Dynamic loading of spur gears with linear tooth profile modification: modelling and experimental comparisons

D V Kalinin
Central Institute of Aviation Motors, Strength Department, 111116, Moscow, Aviamotornaya Street, 2, Russia
dvkalinin@ciam.ru

Abstract. A dynamic model of a gear pair has been developed to investigate methods of decreasing dynamic loads and analysing dynamic stresses of gears in aviation engine drives. The effect of tooth profile modification on dynamic stresses in the system, as well as the effect of main gear parameters is analysed. It is shown that high level of dynamic stresses and vibrations in gears may be caused by using standard parameters of tooth profile modification. An experimental study of the dynamics of gears with various profile modification parameters has been carried out. The paper shows the necessity of changing the approach to calculating parameters of tooth profile modification of teeth.

1. Introduction
One of the major concerns in the design of power transmission gears for geared turbofan engines is the reduction of gear dynamic load. The task of decreasing dynamic forces and vibrations in aviation transmissions and drives continues to be relevant in the development of modern aircraft engines. Dynamic load creates cyclic bending stresses in tooth roots which can lead to fatigue failure as well as cyclic subsurface stresses which can cause tooth surface failure by pitting and scoring. Its levels largely determine the noise and vibrations in the drive as well as the durability of the gears and other elements of the drives, both the gearbox and the engine itself [1, 2]. Due to the dense spectrum of vibrations in the meshing of planetary gearbox, resonant vibrations occur in the compressor blades and other parts, resulting in a longer engine development time. There occur some cases of in-operation destruction of the turbofan disks caused by resonant vibrations from the over-excitation of the gear set. The reasons behind the high level of dynamic loads were the overestimated parameters of the profile modification of the gear teeth.

2. Dynamic model
The dynamic model of gear pair is represented as a 6-degree-of-freedom nonlinear system [3] consisting of:
- gear pair represented by rigid disks with mass $m_1$ and $m_2$ and inertia moments $J_1$ and $J_2$, respectively, and connected by an elastic-damping coupling with variable mesh stiffness $k_z(t)$, directed along the line of action;
- bearing supports of gear shafts characterized by the stiffness of $k_{bx}$ and $k_{by}$ in accordance with the directions of the selected orthogonal coordinate system for each gear.
The system is balanced by torque applied to the gears in opposite directions (figure 1).
The governing equations of motion for the cylindrical gear pair model shown in Figure 1 in the matrix form:

$$[M]\{\ddot{q}\} + [K(t, q)]\{q\} + [C]\dot{\{q\}} = \{F(t)\} + \{F_{fr}(t, q)\}$$  \hspace{1cm} (1)

Matrix $[M] = diag\{J_1, J_2, m_1, m_2\}$ is an inertia diagonal matrix, the elements of which are inertia moments and mass of gears; $[K(t, q)]$ is a symmetric stiffness matrix; $[C]$ is a damping matrix; $\{q\} = \{\varphi_1, \varphi_2, x_1, x_2, y_1, y_2\}^T$ is a column matrix of a system of generalized coordinates which are the angles of rotation, horizontal and vertical movements of the centers of mass of the gears $\{F(t)\} = \{M_1(t), M_2(t), 0, 0, 0, 0\}^T$ is a column matrix of external forces; $\{F_{fr}(t, q)\}$ is a column matrix of frictional forces [4]; $k_z(t, q)$ is a periodically varying mesh stiffness. Time varying mesh stiffness is the main source of kinematic excitation of parametric vibration in a dynamic gear system [5]. The finite element method (FEM) is used to calculate the mesh stiffness function and the transmission error of each gear pair.

Furthermore, the results of the FEM calculations (figure 2) for the static formulation are combined with the results of the analytical system’s solution of the dynamic transmission model (1). The nonlinear differential equation of motion is solved numerically by using a fourth order, variable step Runge-Kutta (Dormand-Prince pair) numerical integration routine available in MATLAB.

Figure 1. Dynamic model of a spur gear pair

Figure 2. Dynamic model of a spur gear pair
Fig. 3 shows graphs of bending stress distribution in the tooth root along the one mesh phase during static calculation and at rotation speed of driven gear of 1000 rpm. This result is of great interest in the evaluation of both static and fatigue strength of the teeth, because existing methods for evaluating stress in gear dynamic allow to obtain the average value for a series of meshing teeth. Strain measurement results do not give a high precision at high speeds. The graph clearly shows the zone of single-tooth meshing corresponding to peak site and zone of double contact meshing with the increase of the root stress at the start of meshing and decreasing at end of meshing as a result of load transfer between the teeth.

Figure 3. Bending stress calculation: FEA results

The most difficult task in gear dynamic modelling is to evaluate the level of dynamic stresses in the transmission elements. The most precise way of obtaining such a solution is by a numerical method for solving dynamic elasticity problems, such as finite element method in a dynamic setting. However, using the finite element method is effective only for the simplest model of a gear pair. Evaluation of the dynamic properties of the entire powertrain is usually carried out by means of analytical models of dynamic systems with lumped parameters. For a more accurate result the properties of each subsystem of such a model are obtained by dynamic simulation in the FEM.

