PHENOMENOLOGICAL BOUNDS ON B TO LIGHT SEMILEPTONIC FORM FACTORS

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Abstract

The form factors for the weak currents between B and light mesons are studied by relating them to the corresponding D form factors at $q^2_{\text{max}}$ according to HQET, by evaluating them at $q^2 = 0$ by QCD sum rules, and by assuming a polar $q^2$ dependence. The results found are consistent with the information obtained from exclusive non-leptonic two-body decays and, with the only exception of $A_1$, with lattice calculations.
The properties of hadrons containing a single heavy quark $Q$ are characterized, in the limit $m_Q \to \infty$, by $SU(2n_Q)$ spin-flavour symmetry of the strong interactions \([1]\). This symmetry allows us to understand the spectroscopy and the decays of heavy hadrons. In particular, for the $B \to D^{(*)} l^- \nu$ transitions all the form factors are proportional to the Isgur-Wise function \(\xi(w)\), where $w = v_B \cdot v_{D^{(*)}}$, which is equal to 1 at zero-recoil point ($w = 1$). Thus the Heavy Quark Effective Theory (HQET) has been successful in obtaining a rather precise determination of the modulus of the CKM matrix element $|V_{ub}|$ from the study of the semileptonic decays \([8]\). This task was obtained also considering symmetry breaking and perturbative corrections \([\ddagger]\).

Another consequence of the Heavy Quark Symmetry (HQS) is the possibility to relate the semileptonic heavy-to-light form factors; indeed the form factors $B \to D$ (L indicates a light meson) are related to the ones of $D \to L$ at the same velocity ($v_B = v_D$) \([\ddagger]\). The symmetry, however, does not help us to fix their normalization as it is the case for the heavy-to-heavy form factors. However, from the phenomenological point of view, the knowledge of the heavy-to-light form factors is fundamental to determine, for example, the magnitude of the $|V_{ub}|$ CKM matrix element \([\ddagger]\). The rare B decays represent an important channel for testing the standard model predictions and also in this case the knowledge of the heavy-to-light form factors is the fundamental ingredient of the investigation \([\ddagger]\).

Here we study the $q^2$ dependence of the heavy-to-light form factors.

The heavy flavour symmetry relates charmed semileptonic form factors to those of the beauty particles \([\ddagger]\). Therefore, in principle, we can evaluate the $B \to L$ form factors from the experimental knowledge of the form factors appearing in the semileptonic decays of the D mesons \([8]\). This approach was used in \([4, 10, 11]\) to estimate the $|V_{ub}|$ CKM matrix element and study the non-leptonic two-body decays of beauty mesons. The authors of Ref. \([11]\) performed also a detailed study of the CP violating asymmetry for the Cabibbo allowed two-body decays of B mesons taking into account annihilation terms and non-factorizable contributions.

For the hadronic matrix elements we adopt the parameterization chosen in Ref. \([12]\). In particular, for the transition between two pseudoscalar mesons, i.e. $P_1(p) \to P_2(p')$, one has:

$$
\langle P_2(p') \mid V^\mu \mid P_1(p) \rangle = (p + p')^\mu F_1(q^2) + \frac{m_1^2 - m_2^2}{q^2} q^\mu \left[ F_0(q^2) - F_1(q^2) \right]
$$

(1)

and for the transition between a pseudoscalar and a vector meson, $P_1(p) \to V_2(\epsilon, p')$:

$$
\langle V_2(\epsilon, p') \mid V_\mu \mid P_1(p) \rangle = \frac{2i}{m_1 + m_2} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu\alpha} p^{*\beta} V(q^2)
$$

(2)

$$
\langle V_2(\epsilon, p') \mid A_\mu \mid P_1(p) \rangle = (m_1 + m_2) \epsilon_\mu^* A_1(q^2) - \frac{\epsilon^* \cdot q}{m_1 + m_2} (p + p')_\mu A_2(q^2) - 2m_2 \frac{\epsilon^* \cdot q}{q^2} q_\mu A_3(q^2)
$$

(3)

where $\epsilon_\mu$ is the polarization vector of $V_2$. The form factor $A_3(q^2)$ is given by the linear combination

$$
A_3(q^2) = \frac{m_1 + m_2}{2m_2} A_1(q^2) - \frac{m_1 - m_2}{2m_2} A_2(q^2)
$$

(4)
and, in order to cancel the poles at \( q^2 = 0 \), the conditions

\[
F_1(0) = F_0(0) \quad A_3(0) = A_0(0)
\]

should be verified.

