Performance of FFT-OFDM versus DWT-OFDM under Compressive Sensing

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Abstract. In this work, we present a comparative study on the performance of Fourier-based OFDM (FFT-OFDM) and wavelet-based OFDM (DWT-OFDM) under compressive sensing (CS). Transmission over FFT-OFDM and DWT-OFDM, which has been made under different baseband modulation schemes such as Binary Phase Shift Keying (BPSK), Quadrature Phase Shift Key (QPSK), Quadrature amplitude modulation (16QAM) and (64QAM) has been considered. From numerical simulation results, it is observed that the Wavelet-based OFDM system outperforms Fourier based OFDM when the Quadrature Amplitude Modulation is 16QAM and 64QAM within the signal to noise ratios range 30 to 40 dB. Although CS is more efficient in compression than classical compression techniques, it introduces more errors over OFDM transmission. Future directions of this work are also suggested.

Keywords. Compressive Sensing, Wavelet, DWT, Fourier, OFDM, DCT.

1. Introduction

In view of the increasing demand for the rapid transfer of data, especially video file, the recently started using Fourier modulation used worldwide in fourth-generation telecommunications systems. This may extend to the fifth generation, but wavelets can also be used for orthogonal modulation. In addition, to support rapid transit, compression modeling emerged in 2007, as data compression occurs during modeling, not beyond.

Communication and some of the modern applications need high data rates, transmission, and system demands to increase the data rate. There are many transmission techniques used to transmit digital signals with different environments. One of the most significant techniques, which proved to be efficient, is the Orthogonal Frequency Division Multiplexing system (OFDM). The OFDM has represented a special form of multi-carrier transmission, since it was first proposed in 1966 [1]. The OFDM is an efficient and flexible modulation technique that is greatly used in wired communication and wireless systems. The OFDM is the basic transmission scheme in fourth-generation (4G) and five-generation (5G) [2], [3].

In multimedia like (voice, image, or video), when we needed to store or communications with other, it is often required to save a large amount of data in a limited space or transmit a large amount of data.
in a short period; therefore, this is one of the challenges for 4G mobile communications. One way to overcome this problem is to compress data efficiently. Therefore, many methods of data compression appeared, but the CS is the best way to the compression, as CS is seen as a form of random sampling, considers the acquisition and reconstruction of sparse or compressible signals at a rate significantly lower than Nyquist, Compact sensor is a new signal processing method that began in 2006 by Donoho, Candes, Romberg, and Tao [1].

In this thesis, we will study the effects of orthogonal (Fourier and wavelet) modulation with compression modeling on the characteristics of signals that are used for medical and security applications, for example, the perception of the speaker or portrait in airports and military applications, in preparation for the fifth generation who is expected to use compression modeling [4].

2. Literature Review

Abdullah & Hussain in (2009) studied the comparison of DWT-OFDM to replace FFT-OFDM with their BER results, showing that DWT-OFDM was better than FFT-OFDM [5]. Abdullah & Hussain in (2011) discussed BER output in DWT-OFDM, The DWT-OFDM features zero padding and transpose vector when transmitting the signal, with impulsive noise effects by varying the Poisson recurrence parameter from small to high, and results show that the impact of impulsive noise on the device is restricted when the value is massive [6]. Parveen, N. et al. (2019) studied Technique for Diversity Using Discrete Wavelet Transform in OFDM Framework DWT-OFDM works better because of avoidance of the cyclic prefix. These also have the feature of a Multicarrier Modulation. Multiple inputs and a different output antenna is used to avoid signal loss and increase the performance of devices. DWT-OFDM, with multiple inputs and multiple output, it used for the future generation [7].

