Top–Quark Production and Flavor Physics

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Abstract
Because of the top quark’s very large mass, about 175 GeV, it now provides the best window into flavor physics. Thus, pair-production of top quarks at the Tevatron Collider is the best probe of this physics until the Large Hadron Collider turns on in the next century. I will discuss aspects of the mass and angular distributions that can be measured in $t\bar{t}$ production with the coming large data samples from the Tevatron and even larger ones from the LHC.

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1. Introduction

For more than a year, persistent rumors have been privately circulating among particle physicists that signatures for the top quark have been found in the CDF experiment at the Tevatron Collider. According to these rumors, the “best-fit” top-quark mass is so large that the rate of the signature events cannot be accounted for by ordinary QCD production of $t\bar{t}$ [1]. These rumors have now been confirmed in two papers from the CDF collaboration [2], [3]. According to these papers, the top mass is $m_t = 174 \pm 10^{+13}_{-12}$ GeV. The data in these papers are based on an integrated luminosity of $19.3 \text{ pb}^{-1}$. When combined with CDF’s efficiencies and acceptances, this yields the measured cross section $\sigma(p\bar{p} \to t\bar{t}) = 13.9^{+6.1}_{-4.8} \text{ pb}$ at $\sqrt{s} = 1800$ GeV. For the central value of the measured top mass, the predicted QCD cross section for this mass, including next-to-leading-log and corrections and and soft-gluon resummation [4], [5], is $\sigma(t\bar{t}) = 5.10^{+0.73}_{-0.43} \text{ pb}$, 2.8 times smaller than the central value of the measured cross section.

The experimental errors on the CDF measurements are large. But so is the discrepancy with QCD, and this is tantalizing—evidence, we all hope, for the long-sought breakdown of the standard model. In any case, it is clear that the top quark is a wide open window into the world of flavor physics. For example, at the top mass of 174 GeV, its Yukawa coupling to a standard Higgs boson is $\Gamma_t = 2^{3/4} G_F^{1/2} m_t = 1.00$. In Refs. [6] and [7], it was stressed that measurements of the $t\bar{t}$ rate and associated distributions at the Tevatron potentially provide the most powerful flavor probes we now have. In particular, top-quark production can be significantly modified from QCD expectations by the resonant production of colored, flavor-sensitive particles with mass in the range 400–500 GeV. In Ref. [6], Hill and Parke proposed that color-octet vector mesons, $V_8$, associated with “top color” [8] are copious sources of $t\bar{t}$. In Ref. [7], Eichten and I stressed that the color-octet $\eta_T$ occurring in multiscale models of walking technicolor [9], [10], [11] is expected in just this mass range and easily could double the $t\bar{t}$ rate expected from QCD. The top quark’s mass is so large that, whatever the nature of flavor physics, it is hard to believe that $m_t$ does not offer some clue to it.

In this paper, we discuss two distributions that may reveal aspects of flavor physics in $t\bar{t}$ production—the invariant mass distribution, $d\sigma/dM_{t\bar{t}}$, and the center-of-mass angular distribution of the top quark, $d\sigma/d\cos\theta$. The magnitude and shape of the invariant mass distribution will reveal whether $t\bar{t}$ production is standard or not, and whether resonances decaying to $t\bar{t}$ exist. We also point out that, for standard QCD production, the $M_{t\bar{t}}$
distribution can provide an independent determination of the top quark’s mass. We apply this to the existing data [2] and find good consistency with the reported mass. We also emphasize the importance of measuring subsystem masses for testing alternative top–production mechanisms.

The angular distribution of top quarks also reflects the underlying production mechanism. Even though most of $t\bar{t}$ production is near threshold, the expectation that it is mainly s–wave can be overturned if there are large parity–violating components in the $q\bar{q} \rightarrow t\bar{t}$ process. We shall compare the angular distributions for standard and nonstandard $t\bar{t}$ production at the Tevatron and at the CERN Large Hadron Collider. We shall see that, because of the much larger $\tau = \hat{s}/s$ of top–quark production at the Tevatron, experiments there have a potential advantage over those at the LHC.

All these tests require much larger data sets than will be available in the next year or two. To realize the full potential of this handle on flavor physics, it is essential that the Tevatron experiments be able to collect samples as large as $1–10 \text{fb}^{-1}$. Such large data sets may even help end what Mark Twain described as “such wholesale returns of conjecture out of such a trifling investment of fact.”

