Topological order in a correlated Chern insulator

Joseph Maciejko\(^1\) and Andreas Rüegg\(^2\)

\(^1\)Princeton Center for Theoretical Science, Princeton University, Princeton, New Jersey 08544, USA

\(^2\)Department of Physics, University of California, Berkeley, California 94720, USA

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We study the effect of electron-electron interactions in a spinful Chern insulator. For weak on-site repulsive interactions at half-filling, the system is a weakly correlated Chern insulator adiabatically connected to the noninteracting ground state, while in the limit of infinitely strong repulsion the system is described by an effective spin model recently predicted to exhibit a chiral spin liquid ground state. In the regime of large but finite repulsion, we find an exotic gapped phase with characteristics partaking of both the noninteracting Chern insulator and the chiral spin liquid. This phase has an integer quantized Hall conductivity \(2e^2/h\) and quasiparticles with electric charges that are integer multiples of the electron charge \(e\), but the ground state on the torus is four-fold degenerate and quasiparticles have fractional statistics.

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Over the past few years, the study of topological insulators [1, 2] has become one of the most active areas of condensed matter research. The prototypical topological insulator is the Chern insulator (CI), first theoretically proposed by Haldane [3], which exhibits the integer quantum Hall effect (QHE) in the absence of any external magnetic fields. Many concrete proposals to realize this new state of matter have followed Haldane’s original idea, culminating in the recent theoretical prediction [4] and experimental discovery [5] of the CI in thin films of Cr-doped (Bi,Sb)\(_2\)Te\(_3\). Although electron-electron interactions are needed to establish fermionic order in these films and break time-reversal symmetry as required for the existence of a QHE, this effect is described by mean-field theory and the resulting CI state is weakly correlated. A burning question in the field of topological insulators is whether strong electron correlation effects can give rise to qualitative changes in their properties [6].

Similar to the spin-polarized \(\nu = 1\) integer QH state, the simplest model of CI such as Haldane’s original model describes noninteracting spinless electrons and has a Hall conductivity \(\sigma_{xy} = Ce^2/h\) where the Chern number \(C = \pm 1\). The simplest type of interaction one can add is a nearest-neighbor density-density interaction. Many recent studies have focused on partially filled bands, which correspond to a metal in the noninteracting limit and lead at certain fillings to a fractional CI [7–11] for strong enough interactions and sufficiently flat band. In this paper we consider a completely filled valence band (half-filling), corresponding to a CI in the noninteracting limit. Analytical and numerical studies [12–14] of the spinless interacting problem show that the CI persists up to a critical interaction strength beyond which the ground state develops charge density wave order. Once the electron spin is considered, two possibilities arise. The first is to combine two copies of the CI with \(C^\uparrow = 1\) for spin up and \(C^\downarrow = -1\) for spin down. The resulting noninteracting model is the time-reversal invariant quantum spin Hall (QSH) insulator [15] in the limit of conserved \(z\) component of spin [16, 17]. In this case, the simplest type of interaction to consider is an on-site Hubbard interaction, and the resulting interacting problem at half-filling has been the focus of intense recent study [6]. The second possibility is to consider two copies of the CI with the same Chern number \(C^\uparrow = C^\downarrow = \pm 1\) for both spins. The resulting noninteracting model at half-filling is a CI with total Hall conductivity \(\sigma_{xy} = \pm 2e^2/h\). In this paper we consider the effect of an on-site Hubbard interaction \(U\) in such a system. This problem, which has received considerably less attention than the interacting QSH insulator, has been studied recently using the slave-rotor technique [18, 19] and the mapping to a spin model [20]. In both cases a ground state with topological order corresponding to the Kalmeyer-Laughlin chiral spin liquid state [21–23] was found in the phase diagram. This result corresponds to the \(U \to \infty\) limit where charge degrees of freedom are frozen and spin-charge separation occurs. Here, motivated by recent work on the interacting QSH insulator [24, 25] we are interested in the possibility of new phases occurring at intermediate \(U\). Besides the weakly correlated CI at small \(U\) and the chiral spin liquid at large \(U\), we find within the \(Z_2\) slave-spin representation [26–28] a new chiral topological phase at intermediate \(U\) which we dub the CI* phase by analogy with the QSH* phase in Ref. 25. The CI* phase is characterized by an emergent deconfined \(Z_2\) gauge field, a four-fold topological ground state degeneracy on the torus, and spin-charge separated excited states with fractional statistics (Fig. 1 and Table I).

