Extracting $\sigma_{\text{eff}}$ from the CDF $\gamma + 3\text{jets}$ measurement

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Abstract: In their 1997 paper, CDF measured $\sigma_{\text{eff}}$, the normalization factor that relates the cross section for double parton scattering to the product of the inclusive cross sections of the two individual scatters, in a model in which they are assumed to be independent. In his 2007 paper, Treleani pointed out that CDF used a non-standard definition, in which the double parton scattering cross section corresponds to exactly two scatters, rather than the more conventional one in which it is the inclusive two-scatter cross section. He also estimated the correction from one definition to the other, to give a corrected value of $\sigma_{\text{eff}}$. Treleani’s form would be correct under the assumption that CDF were able to uniquely identify and count the number of scatters in an event, which is certainly not the case. In this publication we consider CDF’s event definition in more detail to provide an improved correction.
1 Introduction

Due to the composite nature of hadrons, it is possible to have multiple parton scatterings, i.e. events in which two or more distinct parton interactions occur simultaneously in a single hadron-hadron collision. At fixed final state invariant masses, such cross sections tend to increase with collision energy because partons with successively lower momentum fraction $x$, hence rapidly increasing fluxes, are probed. Therefore, the question of multiple parton interactions (MPI) in a single hadronic collision has rapidly moved from a theoretical curiosity, when double parton interactions was for the first time observed by AFS experiment at $\sqrt{s} = 63$ GeV [1] to a critical issue at the LHC. Multiple parton interactions at the LHC give rise to different effects, among others a substantial increase of the unavoidable background to most observables used for the search of new physics [2–7]. For this reason, multiple parton scattering has taken on considerable importance in recent years, since a variety of new and improved Monte Carlo models of underlying event physics rely on it [8–17]. Unfortunately the process cannot be estimated in a straightforward way, due to the lack of knowledge of the non-perturbative physics, therefore these models rely on the experimental input. Especially, the experimental value of $\sigma_{\text{eff}}$, the normalization factor that relates the cross section for double parton scattering to the product of the inclusive cross sections of the two individual scatters, has the potential to act as a strong constraint
on models of multiple parton scattering and, in particular, their models of the transverse-space distribution of partons in hadrons. The value of $\sigma_{\text{eff}}$ is also crucial for calculations of double pair production based on the framework of the $k_t$-factorization approach, see for example [18].

CDF’s measurement of the double parton scattering cross section [19], still one of the best available, therefore has considerable significance not only for MPI models but also as a standard candle for new [20–22] and planned measurements [23]. CDF’s measurement was better than any that came before it, because it avoided almost all reliance on a Monte Carlo description of their final states and on theoretically-calculated cross sections. Instead it made an ingenious direct extraction of $\sigma_{\text{eff}}$ by defining an event selection sensitive to double parton scattering and comparing the rate of these events from beam crossings with a single vertex (assumed to be double parton scattering within one proton–antiproton collision) and with two vertices (assumed to be single-parton scatterings within two independent proton–antiproton collisions). The main assumption that their extraction relies on is that the final state of two scatters in the same proton–antiproton collision is identical to that of two scatters in different proton–antiproton collisions. CDF defined the cross section normalization factor $\sigma_{\text{eff}}$ through the equation

$$
\sigma_{ab;2} = \frac{\sigma_a \sigma_b}{\sigma_{\text{eff},CDF}},
$$

where $\sigma_{ab;2}$ is the cross section for a colliding proton–antiproton pair to have exactly two scatters of types $a$ and $b$. We choose the notation for exclusive cross sections, where the process is described by the subscript containing the type of individual parton processes and the number of scatters after a semi-colon. Here, we assume that the two processes $a$ and $b$ are different, i.e. distinguishable$^1$. $\sigma_{a,b}$ are their inclusive cross sections. To distinguish this from the definition more commonly used in theoretical studies, we write it as $\sigma_{\text{eff},CDF}$ from here on. CDF’s final value was

$$
\sigma_{\text{eff},CDF} = (14.5 \pm 1.7^{+1.7}_{-2.3}) \text{ mb}.
$$

(1.2)

Since they used the definition of (1.1), which refers to exactly two scatters, they made a correction to account for the fact that a fraction of their events (which they estimated to be $17^{+4}_{-8}$%) came from triple-parton scattering events. In their analysis CDF used this estimation to re-scale the number of the accepted events by the factor of $0.83^{+0.08}_{-0.04}$. The ratio of these two numbers is equal to the ratio of the exclusive triple and double parton scattering cross sections producing the same final state:

$$
\frac{\sigma_{ab;3}}{\sigma_{ab;2}} \approx \frac{17}{83}.
$$

(1.3)

From the theoretical point of view, it is more convenient to define $\sigma_{\text{eff}}$ through an analogous formula, but for the inclusive double-scattering cross section,

$$
\sigma_{ab} = \frac{\sigma_a \sigma_b}{\sigma_{\text{eff}}},
$$

(1.4)

$^1$CDF use an experimental method to verify that this is the case with their event selection, even though there could be some overlap in principle.
since, as shown in Section 3, with this definition in the assumption of independence of individual scatters, $\sigma_{\text{eff}}$ depends only on properties of the colliding hadrons and not on the scattering types or cross sections.

The non-standard definition used by CDF has been pointed out for the first time by Treleani in his publication [24]. Based on the theoretically-pure (parton-level) situation, assuming that one could measure, for a given event, whether it came from two scatters of definite types $a$ and $b$ or three, one of type $a$ and two of type $b$, Treleani calculated the correction from $\sigma_{\text{eff,CDF}}$ to $\sigma_{\text{eff}}$. Using the formula in his paper, one obtains the value\(^2\)

$$\sigma_{\text{eff}} = 10.3 \text{ mb}. \quad (1.5)$$

The purpose of this publication is to point out that this extraction is over-simplified – the indirectness of CDF’s measurement means that they are far from being in this pure (parton-level) situation.

