Precision estimation of large space-borne parabolic antenna support structure

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Abstract
An accuracy analysis approach – mosaic equivalence approach which is based on the principle of approximate structure substitution, to a large complex spatial mechanism with three-dimensional (3D) paired bearings support joint (PBS-joint) clearance is presented to effectively estimate the support structure precision of large parabolic space-borne antenna. The analysis suggests when all the PBS-joints equipped with Metric 628/6 bearings in standard clearance, the surface precision of support structure can obtain relatively high accuracy with a 99.73% probability that root mean square (RMS) was kept in (0.3275, 0.7673) mm and peak-to-valley (PV) was kept in (0.8806, 1.8178) mm. The solution of deviation configuration under a large complex spatial mechanism using the proposed mosaic equivalence approach can be transformed into that under a mosaiced structure of its simple sub-mechanisms. As a result, the high-dimensional coupling between the deviation configuration decision variables can be effectively avoided. Besides, the constraint equations of large complex mechanisms with the PBS-joint 3D clearance can be simplified. This method lays a foundation for reducing the manufacturing cost and risk of large-diameter, high-precision satellite antennas. It has essential engineering value.

Keywords
Structure precision, joint clearance, multi-loop mechanism, space-borne antenna, precision estimation

Introduction
In order to transmit the weak signal as well as improve the bandwidth and power of the transmitted signal, the space-borne antenna with large aperture and high precision is required.1 Meanwhile, the parabolic antenna with wide swath and high gain has been applied in the field of large space-borne antenna to cope with the increasingly complex electromagnetic environment.2–4 Therefore, a large parabolic cylindrical deployable truss antenna fulfilling the above requirements is proposed, as shown in Figure 1. The feature that characterises deployable truss antenna is high storage rate, light weight and high surface precision.5 Its reflector is fixed on the support structure, which controls the tensioning of the reflector and maintains the shape. So, the reflector precision is directly determined by the precision of the support structure. There’s an overwhelming necessity for analysis and estimation of the precision of support structure.

There are many factors that affect the precision of the support structure, including joint clearance, machining precision of parts and parts deformation.

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caused by force or temperature. The joint clearance leading to random configuration deviation during the process of support structure manufacture is difficult to be eliminated by employing error compensation for the reason that the support structure contains a large number of joints, as shown in Figure 2. Therefore, only the influence of joint clearance on the support structure is studied in this paper.

At present, the main research on joint clearance is based on plane-joint clearance model, three-dimensional (3D) clearance model. However, the plane-joint clearance model cannot describe the axial and radial deviation of the joint. It needs to project the mechanism into a planar mechanism along its non-moving direction. During the deployed process of support structure, the movement trajectory of each node is a 3D curve, which is difficult to find a suitable plane to project support structure into a planar mechanism. Therefore, the first model is not suitable. The 3D clearance model can describe the spatial characteristics of joint deviation, though geometric construction of joints model of cylinder shell and shaft as general research objects as shown in Figure 3 and Figure 4, is significantly different from the paired bearings support joint (PBS-joint) where used in the support structure, as shown in Figure 5. According to the study of PBS-joint clearance by Lin et al., for the mechanism with 3D motion trajectory, the simplification of PBS-joint into plane-joint will cause serious deviation of precision estimation results.

In addition, the accuracy analysis solutions for serial, single-loop, general parallel, planar multi-loop, and simple spatial multi-loop mechanisms need to solve the variables directly from constraint equation. However, the support structure is a complex spatial mechanism with multi-loop coupled each other, which belongs to the spatial multi-loop coupled mechanism. This kind of mechanism has numerous joints and redundant constraints. The relationships between joint clearance and deviation pose are highly nonlinear and coupled. Moreover, the complexity will skyrocket with the increase of structure scale and interloop constraints. These barriers make analysis very difficult to solve the variables for such a complex system as support structure. Therefore, the above accuracy analysis methods may not be feasible.

In order to predict the precision of large space-borne parabolic antenna support structure more accurately and reduce the cost and risk of manufacturing, an accuracy analysis method of large spatial complex mechanism based on the 3D clearance model of PBS-joint will be presented. And the difficulty of solving the deviation configuration caused by the multiple variables of the 3D clearance model and the complex coupled relationship between the parameters of the mechanism will be overcome.