3. Profile modification

Gear tooth profile modification is introduced in aviation drives to optimize contact patterns and stresses, to compensate for manufacturing errors, and to reduce gear dynamics. Microgeometry modification of involute gear teeth may have a negative effect on static and dynamic performances of the gear system [6].

The problems with requirements for the parameters of profile modification are associated with the lack of theoretical understanding of the influence mechanism of teeth profile modification on improving the quality of gears. In some recommendations it is noted that using profile modification allows to reduce the peak values of contact stresses at the edge contact of the tooth legs, thereby reducing wear of the working surfaces of the teeth. However, in most technical papers devoted to the study of modified gears [6, 7], profile modification is considered as a way to reduce dynamic loads by reducing the transmission error curve to a smoother look.
Figure 4. Tooth profile modification

NASA experimental studies comparing the durability of steel gear wheels with and without tooth tip modification [6, 7] showed that even with a positive effect of tooth tip modification (flanking) on reducing peak contact stresses and reducing the likelihood of the tooth tips chipping, gears without modification had longevity 40 times greater than that of gears with modification under the same conditions.

A decrease of contact ratio $\varepsilon$ due to a decrease in the length of the active portion of the engagement line leads to an increase in the zone of parametric resonance vibrations, accompanied by loss of tooth contact. This is one of the negative effects of introducing profile modification on the dynamic loading of gears. According to the hypothesis on the mechanism of excitation of oscillations in the gear meshing, in order to assess the influence of profile modification on the dynamic loads in the transmission, it is necessary to determine its influence on the function of the mesh stiffness and the transmission error at the load level for which the modification parameters are selected. To obtain the characteristics of the stiffness and transmission error of gears with profile modification, we use an FEM with modeling of the meshing of the teeth, changed in accordance with the parameters of the modified profile of the teeth.

The analysis shows that the choice of the criterion for determining the optimal parameters of the profile modification in order to reduce dynamic loads should be made taking into account the following:
- the profile modification must be selected for a specific load in the transmission;
- the profile modification should be selected taking into account the frequency of rotation of the gears and its operating mode.

Based on the analysis of the results of the calculations, the following general recommendations can be made when choosing the parameters of cylindrical gears profile modification:
- the amplitude of the first harmonic of the mesh stiffness function expansion in the Fourier series for transmissions with modification should not exceed the value of transmissions without modification. These gears include gears with a contact ratio $\varepsilon > 1.7$.
- for gears with a contact ratio $\varepsilon > 2$, it is not recommended to use profile modification;
- the criterion for choosing the optimal parameters for modifying the profile of cylindrical gears can be the level of the amplitudes of the fundamental harmonics from the mesh frequency in the expansion of the mesh stiffness function in Fourier series.
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Figure 5. Mesh stiffness function and the spectrum of gear pairs with different amounts of modification

The choice of the last criterion will make it possible to determine the optimal modification parameters with the construction of amplitude-frequency characteristics of the dynamic loads in the engagement without lengthy calculations. After evaluating the calculation results of dynamic loads in transmissions with different contact ratios as well as for modification parameters, calculated in accordance with the existing recommendations of standards and literature [5], it is proposed to consider the integral parameter calculated through the amplitudes of fundamental harmonics as a criterion characterizing the dynamic loading of a transmission from the mesh frequency in the spectral composition of the meshing stiffness function:

$$A_{k}^{F} = A_{1} + \sqrt{\sum_{i=2}^{6} A_{i}^{2}}$$  \hspace{1cm} (2)

where $A_{i}$ - is the amplitude of the i-th harmonic of the expansion of the mesh stiffness function $k_{z}(t)$ of the transmission under study into Fourier series.

The $A_{k}^{F}$ parameter will be called the effective amplitude of the mesh stiffness. As can be seen from expression (1), it takes into account, first of all, the first harmonic amplitude’s influence of the engagement stiffness function expansion in Fourier series on the dynamic loads in the transmission, which was demonstrated above, since it is the amplitude $A_{1}$ that represents the ratio of the stiffness of the two-pair engagement $k_{Z}^{II}$ to the rigidity of the one-pair link $k_{Z}^{I}$. Minimization of the amplitudes of higher harmonics $A_{i}$ is a secondary task and will affect the reduction of transmission noise by narrowing the width of the active vibration excitation spectrum.

Thus, finding the optimal parameters of profile modification means searching for the minimum of the function $A_{k}^{F}(C_{a}, L_{a}) \rightarrow \text{min}$.

After calculating the optimal parameters of the profile modification according to the criterion of minimum dynamic loading, it is necessary to check the influence of the modification parameters on the distribution of bending stresses in the tooth cavity, due to a change in the gear contact ratio with the introduction of profile modification. Using the FEM in a quasi-static formulation, the dependences of the change in the maximum tensile stresses at the tooth stem for modified gears were obtained.

