Numerical simulation of eigenmodes of ring and race-track optical microresonators

A V Raskhodchikov¹, D V Raskhodchikov¹, S A Scherbak¹,² and A A Lipovskii¹,²

¹ Department of Physics and Technology of Nanoheterostructures, St. Petersburg Academic University RAS, St. Petersburg 194021, Russia
² Institute of Physics, Nanotechnology and Telecommunications, Peter the Great St. Petersburg Polytechnic University, St. Petersburg 195251, Russia

Abstract. We have performed a numerical study of whispering gallery modes of ring and race-track optical microresonators. Mode excitation was considered and their spectra and electromagnetic field distributions were calculated via numerical solution of the Helmholtz equation. We pay additional attention to features of eigenmodes in race-tracks in contrast with ring resonators. Particularly, we demonstrate that modes in race-tracks are not “classic” WGM in terms of total internal reflection from a single boundary, and an inner boundary is essential for their formation. The dependence of effective refractive index of race-tracks modes on the resonator width is shown.

1. Introduction
Optical resonators with disc-like shapes are of particular interest in many areas of applied optics and photonics, e.g. lasing [1, 2, 3], sensing [4, 5], nonlinear frequency mixing [6], frequency comb generation [7], etc. The distinctive feature of disc, ring or alike shaped resonators is the existence of so-called optical whispering gallery modes (WGM) [8, 9], which are strongly localized in a resonator volume and have an extra high quality-factor [10] and therefore very narrow spectral peaks. However, there is a lack of fundamental knowledge about mode structure of resonators of any shape except disc and sphere, where the well-known sets of eigenfunctions [8] are applicable. That is why various numerical methods [11, 12, 13] are being successfully applied for simulation of WGM in resonators of other shapes. In this study we present a numerical analysis of modes of ring and race-track resonators via the finite elements method (FEM).

2. Approach
To analyze spectral and spatial structure of WGM in ring and race-track resonators we used numerical simulation in COMSOL Multiphysics environment. According to COMSOL Application Gallery [14], in the frames of FEM there are two ways to find eigenfrequencies and corresponding field distributions. The first way is to solve an eigenvalue problem directly:

\[
\nabla^2 E_n + k_n^2 E_n = 0.
\]

Despite this is the most obvious and direct choice, this approach has several disadvantages which were mentioned in [14]. First, the results can contain nonphysical modes, e.g. modes of free space. To identify them one has to examine manually the modes obtained. Second, an eigenvalue problem is solved by a memory-intensive direct solver, which can become an issue for complicated 3D problems.
We consider an alternative way [14] which is to excite a structure at different frequencies and to analyze the system response. In accordance with this approach we solve the frequency-dependent Helmholtz equation with specified external sources for each desired frequency:

\[ \nabla^2 \mathbf{E} + k^2 \mathbf{E} = \mathbf{F}(\mathbf{r}, \omega), \]

and calculate integral of electromagnetic energy density over the resonator volume. In the state where excitation frequency is close to an eigenfrequency the energy in the resonator increases greatly. This allows constructing a mode spectrum where only real, physical modes are presented. In our previous paper [15] we simulated experimental results that required solving 3D problem. Here we pay attention mostly to methodology of the problem and consider 2D model to evade calculations overloading.

First, we simulated mode spectra of a microring resonator with inner/outer radii 2.5/3.5 μm using different numbers of dipole antennas (five, four, three, two and one), randomly oriented and randomly scattered throughout the resonator volume. Essentially, we varied the form of source-function \( F(\mathbf{r}, \omega) \) in Eq. (2). The decrease in the number of dipole sources did not lead to considerable degradation of the mode spectra. Then we considered 5 different positions of a single dipole source (see the inset in figure 1): two outside the resonator and three inside (two near the sidewalls and one near the centre). Spectra presented in figure 1 show that placement the antenna outside the resonator (positions 1 and 5) results in significant spectra degradation, whereas internal positioning gives much better outcome. However, the spectrum corresponding position 2 (near the inner sidewall) is worse. Therefore, we conclude that a single dipole antenna is enough for effective excitation of most modes, and its preferable positioning is somewhere between the centre and the outer sidewall of the resonator. Note that we consider two-dimensional problem and the dipoles are also two-dimensional. Therefore, all the modes demonstrated here and below are of TM kind where \( H \)-field is out-of-plane and \( E \)-field is in-plane. Consideration of TE modes does not bring brand new results in the frames of the present study.

![Figure 1](image)

**Figure 1.** Spectra of a microring resonator calculated for different positions (marked by numbers 1-5 in the inset) of a dipole antenna. Spectra 2, 3 and 4 are shifted along y-axis for convenience.

3. Eigenmodes field distribution
In this section we demonstrate results of the simulation of eigenmodes in ring, race-track and stadium-shaped microresonators. The attention is mostly paid to features of WGM and their dependence on geometrical parameters of the resonators.

