The origin of single transverse-spin asymmetries in high-energy collisions

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In this letter, we present, for the first time, a phenomenological analysis that demonstrates single transverse-spin asymmetries in high-energy collisions have a common origin. We perform the first global fit of data from semi-inclusive deep inelastic scattering, Drell-Yan, $e^+e^-$ annihilation into hadron pairs, and proton-proton collisions. Consequently, we are able to extract a universal set of non-perturbative functions that describes the observed asymmetries in these reactions. Furthermore, we achieve the first phenomenological agreement with lattice on the up and down quark tensor charges of the nucleon.

Introduction. For some fifty years, the spin and momentum structure of hadrons has been investigated in terms of their partonic (quark and gluon) content within the theory of Quantum Chromodynamics (QCD). Single transverse-spin asymmetries (SSAs) have played a central role in these studies and continue to pose a number of challenges and puzzles. Early predictions from QCD that SSAs in single-inclusive hadron production should be exceedingly small [1] were in stark contrast with measurements showing large asymmetries [2, 3] that persist in recent experiments [4–18].

A better understanding of SSAs has emerged with the aid of QCD factorization theorems [19–23]. They separate cross sections into short distance, perturbatively calculable scattering contributions and long distance, non-perturbative physics that are encoded in parton distribution functions (PDFs) and fragmentation functions (FFs). QCD factorization theorems constrain the definitions of PDFs and FFs, and they ultimately lead to equations governing how those functions evolve with the energy scale.

For processes with one large measured scale, $Q \gg \Lambda_{\text{QCD}}$, where $\Lambda_{\text{QCD}}$ is a typical hadronic mass scale, experiments are sensitive to the collinear motion of partons. For example, in $p^p \to h X$, the hard scale is set by the hadron transverse momentum $P_{hT}$. In this case, collinear twist-3 (CT3) factorization [19, 20] is valid, and spin asymmetries arise due to the quantum mechanical interference from multi-parton states, such as quark-gluon-quark or tri-gluon [19, 20, 24–32].

For reactions with two well-separated scales $Q_1 \gg Q_2$, experiments probe not only collinear but also intrinsic parton motion that is transverse to the parent hadron’s momentum. For example, in semi-inclusive lepton-nucleon deep inelastic scattering (SIDIS), $\ell N \to \ell h X$, one has $\Lambda_{\text{QCD}} \sim P_{hT} \ll Q$, where $-Q^2$ is the photon virtuality. For such processes, transverse momentum dependent (TMD) factorization [21–23, 33, 34] is valid, and the mechanism responsible for spin asymmetries is encoded in TMD PDFs and FFs (collectively called TMDs) [35–40].

There is theoretical evidence that CT3 and TMD factorization theorems yield a unified picture of spin asymmetries in hard processes [41–46]. This is one of the cornerstones for studying the 3-dimensional structure of hadrons at existing [47–51] and future facilities, including the Electron-Ion Collider [52, 53]. However, it has never been shown that one can simultaneously fit a universal set of non-perturbative functions to SSAs in both types of reactions [54–57]. In this letter, we provide, for the first time, a phenomenological demonstration that SSAs have a common origin. We perform the first simultaneous global analysis of the available data in SIDIS, Drell-Yan (DY), semi-inclusive $e^+e^-$ annihilation (SIA), and proton-proton collisions. Furthermore, we find, for the first time, excellent agreement with lattice QCD for the up and down quark tensor charges.

Theoretical Background. The key observation that makes our analysis possible is that in both the CT3 and TMD formalisms, collinear multi-parton correlations play an important role. A generic TMD PDF $F(x,k_T)$ depends on $x$, the fraction of the nucleon’s longitudinal momentum carried by the parton, and $k_T \equiv |k_T|$, the parton’s transverse momentum. The same TMD when Fourier conjugated into position ($b_T$) space [34, 58–60] exhibits an Operator Product Expansion (OPE) in the limit when $b_T$ is small. TMDs relevant for SSAs can be expressed in terms of CT3 multi-parton correlation functions in this OPE [60–63].

