Multicomponent stress-strength system reliability estimation for generalized exponential-poisson distribution

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Abstract. This study deals with (s-out-of-k) Multicomponent Stress (Y) and Strength (X) System Reliability Estimation. Both stress and strength assumed to have Generalized Exponential-Poisson Distribution with common and known scale parameters (θ and λ). The aim here is to estimate the unknown shape parameters (α and β) for X and Y respectively using two methods of estimation ML and Bayes analysis by one prior with five loss functions. Then estimate Reliability using the same methods and compared the results by Mean square error criteria from simulation study to find the best performance of the estimators. The results show that the best estimator for \(R_{(s,k)}\) is Bayes estimator under Quadratic loss function using Gamma prior function, followed by GD, GW, GP, MLE and GS estimators, respectively.

1. Introduction

A distribution obtained by compounding an exponential distribution with geometric distribution, with decreasing failure rate, known as exponential-geometric distribution is introduced by Adamidis and Loukas (1998) [1]. In the same fashion, Kus (2007) [9] introduced a two-parameter distribution known as exponential-Poisson (EP) distribution, which has decreasing failure rate, by compounding an exponential distribution with a Poisson distribution. The generalization of this distribution is come from Barreto-Souza and Cribari-Neto [3], with failure rate can be decreasing or increasing. The two-parameter exponential-Poisson (EP) with cumulative distribution function (cdf) given as: [2]

\[
F(x) = \frac{1 - \exp[-\lambda(1 - \exp(-\theta x))]^{\alpha}}{1 - \exp(-\lambda)} \quad x > 0; \quad \theta, \lambda > 0
\]

The random variables X & Y have Generalized Exponential-Poisson distribution with parameters (α, λ, θ) and (β, λ, θ) respectively if cdf’s define as:

\[
F(x) = \left[\frac{1 - \exp[-\lambda(1 - \exp(-\theta x))]}{1 - \exp(-\lambda)}\right]^\alpha = \frac{1 - A_x}{1 - e^{-\lambda}} \quad (1)
\]

\[
F(y) = \left[\frac{1 - \exp[-\lambda(1 - \exp(-\theta y))]}{1 - \exp(-\lambda)}\right]^\beta = \frac{1 - A_y}{1 - e^{-\lambda}} \quad (2)
\]

where \(A_x = e^{-\lambda(1 - e^{-\theta x})}\), \(A_y = e^{-\lambda(1 - e^{-\theta y})}\) for \(\alpha, \beta > 0\) the shape parameters, where X, Y are called Generalized Exponential-Poisson distribution(GEPD) random variable with scale parameters (θ, λ). The corresponding probability density functions (pdf’s) are define as:
If X is the strength of a component subjected to a stress Y, then R is a measure of system performance, the system fails if and only if the applied stress is greater than its strength, which referred as the stress-strength parameter. It arises in the context of mechanical reliability of a system. Another example of R discussed by Surles and Padgett [11] and Kotz et al. [8] involves the comparison of carbon strengths at different gauge lengths.

The rest of the paper is organized as follows. Section 2, introduce the obtained mathematical expression for the Multicomponent model reliability. In Section 3, considering two methods for estimating R, [ML and Bayes analysis] estimation methods. In Section 4, comparing the estimators of R by Monte Carlo simulations. Finally, the results conclusions are given in Section 5.

2. Multicomponent S-out of-K Stress-Strength System Reliability

The estimation of the stress-strength parameter R is very common in the statistical literature. Several authors have considered estimation of R with different assumption of distributions (Huang et al. [6], Baklizi [2], Ghitany et al. [5], Bagheri [10]).

