Neutron skins of $^{208}$Pb and $^{48}$Ca from pionic probes

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Abstract

The neutron skin of $^{208}$Pb has received considerable attention in recent years. A variety of strongly-interacting probes depict a rather consistent picture but pionic probes have not been referred to in this context. We present here neutron-skin values from pionic atoms and from total reaction cross sections of $\pi^+$ between 0.7 and 2 GeV/c which fit well into the picture. In addition we show that a neutron skin for $^{48}$Ca can be obtained from existing data on pionic atoms and the result agrees with pion scattering experiments and with the scattering of $\alpha$ particles.

Keywords: pionic atoms of $^{48}$Ca and $^{208}$Pb; total reaction cross sections for $\pi^+$ on Pb; derived $\delta R_{np}$

1. Introduction

The topic of neutron skin of nuclei, or the difference between r.m.s. radii of neutron and proton distributions $\delta R_{np}$, received in recent years considerable attention because it correlates with several quantities of interest both for nuclear physics and astrophysics, see Tsang et al. [1] for a very recent extensive discussion and references. The neutron skin of $^{208}$Pb was the subject of several experiments which used a variety of strong-interaction probes, leading to a rather consistent set of ‘accepted’ values, see, for example, Jastrzębski et al. [2]. A later experiment with 295 MeV polarized protons [3] agreed, within errors, with earlier measurements at 800 MeV and with the above accepted values. In contrast to experiments with strongly-interacting probes which require a model for the derivation of nuclear sizes, the PREX experiment at Jlab. aimed at model-independent extraction of the neutron radius of $^{208}$Pb from parity-violating electron scattering. Unfortunately, it failed, so far, to provide meaningful results due to large statistical errors [4].
A strongly-interacting probe that has not been referred to in this context is the pion. Among the wealth of available experimental results on the interaction of pions with Pb we chose two examples, namely, (i) pionic atoms where one can use the strong interaction level shifts and widths to gain information on nuclear sizes, and (ii) total reaction cross sections of $\pi^+$ mesons on nuclei between momenta of 0.7 and 2 GeV/c. It is shown in the present work that these two examples of pionic probes lead to values for the neutron skin of $^{208}$Pb that have acceptable uncertainties and agree well with the majority of other experiments.

Very recently a new proposal to Jlab. [5] addressed the neutron skin of $^{48}$Ca for similar reasons as for $^{208}$Pb. Here too pion probes have not been discussed and therefore we present below also values of $\delta R_{np}$ obtained from analyses of pionic atoms of $^{48}$Ca and compare the derived values for the neutron skin with other studies with pions. We include also some remarks on elastic scattering of alpha particles as a possible source of information on nuclear radii, arguing that the results of diffraction scattering quoted in Ref.[5] are irrelevant to the present problem, but that, in contrast, experiments extending to large angles that were overlooked so far, do provide neutron skin values in agreement with pionic atoms.

2. Neutron skin of $^{208}$Pb

2.1. Pionic atoms

Strong interaction level shifts and widths of pionic atoms have been measured and interpreted for the past few decades, see Ref.[6] for a review and references. An optical potential that is related to nuclear densities is capable of reproducing the experimental results which, in turn, could supply information on nuclear densities. Garcia-Recio et al. [7] studied pionic atom potentials while varying also radial parameters of the neutron distributions. They presented values of $\delta R_{np}$ from their own analysis as well as from two previous models, leading to a weighted average of $\delta R_{np} = 0.18 \pm 0.05$ fm for $^{208}$Pb.

The present work is based on more recent global fits of potentials to 100 data points across the periodic table, where the enhancement of the isovector s-wave amplitude $b_1$ was taken care of by introducing the chiral-motivated density dependence due to Weise [8, 9]. For more details see Figs. 11 & 12 of Ref.[6]. Figure 1 shows values of $\chi^2$ for $^{208}$Pb obtained with potentials of the global fit, when varying only radial parameters of
Figure 1: $\chi^2$ values for four data points of pionic atoms of $^{208}\text{Pb}$ vs. $\delta R_{np}$ based on pionic atoms potentials from global fits [6]. The data points are shifts and widths of the 3$d$ and 4$f$ levels of pionic atoms of $^{208}\text{Pb}$.

