Computational Results With Non-singular and Non-local Kernel Flow of Viscous Fluid in Vertical Permeable Medium With Variant Temperature

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This present article explores the transversal magnetized flow of a viscous fluid. The flow is confined to a vertical wall, saturated in permeable medium, along with ramped wall temperature. In this study, the conjugate impact of heat and mass transfer with slip and non-slip conditions are considered in the velocity field and energy equation. The dimensionless Atangana-Baleanu fractional governing equations are derived with Laplace transformation. Computational results are expressed graphically with the effect of various physical parameters. Comparative graphical analysis of the Atangana-Baleanu derivative for temperature, concentration and velocity field, with slip and non-slip impact, shows that the memory effects of the Atangana-Baleanu derivative are better than the results that exist in the literature.

Keywords: slip effect, heat and mass transfer, conjugate effect, magnetic effect, Stehfest's algorithm, fractional derivative

1. INTRODUCTION

In nature, heat and mass transfer is a common conjugate phenomenon for chemical reaction, evaporation, and condensation caused by temperature and concentration. Consequently, the behavior of heat transfer exists in different practical applications. The heat transfer mechanism is linked with mass, to jointly produce electrically conducting fluid flow with a conjugate effect. In a preambule surface the process of thermal and mass transfer with a conjugate effect have different applications in the area of nuclear production, industry, oil production, and engineering disciplines\textsuperscript{1, 2}. The conjugate effect with convection flow over an infinite plate in preamble medium, along time dependent velocity, electrically flow with a magnetic effect and have been studied by different researchers. Ramped wall temperatures with thermal radiation have received much interest in convection flow over boundless vertical plates\textsuperscript{3–6}. In literature, Toki and Tokis\textsuperscript{7} studied time dependent boundary conditions on viscous fluid over an infinite plate in preamble medium. Khan et al.\textsuperscript{8} investigated the influence of chemical parameters on viscous fluid over preamble medium with a bounded slip region. Khan et al.\textsuperscript{9} discussed the influence of heat and mass diffusion of a viscous fluid over an oscillating plate. Das et al.\textsuperscript{10} and Narahari and Ishaq\textsuperscript{11} investigated the solution of unsteady Walter's fluids on convection flow over preamble medium with a magnetic effect and
constant suction heat. Recently, Kumar et al. [12] discussed the fractional model for radial fins with heat transfer. Some of the latest results, according to this research, are given in Gupta et al. [13], Khan et al. [14], and Imran et al. [15].

Moreover, the application of magnetic fields is significant, with heat transfer in different situations of flow of an incompressible fluid, for example, geothermal energy, magnetic generator, and metallurgical processes. The influence of the slip and non-slip condition with the magnetic field and chemical reaction of an electrically conducting fluid over a porous surface, have been developed by Boussinesq’s approximation [16]. Jha and Apere [17] and Seth et al. [18] analyzed the ion slip and hall effect boundary conditions on a magnetized electrically conducting fluid between parallel plates. The impact of the current and rotation with heat radiation and mass transfer, on time depending heat observation over a preamble surface, were taken into account. Over the last few years, fractional calculus has played a significant role in viscoelastic models. The derivative of the fractional order can be achieved by constitutive equations of well-known models through time ordinary derivatives. Recently, many fractional time derivative problems have been studied [19, 20]. Different real life problems have been investigated through fractional time operators [21–23]. A modern fractional approach has been presented without a singular kernel. A non-singular kernel is used to find the solution for MHD convection flow with ramped temperature, was investigated by Riaz et al. [24]. Furthermore, Riaz and Saeed [25] discussed the solution of MHD Oldroyd-B fluid using integer and fractional order derivatives with slip effect and time boundary conditions. The Study of natural convection flow with in channel using non-singular kernels is discussed by Saeed et al. [26].

In this paper, we discuss the computational calculation for the magnetized flow of Newtonian fluid with slip and conjugate effect, through a preamble surface. Computational results for the velocity profile, temperature gradient, and concentration field are calculated with the Atangana-Baleanu fractional derivative, through the Laplace transform. Tzou and Stehfest’s algorithm is used to find the inverse Laplace transform. Further, We show the strength of non-singular and non-kernels. Fractional order Atangana-Baleanu (ABC) derivatives are used to analyze fractional parameters (memory effect) on the dynamics of fluid. We conclude that the fractional order model is best for memory effect and flow behavior of the fluid with reference to classical models. ABC is good at highlighting the dynamics of fluid. The influence of transverse magnetic fields are studied for ABC and CF. Moreover, the impact of parameters on the velocity profile are analyzed through numerical simulation and graphs for ABC and CF models. Expression from some limited and special cases were also obtained in terms of the velocity profile with different flow parameters.

