages and has been widely used in both academic study and engineering practice, the drawback of its heavy computation burden prohibits its further development and application. For the sake of reconcile the conflict between modelling efficiency and precision, the equivalent circuit model (ECM) is proposed and used in governing control for PSU [2, 3]. Compared with the distributed-parameter characteristic-line method, ECM is a lumped-parameter model but precisely models each component in the hydraulic system based on the circuit theory.

As for the governing control of PSU, many advanced control theories including fractional-order PID control [4, 5], fuzzy logic control [6], and predictive control [7-10] have been investigated and compared with the classic PID. Among these control schemes, predictive control has been extensively studied for its unique capability of multi-step state prediction and explicit constraint treatment. In order to improve the overall performance of the governing control in PSU, this paper presents a novel multi-variable constraint generalized predictive control (CGPC) strategy to closely track the unit frequency and naturally takes system state constraints into consideration. The CGPC is acted on a lump-parameter equivalent circuit model for the governing system of PSU. The case study has been conducted under the start-up condition at the turbine mode through numerical simulations. The simulation results indicate that the proposed CGPC performs better than the traditional PID in frequency tracking and hydraulic fluctuation damping during the typical transient process.

The rest of the paper is organized as follows. Nonlinear ECM of the governing system of PSU is given in Section 2. Subsequently, the multi-variable CGPC is formulated in Section 3. Then, Section 4 presents the case study and relevant analysis. Finally, conclusions are summarized in Section 5.

2. Nonlinear model of the governing system of PSU
The conventional one-dimensional momentum and continuity equations [7] are often used to describe the hydraulic characteristics of the hydraulic structures including the pumped storage unit. A famous distributed-parameter modelling scheme for the hydraulic power generating unit and pumped-storage unit is the method of characteristics. However, the distributed-parameter model greatly increases the computational complexity although the hydraulic and mechanical dynamics can be clearly reflected. In order to reduce the computation cost while maintain the modelling accuracy to a certain extent, the equivalent circuit model is proposed to describe the hydraulic behaviour of different hydraulic structures in the pumped-storage plant.

2.1 Modeling of the pipeline system
In this modelling approach, the hydraulic pressure and the discharge are analogue to voltage and current, respectively. Besides, the resistance, inductance and capacitance (LFC) in the equivalent circuit are defined as follows,

\[
\begin{align*}
R &= \frac{f |Q|}{2gDA^2} dx \\
L &= \frac{1}{gA} dx \\
C &= \frac{2A}{a^2} dx
\end{align*}
\]

where, \( Q, a, f, D, A \) denote the discharge, water hammer wave velocity, friction coefficient, pipe diameter and pipe sectional area, respectively.
The hydraulic behaviour of the surge tank can also be described with equivalent circuit model. The RLC parameters are expressed as Eq. (2),

\[
\begin{align*}
R_{\text{sur}} &= \frac{f_{\text{im}}(Q_s)|Q_s|}{2gA_{\text{f}}}, \\
C_{\text{im}} &= A(z), \\
L_{\text{sur}} &= \int_{z_{\text{in}}}^{H_s} \frac{1}{gA(z)} dz
\end{align*}
\]

Where, \( z \) denotes the height, \( f_{\text{im}} \) the loss coefficient of the impedance orifice, \( Q_s \) the surge tank’s discharge, \( z_{\text{in}} \) the height of the inlet of the surge tank, \( H_s \) the height of the hydraulic grade line.

The whole pipeline system of the pumped-storage governing system can be expressed with the following Kirchhoff equations in matrix form.

\[
\frac{dX}{dt} + BX = C
\]

where, \( X \) is the state variable vector, \( C \) represents the boundary vector, \( A \) and \( B \) are system parameter matrices. The detailed definitions of system variables in Eq. (3) and the solution of the equation can refer to Reference [3].

