Hexagonal symmetry of magnetic field dependence of Josephson current through triangle shape superconducting junctions

Akiyoshi NAKAYAMA1,2, Susumu ABE1, Tetsuya SHIMOYAMA1, Norimichi WATANABE2, Hsu Jui-Pang1,2 and Yoichi OKABE3

1 Department of Engineering, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama, 221-8686, Japan
2 High-tech Research Centre, Kanagawa University, 3-27-1 Rokkakubashi, Kanagawa-ku, Yokohama, 221-8686, Japan
3 Department of Engineering, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan

E-mail: nakayama@ee.kanagawa-u.ac.jp

Abstract. We have found hexagonal symmetry in the dependence of Josephson current of triangle shape junctions upon external magnetic field. Niobium/aluminum-oxide/niobium junctions are fabricated using magnetron sputtering of niobium and thermal oxidation of aluminum layers. Selective niobium anodization process has been used to define junction areas. In the case of square shaped junction, dependence of Josephson current obtained by two dimensional scanning of the external field parallel to aluminium-oxide layer, have become the product of the two Fraunhofer patterns in the direction $H_x$, $H_y$ parallel to each edge of this square junction area (1). Sub peaks have been observed in four directions at 0, 90, 180 and 270 degree around the main peak in the centre of the $(H_x, H_y)$ plane of the $I_c$-$H$ $(H_x, H_y)$ dependence. We have also fabricated triangle shaped junctions and obtained the magnetic field dependence of Josephson current, where sub peaks can be observed in six directions at 0, 60, 120, 180 240 and 300 degree around the centre of the $(H_x, H_y)$ magnetic plane. This magnetic dependence has hexagonal symmetry. We also simulated the dependence assuming the uniform current flow in the junction area. The gauge-invariant-phase-difference between the two superconducting electrodes is modulated perpendicular to the external magnetic field direction inside the barrier region, where the modulation wavelength is inverse proportional to the magnitude of the applied magnetic field. Quite excellent agreement between the measured data and the simulation result has successfully been obtained. (1)Akiyoshi Nakayama, Susumu Abe, Tatsuyuki Morita, Makoto Iwata, and Yusuke Yamamoto, IEEE Trans.Mag., Vol.36, No.5, pp.3511-3513, Sep., 2000

1. Introduction

It is well known that the modulation of Josephson currents through superconducting junctions can be obtained by applying an external magnetic field. Along the pass across the barrier region of the junction, the gauge-invariant-phase-difference between the two superconducting electrodes can be considered from the sum of the phase difference and the line integral of vector potential of electro-
magnetic field. This gauge-invariant-phase-difference is modulated perpendicular to the external magnetic field direction inside this barrier region, where the modulation wavelength is inverse proportional to the magnitude of the applied magnetic field. From this modulation pattern of the Josephson current by the magnetic field, uniformity of the tunnel barrier can be confirmed [1]. Specially shaped junctions defined by quartic polynomial [2,3] and normal-distribution-function [4] have been fabricated for X-ray spectroscopy. The modulation of the Josephson current $I_c$ of the superconducting junctions has usually been observed by one-dimensional scanning of the applied magnetic field. Recently, the external magnetic field has been scanned in two dimensions and the two-dimensional surfaces of the $I_c-H(H_x, H_y)$ dependence has been measured [5-7].

In this paper, triangle shape junctions are fabricated. The external magnetic field dependence of the Josephson current $I_c-H(H_x, H_y)$ is first obtained by using two pairs of the Helmholz coils. In sec. 2, measurement method for the $I_c-H(H_x, H_y)$ dependence is described. The measured $I_c-H(H_x, H_y)$ dependence is presented in sec. 3. Numerical simulation results are compared with the measured results in sec. 4. Conclusions are presented in sec. 5.

