Improving multi-material structures using topological optimization and the modified SIMP method

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Abstract. This paper proposes to use a modified SIMP method to solve the problem of topological optimization of structures containing components of more than two different materials, which must be distributed in such a way as to obtain the best structural characteristics using one continuous design variable. The paper demonstrates the effectiveness of the proposed method by the example of solving the problem of topological optimization of a structure made of multimaterial. With the help of such a solution, it is possible to obtain reliable structures with improved mechanical characteristics, which can be used to solve real problems of design and manufacture of complex structures.

1. Introduction
Topological optimization methods allow you to get the most efficient distribution of materials in the design area. The topological optimization method proposed by Bendso and Kikuchi [1] is widely used in solving problems of optimizing the mechanical characteristics of structures, as well as in solving other technical problems, for example, problems of thermoelasticity [2], hydrodynamics [3, 4], acoustics [5], wave propagation [6], aerospace object design [7], multifunctional materials design [8–10], Multiphysics systems [11, 12], etc. New materials and structures can also be developed using this method. For example, using topological optimization and homogenization methods [13], piezo composites have been developed. Recently, topological optimization has also been used in medicine, for example, in the development of bone prostheses [14] or implants for specific patients [15]. However, most of the problems in these articles only concern one material and void, although in some cases it is interesting to look at the topology of use with material phases having different characteristics. As a rule, such a multicomponent topological problem is optimized by searching for the optimal distribution of phases of various materials with specified volume fractions within the computational domain. The method represents the topology of the structure according to the physical properties of each of the materials. Currently, the most widely used approaches to solving topological optimization are explicit optimization methods that work in a fixed domain of finite elements; however, instead of a set of elastic properties of the microstructure, each finite element contains only one design variable. This variable often understood as the density of the element material. One of the most well-known methods is the SIMP (Solid Isotropic Penalized Material) method, which uses simple interpolation for the material and uses a void and a single phase for simple structures. The material interpolation rule has the form [16]:

\[ E_e (\rho_e) = \rho_e^p; \quad 0 \leq \rho_{\text{min}} \leq \rho_e \leq 1, \]
where \( \rho \) is the amount of the fine. The variable \( \rho \) is limited to decreasing a small positive constant \( \rho_{\text{min}} \), which is introduced to prevent degeneration of the finite element matrix. Note that for values \( \rho_{\text{min}} \leq \rho_{e} \leq 1 \) and positive \( p \) (usually assumed to be 3 or 5), the modulus \( E_{e} (\rho_{e}) \) is limited to a small value at the density \( \rho_{e} = \rho_{\text{min}} \) and Young’s \( E_{0} \) modulus of the base material phase when \( \rho_{e} = 1 \). The work [16] describes a method for structures made of two-phase materials:

\[
E (\rho) = \rho^{p} E_{1} + (1 - \rho^{p}) E_{2}; \quad 0 \leq \rho \leq 1,
\]

(2)

where \( E_{1} \) and \( E_{2} \) are the values of Young’s moduli for each phase. In this case, emptiness can be obtained by \( E_{2} = 0 \). For materials containing two solid phases and voids, the interpolation scheme can be formulated as:

\[
E (\rho_{1}, \rho_{2}) = \rho_{1}^{p} \left( \rho_{2}^{p} E_{1} + (1 - \rho_{2}^{p}) E_{2} \right).
\]

(3)

Note that describing the three material phases requires the introduction of two design variables \( \rho_{1} \) and \( \rho_{2} \). To solve the optimization problem for a material containing \( m \) phases, it is necessary to use \( m - 1 \) design variables. Such interpolation becomes cumbersome when solving problems of optimizing structures from more than two materials. Most of the currently known approaches to solving problems of topological optimization of structures containing many materials require the introduction of additional design variables, which increases the computational costs. As an alternative method of topological optimization for multimaterial structures, the objective function was used in the materials approximation in [17]. This method does not increase the number of design variables, but the presence of specific points in this method is a potential source of difficulty in moving from one material phase to another. In [18], an algorithm was proposed for solving a multiphase topological optimization problem by dividing it into several traditional topological optimization subproblems for two materials. Nevertheless, for \( m \) materials, it is required to solve \( m (m - 1) / 2 \) traditional problems, which leads to high computational costs. In the article, to solve the problem of multicomponent topological optimization, a modified SIMP method with one calculated variable is used, which makes it possible to take into account the properties of several phases of materials. In this approach, the density of the material is considered as an independent design variable and is selected from a continuous range (including zero density for voids), after which it is separated by discrete values of the densities of each of the phases of the material. The rest of the properties are considered as continuous density functions. Since the proposed method does not require the introduction of additional variables for the interpolation of materials, the estimated costs do not depend on the amount of materials under consideration. The iterative scheme allows for a stable transition from one phase of the material to another. To demonstrate this method, numerical examples are considered in the work. Due to its conceptual simplicity, the proposed modified SIMP method for interpolating multiphase materials can be easily applied to any existing topological optimization problems.

