A competency-driven staff assignment approach to improving employee scheduling robustness

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Abstract

Presented paper concerns the competency-driven staff assignment and scheduling approach to the management of project portfolios subject to perturbations caused by employee absences and/or unexpected arrival of high priority jobs. Proactive strategy is considered, which exploits the concept of employee substitutability to improve the robustness of personnel allocation in the case of occurrence of specific types of disruptions. Solutions obtained using the model of a constraint satisfaction problem developed in this study are validated in series quantitative and qualitative experiments. With a view to future implementation in a Decision Support Systems dedicated to prototyping of proactive personnel allocation, a methodology employing the concept of a competency framework-based robustness measure is proposed. Implemented in a declarative framework, the proposed approach allows one to find a redundant competency framework robust to a given set of disruptions.

Keywords

competency assignment, competency framework, personnel scheduling, robustness measure.

1. Introduction

The objective of manpower planning, factors that determine employment planning, such as workforce allocation and personnel scheduling, are associated with the arrangement of work schedules and the assignment of personnel to shifts, in order to meet the demand for human resources that varies over time. In this context, a pivotal role is played by so-called project-centered planning [6], which is used in companies that divide their work into projects to which they assign different groups of employees. Typical examples of such firms include job production companies such as ship-building, bridge-building, and construction companies; companies that manufacture one-off products (e.g., yachts); businesses that produce handmade craft items like furniture; or engineer-to-order companies, in which employees must be qualified to perform creative tasks [20, 33].

In the literature of the subject [2, 22, 45], competencies are defined as a set comprising theoretical knowledge, practical skills, behaviors, and qualifications that allow workers to successfully execute their tasks. During the scheduling phase, a personnel roster (or work assignment) is constructed by assigning the available personnel resources (employees with specific personal competencies) to specific duties. In other words, planning decisions regard the allocation of project tasks (which require specific employee competencies) to resources (employees with given competencies). Projects are often subject to various disruptions that influence the duration of activities. This means that it is necessary to develop effective approaches that allow for the generation of robust project schedules which are less sensitive to disruptions caused by such uncontrollable factors such as employee absences or the unexpected arrival of a priority job [19]. In order to deal with these uncertainties [36], organizations need to adopt proactive and reactive scheduling strategies to protect the personnel roster and to respond to operational variability, respectively. Methods must then be developed to support decision-makers in situations that require responding to dynamic changes to organizational settings, e.g., frequent changes in the scope and structure of objectives, tasks, and resources. It should be noted, however, that while the existing literature describes many methods for the assessment and determination of competency frameworks [44], the problem of constructing robust personnel rosters has received only limited attention. This is the reason why a proactive approach based on the employee substitutability concept, i.e. taking into account employees specific competencies in the event of disturbances [42], is being proposed.

The considered problem of redundant competency framework synthesis that take into account the specificity of human resources and...
issues concerning projects planning, fits within the framework of the well-known Redundancy Allocation Problem [44].

The present study is a continuation of our previous work, which explored methods of fast prototyping of solutions to workforce allocation and personnel scheduling problems that are robust to a given type of disruptions occurring in the course of the execution of multiple projects [12]. The main contributions of this paper are as follows:

1) Proposed approach to the prototyping of robust competency-driven staff assignments and schedules takes into account both: the projects are subject to disruptions (employee absences) that influence their execution, the redundancy of employee competences affects the efficiency of projects driven enterprises. Thus it allows for the construction of more realistic, i.e., more accurate, models, taking into account proactive strategies that guarantee robust arrangement of work schedules and robust assignment of personnel to a project portfolio.

2) The proposed definition of robustness measure allows to find redundant competency frameworks. Consequently, introducing sufficient conditions for the existence of competency structures resistant to a given type of disturbance, provides an attractive analytical method as an alternative to the currently used simulation based methods.

3) The considered problem is implemented in a constraint programming environment and verified with an example of a real-life project portfolio. Its constraint satisfaction (CSP) model [5, 28] allows one to search for competency-driven staff assignments and schedules robust to employee absences.

In Section 2, the overview of the literature is provided. An example introducing to the competency-driven staff assignment approach is provided in Section 3. A reference model of a CSP which allows one to find competency frameworks robust to a selected set of anticipated types of disruption is presented in Section 4. Evaluation of computational experiments verifying the proposed method is presented in Section 5. In Section 6 the conclusions and directions for further research are presented.

2. Related works

The last two decades have seen a rapidly increasing interest in the problems of workforce allocation and personnel scheduling in reference to the arrangement of work schedules and the assignment of personnel to shifts. There is a fast-growing body of literature on these topics [2, 7, 13, 22, 29, 31, 32, 34], which encompasses nearly all areas associated with production and services management, in particular those regarding the issues of personnel scheduling [43], e.g., crew scheduling, shift scheduling, and personnel assignment [1, 33], e.g., competency-driven staff assignment. This especially refers to settings where a creative task must be performed, for instance in engineering-to-order companies. The interlacing problems of scheduling and manpower assignment involve, allocation of employees with different competences to activities carried out in the given time intervals. Both problems are combinatorially hard [38].

As manufacturers increasingly convert their production systems from make-to-stock production systems to make-to-order or assemble-to-order production systems, in which products or parts are assembled once an order has been received, there is a growing focus on human resource management in these jobbing production environments. Jobbing production, which involves the manufacture of one-off products such as yachts, furniture, and artificial limbs, or software development, tends to be labor intensive, and requires a multi-skilled workforce. In companies that produce custom goods the problem of manpower assignment involve, allocation of employees with different competences to activities carried out in the given time intervals. Both problems are combinatorially hard [38].

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3. Introductory example

This section introduces to the competency-driven staff assignment approach aimed at increasing robustness of the assumed competency framework. In this context a measure of competency framework robustness enables one to improve job production resistance with regard to employees’ absence.

Consider a job production system where two individual jobs are performed: \( Q = \{ Q_1, Q_2 \} \) (Fig. 1a). The following sets of tasks: \( Z_1 = \{ Z_{11}, \ldots, Z_{1n} \} \), \( Z_2 = \{ Z_{21}, \ldots, Z_{2m} \} \), in the event of absences of employees, at time point \( t \).

Assuming that the tasks are non-preemptive and employees working time do not exceed 8 u.t. the answer to the following question is sought: Is it possible to assign tasks to currently available employees, guaranteeing their implementation according to the schedule shown in Figure 2?

