Anisotropic stars with non-static conformal symmetry

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Abstract

We have proposed a model for relativistic compact star with anisotropy and analytically obtained exact spherically symmetric solutions describing the interior of the dense star admitting non-static conformal symmetry. Several features of the solutions including drawbacks of the model have been explored and discussed. For this purpose we have provided the energy conditions, TOV-equations and other physical requirements and thus thoroughly investigated stability, mass-radius relation and surface redshift of the model. It is observed that most of the features are well matched with the compact stars, like quark/strange stars.

Key words: General Relativity; noncommutative geometry; compact stars

1 Introduction

Since the striking idea of \textit{white dwarf} by Chandrasekhar\textsuperscript{11} the study of general relativistic compact objects received a tremendous thrust to carry out research in the field of ultra-dense objects. White dwarfs are composed of one of the

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densest forms of matter known, surpassed only by other compact stars such as neutron stars, quark stars/strange stars, boson stars, gravastars etc.

In the compact stars the matter is found to be in stable ground state where the quarks are confined inside the hadrons. If it is composed of the de-confined quarks then also a stable ground state of matter, known as ‘strange matter’, is achievable which provides a ‘strange star’ [2,3,4,5]. The main theoretical motivation for postulating the existence of strange stars was to explain the exotic phenomena of gamma ray bursts and soft gamma ray repeaters [6,7]. On the observational side, the Rossi X-ray Timing Explorer has now confirmed that SAX J1808.4 – 3658 is one of the candidates for a strange star [8].

As a start of the astrophysical objects of compact nature of the above kind the theoretical investigation has been done by several researchers by using both analytical and numerical methods. However, the density distribution inside the compact stars need not be isotropic and homogeneous as proposed in the TOV model. In the early seventies Ruderman [9] has investigated the stellar models and argued that the nuclear matter may have anisotropic features at least in certain very high density ranges (> $10^{15}$ gm/c.c.), where the nuclear interaction must be treated relativistically. Later on Bowers and Liang [10] gave emphasis on the importance of locally anisotropic equations of state for relativistic fluid spheres. They showed that anisotropy might have non-negligible effects on such parameters like maximum equilibrium mass and surface redshift. Anisotropic matter distribution have been considered recently by several investigators as key feature in the configuration of compact stars (with or without cosmological constant) to model the objects physically more realistic [11,12,13,14,15,16,17,18].

As one of the physical characteristics Mak and Harko [19] have calculated the mass-radius ratio or compactness factor for compact relativistic star however in the presence of cosmological constant. To study mass and radii of neutron star Egeland [20] incorporated the existence of cosmological constant proportionality depending on the density of vacuum. Kalam et al. [18] by using their model calculated mass-radius ratio for Strange Star Her X – 1 of their model and found it as $M_{eff}/R_{max} = 0.336$ which satisfies Buchdahl’s limit and corresponds to the same mass-radius ratio for the observed Strange Star Her X – 1. This physical aspect is very important because of the fact it helps to determine nature of the compact stars, its size, shape, matter contain and several other features.

Now to get more information, several studies have been done on charged or neutral fluid spheres with a spacetime geometry that admits a conformal symmetry, in the static as well as generalized non-static cases. In this line of conformal symmetry we note that there are lots of works available in the literature [21,22,23,24,25]. However, we investigate in the present work a new
anisotropic star admitting non-static conformal symmetry (mathematical formalism is provided in detail in the next Sec. 2).

Under this background and motivation we investigation in the present paper a model for relativistic compact star with anisotropy and find out exact spherically symmetric solutions which describe the interior of the dense star admitting non-static conformal symmetry. The scheme of this study is as follows: We provide mathematical formalism for non-static conformal motion in Sec. 2 whereas the Einstein field equations for anisotropic stellar source are given in Sec. 3. The solutions and general features of the field equations under the Dev-Gleiser energy-density profile have been found out in Sec. 4 along with a special discussion on two prescriptions, in section 5, viz. (5.1) the Misner prescription, and (5.2) the Dev-Gleiser prescription are done to show that mass to size ratio clearly indicates that the model represents a quark/strange compact star. In Sec. 6 we have made some concluding remarks.

