Quantum Depinning of a Magnetic Skyrmion

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We investigate the quantum depinning of a weakly driven skyrmion out of an impurity potential in a mesoscopic magnetic insulator. For small barrier height, the Magnus force dynamics dominates over the inertial one, and the problem is reduced to a massless charged particle in a strong magnetic field. The universal form of the WKB exponent, the rate of tunneling, and the crossover temperature between thermal and quantum tunneling is provided, independently of the detailed form of the pinning potential. The results are discussed in terms of macroscopic parameters of the insulator Cu2OSeO3 and various skyrmion radii. We demonstrate that small enough magnetic skyrmions, with a radius of ∼10 lattice sites, consisting of some thousands of spins, can behave as quantum objects at low temperatures in the mK regime.

Magnetic systems have been theoretically predicted [1–9] and experimentally verified [10–16] to be good candidates for the observation of macroscopic quantum tunneling events and quantum to classical phase transitions [17]. In such systems, a large number of elementary magnetic moments display quantum behavior, as they may coherently tunnel from a metastable configuration to a more stable magnetic state. A particle-like configuration of the classical magnetization field, e.g. a domain wall or a magnetic vortex, supports a collective mode of position that tunnels out of the local minimum through a potential barrier into the classically forbidden region.

Among the various magnetic solitons, skyrmions are in the focus of current research because they appear as attractive candidates for future spintronic devices [18, 19]. Skyrmions are spatially localized two-dimensional (2D) topological magnetic textures, local whirls of the spin configuration in a magnetic material, which can be either metallic [20], a multiferroic insulator [21], or ultrathin metal film on heavy-element substrates [22]. Typically they are classical objects with a size of the order of 50 nm and a dynamics that is governed by the Landau-Lifshitz-Gilbert (LLG) equation [23, 24], although small-size skyrmions of 1 nm (a few lattice constants) have been recently observed [25], inspiring studies on the quantum properties of skyrmions [26–35].

In this Letter, we study the quantum depinning of a magnetic skyrmion out of a potential created by an atomic defect in a magnetic insulator Cu2OSeO3, and find that skyrmions, under certain specified circumstances, can exhibit macroscopic quantum behavior.

To study the macroscopic tunneling of a magnetic texture m from a defect pinning center, we employ the imaginary time formulation for path integrals with a Euclidean action written in the form,

$$ S_E = N_A \int_0^\beta d\tau \left( iSd^2 \int d\Phi(1 - \Pi) + \mathcal{H}/J_0 \right) , $$

where the magnetization density at position r is represented in polar coordinates, \( \mathbf{m}(r) = [\sin \Theta \cos \Phi, \sin \Theta \sin \Phi, \cos \Theta] \). S is the total spin, and \( N_A \) the number of layers. The magnetic Hamiltonian \( \mathcal{H} = J_0 \int d\mathbf{r} F(\mathbf{m}) \) reads,

$$ F(\mathbf{m}) = \sum_{i=x,y} \left( \frac{\partial m_i}{\partial r_i} \right)^2 + \mathbf{m} \cdot \nabla \times \mathbf{m} - \kappa m_z^2 - hm_z . $$

The exchange coupling \( J_0 \) sets the energy scale, while \( \kappa = KJ_0/D_0^2 \) and \( h = g\mu_B H J_0/D_0^2 \) are dimensionless and denote the strength of anisotropy and uniform magnetic field, respectively. \( D_0 \) denotes the Dzyaloshinskii-Moryia (DM) coupling in units of energy, K the anisotropy coupling in units of energy and H the external magnetic field in units of T. Imaginary time \( \tau \) and space \( \mathbf{r} \) variables are given in reduced units. Physical units are restored as \( r' = rad \) and \( \tau' = \tau/J_0 \), where \( d = J_0/D_0 \), and \( \alpha \) is the lattice constant. Also, we set \( h = 1 \). Minimization of the functional Eq. (2) results in a stable skyrmionic solution,
described by $\Phi = \phi + \pi/2$ and the approximate function $\Theta(\rho) = 2\tan^{-1}[(\rho/\rho_0) - \rho/\rho_0]$, with $(\rho, \phi)$ the polar coordinate system, $\rho_0 = \sqrt{2/(2k + h)}$, while $\lambda$, which we obtain numerically from the Euler-Lagrange equation of the stationary skyrmion, is the skyrmion radius [29]. Magnetic skyrmions are characterized by a finite topological charge $Q$, 

$$Q = \frac{1}{4\pi} \int d\mathbf{r} \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m}),$$ (3)

which denotes the mapping from the 2D magnetic system in real space into the 3D spin space.

