On X-Ray Waveguiding in Nanochannels: 
Channeling Formalism

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Abstract

The question on X-ray extreme focusing (smallest reachable spot size) brings us to the idea for using the wave features of X-ray propagation in media. As known, wave features are revealed at propagation in ultra-narrow collimators as well as at glancing reflection from smooth flat and/or strongly curved surfaces. All these phenomena can be described within the general formalism of X-ray channeling.

Key words: X-ray waveguide, channeling, capillary optics, nanostructure
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1 Introduction

The advent of the nanotechnology era has given rise to unprecedented opportunities, challenges and enormous potential for new products and wealth creation. Manipulation of matter on the atomic scale will require new tools for lithography and metrology and radiation at X-ray wavelengths will be fundamental to the development of this area. Progress within the fields of nanotechnology and X-ray propagation will be mutually beneficial. For instance, some types of processes based on the self-organization of materials that have recently attracted considerable interest because of the possibility of preparing fine patterns of nanometer dimensions over larger areas, can be used for the fabrication of X-ray waveguides [12]. Among them the formation of highly-ordered aligned carbon nanotubes and ordered arrays of uniform-sized porous

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in anodic alumina is of great interest. X-ray propagation in $n$-channels\(^1\) is important due to potential applications in X-ray optics. A special feature of these structures is a long, hollow, inner cavity, which could act as a channel for selective radiation penetration, similar to channeling of charged particles in crystals (see [3] and Refs. in); $n$-channels can be considered as capillaries (the base of capillary/polycapillary optical elements [4][5][6]). Research in X-ray propagation in capillary structures shows that diminishing the capillary internal radius from microns to nanometers results in a change of the character of radiation propagation, from the surface channeling in $\mu$-capillaries down to bulk channeling in $n$-capillaries. Numerical simulations [7][8] have shown that carbon nanotubes will act as soft X-ray waveguides and support modes of propagation when coated with various materials [9]. Other researchers have already demonstrated experimentally the coherent propagation of X-rays in a planar waveguide with a tunable air gap [10]. The angular dependence of the intensity of C K$\alpha$ radiation vs the aligned carbon nanotubes’ orientation suggests the possibility of X-ray channeling as well as radiation diffraction on nanotube multiwalls [11].

It is important to clarify the origin of some peculiarities just from the beginning in order to prevent misunderstanding (that time by time takes place in publications) in the processes observed or to be expected. As shown below, one exists a well specified difference in the origin of trapped (bound states) radiation propagation for $\mu$- and $n$-size channel structures\(^2\).

The main criterion for observing the wave features of radiation propagation in media of the length $L$ is very simple - the transverse space where radiation is limited at propagation, does not matter by either a profiled surface or a specific collimation system, should approach by size the transverse radiation wavelength: $X_{\perp} \simeq \lambda_{\perp}$ ($\lambda_{\perp} \simeq \lambda/\vartheta_c$, where $\vartheta_c = \omega_p/\omega$ with $\omega$ as photon energy and $\omega_p = \sqrt{4\pi n_e e^2/m_0}$ - the plasmon energy, $n_e$ is the electron density of the cladding, $e$ and $m_0$ are the electron charge and mass). For X-ray frequencies, at reflection from a flat surface (optimally, we deal with the total external reflection) as well as at X-ray propagation in a planar waveguide, the transverse dimension of a beam can be estimated as $X_{\perp} \simeq L\vartheta_c$; and thus we get very simple and important expression that allows the limits for revealing the wave features at reflection to be evaluated

$$L\vartheta_c^2 \simeq \lambda$$  \hspace{1cm} (1)

It makes evident why even at radiation propagation in $\mu$-channels it is possible to observe the wave features. This relation will be in details examined below.

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\(^1\) Below in the text we use $n$- for nano- and $\mu$- for micro-.

\(^2\) The latter is mainly related to propagation of X-rays in capillary structures (mono- and polycapillaries of various origins and sizes, and optical elements on its base).
from the view point of the both wave equation solution and simple physical base of the phenomenon.

