The di-Ω in the extended quark delocalization, color screening model

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Abstract

The ΩΩ (SIJ=−6,0,0) dibaryon state is studied with the extended quark-delocalization color-screening model including a π meson exchange tail, which reproduces the properties of the deuteron quantitatively. We find the mass of the di-Ω to be about 45 MeV lower than the Ω−Ω threshold. The effect of channel coupling due to the tensor force has been calculated and found to be small in this case. We have also studied the effect of other pseudoscalar meson exchanges and sensitivity to the short-range cutoff radius, \( r_0 \), for the meson exchanges.
I. INTRODUCTION

Quantum ChromoDynamics (QCD) is believed to be the fundamental theory of the strong interaction. However, it is difficult to use to calculate the low energy properties of complicated quark-gluon systems directly, such as the NN interaction and the structure of multi-quark-gluon systems. Various quark models afford a framework to understand the physics in these cases.

In the last 30 years, various QCD inspired models have been developed with both successes and failures in explaining low energy hadron physics. To mention a few but certainly not all, there are the MIT bag model[1], cloudy bag model[2], Friedberg-Lee nontopological soliton model[3], Skyrme topological soliton model[4], the constituent quark model[5], etc. Different models use quite different effective degrees of freedom, which might be indicative of the nature of low-energy QCD. Under different approximations one can ”derive” these models from QCD[5]. Recently there has been a hot debate regarding the proper effective degrees of freedom of the constituent quark model[6].

We performed a phenomenological study of the baryon interactions with three constituent quark models, the Glozman-Riska-Brown quark-meson coupling model[8], the Fujiwara quark-gluon-meson coupling model – one version of the Manohar-Georgi chiral quark model[9], and the quark delocalization, color screening model (QDCSM) developed by ourselves – a modified version of the de Rujula-Georgi-Glashow-Isgur quark gluon coupling model[10, 11, 12], and found that, in about 2/3 of the channels formed from the octet and decuplet baryons, these three models gave qualitatively the same effective baryon-baryon (B-B) interactions[13]. These constituent quark models have the same long range Goldstone boson exchange interaction, (except for the QDCSM,) but have different mechanisms for the intermediate and short range interactions when they are used to study B-B interactions. Present data on baryon spectroscopy and B-B interactions seem to be insufficient to distinguish between these models and they all seem to have an approximate QCD basis[5]. A new generation of hadron spectroscopy should be helpful in disentangling these hadronic models.

There are good reasons, both theoretically and phenomenologically, to believe that a two body confinement potential might be a good approximation for single hadrons. However there is no compelling reason to believe it is also a good approximation for multihadron systems. Dibaryons provide a good testing ground for different confinement mechanisms
and baryon interaction models. Being the unique, stable, B=2 system, and its size, binding energy and quadrupole moment having been measured very precisely, the deuteron plays a vital role in the development of NN interaction models. Since the H-particle, a dibaryon with strangeness $S = -2$, isospin $I=0$ and spin $J=0$, was first predicted by the MIT bag model in 1977\cite{14}, tremendous efforts have been made both experimentally and theoretically to search for it. Up to now there is no experimental evidence for the H.

Other dibaryon candidates have also been discussed in the literature. An $S=0$, $J^P = 0^-$, $M \sim 2.06$ GeV, $\Gamma \sim 0.5$ MeV, $I=0$ or 2 dibaryon, $d'$, was a hot topic in the 1990’s\cite{13, 15}. The S=0, I=0, $J^P = 3^+$ dibaryon, $d^*$, has also been a long standing topic; almost all quark models predict there should be attraction in this channel but with differing strengths in the different models\cite{12, 17, 18, 19, 20}. In 1987, we showed that the $S=-3$, $I=1/2$, $J=2$ dibaryon state might be a narrow resonance in a relativistic quark model \cite{21}. In 1990, V. B. Kopeliovich et al.\cite{22} predicted that there are strong interaction stable dibaryons with high strangeness, such as an $S=-6$, di-Ω within the flavor SU(3) Skyrmion model. A general survey of dibaryon states carried out with both the nonrelativistic QDCSM and the relativistic version, using an adiabatic calculation, found few high strangeness states with masses around the lowest threshold\cite{18, 19}. Zhang et al.\cite{23} predicted that the ΩΩ $I=0$, $J=0$ dibaryon is strong interaction stable using a chiral SU(3) quark model.