2. Invariant Mass Distributions

In QCD production of top–quark pairs, the mean and root–mean–square of the $t\bar{t}$ invariant mass distribution, $\langle M_{t\bar{t}} \rangle$ and $\langle M_{t\bar{t}}^2 \rangle^{1/2}$, are nearly linear functions of the top–quark mass [12]. To understand why this is so, we show in Fig. 1 the $M_{t\bar{t}}$ distribution, $d\sigma(pp \rightarrow t\bar{t} + X)/dM_{t\bar{t}}$, at $\sqrt{s} = 1800 \text{GeV}$ for $m_t = 100–220 \text{GeV}$.

\begin{footnote}
1 These plots and all other calculations in this paper were carried out using lowest–order QCD subprocess cross sections and the EHLQ Set 1 parton distribution functions [13]. We believe that our general conclusions will remain true when higher–order corrections are included. Our $t\bar{t}$ cross sections have been multiplied by a factor of 1.6165. This makes our standard QCD rates as a function of $m_t$ agree to within a per cent with the central values quoted in Ref. [5] over the entire range of top masses of interest. Our numerical results for the linear dependence of $\langle M_{t\bar{t}} \rangle$ and $\langle M_{t\bar{t}}^2 \rangle^{1/2}$ on $m_t$ are accurate so long as the higher–order corrections are well–represented by a simple multiplicative factor. All our parton level calculations ignore transverse motion of the $t\bar{t}$ center–of–mass induced, e.g., by initial–state radiation. While this effect is not large, it can and should be taken into account in more detailed simulations.
\end{footnote}
this mass range produced at the Tevatron, the cross section peaks reasonably sharply at $M_{\text{max}} \simeq 2.1m_t + 10 \text{ GeV}$. At least for the first few moments, then, we expect that

$$\langle M_{tt}^n \rangle = \frac{\int dM_{tt} (d\sigma/dM_{tt}) M_{tt}^n}{\int dM_{tt} (d\sigma/dM_{tt})},$$

(2.1)

is a linear function of $m_t$. Using the lowest–order cross section, we find that, for $100 \lesssim m_t \lesssim 200 \text{ GeV}$, the first two moments are well–fit by the formulae

$$\langle M_{tt} \rangle = 50.0 \text{ GeV} + 2.24 m_t$$

$$\langle M_{tt}^2 \rangle^{1/2} = 58.4 \text{ GeV} + 2.23 m_t.$$  

(2.2)

In the range $m_t \simeq 140–180 \text{ GeV}$, the dispersion in $M_{tt}$ expected for standard QCD production is $\Delta M_{tt} = 70–75 \text{ GeV}$.

In Ref. [2], the top quark mass was determined from a sample of seven $W \to \ell\nu + 4$ jets events by making an overall constrained best fit to the hypothesis $p\overline{p} \to t\overline{t} + X$ followed by the standard top decays $t \to W^+b$ with one $W$ decaying leptonically and the other hadronically. At least one of the $b$–jets was tagged. The CDF paper provides the momentum 4–vectors of all particles in the event before and after the constrained fit. From these, the central values of kinematic characteristics of the seven events may be determined. Table 1 lists the best–fit top–quark masses determined by CDF together with the invariant mass of the events before and after the constrained fit.\footnote{Particle 4–vectors before the constrained fit do have various corrections—e.g., for the jet energy scale—made to them [2]. Only $E_T$ is provided for the neutrino(s) in the before–fit 4–vectors. The biggest change in the before– and after–momenta occurs in $E_T$. We used the $W \to \ell\nu$ 4–momenta determined from the constrained fit in both cases.} We used these $M_{tt}$ to compute the mean and RMS. Both sets of 4–momenta gave essentially identical results. Using 4–momenta from the constrained fit, we found:

$$\langle M_{tt} \rangle = 439 \text{ GeV} \quad \Rightarrow \quad m_t = 173 \text{ GeV}$$

$$\langle M_{tt}^2 \rangle^{1/2} = 443 \text{ GeV} \quad \Rightarrow \quad m_t = 172 \text{ GeV}$$

$$\Delta M_{tt} = 59.5 \text{ GeV}.$$  

(2.3)

These results give some confidence that the measured central value of the top–quark mass, 174 GeV, is accurate. For example, if $m_t = 160 \text{ GeV}$ (for which Ref. [5] predicts $\sigma(t\overline{t}) = 8.2^{+1.3}_{-0.8} \text{ pb}$), we would expect $\langle M_{tt} \rangle = 409 \text{ GeV}$ and $\langle M_{tt}^2 \rangle^{1/2} = 415 \text{ GeV}$, both
well below the values determined above. Thus, if something is going to change in the CDF results from the next large data sample, we expect it will be the cross section—which would need to be two to three times smaller to agree with the standard model.

Nonstandard explanations for the large cross section are not necessarily disfavored by the good agreement between the central values of the measured top mass and the top masses deduced in Eq. (2.3). As an example of nonstandard physics, we show in Figs. 2 and 3 two examples of $t\bar{t}$ production rates with an $\eta_T$ resonance, one for $M_{\eta_T} = 450$ GeV, the other for $M_{\eta_T} = 475$ GeV. The parameters are typical of those used in the calculations of Ref. [7]. They are listed in Table 2 along with the production and kinematic characteristics of the two cases. The top–quark masses inferred from the $M_{t\bar{t}}$ distributions are close to the 175 GeV input to these calculations. We note that the $M_{t\bar{t}}$ dispersion in these examples is $\Delta M_{t\bar{t}} = 50–55$ GeV, characteristic of narrow–resonance production. Although this is closer to the measured dispersion of 60 GeV (Eq. (2.3)) than the QCD expectation of about 75 GeV, the statistics are too low for this agreement to be significant.