2.1. Background

2.1.1. OFDM

Chang was proposed the first system called OFDM in 1966 [1]. To be transmitted the data over the orthogonal carriers will be originating as a stream. This serial data after parallel transformation OFDM is a modulation technique mainly suited for transmitting signals over which dispersive medium. In OFDM, the signals are orthogonal to each other; it means that they are very independent of one another. Hence, there are many advantages to using OFDM. Though there are some disadvantages like errors in frequency caused by a local oscillator at the transmitting end and the receiving end, it will avoid the ISI effect, the efficiency of the bandwidth is high, it will reduce burst errors and frequency selective fading. The wavelet transforms are considered along with the OFDM [8]. In OFDM wideband fading channels are divided into narrowband sub-channels, it means that OFDM divides the complete spectrum into many sub-channels [21].

2.1.2. Fourier-based OFDM

A model diagram of an FFT-OFDM system is shown in Figure 1. The system model for FFT-based OFDM, first the input data as an image, convert to 0 and 1 by the encoder and then being processed by a mapping. M-ary QAM modulator is used for this work to map the raw binary data to appropriate QAM symbols. These symbols are then inputted into the IFFT block. This involves taking N parallel streams of QAM symbols (N being the number of sub-carriers used in the transmission of the data in this paper used PFSK, 4-QAM, 16QAM, and 64QAM) and performing an IFFT operation on this parallel stream. The mathematical format in the time domain shown in the equation. in the receiver side Reflect the process and used FFT[23]:

\[
X = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X e^{i 2\pi km/N} \quad (1)
\]
2.1.3. Wavelet-based OFDM

The difference between FFT-OFDM and DWT-OFDM is that the unit does not need to enter CP into the OFDM code, which reduces bandwidth, and does not lose data and transmit it very quickly [9]. The method uses inverse discrete wavelet transformation (IDWT) and discrete wavelet transformation (DWT) to replace IFFT and FFT in transmitter and receiver, respectively, as shown in Figure 1.

**Discrete Wavelet Transform (DWT) transmitter**

DWT decomposes the input signal into the number of sub-bands defined as approximate coefficients and information. These coefficients provide a much better study of both time and frequency scaling for a signal.

The transform uses the estimated and accurate filter coefficients in LPF and HPF, respectively. Sometimes, the approximated coefficients are referred to as scaling coefficients, while the accurate coefficients are referred to as wavelet coefficients [3][24]. These two filters may often be called sub-band coding because the signals are separated into low and high-frequency sub-signals respectively [9].

**Discrete Wavelet Transform (DWT) receiver**

The DWT receiver consists of a converter used to transform the data from parallel to serial as shown in FFT. After passing from the channel that affects the signal by Additive white Gaussian noise AWGN, next the signal is reconstructed using reconstruction filters that satisfy the wavelet reconstruction [9] as shown in the Figure 3 below[25]. The original signal is obtained by a sequential grouping of the resulting factors (approximate and detailed); therefore, the signal reconstruction, by passing through low pass filter and high pass filter, does the convolution of the approximate coefficient $y_a$ with low pass filter and detail coefficients $y_d$ with High pass Filter, then generate a sequence as the output decomposition the signal as shown in the mathematical relation[7].

![Diagram of FFT/DWT-OFDM](image-url)
2.2. Compressed sensing
Compressed sensing is an exciting, fast-growing field that has attracted considerable interest in signal processing, statistics, and computer science, as well as its importance in the wider scientific community. The new theory called compressive sensing assumes that there are many areas in the frequency representation of the signal that are, i.e., empty areas exist that should not be sampled. In 2008, a new approach in CS was invented based on chaos filters [10]. Compressed sensing (CS), recently been introduced by Candes and Tao [11] and Donoho [12] as a type of random undersampling, allows for the acquisition and reconstruction of sparse/compressible signals at a rate lower than that of Nyquist. First, random linear projection is used to acquire efficient representations of the signals directly. Then, nonlinear techniques are used to The structure of the optimization equation (finding the best value for the vector under certain conditions) is using one of the aggregate (norm) formulas for the vector elements.

2.2.1. Sparse Signal Acquisition
The signal (input image), represented as a column vector, this signal Considers a real, finite, one-dimensional, can represent signal x as SPARES vector by expressed as

\[ x = \sum_{i=1}^{N} \psi_i s_i = \Psi s \]  

(2)

Where \( x \): Signal expressed in the time domain.

