Interaction of a rigid underground pipeline with elastic-viscous-plastic soil

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Abstract. The results of numerical solutions of an axisymmetric two-dimensional problem of a rigid underground pipeline interaction with elastic-viscous-plastic soil are presented. The pipe-soil interaction begins when the underground pipeline moves in an axial (longitudinal) direction. The changes in velocity, displacement, shear stress, shear strains in the direction of the pipeline axis for fixed points of soil along the radial axis are obtained at a linear change in the given longitudinal velocity of the pipeline from zero to a certain constant value. An analysis of numerical results showed that the changes in the interaction force on the contact soil layer occur according to a two-link law with the manifestation of the peak value of shear stresses. When the pipeline moves at a constant velocity, the values of displacement over time increase, and the shear stress values remain constant. Obviously, at this stage, the Coulomb law is fulfilled. This result agrees with the results of known experiments. The obtained theoretical dependences of shear stresses on soil displacement relative to the underground pipeline reveal the formation mechanisms of the conditions for the underground pipelines - soil interaction. This result could not be obtained within the framework of the one-dimensional problem of the pipe - soil interaction.

1. Introduction

Underground trunk pipelines designed to transport liquid and gas substances are the strategically important objects and make a significant contribution to the national economy [1]. The problem of ensuring the safe and uninterrupted operation of such underground utilities in the form of underground pipelines in seismic areas is directly related to the reliability and strength of the pipelines [1, 2]. The strength and reliability of underground pipelines under various external dynamics, including seismic, impacts depend on the behavior of surrounding soil, i.e. on the force of interaction of the underground pipeline with soil [1-13]. If we take into account that all external factors are transmitted to underground pipelines through the soil, then the study of the dynamic behavior of soil medium in the vicinity of the underground pipeline is one of the urgent problems of the underground pipeline - soil interaction [4-11]. When considering the problems of the underground pipelines interaction with external medium, usually the latter is replaced by various kinematic or dynamic relationships [4-5] or interaction conditions [14-15]. As the calculations in [14- 21] show, the behavior and the stress-strain state of soil medium around the underground pipeline directly determine the interaction conditions. The present work is devoted to the study of soil dynamic behavior around an underground pipeline under the axial motion of the pipeline.
The aim of the work is to determine the law of interaction of an underground pipeline with soil under the longitudinal motion of a pipeline from the solution of a two-dimensional axisymmetric wave problem.

The objectives of the study are:
1. The statement of an axisymmetric unsteady-state problem for the "soil medium-underground pipeline" system under the relative longitudinal motion of the pipeline, taking into account the elastic, viscous, and plastic properties of soil.
2. The determination of wave kinematic parameters in the soil around the underground pipeline, depending on the problem parameters.
3. The identification of the effect of elastic, viscous, and plastic properties of soil on the laws of interaction of the underground pipeline with soil.

2. Methods

As is well known, mechanical characteristics of steel pipelines are greater than the characteristics of the surrounding soil, in this regard, the underground pipeline is considered as a rigid body, and the soil medium is modeled as an elastic-viscous-plastic medium. To describe the soil model, the stress components $\sigma_{zz}, \sigma_{rr}, \sigma_{\phi\phi}$ in the cylindrical coordinate system $r, z, \phi$ ( $r$ and $\phi$ are the radial and angular coordinates, $z$ is the axis of underground structure) are written as the sum of the volumetric and deviator parts:

$$\sigma_{zz} = S_{zz} + P, \quad \sigma_{rr} = S_{rr} + P, \quad \sigma_{\phi\phi} = S_{\phi\phi} + P$$  (1)

where $S_{zz}, S_{rr}, S_{\phi\phi}$ and $P$ are the components of the stress deviator and the pressure. Then the elastic-viscous-plastic soil model is described by the relations [15]:

$$\frac{1}{K_D} \frac{dP}{dt} + \frac{P}{K_S} = \frac{d\theta}{dt} + \mu_P \theta,$$  (2)

