The “Quenching” of Nuclear $g_A$ Revisited in Scale-Chiral Effective Field Theory

Mannque Rho

1 Institut de Physique Théorique, CEA Saclay, 91191 Gif-sur-Yvette cédex, France
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More than two decades ago, the quenching factor $q \simeq 0.79$, giving the effective $g_A^{\text{eff}} \simeq 1$ in nuclei, was predicted for the Gamow-Teller coupling constant using an extremely simple argument based on a Landau Fermi-liquid fixed-point approach to nuclear matter, together with the simple explanation of the anomalous orbital gyromagnetic ratio and strongly enhanced axial-charge transitions in nuclei [1,2]. This prediction has received little attention in the current developments in the field. I revisit this old prediction and revalidate it with an unprecedented precision in a modern effective field theory framework that combines chiral symmetry and scale symmetry of QCD, called “scale-chiral symmetry.” I discuss the implication of that prediction in view of the recent beautiful measurements of the superallowed $\beta$ decay in $^{100}$Sn and the current highly powerful effective field theory approaches to nuclear weak processes. It is reconfirmed that modulo possible, most likely small, contributions from Δ-hole contributions, the renormalization of $g_A$ from $n$-body exchange currents with $n > 1$, when fully calculated within the nucleon Hilbert space, could be ignored. It is argued that there is no “intrinsic density” effect of QCD in $g_A$ in a stark contrast to the pion decay constant. I suggest how this set of observables and modern theoretical tools could be exploited to check the fundamental symmetry structure, hidden in QCD, manifested in dense matter. This development will have a significant impact on how to address the nuclear Gamow-Teller matrix elements in neutrinoless double beta decay processes that purport to go beyond the Standard Model.

I. INTRODUCTION

The behavior of the axial-vector coupling constant $g_A$ in nuclear medium has been a long-standing puzzle in nuclear physics, with a strong impact on astrophysics and particle physics. In nuclear physics, since early 1970s [3], there has been the mysterious $\sim 20\%$ universal “quenching” of $g_A$, giving the effective value $g_A^{\text{eff}} \simeq 1$ in shell-model calculations of nuclear beta decay and giant Gamow-Teller resonances. Recently this phenomenon resurfaced prominently as a potentially serious issue due to modern developments in other areas of physics [4]. In particle physics, there is the issue of partial restoration of chiral symmetry, an intrinsic property of the symmetry of QCD, and in astrophysics, a surprising role of first-forbidden beta decay in nucleosynthesis. The main cause of this development is due to both some striking experimental results and the emergence of highly sophisticated theoretical tools anchored on effective field theory approach to nuclear many-body problems.

What stands out most surprising with the so-called quenching of $g_A$ in nuclear medium is that the effective $g_A$ – denoted as $g_A^{\text{eff}}$ – converges very close to 1, $g_A^{\text{eff}} \approx 1$, from light to heavy nuclei. This has led to a variety of ideas as to how the axial-vector coupling gets renormalized in nuclear matter, including the conjecture that $g_A^{\text{eff}} \rightarrow 1$ signals a “precursor” to the chiral symmetry restoration. If one ignores the light up- and down-quark masses – called the chiral limit – then the chiral symmetry imposes that both the vector current $J^\mu_V$ and the axial-vector current $J^\mu_A$ in QCD be conserved, i.e., $\partial_\mu J^\mu_V = \partial_\mu J^\mu_A = 0$. The vector-current conservation implies that the vector-current coupling constant $g_V = 1$. However the axial-current conservation does not imply that the axial coupling constant should also be equal to 1. This is because the axial symmetry is “hidden”: It is realized in the Nambu-Goldstone (NG) mode. Indeed the celebrated Adler-Weisberger (AW) relation that comes from the current algebras leads to the difference $(g_A - 1)$ as an integral over quantities involving $\pi^+ p$ cross sections multiplied by $f^2_\pi$. There is no reason why the integral should vanish, and the pion decay constant is non-zero in the vacuum. Indeed the integral can be calculated reliably, and with the known value of $f_\pi \approx 93$ MeV, the quantity $(g_A - 1)$ comes out to be $\sim 0.24$, close to the currently established value 0.27 [5].

Now in order for the difference $(g_A - 1)$ to tend to zero in nuclear medium, all that is required is that the AW relation applied to nuclear medium is that $f_\pi \rightarrow 0$ in medium. Both $f_\pi$ and $g_A$ figure importantly, clearly intricately connected, in chiral symmetry, so one would think that the property of one, $f_\pi$, would influence that of the other, $g_A$. Since the pion decay constant going to zero signals the chiral restoration, it was not totally absurd to think $g_A^{\text{eff}} \approx 1$ signals a precursor to approaching symmetry

\[ \text{\cite{5}} \]

\footnote{This reference recounts the history of the soft-pion theorems which led to a breakthrough in EFT for strong interactions at low energy and which are the basis of the working of the modern EFTs in nuclear physics in the spirit of Weinberg’s Folk Theorem. Let me also mention that these soft-pion theorems join the deep concept of “soft theorems” in a wide variety of theories.}

\footnote{I put the quotation mark here to stress that there is actually no genuine quenching involved. It is a misnomer. It is perhaps more appropriate to use “nonquenching” as was done in a recent article that will be referred to below. I will however use “quenching” without the quotation mark.}
restoration. But the problem is that there is no indication that the pion decay would fall to zero at finite density. And there is no reason that the integral go precociously to zero either. I will show that it is almost entirely nuclear correlations that are responsible.

An intriguing observation is however that the fundamental axial constant — not the effective one — does have to go to 1 in highly dense nuclear medium in the presence of scale symmetry in what is called “dilaton limit fixed point”. I will discuss below whether the two observations could be related, lurking behind the quenching factor. Whatever the case might be, the two phenomena arise from one symmetry scheme where chiral symmetry and scale symmetry are implemented together.

Since the quenching in the nuclear process involves both a calculated Gamow-Teller matrix element and a measured quantity, it is necessary to give precision to the quenching factor \( q \). It defined by

\[
g_A^{\text{eff}} = g_A \tag{1}
\]

where \( g_A \) is the constant for the free-space neutron beta decay, the most recently determined value of which is \( 1.27755(11) \).

\[
g_A = 1.27755(11) \tag{2}
\]

The quenching factor \( q \) is determined from the ratio of the experimental Gamow-Teller matrix element \( M_{\text{GT}}^{\exp} \) over the theoretical matrix element \( M_{\text{GT}}^{\text{th}} \) calculated with the empirical \( g_A \). Since \( q \) is typically less than 1, it is dubbed “quenching factor.” What \( q \) is depends on how the theoretical matrix element is calculated, which involves the calculating procedure and the accuracy with which the relevant GT operators and wave functions involved are calculated. What makes the recent theoretical development particularly noteworthy is that it anchors on effective quantum field theories (EFTs), consistent with QCD, applied to nuclear systems. The strategy employed is along the line faithful to the “Folk Theorem (FT)” put forward by Weinberg [8]. The title of the theorem evidently betrays certain non-rigorosity, but it is closest to a “first principle approach” to nuclear physics. An extensive discussion on this approach in nuclear processes and nuclear-astrophysical processes can be found in the recent monograph [12].