4. Experimental object and test base

Verification of the dynamic model of cylindrical gear pair is carried out by comparing the results of mathematical modelling with the data of an experimental study performed at the specialized test bench (CIAM, Moscow) for testing cylindrical gears.

Features of the operating conditions of gears in aircraft gearboxes and transmissions associated with high rotation speeds, oil temperature of up to 100°C and a large number of sources of dynamic load excitation complicate the study of dynamic processes on real objects.
Gears from set No. 1 do not have a profile modification of the teeth, and the gears from sets No. 2 and No. 3 are made with a linear profile modification of the tops of the teeth in accordance with GOST 8889-88. Appendix 2. The gears from set No. 2 have a profile modification depth at the top of the teeth of 30 μm, which corresponds to the results of choosing the optimal modification parameters to reduce the excited vibrations. The gears from set No. 3 have a profile modification depth of 62 μm at the top, which is selected in accordance with the recommendations of the ISO 21771 standard [8].

Figure 6. Experimental gear pair and pinion with strain gauges

5. Results and discussion
In the process of gear dynamic tests, their bursting vibrations were recorded in resonant and subresonant modes of operation at various values of the transmitted load. The moment of loss of contact of the gears’ teeth was recorded on the basis of an assessment of the records of deformation signals from strain gauges glued at the cavities of the teeth. It is assumed that as a result of high amplitudes of gear bodies’ torsional vibrations, with the loss of contact of the teeth with working surfaces, the magnitude of tensile stresses at the cavity of the gear tooth should decrease to zero or take a negative value during the period of teeth engagement. This phenomenon was recorded at several frequencies corresponding to the resonant and subresonant modes (Figure 7).
Figure 7. Experimentally measured bending strains in the root of the drive gear teeth at a load of 2000 Nm.

The results of vibrometry of the studied gear sets are evaluated by analyzing the spectral composition of vibration signals measured in the radial direction ar (Figure 8). During the analysis, the components of the tooth harmonic are distinguished in the vibration spectrum, as the main source of vibration excitation in a mechanical system.
Figure 8 shows a comparison of the amplitudes of the first five tooth harmonics in the vibration signal, measured in the radial direction, under different transmission modes. Figure 8 shows a sharp increase in the amplitude of the first toothed harmonic when the transmission is operating in the resonant mode and the absence of significant changes in the amplitudes of the 2nd and highest tooth harmonics of the vibration spectrum in comparison with the results of measurements in the pre- and resonant modes of the transmission. Over the entire frequency range of the gears in the vibration spectrum of the three sets...
of gear pairs, there is a distinct difference in the levels of the amplitudes of the tooth harmonics in the vibration signals depending on the modification of the teeth, predicted by the simulation result. The second set of gears with optimal modification parameters has the lowest vibration activity, and when operating in resonant mode, the vibration amplitude of all tooth harmonics is 40% less (1st harmonic) and higher (64% less for the second harmonic) in comparison with the vibration levels of wheel set No. 1 without profile modification.

The vibration levels for the transmission from set No. 3 with the recommended ISO 21771 parameters of profile modification exceed the levels for sets No. 1 and No. 2 by 2 or more times. Almost over the entire frequency range of transmission in the spectral composition of vibration signals for set No. 3 with overestimated modification parameters, there is a sharp increase in the amplitude of the first tooth harmonic, however, in a narrow range in the pre-resonant mode of operation (1450 - 1520 rpm), there is an increase in the amplitude of the second harmonic with a corresponding decrease in the amplitude of the first harmonic.
The results of the experimental study confirm the conclusions of the dynamic modeling of the cylindrical gears meshing process with various modification parameters. Recommendations for selecting the profile modification parameters that reduce contact stresses are categorically unsuitable for highly loaded gears operating under high dynamic loads. The results of an experimental study show that a gear train with a profile modification, selected on the basis of the developed dynamic model, has the best values of vibrations and dynamic stresses.

6. Conclusions
Spur gears with various profile modifications were tested in the CIAM gear dynamic rig. Dynamic tooth bending strains were recorded for each gear design at 9 operating conditions. The experimental results were compared to examine the influence of the tooth profile on the dynamic behavior of the gears under various operating conditions. The following conclusions were drawn from the data:

1. Experimental study of three sets of gears on a mechanically closed test bench was carried out, which showed the effectiveness of the developed technique for assessing dynamic loads and choosing the optimal parameters of profile modification. Based on the results of calculating the gearing stiffness using FEM, this technique allows, to calculate modification parameters that reduce the amplitudes of fundamental tooth harmonics in the vibration signal excited in the gear meshing.

2. The dynamic loading response of an unmodified gear pair was compared with that of gears with various profile modifications. Correlations were found between various profile modifications and the resulting dynamic loads. An effective error, obtained from frequency domain analysis of the gear mesh stiffness, gave a very good estimation of gear dynamic loading.

3. The hypothesis of the negative effect of teeth profile modification with the parameters selected in accordance with the recommendations of the standardized methods was confirmed.

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