The paper is organised as follows. We begin by recapping the salient points of CDF’s measurement and event selection in Sect. 2. In Sect. 3 we consider how to calculate the two- and three-scatter cross sections with this event selection. We show, in Sect. 4, that further input is needed to relate these to $\sigma_{\text{eff}}$. In Sect. 5 we make Monte Carlo estimates of this input and its uncertainty. One important point to note already is that we only need ratios of closely-related cross sections, so we hope not to be too sensitive to details of the Monte Carlo event generation so that the uncertainty is fairly small. Finally, in Sect. 6 we put this input together with CDF’s measurement to give a final value for $\sigma_{\text{eff}}$. In appendices we give more details backing up our Monte Carlo evaluation of the correction factor and its uncertainty.

2 CDF’s experimental measurement

The intricacies of the clever extraction of $\sigma_{\text{eff}}$ using the single- and double-vertex data will not need to concern us here. What will be important is the definition of the final state containing a direct photon and exactly three jets. The photon was measured within pseudo-rapidity acceptance $|\eta| < 0.9$ and was required to have $E_T \gamma > 16$ GeV. Jets were reconstructed from all objects in the calorimeter region, $|\eta| < 4.2$, using a cone algorithm CDFJetClu with cone radius 0.7 [25]. Jets were ordered according to their $E_T$ so that $E_{T1} > E_{T2} > E_{T3}$ and were required to be separated in $\eta - \phi$ plane from each other by 0.7 and from the photon by 0.8. Events were accepted if all three jets had $E_T > 5$ GeV and simultaneously the second and the third hardest jets had $E_T < 7$ GeV.

In what we will call the pure parton picture, one would therefore have one of the jets also above 16 GeV, equal and opposite to the photon in transverse momentum both coming from one scatter, and the other two jets equal and opposite to each other, from the other scatter. However, with 5 GeV jets, one is far from this theoretically pure situation. CDF spent some time investigating the properties of such low $E_T$ jets and concluded that they are sufficiently correlated with the underlying parton dynamics to enable their measurement\(^2\) which in [24] is written as “$\sigma_{\text{eff}} \approx 11 \text{mb}$.”
but that there was a great deal of smearing and creation of jets ‘from nothing’. We certainly cannot therefore rely on this simple parton picture. In fact, CDF estimated that 75% of their event sample came from events in which the photon and two of the jets came from one scattering and one jet from the other. They stressed throughout that the numerator $\sigma_a\sigma_b$ in Eq. (1.1) is a shorthand for the sum over all separations into two scatters of the source of their three jets. In this spirit, one could write more precisely

$$\sigma_{\gamma+3\text{jets};2} = \frac{\sigma_{\gamma+1\text{jet}}\sigma_{2\text{jets}} + \sigma_{\gamma+2\text{jets}}\sigma_{1\text{jet}}}{\sigma_{\text{eff},CDF}}.$$  \hspace{1cm} (2.1)$$

Note that the contribution containing $\sigma_{\gamma+0\text{jets}}$, which should in principle also appear here, was found to be negligible [19].

The data were corrected for the trigger efficiency, but not for any other detector effects, but we argue below (see Sec. 5) that, since we will only need ratios of closely-related cross sections, they should largely cancel.

### 3 Parton level correction

In order to understand the correction at the parton level from $\sigma_{\text{eff},CDF}$ to $\sigma_{\text{eff}}$, we write down the expression for the cross section $\sigma_{ab;2}$ according to Eq. (2.1) in a simple eikonal model. We assume that the dependence of the parton distributions on the two-dimensional impact parameter, $\vec{b}$, and longitudinal momentum fraction, $x$, factorize. We make extensive use of the overlap function $A(\vec{b})$, normalized such that

$$\int d^2 b A(\vec{b}) = 1.$$  \hspace{1cm} (3.1)$$

To set the scene, we mention that the cross section for exactly $n$ scatters of a type $b$ is given by

$$\sigma_{b;n} = \int d^2 b \frac{1}{n!} \left( \sigma_b A(\vec{b}) \right)^n \exp \left\{ -\sigma_b A(\vec{b}) \right\}.$$  \hspace{1cm} (3.2)$$

That is, the cross section is obtained by integrating over all values of impact parameter the probability of the scatters: each has a probability $\sigma_b A(\vec{b})$; there are $n$ of them and they are independent, giving the power of $n$; and they are all of the same type, giving the $n!$ factor. Finally, the exponential gives the probability that there are no other scatters (of type $b$). It is straightforward to check that the inclusive cross section is given by

$$\sigma_b = \sum_n n \sigma_{b;n}.$$  \hspace{1cm} (3.3)$$

For a total of $n$ scatters: exactly one of type $a$ and $n-1$ of type $b$, with $a$ and $b$ assumed distinguishable, we obtain in the same way

$$\sigma_{ab;n} = \int d^2 b \left( \sigma_a A(\vec{b}) \right) \frac{1}{(n-1)!} \left( \sigma_b A(\vec{b}) \right)^{n-1} \exp \left\{ -(\sigma_a + \sigma_b) A(\vec{b}) \right\}.$$  \hspace{1cm} (3.4)$$

In all that follows, we assume that one of the two scattering types, let us say $a$, has a small cross section, so we can drop $\sigma_a$ from the exponent. We therefore have, for the exactly
two-scatter cross section
\[ \sigma_{ab;2} = \sigma_a \sigma_b \int d^2 b \left( A(\vec{b}) \right)^2 \exp \left\{ -\sigma_b A(\vec{b}) \right\} , \quad (3.5) \]
and hence
\[ \sigma_{\text{eff},CDF} = \frac{1}{\int d^2 b \left( A(\vec{b}) \right)^2 \exp \left\{ -\sigma_b A(\vec{b}) \right\}} . \quad (3.6) \]

If, instead, we define \( \sigma_{\text{eff}} \) through the inclusive two-scatter cross section
\[ \sigma_{ab} = \sum_{n} (n - 1) \sigma_{ab;n} = \sigma_a \sigma_b \int d^2 b \left( A(\vec{b}) \right)^2 , \quad (3.7) \]
we obtain
\[ \sigma_{\text{eff}} = \frac{1}{\int d^2 b \left( A(\vec{b}) \right)^2} , \quad (3.8) \]
independent of the cross sections for the individual processes selected.