\( s \): The vector of coefficients.

\( \Psi \): Discrete cosine transform matrix.

\[ \Psi = \begin{bmatrix} \psi_1 & \psi_2 & \cdots & \psi_N \end{bmatrix} \]

\[ \psi_i = \begin{bmatrix} \psi_{i1} & \psi_{i2} & \cdots & \psi_{iN} \end{bmatrix} \]

\[ \mathbf{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix} \]

\[ \mathbf{S} = \begin{bmatrix} s_1 & s_2 & \cdots & s_N \end{bmatrix} \]

\[ \mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix} \]

\[ \mathbf{\Phi} = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_N \end{bmatrix} \]

\[ \mathbf{\Psi} = \begin{bmatrix} \psi_1 & \psi_2 & \cdots & \psi_N \end{bmatrix} \]

\[ \mathbf{\Theta} = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_N \end{bmatrix} \]

Where \( \Phi \): is defined as the random matrix with Gaussian i.i.d. element.

\( y \): Is the measurement vector.

2.2.2. Reconstruction Signal

A measurement vector \( y \) consisting of \( m < n \) linear vector \( x \) projections. This can be described compactly through the [18][19]

\[ y = \Phi x \]  

(3)

Then, \( y \) can be written, by substitution equation 2 in above as shown

\[ y = \Phi \Psi s = \Theta s \]  

(4)

\[ \mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_m \end{bmatrix} \]

\[ \mathbf{\Phi} = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_n \end{bmatrix} \]

\[ \mathbf{\Psi} = \begin{bmatrix} \psi_1 & \psi_2 & \cdots & \psi_m \end{bmatrix} \]

\[ \mathbf{\Theta} = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_m \end{bmatrix} \]

\[ \mathbf{s} = \begin{bmatrix} s_1 & s_2 & \cdots & s_n \end{bmatrix} \]

\[ \mathbf{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \end{bmatrix} \]
The signal reconstruction algorithm must take the measurements of M in the vector \( y \), the random matrix as well as the basis and reconstruct equivalently the signal x or its sparse coefficient vector \( s \). Equation (4) can be solved by the structure of the optimization equation is using one of the aggregate (norm) formulas for the vector elements[20].

- **Restoration of the minimum \( L_0 \) norm**: The sum of the elements whose value is not equal to zero.
  \[
  \hat{s} = \text{argmin} \| s' \|_0 \tag{5}
  \]
  Can recover a correct sparse signal but needs an exhaustive listing of all possible positions of non-zero inputs in \( s \), hence that numerically unstable and non-complete reconstruction.

- **Reconstruction of the minimum \( L_1 \) norm**: The sum of the absolute values
  \[
  \hat{s} = \text{argmin} \| s' \|_1 \tag{6}
  \]
  Luckily, the optimization is based on the norm \( L_1 \) and can exactly recover K-sparse signals and closely approximate compressible signals with high probability.

**Reconstruction of minimum \( L_2 \) norm**: The square of values as shown down
  \[
  \hat{s} = \text{argmin} \| s' \|_2 \tag{7}
  \]
  The \( L_2 \) norm, tests signals energy and does not signal sparsity, therefore, this norm never finds a sparse solution.

### 2.3. Classical Compression

The term compression refers to reducing the quantity of data used to represent (image, sound, and video) without far reducing the quality of the original data. The main purpose of image compression is to reduce the redundancy and irrelevancy present in the image so that it can be stored and transferred efficiently. The bits in the compressed image is less number of bits compared to the original. Therefore, the required storage size will reduce, so big size images can be stored and it can transfer in a faster way to save time, transmission bandwidth [13].

