Instability of Dirac semimetal phase under strong magnetic field

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The quantum limit can be easily reached in the Dirac semimetals under the magnetic field, which will lead to some exotic many-body physics due to the high degeneracy of the topological zeroth Landau bands (LBs). By solving the effective Hamiltonian, which is derived by tracing out the high energy degrees of freedom, at the self-consistent mean field level, we have systematically studied the instability of Dirac semimetal under a strong magnetic field. A charge density wave (CDW) phase and a polarized nematic phase formed by “exciton condensation” are predicted as the ground state for the tilted and untilted bands, respectively. Furthermore, we propose that, distinguished from the CDW phase, the nematic phase can be identified in experiments by anisotropic transport and Raman scattering.

I. INTRODUCTION

Searching for new states of matter in solid materials is one of the key problems in condensed matter physics, which attracts lots of research interests recently. External magnetic field has provided an additional dimension for such studies, leading to surprisingly rich phenomena and phases in two dimensional electron gas systems already, e.g. the integer1 and fractional2 quantum Hall effects, the Wigner crystal phase as well as the nematic phases3–5. The high degeneracy of Landau levels resulted from the Landau quantization of the electronic wave functions is the main origin of the instability towards the various exotic phases mentioned above. In three dimensional systems, the Landau quantization only happens in the plane perpendicular to the magnetic field and the energy dispersion along the field direction remains unchanged. For ordinary semiconductor system with quadratic band dispersion, the high degeneracy of the LBs leads to almost perfect nesting of the “Fermi surfaces” along the field direction, as illustrated schematically in Fig. (1). Such a nesting effect will be greatly enhanced in the so called quantum limit, where only the lowest LB cuts through the Fermi level, and the field induced symmetry breaking phases such as CDW6–8, spin-density wave9,10, valley-density wave10, will be stabilized as the ground state.

It is very difficult to reach the quantum limit in normal semiconductors and semimetals and the experimental observation of the field induced CDW phase in real materials, i.e. Bi and Sb, is still under debate.11,12 The recently discovered topological semimetals provide a new platform for the search of new exotic phenomena under magnetic field.13–21 For Dirac22–24 or Weyl25–31 semimetals, where the Fermi level is very close to the Dirac or Weyl points, the quantum limit can be easily reached even under a weak magnetic field and more fruitful many-body physics can be realized due to the extra valley and orbital degrees of freedom.22,32 For instance, in strong magnetic field, the Weyl semimetal is found to be stabilized as a chiral-symmetry-breaking CDW state.34,35 In the present Letter, we systematically study the possible instabilities of Dirac semimetal state under the magnetic field in the quantum limit. We find that, besides the CDW phase, a new state, the polarized nematic phase can be stabilized in a large part of the phase diagram. Such an exotic phase is caused by the “exciton condensation” between the two zeroth LBs, which breaks both the rotational symmetry and the inversion symmetry, leading to a number of important physical consequences in transport and optical experiments.

II. MODEL

The Dirac semimetals can be divided into two categories by whether the Dirac points (DPs) are located on high symmetry lines or points24 of the Brillouin zone (BZ). In this Letter, we will focus on the first category, where the DPs are protected by the crystalline symmetry along the high symmetry lines and always appear in pairs due to the presence of the time reversal symmetry. The typical example of such type of materials is Na3Bi25, where the DPs are generated by the crossings of two doubly degenerate bands along the z axis. The low energy physics of such type of Dirac semimetal can be well described by the following k·p model,

\[
H^0 = C(k_z) + \begin{pmatrix}
M(k_z) & -\nu k_+ & \gamma(k) & 0 \\
-\nu k_+ & -M(k_z) & 0 & \gamma(k) \\
\gamma^*(k) & 0 & -M(k_z) & \nu k_- \\
0 & \gamma^*(k) & \nu k_- & M(k_z)
\end{pmatrix}
\]

(1)

