\( \Omega_b \) semi-leptonic weak decays

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Abstract

\( \Omega_b \rightarrow \Omega_c^{(*)} \) semi-leptonic decays are studied in details. Relevant helicity amplitudes are written down. Both unpolarized and polarized \( \Omega_b \) cases are considered. Decay angular distributions, asymmetry parameters and semileptonic decay rates are calculated, with numerical results using leading order results of the large \( N_c \) heavy quark effective theory.

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I. INTRODUCTION

Heavy baryons can be a good application ground of QCD. They reveal some important features of the heavy quark physics. Data on heavy baryons have been accumulating by experiments of LHC and Tevatron, as well as by previously LEP, LEPII and B-factories. Detailed theoretical analysis are necessary. The $\Lambda_b$ baryon has been studied considerably. For an example, the $\Lambda_b \to \Lambda_c$ semileptonic decay was analyzed thoroughly in Refs. \cite{1–4} in terms of decay rates, distributions and various asymmetry parameters.

Although established for over 35 years, QCD’s nonperturbative aspects are still not fully understood, which render us from precise calculations for the hadron physics. For heavy hadrons containing a single heavy quark, the heavy quark effective theory (HQET) \cite{5, 6} is the right QCD, which correctly factorizes the perturbatively calculable part out from hadronic matrix elements of weak currents in a simple and systematic way. The really tough task lies in calculating the nonperturbative part which is the universal Isgur-Wise functions. They can only be calculated by some nonperturbative methods of QCD, like the large $N_c$ QCD \cite{7}.

In this paper, $\Omega_b$ baryon semileptonic weak decays are studied. The $\Omega_b$ baryon was discovered by Tevatron experiments \cite{8}, via its 2-body nonleptonic decay $\Omega_b \to J/\Psi \Omega^-$. In terms of the valence quark content, it is made of $b - s - s$. Unlike B-mesons or charm hadrons, b-baryons cannot be produced at B-factories, they have been only produced at LEP, Tevatron and LHC. It would be a stable particle if the electroweak interaction were shut down. While the process $\Omega_b \to J/\Psi \Omega^-$ is the most appropriate for determining the $\Omega_b$ mass, the weak interaction properties of the $\Omega_b$ baryon cannot be precisely extracted out, because nonleptonic decays are subjected to a large nonperturbative QCD uncertainty. They are a lot cleaner in the semileptonic decays $\Omega_b \to \Omega_c^{(*)} l \nu$ which are not CKM suppressed. In the near future, more data on $\Omega_b$ will be obtained by the Tevatron and LHCb experiments. Furthermore, the planning $Z$ factory \cite{9} can also produce a large amount of $\Omega_b$ data. In the $Z$ factory, $Z$ is polarized, $\Omega_b$ coming out from $Z$ is also polarized. All these make it viable to analyze the $\Omega_b$ semileptonic decays experimentally. Theoretically semileptonic decays are simply parameterized in terms of form factors which contain all the nonperturbative QCD effects. With the help of the HQET, there are only two universal Isgur-Wise functions at the leading order of heavy quark expansion in the $\Omega_b \to \Omega_c^{(*)}$ transitions \cite{10}. These Isgur-Wise
functions can be further calculated in the large $N_c$ QCD [12, 13]. This is partly based on the observation of the light-quark spin-flavor symmetry in the large $N_c$ limit [14].

We will perform a detailed analysis considering polarization effects of the decays. Our analysis follows the way of Körner and Krämer [1] who analyzed $\Lambda_b$ semileptonic decays. The technique of helicity amplitudes is adopted which can be found in [15, 16]. For obtaining detailed information of the $\Omega_b$ decays, all kinds of observables are calculated, although some of them are not practically measurable in the current stage. Nevertheless in such a systematic way, the semileptonic decay branching ratio and spectrum are also obtained at last. In Sect. II, helicity amplitudes are written down for analyzing the $\Omega_b \rightarrow \Omega_c^{(*)}$ weak decays. Decay distributions and various asymmetry parameters are calculated in Sect. III. The decay rates are presented in Sect. IV. In Sect. V, we summarize the results.