Another way to establish the connection between CT3 functions and TMDs is by the use of parton model iden-
ties. One such relation is [64]

$$\pi F_{FT}(x, x) = \int d^2 p_\perp \frac{k_T^2}{2M_h^2} f_{1T}^\perp(x, k_T^2) \equiv f_{1T}^{(1)}(x),$$

where $F_{FT}(x, x)$ is the Quin-Sterman CT3 matrix element, and $f_{1T}^{(1)}(x)$ is the first moment of the TMD Sivers function $f_{1T}^\perp(x, k_T^2)$ [65, 66]. The dependence of this relation on the energy scale is under further investigation since TMDs and CT3 functions have different divergences that make their evolution principally different. As mentioned, here we use parton model identities and do not address their validity beyond leading order [60–63]. We also employ a Gaussian parametrization for the transverse momentum dependence of all TMDs. This assumes that most of the transverse momentum dependence is non-perturbative and is thus related to intrinsic properties of the colliding hadrons rather than to hard gluon radiation.

A central focus of TMD asymmetries has been on the Sivers and Collins SSAs in SIDIS, $A_{UT}^{\text{SIDIS}}$ [67–72] and $A_{UT}^{\text{SIDIS}} \equiv A_{\text{SIDIS}}^\text{Col}$ [68–71, 73]; Sivers SSA in DY, $A_{DY}^{\text{SIDIS}}$, for $W^+/Z^-$ production $\equiv A_N^{W/Z}$ [74] and for $\mu^+\mu^-$ production $\equiv A_{\text{SIDIS}}^{\mu^+\mu^-}$ [75]; and Collins SSA in SIA, $A_{\text{SIA}}^{\text{SIDIS}}$ [76–80]. The relevant TMDs probed by these processes [35–40] are the transverse TMD $h_1(x, k_T^2)$ [81], the Sivers function $f_{1T}^\perp(x, k_T^2)$ [65, 66], and Collins function $H_1^\perp(z, z^2 p_\perp^2)$ [82]. Each of them can be written in terms of a collinear counter-part using the OPE. The function $h_1(x, k_T^2)$ is related to the collinear (twist-2) transversity function $h_1(x)$ [83]; $f_{1T}^\perp(x, k_T^2)$ to the Quin-Sterman function $F_{FT}(x, x)$ [60]; and $H_1^\perp(z, z^2 p_\perp^2)$ to its first $p_\perp$-moment [84],

$$H_1^{(1)}(z) = z^2 \int d^2 p_\perp \frac{p_\perp^2}{2M_h^2} H_1^\perp(z, z^2 p_\perp^2),$$

where $M_h$ is the hadron mass and $p_\perp$ the parton transverse momentum.

The same set of functions, $h_1(x)$, $F_{FT}(x, x)$, $H_1^{(1)}(z)$, that arise in the OPE of TMDs are also the non-perturbative objects that drive the collinear SSA $A_N^h$ in $p^+p \to h X$ [26, 28, 30–32]. In fact, in the CT3 framework, the main cause of $A_N^h$ can be explained by the coupling of $h_1(x)$ to $H_1^{(1)}(z)$ and another multi-parton correlator $\tilde{H}(z)$ [56, 57]. The latter generates the $F_{HT}$-integrated SIDIS $A_{UT}^{\text{SIDIS}}$ asymmetry by coupling with $h_1(x)$ [38].

One can therefore argue that SSAs have a common origin, namely, multi-parton correlations. We present, for the first time, a phenomenological verification of this assertion by simultaneously fitting $A_{\text{SIDIS}}^{\text{SIDIS}}$, $A_{\text{SIDIS}}^{\text{Col}}$, $A_{\text{DY}}^{\text{SIDIS}}$, $A_{\text{SIA}}^{\text{SIDIS}}$, and $A_N^h$, and extracting the non-perturbative functions $h_1(x)$, $F_{FT}(x, x)$, and $H_1^{(1)}(z)$, along with the relevant transverse momentum widths. (Ultimately, $\tilde{H}(z)$ was set to zero in our analysis, as will be explained in the Methodology section.)

We further claim that such an analysis exhibits universal properties for the underlying partonic functions and therefore operates as a consistency test on the validity of the theoretical framework itself. In particular, a set of necessary conditions for a universality test is as follows: 1) The system must be over-constrained. That is, the number of equations relating partonic functions to observables must be larger than the total number of partonic functions. 2) Each function must appear at least twice in such equations. 3) There must be reasonable kinematical coverage between observables. These conditions are satisfied in the present analysis, as summarized in Table I. There is also considerable kinematical overlap in $x$, $z$, and $Q^2$ between the observables. SIDIS covers a region $x \lesssim 0.3$, $0.2 \lesssim z \lesssim 0.6$, and $2 \lesssim Q^2 \lesssim 40 \text{GeV}^2$. SIA data has $0.2 \lesssim z \lesssim 0.8$ and $Q^2 \approx 13 \text{GeV}^2$ or $110 \text{GeV}^2$. For DY data, $0.1 \lesssim x \lesssim 0.35$ and $Q^2 \approx 30 \text{GeV}^2$ or $(80 \text{GeV})^2$. Lastly, $A_N^h$ integrates from $x_{\text{min}}$ to 1 and $z_{\text{min}}$ to 1, where $0.2 \lesssim (x_{\text{min}}, z_{\text{min}}) \lesssim 0.7$, with $1 \lesssim Q^2 \lesssim 13 \text{GeV}^2$.