In this study, we use the extended QDCSM\cite{12} to calculate the eigen energy of the di-Ω state $S,I,J = -6,0,0$. This model allows the quark system to choose its most favorable configuration through its own dynamics in a larger Hilbert space than the other models, and takes into account the possible difference of the confinement interaction inside a single baryon and between two color singlet baryons. In particular, this is the unique model that gives an explanation of the similarity between molecular and nuclear forces. It also explains why nuclei are accurately viewed as a collection of nucleons rather than a single big bag with 3A quarks.

Here, we recalculate the QDCSM result because the extended QDCSM reproduces the deuteron properties well quantitatively, unlike the QDCSM, and because preliminary NN phase shift calculations using it fit the NN, NΛ, NΣ scattering data better than the earlier QDCSM calculations, which fit the data only qualitatively\cite{11, 12}. Consequently, we expect this extended QDCSM calculation will provide a better model estimate of the mass of the di-Ω.
In the following section, we give a short description of the extended QDCSM. The results and a discussion are presented in section III.

II. SHORT DESCRIPTION OF THE EXTENDED QDCSM

The details of the QDCSM and its extension can be found in Refs. [11, 12, 18] and the resonating-group calculation method (RGM) has been presented in Refs. [20, 24]. Here we present only the (complete) extended model Hamiltonian, wave functions and the necessary equations used in the current calculation.

The Hamiltonian for the 3-quark system is the same as the usual quark potential model. For the six-quark system, it is assumed to be

\[
H_6 = \sum_{i=1}^{6} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{i<j}^{6} \left( V_{conf}(r_{ij}) + V_G(r_{ij}) + V_{\chi}(r_{ij}) \right),
\]

\[
V_G(r_{ij}) = \frac{\alpha_s}{4} \left[ \frac{1}{r_{ij}} - \frac{\pi \delta(\vec{r})}{m_im_j} \left( 1 + \frac{2}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j \right) \right] \left( 3(\vec{\sigma}_i \cdot \vec{\hat{r}})(\vec{\sigma}_j \cdot \vec{\hat{r}}) - \vec{\sigma}_i \cdot \vec{\sigma}_j \right),
\]

\[
V_{\chi}(r_{ij}) = \left\{ \sum_{a=1}^{3} V_\pi(r_{ij}) \lambda_i^f \cdot \lambda_j^a + \sum_{b=4}^{7} V_K(r_{ij}) \lambda_i^f \cdot \lambda_j^b + V_\eta(r_{ij}) \lambda_i^f \cdot \lambda_j^8 \right\},
\]

\[
V_{\gamma}(r_{ij}) = \theta(r - r_0) \frac{g_\gamma^2}{4\pi} \frac{\mu_\gamma^2}{12m_im_j} \frac{1}{r} e^{-\mu_\gamma r} \times \left[ \vec{\sigma}_i \cdot \vec{\sigma}_j + \left\{ \frac{3(\vec{\sigma}_i \cdot \vec{\hat{r}})(\vec{\sigma}_j \cdot \vec{\hat{r}})}{r^2} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right\} \left( \frac{3}{(\mu_\gamma r)^2} + \frac{3}{\mu_\gamma r} + 1 \right) \right], \quad \gamma = \pi, K, \eta
\]

\[
\frac{g_{\pi\pi}^2}{4\pi} = \left( \frac{3}{5} \right)^2 \frac{g_{\pi\pi}^2}{4\pi},
\]

\[
\frac{g_{\gamma\pi}^2}{4\pi} = \left( \frac{3}{5} \right)^2 \frac{g_{\gamma\pi}^2}{4\pi} \left( \frac{m_q}{m_N} \right)^2
\]

\[
V_{\text{conf}}(r_{ij}) = -a_{eff} \vec{\lambda}_i \cdot \vec{\lambda}_j \begin{cases} \frac{r_{ij}^2}{1 - e^{-\mu_\gamma^2 r_{ij}}} & \text{if } i, j \text{ occur in the same baryon orbit}, \\ \frac{1}{\mu} & \text{if } i, j \text{ occur in different baryon orbits,} \end{cases}
\]

\[
\theta(r - r_0) = \begin{cases} 0 & r < r_0, \\ 1 & \text{otherwise}, \end{cases}
\]

where \(\vec{\sigma}_i, \vec{\lambda}_i^c\) and \(\vec{\lambda}_i^f\) are, respectively, the spin, color and flavor operators of the \(i^{th}\) quark. The spin operators are represented by the three \(2 \times 2\) Pauli matrices, and the color and flavor operators are represented by the eight \(3 \times 3\) Gell-Mann matrices. The \(\mu_\gamma\) are the masses of the Goldstone bosons \((\gamma = \pi, K, \eta)\). We assume a single quark-meson coupling
constant $g_2^2/4\pi$ for all mesons. The confinement potential $V_{\text{conf}}(r_{ij})$ has been discussed in Refs.\[10, 20].