II. FORM FACTORS AND HELICITY AMPLITUDES

A. Form factors

The hadronic matrix elements of the weak currents $V_\mu \equiv \bar{c}\gamma_\mu b$ and $A_\mu \equiv \bar{c}\gamma_\mu\gamma_5 b$ can be parametrized by fourteen form factors which are defined as below [17],

$$\langle \Omega_c(v', s')|V^\mu|\Omega_b(v)\rangle = \bar{u}(v', s')(F_1\gamma^\mu + F_2v^\mu + F_3v'^\mu)u(v, s);$$

$$\langle \Omega_c(v', s')|A^\mu|\Omega_b(v)\rangle = \bar{u}(v', s')(G_1\gamma^\mu + G_2v^\mu + G_3v'^\mu)\gamma^5 u(v, s);$$

$$\langle \Omega_c^*(v', s')|V^\mu|\Omega_b(v)\rangle = \bar{u}_\lambda(v', s')(N_1v^\lambda\gamma^\mu + N_2v^\lambda v^\mu + N_3v^\lambda v'^\mu + N_4g^{\lambda\mu})\gamma^5 u(v, s);$$

$$\langle \Omega_c^*(v', s')|A^\mu|\Omega_b(v)\rangle = \bar{u}_\lambda(v', s')(K_1v^\lambda\gamma^\mu + K_2v^\lambda v^\mu + K_3v^\lambda v'^\mu + K_4g^{\lambda\mu})u(v, s).$$ (1)
where $u_\lambda$ is the Rarita-Schwinger spinor for the $\Omega^*_c$. It is convenient to redefine some of the form factors as below:

\[
F'_2 = \frac{1}{2} \left( \frac{F_2}{M_1} + \frac{F_3}{M_2} \right), \quad F'_3 = \frac{1}{2} \left( \frac{F_2}{M_1} - \frac{F_3}{M_2} \right);
\]

\[
G'_2 = \frac{1}{2} \left( \frac{G_2}{M_1} + \frac{G_3}{M_2} \right), \quad G'_3 = \frac{1}{2} \left( \frac{G_2}{M_1} - \frac{G_3}{M_2} \right);
\]

\[
N'_2 = \frac{1}{2} \left( \frac{N_2}{M_1} + \frac{N_3}{M_2} \right), \quad N'_3 = \frac{1}{2} \left( \frac{N_2}{M_1} - \frac{N_3}{M_2} \right);
\]

\[
K'_2 = \frac{1}{2} \left( \frac{K_2}{M_1} + \frac{K_3}{M_2} \right), \quad K'_3 = \frac{1}{2} \left( \frac{K_2}{M_1} - \frac{K_3}{M_2} \right),
\]

where $M_1$ is the $\Omega_b$ mass, $M_2$ and $M'_2$ masses of $\Omega_c$ and $\Omega^*_c$ masses, respectively, while $M_1 = 6.071$ GeV, $M_2 = 2.695$ GeV, and $M'_2 = 2.770$ GeV \[18\]. For simplicity, we shall neglect lepton masses. In this case, $F'_3$, $G'_3$, $F'_3$, $N'_3$ and $K'_3$ have no contribution to the decays.

In the HQET, according to the standard tensor method \[10\], we denote the $\Omega^M_Q$ states by $\Omega^M_Q$, where $M = 1$ is for $\Omega_Q$ and $M = 2$ for $\Omega^*_Q$. Then the tensor fields describing the $\Omega^M_Q$ states are $B^M_{\mu}$,

\[
B^1_{\mu}(v, s) = \frac{1}{\sqrt{3}} (\gamma_\mu + v_\mu) \gamma^5 u(v, s), \quad B^2_{\mu}(v, s) = u_\mu(v, s).
\]

To the leading order of heavy quark expansion, the fourteen form factors are reduced into two Isgur-Wise functions \[10\],

\[
\langle \Omega^M_c | \bar{h}^{(c)} \Gamma h^{(b)} | \Omega^N_b \rangle = C B^M_{\mu} B^N_{\nu} \left[ -g^{\mu\nu} \xi_1(\omega) + v^\mu \nu^\nu \xi_2(\omega) \right],
\]

\[
C = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} = 1.1.
\]

