Large-$N_c$ operator analysis of hyperon-nucleon interactions in SU(3) chiral effective field theory

Xuyang Liu, 1 Viroj Limkaisang, 2 Daris Samart, 2, 3, 4, ∗ and Yupeng Yan 5, 3

1 School of Mathematics and Physics, Bohai University, Liaoning, 121013, China
2 Department of Applied Physics, Faculty of Sciences and Liberal arts, Rajamangala University of Technology Isan, Nakhon Ratchasima, 30000, Thailand
3 Center Of Excellent in High Energy Physics & Astrophysics, Suranaree University of Technology, Nakhon Ratchasima, 30000, Thailand
4 Advanced Materials and Renewable Energy Research Unit, Rajamangala University of Technology Isan, Nakhon Ratchasima, 30000, Thailand
5 School of Physics, Suranaree University of Technology, Nakhon Ratchasima, 30000, Thailand

(Dated: October 30, 2017)

We study octet-octet baryon ($J^P = \frac{1}{2}^+$) contact interactions in SU(3) chiral effective field theory by using large-$N_c$ operator analysis. Applying the $1/N_c$ expansion of the Hartee Hamiltonian, we find 15 operators in the octet-octet baryon potential where 4 operators for leading order (LO) and 11 for and net-to-next-to-leading order (NNLO). The large-$N_c$ operator analysis of octet-octet baryon matrix elements reduces the number of free parameters from 15 to 6 at LO of the $1/N_c$ expansion. The application of large-$N_c$ sum rules to the Jülich model of hyperon-nucleon (YN) interactions at the LO of the chiral expansion reduces the model parameters to 3 from 5 at the LO of $1/N_c$ expansion. We find that the values of LECs fitted to YN scattering data in Ref. 20 in the relativistic covariant ChEFT (EG) approach is more consistent with the predictions of large-$N_c$ than in the heavy baryon (HB) formalism approach.

PACS numbers:

I. INTRODUCTION

Chiral effective field theory (ChEFT) 1, 2, based on the approximately and spontaneously broken chiral symmetry of QCD, allows for a systematic way of calculating low-energy hadronic observables. It is very efficient and convenient to use hadrons as basic degrees of freedom rather than quarks and gluons in the ChEFT. Chiral Lagrangian is required to include all possible interactions between hadrons which are constructed in terms of the relevant symmetries of QCD 3. A number of low-energy properties in the strong interaction is very successfully described by using the ChEFT. The ChEFT is also utilized to shed light on the study of nuclear forces (see 4, 5 for reviews). It was demonstrated by Weinberg’s seminal works 6, 7 that one can calculate the nuclear forces systematically by using appropriate power counting scheme. Therefore, loop-corrections and higher order terms can be included for the accuracy of the calculations. Nucleon-nucleon (NN) forces derived in the ChEFT successfully described a huge number of NN experimental data. The NN potentials are composed of the long and short range interactions, where the long range NN force is mainly contributed by the pion exchange while the short range part is encoded by contact term NN interactions with unknown low-energy constants (LECs) to be fitted to experimental data. The higher order contact terms of the NN potentials have been constructed in Refs. 8, 9 at next-to leading order (NLO) and in Refs. 10, 11 for next-to-next-to-leading order (N 3 LO) in terms of chiral expansions.

On the other hand, hyperon-nucleon (YN) and hyperon-hyperon (YY) forces have been less studied compared with the NN forces. YN interactions are keys for understanding hyper-nuclei and neutron stars 12, 13. The contact and meson exchange terms of the YN interactions in the ChEFT were constructed by using the SU(3) flavor symmetry in Ref. 14 at leading order (LO) and extended to NLO in Ref. 15. The most general SU(3) chiral Lagrangians of the octet-octet baryon contact term interactions have been worked out in Ref. 14. The study of the YY interactions was performed in Refs. 17, 19. At the LO of the YN interactions 14, 20, the SU(3) chiral Lagrangian has 15 free parameters (LECs) and the partial-wave expansion analysis leads to 5 LECs which are fixed with YN data. In this work, we will use the large-$N_c$ operator analysis to explore the $N_c$ scales and reduce the number of the unknown LECs in the SU(3) chiral Lagrangians and in the LO YN potential 14, 20.

*Electronic address: daris.sa@rmuti.ac.th
Large-$N_c$ is an approximate framework of QCD and very useful in the study of hadrons at low-energies. The basic idea is that one can consider the number of colors ($N_c$) to be large and expand it in power of $1/N_c$ [21, 22]. By using this framework, a number of simplifications of QCD occurs in the large-$N_c$ limit (see Refs. [23, 24] for reviews). The $1/N_c$ expansion of QCD for the baryon [25, 26] has been applied to the NN potential in [28, 30] and three-nucleon potential in [31]. Moreover, the $1/N_c$ expansion is used to study parity-violating NN potentials in [32, 33] as well as time-reversal violating NN potentials [34]. The study of the large-$N_c$ analysis in the NN system provides the understanding of the $N_c$ scales of the LECs in the NN forces. In addition, the $1/N_c$ expansion also helps us to reduce the independent number of the LECs [33]. However, the octet-octet baryon interactions in SU(3) flavor symmetry have not been investigated in the large-$N_c$ approach. In this work, we will extend the large-$N_c$ operator analysis in Refs. [29, 31] to the SU(3) chiral Lagrangian in Refs. [14, 20]. The large-$N_c$ octet-octet baryon potential is constructed up to NNLO in terms of the $1/N_c$ expansion. We will apply large-$N_c$ sum rules to YN interactions at LO which has been recently investigated in Ref. [21]. Moreover, the results can be applied to the YN at NLO and YY sector.

We outline this work as follows: In section 2 we will setup the matrix elements of the octet-octet baryon potential from the SU(3) chiral Lagrangian. In the next section, the potential of the $1/N_c$ expansion is constructed up to NNLO and large-$N_c$ sum rules for LECs are implied. In section 4, we apply results of the large-$N_c$ sum rules to the LO YY potential. In the last section, we give the conclusion in this work.

