WORMHOLES AND SUPERSYMMETRY

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October 1990

A thesis submitted to the Faculty
of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Science.
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ABSTRACT: Following work by Coleman and Lee using ordinary gravity, we obtain a low-energy effective Lagrangian for a supergravity system, in which superwormholes are represented by local operators in superspace. Supersymmetry is not broken by these new interactions.

RÉSUMÉ: En suivant un argument de Coleman et Lee pour un système avec la gravité ordinaire, nous trouvons un Lagrangien effectif à basse énergie pour un système avec la supergravité. Les effets des “superwormholes” sont représentées par des opérateurs locaux dans la super-espace. La supersymétrie n’est pas brisée spontanément par ces nouvelles interactions.

Note to the reader:

This thesis is written using the “Victorian we” throughout. We do not know where we acquired this affectation, but it seems to be common in scientific papers, even those with only one author. Perhaps it is intended to include the readers, making them feel more involved in the discussion. Perhaps it gives nervous authors a pathetic illusion of safety in numbers. Or perhaps it is only strict accuracy, recognizing that even physicists are composed of many parts and particles.

From all my quarks, to all of yours ...
I. INTRODUCTION

The idea that small-scale geometry might be more than meets the eye seems to pre-date General Relativity\(^1\). Wheeler originated the modern hypothesis that quantum mechanics and gravity together imply that space has non-trivial, fluctuating topology on the small scale\(^2\). The key point in his argument was that the Einstein Lagrangian allows drastic metrical changes on small scales to have small action and so make significant contributions to transition amplitudes. The idea is intriguing, in that by giving space a rich texture as well as large-scale geometry, it promises to endow with explanatory properties the space-time background that our theories all have to have anyway. Economy is an elegance.

The problem with quantum topological fluctuations is that the causality embedded in a Lorentzian geometry forbids them on pain of singularities\(^3\). Thus Wheeler’s idea for a spacetime froth seemed just that, until Hawking’s claim that quantum gravity should use Euclidean geometry\(^4\). Hawking’s arguments for imaginary time are based on the fact that his event horizon thermodynamics, obtained now by many independent approaches, appears very elegantly in a Euclidean gravity formalism where time is analytically continued to imaginary values. Since many calculations that are simply impossible in Lorentzian gravity are rather easy in a Euclidean theory (because functions tend to decay instead of propagating), imaginary time has been a boon to quantum gravity physicists despite its uncertain role in quantum gravity physics.

A less extreme position than Hawking’s that still lets one use Euclidean space is
that in which imaginary time in gravity is compared to imaginary time in conventional second-quantized field theory or quantum mechanics. In such theories, quasi-classical approximations for “tunneling” phenomena can be done by Wick rotating $t \rightarrow t \tau$ and expanding path integrals around Euclidean space saddlepoints known as *instantons.* Since instantons are by definition field configurations that are localized in (Euclidean) space, they are amenable to various convenient non-interacting approximations. In conventional theories such as Yang-Mills, these techniques are irreproachable mathematical tricks; in gravity, they are not so irreproachable, because time is a dynamical variable whose properties are radically changed by analytic continuation.

In this context, actual instanton solutions to Euclidean-space Einstein equations are of interest. Giddings and Strominger found the first one, in a system with an axionic matter field. It is the simplest non-trivial topology, namely a *wormhole.* Wormhole configurations are geometries in which two asymptotically flat spaces are connected by a narrow “throat” region. Others have since appeared, including one found by Coleman and Lee, in which a conserved global charge is manifestly the thing that makes the wormhole, by keeping the throat from collapsing into nothing. (Hawking has also used wormhole geometries that are not instantons, in that they do not satisfy all the equations of motion.)

As the calculable harbingers of quantum gravity, wormhole effects have been invoked to produce the “wild and wonderful” phenomena of fundamental quantum incoherence and the vanishing of the cosmological constant. There has been great controversy over these notions, with disagreements on fundamentals and details alike leading to dramatically different results. The state of the theory of wormholes
is itself rather like a wormhole geometry. On opposite sides are the two asymptotically elegant formalisms of Hawking's Euclidean gravity, and classically causal relativity; they communicate through a channel equally inaccessible to well-grounded theory and to experiment.

In order to have some hope of finding some facts in a morass of speculation, a good thing to do is to find the low-energy effects of wormholes, in an attempt to make contact with observable physics. There is widespread agreement that wormholes should be so small that we never see them explicitly, but they ought to give rise to effective local operators in a low-energy approximation. Even if such operators turn out not to be observable at currently accessible scales, local field theory is well understood (in comparison with quantum gravity), and the better we manage to fit wormholes into a familiar framework, the better is our chance of appreciating their real significance, great or small.

Another interesting but speculative idea is supergravity, which an extension of the Coleman-Mandula theorem (still in flat space) strongly suggests to be the "wildest" quantum theory reasonable in the curved space regime below the Planck scale. There is no experimental evidence whatever for it, but neither is there damning evidence against it. It is mainly of interest because any analysis that works with supergravity is invoking what is presumably the most general symmetry money can buy, and we need powerful symmetries to hope to explain things like the hierarchy problem and the observed particle spectrum.

In this thesis we will combine these two interesting possibilities, thus bringing wormholes further into the arena of conventional theoretical physics. We will con-
struct the effective Lagrangian for a supersymmetric multiplet of matter fields coupled to supergravity, ignoring loops, but including the tree-level effects of superwormhole instantons. We will follow and extend a paper by Coleman and Lee\textsuperscript{9}. Our results will support the common assumption that supersymmetry should suppress the effective scalar self-couplings that wormholes generically induce\textsuperscript{10}.

In the remaining part of this introduction, we will describe the basic features of the argument we will follow. In Section II we will apply this argument to a simple system of a scalar field coupled to Einstein gravity. This will follow part of Reference [9] very closely. Then in Section III we will tackle supergravity, and obtain an original result exactly like that of Section II, but in superspace. Section IV is a brief conclusion. An appendix is attached giving a whirlwind summary of supersymmetry, superspace, and supergravity.

Our goal is to determine the effect at experimentally accessible scales of wormholes that are very much smaller than those scales, yet sufficiently larger than the Planck scale for us to use general relativity (and ultimately supergravity) in our action. There is plenty of room — about 17 orders of magnitude — between those scales to fit in our wormholes. Our approach will take advantage of this fact to insert yet another scale (hereafter "\(r_0\)") that is to be much larger than the wormhole scale and yet also much smaller than the experimental. We will then construct a field configuration in Euclidean four-space in which a 3-ball of radius \(r_0\) is cut out of a flat background space and replaced with a wormhole, matching our fields at the boundary. We will perform a saddle-point approximation to the path integral using this "cut and patch" configuration, and we will find that the leading contributions to the action coming
from the wormhole insertion are boundary terms on the sphere at \( r_0 \). Since \( r_0 \) is small on the laboratory scale, these terms may effectively be taken as point-like terms “at the wormhole.”

There are three steps in this procedure that need justifying. The first is the use of imaginary time to give Euclidean four-space instead of Lorentzian spacetime. There is no real justification for this procedure, but in these days when there is no king in Israel, every one does what seems right in his or her own eyes. There is no compelling argument against imaginary time in gravity, and it works well in flat space tunneling calculations for Yang-Mills theories. Admittedly, time is in no sense a dynamical variable in such theories, but theorists have to earn a living somehow.

The second step is the use of the abrupt “cut and patch” configuration. As we will see, it is not really very abrupt, because all our fields will approach their background values quite rapidly as we move out from the wormhole to the cut-off \( r_0 \).

The third great leap is identifying the three-sphere of radius \( r_0 \) as the point “at the wormhole.” This seems reasonable insofar as \( r_0 \) can easily be very small indeed, and we will not try to justify it any further here. There are depths to the idea, however, that may well reward exploration.

In the end we must confess that in the wormhole geometry of theories about wormholes, we will sit far out in the asymptotically flat space on Hawking’s side of the controversy. We idealize any fundamental conceptual flaws in Euclidean path integrals or instanton approximations as point sources of anguish and regret. Hopefully the loss of coherence will not be too great.
II. THE BOSONIC WORMHOLE

To warm up for the supergravity case, we will determine the effective action incorporating wormholes for the simpler system of a massless, complex scalar field coupled to Einstein gravity. In Euclidean space, we assume the Lagrangian

\[ \mathcal{L}_0 = -\sqrt{g}(M_P^2 R - \nabla_\mu \phi \nabla^\mu \phi^*) \]

\[ = -\sqrt{g}M_P^2 R + \frac{1}{2} \sqrt{g}(\nabla_\mu f \nabla^\mu f + f^2 \nabla_\mu \theta \nabla^\mu \theta), \]

where \( \phi = fe^{i\theta}/\sqrt{2}, \) \( M_P \) is the Planck mass\(^\dagger\) and \( R \) is the Ricci scalar.

