Floquet analysis of self-resonance in single-field models of inflation

Krzysztof Turzyński* and Michał Wieczorek†

Institute of Theoretical Physics, Faculty of Physics, University of Warsaw,
Pasteura 5, 02-093 Warsaw, Poland

August 3, 2018

We review 51 models of single-field inflation, paying special attention to the possibility that self-resonance of the unstable inflaton perturbations leads to reheating. We compute Floquet exponents for the models that are consistent with current cosmological data. We find five models that exhibit a strong instability, but only in one of them – KKLT inflation – the equation of state efficiently approaches that of radiation.

1 Introduction

The concept of cosmological inflation \cite{1-6} has become a vital part of the standard cosmological model, in particular, thanks to providing a mechanism for generating of primordial density perturbations \cite{7-12}. However, after almost four decades of theoretical pursuit inflation remains a very general theory with no direct link to the Standard Model of particle physics. In particular, there is no universally accepted mechanism of reheating, i.e. the transition between inflationary era and radiation domination era. Various plausible scenarios for reheating \cite{14,15}, well embedded in the framework of quantum field theory have been thoroughly investigated (see, e.g., \cite{16,17} for a review). One appealing possibility relies on mode amplification of quantum fluctuation and particle production that can occur when the homogeneous part of the inflaton field oscillates around the minimum of its potential \cite{18,19}. Quite recently, it has been understood that in some models such oscillations can excite fluctuations of the inflaton field to the extent that the Universe starts expanding as radiation dominated. This mechanism, called self-resonance \cite{20,22}, offers an economical and elegant exit from inflation in some inflationary models, greatly reducing theoretical uncertainty associated with reheating \cite{23}. For a given model, one can numerically investigate self-resonance with lattice simulations, but this is both memory- and time-consuming and, so far, has been done only for...
a few inflationary models, see, e.g., [24, 25]. However, before undertaking full nonlinear lattice simulations one can approach the problem at linear order in perturbations and, using Floquet theory, predict whether self-resonance can lead to efficient reheating (see, e.g., [26] for a review).

In this letter, we utilize *Encyclopaedia Inflationaris*, a comprehensive review of single-field models of inflation [27], to identify models which are both consistent with observational data (including the 2018 Planck data release [28]) and admit efficient self-resonance as the mechanism for reheating. To this end, we employ Floquet analysis of inflaton perturbations. The letter is organized as follows. In Section 2 we briefly present the self-resonance mechanism and briefly review Floquet theory. In Section 3 we present the overview of single field inflationary models together with their prospective Floquet analysis. We draw our conclusions in Section 4.

### 2 Floquet analysis of self-resonance

#### 2.1 Inflationary perturbations

Throughout the paper we will consider the models of inflationary universe with one scalar field minimally coupled to gravity and with standard kinetic term, described by the action:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \right],
\]

where \(\phi(t, x) \equiv \phi(t) + \delta \phi(t, x)\).

From now on, we will denote by \(\phi\) the homogeneous part of the field. The metric \(g_{\mu\nu}\) is the perturbed flat FLRW metric which in longitudinal gauge; in the absence of anisotropic stress it reads:

\[
d s^2 = -(1 + 2\Psi)\, dt^2 + a^2(1 - 2\Psi)\, dx^2.
\]

Minimizing the action (1), we obtain the following zeroth-order equations:

\[
H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right],
\]

\[
\ddot{\phi} + 3H \dot{\phi} + V_{,\phi} = 0.
\]

In the first order in perturbations, the relevant degree of freedom is the gauge invariant Mukhanov-Sasaki variable \(Q \equiv \delta \phi + \frac{\dot{\phi}}{H} \Psi\), for which the equation of motion can be written as:

\[
\ddot{Q}_k + 3H \dot{Q}_k + \left(\frac{k^2}{a^2} + \mu_\phi^2\right)Q_k = 0,
\]

2
where $Q(x, t) \equiv \int \frac{d^3k}{(2\pi)^3} Q_k(t) e^{-ikx}$ and

$$
\mu_\phi^2 = \left( V_{,\phi\phi} - \frac{\dot{\phi}_4^2}{2M_p^4H^2} + 3 \frac{\dot{\phi}_2^2}{M_p^2} + 2 \frac{\dot{\phi}_V\phi}{M_p^2H} \right),
$$

(7)

In certain models, solutions of (6) are unstable. Growing amplitudes of the Fourier modes of the inflaton perturbations indicate that energy is transferred from a homogeneous inflaton condensate to the inflaton fluctuations. This is the phenomenon called self-resonance \cite{20–22}. In certain models, the kinetic and gradient energies of the inflaton fluctuations may eventually dominate the potential energy of the inflaton and the equation of state of the universe may approach that of radiation \cite{24,25}.

