Damage development and lifetime prediction of fiber-reinforced ceramic-matrix composites subjected to dwell-fatigue loading at elevated temperatures in oxidizing atmosphere

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In this paper, the damage development and lifetime prediction of fiber-reinforced ceramic-matrix composites subjected to the dwell-fatigue loading at elevated temperatures in oxidizing atmosphere have been investigated using the micromechanics approach. Considering the damage mechanisms of matrix multicracking, fiber/matrix interface debonding, interface wear and interface oxidation and fibers fracture, the damage evolution of the fatigue hysteresis dissipated energy, fatigue hysteresis modulus, fatigue peak strain and broken fibers fraction have been analyzed. The relationships between the fatigue hysteresis-based damage parameters and the internal damage development of matrix multicracking, fiber/matrix interface debonding and sliding and fibers fracture have been established. The experimental dwell-fatigue damage development and fatigue lifetime curves of cross-ply Nicalon®-SiC/MAS and 2D Sylramic™-SiC/SiC composites subjected to the dwell-fatigue loading at elevated temperatures in air and in steam conditions have been predicted.

Key-words : Ceramic-matrix composites (CMCs), Dwell-fatigue, Damage evolution, Oxidation, Matrix multicracking, Interface debonding

1. Introduction

Ceramic-matrix composites (CMCs) possess high strength-to-weight ratio at elevated temperatures, and are being designed and developed for hot section components in commercial aero engine. As new materials, the CMCs need to meet the airworthiness certification requirements, and it is necessary to analyze the degradation, damage and failure mechanisms subjected to cyclic loading at different temperatures and environments.

Many researchers performed experimental and theoretical investigations on the dwell-fatigue behavior of fiber-reinforced CMCs. Zhu investigated the monotonic tension, creep and tension–tension fatigue behavior of 2D Nicalon®–SiC/SiC composite at an elevated temperature of 1300°C in air and argon environment. The slope decreases and the width of the loops increases with increasing of applied cycles. The hysteresis loops move to the right along the strain axis due to the time-dependent damage, i.e., matrix cracking, fiber/matrix interface debonding and oxidation. However, the relationships between the macro stress/strain behavior and microstructure damages inside of SiC/SiC composite have not been established. John et al. investigated the durability of Melt Infiltrated (MI) 2D Hi-Nicalon™-SiC/SiC composite for different loading frequencies, i.e., 1 and 30 Hz, and dwell fatigue and creep loading at elevated temperatures of 815 and 1204°C. It was found that the peridoc loading and unloading cycles degrades the material performance of the dwell-fatigue specimens compared with that of creep specimens. The lifetime under fatigue and creep for different peak stresses were compared and analyzed, however, the micro damage mechanisms which caused the lifetime difference have not been analyzed. Gowayed et al. investigated the accumulation of time-dependent strain under dwell-fatigue of MI 2D Sylramic™-SiC/SiC composites with and without holes at 815 and 1204°C. The time-dependent strain accumulation was observed at the maximum stress level caused by matrix crack opening and oxidation of the BN coating. Upon unloading and reloading, the specimen breaks the seal and allows further ingress of the environment, which degrades the material performance of the composite under dwell fatigue loading. However, the accumulation of the composite time-dependent strain was predicted by data fitting using a 3-parameter curve fitting approach, not the damage-based models or approaches. Ruggles–Wrenn and Sharma investigated the cyclic tension–tension fatigue behavior of 2D Sylramic™-SiC/[SiC+Si3N4] composite at 1300°C in air and in steam atmospheres. At higher fatigue peak stress, the presence of steam caused noticeable degradation.

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in fatigue performance of the composite. The evolution of the peak strain and normalized modulus versus cycle fatigue cycles have been analyzed. Ruggles–Wrenn et al. investigated the cyclic tension–tension fatigue behavior of 2D Hi-Nicalon\textsuperscript{TM}–SiC/SiC composite with an inhibited matrix at 1200 and 1300\degree C in air and in steam conditions. The presence of steam has little influence on the fatigue performance of the composite at 1.0 Hz, but noticeably degrades fatigue lifetimes at 0.1 Hz. The flaw growth mechanisms in SiC fibers controlled the fatigue performance and lifetime of the SiC/SiC composite, which is affected by the testing temperature and environments. Shi et al.\textsuperscript{14} investigated the low cycle tension–tension fatigue behavior of 3D KD-F\textsuperscript{TM}–SiC/SiC composite at 1300\degree C in air condition. The effects of coating and loading frequency on the fatigue performance at elevated temperature were investigated. The strain ratcheting and modulus degradation with applied cycles were correlated with damage evolution, i.e., matrix multicrocking, fiber/matrix interface debonding and oxidation. However, the quantitative relationships between the damage evolution and the loading frequency, fatigue peak stress and testing temperature have not been established. Maillet et al.\textsuperscript{15} developed an acoustic emission based approach to monitor the damage evolution and lifetime of 2D SiC/[Si–B–C] composite with the self-healing matrix under static fatigue loading at 450 and 500\degree C. Singh\textsuperscript{16} developed a novel approach to monitor the crack initiation and growth in interlaminar testing of CMCs using the acoustic emission and direct current potential drop method. The damage modes of matrix crack, fibers breakage, and delamination can be detected. Zhang et al.\textsuperscript{17} investigated the deformation histories and identified the locations of strain concentration for both un-notched and single-edge-notched oxide/oxide CMCs using the digital image correlation (DIC) technique. However, the acoustic emission and DIC approaches have limitations on monitoring the multiple stage damage evolution process under dwell fatigue, thermomechanical fatigue or creep loading. Li\textsuperscript{18,19} established the relationship between the fatigue hysteresis dissipated energy-based damage parameter, internal damage of matrix cracking, fiber/matrix interface debonding and fibers fracture, and predict the damage evolution of unidirectional C/SiC under cyclic tension–tension fatigue loading at 800\degree C in air condition.\textsuperscript{20} Li\textsuperscript{21,22} investigated fatigue damage and lifetime of 2D SiC/SiC composite at elevated temperature without hold time in air and in steam atmosphere, and predicted the fatigue lifetime and fatigue limit stresses for different testing temperatures and environments; and compared the fatigue damage evolution between C/SiC and SiC/SiC composite through the fatigue hysteresis dissipated energy and fiber/matrix interface shear stress degradation rate. Li\textsuperscript{23} developed the hysteresis loops model of fiber-reinforced CMCs considering oxidation subjected to static fatigue loading. Pailler and Lamon\textsuperscript{24} investigated the static fatigue behavior of SiC/SiC micromposite considering the interface degradation at elevated temperature. For the lifetime prediction of CMCs, besides the interface degradation, the fibers strength degradation should also be considered. However, under dwell-fatigue loading at elevated temperature in the oxidative environment, the fiber/matrix interface debonding and sliding between fibers and the matrix are affected by the combination effects of interface oxidation and interface wear. The fatigue hysteresis based damage models of fiber-reinforced CMCs subjected to dwell fatigue at elevated temperature have not been established.