The system when consisting k component and we need the work s of k is called multicomponent stress-strength system (s-out of-k). This system is studied by Bhattacharyya and Johnson [4]. Let the strength of the components X_i, i = 1, ..., k be the kth components strength variable which exposed to common stress random variable Y~GEP(β, θ, λ) independently, the reliability of (s-out of-k) multicomponent stress-strength system is:

\[ R_{(s,k)} = \text{Prob (at least } s \text{ of } X_1, X_2, ..., X_k \text{ exceed } Y) \]

\[ = \sum_{i=s}^{k} C_i^k \int_0^\infty \left[ 1 - \left( \frac{1-A_Y}{1-e^{-\lambda}} \right)^i \right] \beta \lambda \theta e^{-\theta Y} A_Y^{i-1} [1 - A_Y]^{\beta - 1} dy \]

(5)

Using: \((1-x)^n = \sum_{i=0}^{n} C_i^n (-1)^i x^i\), that \(1 - \left( \frac{1-A_Y}{1-e^{-\lambda}} \right)^i = \sum_{j=0}^{i} C_j^i (-1)^i \left( \frac{1-A_Y}{1-e^{-\lambda}} \right)^j\)

Then:

\[ R_{(s,k)} = \sum_{i=s}^{k} C_i^k \sum_{j=0}^{i} C_j^i (-1)^j \frac{\beta \lambda \theta A_Y^{i-1} [1 - A_Y]^{\beta - 1}}{\beta \lambda \theta^i [1-e^{-\lambda}]^{\alpha(i-1)}} \int_0^\infty e^{-\theta Y} A_Y^{(k+j-i)+\beta-1} dy \]

(6)

Where s, k, i, j are integers.
Multicomponent S-out-of-K models; R Since the concept of stress-strength in engineering has been one of the deciding factors of the failure of devices, this study can be applied to engineering situations. The following figures of reliability function $R_{(s,k)}$ in (6) are presented below for different values of shape parameters $(\alpha, \beta)$.

![Figure 1. Multicomponent Reliability against parameter $\alpha$.](image)

In figure (1) and for the strength random variable $X$, show that reliability value increasing by increasing value of strength shape parameter $\alpha$, where in figure (2) below and for the stress random variable $Y$, show that reliability value decreasing by increasing value of stress shape parameter $\beta$, knowing these cases with: $(s, k) = (2, 3)$ and $(3, 4)$. 
3. Estimation Procedures

The estimation of the unknown parameters $\alpha$, $\beta$ and the reliability is done using two estimation methods [MLE, Bayes].

3.1 Maximum Likelihood estimation

Let the two independent $X_1$…$X_n$ and $Y_1$…$Y_m$ are random sample from GEP, and then the likelihood function for equation (3) is:

$$L = a^n(\lambda \theta)^n e^{-\alpha n(1-e^{-\lambda})} e^{-\theta \sum_{i=1}^{n} x_i (\pi_{i=1}^{n} A_{X_i})} e^{-\alpha \sum_{i=1}^{n} (1-A_{X_i})^{-1}} e^{\sum_{i=1}^{n} \ln(1-A_{X_i})^{-1}}$$

Let $k = (\lambda \theta)^n e^{-\theta \sum_{i=1}^{n} x_i (\pi_{i=1}^{n} A_{X_i})} e^{\sum_{i=1}^{n} \ln(1-A_{X_i})^{-1}}$

Then:

$$L = k a^n e^{-\alpha [n \ln(1-e^{-\lambda})+\sum_{i=1}^{n} \ln(1-A_{X_i})^{-1}]}$$

(7)

If:

$$w_x = n \ln(1 - e^{-\lambda}) + \sum_{i=1}^{n} \ln(1 - A_{X_i})^{-1}$$

Then:

$$L = k a^n e^{-\alpha w_x}$$

$$\ln L = \ln k + n \ln \alpha - \alpha w_x$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} - w_x \to 0$$
So the ML’s estimator for the unknown shape parameters $\alpha$, $\beta$ will be as:

$$\hat{\alpha}_{ML} = \frac{n}{w_x}, \quad \hat{\beta}_{ML} = \frac{m}{w_y} \quad (8)$$