We obtained the values of b, $\alpha_s$ and $a_c$ by reproducing the $N-\Delta$ mass difference, the nucleon mass and applying a stability condition. The mass of the u, d quark is assumed to be 1/3 of the nucleon mass to meet the requirements of the resonating group method (RGM) calculation while the strange quark mass $m_s$ is determined by an overall fit to the baryon octet and decuplet. A flavor symmetric overall octet quark-meson coupling constant is assumed for all of the octet mesons ($\pi, K, \eta$).

The color screening parameter $\mu$ is determined by the binding energy of the deuteron. The full set of parameters reproduces other properties of deuteron as well: the size and the D-S wave mixing amplitude. It is worth emphasizing that the extended QDCSM has only one parameter, the color screening constant $\mu$, that needs to be adjusted to obtain the correct binding energy of the deuteron, and that this is sufficient to reproduce well all of the properties of the deuteron. The tensor force of pion exchange plays a vital role in reproducing the deuteron properties quantitatively, but the results are not extremely sensitive to the meson exchange cutoff parameter, $r_0$.

After introducing Gaussian functions with different reference centers $S_i$, i=1...n, which play the role of the generating coordinates in this formalism, and including the wave function for the center-of-mass motion\[25\], the ansatz for the two-cluster wave function used in the RGM can be written as

$$
\Psi_{6q} = A \sum_{k} \sum_{i=1}^{n} \sum_{L_k=0,2} C_{k,i,L_k} \int \frac{d\Omega_{S_i}}{4\pi} \prod_{\alpha=1}^{3} \psi_{\alpha}(S_i, \epsilon) \prod_{\beta=4}^{6} \psi_{\beta}(-S_i, \epsilon) \left( [\eta_{i1k} s_{1k}(B_{1k}) \eta_{i2k} s_{2k}(B_{2k})]^{I S_k Y^{L_k}}(\tilde{S}_i) \right)[\chi_c(B_1) \chi_c(B_2)]^{[\sigma]},
$$

where $k$ is the channel index. For the di-$\Omega$, we have $k = 1, 2$, corresponding to the channels $\Omega\Omega$ S=0, L=0 and S=2, L=2. We consider only these channels as they are the only ones coupled by the tensor interaction.

The delocalized single-particle wave functions used in the QDCSM are

$$
\psi_{\alpha}(S_i, \epsilon) = \left( \phi_{\alpha}(S_i) + \epsilon \phi_{\alpha}(-S_i) \right) / N(\epsilon),
$$

$$
\psi_{\beta}(-S_i, \epsilon) = \left( \phi_{\beta}(-S_i) + \epsilon \phi_{\beta}(S_i) \right) / N(\epsilon),
$$

$$
N(\epsilon) = \sqrt{1 + \epsilon^2 + 2\epsilon e^{-S_i^2/4\mu^2}}.
$$

(2)
\[
\phi_\alpha(\vec{S}_i) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2} (r_\alpha - \vec{S}_i/2)^2} \\
\phi_\beta(-\vec{S}_i) = \left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{1}{2b^2} (r_\beta + \vec{S}_i/2)^2}.
\]
The delocalization parameter, \(\epsilon\), is determined by the dynamics of the quark system rather than being one of the fixed (adjusted) parameters.

From the variational principle, after variation with respect to the relative motion wave-function \(\chi(R)\), one obtains the RGM equation

\[
\int H(\vec{R}, \vec{R}')\chi(\vec{R}')d\vec{R}' = E \int N(\vec{R}, \vec{R}')\chi(\vec{R}')d\vec{R}', \tag{4}
\]

With the above ansatz, the RGM equation, Eq.(4), becomes an algebraic eigenvalue equation,

\[
\sum_{j,k,L} C_{j,k,L} H_{i,j}^{k',L',k,L} = E \sum_{j} C_{j,k,L} N_{i,j}^{k',L',k,L} \tag{5}
\]
where \(N_{i,j}^{k',L'}\), \(H_{i,j}^{k',L',k,L}\) are the (Eq.(2)) wave function overlaps and Hamiltonian matrix elements, respectively. By solving the generalized eigen value problem, we obtain the energies of the 6-quark system and their corresponding wave functions.

