Coulomb drag in compressible quantum Hall states

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We consider the Coulomb drag between two layers of two-dimensional electronic gases subject to a strong magnetic field. We first focus on the case in which the electronic density is such that the Landau level filling fraction $\nu$ in each layer is at, or close to, $\nu = 1/2$. Discussing the coupling between the layers in purely electronic terms, we show that the unique dependence of the longitudinal conductivity on wave-vector, observed in surface acoustic waves experiments, leads to a very slow decay of density fluctuations. Consequently, it has a crucial effect on the Coulomb drag, as manifested in the transresistivity $\rho_D$. We find that the transresistivity is very large compared to its typical values at zero magnetic field, and that its temperature dependence is unique -- $\rho_D \propto T^{4/3}$. For filling factors at or close to $1/4$ and $3/4$ the transresistivity has the same $T$-dependence, and is larger than at $\nu = 1/2$. We calculate $\rho_D$ for the $\nu = 3/2$ case and propose that it might shed light on the spin polarization of electrons at $\nu = 3/2$. We compare our results to recent calculations of $\rho_D$ at $\nu = 1/2$ where a composite fermion approach was used and a $T^{4/3}$-dependence was obtained. We conclude that what appears in the composite fermion language to be drag induced by Chern-Simons interaction is, physically, electronic Coulomb drag.

I. INTRODUCTION AND SUMMARY OF RESULTS

The advancement of fabrication and lithography techniques of semi-conductors have made it possible to investigate bi-layer systems made of two electronic two-dimensional layers at very close proximity (several hundred Angstroms) in a very controlled manner. Among the controllable parameters are the tunneling of electrons between the layers, the density of electrons at each layer, and external parameters such as the magnetic field and temperature. Moreover, it is also possible to make separate electrical contacts to the two layers. Experiments carried out on such systems have revealed a wealth of interesting phenomena, and have also stimulated an active theoretical study.

One of the measurements performed on systems of two electronic layers at close proximity is that of the transresistivity. For that measurement a current $I_1$ is driven in one layer, while the current in the other layer, $I_2$, is kept zero. Due to the interactions between electrons at different layers, momentum is transferred from the current carrying layer to the second one. As a consequence of that momentum transfer, one needs to apply a voltage on the second layer, $V_2$, to keep $I_2$ zero. The ratio $-V_2/I_1$ is defined as the transresistivity, $\rho_D$. The transresistivity was measured experimentally in Ref. 1 and studied theoretically at Ref. 2. Transresistivity in a magnetic field was studied in Ref. 3. It was found that although $\rho_D$ is a dc measurement, it reflects the response of the two layers to driving forces of finite wave vector $q$ and finite frequency $\omega$. In the limit of weak coupling between the layers, the resistivity was found to be mostly due to the Coulomb interaction between electrons at the two layers, and to be given by the following expression, valid both in the absence and presence of a magnetic field.

\[
\rho_D = \frac{1}{2(2\pi)^2} \frac{\hbar}{c^2} \int \frac{dq}{(2\pi)^2} \int_0^{\infty} \frac{d\omega}{\sinh^2 \frac{n_2}{2T}} q^2 |U_{sc}(q, \omega)|^2 \text{Im} \Pi^{(1)}(q, \omega) \text{Im} \Pi^{(2)}(q, \omega) ,
\]

where $U_{sc}$ is the screened inter-layer Coulomb interaction, $\Pi^{(i)}$ is the single-layer density-density response function of the $i$th layer, $n_i$ is the average density of electrons at layer $i$ and $T$ is the temperature. Both $U_{sc}$ and $\Pi$ are defined precisely in the next section, where Eq. (1) is studied in detail. There we also comment on the validity of Eq. (1) in the presence of a magnetic field.

In this paper we consider the transresistivity, in the limit of weak inter-layer coupling, of two systems of two dimensional electron gases (2DEG) in a strong magnetic field, where the Landau level filling fractions of the two layers, $\nu_1$ and $\nu_2$, are smaller than one and the electrons form a compressible state. The most prominent example is that of $\nu_1$ and $\nu_2$ being equal to, or close to, $1/2$. In Section II D we consider the $\nu_1 = \nu_2 = 3/2$ case. Our work is motivated by several experimental and theoretical works that discovered a unique response of the $\nu = 1/2$ state to finite $q, \omega$ driving forces. A preliminary experimental study of Coulomb drag in the regime of a partially filled Landau level was recently carried out by Eisenstein et al. Ref. 4.

Most theoretical studies of the single layer $\nu = 1/2$ state have used the composite fermions approach, as introduced in Refs. 2, 3, to calculate electronic quantities,
such as the electronic linear response functions. In our study of the transresistivity we use the composite fermion approach to obtain information regarding the electronic single layer density-density response function, but discuss the coupling between the layers in purely electronic terms. This approach makes the physical picture underlying the transresistivity quite transparent. In particular, it allows us to point out that the unique features of the transresistivity in the case we study are closely related to the unique $q$-dependence of the electronic longitudinal conductivity, as observed in surface acoustic waves measurements. Both phenomena reflect the slow relaxation of density fluctuations in the $\nu = 1/2$ state.

As discussed in detail below, based on the use of Eq. (3), with the $\Pi$ and $U_{sc}$ appropriate for the case at hand, we find,

1. For $\nu_1 = \nu_2 = 1/2$, the leading order temperature dependence of the transresistivity is

$$
\rho_D = 0.825 \frac{\hbar}{e^2} \left( \frac{T}{T_0} \right)^{3/2} \frac{1}{1 + \frac{12\alpha + 6\alpha^2}{1 + \alpha}} \left( 4\pi n \right)^{1/2} d \left( \frac{T}{T_0} \right)^{3/2} + O(T^2 \log T),
$$

with

$$
T_0 = \frac{4\pi e^2 n d}{\phi^2 \varepsilon} \left( 1 + \alpha \right).
$$

In Eq. (3) $\phi = 2$ for $\nu = 1/2$ and $\alpha^{-1} = \frac{2\pi e^2 d}{\varepsilon} \frac{dn}{d\mu}$, where $\frac{dn}{d\mu}$ is the thermodynamical compressibility of the $\nu = 1/2$ state and $\varepsilon$ is the dielectric constant. Essentially, $\alpha$ is the ratio of the Thomas-Fermi screening length of the $\nu = 1/2$ state to the separation between the layers, $d$. For realistic numbers, $\alpha$ is small compared to one. In other words, the screening length is smaller than $d$. The temperature scale $T_0$ is of the order of $190^\circ K$. A plot of the calculated $\rho_D$ as a function of temperature, at $\nu = 1/2$ is given in Fig. 1.

