Emergent naming conventions in a foraging robot swarm

Roman Miletitch1 · Andreagiovanni Reina1,2 · Marco Dorigo1 · Vito Trianni3

Received: 13 March 2021 / Accepted: 6 April 2022 / Published online: 13 July 2022
© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2022

Abstract

In this study, we investigate the emergence of naming conventions within a swarm of robots that collectively forage, that is, collect resources from multiple sources in the environment. While foraging, the swarm explores the environment and makes a collective decision on how to exploit the available resources, either by selecting a single source or concurrently exploiting more than one. At the same time, the robots locally exchange messages in order to agree on how to name each source. Here, we study the correlation between the task-induced interaction network and the emergent naming conventions. In particular, our goal is to determine whether the dynamics of the interaction network are sufficient to determine an emergent vocabulary that is potentially useful to the robot swarm. To be useful, linguistic conventions need to be compact and meaningful, that is, to be the minimal description of the relevant features of the environment and of the made collective decision. We show that, in order to obtain a useful vocabulary, the task-dependent interaction network alone is not sufficient, but it must be combined with a correlation between language and foraging dynamics. On the basis of these results, we propose a decentralised algorithm for collective categorisation which enables the swarm to achieve a useful—compact and meaningful—naming of all the available sources. Understanding how useful linguistic conventions emerge contributes to the design of robot swarms with potentially improved autonomy, flexibility, and self-awareness.

Keywords Swarm robotics · Foraging · Collective decision making · Language games

Roman Miletitch
roman.miletitch@gmail.com
Andreagiovanni Reina
andreagiovanni.reina@ulb.be
Marco Dorigo
mdorigo@ulb.ac.be
Vito Trianni
vito.trianni@istc.cnr.it

1 IRIDIA, Université Libre de Bruxelles, Brussels, Belgium
2 Department of Computer Science, University of Sheffield, Sheffield, UK
3 ISTC, Italian National Research Council, Rome, Italy
1 Introduction

The development of advanced forms of communication—i.e., a primitive form of language—can help robots in a swarm to share relevant information about the task execution, adapting it to the current activities and environmental contingencies experienced by the robots (Cambier et al., 2020). Indeed, linguistic conventions can be useful to describe the environment and the task execution progress in a compact way, supporting the coordination within the swarm. Among the tasks relevant for swarm robotics, foraging—a task often observed in natural self-organising systems (Bailis et al., 2010; Saleh & Chittka, 2007) is certainly one among the most studied (Ducatelle et al., 2014; Ferrante et al., 2015; Miletitch et al., 2018; Talamali et al., 2020), as it lends itself to represent multiple realistic applications like mining, search-and-rescue or logistics. While foraging, the swarm needs to explore an environment and decide which source to exploit among several available. In such context, linguistic conventions can provide compact ways of uniquely identifying relevant aspects of the environment (e.g., different terms to identify different sources from which to forage), which can evolve to adapt to a changing landscape (e.g., assigning new terms to newly discovered sources, or dropping terms associated with depleted sources), hence maximising the communication efficiency. Moreover, an evolving language can contain sequences of terms, providing swarms the ability to decide on the most useful course of action (e.g., a sequence of sources from which to forage).

To make language evolution possible, however, robots in a swarm need to interact and agree on the terms to be used and their meaning. This is the realm of language games, that is, computational models developed to understand the emergence of language through communication and self-organisation (Steels, 2001; Baronchelli et al., 2010; Spranger, 2013). As in swarm robotics communication is often local and intermittent, complex and dynamical interaction networks among robots emerge. A language game played in these conditions would have its dynamics largely affected by the network topology resulting from the task execution (Loreto et al., 2011). In this paper, we study the correlation between the task-induced interaction network and the evolving language. Indeed, the outcome of the language game can be correlated with both the intrinsic dynamics and outcome of the task itself, and the features of the environment in which the task is carried out. When such correlations are present, the linguistic conventions resulting from the language game are semantically grounded onto the task and its environment and can therefore be exploited for the accomplishment of the task itself. Some experiments have explored semantic connections between language games and the physical spaces in which they are played (Steels, 1995; Spranger, 2013). However, applications in swarm robotics are still limited (Cambier et al., 2020), and only a few experiments with a self-organised aggregation problem can be reported to date (Cambier et al., 2018).

In this paper, we demonstrate how language games can be grounded onto the execution of a foraging task. Specifically, we show that the task-induced interaction network is not sufficient per se in determining the conditions for semantically grounding the emergent linguistic conventions onto the task. However, we show that such grounding is possible when the language game is played by robots actually exploiting a source. The understanding of the language dynamics leads us to define a category game tailored to better represent different sources distributed in space, as long as these are relevant to the foraging task.

The paper is organised as follows. In Sect. 2, we discuss how language games can be meaningfully played by a robot swarm engaged in a source exploitation task. In Sect. 3, we present the experimental setup. In Sect. 4, we show how the dynamics of the interaction
network can lead to emergent linguistic conventions. Then, in Sect. 5, we analyse the properties of the interaction network, suggesting that it meaningfully supports the evolution of useful linguistic conventions. Finally, in Sect. 6 we present the category game introduced to better support self-organised foraging. Finally, Sect. 7 concludes the paper.

2 Language games in foraging robot swarm

In swarm robotics, coordination and self-organisation allow groups of robots to be more efficient than isolated robots in performing a given task (Dorigo et al., 2014, 2020). The collaborative processes designed for robot swarms are often inspired by social insects and other group-living animals (Brambilla et al., 2013; Trianni & Campo, 2015). Communication is one fundamental aspect for self-organisation and can be either indirect (e.g., stigmergy) or direct. Both types of communication are encountered in animal societies, such as the pheromone trails used by ants (Beekman et al., 2001) or the waggle dance used by honey bees (Biesmeijer & de Vries, 2001). These communication mechanisms have been implemented with success in swarm robotics systems, for example, using indirect stigmergic interactions (Holland & Melhuish, 1999; Beckers et al., 2000; Allwright et al., 2014), pheromones (Fujisawa et al., 2014; Talamali et al., 2020) and direct communication (Gutiérrez et al., 2010; Miletitch et al., 2018). While efficient, these communication mechanisms are usually designed for a specific task/environment (e.g., application in warehouses, see Stiefelhagen et al., 2004) and convey specific pieces of information, hence limiting the system flexibility.

Researchers aimed to add more plasticity to the communication process, for instance, by exploiting an evolutionary process to design at the same time signals and adapted responses (Marocco & Nolfi, 2007; Floreano et al., 2007). The resulting communication mechanisms are very well adapted to the tasks and environmental conditions encountered during training and also show some generalisation abilities. However, the characteristics of the obtained communication mechanisms remain very simple, with few signals and responses to signals that cannot easily scale up to more complex environments and/or tasks. A possibility to provide more complex communication abilities to a robotic system comes from models of natural language evolution (Wang & Minett, 2005; Solé et al., 2010).

