Research Article

FaceFilter: Face Identification with Deep Learning and Filter Algorithm

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1. Introduction

Recently, deep neural networks and especially convolutional neural networks (CNNs) have become the most commonly used method for feature representation and have achieved good results in face recognition problems. Face recognition can be divided into two categories: face verification, where two faces are presented and the system needs to verify whether these two faces belong to the same person, and face identification, where a face image is presented with an unknown identity and the system needs to determine this identity.

Most existing works that have focused on face recognition have achieved a high level of success [1–13]. However, if the pose is significantly changed or the face is presented at an angle, the individual cannot be identified.

Previous approaches to face recognition that are based on the discriminative classification model (face identification) are trained on a dataset of known identities, and an intermediate bottleneck layer is used as a representation for recognition. This approach generalizes a very large representation for each face, but some works have tried to reduce this dimensionality using PCA [10].

Another approach used in FaceNet [14] directly trained its output to obtain 128-D embedding using a triplet-based loss function based on LMNN [9]. These triplets comprise two matching faces and a nonmatching face. The aim of the triplet loss function is to separate positive results from negative ones by a certain distance margin.

In contrast, our approach uses an unsupervised learning technique to obtain 128 bytes per face and then passes these bytes to a filter in order to find the most suitable representation for each face. We then reduce the dimensionality of the representation to half of the 128 bytes, to match the original face with other faces to find the identity. This approach can identify a given face in different poses and can identify other faces that are most similar to the original identity.
As an illustration, Figure 1 shows a picture of a single individual at different angles and in different poses.

The remainder of this paper is organized as follows: Section 2 discusses the most important related work in face recognition. Our method is presented in Section 3, including a description of deep neural networks and our algorithm for handling the features. Sections 4 and 5 present some quantitative results and an evaluation of these.

2. Related Works

Our approach is similar to other recent works [3, 10, 14] in that it learns its representation directly from the face. However, instead of using a vector of features for reidentification, we reduce the vector representation to half of the features extracted for each face. We use a deep convolutional neural network architecture inspired by the NN4 FaceNet [14] and OpenFace [15] networks, but we remove the L2 normalization layer and instead use another fully connected layer.

There are an enormous number of studies of face recognition, and we will briefly discuss the most relevant works.

Huang et al. [16] proposed a convolutional deep belief network based on local convolutional restricted Boltzmann machines to learn a face representation. The learning method was unsupervised learning and the training was on an unlabeled natural image dataset. After that, they transfer the learned representation to a face identification through a classification method such as SVM.

Another attempt for face recognition was proposed by Taigman et al. [17]. This approach called DeepFace and it is one of the earlier large-scale applications of a 3D model for face recognition. They extracted the face representation using a nine-layer DeepFace model which mainly consists of two convolutional layers, three locally connected layers, and two fully connected (FC) layers with more than 120 million parameters using several locally connected layers without weight sharing. Their system was trained on 4.4 M 2D facial images of 4,030 identities and they achieved an accuracy of 97.35% on the benchmark LFW [18] dataset.

Schroff et al. proposed a CNN-based approach used for face recognition and clustering. This approach is called FaceNet [14] which is based on eleven convolutional and three FC layers. They have trained a deep convolutional network on a dataset of 200 M faces and 8 M identities and triplet loss function to directly optimize the embedding instead of an intermediate bottleneck layer as in the previous works. They have used triplets of roughly aligned matching/nonmatching face patches using an online triplet mining method, and they achieved the performance of state-of-the-art face recognition with 128 bytes for each face.

Sun et al. proposed another framework called DeepID [5, 6, 10] for face identification and verification. Their approach utilized an ensemble of shallower and smaller deep convolutional networks than DeepFace, i.e., every DCNN has four convolutional layers and uses 39, 31, and 1 patches, respectively, as an input. Their framework was trained on 202,599 images of 10,177 subjects. Their approach is considered as the first approach that achieved results that surpass human performance for face verification on the LFW dataset.

Parkhi et al. [19] collected a face dataset of 2.6 M 2D faces from 2,622 identities by proposing a new method for crawling the faces from the web. They presented a VGG-Face model consisting of 16 convolutional layers and three fully connected (FC) layers. The authors claimed that they achieved 98.95% accuracy on the LFW [18] dataset.