In figure 2 we present spatial distribution of \( |H| \)-field of WGM in the ring resonator with inner/outer radii 3/3.5 μm. These modes are exemplary for further discussion. Refractive index of the resonator material here and below is 3.5, and all the modes considered are in near-infrared range,
around 1250-1300 nm. We denote modes as TM(x,y) where x and y are radial and azimuthal order of a mode, respectively.

**Figure 2.** Spatial distribution of |H|-field of a first radial mode TM(1,52) (a) and a second radial mode TM(2,38) (b) in the ring resonator with inner/outer radii 3/3.5 µm.

Indeed, the modes in figure 2 are very typical whispering gallery modes. The field is distributed evenly along the resonator sidewalls. However, this high-symmetrical distribution degrades if a straight section is added to the resonator (so-called race-track shape). We simulated eigenmodes of race-track resonators with different widths w (the distance between inner and outer side walls) and straight section lengths L. The resonator width was varied from 0.15 to 0.7 µm and straight section length from 1.0 to 5.0 µm. In contrast to a ring resonator, electromagnetic field of the first radial modes in race-tracks is not concentrated near a border, but tends to oscillate between inner and outer sidewalls (so-called zigzagging) as was observed in the experiments [15]. Calculated distributions of |H|-field of first radial modes in race-track resonators with different parameters are shown in figure 3.

**Figure 3.** Field distribution of first radial modes in race-track resonators with inner radius 3 µm. (a) L=5 µm, w=0.5 µm; TM(1,80) mode; the field is distributed almost evenly in a bent section of the resonator and oscillates only in a straight section; (b) L=3 µm, w=0.6 µm; TM(1,71) mode; the wave refracts mainly from the outer sidewall of the resonator in the bent section and from both walls in the straight one; (c) L=1 µm, w=0.5 µm; TM(1,58) mode; the wave refracts from both sidewalls along all the parts of the resonator; (d) L=2 µm, w=0.64 µm; TM(1,67) mode; zigzagging is observed in the bent section only, whereas in the straight section the field is distributed almost without deviations in its direction. Arrows schematically show trajectory of the wavefront of the eigenmodes (guide for eyes only) and white lines indicate straight and bent parts of the resonators.
Here we emphasize that the zigzagging behaviour essentially depends on the resonator parameters, most typical behaviours being presented in figure 3. Knowledge about these distributions can help in optimizing systems using these resonators, like resonator-waveguide directional coupling, etc. Field distributions of second radial modes in two race-tracks differing in parameters are shown in figure 4.

**Figure 4.** Two types of field distribution in second radial modes in race-tracks with inner radius 3 μm: (a) \( L=5 \) μm, \( w=0.5 \) μm; TM(2,61) mode; intensity of energy redistribution between radial maxima is barely noticeable; (b) \( L=4 \) μm, \( w=0.5 \) μm; TM(2,56) mode; the redistribution is well-pronounced. White lines indicate straight and bent parts of the resonators.

A feature of the second radial order modes in the race-track resonators is that they can exhibit energy redistribution between first and second radial maxima. This was also observed in SNOM experiments [15]. However, the power of this redistribution can differ depending on resonator parameters, e.g. it is barely noticeable in figure 4a and well pronounced in figure 4b. Moreover, in wider resonators these modes also tend to the zigzagging. Besides, in the modeling an interesting correlation was also revealed: first radial modes do not exhibit zigzagging behavior if a resonator is narrow enough to forbid second radial modes existence.

Both presented in figures 3 and 4 phenomena are periodic, i.e. these features of eigenmodes have the other characteristic period besides the wavelength. It is worth to note that the results obtained in the 2D case are qualitatively the same as in the 3D case. In this study we demonstrate all the phenomena, which were revealed in [15] where 3D modeling and related experiments were presented. However, in this study we have performed more detailed consideration which became possible because of less computational time and powers required for the 2D modeling.

The features of WGM in race-tracks shown above make us assume that these are not “classic” WGM, which distributed along a single border due to total internal reflection phenomenon. To support this item we show the transformation of mode-field distribution under the elongation of initially disk-shaped (without space in the center) resonator in figure 5. Indeed, the images in figure 5 demonstrate that a “classic” WGM in a disc-shaped resonator (figure 5a) suffers noticeable degradation with inserting straight section and increasing its length (figures 5b,c). For longer sections WGM are almost gone. Therefore, we assume that an inner sidewall of race-track resonators is essential for the eigenmodes formation, in contrast to ring and disc resonators where WGM are distributed along the outer border only.
4. Race-track resonator: effective mode index

In this section we present a dependence of an effective mode index on a resonator width (figure 6). This kind of dependence is convenient to demonstrate what kinds of radial modes are supported by a resonator of a certain width and how well the modes are localized in the resonator volume. An effective index is defined by a proportion of electromagnetic energy in a resonator volume and an outer medium and described by equation:

\[
 n_{\text{eff}} = \frac{1}{V_{\text{res}} + V_{\text{out}}} \left[ n_{\text{res}} \int_{V_{\text{res}}} W_{\text{em}} \, dv + n_{\text{out}} \int_{V_{\text{out}}} W_{\text{em}} \, dv \right],
\]  

(3)

where \( W_{\text{em}} \) - electromagnetic energy density; \( n_{\text{res}} \) and \( n_{\text{out}} \) - refractive indices of the resonator material and the outer medium; \( V_{\text{res}} \) and \( V_{\text{out}} \) - volumes of the resonator and the outer medium, respectively.

