Effects on Optimal Merging Trajectories with Allocation Optimization from the Trade-off between Fuel Consumption and Flight Time

Daichi TORATANI * and Eri ITOH *

Abstract: This study investigates the effects of the trade-off between fuel consumption and flight time on optimal merging trajectories with allocation optimization. A merging optimization method that simultaneously optimizes arrival trajectories and the sequence of aircraft has the potential to improve future decision support systems for air traffic control. In addition, the merging optimization method is required to optimize allocation of the arrival aircraft to parallel runways because most large-scale airports have parallel runways. The merging optimization method with allocation optimization has been developed in previous studies, but only fuel consumption was minimized. Furthermore, in practice, the total flight time must be minimized, and it is possible that the trade-off between minimizing fuel consumption and flight time affects the optimal solution of the merging trajectory. Numerical simulations are performed to demonstrate the variation of the optimal merging trajectory. In particular, this study focuses on the variation of allocation of aircraft, which is a discrete factor in the merging optimization problem. The simulation results show that the allocation of aircraft can change due to the trade-off between minimizing fuel consumption and flight time. The optimality of the allocation is confirmed by comparing with the simulation results with specified allocation.

Key Words: merging optimization, allocation optimization, hybrid system, trade-off, air traffic management.

1. Introduction

The focus of this study is a merging optimization problem that has been studied recently in the field of air traffic management. Figure 1 shows a schematic image of the merging optimization problem. Arrival aircraft merge to land at an airport sequentially. All the aircraft entering sequence have to maintain time separation at the merging point to avoid wake turbulence generated by leading aircraft. The trajectory and sequence are considered closely coupled. Therefore, it is necessary to optimize both the trajectory and sequence simultaneously to derive the optimal merging trajectory.

The potential application of such an optimization method is an arrival manager (AMAN) which is a grand-based air traffic controller advisory tool. The AMAN obtains aircraft information including position and velocity via radar system and calculates optimal arrival sequence and trajectories. Air traffic controllers make instructions to each pilot based on the optimal sequence and trajectories.

In the merging optimization problem, the trajectory is a continuous dynamical system, and the sequence of aircraft is a discrete system. Each optimization problem, trajectory optimization and sequence optimization, has been studied widely by using several optimization methods for continuous [1]–[3] and discrete systems [4]–[6], respectively. The merging optimization problem can be defined as a hybrid dynamical system that includes both continuous dynamical and discrete systems [7]. One of the difficulties of solving the merging optimization problem is attributed to the difficulties of solving the hybrid system optimization problem.

To solve the merging optimization problem, several researchers have formulated the optimization problem as a single system through non-linear optimal control, which is one of the optimization methods for continuous dynamical systems [8],[9]. A hybrid dynamical system has discontinuity, which is one of the difficulties when formulating the merging optimization problem as a non-linear optimal control problem [10]. Other researchers have combined the trajectory optimization method with the arrival sequencing optimization method [11],[12]. The challenges with their research are how to optimize the trajectory and sequence simultaneously. One such merging optimization method is the simultaneous optimization method for trajectory and sequence (SOM-TS), which combines optimal control and mixed integer linear programming (MILP) [13]. The SOM-TS is capable of optimizing the trajectory and sequence simultaneously while using a non-linear optimization method to optimize each trajectory.

In Ref.[13], the aircraft model was formulated as a simple model, such as Dubin’s car, and the simulations for evaluating the SOM-TS was conducted with a simple aircraft model. However, the SOM-TS can use a more realistic model because the SOM-TS optimizes each trajectory through non-linear optimal control. Reference [14] applied the SOM-TS to an arrival

Special Issue on SICE Annual Conference 2017

* Air Traffic Management Department, Electronic Navigation Research Institute, National Institute of Maritime, Port and Aviation Technology, 7-42-23 Jindaijihigashi-machi, Chofu, Tokyo 182-0012, Japan
E-mail: toratani-d@mpat.go.jp, eri@mpat.go.jp
(Received October 30, 2017)
(Revised January 28, 2018)

JCMSI 0003/18/1103–0182 © 2017 SICE
scheduling algorithm. The trajectory was optimized based on the aircraft dynamics and the aircraft performance model [15]. Additionally, meteorological data and actual radar data are taken into account to simulate more realistic arrival air traffic flow.