II. THE POTENTIAL OF THE SU(3) OCTET-OCTET BARYON CONTACT TERM INTERACTIONS

We start with the SU(3) chiral Lagrangian of the octet-octet baryon interactions and it was proposed by Ref. [14]. The SU(3)-flavor symmetry is imposed and the chiral Lagrangian is Hermitian and invariant under Lorentz transformations and the CPT discrete symmetry is implied. The minimal SU(3) invariant chiral Lagrangian with non-derivative is given by,

\[
\mathcal{L}^{(1)} = C_1 \langle B_1 B_2 \Gamma_1 B_2 (\Gamma_1 B_1) \rangle ,
\]

\[
\mathcal{L}^{(2)} = C_2 \langle B_1 (\Gamma_1 B_1 B_2 (\Gamma_1 B_2) \rangle ,
\]

\[
\mathcal{L}^{(3)} = C_3 \langle B_1 (\Gamma_1 B_1) B_2 (\Gamma_1 B_2) \rangle .
\]

Here 1 and 2 denote the label of the particles in the scattering process, the $B$ is the usual irreducible octet representation of SU(3) given by

\[
B = \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda^a B^a = \left( \begin{array}{ccc}
\frac{\sqrt{3}}{3} + \frac{\delta}{\sqrt{6}} & \frac{\Sigma^+}{\sqrt{6}} + \frac{\delta}{\sqrt{6}} & p \\
- \Sigma^- & \frac{-\Sigma^-}{\sqrt{6}} + \frac{\delta}{\sqrt{6}} & n \\
 & & -\frac{\delta}{\sqrt{6}}
\end{array} \right),
\]

where the $\langle \cdots \rangle$ brackets denote taking the trace in the three-dimensional flavor space and the normalization of Gell-Mann matrices $\langle \lambda^a \lambda^b \rangle = 2 \delta^{ab}$ is used. The $\Gamma_i$ are the usual elements of the Clifford algebra

\[
\Gamma_1 = 1, \quad \Gamma_2 = \gamma^\mu, \quad \Gamma_3 = \sigma^{\mu \nu}, \quad \Gamma_4 = \gamma^\mu \gamma_5, \quad \Gamma_5 = i \gamma_5 .
\]

By using the chiral power counting in Ref. [14], it has been shown that we have 15 LO non-derivative terms of the chiral Lagrangian. It has also been demonstrated in Ref. [14] that the above Lagrangians are the minimal set of the contact interaction terms in terms of flavor and spin structures by using Cayley-Hamilton identity and Fierz transformation.

To obtain the potentials, we follow approach in Refs. [37, 38] by imposing relativistic covariant constraints. Letting $\mathcal{H} = -\mathcal{L}$ and taking the approach of the relativistic constraints in [37, 38] into account, one obtains the potential of the octet-octet baryon contact interactions up to the second order of the small momenta of the baryons and it reads,

\[
V^{(1)} = \langle \bar{\chi}_2 d \gamma_1 c | \mathcal{H}^{(1)} | a, \chi_1, b, \chi_2 \rangle
\]

\[
= \left\{ \frac{1}{3} \delta^{cd} \delta^{ba} + \frac{1}{2} (d^{dca} + if^{dca}) (d^{eba} + if^{eba}) \right\}
\]

\[
\times \left\{ c_1^{(1)} \bar{S} + c_1^{(1)} \bar{T} + c_2^{(1)} p_\perp + c_2^{(1)} p_\perp \right\} \delta_{\chi_1 \chi_2} \delta_{\bar{\chi}_1 \bar{\chi}_2} + \left( c_1^{(1)} p_\perp + c_4^{(1)} p_\perp \right) \bar{\sigma}_1 \cdot \bar{\sigma}_2
\]

\[
+ c_5^{(1)} \frac{1}{2} (\bar{\sigma}_1 + \bar{\sigma}_2) \cdot (p_\perp \times p_\perp) + c_6^{(1)} (p_\perp \cdot \bar{\sigma}_1) (p_\perp \cdot \bar{\sigma}_2) + c_7^{(1)} (p_\perp \cdot \bar{\sigma}_1) (p_\perp \cdot \bar{\sigma}_2) \right\},
\]

(4)
where
\[
\hat{O}_S = \delta_{\chi_1 \chi_1} \delta_{\chi_2 \chi_2} + \frac{i}{2M^2} (\vec{p}_+ \times \vec{p}_-) \cdot (\vec{\sigma}_1 - \vec{\sigma}_2),
\]
\[
\hat{O}_T = \vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{i}{2M^2} (\vec{p}_+ \times \vec{p}_-) \cdot (\vec{\sigma}_1 - \vec{\sigma}_2),
\]
and \(\vec{\sigma}_i \equiv \vec{\sigma}_{\chi_i}\) with \(i = 1, 2\). The indices \(a (c), b (d), \chi_1 (\chi_1)\) and \(\chi_2 (\chi_2)\) are flavor and spin indices of incoming (outgoing) baryon number 1 and 2 respectively and \(M\) is the octet baryon mass in the SU(3) flavor symmetry limit. We note that the octet-octet baryon potentials agree with the heavy baryons formulation of ChEFT in Refs. [39, 40] for the spin structures. By using the partial integrations and the baryon equation of motion to eliminate time derivative as shown in Refs. [37, 38], the potential in Eq. (4) is the minimal set of linearly independent operators and it consists of 2 LO and 7 NLO operators (see Appendix A for the detail derivation of the potential). The LECs, \(c_i^{(1)}\) are linear combinations of the couplings \(C_i^{(1)}\) as,
\[
c_S^{(1)} = C_1^{(1)} + C_2^{(1)}, \quad c_T^{(1)} = C_3^{(1)} - C_4^{(1)}, \quad c_1^{(1)} = -\frac{1}{4M^2} (C_2^{(1)} + C_3^{(1)}), \quad c_2^{(1)} = -\frac{1}{2M^2} (C_1^{(1)} - C_2^{(1)}), \quad c_3^{(1)} = -\frac{1}{4M^2} (C_1^{(1)} + C_3^{(1)}), \quad c_4^{(1)} = \frac{1}{4M^2} (C_3^{(1)} - C_4^{(1)}), \quad c_5^{(1)} = -\frac{1}{2M^2} (C_1^{(1)} - 3C_2^{(1)} - 3C_3^{(1)} - C_4^{(1)}), \quad c_6^{(1)} = \frac{1}{4M^2} (C_2^{(1)} + C_3^{(1)} + C_4^{(1)} + C_5^{(1)}), \quad c_7^{(1)} = -\frac{1}{4M^2} (C_3^{(1)} + C_4^{(1)}).
\]
In addition, it is worth to discuss about the chiral power counting \((Q/M)\) where a \(Q\) is typical three momentum of the baryon. If we impose \(M \sim \Lambda\) where \(\Lambda\) is a chiral symmetry breaking scale. Therefore, our power counting rule in this work adopts \(Q/M \sim (Q/\Lambda)^2\) which has been used in Refs. [9, 10] for the NN potentials. The notations of the momentum in this work are defined below
\[
\vec{p}_+ = \frac{1}{2} (\vec{p}' + \vec{p}), \quad \vec{p}_+^2 = \vec{p}_+ \cdot \vec{p}_+, \quad \vec{p}_- = \vec{p}' - \vec{p}, \quad \vec{p}_-^2 = \vec{p}_- \cdot \vec{p}_-, \quad \vec{n} = \vec{p} \times \vec{p}' = \vec{p}_+ \times \vec{p}_-,
\]
where \(\vec{p} (\vec{p}')\) is incoming (outgoing) three-momentum in the c.m. frame and the on-shell condition of the external momenta is given by
\[
\vec{p}_+ \cdot \vec{p}_- = 0.
\]