Our initial task will be to find a wormhole. If we simply look for extrema of the four-space integral of the above Lagrangian we will discover that none of them can be wormholes\(^9\) (unless we allow the \( \theta \) field to be imaginary). This forces us to go back to the fundamental, Hamiltonian form of the path integral, from which the Lagrangian form is derived.

Suppressing the gravitational sector to fit the formula on the page, the fundamental form of the path integral is

\[ \langle F| e^{-\int d^4x} | I \rangle = \int \mathcal{D}f \mathcal{D}Q \mathcal{D}\pi \mathcal{D}Q \ e^{-\int \sqrt{g}(\mathcal{H}(\pi, Q, f, \theta) - \pi \nabla_\pi f - i Q \nabla_\pi \theta) d^4x} \Psi_F^* \Psi_I, \]

where \( \pi \) and \( Q \) are the momenta conjugate to \( f \) and \( \theta \), respectively, and \( \mathcal{H} \) is the Hamiltonian density. We have set \( h \) equal to one and Wick rotated \( i \rightarrow -ir \). \( \Psi_f \) and \( \Psi_F \) are wave functionals for the initial and final quantum states, which we assume to be standard \( N \)-particle states (and hence eigenstates of the total charge operator \( Q \)). Hereafter we will omit \( \Psi \)'s from our formulae, but they are understood to be

\(^\dagger\) \( M_P \equiv (16\pi G)^{-\frac{1}{2}} = 1.6 \times 10^{-7} \text{ kg} \approx 10^{17} \text{ TeV} \), where \( G \) is Newton's constant, and the conversion to TeV assumes \( h = c = 1 \).
implicitly present. Since $\mathcal{H}$ is quadratic in the momenta, the momentum sector of the path integral can be converted into a Gaussian form. This integral can be performed to yield a constant (of an arbitrarily large kind) that is absorbed in the normalization of the whole integral. What is left is the usual path integral involving only the fields and the Lagrangian. Nothing forces us to do this Gaussian integral, however; in this case, we will choose not to do it for some modes of the $\theta$ field and their conjugate momenta.

We will look for a saddle-point having central (three-spherical) symmetry, and so to evaluate the functional integral we will make the radial co-ordinate $r$ discrete, and expand all fields at each value of $r$ in series of orthonormal angular modes. Thus the $\theta$ sector of our path integral will look like

$$\prod_{n=1}^{\infty} \int \prod_{kln} da_{kln}(r = n\Delta r) \ e^{\Delta r \int d^3\Omega A(\theta) = \sum_{klm} a_{klm}(n\Delta r) Y_{klm}(\Omega)}, \tag{3}$$

where we understand that $\Delta r \to 0$. The $f$ and $g_{\mu\nu}$ integrations and the momentum sector are of similar form. ($A \equiv \mathcal{H} - i\pi \nabla_r f - iQ \nabla_r \theta$, and is introduced only for brevity.)

Our next step will be to split up the four-space integral in (3) into a large outer region and an inner ball of radius $r_0$, where $r_0$ is very much smaller than any observable length scale. We will integrate away the momenta just as usual in the large, outer region, to obtain an ordinary, Lagrangian path integral whose action stops short at $r = r_0$. We will also expand the gravitational sector of this integral around the saddlepoint of flat space, and suppress the free gravitons to concentrate on the scalar field. Terms in which fluctuations of the metric (about any saddlepoint, not just flat
space) couple to matter fields are suppressed by inverse powers of the Planck mass. We will call this portion of the path integral

\[ \text{BACKGROUND} = \int \mathcal{D}f \mathcal{D} \theta \ e^{-\int_{a}^{b} r^3 dr d^3 \Omega \mathcal{L}_0(f, \theta)}. \]  

(4)

The other part will be called "WORMHOLE", so that

\[ \langle F | e^{-\int dr^H} | I \rangle = \text{BACKGROUND} \times \text{WORMHOLE}. \]

For this part of the path integral, we will integrate out all the momenta as usual except for the momentum conjugate to \( a \equiv a_{000} \), the angular mode of \( \theta \) that is the constant mode. This particular momentum is the total charge \( Q \). We will now suppress from our formulae all the higher angular modes, in order to search for a stationary point of the path integral that is centrally symmetric. This is of course consistent, because all such modes enter the path integral quadratically, and so a centrally symmetric extremum is stationary against angular variations. The "stripped down" WORMHOLE path integral is then, suppressing gravity,

\[ \int \mathcal{D}Q \mathcal{D}f \mathcal{D}a \ \exp \left\{ -4\pi^2 \int r_0^0 \, dr \left( \frac{1}{2} \sqrt{g} \nabla_r \psi \nabla^r \psi + \frac{1}{2} \frac{Q^2}{f^2 2 f \frac{\sqrt{g}}{\sqrt{g}}} - iQ \nabla_r a \right) \right\} = e^{-4\pi^2 Q^2(r_0)} \int \mathcal{D}Q \mathcal{D}f \mathcal{D}a \ \exp \left\{ -4\pi^2 \int r_0^0 \, dr \left( \frac{1}{2} \sqrt{g} \nabla^2 f \nabla f + \frac{1}{2} \frac{Q^2}{f^2 2 f \frac{\sqrt{g}}{\sqrt{g}}} + \sqrt{1} \nabla f \right) \right\}, \]

(5)

where we have assumed that on the \( r_0 \) scale the background \( \theta \) is very close to constant. We can now do the \( a \) integration, to obtain a \( \delta \)-function which forces \( Q \) to be constant for all \( r < r_0 \). This makes the \( Q \) integration trivial, leaving us with a path integral whose centrally symmetric sector is simply

\[ \text{WORMHOLE} = e^{-4\pi^2 Q^2(r_0)} \int \mathcal{D}g_{\mu \nu} \mathcal{D}f e^{-4\pi^2 \int_{r_0}^{0} dr \left( -M^2 \sqrt{\sqrt{g} \nabla^r f} + \frac{1}{2} \frac{Q^2}{f^2 2 f \frac{\sqrt{g}}{\sqrt{g}}} \right)}, \]

(6)
where we have included the metric sector for the purpose of finding a wormhole saddlepoint.

$Q$ is now merely a constant parameter within $r = r_0$, the (conserved) charge flowing into the wormhole. The constant angular mode $a(r < r_0)$ has disappeared as a degree of freedom, which is all right, since it was never a physical degree of freedom at all, but a cyclic variable.

We have replaced the familiar Lagrangian with a function known to classical mechanics as the Routhian, formed by turning

$$\sqrt{g}f^2\nabla_{r}\theta
\nabla_{r}\theta \quad \text{into} \quad \frac{Q^2}{f^2g^{rr}\sqrt{g}}$$

and adding a surface term. It should be emphasized that this procedure is perfectly legitimate despite the fact that it is not what is usually done with a path integral. It is also worth noting at this point that this treatment is actually equivalent to an approach in which one allows the cyclic mode of $\theta$ to be imaginary, on the grounds that the fields in the path integral are ultimately operators on quantum states, and may have real or imaginary eigenvalues depending on the kind of states between which one is computing the tunneling amplitude. (For tunneling between states of real Lorentzian charge we require imaginary Euclidean charge in the Euclidean path integral.) The argument we are following rather sweeps the issue of quantum states under the rug, but it is of interest as an alternative derivation based on the path integral itself. It is perhaps also easier to digest for any who find that declaring a matter field to be imaginary is even more suspicious than having Wicked time.

We will now find a wormhole saddle-point for our part path integral WORM-
HOLE, and so justify its prophetie name. We let our Euclidean metric be

\[ ds^2 = g^{-2} dr^2 + r^2 d\Omega_3^2 \]

where \( d\Omega_3^2 \) is the line element on a round unit three-sphere.

