Two-dimensional systems in which there is a competition between long-range repulsion and short-range attraction exhibit a remarkable variety of patterns such as stripes, bubbles, and labyrinths. Such systems include magnetic films, Langmuir monolayers, polymers, gels, water-oil mixtures, and two-dimensional electron systems. In many of these systems quenched disorder from the underlying substrate may be present. We examine the dynamics and stripe formation in the presence of both an applied dc drive and quenched disorder. When the disorder strength exceeds a critical value, an applied dc drive can induce a dynamical stripe ordering transition to a state that is more ordered than the originating undriven, unpinned pattern.

The growing interest in nonextensive thermodynamics and statistical mechanics represented in these Proceedings is stimulated by observations (from real experiments or simulations of models) of phenomena such as incomplete (e.g., fractal) coverage of phase space, rare-event statistics, intermittency, stochastic resonance, and so forth. We can hope for, and certainly seek to find, underpinning mechanisms and classifications of such systems. We suggest that among the relevant classifications are (a) systems exhibiting mesoscopic self-assembly (usually from nonlinearity or competing scales at more microscopic scales), and (b) systems with long-range interactions (and often coexisting short-range interactions and entropic forces). Long-range interactions may be in the microscopic variables of the system (e.g., Coulomb fields), or they may be induced at mesoscopic scales because of local constraints or symmetries (e.g., vortices in fluids and magnets, patterns in convection cells and vibrated granular materials, strain-strain interactions in elastic media). It is important to distinguish topological (singularity) mesoscale excitations, which have long-range mutual interactions (such as vortices, dislocations, stress filaments) from mesoscopic collective excitations with short-range interactions (such as solitons, breathers). We expect that the former will exhibit mesoscopic statistics driven by their mutual interactions, whereas the latter should show mesoscopic statistics driven by the (colored, multiplicative) noise bath of the non-collective degrees of freedom. We have studied aspects of anomalous statistics and dynamics in a number of physically motivated models, including: soliton diffusion [1]; relaxation in two and three dimensional complex Ginzburg-Landau equations [2]; multiscale structure and dynamics in elastic media [3]; filamentary transport in superconductor flux lattices [4]; surface morphology and growth [5]; relaxation in magnets with planar spin anisotropy [6]; glassy dynamics near a spinodal in a Van der Waals fluid [7]; charge, spin ordering and dynamics in strongly correlated electronic materials [8]; and large-scale materials deformation (fracture, friction) [9].

Motivated by the similarities in the phenomena exhibited by these (and related) model systems, we have begun studies of a “minimal” model with coexisting short- and long-range interactions. We introduce this model here, and briefly report some of its properties. Recent proposals for ordering in two-dimensional (2D) electron systems consider the idea of an electron liquid crystal system [10–12]. Such a system can form stripe phases...
with varying degrees of disorder: An anisotropic Wigner crystal is well-ordered in both directions; a smectic phase has ordering along only one direction; in a nematic, dislocations are introduced into the smectic phase but there is still an overall preferred orientation for the system; finally, in an isotropic liquid stripe phase, the stripe pattern is filled with dislocations and has no unique orientational direction.

Stripe patterns are very general and are formed in a wide range of systems [13]. For example, the Swift-Hohenberg model produces glassy stripe configurations [14]. Other stripe-forming systems include magnetic films [15], Langmuir monolayers, polymers, gels, and water-oil mixtures [16]. One system in which stripes have attracted a particularly large amount of interest is holes in layered transition metal oxides. In a quasiclassical description, the interaction between two holes consists of both a long-range repulsion generated by the Coulomb interaction, as well as a short-range attraction that arises due to the breaking of the antiferromagnetic bonds surrounding the holes. As a result, the holes can form linear arrays termed stripes. There is experimental evidence for the presence of stripes in several of the lanthanum cuprate oxides, but it is not yet clear whether the high-Tc superconducting compounds such as YBCO contain stripes, nor what the role of the stripes would be if they exist. Experimental detection of stripes is made more difficult by the possible interaction of impurities in the material with the stripes. The effect of disorder on a stripe phase is not well understood. Therefore we have constructed a simple model containing the basic physics of the system in order to gain a better understanding of the ordering and dynamics of a stripe phase.

