Statistical nature of cluster emission in nuclear liquid phase

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The emission of nuclear clusters is investigated within the framework of isospin dependent target gas model and classical molecular dynamics model. It is found that the emission of individual cluster which is heavier than proton is almost Poissonian except near the liquid gas phase transition point and the thermal scaling is observed by the linear Arrhenius plots which is made from the average multiplicity of each cluster versus the inverse of temperature in the nuclear liquid phase. It indicates of a statistical nature of such cluster emission in the models. The "emission barriers" which are the slopes of the Arrhenius plots are extracted as a function of the mass or charge number and fitted by the formula embodied with the contributions of the surface energy and Coulomb interaction. The possible influences of the source size, Coulomb interaction and "freeze-out" density and related physical implications are discussed.

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I. INTRODUCTION

Hot excited nuclei with moderate temperature can be formed by the collisions between heavy ions at low-intermediate energy and they finally de-excite by the different decay modes, such as the light particle evaporation and the emission of multiple intermediate mass fragment (IMF), ie. multifragmentation. Even though extensive studies on multifragmentation have been carried out experimentally and theoretically, it is still an open question to clarify definitely whether the multifragmentation is statistical or dynamical, sequential or simultaneous. Among such efforts, Moretto et al. found that there exists the resilient reducibility and thermal scaling in multiple fragment emission process, which seems to show one a possible interpretation picture to look and understand the multifragmentation. Originally, they observed that the experimental Z-integrated fragment multiplicity distributions $P^m_n$ are binomially distributed, i.e.,

$$P^m_n(p) = \frac{m!}{n!(m-n)!} p^n (1-p)^{m-n},$$  \hspace{1cm} (1)

in each transverse energy ($E_t$) window, where $n$ is the number of emitted fragments and $m$ is interpreted as the number of times the system tries to emit a fragment. The probability of emitting $n$ fragments can be reduced to a single-particle emission probability $p$ which gives linear Arrhenius plots (ie. excitation functions) when $\ln(1/p)$ is plotted vs $1/\sqrt{E_t}$. By assuming a linear relationship between $\sqrt{E_t}$ and temperature $T$, they proposed that the linearity of the observed $\ln(1/p) \propto 1/\sqrt{E_t}$ can be interpreted as a thermal scaling of the multifragment process \cite{1, 2, 3}. In this case, these linear Arrhenius plots suggest that $p$ has the Boltzmann form $p \propto \exp^{-B/T}$ with a common fragment barrier $B$. However, since the binomial decomposition has been performed on the Z-integrated multiplicities, typically associated with $3 \leq Z \leq 20$, the Arrhenius plot generated with the resulting one fragment probability $p$ is an average over a range of $Z$ values. Some comments and criticisms on the above binomial distribution and the thermal scaling were also raised \cite{4, 5, 6, 7}. One paper, that of Tsang and Danielewicz \cite{4}, found that the assumption of $T \propto \sqrt{E_t}$ may be only valid for compound nuclei formed at low-to-moderate temperatures and it will be broken down for the intermediate energy heavy ion collisions such as $^{40}\text{Ar} + ^{197}\text{Au}$ at $E/A = 37$ to 110 MeV due to the mid-rapidity particle emission from the overlap region of projectile and target as well as the delay emission from projectile-like and target-like residues. In this case the above interpretation of thermal scaling is not valid in the point of experimental view. Moreover, they found that the fit with the binomial distribution can be replaced by the Poisson distribution in the constraint of charge conservation. As a consequence, the reducibility of fragment emission to binomial distributions does not imply the fundamental significance for the parameters of $m$ and $p$ thereby extracted. If $p$ is not an elementary emission probability, then the Arrhenius law will fail also.

Later, instead of analyzing for Z-integrated multiplicities, Beaulieu and Moretto et al. analyzed the behavior of individual fragment species of a given $Z$ for higher resolution experimental data and noticed that the $n$-fragment multiplicities $P(n)$ obey a nearly Poisson distribution,

$$P(n) = \frac{<n>^n e^{-<n>}}{n!},$$  \hspace{1cm} (2)

where $n$ is the number of fragments of a given $Z$ and the average value $<n>$ is a function of the total transverse energy $E_t$, and were thus reducible to a single-fragment probability proportional to the average value $<n>$ for each $Z$ \cite{8, 9}. Similarly the $<n>$ is found to be proportional to $\exp^{-B/T}$ providing that $T \propto \sqrt{E_t}$, ie. there
exists also a thermal scaling law. More recently, Elliott and Moretto et al. discovered that the common features of Poissonian reducibility and thermal scaling can also be revealed in percolation and the Fisher droplet model [10, 11].