The presence of a crystal defect at $\mathbf{r} = 0$ alters the exchange and DM couplings as $J/J_0 = 1 - J'e^{-\rho/\lambda_d}$ and $D/D_0 = 1 - D'e^{-\rho/\lambda_d}$, respectively [26], with $J', D'$ being the strength, and $\lambda_d$ the size of the defect. On the classical level, the interactions of skyrmions with atomic defects crucially affect their mobility [38–41]. $J'$ and $D'$ are perturbations with respect to $J_0$ and $D_0$ and thus, the distortion of the skyrmion profile is weak.

The resulting pinning potential $V_p$ as a function of the distance $r_0$ between the center of the defect and the center of the skyrmion can be approximated by the function

$$V_p(r_0) = -\frac{1}{J_0} \frac{V_0(\lambda_d)}{r_0^3 + a(\lambda_d)^2},$$ (4)

where $V_0(\lambda_d) = c_0 N_A V_0^j(\lambda_d)$, $c_0 = (1 - d/d')J'/J_0$ and $d' = J'/D'$ [42]. We take $V_0(\lambda_d) > 0$, in order for the skyrmion to experience an attractive potential. The behavior of $a(\lambda_d)$ and $V_j(\lambda_d)$ is summarized in Fig. 1.

To describe a skyrmion escaping from the potential well by tunneling through the energy barrier, we need to employ a description which isolates the center of mass of the skyrmion, from all other degrees of freedom. This is achieved by promoting the skyrmion center-of-mass to a dynamical variable $\mathbf{R}(\tau)$ and integrate out the magnon degrees of freedom, within a quantum field theory method which makes use of the Faddeev-Popov technique for collective coordinates [29]. Then one finds that the Euclidean action of Eq. (1) takes the form,

$$S_E = \int_0^\beta d\tau [i\dot{\mathbf{Q}}(\dot{\mathbf{X}}) - \dot{\mathbf{Y}}\cdot \dot{\mathbf{X}}] + \frac{1}{2} M \dot{\mathbf{R}}^2 + U(X, \mathbf{Y}),$$ (5)

with $\mathbf{R} = (X, \mathbf{Y})$, and $\dot{\mathbf{Q}} = 2\pi N_A SQ$. Here $U(X, \mathbf{Y}) = V_p(\sqrt{X^2 + \mathbf{Y}^2} - F_{\text{ext}} X)$, where $V_p$ being the pinning potential of Eq. (4), and $F_{\text{ext}}$ is a linear force acting on the skyrmion collective coordinate equal to $F_{\text{ext}} = h_{\text{ext}} \partial / \partial X \int d\mathbf{r} \, \mathbf{m} \cdot (\mathbf{r} - \mathbf{R})$, as the result of an applied out-of-plane magnetic field gradient. We introduce $h_{\text{ext}} = g_{\mu B} N_A H_{\text{ext}}/\sqrt{2}/D_0$ and $B_{\text{ext}}$ is measured in T. Here, $M$ denotes the effective mass which arises from the skyrmion-magnon bath coupling in the presence of a pinning potential [29], while the first term in (5) is a Magnus force acting on the skyrmion proportional to the topological number [43]. A non-negligible mass term gives rise to oscillatory modes in the real-time dynamics of the skyrmion [44, 45], which performs a cyclotron rotation of frequency $\propto \dot{Q}/M$.