We consider a class of 2D electron lattice models of the form

\[
H = \sum_{rr'} \sum_{\alpha} t_{rr'} c_{r\alpha}^\dagger c_{r'\alpha} + U \sum_r \left( \sum_{\alpha} n_{r\alpha} - 1 \right)^2 , \tag{1}
\]

where \(c_{r\alpha}^\dagger (c_{r\alpha})\) is a creation (annihilation) operator for an electron of spin \(\alpha = \uparrow, \downarrow\) at site \(r\), \(n_{r\alpha} = c_{r\alpha}^\dagger c_{r\alpha}\) is...
the number of electrons of spin $\alpha$ on site $r$, $t_{\alpha r'} = t_{\alpha r}$ is a spin-independent hopping amplitude and $U$ is the on-site Hubbard interaction. We consider the half-filled case $\langle \sum_\alpha n_{\alpha r} \rangle = 1$, and choose the hopping amplitude $t_{\alpha r} \rightarrow$ to be that of a spinless two-band CI with (say) Chern number 1 in the valence band. Although we will present numerical results for Haldane’s honeycomb lattice model [3] later on in the paper, we begin with a more general discussion of the possible phases and their expected universal properties which are independent of the details of the lattice model.

The model (1) can be first investigated in two extreme limits. In the limit $U = 0$, the system is a noninteracting CI with total Chern number $C = 2$ and Hall conductivity $\sigma_{xy} = 2e^2/h$. Because this is a gapped state, we expect $\sigma_{xy}$ to remain quantized for nonzero but small $U$, with the ground state evolving adiabatically from a noninteracting CI to a weakly correlated CI. In the limit $U \rightarrow \infty$, one can derive an effective low-energy Hamiltonian in the subspace of one electron per site which, owing to the full $SU(2)$ spin rotation symmetry of the Hamiltonian (1), is an $SU(2)$ invariant $S = 1/2$ spin model. For a particular choice of $t_{\alpha r}$ on the square lattice, such a spin model was recently studied numerically [20] and argued to have a chiral spin liquid ground state.

In order to study the possible phases of (1) at intermediate values of $U$, we make use of the recently introduced $Z_2$ slave-spin theory for correlated electron systems [25–28]. This theory is based on the simple observation that the Hubbard interaction energy in (1) depends only on the total occupation of site $r$ modulo 2, which can be represented by an Ising variable $\tau_r^z$, viz. $\tau_r^z = 1$ for unoccupied or doubly occupied sites and $\tau_r^z = -1$ for singly occupied sites. The electron operator $c^{(i)}_{\alpha r}$ is written as a product of a slave-fermion operator $f^{(i)}_{\alpha r}$ and an Ising slave-spin $\tau_r^z$,

$$\tau_r^z = f^{(i)}_{\alpha r} \tau_r^z,$$

where $\tau_r^z$ and $\tau_r^z$ obey the (anti)commutation relations of the associated Pauli matrices, so that $\tau_r^z$ flips the sign of $\tau_r^z$ since creating/annihilating an electron changes the occupation modulo 2. The Hamiltonian (1) is then written in terms of the slave-fermions and slave-spins as

$$H = \sum_{rr'} \sum_\alpha t_{\alpha r', r} \tau_r^z \tau_{r'}^z f^{(i)}_{\alpha r'} f^{(i)}_{\alpha r} + U \sum_r (\tau_r^z)^2 + 1.$$

While the original Hamiltonian (1) acts in the Hilbert space of physical electron states $\{|\psi_{\text{phys}}\rangle\}$, the Hamiltonian (3) acts in an enlarged Hilbert space which contains unphysical states $\{|\psi_{\text{unphys}}\rangle\}$ such as states with a single slave-fermion $\sum_\alpha f^{(i)}_{\alpha r, f_{\alpha r}} = 1$ but formally zero/double occupancy $\tau_r^z = 1$. When evaluating the partition function, one should sum only over physical states. This is accomplished by introducing a projector $P$ inside the partition sum $Z = \text{Tr} \left( e^{-\beta H} P \right)$ where $P$ is defined such that $P|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$ and $P|\psi_{\text{unphys}}\rangle = 0$. Although the details of the derivation will be given elsewhere [29], $Z$ can be written as the partition function of a Z_2 gauge theory [30] in 3D Euclidean spacetime with bosonic and fermionic matter in the fundamental representation,

