Spin-one ferromagnets with single-ion anisotropy in a perpendicular external field

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Abstract

In this paper, the conventional Holstein-Primakoff method is generalized with the help of the characteristic angle transformation [Lei Zhou and Ruibao Tao, J. Phys. A 27 5599 (1994)] for the spin-one magnetic systems with single-ion anisotropies. We find that the weakness of the conventional method for such systems can be overcome by the new approach. Two models will be discussed to illuminate the main idea, which are the “easy-plane” and the “easy-axis” spin-one ferromagnet, respectively. Comparisons show that the current approach can give reasonable ground state properties for the magnetic system with “easy-plane” anisotropy though the conventional method never can, and can give a better representation than the conventional one for the magnetic system with “easy-axis” anisotropy though the latter is usually believed to be a good approximation in such case. Study of the easy-plane model shows that there is a phase transition induced by the external field, and the low-temperature specific heat may have a peak as the field reaches the critical value.
1 Introduction

Magnetic systems with single-ion anisotropy $D(S^z_i)^2$ have been attracting attentions for years since such kind of anisotropy was found to be very popular in many magnetic materials [1]-[2]. On theoretical side, the spin-wave excitation in such systems are not very easy to handle caused by the off-diagonal effect of the single-ion anisotropy, especially when the spontaneous magnetized direction is not the same as the anisotropic direction. Many theoretical approaches have been developed to deal with such kind of systems. Usually, one first apply a rotating transformation of the spin vectors to determine the ground state, then perform a Holstein-Primakoff (H-P) transformation to study the low-lying spin wave excitations. However, this method was found to be a good approximation only when the anisotropy is the “easy-axis” case (i.e. $D < 0$). In the “easy-plane” case (i.e. $D > 0$), the method was much worse. To understand it, one can study an easy-plane Heisenberg model. If the conventional H-P method is used naively to discuss the ground state and the magnon excitations of such a system, an imaginary value of the excitation energy for “$k = 0$” mode will always be encountered, which implies the failure of this method.

Such deficiency of the conventional method is caused by missing an important quantum effect. Actually, for the single-ion anisotropy (no matter “easy-axis” case or “easy-plane” case), off-diagonal term $D \sin^2 \theta (S^x_i')^2$ will always appear in the Hamiltonian as well as the diagonal terms $D \cos^2 \theta (S^z_i')^2$ after introducing the spin vector rotation. Such off-diagonal term may have the tendency to mix the single-site spin-state $|n\rangle$ with $|n + 2\rangle$ and $|n - 2\rangle$ to form the proper eigenstates, and this spin-states mixing effect is completely a quantum one which is very important in “easy-plane” anisotropy case. Unfortunately, such a quantum effect has been neglected by the conventional H-P method. As the result, the conventional method failed for the magnetic systems with “easy-plane” anisotropy.

On the other hand, many methods have been proposed for the easy-plane magnetic systems [3]-[10]. The matching of the matrix elements (MME) method [3]-[4] was one which can be used to consider the spin-states mixing effect perturbatively so that it can give a reasonable representation for an easy-plane ferromagnet when the single-ion anisotropy is small [3]-[10], and some numerical methods were developed for an easy-plane spin-one ferromagnet [3]-[10]. Recently, another method - the characteristic angle (CA) method was proposed for the easy-plane spin-one ferromagnet which could be applied to describe such spin-states mixing effect by a variation parameter through a spin operator transformation [11]. The magnetic properties had been investigated for such a system in zero field, and the results seemed to be closer to the numerical results than those of the MME method [11].

The present work is focused on generalizing the conventional H-P method with the help of CA transformation for the spin-one magnetic systems with single-ion anisotropies. Two particular models will be studied as the illustration of the CA approach, although the latter is certainly not limited to such models. The difficulties faced by
the conventional H-P method are overcome for such systems with the help of the new approach.

This paper is organized as follows. In the next section, the easy-plane model is studied using the CA approach. Detailed comparisons of the CA approach with the conventional method is made in Section 3. Section 4 is devoted to an easy-axis model, and the conclusions are summarized in the last section.