With the same manner, the octet-octet baryon potentials for \(C_i^{(2)}\) and \(C_i^{(3)}\) are written by
\[
V^{(2)} = \langle \bar{\chi}_2, d; \chi_1, c | H^{(2)} | a, \chi_1 ; b, \chi_2 \rangle
= \frac{1}{3} \delta^{ab} \delta^{cd} + \frac{1}{2} \left( d^{ac} + i f^{ac} \right) (d^{db} + i f^{db}) \left\{ c_S^{(2)} \hat{O}_S + c_T^{(2)} \hat{O}_T + \left( c_1^{(2)} p_+^2 + c_2^{(2)} p_+^2 + c_3^{(2)} p_-^2 + c_4^{(2)} p_-^2 \right) \delta_{\chi_1 \chi_1} \delta_{\chi_2 \chi_2} + \left( c_3^{(2)} p_-^2 + c_4^{(2)} p_-^2 \right) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\
+ c_5^{(2)} \frac{i}{2} \left( \vec{\sigma}_1 + \vec{\sigma}_2 \right) \cdot \left( \vec{p}_+ \times \vec{p}_- \right) + c_6^{(2)} \left( \vec{p}_- \cdot \vec{\sigma}_1 \right) \left( \vec{p}_- \cdot \vec{\sigma}_2 \right) + c_7^{(2)} \left( \vec{p}_+ \cdot \vec{\sigma}_1 \right) \left( \vec{p}_+ \cdot \vec{\sigma}_2 \right) \right\},
\]
and
\[
V^{(3)} = \langle \bar{\chi}_2, d; \chi_1, c | H^{(3)} | a, \chi_1 ; b, \chi_2 \rangle
= \delta^{ab} \delta^{cd} \left\{ c_S^{(3)} \hat{O}_S + c_T^{(3)} \hat{O}_T + \left( c_1^{(3)} p_+^2 + c_2^{(3)} p_+^2 + c_3^{(3)} p_-^2 + c_4^{(3)} p_-^2 \right) \delta_{\chi_1 \chi_1} \delta_{\chi_2 \chi_2} + \left( c_3^{(3)} p_-^2 + c_4^{(3)} p_-^2 \right) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\
+ c_5^{(3)} \frac{i}{2} \left( \vec{\sigma}_1 + \vec{\sigma}_2 \right) \cdot \left( \vec{p}_+ \times \vec{p}_- \right) + c_6^{(3)} \left( \vec{p}_- \cdot \vec{\sigma}_1 \right) \left( \vec{p}_- \cdot \vec{\sigma}_2 \right) + c_7^{(3)} \left( \vec{p}_+ \cdot \vec{\sigma}_1 \right) \left( \vec{p}_+ \cdot \vec{\sigma}_2 \right) \right\},
\]
where the LECs in Eqs. (9) and (10) are the linear combinations of the couplings as in Eq. (4) by replacing \(c_i^{(1)} \rightarrow c_i^{(2,3)}\) and \(C_i^{(1)} \rightarrow C_i^{(2,3)}\). By using relativistic reductions as in Refs. [37, 38], we obtain the minimal set of the SU(3) octet-octet baryon potentials and there are 27 operators totally. Moreover, Fierz identities for the Gell-mann matrices \(\lambda^a\) are also taken into account for the calculations of the potentials in Eqs. (4), (9) and (10). We found that there is no the redundant terms of the SU(3) flavor structures. We obtain 6 and 21 operators at LO and NLO of the small momentum scale expansion \((Q/M)\). At the LO, the operators from the couplings \(C_{1,2,3,4}^{(1,2,3)}\) enter to contribute the potential but
the couplings $C_5^{(1,2,3)}$ start at NLO. We will reduce the independent number of the LECs of the SU(3) octet-octet baryon interactions in the ChEFT by using the large-$N_c$ operator analysis in the next section.

III. THE $1/N_c$ OPERATOR PRODUCT EXPANSION ANALYSIS OF THE TWO-BARYON MATRIX ELEMENTS

A. The $1/N_c$ expansion octet-octet baryon ansatz

In this section, we are going to study the $1/N_c$ expansion for the octet-octet baryon matrix elements. According to Witten’s conjecture \[22\], the matrix elements of baryon-baryon scattering should scale like $N_c$, i.e. \[28, 29\],

$$N_c \langle B_1 | \hat{O}_1^i | B_1 \rangle \langle B_2 | \hat{O}_2^i | B_2 \rangle,$$

where $\hat{O}_1^i$ and $\hat{O}_2^i$ operators are the $i$- and $i'$-quark current operators on the first and the second baryon. It has proven in the Ref. \[27\] that the matrix elements for one baryon in SU(3) flavor symmetry has the $N_c$ scaling as,

$$\langle B_i | \hat{O}_j^i | B_j \rangle \lesssim N_c^0,$$

with $j = 1, 2$. This holds for the matrix elements of the second baryon as well. One can expand the matrix elements in terms of effective quark operator and effective spin-flavor baryon states in $1/N_c$ expansion as \[21\] \[27\]. The $O'^r$ are the $r$-body operators which comprises of the effective quark operators \[28, 29\],

$$\left( \frac{O}{N_c} \right)^r = \left( \frac{J}{N_c} \right)^I \left( \frac{T}{N_c} \right)^m \left( \frac{G}{N_c} \right)^n, \text{ with } l + m + n = r. \quad (14)$$

The operators $J$, $T$ and $G$ are spin, flavor and spin-flavor operators, respectively and they are defined by \[20\] \[33\],

$$1 = q^\dagger (1 \otimes 1) q, \quad J_i = q^\dagger \left( \frac{\sigma_i}{2} \otimes 1 \right) q, \quad T^a = q^\dagger \left( 1 \otimes \frac{\lambda_a}{2} \right) q, \quad G^a_i = q^\dagger \left( \frac{\sigma_i}{2} \otimes \frac{\lambda_a}{2} \right) q,$$

where $q$ and $q^\dagger$ are quark annihilation and creation operators respectively. According to the fully antisymmetry and Fermi statistics of the SU($N_c$) color group, the spin and flavor of baryonic ground state of the $N_c$ quarks have to be completely symmetric representation. Therefore one can consider quark operators $q$ and $q^\dagger$ as bosonic operators with the commutation relation $[q, q^\dagger] = 1$. The $N_c$ scaling of the $r$-body operator $O'^r$ and the the coefficient $c^{(i)}$ scale like \[28, 29\],

$$\langle B | O'^r | B \rangle \lesssim N_c^r, \quad c^{(i)} \sim N_c^0. \quad (16)$$

In addition, The one-baryon matrix elements of the operators $J$, $T$ and $G$ in SU(3) flavor symmetry framework have $N_c$ scaling in the following way \[20\],

$$\langle B | J^i | B \rangle \sim N_c^0, \quad \langle B | 1 | B \rangle \sim N_c, \quad \langle B | T^a | B \rangle \lesssim N_c, \quad \langle B | G^a | B \rangle \lesssim N_c. \quad (17)$$