The effect of disorder is neglected in the derivation of Eq. (3), and this neglect is justified as long as $\rho_{xx} \ll \frac{1}{e^2} (T/T_0)^{1/3}$, where $\rho_{xx}$ is the single layer diagonal resistivity, and as long as $(T/T_0)^{1/3} \ll 1$.

The transresistivity at $\nu = 1/2$, as given by Eq. (3), is much larger than at $B = 0$. The latter is given by

$$
\rho_D(B = 0) = 0.118 \frac{\hbar}{e^2} \left( \frac{T\varepsilon}{e^2 k_F^2 d^2} \right)^2,
$$

where $k_F$ is the Fermi wavevector. Interestingly, in the limit of strong screening, both $\rho_D$ at $B = 0$ and $\rho_D$ at $\nu = 1/2$ are determined by a combination of geometrical factors (interlayer distance and inter-particle distance) and the strength of the Coulomb interaction. Both are independent of the mass, whether bare or effective.

Eq. (2) holds also for the $\nu = 1/4$ and $\nu = 3/4$ cases, with one modification, namely $\phi = 4$. Thus, at low $T$, if the filling fraction is varied from $\nu = 1/2$ to $\nu = 1/4$ or $\nu = 3/4$, keeping the electron density fixed, the transresistivity is expected to increase by a factor smaller, but close to, $4^{4/3}$.

While this paper was in preparation, two other related works were submitted for publication [14, 15]. Both have considered the case $\nu_1 = \nu_2 = 1/2$ and have found a temperature dependence of $\rho_D \propto T^{4/3}$. Our analysis agrees with this result. Both works have used a composite fermions approach to analyze the transresistivity, and in one the unique temperature dependence was interpreted as reflecting the non-Fermi liquid properties of the composite fermions. As explained below, our understanding of this temperature dependence is different.

2. For $\nu_1 = \nu_2 = 1/2$ we study the temperature dependence of $\rho_D$ for electron-electron interaction of the form $V(r) \propto \frac{1}{|r|^\eta}$. Although this study is mostly of theoretical value, it was proven to be of interest in the subject of the composite fermion Fermi liquid theory [3, 4]. We find the leading term to be,

$$
\rho_D \propto \begin{cases} 
T^{4/3} & \text{for } 0 < \eta < 1 \\
T^{2} & \text{for } 1 < \eta < 2.
\end{cases}
$$

FIG. 1. The transresistivity as a function of temperature for $\nu = 1/2$, $n = 1.4 \times 10^{11}$ cm$^{-2}$, and $d = 300\AA$. The solid lines are calculations assuming $m^* = 4m_b$, while the dashed lines assume $m^* = 12m_b$. The corresponding values of $\alpha$ are 0.11 and 0.037. The dotted lines are numerical calculations, while the undotted ones are plots of Eq. (3)
3. For the case of \( \nu_1 = \nu_2 \) and both near 1/2 we find the leading temperature dependence of \( \rho_0 \) to be \( T^{4/3} \). Based on a semiclassical approximation, we find that this statement holds as long as the composite fermion cyclotron radius in each layer, \( R^c_n \propto |\nu - 1/2|^{-1} \), is larger than the distance a composite fermion traverses without being scattered by impurities. The effect of fermion-fermion scattering on \( \rho_0 \) is presently under study.

4. For two layers of densities \( n_1 = \bar{n} + \frac{1}{2}\Delta n \) and \( n_2 = \bar{n} - \frac{1}{2}\Delta n \), with the average density corresponding to \( \nu = 1/2 \), we find that for low temperature and small \( \Delta n \),

\[
\rho_0(\Delta n) = \rho_0(\Delta n = 0) \left[ 1 + \frac{7}{48} \left( \frac{\Delta n}{\bar{n}} \right)^2 \right].
\] (6)

5. Finally, we study the case of \( \nu_1 = \nu_2 = 3/2 \). The spin polarization of the single layer \( \nu = 3/2 \) state is, at the moment, not well understood \([17]\). We calculate \( \rho_0 \) for the cases of complete spin polarization and zero spin polarization, and find that the transresistivity for a completely unpolarized \( \nu = 3/2 \) state is larger by a factor close to \( 2^{2/3} \) compared to the transresistivity of a fully polarized \( \nu = 3/2 \) state.

Thus, if the picture emerging from Ref. \([17]\) is correct, and the \( \nu = 3/2 \) is not spin-polarized, but may become polarized by an application of a magnetic field \( B_{\|} \), parallel to the layer, a measurement of \( \rho_0 \), as a function of \( B_{\|} \) should be sensitive to that change of polarization.

By passing, we also note that a change of polarization should be reflected also in surface acoustic waves measurements similar to the ones reported by Willet \([18]\).

Before turning to describe the physical picture emerging from our study, we comment on the approximations made. Eq. (1) is derived under the assumption of weak inter-layer coupling. All throughout this paper we neglect the finite thickness of the two layers. Since our main findings result from small-\( q \) behavior, we expect the finite thickness not to affect any of the qualitative features outlined above, but rather to modify prefactors only. Both of these approximations are conventionally taken in Coulomb drag studies \([1]\).

Approximations special to the case at hand are taken when the single layer \( \Pi(q, \omega) \) is calculated. As explained below, we carry our calculations in the modified RPA (mRPA) approach to the composite fermions Fermi liquid theory \([19]\). We find, however, that our main findings turn out to be independent of the composite fermion effective mass \( m^* \) and Landau parameter \( f_1 \) introduced within mRPA, and thus can be expected to be insensitive to the details of the approximation.

The structure of the paper is as follows: in Section II we describe the physical picture behind our results. In Section III we give the detailed calculations. Section IV summarizes the paper. In the appendix we discuss the relation of our approach to the composite fermion approach to the problem, as employed in \([14]\) and \([15]\).

II. THE PHYSICAL PICTURE

In this section we describe the physical picture behind the results summarized above. We try to point out clearly what features of our analysis are independent of the magnetic field, and, in contrast, what is unique to \( \nu = 1/2 \) and \( \nu \) close to 1/2. To this end, we start by reviewing the general theoretical considerations leading to Eq. (1) and the precise definition of each term in this equation. After doing that, we focus on the \( \nu = 1/2 \), and show that the unique features we find in the transresistivity result from the slow relaxation of density fluctuations in that state.