A popular approach to the study of language dynamics is represented by language games played by a population of agents/robots, with the purpose of mimicking real-world linguistic interactions leading to the emergence of a structured language. Various kinds of language games have been proposed to date, from imitation games (Billard & Hayes, 1997) to guessing games (Steels, 2001) and category games (Puglisi et al., 2008; Baronchelli et al., 2010). One game in particular has received a lot of attention: the naming game (Steels 1995; 2003). In this game, two or more robots interact to assign a unique name to a set of objects. At each interaction, one robot is chosen as a speaker and another as a listener. The speaker chooses a referring object and an associated word from its vocabulary—or invents one when no word is available—and then transmits it to the listener. If the listener knows the word, then the game is a success, and both agents remove all other words associated with the chosen object from their vocabulary, keeping only the shared word. If instead the listener does not know the received word, then the game fails, and the listener adds this new word to its vocabulary. We use in our study a specific version of this game: the minimal naming game (MNG, see Baronchelli et al., 2006b). Here, focus is given only to reaching consensus on a single world within a population of communicating agents. Specifically,
we consider an implementation in which the speaker broadcasts its word to all agents in his
neighbourhood, while the listener is the only agent that updates the vocabulary upon suc-
cess or failure of a game (Baronchelli, 2011).

As naming games are based on interactions between pairs of speaker and listener agents,
the time to achieve consensus and the underlying dynamics are directly linked to the topol-
ogy of the interaction network. In non-embodied implementations, the link between topol-
ogy and language dynamics has been extensively studied (e.g., fully connected regular,
small world or random geometric networks, see Baronchelli et al., 2007; Lu et al., 2008).
Embodied implementations can be divided in two categories. On the one hand, a popu-
lation of virtual agents can use a small number of robots (sometimes reduced to two, as
in Spranger, 2013) to play the naming game, so that at each iteration, agents are selected
and assigned to robots in order to record physical interactions among them. On the other
hand, the naming game can be played among a population of embodied mobile agents
(Baronchelli & Diaz-Guilera, 2012; Trianni et al., 2016) that interact locally with each
other according to a topology of interactions that is the direct result of the mobility pattern
of the agents induced from the task being executed.

In this study, the MNG is played on top of a self-organised foraging task. When forag-
ing, a swarm needs to explore the environment, identify and evaluate the available sources
and make decisions on which source to exploit, going through different transitory states
before reaching an equilibrium (e.g., convergence on one single source to exploit or split/
load-balance among many, as in Miletitch et al., 2018). Similar behaviours provide a com-
plex and time varying interaction network among robots, which can be exploited to support
linguistic interactions among agents. Our main goal is to study whether the dynamics of the
interaction network are sufficient to determine language dynamics that represent features of
the task execution (e.g., choice of one or the other source), of the environment (e.g., the
presence of more than one sources, each associated with a different word), or both. To this
end, we run experiments with two versions of the MNG. Beside the classic MNG, we play
a version where the creation of words is linked with the discovery of sources by exploring
robots. In this setup, we study how well the robots manage to have an accurate description
of their surroundings that is both complete (a word for each source) and correct (no mis-
nomer) for as long as each source is relevant to the swarm, where relevance is measured as
the number of robots actively foraging from the source (see Sect. 3). Our goal is to under-
stand how the swarm interaction topology influences the language dynamics and how the
creation of words is correlated with the robots foraging from a source.

3 Experimental setup

In this study, the goal of the swarm is to play a MNG while identifying and exploiting
either of two sources (referred to as source A and source B) placed at the opposite side of
a home area (referred to as nest, see Fig. 1). The environment is a 2D infinite plane with-
out obstacles, and both nest and sources have circular shape with radius $R = 0.3$ m. Each
source is located at the same distance $d = 2.5$ m from the nest.

3.1 Robots and simulations

Experiments are run in simulation using ARGoS (Pincirolì et al., 2012). In our study, we
use this simulator to model a swarm of 50 e-puck robots (Mondada et al., 2009). E-pucks
have a differential drive motion with a maximum linear speed of $v = 0.1 \text{ m/s}$, and the wheels’ rotation is measured by an encoder. Avoidance of other robots is done at short range ($\approx 10 \text{ cm}$) using infrared proximity sensors and at longer range ($\approx 1 \text{ m}$) using the infrared range and bearing system (Gutierrez et al., 2009). The obstacle avoidance behaviour has been optimised to minimise the effects of robot density and congestion and to support the ability to navigate back and forth between sources, as detailed in a previous study (Miletitch et al., 2013). Robots perceive nest and sources only when they are located in the corresponding areas by means of infrared ground sensors that robots use to differentiate between the white colour of the floor, the grey colour of the sources and the black colour of the nest. We assume here that robots start from the nest without any knowledge about sources, which need to be located through exploration. Robots can locally broadcast short messages through the infrared range and bearing system within a range that is limited to $d_I = 0.2 \text{ m}$ (indicated by the dotted circle around the robot in Fig. 1). Robots can broadcast a message at regular intervals of 0.1 s with no re-broadcast of information received (no multi-hop communication). They keep track of the position of nest and known sources through odometry. The error on positioning produced through this tracking method can be efficiently compensated through social odometry (Gutiérrez et al., 2010; Miletitch et al., 2013). Owing to this, in this study we neglect odometry errors and focus on the interplay between motion and language dynamics.

At the beginning of the experiment, robots are uniformly distributed within a 0.8 m side square centred on the nest. During the first 200 s, robots perform a blind random walk during which they do not communicate or search for sources. This allows us to neglect the initial transitory phase in which robots are too densely distributed around the nest, allowing us to study the system dynamics after the robots spread out in the environment according to their search pattern. This assures that—whatever the experimental condition—the initial distribution of robots does not severely impact the final outcome. In the following experiments, unless mentioned otherwise, we perform 100 runs for each experimental setup. These runs last until language convergence, which, depending on internal parameters, can take up to 12000 s.

### 3.2 Individual and collective behaviour

#### 3.2.1 Source exploitation

The desired swarm behaviour (localisation and exploitation of sources) takes inspiration from the decision-making process displayed by house-hunting honeybees—also known as nest-site selection (NSS, see Pais et al., 2013; Seeley et al., 2012; Reina et al., 2013).
The spatial dynamics during foraging resulting from the NSS process have been studied by Reina et al. 2015a and Miletitch et al. 2018. Here, we make use of the individual robot behaviour from the former (Reina et al., 2015a), which was designed for the e-puck robots following a design pattern based on the NSS process (Reina et al., 2015b). According to this design pattern, a robot is considered to be committed to a source when it knows its location, and hence moves back and forth between the source and the nest. Otherwise, a robot is considered uncommitted and explores the arena searching for a source. Robots committed to source A (B) are considered to belong to the population $P_A$ ($P_B$), while uncommitted robots belong to the population $P_U$, all summing up to $N$ robots: $|P_A| + |P_B| + |P_U| = N$.

