Efficient Error-tolerant Search on Knowledge Graphs

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ABSTRACT

Edge-labeled graphs are widely used to describe relationships between entities in a database. Given a query subgraph that represents an example of what the user is searching for, we study the problem of efficiently searching for similar subgraphs in a large data graph, where the similarity is defined in terms of the well-known graph edit distance. We call these queries error-tolerant exemplar queries since matches are allowed despite small variations in the graph structure and the labels. The problem in its general case is computationally intractable, but efficient solutions are reachable for labeled graphs under well-behaved distribution of the labels, commonly found in knowledge graphs. We propose two efficient exact algorithms, based on a filtering-and-verification framework, for finding subgraphs in a large data graph that are isomorphic to a query graph under some edit operations. Our filtering scheme, which uses the neighbourhood structure around a node and the presence or absence of paths, significantly reduces the number of candidates that are passed to the verification stage. Moreover, we analyze the costs of our algorithms and the conditions under which one algorithm is expected to outperform the other. Our analysis identifies some of the variables that affect the cost, including the number and the selectivity of query edge labels and the degree of nodes in the data graph, and characterizes their relationships. We empirically evaluate the effectiveness of our filtering schemes and queries, the efficiency of our algorithms and the reliability of our cost models on real datasets.

1. INTRODUCTION

Motivation. Graphs are widely used to model relationships, for example between entities in a knowledge graph, chemical compounds and organisms, objects and scenes in images, entities in RDF, functions and subroutines in a piece of software, etc. An important problem that arises in many of these domains is finding graph structures that are similar to a query graph.

Searching for similar, rather than exact, matches of a query is more desirable when data is noisy or inconsistencies are allowed. For example, in computational biology, the data can be highly noisy because of possible errors in data collection, different thresholds used in experiments, as well as the difficulty in cleaning the data. Despite the noise, searching for similar biological structures may enable a biologist to learn more about a new organism [6]. In molecular chemistry, identifying similar molecular structures of a target molecule may enable a chemist to design new molecular structures [4]. In social network analysis, searching for similar subgraphs may help to identify communities and to predict the network dynamics [10].

Similarity Search. In all aforementioned scenarios, one needs to identify the induced subgraphs in a data graph that are similar to a query graph. A number of similarity measures have been proposed [4, 15], of which the graph edit distance [8] is the most general and widely accepted. Graph edit distance is defined by the number of edit operations (i.e. the deletion, insertion and substitution of nodes or edges) that is needed to transform one graph into another. A valuable feature of graph edit distance is its error tolerance to noise and distortion, allowing user information needs to be captured. This paper uses the graph edit distance as its similarity measure between graphs.

There is a large body of work on subgraph similarity search. TALE [17], which indexes each node of a data graph with the node neighbourhood information (such as adjacent node labels, degrees, etc.), allows matching a subset of query nodes before progressively extending these matches. SAPPER [20] takes advantage of pre-generated random spanning trees and a carefully designed graph enumeration order to find approximate subgraph matches. However, these techniques are not applicable in many scenarios where knowledge graphs are queried. For example, consider searching for isomorphic matchings of the query “Ruby influenced Swift” in the Freebase knowledge graph [3]. This search in TALE will return the query itself. The same query in SAPPER will return the nodes “Ruby” and “Swift”, providing information that users already know about. Our focus in this paper is on exemplar queries [13, 14], where a query is an example of the expected query answer. More formally, an exemplar query returns all isomorphic matches where the matching edges have the same labels. For instance, the exemplar query “Ruby influenced Swift” (as shown in Figure 1) returns all matches of the form “A influenced B”, e.g., “D influenced Swift”, “Java influenced Closure”, etc. However, perfect matching of the labels of all query edges can be
too strong a constraint, and may not retrieve many desired matches.

**Error-Tolerant Exemplar Queries.** In this paper, we propose a framework that overcomes the problems mentioned above, through the use of graph edit distance operations. Introducing edit operations in exemplar queries may significantly expand the search space: on a data graph with \( L \) distinct labels, a naive solution is to run an exemplar query for every edit, i.e. \( O(L^t|E_0|) \), exemplar queries for a query with \( |E_0| \) edges and edit distance threshold \( t \), which would be prohibitively expensive. Therefore, novel techniques are necessary in order to provide efficient and scalable solutions. Since edges with matching labels can match when edit distance operations are allowed, we call our queries error-tolerant exemplar queries (ETEQ). A detailed motivating example is given in Section 2.

Given that ETEQ generalizes exemplar queries, the queries in ETEQ are applicable in many domains where one does not have a clear idea of what is being searched, but has a starting element in the result set. For example, the query “Ruby and D influenced Swift” with ETEQ can give the influence relationships between programming languages as well as other relevant relationships that may be retrieved when edit operations are allowed. In question answering, ETEQ may be used to find other instances in a knowledge graph that are similar to a given answer instance (see Section 6.4). Also, ETEQ can help existing search engine services improve in two ways. First, search engines can append the results of ETEQ to their results, for example, to increase their recall or better capture the users’ information needs. Second, the results of ETEQ can be considered related, and may help with query suggestions. For instance, when a user searches for “relationship between Ruby and Swift”, current search engines will show the results that mention the relationship. ETEQ can provide relationships of other programming languages, e.g. Java and Closure, Swift and Ruby, etc.

In this work, we identify the challenges related to efficiently evaluating ETEQ. First, the number of joins for a query with \( |E_0| \) edges is in the order of \( O(|E_0|) \). This becomes a computationally intensive process for large values of \( |E_0| \). Second, allowing edit operations further increases the size of the search space, as well as the space overhead for the intermediate results. We address these challenges by (1) proposing efficient indexes and sketches for filtering candidates; and (2) developing novel, accurate estimates for query selectivity and cost. Accordingly, we describe two new algorithms for efficiently evaluating ETEQ: these algorithms explore the overlap among query transformations under different edit operations, and can effectively reduce the search space and minimize the overall cost.

**Contributions.** Our contributions are as follows:

- We propose exemplar queries with edit distance operations, which enable error-tolerant searches on graph data.
- We describe two efficient algorithms for ETEQ based on a filtering-and-verification framework, and study efficient pruning strategies that use the neighbourhood structure and the paths to filter unqualified results.
- We develop novel cost models that allow us to compare the cost of our algorithms across different queries, without actually running the algorithms.
- We analyze our algorithms using our cost models, and study the conditions under which one algorithm is expected to outperform the other.
- We perform a thorough experimental evaluation, using real data, of the effectiveness of our filtering schemes, the performance and the scalability of our algorithms, as well as the reliability of our cost model. The results demonstrate the efficiency and effectiveness of the proposed approach.

### 2. MOTIVATING EXAMPLE

Consider a search scenario where one wants to find more information about programming languages. The user, if not familiar with the area, may try “programming languages’ basic information”. But this query will most likely return documents discussing programming languages in general terms. The user may instead provide an example result. Thus, she can formulate a query with all basic information about Swift as shown in Figure 1. This query, typed into a search engine, will return results about Swift (or maybe Ruby, Scala, etc.), but no results covering other languages.

Using exemplar queries allows us to find relevant answers matching all query edges. However, the given relationships in the query only holds for Swift. In other words, there is no other relevant answer that perfectly matches all query edges and their labels. Consider the candidate answer shown in Figure 1, denoting the Closure programming language. Although the relationship between Closure and Rich Hickey (“developer”) does not appear in the query graph and there are only two edges labeled “influenced” in the candidate (compared to three such edges in the query graph), the candidate has very similar structure to the query and is very likely an answer that users will find relevant. Exemplar queries cannot find such relevant answers. Also, it might be difficult for the users to describe the query with accurate relationships between entities. Thus, there is a need for efficiently retrieving relevant answers to a given example in the presence of small errors and mismatches in labels.

### 3. PROBLEM DEFINITION

Data sources that are in the form of entities and relationships may be encoded as labeled directed graphs using nodes to represent entities and edges to represent the relationships.

**Definition 1.** *(Knowledge Graph)* A knowledge graph \( G = (V, E, L) \) is a directed labeled graph, where \( V \) denotes a set of nodes, \( E \subseteq V^2 \) is a set of edges, and \( L \) is a labeling function that maps each node in \( V \) and each edge in \( E \) to a label.