There are many papers in the literature on the quenching factor and its impact on fundamental issues reviewed in [3]. Here, avoiding confusing details, I will pick one recent paper [10] which extensively and objectively covers the theoretical matter, highlighting an extremely elegant experiment on superallowed Gamow-Teller \( \beta \) decay of the doubly magic nucleus \(^{100}\text{Sn}\) [11]. There is a parallel development in realistic shell-model calculations employing renormalization-group flow using the \( V_{\text{lowk}} \) formalism [12].

I will sketch below as succinctly as I can the theoretical procedure involved in the EFT to be employed. It will clarify what is involved in the approach of [10] and the difference from the approach I will adopt in addressing the GT decay in \(^{100}\text{Sn}\). Here I will focus on what is referred to in the literature as “the total renormalized Gamow-Teller strength of the extreme single-particle estimate” denoted as \( B_{\text{GT,ESPM}} \) [11]. This quantity will be found to be directly connected to what I identify as “Landau Fermi-liquid fixed point (FLFP)” quantity.

The nucleus \(^{100}\text{Sn}\) measured in [11] is a doubly magic nucleus with \( N = Z = 50 \), particularly suited to both experimental and theoretical studies of GT transitions. The GT transition takes place dominantly to a single state with little fragmentation of the GT strength. This makes particularly relevant the extreme single-particle picture where the GT transition is the decay of a proton in the filled \( g_{9/2} \) shell to a neutron in the empty \( g_{7/2} \) shell. The quenching factor \( q \) extracted from \( B_{\text{GT,ESPM}} \) of this experiment (which is in agreement with what’s given in [13]) is in the range

\[
q = 0.78 - 0.82. \tag{3}
\]

This factor implies (with [2])

\[
g_A^{\text{eff}} = 1.00 - 1.05. \tag{4}
\]

For simplicity, I will simply take \( g_A^{\text{eff}} = 1.0 \) for \( q_{\text{ESPM}} \).

To what extent this can be taken seriously will be discussed below. An important point here that this result has a far-reaching implication on the symmetry structure of baryonic matter with chiral symmetry and scale symmetry combined together, an issue most likely relevant in search of BSM (beyond the Standard Model of particle physics), e.g., neutrinoless double-beta decay processes [4] and dilatonic Higgs model. In [10], nuclear EFT to N\(^4\)LO with two-body currents (2BC) is applied to \(^{100}\text{Sn}\) and lighter nuclei. This approach requires the precise definition of the underlying strategy of the Folk Theorem adopted, which will be taken up in what follows. I come to the same conclusion as [10] (with one caveat). “For beyond-standard-model searches of new physics ... it suggests that a complete and consistent calculation without a phenomenological quenching of the axial-coupling \( g_A \) is called for.”

I will state the caveat here — which I will explain in what follows — with respect to the conclusion of [10] stated above: The Fermi-liquid approach used here strongly suggests that for low-momentum transfer processes either (a) the 2BC calculation should be simply left out or (b) it is treated more fully including all N\(^4\)LO and higher-order terms ignored in [14] or (c) if the corrections of the type (b) persist with the strength of \( >10\% \) relative to the LO term, then the power counting scheme cannot be trusted. Then a basically different power-counting than Weinberg’s will be called for.

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3 Jumping ahead, these states will correspond to the quasiparticle states with the renormalization-group \( \beta \) function going to zero for the interactions that enter in the Fermi-liquid fixed point theory discussed below.
II. EFFECTIVE QUANTUM FIELD THEORY FOR NUCLEAR MATTER

In performing EFT calculations faithful to the Folk Theorem, one first has to pick an efficient set of scales and relevant degrees of freedom involved in doing Wilsonian renormalization-group (RG) decimations. In nuclear interactions, multiple energy/length scales are involved, with no clear sharp delineations. For the sake of maximum simplicity, one in practice limits to two scales, i.e., what’s referred to as “double decimation” \(^1\). Since QCD is the fundamental (or microscopic) theory, the EFT needs to be matched at some scale to QCD. This can be done, for nuclear dynamics, at the chiral scale \(\Lambda_{\chi}\) \(\sim\) \(4\pi f_\pi \sim 1\) GeV \(^2\). At this scale EFT is constructed – via matched correlators – in terms of QCD variables, specifically the QCD condensates, such as quark, gluon and other condensates, that encode the nonperturbative properties of QCD. The cutoff scale dictates the relevant degrees of freedom in nuclear physics, which are, apart from the pions and nucleons, that encode the nonperturbative \(\chi^EFT\) variables. Left with the pions and nucleons, \(\Delta\) etc. Unfortunately an EFT with the cutoff set at \(\Lambda_{\chi}\) is at present totally unworkable because of the too large number of parameters of the effective Lagrangian involved. One therefore lowers the cutoff from \(\Lambda_{\chi}\) to a lower scale, typically (a) just above the scale of \(\Lambda_V \sim m_V \sim 700\) MeV or (b) \(\Lambda_{\chi\text{nucl}} \sim (400–600)\) MeV from which calculations are performed à la FT.

I will return below to the case (a) which will be the approach leading to the prediction made in \(^1\,2\). Here I first elaborate on the case (b), currently the most actively followed in the literature, exemplified by \(^10\) and \(^12\). In what follows, the EFTs based on (b) will be referred to as “standard chiral effective field theory (S\(\chi\)EFT). They will be distinguished from the scheme adhering to (a) called below as \(bs\)HLS.

A. Standard (nuclear) chiral EFT

With the cutoff set below the vector or scalar masses, all heavy objects including the \(\Delta\)s are to be integrated out. Their effects are then incorporated into higher-order counter terms in the EFT Lagrangian with a suitable power counting scheme. Left with the pions and nucleons only, among a variety of power-counting rules for systematic expansion, Weinberg’s counting rule \(^16,\,17\) or improved version thereof is employed in all highly sophisticated EFT approaches to nuclear structure calculations presently in vogue. The current state of art in the calculation represented, e.g., by \(^10\) is up to next-to-4th-to-leading order (N\(^4\)LO) in the nuclear potentials – including 3-nucleon potential – and to N\(^3\)LO (and part of N\(^4\)LO) in the electro-weak (EW) currents.