Four concurrent processes determine the individual behaviour, two for transitions between uncommitted and committed states, and two for the opposite. An uncommitted robot turns committed either through discovery or through recruitment. The former takes place when the robot enters the area of a source. The latter takes place with probability $P_d$ when a robot receives the information about a source known by a committed neighbour. Conversely, a committed robot turns uncommitted either through abandonment or through cross-inhibition. The former takes place anytime with a fixed probability $P_a$ per time step. The latter takes place with probability $P_c$ upon interaction with a neighbouring robot committed to a different source. Cross-inhibition introduces a negative feedback loop that helps the system break the symmetry and leads to a choice between two identical sources (for more details, see Reina et al., 2015a, b). In our study, recruitment and cross-inhibition happen only upon communication with other robots when located into the nest. Differently from Reina et al. 2015a, we set the probability of abandonment $P_a$ to zero, so that the only way for robots to become uncommitted is through cross-inhibition. This favours the attainment of a consensus state in which all robots within the swarm are committed to the one or the other source (Reina et al., 2015b).

The actual movements of the robots are governed by the following basic behaviours. When uncommitted, the robots explore the arena, performing a correlated random walk (Dimidov et al., 2016), and have a fixed and small probability at every control step to return to the nest. When committed, the robots enter an exploitation loop where they move back and forth between the known source and the nest (for a detailed description, see Reina et al., 2015a).

Depending on the value of $P_d$ and $P_a$, the swarm displays different dynamics and different final distributions of robots among the populations $P_U$, $P_A$ and $P_B$. In this study, we focus on two specific cases: strong cross-inhibition and weak cross-inhibition. In the strong case ($P_a = 0.7$, Fig. 2 top row) the swarm rapidly converges to a consensus for the one or the other source, whereas the weak case ($P_a = 0.1$, see Fig. 2 bottom row) leads to slower dynamics (Reina et al., 2018). Given enough time, the swarm would end up converging to a consensus for a single source. However, over the duration of our experiments, the swarm did not break the symmetry but splits between the two sources (see Fig. 2, bottom row). At any time, with or without consensus, we define the source with the highest number of committed robots (relative majority) as the “selected” source. We define $O \in \{A, B\}$ as the selected source and $X \in \{A, B\}$ as the non-selected source, and $P_O$ and $P_X$ as the respective populations, with $P_O \geq P_X$. 

\[ \text{Springer} \]
3.2.2 Minimal naming game

The language game played by the robots in our study is an implementation of the minimal naming game (MNG) for mobile agents/robots (Baronchelli et al., 2006b; Baronchelli & Díaz-Guilera, 2012; Trianni et al., 2016). Each robot starts with an empty inventory. At each time step (of length $\tau_s = 100$ ms), each robot has a probability $P_s$ of becoming a speaker (here, $P_s \in \{0.0003, 0.0006, 0.001, 0.002\}$). These values of $P_s$ were selected so that foraging dynamics and language dynamics would share comparable time scales. The language game is played as follows: the speaker robot selects a word from its inventory and broadcasts it to its neighbors. At each time step, if a robot receives at least one message, it becomes a hearer robot. The hearer selects one (and only one) word at random among those received and checks it against its own inventory. If the hearer finds the selected word in its inventory, the hearer keeps only that word in the inventory while deleting all the others. If instead the hearer does not find the selected word in its inventory, it updates its inventory by adding the word (for more details, see Trianni et al., 2016).

In this study, we consider two variants of the MNG, which differ in the way in which words are generated. In one case (referred to as classic game), the robots create a new word when becoming speaker with an empty vocabulary. In the second variant (referred to as spatial game), the robots create a new word when encountering a source with an...
empty vocabulary. In both cases, we associate each word with the closest source to the robot at the time of the word creation, and we define \( W_A \) (\( W_B \)) the set of words associated with source \( A \) (\( B \)). Note that, by construction, \( W_A \cap W_B = \emptyset \). Robots having in their inventory any word \( w \in W_A \) (\( W_B \)) constitute population \( P_{W_A} \) (\( P_{W_B} \)). Robots with no words constitute population \( P_{WO} \). In Fig. 3, we depict a possible partition of robots among different populations, both with respect to the commitment state and to their vocabulary. Since a robot can have at a given time an inventory with words originating in both source \( A \) and \( B \), the propriety \( P_{W_A} \cap P_{W_B} = \emptyset \) is not always verified. Similarly, through exchanges of words and robots between the different populations, at a given time the inventory of robots committed to one source might contain a word associated with the other source (resulting in \( P_A \neq P_{W_A} \)). At any time, we can look at the population of robots that know words associated with the source they are committed to, that is:

\[
P_M = (P_{W_A} \cap P_A) \cup (P_{W_B} \cap P_B).
\]  

(1)

Conversely, we can define the population of committed robots that know words from a non-matching source:

\[
P_S = (P_A \cap P_{W_B}) \cup (P_B \cap P_{W_A}).
\]  

(2)

Corresponding to the collectively selected source \( O \) (see definition above), we define the set of matching words \( W_O \) and non-matching words \( W_X \) as follows:

\[
W_O = \{w|(w \in W_A \land P_A > P_B) \lor (w \in W_B \land P_B > P_A)\}
\]  

(3)

\[
W_X = \{w|(w \in W_A \land P_B > P_A) \lor (w \in W_B \land P_A > P_B)\}
\]  

(4)

We define:

- **polarisation**, the condition in which committed robots know only words associated with the source they are committed to, that is, when \( P_S = \emptyset \);
- **vocabulary matching**, the condition in which only words associated with the selected source are retained within the swarm vocabulary, that is \( W_X = \emptyset \) and \( W_O \neq \emptyset \);
- **vocabulary completeness**, the condition in which exactly one word associated with each source is retained within the swarm vocabulary, that is \( |W_O| = 1 \) and \( |W_X| = 1 \).
Given a sufficiently connected swarm, the MNG dynamics ensure that the swarm will eventually converge to a final single-word vocabulary, albeit after a very long time (Baronchelli et al., 2006b; Baronchelli & Díaz-Guilera, 2012; Trianni et al., 2016). According to the previous definitions, the final vocabulary can be matching or not the selected source.

4 Correctness and completeness of the swarm vocabulary

In this section, we focus on the evolution of the swarm’s vocabulary, looking in particular to the provenance of the last words and their relation to the selected source. As already discussed (see Fig. 2), the foraging dynamics lead to either the quick selection of a single source, or to the swarm being split between the two sources, possibly for a long time. This means that, apart for a few cases and random fluctuations, there will always be a source that is selected—albeit temporarily—by the swarm. In certain settings, the swarm may forage from both sources for a long time; hence, vocabulary completeness may be observed. In other cases, the swarm will quickly converge to exploit a single source, and vocabulary matching is expected. In any case, interactions between different populations of robots are frequent, ensuring that the language dynamics always converge to a single-word vocabulary.