In real operations, a trajectory of aircraft is optimized by the flight management system (FMS) based on cost index (CI) which is the trade-off parameter between fuel consumption and flight time [16]. With setting a small CI for the FMS, the aircraft flies in order to reduce fuel consumption more than it does flight time. With setting a large CI, the aircraft flies faster than the aircraft with small CI, even with increasing fuel consumption. However, in Refs. [13] and [14], the SOM-TS minimizes only the total value of the criterion that corresponds to fuel consumption. Reference [17] investigated the effects on optimal merging trajectories from the trade-off between criterion and terminal time, namely, the trade-off between fuel consumption and flight time. The simulation results showed that by prioritizing reduction of terminal time, all the aircraft arrive at the merging point earlier than when the merging trajectory is prioritizing reduction of fuel consumption. In addition, the results showed that not only optimal terminal time but also optimal arrival sequence changes as a result of the trade-off.

As an extended contribution, this study investigates effects of the trade-off between criterion and terminal time on the merging optimization problem with allocation optimization. Reference [17] addressed the merging optimization problem with a single merging point. However, most large-scale airports have parallel runways that have two merging points. In the merging optimization problem with multiple merging points, allocation of aircraft to the multiple merging points has to be optimized as well as trajectory and sequence. The allocation of aircraft to the multiple merging points is a discrete system as well as an arrival sequence. Thus, the merging optimization problem with multiple merging points is a more complex hybrid system optimization problem than that with a single merging point. This study shows how optimal trajectory, sequence, and allocation to the multiple merging points change as a result of the trade-off between fuel consumption and flight time by introducing this trade-off to the merging optimization problem with multiple merging points.

The rest of this paper proceeds as follows. Section 2 presents a summary of the SOM-TS which is the merging optimization method. This section also presents the non-linear optimal control-based trajectory optimization method, which is a part of the SOM-TS. Section 3 explains practical problems for the merging optimization problem. This section also explains a method to introduce allocation optimization to the SOM-TS and to take into account the trade-off. Section 4 presents the simulation results, demonstrating variation of the optimal merging trajectory as a result of the trade-off between fuel consumption and flight time. The final section concludes.

### 2. Merging Optimization Method

#### 2.1 Summary of SOM-TS

The SOM-TS uses a criterion function that expresses the relationship for trajectory optimization between the terminal time \( t_f \) and the value of the criterion \( J \), such as fuel consumption. Figure 2 shows three trajectories and the criterion functions.

The optimal terminal time for each aircraft is \( t_f \) minimizing \( J \). However, all the aircraft have to maintain minimum time separation \( \Delta t_{min} \) at the merging point in the merging optimization problem. In the representation of the criterion function, the merging optimization problem can be formulated using Eq. (1).

\[
\min_{t_f} \sum_{i=1}^{NAC} J_i \\
\text{subject to:} \quad J_i = f_p(t_f) \quad (i = 1, \ldots, NAC), \quad |t_{fp} - t_{fq}| \geq \Delta t_{min}, \quad p, q = \{1, \ldots, NAC \mid p < q\}.
\]

Eq. (1) indicates that the optimization problem that minimizes the sum of \( J \) is subject to the criterion functions and time-separation constraints. Trajectory optimization problems are not included explicitly in the merging optimization problem. However, by deriving the criterion functions using the trajectory optimization method, the SOM-TS can evaluate the trajectory and sequence simultaneously.

