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Theoretical investigation of the correlation between perturbations of quantum optical circuit parameters and its performance

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Abstract. In this paper described the operability of some quantum optical circuits which depends on their constructions, such as reflectivity of beam splitters in the linear optical controlled-NOT gate in the coincidence basis.

1. Introduction
In the present there is an active development of such direction of quantum information as quantum teleportation [1]. Experimental and theoretical research of quantum teleportation, and its practical application, is one of the main directions in the field of quantum information. As a result, there were quite a number of experimentally realized schemes, ranging from those using photons [2] and the atoms [3], various hybrid systems and ensembles of particles [4]. The CNOT gate is one of the most important elements in quantum information theory and particularly in quantum computing. There are different realizations of this gate, both linear and non-linear. But all of them have one major problem – it operates with certain probability. However, furthermore its work is also influenced by inaccuracies of the constructions which may have a significant impact on the performance of the gate.

2. Methods
In this work we investigate how these quantum optical circuits work with some perturbed parameters such as reflectivity (transmittivity) of beam splitters. In the linear optical CNOT gate in the coincidence basis [5] circuit there are five beam splitters – two of them have the reflectivity of one-half and others have reflectivities of one-third.

In spite of this we want to see how it operates with the arbitrary values of reflectivity of the beam splitters. By using the Heisenberg equation for beam splitters

\[
a_{out} = \sqrt{\eta}a_{in} + \sqrt{1-\eta}b_{in},
\]

\[
b_{out} = \sqrt{1-\eta}a_{in} - \sqrt{\eta}b_{in},
\]

(1)

where \(\eta\) and \((1-\eta)\) is the reflectivity and transmittivity of the beam splitters, we can obtain the relation of control and target input modes to their corresponding output mode operators. It is necessary for calculating the output state of the system in the Schrödinger picture. We can calculate it through...
substituting input mode operators for the output operators, which we already derived. These operators are
\[ c_{H_0} = \sqrt{\eta_1} c_H + \sqrt{1 - \eta_1} v_c, \]
\[ c_{V_0} = -\sqrt{\eta_2} c_V + \sqrt{\eta_3 (1 - \eta_2)} t_H + \sqrt{(1 - \eta_3)(1 - \eta_2)} t_V, \]
(2)
\[ t_{H_0} = \sqrt{\eta_4 (1 - \eta_2)} c_V + \sqrt{\eta_3 (1 - \eta_2) \eta_4 + (1 - \eta_3)(1 - \eta_2)(1 - \eta_4)} t_H + \sqrt{(1 - \eta_4)(1 - \eta_2)} v_T, \]
\[ t_{V_0} = \sqrt{(1 - \eta_4)(1 - \eta_2)} c_V + \sqrt{\eta_3 (1 - \eta_5) \eta_4 + (1 - \eta_3)(1 - \eta_2)(1 - \eta_4)} t_V - \sqrt{\eta_4 (1 - \eta_5)} v_T. \]

Thereby we obtain the function of the output state for the circuit with the arbitrary beam splitter ratios, substituting these parameters (2) in next equation [5]:
\[ |\psi_{out}\rangle = (\alpha c_{H_0}^\dagger t_{H_0}^\dagger + \beta c_{H_0}^\dagger t_{V_0}^\dagger + \gamma c_{V_0}^\dagger t_{H_0}^\dagger + \delta c_{V_0}^\dagger t_{V_0}^\dagger) 0000 \rangle 00\rangle. \]
(3)

After substituting we obtain several new summands, which reduced when the optimal parameters used. These summands have an important influence on the circuit’s operability, due to the fact that with them this circuit operates as identity operator but not like the CNOT gate. Thus we can analyze how this gate operates and what the error probabilities are.

Also in this work we have calculated and plotted density matrices for certain values of the deviations of the reflectivity in the beam splitters from the optimal values by using next equation:
\[ \rho = |\psi_{out}\rangle \langle \psi_{out}|. \]
(4)

Here presented density matrices of output state for different qubits with optimal values of reflectivity in the beam splitters and certain deviations from them.

\[ \alpha=1, \beta=\gamma=\delta=0 \]

Figure 1. a. Output state density matrix for optimal values for beam splitters \( \eta_1 = \eta_2 = \eta_5 = 0.33 \) and \( \eta_3 = \eta_4 = 0.5 \). b. Output state density matrix for arbitrary deviations of values for beam splitters \( \eta_1 = \eta_2 = \eta_5 = 0.37 \) and \( \eta_3 = \eta_4 = 0.58 \).
Figure 2. a. Output state density matrix for optimal values for beam splitters \( \eta_1 = \eta_2 = \eta_5 = 0.33 \) and \( \eta_3 = \eta_4 = 0.5 \). b. Output state density matrix for arbitrary deviations of values for beam splitters \( \eta_1 = \eta_2 = \eta_5 = 0.37 \) and \( \eta_3 = \eta_4 = 0.58 \).

Figure 3. a. Output state density matrix for optimal values for beam splitters \( \eta_1 = \eta_2 = \eta_5 = 0.33 \) and \( \eta_3 = \eta_4 = 0.5 \). b. Output state density matrix for arbitrary deviations of values for beam splitters \( \eta_1 = \eta_2 = \eta_5 = 0.37 \) and \( \eta_3 = \eta_4 = 0.58 \).

Figure 4. a. Output state density matrix for optimal values for beam splitters \( \eta_1 = \eta_2 = \eta_5 = 0.33 \) and \( \eta_3 = \eta_4 = 0.5 \). b. Output state density matrix for arbitrary deviations of values for beam splitters \( \eta_1 = \eta_2 = \eta_5 = 0.37 \) and \( \eta_3 = \eta_4 = 0.58 \).
In order to have a better representation of this difference we plotted some graphics to show it.

![Graphs showing difference in density matrices of output state](image)

**Figure 5.** Difference between density matrices of output state with $\eta_1 = \eta_2 = \eta_5 = 0.33$ and $\eta_3 = \eta_4 = 0.5$ and density matrices of output state with $\eta_1 = \eta_2 = \eta_5 = 0.37$ and $\eta_3 = \eta_4 = 0.58$.

a. For $\alpha=1, \beta=\gamma=\delta=0$. b. For $\beta=1, \alpha=\gamma=\delta=0$. c. For $\gamma=1, \alpha=\beta=\delta=0$. d. For $\delta=1, \alpha=\beta=\gamma=0$.

There are different realizations of this gate with various type and amount of elements, in particular beam splitters, but despite this we can apply similar technique and methods to obtain the function of output states for these circuits.

### 3. Results and discussion

In the presented work, we have derived the analytical expression for the output state function of the system for realized circuit of control-NOT gate, such as linear optical CNOT gate in the coincidence basis. Furthermore we have calculated output state function for this circuit with arbitrary deviations in beam splitter ratios. Also we have obtained and plotted density matrices for certain values of the deviations of the reflectivity in the beam splitters from the optimal values. Thus it was investigated, that with $\eta_1 = \eta_2 = \eta_5 = 0.33 \pm 0.03$ and $\eta_3 = \eta_4 = 0.5 \pm 0.05$ the circuit operates good and the error probability is near 1 per cent, but starting from $\eta_1 = \eta_2 = \eta_5 = 0.33 \pm 0.04$ and $\eta_3 = \eta_4 = 0.5 \pm 0.08$ the probability to find photon on some outputs can be very different from optimal values.

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