2. MATHEMATICAL MODEL WITH STATEMENT OF THE PROBLEM

In this article, we assumed the slip effect between fluid and a wall. After $t = 0^+$, the temperature on the plate is enhanced or reduced to $\theta_\infty + (\theta_\infty - \theta_\infty) \frac{t}{t_0}$ when $t \leq t_0$, and therefore, for $t > t_0$, is retained at a constant temperature $\theta_\infty$, and the concentration is enhanced to $C_\infty$. The set of governing equations are given in [27]:

$$v_t = v_{\xi \xi} + G_\xi \theta + G_m C - K_p v - M^2 v,$$

$$\left( p_{\text{eff}} \right) \theta_t = \theta_{\xi \xi},$$

$$S_C C_t = C_{\xi \xi}.$$  

With suitable conditions

$$v(\xi, 0) = 0, \theta(\xi, 0) = \theta_\infty, C(\xi, 0) = C_\infty, \forall \xi \geq 0.$$  

![FIGURE 1](image1.jpg) Variations in temperature with altered values of $\alpha$ and other parameters are $k_p = 1.5$, $G_r = 2$, $P_{\text{eff}} = 0.1$, $G_m = 0.75$, $M = 0.9$, and $S_c = 0.5$.

![FIGURE 2](image2.jpg) Variations in concentration with altered values of $\alpha$ and other parameters are $k_p = 1.5$, $G_r = 2$, $P_{\text{eff}} = 0.1$, $G_m = 0.75$, and $M = 0.9$. 

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\[ v(\xi, t) - L(v_{\xi} \big|_{\xi=0}) = \frac{f(t)}{\mu}, C(0, t) = C_{\infty}, \quad t > 0, \quad (5) \]

\[ \theta(\xi, 0) = \theta_{\infty} + (\theta_{w} - \theta_{\infty}) \frac{t}{t_{o}}, \quad 0 < t < t_{o}, \quad (6) \]

\[ v(y, t) = \theta_{\infty}, C(y, t) = C_{\infty}, \quad t > 0, \]
\[ \theta(y, t) = \theta_{\infty}, \quad t > t_{o}, \quad y \to \infty. \quad (7) \]

We arrive at the governing equations, in terms of the Atangana-Baleanu fractional derivative, as:

\[ ABC^{\alpha} D^{\alpha}_{t} v = v_{\xi \xi} + G_{r} \theta + G_{m} C - k_{p} v - M^{2} v, \quad (8) \]

\[ ABC^{\alpha} D^{\alpha}_{t} \theta = \left( \frac{1}{P_{\text{reff}}} \right) \theta_{\xi \xi}, \quad (9) \]

\[ ABC^{\alpha} D^{\alpha}_{t} C = \left( \frac{1}{S_{c}} \right) C_{\xi \xi}, \quad (10) \]

where \( ABC^{\alpha} D^{\alpha}_{t} \) is the fractional differential operator of order \( 0 < \alpha < 1 \) called the Atangana-Baleanu fractional operator as defined by [21, 28]:

\[ ABC^{\alpha} D^{\alpha}_{t} f(\xi, \tau) = \frac{M(\alpha)}{1 - \alpha} \int_{0}^{\tau} E_{\alpha} \left( -\frac{\alpha(t - \tau)^{\alpha}}{1 - \alpha} \right) \frac{\partial f(\xi, \tau)}{\partial \tau} d\tau, \]

\[ \text{with } \sum_{m=0}^{\infty} (-t)^{\alpha m} = E_{\alpha}(-t)^{\alpha}, \quad (11) \]

where \( M(\alpha) \) denotes a normalization function obeying \( M(0) = M(1) = 1 \).

The Laplace transform of Equation (11) is as follows [26]:

\[ L \left[ ABC^{\alpha} D^{\alpha}_{t} f(\xi, \tau) \right] = \frac{s^{\alpha} L[f(\xi, \tau)] - s^{\alpha - 1} f(\xi, 0)}{(1 - \alpha)s^{\alpha} + \alpha}. \quad (12) \]

The appropriate initial and boundary conditions are:

\[ v(\xi, 0) = \theta(\xi, 0) = C(\xi, 0) = 0, \quad \forall \xi \geq 0, \quad (13) \]

\[ v(\xi, t) - h v_{\xi} \big|_{\xi=0} = Z(t), \quad (14) \]

\[ C(0, t) = 1, \quad C(\infty, t) = 0, \quad t > 0, \quad (15) \]

\[ \theta(\infty, t) = 0, \quad v(\infty, t) = 0, \quad t > 0, \quad (16) \]

\[ \theta(0, t) = t, \quad 0 < t \leq 1, \quad \theta(0, t) = 1, \quad t > 1. \quad (17) \]