Substation the formulas of equation (8) in the equation (6), the ML estimator for $R_{(s,k)}$ say $\hat{R}_{MLE}$ can obtained as:

$$\hat{R}_{MLE} = \sum_{i=s}^{k} \sum_{j=0}^{l} C_i^k C_j^l (-1)^j \frac{\hat{\beta}_{MLE}}{\hat{\beta}_{MLE} + \hat{\alpha}_{MLE}(k+j-i)} \quad (9)$$

### 3.2 Bayes analysis

In this section, consider the Bayes estimation of the unknown parameters $\alpha$, $\beta$ and reliability with assumed that these parameters as random variables under gamma prior as:

$$g(\alpha) = \frac{b^\alpha}{\Gamma(\alpha)} \alpha^{\alpha-1} e^{-ab} \quad \alpha > 0; \ b > 0 \quad (10)$$

The posterior function is:

$$p(\alpha|x) = \frac{L(x|\alpha) g(\alpha)}{\int_0^\infty L(x|\alpha) g(\alpha) \ d\alpha} \quad (11)$$

where: $L(x|\alpha)$ is the likelihood for the density function, $g(\alpha)$ is the prior function for the shape parameter. Now using equations (7), (10) in (11), we get:

$$p(\alpha|x) = \frac{ka^a e^{-awx} (\Gamma(a) \alpha^{a-1} e^{-ab})}{\int_0^\infty ka^ae^{-awx} (\Gamma(a) \alpha^{a-1} e^{-ab}) \ d\alpha} \cdot p(\alpha|x) = \frac{(w_x+b)^{n+a}}{\Gamma(n+a)} e^{n+a-1} e^{-a[w_x+b]} \quad (12)$$

#### 3.2.1. Squared error loss function:

$$\hat{a}_s = E(\alpha|x) = \int_0^\infty \alpha p(\alpha|x) \ d\alpha = \frac{(w_x+b)^{n+a}}{\Gamma(n+a)} \int_0^\infty \alpha^{n+a} e^{-\alpha[w_x+b]} \ d\alpha$$

Then:

$$\hat{a}_s = \frac{n+a}{w_x+b}, \quad \hat{\beta}_s = \frac{m+a}{w_y+b} \quad (13)$$

and the reliability estimation function in equation (6) given by:

$$\hat{R}_{BS} = \sum_{i=s}^{k} \sum_{j=0}^{l} C_i^k C_j^l (-1)^j \frac{\hat{\beta}_s}{\hat{\beta}_s + \hat{\alpha}_s(k+j-i)} \quad (14)$$

#### 3.2.2 Precautionary loss function

$$\hat{a}_p = \sqrt{E(\alpha^2|x)}$$

Since $E(\alpha^2|x) = \int_0^\infty \alpha^2 p(\alpha|x) \ d\alpha = \frac{(w_x+b)^{n+a}}{\Gamma(n+a)} \int_0^\infty \alpha^{n+a+1} e^{-\alpha[w_x+b]} \ d\alpha$

$$\therefore \hat{a}_p = \frac{(n+a)(n+a+1)}{(w_x+b)^2}, \quad \hat{\beta}_p = \frac{m+a(m+a+1)}{(w_y+b)^2} \quad (15)$$

and the reliability estimation function in equation (6) given by:

$$\hat{R}_{BP} = \sum_{i=s}^{k} \sum_{j=0}^{l} C_i^k C_j^l (-1)^j \frac{\hat{\beta}_p}{\hat{\beta}_p + \hat{\alpha}_p(k+j-i)} \quad (16)$$

#### 3.2.3. De-Groot loss function

$$\hat{a}_p = \frac{E(\alpha^2|x)}{E(\alpha|x)}$$
and the reliability estimation function in equation (8) given by:

$$\hat{R}_{BD} = \sum_{i=0}^{k} C_i^k \sum_{j=0}^{l} C_j^l (-1)^j \frac{\hat{\beta}_D}{\hat{\beta}_D + \hat{\alpha}_D(k+j-i)}$$