$$\frac{1}{2G_D} \frac{dS_{zz}}{dt} + \mu_S \left( \frac{S_{zz}}{2G_S} - e_{zz} \right) + \lambda S_{zz} = \frac{de_{zz}}{dt}, \quad \frac{1}{2G_D} \frac{dS_{rr}}{dt} + \mu_S \left( \frac{S_{rr}}{2G_S} - e_{rr} \right) + \lambda S_{rr} = \frac{de_{rr}}{dt},$$

$$\frac{1}{2G_D} \frac{dS_{\phi\phi}}{dt} + \mu_S \left( \frac{S_{\phi\phi}}{2G_S} - e_{\phi\phi} \right) + \lambda S_{\phi\phi} = \frac{de_{\phi\phi}}{dt},$$  (3)

where $K_D, K_S$ and $G_D, G_S$ are the dynamic and static compression and shear moduli; $\mu_P$ and $\mu_S$ are the parameters of volume and shear viscosity of soil, determined through dynamic viscosity $\eta$:

$$\mu_P = \frac{K_D}{(K_D - K_S)} \eta, \quad \mu_S = \frac{G_D G_S}{(G_D - G_S)} \eta$$  (4)

$e_{zz}, e_{rr}, e_{\phi\phi}, e_{\phi r}$ are the components of the strain deviator related to soil particles velocity by the Cauchy relations:

$$\frac{de_{zz}}{dt} = \frac{dv_z}{dt} - \frac{1}{3} \frac{d\theta}{dt}, \quad \frac{de_{rr}}{dt} = \frac{dv_r}{dt} - \frac{1}{3} \frac{d\theta}{dt}, \quad \frac{de_{\phi\phi}}{dt} = \frac{v_r - 1}{r} \frac{d\theta}{dt}, \quad \frac{de_{\phi r}}{dt} = \frac{dv_z}{\partial r} + \frac{dv_r}{\partial z};$$  (5)

$v_z, v_r$ are the components of soil velocity; $\theta$ is volume strain; $d\theta/dt = \text{div} \mathbf{v}$. In the soil model equation, the functional $\lambda$ is nonzero in the plastic region of strain, i.e.

$$\lambda J_2 < \frac{Y^2(P)}{3}, \quad \lambda = 0,$$  (6)

otherwise $\lambda = \frac{1}{2G_D J_2} \left( G_D W - \frac{1}{2} \frac{dJ_2}{dt} - \mu_S \frac{G_D - G_S}{G_S} J_2 \right)$.
where 
\[ J_2 = S_{zz}^2 + S_{rr}^2 + S_{\phi\phi}^2 + 2\sigma_{zz}', \quad W = S_{zz} \frac{de_{zz}}{dt} + S_{rr} \frac{de_{rr}}{dt} + S_{\phi\phi} \frac{de_{\phi\phi}}{dt} + 2\sigma_{zz} \frac{de_{zz}}{dt}, \]
\[ Y(P) = C + \mu P, \quad C \text{ and } \mu \text{ are the cohesion and the coefficient of the angle of internal friction of soil.} \]

At the pipeline and soil contact, we accept the condition of complete cohesion of soil to the underground pipeline. Under the longitudinal motion of the underground pipeline, the soil particles undergo a shear and begin to move. Due to axial symmetry, these motions are described by the equations:

\[ \rho \frac{dv_z}{dt} = \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{zx}}{\partial r} + \frac{\sigma_{zz}}{r}, \quad \rho \frac{dv_r}{dt} = \frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{zx}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\phi\phi}}{r} \]

where \( \rho \) is the soil density. To close the system of equations (1)-(7) the continuity equations should be added in the form

\[ \frac{d\rho}{dt} + \rho \text{div} \vec{v} = \frac{d\rho}{dt} + \rho \frac{d\theta}{dt} = \frac{d\rho}{dt} + \rho \left( \frac{\partial v_z}{\partial z} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} \right) = 0. \]