2 Easy-plane case

The first model we will study is an easy-plane spin-one ferromagnet in an external magnetic field which is applied perpendicular to the "easy-plane". The Hamiltonian of this system can be given as:

\[ H = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + D \sum_i (S_i^z)^2 - h \sum_i S_i^z, \]  

where the first term is the exchange interaction, and the second one is the single-ion anisotropy. The anisotropy parameter \( D \) is positive so that \( x-y \) plane is the so-called "easy-plane" and \( z \) axis is the "hard axis". An external magnetic field \( h \) is applied along the "hard axis".

Although the single-site part of Hamiltonian \( D(S_i^z)^2 - hS_i^z \) has already been a diagonalized form, it is still unreasonable to apply a H-P transformation naively to discuss the magnetic properties of such a system by assuming the ground state to be the ordinary ferromagnetic state. Actually if we do that, we will easily find that the magnon excitation energy of \( \mathbf{k} = 0 \) mode will be always negative in the case of \( h < D \). That is because the "starting point" based on which the spin deviations are discussed is wrong.

One must be very careful in finding a reasonable "starting point". Actually, in such a system, on the one hand, the spins are forced into the "easy-plane" by the single-ion anisotropy, on the other hand, they have the tendency to point along the "hard axis" caused by the external field. As the result, this two effects must compete with each other and a new direction \( z' \) axis would be optimized to describe the spontaneous magnetized direction. So, it’s desirable to introduce a new coordinates system \((x',y',z')\) in which the spin components are related to those in the original coordinates by the following transformation:

\[ S^z_i = \cos \theta_r S^z_i' - \sin \theta_r S^x_i', \]  

\[ S^x_i = \cos \theta_r S^x_i' + \sin \theta_r S^z_i', \]  

\[ S^y_i = S^y_i'. \]  

Applying the above transformation to Hamiltonian (1), we have

\[ H = -J \sum_{\langle i,j \rangle} S'_i \cdot S'_j + D \cos^2 \theta_r \sum_i (S'_i^z)^2 + D \sin^2 \theta_r \sum_i (S'_i^x)^2 - h \cos \theta_r \sum_i S'_i^z. \]
In a classical view, we can always determine $\theta_r$ based on variation method assuming all spins are aligned along the $z'$ direction in the ground state. However, one should be careful in quantum case, especially in the current “easy-plane” anisotropy case. Actually, if we apply the H-P transformation naively to investigate the spin-waves excitation in such system, an imaginary value of the magnon excitation energy for “$k = 0$” mode will always exist. In fact, since

$$D \sin \theta_r \cos \theta_r \sum_i (S_i^{x'} S_i^{x'} + S_i^{z'} S_i^{z'}) + h \sin \theta_r \sum_i S_i^{z'},$$

(5)

if the H-P transformation is applied naively to the Hamiltonian (5), one may find that the off-diagonal terms $D \sin \theta_r (S_i^{x'} S_i^{x'} + S_i^{z'} S_i^{z'})$ in the above equation have no contribution to the constant term of the transformed Hamiltonian. That means the spin-states mixing effect has already been neglected by the conventional H-P method. Unfortunately, such effect is very important and must be considered in such case. The characteristic angle (CA) transformation \[10\] was developed to describe the spin-state mixing effect in spin-one case by introducing another variation parameter $\theta_c$:

$$S_j^{+'} = \cos \theta_c \tilde{S}_j^+ + \sin \theta_c \exp(i \pi \tilde{S}_j^z),$$

(7)

$$S_j^{-'} = \cos \theta_c \tilde{S}_j^- + \sin \theta_c \exp(-i \pi \tilde{S}_j^z) \tilde{S}_j^+, \quad (8)$$

$$S_j^{z'} = (1/2)[S_j^{+'}, S_j^{-'}]. \quad (9)$$

The spin operators are transformed to a new set of quasi-spin operators $(\tilde{S}_j^+, \tilde{S}_j^z)$ which had been proved to obey all spin-one operator’s commutation rules \[10\]. After the CA transformation, we can apply a H-P transformation to transform the quasi-spin operator to Bose one,

$$\tilde{S}_i^z \rightarrow 1 - a_i^+ a_i,$$

(10)

$$\tilde{S}_i^+ \rightarrow \sqrt{2} \sqrt{1 - (a_i^+ a_i/2)} a_i,$$

(11)

$$\tilde{S}_i^- \rightarrow \sqrt{2} a_i^+ \sqrt{1 - (a_i^+ a_i/2)}.$$

(12)