Unless explicitly stated otherwise, the terms graph, knowledge graph and edge-labeled graph are used interchangeably in this paper.
Definition 2. (Edge-preserving Isomorphism) A graph $G$ is edge-preserving isomorphic to a graph $G'$, denoted as $G \simeq G'$, if there is a bijective function $\mu$ from the nodes of $G$ to the nodes of $G'$ such that for every edge $n_1 \rightarrow n_2$ in $G$, the edge $\mu(n_1) \rightarrow \mu(n_2)$ in $G'$.

Definition 3. (Edge-preserving Edit Distance) The edit distance between two non-isomorphic graphs $G$ and $G'$ is the minimum number of edit operations that make $G \simeq G'$.

Definition 4. (Error-tolerant Exemplar Query) An error-tolerant exemplar query is a pair $(Q,t)$ where $Q$ is a connected graph and $t \in R$ is a threshold. The answer to query $(Q,t)$ on a database $D$ is the set of all subgraphs $D_{sub}$ in $D$ such that $D_{sub}$ becomes edge-isomorphic to $Q$ after applying some edit operations to $Q$, $D_{sub}$ or both, and the cost of those operations does not exceed the threshold $t$.

Edit operations include insertion, deletion and substitution of edge or node labels, and these operations can be applied to both query and data graphs. Without loss of generality, we may limit the edit operations only to the queries. On the other hand, not all edit operations are applicable to exemplar queries. In particular, inserting an edge to the query graph is not a meaningful operation under subgraph isomorphism; if there is no subgraph matching a query, adding an edge to the query is not going to change that. Also, the label substitutions are limited to the edge labels since the node labels are ignored in exemplar queries (this is because we are interested in answers that have the same structure as the query, but do not necessarily involve the same nodes [14]). This reduces the edit operations to edge deletion and edge label substitution. In general, each edit operation may have a different cost. For example, substituting a label may be less costly when the two labels are synonyms. That said, for the sake of simplicity of our presentation, we assume all edit operations have the same cost, and may sometime refer to the edit threshold $t$ as the number of edit operations that are allowed.

In the rest of the paper, we will refer to error-tolerant exemplar queries simply as queries. By query cost model, we mean a parametric equation that estimates the cost of an algorithm or a query plan, in terms of the number of operations (I/O and CPU) that is needed to evaluate the query.

Problem Statement: We aim to address the following two problems. (1) Given an ETEQ in the form of a query graph $q$ and an edit distance threshold $t$, we aim to efficiently retrieve all relevant answers in a data graph that are edge-preserving isomorphic to $q$ with at most $t$ edit operations. (2) Given two algorithms for the problem in (1), we aim to compare their costs in terms of a cost model and find out which one algorithm outperforms the other.

4. PROPOSED APPROACH

4.1 Basic Algorithm (EXED)

Given a data graph $G = (V, E)$, a query $Q$ and the edit distance threshold $t$, a naive approach to find subgraphs that are within $t$ edit distances of the query $q$ is to compare the query with every subgraph in the data graph $G$. Our basic algorithm for exemplar queries with an edit distance constraint (referred to as EXED) randomly chooses one node $n_q$ from the query as a seed, instead of comparing the query with an exponential number of subgraphs in the data graph. Subsequently, it considers all nodes of the data graph one by one as possible mappings of the node $n_q$. For each such node $n_q$ in $V$, it checks if there exists a subgraph that contains $n_q$ and is isomorphic to the query with at most $t$ edit operations. All matching subgraphs are added into the result set. Algorithm 1 describes the above steps in pseudocode.

Algorithm 1 EXED

Input: Data graph $G = (V, E)$, query graph $Q$
Input: Threshold $t$
Output: Set of answers $S$
1: $S \leftarrow \emptyset$
2: $n_q \leftarrow$ chooseARandomNode($Q$)
3: for each node $n_q \in V$
do: 4: $s = \text{SearchSimilarSubGraph}(G, Q, n_q, u, t)$
5: if $s \neq \emptyset$ then
6: Add $s$ to answer set $S$.
7: end if
8: end for
9: return $S$

4.2 Neighbourhood-based Pruning

In EXED, every node $n_q$ of the data graph is considered a possible match of the query node $n_q$ and as a seed to start the search for relevant answers. This is highly inefficient since only a small fraction of data nodes are true candidates. To reduce this search space, one has to reduce the number of unnecessary data nodes from which the search for similar subgraphs starts. Inspired by [10], we propose a method called NEIGHBOURHODPRUNING to prune the search space.

Definition 5. (d-neighbour) Let $n \in V$ be a node of the data graph $G = (V, E)$, The node $n_1 \in V$ is a $d$-neighbour of $n$ if there exists a path from $n$ to $n_1$ of length at most $d$. The $d$-neighbourhood nodes of $n$, denoted as $N_d(n)$, is the set of all $d$-neighbours of $n$, and the $d$-neighbourhood labels of $n$, denoted as $L_d(n)$, is the set of edge labels on paths of length at most $d$ from $n$ to its $d$-neighbourhood nodes.

NEIGHBOURHODPRUNING compares data nodes with query nodes using their neighbourhood information, and filters out those data nodes that requires more than $t$ edit operations to match the query node’s neighbourhood. Let $T_{n,k,i}$ denotes those neighbour nodes of $n$ which are reachable from $n$ in a path of length $k$ and $i$ is the last label in the path, i.e.,

$$T_{n,k,i} = \{ n_1 | n_1 \rightarrow n_2 \cup \ldots \cup n_{k-1} | n_k \in N_{k-1}(n) \}.$$

Since keeping the table of neighbour nodes for every data nodes is expensive in terms of space, we only keep the cardinality of $T_{n,k,i}$. Also, to efficiently retrieve candidates matching a query node, we implement an inverted index which stores a list of nodes for every label, every cardinality and every distance. In other words, the index allows
us to efficiently find data nodes that have a label \( l \) at their \( k \)-neighbourhood with a certain cardinality.

Once the neighbourhood tables \( T_{n,k,l} \) of both data and query nodes are computed for each label \( l \) and path length \( k \leq d \), we can compare the neighbourhood of a query node to that of a data node and filter out unqualified data nodes. The edit distance between data node \( n_q \) and query node \( n_d \) for label \( l \) at \( k \)-neighbourhood can be written as

\[
dist_{k,l}(n_q, n_d) = \begin{cases} 0 & \text{if } |T_{n_d,k,l}| - |T_{n_q,k,l}| \\ \infty & \text{otherwise.} \end{cases}
\]

Given an edit distance threshold \( t \), \( n_q \) is considered a candidate for the query node \( n_d \) when the distance between the \( d \)-neighbourhoods of the two nodes does not exceed \( t \), i.e.,

\[
\sum_{i=1}^{d} \sum_{l \in L_i(n_q)} \dist_{i,l}(n_q, n_d) \leq t.
\]

Note that this filtering may introduce false positives, because neighbourhood-based pruning cannot identify if the labels are in the same path. For example, the neighbourhood-based distance between \( q_1 \) and \( g_1 \) in Fig. 2 is 0 whereas the actual edit distance is 6. It may be noted that the more correlated the edge labels in a query’s path are, the less false positives the neighbourhood-based pruning can produce. This summarized representation of a neighbourhood is highly effective at pruning nodes without actually visiting their neighbourhood. False positives can be removed at the verification stage which takes the previous comparisons of the nodes into the consideration.

![Figure 2: Query graph and data graph](image)

**Definition 6. (Simulation)** Let \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \) be two graphs. \( G_2 \) simulates \( G_1 \) if there exists a relation \( R \) such that, for every node \( n_1 \in N_1 \) and \( n_2 \in N_2 \) for which \( (n_1, n_2) \in R \) and \( n_1 \xrightarrow{1} n'_1 \), there exists a \( n'_2 \) such that \( n_2 \xrightarrow{1} n'_2 \) and \( (n_1, n_2) \in R \).

Verifying a simulation can be done more efficiently since \( n'_2 \in V_2 \) is a possible match of \( n'_1 \in V_1 \) only if in a previous comparison of nodes \( n_2 \) and \( n_1 \) the edge between \( n_2 \) and \( n_2' \) has label \( i \) and a corresponding edge with label \( i \) between \( n_1 \) and \( n'_1 \). With this observation, we only need to examine adjacent nodes of previously matched data nodes rather than all data nodes to find possible matches of a query node.

The EXED algorithm randomly chooses a query node \( n_q \) as a seed (starting node) and starts the search from the seed node. However, when we take the previous comparisons of the nodes into consideration, the choice of the starting node can affect the performance of the algorithm. The fewer previously matched data nodes are, the less comparisons we need to do in the following steps of the simulation. We capture this by introducing the selectivity of query nodes and labels into our algorithm.