The objective of this paper is to investigate the damage development and fatigue lifetime prediction of two different fiber-reinforced CMCs subjected to the dwell-fatigue loading at elevated temperatures in oxidizing atmosphere. The fatigue hysteresis based damage models under dwell fatigue at elevated temperature are developed considering the fiber/matrix interface debonding, sliding, wear and oxidation. The fibers failure probability is determined in the fiber/matrix interface debonded region and interface bonded region considering fiber/matrix interface wear and oxidation. The relationships between the damage parameters and internal damage development of matrix multicrocking, fiber/matrix interface debonding and sliding, and fibers fracture are established. The experimental damage development and fatigue lifetime curves of cross-ply Nicalon\textsuperscript{TM}–SiC/MAS and 2D Sylramic\textsuperscript{TM}–SiC/SiC composites subjected to dwell-fatigue loading at elevated temperature in air and in steam conditions are predicted.

2. Theoretical analysis

The unit cell contained a single fiber surrounded by a hollow cylinder of matrix is extracted from the ceramic composite system, as shown in Fig. 1. The fiber radius is $r_f$, and the matrix radius is $R = r_f / F_f^{1/2}$, as shown in Fig. 1. At elevated temperatures, the oxidative gas can enter into the internal of the composite through matrix crackings and oxidize the fiber/matrix interphase and fibers, as shown in Fig. 2. The opening of matrix crackings will be increased subjected to cyclic fatigue loading, which greatly reduced the resistance to oxidation. Under cyclic fatigue loading, the fiber/matrix interface shear stress decreases with applied cycles due to the interface wear and interface oxidation at elevated temperature.\textsuperscript{25–34} Fantozzi and Reynaud\textsuperscript{35} found that the fiber/matrix interface shear stress degradation in 2D SiC/SiC and C/ [Si–B–C] composites at elevated temperatures due to the oxidation of fiber/matrix PyC interface. Li\textsuperscript{26} compared the fiber/matrix interface shear stress degradation rate between C/SiC and SiC/SiC composites with different fiber preforms, i.e., unidirectional, cross-ply, 2D and 2.5D woven, and 3D braided, under cyclic fatigue loading at room and elevated temperatures. It was found that the fiber/matrix interface shear stress degradation rate is the highest for 3D braided SiC/SiC composite at 1300\degree C in air environment due to the fiber/matrix interface oxidation, and the lowest for 2D C/SiC composite at room temperature. The fiber/matrix interface debonded region can be divided into two regions, including: (1) the fiber/matrix interface oxidation region [i.e., $x \in [0, \xi]$], $\tau_f(x) = \tau_f$, and $\xi$ denotes the interface oxidation length; and (2) the
fiber/matrix interface wear region [i.e., \(x \in [\xi, l_d]\)] \(\tau(x) = \tau(N)\), and \(l_d\) denotes the fiber/matrix interface debonded length. The degradation of the fiber/matrix interface shear stress in the interface wear region can be determined using the following equation.37)

\[
\frac{\tau(N) - \tau_s}{\tau_0 - \tau_s} = \left(1 + b_0 \left(1 + b_0 N^j\right)^{-1}\right) \quad (1)
\]

where \(\tau(N)\) denotes the fiber/matrix interface shear stress at the \(N\)th applied cycle; \(\tau_0\) denotes the initial fiber/matrix interface shear stress; \(\tau_s\) denotes the steady-state fiber/matrix interface shear stress; \(b_0\) is a coefficient; and \(j\) is an exponent which determines the rate at which the fiber/matrix interface shear stress drops with the number of cycle \(N\).

2.1 Dwell-fatigue damage development models

Under dwell-fatigue loading, the damage development inside of the fiber-reinforced CMCs depends upon the dwell-fatigue time, cyclic number and fatigue peak stress. In the present analysis, the combining effects of the fiber/matrix interface oxidation and interface wear in the interface debonded region on the interface debonding, interface sliding and fibers fracture in different damage regions are considered. The relationships between the fatigue hysteresis-based damage parameters, dwell-fatigue time, cycle number and fatigue peak stress are established based on the damage development models. Based on the fiber/matrix interface debonding and interface sliding between the fiber and the matrix inside of composite, as shown in Fig. 3, the fatigue hysteresis loops of fiber-reinforced CMCs subjected to dwell-fatigue loading can be divided into four different cases, including:

1. Case 1: the fiber/matrix interface oxidation region and the interface wear region are less than matrix crack spacing, and the fiber/matrix interface counter-slip and the fiber/matrix interface new-slip lengths are equal to the fiber/matrix interface debonded length.

2. Case 2: the fiber/matrix interface oxidation region and the fiber/matrix interface wear region are less than matrix crack spacing, and the fiber/matrix interface counter-slip and the interface new-slip lengths are less than the fiber/matrix interface debonded length.

3. Case 3: the fiber/matrix interface oxidation region...
and the interface wear region are equal to matrix crack spacing, and the fiber/matrix interface counter-slip and the interface new-slip lengths are less than matrix crack spacing.

(4) Case 4: the fiber/matrix interface oxidation region and the interface wear region are equal to matrix crack spacing, and the fiber/matrix interface counter-slip and the interface new-slip lengths are equal to matrix crack spacing.

When the fiber/matrix interface oxidation region and the interface wear region are less than matrix crack spacing, the unloading and reloading stress-strain relationships are determined using the following equation.

\[
\varepsilon_{\text{un}} = \frac{2\sigma_{\text{f}}}{V_1E_{\text{f}}l_{\text{c}}} + \frac{2\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} \xi^2 + \frac{4\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} (l_\text{d} - \xi) + \frac{4\varepsilon_{\text{f}}}{E_{\text{f}}l_{\text{c}}} (y - \xi)^2 - \frac{2\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} (2y - \xi - l_\text{d})^2 + \frac{2\sigma_{\text{m}}}{E_{\text{m}}} \left( \frac{l_\text{c}}{2} - l_\text{d} \right) + \frac{2r_{\text{f}}}{\rho E_{\text{f}}} V_\text{m} \sigma_{\text{mo}} + \frac{2\tau_{\text{f}}}{r_{\text{f}}} \xi \\
\times \left[ 1 - \exp \left( -\rho \frac{l_\text{c}/2 - l_\text{d}}{r_{\text{f}}} \right) \right] - (\alpha_c - \alpha_\xi) \Delta T
\]

(2)

\[
\varepsilon_{\text{re}} = \frac{2\sigma_{\text{f}}}{V_1E_{\text{f}}l_{\text{c}}} l_\text{d} - \frac{4\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} (2z - \xi)^2 - \frac{4\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} (l_\text{d} - \xi) + \frac{4\varepsilon_{\text{f}}}{E_{\text{f}}l_{\text{c}}} (y - \xi)^2 - \frac{2\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} (2y - \xi - l_\text{d})^2 + \frac{2\sigma_{\text{m}}}{E_{\text{m}}} V_\text{m} \sigma_{\text{mo}} + \frac{2\tau_{\text{f}}}{r_{\text{f}}} \xi \\
+ \frac{2r_{\text{f}}}{\rho E_{\text{f}}} V_\text{m} \sigma_{\text{mo}} + \frac{2\tau_{\text{f}}}{r_{\text{f}}} \xi \\
\times \left[ 1 - \exp \left( -\rho \frac{l_\text{c}/2 - l_\text{d}}{r_{\text{f}}} \right) \right] - (\alpha_c - \alpha_\xi) \Delta T
\]

