Energy evaluation of steam-water cycle operation with mathematical modelling application

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Abstract  In recent years, we can observe the development of the thermal diagnosis and operating control systems based on measuring techniques and mathematical modelling of processes improvement. Evaluation of the actual operating state is insufficient to make an optimal operating decisions. Thus, information about the influence of the operating parameters’ deviations from the reference state on indicators describing energy consumption of the process (for example specific heat consumption or specific energy consumption) is also necessary. The paper presents methods for generation the information about the influence of the steam-water cycle operating parameters on specific heat consumption in a turbine’s cycle. A mathematical model of steam-water cycle for a CHP (Cogeneration – also Combined Heat and Power) unit is being worked out. Methods for calculation of operating deviations with the application of correction curves and a mathematical model are described. Exemplary calculation results are presented.

Keywords:  CHP unit; Steam-water cycle; Bleed-condensing turbine; Mathematical model; Correction curves; Deviations of specific heat consumption

Nomenclature

\[ A \] – empirical coefficient
\[ B \] – empirical coefficient
\[ c \] – specific heat capacity, J/(kgK)

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\(C\) – empirical coefficient
\(D\) – empirical coefficient
\(E\) – empirical coefficient
\(\dot{G}\) – mass flow, kg/s
\(i\) – specific enthalpy, kJ/kg
\(K\) – correlation coefficient, %
\(N\) – power, MW
\(p\) – pressure, MPa
\(q\) – specific heat consumption, kJ/kWh
\(\dot{Q}\) – thermal power, MW
\(t\) – temperature, \(^\circ\)C
\(T\) – temperature, K
\(v\) – specific volume, m\(^3\)/kg
\(x\) – operating parameter

Greek symbols

\(\beta\) – specific energy consumption, kJ/kWh
\(\delta\) – losses
\(\eta\) – efficiency

Subscripts

\(b\) – base
\(B\) – boiler
\(bl\) – bleed
\(c\) – condensate
\(cal\) – calculated
\(ch\) – chemical
\(cor\) – corrected
\(el\) – electrical
\(g\) – applies to gland
\(G\) – generator
\(i\) – internal
\(i\) – number of special measurements
\(in\) – inlet
\(j\) – number of operating parameters
\(m\) – mechanical
\(me\) – electromechanical
\(mea\) – measured
\(nom\) – nominal
\(out\) – outlet
\(p\) – applies to pipelines
\(q\) – applies to specific heat consumption
1 Introduction

Contemporary thermal diagnostics and operating control systems of power units require advanced calculation modules [10]. Besides energy evaluation of operation such systems should generate information which allow to localise excessive energy consumption sources [11]. However, it involves the reference operating parameters assignment. Moreover, the influence of operating parameters’ deviations from the reference values on specific fuel chemical energy consumption has to be determined. It can be obtained basing on a mathematical model or correction curves and boiler energy characteristics.

The paper presents a semi-empirical model of the steam-water cycle for 70 MW_{el} CHP (Cogeneration – also Combined Heat and Power) unit with the bleed-condensing turbine. The model contains mass and energy balances, a model of the steam expansion line in turbine, an empirical model of the heat transfer in heat exchangers and auxiliary empirical functions. Correction curves are reviewed. Algorithm for determination the influence of operating parameters’ deviations from the reference values on specific heat consumption in the turbine’s cycle is described. Exemplary calculation results are discussed.

2 Mathematical model of steam-water cycle

Figure 1 presents a diagram of the analysed CHP unit. Typical points of its cycle are denoted. Denotations of typical points are in accordance with the ones containing special and commissioning measurements. The analysed steam-water cycle comprises a turbine (T), a condenser (COND), a district heat exchanger (DHE), a high- and low-pressure heat regeneration system (including heat exchangers HP2, HP1, LP2 and LP1), a feed-water tank (FWT) with a degasifier, a vapour cooler and pumps. The mathematical model contains a balance model, a model of a steam expansion line in a turbine and an empirical model of a heat transfer in the exchangers.

\[
\begin{align*}
s & \quad \text{saturation} \\
swc & \quad \text{applies to steam-water cycle} \\
T & \quad \text{applies to turbine} \\
v & \quad \text{applies to valve} \\
w & \quad \text{water}
\end{align*}
\]
Figure 1. Diagram of the 70 MW el CHP unit with bleed-condensing turbine.

