Unstable particles in matter at a finite temperature: the rho and omega mesons

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Unstable particles (such as the vector mesons) have an important role to play in low mass dilepton production resulting from heavy ion collisions and this has been a subject of several investigations. Yet subtleties, such as the implications of the generalization of the Breit-Wigner formula for non-zero temperature and density, e.g. the question of collisional broadening, the role of Bose enhancement, etc., the possibility of the kinematic opening (or closing) of decay channels due to environmental effects, the problem of double counting through resonant and direct contributions, are often given insufficient emphasis. The present study attempts to point out these features using the rho and omega mesons as illustrative examples. The difference between the two versions of the Vector Meson Dominance Model in the present context is also presented. Effects of non-zero temperature and density, through vector meson masses and decay widths, on dilepton spectra are studied, for concreteness within the framework of a Walecka-type model, though most of the basic issues highlighted apply to other scenarios as well.

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I. INTRODUCTION

Heavy ion collisions at high energies produce matter far above the ground state providing thereby a rich arena for the study of hot hadronic matter, possibilities of chiral symmetry restoration, transition to a quark gluon plasma etc. However, hadronic signals are generally unsuitable for the task of uncovering the information on the underlying occurrences, since the history of strongly interacting particles entail layers of complicated dynamics which mask the basic issues. As such, electromagnetic signals, as manifested through emitted photon and dilepton spectra, are relatively cleaner, since electromagnetic quanta couple but weakly to hadronic matter. Final spectra exhibit resonance structures, which, in the low mass region, include the rho and omega mesons. Consequently, details of their creation, propagation and decay in the medium (or outside) are of paramount importance for the analysis of the resultant spectra observed. The general framework for such an investigation has been provided by Weldon through a beautiful and lucid exposition on the Breit-Wigner (BW) formula at non-zero temperature and density.

Accordingly section II, devoted to the underlying principles, begins with a quick review of the main results of Weldon’s paper, followed by a brief discussion of the basic ideas of vector meson dominance (VMD) which enables a phenomenological introduction of the coupling of photons to hadrons and hence to lepton pairs. We close our presentation of basics through an outline of a Walecka-type model which is merely used, in the present context, to provide a setting wherein we can formulate the hadronic scenario allowing us to estimate the differences that can accrue if insufficient emphasis is placed on the subtleties. We go on in section III to present the necessary ingredients to evaluate the dilepton emission rate from vector meson decays and the pion annihilation process in hot and dense hadronic surroundings. The last section is devoted to a discussion of results and conclusions.

II. FORMALISM

IIa. Generalised Breit-Wigner formula for unstable particles in a thermal bath

Different species of hadrons in thermalised matter exist with equilibrium distributions determined by temperature $T$, the chemical potential $\mu$ and the statistics obeyed by that species. Consider an unstable hadron (R) in such a heat bath. If the decay products themselves are hadrons then they thermalise in the bath and no distinctive decay characteristic can possibly be discerned out. Thus we are interested in hadrons (R) that decay in the heat bath (one assumes that the collision volume can be so described) and decay into leptons and photons (described by the state
vector \( |f\rangle \), say) that escape without thermalization. With \( q \) the total four momenta of the non-hadronic final state \( |f\rangle \) the resonance peak should appear in the invariant mass \( (M) \) plot for say, the number of lepton pair events versus \( q^2 = M^2 \) at \( M = m_R \). The mass \( m_R \) of the resonance \( R \) is the mass in the heat bath, theoretical estimation of which will of course depend on the model of hadronic interactions adopted. The central result of Weldon’s paper is the generalization of the BW formula:

\[
\frac{dN_f}{d^4xd^4q} = (2J + 1) \frac{M^2}{4\pi^4} \frac{\Gamma_{\text{all} \rightarrow R} \Gamma_{\text{vac} R \rightarrow f}(M)}{M^2 - m_R^2 + (m_R \Gamma_{\text{tot}})^2}
\]  

(1)

with \( dN_f \) the number of lepton pair events, say in the space-time and four momentum element \( d^4xd^4q \), \( J \) being the spin of the resonance, \( \Gamma_{\text{all} \rightarrow R} \) is the formation width, \( \Gamma_{\text{tot}} \) is the total width and \( \Gamma_{\text{vac} R \rightarrow f}(M) \) is the partial decay width for off-shell \( R \) (i.e. of mass \( M \)) to go into the non-hadronic state \( |f\rangle \). While this result is deceptively similar to the usual BW formula, it must be realised that the thermal distribution is implicitly contained through the ‘entrance’ width \( \Gamma_{\text{all} \rightarrow R} \) as

\[
\Gamma_{\text{all} \rightarrow R} = \frac{\Gamma_{\text{tot}}}{\exp[\beta(E - \mu)]} \pm 1, 
\]  

(2)

and also through the interpretation of \( \Gamma_{\text{tot}} \) for which a bosonic resonance \( R \) (our present concern) is given by the loss minus the gain,

\[
\Gamma_{\text{tot}} = \Gamma_{R \rightarrow \text{all}} - \Gamma_{\text{all} \rightarrow R},
\]  

(3)

which is actually the rate at which particles equilibrate and relax to chemical equilibration. Here it is important to emphasize that the width of the invariant mass plots is related to the thermal damping rate \( \Gamma_{\text{tot}} \) and this result generalises collision broadening treated by Van Vleck and Weisskopf in the context of molecular spectroscopy.