These form factors, using the Heavy-flavour symmetry, can be related to the corresponding form factors for \( D \to K(K^*) \) transitions [3]. The relations were firstly obtained by Isgur and Wise in a different parameterization for the hadronic matrix elements:

\[
\begin{align*}
(f_+ + f_-)^{b \to q} &= C_{bc} \sqrt{\frac{m_D}{m_B}} (f_+ + f_-)^{c \to q} \\
(f_+ - f_-)^{b \to q} &= C_{bc} \sqrt{\frac{m_D}{m_B}} (f_+ - f_-)^{c \to q} \\
(a_+ + a_-)^{b \to q} &= C_{bc} \sqrt{\frac{m_D}{m_B}} (a_+ + a_-)^{c \to q} \\
(a_+ - a_-)^{b \to q} &= C_{bc} \sqrt{\frac{m_D}{m_B}} (a_+ - a_-)^{c \to q} \\
f^{b \to q} &= C_{bc} \sqrt{\frac{m_D}{m_B}} f^{c \to q},
\end{align*}
\]

where \( C_{bc} = \left( \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-6/25} \approx 1.12 \) and \( q = u, d, s \). These form factors are related to the ones defined in Eqs. (1)-(3) by the following expressions:

\[
\begin{align*}
\label{eq:6}
f_+ &= F_1 \\
f_- &= \frac{m_1^2 - m_2^2}{q^2} (F_0 - F_1) \\
a_+ &= -\frac{A_2}{m_1 + m_2} \\
a_- &= -\frac{1}{q^2} \left[ (m_1 + m_2)A_1 - (m_1 - m_2)A_2 - 2m_2A_0 \right] \\
f &= (m_1 + m_2)A_1.
\end{align*}
\]

The reliability of the relations in Eq. (3) is extensively discussed in the literature. Quark model calculations (cfr [13]) are consistent with them in all the semileptonic allowed kinematical range. Corrections of order of 15% are found for \( A_1 \) at zero recoil point [14]. Similar conclusions have been obtained by Neubert et al. [15] for the form factor \( F_1 \). Certainly the relations in Eq. (3) are more reliable in the zero-recoil point, where \( 1/m_Q \) corrections are estimated to be small. We take advantage of this observation to get the normalization of all \( B \to L \) form factors at \( q_{\text{max}}^2 \) from the more recent fit of \( D \to K(K^*) \) semileptonic transitions [16]. For the sake of completeness, we report in Tab. 1 the values at \( q^2 = 0 \) and at \( q_{\text{max}}^2 \) of the experimentally determined values of the \( D \to K(K^*) \) form factors. In the same table the \( B \to K(K^*) \) form factors at zero-recoil point (\( q_{\text{max}}^2 \)) are also reported.
The inclusion of SU(3) symmetry breaking terms allows to obtain vector form factors for $B \to K$ different from $B \to \pi$. The agreement with the lattice results is good for the vector form factors $F_1$ and $F_0$ and quite good for $A_0$. Large deviations appear in the axial $A_1$ form factor: the lattice results seem to suggest large deviations from the predictions of HQS, in contrast with the calculations of quark models.

At low $q^2$ one has the kinematical constraints (at $q^2 = 0$) in Eq. (5) and the QCD sum rule calculations [17, 18, 19]. We fix the $B_q$ values for the form factors over a limited region near $q_{\text{max}}$. Lattice calculations, in fact, provide results obtained with Light-cone sum rule approach [18], while different $q^2$ behaviours are predicted. Therefore, extra assumptions are needed to cover the full $q^2$ range and we shall compare them with the results of lattice calculations of the same form factors. Lattice calculations, in fact, provide values for the form factors over a limited region near $q_{\text{max}}$. For $B$ decays in light mesons we assume a pole dominated model for the $q^2$ behaviour of all form factors. In Fig. 1, we plot the allowed range of values for them compared with the UKQCD results for $B \to \pi(\rho)$ transitions [20]. The agreement with the lattice results is good for the vector form factors $F_1$ and $F_0$ and quite good for $A_0$. Large deviations appear in the axial $A_1$ form factor: the lattice results seem to suggest large deviations from the predictions of HQS, in contrast with the calculations of quark models [14].