Subsystem invariant masses may be as interesting as the total invariant mass. For example, in multiscale technicolor, it is possible that a color–octet technirho is produced and decays as $\rho_T \rightarrow W^\pm \pi_T^\pm$, with $\pi_T^\pm \rightarrow t\bar{b} \rightarrow W^+ b\bar{b}$, the same final state as in $t\bar{t}$ production [10]. Searches for processes such as these, using a constrained–fit procedure analogous to that employed by CDF for the $t\bar{t}$ hypothesis, should be carried out. All this will require much more data from the Tevatron, probably 1 fb$^{-1}$ or more. At the expense of increasing backgrounds, larger data samples may be had by using appropriately selected events without a tagged $b$–jet. This was done already in Ref. [2] and was found to give an excess of events with constrained–fit $m_t$ above 160 GeV.

To summarize: The invariant mass distributions that can be formed in top–quark production provide incisive probes for distinguishing between standard and non–standard mechanisms. In standard QCD, the mean and RMS of $M_{t\bar{t}}$ provide an independent measure of $m_t$ which should agree with the directly–measured mass. In QCD, the variance $\Delta M_{t\bar{t}}$ is expected to be about 75 GeV. The total–system invariant mass can reveal the presence of $t\bar{t}$ resonances such as the $\eta_T$ [7],[9] and the top–color vectors $V_8$ discussed by Hill and Parke [6],[8]. Subsystem invariant masses can be studied to test for alternative explanations of the top–production data. In this regard, we emphasize that it is dangerous to use the standard QCD $t\bar{t}$ production model to select top–candidate events.

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3 The $\eta_T$ contribution in the 475 GeV case was multiplied by 2.25 instead of 1.62. This is consistent with the higher–order corrections to $gg \rightarrow t\bar{t}$ processes predicted in Ref. [5].
example, a resonance in $t\bar{t}$ production can distort the summed scalar–$E_T$ and sphericity or aplanarity distributions of candidate events from their QCD expectation.

3. Angular Distributions

The angular distribution of top quarks provides important information about their production mechanism. In hadron colliders, the $t\bar{t}$ pair is produced by $q\bar{q}$ annihilation and $gg$ fusion. For pure QCD production, the former process is dominant at the Tevatron—providing about 90% of the $t\bar{t}$ rate. Gluon fusion dominates by the same amount at much higher energy machines such as the LHC. Resonances such as the $\eta_T$ [9], [7] and the “top–color” color–octet $V_8$ vector boson [6] can change these proportions and the expected angular distributions.\(^4\)

By Bose symmetry, the center–of–mass angular distribution in $gg \rightarrow t\bar{t}$ is forward–backward symmetric. Although this is also true in lowest–order QCD for $q\bar{q} \rightarrow t\bar{t}$, there is no reason that it need be so for non–standard production mechanisms. For example, if a $V_8$ exists and couples only to left–handed quarks, the angular distribution in the subprocess c.m. will be $(1 + \beta \cos \theta)^2$, where $\theta$ is the angle between the incoming light quark and the outgoing top quark and $\beta$ is the top quark’s velocity.

In the study of top angular distributions, the Tevatron has a distinct advantage. In $p\bar{p} \rightarrow t\bar{t}$ at $\sqrt{s} = 1800$ GeV, the direction of the incoming quark is the same as that of the proton practically all the time. Thus, if we denote by $\theta^*$ the angle between the proton direction and the top–quark direction in the subprocess c.m., this angle is almost always the same as $\theta$.\(^5\)

In $pp$ collisions, the direction of the incoming quark can be inferred with confidence only for events with high boost rapidity, $\eta_B$, or large fractional subprocess energy, $\tau = \hat{s}/s$. For large $\tau$, the direction of the quark tends to be the same as the boost of the c.m., even if $\eta_B$ is small [12]. (In the case of $pp$ collisions, $\theta^*$ will refer to the angle between the direction of the boost and that of the top quark in the subprocess c.m.) Thus, angular information on top production is doubly difficult to come by in $t\bar{t}$ production at the LHC: The process

\(^4\) Hill and Parke [6] consider only $V_8$ bosons which have vector couplings to quarks. Such a $V_8$ will not induce a significant change in the shape of the expected QCD angular distribution at the Tevatron and LHC. For this reason, we consider below a $V_8$ which couples only to one chirality of the quarks. The effect of this can be dramatic.

