Eddy currents in fine polycrystalline metal particle

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Abstract. The magnetic dipole absorption in fine metal polycrystalline particle is considered. The kinetic equation with a collision integral that takes into account the influence of electron scattering at the grain boundaries on the properties of the metal has been solved. We consider the case of diffuse reflection of electrons from the sample surface. Analysis of the dependence of the absorption cross section of particles from the parameter describing the degree of influence of the scattering on the grain boundaries of polycrystalline particles on the kinetic processes has been considered.

1. Introduction
Small metal particles are characterized by unique optical characteristics. These characteristics cannot be described by the macroscopic theory of the interaction of electromagnetic radiation with a metal particle of spherical shape. These features are characteristic for particles whose size $R$ is comparable to the mean free path $\lambda$ of electrons in the metal [1]. A fine particle refers to a particle size where $R$ is much smaller than the wavelength of incident radiation $R \ll \lambda$. To analyze such a case we use the kinetic equation in $\tau$-approximation [2]. While the calculations are still not taken into account the possible influence of scattering at grain boundaries for polycrystalline particles. But with such particles one often have to deal with in the real cases.

It is known that for fine metal particles the dominant contribution to the absorption carry in the magnetic dipole absorption. This absorption is caused by the occurrence of eddy currents in the particle. To describe this phenomenon often applies the standard kinetic theory of a degenerate Fermi gas of conduction electrons in the metal. The particle radius $R$ is assumed small compared to the characteristic depth of the skin layer [2]. Therefore, the skin effect is not taken into account. Then the magnetic field inside the particles can be considered homogeneous $\exp(-i\omega t)$. Then vortex electric field in the particle has the form [9]:

$$E = \frac{\omega}{2ci} H_0 \times r \exp(-i\omega t),$$

where $\omega$ is the angular wave frequency, $c$ - the speed of light, $H_0$ is the amplitude of the magnetic field of the wave, $t$ is time, $r$ is the radius-vector in spherical coordinate system. The origin is in the center of the particle; $\varphi$ and $\theta$ - polar and azimuthal angles. From equation (1) ensue that the electric field has only a $\varphi$-th component. Required cross-section $\sigma$ of the magnetic absorption is determined by the formula [3]:

$$\sigma = \frac{1}{2} \left( \frac{8\pi}{cH_0^2} \right) \text{Re} \int j_\varphi E_{\varphi} d^3r.$$
Here $j_\varphi$, $E^*_\varphi$ and are the components of the current density and the complex conjugate value of the electric field in the particle, respectively. Re is the real part of the number. For finding the response of the electrons in the electric field we use the moment method of solving the kinetic Boltzmann equation for electrons. The Boltzmann equation with the collision integral in the $\tau$-approximation [2] is as follows:

$$-i\omega f_1 + V \frac{\partial f_1}{\partial r} + eVE \frac{\partial f_0}{\partial \varepsilon} = -\frac{f_1}{\tau}. \quad (3)$$

Here $f_0$ is the Fermi-Dirac equilibrium distribution function, $f_1$ - deviation of the distribution function $f$ from the value $f_0 : f = f_0 + f_1$, $e$ and $V$ the charge and velocity of the electrons, respectively; $\varepsilon$ is the electron energy, $\tau$ is the relaxation time. Such a representation is possible for sufficiently small external fields, when linearization is possible.

This kinetic equation corresponds to the case when the electron scattering is purely isotropic in nature. However, this $\tau$-model does not account for the possibility of scattering of electrons on grain boundaries of polycrystalline metal. Therefore, it is necessary to change the right side of equation (3) modeling the collision integral to describe a situation in which electron scattering is not purely isotropic. The need for such changes is caused, in particular, by taking into account the scattering of electrons at the boundaries of crystallites.

The cross-section of magnetic absorption in fine metal particles with using the kinetic equation (3) was previously considered in [4-6].

### 2. Formulation of the problem

The cross section $\sigma$ of magnetic absorption in accordance with equation (2) depends on the current density in the particle. The current density is determined by the function $f_1$ [1,2]

$$j = 2 \left(\frac{m}{h}\right)^3 \int V f_1 d^3V.$$

Here $h$ is the Planck constant, $e$ -electron charge.

