Frequency Coupling Admittance Modeling of Quasi-PR Controlled Inverter and Its Stability Comparative Analysis Under the Weak Grid

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ABSTRACT This paper intends to comparatively study the stabilities of grid-connected inverters with three closely related controllers: quasi-proportional resonance (quasi-PR), proportional integral (PI), and proportional resonance (PR) under the weak grid. Firstly, considering the influence of frequency coupling characteristic, a frequency coupling admittance model of quasi-PR controlled inverter is established. Then, the admittance characteristics of the quasi-PR, PI and PR controlled inverters are compared. Admittance characteristics of the PI and PR controlled inverters are similar while the quasi-PR controlled inverter is quite different: the amplitude of the quasi-PR controlled inverter is larger than that of the PI controlled inverter and the phase difference between the two inverters is obvious in the mid-high frequency areas, which are mainly caused by the resonance bandwidth of the quasi-PR controller. Furthermore, the stabilities of the quasi-PR, PI and PR controlled inverters are analyzed. The stabilities of the PI and PR controlled inverters are similar but the quasi-PR controlled inverter is more sensitive to weak grid and high inverter output power. To achieve the same system stability, the voltage outer-loop bandwidth of the quasi-PR controlled inverter should be designed narrower than that of the PI and PR controlled inverters. Finally, experiments verify the correctness of the analyses.

INDEX TERMS Frequency coupling, admittance modeling, quasi-PR control, stability analysis.

I. INTRODUCTION

With the development of renewable energy generation, more and more power electronic converters are connected to the grid [1]–[3]. Inverters are used as the interface for renewable energy to connect to the grid, thus its control method has received a lot of attention [4], [5].

The PI control method in dq-domain (the synchronous reference frame) [6], [7], the PR and quasi-PR control methods in phase-domain (the stationary reference frame) [8], [9] are often used in the inner-loop control of grid-connected inverters. As a typical grid-connected inverter control method, the stability problem of the PI controlled inverter has been widely studied in [10], [11]. A small-signal transfer function model for the PI controlled inverter to analyze system stability has been established in [10] and the stability of the PI controlled inverter through the impedance-based method has been analyzed [11]. However, since the zero steady-state error performance of the PI control method should be achieved in dq-domain, several times of coordinate transformation are required during the control process [12]. Therefore, the conversion relationship between the control in dq-domain and the control in phase-domain is established, and the PR control method in phase-domain which is equivalent to the PI control method in dq-domain is proposed [13]. To reduce the sensitivity of
the control system to slightly frequency deviation in the grid, quasi-PR control method, which is approximately equivalent to PR control method, is often applied in practice [14]. The quasi-PR control method has a relatively large gain at the fundamental frequency, which is helpful for the controller to nearly realize zero steady-state error performance. What’s more, the quasi-PR control method can be directly used in phase-domain, which can reduce the times of coordinate transformation [15]. Therefore, the quasi-PR control method has attracted many attentions [17]–[21]. The effect of the quasi-PR controller gain on the control performance was analyzed [17], and an adaptive quasi-PR controller was proposed to obtain the controller parameters online when AC-side inductance is unknown [18]. An optimized design method was proposed in [19], which can effectively increase the proportional coefficient and integral coefficient gains of the quasi-PR controller. Quasi-PR controller is also applied to the control loops for sub-synchronous oscillation suppression of DFIG [20] and resonance suppression of the system while inverters are connected to the grid with the long transmission lines [21].