The modes of radiation propagation in a waveguide are revealed at interference between the incident and reflected waves forming a standing wave pattern [1]. However, it becomes constructive just for specific angles. This phenomenon, valid for reflection from a flat surface, takes place just in the vicinity of the surface. Similar phenomenon can be observed at radiation reflection from the curved surface (so called "whispering modes") [12]. Strong radiation redistribution takes also place behind capillary systems (which is actually a simple example of the curved surface system); some structural features in the distribution are due to the spatial geometry of the system (typically, hexagon type in the transverse cross section). However, some fine features could not be interpreted by the ray optics, and require solution of the wave equation of radiation propagation.

History of interference phenomena observed for X-rays, from various sources, propagating in capillary structures counts more than 10 years. First theoretical note regarding such a possibility for capillary optical systems, where the X-ray interference behind capillaries, was published as an internal note [13]. After that, during 1993-1994 this phenomenon was observed in a set of experiments with polycapillary optical elements (channels sizes of $\sim 100 \, \mu m$) in the beams of synchrotron radiation ($\lambda \approx 10 \, \AA$, S-60 LPI). Then, the first joint paper appeared [14] with the wave interpretation of the features recorded. Phenomenology of the phenomenon given in [15] has shown that the fine features of X-ray propagation in $\mu$-size channels, which have been observed behind capillary structures, can be explained in view of the radiation interference due to the various channel's curvatures. Later the trapped radiation propagation in the very vicinity of a surface was carefully studied in a number of works [16,17,18,19,20,21,22,23] where the wave theory of X radiation propagation along a curved surface was developed (for complete citation, please, see Refs. in [3]).

Regardless of the research on radiation transmission by capillary systems, which represent circular guide systems, nowadays, considerable progress in studying of X-ray waveguiding in the planar structures (as specially fabricated waveguides consisting of the guiding and cladding layers, or ultra narrow slits and collimators with air gap, etc.) is achieved [10,11,12,13,14,25,26,27,28,29,30,31]. Additional to the planar case, in paper [27] the authors have proposed a wave theory of X-ray propagation through a circular guide. It is notable for this paper that the wave theory of X radiation propagation in capillaries (circular shaped guides!) was developed in many previous papers as a general theory of radiation propagation for both $\mu$-size and $n$-size capillaries. Detailed analysis presented in many articles is based on the transverse wavelength approach as a main criterion (see the review [3]). The authors of the paper [27], thanking D.
Bilderback for the information on possibility to observe an interference picture behind capillary/polycapillary systems, have forgotten to make any reference to previously published results; the adequate citation were done just for the case of planar waveguides. Since 1994 new coherent features of X radiation propagation in capillary structures have been discussed at many meetings: SPIE annual conferences, specially organized meetings on capillary optics and its applications (International Capillary Optics Meeting - Antwerp 2001; International Conference on Capillary X-Ray and Neutron Optics - Zvenigorod 2001 and 2003, International Conference on Charged and Neutral Particles Channeling Phenomena - ”Channeling 2004 ” and ”Channeling 2006 ” Frascati), practically all conferences on X-ray physics.

In this work after the introduction to physics of coherent phenomena of radiation scattering in $\mu$– and $n$-structures and X-ray channeling in hollow channels of various origins, the results obtained within international collaborations will be presented.

2 General theory of X-ray channeling

As shown recently, the propagation of X-ray photons through the narrow guides exhibits a rather complex character [14,26,27]. Not all the features shown experimentally can be explained within the geometrical (ray) optics approximation [18,21,34]. On the contrary, application of the wave optics methods allows the processes of radiation transmission by the guides to be described in details.

The passage of X radiation through the guides is mainly defined by its interaction with the inner guide walls. In the ideal case, when the boundary between hollow channels and walls represents a smooth edge, the beam is split in two components: the mirror-reflected and refracted ones. The latter appears sharply suppressed in the case of total external reflection. The characteristics of scattering inside the structures of ultra-small holes of various shapes can be evaluated from solution of the Helmholtz equation. In the first order approximation, propagation of X radiation through specially designed guides, the cladding material of which is characterized by the refractive index $n = 1 − \delta(r) + i\beta(r)$ defining by the guide geometry, is described by a wave

\[ n = 1 − \delta(r) + i\beta(r) \]