\(^5\) The distinction between $t$ and $\bar{t}$ is based on the sign of the charged lepton in $W$–decay.
is dominated by gluon fusion, potentially obscuring interesting $\cos \theta$ dependence, and $\tau$ is small, making it hard to distinguish $\theta$ from $\pi - \theta$. As we shall see, the Tevatron’s analyzing power would be significantly greater if the luminosity of the Tevatron were increased to $10^{33}\text{cm}^{-2}\text{s}^{-1}$ or more and its detectors upgraded to handle (and survive) this luminosity.\footnote{The reader may have noticed that I did not mention high–energy $e^+e^-$ colliders such as the 500 GeV or so NLC. It is clear from the discussion here that lepton machines cast no light on such strongly–coupled flavor physics aspects of $t\bar{t}$ production as the $\eta_T$ and $V_8$. The higher rates possible at hadron machines also make them ideal for searches for new particles in top–quark decays.}

To illustrate the ability of high–luminosity hadron collider experiments to distinguish among different production mechanisms, we compare the $\cos \theta^*$ distributions for QCD, an $\eta_T$ of mass 450 GeV with isotropic production, and a 475 GeV color–octet $V_8$ that couples only to left–handed quarks. The angular distributions for the subprocesses $q\bar{q} \rightarrow t\bar{t}$ and $gg \rightarrow t\bar{t}$ in lowest–order QCD are:

\[
\frac{d\hat{\sigma}(q\bar{q} \rightarrow t\bar{t})}{dz} = \frac{\pi \alpha_s^2 \beta}{9\hat{s}} (2 - \beta^2 + \beta^2 z^2),
\]

\[
\frac{d\hat{\sigma}(gg \rightarrow t\bar{t})}{dz} = \frac{\pi \alpha_s^2 \beta}{6\hat{s}} \left\{ \frac{1 + \beta^2 z^2}{1 - \beta^2 z^2} - \frac{(1 - \beta^2)^2 (1 + \beta^2 z^2)}{(1 - \beta^2 z^2)^2} \right\} - \frac{9}{16} (1 + \beta^2 z^2),
\]

where $z = \cos \theta$ and $\beta = \sqrt{1 - 4m_t^2/\hat{s}}$. For $\hat{s} \gg 4m_t^2$, these cross sections—especially the gluon fusion one—are forward–backward peaked. But, at the modest $\hat{s}$ at which QCD $t\bar{t}$ production is large, the cross sections are fairly isotropic.

If there exists an $\eta_T$ with decay rates to gluon and quark pairs given by \cite{7}

\[
\Gamma(\eta_T \rightarrow gg) = \frac{5 \alpha_s^2 N_T C_{\eta_T}}{384 \pi^3 F_Q^2} M_{\eta_T}^3,
\]

\[
\Gamma(\eta_T \rightarrow \bar{q}q) = \frac{2C_q^2 m_q^2 M_{\eta_T} \beta_q}{16 \pi F_Q^2},
\]
the gluon fusion cross section for $t\bar{t}$ production has the following additional terms:

$$
\frac{d\sigma(gg \to \eta_T \to t\bar{t})}{dz} = \frac{\pi}{4} \frac{\Gamma(\eta_T \to gg) \Gamma(\eta_T \to \bar{t}t)}{(\hat{s} - M_{\eta_T}^2)^2 + \hat{s} \Gamma^2(\eta_T)}
$$

$$
+ \frac{5\sqrt{2} \alpha_s^2 N_{TC} C_t m_t^2 \beta}{768\pi F_Q^2} \frac{\hat{s} - M_{\eta_T}^2}{(\hat{s} - M_{\eta_T}^2)^2 + \hat{s} \Gamma^2(\eta_T)} \frac{1 - 2\beta^2 z^2}{1 - \beta^2 z^2}.
$$

In these expressions, it is assumed that the $\eta_T$ is composed from a single doublet of techniquarks $Q = (U, D)$ in the $N_{TC}$ representation of $SU(N_{TC})$; $F_Q$ is the decay constant of technipions in the $Q\bar{Q}$ sector; and $C_q$ is a dimensionless factor of $O(1)$ in the Yukawa coupling of $\eta_T$ to $\overline{q}q$. The second (interference) term in the $\eta_T$ angular distribution is never very important, but we include it for completeness.

The color–octet vector boson, $V_8$, is assumed to couple to $\overline{q}q$ as follows:

$$
A(V_8^q(p, \lambda) \to q(p_1) \overline{q}(p_2)) = g_s \xi_q \epsilon^\mu(p, \lambda) \overline{u}_q(p_1) \frac{\lambda_a}{2} \gamma_\mu \left( \frac{1 - \gamma_5}{2} \right) v_q(p_2),
$$

(3.4)

where, following Ref. [6], $g_s$ is the QCD coupling and $\xi_t = \xi_b = \pm 1/\xi_q$ ($q = u, d, c, s$).