2.2 Floquet theorem

In order to treat the possible growth of amplitude quantitatively, we will use Floquet analysis along the lines of Ref. \cite{26}. To this end, we write the equation (6) as a set of two first-order equations:

$$
\begin{pmatrix}
\dot{Q}_k \\
\dot{\Pi}_k
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
-\left( \frac{k^2}{a^2} + \mu_\phi^2 \right) & -3H
\end{pmatrix}
\begin{pmatrix}
Q_k \\
\Pi_k
\end{pmatrix},
$$

(8)

After the end of inflation the field $\phi$ begins to oscillate around the minimum of the potential. These oscillations are almost periodic, as the changes of the period and the amplitude of the oscillations are slow compared to the time scale of the oscillations. Moreover, the changes of $a(t)$ and $H(t)$ are also slow in this sense. Therefore, the matrix on the r.h.s of eq. (8) can be regarded as periodic. Then, by Floquet theorem, the fundamental matrix $O(t, t_0)$ of the solutions of the equation (8) can be written as:

$$
O(t, t_0) = P(t, t_0) \exp \left[ (t - t_0) \Lambda(t_0) \right],
$$

(9)

where $P(t, t_0)$ is a periodic matrix with the same period as the matrix in eq. (8) and $P(t_0, t_0)$ is the identity matrix; $\Lambda(t_0)$ is a constant matrix with eigenvalues $\mu_k^{(i)}$. A rule-of-thumb criterion for unstable growth of the amplitude of the inflaton perturbation is

$$
\mu_k^{(\text{max})} = \max_i \{|\text{Re}(\mu_k^{(i)})|\} \gg H.
$$

For brevity, we will from now on call $\text{Re}(\mu_k^{(i)})$ Floquet exponents.\footnote{Strictly speaking, the name Floquet exponents should refer to the generically complex eigenvalues $\mu_k^{(i)}$. However, in single-field inflation, there are just two Floquent exponents, which are real numbers if there is an instability.}

2.3 Basics of Floquet analysis of the inflaton perturbations

Eq. (9) suggests that it is the maximal value of the Floquet exponent $\mu_k^{(\text{max})}$ that is responsible for the growth of the perturbations, which increase as $\exp \left( \mu_k^{(\text{max})}(t - t_0) \right)$.\footnote{Strictly speaking, the name Floquet exponents should refer to the generically complex eigenvalues $\mu_k^{(i)}$. However, in single-field inflation, there are just two Floquent exponents, which are real numbers if there is an instability.}
However, in the inflationary context one also needs to take into account the width of the resonance band (the interval of wave numbers for which the Floquet exponents are positive) and the duration of the reheating era. To quantitatively analyze this problem, we have to consider the evolution of perturbations during many periods of background oscillations, so the expansion of the Universe can no longer be neglected. Moreover, one has to take into account that the amplitude of the oscillations of the homogeneous inflaton field decreases due to Hubble friction. Therefore, considering the evolution of the perturbations through many background oscillations, one has to include the time dependence of Floquet exponents. It boils down to the following change of the function describing the growth of the amplitude:

$$\exp \left[ \text{Re}(\mu_k) \cdot (t - t_0) \right] \rightarrow \exp \left[ \int_{t_0}^{t} dt' \text{Re}(\mu_k(t')) \right],$$

where $t_0$ now corresponds to the onset of the instability.

For an inflationary potential, which can be approximated as $V(\phi) \propto \phi^{2n}$ near its minimum, the time evolution of background oscillations amplitude can be described by the relation $\dot{\phi} \propto a^{-3/(n+1)}$. Simultaneously, in the expanding Universe the effective wave number decreases and satisfies $k_{\text{eff}} = k/a$. Therefore, to track the time evolution of Floquet exponents, we can compute them for a range of values of the amplitude $\phi$ and different wave numbers $k$. On the plane $(k, \phi)$, the end of inflation corresponds to a particular value of the amplitude of the homogeneous inflaton field. As the Universe expands, the time evolution of the given mode corresponds to a particular path on the plane $(k, \phi)$ described by the relation $\phi \propto k^{3/(n+1)}$. In Section 3, we will present Floquet exponents calculated for various values of $k$ and $\phi$, the examples of the paths described above are marked with white lines and the end of the inflation corresponds to red lines. The amplitude of a given mode will then grow according to (10), in which Floquet exponents have to be evaluated at appropriate points along a path; for significant growth, the relation $\text{Re}(\mu_k(t)) \gg H$ needs to be satisfied for approximately one Hubble time. Empirically, we should usually require $\text{Re}(\mu_k) \gtrsim 10H$ (see, e.g., [25]), which corresponds to a typical time during which a path crosses the instability patch.

The values of Floquet exponents are presented in units $M^2/M_P$. These units are natural for Floquet exponents for two reasons. First, the entire dependence of Floquet exponents on the scale $M \sim V^{1/4}$ is factored out. Second, the Hubble rate is of order of $M^2/\sqrt{3}M_P$ at the end of inflation. Therefore, values of Floquet exponents describe naturally the rate of the exponential growth of the amplitude of a given mode: with $\tilde{\mu}_k \equiv \mu_k^{\text{(max)}} M^2/M_P$, the amplitude grows roughly by $\sim e^{\sqrt{3} \tilde{\mu}_k}$ during one Hubble time. In these units the requirement for the strong resonance can be expressed as $\text{Re}(\mu_k^{\text{(max)}}) \gtrsim 5M^2/M_P$.

It is known that a strong growth of perturbations alone does not suffice to reheat the Universe, since the fragmented inflaton does not always acquire the equation of state characteristic for radiation. For example, very often oscillons are created [20–22, 24, 25, 26], which constitute the dominant part of the energy in the Universe and yield the equation of state of non-relativistic matter. The persistence of oscillon-like solutions is
typical for many inflationary potentials, which are quadratic near their minimum and flatten outside this region. As suggested in [25], this fact seems to be quite general and is connected with the vanishing Floquet instability bands for $\phi \to 0$ in such models. Therefore, in our analysis we will focus on both the magnitude of Floquet exponents and the shape of the corresponding instability bands.