Then, the Hamiltonian will have the following form:

$$H = U_0 + H_1 + H_2 + \cdots \quad (13)$$

where

$$U_0 = N[-JZ \cos^2 2\theta_c + D - \frac{D}{2} \sin^2 \theta_r (1 + \sin 2\theta_c) - h \cos \theta_r \cos 2\theta_c] \quad (14)$$

$$H_1 = -\frac{\sqrt{2}}{2} \sum_i [D \sin \theta_r \cos \theta_r (\cos \theta_c + \sin \theta_c) - h \sin \theta_r (\cos \theta_c - \sin \theta_c)]$$

$$(a_i^+ + a_i) \quad (15)$$
and $H_2$ can be written in momentum $k$ space as follows

$$H_2 = \sum_k A_k a^+_k a_k + \sum_k B_k (a^+_k a^-_k + a^-_k a_k)$$

$$A_k = 2JZ (\cos^2 \theta_c - \gamma_k) + \frac{D}{2} \sin^2 \theta_r (1 + \sin 2 \theta_c) + h \cos \theta_r \cos 2 \theta_c$$

$$B_k = \frac{\sqrt{2}}{2} \left[-JZ \sin 4 \theta_c + \frac{D}{2} \sin^2 \theta_r \cos 2 \theta_c - h \cos \theta_r \sin 2 \theta_c \right]$$

$$+ JZ \sin 2 \theta_c \gamma_k.$$  

Based on the variation method we understand that the two parameters $\theta_r, \theta_c$ should be determined by minimizing the ground state energy. As a first order approximation, we may obtain:

$$\frac{1}{N} \frac{d}{d \theta_r} U_0(\theta_r, \theta_c) = -D \sin \theta_r \cos \theta_r (1 + \sin 2 \theta_c) + h \sin \theta_r \cos 2 \theta_c = 0$$

$$\frac{1}{N} \frac{d}{d \theta_c} U_0(\theta_r, \theta_c) = 4JZ \sin 2 \theta_c \cos 2 \theta_c - D \sin^2 \theta_r \cos 2 \theta_c + 2h \cos \theta_r \sin 2 \theta_c = 0$$

Eq. (19) is just the same as the condition $H_1 = 0$ and Eq. (20) can cancel most of the off-diagonal terms which are in the square bracket in the expression of $B_k$. If we substitute the solution of the above non-linear equations into to the Hamiltonian (13), then diagonalize the harmonic part of Hamiltonian $H_2$ by a usual Bogolyubove transformation, the total Hamiltonian will be:

$$H = U'_0 + \sum_k E_k \alpha_k^+ \alpha_k + \cdots$$

where

$$U'_0 = U_0 - \frac{1}{2} \sum_k A_k + \frac{1}{2} \sum_k \sqrt{A_k^2 - 4B_k^2}$$

$$E_k = \sqrt{A_k^2 - 4B_k^2}.$$  

The ground state in such a method can be defined by

$$\alpha_k |0 \rangle = 0$$

Then the induced magnetization $M(h)$ is derived in the harmonic approximation as follows:

$$M(h) = \frac{1}{N} \sum_i \langle 0 | S_i^z | 0 \rangle \simeq \frac{1}{N} \sum_i \cos \theta_r \langle 0 | S_i^z | 0 \rangle$$
\[ \simeq \frac{1}{N} \sum_i \cos \theta_r \left\{ \cos 2 \theta_c \langle 0 | \tilde{S}_i^+ | 0 \rangle + \sin \theta_c \cos \theta_c \langle 0 | (\tilde{S}_i^+)^2 + (\tilde{S}_i^-)^2 | 0 \rangle \right\} \]