Deep 3D face recognition results have been represented by Kim et al. [20]. They fine-tuned the VGG-Face network [19] on 3D depth images. After that, they reported their results on three public datasets. They used an augmented dataset of 123,325 depth images to fine-tune VGG-Face. After that, they tested the model on Bosphorus [21], BU3DFE [5], and 3D-TEC (twins) [22] datasets. But their
results do not perform as the state-of-the-art results of the
convolutional methods.

3. Method

3.1. Deep Convolutional Networks. We used a deep neural
network structure called an NN4 neural network. Before they
were input to the network, we resized all images to a size of
96 × 96 × 3. These were used as input to the first convolutional
layer, which has 64 kernels of size 7 × 7 × 3 with stride 2. The
second convolutional layer has 64 kernels of size 1 × 1 × 3 with
stride 2, and in the third convolutional layer, 192 kernels are
used with size 3 × 3 × 3 and stride 2. After these layers, an
inception architecture was used in which there were six blocks
labeled inception 3a, inception 3b, inception 3c, inception 4a,
inception 5b, and inception 5c [23].

Since the input of the network was 96 × 96 × 3 and the
receptive field was small, the computational requirement
was drastically reduced. The total number of parameters was
3,743,925, and the number of trainable parameters was
3,734,613, with 9,312 nontrainable parameters. We trained
the network using a stochastic gradient descent (SGD) al-
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3.2. Face Reidentification Equations. Each original image \( x \)
that we want to predict is represented by \( f(x) \in \mathbb{R} \) as a
vector of 128 bytes indexes from 1 to 128. This can be
expressed as in (1):

\[
f(x_o) = (v^1, v^2, v^3, \ldots, v^n), \quad \text{where } n = 128,
\]

where \( x_o \) is the original image that we want to predict and \( n \)
is the number of features \( v \) in that vector. The vectors of the
identities in the dataset will also be extracted, as it is
expressed as in (2), and kept in a separate model file:

\[
f(x_{id}^i) = (v_{id}^1, v_{id}^2, v_{id}^3, \ldots, v_{id}^n),
\]

where \( n = 128 \) and \( x_{id}^i \) is the image of a particular identity \( id \)
and \( i \) refers to the number of that identity in the dataset. After
extracting the vectors, we will pass the vector of the original
image to a filter to extract the most important values that can
represent the original image. The filter works as a net to select
the highest values among the features in the vector of the
original image. It takes the values greater than zero with their

\[
\text{val, ind} = \sum_{i} f(x_{id}^i, \text{index}_{id}^i), \quad \text{where } x_{id}^i > 0,
\]

where \( n \) is the number of features \( v_{id}^i \) in the vector of the
original image, i.e., 128, and \( i \) is the index of each feature in
the vector. The selected features \( v_{id}^i \) which have values greater
than zero will be stored in \( \text{val} \) while their corresponding
indices will be stored in \( \text{ind} \). So, we can select all features of
each image in the identities of the dataset with the same
indices of the selected features of the original image:

\[
\text{val}_{id}^{\text{val}} = \sum_{i=1}^{\text{mid}} f(x_{id}^i, \text{index}_{id}^i), \quad \text{for each } \text{index}_{id}^i = \text{index}_{id}^o,
\]

where \( id \) refers to a particular identity in the set of identities \( i \)
and \( \text{mid} \) is the number of features in each image of the
identities. The selected features \( v_{id}^i \) of the identity \( id \) will be
chosen if its indices \( \text{index}_{id}^i \) is equal to the indices of the selected
features of the original image \( \text{index}_{id}^o \) and will be stored in \( \text{val}_{id}^{\text{val}} \).
Here, we do not need to select the values greater than zero for
each identity in the dataset; rather, we just take the values

\[
\text{idem} = \min \left( \begin{array}{c}
\text{val} \\
\text{val}_{id}^{\text{val}} \\
\vdots \\
\text{val}_{id}^{\text{mid}}
\end{array} \right)
\]

where \( i \) refers to the number of identities. It should be noticed that we have only weights of the images of all
identities obtained by the model where these weights
have been kept in another model file called acknowledge base.

