Longitudinal conductivity and transverse charge redistribution in coupled quantum wells subject to in-plane magnetic fields

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In double quantum wells electrons experience a Lorentz force oriented perpendicular to the structure plane when an electric current is driven perpendicular to the direction of an in-plane magnetic field. Consequently, the excess charge is accumulated in one of the wells. The polarization of a bilayer electron system and the corresponding Hall voltage are shown to contribute substantially to the in-plane conductivity.

75.50.Pp, 75.30.Ds

I. INTRODUCTION

A finite size, in the growth direction, of bilayer electron systems confined in double quantum wells leads to strong orbital effects of an applied in-plane magnetic field. As a result, an oscillation appears in the field dependent density of states, corresponding to the depopulation of the antibonding subband at a critical field \( B_1 \), and to the splitting of the Fermi see into two separated electron sheets at a second critical field \( B_2 \). The variation of the density of states with \( B_1 \) is reflected in magnetoresistance traces recorded as functions of \( B_1 \). Critical fields \( B_1 \) and \( B_2 \) can, therefore, be determined experimentally.

The connection between the magnetoresistance \( \rho(B_1) \) and the density of states \( g(B_1) \) employed in this papers is based on an assumption that the scattering rate is proportional to the density of states. Unlike \( g(B_1) \), which is determined exclusively by the electron energy spectra, the \( \rho(B_1) \) depends also on the electron scattering mechanism and on the mutual orientation of the electric current and the magnetic field. Therefore, the proportionality between \( \rho(B_1) \) and \( g(B_1) \) is only approximate.

A conventional approach describing transport in double quantum wells subject to in-plane magnetic fields relies on one-electron approximation and semiclassical Boltzmann equation. This approach is justified in the case of “parallel” conductivity \( \sigma_1 \) related to magnetoresistance by \( \rho_1 = 1/\sigma_1 \), i.e., when an electric current is driven along the magnetic field \( \mathbf{j} \perp \mathbf{B} \) and no Lorentz force acts on electrons.

II. ELECTRONIC STRUCTURE AND TRANSPORT COEFFICIENTS

We consider two coupled, strictly two-dimensional (2D) electron layers confined in very narrow potential wells at the distance \( d \) (see Fig. 1). The vector potential, \( \mathbf{A} = (zB, 0, 0) \), is used to describe the influence of the in-plane magnetic field, \( \mathbf{B} = (0, B, 0) \), on the electronic structure.

![Fig. 1. Schematic picture of a double-layer system. Directions of applied electric and magnetic fields are indicated.](image)

Only the lowest bound states of the left and right wells are used to construct the model Hamiltonian \( \hat{H}_0 \) of the system. The Hamiltonian is diagonal in the wavevector \( \mathbf{k} = (k_x, k_y) \) and takes a matrix form...
\[ \langle \tilde{k} | H_0 | \tilde{k}' \rangle = \delta_{\tilde{k}, \tilde{k}'} \left( \epsilon_L(\tilde{k}) \frac{t}{t} \epsilon_R(\tilde{k}) \right), \]  
(1)

where \( \epsilon_L(R)(\tilde{k}) \) are single-well eigenenergies,
\[ \epsilon_L(R)(\tilde{k}) = \frac{\hbar^2}{2m^*} (k_x - k_0)^2 + \frac{\hbar^2 k_0^2}{2m^*}, \]  
(2)

and \( k_0 = \pm |e| Bd/2\hbar \). The hopping integral \( t \) is given by a matrix element of the well potential between lowest eigenstates of the left and right wells. The diagonalization of the matrix \( \langle \rangle \) yields the bonding and antibonding eigenstates.

We employ a minimal model that accounts for the inter-well polarization effect in the Hartree approximation. Transport coefficients are obtained using the random phase approximation and Kubo formalism of linear response theory. In our geometrical arrangement the “parallel” conductivity \( \sigma_\parallel \) relates the current component \( j_y \) to the external electric field component \( E_y \) in a standard way where \( j_y = \sigma_\parallel E_y \). The structure of \( \sigma_\parallel \) is more complicated as it involves the electron-electron interaction contribution. The basic scheme of our calculation is outlined below: for more details see Ref. The “perpendicular” conductivity is composed of \( \sigma_\perp \), which is the response to the applied field component \( E_x \), and \( \sigma_\perp^{(2)} \) corresponding to the “Hall” electric field \( E_z \), i.e., \( j_x = \sigma_\perp^{(1)} E_x + \sigma_\perp^{(2)} E_z \). The “Hall” field appears as a consequence of the polarization of the sample and is related to the inter-well “Hall” potential \( U_H \) as \( E_z = -U_H/d \). It is calculated in a self-consistent way from the non-equilibrium excess charge \( \delta Q \) as a solution to the Poisson equation. In our model, \( \delta Q \) and the potential \( U_H \) are related by a particularly simple formula corresponding to the parallel plate capacitor,
\[ \delta Q = \varepsilon \frac{U_H}{d}, \]  
(3)