**Figure 5.** Field distribution of an eigenmode in: (a) disc resonator; (b) stadium-shaped microresonators with straight section lengths of 0.5 µm; (c) stadium-shaped microresonators with straight section lengths of 1.5 µm. Radius of the resonators is 3.5 µm.

**Figure 6.** The modes effective refractive index vs width of a race-track resonator. Straight section length is 2 µm, inner radius is 3 µm. The dashed lines correspond to the case when a mode is barely seen in a calculated spectrum. The inset: spectral shape of the first radial modes corresponding to the dashed line; resonator widths in nm are labeled near the curves; the spectra are normalized and centered relatively their resonant wavelengths.
In a resonator narrower than 150 nm almost no modes are presented, second radial modes can be excited in a resonator wider than 350 nm and third radial – wider than 550 nm. In figure 6 areas where corresponding modes are barely seen are marked as dashed lines. At these widths the modes suffer an abrupt decrease in their quality factor – see the inset in figure 6. A spectral shape of the mode at width \( w=170 \) nm qualitatively does not differ from ones of wider resonators. Whereas even slightly decrease in the width to \( w=165 \) nm considerably broadens the resonant curve. Further narrowing of the resonator results in mode disappearance – this approximately corresponds to the area where the dashed lines break. We believe that this relates to radiative losses caused by the bent section [16].

Generally, the relation of the effective index and material index describes the part of energy propagating in the resonator material. One can see that above 80% of the first radial modes energy are localized in the resonator volume if the resonator width exceeds 170 nm. The curves in the figure 6 saturate quickly, and the saturation level is close to 3.5, which is the resonator material refractive index. This means that for wider resonators the modes are strongly localized in the resonator volume.

5. Conclusion

We numerically simulated mode spectra and corresponding electromagnetic field distributions of microring and race-track resonators and effective mode indices of a race-track using 2D model. So-called zigzagging phenomenon in the race-track resonators, when the optical field is not concentrated near one boundary but tends to oscillate between sidewalls, was considered. It was shown that the appearance, behaviour and power of the zigzagging strongly depend on the resonator geometry. It was demonstrated that second radial modes exhibit energy redistribution between first and second radial maxima. We have shown that WGM in stadium-shaped resonators drastically degrades with increasing straight section length. This makes us presume that WGM in race-tracks are not “classic” WGM and an inner sidewall of a resonator is essential for their formation.

Acknowledgements

This work was supported by Russian Foundation for Basic Research, project #16-29-03111.

References

[1] Ide T, Baba T, Tatebayashi J, Iwamoto S, Nakaoka T and Arakawa Y 2005 Opt. Express 13 (5) 1615
[2] Nadtochiy A et al. 2013 Tech. Phys. Lett. 35 830
[3] McCall S, Levi A, Slusher R, Pearton S and Logan R 1992 Appl. Phys. Lett. 60 289–91
[4] Boyd R and Heebne J 2001 Appl. Opt. 40 (31) 5742
[5] Foreman M, Swaim J and Vollmer F 2015 Adv. Opt. Photon. 7 168-240
[6] Sinha R, Karabiyik M, Ahmadivand A, Al-Amin C, Vabbina P, Shur M and Pala N 2016 J Infrared Milli Terahz Waves 37 230
[7] Del’Haye P, Schliesser A, Arcizet O, Wilken T, Holzwarth R and Kippenberg T 2007 Nature 450 1214-17
[8] Rayleigh L 1910 Philos. Mag 20 1001–04
[9] Mie G 1908 Annalen der Physik 25 377
[10] Grudinin I, Ilchenko V and Maleki L 2006 Phys. Rev. A 74 063806
[11] Borisikina S, Benson T, Sewell P and Nosich A 2003 Opt. and Quant. Electron. 35 545–59
[12] Ctyroky J, Prkna L and Rigorous M 2004 Proc. 6th Int. Conf. on Transparent Optical Networks (Wroclaw, Poland) vol 2 (IEEE) pp 281–286
[13] Mintairov A, Chu Y, He Y, Blokhin S, Nadtochiy A, Maximov M, Tokranov V, Oktaybrsky S and Merz J 2008 Phys. Rev. B 77 195322
[14] https://www.comsol.com/model/download/344161/models.woptics.fabry_perot.pdf
[15] Kryzhnovskaya N et al. 2017 J. Appl. Phys. 121 043104
[16] Unger H G 1977 Planar Optical Waveguides and Fibres (Oxford: Oxford University Press)