Reverting back to the problem at hand it is helpful to have a rough qualitative picture of decays for which the collision broadened BW is applicable, assuming that the thermalised hadron fluid lasts for a time \( \tau_f \) (after which it freezes out). The amplitude for a hadron to survive at time \( t \) can be modeled, albeit noncovariantly in the frame of the medium (bath), by

\[
A(t) = \exp(-iE^* t - m_R^* \Gamma_{\text{tot}} t/2E^*) \quad 0 < t < \tau_f
\]

\[
\exp(-iE t - m_R \Gamma_{\text{vac}} t/2E) \quad \tau_f < t < \infty
\]  

(4)

where \( E \) and \( m_R \) denote the energy and mass of the unstable particle and the asterisks represent the same quantities as modified by the medium. Particles for which \( \Gamma_{\text{vac}} \tau_f >> 1 \), the relevant portion of its history is from the early period \( (0 < t < \tau_f) \) as it damps out at later times, and hence medium modifications determine its properties. On the other hand for particles with \( \Gamma_{\text{vac}} \tau_f << 1 \) and \( 1 > \tau_f \Gamma_{\text{tot}} >> \tau_f \Gamma_{\text{vac}} \) it is the second term that dominates and in such cases the thermal effects on the mass and width are negligible.

Another noteworthy feature is the fact that the decay width \( \Gamma_{\text{vac} R \rightarrow f}(M) \), to be evaluated for an off-shell \( R \), occurs in the formula and not \( \Gamma_{\text{vac} R \rightarrow f}(M = m_R) \) though of course the point \( M = m_R \) gets weighted most heavily (at the peak) due to the occurrence of the ‘Breit-Wigner’ denominator. However, for broad resonances this aspect does lead to some discernible differences as shall be illustrated later.

Lastly, it is necessary to re-emphasize that for a particle which is sufficiently short lived to decay within the medium, the width of the dilepton spectra is actually the rate at which it equilibrates \( (\Gamma_{\text{tot}} = \Gamma_{R \rightarrow \text{all}} - \Gamma_{\text{all} \rightarrow R}) \), involving in principle various processes in which \( R \) participates. However it may be observed that elastic scattering does not enter into \( \Gamma_{\text{tot}} \) as it cancels, contributing equally as it does to \( \Gamma_{R \rightarrow \text{all}} \) and \( \Gamma_{\text{all} \rightarrow R} \). This is however, in contradiction to earlier observations made in this context. It may be borne in mind that although the elastic scattering contributes to the kinetic equilibrium it has no direct effect on the chemical equilibrium of the system while of course such elastic processes are of importance in phenomena such as viscosity etc. Indeed elastic scattering changes the momentum of the colliding particles but the nature of the particles remains unaltered and hence this process does not contribute to the decay life time in the bath.

III. Vector Meson Dominance

The second important element in our framework is to have a robust phenomenological description of the electromagnetism of hadrons (in particular rho and omega mesons) as this shall enter into the decay channels of interest.
Such a setting provided, for instance, by the Vector Meson Dominance (VMD) model proposed by Nambu and developed by Sakurai [3], which assumes that the photon interacts with physical hadrons through vector mesons. Thus the cross section for the process \( \pi^+ \pi^- \rightarrow l^+ l^- \) (where \( l = e \) or \( \mu \)) will involve the coupling of the photon to the pion which is expressed in terms of the pion form factor \( F_\pi(q^2) \) occurring in the matrix element of the electromagnetic current between pion states \( \langle \pi(p) | j_\mu | \pi(p') \rangle = F_\pi(q^2) (p'^\mu - p^\mu) \), where the four momentum transfer is \( q = p' - p \). While the photon can couple directly to the pion through its electromagnetic current \( eJ_\mu = ie(\pi^- \partial_\mu \pi^+ - \pi^+ \partial_\mu \pi^-) \), the photon can also couple to the pion through a vector meson, which in this case, must also be an isovector. This is taken to be the rho meson. Based on such notions Sakurai enunciated the VMD model which has two formulations often referred to as VMD1 and VMD2 [7]. The photon and isovector meson part of the effective Lagrangian in the first representation is

\[
L_{\text{VMD}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} \rho^{\mu\nu} \rho_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - g_{\rho\pi\pi} \rho_\mu J_\mu - \frac{e}{2 g_\rho} F^{\mu\nu} \rho_{\mu\nu},
\]

where, \( \rho^{\mu\nu} \) is the field tensor for the rho field constructed analogously to the electromagnetic field tensor \( F^{\mu\nu} \), \( g_{\rho\pi\pi} \) may be determined from the decay \( \rho \rightarrow \pi \pi \), and \( g_\rho \) from fits to the process \( e^+ e^- \rightarrow \pi^+ \pi^- \). Thus here we have a direct photon-matter coupling as well as a photon-rho coupling which vanishes at \( q^2 = 0 \) due to the occurrence of the derivatives in the last term above. This leads, after provision is made for the finite width of the unstable rho (emanating ostensibly from the imaginary part of the pion loop in the rho self energy), to the following expression for the rho dominated pion form factor,

\[
F_\pi^{\text{VMD1}}(q^2) = 1 - q^2 \frac{g_{\rho\pi\pi}}{g_\rho} \frac{1}{q^2 - m_\rho^2 + i m_\rho \Gamma_\rho}.
\]