3 Floquet analysis of single-field inflationary models

In the decades following the formulation of the inflationary scenario, numerous models of inflation have been proposed. It is a nearly Herculean task to describe them all comprehensively, because of both the huge number of authors who have worked on inflation and the shifting theoretical focus. In our analysis of single-field inflationary models, we relied on a review by Martin, Ringeval and Vennin, aptly titled Encyclopaedia Inflationaris [27], which provides a useful classification and parametrization of these models, as well as discussion about theoretical limits of their validity. The computation of the Floquet exponents has been done for all the models that are in agreement with current cosmological data [28] for at least some values of the parameters. Some inflationary models spurred theoretical interest in the past, but were later found to be inconsistent with data; adhering closely to Ref. [27], we list those models in Appendix A.

In order to facilitate orientation in the large number of models, we provide a summary table in Appendix B, in which we indicate whether the Floquet exponents (denoted FE therein) are large enough to enable exponential growth of perturbations.

3.1 Models for which Floquet analysis is impossible or irrelevant

A number of inflationary models purports to describe only a limited part of the field range which is relevant for the generation of the perturbations that can be detected via CMB measurements. In these models, a graceful exit from these models of inflation must be provided by some additional mechanisms. One of possible reason may be that the corresponding potential does not have a minimum with a vanishing value of the potential. Alternatively, a model may exhibit a troubling amount of fine-tuning or require the presence of other fields to end inflation. In this Section, we shall list such models and, where appropriate, briefly explain the reasons for excluding them from our Floquet analysis.

3.1.1 Models without potential minimum at zero

We list such zero-, one- and two-parameter models in Tables 1, 2 and 3, respectively. HI is the only zero-parameter model in this group. It postulates, that the inflaton field is the Higgs field with non-minimal coupling to gravity. The potential of this model has no minimum and there are strong theoretical arguments against this model [30].
Higgs inflation (HI) \quad V(\phi) = M^4 \left( 1 - e^{-\sqrt{2/3} \phi/M_P} \right)

Table 1: Models with no free parameters and without a minimum of the potential at zero

| name and acronym            | potential                                                                 |
|-----------------------------|---------------------------------------------------------------------------|
| Exponential SUSY inflation  | V(\phi) = M^4 \left( 1 - e^{-q\phi/M_P} \right)                          |
| Power law inflation (PLI)   | V(\phi) = M^4 \left( e^{-\alpha\phi/M_P} \right)                        |
| Loop inflation (LI)         | V(\phi) = M^4 \left( 1 + \alpha \ln \left( \frac{\phi}{M_P} \right) \right) |
| Arctan inflation (AI)       | V(\phi) = M^4 \left( 1 - \frac{\alpha}{\pi} \arctan \left( \frac{\phi}{M_P} \right) \right) |
| Constant n_s A inflation    | V(\phi) = M^4 \left( 3 - \left( 3 + \alpha^2 \right) \tanh^2 \left( \frac{\alpha}{\sqrt{2} M_P} \phi \right) \right) |
| Constant n_s B inflation    | V(\phi) = M^4 \left( 3 - \alpha^2 \right) \tan^2 \left( \frac{\alpha}{\sqrt{2} M_P} \frac{\phi}{M_P} \right) - 3) |

Table 2: Models with one free parameter and without a minimum of the potential at zero
| name and acronym         | potential                                                                 |
|--------------------------|---------------------------------------------------------------------------|
| Small field inflation (SFI) | \[ V(\phi) = M^4 \left( 1 - \left( \frac{\phi}{\mu} \right)^p \right) \] |
| Intermediate inflation (II) | \[ V(\phi) = M^4 \left( \left( \frac{\phi - \phi_0}{M_P} \right)^{-\beta} - \frac{\beta^2}{6} \left( \frac{\phi - \phi_0}{M_P} \right)^{-\beta - 2} \right) \beta > 0 \] |
| Logamediate inflation (LMI) | \[ V(\phi) = M^4 \left( \frac{\phi}{M_P} \right)^\alpha e^{-\beta (\phi/M_P)} \] |
| Brane SUSY breaking inflation (BSUSYBI) | \[ V(\phi) = M^4 \left( e^{\sqrt{6}(\phi/M_P)} + e^{\sqrt{6}(\phi/M_P)} \right) \] |
| \(\beta\)-exponential inflation (BEI) | \[ V(\phi) = M^4 \exp_{1-\beta} \left( -\lambda \frac{\phi}{M_P} \right) \] |
| Pseudo natural inflation (PSNI) | \[ V(\phi) = M^4 \left( 1 + \alpha \ln \left( \frac{\phi}{f} \right) \right) \] |
| Non-canonical Kahler inflation (NCKI) | \[ V(\phi) = M^4 \left( 1 + \alpha \ln \left( \frac{\phi}{f} \right) + \beta \left( \frac{\phi}{f} \right) \right) \alpha > 0 \] |
| Constant \( n_s \) C inflation (CNCI) | \[ V(\phi) = M^4 \left( 3 + \alpha^2 \coth^2 \left( \frac{\alpha \phi}{\sqrt{2} M_P} \right) - 3 \right) \] |
| Inverse monomial inflation (IMI) | \[ V(\phi) = M^4 \left( \frac{\phi}{M_P} \right)^{-p} \] |
| Brane inflation (BI) | \[ V(\phi) = M^4 \left( 1 - \left( \frac{\phi}{M_P} \right)^{-p} \right) \] |
| Kahler moduli inflation II (KMIII) | \[ V(\phi) = M^4 \left( 1 - \alpha \left( \frac{\phi}{M_P} \right)^{\frac{4}{3}} e^{-\beta \left( \frac{\phi}{M_P} \right)^{\frac{2}{3}}} \right) \] |