**Definition 7. (Selectivity)** The selectivity of a query node \( n \) in a data graph \( G \) is the probability that a node of \( G \) matches \( n \). The selectivity of a label \( l \), denoted as \( sel(l) \), is the probability that an arbitrary edge of \( G \) is labeled \( l \), and is computed as the ratio of the frequency of label \( l \) to the number of edges in \( G \).

As the actual selectivity of a query node may be known only after finding its matches, we devise a method to estimate the selectivity in advance (see Sec. 5 for details).

**Algorithm 2 NeighbourhoodPruning**

**Input:** Data graph \( G = (V, E) \), query graph \( Q = (V_q, E_q) \)

**Input:** Threshold \( t \)

**Output:** Set of candidate mappings \( \mu \subset V_q \times N_q \)

1: \( L_d \leftarrow \) \( d \)-neighbour labels of \( Q \)
2: \( Vis \leftarrow \emptyset \)
3: \( n_{min} \leftarrow \arg \min_{n \in V_q} sel(n) \)
4: \( N_q \leftarrow (V_q, \emptyset) \)
5: \( \mu(n_{min}) \leftarrow N_q \)
6: \( Q \leftarrow \{n_{min}\} \)
7: for each \( n_q \in Q \) do
8: \( \text{if } \langle n_q, n_q' \rangle \in E_q \text{ and } n_q' \notin Vis \text{ then} \)
9: \( \text{Update edit distance of nodes in } \mu(n_q), \mu(n_q') \)
10: \( \text{Remove nodes that exceed threshold.} \)
11: \( \text{end if} \)
12: \( Q \leftarrow Q \cup \{n_q' \mid n_q \xrightarrow{1} n_q' \land \mu(n_q') \} \)
13: \( Q \leftarrow Q \setminus \{n_q\} \)
14: \( Vis \leftarrow Vis \cup \{n_q\} \)
15: \( \text{end for} \)

Let \( n_{min} \) be a query node with the minimum selectivity. Our Algorithm 2 initially takes the set of all data nodes as candidate mappings of \( n_{min} \). For each query node \( n_q \) that has not been visited yet, the algorithm checks if each data node \( n \in V_q \) has the matching edges (i.e. edges with the same label and direction) for each adjacent edges of \( n_q \). If it does not match and the edit distance \( t \) has already reached the threshold, \( (n, t) \) is removed from \( \mu(n_q) \). If it does not match and \( t \) has not reached the threshold, a node \( n' \) adjacent to \( n \) is considered a candidate for the query node \( n_q \). If it does not match and \( t \) has reached the threshold, the node \( n' \) is inserted into \( \mu(n_q') \). Otherwise, \( n' \) is a candidate match of \( n_q \) with no edit penalty and the entry \( (n', t + 1) \) is inserted into \( \mu(n_q') \). Finally, the query node \( n_q \) is marked as visited and is removed.

### 4.3 Improving Neighbourhood-based Pruning

The neighbourhood-based pruning may introduce false positives, because two matching labels may not be under the same path or have the same direction. In this section, we introduce a path-based filtering algorithm to prune out some of the false positives.

The path-based filtering algorithm compares data nodes with the query node in terms of their paths and filters out those data nodes that requires more than \( t \) edit operations to match the query node. However, keeping every path for every node can be very expensive in terms of space. For a graph with average degree \( D \) and \( d \)-edge path indexes,
We set the false positive rate to 1%. The optimal number of elements, one needs to find the false positive. The error rate depends on $m$, $|N|$, and $k$. We set the false positive rate to 1%. The optimal number of hash functions is approximately $0.7m/|N|$, and the optimal number of bits $m$ is approximately $|N|\ln p/\ln^2 2$. The number of inserted elements can be estimated by $D^d$, where $D$ is the average degree of the data graph. The Bloom filter based path filtering allows us to control the false positives at a low rate with a compact storage and an efficient access time. Moreover, it has no false negatives.

To insert a path into the Bloom filter, we concatenate the labels in the path to form a string that is inserted into the Bloom filter. To encode the direction of an edge, a sign symbol is added to each label to distinguish between incoming and outgoing edges. In addition, the count of each path is described by preceding the label sequence and separated from the rest of string by “P”. For example, the string “2P+1-2” describes two paths that have one outgoing edge labeled with value 1 and one incoming edge labeled with value 2. Since all labels in a path are encoded into one string, an unmatched path can have up to $d$ unmatched labels. To avoid filtering out false negatives, we consider the lower bound of the edit distance for an unmatched path, which is 1. This also introduces false positives if we only use path filtering. However, these false positives can be removed by considering the neighbourhood filter.

Our experiments show that the two filtering schemes work nicely, complementing each other. Path filtering can identify if multiple labels are in the same path and if the matching edges with the same labels have the same direction which neighbourhood filter cannot do; on the other hand, neighbourhood filtering can identify the level of mismatched labels which cannot be done by a path-based filtering.

### 4.4 The WCED Algorithm

The main problem with EXED is that the number of intermediate results can become huge, especially for large edit distance thresholds and large node degrees of the data graph. Most of those intermediate results need to be kept until a very late stage of the searching.

To reduce the number of intermediate results, we develop a new algorithm referred to as wildcard queries with edit distance constraint (WCED). The approach taken in this algorithm is to map the subgraph edit distance problem instance into subgraph isomorphism problem instances without missing any relevant answers. This is done by introducing wildcard labels. A wildcard label is a label that can substitute for any other label in graph matching.

The main idea is to perform multiple subgraph isomorphism searches based on the original query and merge the retrieved answers to obtain the final results. This approach has two phases: query pre-processing and subgraph search and answer mergence.

In the query preprocessing phase, we choose $t$ edges from $|E_q|$ query edges, where $t$ is the edit distance threshold and set their labels to the wildcard for edge label substitution (see our discussion in this section for other edit operations). This gives us $O(\binom{|E_q|}{t})$ wildcard queries assuming that $t \leq |E_q|$. For example, Figure 3 shows a two-edge query and its wildcard queries with edit distance threshold 1.

In the next phase, we run subgraph isomorphism searches on those generated wildcard queries. For this, we directly adopt EXED with edit threshold set to 0. This returns the subgraphs where the wildcard matches any label. For example, searching for the first wildcard query in Figure 3 will give us all subgraphs which have an edge labelled $l_1$ and an edge with any label, both under the same parent node. Finally, duplicates due to possible overlaps between wildcard queries are removed.

The WCED algorithm reduces the number of intermediate results by converting the subgraph edit distance into subgraph isomorphism. This is for the cost of running EXED $\binom{|E_q|}{t}$ times with edit distance threshold 0.

**Other edit operations** Supporting edge deletion is similar to substitution except that the edges are removed instead of being labeled with a wildcard. The only exception is that deleting an edge can result in a disconnected query graph, hence deletion may be applied to a subset of the edges whereas substitution can be applied to all edges. With $|E_q|$ query edges and edit distance threshold $t$, there are at most $\binom{|E_q|}{t}$ possible choices for deletion, each leading to a subgraph isomorphism search. As discussed in Section 3, edge insertion does not arise in exemplar queries since we are searching for subgraphs of the query graph, and insertions are already supported at no cost. For example, the data graph can have any number of additional edges, and those edges are ignored in a subgraph search.

### 5. Algorithm Cost Analysis

We have presented two algorithms for exemplar queries with edit distance constraints, each with some advantages over the other. WCED has much less number of intermediate results (meaning less space usage), while EXED only needs to be run once and has no duplicate answers. To determine which algorithm has the least cost for a given query and data graph (without actually running the algorithms), one needs an accurate cost estimation. This is the problem addressed in this section.

EXED consists of three parts: starting node selection, neighbourhood-based pruning and subgraph verification. The time cost of starting node selection and neighbourhood-based pruning are linear in the number of query nodes and number of data graph nodes respectively, while the time cost of subgraph verification grows exponentially with the edit distance threshold and the number of query edges. WCED consists of three phases: query pre-processing, subgraph isomorphism
search and answer mergence. Subgraph isomorphism search uses EXED with the edit distance threshold 0, the cost of which also grows exponentially with the number of query edges. The time cost of query pre-processing depends on the number of query edges and the edit distance threshold. The time cost of answer mergence is linear in the number of answers. Both of them are relatively low and negligible compared to the cost of subgraph isomorphism search. Therefore, we focus on the cost of verification of two algorithms. The cost depends on the number of data nodes (candidates) matching the query starting node and the cost of verifying each candidate.