As stressed by Weinberg \(^5\), the main reason why the nuclear EFT anchored on S\(\chi\)PT\(^4\) is expected to work well at low energy is because nuclear interactions are “soft” even though the nucleon mass is “hard.” If the interactions were not soft, then FT would lose its predictive power in nuclear physics. This feature has been interpreted as a “chiral filter” mechanism encoded in soft-pion theorems \(^18\). This is a crucial point in my thesis. It exposes why the two-body current (referred to as “2BC” in \(^10\)) cannot play an important role for the quenching of \(g_A\). It also illustrates where and how the pionless EFT could lose its predictive power as a nuclear EFT.

It is important to clearly understand, for the \(g_A\) problem, in what way “soft pions” buried in the chiral expansion figure in the axial current in S\(\chi\)EFT. Since the pion is the only degree of freedom here, the pion exchange is the only mesonic current involved. It is straightforward to make a systematic power counting in S\(\chi\)EFT involved in the EW current. However it is extremely instructive to see how the current algebras of chiral symmetry and soft-pion theorems associated with them enter in the process.

The axial current does not couple to the pion exchanged between nucleons, so the quantity involved in the many-body currents is the amplitude \(A = N + \pi_{\text{Ext}} \rightarrow N + \pi_{\text{In}}\) where \(\pi_{\text{Ext}}\) is the pionic component of the axial current and \(\pi_{\text{In}}\) is the propagating pion exchanged between nucleons. If \(\pi_{\text{Ext}}\) is soft and \(\pi_{\text{In}}\) is hard or vice-versa, then the Adler’s single soft-pion theorem suggests \(A\) is strongly suppressed, even if it is not zero for non-zero momentum. On the other hand, if both \(\pi\)s are soft, then the double soft-pion theorem gives a unique non-zero \(O(1)\) prediction for the amplitude \(A\). One deduces from this observation that the axial charge operator must have an \textit{enhanced} two-body contribution whereas the Gamow-Teller transition cannot. One can see this clearly in the power expansion, first worked out in \(^19\) and summarized with corrections in \(^20\). Here on I will take the single-particle operator as the leading order, denoted as LO, and compare the higher-order terms relative to the LO.

1. Axial charge operator: The two-body correction to the leading-order axial charge (single-particle) operator comes at NLO with the next corrections coming at two orders down in power-counting, at N\(^3\)LO. This turns out, as we will see numerically, to make the 2BC correction to be big, nearly 100%
of the LO. It is by far the largest and most unambiguous mesonic effect found in nuclear processes, offering a beautiful illustration of how the FT works in nuclei.

2. Gamow-Teller operator: Here the situation is drastically different. The Gamow-Teller operator effectively plays the role of a “hard pion.” The power counting then shows that the leading two-body current contribution comes highly suppressed at N^3LO. It is therefore expected that the many-body current corrections must be “naturally” suppressed in the sense of the FT unless there is an accidental suppression of the LO term by symmetry considerations or kinematics. Should the N^3LO contribution come out to be substantial numerically, not suppressed by the two orders of counting, then there is absolutely no reason to believe that one can stop at that order. One would then have to go to the next order, i.e., N^4LO or even higher. At that order, not only higher-body terms but also complicated many-body terms that appear in non-covariant counting inevitable in many-body problems, such as a variety of recoil terms, required by in-medium Ward identities, must be taken into account. This would render the SχEFT power counting most likely unpredictable. In fact this argument of chiral filtering has received support from some of the recent papers employing powerful numerical techniques, i.e., the recent Monte Carlo calculations in A = 6–10 nuclei \[23\]. As I will argue below, a reasoning based on Landau Fermi liquid theory, that implements the Wilsonian RG strategy, strongly supports this result.

B. Scale-chiral symmetry in nuclear matter.

The SχEFT discussed above has the cutoff scale typically below the vector meson masses \( m_V \sim 700 \text{ MeV} \). The physics of those massive degrees of freedom, suitably integrated out, is to be encoded in the counter terms, so for low-energy processes, high-order loop corrections with the appropriate counter terms could simulate those effects of integrated-out degrees of freedom. The question is then to what order in the expansion one has to go to capture the relevant physics. Presently nuclear theorists work up to N^4LO, but it is not clear whether one can go any further in practice. This issue will be relevant, as I will mention later, for Gamow-Teller matrix elements indispensable in neutrinoless beta decay. To address this issue, I will consider the scale that makes the vector mesons \( V_\mu \) and the scalar meson \( \sigma \) explicitly relevant degrees of freedom. For this, the cutoff will be set just above the vector mass, \( \Lambda_V \gtrsim m_V \).

1. Hidden symmetries

Now the question is how to bring the vectors \( V_\mu \) and the scalar \( \sigma \) – that I will refer to as “dilaton” for the reason to be explained – into an EFT framework. For this, let me first describe hidden symmetries in the strong interaction physics: Hidden scale symmetry associated with \( \sigma \) and hidden local symmetry (HLS) associated with \( V_\mu = (\rho, \omega) \).

Let me first discuss hidden local symmetry. I will focus on the \( \rho \) meson, although the \( \omega \) can be treated on the same footing. Up to the density in the vicinity of \( n_0 \), one can assume the \( U(2) \) gauge symmetry for \( V_\mu \). The symmetry breaks down at high density – as will be encountered below in connection with the dilaton-limit fixed point – which does not concern the problem in question.

A highly portent way of introducing the vector meson \( \rho \) in chiral dynamics is to have it appear as a nonabelian gauge boson. It was introduced \[15\] in hidden local symmetry “gauge equivalent” to nonlinear sigma model. Such a vector meson could be considered in chiral effective field theory with pions and nucleons only as a composite of the matter particles, fermions or bosons. Then a theorem by Suzuki \[22\] states: “If a gauge-invariant Lagrangian is written in terms of matter fields alone, there must be a composite gauge boson or bosons made of the matter particles.” In the present case, when considered in nuclear medium, the matter particles could be the pions and nucleon particle-holes. This means it is plausible in SχEFT where \( \rho \) is absent that the physics of \( \rho \) could be generated at high orders in the chiral expansion at high density. Given the vector meson mass \( m_\rho \gg m_\pi \), the gauge symmetry is not visible in nature in the vacuum. If it is present, it must be hidden. It turns out, to one’s surprise, that one can perform a chiral expansion with \( m_\rho \) taken on the same footing as \( m_\pi \) \[15\]. An extremely challenging question in nuclear physics is whether the physics of \( \rho \) generated at high order in SχEFT can be equivalent to that of HLS in dense nuclear matter. To proceed, I will suppose it makes sense to think of the \( \rho \) meson massless in some limit.\[6\]