**Support structure configuration and its working principle**

**Configuration of support structure**

Figure 2 shows the structural configuration of the support structure. Its folded state is a rectangular structure, and expanded state is a cylindrical fitting structure. It consists of 32 basic loops as shown in Figure 6. The
basic ring consists of three basic units, that is, the basic unit on the directrix (BUD), the main basic unit on the generatrix (MBUG) and the assistant basic unit on the generatrix (ABUG). MBUG is parallel to ABUG and the upper and lower sides are opposite. MBUG and BUD share the same slider and are perpendicular to each other.

**Working principle of support structure**

The support structure effectively solves the contradiction between the small payload space of the carrier rocket and the large aperture of the antenna. In the folded state, the antenna can be stored in the narrow rocket payload space. When the satellite reaches the predetermined orbit, the antenna can be deployed into the predetermined shape driven by the support structure and get into the working state.

Figure 7 is the schematic diagram of the basic unit. The antenna reflector is installed on the downside of BUD and MBUG (fixed at A and B) and on the upside of ABUG (fixed at C and D). After the satellite reaches the predetermined orbit, the springs of all the basic units in the support structure simultaneously push the sliders, and the sliders drive each link to move until link 14 and link 45 contacts at the mechanical limit point H. At this point, the support structure stops moving.
and gets self-locked. The antenna reflector is pulled by A, B, C and D to form the desired surface shape and maintain it.

Analysis model of surface precision

3D model of joint clearance

The joints of the support structure are all PBS-joints. According to the analysis of Lin et al., the deviation configuration of PBS-joint can be described as: (1) the offset static (the shaft axis is parallel to the axis of outer-ring), (2) deflection static (the shaft axis intersects with the axis of outer-ring), as shown in Figure 8.

In the 3D space, the relationship of each reference frame that represents the deviation configuration of PBS-joint can be described as in Figure 8, reference frame $O_{ij} - x_{ij}^y y_{ij}^z$ is set at the centre point of the centre surface defined by the outer rings of the two bearings, reference frame $O_{ij} - x_{ij}^y y_{ij}^z$ is set at the centre of the shaft, shown as in Figure 5, reference frame $O^P - x^P y^P z^P$ and $O^R - x^R y^R z^R$ are adopted to denote the reference frame $O_{ij} - x_{ij}^y y_{ij}^z$ in the offset static and deflection static respectively.

According to the relationship shown in Figure 9, the 3D model of PBS-joint clearance can be obtained by equation (1).

$$
\begin{align*}
&T_P = \text{Rot}(z, \alpha) \text{Trans}(r_{OP}) \\
&T_R = \text{Rot}(z, \alpha) \text{Trans}(r_{OP}) \text{Rot}(y, \Delta \theta)
\end{align*}
$$

where, $^T_P$ and $^T_R$ are the homogeneous matrix of shaft that corresponds to the offset static and the deflection static, respectively. $\text{Rot}$ and $\text{Trans}$ are rotation and translation operators, respectively. $\alpha$ is the angle between plane $O_j - x_j^y z_j$ and plane $O_j - x_j^y z_j$. $r_{OP}$ is the vector from the $O_j$ to $O^P$. $\Delta \theta$ is the deflection angle of axis $z_j^R$ relative to axis $z_j$.

Deviation configuration analysis of basic loop

The basic loop shown in Figure 6 is the same as the calculation case studied in literature, so the calculation case studied in literature, so the calculation
method in it is still adopted for analysing the deviation configuration of the basic loop. The analysis process is as follows:

(1) Establish the reference frames on the fixed points of the reflector (points A and B of BUD and MBUG and C and D of ABUG), as shown in Figure 7. The axis x of each reference frame coincides with link AC or link BD, and its direction is pointed to the side where the reflector is fixed. The direction of axis y is show in the Figure 7 and the direction of axis z is determined by right-hand law. Then, take $O_A-x_Ay_Az_A$ of BUD and MBUG and $O_C-x_Cy_Cz_C$ of ABUG as the characteristic reference frames (CRFs) to describe the deviation configuration of the basic unit.