### 3.2.4. Quadratic loss function

$$\hat{\alpha}_Q = \frac{E(\alpha^{-1}|x)}{E(\alpha^{-2}|x)}$$

$$E(\alpha^{-1}|x) = \int_0^{\infty} \alpha^{-1} p(\alpha|x) d\alpha = \frac{\Gamma(a+n)}{\Gamma(n+a)} \int_0^{\infty} \alpha^{-n-a+1} e^{-\alpha [w_x+b]} d\alpha = \frac{w_x+b}{\Gamma(n+a)}$$

$$E(\alpha^{-2}|x) = \int_0^{\infty} \alpha^{-2} p(\alpha|x) d\alpha = \frac{\Gamma(a+n)}{\Gamma(n+a)} \int_0^{\infty} \alpha^{-n-a-2} e^{-\alpha [w_x+b]} d\alpha = \frac{w_x+b}{(n+a-1)(n+a-2)}$$

$$\hat{\beta}_Q = \frac{m+a-2}{|w_y+b|}$$

### 3.2.5. Weighted loss function

$$\hat{\alpha}_W = \frac{1}{E(\alpha^{-1}|x)}$$

$$\hat{\alpha}_W = \frac{n+a-1}{|w_x+b|}, \hat{\beta}_W = \frac{m+a-1}{|w_y+b|}$$

and the reliability estimation function in equation (8) given by:

$$\hat{R}_{BW} = \sum_{i=0}^{k} C_i^k \sum_{j=0}^{l} C_j^l (-1)^j \frac{\hat{\beta}_W}{\hat{\beta}_W + \hat{\alpha}_W(k+j-i)}$$

### 4. Simulation Study

Here, we present results of some numerical three experiments, results based on Monte Carlo simulation to compare the performance of different estimators proposed in the previous sections and different sample sizes \((n,m)\) - \((15,25), (25,15), (25,50), (25,70), (70,25), (50,70), (70,50)\) and the parameters values: \((a=0.4, b=1.2)\) for Gamma prior, \([\alpha=0.2, 0.9), (\beta=0.5, 0.7), (\theta=0.5, 0.7), (\lambda=0.7, 1), (s1,k1,s2,k2)=2, 3, 3, 4\) for \(R_{s,k}\). MSE of reliability estimates over the 1000 replications are given in six tables (1 to 6).

### Table 1. The best estimate for \(R_{s,k}\) when \((\alpha, \beta, \theta, \lambda)\) = \((0.2, 0.5, 0.5, 0.7)\) and \(s=2, k=3\), \(R = 0.2424\).

| \(n, m\) | RML | RGS | RGP | RGD | RGQ | RGW | BEST |
|----------|-----|-----|-----|-----|-----|-----|-------|
| 15,25    |     |     |     |     |     |     |       |
| Mean     | 0.2738 | 0.2775 | 0.2806 | 0.2837 | 0.2633 | 0.2708 | RGQ   |
| MSE      | 0.0073 | 0.0074 | 0.0077 | 0.0080 | 0.0063 | 0.0068 |       |
Table 2. The best estimate for $R_{(s,k)}$ when $(\alpha, \beta, \theta, \lambda) = (0.2, 0.5, 0.7)$ and $s=3$, $k=4$, $R = 0.1492$.