Here, we first focus on the patterns observed when the vocabulary converges to one or two words, to determine if matching and completeness are achieved. First, we analyse the provenance of the final word \( w_f \) to determine if it matches the selected source or not (i.e., \( w_f \in W_O \)). As the distribution of robots among sub-populations may sometimes change even after convergence to a single-word dictionary (e.g., if the language dynamics are much faster than the source selection dynamics), the final selected source may also change. Hence, we consider the source selected at the time of convergence to the final word \( w_f \), no matter what happens later to the population distribution. Similarly, we consider also the second-last word \( w_e \), to determine whether it was also matching the selected source or not at the time in which only two words remained within the whole swarm. Given such definitions, every run can end up in one of the following four possibilities:

\[
\begin{align*}
\text{OO} & : w_f \in W_O \land w_e \in W_O \\
\text{OX} & : w_f \in W_O \land w_e \in W_X \\
\text{XO} & : w_f \in W_X \land w_e \in W_O \\
\text{XX} & : w_f \in W_X \land w_e \in W_X
\end{align*}
\]

In case OO or OX is observed, the swarm has identified a final word that matches the currently selected source, although in the OX case the second-last word was associated with the non-selected source. The XO case represents a missed opportunity of matching, as a matching word was still existing in the vocabulary and could have been chosen. The XX case instead suggests that the association of words to source does not reflect the current state of the source selection. Both middle cases (OX and XO) indicate a complete vocabulary up until convergence on one word.
Given these definitions, we study the influence of the language game and the foraging dynamics over the provenance of the last two words of the vocabulary. Fig. 4 shows the frequency of each case out of the 100 runs performed for each different experimental condition. When playing the classic game (top row in Fig. 4), the swarm shows no tendency to favour a specific provenance for the final two words, and a distribution close to uniform across the four possible cases is observed. On the other hand, when playing the spatial game (bottom row in Fig. 4), the swarm favours words that match the selected source, both for the last and second-last word. In particular, the $\text{OO}$ state is strongly favoured for both weak and strong cross-inhibition, and the $\text{XX}$ state is especially disfavoured when the weak cross-inhibition leads to slower decision dynamics. In conclusion, we clearly find that the spatial game, by making the creation of words conditional to the discovery of sources, determines a strong tendency to converge towards words that represent the source that is ultimately selected. The naming process is “correct” as it best represents the source that is the most relevant for the swarm. In about 40% of the cases ($\text{OX} + \text{XO}$), the naming is “complete” as the last two words represent “names” for both the available sources. This remains valid for different values of the probability of speaking $P_s$, as shown in the supplementary Figure S1, suggesting that the spatial game is resilient to variations in the timescale of the language game.

To better understand the relationship between source selection and naming dynamics, in Figure 5 we show how the distribution of agents between sources relates with the provenance of the last two words in the swarm vocabulary. Indeed, there is a large difference between a swarm that forages from a single source and one that instead is evenly split between the two sources. In the former, we expect vocabulary matching, that is, only words from the selected source are retained (hence, case $\text{OO}$ and to some extent...
In the latter, we instead expect vocabulary completeness, that is, words coming from both sources are present (hence, cases OX and XO) because both sources are still exploited by the swarm and the selected source can change over time. Indeed, the swarm does not clearly favour the exploitation of any source, to the point of possibly changing its selected source overtime, and multiple times.\footnote{\footnotesize{Recall that the distribution of robots can change over time and always converges to the selection of one source, although after a very long time as discussed in Sect. 3.2. Here, we consider the distribution at the time of convergence of the naming dynamics, which is determined by the probability of speaking \( P_s \). Hence, an even distribution of robots among the sources is observable not only with weak cross-inhibition \( (P_s = 0.1, \text{see Fig. 2}) \), but also for strong cross-inhibition when high values of \( P_s \) cause a quick convergence of the vocabulary.}}

When the classic game is played, the distribution of robots across sources has little to no impact on the provenance of the last two words (top row of Fig. 5, see also the supplementary Figure S2 for other values of \( P_s \)). For the spatial game, instead, vocabulary matching is observed when the swarm has clearly selected one of the sources. Conversely, vocabulary completeness is more often observed with swarms that are still exploiting two sources. This is evident in case of weak cross-inhibition that entails slower dynamics in the source selection process. With strong cross-inhibition, the swarm quickly converges to exploiting a single source, and the cases in which the swarm is exploiting both sources at the time of convergence are very rare. Only when the language dynamics are particularly fast, we can observe cases of vocabulary completion for strong cross-inhibition, as shown in supplementary Figure S2 for \( P_s = 0.002 \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Empirical distribution over 100 runs of the occurrence of the last two words in the vocabulary (see Fig. 4) detailed for different distribution of the foraging swarm across the two sources, computed at the time of vocabulary convergence with \( P_s = 0.001 \). Each stacked histogram corresponds to a specific distribution of robots over the non-selected source \( \frac{P_s}{P_s+P_x} \). Bars are colour-coded as in Fig. 4. Over each histogram, the number of runs that resulted in the specified range is displayed. All tested values of \( P_s \) present similar results, shown in Figure S2. In the rare case of an equally split swarm \( (P_0 = P_x) \), there is no notion of matching a non-matching word. In that case, we redistribute AA and BB equally between OO and XX (one half each). Similarly, AB and BA are redistributed equally to OX and XO. Top row: classic game. Bottom row: spatial game.}
\end{figure}
From this analysis, we can conclude that the spatial game leads to language dynamics that correctly represent the sources relevant to the swarm, that is, those from which the swarm is currently foraging. This is obtained solely by the creation of words, which is strongly correlated with the source discovery. The interplay between language and foraging dynamics preserves such correlation despite the high number of interactions between robots from different populations and with different vocabularies. In the next section, we study how this is possible by looking at the interaction patterns between robots.

5 A study of the swarm's spatial characteristics

There are two extremes for the swarm to reach convergence on a final word. Either the swarm converges as a whole—homogeneously—on this final word, or sub-populations foraging from different sources first converge towards a word representing their source, and then a competition between these two words determines the final outcome. In this section, we look at how robots create and share their words, and how they exchange words within and across foraging sub-populations.

5.1 Impact of spatial word creation

First of all, we look at the initial phases of the naming game, when robots create and share new words. Indeed, the difference between the classic and the spatial game is solely related to this phase. Besides word creation, robots can fill their vocabulary with words shared by others. To better understand how robots obtain their first word, we plot in Fig. 6 the cumulative number of robots with at least one word in their vocabulary for the case of weak cross-inhibition.2 We highlight whether the first word was created by the robot itself or

---

2 Results for strong cross-inhibition are very similar and are displayed in Figure S3.

Springer
received from other robots upon playing the naming game. Finally, we distinguish between robots being uncommitted and exploring, or robots committed and exploiting one source. Uncommitted robots are particularly relevant, as they can get committed to any source, despite having a word associated with one or the other: they do carry a naming information that may not correspond to the source they will become committed to.

