Forces on hockey players and conservation laws: on the theoretical efficiency of different techniques

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Abstract

With a more comprehensive, yet still idealized, take on the analysis by Nässén et al (2019 Eur. J. Phys. 40 065005) regarding the efficiency of different ice skating strides, we find no superiority of the ‘angular momentum skating’ technique within their model. The fundamental reason is that for the ‘angular momentum stride’, there is a force component opposing the forward motion during the latter part of the stride, but not so for the traditional skating technique. We conclude with a short pedagogical discussion regarding conservation laws in physics.

Keywords: skating, mechanics, angular momentum, conservation law, efficiency

1. Introduction

In general, ice or roller skating is a form of bipedal locomotion that is more efficient than walking or running, thanks to the much smaller frictional forces. This trivial observation aside, just as cross-country skiing knows different techniques of varying efficiency and power [1] to the extent that there are different types of competitive rules (‘classical’ and ‘free-style’), it is interesting to investigate the analogous case for hockey skating. The kinematics and dynamics of hockey players on ice have been extensively studied in the biomechanics literature [2–8]. These contributions are nevertheless not optimal as starting points for physics instruction, where focus is typically on idealized models that maximally highlight the physical principles at play and while there have appeared papers [9, 10] in the pedagogical literature on ice skating, they are

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typically concerned with figure skating and not with the type of fast linear movement associated with the game of ice hockey where efficient locomotion rather than graceful movement is of the essence. When it comes to the game of hockey, the pedagogical literature is richer in technical aspects of the behavior of the hockey puck [11–15] than of the players.

To fill this lacuna in the pedagogical literature, Nässén et al [16] published in this journal an elementary analysis of two different types of ‘ice skating strides’ in which they conclude that the traditional skating technique used by hockey players is of suboptimal efficiency while their anecdotal experience indicates that their alternative technique—dubbed ‘angular momentum skating’ (AMS)—gains its increased efficiency at no cost to the maximum power output. The conclusion of their analysis was criticized by Bracko [17] on empirical grounds as unphysical; I believe their analysis to be correct in every theoretical respect that they consider, but that they unduly neglect some important factors in their analysis, justifying in part Bracko’s critique even if he did not identify them. It is the purpose of this paper to offer a pedagogical extension of the analysis provided by Nässén et al [16].

Obviously, the scientific problem is of such a complexity that anything short of an experimental study is unlikely to yield any valid solution. Nevertheless, this being a pedagogical journal, the purpose is not necessarily to reach an indubitable theoretical result, but rather to focus on a discussion of the physical effects at play, and in this respect I believe the analysis offered in this contribution provides much pedagogical benefit for the beginning undergraduate student. In particular, unlike [16], we do not limit ourselves to a purely mechanical analysis, but also include—albeit at an extremely elementary level—basic physiological considerations within the confines of the typical physics curriculum.

2. Preliminary considerations

We start by recapitulating some of the basic tenets given in [16] and we shall use the same symbols for maximum clarity.

Because of the virtually vanishing friction force parallel to the blades, for the ice skater to propel forward, she needs to apply force to the ice at right angles to the blade of the skates to incur a sufficient reaction force with no slipping. In a two-dimensional Cartesian coordinate system in the plane of the ice, where the y-axis is aligned with the average direction of motion (or at least intended direction of motion), we denote by $\theta'$ the angle of the blade of the skate with respect to the y-axis. As the skater propels herself forward, her center-of-mass is displaced at an angle $\theta$ from the y-axis. In general $\theta \neq \theta'$, and the precise relation between them will depend on the type of hockey stride and also on the reference frame (vide infra).

We shall assume that the ice skater uses both legs for propulsion, although not necessarily concurrently. Otherwise, she could align the blade of one skate parallel to the intended direction of motion ($\theta') = 0$, and keep the other one perpendicular to the same. She could then propel herself forward with one leg while gliding essentially without friction on the skate of the other. In this case, $\theta = \theta' = 0$ and this mode of propulsion would lead to comparatively little dissipation but at approximately half the total power output of using both legs for propulsion. If both legs are used for propulsion $\theta'$ cannot be zero if it is assumed that the blade cannot change orientation while in contact with the ice.

