Nonequilibrium magnetooscillations in spatially non-uniform quantum Hall systems

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Abstract. A theory of nonequilibrium magnetotransport in high Landau levels of inhomogeneous quantum Hall systems is presented. A nonlinear current is calculated in response to given local gradients of the chemical and electrostatic potential in the presence of moderately strong microwave radiation. In the regime of high temperature, theory generalizes previously obtained results describing microwave- and Hall-induced oscillations of magnetoresistivity for the case of spatially non-uniform systems. Additionally, the regime of low temperatures is studied, where strong modification of Shubnikov-de Haas oscillations by the ac and dc fields is demonstrated.

1. Introduction
Over the past decade, a family of fascinating nonequilibrium phenomena has been discovered in 2D electron gas (2DEG) including the microwave-induced (MIRO) [1–18], Hall-induced (HIRO) [19–32], and phonon-induced (PIRO) [33–39] resistance oscillations. Particularly interesting are the zero-resistance states (ZRS) [40–56] which form in the minima of MIRO at sufficiently high microwave power. The magnetoresistivity in these structures is governed by spectral and spatial resonances set by the cyclotron frequency \( \omega_c = eB/mc \) and the cyclotron diameter \( R_c = v_F/\omega_c \) in various combinations with the parameters of the external fields. All these effects were observed in moderately strong transverse magnetic fields \( B \parallel e_z \) where the dynamics of electrons remains essentially quasiclassical. At the same time, Landau quantization of the orbital motion can be pronounced and leads to the emergence of novel spectral and spatial resonances. This paper extends existing theories of these phenomena [54–92] to include the case of spatial inhomogeneity as well as effects of strong dc and ac fields on Shubnikov-de Haas oscillations (SdHO).

2. Nonlinear magnetotransport in irradiated inhomogeneous 2DEG
For a 2DEG in a classically strong magnetic field \( (\omega_c \tau_{tr} \gg 1) \), where \( \tau_{tr} \) is the transport scattering time, and in high Landau levels (chemical potential \( \mu(r) \gg \omega_c \)), the local dissipative current is calculated in response to a moderately strong microwave illumination as well as to local gradients of the chemical potential and electrostatic potential, \( eE_\mu = -\nabla \mu \) and \( E = -\nabla \phi(r) \). Local gradient of the electrochemical potential \( \eta(r) = \mu(r) + e\phi(r) \), given by \( E_\eta = -\nabla \eta/e = E_\mu + E \), is zero in equilibrium, in the absence of microwaves and external bias. In a given nonequilibrium situation, the solution of electrostatic problem which provides the relation between the local...
electron density \( n(r) \) and electrostatic potential should be found self-consistently using 
setup-specific boundary conditions.

In the classical limit \( \omega_c\tau_q \ll 1 \), where \( \tau_q \) is the quantum scattering time, the density of 
electronic states (DoS) in a 2D parabolic band is constant, \( \nu = \nu_0 = m/2\pi \) (we use \( \hbar = 1 \)). Provided interaction and memory effects [89] can be ignored, the local direct current 
\( j(r) = en(r)v_H + \sigma_D E_\eta \). Here the Hall velocity \( v_H = eE \times B/B^2 \), and Drude conductivity 
\( \sigma_D = e^2n(r)/m\omega_c^2\tau_q \) is insensitive to the strength of dc fields, to the microwave irradiation,
and to the details of the energy distribution of electrons [68].

We will see that even weak modulation of the DoS due to Landau quantization,
\( \tilde{\nu}(\varepsilon) \equiv \nu(\varepsilon)/\nu_0 = 1 - 2\lambda \cos(2\pi\varepsilon/\omega_c), \quad \lambda = \exp(-\pi/\omega_c\tau_q) \ll 1 \) (1)
drastically changes this picture and makes the transport properties of 2DEG very sensitive both
to external perturbations and to internal inhomogeneities.

Being expressed in terms of migration of the guiding center \( R \) of the cyclotron orbits, the current in translationally invariant system with given gradients of the chemical and electrostatic potentials reads
\[
j(r) = en(r)v_H + 2e\nu_0 \int d\varepsilon \langle \Delta R_{\varphi\varphi'}(\varepsilon) \rangle_{\varphi',\varphi},
\]
(2)
where \( \varphi \) and \( \varphi' \) denote the angle of momentum before and after the scattering, \( \Delta R_{\varphi\varphi'} = R'_e e_z \times (n_{\varphi'} - n_{\varphi}) \) with
\( n_{\varphi} = (\cos \varphi, \sin \varphi) \) is the corresponding shift of the guiding center, \( \Pi_{\varphi\varphi'}(\varepsilon) \) stands for the probability of such scattering event for electron with initial kinetic energy \( \varepsilon \), and angular brackets denote the angular average. In the presence of disorder, characterized by the scattering rate \( \tau_{\varphi}^{-1} = \sum \tau^{-1} e^{in\theta} \), and microwave radiation of (cyclic) frequency \( \omega \),
\( \Pi_{\varphi\varphi'} = \Pi_{\varphi\varphi'}^{(ed)} + \sum_{\pm} \Pi_{\varphi\varphi'}^{(\pm \omega)} \),

