Physical description of the blood flow from the internal jugular vein to the right atrium of the heart: new ultrasound application perspectives

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Ikken hissatsu (拳必殺) means something like to annihilate at one blow. This document is part of a series of notes each one targeting a single goal. Each note has to annihilate at one blow!

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BACKGROUND

The brain drainage is due to the venous blood flow directed from the brain to the heart through the internal jugular veins (IJVs), epidural veins and vertebral veins (see Fig. 1). The brain-heart direction indicates that there must be a negative pressure gradient driving the blood flow (i.e. the pressure in the brain is higher than the pressure in the right atrium (RA)). The pressure, where the IJVs begin, is the residual arterial pressure and, despite its pulsatile nature, in this study it is considered to be constant.

The pressure in the RA varies according to the cardiac cycle. Its trace presents two wave peaks called a and v and two waves minima called x and y. Such waves have a precise phase relationship with the ECG waves PQRST [Applefeld(1990)], see Fig. 2. The waves a, x, v and y are also detectable at level of the neck due to the internal jugular vein (IJV) pulsation [Mackenzie(1902)].

The pressure changes generated in the RA are indeed transmitted to the IJV and affect the velocity of the blood in two ways i) modifying the pressure gradient so that it is no longer constant over time and generates a non-steady flow [Sisini et al.(2016)], [Kalmanson et al. (1972)], ii) cyclically varying the CSA of the IJV [Sisini et al.(2015)]. Since the walls of the IJVs are neither rigid nor collapsed, the pressure variations generated in the RA are transmitted to the IJV as a pressure wave with finite
propagation velocity $c$. Such pressure waves are responsible for the modulated component of the velocity of blood in the IJV. For this reason, their wave equation can be used to derive the instantaneous velocity of the blood even in the absence of a direct measurement of such parameter obtained, for example, by using an ultrasound Doppler scanner. A first attempt to this goal was presented in [Sisini et al. (2015b)], where the instantaneous velocity of the blood in the IJV was determined qualitatively, for one cardiac cycle, using the Womersley equation [Womersley (1955a), Womersley (1955b), Womersley (1955c)].

The solution of Eq.1 and the complete procedure to calculate it are explained in detail in [Sisini et al. (2015b)]. However the solution is there defined up to the multiplicative constant $c > w$. As a consequence, the instantaneous velocity trace, calculated in this way, is proportional but not equal to the actual blood velocity.

**Pressure waves propagation**

The pressure is transmitted along the direction AR-IJV according to the following equation

$$ \frac{\partial^2 w(r, z, t)}{\partial r^2} + \frac{1}{r} \frac{\partial w(r, z, t)}{\partial r} - \frac{\partial w(r, z, t)}{\partial t} = \frac{1}{\mu} \frac{\partial p(t, z)}{\partial z} \quad (1) $$

The radial component of the blood velocity and the nonlinear terms of the equation are negligible for $c >> w$. The solution of Eq.1 and the complete procedure to calculate it are explained in detail in [Sisini et al. (2015b)]. However the solution is there defined up to the multiplicative constant $c$ and up to the additive constant $w_0$. As a consequence, the instantaneous velocity trace, calculated in this way, is proportional but not equal to the actual blood velocity.

**Physical description of the RA-IJV segment**

The RA-IJV system is represented in Fig. 1 as a tube with a circular section. $G$ indicates a reference point on the right IJV. The $w(z, r, t)$ function represents the velocity of the blood in the $z$ direction. It depends on $z$ coordinate, on $r$ (the distance from the axis $z$) and on $t$ (the time). The symbol $\overline{w}(z, t)$ is used to indicate the blood velocity averaged over the CSA. The pressure at the right end of the tube ($z = 0$) is the residual arterial pressure and it is assumed to have a constant value $p_r$ while at the left end of the tube ($z = z_{RA}$) the pressure feels the effects of the RA activity varying periodically with the cardiac cycle. The pressure at $z = z_{RA}$ (see Fig. 1) is supposed to be the sum of a component $p_c$ constant in time and of a periodic component $p_a(t)$ which varies according to the atrial cardiac activity. The blood flow is due to the pressure gradient between the ends of the cylinder. It produces a velocity $\overline{w}_T(t)$ that results from a constant component $w_0$ due to the constant gradient $\frac{p_r}{L}$ and a time varying component $\overline{w}(t)$ due to the time varying $\overline{p}(t) - p_c$. This system can be described using the Womersley equation that is a linear differential equation where the unknown is the function $\overline{w}$ and the pressure gradient $(\partial p(t)/\partial z)$ is the source term:

$$ \frac{\partial^2 w(r, z, t)}{\partial r^2} + \frac{1}{r} \frac{\partial w(r, z, t)}{\partial r} - \frac{\partial w(r, z, t)}{\partial t} = \frac{1}{\mu} \frac{\partial p(t, z)}{\partial z} \quad (1) $$

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![Geometrical model of the jugular-heart segment.](image)

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Figure 2: The figure a) shows the JVP trace of pressure in the IJV, measured in G, together with the ECG trace. The interval $\Delta t_{aQP}$ is represented over the curves a and QP. In figure b) it is shown the pressure along the z-axes at instant $t_{QP}$.