Before the underground pipeline motion, the surrounding soil medium is considered stress-free and at rest, i.e. all kinematic parameters and components of stresses and strains are considered equal to zero. According to the condition of complete cohesion on the pipe – soil contact surface, the relationship describing the pipeline motion corresponds to the boundary condition (9) for the soil under study. Let the pipeline moves from the reference point at a set velocity

\[ v_z = V(t), \quad v_r = 0 \text{ at } r = R_0, \quad t \geq 0. \]

Thus, the problem of determining the soil behavior around an underground pipeline at its relative shear is reduced to solving the system of equations (1)-(8) with boundary condition (9) and zero initial conditions. The given system of equations cannot be solved by analytical methods. The problem is solved numerically using the finite difference method. For a numerical solution, the Wilkins difference scheme is applied [22], which was successfully used in solving two-dimensional problems [22,23] to study the behavior of the contact layer of soil interacting with the external medium. The finite-difference relations for the problem under consideration are similar to those given in [22].

3. Results and discussion

Consider the results of numerical solutions to the problem. The following data are taken as the initial values of the physical quantities: initial soil density - 2000 kg m\(^{-3}\); dynamic modulus of elasticity 200 MPa, static modulus of elasticity 100 MPa; Poisson’s ratio - 0.3; the value of soil dynamic viscosity 106 Pa·s; soil cohesion 0.0768 MPa; the coefficient of the angle of internal friction of soil 0.42. The outer radius of the underground pipeline is 0.2 m. Elastic-plastic and elastic-viscous-plastic strains of soil (2)-(3) are considered in various options of computer calculations.

It should be noted that here only the kinematic parameters of a shear wave in the soil are considered to reveal the law of interaction of the underground pipeline with soil. At the boundary contact between the pipeline and soil, the conditions of complete cohesion are accepted. At some distances from the outer surface of the pipeline in soil, along the radial coordinate \( r \), we consider the changes in time velocity, displacement, strain, and shear stress.

Using changes in these parameters for fixed soil points in the radial direction, the dependence of the shear stresses at these points on the soil relative motion can be constructed. These dependencies are the laws of interaction of an underground pipeline with soil, obtained theoretically.

Figures 1 and 2 show the changes in soil particles velocity over time. Curves 1–7 correspond to fixed points of radial coordinate \( r = 0.2; 0.205; 0.225; 0.255; 0.305; 0.405 \) and 0.7 m, which correspond to the distances from the outer surface of the underground pipeline 0; 0.5; 2.5; 5.5; 10.5; 20.5 and 50 cm in the radial direction. These results were obtained at the longitudinal velocity of the
pipeline $V(t) = V_{\text{max}} \cdot t / T$ at $t \leq T$, $V(t) = V_{\text{max}}$ at $t > T$, according to equation (9), where $V_{\text{max}} = 0.5 \, \text{m} \cdot \text{s}^{-1}$ and $T = 0.002 \, \text{sec}$.

As seen from Figures 1 and 2, the soil motion in the pipeline axis direction begins with the arrival of a shear wave induced by underground pipeline motion. On the pipeline – soil surface, soil particles velocity, according to the condition of complete cohesion, coincides with the pipeline velocity (curve 1). Further, with increasing distance from the pipeline, the maximum values of the particle velocity decrease. Over time, in fixed particles in the soil, the velocity decreases. With an account for plastic properties, the viscous and plastic characteristics of soil affect the intensity of a decrease in soil particles velocity over time. An account for viscous properties of soil leads to a slight decrease in the particle velocity at the initial points in time behind the wavefront, but over time the difference in results of these two calculation options becomes significant.
Figures 3 and 4 show the changes in shear stresses over time for the same soil points as in Figures 1 and 2. Here, the results shown in figure 3 correspond to the elastic-plastic strain of soil, and in figure 4 to the elastic-viscous-plastic strain of soil. A decrease in strain with distance is observed in shear strain changes over time at fixed points of soil. Over time, the shear behavior in fixed sections occurs differently. When considering the elastic-plastic properties of soil, a slight decrease in shear strain is observed with a distance from the contact surface of the underground pipeline. At distances of 0.5 m or more, this decrease over time becomes significant. In the case of considering the viscous properties of soil (figure 4), an increase in shear strain over time is observed behind the shear wavefront in fixed sections of soil. A significant increase in soil shear strain is observed near the contact surface of the underground pipeline.