The PR control method in phase-domain can be derived from the PI control method in dq-domain, thus they can make the grid-connected inverters achieve a similar performance in different coordinate systems. However, the quasi-PR control method is obtained from the PR control method through introducing resonance bandwidth, which may change the control characteristics. It is worth to be studied whether the stability of the quasi-PR controlled inverter is consistent with that of the PR and PI controlled inverters. The sequence-impedance model of the quasi-PR and the PI controlled inverters was established in [22]. However, the built model does not consider the effects of the voltage outer-loop [23], [24] and the frequency coupling characteristic [25], which may lead to inaccurate analysis results. The effects of the PI control method in dq-domain and quasi-PR control method in phase-domain on the stability of the two-level VSC-HVDC system was compared in [26]. It pointed out that under the appropriate parameters, the PI controlled and the quasi-PR controlled systems have a similar dynamic performance. However, the above comparative analysis was performed when different control parameters were selected for the PI and quasi-PR controller without considering the relationship between these two control methods. Therefore, in this paper, considering the effects of voltage outer-loop, current inner-loop, phase-locked loop (PLL), control delay and frequency coupling characteristic, a frequency coupling admittance model of the quasi-PR controlled inverter is established, and the differences of the stability for the quasi-PR, PI and PR controlled systems are explored under the same operating conditions.

The rest of the paper is organized as follows. In Section II, the relationship between the PI control method in dq-domain and the PR and quasi-PR control methods in phase-domain is introduced firstly. Then, the frequency coupling admittance model of the quasi-PR controlled inverter is built in Section III. The admittance characteristics of the quasi-PR, PI and PR controlled inverters are compared in Section IV, and the stabilities of the grid-connected inverters under these three control methods are analyzed through the impedance-based method and generalized Nyquist criterion in Section V. The experiments are carried out in Section VI. Finally, some conclusions are made in Section VII.

II. PI CONTROL METHOD IN DQ-DOMAIN AND QUASI-PR AND PR CONTROL METHODS IN PHASE-DOMAIN

Fig. 1(a) presents the topology of the grid-connected system, $v_{dc}$ is the DC-side voltage; $e_a$, $e_b$ and $e_c$ are mid-point voltages of the grid-connected inverter; $v_a$, $v_b$, $v_c$, $i_a$, $i_b$, and $i_c$ are the output voltages and inductor currents of the grid-connected inverter; $L_f$, $C_f$ and $R_f$ are the filter inductor, capacitor, and damping resistance, respectively; $L_g$ and $R_g$ are the equivalent line inductance and resistance of the grid, respectively.

![FIGURE 1. Topology and control methods. (a) Topology of the grid-connected inverter system. (b) PI control method. (c) Quasi-PR control method.](image-url)
where \( k_{q,y}, k_{q,i} \) and \( k_{p} \) are proportional coefficients; \( k_{r,y}, k_{r,i} \), and \( k_{r} \) are integral coefficients; \( \omega_{r} \) is resonance bandwidth of quasi-PR controller; the center frequency of the quasi-PR controller \( \omega_{0} = 2\pi f_{1} \); \( f_{1} \) is the fundamental frequency.

\[
G_{PR}(s) = G_{pl} \left( \frac{s^2 + \omega_0^2}{2s} \right) = k_{pr} + \frac{2k_{r}s}{s^2 + \omega_0^2} \tag{3}
\]

Existing research shows that, after the conversion in (3), the equivalent controller, PR controller \( G_{PR}(s) \) in AC system, can be obtained from the PI controller \( G_{pl}(s) \) in DC system [13].

The bode plot for the transfer function of the PR controller is shown with the dotted orange curve in Fig. 2. The PR control method can realize the zero steady-state error for tracking the fundamental signal.

The conversion from the PR controller to the quasi-PR controller is shown as the solid blue curve in Fig. 2. The PR control method is obtained from the PI controller \( G_{pl}(s) \) and \( s \) in AC system, can be obtained from the PI controller \( G_{pl}(s) \) in DC system [13].