Here \(C(k_z) = C_0 (\cos a_0 k_z - \cos a_0 k_c)\), \(M(k_z) = M_0 (\cos a_0 k_z - \cos a_0 k_c)\), \(k_\pm = k_x \pm ik_y\), \(\nu\) is the velocity in xy plane, \(a_0\) is the lattice along \(k_z\), and \(\pm k_c\) are the locations of DPs. The bases of the k·p model can be labeled by their main orbital characters as \(|P_{\frac{3}{2}}\rangle\), \(|S_{\frac{1}{2}}\rangle\), \(|S - \frac{1}{2}\rangle\), \(|P - \frac{3}{2}\rangle\), respectively. The first term in Eq. (1) plays an important role in the formation of type II Weyl points.36,37 While as long as \(|C_0| < |M_0|\), that is the case
we focus on, the $C(k_z)$ term will just tilt the DPs and change the ellipsoidal Fermi surface to a pyriform one. Even so, as shown in the following, this term will play an important role in determining whether the CDW or nematic phase will be stabilized. The high order term $\gamma(k)$ won’t play any important role for the physics discussed here and so will be neglected in the rest of the Letter.

The external magnetic field $B$ is applied along the $z$ direction. Adopt the Landau gauge $\mathbf{A} = (-yB, 0, 0)$, which leaves $k_x$ and $k_z$ still good quantum numbers, the LB eigenenergies and eigenstates can be solved analytically (see appendix A). As shown in Fig. (1), the two zeroth LBs disperse linearly and cross with each other at the DPs. The quantum limit can be reached by increasing the magnetic field such that only the zeroth LBs cuts through the Fermi level. In the present work we are only interested in the instability in the quantum limit, therefore we only keep the zeroth LBs in the non-interacting Hamiltonian

$$\hat{H}_0 = \sum_{ak_xk_z} \epsilon_{k_xa} \psi_{k_xk_z,a}^\dagger \psi_{k_xk_z,a}$$

$$\epsilon_{k_xa} = \begin{cases} 
C(k_z) - M(k_x) & a = c \\
C(k_z) + M(k_x) & a = v 
\end{cases}$$

Here $c$ and $v$ represent the conduction band (red band in Fig. (1)) and valence band (blue band in Fig. (1)), which are formed by $|S\frac{1}{2}\rangle$ and $|P - \frac{3}{2}\rangle$ states respectively. Since they belong to different eigenvalues of $C_6$, the crossings at $\pm k_z$ are protected by rotational symmetry and will persist even if nonzero $\gamma(k)$ presents.

Notice that the Zeeman’s coupling between the magnetic field and the field-free orbitals is neglected here. In a first principle study of the effective $g$ factor,\textsuperscript{35} we show that the Zeeman’s splitting in a typical Dirac semimetal $||{\hat{c}}||$ tries and will persist even if nonzero $\gamma(k)$ presents.

III. EFFECTIVE INTERACTION

To explore the stability of the above system under Coulomb repulsive interaction, we need to derive an effective interaction for the zeroth LBs by tracing out all the high LBs. Take the random phase approximation (RPA), we get

$$\hat{H}_\text{int}^\text{eff} = \frac{1}{2\Omega} \sum_{\mathbf{q}\neq 0} \sum_{k_x} \sum_{p_x} \sum_{p_z} e^{i\mathbf{q}\cdot\mathbf{r}} \epsilon_{k_x, p_x} \psi_{k_x, p_x, p_z,a}^\dagger \psi_{k_x, p_x, p_z,a} \times \psi_{k_x+p_x, k_z+p_z,a}^\dagger \psi_{k_x+p_x, k_z+p_z,a}$$

$$W(\mathbf{q}) = \frac{e^2}{\epsilon_0 \kappa(\mathbf{q})} e^{-\frac{1}{2}i^2\mathbf{q}_\perp^2}$$

FIG. 1: Band structure and instability channels. The two zeroth LBs, referred as the conductance and valence bands, are plotted in red and blue respectively, and the high energy LBs are plotted in gray. (a) shows the nematic phase channel (green arrows) and three CDW phase channels (orange arrows) in an untilted band structure ($C_0 = 0$). As argued in the text, only the nematic phase and the $2k_c$ CDW phase can be realized in low density limit. Here the dashed colored lines represent the folded bands in the $2k_c$ CDW phase. (b) shows the two channels in a tilted band structure ($C_0 \neq 0$). It is apparent that the kinetic energy cost of the $2k_c$ CDW order will be significantly lowered by the tilting.