Moreover, in the paper there is no citation to the Bilderback's paper [32] published as a review on the discussions at the ICOM’2001 meeting (International Capillary Optics Meeting, Antwerp, 17-21 June 2001), where the wave features of X radiation propagation in capillary systems, theoretically evaluated and experimentally proved, were presented and discussed. These results were published in the same volume of X-Ray Spectrometry issued as a special volume of the ICOM’2001 proceedings [23,33].
The propagation equation

\[
(\Delta + k^2 n^2(r)) E(r) = 0, \quad n \equiv \begin{cases} 
1, & \text{hollow core} \\
1 - \delta_0 + i \beta_0, & \text{cladding}
\end{cases}
\]

(2)

for the electromagnetic field amplitude \( E \), where \( k \equiv (k_\parallel, k_\perp) \) is the wave vector of radiation, \( k = 2\pi/\lambda \), \( \Delta \equiv \partial^2/\partial r^2_\perp + \partial^2/\partial z^2 \) is the Laplacian, \( r \equiv (r_\perp, z) \).

Separating a transverse part of the radiation field as \( E(r) = E(r_\perp)e^{ik_\parallel z} \), the Helmholtz equation can be reduced to

\[
\nabla^2_\perp E (r_\perp) = \left(2k^2\delta - k^2_\perp\right) E (r_\perp),
\]

(3)

where the right side term in brackets is a potential of interaction \( V_{eff} \). Due to the fact that the transverse wave vector \( k_\perp \approx k\vartheta \) under the grazing wave incidence (\( \vartheta \ll 1 \)), an "effective interaction potential" is estimated by the expression

\[
V_{eff}(r_\perp) = k^2 \left(2\delta(r_\perp) - \vartheta^2\right) = \begin{cases} 
-k^2 \vartheta^2, & \text{guiding channel} \\
k^2 (2\delta_0 - \vartheta^2), & \text{cladding}
\end{cases}
\]

(4)

From the latter the phenomenon of total external reflection at \( V_{eff} = 0 \) follows, when \( \vartheta \equiv \vartheta_c \simeq \sqrt{2\delta_0} \) is the Fresnel’s angle. One can see that Eq.(3) with the effective potential (4) corresponds to the Schrödinger equation for a massive particle motion, \( \nabla^2_\perp \equiv p^2/(2m) \), in a specified potential well \( V_{eff} \).

That’s why it becomes much more convenient to use below the terminology of "channeling" [35], where a channel is formed by the effective potential of radiation interaction in a guide (a quantum well)\(^4\). As well known from quantum mechanics, any well is able to support at least one quantum bound state (channeling state); the number of the states can be estimated from the expression for the potential (4). The equation (3) for radiation propagation in a media with the potential (4) can be solved for the case of \( \mu \)-channels as well as for the \( n \)-guides. It is important to underline the main difference of the radiation propagation in \( \mu \)- and \( n \)-channels that is defined by the ratio between the effective channel size and the transverse wavelength of radiation. Moreover, in \( \mu \)-channels the main parameter of the guide is its shape (collimation profile or surface curvature) whereas for \( n \)-channels it is the transverse channel size.

However, there is another very interesting case to be examined, namely, when the radiation is propagating along the curved surface (so called multiple reflection regime). It is of strong importance in case of the circular \( \mu \)-guides.

\(^4\) It is well known that channeling of charged particles may take place when the small divergent beams are transversing crystals near the main crystallographic planes or axes at the angles less than some critical one \( \varphi_L \) known as the Lindhard angle of channeling (\( \varphi_L \simeq \sqrt{V_{eff}/\varepsilon} \), \( \varepsilon \) is the particle energy).
When the reflecting surface is not more flat but curved, the effective potential reveals an additional contribution. To describe new expected features due to the new surface profile, we have to take into account the fact that reflection of electromagnetic wave occurs on extended area. The minimal size of it at sliding angles is defined by $(\Delta d)_{\parallel\text{min}} \sim (4\pi c/\omega_p) \vartheta^{-1} (c$ is the light velocity) that is much greater than the atomic distances. Due to this fact, it is possible to consider that interaction takes place in a macro field characterized by a macroscopic dielectric permittivity $\varepsilon_0 \equiv n_0^2$. The last determines reflection and absorption characteristics of interaction. Thus, the reflection coefficient is defined by polarization and absorption in the layer of electric field penetration $(\Delta d)_{\perp} \sim 2\pi c/\omega_p$. General analysis shows, that at sliding angles less than the Fresnel’s angle, TER is observed. Within the geometrical optics approximation, the coherent scattering occurs in a specular (mirror) direction, while the incoherence is taken into account by the radiation absorption in the reflecting layer. In other words, a new term in the potential of interaction can be considered as corresponding to the additional "potential energy". Due to the reflecting surface curvature a photon "receives" an angular momentum $k r_{\text{curv}} \varphi$, where $r_{\text{curv}}$ is a curvature radius of the photon trajectory. The latter is supplied by the "centrifugal potential energy" $-2k^2 r_{\perp} / (r_{\text{curv}})$ [20].