2.1 A model of the turbine

A model of the turbine contains mass and energy balances and a model of steam expansion line for each group of stages. Mass and energy balances are (Fig. 1):

\[
\dot{G}_2 + \dot{G}_3 + \dot{G}_4 + \dot{G}_5 + \dot{G}_6 + \dot{G}_7 + \dot{G}_{g47} + \dot{G}_{g48} + \dot{G}_{g49} + \\
\dot{G}_{g57} + \dot{G}_{v72} + \dot{G}_{v74} = \dot{G}_0 + \dot{G}_{d62} ,
\]

(1)

\[
\dot{G}_{1-2}(i_1 - i_2) + \dot{G}_{2-3}(i_2 - i_3) + \dot{G}_{3-4}(i_3 - i_4) + \dot{G}_{4-5}(i_4 - i_5) + \\
\dot{G}_{5-6}(i_5 - i_6) + \dot{G}_{6-7}(i_6 - i_7) = N_{el}/\eta_{me} ,
\]

(2)

where steam mass flows in particular groups of stages are calculated from relations:

\[
\dot{G}_{1-2} = \dot{G}_1 - \dot{G}_{g46} - \dot{G}_{g47} + \dot{G}_{g48} + \dot{G}_{g49} ,
\]

(3)
and electromechanical efficiency of turbine $\eta_{me}$ from:

$$\eta_{me} = \eta_{mT} \eta_G .$$

It is assumed, that electromechanical efficiency of the turbine in whole range of the analysed operation is constant: $\eta_{me} = 96.8\%$ [13,16]. Leaks from valves $\dot{G}_{v72}, \dot{G}_{v74}$ and steam and vapour mass flows from the external glands $\dot{G}_{g46}, \dot{G}_{g47}, \dot{G}_{g48}, \dot{G}_{g49}, \dot{G}_{g57}, \dot{G}_{g59}, \dot{G}_{g62}$ are calculated on the basis of data provided by the turbine’s producer.

A course of the steam expansion line for particular groups of stages is modelled with the application of a steam flow capacity equation and internal efficiency equation. Different forms of the steam flow capacity equation [1,4,5,7–10,13,15] and internal efficiency equation [7,9,12,13] were analysed. One of the most common form of the steam flow capacity equation is Flügel’s equation which combines pressure, temperature and the mass flow of steam in group of stages. This equation is often modified adding constant element [1,10,13]. One can obtain:

$$\dot{G}^2 \frac{v_{in}}{p_{in}} = A_0 + A_1 \left[ 1 - \left( \frac{p_{out}}{p_{in}} \right)^2 \right] ,$$

where:
- $G$ – steam mass flow in group of stages,
- $p_{in}, v_{in}$ – steam pressure and specific volume at the inlet of the group of stages,
- $p_{out}$ – steam pressure at the outlet of the group of stages, $A_0,$
- $A_1$ – empirical coefficients.
Internal efficiency $\eta_i$ of the adiabatic process is modelled on the basis of the empirical functions presented in literature [7,12,13]. Among the analysed forms of the internal efficiency equation for group of stages, satisfying calculation results brings the relation:

$$\eta_i = B_0 + B_1 \frac{p_{out}}{p_{in}} + B_2 \left(\frac{p_{out}}{p_{in}}\right)^2,$$

(12)

where $B_0$, $B_1$ and $B_2$ are empirical coefficients.

An objective function in the procedure of estimation the coefficients from Eqs. (11) and (12) is assumed:

$$\sum_{k=1}^{6} \sum_{i=1}^{25} \left[ \left( \frac{\eta_{mea,i,k} - \eta_{cal,i,k}}{\eta_{mea,i,k}} \right)^2 + \left( \frac{p_{mea,i,k} - p_{cal,i,k}}{p_{mea,i,k}} \right)^2 \right] \to \min,$$

(13)

where index $k$ denotes outlet of the group of stages and index $i$ is number of special measurement.

For statistical evaluation of quality prediction for the elaborated model, a coefficient of determination $R$ and a mean squared error $\delta$ for pressure and specific enthalpy at the outlet of the group of stages are used:

$$R = \frac{\sum_{i=1}^{n} (Y_i - \bar{Y})(\hat{Y}_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (Y_i - \bar{Y})^2 \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2}},$$

(14)

$$\delta = \sqrt{\frac{\sum_{i=1}^{n} (\hat{Y}_i - Y_i)^2}{n - m - 1}},$$

(15)

where:

- $Y_i$ – measurement,
- $\bar{Y}$ – mean value,
- $\hat{Y}_i$ – estimated value,
- $n$ – number of measurements,
- $m$ – number of estimated coefficients, $Y = p, i$.