$$Z = \int Df_{\alpha r} D\tau_{\alpha r} \sum_{\{\tau_r^\pm\}} \sum_{\{\tau_{\alpha r}^\pm\}} e^{-S_{\tau, \psi}[f_{\alpha r}, \tau_r^\pm, \psi]},$$

with an action $S = S_{\tau, \psi} + S_f + S_B$ such that

$$S_{\tau} = -\kappa \sum_{ij} \tau_i^x \sigma_{ij} \tau_j^x,$$

$$S_f = -\sum_{ij} t_{ij} f_{i\alpha} \sigma_{ij} f_{j\alpha},$$

$$e^{-S_B} = \prod_{i,j=\ell^{-\mp}} \sigma_{ij},$$

where $\sigma_{ij} = \pm 1$ is a spacetime $Z_2$ gauge field, $i, j$ are sites on a 3D spacetime lattice, and $t_{ij}$ is proportional to $t_{\alpha r}$ on spatial links and equal to $-1$ on temporal links. $S_B$ is a Berry phase term [31] which corresponds to a background static $Z_2$ charge on every site [32]. The constant $\kappa$ depends on the Hubbard interaction strength $U$,

$$\kappa = \frac{1}{2} \ln \cosh \left( \frac{\epsilon U}{2} \right),$$

where $\epsilon$ is a short-time cutoff [33].

We now discuss the possible phases of the gauge theory (4). In the noninteracting limit $U = 0$, Eq. (5) implies that $\kappa \rightarrow \infty$. In this limit, the gauge fields are frozen [29, 34] and can be gauged away, e.g., $\sigma_{ij} = 1$. The limit $U \rightarrow \infty$.
The only configurations which contribute to the partition function in the \( \kappa \to \infty \) limit are those with the slave-spins ferromagnetically ordered, e.g., \( \tau_f^a = 1 \) on every site (or gauge equivalent configurations). Equation (2) then implies that the electron and slave-fermion operators are proportional, and we recover the noninteracting CI. In the \( U \to \infty \) limit, Eq. (5) implies that \( \kappa \to 0 \). In this limit the slave-spins can be integrated out. Tracing over \( \sigma_{ij} \) on spatial links generates interactions between slave-fermions,

\[
\delta S_{\text{eff}}^\tau = -\frac{1}{2} \sum_{rr'} \sum_{\alpha} (t_{rr'})^2 \bar{f}_{i,\alpha \tau} f_{j,\alpha \tau} \bar{f}_{i,\beta \tau} f_{j,\beta \tau} + \ldots ,
\]

where \( \ldots \) involves higher multiples of four slave-fermion operators, while tracing over \( \sigma_{ij} \) on temporal links imposes the local constraint \( \sum_{\alpha} f_{i,\alpha f} r_{ra} = 1 \) on every site \( r \). Therefore the \( U \to \infty \) limit corresponds as expected to a \( S = 1/2 \) spin model, which can exhibit a variety of ground states depending on the form of \( t_{rr'} \). Ref. 20 argues that a special choice of \( t_{rr'} \) on the square lattice can give rise to a chiral spin liquid.

We now discuss the regime of large but finite \( U \). In general, using the Poisson summation formula we can rewrite a \( \mathbb{Z}_p \) gauge theory (here \( p = 2 \)) as a compact \( U(1) \) gauge theory coupled to a charge-\( p \) integer-valued link variable \( n_{ij} \) with no dynamics [35],

\[
Z = \int D\bar{f}DfDn_{ij} \prod_{\{\tau_1^a\}\{n_{ij}\}} e^{-S[f,\bar{f},\tau,\sigma_{ij}=e^{i\sigma_{ij} n_{ij}}]},
\]

where \( S = S_{\tau} + S_f + S_n + S_B \) with

\[
S_{\tau} = -\kappa \sum_{ij} \tau_i^a \bar{e}^{ia_{ij}} \tau_j^a,
\]

\[
S_f = -\sum_{ij} \sum_{\alpha} t_{ij} \bar{f}_{i,\alpha} e^{i(a_{ij} + eA_{ij})} f_{j,\alpha},
\]

\[
S_n = -ip \sum_{ij} n_{ij} a_{ij},
\]

\[
e^{-S_B} = \prod_{i,j=1} e^{ia_{ij}},
\]

where \( a_{ij} \) is a compact \( U(1) \) gauge field and we have introduced the external \( U(1) \) electromagnetic gauge potential \( A_{ij} \) which couples to the electrons with charge \( e \). The summation over \( n_{ij} \) has the effect of discretizing \( a_{ij} \) in integer multiples of \( 2\pi/p \). For large but finite \( U \), \( \kappa \) is small. The slave-spins are gapped and can be integrated out perturbatively [34], resulting in a lattice version of the Maxwell term for \( a_{ij} \). Because the band structure of the slave-fermions is gapped, we expect on general grounds [30] that the resulting gauge theory admits a gapped deconfined phase for large but finite values of \( U \). In the rest of the paper we focus on the physics of this phase which we denote the CI* phase.