Eq. (1) is transformed into Eqs. (2), (3), and (4) to solve the merging optimization problem expressed in the criterion function by using MILP, which is well used to solve the hybrid system optimization problem [18],[19].

\[
\min_{t_f} \sum_{i=1}^{NAC} J_i, \\
\text{subject to:} \quad -a_{ij}t_{fj} + J_i - Mc_{ij} \leq b_{ij} \\
\quad a_{ij}t_{fj} - J_i - Mc_{ij} \leq -b_{ij} \\
\quad -t_{fj} - Mc_{ij} \leq -t_{fij} \\
\quad t_{fj} - Mc_{ij} \leq t_{fij} \\
\quad \sum_{j=1}^{Ndiv} c_{ij} = N_{div} - 1 \\
\quad (i = 1, \ldots, NAC), (j = 1, \ldots, N_{div} + 1),
\]

\[
\text{subject to:} \quad t_{fp} - t_{fq} - Me_1 \leq -\Delta t_{min} \\
\quad t_{fp} + t_{fq} - Me_2 \leq -\Delta t_{min} \\
\quad \sum_{k=1}^{2} e_k = 1 \\
\quad p, q = \{1, \ldots, NAC \mid p < q\}.
\]

Here, \( M \) is a number that is sufficiently larger than any parameters in the optimization problem, and \( c \) and \( e \) are binary variables. Eq. (2) is the objective function of the optimization problem. Eq. (3) represents the piecewise-linearized criterion function, as shown in Fig. 3. Furthermore, \( a \) and \( b \) are the slope and intercept of the piecewise-linearized functions, respectively, and \( N_{div} \) is the division number of the criterion function. Eq. (4) shows the transformed time-separation constraints. The MILP cannot treat a non-linear function as a criterion function and formulate the time-separation constraints in Eq. (1) directly due to the OR operator. By the transformation.
into Eqs. (3) and (4), known as the Big-M method [20], MILP can solve the merging optimization problem, as shown in Fig. 2, and calculate the optimal terminal times of all aircraft minimizing the sum of the criterion while maintaining time separation. The SOM-TS yields the optimal merging trajectories by optimizing the trajectories with these optimal terminal times. Further details on the SOM-TS can be found in Ref. [21].

2.2 Trajectory Optimization

In the SOM-TS, the trajectory optimization method based on non-linear optimal control is used to derive the criterion function. The aircraft model is formulated as shown in Fig. 4. All the parameters are non-dimensionalized. The trajectory optimization problem is formulated as the optimal control problem using Eq. (5), Eq. (6), and Table 1:

$$\frac{d}{dt} \begin{pmatrix} x \\ y \\ V \\ \theta \\ acc \\ u \end{pmatrix} = \begin{pmatrix} V \cos \theta \\ V \sin \theta \\ acc \\ u \end{pmatrix}, \quad (5)$$

$$J = \int_{0}^{t_f} \left( \frac{w_{acc}}{2} acc^2 + \frac{wu}{2} u^2 \right) dt, \quad (6)$$

where \(w_{acc}\) and \(wu\) are the weighting factors for the acceleration \(acc\) and the time derivative of azimuth angle \(u\), respectively. The initial and terminal boundary conditions are specified, except for the terminal time. The optimal control problem minimizes the total control effort due to setting the objective function in accordance with Eq. (6) while satisfying Eq. (5) and boundary conditions. To solve the optimal control problem, FALCON.m is used. FALCON.m is a MATLAB class library that sets up and solves non-linear optimal control problems. With setting an optimal control problem, the FALCON.m transforms an optimal control problem to a non-linear programming problem through direct collocation method and solves the non-linear programming problem by using IPOPT or SNOPT. [22].

Figure 5 shows the example of the trajectory optimization with several \(t_f\). The boundary conditions are set as shown in Table 2. Figure 6 shows the corresponding criterion function. With the criterion functions for all the aircraft, it is possible to use the SOM-TS to calculate the optimal merging trajectory.

3. Allocation Optimization and Trade-off

3.1 Problem Overview

Figure 7 shows aircraft trajectories arriving at Haneda Airport. Aircraft fly from west and north directions and land at two runways, RWY34L and RWY34R. Intuitively, it is efficient for the aircraft flying from west and north directions to land at RWY34L and RWY34R, respectively. However, the number of aircraft from the west is more than that from the north. With uneven air traffic flow, it is possible that it is more optimal in the merging optimization problem to optimize the allocation of arrival aircraft to runways. This problem can be defined as the merging optimization problem with allocation optimization, as shown in Fig. 8. In this problem, only aircraft arriving at the same merging point have to maintain time separation. For the merging optimization problem, the allocation is an additional discrete factor. Therefore, to solve the merging optimization problem with allocation optimization is more

| Symbol | Nomenclature | Min | Max |
|--------|--------------|-----|-----|
| \(t\)  | Time         | –   | –   |
| \(x\), \(y\)  | Position     | –   | –   |
| \(V\)  | Velocity     | 0.75 | 1.25 |
| \(\theta\)  | Azimuth angle | –180 | 180  |
| \(acc\)  | Acceleration | –5   | 5   |
| \(u\)  | Time derivative of azimuth angle | –90 | 90  |

