TWO PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS AT INTERMEDIATE ENERGIES

L. ALVAREZ-RUSO\textsuperscript{a}, E. OSET\textsuperscript{a}, E. HERNÁNDEZ\textsuperscript{b}

\textsuperscript{a}Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, 46100 Burjassot (Valencia), Spain
\textsuperscript{b}Grupo de Física Nuclear, Facultad de Ciencias, Universidad de Salamanca, 37008 Salamanca, Spain

We have developed a model for the $NN \rightarrow NN\pi\pi$ reaction and evaluated cross sections for the different charged channels. The low energy part of those channels where the pions can be in an isospin zero state is dominated by $N^*(1440)$ excitation, driven by an isoscalar source followed by the decay $N^* \rightarrow N(\pi\pi)^{T=0}_{S-wave}$.

At higher energies, and in channels where the pions are not in $T=0$, $\Delta$ excitation mechanisms become relevant.

Pion production in $NN$ collisions is one of the sources of information on nucleon-nucleon interaction and resonance properties. Nowadays, two pion production in $pp$ collisions is a subject of experimental research at the CELSIUS storage ring by the WASA/PROMICE collaboration\textsuperscript{1}. A direct comparison of our theoretical results with the measured total cross sections and invariant-mass distributions will certainly provide useful information about the mechanisms governing this reaction.

Our model can also be relevant to understand certain features observed in other two pion production reactions; namely, $n + p \rightarrow d + (\pi\pi)^0$ and $p + d \rightarrow ^3He + \pi^+\pi^-$. The first of them was experimentally studied at LAMPF for a neutron beam energy of 800 MeV\textsuperscript{2}, and is also an important ingredient of the deuteron spectra measured with a 1160 MeV neutron beam at Saturne\textsuperscript{3}, where the ABC effect was first observed in the free nucleon reaction. The second is being studied at COSY in the MOMO experiment\textsuperscript{4}.

Finally, the present model has also repercussions for the reaction $p + p \rightarrow p + p + \pi^0$ since a possibly relevant mechanism can be obtained from $p + p \rightarrow p + p + \pi^0\pi^0$ when one of the $\pi^0$’s is emitted and the other is absorbed\textsuperscript{5}.

In order to build the model, we have closely followed the guidelines of a previous model for the $\pi N \rightarrow \pi\pi N$ reaction\textsuperscript{6}. A relevant finding of that work is that, even at threshold, the Roper resonance excitation and subsequent decay into $N(\pi\pi)^{T=0}_{S-wave}$ is a very important mechanism. In our case, the situation is similar, but more involved because one has to deal with the $NN \rightarrow NN^*$ transition, which is poorly known. Below, we present a brief description of the ingredients of the model. Details can be found in Ref. 7.

Diagrams (1), (2) and (3) of Fig. 1 can be derived from the lowest order
SU(2) chiral Lagrangians containing both pions and nucleons at tree level

\[ \mathcal{L} = \mathcal{L}_{\text{2}} + \mathcal{L}_{\text{1}}^{(B)}. \]

The expressions for these Lagrangians are given, for instance, in Ref 8. With these ingredients we generate only the isovector part of the s-wave \( \pi N \) amplitude, needed for diagram (3). The much smaller isoscalar part appears at higher order. We just consider it empirically

\[ \mathcal{L} = -4\pi \frac{\lambda_1}{m_\pi} \bar{\psi} \phi^2 \psi, \quad \lambda_1 = 0.0075. \]

In spite of the fact that these amplitudes do not vanish at threshold, their contribution to the total cross section is very small, even at threshold, in most channels (see Fig. 2).