\[ \simeq \frac{1}{N} \sum_i \cos \theta_r \left\{ \cos 2 \theta_c \langle 0 | 1 - a_i^+ a_i | 0 \rangle + \sqrt{2} \sin \theta_c \cos \theta_c \langle 0 | a_i^+ a_i^2 + a_i^2 | 0 \rangle \right\} \]

\[ \simeq \frac{3}{2} \cos \theta_r \cos 2 \theta_c - \frac{1}{2N} \sum_k \cos \theta_r \cos 2 \theta_c A_k + 2 \sqrt{2} \cos \theta_c \sin 2 \theta_c B_k. \tag{25} \]

Thus, put the solution of Eqs. (19)-(20) into Eqs. (22)-(23), (25), such physical properties as the ground state energy, the magnon dispersion relation and the induced magnetization can be obtained. However, since it is very difficult to solve the nonlinear equations analytically, numerical calculations are carried out. The system with anisotropy parameter \( D/4JZ = 0.6 \) has been studied as an example.

\( \theta_r \) and \( \theta_c \) as the functions of the external field have been drawn together in figure 1 from which one can find that they are both the decreasing function of the external field. It is understood that \( \theta_r \) is used to describe the spontaneous magnetized direction and \( \theta_c \) the spin-states mixing effect, in zero applied field case, the spontaneous magnetized direction will be the \( x \) axis (\( \theta_r = 90^\circ \)) and the spin-states mixing effect should be the strongest since the off-diagonal term \( D \sin^2 \theta_r (S_i^x) \) is the strongest, and the value of \( \theta_c \) is consistent with Ref. [10] where \( h = 0 \) case has already been discussed. While the external magnetic field is strengthened, the spins will point along a direction which is closer to the \( z \) axis due to the interaction with the external field so that \( \theta_r \) will decrease, at the same time, the spin-states mixing effect is also weakened since the off-diagonal interactions in the total Hamiltonian will turn smaller along with the decrement of \( \theta_r \). However, when \( h \) reaches a critical value \( h_c = D \), the external magnetic field is so strong that the spins will not be rotated any longer, and the off-diagonal term comes to zero either, as the result, \( \theta_r \) and \( \theta_c \) will vanish simultaneously.

### 3 Comparisons and Discussions

In this section, we will compare the CA method with the conventional H-P method in details and discuss the magnetic properties of the above mentioned system.

First, one may find that more quantum effects have been comprised in the constant term of the Hamiltonian by the new approach. Introducing the H-P transformation naively to Hamiltonian (5), the constant term can be found as:

\[ U_0^{HP} = N(-JZ + D \cos^2 \theta_r - h \cos \theta_r) + N \frac{D}{2} \sin^2 \theta_r \]

\[ = U_0^C + N \frac{D}{2} \sin^2 \theta_r. \tag{26} \]
where \( U_0^C \) is the ground state energy obtained by a classical rotating transformation.

After applying the CA transformation, the ground state energy is Eq. (14) which can be rewritten as:

\[
U_0 = U_0^{HP} + U_1
\]  
(27)

where

\[
U_1 = N(JZ \sin^2 2\theta_c - \frac{D}{2} \sin^2 \theta_r \sin 2\theta_c + 2h \cos \theta_r \sin^2 \theta).
\]  
(28)

\( U_1 \) is an additional term introduced by the CA transformation which will vanish as \( \theta_c = 0 \).

The conventional H-P method is a semi-classical one. The only quantum effect in \( U_0^C \) is the term \( N\frac{D}{2} \sin^2 \theta_r \) which comes from the contribution of \( \frac{D}{2} \sin^2 \theta_r (S_{i}^{x}S_{i}^{x} + S_{i}^{y}S_{i}^{y}) \) in Eq. (8), and other terms which have been collected in \( U_0^C \) of Eq. (26) can be easily recovered by a classical method. However, after applying the CA transformation, it can be clearly found that there is an additional contribution \( U_1 \) in the expression of \( U_0 \) which describes the single-site spin-states mixing effect through the variation parameter \( \theta_c \). Such an effect is completely a quantum one which has no classical counterpart, and it is expressed as the competition of the exchange term \( (JZ) \) and the external term \( (h) \) with the single-ion anisotropy term \( (D) \).