The admittance characteristics of grid-connected inverters can be affected by control loops, control delay and frequency coupling characteristic. Thus, the admittance model of quasi-PR controlled inverter will be built considering these factors. A positive-sequence disturbance voltage at the disturbance frequency \( f_{p} \) being injected into the quasi-PR controlled inverter system. A positive-sequence disturbance response current at \( f_{p} \) and a negative-sequence response current at coupling frequency \( f_{p1} = f_{p} - 2f_{1} \) will be generated in the output current of the grid-connected inverter due to frequency coupling characteristic. The negative-sequence response current flowing through the grid impedance will generate the negative-sequence voltage at \( f_{p1} \). Thus, \( i_{a} \) and \( v_{a} \) can be expressed in frequency-domain as below.

\[
I_{a}[f] = \begin{bmatrix} I_{1}, & f = \pm f_{1} \\ I_{p}, & f = \pm f_{p} \\ I_{p1}, & f = \pm f_{p1} \end{bmatrix} \quad V_{a}[f] = \begin{bmatrix} V_{1}, & f = \pm f_{1} \\ V_{p}, & f = \pm f_{p} \\ V_{p1}, & f = \pm f_{p1} \end{bmatrix} \tag{4}
\]

where \( I_{1} = (I_{L}/2)e^{\pm j\varphi_{p1}}, I_{p} = (I_{p}/2)e^{\pm j\varphi_{p}}, I_{p1} = (I_{p1}/2)e^{\pm j\varphi_{p1}}, V_{1} = V_{1}/2, V_{p} = (V_{p}/2)e^{\pm j\varphi_{p}}, V_{p1} = (V_{p1}/2)e^{\pm j\varphi_{p1}}; I_{1}, I_{p}, I_{p1} \) are the amplitudes of the currents, respectively; \( V_{1}, V_{p} \) and \( V_{p1} \), are the amplitudes of the voltages, respectively; \( \varphi_{p1}, \varphi_{p}, \varphi_{p1} \) and \( \varphi_{p1} \) are the initial phase angles of the corresponding current and voltage components, respectively.

**III. FREQUENCY COUPLING ADMITTANCE MODELING OF QUASI-PR CONTROLLED INVERTER**

Admittance characteristics of grid-connected inverters can be affected by control loops, control delay and frequency coupling characteristic. Thus, the admittance model of quasi-PR controlled inverter will be built considering these factors. After a positive-sequence disturbance voltage at the disturbance frequency \( f_{p} \) being injected into the quasi-PR controlled inverter system. A positive-sequence disturbance response current at \( f_{p} \) and a negative-sequence response current at coupling frequency \( f_{p1} = f_{p} - 2f_{1} \) will be generated in the output current of the grid-connected inverter due to frequency coupling characteristic. The negative-sequence response current flowing through the grid impedance will generate the negative-sequence voltage at \( f_{p1} \). Thus, \( i_{a} \) and \( v_{a} \) can be expressed in frequency-domain as below.

\[
I_{a}[f] = \begin{bmatrix} I_{1}, & f = \pm f_{1} \\ I_{p}, & f = \pm f_{p} \\ I_{p1}, & f = \pm f_{p1} \end{bmatrix} \quad V_{a}[f] = \begin{bmatrix} V_{1}, & f = \pm f_{1} \\ V_{p}, & f = \pm f_{p} \\ V_{p1}, & f = \pm f_{p1} \end{bmatrix} \tag{4}
\]

where \( I_{1} = (I_{L}/2)e^{\pm j\varphi_{p1}}, I_{p} = (I_{p}/2)e^{\pm j\varphi_{p}}, I_{p1} = (I_{p1}/2)e^{\pm j\varphi_{p1}}, V_{1} = V_{1}/2, V_{p} = (V_{p}/2)e^{\pm j\varphi_{p}}, V_{p1} = (V_{p1}/2)e^{\pm j\varphi_{p1}}; I_{1}, I_{p}, I_{p1} \) are the amplitudes of the currents, respectively; \( V_{1}, V_{p} \) and \( V_{p1} \), are the amplitudes of the voltages, respectively; \( \varphi_{p1}, \varphi_{p}, \varphi_{p1} \) and \( \varphi_{p1} \) are the initial phase angles of the corresponding current and voltage components, respectively.

**A. MODELING OF THE VOLTAGE OUTER-LOOP**

The voltage outer-loop control is the important part of the control strategy of the grid-connected inverter in Fig. 1. To accurately describe the admittance characteristics of the grid-connected inverter, the voltage outer-loop control should be considered.