A. General theoretical arguments regarding the transresistivity

The physical picture behind Eq. (1) was discussed by various authors \([1] \). Here, we elaborate on some points that are essential to our discussion. Drag resistivity stems from scattering events between electrons of different layers. This scattering results from the screened inter-layer Coulomb interaction \( U_{sc} \). Scattering events transfer momentum \( \hbar q \) and energy \( \hbar \omega \) between the layers. Phase space availability effectively constrains the energy transfer to be smaller than the temperature, as implied by the \( 1 \) factor. The function \( \Pi(q, \omega) \) is the single layer density-density response function, irreducible with respect to the Coulomb interaction. It is defined in the following manner: suppose that one applies a weak scalar potential \( V_{ext}(q, \omega) \) on a single layer (in the absence of a second one). In linear response this potential leads to a particle density response \( \rho(q, \omega) \), which induces a Coulomb potential \( V_{ind} = \frac{2\pi e^2}{q} \rho(q, \omega) \). The response function \( \Pi(q, \omega) \) is the ratio of the induced density to the total electric potential \( V_{tot} \equiv V_{ext} + V_{ind} \).

\[
\rho(q, \omega) = -\Pi(q, \omega) V_{tot}(q, \omega).
\] (7)

Since current conservation implies \( \omega \rho = q \cdot J \) and the electric field is related to the potential by a gradient operator, the longitudinal conductivity \( \sigma(q, \omega) \), relating charge current to the electric field, is related to \( \Pi(q, \omega) \) through

\[
\sigma(q, \omega) = -\frac{i}{q^2} \frac{\omega}{\hbar} \Pi(q, \omega).
\] (8)

The response function \( \Pi(q, \omega) \) also determines the screening of the Coulomb potential, both inter-layer and
intra-layer. We denote the bare intra-layer Coulomb interaction by $V_b(q)$ and the bare inter-layer one by $U_b(q)$. It is convenient to describe the interaction by $2 \times 2$ matrices, where the two entries correspond to the two layers. The interlayer screened Coulomb interaction is the off-diagonal element of the matrix $\hat{V}_{sc}$. The latter satisfies the equation

$$\hat{V}_{sc} = \hat{V}_{bare} \frac{1}{1 + \Pi \hat{V}_{bare}},$$

with

$$\hat{V}_{bare} \equiv \begin{pmatrix} V_b & U_b \\ U_b & V_b \end{pmatrix} \quad \text{and} \quad \Pi \equiv \begin{pmatrix} \Pi_1 & 0 \\ 0 & \Pi_2 \end{pmatrix}.$$  

(9, 10)

Note the notation: $\hat{V}, \hat{V}$ are matrices, $\Pi_{1(2)}$, $V_b$ and $U_b$ are scalar functions.

We now limit ourselves to the case of two identical layers, which is our main focus in this paper. For that case $\Pi_1 = \Pi_2$, such that the subscript may be omitted. Then, the screened inter-layer potential can be written as

$$U_{sc} = \frac{1}{2} \left( \frac{V_b + U_b}{V_b + U_b} - \frac{V_b - U_b}{2} \right).$$

(11)

Eq. (11) can be understood in the following way. The screened potential is the potential induced in one layer by a test charge of magnitude one in the other layer, together with the screening cloud it creates. One can view a test charge in one layer as a sum of two charge distributions—a symmetric distribution, with the same test charge in both layers, and an antisymmetric distribution, with opposite test charges in the two layers. The bare Coulomb potential induced by a symmetric distribution is $V_b + U_b$, while the bare potential induced by an anti-symmetric distribution is $V_b - U_b$. The two terms in (11) are the screened Coulomb potentials induced by the symmetric and anti-symmetric distributions. For the case we consider here, of two coupled two-dimensional systems, $V_b = \frac{2\pi e^2}{q}$ and $U_b = \frac{2\pi e^2}{q} e^{-qd}$, such that, for $qd \ll 1$, $V_b + U_b \approx 2V_b$ and $V_b - U_b \approx 2V_b qd$. A symmetric distribution induces a screened Coulomb potential, while an anti-symmetric one induces a screened Coulomb potential of electric dipoles, whose dipole moment is $ed$.

We now focus our attention on the product $\text{Im} \Pi(q, \omega)|U_{sc}(q, \omega)|$, a product that appears in Eq. (1). Using Eq. (11), and suppressing, for brevity, the $q, \omega$ arguments of $V_b, U_b$ and $\Pi$, this product may be written as

$$\text{Im} \Pi(q, \omega)|U_{sc}(q, \omega)| = -\text{Im} \Pi^{-1} \left| \frac{U_b}{(\Pi^{-1} + V_b + U_b)(\Pi^{-1} + V_b - U_b)} \right|. $$

(12)

By Eq. (11), the transresistivity will be determined by $q, \omega$ for which the product $\text{Im} \Pi(q, \omega)|U_{sc}(q, \omega)|$ is large. Thus, it is interesting to examine its poles. As we see below, $\text{Im} \Pi^{-1}(q, \omega)$ does not have any poles, and therefore the only poles are the solutions to the equations,

$$i\omega - \frac{q^2}{e^2} \sigma(q, \omega)(V_b(q) + U_b(q)) = 0$$

(13)

$$i\omega - \frac{q^2}{e^2} \sigma(q, \omega)(V_b(q) - U_b(q)) = 0.$$  

(14)

To illuminate the physical significance of the poles, we expressed $\Pi$ in terms of the conductivity $\sigma$. The solutions to (13) and (14) are the dispersion relations for charge density modes. Suppose that at time $t = 0$ one sets up a symmetric charge modulation, $\rho_0(q)$ in each of the two layers, and observes its time evolution. This charge modulation induces, in each of the layers, an electric field given by $E = -i\sigma(q, \omega)(V_b(q) + U_b(q))$. The electric field generates, in turn, a current $\mathbf{j} = \sigma E$ in each of the layers. Since $\frac{\partial \rho}{\partial t} = -i\mathbf{j} \cdot \mathbf{E}$, the current leads to a decay of the charge modulation. The dispersion relation corresponding to that decay is the one given in Eq. (13). The second pole results from an anti-symmetric charge distribution, i.e., a distribution in which at $t = 0$ the charge modulation in the two layers is $\pm \rho(q)$. In that case, the electric field induced in the two layers is $\pm E = -i\sigma(q, \omega)(V_b(q) - U_b(q))$. Again, this electric field leads to a decay of the charge modulation, following the dispersion relation given in Eq. (14).

In both cases, when $\sigma$ is real, the density modes are damped, and density modulations decay in time. When $\sigma$ is purely imaginary, the solutions to Eqs. (13) and (14) are the plasma modes of the double layer system.