**Methodology.** In order to perform our global analysis, we must postulate a functional form for the non-perturbative functions. Since we use the parton model relations between CT3 and TMD functions, for the TMDs we will employ a simple Gaussian parametrization for the transverse momentum dependence. This is a standard approach within the literature – see, e.g., Refs. [85–87]. The dependence of the TMDs on the parton longitudinal momentum fraction is constructed from the collinear functions that arise in the OPE.

This type of parametrization does not have the complete features of TMD evolution, in particular the broadening of the transverse momentum widths. However, it was shown in Refs. [88, 89] that utilizing such a parametrization gives results that are compatible with full TMD evolution [84, 90–93]. In addition, asymmetries are ratios of cross sections where evolution effects tend to cancel out [93].

For the unpolarized and transversity TMDs we have

$$f^q(x, k_T^2) = f^q(x) \mathcal{G}_f^q(k_T^2),$$

where the generic function $f^q = f_1^q$ or $h_1^q$, and

$$\mathcal{G}_f^q(k_T^2) = \frac{1}{\langle k_T^2 \rangle_f} \exp \left[ -\frac{k_T^2}{\langle k_T^2 \rangle_f} \right].$$

Using the relation $\pi F_{FT}(x, x) = f_{1T}^{(1)}(x)$ [64], the Sivers function reads

$$f_{1T}^{\perp}(x, k_T^2) = \frac{2M^2}{(k_T^2)^2} \pi F_{FT}(x, x) \mathcal{G}_f^q (k_T^2).$$

The transverse momentum widths $\langle k_T^2 \rangle_f^q$ are in general flavor dependent, and can be functions of $x$, although here we assume there is no $x$ dependence.
For the TMD FFs, the unpolarized function is parametrized as
\[ D_1^{h/q}(z, z^2 p_\perp^2) = D_1^{h/q}(z) \, g_{D_1^{h/q}}(z^2 p_\perp^2), \]
(6) while the Collins FF reads
\[ H_{1h/q}^{+}(z, z^2 p_\perp^2) = \frac{2z^2 M_h^2}{g_{D_1^{h/q}}(z) H_{1h/q}^{+}(z^2 p_\perp^2)}, \]
where we have explicitly written its dependence in terms of its first moment \( H_{1h/q}^{+}(z) \). For \( f_2^{\perp}(x) \) and \( D_2^{\perp}(x) \) we use the leading order CJ15 [94] and DSS [95] functions. The pion PDFs are taken from Ref. [96].

Note Eqs. (3), (5), (7) make clear that the underlying non-perturbative functions, \( h_1(x) \), \( F_{FT}(x, x) \), \( H_{1h/q}^{+}(z) \), that drive the (TMD) SSAs \( A_{SIDIS}^{Col}, A_{SIDIS}^{SIV}, A_{SIDIS}^{SIV} \), and \( A_{Col}^{SIV} \) (along with \( H(z) \)). We generically parametrize these collinear functions as
\[ F^q(x) = \frac{N_q \, x^{\alpha_q} (1 - x)^{b_\perp} (1 + \gamma_q \, x^{\alpha_q} (1 - x)^{b_\perp})}{B[a_q + 2, b_\perp + 1] + \gamma_q B[a_q + \alpha_q + 2, b_\perp + \beta_\perp + 1]}, \]
where \( F^q = h_1^q \), \( \pi F^{\perp}_{FT} \), \( H_{1h/q}^{+} \) (with \( x \rightarrow z \) for the Collins function), and \( B \) is the Euler beta function. In the course of our analysis, we found that \( H(z) \) was consistent with zero within error bands. Moreover, if one considers the relative error of the moment \( F^{(1)} \equiv \int_0^1 dx \, x \, F(x) \) of the various functions in our fit, \( h_1(x) \), \( \pi F^{\perp}_{FT}(x, x) \), and \( H_{1h/q}^{+}(z) \) all have \( \delta F^{(1)}/F^{(1)} \lesssim 1.5 \), whereas for \( H(z) \), \( \delta F^{(1)}/F^{(1)} \gg 1.5 \). This indicates that there is no significant signal for \( H(z) \) from \( A_{SIV}^{h/q} \) data alone, and the function simply emerges as noise in our fit. Therefore, data on the aforementioned (\( P_{T}\)-integrated) \( A_{SIDIS}^{h/q} \) asymmetry in SIDIS is needed to properly constrain \( H(z) \). For now, we set \( H(z) \) to zero, which is consistent with preliminary data from HERMES [97] and COMPASS [98] showing a small \( A_{SIDIS}^{h/q} \).