2 NON-STATIC CONFORMAL SYMMETRY

Now, we consider the interior of a star under conformal motion through non-static Conformal Killing Vector as  

\[ L_\xi g_{ij} = g_{ij;k}\xi^k + g_{kj}\xi^i_k + g_{ik}\xi^j_k = \psi g_{ij}, \]  

where \( L \) represents the Lie derivative operator, \( \xi \) is the four vector along which the derivative is taken, \( \psi \) is the conformal factor and \( g_{ij} \) are the metric potentials \[26,27,28,29,25,30\]. It is to be noted that for \( \psi = 0 \), the equation above yields the killing vector; whereas for \( \psi = \text{const.} \) corresponds to the homothetic vector. So, in general for \( \psi = \psi(x,t) \) we obtain conformal vectors. In this manner, one can perform a more general study of the spacetime geometry by using Conformal Killing Vector.

The static spherically symmetric spacetime is given by the line element (with \( G = 1 = c \) in geometrized units) \[31,32\]

\[ ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \]  

where \( \nu(r) \) and \( \lambda(r) \) are the metric potentials and functions of radial coordinate \( r \) only.

The proposed charged fluid spacetime is mapped conformally onto itself along the direction \( \xi \). Here following Herrera et al. \[33,34\] we assume \( \xi \) as non-static
but $\psi$ to be static as follows:

$$\xi = \alpha(t, r)\partial_t + \beta(t, r)\partial_r, \quad (3)$$

$$\psi = \psi(r). \quad (4)$$

The above set of equations (1)-(4) give the following expressions\[26,27,28,29,15,30\] for $\alpha$, $\beta$, $\psi$ and $\lambda$:

$$\alpha = A + \frac{1}{2}kt, \quad (5)$$

$$\beta = \frac{1}{2}Bre^{-\frac{\lambda}{r}}, \quad (6)$$

$$\psi = Be^{-\frac{\lambda}{r}}, \quad (7)$$

$$e^\nu = C^2r^2exp \left[-2kB^{-1}\int \frac{e^{\frac{\lambda}{r}}}{r}dr \right], \quad (8)$$

where $k$, $A$, $B$ and $C$ are arbitrary constants. According to Maartens and Maharaj\[26\] one can set $A = 0$ and $B = 1$ so that by rescalling we can get

$$\alpha = \frac{1}{2}kt, \quad (9)$$

$$\beta = \frac{1}{2}re^{-\frac{\lambda}{r}}, \quad (10)$$

$$\psi = e^{-\frac{\lambda}{r}}, \quad (11)$$

$$e^\nu = C^2r^2exp \left[-2k\int \frac{e^{\frac{\lambda}{r}}}{r}dr \right]. \quad (12)$$

However, one can note that the above considerations, i.e. $A = 0$ and $B = 1$, do not loss any generality. This is because the rescalling $\xi$ and $\psi$ in the following manner, $\xi \rightarrow B^{-1}\xi$ and $\psi \rightarrow B^{-1}\psi$, leaves Eq. (1) invariant.

3 THE FIELD EQUATIONS FOR ANISOTROPIC STELLAR SOURCE

In the present investigation we consider the most general energy-momentum tensor compatible with spherically symmetry in the following form:

$$T_\mu^\nu = (\rho + p_r)u_\mu u_\nu + p_r g_\mu^\nu + (p_t - p_r)\eta_\mu^\nu, \quad (13)$$
with $u^\mu u_\mu = -\eta^\mu \eta_\mu = 1$, where $\rho$ is the matter density, $p_r$ and $p_t$ are radial and the transverse pressures of the fluid respectively.