\[
\Pi_{\varphi\varphi'}^{(ed)}(\varepsilon) = \tau_{\varphi}^{-1} \varphi'[1 - \mathcal{P}_\omega \sin^2(\varphi/2 - \varphi'/2)]\mathcal{M}(\varepsilon, eE\Delta R_{\varphi\varphi'}, eE_\eta \Delta R_{\varphi\varphi'})
\]
(3)
accounts for elastic scattering modified by the microwave radiation, and

\[
\Pi_{\varphi\varphi'}^{(\pm \omega)}(\varepsilon) = [\mathcal{P}_\omega/2\tau_{\varphi\varphi'}] \sin^2(\varphi/2 - \varphi'/2)]\mathcal{M}(\varepsilon, eE\Delta R_{\varphi\varphi'} \pm \omega, eE_\eta \Delta R_{\varphi\varphi'} \pm \omega)
\]
(4)
gives the rate of microwave photon absorption and emission. In equations above we introduced the dimensionless microwave power,
\( \mathcal{P}_\omega = [ev_F E_\omega/\omega(\omega \pm \omega_c)]^2 \), where \( E_\omega \) is the (screened [13; 84])
microwave field and + (−) sign corresponds to the passive (active) circular polarization [93].

\[
\mathcal{M}(x, y) = \tilde{\nu}(\varepsilon)\tilde{\nu}(\varepsilon + x)f(\varepsilon, \mu)[1 - f(\varepsilon + x, \mu + x - y)]
\]
(5)
contains the product of initial and final DoS, and occupation of initial and final states given by
the the local energy distribution function \( f(\varepsilon, \mu) \). In equilibrium, the distribution function
\( f^{(T)}(\varepsilon, \mu) = 1/(\exp[\varepsilon/T - \mu/T] + 1) \) is position-independent for a given total energy of electron
\( \varepsilon^{(tot)} = \varepsilon + e\phi \), since it is a function of the difference \( \varepsilon^{(tot)} - \eta \) with \( \eta = \text{const}(r) \), giving rise to
vanishing dissipative current.

In the presence of external electric field, \( E_\eta \neq 0 \), and/or microwave radiation, the stationary
distribution function obeys the kinetic equation
\[
-\tau_{\varphi}^{-1}\tilde{\nu}(\varepsilon)f(\varepsilon, \mu) - f^{(T)}(\varepsilon, \mu) = \langle \Pi_{\varphi\varphi'}(\varepsilon) - \Pi_{\varphi\varphi'}^{(\pm \omega)}(\varepsilon) \rangle_{\varphi',\varphi}
\]
(6)
which balances the inelastic relaxation (described by the inelastic scattering rate \( \tau_{\varphi}^{-1} \)) and
redistribution of electrons due to spectral diffusion and absorption/emission of the microwave
quantum ($\Pi$) is the probability of the inverse process with the final state \(\{\varepsilon, \varphi\}\). In the classical limit \(\lambda \to 0\), Eq. (6) has a solution of the Fermi-Dirac form with temperature \(T\) increased due to heating by the dc field and microwave radiation. Much more interesting is the oscillatory correction due to Landau quantization, which at linear order \(\mathcal{O}(\lambda)\) has the form

\[
f(\varepsilon, \mu) = f^{(T)}(\varepsilon, \mu) + \pi^{-1} \omega_c \lambda A \sin(2\pi \varepsilon/\omega_c) \partial_{\varepsilon} f^{(T)}(\varepsilon, \mu),
\]

\[
f^{(T)}(\varepsilon, \mu) = \mathcal{P}_\omega \bar{\omega}^2 \sin \bar{\omega} \gamma(\zeta) + \zeta \cos(E_{0,\mu}E) \partial_{\zeta} \left[ 2\mathcal{P}_\omega \bar{\omega}^2 \sin^2(\bar{\omega}/2) \gamma(\zeta) - \gamma(\zeta) \right].
\]

In terms of the Bessel function \(J_n(x)\) and \(\tau_n^{-1} = \tau_n^{-1} - \tau_{n-1}^{-1}/2 - \tau_{n+1}^{-1}/2\), we used here

\[
\bar{\omega} = \frac{2\pi \omega}{\omega_c}; \quad \{\zeta, \zeta, \zeta\} = \frac{2\pi e R_c}{\omega_c} \{E, E, E\}; \quad \gamma(x) = \sum \tau_n^{-1} J_n^2(x); \quad \bar{\omega}(x) = \sum \tau_n^{-1} J_n^2(x).
\]

In the homogeneous case \(E = E_0,\ E_\mu = 0\), Eqs. (7), (8) reproduce Eq. (4.5) of Ref. [90].