For this chiral coupling, the $q\overline{q} \to t\bar{t}$ angular distribution in Eq. (3.1) is modified by the addition of

$$
\frac{d\sigma(q\overline{q} \to V_8 \to t\bar{t})}{dz} = \frac{\pi \alpha_s^2 \beta}{36\hat{s}} (1 + \beta z)^2 \left\{ 1 + \xi_q \xi_t \frac{\hat{s}}{\hat{s} - M_{V_8}^2 + i\sqrt{\hat{s}} \Gamma(V_8)} \right\}^2 - 1 \right\},
$$

(3.5)

where, ignoring the mass of all quarks except the top’s, the $V_8$ width is

$$
\Gamma(V_8) = \frac{\alpha_s M_{V_8}}{12} \left\{ 4\xi_q^2 + \xi_t^2 \left( 1 + \beta(1 - m_t^2/M_{V_8}^2) \right) \right\}.
$$

The $\cos \theta^*$ distributions we present below are an integral over $t\bar{t}$ invariant mass of $d\sigma(pp \to t\bar{t})/dM_{t\bar{t}} d\cos \theta^*$. The integration region is centered on the peak of the invariant mass distribution and is approximately the width of the resonance. For the $\eta_T$, we used $M_{\eta_T} = 450$ GeV, $N_{TC} = 5$, $F_Q = 30$ GeV and $C_t = -1/3$. Then,

$$
\Gamma(\eta_T) \cong \Gamma(\eta_T \to \bar{t}t) + \Gamma(\eta_T \to gg) = 21 \text{ GeV} + 11 \text{ GeV} = 32 \text{ GeV},
$$

and $\sigma(pp \to t\bar{t}) \cong 14$ pb at $\sqrt{s} = 1800$ GeV. For the $V_8$, we took $M_{V_8} = 475$ GeV and $\xi_t = \sqrt{40/3}$ (see [6]). Then,

$$
\Gamma(V_8) \cong \Gamma(V_8 \to b\bar{b}) + \Gamma(V_8 \to \bar{t}t) = 85 \text{ GeV}
$$

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and \( \sigma(p\bar{p} \to t\bar{t}) \simeq 15 \text{ pb (11 pb)} \) for \( \xi_q = -1/\xi_t \) (\( \xi_q = 1/\xi_t \)). Note that this \( V_8 \) model predicts a large enhancement in \( q\bar{q} \to b\bar{b} \), providing a nice way to test it.\(^7\)

The mass distribution for the 450 GeV \( \eta_T \) was shown in Fig. 2 for the Tevatron. Figures 4 and 5 show the \( V_8 \) mass distribution at the Tevatron for \( \xi_q = -1/\xi_t \) and \( \xi_q = 1/\xi_t \), respectively. The effect of the \( V_8 \)’s interference with the \( s \)-channel gluon in \( q\bar{q} \to t\bar{t} \) and its relation to the sign of \( \xi_q \xi_t = \mp 1 \) are quite clear. The mean and RMS \( t\bar{t} \) invariant masses for the two cases are, respectively:

\[
\langle M_{t\bar{t}} \rangle = 440 \text{ GeV} \quad \Rightarrow \quad m_t = 174 \text{ GeV} \\
\langle M_{t\bar{t}}^2 \rangle^{1/2} = 444 \text{ GeV} \quad \Rightarrow \quad m_t = 173 \text{ GeV}
\]

\( \Delta M_{t\bar{t}} = 53.4 \text{ GeV} \); \(^{(3.7)}\)

and

\[
\langle M_{t\bar{t}} \rangle = 482 \text{ GeV} \quad \Rightarrow \quad m_t = 193 \text{ GeV} \\
\langle M_{t\bar{t}}^2 \rangle^{1/2} = 487 \text{ GeV} \quad \Rightarrow \quad m_t = 192 \text{ GeV}
\]

\( \Delta M_{t\bar{t}} = 66.6 \text{ GeV} \). \(^{(3.8)}\)

The \( \cos \theta^* \) distributions in \( p\bar{p} \) and \( pp \) collisions, defined as described above, are shown for the \( \eta_T \) and \( V_8 \) models in Figs. 6–9 and their global features summarized in Table 3. In all cases, the pseudorapidities of the \( t \) and the \( \bar{t} \) were required to be less than 1.5 to allow for the mobility of their decay products and the finite coverage of Tevatron and LHC detectors.\(^8\) We discuss them in turn:

1.) Figure 6 shows the \( q\bar{q} \to t\bar{t} \), \( gg \to t\bar{t} \) and \( gg \to \eta_T \to t\bar{t} \) components of the top–production \( \cos \theta^* \) distribution expected at the Tevatron. The \( M_{t\bar{t}} \) integration region is 430 to 470 GeV. The QCD contribution is flat, the forward–backward peaking diminished by the proximity of threshold. The \( \eta_T \) contribution is also flat, of course, and makes up about 80% of the total cross section. The falloff near \( \cos \theta^* = \pm 0.90 \) is due to the rapidity cut, \( |\eta_{t,\bar{t}}| < 1.5 \). (For the fun of it, we computed the \( \cos \theta^* \) distribution of the seven \( 7 \) \( t\bar{t} \) candidate events. The results, along with the c.m. velocity \( \beta \), are listed in Table 1. They form a perfectly flat distribution.) Table 3 lists the total \( t\bar{t} \) cross section as well

\(^7\) Radiative corrections were approximated as above, by multiplying the total lowest–order cross section by 1.62. This may overestimate the \( q\bar{q} \to t\bar{t} \) contribution.