Figure 3a illustrates, a task assignment for the case of an absence of employee \( P_{k} \). Absences of a larger number of employees are presented in Figure 3b–d: cases of absence of two \( (P_1, P_2) \), three \( (R_1, P_3, P_5) \), and four \( (R_1, P_2, P_3, P_5) \) employees. All of these scenarios ensure that the portfolio of projects \( P \) is completed by the available staff within the given time horizon of 16 u.t. In the general case, however, e.g., when employees \( (R_1, P_2) \) are absent, there are no suitable replacements able to take over their duties.

In the examples considered above, it is assumed that cases/types of absence are known before the projects in portfolio \( P \) are executed. Assuming that in the example in Figure 1 a disruption occurs \( t = 0 \) with employees \{1, …, 4\} being absent from work at any time during the execution of the project portfolio \( P \) due to accidents, illness, etc. This means that, depending at which time point they occur, absences may have a different effect on the timely execution of jobs.

An example of a schedule of job is presented in Figure 2. It is assumed that only one employee can be assigned to each task \( Z_i \). For example, tasks \( Z_2 \) and \( Z_{11} \) have been assigned to employee \( P_2 \).

In order to assess the robustness of the staff of employees \( P \) implementing the project portfolio \( Q \) to the simultaneous absenteeism of \( \omega \) employees the following concept of Robustness of a Competency Framework is used:

\[
R^Q_P(\omega, t) = \frac{\sum_{\omega} P_{\omega,t}}{\omega},
\]

where:

- \( U_\omega \) – family of \( \omega \)-element employee absence scenarios:
  \( U_\omega = \{ u_i | u_i \in P : |u_i| = \omega \} \).
- \( L P_{\omega,i} \) – subset of set \( U_\omega \) containing scenarios \( u_i \) which guarantee timely completion of the portfolio of projects \( Q \) in the event of absences of employees, at time point \( t \).

The values of function \( R^Q_P(\omega, t) \) belong to the range \([0,1] \subset \mathbb{R} \), where:

- \( R^Q_P(\omega, t) = 0 \) – means no robustness, i.e., there is no scenario \( u_i \) for which the replacement guaranteeing timely completion of the planned project portfolio \( Q \) exists.
- \( R^Q_P(\omega, t) = 1 \) – means full robustness, i.e., for each scenario \( u_i \), there exists at least one replacement guaranteeing timely completion of the project portfolio \( Q \).

Assuming that in the example in Figure 1 a disruption occurs at time point \( t = 0 \) with \( \omega = \{1, \ldots, 4\} \) employees being absent from work, the values of \( R^Q_P(\omega, 0) \) determined from equation (1) are:

\[
R^Q_P(\omega, 0) = 0.675; R^Q_P(2, 0) = 0.71; R^Q_P(3, 0) = 0.53; R^Q_P(4, 0) = 0.2.
\]

This means that for seven scenarios of one employee absence (\( R^Q_P(1, 0) \)), there exists the replacement guaranteeing completion of the portfolio of projects \( Q \). Similarly, in the case of a simultaneous absence of four employees \( R^Q_P(4, 0) \), project portfolio \( Q \) can be completed on time in 14 out of the 70 possible absence scenarios. Values of \( R^Q_P(\omega, t) \) have been deter-
It is easy to see that the value of robustness \( R^Q_P (\omega, t) \) of the adopted competency framework varies depending on the time point \( t \) at which the disruption occurs. The changes in robustness are shown in graphic form in the radar charts in Figures 4–6. In these figures, robustness \( R^Q_P (\omega, t) \) of the project portfolio \( Q \) is marked in blue, while green and yellow mark robustness values for the individual jobs \( Q_1, Q_2 \). It is worth noting that the robustness of the portfolio \( Q \) corresponding to different cases of absence (\( \omega = 1...4 \)) of employees increases monotonically with time to completion of the portfolio. In addition, the values of robustness of jobs \( Q_1, Q_2 \) are not less than the robustness of the entire portfolio \( Q \); this is due to the fact that an absence of employees can disrupt the execution of only one of the jobs without affecting the robustness of other jobs to be completed as part of the portfolio.

It is noteworthy that in the case under study the differences between robustness values \( R^Q_P (\omega, t) \) and at the beginning and end of time horizon \( H \) are relatively large. This observation naturally raises the question of whether it is possible to restructure the competency framework \( G \) in such a way as to ensure that robustness values change in a predetermined manner.

The essence of this question is illustrated in Figures 5 and 6, which show the expected robustness values (so called robustness thresholds *\( R^Q_P (\omega, t) \) ), see the red line) that guarantee timely implementation of the considered portfolio of projects along the entire time horizon \( H \).

The value 0.8 is reachable only for the entire portfolio \( Q \) at time point \( t = 14 \) u.t. (from \( t = 11 \) u.t. for job \( Q_1 \) and from \( t = 14 \) u.t. for job \( Q_2 \)). Which employee should acquire which competencies in the new competency framework ?

In general, the robustness can be expressed by matrix , and the corresponding matrix of thresholds robustness describing different thresholds of robustness for different number of absent employers () and different time (). In that context the required level of an individual employee’s absenteeism may be higher than of two absent employers and so on. The considered problem of robust competency frameworks synthesis boils down to the following question:

**Does there exist, for the given portfolio of projects \( Q \) executed by a staff of employees \( P \), a competency framework \( G \), which, at any time point along the adopted time horizon \( H \) guarantees robustness values \( R^Q_P (\omega, t) \geq R^Q_P * (\omega, t) \)?**

In this context, since the selection of redundant competencies that ensure the \( R^Q_P * \) value at a given \( R^Q_P \) level enables the protection of the execution of given production orders against the effects of specific disruptions, hence the fulfillment of condition \( R^Q_P (\omega, t) \geq R^Q_P * (\omega, t) \) guarantees the existence of the sufficient solution ensuring the timely execution of considered order.

4. Modelling and problem description

The formalism of the CSP seems to be best suited for modelling the robust competency frameworks synthesis problem. Moreover, it can be implemented in a constraints programming environment to generate feasible scenarios of execution of the projects portfolio in terms of appropriate workforce allocation and personnel scheduling.

4.1. A reference model

An organization’s production potential and the requirements posed by the production orders placed (hereinafter referred to as the “project portfolio”) can be represented as part of the reference model, which consists of a model of the portfolio of projects executed in the system and a model of the framework of the competencies possessed by the organization’s personnel.