Figure 3. Change in shear strain of soil over time under elastic-plastic strain

![Figure 3](image)

Figure 4. Change in shear strain of soil over time under elastic-viscous-plastic strain

![Figure 4](image)

Figure 5 shows the changes in shear stresses over time at the same fixed points in soil. Note that here, in the considered calculation options, similar results were obtained for shear stresses under elastic-plastic and elastic-viscous-plastic strains of soil. For other values of the coefficients of volume and
shear viscosity, the values of shear stresses vary significantly. In this case, in figure 5, there is a decrease in shear stresses over time with increasing distance from the contact surface of the pipeline.

Figures 6 and 7 show the changes in soil particles motion along the underground pipeline axis in the case of elastic-plastic (Figure 6) and elastic-viscous-plastic soil (Figure 7). Here, curves 1 and 7 correspond to soil particles located at distances 0; 0.5; 2.5; 5.5; 10.5; 20.5 and 50 cm, from the contact surface of the underground pipeline in the radial direction. Figures 6 and 7 show that soil displacement begins with the arrival of a shear wave, and then it increases over time. The intensity of the increase in displacements over time, at fixed points of soil, decreases with the distance from the contact surface of the underground pipeline in the radial direction. When taking into account the viscous properties of an elastic-plastic strain of soils, the attenuation of the particle velocity decreases over time at fixed points of soil (figures 1 and 2), as a result, the shear strain (figures 4 and 5) and the soil particles displacement (figures 6 and 7) exceed the corresponding parameter values of elastic-plastic soil without viscous properties. The soil motion on the contact surface (curve 1) exceeds (by 3-4 times in considered time interval) the displacements of the remaining points considering the plastic properties of soil. This indicates the formation of a contact layer of soil near the outer surface of the underground pipeline.

![Figure 5. Change in shear stress over time](image1)

![Figure 6. Change in soil particles motion over time under elastic-plastic strain](image2)
In the theory of underground pipelines – soil interaction, a special place is occupied by changes in shear stresses from soil displacement relative to the pipeline, which is a model (a condition) of interaction in [21]. Figure 8 shows such a change, i.e. the dependence of shear stress on relative displacement $\sigma_{\tau z}(u)$, $u = u_z - \int_0^t V(t) \, dt$ under the elastic-viscous-plastic strain of soils. Curves 1-6 in figure 8 correspond to shear stresses and displacements of soil relative to the pipeline at fixed points at a distance of 0.5; 2.5; 5.5; 10.5; 20.5 and 50 cm in the radial direction. Here, the qualities of the dependence of shear stress on relative displacement near the contact of the underground pipeline with soil coincide with the results given in [21] and reflect two stages of interaction: the first stage is an intensive increase in shear stress depending on relative displacement and the second stage is the transition to Coulomb friction and it does not depend on relative displacement. As seen from figure 8, the diagrams "shear stress - relative displacement" near the contact surface of an underground pipeline with soil are the closest to the conditions of interaction [21] (curves 1-3).
Thus, the above results show that when using the law of strain considering viscous and plastic properties of soils, there is no need to use the complex laws of interaction proposed in [21]. The laws of soil strain (1) - (3), fully ensure the implementation of the laws of interaction proposed in [21], on the contact layer of soil around the underground pipeline.

4. Conclusions
1. The statement of a two-dimensional axisymmetric problem for studying longitudinal interaction of an underground pipeline with soil considering elastic-plastic and elastic-viscous-plastic strain of soil is given.
2. The change in kinematic parameters (particle velocity, displacement, shear strain) of shear waves in soil over time induced by the longitudinal (axial) motion of the underground pipeline is obtained by numerical solution to the considered problem by the finite difference method according to the Wilkins scheme.
3. An account for viscous and plastic properties of soil during the underground pipeline – soil interaction, under conditions of hard contact on the outer surface of the pipe, leads to the formation of the contact layer of soil around the underground pipeline.

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