#### 2.3.1. Discreet Cosine Transfer

Proposed the DCT by Ahmed et al. [14]. Discrete Cosine Transform used to process single or two-dimensional data. It is a widely used unitary transform for video and image applications. DCT, such as DFT provides information of the signal in the frequency domain [15]. The important feature of the DCT, the feature that makes it so useful in data compression, is that it takes correlated input data and concentrates its energy in just the first few transform coefficients. It is used cosine function wave among many waves as basis function and deals with real-valued signals and spectral coefficients [4]. The DCT represents the transformation of the original image \( x(n,m) \) pixels to their DCT coefficients; the coefficients equal the original image pixels in number and, for 2D image can be defined as mathematical form [4][22]

\[
(r, k) = \frac{pr pk}{N} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} x(n, m) \cos \frac{\pi (2n + 1)r}{2N} \cos \frac{\pi (2m + 1)k}{2N} \tag{8}
\]

Where:

\[0 \leq r, k, n, m \leq N\]

\[p_v = \begin{cases} 
1 & v = 0 \\
\sqrt{2} & v \neq 0 \end{cases}\]

#### 2.3.2. Discrete Wavelet Transform

Wavelets are useful for compressing signals. Wavelets can be used to process, compress and improve signals, in fields such as medical imaging where image smashing when is not bear. It can be used to remove noise from an image [16]. The wavelet transform provides a representation of the time and frequency of the signal [17]. DWT has widely used in several areas such as engineering, computer
science, and mathematics. DWT based on a group of discrete functions or filters, which convolved with the input data, signal to affect translation. It is effectively stretched by sub-sampling the signal as it is passed out of each scale [15]. The way to compress data using wavelet is to assume a threshold then cut all coefficients below this threshold, coefficients It is given by [4].

\[ C(l, j) = \sum x(n). \psi_{l,j}(n) \]  

Where: \(C(l, j)\) represent the coefficients ,  
\(\Psi_{l,j}(n):\) The wavelet.

3. Research Method

At the transmitter, the data signals (image) that were compressed in three ways classical compression (DCT, WAV) and compressive sensing to be transmit using FFT-OFDM and DFT-OFDM techniques. After modulation by rectangular modulation with different degrees, data are converted from serial to parallel then converted from the frequency domain to the time domain by using IFFT if used FFT-OFDM and IDWT when used DWT-OFDM. A cyclic prefix (CP) is inserted into each symbol to prevent the inter symbol interference (ISI) but this step is not needed for dealing with DWT-OFDM, because it does not require the addition of a periodic prefix, and then converted from parallel to serial. The OFDM signal is ready for transmission. At the receiver, all stages of the transmitter will be reversed, which include analog to digital converter, converting from serial to parallel, cyclic prefix removal, converting the data from the time domain to the frequency domain by using FFT and DWT, applying demodulation of rectangular modulation, and converting from parallel to serial. After these stages, Mean squared error (MSE) measures are applied to know which properties of signals have been affected of the transmission process through the noisy OFDM system and the comparison between two systems is, FFT-OFDM and DWT-OFDM to know which is better in the transmission.

![Figure 4. FFT/DWT-OFDM Transmission System with three Type of Compression.](image)

4. Results and Discussion

Using MATLAB, we implemented FFT-OFDM and DWT-OFDM with (BPSK, QPSK,16QAM, 64QAM) modulation, on the compressed image in three methods (Discrete Cosine Transform, Discrete Wavelet Transform, and Compressive Sensing) with compression ratio (CR) equal two, where the signal to noise ratio (SNR) ranges from -40 dB to 40 dB; assuming wired communication system, the compressed signal passed on only via one path. After compressed signals are receiving from FFT/DWT-OFDM, The mean squared error (MSE) of the sent and received image is calculated. Figures 1-2 show the DCT-Compressed Image Transmission over FFT-OFDM.
The compressed image by DCT when sending over FFT-OFDM with different QAM modulation shown in the figure below

Through the above Figure, it was found that when using different degrees of QAM while sending compressed images via FFT-OFDM, the higher the QAM degree, the faster the speed of transmission with the greater the error rate. While Figures 7 show the DCT-Compressed Image Transmission over DWT-OFDM
Figures below show compressing the image by wavelet and sending it through the DWT-OFDM system.