Table 3: Models with two free parameters and without a minimum of the potential at zero
| name and acronym                          | potential                                                                 |
|-------------------------------------------|---------------------------------------------------------------------------|
| Twisted inflation (TWI)                   | \[ V(\phi) = M^4 \left( 1 - A \left( \frac{\phi}{\phi_0} \right)^2 e^{-\left( \frac{\phi}{\phi_0} \right)} \right), \ A \simeq 0.33 \] |
| Generalized MSSM inflation (GMSSMI)       | \[ V(\phi) = M^4 \left( \left( \frac{\phi}{\phi_0} \right)^2 - \frac{2}{3} \alpha \left( \frac{\phi}{\phi_0} \right)^6 + \frac{1}{5} \alpha \left( \frac{\phi}{\phi_0} \right)^{10} \right) \] |

Table 4: Excluded models with two parameters

In the formula for BEI potential, the function \( \exp_{1-\beta} \) is defined as:

\[
\exp_{1-\beta}(f) \equiv \begin{cases} 
(1 + \beta f)^{(1/\beta)} & \text{for } (1 + \beta f) > 0 \\
0 & \text{for } (1 + \beta f) \leq 0 
\end{cases}
\] (11)

In the formula for LMI potential, the parameters \( \alpha, \beta \) and \( \gamma \) are the functions of two parameters \( A \) and \( \lambda \) defined as \( \alpha = 4 \frac{\lambda-1}{\lambda+1}, \beta = 2 \left( \frac{\lambda+1}{2\sqrt{4A\lambda}} \right)^{\frac{2}{\lambda+1}}, \gamma = \frac{2}{\lambda+1} \). Constraints on the parameters for these two models imply that neither of their potentials has a minimum suitable for Floquet analysis.

3.1.2 Other models excluded from Floquet analysis

Two- and three-parameter models belonging to this category are listed in Tables 4 and 5 respectively. In the case of TWI, the slow-roll parameter \( \epsilon = -\dot{H}/H^2 \) satisfies \( \epsilon < 1 \) for all values of the inflaton field, so inflation cannot end. Therefore one has to go beyond that model and possibly resort to additional fields to make the exit from inflation possible. Therefore, performing Floquet analysis in this case would be physically irrelevant.

In order to obtain sufficient amount of inflation for GMSSMI, we need to choose a very fine-tuned value of the parameter \( \alpha \), i.e., it has to satisfy \( 0 < 1 - \alpha \leq 10^{-20} \). There are no strong theoretical arguments behind this fine-tuning, see, e.g., [31,32]. Moreover, even in case of the proper value of parameter \( \alpha \), the predictions of this model are on the boundary of 2\( \sigma \) confidence level. With these two arguments against this model, we decided to forgo its Floquet analysis.

Similarly to TWI, in case of RMI, the inflation can end by violation of slow-roll only in the regime where additional unknown corrections arise, so we did not perform Floquet analysis of this model. The same argument applies to VHI, in which one has to rely in fields other than the inflaton to end inflation.

We also decided not to include \( \alpha \)-attractor T-models into our analysis, despite the possibility of efficient reheating pointed out in [24,25], because in this class of models two-field effects cannot be entirely neglected at the end of inflation [33].
Running-mass inflation (RMI) \( V(\phi) = M^4 \left( 1 - \frac{c}{2} \left( -\frac{1}{2} + \log \frac{\phi}{\phi_0} \right) \frac{\phi^2}{M_P^2} \right) \)

Valley hybrid inflation (VHI) \( V(\phi) = M^4 \left( 1 + \left( \frac{\phi}{\phi_0} \right)^p \right) \)

Constant ns D inflation (CNDI) \( V(\phi) = M^4 \left( 1 + \beta \cos \left( \frac{\phi - \phi_0}{M_P} \right) \right)^{-2} \)

| name and acronym | potential | values of parameters |
|------------------|-----------|----------------------|
| \( R + R^{2p} \) inflation (RPI) | \( V(\phi) = M^4 e^{-\sqrt{\frac{3}{2}} \frac{\phi}{M_P}} e^{\sqrt{\frac{3}{2}} \frac{\phi}{M_P}} - 1 \) | \( p = 1.01 \) |
| Large field inflation (LFI) | \( V(\phi) = M^4 \left( \frac{\phi}{M_P} \right)^p \) | \( p = 2 \) |
| Mixed Large field inflation (MLFI) | \( V(\phi) = M^4 \left( \frac{\phi}{M_P} \right)^2 \left( 1 + \alpha \frac{\phi^2}{M_P} \right) \) | \( \alpha = 5 \times 10^{-4} \) |
| Radiatively corrected massive inflation (RCMI) | \( V(\phi) = M^4 \frac{\phi}{M_P} \left( 1 - 2\alpha \frac{\phi^2}{M_P} \ln \left( \frac{\phi}{M_P} \right) \right) \) | \( \alpha = 10^{-4} \) |
| Natural inflation (NI) | \( V(\phi) = M^4 \left( 1 + \cos \left( \frac{\phi}{f} \right) \right) \) | \( f = 100M_P \) |
| Kahler Moduli inflation I (KMII) | \( V(\phi) = M^4 \left( 1 - \alpha \frac{\phi}{M_P} e^{-\phi/M_P} \right) \) | \( \alpha = e \) |
| Double well potential inflation (DWI) | \( V(\phi) = M^4 \left( \frac{\phi}{\phi_0} \right)^2 - 1 \) | \( \phi_0 = 25M_P \) |
| Fibre inflation (FI) | \( V(\phi) = M^4 \left( (3 - R) - 4(1 - \frac{R}{6}) e^{-\frac{2\phi}{\sqrt{3}M_P}} + (1 + \frac{2R}{3}) e^{-\frac{2\phi}{\sqrt{3}M_P}} + Re\left( \frac{\phi}{\sqrt{3}M_P} \right) \right) \) | \( R = 10^{-6} \) |