Our simple model consists of particles with competing long-range and short-range interactions. The particles experience a long-range repulsion of a Coulomb form, and a short-range attraction which we assume to be a simple exponential form. When these two interactions are combined, a new length scale emerges which depends on the density of the particles and the strength of the short-range attractive term. We further add to our model quenched disorder to represent impurities in the material, as well as an external field, which would be an electric field in the case of holes.

The particles are assumed to obey overdamped Langevin dynamics given by

$$\mathbf{f}_i = -\sum_{j=1}^{N_u} \nabla U(\rho) + f^T + f^p + f^d = \eta \mathbf{v}_i.$$  (1)

The potential between two particles is

$$U(\rho) = \frac{q}{\rho} - B \exp(-\rho/\xi),$$  (2)

where \(q = 1\) is the charge on each particle, and \(\xi\) is the magnetic screening length. The long range interactions are treated as in Ref. [17].

Temperature is represented by random thermal kicks which have the property, \(< f(t) f^T(t') > = 0\), \(< f(t) f(t')^T > = 2\eta k_B T \delta(t-t')\). The disorder is modeled as random, point-like pins, represented by parabolic traps. The pinning force is given by

$$f^p = \sum_{k=1}^{N_p} f_p \frac{\xi_p}{\xi} |\mathbf{r}_i - \mathbf{r}_k^{(p)}| \Theta (\xi_p - |\mathbf{r}_i - \mathbf{r}_k^{(p)}|) \mathbf{r}_{i,k},$$  (3)

where \(f_p = 0.2\) is the pinning force, \(\xi_p = 0.125\xi\) is the pin radius, and \(\Theta\) is the Heaviside step function. The force from an applied field (voltage) is modeled as a uniform drive, \(f_d = f_d \hat{\mathbf{d}}\). We measure the velocity signature \(< V_x >\) of the system, which would correspond to a current.

We initially consider a system with no pinning or external drive. We vary the strength of the short range attraction by changing the value of \(B\), while holding the particle density fixed at \(n = 0.64/\xi^2\). As shown in Fig. 1(a), when \(B\) is small, the long range repulsion dominates and the system forms a Wigner crystal. At high values of \(B\), Fig. 1(c), the short range attraction dominates and the particles form clumps which become larger as the value of \(B\) is further increased. The clumps organize into a superstructure of a triangular lattice of clumps. At intermediate values of \(B\), Fig. 1(b), the short and long range interactions are of comparable strength, and the system forms a disordered stripe structure. We observe the same phases if we fix the value of \(B\).
and vary the density of the system. This is illustrated in Fig. 2.

We have measured the melting temperature of the three phases by computing the particle diffusion at each temperature. We define the melting temperature to be the temperature at which the particles first diffuse more than a lattice constant. The melting temperature $T_m$ as a function of $B$ is plotted in Fig. 3. $T_m$ decreases in the crystal phase as $B$ increases since the short range attractive term introduces distortions in the crystal, allowing it to melt more easily. The stripe phase has the lowest melting temperature, but $T_m$ increases dramatically upon entering the clump phase. This is because the clump phase contains a large amount of elastic energy. Within each clump there is triangular ordering of the particles, and additionally the clumps form an ordered superstructure. We find that the clump phase simultaneously melts and dissociates for the parameters considered here. In contrast, in the stripe phase melting first occurs along the length of each stripe, as illustrated in Fig. 4. Particles diffuse freely within the stripes but do not enter the regions between the stripes until higher temperatures are applied. Thus we find evidence for a two-stage melting of the stripe phase.