(3)

where \(\sigma\) denotes the applied stress; \(V_1\) and \(V_\text{m}\) denote the fiber and matrix volume content, respectively; \(E_{\text{f}}\) denotes the fiber elastic modulus; \(l_\text{c}\) denote the matrix crack spacing; \(\tau_{\text{f}}\) denotes the fiber/matrix interface shear stress in the interface oxidation region; \(\sigma_{\text{m}}\) and \(\sigma_{\text{mo}}\) denote the fiber and matrix axial stress in the fiber/matrix interface bonded region, respectively; \(\rho\) denotes the shear-lag model parameter; \(\zeta_\text{d}\) denotes the fiber/matrix interface debonded energy; \(\alpha_\xi\) and \(\alpha_c\) denote the fiber and composite thermal expansion coefficient, respectively; and \(\Delta T\) denotes the temperature difference between the fabricated temperature \(T_0\) and testing temperature \(T_1\) (\(\Delta T = T_1 - T_0\)). The fiber/matrix interface counter-slip length (y) and interface new slip length (z) considering the combining effects of the fiber/matrix interface oxidation and interface wear are determined using the following equation when the fiber/matrix interface partially debonds.

\[
y = \frac{1}{2} \left( l_\text{d} + \left[ 1 - \frac{\tau_{\text{f}}}{\tau_{\text{i}}(N)} \right] \xi \right) = \frac{r_{\text{f}}}{2} \left[ V_\text{m}E_{\text{f}} \sigma \frac{\sigma}{V_1E_{\text{f}} \tau_{\text{i}}(N)} - 1 \right] \rho
\]

\[
- \frac{\tau_{\text{f}}}{2\rho} \frac{l_\text{c}}{2} + \frac{r_{\text{f}}V_\text{m}E_{\text{f}}}{E_{\text{f}} \tau_{\text{i}}(N)} \zeta_\text{d} \right)
\] (4)

\[
z = \frac{\tau_{\text{f}}(N)}{\tau_{\text{i}}(N)} \left( y - 1 \right) \left( l_\text{d} + \left[ 1 - \frac{\tau_{\text{f}}}{\tau_{\text{i}}(N)} \right] \xi \right) = \frac{r_{\text{f}}}{2} \left( V_\text{m}E_{\text{f}} \sigma \frac{\sigma}{V_1E_{\text{f}} \tau_{\text{i}}(N)} - 1 \right) \rho
\]

\[
- \left[ \frac{r_{\text{f}}}{2\rho} \frac{l_\text{c}}{2} + \frac{r_{\text{f}}V_\text{m}E_{\text{f}}}{E_{\text{f}} \tau_{\text{i}}(N)} \zeta_\text{d} \right] \right)
\] (5)

When the fiber/matrix interface oxidation region and the interface wear region are equal to matrix crack spacing, the unloading and reloading stress-strain relationships are determined using the following equation.

\[
\varepsilon_{\text{un}} = \frac{\sigma}{V_1E_{\text{f}}} - \frac{2\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} \xi^2 + \frac{2\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} \xi + \frac{4\varepsilon_{\text{f}}}{E_{\text{f}}l_{\text{c}}} (y - \xi)^2 - \frac{2\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} (2y - \xi - l_\text{d})^2 + \frac{2\sigma_{\text{m}}}{E_{\text{m}}} V_\text{m} \sigma_{\text{mo}} + \frac{4\tau_{\text{f}}}{r_{\text{f}}E_{\text{f}}l_{\text{c}}} (y - \xi)^2
\]

\[
\times \left[ 1 - \exp \left( -\rho \frac{l_\text{c}/2 - l_\text{d}}{r_{\text{f}}} \right) \right] - (\alpha_c - \alpha_\xi) \Delta T
\] (6)
where the fiber/matrix interface counter-slip length and interface new slip length considering the combining effects of the interface oxidation and interface wear are calculated using the following equation, corresponding to the fiber/matrix interface completely debonding.

\[
y = \left[ 1 - \frac{\tau_{f}}{\tau_{f}(N)} \right] \frac{\xi}{r_{f}V_{m}E_{m}} + \frac{r_{f}V_{m}E_{m}}{4V_{f}E_{f}r_{f}(N)} (\sigma_{\text{max}} - \sigma)
\]

\[
z = y(\sigma_{\text{min}}) - \frac{r_{f}V_{m}E_{m}}{4V_{f}E_{f}r_{f}(N)} (\sigma_{\text{max}} - \sigma)
\]

where \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) denote the peak and valley stress, respectively.

The fatigue hysteresis dissipated energy \( (U_{c}) \) is defined using the following equation.

\[
U_{c} = \int_{\sigma_{\text{min}}}^{\sigma_{\text{max}}} \left[ \varepsilon_{c,\text{unload}}(\sigma) - \varepsilon_{c,\text{reload}}(\sigma) \right] d\sigma
\]

where \( \varepsilon_{c,\text{unload}} \) and \( \varepsilon_{c,\text{reload}} \) denote the unloading and reloading strain, respectively.

### 2.2 Dwell-fatigue lifetime prediction models

Under dwell-fatigue loading, fibers failure in fiber-reinforced CMCs depends upon the degradation of fibers strength in different damage regions, i.e., the fiber/matrix interface oxidation region, interface wear region and interface debonded region. In the present analysis, the fibers failure probabilities in different damage regions have been determined considering the dwell-fatigue time, cyclic number and fatigue peak stress. The fibers broken fraction versus cycle number curves and fatigue lifetime subjected to dwell-fatigue loading are predicted using the present dwell-fatigue lifetime prediction models. The Global Load Sharing (GLS) assumption is used to determine the load carried by intact and fracture fibers.

\[
\frac{\sigma}{V_{f}} = \left[ 1 - P_{t} \left( 1 + \frac{2l_{f}}{l_{c}} \right) \right] T + P_{t} \frac{2l_{f}}{l_{c}} (T_{b})
\]

where \( l_{f} \) denotes the slip length over which the fiber stress would decay to zero if not interrupted by the far-field equilibrium stresses; \( T_{b} \) denotes the average stress carried by broken fibers; \( P_{t} \) denotes the total fiber failure probability; and \( P_{f} \) denotes the fiber failure probability in the interface debonded region and interface bonded region.