It is clear that if the cross section for process \( b \) is small, the two definitions, Eqs. (3.6) and (3.8), become identical. Keeping the first correction to this, we obtain
\[ \sigma_{\text{eff},CDF} \approx \sigma_{\text{eff}} + \mathcal{R} \sigma_b , \quad (3.9) \]
where
\[ \mathcal{R} = \frac{\int d^2 b \left( A(\vec{b}) \right)^3}{\left[ \int d^2 b \left( A(\vec{b}) \right)^2 \right]^2} \quad (3.10) \]
is a function only of the shape, but not the size, of the overlap function. For example, for a Gaussian matter distribution we have \( \mathcal{R} = 1.33 \), for a matter distribution based on the electromagnetic form factor, as used in Refs.[10, 15], we have \( \mathcal{R} = 1.46 \); for an exponential matter distribution, we have \( \mathcal{R} = 1.78 \) and for a ‘black disk’ we have \( \mathcal{R} = 1.26 \) (we mention these numbers for illustration, but they are not needed for our extraction).

Treleani’s idea is to obtain \( \mathcal{R} \sigma_b \) term from the rate of 3-scatter events,
\[ \sigma_{ab;3} = \int d^2 b \left( \sigma_a A(\vec{b}) \right) \frac{1}{2!} \left( \sigma_b A(\vec{b}) \right)^2 \exp \left\{ -\sigma_b A(\vec{b}) \right\} . \quad (3.11) \]
With the same accuracy as we have just used to obtain Eq. (3.9), we can replace the exponential by unity and obtain
\[ \sigma_{ab;3} \approx \frac{1}{2!} \sigma_a \sigma_b^2 \int d^2 b \left( A(\vec{b}) \right)^3 \approx \frac{1}{2!} \sigma_{ab;2} \frac{\mathcal{R} \sigma_b}{\sigma_{\text{eff}}} . \quad (3.12) \]
By insertion of \( \mathcal{R} \) from (3.12) into (3.9), we completely reproduce the correction suggested in [24]:
\[ \sigma_{\text{eff},CDF} \approx \sigma_{\text{eff}} \left( 1 + 2 \frac{\sigma_{ab;3}}{\sigma_{ab;2}} \right) . \quad (3.13) \]
4 Jet level correction

We turn now to a calculation of the same quantities, with the same accuracy, but at the jet level using the event definition of CDF. By jet level, we mean that we take account of the subsequent evolution of the parton-level event into a multi-parton system and thence into the hadrons that are reconstructed as jets in the detector. These steps smear the results considerably, due to the emission of extra jets into the event, the recoil of the primary jets from them, the smearing of jet momenta by hadronization effects, the loss of hadronic energy out of the jet, the merging of nearby jets, etc. For jet physics with a threshold of 5 GeV, all of these effects are large. Not only are the jets smeared, but it becomes impossible to match them up and know which jets came from which scatters.

Returning to Eq. (2.1), i.e. including the fact that the observed three jets can come from two scatters in one of two ways, we obtain

$$\sigma_{\gamma+3\text{jets};2} = \int d^2b \left( \sigma_{\gamma+1\text{jet}} A(\vec{b}) \right) \left( \sigma_{2\text{jets}} A(\vec{b}) \right) \exp\left\{ -\sigma_{\text{jets}} A(\vec{b}) \right\}. \quad (4.1)$$

Note that the cross section in the exponent is a third type of scattering: in both separations of events, we veto all scatters with any jets above 5 GeV. Therefore $\sigma_{\text{jets}}$ is the cross section for a scatter to produce at least one jet with $E_T > 5$ GeV. With the same accuracy as the argumentation made above, we therefore have

$$\sigma_{\text{eff},\text{CDF}} \approx \sigma_{\text{eff}} + R\sigma_{\text{jets}}. \quad (4.2)$$

On the other hand, the three-scatter contribution to this cross section is given by

$$\sigma_{\gamma+3\text{jets};3} = \frac{1}{2!} \int d^2b \left( \sigma_{\gamma+1\text{jet}} A(\vec{b}) \right) \left( \sigma_{1\text{jet}} A(\vec{b}) \right) \left( \sigma_{1\text{jet}} A(\vec{b}) \right) \exp\left\{ -\sigma_{\text{jets}} A(\vec{b}) \right\}. \quad (4.3)$$

That is, we continue to neglect the $\gamma+0$ jets term, so three jets from three scatters can only come about from each of the extra scatters contributing exactly one jet. Approximating this in the same way as above and putting everything together we obtain

$$\sigma_{\text{eff},\text{CDF}} \approx \sigma_{\text{eff}} + 2 \sigma_{\text{jets}} \left( \frac{\sigma_{2\text{jets}}}{\sigma_{1\text{jet}}} + \frac{\sigma_{\gamma+2\text{jets}}}{\sigma_{\gamma+1\text{jet}}} \right) \left( \frac{\sigma_{\text{jets}}}{\sigma_{1\text{jet}}} \right) \quad (4.4)$$

$$= \sigma_{\text{eff}} \left( 1 + 2 \frac{\sigma_{\gamma+3\text{jets};3}}{\sigma_{\gamma+3\text{jets};2}} \frac{\sigma_{2\text{jets}}}{\sigma_{1\text{jet}}} + \frac{\sigma_{\gamma+2\text{jets}}}{\sigma_{\gamma+1\text{jet}}} \right) \left( \frac{\sigma_{\text{jets}}}{\sigma_{1\text{jet}}} \right) \quad (4.5)$$

$$= \sigma_{\text{eff}} \left( 1 + 2 \frac{\sigma_{\gamma+3\text{jets};3}}{\sigma_{\gamma+3\text{jets};2}} f \right), \quad (4.6)$$

where in the last equation we define the correction factor

$$f = \left( \frac{\sigma_{2\text{jets}}}{\sigma_{1\text{jet}}} + \frac{\sigma_{\gamma+2\text{jets}}}{\sigma_{\gamma+1\text{jet}}} \right) \frac{\sigma_{\text{jets}}}{\sigma_{1\text{jet}}}. \quad (4.7)$$

Note that at the parton level, in which scatters can be identified and counted perfectly and jets are replaced by partons, the correction factor $f$ is equal to unity since each of the two
new ratios that have appeared in Eq. (4.4) is equal to one, since in this case \( \sigma_{1\text{jet}} \), \( \sigma_{2\text{jets}} \) and \( \sigma_{\text{jets}} \) are all equal and \( \sigma_{\gamma+2\text{jets}} \) is negligible. In general, however, the first new factor can be expected to be smaller than unity and the second one larger than unity so it is not clear the direction of the overall effect.