**Project Portfolio \( Q \).** The portfolio is assumed to include projects that are executed at a customer’s order or are the organization’s own undertakings (e.g., modernization or execution of production orders). A formula is adopted in which \( Q = \{ Q_1, Q_2, ..., Q_n \} \) stands for a project portfolio, where \( Q_j \) is the \( j \)-th job that involves a set of tasks (activities) \( Z_j \), and \( Z \) is a set of tasks \( Z_i \) to be executed by the organization. A task \( Z_i \) is defined as follows:
where:

\( y_i \): starting time of task \( Z_i \),

\( l_i \): duration of task \( Z_i \),

\( w_i \): set of tasks that exclude the execution of task \( Z_i \), \( w_i \subseteq Z \); task \( Z_i \) and task \( Z_a \in w_i \) are said to be mutually exclusive when they cannot be performed by the same employee,

\( \varphi_i \): number of employees necessary to complete the task \( Z_i \).

It is assumed that job \( Q_j \) is characterized by a network of tasks that can be represented as a Task-on-Node (TN) network diagram in which tasks \( Z_i \) are assigned to nodes, and precedence relationships are represented by arcs (see Fig. 1). The task network can be represented as the digraph \( DG_j = (Z_j, E_j) \) where \( Z_j \) refers to the set of tasks of job \( Q_j \) and \( E_j \subseteq Z_j \times Z_j \) are the set of arcs.
In addition, it is assumed that:
• project portfolio \( Q \) is completed (i.e., all tasks in portfolio \( Q \) are to be completed) in a given horizon time \( H \);
• tasks are indivisible in time, i.e., once started, a task cannot be interrupted until it has been completed;
• tasks are completed by a staff of \( \Phi_i \) competent employees.

Staff. Set \( P = \{P_1, \ldots, P_m\} \) represents a group of employees, where \( P_k \) is a pair:
\[
P_k = (s_k, z_k)
\]
where \( s_k \) and \( z_k \) determine the minimum/maximum working hours of \( P_k \).

For the set \( P \) the competency framework \( G \) is defined:
\[
G = \{g_{k,i}|k=1, \ldots, m; i=1, \ldots, n\}
\]
where:
\[
g_{k,i} = \begin{cases} 1 & \text{when employee } P_k \text{ has the competencies to execute task } Z_i, \\ 0 & \text{in remaining cases} \end{cases}
\]

Assignment \( X \) defined by the following matrix determines the tasks assigned to employees from the set \( P \):
\[
X = \{x_{k,i}|k=1, \ldots, m; i=1, \ldots, n\}
\]
where:
\[
x_{k,i} = \begin{cases} 1 & \text{when task } Z_i \text{ is executed by employee } P_k, \\ 0 & \text{in remaining cases} \end{cases}
\]

For example, assignment \( X \) corresponding to the plan from Figure 2, has the following form:

Disruptions. Considered type of disruptions is characterized by the set \( U_\omega = \{u_i | u_i \subseteq P, \omega = \omega, \} \) imposing ensuing from this the set of \( \omega \)-element scenarios of employee absences. The disruptions occurrence makes the search for assignment \( X \) guaranteeing timely completion of considered projects portfolio \( Q \). Consequently, the concept of robustness \( R^G_\omega (\omega, t) \) (1) is used to evaluate this kind of assignment.

To put this type of problems into formal terms, the following reference model is introduced:

Sets:
\[
Z: \text{ tasks executed as part of the project portfolio } Q \subseteq \{Z_1, \ldots, Z_n\}, \quad H: \text{ horizon of completion of project portfolio } Q \subseteq \{0, 1, \ldots, h\}, \quad \mathcal{P}: \text{ set of employees } \mathcal{P} = \{P_1, \ldots, P_m\}, \quad U_\omega: \text{ family of } \omega \text{-element employee absence scenarios:}
\]
\[
U_\omega = \left\{ u_{f} | u_{f} \subseteq P; f = 1, \ldots, |P| \right\},
\]
where \( u_{f} \) is a set of absence employee (employee absence scenario), \( \omega \) - expected robustness of competency framework, given by matrix:
\[
\omega = \left[ R^G_{\omega} (\omega, t) \right]_{\omega \times t}
\]

- expected robustness to the absence of \( \omega \) employees at time point \( t \).

Parameters:
\[
n, m, \omega: \text{ number of tasks executed as part of the project portfolio } Q, \quad \omega \text{ number of employees of staff } \mathcal{P}, \quad \omega < m, \quad l_i, \omega: \text{ duration of task } Z_i, \quad \omega: \text{ number of employees needed to execute task } Z_i, \quad \omega, z_i, s_k, \omega, \omega: \text{ minimum limit of working time of } P_k, \quad \omega, z_i, s_k, \omega, \omega: \text{ maximum limit of working time of } P_k, \quad \omega, w_i, \omega: \text{ set of tasks that exclude the execution of task } Z_i, \quad \omega, \omega, \omega, \omega: \text{ expected robustness of the competency framework, given by matrix:}
\[
\omega = \left[ R^G_{\omega} (\omega, t) \right]_{\omega \times t}
\]

- expected robustness to the absence of \( \omega \) employees at time point \( t \).

Decision variables:
\[
G: \text{ competency framework given by matrix } G = \left[ g_{k,i} \right]_{k=1, \ldots, m; i=1, \ldots, n} \text{ where:}
\]
\[
g_{k,i} = \begin{cases} 1 & \text{when employee } P_k \text{ has the competencies to execute task } Z_i, \\ 0 & \text{in remaining cases} \end{cases}
\]

robustness of competency framework \( G \), given by matrix:
\[
R^G_{\omega} = \left[ R^G_{\omega} (\omega, t) \right]_{\omega \times t}
\]

- expected robustness to the absence of \( \omega \) employees at time point \( t \).

\[
G^u_f: \text{ competency framework taking into account absences from the employee absence scenario } u_{f} \subseteq U_\omega:
\]
\[
G^u_f = \left[ g^u_{k,i} \right]_{k=1, \ldots, m; i=1, \ldots, n}
\]

- expected robustness to the absence of \( \omega \) employees at time point \( t \).

\[
X: \text{ assignment of tasks in the portfolio } Q \text{ to employees of staff } \mathcal{P},
\]
\[
X = \left[ x_{k,i} \right]_{k=1, \ldots, m; i=1, \ldots, n}
\]

- expected robustness to the absence of \( \omega \) employees at time point \( t \).
assignment taking into account absences from the employee absence scenario \( u_f \in U_{ao} \): \( X^{u_f} = \left[ \begin{array}{c} x_{k,i} \\ \vdots \\ x_{k,n} \end{array} \right]_{k=1 \ldots m; i=1 \ldots n} \) where:

\[ s_{k,j}^\Phi = \begin{cases} 1 & \text{when task } Z_i \text{ is executed by employee } P_k, \\ 0 & \text{in remaining cases} \end{cases} \]

\( c^{u_f} \): a variable that specifies whether there exists assignment \( X^{u_f} \) ensuring timely completion of project portfolio \( O \).