For the metric given in Eq. (2), the Einstein field equations are [35]

$$e^{-\lambda} \left[ \frac{\nu'}{r} + \frac{1}{r^2} \right] - \frac{1}{r^2} = 8\pi p_r,$$

$$\frac{1}{2} e^{-\lambda} \left[ \frac{1}{2} (\nu')^2 + \nu'' - \frac{1}{2} \lambda' \nu' + \frac{1}{r} (\nu' - \lambda') \right] = 8\pi p_t.$$  

Imposing the conformal motion, one can write the stress energy components in terms of conformal function as follows:

$$8\pi \rho = \frac{1}{r^2} (1 - \psi^2 - 2r \psi \psi'),$$

$$8\pi p_r = \frac{1}{r^2} (3\psi^2 - 2k \psi - 1),$$

$$8\pi p_t = \frac{1}{r^2} \left[ (\psi - k)^2 + 2r \psi \psi' \right].$$  

4 THE SOLUTIONS AND FEATURES OF THE FIELD EQUATIONS FOR DEV-GLEISER ENERGY-DENSITY PROFILE

Now our task is to find out solutions for the above set of the modified Einstein equation in terms of the conformal motion. However, to do so we would like to study the following case for specific energy density as proposed by Dev and Gleiser [21] for stars with the energy density in the form

$$\rho = \left( \frac{a}{r^2} + 3b \right) / 8\pi,$$

where $a$ and $b$ are constants. It is to be noted that the choice of the values for $a$ and $b$ is dictated by the physical configuration under consideration, e.g. $a = 3/7$ and $b = 0$, corresponds to a relativistic Fermi gas for ultradense cores of neutron stars [36] whereas the values $a = 3/7$ and $b \neq 0$ represent a relativistic Fermi gas with core immersed in a constant density background [21]. It can easily be observed from Eq. (20) that for large $r$ the constant density term dominates ($r_c^2 \gg a/3b$) so that the corresponding star can be modeled with shell-type feature surrounding the core.
Due to use of the above energy density, Eq. (17) will take the following form:

\[
\left( \frac{a}{r^2} + 3b \right) = \frac{1}{r^2} \left( 1 - \psi^2 - 2r\psi' \psi' \right),
\]

(21)

Solving this Eq. (21), we get

\[
\psi^2 = 1 - a - br^2 + \frac{d}{r} = g(r) \text{ (say)},
\]

(22)

where \( d \) is an integration constant.

Substituting the value of \( \psi \) in Eqs. (18) and (19), expressions for radial and tangential pressures can be obtained as

\[
p_r = \frac{1}{8\pi r^2} \left[ 2 - 3a - 3br^2 + \frac{3d}{r} - 2k\sqrt{g(r)} \right],
\]

(23)

\[
p_t = \frac{1}{8\pi r^2} \left[ 1 - a - 3br^2 + k^2 - 2k\sqrt{g(r)} \right].
\]

(24)

Fig. 1. Variation of radial pressure, \( p_r \), with respect to the parameters \( a \) and \( b \) for a fixed radial distance \( r = 0.4 \) km

In Figs. 2 - 3 we have shown the behaviour of radial and tangential pressures and whereas Fig. 1 represents the 3-dimensional presentation indicating the variation of radial pressure with respect to the parameters \( a, b \) for a fixed radius. All the curves are in accordance to the physical feasibility within the acceptable range.

Let us now consider a simplest form of barotropic equation of state (EOS) as \( p = \omega_i \rho \), where \( \omega_i \) are the EOS parameters along radial and transverse directions. Here at present as such we are not considering different forms of \( \omega \) with it’s possible space and time dependence as are available in the literature.
Thus, the EOS parameters $\omega$ can straightly be written in the following forms:

$$\omega_r = \frac{p_r}{\rho} = \frac{1}{(a + 3br^2)} \left[ 2 - 3a - 3br^2 + \frac{3d}{r} - 2k\sqrt{g(r)} \right], \quad (25)$$

$$\omega_t = \frac{p_t}{\rho} = \frac{1}{(a + 3br^2)} \left[ 1 - a - 3br^2 + k^2 - 2k\sqrt{g(r)} \right]. \quad (26)$$

In Figs. 4 and 5 we plot variations of radial and transverse EOS parameter with respect to radial distance. In which radial EOS represents expected nature with a decreasing pattern. However, transverse EOS parameter indicates that it lies between $\left(-\frac{1}{3}, 0\right)$. It is negative like exotic matter in nature, however, later we will see that all energy conditions are satisfied for real matter situation.
4.1 Anisotropic behavior