**FIGURE 3** | Velocity profiles with altered values of \( \alpha \) and other parameters are \( k_{p} = 1.5, G_{r} = 2, P_{\text{reff}} = 0.1, G_{m} = 0.75, M = 0.9, \) and \( S_{c} = 0.5. \)
3. SOLUTION OF THE PROBLEM

3.1. Distribution of Temperature Gradient With Fractional Model 0 < α < 1

In order to find the solution of fractional concentration distribution, we employ Equation (12) into Equation (9), and obtain:

\[
\tilde{\theta}(\xi, s) = \frac{1 \cdot \frac{s^\alpha}{(1 - \alpha)s^\alpha + \alpha}}{P_{\text{reff}} \left( \frac{\mu}{(1 - \alpha)s^\alpha + \alpha} \right) + c_2 e^{\xi \sqrt{\mu \left( \frac{(1 - \alpha)s^\alpha + \alpha}{P_{\text{reff}}} \right)}}}
\]

with the help of (13)–(17), we find the values of constants \(c_1\) and \(c_2\), and we have.

\[
\tilde{\theta}(\xi, s) = \left( \frac{1}{s^2} \right) e^{-\xi \sqrt{\mu \left( \frac{(1 - \alpha)s^\alpha + \alpha}{P_{\text{reff}}} \right)}}.
\]

3.2. Distribution of Concentration Gradient With Fractional Model 0 < α < 1

In order to find the solution of fractional concentration distribution, we employ Equation (12) into Equation (10), and obtain:

\[
\tilde{C}(\xi, s) = \frac{1 \cdot \frac{s^\alpha}{(1 - \alpha)s^\alpha + \alpha}}{S_c \left( \frac{\mu}{(1 - \alpha)s^\alpha + \alpha} \right) + c_2 e^{\xi \sqrt{\mu \left( \frac{(1 - \alpha)s^\alpha + \alpha}{S_c} \right)}}}
\]

with the help of (13)–(17), we find the values of constants \(c_1\) and \(c_2\), and we have.

\[
\tilde{C}(\xi, s) = \left( \frac{1}{s} \right) e^{-\xi \sqrt{\mu \left( \frac{(1 - \alpha)s^\alpha + \alpha}{S_c} \right)}}.
\]

3.3. Distribution of Velocity Field With Fractional Model 0 < α < 1

In order to find the solution of the fractional concentration distribution, we employ Equation (12) into Equation (8), and obtain:

\[
\tilde{v}(\xi, s) = \frac{1 \cdot \frac{s^\alpha}{(1 - \alpha)s^\alpha + \alpha}}{G_r \tilde{\theta}(\xi, s) + G_m \tilde{C}(\xi, s) - k_p \tilde{v}(\xi, s) - M^2 \tilde{v}(\xi, s)}
\]

FIGURE 4 | Velocity profiles with altered values of \(k_p\) and other parameters are \(G_r = 2, \alpha = 0.5, P_{\text{reff}} = 0.1, G_m = 0.75, M = 0.9,\) and \(S_c = 0.5.\)
The solution of the homogeneous part of the second order partial differential equation say that (24) is,

\[ \bar{v}(\xi, s) = c_1 e^{-\xi} \sqrt{\frac{\mu'}{(1-\alpha)^2 + \alpha}} + k_p M^2 + c_2 e^{\xi} \sqrt{\frac{\mu'}{(1-\alpha)^2 + \alpha}} + k_p M^2. \]  

(25)

The general solution can be give as follows, after making use of \( \dot{\theta}(\xi, s) \) and \( \dot{C}(\xi, s) \),

\[ \bar{v}(\xi, s) = c_1 e^{-\xi} \sqrt{\frac{\mu'}{(1-\alpha)^2 + \alpha}} + k_p M^2 + c_2 e^{\xi} \sqrt{\frac{\mu'}{(1-\alpha)^2 + \alpha}} + k_p M^2 - \frac{G_r(1 - e^{-\xi})(1 - \alpha) \mu' + \alpha}{s^2 (1 - \alpha)^2 + \alpha (P_{\text{reff}} - 1) - (k_p + M^2)} - \frac{G_m (1 - e^{-\xi})}{s (1 - \alpha)^2 + \alpha (S_c - 1) - (k_p + M^2)} \]