In contrast to the SU(2) flavor symmetry, there is only one operator that can suppress rising of the $N_c$ for one-baryon matrix elements i.e. the $J$ whereas all the rest of the effective operators rises the $N_c$ factor. However, the symbol, $\lesssim$ is used for saturating the maximum of the $N_c$ scaling for the $\langle B | T^a | B \rangle$ and $\langle B | G^a | B \rangle$ because the matrix elements of the $T^a$ operator scales like $N_c^0$ for $a = 1, 2, 3$, but as $\sqrt{N_c}$ when $a = 4, 5, 6, 7$ and as $N_c$ when $a = 8$. On the other hand, the matrix elements of the $G^a$ scales like $N_c$ for $a = 1, 2, 3$, as $\sqrt{N_c}$ when $a = 4, 5, 6, 7$ and as $N_c^0$ when $a = 8$ \[20\]. These are the differences of the effective operators between SU(2) and SU(3) flavor symmetries. Moreover, it is
worth to discuss about the $N_c$ scaling of the external momentum variables. Here we consider all momenta in c.m. frame as we discussed in the previous section. One recalls the $N_c$ scaling of the momentum variables in \( \text{Eq. (7)} \), it reads \( p_+ \sim 1/N_c, \quad p_- \sim N_0^0 \).

In a meson exchange picture, the $p_+$ can only appear in the baryon-baryon potential as a relativistic correction (i.e., a velocity dependent term). Therefore, the $p_+$ always come with the factor $1/M$. Since $M \sim N_c$, this gives $p_+ \sim 1/N_c$ (for more detail discussions see \( [29, 31, 33] \)). The baryon-baryon potential in terms of $1/N_c$ takes the following form, it takes the following form, \( \hat{H} = N_c \sum_r \sum_{lm} c_{r,lm} \left( \frac{J}{N_c} \right)^l \left( \frac{T}{N_c} \right)^m \left( \frac{G}{N_c} \right)^{r-l-m} \), (19)

where again the $c_{r,lm}$ coefficient function has scale $N_0^0$. It is well known that, at the large-$N_c$ limit, the spin-1/2 and 3/2 baryons are degeneracy states. In this work, we project the Hamiltonian $\hat{H}$ to the octet (spin-1/2) baryon sector only. This has been discussed extensively in \( [29] \). We will construct the Hamiltonian in order of $1/N_c$ expansion. Then the leading-order (LO) is given by

\[ \hat{H}_{\text{LO}} = U_1^{\text{LO}}(p_+^2) \cdot 1 \cdot 1_2 + U_2^{\text{LO}}(p_+^2) \cdot T_1 \cdot T_2 + U_3^{\text{LO}}(p_+^2) \cdot G_1 \cdot G_2 + U_4^{\text{LO}}(p_+^2) \cdot (\sigma_1^i \cdot \sigma_2^j \cdot \delta_{ij} A \cdot B), \]

and then

\[ (p_+^2 \cdot \sigma_1^i) \cdot \text{(18)} \]

In this work, we terminate the $1/N_c$ expansion at the $1/N_c^2$ order. Then, the octet-octet baryon Hamiltonian at NNLO takes the following form,

\[ \hat{H}_{\text{NNLO}} = U_1^{\text{NNLO}}(p_+^2) \cdot 1 \cdot 1_2 + U_2^{\text{NNLO}}(p_+^2) \cdot J_1 \cdot J_2 + U_3^{\text{NNLO}}(p_+^2) \cdot J_1 \cdot J_2 \cdot T_1 \cdot T_2 + U_4^{\text{NNLO}}(p_+^2) \cdot J_1 \cdot J_2 \cdot T_1 \cdot T_2 + U_5^{\text{NNLO}}(p_+^2) \cdot G_1 \cdot G_2 + U_6^{\text{NNLO}}(p_+^2) \cdot (\sigma_1^i \cdot \sigma_2^j \cdot \delta_{ij} A \cdot B), \]

and then

\[ (p_+^2 \cdot \sigma_1^i) \cdot \text{(19)} \]

Here the $1/N_c$ scale factor is implied on each effective operator, $1$, $J$, $T$ and $G$ implicitly. The functions $U_1^{\text{LO}}(p_+^2)$ and $U_1^{\text{NNLO}}(p_+^2)$ have $N_0^0$ scale. Noting that there are no $p_+^2 \cdot J_1 \cdot J_2$ and $(p_+^2 \cdot p_+^i) \cdot (J_1 J_2^j)$ structures because these operators have a further suppression in order $1/N_c^4$.

Let’s us discuss comparisons between the octet-octet baryon potential and the nucleon-nucleon potential in the $1/N_c$ expansion. In the case of the SU(3) flavor symmetry, we find addition operator $T_1 \cdot T_2$ at LO instead of NNLO because \( T^8 T^8 / N_c \sim N_c \) while there is no such operator in nucleon-nucleon potential. Superficially, the two-body operator, \( T^8 G^a / N_c \) should scale like $N_c$ by using the $N_c$ scale counting rules in \( \text{Eq. (7)} \). But if we consider the operator more carefully then we find \( T^8 G^a / N_c \sim N_c \) because \( T^{4,6,\cdots} G^{5,3,\cdots} / N_c \sim T^{4,6,\cdots} G^{4,5,6,\cdots} / N_c \sim T^8 G^8 / N_c \sim N^0_0 \). Surprisingly, the SU(3) octet-octet potential has the same structures as the nucleon-nucleon potential in SU(2) flavor symmetry i.e. there is no NLO term in the $1/N_c$ expansion. The extension of the flavor symmetry from SU(2) to SU(3) does not change the profile of the $1/N_c$ potential. Before closing this section, we would like to summarize the $1/N_c$ expansion octet-octet baryon Hamiltonian. There are 4 LO operators. At the NNLO of $1/N_c$ expansion, we obtain 11 operators. We totally have 15 operators of $1/N_c$ expansion for octet-octet baryon potential.
where \( \Lambda/N \) and the 1\ freedom in this work. Before matching operators, we make ansatz for the arbitrary functions

\[ V = (\chi_2, d; \chi_1, c | \hat{H} | a, \chi_1; b, \chi_2), \]

(24)

where \( a (c), b (d), \chi_1 (\bar{\chi}_1) \) and \( \chi_2 (\bar{\chi}_2) \) are flavor and spin indices of incoming (outgoing) baryon number 1 and 2 respectively. After that we will do matching the octet-octet baryon potential and 1/\( N_c \) operator product expansion to correlate the LECs from the chiral Lagrangian in Eq. (1). First of all, we recall the action of the effective operators on the effective baryon states at \( N_c = 3 \) as

\[ 1 \left| a, \chi \right> = 3 \left| a, \bar{\chi} \right>, \]

\[ J_i \left| a, \chi \right> = \frac{1}{2} \delta_{ii} \left| a, \chi \right>, \]

\[ T^a \left| b, \chi \right> = i f^{bca} \left| c, \chi \right>, \]

\[ G^a \left| b, \chi \right> = \frac{1}{2} (d^{bca} + i f^{bca}) \left| c, \chi \right> + \cdots, \]