The derivation of such dielectric constants is given in appendix B. As shown below, the long wave part of the interaction contributes the most in both of the possible instabilities, thus we can approximate $\kappa(\mathbf{q})$ by a dielectric constant

$$\kappa_z \approx \frac{e^2}{3\pi^2\epsilon_0\epsilon^2\hbar} \left( 0.9 + \ln \left( \frac{M_0 l_B}{v\hbar} \right) \right)$$

$$\kappa_{xy} \approx \frac{e^2}{4\pi^2\epsilon_0\epsilon^2\hbar} \left( 0.6 + \ln \left( \frac{M_0 l_B}{v\hbar} \right) \right)$$

where $\mathbf{q}_\perp = (q_x, q_y)$, $l_B = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length, $\kappa(\mathbf{q})$ is the effective dielectric function, and $\Omega$ is the sample volume. Details of the RPA derivation and the discussion of the dielectric function are given in appendix B. As shown below, the long wave part of the interaction contributes the most in both of the possible instabilities, thus we can approximate $\kappa(\mathbf{q})$ by a dielectric constant

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are the dielectric constants from high LBs, $\kappa_0$ is the dielectric constant from the core electrons, and $u = \frac{1}{2} M_0 a_0 \sin \left( a_0 k_z \right)$ is the Dirac velocity along $z$ direction. The derivation of such dielectric constants is given in appendix C. It should be aware that the results in Eq. (6) (7) are not only applied to this particular model, in fact it is universal for all the Dirac/Weyl semimetals. One of the important features for the above effective interaction is that its strength can be tuned by external magnetic field, which is a bit unusual in condensed matter physics. The mechanism is easy to be understood, that is, the energy gap between the zeroth and high LBs increases with the field strength, which weakens the screening effect.
IV. COHSEX METHOD

It is well known that the direct Hartree-Fock mean field approximation for metals with long range Coulomb interaction leads to a singular Fermi velocity because of a logarithmic divergence in the exchange channel. To handle this problem, we adopt the “Coulomb hole plus screened exchange” (COHSEX) method, which is a simplified version of the GW method\(^\text{19}\). Applying this method to our model, the self energy consists of a direct Hartree term \(\Sigma^H\) and a screened exchange term \(\Sigma^E\) where the interaction is not only screened by high LB electrons but also the zeroth LB electrons. As explained in the next section, in low carrier density limit, the system has a CDW instability at \(Q = 2k_c\). For convenience of calculation, we take the commensurate limit by setting \(k_c = \frac{\pi}{D_c a_0}\), where \(D_c\) is an integer. The BZ will be folded \(D_c\) times if the CDW order presents. Thus, in general, we can define the Green’s function as \(G_{an,bm}(k_x,k_z,t) = \langle T\psi_{an,k_x,k_z+\eta}c(t)\bar{\psi}_{bm,k_x,k_z}^\dagger(0)\rangle\), where \(n,m = 0 \cdots D_c - 1\) is the sub-BZ index and \(k_z\) takes value in the reduced BZ: \(0 \leq k_z < Q\). Then the self energy can be expressed as (Fig. (2b))

\[
\Sigma_{an,bm}^H(k_x,k_z) = \delta_{ab}W(0,0,(n-m)Q) \int \frac{dp_z dp_x d\omega}{(2\pi)^3} \times \sum_c G_{cn,n'c,n'}(p_x,p_z,\omega) e^{i\omega t} \tag{8}
\]

\[
\Sigma_{an,bm}^E(k_x,k_z) = \sum_n \int \frac{d^2p_z}{(2\pi)^2} \int \frac{dp_z d\omega}{(2\pi)^2} \times W^S(\omega = 0, q, k_z - p_z + n'Q) \times G_{an,n'-b,m-n'}(k_x,q_z,p_z,\omega) e^{i\omega t} \tag{9}
\]

where \(W^S(\omega = 0)\) is the static screened interaction. Here we approximate the Green’s function screening \(W^S\) by the free Green’s function at zero doping, as shown in Fig. (2c). Such an approximated screened interaction can be derived analytically

\[
W^S(\omega = 0, q) = \frac{e^2}{\epsilon_0} \cdot \frac{e^{-\frac{1}{2}q^2\sigma^2}}{\kappa q^2 + q^2_{TF}(q_z) e^{-\frac{1}{2}q^2\sigma^2}} \tag{10}
\]

where \(q_{TF}(q_z)\) is the effective Thomas-Fermi wavevector

\[
q_{TF}^2(q_z) = \frac{e^2 M_0}{2\epsilon_0 \pi^2 \rho^2} \ln \frac{\sin ak_z + \sin \frac{\pi}{2} \sin ak_z - \sin \frac{\pi}{2}}{\sin ak_z - \sin \frac{\pi}{2} a \sin \frac{2\pi}{2}} \tag{11}
\]

We have checked this approximation by comparing it with full self-consistent calculations, where \(W^S\) is calculated from \(G\) self-consistently, and find that the correction on results is very small.