Our field equations therefore become

\[ \nabla_r (gr^3 \nabla_r f) = -\frac{Q^2}{gr^3 f^3} \]

\[ -4r^6 M_p^2 G_r' = 12M_p^2 r^4 (1 - g^2) = \frac{Q^2}{f^2} - (gr^3 \nabla_r f)^2 \]  \hspace{0.5cm} (7)

\[ -4r^6 M_p^2 G_\Omega = M_p^2 r^4 (1 - g^2 - 2gr \nabla_r g) = (gr^3 \nabla_r f)^2 + \frac{Q^2}{f^2}. \]  \hspace{0.5cm} (8)

Differentiating (8) and applying (7) yields

\[ \nabla_r \left( r^4 (1 - g^2) \right) = 0, \text{ which implies } 1 - g^2 = \frac{L^4}{r^4}. \]  \hspace{0.5cm} (10)

(This metric is exactly the same as that found by Giddings and Strominger, \(^6\) with a different matter field.)

Evidently \( r \) is restricted to the range \( L < r < r_0 \). The apparent singularity in the metric at \( r = L \) may be removed by changing to co-ordinates \( y = \arccos(L^2/r^2) \), which range from \( y = \frac{\pi}{2} \) at \( r = \infty \), decreasing with \( r \) through \( y = 0 \) at \( r = L \), to \( y = -\frac{\pi}{2} \) at another asymptotically flat infinity on the other side of what is clearly a wormhole. Since the volume of a three-sphere of constant \( r \) decreases from infinity at \( y = \frac{\pi}{2} \) to a minimum of \( 2\pi^2 L^3 \) at \( y = 0 \) and then increases to infinity again at \( y = -\frac{\pi}{2} \), we can see that \( L \) is the radius of the wormhole. Since the \( r \) co-ordinates match asymptotically onto flat-space polar co-ordinates we will retain them, but the complete wormhole is covered by two identical co-ordinate patches with \( r_\pm \) both ranging from \( L \) to \( r_0 \).
Putting our wormhole metric (10) into (8) yields
\[ 2f \nabla_r f = \pm \frac{2\sqrt{Q^2 - 12M_p^2L^4 f^2}}{r^3 \sqrt{1 - \frac{L^4}{r^4}}} \]

This can be integrated by re-arranging it as
\[ \frac{L^2 d(f^2)}{\sqrt{Q^2 - 12M_p^2L^4 f^2}} = \pm d \arccos \left( \frac{L^2}{r^2} \right) \]

to reveal
\[ \sqrt{Q^2 - 12M_p^2L^4 f^2} = \pm 6M_p^2L^2 \left( \arccos \left( \frac{L^2}{r^2} \right) + C \right) \]
for any real constant C. Defining
\[ x \equiv 6M_p^2L^2 \left( \arccos \left( \frac{L^2}{r^2} \right) + C \right) \]
\[ = 6M_p^2L^2 \left( \frac{\pi}{2} + C - \frac{L^2}{r^2} + \mathcal{O} \left( \frac{L}{r^4} \right)^4 \right) \] on one side of the throat
\[ = 6M_p^2L^2 \left( -\frac{\pi}{2} + C + \frac{L^2}{r^2} + \mathcal{O} \left( \frac{L}{r^4} \right)^4 \right) \] on the other side.

we therefore have
\[ f = \frac{\sqrt{Q^2 - x^2}}{2\sqrt{3}M_p L^2}. \] (11)

At this point we can recall (9) and note that it is satisfied by the solutions in (10) and (11). The remaining components of the Einstein equations all vanish trivially.

We have now to choose boundary conditions to fix our constants of integration \( L \) and \( C \) by matching \( f \) to the two BACKGROUND values \( f_\pm \) at \( r_\pm = r_0 \). In doing so we will neglect all powers of \( \frac{L^2}{r_0^4} \), since this quantity can easily be of order \( 10^{-30} \) for \( L \) near the Planck length. We will, however, keep terms up to second order in the ratio of the boundary values of \( f \) to the Planck mass, because we will need them to obtain
non-trivial results in Section III.

\[ f_{\pm}(r_0)^2 = \frac{Q^2 - 36M_p^4L^4(\arccos(L^2/r_0^2) + C)^2}{12M_p^2L^4} \]

\[ \simeq \frac{Q^2 - 36M_p^4L^4(\frac{\pi}{2} \pm C)^2}{12M_p^2L^4}, \]  

where in the second line we drop terms of order \( \frac{L^2}{r_0^2} \). We therefore fix \( C \) by requiring

\[ f_-^2 - f_+^2 = 6\pi M_p^2 C. \]  

(13)

We also have

\[ f_+^2 + f_-^2 = \frac{Q^2 - 36M_p^4L^4(\frac{\pi}{4} + C^2)}{6M_p^2L^4}, \]

which with (13) implies

\[ \frac{Q^2}{L^4} = 6M_p^2(f_-^2 + f_+^2) + \frac{(f_-^2 - f_+^2)^2}{\pi^2} + 9M_p^4\pi^2. \]  

(14)

(For small \( Q \), (14) may be pushing \( L \) too close to the Planck scale for comfort, but we will continue undaunted anyway. The only alternative is to take up string theory.)

We can now evaluate the WORMHOLE sector of the path integral with tree-level accuracy by ignoring fluctuations around the wormhole saddlepoint. We will call this contribution \( R_0 \), because it is not an action but the integral of a Routhian, and
because it is really a leading order term in a series in powers of $\hbar$.

$$\mathcal{R}_0 = 4\pi^2 \int_{-}^{+} dr \left( -\frac{6M_p^2(1-g^2)r}{g} + 6M_p^2r^2\nabla_r g + \frac{1}{2}gr^3(\nabla_r f)^2 + \frac{Q^2}{2gr^3f^2} \right)$$

$$= 4\pi^2 \int_{-}^{+} dr \left( \frac{6M_p^2L^4}{gr^3} + \frac{1}{2}gr^3(\nabla_r f)^2 + \frac{Q^2}{2gr^3f^2} \right)$$

$$= 4\pi^2 \int_{-}^{+} \frac{dx}{Q^2 - x^2}$$

$$= 2\pi^2 Q \ln \left( \frac{Q + x}{Q - x} \right)_{+}^{-},$$

where the limits of integration $+$ and $-$ stand for $r_{\pm} = r_0$. We have neglected gravitational boundary terms that are of order $(L/r_0)^4$.

We choose $Q$ to be positive at this point without loss of generality, because if it were negative, $Q \pm x$ would simply interchange and the two ends of the wormhole be switched.

Using

$$(Q \pm x)^2 \simeq 12M_p^2L^4(3\pi^2M_p^2 + 2f^2),$$

we can approximate as follows:

$$2\pi^2 Q \ln \left( \frac{(Q + x)^2}{(Q + x)(Q - x)} \right)_{r_+ = r_0} \simeq 2\pi^2 Q \ln \left( \frac{3M_p^2\pi^2 + 2f^2}{f_+^2} \right)$$

$$= -4\pi^2 Q \ln \left( \frac{f_+}{\sqrt{3M_p\pi}} \right) + 4\pi^2 Q \left( \frac{f_-}{\sqrt{3M_p\pi}} \right)^2,$$

and similarly, with $+$ and $-$ interchanged, for the other end's term. We therefore have

$$\frac{\mathcal{R}_0}{4\pi^2 Q} \simeq -\ln \left( \frac{f_+}{\sqrt{3\pi M_p \sqrt{3\pi M_p}}} \right) + \left( \frac{f_+}{\sqrt{3\pi M_p} \sqrt{3\pi M_p}} \right)^2 + \left( \frac{f_-}{\sqrt{3\pi M_p}} \right)^2. \quad (16)$$
Thus the total path integral including the tree-level effects of a single wormhole is, taking subscripts $\pm$ to mean evaluation at $r_{\pm} = r_0$,

$$BACKGROUND \times e^{-\mathcal{R}_0 e^{4i\pi^2Q_0^+} e^{-4i\pi^2Q_0^-}}$$

$$= BACKGROUND \times A^q \phi^q_+ \phi^- e^{-\frac{1}{2}Aq(f_+^2+f_-^2)}$$

$$\simeq BACKGROUND \times A^q \phi^q_+ (1 - \frac{1}{2}Aq f_+^2) \phi^- (1 - \frac{1}{2}Aq f_-^2),$$

where $q \equiv 4\pi^2Q$, and $A \equiv \frac{2}{3\pi^2M_p^2}$.