**Constraints:**

The matrix \( G^{u_f} \) takes the value 0 for cells \( g_{k,j}^{u_f} \) corresponding with absent employees (\( P_k \in u_f \)):

\[ g_{k,j}^{u_f} = \begin{cases} g_{k,j} & \text{when } P_k \notin u_f, \\ 0 & \text{when } P_k \in u_f. \end{cases} \] (6)

Employees who have the appropriate competencies can execute the tasks:

\[ x_{k,j}^{u_f} \leq g_{k,j}^{u_f}, \quad \text{for } k=1 \ldots m; j=1 \ldots n; \quad u_f \in U_{ao}. \] (7)

At a given time point, an employee executes at most one task:

\[ \left( y_\alpha + l_\beta \leq y_\beta \right) \lor \left( y_\beta + l_\beta \leq y_\alpha \right) \Rightarrow \left( x_{\alpha,\beta}^{u_f} + x_{\beta,\alpha}^{u_f} \leq 1 \right). \]

\( \alpha, \beta = 1 \ldots n; k = 1 \ldots m; \quad u_f \in U_{ao}. \) (8)

Each task is executed by exactly \( \Phi \) employees:

\[ \left( \sum_{k=1}^m x_{k,j}^{u_f} = \Phi \right) \Leftrightarrow \left( c_{j}^{u_f} = 1 \right), \quad \text{for } i=1 \ldots n; \quad u_f \in U_{ao}. \] (9)

Workload of \( P_k \) should not to exceed the minimum/maximum limit \( s_k / z_k \):

\[ \sum_{i=1}^n x_{k,j}^{u_f} \cdot l_i \geq s_k \Leftrightarrow \left( c_{f}^{u_f} = 1 \right), \quad \text{for } P_k \in D \setminus u_f; \quad u_f \in U_{ao}. \] (10)

\[ \sum_{i=1}^n x_{k,j}^{u_f} \cdot l_i \leq z_k \Leftrightarrow \left( c_{f}^{u_f} = 1 \right), \quad \text{for } P_k \in D \setminus u_f; \quad u_f \in U_{ao}. \] (11)

Execution of mutually exclusive tasks:

\[ \left( Z_i \in w_j \right) \Rightarrow \left( x_{k,i}^{u_f} + x_{k,j}^{u_f} \leq 1 \right), \quad \text{for } i=1 \ldots n; \quad k=1 \ldots m; \quad u_f \in U_{ao}. \] (12)

According to (1) the robustness \( R_G^\omega (\omega, t) \) is calculated as the following ratio:

\[ R_G^\omega (\omega, t) = \frac{\left[ L_{P_{\omega,t}} \right]}{\left[ V_{\omega} \right]} \geq R_G^\omega (\omega, t) \cdot t \in H, \] (13)

\[ \left[ L_{P_{\omega,t}} \right] = \sum_{u_f \in \omega} c^{u_f}, \] (14)

The open structure of the proposed model, allowing it to be easily expanded by various combinations of various criteria and restrictions that occur in practice, implies a choice of constraint programming (CP) formalism implementing the paradigm of declarative modeling, the essence of the CSP problem formulation. In this context, the original element of research is the proposed measure \( R_G^\omega (\omega, t) \) of robustness of competency framework \( G \) to the absences of \( \omega \) employees. A feature of the measure that has been recognized as a result of the research is its monotonic course, which increases with the approaching project completion date. This fact finds its practical use in computer-aided interactive resource allocation planning systems.

### 4.2. Problem definition

The structure of the adopted model allows one, in a natural way, to formulate the synthesis problem of robust competency framework \( G \) as a Constraints Satisfaction Problem:

\[ CS = \left( \left( V, D \right), C \right), \] (16)

where:

\[ V = \left\{ G, G^{u_f} \cdot u_f, X^{u_f} \cdot u_f, R_G^\omega \right\} \] – set of decision variables, including \( G \) and competency subframeworks \( G^{u_f} \cdot u_f \) corresponding to a situation of simultaneous absence of \( \omega \) employees, assignments \( X^{u_f} \cdot u_f \), and \( R_G^\omega \).

\( D \) – a finite set of decision variable domains \( \left\{ G, G^{u_f} \cdot u_f, X^{u_f} \cdot u_f, R_G^\omega \right\} :\)

\[ r_{k,i} \in \left\{ 0,1 \right\}, \quad g^{u_f}_{k,j} \in \left\{ 0,1 \right\}, \quad x^{u_f}_{k,j} \in \left\{ 0,1 \right\}, \quad R_G^\omega (\omega, t) \in \left( 0,1 \right]. \]

\( C \) – a set of constraints specifying the relationships among the variables \( G, Z, R_G^\omega \) (constraints (6)–(15)).

To solve \( CS \) problem (16), we have to find the values of variables \( G \) (personnel competency framework), \( X^{u_f} \) (assignment), and \( R_G^\omega \), for which all the constraints given in set \( C \) are satisfied. In other words, the solution to \( CS \) is a variant of competency framework \( G \) which guarantees the given value of \( R_G^\omega \) for a given type of disruptions.