According to the law of energy conservation, the power on the DC-side is equal to that on the AC-side.

\[
(\dot{V}_{pv} - sC_{dc}V_{dc})V_{dc} = (v_{a} + sL_{q}i_{a})I_{a} + (v_{b} + sL_{q}i_{b})I_{b} + (v_{c} + sL_{d}i_{c})I_{c} \tag{5}
\]

The expression of \( V_{dc} \) at \( f = \pm (f_{p} - f_{1}) \) can be obtained from (5) after the convolution calculation in frequency-domain.

\[
V_{dc} = 3 \left[ I_{1}V_{p1} + (sL_{q}I_{1} + V_{1})I_{p1} + (V_{1}^{*} + sL_{d}I_{1}^{*})I_{p} + I_{1}^{*}V_{p} \right] i_{pv}^{-sC_{dc}V_{dc}} \tag{6}
\]

where the superscript “*” represents the conjugated variable; \( V_{dc} \) is the steady-state component of the DC-side voltage.
In order to simplify the expression, we can define
\[ v_{dc} [f] = F_1 I_p + F_1 I_p + F_v V_p + F_v V_p \] (7)
where
\[
\begin{bmatrix}
F_i & F_{i\oplus} \\
F_v & F_{v\oplus}
\end{bmatrix}
= \begin{bmatrix}
3 (V_1 + s L_i I_1) & 3 (V_1 + s L_i I_1) \\
I_p - s C_{dc} V_d c & I_p - s C_{dc} V_d c \\
I_p - s C_{dc} V_d c & I_p - s C_{dc} V_d c \\
I_p - s C_{dc} V_d c & I_p - s C_{dc} V_d c
\end{bmatrix}.
\]

It is defined that \( s = \pm 2 j \pi f_1, s_2 = \pm (2 \pi f_2 - j 4 \pi f_1), \) and \( s_p1 = \pm (2 \pi f_2 - j 4 \pi f_1) \).

The reference value of the output current in dq-domain \( i_{dr} \) can be expressed as shown below.
\[ i_{dr} = (v_{dc} - V_{dc}) G_v (s_2) \] (8)

Submitting (6) into (8), it can be obtained
\[ i_{dr} [f] = \begin{cases} I_{dr} & \text{dc} \\
G_v (s_2) G_{v\alpha} (s_2) v_{dc}, & f = \pm (f_p - f_1) \end{cases} \] (9)

**B. MODELING OF THE PLL**

Considering the disturbance, the PLL output can be expressed as \( \theta_{PLL} (t) = \Delta \theta_{PLL} (t) + \theta (t) \). According to the derivation method in [5], the relationship between \( \Delta \theta_{PLL} \) and the disturbance voltage in PCC at \( f = \pm (f_p - f_1) \) can be obtained. The PLL transfer function \( G_{PLL} (s_2) = (k_p_{PLL} + k_{i\oplus}/s_2) \).

\[ \Delta \theta_{PLL} [f] = T_p (s_2) G_v (s_2) V_p + T_p (s_2) G_v (s_2) V_p \] (10)

\[ T_p (s_2) = \frac{\pm j G_{PLL} (s_2)}{1 + V_1 G_{PLL} (s_2)}, \quad T_p (s_2) = \frac{\pm j G_{PLL} (s_2)}{1 + V_1 G_{PLL} (s_2)} \] (11)

The output current reference value in phase A \( i_{ar} \) can be obtained by the following formula.
\[ i_{ar} [f] = \cos (\theta_1 [f]) \otimes (i_{dr} [f] - \Delta \theta_{PLL} [f] \otimes i_{dr} [f] + i_{ar} [f]) \] (12)

where the steady-state component of \( i_{ar} \) is zero, the symbol “\( \otimes \)” represents a convolution calculation.