$$V_{\text{eff}}(r_{\perp}) = k^2 \left(2\delta(r_{\perp}) - \vartheta^2 - 2 \frac{r_{\perp}}{r_{\text{curv}}} \right).$$ (5)

Because of variation in the system spatial parameters, the interaction potential has been changed from the step potential with the potential barrier of $2k^2 \delta_0$ to the well potential, with the depth and width defined by the channel characteristics.

Above we have described a general rule to be fulfilled in order to reveal the radiation wave behaviours. Obviously, it should be valid for any optical scheme regardless both its size and shape (we can consider just two important parameters of any device: guide channel and its shape, which is actually defined by the interacting surface curvature). Let us now to estimate the limit in the curvature radius $r_{\text{curv}}$, at which the wave behaviours are displayed under propagation of radiation in channels [13] (in the case of capillaries or system of capillaries it is defined by its channel diameter and bending). Let consider a photon with the wave vector $k$ propagating along a curved surface with the curvature radius $r_{\text{curv}}$ (at the angles less than the critical one $\vartheta_c$, it happens as surface channeling, the bound states will be evaluated below in the text). At small glancing angles, $\vartheta < \vartheta_c$, the change of the longitudinal wave vector, $k_{\parallel}$, under reflection from a surface is negligibly small; but one mainly changes the transverse wave vector $k_{\perp}, k_{\perp} \approx k \vartheta_c (\vartheta_c \ll 1)$. Correspondingly, it follows that the transverse wavelength will much exceed the longitudinal wavelength that provides the interference effects to be observable even for very short wavelengths. Indeed, $\lambda_{\perp} \approx \lambda/\vartheta_c >> \lambda$, and quantum mechanical principles postulate that, in order to display the wave properties of a channeling photo-
ton, it is necessary that typical transverse sizes of an "effective space" $\delta i$, in which waves have been propagating, be commensurable with the transverse wavelength, i.e. $\delta_i(\vartheta) \equiv \lambda_\perp$. This condition may be rewritten in the following form:

$$r_{\text{curve}}\vartheta^3 \simeq \lambda$$

(6)

So, from this simple estimate we can conclude that the relation (6) provides a specific dependence for surface bound state propagation of X-rays - *surface channeling* - along the curved surfaces (for instance, in capillary systems). Taking into account that in approximation of the glancing angles $\vartheta \ll 1$ one defines the longitudinal size of the curved reflecting surface as $L \simeq r_{\text{curve}}\vartheta$, the expression (6) is in rather good agreement with a general condition (1) to be satisfied at any optical device for observing wave character of radiation propagation.

In the following we briefly discuss a solution of the wave equation in the case of an ideal reflecting surface (i.e. without roughness), when the reflected beam is basically determined by the coherently scattered part of radiation (for details see [19]). Evaluating the wave equation with the boundary conditions of a channel shows that X-radiation may be distributed over the bound state modes defined by the channel potential. It is important to note here that the channel potential acts as an effective reflecting barrier, and then, the effective transmission of X-radiation by the hollow guides is observed. While the main portion of radiation undergoes incoherent diffuse scattering, the remaining contribution (usually small) is due to coherent scattering that represents a special phenomenon, extremely interesting to observe and clarify [3,26,36].