| \( l_i \) | \( \theta_{l_{ij}} \) | \( \theta_{l_{ij'}} \) |
|---|---|---|
| \( (1, 0, 0, 0) \) | \(-e\) | \( \pi/2 \) |
| \( (0, -1, 0, 0) \) | \( e \) | \(-\pi/2 \) |
| \( (1, -1, 0, 0) \) | \( 0 \) | \( 0 \) |
| \( (0, 0, 1, 0) \) | \( e \) | \( 1/2 \pi \) |
| \( (0, 0, 0, 1) \) | \( -e \) | \(-1/2\pi \) |

**TABLE I.** Electromagnetic charge \( Q \), spin \( S_z \), self-statistics \( \theta_{l_{ij}} \) and nontrivial mutual statistics \( \theta_{l_{ij'}} \) of quasiparticles in the CI* phase, where \( l_i \) is the gauge charge vector [36] of quasiparticle \( i \).

In the CI* phase, the gauge field \( a_{ij} \) is essentially free and we can take the continuum limit \( a_{ij} \to a_{\mu} \). Following Senthil and Fisher [31], we expect that the Berry phase term \( S_B \) can affect the position of the phase boundary for the deconfined phase but not the universal properties of this phase. On the other hand, \( S_n \) is essential to remember the \( \mathbb{Z}_2 \) nature of the gauge theory in the continuum \( U(1) \) description. The link variable \( n_{ij} \to n_u \) obeys the constraint \( \partial_\tau n_{u} = 0 \) to preserve the gauge invariance of the action under a \( U(1) \) gauge transformation of \( a_{\mu} \). Solving the constraint by \( n_u = \frac{1}{2\pi} \epsilon_{\mu \nu \lambda} b_{\mu} \partial_{\nu} a_{\lambda} \), where \( b_{\mu} \) is a compact \( U(1) \) gauge field and passing to a real time description, we obtain

\[
S_n \to \frac{p}{2\pi} \int d^3 x \epsilon_{\mu \nu \lambda} b_{\mu} \partial_{\nu} a_{\lambda},
\]

i.e., the \((2+1)\)D level-\( p \) BF term [37, 38]. Finally, we consider the slave-fermion action \( S_f \). In a hydrodynamic approach [36], we introduce a conserved \( U(1) \) current \( j_\mu^a = \frac{1}{2\pi} \epsilon_{\mu \nu \lambda} \partial_{\nu} a_{\lambda}^a \) for slave-fermions of each spin \( \sigma = \uparrow, \downarrow \) [39], where \( a_\uparrow^a, a_\downarrow^a \) are compact \( U(1) \) gauge fields. Considering that slave-fermions of each spin form a CI with Chern number \( C_l = C_1 = 1 \), we obtain the effective Lagrangian of the CI* phase,

\[
\mathcal{L}_{\text{CI}*} = \frac{1}{\pi} \epsilon_{\mu \nu \lambda} b_{\mu} \partial_{\nu} a_{\lambda} + \frac{e}{2\pi} \epsilon_{\mu \nu \lambda} A_{\mu} \partial_{\nu} (a_\uparrow^a + a_\downarrow^a) + \sum_{\sigma=\uparrow, \downarrow} \left( \frac{1}{4\pi} \epsilon_{\mu \nu \lambda} a_\uparrow^\mu \partial_{\nu} a_\downarrow^\lambda + \frac{1}{2\pi} \epsilon_{\mu \nu \lambda} a_\downarrow^\mu \partial_{\nu} a_\uparrow^\lambda \right),
\]

which is the main result of this work. From Eq. (6) we can extract all the universal topological properties of the CI* phase, such as the \( K \)-matrix [36, 40],

\[
K = \begin{pmatrix}
0 & 2 & 1 & 1 \\
2 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1
\end{pmatrix},
\]