In the present work, we would like to make a theoretical reexamination on the Poissonian reducibility and its thermal scaling rather than the binomial reducibility and its thermal scaling. Unlike in experiment, we will adopt the true temperature to check the Poisson reducibility and thermal scaling in the frameworks of the isospin dependent lattice gas model (I-LGM) [12, 13] and followed by the isospin dependent classical molecular dynamics (I-CMD) of Das Gupta and Pan [14]. Of course, we should keep in mind that the assumption of integrable dynamics (I-CMD) of Das Gupta and Pan [14].

One of the results of this work is the application of the mean field approximation [22]. In this work, we will adopt the lattice gas model which was developed by Das Gupta et al. [2, 13]. In addition, a classical molecular dynamical model of Das Gupta et al. [14] is also used to compare with the results of lattice gas model. For completeness of the paper, here we make a brief description for the models.

In the lattice gas model, $A (= N + Z)$ nucleons with an occupation number $s_i$, which is defined $s_i = 1 (-1)$ for a proton (neutron) or $s_i = 0$ for a vacancy, are placed on the $L$ sites of lattice. Nucleons in the nearest neighboring sites have interaction with an energy $\epsilon_{s_i s_j}$. The Hamiltonian is written as

$$E = \sum_{i=1}^{A} \frac{p_i^2}{2m} - \sum_{i<j} \epsilon_{s_i s_j} s_i s_j,$$  \hspace{1cm} (3)

where $\mu_i$ is the momentum of the nucleon and $m$ is its mass. The interaction constant $\epsilon_{s_i s_j}$ is chosen to be isospin dependent and be fixed to reproduce the binding energy of the nuclei [14]:

$$\begin{align*}
\epsilon_{nn} &= \epsilon_{pp} = 0.6 MeV, \\
\epsilon_{pn} &= -5.33 MeV. 
\end{align*}$$

Three-dimension cubic lattice with $L$ sites is used which results in $\mu_i$ of an assumed freeze-out density of disassembling system, in which $\rho_0$ is the normal nuclear density. The disassembly of the system is to be calculated at $\mu_f$, beyond which nucleons are too far apart to interact. Nucleons are put into lattice by Monte Carlo Metropolis sampling. Once the nucleons have been placed we also ascribe to each of them a momentum by Monte Carlo samplings of Maxwell-Boltzmann distribution.

Once this is done the I-LGM immediately gives the cluster distribution using the rule that two nucleons are part of the same cluster if

$$P_r^2/2\mu - \epsilon_{s_i s_j} s_i s_j < 0,$$  \hspace{1cm} (5)

where $P_r$ is the relative momentum of two nucleons and $\mu$ is their reduced mass. This prescription is evidenced to be similar to the Coniglio-Klein’s prescription [23] in condensed matter physics and be valid in I-LGM [12, 13]. To calculate clusters using I-CMD we propagate the particles from the initial configuration for a long time under the influence of the chosen force. The form of the force is chosen to be also isospin dependent in order to compare with the results of I-LGM. The potential for unlike nucleons is

$$\begin{align*}
v_{pp}(r)\left(\frac{r}{r_0} < a\right) &= A \left[ B \left(\frac{r}{r_0}\right)^p - \left(\frac{r}{r_0}\right)^q \right] \exp \left(\frac{1}{\frac{r}{r_0} - a}\right), \\
v_{pp}(r)\left(\frac{r}{r_0} > a\right) &= 0.
\end{align*}$$

In the above, $r_0 = 1.842 fm$ is the distance between the centers of two adjacent cubes. The parameters of the potentials are $p = 2, q = 1, a = 1.3, B = 0.924$, and $A =$$
1966 MeV. With these parameters the potential is minimum at \( r_0 \) with the value -5.33 MeV, is zero when the nucleons are more than 1.3\( r_0 \) apart and becomes stronger repulsive when \( r \) is significantly less than \( r_0 \). The potential for like nucleons is written as

\[
\begin{align*}
v_{pp}(r < r_0) &= v_{np}(r) - v_{np}(r_0), \\
v_{pp}(r > r_0) &= 0.
\end{align*}
\]

(7)

This means there is a repulsive core which goes to zero at \( r_0 \) and is zero afterwards. It is consistent with the fact that we do not put two like nucleons in the same cube. The system evolves for a long time from the initial configuration obtained by the lattice gas model under the influence of the above potential. At asymptotic times the configuration obtained by the lattice gas model under the freeze-out density. 1000 events are simulated for each \( \sigma \) Eq.(2) is the ratio \( < n_i > \) for the clusters classified with different masses \( (A) \), light isotopes \( (ISO) \) and atomic numbers \( (Z) \) for the disassembly process.