5.1 An Exact Cost Model

Both algorithms EXED and WCED start with a set of candidate nodes in data graph \( D \) that are likely to match a query node \( n_q \); those candidates may be selected based on a filtering scheme such as the neighbourhood or the path filtering. Given a candidate node in \( G \), we must check if there is a subgraph in \( G \) that simulates the query graph in which the candidate node matches \( n_q \). The cost of this process depends on two factors: the number of candidates matching the query node and the cost of verifying each candidate.

5.1.1 A cost model for WCED

Given a query and an edit distance threshold that is larger than zero, the WCED algorithm generates a set of wildcard queries based on the edit distance threshold, hence it has to perform multiple subgraph isomorphism searches on those wildcard queries. The cost is the sum of the costs of those searches. It should be noted that a wildcard query is like any query except that some edges are labeled with wildcards and those wildcards can match any label.

Estimating the number of candidates: Given a seed \( n_q \), we want to estimate the probability that a data node is a candidate for \( n_q \).

**Lemma 1.** Given a query node and its adjacent edge labels \( l_1, \ldots, l_k \), and assuming independence between the labels, the probability that a data node with \( D \) adjacent labels has all query labels is

\[
P_D(l_1, l_2, \ldots, l_k) = \sum_{i+j+1}^{k+1} (-1)^{i+1} P_D(-l_{j+1}, \ldots, -l_{i+1}, l_i) + \sum_{i+j+1}^{k+1} (-1)^{i+1} P_D(1 - l_{j+1}, \ldots, 1 - l_i).
\]

Proof. See the Appendix.

Lemma 1 directly gives the selectivity of a query node based on its 1-neighbourhood. Let \( L_v(n_q) \) denote the set of labels at the \( i^{th} \) neighbourhood of a query node \( n_q \). The probability that the neighbourhood of a data node matches that of a query node at levels 1, \ldots, \( d \) can be written as

\[
P(n_q) = \prod_{m=1}^{d} P_{D_m}(L_m(n_q)),
\]

where \( P_{D_m}(L_m(n_q)) \) is as defined in Lemma 1 and \( D_m \) is the number of edges at the \( m^{th} \) neighbourhood of a data node. We generally do not know \( D_m \) when estimating our probabilities in Equations 1. Assuming that each data node has the same degree \( \hat{D}, D_m = \hat{D}^m \). Then, the number of candidates matching query node \( n_q \) is \( |C(n_q)| = |V_q| \cdot P(n_q) \).

**Estimating the cost of verifying each candidate:** For each candidate of the starting node, the algorithm starts from a graph \( g \) with only one node (i.e. the candidate node) and iteratively adds new edges to \( g \) until either \( g \) simulates the query, or no such simulation is found. The cost of adding each new edge depends on the expected number of matching edges of a query edge and the number of subgraphs to which the edges are added. Let \( \hat{D} \) denote the expected degree of a data node. For a query label \( l_i \), we expect \( \hat{D} \cdot \text{Sel}(l_i) \) edges in the data graph to match \( l_i \). For a fixed candidate node in the data graph, the expected number of subgraphs (partial matchings) that can be constructed starting from the candidate and simulating the query subgraph rooted at the seed with labels \( l_1, \ldots, l_k \) is \( \prod_{m=1}^{k} \hat{D} \cdot \text{Sel}(l_i) \) and the total expected cost of verifying a candidate \( n \) is

\[
\sum_{i=1}^{|E_q|} \hat{D} \cdot \text{Sel}(l_i).
\]

Note that this is based on the assumption that a search starting from a candidate node will not stop early if the simulation exceeds the edit distance threshold. The total cost of verifying \( |C(n_q)| \) candidates is

\[
\text{Cost}(q) = |C(n_q)| \cdot \sum_{i=1}^{|E_q|} \prod_{j=1}^i \hat{D} \cdot \text{Sel}(l_j).
\]

Since we have replaced a query graph with \( \binom{|E_q|}{t} \) graphs each with \( t \) wildcards, the total cost is the sum of the costs of verifying those wildcard queries.

5.1.2 EXED Cost Model

To estimate the cost for EXED, we also need to estimate the number of candidates in the data graph matching a query seed node and the cost of verifying each candidate.

**Estimating the number of candidates:** Since a data node is allowed to have up to \( t \) edit operations in its neighbourhood, directly estimating the probability that a data node is a qualified candidate is difficult. Therefore, we estimate the number of candidates for a set of wildcard queries where the labels are all fixed. By summing up the number of candidates for these wildcard queries and removing the repetitive candidates due to overlaps between queries, the number of candidates for \( n_q \) in EXED can be written as

\[
|C(n_q)| = \sum_{i=1}^{|E_q|} |V_q| \cdot P(n_{w_i(q,t)}) - \binom{|E_q|}{t} \cdot |V_q| \cdot P(n_q),
\]

where \( w_i(q,t) \) is a wildcard query constructed from \( q \) by replacing \( t \) edge labels with wildcards and \( P(n_q) \) is as in Equation 2. The last term gives the number of double-count candidates for \( \binom{|E_q|}{t} \) wildcard queries.

**Estimating the cost of verifying each candidate:** To estimate the cost of verifying each candidate, we need to estimate the number of partial matchings. There are two kinds of partial matchings in EXED: (1) matchings that have reached the edit distance threshold, and (2) matchings that have not reached the threshold. For (1), edges with any label can
be added to the matching in the next step of the simulation. However, for (2), only edges with matching labels can be added. In this case, the next step of a simulation costs only edges with a matching label can be added. In this case, whereas for (2), only edges with matching labels can be added to the matching in the next step of the simulation.

Given query labels \( l_1, \ldots, l_m \), we generally do not know in advance which labels will mismatch and need to check all choices of \( (\begin{array}{c} m \end{array}) \) sets of labels. The number of partial matchings that needs to be verified is

\[
S_l(q, m) = \begin{cases} 
0 & \text{if } t > m \\
\hat{D}^m \prod_{i=1}^{m-k} Sel(l_i) \prod_{j=1}^{k} (1 - Sel(l_j)) & \text{if } t = 0 \\
\sum_{k=1}^{\hat{D}^m} \prod_{i=1}^{m-k} Sel(l_{k,i}) \prod_{j=1}^{t} (1 - Sel(l_{k,j})) & \text{if } t < m.
\end{cases}
\]  

For any partial matching that have not reached the threshold \( t \), any edge can be added into the matching in the next step of the simulation. In this case, the next step of simulation costs: \( \sum_{t=0}^{\hat{D}^{-1}} S_l(q, m) * \hat{D} \).

For any partial matching that have reached the threshold, only edges with a matching label can be added. In this case, the next step of a simulation costs \( S_l(q, m) * \hat{D} * Sel(l_{m+1}) \).

The cost of verifying each candidate in EXED is

\[
\text{Cost}(q) = \sum_{i=0}^{|E_q|-1} (S_l(q, i) * \hat{D} * Sel(l_{i+1}) + \sum_{j=0}^{t-1} S_l(q, i) * \hat{D}),
\]

and the total cost of EXED is the product of the number of candidates (as given in Equation 5) and the cost of verifying a candidate (as given above): \( \text{Cost}_{ex} = |C(n_q)| * \text{Cost}(q) \).

### 5.1.3 Cost Model Comparison

We want to compare the costs of verifying the candidates for EXED and WCED and identify the conditions under which one outperforms the other. Our cost comparison assumes that the threshold \( t \) is less than the number of query edges; otherwise, the problem is subgraph isomorphism with no label constraints, which is not addressed in this paper.

For the edit distance threshold larger than zero, the cost of verifying a candidate in EXED is higher than that in WCED, because edit operations can happen on any label in EXED while they are fixed in WCED. Hence if the number of candidates for WCED and EXED are roughly the same, WCED will outperform EXED. In other words, WCED outperforms EXED if the number of candidates for the original query is small (See Equation 5). This is a more plausible scenario for our queries; otherwise edit operations are less likely to be considered. The next lemma shows what happens when this condition does not hold.

**Lemma 2.** Given a data graph with expected node degree \( \hat{D} \), a query graph \( q \) with at least 2 edges and the edit distance threshold set to 1, the cost of verifying a candidate in EXED is less than the sum of the cost of verifying a candidate for every wildcard queries in WCED when

\[
Sel(l_i) > \frac{1}{|l_i\sqrt{n_i}}.
\]

where \( l_i \) is a query label that has the highest selectivity (i.e. the smallest value of Sel(l_i)).