If we assume that the three-spheres $r_{\pm} = r_0$ can be taken as the “effective points” $x_{\pm}^\mu$ in Cartesian co-ordinates, then the effective path integral for a single wormhole is

$$\langle F|e^{-\int Hdr} |I\rangle = \int Df^+ D\theta e^{-\int d^4 x \mathcal{L}_0} A^q \phi(x_+^\mu q)(1 - \frac{1}{2}Aq f_+^2) \phi^*(x_-^\mu q) (1 - \frac{1}{2}Aq f_-^2).$$

It is quite evident that translations of $x_{\pm}^\mu$ are zero modes of this system, and so will be integrated over upon evaluating quadratic fluctuations in the saddlepoint approximation. We introduce an unknown normalization constant $B_q^2$, which contains the 1-loop determinant for a wormhole of charge $q$, and which we assume has the correct mass dimension of 4 to make path integral dimensionless.

$$\langle F|e^{-\int Hdr} |I\rangle = \int Df^+ D\theta e^{-\int d^4 x \mathcal{L}_0}$$

$$\times B_q^2 \int d^4 x_+ A^q / 2 \phi^q (1 - Aq \phi^\star \phi) \int d^4 x_- A^q / 2 \phi^* \phi (1 - Aq \phi^\star \phi).$$

A further improvement we can make in our result is to include configurations with some number $N$ of wormholes just like the one we have been studying. As long as their ends are all separated by at least $r_0$ from each other, we can simply add their action contributions together. When we integrate over all their possible locations we are actually counting some arrangements in which they are closer than this, but
since \( r_0 \) is very small, this discrepancy can safely be neglected. Of course, the path integral contribution for each N-wormhole configuration must be divided by N! to make up for the zero mode integration having counted separately the N! identical configurations where the wormholes are merely permuted. Thus the effective path integral contribution from all N-wormhole saddlepoints is

\[
\int Df D\theta \frac{e^{-\int d^4x L_0}}{N!} \left( B_i^2 \int d^4x_+ A^{\alpha/2} \phi^{q(1 - Aq\phi^* \phi)} \right) \left( d^4x_- A^{\alpha/2} \phi^{*q(1 - Aq\phi^* \phi)} \right)^N.
\]

We can add up all such contributions from N=0 to \( \infty \) to get an exponential. We must also sum over all positive \( q \) to count all different types of wormholes. Since \( \theta \) is an angular variable, its constant mode \( a \) takes values on a circle and the conjugate momentum \( q \) is quantized. Letting \( Q \) be a canonical momentum operator again instead of a constant, we have

\[
[\theta(r, \Omega), Q(r, \Omega')] = i\hbar \delta^3(\Omega - \Omega'),
\]

where the operators are evaluated on equal-\( r \) three-spheres of the skeletonization given by (3). We can integrate over both \( \Omega \)s to obtain \([a, q] = i\hbar\), which makes the eigenvalues \( q = n\hbar \) for integers \( n \); \( q \) is like the momentum of a particle in a box of length \( 2\pi \), because \( a \) is periodic.

Thus we finally obtain

\[
\langle F|e^{-\int Hdt}|I \rangle = \int Df D\theta e^{\int d^4x L_0} \times \exp \sum_{q>0} \left[ \int d^4x_+ \int d^4x_- B_i^2[A^{\alpha/2} \phi^{q(1 - Aq\phi^* \phi)}]_{x_+} [A^{\alpha/2} \phi^{*q(1 - Aq\phi^* \phi)}]_{x_-} \right].
\]

(18)

This result is essentially the same as that of Reference [9], except that we show the effects of both ends of the wormhole explicitly, and we go to higher order in
inverse Planck mass. We have a bi-local expression. We can make it into a local operator, however, by introducing a Gaussian integral over complex dummy variables $\alpha$, following Reference [18]. The last exponential in (18) is equal to

$$\prod_{q>0} \frac{1}{\pi} \int d^2 \alpha_q \left[ e^{-\alpha_q^* \alpha_q} e^{-B_q \int d^4 x \left[ \alpha_q A^{q/2} \phi^* \phi + \alpha_q^* A^{q/2} \phi^* \phi + \frac{1}{A^{q/2}} \phi^* \phi \right]} \right].$$

(19)

This gives us an effective Lagrangian which is local and real, but which includes some new $\alpha_q$ parameters that may be thought of as creation/annihilation operators for baby universes carrying global charge $q$.

The wormhole throat may be regarded as the “history” of a small, closed “baby universe” which pinches off from a large, flat parent universe. The other side of the wormhole may be another parent universe which the baby universe breaks into, or it may be a distant region of the original parent universe which the baby universe rejoins. One can even cut the wormhole at its narrowest part and have the baby universe remain separate from any parent. The details of these geometries are unimportant in the low-energy effective Lagrangian picture, where we are concerned with tunneling between states defined in one large parent universe. We will therefore consider (19) to give a low energy effective potential which breaks the global $U(1)$ symmetry of the original Lagrangian, but whose coupling constant depends on baby universe dynamics.

Note that in the limit where $M_P \to \infty$, which is implied by our decoupling of gravity in the BACKGROUND, $A$ vanishes. Thus our result really presupposes that if we take $M_P$ to be large we must also allow $\alpha_q$ to be large so that $B_q A^{q/2} \alpha_q$ remains finite. Even in this limit, the $A\phi^* \phi$ factor in (19) becomes vanishingly small, but we retain it because it will be important in Section III. This concludes our study of the bosonic wormhole.
III. THE SUPERWORMHOLE

We will now attack wormholes and supersymmetry. To do this, we will take the simplest supersymmetric matter multiplet, and couple it to supergravity. At first we will simply study supersymmetric Lagrangians; when we have built up the necessary background of formalism, we will insert a wormhole into the discussion. For a thorough treatment of supergravity, the reader may consult such authors as Wess and Bagger or van Nieuwenhuizen. A brief summary of supersymmetry appears in the Appendix. Before we bring in supergravity, let us examine a supersymmetric model in flat Lorentzian spacetime.

The simplest supersymmetric system is the massless "scalar multiplet" consisting of a real scalar $A$, a real pseudo-scalar $B$, and a spinor $\chi$. The scalar and pseudo-scalar are grouped together into a complex field $\phi \equiv (A + iB)/\sqrt{2}$. The simplest Lagrangian involving these fields is

$$\mathcal{L}_0 = -\nabla_\mu \phi \nabla^\mu \phi^* - \frac{1}{2} \bar{\chi} \chi' \chi$$,  \hspace{1cm} (20)

where $\gamma^\mu S_\mu$ for any vector $S_\mu$. We take the trace +2 metric.\footnote{One may, for example, choose the spinor representation of the $\gamma$ matrices:

$$\gamma^k = \begin{pmatrix} 0 & -i \sigma_k \\ i \sigma_k & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$, \hspace{1cm} (21)

where $k \in \{1, 2, 3\}$. We define the chiral projection operators $P_L = \frac{1}{2}(1 + \gamma_5)$ and $P_R = \frac{1}{2}(1 - \gamma_5)$.}
transformation

\[ \delta \phi = \sqrt{2} \epsilon P_L \chi; \]
\[ \delta \chi = \sqrt{2}(P_L \nabla \phi + P_R \nabla \phi^*) \epsilon, \tag{22} \]

where \( \epsilon \) is a constant, infinitesimal, Grassman, Majorana spinor with mass dimension \(-\frac{1}{2}\). It is to be noted that these transformations only satisfy the proper supersymmetry commutation relations (see Appendix) if we employ the equations of motion. This is a general feature of supersymmetric theories. A theory with a symmetry relating bosons and fermions that does not satisfy the supersymmetry CRs is known by Reference [16] to violate such basic principles as causality and locality, so satisfying them is important.

This will pose a special problem for us, however, in that we plan ultimately to construct an effective Lagrangian in which wormhole terms are added to (20). We expect the low-energy action to be supersymmetric, inasmuch as supergravity is a gauged symmetry and thus generically unbroken by wormholes.\(^{11}\) Since the equations of motion will change in going to the effective theory, though, the supersymmetry transformations may differ from (22). Finding the proper supersymmetry transformations could mean some awkward guessing.