(26)

with the help of Equations (13)–(17), we find the values of constants \( c_1 \) and \( c_2 \) for the velocity equation:

\[ \bar{v}(\xi, s) = \left[ 1 + \frac{1}{1 + h \sqrt{\frac{\mu'}{(1-\alpha)^2 + \alpha}} + k_p + M^2} \right] \]

\[ \left\{ \begin{array}{c}
\frac{G_r(1 - e^{-\xi})}{s^2 (1 - \alpha)^2 + \alpha (P_{\text{reff}} - 1) - (k_p + M^2)} \\
+ \frac{G_m}{s} \left( \frac{1 + h \sqrt{\frac{\mu'}{(1-\alpha)^2 + \alpha}}}{S_c - 1} - (k_p + M^2) \right) \\
+ Z(s) \end{array} \right\} \]

(27)

The skin friction is defined as:

\[ \tilde{\tau}(\xi, s) = \frac{\partial \bar{v}(\xi, s)}{\partial \xi} \bigg|_{\xi=0}, \] 

(28)

**FIGURE 5** | Velocity profiles with altered values of \( G_r \) and other parameters are \( k_p = 1.5 \), \( G_m = 0.75 \), \( P_{\text{reff}} = 0.1 \), \( \alpha = 0.5 \), \( M = 0.9 \), and \( S_c = 0.5 \).
\[ \bar{v}(\xi, s) = \left[ \frac{1}{1 + h\sqrt{(1-\alpha)s^\mu + \alpha}} + k_p + M^2 \right] \\
\left\{ \frac{G_r(1 - e^{-s})}{s^2} \left( \frac{1 + h\sqrt{s^\mu P_{reff}}}{(P_{reff} - 1) - (k_p + M^2)} \right) \right. \\
\left. + \frac{G_m}{s} \left( \frac{1 + h\sqrt{s^\mu S_c}}{(S_c - 1) - (k_p + M^2)} \right) + Z(s) \right] \\
\left( \sqrt{(1-\alpha)s^\mu + \alpha} + k_p + M^2 \right) \\
\frac{G_r(1 - e^{-s})}{s^2} \left( \frac{1 + h\sqrt{s^\mu P_{reff}}}{(P_{reff} - 1) - (k_p + M^2)} \right) \\
\frac{G_m}{s} \left( \frac{1 + h\sqrt{s^\mu S_c}}{(S_c - 1) - (k_p + M^2)} \right) - G_r(1 - e^{-s}) \\
\left( \frac{e^{-\xi} \sqrt{s^\mu P_{reff}}}{(P_{reff} - 1) - (k_p + M^2)} \right) - \frac{G_m}{s} \left( \frac{e^{-\xi} \sqrt{s^\mu S_c}}{(S_c - 1) - (k_p + M^2)} \right). \tag{30} \]

4. LIMITING CASES

A comparative study of the existing literature and the Atangana-Baleanu derivative for some limiting cases are recovered from the general solution of (Equation 30, [27]) and the general solution of the given problem at Equation (27) are both discussed in this section.

4.1. Results With Ramped Wall Temperature and Without Porosity Effect \( (k_p \to 0) \)

The velocity profile with the Atangana-Baleanu derivative is expressed for a general solution of the given problem at Equation (27) is given as:

\[ \bar{v}(\xi, s) = \left[ \frac{1}{1 + h\sqrt{(1-\alpha)s^\mu + \alpha}} + M^2 \right] \\
\left\{ \frac{G_r(1 - e^{-s})}{s^2} \left( \frac{1 + h\sqrt{s^\mu P_{reff}}}{(P_{reff} - 1) - (k_p + M^2)} \right) \right. \\
\left. + \frac{G_m}{s} \left( \frac{1 + h\sqrt{s^\mu S_c}}{(S_c - 1) - (k_p + M^2)} + Z(s) \right) \right] \\
\left( \sqrt{(1-\alpha)s^\mu + \alpha} + k_p + M^2 \right) \\
\frac{G_r(1 - e^{-s})}{s^2} \left( \frac{1 + h\sqrt{s^\mu P_{reff}}}{(P_{reff} - 1) - (k_p + M^2)} \right) \\
\frac{G_m}{s} \left( \frac{1 + h\sqrt{s^\mu S_c}}{(S_c - 1) - (k_p + M^2)} \right) - G_r(1 - e^{-s}) \\
\left( \frac{e^{-\xi} \sqrt{s^\mu P_{reff}}}{(P_{reff} - 1) - (k_p + M^2)} \right) - \frac{G_m}{s} \left( \frac{e^{-\xi} \sqrt{s^\mu S_c}}{(S_c - 1) - (k_p + M^2)} \right). \]
4.2. Results Without Thermal Radiation
\((N_r \to 0)\)
The velocity profile is obtained with the Atangana-Baleanu derivative for the general solution of the given problem at Equation (27) is given as:

\[
\tilde{v}(\xi, s) = \left[ \frac{1}{1 + h\sqrt{\left(\frac{\mu}{(1-\alpha)\mu + \alpha}\right)^2 + k_p + M^2}} + G_r (1 - e^{-\xi}) \right. \\
+ \frac{G_r (1 - e^{-\xi})}{s^2} \left( \frac{1 + h\sqrt{s^2 P_r}}{(1-\alpha)\mu + \alpha} (P_r - 1) - (k_p + M^2) \right) \\
- \frac{G_m}{s} \left( \frac{e^{-\xi\sqrt{P_r}}}{(1-\alpha)\mu + \alpha} (S_c - 1) - (k_p + M^2) \right) \\
\left. + G_m \left( \frac{1 + h\sqrt{s^2 S_c}}{(1-\alpha)\mu + \alpha} (S_c - 1) - (k_p + M^2) \right) \right] \cdot (31)
\]

4.3. Result Without Magnetic Parameter
\((M \to 0)\)
The velocity profile is obtained with the Atangana-Baleanu derivative for the general solution of the given problem at Equation (27) is given as:

\[
\tilde{v}(\xi, s) = \left[ \frac{1}{1 + h\sqrt{\left(\frac{\mu}{(1-\alpha)\mu + \alpha}\right)^2 + k_p}} + G_r (1 - e^{-\xi}) \right. \\
+ \frac{G_r (1 - e^{-\xi})}{s^2} \left( \frac{1 + h\sqrt{s^2 P_{reff}}}{(1-\alpha)\mu + \alpha} (P_{reff} - 1) - k_p \right) \\
- \frac{G_m}{s} \left( \frac{e^{-\xi\sqrt{P_{reff}}}}{(1-\alpha)\mu + \alpha} (S_c - 1) - k_p \right) \\
\left. + G_m \left( \frac{1 + h\sqrt{s^2 S_c}}{(1-\alpha)\mu + \alpha} (S_c - 1) - k_p \right) \right] \cdot (32)
\]

![Figure 7](image-url)
5. SPECIAL CASES

For validation and to check our general results in this section, we will discuss some special cases by customizing the value of \( f(t) \). Moreover, our aim is to provide a comparison of our results with the Caputo-Fabrizio (CF) time fractional derivative.

5.1. Case-I

By putting \( z(t) = t \) into Equation (27), we obtain a suitable result for the velocity profile:

\[
\bar{v}(\xi, s) = \left[ \frac{1}{1 + \frac{k_p + M^2}{\sqrt{1-\alpha}}} + \frac{G_r(1 - e^{-t})}{s^2} \left( \frac{1 + \frac{k_p + M^2}{\sqrt{1-\alpha}}}{P_r - 1 - (k_p + M^2)} \right) + \frac{G_m}{s} \left( \frac{1 + \frac{k_p + M^2}{\sqrt{1-\alpha}}}{S_r - 1 - (k_p + M^2)} \right) \right] \left( e^{-\xi \sqrt{1-\alpha}} + k_p + M^2 \right)
\]

The analogs of the velocity profile are obtained by (Equation 40, [27]) using the CF operator:

\[
\bar{v}(\xi, s) = \left[ \frac{1}{1 + \frac{k_p + M^2}{\sqrt{1-\alpha}}} + \frac{G_r(1 - e^{-t})}{s^2} \left( \frac{1 + \frac{k_p + M^2}{\sqrt{1-\alpha}}}{P_r - 1 - (k_p + M^2)} \right) + \frac{G_m}{s} \left( \frac{1 + \frac{k_p + M^2}{\sqrt{1-\alpha}}}{S_r - 1 - (k_p + M^2)} \right) \right] \left( e^{-\xi \sqrt{1-\alpha}} + k_p + M^2 \right)
\]
Graphs for the profiles of the velocity for both operators for the variation of physical parameters \( \alpha, P_{\text{reff}}, M, G_r, G_m, S_C, \) and \( k_p \) are prepared. Moreover, the slip and no slip effects are significant. It is noted that the memory effects obtained by the Atangana-Baleanu derivative express more significant results than the results recovered by the Caputo-Fabrizio derivative.