### 2.2 A model of heat exchangers

Models of high- and low-pressure regenerative heat exchangers contain: mass and energy balances, empirical functions describing heat transfer in
the exchangers, empirical relations determining pressure losses between the bleed and exchanger, and empirical relations describing condensate subcooling. Figure 2 presents a diagram of a heat exchanger.

![Figure 2. Diagram of a heat exchanger.](image)

The basic equation in the mathematical model of the heat exchanger is the energy balance. It has a form (Fig. 2):

$$\dot{G}_{bl}(i_{bl} - i_c)\eta = \dot{G}_w c_w (T_{w out} - T_{w in}) .$$

(16)

It has been assumed that efficiency of the heat exchanger $\eta$ is 99% [6].

The analytical description of heat transfer in a real exchanger is difficult. For thermal diagnostics needs, instead of analytical determination, a thermal load method can be applied [2,3,9,13,14]. The description of the heat transfer process is replaced by a so-called load factor $\Phi$. For the heat exchanger, where steam condensation proceeds (Fig. 2) a relation for the load factor $\Phi$ has a form:

$$\Phi = \frac{T_{w out} - T_{w in}}{T_a - T_{w in}} .$$

(17)

Using a load factor, the water temperature at outlet of the heat exchanger $T_{w out}$ at variable load of heat exchanger can be determined:

$$T_{w out} = T_{w in} + \Phi(T_a - T_{w in}) .$$

(18)

There is linear function dependent on heated water mass flow $\dot{G}_w$ applied to approximation of the load factor

$$\Phi = C_0 + C_1 \dot{G}_w ,$$

(19)
where \( C_0 \) and \( C_1 \) are empirical coefficients.

Next equation in the model of the heat exchanger is a relation describing pressure losses \( \delta p \) between the turbine’s bleed and exchanger. It can be written down in a form \([13]\):

\[
\delta p = D_0 \nu_{\text{bl}} \dot{G}_{\text{bl}}^2 ,
\tag{20}
\]

where \( D_0 \) is empirical coefficient.

Condensate temperature at the outlet of the heat exchanger \( T_c \) is lower than saturation temperature \( T_s \) for pressure in exchanger about a so-called condensate subcooling \( \Delta T \). Linear empirical relation describing a condensate subcooling as a function of thermal power of an exchanger \( \dot{Q} \) is applied \([13]\):

\[
\Delta T = E_0 + E_1 \dot{Q} ,
\tag{21}
\]

where \( E_0, E_1 \) are empirical coefficients.

There are unknown empirical coefficients in equations (19), (20) and (21). They are estimated with the application of the least squares method on the basis of special measurements. As an objective function, it is assumed:

\[
\sum_{i=1}^{25} \left[ \left( \frac{\dot{G}_{\text{mea},i}^{\text{bl}} - \dot{G}_{\text{cal},i}^{\text{bl}}}{\dot{G}_{\text{mea},i}^{\text{bl}}} \right)^2 + \left( \frac{T_{\text{mea},i}^{\text{out}} - T_{\text{cal},i}^{\text{win}}}{T_{\text{mea},i}^{\text{out}}} \right)^2 \right] \rightarrow \min .
\tag{22}
\]

For statistical evaluation of prediction for obtained models of the heat exchangers, a coefficient of determination \( R \) and a mean-squared error \( \delta \) of bleed steam mass flow and heated water temperature are used.

3 Correction curves

Performing commissioning measurements for a turbine, the aim is to obtain values of operating parameters similar to the nominal ones for current load of power unit. During actual work of a unit, operating parameters usually differ from nominal values. To calculate the influence of operating parameters deviations from nominal values on factors, describing energy consumption of the process (for example specific heat consumption in a turbine’s cycle), correction curves, provided by turbine’s producer, are applied.