Fortunately there is an elegant way out of this difficulty. We can add to our system an extra, non-physical complex field \( F \) whose sole purpose is to keep track of the Lagrangian-dependence of the supersymmetry. The Lagrangian becomes

\[ \mathcal{L}_0 = -\nabla_\mu \phi \nabla^\mu \phi^* - \frac{1}{2} \bar{\chi} \nabla \chi + F^* F. \tag{23} \]
We change the transformation rules to
\[ \delta \phi = \sqrt{2} \epsilon P_L \chi; \]
\[ \delta \chi = \sqrt{2} P_L (\nabla \phi + F) \epsilon + \sqrt{2} P_R (\nabla \phi^* + F^*) \epsilon; \]
\[ \delta F = \sqrt{2} \epsilon \nabla P_L \chi. \]

(24)

\( F \) is obviously not a physical degree of freedom; it can be removed entirely from a path integral by a Gaussian integration, and it does not propagate. Now, though, the transformations (24) represent supersymmetry regardless of the equations of motion for \( \phi, \chi, \) and \( F \). We need only find an effective Lagrangian involving \( \phi, \chi, \) and \( F \) that is invariant under (24) to have, and know we have, a supersymmetric effective Lagrangian.

Another convenient technique we can use is to construct superspace. (Again, see the Appendix for more details.) We define the scalar (or chiral) superfield
\[ \Phi \equiv \phi + \sqrt{2} \bar{\eta} P_L \chi + (\bar{\eta} P_L \eta) F + \frac{1}{2} (\bar{\eta} \gamma_\alpha \gamma^\mu \eta) \nabla_\mu \phi + \frac{1}{\sqrt{2}} (\bar{\eta} P_L \eta) \bar{\eta} \nabla P_L \chi + \frac{1}{8} (\bar{\eta} \eta)^2 \Box \phi, \]
where \( \eta \) is a constant, Majorana, Grassman spinor of mass dimension \(-\frac{1}{2}\).

This superfield is to be thought of as a function defined on superspace, the union of ordinary four-space and the Grassman four-space spanned by \( \eta \). Since all powers of Majorana Grassman 4-spinors higher than the fourth vanish identically, the expansion in \( \eta \) of (25) completely defines \( \Phi \). Our supersymmetry transformation (24) corresponds to
\[ \delta \Phi = \bar{\epsilon} (\frac{\partial}{\partial \bar{\eta}} - \nabla \eta) \Phi. \]

A property of \( \Phi \) that will be very convenient in our superwormhole argument is that it is equal to the result of a finite supersymmetry transformation of parameter \( \eta \) acting on \( \phi \).
We can define $\Phi^*$ to be just the complex conjugate of $\Phi$. We may then note that

$$\Phi^*\Phi = \ldots + \eta_1 \eta_2 \eta_1^* \eta_2^* \left( -2\chi \nabla \chi - 2\nabla_\mu \phi \nabla^\mu \phi^* + \phi^* \Box \phi + \phi \Box \phi^* + 4F^* F \right).$$

Thus its $\eta^4$ component is four times the Lagrangian (23), up to a divergence. Using the convention that Grassman integration obeys $\int d\eta_i (a\eta_i + b) \equiv a$, for $a$ and $b$ ordinary numbers, we can therefore write

$$S_o = \int d^4x d^4\eta \frac{1}{4} \Phi^* \Phi. \quad (26)$$

In fact, any function of $\Phi$ and $\Phi^*$ provides a superspace Lagrangian which is supersymmetric, although not necessarily one having canonical kinetic terms.

We can now indicate why it was worth taking the trouble in Section II to extend the $\phi^a$ result of Reference [9] to the next higher order in $\phi$. The generalization to superspace of the Coleman-Lee effective vertex would be $\Phi^q$. This vertex is itself a scalar superfield, and by comparison with (25) we see that its $\eta^4$ component is just a total derivative.

We will now start our analysis by coupling our supersymmetric matter to supergravity, which is the gauge theory of local supersymmetry. (We cannot combine supersymmetry and gravity without supergravity; see the Appendix for why this is so.) If we restrict ourselves to the $(0^+, 0^-, \frac{1}{2})$ particle content of our simple example above, we find that the simplest locally supersymmetric Lagrangian is the special case $K = \Phi^* \Phi$ of the non-linear sigma model discussed by Bagger in Reference [21]. (This case is simplest in that it is the unique Lagrangian that yields canonical kinetic
Thus, in Lorentzian spacetime, we take

\[ \mathcal{L} = -M_P^2 \sqrt{g} R - \sqrt{g} \nabla_{\mu} \phi \nabla^{\mu} \phi^* - \frac{1}{2} \sqrt{g} \chi \nabla \chi - \frac{1}{2} \psi_\mu \gamma_\nu \nabla_\rho \psi_\sigma \epsilon^{\mu \nu \rho \sigma} + \frac{1}{16M_P^2}(\phi^* \nabla_\mu \phi - \phi \nabla_\mu \phi^*)(\overline{\psi}_\nu \gamma_\rho \psi_\sigma \epsilon^{\mu \nu \rho \sigma} - \sqrt{g} \chi \gamma_\mu \chi) + \frac{1}{2M_P} \sqrt{g} \chi \gamma^\nu \gamma^{-} \nabla_\nu (P_L \phi^* + P_R \phi) \psi_\mu \]

\[ + \frac{1}{128M_P^2} \chi_\gamma \gamma_\sigma \chi (4 \overline{\psi}_\mu \gamma_\nu \psi_\rho \epsilon^{\mu \nu \rho \sigma} - 4 \sqrt{g} \overline{\psi}_\mu \gamma_\sigma \chi - \sqrt{g} \chi \gamma_\sigma \chi), \]

where \( \psi_\mu \) is the gravitino, a Majorana spinor-vector that is the gauge field for local supersymmetry, and \( g \equiv - \det g_{\mu \nu} \). \( F \), as well as the auxiliary fields required by the graviton-gravitino multiplet, have been eliminated by Gaussian integration.

This Lagrangian is invariant — up to a total derivative — under the following supergravity transformations:

\[ \delta \phi = \sqrt{2} \epsilon P_L \chi; \]

\[ \delta \chi = \sqrt{2} (\nabla_\mu \phi P_L + \nabla_\mu \phi^* P_R) \gamma^\mu \epsilon + \frac{1}{4 \sqrt{2} M_P^2} (\chi (\phi^* P_L - \phi P_R) \gamma_5 \chi + \frac{1}{2 \sqrt{2} M_P} (\overline{\psi}_\mu \chi) + \gamma_5 (\overline{\psi}_5 \gamma_5 \chi)) \gamma_\mu \epsilon; \]

\[ \delta g_{\mu \nu} = \frac{1}{\sqrt{2} M_P} (\bar{\epsilon} \gamma_\mu \psi_\nu + \bar{\epsilon} \gamma_\nu \psi_\mu); \]

\[ \delta \psi_\mu = 2 \sqrt{2} M_P \nabla_\mu \epsilon - \frac{1}{4 \sqrt{2} M_P^2} (\chi (\phi^* P_L - \phi P_R) \gamma_5 \psi_\mu + \frac{1}{2 \sqrt{2} M_P} (\frac{1}{2} \sigma_\mu \chi_5 \gamma_5 \chi + (\phi^* \nabla_\mu \phi - \phi \nabla_\mu \phi^*) \gamma_5 \epsilon), \]

for \( \epsilon \) an infinitesimal Grassman Majorana spinor field.*

* These transformation laws are more complicated than those obtained by just adding gravitino terms to (24). This is because some field redefinitions have been done. The original fields obeyed much simpler transformation rules, but the kinetic terms in the Lagrangian were not of canonical form in those fields.
Our system is invariant under the global $U(1)$ transformation

\[ \phi \rightarrow e^{ia} \phi \]

\[ \chi \rightarrow e^{ia\gamma_5} \chi, \]

for $a$ an ordinary real number. This symmetry provides a generalization of the charge that was so important in Section II, most easily expressed through a newly defined $\theta$ field. Let

\[ \phi = \frac{1}{\sqrt{2}} fe^{i\theta} \]

\[ \chi = e^{i\gamma_5 \theta} \lambda. \]

We choose $f$ to be real, but this still leaves $\lambda$ as an unconstrained Majorana spinor.