Pressure in the IJV, measured in G, can be obtained simultaneously with the acquisition of the ECG trace [Sisini et al.(2016)]. On this trace, that reports both JVP and ECG waves, the time interval $\Delta t_{aQP}$ between $t_P$ and $t_a$ is given by:

$$\Delta t_{aQP} = t_a - t_{QP} = \frac{l_G}{c}$$

where $t_a$ is the instant corresponding the a wave.

The parameter $\Delta t_{aQP}$ is the time interval between the instant when the pressure is maximum at point G (instant $t_a$) and the instant when the pressure is maximum at point RA (instant $t_{QP}$).

METHODS

The applicability of the relationship expressed in Eq. 1 can be tested by comparing the instantaneous velocity ($w(t)$) of the blood in the IJV calculated according to the equation shown above, with the instantaneous blood velocity ($w_D(t)$) measured with a Doppler scanner. However, the data and the results presented here are for illustration only and are not to be considered the result of a scientific study.

Pressure waves velocity calculation

The delay $\Delta t_{aQP}$ is measured over the JVP+ECG trace as shown in Fig. 2. The distance $l_G$ is measured on the volunteer’s chest. The velocity $c$ is the ratio between $l_G$ and $\Delta t_{aQP}$.

Compliance calculation

From the Moens-Korteweg equation we obtain the compliance per unit length:

$$C' = \frac{CSA_x}{\rho c^2}$$

where $CSA_x$ is the CSA of the IJV measured in G at the x wave.

Pressure gradient calculation

The pressure inside the IJV is calculated from the instantaneous CSA as follows:

$$p(t) = \frac{1}{C'} CSA(t)$$

substituting the expression of $p(t)$ obtained above into Eq. 2 we obtain the expression for the pressure gradient:

$$\frac{\partial p}{\partial z} = \frac{1}{cC'} \frac{\partial CSA}{\partial t}$$

Flow velocity calculation $w(t, z)$

The function $w(t, z)$ is calculated by inserting into the Eq. 1 the expression for the pressure gradient found in Eq. 7. The mathematical details are given in Sisini et al.(2015b).
RESULTS EXAMPLE

Pressure waves velocity calculation

The measured delay ($\Delta t_{aQP}$) was 0.14 s and the distance $l_G$ was 23 cm. As a consequence the velocity $c$ was 164 cm/s.

Compliance calculation

The minimum value of the CSA ($CSA$) during the cardiac cycle was 0.2 cm$^2$ and the compliance for unit of length was $9.8 \times 10^{-3}$ cm$^2$/mmHg.

Flow velocity calculation $\mathbf{w}(t, z)$

The velocity was calculated following Eq. (1) and its trace is shown in Fig. 3 together with the experimental trace obtained by the Doppler examination. The two traces in agreement, nevertheless, since the velocity $\mathbf{w}(t, G + l)$ was calculated up to an additive constant, only the wave amplitude has physical meaning whereas its shift along the velocity axis is not significant.

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REFERENCES

[Applefeld(1990)] Applefeld MM. The Jugular Venous Pressure and Pulse Contour. In: Source Clinical Methods: The History, Physical, and Laboratory Examinations. Boston: Butterworths, 1990. Chapter 19.

[Beulen et al.(2011)] Beulen, B. et al. Toward noninvasive blood pressure assessment in arteries by using ultrasound. Ultrasound in Medicine; 2011: 37(5), 788–797.

[Kalmanson et al. (1972)] Kalmanson D, Veyrat C, Deraï C, Savier CH, Berkman M, Chiche P. Non-invasive technique for diagnosing atrial septal defect and assessing shunt volume using directional Doppler ultrasound. Correlations with phasic flow velocity patterns of the shunt. British Heart Journal. 1972;34:981-991.

[Mackenzie(1902)] Mackenzie J. The Study of the Pulse, Arterial, Venous, and Hepatic and of the Movements of the Heart. Edinburgh: Young J Pentland, 1902.

[Sisini et al.(2015)] Sisini F, Tessari M, Gadda G, Di Domenico G, Taibi A, Menegatti E, Gambaccini M, Zamboni P, Sisini F, Tessari M, Gadda G, Di Domenico G, Taibi A, Menegatti E, Gambaccini M, Zamboni P. An Ultrasonographic Technique to Assess the Jugular Venous Pulse: a Proof of Concept. Ultrasound Med Biol, 2015;41:1334–1341

[Sisini et al.(2015b)] Sisini F, Toro E, Gambaccini M, and Zamboni P, “The Oscillating Component of the Internal Jugular Vein Flow: The Overlooked Element of Cerebral Circulation,” Behavioural Neurology. 2015; Article ID 170756

[Sisini et al.(2016)] Sisini F, Tessari M, Menegatti E, Vannini ME, Gianesini S, Tavoni V, Gadda G, Gambaccini G, Taibi A, Zamboni P. Clinical applicability of the assessment of the jugular flow over the individual cardiac cycle compared with current ultrasound methodology. Ultrasound Med Biol, In press. DOI: 10.1016/j.ultrasmedbio.2016.03.002

[Womersley(1955a)] Womersley, J. . Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known. The Journal of Physiology, 1955;2:553–563.

[Womersley(1955b)] Womersley J R Oscillatory Flow in Arteries: the Constrained Elastic Tube as a Model of Arterial Flow and Pulse Transmission. Phys Med Biol 1957;2:178