5.2. Case-II

By putting \( z(t) = t^\alpha \) into Equation (27), we obtain a suitable result for the velocity profile:

\[
\tilde{v}(\xi, s) = \left[ \frac{1}{1 + h \sqrt{(1-\alpha)^{2s^2} + k_p + M^2}} \left\{ G_r(1 - e^{-\frac{1}{s^2}}) \left( 1 + h \sqrt{s^2 P_{\text{reff}} - (P_{\text{reff}} - 1) - (k_p + M^2)} \right) \right. \right.
\]
\[
+ \frac{G_m}{s} \left( 1 + h \sqrt{s^2 S_C - (S_C - 1) - (k_p + M^2)} \right) \right\}
\]
\[
\left. \frac{1}{1 + \frac{1}{s^2} \left( e^{-\xi \sqrt{\frac{s^2 P_{\text{reff}}}{1 - \alpha} + k_p + M^2}} \right) - \frac{G_r(1 - e^{-\frac{1}{s^2}})}{s^2} \left( e^{-\xi \sqrt{s^2 P_{\text{reff}} - (P_{\text{reff}} - 1) - (k_p + M^2)}} \right) - \frac{G_m}{s} \left( e^{-\xi \sqrt{s^2 S_C - (S_C - 1) - (k_p + M^2)}} \right) \right] \right].
\] (35)

The analogs of the velocity profile are obtained by (Equation 42, [27]) using CF operator:

\[
\tilde{v}(\xi, s) = \left[ \frac{1}{1 + h \sqrt{(1-\alpha)^{2s^2} + k_p + M^2}} \right. \left\{ G_r(1 - e^{-\frac{1}{s^2}}) \right. \right.
\]
\[
\left. \frac{1}{s^2} \left( s P_{\text{reff}} - \frac{1}{(1-\alpha)^{2s^2} + k_p + M^2} \right) \right. \right.
\]
\[
+ \frac{G_m}{s} \left( s S_C - \frac{1}{(1-\alpha)^{2s^2} + k_p + M^2} \right) \right\}
\]
\[
\left. \left( e^{-\xi \sqrt{(1-\alpha)^{2s^2} + k_p + M^2}} \right) \right. \right.
\]
\[
- \frac{G_r(1 - e^{-\frac{1}{s^2}})}{s^2} \left( e^{-\xi \sqrt{s^2 P_{\text{reff}} - (P_{\text{reff}} - 1) - (k_p + M^2)}} \right) - \frac{G_m}{s} \left( e^{-\xi \sqrt{s^2 S_C - (S_C - 1) - (k_p + M^2)}} \right) \right].
\] (36)

**FIGURE 9** | Velocity profiles with altered values of \( S_C \) and other parameters are \( \alpha = 0.5, G_r = 2, P_{\text{opt}} = 0.1, G_m = 0.75, k_c = 1.5, \) and \( M = 0.9. \)
5.3. Case-III
By putting \( z(t) = \sin(\omega t) \) into Equation (27), we obtain a suitable result for the velocity profile:

\[
\bar{v}(\xi, s) = \left[ \frac{1}{1 + h\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} + k_p + M^2 \right] \\
\left\{ \frac{G_s(1 - e^{-s})}{s^2} \left( \frac{1 + h\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} + k_p + M^2 \right) \\
+ \frac{G_m}{s} \left( \frac{1 + h\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} + k_p + M^2 \right) \right\} \\
\left( e^{-s\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} \right) + G_s(1 - e^{-s}) \\
- \frac{G_m}{s} \left( e^{-s\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} \right). \tag{37} \]

The analogs of the velocity profile are obtained by (Equation 44, [27]) using CF operator:

\[
\bar{v}(\xi, s) = \left[ \frac{1}{1 + h\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} + k_p + M^2 \right] \\
\left\{ \frac{G_s(1 - e^{-s})}{s^2} \left( \frac{1 + h\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} + k_p + M^2 \right) \\
+ \frac{G_m}{s} \left( \frac{1 + h\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} + k_p + M^2 \right) \right\} \\
\left( e^{-s\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} \right) + G_s(1 - e^{-s}) \\
- \frac{G_m}{s} \left( e^{-s\sqrt{s\alpha - \frac{k_p + M^2}{s(1 - \alpha)\alpha + \alpha}}} \right) \tag{38} \].

**FIGURE 10** | Velocity profiles with altered values of \( \alpha \) and other parameters are \( \omega = 0.2, G_s = 2, P_{ref} = 0.1, G_m = 0.75, k_p = 1.5, S_c = 0.5, \) and \( M = 0.9. \)
5.4. Case-IV