Table 5: Excluded models with three parameters

Table 6: Models with one parameter and negative or negligible Floquet exponents
name and acronym | potential
---|---
Supergravity brane inflation (SBI) | $V(\phi) = M^4 \left( 1 + \left( -\alpha + \beta \ln \left( \frac{\phi}{M_P} \right) \right) \left( \frac{\phi}{M_P} \right)^4 \right)$
Spontaneous symmetry breaking inflation (SSBI) | $V(\phi) = M^4 \left( 1 + \alpha \left( \frac{\phi}{M_P} \right)^2 + \beta \left( \frac{\phi}{M_P} \right)^4 \right)$

Table 7: Models with two parameters and negative or negligible Floquet exponents

| name and acronym | potential |
|---|---|
| Logarithmic potential inflation (LPI) | $V(\phi) = M^4 \left( \frac{\phi}{\phi_0} \right)^p \left( \log \frac{\phi}{\phi_0} \right)^q$ |

Table 8: Models with three parameters and negative or negligible Floquet exponents

3.2 Models with negative or negligible Floquet exponents

Predictions of the one-parameter models listed in Table 6 agree with Planck data at 2\(\sigma\) confidence level and we can perform Floquet analysis for the inflaton field oscillating around the minimum of the potential. We used the values of the parameters that are favored by the data and justified by theoretical derivation of the models. We also extended the list of models beyond Ref. [27], including fibre inflation put forth in Ref. [34]. For all the models in this group, the obtained Floquet exponents are negative or negligible.

Models with two parameters are listed in Table 7. In SBI, the parameter $\beta \approx 10^{-4}$ gives the best agreement with Planck data, at the border of 2\(\sigma\) confidence level. To obtain the minimum of the potential at zero, we have to take $4\alpha = \beta(1 - \ln(\beta/4))$, so there remains just one adjustable parameter. We performed Floquet analysis with these parameters values. For the SSBI model, it is convenient to divide the analysis into the following cases:

1) $\alpha > 0$, $\beta > 0$
2) $\alpha < 0$, $\beta < 0$
3) $\alpha > 0$, $\beta < 0$, $\phi_{\text{start}}^2 < -\alpha/2\beta$
4) $\alpha > 0$, $\beta < 0$, $\phi_{\text{start}}^2 > -\alpha/2\beta$
5) $\alpha < 0$, $\beta > 0$, $\phi_{\text{start}}^2 < -\alpha/2\beta$
6) $\alpha < 0$, $\beta > 0$, $\phi_{\text{start}}^2 > -\alpha/2\beta$

where $\phi_{\text{start}}$ denotes the value of the inflaton field at the beginning of observable inflation. We performed Floquet analysis for cases 3) and 5) and obtained negative or negligible Floquet exponents. Other cases either do not agree with Planck data at 2\(\sigma\) confidence level or the potential has no minimum at zero.

There is just a single three-parameter model, LPI, in this category; we present it in Table 8. There are different regions of this potential, where inflation can hold. However all of them are either disfavored by Planck data or the obtained Floquet exponents are
name and acronym & potential & values of parameters \\
Mutated hilltop inflation (MHI) & $V(\phi) = M^4 \left(1 - \text{sech} \left(\frac{\phi}{\mu}\right)\right)$ & $\mu = 0.02 M_P$ $\mu = 0.04 M_P$ \\
Radion gauge inflation (RGI) & $V(\phi) = M^4 \left(\frac{\phi}{M_P}\right)^2 \alpha + \left(\frac{\phi}{M_P}\right)^2 \alpha$ & $\alpha = 0.001$ $\alpha = 0.002$ \\
Witten-O’Raifeartaigh inflation (WRI) & $V(\phi) = M^4 \ln^2 \left(\frac{\phi}{\phi_0}\right)$ & $\phi_0 = 0.01 M_P$ $\phi_0 = 0.02 M_P$ \\

| Table 9: Models with one parameter and positive Floquet exponents for given values of parameters |

| $\mu = 0.04$ | $\mu = 0.02$ |
|---|---|
| $\phi$ vs $M_i$ | $\phi$ vs $M_i$ |

| $\mu = 0.04$ | $\mu = 0.02$ |
|---|---|
| $\phi$ vs $M_i$ | $\phi$ vs $M_i$ |

Figure 1: Floquet exponents for the model MHI for two different parameter choices consistent with Planck data.

negligibly small to support self-resonance reheating in case of this model.

3.3 Models for which the Floquet exponents are positive.

The models discussed in this Section are consistent with Planck data at 2\(\sigma\) confidence level and, since the potential vanishes at the minimum, we can perform Floquet analysis. We did it for appropriately chosen parameters and we list our results for one- and two-parameter models in Tables 9 and 10, respectively.

