Linking Dynamical and Thermal Models of Ultrarelativistic Nuclear Scattering

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To analyse ultrarelativistic nuclear interactions, usually either dynamical models like the string model are employed, or a thermal treatment based on hadrons or quarks is applied. String models encounter problems due to high string densities, thermal approaches are too simplistic considering only average distributions, ignoring fluctuations. We propose a completely new approach, providing a link between the two treatments, and avoiding their main shortcomings: based on the string model, connected regions of high energy density are identified for single events, such regions referred to as quark matter droplets. Each individual droplet hadronizes instantaneously according to the available n-body phase space. Due to the huge number of possible hadron configurations, special Monte Carlo techniques have been developed to calculate this disintegration.

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Studying nuclear collisions at ultrarelativistic energies ($E_{\text{cm}}$/nucleon $\gg$ 1 GeV) is motivated mainly by the expectation that a thermalized system of quarks and gluons (quark–gluon plasma) is created [1]. There are essentially two directions for modelling such interactions: dynamical and thermal approaches. The former ones refer to string models [2–7] or related methods [8], supplemented by semihard interactions at very high energies [9–12]. Here, a well established treatment of hadron-hadron scattering, based on Pomerons and AGK rules [13], is extended to nuclear interactions. Thermal methods [14–19] amount to assuming thermalization after some initial time $\tau_0$, with evolution and hadronization being mostly based on ideal gas assumptions.

![Image of high density regions](attachment:high_density_regions.png)

FIG. 1. High density regions in the $x$-$\zeta$ plane for a typical event.

Both methods have serious theoretical drawbacks. Even for nuclei as light as sulfur the string models produce particle densities that high that the hadrons are overlapping. So the independent string model is certainly too simplistic, and also considering secondary interactions as binary collisions among hadrons can theoretically not be justified. On the other hand it is also not realistic to consider a homogeneous plasma occupying the whole available volume, what is assumed in thermodynamic models. To illustrate this, we show in fig. 1 a typical event of a string model simulation. We consider a central S+S collision at 200 GeV ($E_{\text{cm}}$/A $\approx$ 10 GeV), the transverse coordinates being $x$ and $y$, the longitudinal one (= beam direction) being $z$. It is useful to consider space-time rapidity $\zeta = \ln(t + z)/(t - z)$ rather than $z$.

In this paper we introduce a completely new approach, more realistic than the string model and more realistic than thermal approaches, providing a link between the two. Based on the string model, we first determine connected regions of high energy density ($\epsilon \geq \epsilon_c$, for a given $\epsilon_c$). These regions are referred to as quark matter (QM) droplets. For such regions, the initially produced hadrons serve only as a mean to produce the proper fluctuations in the energy density. Presently, a purely longitudinal expansion of the QM droplets is assumed. Once the energy density falls beyond $\epsilon_c$, the droplet $D$ decays instantaneously into an $n$-hadron configuration $K = \{h_1, h_2, \ldots, h_n\}$ with a probability proportional to $\Omega$, with $\Omega$ representing...
the microcanonical partition function of an $n$-hadron system. Due to the huge configuration space, sophisticated methods of statistical physics [21,22] have to be employed to solve the problem without further approximations.

Our hadronization scenario is referred to as “microcanonical hadron gas (MHG) scenario”. It is certainly not the only one and probably not the most realistic one. However, we start with this scenario for a couple of reasons: the hadron gas scenario is a benchmark, widely used in the literature (in a simplified fashion though); the MHG scenario can be solved exactly; for massless hadrons even an analytical treatment is possible, providing very useful tests for the complicated numerical procedures. After having gained experience in the techniques to solve the MHG scenario, we plan to investigate alternatives as well. So the purpose of this paper is not so much to promote this particular scenario, but rather to show how a dynamical and a statistical treatment can be combined.