For the collinear PDFs \( h_1^{\perp}(x) \) and \( \pi F^{\perp}_{FT}(x, x) \), we only allow \( q = u, d \) and set anti-quark functions to zero. For both functions we also set \( b_u = b_d \). For the collinear FF \( H_{1h/q}^{+}(z) \), we allow for favored \( (f_{av}) \) and unfavored \( (uf) \) parameters. We also found that the set of parameters \( (\gamma, \alpha, \beta) \) is needed only for \( H_{1h/q}^{+}(z) \), due to the fact that the data for \( A_{SIV}^{Col} \) has a different shape at smaller versus larger \( z \). Since those data (and the ones for \( A_{SIDIS}^{SIV} \)) are at \( z \gtrsim 0.2 \), we set \( \alpha_{f_{av}} = \alpha_{uf} = 0 \), similar to what has been done in fits of unpolarized collinear FFs [99]. This gives us a total of 20 parameters for the collinear functions. There are also 4 parameters for the transverse momentum widths associated with \( h_1 \), \( f_1^{\perp} \), and \( H_{1h/q}^{+} \): \( \langle k_T^2 \rangle_{f_1^{\perp}} \), \( \langle k_T^2 \rangle_{H_{1h/q}^{+}} \), \( \langle k_T^2 \rangle_{f_1^{\perp}} \), \( \langle k_T^2 \rangle_{H_{1h/q}^{+}} \), \( \langle k_T^2 \rangle_{f_1^{\perp}} \), \( \langle k_T^2 \rangle_{H_{1h/q}^{+}} \), \( \langle k_T^2 \rangle_{f_1^{\perp}} \), \( \langle k_T^2 \rangle_{H_{1h/q}^{+}} \). The pion PDF widths are taken to be the same as those for the proton. We include normalization parameters for each data set that vary within the quoted experimental normalizations uncertainties. This results in an additional 77 “mismeasure” parameters.

We use Bayesian inference in order to sample the posterior distribution for all parameters. Due to the large dimensionality of the parameter space, we use the multistep strategy in the Monte Carlo framework developed in Ref. [100]. Our partonic distributions are inferred from about 1000 Monte Carlo samples drawn from the Bayesian posterior distribution.

We also implement a DGLAP-type evolution of the collinear functions analogous to Ref. [101], where a double-logarithmic \( Q^2 \)-dependent term is explicitly added to the parameters. Note that the transverse momentum widths do not vary with \( Q^2 \). We leave a more rigorous treatment of the complete TMD and CT3 evolution for future work.

**Phenomenological Results.** Using the above methodology, we fit SSA data from HERMES [67, 73], COMPASS [68, 70, 75], Belle [76], BaBar [77, 78], BESIII [79], BRAHMS [9], and STAR [7, 10, 13, 74]. For \( A_{SIDIS}^{SIV}, A_{SIDIS}^{SIV}, A_{SIDIS}^{SIV}, \) and \( A_{SIV}^{h/q} \) we focus on pion production data, while for \( A_{SIDIS}^{h/q} \) we use both the \( \mu^+ \mu^- \) pair production data