We do not believe it is possible to extract these ratios from numbers in the CDF paper alone. Nevertheless, since they are ratios of closely-related cross sections, we may hope that they are considerably better predicted than the cross sections themselves. In the next section we use Monte Carlo data to extract the correction factor and make an estimate of its uncertainty.

5 Estimate of the jet correction factor \( f \)

Before proceeding to the numerical estimate, we comment on the accuracy required. Since the ratio (1.3) has an uncertainty of \( \sim \frac{+30}{-50} \% \), an uncertainty of the correction factor \( f \) that multiplies it of order 20% or less would be ample, leaving the result for the effective cross section dominated by the experimental uncertainties. It would be unreasonable to aim for a significantly higher accuracy than this, since \( \alpha_S \langle 5 \text{ GeV} \rangle \sim 0.2 \) and this analysis relies on the leading order Monte Carlo generators.

In order to calculate the correction factor \( f \) and more reliably estimate its accuracy, we used three different Monte Carlo event generators \texttt{Herwig++} 2.5.2 [26, 27], \texttt{fHerwig} 6.510 [28, 29] and \texttt{Pythia} 6.4.26 [30] to produce events of both \( \gamma + \text{jets} \) and pure QCD jets types. Each generated event was handed over to the Rivet package [31] to be analyzed. This ensured that the computation of observables is exactly the same for each generator. Since the CDF analysis was not available among the standard Rivet’s analyses we have implemented its event selection criteria (see Section 2) into the package. For jet finding, we used the default settings of \texttt{CDFJetClu} jet algorithm as implemented in \texttt{FastJet} 2.4.2 [32, 33] with a cone radius \( R = 0.7 \), which according to [25] was used in the original CDF analysis.

In the CDF analysis, jet cuts depend on the order of jet in the event with respect to its transverse energy. The fact that the leading jet could be above or below the 7 GeV, while the two trailing jets have to be below this threshold, induces an additional correlation between the two scatters, in principle. The sum over the two divisions of the jet origin between the two scatters should be extended to include a sum over all assignments of the leading jet between the two scatters, and whether it is above or below 7 GeV. However, the part of the cross section coming from events in which the leading jet comes from the QCD scattering and not from the photon production was found to be tiny, about 1.5%. Moreover, it is about the same fraction in the numerator and the denominator for the appropriate ratio, so neglecting these events really has a negligible effect. Therefore, the highest \( E_T \) jet in \( \gamma + \text{jets} \) is required to be above 5 GeV and all other jets to be between 5 GeV and 7 GeV.

Since we want to make predictions for the final state of a single scattering, we switch off all MPI effects, but leave all other generator options on and at their default values. As we have emphasised, jets of such low transverse momentum are strongly smeared and only weakly correlated with the partonic scattering that produced them. Correspondingly, there is a significant probability that jets that enter our acceptance may be initiated by partonic
scatterings with transverse momentum well below that acceptance. In order to estimate the amount of this migration, which also gives an idea of the contamination of the jet sample by very soft physics, we divide the calculated cross sections into two separate parts, which we call Soft and Hard, according to the matrix element $\hat{p}_t$. For the jets ($\gamma + \text{jets}$, respectively) production cross sections the Hard part is defined with $\hat{p}_t > 2 \text{ GeV}$ ($\hat{p}_t > 10 \text{ GeV}$) and the Soft$^3$ with $0.5 < \hat{p}_t < 2 \text{ GeV}$ ($5.0 < \hat{p}_t < 10 \text{ GeV}$).

The obtained cross sections, their ratios and values of the correction factor for each generator are presented in Table 1$^4$. The first observation is that the $\gamma + \text{jets}$ sample for $E_{T\gamma} > 16 \text{ GeV}$ is well behaved: there is almost no migration from hard processes below 10 GeV. However, in the case of the QCD scattering cross sections we see a significant amount of migration from below 2 GeV. We conclude that these events are somewhat less likely to be well modeled, which is also reflected by significantly different cross sections obtained from different generators. Nevertheless, the amount of migration is similar in each of these cross sections, therefore the mis-modeling cancels to some extent in their ratios and hence the correction factor is reasonably predicted. This is particularly evident when comparing the correction factors obtained from Herwig++ and Pythia 6. Despite significant differences in the cross sections, the values of the correction factors are very close. The reason why this coefficient is different in the case of fHerwig will be explained later in this section.

| $\sigma$ [mb] | Herwig++ | fHerwig | Pythia 6 |
|---------------|-----------|---------|---------|
| $\sigma_{1\text{jet}}$ | 9.16 | 3.16 | 12.32 |
| $\sigma_{2\text{jets}}$ | 0.62 | 0.15 | 0.77 |
| $\sigma_{\text{jets}}$ | 13.87 | 3.70 | 17.57 |
| $\sigma$ [nb] | | | |
| $\sigma_{\gamma+1\text{jet}}$ | 5.66 | 0.03 | 5.69 |
| $\sigma_{\gamma+2\text{jets}}$ | 1.46 | 0.01 | 1.47 |
| $\sigma_{2\text{jets}}$ | 0.063 | 0.103 | 0.076 |
| $\sigma_{1\text{jet}}$ | 1.426 | 1.426 | 1.383 |
| $\sigma_{\text{jets}}$ | 0.258 | 0.300 | 0.246 |
| $\sigma_{\gamma+1\text{jet}}$ | 0.458 | 0.575 | 0.445 |
| $f$ | 0.493 | | |

Table 1. The calculated cross sections, their ratios and values of the correction factor for the default settings of each generator Herwig++, fHerwig, and Pythia 6.