We next consider the effect of disorder on the three phases. We add random point-like disorder to the system, and then apply a driving force to the particles. At small drive strengths, the particles are trapped by the disorder and do not move. We measure the threshold force required to depin the particles in each phase. As shown in Fig. 5, the depinning force increases with increasing $B$ in the crystal phase, is highest in the stripe state, and then drops dramatically upon entering the clump phase. This is the same behavior, but inverted, that was observed for the melting transition in Fig. 3. This similarity is due to the fact that the softness of the stripe phase, which allows it to melt at lower temperature, also allows it to be well pinned by the randomly located pinning sites. The stripe state is disordered and can readily readjust to take advantage of the maximum number of pinning sites. In contrast, in the highly elastic and stiff clump phase, the particles are so constrained by the ordering of the phase that it is very difficult for them to adjust in order to find the lowest energy pinning location. As a result many of the pins are ineffective in the clump phase and the depinning force is low. This similarity between the melting and depinning signatures is particularly interesting as it implies that information about a thermal transition (melting) can be obtained from a dynamical experiment at constant temperature (depinning).

At high applied drives, it is possible to (dynamically) reorder the phases, which are distorted by the presence of the pinning at low drives. Dynamical reordering of a triangular lattice of repulsively interacting particles has been observed previously in several systems, including superconducting vortices [18,19] and Wigner crystals [20]. At high drives, the force from the pinning begins to resemble a temperature which decreases as the drive increases, until at infinite drive the effective temperature is zero, leading to the concept of dynamical freezing [18]. Dynamical freezing has already been observed in the Wigner crystal phase. We find that the clump phase also undergoes a dynamical freezing transition, and that at high drives the clump phase in the presence of disorder has the same form as the static clump phase in the absence of disorder, as shown in Fig. 6.

When we consider the dynamical reordering of the stripe phase, illustrated in Fig. 7, we find that the final ordered stripe phase is more ordered than the corresponding stripe phase that appears in a static system without pinning. The stripes are aligned due to the driving force, rather than being randomly aligned as was the case in the thermally annealed system. This indicates that, in a stripe forming system, it may be possible to use a combination of quenched disorder and an external drive to produce well-ordered stripes. The reordering of the stripe phase occurs only if the stripe is sufficiently strong to cause the stripe to undergo disordered, plastic flow at low drives. The original ordering of the stripe must be destroyed by the disorder in order to allow for a new ordering of the stripe to form aligned with the drive. As a result of the reordering of the stripe phase, only this shows hysteresis if the driving force is increased and then decreased again. The other two phases, clump and Wigner, do not exhibit
any hysteresis.

In summary, the competition between short range attraction and long range repulsion leads to the formation of Wigner crystal, stripe, and clump phases as a function of either the strength of the short range term or the particle density. The melting temperature is lowest in the stripe phase, while the depinning force is highest in the stripe phase, and lower in the Wigner and clump phases. Quenched disorder distorts the ordered phases, but the structure of all three phases can be regained through dynamical reordering at high drives. The dynamically reordered stripe phase is more ordered than the equilibrium stripe phase in a clean system, and as a result the stripe phases exhibit hysteretic signatures which could be observed experimentally.

In the context of “nonextensive thermodynamics,” it is now clearly of interest to study the various mesoscopic phases of our model (and their dynamics), and to compare their statistics and responses with predictions of Tsallis or other superstatistics.

This work was supported by the US Department of Energy under contract W-7405-ENG-36.

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Figure 1. Static phases as a function of increasing strength of the short range attraction, $B$. (a) Wigner crystal phase at low $B$. (b) Stripe phase at intermediate $B$. (c) Clump phase at high $B$.

Figure 2. Static phases for increasing particle density with fixed $B$.

Figure 3. Melting temperature for the three phases as a function of $B$.

Figure 4. Snapshots at different temperatures illustrating the melting of the stripe phase. Black dots are the particles; lines indicate the trajectories of the particles over a fixed interval of time which is the same for all panels.
Figure 5. Depinning force $F_{dp}$ for the three phases as a function of $B$.

Figure 6. Dynamic reordering of the clump phase. The external driving force increases from panel (a) to panel (f).

Figure 7. Dynamic reordering of the stripe phase. The external driving force increases from panel (a) to panel (f).