When matrix cracking and interface debonding occur subjected to dwell-fatigue loading, the micromechanical stress distribution in fibers change along the fiber length in different damage regions. Due to the characteristic of statistical failure for fibers, the total fibers failure probability \( P_{f} \) is determined as a sum of fibers failure probability in the interface oxidation region, interface debonded region and interface bonded region, following the treatment of fibers fracture probability in different damage regions by Oh and Finnie,48 Curtin,39 Liao and Reifsnider,40 and Li.41 The total fiber failure probability \( P_{f} \) and the fiber failure probability in the interface debonded region and interface bonded region \( P_{f} \) are determined using the following equations.

\[
P_{f} = \chi[\zeta P_{fa} + (1 - \eta)P_{fb}] + P_{fe} + P_{ld}
\]

\[
P_{fa} = P_{fe} + P_{ld}
\]

where \( P_{fa}, P_{fb}, P_{fe} \) and \( P_{ld} \) denote the fiber failure probability of oxidized fibers in the oxidation region, unoxidized fibers in the oxidation region, fibers in the interface debonded region and interface bonded region, respectively; \( \zeta \) denotes the oxidation fibers fraction in the oxidized region; and \( \chi \) denotes the fraction of oxidation in the multiple matrix cracks.

\[
P_{fa}(T) = 1 - \exp \left\{ -2 \frac{l_{f}}{l_{0}} \frac{T}{\sigma_{0}(T)} \right\}
\]

\[
P_{fb}(T) = 1 - \exp \left\{ -2 \frac{l_{f}}{l_{0}} \frac{T^{m}}{\sigma_{0}(T)} \right\}
\]

\[
P_{fe}(T) = 1 - \exp \left\{ - \frac{r_{f}T^{m+1}}{l_{0}[\sigma_{0}(N)]^{m+1}} \cdot \tau_{f}(N)(m + 1) \right\}
\]

\[
P_{ld}(T) = 1 - \exp \left\{ - \frac{2r_{f}T^{m}}{\rho L_{0}[\sigma_{0}(N)]^{m+1}(m + 1)} \right\}
\]

where \( T \) denotes the load carried by intact fibers; \( \sigma_{0} \) denotes the fiber stress in the interface bonded region; \( l_{f} \) denotes the slip length over which the fiber stress would decay to zero if not interrupted by the far-field equilibrium stresses. The time-dependent fiber stress is controlled by surface defects resulting from oxidation.42

\[
\begin{align*}
\sigma_{0}(t) &= \sigma_{0}, & t &\leq \frac{k}{Y} \left( \frac{K_{IC}}{Y\sigma_{0}} \right)^{4} \\
\sigma_{0}(t) &= \frac{K_{IC}}{Y\sqrt{k}}, & t &> \frac{k}{Y} \left( \frac{K_{IC}}{Y\sigma_{0}} \right)^{4}
\end{align*}
\]

where \( K_{IC} \) denotes the critical stress intensity factor; \( Y \) is a geometric parameter; and \( k \) is the parabolic rate constant.

With increasing of applied cycle number, the fiber/matrix interface shear stress and fibers strength decrease due to the fiber/matrix interface wear and interface oxidation. The fibers failure probability in the fiber/matrix...
interface oxidation region, interface debonded region and interface bonded region can be obtained by combining the interface wear model, interface oxidation model and fiber strength degradation model with Eqs. (11)-(18). The evolution of fibers failure probability versus applied cycle number curves can be obtained. When the fibers broken fraction approaches to the critical value, the composite fatigue fractures. The fatigue limit stress is calculated when the fracture applied cycles approach to the maximum cycle number.

3. Experimental comparisons and discussions

The damage development and fatigue lifetime of cross-ply Nicalon™-SiC/MAS and 2D Sylramic™-SiC/SiC composites at elevated temperatures are predicted. The fatigue hysteresis-based damage parameters of the fatigue hysteresis dissipated energy ($U_e$), fatigue hysteresis modulus ($E_{NOR}$), and fatigue peak strain ($\varepsilon_p$) versus applied cycles are analyzed. For the SiC/SiC and SiC/MAS composites, cracks can form in the matrix material at a relatively low stresses. Oxygen from the atmosphere travels through the matrix cracks to the fiber coating. The carbon coatings can begin oxidizing at 450°C. Once the carbon is removed, the oxygen reacts with the fiber to form a silica layer. The silica (SiO$_2$) layer weakens the fiber strength.

3.1 Cross-ply Nicalon™-SiC/MAS composite at 566 and 1093°C in air

Grant$^{(43)}$ investigated the cyclic tension–tension fatigue behavior of cross-ply Nicalon™-SiC/MAS composite under dwell-fatigue loading at elevated temperatures of 566 and 1093°C in air atmosphere. The fatigue tests were conducted under the load control. The loading frequency was 1 Hz, and the fatigue load ratio (i.e., minimum to maximum stress) was 0.1. At 566°C in air condition, the composite tensile strength was approximately 292 MPa, and the fatigue peak stresses were 137 and 103 MPa with the dwell-fatigue time of $t=1$, 10 and 100 s; and at 1093°C in air condition, the composite tensile strength were approximately 209 MPa, and the fatigue peak stresses were 137 and 103 MPa with the dwell-fatigue time of $t=1$, 10 and 100 s. The material properties of cross-ply Nicalon™-SiC/MAS composite are given by:$^{(44)}$ $V_f=20\%$, $r_f=7.5\mu m$, $E_f=200\ GPa$, $E_m=138\ GPa$, $\alpha_f=4\times10^{-6}/^\circ C$, $\alpha_m=2.4\times10^{-6}/^\circ C$, $\Delta T=-1000^\circ C$, $\tau_f=20\ MPa$, $\tau_f=5\ MPa$, $\varepsilon_f=0.11/m^2$, $m=4.4$, $\sigma_0=2.6\ GPa$ and $L_0=100\ nm$. $^{(44)}$ It should be noted that the material properties listed above are assumed constant with respect to temperature.

3.1.1 566°C in air condition

Under dwell-fatigue peak stress of $\sigma_{max}=137\ MPa$ with the dwell-fatigue time of $t=1$, 10 and 100 s, the experimental fatigue hysteresis dissipated energy ($U_e$) and the normalized fatigue hysteresis modulus ($E_{NOR}$) decrease with applied cycles, and the fatigue peak strain ($\varepsilon_p$) increases with applied cycles, as shown in Fig. 4 and Table 1.
ical computational values; the degradation of the fiber/matrix interface shear stress is simulated using the Evans model,\(^37\) in which the model parameters are given by: \(\tau_0 = 20\ \text{MPa}, \ \tau_s = 5.0\ \text{MPa}, \ b_0 = 2.0\) and \(j = 0.12\); and the time-dependent fiber strength is determined using the Eq. (18), and the model parameters are given by:\(^42\) \(k = \exp[11.383 - (8716/T_{\text{em}})] \times 10^{-18}/60\ \text{m}^2/\text{s}, \ K_{IC} = 0.5\ \text{MPa}/\text{m}^{1/2}\) and \(Y = 1\). The experimental fatigue hysteresis dissipated energy \(U_e\) decreases from 11.8 kJ/m\(^3\) at the first applied cycle to 7.8 kJ/m\(^3\) at the 265th applied cycle due to matrix multicrocking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue hysteresis dissipated energy \(U_e\) decreases from 11.9 kJ/m\(^3\) at the first applied cycle to 8.2 kJ/m\(^3\) at the 300th applied cycle, corresponding to the fiber/matrix interface slip Case 4, i.e., the fiber/matrix interface completely debonding and the fiber completely sliding relative to the matrix in the interface debonded region, as shown in Fig. 4(a) and Table 1. The normalized experimental fatigue hysteresis modulus \(E_{\text{NOR}}\) decreases from 1.0 upon first loading to 0.55 at the 265th applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue hysteresis modulus \(E_{\text{NOR}}\) decreases from 1.0 upon first loading to 0.56 at the 300th applied cycle, as shown in Fig. 4(b) and Table 1. The experimental fatigue peak strain \(\varepsilon_p\) increases from 0.342% at the first applied cycle to 0.453% at the 265th applied cycle due to matrix cracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue peak strain \(\varepsilon_p\) increases from 0.349% at the first applied cycle to 0.443% at the 3000th applied cycle, as shown in Fig. 4(c) and Table 1.