the neutron distribution. The experimental results are from de Laat et al. [10]. Since the r.m.s. radius does not determine uniquely a distribution we used two extreme shapes for the neutron distribution where the excess r.m.s. radius over the corresponding value for the proton distribution is either due to an increased half-density radius (S) or increased diffuseness of the surface (H), referred to as ‘skin’ and ‘halo’, respectively, by Trzcińska et al. [11]. The figure shows results of fits to four data points when varying one parameter so that the minimal $\chi^2$ achieved of 5.5 per degree of freedom is somewhat disappointing compared to the corresponding value for the global fit to 100 data points which is only 1.9. This is obviously reflected in the uncertainties and the derived values for $\delta R_{np}$ are 0.15±0.08 fm for the ‘S’ shape for the neutron distribution and 0.14±0.10 fm for the ‘H’ shape. Both results are in full agreement with other derivations of $\delta R_{np}$ for $^{208}\text{Pb}$ from strong-interaction experiments.
2.2. Total reaction cross sections of $\pi^+$

Interactions of high energy particles with nuclei have been a rich source of information on nuclear sizes [12]. In the present work we used total reaction cross sections for $\pi^+$ mesons on nuclei at six momenta between 0.7 and 2.0 GeV/c [13]. The method was first to ‘calibrate’ the potential with total reaction cross sections $\sigma_R$ for C and Ca, measured together with Pb in the same experiment, where the neutron skin vanishes. Then we included in the analysis the cross sections for Pb and repeated the process varying the neutron skin $\delta R_{np}$ value.

Cross sections were calculated by solving the Klein-Gordon equation [6] with a ‘$t\rho$’-type optical potential constructed from the relevant forward pion-proton c.m. scattering amplitudes $f$:

$$f^\pm(0) = \frac{k_0}{4\pi} \sigma^\pm_T (i + \alpha^\pm)$$

(1)

where $k_0$ is the pion-proton c.m. momentum at the relevant energy and $\sigma^\pm_T$ are the pion-proton total cross sections. $\alpha^\pm$ are the ratios of real to imaginary parts of the forward scattering amplitudes. The scattering amplitudes near the forward direction were written as function of the momentum-transfer $q$ in the usual way,

$$f^\pm(q) = f^\pm(0)e^{-\beta^\pm q^2/2}$$

(2)

and the slope parameters $\beta^\pm$ were then used to fold-in finite-range Gaussian interaction into the potential in coordinate space [13]. We compared values of $\sigma^\pm_T$ from the 2008 SAID analysis [14] with the corresponding values of Carter et al. [15] used in Ref. [13]. Since the differences were very small and in any case one needs Fermi-averaged values of $\sigma^\pm_T$ for constructing the potentials, we used throughout the present work the Fermi-averaged $\pi^-$-p total cross sections of Allardyce et al. [13] together with their values of $\beta^\pm$ and of $\alpha^\pm$.

Note that the dependence of calculated $\pi^\pm$-nucleus reaction cross sections on the real potential is extremely small. Since ‘$t\rho$’-type optical potentials usually refer to the pion-nucleus system, the above c.m. forward amplitudes Eq. (1) were transformed into the pion-nucleus system, see e.g. Eq. (4)-(7) of Ref. [6].

A major point of the present analysis was the estimate of uncertainties. The first step was to see how well can the above parameter-free potential reproduce the $\pi^+$ reaction cross sections on C and Ca, where one
may safely assume that there are essentially no neutron skins. In order to achieve agreement with experiment we added an empirical imaginary term to the potential, proportional to the square of the density. Adjusting this single parameter (B), $\chi^2$ values for the two data points were between less than 1.0 and up to 7, depending on momentum. The values of B meant corrections of the order of $\pm 10\%$ to the potentials. Following this calibration we then included, at each momentum, the $\pi^+$ reaction cross section for Pb and repeated the process, varying radial parameters of the neutron distribution for Pb. Figure 2 shows results of simultaneous fits to C, Ca and Pb. It is reassuring to note that for each momentum the empirical value of B turned out, at the minimum of $\chi^2$, to be the same as the corresponding value found in the calibration. Moreover, values of minima of $\chi^2$ shown in the figure are close to the corresponding minima achieved in the calibration with the latter obviously affecting the uncertainties of the derived values of $\delta R_{np}$ for Pb.
Table 1: Values of $\delta R_{np}$ of Pb from $\pi^+$ reaction cross sections at different momenta.

| $p_\pi$ (GeV/c) | $\chi^2/df$ | $\delta R_{np}$ (fm) |
|-----------------|-------------|----------------------|
| 0.71            | 2.3         | 0.151±0.097          |
| 0.84            | 0.2         | 0.162±0.066          |
| 1.00            | 0.0         | 0.073±0.057          |
| 1.36            | 0.7         | 0.114±0.046          |
| 1.58            | 3.6         | 0.126±0.109          |
| 2.00            | 3.4         | 0.084±0.085          |

This is simply because the uncertainties are determined not only by the curvature of the $\chi^2$ function but are also scaled by the square root of the $\chi^2$ per degree of freedom, $\chi^2/df$, when it is larger than 1. In this way we included the contributions of the calibration errors to the final uncertainties. Table I summarizes the results, and the weighted average of these is $\delta R_{np}=0.11±0.06$ fm. Note that no correlations were found between the empirical parameter B, the $\chi^2$ values achieved and the skin values $\delta R_{np}$ obtained in the present analysis. With the excellent consistency between the calibration on C and Ca and the eventual application to Pb, the present results seem to be quite reliable.