**Proof.** See the Appendix. \( \square \)

When the number of candidates for the original query is large (more precisely, roughly equal to the number of candidates for a wildcard query), the cost of EXED and WCED can both be approximated based on the number of candidates for the original query. In this case, EXED can outperform WCED given the condition of the lemma.

### 5.2 An Upper Bound Cost Model

The exact cost model is based on two assumptions: (1) labels are evenly distributed, and (2) labels are pairwise independent. These assumptions may not hold in real-world data graphs. This is a problem especially for large queries since the error can accumulate and become significant as the number of query edges increases. In this section, we present a cost model that gives an upper bound of the actual cost but is more accurate for larger query graphs.

**Estimating the number of candidates:** To estimate the upper bound for the number of candidates, two weaker assumptions of label independence are considered: (1) the labels of the adjacent edges of a data node are independent whereas labels, which are in a path starting from a node, are correlated; (2) the labels of the adjacent edges of a data node are correlated whereas labels, which are in a path starting from the node, are independent. For two or more correlated labels, the selectivity of the label with the least selectivity provides an upper bound of the selectivity of the set.

Under the first assumption, the selectivity of the label with the minimum selectivity in each path is used to estimate the selectivity upper bound of the path. This reduces each path in the query to an edge (with the minimum selectivity), and as a result the query becomes a node with a set of adjacent edges (i.e. a tree with only one level). Assuming independence between the labels of these edges, Lemma 2 will give an upper bound of the probability that a data node is a candidate for a query node. Note that \( \hat{D} \) in the Lemma is set to the number of paths in the d-neighbourhood.

Under the second assumption, all edges under a node are collapsed into a single edge, which is labeled with a label from the set that has the least selectivity. Since the edge labels of the resulting query are all independent, Equation 2 can be used to estimate the upper bound.

**Estimating the cost of verifying each candidate:** To estimate the upper bound for the cost of verifying each candidate, the maximum frequency of each label under a node is used to upper bound the number of matching label in each step of the simulation. Let \( N(l_i) \) denote the maximum frequency of label \( l_i \) in the adjacent edges of a node.

In our exact cost model, the number of matching labels for a label \( l_i \) is \( \hat{D} \times Sel(l_i) \) assuming that every label is uniformly distributed on the adjacent edges of a node. Replacing \( \hat{D} \times Sel(l_i) \) in Equations 4 and 7 by \( N(l_i) \) will give us an upper bound of the cost of verifying each candidate in WCED and EXED respectively.
6. EXPERIMENTAL EVALUATION

This section presents an experimental evaluation of our algorithms and cost models. All our experiments were performed on a 2.4 GHz 4 Core CPU with 60G memory running Linux. The algorithms are implemented in Java 1.8. Unless explicitly stated otherwise, the path length $d$ in our filtering scheme is set to 3.

**Dataset:** We downloaded a full dump of Freebase\(^1\) in May 2015. We removed the triples that were used as internal specification for the community (e.g., user and group data and discussion topics) obtaining a fully connected graph of 84 million nodes and 335 million edges. Since the entire Freebase is too large for our machine (occupies approximately 90G of memory when fully loaded), we extract subgraphs from Freebase with different parameters. The subgraphs are extracted using a breadth first traversal of the graph from a randomly selected starting node and randomly choosing new edges to be included in the data graph. Unless explicitly stated otherwise, the data graphs are randomly generated from Freebase with the number of nodes set to 10K and average node degree set to 15.

**Queries:** Three types of queries are used in our experiments: (1) a set of real queries from the AOL query log, manually mapped to the data graph, (2) a set of real queries from QALD-4\(^2\), a benchmark for evaluating question answering over linked data, and (3) randomly selected subgraphs of the data graph. These queries vary in their number of edges and the selectivity of their labels. Unless explicitly stated otherwise, our experiments use 100 randomly selected queries, each a subgraph of the data graph.

**Summary of our experiments:** Our cost models are evaluated in Section 6.1 and the effectiveness of our filtering schemes under different settings and combinations is evaluated in Sections 6.2 and 6.3. The impact of our filtering schemes on the performance of our algorithms and improvements over existing algorithms are evaluated in Section 6.4.

### 6.1 Effectiveness of Our Cost Models

In this section, we evaluate our cost models in terms of the correlation between our estimates and the actual costs. Our results show that: (1) the selectivity estimation is reliable when the number of query edges $|E_q| \leq 10$ (See Fig. 4 and its discussions); (2) there is a linear relationship between our exact cost model and the real cost for $|E_q| \leq 3$, which allows us to estimate the running time of our algorithms (See Fig. 5 and its discussions); (3) the exact cost is reliable for the comparison of our algorithms when $|E_q| \leq 6$ (See Fig. 7 and its discussions); (4) the exact cost is reliable for the comparison of query costs when $|E_q| \leq 8$ (See Fig. 8 and its discussions).

**Effectiveness of the selectivity estimation** Since selectivity estimation is a core component of our cost model, we first assess the quality of our selectivity estimation. To do so, we measure the correlation between the actual number of candidates and the estimated number of candidates based on our selectivity estimates. In our case, the selectivity is used in choosing a query starting node and for cost comparisons, hence, a relative ordering of the selectivity values is sufficient in these cases. Therefore, we chose Spearman’s rank correlation between estimate and actual selectivities, which shows the monotonic relationship of the two variables. The experiments are in the context of WCED and EXED algorithms. Let “exact” denote the exact selectivity estimation, “ub-path” and “ub-adj” denote the upper bounds of the selectivity estimations respectively assuming that path labels and adjacent labels are independent. Figure 4 shows that although “exact” has a better correlation (0.96) than “ub-adj” and “ub-path” for queries with 2 edges, the correlation decreases rapidly for queries with a large number of edges, with a correlation at 0.55 for queries with 10 edges. In contrast, the upper-bound selectivity estimates remain stable with different number of query edges in both WCED and EXED (with correlation between 0.7 and 0.96 and significance level between 3.08E – 64 and 5.38E – 16). We conclude that there is strong positive correlation between the selectivity estimation and the actual selectivity and that we can safely use this selectivity estimation for the choice of a query starting node and cost comparisons.

**Exact cost model evaluation** In this set of experiments, we examine the linear relationship between our estimated cost and the actual number of operations using Pearson correlation. The larger the absolute value of the coefficient, the stronger the relationship between the actual cost and the estimated cost. For relatively large values of the correlation coefficient, one can predict with a good accuracy the actual cost from our estimated cost, using a simple linear regression. Figure 5 shows that for small queries (with up to 3 edges), the correlation coefficient is over 0.7 and 0.6 for WCED and EXED respectively. However, the correlation coefficient drops sharply as the number of edges increases. These results are expected because the exact cost model is based on the assumption that the labels are evenly distributed and that they are independent.

**Upper-bound cost model evaluation** In this set of ex-
6.2 Effectiveness of Our Filtering Strategies

To evaluate the pruning power of our filtering schemes, the number of nodes in the data graph was set to 10K. Let “neighbour” denote the neighbourhood-based filtering strategy, “path” denote the path-based pruning strategy and “both” denote the case where both schemes were used.

Varying the number of query edges: For this experiment, we varied the number of query edges from 2 to 10 and set the edit distance threshold to 1. Figure 8 shows the fraction of candidates that are pruned in EXED and WCED as we vary the number of query edges. As shown for WCED and EXED, “path” can filter out respectively up to 99.4% and 99.1% of the data nodes on average, while “neighbour” can filter out respectively up to 99.0% and 97.3% of the data nodes on average. The pruning power does not increase by more than 1% when both strategies are used. However, considering the large number of data nodes and the high cost of verifying each candidate, even a small improvement in the pruning stages is amplified and positively affects the performance of the algorithms (See Figure 13).

Varying the edit distance threshold: For this experiment, the number of query edges was fixed at 8 with the edit distance threshold varied from 1 to 5. When the edit distance threshold is equal to or exceeds the number of query edges, the labels become irrelevant and the problems becomes subgraph isomorphism on unlabeled graphs, which is not the problem addressed in this paper. Figure 9 shows that both “neighbour” and “path” have good pruning power (over 78%) under different distance thresholds, and it becomes more effective to apply both filtering schemes as the edit distance threshold increases. This is because “neighbour” scheme does not encode edge direction in its indexes and higher edit distance threshold introduces more false positives with wrong edge direction, while adding “path” on top of “neighbour” can effectively prune out those false positives.