With the above approximation, the Dyson’s equation \((\hat{G}^{0-1} - \Sigma)\hat{G} = \mathbb{1}\) (Fig. (2a)) and the equations (8) and (9) set up a self-consistent loop to determine the possible symmetry breaking phases at zero temperature by assuming different non-diagonal matrix elements in the self energy matrix. For convenience, we define the order parameter as \(\Delta_{an,bm}(k_x,k_z) = \langle \psi_{an}(k_x,k_z)\bar{\psi}_{bm}(k_x,k_z) \rangle\), whose non-diagonal elements in the band index \(a, b\) and sub-BZ index \(n, m\) denote the appearance of the nematic phase and the CDW phase respectively.

V. CDW PHASE

CDW phase acquires its instability from the Fermi surface nesting in the quasi one dimensional band structure (Fig. (1)). At first sight, it seems that the CDW should occur simultaneously at \(Q = 2k_c + 2k_F\) and \(Q = 2k_c - 2k_F\) channels for conduction and valence bands respectively. However, the interband Hartree energy can lock the CDWs in different bands to same \(Q = 2k_c\), at least for low enough carrier density. This conclusion can be reached by simply comparing the energy difference between the CDW phases with \(Q = 2k_c + 2k_F\) and \(Q = 2k_c\). According to Eq. (8), the \(Q = 2k_c\) phase gains an extra interband Hartree energy of \(\sim W(0)\) \(\Re\{\Delta_{\delta=1}^{CDW}\Delta_{\delta=-1}^{CDW}\})\), which reaches a negative constant as \(k_F\) approaching zero if \(\Delta_{\delta=1}^{CDW} = -\Delta_{\delta=-1}^{CDW}\). While the kinetic energy and exchange energy (Eq. (9)) difference between the \(Q = 2k_c\) and the \(Q = 2k_c \pm 2k_F\) phases will vanish with \(k_F\) approaching zero. Therefore as long as \(k_F\) is small enough, the CDW phase with \(Q = 2k_c\) for both bands will be stabilized.

The numerical calculation is performed with the initial condition \(\Delta_{an,bm}^{CDW}(k_x,k_z) = \delta_{n,m+1}\eta_{n,b}(k_z) + \delta_{n+1,m}\eta_{a,n}(k_z)\), where \(\eta(k_z)\) is a random matrix. The parameters are set as \(\kappa_0 = 5\), \(a_0 = 9.66\AA\), \(M_0a_0 = \)

![FIG. 2: Feynman diagrams for the COHSEX method. The full and free Green’s functions are represented by thick and thin lines, respectively. (a) is the diagram for the Dyson’s equation. (b) is the diagram for the self energy. (c) is the diagram for the screened effective interaction, where the Green’s functions participating the screening are approximated by the free Green’s functions.](image-url)
2.3eV · Å, ħν = 2.0eV · Å and Dc = 4, which give the same Dirac velocity for Na3Bi with the first principle results. We set C0 as C0 = −tM0, where t ∈ [0, 1) is the tilting ratio describing how much the bands are tilted. In Fig. (3b), we plot the band gaps and order parameters at various tilting ratios and magnetic fields. It shows that the tilting can significantly enlarge the CDW order, which is a direct consequence of saving the kinetic energy, as sketched in Fig. (1b).

