On Coded Caching with Private Demands

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Abstract

Caching is an efficient way to reduce network traffic congestion during peak hours by storing some content at the user’s local cache memory without knowledge of later demands. For the shared-link caching model, Maddah-Ali and Niesen (MAN) proposed a two-phase (placement and delivery phases) coded caching strategy, which is order optimal within a constant factor. However, in the MAN scheme, each user can obtain the information about the demands of other users, i.e., the MAN scheme is inherently prone to tampering and spying the activity/demands of other users. In this paper, we formulate a shared-link caching model with private demands. Our goal is to design a two-phase private caching scheme with minimum load while preserving the privacy of the demands of each user with respect to other users. Two private caching schemes are proposed with some (order) optimality results for high memory regime. We also propose a private scheme to further reduce the transmitted load for the case where each user has a distinct request, e.g., in practical asynchronous streaming users always have distinct demands. Finally, this scheme is extended to the general demand case, which is order optimal for any memory size, but with an extremely high sub-packetization level. While the ensemble of our results essentially close the problem of (order) optimal coded caching subject to the privacy of users’ demands, the problem of how to achieve this optimality in the small memory regime without an exponentially large expansion of the sub-packetization order remains as an interesting open problem for future investigation.

I. INTRODUCTION

A. Brief Review of Coded Caching

Recent years have witnessed a steep increase of wireless devices connected to the Internet, leading to a heavy network traffic because of multimedia streaming, web-browsing and social networking. Furthermore, the high temporal variability of network traffic results in congestions
During peak-traffic times and underutilization of the network during off-peak times. *Caching* is a promising technique to reduce peak traffic by taking advantage of memories distributed across the network to duplicate content during off-peak times [1]. With the help of caching, network traffic could be shifted from peak to off-peak hours in order to smooth out the traffic load and reduce congestion. In the seminal paper [2], an information-theoretic and network-coding theoretic model for caching was proposed. In this model, two phases are included in a caching system: i) *placement phase*: each user equipped with cache stores some bits in its cache component without knowledge of later demands; ii) *delivery phase*: after each user has made its request and according to cache contents, the server transmits packets such that each user can recover its desired file(s). The goal is to minimize the number of transmitted bits (referred to as load in this paper).

Coded caching strategy was originally proposed in [2] for the shared-link broadcast networks where a server with a library of \( N \) files, of \( B \) bits each, is connected to \( K \) users (each of which is with a cache of \( MB \) bits) through a shared error-free broadcast link. Each user requests one file independently in the delivery phase. Maddah-Ali and Niesen (MAN) proposed a coded caching scheme that utilizes an uncoded combinatorial cache construction in the placement phase and a binary linear network code to generate multicast messages in the delivery phase. For \( M = tN/K \) with \( t \in [0:K] \), the transmitted load is \( K(1 - tM/N)/(1 + KM/N) \). For other memory size, the lower convex envelope of the above memory-load tradeoff points is achievable by memory-sharing between schemes for integer values of \( t = KM/N \). Compared to the conventional uncoded caching scheme which lets each user store \( MB/N \) bits of each file in the placement phase and broadcasts the uncached part of each desired file during the delivery phase with the transmitted load \( K(1 - M/N) \), the MAN scheme has an additional coded caching gain (i.e., load reduction factor) equal to \( 1 + KM/N \). It was proved in [3] that the worst-case load achieved by the MAN scheme among all possible demands is optimal under the constraint of uncoded placement (i.e., each user directly stores a subset of bits in the library) and \( N \geq K \). When \( N \geq K \), the MAN scheme was also proved in [4] to be generally order optimal within a factor of 2. For any \( N \) and \( K \), a factor of 4 for the order optimality of the MAN scheme was proved in [5]. By observing that some MAN multicast messages are redundant for the case \( N < K \), the authors in [6] proposed an improved delivery scheme which is optimal under the constraint of uncoded cache placement for any \( N \) and \( K \), and optimal within a factor of 2 over all possible placement strategies.

In the MAN caching model, each user requests only one file, which may not be practical. The
caching problem with multi-request was originally considered in [7] where each user demands L files from the library. With the MAN placement, to divide the delivery phase into L rounds where in each round the MAN scheme in [2] (referred to as L-round MAN caching scheme in this paper, which reduces to the MAN scheme when L = 1) is used to let each user decode one file, can achieve a generally order optimal worst-case load within a factor of 18 [7]. By further tightening the converse bound, this order optimality factor was reduced to 11 in [8].

The MAN coded caching strategy was also used in a number of extended models, such as decentralized caching where users must fill their caches independently of other users [9], device-to-device (D2D) caching systems where users communicate among each other during the delivery phase [10], cache-aided topological networks where the server communicates with the users through some intermediate relays [11]–[13], etc. However, these extended models will not be considered in our paper, and thus we do not go into details.

B. Existing Secure Coded Caching Schemes

Soon after the appearance of [2], various ‘secure’ versions of the caching problem have been proposed. Secure coded caching was originally considered in [14], where there are some wiretappers who can also receive the broadcasted packets from the server. To prevent the wiretappers from obtaining any information about the files in the library, the authors in [14] let each user store not only the content about the library in its cache, but also some ‘keys’. In the delivery phase, each multicast message is generated by taking XOR of the MAN multicast message and some key in order to ‘lock’ the multicast messages such that only the intended users can unlock it. This secure caching scheme against wiretappers was proved in [15] to be optimal under the constraint of uncoded cache placement. Another secure shared-link caching model was proposed in [16]. In this case, the objective is to avoid each user to get any information about the files not required by that user. The placement and delivery phases were designed based on the MAN caching scheme with an additional secret sharing precoding [17] on each file (i.e., by encoding a message with (n, t) secret sharing code where n > t, any t shares do not reveal any information about the message and the message can be reconstructed from all the n shares). In addition, the secure caching scheme in [16] could also successfully prevent external wiretappers, because each multicast message is also ‘locked’ by a ‘key’. The above strategies to prevent external wiretappers and internal malicious users from retrieving information about the
library, were then used in extended models, such as D2D caching systems [18], [19], topological cache-aided relay networks [20], erasure broadcast channels [21], etc.

C. Coded Caching with Private Demands

The existing secure caching schemes are based on the MAN caching scheme (with or without a secure precoding on each file) and then generate ‘locked’ MAN multicast messages. However, a malicious user could simply use the MAN multicast messages (or ‘locked’ MAN multicast messages) in order to learn the requests of other users, e.g., to perform some survey on user preferences, which is not good in terms of privacy. Shared-link caching problem with single request to preserve the users demands from other users was originally considered in [22]. The caching scheme proposed in [22] generates ℓ virtual users each of which randomly demands one file, such that each user cannot match the exact request to any other user. However, this caching scheme is not completely private from an information theoretic viewpoint. For example, if there exists undemanded file by any real or virtual user, each user will know this file has not been demanded such that it can get some information about the users demands from the transmission. In this paper, we formulate an information-theoretic caching problem which aims to preserve the privacy of the demands of each user with respect to other users during the transmission.

Let us focus on a toy example with K = 2, N = 3 and M = 2N/3 = 2. In this example, t = KM/N = 4/3, which is not an integer, and thus we should use the memory-sharing between M_1 = Nt_1/K = 3/2 with t_1 = 1 and M_2 = Nt_2/K = 3 with t_2 = 2. By the MAN placement, we divide each file into three equal-length and non-overlapping subfiles, the i-th file, denoted by F_i, has three subfiles F_i,\{1\}, F_i,\{2\}, and F_i,\{1,2\}. User 1 caches F_i,\{1\} and F_i,\{1,2\}, while user 2 caches F_i,\{2\} and F_i,\{1,2\}.

In the delivery phase, we consider two demands:

- if the demand is (1, 2), i.e., user 1 demands file F_1 and user 2 demands file F_2, we transmit the MAN multicast message F_1,\{2\} ⊕ F_2,\{1\}, where ⊕ represents the XOR operation, such that user 1 can recover F_1,\{2\} and user 2 can recover F_2,\{1\}. However, for the sake of successful decoding, user 1 needs to know that F_2,\{1\} is contained by the multicast message, and thus it knows user 2 demands F_2. Similarly, user 2 will know the demand of user 1.
- if the demand is (1, 1), i.e., both users 1 and 2 demand file F_1, we transmit the MAN multicast message F_1,\{2\} ⊕ F_1,\{1\}, such that user 1 can recover F_1,\{2\} and user 2 can recover
However, from the transmission, user 1 knows user 2 demands $F_1$, while user 2 knows the demand of user 1.

Instead of using the MAN scheme which is inherently prone to tampering and spying the activity/demands of other users, in this paper, we design the following scheme which can preserve the privacy of the users demands. In the placement phase, we encode each file $F_i$ by a $(4,3)$ MDS code (i.e., each file $F_i$ is split into 3 blocks of $B/3$ bits each, which are then encoded by a $(4,3)$ MDS code such that each of the four MDS coded symbols has $B/3$ bits). Each file can be reconstructed by any three MDS coded symbols. The four MDS coded symbols are denoted by $S_{i,1}, S_{i,2}, S_{i,3}, S_{i,4}$. We randomly generate a permutation of $\{1,2,3,4\}$, denoted by $p_i = (p_{i,1}, p_{i,2}, p_{i,3}, p_{i,4})$ and let $F_{i,\emptyset} = S_{p_{i,1}}^i$, $F_{i,\{1\}} = S_{p_{i,2}}^i$, $F_{i,\{2\}} = S_{p_{i,3}}^i$, and $F_{i,\{1,2\}} = S_{p_{i,4}}^i$. We let user 1 cache $F_{i,\{1\}}$ and $F_{i,\{1,2\}}$, and user 2 cache $F_{i,\{2\}}$ and $F_{i,\{1,2\}}$. Notice that, for the sake of successful decoding, user 1 knows the compositions of $F_{i,\{1\}}$ and $F_{i,\{1,2\}}$ (i.e., it knows from which code on which bits $F_{i,\{1\}}$ and $F_{i,\{1,2\}}$ are generated), but it does not know $p_i$, i.e., it does not know which one of $F_{i,\{1\}}$ and $F_{i,\{1,2\}}$ is cached by user 2. Hence, $F_{i,\{1\}}$ and $F_{i,\{1,2\}}$ are equivalent from the viewpoint of user 1. Similarly, $F_{i,\{2\}}$ and $F_{i,\emptyset}$ are also equivalent from the viewpoint of user 1.

In the delivery phase, we also consider two demands:

- if the demand is $(1,2)$, we transmit $F_{1,\{2\}} \oplus F_{2,\{1\}} \oplus F_{3,\{1,2\}}$, such that user 1 can recover $F_{1,\{2\}}$ and user 2 can recover $F_{2,\{1\}}$. Notice that each user only knows the composition of each MDS coded symbol in the sum, without knowing whether the other user caches it or not. From the viewpoint of user 1, in the sum there is one MDS coded symbol from each file and among these MDS coded symbols it caches the ones from the non-demanded files (i.e., $F_2$ and $F_3$).
- if the demand is $(1,1)$, we transmit $F_{1,\emptyset} \oplus F_{2,\{1,2\}} \oplus F_{3,\{1,2\}}$, such that users 1 and 2 can recover $F_{1,\emptyset}$. Again, from the viewpoint of user 1, in the sum there is one MDS coded symbol from each file and among these MDS coded symbols, it caches the ones from the non-demanded files.

For the above two demands, from the viewpoint of user 1, the delivery phases are equivalent. Hence, user 1 cannot know any information about the request of user 2. A symmetric situation holds for user 2 and for all other possible demands.

To prevent users from retrieving information about the non-demanded files, as considered
in the mentioned literature, may not be practical because if a user wants to obtain a specific file, it may simply demand this file directly. In practice, it may be important to preserve the privacy of the users demands. The above example motivates the following question: what is the fundamental coded caching gain subject to such strict privacy constraint on the users demands? In this paper, we focus on the private shared-link caching model with multiple requests from an information-theoretic viewpoint, where each user requests $L$ files. The objective is to design a private caching scheme with minimum load in the delivery phase, in order to maintain the successful decoding for each user and also to prevent each user from getting any information about other users’ demands.

\textbf{D. Relation to Private Information Retrieval}

The privacy of the users demands was originally considered as the Private Information Retrieval (PIR) problem in [23]. In this setting, a user wants to retrieve a desired message from some distributed non-colluding databases (servers), and the objective is to prevent any server from retrieving any information about the user demand. Recently, the authors in [24] characterized the information-theoretic capacity of the PIR problem by proposing a novel converse bound and a coded PIR scheme based on an interference alignment idea.

Later, models combining the PIR problem with some caching component were proposed in [25]–[30]. In [25], the user randomly caches some files in the library and its side information is unknown to the servers. The capacity region of the rate in terms of the number of cached files was characterized in [25]. The authors extended the model in [25] to the single-server multi-user case, where each user caches some files and knows the demands of other users. A caching scheme based on Maximum Distance Separable (MDS) code was proposed. In [27]–[29], for the single-user PIR problem with end-user-cache, instead of caching the whole files, the user can choose any bits to store as in the coded caching model. Novel converse and achievable bounds were proposed in [27]–[29] for the cases where the user’s cache is known, partially known, and unknown to the servers, respectively. The authors in [30] considered the single-user PIR problem with end-database-caches, where each server can choose any bits to store instead of being able to access to the whole library. Under the constraint of uncoded cache placement, the optimal PIR scheme was given in [30]. The PIR problem was then generalized to the Private Computation (PC) problem in [31], where the user should compute a function on the library instead of directly retrieving one message.
The considered coded caching problem with private demands aims to preserve the privacy of the demands of each user from other users, while the cache-aided PIR (or PC) problems aim to preserve the privacy of the users demands from the databases. Hence, the main challenge of the considered problem is to design multicast messages transmitted from the server such that each user can decode its desired files without getting any information of other users’ demands. The objective is to design a private two-phase caching scheme with the largest multicasting gain (i.e., coded caching gain) while preserving the privacy.

E. Contributions

Our main contributions are as follows.

- We formulate an information-theoretic shared-link coded caching model with multiple requests, and the constraints on the privacy of the users demands from other users.
- We propose a baseline private caching scheme, which either serves users’ demands by separate unicast messages without multicasting gain or lets each user recover the whole library by multicast messages.
- In order to maintain the demands’ privacy while also achieving a non-trivial coded caching gain, we propose a novel improved caching scheme with novel cache placement and delivery phases. The main idea is to transmit symmetric multicast messages, each of which involves the same number of bits of each file in the library (as the scheme in the above toy example), such that for different demands, the compositions of the received messages are equivalent from the viewpoint of each user. By comparing the proposed scheme to the existing converse bounds for the shared-link caching model without privacy constraint, we proved that when \( M \geq N/2 \), the improved scheme is order optimal within a factor of 2. In other words, we can maintain the order optimality while preserving users’ demands. When \( M \geq \min \left\{ \frac{2K-1}{2^K}, \frac{2^{K-1}}{2^{K-1} + 1} \right\} N \), the improved scheme is exactly optimal. In other words, we can maintain the exact optimality while preserving users’ demands.
- We then focus on the case where each user demands a distinct file. Practical applications of this model includes asynchronous video streaming, where each video streaming is composed of multiple chunks and users make their demands asynchronously, and distributed computation paradigm, where some distributed computing platforms/workers with memories such as Google Cloud and Amazon Web Services Cloud (as the users in caching) are asked to compute a task in a distributed way. The workers store some information of data units in
advance (as the cache placement phase in caching), and after the task is revealed (as the delivery phase in caching), each worker is asked to compute a sub-task which is a function of some distinct data units (as a function of a distinct file in caching). The master (as the server in caching) broadcasts some packets such that each worker can recover the input data units of its assigned sub-task by the help of its stored contents. Besides successfully computing the task, the master also wants to hide the other sub-tasks not assigned to each given worker.