In order to obtain the amplitudes for diagrams (9)-(15) we need the following phenomenological Lagrangians

\[ \mathcal{L}_{\Delta N \pi} = f^* \frac{\psi_\Delta S_\lambda^i (\partial_i \phi^\lambda) T^{\lambda i} \psi_N + h.c.}{m_\pi} \]

\[ \mathcal{L}_{\Delta \Delta \pi} = f^\lambda \frac{\psi_\Delta S_{\Delta i} (\partial_i \phi^\lambda) T^\lambda_{\Delta i} \psi_N + h.c.}{m_\pi} \]

where \( S^i \) \( (T^i) \) and \( S_\Delta \) \( (T_\Delta) \) are the spin (isospin) 1/2 \( \rightarrow \) 3/2 transition operator and spin (isospin) 3/2 operator respectively. The coupling constant \( f^* = 2.13 \) is obtained from the \( \Delta \rightarrow N \pi \) partial width \( 9 \) and \( f_\Delta = 4/5 f_{NN} \) comes from SU(6) quark model. Our \( T = 1 \) exchange potential includes \( \pi \) and \( \rho \) exchange.
and short range correlations$^9$. All these terms are quite small except diagram (12), which becomes relevant at energies $T_p > 1$ GeV.

We use the information about the $N^*(1440)$ properties contained in the PDG book$^8$ in order to extract the couplings required for diagrams (4)-(8):

$$L_{N^*N\pi} = \tilde{f} \psi^\dagger_{N^*} \sigma_i (\partial_i \phi) \tau \psi_N + h.c. \quad (5)$$

$$L_{N^*\Delta\pi} = \frac{g_{N^*\Delta\pi}}{m_\pi} \psi^\dagger_\Delta S_i^{\dagger} (\partial_i \phi^\lambda) T^{\dagger \lambda} \psi_{N^*} + h.c. \quad (6)$$

Here, the couplings $\tilde{f} = 0.477$ and $g_{N^*\Delta\pi} = 2.07$ are obtained from the corresponding partial widths assuming branching ratios of 65 % and 25 % respectively, and a total width of 350 MeV.

The Lagrangian for the $N^* \rightarrow N(\pi\pi)_S$ wave decay channel is given in Ref. 12. After expanding in pion fields one gets

$$L_{N^*N\pi\pi} = -c_1 m_\pi^2 \bar{\psi}_{N^*} \phi^2 \psi_N - c_2 \frac{1}{f^2 M^2} (\bar{\psi}_{N^*} (\tau \partial_{\mu} \phi)(\tau \partial_{\mu} \phi) \psi_N + h.c. \quad (7)$$

In this case, the free parameters $c_1$ and $c_2$ can not be both obtained from the partial decay width. They can just be constrained to an ellipse$^8$. In order to further constrain these parameters we use the model of Ref. 6 for the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$. The best agreement with the experiment is obtained with $c_1 = -7.27 GeV^{-1}$ and $c_2 = 0$ ( Set I ) but the experimental errors are also compatible with $c_1 = -12.7 GeV^{-1}$ and $c_2 = 1.98 GeV^{-1}$ ( Set II).

In this case, the $T=1$ exchange is similar to the one for the $\Delta$ terms, but in addition we must consider an exchange in the $T=0$ channel. In a recent analysis of the $(\alpha, \alpha')$ reaction on a proton target$^{12}$, the strength of the isoscalar $NN \rightarrow NN^*$ transition was extracted by parameterizing the transition amplitude in terms of an effective “$\sigma$”, which couples to $NN$ as the Bonn model $\sigma$$^{13}$

$$L_{\sigma NN} = g_{\sigma NN} \bar{\psi} \phi \psi, \quad g_{\sigma NN}^2 / 4\pi = 5.69 \quad (8)$$

and couples to $NN^*$

$$L_{\sigma NN^*} = g_{\sigma NN^*} \bar{\psi}_{N^*} \phi \psi + h.c. \quad (9)$$

with a strength provided by a best fit to the data: $g_{\sigma NN^*}^2 / 4\pi = 1.33$, and such that $g_{\sigma NN}$ and $g_{\sigma NN^*}$ have the same sign.