Combining (9), (10) and (12), the specific expression of \( i_{ar} \) can be derived as below.
\[ i_{ar} [f] = \begin{cases} 0.5 (i_{dr} [s_2] \pm j i_{dr} (\Delta \theta_{PLL} [s_2])), & f = \pm f_p \\
0.5 (i_{dr} [s_2] \pm j i_{dr} (\Delta \theta_{PLL} [s_2])), & f = \pm (f_p - 2 f_1) \end{cases} \] (13)

**C. MODELING OF THE QUASI-PR CONTROLLED INVERTER**

According to the control block diagram shown in Fig. 1(c), the expression of the modulation signal \( m_1 \) can be obtained.
\[ m_1 = \frac{V_p + j 2 \pi f_1 L_i i_1}{K_m V_{dc} G_{del} (s_1)} \] (14)

where the control delay can be expressed as \( G_{del} (s) = e^{-1.5 T_s s}, \) and \( T_s \) is the sampling period.

From Fig. 1(c), the relationship between the output voltage and inductor current of the grid-connected inverter can be obtained as follows.
\[ v_{L1} = K_{PWM} G_{del} v_{dc} \otimes [(i_{ar} - i_a) G_{QPR} (s) + K_{i v_a}] - v_a \] (15)

Combining (5), (13) and (15), the admittance matrix of the grid-connected inverter can be calculated as
\[ \begin{bmatrix} J_1 \\ J_{i1} \end{bmatrix} = \begin{bmatrix} -V_{dc} G_{QPR} (s) G_{if} (s) \\ -V_{dc} G_{QPR} (s) G_{if} (s) \end{bmatrix} \] (16)

The expressions of \( Y_{11}, Y_{12}, Y_{21} \) and \( Y_{22} \) in the admittance matrix are shown in (17) to (20). Those variables in (17) to (20) as shown at the bottom of the page.
\[
D_{v1} = (\pm 0.5 j \omega L_d G_p (s_2) G_{QPR} (s) + K_I V_{dc} G_{Vf} (s)) \\
D_{v2} = \pm 0.5 j \omega L_d V_{dc} G_{Vf} (s) G_p (s_2) G_{QPR} (s) \\
D_{v3} = \mp 0.5 j \omega L_d V_{dc} G_{Vf} (s) G_p (s_2) G_{QPR} (s_1) \\
D_{v4} = (\mp 0.5 j \omega L_d G_p (s_2) G_{QPR} (s_1) + K_I V_{dc} G_{Vf} (s_1)) 
\]

(23)

### IV. COMPARISON OF ADMITTANCE CHARACTERISTICS OF THE QUASI-PR, PI AND PR CONTROLLED INVERTERS

The parameters of the quasi-PR controlled inverter studied in this paper are shown in Table 1. In order to verify the correctness of the established admittance model, an admittance measurement platform was established in Matlab/Simulink. The admittance measurement results of the quasi-PR controlled inverter are shown in Fig. 3. The solid red line indicates the established frequency coupling admittance model of the quasi-PR controlled inverter, and the red circle represents the admittance measurement result. It can be seen from Fig. 3 that the admittance measurement results essentially agree with the established frequency coupling admittance model, which proves the correctness of the established frequency coupling admittance model of the quasi-PR controlled inverter.

**FIGURE 3.** Admittance characteristic curves of the grid-connected inverter under these three control methods.

The frequency coupling admittance models of the PI and PR controlled inverters are shown as (24)-(25).

\[
\begin{bmatrix}
I_p \\
I_{p1}
\end{bmatrix} =
\begin{bmatrix}
Y_{11, \, PI} & Y_{12, \, PI} \\
Y_{21, \, PI} & Y_{22, \, PI}
\end{bmatrix}
\begin{bmatrix}
V_p \\
V_{p1}
\end{bmatrix} 
\]

(24)

\[
\begin{bmatrix}
I_p \\
I_{p1}
\end{bmatrix} =
\begin{bmatrix}
Y_{11, \, PR} & Y_{12, \, PR} \\
Y_{21, \, PR} & Y_{22, \, PR}
\end{bmatrix}
\begin{bmatrix}
V_p \\
V_{p1}
\end{bmatrix} 
\]

