Problems of thermal conductivity for storage tanks of liquefied gases and oil products

Y V Nemirovsky ¹ and A S Mozgova²

¹Physics of Fast Processes Laboratory, Khristianovich Institute of Theoretical and Applied Mechanics Siberian Branch of the Russian Academy of Sciences, Institutskaya str., 4/1, Novosibirsk, 630090, Russia
²Construction Faculty, I. Ulianov Chuvash State University, 15, Moskovskiy pr., Cheboksary, Chuvash Republic, 428015, Russia

E-mail: mozgova-energo@yandex.ru

Abstract. Spherical and cylindrical tanks for storage of liquefied gases and oil products are considered in the article. Tanks are represented as a multilayered structure each layer of which has its own physical properties. In this work the stationary problem of thermal conductivity of multilayered tanks is considered. In the solution of the problem the coefficient of thermal conductivity is not a constant, and is presented in the function form.

1. Introduction

Methane, ethane and ethylene are effectively compressed for convenient storage and transportation. Liquefaction, storage and transportation of methane, ethane and ethylene is usually carried out under pressure close to atmospheric, but at negative temperatures, from -161 °C to -90 °C [1]. There are two ways to store the liquefied gases: under high pressure and at ambient temperature; under pressure close to atmospheric, and at appropriate negative temperature. At present, cylindrical and spherical steel tanks [1] are used for storage of liquefied gases. Spherical tanks in comparison with cylindrical ones have a perfect geometrical shape. Vessels of this shape require less metal consumption per unit of capacity due to reduction of wall thickness. This is possible because of the uniform distribution of stresses throughout the entire shell and in the welds [1]. Depending on the location on the area, the tanks can be ground and underground [2].

For storage of viscous oil products and to prevent thickening of the stored liquid, tanks are equipped with heating devices: steam-water, electric or therm-oil heaters. Thermal insulation can be installed on the case of the tank to maintain the temperature condition inside it [1]. Heating of oil products in tanks allows to solve a complex problem connected with the operations of discharge and filling of the product. At decreasing air temperature oil and oil products become more viscous and their transportation without heating becomes impossible [3]. The main task of storage of oil and oil products is maintenance of the preassigned temperature conditions. If the temperature of heating of oil exceeds the set limits, it leads to decomposition of oil on fractions that leads to decline in the quality of oil as raw material for oil refining and there can be irreversible processes, such as explosion [4]. Therefore, it is necessary to count distribution of temperature of the stored products in the tank. Based on the laws of thermal conductivity,
it is possible to determine the non-uniform temperature distribution in each layer of the tank, considering that each layer has the physical properties. The thermal conductivity of homogeneous spherical and cylindrical structures is widely considered in the works [5], [6], [7], [8]. In these works the coefficient of thermal conductivity is a constant. The coefficient of thermal conductivity generally depends on the coordinate $r$ in consideration of inhomogeneous layers of tanks. The purpose of this work is to obtain the solution of the stationary problem for the spherical and cylindrical multilayered tank with any quantity of layers and random heterogeneity of thermophysical properties.

2. Methods

The equation of thermal conductivity in spherical coordinates with heat sources has the form [7]:

$$c_j \rho_j \frac{\partial T_j}{\partial t} = div(\lambda_j \nabla T_j) + Q_j(t)$$  \hspace{1cm} (1)

$$div(\lambda_j \nabla T_j) = \frac{\partial \lambda_j}{\partial r} \frac{\partial T_j}{\partial r} + \frac{1}{r^2} \frac{\partial \lambda_j}{\partial \theta} \frac{\partial T_j}{\partial \theta} + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial \lambda_j}{\partial \phi} \frac{\partial T_j}{\partial \phi} +$$

$$+ \lambda_j \left( \frac{1}{r^2} \frac{\partial T_j}{\partial r} + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial}{\partial \theta} \left( \frac{\partial T_j}{\partial \theta} \sin(\theta) \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2 T_j}{\partial \phi^2} \right)$$

$$\hspace{1cm} (j = 1, m)$$

Here $r, \theta, \phi$ are coordinates; $t$ is time; $T_j = T_j(r, \theta, \phi, t)$ - a temperature field of $j$-layer of the sphere; $c_j$ is specific thermal capacity of $j$-layer of the sphere; $\rho_j$ is density of $j$-layer of the sphere; $\lambda_j$ is a thermal conductivity coefficient of $j$-layer of the sphere; $Q_j$ is internal heat sources.

In the case when the temperature field has spherical symmetry, the coefficient of thermal conductivity depends on the coordinate $r$, the equation (1) takes the form:

$$c_j \rho_j \frac{\partial T_j(r, t)}{\partial t} = \frac{\partial \lambda_j(r)}{\partial r} \frac{\partial T_j(r, t)}{\partial r} + \lambda_j(r) \left( \frac{1}{r^2} \frac{\partial T_j(r, t)}{\partial r} \right) + Q_j(r, t)$$  \hspace{1cm} (2)

$$\hspace{1cm} (j = 1, m)$$

Here $T_j = T_j(r, t)$ is a temperature field of $j$-layer of the sphere.