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6 Whether the \( \rho \) becomes massless under extreme conditions such as at high temperature or at high density has been a big issue in hadron physics. The dilepton experiments purporting to look for such massless \( \rho \) at high temperature in relativistic heavy-ion collisions failed to “see” it, not because it is “ruled out” in nature but because even if it existed in the process, it would be like seeing a needle in the giant haystack in the dilepton production and hence could not be isolated from the huge background of on-shell production. Up to now there is no evidence either for or against the \( \rho \) becoming (nearly) massless. It turns out, however, that the limit at which the \( \rho \) can become massless, termed “vector manifestation fixed point” \[13\], has a very interesting and appealing implication in the equation of state for massive compact stars, such as the recent LIGO/Virgo gravitational observation et c. A brief remark will be made at the end of this article.
In a somewhat different way, a scale symmetry could also be hidden in QCD \cite{26}. In the absence of quark masses (in the chiral limit), the classical QCD Lagrangian is invariant under scale transformation. However because of the trace anomaly which is a quantum effect, the scale symmetry is broken in the vacuum of QCD. In EFT, one can go, by dialing a constant from strong coupling to weak coupling, from a nonlinear sigma model to a scale-invariant model, modulo a potential – that I denote $V_{\text{dilaton}}$ (to be defined below) – that breaks scale symmetry, both explicitly and spontaneously. There is no sign of scale symmetry in the nonlinear sigma model on which S\_\chi EFT is anchored. In nuclear interactions under normal conditions, whether scale symmetry is active or not is not evident. It is actually hidden, buried in the kinetic energy term, which gets “un-hidden” in the weak coupling limit \cite{26}.

There is, however, the scalar $f_0(500)$ in nature which is relatively light, and a light scalar of mass $\sim 600$ MeV is needed in nuclear dynamics in certain models such as Walecka’s mean field model \cite{27} and also in phenomenological (OBEP) nucleon-nucleon potentials. That scalar meson cannot be identified with the fourth component of the chiral four-vector in the Gell-Man-Lévy linear sigma model or with a scalar glueball. The only plausible possibility, which I propose to adopt, is that it is the dilaton, an NG boson that arises from the spontaneous breaking of scale symmetry encoded in the potential $V_{\text{dilaton}}$. Whether the NG boson of this nature can be accommodated in QCD with the number of flavors $N_f \sim 2$ or 3 is unknown. There seems to be a consensus among particle theorists that an IR fixed point in QCD at $N_f \sim (2 - 3)$ is highly unlikely. At present, however, there is no firm (nonperturbative) QCD argument, e.g., lattice, for it. Neither is there no-go theorem.\cite{10} For the purpose of the discussion made here, whether QCD has an IR fixed point for small number of flavors is not really crucial in nuclear medium. It is analogous to asking whether local (flavor) gauge symmetry exists for the vector meson $\rho$ in QCD proper. The point of view taken here is that in nuclear medium an approximate scale symmetry can emerge via strong nuclear correlations. The effect of scalar degree(s) of freedom could plausibly be generated in high-order loop effects in S\_\chi EFT approach, much like higher-order loop effects in the $\rho$ channel. This must be so since nuclear interactions are fairly well described in chiral perturbation expansions to some orders (as reviewed in \cite{28}) and also in Walecka mean-field model. The appearance of a scalar captured by the dilaton $\sigma$ in nuclear interactions can be viewed as “un-hiding” of the hidden scale symmetry in QCD. Here the density could do the role of tuning the relevant parameter to the weak coupling regime in the linear sigma model of the nature discussed in \cite{26}.

Considering the dilaton as a (pseudo-)NG boson of scale symmetry, one can write down a generic soft-sigma Lagrangian possessing low-energy theorems in analogy to the pions – the (pseudo-)NG bosons of chiral symmetry. Recently this strategy has been used to write a generalized scale-chiral Lagrangian going from extended chiral perturbation theory to “nearly conformal” gauge theories that address a composite Higgs boson.\cite{9,15} For this purpose, a linear sigma model is appropriate, since the situation is near the sigma-model region regardless of whether it is an $SU(3)$ gauge theory with $N_f = 8$ in the fundamental representation of the gauge group or an $SU(3)$ gauge theory with two flavors in the symmetric sextet representation or some other gauge groups. What seems significant in terms of the generic low-energy theory effective Lagrangian with an IR structure is that different gauge structures in the conformal window manifest in different parameters of the same Lagrangian. This suggests that in many-body systems with the “vacua” modified by density, a similar generic structure can be envisaged where the symmetry is buried “hidden” regardlessly of how precisely the scale symmetry figures in the dynamics. This notion, essential for compact-star matter, I will argue, is found to work very well also at normal matter density appropriate for the Gamow-Teller transitions.

2. Scale-symmetric baryon hidden local symmetry (bsHLS)

For nuclear interactions at densities in the vicinity of nuclear matter density, it is the nonlinear realization of chiral symmetry that is applicable. Thus as mentioned, it is more appropriate to start with the nonlinear sigma model and implement the vector mesons – via HLS – to the baryonic nonlinear chiral Lagrangian.\cite{12} This can be readily done by coupling the vector-meson field $V_\mu$ to the baryon and pion fields in terms of covariant derivatives (see \cite{12} for notations). Let me call the Lagrangian so constructed $bHLS$, with $b$ standing for baryons. One writes down the Lagrangian by using gauge-covariant Maurer-Cartan 1-forms to any order in the power counting generalized from non-linear sigma model with a suitable power counting for the vector mesons. For the purpose of this discussion, one can limit to the leading order, which is $O(p)$ in the baryon sector and $O(p^2)$ in the mesonic sector. Explicit forms are not very illuminating, so I won’t write them down. They can be found in \cite{12}.

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7 What is known by lattice calculations is that there can be an infrared (IR) fixed point (with a vanishing $\beta$ function) for large $N_f$, say, $N_f \sim 8$. There is an activity going on working in the conformal window near that IR fixed point for a composite Higgs boson going beyond the Standard Model.

8 For recent references, see \cite{29,30} where further references can be found. Such linear sigma-model-type Lagrangian dates way back to 1969.

9 I will come to the linear sigma model structure below in dense nuclear matter in connection with the “dilaton-limit fixed point” first discussed in \cite{31}.
Given the HLS and bHLS Lagrangians, it remains to incorporate, also systematically, scale symmetry. Here not knowing what the IR structure of QCD is for small number of flavors, the matter is extremely intricate, non-unique and controversial even among the experts. There are several different ways of systematically writing down the power counting for the Lagrangian that combines both chiral symmetry and scale symmetry. It turns out, however, that to the leading order of scale symmetry (dubbed “LOSS”), i.e., the lowest order in the “small” mass of the dilaton analogous to the small pion mass for chiral symmetry, the different ways of writing down the scale-chiral Lagrangian are found to be equivalent \[32\]. This can be taken as the same generic structure of the “soft-sigma” low-energy Lagrangian independent of the IR structure, mentioned above.

