Computation and structural methods to expand feed channels in planetary hydraulic machines*

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Abstract. Planetary Rotor Hydraulic Machines (PRHM) with wave-shaped central gearwheels and floating satellites show notable engineering performance, at the same time, this subject lack in-depth study. A methodology of computing cross-sections of feed channels that supply power fluid has been devised to embrace a variety of modifications of planetary hydraulic machines. Design formulas have been proposed, and a comparative assessment has been performed related to the channel cross-section criterion. Locating channels in the sun gearwheel (having a lesser number of waves $M$) contributes to expanding channels. In the cases when channels are in the epicyclic gearwheel ($N$ being the wave number), the following configurations $M\times N$ have been discovered as preferential: 6×8; 4×6; 2×4. A design concept to expand channel cross-sections by means of sequentially joining two PRHM stages of the same type has been studied. This concept is more efficient in the cases when configurations have the same number $M=N$ and their sun gearwheel and epicycle are related as 2×2; 3×3.

1. Introduction

Volume hydraulic machines: pumps and hydraulic engines, are one of the most important and integral elements of the modern equipment. Among generally known types of positive-displacement hydraulic machines, planetary rotor hydraulic machines (PRHM) equipped with floating satellites possess a number of principal advantages. They have neither valves nor loaded sliding kinematic couples. When applied as hydraulic motors, they produce better power-to-weight indicators outdoing any other types of hydraulic machines manufactured to date [1; 2]. Operating in pumping mode they excel in efficiency at pressures below 20 MPa [3]. The problems faced by the widespread use of the PRHM are due to the presence of non-circular gears. One of them is connected with the need to use fairly complex methods for calculating the geometry of non-circular toothed links [3, 4, 5, 6]. Currently, the issues of profiling units of the PRHM are simplified by the use of computer graphics methods [7, 8, 9]. The second, no less important problem is the technology of manufacturing non-circular gears. Mechanical methods of processing of such wheels [10] are too expensive. The prospect of profitable production of PRHM is provided by modern 2D-technologies [11].

Thus, at present, the possibility of production of PRHM becomes real, therefore, more detailed study of the properties and optimization of the structures of such hydraulic machines become relevant.

2. Problem formulation

An important characteristic of the PRHM is the cross-section area in the channels that deliver power fluid. For a system to operate effectively it is necessary that this area should be commensurable with the cross-section area of the pipeline. Until now the dimensions of the channels in the face walls of these hydraulic machines have been determined graphically [12, 13, 14]. Graphical methods when selecting a configuration of a PRHM and making a preliminary estimation of its performance at earlier stages of development are extremely labour-consuming. The first task of this research consisted in

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developing a universal methodology to calculate channel cross-sections in various configurations of PRHM’s. The second task was to find engineering solutions contributing to expanding the supply channels.

3. Theory

Representative configurations of PRHM’s [15; 16] are shown in figure 1. In these machines power fluid is enclosed in the spaces between the wave-shaped central wheel 1, which has external teeth (sun gearwheel), the wave-shaped central wheel 2 with internal teeth (epicycle), and the circular satellites 3. The volume in the work spaces periodically changes as one of the central gearwheels turns. The number of satellites matches the sum of the number \( M \) of the waves in the sun gearwheel 1 and the number \( N \) of the waves in the epicycle 2: \( V = M + N \).

Commutation of power fluid occurs through the input and output channels 4, which are periodically closed by the satellites. The channels may be incorporated into the flat faces 5 in the epicycle, as is shown in figure 1a, or in the flat end walls rigidly attached to the sun gearwheel or in the cylindrical functional surface of the sun gearwheel, as is shown in figure 1b. The total number of the channels \( K \) is dependent on the number \( G \) of the wheel waves, into which the channels are incorporated: \( K = 2G = 2M \) or \( K = 2G = 2N \). The configuration of the PRHM with a rotating rotor, presented in figure 1, can be designated in the following way: \( 2 \times 4_f \) and \( 1^c \times 2_f \). This designation is based on the relation \( M \times N \) of the wave number, the lower symbol «\( f \)» shows the stationary unit, while the upper symbol «\( C \)» indicates the unit that has channels on the cylindrical functional surface.