In general the CSP (16) can be treated as an optimization constraint optimization problem (COP) [44] given by the formula:

\[ CO = \left( \left( V, D \right), C, F \right), \] (17)

where: \( V, D, C \) are defined as in (16), and \( F \) is the objective function:

\[ F(G) = \sum_{k=1}^{i=1, n+k} g_{k,j}. \] (18)

To solve \( CO \), one has to determine such values of decision variable \( G_{OPT} \) for which all constraints given in the set \( C \) are satisfied and for which function \( F \) has a minimum value (a minimum number of changes have to be made to the original competency framework \( G \) ) or, stated differently, returns a minimum competency framework. In general, \( CO \) (17) allows one to synthesize (minimum) robust competency frameworks. In addition to the aforementioned benefits resulting from the adoption of the declarative modeling paradigm (enabling, among others, the implementation of the introduced measure of competency framework robustness \( R_G^\omega (\omega, t) \)) another of its advantages is the possibility to simultaneously evaluate all employee absence scenarios with a given (currently analyzed) variant of
the set of competences. The proposed approach is illustrated in Figure 7 (corresponding to the $\omega = 1$ instance). The adopted approach assumes that each considered instance of competency framework $G$ corresponds to the set of competency frameworks $G_{\omega}^{t}$ (representing subsequent cases of employee absenteeism) and the corresponding $X_{\omega}^{t}$ allocation. This means that the mechanisms (implemented in CP environments) used to search for solutions by specifying the values of subsequent elements $g_{i}^{j}$ (see the red line in Fig. 7) of competency framework $G$ determine the degree of compliance with restrictions for each case of absence, which allows the determination of the value of robustness level $R_{G}^{t}(\omega, t)$ (13). In this manner it becomes possible to determine the competency framework $G_{OPT}$ guaranteeing robustness value $R_{OPT}$ without iterative necessity, determining the $R_{G}^{t}(\omega, t)$ values characteristic of imperative programming methods.

In other words, the space for potential solutions is screened against the criterion of meeting (at a given level determined by $R_{OPT}$) the constraints ((6)–(15)) for all variants of absence of the considered instance of the problem. The benefit of this fact is that once obtained, confirmation of existence of an admissible assignment $X_{\omega}^{t}$ for the absence scenario $P_{1}$ (answer YES in Fig. 7) does not need to be confirmed again in the further synthesis process of the competency framework $G$. Consequently, this allows the search process to be limited to those scenarios for which there is no allocation of $X_{\omega}^{t}$ that meets the restrictions (6)–(15). The limitation of the search space is implemented by mechanisms of constraints propagation and variables distribution implemented in CP environments.

5. Computer experiments

The quantitative and qualitative evaluation of the effectiveness of the proposed method of synthesizing robust competency frameworks was verified in a series of experiments. In the constraint optimization problem from (17) being solved the input data used was an archival data collected from selected project-driven organizations.

5.1. Qualitative assessment

Consider the project portfolio shown in Figure 1, which is executed by a staff of $P = \{P_{1}, \ldots, P_{k}\}$ employees. The method of planning tasks for the individual jobs is shown in the schedule in Figure 2. As Figures 4 and 5 clearly demonstrate, the adopted competency framework $G$ (Figure 1b) does not guarantee the required level of robustness $R_{G}^{t}(\omega, t) \geq 0.8$ to the absences of $\omega \in \{1, \ldots, 4\}$ employees within time horizon $H = \{0, 1, \ldots, h\}$.

Which employee should acquire which competencies from the new competency framework $G$ to guarantee $R_{G}^{t}(\omega, t) \geq 0.8$?

To answer this question, the problem CO (17) was solved (implementation in the GUROBI/Intel i7-4770, 8GB RAM). The obtained minimum competency framework $G_{OPT}$ (time computation = 1s.) is shown in graphic form in the Table 1.

This shows that employees must improve their qualifications by acquiring nine new redundant competencies: employee $P_{1}$ should acquire the competencies necessary to execute tasks $Z_{3}$ and $Z_{6}$; $P_{2}$ competencies for tasks $Z_{4}$ and $Z_{9}$; $P_{3}$ competencies for task $Z_{2}$; $P_{4}$ competencies for tasks $Z_{1}$ and $Z_{6}$; and $P_{5}$ competencies for tasks $Z_{2}$ and $Z_{8}$. Acquisition of these competencies guarantees robustness $R_{G}^{t}(\omega, t) \geq 0.8$ across the time horizon $H$. Robustness $R_{G}^{t}(\omega, t)$ for time points $t = 0 \ldots 16$ along time horizon $H$ takes the following values (see Fig. 8):

$$R_{G}^{t}(1, 1) = 1; \quad R_{G}^{t}(2, 1) = 1; \quad R_{G}^{t}(3, t) = 0.98; \quad R_{G}^{t}(4, t) = 0.8 \quad \text{for} \quad t = 0 \ldots 8,$$

$$R_{G}^{t}(1, 1) = 1; \quad R_{G}^{t}(2, 1) = 1; \quad R_{G}^{t}(3, t) = 0.98; \quad R_{G}^{t}(4, 0.87) = 0.87 \quad \text{for} \quad t = 9, 10,$$

$$R_{G}^{t}(1, 1) = 1; \quad R_{G}^{t}(2, 1) = 1; \quad R_{G}^{t}(3, 0.98) = 0.98; \quad R_{G}^{t}(4, 0.89) = 0.9 \quad \text{for} \quad t = 11,$$

$$R_{G}^{t}(1, 1) = 1; \quad R_{G}^{t}(2, 1) = 1; \quad R_{G}^{t}(3, 0.9) = 0.9 \quad \text{for} \quad t = 12, 13,$$

$$R_{G}^{t}(1, 1) = 1; \quad R_{G}^{t}(2, 1) = 1; \quad R_{G}^{t}(3, 0.9) = 0.9 \quad \text{for} \quad t = 14, 15,$$

$$R_{G}^{t}(1, 1) = 1; \quad R_{G}^{t}(2, 1) = 1; \quad R_{G}^{t}(3, 0.9) = 0.9 \quad \text{for} \quad t = 16.$$

The approach proposed in this paper has been verified in several experiments involving: 10–100 employees and 2–4 jobs (consisting of a different number of tasks (10–100)). Calculations were made to determine the time needed to synthesize a competency framework (robust to the absences of employees across the time horizon which results from the critical path of the jobs being executed). The obtained results (Table 2) show when the size of the problem does not exceed three jobs and 60 tasks, can be found in less than one hour.

It is worth noting that in real-life settings, project portfolios are executed in parallel with other jobs run simultaneously, often involving the same employees. This means that some of the workers can be engaged in executing a given project portfolio only during certain periods along time horizon $H$, which strongly limits the possibility of finding replacements for absent staff members. Figure 9 shows a schedule for project portfolio...
Q, which incorporates examples of employee unavailability intervals for a staff of employees P (the set of unavailability intervals is hereafter referred to as a mask M – set of additionally tasks with fixed assignment).