(25)

where \( \cdots \) stands for a relevant structure of spin-\( \frac{1}{2} \) baryons \[35\] but we do not consider the spin-\( \frac{1}{2} \) baryons degree of freedom in this work. Before matching operators, we make ansatz for the arbitrary functions \( U^{LO}_i \) and \( U^{NLO}_i \) that they are

\[ U^{LO}_i (p^2) = g_i, \quad U^{NLO}_i (p^2) = h_i. \]

(26)

Using Eq. (25) in Eqs. (20) and (23), the potential in terms of the large-\( N_c \) operators at the LO is given by,

\[ V^{LO} = - g_1 \delta_{\chi_1,1} \delta_{\chi_2,2} \Delta^{ace} \delta^{bde} + g_2 i 2 f^{ace} f^{bde} \delta_{\chi_1,1} \delta_{\chi_2,2} + g_3 \delta_{1} \cdot \bar{\delta}_{2} \left( \frac{1}{2} d^{ace} + \frac{i}{3} f^{ace} \right) \left( \frac{1}{2} d^{bde} + \frac{i}{3} f^{bde} \right), \]

(27)

and at the NNLO of the 1/\( N_c \) expansion takes form,

\[ V^{NNLO} = - g_1 h_1 p_1^2 \delta_{\chi_1,1} \delta_{\chi_2,2} \Delta^{ace} \delta^{bde} + \frac{1}{4} h_2 \delta_{1} \cdot \bar{\delta}_{2} \Delta^{ace} \delta^{bde} + \frac{1}{4} h_3 \delta_{1} \cdot \bar{\delta}_{2} i 2 f^{ace} f^{bde} + h_4 p_1^2 i 2 f^{ace} f^{bde} \delta_{\chi_1,1} \delta_{\chi_2,2} \]

\[ + h_5 p_1^2 \delta_{1} \cdot \bar{\delta}_{2} \left( \frac{1}{2} d^{ace} + \frac{i}{3} f^{ace} \right) \left( \frac{1}{2} d^{bde} + \frac{i}{3} f^{bde} \right) + \frac{3}{2} i h_6 \left( \bar{p}_1^2 \cdot \bar{\delta}_{1} \cdot \bar{\delta}_{2} \cdot \delta^{ace} \delta^{bde} \right) \]

\[ + \frac{1}{2} h_7 \left( \bar{p}_1^2 \cdot \bar{\delta}_{1} \cdot \bar{\delta}_{2} \cdot \delta^{ace} \delta^{bde} \right) \left( \frac{1}{2} d^{ace} + \frac{i}{3} f^{ace} \right) \left( \frac{1}{2} d^{bde} + \frac{i}{3} f^{bde} \right). \]

(28)

We note that the \( N_c \) scales of the above potentials are \( V^{LO} \sim N_c \) and \( V^{NNLO} \sim N_{c}^{-1} \).

By using Eqs. (4), (11), (31), (27) and (28), the \( N_c \) scaling relations of the LECs can be extracted,

\[ C^{(1)}_{1,2} \sim C^{(2)}_{1,2} \sim C^{(3)}_{1,2} \sim N_c, \quad C^{(1)}_{3,4,5} \sim C^{(2)}_{3,4,5} \sim C^{(3)}_{3,4,5} \sim N_{c}^{-1}, \]

(29)

where \( \Lambda \sim N_{c}^{0} \) \[29,32,33\] is implied. Note that the couplings \( C^{(1)}_{1,2,3}, C^{(2)}_{1,2,3,4,5}, C^{(3)}_{1,2,3} \) are LO of order \( N_c \) while the \( N_c \) scaling of the \( C^{(1)}_{3,4,5}, C^{(2)}_{3,4,5} \) and \( C^{(3)}_{3,4,5} \) are further suppressed by order \( 1/N_{c}^{2} \). We found that there is no NLO of the LECs in the 1/\( N_c \) expansion.

Matching the spin and flavor structures between the octet-octet baryon potential of the SU(3) chiral Lagrangian and the 1/\( N_c \) expansion up to NNLO, the large-\( N_c \) operator analysis leads to the relations between the LECs of the SU(3) baryon contact interaction and we find the following results,

\[ C^{(2)}_{1} = C^{(1)}_{1} + g_2 - 4 h_4 \Lambda^2, \]
\[ C_2^{(2)} = C_2^{(1)} + g_2 + 4 h_4 \Lambda^2, \]

\[ C_3^{(2)} = C_3^{(1)} - \frac{1}{2} g_2 + \frac{1}{8} h_3 - 4 h_4 \Lambda^2 + 2 h_+ \Lambda^2, \]

\[ C_4^{(2)} = C_4^{(1)} - \frac{1}{2} g_2 - \frac{1}{8} h_3 - 4 h_4 \Lambda^2 + 2 h_+ \Lambda^2, \]

\[ C_5^{(2)} = C_5^{(1)} + \frac{1}{4} h_3 + 4 h_4 \Lambda^2 - 4 h_+ \Lambda^2 + 2 h_{10} \Lambda^2, \]

\[ C_1^{(3)} = \frac{1}{3} C_1^{(1)} + \frac{9}{2} g_1 - \frac{1}{3} g_2 - 18 h_1 \Lambda^2 + \frac{4}{3} h_4 \Lambda^2, \]

\[ C_2^{(3)} = \frac{1}{3} C_2^{(1)} + \frac{1}{3} g_1 - \frac{1}{3} g_2 + 18 h_1 \Lambda^2 - \frac{4}{3} h_4 \Lambda^2, \]

\[ C_3^{(3)} = \frac{1}{3} C_3^{(1)} - \frac{9}{4} g_1 + \frac{1}{6} g_2 - 18 h_1 \Lambda^2 + \frac{1}{16} h_2 - \frac{1}{24} h_3 + \frac{4}{3} h_4 \Lambda^2 + \frac{3}{2} h_6 \Lambda^2 - \frac{2}{3} h_+ \Lambda^2, \]

\[ C_4^{(3)} = \frac{1}{3} C_4^{(1)} - \frac{9}{4} g_1 + \frac{1}{6} g_2 - 18 h_1 \Lambda^2 - \frac{3}{16} h_2 + \frac{1}{8} h_3 + \frac{4}{3} h_4 \Lambda^2 + \frac{3}{2} h_6 \Lambda^2 - \frac{2}{3} h_+ \Lambda^2, \]

\[ C_5^{(3)} = \frac{1}{3} C_5^{(1)} + \frac{1}{8} h_2 - \frac{1}{12} h_3 - \frac{4}{3} h_4 \Lambda^2 - 3 h_6 \Lambda^2 + 3 h_9 \Lambda^2 + \frac{4}{3} h_+ \Lambda^2 - \frac{2}{3} h_{10} \Lambda^2, \]

where \( h_+ = 2 h_7/3 + 3 h_8 \). Note that the Jacobi identities for the \( f \) and \( d \) symbols,

\[ f^{abc} f^{eed} + f^{bec} f^{ead} + f^{cae} f^{ebd} = 0, \]

\[ d^{abc} f^{eed} + d^{bec} f^{ead} + d^{cae} f^{ebd} = 0 \]

have been used in the matching procedure.