For the classic game (top row in Fig. 6), we note that the word creation dynamics is rather fast and solely depends on the probability of speaking \( P_s \). Additionally, uncommitted robots represent the large majority, meaning that word creation is strongly uncorrelated from source selection: even if a word is created closer to a source, it is generally associated with an uncommitted robot that may eventually get committed to any source, due to recruitment or discovery.\(^3\)

In the spatial game instead (see bottom row in Fig. 6), the dynamics of word creation is independent of \( P_s \) because it is determined by robots encountering a source. Specifically, \( P_s \) does not impact the number of robots that create a word when uncommitted, as these robots individually discover a source following the foraging dynamics. However, \( P_s \) determines the share of robots that create a word when committed or that receive a word when uncommitted. The former is higher when \( P_s \) is small, as the foraging dynamics are faster than the language game dynamics, meaning that several robots get recruited first and encounter a source while still having an empty vocabulary. These robots have a naming information that is strongly correlated with the source they are exploiting. Conversely, with high \( P_s \) the number of uncommitted robots that receive a word from other robots grows. These robots potentially have a naming information that differs from the source they will exploit, leading to lower spatial correlation. As a matter of fact, matching and completeness are slightly worse for this case, as can be observed in supplementary Figures S1 and S2.

### 5.2 Communication topology and interactions within the swarm

Once words have been generated, the MNG imposes a selection process until a single one is selected. This process takes place through speaker–hearer interactions and can be strongly influenced by the communication topology (Baronchelli et al., 2006a; Moretti et al., 2013). The latter is determined by the distribution of robots in space, which is a result of the foraging task the robots carry out. To understand how different sub-populations of the swarm interact, we performed an experiment with locked-size populations, forcing all robots in a pre-defined committed state. We measure the size of the neighbourhood \( N \) with which robots can potentially interact anytime, and we further distinguish between neighbours belonging to the same or to a different population. In Fig. 7, the probability of observing a neighbourhood of a given size is displayed for each possible partition \( \mathcal{P} \), between sub-populations, where \( \mathcal{P}_j = p \) indicates that \( |\mathcal{P}_A| = p \) and \( |\mathcal{P}_B| = N - p \) (in these tests, \( \mathcal{P}_U = \emptyset \)). Additionally, we also consider the case in which \( |\mathcal{P}_U| = N \), where robots are forced in the random exploration state. Given a population of agents \( \mathcal{P} \), the probability of observing a given neighbourhood of size \( n \) exclusively composed of agents from population \( \mathcal{P}' \) is computed as follows:

\(^3\) Recall that robots periodically return to the home location, where they can get recruited by any other robot, or they can start a new exploration trip in a totally different direction from the previous one. Hence, an uncommitted robot that creates a word near one source may get recruited to the other source or discover it in the following exploration trip.
where $H_r(n, t, \mathcal{P})$ counts the timesteps $t \leq T$ in which robot $r$ has neighbourhood of size $n$ limited to robots belonging to population $\mathcal{P}$. Hence, this probability is strictly dependent on the size of the populations $\mathcal{P}$ and $\mathcal{P}'$ that are being considered.

For small values of $|\mathcal{P}_s|$, one of the sub-populations is large and interactions within sub-population dominate (see Fig. 7, left panels). The neighbourhood size can take large values (e.g., more than 5 robots), even larger than the case of randomly exploring robots (see Fig. 7, bottom-left panel). Contrarily, interactions between sub-populations are practically absent, the typical neighbourhood size being $|\mathcal{N}| = 0$ (see Fig. 7, top-right panel). The more the partition among sub-populations is even, the more frequent the interactions among sub-populations become. Still, robots more likely interact within the same population, and only few cross-population interactions are recorded (see Fig. 7, bottom-right panel). This confirms that, if the swarm leans towards selecting a single source, the language dynamics are played mostly within the same population, reinforcing the correlation between words and sources in favour of matching. At the same time, the small number of interactions between sub-populations also favour completeness, with each sub-population having the chance to converge on its own word.

It is worth recalling that, besides communications between sub-populations, a mismatching word can enter a sub-population also when it is physically carried by a robot.

$$P_\Sigma(|\mathcal{N}| = n|\mathcal{P}) = \frac{1}{T|\mathcal{P}|} \sum_{r \in \mathcal{P}'} \sum_{t \leq T} H_r(n, t, \mathcal{P}') ,$$

(9)
changing from one to the other population. In order to understand how relevant the movements of robots between sub-populations are for the spreading of words, we measured the rate at which these movements take place, and compared it with the rates of interactions within and between populations during a standard experiment (see Fig. 8). The results indicate that movements between sub-populations are not as frequent as the interactions via message exchange, especially when the probability of speaking $P_s$ is high (see also Figure S4). Indeed, the rate at which messages are exchanged within and between populations increases with $P_s$, and is generally larger for intra-population interactions, confirming our previous analysis. Conversely, the rate at which robots move from one population to the other does not depend on $P_s$, and is higher when cross-inhibition is strong. We infer that the movements of robots between sub-populations do not have a relevant impact on the language dynamics in this specific experimental setup.

In the light of the presented results, we can conclude that the pattern of interactions between robots favours the segregation between sub-populations. This means that different words are likely selected within each sub-population, resulting in the vocabulary

Fig. 8 Evolution over time of the rate of communications within and between sub-populations exploiting different sources, and of the rate of robot movements between sub-populations. Each graph has been plotted for the spatial game. Similar dynamics are displayed by the classic game (see supplementary Figure S4). Top row: strong cross-inhibition. Bottom row: weak cross-inhibition.
completeness. At the same time, vocabulary matching is possible thanks to the strong correlation between word creation and source exploitation by committed robots, as discussed above. While the vocabularies well represent the environmental features and their relevance for the swarm, we note that completeness is a transient property. Indeed, the MNG dynamics determine the convergence towards a single word shared by the swarm, loosing information about previously exploited sources. To avoid this, we present in the next section a proof of concept of a language game to preserve matching and complete vocabularies.

6 Emergence of spatial categories for foraging swarms

Keeping a complete description of the environment with all its sources requires the ability to distinguish between different regions in space, leading to the construction of spatial categories. We consider a spatial category as a set of possible words, associated with an area representing the region covered by the category (here, a circle defined by its radius and its centre, the latter determining the prototype location of the spatial category). Speaking in general terms, any location in space can belong to one category, to multiple ones (in case of overlapping categories) or to none (in the case of a nonexhaustive partition of the space). The same robot can potentially hold multiple words (synonyms) referring to a given category. As a consequence, the set of the categories known to a robot—and, by extension, to the swarm—results in a kind of thesaurus. In this section, we propose a language game based on word-location pairs with the goal of representing the landscape of available sources. The language game is now first played on categories and then on words, making it more similar to a category game (Baronchelli et al., 2010).