2.2 Models of pump-turbine, generator-motor and speed governor

In engineering practice, complete characteristic curves are utilized to calculate the torque and discharge of the pump turbine along with the characteristic method for the water delivery system. The detailed calculation process for torque and discharge has been clearly given in [7, 10]. The characteristic curves of pump turbine in this study are shown in Figure 1 [10].

![Figure 1. The characteristic curves of the pump-turbine](image)

The block diagram of the servo system transfer function is depicted in Figure 2, where \( k, T_{11} \) denote the gain and time constant of the main pressure value and \( T_s \) is the time constant of servomotor. The whole system comprises dead zone, pressure valve, speed limit, servomotor and saturation segments.

![Figure 2. Block diagram of servo system transfer function](image)

Since the electro-magnetic effect is often ignored in the study of governing control, the mathematic model of the generator-motor can be simplified as a first-order rotor motion equation in Eq. (4),
\[
T \frac{dn}{dr} + e_n (n-n_0) = m_i - m_e
\]

where, \(m_i\) and \(m_e\) represent the relative mechanical torque deviation and the relative damping torque deviation, respectively. \(n\) and \(n_0\) denote the relative speed and the rated speed, respectively. And \(e_n\) is the self-regulation coefficient for the PSU.

3. Multi-Variable Constrained Generalized Predictive Control

The well-known CARIMA model, which describes the linear relationship between system’s output and input sequences in a parametric approach, is widely used as the predictive model in CGPC and its modifications. In order to consider the constraints on system states, multi-variable CARIMA model are introduced in this study to describe the dynamic characteristic of the state variables of the system.

3.1 Multi-variable CARIMA model

The multi-variable CARIMA model can be stated as Eq.(5),

\[
\begin{bmatrix}
A(z^{-1}) & 0 \\
0 & A(z^{-1})
\end{bmatrix}
\begin{bmatrix}
f(k) \\
y(k)
\end{bmatrix}
= z^{-d}
\begin{bmatrix}
B_1(z^{-1}) \\
B_2(z^{-1})
\end{bmatrix}
\begin{bmatrix}
u(k) \\
y(k)
\end{bmatrix} + \frac{1}{\Delta}
\begin{bmatrix}
\xi_i(k) \\
\xi_j(k)
\end{bmatrix}
\]

where, \(d\) represents the length of time delay; \(\Delta = 1 - z^{-1}\) is difference operator, \(u(k), f(k), y(k)\), \(\xi(k)\) denote the control input, frequency, guide vane opening and white noise of the pumped-storage governing system, respectively. \(A(z^{-1}), B_i(z^{-1})\) \(i=1, 2\) can be expressed with Eq.(6) as follows,

\[
A(z^{-1}) = a_0 + a_1 z^{-1} + \cdots + a_n z^{-n} \\
B_i(z^{-1}) = b_{i0} + b_{i1} z^{-1} + \cdots + b_{in} z^{-n}
\]

Eq.(6) can be simplified as Eq.(7) by removing the difference operator in the denominator on the right side,

\[
\begin{bmatrix}
\bar{A}(z^{-1}) & 0 \\
0 & \bar{A}(z^{-1})
\end{bmatrix}
\begin{bmatrix}
f(k) \\
y(k)
\end{bmatrix}
= \begin{bmatrix}
\bar{B}_1(z^{-1}) \\
\bar{B}_2(z^{-1})
\end{bmatrix}\Delta u(k-d) + \begin{bmatrix}
\bar{\xi}_i(k) \\
\bar{\xi}_j(k)
\end{bmatrix}
\]

where, \(\bar{A}(z^{-1}) = A(z^{-1})\Delta = 1 + \bar{a}_0 z^{-1} + \cdots + \bar{a}_n z^{-n}\), \(n_z = n_a + 1\), \(\bar{a}_{in} = -a_{in}\), \(\bar{a}_j = a_j - a_{-i,j}\) and \(1 \leq j \leq n_a\). Besides, values of the system parameters in \(A(z^{-1}), B_i(z^{-1})\) are updated in each sampling instant according to the system current and historical states.