We have already begun to address the question of how the results depend on the Monte Carlo modeling using three different event generators with their default settings. In addition, we use the Herwig++ generator with the default settings altered in order to determine how the details of the simulation affect the results. More details are given in the

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$^3$It is worth noting that default setting of Pythia 6 does not produce any events with $\hat{p}_t < 1.0 \text{ GeV}$.

$^4$The statistical errors are negligible, therefore we suppress them in the Table.
appendices. The most important effects for our studies are the order of $\alpha_S$ (1-loop and 2-loops) used in the simulation, width of the Gaussian distribution of the intrinsic transverse momentum $k_T$ of the interacting partons (we studied three values 0, 1 and 2 GeV) and the parton distribution functions. In addition, since CDF did its studies at calorimeter level and we do it at particle level, we believe that there is value in using a different jet algorithm (CDFJetClu, PxCone\cite{34} and Anti-$k_T$\cite{35} as implemented in FastJet 2.4.2), to see how dependent we are on these fine details. In Table 2 we present results obtained using three different PDF sets (MRST98\cite{36}, CTEQ6L1\cite{37} and MRST LO**\cite{38}) and two different orders of $\alpha_S$. We can see that the impact of the PDF on the result is small. Similarly, the order of $\alpha_S$ has little effect on the $f$ factor. The jet clustering algorithms, see

| PDF    | MRST98 | CTEQ6L1 | MRST LO** | $\alpha_S$ | 1-loop | 2-loops |
|--------|--------|---------|-----------|------------|--------|---------|
| $f$    | 0.477  | 0.447   | 0.458     |            | 0.476  | 0.458   |

Table 2. The correction factors obtained using Herwig++ with three different PDF sets MRST98, CTEQ6L1 and MRST LO** (default in Herwig++) and two different orders of $\alpha_S$, 1-loop and 2-loops (default in Herwig++).

Table 3, have slightly bigger influence on the outcome but still smaller then the uncertainty coming from the different Monte Carlo models, see Table 1.

| Jet algorithm | CDFJetClu | PxCone | Anti-$k_T$ |
|---------------|-----------|--------|------------|
| $f$           | 0.458     | 0.512  | 0.525      |

Table 3. The correction factors obtained using Herwig++ with three different jet algorithms PxCone, Anti-$k_T$ and CDFJetClu (used in the CDF analysis).

By far, the dominant effect is due to the intrinsic $k_T$ modeling, therefore we have studied its influence in more detail using all three generators. The results from Table 4 explain why the results obtained using default settings of Pythia 6 and Herwig++ are very similar (see Table 1). This is because the intrinsic momentum in both generators was tuned to experimental data and have by default similar value $k_T \sim 2$ GeV, while in fHerwig it was not tuned to the data and by default is equal to 0 GeV. Therefore, in this respect results from Herwig++ and Pythia 6 should be trusted more then from fHerwig. We also see (Table 4) that all generators provide a similar value of $f$ for the same $k_T$ value.

| $f$ | $k_T = 0.0$ GeV | $k_T = 1.0$ GeV | $k_T = 2.0$ GeV |
|-----|----------------|----------------|----------------|
| Herwig++ | 0.648 | 0.582 | 0.465 |
| fHerwig | 0.575 | 0.619 | 0.564 |
| Pythia 6 | 0.620 | 0.590 | 0.445 |

Table 4. The correction factors obtained using three generators and three different values of intrinsic $k_T$.

\footnote{The minimal transverse energy threshold was changed from the default value of 0.5 GeV to 0.1 GeV.}
The more detailed results including the cross sections and their ratios are included in Appendix A.

As an estimate of the correction factor we take the average value of $f$ obtained from the three different event generators with their default settings (see Table 1). The systematic error of the estimation is taken as a half of the difference between the maximum and minimum value of $f$ caused by the effects studied in this section. Therefore, the final result is

$$f_{\text{avg.}} = 0.49 \pm 0.10 .$$

(5.1)

Which indeed, as we anticipated from the estimation of $\alpha_s(5 \text{ GeV})$, is around 20% of the correction factor.

As a final cross-check, we calculate the fraction of $\gamma + 3\text{jets}$ events that come from a $\gamma + 2\text{jets}$ collision plus a 1jet collision for which the experimental value was quoted as $\approx 75\%$[19]. We obtain 80% in Herwig++, 74% in fHerwig and 76% in Pythia 6. Taking into account the inherent uncertainties in jet physics at 5 GeV, the fact that we are working at particle level and they work at uncorrected detector level, and the accuracy we are aiming for in the final correction factor, we consider this to be very good agreement with the experimental number.

6 Result

Using the result of the previous section and the numbers in CDF’s paper, we are ready to extract a value for $\sigma_{\text{eff}}$. However, we first make one final comment, concerning the systematic errors. In CDF’s analysis, the correction for triple-scattering events makes one of the biggest single contributions to the final systematic error, since they subtract the roughly estimated number of triple-scattering events off the number of double-scattering events. We revise CDF’s method and define a new effective cross section, $\bar{\sigma}_{\text{eff},\text{CDF}}$, which is the value of $\sigma_{\text{eff}}$ they would have obtained by keeping triple-scattering events in their sample and thus we avoid this source of the systematic error. Their master formula is

$$\sigma_{\text{eff},\text{CDF}} = \frac{N_D I}{N_D P} \left( \frac{A_{D P}}{A_{D I}} \right) R_c \sigma_{N S D},$$

(6.1)

with

$$N_D I = 1060 \pm 110 \pm 110,$$

(6.2)

$$N_{D P+T P} = 8865 \pm 430 \pm 150,$$

(6.3)

$$R_{D P} = 0.83 \pm 0.08 \pm 0.08 ,$$

(6.4)

$$N_{D P} = N_{D P+T P} \times R_{D P} = 7360 \pm 360_{-380}^{+720},$$

(6.5)

$$A_{D P}/A_{D I} = 0.958 \pm 0 \pm 0,$$

(6.6)

$$R_c = 2.06 \pm 0.02^{+0.01}_{-0.13} ,$$

(6.7)

$$\sigma_{N S D} = (50.9 \pm 0 \pm 1.5) \text{ mb} ,$$

(6.8)

where we have consistently written the first error as statistical and the second as systematic, even when they are assumed to be zero. $N_{D I}$ stands for the number of double hadron
interactions (DI) identified using the vertex detector, \( N_{DP} \) is the number of pure exclusive double parton scattering (DP) events, whose fraction within all measured multiple-scattering events \( (N_{DP+TP}) \) is \( R_{DP} \). Factors \( A_{DP} \) and \( A_{DI} \) characterize the acceptances of the appropriate kinematic selections, except the requirements connected to vertex reconstruction. Coefficient \( R_c \) represents the ratio between the number of beam crossings in which the detector found only one vertex and the number of beam crossings in which the detector reconstructed exactly two vertices. All beam crossings are taken into consideration in which any kind of non-single-diffractive (NSD) inelastic proton–antiproton collision was detected. The appropriate cross section \( \sigma_{NSD} \) is also given above.