(25)

In order to compare and analyze the admittance characteristics of the quasi-PR, PI and PR controlled inverters more conveniently, the admittance model and measurement results of the PI and PR controlled inverters are also drawn in Fig. 3. The solid red line indicates the established frequency coupling admittance model of the quasi-PR controlled inverter, and the red circle represents the admittance measurement result. It can be seen from Fig. 3 that the admittance measurement results essentially agree with the established frequency coupling admittance model, which proves the correctness of the established frequency coupling admittance model of the quasi-PR controlled inverter.

**FIGURE 4.** Admittance differences between quasi-PR and PI controlled inverters.

To describe this difference clearly, the admittance differences between the quasi-PR and PI controlled inverters are shown in Fig. 4. The different colors and curves are used to present the admittance differences of \(Y_{11}, Y_{12}, Y_{21}, \) and \(Y_{22}\). In the frequency areas 350 Hz to 1300 Hz, the amplitude difference is larger than 3.5 dB. In the frequency areas 400 Hz to 500 Hz and 600 Hz to 1100 Hz, the amplitude difference is

**TABLE 1.** Parameters of the Quasi-PR controlled inverter.

| Symbol | Quantity | Values |
|--------|----------|--------|
| \(L\)  | filter inductance | 3 mH |
| \(C\)  | filter capacitor | 20 µF |
| \(R\)  | damping resistance of filter capacitor | 1.5 Ω |
| \(V_{dc}\) | DC-side voltage | 760 V |
| \(f_1\) | grid voltage | 311 V |
| \(f_0\) | fundamental frequency | 50 Hz |
| \(K_{p_{QPR}}\) | modulation coefficient | 0.5 |
| \(k_{dc}\) | the DC-side current | 14.29 A |
| \(T_s\) | the sampling period | 5\times 10^{-3} s |
| \(k_{p_{P1}}\) | proportional coefficient in \(G_{P1}(s)\) | 0.2639 |
| \(k_{i_{P1}}\) | integral coefficient in \(G_{P1}(s)\) | 10.9988 |
| \(\omega_r\) | resonance bandwidth of QPR controller | 2π rad/s |
| \(k_{p_{QPR}}\) | proportional coefficient of QPR controller | 0.035 |
| \(k_{i_{QPR}}\) | integral coefficient of QPR controller | 5 |
| \(k_{p_{Q1}}\) | proportional coefficient in \(G_{Q1}(s)\) | 8.3398 |
| \(k_{i_{Q1}}\) | integral coefficient in \(G_{Q1}(s)\) | 698.5 |
| \(\omega_{c_{Q1}}\) | the cut-off frequency of the sampling filters for the voltage | 2π \times 4000 rad/s |
| \(\omega_{c_{Q2}}\) | the cut-off frequency of the sampling filters for the current | 2π \times 4000 rad/s |
| \(K_I\) | voltage feed forward coefficient | 1/350 |
larger than 6 dB which indicates the amplitude of the quasi-PR controlled inverter is more than twice as large as that of the PI controlled inverter. In 250 Hz to 2000 Hz, the phase difference is larger than 20 degrees.

Therefore, the admittance differences between the quasi-PR and PI controlled inverters are quite large.

In order to further explore the factor caused the above admittance differences, we try to focus on the only different parameter \( \omega_r \) between the PI and quasi-PR controllers. The admittance characteristics of the quasi-PR controlled inverter when \( \omega_r \) is changed are observed and their admittance characteristic differences are compared as shown in Fig. 5.

According to (26), \( \lambda_1 \) and \( \lambda_2 \) are defined as the eigenvalues of the return matrix. The system stability is analyzed by the Nyquist diagrams of \( \lambda_1 \) and \( \lambda_2 \) which are indicated by the solid and dotted line, respectively. 

\[
\lambda_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \lambda_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\]  

(28)

In Fig. 5, the change of \( \omega_r \) has a great impact on admittance characteristics in the mid-high frequency areas. As \( \omega_r \) increases, the admittance peaks become sharper. The smaller the \( \omega_r \), the closer the admittance characteristics of the quasi-PR and PI controlled inverters.