Varying path label correlation: For this experiment, the neighbourhood filtering scheme is considered as a baseline, on top of which we added our path filtering scheme and monitored the improvement in pruning power. We fixed the number of query edges at 8 and set the edit distance threshold to 1. As shown in Figure 10, the improvement in pruning power by adding “path” in both EXED and WCED drops with more correlation. This meets our expectation since the more correlated the labels are, the less false positives the neighbourhood-based pruning can produce and the less room for “path” filtering improvements.

6.3 Combining Filtering Schemes

In this set of experiments, we evaluate the impact of adding path filtering on top of the neighbourhood filtering. In order to show the impact of using both filtering schemes, we consider EXED with the neighbourhood filtering scheme as our baseline and compare it against EXED with both filtering schemes, WCED with the neighbourhood filtering scheme and WCED with both filtering schemes. We denote EXED and WCED with the neighbourhood filtering scheme as “neighbour-EXED” and “neighbour-WCED” respectively, WCED and EXED with both filtering schemes as “both-WCED” and “both-EXED” respectively.

Varying the edit distance threshold: We varied the edit distance threshold $t$ from 1 to 5 and fixed the number of query edges at 8. Figure 11 shows that “neighbour-WCED” outperforms “neighbour-EXED” by a factor of 1.5 when $t = 5$, reducing the search time more than half in this particular experiment. Comparing “neighbour-WCED”
and “both-WCED”, we find that even though there is no clear speedup for adding path filtering on top of the neighborhood filtering scheme at small thresholds \( t \leq 2 \), the performance gap becomes wider at larger thresholds with around 200 seconds saved when \( t = 5 \). This meets our expectation because the benefits of using both schemes over “neighbour” in pruning power becomes clear when the edit distance threshold increases (See Figure 9).

Varying average degree of data graph: In another experiment, we varied the average degree of a node from 5 to 25. Figure 12 shows that “both-WCED” has a greater advantage in a data graph with larger average degrees, outperforming “neighbour-WCED” and “neighbour-EXED”. This is because the cost of verifying each candidate depends on the average degree of the data graph, and a larger average degree results in a higher cost of verifying each candidate and thus a wider gap between “both-WCED” and the others.

Varying the number of query edges: In another experiment, we varied the number of query edges from 2 to 10 with the edit distance threshold fixed at 1. Figure 13 shows that the gap between “neighbour-WCED” and “both-WCED” (and similarly between “neighbour-EXED” and “both-EXED”) widens as we increase the number of edges. This meets our expectation, because the cost of verifying each candidate grows exponentially with the number of edges.

Our experiments in this section reveals that adding path filtering improves the performance of both algorithms EXED and WCED under one or more of these conditions: (1) the data graph has high average degree; (2) the edit distance threshold \( t \geq 2 \); (3) the query has over 5 edges.

6.4 Algorithms Comparison

To evaluate the performance of our algorithms against the competitors, we selected two recent algorithms from the literature: (1) SAPPER [20], which is an algorithm for indexing and approximate matching in large graphs, and (2) the work of Mottin et al. [13], which is similar to our work but is limited to edit distance threshold zero.

Comparing against SAPPER: In this set of experiments, we compare the scalability of our algorithms against SAPPER. In the first experiment, we varied the number of nodes in the data graph from 10K to 1M while the edit distance threshold was set to 1. This is consistent with the settings by the authors of SAPPER except that their largest data graph had only 10K nodes. Since SAPPER only supports edge deletion (missing edges), we modified SAPPER to support edge label substitutions. Figure 14 shows the running time for retrieving the first 100 answers. The results show that SAPPER is slower by an order of magnitude, and it becomes slower as the data graph size increases. In contrast, our algorithms are not much sensitive to the size of the data graph and scale gracefully to large data graphs, exhibiting an almost constant behavior.

In our second experiment, we use the same setting as the first one, except that we do not limit the number of answers; instead we set a 1500 seconds time limit on each algorithm. The running times for SAPPER and our algorithms are reported in Figure 15. The graph shows that SAPPER is not a viable solution to our problem for data graphs of realistic sizes: the running time of SAPPER grows much faster than our algorithms, and quickly hits the 1500 seconds time limit in the data graph with 100K nodes. WCED with both filtering schemes exhibits the best performance. We also observe that the time cost of our algorithms grows linearly with the number of nodes in the data graph. This meets our expectation, because the number of relevant answers increases rapidly (See Figure 16) as the number of data nodes increases.

Comparing against Exemplar queries: To compare the effectiveness of our queries to that of exemplar queries, we ran experiments using queries from both AOL query log and the QALD4 benchmark. From the AOL query log, we chose 10 queries (see the Appendix) and manually mapped them to subgraphs in freebase. For each query, we introduce errors
by randomly selecting an edge and replacing its label with a label randomly selected from the data graph. The number of query edges ranged between 6 and 8. To control the size of the answer set (and to avoid a blow-up), we varied the edit distance threshold from 0 to 2. When the edit distance threshold is 0, our queries are identical to exemplar queries. As expected and shown in Figure 17, exemplar queries fail to return any answer for queries with errors, whereas ETEQ retrieves the answers despite the error, and the larger the edit distance thresholds are, the more answers are returned.

To evaluate the quality of ETEQ answers, we conducted the following user study. We asked 10 users (students at the University of Alberta) to evaluate our system. For each query in the test set, we provided an explanation of the topic, the query intention, and our answer set with different edit distances. We asked each user to rate each result as irrelevant, weakly related, or very related with respect to the topic and the expressed query intent. Due to the large size of the answer sets, for each answer set and each edit distance, we randomly chose up to 10 answers for evaluation. We observe in Figure 18 that the relevant set has many answers with edit distances 1 and 2. These answers cannot be returned by exemplar queries.

We also evaluated the effectiveness of our ETEQ queries over the QALD4 benchmark, a collection of natural language questions over linked data. To adapt this benchmark to our framework, we focused on list questions; we selected one answer for each question (the first one given in the benchmark) and collected all predicates from Dbpedia that had that answer (as a subject, or an object) and a term from the question. For example, for the question “which books by Kerouac were published by Viking Press?”, the predicates “*Kerouac, notableWork, X”, “X, author, *Kerouac” and “X, publisher, Viking_Press” were collected, where X indicates the initial given answer and * indicates a wild card; we also collected pairs of predicates that could be joined, giving a path of length 2, and the path had both the answer and a question word. For example, for the question “Give me all actors starring in movies directed by William Shatner.” the predicates “a, starring, X” and “a, director, William_Shatner” were collected. Our goal was to find more answers matching X from a given example. We had to make a slight change in our algorithm to force matches on node labels that were present, such as “*Kerouac” and “Viking_Press”. Since the answers to questions were given, we could track at each edit distance the answers that were returned. As shown in Table 1 for 20 queries from this benchmark only 37% of the answers are at edit distance zero and can be returned using exemplar queries, whereas the rest of the answers are at larger edit distances and can only be returned using our ETEQ queries.

We are not comparing the efficiency of our algorithms against exemplar queries of Mottin et al. [14] because (1) their framework does not support edit distance thresholds

| Edit distance | 0   | 1   | 2   | 3   | 4   |
|---------------|-----|-----|-----|-----|-----|
| % of answers (mean) | 0.37 | 0.23 | 0.25 | 0.06 | 0.04 |
| % of answers (std)   | 0.35 | 0.25 | 0.32 | 0.11 | 0.10 |

Table 1: The fraction of answers at each edit distance for 20 queries from QALD4 benchmark.

3The selected queries were the first set of list queries with 3 or more answers, and excluded more trivial questions that only matched one predicate. Both the queries and the predicates can be found in the Appendix.
larger than zero, and (2) our EXED algorithm becomes identical to their exemplar queries when the edit distance threshold is zero and we have extensively evaluated EXED with different edit distance thresholds. Also since it is shown that their exemplar queries outperforms NeMa \cite{11}, we are not comparing our queries to NeMa.