6.1 Experimental Setup

Similar to the spatial game discussed above, categories—and associated names—are spontaneously created when a robot encounters a source at a location that is not represented by any available category. Even if a category exists for the same source, a robot may enter from a location that is not covered by the current category description. This leads to an initial proliferation of categories, which are subsequently pruned by a merging mechanism (see below).

With probability $P_s$, a robot knowing at least one category takes the speaker role: it first selects one of its known categories, followed by a word belonging to this category’s inventory. The speaker will share with the neighbours the selected word paired with the category prototype’s location. In order to maintain a correspondence between the foraging behaviour and the language game, the selection of the category is determined by the commitment status: the speaker always selects the category corresponding to the sources it is foraging from. For uncommitted robots, the category is selected randomly. On the hearer side, first a match of the received word-location pair must be found with the known categories. If the location does not belong to any known category, the hearer creates a category centred on that location, with a default starting radius of $r_0 \in \{0.2, 0.3, 0.4\}$, and add the received word to this category. If the word is fitting multiple categories, these are merged into one, and then the MNG is played with respect to the resulting category’s inventory. Categories are merged two by
two, with the resulting category being the smallest possible circle containing each original category’s circle. The merged vocabulary is the union of each category’s vocabulary.

To evaluate the ability of the swarm to generate shared spatial categories that correctly represent the available source landscape, we performed a series of experiments varying both the probability of speaking $P_s$ and the value of the initial category radius $r_0$. We introduce no change in the physical layout of the arena (see Fig. 1). Experiments are run for longer times, and are stopped once convergence is reached on both categories and number of words in each category. The additional complexity introduced by categories entails a slower language dynamics with respect to the simple naming game described before. To study the ability of the foraging swarm to correctly represent both sources, we prevent the selection of a single one by forcing $P_\sigma = 0$. In this way, the robots will find and exploit both sources (possibly with an uneven distribution across the two), and no robot will ever change source. As we observed in Sect. 5, the effects on the language dynamics of robots physically moving from one to the other source are anyway negligible.

### 6.2 Results

The evolution over time of the number of words and of categories is shown in Fig. 9 for $P_s = 0.001$ (see Figure S6 for other values). Both words and categories follow a similar pattern, with an initial fast proliferation and a following convergence towards the minimum number of elements: one single category for each source, and one single word per category. The radius $r_0$ determines the likelihood that a new category is created: when the radius is large enough, the initial category easily covers the whole source, and creation of new categories for the same source is unlikely. As a consequence, also the number of words generated is lower, because different words are generated for different categories, and the vocabularies are preserved by the category merging. In any case, the system tends to converge to the minimum number of words/categories for each value of $r_0$.

We note that the actual convergence on two categories (and hence two words) is not always permanent, as new categories can emerge after convergence on two categories. These rare events are unlikely to have long lasting impact as the swarm can recover quickly. Under these conditions, we define as time of convergence (over two categories or two words) the first time the whole swarm reaches the minimum number of words/categories.
Both category and word convergence times depend heavily on $r_0$, but also on the probability of speaking $P_s$ (see the top-left panel in Fig. 10). When $r_0$ is intermediate-small, the large proliferation of categories requires several merging operations, and having more variability in each category does not give an advantage. On the other hand, for large $r_0$ few categories are formed, and a high $P_s$ helps in quickly converging. These dynamics are confirmed also by the time of convergence to a single word per category (Fig. 10 top right), which always decreases when $P_s$ increases, with a larger effect for larger $r_0$.

Apart from the speed of convergence, another relevant aspect concerns the accuracy with which the emerging categories describe the sources to which they are associated. To measure this, we consider the position error as the distance between the centre of the category and the centre of the source (see Fig. 10 bottom left) and the average radius of the final category (Fig. 10 bottom right). When the initial radius is smaller, the error in the position of the prototype is very small, as it results from the average of many categories defined all around the source. With larger $r_0$, the position error increases because fewer categories are generated. Large values of the probability of speaking $P_s$ result in even fewer generated categories: as robots receive their initial category from other robots, larger errors are made. For what concerns the final radius of the emergent categories, smaller values are observed for small $r_0$. However, the relative increase in the final radius with respect to the initial $r_0$ is much larger for small $r_0$ than for large $r_0$ because many different categories are merged together.

7 Conclusion

In this article, we studied how the language game dynamics are influenced by the evolving topology of a swarm engaged in a decision-making and foraging task. In particular, we studied how well the swarm could maintain a description of its whole environment that is
at the same time correct and complete, with the vocabulary containing only words that are relevant to the swarm, that is, those associated with sources under exploitation. We focused on such a compelling research question, without questioning properties commonly studied in swarm robotics such as robustness or scalability. Such properties have been largely studied for foraging, language dynamics and decision-making in previous studies and in conditions very close to the ones discussed here (Trianni & Campo, 2015; Reina et al., 2015a). Hence, they are no further debated, allowing us to focus on the interplay between language and decision dynamics.

We began by comparing two variations in the MNG: one that binds the creation of words with the sources available in the environment (spatial game), the other without such spatial correlation (classic game), where words are used as simple tokens. Note that a mild spatial correlation is available between words and sources also in the classic game, given that words are created by robots at locations that are always closer to one of the sources. However, this was not sufficient to guarantee the emergence of a correct and complete language as in the spatial game. Indeed, the stronger correlation between creation of words and source location granted by the spatial game is not the only reason for the better matching and completeness. We observed that a major difference is given by the role of uncommitted, exploring robots into the creation and sharing of words. These robots can end up choosing any source, bringing words created near one source to the population exploiting the other. Additionally, we observed that the topology of the robot’s interaction network—determined by the robot’s movements during the foraging activity—consists of two almost segregated sub-populations, with sporadic interactions constrained to the central nest area. Such segregation creates the conditions for the maintenance of one word for each source, supporting completeness of the evolving vocabulary. In order for the swarm to maintain a complete description of the environment even when sources are not relevant any more, we proposed as a proof of concept a simple version of a category game embedded in space. In this setup, the swarm creates different categories for each source, and ends up retaining an exhaustive description that can also be sufficiently precise to potentially support the foraging activities.

One potential drawback of language evolution as observed in our experiments is related to the time required for emergent conventions to settle, which can be very large if interactions are sporadic, as well as the possibility that new conventions enter the population and destabilise the language dynamics. In this respect, it is important to note that linguistic conventions do not have an intrinsic value (e.g., every name can be equivalent as long as it is understood), but are more valuable when they are largely shared within a population, favouring coordination and avoiding misunderstandings. Hence, it is possible to speed up convergence towards a shared convention within a population by means of positive feedback mechanisms that favour the conventions more commonly found within the population. For instance, the simple rules of the naming game could be enhanced with estimates of the frequency of words in the population, allowing to favour the selection of more frequent words when speaking, hence speeding up convergence. Additionally, decentralised quorum sensing approaches can be exploited to determine a final convention, avoiding that noise is added by new alternatives when a largely shared one is already present. These and similar mechanisms can reduce the number of interactions required to achieve language convergence within a population, making language games practicable in realistic settings beyond the abstract scenario studied in this paper.