| n,m   |   |   |   |   |   |   |   |   |
|-------|---|---|---|---|---|---|---|---|
| 15,25 | Mean | 0.1721 | 0.1754 | 0.1780 | 0.1808 | 0.1630 | 0.1695 | RGD |
| MSE   |   | 0.0047 | 0.0048 | 0.0051 | 0.0053 | 0.0040 | 0.0044 | RGD |
| 25,15 | Mean | 0.1732 | 0.1763 | 0.1736 | 0.1710 | 0.1891 | 0.1823 | RGD |
| MSE   |   | 0.0053 | 0.0053 | 0.0051 | 0.0049 | 0.0066 | 0.0059 | RGD |
| 25,50 | Mean | 0.1743 | 0.1762 | 0.1783 | 0.1804 | 0.1671 | 0.1717 | RGD |
| MSE   |   | 0.0038 | 0.0039 | 0.0041 | 0.0043 | 0.0033 | 0.0036 | RGD |
| 50,25 | Mean | 0.1615 | 0.1633 | 0.1613 | 0.1593 | 0.1722 | 0.1676 | RGD |
| MSE   |   | 0.0035 | 0.0035 | 0.0034 | 0.0033 | 0.0041 | 0.0038 | RGD |
| 50,70 | Mean | 0.1749 | 0.1767 | 0.1794 | 0.1822 | 0.1650 | 0.1709 | RGD |
| MSE   |   | 0.0035 | 0.0035 | 0.0038 | 0.0040 | 0.0028 | 0.0032 | RGD |
| 70,25 | Mean | 0.1644 | 0.1660 | 0.1633 | 0.1608 | 0.1776 | 0.1716 | RGD |
| MSE   |   | 0.0033 | 0.0033 | 0.0031 | 0.0030 | 0.0041 | 0.0037 | RGD |
| 50,70 | Mean | 0.1661 | 0.1672 | 0.1678 | 0.1684 | 0.1647 | 0.1660 | RGD |
| MSE   |   | 0.0026 | 0.0027 | 0.0027 | 0.0027 | 0.0025 | 0.0026 | RGD |
| 70,50 | Mean | 0.1656 | 0.1666 | 0.1660 | 0.1654 | 0.1692 | 0.1679 | RGD |
| MSE   |   | 0.0025 | 0.0028 | 0.0027 | 0.0026 | 0.0028 | 0.0027 | RGD |

Table 3. The best estimate for $R_{(s,k)}$ when $(\alpha, \beta, \theta, \lambda) = (0.9, 1.2, 0.7, 1)$ and $s=2$, $k=3$, $R = 0.4154$.

| n,m   |   |   |   |   |   |   |   |   |
|-------|---|---|---|---|---|---|---|---|
| 15,25 | Mean | 0.2619 | 0.2656 | 0.2626 | 0.2596 | 0.2799 | 0.2723 | RGD |
| MSE   |   | 0.0064 | 0.0064 | 0.0062 | 0.0060 | 0.0076 | 0.0069 | RGD |
| 25,15 | Mean | 0.2656 | 0.2678 | 0.2702 | 0.2725 | 0.2575 | 0.2628 | RGD |
| MSE   |   | 0.0055 | 0.0056 | 0.0058 | 0.0060 | 0.0049 | 0.0053 | RGD |
| 50,25 | Mean | 0.2604 | 0.2624 | 0.2600 | 0.2577 | 0.2727 | 0.2647 | RGD |
| MSE   |   | 0.0049 | 0.0049 | 0.0048 | 0.0046 | 0.0057 | 0.0052 | RGD |
| 25,70 | Mean | 0.2663 | 0.2684 | 0.2715 | 0.2746 | 0.2551 | 0.2619 | RGD |
| MSE   |   | 0.0041 | 0.0042 | 0.0044 | 0.0046 | 0.0035 | 0.0038 | RGD |
| 70,25 | Mean | 0.2546 | 0.2564 | 0.2534 | 0.2505 | 0.2695 | 0.2628 | RGD |
| MSE   |   | 0.0049 | 0.0049 | 0.0048 | 0.0046 | 0.0058 | 0.0053 | RGD |
| 50,70 | Mean | 0.2641 | 0.2654 | 0.2661 | 0.2668 | 0.2625 | 0.2639 | RGD |
| MSE   |   | 0.0036 | 0.0036 | 0.0037 | 0.0037 | 0.0035 | 0.0036 | RGD |
| 70,50 | Mean | 0.2621 | 0.2633 | 0.2626 | 0.2619 | 0.2662 | 0.2647 | RGD |
| MSE   |   | 0.0043 | 0.0043 | 0.0043 | 0.0042 | 0.0045 | 0.0044 | RGD |