The first stage of our approach is the identification of high energy density regions, based on the string model, which is already discussed elsewhere [20]. Due to the empirically found correlation, $\hat{y} = \zeta$, between the average rapidity $\bar{y}$ and space-time rapidity $\zeta$, a hypersurface $H_{\zeta}$ of constant proper time $\tau$ may be introduced, in the central region simply defined by $t^2 - z^2 = \tau^2$. Appropriate coordinates on $H_{\zeta}$ are the space-time rapidity $\zeta = 0.5 \ln(t + z)/(t - z)$ and the transverse coordinates $x$ and $y$. After having used the string model (VENUS 5.08) to get complete information of hadron trajectories in space and time, we may now, for given $\tau$, determine energy densities on $H_{\zeta}$ and thus locate high density regions on $H_{\zeta}$ (with $\varepsilon \geq \varepsilon_{c,1}$), as shown in figure 1 for a typical example.

High density regions are considered as QM droplets, presently it is assumed that they expand purely longitudinally. Whenever other clusters or hadrons cross their way, the two objects fuse to form a new, more energetic cluster. Due to the expansion, the energy density of a cluster will at some stage drop below $\varepsilon_c$, which causes an instantaneous decay.

We employ the “microcanonical hadron gas (MHG) scenario” for the hadronization: the probability of a droplet $D$ — characterized by the invariant mass $E$, the volume $V$, and the flavour $Q = (Q_1, Q_2, \ldots)$ — to decay into a hadron configuration $K = \{h_1, \ldots, h_n\}$ of hadrons $h_i$ is given as

$$\text{prob}(D \to K) = \Omega(K),$$

with $\Omega(K)$ being the microcanonical partition function of an ideal, relativistic gas of the $n$ hadrons $h_i$,

$$\Omega(K) = C_{\text{vol}} C_{\text{deg}} C_{\text{ident}} \phi,$$

with

$$C_{\text{vol}} = \frac{V^n}{(2\pi)^{3n}}, \quad C_{\text{deg}} = \prod_{i=1}^{n} g_i, \quad C_{\text{ident}} = \prod_{\alpha \in \mathcal{S}} \frac{1}{n^\alpha!}.$$

Here, $C_{\text{deg}}$ accounts for degeneracies ($q_i$ is the degeneracy of particle $i$), and $C_{\text{ident}}$ accounts for the occurrence of identical particles in $K$ ($n_\alpha$ is the number of particles of species $\alpha$). The last factor

$$\phi = \int \prod_{i=1}^{n} d^3 p_i \delta(E - \Sigma \varepsilon_i) \delta(\Sigma p_i^\mu) \delta_Q, y_{Q, \Sigma}, \quad \text{(4)}$$

is the so-called phase space integral, with $\varepsilon_i = \sqrt{\frac{1}{2} p_i^2}$ being the energy and $p_i$ the 3-momentum of particle $i$. The vector $q_i = (q_{i1}, q_{i2}, \ldots)$ represents the flavour content of hadron $i$. The expression eq. (4) is valid for the centre-of-mass frame of the droplet $D$.

We have to define a set $\mathcal{S}$ of hadron species; we include the pseudoscalar and vector mesons ($\pi, K, \eta, \eta', \rho$, $K^*, \omega, \phi$) and the lowest spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons ($N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega$) and the corresponding antibaryons. A configuration is then an arbitrary set $\{h_1, \ldots, h_n\}$ with $h_i \in \mathcal{S}$.

We are interested in droplet masses from few GeV up to $10^3$ GeV, corresponding to particle numbers $n = |K|$ between 2 and $10^3$. So we have to deal with a huge configuration space, which requires to employ Monte Carlo techniques, well known in statistical physics. The method at hand is to construct a Markov process, specified by an initial configuration $K_0$, and a transition probability matrix $p(K_{t+1} \to K_t)$, generating a sequence $K_0, K_1, K_2, \ldots$ two fundamental issues have to be paid attention at:

1. **initial transient**: starting usually off equilibrium, it takes a number of iterations, $I_{eq}$, before one reaches equilibrium;
2. **autocorrelation in equilibrium**: even in equilibrium, subsequent configurations, $K_0$ and $K_{t+i}$, are correlated for some range $I_{auto}$ of $i$.