3.3. Image Aligning with Face Reidentification. Face detection and recognition still have many problems to identify the face especially when the face is aligned to down or to any other angle in the image. This problem can be solved by searching for a face in the image. If the image does not have a face, we will rotate the image step by step from 0 to 360°, where each step is rotated for 14°, until we find a face in that image and pass it as a new image. Therefore, the total number of steps is 25. In case we could not find a face in the image after rotating it, we will pass the image without

| Layer                      | Size in  | Size out   | Kernel      | Feature map | No. of parameters |
|----------------------------|----------|------------|-------------|-------------|-------------------|
| Input                      | 96 × 96  | 102 × 102  | 7 × 7 × 3   | 2           | 9472              |
| ZeroPadding2D              | 96 × 96  | 102 × 102  | 7 × 7 × 3   | 2           | 9472              |
| Conv1                      | 102 × 102| 48 × 48    | 7 × 7 × 3   | 2           | 4160              |
| Norm                       | 48 × 48  | 50 × 50    | 1 × 1 × 3   | 2           |                   |
| Max pool                   | 50 × 50  | 24 × 24    | 7 × 7 × 3   | 2           |                   |
| Conv2                      | 24 × 24  | 24 × 24    | 1 × 1 × 3   | 2           |                   |
| ZeroPadding                | 24 × 24  | 26 × 26    |             |             |                   |
| Conv3                      | 26 × 26  | 24 × 24    | 3 × 3 × 2   | 2           | 192               |
| Norm                       | 24 × 24  | 24 × 24    |             |             |                   |
| ZeroPadding                | 24 × 24  | 26 × 26    |             |             |                   |
| Max pool                   | 26 × 26  | 12 × 12    | 3 × 3 × 3   | 2           |                   |
| Inception 3a               | 12 × 12  | 12 × 12    | 3 × 3 × 3   | 2           |                   |
| Inception 3b               | 6 × 6    | 6 × 6      | 3 × 3 × 3   | 2           |                   |
| Inception 3c               | 24 × 24  | 24 × 24    | 3 × 3 × 3   | 2           |                   |
| Inception 4a               | 3 × 3    | 3 × 3      |             |             |                   |
| Average pool               | 3 × 3    | 1 × 1      | 3 × 3 × 3   | 1           |                   |
| fc1                        | 1 × 1    | 128        |             |             |                   |
| Fc                          | 128      | 1          |             |             |                   |

Figure 2: The structure of the model diagram.
rotating because there may be a face in the image where the face is in different poses and cannot be detected. Figure 4 shows an image with a face that the face detector cannot detect it, but after the rotations, we find a face while Figure 5 shows a face that cannot be detected after 360 rotations, so the original image will not be changed.

**4. Evaluation**

We used a neural network to extract the features of the faces. Feature extraction takes 128 bytes for each face and then finds the weights greater than zero from the original image with their corresponding indices and finds the other weights.

![Inception Structure Diagram](image-url)
of the identities with the corresponding indices of the original image. The process of selecting weights larger than zero with their corresponding indices is called a filter process where the dimensional of the vector will be reduced to half of 128 bytes. After that, the distance of the filtered bytes of the original image with the bytes of each identity in the same indices of the original image is calculated to find the minimum number. The minimum number will refer to the identity of the face in rotate 16 has been recognized effectively by the system.

\[
\text{ind} = \text{total} \times \frac{\text{weights}}{128}.
\]

(7)

For the original image that we want to identify it, we extracted its 128 features using our model and passed these features to the filter to find the most important features for representation and reduce the dimensionality to the half. After choosing the positive values of each feature of the original image and taking their corresponding indexes, we will extract the features of each block from the features of the dataset according to the indexes of the positive features of the original image as in the following equation:

\[
TW = \sum_{i=0}^{128} W_{oi}^{dn}, \quad \text{if } o_2 = i dn_i,
\]

(8)

where \(o_2\) is the indexes of the filter weights of the original image and the values of \(o_2\) are indexed from 0 to half of 128. \(W\) is the weight of the identity \(i dn\). Finally, we applied (6) to identify the image.

5. Experiments

5.1. Dataset. We have used Celeb Faces Attributes Dataset (CelebA) as the training faces in our training. It consists of 10,177 identities and 202,599 faces. Before training, we extract the face of each image in the dataset using a face detector and then we resized it to the input size of our neural networks which is 96 x 96 x 3 pixels.