GRIPI is in agreement with Planck data at 2\(\sigma\) confidence level for $\phi_0/M_P \leq 1$ and $0 < 1 - \alpha < 10^{-9}$. The fine-tuning of parameter $\alpha$ is necessary to obtain sufficient amount of inflation, but it is significantly less severe than in case of GMSSMI, so we decided to not to discard GRIPI from Floquet analysis. We would also like to mention that the KKLTI with $p = 2$ is equivalent to RGI; in particular, our chosen parameter $\mu = 0.01 M_P$ corresponds to $\alpha = 0.0001$, so we in fact consider two different examples. In the following, we will formally distinguish between these two models, but one has to remember that this distinction is artificial.
\[ \alpha = 0.004 \]
\[ \alpha = 0.002 \]

Figure 2: Floquet exponents for the model RGI for two different parameter choices consistent with Planck data.

\[ \phi_0 = 0.01 \]
\[ \phi_0 = 0.02 \]

Figure 3: Floquet exponents for the model WRI for two different parameter choices consistent with Planck data.

| name and acronym | potential | values of parameters |
|------------------|-----------|----------------------|
| Generalized renormalizable point inflation (GRIPI) | \( V(x\phi_0) = M^4 \left( x^2 - \frac{2\alpha}{3} x^3 + \frac{\alpha}{2} x^4 \right) \) | \( 1 - \alpha = 10^{-11}, \phi_0 = 0.05M_P \) \( 1 - \alpha = 10^{-11}, \phi_0 = 0.07M_P \) |
| KKLT inflation (KKLTI) | \( V(\phi) = M^4 \left( 1 + \left( \frac{\phi}{\mu} \right)^{-p} \right)^{-1} \) | \( \mu = 0.01M_P, p = 2 \) \( \mu = 0.01M_P, p = 3 \) \( \mu = 0.01M_P, p = 4 \) \( \mu = 0.04M_P, p = 3 \) |

Table 10: Models with one parameter and positive Floquet exponents
As already mentioned, even in inflationary models exhibiting a strong instability of the perturbations there is no guarantee of efficient reheating, especially if the inflationary potential is approximately quadratic near the minimum and has a plateau or flattens away from the minimum. In this case, long-lived oscillons are created \[22,24,25,29\] and the equation of state of the Universe is that of pressureless dust. Our short investigation of the models with positive Floquet exponents suggests that the creation and domination of the long-lived oscillons can be expected for the following models: GRIPI, MHI, RGI, WRI and KKLTI with \( p = 2 \). Thus our analysis points towards KKLTI with \( p \neq 2 \) as the only new candidate of inflationary model for which the self-resonance can be responsible for reheating.

In order to corroborate this conclusion, we performed numerical lattice simulations of the evolution of the perturbations of the inflaton field in KKLTI with \( p = 2 \) and \( p = 3 \), using a code described in \[33\]. We used cubic lattices of a linear size \( N_{\text{lattice}} = 64 \) with the momentum space cutoff \( k_{\text{max}} = 1500 M^2/M_p \); we expect that the latter is a good trade-off between granularity in Floquet instability regions and the necessity of including high frequency modes. Our results for the evolution of the barotropic parameter \( w = p/\rho \) are shown in Figure 6. Indeed, we found that the equation of state of the Universe remains close to that of pressureless dust \((w = 0)\) for \( p = 2 \), while it quickly approaches that of radiation \((w = 1/3)\) for \( p = 3 \). Taken together, our findings suggest that efficient reheating via self-resonance is rather rare among inflationary models.

### 4 Summary

We performed Floquet analysis of the evolution of the perturbations of the inflaton after the end of inflation in a number of theoretically justified and observationally favored single-field inflationary models considered by other authors. We showed that these perturbations can be unstable and lead to the fragmentation of inflaton condensate in
Figure 5: Floquet exponents for the model KKLTI for four different parameter choices consistent with Planck data.
\( p = 2, \mu = 0.01M_P \)

\( p = 3, \mu = 0.01M_P \)

**Figure 6:** Barotropic parameter \( w = p/\rho \) for two realizations of KKLTI as a function of the number of e-folds after the end of inflation. Horizontal red lines correspond to the equation of state characteristic of radiation.

five of investigated models. However, we found that only in case of KKLT inflation with \( p \neq 2 \), self-resonance can be a good scenario for reheating, i.e. describe the transition of the equation of state of the Universe close to that of radiation. In the remaining cases, the instability leads to creation of long-lived oscillons, for which the typical equation of state is that of non-relativistic matter. Therefore, in those cases, other scenario of reheating are necessary.

**Acknowledgements**

The authors are supported by grant No. 2014/14/E/ST9/00152 from the National Science Centre (Poland).