An attempt to describe the processes of scattering of electrons on impurities and on the border of the crystallites [7] by the mechanical addition of these effects is inconsistent. This representation can be used to describe the conductivity of metals in some approximation only for the homogeneous case. There have been attempts to apply this approach for description of the electrical properties of thin films [7,8]. However, in areas where the effect of the metal borders is significantly, this approach seems inconsistent. In this case, to separate the influence to separate the contribution to the conductivity of the scattering by impurities (or phonons) and scattering at the boundaries of crystallites is impossible. Description of these processes requires an integrated approach.

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To account for the scattering of electrons at the boundaries of the crystallites, we will use a method similar to that previously proposed in [11] to account for electron-electron collisions. In both cases, the distribution function of electrons after scattering is partially retains information about its state before the scattering. Namely, the velocity of the electrons after scattering.
will depend on the velocity of the electrons before the scattering. But in the case of electron-electron collisions the electron velocity after scattering is reduced, but retains its sign. In the case of scattering of electrons at the boundaries of crystallites, the direction of electron motion is reversed. That is, the electrons are mainly scattered back. This leads to a change of sign of the parameter which describes these processes.

Then the modified linearized Boltzmann equation for electrons, which takes into account electron scattering at the boundaries of crystallites will be:

\[ -i\omega f_1 + V \frac{\partial f_1}{\partial r} + eV E \frac{\partial f_0}{\partial \varepsilon} = -\frac{1}{\tau} \left( f_1 - \frac{3\beta m}{4\pi V_F^2} \frac{\partial f_0}{\partial \varepsilon} \right) \int V' f_1 \partial^3 V', \]  

where \( V_F \) is the Fermi velocity for an electron with effective mass \( m \), where \( V_F \) is the Fermi velocity for an electron with effective mass \( m \), \( \beta \) is a numeric parameter. In the case of electron-electron collisions \( \beta > 0 \). In the scattering at the boundaries of crystallites \( \beta < 0 \).

Static conductivity of polycrystalline metal \( \Sigma \) is defined by the expression:

\[ \Sigma = \frac{\Sigma_0}{1 - \beta}. \]

Here \( \Sigma_0 \) is a static conductivity of the metal in the absence of scattering at the boundaries of crystallites. This value is equal to \( \Sigma_0 = e^2 n \tau / m \), \( n \) is the concentration of conduction electrons in the metal.

Using the moment method one can search for the function \( f_1 \) in the form of a combination of moments \( C_{\varphi} \) and \( C_r C_{\varphi} \):

\[ f_1 = (a_1(r) C_{\varphi} + a_2(r) C_r C_{\varphi}) \delta(\varepsilon - \varepsilon_F) \sin \theta \exp(-i\omega t). \]

Here

\[ \delta(\varepsilon - \varepsilon_F) = \frac{\delta(V - V_F)}{m V_F} = \frac{\delta(C - 1)}{m V_F^2}, \]

where \( \delta(x) \) is the Delta Dirac function; \( a_1(r), a_2(r) \) - coefficients at the respective moments, \( C = V/V_F, C_r = V_r/V_F, C_{\varphi}/V_F, - \) dimensionless components of the velocity vector.

3. Solution

Successively multiply the kinetic equation (4) on the moments \( C_{\varphi} \) and \( C_r C_{\varphi} \). Then integrate the resulting expression over the entire space of velocities. We introduce the following dimensionless quantities

\[ \alpha_1 = \frac{a_1}{eRH_0}, \quad \alpha_2 = \frac{a_2}{eRH_0}, \quad \xi = \frac{r}{R}, \quad x = \frac{R}{\tau V_F}, \quad y = \frac{R\omega}{V_F}, \quad z = x - iy, \quad z_0 = (1 - \beta)x - iy. \]

As a result, we obtain the following system of equations:

\[ \begin{cases} 10z_0 \alpha_1 + 6 \frac{\alpha_2}{\xi} + 2 \frac{\partial \alpha_2}{\partial \xi} = \delta \frac{i y \xi V_F}{c}, \\ \frac{\alpha_1}{\xi} - \frac{\partial \alpha_1}{\partial \xi} - \alpha_2 = 0. \end{cases} \]

