Moving particle semi-implicit method for fluid simulation with implicitly defined obstacles

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Abstract. We developed a particle-based fluid simulation method with obstacles defined in implicit form. Our fluid simulation is based on the Moving Particle Semi-implicit (MPS) method, a typical particle-based algorithm achieving incompressible flow. In general, the particle-based fluid simulation is performed with obstacles that are defined as a set of particles located along the boundaries. We employ the implicit representation for the geometric information of obstacles which requires a new formulation of particle motion at the vicinity of boundaries. The main difficulty of MPS-based simulation with implicit obstacles is the lack of boundary particles that are required for the computation of two quantities: particle number density and particle force determined by pressure field. In our formulation, new definitions of the two quantities giving good estimation of the original ones is developed and incorporated in the MPS algorithm. In addition, we provide a modified algorithm for the construction of discrete linear system specific to the implicit representation for the computation of particle pressure. Our numerical tests show that the proposed approximation techniques provide adequate particle motion at the vicinity of the implicitly defined obstacles.

1. Introduction
In this paper, we present a technique for incompressible fluid simulation with obstacles defined in implicit form. Our method is performed under the following conditions: the incompressible fluid simulation is performed using the Moving Particle Semi-implicit (MPS) method [1] and each obstacle in the domain has implicit representation that defines the geometry, i.e., the boundary of each obstacle is defined as the solution of $f(x) = 0$.

The main problem to be addressed for the purpose above is the formulation of particle motion in the vicinity of obstacles. Since implicitly defined obstacles do not possess boundary particles, a new formulation specific to the implicit representation is required. The quantities such as the particle number density and particle force of each fluid particle in the vicinity of boundaries need to be determined depending on the implicit form. In our formulation, we define the former quantity by assuming hypothetical particles located along the boundaries and the latter one is defined by assuming an artificial repulsive force acting between obstacles and fluid particles.

The particle-based fluid simulation method called the Smoothed Particle Hydrodynamics (SPH) is one of the most popular approaches that has been used for scientific computation [2].
and for computer graphics [3]. The weakly compressible SPH [4] is one advanced method achieving nearly incompressible flow, a key requirement for many practical applications. The MPS is another approach that accomplishes the incompressibility with high accuracy and has been successfully applied to engineering applications.

In many particle-based simulations, the geometry of obstacles is assumed to be represented by a set of boundary particles. Another approach is polygonal representation of boundaries [5]. In our formulation, obstacles are assumed to have geometry defined in implicit form. In computer graphics, a number of techniques for generating implicit surface have been proposed [6, 7]. In addition, the implicit representation is suitable for representation of time-varying shapes [8].

Our method is performed by combining the two techniques: the MPS and the implicit representation. This new formulation helps to simulate incompressible fluid with time-varying smooth obstacles.

2. MPS with boundary particles

The MPS is a method for numerical analysis of incompressible flow with free surfaces. The governing equations consist of the Navier-Stokes equations, mass conservation law and the incompressibility condition:

\[
\frac{Dv}{Dt} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 v + g, \quad \frac{D\rho}{Dt} + \rho \nabla \cdot v = 0, \quad \frac{Dp}{Dt} = 0,
\]

where \(v\), \(\rho\), \(P\), \(\nu\) and \(g\) are the velocity, density, pressure, kinematic viscosity coefficient and the gravity, respectively. The pressure term \(\mathbf{a}_i^{\text{press}}\) is obtained using the following model:

\[
\mathbf{a}_i^{\text{press}} = -\frac{D}{\rho n_i^0} \sum_{j \neq i} \left[ \frac{\hat{p}_j^{k+1} - \hat{P}_i^{k+1}}{\| \mathbf{r}_j^* - \mathbf{r}_i^* \|^2} \left( \mathbf{r}_j^* - \mathbf{r}_i^* \right) \right] w \left( \left( \| \mathbf{r}_j^* - \mathbf{r}_i^* \| \right) \right),
\]

where \(D\), \(n_i^0\), \(\hat{P}_i^{k+1}\) and \(\mathbf{r}_i^*\) are the number of spatial dimensions, the particle number density \(n_i\) at the initial state, the minimum pressure in the effective radius and the particle position, respectively. The particle number density \(n_i\) and the weight function \(w(r)\) are evaluated as:

\[n_i = \sum_{j \neq i} w \left( \| \mathbf{r}_j - \mathbf{r}_i \| \right), \quad w(r) = \begin{cases} \frac{r^2}{r^2 - r^2} - 1 & (0 \leq r < r_e) \\ 0 & (r_e \leq r) \end{cases},\]

where \(r_e\) is the effective radius. The pressure \(P_i^{k+1}\) is obtained as the solution of the linear system:

\[-\frac{\rho_0}{\Delta t^2} (n_i^e - n_i^0) - \alpha P_i^{k+1} = \frac{2D}{\lambda n_i^0} \sum_{j \neq i} \left[ \left( P_j^{k+1} - P_i^{k+1} \right) w \left( \left( \| \mathbf{r}_j^* - \mathbf{r}_i^* \| \right) \right) \right],\]

where \(\alpha\) is the compressibility of the fluid and \(\lambda\) is the coefficient determined statistically.