**A Models inconsistent with Planck data.**

For completeness of our presentation, we mention inflationary models whose predictions are inconsistent with the latest Planck data for all parameters ranges allowed by their theoretical background. These models with one, two and three parameters are listed in Tables [11], [12] and [13] respectively.
| name and acronym | potential |
|------------------|-----------|
| Radiatively corrected Quartic inflation (RCQI) | $V(\phi) = M^4 \left( \frac{\phi}{M_P} \right)^4 \left( 1 - \alpha \ln \left( \frac{\phi}{M_P} \right) \right)$ |
| Horizon flow inflation at first order (HF1I) | $V(\phi) = M^4 \left( 1 + A_1 \frac{\phi}{M_P} \right)^2 \left( 1 - \frac{2}{3} \frac{A_1}{1 + A_1 (\phi/M_P)} \right)$ |
| Radiatively corrected Higgs Inflation (RCHI) | $V(\phi) = M^4 \left( 1 - 2e^{-2\phi/(\sqrt{3}M_P)} + \frac{A_I}{16\pi^2} \frac{\phi}{\sqrt{6}M_P} \right)$ |
| MSSM inflation (MSSMI) | $V(\phi) = M^4 \left( \frac{\phi}{\phi_0} \right)^2 - \frac{2}{3} \left( \frac{\phi}{\phi_0} \right)^6 + \frac{1}{5} \left( \frac{\phi}{\phi_0} \right)^{10}$ |
| Renormalizable inflection point inflation (RIPI) | $V(\phi) = M^4 \left( \frac{\phi}{\phi_0} \right)^2 - \frac{4}{3} \left( \frac{\phi}{\phi_0} \right)^3 + \frac{1}{4} \left( \frac{\phi}{\phi_0} \right)^4$ |
| Open string tachyonic inflation (OSTI) | $V(\phi) = -M^4 \left( \frac{\phi}{\phi_0} \right)^2 \ln \left( \frac{\phi}{\phi_0} \right)^2$ |
| Coleman Weinberg inflation (CWI) | $V(\phi) = M^4 \left( 1 + 4e^{\frac{\phi}{Q}} \ln \left( \frac{\phi}{Q} \right) \right)$ |

Table 11: Models with one parameter, inconsistent with Planck data

| name and acronym | potential |
|------------------|-----------|
| Constant spectrum inflation (CSI) | $V(\phi) = M^4 \left( 1 + \alpha \left( \frac{\phi}{M_P} \right) \right)^{-1}$ |
| Orientifold inflation (OI) | $V(\phi) = M^4 \left( \frac{\phi}{\phi_0} \right)^4 \left( \ln \left( \frac{\phi}{\phi_0} \right)^2 - \alpha \right)^{-1}$ |
| Tip inflation (TI) | $V(\phi) = M^4 \left( 1 + \cos \left( \frac{\phi}{\rho} \right) + \alpha \sin^2 \left( \frac{\phi}{\rho} \right) \right)$ |

Table 12: Models with two parameters, inconsistent with Planck data

| name and acronym | potential |
|------------------|-----------|
| Dynamical supersymmetric inflation (DSI) | $V(\phi) = M^4 \left( 1 + \left( \frac{\phi}{\rho} \right)^p \right)^{\phi_{\text{end}}}$ |
| Generalized mixed inflation (GMLFI) | $V(\phi) = M^4 \left( \frac{\phi}{M_P} \right)^p \left( 1 + \alpha \left( \frac{\phi}{M_P} \right)^3 \right)$ |

Table 13: Models with three parameters, inconsistent with Planck data
## B List of presented models of inflation.

| model                                      | # parameters | Planck data OK? | large FE? |
|--------------------------------------------|--------------|-----------------|----------|
| Higgs inflation (HI)                       | 0            | Yes             | -        |
| Exponential SUSY inflation (ESI)           | 1            | Yes             | -        |
| Power law inflation (PLI)                  | 1            | No              | -        |
| Loop inflation (LI)                        | 1            | Yes             | -        |
| Arctan inflation (AI)                      | 1            | Yes             | -        |
| Constant $n_s$ A inflation (CNAI)          | 1            | Yes             | -        |
| Constant $n_s$ B inflation (CNBI)          | 1            | Yes             | -        |
| Radiatively corrected Quartic inflation (RCQI) | 1          | No              | -        |
| Horizon flow inflation at first order (HF1I) | 1            | No              | -        |
| Radiatively corrected Higgs Inflation (RCHI) | 1            | Yes             | -        |
| MSSM inflation (MSSMI)                     | 1            | Yes             | -        |
| Renormalizable inflection point inflation (RIPI) | 1          | Yes             | -        |
| Open string tachyonic inflation (OSTI)     | 1            | Yes             | -        |
| Coleman Weinberg inflation (CWI)           | 1            | Yes             | -        |
| Large field inflation (LFI)                | 1            | Yes             | No       |
| Mixed Large field inflation (MLFI)         | 1            | Yes             | No       |
| Radiatively corrected massive inflation (RCMI) | 1          | Yes             | No       |
| Natural inflation (NI)                     | 1            | Yes             | No       |
| Kahler Moduli inflation I (KMI)            | 1            | Yes             | No       |
| Double well potential inflation (DWI)      | 1            | Yes             | No       |
| Fibre inflation (FI)                       | 1            | Yes             | No       |
| $R + R''$ inflation (Rpi)                  | 1            | Yes             | No       |
| Mutated hilltop inflation (MHI)            | 1            | Yes             | No       |
| Radion gauge inflation (RGI)               | 1            | Yes             | No       |
| Witten-O'Raifeartaigh inflation (WRI)      | 1            | Yes             | No       |
| Small field inflation (SFI)                | 2            | Yes             | -        |
| Intermediate inflation (II)                | 2            | No              | -        |
| Logamediate inflation (LMI)                | 2            | Yes             | -        |
| Brane SUSY breaking inflation (BSUSYBI)     | 2            | No              | -        |
| $\beta$-exponential inflation (BEI)        | 2            | Yes             | -        |
| Pseudo natural inflation (PSNI)            | 2            | Yes             | -        |
| Non canonical Kahler inflation (NCKI)      | 2            | Yes             | -        |
| Constant $n_s$ C inflation (CNCI)          | 2            | No              | -        |
| Inverse monomial inflation (IMI)           | 2            | No              | -        |
| Brane inflation (BI)                       | 2            | Yes             | -        |
| Kahler moduli inflation II (KMII)          | 2            | Yes             | -        |
| Twisted inflation (TWI)                    | 2            | No              | -        |
| Generalized MSSM inflation (GMSSMI)        | 2            | Yes             | -        |
| Constant spectrum inflation (CSI)          | 2            | No              | -        |
| Orientifold inflation (OI)                 | 2            | No              | -        |
| Tip inflation (TI)                         | 2            | Yes             | -        |
| Supergravity brane inflation (SBI)         | 2            | Yes             | No       |
| Spontaneous symmetry breaking inflation (SSBI) | 2          | Yes             | No       |
| Generalized renormalizable point inflation (GRIPI) | 2          | Yes             | Yes      |
| KKLT inflation (KKLTI)                     | 2            | Yes             | Yes      |
| Running-mass inflation (RMI)               | 3            | Yes             | -        |
| Valley hybrid inflation (VHI)              | 3            | Yes             | -        |
| Constant $n_S$ D inflation (CNDI)          | 3            | Yes             | -        |
| Logarithmic potential inflation (LPI)       | 3            | Yes             | No       |
| Dynamical supersymmetric inflation (DSI)   | 3            | No              | -        |
| Generalized mixed inflation (GMLFI)        | 3            | No              | -        |
References