| Observable | Reactions | Non-Perturbative Function(s) | $\chi^2/N_{\text{pts.}}$ | Exp. Refs. |
|------------|-----------|-------------------------------|-------------------------|-----------|
| $A_{SIDIS}^{Col}$ | $e^+ (p, d)^+ \rightarrow e^- (\pi^+, \pi^-, \rho^0) + X$ | $f_1^\perp(x, k_T^2)$, $H_{1h/q}^{+}(z, z^2 p_\perp^2)$ | 150.0/162 = 1.19 | [67, 68, 70] |
| $A_{SIDIS}^{SIV}$ | $e^+ (p, d)^+ \rightarrow e^- (\pi^+, \pi^-, \rho^0) + X$ | $h_1(x, k_T^2), H_{1h/q}^{+}(z, z^2 p_\perp^2)$ | 111.3/162 = 0.88 | [68, 70, 73] |
| $A_{SIV}^{h/q}$ | $e^+ e^- \rightarrow \pi^+\pi^- (U, U)^+ \pi^+\pi^- (U, U)$ | $H_{1h/q}^{+}(z, z^2 p_\perp^2)$ | 154.5/162 = 0.88 | [76-79] |
| $A_{SIDIS}^{SIV}$ | $\pi^+ + p \rightarrow \mu^+ \mu^- + X$ | $f_1^\perp(x, k_T^2)$, $H_{1h/q}^{+}(z, z^2 p_\perp^2)$ | 5.96/12 = 0.50 | [75] |
| $A_{SIDIS}^{SIV}$ | $p^+ + p \rightarrow (W^+, W^-, Z) + X$ | $f_1^\perp(x, k_T^2)$, $H_{1h/q}^{+}(z, z^2 p_\perp^2)$ | 31.8/17 = 1.87 | [74] |
| $A_{SIV}^{h/q}$ | $\pi^+ + p \rightarrow (\pi^+, \pi^-, \rho^0) + X$ | $h_1(x), F_{FT}(x, x) = \frac{1}{2} f_1^\perp(x, k_T^2), H_{1h/q}^{+}(z, z^2 p_\perp^2)$ | 66.5/60 = 1.11 | [7, 9, 10, 13] |
from COMPASS and the weak gauge boson production data from STAR. For $A_{\Sigma A}^{\text{Col}}$ we have only included the so-called $A_0$ asymmetry since this observable has a TMD factorization theorem. We only include $A_N^S$ data with $P_{hT} > 1$ GeV in order to stay within the regime where the CT3 formalism is applicable. Similarly, we do not include low-energy SSA data from JLab due to concerns about the pion production mechanism at relatively low energies [105–107]. The standard cuts [108] of $0.2 < z < 0.6$, $Q^2 > 1.63$ GeV$^2$, and $0.2 < P_{hT} < 0.9$ GeV have been applied to all SIDIS data sets, giving us a total of 517 SSA data points in the fit along with 807 HERMES multiplicity [99] data points.

The extracted functions [109] and their comparison to other groups are shown in Fig. 1. We obtain a good agreement between theory and experiment, as illustrated in Figs. 2–4. Specifically we find $(\chi^2/N_{\text{pts}})_{\text{SSA}} = 520/517 = 1.01$ for SSA data alone, and $\chi^2/N_{\text{pts}} = 1373/1324 = 1.04$ for all data, including HERMES multiplicities.
The tensor charges $\delta u$, $\delta d$, and $g_T$ are in excellent agreement with lattice data. We stress that the inclusion of $A_N^T$ is crucial in order to achieve the agreement between our results $\delta u = 0.72(19)$, $\delta d = -0.15(16)$ and those from lattice. We emphasize that future experiments will be essential to reduce the uncertainty associated with extrapolation beyond regions constrained by current measurements.

**Conclusions.** In this letter we have performed the first global analysis of the available SSA data in SIDIS, DY, $e^+e^-$ annihilation, and proton-proton collisions. The predictive power exhibited by the combined analysis suggests a common physical origin of SSAs. Namely, they are due to the intrinsic quantum-mechanical interference from multi-parton states. The success achieved with a Gaussian ansatz for the transverse momentum dependence further implies that the effects are dominantly non-perturbative and intrinsic to hadronic wavefunctions. We also observe that the extracted up and down quark tensor charges are in excellent agreement with lattice QCD. Moreover, the future data coming from Jefferson Lab 12 GeV, COMPASS, an upgraded RHIC, Belle II, and the Electron-Ion Collider will help to reduce the uncertainties of the extracted functions and ultimately lead to a better understanding of hadronic structure.

**Acknowledgments.** This work has been supported by the NSF under Grants No. PHY-1623454 (A.P.), No. PHY-1720486 (Z.K.), the DOE Contracts No. DE-AC05-06OR23177 (L.G.), No. DE-AC05-06OR23177 (A.P., N.S., T.R.) under which JSA, LLC operates JLab, a LVC Arnold Student-Faculty Research Grant (J.A.M. and D.P.), and within the framework of the TMD Topical Collaboration.

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