The procedure to make the HLS and bHLS Lagrangians scale symmetric in the LOSS approximation can be phrased as a simple recipe: Take the scale dimension of the meson field \([M] = 1\) and that of the fermion field \([\psi] = 3/2\). They are denoted as \([M] = 1\) and \([\psi] = 3/2\). The chiral symmetry-breaking mass term \(M\) is taken as a spurion field of \([M] = 1\). The derivative \(\partial\) and momentum \(p\) have the scale dimensions \([\partial] = [p] = 1\). Next define the “conformal compensator field” \(\chi\) that is of mass dimension 1, and transforms linearly under scale transformation, i.e., \(|\chi| = 1\),

\[
\chi = f e^{\sigma/f}
\]

where \(f\) is a constant of mass dimension 1 which will later be identified with the dilaton decay constant \(f_\rho\) in the vacuum. As written the \(\sigma\) transforms nonlinearly under scale transformation as the \(\pi\) does under chiral transformation.

Given these quantities, the recipe is to multiply the matter-field terms in the Lagrangian by powers of \(\chi/f\) to make the scale dimension to be 4. The action so constructed will be scale-invariant. The scale symmetry breaking, present in the vacuum, is lumped into the \(V_{\text{dilaton}}(\chi)\) \[32\] given in the form \(V_{\text{dilaton}} \approx m_\sigma^2 \left(\frac{\chi}{f}\right)^6 \ln \left(\frac{\chi}{f} - \frac{1}{2}\right)\) where \(m_\sigma\) is the dilaton mass in the matter-free vacuum. This expression makes sense in the sense that the dilaton mass \(m_\sigma\) is taken to be small of order of \(\sim m_\pi\) so that low-energy theorems are applicable. This is somewhat like the \(\rho\) mass to be considered of order \(\sim m_\pi\) in the chiral expansion including the vector meson \[15\]. Although both may be questionable in the matter-free space, they could be valid in nuclear medium where the \(\sigma\) mass decreases (as \(m_\sigma\) does due to the VM fixed point). This is an assumption and it needs to be verified ultimately\[14\] I call the resulting Lagrangian sHLS for the mesonic part and bsHLS for the baryonic part. The sum of the two will be denoted HLS\(_{SS}\).

C. bsHLS in nuclear medium

Working with HLS\(_{SS}\) with the cutoff \(\Lambda_V\) set slightly above the vector meson mass \(m_V \sim 700\) MeV, one assumes that there is no flow of scale in the “bare” parameters of the Lagrangian from \(\Lambda_V\) where the matching between EFT and QCD is made to \(\Lambda_V\) where the bsHLS is defined, so the bare parameters of bsHLS are taken to remain unchanged from what’s measured in experiments.

Next embedded in medium at a density \(n\), the parameters of the EFT Lagrangian defined at the scale \(\Lambda_V\) carry the condensates inherited from QCD evaluated at the density relevant to the “vacuum” modified by the density. The most direct quantity is the medium-modified vacuum expectation value \(f^*_\sigma \equiv \langle \chi^* \rangle\) where * stands for in-medium at the density \(n\). This means that the dilaton condensate directly, and in a simple way, affects only those matter terms multiplied by the compensator field \(\chi\). This property comes from the scale symmetry hidden in medium but emerging under certain conditions tied with the dilaton-limit fixed point (DLFP) alluded to above.

There is another property that QCD impacts on dense matter, and that is associated with hidden local symmetry emergent at high density. It is connected to the “vector manifestation (VM)” which indicates an UV fixed point at which the hidden gauge coupling \(g_\rho\) is fractionally smaller than the \(\rho\) mass goes to zero, but not to zero, hence the \(\rho\) mass goes to zero\[11\].

Those two properties are intrinsic to the fundamental non-perturbative aspects of the microscopic theory, QCD, so the associated density dependence is called “intrinsic density dependence (IDD)”\[13\] to be distinguished from mundane many-body correlations involving the macroscopic (effective) degrees of freedom. It is to be noted that this IDD is missing in the \(S_\chi\)EFT, although part of them may be buried in the parameters of the Lagrangian that are determined from phenomenology in the decimation process. How the DLFP for the dilaton and VM fixed points affect nuclear dynamics depends on density. What matters primarily for nuclear properties near the equilibrium density \(n_0 \simeq 0.16\) fm\(^{-2}\) is the dilaton condensate whereas the VM influences matter at higher density \(n \gtrsim 2n_0\). The latter, explained in detail in the monograph \[9\], is found to be crucially important for compact-star physics, but will have no direct influence on what I am discussing here, focused on density \(n \sim n_0\).

In sum, the IDDs in HLS\(_{SS}\) = sHLS + bsHLS are

\[10\] It would be worth pointing out that in dilatonic Higgs models in the vicinity of a conformal window, the dilaton mass is go down even lower than the pion mass. For our case, it is an assumption in applying the model to nuclear medium at high density that \(m_\sigma\) is comparable to \(m_\pi\), not necessarily lighter, until one reaches the dilaton limit fixed point.

\[11\] This is due to the KSRF-type relation \(m_V^2 = af_g^2 g_V^2\) which is exact to all loop orders in HLS and \(g_V \to 0\) at the VM fixed point \[16\]. Note however that at the VM fixed point, the \(\rho\) mass can go to zero independently of what \(f_\rho\) is. This means that at high density, say, \(n \gtrsim 2n_0\), the flavor \(U(2)\) symmetry can be broken before the DLFP is reached with \(f_\rho \approx f_\pi \to 0\).
The quantity obtained in the Fermi-liquid approach is

\[ g_L / g_A \approx g_{NN} / g_{\sigma NN} \approx g_{NN} / g_{\sigma NN} \approx 1 \] (7)

where \( m = \rho, \omega, \sigma \) and \( V = \rho, \omega \). The IDD for the pion mass is special because of the anomalous dimension \( \gamma \) of \( qq \) mass. It is given by \( m^2 / m_0 = (f_{\pi} / f_\sigma)^2(1 - \gamma) \). The anomalous dimension is not known in QCD for \( N_f \sim (2 - 3) \). If one takes \( \gamma = 1 \) for the sake of simplicity, then the pion mass will be intrinsically non-scaling. Of course there will be nuclear correlation effects in medium, which should not be confused with the “intrinsic” property. Assuming that the “tiny” pion mass is well protected at low density by chiral symmetry, the pion mass will not be scaled in the discussion that follows.