We consider the stationary problem of determining the temperature field in a multilayer sphere with heat sources when the temperature at all points of space does not depend on time, and the coefficient of thermal conductivity depends on the coordinate $r$. In this case, the equation (2) takes the form:

$$\frac{d\lambda_j(r)}{dr} \frac{dT_j(r)}{dr} + \lambda_j(r) \left( \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT_j(r)}{dr} \right) \right) + Q_j(r) = 0$$  \hspace{1cm} (3)

$$\hspace{1cm} (j = 1, m)$$
3. Results

The equation (3) can be written in the form:

$$\lambda_j(r)T_j''(r) + \frac{\lambda_j(r)}{r^2} \left( r^2 T_j'(r) \right)' + Q_j(r) = 0$$

$$\left( \lambda_j(r)U_j(r) \right)' = -Q_j(r)r^2$$

(4)

where \( U_j(r) = r^2 T_j'(r) \), and the prime denotes the derivative of \( r \).

We integrate the equation (4) in the range from \( r_{j-1} \) to \( r_j \):

$$\lambda_j(r)r^2 \frac{dT_j}{dr} = \lambda_j(r_{j-1})r_{j-1}^2 \frac{dT_j}{dr} \bigg|_{r=r_{j-1}} - F_j(r)$$

(5)

$$\lambda_j(r)r^2 \frac{dT_j}{dr} = C_{j-1} \lambda_j(r_{j-1}) r_{j-1}^2 - F_j(r)$$

(6)

where \( C_{j-1} = \left( \frac{dT_j}{dr} \right)_{r=r_{j-1}} \), \( F_j(r) = \int_{r_{j-1}}^{r_j} Q_j(r) r^2 dr \).

From the equation (6) we obtain:

$$\frac{dT_j}{dr} = \frac{1}{\lambda_j(r)r^2} \left[ C_{j-1} \lambda_j(r_{j-1}) r_{j-1}^2 - F_j(r) \right], \quad r_{j-1} \leq r \leq r_j$$

(7)

$$\frac{dT_j}{dr} \bigg|_{r=r_j} = \frac{C_{j-1} \lambda_j(r_{j-1}) r_{j-1}^2 - F_j(r)}{\lambda_j(r_j)r_j^2}$$

(8)

Then we integrate the equation (7) within the limits from \( r_{j-1} \) to \( r_j \), we obtain:

$$T_j(r) = C_{j-1} \lambda_j(r_{j-1}) r_{j-1}^2 \int_{r_{j-1}}^{r_j} \frac{dr}{\lambda_j(r)r^2} - \int_{r_{j-1}}^{r_j} \frac{F_j(r)}{\lambda_j(r)r^2} dr + D_{j-1}$$

(9)

where \( D_{j-1} = T_j(r_{j-1}) \).

Similarly, for the \((j+1)\)-layer we have the formulas:

$$\frac{dT_j}{dr} = \frac{C_j \lambda_j(r_j)r_j^2}{\lambda_j(r_{j+1})r_{j+1}^2} - \frac{F_{j+1}(r)}{\lambda_j(r_{j+1})r_{j+1}^2}, \quad r_j \leq r \leq r_{j+1}$$

(10)

$$T_j(r) = C_j \lambda_j(r_j)r_j^2 \int_{r_{j+1}}^{r_{j+1}} \frac{dr}{\lambda_j(r)r^2} - \int_{r_{j+1}}^{r_{j+1}} \frac{F_{j+1}(r)}{\lambda_j(r)r^2} dr + D_j$$

(11)

where \( D_j = T_j(r_j) \).

The connection between the constants \( C_j, C_{j-1}, D_j \) and \( D_{j-1} \) is established by conjunction condition on the boundary \( r = r_j \):

$$T_j \bigg|_{r=r_j} = T_{j+1} \bigg|_{r=r_j}$$

$$\lambda_j(r_j) \frac{dT_j}{dr} \bigg|_{r=r_j} = \lambda_{j+1}(r_j) \frac{dT_{j+1}}{dr} \bigg|_{r=r_j}$$

The remaining constants are determined depending on the statement of the problem from conditions on the inner layer of the spherical tank at \( r = r_0 \) and on the outer one \( r_n \).
In some cases of operation, it can be necessary to provide the set increase or decrease of temperature on the radius (of the given function \( \frac{dT}{dr} \)). In this case, from the formula (6), we define the necessary law of variation of thermal conductive characteristics with respect to \( r \). The obtained dependence can be approximated by a close piecewise constant function, which will allow to choose layers from existing real materials in practice.

Thus, the solution of the stationary problem for a spherical multilayer tank with any number of layers and arbitrary inhomogeneity of thermophysical properties is obtained. On the basis of the received stationary solution, it is possible to construct procedure of numerical or approximate analytical methods of the solution of a nonstationary problem. For this purpose, it is possible to use, for example, known finite-difference schemes or methods based on the procedure of finite integral transformations [8], [9]. The description of details of these procedures goes beyond the scope of the article.