(2) Establish the geometric constraint equations of each loop in the basic unit step by step from inside to outside according to the inclusion relation between single loops. Solving the geometric constraint equations to obtain the full configurations of the CRFs. These configurations are the deviation configurations of the base unit.

(3) The relationship between CRFs is equivalent to a characteristic link, and using the characteristic link describe the deviation transfer characteristics. Meanwhile, the basic loop can be equivalent to a single loop mechanism described by characteristic links. Then, the geometric constraint equations of the equivalent single loop mechanism can be established. Solving the geometric constraint equations obtain the full configurations of the CRFs. These configurations are the deviation configurations of the base loop.

The analysis process starts from working out the deviation configuration of the lower-layer single loop structure in the basic loop to the step-by-step outward calculation of the deviation configuration of the higher-layer single loop structure. The deviation configuration of the higher-layer single loop should be solved on the basis of the deviation configuration of the lower-layer single loop, so this method was named as cascade approach.

**Deviation configuration analysis of support structure**

The support structure consists of 32 basic loops, each link is coupled complexly, as shown in Figure 2. Its equivalent model assembled by characteristic links is shown in Figure 10, where, the sequence Numbers on the left and upper sides are used to number the nodes $p_{ij}$ and right and bottom sides are used to number the basic loops $L_{ij}$. Due to the complexity of the structure, the direct establishment of the closed geometric equations is difficult to determine all the independent variables and gain the solution of deviation configuration. According to the cascade approach, the unconstrained deviation configurations of the support structure should be obtained first by step-by-step outward calculation of the pose of the CRF on each node from the node where the fixed reference frame is located, and then the effective deviation configuration of the support structure can be selected according to the constraint conditions shown in equation (32). In fact, the solution space formed by unconstrained deviation configuration will increase sharply with the length scale of deviation transmission path. However, as the number of closed
loops increases, the number of geometrical closure constraints will also increase, and the solution space of the effective deviated configuration will become smaller. Therefore, the cascade approach to obtain the deviation configuration needs to search from the vast unconstrained solution space. For the mechanism with large configuration needs to search from the vast unconstrained solution space. For the mechanism with large constraint, this process is like looking for a needle in a haystack, and the computational effort is often huge.

To avoid the above problems, using the approximate support structure obtained by the independent basic loop mosaics simulate the actual deviation configuration of support structure (known as mosaic equivalence approach):

1. Use the cascade approach to generate N basic loops with different deviation configurations.
2. Randomly select a basic loop from the samples as the starting loop, this paper assumes that the basic loop is $L_{31}$.
3. Determine the registration boundary according to the mosaic order, that is assembly following red arrow from left to right as shown in Figure 10.
4. Select the basic loop whose CRFs poses are the smallest difference with that on the registration boundary from the samples, that is, select the basic loop with the smallest registration error.
5. Select a CRF on the boundary and make the corresponding CRF on the selected basic loop to coincide with it to complete the mosaics, as shown in Figure 11.
6. Finally, according to the mosaic order to determine the next registration boundary, and repeat the above steps until the structure forms the support structure.

In order to quantify the registration error, the pose matrix $B_{mi}$ is constructed to describe the pose characteristic of CRF on node $m$ in basic loop $i$, as expressed in equation (2), and the cosine distance of the matrix is used to evaluate the similarity of the pose characteristic matrices. The calculation method is shown in equation (3). However, there is more than one node on the registration boundary, and when the registration error is small, the cosine distance of the pose matrices at the same registration point is very close to 1. Therefore, the mean deviation of cosine distances as shown in equation (4) is used to quantitatively evaluate the registration accuracy. The closer the value is to 0, the higher the registration accuracy is.

$$B_{mi} = \begin{bmatrix} P_{mi} \\ \varphi_{mi} \end{bmatrix}$$

where, $P_{mi}$ is the position vector of CRF; $\varphi_{mi} = [\varphi_x, \varphi_y, \varphi_z]$ is the Euler Angle vector which can be obtained from the following equations,

$$\varphi_x = \arctan \frac{R_{mi}(3,2)}{R_{mi}(3,3)};$$
$$\varphi_y = \arctan \frac{-R_{mi}(3,1)}{\sqrt{[R_{mi}(3,2)]^2 + [R_{mi}(3,3)]^2}};$$
$$\varphi_z = \arctan \frac{R_{mi}(2,1)}{R_{mi}(1,1)};$$