7. RELATED WORK

In the special case where the edit distance threshold is zero, the problem of graph edit distance becomes subgraph isomorphism, which is NP-complete; most existing methods adopt a filter-and-verification framework. Wang et al. \cite{18} propose an efficient index for sparse data graphs. They decompose graphs to small grams (organized by k-Adjacent Tree patterns) and use these tree patterns to estimate a lower bound of their edit distance for candidate filtering. Zeng et al. \cite{19} propose a method to compute the edit distance by transforming a graph to a multi-set of star structures and using a path-based index for candidates filtering. These algorithms target data graphs that are small (< 10K nodes) and sparse, and they do not exploit the labels in large knowledge and RDF graphs.

Approximate graph matching or similarity-based search in large graphs has been studied in the past under various settings. TALE \cite{17} introduces a neighbourhood based index (NH-Index) where it matches important vertices of a query graph first before extending the match progressively. SAPPER \cite{20} takes advantage of pre-generated random spanning trees and a carefully designed graph enumeration order to find approximate subgraph matches. Mongiovi et al. \cite{12} introduce a set-cover based inexact subgraph matching technique, called SIGMA. These algorithms are not exact and can miss qualifying answers.

NeMa \cite{11} introduces a similarity measure preserving proximity of node pairs and label information. Based on this similarity measure, the authors propose a heuristic for the problem of minimum cost subgraph matching, avoiding the costly subgraph isomorphism and edit distance computation. We are not comparing our algorithms directly to NeMa but to exemplar queries which are shown to outperform NeMa \cite{13}. Jayaram et al. \cite{9} present an algorithm that takes a set of entities (instead of a graph) and finds the best matching subgraph that includes those entities. The resulting subgraph may be used as an exemplar query. This work is orthogonal to ours and may be combined with ETEQ, for more efficient query formulations.

Our query node neighbourhood filtering is inspired by the work of Khan et al. \cite{10}; we study the interactions between neighbourhood filtering and path filtering and show that combining them performs better than either index alone. Finally compared to the exemplar queries of Mottin et al. \cite{14}, which can only find relevant answers that are edge-isomorphic to a query, our algorithm can find relevant subgraphs that are edge-preserving isomorphic to the query after some edit operations.

8. CONCLUSIONS

This paper studies the problem of error-tolerant exemplar queries on knowledge graphs. Unlike exemplar queries that support only exact matching of the labels, our developed algorithms in this paper allow errors in both query and data graphs. Two filtering techniques (neighbourhood and path filtering) and two algorithms (EXED and WCED) are developed to efficiently support our ETEQ queries and their costs are estimated and analyzed. Through a comprehensive experimental evaluation on real and synthetic datasets, we have shown that our algorithms are both efficient and effective, outperforming existing algorithms. As a future direction, we plan to efficiently support Top-k ETEQ queries.

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APPENDIX

A. PROOF OF LEMMA

A.1 Proof of Lemma 1

Proof. This lemma can be proved using the probability subtraction rule:

\[ P_D(l_1, l_2, \ldots, l_k) = P_D(l_2, \ldots, l_k) - P_D(-l_1, l_2, \ldots, l_k) = P_D(l_2, \ldots, l_k) - (P_D(-l_1, l_3, \ldots, l_k) - P_D(-l_1, -l_2, \ldots, l_k)) = P_D(l_2, l_3, \ldots, l_k) - P_D(-l_1, l_4, \ldots, l_k) - P_D(-l_2, -l_3, -l_4, \ldots, l_k) = \cdots \]

\[ \sum_{i=2}^{k} (-1)^{i-1} P_D(-l_j, \ldots, -l_{i-1}, l_{i+1}, \ldots, l_k) + (-1)^k P_D(-l_1, -l_2, \ldots, -l_k) + P_D(l_2, l_3, \ldots, l_k). \]  

(9)

If we expand \( P_D(l_2, \ldots, l_k) \) further using the equation above, we will have a set of terms that look similar to the first and the second terms in Eq. 8 and the base case \( P_D(l_k) \).

For the base case, we have \( P_D(l_k) = \prod_{l=1}^{k} \tau \cdot \text{Sel}(l_k,j) \).

We also know that \( P_D(-l_j, \ldots, -l_k) = (1 - \sum_{l_{j-1}=1}^{\tau} \text{Sel}(l_i))^{D_l} \) assuming independence. Putting these pieces together will give the statement of the lemma.

\[ \square \]

A.2 Proof of Lemma 2

Proof. Using Equations 3, the cost of verifying each wildcard query for WCED with edit distance 1 can be written as

\[ \text{Cost}_{wc} = \sum_{k=1}^{\tau} \sum_{l=1}^{i} D \cdot \text{Sel}(l_k, j) \]

Let \( l_1, \ldots, l_k \) denotes the labels in increasing order of selectivities. Since the labels in a query are verified in increasing order of selectivities, for those wildcard queries where \( l_j (1 \leq j \leq i) \) is not set to wildcard, the edge with label \( l_j \) is verified at \( i^{th} \) step of the simulation, the cost of verifying the edge is \( D^i \prod_{l=1}^{j} \text{Sel}(l_i) \), where \( l_j \neq l_m \).

Let \( T_i \) be

\[ T_i = \sum_{k=1}^{i} \prod_{m=1}^{k} \text{Sel}(l_{k,m}) \text{ where } l_{k,m} \neq l_k. \]  

(10)

The sum of verifying costs for those wildcard queries that \( l_m (1 \leq m \leq i) \) is set to wildcard at \( i^{th} \) step of the simulation is \( D^i (T_{i-1} - D^i \prod_{l=1}^{j} \text{Sel}(l_j)) \). Then, the sum of verifying costs for WCED can be written as

\[ \text{Cost}_{wc} = \sum_{i=1}^{\tau} D^i \left( (|E_q| - i - 1) \prod_{j=1}^{i} \text{Sel}(l_j) + T_{i-1} \right) \]

\[ + D^i |E_q| T_{i-1}. \]

Using Equations 6 and 7, the verifying cost of EXED with
We also know that the derivative of \( F_k(x) \) is
\[
\frac{\partial F_k}{\partial x} = -k - k(k - 1)\hat{D}^{k-1}x^{k-1} - \sum_{i=2}^{k} i(k + i - 1)x^{i-1} < 0.
\]

Note that the derivative of \( F_k(x) \) is
\[
F_k(x) = \hat{D}(1 - kx) - (k - 1)\hat{D}^{k-1}x^{k-1}
+ \sum_{i=2}^{k-1} \hat{D}^i(i - (k + i - 1)x^i) < 0
\text{ subject to } x > \frac{1}{\sqrt{D}}.
\]

Using the both inequalitis above, the upper bound of \( \Delta \)
\[
\Delta \leq \hat{D}(1 - |E_q|Sel(l_1)) - (|E_q| - 1)\hat{D}^{|E_q|Sel(l_1)|}\]  
\[
+ \sum_{i=2}^{n} \hat{D}^i((|E_q| - 1)Sel(l_1|) + i - 1)Sel(l_1)^i).
\]

Let \( F_n(x) \) denote the upper bound of \( \Delta \) using \( x \) to denote \( Sel(l_1) \) and \( n \) to denote the number of query edges. To show the correctness of the Lemma 2 we prove that \( F_{|E_q|}(x) \leq 0 \) with different number of edges when the conditions in the Lemma holds using mathematical induction.

**Basis:** \( n = 2 \): \( F_2(x) \) can be written as
\[
F_2(x) = \hat{D}(1 - 2x) - \hat{D}^2 x^2.
\]

When \( x = \frac{1}{\sqrt{D}} \), we have \( F_2(x) \).
\[
\hat{D}(1 - 2x) - \hat{D}^2 \left( \frac{1}{\sqrt{D}} \right)^2 = \hat{D}(-2x) < 0.
\]

We also know that the derivative of \( F_2(x) \) is
\[
\frac{\partial F_2}{\partial x} = -2\hat{D} - 2\hat{D}^2 x < 0.
\]

Combining two facts above, we know that \( F_2(x) < 0 \) when \( Sel(l_1) > \frac{1}{\sqrt{D}} \).