VI. NEMATIC PHASE

As shown in Fig. (1), if the chemical potential is close enough to the DPs a rotation broken phase, i.e. the nematic phase, can be stabilized. Since the nematic phase doesn’t break the translational symmetry, its order parameter can be expressed in the full BZ as \( \Delta_{\text{NM}}(k_x, k_z) = \delta_{\bar{a}b}\eta(k_z) \), where \( -\pi \leq k_z < \pi \), and \( \bar{a} = v(c) \) for \( a = c(v) \). Two different types of \( \eta \) can be got: odd or even with respect to \( k_z \). According to the definition of LB wave function, inversion operator acts on it as

\[
\mathcal{P} \bar{\psi}_{k_x, k_z, a}^\dagger \psi_{k_x, k_z, a} \mathcal{P}^{-1} = -\bar{\psi}_{k_x, -k_z, c}^\dagger \psi_{-k_x, k_z, c},
\]

thus the even and odd \( \eta \) will respectively break and maintain the inversion symmetry. As will be discussed in the next paragraph, the inversion broken phase, which will be referred as the polarized nematic phase in the following, is always more favored.

The band gaps and order parameters of the polarized nematic phase, and the phase diagram consisting of the phases mentioned above are calculated with the same parameters used for the CDW phase and shown in Fig. (3a) and (4) respectively, which indicates that the polarized nematic phase is more favoured in untilted bands while the CDW phase is more favoured in tilted bands. This can be understood as a result of competition between kinetic energy and interaction energy. One one hand, as will be explained latter, the polarized nematic phase has a lower interaction energy; on the other hand, as shown in Fig. (1b), the tilting will significantly lower the kinetic energy cost in the CDW phase. Therefore, as shown in Fig. (4), the area of the polarized nematic phase in phase diagram will shrink and eventually vanish with an increasing tilting. Now let us explain why the polarized nematic phase has a lower interaction energy. Since its Hartree energy reaches zero, i.e. the minimum, we only need to compare the exchange energies. Eq. (9) suggests that the exchange energy in the CDW phase is approximately \( -W_S(q, 0) |\Delta_{\text{CDW}}^{\text{NM}}(\pm k_z)|^2 \). While the exchange energy in the nematic phase consists of three parts: two intravalley parts \( -\frac{1}{2} W_S(q, 0) |\Delta_{\text{NM}}^{\text{NM}}(\pm k_z)|^2 \), which equal to the CDW one; and an intervalley part \( -W_S(q, Q) \text{Re}(\Delta_{\text{NM}}^{\text{NM}}(k_c)\Delta_{\text{NM}}^{\text{NM}}^*(\pm k_c)) \), which is negative in the polarized nematic phase \( |\Delta_{\text{NM}}^{\text{NM}}(k_z)| = |\Delta_{\text{NM}}^{\text{NM}}(\mp k_z)| \). Here we have omitted the summation over index symbols for brevity. Thus we conclude that the polarized nematic order has lower interaction energy than the inversion symmetric nematic order and the CDW order.

Another aspect to understand this nematic order is to view it as a “pairing order” between electrons in the conduction band and holes in the valence band, which is the “exciton condensation” state in the mean field
Raman shift and $M_{F,I}$ is the light scattering matrix element:

$$M_{F,I} = e_i \cdot e_s \langle F|\hat{\rho}|G\rangle + \frac{1}{m} \sum_J \frac{\langle F|\hat{\pi}^s|J\rangle \langle J|\hat{\pi}^i|G\rangle + \langle F|\hat{\pi}^i|J\rangle \langle J|\hat{\pi}^s|G\rangle}{E_G - E_J + \omega_i}.$$

(VIII) SUMMARY

In summary, we have systematically studied the instabilities of Dirac semimetal phase in the quantum limit due to the Coulomb interaction. The high LB electrons far away from the Fermi level are considered as a background to screen the interaction by an effective dielectric constant in the long wavelength limit. All the possible instabilities on the zeroth LBs, i.e. the inter/intra-valley and inter/infra-band channels, are treated within the so called COHSEX method. By numerical calculations, we have shown that a polarized nematic phase breaking both the rotational and inversion symmetry and a CDW phase breaking translational symmetry will be stabilized depending on the strength of the tilting terms for the Dirac cones. Relevant experiments, including transport and Raman scattering, are also proposed to verify the existence of such phases. Further theoretical studies on the physical properties like magneto-transport in these exotic phases are also strongly encouraged.