At this point we briefly mention the coefficient $\alpha$ from [16]. The square of this coefficient is the ratio of non-dissipated energy to the total energy of a skating stride. Thus, it assumes values in the interval $\alpha \in [0, 1]$, where $\alpha = 1$ corresponds to motion without friction and drag (no dissipation) and $\alpha = 0$ corresponds essentially to ice skating on dry ground in that all kinetic energy is dissipated as heat in every stride. In the absence of a precise estimate of $\alpha$, it
is reasonable to take $\alpha = 1$ for realistic ice skating although it is clear that $\alpha$ will be a speed-dependent function that decreases monotonically as the skater accelerates due to increased drag.

2.1. Ice-skating efficiency

The quickest displacement between two points being that along a straight line, it follows that the most efficient forward skating stride is that which minimizes $|\theta|$. Propulsion along this line means both that the power is focused on propulsion in the right direction (minimizing unneeded losses) and that trajectory time between the two points is minimized for a given kinetic energy.

We will now consider the definition of ‘efficiency’ more precisely. The model that we adopt from [16], neglects all biomechanical complications and relies on a two-dimensional point-particle analysis of the center-of-mass motion of the hockey player. Staying within the confines of said model and taking the instantaneous mechanical efficiency as the ratio of ‘useful power’ to ‘total power’, we define

$$\eta' = \frac{f_y v_y}{|f_x v_x| + |f_y v_y|},$$

where $f_i$ is the $i$-component of the propulsion force on the center-of-mass and $v_i$ is the velocity analog. The numerator is the power by the work–energy theorem in the $y$-direction, which we take to be useful power as it propels the center-of-mass of the hockey player forward. The denominator is the total power corresponding to displacements both in the $x$- and $y$-directions. These are taken as absolute values, so that we do not distinguish positive from negative contributions to the instantaneous total work. The rationale is that both positive and negative changes to the center-of-mass energy require physiological exertion on part of the hockey player. It is important to stress that we do not consider the net force in equation (1), but only the force in the absence of friction and drag, which is the force generated directly by the skater. Consequently, even if the velocity is constant because of air resistance and friction, does equation (1) provide a well-defined efficiency, as long as the skater is generating muscular work.

Equation (1) provides an instantaneous efficiency, for which $\eta' \in (-1,1)$. Not only is a comparison between different techniques impossible if there is no unique way of defining their efficiency, as equation (1) will provide different values at different instants, but the concept of ‘negative efficiency’ may also appear strange. However, these negative efficiencies arise when the strides do not accelerate the hockey player in the positive $y$-direction (motive force), but rather act as a decelerating (braking) force. Seen this way, instantaneous negative efficiencies appear logical when physically justified, even if the single number that we will introduce presently to characterize the efficiency cannot be negative.

While arguably the most straightforward way to obtain a single number is to take a time-average for $\eta'$, such an average cannot be physically interpreted as a fraction of ‘useful’ to ‘total’ energy expenditure, which is how efficiencies for engines are typically defined. Therefore, we instead define

$$\eta = \frac{\int f_y v_y \, dt}{\int \{|f_x v_x| + |f_y v_y|\} \, dt'},$$

[1] In fact, even static forces require physiological energy because of the microscopic displacements and work inside the muscle fibers. Consequently, all the efficiencies calculated in this contribution are overestimates.
where the time integrals are taken over a complete cycle of a hockey stride. Here the numerator is to be interpreted as the net useful work done during one stride and the denominator as the total energy expenditure. Note that here the modulus functions imply that ‘work cannot be undone’. This is an important point where we depart in our analysis from that of Nässén et al [16], and we justify this as a direct consequence of thermodynamic irreversibility, which is otherwise lacking in the mechanical model. Clearly, for any stride that contributes to the forward motion of the hockey player, the integral in the numerator cannot be negative. For other types of ‘strides’ (loosely defined), say the maneuver performed when braking, equation (2) may well turn negative, but this is without any implications for our comparisons of ‘motive strides’.

These equations for the efficiency allow meaningful comparisons for all values of \( \alpha \), provided it be constant, but only in the case where there is no energy dissipation and no change of the sign of the products \( f_x v_x \) and \( f_y v_y \) during the completion of a stride, do they represent pure ratios of kinetic energies of the center-of-mass (‘forward’ kinetic energy and ‘total’ kinetic energy, respectively). Otherwise, they are best understood as ratios of kinetic and dissipated energy in the form of heat, but importantly the equations capture the ratio of heat due to propulsion along the \( y \)-axis (unavoidable within the model) to that due to lateral motion (avoidable). Note also that \(|v_x|\) is bounded for any reasonable hockey strides meant to propel along the \( y \)-direction, whereas \(|v_y|\) is not. Consequently, the instantaneous efficiency increases as the hockey player picks up speed along the \( y \)-direction, and asymptotically approaches unity ceteris paribus.