**B. What is unique in the transresistivity of the $\nu = 1/2$ case?**

As pointed out by Zheng and MacDonald [8], Eq. (1) is valid both in the absence and in the presence of a magnetic field. A simple way of understanding that point is by following the derivation of (1), as given by Zheng and MacDonald. Upon doing that, one finds that the derivation is unchanged in the presence of a magnetic field. Thus, the study of drag resistivity for the system at hand is reduced to a study of the density-density response function $\Pi$ for electrons at $\nu = 1/2$. And it is there where the behavior of electrons at $\nu = 1/2$ differs strongly from that of electrons at $B = 0$.

The calculation of $\Pi(q, \omega)$ is carried out, following a similar calculation by Halperin, Lee and Read (HLR) [13], in the next section, and is summarized by Eq. (15). For the description of the physical picture it is enough to quote the result for $q \ll k_F$ and $\omega \ll qv_F$ (with $v_F$ being the Fermi velocity):

$$\Pi(q, \omega) \approx \frac{q^3}{q^3 \left( \frac{d\rho}{dt} \right)^{-1} - 2\pi i h \delta^2 \omega k_F},$$

(15)
where $\Pi(q \to 0, \omega = 0) \equiv \frac{4\pi}{\omega}$ is, by definition, the thermodynamic electronic compressibility of the $\nu = 1/2$ state, and $k_F \equiv \sqrt{4\pi n}$.

This form of $\Pi$ is unique, and deserves a few comments: firstly, it is similar to the form of $\Pi$ for a diffusive system at $B = 0$, namely, $\frac{d\sigma}{d\omega} \propto \frac{D}{D^2 q}$, but with the effective “diffusion constant” being linear in $q$. Eq. (13) describes, then, a very slow diffusion, where the diffusion constant vanishes for $q \to 0$. This slow diffusion affects the dispersion relation of the two density relaxation poles of the screened Coulomb potential, defined in Eqs. (13) and (14). Those become $\omega \propto q^2[B_0(q) + B_0(q)]$ and $\omega \propto q^2[B_0(q) - B_0(q)]$, and describe a slow relaxation of density fluctuations, that eventually leads to a large drag resistivity. Secondly, as revealed by surface acoustic waves experiments, explained by the composite fermion theory, and manifested in Eq. (15), the $q$-dependence of the longitudinal conductivity of electrons at $\nu = 1/2$ is very different from that of electrons at zero magnetic field. At $B = 0$ the electronic conductivity is usually at its maximum at $q = 0$, and decreases with increasing $q$. At $\nu = 1/2$, in the absence of disorder, the longitudinal conductivity vanishes at $q = 0$, and increases linearly with increasing $q$, as long as $q$ is not too large. Again, this small longitudinal conductivity leads to a slow decay of density fluctuations, and thus to a large transresistivity. Thirdly, Eq. (13) describes a strong suppression of $\text{Im}\Pi^{-1} \propto 1/q^3$ for large values of $q$. Consequently, the $q$ integral in (1) is not dominated by the upper cut-off $q \sim d^{-1}$, as is the case at $B = 0$, but rather by the solution $q_0(\omega)$ of Eq. (14). Since the $\omega$ integral is dominated by $\omega \approx T$, the most important contribution to the transresistivity comes from the region

$$q \approx q_0(T) \approx k_F \left( \frac{T}{T_0} \right)^{1/3}.$$  

We now comment on the temperature dependence of the transresistivity when the electron density in the two layers is varied such that the filling fraction is slightly away from $\nu = 1/2$. The transresistivity is again determined by Eq. (16), with the crucial ingredient being the density-density response function $\Pi$. For the analysis of $\Pi(q, \omega)$ we resort to the composite fermion approach.

For $\nu$ slightly away from $1/2$, a new length, the composite fermion cyclotron radius, $R^*_c$, and a new frequency scale, the composite fermion cyclotron frequency, $\omega^*_c$ are introduced. As long as $R^*_c$ is larger than the transport mean free path $l_{tr}$, introduced by disorder, the composite fermion dynamics is, to a large extent, unaffected by the effective magnetic field they are subject to. The qualitative $q, \omega$ dependence is then similar to that of the $\nu = 1/2$ case, and we expect $\rho_D \propto T^{4/3}$. Moreover, even when $R^*_c$ is smaller than the disorder-induced mean free path, scattering events between composite fermions might suppress the sensitivity of the composite fermions to the effective magnetic field they are subject to. We thus expect that the $T^{4/3}$ temperature dependence may even extend to the region $R^*_c < l_{tr}$.

III. CALCULATIONS

A. Transresistivity for $\nu_1 = \nu_2 = 1/2$

The response function $\Pi$ for a single layer at $\nu = 1/2$ was calculated by HLR within the random phase approximation (RPA). Improvements to that approximations were developed in Refs. [19,20]. We now review the RPA calculation and its improvements.

The calculation of the electronic density-density response function necessitates a notation for single layer response functions. The single layer response functions are $2 \times 2$ matrices, whose entries correspond to density and transverse current. We urge the reader to distinguish single layer response functions from the $2 \times 2$ matrix $\Pi$ (defined in Eq. (1)). In the latter the two entries correspond to densities in the two layers. To help this distinction, we denote single layer matrices by tildes, while double layer matrices are denoted by hats.

The single layer electronic density-density response function is the density-density element of the electronic single layer response function $\Pi^e$. The following discussion, concluded by Eq. (22), is devoted to a calculation of that element. Consequently, the two matrices we are about to define now, $\tilde{C}$ and $\Pi^\text{CF}$, are single layer matrices, with entries corresponding to density and transverse current.

Defining the Chern-Simons interaction matrix,

$$\tilde{C} = \begin{pmatrix} 0 & \frac{2\pi h\phi}{q} \\ -\frac{2\pi h\phi}{q} & 0 \end{pmatrix},$$

we write
where $\tilde{\Pi}_{\text{CF}}$ is the composite fermion density-density response function, describing the response of the composite fermions to the total scalar and vector potentials they are subject to, including external, Coulomb and Chern-Simons contributions. Diagrammatically, $\Pi_{\text{CF}}$ is the sum of all diagrams irreducible with respect to a single Chern-Simons or Coulomb interaction line.