Our new effective cross section has the same definition as in (6.1) but with \( N_{DP} \) replaced by \( N_{DP+TP} \),

\[
\sigma_{\text{EFF},CDF} = \frac{N_{DI}}{N_{DP+TP}} \left( \frac{A_{DP}}{A_{DI}} \right) R_c \sigma_{NSD} = (12.0 \pm 1.4^{+1.3}_{-1.5}) \text{ mb}. \tag{6.9}
\]

Note that the fractional systematic errors are indeed significantly smaller.

Finally, we are ready to calculate \( \sigma_{\text{eff}} \). In terms of \( \sigma_{\text{EFF},CDF} \) it is given by

\[
\sigma_{\text{eff},CDF} = \sigma_{\text{eff}} \left( 1 + \frac{\sigma_{\gamma+3\text{jets};3}}{\sigma_{\gamma+3\text{jets};2} + \sigma_{\gamma+3\text{jets};3}} [2f_{\text{avg}} - 1] \right). \tag{6.10}
\]

Given that correction factor \( f_{\text{avg}} \) is 0.49 \pm 0.10, the factor in square brackets turns out to be very close to zero indicating that the difference between the inclusive measurement and the true inclusive cross section for double parton scattering is very small. The uncertainty on the triple-scattering event fraction 0.17\(^{+0.04}_{-0.08}\) can be neglected with respect to the uncertainty of the \( 2f_{\text{avg}} - 1 \) term. Our final result for the effective cross section is

\[
\sigma_{\text{eff}} = (12.0 \pm 1.4^{+1.3}_{-1.5}) \text{ mb}. \tag{6.11}
\]

We see that despite the additional uncertainty coming from the correction factor, the systematic uncertainty is smaller and more symmetrical than on the CDF’s final value.

7 Conclusions

The CDF measurement of double parton scattering [19] used a non-standard definition of \( \sigma_{\text{eff}} \) that makes this important quantity process dependent. Therefore, the value provided by the experiment \( \sigma_{\text{eff,CDF}} = (14.5 \pm 1.7^{+1.7}_{-2.3}) \text{ mb} \) is not suitable for comparisons with other measurements or as input for theoretical calculations or Monte Carlo models. The non-standard definition used by CDF has been pointed out and corrected for the first time by Treleani in his publication [24]. Based on the theoretically-pure (parton-level) situation, [24] estimated the inclusive (process independent) \( \sigma_{\text{eff}} = 10.3 \text{ mb} \). This result would be correct under the assumption that CDF were able to uniquely identify and count the number of scatters in an event, which is certainly not the case. In this publication we have considered CDF’s event definition in more detail to provide an improved correction leading to

\[
\sigma_{\text{eff}} = (12.0 \pm 1.4^{+1.3}_{-1.5}) \text{ mb}. \tag{7.1}
\]
It is worth noting that both statistical and systematic uncertainties have decreased, since the additional uncertainty of our correction factor is much smaller than the avoided uncertainty stemming from the triple scattering removal done originally by CDF.

The obtained value of $\sigma_{\text{eff}}$ serves as a constraint on the Monte Carlo models since the recent tunes of MPI models to the LHC data predict its value to be between $25 - 42 \text{ mb}$ [39]. This inconsistency between theory and experiment can also be seen from Eq. 4.2, if one uses the $\sigma_{\text{jets}}$ around $15 \text{ mb}$, see Table 1, and $R$ from Section 3. This behavior suggests that the overlap function used in the MC models is oversimplified and should be improved, for example, by including $x$-dependence [40–42]. This value can also help to understand the recent results from the LHCb experiment [20](see page 23, Fig 10). The experimental results for $\sigma_{\text{eff}}$ extracted from the production of $J/\psi$ mesons together with an associated open charm hadron and from double open charm hadron production are different by a factor of between two and three.

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## A Detailed Results

### A.1 Intrinsic $k_T$ dependence

|                | Herwig++ | fHerwig | Pythia 6 |
|----------------|----------|---------|----------|
|                | Hard     | Soft    | Hard     | Soft    | Hard     | Soft    |
| $\sigma_{1jet}$ | 5.13     | 1.40    | 5.33     | 6.61    | 4.79     | 0.06    |
| $\sigma_{2jets}$| 0.65     | 0.26    | 0.54     | 0.70    | 0.66     | 0.00    |
| $\sigma_{jets}$ | 8.67     | 2.05    | 8.72     | 8.31    | 8.04     | 0.06    |
| $\sigma$ [nb]  |          |         |          |         |          |         |
| $\sigma_{\gamma+1jet}$ | 5.38    | 0.06    | 3.41     | 0.16    | 4.46     | 0.08    |
| $\sigma_{\gamma+2jets}$ | 1.39    | 0.01    | 1.02     | 0.04    | 0.92     | 0.15    |
| $f$             | 0.139    | 0.103   | 1.641    | 1.426   | 1.668    |         |
| $\frac{\sigma_{2jets}}{\sigma_{1jet}}$ | 0.136    |         |         |         |          |         |
| $\frac{\sigma_{jets}}{\sigma_{1jet}}$ | 1.668    |         |         |         |          |         |
| $\frac{\sigma_{\gamma+2jets}}{\sigma_{\gamma+1jet}}$ | 0.236    |         |         |         |          |         |