As it turns out, when additional tasks M are taken into consideration, the robustness \( R^Q_P (\omega, t) \) of portfolio Q is affected. For example, consider a situation in which the tasks scheduled as in Figure 9 are executed by employees P who have competencies defined by framework \( G_{OPT} \) of Table 1.

Does competency framework \( G_{OPT} \) guarantee robustness \( R^Q_P (\omega, t) \geq 0.8 \) of portfolio Q when mask M, which specifies the unavailability of employees over time, is considered (as in Fig. 9)? The following values of \( R^Q_P (\omega, t) \) are obtained:

Table 1. Competency framework which guarantees \( R^Q_P (\omega, t) \geq 0.8 \) (the competencies the employees have to acquire are given in bold)

| G_{OPT} | Z1 | Z2 | Z3 | Z4 | Z5 | Z6 | Z7 | Z8 | Z9 | Z10 | Z11 | Z12 |
|---------|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| P1      | 1  | 0  | 1  | 1  | 0  | 0  | 1  | 0  | 1  | 0   | 1   | 0   |
| P2      | 1  | 0  | 1  | 1  | 0  | 0  | 1  | 0  | 1  | 0   | 1   | 0   |
| P3      | 0  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 0   | 1   | 0   |
| P4      | 1  | 0  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 0   | 1   | 0   |
| P5      | 0  | 1  | 1  | 1  | 1  | 0  | 0  | 1  | 0  | 0   | 1   | 0   |
| P6      | 1  | 1  | 1  | 0  | 0  | 0  | 1  | 0  | 1  | 0   | 1   | 0   |
| P7      | 0  | 0  | 0  | 1  | 1  | 1  | 0  | 1  | 0  | 0   | 1   | 0   |
| P8      | 0  | 1  | 0  | 0  | 1  | 1  | 1  | 0  | 0  | 0   | 1   | 0   |

Table 2. Calculation times for \( G_{OPT} \) guaranteeing robustness \( R^Q_P (\omega, t) = 1 \) for \( \omega \in \{1, \ldots, 4\} \) and \( t \in H \)

| Number of jobs | Number of tasks | Number of employees | Calculation time for \( G_{OPT} \) for \( \omega = 1 \) | \( \omega = 2 \) | \( \omega = 3 \) | \( \omega = 4 \) |
|----------------|-----------------|---------------------|----------------|-------------|-------------|-------------|
| 2              | 10              | 5                   | T1($)          | T2($)       | T3($)       | T4($)       |
| 2              | 20              | 10                  | 1              | 1           | 34          | 68          |
| 3              | 30              | 15                  | 2              | 3           | 56          | 234         |
| 3              | 40              | 20                  | 4              | 23          | 340         | 1245        |
| 3              | 50              | 25                  | 7              | 54          | 896         | 3245        |
| 3              | 60              | 30                  | 10             | 78          | 1356        | >5000       |
| 4              | 70              | 35                  | 18             | 124         | 2755        | >5000       |
| 4              | 80              | 40                  | 23             | 198         | >5000       | >5000       |
| 4              | 90              | 45                  | 35             | 234         | >5000       | >5000       |
| 4              | 100             | 50                  | 50             | 345         | >5000       | >5000       |

Fig. 8. Graphs of observed and expected changes in robustness \( R^Q_P (\omega, t) \) of competency framework \( G_{OPT} \) from Table 1

\( R^Q_P (1, t) = 1; R^Q_P (2, t) = 0.86; R^Q_P (3, t) = 0.39; R^Q_P (4, t) = 0.06 \) for \( t = 0 \ldots 4 \), \( R^Q_P (1, t) = 1; R^Q_P (2, t) = 0.89; R^Q_P (3, t) = 0.49; R^Q_P (4, t) = 0.12 \) for \( t = 5 \ldots 7 \), \( R^Q_P (1, t) = 1; R^Q_P (2, t) = 0.93; R^Q_P (3, t) = 0.49; R^Q_P (4, t) = 0.13 \) for \( t = 8 \), \( R^Q_P (1, t) = 1; R^Q_P (2, t) = 0.93; R^Q_P (3, t) = 0.53; R^Q_P (4, t) = 0.22 \) for \( t = 9 \ldots 10 \), \( R^Q_P (1, t) = 1; R^Q_P (2, t) = 0.96; R^Q_P (3, t) = 0.73; R^Q_P (4, t) = 0.46 \) for \( t = 11 \ldots 12 \), \( R^Q_P (1, t) = 1; R^Q_P (2, t) = 0.96; R^Q_P (3, t) = 0.88; R^Q_P (4, t) = 0.78 \) for \( t = 14 \ldots 15 \), \( R^Q_P (1, t) = 1; R^Q_P (2, t) = 1; R^Q_P (3, t) = 1; R^Q_P (4, t) = 1 \) for \( t = 16 \).

Figure 10 depicts the results in graphic form. It is easy to see that the constraints which take into account the structure of M significantly change the values of robustness \( R^Q_P (\omega, t) \). The expected value of 0.8 is obtained as late as the last time unit (\( t = 16 \)) of job execution.

This observation naturally leads to another question: Is it possible to further extend competency framework \( G_{OPT} \) in such a way as to guarantee the expected value of robustness \( R^Q_P (\omega, t) \) across time horizon H? When an appropriate synthesis problem CO (17) was solved, a negative result was obtained. The maximum values of \( R^Q_P (\omega, t) \) determined for the so-called full competency framework (each employee has the competency to execute each task: \( g_{k,i} = 1 \)) were as follows:

\[ R^Q_P (1, t) = 1; R^Q_P (2, t) = 1; R^Q_P (3, t) = 0.8; R^Q_P (4, t) = 0.41 \] for \( t = 0 \ldots 4 \),
\[ R^Q_P (1, t) = 1; R^Q_P (2, t) = 1; R^Q_P (3, t) = 0.8; R^Q_P (4, t) = 0.49 \] for \( t = 5 \ldots 10 \),
\[ R^Q_P (1, t) = 1; R^Q_P (2, t) = 1; R^Q_P (3, t) = 1; R^Q_P (4, t) = 0.93 \] for \( t = 11,12,13 \),
\[ R^Q_P (1, t) = 1; R^Q_P (2, t) = 1; R^Q_P (3, t) = 1; R^Q_P (4, t) = 1 \] for \( t = 14,15,16 \).