We then have the conserved current

\[ j^\mu = \frac{i}{2} \sqrt{g} \lambda \gamma^\mu \lambda - \sqrt{g} f^2 \nabla^\mu \theta \]

\[ + i \frac{f^2}{16M_P^2} (\bar{\psi}_\nu \gamma_\rho \psi_\sigma e^{\mu \nu \rho \sigma} - \sqrt{g} \lambda \gamma_\mu \lambda) - i \frac{f}{2\sqrt{2}M_P} \lambda \gamma^\nu \gamma^\mu \gamma_5 \psi_\nu \]

which appears in the re-written Lagrangian

\[ \mathcal{L} = -M_P^2 \sqrt{g} R - \frac{1}{2} \sqrt{g} \nabla_\mu f \nabla^\mu f - \frac{1}{2} \sqrt{g} \lambda \nabla_\lambda \lambda - \frac{1}{2} \bar{\psi}_\mu \gamma_\nu \gamma_\sigma \nabla_\rho \psi_\sigma e^{\mu \nu \rho \sigma} \]

\[ + \frac{1}{2} \sqrt{g} f^2 \nabla_\mu \theta \nabla^\mu \theta + j^\mu \nabla_\mu \theta \]

\[ + \frac{1}{2\sqrt{2}M_P} \sqrt{g} \lambda \gamma^\mu \gamma^\nu \psi_\mu \nabla_\nu f \]

\[ + \frac{1}{128M_P^2} \bar{\lambda} \gamma_5 \gamma^\sigma \lambda (4 \bar{\psi}_\mu \gamma_5 \gamma^\nu \psi_\rho e^{\mu \nu \rho \sigma} - 4 \sqrt{g} \bar{\psi}_\mu \gamma_5 \gamma^\sigma \psi_\mu - \sqrt{g} \bar{\lambda} \gamma_5 \gamma^\sigma \lambda). \]

Now to deal with wormholes in our supersymmetric theory, one must come to grips with the euclideanization of this action. While this is a trivial exercise for the purely bosonic part of the action, it is somewhat more subtle for the fermions because there are no Majorana spinors in four Euclidean dimensions. This problem has been resolved\textsuperscript{23,24,25}. The essential point is to surrender the usual definition of adjoint
spinors and use instead Majorana conjugation\textsuperscript{23}. We leave this construction and the details of the supersymmetry zero modes (see below) to elsewhere\textsuperscript{26}. We will be able to obtain our result with minimal calculation, by relying on supersymmetry itself to give us a supersymmetric generalization of the bosonic result that we already have.

It would be straightforward to define the Routhian analogous to that of Section II. Since both the charge and the Lagrangian (30) reduce to those of Section II when all fermion fields vanish, so the Routhian must also be identical in the absence of fermions. Also, since the fermions must be introduced in bilinears of spinor fields, it is clear that a bosonic saddlepoint is stationary against fermionic variations. Consequently we know that the bosonic wormhole solution of Section II is a saddlepoint of the supergravity Routhian in Euclidean space. Having determined that a wormhole instanton exists for our supergravity system, we will construct an effective Lagrangian incorporating a vertex representing the wormhole. We will do the same things we did in Section II, namely patch a WORMHOLE saddlepoint cut off at a radius \( r_0 \) into a flat BACKGROUND. Eq. (28) describes our WORMHOLE sector; we should now consider the BACKGROUND.

As in Section II, we want to expand around a flat space saddlepoint and concentrate only on the matter sector, while disregarding the supergravity fields. We cannot, however, simply follow Section II in taking as BACKGROUND Lagrangian the full supergravity Lagrangian (27) in the limit where \( M_p \) becomes infinite. That limit yields the simple Lagrangian (20), but it requires discarding terms that will be of interest. To be precise, if we take the limit \( M_p \rightarrow \infty \) we would find that the lowest order \( U(1) \) symmetry-breaking effective wormhole vertex vanished. So instead we
return to the superspace matter action that appears in the construction of (27) and for the present ignore the supergravity sector for simplicity. This leaves the globally supersymmetric matter action

\[
S_{\text{matter}} = -\frac{3}{2} M_P^2 \int d^4x d^4\eta \ e^{-\frac{1}{6M_P^2} \Phi^* \Phi}.
\]  

(31)

This is the special case \( K = \Phi^* \Phi \) of the general sigma model obtained in Reference [21]. Note that after the Grassman integration has been done and the auxiliary field \( F \) eliminated, this action has non-renormalizable vertices and non-canonical kinetic terms:

\[
\mathcal{L}_\sigma = e^{-\frac{\phi^* \phi}{6M_P^2}} \left( \left( 1 - \frac{\phi^* \phi}{6M_P^2} \right) \left( -\nabla_\mu \phi \nabla^\mu \phi^* - \frac{1}{2} \bar{\chi} \nabla \chi \right) \right)
\]

\[
- \frac{1}{24M_P^2} \left( 2 - \frac{\phi^* \phi}{6M_P^2} \right) \bar{\chi} \gamma^\mu \gamma^\nu \chi \left( \phi^* \nabla_\mu \nabla_\nu \phi \right)
\]

\[
- \frac{(\bar{\chi} \chi)}{48M_P^2} \frac{2 - \frac{\phi^* \phi}{3M_P^2} + \frac{\phi^* \phi}{36M_P^2}}{1 - \frac{\phi^* \phi}{6M_P^2}}.
\]

(32)

It is only in the limit that \( M_P \to \infty \) that one recovers (20). It is also only in this limit that we can really say that supergravity is decoupled from the matter. Since as we have explained we do not wish to take this limit, we must accept that the supergravity sector needs to be included in a full analysis of wormhole effects, which may modify our final result by the addition of some new interactions. For simplicity we will nevertheless continue to ignore it here, and relegate the full analysis to a future investigation\(^{26}\). In the meantime, we may seem somewhat inconsistent in discarding some terms suppressed by inverse Planck masses, but keeping others. Our justification will be that the terms we keep will be leading order symmetry breaking terms, whose presence is significant even when they are very small.

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The fermion in (32) is related to that of (27) by a field redefinition involving the scalar and the gravitino; the (fixed) metric is related to that of (27) by a Weyl rescaling involving the scalar field. We emphasize, however, that while the field redefinitions used to obtain (27) from coupling (31) to supergravity rescaled the metric and altered $\chi$, they did not affect the scalar field $\phi$. This will be important, because as we know from Section II, the bosonic wormhole saddlepoint of (28) will yield an effective vertex that is a function of the scalar field. Thus we will be able to compute the \textsc{Wormhole} contribution to the path integral using the redefined fields of (28), and then interpret the result as a function of the \textsc{Background} scalar field.

Supergravity being the gauge theory of supersymmetry (as well as diffeomorphisms), one must fix the gauge when evaluating a path integral. A convenient convention is fix local supersymmetry transformations by imposing $\gamma^\mu \psi_\mu = 0$, for this makes the gravitino a pure spin-3/2 field just as we make gravitons and photons transverse. As with diffeomorphisms, however, there remains some residual supersymmetry after gauge-fixing. In the case of an asymptotically flat space such as our wormhole configuration, this residual symmetry does not vanish at infinity, but tends to a rigid, global supersymmetry. Such a symmetry constitutes a zero mode of a wormhole instanton: the path integral is invariant along the subspace of field configuration space spanned by the symmetry generators. Just as the translational zero modes of the bosonic wormhole in Section II led to an integration over position of the effective wormhole operator, so the translational and supersymmetric zero modes of the supergravity wormhole lead to an integration over superspace "position" of the superwormhole effective operator. Just as we had independent four-spaces of transla-
tional modes for each end of the wormhole in Section II, so additionally we have four independent supersymmetry modes for each end of the superwormhole\textsuperscript{26}.

In the no-fermion limit, the superwormhole is identical to the bosonic wormhole of Section II, and so yields an effective operator

\[
A^{q/2} \phi^q (1 - Aq\phi^* \phi) 
\]

plus a complex conjugate term for the other wormhole end. (Recall that \( A \equiv 2/(3\pi^2 M_p^2) \).) This expression is identically equal to the superfield operator

\[
A^{q/2} \phi^q (1 - Aq\Phi^* \Phi)|_{\eta=0}. 
\]

Because of the fact (noted above) that a superfield is equal to the value of the scalar field after a finite supersymmetry transformation of measure \( \eta \), we can easily write the effective operator for the superwormholes related to the bosonic one by a global supersymmetry transformation as

\[
A^{q/2} \phi^q (1 - Aq\Phi^* \Phi). 
\]

Since translation in \( \eta \) is a zero mode of the path integral, as with ordinary translation in \( x^\mu \) in Section II (and here) we must integrate over it to obtain tree-level accuracy in our saddlepoint approximation. Thus the one-superwormhole end effective operator looks just like a superspace action term

\[
B_q A^{q/2} \int d^4x d^4\eta \, \Phi^{q+1}\Phi^*, 
\]

where we have dropped the \( \Phi^q \) term because it gives the total derivative \( \Box \phi^q \) after Grassman integration, and adjusted the power of \( A \) to give the correct mass dimension

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of 2 for a superspace Lagrangian. (That is, we assume that the zero mode functional integrations involve a normalization constant of the correct mass dimension for $B_i$ to be dimensionless.) Apart from the suppression of the leading order term, we have merely translated our bosonic result into superspace in the most obvious way.