For bleed-condensing turbines, there are usually correction curves for electrical power, for specific heat consumption in a turbine’s cycle and for
district heating exchanger’s provided by a producer. Corrected values of specific heat consumption $q_{T}^{\text{cor}}$ are determined on the basis of relation [13]:

$$q_{T}^{\text{cor}} = q_{T}^{\text{mea}} / K_{q}.$$  \hspace{1cm} (23)

Correlation coefficient for specific heat consumption in a turbine’s cycle $K_{q}$ is calculated using the formula:

$$K_{q} = \prod_{j} (1 + 0,01K_{qj}).$$  \hspace{1cm} (24)

Values of correlation coefficients are determined according to a turbine’s operation mode. For example in a condensing mode of the analysed turbine the relation is:

$$K_{k} = (1 + 0,01K_{p0})(1 + 0,01K_{t0})(1 + 0,01K_{G60})(1 + 0,01K_{t60}),$$  \hspace{1cm} (25)

where $K_{p0}, K_{t0}$ are correlation coefficients for live steam pressure and temperature, while $K_{G60}, K_{t60}$ are correlation coefficients for cooling water mass flow and temperature at the inlet of a condenser.

Figure 3. Correction curve of specific heat consumption in bleed-condensing turbine on live steam temperature – condensing mode.

Figures 3–6 present correction curves for specific heat consumption in the turbine’s cycle in condensing mode dependent on deviations of live steam temperature, live steam pressure, cooling water temperature at the inlet of
a condenser and cooling water mass flow. Particular curves in Figs. 3–6 concern different live steam mass flows: curve $a$ – 40 kg/s, $b$ – 56 kg/s, $c$ – 72.2 kg/s, $d$ – 75 kg/s. Curve $e$ applies to full-opened flow regulating valves. Nominal operating parameters are: $t_0^{\text{nom}} = 535^\circ\text{C}$, $p_0^{\text{nom}} = 13 \text{ MPa}$, $t_{60}^{\text{nom}} = 17^\circ\text{C}$, $\dot{G}_{60}^{\text{nom}} = 2700 \text{ kg/s}$. 

Figure 4. Correction curve of specific heat consumption in bleed-condensing turbine on live steam pressure – condensing mode.

Figure 5. Correction curve of specific heat consumption in bleed-condensing turbine on cooling water mass flow – condensing mode.
Correction curves of specific heat consumption in the analysed bleed- condensing turbine's cycle on live steam temperature and live steam pressure, presented in Figs. 3 and 4, have the course close to linear, whereas correction curves of specific heat consumption on cooling water mass flow and temperature, presented in Figs. 5 and 6, have the course closer to polynomial with order 2.

4 Algorithm for calculation the operating deviations of specific heat consumption

A gross specific energy fuel consumption index is described by relation:

$$\beta = \frac{\dot{E}_{ch}}{N_{el}} = \frac{\dot{Q}_T}{\eta_{EK}\eta_p N_{el}} = \frac{q_T}{\eta_{EB}\eta_p},$$  \hspace{1cm} (26)$$

where:

$\dot{E}_{ch}$ – fuel chemical energy,

$N_{el}$ – electrical power,

$\dot{Q}_T$ – heat transferred in boiler,
\( \eta_{EB} \) – energy efficiency of boiler,
\( \eta_p \) – efficiency of pipelines (taking into account leaks and heat losses in pipelines),
\( q_T \) – specific heat consumption in turbine’s cycle.

Specific heat consumption in a turbine’s cycle \( q_T \) is a function of operating parameters of a steam-water cycle \( x_{swc} \):

\[
q_T = q_T(x_{swc1}, x_{swc2}, \ldots, x_{swcn}).
\] (27)

Expanding (27) in Taylor series, surrounded by the reference values of operating parameters, omitting higher derivatives one gets:

\[
q_T - q_T^b = \sum_{i=j}^n \left( \frac{\partial q_T}{\partial x_{swcj}} \right)^b (x_{swcj} - x_{swcj}^b) = \sum_{j=1}^n \Delta q_{Tj},
\] (28)

where \( \Delta q_{Tj} \) denotes deviation of specific heat consumption in a turbine’s cycle caused by deviation of \( j \)-th steam-water cycle operating parameter from a reference value.

In algorithm with the mathematical model of the steam-water cycle application, Eq. (28) is used directly, whereas partial derivative \( \frac{\partial q_T}{\partial x_{swcj}} \) is replaced by the difference quotient in a form:

\[
\frac{\partial q_T}{\partial x_{swcj}} = \frac{q_T(x_{swcj}) - q_T^b(x_{swcj}^b)}{x_{swcj} - x_{swcj}^b},
\] (29)

where \( q_T(x_{swcj}) \) is specific heat demand in turbine’s cycle calculated using mathematical model when value of \( j \)-th steam-water cycle operating parameter differs from base value \( x_{swcj}^b \) on actual value \( x_{swcj} \).