When the dwell-fatigue time is \(t = 100\ s\), the fiber/matrix interface shear stress corresponding to different applied cycle numbers can be determined through comparing experimental fatigue hysteresis dissipated energy with theoretical computational values; the degradation of the fiber/matrix interface shear stress is simulated using the Evans model,\(^37\) in which the model parameters are given by: \(\tau_0 = 20\ \text{MPa}, \ \tau_s = 5.0\ \text{MPa}, \ b_0 = 2.0\) and \(j = 0.18\); and the time-dependent fiber strength is determined by the Eq. (18), and the model parameters are given by:\(^42\) \(k = \exp[11.383 - (8716/T_{\text{em}})] \times 10^{-18}/60\ \text{m}^2/\text{s}, \ K_{IC} = 0.5\ \text{MPa}/\text{m}^{1/2}\) and \(Y = 1\). The experimental fatigue hysteresis dissipated energy \(U_e\) decreases from 11.8 kJ/m\(^3\) at the first applied cycle to 7.8 kJ/m\(^3\) at the 265th applied cycle due to matrix multicrocking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue hysteresis dissipated energy \(U_e\) decreases from 11.9 kJ/m\(^3\) at the first applied cycle to 8.2 kJ/m\(^3\) at the 300th applied cycle, corresponding to the fiber/matrix interface slip Case 4, i.e., the fiber/matrix interface completely debonding and the fiber completely sliding relative to the matrix in the interface debonded region, as shown in Fig. 4(a) and Table 1. The normalized experimental fatigue hysteresis modulus \(E_{\text{NOR}}\) decreases from 1.0 upon first loading to 0.55 at the 265th applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue hysteresis modulus \(E_{\text{NOR}}\) decreases from 1.0 upon first loading to 0.56 at the 300th applied cycle, as shown in Fig. 4(b) and Table 1. The experimental fatigue peak strain \(\varepsilon_p\) increases from 0.342% at the first applied cycle to 0.453% at the 265th applied cycle due to matrix cracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue peak strain \(\varepsilon_p\) increases from 0.349% at the first applied cycle to 0.443% at the 3000th applied cycle, as shown in Fig. 4(c) and Table 1.

When the dwell-fatigue time is \(t = 100\ s\), the fiber/matrix interface shear stress corresponding to different applied cycle numbers can be determined through comparing experimental fatigue hysteresis dissipated energy with theoretical computational values; the degradation of the fiber/matrix interface shear stress is simulated using the Evans model,\(^37\) in which the model parameters are given by: \(\tau_0 = 20\ \text{MPa}, \ \tau_s = 5.0\ \text{MPa}, \ b_0 = 2.0\) and \(j = 0.24\); and the time-dependent fiber strength is determined by the Eq. (18), and the model parameters are given by:\(^42\) \(k = \exp[11.383 - (8716/T_{\text{em}})] \times 10^{-18}/60\ \text{m}^2/\text{s}, \ K_{IC} = 0.5\ \text{MPa}/\text{m}^{1/2}\) and \(Y = 1\). The experimental fatigue hysteresis dissipated energy \(U_e\) decreases from 11.8 kJ/m\(^3\) at the first applied cycle to 7.8 kJ/m\(^3\) at the 265th applied cycle due to matrix multicrocking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue hysteresis dissipated energy \(U_e\) decreases from 11.9 kJ/m\(^3\) at the first applied cycle to 8.2 kJ/m\(^3\) at the 300th applied cycle, corresponding to the fiber/matrix interface slip Case 4, i.e., the fiber/matrix interface completely debonding and the fiber completely sliding relative to the matrix in the interface debonded region, as shown in Fig. 4(a) and Table 1. The normalized experimental fatigue hysteresis modulus \(E_{\text{NOR}}\) decreases from 1.0 upon first loading to 0.55 at the 265th applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue hysteresis modulus \(E_{\text{NOR}}\) decreases from 1.0 upon first loading to 0.56 at the 300th applied cycle, as shown in Fig. 4(b) and Table 1. The experimental fatigue peak strain \(\varepsilon_p\) increases from 0.342% at the first applied cycle to 0.453% at the 265th applied cycle due to matrix cracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue peak strain \(\varepsilon_p\) increases from 0.349% at the first applied cycle to 0.443% at the 3000th applied cycle, as shown in Fig. 4(c) and Table 1.

When the dwell-fatigue time is \(t = 100\ s\), the fiber/matrix interface shear stress corresponding to different applied cycle numbers can be determined through comparing experimental fatigue hysteresis dissipated energy with theoretical computational values; the degradation of the fiber/matrix interface shear stress is simulated using the Evans model,\(^37\) in which the model parameters are given by: \(\tau_0 = 20\ \text{MPa}, \ \tau_s = 5.0\ \text{MPa}, \ b_0 = 2.0\) and \(j = 0.18\); and the time-dependent fiber strength is determined by the Eq. (18), and the model parameters are given by:\(^42\) \(k = \exp[11.383 - (8716/T_{\text{em}})] \times 10^{-18}/60\ \text{m}^2/\text{s}, \ K_{IC} = 0.5\ \text{MPa}/\text{m}^{1/2}\) and \(Y = 1\). The experimental fatigue hysteresis dissipated energy \(U_e\) decreases from 11.8 kJ/m\(^3\) at the first applied cycle to 7.8 kJ/m\(^3\) at the 265th applied cycle due to matrix multicrocking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue hysteresis dissipated energy \(U_e\) decreases from 11.9 kJ/m\(^3\) at the first applied cycle to 8.2 kJ/m\(^3\) at the 300th applied cycle, corresponding to the fiber/matrix interface slip Case 4, i.e., the fiber/matrix interface completely debonding and the fiber completely sliding relative to the matrix in the interface debonded region, as shown in Fig. 4(a) and Table 1. The normalized experimental fatigue hysteresis modulus \(E_{\text{NOR}}\) decreases from 1.0 upon first loading to 0.55 at the 265th applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue hysteresis modulus \(E_{\text{NOR}}\) decreases from 1.0 upon first loading to 0.56 at the 300th applied cycle, as shown in Fig. 4(b) and Table 1. The experimental fatigue peak strain \(\varepsilon_p\) increases from 0.342% at the first applied cycle to 0.453% at the 265th applied cycle due to matrix cracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue peak strain \(\varepsilon_p\) increases from 0.349% at the first applied cycle to 0.443% at the 3000th applied cycle, as shown in Fig. 4(c) and Table 1.
0.5 MPa/m^{1/2} and Y = 1. The experimental fatigue hysteresis dissipated energy \( U_c \) decreases from 13 kJ/m^3 at the first applied cycle to 9.8 kJ/m^3 at the 14th applied cycle due to matrix multicracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue hysteresis dissipated energy \( U_{c, p} \) decreases from 13 kJ/m^3 at the first applied cycle to 4.8 kJ/m^3 at the 100th applied cycle, corresponding to the fiber/matrix interface slip Case 4, i.e., the fiber/matrix interface completely debonding and the fiber completely sliding relative to the matrix in the interface debonded region, as shown in Fig. 4(a) and Table 1. The normalized experimental fatigue hysteresis modulus \( E_{f, int}^{NOR} \) decreases from 1.0 upon first loading to 0.51 at the 20th applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue hysteresis modulus \( E_{f, int}^{p} \) decreases from 1.0 upon first loading to 0.53 at the 30th applied cycle, as shown in Fig. 4(b) and Table 1. The experimental fatigue peak strain \( \varepsilon_{p} \) increases from 0.303% at the first applied cycle to 0.448% at the 14th applied cycle due to matrix multicracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue peak strain \( \varepsilon_{p}^{p} \) increases from 0.307% at the first applied cycle to 0.407% at the 300th applied cycle, as shown in Fig. 4(c) and Table 1.