where $\omega = v \cdot v'$, and $C$ is the QCD perturbative leading logarithm correction, which has been evaluated at the scale $\mu = m_c$. The fourteen form factors are then expressed as below.
\[ F_1 = \frac{-\omega}{3} \xi_1 + \frac{\omega^2 - 1}{3} \xi_2, \quad G_1 = \frac{-\omega}{3} \xi_1 + \frac{\omega^2 - 1}{3} \xi_2 \]
\[ F_2 = \frac{2}{3} \xi_1 + \frac{2(1 - \omega)}{3} \xi_2, \quad G_2 = \frac{2}{3} \xi_1 + \frac{-2(1 + \omega)}{3} \xi_2 \]
\[ F_3 = \frac{2}{3} \xi_1 + \frac{2(1 - \omega)}{3} \xi_2, \quad G_3 = \frac{-2}{3} \xi_1 + \frac{2(1 + \omega)}{3} \xi_2 \]
\[ N_1 = \frac{-1}{\sqrt{3}} \xi_1 + \frac{\omega - 1}{\sqrt{3}} \xi_2, \quad K_1 = \frac{-1}{\sqrt{3}} \xi_1 + \frac{\omega + 1}{\sqrt{3}} \xi_2 \]  
\[ N_2 = 0, \quad K_2 = 0 \]
\[ N_3 = 0 + \frac{2}{\sqrt{3}} \xi_2, \quad K_3 = 0 + \frac{-2}{\sqrt{3}} \xi_2 \]
\[ N_4 = \frac{-2}{\sqrt{3}} \xi_1 + 0, \quad K_4 = \frac{2}{\sqrt{3}} \xi_1 + 0, \tag{6} \]

It is at this stage that nonperturbation methods are needed. In the large \( N_c \) limit, these two Isgur-Wise functions are related to that of \( \langle \Lambda_c|\bar{h}^{(c)} \Gamma h^{(b)}|\Lambda_b \rangle \). While \( \langle \Lambda_c|\bar{h}^{(c)} \Gamma h^{(b)}|\Lambda_b \rangle = \eta \bar{u}_c \Gamma u_b \), the relations are \([11, 12]\):

\[ \eta(\omega) = \xi_1(\omega) = (\omega + 1) \xi_2(\omega). \]  
\[ \tag{7} \]

Furthermore, in the large \( N_c \) limit, \( \eta \) is predicted as \([13]\):

\[ \eta(\omega) = 0.99 \exp[-1.3(\omega - 1)]. \]  
\[ \tag{8} \]

**B. Helicity amplitudes**

Following the way of Ref. \([1]\) for \( \Lambda_b \to \Lambda_c l\bar{\nu} \) decays, we analyze \( \Omega_b \to \Omega_c^{(*)} l\bar{\nu} \) semileptonic decays. It is convenient to regard the decay as two-successive decays \( \Omega_1 \to \Omega_2 + W_{\text{off-shell}} \) and \( W_{\text{off-shell}} \to \ell + \bar{\nu} \). We denote helicity amplitudes of \( \Omega_b \to \Omega_c + \ell + \bar{\nu} \) as \( H^{V,A}_{\lambda_2 \lambda_W} \), and that of \( \Omega_b \to \Omega_c^{(*)} + \ell + \bar{\nu} \) as \( H^{V,A}_{\lambda_2 \lambda_W} \), where \( \lambda_2 \) and \( \lambda_W \) are helicities of the daughter baryon and the off-shell \( W \)-boson. These amplitudes can be expressed by our redefined form factors as:

\[ \sqrt{q^2} H^{V}_{1/2 \, 0} = \sqrt{Q_-}[(M_1 + M_2)F_1 + F'_2Q_+], \quad H^{V}_{1/2 \, 1} = -\sqrt{2Q_-}F_1; \]
\[ \sqrt{q^2} H^{A}_{1/2 \, 0} = \sqrt{Q_+}[(M_1 - M_2)G_1 - G'_2Q_-], \quad H^{A}_{1/2 \, 1} = -\sqrt{2Q_+}G_1; \]  
\[ \tag{9} \]
and

\[
\sqrt{q^2} H_{1/2}^V = \frac{2}{3} \frac{p'}{M_2} \sqrt{Q_+}(M_1 - M'_2)N_1 - N'_2Q_- \\
- \frac{2}{3} \sqrt{Q_-} \left[ \frac{Q_-}{2M_2} + (M_1 - M'_2) \right] N_4 \\
\sqrt{q^2} H_{1/2}^A = \frac{2}{3} \frac{p'}{M_2} \sqrt{Q_+}(M_1 + M'_2)K_1 + K'_2Q_+ \\
+ \frac{2}{3} \sqrt{Q_-} \left[ \frac{Q_+}{2M_2} - (M_1 + M'_2) \right] K_4
\]