The classic compression that includes DCT and WAV have been used. Now we will compress the image using the compressive sensing method with compression ratio (CR), which is a modern method.

Now shown CS over DWT-OFDM
Figure 11. CS over DWT-OFDM.

Future directions:
Compressive sensing is getting attention in recent years. Hence, the research performed in this paper could be applied to Nakagami channels [26]. Power savings for OFDM is also an important direction to consider with CS [27]. Impulsive noise and power line communications would also be good to consider [28].

5. CONCLUSIONS
In this paper, we used OFDM system with FFT-OFDM and DWT-OFDM types to transfer compressed images in three ways (DCT, WAV, and Compressive Sensing) under White Gaussian Noise with BPSK, QPSK, 16QAM, and 64QAM modulation and show the following:

DWT-OFDM performs better than FFT-OFDM because of DWT-OFDM the cyclic prefix avoidance, which leads to the utilization of Bandwidth. Unlike FFT-OFDM, and it observed that with the same SNR dB and compression ratio, DWT-OFDM provides low Mean Square Error (MSE) as compared to FFT-OFDM. Therefore, The Wavelet-based OFDM system is better than Fourier based OFDM when the Quarter Amplitude Modulation is 16QAM and 64QAM and the signal ratio ranges from 20 to 40. Finally, the performance of the methods that implement the wavelet has overcome the performance of the methods that implement the Fourier of the same signal (image).

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References
[1] Hwang, Taewon and Yang, Chenyang and Wu, Gang and Li, Shaoqian and Li, Geoffrey Ye, "OFDM and its wireless applications: A survey," IEEE transactions on Vehicular Technology, vol. 58, no. 4, pp. 1673-1694, 2008.
[2] Dahlman, Erik and Parkvall, Stefan and Skold, Johan, 5G NR: The next generation wireless access technology, Academic Press, 2018.
[3] Correia, Luis M, Mobile broadband multimedia networks: techniques, models and tools for 4G, Elsevier, 2010.
[4] Hussain, Zahir M and Sadik, Amin Z and O’Shea, Peter, Digital signal processing: an introduction with MATLAB and applications, Springer Science & Business Media, 2013.
[5] Abdullah, Khaizuran and Hussain, Zahir M and others, "Studies on dwt-ofdm and fft-ofdm systems," in IEEE International Conference on Communication, Computer and Power, 2009, pp. 15--18.
[6] Abdullah, Khaizuran and Hussain, Zahir M, "Simulation of models and BER performances of DWT-OFDM versus FFT-OFDM," Discrete Wavelet Transforms-Algorithms and Applications, 2011.
[7] Parveen, Nagma and Abdullah, Khaizuran and Islam, Rafiqul and Boby, Rounakul Islam, "Diversity Technique Using Discrete Wavelet Transform In OFDM System," Int. J. Eng. Adv. Technol., vol. 8, no. 2S2, pp. 284--287, 2019.
[8] Bodhe, Rohit and Narkhede, Satish and Joshi, Shirish, "Design of Simulink Model for OFDM and Comparison of FFT-OFDM and DWT-OFDM," International Journal of Engineering Science and Technology, vol. 4, no. 5, pp. 1914--1924, 2012.

[9] Naveen, H and Sreerama Reddy, GM, "Performance Analysis of FFT based OFDM and DWT based OFDM for Underwater Acoustic Communication," Indian Journal of Science and Technology, vol. 10, no. 36, 2017.

[10] Van Phong, Dinh and Linh-Trung, Nguyen and Tan, Tran Duc and Le, Ha Vu and Do, Minh N, "Fast image acquisition in magnetic resonance imaging by chaotic compressed sensing," in 2011 IEEE International Symposium on Biomedical Imaging: From Nano to Macro, IEEE, 2011, pp. 85--88.