\(^8\) This may be a bit unfair to the LHC whose detectors ought to be much more hermetic and have somewhat greater rapidity coverage for jets, electrons and muons than the Tevatron detectors.
as the cross sections $\sigma_F$ for $\cos\theta^* > 0$ and $\sigma_B$ for $\cos\theta^* < 0$. The forward-backward asymmetry is calculated as

$$A_{FB} = \frac{N_F - N_B}{N_F + N_B} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}.$$  \hspace{1cm} (3.9)

The statistical error on $A_{FB}$ is

$$(\Delta A_{FB})_{\text{stat}} = 2\sqrt{\frac{N_F N_B}{(N_F + N_B)^3}} = 2\sqrt{\frac{\sigma_F \sigma_B}{(\sigma_F + \sigma_B)^3}} \int L dt,$$  \hspace{1cm} (3.10)

where $\epsilon_{t\bar{t}}$ is the overall efficiency, including branching ratios, for identifying and reconstructing $t\bar{t}$ events. For the CDF experiment at the Tevatron, we can infer from Ref. [2] that $\epsilon_{t\bar{t}}$(CDF) $\simeq$ 5–10 events/(19 pb$^{-1} \times 14$ pb) = 2–4%. We use $\epsilon_{t\bar{t}}$(TEV) = 3%. It is difficult to say what value of the efficiency is appropriate for LHC experiments; detailed simulations are needed (see e.g., Ref. [12]). We shall assume $\epsilon_{t\bar{t}}$(LHC) = 5%, although it turns out not to matter in the examples we consider.

The components of the $\cos\theta^*$ distribution expected at the LHC are shown in Fig. 7. Because of the small $\tau$ values involved, the roles of gluon fusion and $q\bar{q}$ annihilation are reversed, with gluon fusion making up about 90% of the QCD rate. The enormous $\eta_{t\bar{t}} \rightarrow t\bar{t}$ rate is due to the very large $gg$ luminosity at small $\tau$ [13]. The central bowing of the $\cos\theta^*$ distribution is due to the top–rapidity cut. At the LHC energy, such large boost rapidities occur that events at large c.m. rapidity and $\cos\theta^*$ are depleted.

2.) Figure 8 shows the components of the $\cos\theta^*$ distribution at the Tevatron for the 475 GeV $V_8$ model coupling to left–handed quarks with relative strengths $\xi_t = -1/\xi_q = \sqrt{40}/3$. The $M_{t\bar{t}}$ integration region is 400–500 GeV. The effect of the chiral coupling is evident, though somewhat diminished by the $\eta_{t\bar{t}}$ cut. The forward–backward asymmetry of 0.30 could be measured at the 4$\sigma$ (statistical) level with an integrated luminosity of 1 fb$^{-1}$. For this luminosity, the statistical errors on $d\sigma/d\cos\theta^*$ in six bins 0.30 units wide would range from 30% down to 15%. This example shows how useful it would be if the Tevatron luminosity could be upgraded to $10^{33}$ cm$^{-2}$ s$^{-1}$.

The $\cos\theta^*$ distributions expected at the LHC for this $V_8$ are shown in Fig. 9. In this example, the contribution of the $V_8$ is about 20% of the total and it is polluted by the $q \leftrightarrow \bar{q}$ ambiguity, so that the rise in the cross section with $\cos\theta^*$ is invisible. The asymmetry is only 1%. This illustrates the dominance of $gg$ processes and the uncertainty
in determining the quark direction at small $\tau$ in a high-energy $pp$ collider that we mentioned earlier. Essentially similar results were obtained for the $\xi_t = 1/\xi_q$ case (see Table 3).

The pronounced central peaking in Fig. 9 is an artifact of the rather tight $\eta_t,\tau$ cut. It goes away for a looser cut, as seen in Fig. 10 for $|\eta_{t,\tau}| < 2.5$. The asymmetry in $\cos\theta^*$ is still invisible, however, and the error in $A_{FB} = 0.025$ probably would be dominated by systematic effects. We also found that there is nothing to be gained at the LHC by limiting the $M_{t\bar{t}}$ integration region to a narrow band about $M_{V_8}$ or by selecting events produced at large boost rapidity.

To summarize this section: The dominance of $q\bar{q}$ annihilation in top–quark production processes at the Tevatron collider gives it an advantage over the LHC for studying angular distributions. However, measurements of these distributions would benefit greatly from a significant upgrade of the collider and its detectors so that data samples of $O(10 \text{ fb}^{-1})$ can be collected. The studies carried out here have all been at the most naive parton level; it is hoped that detailed, detector–specific simulations will be undertaken in the not–too–distant future.