For the considered coded caching problem with distinct private requests, we propose a non-trivial private caching scheme by creating some virtual users demanding the unrequested files. This scheme is proved to be order optimal within a factor of 4, and exactly optimal when $M \geq \frac{K}{K+1}N$.

- Finally, we extend the proposed scheme with virtual users to general demands, which is order optimal within a factor of 8 if $L = 1$ and a factor of 22 if $L > 1$, but with an extremely high sub-packetization level (exponential to $\binom{N}{L}K$, i.e., $O(2^{(N/2)K})$).

F. Paper Organization

The rest of this paper is organized as follows. Section II formulates the considered shared-link caching model with private demands. Section III presents our proposed achievable schemes and some (order) optimality results for general demands. Section IV presents a novel scheme for the case of distinct requests and its extension. Section V provides some numerical evaluations for the proposed caching schemes. Section VI concludes the paper and some proofs are given in the Appendices.

G. Notation Convention

Calligraphic symbols denote sets, bold symbols denote vectors, and sans-serif symbols denote system parameters. We use $|\cdot|$ to represent the cardinality of a set or the length of a vector; $[a : b] := \{a, a+1, \ldots, b\}$ and $[n] := [1, 2, \ldots, n]$; $\oplus$ represents bit-wise XOR; $E[\cdot]$ represents the expectation value of a random variable; $[a]^+ := \max\{a, 0\}$; we let $\binom{x}{y} = 0$ if $x < 0$ or $y < 0$ or $x < y$; we denote the power set of $[a]$ by Pow($a$), and sort all sets in lexicographic order. Pow($a$, $j$) denotes the $j^{th}$ set. For example,

$$\text{Pow}(3) = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 3\}, \{1, 3\}, \{2\}, \{2, 3\}, \{3\}\},$$

and Pow(3, 1) = $\emptyset$, Pow(3, 2) = $\{1\}$, etc.
II. SYSTEM MODEL AND RELATED RESULTS

A. System Model

A \((K, N, M, L)\) shared-link caching system with private demands is defined as follows. The system contains a server with access to a library of \(N\) independent files, denoted by \((F_1, F_2, \ldots, F_N)\), where each file is composed of \(B\) i.i.d. bits. As in [2], we assume that \(B\) is sufficiently large such that any sub-packetization of the files is possible. The server is connected to \(K\) users through an error-free shared-link. The caching system operates in two phases.

Placement Phase. During the placement phase, user \(k \in [K]\) stores content in its cache of size \(M\) MB bits without knowledge of later demands, where \(M \in [0, N]\). We denote the content in the cache of user \(k \in [K]\) by \(Z_k \cup \{T_S : S \subseteq [K], k \in S\}\), where \(T_S\) represents the ‘key’ shared by the users in \(S\). \(Z_k\) contains two parts

\[
Z_k = (\mathcal{M}(C_k), C_k),
\]  

where \(C_k\) represents cached content from the \(N\) files,

\[
C_k = \phi_k(F_1, \ldots, F_N, \mathcal{M}(C_k)),
\]

and \(\mathcal{M}(C_k)\) represents the metadata/composition of \(C_k\) (i.e., from which code on which bits, \(C_k\) are generated). Notice that for any bit in \(C_k\), the metadata of this bit does not reveal which of the other users cache it.

We assume that the total length of \(\mathcal{M}(C_k)\) and \(\{T_S : S \subseteq [K], k \in S\}\) is negligible compared to the file length \(B\) such that, for simplicity, the relevant cache size constraint is

\[
\frac{H(Z_k \cup \{T_S : S \subseteq [K], k \in S\})}{B} = \frac{H(C_k)}{B} \leq M, \forall k \in [K]. \quad \text{(Memory size)}
\]

We also denote by \(Z := (Z_1, \ldots, Z_K)\) the content of all \(K\) caches.

Delivery Phase. During the delivery phase, each user demands \(L\) files, where \(L \in [N]\). In this paper, we consider \(N \geq L\) to ensure each user has \(L\) demands. The demand vector of user \(k \in [K]\) are denoted by \(d_k := (d_{k,1}, d_{k,2}, \ldots, d_{k,L})\), where \(1 \leq d_{k,1} < d_{k,2} < \cdots < d_{k,L} \leq N\). The demand matrix of all \(K\) users is denoted by \(D := [d_1; d_2; \ldots; d_K]\). In addition, we define \(D_{\setminus\{k\}}\) for each \(k \in [K]\) as the demand vectors of all users except user \(k\), where

\[
D_{\setminus\{k\}} := [d_1; \ldots; d_{k-1}, d_{k+1}, \ldots, d_K].
\]

We also denote the set of all possible demand matrices by

\[
\mathcal{D} := \{D : 1 \leq d_{k,1} < d_{k,2} < \cdots < d_{k,L} \leq N, \forall k \in [K]\}.
\]
We assume that the metadatas of users’ caches, the keys, users’ demands, and the library contents are independent,

\[
H\left(F_1, F_2, \ldots, F_N, \{\mathcal{M}(C_k) : k \in [K]\}, \{T_S : S \subseteq [K]\}, \mathbb{D}\right)
= NB + H(\{\mathcal{M}(C_k) : k \in [K]\}) + \sum_{S \subsetneq [K]} H(T_S) + H(\mathbb{D}).
\] (6)

Given the demand matrix \(\mathbb{D}\) and the users’ caches \(Z\), for each non-empty subset of users \(S \subseteq [K]\) where \(|S| > 0\), the server broadcasts a packet \(X_S\) which includes three parts (Header, locked Metadata, and Payload) as illustrated in Fig. 1. The header of \(X_S\) provides information (e.g., protocols, source, destination, etc.) to ensure that all users in \([K]\) can receive successfully the broadcast packet \(X_S\). In addition, it is also indicated in the header that the \(X_S\) is useful to users in \(S\). To ensure the successful decoding on the payload, the metadata \(\mathcal{M}(P_S)\) represents the composition of the payload \(P_S\). In addition, the metadata \(\mathcal{M}(P_S)\) is ‘locked’ by the ‘key’ \(T_S\) (i.e., the ‘locked’ Metadata is \(\mathcal{M}(P_S) \oplus T_S\)). Knowing \(X_S\) is useful to users in \(S\), each user in \(S\) can use \(T_S\) to ‘unlock’ the metadata \(\mathcal{M}(P_S)\). The payload contains the coded packets from the \(N\) files,

\[P_S = \psi_S(F_1, \ldots, F_N, \mathcal{M}(P_S)).\] (7)

We also assume that the total length of the header and ‘locked’ metadata are negligible compared to the payload, such that we have

\[R_S := H(X_S)/B = H(P_S)/B,\] (8)

where \(R_S\) represents the load (i.e., normalized number of total transmitted bits) of \(X_S\). Moreover, other users in \([K] \setminus S\) without knowledge of \(T_S\) cannot ‘unlock’ the metadata \(\mathcal{M}(P_S)\). Hence, we assume that each user in \([K] \setminus S\) cannot get any information about \(P_S\). Since the ‘key’ \(T_S\) is used to avoid other users in \([K] \setminus S\) to get any information about \(P_S\), in the rest of this paper we directly assume each user \(k \in [K]\) can only receive \(X_S\) where \(S \subseteq [K]\) and \(k \in S\), and assume the metadata in each multicast message is ‘unlocked’. Hence, we do not need to consider the ‘keys’ in the rest of this paper.

\[1^\text{This assumption may not hold from the information theory viewpoint, because in \cite{17} it was shown that to completely secure a message with length } l \text{ needs a ‘key’ with length } l. \text{ However, in practice, if the metadata of one coded message is unknown to one user, it is extremely hard for this user to get any information about it.}\]
Fig. 1: The delivery packet of \( X_S \), where ‘H’ represents Header, ‘LM’ represents ‘Locked’ Metadata, ‘P’ represents Payload.

The constraints on the decoding of the demanded files by each user while maintaining the privacy is given as follows. For each user \( k \in [K] \), it must hold that

\[
H(\{F_i : i \in d_k\} | \{X_S : S \subseteq [K], k \in S\}, Z_k, d_k) = 0, \ \forall k \in [K]. \text{ (Decodability)} \tag{9}
\]

In addition, given \( d_k \), user \( k \) cannot get any information about the demands of other users from \( (\{X_S : S \subseteq [K], k \in S\}, Z_k) \), i.e.,

\[
I(\mathbb{D}_{\backslash\{k\}}; (X_S : S \subseteq [K], k \in S), Z_k | d_k) = 0, \ \forall k \in [K]. \text{ (Privacy)} \tag{10}
\]

Since \( Z_k \) is independent of \( \mathbb{D} \), the privacy constraint in (10) can be also written as

\[
I(\mathbb{D}_{\backslash\{k\}}; (X_S : S \subseteq [K], k \in S) | Z_k, d_k) = 0, \ \forall k \in [K]. \text{ (Privacy)} \tag{11}
\]

**Objective.** We denote the load transmitted in the delivery phase by

\[
R := \sum_{S \subseteq [K] : |S| > 0} R_S. \tag{12}
\]

By the constraint of privacy, we can see the transmitted loads for different demand matrices should be the same; otherwise, the transmitted load which can be counted by each user will reveal information about the users demands. The memory-load tradeoff \((M, R)\) is said to be achievable for the memory constraint \( M \), if there exist a two-phase private caching scheme as defined above such that all possible demand matrices can be delivered with load at most \( R \) while the decodability and privacy constraints in (9) and (11) are satisfied. The objective is to determine, for a fixed \( M \in [0, N] \), the minimum load \( R^* \).

**B. MAN Caching Scheme**

In the following, we recall the MAN shared-link caching scheme proposed in [2] and show this scheme cannot preserve the privacy of the users demands. We first focus on \( L = 1 \), i.e., each user requests one file.
**Placement Phase.** Let $M = \frac{Nt}{K}$ where $t \in [0 : K]$. Each file $F_i$ where $i \in [N]$ is divide into $\binom{K}{t}$ non-overlapping and equal-length subfiles, $F_i = \{F_{i,W} : W \subseteq [K], |W| = t\}$, while each user $k \in [K]$ caches $F_{i,W}$ where $k \in W$. In other words,

$$C_k = \{F_{i,W} : i \in [N], W \subseteq [K], |W| = t, k \in W\}, \forall k \in [K].$$

(13)

Hence, each user caches $NB\frac{\binom{K-1}{t}}{\binom{K}{t}} = MB$ bits.

**Delivery Phase.** For each $S \subseteq [K]$ where $|S| = t + 1$, the server generates an MAN multicast message

$$P_S = \bigoplus_{k \in S} F_{d_k,1,S \setminus \{k\}},$$

(14)

and transmit $X_S = (\mathcal{M}(P_S), P_S)$. We define the composition of an XOR message of subfiles (or MDS coded symbols) as the containing subfiles (or MDS coded symbols) in this message. It can be seen that in the composition of $P_S$, each user in $S$ caches all subfiles except $F_{d_k,1,S \setminus \{k\}}$ such that it can recover $F_{d_k,1,S \setminus \{k\}}$. Considering all $S \subseteq [K]$ where $|S| = t + 1$, each user can recover its desired file, i.e., the decodability constraint in (9) is satisfied.

When $L > 1$, the transmission is divided into $L$ rounds, where in each round we serve one demand of each user. By using the above MAN delivery scheme by $L$ times, the achieved load by the $L$-round MAN scheme is as follows,

$$(M_{MAN}, R_{MAN}) = \left(\frac{Nt}{K}, L\frac{K - t}{t + 1}\right), \forall t \in [0 : N].$$

(15)

For other memory sizes, we can take the lower convex envelope of the corner points in (15).

However, consider one $S \subseteq [K]$ where $|S| = t + 1$, each user knows the metadata of each subfile in $P_S$ from $\mathcal{M}(P_S)$. Hence, it knows the composition of $P_S$, i.e, it knows the union set of the demanded files by users in $S$ is $\cup_{k \in S}\{d_k,1\}$, which contradicts the privacy constraint in (11).

In the following section, we will propose two caching schemes which can satisfy both of the decodability and privacy constraints.

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2 Notice that for the MAN caching scheme, even if we can hide the identity of the intended users of each multicast messages (i.e., each user $k \in S$ does not know that $P_S$ is useful to users in $S \setminus \{k\}$), user $k$ still knows that the files in $\cup_{k \in S}\{d_k,1\}$ are demanded by some users. Hence, the privacy constraint in (11) cannot hold.
III. CODED CACHING WITH PRIVATE DEMANDS

A. Main Results

We first provide the achieved loads of the proposed two achievable private caching schemes with some optimality results. The detailed descriptions of the proposed schemes are given in the following subsections.

The first scheme is treated as the baseline scheme, which trivially transmits either uncoded unicast packets to each user or transmit multicast messages to users such that each user could recover the whole library. The detailed description can be found in Section III-B.

**Theorem 1** (Baseline Scheme). For the \((K, N, M, L)\) shared-link caching system with private demands, \(R^*\) is upper bounded by

\[
R^* \leq R_{\text{base}} = \min\{LK, N\} \left(1 - \frac{M}{N}\right).
\]

We then propose an improved caching scheme based on MDS precoding in the placement phase, which transmit symmetric multicast messages in the delivery phase. The improved scheme ensures that each multicast message contains identical number of bits from each file and the compositions of the received messages for different demands are equivalent from the viewpoint of each user (i.e., the compositions of the received messages are independent of the demands of other users from the viewpoint of each user). The detailed description of the improved scheme can be found in Sections III-C and III-D.

**Theorem 2** (New Proposed Scheme). For the \((K, N, M, L)\) shared-link caching system with private demands, \(R^*\) is upper bounded by the lower convex envelope of the following memory-load pairs

\[
(M, R_{\text{new}}) = \left(\frac{2^{K-1}}{2^{K-1} + (K-1) \frac{K}{t} + \cdots + (K-1) \frac{K}{K-1}} N, L \frac{2^K - \binom{K}{0} - \cdots - \binom{K}{t}}{2^{K-1} + (K-1) \frac{K}{t} + \cdots + (K-1) \frac{K}{K-1}} \right), \forall t \in [0 : K],
\]

and

\[
(M, R_{\text{new}}) = \left(\frac{2K - 1}{2K} N, \frac{L}{2K} \right).
\]

From Theorem 2, we can see that the memory size for the first corner point (i.e., \(t = 0\) in (17)) is \(N/2\), instead of \(M = 0\) as in (16). This is because for the improved scheme, each multicast message \(X_S\) which is useful to users in \(S\) and composed of \(L\) linear combinations, contains one MDS coded symbol of each file in the library. Among all MDS coded symbols in \(X_S\), each user
knows all except $L$ ones such that it can recover these $L$ coded symbols. In addition, for each
file which is not demanded by any users in $S$, the corresponding MDS coded symbol should be
cached by all users in $S$. Hence, the minimum memory sizes of the improved scheme cannot
be 0.

However, to have a complete memory-load region, we can take the lower convex envelope
of the achieved memory-load corner points by the proposed schemes in (16), (17), and (18) by
memory-sharing. We define $R_{\text{com}}$ as the achieved load by the combined scheme for $M \in [0, N]$.

When $M = N/2$ and $N \geq LK$, the load $R_{\text{base}}$ is $LK/2$, which is proportional to $K$. However, the
achieved load $R_{\text{new}}$ is $L(2^{K-1}/2^K) \leq L$, which is not proportional to $K$, such that the coded caching
gain is larger than $K/2$. In addition, $(M, R_{\text{new}}) = (N, 0)$ is also a corner point in (17). Hence,
by taking the memory-sharing between $M_1 = N/2$ and $M_2 = N$, for any $N/2 \leq M \leq N$, the
achieved load is not proportional to $K$.