Table 3. The best estimate for $R_{(s,k)}$ when $(\alpha, \beta, \theta, \lambda) = (0.9, 1.2, 0.7, 1)$ and $s=2$, $k=3$, $R = 0.4154$. 
Table 4. The best estimate for $R_{(\alpha, \beta, \theta, \lambda)}$ when $(\alpha, \beta, \theta, \lambda) = (0.9, 1.2, 0.7, 1)$ and $s=3$, $k=4$, $R = 0.3115$.

| n,m   | RML   | RGS   | RGP   | RGD   | RGQ   | RGW   | BEST |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 15,25 | Mean  | 0.3253 | 0.3229 | 0.3264 | 0.3299 | 0.3064 | 0.3151 |
|       | MSE   | 0.0098 | 0.0085 | 0.0086 | 0.0088 | 0.0081 | 0.0083 |
| 25,15 | Mean  | 0.3152 | 0.3268 | 0.3233 | 0.3197 | 0.3436 | 0.3347 |
|       | MSE   | 0.0081 | 0.0074 | 0.0072 | 0.0071 | 0.0083 | 0.0078 |
| 25,50 | Mean  | 0.3158 | 0.3137 | 0.3165 | 0.3192 | 0.3016 | 0.3078 |
|       | MSE   | 0.0049 | 0.0046 | 0.0046 | 0.0047 | 0.0045 | 0.0047 |
| 50,25 | Mean  | 0.3094 | 0.3176 | 0.3148 | 0.3121 | 0.3299 | 0.3235 |
|       | MSE   | 0.0048 | 0.0045 | 0.0045 | 0.0044 | 0.0049 | 0.0047 |
| 25,70 | Mean  | 0.3212 | 0.3170 | 0.3206 | 0.3243 | 0.3016 | 0.3095 |
|       | MSE   | 0.0052 | 0.0048 | 0.0048 | 0.0050 | 0.0047 | 0.0049 |
| 70,25 | Mean  | 0.3082 | 0.3176 | 0.3141 | 0.3105 | 0.3334 | 0.3253 |
|       | MSE   | 0.0045 | 0.0043 | 0.0043 | 0.0042 | 0.0048 | 0.0045 |
| 50,70 | Mean  | 0.3127 | 0.3130 | 0.3138 | 0.3146 | 0.3095 | 0.3113 |
|       | MSE   | 0.0029 | 0.0028 | 0.0028 | 0.0028 | 0.0027 | 0.0028 |
| 70,50 | Mean  | 0.3173 | 0.3204 | 0.3196 | 0.3188 | 0.3239 | 0.3221 |
|       | MSE   | 0.0026 | 0.0025 | 0.0024 | 0.0024 | 0.0026 | 0.0026 |
Table 5. The best estimate for $R_{(s,k)}$ when $(\alpha, \beta, \theta, \lambda) = 1.2, 0.8, 1.5, 0.9$ and $s=2$, $k=3$, $R = 0.6136$.