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Appendix A: Solution of the free Hamiltonian

The eigenenergies and eigenstates of our model Hamiltonian can be explicitly derived as:

$$
\epsilon_{k,\alpha} = \begin{cases} 
C(k_z) + \sqrt{M^2(k_z) + 2\hbar^2 e^2 I_B^{-1}} & \alpha > 0 \\
C(k_z) - sM(k_z) & \alpha = 0 \\
C(k_z) - \sqrt{M^2(k_z) + 2\hbar^2 e^2 I_B^{-1}} & \alpha < 0
\end{cases}
$$

(A1)

where \(\kappa_0\) is the dielectric constant contributed by the core electron states. By a representation transformation, the interaction can be written on the LB bases

$$
\hat{H}_{\text{int}} = \frac{1}{2\Omega} \sum_{\alpha} \sum_{\beta} \sum_{\alpha'} \sum_{\beta'} \sum_{\mathbf{p}} \sum_{q} e^{i\mathbf{p}\cdot\mathbf{q}} \psi_{\mathbf{p}+\mathbf{q}}(\mathbf{k}_z,\mathbf{p}) \psi_{\mathbf{q}}(\mathbf{k}_z,\mathbf{p}) \hat{U}_{k_z\kappa_{\alpha\alpha'}}(\mathbf{q}) \hat{U}_{\mathbf{k}_z\kappa_{\alpha\alpha'}}(\mathbf{q}) \hat{U}_{\mathbf{p}_x\kappa_{\beta\beta'}}(\mathbf{q}) \hat{U}_{\mathbf{p}_x\kappa_{\beta\beta'}}(\mathbf{q})
$$

(B2)

where \(\mathbf{q}\) is the well known form factor of Landau levels, which is defined as

$$
\hat{F}_{\alpha,\beta}(\xi) = \frac{\beta^3}{\alpha^3} e^{-\frac{1}{2} \xi^2} L_{\beta}(\alpha - \beta) \left| \xi \right|^2
$$

(B5)

for \(\alpha \geq \beta\) and \(\hat{F}_{\alpha,\beta}(\xi) = \hat{F}_{\beta,\alpha}(-\xi)\) for \(\alpha < \beta\), and \(L_{\beta}(\alpha - \beta)\) is the Laguerre polynomial.

In the Feynman’s diagram representation, the RPA effective interaction on the zeroth LBs can be interpreted as the “dressed” interaction, that has been inserted with bubble diagrams concerning high LBs (Fig. (6)). Thus the static effective interaction satisfy

$$
W(\mathbf{q}) = U(\mathbf{q}) + U(\mathbf{q}) \chi^{0\beta}(0,\mathbf{q}\cdot\mathbf{z}) W(\mathbf{q})
$$

(B6)

where the matrix subscripts are omitted. \(\chi^{0\beta}\) is the bare susceptibility of high LBs:

$$
\chi^{0\beta}_{k_zk'_z\kappa_{\alpha\alpha'}k'_{\beta\beta'}}(\omega,0) = \delta_{k_zk'_z} \delta_{\kappa_{\alpha\alpha'}k'_{\beta\beta'}} \delta_{\alpha\alpha'} \delta_{\beta\beta'}
$$

(B7)

FIG. 6: RPA diagrams of the effective interaction on the zeroth LBs. The solid wavy line represents the effective interaction, while the dashed wavy line represents the bare interaction. The dashed straight line represents the Green’s functions in high LBs.

Appendix B: Effective interaction on the zeroth LBs

In this section, we will derive the effective interaction on the zeroth LBs by tracing out the high LBs in RPA. The long range Coulomb interaction can be written as

$$
\hat{H}_{\text{int}} = \frac{1}{2\Omega} \sum_{j} \sum_{j'} d^3r d^3r' \frac{e^2}{4\pi e \kappa_0 |r-r'|} \psi_j^T(r) \psi_{j'}^T(r') \psi_j(r) \psi_{j'}(r)
$$

(B1)
Therefore the effective interaction can be derived as

$$W_{k_{z,s,s'}, s'} (q) = \left[ U(q) \left( 1 - \frac{\chi^{0>}_z(q)}{\Omega} \right) \right]_{k_{z,00},s,s',00} = \frac{e^2}{\epsilon_0 \kappa(q) q^2} e^{-\frac{iq^2 a^2}{2}}$$

(B8)

where

$$\kappa(q) = \kappa_0 - \frac{e^2}{2\pi \epsilon_0 l_B^2} q^2 e^{-\frac{iq^2 a^2}{2}} \sum_{s,\alpha,\beta}$$

$$\times \int \frac{dk_{z'}}{2\pi} \Lambda_{k_{z,s},s',\alpha} (q) \chi^{0>}_{k_{z,s},\alpha\beta} (q_z) \Lambda_{k_{z's'},\alpha\beta} (q')$$

(B9)

is the effective dielectric function. For brevity, here we use $\chi^{0>}_{k_{z,s},\alpha\beta} (\omega q_z)$ to represent the diagonal elements of $\chi^{0>}_z$. As $W_{k_{z,s,s'}, s'} (q)$ does not depend on its subscripts, we will denote it as $W(q)$ in the paper.