Moreover, from the fact that any converse bound on the worst-case load for the MAN shared-
link caching model (with single or multiple request(s)) is also a load lower bound in our problem,
we can derive the following order optimality results of $R_{\text{new}}$ for $M \geq N/2$.

**Theorem 3** (Order Optimality). For the $(K, N, M, L)$ shared-link caching system with private
demands, when $M \geq N/2$, $R_{\text{new}}$ is order optimal within a factor of 2.

*Proof: Converse.* We use the existing converse bound in [32] for the shared-link caching
model without privacy, which obviously provides a load lower bound for the shared-link caching
model with private demands. From [32, Theorem 1] with $s = 1$, we have

$$R^* \geq L \left( 1 - \frac{M}{N} \right).$$

\hspace{1cm} (19)

**Achievability.** When $M_1 = N/2$, from (17) with $t = 0$ achieved by the improved scheme, we have

$$R_{\text{new}} = L \left( \frac{2^K - 1}{2^K} \right) \leq L.$$ 

\hspace{1cm} (20)

Hence, by memory-sharing between $M_1 = N/2$ with load less than $L$ and $M_2 = N$ with load
equal to 0, we have for any $M \in [N/2, N]$,

$$R_{\text{new}} \leq 2L \left( 1 - \frac{M}{N} \right) \leq 2R^*,$$

\hspace{1cm} (21)
where (21) comes from (19).

By comparing the achievable bounds in (17) with \( t = K - 1 \) and (18) with the converse bound for the MAN shared-link caching model with multiple requests in [32], we have the following exact optimality result.

**Theorem 4 (Exact Optimality).** For the \((K, N, M, L)\) shared-link caching system with private demands where \( M \geq \min \left\{ \frac{2K-1}{2^{K-1}}, \frac{2K-1}{2^{K-1}+1} \right\} N \), we have

\[
R^* = R_{\text{new}} = L \left(1 - \frac{M}{N}\right).
\]

(22)

**Proof: Converse.** From [32, Theorem 1] with \( s = 1 \), the MAN coded caching model with multiple requests (i.e., each user has \( L \) requests), when \( N \geq L \), it should satisfy

\[
R^* \geq L \left(1 - \frac{M}{N}\right).
\]

(23)

**Achievability.** When \( M \geq \frac{2K-1}{2^{K-1}} N \), by memory-sharing the corner points between \( t = K - 1 \) and \( t = K \) in (17), we can achieve the load \( L \left(1 - \frac{M}{N}\right) \).

When \( M_1 \geq \frac{2K-1}{2^{K-1}} N \), by memory-sharing the corner points between (18) and \( t = K \) in (17), we can achieve the load \( L \left(1 - \frac{M_1}{N}\right) \). This proves Theorem 4.

It can be seen that for the considered large memory size regime in Theorem 4, our proposed scheme can maintain the exact optimality for the shared-link caching model with multiple requests, while preserving the privacy of the users demands (which is not possible for the MAN caching scheme nor the L-round MAN caching scheme).

**B. Baseline Scheme**

We now introduce the baseline private caching scheme with the load in Theorem 1.

**Placement Phase.** Each user caches the same \( MB/N \) bits of each file \( F_i \) where \( i \in [N] \). We denote the cached part of \( F_i \) by \( F_i^c \) and the uncached part by \( F_i^u \). Since users have the same cached content, each user knows the cached content of other users.

**Delivery Phase.** We consider two cases.

1) \( L K < N \). For each user \( k \in [K] \), the server transmits \( X_{\{k\}} = (\mathcal{A}(P_{\{k\}}), P_{\{k\}}) \) to user \( k \), where \( P_{\{k\}} = \{ F_i^u : i \in d_k \} \). For the decodability, each user has received uncached part of each desired file. For the privacy, other users in \([K] \setminus \{k\}\) cannot get any information about \( P_{\{k\}} \). Hence, the demands of user \( k \) are preserved.
2) $L K \geq N$. The server transmits $X_{[K]} = (\mathcal{M}(P_{[K]}), P_{[K]})$ to all users, where $P_{[K]} = \{F_i^u : i \in [N]\}$. For the decodability, each user has received uncached part of each file in the library, which includes its desired files. For the privacy, since we transmit the the uncached parts of all files, each user cannot know which files among them are desired by other users. Hence, the privacy of the users demands is preserved.

**Performance.** The normalized length of the uncached part of each file is $1 - \frac{M}{N}$. Hence, the achieved load is $\min\{LK, N\} (1 - \frac{M}{N})$ as in Theorem 1.

C. **New Scheme to achieve (17)**

It was shown that the baseline private caching scheme can provide a successful transmission with privacy. Notice that the load in Theorem 1 is the same as the load achievable with the conventional (uncoded) caching scheme. In other words, the privacy constraints incur no loss with respect to uncoded caching. In the following, we introduce a new improved scheme which effectively achieves some coded caching gain. We first use a more complicated example than the toy example in Section I-C to highlight more insights.

**Example 1** $(K = 3, N = 6, M = 3, L = 2)$. Consider a $(K, N, M, L) = (3, 6, 3, 2)$ shared-link caching problem with private demands. By the baseline private caching scheme, the achieved load is $\min\{LK, N\} (1 - \frac{M}{N}) = 3$. In the following, we introduce the new scheme for this example.

**Placement Phase.** From (17), we can compute $t = 0$ in this example. Each file $F_i$ where $i \in [N]$ is divided into $\binom{K}{0} + \cdots + \binom{K}{K} = 2^K = 8$ non-overlapping and equal-length pieces, denoted by $S_{i,1}^i, \ldots, S_{i,2^K}^i$. In this example where $t = 0$, we do not need the MDS precoding in the placement, which is necessary for $t > 1$ and will be clarified in the next example. We randomly generate a permutation of $[2^K]$, denoted by $p_i = (p_{i,1}, \ldots, p_{i,2^K})$, independently and uniformly over the set of all possible permutations. We then assign each piece to a subfile according to $p_i$ as follows,

$$
\begin{align*}
&f_{i,0} = S_{p_{i,1}}^i, \ f_{i,1} = S_{p_{i,2}}^i, \ f_{i,1,2} = S_{p_{i,3}}^i, \ f_{i,1,2,3} = S_{p_{i,4}}^i, \\
&f_{i,1,3} = S_{p_{i,5}}^i, \ f_{i,2} = S_{p_{i,6}}^i, \ f_{i,2,3} = S_{p_{i,7}}^i, \ f_{i,3} = S_{p_{i,8}}^i.
\end{align*}
$$

Each user $k \in [K]$ caches $f_{i,W}$ if $k \in W$, i.e., the cached contents of the three users for each file $F_i$ are as follows:

- User 1 stores $f_{i,1}, f_{i,1,2}, f_{i,1,3}$, and $f_{i,1,2,3}$.
• User 2 stores \( f_{i,\{2\}}, f_{i,\{1,2\}}, f_{i,\{2,3\}}, \) and \( f_{i,\{1,2,3\}}. \)
• User 3 stores \( f_{i,\{3\}}, f_{i,\{1,3\}}, f_{i,\{2,3\}}, \) and \( f_{i,\{1,2,3\}}. \)

Hence, each user caches 4 subfiles (each of which has B/8 bits) for each file in its cache, and thus it totally caches \( 3B = MB, \) satisfying the memory size constraint. In addition, for each subfile of \( F_i \) cached by user \( k \in [K], \) since the random permutation \( p_i \) is unknown to user \( k, \) it does not know the other users who also cache it. Hence, each cached subfile of \( F_i \) is equivalent from the viewpoint of user \( k. \) Similarly, each uncached subfile of \( F_i \) is also equivalent from the viewpoint of user \( k. \)

For the delivery phase, we do not consider all possible non-equivalent demand configurations, for the sake of brevity. Instead, we give two explicit examples of the construction of the delivery phase and then extract some general properties that demonstrate the privacy.

Delivery Phase for \( \mathbb{D} = [1, 2; 3, 4; 5, 6]. \) For this demand matrix, user 1 demands \( F_1 \) and \( F_2, \) user 2 demands \( F_3 \) and \( F_4, \) and user 3 demands \( F_5 \) and \( F_6. \) For each file \( F_i \) where \( i \in [N], \) we define

\[
Q_i := \{k \in [K] : i \in d_k\},
\]

as the set of users demanding \( F_i. \) For \( \mathbb{D} = [1, 2; 3, 4; 5, 6], \) we have \( Q_1 = Q_2 = \{1\}, Q_3 = Q_4 = \{2\}, \) and \( Q_5 = Q_6 = \{3\}. \)

For each subset \( S \subseteq [K] \) where \( |S| \geq t+1 = 1, \) we let the server transmit \( X_S = (\mathcal{M}(P_S), P_S). \) Our purpose is to let \( P_S \) be \( L = 2 \) linear combinations of \( N \) subfiles, where each file has one subfile in \( P_S \) and each user in \( S \) caches \( N - L \) subfiles from the files which it does not request. In addition, the \( L = 2 \) linear combinations are generated by the \( L \times N = 2 \times 6 \) parity-check matrix of the \([N, N - L, L + 1]\) MDS code (or an \( L \times N \) Cauchy matrix) denoted by \( G_{L \times N}, \) such that each \( L \) columns are linearly independent (see \( [33] \)) and each user in \( S \) can recover the \( L \) uncached subfiles. For each file \( F_i \) where \( i \in [N], \) the subfile of \( F_i \) in \( P_S \) is \( f_{i, S \cup Q_i \setminus (S \cap Q_i)} \) (the motivation of this construction will be explained in Footnote \([3]\), which is cached by each user in \( S \) not requesting \( F_i, \) and not cached by each user in \( S \) requesting \( F_i. \)

We first consider \( S = \{1\}, \) which only contains one user. It can be easily checked that

\[
P_{\{1\}} = G_{2 \times 6} \begin{bmatrix} f_{1,\emptyset}; f_{2,\emptyset}; f_{3,\{1,2\}}; f_{4,\{1,2\}}; f_{5,\{1,3\}}; f_{6,\{1,3\}} \end{bmatrix}.
\]

From \( P_{\{1\}}, \) user 1 caches all except \( f_{1,\emptyset} \) and \( f_{2,\emptyset}, \) such that it can recover those two subfiles in \( P_{\{1\}} \) (recall each two columns of \( G_{2 \times 6} \) are independent). Similarly, we have

\[
P_{\{2\}} = G_{2 \times 6} \begin{bmatrix} f_{1,\{1,2\}}; f_{2,\{1,2\}}; f_{3,\emptyset}; f_{4,\emptyset}; f_{5,\{2,3\}}; f_{6,\{2,3\}} \end{bmatrix}.
\]
\[ P_{\{3\}} = \mathbb{G}_{2 \times 6} [f_{1,\{1,3\}}; f_{2,\{1,3\}}; f_{3,\{2,3\}}; f_{4,\{2,3\}}; f_{5,\emptyset}; f_{6,\emptyset}]. \]  

(28)

We then consider \( S = \{1,2\} \), which contains two users. It can be easily checked that

\[ P_{\{1,2\}} = \mathbb{G}_{2 \times 6} [f_{1,\{2\}}; f_{2,\{2\}}; f_{3,\{1\}}; f_{4,\{1\}}; f_{5,\{1,2,3\}}; f_{6,\{1,2,3\}}]. \]  

(29)

From \( P_{\{1,2\}} \), user 1 caches all except \( f_{1,\{2\}} \) and \( f_{2,\{2\}} \), such that it can recover those two subfiles in \( P_{\{1,2\}} \). In addition, user 2 can recover \( f_{3,\{1\}} \) and \( f_{4,\{1\}} \) from \( P_{\{1,2\}} \). Similarly, we have

\[ P_{\{1,3\}} = \mathbb{G}_{2 \times 6} [f_{1,\{3\}}; f_{2,\{3\}}; f_{3,\{1,2,3\}}; f_{4,\{1,2,3\}}; f_{5,\{1\}}; f_{6,\{1\}}]. \]  

(30)

\[ P_{\{2,3\}} = \mathbb{G}_{2 \times 6} [f_{1,\{1,2\}}; f_{2,\{1,2\}}; f_{3,\{3\}}; f_{4,\{3\}}; f_{5,\{2\}}; f_{6,\{2\}}]. \]  

(31)

Finally we consider \( S = \{1,2,3\} \), which contains three users. It can be easily checked that

\[ P_{\{1,2,3\}} = \mathbb{G}_{2 \times 6} [f_{1,\{2,3\}}; f_{2,\{2,3\}}; f_{3,\{1,3\}}; f_{4,\{1,3\}}; f_{5,\{2,3\}}; f_{6,\{2,3\}}]. \]  

(32)

From \( P_{\{1,2\}} \), user 1 caches all except \( f_{1,\{2,3\}} \) and \( f_{2,\{2,3\}} \), such that it can recover those two subfiles in \( P_{\{1,2,3\}} \). In addition, user 2 can recover \( f_{3,\{1,3\}} \) and \( f_{4,\{1,3\}} \) while user 3 can recover \( f_{5,\{1,2\}} \) and \( f_{6,\{1,2\}} \).

Hence, each user can recover its desired files in the delivery phase. For the privacy constraint in (11), we let then focus on the demand matrix \( \mathbb{D} = [1,2; 1,3; 1,4] \), and show the compositions of the received multicast messages by each user are equivalent from its viewpoint to the ones for the demand matrix \( \mathbb{D} = [1,2; 3,4; 5,6] \).