For the model under consideration the measure of anisotropy in pressures can be obtained, from Eqs. (23) and (24), as follows:

$$\Delta \equiv (p_t - p_r) = \frac{1}{8\pi r^2} \left[ k^2 + 2a - \frac{3d}{r} - 1 \right]. \quad (27)$$

It can be seen that the ‘anisotropy’ will be directed outward when $p_t > p_r$ i.e. $\Delta > 0$, and inward when $p_t < p_r$ i.e. $\Delta < 0$. This feature is obvious from Fig. (5) of our model that a repulsive ‘anisotropic’ force ($\Delta < 0$) allows the construction of a more massive stellar distribution [41].
4.2 Energy conditions

We now put and verify different energy conditions of the stellar configuration as follows:

\[
\rho \geq 0, \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \quad \rho + p_r + 2p_t \geq 0,
\]

\[
\rho > |p_r| \quad \text{and} \quad \rho > |p_t|.
\]

It is seen that Null Energy Condition (NEC), Weak Energy Condition (WEC), Strong Energy Condition (SEC) and Dominant Energy Condition (DEC) i.e. all the energy conditions for our particular choices of the values of mass and radius are satisfied which can also be observed from Fig. 7.
4.3 TOV Equation

To search equilibrium situation of this anisotropic star under different forces, we write the generalized Tolman-Oppenheimer-Volkoff (TOV) equation as [14]

\[- \frac{M_G(\rho + p_r)}{r^2} e^{\frac{\nu}{2}} - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0, \tag{28}\]

where \(M_G = M_G(r)\) is the effective gravitational mass inside a sphere of radius \(r\) which can be derived from the Tolman-Whittaker formula as

\[M_G(r) = \frac{1}{2} r^2 e^{\frac{\nu - \Delta}{2}}. \tag{29}\]

The above equation explains the equilibrium conditions of the fluid sphere due to the following hydrostatic, anisotropic and gravitational forces:

\[F_h = -\frac{dp_r}{dr} = \frac{1}{8\pi} \left[ \frac{4}{r^3} - \frac{6a}{r^3} + \frac{9d}{r^4} - \frac{4k\sqrt{g(r)}}{r^3} - \frac{k(2br^2 + d)}{r^3} \right], \tag{30}\]

\[F_a = \frac{2}{r}(p_t - p_r) = \frac{1}{8\pi} \left[ -\frac{2}{r^3} + \frac{4a}{r^3} + \frac{2k^2}{r^3} - \frac{6d}{r^4} \right], \tag{31}\]

\[F_g = -\frac{\nu'}{2}(\rho + p_r) = -\frac{1}{8\pi} \left[ \frac{2}{g(r)r} \left\{ -\frac{12a}{r^2} + \frac{2}{r^2} + \frac{15d}{r^3} - \frac{10k\sqrt{g(r)}}{r^2} + \frac{10ak\sqrt{g(r)}}{r^3} + \frac{18d^2}{r^4} + \frac{6a^2}{r^2} \right\} \right. \]

\[- \frac{15ad}{r^3} + 6ab - 6b - \frac{9bd}{r} + 6bk\sqrt{g(r)} - \frac{12dk\sqrt{g(r)}}{r^3} + \frac{4k^2 g(r)}{r^2} + \frac{a}{r^3} - \frac{1}{r^3} - \frac{3d}{2r^4} + \frac{k\sqrt{g(r)}}{r^3} \tag{32}\]

The Eq. (28) can be rewritten in the form

\[F_g + F_h + F_a = 0. \tag{33}\]

The profiles of different forces \(F_g, F_h, F_a\) are shown in Fig. 8. The figure exposes that our model of anisotropic star is in static equilibrium under the gravitational \((F_g)\), hydrostatics \((F_h)\) and anisotropic \((F_a)\) forces.
Fig. 8. The dark energy star is in static equilibrium under gravitational ($F_g$), hydrostatics ($F_h$) and anisotropy ($F_a$) forces.