In algorithm with the correction curves application, deviation of specific heat consumption in turbine’s cycle \( \Delta q_{Tj} \) is calculated on the basis of relations (23) and (24). Equations (23) and (24) can be written down in a form:

\[
q_T = q_T^b \prod_{j=1}^n (1 + 0.01K_q^j).
\] (30)

Expanding (30) in Taylor series, surrounded by the base values of steam-water cycle operating parameters, omitting higher derivatives one gets

\[
q_T - q_T^b = \sum_j \frac{0.01K_q^b}{1 + 0.01K_q^j} \left( \frac{\partial K_q^j}{\partial x_{obj}} \right)^b (x_{swcj} - x_{swcj}^b) = \sum_j \Delta q_{Tj}.
\] (31)
Taking into account Eq. (28) and (30), deviation of specific heat consumption in the turbine’s cycle $\Delta q_{Tj}$ can be calculated from:

$$\Delta q_{Tj} = q_T \frac{0.01K^b_q}{1 + 0.01K^b_q} \left( \frac{\partial K_{qj}}{\partial x_{swej}} \right)^b (x_{swej} - x_{swej}^b).$$

(32)

Partial derivatives $\partial K_{qj}/\partial x_{swej}$ can be calculated, using difference quotient in a form:

$$\frac{\partial K_{qj}}{\partial x_{swej}} = \frac{K_{qj}(x_{swej}) - K_{qj}^b(x_{swej}^b)}{x_{swej} - x_{swej}^b}.$$  

(33)

Correlation coefficients $K_{qj}$ for a base and reference state of operation are determined with the application of correction curves, presented in part 3.

5 Calculation results

Using the worked out method for determining the operating deviations, the influence of live steam temperature, cooling water mass flow and cooling water temperature at the inlet of condenser on specific heat consumption in the bleed-condensing turbine’s cycle (in condensing mode) is calculated. Exemplary calculation results are presented in Figs. 7–9. Solid line presents results obtained with the application of correction curves, whereas points present results from the mathematical model. Moreover, the base and reference states are denoted.

Basing on Figs. 7–9 it can be stated that deviations of specific heat consumption in the analysed bleed-condensing turbine in a condensing mode on all concerned operating parameters obtained with the application of the mathematical model have high accordance with the calculations based on the correction curves. It applies to each analysed variation range of each operating parameter. The greatest differences, 14 kJ/kWh, appear at cooling water temperature at the inlet of the condenser variations. The influence of live steam temperature in the analysed range is lower and it does not exceed 2.5 kJ/kWh. In case of cooling water mass flow, the differences are minimal (below 1 kJ/kWh).
Figure 7. Deviation of specific heat consumption $\Delta q_{T_o} = q_{o} - q_{b}^o$ caused by deviation of live steam temperature from base state in turbine’s condensing mode.

Figure 8. Deviation of specific heat consumption $\Delta q_{GCCW} = q_{GCCW} - q_{GCCW}^o$ caused by deviation of cooling water mass flow from base state in turbine’s condensing mode.

6 Conclusion

For evaluation the energy consumption in heat and power generation process, a specific fuel energy consumption factor is applied. For each operating state of a CHP unit, values of operating parameters, for which fuel consumption is minimal, can be determined. This operating state is called
the reference state. During actual operation of a CHP unit values of operating parameters differ from the reference ones. For decreasing energy consumption of the heat and power generation process important is information about the influence of operating parameters’ deviations obtained from the reference state on deviation of specific energy consumption factor.

Broadest information brings the application of the mathematical model. However, construction of the model demands knowledge and skills in mathematical modelling and it is also a time-consuming process. If there is information about correction curves and boiler energy characteristics, both can be applied instead of a mathematical model. Nevertheless, a set of operating parameters, for which influence on energy consumption can be determined is limited significantly. The limitations result from a set of correction curves provided by a producer of a turbine and corresponding boiler characteristics. Thus, operating deviations calculations on the basis of a mathematical model generate more useful information for operation and maintenance (OM) services of CHP units.
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