**Delivery Phase for** \( \mathbb{D} = [1,2; 1,3; 1,4] \). For \( \mathbb{D} = [1,2; 1,3; 1,4] \), we have \( Q_1 = \{1,2,3\} \), \( Q_2 = \{1\} \), \( Q_3 = \{2\} \), \( Q_4 = \{3\} \), and \( Q_5 = Q_6 = \emptyset \). From the same way to construct multicast messages as described above, for \( \mathbb{D} = [1,2; 1,3; 1,4] \) we have

\[ P_{\{1\}} = \mathbb{G}_{2 \times 6} [f_{1,\{2,3\}}; f_{2,\emptyset}; f_{3,\{1,2\}}; f_{4,\{1,3\}}; f_{5,\{1\}}; f_{6,\{1\}}]. \]  

(33)

\[ P_{\{2\}} = \mathbb{G}_{2 \times 6} [f_{1,\{1,3\}}; f_{2,\{1,2\}}; f_{3,\emptyset}; f_{4,\{2,3\}}; f_{5,\{2\}}; f_{6,\{2\}}]. \]  

(34)

\[ P_{\{3\}} = \mathbb{G}_{2 \times 6} [f_{1,\{1,2\}}; f_{2,\{1,3\}}; f_{3,\{2,3\}}; f_{4,\emptyset}; f_{5,\{3\}}; f_{6,\{3\}}]. \]  

(35)

\[ P_{\{1,2\}} = \mathbb{G}_{2 \times 6} [f_{1,\{3\}}; f_{2,\{2\}}; f_{3,\{1\}}; f_{4,\{1,2,3\}}; f_{5,\{1,2\}}; f_{6,\{1,2\}}]. \]  

(36)

\[ P_{\{1,3\}} = \mathbb{G}_{2 \times 6} [f_{1,\{2\}}; f_{2,\{3\}}; f_{3,\{1,2,3\}}; f_{4,\{1\}}; f_{5,\{1,3\}}; f_{6,\{1,3\}}]. \]  

(37)

\[ P_{\{2,3\}} = \mathbb{G}_{2 \times 6} [f_{1,\{1\}}; f_{2,\{1,2,3\}}; f_{3,\{3\}}; f_{4,\{2\}}; f_{5,\{2,3\}}; f_{6,\{2,3\}}]. \]  

(38)

\[ P_{\{1,2,3\}} = \mathbb{G}_{2 \times 6} [f_{1,\emptyset}; f_{2,\{2,3\}}; f_{3,\{1,3\}}; f_{4,\{1,2\}}; f_{5,\{1,2,3\}}; f_{6,\{1,2,3\}}]. \]  

(39)
Privacy. For any demand matrix (we do not list the transmission for all demand matrices for sake of simplicity), we can summarize three common points:

1) for any $i \in [N]$, each subfile of $F_i$ cached by user 1 is equivalent from the viewpoint of user 1; each subfile of $F_i$ not cached by user 1 is also equivalent from the viewpoint of user 1;

2) in each of $P\{1\}, P\{1,2\}, P\{1,3\}, P\{1,2,3\}$ received by user 1, there is exactly one subfile of each file while user 1 caches one subfile from each of its unrequested files;

3) there does not exist any subfile appearing in two of $P\{1\}, P\{1,2\}, P\{1,3\}, P\{1,2,3\}$ by the construction, which ensures both the decodability and privacy.\footnote{This is the main reason to transmit $f_{i,S\cup Q_i\setminus(S\cap Q_i)}$ for the file $F_i$ in $P_S$, instead of $f_{i,S\setminus Q_i}$. We assume $f_{i,S\setminus Q_i}$ is transmitted in $P_S$. For each demand matrix where users request different files, it is clear to see that there does not exist any subfile appearing in two multicast messages. However, if $F_1$ is demanded by both users 1 and 2, it can be seen that $f_{1,(3)}$ in both $P\{1,3\}$ and $P\{2,3\}$. Hence, user 3 can get some information about the demands of users 1 and 2 by checking whether one subfile appears in multiple multicast messages or not, which contradicts the privacy constraint in (11).}

Hence, the composition of each multicast message in $(P\{1\}, P\{1,2\}, P\{1,3\}, P\{1,2,3\})$ (i.e., the subfiles in each XOR multicast message), are symmetric for different demand matrices in which $d_1 = (1, 2)$. In other words, knowing $d_1$ and $Z_1$, the probability that $(P\{1\}, P\{1,2\}, P\{1,3\}, P\{1,2,3\})$ is generated for any demand matrix $\mathbb{D}\setminus\{1\}$, is identical. Since the compositions of the received messages for different demand matrices are equivalent from the viewpoint of user 1, user 1 cannot get any information about the demands of other users such that the compositions of the received messages are independent of the demands of other users from the viewpoint of user 1. Similarly, for any user in $[K]$, it cannot get any information about the demands of other users neither. The formal information-theoretic proof on the privacy constraint in (11) of the new private caching scheme can be found in Appendix B.

Performance. For any demand matrix, we transmit $\binom{K}{1} + \cdots + \binom{K}{K} = 2^K - 1 = 7$ multicast messages, each of which contains $L = 2$ linear combinations of subfiles. Since each subfile has $B/8$ bits, the load in the delivery phase is $14/8 = 1.75$, which is less than 3 achieved by the baseline scheme.

\[\square\]

In the following example, we also consider $K = 3$, $N = 6$, $L = 2$, but with $M = 24/7$ which leads $t = 1$ in (17). For $t \geq 1$, the new private caching scheme needs an MDS precoding in the
placement phase.

**Example 2** (K = 3, N = 6, M = 24/7, L = 2). From (17), we can compute t = 1.

**Placement Phase.** Each file $F_i$ where $i \in [N]$ is divided into $2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-t-1} = 7$ non-overlapping and equal-length pieces, which are then encoded by a $(9K, 2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-t-1}) = (8, 7)$ MDS code (the parameters of the MDS code will be explained later). Each MDS coded symbol has $B/7$ bits. By the property of the MDS code, any MDS coded symbols of $F_i$ are the same as (29)–(32). From $P_{1,2}, P_{1,3}, P_{1,2,3}$, user 1 can recover $\binom{K-1}{t} + \cdots + \binom{K-1}{K-t-1} = 3$ MDS coded symbols for each of its desired files. Since it caches $2^{K-1} = 4$ MDS coded symbols for each file, it can recover each of its desired files by the $4 + 3 = 7$ MDS coded symbols.

In short, the $P_S$’s where $|S| < t + 1$ are not transmitted in the delivery phase and thus each user cannot recover all subfiles of its desired files. Hence, we need the MDS precoding for $t \geq 1$.

**Privacy.** By the same reason as Example 1, the new private scheme for $t = 1$ also satisfies the privacy constraint.

**Performance.** For any demand matrix, we transmit $\binom{K}{t+1} + \cdots + \binom{K}{K} = 4$ multicast messages, each of which contains $L = 2$ linear combinations of subfiles. Since each subfile has $B/7$ bits, the load in the delivery phase is $8/7 \approx 1.14$, which is less than $\min\{|LK, N\} \left(1 - \frac{M}{N}\right) = 18/7 \approx 2.57$ achieved by the baseline scheme.

\[\square\]

We are now ready to generalize Examples 1 and 2. We focus on the memory size $M = \frac{2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-t-1}}{N}$, where $t \in [0 : K - 1]$. Notice that if $t = K$, we have $M = N$ and each user can store the whole library in its cache, such that the server needs not to transmit any packet in the delivery phase.

\(^4\)When $t = 0$, it can be seen that $2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-t-1} = 2^K$. So we do not need the MDS precoding.
**Placement Phase.** Each file $F_i$ where $i \in [N]$ is divided into $2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-1}$ non-overlapping and equal-length pieces, which are then encoded by a $\left(2^K, 2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-1}\right)$ MDS code. Each MDS coded symbol has $\frac{2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-1}}{t}$ bits, and the MDS coded symbols of $F_i$ is denoted by $S^i_1, \ldots, S^i_{2^K}$. We randomly generate a permutation of $[2^K]$, denoted by $p_i = (p_{i,1}, \ldots, p_{i,2^K})$, independently and uniformly over the set of all possible permutations. Recall that Pow$(a, j)$ denotes the $j^{th}$ set in the power set of $[a]$ with a lexicographic order. For each $j \in [2^K]$, we generate one subfile

$$f_{i,\text{Pow}(2^K,j)} := S^i_{p_{i,j}}.$$  

Any $2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-1}$ subfiles of $F_i$ can reconstruct $F_i$. For each $\mathcal{W} \subseteq [K]$, user $k \in [K]$ caches $f_{i,\mathcal{W}}$ if $k \in \mathcal{W}$. It can be seen that each user caches $2^{K-1}$ subfiles of each file. Hence, each user totally caches

$$\frac{2^{K-1}}{2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-1}} \times N \cdot B = MB \text{ bits in its cache, satisfying the memory size constraint.}$$

**Delivery Phase for $D$.** Recall that $Q_i$ where $i \in [N]$ denotes the set of users demanding $F_i$. For each subset $S \subseteq [K]$ where $|S| \geq t + 1$, we let the server transmit $X_S = (\mathcal{M}(P_S), P_S)$, where

$$P_S = \mathcal{G}_{L \times N} \left[ f_{1,S \cup Q_1 \setminus (S \cap Q_1)}; f_{2,S \cup Q_2 \setminus (S \cap Q_2)}; \cdots; f_{N,S \cup Q_n \setminus (S \cap Q_n)} \right].$$  

$P_S$ contains $L$ linear combinations and in $P_S$, each user $k \in S$ caches all subfiles except $f_{i,S \cup Q_i \setminus (S \cap Q_i)}$ where $i \in d_k$. By the property of $\mathcal{G}_{L \times N}$ (each $L$ columns are linearly independent), user $k$ can recover $f_{i,S \cup Q_i \setminus (S \cap Q_i)}$ where $i \in d_k$.

**Decodability.** We first introduce the following lemma, which will be proved in Appendix A.

**Lemma 1.** For any demand matrix $D \in \mathcal{D}$, there is no subfile transmitted in more than one multicast message of the scheme in Section III-C.

We focus on user $k \in [K]$ and file $F_i$ where $i \in d_k$. For each subset $S \subseteq [K]$ where $|S| \geq t + 1$ and $k \in S$, user $k$ can recover one uncached subfile of $F_i$ from the multicast message $P_S$. Considering all such subsets, user $k$ can recover $\binom{K-1}{t} + \cdots + \binom{K-1}{K-1}$ uncached subfiles of $F_i$. By Lemma 1, these subfiles are distinct. Hence, user $k$ can totally obtain $2^{K-1} + \binom{K-1}{t} + \cdots + \binom{K-1}{K-1}$ subfiles of $F_i$ from the placement and delivery phases, such that it can recover $F_i$.

**Privacy.** Let us focus on user $k$. Intuitively, for each subfile of $F_i$ cached by user $k$, since the random permutation $p_i$ is unknown to user $k$, it does not know the other users who also cache
it, and thus each cached subfile of $F_i$ is equivalent from the viewpoint of user $k$. Similarly, each uncached subfile of $F_i$ is equivalent from the viewpoint of user $k$. For any demand matrix, there is exactly one subfile of each file in each multicast message $P_S$ where $k \in S$. Among the $N$ subfiles in $P_S$, user $k$ caches all except the $L$ subfiles from its demanded files. In addition, by Lemma 1, there does not exist any subfile transmitted in more than one multicast messages. Hence, the compositions of the multicast messages $(P_S : k \in S)$ for different demand matrices in which $d_k$ is the same, are equivalent from the viewpoint of user $k$.

In Appendix B, we will prove the privacy in a formal information-theoretic way.

**Performance.** For any demand matrix, we transmit $\binom{K}{K-t} + \cdots + \binom{K}{K}$ multicast messages, each of which contains $L$ linear combinations of subfiles. Since each subfile has $2^{B - \binom{K}{K-t} \cdots - \binom{K}{K}}$ bits, the achieved load is $L \frac{2^{B - \binom{K}{K-t} \cdots - \binom{K}{K}}}{2^{K-1+(K-1)\cdots+(K-1)}} = L \frac{2^{\binom{K}{K-t} \cdots - \binom{K}{K}}}{2^{K-1+(K-1)\cdots+(K-1)}}$, as in (17).

**Remark 1.** The proposed placement precoding which leads that each cached subfile of one file by one user is equivalent from the viewpoint of this user, is the key to preserve the privacy of the demands of other users from this user. We refer this precoding as to Private Placement Precoding, which can be generalized as follows.

We focus on a caching placement with a $(n,k)$ MDS precoding where $n \geq k$. Each file $F_i$ is divided into $k$ non-overlapping and equal-length pieces, which are then encoded by a $(n,k)$ MDS code. The MDS coded symbols of $F_i$ is denoted by $S_{i1}^i, \ldots, S_{in}^i$, each of which contains $B/k$ bits. We randomly generate a permutation of $[n]$, denoted by $p_i = (p_{i,1}, \ldots, p_{i,n})$, independently and uniformly over the set of all possible permutations. For each $j \in [n]$, we generate one subfile of each file $F_i$,

$$f_{i,W_j} := S_{p_{i,j}}^i,$$

where $W_j \subseteq [K]$ and we let each user in $W_j$ cache $f_{i,W_j}$. As a result, from the viewpoint of user $k$, each cached subfile of $F_i$ is equivalent from the viewpoint of user $k$, while each uncached subfile of $F_i$ is also equivalent. It is obvious that when $n = k$, the placement is uncoded. Hence, the proposed private placement precoding can be used with any uncoded cache placement.

Even if we use the proposed private precoding for the MAN caching scheme described in Section II-B, the privacy constraint does not hold because the compositions of the MAN multicast messages are not symmetric for different demand matrices.
D. New Scheme to achieve (18)

When \( M \geq \frac{2^{K-1}}{2K}N \), by memory-sharing between the corner points \( t = K - 1 \) and \( t = K \) in (17), the new improved scheme in Section III-D achieves the load \( L \left( 1 - \frac{M}{N} \right) \), which coincides the converse bound for the MAN caching model with multiple request in [32].

In the following, we will introduce another private caching scheme for \( M = \frac{2^{K-1}}{2K}N \). By memory-sharing between the corner points \( M = \frac{2^{K-1}}{2K}N \) and \( M_2 = N \), for any \( M_1 \geq \frac{2^{K-1}}{2K}N \), the load \( L \left( 1 - \frac{M_1}{N} \right) \) is achievable. Hence, if \( \frac{2^{K-1}}{2K}N \leq \frac{2^{K-1}}{2K}N \) (i.e., \( K \geq 4 \)), we can replace the corner point in (17) with \( t = K - 1 \) by the corner point in (18).

**Placement Phase.** Each file \( F_i \) where \( i \in [N] \) is divided into \( \binom{K}{K-1} + \binom{K}{K} = 2K \) non-overlapping and equal-length pieces, denoted by \( S_1^i, \ldots, S_{2K}^i \), where each piece has \( \frac{B}{2K} \) bits. We randomly generate a permutation of \( [2K] \), denoted by \( p_i = (p_{i,1}, \ldots, p_{i,2K}) \), independently and uniformly over the set of all possible permutations. For each \( k \in [K] \), we generate one subfile \( f_{i,k} = S_{p_i,k}^i \). In addition, for each \( q \in [K] \), we also generate one subfile \( f_{i,[K],q} = S_{p_i,k+q}^i \).

Each user \( k \in [K] \) caches \( f_{i,W} \) where \( W \subseteq [K] \) and \( |W| = K - 1 \), if \( k \in W \). User \( k \) also caches \( f_{i,[K],q} \) for each \( q \in [K] \). Hence, each user totally caches \( \frac{K-1}{2K} + \frac{K}{2K} = \frac{1}{2K}NB = MB \) bits in its cache, satisfying the memory size constraint.

**Delivery Phase for \( D \).** Notice that each user caches \( 2K - 1 \) subfiles of each file, and thus it needs to recover one subfile of each of its desired files.

In the delivery phase, only one multicast message is generated and transmitted by the server, \( X_{[K]} = (\mathcal{M}(P_{[K]}), P_{[K]}) \), where

\[
P_{[K]} = \mathcal{G}_{L \times KN} \mathcal{g}_D.
\]

\( \mathcal{g}_D \) is a vector containing \( KN \) pieces. Define \( \mathcal{g}_D(j) \) as the \( j \)th piece of \( \mathcal{g}_D \), where \( j \in [KN] \). For each \( i \in [N] \) and each \( k \in [K] \),

- if \( i \in d_k \) (i.e., user \( k \) demands \( F_i \)), we let \( \mathcal{g}_D((i - 1)K + k) = f_{i,[K]\{k\}} \);
- otherwise, we let \( \mathcal{g}_D((i - 1)K + k) = f_{i,[K],k} \).

**Decodability.** Among the KN subfiles in \( P_{[K]} \), each user \( k \in [K] \) caches all except \( f_{i,[K]\{k\}} \) where \( i \in d_k \). By the property of \( \mathcal{G}_{L \times KN} \) (each \( L \) columns are linearly independent), user \( k \) can recover these \( L \) subfiles. Hence, we prove the decodability.

**Privacy.** Let us focus on user \( k \). Intuitively, for any demand matrix, there are exactly \( K \) subfiles of each file in \( P_{[K]} \). Among the \( K \) subfiles of each file demanded by user \( k \), user \( k \) caches \( K - 1 \) subfiles, while among the \( K \) subfiles of each file not demanded by user \( k \), user \( k \) caches \( K \)
subfiles. In addition, for any \( i \in [N] \), each subfile of \( F_i \) cached by user \( k \) is equivalent from the viewpoint of user \( k \). Hence, the multicast message \( P_{[k]} \) for different demand matrices in which \( d_k \) is the same, are equivalent from the viewpoint of user \( k \).