At this point, let us briefly digress and discuss a subtle point. Note that \( \theta \), like \( \eta \), is in general not invariant between the ice frame (stationary) and ‘body’ frame (comoving in the \( y \)-direction, stationary in \( x \), ‘comoving’ being loosely interpreted in the case of a point-body), that is

\[
[\theta]_{\text{ice}} \neq [\theta]_{\text{body}}. \tag{3}
\]

Keeping \([\theta]_{\text{body}}\) constant between strides, leads to an asymptotic decrease of \([\theta]_{\text{ice}}\) toward zero as the velocity increases in the \( y \)-direction and a corresponding increase in \([\eta]_{\text{ice}}\). If one instead keeps \([\theta]_{\text{ice}}\) constant, \([\theta]_{\text{body}}\) will increase keeping \([\eta]_{\text{ice}}\) constant. This direct consequence of the Galilei transformations serves as a reminder that when comparing different skating techniques, we must inter alia make sure that the comparisons are in the same frame of reference. Moreover, that real-world hockey skaters change their stride technique as they pick up speed [5] is possibly a consequence of this kinematical constraint. For simplicity, we will restrict our calculations to the inertial reference frame of the ice in all that follows.

3. Comparison of two different skating strides

In our analysis, we shall pay no heed to the angle \( \theta' \), but only to the angle \( \theta \). The precise relation between these two angles will depend on the biomechanical model one uses to model the hockey player, but the mathematical equations that we consider depend only on the center-of-mass displacement and hence only on \( \theta \). Given that we cannot determine \( \theta' \) a priori, there is no added benefit in trying to estimate \( \theta \) from an uncertain \( \theta' \) when one can just use an (equally uncertain) estimate of \( \theta \) directly.

\[\text{As a simple illustration, if a person rolls a ball up a hill, she does not regain the biochemical energy expended when the ball rolls back down, even if the ball rolls without any energy losses, and the use of the modulus function is meant to capture this irreversibility. However, this assumption of (biochemical) irreversibility is not strictly necessary for the general conclusion.}\]
3.1. Traditional strides

Now consider the traditional, straight hockey strides within the model. Assume for simplicity that \( \theta = \pi/4 \), in which case we have

\[
\eta'_\text{TS} = \frac{f v_y \cos \pi/4}{|f v_x \sin \pi/4| + |f v_y \cos \pi/4|} = \frac{v_y}{|v_x| + |v_y|},
\]

where \( \text{TS} \) denotes ‘traditional strides’ and \( f = \sqrt{f_x^2 + f_y^2} \). If the starting velocity is zero, then \( v_x = v_y \geq 0 \) during the first stride, and its efficiency may be expressed as

\[
\eta'_{\text{TS}} = \eta_{\text{TS}} = \frac{\int v_y \, dt}{\frac{1}{2} \int |v_y| \, dt} = \frac{1}{2}.
\]

However, as mentioned in the brief digression at the end of section 2.1, \( \eta_{\text{TS}} \) will increase thereafter as more strides are taken so that \( v_y \) grows (this is known as the ‘Ober effect’ in rocketry [18]), unless \( \alpha = 0 \), for which it remains fixed at 1/2 since then all of the skater’s momentum is lost after each stride.

Nevertheless, the maximum efficiency is obtained when \( \theta = 0 \), in which case \( \eta'_\text{TS} = \eta_{\text{TS}} = 1 \). This highlights the inadequacies of the point-particle model, and is not a spurious result confined to \( \alpha = 1 \) as Nässén et al seem to imply [16]. No matter the value of \( \alpha \), the most efficient locomotion along the \( y \)-axis is obtained with the net propulsion force parallel to said axis. Because of effects not taken into account by the simple point-particle model, it is reasonable to assume (from, say, observation of actual hockey skaters) that \( \theta = \pi/4 \) represents a passable estimate of the smallest possible angle and hence to restrict the analysis to this case in an ad hoc manner.