In general, both $I_{eq}$ and $I_{auto}$ should be as small as possible.

We are going to proceed as follows: for a given droplet $D$ with mass $E$, volume $V$, and flavour $Q$, we start from some initial configuration $K_0$, and generate a sequence $K_0, K_1, \ldots, K_{eq}$, with $I_{eq}$ being sufficiently large to have reached equilibrium. If we repeat this procedure many times, getting configurations $K_{eq}^{(1)}, K_{eq}^{(2)}, \ldots$, these configurations are distributed as $\Omega(K)$. So for our problem, we have only to deal with the initial transient, not with the autocorrelation in equilibrium. We have to find a transition probability $p$ such that it leads to an equilibrium distribution $\Omega(K)$, with the initial transient $I_{eq}$ being as small as possible.

**Sufficient for the appropriate convergence to $\Omega(K)$ is the detailed balance condition,**

$$\Omega(K_a) p(K_a \to K_b) = \Omega(K_b) p(K_b \to K_a), \quad \text{(5)}$$

and ergodicity, which means that for any $K_a, K_b$ there must exist some $r$ with the probability to get in $r$ steps
from $K_a$ to $K_b$ being nonzero. Henceforth, we use the
abbreviations
\[ \Omega_a := \Omega(K_a); \quad p_{ab} := p(K_a \rightarrow K_b). \]  
(6)

Following Metropolis [21], we make the ansatz
\[ p_{ab} = w_{ab} u_{ab}, \]  
with a so-called proposal matrix $w$ and an acceptance
matrix $u$. Detailed balance now reads
\[ \frac{u_{ab}}{u_{ba}} = \frac{\Omega_b w_{ba}}{\Omega_a w_{ab}}, \]  
which is obviously fulfilled for
\[ u_{ab} = F \left( \frac{\Omega_b w_{ba}}{\Omega_a w_{ab}} \right), \]  
with some function $F$ fulfilling $F(z)/F(z^{-1}) = z$. Following Metropolis [21], we take
\[ F(z) = \min(z, 1). \]  
(10)

The power of the method is due to the fact that an arbit-
ary $w$ may be chosen, in connection with $u$ being given
by eq. (9). So our task is twofold: we have to develop an
efficient algorithm to calculate, for given $K$, the weight
$\Omega(K)$, and we have to find an appropriate proposal
matrix $w$ which leads to fast convergence (small $I_{eq}$).
The first task can be solved, a detailed description will be
published soon [23]. In the following we discuss about
constructing an appropriate matrix $w$.

Most natural, though not necessary, is to consider sym-
metric proposal matrices, $u_{ab} = u_{ba}$, which simplifies
the acceptance matrix to $u_{ab} = F(s_b/s_a)$. This is usually
referred to as Metropolis algorithm. Whereas for spin
systems, it is obvious how to define a symmetric matrix
$w$, this is not so clear in our case. We may take spin
systems as guidance. A configuration $K$ is def ined as a
set of hadrons $\{h_1, \ldots, h_n\}$ with the ordering not being
relevant, so $\{\pi^0, \pi^0, p\}$ is the same as $\{p, \pi^0, \pi^0\}$. We intro-
duce “microconfigurations” to be sequences $\{h_1, \ldots, h_n\}$
of hadrons, where the ordering does matter. So for a
given configuration $K_a = \{h_1, \ldots, h_n\}$ there exist several
microconfigurations $K_{aj} = \{h_{\pi^{-1}(1)}, \ldots, h_{\pi^{-1}(n)}\}$, with $\pi_j$
representing a permutation. The weight of a microconfig-

\[ \Omega(K_{aj}) = \frac{1}{n!} \left\{ \prod_{\alpha \in \mathcal{S}} n_{a,1} \right\} \Omega(K_a), \]  
(11)

with $n_{a,1}$ being the number of hadrons of type $\alpha$. Taking
for example $K = \{p, \pi^0, \pi^0\}$, there are three microcon-
figurations $\{p, \pi^0, \pi^0\}$, $\{\pi^0, p, \pi^0\}$ and $\{\pi^0, \pi^0, p\}$, with
weight $\Omega(K)/3$.