Thus, the resonance bandwidth of the quasi-PR controller is an important factor that causes differences in admittance characteristics of the grid-connected inverters under the quasi-PR and PI control methods.

V. COMPARATIVE ANALYSES OF SYSTEM STABILITY

Based on the established frequency coupling admittance model and generalized Nyquist criterion, the influence of the grid impedance, inverter output power \( P_s \) and the outer-loop bandwidth \( BW_v \) on system stability are studied and the system stabilities for the quasi-PR, PI and PR controlled inverters are compared. The equivalent return matrix of the grid-connected inverter system in Fig. 1(a) can be defined as below.

\[
L = Z_g \cdot Y_{\text{inv}} = \begin{bmatrix}
Z_{g11} \cdot Y_{11} & Z_{g11} \cdot Y_{12} \\
Z_{g22} \cdot Y_{21} & Z_{g22} \cdot Y_{22}
\end{bmatrix}
\]

(26)

where \( Z_{g11} = Z_g(s) \), \( Z_{g22} = Z_g(s - j4\pi f_1) \).

The filter capacitor branch is not included in the control of the grid-connected inverter. Thus, the filter capacitor branch can be regarded as a part of the grid impedance:

\[
Z_g(s) = \frac{(sL_g + R_g)\left[ R_f + 1/(sC_f) \right]}{sL_g + R_g + R_f + 1/(sC_f)}
\]

(27)

A. THE EFFECTS OF THE GRID IMPEDANCE ON SYSTEM STABILITY

To comparatively study system stability when \( L_g \) is changed, the Nyquist diagrams in Fig. 6 are analyzed. In Fig. 6(a), when the \( L_g \) is 1.6 mH, 2.4 mH, or 3.2 mH, the Nyquist curves of the quasi-PR controlled inverter have surrounded the point \((-1, j0)\), which indicates the system is unstable. As shown in Fig. 6(b) and Fig. 6(c), the Nyquist curves of the PI and PR controlled inverters do not surround the point \((-1, j0)\) even the \( L_g \) is 3.2 mH, which indicates the systems can remain stable.

Therefore, under the same conditions, the PI and PR controlled inverters have a similar system stability while the
quasi-PR controlled inverter is more sensitive to the weak grid.

As shown in Fig. 8(a), when $BW_v$ is 140 Hz, the Nyquist curve of the quasi-PR controlled inverter has surrounded the point $(-1, j0)$, which indicates the system is unstable. In Fig. 8(b) and Fig. 8(c), the Nyquist curves of the PI and PR controlled inverters do not surround the point $(-1, j0)$, which indicates the systems are stable.

Thus, under the same operating conditions, compared with the PI and PR controlled inverters, the quasi-PR controlled inverter is more sensitive to high $P_s$.

D. THE EFFECTS OF THE RESONANCE BANDWIDTH ON SYSTEM STABILITY

To analyze the system stability when $\omega_r$ is changed, the Nyquist diagram in Fig. 9 is studied while $L_g$ is 1.6 mH and $P_s$ is 0.6 pu.

In Fig. 9, when $\omega_r$ is increased to $2.8\pi$ rad/s, the Nyquist curve of the quasi-PR controlled inverter has surrounded the point $(-1, j0)$, which indicates that the system is unstable. Therefore, with the increasing of $\omega_r$, the quasi-PR controlled inverter system tends to be unstable.
From the above analysis, it is can be seen that the stabilities of the PI and PR controlled inverters are similar while the stability of the quasi-PR controlled inverter is worse than that of the PI and PR controlled inverters.

In the quasi-PR controlled inverter system, the $\omega_r$ is an important factor that affects the system stability. To reduce the impact on system stability, the $\omega_r$ should be set as small as possible in the area which can meet the control requirement.