### 3 Surface channeling: $\mu$-channels

#### 3.1 Planar guide

As mentioned above, in $\mu$-channels the propagation features are defined by the radiation interaction with a surface. Taking into account that a skin layer, where the reflection is formed, is negligible thin in respect with the guide wall thickness, the amplitude of the guiding wave can be presented as

$$E_m(r) = u_m(r_\perp, z) \, e^{-ik_{\parallel m}z},$$

(7)

where $r_\perp \equiv xe_x$ for a planar, 1D waveguide and $r_\perp \equiv xe_x + ye_y$ for a 2D waveguide, $|e_x|^2 = |e_y|^2 = 1$, $k_{\parallel m}$ is the propagation constant. Neglecting absorption, in case of a planar waveguide with the guide channel diameter $d$,
the solution of the wave equation \((3)\) gives for \(\vartheta < \vartheta_c (\ll 1)\) \[10\]

\[
    u_m(x) \propto \begin{cases} 
        \sin(k\vartheta_m x), & |x| \leq \frac{d}{2} \\
        0, & |x| > \frac{d}{2} 
    \end{cases},
\]

where \(k|_m = k \cos \vartheta_m \simeq k \left(1 - \frac{\vartheta_m^2}{2}\right)\) with \(\vartheta_m = \frac{\pi (m+1)}{kd}\). Hence we can define also a maximum number of the waveguide modes for the fixed wave number \(k\) in respect with geometrical parameters of the waveguide as \(m_{\text{max}} = \frac{2d}{\lambda} \vartheta_c - 1\). From the latter the condition for a single mode propagation can be reduced as \(\lambda_\perp \equiv d\). Hence, as seen from the solution of Helmholtz equation for a planar guide, we have obtained the same criterion for observing wave features of radiation propagation as obtained by the phenomenology above presented (relation \([1]\), where \(L \vartheta_c \simeq d\), and \(\lambda_\perp \simeq \lambda/\vartheta_c\)).

Situation with a bent planar guide can be considered as a limit case of the circular guide, analysis of which is given below via example of a capillary system.

### 3.2 Circular guide

Since in the case of capillary systems, the principal waveguide is a hollow cylindrical tube (a circular guide), the interaction potential, in which the waves are propagating, is determined by Eq.\([5]\) with the radiation polarizability parameter \(\delta_0 \simeq \vartheta_c^2/2\) (for simplicity the absorption is considered to be negligible \(\beta_0 \ll 1\)). Solving the wave equation for the capillaries with \(\mu\)-size holes we are mainly interested in the surface propagation, which, in fact, defines a wave guiding character inside the channel \((r_\perp \simeq r_1, \rho \ll r_1)\) \([17][23]\)

\[
    E_n(r) \simeq \sum_m C_m u_m(\rho) e^{i(k_\parallel z + n\varphi)},
\]

\[
    u_m(\rho) \propto \begin{cases} 
        Ai_m(\rho), & \rho > 0 \text{ - guiding channel} \\
        \alpha Ai'_m(0) e^{\alpha \rho}, & \rho < 0 \quad (\alpha > 0) \text{ - cladding} 
    \end{cases},
\]

where \(Ai_m(x)\) is the Airy function, and \(\alpha\) is the arbitrary unit characterizing the capillary substance. Evidently, these expressions are valid only for the lower-order modes and in the vicinity of a channel surface. The expression \([9]\) characterizes the waves that propagate close to the waveguide wall, or in other words, the equation describes the grazing modal structure of the electromagnetic field inside a capillary (surface X-ray channeling states). The solution shows also that the wave functions are damped both inside the channel wall and moving from the wall towards the center. It should be underlined here that the bound modal propagation takes place without the wave front
distortion that is important for explaining the interference behind capillary systems (in multiple reflection optics, [3] and Refs. in).

The analysis of these expressions allows us also to conclude that almost all radiation power is concentrated in the hollow region and, as a consequence, a small attenuation along the waveguide walls is observed. However, evaluating the solution of the wave equation inside the cladding, we can estimate the tunneling effect (penetration of radiation deep into the cladding), which becomes significant for the guide acceptance and propagation in both thin wall and long length guides.

As for the supported modes of the electromagnetic field, estimating a characteristic radial size of the main grazing mode \( m = 0 \) results in

\[
2\pi^2 \pi_0^2 \simeq \lambda^2 r_1,
\]

and we can conclude that the typical radial size \( \pi_0 \) may overcome the wavelength \( \lambda \), whereas the curvature radius \( r_1 \) in the trajectory plane exceeds the inner channel radius, \( r_0 \); \( \pi_0 \gg \lambda \) (for example, \( \pi_0 \gtrsim 0.1 \, \mu \text{m} \) for a capillary channel with the radius \( r_0 = 10 \, \mu \text{m} \)).