By putting $z(t) = t \sin(\omega t)$ into Equation (27), we obtain a suitable result for the velocity profile:

$$
\bar{v}(\xi, s) = \left[ \frac{1}{1 + h \sqrt{1 - \alpha} + k_p + M^2} \right] \\
\left\{ \frac{G_r(1 - e^{-s})}{s^2} \left( \frac{1 + h \sqrt{1 - \alpha} (P_{ref} - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right) \\
+ \frac{G_m}{s} \left( \frac{1 + h \sqrt{1 - \alpha} (S_c - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right) \\
- \frac{G_r(1 - e^{-s})}{s^2} \left( \frac{e^{-s} \sqrt{1 - \alpha} (P_{ref} - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right) \\
- \frac{G_m}{s} \left( \frac{e^{-s} \sqrt{1 - \alpha} (S_c - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right) \right\} \\
\left( e^{-\xi \sqrt{1 - \alpha} + k_p + M^2} \right) \\
- \frac{G_r(1 - e^{-s})}{s^2} \left( \frac{e^{-s} \sqrt{1 - \alpha} (P_{ref} - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right) \\
- \frac{G_m}{s} \left( \frac{e^{-s} \sqrt{1 - \alpha} (S_c - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right). \quad (39)
$$

The analogs of the velocity profile are obtained by (Equation 46, [27]) using CF operator:

$$
\tilde{v}(\xi, s) = \left[ \frac{1}{1 + h \sqrt{1 - \alpha} + k_p + M^2} \right] \\
\left\{ \frac{G_r(1 - e^{-s})}{s^2} \left( \frac{1 + h \sqrt{1 - \alpha} (P_{ref} - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right) \\
+ \frac{G_m}{s} \left( \frac{1 + h \sqrt{1 - \alpha} (S_c - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right) + \frac{2s\omega}{s^2 + \omega^2} \right\} \\
\left( e^{-\xi \sqrt{1 - \alpha} + k_p + M^2} \right) \\
- \frac{G_r(1 - e^{-s})}{s^2} \left( \frac{e^{-s} \sqrt{1 - \alpha} (P_{ref} - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right) \\
- \frac{G_m}{s} \left( \frac{e^{-s} \sqrt{1 - \alpha} (S_c - 1) - (k_p + M^2)}{1 - h \sqrt{1 - \alpha} + k_p + M^2} \right). \quad (40)
$$

By making $\alpha \to 1$ in Equations (20), (23), and (27) we obtain a result for a classical model, the same as that discussed by Ghalib et al. [27]. This validates our obtained results. In our flow models,

FIGURE 11 | Velocity profiles with altered values of $k_p$ and other parameters are $\omega = 0.2$, $G_r = 2$, $P_{ref} = 0.1$, $G_m = 0.75$, $\alpha = 0.5$, $S_c = 0.5$, and $M = 0.9$. 
we use the Laplace transform technique to solve this model, using the definition of the ABC model. In order to find the inverse, we use Stehfest’s algorithms [29] for semi-analytical solutions. Stehfest’s algorithms are used for the verification of our inverse Laplace transformation

\[
v(y, t) = \frac{\ln(2)}{t} \sum_{j=1}^{2m} d_j \bar{v}(y, j \ln(2)),
\]

\[
d_j = (-1)^{j+m} \sum_{i=\left\lfloor \frac{j}{2} \right\rfloor}^{\min(j, m)} \frac{i^m(2i)!}{(m - i)!i!(i - j)!(2i - j)!}.
\]

6. RESULTS AND DISCUSSION

The physical aspects of the CF and ABC time derivative are discussed in the given problem. Numerical results for \( T, C, \) and \( v \) are plotted using MATHCAD for embedded physical parameters, such as \( M, k_p, P_{\text{eff}}, G_r, G_m, S_c, \) and slip parameter \( h.\) Figure 1 shows the behavior of \( \alpha \) on temperature. It is shown that the value of \( \alpha \) increases, while the temperature of the fluid decreases. The memory effect is explained well with the ABC derivative in comparison to the CF derivative.