So, more quantum effects have been considered by the CA approach than the conventional H-P method even in the constant term of the Hamiltonian. Furthermore, considering such quantum effect will lead to a lower ground state energy since the ground state in CA method is selected to be the minimum point of function \( U_0 \) although that in the conventional H-P method is not so.

Now, one can compare the CA method with the conventional H-P method for the elementary excitation of such system. Put the solution of \( \theta_r \) and \( \theta_c \) into Eq. (23), the magnon excitation gap can be calculated with respect to the external magnetic field and the result has been shown in figure 2, where one can find that the magnon excitation gap obtained by the CA method will always be positive or zero.

However, based on the conventional H-P method, one should obtain the variation parameter \( \theta_r \) by minimizing \( (26) \) which yields:

\[
\frac{1}{N} \frac{d}{d\theta_r} U_0^{HP} = -D \sin \theta_r \cos \theta_r + h \sin \theta_r = 0
\]  
(29)

then substitute the solution of the above equation into Eqs. (17-18), (23), one can easily find that the magnon excitation gap in H-P method will be

\[
\Delta^{HP} = \sqrt{\frac{h^2 - D^2}{2D}}.
\]  
(30)
Of course, the excitation gap will never be real when \( h < D \). That is to say the conventional H-P method can not be applied naively to study the magnetic systems with “easy-plane” anisotropy. However, the CA method has overcome this difficulty as shown in figure 2.

The induced magnetization as the function of the external magnetic field has been drawn in figure 3. From figures 1-3, one may find that the point \( h_c = D \) is very strange and there seems to be a phase transition in such a point. As the external field is strengthened across \( h_c \), the system transits to a phase in which the spins are not rotated any longer. Actually, this phase transition can be clearly shown by calculating the low-temperature specific heat \( C_v \).

Suppose the system is at low temperature, and only the low energy excitation is considered, then the inner energy will be:

\[
E(T) = E_0 + \sum_k E_k \frac{1}{\exp\left(\frac{E_k}{k_B T}\right) - 1}
\]

So the specific heat can be obtained:

\[
C_v = \frac{dE(T)}{dT} = \frac{1}{N} \sum_k \frac{(E_k)^2 \exp\left(\frac{E_k}{k_B T}\right)}{\left(\exp\left(\frac{E_k}{k_B T}\right) - 1\right)^2} N k_B
\]

The specific heat of the system as the function of the external magnetic field at the temperature \( k_B T/JZ = 0.1 \) has been shown in figure 4, in which a peak can be apparently found in the critical point \( h_c = D \). The physics can be understood as follows: in the vicinity of the phase transition point \( h_c \), the magnons will be excited without a gap (fig. 2) so that the thermal fluctuations are strong.

4 Easy-axis case

Now we will study another model which is an “easy-axis” spin-one ferromagnet in an external magnetic field whose direction is perpendicular to the “easy-axis”. The Hamiltonian of such system can be given as:

\[
H = -J \sum_{\langle ij \rangle} S_i \cdot S_j - D \sum_i (S_i^x)^2 - h \sum_i S_i^z,
\]

where the single-ion anisotropy makes \( x \) axis an easy-axis, and an external magnetic field is applied along \( z \) direction.

At a first glance, this Hamiltonian has a same classical picture as the last model - the anisotropic interaction and the interaction with external field have to compete with each other and will be balanced at some angle \( \theta_r \). So a rotating transformation of the
spin vectors is helpful. In fact, many authors have used this method to discuss various kinds of magnetic systems with single-ion anisotropy and believed this approximation will work well when the single-ion anisotropy is “easy-axis” case. After the rotating transformation (4)-(11), the Hamiltonian will be:

\[
H = -J \sum_{(i,j)} \mathbf{S}'_i \cdot \mathbf{S}'_j - D \sin^2 \theta_r \sum_i (S'^z_i)^2 - D \cos^2 \theta_r \sum_i (S'^z_i)^2 - h \cos \theta_r \sum_i S'^z_i
\]

\[
= -D \sin \theta_r \cos \theta_r \sum_i (S'_i S'^z_i + S'_i S'^z_i) + h \sin \theta_r \sum_i S'^z_i
\]

(34)

there are still off-diagonal terms \((-D \cos^2 \theta_r \sum_i (S'^z_i)^2\)), in the Hamiltonian, and this off-diagonal interactions may be important in some cases. So it is helpful to apply the CA transformation to get a more reasonable representation.