For small values of the magnetic field $F_{\text{ext}}$, the skyrmion is trapped at its minimum position. As the field grows, the barrier (4) is lowered and the skyrmion eventually gets depinned at the coercive force $F_c$. However, even for $F_{\text{ext}} < F_c$, the position of the skyrmion at the pinning center becomes metastable and can tunnel out of the local minimum, as long as $0 < \epsilon \equiv 1 - F_{\text{ext}}/F_c \ll 1$ [2, 8]. The coercive force is given by $F_c = V''(R_i)$, where $R_i$ is the inflection point close to a local minimum, calculated by requiring $V''(R_i) = 0$, $V'_y(R_i) > 0$ and $V'_x(R_i) < 0$. For the potential of Eq. (4), we find $R_i = a/\sqrt{3}$.

Within second order perturbation theory with respect to the amplitude of the potential $U$, the effective mass in dimensionless units is $M \propto N_A S U_{\text{eff}}^2$, where $U_0$ is the height of the barrier [29]. Motivated by the fact that the optimum condition for the observability of tunneling events is when the potential barrier is small and narrow, it is convenient to separate the fast cyclotron rotation of frequency, $\dot{Q}/M \gg 1$, from the slow motion of the guiding center [46]. This is achieved by considering the real time Lagrangian $L$, obtained upon replacing imaginary time $\tau$ with real time $t = -i\tau$ in the imaginary time Lagrangian $\mathcal{L}$, with $\mathcal{S}_E = \int_0^\beta d\tau L$ and $\mathcal{S}_E$ given in Eq. (5). The Hamiltonian $H$ that corresponds to $L$ is given by

$$H = \frac{1}{2M} [(P_x + \dot{Q}Y)^2 + (P_y - \dot{Q}X)^2] + U(X, Y),$$ (6)

with $P_x = M \dot{X} - \dot{Q}Y$ and $P_y = M \dot{Y} + \dot{Q}X$. Following Refs. [37, 46], instead of the original coordinates $X, Y$ and conjugated momenta $P_x, P_y$, we define new operators
where \( \omega_e \) is defined as \( (\Psi_0|H|\Psi_0) = (2\pi/Q)^{-1/4}e^{-i\Pi Y^2/4} \), describes the cyclotron motion of the skyrmion at the zero Landau level with ground state energy equal to \( \omega_e/2 \). By averaging over the fast rotation, \( \langle \Psi_0|H|\Psi_0 \rangle \), and taking the zero Landau level as a reference point for energy, \( H = \langle \Psi_0|H|\Psi_0 \rangle - \omega_e/2 \), we obtain \( H = U(X,Y) \), with \( [X,Y] = -i/2Q \). This approximation holds as long as \( l < \alpha \), where \( l \) is the magnetic length \( l = aQ^{-1/2} \), while in this limit \( [X,Y] \to 0 \).

With these preparations, the problem is reduced to a problem equivalent to that of the motion of a massless charged particle in a strong magnetic field, with an imaginary-time Euclidean action of the form

\[
S_E = \int_0^\beta d\tau \left[i\dot{Q}(XY - \dot{Y}X) + U(X,Y)\right],
\]

and a saddle point solution which is in general complex. We introduce a normalized potential of the form \( U(X,Y) = V_p(X,Y) - F_{\text{ext}}X \), with \( V_p(X,Y) = V_0(\alpha \lambda_d)(1/\alpha \lambda_d)^2 - 1/(X^2 + Y^2 + \alpha(\lambda_d)^2) \), and \( V_0, \alpha \) as in (4).

We further consider the potential in shifted coordinates \( U(X,Y) \to U(X + X_{\text{min}}, Y) - U(X_{\text{min}},0) \), where \( X_{\text{min}} \) is defined as \( \partial U(X)/\partial X|_{X=X_{\text{min}}} = 0 \). In Fig. 2 we plot the potential energy \( U \) for \( F_{\text{ext}} = 0 \) and \( 0 < F_{\text{ext}} < F_c \). The analysis is significantly simplified if we expand around the inflection point \( X_1 \), defined as \( \partial^2 U(X,Y)/\partial X^2|_{X=X_1,Y=0} = 0 \). The resulting expression is

\[
\dot{U}(X,Y) \approx V_{\text{max}}^x X^2 X_{\text{min}}^2 (1 - X/X_{\text{min}}^2) + V_{\text{max}}^y Y^2 Y_{\text{min}}^2 (1 - Y/Y_{\text{min}}^2).
\]