3.2. Angular momentum skating strides

For the ‘AMS’ strides, the analysis becomes a bit more involved. We follow Nässén et al [16] and assume that each stride follows gradually narrowing circular arcs. Let us denote by \( \phi \) the angle of the radius vector of such a circular arc with respect to the \( y \)-axis. Clearly, we have \( \theta = \phi - \pi/2 \) and hence,

\[
f_y = \frac{v_y^2 + v_z^2}{\rho(\phi)} \sin(\phi - \pi/2)
\]

and

\[
f_z = \frac{v_y^2 + v_z^2}{\rho(\phi)} \cos(\phi - \pi/2),
\]

where \( \rho(\phi) \) is the radius of curvature, expressed parametrically as a function of \( \phi \). To ensure consistency with the traditional strides, we shall assume—although it is not mandated by our analysis—that the initial angle of the motion with respect to the \( y \)-direction at the start of the circular arc is \( \theta = \pi/4 \) (\( \phi = 3\pi/4 \)), and at the end of the stride \( \theta = -\pi/4 \) (\( \phi = \pi/4 \)). These values seem reasonable judging from the tracks in the photograph of figure 8 in [16], and they

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3 It is of course not physically necessary to assume that the arcs are concentric or strictly circular, but relaxing both of these assumptions would lead to a much more complicated analysis.

4 If \( \rho(\phi) \) is constant, then obviously \( \eta_{\text{AMS}} = 0 \), but no work is performed.
also simplify the mathematics further on. To bring our notation in line with reference [16], we let \( R = \rho(3\pi/4) \) and \( r = \rho(\pi/4) \).

If we assume \( \alpha = 1 \), there is no dissipation and the evaluation of the integrals in equation (2) is greatly aided by interpreting them as work–energy relations. The total work performed in one stride by the AMS skater is [16, page 8]

\[
\frac{mv_0^2}{2} \left[ \left( \frac{R}{r} \right)^2 - 1 \right],
\]

where \( m \) is the mass of the hockey player and \( v_0 \) the speed at the start of the stride. For our judicious choice of \( \theta \) for the initial and final angles of the motion in the arcs, it is clear from symmetry that half of this work goes into the motion along the \( y \)-direction. Hence, we have

\[
\int f_y v_y \, dt = \frac{mv_0^2}{4} \left[ \left( \frac{R}{r} \right)^2 - 1 \right]
\]

which is the change in kinetic energy along this direction by the work-energy theorem. This is the net change stemming both from positive and negative contributions from the integrand.

During the AMS stride, the velocity along the \( x \)-axis is inverted but the net change of the corresponding kinetic energy is the same as for the \( y \)-direction by symmetry as indicated above. This process can be imagined as proceeding in two disjunct steps. First, work of magnitude equal to the initial kinetic energy, \( mv_0^2/4 \), along the \( x \)-axis is necessary to bring this component of the motion to a halt. Second, work equal to the final kinetic energy along said axis is needed to reach the final velocity in the opposite direction. Hence,

\[
\int |f_x v_x| \, dt = \frac{mv_0^2}{4} + \frac{mv_0^2}{4} \left( \frac{R}{r} \right)^2 = \frac{mv_0^2}{4} \left[ \left( \frac{R}{r} \right)^2 + 1 \right]
\]

Now, for the remaining integral along the \( y \)-axis, we note the inequality

\[
\int |f_y v_y| \, dt \geq \int f_y v_y \, dt
\]

and noncommittally write,

\[
\int |f_y v_y| \, dt = \frac{\gamma(R/r)mv_0^2}{4} \left[ \left( \frac{R}{r} \right)^2 - 1 \right]
\]

where \( \gamma(R/r) \geq 1 \) is an undetermined function of \( R/r \).

3.3. Conclusion

From equations (8), (9) and (11) inserted into equation (2), we obtain

\[
\eta_{\text{AMS}} = \frac{(R/r)^2 - 1}{\gamma(R/r)[(R/r)^2 - 1] + (R/r)^2 + 1}
\]

independent of both \( m \) and \( v_0 \) (the latter independence is a result of having fixed the angles \( \theta \) and \( \phi \) in the ice frame), but crucially dependent on the ratio of the radii of curvature of
the initial and final circular arcs. It is nevertheless clear that $\eta_{\text{AMS}} \leq 1/2$. For completeness, note also that if one removes the modulus functions in equation (2)—with the concomitant changes to equations (9) and (11)—and thereby takes the total work (the denominator) to be reversible, one obtains instead of equation (12), $\eta_{\text{AMS}} = 1/2$ while equation (5) remains unchanged. Clearly then, under our assumptions, $\eta_{\text{AMS}} \leq \eta_{\text{TS}}$, indicating no advantage of the AMS technique.