3. MPS with implicitly defined obstacles

The standard MPS is formulated assuming a set of particles located inside obstacles and along their boundary. In this study, the particle motion needs to be formulated without using the particles of obstacles and, instead of using them, we divide the computation of the physical quantities at each particle into the quantities determined by surrounding fluid particles and those by obstacles. Assuming a set of boundary particles, the quantities determined by the obstacles defined by the following equations:

\[
\begin{align*}
\mathbf{a}_i^{\text{press}} &= -\frac{D}{\rho n_i^0} \sum_{j \in \text{obstacle}} \left[ \frac{p_j^{k+1} - \hat{p}_i^{k+1}}{\| \mathbf{r}_j^* - \mathbf{r}_i^* \|^2} \left( \mathbf{r}_j^* - \mathbf{r}_i^* \right) \right] w \left( \left( \| \mathbf{r}_j^* - \mathbf{r}_i^* \| \right) \right), \\
\langle \nabla^2 P \rangle_i^{k+1} &= \frac{2D}{\lambda n_i^0} \sum_{j \in \text{obstacle}} \left[ \left( p_j^{k+1} - p_i^{k+1} \right) w \left( \left( \| \mathbf{r}_j^* - \mathbf{r}_i^* \| \right) \right) \right].
\end{align*}
\]
In order to replace these equations with expressions without boundary particles, we apply the following two models. The first one is to assume hypothetical boundary particles that determine quantities at fluid particles affected by the obstacles. The second one is to define an artificial pressure term affected from obstacles to each fluid particle. In this study, we assume that the hypothetical boundary particles are located along the tangent plane at the closest boundary point on the implicit surface.

By applying the first model to the weight function, we obtain

$$w_{\text{obstacle}}(d_i) \approx \sum_{j \text{ in obstacle}} w(\|r_j - r_i\|),$$

where $d_i$ is the distance from $i$-th fluid particle to the closest point on the boundary and $r_j$ is the hypothetical boundary particle. Note that the weight function is free of the direction of the tangent plane and, as a result, the weight function can be determined as a function of $d_i \geq 0$ in advance of the time-step computation in practical computation. For the pressure term in (1), we use the second model, i.e., we define an artificial force that acts on each fluid particle as following:

$$a_{\text{press}, \text{obstacle}}(d_i) \equiv \beta n_i w_{\text{obstacle}}(d_i) \frac{d_i^2}{d_i^2} d_i r_i,$$

where $\beta$ is the coefficient for force level and $d_i$ is the direction from the fluid particle to the nearest boundary (see Figure 1). The bottom equation in (1) is obtained by

$$\left\langle \nabla^2 P \right\rangle_{i, \text{obstacle}}^{k+1} \approx \frac{2m_i w_{\text{obstacle}}(d_i) \|d_i\|}{\lambda d_i}.$$ 

The quantities undefined in the above discussion are $d_i$, the distance from a fluid particle to the closest boundary point, and $d_i$, the normal direction of the corresponding tangent plane at the closest boundary point. We obtain the distance $d_i$ by the Taubin's distance approximation.

For the normal $d_i$, we replace the direction with the gradient of the implicit function at the corresponding fluid particle:

$$d_i \approx \|f(r_i)\|/\|\nabla f(r_i)\|, \quad d_i \approx (r_{\text{init}} - d_i) \cdot \nabla f(r_i)/\|\nabla f(r_i)\|,$$

where $r_{\text{init}}$ is the distance of initial particles.

Figure 2 shows the results of our MPS computation with implicit surfaces. The simulation is performed with 7,900 fluid particles and the obstacles defined by $\left(\sqrt{x^2 + y^2} - R\right)^2 - r^2 = 0$. The results show that adequate particle motion can be achieved in the vicinity of the boundary by our MPS-based formulation with implicit expression.
4. Conclusions
We have presented a fluid simulation method with implicit function form obstacles. The formulation is performed by giving two models specific to implicit representation: one is to assume hypothetical particles along boundaries and the other is to define a new model of the pressure term component affected by obstacles. Some numerical tests have shown that our method gives adequate particle motion with implicitly defined obstacles.

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