In Appendix C, we will prove the privacy in a formal information-theoretic way.

*Performance.* For any demand matrix, \( P_{[k]} \) contains \( L \) linear combinations of subfiles. Since each subfile has \( \frac{B}{2K} \) bits, the achieved load is \( \frac{1}{2K} \), as in (18).

IV. CODED CACHING WITH PRIVATE AND DISTINCT DEMANDS

The proposed baseline and new improved private caching schemes can preserve users’ demands for any possible demand matrix. However, the main limitation of the baseline scheme is the lack of coded caching gain, while the main limitation of the new scheme is the need for high memory size. In the following, we will focus on the shared-link caching model with private and distinct demands, where \( L = 1 \) and there does not exist any file demanded by more than one user.

A practical application of this model is a distributed computation paradigm, as described in Section I-E. Another example where users have (with overwhelming high probability) distinct requests is the case of asynchronous video streaming. As already introduced by MAN in [2], we can consider a streaming session as the sequential and independent video chunks, where each chunk has \( B \) bits. We can identify the chunks with ‘files’. Users can start their streaming session at any time. Assuming the number of chunks contained in one video is large, even if the number of videos in the library is not large, the probability that two users request the same chunk at the same is extremely small. It goes to 0 if the number of chunks in one video goes to infinity and the number of users \( K \) is limited.

For this model, we will propose a novel private coded caching scheme for this model, which does not need a high memory size.

A. Main Results

We now focus on a \((K, N, M)\) shared-link caching system with private and distinct demands where \( N \geq K \). During the delivery phase, each user demands one file \( d_k \in [N] \) and we denote the demand vector of the \( K \) users by \( d = (d_1, \ldots, d_K) \). We assume that each user demands a distinct file, i.e., \( |\{d_k : k \in [K]\}| = K \). In addition, we define \( d_{\backslash \{k\}} \) for each \( k \in [K] \) as the demands of all users except user \( k \), where

\[
d_{\backslash \{k\}} := [d_1; \ldots; d_{k-1}, d_{k+1}, \ldots, d_K].
\]
Now the decodability and privacy constraints in (9) and (11) become
\[
H(F_{d_k}|\{X_S: S \subseteq [K], k \in S\}, Z_k, d_k) = 0, \forall k \in [K]; \quad \text{(Decodability)} \tag{45}
\]
\[
I(d_{\{k\}}; \{X_S: S \subseteq [K], k \in S\}|Z_k, d_k) = 0, \forall k \in [K]. \quad \text{(Privacy)} \tag{46}
\]

The objective is to determine, for a fixed \(M \in [0, N]\), the minimum achievable load \(R_d^*\).

For the shared-link caching systems with private and distinct demands, we propose a non-trivial private caching scheme for this model by creating some virtual users demanding the unrequested files, whose detailed description can be found in Section IV-B.

**Theorem 5.** For the \((K, N, M)\) shared-link caching system with private and distinct demands where \(N \geq K\), \(R_d^*\) is upper bounded by the lower convex envelope of \((M, R_{d,new}) = (0, K)\) and the following memory-load pairs
\[
(M, R_{d,new}) = \left(\frac{(N-1)}{(t-1)} - \frac{(N-K-1)}{(t-K-1)}, \frac{N}{(t)} - \frac{(N-K-1)}{(t-K-1)}\right), \forall t \in [N]. \tag{47}
\]

Since for the shared-link caching systems with private and distinct demands, we can directly use any converse bound on the worst-case load for the MAN shared-link caching model as in Theorems 3 and 4. Hence, the (order) optimality results in Theorems 3 and 4 with \(L = 1\) also holds for the model considered in this section. In addition, by comparing the proposed scheme in Theorem 5 and the converse bound for the MAN shared-link caching model in [4], we have the following order optimality results, which will be proved in Appendix D.

**Theorem 6 (Order Optimality).** For the \((K, N, M)\) shared-link caching system with private and distinct demands where \(N \geq K\), \(R_{d,new}\) is order optimal within a factor of 4.

Finally, by comparing the achievable bound in (47) with \(t = N - 1\) with the converse bound for the MAN shared-link caching model in [2], we have the following exact optimality result.

**Theorem 7 (Exact Optimality).** For the \((K, N, M)\) shared-link caching system with private demands where \(N \geq K\) and \(M \geq \frac{K}{K+1}N\), we have
\[
R_d^* = R_{d,new} = 1 - \frac{M}{N}. \tag{48}
\]

**Proof: Converse.** From [2, Theorem 2] with \(s = 1\), we have
\[
R_d^* \geq 1 - \frac{M}{N}. \tag{49}
\]
Achievability. When \( M \geq \frac{K}{K+1} N \), by memory-sharing the corner points between \( t = K - 1 \) and \( t = K \) in (47), we can achieve the load \( 1 - \frac{M}{N} \).

\[ \Box \]

B. Novel Coded Caching Scheme with Private and Distinct Demands

We first use an example to highlight the key ideas.

Example 3 \( (K = 2, N = 6, M = 54/19) \). Consider a \((K, N, M) = (2, 6, 54/19)\) shared-link caching problem with private and distinct demands. By the baseline private caching scheme, the achieved load is \( R_{\text{base}} = \min\{K, N\} (1 - M/N) = 20/19 \approx 1.05 \). Since \( M < N/2 \), we cannot directly use the new scheme in Theorem 2. By memory-sharing between the corner point in (17) with \( t = 0 \) and the corner point in (16) with \( M_1 = 0 \), the combined scheme can achieve \( R_{\text{com}} = 31/38 \approx 0.82 \). In the following, we introduce the novel scheme in Theorem 5 for this example.

Placement Phase. From (47), we can compute \( t = 3 \) in this example. Each file \( F_i \) where \( i \in [N] \) is divided into \( \binom{N}{t} - \binom{N-K-1}{t-K-1} \) 19 non-overlapping and equal-length pieces, denoted by \( S_{i1}^i, \ldots, S_{i19}^i \), where each piece has \( B/19 \) bits. We randomly generate a permutation of \([19]\), denoted by \( p_i = (p_{i1}, \ldots, p_{i19}) \), independently and uniformly over the set of all possible permutations. For each set \( \mathcal{W} \subseteq [N] \) where \( |\mathcal{W}| = t \) and \( \{1, 2\} \notin \mathcal{W} \), we generate a subfile \( f_{i,\mathcal{W}} \) of \( F_i \) according to \( p_i \) as follows,

\[
\begin{align*}
  f_{i,\{1,3,4\}} &= S_{p_{i1}}^i, \\
  f_{i,\{1,3,5\}} &= S_{p_{i2}}^i, \\
  f_{i,\{1,3,6\}} &= S_{p_{i3}}^i, \\
  f_{i,\{1,4,5\}} &= S_{p_{i4}}^i, \\
  f_{i,\{1,4,6\}} &= S_{p_{i5}}^i, \\
  f_{i,\{1,5,6\}} &= S_{p_{i6}}^i, \\
  f_{i,\{2,3,4\}} &= S_{p_{i7}}^i, \\
  f_{i,\{2,3,5\}} &= S_{p_{i8}}^i, \\
  f_{i,\{2,3,6\}} &= S_{p_{i9}}^i, \\
  f_{i,\{2,4,5\}} &= S_{p_{i10}}^i, \\
  f_{i,\{2,4,6\}} &= S_{p_{i11}}^i, \\
  f_{i,\{2,5,6\}} &= S_{p_{i12}}^i, \\
  f_{i,\{3,4,5\}} &= S_{p_{i13}}^i, \\
  f_{i,\{3,4,6\}} &= S_{p_{i14}}^i, \\
  f_{i,\{3,5,6\}} &= S_{p_{i15}}^i, \\
  f_{i,\{4,5,6\}} &= S_{p_{i16}}^i.
\end{align*}
\]  

(50)

There are \( \binom{N}{t} - \binom{N-K}{t-K} \) subfiles in (50). Each user \( k \in [K] \) caches \( f_{i,\mathcal{W}} \) if \( k \in \mathcal{W} \). Hence, each user in \([2]\) caches \( \binom{N-1}{t-1} - \binom{N-K}{t-K} = 6 \) subfiles in (50). In addition, we also generate the following subfiles,

\[
\begin{align*}
  f_{i,\{1,2\},1} &= S_{p_{i17}}^i, \\
  f_{i,\{1,2\},2} &= S_{p_{i18}}^i, \\
  f_{i,\{1,2\},3} &= S_{p_{i19}}^i.
\end{align*}
\]  

(51)

Each user \( k \in [K] \) caches all

\[
\binom{N}{t} - \binom{N-K-1}{t-K-1} - \left( \binom{N}{t} - \binom{N-K}{t-K} \right) = \binom{N-K-1}{t-K} = 3
\]

\[ \Box \]
In this example, we have we define an integer subfiles in (51).

Hence, each user caches

$$
\binom{N-1}{t-1} - \binom{N-K}{t-K} + \binom{N-K-1}{t-K} = \binom{N-1}{t-1} - \binom{N-K-1}{t-K} = 9
$$

subfiles of each file, and thus it totally caches \(6^{9B}/19 = \) MB, satisfying the memory size constraint. In addition, for each subfile of \(F_i\) cached by user \(k \in [K]\), user \(k\) does not know the other user(s) who also cache(s) it. Hence, each cached subfile of \(F_i\) is equivalent from the viewpoint of user \(k\). Each uncached subfile of \(F_i\) is also equivalent from the viewpoint of user \(k\).

**Delivery Phase** for \(d = (3, 4)\). We generate a permutation of \([N] \setminus \cup_{k \in [K]} \{d_k\} = \{1, 2, 5, 6\}\), denoted by \(u = (u_1, \ldots, u_{N-K})\), where \(u_1 < u_2 < \ldots < u_{N-K}\). In this example, we have \(u = (1, 2, 5, 6)\). For each \(k' \in [K+1: N]\), we let

$$
d_{k'} = u_{k'-K},
$$
i.e., \(d_3 = 1, d_4 = 2, d_5 = 5, \) and \(d_6 = 6\). In one word, we generate \(N-K\) virtual users, demanding the remaining \(N-K\) files which are not requested by the \(K\) users.

For each \(k' \in [K+1: N]\), we sort all sets \(S_k' \subseteq [N]\) where \(|S_k'| = t+1 = 4\) and \(\{1, 2, k'\} \subseteq S_k'\), in a lexicographic order, denoted by \(S_k'(1), \ldots, S_k'\left(\left(\begin{array}{c}N-K-1 \\ t-K \end{array}\right)\right)\). For each \(j \in \left[\left(\begin{array}{c}N-K-1 \\ t-K \end{array}\right)\right] = [3]\), we define an integer

$$
q_{k',S_k'(j)} := j.
$$

In this example, we have

\begin{align*}
q_{3,\{1,2,3,4\}} &= 1, & q_{3,\{1,2,3,5\}} &= 2, & q_{3,\{1,2,3,6\}} &= 3; \\
q_{4,\{1,2,3,4\}} &= 1, & q_{4,\{1,2,4,5\}} &= 2, & q_{4,\{1,2,4,6\}} &= 3; \\
q_{5,\{1,2,3,5\}} &= 1, & q_{5,\{1,2,4,5\}} &= 2, & q_{5,\{1,2,5,6\}} &= 3; \\
q_{6,\{1,2,3,6\}} &= 1, & q_{6,\{1,2,4,6\}} &= 2, & q_{6,\{1,2,5,6\}} &= 3.
\end{align*}

For each set \(S \subseteq [N]\) where \(|S| = t+1 = 4\), the server generates a binary sum of \(t+1 = 4\) subfiles. We focus on the following two cases.

- \([K] \not\subseteq S\). The server generates

$$
W_S = \bigoplus_{k \in S} f_{d_k,S \setminus \{k\}}.
$$
In this example, we have

\[ W_{\{1,3,4,5\}} = f_3,\{3,4,5\} \oplus f_1,\{1,4,5\} \oplus f_2,\{1,3,5\} \oplus f_5,\{1,3,4\}, \]  
\[ W_{\{1,3,4,6\}} = f_3,\{3,4,6\} \oplus f_1,\{1,4,6\} \oplus f_2,\{1,3,6\} \oplus f_6,\{1,3,4\}, \]  
\[ W_{\{1,3,5,6\}} = f_3,\{3,5,6\} \oplus f_1,\{1,5,6\} \oplus f_5,\{1,3,6\} \oplus f_6,\{1,3,5\}, \]  
\[ W_{\{1,4,5,6\}} = f_3,\{4,5,6\} \oplus f_2,\{1,5,6\} \oplus f_5,\{1,4,6\} \oplus f_6,\{1,4,5\}, \]  
\[ W_{\{2,3,4,5\}} = f_4,\{3,4,5\} \oplus f_1,\{2,4,5\} \oplus f_2,\{2,3,5\} \oplus f_5,\{2,3,4\}, \]  
\[ W_{\{2,3,4,6\}} = f_4,\{3,4,6\} \oplus f_1,\{2,4,6\} \oplus f_2,\{2,3,6\} \oplus f_6,\{2,3,4\}, \]  
\[ W_{\{2,3,5,6\}} = f_4,\{3,5,6\} \oplus f_1,\{2,5,6\} \oplus f_5,\{2,3,6\} \oplus f_6,\{2,3,5\}, \]  
\[ W_{\{2,4,5,6\}} = f_4,\{4,5,6\} \oplus f_2,\{2,5,6\} \oplus f_5,\{2,4,6\} \oplus f_6,\{2,4,5\}, \]  
\[ W_{\{3,4,5,6\}} = f_1,\{1,5,6\} \oplus f_2,\{3,5,6\} \oplus f_5,\{3,4,6\} \oplus f_6,\{3,4,5\}. \]  

\* \([K] \subseteq S\). The server generates

\[ W_S = \bigoplus_{k \in [K]} f_k,\{S \setminus \{k\}\} \oplus \bigoplus_{k' \in S \setminus [K]} f_{d_{k'},\{K\} \setminus \{k'\}}. \]  

In this example, we have

\[ W_{\{1,2,3,4\}} = f_3,\{2,3,4\} \oplus f_4,\{1,3,4\} \oplus f_1,\{1,2\},1 \oplus f_2,\{1,2\},1, \]  
\[ W_{\{1,2,3,5\}} = f_3,\{2,3,5\} \oplus f_4,\{1,3,5\} \oplus f_1,\{1,2\},2 \oplus f_5,\{1,2\},1, \]  
\[ W_{\{1,2,3,6\}} = f_3,\{2,3,6\} \oplus f_4,\{1,3,6\} \oplus f_1,\{1,2\},3 \oplus f_6,\{1,2\},1, \]  
\[ W_{\{1,2,4,5\}} = f_3,\{2,4,5\} \oplus f_4,\{1,4,5\} \oplus f_2,\{1,2\},2 \oplus f_5,\{1,2\},2, \]  
\[ W_{\{1,2,4,6\}} = f_3,\{2,4,6\} \oplus f_4,\{1,4,6\} \oplus f_2,\{1,2\},3 \oplus f_6,\{1,2\},2, \]  
\[ W_{\{1,2,5,6\}} = f_3,\{2,5,6\} \oplus f_4,\{1,5,6\} \oplus f_5,\{1,2\},2 \oplus f_6,\{1,2\},3. \]  

Finally we sort all sets \( S \subseteq [N] \) where \( |S| = t + 1 \), denoted by \( S(1), \ldots, S(\binom{N}{t+1}) \), where

\[ \left( \bigcup_{k \in S(1)}\{d_k\}, \ldots, \bigcup_{k \in S(\binom{N}{t+1})}\{d_k\} \right) \]

is in a lexicographic order. The server then transmits \( X_{[K]} = (M(P), P) \), where

\[ P = \left( W_{S(1)}, \ldots, W_{S(\binom{N}{t+1})} \right). \]  

Decodability. User 1 has cached 9 subfiles of \( F_3 \) in its cache. Hence, it needs to recover the remaining 10 subfiles in the delivery phase. For each XOR message in (55)-(58) and (65)-(70), from the metadata \( M(P) \), user 1 knows that there exists one distinct subfile of \( F_3 \) in this XOR
message. Except this subfile, the other subfiles in the XOR message is cached by user 1 and thus it can recover this subfile. Hence, user 1 can totally recover 10 different subfiles which are not cached by itself. Hence, it can recover $F_3$ in the delivery phase. Similarly, user 2 can also recover $F_4$ in the delivery phase.