[1] A. A. Starobinsky, Phys. Lett. B 91 (1980) 99.
[2] K. Sato, Mon. Not. Roy. Astron. Soc. 195 (1981) 467
[3] A. H. Guth, Phys. Rev. D 23 (1981) 347
[4] A. D. Linde, Phys. Lett. B 108 (1982) 389
[5] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220
[6] A. D. Linde, Phys. Lett. B 129 (1983) 177
[7] A. A. Starobinsky, JETP Lett. 30 (1979) 682
[8] V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33 (1981) 532
[9] S. W. Hawking, Phys. Lett. B 115 (1982) 295
[10] A. A. Starobinsky, Phys. Lett. B 117 (1982) 175
[11] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49 (1982) 1110
[12] J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. D 28 (1983) 679
[13] V. F. Mukhanov *Physical foundations of cosmology*, Cambridge University Press 2005
[14] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D 42 (1990) 2491.
[15] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. 73 (1994) 3195 [hep-th/9405187].
[16] B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. 78 (2006) 537 [astro-ph/0507632].
[17] R. Allahverdi, R. Brandenberger, F. Y. Cyr-Racine and A. Mazumdar, Ann. Rev. Nucl. Part. Sci. 60 (2010) 27 [arXiv:1001.2600 [hep-th]].
[18] Y. Shtanov, J. H. Traschen and R. H. Brandenberger, Phys. Rev. D 51 (1995) 5438 [hep-ph/9407247].
[19] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D 56 (1997) 3258 [hep-ph/9704452].
[20] M. A. Amin [arXiv:1006.3075 [astro-ph.CO]].
[21] M. A. Amin, R. Easther, H. Finkel, JCAP 1012 (2010) 001 [arXiv:1009.2505 [astro-ph.CO]].
[22] M. A. Amin, R. Easther, H. Finkel, R. Flauger, M. P. Hertzberg, Phys. Rev. Lett. 108 (2012) 241302 [arXiv:1106.3335 [astro-ph.CO]].

[23] J. Martin, Ch. Ringeval, Phys. Rev. D 82 023511 [arXiv:1004.5525 [astro-ph.CO]].

[24] K. D. Lozanov and M. A. Amin, Phys. Rev. Lett. 119 (2017) no.6, 061301 [arXiv:1608.01213 [astro-ph.CO]].

[25] M. A. Amin, K. D. Lozanov, Phys. Rev. D 97 023533 [arXiv:1710.06851 [astro-ph.CO]].

[26] M. A. Amin, M. P. Hertzberg, D. I. Kaiser and J. Karouby, Int. J. Mod. Phys. D 24 (2014) 1530003 [arXiv:1410.3808 [hep-ph]].

[27] J. Martin, Ch. Ringeval, V. Vennin, Phys. Dark Univ. 5-6 (2014) 75-235 [arXiv:1303.3787 [astro-ph.CO]].

[28] Planck Collaboration, [arXiv:1807.06209 [astro-ph.CO]].

[29] M. A. Amin, D. Shirokoff, Phys. Rev. D 81 085045 [arXiv:1002.3380 [astro-ph.CO]].

[30] M. P. Hertzberg, JHEP 1011 (2010) 023 [arXiv:1002.2995 [hep-ph]].

[31] K. Enqvist, L. Mether and S. Nurmi, JCAP 0711 (2007) 014 [arXiv:0706.2355 [hep-th]].

[32] Z. Lalak and K. Turzynski, Phys. Lett. B 659 (2008) 669 [arXiv:0710.0613 [hep-th]].

[33] T. Krajewski, K. Turzyński and M. Wieczorek, arXiv:1801.01786 [astro-ph.CO].

[34] M. Cicoli, C. P. Burgess and F. Quevedo, JCAP 0903 (2009) 013 [arXiv:0808.0691 [hep-th]].