Table 2 Boundary conditions.

| Symbol | Initial | Terminal |
|--------|---------|----------|
| \(x\) (\(\cos(2\pi/3)\)) | -0.02 | |
| \(y\) (\(\sin(2\pi/3)\)) | 0 | |
| \(V\) | 1 | 1 |
| \(\theta\) (\(\circ\)) | -60 | -90 |
complex than that with a single merging point. Grütter et al. applied a potential function to the optimal control-based merging optimization method [23]. The potential function requires aircraft to arrive at either merging point and allows the merging optimization method to optimize the allocation of aircraft to merging points. We propose the SOM-TS with allocation optimization problem by improving the setting of the criterion function [24]. Numerical simulations are performed to show that the improved SOM-TS can optimize the trajectory, sequence, and allocation simultaneously.

It is important for the merging optimization problem with allocation optimization to investigate the trade-off between fuel consumption and flight time. Reference [17] shows that regarding the merging optimization problem with a single merging point, not only do aircraft arrive earlier but also the arrival sequence can change by prioritizing terminal time. The sequence change is crucial from several points of view. To change the sequence is a discrete event for the merging trajectory as a hybrid system. Accordingly, the behavior of the merging trajectory discretely changes before and after the sequence change. In addition, the merging optimization method can be applied to AMAN. In practice, arrival sequencing is one of the most difficult problems for air traffic controllers. Sequence change due to the trade-off is important from an applied perspective. In addition, the allocation of aircraft to merging points is a discrete event as well as a sequence change and crucial point for the AMAN. Therefore, it is necessary to investigate weather allocation changes as a result of the trade-off between fuel consumption and flight time. In particular, when either runway has congestion, as shown in Fig. 7, it is possible for allocation optimization to yield a more optimal merging trajectory. The next two subsections explain the merging optimization method with allocation optimization and the method for deriving the trade-off between fuel consumption and flight time.

### 3.2 Allocation Optimization

The top figure of Fig. 9 shows the criterion functions for the merging optimization problem with allocation optimization, as shown in Fig. 8. The solid and dotted lines show the criterion functions for merging points 1 and 2, respectively. Hereafter, merging points 1 and 2 are denoted as MP1 and MP2, respectively. The trajectory changes discretely when the aircraft chooses different merging points. Therefore, the criterion functions of a single aircraft for merging points 1 and 2 are separated. Each aircraft has to choose either criterion function to arrive at one merging point. To formulate the optimization problem shown in Fig. 9, the constraints regarding the piecewise-linearized criterion functions as shown in Eq. (3) is modified to Eq. (7).

**subject to:**

\[
\begin{align*}
- a_{ij} t_{fi} + J_i - M c_{ijk} &\leq b_{ijk} \\
- a_{ij} t_{fi} - J_i - M c_{ijk} &\leq -b_{ijk} \\
- f_{fi} - M c_{ijk} &\leq -t_{f(i)jk} \\
- f_{fi} - M c_{ijk} &\leq t_{f(U)jk} \\
N_{MP} \sum_{k=1}^{Nsec} c_{ijk} &\leq N_{MP} N_{div} - 1 \\
(i = 1, \ldots, N_{MC}), & \\
(j = 1, \ldots, N_{sec} + 1), & \\
(k = 1, \ldots, N_{MP}), &
\end{align*}
\]

where $N_{MP}$ denotes the number of merging points. With Eq. (7), the aircraft moves on either criterion function, and the allocation of aircraft to merging points can be optimized. However, with only this improvement, the time-separation constraints expressed as Eq. (4) are active even for aircraft arriving at different merging points. To avoid this problem, the calculation process shown in Fig. 9 is added. The criterion functions
for merging point 2 are shifted so as not to interfere with aircraft arriving at merging point 1, and the optimization problem is solved by using MILP. After solving the optimization problem, the criterion functions are re-shifted, and the optimal terminal times can be derived. By the additional calculation process, the time-separation constraints for aircraft arriving at different merging points can be inactive, while the time-separation constraints for aircraft arriving at the same merging point are active.