Table 5. The calculated cross sections, their ratios and the final correction factors for the intrinsic $k_T$ RMS = 0.0 GeV for three MC generators. CDFJetClu jet algorithm was used.

|                | Herwig++ | fHerwig | Pythia 6 |
|----------------|----------|---------|----------|
|                | Hard     | Soft    | Hard     | Soft    | Hard     | Soft    |
| $\sigma_{1jet}$ | 5.72     | 0.86    | 5.60     | 6.96    | 4.78     | 0.10    |
| $\sigma_{2jets}$| 0.64     | 0.08    | 0.54     | 0.72    | 0.75     | 0.00    |
| $\sigma_{jets}$ | 9.37     | 1.11    | 9.01     | 8.88    | 8.01     | 0.10    |
| $\sigma$ [nb]  |          |         |          |         |          |         |
| $\sigma_{\gamma+1jet}$ | 5.50    | 0.09    | 3.46     | 0.11    | 4.48     | 0.08    |
| $\sigma_{\gamma+2jets}$ | 1.43    | 0.01    | 1.13     | 0.06    | 0.92     | 0.07    |
| $f$             | 0.109    | 0.100   | 1.591    | 1.424   | 1.668    |         |
| $\frac{\sigma_{2jets}}{\sigma_{1jet}}$ | 0.136    |         |         |         |          |         |
| $\frac{\sigma_{jets}}{\sigma_{1jet}}$ | 1.668    |         |         |         |          |         |
| $\frac{\sigma_{\gamma+2jets}}{\sigma_{\gamma+1jet}}$ | 0.218    |         |         |         |          |         |

Table 6. The calculated cross sections, their ratios and the final correction factors for the intrinsic $k_T$ RMS = 1.0 GeV for three MC generators. CDFJetClu jet algorithm was used.
|       | Herwig++ | fHerwig | Pythia 6 |
|-------|----------|---------|----------|
| $\sigma$ [mb] | Hard | Soft | Hard | Soft | Hard | Soft |
| $\sigma_{1\text{jet}}$ | 9.73 | 3.79 | 6.23 | 10.74 | 6.93 | 2.51 |
| $\sigma_{2\text{jets}}$ | 0.65 | 0.13 | 0.72 | 0.81 | 0.72 | 0.00 |
| $\sigma_{\text{jets}}$ | 14.73 | 4.38 | 10.87 | 12.99 | 10.54 | 2.52 |
| $\sigma$ [nb] |       |       |       |       |       |       |
| $\sigma_{\gamma+1\text{jet}}$ | 5.51 | 0.05 | 3.43 | 0.17 | 4.47 | 0.08 |
| $\sigma_{\gamma+2\text{jets}}$ | 1.48 | 0.03 | 1.08 | 0.05 | 1.05 | 0.07 |
| $\frac{\sigma_{2\text{jets}}}{\sigma_{1\text{jet}}}$ | 0.058 | 0.090 |       |       | 0.076 |       |
| $\frac{\sigma_{2\text{jets}}}{\sigma_{\text{jets}}}$ | 1.414 | 1.404 |       |       | 1.383 |       |
| $\frac{\sigma_{\gamma+2\text{jets}}}{\sigma_{\gamma+1\text{jet}}}$ | 0.271 | 0.312 |       |       | 0.246 |       |
| $f$ | 0.465 | 0.564 |       |       | 0.445 |       |

Table 7. The calculated cross sections, their ratios and the final correction factors for the intrinsic $k_T$ RMS $= 2.0$ GeV for three MC generators. CDFJetClu jet algorithm was used.

### A.2 Jet algorithm dependence

|       | CDFJetClu | PxCone | Anti-$k_T$ |
|-------|-----------|--------|------------|
| $\sigma$ [mb] | Hard | Soft | Hard | Soft | Hard | Soft |
| $\sigma_{1\text{jet}}$ | 9.16 | 3.16 | 10.90 | 3.91 | 8.89 | 3.21 |
| $\sigma_{2\text{jets}}$ | 0.62 | 0.15 | 0.88 | 0.24 | 0.70 | 0.24 |
| $\sigma_{\text{jets}}$ | 13.87 | 3.70 | 16.02 | 4.56 | 13.38 | 3.98 |
| $\sigma$ [nb] |       |       |       |       |       |       |
| $\sigma_{\gamma+1\text{jet}}$ | 5.66 | 0.03 | 5.28 | 0.04 | 2.89 | 0.01 |
| $\sigma_{\gamma+2\text{jets}}$ | 1.46 | 0.01 | 1.53 | 0.02 | 0.83 | 0.00 |
| $\frac{\sigma_{2\text{jets}}}{\sigma_{1\text{jet}}}$ | 0.063 | 0.076 |       |       | 0.078 |       |
| $\frac{\sigma_{2\text{jets}}}{\sigma_{\text{jets}}}$ | 1.426 | 1.390 |       |       | 1.434 |       |
| $\frac{\sigma_{\gamma+2\text{jets}}}{\sigma_{\gamma+1\text{jet}}}$ | 0.258 | 0.292 |       |       | 0.288 |       |
| $f$ | 0.458 | 0.512 |       |       | 0.525 |       |

Table 8. The calculated cross sections, their ratios and the final correction factors in dependence on the jet clustering algorithm using Herwig++ generator.
| | CDFJetClu | PxCone | Anti-\(k_T\) |
|---|---|---|---|
| \(\sigma\) [mb] | Hard | Soft | Hard | Soft | Hard | Soft |
| \(\sigma_{1\text{jet}}\) | 5.33 | 6.61 | 6.18 | 8.30 | 5.73 | 9.58 |
| \(\sigma_{2\text{jets}}\) | 0.54 | 0.70 | 0.78 | 1.25 | 0.71 | 1.96 |
| \(\sigma_{\text{jets}}\) | 8.72 | 8.31 | 9.85 | 10.72 | 9.27 | 13.84 |
| \(\sigma\) [nb] | | | | | | |
| \(\sigma_{\gamma+1\text{jet}}\) | 3.41 | 0.16 | 3.07 | 0.11 | 1.64 | 0.02 |
| \(\sigma_{\gamma+2\text{jets}}\) | 1.02 | 0.05 | 1.05 | 0.06 | 0.45 | 0.03 |
| \(\frac{\sigma_{2\text{jets}}}{\sigma_{1\text{jet}}}\) | 0.103 | 0.140 | | | | |
| \(\frac{\sigma_{\text{jets}}}{\sigma_{\gamma+1\text{jet}}}\) | 1.426 | 1.421 | 1.51 | 1.60 | | |
| \(\frac{\sigma_{\gamma+2\text{jets}}}{\sigma_{\gamma+1\text{jet}}}\) | 0.300 | 0.348 | 0.291 | | | |
| \(f\) | 0.575 | 0.693 | 0.704 | | | |