Adding the complex conjugate term for the other wormhole end — with its own four independent supersymmetry zero modes, summing over all N-superwormhole configurations to obtain an exponential, summing over positive $q$, and introducing dummy $\alpha$ parameters to obtain a local action all proceed exactly as in Section II. We obtain

$$S_{\text{eff}} = \int d^4x d^4\eta \left( -\frac{3}{2} M_p^2 e^{-\frac{1}{\alpha M_p^2} } \Phi^* \Phi + \sum_{q>0} B_q A^{q/2} \left[ \alpha^* \phi^{q+1} \Phi^* + \alpha_q \phi^{*q+1} \Phi \right] \right)$$

(33)

as the tree-level action incorporating the effects of superwormholes. Doing the Grassman integration, and defining

$$V(\phi, \phi^*) \equiv \sum_{q>0} B_q A^{q/2} \left[ \alpha^* \phi^{q+1} \Phi^* + \alpha_q \phi^{*q+1} \Phi \right] - \frac{3}{2} M_p^2 e^{-\frac{1}{\alpha M_p^2} } \Phi^* \Phi$$

$$DV \equiv \frac{dV}{d\phi} \quad D^* V \equiv \frac{dV}{d\phi^*}$$

we obtain

$$\mathcal{L}_{\text{eff}} = D^* D V \left( -4 \nabla_{\mu} \phi \nabla^\mu \phi^* - 2 \bar{\chi} \nabla \chi + 4 F^* F \right)$$

$$+ \bar{\chi} \gamma_5 \gamma^\mu \chi \left( D^* D^2 V \nabla_{\mu} \phi - D^* D V \nabla_{\mu} \phi^* \right)$$

$$- 2 (D^* D V F \bar{\chi} P_{\chi} + D^* D^2 V F^* \bar{\chi} P_{\chi}) + \frac{1}{2} D^* D^2 V (\bar{\chi} \chi)^2.$$

We can finally eliminate $F$ by using its non-propagating field equation,

$$F = \frac{1}{2} \bar{\chi} P_{\chi} \frac{D^* D^2 V}{D^* D V}.$$
This gives the ultimate result

\[ \mathcal{L}_{\text{eff}} = D^* D V (-4 \nabla_\mu \phi \nabla^\mu \phi^* - 2 \bar{\chi} \nabla \chi) \]

\[ + \bar{\chi} \gamma_\gamma \gamma^\mu \chi (D^* D^2 V \nabla_\mu \phi - D^* D V \nabla_\mu \phi^*) \]

\[ + \frac{1}{2} \left( D^* D^2 V - \left| \frac{D^* D^2 V}{D^* D V} \right|^2 \right) (\bar{\chi} \chi)^2. \]  

(34)

Since this action was written in superspace, it is manifestly supersymmetric; using the \( F \) equation of motion above we can write its supersymmetry in terms of physical fields only as

\[ \delta \phi = \sqrt{2} \bar{\epsilon} P_L \chi; \]

\[ \delta \chi = \sqrt{2} P_L \left( \bar{\gamma} \phi + \frac{1}{2} \bar{\chi} P_L \chi \frac{D^* D^2 V}{D^* D V} \right) \epsilon + \sqrt{2} P_R \left( \bar{\gamma} \phi^* + \frac{1}{2} \bar{\chi} P_R \chi \frac{D^* D^2 V}{D^* D V} \right) \epsilon. \]  

(35)

To see the low-energy behaviour of this system, we let the matter fields be small in comparison with the Planck scale. We will look for the lowest order \( U(1) \) symmetry-breaking term. To find it, we will need to keep wormhole terms having co-efficients as small as \( M_p^{-3} \). The justification for keeping these terms but suppressing others of similar dimension that do not carry \( U(1) \) charge is that violation of charge conservation is a drastic effect significant even if highly suppressed. We therefore approximate

\[ \mathcal{L}_{\text{eff}} \approx - \nabla_\mu \phi \nabla^\mu \phi^* - \frac{1}{2} \bar{\chi} \nabla \chi \]

\[ + \sum_{q=1}^{3} B_q (q + 1) A^{q/2} \left( (\alpha_q \phi^q + \alpha_q^* \phi^{*q}) (-4 \nabla_\mu \phi \nabla^\mu \phi^* - 2 \bar{\chi} \nabla \chi) \right. \]

\[ + \bar{\chi} \gamma_\gamma \gamma^\mu \chi \nabla_\mu (\alpha_q \phi^q - \alpha_q^* \phi^{*q}) \]

\[ + \frac{\sqrt{2} B_A}{3 \sqrt{3} M_p^2 \pi} (\alpha \phi + \alpha^* \phi^*) (\bar{\chi} \chi)^2 + \mathcal{O}(\frac{1}{M_p^4}). \]  

(36)

To interpret this Lagrangian we need to make its kinetic terms canonical, by
defining

\[ \varphi = \phi + 4 \sum_{q=1}^{3} B_q A^{q/2} \alpha_q^* \phi^{q+1} \]

\[ \psi = [1 + 4 \sum_{q=1}^{3} B_q(q + 1) A^{q/2}(\alpha_q^* \phi^q P_L + \alpha_q \phi^q P_R)] \chi. \]

This field redefinition does not affect the S-matrix in perturbative expansion in \( \alpha \)'s because it is invertible to any given order in \( \alpha \)'s. Eq. (36) then becomes, to linear order in \( \alpha \)'s, and keeping only lowest dimension terms,

\[ \mathcal{L}_{\text{eff}} = -\nabla_\mu \varphi \nabla^\mu \varphi^* - \frac{1}{2} \bar{\psi} \gamma \psi + \frac{\sqrt{2} B_1}{3 \sqrt{3} M_P^2} (\alpha_1^* \varphi + \alpha_1 \varphi^*) (\bar{\psi} \psi)^2 + \ldots \]  

(37)

Thus we obtain a four-fermion vertex coupled to baby universe creation/annihilation operators which carry off or inject one unit of scalar field charge. This is what one would expect an instanton with four fermionic zero modes to reduce to in the low-energy limit, because those zero modes will make all Feynman diagrams without four external fermionic lines vanish in the presence of a wormhole. Note that while the new vertex of (37) does break the scalar field's U(1) symmetry, it does not break supersymmetry. As we know from (35) above, the full theory is supersymmetric when we include all of the higher order terms.
IV. CONCLUSION

We have found that, to tree level accuracy, the effect of wormholes in a supergravity system is to add a new operator which is a local operator in superspace, and is in fact a simple generalization to superspace of a purely bosonic expression. The leading bosonic term found by Coleman and Lee\textsuperscript{9} is cancelled by supersymmetry. This is an example of supersymmetric suppression of wormhole-induced scalar vertices as suggested by Hawking\textsuperscript{10}. There was no new dynamics in our derivation. Our answer follows from the bosonic result by transformation properties alone.

How much one should believe in this answer? Right now, the only voice that can be heard on this issue is that of aesthetics. Insofar as our analysis has been smooth and "inevitable," it is another piece on the pile of evidence that wormholes should be part of nature because they are so nice. Insofar as we have had to beg some serious questions, we remain open to the criticism that wormholes are only nice because we make them that way.

The effective operator that we have found cannot be used to break supersymmetry spontaneously. The potential term in (36) — the term left when all fields are set to be constant — has a minimum value of zero; this means that supersymmetry remains unbroken. (See the Appendix for an explanation of why this is so.)

Our result could be improved by computing the one-loop term found by doing the Gaussian integral of the wormhole action to second order in fluctuations about the saddlepoint. Unless we believe with Hawking that quantum gravity is fundamentally Euclidean, going any further than this would be to exceed the limits of validity of the
tunneling approximation that put us into Euclidean space in the first place, and so would probably be self-defeating.

One could also improve the dilute gas approximation by considering the classical interactions between wormholes. Since on shell the scalar field falls of as $1/r^3$ near one of our wormhole points, and since the scalar field action is just like that for an electromagnetic potential, we can see that the classical interactions turn our dilute gas into a Coulomb gas in four-space. The Coulomb gas partition function can thus be used to improve our accuracy. Note, however, that this improvement is only the same improvement made when calculating quantum corrections to any classical Lagrangian. As a tool for obtaining Feynman rules, our Lagrangian needs no Coulomb gas corrections.