Within RPA, the composite fermion response function $\Pi_{\text{CF}}$ is that of non-interacting fermions at zero magnetic field. It is therefore a diagonal matrix. To lowest order in $\frac{q}{q_F}$ and $\omega/(q_F v_F)$ (with $v_F$ being the Fermi velocity) its components are,

$$
\Pi_{00}^{\text{CF}} \approx \frac{m}{2\pi \hbar^2} \\
\Pi_{11}^{\text{CF}} \approx -\frac{q^2}{24\pi m^*} + \frac{i \omega k_F}{2\pi \hbar q},
$$

where $m$ is the bare mass. The finite value of $\Pi_{11}^{\text{CF}}$ in that limit reflects the compressibility of the system, while the limit of $\Pi_{00}^{\text{CF}}$ reflects Landau diamagnetism (the real part) and Landau damping (the imaginary part).

Improvements to RPA include the modified RPA (mRPA), in which the bare mass is replaced by a renormalized mass $m^*$, and a Landau interaction parameter $f_l$ is introduced, and the magnetized modified RPA (mmRPA), in which an orbital magnetization is attached to each fermion.

Within mRPA, Eq. (21) is replaced by

$$
\Pi_{00}^{\text{CF}} \approx \frac{m^*}{2\pi \hbar^2} \\
\Pi_{11}^{\text{CF}} \approx -\frac{q^2}{24\pi m^*} + \frac{i \omega k_F}{2\pi \hbar q}.
$$

The attachment of magnetization does not affect the component $\Pi_{00}$ of the matrix $\Pi$, which is the only one relevant for Coulomb drag. Thus, we use Eqs. (14) and (22) to calculate the electronic density-density response function $\Pi = \Pi_{\text{CF}}$ obtaining,

$$
\Pi(q, \omega) = \frac{\Pi^{\text{CF}}_{00}}{1 - \Pi_{00}^{\text{CF}} \Pi^{\text{CF}}_{11} \left( \frac{2\pi \hbar}{q} \right)^2} \\
= \frac{1 + \frac{m^*}{2\pi \hbar^2} \left( \frac{q^2}{24\pi m^*} - \frac{i \omega k_F}{2\pi \hbar q} \right)}{1 + \frac{m^*}{2\pi \hbar^2} \left( \frac{2\pi \hbar}{q} \right)^2}
$$

which is Eq. (15). Note that the thermodynamic compressibility is $\frac{dn}{n} \approx \frac{m^*}{2\pi \hbar} \left( 1 + \frac{q^2}{2} \right)$. Substituting Eq. (23) in Eqs. (14) and (1) we find,

$$
\rho_D = \frac{\Gamma \left( \frac{5}{3} \right) \zeta \left( \frac{5}{3} \right) h}{3\sqrt{3} \, e^2} \left( \frac{T}{T_0} \right)^{\frac{1}{3}} \\
\quad - \frac{\Gamma \left( \frac{5}{3} \right) \zeta \left( \frac{5}{3} \right) h}{18 \, e^2} \left[ 1 + 12\alpha + 6\alpha^2 (4\pi n)^{1/2} d \left( \frac{T}{T_0} \right) \right] \left( \frac{T}{T_0} \right)^{\frac{1}{3}} + \mathcal{O}(T^2 \log T).
$$

The origin of the $T^{5/3}$ term is in the second order expansion of the $e^{-q^d}$ factor in the bare inter-layer Coulomb interaction.

A numerical evaluation of Eq. (1) using (22) yields the temperature dependence plotted in Fig. (1). The parameters used in the numerical calculation are $n = 1.4 \times 10^{11}$ cm$^{-2}$, $d = 300$ A, and $\nu = 1/2$. The numerical calculation uses Eq. (22) for $\Pi$, but uses the exact value of $U_b$, rather than a small $qd$ expansion.

The transresistivity at $\nu = 1/4$ is obtained directly from Eqs. (13) and (3). The transresistivity at $\nu = 3/4$ is obtained by regarding the $\nu = 3/4$ state as a superposition of a $\nu = 1/4$ state of holes and a $\nu = 1$ state of electrons.

**B. Transresistivity for $\nu_1 = \nu_2 = 1/2$, as a function of the range of interactions**

In the study of the Chern-Simons Fermi liquid theory formed by composite fermions in the $\nu = 1/2$ state it is useful to consider electron-electron interactions of the form

$$
V(r) = \lambda |r|^{-\eta}
$$

with $0 < \eta < 2$. For $\eta < 1$, the composite fermions form a conventional Fermi liquid—the effective mass and quasi-particle residues are finite. At $\eta = 1$, Coulomb interaction, the Fermi liquid is “marginal”, i.e., both quantities are logarithmically singular. And for $\eta > 1$, the effective mass strongly diverges and the quasi-particle residue vanishes at the Fermi level. In all cases, however, the compressibility of the $\nu = 1/2$ state is not singular.

The small $q$ behavior of the potential (24) is $V(q) \propto \frac{1}{q^{\eta-2}}$, and consequently the bare potential induced by anti-symmetric density modulations is $V_b - U_b \propto q^{\eta-1}$. The form of $\Pi(q, \omega)$ given in Eqs. (14) and (22) does not depend on the interaction potential. Thus, in the calculation of the drag resistivity one has to distinguish between two cases:

1. Long range interactions where $0 < \eta < 1$: In that case the range of the interaction determines the dispersion relation of the anti-symmetric density relaxation modes (14) to be $\omega \propto q^{2+\eta}$. The leading temperature dependence of the transresistivity is then $\rho_D \propto T^{2+\eta}$. Although the composite fermions form a Fermi liquid, $\rho_D$ does not exhibit the $T^2$ temperature dependence characteristic of non-interacting electrons at zero magnetic field. The difference stems from the slow decay of density modulations.
2. Short range interactions where \(1 < \eta < 2\): In that case the dispersion relation of anti-symmetric density relaxation modes (14) is not determined by the interaction, but rather by the compressibility—a density modulation induces a spatial dependence in the chemical potential, which creates a current, which relaxes the modulation of the density. The dispersion relation is \(\omega \propto q^3\), independent of \(\eta\). Consequently, the temperature dependence of \(\rho_d\) is not determined by \(\eta\), and is \(\rho_d \propto T^{4/3}\).