We also introduce \( X_p = c_1/c_2, \ X_d = c_4/c_3, V_{\text{max}}^x = c_3^2/c_2^2, V_{\text{max}}^y = c_3^2/c_2^2 \), where \( c_1 = (1/2)V_{(0,2)} + (eF_c V_1^2)/(-2V_{(3,0)})^{1/2}, \ c_2 = -(1/2)V_{(1,2)}, \ c_3 = -(1/6)V_{(3,0)}, \ c_4 = -(1/2)eF_c V_{(3,0)}^{1/2} \). Further, derivatives are denoted as \( V_{(i,j)} = V_{(i,j)}^x(X_1,0) \), where \( V_{(3,0)}^x(X_1,0), V_{(1,2)}^y(X_1,0) < 0 \). For the particular choice of the pinning potential, the parameters simplify as \( c_1 = c(1/\sqrt{3} + 2\sqrt{2}/3), c_2 = c/a, c_3 = c_2/2, c_4 = c\sqrt{2}, X_d = 2a\sqrt{2} \), and \( V_{\text{max}}^x = 4a^2 c^3/2 \) with \( c = 9\sqrt{2}\text{V}_0/\hbar 16a^4 \).

To study the imaginary time trajectories and obtain a real problem from the action (8), we perform the additional transformation \( Y \to iY \), provided that the condition \( S[U(X,iY)] = 0 \) holds. The instanton trajectories \( (X_I,Y_I) \) are the classical solutions of the equations of motion in Euclidean time, \( 2\dot{Q}Y_I + \partial U/\partial X = 0 \) and \( -2\dot{Q}X_I + \partial U/\partial Y = 0 \). By integrating the first (second) equation with respect to \( X_I \) (\( Y_I \)), we arrive at the condition \( U(X_I,Y_I) = 0 \), which we also took into account that the energy along the trajectory has to vanish, since it is conserved by the dynamics [37]. Then one finds that \( Y_I = \mathcal{J}(X_I) \), which takes the following simplified form for the expanded potential (9),

\[
\mathcal{J}(X_I) = \sqrt{X_I^2(c_4 - c_3 X_I)}/c_1 - c_2 X_I.
\]

The \( X_I \) variable ranges from zero up to the turning point \( X_d \), calculated by the requirement \( \mathcal{J}(X_d) = 0 \). We find the relative simple expression \( X_d = 2a\sqrt{c} \) for the expanded potential \( \dot{U} \) (9). The instanton trajectories, defined by the equipotential lines \( U(X_I,Y_I) = 0 \), are illustrated in Fig.3-(a), for a magnetic skyrmion of radius \( \lambda = 1.86 \), pinned by a pinning center of radius \( \lambda_d = 2.5\lambda \), and for \( \epsilon = 0.096 \), with \( F_c = 0.13 \). Fig.3-(b) compares the values of \( X_d \), derived by both the potential \( U \) and \( \dot{U} \) as a function of \( \epsilon \), and implies that \( \dot{U} \) is a good approximation of \( U \), as long as \( \epsilon \lesssim 0.05 \).

The quantum tunneling of the particle into the classically forbidden region is achieved by the nontrivial instanton solution \( (X_I,Y_I) \), in which the skyrmion starts at \( X = 0 \) at \( \tau = -\infty \), reaches \( X = X_d \) at \( \tau = 0 \), and then returns to \( X = 0 \) at \( \tau = \infty \). This motion occurs with a characteristic tunnel frequency

\[
\omega_{\tau} = \sqrt{V_0^x V_{\text{max}}^x}/|Q| X_d X_y = 9V_0(3\epsilon)^{1/4}(\alpha d)^2/16\hbar|Q|a^4.
\]
where for the rest of the paper units are restored. The probability of tunneling is governed by the temperature-independent WKB exponent, $e^{-S_0}$, with the tunneling action $S_0 = S[X_I, Y_I]$ given by

$$S_0 = 2\hbar \tilde{Q} \int_0^{X_d} dX_I [J(X_I) - J'(X_I)X_I] \simeq \frac{16V_x^x J_0}{15\omega_{s}/\epsilon},$$