D. Going with \( b \)HLS to Landau Fermi-liquid fixed point theory

It is well recognized that relativistic mean-field (RMF) theory, as a variant of energy density functional theory, works fairly well in nuclear physics both in the vicinity of nuclear matter and at higher density going toward compact stars. In fact the rationale for its working is that Walecka’s mean-field approach, on which the RMF theory is anchored, is equivalent to Landau Fermi liquid theory \( \text{[32]} \). Given a reasonable EFT Lagrangian (in the sense of Weinberg’s Folk Theorem) for nuclear many-body systems, one can formulate, following \( \text{[33]} \), a renormalization group approach for an effective Lagrangian on Fermi sphere given by Fermi-liquid fixed point quantities such as the Landau mass and Landau(-Migdal) quasiparticle interactions as \( \bar{N} = k_F / (\Lambda_N - k_F) \to \infty \). This was worked out non-relativistically in \( \text{[1, 2]} \). It should not be difficult to formulate it relativistically as it is done in covariant RMF theory. This NR formulation has been applied to the \( b \)HLS Lagrangian for densities near \( n_0 \) as well as in compact-star matter at \( n \sim (5 - 7)n_0 \), which in certain simplifying approximations reduces to the form analyzed in \( \text{[1, 2]} \).

1. Gamow-Teller transitions

Here I first apply the Fermi-liquid fixed point theory obtained, as I described above, from the \( b \)HLS Lagrangian to the derivation of the GT quenching factor. The quantity obtained in the Fermi-liquid approach is

\[ q_L = g_L^f / g_A \] where \( g_L^f \) is what I will interpret as a Fermi-liquid fixed point constant that multiples the zero-momentum-transfer matrix element \( K = \sum i \tau_i \sigma_i \) for the quasi-particle on top of the Fermi sea making the GT transition, \( M_{GT} = q_{L} g_{A} K \), which then should be compared with the experimental value. This \( q_L \) can be directly related to the quenching factor associated with the \( B_{\text{GT,EM}} \) in \( ^{100} \)Sn \( \text{[11]} \). The superallowed GT transition makes this connection optimally suited.

The part of \( bs \)HLS relevant for the calculation is

\[ \mathcal{L} = i \bar{N} \gamma^\mu \partial_\mu N - \frac{\chi}{f_\sigma} m_N \bar{N}N + g_A \bar{N} \gamma^\mu \gamma_5 \tau_\alpha N A^\alpha_\mu + \cdots \] (8)

where \( A_\mu \) is the external axial field. The part given by \( \cdots \) is not directly relevant. The point to note is that the axial response is scale-invariant, whereas the nucleon mass term is linear in the conformal compensator. This means that embedded in nuclear medium, \( g_A \) is independent of IDDs, whereas the nucleon mass does scale “intrinsically” as already indicated in \( \text{[3]} \). What is not obvious is why \( g_A \) is free of the intrinsic scaling while \( f_\pi \) is crucially dependent on it, although both are intricately tied to each other in chiral symmetry.

Shifting \( \chi = (\chi)^* + \chi' = f_\pi^* + \chi' \) in medium, the resulting Lagrangian with the IDDs implemented can then be treated either in the way formulated in \( \text{[1]} \), that is, put in Landau-Fermi liquid fixed-point theory with the \( b \)HLS Lagrangian, that is, ignoring \( 1 / N \) corrections, or in the renormalization-group \( V_{\text{land}} \) formalism leading to Fermi-liquid theory which goes beyond the fixed point approximation \( \text{[33]} \). Here I will follow the former. It is much simpler – and accurate enough for the purpose.

As given in \( \text{[12]} \), the key quantity is the Landau mass for the quasiparticle, which is a Fermi-liquid fixed point quantity,

\[ \frac{m_L}{m_N} = 1 + \frac{F_1}{m_N/m_L} = \left( 1 - \frac{\bar{F}_1}{3} \right)^{-1} \approx \Phi^* \sqrt{g_L^f / g_A} \] (9)

where \( \bar{F}_1 \) is related to the Landau parameter \( F_1 \) by \( \bar{F}_1 = (m_N/m_L)F_1 \). Here \( \Phi^* \) stands for the density dependence defined in \( \text{[4]} \) given by the pion decay constant. The fixed point quantities become more reliable the larger the density as the loop corrections, ignored, go to zero as \( \bar{N} \to k_F / (\Lambda_N - k_F) \to \infty \text{[34]} \). The last relation in \( \text{[9]} \), which is approximate, follows in the large \( N_c \) approximation in addition to the large \( \bar{N} \) limit in the description of dense matter as a skyrmion matter. This step requires a far-from-rigorous argument that I prefer to put in footnote\( \text{[35]} \).
\textbf{Now in order to evaluate }$g_A^L$, one can rewrite (2), using the Landau mass formula, as
\[ g_A^L/g_A \approx (1 - \frac{1}{3} \Phi^*\tilde{F}_1) - (10) \]
where $\tilde{F}_1$ is the pion Fock term contribution to the Landau parameter $\tilde{F}_1$. The Fock term is a loop contribution, so naively $O(1/N)$. But the pion being “soft,” it plays an indispensable role (as it does for the anomalous orbital gyromagnetic ratio $\delta g^p_\mu$ – to be described below). Note that were the pionic term absent, one would wind up with no quenching at all! Does this mean that it is the pion that does the quenching?

As mentioned, the pion mass should be protected by chiral symmetry, so should not get affected by density. Hence $\tilde{F}_1$ is known precisely for any density near $n_0$. As for the scaling function $\Phi^*$, one can calculate it in S\chi PT up to $\sim n_0$ backed by experiments (e.g. deeply bound pionic nuclear systems). Surprisingly it is found that the decrease of $\Phi^*$ in density is almost completely cancelled by the increase in $\tilde{F}_1$, with the consequence that the product $\tilde{F}_1^*\Phi^*$ remains surprisingly constant from $\sim n_0/2$ to $\sim n_0$.

The calculation gives the quenching factor
\[ g_A^L \equiv g_A^L/g_A \simeq 0.79 \rightarrow g_A^L \simeq 1.0, \] (11)

exactly what was found in [1]. This predicts that $g_A^\text{eff} \simeq 1.0$ in light as well as heavy nuclei – including nuclear matter.

This result with no N$^\text{LO}$ current operators (for $n > 1$) is to account for all nuclear correlations up to the energy scale $\lesssim (m_\Delta - m_N) \approx 300$ MeV.

The prediction (11) is more or less consistent also with what’s observed in light nuclei. See review, e.g., by Suohonen [4].

I will come back to the possible role of $\Delta$ resonance to the quenching factor, which is the only potential contribution that remains to be figured out.

2. $g_A = 1$ at the dilaton limit fixed point

Here is a perhaps coincidental observation that $g_A \rightarrow 1$ can also be arrived by tuning $f_\pi$ to zero by density, i.e., the dilaton-limit fixed point. It is obviously not directly related to the strong nuclear correlations that led to $g_A^L \simeq 1$.