This means that a full competency framework does not ensure the expected level of robustness when \( \omega = 4 \) employees are absent (see Fig. 11). Robustness \( R^Q_P (4, t) \) at the initial stage of execution of job (\( t = 0 \ldots 4 \)) does not exceed 0.41. This means that the available staff of employees P, despite having all the necessary competencies, are not able to secure the completion of the project portfolio in the event of an absence of four employees.

Again, the question naturally comes to mind whether it is possible to enlarge the existing staff of employees (by hiring additional employees) in such a way as to build a competency framework that guarantees an expected level of robustness \( R^Q_P (\omega, t) \) across time horizon H. In order to answer this question, CO (17) was solved. The solution we obtained is shown in Table 3 and Figure 12.

The minimum competency framework \( G_{OPT} \) (Table 3) shows that staff P should be supplemented with two new employees, \( P_3 \) and \( P_4 \), with eight competencies between them. In addition, the existing employees must improve their qualifications by acquiring five new competencies: employee \( P_1 \) should acquire the competency to execute task \( Z_{11} \); \( P_2 \) competencies for tasks \( Z_1 \), \( Z_4 \), and \( Z_{11} \); and \( P_3 \) the competency for task \( Z_{12} \). The acquisition of these competencies guarantees robustness \( R^Q_P (\omega, t) \geq 0.8 \) across the time horizon H. More specifically, robustness \( R^Q_P (\omega, t) \) for time points \( t = 0 \ldots 16 \) along time horizon H obtains the following values (see Fig. 8):
The results of the experiments demonstrate the competitiveness of the adopted model (which allows one to solve strongly non-linear combinatorial optimization problems) and the computational efficiency of the constraints programming techniques used to analyze it.

The examples provided above illustrate selected options for formulating questions related to different situations in workforce allocation and personnel scheduling processes. The solutions presented, which focus on variants of robust personnel allocation and scheduling, show that the model can be used to design competency frameworks robust to absences of employees for a portfolio of up to four projects. It is worth noting that the concept of mask, introduced in the last example, in addition to solutions robust to employee absenteeism, allows one to search for solutions robust to disruptions caused by the arrival of new jobs during the execution of planned ones.

### 5.2. Quantitative assessment

The proposed approach was evaluated using data of project-driven company carrying out different orders at the same time. The case under consideration relates to a situation in which 49 employees are recruited for six production orders forming the project portfolio \( Q \). More precisely, the portfolio consists of six jobs: \( Q = \{Q_1, \ldots, Q_6\} \) including \( n = 214 \) tasks: \( \mathcal{Z} = \{Z_1, Z_2, \ldots, Z_{214}\} \) with a total of 14,100 hours.

The example of parameters \( f_j \) (starting time of task), \( l_j \) (duration of task), and \( w_i \) (set of excluded tasks) are collected in Table 5, and the project portfolio schedule determined by them in Figure 13. Due to the scale of the network of activities describing the order of implementation of individual tasks from among all jobs \( Q_1, \ldots, Q_6 \), their graphical representations are omitted in Figure 13. The project portfolio \( Q \) should be completed within a time horizon of 77 days.

Table 5. Competency framework which guarantees \( R^Q(\omega, t) \geq 0.8 \) (the competencies that the employees need to acquire are given in bold)

| \( G_{OPT} \) | Z1 | Z2 | Z3 | Z4 | Z5 | Z6 | Z7 | Z8 | Z9 | Z10 | Z11 | Z12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| P1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| P2 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| P3 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| P4 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| P5 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| P6 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| P7 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| P8 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| P9 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| P10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |

A large number of competences means that an employee can carry out many similar tasks - for example: \( Z_1 \) “assembly and tacking 1”; \( Z_2 \) “welding in the vehicle 1”; \( Z_6 \) “assembly and tacking 2”; \( Z_{74} \) “welding in the vehicle 2”; \( Z_{12} \) “welding in the vehicle 3”, etc. Due to the requirements imposed by the General Data Protection Regulation, data pseudonymisation has been introduced.
In addition, a lower \( s_k \) and upper \( z_k \) limit of working time assigned to each employee is collected in Table 7. Assignment of tasks \( X \) sufficient for completion of the given set of tasks \( Z \) following the schedule from Figure 13 is presented in Table 8 and meets the requirements assuming that:

- task \( Z_i \) can only be executed by a competent employee,

\[
\begin{array}{|c|c|c|c|}
\hline
Z_i & y_i \text{ [days]} & l_i \text{ [hours]} & w_i \\
\hline
Z_1 & 0 & 80 & \{Z_3, Z_5, Z_{12}, Z_{40}\} \\
Z_2 & 10 & 25 & \{Z_4, Z_{10}, Z_{42}, Z_{50}, Z_{61}\} \\
Z_3 & 0 & 60 & \{Z_1, Z_5, Z_{12}, \ldots, Z_{62}\} \\
Z_4 & 15 & 45 & \{Z_2, Z_{10}, Z_{42}, Z_{50}, Z_{61}, Z_{65}\} \\
Z_5 & 9 & 100 & \{Z_1, Z_3, Z_{12}, Z_{40}\} \\
\ldots & \ldots & \ldots & \ldots \\
Z_{213} & 10 & 30 & \{Z_{192}, Z_{201}, Z_{207}, \ldots, Z_{212}\} \\
Z_{214} & 6 & 30 & \{Z_{188}, Z_{199}, Z_{210}, Z_{211}\} \\
\hline
\end{array}
\]

The above data was used to conduct the analysis of robustness competency framework \( G \) and to determine the competency frameworks protecting the company against selected types of disruptions.

**Analysis of robustness of competency framework \( G \)**

The subject of the analysis is the evaluation of robustness level \( R^G_{QR}(\omega, t) \) of competency framework \( G \) (Table 6) while implementing a given project portfolio \( Q \) (Fig. 13) and disturbances resulting in simultaneous absence of one to four employees \( \omega = 1 \ldots 4 \). What is the current value of robustness \( R^G_{QR}(\omega, t) \) of competency framework \( G \) to the absences of \( \omega = 1 \ldots 4 \) employees?