Figure 2 examines the behavior of \( \alpha \) on concentration. It reduces as the value of \( \alpha \) increases. The Antangna-Baleanu derivative shows significant behavior in comparison to the Caputo-Febrizio derivative for different values of \( \alpha.\) Graphs for the velocity, with function \( f(t) = t, \) are shown in Figures 3–9, and with function \( f(t) = \sin(\omega t), \) as shown in Figures 10–16. Fluid velocity decreases with the increase of \( \alpha \) as well as for slip and non-slip boundary conditions. Figure 3 shows that the memory effects of the Antangna Baleanu derivatives, with short and long time for the velocity profile, as well as with slip and non-slip conditions, are more significant than the memory effects of the Caputo Febrizio derivatives. For longer times, the graphical representation of the velocity shows inverse behavior, as the velocity increases with the increase of the value of \( \alpha, \) for both the velocity profile and slip and non-slip velocity. Variation in fluid velocity with respect to the porosity coefficient is displayed in Figures 4, 11. It represents the increase in the porosity coefficient, resulting in the decrease in the velocity profile, as well as the velocity with slip and non-slip boundary conditions for both a short and long time. The representation of the velocity profile with the Antangna Baleanu derivatives, for a short and long time, as well as fluid velocity with the slip and non-slip effect is more significant than the velocities recovered with the Caputo-Fabrizio derivatives. Figures 5, 12 illustrate the influence of the Grashof
number $G_r$ on the fluid velocity, which increases with the increase of the Grashof number $G_r$ for a short time as well as for a long time, both in the case of the slip and non-slip effects, because the thermal buoyancy forces tend to accelerate the fluid velocity for different times. The memory effects of the Antangna Baleanu derivatives for the variation of $G_r$ with a short time and long time, uncover more significant memory effects than the Caputo Febrizio derivatives. The velocity profile for different values of the effective Prandtl number $P_{reff}$ are shown in Figures 6, 13. Fluid velocity decreases with the increase of $P_{reff}$ for different times, also in the case of slip and non-slip boundary conditions. Graphical representation for various values of $P_{reff}$ with the Antangna-Baleanu derivative is more impressive for short and long times as well as for slip and non-slip boundary conditions, than it is for the caputo-Fabrizio derivatives. Figures 7, 14 display the influence of the variation of a modified Grashof number $G_m$, and the fluid velocity increases with the increase of $G_m$ for various times, as well as with the slip and non-slip parameters. Memory effects with the Antangna-Baleanu derivatives are better than with the Caputo-Fabrizio derivatives. The velocity profile for different values of magnetic field $M$ are given in Figures 8, 15. Fluid velocity shrinks on a large value of $M$ with a short time as well as with long time. It also displays the same behavior for both slip and non-slip boundary conditions, particularly, on increasing the value of $M$ causes to enhance the frictional force which tends to resist the flow of fluid and, thus, velocity ultimately decreases. Moreover, we observed that the fluid velocity obtained with the Atangana-Baleanu derivatives for the variation of $M$, in case of both a short and long time, is more significant than the velocity obtained with the Caputo-Fabrizio derivatives. In Figures 9, 16 velocity profiles with variations of $S_c$ are shown. It was found that the velocity decreases when increasing the value of $S_c$ for both short and long times, as well as for slip and non-slip parameters. The velocity profile of different values of $S_c$ with the Atangana-Baleanu derivatives for various times, are more expressive than the velocity that is obtained with the Caputo-Fabrizio derivatives. In Figure 10 fluid velocity reduces with enlarged values of $\alpha$. It also shows the same behavior with slip as well as non-slip fluid flow conditions, and it shows the same behavior for short and long times. Memory effects show better results with the Atangana-Baleanu derivative in comparison to the Caputo-Fabrizio derivative.

7. CONCLUSION

Ramped wall velocity and temperature conditions had a significant impact on MHD fractional Oldroyd-B fluid over
FIGURE 14 | Velocity profiles with altered values of $G_m$ and other parameters are $\omega = 0.2$, $Gr = 2$, $P_{ref} = 0.1$, $\alpha = 0.5$, $k_c = 1.5$, $S_c = 0.5$, and $M = 0.9$.

FIGURE 15 | Velocity profiles with altered values of $M$ and other parameters are $\omega = 0.2$, $Gr = 2$, $P_{ref} = 0.1$, $G_m = 0.75$, $\alpha = 0.5$, $S_c = 0.5$, and $k_c = 1.5$. 
a infinite vertical plate on a permeable surface. Fractional derivative operators are used to find the analytical solution using the Laplace transformation and inversion algorithm. Fluid velocity was analyzed through graphical results with the effect of different physical parameters. The main points of this problem are:

- The ABC fractional derivative is more significant compared to the classical model and other fractional models.
- The magnitude of the velocity increases with an increase in the fractional parameter $\alpha$.
- The relationship between fractional parameters $\alpha$ and $\gamma$ are reversed.
- Retardation time and relaxation time have a strong impact on the motion of fluid velocity.
- Velocity enhances with an increase in the value of $\lambda_r$.
- The relationship between $\lambda$ and $\lambda_r$ is the opposite to each other.
- The fluid velocity decreases with a large value of $Pr$.
- In the velocity field, the velocity reduces with the expansion of $M$.

**DATA AVAILABILITY STATEMENT**

All datasets generated for this study are included in the article/supplementary material.

**AUTHOR CONTRIBUTIONS**

MR and DB: conceptualization, investigation, and final review and editing. MR, DB, and SS: methodology. MG: software. SS: formal analysis. SS and MG: resources and original draft preparation. All authors contributed to the article and approved the submitted version.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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