[11] Candès, Emmanuel J and Romberg, Justin and Tao, Terence, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," IEEE Transactions on Information Theory, vol. 52, pp. 489--509, 2006.

[12] Donoho, David L, "Compressed sensing," IEEE Transactions on Information Theory, vol. 52, pp. 1289--1306, 2006.

[13] Singh, Harjeetpal and Sharma, Sakhi, "Hybrid image compression using DWT, DCT & Huffman encoding techniques," International Journal of Emerging Technology and Advanced Engineering, vol. 2, pp. 2250--2459, 2012.

[14] Ahmed, Nasir and Natarajan, T. and Rao, Kamisetty R, "Discrete cosine transform," IEEE transactions on Computers, vol. 100, no. 1, pp. 90--93, 1974.

[15] Bull, David R, Communicating pictures: A course in Image and Video Coding, Academic Press, 2014.

[16] Deshlahra, Archana and Shirnewar, GS and Sahoo, AK, "A comparative study of DCT, DWT & hybrid (DCT-DWT) transform," 2013.

[17] Vyas, Aparna and Yu, Soohwan and Paik, Joonki, "Wavelets and Wavelet Transform," in Multiscale transforms with application to image processing, Springer, 2018, pp. 45--92.

[18] Valls, Javier and Sansaloni, Trini and Perez-Pascual, Asun and Torres, Vicente and Almenar, Vicen, "The use of CORDIC in software defined radios: A tutorial," IEEE communications magazine, vol. 44, pp. 46--50, 2006.

[19] Dick, Chris and Harris, Fred and Rice, Michael, "Synchronization in software radios. Carrier and timing recovery using FPGAs," in Proceedings 2000 IEEE Symposium on Field-Programmable Custom Computing Machines (Cat. No. PR00871), 2000.

[20] Baraniuk, Richard G, "Compressive sensing [lecture notes]," IEEE Signal Processing Magazine, vol. 24, pp. 118--121, 2007.

[21] Dihin, Rashsa A; Hussain, Zahir M, "Face recognition over FFT-OFDM computer networks," International Journal of Applied Engineering Research, vol. 12, pp. 14764--14773, 2017.

[22] ABOSINNEE, ALIS; HUSSAIN, ZAHIR M, "STATISTICAL VS. INFORMATION-THEORETIC SIGNAL PROPERTIES OVER FFT-OFDM," Journal of Theoretical and Applied Information Technology, vol. 97, 2019.

[23] E. Dahlman, S. Parkvall, and J. Sköld, 4G LTE/LTE-Advanced for Mobile Broadband. Academic Press, 2011.

[24] Hariprasad, N and Sundari, G, “Performance comparison of DWT OFDM and FFT OFDM in presence of CFO and Doppler effect,” 2014 International Conference on Control, Instrumentation, Communication and Computational Technologies (ICCICCT) “567—570, 2014, IEEE.

[25] D. Krishna and M. S. Anuradha, “Image Transmission through OFDM System under the Influence of AWGN Channel”, IOP Conference, Series: Materials Science and Engineering, vol., 225, no. 1, 2017.

[26] Gurung, Arun K., et al, “Performance analysis of amplify-forward relay in mixed Nakagami-m and Rician fading channels,” IEEE International Conference on Advanced Technologies for Communications (ATC 2010), Vietnam, 2010.
[27] Gurung, Arun K., et al, "Power Savings Analysis of Clipping and Filtering Method in OFDM Systems," Australasian Telecommunication Networks and Applications Conference (ATNAC’08), Adelaide, 2008.

[28] Al-Mawali, Khalifa S., et al, “Time-Domain Techniques for Impulsive Noise Reduction in OFDM-Based Power Line Communications,” International Conference on Communication, Computer and Power (ICCCP’09), Muscat, Oman, 2009.

[29] Al-asadi TA, Obaid AJ (2016) Discovering similar user navigation behavior in Web log data. Int J Appl Eng Res 11(16):8797–8805.