The obtained values of robustness levels \( R^G_{QR}(\omega, t) \) (GUROBI/Intel i7-4770, 8 GB RAM, 10s.) are illustrated in Figure 14. The company is not protected against the threat of employee absence from the entire project portfolio horizon (the expected values of robustness levels are not less than 0.8). Further, the figure shows that just as was the case in the experiments presented in subsection 5.1, robustness value \( R^G_{QR}(\omega, t) \) increases monotonically with the passage of time and the expected value of 0.8 is reached:

- for \( \omega = 1 \) after 38 days of portfolio \( Q \) realization,
- for \( \omega = 2,3 \) after 50 days of portfolio \( Q \) realization (after job \( Q_6 \) completion),
- for \( \omega = 4 \) on the last day (day 77) of portfolio realization \( Q \).

In an extreme case (from 1–12 days), robustness to simultaneous absence from four employees is below 0.15. This means that 85% of four employees’ absence scenarios in the period from the first to the 12th day will result in non-compliance with accepted deadlines for implemented orders.

**Synthesis of competency frameworks robust to simultaneous absence of \( \omega = 1, \ldots, 4 \) employees**

Due to the fact that the expected value of robustness level \( R^G_{QR}(\omega, t) \geq 0.8 \) has not been achieved since the beginning of the project portfolio \( Q \) realization, an attempt was made to synthesize the competence framework guaranteeing this value. Two robustness thresholds were adopted: \( R^G_{QR}(\omega, t) = 0.8 \) and \( R^G_{QR}(\omega, t) = 1.0 \). The answer to the following question was sought: Is it possible to enlarge the existing staff of employees (by hiring additional employees) in such a way as to build a redundant competency framework that guarantees an expected level of robustness across time horizon \( H \)?

The obtained values of robustness levels \( R^*_G(\omega, t) \) for both considered thresholds are illustrated in Figures 15 and 16. The presented robustness values are conditioned by the need to employ additional staff:

- for \( R^*_G(\omega, t) \geq 0.8 \) (Fig. 15) a team of six employees with 56 additional competences \( \{Z_3, Z_4, Z_{10}, Z_{22}, \ldots, Z_{212}\} \) should be employed,
### Table 6. Competency framework $G$ of staff $P$

| $G$ | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ | $Z_5$ | ... | $Z_{28}$ | $Z_{29}$ | $Z_{30}$ | $Z_{31}$ | $Z_{32}$ | $Z_{33}$ | $Z_{34}$ |
|-----|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-------|-------|
| $P_1$: Mills | 1 | 1 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 1 | 0 | 0 |
| $P_2$: Garner | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_3$: Ray | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_4$: MacPherson | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_5$: Burnham | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_6$: Davis | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_7$: Crockett | (0.1) | (0.1) | (0.1) | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_8$: Hudson | (0.1) | (0.1) | (0.1) | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_9$: Roach | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |

### Table 7. Limit of hours of team $P$ members

| $G$ | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ | $Z_5$ | ... | $Z_{28}$ | $Z_{29}$ | $Z_{30}$ | $Z_{31}$ | $Z_{32}$ | $Z_{33}$ | $Z_{34}$ |
|-----|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-------|-------|
| $P_1$: Mills | 1 | 1 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 1 | 0 | 0 |
| $P_2$: Garner | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_3$: Ray | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_4$: MacPherson | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_5$: Burnham | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_6$: Davis | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_7$: Crockett | (0.1) | (0.1) | (0.1) | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_8$: Hudson | (0.1) | (0.1) | (0.1) | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_9$: Roach | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |

### Table 8. Assignment $X$ determined by the schedule from Figure 13

| $G$ | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ | $Z_5$ | ... | $Z_{28}$ | $Z_{29}$ | $Z_{30}$ | $Z_{31}$ | $Z_{32}$ | $Z_{33}$ | $Z_{34}$ |
|-----|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-------|-------|
| $P_1$: Mills | 1 | 1 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 1 | 0 | 0 |
| $P_2$: Garner | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_3$: Ray | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_4$: MacPherson | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_5$: Burnham | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_6$: Davis | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_7$: Crockett | (0.1) | (0.1) | (0.1) | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_8$: Hudson | (0.1) | (0.1) | (0.1) | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |
| $P_9$: Roach | 0 | 0 | 0 | 0 | 0 | ... | 0 | ... | 0 | ... | 0 | 0 | 0 |

### Fig. 14. Robustness $R^G_{0.1}$ of competency framework $G$ from Table 6
for $\tau(t) = \sum_{i=1}^{124} \theta_i$ (Fig. 16), a team of 11 employees with 124 additional competencies $\{Z_6, Z_7, Z_9, Z_{32}, \ldots, Z_{214}\}$ should be employed.

The above solutions are examples of selected variants from the set of acceptable solutions received. According to the received solutions, securing the company against the effects of employee absence (absence of up to four employees at the same time) is conditioned by increasing the staff by six (robust to 80% of possible absence scenarios) and 11 (robust to 100% of possible absence scenarios) additional employees. In the considered case, the process of synthesizing resistant competency framework required more than four hours of calculation. Considering the scale of the project portfolio, this duration is acceptable to the company.

6. Conclusions

The proposed method allows one to plan the allocation of production jobs to resources in situations in which the disruptions are caused by employee absences. According to this method, it is necessary to determine which additional (redundant) competencies organizations need to possess in order to compensate for competencies lost as a result of employee absenteeism. The experiments have shown that the method can be effectively used in an online mode to solve small-scale problems in organizational units of up to 30 employees and 60 tasks. It may be possible to increase the scale of the problems solved by using hybrid methods [44] dedicated to models that use sparse data structures.

The conducted experiments were limited to a selected class of competencies occurring in the industrial environment. In general, the proposed model can also be used in other areas requiring management competencies, maintenance management competencies, software skills, and so on. Assessment of the possible implementation of the proposed approach in such areas will be the subject of further research.

The proposed approach can be implemented for example in Decision Support Systems (DSS), Enterprise Resource Planning (ERP) systems [30], used in the online task assignment. Our future work will focus on developing the computational module which can be used as a software overlay for commercially available decision support systems used in human resources management. The functionalities discussed are solutions falling within the scope of human resource controlling [9] aimed at effective staff management while creating transparent rules and procedures for planning, monitoring, and control. It is easy to notice that from the controlling perspective, our method can be used in a broader sense of a digital twin concept [24].

A topic worth considering in terms of the future modification of the model is the assessment of the cost and time consumption of changes in the competency framework. The presented model assumes that the cost/time of each acquired competence is the same. By introducing appropriate cost and time parameters, it will be possible to search for variants of competency frameworks that can also find their economic justification.

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