III. RESULTS AND DISCUSSIONS

In this paper we choose the medium size nuclei \(^{129}\text{Xe}^\) as a main example to analyze the behavior of individual fragment emission during nuclear disassembly with the help of I-LGM and I-CMD. In addition, the systems with \( A_{sys} = 80 \) \( (Z_{sys} = 33) \) and 274 \( (Z_{sys} = 114) \) are also studied to investigate the possible source size dependence. In most case, \( \rho_f \) is chosen to be about 0.38 \( \rho_0 \), since the experimental data can be best fitted by \( \rho_f \) between 0.3\( \rho_0 \) and 0.4\( \rho_0 \) in the previous LGM calculations [12, 22], which corresponds to \( 7^3 \) cubic lattice is used for \( \text{Xe}^\) 6\(^3\) for \( A_{sys} = 80 \) and 9\(^3\) for \( A_{sys} = 274 \). In the condition of the fixed freeze-out density, the only input parameter of the models is the temperature \( T \). In the I-LGM case, \( \rho_f \) can be thought as the freeze-out density but in the I-CMD case \( \rho_f \) is, strictly speaking, not a freeze-out density but merely defines the starting point for time evolution. However since classical evolution of a many particle system is entirely deterministic, the initialization does have in it all the information of the asymptotic cluster distribution, we will continue to call \( \rho_f \) as the freeze-out density. 1000 events are simulated for each \( T \) which ensures enough statistics.

A. Poissonian Reducibility

One of the basic characters of the Poisson distribution Eq.(2) is the ratio \( \sigma_i^2/ < n_i > \rightarrow 1 \) where \( \sigma_i^2 \) is the variance of the multiplicity distribution and \( < n_i > \) is the mean value of the multiplicity distribution. The first step we are showing is this ratio. We obtain these ratios for clusters classified with different masses \( (A) \), light isotopes \( (ISO) \) and atomic numbers \( (Z) \) for the disassembly of \(^{129}\text{Xe}^\) as a function of temperature in the framework of I-LGM and I-CMD with Coulomb in Fig. 1. Obviously, most of the ratios are close to one, which indicates that basically these cluster production obeys the Poisson distributions, i.e., a cluster is formed independently from one another. Of course, we also notice that the values of \( \sigma_i^2/ < n_i > \rightarrow 1 \) is almost lower than the unique, i.e., it is narrower than the Poisson distribution. This could be due to protons can be easily produced by an ”evaporation”-like mechanism in the models, i.e., protons can be easily separated from some unstable multi-nucleon clusters. In this case, this kind of proton production is obviously related to the parent cluster and then a narrower distribution of mixed protons could reveal. On the contrary, some points are slightly larger than 1 and this behavior becomes obvious in the medium temperature range, which could be related to the onset of phase transition in such moderate thermal excitation [21], where the large critical fluctuation [23] makes the Poisson reducibility broke-down.

Besides the above Poisson condition is basically sustained, the excitation function of the average multiplicity of \( n \)-multiple individual cluster emission can be well fitted with the Poisson distribution. For some examples,
The excitation functions of $n$-multiple cluster emission probability ($P_n$) for elements with $Z \geq 2$ emission from the source $^{129}$Xe in the I-LGM calculation. The lines show the expected values in different temperature with the Poisson assumption according to Eq.(2).

Fig. 3 shows the quality of the Poisson fits to the average multiplicity of $n$-multiple individual cluster emission in the different temperature for $^{129}$Xe in the I-LGM case. In each panel of this figure, we first plot the probability of $n$-multiple emission species ($P(n)$) as a function of temperature (as shown by the open symbols), and then we connect the Poisson probabilities in the different temperature as lines in terms of Eq.(2) due to we know $n$ and its average value $< n >$ over all $n$-multiple emission in each temperature (as shown by the lines). Obviously, these Poisson fits are quite good for almost $Z \geq 2$ over the entire range of $T$. The similar good Poisson fit is overall obtained in the cases of I-CMD. As a consequence, we think that Poissonian reducibility is valid in the thermal-equilibrium lattice gas model or molecular dynamics, which illustrates that the cluster production is almost independent each other in the studied models.