Looking at the Tab. 1, we observe that the masses of the resonances for the $B = S = \pm 3$ mesons with the proper $J^P$ quantum numbers agree with the mass of the lowest state in the case of negative parity $Q\bar{q}$ mesons ($0^-$ and $1^-$) as suggested by Vector Meson Dominance hypothesis. Instead, they are larger for the positive parity ones ($0^+$ and $1^+$). Another test of the compatibility of our predicted range for the form factors can be obtained by studying the exclusive non-leptonic decays. To this extend we consider the Cabibbo allowed non-leptonic two-body decays of B mesons. For the evaluation of the theoretical rates we adopt the following procedure:

Table 1: Experimental semileptonic $D \to K(K^*)$ form factors [16] and the values for the used resonances. The values of the $B \to K(K^*)$ form factors as results in Ref. [19] are reported. In the last column the values, with their errors, of the resonances compatible with the allowed ranges (see text). Note that $A_0(0)$ is obtained by the Eq. (5).

| $q^2 = q_{\text{max},D}$ | $M_R^{(cs)}$ (GeV) | $q^2 = q_{\text{max},B}$ | $q^2 = 0$ | $M_R^{(bs)}$ (GeV) |
|-----------------|-----------------|-----------------|----------|-----------------|
| $F_1^{(cs)}$    | 0.74 ± 0.03     | 2.11            | 1.89 ± 0.08 | 5.14 ± 0.05     |
| $A_1^{(cs)}$    | 0.55 ± 0.03     | 2.54            | 0.54 ± 0.03 | 7.82 ± 0.91     |
| $A_2^{(cs)}$    | 0.40 ± 0.08     | 2.54            | 0.71 ± 0.08 | 6.64 ± 0.63     |
| $F_0^{(cs)}$    | 0.74 ± 0.03     | 2.57            | 0.80 ± 0.03 | 5.77 ± 0.16     |
| $A_0^{(cs)}$    | 0.63 ± 0.06     | 1.97            | 1.4 ± 0.2   | 4.95 ± 0.34     |

*Very recently a new determination of $B \to K$ form factors was obtained in the framework of light-cone sum rules [21]. The inclusion of SU(3) symmetry breaking terms allows to obtain vector form factors for $B \to K$ different from $B \to \pi$.

†Note that, using the results in Ref. [22], the agreement with the UKQCD for $F_1$ becomes better, while for $F_0$ get worse.
1. use the factorization approximation in the evaluation of the amplitudes;

2. neglect final state interactions \(^\dagger\);

3. choose the following expressions for the Isgur-Wise function \(^\dagger\)
   \[
   \begin{align*}
   \xi_1(w) &= 1 - \rho_1(w - 1) \\
   \xi_2(w) &= \exp\{ -\rho_2(w - 1) \} \\
   \xi_3(w) &= \frac{2}{w + 1} \exp\left\{ (1 - 2\rho_3) \frac{w - 1}{w + 1} \right\}
   \end{align*}
   \] (8)

   with
   \[
   \begin{align*}
   \rho_1 &= 0.91 \pm 0.21 \\
   \rho_2 &= 1.27 \pm 0.41 \\
   \rho_3 &= 1.53 \pm 0.50 ;
   \end{align*}
   \] (9)

4. \(a_1^{\text{eff}}\) and \(a_2^{\text{eff}}\) are considered as free parameters and fitted by the experimental data;

5. we fit the experimental data on these decays allowing for a variation of the involved form factors in the previously determined ranges. An SU(3) symmetry breaking effect in \(F_1\) was considered.

It is worth noticing that the form factors appearing in the amplitudes correspond, with the only exception of \(B \to KJ/\Psi\) one, to the \(b \to (u, d)\) transitions. However, we allow for SU(3) breaking effects but assume that they are small enough to be included in the obtained ranges.