(10)

\[
H_{1/2}^V = \sqrt{\frac{1}{3}} \sqrt{Q_-} \left[ N_4 - N_1 \frac{Q'_+}{M_1 M_2} \right] \\
H_{1/2}^A = \sqrt{\frac{1}{3}} \sqrt{Q_+} \left[ K_4 - K_1 \frac{Q'_-}{M_1 M_2} \right] \\
H_{3/2}^V = -N_4 \sqrt{Q_-} \equiv H_{3/2}^V \\
H_{3/2}^A = K_4 \sqrt{Q_+}
\]

where \( Q_\pm = (M_1 \pm M'_2)^2 - q^{(o)}_2 \) and \( q^{(o)}_2(W) = (q^{(o)}_0, 0, 0, -p^{(o)}) \) while \( p^{(o)} = \sqrt{Q_+^o Q_-^o}/2M_1 \) and \( q^{(o)}_0 = (M_1^2 - M'_2^2 + q^{(o)}_2^2)/2M_1 \). Other helicity amplitudes can be obtained via using the parity relations:

\[
H_{V(A)}^{\lambda_2 - \lambda_w} = \pm(\pm) H_{\lambda_2 \lambda_w}^{V(A)}.
\]

III. ANGULAR DISTRIBUTIONS AND ASYMMETRY PARAMETERS

Unpolarized and polarized \( \Omega_b \) decays will be considered, respectively. And in case of the \( \Omega_b \rightarrow \Omega_c \) transition, the cascade nonleptonic weak decay \( \Omega_c \rightarrow a + b \) (for example \( \Omega_c \rightarrow \Omega + \pi \) \[18\]) will be taken into account, where \( a \) has spin 1/2, and \( b \) is a spin zero particle. While in the case of \( \Omega_b \rightarrow \Omega^*_c \) transition, we will not further consider \( \Omega^*_c \) cascade decays which are either strong or radiative decays \[18\] and therefore will not produce the asymmetry factors.
A. Unpolarized $\Omega_b$ decay

For that $\Omega_b$ is unpolarized, it is convenient to introduce the correlation density matrix first, which is given by

$$
\rho_{\lambda \lambda' W} = H_{\lambda \lambda' W}^* \cdot \rho_{\lambda' \lambda W}.
$$

With this density matrix, using the methods of Refs. [15, 16, 19] and ignoring lepton masses, we obtain the angular distribution for the whole decay $\Omega_b \rightarrow \Omega_c(\rightarrow a + b) + W(\rightarrow \ell + \bar{\nu})$:

$$
\frac{d\Gamma}{d\omega d\cos \Theta d\chi d\cos \Theta_{\Omega}} = Br(\Omega_c \rightarrow a + b) \frac{G^2}{(2\pi)^4} |V_{cb}|^2 q^2 \sqrt{\omega^2 - M^2_{\Omega_c}} \frac{M^2}{24 M_1} \times \left( \frac{3}{8} (1 + \cos \Theta)^2 |H_{1/2 \, 1}|^2 (1 + \alpha_\Omega \cos \Theta_{\Omega}) + \frac{3}{8} (1 - \cos \Theta)^2 |H_{-1/2 \, -1}|^2 (1 - \alpha_\Omega \cos \Theta_{\Omega}) + \frac{3}{4} \sin \Theta^2 |H_{1/2 \, 0}|^2 (1 + \alpha_\Omega \cos \Theta_{\Omega}) + \frac{3}{4} \sin \Theta^2 |H_{-1/2 \, 0}|^2 (1 - \alpha_\Omega \cos \Theta_{\Omega}) - \frac{3}{2\sqrt{2}} \alpha_\Omega \cos \chi \sin \Theta \sin \Theta_{\Omega} [(1 + \cos \Theta) \text{Re}(H_{-1/2 \, 0} H_{1/2 \, 1}^*)] - \frac{3}{2\sqrt{2}} \alpha_\Omega \cos \chi \sin \Theta \sin \Theta_{\Omega} [(1 - \cos \Theta) \text{Re}(H_{1/2 \, 0} H_{-1/2 \, -1}^*)],
\right)

(13)

where the polar angle $\Theta$ is for $l$, $\Theta_{\Omega}$ for $a$, and $\chi$ is the azimuthal angle. These angles are illustrated in Fig.1 and Fig.2. $G$ is the Fermi coupling and $V_{cb}$ is the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix element. The daughter baryon $\Omega_c$ decays into $a$ and $b$ with a branching ratio $Br(\Omega_c \rightarrow a + b)$ and decay asymmetry parameter $\alpha_\Omega$. $p$ is the momentum of $\Omega_c$ in the rest reference frame of $\Omega_b$. According to the results of [20], in Eq.(13) we have assumed all the helicity amplitudes are real, since otherwise we will have to include the effects of $CP$-violation.