To tune $g_A$ to 1, redefine the fields as $\bar{\sigma} \equiv \langle \chi \rangle^*/f_\sigma$ and $\bar{\pi} \equiv \langle \chi \rangle^*/f_\pi$. The resulting Lagrangian from bsHLS with the redefined fields inserted has a Walecka-type form with the IDDs at the leading order. The key point to note here is that there are singular terms when $\langle \chi \rangle^*$ goes to zero [39]

\[ \mathcal{L}_{\text{singular}} = \frac{f_\sigma^2 - f_\pi^2}{\langle \chi \rangle^*} \bar{A}(\bar{\sigma}, \bar{\pi}) + \frac{g_A - 1}{\langle \chi \rangle^*} B(\bar{\sigma}, \bar{\pi}, N) \] (12)
arising from the non-linear terms of $N$, $\bar{\sigma}$ and $\bar{\pi}$. Avoiding the singularities requires
\[ g_A = 1 \] (13)
and
\[ f_\sigma^* = f_\sigma^*. \] (14)

Note that the relation (14) is for the fundamental axial constant in QCD untouched by $\Phi^*$ (IDD), whereas (13) is the medium relation impregnated with the IDD. In approaching this limit, one arrives at a Gell-Mann-Lévy (GML)-type linear sigma model with the degenerate $O(4)$ multiplet for the scalar and pseudo-scalar fields (massless in the chiral limit) [2, 31].

What is remarkable here is that the $\rho$ meson is decoupled from the nucleons while the $\omega$ remains coupled gauge-invariantly. This leads to the breaking at high density of the flavor $U(2)$ symmetry for the vector mesons ($\rho$, $\omega$), a crucial feature indispensable for compact-star physics.

As $f_\sigma^* \propto \langle \chi \rangle^* \rightarrow 0$, one arrives at scale-chiral invariance (in the chiral limit). This is a symmetry which is not in QCD in the sense that one cannot zero-in on the infrared (IR) fixed point with the anomalous dimension $\beta'$ going to zero.

It is an extremely intriguing question to ask how $g_A^\text{eff} \simeq 1$ at $n \sim n_0$ goes over to $g_A = 1$ at $n \rightarrow n_0^-$? Is there a connection or is it just a coincidence?

Another mystery would be how ($g_A - 1) \approx 0.28$ in the Adler-Weisberger relation in the vacuum – which is not touched by the dilaton condensate – disappear as the DLFP is approached. It is possible that this phenomenon is correlated with the VM where the hidden gauge coupling $g$, unaffected by the dilaton condensate, goes to zero.

3. Axial charge transitions

In a stark contrast to the Gamow-Teller transition, the axial-charge transition
\[ A(J^+) \leftrightarrow B(J^-) \] (15)
with change of one unit of isospin $\Delta T = 1$ is predicted to be strongly enhanced by the soft-pion two-body exchange current, with corrections from higher-order terms
suppressed by several orders in the power counting. This is one of the “chiral filter-protected” processes that represent the other side of the same coin of the Gamow-Teller operator.

One can formulate the soft-pion effect along the same line of Fermi-liquid theory as done for the Gamow-Teller process \[2\]. Denoting the Fermi-liquid matrix element of the axial-charge operator that consists of the one-body and soft-pion-exchange two-body operators as \(\delta^A\), the total axial charge operator can be written as

\[
A^a_{0L} = \frac{g_A}{m_N} \vec{\sigma} \cdot \vec{k} \epsilon_L
\]

where

\[
\epsilon_L = (1 + \delta^L)/\Phi^*
\]

is the factor that multiplies the leading-order single-particle charge operator. It incorporates the IDDs associated with the pion decay constant and the nucleon mass. The factor \(\delta^L\) can be readily calculated using non-interacting quasiparticles. At the normal matter density \(n_0\), it comes out to be \[2\]

\[
\epsilon_L(n_0) \simeq 2.1.
\]

Since the two-body operator is highly reliably given with little, if any, higher-order corrections, one can compute its effect by simply using the soft-pion term without resorting to a Fermi-liquid formalism. The result turns out to be very close to \[15\], differing by less than 5%, implying that the nuclear correlations that are responsible for the \(g_A\) quenching are unimportant for the axial charge matrix element. The prediction \[15\] is in agreement with the Warburton measurement in the Pb region, \(\epsilon_{\text{exp}} = 2.01 \pm 0.05 \ [16]\), and also with measurements in light nuclei \[11\]. I should caution however that there is a caveat in comparing \(\epsilon_L\) with the experimental value. Unlike the GT transition, extracting \(\epsilon\) would require accounting for many other transition operators than the known leading axial-charge operators (single-particle and soft-pion 2BC) contributing to the first-forbidden transition. The contributions from those extra terms, the operators of which can in principle be readily obtained—though a tedious task—in a systematic non-relativistic framework, need to be carefully calculated and subtracted from the experimental value to extract the \(\epsilon_{\text{exp}}\). This should be feasible for those approaches claimed to be anchored on “first principles.”

It is significant that in the present approach, the Gamow-Teller matrix element is entirely governed by nuclear correlations with no IDDs and no multi-body currents, whereas the axial-charge matrix element is entirely governed by the well-controlled one-body and soft-pion two-body currents with little, if any, nuclear correlations.

4. Anomalous orbital gyromagnetic ratio for the proton \(\delta g^p_I\)

Though somewhat less spectacular than the axial-charge transition, soft pions make also an important contribution to the EM vector current. This aspect is generally ignored in the nuclear physics community studying magnetic response functions, particularly magnetic moments, of nuclei. It is in fact the soft-pion contribution that explained for the first time the thermal \(np\) capture process \(n + p \rightarrow d + \gamma \ [19]\).

The Fermi-liquid approach gives the Migdal formula for the orbital current \[43\]

\[
\vec{J} = \frac{\vec{k}}{m_N} g_l
\]

where

\[
g_l = \frac{1 + \tau_3}{2} + \delta g_l
\]

\[
\delta g_l = \frac{1}{6} (\tilde{F}_1^T - \tilde{F}_1) \tau_3.
\]

This formula is exactly reproduced by the Fermi-liquid formulation with the \(bs\)HLS Lagrangian, including the dependence on the vacuum value of the nucleon mass \(m_N\) not on \(m_N^*\) or \(m_L\) required by the current conservation, i.e., the Kohn theorem. Substituting the Landau parameters obtained from \(bs\)HLS, one gets \[2\]

\[
\delta g_l(n_0) \simeq 0.21
\]

which agrees with what’s measured in the Pb region, \(\delta g^\text{proton}_l = 0.23 \pm 0.03 \ [14]\). As far as I am aware, this quantity fails to be explained by \(S\chi\)EFT. The calculation that I am familiar with \[14\] gives 0.07 \pm 0.02, far short of the experimental value. It would be interesting to understand why the approach \(S\chi\)EFT (where the vector mesons and the dilaton are integrated out) fails so badly. Perhaps the IDDs missing at low orders in \(S\chi\)EFT?