Table 9. The calculated cross sections, their ratios and the final correction factors in dependence on the jet clustering algorithm using \(f\)Herwig generator.

| | CDFJetClu | PxCone | Anti-\(k_T\) |
|---|---|---|---|
| \(\sigma\) [mb] | Hard | Soft | Hard | Soft | Hard | Soft |
| \(\sigma_{1\text{jet}}\) | 9.62 | 2.88 | 11.41 | 3.98 | 8.84 | 2.50 |
| \(\sigma_{2\text{jets}}\) | 1.17 | 0.00 | 1.51 | 0.00 | 1.16 | 0.00 |
| \(\sigma_{\text{jets}}\) | 15.04 | 2.92 | 17.20 | 4.02 | 13.81 | 2.55 |
| \(\sigma\) [nb] | | | | | | |
| \(\sigma_{\gamma+1\text{jet}}\) | 4.47 | 0.08 | 4.21 | 0.07 | 1.90 | 0.02 |
| \(\sigma_{\gamma+2\text{jets}}\) | 1.05 | 0.07 | 1.04 | 0.07 | 0.45 | 0.07 |
| \(\frac{\sigma_{2\text{jets}}}{\sigma_{1\text{jet}}}\) | 0.076 | 0.085 | 0.075 | | | |
| \(\frac{\sigma_{\text{jets}}}{\sigma_{\gamma+1\text{jet}}}\) | 1.383 | 1.338 | 1.378 | | | |
| \(\frac{\sigma_{\gamma+2\text{jets}}}{\sigma_{\gamma+1\text{jet}}}\) | 0.246 | 0.258 | 0.270 | | | |
| \(f\) | 0.445 | 0.459 | 0.475 | | | |

Table 10. The calculated cross sections, their ratios and the final correction factors in dependence on the jet clustering algorithm using Pythia 6 generator.
## A.3 PDF dependence

| σ [mb] | MRST98 | CTEQ6L1 | MRST LO** |
|--------|--------|---------|-----------|
|        | Hard   | Soft    | Hard      | Soft      | Hard    | Soft     |
| σ_{1jet} | 5.54   | 1.71    | 6.39      | 1.73      | 9.16    | 3.16     |
| σ_{2jets} | 0.36   | 0.07    | 0.41      | 0.07      | 0.62    | 0.15     |
| σ_{jets}  | 8.46   | 1.98    | 9.59      | 1.98      | 13.87   | 3.70     |

| σ [nb]   |         |         |           |           |
|----------|---------|---------|-----------|
| σ_{γ+1jet} | 3.77    | 0.06    | 4.21      | 0.04      | 5.66    | 0.03     |
| σ_{γ+2jets} | 1.00    | 0.04    | 1.06      | 0.03      | 1.46    | 0.01     |

| $\frac{σ_{2jets}}{σ_{1jet}}$ | 0.060   | 0.059   | 0.063     |
| $\frac{σ_{1jets}}{σ_{1jet}}$ | 1.440   | 1.423   | 1.426     |
| $\frac{σ_{γ+2jets}}{σ_{γ+1jet}}$ | 0.271   | 0.255   | 0.258     |
| $f$ | 0.477   | 0.447   | 0.458     |

**Table 11.** The calculated cross sections, their ratios and the final correction factors in dependence on the parton distribution function used in Herwig++ generator.

| σ [mb] | CTEQ5L | MRST LO** |
|--------|--------|-----------|
|        | Hard   | Soft    | Hard      | Soft     |
| σ_{1jet} | 9.62   | 2.88    | 9.62      | 2.88     |
| σ_{2jets} | 1.17   | 0.00    | 1.17      | 0.00     |
| σ_{jets}  | 15.04  | 2.92    | 15.04     | 2.92     |

| σ [nb]   |         |         |           |           |
|----------|---------|---------|-----------|
| σ_{γ+1jet} | 4.47    | 0.08    | 6.32      | 0.09     |
| σ_{γ+2jets} | 1.05    | 0.07    | 1.29      | 0.19     |

| $\frac{σ_{2jets}}{σ_{1jet}}$ | 0.076   | 0.094   |
| $\frac{σ_{1jets}}{σ_{1jet}}$ | 1.383   | 1.436   |
| $\frac{σ_{γ+2jets}}{σ_{γ+1jet}}$ | 0.246   | 0.232   |
| $f$ | 0.445   | 0.468   |

**Table 12.** The calculated cross sections, their ratios and the final correction factors in dependence on the parton distribution function used in Pythia 6 generator.
A.4 Order of $\alpha_S$ dependence

|       | 1-loop $\alpha_S$ | 2-loops $\alpha_S$ |
|-------|-------------------|--------------------|
| $\sigma$ [mb] | Hard | Soft | Hard | Soft |
| $\sigma_{1\text{jet}}$ | 13.99 | 5.23 | 9.16 | 3.16 |
| $\sigma_{2\text{jets}}$ | 0.95 | 0.24 | 0.62 | 0.15 |
| $\sigma_{\text{jets}}$ | 20.96 | 6.19 | 13.87 | 3.70 |

| $\sigma$ [nb] |  |  |
| $\sigma_{\gamma+1\text{jet}}$ | 6.53 | 0.05 | 5.66 | 0.03 |
| $\sigma_{\gamma+2\text{jets}}$ | 1.77 | 0.04 | 1.46 | 0.01 |

Table 13. The calculated cross sections, their ratios and the final correction factors in dependence on the order of $\alpha_S$ used in Herwig++ generator.

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