The experimental and theoretical predicted fatigue lifetime curves and the theoretical predicted broken fibers versus applied cycles curves under fatigue peak stresses of \( \sigma_{\max} = 137 \) MPa of cross-ply Nicalon\textsuperscript{TM}-SiC/MAS composite with the dwell-fatigue time of \( t = 1, 10 \) and 100 s at 566°C in air condition are illustrated in Fig. 5. With increasing of dwell-fatigue time, the fatigue lifetime under the high fatigue peak stress rapidly decreases, and the fatigue limit stress decreases, as shown in Fig. 5(a). The experimental and predicted failure cycles of cross-ply Nicalon\textsuperscript{TM}-SiC/MAS composite under dwell-fatigue loading at 566°C in air condition is given in Table 2.

By combining the fiber/matrix interface wear model, interface oxidation model and fiber strength degradation model with Eqs. (11)–(18), the fibers failure probability in the fiber/matrix interface oxidation region, interface debonded and interface bonded region can be determined. The evolution of the fibers failure probability versus applied cycle number curves can be obtained. When the dwell-fatigue time is \( t = 1 \) s, the theoretical predicted broken fibers fraction increases from 0.5% at the first applied cycle to 31.1% at the 359th applied cycle; and when the dwell-fatigue time is \( t = 100 \) s, the theoretical predicted broken fibers fraction increases from 0.6% at the first applied cycle to 30.8% at the 22th applied cycle, as shown in Fig. 5(b). In the present analysis, the GLS criterion was adopted to determine the load carrying between intact and broken fibers, leading to the high fiber failure probability under cyclic fatigue loading.

### 3.1.2 1093°C in air condition

Under dwell-fatigue peak stress of \( \sigma_{\max} = 103 \) MPa with the dwell-fatigue time of \( t = 1 \) and 10 s, the experimental fatigue hysteresis dissipated energy \( U_c \) and norm-

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**Table 2.** The experimental and predicted failure cycles of cross-ply Nicalon\textsuperscript{TM}-SiC/MAS composite subjected to dwell-fatigue loading at 566°C in air condition

| Items   | Experiment | Theory | Experiment | Theory | Experiment | Theory |
|---------|------------|--------|------------|--------|------------|--------|
|         |           |        | \( t = 1 \) s |        | \( t = 10 \) s |        | \( t = 100 \) s |        |
| \( \sigma_{\max} \) (MPa) | \( \sigma_{\max} \) (MPa) | \( \sigma_{\max} \) (MPa) | \( \sigma_{\max} \) (MPa) | \( \sigma_{\max} \) (MPa) | \( \sigma_{\max} \) (MPa) | \( \sigma_{\max} \) (MPa) |
| Cycles  | 1195       | 101834 | 1934       | 1181930 | 265        | 91739  | 362        | 92868  | 14         | 5135   | 24         | 11983   |

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**Fig. 5.** (a) The experimental and theoretical predicted fatigue life S–N curves; and (b) the theoretical predicted broken fibers fraction versus applied cycles curves under the fatigue peak stress of \( \sigma_{\max} = 137 \) MPa of cross-ply Nicalon\textsuperscript{TM}-SiC/MAS composite with the dwell-fatigue time of \( t = 1, 10 \), and 100 s at 566°C in air condition.
Fig. 6. (a) The experimental and theoretical predicted fatigue hysteresis dissipated energy versus applied cycles; (b) the normalized experimental and theoretical predicted fatigue hysteresis modulus versus applied cycles; and (c) the experimental and theoretical predicted fatigue peak strain versus applied cycles of cross-ply Nicalon™-SiC/MAS composite under the fatigue peak stress of $\sigma_{\text{max}} = 103 \text{MPa}$ at 1093°C with dwell-fatigue time of $t = 1$ and 10 s.

Normalized fatigue hysteresis modulus ($E_{\text{NOR}}$) decrease with applied cycles, and the fatigue peak strain ($\epsilon_p$) increases with applied cycles, as shown in Fig. 6 and Table 3.

When the dwell-fatigue time is $t = 1$ s, the fiber/matrix interface shear stress corresponding to different applied cycle numbers can be determined through comparing experimental fatigue hysteresis dissipated energy with theoretical computational values; the degradation of the fiber/matrix interface shear stress is simulated using the Evans model, in which the model parameters are given by: $\tau_0 = 20 \text{MPa}$, $\tau_s = 5 \text{MPa}$, $b_0 = 2.0$ and $j = 0.13$; and the time-dependent fiber strength is determined by the Eq. (18), and the model parameters are given by: $k = \exp[1.383 - (8716/T_{\text{em}})] \times 10^{-18}/60 \text{m}^2/\text{s}$, $K_\text{IC} = 0.5 \text{MPa}/\text{m}^{1/2}$ and $Y = 1$. The experimental fatigue hysteresis dissipated energy ($U_e$) decreases from 16.9 kJ/m$^3$ at the first applied cycle to 7.5 kJ/m$^3$ at the 6017th applied cycle due to matrix multicracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue hysteresis dissipated energy ($U_e$) decreases from 16.7 kJ/m$^3$ at the first applied cycle to 6.9 kJ/m$^3$ at the 6017th applied cycle, corresponding to the fiber/matrix interface slip Case 4, i.e., the fiber/matrix interface completely debonding and the fiber completely sliding relative to the matrix in the interface debonded region, as shown in Fig. 6(a) and Table 3. The normalized experimental fatigue hysteresis modulus ($E_{\text{NOR}}$) decreases rapidly at the initial applied cycles due to matrix cracking and fiber/matrix interface debonding, i.e., from 1.0 upon first loading to 0.89 at the 4th applied cycle, and the normalized theoretical predicted fatigue hysteresis modulus ($E_{\text{NOR}}$) decreases from 1.0 upon first loading to 0.73 at the 600th applied cycle, as shown in Fig. 6(b) and Table 3. The experimental fatigue peak strain ($\epsilon_p$) increases from 0.484% at the first applied cycle to 0.633% at the 6017th applied cycle due to matrix multicracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue peak strain ($\epsilon_p$) increases from 0.487% at the first applied cycle to 0.55% at the 6017th applied cycle, as shown in Fig. 6(c) and Table 3.