where \( \varepsilon \) denotes the dielectric constant of the barrier. The excess charge \( \delta Q \) is obtained as a sum of components \( \delta Q^{(1)} \) calculated from the Kubo formula as a response to the fields \( E_x \) and \( E_z \), respectively. Writing
\[ \delta Q^{(1)} = \varepsilon^{(1)} E_x, \quad \delta Q^{(2)} = \varepsilon^{(2)} E_z, \]  
(4)

where \( \varepsilon^{(1)} \) and \( \varepsilon^{(2)} \) are the generalized dielectric functions, \( E_z \) is the solution to the self-consistent equation \( \varepsilon \) and takes the form
\[ E_z = \frac{\varepsilon^{(1)}}{\varepsilon^{(2)} + \varepsilon} E_x. \]  
(5)

This formula relates the self-consistent Hartree field of electrons to the \( x \) component of the applied external electric field \( \tilde{E} \). Using Eq. \( \varepsilon \), the perpendicular conductivity can be written in a form
\[ \sigma_\perp = \sigma_\perp^{(1)} + \frac{\varepsilon^{(1)}}{\varepsilon^{(2)} + \varepsilon} \sigma_\perp^{(2)}. \]  
(6)

In this equation the first term \( \sigma_\perp^{(1)} \) corresponds to the conventional solution of the Boltzmann equation while the second term represents the novel “Hall” correction to the conductivity.

### III. SHORT-RANGE SCATTERERS IN THE BORN APPROXIMATION

We have performed numerical calculations of transport coefficients for the realistic parameters of a bilayer 2D system. Since we have only a limited knowledge of the nature of scatterers and their distribution in the sample, we assume that electrons are scattered by impurities distributed randomly in both left and right wells. To be more specific, the scattering on an individual impurity is considered to be intra-well (diagonal in the layer index) and isotropic in the \( \tilde{k} \) space. The concentration of impurities is assumed very low and the weak scattering on an individual impurity is treated in the non-self-consistent version of the Born approximation. The strength of the scattering on an individual impurity and the impurity concentrations are taken as adjustable parameters.

In the course of calculation we introduced the resolvents in expressions for the dielectric functions and the conductivity components. The finite quasiparticle lifetime and the transport relaxation time are related to the imaginary part of the self-energy and to the vertex corrections to velocity components, respectively. The quantities result from replacing the resolvents and their products, characteristic for a given configuration of scatterers, by expressions averaged over all possible configurations.

To proceed further, we adopt additional simplifications justified by the very low scattering rate in high-mobility samples. Only terms of the lowest order in the scattering rate are kept in all expressions. While \( \sigma_\parallel \), \( \sigma_\perp^{(1)} \), and \( \varepsilon^{(1)} \) would diverge in samples without impurities, the leading terms, \( \sigma_\perp^{(2)} \) and \( \varepsilon^{(2)} \) of the expansions obtained as a response to \( U_H \), are finite in this limit. Note that, in spite of finite \( \sigma_\perp^{(2)} \), both terms of \( \sigma_\perp \) are inversely proportional to the scattering rate, as the second term is proportional to \( \varepsilon^{(1)} \) according to Eq. \( \varepsilon \).

The vertex corrections vanish for the velocity component \( v_y \) as expected for randomly distributed short-range scatterers. This is not true for \( v_x \). Due to the anisotropy of the scattering induced by the in-plane magnetic field, the vertex corrections have to be taken into account.

### IV. RESULTS AND DISCUSSION

The in-plane magnetic field changes qualitatively the topology of Fermi contours. At zero field, the Fermi contours are two concentric circles. The in-plane field shifts the centers of Fermi circles in opposite directions in \( k \)-space and gradually changes the contour shapes. At the
critical field $B_{c,1}$, the Fermi line of an antibonding subband vanishes and the transition from a two-component to a one-component system occurs. The Fermi contour of a bonding subband splits at the higher critical field $B_{c,2}$ into two parts and the left and right electron layers are decoupled.

In Fig. 2, the curves are strikingly similar below $B_{c,1}$, except of $\rho_\perp$. The components $\rho_\parallel$ and $\rho_\perp^{(1)}$, which can be obtained by solving the Boltzmann equation, exactly coincide with $g$ for our simplified model. It is the Hall contribution which quantitatively changes the field dependence of $\rho_\perp$ to negative magnetoresistance in this region. Between $B_{c,1}$ and $B_{c,2}$ only the bonding subband is occupied and the difference between $\rho_\parallel$ and $\rho_\perp$ originates from the Fermi contour anisotropy and the related anisotropy of the effective mass and the relaxation time. The anisotropy of the transport relaxation time (the vertex correction to $v_x$) plays a role also above $B_{c,2}$.

To conclude, we have shown that the charge redistribution within a double-layer system gives rise to the Hall-like contribution to the perpendicular magnetoresistance component $\rho_\perp$ which is so strong that it cannot be omitted in the analysis of bilayer 2D system magnetotransport.

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