It should be clear already without any explicit calculations that the AMS technique, as analyzed here, has a force component from the ice on the skater during the latter part of the stride that is diametrically opposite to the direction of motion. In our analysis, an optimal technique should not have such a force component at any time during the stride, or indeed any force component that is not parallel to the average direction of motion. Nevertheless, I remind the reader again that the overly simplistic model and the assumptions that we have used are very ‘shaky’ for such a complicated system as a human hockey player. The problem of which type of skating stride is the most efficient is likely insoluble ‘from the armchair’ as we have attempted here, but I hope the exposition has nonetheless been instructive in highlighting the different physical effects in play.

4. Short pedagogical discussion

Conservation laws—in the broadest sense possible that the ‘conserved quantity’ assumes the same numerical value under at least two different conditions (not necessarily separate in time)—clearly predate Newtonian physics, beginning in classical antiquity with Archimedes’s ‘law of the lever’. While this law is in modern times subsumed by the ‘law of conservation of angular momentum,’ it was extended during the Renaissance into the ‘golden rule of mechanics’ (‘what is gained in power or force is lost in displacement’) which paved the way for the discovery of the ‘conservation of mechanical energy’ with which all studious physics students are familiar from a young age. It was under the assumption that entropy (then called ‘caloric’) was conserved that Carnot proved his theorem on the efficiency of heat engines. Since Clausius, we know that like the case of mechanical energy, entropy conservation fails under some circumstances, but rather than deplete like the mechanical energy, entropy increases when the conditions for its conservation are not fulfilled (irreversible process). Even in modern times, violation of conservation laws presents interesting research physics, especially when studied in non-esoteric settings such as an atom.

Given the ubiquity of conservation laws in physics, and paralleling their historical development, a physics student is likely tempted to apply them beyond their domain of validity. There is therefore an educational imperative to stress their limits. For instance, for non-rigid bodies, the consequences of the law of conservation of angular momentum are not as strong as for rigid ones. Similarly, in the present case, the irreversible nature of the functioning of the biological movement apparatus leads to macroscopic violations of the ‘golden rule’. Also, non-conservative forces may lead to the violation of linear momentum conservation in collisions. On the other hand, when analyzed in a different way, the same conservation law will be trivially satisfied under all conditions. For instance, the linear momentum of

5 There are likely implicit limits on the relation between $v_0$ and the radii of curvature set by the physical strength of the skater.

6 The anonymous ‘reviewer 2’ says that the centripetal acceleration for the AMS stride comes primarily from plyometric muscle work (of the quadriceps and gastrocnemius muscles), as opposed to the mainly concentric muscle work of traditional skating strides, and that this gives rise to differences in the perceived energy expenditure.
combined system ‘hockey player and Earth’ is conserved no matter the accelerations of the hockey player; the Earth is simply pushed back in the opposite direction. Seen this way, the ‘law of conservation of linear momentum’ does not restrict the movement of the hockey player in any way.

The problem of the most efficient ice skating stride as given in [16], as well as the ensuing exchange [17, 24], illustrates some of these pedagogical pitfalls nicely. The gist of the argument presented in [16] is that for the traditional strides the linear momentum of the skater is conserved in a direction oblique to the one optimal for forward propulsion and it takes incessant physical work to change it through subsequent strides. For the AMS technique, however, ‘orbital’ angular momentum (not linear momentum) of the skater is conserved and these changes of direction can be performed without work (at constant energy). Any actual, positive work performed is then excess that goes into increasing the kinetic energy and speed of the skater. As briefly mentioned in one sentence of their open reply to Bracko [24]—

[Bracko] fails to distinguish between work changing the horizontal motion of the center of mass and changes in the internal energy of the body (page 2)—

this conservation argument neglects the internal degrees of freedom of the skater. We have extended their argument by taking them partially into account. Moreover, we have noted that increasing the speed of the skater is not necessarily optimal if the velocity is not increased in the direction of intended motion.

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