Note that the charge normalization constraint \( F_\pi(q^2 = 0) = 1 \) is automatically built in. Furthermore, it may be remarked that often a gauge-like argument is advanced to the effect that rho couples universally (with the same strength) to all hadrons. However, the experimental fits reveal that universality is not exact; and indeed one finds

\[
\frac{g_{\rho\pi\pi}}{g_\rho} = 1 + \epsilon,
\]

with \( \epsilon = 0.2 \) [3]. Sakurai also outlined an alternative formulation (VMD2), which though not as elegant as the first (having for instance a photon mass-term in the Lagrangian), enjoys considerable popularity. Here

\[
L_{\text{VMD}2} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} \rho^{\mu\nu} \rho_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - g_{\rho\pi\pi} \rho_\mu J_\mu - \frac{e m_\rho^2}{g_\rho} \rho^\mu A_\mu + \frac{e^2 m_\rho^2}{2 g_\rho^2} A_\mu A_\mu,
\]

and accordingly

\[
\rho - \gamma \text{ vertex} = -\frac{i e m_\rho^2}{g_\rho}.
\]

Furthermore

\[
F_\pi^{\text{VMD2}}(q^2) = \frac{m_\rho^2}{q^2 - m_\rho^2 + i m_\rho \Gamma_\rho(q^2)} \frac{g_{\rho\pi\pi}}{g_\rho},
\]

where

\[
\Gamma_\rho(q^2) = \Gamma_\rho(m_\rho^2) \frac{(q^2 - 4 m_\rho^2)^{3/2}}{m_\rho} \frac{m_\rho^2}{M} \Theta(q^2 - 4 m_\rho^2),
\]

and in order to maintain the condition \( F_\pi(q^2 = 0) = 1 \), it is necessary here to impose the universality condition viz. \( g_{\rho\pi\pi} = g_\rho \), whereas in VMD1 the charge normalization constraint is automatically maintained. Insertion of a momentum dependent width for the unstable vector meson in the VMD2 form factor maintains the condition \( F_\pi(q^2 = 0) = 1 \). The momentum dependence originates on the one hand from the condition of the lowest mass state into which the rho meson can decay namely into two pions (hence the Heaviside theta function), and on the other hand due to the p-wave decay (therefore, the third power of the three momentum). We prefer VMD1 for reasons to be given later, the relevant results for VMD2 are also discussed as this version is used by several authors.
to the isoscalar part of the electromagnetic interactions of hadrons this shall analogously be taken to be dominated by the isoscalar vector meson $\omega$. The relevant part of the effective Lagrangian density involving $\omega$ is taken to be
\[
\mathcal{L}_\omega^{\text{relevant}} = \frac{e m^2_\omega}{g_\omega} \omega^\mu A_\mu + \frac{g_\omega \pi}{m_\pi} \epsilon_{\mu\nu\rho} \partial^\mu \omega^\nu \rho^\rho \cdot \pi.
\] (12)

The coupling of the omega to the photon (coefficient of $\omega^\mu A_\mu$ above) has long been considered to be approximately one third of that for the rho to the photon (coefficient of $\rho^\mu A_\mu$ in Eq. (8)), which yields reasonable agreement with the ratio of the observed partial widths $\Gamma(\rho \rightarrow e^+ e^-)/\Gamma(\omega \rightarrow e^+ e^-)$. Furthermore this is also supported by a recent QCD based study [13]. The coupling $g_\omega \pi$ may be determined by using this term to calculate the observed $\omega \rightarrow \pi^0 \gamma$ decay through the use of the rho-photon coupling already introduced in Eq. (9). However, the zero-ranged $\omega - 3\pi$ vertex can also be obtained from the Lagrangian [14],
\[
\mathcal{L}_{\omega \gamma \pi} = f_\omega \epsilon_{\mu\nu\rho} \omega^\mu \epsilon^{ijk} \pi_i \partial^\nu \pi_j \partial^\rho \pi_k,
\] (13)

the latin indices referring to isospin.

IIc. The Walecka model and vector mesons in hot and dense hadronic matter

The third ingredient needed, in order to discuss the characteristics of the rho and omega mesons in hadronic matter at a finite temperature ($T$) and density ($n_B$) is to have an underlying model. Temperatures in the range $\sim 150 - 200$ MeV and/or baryon densities $n_B$ a few times nuclear matter density are of relevance. As a result the study of hadronic interactions leading to changes in their masses and decay widths under such conditions assumes great significance. Various investigations have addressed this issue over the past several years. Hatsuda and collaborators [12] and Brown [13] have used the QCD sum rules at finite temperature and density to study the effective masses of the hadrons. Brown and Rho [14] also argued that requiring chiral symmetry (in particular addressing the QCD trace anomaly) yields an approximate scaling relation between various effective hadron masses, which implies, that all hadronic masses decrease with temperature. The gauged linear sigma model [15], however, shows the opposite trend, i.e. $m^*_\rho$ increasing with temperature. In the present study we choose a Walecka-type model to be delineated below to provide the medium effects on the vector meson essentially for the sake of illustration. Many of our remarks and subtleties shall apply in some aspects to other scenario as well.