(12)

where for the last approximate equality we used the expanded potential (9), and is further simplified as $S_0 \simeq 5.6\hbar \tilde{Q} a^2 e^{5/4}/(a\epsilon)^2$. We note that the tunneling action depends on the width of the pinning potential $a$, but is independent of its height $V_0$ [38], and the coercive force $F_c$, in contrast to the tunneling exponent of domain walls in ferromagnets [8]. The dependence of $S_0$ on $\epsilon$ is depicted in Fig. 3-(c), for a skyrmion with radius $\lambda = 1.86\alpha d$, while the inset summarizes the dependence of $S_0$ from the skyrmion size $\lambda$. The decay rate $\Gamma$ at zero temperature is calculated as [47],

$$\Gamma \simeq \frac{\omega_{s}}{2\pi} e^{-S_0/\hbar} \simeq \frac{9V_0(3\epsilon)^{1/4}(a\epsilon)^2}{32\pi \hbar |\tilde{Q}| a^4} e^{-S_0/\hbar}.$$

(13)

To make quantum effects observable, two conditions must be satisfied. First, the inverse escape rate $\Gamma^{-1}$ must not exceed a few hours [8], and second, the thermal activation events over the barrier do not dominate over the quantum tunneling-induced transitions. The decay rate becomes determined solely by quantum effects below a characteristic temperature $T_c = \hbar U_0/k_B S_0 = 5\omega_{s}/36k_B$ [48], where $U_0 = 4V_x^x /27$ is the height of the potential barrier.

Table I summarizes typical values of the tunneling exponent $S_0$, the oscillation frequency $\omega_{s}$, the inverse tunneling rate $\Gamma^{-1}$, and the crossover temperature $T_c$ for various skyrmion radii and values of $\epsilon$ for the chiral magnetic insulator Cu$_2$OSeO$_3$, which is known to support stable skyrmions [50]. For sufficiently small skyrmions with a radius of a few lattice sites, coherent tunneling out of a pinning potential is expected to take place involving some thousands of spins, within a few seconds, in the mK temperature regime.

For the applicability of the approach used, we estimate the range of parameters that satisfy the assumptions made. The action (1) assumes a quasi-2D behavior, established when the transverse degrees of freedom are frozen out due to the finite number of layers $N_A$. The transverse magnon excitations of a bulk sample, with energy $\omega(k_z) = A(k_z + l_0^{-1})^2 + g\mu_B H + 2K - A l_0^{-2}$ [49], where $l_0 = 2a\lambda$, acquire an additional finite size gap for finite sample widths $w = N_A\alpha$, which arises since $k_{z_{min}} = \pi/w$. We introduce $A = 2J_0g\mu_B/\alpha M_s$, with $M_s$ the saturation magnetization. Thus, for a given temperature $T$, all transverse excitations freeze out below a critical width given by $w(T) = \pi g/(k_B T - g\mu_B H - 2K)$, with $g = A l_0^{-1} + (A\mu_B T + A l_0^{-2} + A[g\mu_B H + 2K])^{1/2}$. To make an estimate of the range of this quasi-2D behavior for Cu$_2$OSeO$_3$, we use the parameters summarized in Table I, and a choice of $H = 345.6$ mT. A minimization of the free energy $F$, Eq. (2), yields an energetically stable skyrmion with radius $\lambda = 8.31\alpha$, and for a freezing temperature of $T = 2.2$ K, we find $N_{A_{max}}^H = w(T)/\alpha = 114$ layers.