Actually, after almost the same procedure as that for the easy-plane model, the Hamiltonian can be transformed to:

\[
H = U_0 + H_1 + H_2 + \cdots
\]

(35)

where

\[
U_0 = N[-JZ \cos^2 2\theta_c - D + \frac{D}{2} \cos^2 \theta_r(1 + \sin 2\theta_c) - h \cos \theta_r \cos 2\theta_c]
\]

(36)

\[
H_1 = -\frac{\sqrt{2}}{2} \sum_i \left[-D \sin \theta_r \cos \theta_r(\cos \theta_c + \sin \theta_c) + h \sin \theta_r(\cos \theta_c - \sin \theta_c)\right] (a^+_i + a_i)
\]

(37)

and

\[
H_2 = \sum_k A_k a^+_k a_k + \sum_k B_k (a^+_k a^-_k + a^-_k a^+_k),
\]

(38)

\[
A_k = 2JZ(\cos^2 2\theta_c - \gamma_k) - \frac{D}{2} \cos^2 \theta_r(1 + \sin 2\theta_c) + h \cos \theta_r \cos 2\theta_c
\]

\[
+ D \sin^2 \theta_r,
\]

(39)

\[
B_k = \frac{\sqrt{2}}{2} \left[-JZ \sin 4\theta_c - \frac{D}{2} \cos^2 \theta_r \cos 2\theta_c - h \cos \theta_r \sin 2\theta_c\right]
\]

\[
+ JZ \sin 2\theta_c \gamma_k.
\]

(40)

The two variational parameters \(\theta_r, \theta_c\) satisfy:

\[
\frac{1}{N} \frac{d}{d \theta_r} U_0(\theta_r, \theta_c) = -D \sin \theta_r \cos \theta_r(1 + \sin 2\theta_c) + h \sin \theta_r \cos 2\theta_c = 0
\]

(41)

\[
\frac{1}{N} \frac{d}{d \theta_c} U_0(\theta_r, \theta_c) = 4JZ \sin 2\theta_c \cos 2\theta_c + D \cos^2 \theta_r \cos 2\theta_c + 2h \cos \theta_r \sin 2\theta_c
\]

\[
= 0
\]

(42)
where Eq. (41) will cancel the $H_1$ part of the Hamiltonian.

Physical properties such as magnon excitation, the induced magnetization have the same forms as which in last model (i.e. Eqs. (22),(23)) except the concrete expression of the functions $A_k$ and $B_k$.

Very similar to the “easy-plane” case, the constant term in the Hamiltonian can further be divided into two two terms:

$$U_0 = U_0^{HP} + U_1$$

where

$$U_0^{HP} = N(-JZ - D \sin^2 \theta_r - h \cos \theta_r) - N \frac{D}{2} \cos^2 \theta_r$$

$$U_1 = N(JZ \sin^2 2\theta_c + \frac{D}{2} \cos^2 \theta_r \sin 2\theta_c + 2h \cos \theta_r \sin^2 \theta_c).$$

$U_0^{HP}$ is the contribution of the conventional H-P method, and $U_1$ is an additional term which describes the quantum effect of spin-states mixing in single-site. All the discussions are similar to the first model: more quantum effects have been comprised in the constant term of the Hamiltonian by the CA method, and considering such effect in CA method will lead to a lower ground state energy than that in the conventional H-P method. Actually, as shown in figure 5 where $U_0$ and $U_0^{HP}$ are drawn together with respect to the external magnetic field for an easy-axis spin-one ferromagnet, $U_0$ is always found to be lower than $U_0^{HP}$.