The relevant interaction Lagrangian in the Walecka model, which we have considered, and comprising of the iso-scalar sigma, the rho, the omega and the nucleon, is given by,
\[
\mathcal{L}_{\text{int}}^{\text{relevant}} = g_\sigma \bar{N} \phi_\sigma N - g_\rho NN \left( \bar{N} \gamma_\mu \gamma_5 N - i \frac{\kappa_\rho}{2m_N} \bar{N} \gamma_\mu \gamma_5 N \gamma_\nu \gamma_5 \right) \cdot \rho^\nu - g_\omega NN \bar{N} \gamma_\mu N \omega^\mu.
\] (14)

It may be observed that the rho has been taken to couple both minimally and through the Pauli tensor coupling to the nucleon. We shall adopt the value $m_\sigma = 450$ MeV for the sigma mass, and the coupling constants shall be taken to be $g_\rho NN = 10$, $g_\omega NN \sim 2.6$, $\kappa_\rho \sim 6.1$, $\kappa_\omega = 0$ and $g_\omega \sim 7.4$, chosen so as to reproduce the saturation density and the binding energy per nucleon in nuclear matter [16].

In the mean field approximation where the sigma meson field operator is replaced by its (classical) ground state expectation value $\langle \phi_\sigma \rangle \neq 0$ the Lagrangian (14) immediately yields a medium dependent reduction in the nucleon mass. This is calculated from the nucleon propagator modified by a tadpole with a nucleon loop as its head and its tail emerging from the propagating nucleon line. Here we also include in the Relativistic Hartree Approximation (RHA), the properly renormalized contribution to the baryon self energy from the Dirac sea as well. This leads [17] to a substantial reduction in the nucleon mass ($m^*_N$) in the medium [18]. It is in this setting that we consider the vector mesons.

To compute in-medium meson propagators one solves Dyson’s equation by essentially summing an infinite geometric series whose common ratio is the lowest order proper polarization which comprises of the nucleon loop (with in-medium mass $m^*_N$) containing both the particle-hole (Fermi sea) and nucleon-antinucleon (Dirac sea) contributions. The effective mass of the vector meson ($m^*_V$) in nuclear matter is obtained by finding the value of energy $q_0$ going to the limit $q \to 0$ for which the imaginary part of the propagator attains its maximum or equivalently by solving the full dispersion relation
\[
q_0^2 - q^2 - m^2_V + \text{Re} \Pi^D_{\omega \gamma \pi}(q_0, q) + \text{Re} \Pi^F(q^2) = 0,
\] (15)
in the limit $\mathbf{q} \to 0$ in the region where $\text{Im} \Pi = 0$. $\Pi_L^{(T)}$ are the longitudinal (transverse) components of the in-medium self energy for thermal nucleon loop. $\Pi_F$ is the vacuum self energy of the vector meson with modified nucleon mass due to sigma tadpole diagram. The expressions for $\Pi_L^{(T)}$ and $\Pi_F$ are given in the appendix of Ref. [18].

It has been shown earlier [18,19] that the change in the rho mass due to rho pion interaction is negligibly small in the rest frame of the medium. In a different model calculation, it has been shown by Klingl et al [20] that to leading order in density the shift in the rho mass is very small. Therefore the change in rho meson mass due to $\rho - \pi - \pi$ interaction is neglected here. However, at non-zero density the in-medium modification in the spectral function of the rho meson was studied [21–23] by including the medium effects in the internal nucleon loop in the rho and omega self energy, i.e. the internal nucleon loop in the rho and omega self energy are modified due to tadpole diagram only. The inclusion of vertex corrections and modification of the pion propagator due to delta-nucleon hole excitation will take us beyond MFT and hence are not considered here for the sake of self consistency. Moreover, we do not include the delta baryon in the present work, as we have simply adopted a particular model for the sake of illustration.

### III. DILEPTON PRODUCTION

The dilepton production rate due to processes occurring in a thermalised hadronic environment is obtained by folding the in-medium cross-section with the thermal distribution of the participants. In this article we consider dilepton production from pion annihilation ($\pi^+\pi^- \rightarrow e^+e^-$) and the rho decay ($\rho \rightarrow e^+e^-$). The thermal production rate per unit four-volume for lepton pairs is related to the imaginary part of the one-particle irreducible photon self energy by [24,25],

$$\frac{dR}{d^4q} = \frac{\alpha g^{\mu\nu} \text{Im} \Pi_{\mu\nu}(q)}{12\pi^4 q^2 (e^{2m_\pi q_0} - 1)},$$

where, $q^\mu = (q_0, \mathbf{q})$ is the four momentum of the virtual photon and $\Pi_{\mu\nu}(q)$ is the one particle irreducible photon self energy. In the low invariant mass region, the pion annihilation channel is known to be the dominant one. The invariant mass distribution of the lepton pair in the case of pion annihilation is given by,

$$\frac{dR}{dM} = \frac{M^3}{2(2\pi)^4} (1 - 4m_\pi^2 / M^2) \int M_T dM_T dy \sigma(q_0, \mathbf{q}) \exp(-M_T \cosh y / T)$$

(17)

Here, $q_0 = M_T \cosh y$, $|\mathbf{q}| = \sqrt{q_0^2 - M^2}$; $\sigma(q_0, \mathbf{q})$ is the cross-section for the pion annihilation calculated using VMD1 and VMD2 described by Eqs. (5) and (8) with appropriate modifications at finite temperature and density and is given by,

$$\sigma(q_0, \mathbf{q}) = \frac{4 \pi \alpha^2}{3M^2} \sqrt{1 - 4m_\pi^2 / M^2} \sqrt{1 - 4m_\pi^2 / M^2 (1 + 2m_\pi^2 / M^2)} |F_\pi(q_0, \mathbf{q})|^2.$$

(18)

The $q_0$ and $\mathbf{q}$ dependence of the pion form factors in the case of VMD1 and VMD2 originate from the real and imaginary part of the rho self energy in the medium. In the rest frame of the rho meson, Eq. (17) reduces to the well known form

$$\frac{dR}{dM} = \frac{\sigma(M)}{(2\pi)^4} M^4 T \sum_n K_1(nM/T) (1 - 4m_\pi^2 / M^2),$$

(19)

where $K_1$ is the modified Bessel function, and $M$ is the invariant mass of the lepton pair.