**Privacy.** We focus on user 1. For any demand vector, we have

1) for any $i \in [N]$, each subfile of $F_i$ cached by user $k$ is equivalent from the viewpoint of user 1. In addition, each subfile of $F_i$ not cached by user $k$ is also equivalent from the viewpoint of user 1.

2) In $P$, for each $T \subseteq [N]$ where $|T| = t + 1$, there is one XOR message including one subfile of each file in $T$. If $d_1 \in T$, in this XOR message there are $t$ subfiles cached by user 1 and one subfile of $F_{d_1}$ not cached by user 1; otherwise, in this XOR message all $t + 1$ subfiles are not cached by user 1.

3) There does not exist any subfile appearing in two XOR messages of $P$ by the construction. Hence, user 1 cannot distinguish which users are virtual users such that it cannot get any information about the demands of other users. Similarly, user 2 cannot get any information about the demands of other users either. The formal information-theoretic proof on the privacy constraint in (46) of the proposed scheme can be found in Appendix E.

**Performance.** For any demand vector, we transmit $\binom{N}{t+1} = 15$ XOR messages, each of which contains $B/19$ bits. Hence, the achieved load is $15/19 \approx 0.79$, which is less than $R_{\text{base}} \approx 1.05$ achieved by the baseline scheme and $R_{\text{com}} \approx 0.82$ achieved by memory-sharing between the baseline scheme and the new scheme in Theorem 2.

\[ \square \]

**Remark 2.** There is a straightforward way to design a private scheme by creating $N - K$ virtual users demanding the remaining $N - K$ files.

Fix one $t' \in [N]$. In the placement phase, for each file $i \in [N]$ and each $W \subseteq [N]$ where $|W| = t'$, we generate a subfile $f_{i,W}$, which should be cached by users in $W$. Hence, the memory size is $M' = N\frac{t'}{N} = t'$.

In the delivery phase, we let each virtual user demand one file as in (52). For each set $S \subseteq [N]$ where $|S| = t' + 1$, we generate $W'_S$ as the MAN multicast message, $W'_S = \bigoplus_{k \in S} f_{d_k,S \setminus \{k\}}$. The broadcast message from the server is $X[K] = \{W'_S : S \subseteq [N], |S| = t' + 1\}$. Hence, the achieved load is $\frac{N-t'}{t'+1}$. In Example 3 this straightforward scheme achieves $16/19 \approx 0.84$, which is larger
than the proposed scheme in Example 3.

Each subfile \( f_{i,W} \) where \([K] \subseteq W\) is cached by all users in \([K]\). If \( F_i \) is demanded by some user in \([K]\), we need not to transmit any subfile of \( F_i \) cached by all users in \([K]\) in the delivery phase; otherwise, in the delivery phase we transmit \( \binom{N-K-1}{t-K-1} \) subfiles of \( F_i \) cached by all users in \([K]\). However, in the placement, we have generate \( \binom{N-K}{t-K} \) subfiles of \( F_i \) cached by all users in \([K]\). In other words, some of such subfiles are redundant. By removing the redundant subfiles, we can obtain the proposed scheme in Example 3.

We are now ready to generalized the proposed scheme in Example 3. When \( M = 0 \), the server transmits the demanded file to each user individually to guarantee the decodability and privacy, i.e., \( X_{\{k\}} = F_{d_k} \) for each \( k \in [K] \). We then focus on the memory size \( M = \binom{N-1}{t-K-1} - \binom{N-K-1}{t-K-1} \) in (47), where \( t \in [N - 1] \). Notice that if \( t = N \), we have \( M = N \) and each user can store the whole library in its cache, such that the server needs not to transmit any packet in the delivery phase.

**Placement Phase.** Each file \( F_i \) where \( i \in [N] \) is divided into \( \binom{N}{t} - \binom{N-K-1}{t-K-1} \) non-overlapping and equal-length pieces, denoted by \( S_{i,1}, \ldots, S_{i,\binom{N}{t}-(N-K-1)} \), where each piece has \( \frac{B}{\binom{N}{t}-(N-K-1)} \) bits. We randomly generate a permutation of \( \left[ \binom{N}{t} - \binom{N-K-1}{t-K-1} \right] \), denoted by \( p_i = (p_{i,1}, \ldots, p_{i,\binom{N}{t}-(N-K-1)}) \), independently and uniformly over the set of all possible permutations. We sort all sets \( W \subseteq [N] \) where \( |W| = t \) and \( [K] \not\subseteq W \), in a lexicographic order, denoted by \( W(1), \ldots, W(\binom{N}{t} - \binom{N-K}{t-K}) \). For each \( j \in \left[ \binom{N}{t} - \binom{N-K}{t-K} \right] \), we generate a subfile

\[
f_{i,W(j)} = S_{p_{i,j}}^j.
\]

In addition, for each \( j_1 \in \left[ \binom{N-K-1}{t-K} \right] \), we generate a subfile

\[
f_{i,[K],j_1} = S_{p_{i,j_1},j_1}^{j_1}.
\]

Each user \( k \in [K] \) caches \( f_{i,W} \) where \( W \subseteq [N], |W| = t, [K] \not\subseteq W \), and \( k \in W \). In addition, user \( k \) caches all subfiles \( f_{i,[K],j_1} \) where \( j_1 \in \left[ \binom{N-K-1}{t-K} \right] \).

Hence, each user caches \( \binom{N}{t-K} - \binom{N-K-1}{t-K} + \binom{N-K-1}{t-K} = \binom{N-1}{t-K} - \binom{N-K-1}{t-K} \) subfiles of each file, and thus it totally caches \( \binom{N}{t} - \binom{N-K-1}{t-K} \) subfiles of each file, satisfying the memory size constraint.

**Delivery Phase for d.** We generate a permutation of \( [N] \setminus \cup_{k \in [K]} \{d_k\} \), denoted by \( u = (u_1, \ldots, u_{N-K}) \), where \( u_1 < u_2 < \ldots < u_{N-K} \). For each \( k' \in [K+1: N] \), we let \( d_{k'} = u_{k'-K} \) as in (52). We sort all sets \( S_{k'} \subseteq [N] \) where \( |S_{k'}| = t + 1 \) and \( ([K] \cup \{k'\}) \subseteq S_{k'} \), in a lexicographic
generates \( \ell \) virtual users. We use the an \( \ell \) of each user contains \( q \) order, denoted by \( q_{k', \ell}(1), \ldots, q_{k', \ell} \left( \binom{N-K-1}{t-K} \right) \). For each \( j \in \left[ \binom{N-K-1}{t-K} \right] \), we define an integer \( q_{k', \ell}(j) := j \), as in (53).

For each set \( S \subseteq [N] \) where \( |S| = t + 1 \), the server generates a binary sum of \( t + 1 \) subfiles. We focus on the following two cases.

- \([K] \not\subseteq S\). The server generates \( W_S = \bigoplus_{k \in S} f_{d_k, S \setminus \{k\}} \), as in (54).
- \([K] \subseteq S\). The server generates \( W_S = \bigoplus_{k \in [K]} f_{d_k, S \setminus \{k\}} \bigoplus \bigoplus_{k' \in S \setminus [K]} f_{d_{k'}, [K], q_{k', \ell}}, \) as in (64).

Finally we sort all sets \( S \subseteq [N] \) where \( |S| = t + 1 \), denoted by \( S(1), \ldots, S \left( \binom{N}{t+1} \right) \), where \( \left( \bigcup_{k \in S(1)} \{d_k\}, \ldots, \bigcup_{k \in S \left( \binom{N}{t+1} \right)} \{d_k\} \right) \) is in a lexicographic order. The server then transmits \( X_{[K]} = (\mathcal{M}(P), P) \), where \( P = \left( W_{S(1)}, \ldots, W_{S \left( \binom{N}{t+1} \right)} \right) \), as in (71).

**Decodability.** We focus on user \( k \in [K] \). From the metadata \( \mathcal{M}(P) \), user \( k \) knows that among all \( \binom{N}{t+1} \) XOR messages, \( W_S \) is useful to user \( k \) if \( W_S \) contains one subfile of \( F_{d_k} \). In addition, \( W_S \) is a XOR message of \( t + 1 \) subfiles while user \( k \) caches all except \( f_{d_k, S \setminus \{k\}} \), such that user \( k \) can recover \( f_{d_k, S \setminus \{k\}} \). Hence, user \( k \) can recover all uncached subfiles of \( F_{d_k} \) from \( P \) transmitted in the delivery phase.

**Privacy.** The intuition was given in Example 3. The information-theoretic proof can be found in Appendix E.

**Performance.** For any demand vector, we transmit \( \binom{N}{t+1} \) XOR messages, each of which contains \( B \) bits. Hence, the achieved load is \( \frac{B}{\binom{N}{t+1} - \binom{N-K-1}{t-K-1}} \), as shown in (47).

C. Extension to General Demands

In the following, we extend the proposed scheme in Theorem 5 with virtual users to the case of general demands formulated in Section II. The main ingredient of the scheme is as follows. Since in the library there are \( N \) files while each user demands \( L \) among them (i.e., the demand set of each user contains \( L \) files), it can be seen that there are totally \( \binom{N}{L} \) possibilities of demand sets, each of which is requested by at most \( K \) users. Hence, following the strategy based on virtual users in Theorem 5, we can generate \( \binom{N}{L} K - K \) virtual users such that the system contains totally \( \binom{N}{L} K \) real or virtual users and each possible demand set is requested by exactly \( K \) real or virtual users. We use the an \( L \)-round MAN caching scheme for this system. From a similar intuition as the proposed scheme in Theorem 5, each real user cannot distinguish which users are virtual users such that it cannot get any information about the demands of other real users.

The proposed caching scheme in [22] focuses on the case of single request (i.e., \( L = 1 \)) and generates \( \ell \) virtual users each of whom randomly demands one file, which cannot guarantee the
information-theoretic privacy constraint in (10). Our scheme needs a finite and fix number of virtual users which depends on the system parameters and also proposes a determinate scenario to choose one demand set for each virtual user, such that the the information-theoretic privacy constraint in (10) holds.

We then introduce the performance of the extended scheme, whose detailed description could be found in Appendix F.

**Theorem 8.** For the \((K, N, M, L)\) shared-link caching system with private demands, \(R^*\) is upper bounded by the lower convex envelope of \((M, R_{\text{base}}) = (M, \min\{LK, N\}(1 - M/N))\) and the following memory-load pairs

\[
(M, R_e) = \left(\frac{t}{t + 1}N, L \frac{N}{L} K - t\right), \quad \forall t \in \left[\binom{N}{L} : K\right].
\]

(74)

**Theorem 9 (Order Optimality).** For the \((K, N, M, L)\) shared-link caching system with private demands, the proposed scheme in (8) is order optimal within a factor of 8 for \(L = 1\) and a factor of 22 if \(L > 1\).

**Proof: Converse.** We use the existing converse bound in [5], [8] for the shared-link caching model without privacy, which obviously provides a load lower bound for the shared-link caching model with private demands. It was proved in [5], [8] that the lower convex envelope of \((M, N - M)\) and \(L^{K - t}/(t + 1)\) where \(t \in [0 : K]\) is order optimal within a factor of 4 if \(L = 1\) and a factor of 11 if \(L > 1\), respectively.

**Achievability.** We can use the same method as in Appendix D to prove that the multiplicative gap between the convex envelope \(\left(\frac{t'}{t' + 1}N, L \frac{N}{L} K - t'\right)\), where \(t' \in \left[\binom{N}{L} : K\right]\) and the convex envelop \(L^{K - t}/(t + 1)\) where \(t \in [K]\) is at most two. In addition, the proposed scheme in Theorem 8 achieves \((M, \min\{LK, N\}(1 - M/N))\).

In conclusion, we prove the scheme in Theorem 8 is order optimal within a factor of \(4 \times 2 = 8\) if \(L = 1\) and a factor of \(11 \times 2 = 22\) if \(L > 1\), respectively.

The caching scheme in Theorem 8 is equivalent to the \(L\)-round MAN scheme with \(\binom{N}{L} K\) users, and thus the needed sub-packetization level is exponential to \(\binom{N}{L} K\) (i.e., \(O(2^{\binom{N}{L} K})\)), which is much higher than the sub-packetizations levels of the \(K\)-user MAN scheme without virtual users (exponential to \(K\), i.e., \(O(2^K)\)), the proposed scheme in Theorem 2 (exponential to \(K\), i.e., \(O(2^K)\)), and the proposed scheme in Theorem 5 (exponential to \(N\), i.e., \(O(2^N)\)). While the scheme in Theorem 8 solves the problem within a constant factor in an information theoretic
Fig. 2: \((M,R)\) tradeoff for the \((K,N,M,L)\) shared-link caching system with private demands.

sense without sub-packetization (finite length) limits, it still requires a sub-packetization order quite larger than even the baseline Maddah-Ali and Niesen (MAN) scheme, and therefore the problem of preserving the privacy of the demands in the regime \(M < N/2\) with small sub-packetization (at least not exponentially larger than the original MAN scheme remains open.

V. Numerical Evaluations

We first provide numerical evaluations of the proposed private caching schemes for the \((K,N,M,L)\) shared-link caching system with private demands. In Fig. 2, we let \(L = 1\) and use the converse bound in [4] for the shared-link caching model with single request, as the converse bound in our problem. In Fig. 2a, we let \((K,N) = (10, 20)\) and in Fig. 2b we let \((K,N) = (10, 5)\). Both of the figure shows that the combination of the baseline scheme in Theorem 1 and the new scheme in Theorem 2 outperforms the baseline scheme in Theorem 1. In addition, when \(M \leq N/2\), the proposed scheme in Theorem 74 with load \(R_{d,new}\) improves the combined scheme with load \(R_{com}\), while when \(M \geq N/2\), the loads achieved by these two schemes are approximately equal.

We then provide numerical evaluations of the proposed private caching schemes for the \((K,N,M)\) shared-link caching system with private and distinct demands, in Fig. 3. We also plot the converse bound in [4] for the shared-link caching model with single request, as the converse bound in our problem. In Fig. 3a, we let \((K,N) = (5, 10)\) and in Fig. 3b we let \((K,N) = (5, 20)\).
VI. CONCLUSIONS

In this paper, we introduced a novel shared-link caching model with private demands, while the objective is to design a two-phase caching scheme with minimum load while preserving the privacy of the users demands. We proposed both a baseline private caching scheme and a new improved private caching scheme. Compared to the existing converse bounds for the shared-link caching model without privacy constraint, the new scheme was proved to be order optimal within a factor of 2 when $M \geq N/2$. In addition, for some large memory size regime, the new scheme is exactly optimal, which shows that we can maintain optimality while preserving the privacy of the users demands. We then focused on the case where each user has a single and distinct demand. We proposed a novel private caching scheme specially for this model, which was shown to be order optimal within a factor of 4. Finally, we extended this scheme to general demands which was order optimality within a factor of $2^2$, but with an extremely high sub-packetization level.