2. Experiment
Magnetron sputtering systems with load-lock chambers for preparing niobium and aluminum layers have been used for fabricating triangle shape Nb/AlO$_x$/Nb tunnel junctions. Aluminum-oxide tunnel barriers are formed by a natural oxidation of Al layer in pure oxygen. Two-dimensional magnetic field dependence of the Josephson current has been measured by using two pairs of Helmholz coils as shown in Fig.1. Two current sources for driving these Helmholtz coils have been controlled by GPIB system in order to obtain magnetic field dependences automatically [7].

![Fig.1 Measuring system for the magnetic field dependence of a triangle shape junction.](image)

3. Experimental results
Josephson currents through Nb tunnel junctions have been modulated by two-dimensional scanning of the applied magnetic field. In the case of square shape junction, dependence of Josephson current obtained by 2-D scanning of the external field parallel to aluminium-oxide layer, have become the product of the two Fraunhofer patterns in the direction $H_x, H_y$ parallel to each edge of this square junction area [5,6]. Sub peaks have also been observed in four directions at 0, 90, 180 and 270 degree around the main peak in the center of the $(H_x, H_y)$ plane of the $I_c-H(H_x, H_y)$ dependence. We have first fabricated triangle shape junctions. Figure 2 shows the magnetic field dependence of the Josephson current through a triangle shape Nb/AlO$_x$/Nb junction by two-dimensional (2-D) scanning of the external magnetic field. In the obtained magnetic field dependence of Josephson current, sub peaks
can be observed in six directions at 0, 60, 120, 180, 240, and 300 degree around the center of the \((H_x, H_y)\) magnetic plane. This magnetic dependence has hexagonal symmetry.

4. Numerical simulation

In this section, magnetic field dependence of the Josephson current through superconducting junction is numerically obtained. No magnetic field by the junction current is considered in this calculation. A distribution of the gauge-invariant-phase-difference (hereafter phase difference) between the two electrodes has to obey following conditions. First, at each point in the junction area, superconducting current is proportional to the sin function of the phase difference. Secondly, the phase difference should change perpendicular direction to the external magnetic field as shown in Fig. 3, where the wavelength \(L\) of this modulation is inversely proportional to the absolute value of this field. Finally, in the phase difference there is an arbitral phase factor.

![Fig. 2](image)

**Fig.2** Dependences of the Josephson current upon the external magnetic field. (left) simulation results (right) measured data.

There are two states: no voltage state and finite voltage state between the two electrodes of junctions. The condition that the total current flowing through the junction area must be equal to the bias current, has to be satisfied by changing the arbitral phase factor. If exists a distribution of a certain time-independent phase difference satisfying the total current condition, the junction shows no voltage drop. There is a threshold value of the current of this no voltage state. Above this threshold value, i.e. maximum superconducting current, no distribution of time-independent phase difference can satisfy the total current condition even if an arbitral phase factor changes. In this case the junction
shows voltage state. Numerical simulation for the magnetic field dependence of the superconducting current can be performed by this method. As shown in Fig. 2, the simulated $I_c - H(H_x, H_y)$ dependence has sub peaks in six directions at 0, 60, 120, 180, 240 and 300 degree around the center of the $(H_x, H_y)$ magnetic plane. The numerical results have also hexagonal symmetry and shows good agreement with the measured results.

Fig. 3 Change of the phase difference perpendicular direction to the external magnetic field.

5. Conclusions
Triangle shape superconducting junctions have been fabricated using dc magnetron sputtering of niobium and aluminum layers. Dependences of Josephson currents through triangle shape Nb/AlO$_x$/Nb junctions upon the external magnetic field $H(H_x, H_y)$ have first been measured by 2-D scanning of magnetic field in a plane parallel to the tunnel oxide using two pairs of the Helmholz coils. In the obtained $I_c - H(H_x, H_y)$ dependence sub peaks can be observed in six directions at 0, 60, 120, 180, 240 and 300 degree around the center of the $(H_x, H_y)$ magnetic plane. This magnetic dependence has hexagonal symmetry. The numerical results have good agreement with the measured results.

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