In the same way, the invariant mass distribution of lepton pairs from the decays of vector mesons propagating with a momentum $\mathbf{q}$ is obtained using Eq.(16), as

$$\frac{dR}{dM} = \frac{2J + 1}{2\pi^3} M \int M_T dM_T dy \exp(-M_T \cosh y) \times \frac{\omega \Gamma_{\text{tot}}}{(M^2 - m_\nu^2 + \text{Re}\Pi)^2 + \omega^2 \Gamma_{\text{tot}}^2} \omega \Gamma^{\nu \rightarrow e^+e^-}_{\nu \rightarrow e^+e^-}.$$  

(20)

In the rest frame of the vector meson this reduces to
where the elastic channels involving the omega were also included. However, due to reasons mentioned earlier, we have due to which the depletion rate of the omega increases substantially. This issue has also been discussed by Haglin [6].

So we have adopted the usual approach of keeping the pion mass constant. This is simply a consequence of the Nambu-Goldstone theorem in a medium [27]. So we have adopted the usual approach of keeping the pion mass constant. On the other hand, in models with chiral symmetry such as for instance the Nambu-Jona-Lasinio model, and the linear sigma model with nucleons, it is known that the pion mass is almost unchanged as long as it is in the Nambu-Goldstone phase. This is simply a consequence of the Nambu-Goldstone theorem in a medium [27].

From Fig. (2) it is clear that the decay rate of a particle propagating in a medium with momentum \( q \) can be obtained from its rest frame value by Lorentz transformation only. This is because of the preferred frame of the medium. For a rho meson propagating in a medium with energy \( \omega \) this is given by

\[
\Pi_T - \Pi_L = \frac{2 g_{\rho NN}^2}{\pi^2} \left(1 - \frac{q^2}{2M} \right)^2 \int \frac{k^2 \, d\kappa \, (\cos \theta)}{\sqrt{k^2 + M^2}} \left[ f_D + \bar{f}_D \right] \times \left[ \frac{u \cos^2 \theta - v \cos \theta + w}{C + 8q_0q_0|k| q \cos \theta - 4k^2 q^2 \cos^2 \theta} \right]
\]

where,

\[ u = 3q_0^2k^2 - q^2k^2 \ ; \ v = 4q_0k_0|k|q \ ; \ w = 2k_0^2q^2 + q^2k^2 - q_0^2k_0 \] \& \[ C = q^4 - q_0^2k_0^2 \] \& \[ f_D(\bar{f}_D) \] is the fermi distribution for nucleons (antinucleons). The imaginary part of the rho self energy is related to the probability of its survival in a medium. For a rho meson propagating in a medium with energy \( \omega \) this is given by

\[
\Gamma(\omega) = \frac{g_{\rho NN}^2}{48\pi} W^3(s) \frac{2T}{\omega} \ln \left\{ \frac{1 - \exp\left[\frac{-\omega}{2}(\omega + W(s)\sqrt{\omega^2 - s})\right]}{1 - \exp\left[\frac{-\omega}{2}(\omega - W(s)\sqrt{\omega^2 - s})\right]} \right\}
\]

where \( s = q^2 = \omega^2 - q^2 \) and \( W(s) = \sqrt{1 - 4m_N^2/s} \). In the limit \( |q| \to 0 \), the above expression reduces to the in-medium decay width (with the decaying particle at rest in the medium)

\[
\Gamma_{\rho \to \pi \pi} = \frac{g_{\rho NN}^2}{48\pi} \omega W^3(\omega) \left[ \left(1 + f(\frac{\omega}{2})\right) \left(1 + f(\frac{\omega}{2})\right) - f(\frac{\omega}{2})f(\frac{\omega}{2}) \right]
\]

It is interesting to contrast the rate given by Eq. (23) with the one obtained purely from considerations of relativistic time dilation from equation (24). In this case the decay rate of an unstable particle in a general frame is given by

\[
\Gamma_{\rho \to \pi \pi}^{\text{Lor}} = \frac{\sqrt{s}}{\omega} \Gamma_{\rho \to \pi \pi}^{\text{restframe}}
\]

From Fig. (3) it is clear that the decay rate of a particle propagating in a medium with momentum \( k \) cannot be obtained from its rest frame value by Lorentz transformation only. This is because of the preferred frame of the medium.

It is well known that the naive Walecka model does not possess chiral symmetry, therefore, it is rather difficult to predict anything reliable on the pion mass. On the other hand, in models with chiral symmetry such as for instance the Nambu-Jona-Lasinio model, and the linear sigma model with nucleons, it is known that the pion mass is almost unchanged as long as it is in the Nambu-Goldstone phase. This is simply a consequence of the Nambu-Goldstone theorem in a medium [27].