$R_{mi}(i,j)$ is the element in the rotation matrix of CRF whose position is at column $j$, row $i$.

$$\cos(\varphi_m) = \frac{\langle B_{mi}, B_{mj} \rangle}{\|B_{mi}\| \|B_{mj}\|}$$

where, $\langle B_{mi}, B_{mj} \rangle = \text{tr}(B_{mi}^T B_{mj})$, $\text{tr}(\cdot)$ denotes the sum of the elements on the main diagonal of the square matrix; $\|B_{mi}\| = \sqrt{\langle B_{mi}, B_{mj} \rangle}$, $\cos(\varphi_m)$ is within $[-1,1]$, the more similar the matrix, the closer it is to 1.

$$\rho = \frac{n}{\sum_{m=1}^{n} [1 - \cos(\varphi_m)]}$$

where, $n$ is the number of nodes on the registration boundary.

However, starting to mosaic from the randomly selected initial basic loop may not ensure that corresponding individuals with making the registration boundaries highly accurate were found from the generated basic loop samples. Therefore, the registration accuracy threshold $\epsilon_\rho$ is needed, and if the individual making $\rho \leq \epsilon_\rho$ cannot be found, then restart to select of initial basic loop, which means the current selected initial basic loop cannot be practicable in the actual support structure.
To evaluate the surface precision, the root mean square (RMS) and peak-to-valley (PV) were determined by geometric parameters of BUD.

The radiuses of cylinders fitted by the odd and even columns of support structure, respectively. They are constants of joint configuration and the fitted cylinder surface can directly reflect the forming accuracy. According to the general equation for a cylinder, the distance from node $p_{ij}$ to the centre axis of the fitted cylinder is,

$$U^2 + V^2 + W^2 = r_{ij}^2$$

where,

$$U = c(y_{ij} - y_0) - b(z_{ij} - z_0);$$

$$V = a(z_{ij} - z_0) - c(x_{ij} - x_0);$$

$$W = b(x_{ij} - x_0) - a(y_{ij} - y_0);$$

$s(a, b, c)$ is the direction vector of the cylinder centre axis;

$O_5(x_0, y_0, z_0)$ is the starting point of the cylinder centre axis.

Therefore, the distance from node $p_{ij}$ to the fitted cylinder surface can be obtained by,

$$d_{ij} = r_{ij} - r'$$

where, if $j$ is odd, $r' = r_1'$, if $j$ is even, $r' = r_2'$, $r_1'$ and $r_2'$ are the radiuses of cylinders fitted by the odd and even columns of support structure, respectively. They are constants determined by the geometric parameters of BUD.

In order to directly represent the surface precision, using root mean square (RMS) and peak-to-valley (PV) of the error to evaluate the surface precision, calculated as follows,

$$\text{RMS} = \sqrt{\frac{\sum_{i=1}^{9} \sum_{j=1}^{9} d_{ij}^2}{81}},$$

$$\text{PV} = \max(d_{ij}) - \min(d_{ij})$$

Table 1. The parameters of support structure.

| Parameter | Value  | BUD (In $O_A - x_A y_A z_A$) | MBUG (In $O_B - x_B y_B z_B$) | ABUG (In $O_C - x_C y_C z_C$) |
|-----------|--------|-------------------------------|------------------------------|-------------------------------|
| $G_r$ ($\mu m$) | 2~13 | (-22.0000, -35.0000) | (-22.0000, -35.0000) | (-433.0000, -35.0000) |
| $D_{pw}$ (mm) | 9.5 | (-47.8002, -141.9314) | (-57.6081, -144.4535) | (-401.1077, -143.7217) |
| $D_w$ (mm) | 2.381 | (-154.0000, -35.0000) | (-154.0000, -35.0000) | (-301.0000, -35.0000) |
| $R_i$ (mm) | 1.226 | (-90.8476, -320.3417) | (-118.6617, -332.1221) | (-353.3192, -306.6269) |
| $R_e$ (mm) | 1.238 | (-388.4660, -1468.5658) | (-500.0000, -1440.0000) | (0.0000, -1440.0000) |
| $D_j$ (mm) ($j = 1,9$) | 31 | (-386.4324, -1493.4832) | (-500.0000, -1465.0000) | (0.0000, -1465.0000) |
| $D_j$ (mm) ($j = 2$~8) | 17 | (60.3533, -1384.7622) | (0.0000, -1398.0000) | (-455.0000, -1398.0000) |

where, $D_j$ denotes the $D$ of joint $j$.