Various angular distribution and asymmetry parameters of $\Omega_b$ semileptonic decays can now be obtained. First, from Eq.(13), by integrating other angles, the polar angle distribu-
tion of the successive decay $\Omega_c \rightarrow a + b$ is

$$\frac{d\Gamma}{d\omega d\cos \Theta} \propto 1 + \alpha_1 \alpha_\Omega \cos \Theta_\Omega,$$

(14)

where the asymmetry parameter $\alpha_1$ is defined as

$$\alpha_1 = \frac{|H_{1/2 \ 1}|^2 - |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 - |H_{-1/2 \ 0}|^2}{|H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2},$$

(15)

and the polar angle distribution of the decay $W \rightarrow \ell + \bar{\nu}$ is

$$\frac{d\Gamma}{d\omega d\cos \Theta} \propto 1 + 2\alpha_2 \cos \Theta + \alpha_3 \cos^2 \Theta,$$

(16)

where the parameters $\alpha_2$ and $\alpha_3$ are

$$\alpha_2 = \frac{|H_{1/2 \ 1}|^2 - |H_{-1/2 \ -1}|^2}{|H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 + 2(|H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2)},$$

(17)

$$\alpha_3 = \frac{|H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 - 2(|H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2)}{|H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 + 2(|H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2)},$$

(18)

and the $\chi$ distribution is

$$\frac{d\Gamma}{d\omega d\chi} \propto 1 - \frac{3\pi^2}{32\sqrt{2}} \gamma \alpha_\Omega \cos \chi,$$

(19)

where

$$\gamma = \frac{2 \text{Re}(H_{-1/2 \ 0} H_{1/2 \ 1}^* + H_{1/2 \ 0} H_{-1/2 \ -1}^*)}{|H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2}. $$

(20)

Up to now, all of the analysis in this section are model independent. With the help of the large $N_c$ Isgur-Wise function given in Sec.II, we can calculate all these asymmetry parameters numerically, the results are listed in Table I.

Next, let us turn to the analysis of the decay $\Omega_b \rightarrow \Omega_c^* + W(\rightarrow \ell + \bar{\nu})$, the procedure is analogous to the analysis of $\Omega_b \rightarrow \Omega_c(\rightarrow a + b) + W(\rightarrow \ell + \bar{\nu})$, we can get the angular
FIG. 1: Definition of polar angles $\Theta_\Omega$ and $\Theta$, both angles are defined in rest frames of the decaying particles.

FIG. 2: Definition of the azimuthal angle $\chi$ which is the one between two cascade decay planes.

distribution as the following:

$$\frac{d\Gamma'}{d\omega d\cos\Theta} = \frac{G^2}{(2\pi)^3}|V_{cb}|^2 q^2 \sqrt{\omega^2 - 1} \frac{M_2^2}{12M_1}$$

$$\times \left( \frac{3}{8}(1 + \cos\Theta)^2 |H'_{3/2_1}|^2 + \frac{3}{8}(1 - \cos\Theta)^2 |H'_{-3/2_-1}|^2 \right.$$

$$+ \frac{3}{8}(1 + \cos\Theta)^2 |H'_{1/2_1}|^2 + \frac{3}{8}(1 - \cos\Theta)^2 |H'_{-1/2_-1}|^2$$

$$+ \frac{3}{4}\sin^2\Theta |H'_{1/2_0}|^2 + \frac{3}{4}\sin^2\Theta |H'_{-1/2_-0}|^2 \right), \quad (21)$$

where the angle $\Theta$ has the same meaning as before. Again we can get some asymmetry parameters. The polar angular distribution of the cascade decay of $W \to \ell + \bar{\nu}$ is