The most significant process which contributes to the broadening of the omega in the thermal bath is \( \omega \pi \leftrightarrow \pi \pi \), due to which the depletion rate of the omega increases substantially. This issue has also been discussed by Haglin [8] where the elastic channels involving the omega were also included. However, due to reasons mentioned earlier, we have

\[
\frac{dR}{dM} = \frac{2J + 1}{\pi^2} M^2 T \sum_n K_1(nM/T) \times \frac{m_V^4 \Gamma_{\text{tot}}/\pi}{(M^2 - m_V^2)^2 + m_V^4 \Gamma_{\text{tot}}^2} m_V^4 \Gamma_{\nu \to e^+ e^-},
\]

where \( \Gamma_{\text{tot}} \) is defined by Eq. (3), and \( \Gamma_{\nu \to e^+ e^-} \) is the partial width for the leptonic decay mode for the off-shell vector particles.

IV. RESULTS AND DISCUSSIONS

Within the ambit of the hadronic model adopted by us, the effect of finite temperature \( T \) and density \( n_B \) on the self energies of the vector mesons reveals that the mass of the rho meson \( (m_\rho) \) decreases more rapidly with increasing \( T \) and/or \( n_B \) than that of the omega \( (m_\omega) \) as shown in Fig. (4). One observes a small difference between the longitudinal(L) and transverse(T) modes in case of the omega meson but in case of the rho this splitting is negligible (attributable to the smaller vector coupling constant). We have observed that the quantity \( q_0^2 - q^2 \) along the dispersion curve remains almost constant(\( \sim m_\pi^2 \)) which is defined as \( q_0^2 \) at \( q = 0 \) on the mass hyperboloid). This means that a simple pole approximation of the rho and omega propagator at \( k^2 = m_V^2 \) is good enough for our calculations. The splitting between the transverse and longitudinal components of the self energy of vector mesons with both vector and tensor interactions can be shown to be (see also ref. [26]),

\[
\Pi_T - \Pi_L = \frac{2 g_{\rho NN}^2}{\pi^2} \left(1 - \frac{q_0^2}{2M} \right)^2 \int \frac{k^2 \, d\kappa \, (\cos \theta)}{\sqrt{k^2 + M^2}} \left[ f_D + \bar{f}_D \right] \times \left[ \frac{u \cos^2 \theta - v \cos \theta + w}{C + 8q_0q_0|k| q \cos \theta - 4k^2 q^2 \cos^2 \theta} \right]
\]

where,

\[ u = 3q_0^2k^2 - q^2k^2 \ ; \ v = 4q_0k_0|k|q \ ; \ w = 2k_0^2q^2 + q^2k^2 - q_0^2k_0 \] \& \[ C = q^4 - q_0^2k_0^2 \] \& \[ f_D(\bar{f}_D) \] is the fermi distribution for nucleons (antinucleons). The imaginary part of the rho self energy is related to the probability of its survival in a medium. For a rho meson propagating in a medium with energy \( \omega \) this is given by

\[
\Gamma(\omega) = \frac{g_{\rho NN}^2}{48\pi} W^3(s) \frac{2T}{\omega} \ln \left\{ \frac{1 - \exp\left[\frac{-\omega}{2}(\omega + W(s)\sqrt{\omega^2 - s})\right]}{1 - \exp\left[\frac{-\omega}{2}(\omega - W(s)\sqrt{\omega^2 - s})\right]} \right\}
\]

where \( s = q^2 = \omega^2 - q^2 \) and \( W(s) = \sqrt{1 - 4m_N^2/s} \). In the limit \( |q| \to 0 \), the above expression reduces to the in-medium decay width (with the decaying particle at rest in the medium)

\[
\Gamma_{\rho \to \pi \pi} = \frac{g_{\rho NN}^2}{48\pi} \omega W^3(\omega) \left[ \left(1 + f(\frac{\omega}{2})\right) \left(1 + f(\frac{\omega}{2})\right) - f(\frac{\omega}{2})f(\frac{\omega}{2}) \right]
\]

It is interesting to contrast the rate given by Eq. (23) with the one obtained purely from considerations of relativistic time dilation from equation (24). In this case the decay rate of an unstable particle in a general frame is given by

\[
\Gamma_{\rho \to \pi \pi}^{\text{Lor}} = \frac{\sqrt{s}}{\omega} \Gamma_{\rho \to \pi \pi}^{\text{restframe}}
\]
only considered the above (inelastic) channel to study the in-medium depletion of omega. Due to this process alone
the decay rate of the omega comes out to be \( \sim 80 \text{ MeV} \). We now focus on an interesting possibility which becomes
realisable (when \( m_\omega^* > m_\rho^* + m_\pi^* \)), in that the decay \( \omega \rightarrow \rho \pi \), which is closed in free space (as \( m_\omega < m_\rho + m_\pi \)),
becomes an open channel. Thus whereas the omega meson which in usual circumstances decays through the three
particle channel \( \omega \rightarrow 3\pi \), could, given the appropriate environment, decay by the two particle (\( \rho \pi \)) mode. Under
these conditions the in-medium width for \( \omega \rightarrow \rho \pi \) is readily found by applying the finite temperature cutting rules
to yield

\[
\Gamma_{\omega \rightarrow \rho \pi} = \frac{g_{\omega \pi \rho}^2}{32\pi m_\omega^3 m_\rho^2 m_\pi^2} \lambda^{3/2}(m_\omega^2, m_\rho^2, m_\pi^2) \left[ 1 + f(E_\pi) + f(E_\rho) \right],
\]