4 Bulk channeling: \( n \)-channels

Above we have considered the transmission of X-ray beams by the guides of \( \mu \)-size channels. As shown, in that case we deal with the surface channeling of radiation due to the fact that the channel sizes are much larger than the radiation wavelength and even than the transverse wavelength. However, the situation sharply changes in the case when the sizes of channels become comparable with the radiation transverse wavelength. In practice it means, that the angle of diffraction for the given wave, determined as \( \vartheta_d = \lambda/d \) (\( d \) is the guide transverse size), becomes comparable with a critical angle of total external reflection; and the transverse wavelength of a photon approaches the guiding gap: \( \lambda_\perp/d \sim 1 \). In this case, at the specific conditions, channeling of photons (note, not surface channeling!) in channels of capillary systems may take place, i.e. actually, in other words, we deal with X-ray waveguiding similar to light waveguiding in fiber optics. Situation is similar to channeling of charged particles in crystals, that’s why it may be considered as bulk channeling (the quantum well is formed by the bulk properties of a guiding channel) [37].

Of course, in this limit the radiation penetration into the cladding becomes to be important (significant) for resolving propagation problem that speaks on the necessity of taking into account solution of the wave equation inside the cladding.


4.1 Planar guide

Solving Eq. (3) for a planar 1D waveguide, we obtain the amplitudes of channeling states inside the guiding core \( |x| \leq d \) as

\[
E_m(x) \propto \begin{cases} 
\cos(k\vartheta_m x), & \text{even mode} \\
\sin(k\vartheta_m x), & \text{odd mode}
\end{cases},
\]

and inside the cladding \( |x| > d \) -

\[
|E_m(x)| \propto e^{-k\sqrt{\vartheta^2_m \varrho^2 - \vartheta^2_m}|x|},
\]

and the dispersion equations for the channeling quantum states -

\[
\tan(k\vartheta_m d) = \begin{cases} 
\left(\frac{\vartheta^2_m}{\varrho^2_n} - 1\right)^{1/2}, & \text{even mode} \\
-\left(\frac{\vartheta^2_m}{\varrho^2_n} - 1\right)^{-1/2}, & \text{odd mode}
\end{cases}
\]

The dispersion equations for a planar guide are defining the modal structure of radiation propagation, i.e. the quantum states of channeling. For instance, the phase \( \Delta\varphi \equiv k\vartheta_m d \) determines character of radiation transmission: if the number of quantum states of channeling is large, \( d/\lambda_{\perp} \gg 1 \), then we obtain \( \Delta\varphi \gg 2\pi \) that speaks on possibility of using the ray optics approach for describing the radiation propagation.

Evidently, there is a special interest to consider a waveguide, which is formed by the wall of periodic \( n \)-channels (multilayer planar guides or narrow multislit system), with a central guiding air gap \( d_0 \) and the distance \( d \) between the layers composing a waveguide wall \[38\]. For the sake of simplicity, interaction potential in such a waveguide system may be presented as follows:

\[
V(x) = \sum_n V_n(x) = k_{\perp}^2 \left[ 1 + \Delta \sum_n \delta \left( |x| - \frac{d_0}{2} - nd \right) \right],
\]

where \( \Delta \equiv \varrho_0 d \) is the spatially averaged polarizability of the wall cladding.

Taking into account the boundary conditions and because of the potential symmetry, one may conclude that for the central channel \( |x| \leq d_0/2 \), solution of the propagation equation in the transverse plane will be defined as

\[
E_0(x) \propto \begin{cases} 
\cos(k_{\perp} x), & \text{even mode} \\
\sin(k_{\perp} x), & \text{odd mode}
\end{cases}
\]