We believe that preserving the privacy of the users demands from other users that legitimately use the caching/content delivery system is an important problem that differs conceptionally from
previously proposed models with eavesdroppers or private information retrieval (PIR), as shortly outlined in Section I.

On-going/future work includes deriving a converse bound for this caching model with privacy and designing improved private caching schemes with reduced sub-packetization level for low memory size regime.

APPENDIX A

PROOF OF LEMMA 1

It is equivalent to prove for any two sets \( S_1 \subseteq [K] \) and \( S_2 \subseteq [K] \) where \( S_1 \neq S_2 \), we have
\[
(S_1 \cup Q_i) \setminus (S_1 \cap Q_i) \neq (S_2 \cup Q_i) \setminus (S_2 \cap Q_i), \forall i \in [N].
\]
(75)

In addition, for each \( j \in \{2\} \), we let \( S_j \subseteq S_{j,1} \cup S_{j,2} \) where \( S_{j,1} \subseteq Q_i \) and \( S_{j,2} \cap Q_i = \emptyset \).

Without loss of generality, we assume \( |S_1| \geq |S_2| \). We focus on two cases:

1) \( S_{1,2} \neq S_{2,2} \). It can be seen that \( S_{j,2} \subseteq ((S_j \cup Q_i) \setminus (S_j \cap Q_i)) \) for each \( j \in \{2\} \). Hence, (75) holds for this case.

2) \( S_{1,2} = S_{2,2} \) and \( S_{1,1} \neq S_{2,1} \). Since \( |S_1| \geq |S_2| \) and \( S_{1,1} \neq S_{2,1} \), there exists at least one user in \( Q_i \) (assume to be \( k \)) who is in \( S_{1,1} \setminus S_{2,1} \). Hence, this user \( k \) is in \( (S_2 \cup Q_i) \setminus (S_2 \cap Q_i) \) but not in \( (S_1 \cup Q_i) \setminus (S_1 \cap Q_i) \). Hence, (75) holds for this case.

In conclusion, we prove Lemma 1.

APPENDIX B

PROOF OF THE PRIVACY FOR THE NEW SCHEME IN (17)

We consider \( t = 0 \) in (17). It can be seen when \( t > 0 \), the transmitted multicast messages are included in the the transmitted multicast messages for \( t = 0 \). Hence, if we prove the privacy for \( t = 0 \), the privacy for \( t > 0 \) can also be proved.

For the new scheme in (17), we want to prove the privacy constraint in (11),
\[
I(\mathbb{D}_{\{k\}}; (X_S : S \subseteq [K], k \in S)|Z_k, d_k) = 0, \forall k \in [K].
\]
(76)

We now focus on one user \( k \), one demand vector \( d_k \), and one cache realization \( z_k \). Assume \((x_S : S \subseteq [K], k \in S)\) is a possible realization of \((X_S : S \subseteq [K], k \in S)\), given \( d_k \) and \( z_k \). We want to prove for any demand matrix \( \mathbb{D}_{\{k\}} \), the probability
\[
\Pr\{(X_S : S \subseteq [K], k \in S) = (x_S : S \subseteq [K], k \in S)|d_k, z_k, \mathbb{D}_{\{k\}}\}
\]
does not depend on $\mathbb{D}_{\{k\}}$.

For each $S \subseteq [K]$ and each $i \in [N]$, we denote the coded MDS symbol of $F_i$ in $X_S$ by $X_{S,i}$. We have

$$
\Pr\{ (X_S : S \subseteq [K], k \in S) = (x_S : S \subseteq [K], k \in S)|d_k, z_k, \mathbb{D}_{\{k\}} \} = \Pr\{ (X_{S,i} : S \subseteq [K], k \in S, i \in [N]) = (x_{S,i} : S \subseteq [K], k \in S)|d_k, z_k, \mathbb{D}_{\{k\}} \}
$$

$$
= \prod_{i \in [N]} \Pr\{ (X_{S,i} : S \subseteq [K], k \in S) = (x_{S,i} : S \subseteq [K], k \in S)|d_k, z_k, \mathbb{D}_{\{k\}} \},
$$

(77)

where (77) comes from that the placement permutations $p_1, \ldots, p_N$ are independent.

We then focus on two cases:

- $i \in d_k$. It is claimed in Lemma [1] that there does not exist any subfile appearing two multicast messages. Given $z_k$, there are $2^{K-1}$ MDS coded symbols of $F_i$ not in $z_k$, each of which should be in one different $X_S$ where $S \subseteq [K]$ and $k \in S$. Hence, we have

$$
\Pr\{ (X_{S,i} : S \subseteq [K], k \in S) = (x_{S,i} : S \subseteq [K], k \in S)|d_k, z_k, \mathbb{D}_{\{k\}} \} = \frac{1}{2^{K-1}!},
$$

(78)

where $!$ represents the factorial operation.

- $i \notin d_k$. It is claimed in Lemma [1] that there does not exist any subfile appearing two multicast messages. Given $z_k$, there are $2^{K-1}$ coded MDS symbols of $F_i$ in $z_k$, each of which should be in one different $X_S$ where $S \subseteq [K]$ and $k \in S$. Hence, we have

$$
\Pr\{ (X_{S,i} : S \subseteq [K], k \in S) = (x_{S,i} : S \subseteq [K], k \in S)|d_k, z_k, \mathbb{D}_{\{k\}} \} = \frac{1}{2^{K-1}!}.
$$

(79)

It can be seen both of the probabilities in (78) and (79) are independent of $\mathbb{D}_{\{k\}}$. Hence, we can prove the probability in (77) is also independent of $\mathbb{D}_{\{k\}}$. In conclusion, we prove the privacy constraint in (11).
APPENDIX C

PROOF OF THE PRIVACY FOR THE NEW SCHEME IN \[18\]

For the new scheme in \[18\], there is only one multicast message, which is \(X_K\). We also want to prove the privacy constraint in \[11\].

\[
I(\mathbb{D}_{\{k\}}; X_{[K]}|Z_k, d_k) = 0, \quad \forall k \in [K]. \tag{80}
\]

We also focus on one user \(k\), one demand vector \(d_k\), and one cache realization \(z_k\). Assume \(x_{[K]}\) is a possible realization of \(X_{[K]}\), given \(d_k\) and \(z_k\). We want to prove for any demand matrix \(\mathbb{D}_{\{k\}}\), the probability

\[
\Pr\{ X_{[K]} = x_{[K]} \mid d_k, z_k, \mathbb{D}_{\{k\}} \}
\]

does not depend on \(\mathbb{D}_{\{k\}}\).

In \(X_{[K]}\), there are \(K\) pieces of each file \(F_i\). Hence, \(X_{[K],i}\) now denotes the set of \(K\) pieces of \(F_i\) in \(X_{[K]}\). Since the placement permutations \(p_1, \ldots, p_N\) are independent, we have

\[
\Pr\{ X_{[K]} = x_{[K]} \mid d_k, z_k, \mathbb{D}_{\{k\}} \} = \prod_{i \in [N]} \Pr\{ X_{[K],i} = x_{[K],i} \mid d_k, z_k, \mathbb{D}_{\{k\}} \}. \tag{81}
\]

We also focus two cases:

- \(i \in d_k\). Notice that \(z_k\) contains \(2K - 1\) pieces of \(F_i\) while \(F_i\) contains \(2K\) pieces. In addition, in \(X_{[K],i}\) there are \(K - 1\) pieces of \(F_i\) cached in \(z_k\) and one piece of \(F_i\) not cached in \(z_k\). Hence, we have

\[
\Pr\{ X_{[K],i} = x_{[K],i} \mid d_k, z_k, \mathbb{D}_{\{k\}} \} = \frac{1}{2K-1}. \tag{82}
\]

- \(i \notin d_k\). In \(X_{[K],i}\) there are \(K\) pieces of \(F_i\) cached in \(z_k\). Hence, we have

\[
\Pr\{ X_{[K],i} = x_{[K],i} \mid d_k, z_k, \mathbb{D}_{\{k\}} \} = \frac{1}{K} = \frac{1}{2K-1}. \tag{83}
\]

It can be seen both of the probabilities in \[82\] and \[83\] are independent of \(\mathbb{D}_{\{k\}}\). Hence, we can prove the probability in \[81\] is also independent of \(\mathbb{D}_{\{k\}}\). In conclusion, we prove the privacy constraint in \[11\].
Appendix D

Proof of Theorem 6

Converse. It was proved in [4] that for the shared-link caching model with $N \geq K$ and $L = 1$, the lower convex envelope $(M_{\text{MAN}}, R_{\text{MAN}}) = \left( \frac{N}{K}, \frac{K-t}{t+1} \right)$, where $t \in [0 : K]$, achieved by the MAN scheme is order optimal within a factor of 2.

We will also use this converse in our model. Hence, for $M \in [0, N]$, $R_d^*$ is lower bounded by the lower convex envelope $(M_{\text{MAN}}, R_{\text{MAN}}/2) = \left( \frac{N}{K}, \frac{K-t}{2(t+1)} \right)$, where $t \in [0 : K]$.

Achievability. When $M = 0$, the proposed scheme in Theorem 5 can trivially achieve the memory-load pair $(M, R_d) = (0, K)$ by transmit the demanded file of each user individually. We then prove the proposed scheme in Theorem 5 can cover the corner points in Remark 2 achieved by directly creating $N - K$ virtual users demanding the unrequestes files, $(M', R') = \left( t', \frac{N-t'}{t'+1} \right)$, where $t' \in [N]$.

Recall that the achieved corner points in Theorem 5 are, $(M, R_{\text{d,new}}) = \left( \frac{(N-1)(N-K-1)}{(t')^2} - \frac{(N-K-1)}{(t'-K-1)} \right) N, \frac{(N-K-1)}{(t')^2} - \frac{(N-K-1)}{(t'-K-1)}$, $\forall t \in [N]$. Hence, we can directly achieve $(M', R')$ where $t' \in [K] \cup \{N\}$ by letting $t = t'$. In the following, we focus on each $t' \in [K+1 : N-1]$.

We interpolate the proposed scheme in Theorem 5 between $M_1 = \frac{(N-1)(N-K-1)}{(t')^2} - \frac{(N-K-1)}{(t'-K-1)} N$ and $M_2 = N$ to match $(M', R') = \left( t', \frac{N-t'}{t'+1} \right)$ where $t' \in [K+1 : N-1]$. More precisely, by memory-sharing between $\left( M_1, \frac{(N-1)(N-K-1)}{(t')^2} - \frac{(N-K-1)}{(t'-K-1)} \right)$ and $(M_2, 0)$ with coefficient $\alpha = \frac{(N-t')-(N-K-1)}{(t')^2}$, we get

$$M_3 = \alpha N \frac{(N-1)}{(t')} - \frac{(N-K-1)}{(t'-K-1)} + (1 - \alpha) N = \frac{(N-1)}{(t')} N = M';$$

$$R_3 = \alpha \frac{N}{(t'+1)} - \frac{(N-K-1)}{(t'-K-1)} = \frac{N}{(t'+1)} = R'.$$

Hence, we prove that all corner points in $(M', R') = \left( t', \frac{N-t'}{t'+1} \right)$ where $t' \in [N]$ can be achieved by the proposed scheme in Theorem 5.
We will prove that from \((M', R') = (t', \frac{N-t'}{t'+1})\) where \(t' \in [N]\), we can achieve \((M_{\text{MAN}}, 2R_{\text{MAN}}) = (\frac{Nt}{K}, \frac{2(K-t)}{t+1})\), where \(t \in [K]\). Hence, with the achieved point \((M, R_d) = (0, K)\), we can prove when for \(M \in [0, N]\), the proposed scheme in Theorem 5 is order optimal within a factor of 4.

We now focus on one \(t \in [K]\). We consider the following two cases.

- \(\frac{Nt}{K}\) is an integer. Since \(\frac{Nt}{K} \leq N\), we can let \(t' = \frac{Nt}{K}\). With \(t' = \frac{Nt}{K}\), we can achieve

\[
R' = \frac{N - t'}{t' + 1} = \frac{N}{K} \left( K - \frac{t'K}{N} \right)
\]

\[
= \frac{K - t}{t + \frac{K}{N}}
\]

\[
\leq \frac{2(K - t)}{t + 1},
\]

where (86) comes from \(\frac{t+1}{t+K/N} \leq 2\) when \(t \geq 1\) and \(N \geq K\).

- \(\frac{Nt}{K}\) is not an integer. We let \(a = \left\lfloor \frac{Nt}{K} \right\rfloor\) and \(b = \left\lceil \frac{Nt}{K} \right\rceil\). By memory-sharing between \((M'_1, R'_1) = (a, \frac{N-a}{a+1})\) and \((M'_2, R'_2) = (b, \frac{N-b}{b+1})\) with coefficient \(\alpha = \frac{b-Nt/K}{b-a}\), we get

\[
M'_3 = \alpha a + (1 - \alpha) b = \frac{Nt}{K};
\]

\[
R'_3 = \alpha \frac{N - a}{a + 1} + (1 - \alpha) \frac{N - b}{b + 1}
\]

\[
= \frac{b - Nt/K}{a + 1} + \frac{Nt/K - a}{b + 1}
\]

\[
\leq \frac{1}{a + 1} \left[ N - \left( \frac{b - Nt/K}{b - a} + \frac{Nt/K - a}{b - a} \right) \right]
\]

\[
= \frac{1}{a + 1} \left( N - \frac{Nt/K}{b - a} \right)
\]

\[
= \frac{K - t}{a + \frac{K}{N}}
\]

\[
\leq \frac{(\frac{Nt}{K} - 1)}{\frac{K}{N} + \frac{K}{N}} \frac{K - t}{t}
\]

\[
\leq \frac{2(K - t)}{t + 1},
\]

where (87) comes from \(t \geq 1\).

Hence, we conclude the proof of Theorem 6.
APPENDIX E

PROOF OF THE PRIVACY FOR THE PROPOSED SCHEME IN THEOREM 5

We focus one $t \in [N]$ in Theorem 5. For the proposed scheme in Theorem 5, we want to prove the privacy constraint in (46).

$$I(d_{\backslash\{k\}}; X[k] | Z_k, d_k) = 0, \ \forall k \in [K]. \ \ \ \ (88)$$

We now focus on one user $k$, one demand vector $d_k$, and one cache realization $z_k$. Assume $x[k]$ is a possible realization of $X[k]$, given $d_k$ and $z_k$. We want to prove for any demand matrix $d_{\backslash\{k\}}$, the probability

$$\Pr\{X[k] = x[k] | d_k, z_k, d_{\backslash\{k\}}\}$$

does not depend on $d_{\backslash\{k\}}$.

Recall that $u = (u_1, \ldots, u_{N-K})$ is a permutation of $[N] \setminus \bigcup_{k \in [K]} \{d_k\}$, where $u_1 < u_2 < \ldots < u_{N-K}$. Hence $u$ is a function of $d$. For each $k' \in [K+1:N]$, we let $d_{k'} = u_{k'-K}$ as in (52). Hence, we have

$$\Pr\{X[k] = x[k] | d_k, z_k, d_{\backslash\{k\}}\} = \Pr\{X[k] = x[k] | d_k, z_k, d_{\backslash\{k\}}, u\}. \ \ \ \ (89)$$

Recall that $X[k]$ is a vector of linear combinations $(W_{S(1)}, \ldots, W_{S(\binom{N}{t+1})})$ as in (71), where $(\bigcup_{k \in S(1)} \{d_k\}, \ldots, \bigcup_{k \in S(\binom{N}{t+1})} \{d_k\})$ is in a lexicographic order. We define the $j^{th}$ linear combination in $X[k]$ is $X[k](j)$. For each file $F_i$ where $i \in [N]$, the piece of $F_i$ contained in $X[k](j)$ is denoted by $X[k](j,i)$.