3.2 Rolling optimization of CARIMA

By transforming the white noise as \(\xi(k) = \bar{A}y(k) - B\Delta u(k-1)\), the prediction equations for the two output variables can be simplified as Eq.(8),

\[
\begin{align*}
\hat{f}(k+j) &= E_{ij}\xi_i(k+j) + F_{ij}\Delta u(k+j-1) + G_{ij}f(k) \\
\hat{y}(k+j) &= E_{ij}\xi_j(k+j) + F_{ij}\Delta u(k+j-1) + G_{ij}y(k)
\end{align*}
\]

where,

\[
\begin{align*}
E_{ij}(z^{-1}) &= g_{i0} + g_{i1}z^{-1} + \cdots + g_{in_z}z^{-n_z} \\
F_{ij}(z^{-1}) &= f_{i0} + f_{i1}z^{-1} + \cdots + f_{in_z}z^{-n_z}
\end{align*}
\]

\(n_{zj} = j - 1\), \(n_{qj} = n_a - 1\), \(n_{i} = n_{q} + j - 1\).

The output trajectories in Eq.(8) of the system can be obtained iteratively by solving the well-known Diophantine equations [7]. Due to space cause, the detailed procedure is omitted here. Taking the overall performances of output trajectory within prediction horizon and control movement trajectory within control horizon into consideration, the general cost function \(J(k)\) of CGPC can be formulated as Eq.(19) in the matrix format,
\[ J(k) = \min E \left\{ \sum_{i=1}^{N_u} \left[ f(k+i | k) - f_s(k+i | k) \right]^2 + \sum_{i=1}^{N_y-1} r_i \Delta u(k+i | k)^2 \right\} \]  

(9)

Subject to

\[ \Delta U_{\text{min}} \leq \Delta U \leq \Delta U_{\text{max}} \]  

(10a)

\[ U_{\text{min}} \leq U \leq U_{\text{max}} \]  

(10b)

\[ y_{\text{min}} \leq y \leq y_{\text{max}} \]  

(10c)

\[ f_{\text{min}} \leq f \leq f_{\text{max}} \]  

(10d)

where, the weighting matrix of the control increments. The vectors \( \Delta U_{\text{max}} = [\Delta u_{\text{max}} \ldots \Delta u_{\text{max}}]^T \), \( U_{\text{max}} = [u_{\text{max}} \ldots u_{\text{max}}]^T \), \( U_{\text{min}} = [u_{\text{min}} \ldots u_{\text{min}}]^T \), the dimensions of the four vectors are all \( N_u \times 1 \). \( y_{\text{max}} = [y_{\text{max}} \ldots y_{\text{max}}]^T \), \( y_{\text{min}} = [y_{\text{min}} \ldots y_{\text{min}}]^T \), \( f_{\text{max}} = [f_{\text{max}} \ldots f_{\text{max}}]^T \), \( f_{\text{min}} = [f_{\text{min}} \ldots f_{\text{min}}]^T \) the dimensions of the four vectors are all \( N_p \times 1 \).

The aforementioned cost function of the CGPC can be solved as a quadratic programming (QP) problem. The extensively studied active set method is applied here to obtain the optimal solution of the system control input.

3.3 Constraint treatments

The GPC has been proved to be capable of explicitly treat different types of system constraints. In many existing publications, the input and output constraints are naturally converted to the standard form of the inequality constraints in the cost function. However, the state constraints in GPC is barely mentioned in the related research. In this paper, the constraints on the opening and closure velocity of the guide vane is considered. This type of constraint is treated as state inequality constraints with multi-variable CARIMA equations.