C. Transresistivity close to \(\nu = 1/2\)

When the two layers are at identical filling fractions close to \(\nu = 1/2\), the density-density response functions are identical \(\Pi^{(1)} = \Pi^{(2)}\). Within the composite fermion approach, both can be calculated by Eq. (13). The function \(\Pi^{CF}\) is, in that case, the response function of composite fermions at an effective magnetic field of \(\Delta B = B - 2\Phi_0 n\). We limit ourselves here to relatively high temperatures, \(T \gg \hbar \omega_c^*\). In that regime a semiclassical random phase approximation may be employed for a calculation of \(\Pi(q, \omega)\), as was done, e.g., by HLR and by Simon and Halperin [19]. The semiclassical composite fermion response function \(\Pi^{CF}\) is [13,21],

\[
\Pi_{00}^{CF} = \frac{m}{2\pi \hbar^2} \frac{1 - \frac{\pi W}{\sin \pi W} J_W(q R_c^*) J_{-W}(q R_c^*)}{1 - \frac{\pi}{\omega_c^*} \frac{\pi}{\sin \pi W} J_W(q R_c^*) J_{-W}(q R_c^*)},
\]
\[
\Pi_{01}^{CF} = -i v_F \frac{m}{4\pi \hbar^2} \frac{\frac{\omega}{\omega_c^*}}{\sin \pi W} J_1(q R_c^*) J_{-W}(q R_c^*),
\]
\[
\Pi_{11}^{CF} = \frac{\omega^2}{q^2} \Pi_{00}^{CF},
\]

where \(W = \frac{\omega}{\omega_c^*} + \frac{1}{\omega_c^*}\), \(J_W\) is the Bessel function of order \(W\), and \(\nu\) denotes differentiation with respect to the argument.

In the limit of \(\omega_c^* \tau_r \ll 1\) (or, equivalently \(R_c > l_r\)) we find, using asymptotic expansion of the Bessel functions, that the response functions (25) reduce, at the relevant range of \(q, \omega\) to the zero effective magnetic field response functions (24). Thus, in that regime the leading temperature dependence of the transresistivity is expected to be approximately equal to that of the \(\nu = 1/2\) case.

There are several contributions to the composite fermion scattering rate. The first is impurity scattering, which is independent of energy. The second is mutual composite fermion scattering, which does depend linearly on energy (at least at the \(\nu = 1/2\) case [21]). The precise effect of the latter on the response at finite \(q\) is a delicate subject, which is presently under investigation [21].

We now turn to consider two layers of densities \(n_1 = \bar{n} + \frac{1}{4} \Delta n\) and \(n_2 = \bar{n} - \frac{1}{4} \Delta n\), where the density \(\bar{n}\) corresponds to a filling fraction \(\nu = 1/2\). Measurements of the dependence of the transresistivity on a density bias between the two layers were done in the past as a way of separating out Coulomb drag and phonon drag mechanisms. A calculation of the dependence of \(\rho_d\) on such a density bias could therefore be useful for such measurements at \(\nu = 1/2\).

Since the two layers are not identical, the screened inter-layer Coulomb potential cannot be put in the form (14), and is rather given by

\[
U_{sc} = \frac{U_b}{[1 + \Pi(V_b + U_b)][\Pi(V_b - U_b)] - \delta \Pi^2(V_b^2 - U_b^2)},
\]

where \(\Pi^{(1)} = \bar{\Pi} \pm \delta \Pi\).

The density bias \(\Delta n\) changes the density-density response function \(\Pi\) in two ways. The first, trivial, way, is through the dependence of \(\Pi\) on the electron density. The second way is by changing the filling fraction. However, as shown above, at least within the semiclassical approximation, if \(R_c^*\) is larger than the mean free path the composite fermion response functions at the relevant range of \(q, \omega\) are approximately insensitive to the effective magnetic field \(\Delta B\). Thus, the main effect of \(\Delta n\) comes through the dependence of \(\Pi\) on the density. Using (14) and (24) with two different response functions for the two layers, we find that for small \(\Delta n\), and to leading order in the temperature, the transresistivity is

\[
\rho_d(\Delta n) = \rho_d(\Delta n = 0) \left[ 1 + \frac{7}{48} \left( \frac{\Delta n}{\bar{n}} \right)^2 \right].
\]

D. Transresistivity for \(\nu_1 = \nu_2 = 3/2\)

We now analyze the transresistivity for two layers of \(\nu = 3/2\) each. Our motivation for studying this case stems from the ambiguous experimental picture of spin polarization at the \(\nu = 3/2\) state. While transport measurements at tilted magnetic fields indicate only a partial spin polarization of the \(\nu = 3/2\) state, surface acoustic waves measurements do not seem to support such a picture. We therefore attempt to find out whether a measurement of transresistivity can shed light on the spin polarization of the \(\nu = 3/2\) state.

We regard the \(\nu = 3/2\) state as a superposition of a \(\nu = 2\) state of electrons and a \(\nu = 1/2\) state of holes, a picture which is valid when the electronic Landau level separation \(\hbar \omega_c\) is much larger than the temperature and the Coulomb energy scales, such that Landau level mixing can be neglected. Under these conditions the contribution of the electronic \(\nu = 2\) state to the single-layer density-density response function \(\Pi\) is exponentially small for the relevant frequency range \(\hbar \omega < T\), due to the
energy gap between the $n = 0$ and $n = 1$ electronic Landau levels. The transresistivity is therefore almost solely due to the $\nu = 1/2$ state of the holes.

The spin polarization of the $\nu = 1/2$ hole state may be anywhere between zero and full polarization. So far we considered only fully polarized states. When considering a partially polarized state, one expects all relevant functions ($\Pi, V_{\text{bare}}, \hat{V}_{\text{cd}}$ of Eqs. (9) and (10)) to acquire spin indices. As a first step, then, a convenient choice of a basis may be helpful. This choice emerges naturally when considering the bare Coulomb interaction, even within one layer. We write the intra-layer bare interaction as a $2 \times 2$ matrix, where the two entries are for densities of electrons with two spin states.

In the basis of eigenvectors of $\sigma_z$,

$$V_{\text{bare}} = \frac{2\pi e^2}{q} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(28)

since the Coulomb interaction does not depend on the spin direction. The convenient basis is the basis of charge density $\rho_\uparrow + \rho_\downarrow$ and spin density $\rho_\uparrow - \rho_\downarrow$ (where $\rho_\uparrow$ ($\rho_\downarrow$) denote the density of electrons whose spin is parallel (antiparallel) to the magnetic field). In that basis,

$$V_{\text{bare}} = \frac{2\pi e^2}{q} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$  

(29)

The screened Coulomb interaction has a structure similar to (29), since it, too, couples only to charge densities, and does not depend on spin densities. Consequently, the only component of the response function $\Pi$ that affects the transresistivity is the one coupling charge density to charge density. For the fully polarized case, it is given by Eq. (15). We now turn to discuss this component in the other extreme case, that of zero spin polarization.