3.3 The Trade-off between Fuel Consumption and Flight Time

The objective function of the SOM-TS expressed as Eq. (2) is modified as Eq. (8) in order to investigate the effects from the trade-off between fuel consumption and flight time:

$$\min_{t_f} \sum_{i=1}^{N_{AC}} \left( wJ_i + \frac{tf_i}{w} \right),$$

where $w$ denotes the weighting factor for the trade-off. In this study, the criterion of the trajectory optimization $J$ is assumed as the fuel consumption, even though $J$ is set as the total control effort as Eq. (6), because both the total control effort and fuel consumption are convex functions against $tf$. In Eq. (8), by increasing $w$, the effect of $J$ is dominant in the objective function, and the SOM-TS reduces $J$ more than $tf$. Conversely, with decreasing $w$, $tf$ is reduced more than $J$ is. By adjusting $w$, the merging optimization method can yield the trade-off between fuel consumption and flight time.

4. Simulations

4.1 Merging Optimization with Allocation Optimization

Numerical simulations are performed to demonstrate effects on the merging optimization with allocation optimization from the trade-off between fuel consumption and flight time. Figure 10 and Table 3 show the simulation conditions. Five aircraft enter the simulation area from the left side, and a single aircraft enters from the right side at the same initial time. Here, $\Delta t_{min}$ is set as 0.05 at the merging point. MP1 and MP2 are set as $(x, y) = (-0.02, 0)$ and $(0.02, 0)$, respectively. The initial and terminal velocity and the terminal azimuth angle are set in accordance with Table 2. The initial azimuth angle $\theta_0$ and initial position are set as shown in Table 3.

Figures 11 and 12 are the optimal merging trajectory and the criterion function in case $w = 10^6$, respectively. All the aircraft can merge while maintaining sufficient time separation 0.05 with the sequence 5-4-3-2-1 for MP1 and only AC6 for MP2. In this case, all the aircraft from the left side arrive at MP1, and the aircraft from the right side arrives at MP2. The simulation is performed by using a standard desktop computer equipped with Intel® Core™ i7-6700CPU@3.40 GHz, 64.0 GB RAM, and Windows 10 Pro 64-bit OS. To solve the trajectory optimization problem and MILP problem, FALCON.m 1.01 and IBM® ILOG® CPLEX® 12.7.1 are used respectively. The average computational time of 10 trials is 106.84 s including all the phases of the SOM-TS, such as generating the criterion functions, solving the optimization problem in the criterion function through MILP, and re-optimizing trajectories. Figures 13 and 14 show the results in case $w = 10^{-6}$. In this case, all the aircraft arrive earlier than the case $w = 10^6$, denoting that the SOM-TS reduces $tf$ more than $J$ owing to smaller $w$. In addition, AC3 and AC5 arrive at MP2 even though they enter from the left side. This result is derived by allocation optimization.
Fig. 13 Optimal merging trajectory with $w = 10^{-0.6}$.  

Fig. 14 Criterion function with $w = 10^{-0.6}$ (top: MP1; bottom: MP2).

The SOM-TS calculates that the objective function expressed as Eq. (8) can be minimized by allocating AC3 and AC5 to MP2.

4.2 Effects from Trade-off

The optimization results shown in Section 4.1 indicate that not only $t_f$ but also the allocation of aircraft to merging points can be changed as a result of the trade-off between $J$ and $t_f$. To investigate the trade-off with a wider range, Fig. 15 shows the variation of the terminal time, sequence, and allocation of aircraft to merging points with a changing weighting factor. This figure shows that, for example, in case $w = 10^{-0.23}$, AC1 to AC4 arrive at MP1 in the sequence 4-3-2-1, and AC5 and AC6 arrive at MP2 in the sequence 6-5.