When the dwell-fatigue time is $t = 10$ s, the fiber/matrix interface shear stress corresponding to different applied cycle numbers can be determined through comparing experimental fatigue hysteresis dissipated energy with theoretical computational values; the degradation of the fiber/matrix interface shear stress is simulated using the Evans model in which the model parameters are given by: $\tau_0 = 20 \text{MPa}$, $\tau_s = 5 \text{MPa}$, $b_0 = 2.0$ and $j = 0.2$; and the time-dependent fiber strength is determined by Eq. (18), and the model parameters are given by: $k = \exp[1.383 - (8716/T_{\text{em}})] \times 10^{-18}/60 \text{m}^2/\text{s}$, $K_\text{IC} = 0.5 \text{MPa}/\text{m}^{1/2}$ and $Y = 1$. The experimental fatigue hysteresis dissipated energy ($U_e$) decreases from 12.4 kJ/m$^3$ at the first applied cycle to 6.3 kJ/m$^3$ at the 216th applied cycle due to matrix multicracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue hysteresis dissipated energy ($U_e$) decreases from 12.4 kJ/m$^3$ at the first applied cycle to 5.4 kJ/m$^3$ at the 216th applied cycle, corresponding to the fiber/matrix interface slip Case 4, i.e., the fiber/matrix interface completely debonding and the fiber completely sliding relative to the matrix in the interface debonded region, as shown in Fig. 6(a) and Table 3. The normalized experimental
fatigue hysteresis modulus ($E_{NOR}$) decreases from 1.0 upon first loading to 0.55 at the 263rd applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue hysteresis modulus ($E_{NOR}$) decreases from 1.0 upon first loading to 0.52 at the 316th applied cycle, as shown in Fig. 6(b) and Table 3. The experimental fatigue peak strain ($\epsilon_p$) increases from 0.457% at the first applied cycle to 0.532% at the 216th applied cycle due to matrix multicracking and fiber/matrix interface debonding and sliding, and the theoretical predicted fatigue peak strain ($\epsilon_p$) increases from 0.457% at the first applied cycle to 0.525% at the 216th applied cycle, as shown in Fig. 6(c) and Table 3.

The experimental and theoretical predicted fatigue lifetime curves and the theoretical predicted broken fibers fraction versus applied cycle curves of cross-ply Nicalon$^{TM}$–SiC/MAS composite under fatigue peak stress of $\sigma_{max} = 103$ MPa with the dwell-fatigue time of $t = 1$, 10 and 100 s at 1093°C in air condition are illustrated in Fig. 7. With increasing of dwell-fatigue time, the fatigue lifetime under high fatigue peak stress rapidly decreases, and the fatigue limit stress decreases. When the dwell-fatigue time is $t = 1$ s, the fatigue limit stress approaches to 28% tensile strength; when the dwell-fatigue time is $t = 10$ s, the fatigue limit stress approaches to 10% tensile strength; and when the dwell-fatigue time is $t = 100$ s, the fatigue limit stress approaches to 8% tensile strength, as shown in Fig. 7(a). The experimental and predicted failure cycles of cross-ply Nicalon$^{TM}$–SiC/MAS composite subjected to dwell-fatigue loading at 1093°C in air condition is shown in Table 4.

By combining the fiber/matrix interface wear model, interface oxidation model and fiber strength degradation model with Eqs. (11)–(18), the fibers failure probability in the interface oxidation region, interface debonded and interface bonded region can be determined. The evolution of the fibers failure probability versus applied cycle number curves can be obtained. Under fatigue peak stress of $\sigma_{max} = 103$ MPa, the theoretical predicted broken fibers fraction increases from 0.7% at the first applied cycle to 31.2% at the 3076th applied cycle when the dwell-fatigue time is $t = 1$ s; when the dwell-fatigue time is $t = 10$ s,
Table 4. The experimental and predicted failure cycles of cross-ply Nicalon™–SiC/MAS composite subjected to dwell-fatigue loading at 1093°C in air condition

| Items | $\tau = 1\ s$ | $\tau = 10\ s$ | $\tau = 100\ s$ |
|-------|----------------|----------------|-----------------|
|       | Experiment | Theory | Experiment | Theory | Experiment | Theory |
| $\sigma_{\max}$ | $\delta_{\max}$ | $\sigma_{\max}$ | $\delta_{\max}$ | $\sigma_{\max}$ | $\delta_{\max}$ | $\sigma_{\max}$ | $\delta_{\max}$ |
| MPa | MPa | MPa | MPa | MPa | MPa | MPa | MPa | MPa |
| 137 | 103 | 137 | 103 | 137 | 103 | 137 | 103 |
| Cycles | 18 | 6017 | 29 | 3237 | 11 | 216 | 13 | 240 | 4 | 10 | 5 | 24 |

22th applied cycle, as shown in Fig. 7(b). In the present analysis, the GLS criterion was adopted to determine the load carrying between intact and broken fibers, leading to high fiber failure probability under cyclic fatigue loading.

3.2 2D Sylramic™–SiC/SiC composite at 1204°C in air condition

Gowayed et al. investigated the accumulation of time-dependent strain during dwell-fatigue loading of 2D Sylramic™–SiC/SiC composites at 1204°C. The fatigue stress levels were 110.4, 165.6, 193.2 and 220.8 MPa at a stress ratio of 0.05, and the dwell-fatigue time was set to be 2 h. The composite tensile strength at 1204°C was about 378 MPa. At elevated temperatures above 1200°C, a constant tensile load produces an instantaneous strain response followed a time dependent (creep) strain. The creep strain is transient, and continuously decreasing strain rate (primary stage) appears at first. Then it goes to a steady state (constant strain rate, secondary) stage, at last accelerating (tertiary stage) to rupture. The existence of one or two or three stages depends on the stress levels and temperature conditions, and also on materials. Evans and Weber investigated the high-temperature creep behavior of fiber-reinforced CMCs composite using a creep mismatch ratio (CMR), i.e., the ratio of the creep rate of the fiber to that of the matrix. When CMR < 1, the main damage mechanism is periodic fiber fracture and the creep behavior of the composite is controlled by the embedded fibers; and when CMR > 1, the main damage mechanism is matrix microcracking and the creep behavior of the composite is controlled by bridging fibers. In the SiC/SiC composite, the matrix has greater creep resistance than the fibers, and the creep behavior of the SiC/SiC composite was greatly affected by the strength of the fibers and the fiber/matrix interphase. Morscher et al. investigated the tensile creep behavior of 2D SiC/SiC composite at 1204°C in air condition. The creep failure of the SiC/SiC composite is attributed to the oxidation-induced unbridged crack growth and the degradation of the fibers strength. The consumption of the PyC interface resulted in a weak interface bonding between the fibers and the matrix. In the present analysis, the damage mechanisms of the matrix microcracking, fiber/matrix interface debonding, interface wear and fibers strength degradation at 1204°C in air condition have been considered to analyze the damage development and lifetime prediction; however, the creep-controlled matrix cracking propagation mechanism has not been considered. The material properties of 2D Sylramic™–SiC/SiC composite are given by: $V_f = 15\%, r_f = 7.5\, \mu m$, $E_f = 230\, GPa$, $E_m = 350\, GPa$, $\alpha_f = 4 \times 10^{-6}/^\circ C$, $\alpha_m = 2 \times 10^{-6}/^\circ C$, $\tau_f = 25\, MPa$, $\tau_f = 1\, MPa$, $\zeta_\delta = 0.3\, J/\, m^2$, $m = 6$, $\sigma_0 = 3.2\, GPa$ and $L_\delta = 25\, mm$. It should be noted that the material properties listed above are assumed constant with respect to temperature.