In the special case of a separable pinning potential $U(X, Y) = U_1(X) + U_2(Y)$, the problem can be reduced to a one-dimensional massive particle by integrating out the $Y$ variable from Eq. (8) [52]. Nevertheless, the tunneling properties Eqs. (11)-(13) remain unaffected. Mesoscopic systems are unavoidably coupled to their environment, a source of dissipation that could suppress the
probability of the tunneling process. Ohmic couplings are expected to have the most detrimental effect on the tunneling rate of mesoscopic systems [53], while super-Ohmic interactions have a very small contribution [4]. In insulators and for low temperatures below the energy gap of the magnon modes of the underlying ferromagnet, which is about 10 K for the parameters summarized in Table I, the super-Ohmic skyrmion-magnon interaction is the main source of dissipation [30]. In the presence of a pinning potential, the lowest Landau level (LLL) splits into quantized levels with spacing proportional to the potential height [26]. Quantum tunneling processes that involve the excitation of the skyrmion from the LLL to the next higher one, mediated by thermal excitations, are expected to provide a better estimation of $T_c$ [55]. Such processes require a detailed understanding of the rates of the thermal excitations, and we thus leave it for future work. In view of the increasing interest on new insulating materials that enable the stabilization of skyrmions [54], we anticipate that our results will initiate experimental studies towards the possibility of observing a quantum mechanical behavior at a mesoscopic scale for a topological particle.

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**TABLE I.** Tunneling quantities for the chiral magnetic insulator Cu$_2$OSeO$_3$, with $J_0 = 3.34$ meV, $D = 0.79$ meV, $K = 6.8 \times 10^{-2}$ meV, $M_s = 111.348$ kA m$^{-1}$, $\alpha = 8.911$ Å, $S = M_s \alpha^2 / g \mu_B$ [54], and $Q = 1$, $\lambda = \lambda$, $J^* / J_0 = 0.3$, $D' = 0$, and $N_A = 30$.

| $\lambda$ (nm) | $N$ | $\epsilon$ | $X_d$ | $\omega_\Gamma$ | $S_0 / h$ | $\Gamma^{-1}$ | $T_c$ (mK) |
|----------------|-----|------------|-------|-------------|--------------|-------------|-------------|
| 4.3            | $8.8 \times 10^2$ | 5 x 10$^{-2}$ | 3.02 nm | 2.90 $\times 10^{-3}$ s$^{-1}$ | 288.76 s$^{-1}$ | 5.51 $\times 10^{13}$ s$^{-1}$ | 30.76 mK |
| 2 x 10$^{-3}$  | 0.60 nm | 1.30 $\times 10^{-3}$ s$^{-1}$ | 5.16 s$^{-1}$ | 8.48 $\times 10^{-8}$ s$^{-1}$ | 13.76 mK |
| 5 x 10$^{-4}$  | 0.30 nm | 9.17 $\times 10^{-3}$ s$^{-1}$ | 0.91 s$^{-1}$ | 1.71 $\times 10^{-9}$ s$^{-1}$ | 9.73 mK |
| 7.4            | $2.61 \times 10^3$ | 5 x 10$^{-2}$ | 5.3 nm | 3.54 $\times 10^{-3}$ s$^{-1}$ | 886.59 s$^{-1}$ | 1.96 $\times 10^{17}$ s$^{-1}$ | 3.76 mK |
| 2 x 10$^{-3}$  | 1.06 nm | 1.58 $\times 10^{-3}$ s$^{-1}$ | 15.86 s$^{-1}$ | 0.03 s$^{-1}$ | 1.68 mK |
| 5 x 10$^{-4}$  | 0.5 nm | 1.12 $\times 10^{-3}$ s$^{-1}$ | 2.80 s$^{-1}$ | 9.25 $\times 10^{-8}$ s$^{-1}$ | 1.19 mK |
| 10.3           | $5.05 \times 10^3$ | 5 x 10$^{-2}$ | 7.40 nm | 1.04 $\times 10^{-3}$ s$^{-1}$ | 1731.46 s$^{-1}$ | 5.56 $\times 10^{14}$ s$^{-1}$ | 1.10 mK |
| 2 x 10$^{-3}$  | 1.48 nm | 4.66 $\times 10^{-3}$ s$^{-1}$ | 30.97 s$^{-1}$ | 38.15 $\times 10^{-8}$ s$^{-1}$ | 0.49 mK |
| 5 x 10$^{-4}$  | 0.74 nm | 3.29 $\times 10^{-3}$ s$^{-1}$ | 5.47 s$^{-1}$ | 4.55 $\times 10^{-5}$ s$^{-1}$ | 0.35 mK |
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