Solution for the 1st layer \( d_0/2 \leq |x| \leq d_0/2+d \) is superposition of the oppositely directed waves \( E(r) = b \ e^{ik_{\perp}r} + c \ e^{-ik_{\perp}r} \). Then, we impose on the solutions
the requirements that the wave function $E$ and its transverse derivative $E'$ be continuous at the wall-channel boundary taking into account the Bloch theorem $E(x + d) = e^{i\kappa d}E(x)$ for the periodical potential function $V(x) = V(x + d)$. From these expressions we obtain the dispersion relations for even and odd states

\[
\begin{cases}
\tan \frac{k_\perp d_0}{2} = \left\{-\frac{k^2\Delta}{k_\perp} + \frac{\cos(k_\perp d) - e^{i\kappa d}}{\sin(k_\perp d)}\right\} \\
\cot \frac{k_\perp d_0}{2} = \left\{\frac{k^2\Delta}{k_\perp} - \frac{\cos(k_\perp d) - e^{i\kappa d}}{\sin(k_\perp d)}\right\}
\end{cases}
\] (16)

that allow the eigenvalue problem to be solved. Finally the wave functions of the supported modes for the narrow channel $\{k_\perp d_0, k_\perp d\} \ll 1$ can be presented by the following

\[
E_n(x) \simeq \begin{cases}
\cos(k_\perp x) e^{ikz}, & |x| \leq \frac{d_0}{2} \\
\cos(k_\perp \bar{x}) \frac{e^{ik_x}}{\sin(k_\perp d)} e^{i\kappa nd + kz}, & n\text{-th layer}
\end{cases}
\] (17)

where $|\bar{x}| \equiv |x| - d_0/2 - nd$, and we see that the Eqs. (16) may be solved only for the even modes. However, it is more important to underline that the even mode exists for any ratio between the channel size and the layer distance. The spatial distribution of the mode has a maximum at the channel center, and due to the leak through the potential barrier of wall layers (the tunneling) we observe the propagation of radiation in the cladding. The radiation intensity for the successive layer decreases following an exponential law and is characterized by a local maximum far from the layer wall. Owing strong tunneling of radiation the acceptance of such a waveguide system can much exceed the critical angle $\theta_c$ [39].

### 4.2 Circular guide

Well known solution of the wave equation for a circular guide with the channel diameter $d$ (that corresponds to X radiation propagation in a $n$-capillary) can be reduced if to write the wave function of electromagnetic field as $E(r_\perp) = u(\rho)e^{im\varphi}$, where $\rho$ is the radial coordinate and $\varphi$ is the azimuthal angle. In this case the wave equation for the radial part of the field amplitude becomes

\[
\rho^2 \frac{d^2 u}{d\rho^2} + \rho \frac{du}{d\rho} + \left(k^2(\vartheta^2 - \vartheta_c^2)\rho^2 - m^2\right)u = 0 ,
\] (18)
from which the solutions of (18) in both a hollow part of the guide and the
guide wall cladding -

\[ u(\rho) \propto \begin{cases} 
J_m(k\vartheta_m\rho), & \rho \leq d \\
K_m\left(k\rho \sqrt{\vartheta^2_c - \vartheta^2_m}\right), & \rho > d
\end{cases} \]

(19)

and the dispersion equations -

\[ \frac{(\partial_{\rho}J_m/J_m)_{\rho=d}}{(\partial_{\rho}K_m/K_m)_{\rho=d}} = \left(\frac{\vartheta^2_c}{\vartheta^2_m} - 1\right)^{1/2} \]

(20)
can be obtained.

Here \( J_m(y) \) and \( K_m(y) \) are the 1st kind and the modified 2nd kind Bessel functions, respectively, giving us the bound states for X-ray channeling in a hollow core of the waveguide. The constants of the proportionality can be obtained using the matching and normalization conditions. It is more important to note the asymptotic behaviours of the Bessel functions specified. Namely, in the guide core we can use \( J_m(\rho) \rightarrow \rho^{-1/2} \cos \alpha \rho \) that proves the modal character of propagation (different bound channeling states) with the mode amplitudes decrease from the maximum at the guide center \( \rho = 0 \) to the minimum at the guide wall \( \rho = d \). In the same time we can estimate suppression of the radiation penetration in the guide cladding for \( \rho > d \) as \( K_m(\rho) \rightarrow \rho^{-1/2}e^{-\rho} \). However, being suppressed the radiation penetrates rather deep in the cladding resulting finally in the strong tunneling. This phenomenon, first seems to be useless, provides, in turn, essential increase in the waveguide acceptance.