LFW and YTF datasets used in the evaluation process. LFW is a database of face photographs for studying the problem of unconstrained face recognition. This database contains 13,233 images of 5,749 people detected and centered by the Viola–Jones face detector and collected from the web. YTF is a database of face videos designed to be used for

| Table 2: Face verification of different methods on LFW. |
|-----------------|-----------|
| Method          | Accuracy (%) |
| DeepFace [17]   | 97.35     |
| FaceNet [14]    | 99.63     |
| High-dim LBP [27] | 95.17     |
| TL Joint Bayesian [28] | 96.33     |
| GaussianFace [29, 30] | 98.52     |
| DeepID [5]      | 97.45     |
| DeepID2 [6]     | 99.15     |
| DeepID2+ [10]   | 99.47     |
| DeepID3 [30]    | 99.53     |
| Our method      | 99.70     |

| Table 3: Face verification of different methods on YTF. |
|-----------------|-----------|
| Method          | Accuracy (%) |
| Face reidentification [12] | 90.41     |
| DeepFace [17]   | 91.4      |
| Face representation [11] | 92.24     |
| Deeply learned face [10] | 93.2      |
| FaceNet [14]    | 95.12     |
| Our method      | 94.02     |

**Figure 5:** The first image is the original image. The detector could not detect the face, so we rotated it. After two steps of rotations (rotate 1, rotate 2), the detector still could not detect the face. After 16 steps of rotations (rotate 25), the detector still could not detect the face, and therefore, the original image will remain the first image. The identity of the face in rotate 16 has been recognized effectively by the system.
studying the problem of unconstrained face recognition in videos. This dataset contains 3,425 videos of 1,595 different people downloaded from YouTube. The shortest video contains 48 frames while the longest video contains 6,070 frames.

5.2. Various Dimensionalities. Various embedding dimensionalities were explored in previous studies [14], and accordingly, the dimension 128 has been selected as it gives the best accuracy. The comparison between four embedding dimensionalities, 64, 128, 256, and 512, shows that the difference in the performance is small. In this study, we explored the best dimensionality, i.e., 128, before and after applying the filter. After applying the filter to the dimension of 128, the dimensionality has reduced to the half of 128 with higher accuracy of the dimension of 128 by using our new algorithm.

5.3. Acknowledge Base Identities. In order to increase the number of identities without looking at the picture of any identity in the dataset again, acknowledge base model has been created to save the features of each identity. The features of any new identity will be saved in the acknowledge base model. This acknowledge base model will be used to know any other unseen face picture to predict the identity.

5.4. Effect of Face Detection. Most of the face detection frameworks have shown good results in face detection, whereas
there are still some limitations. Many faces have not been detected incorrectly using the most widely framework used for face detection. This limitation can affect negatively the results. Therefore, face detection still needs some improvements.

5.5. Performance on LFW and YTF. During the evaluation, the feature of every identity is extracted and kept in the acknowledge base. Any other extraction for any identity will be added to the acknowledge base with its corresponding label of that identity. Every time in the evaluation step, we took 200k images for test and kept their features with their corresponding labels in the acknowledge base with any previous features extracted for any identity. That means the acknowledge base model can store the features of all images in the dataset and it can find the single identity of any face among all these identities. We achieved a classification accuracy of 99.70% on the LFW dataset and 94.02% on the YTF dataset. Table 2 and Table 3 show the classification accuracy with some methods as compared to our classification accuracy on LFW and YTF. Figures 6 and 7 demonstrate a comparison chart for previous studies with LFW and YTF.

6. Conclusions

Deep neural network is used in this paper for face re-identification. The filter technique is used to select the most important features from the features extracted by the model. This method can identify the face in different poses and different levels of illumination. The rotation technique for 360° is used for the images that have the face in different angles, while this kind of rotation cannot be done in the augmentation method in deep learning.

We noticed that deep learning is very important to extract the features, but with well-prepared mathematical operations on the extracted features from the deep learning, it can increase the accuracy of the model.

Data Availability

The model, extraction of weight code, features saved in acknowledge base, and the equations for evaluation code are available in the following URL: [https://drive.google.com/open?id=1pXMkhA0x9zV4n8ynmer2xdF5lLeQZ3Rz](https://drive.google.com/open?id=1pXMkhA0x9zV4n8ynmer2xdF5lLeQZ3Rz).

Disclosure

The funding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; and in the decision to publish the results.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors’ Contributions

Mohammed Al-Ghaili performed programming and wrote the manuscript. Zhiyong Li supervised the study. Hamdi A.R. Ali proofread the article.

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