The following functions will be the solution to this system of equations:
\[ \alpha_1 = \frac{(N + N^2 \xi) \exp(-N \xi)}{\xi^2} C_1 + \frac{(N - N^2 \xi) \exp(N \xi)}{\xi^2} C_2 + \frac{i \xi B}{5 z_0}, \]  

(5)

where \( N = \sqrt{\frac{z}{z_0}} \), \( B = \frac{5g V_F}{2c} \), and \( C_1 \) and \( C_2 \) are the coefficients determined from boundary conditions. Value \( \alpha_2 \) has the following form

\[ \alpha_1 = \frac{(3N^2 \xi - 3N - N^3 \xi^2)}{z \xi^3} C_1 + \frac{(3N^2 \xi + 3N + N^3 \xi^2) \exp(-N \xi)}{z \xi^3} C_2. \]  

(6)

In the case when scattering of electrons on the surface of the particles is diffuse [12] we have

\[ f_1(r, V) = 0, \quad \text{when} \quad r = R, r V \leq 0. \]

Then the moment boundary conditions have the following form [11]:

\[ \alpha_2 = \sqrt{3} \alpha_1 \quad \text{when} \quad \xi = 1. \]

Moreover, when \( \xi = 0 \) values \( \alpha_1 \) and \( \alpha_2 \) should not be divergent functions. This gives an additional condition \( C_1 = -C_2 = C \).

For a value \( C \), we have:

\[ C = \frac{i V_F y}{2c z_0} \frac{\sqrt{3}}{A_{21} + A_{22} - \sqrt{3}(A_{11} + A_{12})} \]

where

\[ A_{11} = (N^2 + N) \exp(-N), \quad A_{12} = (N^2 - N) \exp(N), \]

\[ A_{21} = \frac{3N^2 - 3N - N^3}{z} \exp(N), \quad A_{21} = \frac{3N^2 + 3N + N^3}{z} \exp(-N). \]

The current density is determined by the following expression

\[ j_\varphi = \frac{8\pi m^2 e^2 V_F^2 R H_0}{3h^3} \alpha_1 \sin(\theta) \exp(-i\omega t). \]

The absorption cross-section will be presented as \( \sigma(x, y, \beta) = \sigma_0 F(x, y, \beta) \), where

\[ \sigma_0 = \frac{\pi^2 e^2 n V_F R^4}{2mc^2}. \]

Here \( F(x, y, \beta) \) is the dimensionless absorption cross-section. It can be represented as:

\[ F(x, y, \beta) = \frac{16g^2}{3} Re \left\{ \frac{\sqrt{3} \left[ \exp(N)(N^2 - 3N - 3) - \exp(-N)(N^2 + 3N + 3) \right]}{N z_0 (A_{21} + A_{22} - \sqrt{3}A_{11} - \sqrt{3}A_{12})} + \frac{1}{5 z_0} \right\}. \]
Figure 1. The dependence of the dimensionless absorption cross-sections of fine metal particles from dimensionless frequency for different values of the parameter $\beta$, and $x=0.1$.

4. Analysis of the results
The absorption cross-section $\sigma$ depends on three parameters. They are the dimensionless mean free path length of electrons $x$, the dimensionless frequency of electromagnetic radiation $y$, and the parameter $\beta$. The last parameter describes the effect of scattering of electrons on the surface of the crystallites on kinetic processes in polycrystalline metal.

Fig.1 presents graphs of the dimensionless absorption cross-sections $F(x,y,\beta)$ from the dimensionless frequency $y$ at various values of the parameter $\beta$. The value $x$ is equal to $x=0.1$. The graph shows that with increasing the scattering at the boundaries of crystallites (i.e., with increase in absolute value of parameter $\beta$) the absorption cross-section decreases. With increasing frequency the influence of parameter $\beta$ on the absorption cross-section increases.

5. Conclusion
The paper considers the influence of electron scattering at the boundaries of crystallites of the polycrystalline metal on the absorption cross section of fine metal particles. Was used the Boltzmann equation in the modified $\tau$-approximation. For the solution of kinetic equation was used the moment method. It has been shown that taking into account the influence of electron scattering at the boundaries of the crystallites leads to a significant correction of the magnitude of the absorption cross-section.

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