3.3.1 Constraints on control movement. The constraints on the control movement within prediction horizon \( N_p \) indicates the control energy cost of the system, as stated in Eq.(11).

\[ \left[ \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] \Delta u_{\text{min}} \leq I_{N_u \times N_u} \Delta U \leq \left[ \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] \Delta u_{\text{max}} \]  

(11)

3.3.2 Constraints on the amplitude of control signal. The constraints on the amplitude of the control trajectory determine its upper and lower boundary. The control signal amplitude constraint is stated as Eq.(12),

\[ \left[ \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] u_{\text{min}} - u(k-1) \leq \left[ \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] \Delta U \leq \left[ \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] u_{\text{max}} - u(k-1) \]  

(12)

3.3.3 Constraints on system frequency. Constraints on the predicted system frequency can also be converted to the constraints on system input, the system output trajectory can be deduced as Eq.(13),

\[ \left[ \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] f_{\text{min}} - F_{12} \Delta U(k) - G_i f(k) \leq F_i \Delta U \leq \left[ \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] f_{\text{max}} - F_{12} \Delta U(k) - G_i f(k) \]  

(13)

3.3.4 Constraints on guide vane opening and closing velocity. The treatment of the constraints on guide vane opening and closing velocity is similar to that of the constraints on the amplitude of system frequency. It utilizes the other CARIMA equation in the CGPC. In addition, the standard form for QP is presented in Eq.(14) for it is the change rate of the corresponding system state.
\[
\begin{bmatrix}
1 & \cdots & 1
\end{bmatrix}^T \hat{y}_{\text{min}} - C_{\alpha} F_{22} \Delta U(k) - C_{\alpha} G_{2} y(k) + \begin{bmatrix} y(k) & 0 & \cdots & 0 \end{bmatrix} \leq C_{\alpha} F_{21} \Delta U
\]
\[
C_{\alpha} F_{31} \Delta U \leq \begin{bmatrix}
1 & \cdots & 1
\end{bmatrix}^T \hat{y}_{\text{max}} - C_{\alpha} F_{32} \Delta U(k) - C_{\alpha} G_{3} y(k) + \begin{bmatrix} y(k) & 0 & \cdots & 0 \end{bmatrix}
\]
(14)

where, the covert matrix \( C_{\alpha} = \begin{bmatrix} -1 & 1 & \cdots & -1 & 1 \end{bmatrix} \).

4. Case study and result analysis

The turbine start-up process is taken as the test scenario in this study. The working head of the pump-turbine locates at \( H_r = 546 \text{ m} \) in the numerical simulation, which is close to the rated head \( H_{tr} = 540 \text{ m} \). In this case, the PSGS is exactly running on a relevantly steady operation area that the engineering practitioner expects. The detailed parameters of the studied system plant can be obtained in Reference [10]. In addition, the parameter settings of the two controllers are given in Table 1 below.

Table 1. Parameter settings of the two controllers

| Controller | \( K_p \) | \( K_i \) | \( K_d \) | \( N_1 \) | \( N_u \) | \( N_p \) | \( \gamma \) | \( \alpha \) |
|------------|--------|--------|--------|-------|-------|-------|-------|-------|
| PID        | 0.52   | 0.41   | 0.23   | -     | -     | -     | -     | -     |
| CGPC       | -      | -      | -      | 1     | 8     | 18    | 4     | 0.15  |

The simulated trajectories of the rotation speed, discharge and torque in the complete characteristic curves of the pump-turbine are displayed in Figure 3, respectively. In these two figures, it’s found that the turbine’s operating points start from the origin and end at no-load operation region in \( N_{11}-Q_{11} \) and \( N_{11}-M_{11} \) curves. Since that area is in the vicinity of the “S” shaped area, the dynamic balance of the unit is vulnerable to be affected for the intrinsic unstable “S” characteristic in pump-turbine. The aim of the proposed CGPC control scheme is to improve the comprehensive control performance in this process.