In principle, one may envision several Chern-Simons transformations that might be applicable to the fully unpolarized $\nu = 1/2$ state. At $\nu = 1/2$ there are two flux quanta per a hole. One possible transformation attaches four flux quanta to each hole, and allows an interaction only between same-spin composite fermions. This transformation creates a correlated wave function of same-spin holes, and reduces the probability of finding two same spin holes very close to one another (23). It does not, however, do the same for holes of different spin directions. Thus, while the Coulomb energy cost associated with holes of identical spins is reduced, that of holes of different spin directions is unaffected. An energetically favorable transformation, that reduces the Coulomb energy of both types of interactions, is one that attaches two flux quanta to each hole, and allows an interaction that is independent of the spin direction. The correlations induced by this transformation are, at least on the RPA level, spin-independent. We use the second transformation.

The calculation of $\Pi$ is similar to that leading to Eq. (23). Eq. (19) still holds, although $\Pi^e, \tilde{C}$ and $\Pi^\text{CF}$ become single layer $4 \times 4$ matrices, with the entries corresponding to densities and transverse current of two spin states. Within mRPA, in the basis of eigenstates of $\sigma_z$, the response function $\Pi^\text{CF}$ is a block-diagonal matrix with two pairs of identical blocks, denoted by $\chi^\text{CF} \equiv \Pi^\text{CF} = \tilde{\Pi}^\text{CF}$ (Note that $\chi^\text{CF}, \tilde{\Pi}^\uparrow\uparrow, \tilde{\Pi}^\downarrow\downarrow$ are $2 \times 2$ matrices with entries corresponding to density and transverse current). Since in our choice of the Chern-Simons transformation the Chern-Simons interaction is independent of the spin direction, it is again more convenient to transform the spin states from eigenstates of $\sigma_z$ to $\rho_\uparrow + \rho_\downarrow$ (and similarly for transverse currents). The components relevant for our calculation are the ones coupling charge density and currents (i.e., spin density and currents are irrelevant), and they are given by the $2 \times 2$ matrix $2\chi^\text{CF}$. Using Eq. (23) and (21), we then find the following expression for the electronic density-density response function, to be compared with Eq. (22),

$$\Pi(q, \omega) = \frac{2\chi_{\text{CF}}^e}{1 - 4\chi_{\text{CF}}^e \chi_{\text{CF}}^\text{res} \chi_{\text{CF}}^\text{res}} \left( \frac{2\pi e^2}{q} \right)^2.$$ 

(30)

Qualitatively, Eq. (30) is very similar to Eq. (22), the corresponding response function for the polarized state. Qualitatively, there are three differences: a factor of 2 in the numerator resulting from the presence of two spin directions, a factor of 4 originating from the interaction of each composite fermion with flux tubes carried by fermions of two spin directions, and a factor $1/\sqrt{2}$ difference between $\text{Im}\chi_{\text{CF}}$ and $\text{Im}\Pi_{\text{CF}}$, originating from the difference between the Fermi wave-vector at the polarized and unpolarized states.

We note, by passing, that this quantitative difference between $\Pi$ of the fully unpolarized state and $\Pi$ of the fully polarized state should be observable in measurements of the $q$-dependent conductivity using surface acoustic waves. For example, one might imagine a measurement of surface acoustic waves attenuation in a $\nu = 3/2$ state as a function of a magnetic field applied parallel to the two dimensional electronic system. If the parallel magnetic field modifies the spin polarization of the $\nu = 3/2$ state, a significant change in $\sigma_{xx}(q)$ should be observable.

Coming back to the transresistivity, an examination of Eq. (29), Eq. (12) and Eq. (1) reveals that the drag transresistivity in the unpolarized case can be obtained from that of the polarized case by three modifications: firstly, $\phi$ should be multiplied by 2, to account for the factor 4 in the denominator of (30). Secondly, the bare interaction should be multiplied by 2, to account for the factor 2 in the numerator of (30). And thirdly, the Fermi moment $k_F$ appearing in the electronic density-density response function (22) should be divided by $\sqrt{2}$. Neglecting $\alpha$ compared to 1, we then find that the transresistivity in the unpolarized case is larger, by a factor of $2^{2/3}$ compared to that of the polarized case.
IV. SUMMARY

In this paper we considered the Coulomb drag between two layers of two dimensional electron gases in a strong magnetic field, when the filling fraction in each layer is at, or close, to $\nu = 1/2$, $\nu = 1/4$ and $\nu = 3/4$, and the electrons in each layer form a compressible state. Using the composite fermion approach to analyze the properties of each layer, and an electronic approach to analyze the coupling between the layers, we find that the unique linear $q$-dependence of the longitudinal conductivity, $\sigma(q) \propto q$, at and close to even denominator filling fractions, lead to a unique temperature dependence of the transresistivity between two layers, $\rho_D \propto T^{4/3}$. In contrast to previous works, we do not associate this temperature dependence to the nature of the liquid formed by composite fermions, but rather interpret it as a result of the slow relaxation of density fluctuations in a compressible partially filled Landau level.

We examine the transresistivity at $\nu = 3/2$ and find that it depends significantly on the spin polarization of the $\nu = 3/2$ state. We therefore propose that measurements of $\rho_D$ at $\nu = 3/2$ might shed light on that polarization, which so far is not well understood.

Our results rely on the assumption of weak coupling between the layers and neglect the finite thickness of each layer. The properties of each layer are analyzed using approximation schemes for the composite fermion problem. We find that $\rho_D$ depends mostly on the strength of the Coulomb interaction and on geometrical factors, such as the inter-layer and inter-electron distance. Its dependence on masses, whether bare or effective, is rather weak. We believe that these observations make the results we obtain mostly independent of the fine details of the approximation we use.

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APPENDIX A: THE RELATION OF COMPOSITE FERMION DRAG TO ELECTRON DRAG

In the main body of the paper we discussed the transresistivity from a purely electronic point of view, showing that it originates from the Coulomb drag between electrons at the two layers. In this appendix we relate the electronic calculation to a calculation in terms of composite fermions, which is the approach taken in Refs. [14] and [15].

First, we note that as long as the inter-layer coupling is weak, composite fermions in one layer do not interact with the Chern-Simons field of the other layer (an inter-layer Chern-Simons interaction gives rise to 331-type states). Under these conditions, the electronic drag resistivity is identical to the composite fermion drag resistivity,

$$\rho_D^e = \rho_D^{CF}. \quad (A1)$$

In a single layer it is well known [13] that the composite fermion resistivity matrix $\rho^{CF}$ and the electronic resistivity matrix $\rho^e$ are related by

$$\rho^e = \rho^{CF} + \frac{e}{2} \left( \begin{array}{cc} 0 & \tilde{\phi} \\ -\tilde{\phi} & 0 \end{array} \right). \quad (A2)$$

Eq. (A1) results from an extension of Eq. (A2) to the $4 \times 4$ double layer resistivities. Similar to the single layer case, electronic longitudinal resistivities are equal to those of the composite fermions. The electronic (measurable) drag conductivity is not identical to the composite fermion drag conductivity.