To define the shape and sizes of the channels in a PRHM’s face walls (figure 1) the contours of the two satellites are superimposed one over the other, one of them corresponding to the beginning of the cycle in the work chamber, the other – to its end [12, 13, 14]. In a PRHM with one section positions of the centres «\( A \)» and «\( B \)» of these satellites correspond to the minimum and maximum volumes in the work chamber. To calculate the cross-section area of the channel the distance \( l_{AB} \) is to be found:

\[
l_{AB} = |\delta \cdot R|,
\]

where \( R \) is the mean radius of the trajectory of the satellites; \( \delta \) is the angular distance the points \( A \) and \( B \):

\[
\delta = \gamma - \tau
\]

\( \gamma \) is the angle between the adjacent channels:

\[
\gamma = \frac{2\pi}{K} = \frac{\pi}{G}.
\]

\( \tau \) is the angle between the adjacent satellites:
\[ \tau = 2\pi /V = 2\pi / (N + M) \]

For one-section PRHM’s the expression (2) is transformed:

\[ \delta = \gamma - \tau = \pi \left( \frac{1}{G} \frac{2}{N + M} \right). \]

Results of calculating the angle \( \delta \) for various configurations of PRHM’s are given in table 1. For channels in the epicycle see \( \delta_N \); for channels in the sun gearwheel – \( \delta_M \), for the cases of two-section machines with channels in the epicycle – \( \delta_{2N} \).

**Table 1.** Angular length of channels measured on satellite centres.

| M\(\times\)N | 6×8 | 4×6 | 3×4 | 2×4 | 2×3 | 1×3 | 1×2 | 3×3 | 2×2 | 1×1 |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \(\delta_N\) | -3.2° | -6° | -6.4° | -15° | -12° | -30° | -30° | 0° | 0° | 0° |
| \(\delta_M\) | 4.3° | 9° | 8.6° | 30° | 18° | 60° | 60° | 0° | 0° | 0° |
| \(\delta_{2N}\) | 6.4° | 6° | 12.9° | 0° | 12° | -15° | 0° | 30° | 45° | 90° |

In PRHM’s that have channels in face walls the channel cross-section area, in addition to the angle \( \delta \), is also dependent on the radius of the circumference in the recesses of the satellite \( R_3 \). If to apply the formula (3) for the configurations with channels in the epicycle 2, the angle \( \delta \) is negative; with channels in the sun gearwheel 1 it is positive. Consequently, in the first case (figure 2a) the cross-section area of a channel is equal to the difference of the area \( S_3 = \pi (R_3^2) \) of the circumference in the recesses of the satellites and the area \( S_{AB} = 2l_{AB} \cdot R_3 \) of the rectangle with the sides \( l_{AB} \) and \( 2R_3 \), in the second case it is equal to the sum of the areas of these figures (figure 2b). The radius of the recesses in the satellite is \( R_{3f} = f \cdot R_3 \), where the coefficient \( f \) allows for the representative relation of the radius \( R_{3f} \) of the recesses and the initial radius \( R_3 \). For preliminary computation \( f = 0.7 \ldots 0.8 \) is recommended.

![Figure 2](image_url)

**Figure 2.** Computational diagram for finding the section area in PRHM channels.

a – computational diagram for defining channels incorporated in the epicycle;
b – computational diagram for defining channels incorporated in the sun gearwheel

To unify computation the relative value of the channel area should be applied; its actual area is divided by the area of the circumference \( S_2 = \pi (R_2^2) \), which is contained inside the pitch line of the epicycle 2. The computational relative area \( S_i \) of the section of one channel can be determined by the formula:

\[ S_i = \frac{S_3 \pm S_{AB}}{S_2} = \frac{\pi \cdot (f \cdot R_3)^2 \pm l_{AB} \cdot 2 \cdot f \cdot R_3}{\pi \cdot (R_2)^2} \]

(4)

For an one-section PRHM that has the number of waves \( M \neq N \) and channels in the face walls in one of its central wheels the expression (4) after adequate substitutions will look as follows:
\[ S_i = \frac{f}{4 \cdot G \cdot (N)^2} \cdot (N - M)^2 \cdot (f \cdot M \pm 2) \] (5)

The sign «+» is applicable in the case with channels located in the faces of the sun gearwheel; «-» – in the case with channels in the epicycle.

For an one-section PRHM that has the number of waves \( M = N \) the angular distance \( \delta \) applying the formula (3) is equal to zero \((\delta = 0)\), and the design formula for calculating the channel cross-section area will be as follows:

\[ S_i = 0.01f_2 \] (6)

The values of the combined relative area \( S_G^p = S_i \cdot G \) of input (or output) channels calculated for different configurations of PRHM’s are shown in the diagram in figure 3.