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Run—Event | $m_t$ | $\mathcal{M}_{t\bar{t}}$(before fit) | $\mathcal{M}_{t\bar{t}}$(after fit) | $\beta$(after fit) | $\cos \theta^*$
--- | --- | --- | --- | --- | ---
40758–44414 | 172 ± 11 | 523 | 526 | 0.757 | 0.404
43096–47223 | 166 ± 11 | 533 | 511 | 0.760 | 0.820
43351–266423 | 158 ± 18 | 440 | 460 | 0.727 | 0.512
45610–139604 | 180 ± 9 | 338 | 366 | 0.180 | −0.0011
45705–54765 | 188 ± 19 | 440 | 431 | 0.489 | −0.348
45879–123158 | 169 ± 10 | 411 | 412 | 0.572 | −0.767
45880–31838 | 132 ± 8 | 384 | 365 | 0.691 | −0.682

TABLE 1: Best fit top–quark masses (Ref. [2]) and kinematic characteristics of the $t\bar{t}$ candidate events.

Masses are in GeV. Transverse motion of the subprocess c.m. was neglected in determining the top–quark velocity $\beta$ and scattering angle $\theta^*$.

| $M_{\eta T}$ | $\sigma(t\bar{t})$ | $\sigma_{\eta T}(t\bar{t})$ | $\langle \mathcal{M}_{t\bar{t}} \rangle$ | $m_t(\langle \mathcal{M}_{t\bar{t}} \rangle)$ | $\langle \mathcal{M}_{t\bar{t}}^2 \rangle^{1/2}$ | $m_t(\langle \mathcal{M}_{t\bar{t}}^2 \rangle^{1/2})$
--- | --- | --- | --- | --- | --- | ---
450 | 13.5 | 8.53 | 432 | 171 | 435 | 169
475 | 13.9 | 8.95 | 442 | 175 | 445 | 173

TABLE 2: $p\bar{p} \rightarrow \eta_T \rightarrow t\bar{t}$ cross sections (in pb) and their kinematic characteristics.

In the notation of Ref. [7], $N_{TC} = 5$, $F_Q = 30$ GeV, and the coupling of the $\eta_T$ to $t\bar{t}$ is $C_t = -1/3$. To account for QCD radiative corrections, cross sections were multiplied by 1.62. For $M_{\eta_T} = 475$ GeV, the $\eta_T$–contribution was multiplied by 2.25. The QCD cross section is 4.96 pb [5]. The dispersion in $\mathcal{M}_{t\bar{t}}$ is $\Delta \mathcal{M}_{t\bar{t}} = 52$ GeV for $M_{\eta_T} = 450$ GeV and 55 GeV for $M_{\eta_T} = 475$ GeV.
| Model       | $\mathcal{M}_{t\bar{t}}$ range | Collider | $\sigma(t\bar{t})$ | $\sigma_F$ | $\sigma_B$ | $A_{FB}$ |
|-------------|-------------------------------|-----------|--------------------|-----------|-----------|---------|
| $\eta_T$   | 430 – 470                     | TEV       | 4.03               | 2.01      | 2.01      | 0       |
| $\eta_T$   | 430 – 470                     | LHC       | 2800               | 1400      | 1400      | 0       |
| $V_8 (\xi_q = -1/\xi_t)$ | 400 – 500                     | TEV       | 6.43               | 4.20      | 2.24      | 0.30    |
| $V_8 (\xi_q = -1/\xi_t)$ | 400 – 500                     | LHC       | 175                | 88        | 86        | 0.010   |
| $V_8 (\xi_q = 1/\xi_t)$  | 425 – 525                     | TEV       | 4.80               | 3.30      | 1.50      | 0.37    |
| $V_8 (\xi_q = 1/\xi_t)$  | 425 – 525                     | LHC       | 159                | 80        | 178       | 0.012   |

**TABLE 3:** Angular dependences of $t\bar{t}$ production in the $\eta_T$ and $V_8$ resonance models.

Top quarks are produced with pseudorapidity $|\eta_{t,\bar{t}}| < 1.5$ and cross sections (in pb) have been multiplied by 1.62. The $V_8 t\bar{t}$ couplig is $\xi_t = \sqrt{40/3}$. 
Figure Captions

[1] The $t\bar{t}$ invariant mass distributions, in $p\bar{p}$ collisions at $\sqrt{s} = 1800\text{ GeV}$, for $m_t = 100 - 220\text{ GeV}$ in 20 GeV increments. EHLQ Set 1 distribution functions were used and the cross sections were multiplied by 1.62 as explained in the text. No rapidity cut is applied.