To the LO contributions of the \( 1/N_c \) expansion, one can reduce the number of the free parameters with \( O(1/N_c^2) \) \( \equiv h_i \) corrections. 9 sum rules of the LECs of the SU(3) octet-octet baryon contact interactions in the ChEFT are derived

\[ C_1^{(1)} = C_1^{(2)} = -3 C_1^{(3)} - 2 C_4^{(2)} + 6 C_4^{(3)}, \quad C_2^{(1)} = C_2^{(2)} = -3 C_2^{(3)} - 2 C_4^{(2)} - 6 C_4^{(3)}, \]

\[ C_3^{(1)} = C_3^{(2)} = -3 C_3^{(3)} + C_4^{(2)} + 3 C_4^{(3)}, \quad C_4^{(1)} = C_4^{(2)} = -3 C_3^{(3)} - 3 C_4^{(2)} + 6 C_4^{(3)} - 3 C_5^{(3)}. \]

We find that there are 6 free parameters of the SU(3) octet-octet baryon contact interactions in the ChEFT from the large-\( N_c \) operator analysis. At \( N_c = 3 \), these sum rules are held up to corrections of the \( 1/N_c^2 \approx 10\% \) approximately. In order to see the application of the 9 large-\( N_c \) sum rules, we will apply our results to YN interactions in next section.

## IV. APPLICATION OF THE LARGE-\( N_c \) SUM RULES TO THE JÜLICH HYPERON-NUCLEON CONTACT INTERACTIONS AT THE LO

In this section, we will apply the large-\( N_c \) sum rules to the Jülich hyperon-nucleon contact interactions at LO [14]. The LO contact terms of the chiral Lagrangians in Eq. (1) with the large component of the baryon spinors have 6 free parameters. They read, [14],

\[ C_S^{(1)}, \quad C_S^{(2)}, \quad C_S^{(3)}, \quad C_T^{(1)}, \quad C_T^{(2)}, \quad C_T^{(3)}. \]

The \( C_{S,T}^{(1,2,3)} \) are linear combinations of the coupling constants in Eq. (1) as

\[ C_S^{(1,2,3)} = C_S^{(1,2,3)} + C_2^{(1,2,3)} + C_T^{(1,2,3)} = C_S^{(1,2,3)} + C_T^{(1,2,3)}. \]

The operator from the couplings, \( C_5^{(1,2,3)} \) does not contribute to the YN potentials at the LO of the chiral expansion. Applying the large-\( N_c \) sum rules in Eq. (32), we find 3 sum rules i.e.,

\[ C_S^{(1)} = C_S^{(2)} = C_T^{(1)} = C_T^{(2)} = C_T^{(3)} = -3 C_T^{(3)}. \]

Above sum rules give only 3 free parameters and the \( N_c \) scalings of those parameters are given by

\[ C_S^{(1,2,3)} \sim N_c, \quad C_T^{(1,2,3)} \sim N_c^{-1}. \]
parameters are read only 5 parameters (potentials) which are used to fit the experimental data of the hyperon-nucleon scattering. The hyperon-nucleon partial wave potentials at LO have been constructed and studied in Ref. [14] and Eq. (35) are useful for calculating the partial wave potentials at the LO in the chiral expansion of the hyperon-nucleon scattering.

It is interesting to note that the Jülich model of the LO hyperon-nucleon potentials are written in terms of the couplings $C^{1,2,3}$ in Eq. (39) agree with the NN case [29, 31]. The sum rules in Eq. (35) are useful for calculating the partial wave potentials at the LO in the chiral expansion of the hyperon-nucleon scattering. The authors of the Ref. [14] find that there are only 5 parameters (potentials) which are used to fit the experimental data of the hyperon-nucleon scattering. The parameters are read

$$C^{\Lambda\Lambda}_{150} \equiv V^{\Lambda\Lambda}_{150}, \quad C^{\Sigma\Sigma}_{3451} \equiv V^{\Sigma\Sigma}_{3451}, \quad C^{\Sigma\Sigma}_{150} \equiv V^{\Sigma\Sigma}_{150}, \quad C^{\Lambda\Lambda}_{3451} \equiv V^{\Lambda\Lambda}_{3451}, \quad C^{\Lambda\Lambda}_{3451} \equiv V^{\Lambda\Lambda}_{3451},$$

where the Jülich model of the LO hyperon-nucleon potentials are written in terms of the couplings $C^{1,2,3}$ in the following forms [14],

$$V^{\Lambda\Lambda}_{150} = 4\pi \left[ \frac{1}{6} \left( C^{(1)}_{\Lambda} - 3C^{(1)}_{T} \right) + \frac{5}{3} \left( C^{(2)}_{\Lambda} - 3C^{(2)}_{T} \right) + 2 \left( C^{(3)}_{\Lambda} - 3C^{(3)}_{T} \right) \right],$$

$$V^{\Lambda\Lambda}_{3451} = 4\pi \left[ \frac{3}{2} \left( C^{(1)}_{\Lambda} + C^{(1)}_{T} \right) + \left( C^{(2)}_{\Lambda} + C^{(2)}_{T} \right) + 2 \left( C^{(3)}_{\Lambda} + C^{(3)}_{T} \right) \right],$$

$$V^{\Sigma\Sigma}_{150} = 4\pi \left[ 2 \left( C^{(2)}_{\Sigma} - 3C^{(2)}_{T} \right) + 2 \left( C^{(3)}_{\Sigma} - 3C^{(3)}_{T} \right) \right],$$

$$V^{\Sigma\Sigma}_{3451} = 4\pi \left[ -2 \left( C^{(2)}_{\Sigma} + C^{(2)}_{T} \right) + 2 \left( C^{(3)}_{\Sigma} + C^{(3)}_{T} \right) \right],$$

$$V^{\Lambda\Lambda}_{3451} = 4\pi \left[ \frac{3}{2} \left( C^{(1)}_{\Lambda} + C^{(1)}_{T} \right) + \left( C^{(2)}_{\Lambda} + C^{(2)}_{T} \right) \right].$$

Using the sum rules in Eq. (38) to the 5 free parameters in Eq. (39), one finds at LO of the $1/N_c$ expansion,

$$C^{\Sigma\Sigma}_{150} = 8 \cdot \frac{C^{\Lambda\Lambda}_{150}}{7} - \frac{11}{7} C^{\Lambda\Lambda}_{3451} - \frac{21}{24} C^{\Sigma\Sigma}_{3451}, \quad C^{\Sigma\Sigma}_{3451} = C^{\Lambda\Lambda}_{3451} + 9 C^{\Lambda\Lambda}_{3451}.$$  

Note that all of the LECs has the same $N_c$ scaling as $N_c$. The large-$1/N_c$ analysis of the LO YN potentials predicts that there are 3 free parameters at the LO of $1/N_c$ expansion with $O(1/N_c^2)$ corrections. With the same manner of the large-$N_c$ analysis of the LO YN potentials, one can apply the sum-rules in Eq. (32) for the partial-wave analysis in the YN potentials at NLO in Ref. [15] as well as for the YY sector in Refs. [13, 17].