Calculation of the current, Eqs. (2)-(5), with known distribution function, Eqs. (7), (8), is straightforward. At order \(\mathcal{O}(\lambda)\) the result reads

\[
j(r) = en(v) \mathbf{v}_H + \sigma_D E_\eta - \sigma_D S_{SdH}[\mathcal{K}(E_\eta) + \mathcal{K}(E) + \mathcal{K}(E_\mu)] + 2\sigma_D S_{SdH} \mathcal{P}_\omega \sin^2(\bar{\omega}/2)[\mathcal{K}(E_\eta) + \mathcal{K}(E)] + \sigma_D S_{SdH} A \mathcal{K}(E_\mu),
\]

where \(\mathcal{K}(E) = \frac{E_{\eta, \mu} \partial_{\eta}(\zeta)}{2\zeta} \cdot \frac{E_{\eta, \mu} \partial_{\mu}(\zeta)}{2\zeta} [\text{in } \mathcal{K}(E_\eta)\text{ one should make the replacement } E \to E_\eta \text{ and } \zeta \to \zeta_\eta \text{ etc.}]\), and \(S_{SdH} = -4\lambda \cos[2\pi \mu(\varepsilon)/\omega_c] X_T/\sinh X_T \text{ with } X_T = 2\pi^2 T/\omega_c\).

It is interesting to mention that the inelastic correction to SdHO, represented by the last term in Eq. (10), vanishes in the homogeneous case \(E_\mu = 0\). In the limit \(\zeta, \zeta_\eta \to 0\) of weak electric fields, Eq. (10) gives ordinary SdHO with first-order microwave-induced corrections

\[
j(r) = en(r) \mathbf{v}_H + \sigma_D E_\eta + \sigma_D S_{SdH} E_\eta[1 - (\tau_n/\tau_s) \mathcal{P}_\omega \sin^2(\bar{\omega}/2)]
\]

\[
- \frac{1}{2} \sigma_D S_{SdH} E_\mu \mathcal{P}_\omega \left[ \frac{\tau_n}{\tau_s} \sin^2 \frac{\bar{\omega}}{2} + \frac{\bar{\omega} \sin \bar{\omega}}{\tau_n/\tau_s + 2\mathcal{P}_\omega \sin^2(\bar{\omega}/2)} \right],
\]

where \(\tau_s^{-1} = 3/\tau_0 - 4/\tau_1 + 1/\tau_2\) (in terms of \(\tau_n, \tau_0 = \tau_0, \tau_1 = 1/\tau_0\) and \(\tau_1 = 1/\tau_0 - 1/\tau_1\)). Another interesting case is the limit of strong electric field \(\zeta_\eta \gg 1\) in homogeneous 2DEG, \(E = E_\eta\), where

\[
j(r) = en(r) \mathbf{v}_H + \sigma_D E_\eta - \sigma_D S_{SdH} E_\eta \frac{2\tau_n}{\tau_s} \cos \frac{\zeta}{\tau_s} \left[ 1 - 4\mathcal{P}_\omega \sin^2 \frac{\bar{\omega}}{2} \right].
\]

Here \(\tau_s^{-1} = \sum \tau_n^{-1} \cos \eta\) is the backscattering rate. This expression shows periodic change of the phase of SdHO (modified by the microwave radiation) with increasing dc bias.

At temperature higher than the Dingle temperature \(T_D = (2\pi \nu_{\eta})^{-1}\), first-order corrections \(\propto S_{SdH}\) to the Drude result become exponentially suppressed. Neglecting the first-order terms and collecting second-order corrections which do not oscillate with \(\mu/\omega_c\) and, therefore, survive high \(T > T_D\), one obtains

\[
j(r) = en(r) \mathbf{v}_H + 8\lambda^2 \sigma_D A \mathcal{K}(E) + 4\lambda^2 \sigma_D \bar{\omega} \sin \bar{\omega} \mathcal{P}_\omega \mathcal{K}(E) + 2\lambda^2 \sigma_D \tau_n \left[ \frac{E_\eta E_\mu}{E^2} \partial^2_{\eta} + \frac{\mathbf{v}_H E_\eta E_\mu}{v_H^2} \partial_{\zeta} \right] \left\{ 2\sin^2(\bar{\omega}/2) \mathcal{P}_\omega \gamma(\zeta) - \gamma(\zeta) \right\}.
\]

In different limits, Eq. (13) reproduces known results for MIRO and HIRO obtained in Refs. [65; 67; 71; 73; 79; 81; 87; 90] for various models of disorder potential, as well as results...
for photovoltage and photocurrent oscillations which arise in inhomogeneous system due to nonequilibrium violation of the Einstein relation. Furthermore, Eqs. (10) and (13) predict new effects in the nonlinear response of irradiated inhomogeneous systems, which deserve further investigation.

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In the general case of elliptic polarization, the results usually (see, e.g., Ref. [81]) change only slightly but the analytic expressions are much more complex.