Thus our result is a tree-level result in two senses. We have neglected fluctuation effects in the quantum gravity saddlepoint approximation, and we have neglected loops between effective wormhole vertices. Improvements could be made in both areas, in tedious but straightforward ways.

A still harder task would be to examine non-centrally-symmetric wormholes. In our superwormhole, based as it is on the centrally symmetric wormhole presented in Section II, the fermion never carries any of the U(1) charge supporting the wormhole. This possibility would be a new feature of non-spherical wormholes.

If what one most wants out of superwormholes is a method for breaking supersymmetry spontaneously, however, it is unlikely that any amount of accuracy will satisfy the requirement. This is because superwormholes, even non-spherical ones, have four supersymmetry zero modes. Consequently the effective operators are generically “D-
terms" — $\eta^4$ terms like our result. D-terms are kinetic terms; they do not give rise to non-zero vacuum energies. To break supersymmetry, an "F-term" is needed. F-terms are $\eta^2$ terms; to obtain one from a superwormhole we would need somehow to kill off two zero modes. In a string motivated model, Park, Srednicki and Strominger are able to do just this.$^8$ It would be interesting to determine what general features of their model lead to this result.
APPENDIX: REVIEW OF
SUPERSYMMETRY, -SPACE, AND -GRAVITY

The following very brief introduction to supersymmetry presents knowledge gained
largely from a course taught by C.P. Burgess at McGill in 1989. Reference [19] pro-
vides a concise introduction to supersymmetry and supergravity in the ein- gant sup-
erspace approach. It uses Weyl spinors. Reference [20] is an exhaustive review of
supergravity, presenting many formalisms.

Supersymmetry is a symmetry relating bosons and fermions. SuSy (for short) is
therefore generated by fermionic operators, which obey anti-commutation relations.
Grassman algebra is therefore inevitable, since Grassman variables are numbers de-
dined to anti-commute with each other and with any fermionic field.

SuSy is a quite specific fermionic transformation. A very powerful theorem (Refer-
ce [16]) states that, under some very mild assumptions — such as causality, locality
etc. — the most general symmetry group of a field theory in flat space is

$$(\text{Poincaré}) \times (\text{SuSy}) \times (\text{internal symmetries}),$$

where $(\text{Poincaré}) \times (\text{SuSy})$ has the following (anti-)commutation structure, using

$[A, B]_\pm \equiv AB \pm BA.$

\[
\begin{align*}
[J_{\mu \nu}, J_{\lambda \rho}]_- &= -i g_{\mu \rho} J_{\nu \lambda} + \text{permutations} \\
[J_{\mu \nu}, Q]_- &= \frac{i}{4} [\gamma_\mu, \gamma_\nu]_- Q \\
[J_{\mu \nu}, P_\lambda]_- &= i (g_{\mu \lambda} P_\nu - g_{\nu \lambda} P_\mu) \\
[P_\mu, Q]_- &= 0 \\
[P_\mu, P_\nu]_- &= 0 \\
[Q, Q]_+ &= -2 \gamma^\mu P_\mu
\end{align*}
\]

(38)

Here $J_{\mu \nu}$ are the Lorentz generators, $P_\mu$ generate translations, and $Q$ are the spinor
generators of SuSy. (Actually, the Poincaré group is a sub-group of this larger group usually called the SuSy group, but SuSy is also taken to refer only to the $Q$s.)

SuSy is evidently more like a spacetime symmetry than it is like an internal symmetry. The fact that two $Q$s can anti-commute to make a translation is remarkable, to say the least.

We can highlight this fact and represent SuSy in a way that gains elegance at the price of abstraction by constructing superspace. Superspace is the union of ordinary four-space and a four-space of Grassman variables $\eta$, where the $\eta$'s form a Majorana spinor just as $x^\mu$ form a vector (in flat space). The superspace co-ordinate transformation

$$x^\mu \rightarrow x^\mu + \bar{\eta} \gamma^\mu \epsilon;$$
$$\eta \rightarrow \eta + \epsilon$$

represents SuSy.

Fields defined on superspace are called superfields. They can be expanded in a series of powers of $\eta$ up to the fourth, with the series co-efficients being ordinary fields on four-space. The general superfield turns out to be a reducible representation of SuSy. We can constrain the co-efficient fields in three independent ways to obtain "scalar superfields" and "vector superfields." (A scalar superfield and its complex conjugate are distinct SuSy representations; they are also called left and right chiral superfields.) For example, a complex scalar (ordinary) field, a Majorana spinor, and a complex auxiliary field are sufficient independent co-efficients to form a scalar superfield.

The $\eta^4$ components of the product $\Phi^* \Phi$ of any scalar and conjugate scalar superfield happen to transform into four-space divergences under SuSy. Such components
— known as “D-terms” — are therefore suitable candidates for supersymmetric Lagrangians. Since Grassman integration makes \( \int d^4 \eta \) pick out just such components of a superfield, we can write supersymmetric actions as integrals over superspace of Lagrangian superfields. \( \Phi^* \Phi \) turns out to give a kinetic term.

The \( \eta^2 \) components of either a scalar or a conjugate scalar superfield likewise transform into 4-divergences, and so these “F-terms” are also suitable for Lagrangian purposes, although they do not work as neatly in superspace integrals because they are “picked out” of the superfields by only two Grassman integrations. It can be shown that any function of scalar superfields (but not of mixed scalars and scalar conjugates!) is a scalar superfield. (That is, the co-efficient fields of the product obey the proper restrictions for a scalar superfield.) Consequently \( \int d^2 \eta f(\Phi) \) and \( \int d^2 \eta f(\Phi^*) \) are also Lagrangian candidates, and these provide potential terms (“superpotentials”).

The auxiliary fields that appear as components in superfields are those fields that turn out to have non-propagating field equations when we Grassman-integrate these Lagrangian superfields into ordinary Lagrangians. They can therefore be eliminated and replaced by combinations of other fields, and do not represent physical degrees of freedom. There are always a few of them around. This is implies that, once we get back to ordinary space, the exact form of the symmetry transformations on the fields of a supersymmetric system depends on the Lagrangian of the system. This fact is a feature of SuSy itself, and not a defect in the superspace approach. In fact the chief virtue of the superspace approach is that we never have to guess what the SuSy transformation looks like, or whether it exists as a symmetry. The chief drawback of the superspace formalism is that we only know these things in terms of superfields.
which contain auxiliary fields, and eliminating these is usually tedious.

For spontaneous breaking of supersymmetry, we need a vacuum which is not annihilated by a $Q$. The anti-commutator in (38) implies, however, that

$$\sum_{i=1,2} \|Q_i|0\|^2 + \|Q_i^*|0\|^2 = \langle 0|([Q_1^*, Q_1]_+ + [Q_2^*, Q_2]_+)|0\rangle = 4\langle 0|E|0\rangle,$$

where $Q_i$ are the components of the spinor $Q$ and $E$ is the energy operator $P^0$. This means that SuSy is spontaneously broken if and only if the vacuum energy is not zero. Accordingly, if SuSy remains unbroken down to a low energy scale, it would solve the cosmological constant problem.

Unbroken SuSy also turns out to imply that bosons and fermions always come in pairs of equal mass, because $[Q, P^2]_− = 0$. Since this is manifestly not the case, SuSy must either be spontaneously broken at some scale, or else be broken in the strict, old-fashioned sense of not really working at all. The bad news implicit in the last paragraph is that breaking supersymmetry without inducing a cosmological constant is difficult. It is, however, possible to break supergravity and yet maintain $\Lambda = 0^{22}$.

Supergravity is local supersymmetry. The gravitino, a spin 3/2 field, is the gauge field. The name “supergravity” is justified because, since $[Q, P]_+ \sim P$, gauging $P$ forces $Q$ to be gauged, and vice versa. Supergravity can be expressed as a geometric theory of curved superspace. Since the rigid SuSy transformation (39) is not quite a mere translation in superspace, the supergravity vacuum is not just flat superspace; it has no curvature, but it does have torsion. A further complication that arises is that general relativity in superspace only reduces to supergravity after one imposes several constraints on the various fields involved. There is no natural way of deriving
these constraints from superspace geometry alone; one has to know the gauge theory of local supersymmetry and choose those constraints that obtain it. Thus the superspace formalism betrays itself in the supergravity case as an often convenient construction, but not a fundamental one.
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