Surface precision calculation

As the support structure is a cylindrical fitting structure, the distance between each node under the deviation configuration and the fitted cylinder surface can directly reflect the forming accuracy. According to the geometric parameters of BUD.

The parameters of support structure.

Table 1.

Theory position (mm) - $O_A - x_A y_A z_A$, $O_B - x_B y_B z_B$, $O_C - x_C y_C z_C$

Figure 12. Boundary registration accuracy in 1/8 support structure.

PV = max($d_{ij}$) - min($d_{ij}$)

Results analysis

The size of the joint clearances and the static configurations of joints at any time are simulated by Monte Carlo method, and the various deviation configurations of the support structure are calculated by mosaic equivalence approach. All bearing types in the support structure are metric 628/6 with standard clearance. Geometric parameters of each revolute joint and theoretical configuration parameters of each basic unit are shown in Table 1.

In order to determine the appropriate registration accuracy threshold, taking the 1/8 support structure composed of basic loops $L_{31}$, $L_{32}$, $L_{51}$ and $L_{52}$ as an example, and calculating its surface precision by
cascade approach shown in Section 3.2 and mosaic equivalence approach shown in Section 3.3, respectively. Figure 12 is the registration accuracy distribution of three registration boundaries in the structure, where, the sample size of the basic loop is N, the simulation number is $1 \times 10^4$. The first registration boundary (the registration boundary of $L_{31}$ and $L_{51}$, there are two registration nodes) obtained the highest registration accuracy, its worst accuracy was even up to $2.22 \times 10^{-10}$. The worst accuracy is the third boundary (the registration boundary of $L_{32}$, $L_{51}$ and $L_{52}$, there are four registration nodes), its worst registration accuracy is $9.35 \times 10^{-11}$. The main reason for this difference is the registration order and the number of nodes on the registration boundary. The later the order is, the more nodes need to be registered on the boundary. So, the registration will be more difficult and will rise the probability of obtaining of lower registration accuracy.

Figure 13 shows the RMS and PV differences obtained by the mosaic equivalence approach and the cascade approach. The ordinate denotes the deviation percentage ratio that relative to the RMS or PV obtained by the cascade approach. $\Delta \mu_{\text{RMS}}$ and $\Delta \sigma_{\text{RMS}}$ are the deviations of RMS mean and standard deviation, respectively. $\Delta \mu_{\text{PV}}$ and $\Delta \sigma_{\text{PV}}$ are the deviations of PV mean and standard deviation, respectively. Each indicator shows that RMS has the smallest mean difference and the largest standard deviation difference. When the registration accuracy is not limited, $\Delta \mu_{\text{RMS}}$ is only 0.26%, but the $\Delta \sigma_{\text{RMS}}$ is as high as 10.94%. Moreover, except the $\Delta \mu_{\text{PV}}$, the remaining indexes all decrease with the decrease of registration accuracy threshold. And when the threshold is lowered to a certain value, due to the difficulty to find a large number of basic loop individuals in the sample that meet the requirements, the calculation results have lost diversity, and the indicators have rebounded. Since the changes of each index with the registration accuracy threshold are different, using $\eta$ calculated from equation (9) evaluate the comprehensive performance of various indexes under different registration accuracy threshold. The lower the value of $\eta$, the better the comprehensive performance. Figure 14 shows the change of $\eta$ with the registration accuracy threshold $\epsilon_p$. It shows that the best comprehensive performance appears at $\epsilon_p = 1.5 \times 10^{-11}$. Moreover, from Figure 15, when $\epsilon_p = 1.5 \times 10^{-11}$, the probability distributions of RMS and PV obtained by the two approaches are very similar. Therefore, selecting this value as the registration accuracy threshold analyse the surface precision of the complete support structure.