where \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx) \), arises from the phase space considerations, while \( f \) is the Bose-
Einstein distribution for the pions and the rho mesons in equilibrium. The coupling constant \( g_{\omega \pi \rho} \sim 2 \) appearing
in the expression \( \Gamma_{\omega \rightarrow \rho \pi} \) can be deduced from the observed decay \( \omega \rightarrow \pi^0 \gamma \), using the vector dominance model
of Sakurai \[3\] for the \( \rho \gamma \) vertex, taking the process to occur through a virtual rho converting to the photon. The
concomitant three body process, \( \omega \rightarrow 3\pi \), is estimated from the phenomenological effective term in the Lagrangian
shown in Eq. (13), averting for this purpose the Gell-Mann-Sharp-Wagner \[9\] model where the decay proceeds through
a virtual rho \( \omega \rightarrow [\rho \pi] \rightarrow \pi \pi \pi \), in order to avoid the possibility of double counting when the threshold for the
two body decay is crossed. In view of the above discussion one must include the processes \( \omega \leftrightarrow 3\pi, \omega \leftrightarrow \rho \pi \)
and \( \omega \pi \rightarrow \pi \pi \) as these are the most important among many other possible processes which can contribute to the
broadening of omega in the medium. Taking all these ramifications into account the resulting width as a function
of temperature at zero baryon density and at normal nuclear density (\( n_0^B \)) is depicted in Fig. (3). We have noticed
that the two body channel (\( \omega \rightarrow \rho \pi \)) opens up in the former case at a temperature \( \sim 190 \text{ MeV} \), while in the latter
situation (normal nuclear densities) this channel remains open even at zero temperature. We observe a value of \( \sim 86 \text{ MeV} \)
for \( \Gamma_\omega \) at a temperature of 200 MeV and normal nuclear density. As a result the lifetime of the omega reduces to
\( \sim 2.3 \text{ fm/c} \) which should be compared to that of rho (\( \sim 2.1 \text{ fm/c} \) under the same condition. This is in contradiction
to the commonly held notion that the omega is too long-lived \[28\] to convey any information on the fire ball in heavy
ion collisions.

While pointing out a potential phenomenon interesting in itself, we go on to emphasize the need in general to
consider the possibility of the opening (or closing) of channels due to the effects of finite temperature or non-zero
chemical potential. Modification of both the mass and width of the thermal omega due to the inclusion of anomalous
interactions has also been discussed by Pisarski \[15,29\].

FIG. 1. Transverse and longitudinal dispersion relations of rho and omega mesons. The solid and dashed curves pertains to
the transverse and longitudinal modes respectively.
FIG. 2. Decay rate of rho meson as a function of three momentum. Solid and dashed lines correspond to the in-medium width [Eq. (23)] and the boosted width [Eq. (25)] respectively.

FIG. 3. In-medium decay rate of omega meson comprising the processes $\omega \to \rho \pi$, $\omega \to 3\pi$ and $\omega \pi \to \pi \pi$ at $n_B = n_B^0$ (dashed line) and $n_B = 0$ (solid line) as a function of temperature.

It has been emphasized earlier that both the forward and backward processes should contribute to the probability of propagation of an unstable particle in a medium. This is manifested through the phase space factors which appear in the evaluation of such a quantity (which is neglected by some authors e.g. [6]). In this respect we discuss the term $[1 + f(E_\pi) + f(E_\rho)] = (1 + f(E_\pi)) (1 + f(E_\rho)) - f(E_\rho) f(E_\pi)$, which occurs in the expression for the in-medium $\omega \leftrightarrow \rho \pi$ width given by Eq. (26). This appears naturally in a calculation based on finite temperature field theory [30]. Its physical significance resides in Bose-Enhancement (BE), which implies that the decay rate would increase because of stimulated emission in a gas already containing the decay products in equilibrium. Indeed the significant effect of this feature on the dilepton spectra has been demonstrated in a previous calculation [18]. This enhancement mechanism, of course, operates also for the $\rho \to \pi \pi$, $\omega \pi \to \pi \pi$ and $\omega \to 3\pi$ decays and has been incorporated in our calculations of decay width.
FIG. 4. Invariant mass distribution of lepton pairs from $\rho \rightarrow e^+ e^-$. The almost overlapping solid and dashed lines correspond to transverse and longitudinal rho with non zero three momentum. Dot-dashed line indicates the yield in the rest frame of the rho.