4.4 Stability issue

The velocity of sound should follow the condition $v_s^2 = dp/d\rho < 1$ for a physically realistic model [42,43]. We therefore calculate the radial and transverse sound speed for our anisotropic model as follows [44]:

\[ v_{rs}^2 = \frac{dp_r}{d\rho} = \frac{1}{2a} \left[ 4 - 6a + \frac{9d}{r} - 4k\sqrt{g(r)} - \frac{k(2br^2 + d)}{r} \right], \]  
\hspace{1cm} (34)

\[ v_{ts}^2 = \frac{dp_t}{d\rho} = \frac{1}{2a} \left[ 2 - 2a + 2k^2 - 4k\sqrt{g(r)} - \frac{k(2br^2 + d)}{r} \right]. \]  
\hspace{1cm} (35)

Fig. 9. Variation of radial sound velocity with respect to radial distance.

Let us check whether the sound speeds lie between 0 and 1. For this requirement we plot the sound speeds in Figs. 8 and 9. It is observed that numerical
values of these parameters satisfy the inequalities $0 \leq v_{rs}^2$, $|v_{ts}^2| \leq 1$ everywhere within the stellar object. However, for radial sound velocity there is a prominent change over after attaining certain higher value which gradually goes down. Since the transverse sound velocity is negative (due to negative transverse pressure), we use numerical value. As sound speeds lie between 0 and 1, we should have $|v_{ts}^2| - v_{rs}^2 \leq 1$ as evident from Fig. 10.

Now, one can go through a technique for stability check of local anisotropic matter distribution as available in the literature [42]. This technique is known as the cracking concept of Herrera and states that the region for which radial speed of sound is greater than the transverse speed is a potentially stable region. Fig. 10 indicates that there is a change of sign for the term $|v_{ts}^2| - v_{rs}^2$ within the specific configuration and thus confirming that the model has a transition from unstable to stable configuration. The present stellar model gradually gets stability with the increase of the radius. However, in terms of the maximum allowable compactness (mass-radius ratio) for a fluid sphere as given by Buchdahl [45], the stability issue will be discussed later on.
4.5 Surface redshift

One can calculate the effective gravitational mass in terms of the energy density $\rho$ as

$$M_{\text{eff}} = 4\pi \int_0^R \rho r^2 dr = \frac{1}{2} R \left[ a + bR^2 \right].$$

(36)

Fig. 12. Mass vs radius relation is shown in the plot for the specified range

Therefore, the compactness of the star is given by

$$u = \frac{M_{\text{eff}}}{R} = \frac{1}{2} \left[ a + bR^2 \right].$$

(37)

The nature of variation of the above expression for compactness of the star can be seen in the Fig. 9 which is gradually increasing. We now define the surface redshift corresponding to the above compactness as

$$1 + z_s = [1 - 2u]^{-1/2} = \left[ 1 - (a + bR^2) \right]^{-1/2},$$

(38)

so that the surface redshift can be expressed as follows:

$$z_s = \left[ 1 - (a + bR^2) \right]^{-\frac{1}{2}} - 1.$$

(39)

We plot surface redshift in Fig. 9. Likewise compactness factor $u$ this has also a behaviour of gradual increase. This feature is also observed from the Table 1. The maximum surface redshift for the present stellar configuration of radius 0.62 km turns out to be $z_s = 0.35280$ (see Table 1).
Fig. 13. Variation of redshift $z$ with radial coordinate $r$

Table 1
Calculation of masses and hence compactness factors for different values of constant $b$ with $a = 3/7$ and $R = 0.62$ km (with the conversion $1 \text{ km} = 1.475 \ M_\odot$)

| $b$   | 0.0  | 0.01 | 0.02 | 0.03 |
|-------|------|------|------|------|
| $M_{\text{eff}}$ (in $\ M_\odot$) | 0.19596 | 0.19952 | 0.20335 | 0.20717 |
| $u$   | 0.21428 | 0.21817 | 0.22236 | 0.22653 |
| $z_s$ | 0.32279 | 0.33257 | 0.34257 | 0.35280 |

5 SPECIAL PHYSICAL ANALYSIS OF THE MODEL BASED ON MATTER CONTAINED

If we look at Eq. (35) as well as Eq. (36) then it will at once reveal that there are lots of information inherently hidden inside these two equations. Let us therefore examine the present model for the specified values of $a$ and $b$, appearing in Eq. (35) and Eq. (36), as follows:

5.1 The Misner prescription

For the prescription of Misner [36] i.e. $a = 3/7$ and $b = 0$, we get the effective gravitational mass as $M_{\text{eff}} = (3/14)R$ which numerically comes out to be $0.195 \ M_\odot$ so that the compactness factor becomes $u = M_{\text{eff}}/R = 0.21428$ as can be seen from Table 1. Here to estimate mass we have taken radius of the star $R = 0.62$ km by solving $p_r = 0$ at $r = R$ graphically from Fig. 11. This therefore represents a relativistic Fermi gas for ultradense cores of neutron stars as noticed in the Ref. [36].
Fig. 14. Radius of the star where \( p_r = 0 \) increases with the increase of the parameter \( d \).

5.2 The Dev-Gleiser prescription

In this prescription \(^{[21]}\), by substituting the values \( a = \frac{3}{7} \) and \( b \neq 0 \) one can get the expression for the effective gravitational mass as \( M_{\text{eff}} = \frac{1}{2}R[(\frac{3}{7}) + bR^2] \). For this case we provide here a data sheet by choosing different values for \( b \) (for Case A, \( b = 0 \) whereas for Case B, \( b \geq 0 \)) in Table 1. This corresponds to a relativistic Fermi gas with core immersed in a constant density background \(^{[21]}\).

6 CONCLUDING REMARKS

In the investigations we have observed some interesting points which are as follows:

(1) Note that though transverse pressure is negative, however, all energy conditions are satisfied. This indicates that matter distribution of the star is real i.e. star is comprising of normal matter.

(2) It is observed that the transverse sound velocity is negative (due to negative transverse pressure) so that we use numerical value. As sound speeds lie between 0 and 1, we should have \( |v^2_{ts} - v^2_{rs}| \leq 1 \) (Fig. 10).

(3) In connection to isotropic case and in the absence of the cosmological constant it has been shown for the surface redshift analysis that \( z_s \leq 2 \) \(^{[45, 46, 47]}\). On the other hand, Böhmer and Harko \(^{[47]}\) argued that for an anisotropic star in the presence of a cosmological constant the surface redshift must obey the general restriction \( z_s \leq 5 \), which is consistent with the bound \( z_s \leq 5.211 \) as obtained by Ivanov \(^{[48]}\). Therefore, for an anisotropic star without cosmolog-
ical constant the above value \( z_s = 0.3 \) seems to be within an acceptable range (Fig. 12).

(4) The radius of the star depends on the parameter \( d \), as appears in Eq. (23), with an increasing mode (Fig. 13).

(5) It is well known that the anisotropic factor \( \Delta = p_t - p_r \) should vanish at the origin. However, in the present model the energy density under consideration has an infinite central density. According to Misner and Zapolsky [36] one may use it as an asymptotic solution with a cut-off density below which the Eq. (27) will not valid.

(6) In the present model, the stability of the matter distributions comprising of the anisotropic star has been attained. We have come to conclude this by analyzing the TOV equation which describes the equilibrium condition for matter distribution subject to the gravitational force \( F_g \), hydrostatic force \( F_h \) and another force \( F_a \) due to anisotropic pressure (Fig. 8).

(7) For an overall view of the present study and also to have a physically viable model we put here the data which are available on some compact stars, e.g. Her X – 1 and SAX J 1808.4 – 3658 and Strange Star – 4U 1820 – 30 (see Table 1 of the Ref. [18]). The compactness factor \( u \) for these stars are 0.168, 0.299 and 0.332 respectively. In comparison to these data we can conclude that our model represents a star with moderate compactness and also indicates that the star might be a compact star of Strange/Quark type rather than Neutron star. This is because, in general, superdense stars with mass-size ratio exceeding 0.3 are expected to be composed of strange matter [49].

Now according to Buchdahl [45], the maximum allowable compactness for a fluid sphere is given by \( \frac{2M}{R} < \frac{8}{9} \). Our compactness values for the chosen model (see Table 1) within this acceptable range and hence provides a stable stellar configuration.

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