in the basis \( (a_\mu, b_\mu, a_\uparrow^\mu, a_\downarrow^\mu) \). The ground state degeneracy on the torus is \( |\det K| = 4 \), as for phases with \( \mathbb{Z}_2 \) topological order [41]. Unlike such phases however, the CI* phase is a chiral topological phase with Hall conductivity

\[
\sigma_{xy} = e^2 \hbar T K^{-1} t - \frac{2e^2}{\hbar},
\]

where \( T \) is the temperature and \( t \) is the lattice spacing.
π behaves as an anyon with statistical angle \( \theta \) and statistics can be understood as follows. An applied \( \phi \) to these excitations are electrically charged. Their charge quantum numbers can be read off from the quasiparticles in Table I. The quasiparticle excitations in the CI* phase and their properties of the CI* are reminiscent of noninteracting topological phases (integer quantized Hall conductivity \( \nu \)) for some numerical results, we studied the spinful Haldane-Hubbard model on the honeycomb lattice with \( U = 2 t_0 \) and a complex second-neighbor hopping \( t_2 = i t_1 \) where \( t_1 \) is the nearest-neighbor hopping. Although the Hall conductivity \( \nu \) of the CI* is the same as in the weakly interacting CI, interesting differences appear beyond the linear response regime (Fig. 2). We study the induced charge in a charge-pumping setup where an external flux \( \phi \) is threaded through one hexagon and a flux \( -\phi \) through another hexagon separated by 8 unit cells. While for small fluxes the pumped charge follows the expected relation \( \delta Q = C \delta \phi / \phi_0 \) with \( C = 2 \), we observe a sudden jump of \( \Delta Q = e \) for \( \phi \) slightly larger then \( \phi_0 / 4 \). This jump is associated with the spontaneous creation of a \( Z_2 \) vortex pair (semion/antimemion) which is localized at the flux-pierced hexagons and partially screens the external fluxes. A second jump of \( \Delta Q = e \) occurs at a flux slightly less than \( 3 \phi_0 / 4 \). This behavior is in sharp contrast to the charge pumping observed in the conventional CI (dashed curve) where the screening of the fluxes by \( Z_2 \) vortices cannot occur.

In summary, we predict that a new chiral topological phase of matter, the CI*, can arise when a spinful CI is subjected to a strong on-site Hubbard interaction. Some properties of the CI* are reminiscent of noninteracting topological phases (integer quantized Hall conductivity \( \nu \) and integer charge of quasiparticles) while others indicate the presence of topological order (nonzero ground state degeneracy on the torus and fractional statistics of quasiparticles). To test these predictions, a determination of the phase diagram of Hamiltonian (1) via exact numerical diagonalization or density-matrix renormalization group studies is desirable. The interesting question of whether the chiral spin liquid of Ref. 20 can be reached via a continuous transition from the CI* can perhaps be

\[
K \rightarrow W^T K W = \text{diag}(2, -2, 1, 1),
\]

\[
t \rightarrow W^T t = (-2, 2, 1, 1),
\]

which describes the direct sum of the double semion model \[42\] and a CI with Chern number \( C_1 = C_2 = 1 \). The quasiparticle excitations in the CI* phase and their quantum numbers can be read off from the \( K \)-matrix (Table I). The quasiparticles \( l_1 \) (semion), \( l_2 \) (antimemion), and \( l_3 \) (semion-antimemion bound state) have the same statistics as the excitations of the double semion model, but in contrast to their occurrence in spin models, here these excitations are electrically charged. Their charge and statistics can be understood as follows. An applied \( \pi \) flux in a \( \nu = 1 \) integer QH state traps a fermion mode with \( Q = \pm e/2 \), and the resulting flux-charge composite behaves as an anyon with statistical angle \( \pi/4 \) \[43\]. The semions/antimions in our model likewise arise from in-gap modes of the slave-fermions with total Chern number 2, which are bound to dynamical \( Z_2 \) vortices. The quasiparticles \( l_4 \) and \( l_5 \) are identified with the original electrons, and are topologically equivalent to the vacuum \[44\]. Although the quasiparticle spin \( S_z \) in Table I is only meaningful in the presence of \( U(1) \) spin rotation symmetry, the other properties of the CI* phase are robust under perturbations which break this symmetry but do not close the bulk gap.
studied analytically [47] using the topological field theory
(6) as a starting point, and is left for future work.

After the completion of this work, we learned of a re-
lated study of the CI* phase by Zhong et al. [48].

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