In most cases, according to operating conditions, the cylindrical tanks are made of orthotropic and generally inhomogeneous materials. The general equation of nonstationary thermal conductivity for \( j \)-layer has the form:

\[
\frac{c_j \rho_j}{\partial t} \frac{\partial T_j}{\partial r} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_j}{\partial r} \right) + \lambda_{rj} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_j}{\partial r} \right) + \frac{\lambda_{zj}}{r^2} \frac{\partial T_j}{\partial z} + \frac{\lambda_{\theta j}}{r^2} \frac{\partial T_j}{\partial \theta} + \lambda_{2j} \frac{\partial^2 T_j}{\partial z^2} + Q_j(r, z, \theta, t)
\]

According to the rules of tank operation, the working conditions often correspond to the axially symmetric state, then the equation of stationary thermal conductivity has the form:

\[
\frac{\partial \lambda_{rj}}{\partial r} \frac{\partial T_j}{\partial r} + \frac{\partial \lambda_{zj}}{\partial z} \frac{\partial T_j}{\partial z} + \lambda_{rj} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_j}{\partial r} \right) + \frac{\lambda_{zj}}{r^2} \frac{\partial^2 T_j}{\partial z^2} + Q_j(r, z) = 0
\]

The nomenclature distinguishes short cylindrical tanks with a large diameter and long tanks with a significant excess of diameter over length, as well as thin-walled and thick-walled tanks. The choice of different types of tanks can lead to certain simplifications of the thermal conductivity problem, in particular, to the equation:

\[
\frac{d\lambda_{rj}(r)}{dr} \frac{dT_j}{dr} + \lambda_{rj}(r) \frac{1}{r} \frac{d}{dr} \left( r \frac{dT_j}{dr} \right) + Q_j(r) = 0
\]

The equation (12) is solved similarly to equation (3):

\[
T_j(r) = C_{j-1} \lambda_{j}(r_{j-1}) r_{j-1} \int_{r_{j-1}}^{r} \frac{dr}{\lambda_j(r)r} - \int_{r_{j-1}}^{r} \frac{F_j(r)}{\lambda_j(r)} dr + D_{j-1}, \quad r_{j-1} \leq r \leq r_j
\]

where \( F_j(r) = \int_{r_{j-1}}^{r} Q_j(r) rdr \), \( D_{j-1} = T_j(r_{j-1}) \).

And for \((j+1)\)-layer we have a formula:

\[
T_j(r) = C_j \lambda_{j}(r_j) r_j \int_{r_j}^{r} \frac{dr}{\lambda_j(r)r} - \int_{r_{j-1}}^{r} \frac{F_{j+1}(r)}{\lambda_j(r)} dr + D_j
\]

where \( F_{j+1}(r) = \int_{r_j}^{r} Q_{j+1}(r) rdr \), \( D_j = T_j(r_j) \).
Underground horizontal cylindrical tanks can be single-chamber and multi-chamber. In practice multi-chamber tanks can have certain advantages, as they allow to store several different types of liquids at the same time. While, the radially single-chamber tanks can exist, for which you can use the solution (13) and (14). In the case of axial multi-chamber vessels with a single axial line, the corresponding equation for the \( j \)-chamber can be represented as:

\[
\frac{\partial \lambda_j(z_j)}{\partial z} \frac{\partial T_j(z)}{\partial z} + \lambda_j(z_j) \frac{\partial^2 T_j(z)}{\partial z^2} + Q_j(z) = 0
\]

(15)

The equation (15) is solved similarly to equation (3):

\[
T_j(z) = C_{j-1} \lambda_j(z_{j-1}) \int_{z_{j-1}}^{z_j} \frac{dz}{\lambda_j(z)} - \int_{z_{j-1}}^{z_j} \frac{F_j(z)}{\lambda_j(z)} dz + D_{j-1}, \quad z_{j-1} \leq z \leq z_j
\]

(16)

where \( F_j(z) = \int_{z_{j-1}}^{z_j} Q_j(z) dz \), \( D_{j-1} = T_j(z_{j-1}) \).

For \((j + 1)\) -layer we have a formula:

\[
T_j(z) = C_j \lambda_j(z_j) \int_{z_j}^{z} \frac{dz}{\lambda_j(z)} - \int_{z_j}^{z_j} \frac{F_{j+1}(z)}{\lambda_j(z)} dz + D_j
\]

(17)

where \( F_{j+1}(z) = \int_{z_j}^{z} Q_{j+1}(z) dz \), \( D_j = T_j(z_j) \).

4. Conclusions

The solution of the stationary problem for the spherical and cylindrical multilayered tank with any quantity of layers and random heterogeneity of the thermophysical properties is deduced. By means of simple transformations it is possible to obtain solutions for various boundary conditions, without forming a system of the equations for finding the unknown coefficients. The presented solution is also common for any functions of the thermal conductivity coefficient. The advantage of the proposed solution to the problem is that it is possible to obtain a set of solutions for any type of material by setting the function of thermal conductivity coefficient, which is important for heterogeneous materials.

5. Acknowledgments

The reported study was funded by RFBR and Chuvash Republic according to the research project 17-41-210272

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