2. Modified SIMP method for optimization of structures made of multimaterials

Figure 1 shows a general view of the structure and boundary conditions for multicomponent topological optimization. The design contains optimization, which includes two or more materials; the design can contain technological holes and inclusions. Force loads are applied to the boundaries or rigid fixing conditions are applied, it can also be free of load, depending on the problem under consideration.

In the proposed method, materials are sorted in ascending order of the normalized density of the material \( \rho_{i}^{T} \):

\[
\rho_{i} = (1 - \rho_{i}^{T}) / \rho_{\text{max}}, \quad (i = 1, 2, 3, \ldots, m),
\]

(4)

where \( \rho_{\text{max}} \) — maximum density of all optimized materials, \( \rho_{i}^{T} \) — original material densities, \( m \) — number of materials to be optimized, \( i \) — sorting material number. Using the density
representation, the classical power interpolation of the material for the case of multicomponent optimization takes the form:

$$E_e(\rho_e) = A_E \rho_e^p + B_E,$$

where the coefficients $A_E$ and $B_E$ for $\rho_e \in [\rho_i, \rho_{i+1}]$ expressed as

$$A_E = \frac{E_i - E_{i+1}}{\rho_i^p - \rho_{i+1}^p}, \quad B_E = E_i - A_E \rho_i^p$$

and $E_i, E_{i+1}$ – Young’s moduli for materials in sorting with numbers $i$ and $i + 1$ respectively.

3. **Statement of the problem of topological optimization for multiphase materials**

Typically, for linear elastic problems, the practice is to place the stiffer material at locations with large displacements or stresses. The algorithm in this work minimizes strain energy by increasing density in areas of higher sensitivity while respecting the limits on the amount of each of the materials. The optimization task is to minimize the total deformation energy, that is, to obtain a structure with maximum rigidity

$$\min \frac{1}{W_0} \int_\Omega W(x) d\Omega,$$

where $\Omega$ – optimized domain and $\frac{1}{W_0}$ – normalizing factor. In this case, the following restriction must be met:

$$0 \leq \int_\Omega \rho_i(x) d\Omega \leq \gamma_i A,$$

where $\gamma_i$ – the permissible proportion of each of the materials with a density $\rho_i$, $A$ – optimization area $\Omega$. To eliminate the checkerboard effect in the optimal microstructure, a penalty function is introduced in the form:

$$\frac{h_0 h_{\text{max}}}{A} \int_\Omega |\nabla \rho(x)|^2 d\Omega,$$

where $h_0$ is the specified initial size of elements in the split, $h_{\text{max}}$ sets the current size of elements. The penalty function is dimensionless for the worst possible solution on the order of

![Figure 1](image1.png) **Figure 1.** Schematic representation of the structure and boundary conditions for multicomponent topological optimization

![Figure 2](image2.png) **Figure 2.** Ordered multicomponent SIMP interpolation of the elastic modulus of three materials
unity. The dimensionless objective function must be consistent, for example, in the form of a linear combination of the objective function and the penalty function with a given parameter $q$:

$$
f = \frac{1 - q}{W_0} \int_{\Omega} W(x) \, d\Omega + q \frac{h_0 h_{\text{max}}}{A} \int_{\Omega} |\nabla \rho(x)|^2 \, d\Omega. \tag{10}
$$