The amplitudes corresponding to diagrams (4), (5) give a non-vanishing contribution at threshold, which is by far the dominant one below 1 GeV for
those channels where the pions can be in $T=0$. The contribution of the $T=0$ exchange is an order of magnitude larger than the $T=1$ exchange, which is diminished due to the short range correlations.

![Graph](image)

**Figure 2:** Total cross sections for two of the channels, as a function of the incoming proton kinetic energy in lab. frame. Solid line, total (line labelled 1 for set I and 2 for set II); long-short dashed line, $N^* \to N(\pi\pi)^{T=0}$ $S$-wave excitation mechanisms; dash-dotted line, $\Delta$ excitation mechanisms; short-dashed line, non-resonant terms from diagrams (1)-(3). The partial contributions are calculated with set I. Experimental data are taken from Ref. 14.

We implement the final state interaction (FSI) in a simplified version of the model, meant to work at $T_p < 900$ MeV and for the $pp \to pp\pi^+\pi^-$, where we can assume that the total cross section is clearly dominated by the $N^*$ excitation driven by an isoscalar source. Since the energy of the incoming nucleons must be high enough to produce two pions, we neglect initial state interaction. For the configuration of the figure, the amplitude is

$$M^\text{FSI} = \int \frac{dq}{(2\pi)^3} M^\text{free}(q,p_{N^*}) \tilde{\varphi}_k(P)$$

(10)

where

$$k = \frac{p_4 - p_3}{2}, \quad P = q + \frac{p_5 + p_6 + p_2 - p_1}{2}, \quad p_{N^*} \approx p_5 + p_6 + p_3,$$

(11)

For the wave function in momentum space $\tilde{\varphi}_k(P)$ we use an analytic expression obtained for a separable non-local potential that takes into account the repulsion\(^3\). The parameters are chosen\(^3\) to fit the $^1S_0$ $np$ phase shifts.
Figure 3: Dominant mechanism for $pp \rightarrow pp\pi^+\pi^-$ at low energies with FSI included and total cross sections obtained with (dotted lines) and without (solid lines) FSI for both sets I and II.

Acknowledgments

This work has been partially supported by DGYCIT contract PB 96-0753. L.A.R. acknowledges financial support from the Generalitat Valenciana.

References

1. J. Johanson, in these Proceedings.
2. C. L. Hollas et al., Phys. Rev. C 25 (1982) 2614.
3. F. Plouin, J. Duflo and L. Goldzahl, Nucl. Phys. A 302 (1978) 413.
4. http://merlin.iskp.uni-bonn.de/momo/momo.html
5. E. Hernández and E. Oset, nucl-th/9808017.
6. E. Oset and M.J. Vicente Vacas, Nucl. Phys. A 446 (1985) 584.
7. L. Alvarez-Ruso, E. Oset, E. Hernández Nucl. Phys. A 633 (1998) 519.
8. V. Bernard, N. Kaiser, U. G. Meissner, Int.J.Mod.Phys. E4 (1995) 193.
9. C. Caso et al., The European Physical Journal C 3 (1998) 1.
10. E. Oset and W. Weise, Nucl. Phys. A 319 (1979) 365.
11. V. Bernard, N. Kaiser, U.G. Meissner, Nucl. Phys. B 457 (1995) 147.
12. S.Hirenzaki,P.Fernández de Córdoba, E.Oset,Phys.Rev. C53(1996)277.
13. R. Machleidt, K. Holinde, Ch. Elster, Phys. Rep. 149 (1987) 1.
14. D. C. Brunt, M. J. Clayton, B. A. Westwood, Phys. Rev. 187 (1969) 1856; F. Shimizu et al., Nucl. Phys. A 386 (1982) 571; L. G. Dakhno et al., Sov. J. Nucl. Phys. 37 (1983) 540.
15. J. H. Naqvi, Nucl. Phys. 58 (1964) 289.