**Induction hypothesis:** Assume the Lemma holds when the query has \( k \) edges.
\[
F_k(x) = \hat{D}(1 - kx) - (k - 1)\hat{D}^{k-1}x^{k-1}
+ \sum_{i=2}^{k-1} \hat{D}^i(i - (k + i - 1)x^i) < 0
\text{ subject to } x > \frac{1}{\sqrt{D}}.
\]

**Induction:** Using \( F_k(x) \) to substitute some terms in \( F_{k+1}(x) \), \( F_{k+1}(x) \) can be written as
\[
F_{k+1}(x) = \hat{D}(1 - (k + 1)x) - k\hat{D}^k x^k + k\hat{D}^{k+1} x^{k+1}
\]
\[
+ \sum_{i=2}^{k} \hat{D}^i((k + i - 1)x^i) = \hat{D}(-k - 1)x + \sum_{i=2}^{k} \hat{D}^i(k + i - 1)x^i + k\hat{D}^{k+1} x^{k+1}.
\]

When \( x = \frac{1}{\sqrt{D}} \), after replacing the \( x \) with the value in the last term and combining the last two terms, \( F_{k+1}(x) \) can be written as
\[
F_{k+1}(x) = \hat{D}(-k - 1)x + \sum_{i=2}^{k} \hat{D}^i(k + i - 1)x^i + k\hat{D}^{k+1} x^{k+1}.
\]

Since \( x = \frac{1}{\sqrt{D}} > \frac{1}{\sqrt{D}} \), \( F_{k+1}(x) < 0 \) and the rest of terms are also negative, we have
\[
F_{k+1}(x) < 0.
\]

We also know that the derivative of \( F_{k+1}(x) \) is negative.
\[
\frac{\partial F_{k+1}(x)}{\partial x} = \hat{D}(-k - 1)x - \sum_{i=2}^{k} \hat{D}^{k-1}x^{i-1} - k(k + 1)\hat{D}^{k} x^{k-1} < 0.
\]

Combining two facts above, we know that \( F_{k+1}(x) < 0 \) when \( Sel(l_1) > \frac{1}{\sqrt{D}} \).
Stress diseases Myocardial infarction;
Stress associated Land_cover s Graves’ disease;
Peptic ulcer risk_factors Stress;
Graves’ disease symptoms Anxiety;
Stress diseases Conversion disorder.

3. Going Upriver executive_produced_by Marc Abrams;
The Main Event writer Michael Benson;
Sun Also Rises commanders Michael Benson;
Sun Also Rises writer Marc Abrams;
Marc Abrams episodes_written Sun Also Rises;
Marc Abrams episodes_written The Main Event.

4. Going Upriver executive_produced_by Marc Abrams;
The Main Event writer Michael Benson;
Sun Also Rises commanders Michael Benson;
Sun Also Rises writer Marc Abrams;
Marc Abrams episodes_written Sun Also Rises;
Marc Abrams episodes_written The Main Event.

5. Frederick County contains Ole Orchard Estates;
Frederick County events Second Battle of Winchester;
Frederick County buildings_occupied North Mountain;
Frederick County contains Echo Village;
Frederick County people_born_here James Brenton (1740˘A¸ S1782);
Frederick County contains Green Acres;
Frederick County contains US Census 2000 Tract 51069050100.

6. Research subject_of Carnegie Moscow Center;
Research works Hot talk, cold science;
Research works Person or Persons Unknown;
Research address Stanford University School of Medicine;
Research schools_of_this_kind Indian Institute of Forest Management;
Research organizations_of_this_type Stanford Radiology.

7. Valve Corporation games_developed Half-Life 2;
Valve Corporation games_published Wolfenstein 3D;
Valve Corporation games_published The Maw;
Valve Corporation games_developed CS Online;
Valve Corporation games_published Half-Life 2;
Valve Corporation is_reviewed Place founded;
Valve Corporation games_published CS Online.

8. Scheme influenced Haskell;
Scheme influenced Clojure;
Scheme influenced LFE;
Scheme influenced Dylan;
Scheme influenced_Lisp;
Scheme parent_language Lisp.

9. NetBSD supported_architectures x86;
x86 manufacturers United Microelectronics Corporation;
NetBSD supported_architectures ARM architecture;
Great Giana Sisters game The Great Giana Sisters;
The Great Giana Sisters governing_body NetBSD;
x86 manufacturers Cyrix;
NetBSD parent_os 386BSD. The Great Giana Sisters platforms Dreamcast

10. Xbox 360 games_on_this_platform Garret the Slug;
NBA 2K11 platforms Xbox 360;
Xbox 360 games_on_this_platform Deus Ex: Human Revolution;
Halo 3 platform Xbox 360;
Microsoft Corporation games_published The Maw;
Xbox 360 games_on_this_platform Rainy Woods;
Halo 3 publisher Microsoft Corporation;
Xbox 360 games_on_this_platform Halo 3.

The QALD4 queries used in our evaluation are as follows:

1. Which books by Kerouac were published by Viking Press?
   e.g. On_the_Road
   predicates (*Kerouac, notableWork, X), (X, author, *Kerouac), (X, publisher, Viking_Press)

2. Which states of Germany are governed by the Social Democratic Party?
   e.g. Berlin
   predicates (X, leaderParty, Social_Democratic_Party_of_Germany),
   (Social_Democratic_Party_of_Germany, headquarter, X),
   (X, leader, a), (a, party, Social_Democratic_Party_of_Germany),
   (*, state, X)

3. Which television shows were created by Walt Disney?
   e.g. The_Mickey_Mouse_Club
   predicates (X, creator, Walt_Disney), (X, company, *Walt_Disney*), (X, format, *television*), (X, format, *show)

4. Which actors were born in Germany?
   e.g. Briana_Banks
   predicates (X, birthPlace, Germany), (X, ethnicity, German*), (X, numberOfFilms, *), (X, weight, *), (X, height, *), (*, starring, X), (X, birthDate, *)

5. Give me all people that were born in Vienna and died in Berlin.
   e.g. Hilde_K%C3%B6rber
   predicates (X, birthPlace, Vienna), (X, deathPlace, Berlin)

6. Which companies work in the aerospace industry as well as in medicine?
   e.g. Makino
   predicates (X, industry, Aerospace), (X, industry, Medicine),
   (X, type, *company)

7. Which languages are spoken in Estonia?
   e.g. Estonian_Language
   predicates (X, spokenIn, Estonia), (Estonia, official-Language, X), (Estonia, language, X)

8. Give me all soccer clubs in Spain.
   e.g. Albacete_Balomp%3%A9
   predicates (X, ground, Spain), (*, managerClub, X)

9. Which countries adopted the Euro?
   e.g. Andorra
   predicates (X, currency, Euro), (*, country, X)

10. In which military conflicts did Lawrence of Arabia participate?
    e.g. Arab_Revolt
    predicates (T_E_Lawrence, battle, X), (X, commander, T_E_Lawrence), (X, isPartOfMilitaryConflict, *),
    (*, isPartOfMilitaryConflict, X)

11. Give me the capitals of all countries in Africa.
    e.g. Luanda
    predicates (*, capital, X), (*Africa*, location, X),
    (*Africa*, city, X)

12. Give me all islands that belong to Japan.
    e.g. Kyushu
    predicates (X, country, Japan), (*Japan*, place, X),
    (*X*, location, Japan)
13. Which airports are located in California, USA?
   e.g. Moffett_Federal_Airfield
   predicates (X, location, California), (X, iataLocationIdentifier, *), (X, icaoLocationIdentifier, *), (X, faaLocationIdentifier, *)

14. Which Chess players died in the same place they were born in?
   e.g. Paul_Morphy
   predicates (*Chess*, editor, X), (X, worldChampionTitleYear, *), (X, birthPlace, a), (X, deathPlace, a)

15. Which capitals in Europe were host cities of the summer olympic games?
   e.g. Amsterdam
   predicates (a, capital, X), (*Europe*, location, a), (*, city, X)

16. Give me all cars that are produced in Germany.
   e.g. Porsche_928
   predicates (X, assembly, Germany), (X, manufacturer, *), (X, productionStartYear, *), (X, productionEndYear, *), (X, class, *), (X, bodyStyle, *), (X, engine, *), (X, transmission, *), (X, wheelbase, *), (X, length, *), (X, width, *), (X, weight, *)

17. Give me all actors starring in movies directed by William Shatner.
   e.g. Leonard_Nimoy
   predicates (a, starring, X), (a, director, William_Shatner)

18. Give me all actors starring in Last Action Hero.
   e.g. Arnold_Schwarzenegger
   predicates (Last_Action_Hero, producer, X), (Last_Action_Hero, starring, X)

19. Give me all video games published by Mean Hamster Software.
   e.g. Myst
   predicates (Mean_Hamster_Software, product, X), (X, publisher, Mean_Hamster_Software), (X, genre, *game)

20. Who produced films starring Natalie Portman?
   e.g. Patrice_Ledoux
   predicates (a, producer, X), (a, starring, Natalie_Portman)