The compatibility of the ranges obtained before with the values of the form factors appearing in the amplitudes is good. In Tab. 2 we report, for the different choices of the Isgur-Wise function, the values of the parameter \(a_1^{\text{eff}}\) and \(a_2^{\text{eff}}\) and the resulting branching ratios for the considered decays. These phenomenological parameters contain, as discussed in Ref. \([24]\), the contribution of the non-factorizable terms. In our approach, \(a_1^{\text{eff}}\) results to be very near to the value predicted by the analysis in Ref. \([23]\) (\(\sim 1\)); \(a_2^{\text{eff}}\), instead, results larger than the value predicted by the same analysis (\(\sim 0.3\)). This means that, in our model for the form factors, larger non-factorizable contributions in class II and III are found. In particular our larger values of \(a_2^{\text{eff}}\) depend on the fact that the form factor \(F_1(m^2_{J/\Psi})\) is smaller than the one in the model in Ref. \([23]\) so, to account for the experimental value on the \(B \to KJ/\Psi\) decay, large non factorizable contributions are needed.

In conclusion our determination of the B form factors, based on the relationship to the experimentally measured D form factors at \(q^2_{\text{max}}\), the QCD sum rules at \(q^2 = 0\) and the pole dependence, are consistent with the non-leptonic exclusive ratios and, with the exception of \(A_1\), with the lattice calculations. However, rather large non-factorizable contributions to \(a_2^{\text{eff}}\) are required. The masses found for the \(B = S = \pm 1\) mesons, which dominate the different form factors, agree with the masses of the lowest mesons (\(Q\bar{q}\) states) for the \(l = 0\), while they are larger for the \(l = 1\) states.

\(^\dagger\) Cfr the discussion in Ref. \([22]\) for the validity of this approximation in the two-body non-leptonic Cabibbo allowed decays.
Figure 1: The allowed ranges for $F_1(b\rightarrow q)(q^2)$ (solid line), $F_0^{(b\rightarrow q)}(q^2)$ (dashed line), $A_1^{(b\rightarrow q)}(q^2)$ (dotted line) and $A_0^{(b\rightarrow q)}(q^2)$ (dashed-dotted line) are plotted together with UKQCD numerical results.

We thank F. Buccella for useful discussions. One of us (P.S.) thanks L. Del Debbio for providing us the numerical results of UKQCD Collaboration.

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| $a_1^{\text{eff}}$   | Experimental data (in %) | $\xi_1(w)$ | $\xi_2(w)$ | $\xi_3(w)$ |
|----------------------|--------------------------|-------------|-------------|-------------|
| $a_2^{\text{eff}}$   |                          | 0.476       | 0.443       | 0.441       |
| $Br(B^- \to D^0\pi^-)$ | 0.53 ± 0.05              | 0.497       | 0.509       | 0.515       |
| $Br(B^- \to D^0\rho^-)$ | 1.34 ± 0.18              | 1.28        | 1.28        | 1.29        |
| $Br(B^- \to D^{*0}\pi^-)$ | 0.52 ± 0.08              | 0.589       | 0.562       | 0.551       |
| $Br(B^- \to K^- J/\Psi)$ | 0.101 ± 0.014            | 0.0943      | 0.0943      | 0.0943      |
| $Br(B^0 \to D^+\pi^-)$ | 0.30 ± 0.04              | 0.260       | 0.276       | 0.282       |
| $Br(B^0 \to D^+\rho^-)$ | 0.78 ± 0.14              | 0.670       | 0.684       | 0.690       |
| $Br(B^0 \to D^{*0}\pi^0)$ | $< 4.8 \times 10^{-2}$  | 0.0169      | 0.0156      | 0.0151      |
| $Br(B^0 \to D^{*0}\rho^0)$ | $< 9.7 \times 10^{-2}$  | 0.0177      | 0.0163      | 0.0158      |
| $Br(B^0 \to D^{*+}\pi^-)$ | 0.26 ± 0.04              | 0.320       | 0.309       | 0.303       |
| $Br(B^0 \to D^0\rho^0)$ | $< 5.5 \times 10^{-2}$  | 0.0435      | 0.0412      | 0.0403      |
| $Br(B^0 \to \bar{K}^0 J/\Psi)$ | 0.075 ± 0.021            | 0.0907      | 0.0907      | 0.0907      |

Table 2: Results of the fit of $a_1^{\text{eff}}$ and $a_2^{\text{eff}}$ to non-leptonic Cabibbo allowed two-body decays of B mesons for different choices of Isgur-Wise function. The resulting branching fractions are also reported.