B. Thermal Scaling

Naturally, we want to know if there exists a thermal scaling law in the thermal-equilibrium LGM and CMD models. To this end, the temperature dependence of the mean yield ($< n >$) of individual clusters is investigated. In order to compare a recent well-known thermal Arrhenius-type plot in nuclear multifragmentation phenomenon [3, 4], we plot $ln < n >$ versus $1/T$. Fig. 3 gives a family of these plots for the disassembly of $^{129}$Xe within the framework of I-LGM (left panels) and I-CMD with Coulomb interaction (right panels). Again, as Fig. 3, the clusters are classified according to their masses (upper panels), the light isotopes (the middle panels) and the charge numbers (the lower panels). For all the panels, the obtained Arrhenius plots are linear for the lower $T$ side, and their slopes generally increase with increasing $A$ or $Z$ value. Generally, the thermal scaling is expected when the yields, for a fixed nucleon number system, are dominated by fragment binding. This is the case when the temperatures are low compared to the binding energy per particle. At these temperatures, one can anticipate one large fragment surrounded by many small clusters. However, contrary tendency reveals in the high $T$ side where $ln < n >$ increases with $1/T$, i.e. decreases with increasing $T$. In this case, nuclear Arrhenius plots of $< n >$ with $1/T$ are not valid but the Poissonian reducibility still remains (see Fig. 3). This behavior of $< n >$ at higher $T$ is related to the branch of the fall of the multiplicity of IMF ($N_{IMF}$) with $T$ where the disassembling system is in vaporization [26, 27, 28] and hence only the lightest clusters are dominated and the heavier clusters become fewer and fewer with increasing $T$. The temperature where the Arrhenius-type plot begins to deviate from the linearity just indicates the onset of transition from the Fermi liquid phase to gas phase in I-LGM and I-CMD [21]. In other words, the Arrhenius law looks valid only below the critical phase transition point, i.e. in the nuclear liquid phase. Recognizing this phenomenon, in the following sections we only focus on the branch of lower temperature (i.e. the liquid phase) where the thermal scaling exists to discuss the Arrhenius law and their slopes.

C. “Emission Barriers”

1. Model dependence

From Fig. 3 the slope parameter can be directly extracted in the lower $T$ side as a function of $Z$ or $A$. In Ref. [8] Moretto et al. has interpreted these slope parameters as “emission barriers” of specific individual fragments. Fig. 4 gives the emission barrier of individual fragments with different $A$, ISO and $Z$ in the framework of I-LGM, I-CMD with/without Coulomb interaction. The error bar in the figure represents the error in the extraction of the slope parameter. The first indication from this figure is that the emission barrier in the I-LGM case is the nearly same as the I-CMD case without Coulomb force, which supports that I-LGM is equivalent to I-CMD without Coulomb interaction rather well when the nuclear
potential parameter is moderately chosen, but I-LGM is a quick model to analyze the behavior of nuclear dissociations. The inclusion of long-range Coulomb interaction makes the emission barrier of individual fragments much lower since the repulsion of Coulomb force reduces the attractive role of potential and hence make clusters escape easily. The second indication is that the emission barriers increase with \( A(Z) \) at low \( A(Z) \) values and tend to be saturated at high \( A(Z) \) ones. Similar experimental results have been observed for individual fragments with different \( Z \) in Ref. [8] or different \( A \) in Ref. [10]. However, the middle panel of Fig. 4 shows that bare dependence of emission barrier of \( ISO \) on \( A \) in the fixed atomic number \( Z \), which indicates that the \( Z \) dependence of barrier is perhaps more intrinsic and the \( A \) dependence is basically due to the average effect over the species with the same \( A \) but different \( Z \).

2. Source size and Coulomb interaction dependence

On the origin of these barrier, the surface energy and Coulomb energy would play the roles. If the cluster emission is mainly controlled by its surface energy, it would suggest barriers proportional to \( Z^{2/3}(A^{2/3}) \). In the case of I-LGM and I-CMD without Coulomb, we can try to fit the barrier for the particles with different mass number by

\[
B_{\text{Coul.off}} = c_1 \times A_1^{2/3},
\]

or for the particles with different charge number by

\[
B_{\text{Coul.off}} = c_1 \times ((A/Z)_{fit} \times Z_i)^{2/3},
\]

where \((A/Z)_{fit}\) is a fit coefficient of \( A/Z \) for emitted particles, and \( A_1 \) \((Z_i)\) is the mass (charge) of particle. \( c_1 \) is the fit constant for surface energy term. The solid line in the Fig. [8] is a function of Eq.(8) with \( c_1 = 8.469 \) and
the solid line in the Fig. 4 is a function of Eq. (9) with \( c_1 = 8.469 \) and \( (A/Z)_{fit} = 1.866 \). These excellent fits imply that the surface energy play a major role in controlling the cluster emission when the long range Coulomb force is not considered. However for the cluster emission with the Coulomb field, we can assumed that the barrier is mainly constituted by the surface energy term and an additional Coulomb term as