VI. EXPERIMENTAL RESULTS
To verify the correctness of the above analysis, experiments were carried out on the controller hardware-in-the-loop experimental platform, as shown in Fig. 10. Experimental parameters are consistent with Section V.

After discretizing, (31) is shown as below.

$$
\begin{align*}
  & y(k) = y(k-1) + T_P \left[ k_{pr} u(k-1) + (2k_r u(k-1) - v(k-1)) \right] \\
  & v(k) = v(k-1) + \omega_0^2 T_P y(k)
\end{align*}
$$

where $T_P$ is PWM carrier period.

According to the discrete expressions of $y$, $G_{pr}(s)$ can be realized in the TI DSP TMS320F2812 control board.

A. THE EFFECTS OF GRID IMPEDANCE ON SYSTEM STABILITY
Fig. 12 shows the experimental results when $L_g$ is increased from 0.8 mH to 2.4 mH. (a) The quasi-PR controlled inverter. (b) The PI controlled inverter. (c) The PR controlled inverter.
When $L_g$ is increased to 2.4 mH, the quasi-PR controlled inverter is unstable while the PI and PR controlled inverters can still maintain stable. Therefore, under the same operating conditions, the PI and PR controlled inverters have a similar system stability, but the quasi-PR controlled inverter is more sensitive to the weak grid.

**B. THE EFFECTS OF THE INVERTER OUTPUT POWER ON SYSTEM STABILITY**

The experimental results when $P_s$ is changed are shown in Fig. 13. When $P_s$ is increased from 0.6 pu to 1 pu, the quasi-PR controlled inverter is unstable while the PI and PR controlled inverters can keep stable.

Thus, under the same operating conditions, compared with PI and PR controlled inverters, the quasi-PR controlled inverter is more sensitive to high inverter output power.

**C. THE EFFECTS OF THE OUTER-LOOP BANDWIDTH ON SYSTEM STABILITY**

The experimental results reflecting the influence of $BW_v$ are shown in Fig. 14. When $BW_v$ is increased from 100 Hz to 140 Hz, the quasi-PR controlled inverter is unstable while the PI and PR controlled inverters can still maintain stable.

Therefore, under the same conditions, $BW_v$ of the quasi-PR controlled inverter should be designed narrower than that of the PI and PR controlled inverters to keep the system stable.
D. THE EFFECTS OF THE RESONANCE BANDWIDTH ON SYSTEM STABILITY

The experimental result when \( \omega_r \) is changed is shown in Fig. 15. When \( \omega_r \) is increased from 2\( \pi \) rad/s to 2.8\( \pi \) rad/s, the quasi-PR controlled inverter system oscillates. The experimental result is consistent with the theoretical analysis in Fig. 9.

Therefore, the stability of the quasi-PR controlled inverter is also affected by the resonance bandwidth of the quasi-PR controller. The theoretical analysis is verified by the experimental results.

VII. CONCLUSION

In this paper, considering the influence of frequency coupling characteristic, a frequency coupling admittance model of the quasi-PR controlled inverter is established, and its stabilities are compared with the PI and PR controlled inverters. Following conclusions are drawn:

1) Admittance characteristics of the PI and PR controlled inverters are similar while the quasi-PR controlled inverter is quite different: the amplitude of the quasi-PR controlled inverter is larger than that of the PI controlled inverter and the phase difference between the two inverters is obvious in the mid-high frequency areas.

2) The resonance bandwidth of the quasi-PR controller is an important factor that causes the mentioned admittance difference. With the increasing of the resonance bandwidth, the quasi-PR controlled inverter system tends to be unstable. To reduce the impact on system stability, the resonance bandwidth should be set as small as possible in the area which can meet the control requirement.

3) The admittance difference has a great impact on system stability. Compared with the PI and PR controlled inverters, the quasi-PR controlled inverter is more sensitive to weak grid and high inverter output power. To achieve the same system stability, the outer-loop bandwidth of the quasi-PR controlled inverter should be designed narrower than that of the PI and PR controlled inverters.

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