Overall, we believe that merging language dynamics with the self-organising behaviour of robot swarms can have a high potential, as the robot behaviour can exploit the emergent descriptions of the environment in a way that is dependent on the features relevant for the
swarm. Our experiments demonstrate a possible way to obtain a meaningful link between tasks and the evolving language, supporting future research activities. The link between language and behaviour was relegated here to the creation of words/categories. However, stronger links can be built if behavioural decisions can be determined by the evolving language. This also allows to adapt the language to the environmental contingencies encountered, possibly enabling more flexibility in the swarm behaviour with respect to changing environmental conditions (Cambier et al., 2021).

In future studies, besides describing the relevant features of the environment, linguistic conventions can be exploited also to agree on the best course of action for the swarm. For instance, robots would share short term plans described as a sequence of linguistic elements, creating and merging them following shared compositional strategies. In this sense, the possibilities offered by language evolution are vast, allowing robot swarms to autonomously find sentence-like solutions to complex tasks made of several spatially distributed and temporally dependent sub-tasks.

Acknowledgements A. Reina acknowledges the financial support from the Belgian F.R.S.-FNRS, of which he is a Postdoctoral Researcher, and from the Office of Naval Research Global (ONRG) through grant no. 12547352. M. Dorigo acknowledges support from the Belgian F.R.S.-FNRS, of which he is a Research Director. V. Trianni acknowledges partial support from the Office of Naval Research Global (ONRG) through grant no. N62909-18-1-2093 and from the European project TAILOR (H2020-ICT-48 GA: 952215).

References

Allwright, M., Bhalla, N., El-faham, H., Antoun, A., Pinciroli, C., & Dorigo, M. (2014). SRoCS: Leveraging stigmergy on a multi-robot construction platform for unknown environments. In M. Dorigo et al. (Eds.) Swarm Intelligence: 9th International Conference, ANTS 2014 (LNCS Vol. 8667, pp. 158–169). Springer.

Bailis, P., Nagpal, R., & Werfel, J., et al. (2010). Positional communication and private information in honeybee foraging models. In M. Dorigo et al. (Eds.) Swarm Intelligence: 7th International Conference, ANTS 2010 (LNCS Vol. 6234, pp. 263–274). Springer.

Baronchelli, A. (2011). Role of feedback and broadcasting in the naming game. Physical Review E, 83(4), 046103.

Baronchelli, A., & Díaz-Guilera, A. (2012). Consensus in networks of mobile communicating agents. Physical Review E, 85(1), 016113.

Baronchelli, A., Dall’Asta, L., Barrat, A., & Loreto, V. (2006a). Topology-induced coarsening in language games. Physical Review E, 73(1), 015102.

Baronchelli, A., Felici, M., Loreto, V., Caglioti, E., & Steels, L. (2006b). Sharp transition towards shared vocabularies in multi-agent systems. Journal of Statistical Mechanics: Theory and Experiment, 2006(06), P06014.

Baronchelli, A., Dall’Asta, L., Barrat, A., & Loreto, V. (2007). The role of topology on the dynamics of the naming game. The European Physical Journal Special Topics, 143(1), 233–235.

Baronchelli, A., Gong, T., Puglisi, A., & Loreto, V. (2010). Modeling the emergence of universality in color naming patterns. Proceedings of the National Academy of Sciences, 107(6), 2403–2407.

Beckers, R., Holland, O. E., & Deneubourg, J.-L. (2000). From local actions to global tasks: Stigmergy and collective robotics. Prerational intelligence: Adaptive behavior and intelligent systems without symbols and logic, volume 1, volume 2 Prerational intelligence: Interdisciplinary perspectives on the behavior of natural and artificial systems, volume 3 (pp. 1008–1022). Studies in Cognitive Systems, vol 26. Dordrecht:Springer.

Beekman, M., Sumpter, D. J., & Ratnieks, F. L. (2001). Phase transition between disordered and ordered foraging in Pharaoh’s ants. Proceedings of the National Academy of Sciences, 98(17), 9703–9706.

Biesmeijer, J. C., & de Vries, H. (2001). Exploration and exploitation of food sources by social insect colonies: a revision of the scout-recruit concept. Behavioral Ecology and Sociobiology, 49(2–3), 89–99.
Billard, A., & Hayes, G. (1997). Learning to communicate through imitation in autonomous robots. In: Gerstner, W., Germond, A., Hasler, M., Nicoud, J.D. (eds) Artificial Neural Networks — ICANN 1997. Lecture Notes in Computer Science, vol 1327. (pp. 763–768). Berlin: Springer.

Brambilla, M., Ferrante, E., Birattari, M., & Dorigo, M. (2013). Swarm robotics: a review from the swarm engineering perspective. Swarm Intelligence, 7(1), 1–41.

Cambier, N., Frémont, V., Trianni, V., & Ferrante, E. (2018). Embodied evolution of self-organised aggregation by cultural propagation. In M. Dorigo et al. (Eds), Swarm Intelligence, 11th International Conference, ANTS 2018 (LNCS Vol. 11172, pp. 351–359). Springer.

Cambier, N., Miletitch, R., Frémont, V., Dorigo, M., Ferrante, E., & Trianni, V. (2020). Language evolution in swarm robotics: A perspective. Frontiers in Robotics and AI, 7, 12.

Cambier, N., Albani, D., Frémont, V., Trianni, V., & Ferrante, E. (2021). Cultural evolution of probabilistic aggregation in synthetic swarms. Applied Soft Computing, 113, 108010.

Dimidov, C., Oriolo, G., & Trianni, V. (2016). Random walks in swarm robotics: An experiment with kilobots. In M. Dorigo et al. (Eds.), Swarm Intelligence, 10th International Conference, ANTS 2016 (LNCS Vol. 9882, pp. 185–196). Springer.

Dorigo, M., Birattari, M., & Brambilla, M. (2014). Swarm robotics. Scholarpedia, 9(1), 1463.

Dorigo, M., Theraulaz, G., & Trianni, V. (2020). Reflections on the future of swarm robotics. Science Robotics, 5(49), eabe4385.

Ducatelle, F., Di Caro, G. A., Forster, A., Bonani, M., Dorigo, M., Magenat, S., et al. (2014). Cooperative navigation in robotic swarms. Swarm Intelligence, 8(1), 1–33.

Ferrante, E., Turgut, A. E., Duéñez-Guzmán, E., Dorigo, M., & Wenseleers, T. (2015). Evolution of self-organized task specialization in robot swarms. PLoS Computational Biology, 11(8), e1004273.

Floreano, D., Mitri, S., Magenat, S., & Keller, L. (2007). Evolutionary conditions for the emergence of communicatory in robots. Current biology, 17(6), 514–519.