\[
B_{\text{Coul.on}} = c_2 \times A^{2/3} - \frac{1.44 \times A_i/(A/Z)_{fit} \times Z_{res}}{r_{\text{Coul}}(A_i^{1/3} + ((A/Z)_{fit} \times Z_{res})^{1/3})}
\]  

for the particles classified with different charge number, or

\[
B_{\text{Coul.on}} = c_2 \times ((A/Z)_{fit} \times Z_i)^{2/3} - \frac{1.44 \times Z_i \times Z_{res}}{r_{\text{Coul}}((Z_i \times (A/Z)_{fit})^{1/3} + (Z_{res} \times (A/Z)_{fit})^{1/3})}
\]

for the particles classified with different mass number, where \( c_2 \) is a fit constant for surface term and \( r_{\text{Coul}} \) is chosen to be 1.22 fm. \( Z_{res} \) is a fitted average charge number of the residue. \( (A/Z)_{fit} \) is chosen to be 1.866, as taken from the fits for I-LGM. The overall fits for \( A \) and \( Z \) dependent barrier in the case of I-CMD with Coulomb force give \( c_2 = 12.921 \) and \( Z_{res} \sim 41 \) with the dot-dashed line in Fig. 4a and 4c. The excellent fit supports that the Coulomb energy plays another important role in the cluster emission.

In the case of I-LGM and I-CMD without Coulomb, one would expect the barrier for each \( Z \) \( A \) to be nearly independent of the system studied if only the surface energy is substantial to the emission barrier. The left panel of the Fig. 3 shows the results for \( B_A, B_{ISO} \) and \( B_Z \) for three different systems in the I-LGM case. The same freeze-out density of \( 0.38\rho_0 \) and the same \( N/Z \) is chosen for the systems of \( A_{sys} = 80 \) and \( A_{sys} = 274 \). Actually, it appears to have no obvious dependence of emission barrier on source size as expected for the role of surface energy. The solid line in the figure is the same as in Fig. 2. However, when the long-range Coulomb interaction is considered, the emission barrier reveals a source size dependence. The right panel of Fig. 3 gives the emission barriers \( B_A, B_{ISO} \) and \( B_Z \) in the case of I-CMD with Coulomb force. It looks that the barrier increase with the decreasing of charge of system, which can be explained with the Eq. (10) and (11) where the decreasing of the residue \( Z_{res} \) will result in the decreasing of the Coulomb barrier and hence the increasing of the emission barrier. The lines represent the fits with the Eq. (10) and (11) for three different mass systems.

IV. CONCLUSION

In conclusion, the Poisson reducibility and thermal scaling of the emitted clusters is explored in the lattice gas model and molecular dynamical model. It indicates of a statistical nature of such cluster emission. The Poisson reducibility illustrates that the clusters are produced independently each other and stochastic. But near the liquid gas phase transition, the large fluctuation breaks down the Poisson reducibility. The thermal scaling is existed when the temperatures are low compared to the binding energy per particle. At these temperatures, one can anticipate one large fragment surrounded by many small clusters, \( ie. \) the nuclei is in the liquid phase. The calculations looks qualitatively consistent with the recent experimental observation of Poisson reducibility and thermal scaling by Moretto/Wozniak's
group even though the system studied is different and the temperature was supposed to be proportional to the total transverse energy in their experiments. This also supports somehow that the lattice gas model and classical molecular dynamics is a useful tool to simulate the nuclear disassembly.

Further, based on the Arrhenius law in the liquid phase, we extracted the emission barrier for clusters with the different mass, light isotope mass and charge number. Also, the systematic fits with the formula embodied with the surface energy and Coulomb interaction were performed and the overall good fits were reached. The results suggest that the cluster emission is mainly controlled by both the surface energy and the Coulomb interaction. In the framework of the lattice gas model and molecular dynamics model without the Coulomb interaction, the emission barrier relies on the cluster charge with the $Z^{2/3} (A^{2/3})$ law and it does not depend on the source size and freeze-out density, which indicates that the surface energy play a basic dominant role to control the cluster emission. However, in the framework of molecular dynamics model with the Coulomb force, the emission barrier will decrease strongly according to the Eq.(10) and (11) and it decreases with the increasing of the source size, illustrating that the Coulomb interaction also play another weighty role to control the cluster emission.

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