So far we deal with sequences $\{h_1, \ldots, h_n\}$ of arbit-

\[ v(K_a \rightarrow K_b) = \frac{2}{L(L-1)} \left\{ \prod_{i=1 \atop k \in \{j\}}^{n} \delta_{h_i^k h_j^k} \right\} v(h_i^k h_j^k \rightarrow h_i^k h_j^k), \]  
(13)

with
\[ v(h_i^k h_j^k \rightarrow h_i^k h_j^k) = \begin{cases} |\mathcal{P}(h_i^k h_j^k)|^{-1} & \text{if } h_i^k h_j^k \in \mathcal{P}(h_i^k h_j^k) \\ 0 & \text{else} \end{cases}, \]  
(14)

where $\mathcal{P}(h_i^k h_j^k)$ is the set of all pairs $(h_i h_j)$ with the same
total flavour as the pair $(h_i^k h_j^k)$. The symbol $|\mathcal{P}|$ refers
to the number of pairs of $\mathcal{P}$. The term $\{\}$ in eq. (13)
makes sure that up to one pair all hadrons in $K_a$ and
$K_b$ are the same, the term $2/L(L-1)$ is the probabil-
ity to randomly choose some pair of lattice indices $i$ and $j$. So our proposal matrix amounts to randomly
choosing a pair in $K_a$, and replacing this pair by some
pair with the same flavour, with all possible replacements
having the same weight. The proposal matrix is obvi-
ously symmetric, since $v$ is symmetric (the symmetry of $v$
is crucial!). We have now fully defined an algorithm,
which due to general theorems will converge, but how
fast, i.e., how large is $I_{eq}$? Considering particle ratios,
lke $n_{\pi}/n_{\pi^0}$, we find immediately that we have a very
slow convergence, so $I_{eq}$ is too large for the method to
be of practical importance. This is obvious, since the
method is not very democratic: flavourless particles like \( \pi^0, \rho^0 \) or also zeros are much more frequently proposed than all the rest. This shortcoming can be fixed by defining \( w \) such that two pairs are exchanged rather than one, the first pair being replaced by a completely arbitrary pair, the second one by some pair to guarantee flavour conservation. In addition it is necessary to weight the “zeros” differently than the hadrons. This improved method violates the symmetry of \( w \), however, the asymmetry \( w_{ab}/w_{ba} \) can be calculated and properly taken into account. Further details of the “asymmetric algorithm” will be published elsewhere [23].

The asymmetric method converges quite fast. As a check, we consider massless hadrons, where analytical results exist. In fig. 2 we plot the iterated total multiplicity \( N \) for a droplet with \( E = 10 \text{ GeV} \) and \( V = 10 \text{ fm}^3 \), compared to the average multiplicity \( \bar{N} \) from the analytical calculation (dashed line). The “equilibration time” \( L_{eq} \) is roughly given as

\[
L_{eq} / \bar{N} \approx 10 ,
\]

with the initial configuration being two \( \pi_0 \)'s.

A major advantage of our method is the fact that it is parameterfree. The final distribution of particles from the decay of a QM droplet is given by the phase space only. The necessary technical parameters \( L_{eq} \) and \( L \) are presently systematically studied. For all types of clusters (different \( E, V \), flavour) they have to be chosen as small as possible but large enough to not affect the results. Preliminary calculations have shown that the method is sufficiently fast (1 min per \( S+S \) event on a DEC Alpha) to be of use for the investigation of ultrarelativistic heavy ion collisions. Another systematic study aims at a comparison of our method with a canonical hadron gas, which may be useful to optimize \( L \) and the starting configuration.

Being parameterfree, and with the hadron gas scenario representing a benchmark, it will be very interesting to simulate nuclear collisions and compare with data. The scope of our method, however, is much larger. Our main intention is to introduce a completely new way to describe ultrarelativistic nuclear collisions, by linking dynamical and thermal approaches, and to show that such an approach is technically feasible.

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