In the cylindrical work surface of the central gearwheel channels cannot be made unless the angle \( \delta \) is positive. Concerning one-section PRHM’s it means that it can achieved only when channels are part of the sun gearwheel \( G = M \) in such a case the relative section area of a channel is:

\[ S^C_i = \frac{q \cdot \delta \cdot R_2 \cdot b'}{\pi \cdot (R_2)^2} = \frac{q \cdot \delta \cdot b'}{\pi} \] (7)

where \( b \) is the axial dimension of a PRHM section; \( b' \) is the relative axial dimension of the section \( b' = b/R_2 \);

\( q \) is the coefficient of the axial dimension fraction of one channel.

By making substitutions the final formula is obtained for calculating the relative area of all the \( G = M \) input (or output) channels.

\[ S^C_G = q \cdot b' \cdot \frac{N - M}{N + M} \] (8)

For comparative estimation it is assumed: \( q = 0.5 \); \( b' = 0.3 \). See figure 3 for the result of the calculation.

4. Calculation results

![Diagram of relative channel section area.](image)

**Figure 3.** Diagram of relative channel section area.

SM – channels are located in the faces of the sun gearwheel, SN – channels are located in the end faces of the epicycle;
SMC– channels are located in the cylindrical work surface of the sun gearwheel of $M\neq N$ configurations;
SMC– channels are located in the cylindrical work surface of the sun gearwheel of $M\neq N$ configurations;
S2NC– channels are located in the cylindrical work surface of the epicycle of two-section configurations $M=N$.

The calculations that have been made applying the above methods conform to the measurements of the channel cross-sections graphically made with an accuracy reaching 10 percent.

5. Discussion of results

Provision of channels in the cylindrical work surface of the sun gearwheel proves to be the most efficient in the configurations 1×3, 2×4 and 1×2 (see figure 3); these configurations have the maximum value of the angle $\delta_M$. With lesser $\delta_M$ angles the satellite obstructs the channel with its cylindrical toothed surface.

Channel cross-sections in the faces of the sun gearwheel are always bigger than the ones in the faces of the epicycle. Problems with channels incorporated in the sun gearwheel are related to the design complexity of PRHM’s. This holds true for the configurations with a stationary sun gearwheel [17] and for the configurations, in which medium is commuted the rotating rotor [16]. Configurations of PRHM’s, in which channels, as is shown in figure 1a, are fixedly tied to the epicycle, are a lot simpler. At the same time, in this case channels are more or less wide only in the configurations: 6×8; 4×6; 2×4; 1×3. Application of the configuration 1×3 has to be rejected for the reason that there is no symmetry of imposing forces.

An opportunity to expand channel cross-sections in PRHM’s with channels in the epicycle lies in sequential hydraulic joining of two sections [18]. The essence of this design is that with sequential joining the active phase of functioning can be equally shared by the sections in operation. While the active section is functioning, the passive one lets through and vice versa. The input and output channels in the passive section are simultaneously open. This condition both enables and requires increasing the angular length $\delta$ of channel cross-sections. As a result hydraulic resistance in the channels goes down while the potential output of a PRHM goes up. Sequential hydraulic joining of sections can be provided by both the channels in the flat faces of the epicycle (figure 4a) and the channels in its cylindrical work surfaces (figure 4b).

![Figure 4. Two-section PRHM.](image)

a – configuration $2\times2^p_f$ [18]; b – configuration $2\times2^C_f$ [19]

Computation of channel cross-sections for two-section PRHM is done by employing formulas similar to the ones used for single-section machines. The major difference is the magnitude of the angle $\delta$. In the general case it is:
where \( n \) is the number of sections. The case under consideration is \( n = 2 \) (see example table 1).

The diagram (figure 3) presents the results of applying the formula (10) to calculate channels in the flat faces. In comparison to single section PRHM's, the channel cross-section areas rise nearly by an order of magnitude. Almost the same result is obtained in the case channels located in the cylindrical work surface of the epicycle.

6. Implications and conclusion

Methods of design calculations have been developed to determine cross-sections of incoming channels in PRHM's of various configurations. Design formulas have been derived. A comparative estimation of different modifications of PRHM's has been done based on the criterion of channel cross-section area. Provision of channels in the sun gearwheel contributes to expanding channel cross-section areas nearly by an order of magnitude. The case when channels are located in the epicyclic gear, the configurations 6×8; 4×6; 2×4 should be preferred. A design technique to expand cylindrical work surface of the epicycle by means of sequential jointing of two PRHM sections of the same type has been studied. This technique proves to be most effective for the configurations 2×2; 3×3.

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