$$\frac{d\Gamma'}{d\omega d\cos\Theta} \propto 1 + 2\alpha_1' \cos\Theta + \alpha_2' \cos^2\Theta, \quad (22)$$
where
\[
\alpha' = \frac{|H'_{3/2 1}|^2 - |H'_{-3/2 -1}|^2 + |H'_{1/2 1}|^2 - |H'_{-1/2 -1}|^2}{|H'_{3/2 1}|^2 + |H'_{-3/2 -1}|^2 + |H'_{1/2 1}|^2 + |H'_{-1/2 -1}|^2 + 2(1 + |H'_{1/2 0}|^2 + |H'_{-1/2 0}|^2)}.
\]

\[
\alpha'' = \frac{|H'_{3/2 1}|^2 + |H'_{-3/2 -1}|^2 + |H'_{1/2 1}|^2 + |H'_{-1/2 -1}|^2 - 2(1 + |H'_{1/2 0}|^2 + |H'_{-1/2 0}|^2)}{|H'_{3/2 1}|^2 + |H'_{-3/2 -1}|^2 + |H'_{1/2 1}|^2 + |H'_{-1/2 -1}|^2 + 2(1 + |H'_{1/2 0}|^2 + |H'_{-1/2 0}|^2)}.
\]

All the numerical results of these asymmetry parameters are listed in Table I.

B. Polarized \(\Omega_b\) decay

In this sub-section, the decays of a polarized \(\Omega_b\) will be analyzed, since in the proposed Z-factory [9], the produced bottom quarks will be polarized. It is reasonable to assume the \(\Omega_b\) will also be polarized in Z-factory. Two new decay angles will be introduced, \(\Theta_P\) and \(\chi_P\), where \(P\) denotes the polarization vector of the parent baryon \(\Omega_b\), the angles involved are shown in Fig.3 and Fig.4.

For the decay \(\Omega_b \rightarrow \Omega_c (\rightarrow a + b) + W (\rightarrow \ell + \bar{\nu})\), the density matrix is now the following,

\[
\rho_{1/2 1/2} = |H_{1/2 1}|^2 (1 - P \cos \Theta_P) + |H_{1/2 0}|^2 (1 + P \cos \Theta_P),
\]

\[
\rho_{1/2 -1/2} = \rho_{-1/2 1/2} = P \sin \Theta_P \text{Re}(H_{1/2 0} H^*_{-1/2 0}),
\]

\[
\rho_{-1/2 -1/2} = |H_{-1/2 -1}|^2 (1 + P \cos \Theta_P) + |H_{-1/2 0}|^2 (1 - P \cos \Theta_P).
\]
After integrating the angles of the leptons out, the whole angle distribution is obtained:

\[
\frac{d\Gamma}{d\omega \cos \Theta \cos \Theta} = Br(\Omega_c \rightarrow a + b) \frac{G^2}{(2\pi)^4} |V_{cb}|^2 \sqrt{q^2 - 1} \frac{M_2^2}{48M_1} \\
\times \left( |H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2 \right) \\
+ \alpha_\Omega \cos \Theta_\Omega \left( |H_{1/2 \ 1}|^2 - |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 - |H_{-1/2 \ 0}|^2 \right) x \\
+ P \alpha_\Omega \cos \Theta_\Omega (-|H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 - |H_{-1/2 \ 0}|^2) \\
+ P \alpha_\Omega \cos \Theta_\Omega \cos \Theta_P (-|H_{1/2 \ 1}|^2 - |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2) \\
+ P \alpha_\Omega \sin \Theta_\Omega \sin \Theta_P \cos \chi_P 2\text{Re}(H_{1/2 \ 0}H_{-1/2 \ 0}^*) \right). \tag{26}
\]

Then the \( \Theta_P \) angle distribution is

\[
\frac{d\Gamma}{d\omega \cos \Theta_P} \propto 1 - \alpha_P \cos \Theta_P, \tag{27}
\]

where

\[
\alpha_P = \frac{|H_{1/2 \ 1}|^2 - |H_{-1/2 \ -1}|^2 - |H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2}{|H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2}. \tag{28}
\]

And the \( \chi_P \) distribution is

\[
\frac{d\Gamma}{d\omega d\chi} \propto 1 - \frac{\pi^2}{16} P \gamma_P \alpha_P \cos \chi, \tag{29}
\]

where

\[
\gamma_P = \frac{2\text{Re}(H_{1/2 \ 0}H_{-1/2 \ 0}^*)}{|H_{1/2 \ 1}|^2 + |H_{-1/2 \ -1}|^2 + |H_{1/2 \ 0}|^2 + |H_{-1/2 \ 0}|^2}. \tag{30}
\]
The numerical results of these asymmetry parameters are shown in Table I.

