Correlation functions between topological objects – field theoretic versus geometric definitions

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We analyze topological objects in pure gluonic $SU(2)$ lattice gauge theory and compute correlation functions between instantons and monopoles. Concerning the instantons we use geometric and field theoretic definitions of the topological charge. On a $12^3 \times 4$ lattice it turns out that topological quantities have a non-trivial local correlation. The auto-correlation functions of the topological charge depend on cooling for both definitions. We fit the correlation functions to exponentials and obtain screening masses.

1. Introduction and Theory. Classical gauge field configurations with a non-trivial topology are believed to play an essential role in the confinement mechanism. In the scenario of the dual superconductor abelian monopoles condense leading to confinement. Large and interacting instantons could also produce confinement if they form an instanton liquid. Because of the distinctness of these two pictures the interesting question arises, whether instantons and monopoles are related to each other. In a recent study we computed correlation functions between monopoles and instantons using naive definitions of the topological charge [1]. We found a non-trivial local correlation. Now we investigate if this also holds for a geometric charge definition.

There exist several definitions of the topological charge on the lattice. We use a field theoretic and a geometric charge definition. The field theoretic prescription is a straightforward discretization of the continuum expression. To get rid of the renormalization constants we apply the “Cabbibo-Marinari cooling method” with a cooling parameter $\delta = 0.05$. The geometric charge definitions interpolate the discrete set of link variables to the continuum and then calculate the topological charge directly. In our studies in $SU(2)$ we employ the hypercube prescription for the field theoretic definition 2 and the locally gauge invariant Lüsher charge definition 3.

After fixing the gluon field to the maximum abelian gauge, abelian color magnetic monopole currents $m(x, \mu)$ are evaluated over elementary cubes $[4]$. The quantity of further interest is the monopole density being defined as $\rho(x) = \frac{1}{V} \sum_\mu |m(x, \mu)|$.

We calculate the correlation functions $\langle q(0)q(r) \rangle$ and $\langle |q(0)|\rho(r) \rangle$ for the geometric and the field theoretic definition of the topological charge and normalize them to one after subtracting the corresponding cluster values.

2. Results. Our simulations were performed on a $12^3 \times 4$ lattice with periodic boundary conditions using the Metropolis algorithm. The observables were studied in pure $SU(2)$ both in the confinement and the deconfinement phase at inverse gluon coupling $\beta = 4/g^2 = 2.25$ and 2.4, respectively, employing the Wilson plaquette action. For each run we made 10000 iterations and measured our observables after every 100th iteration. Each of these 100 configurations was first cooled and then subjected to 300 gauge fixing steps enforcing the maximum abelian gauge.

The auto-correlation functions of the topological charge densities for the hypercube definition (l.h.s.) and for the Lüsher method (r.h.s.) are displayed in Fig. 1 for 0, 5, 20 cooling steps in the confinement phase at $\beta = 2.25$. Without cooling both auto-correlation functions are $\delta$-peaked due to the dominance of quantum fluctuations. The auto-correlations become broader with cooling reflecting the existence of extended instantons, whose core sizes behave rather similar in the deconfinement phase at $\beta = 2.4$. 


Figure 1. Auto-correlation functions of the topological charge density using the hypercube definition (l.h.s.) and the Lüscher definition (r.h.s.) in the confinement phase for 0, 5, 20 cooling steps. Both auto-correlations grow with cooling indicating the existence of extended instantons.

Figure 2. Correlation functions between the monopole density and the absolute value of the topological charge density for the hypercube definition and the Lüscher definition in the confinement (l.h.s.) and the deconfinement phase (r.h.s.) for 0, 5, 20 cooling steps. For both definitions the monopole-instanton correlation functions are almost invariant under cooling and extend over approximately two lattice spacings. The correlation functions hardly change across the phase transition.
In Fig. 2 the correlation functions between the monopole density and the absolute value of the topological charge density for the hypercube and the Lüscher definition are depicted in the confinement (l.h.s.) and the deconfinement phase (r.h.s.) for several cooling steps. Both definitions yield qualitatively the same result. For each charge definition the correlation functions are almost independent of cooling and extend over approximately two lattice units. This indicates that there exists a non-trivial local correlation between these topological objects and that the probability for finding monopoles around instantons is clearly enhanced. The ρ|q|-correlations seem to be hardly influenced by the phase transition.

To gain a more quantitative insight, we analyze the correlation functions discussed above by fitting them to an exponential function. The resultant screening masses in lattice units are presented in Table 1 in both phases for 5 and 20 cooling steps. The screening masses computed from the correlations between monopoles and instantons turn out not very sensitive to cooling and to the phase transition. The error bars of the masses are large reflecting the large errors in the raw data not shown for clarity of plots.

Finally we add in Table 2 a result concerning the ratio $R$ of spatial to time-like monopole densities as a function of cooling. It has been reported that this quantity sharply decreases across the deconfinement phase transition and that it might serve as a reasonable order parameter $\beta$. We observe that the same quantity also decreases as a function of cooling yielding some doubt on the quality of this quantity as an order parameter.

| Cool step | $\beta = 2.25$ | $\beta = 2.40$ |
|-----------|--------------|--------------|
| 0         | 0.996        | 0.898        |
| 5         | 0.975        | 0.571        |
| 20        | 0.268        | 0.037        |

Table 2. Ratio of spatial to time-like monopole density $R = \frac{\rho_s}{\rho_t}$ in the confinement and deconfinement for various cooling steps.

3. Conclusion. We calculated correlation functions between monopoles and instantons and found a non-trivial local correlation indicating an enhanced probability for finding monopoles around instantons. This finding does not depend on the topological charge definition used and on the phase of the theory. For a deeper understanding of this relationship we visualize monopoles and instantons for specific gauge field configurations (see these proceedings).

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