In the upper side of Fig. 15, the total $t_f$ decreases in the same sequence and allocation by decreasing $w$. The allocation changes between $w = 10^{-0.23}$ and $w = 10^{-0.24}$. In the case $w = 10^{-0.23}$, AC1 to AC5 are allocated to MP1, and AC6 is allocated to MP2. Intuitively, the allocation in the case $w = 10^{-0.24}$ does not change from that of the case $w = 10^{-0.23}$. It is presumed that this allocation occurs by the result of the allocation optimization. To check the optimality of this allocation change, Table 4 shows the value of the objective function expressed in Eq. (8) in the case $w = 10^{-0.24}$. The top table shows the merging optimization results with allocation optimization. AC1 to AC4 are allocated to MP1, and AC5 and AC6 are allocated to MP2. The bottom table shows the optimization results with specified allocation according to the optimal allocation in the case $w = 10^{-0.23}$. In the bottom simulation, only the trajectory and sequence are optimized, and the allocation is specified such that only AC6 arrives at MP2, and the remaining aircraft arrive at MP1. Comparing those two cases, the objective function with allocation optimization is lower than that with specified allocation. This result indicates that with allocation optimization, the SOM-TS can yield a more optimal solution than the SOM-TS without allocation optimization in the case $w = 10^{-0.24}$.

In a similar way to the allocation change between $w = 10^{-0.23}$ and $w = 10^{-0.24}$, the optimality check in the case $w = 10^{-0.23}$ has to be also checked. Table 5 shows the optimality check in the case $w = 10^{-0.23}$. The top table shows the merging optimization results with allocation optimization, and the bottom table shows the merging optimization results with specified allocation according to the optimal allocation in the case $w = 10^{-0.24}$. The objective function with allocation optimization is also lower than that with specified allocation in this case. By Tables 4 and 5, it is confirmed that the allocation change between $w = 10^{-0.23}$ and $w = 10^{-0.24}$ is optimal.
The allocation can change as a result of the trade-off. Reference [17] showed that the arrival sequence can change as can the terminal time owing to the trade-off between fuel consumption and flight time. Furthermore, the allocation can change depending on the trade-off.

The optimality of the allocation was confirmed. The results of the merging optimization method with allocation optimization were compared with those of specified allocation. The comparisons confirmed that allocation optimization can reduce the value of the objective function more than specified allocation can.

Future work on this topic of research should investigate the trade-off in more realistic simulation conditions. With the aircraft model, meteorological data, and radar data, the actual fuel consumption and flight time can be estimated. Then, the trade-off in real operations would be discussed.

**References**

[1] M. Soler, M. Kamgarpour, and J. Lygeros: A numerical framework and benchmark case study for multi-modal fuel efficient
aircraft conflict avoidance. *Proceedings of the 6th International Council of the Aeronautical Sciences (ICAS)*, 605, 2014.

[2] N.K. Wickramasinghe, Y. Matsuno, A. Harada, T. Kozuka, S. Shigetomi, Y. Miyazawa, M. Brown, and Y. Fukuda: Flight trajectory optimization for operational performance analysis of jet passenger aircraft, *Transactions of JSASS, Aerospace Technology Japan*, Vol. 12, No. 12, APISAT-2013, pp. a17–a25, 2014.

[3] B. Sridhar, H.K. Ng, and N. Chen: Aircraft trajectory optimization and contrails avoidance in the presence of winds, *Proceedings of the 10th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference*, AIAA 2010-9139, 2010.

[4] J.E. Beasley, M. Krishnamoorthy, Y.M. Sharaiha, and D. Abramson: Scheduling aircraft landings: The static case, *Transportation Science*, Vol. 34, No. 2, pp. 180–197, 2000.

[5] H. Balakrishnan and B.G. Chandran: Algorithms for scheduling runway operations under constrained position shifting, *Operations Research*, Vol. 58, No. 6, pp. 1650–1665, 2010.