Next we will compare the prediction of the large-$N_c$ sum rules in Eq. (39) with the best fitted values of the LECs from YN scattering data in Ref. [20]. This reference has performed the partial wave analysis of the YN s-wave scattering by using the same chiral Lagrangian as in our work. Authors in Ref. [20] have used two approaches to solve scattering amplitudes via Kadyshevsky equation with the relativistic covariant ChEFT (referred as EG) and Lippmann-Schwinger equation with the heavy-baryon formalisms (referred as HB). The relativistic covariant ChEFT (EG) approach is also used to study NN interactions in [41]. The best fitted values of the LECs are shown in Tab. [1]. We will use the LECs, $C^{\Lambda\Lambda}_{150}$, $C^{\Sigma\Sigma}_{3451}$ and $C^{\Lambda\Lambda}_{3451}$ as input values in Eq. (39) and the large-$N_c$ sum rules predict that

$$C^{\Sigma\Sigma}_{150,EG} = -0.06327, \quad C^{\Sigma\Sigma}_{3451,EG} = 0.1271,$$

$$C^{\Lambda\Lambda}_{150,H} = -0.04333, \quad C^{\Lambda\Lambda}_{3451,HB} = -0.0176.$$  

Comparing the LECs, $C^{\Sigma\Sigma}_{150}$ and $C^{\Sigma\Sigma}_{3451}$ from the large-$N_c$’s predictions with the best fitted values in Tab. [1] we found that $C^{\Sigma\Sigma}_{150}$ and $C^{\Sigma\Sigma}_{3451}$ from large-$N_c$ are in the same order as the best fitted values and with the same relative sign in EG approach. On the other hand, for the HB formalisms, the $C^{\Sigma\Sigma}_{3451}$ is also in the same order as the large-$N_c$ value and with the same relative sign. But for the $C^{\Lambda\Lambda}_{3451}$ value in HB approach, it is different in order of magnitude of 1 with the large-$N_c$ prediction and with different relative sign. One notes that the LECs best fitted values from EG and HB approaches have statistical uncertainties at 68% (one sigma) level. While Ref. [20] concluded that there is

| Parameters | EG | HB |
|------------|----|----|
| $C^{\Lambda\Lambda}_{150}$ | $-0.04795(151)$ | $-0.03894(1)$ |
| $C^{\Sigma\Sigma}_{150}$ | $-0.07546(81)$ | $-0.07657(1)$ |
| $C^{\Lambda\Lambda}_{3451}$ | $-0.01727(124)$ | $-0.01629(13)$ |
| $C^{\Sigma\Sigma}_{3451}$ | $0.36367(30310)$ | $0.20029(14050)$ |
| $C^{\Lambda\Lambda}_{3451}$ | $0.01271(471)$ | $-0.00176(304)$ |

TABLE I: Best-fitted values of $YN$ s-wave LECs (in units of $10^4 \text{ GeV}^{-2}$) for cut-off, $\Lambda = 600$ MeV in the EG and HB approaches [20].
not much difference between two approaches. But the large-$N_c$ sum rules in this work can show that the LECs from EG approach is more consistent with the predictions of large-$N_c$ than the HB formalism.

V. CONCLUSIONS

In this work, we studied the large-$N_c$ operator analysis of the octet-octet baryon potential from the SU(3) ChEFT. The minimal set of the octet-octet baryon potential is derived by using the relativistic constraints as suggestion in Refs. \[37, 38\] as well as the Claley-Hamilton identity and Fierz rearrangement to eliminate the redundant operators as shown in Ref. \[14\]. Up to NLO of $Q/\Lambda$ expansion, we found 27 operators for the octet-octet baryon potential in SU(3) flavor symmetry, 6 in LO and 21 in NLO of the small momentum scale.

The octet-octet baryon potential in the at LO in The 1/$N_c$ expansion is of order $N_c$ and there are 4 operators while he NNLO potential is of order 1/$N_c$ and we found 11 operators. The LECs of the ChEFT have two $N_c$ scalings, namely $N_c$ and 1/$N_c$ orders as shown in Eq. \[29\]. Interestingly, the extension of the flavor symmetry from SU(2) to SU(3) in the large-$N_c$ operator analysis does not change the profile of the potential in terms of the 1/$N_c$ expansion. There is no NLO for the SU(3) octet-octet baryon potential as for the NN potential \[29, 31\].

The matching between the octet-octet baryon potential and the 1/$N_c$ operator expansion leads to 6 free parameters of the LECs from the SU(3) chiral Lagrangian at the LO of the 1/$N_c$ expansion with $\mathcal{O}(1/N_c^2) \approx 10\%$ correction. The application of the sum rules in Eqs. \[32\] from the large-$N_c$ constraint to the partial-wave potential of the YN interactions at LO of the chiral expansion reduces the LECs of the YN potential to 3 from 5.

The comparison of the large-$N_c$ predictions of the LECs with the best fitted values from the YN s-wave scattering reveals that the large-$N_c$ prediction of the LECs is more consistent with the EG results than the HB formalisms. Noted that The theoretical results from the EG and HB approaches in Ref. \[20\] are quantitatively similar in describing the YN scattering experimental data.

The large-$N_c$ sum rules in this work can also be applied to the NLO of the YN interactions and extended to the ChEFT potential of the YY sector. In addition, we expect that future lattice QCD calculations may check the hierarchy of the $N_c$ scalings of the LECs and the large-$N_c$ sum rules predicted in this work.

Acknowledgments

We would like to thank Daniel Phillips and Carlos Schat for carefully reading manuscript and useful comments. We also thank Li-Shen Geng for explaining detail of the best fitted values of LECs from YN scattering data. XL acknowledges support by National Natural Science Foundation of China (Project No. 11547182), and the Doctoral Scientific Research Foundation of Liaoning Province (Project No. 201501197). This work is partly supported by Thailand Research Fund (TRF) under contract No. MRG5980255 (DS). YY and DS acknowledge support from, Suranaree University of Technology (SUT) and the Office of the Higher Education Commission under NRU project of Thailand (SUT-COE: High Energy Physics & Astrophysics). DS thanks Chamaipawn Jaipang for supporting useful references.