Now it is interesting to compare the elementary excitations calculated by the CA method with those by the conventional H-P method. In the latter case, $\theta_C = 0$ and $\theta_r$ is obtained by minimizing $U_0^{HP}$ which yields:

$$\frac{1}{N} \frac{d}{d\theta_r} U_0^{HP} = -D \sin \theta_r \cos \theta_r + h \sin \theta_r = 0$$

So, substitute the solution of the above equation back into Eqs. (23) then into Eq. (23), the elementary excitation in the conventional H-P method can be calculated readily.

The magnon excitation gaps have been calculated by both methods with respect to the external magnetic field, and the results are presented in figure 6 where the solid line is by CA method and the dotted line is by the conventional H-P method. From the figure one may find that: when the external field is close to the anisotropy parameter $D$, there is a small region where the magnon excitation gap calculated by the H-P method will be imaginary, which indicates that this approximation is poor in such area. However, the solid line in the figure tells us that CA method has overcome this difficulty and the magnon excitation gap will always be real and positive in CA method.
Actually, as shown in figure 7 where the values of the two variation parameters $\theta_c, \theta_r$ are drawn together with respect to the external field, when $h$ is close to $D$, $\theta_r$ comes to zero and $\theta_c$ becomes somewhat larger indicating that the spin-state mixing effect caused by the off-diagonal interaction may be very strong. So, we must consider such effect with the help of CA transformation in such case, otherwise, the “starting point” may be unreasonable and will lead to an imaginary minimum excitation energy. Outside this region, the off-diagonal terms are not so strong comparing to the diagonal parts, as the result, the spin-states mixing effect is not very drastic and the conventional H-P method might be a reasonable approximation as many authors believed. However, the CA transformation may always be helpful to get a more reasonable representation for such a system.

5 Conclusions

To summarize, in this paper, the conventional method has been generalized with the help of the characteristic angle transformation for the spin-one magnetic systems. The difficulties faced by the conventional H-P method for magnetic systems with single-ion anisotropy has been overcome by the new approach. Two models have been discussed to illuminate the main ideas, of which one is an easy-plane spin-one ferromagnet in an external field applied perpendicular to the “easy-plane”, and the other is an easy-axis spin-one ferromagnet in an external field applied perpendicular to the “easy-axis”. Comparisons between the new approach and the old one show: more quantum effects have been considered by the CA method, as the result, CA method can examine the ground state properties of the “easy-plane” spin-one ferromagnet although the old method never can, and CA method can give an improved representation for the “easy-axis” spin-one ferromagnet although the conventional H-P method is usually believed to be a good approximation in such case. Also, study of the easy-plane model shows that a phase transition may take place induced by the applied field, and the low-temperature specific heat is found to have a peak when the external field reaches the critical value.

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Captions:

Figure 1: $\theta_r, \theta_c$ as the functions of the external field for the easy-plane spin-one ferromagnet with $D/4JZ = 0.6$.

Figure 2: Magnon excitation gap as the function of the external field for the easy-plane spin-one ferromagnet with $D/4JZ = 0.6$.

Figure 3: Induced magnetization as the function of the external field for the easy-plane spin-one ferromagnet with $D/4JZ = 0.6$.

Figure 4: Specific heat as the function of the external field at the temperature $K_BT/JZ = 0.1$ for the easy-plane spin-one ferromagnet with $D/4JZ = 0.6$.

Figure 5: Ground state energy with respect to the external magnetic field in the case of using the conventional H-P method $U_0^{HP}$ and using the CA method for an easy-axis spin-one ferromagnet with anisotropic parameter $D/4JZ = 0.3$.

Figure 6: Magnon excitation gaps of the easy-axis spin-one ferromagnet with $D/4JZ = 0.3$ as the functions of the external magnetic field for the case using the CA method (solid line) and using the conventional H-P method (dotted line).

Figure 7: $\theta_r, \theta_c$ as the functions of the external field for the easy-axis spin-one ferromagnet with $D/4JZ = 0.3$. 