Thus, in a composite fermion approach, we calculate $\rho_D^{CF}$. The following derivation is limited to the case of two identical layers at $\nu = 1/2$, and assumes that $\Pi^{CF}$ is diagonal. Extensions will be discussed elsewhere [21]. Similarly to the usual case, the transresistivity is given by [13,14,17],

$$\rho_D^{CF} = \frac{1}{2(2\pi)^2} \frac{h c}{e^2 T n^2} \int \frac{dq}{(2\pi)^2} q^2 \int_0^\infty \frac{hd\omega}{\sinh^2 \frac{\omega}{2T}} \sum_{ij} \text{Im}\Pi^{CF}_{ij}(q, \omega) \left( \text{Im}\Pi^{CF}_{j,i}(q, \omega) \right)^\ast \left( U_{sc}^{CF} \right)_{ij}(q, \omega), \quad (A3)$$

where $i, j$ get the values 0 or 1, corresponding to density and transverse currents. The summation over both density and transverse current components is needed since the screened interaction couples composite fermions both through their densities and their transverse currents. In Eq. (A3) the composite fermion transresistivity is given in terms of composite fermion response functions. We now turn to express it in electronic terms.
We first relate the screened interlayer interaction of composite fermions $U_{sc}^{CF}$ to the electronic screened interlayer interaction $U_{se}$ (the former is a $2 \times 2$ matrix, with density and transverse current indices, while the latter is not, since electrons interact only through the Coulomb interaction). By comparing the two, one finds

$$U_{sc}^{CF} = \begin{pmatrix} (1 - C_{01} \Pi_{10}) U_{se}^{CF} (1 - \Pi_{10}) C_{10} & -(1 - C_{01} \Pi_{10}) U_{se}^{CF} \Pi_{10} C_{01} \\ -C_{10} \Pi_{00} U_{se}^{CF} (1 - \Pi_{10}) C_{10} & C_{10} \Pi_{00} U_{se}^{CF} \Pi_{10} C_{01} \end{pmatrix},$$

where the Chern-Simons interaction matrix $\hat{C}$ is defined in Eq. (18). This relation may be observed diagrammatically as in Fig. 2 and may be derived from the expression for $U_{se}^{CF}$ [Eq. (13)],

$$U_{se}^{CF} = \frac{U_b}{(1 + \Pi_{00}^{se} V_b)^2 - (\Pi_{00}^{se} U_b)^2},$$

where $V_b$ and $U_b$ are the intralayer and interlayer bare Coulomb interactions. Inserting Eq. (A5) in the right hand side of Eq. (A4), and using [see Eq. (19)]

$$\Pi^{CF} = \frac{1}{1 - \Pi_{00}^{CF} \Pi_{11}^{CF} |C_{01}|^2} \begin{pmatrix} \Pi_{00}^{CF} & -\Pi_{00}^{CF} U_{01}^{CF} C_{10} \\ -\Pi_{00}^{CF} U_{10}^{CF} C_{01} & \Pi_{11}^{CF} \end{pmatrix},$$

we obtain for the right hand side of Eq. (A4),

$$\frac{1}{(1 + \Pi_{00}^{CF} V_b - \Pi_{10}^{CF} \Pi_{11}^{CF} |C_{01}|^2)^2 - (\Pi_{00}^{CF} U_b)^2} \begin{pmatrix} U_b & -U_b \Pi_{00}^{CF} C_{01} \\ -C_{10} \Pi_{00}^{CF} U_{10}^{CF} C_{01} & C_{10} \Pi_{00}^{CF} U_{01}^{CF} C_{01} \end{pmatrix}.$$ (A7)

This is exactly the expression for $U_{sc}^{CF}$ [Eq. (A4)].

Using the relation (A4), the expression for the drag resistivity becomes

$$\rho_D^{CF} = \frac{1}{2(2\pi)^2} \frac{h}{2 T n^2} \int \frac{dq}{(2\pi)^2} q^2 \int \frac{d\omega}{\sinh^2 \frac{\omega}{T}} |\Xi(q, \omega)|^2 |U_{se}^{CF}(q, \omega)|^2,$$ (A8)

where

$$\Xi = |1 - C_{01} \Pi_{10}|^2 \text{Im} \Pi_{00}^{CF} + |C_{10} \Pi_{00}^{CF}|^2 \text{Im} \Pi_{01}^{CF}.$$ (A9)

We now show that $\Xi$ is, in fact, the imaginary part of the electronic density-density response function, $\Pi_{00}$. From Eq. (A6) we have $\Pi_{00} = \Pi_{00}^{CF} |\Pi_{00}^{CF}|^2 - (\Pi_{01}^{CF})^2 / (\Pi_{11}^{CF})^2$. Using $\text{Im} z^{-1} = -|z|^{-2} \text{Im} z$, we obtain

$$\text{Im} \Pi_{00} = \frac{|\Pi_{00}^{CF}|^2}{|\Pi_{00}^{CF}|^2} \text{Im} \Pi_{00}^{CF} + \frac{|\Pi_{01}^{CF}|^2}{|\Pi_{11}^{CF}|^2} \text{Im} \Pi_{11}^{CF}.$$ (A10)

On comparing (A9) with (A10), it is left to show that

$$|\Pi_{00}^{CF}|^2 = |1 - C_{01} \Pi_{10}|^2, \quad |\Pi_{01}^{CF}|^2 = |C_{10} \Pi_{00}^{CF}|^2.$$ (A11)

Both relations are verified using (A6).

The expression for drag resistivity is therefore written in terms of the electronic properties, in exactly the way it is written for two identical layers at $B = 0$ [Eq. (1)].

$$\rho_D^{CF} = \rho_D = \frac{1}{2(2\pi)^2} \frac{h}{2 T n^2} \int \frac{dq}{(2\pi)^2} \int_0^{\infty} \frac{h d\omega}{\sinh^2 \frac{\omega}{T}} q^2 |\text{Im} \Pi(q, \omega)|^2 |U_{se}(q, \omega)|^2.$$ (A12)
The only response function which enters this formula is the electron density-density response function $\Pi = \Pi^{00}$, and this is the only place where properties of the half-filled Landau level affect the drag resistivity.

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