III. DISCUSSIONS AND CONCLUSION

Here I add some pertinent remarks and some suggestions for future work for making a potential progress towards a genuine “first-principle approach” to nuclear physics.

The quenched \(g^T_A\) ≃ 1.0 for all nuclei from light to heavy relies on the assertion that \(1\) \(n\)-body currents for \(n \geq 2\) for superallowed GT transitions should be left out and that \(2\) possible contributions from \(\Delta\)-hole contributions could be small.

(1) I admit that there is nothing rigorous known to suggest that multi-body currents should always be ignored. They are of course there in the presently popular \(S\chi\)EFT, but cannot be trusted if not “suppressed” in consistency.
with the power counting. The point is that unless the LO term is accidentally suppressed, if the N^aLO terms for n ≥ 3 come out non-negligible, then there is no reason to stop at that order. There can be substantial cancellations with higher-order terms. In fact this feature has been observed in a powerful calculation in [25]. Furthermore in non-relativistic treatments, as mentioned, there can be many extra terms that are absent in covariant treatments, such as one- or more-body recoil terms in addition to relativistic corrections, which are all of the same order. Partial account of them can be highly misleading. An example is the GT matrix element in the ^14C decay which is accurately explained without 2BC effect [14].

The chiral filter argument as well as the power-counting made for the GT transitions is valid for small momentum transfers. It cannot apply to the momentum transfer that can intervene in neutrinoless double beta decays where the momentum transfers can be of order ∼ 100 MeV. Applying a quenching factor for the GT constant in such processes, as is done by some authors in the literature, cannot be trusted.

(2) In the drastic approximation to ignore all nuclear excitations to the energy scale of ∼ 300 MeV, it was considered that the quenching factor q could be gotten from the Landau-Migdal interaction g_0 in the Δ-hole channel with the strength in universality with that of g_0 in the nucleon-hole channel. The argument relies on the Ericson-Ericson-Lorentz-Lorenz effect in pion-nuclear interactions. This could be invalidated by the tensor force structure intervening in the Δ-hole interaction at increasing density [33]. The tensor force tends to go to zero at a density ∼ (2 − 3)n_0 for r ≥ 0.7 fm because of the VM fixed point that makes the hidden local gauge coupling get weaker at increasing density. The dropping of the strength of the tensor force at n ∼ n_0 must make the Δ-hole coupling weakened strongly. It is suggested that the universality relation in the g_0 interactions cannot hold in dense medium and that (g_0)_{NΔ} ≪ (g_0)_{NN}. There is an indication in nuclear dynamics that g_0 is by far the strongest nuclear interaction and combined with the tensor force, governs nuclear dynamics [18]. But there is no indication for such strong correlations in ΔN and ΔΔ channels. This is an open issue, requiring enlarging the Hilbert space for quasiparticle interactions.

(3) Since the sing-particle isovector magnetic dipole (M1) operator τ_2 σ is just isospin-rotated from the GT operator τ_2 σ, it is natural that the matrix elements with full nuclear correlations, with no IDDs, are related to each other. This does not mean that the full nuclear isovector magnetic moment is simply related to the GT matrix element. While the latter has no contribution from soft pions – as described above, it is known since early 1970s [49] that the former has an important soft-pion 2BC contribution as in the thermal np capture and also in the isovector magnetic form factors. It is therefore incorrect to apply, as often found in the literature, the same quenching factor to both magnetic moments and GT matrix element.

It should be stressed that the modern development with the powerful tool to access many-body systems underpins the long-standing GT problem together with the axial-charge process anchored on effective quantum field theories. It will bring an understanding of the chiral filter mechanism, the deep property of soft-pion processes, that goes beyond standard chiral effective field theories. A possible strategy could be to pick a set of light nuclei that can be treated accurately and explore the range of problems posed, including the role of the tensor forces and Δ-hole configurations.

There are two issues that are not directly related to the main problem of this paper but connected to the hidden symmetries of QCD.

It has been argued that the bsHLS Lagrangian applied in V_{low}RG to compact stars pins down the possible phase structure of dense matter at n ∼ (2 − 4)n_0 at which a smooth change of degrees of freedom from macroscopic (hadrons) to microscopic (QCD) can be translated into a topology change, which can explain all observed properties of massive compact stars of ∼ 2M_⊙, including in particular the recent LIGO/Virgo gravitational wave data for coalescing neutron stars [39]. What figures importantly here are both hidden local symmetry and scale symmetry emerging at a density ∼ 2n_0.

The other, which is perhaps farfetched, is that corrections to the LOSS (leading order scale symmetry) approximation which underlies the predictions described in this paper could be obtained from a precise determination of g_A^β. When corrections to LOSS are included in the expression for g_A^β, it has the form [50]:

\[ g_A(\beta', c_A) = (c_A + (1 - c_A)(\Phi^*)^\beta')g_A^L,\quad 0 \leq c_A \leq 1 \]

where β' is the slope of the anomalous dimension of the gluon stress tensor. In the LOSS for which β' ∼ 1, one gets g_A^β ≈ 1. But c_A is unknown. This relation obtained in a particular scenario of IR structure of scale symmetry is just an indication of what could be involved in further developments.

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14 In this treatment, one can think of the short-range part of the three-body force having been integrated out with the effect going into the scaling parameter Φ^∗. This is sensible since the ω-range three-body interaction involves the scale above the cutoff used in [15] and hence should be simulated in the force structure employed in the V_{low}RG calculation. The integrating-out procedure modifies, among others, the parameter c in the scaling parameter Φ^∗ that brings in non-intrinsic density dependence.

15 This expression is obtained in the version of the scale-chiral symmetric Lagrangian written down in [5]. It could be different in other versions with different IR structures from that of [5]. I discuss this matter to illustrate to what extent we are ignorant in addressing scale symmetry in nuclear matter. This is highly disturbing given that the role of a scalar – whatever that may be – in nuclear physics has been a mystery since forever.
Finally a word on this review note:

This paper was prompted by the intriguing role of hidden symmetries of QCD in nuclear physics encountered in a recent work on compact-star matter performed in collaboration with Yong-Liang Ma. The mechanism for the quenched $g_A$ problem in nuclei, found a long time ago, turns out to be an unexpected spin-off from the effort to go to high density inaccessible from QCD proper and provides a possible insight into what is heralded “first-principle approach” to nuclear theory. I would like to thank Yong-Liang for this collaboration. Some of Yong-Liang’s ideas are reflected in what’s described here, but I must say that he should not be held responsible for whatever mistakes I may have made in my somewhat idiosyncratic approach. I decided to write this up to clarify the persistent and unfounded critiques with gross misunderstandings on the principal results of the work by certain workers in the field, including some Phys. Rev. Lett. referees.

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