[6] A. Andreeva-Mori: A study on finding a substitute to the first come: First served rule applied to aircraft sequencing, *Proceedings of the 28th International Council of the Aeronautical Sciences (ICAS)*, 10.8.2, 2012.

[7] P.J. Antsaklis: A brief introduction to the theory and applications of hybrid systems, *IEEE Special Issue on Hybrid Systems: Theory and Applications*, Vol. 88, No. 7, pp. 879–887, 2000.

[8] M. Harada and S. Watanabe: Trajectory optimization considering arrival time at the merging point, *Proceedings of the 46th JSASS Annual Meeting*, 1096, 2015 (in Japanese).

[9] A. Okubo, S. Watanabe, and M. Harada: Three-dimensional trajectory optimization considering arrival time at the merging point, *Proceedings of the 4th ENRI International Workshop on ATM/CNS (EJWAC2015)*, EN-A-036, 2015.

[10] M. Bittner, M. Rieck, B. Grüter, and F. Holzapfel: Optimal approach trajectories for multiple aircraft considering disturbances and configuration changes, *Proceedings of the 30th International Council of the Aeronautical Sciences (ICAS)*, 0051, 2016.

[11] A. Michelin, M Idan, and J.L. Speyer: Merging of air traffic flows, *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 1, pp. 13–28, 2011.

[12] B. Grüter, M. Bittner, M. Rieck, J. Diepolder, and F. Holzapfel: Optimal sequencing in ATM combining genetic algorithms and gradient based methods to a bilevel approach, *Proceedings of the 30th International Council of the Aeronautical Sciences (ICAS)*, 0287, 2016.

[13] D. Toratani, S. Ueno, and T. Higuchi: Simultaneous optimization method for trajectory and sequence for receding horizon guidance in terminal area, *SICE Journal of Control, Measurement, and System Integration*, Vol. 8, No. 2, pp. 144–153, 2015.

[14] D. Toratani, E. Itoh, and N.K. Wickramasinghe: Applying merging optimization method to arrival scheduling algorithm for single runway, *Proceedings of the 2016 Asia-Pacific International Symposium on Aerospace Technology (APISAT)*, Q1-3, 2016.

[15] EUROCONTROL Experiment Center: User manual for the base of aircraft data (BADA), Rev. 3.12, EEC Technical/Scientific Report, No. 14/04/24-44, 2014.

[16] B. Roberson: Fuel conservation strategies: Cost index explained, *Aero, QTR_02*, pp. 26–28, 2007.

[17] D. Toratani and E. Itoh: Effects on optimal merging trajectories from trade-off between fuel consumption and terminal time, *Proceedings of the SICE Annual Conference 2017*, WeA03.2, 2017.

[18] A. Bemporad and M. Morari: Control of systems integrating logic, dynamics, and constraints, *Automatica*, Vol. 35, No. 3, pp. 407–427, 1999.

[19] A. Richards and J. How: Decentralized model predictive control of cooperating UAVs, *Proceedings of the 43rd IEEE Conferece on Decision and Control*, pp. 4286–4291, 2004.

[20] I. Griva, S.G. Nash, and A. Sofer: Linear and nonlinear optimization, *Society for Industrial and Applied Mathematics*, 2nd edition, ch. 5, sec. 4, pp. 156–162, SIAM, 2009.

[21] D. Toratani: *Study on Simultaneous Optimization Method for Trajectory and Sequence of Air Traffic Management*, Ph.D. dissertation, Yokohama National University, 2016.

[22] M. Rieck, M. Bittner, B. Grüter, and J. Diepolder: FALCON.m: user guide, 2016.

[23] B. Grüter, M. Bittner, M. Rieck, F. Holzapfel, and A. Harada: Allocation, sequencing and trajectory generation for aircraft using superimposed navigation functions and optimal control, *Proceedings of the 4th ENRI International Workshop on ATM/CNS (EJWAC2015)*, EN-A-045, 2015.

[24] D. Toratani and E. Itoh: Study on merging optimization method for multiple merging points, *Proceedings of the 59th Japan Joint Automatic Control Conference (JJACC)*, pp. 126–131, 2016 (in Japanese).