Figure 3. The operation trajectories of the unit during start-up process

(a) Operation trajectory in \( N_{11}-M_{11} \) Curve    (b) Operation trajectory in \( N_{11}-Q_{11} \) Curve

To investigate the effectiveness of the proposed CGPC method and reveal its superiority over the traditional control PID strategy, the comparative study has been conducted. The time evolution of guide vane opening, unit frequency, torque, discharge and hydraulic pressure at draft tube during the start-up process in the simulation are shown in Figure 4 (a)-(f), respectively.
From the comparative results of the simulation, it’s obvious that the proposed CGPC achieves better control performance in this typical transient process of the pumped-storage unit. The oscillation in unit...
frequency of CGPC decays faster and the hydraulic fluctuations in hydraulic pressure and discharge are more prone to be damped compared to those of PID. In general, the advantages of the CGPC lie in the following two aspects: the stronger hydraulic fluctuation damping and rapid frequency tracking. Considering that the optimal parameters of the PID controller are always sensitive to the operation condition and its control performances tend to deteriorate in large-scale operation conversions or low head conditions, the proposed CGPC scheme with online system parameter updating mechanism possesses better adaptivity in the complicated system plant.

5. Conclusions
This paper presents a novel multi-variable constraint generalized predictive control (CGPC) strategy to closely track the unit frequency and naturally different take system state constraints into consideration. The CGPC is acted on a lumped-parameter equivalent circuit model for the governing system of PSU. The case study has been conducted under the turbine start-up condition with numerical simulation techniques. The simulation results indicate that the proposed CGPC performs better than the traditional PID in both frequency tracking and fluctuation damping during the typical transient process.

Acknowledgment
This work was supported by National Natural Science Foundation of China (NSFC) (grant number: 52009096).

References
[1] Chazarra, M, Perez-Diaz, J, Garcia-Gonzalez, J. Optimal Joint Energy and Secondary Regulation Reserve Hourly Scheduling of Variable Speed Pumped Storage Hydropower Plants, IEEE T. Power Syst., 2018, 33, pp. 103-115
[2] Zhou J, Zhao Z, Zhang C, et al. A real-time accurate model and its predictive fuzzy PID controller for pumped storage unit via error compensation. Energies, 2017;11:35.
[3] Zhao Z, Yang J, Yang W, et al. A coordinated optimization framework for flexible operation of pumped storage hydropower system: Nonlinear modeling, strategy optimization and decision making. Energ. Convers. Manage., 2019, 194:75-93.
[4] Xu, Y, Zheng, Y, Du, Y, et al.: Adaptive condition predictive fuzzy PID optimal control of start-up process for pumped storage unit at low head area, Energ. Convers. Manage., 2018, 177, pp. 592-604
[5] Li, C, Zhang, N, Lai X, et al.: 'Design of a fractional-order PID controller for a pumped storage unit using a gravitational search algorithm based on the Cauchy and Gaussian mutation', Inf. Sci., 2017, 396, pp. 162-181
[6] Shi, K, Wang, B, Chen, H: Fuzzy generalised predictive control for a fractional-order nonlinear hydro-turbine regulating system, IET Renew. Power Gener., 2018, 12, pp. 1708-1713
[7] Li, C, Mao, Y, Yang, J, et al.: A nonlinear generalized predictive control for pumped storage unit, Renew. Energ., 2017, 114, pp. 945-959.
[8] Munoz-Hernandez, GA, Gracios-Marin, CA, Jones, DI, et al.: 'Evaluation of gain scheduled predictive control in a nonlinear MIMO model of a hydropower station', Int. J. Elec. Power, 2015, 66, pp. 125-132.
[9] Xiong, M, Gao, F, Liu, K, et al. Optimal Real-Time Scheduling for Hybrid Energy Storage Systems and Wind Farms Based on Model Predictive Control, Energies, 2015, 8, pp. 8020-8051
[10] Zheng, Y, Chen, Q, Xu Y, et al. Hierarchical MPC scheme for the speed governing of PSU with complex conduit system. IET Gen. Trans. Disb, 2020, 14,pp.316-329.