So we have

$$\Pr\{X[k] = x[k] | d_k, z_k, d_{\backslash\{k\}}, u\}$$

$$= \Pr\{X[k](1), \ldots, X[k] \left(\binom{N}{t+1}\right)\} = \left(x[k](1), \ldots, x[k] \left(\binom{N}{t+1}\right)\right) | d_k, z_k, d_{\backslash\{k\}}, u\}$$

$$= \prod_{i \in [N]} \Pr\{X[k](1,i), \ldots, X[k] \left(\binom{N}{t+1}, i\right)\} =$$

$$\left(x[k](1,i), \ldots, x[k] \left(\binom{N}{t+1}, i\right)\right) | d_k, z_k, d_{\backslash\{k\}}, u\}, \ \ \ \ (90)$$

where (90) comes from that $p_1, \ldots, p_N$ are independent.

We then focus on two cases:

5 If $X[k](j)$ does not contain any piece of $F_i$, we have $X[k](j, i) = \emptyset$. 
• \( i = d_k \). Notice that there does not exist any subfile appearing two multicast messages. Given \( z_k \), there are \( \binom{N-1}{t} \) pieces of \( F_i \) which are not in \( z_k \), each of which should be in one different \( W_S \) where \( S \subseteq [N], |S| = t + 1, k \in S \). Hence, we have

\[
\Pr\left\{ \left( X_{[K]}(1, i), \ldots, X_{[K]}\left( \binom{N}{t+1}, i \right) \right) = \\
\left( x_{[K]}(1, i), \ldots, x_{[K]}\left( \binom{N}{t+1}, i \right) \right) \mid d_k, z_k, d_{\{k\}}, u \right\} = \frac{1}{\binom{N-1}{t-1}!}.
\] (91)

• \( i \neq d_k \). Notice that there does not exist any subfile appearing two multicast messages. Given \( z_k \), there are \( \binom{N-1}{t-1} - \binom{N-K-1}{t-1} \) pieces of \( F_i \) which are in \( z_k \). In each of \( W_S \) where \( S \subseteq [N], |S| = t + 1, \{d_k, i\} \subseteq \cup_{j \in S}\{d_j\} \), there is one piece of \( F_i \) cached by user \( k \). In each of \( W_S \) where \( S \subseteq ([N] \setminus \{d_k\}), |S| = t + 1, i \in \cup_{j \in S}\{d_j\} \), there is one piece of \( F_i \) not cached by user \( k \). Hence, we have

\[
\Pr\left\{ \left( X_{[K]}(1, i), \ldots, X_{[K]}\left( \binom{N}{t+1}, i \right) \right) = \\
\left( x_{[K]}(1, i), \ldots, x_{[K]}\left( \binom{N}{t+1}, i \right) \right) \mid d_k, z_k, d_{\{k\}}, u \right\} = \\
\frac{1}{\text{Perm} \left( \binom{N-1}{t-1} - \binom{N-K-1}{t-1}, \binom{N-2}{t-1} \right)} \frac{1}{\text{Perm} \left( \binom{N-1}{t}, \binom{N-2}{t} \right)}.
\] (92)

It can be seen both of the probabilities in (91) and (92) are independent of \( d_{\{k\}} \). Hence, we can prove the probability in (90) is also independent of \( d_{\{k\}} \). In conclusion, we prove the privacy constraint in (46).

APPENDIX F
PROOF OF THEOREM 8

In the following, we describe the proposed caching scheme which achieves the memory-load tradeoff in (74). We focus on one \( t \in \left[ \binom{N}{t} \right] \). We define that \( U := \binom{N}{t} \).

**Placement Phase.** Each file \( F_i \) where \( i \in [N] \) is divided into \( \binom{U}{t} \) non-overlapping and equal-length pieces, denoted by \( S_i^1, \ldots, S_i^{\binom{U}{t}} \), where each piece has \( \frac{8}{\binom{U}{t}} \) bits. We randomly generate a permutation of \( \binom{U}{t} \), denoted by \( \mathbf{p}_i = (p_{i,1}, \ldots, p_{i,\binom{U}{t}}) \), independently and uniformly over the set of all possible permutations. We sort all sets \( W \subseteq [U] \) where \( |W| = t \), in a lexicographic order, denoted by \( W(1), \ldots, W\left( \binom{U}{t} \right) \). For each \( j \in \left[ \binom{U}{t} \right] \), we generate a subfile

\[
f_{i,W(j)} = S_{p_{i,j}}^j.
\] (93)
Each user $k \in [K]$ caches $f_{i,W}$ where $W \subseteq [U]$, $|W| = t$, and $k \in W$. Hence, each user caches $\binom{U-1}{t-1}$ subfiles of each file, and thus it totally caches $\frac{U-1}{(t-1)}N_B = \frac{L}{t}N_B = MB$ bits, satisfying the memory size constraint in (74). It can be checked that the above placement is with a private placement precoding as described in Remark 1. Hence, from the viewpoint of user $k \in [K]$, each cached subfile of $F_i$ is equivalent from the viewpoint of user $k$, while each uncached subfile of $F_i$ is also equivalent.

**Delivery Phase for $D$.** Recall that for one possible demand vector by one user $d := (d_1, d_2, \ldots, d_L)$, we should have $1 \leq d_1 < d_2 < \cdots < d_L \leq N$. Hence, there are totally $\binom{N}{L}$ possible demand vectors, denoted by $d^1, \ldots, d^{(N)}$. We define that

$$n_j := |\{k \in [K] : d_k = d^j\}|$$

(94)

where $j \in \binom{N}{L}$, representing the number of real users demanding the demand vector $d^j$. We then allocate one demand vector to each of the $U - K$ virtual users as follows. For each $j \in \binom{N}{L}$, we let $d_{1+j1} = \cdots = d_{(j+1)K-\sum_{q \in [j]} n_q} = d^j$. For example, when $j = 1$, we let $d_{K+1} = \cdots = d_{2K-n_1} = d^1$; when $j = 2$, we let $d_{2K-n_1+1} = \cdots = d_{3K-n_1-n_2} = d^2$. Hence, by this way, each possible demand vector is requested by $K$ real or virtual users.

Recall for any $k \in [U]$, we define $d_k = (d_{k,1}, \ldots, d_{k,L})$. For each set $S \subseteq [U]$ where $|S| = t+1$, we generate

$$Q_S = \mathcal{G}_{L \times L(t+1)} \left[ f_{k_1,S \setminus \{k_1\}}; f_{k_2,S \setminus \{k_2\}}; \cdots; f_{k_{t+1},S \setminus \{k_{t+1}\}} \right].$$

(95)

containing $L$ combinations where $(k_1, \ldots, k_{t+1})$ is a random permutation of $S$ independently and uniformly over the set of all possible permutations, and we define

$$f_{k_j,S \setminus \{k_j\}} := [f_{d_{k_j,1},S \setminus \{k_j\}}; f_{d_{k_j,2},S \setminus \{k_j\}}; \cdots; f_{d_{k_j,L},S \setminus \{k_j\}}], \quad \forall j \in [t+1].$$

(96)

Finally, we randomly generate a permutation of $\binom{U}{t}$, denoted by $q = (q_1, \ldots, q_{(U)})$, independently and uniformly over the set of all possible permutations. We sort all sets $S \subseteq [U]$ where $|S| = t+1$, in a lexicographic order, denoted by $S(1), \ldots, S\left(\binom{U}{t}\right)$. The server transmits $X_{[K]} = (\mathcal{M}(P), P)$, where

$$P = \left( Q_{S(q_1)}, \ldots, Q_{S\left(\binom{U}{t}\right)} \right).$$

(97)

**Decodability.** We focus on user $k \in [K]$. From the metadata $\mathcal{M}(P)$, for each $j \in \binom{U}{t}$, user $k \in [K]$ checks $Q_{S(q_j)}$. If $Q_{S(q_j)}$ contains $Lt$ cached subfiles and $L$ requested subfiles, user $k$
knows $Q_{S(q_j)}$ is useful to it and decodes the requested the $L$ requested subfiles from the $L$ linear combinations in $Q_{S(q_j)}$. After considering all transmitted packets in $P$, user $k \in [K]$ can recover all requested subfiles to reconstruct its requested files.

Privacy. Intuitively, by a similar reason as the proposed caching scheme in Theorem 5, by the symmetric construction, the composition of $X_{[K]}$ is totally equivalent for different demand matrices from the viewpoint of each user $k \in [K]$, such that user $k$ cannot distinguish the $K - 1$ real users among all $U - 1$ users. The information-theoretic proof on the privacy is also similar to the one for the proposed caching scheme in Theorem 5. For sake of conciseness, we do not repeat the proof.

Performance. For any demand matrix, we transmit $\binom{U}{t+1}$ messages, each of which contains $\frac{LB(U_t)}{(U)}$ bits. Hence, the achieved load is $L = \frac{\binom{U}{t+1}}{U_t} = \frac{U - t}{t + 1}$, as shown in (74).

REFERENCES

[1] E. Bastug, M. Bennis, and M. Debbah, “Living on the edge: The role of proactive caching in 5g wireless networks,” IEEE Communications Magazine, vol. 52, pp. 82–89, Aug. 2014.
[2] M. A. Maddah-Ali and U. Niesen, “Fundamental limits of caching,” IEEE Trans. Infor. Theory, vol. 60, no. 5, pp. 2856–2867, May 2014.
[3] K. Wan, D. Tuninetti, and P. Piantanida, “On the optimality of uncoded cache placement,” in IEEE Infor. Theory Workshop, Sep. 2016.
[4] Q. Yu, M. A. Maddah-Ali, and S. Avestimehr, “Characterizing the rate-memory tradeoff in cache networks within a factor of 2,” in IEEE Int. Symp. Inf. Theory, Jun. 2017.
[5] A. G. Ghassemi and A. Ramamoorthy, “Improved lower bounds for coded caching,” IEEE Trans. Infor. Theory, vol. 63, no. 7, pp. 4388–4413, May 2017.
[6] Q. Yu, M. A. Maddah-Ali, and S. Avestimehr, “The exact rate-memory tradeoff for caching with uncoded prefetching,” in IEEE Int. Symp. Inf. Theory, Jun. 2017.
[7] M. Ji, A. Tulino, J. Llorca, and G. Caire, “Caching-aided coded multicasting with multiple random requests,” in Proc. IEEE Inf. Theory Workshop (ITW), May. 2015.
[8] A. Sengupta and R. Tandon, “Improved approximation of storage-rate tradeoff for caching with multiple demands,” IEEE Trans. Commun., vol. 65, no. 5, pp. 1940–1955, May. 2017.
[9] M. A. Maddah-Ali and U. Niesen, “Decentralized coded caching attains order-optimal memory-rate tradeoff,” IEEE/ACM Trans. Networking, vol. 23, no. 4, pp. 1029–1040, Aug. 2015.
[10] M. Ji, G. Caire, and A. Molisch, “Fundamental limits of caching in wireless d2d networks,” IEEE Trans. Inf. Theory, vol. 62, no. 1, pp. 849–869, 2016.
[11] S. P. Shariatpanahi, S. A. Motahari, and B. H. Khalaj, “Multi-server coded caching,” IEEE Trans. Infor. Theory, vol. 62, pp. 7253 – 7271, Dec. 2016.
[12] M. Ji, A. M. Tulino, J. Llorca, and G. Caire, “Caching in combination networks,” 49th Asilomar Conf. on Sig., Sys. and Comp., Nov. 2015.
[13] K. Wan, M. Ji, P. Piantanida, and D. Tuninetti, “Caching in combination networks: Novel multicast message generation and delivery by leveraging the network topology,” in IEEE Intern. Conf. Commun (ICC 2018), May 2018.

[14] A. Sengupta, R. Tandon, and T. C. Clancy, “Fundamental limits of caching with secure delivery,” IEEE Trans. on Information Forensics and Security, vol. 10, no. 2, pp. 355–370, 2015.

[15] M. Bahrami, M. A. Attia, R. Tandon, and B. Vasic, “Towards the exact rate-memory trade-off for uncoded caching with secure delivery,” in 55th Annual Allerton Conf. on Commun., Control, and Computing (Allerton), Oct. 2017.

[16] V. Ravindakumar, P. Panda, N. Karamchandani, and V. M. Prabhakaran, “Private coded caching,” IEEE Trans. on Information Forensics and Security, vol. 13, no. 3, pp. 685–694, 2018.

[17] A. Shamir, “How to share a secret,” Commun. ACM, vol. 22, no. 11, pp. 612–613, 1979.

[18] A. A. Zewail and A. Yener, “Device-to-device secure coded caching,” arXiv:1809.06844, Sep. 2018.

[19] Z. H. A. and R. Mathar, “Bounds on caching d2d networks with secure delivery,” in 15th Int. Symp. Wireless Commun. Sys. (ISWCS), Aug. 2018.

[20] A. A. Zewail and A. Yener, “Combination networks with or without secrecy constraints: The impact of caching relays,” in IEEE Journal on Selected Areas in Communications, vol. 36, no. 6, pp. 1140–1152, 2018.

[21] S. Kamel, M. Sarkiss, M. Wigger, and G. R. Othman, “Secrecy capacity-memory tradeoff of erasure broadcast channels,” IEEE Trans. Inf. Theory, vol. 65, no. 8, pp. 5094–5124, 2019.

[22] F. Engelmann and P. Elia, “A content-delivery protocol, exploiting the privacy benefits of coded caching,” 2017 15th Intern. Symp. on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), May 2017.

[23] B. Chor, O. Goldreich, E. Kushilevitz, and M. Sudan, “Private information retrieval,” in Proceedings of the 36th Annual Symposium on Foundations of Computer Science, pp. 41–50, 1995.

[24] H. Sun and S. A. Jafar, “The capacity of private information retrieval,” IEEE Trans. Inf. Theory, vol. 63, no. 7, pp. 4075–4088, 2017.

[25] Z. Chen, Z. Wang, and S. Jafar, “The capacity of private information retrieval with private side information,” available at arXiv:1709.03022, Sep. 2017.

[26] S. Li and M. Gastpar, “Single-server multi-user private information retrieval with side information,” in IEEE Int. Symp. Inf. Theory, Jun. 2018.

[27] R. Tandon, “The capacity of cache aided private information retrieval,” in 55th Allerton Conf. Commun., Control, Comp., Oct. 2017.

[28] Y.-P. Wei, K. Banawan, and S. Ulukus, “Cache-aided private information retrieval with partially known uncoded prefetching: Fundamental limits,” available at arXiv:1712.07021, Dec. 2017.

[29] ——, “Fundamental limits of cache-aided private information retrieval with unknown and uncoded prefetching,” available at arXiv:1709.01056, Sep. 2017.

[30] M. A. Attia, D. Kumar, and R. Tandon, “The capacity of private information retrieval from uncoded storage constrained databases,” available at arXiv:1805.04104, May 2018.

[31] H. Sun and S. A. Jafar, “The capacity of private computation,” in IEEE Intern. Conf. Commun (ICC 2018), May 2018.

[32] K. Wan, D. Tuninetti, M. Ji, and G. Caire, “Novel inter-file coded placement and d2d delivery for a cache-aided fog-ran architecture,” arXiv:1811.05498, Nov. 2018.

[33] K. Wan, D. Tuninetti, and P. Piantanida, “On caching with more users than files,” in IEEE Int. Symp. Inf. Theory, Jul. 2016.