$$\eta = \sum_{j=1}^{4} \frac{\Delta_i(j)}{\sum_{j=1}^{4} \Delta_i(j)}$$

\[ (9) \]

where, $\Delta_i(j)$, $j = 1-4$ are corresponding to $\Delta \mu_{\text{RMS}}$, $\Delta \sigma_{\text{RMS}}$, $\Delta \mu_{\text{PV}}$ and $\Delta \sigma_{\text{PV}}$; $n$ is the number of sampling points.

The RMS and PV distributions of the complete support structure obtained by mosaic equivalence approach are shown in Figure 16. They all follow normal distribution, where, the mean value of RMS is 0.5456 mm, the standard deviation is 0.0727 mm, the mean value of PV is 1.3492 mm, and the standard deviation is 0.1562 mm. According the $3\sigma$ rule, there are 99.73% probability that the RMS is kept within (0.3275, 0.7673) mm, and the PV is ensured within (0.8806, 1.8178) mm. Therefore, when the metric 628/6 bearings with standard clearance are installed into all the PBS-joints, a support structure with high surface repetition precision can be obtained. It is worth noting that the above precision distribution law is determined by the relative pose uncertainty between the links, and
the floating range of this uncertainty is determined by
the joint clearance. It is difficult to further improve the
mechanism precision by adjusting the cable or improv-
ing the machining and assembly accuracy of parts
except for bearings. If the analysis results do not meet
the requirements, it is necessary to use smaller clear-
ance bearings instead.

Conclusions
A mosaic equivalence approach to analyse the
large spatial multi-loop coupled mechanism with 3D
PBS-joint clearance was proposed. And the estimated
precision of the support structure on large parabolic
space-borne antenna was obtained. The analysis results
suggested that each PBS-joint under the metric 628/6
bearings with standard clearance can ensure the sup-
port structure to keep the RMS at (0.3275, 0.7673) mm
and the PV at (0.8806, 1.8178) mm with a probability
of 99.73%. This result meets the design requirements.

By the mosaic equivalence approach, the deviation con-
figuration solution of the large complex spatial mechan-
ism can be transformed into a mosaiced structure of its
simple sub-mechanisms. Benefit from it, there is no need
to construct and solve the constraint equations of high-
dimensional coupling between deviation configuration
decision variables and the precision of the large complex
spatial mechanism can be analysed easily. It is of great
significance to realise the manufacture of satellite
antenna with large diameter and high precision.

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**Appendix**

**Notation**

| Abbreviation | Definition |
|--------------|------------|
| 3D | Three-dimensional |
| PBS-joint | Paired bearings support joint |
| RMS | Root mean square |
| PV | Peak-to-valley |
| BUD | Basic unit on the directrix |
| MBUG | Main basic unit on the generatrix |
| ABUG | Assistant basic unit on the generatrix |
| $\mathbf{T}_p$ | Homogeneous matrix of shaft that corresponding to the offset static |
| $\mathbf{T}_r$ | Homogeneous matrix of shaft that corresponding to the deflection static |
| Rot | Rotation operator |
| Trans | Translation operator |
| CRFs | Characteristic reference frames |
| $\mathbf{B}_{mi}$ | Pose matrix of CRF on node $m$ in basic loop $i$ |
| $\mathbf{P}_{mi}$ | Position vector of CRF |
| $\varphi_{mi} = [\varphi_x, \varphi_y, \varphi_z]$ | Euler Angle vector |
| $\mathbf{R}_{mi}(i,j)$ | The element in the rotation matrix of CRF whose position is at column $j$, row $i$. |
| $\Delta \mu_{RMS}$ | Deviations of RMS mean |
| $\Delta \sigma_{RMS}$ | Deviations of RMS standard deviation |
| $\Delta \mu_{PV}$ | Deviations of PV mean |
| $\Delta \sigma_{PV}$ | Deviations of PV standard deviation |
| $\eta$ | Registration quality evaluation indicators |
| $\epsilon_p$ | Registration accuracy threshold |