| n,m  | RML  | RGS  | RGP  | RGD  | RGQ  | RGW  | BEST |
|------|------|------|------|------|------|------|-------|
|      | Mean |      |      |      |      |      |       |
| 15,25| 0.6092 | 0.5987 | 0.6019 | 0.6051 | 0.5835 | 0.5915 | RGD   |
|      | 0.0063 | 0.0059 | 0.0058 | 0.0057 | 0.0068 | 0.0062 |       |
| 25,15| 0.6019 | 0.6017 | 0.5984 | 0.5952 | 0.6166 | 0.6087 |       |
|      | 0.0060 | 0.0054 | 0.0055 | 0.0056 | 0.0051 | 0.0052 |       |
| 25,50| 0.6099 | 0.6029 | 0.6054 | 0.6079 | 0.5918 | 0.5976 |       |
|      | 0.0036 | 0.0035 | 0.0034 | 0.0035 | 0.0040 | 0.0037 |       |
| 50,25| 0.6036 | 0.6043 | 0.6017 | 0.5992 | 0.6152 | 0.6096 |       |
|      | 0.0038 | 0.0036 | 0.0036 | 0.0037 | 0.0035 | 0.0034 |       |
| 25,70| 0.6086 | 0.6009 | 0.6041 | 0.6074 | 0.5866 | 0.5939 |       |
|      | 0.0030 | 0.0030 | 0.0029 | 0.0028 | 0.0036 | 0.0032 |       |
| 70,25| 0.5999 | 0.6020 | 0.5988 | 0.5955 | 0.6160 | 0.6089 |       |
|      | 0.0036 | 0.0033 | 0.0035 | 0.0036 | 0.0031 | 0.0032 |       |
| 50,70| 0.6013 | 0.5985 | 0.5992 | 0.6000 | 0.5953 | 0.5969 |       |
|      | 0.0020 | 0.0021 | 0.0021 | 0.0021 | 0.0023 | 0.0022 |       |
| 70,50| 0.6079 | 0.6071 | 0.6064 | 0.6057 | 0.6102 | 0.6087 |       |
|      | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.0019 | 0.0021 |       |

Table 6. The best estimate for $R_{(s,k)}$ when $(\alpha, \beta, \theta, \lambda) = 1.2, 0.8, 1.5, 0.9$ and $s=3$, $k=4$, $R = 0.5260$.

| n,m  | RML  | RGS  | RGP  | RGD  | RGQ  | RGW  | BEST |
|------|------|------|------|------|------|------|-------|
|      | Mean |      |      |      |      |      |       |
| 15,25| 0.5134 | 0.5019 | 0.5055 | 0.5091 | 0.4847 | 0.4938 | RGD   |
|      | 0.0082 | 0.0078 | 0.0076 | 0.0074 | 0.0090 | 0.0083 |       |
| 25,15| 0.5151 | 0.5146 | 0.5110 | 0.5074 | 0.5315 | 0.5226 |       |
|      | 0.0080 | 0.0072 | 0.0073 | 0.0075 | 0.0071 | 0.0070 |       |
| 25,50| 0.5212 | 0.5132 | 0.5161 | 0.5189 | 0.5007 | 0.5072 |       |
|      | 0.0046 | 0.0044 | 0.0043 | 0.0042 | 0.0049 | 0.0046 |       |
| 50,25| 0.5135 | 0.5143 | 0.5114 | 0.5086 | 0.5267 | 0.5203 |       |
|      | 0.0045 | 0.0042 | 0.0043 | 0.0044 | 0.0040 | 0.0041 |       |
| 25,70| 0.5189 | 0.5102 | 0.5139 | 0.5175 | 0.4940 | 0.5023 |       |
|      | 0.0038 | 0.0038 | 0.0037 | 0.0036 | 0.0046 | 0.0041 |       |
| 70,25| 0.5129 | 0.5153 | 0.5116 | 0.5079 | 0.5312 | 0.5230 |       |
|      | 0.0047 | 0.0043 | 0.0044 | 0.0046 | 0.0042 | 0.0043 |       |
| 50,70| 0.5157 | 0.5124 | 0.5132 | 0.5141 | 0.5089 | 0.5107 |       |
|      | 0.0030 | 0.0029 | 0.0029 | 0.0028 | 0.0031 | 0.0030 |       |
| 70,50| 0.5199 | 0.5190 | 0.5182 | 0.5173 | 0.5225 | 0.5207 |       |
|      | 0.0028 | 0.0027 | 0.0027 | 0.0027 | 0.0025 | 0.0026 |       |
5. Conclusion

From the tables (1 to 6), we have observed that:

- MSE value decreasing by increasing sample size \((n, m)\) for MLE and Bayes estimators.
- The value of \(R_{(s,k)}\) decreases as \((k)\) value decreases.
- In general, the best performance was being in Bayes method under Quadratic loss function estimator, followed by GD, GW, GP and GS estimators, respectively.

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