**Appendix C: Long wave behavior of the effective interaction**

In this section, we intend to get a more explicit expression of the dielectric function in the long wavelength limit. Expand $\Lambda_{k'_{z',\alpha\beta}} (q) \chi^{0>}_{k'_{z'},\alpha\beta} (q_z) \Lambda_{k'_{z'},\alpha\beta}$ to second order of $q$, we have

$$\sum_{\alpha\beta} \Lambda_{k'_{z'},\alpha\beta} (q) \chi^{0>}_{k'_{z'},\alpha\beta} (q_z) \Lambda_{k'_{z'},\alpha\beta} (q)$$

$$\approx \sum_{\alpha} \chi^{0>}_{k_{z'},\alpha \alpha \beta} (q_z) \left[ \frac{1}{4} \left( \frac{\partial \kappa_z}{\partial q_z} q_z^2 \right) + \frac{\ell_B^2 (q_z^2 + q_0^2 \sin^2 \theta_\alpha)}{16 |\alpha|} \right]$$

(C1)

Substitute the definition of the auxiliary angle $\theta_\alpha$ in, we get

$$\kappa(q) \approx \kappa_0 + \kappa_z \cos^2 \langle q, B \rangle + \kappa_{xy} \sin^2 \langle q, B \rangle$$

(C2)

where

$$\kappa_z = \frac{e^2}{8\pi^2 \epsilon_0 l_B^2} \sum_{\alpha=1}^\infty \int \frac{2v^2 l_B^2}{\left( M^2 (k_z) + 2v^2 l_B^2 \cos^2 \theta_\alpha \right)^{3/2}} \frac{dk_z'}{\sqrt{2\pi}}$$

(C3)

$$\kappa_{xy} = \frac{e^2}{8\pi^2 \epsilon_0 l_B^2} \sum_{\alpha=1}^\infty \int \frac{2v^2 l_B^2}{\left( M^2 (k_z) + 2v^2 l_B^2 \cos^2 \theta_\alpha \right)^{3/2}} \frac{dk_z'}{\sqrt{2\pi}}$$

(C4)

and $\langle q, B \rangle$ is the angle between $q$ and the magnetic field. Eq. (C3)-(C4) may be simplified further. Firstly, as the main contribution in the $k_z'$ integral comes from small $M (k_z)$, we can expand $M (k_z)$ to linear order of $k_z$ around each DP. Secondly, the limit $v l_B \leq M_0$ is assumed such that the Landau level splitting is significantly smaller than the bandwidth and so the summation over $\alpha$ can be approximated by integral. Therefore, we achieve the following formula

$$\kappa_z \approx \frac{e^2 u}{3\pi^2 \epsilon_0 v^2} \left( 0.9 + \ln \left( \frac{M_0 l_B}{v^2} \right) \right)$$

(C5)

$$\kappa_{xy} \approx \frac{e^2 u}{4\pi^2 \epsilon_0 v^2} \left( 0.6 + \ln \left( \frac{M_0 l_B}{v^2} \right) \right)$$

(C6)

where $u = \frac{1}{2} M_0 a_0 \sin (a_0 k_z)$ is the Dirac velocity along $z$ direction, and the coefficients 0.9 and 0.6 are got by fitting Eq. (C5)-(C6) with Eq. (C3)-(C4) numerically. Indeed, Eq. (C5)-(C6) give very good approximations for Eq. (C3)-(C4) in a quite wide range. In Fig. (7), we compare the two equations with the parameters used in the paper.

In the end, if we neglect the dependence of $\kappa$ on the direction of $q$, a dielectric constant can be got by an average on the solid angle:

$$\kappa \approx \kappa_0 + \frac{1}{3} \kappa_z + \frac{2}{3} \kappa_{xy}$$

(C7)

1. K. Klitzing, G. Dorda, and M. Pepper, Physical Review