Appendix A: The non-relativistic reductions of the chiral Lagrangian

In this appendix, we derive the non-relativistic reductions of the chiral Lagrangian in Eq. \[11\]. Here we follow the derivation from Ref. \[37, 38\] and focus for the spin (Dirac) structures of the chiral Lagrangian only. The chiral Lagrangian can be re-written in terms of operator as

\[
\begin{align*}
\tilde{O}_1 &\equiv (BB)(\overline{BB}), \\
\tilde{O}_2 &\equiv (B\gamma_\mu B)(B\gamma^\mu B), \\
\tilde{O}_3 &\equiv (B\sigma_{\mu\nu} B)(B\sigma^{\mu\nu} B), \\
\tilde{O}_4 &\equiv (B\gamma_\mu\gamma_5 B)(B\gamma^\mu\gamma_5 B), \\
\tilde{O}_5 &\equiv (B\gamma_5 B)(B\gamma_5 B).
\end{align*}
\]  

(A1)

The relativistic fermion field $B(x)$ can be expanded to the positive energy components $\varphi_B(x)$ in the following from
TABLE II: Operators of the LO and NLO contact term interactions [9], the left (right) arrow on $\nabla$ indicates that the gradient operates on the left (right) field. Normal-ordering of the field operator products is implied.

\[ B(x) = \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2M} \begin{pmatrix} 0 & \sigma \cdot \nabla \\ \nabla \times \sigma \times \nabla \end{pmatrix} + \frac{1}{8M^2} \begin{pmatrix} \nabla^2 & 0 \\ 0 & 0 \end{pmatrix} \right] \varphi_B(x) + \mathcal{O}(Q^3), \quad (A2) \]

where $M$ and $Q$ are baryon mass in SU(3) flavor symmetry limit and small momentum scale respectively. Up to order $Q^5$, the non-relativistic reductions of the operators in Eq. (A1) are given by

\[
\begin{align*}
\tilde{O}_1 \overset{\text{NR}}{\simeq} & \quad O_S + \frac{1}{4M^2}(O_1 + 2O_2 + 2O_3 + 2O_5), \\
\tilde{O}_2 \overset{\text{NR}}{\simeq} & \quad O_S + \frac{1}{4M^2}(-4O_2 - 2O_5 + 4O_6 + O_7 - O_9 + 2O_{10} - 2O_{12}), \\
\tilde{O}_3 \overset{\text{NR}}{\simeq} & \quad O_T + \frac{1}{4M^2}(-O_1 - 2O_2 - 4O_5 + 4O_6 + O_7 - 2O_8 + 2O_{10} - 4O_{12} - 2O_{13}), \\
\tilde{O}_4 \overset{\text{NR}}{\simeq} & \quad -O_T - \frac{1}{4M^2}(-2O_6 + O_7 - O_9 - 2O_{10} - 2O_{12} + 2O_{13} - 2O_{14}), \\
\tilde{O}_5 \overset{\text{NR}}{\simeq} & \quad \frac{1}{4M^2}(O_7 + 2O_{10}), \\
\end{align*}
\]

where we took the above results from Refs. [37, 38] and the operators $O_i$ $(i = 1, ..., 14)$ are listed in Tab. II. By using partial integrations, Ref. [39] has been shown that there are only 12 operators are independent with the following constraints,

\[ O_7 + 2O_{10} = O_8 + 2O_{11} \quad \text{and} \quad O_4 + O_5 = O_6. \quad (A4) \]

Next step, one re-writes the non-relativistic reductions in Eq. (A3) in terms of the basis in Eqs. (4, 9, 10) as [37],

\[
\begin{align*}
A_S & \equiv \tilde{O}_S = O_S + \frac{1}{4M^2}(O_1 + O_3 + O_5 + O_6), \\
A_T & \equiv \tilde{O}_T = O_T - \frac{1}{4M^2}(O_5 + O_6 - O_7 + O_8 + 2O_{12} + 2O_{14}), \\
A_1 & \equiv p^2 \delta_{iX_1} \delta_{jX_2} = O_1 + 2O_2, \\
A_2 & \equiv p^2 \delta_{iX_1} \delta_{jX_2} = 2O_2 + O_3, \\
\end{align*}
\]
\begin{align}
A_3 & \equiv \vec{p}_\perp \cdot \vec{\sigma}_1 \cdot \vec{p}_\perp \cdot \vec{\sigma}_2 = O_9 + 2 O_{12}, \\
A_4 & \equiv \vec{p}_\perp \cdot \vec{\sigma}_1 \cdot \vec{p}_\perp \cdot \vec{\sigma}_2 = O_9 + O_{14}, \\
A_5 & \equiv i (\vec{p}_\perp \times \vec{p}_\perp) \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2)/2 = O_5 - O_6, \\
A_6 & \equiv (\vec{p}_\perp \cdot \vec{\sigma}_1)(\vec{p}_\perp \cdot \vec{\sigma}_2) = O_7 + 2 O_{10}, \\
A_7 & \equiv (\vec{p}_\perp \cdot \vec{\sigma}_1)(\vec{p}_\perp \cdot \vec{\sigma}_2) = O_7 + O_8 + 2 O_{13}. \\
\end{align}

By using above relations, we obtain the non-relativistic reductions of the chiral Lagrangian in Eq. (1) in terms of the operators \( A_i \) as,

\begin{align}
\tilde{O}_1 & \simeq A_S + \frac{1}{4 M^2} (A_2 - A_5), \\
\tilde{O}_2 & \simeq A_S - \frac{1}{4 M^2} (A_1 + A_2 + A_3 - 3 A_5 - A_6), \\
\tilde{O}_3 & \simeq A_T - \frac{1}{4 M^2} (A_1 + A_2 + A_3 - A_4 - 3 A_5 - A_6 + A_7), \\
\tilde{O}_4 & \simeq - A_T + \frac{1}{4 M^2} (A_4 + A_5 + A_6 - A_7), \\
\tilde{O}_5 & \simeq \frac{1}{4 M^2} A_6. \\
\end{align}
[33] M. R. Schindler, R. P. Springer and J. Vanasse, Phys. Rev. C 93, no. 2, 025502 (2016) [arXiv:1510.07598 [nucl-th]].
[34] D. Samart, C. Schat, M. R. Schindler and D. R. Phillips, Phys. Rev. C 94, no. 2, 024001 (2016) [arXiv:1604.01437 [nucl-th]].
[35] M. F. M. Lutz and A. Semke, Phys. Rev. D 83, 034008 (2011) [arXiv:1012.4365 [hep-ph]].
[36] M. F. M. Lutz, D. Samart and A. Semke, Phys. Rev. D 84, 096015 (2011) [arXiv:1107.1324 [hep-ph]].
[37] L. Girlanda, S. Pastore, R. Schiavilla and M. Viviani, Phys. Rev. C 81, 034005 (2010) [arXiv:1001.3676 [nucl-th]].
[38] L. Girlanda and M. Viviani, Few Body Syst. 49, 51 (2011).
[39] S. Pastore, L. Girlanda, R. Schiavilla, M. Viviani and R. B. Wiringa, Phys. Rev. C 80, 034004 (2009) [arXiv:0906.1800 [nucl-th]].
[40] E. Epelbaum, W. Gloeckle and U. G. Meissner, Nucl. Phys. A 637, 107 (1998) [nucl-th/9801064].
[41] X. L. Ren, K. W. Li, L. S. Geng, B. W. Long, P. Ring and J. Meng, [arXiv:1611.08475 [nucl-th]].