Speaking on circular guide systems it would be important also to consider natural nanotube systems (carbon or carbon based nanotubes, alumina porous membranes) \([40,41]\). Due to the nanotube morphology, i.e. presence of inner cavity, that speaks on possibility of the efficient transmittance of X-ray, thermal neutron and charged particle beams, a nanotube can be considered as a capillary of very small inner diameter and wall thickness \([7,8]\). However, there is a strong difference between typical glass capillary and nanotube from the point of view of radiation propagation through these structures. First of all, the dielectric function as a function of the distance from the center of glass microcapillary channel varies by a step-law from zero for the inner hollow cavity to the constant value defined by the substance, for the channel wall. On the contrary, in case of nanotube channel, we have continuous change of the dielectric parameter value. However, the main factor, which defines the character of radiation propagation inside nanotubes, remains the same as for \( n \)-channels; it is the ratio \( \lambda_{\perp}/d \).

There is another very important behaviour to pay attention too. Because of the small wall thickness of nanotube channels (less than \( \lambda_{\perp} \lesssim 100 \, \text{Å} \)) we have
to note that part of the radiation, channeling inside a nanotube structure, will undergo “tunneling” through the potential wall barrier. A simple analysis of the radiation propagation in systems both for the case of macroscopic channel and for the case of totally isotropic spatial structure, shows the presence of the main channeling mode (the main bound state) for any structure, whereas the high modes may be suppressed for specific channel sizes. Hence, nanotubes present a special interest as waveguides, which allow the supported modes to be governed [9]. Moreover, there is a special interest in studying the dispersion of radiation in a nanosystem with a multilayered wall. As follows from the analysis of the general equation of radiation propagation considered above, at any correlation between the channel size and the interlayer distance at least one mode (bound state) should be formed in such a structure. In that case the diffraction of waves reflected from various layers of the channel wall should be observed, hence affecting the radiation distribution at the exit of system.

Evidently, the efficiency of these structures for applications have to be analyzed, despite the importance of the nanotube X-ray waveguide phenomenon from the fundamental point of view. The problems associated with X-ray and neutron channeling in capillary nanotubes (single- and multi-wall systems) present a special interest. The first observation of X-ray channeling in a forest of multiwall carbon nanotubes and successive analysis of the results based on X-ray channeling theory [11,42] has shown that this process is accompanying by X-ray diffraction on the multilayer wall. The latter takes place due to the strong tunneling of radiation through the nanotube wall cladding.

5 Resume

Solution of Maxwell equations describing propagation of electromagnetic waves in media, where the index of refraction changes as a step function, results in forming a discrete set of the modes [13]. Presently, the waveguides for various types radiation are in wide use. Various kinds of them enable to shape the beams of radiation in different energy ranges that makes the waveguides to be extremely attractive for applications. For instance, nowadays it is difficult to imagine any high tech instruments without various cavities for µ- and radio waves, optical fibers, etc. Among them X-ray waveguides are mainly in the research stage of development.

Analysis of radiation propagation through the guides of various shapes, above presented, has shown that all the observed features can be described within an unified theory of X-ray channeling: *surface channeling in η-size guides and bulk channeling in n-size guides*. The main criterion defining character of radiation propagation is the ratio between the transverse wavelength of radiation and the effective size of a guide, i.e. $\lambda_\perp/d \equiv \vartheta_d/\vartheta_c$, in other words, the ratio
between the diffraction and Fresnel angles. When this ratio is rather small, i.e. when the number of bound states is large, the ray optics approximation is valid. In turn, when $\lambda_\perp \simeq d$, a few modes will be formed in a quantum well; and just a single mode - for $\lambda_\perp \gg d$. Obviously, the latter requires solution of the wave equation for describing all the features of radiation propagation in such guides.

Recently, it was shown that at the center of a guide the flux peaking of X radiation, i.e. the increase of the channeling state intensity at the center of a guide, should take place \[39\]. This feature is a proper channeling effect that can be explained only by the modal regime of radiation propagation, and may find an interesting application for the purposes of extreme focusing.

It is also important to note that all the considerations taken for X-rays should be valid for thermal neutrons.

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