However, as mentioned earlier, the hadronic decay modes of rho and omega in the fireball are not very informative and it is through their leptonic decay modes that they become experimentally `visible'. Therefore, it is more relevant to examine the dilepton emission rate from rho in the medium using Eq. (23) which incorporates the generalised in-medium BW formula. Here we focus our attention on $\Gamma_{\text{tot}}$. It is re-emphasized that though elastic processes such as $\rho \pi \leftrightarrow \rho \pi$ are there, they do not contribute to $\Gamma_{\text{tot}}$. Indeed it should be borne in mind that though elastic collisions contribute to kinetic equilibration they do not contribute to the approach to chemical equilibrium, as indicated by $\Gamma_{\text{tot}} = \Gamma_{R\rightarrow all} - \Gamma_{all\rightarrow R}$. For the case of the rho meson the processes $\rho \leftrightarrow \pi \pi$ and $\rho \pi \leftrightarrow \omega$ are considered for the evaluation of $\Gamma_{\text{tot}}$ for rho. From phase space considerations it is clear that the mode $\rho \rightarrow \pi \pi$ contributes dominantly to $\Gamma_{\text{tot}}$. The resulting dilepton spectra from the decay of rho meson is shown in Fig. (4) at $T=200$ MeV and normal nuclear matter density. The notable feature here is the large shift of the rho peak towards lower invariant mass. This is due to the huge reduction in its mass ($m_\rho^* \sim 430$ MeV). The solid and dashed curve show the resulting invariant mass distribution of the lepton pair from the decay of a rho propagating with momentum $q$ in the thermal bath. The dot-dashed curve indicates the results when the decay $\rho \rightarrow e^+ e^-$ is considered in its rest frame. A broader distribution in this case originates from the larger width of the rho in its rest frame as depicted in Fig. (2). However, it has been observed that the effect of collisional broadening due to $\rho \pi \leftrightarrow \omega$ on the dilepton yield from rho decay is insignificant.

FIG. 5. Invariant mass distribution of lepton pairs from $\rho \rightarrow e^+ e^-$ at $T = 200$ MeV and $n_B = n_B^0$. Solid and dashed lines correspond to the case when $\Gamma_{\text{vac}}^{\rho \rightarrow e^+ e^-}$ is evaluated for off-shell and on-shell rho (with non-zero $q$) respectively.

We pass on to the discussion of the effect of off-shellness of the broad rho resonance on its dilepton decay mode as exemplified by the occurrence of $\Gamma_{\text{vac}}^{\rho \rightarrow e^+ e^-}(M)$ in Eq. (1) and again in Eq. (20) as contrasted width $\Gamma_{\text{vac}}^{\rho \rightarrow e^+ e^-}(m_\rho^*)$. Of course, in the narrow resonance limit when the BW structure reduces to a delta function peaked at $m_\rho^*$, this effect is irrelevant. In Fig. (5) the solid curve indicates the dilepton yield when $\Gamma_{\text{vac}}^{\rho \rightarrow e^+ e^-}$ is evaluated at $M$ (off-shell). The
on-shell result (dashed curve) shows a marked difference away from the rho peak (at the rho peak, of course, they must coincide). The off-shellness in $\Gamma_{\text{vac}}^{\pi^+\pi^- \to e^+e^-}$ is calculated in the framework of VMD1. It is relevant to remark here that the in-medium $\gamma-\rho$ vertex is taken as $e\rho^2/\rho\pi\pi$ in VMD2 in the work of Li et al. We devote this paragraph to the process $\pi^+\pi^- \to e^+e^-$, which could proceed through a photon coupled directly to the charge of the pion (ignoring its structure) and for $q^2 \neq 0$ it would begin to see its structure which is modeled here by the intermediary rho meson. This separation is clearly manifested in VMD1 as can be seen from Eq. (6) where the former mechanism is expressed through the occurrence of unity and latter exhibited by the rho-pole term. In VMD2, however, this feature is not at all manifest. A comparison of VMD1 and VMD2 results vis-a-vis dilepton yield from pion annihilation at temperature 200 MeV with $n_B = 0$ and $n_B = 2n_B^0$ is shown in Fig. (6) (the momentum dependence of the rho decay width is neglected here). The yield in the two cases is similar near the rho peak because the form factors around $q^2 = M^2 \sim m^*_\rho^2$ become quite similar. However away from the peak $M > m^*_\rho$ the dilepton yield in the case of VMD1 dominates. For the reasons explained here and also in the introduction we have used VMD1 to evaluate the dilepton yield from pion annihilation and vector meson decays.

Finally in order to see in what way the different issues considered above affect the low mass dilepton spectra, we plot in Fig. (7) the invariant mass distribution of lepton pairs from $\rho \to e^+e^-$ and $\pi\pi \to e^+e^-$. We have used VMD1 to evaluate the contribution from pion annihilation.

In this work we have investigated the in-medium effects on dilepton production from pion annihilation and from the decay of unstable particles such as the rho meson (taken for illustration). Subtleties arising due to the presence of a thermal bath, generalisation of the BW formula in the medium, collisional broadening and possibilities of double counting have been discussed. The variation of effective masses and decay widths of nucleons and vector mesons at
non-zero temperature and baryon density have been calculated within the framework of Walecka model. The BE effect in the decay width and the reduction in the mass of the rho meson is found to affect the dilepton emission rate quite substantially in the low invariant mass region. Moreover, in the presence of nuclear matter at finite temperature a substantial number of omega mesons could decay inside the reaction volume and thus can act as a viable probe for hot hadronic matter formed in relativistic heavy ion collisions. A comparison of the two versions of the vector dominance model has also been presented.

Detailed measurement of photoproduction of lepton pairs should provide invaluable insights into the formation, propagation and decay of vector mesons inside the nuclear medium. Changes in the rho (and also omega) masses would reflect directly in the dilepton invariant mass spectrum due to the quantum interference between rho and omega mediated processes in the photoproduction of lepton pairs (CEBAF Experiment). CERES collaboration [32] has also planned to upgrade their experiment to improve the mass resolution such that rho and omega may be disentangled. Various aspects of the subtleties mentioned above on the observables with the inclusion of space time dynamics are under study, and are important for careful analysis of such experiments.

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