For the decay $\Omega_b \to \Omega_c^* + W(\to \ell + \bar{\nu})$, after integrating the lepton angles out, there are no such two asymmetry factors.

| Table I: Asymmetry parameters |
|--------------------------------|
| $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_P$ | $\gamma$ | $\gamma_P$ | $\alpha_1'$ | $\alpha_2'$ |
| $\omega = 1$ | 0 | 0 | 0 | 0 | 0.943 | -1/3 | 0 | 0 |
| mean-value | 0.522 | -0.04 | -0.751 | -0.626 | 0.478 | 0.468 | -0.132 | -0.363 |

IV. THE DECAY RATES

To be more concrete, we can now calculate the differential decay rates. Neglecting the lepton mass, the $\Omega_b \to \Omega_c \ell \bar{\nu}$ differential decay rate can be expressed in terms of the helicity amplitudes as

$$
\frac{d\Gamma(\omega)}{d\omega} = \frac{G^2}{(2\pi)^3} |V_{cb}|^2 q^2 \sqrt{\omega^2 - 1} \frac{M_2^2}{12M_1} C^2 
\times \left[ |H'_{1/2 1}|^2 + |H'_{-1/2 -1}|^2 + |H'_{1/2 0}|^2 + |H'_{-1/2 0}|^2 \right],
$$

where $q^2 = M_1^2 + M_2^2 - 2M_1 M_2 \omega$.

For the decay of $\Omega_b \to \Omega_c^* \ell \bar{\nu}$, we have:

$$
\frac{d\Gamma'(\omega)}{d\omega} = \frac{G^2}{(2\pi)^3} |V_{cb}|^2 q^2 \sqrt{\omega^2 - 1} \frac{M_2^2}{12M_1} C^2 
\times \left[ |H'_{3/2 1}|^2 + |H'_{-3/2 -1}|^2 + |H'_{1/2 1}|^2 + |H'_{-1/2 -1}|^2 + |H'_{1/2 0}|^2 + |H'_{-1/2 0}|^2 \right],
$$

where $q^2 = M_1^2 + M_3^2 - 2M_1 M_3 \omega$, and the above distributions are plotted in Fig. 5 and Fig. 6. All the results are consistent with [21–23] when expressed in terms of form factors.

By inputting the form factors discussed in Sect. II, numerical results can be obtained. We have taken $G = 1.16637 \times 10^{-5}$ GeV$^{-2}$ and $|V_{cb}| = 40.6 \times 10^{-3}$ [18]. The results are:

$$\Gamma(\Omega_b \to \Omega_c \ell \bar{\nu}) = 1.686 \times 10^{-14} \text{GeV},$$
$$B(\Omega_b \to \Omega_c \ell \bar{\nu}) = 2.82\%.$$ (33)
FIG. 5: The differential decay rate of $\Omega_b \to \Omega_c \ell \bar{\nu}$.

FIG. 6: The differential decay rate of $\Omega_b \to \Omega_c^* \ell \bar{\nu}$.

\[ \Gamma(\Omega_b \to \Omega_c^* \ell \bar{\nu}) = 3.482 \times 10^{-14}\text{GeV} , \]

\[ B(\Omega_b \to \Omega_c^* \ell \bar{\nu}) = 5.82\% . \]  \hspace{1cm} (34)

The second width is about twice as large as the first one, this can be understood easily when we consider the Clebsch-Gordan coefficients. Note that we have obtained the above results by taking two approximations: heavy quark limit and large $N_c$ limit. In the near future, these results can be tested at the LHCb experiment.

V. SUMMARY

In this paper, we have calculated $\Omega_b \to \Omega_c^{(*)}$ semileptonic decays. Relevant helicity amplitudes have been written down. Both unpolarized and polarized $\Omega_b$ baryon cases have
been considered. Decay angular distributions, asymmetry parameters and semileptonic decay rates have been calculated, with numerical results using leading order results of HQET. The large $N_c$ QCD result for Isgur-Wise functions have been used. The numerical results (especially the zero-recoil values) can be checked by the experiment at the LHCb.

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