Fujisawa, R., Dobata, S., Sugawara, K., & Matsuno, F. (2014). Designing pheromone communication in swarm robotics: Group foraging behavior mediated by chemical substance. Swarm Intelligence, 8(3), 227–246.

Gutiérrez, A., Campo, A., Dorigo, M., Donate, J., Monasterio-Huelin, F., & Magdalena, L. (2009). Open e-puck range & bearing miniaturized board for local communication in swarm robotics. In 2009 IEEE International Conference on Robotics and Automation (pp. 3111–3116).

Gutiérrez, A., Campo, A., Monasterio-Huelin, F., Magdalena, L., & Dorigo, M. (2010). Collective decision-making based on social odometry. Neural computing & applications, 19(6), 807–823.

Holland, O., & Melhuish, C. (1999). Stigmergy, self-organization, and sorting in collective robotics. Artificial life, 5(2), 173–202.

Loreto, V., Baronielli, A., Mukherjee, A., Puglisi, A., & Tria, F. (2011). Statistical physics of language dynamics. Journal of Statistical Mechanics: Theory and Experiment, 2011(04), P04006.

Lu, Q., Korniss, G., & Szymanski, B. K. (2008). Naming games in two-dimensional and small-world-connected random geometric networks. Physical Review E, 77, 016111.

Marocco, D., & Nolfi, S. (2007). Emergence of communication in embodied agents evolved for the ability to solve a collective navigation problem. Connection Science, 19(1), 53–74.

Miletitch, R., Trianni, V., Campo, A., Dorigo, M. (2013). Information aggregation mechanisms in social odometry. In Proceedings of the Twelfth European Conference on Artificial Life (ECAL 2013) (pp. 102–109). MIT Press, Cambridge, MA.

Miletitch, R., Dorigo, M., & Trianni, V. (2018). Balancing exploitation of renewable resources by a robot swarm. Swarm Intelligence, 12(4), 307–326.

Mondada, F., Bonani, M., Raemy, X., Pugh, J., Cianci, C., Klaptocez, A., Magenat, S., Zufferey, J.-C., Floreano, D., & Martinoli, A. (2009). The e-puck, a robot designed for education in engineering. In Proceedings of the 9th Conference on Autonomous Robot Systems and Competitions (pp. 59–65). IPCB: Instituto Politécnico de Castelo Branco, Portugal.

Moretti, P., Baronielli, A., Starnini, M., & Pastor-Satorras, R. (2013). Generalized voter-like models on heterogeneous networks. In A. Mukherjee, M. Choudhury, F. Peruani, N. Ganguly, & B. Mitra (Eds.), Dynamics on and of complex networks, applications to time-varying dynamical systems (Vol. 2, pp. 285–300). London: Springer.

Pais, D., Hogan, P. M., Schlegel, T., Franks, N. R., Leonard, N. E., & Marshall, J. A. R. (2013). A mechanism for value-sensitive decision-making. PLoS ONE, 8(9), e73216.

Pinciroli, C., Trianni, V., O’Grady, R., Pini, G., Brutschy, A., Brambilla, M., Mathews, N., Ferrante, E., Di Caro, G. A., Ducatelle, F., Birattari, M., Gambardella, L. M., & Dorigo, M. (2012). ARGoS: a modular, parallel, multi-engine simulator for multi-robot systems. Swarm Intelligence, 6(4), 271–295.
Puglisi, A., Baronchelli, A., & Loreto, V. (2008). Cultural route to the emergence of linguistic categories. *Proceedings of the National Academy of Sciences, 105*(23), 7936–7940.

Reina, A., Miletich, R., Dorigo, M., & Trianni, V. (2015a). A quantitative micro-macro link for collective decisions: the shortest path discovery/selection example. *Swarm Intelligence, 9*(2–3), 75–102.

Reina, A., Valentini, G., Fernández-Oto, C., Dorigo, M., & Trianni, V. (2015b). A design pattern for decentralised decision making. *PLoS ONE, 10*(10), e0140950.

Reina, A., Marshall, J. A. R., Trianni, V., & Bose, T. (2017). Model of the best-of-N nest-site selection process in honeybees. *Physical Review E, 95*(5), 052411.

Reina, A., Bose, T., Trianni, V., & Marshall, J. A. R. (2018). Effects of spatiality on value-sensitive decisions made by robot swarms. In Distributed Autonomous Robotic Systems. Springer Proceedings in Advanced Robotics (Vol. 6, pp. 461–473). Cham: Springer.

Saleh, N., & Chittka, L. (2007). Traplining in bumblebees (*Bombus impatiens*): A foraging strategy’s ontogeny and the importance of spatial reference memory in short-range foraging. *Oecologia, 151*(4), 719–730.

Seeley, T. D., Visscher, P. K., Schlegel, T., Hogan, P. M., Franks, N. R., & Marshall, J. A. R. (2012). Stop signals provide cross inhibition in collective decision-making by honeybee swarms. *Science, 335*(6064), 108–111.

Solé, R. V., Corominas-Murtra, B., & Fortuny, J. (2010). Diversity, competition, extinction: the ecophysics of language change. *Journal of The Royal Society Interface, 7*(53), 1647–1664.

Spranger, M. (2013). Evolving grounded spatial language strategies. *Künstliche Intelligenz, 27*(2), 97–106.

Steels, L. (1995). A self-organizing spatial vocabulary. *Artificial life, 2*(3), 319–332.

Steels, L. (2001). Language games for autonomous robots. *IEEE Intelligent systems, 16*(5), 16–22.

Steels, L. (2003). Evolving grounded communication for robots. *Trends in Cognitive Sciences, 7*(7), 308–312.

Stiefelhagen, M., Meijer, B. R., van der Werff, K., & Tomiyama T. (2004). Distributed autonomous agents, navigation and cooperation with minimum intelligence in a dynamic warehouse application. In 2004 IEEE International Conference on Systems, Man and Cybernetics (Vol. 6, pp. 5573–5578).

Talamali, M. S., Bose, T., Haire, M., Xu, X., Marshall, J. A. R., & Reina, A. (2020). Sophisticated collective foraging with minimalist agents: a swarm robotics test. *Swarm Intelligence, 14*(1), 25–56.

Trianni, V., & Campo, A. (2015). Fundamental collective behaviors in swarm robotics. In J. Kacprzyk & W. Pedrycz (Eds.), *Springer Handbook of Computational Intelligence* (pp. 1377–1394). Springer.

Trianni, V., De Simone, D., Reina, A., & Baronchelli, A. (2016). Emergence of consensus in a multi-robot network: From abstract models to empirical validation. *IEEE Robotics and Automation Letters, 1*(1), 348–353.

Wang, W. S., & Minett, J. W. (2005). The invasion of language: Emergence, change and death. *Trends in ecology & evolution, 20*(5), 263–269.

**Publisher’s Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.