[2] The $t\bar{t}$ invariant mass distribution in the presence of an $\eta_T$, in $p\bar{p}$ collisions at $\sqrt{s} = 1800\text{ GeV}$, for $m_t = 175\text{ GeV}$ and $M_{\eta_T} = 450\text{ GeV}$, $F_Q = 30\text{ GeV}$ and $C_t = -1/3$. The QCD (dotted curve), $\eta_T \to t\bar{t}$ and its interference with the QCD amplitude (dashed), and total (solid) rates have been multiplied by 1.62 as explained in the text. No rapidity cut is applied to the top quarks.

[3] The $t\bar{t}$ invariant mass distribution in the presence of an $\eta_T$, in $p\bar{p}$ collisions at $\sqrt{s} = 1800\text{ GeV}$, for $m_t = 175\text{ GeV}$ and $M_{\eta_T} = 475\text{ GeV}$, $F_Q = 30\text{ GeV}$ and $C_t = -1/3$. The QCD contribution was multiplied by 1.62 and the $\eta_T$ contribution by 2.25. Curves are labeled as in Fig. 2.

[4] The $t\bar{t}$ invariant mass distribution in the presence of a $V_8$, in $p\bar{p}$ collisions at $\sqrt{s} = 1800\text{ GeV}$, for $m_t = 175\text{ GeV}$ and $M_{V_8} = 475\text{ GeV}$, $\xi_t = \xi_b = 1/\xi_q = \sqrt{40/3}$. The QCD (dotted curve) and the total (solid) rates have been multiplied by 1.62 as explained in the text. No rapidity cut is applied to the top quarks.

[5] The $t\bar{t}$ invariant mass distribution in the presence of a $V_8$, in $p\bar{p}$ collisions at $\sqrt{s} = 1800\text{ GeV}$, for $m_t = 175\text{ GeV}$ and $M_{V_8} = 475\text{ GeV}$, $\xi_t = \xi_b = 1/\xi_q = \sqrt{40/3}$. The curves are labeled as in Fig. 4.

[6] The cos $\theta^*$ distribution for $p\bar{p} \to t\bar{t}b$ at $\sqrt{s} = 1800\text{ GeV}$, as defined in the text, in the presence of a 450 GeV $\eta_T$ with parameters as in Fig. 2; $430 < M_{t\bar{t}} < 470\text{ GeV}$. The components are standard QCD $gg \to t\bar{t}$ (dot-dash), $q\bar{q} \to t\bar{t}$ (long dashes), total QCD (dots), $gg \to \eta_T \to t\bar{t}$ and interference with QCD (short dashes), and the total $d\sigma/d\cos\theta^*$ (solid). EHLQ Set 1 distribution functions were used and all cross sections were multiplied by 1.62. The top quarks are required to have pseudorapidity $|\eta_{t,\bar{t}}| < 1.5$.

[7] The cos $\theta^*$ distribution for $pp \to t\bar{t}$ at $\sqrt{s} = 15\text{ TeV}$, as defined in the text, in the presence of a 450 GeV $\eta_T$ with parameters as in Fig. 2; $430 < M_{t\bar{t}} < 470\text{ GeV}$. The components are labeled as in Fig. 6. The top quarks are required to have pseudorapidity $|\eta_{t,\bar{t}}| < 1.5$.

[8] The cos $\theta^*$ distribution for $p\bar{p} \to t\bar{t}$ at $\sqrt{s} = 1800\text{ GeV}$ in the presence of a $475\text{ GeV}$ $V_8$ with parameters as in Fig. 4; $400 < M_{t\bar{t}} < 500\text{ GeV}$. The components are standard QCD $gg \to t\bar{t}$ (dot-dash), $q\bar{q} \to t\bar{t}$ (long dashes), total QCD (dots), $q\bar{q} \to V_8 \to t\bar{t}$ and
interference with QCD (short dashes), and the total $d\sigma/\cos\theta^*$ (solid). EHLQ Set 1 distribution functions were used and all cross sections were multiplied by 1.62. The top quarks are required to have pseudorapidity $|\eta_{t\bar{t}}| < 1.5$.

[9] The $\cos\theta^*$ distribution for $pp \to t\bar{t}$ at $\sqrt{s} = 15$ TeV in the presence of a 475 GeV $V_8$ with parameters as in Fig. 4; $400 < \mathcal{M}_{t\bar{t}} < 500$ GeV. The components labeled as in Fig. 8. The top quarks are required to have pseudorapidity $|\eta_{t\bar{t}}| < 1.5$.

[10] The $\cos\theta^*$ distribution for $pp \to t\bar{t}$ at $\sqrt{s} = 15$ TeV in the presence of a 475 GeV $V_8$ with parameters as in Fig. 4; $400 < \mathcal{M}_{t\bar{t}} < 500$ GeV. The components labeled as in Fig. 8. The top quarks are required to have pseudorapidity $|\eta_{t\bar{t}}| < 2.5$. 
\[(q_d) \left( \cos \theta \right) \frac{dp}{dp} \]