**III. RESULTS AND DISCUSSION**

Our model parameters, which have been fixed by matching baryon and deuteron properties and using only \(\pi\) meson exchange, are given in Table I. For comparison, the results of our earlier calculation \[18\] without \(\pi\) meson exchange are also included in Table I. In order to study the dependence of the result on the short range cutoff radius, we choose three typical values: 0.6, 0.8 and 1.0 fm, for the short-range cutoff of the \(\pi\) meson exchange potential. This cutoff is necessary in our model approach because all short and intermediate range effects are already represented by the quark delocalization and color screening: The short range repulsion of the NN interaction is provided by a combination of the color magnetic interaction due to gluon exchange and the Pauli principle enforced by the quark structure of the nucleon; the intermediate range attraction conventionally modeled by heavy meson and multimeson exchange is also provided by quark delocalization and color screening. If the contact term of single \(\pi\) meson exchange, \(\delta(\vec{r})\), or its smeared version, \(\exp(-\Lambda r)/(\Lambda r)\), were used, double counting \[12\] would occur. In all cases, we found that the contribution
of $\pi$ meson exchange to the dibaryon mass is not large (about 10 MeV or less), primarily due to the effect of the short-range cutoff; correspondingly, the deviations from the original QDCSM parameters are also quite small.

### Table I. Model parameters and results calculated for di-$\Omega$.

|                  | including $\pi$ meson exchange | excluding meson exchange |
|------------------|-------------------------------|-------------------------|
| $r_0 = 0.6$ fm   |                               |                         |
| $r_0 = 0.8$ fm   |                               |                         |
| $r_0 = 1.0$ fm   |                               |                         |
| dynamical cal.   |                               |                         |
| adiabatic cal.   |                               |                         |
| $m_{d,m_s}$ (MeV) | 313 , 634                    | 313 , 634               |
| $b$ (fm)         | 0.6010                        | 0.6015                  |
| $a_c$ (MeV fm$^{-2}$) | 25.40                       | 25.14                   |
| $\alpha_s$       | 1.573                         | 1.5585                  |
| $\mu$ (fm$^{-2}$) | 0.75                         | 0.85                    |
| Mass$_{sc}$ (MeV) | 3290.3                      | 3298.2                  |
| Mass$_{cc}$ (MeV) | 3290.2                      | 3297.9                  |

(sc refers to a $k = 1$ single channel calculation, cc refers to the coupled channel calculation.)

Comparing the last two columns in Table I, where the $\pi$ meson exchange tails have not been included, we see that the dynamic calculation modifies the results of the previous adiabatic calculation significantly. The mass of the $\Omega\Omega$, which is close to the threshold (3344.9 MeV) in the adiabatic approximation, is reduced below the threshold by about 33 MeV. Several effects contribute to this difference, but the most important one is that the resonating group method calculates the relative motion between the two quark clusters rigorously, greatly improving over the rough estimate of the kinetic energy by the zero-point oscillation energy used in the adiabatic approximation.

We note that one should not expect all baryon-baryon (B-B) states to be modified significantly by the dynamical calculation. If the effective B-B interaction has a narrow minimum with respect to the separation variable, then the zero-point oscillation energy provides a good approximation to the kinetic energy of the relative motion of the two clusters. For example, the $d^*$ mass does not change very much between the adiabatic and dynamic calculations. However, if the effective B-B interaction is flat, then the zero-point oscillation energy may be not a good approximation for the kinetic energy of the relative motion; the di-$\Omega$ is such a case.[13]

The first three columns show the dynamical mass of the di-$\Omega$ with (only) $\pi$ meson exchange included but with different cutoff parameters. Comparing these three results with
the fourth column, one can see that the di-Ω mass is reduced by between 9 and 22 MeV. This is due to the fact that the parameters of the model are modified by inclusion of the pion even though the pion is not exchanged between s quarks. The binding energies vary from 55 MeV to 40 MeV with different cutoffs. The larger the cut off, the smaller is the effect on the di-Ω mass.

The last two rows show that the effect of channel coupling caused by the meson tensor force is quite small here. The S and D wave coupled channel masses (cc) are almost the same as those of the single channel approximation (sc). In the deuteron, the effect on the mass of the state, although almost an order of magnitude larger and more significant there, was also found to be small. By contrast, as shown below, the D-S wave mixing in both cases is comparable.