4. Numerical results

To test the proposed approach, multicomponent topological optimization problems were solved for a number of structures. In what follows, all quantities are taken in dimensionless form. External force is present in all examples. The problems were solved using the finite element method. All design areas are divided into triangular finite elements with a maximum size $h_{\text{max}} = 0.03$ over the entire area. When building models, dummy materials are used, whose properties are listed for each of the examples. Based on the performed numerical experiments, the penalty coefficient $p$ was chosen equal to 5. When solving optimization problems, the method of sliding asymptotes (MMA) was used. As an example, consider a beam, boundary conditions and dimensions shown in figure 3. The length of the beam – $l = 6$, height $h = 1$. At the upper limit at the center of the load intensity is $F$. The bottom corners of the beam are fixed in the vertical direction. In the field of optimizing the quality of materials with Young’s moduli $E_1 = 1 \times 10^{-9}$, $E_2 = 150$, $E_3 = 300$ and also set limits on the amount of each material $\gamma_1$, $\gamma_2$, $\gamma_3$ respectively. The number of finite elements in the calculation of this structure is 17,000 and boundary elements – 540. The number of degrees of freedom is approximately 35,000.

Figure 3. Boundary conditions for the beam

![Figure 3](image)

Figures 4, 5 show the optimal microstructures of the obtained beams. A hard material with Young’s modulus $E_3 = 300$ is indicated in red, green is a softer material $E_2 = 150$, and blue is a material in this area that is close to void $E_1 = 1 \times 10^{-9}$. Figure 4 shows topologies for different $\gamma_2$ with constraints $\gamma_3 = 0.25$, in figure 4 - optimal topologies for different $\gamma_3$ for $\gamma_2 = 0.5$.  

![Figure 4](image)  ![Figure 5](image)

Figure 4. Optimal beam topologies for various constraints $\gamma_2$  
Figure 5. Optimal beam topologies for various constraints $\gamma_3$

Table 1 shows the values of the deformation energy $W$ for the obtained optimal microstructures of the beam under various restrictions on the number of materials. The value is calculated as $\gamma_1 = 1 - \gamma_2 - \gamma_3$.

As can be seen from this table, the value of the deformation energy for optimal designs decreases with an increase in the proportion of hard material, that is, the design becomes more
rigid and increases with an increase in the proportion of softer material. Consider another beam design (figure 6). The left border of the beam is fixed, on the right border in the center a downward load $F$ is applied. Length of the beam $l = 3$, height $h = 1$. The calculations for this structure were carried out to divide into 8479 finite and 269 boundary elements. The number of degrees of freedom is approximately 17500.

Table 1. Deformation energy values $W$ for optimal designs

| $\gamma$ | $W_{\gamma=0.4}$ | $W_{\gamma=0.5}$ | $W_{\gamma=0.6}$ |
|----------|------------------|------------------|------------------|
| $0.15$   | 27.329           | 23.682           | 23.682           |
| $0.25$   | 33.088           | 25.560           | 22.387           |
| $0.35$   | 30.805           | 24.358           |                 |

Optimal beam designs were calculated from two materials obtained using the classical interpolation scheme SIMP (1) (figure 7 (a)) and a structure consisting of three materials (figure 7 (b)) obtained using a modified interpolation scheme SIMP. Figure 7 (a) shows the optimal microstructure of a beam made of materials (white), (red) limiting the amount of material. Figure 7 (b) shows the optimal topology for materials with Young’s modulus (white), (green), (red), constraints on the amount of each of the materials. Comparing the figures, it can be seen that the designs are fundamentally similar, from this we can conclude that optimization of the topology with several materials gives an adequate result.

Figure 8 shows the von Mises stresses for the structure shown in Figure 7 (b). Blue means minimum voltage, red means maximum. The figures show that during optimization, the areas with the highest stresses are filled with more rigid materials.

For this design, the optimal topologies were obtained for different ratios and with the preservation of the total amount of these materials.

The results show that the dependences obtained for two different problems coincide and the method can be applied to different structures under different conditions of mechanical loading.

5. Conclusion
The examples considered showed that the proposed method allows one to effectively solve topological optimization problems for several materials without a significant increase in computational costs compared to the classical SIMP method for one material. This method can be easily integrated into any existing optimization algorithm based on the classical SIMP method and allows solving more complex structural optimization problems. Applying a multi-component concept to topology optimization complicates the process of finding a solution, but also opens up new opportunities for engineers by introducing new potential design alternatives that are difficult to predict intuitively.
Figure 8. Von Mises stress

Figure 9. Optimal beam topologies for different values

Acknowledgments

This work was supported by RFBR, project number 19-31-90064/19.

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