The experimental and theoretical predicted fatigue peak strain ($\epsilon_{\max}$) versus applied cycles curves are illustrated in Fig. 8(a) and Table 5. When the fatigue peak stress is $\sigma_{\max} = 110.4\, MPa$, the degradation of the fiber/matrix interface shear stress is simulated using the Evans model, in which the model parameters are given by: $\tau_0 = 25\, MPa$, $V_f = 15\%, \tau_f = 7.5\, \mu m$, $E_f = 230\, GPa$, $E_m = 350\, GPa$, $\alpha_f = 4 \times 10^{-6}/^\circ C$, $\alpha_m = 2 \times 10^{-6}/^\circ C$, $\tau_f = 25\, MPa$, $\tau_f = 1\, MPa$, $\zeta_\delta = 0.3\, J/\, m^2$, $m = 6$, $\sigma_0 = 3.2\, GPa$ and $L_\delta = 25\, mm$. It should be noted that the material properties listed above are assumed constant with respect to temperature.
When the fatigue peak stress is $\sigma_{\text{max}} = 165.6\, \text{MPa}$, the degradation of the fiber/matrix interface shear stress is simulated using the Evans model,\((2)\) in which the model parameters are given by:\(k = \exp[11.383 - (8716/\text{T}_{\text{em}})] \times 10^{-18}/\text{m}^2/\text{s} \), $K_{\text{IC}} = 0.5\, \text{MPa}\cdot\text{m}^{1/2}$ and $Y = 1$; the experimental fatigue peak strain ($\varepsilon_p$) increases from 0.06% at the 20th applied cycle to 0.07% at the 67th applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue peak strain ($\varepsilon_p$) increases from 0.04% at the first applied cycle to 0.075% at the 70th applied cycle, as shown in Fig. (8a) and Table 5.

When the fatigue peak stress is $\sigma_{\text{max}} = 193.2\, \text{MPa}$, the degradation of the fiber/matrix interface shear stress is simulated using the Evans model,\((3)\) in which the model parameters are given by: $\tau_0 = 25\, \text{MPa}$, $\tau_1 = 1\, \text{MPa}$, $b_0 = 2.0$ and $j = 0.22$, and the time-dependent fiber strength is determined by Eq. (18), and the model parameters are given by:\(k = \exp[11.383 - (8716/\text{T}_{\text{em}})] \times 10^{-18}/\text{m}^2/\text{s} \), $K_{\text{IC}} = 0.5\, \text{MPa}\cdot\text{m}^{1/2}$ and $Y = 1$; the experimental fatigue peak strain ($\varepsilon_p$) increases from 0.11% at the 13th applied cycle to 0.147% at the 72th applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue peak strain ($\varepsilon_p$) increases from 0.058% at the first applied cycle to 0.143% at the 80th applied cycle, as shown in Fig. (8a) and Table 5.

When the fatigue peak stress is $\sigma_{\text{max}} = 220.8\, \text{MPa}$, the degradation of the fiber/matrix interface shear stress is simulated using the Evans model,\((4)\) in which the model parameters are given by: $\tau_0 = 25\, \text{MPa}$, $\tau_1 = 1\, \text{MPa}$, $b_0 = 2.0$ and $j = 0.25$, and the time-dependent fiber strength is determined by Eq. (18), and the model parameters are given by:\(k = \exp[11.383 - (8716/\text{T}_{\text{em}})] \times 10^{-18}/\text{m}^2/\text{s} \), $K_{\text{IC}} = 0.5\, \text{MPa}\cdot\text{m}^{1/2}$ and $Y = 1$; the experimental fatigue peak strain ($\varepsilon_p$) increases from 0.19% at the 17th applied cycle to 0.24% at the 101st applied cycle due to matrix cracking and fiber/matrix interface debonding, and the theoretical predicted fatigue peak strain ($\varepsilon_p$) increases from 0.096% at the first applied cycle to 0.242% at the 100th applied cycle, as shown in Fig. (8a) and Table 5.

The experimental and theoretical predicted fatigue lifetime curves of 2D Syrlamic™-SiC/SiC composite are illustrated in Fig. (8b). It can be found that the fatigue lifetime decreases rapidly due to the increase of dwell-fatigue time. The experimental and predicted failure cycles of 2D Syrlamic™-SiC/SiC composite subjected to dwell-fatigue loading at 1204°C in air condition is shown in Table 6.

### Table 5. The damage development of 2D Syrlamic™-SiC/SiC composite subjected to dwell-fatigue loading at 1204°C in air condition

| Items | $\sigma_{\text{max}} = 110.4\, \text{MPa}$ | $\sigma_{\text{max}} = 165.6\, \text{MPa}$ | $\sigma_{\text{max}} = 193.2\, \text{MPa}$ | $\sigma_{\text{max}} = 220.8\, \text{MPa}$ |
|-------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
|       | Experiment | Theory | Experiment | Theory | Experiment | Theory | Experiment | Theory |
| $N = 20$ | $N = 67$ | $N = 1$ | $N = 70$ | $N = 38$ | $N = 80$ | $N = 5$ | $N = 120$ | $N = 1$ | $N = 100$ |
| $\varepsilon_p$(%) | 0.06 | 0.07 | 0.04 | 0.075 | 0.11 | 0.147 | 0.058 | 0.143 | 0.13 | 0.21 | 0.079 | 0.2 | 0.198 | 0.24 | 0.096 | 0.242 |

### Table 6. The experimental and predicted failure cycles of 2D Syrlamic™-SiC/SiC composite subjected to dwell-fatigue loading at 1204°C in air condition

| Items | $\sigma_{\text{max}} = 110.4\, \text{MPa}$ | $\sigma_{\text{max}} = 165.6\, \text{MPa}$ |
|-------|------------------------------------|------------------------------------|
|       | Experiment | Theory | Experiment | Theory |
| Cycles | 125 | 167 | 50 | 65 |

### 4. Conclusions

In this paper, the damage development and fatigue lifetime prediction of fiber-reinforced CMCs subjected to dwell-fatigue loading at elevated temperatures in oxidizing atmosphere have been investigated. The relationships between the fatigue hysteresis-based damage parameters and internal composite damage development of matrix multicrocking, fiber/matrix interface debonding and sliding and fibers fracture have been established. The experimental fatigue life curves of cross-ply Nicalon™-SiC/MAS and 2D Syrlamic™-SiC/SiC composites under dwell-fatigue loading at elevated temperatures in air and in steam conditions have been predicted. With increasing of dwell-fatigue time, the degradation rate of fatigue hysteresis dissipated energy and fatigue hysteresis modulus, and the increasing rate of fatigue peak strain all increase; however, the fatigue lifetime decreases. The comparisons between the theoretical models proposed in the present analysis and experimental results proved the validity of the damage evolution (i.e., hysteresis dissipated energy, hysteresis modulus and peak strain) and the fatigue lifetime prediction for different CMCs.

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528 JCS-Japan

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