The quark spin-orbit interaction has been neglected in the Hamiltonian (1). We would expect it to have a similarly significant effect on the D-wave channel. However, its effect on the di-Ω mass is expected to be small because the D-wave channel coupling itself only affects the di-Ω mass slightly.

Fig.1 shows the relative motion wave functions for two different cutoff parameters. They are not sensitive to the short-range cutoff parameter, $r_0$. Fig.2 shows the effective $\Omega - \Omega$ potential corresponding to the cutoff $r_0 = 0.8 fm$. The other cutoffs give almost indistinguishable effective potentials.

As has been mentioned above, the effects of short and intermediate range interaction arising from $\pi$ meson exchange are represented by quark delocalization and color screening, so the only long range effect in our model comes from $\pi$ exchange. The effects of heavy pseudoscalar meson ($K$, $\eta$) exchange on the mass of the di-Ω have been checked with a representative cutoff of $r_0 = 0.8 fm$. The results are presented in Table II.
Table II. Mass of di-Ω for $r_0 = 0.8 \text{fm}$ and full octet pseudoscalar meson exchange.

|                      | only $\pi$-exchange | $\pi, K, \eta$-exchange |
|----------------------|---------------------|-------------------------|
| $m_u, m_s$           | 313,634             | 313,634                 |
| $b (\text{fm})$      | 0.6015              | 0.6022                  |
| $a_c (\text{MeV fm}^{-2})$ | 25.14             | 25.03                   |
| $\alpha_s (\text{fm}^{-2})$ | 1.5585           | 1.555                   |
| $\mu$                | 0.85                | 0.9                     |
| $\sqrt{\langle r^2 \rangle}$ | 2.0                | 1.92                    |
| $P_d$                | 4.53%               | 4.92%                   |
| $\text{Mass}_{\text{Deuteron}} (\text{MeV})$ | 1876               | 1875.8                  |
| $\text{Mass}_{\text{Di-Ω}} (\text{MeV})$   | 3298.2             | 3300.0                  |

Repeating the same process mentioned above to fix the parameters, we found that adding $K$ and $\eta$-exchange affects the parameters ($b$, $a_c$, $\alpha_s$) only slightly; the properties of the deuteron can be reproduced just as well by a very small readjustment the color screening parameter, $\mu$. Comparing the results in Table I and II, we find the mass of the di-Ω is almost unaffected by the addition of $K$ and $\eta$ exchange. This confirms our expectation that heavier (shorter ranged) meson exchanges are already represented by quark effects in our model, so that explicit representation of such exchanges beyond a cut-off scale are not important in our approach.

In summary, within the framework of the extended QDCSM, using a resonating group method, we estimate the mass of the di-Ω to be about 45 MeV lower than the $\Omega - \Omega$ threshold. This mass is almost unaffected by channel coupling, the value of the short range cutoff length, $r_0$, and heavier pseudoscalar meson exchanges. The inclusion of single pion or heavier meson exchange has only a minor effect on the di-Ω mass. These results are consistent with the calculations of Kopeliovich and of Zhang et al.\cite{22, 23}.

We should, however, take note that the precision of any prediction of the di-Ω mass is limited by that of the model itself. For example, in our model approach, the mass of the $\Omega$ itself is found to be 1651 MeV, or 21 Mev less than the measured value. It could well be argued that this theoretical mass of the $\Omega$ should be used to calculate the threshold. We would then conclude that the di-Ω mass is no more than 5 MeV below threshold. From the model deviations from experimental values for single baryon masses, we estimate a total systematic uncertainty of order 30 MeV as a reasonable value to assign to our model.
Moreover, it is difficult to estimate how much of the model uncertainty is carried over to the dibaryon calculation for such a high strangeness quark system, because the model parameters are mainly determined by the experimental data of the nonstrange sector. Therefore, when designing a detector system to hunt for the di-Ω state, it is theoretically advisable to take into account the possibility that the di-Ω may be either stable or unstable with respect to the strong interaction.

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Figure Captions

Fig. 1. Cluster relative motion wave function of di-Ω.
Fig. 2. S-wave $\Omega - \Omega$ effective potential.
FIG. 1: Cluster relative motion wave function of di-Ω.
FIG. 2: S-wave $\Omega - \Omega$ effective potential.

$\text{Effective potential (MeV)}$

Separation $R$ (fm)

$r_0=0.8\text{fm}, \mu=0.85$