The above results were obtained from the experimental cross sections of [13] for natural isotopic mixture of Pb. At three momenta cross sections are available also for a target of $^{208}$Pb although with increased uncertainties. Repeating the analysis for those three cross sections we obtained values of $\delta R_{np}$ that are consistent with the results of Table I but with increased uncertainties.

Finally a comment on the work of Allardyce et al. [13] is in order. The experiment reported in Ref.[13] was intended to obtain information on neutron densities in nuclei from ratios of total reaction cross sections for $\pi^+$ and $\pi^-$. The conclusions of that project of almost vanishing neutron skin in heavy nuclei had been a puzzle for a long time. An emerging explanation is that the so-called elastic scattering corrections to the data could have had small systematic errors. The corrections are the integrated elastic scattering cross sections for angles beyond the angle subtended by the transmission counters, to be subtracted from the measured beam-loss cross sections, before extrapolations to zero solid angle provide the total reaction cross section.
The corrections are usually calculated with an optical potential, and the program used during the initial phases of the experiment [16] was found later to have some approximations which were inadequate. For the smallest angles measured [13] the elastic scattering corrections for $\pi^+$ are up to an order of magnitude smaller than the corresponding corrections for $\pi^-$. It is therefore believed that the cross sections for $\pi^+$ used in the present work are reliable, which is not the case for $\pi^-$. Unfortunately the raw data were no longer available to make a re-analysis possible [17].

3. Neutron skin of $^{48}$Ca

![Figure 3: $\chi^2$ values for the shift and width of the 2p level of pionic atoms of $^{48}$Ca vs. $\delta R_{np}$ based on pionic atoms potentials from global fits [6].](image)

In the present work we analyzed the $^{48}$Ca pionic atoms data of Powers et al. [18] in a similar way to the analysis presented above for pionic atoms of $^{208}$Pb using the potentials of the latest global analysis [6]. Figure 3 shows $\chi^2$ values for the shift and width of the 2p level of pionic atoms of $^{48}$Ca vs. $\delta R_{np}$ based on pionic atoms potentials from global fits [6].
values for the strong interaction $2p$ level shift and width varying only the radial extent of the neutron distribution. As for $^{208}$Pb we used both the ‘S’ and the ‘H’ versions of the neutron density and the best-fit values for the neutron skin are $\delta R_{np}=0.13\pm0.06$ fm for the ‘S’ version and $\delta R_{np}=0.16\pm0.07$ fm for the ‘H’ version. Note that pionic atoms of $^{48}$Ca have not been included in the analysis of Ref.\[7\].

An extensive study of r.m.s. radii of Ca isotopes with pionic probes was carried out by Gibbs and Dedonder \[19\] who analyzed scattering of $\pi^+$ and $\pi^-$ across the (3,3) resonance. Our pionic atoms results are in very good agreement with their value of $\delta R_{np}=0.11\pm0.04$ fm for $^{48}$Ca. Earlier application of pions to study radii of Ca isotopes yielded similar result \[20\].

4. Discussion

Pionic probes have been shown to provide information on neutron skins in nuclei, $\delta R_{np}$, which is a topic of current interest. For $^{208}$Pb we have obtained values for $\delta R_{np}$ from the latest global analysis of pionic atoms across the periodic table and from analysis of total reaction cross section of $\pi^+$ in the 0.7 to 2 GeV/c momentum range. The results are in agreement with each other and with the majority of other results from strongly-interacting probes.

The neutron skin of $^{48}$Ca is receiving renewed attention in connection with the C-REX proposal to Jlab. \[5\] which is analogous to the PREX experiment \[4\]. The present work demonstrates that the pionic atoms method yields values for the neutron skin which agree with results from pion scattering by $^{48}$Ca across the (3,3) resonance. It is worth mentioning that radial parameters of the Ca isotopes have been studied in great detail by scattering of 104 MeV $\alpha$ particles under the necessary conditions for obtaining such information, namely, scattering to large angles well beyond the diffraction region. As an example we quote one result for $^{48}$Ca \[21\] with $\delta R_{np}=0.17\pm0.05$ fm, in very good agreement with the present work. It will therefore be possible to compile strong-interaction values for $\delta R_{np}$ also for $^{48}$Ca in anticipation of a forthcoming C-REX experiment \[5\].

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