Calculation of shear stresses in the soil of the subgrade using empirical plasticity conditions

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Abstract. The aim of the work is to replace the criterion of the original Mohr – Coulomb criterion with an empirical plasticity condition in which the limit state arises at deformation \( \varepsilon_{\text{lim}} \leq 15 \% \). As a result of the analysis of experimental data on testing soils by triaxial compression, an empirical plasticity criterion is obtained. The limiting state by the empirical criterion occurs when the sample is deformed by 10–12%. The shear stresses calculated by this plasticity condition exceed the shear stresses calculated by the Mohr – Coulomb criterion. Therefore, the application of the proposed criterion in the calculation of pavement by shear resistance in the soil of the subgrade requires an increase in the thickness of the structural layers, compared with the traditional calculation, which is based on Mohr – Coulomb.

1. Introduction

When calculating pavement by shear resistance in the subgrade soil, a multilayer structure leads to a two-layer system. The top layer of the system with a thickness equal to the sum of the thicknesses of the pavement layers located above the subgrade checked for shear is characterized by an elastic modulus averaged over the thickness of the layers. The bottom layer of the system is the subgrade, characterized by the modulus of elasticity, cohesion and the angle of internal friction of the soil. The calculation sequence consists in calculating the shear stresses from the transport load, the limit shear stresses and checking the shear resistance criterion. The formulas for performing these three sequential mathematical operations are given in table 1.
# Table 1. The sequence of operations when calculating the shear in the ground.

| Name of characteristic | Formula |
|------------------------|---------|
| Shear stress from transport load, $T$ | $T = \frac{1}{\cos \varphi_N} \left( \frac{\sigma_1 - \sigma_3}{2} - \tan \varphi_N \cdot \frac{\sigma_1 + \sigma_3}{2} \right)$ |
| where $\sigma_1$ and $\sigma_3$ – maximum and minimum main stresses, Pa; $\varphi_N$ – angle of internal friction, the value of which takes into account the number of repeated loads applied (the greater $N$, the less $\varphi_N$). |
| Limit value of the shear stress, $T_{\text{lim}}$ | Preliminary National Standard Industry Road Standards |
| | $T_{\text{lim}} = k(c_N + 0.1 \gamma Z \tan \varphi_u)$ | $T_{\text{lim}} = k c_N + 0.1 \gamma Z \tan \varphi_{\text{st}}$ |
| $k$ – coefficient taking into account the stress state of the structure at the interface of the sand layer with the lower layer of the base of the pavement; $\gamma$ – weighted average specific gravity of constructive layers located above the test layer, kg/cm$^3$; $Z$ – depth location of the surface of the layer tested for shear resistance, from the top of the structure, cm; $\varphi_u$ – the value of the angle of internal friction under the static action of the load, which is taken as a single impact of the load, that is $\varphi_u = \varphi_1 < \varphi_N$ (at $N > 1$); |
| Shear resistance condition | $T \leq T_{\text{lim}} / K_{\text{str}}$ |
| $K_{\text{str}}$ – strength coefficient at calculation shear, taken depending on the level of reliability |

From the analysis of the formulas of the table 1 it follows that the shear stress and its limiting value are determined through different values of the angle of internal friction. Known methods for calculating the angle of inclination ($\alpha_{\text{shear1}}$ and $\alpha_{\text{shear3}}$) of sliding surface to the main axes [1–6] can be divided into 4 methods. The calculation scheme and the basic formulas used to determine the angle of inclination of the platform of shear to the main axes are given in table 2.

The first method is the traditional method for calculating the $\alpha_{\text{shear1}}$ and $\alpha_{\text{shear3}}$ angles through the angle of internal friction [1, 2]. In this case, to each of the many values of the angle of internal friction corresponds to its own angles of inclination of sliding surface to the main axes. Along the main axes, the main stresses act, they are applied to the main platforms located perpendicular to the lines of action $\sigma_1$ and $\sigma_3$.

The second method is based on determining the angle of inclination of sliding surface to the main axes through the dilatancy angle $\psi$ [4]. Analyzing the rules for calculating the angle of dilatancy proposed by M.D. Bolton [5] and used to date [6–8], it is easy to verify that this angle can be calculated through the critical value of the angle of internal friction. In the Hardening Soil model, the dilatation angle of medium density sands is determined by the angle of internal friction minus 30 degrees. From this it follows that the inclination of platform of shear is determined by a strictly defined value of the dilatancy angle, which can be calculated using only one critical value of the angle of internal friction.
Table 2. Calculation of the angle of inclination of sliding surface to the main axes.

| Calculation scheme | Name of the method | Formulas |
|--------------------|-------------------|----------|
|                    | Using the angle of internal friction $\varphi$ [1, 2] | $\alpha_{sh1} = \frac{\pi}{4} + \frac{\varphi}{2} = 45^\circ + \frac{\varphi}{2}$ |
|                    |                    | $\alpha_{sh3} = \frac{\pi}{4} + \frac{\varphi}{2} = 45^\circ - \frac{\varphi}{2}$ |
|                    | Using the angle of dilatancy $\psi$ [4] | $\alpha_{sh1} = \frac{\pi}{4} + \frac{\psi}{2} = 45^\circ + \frac{\psi}{2}$ |
|                    |                    | $\alpha_{sh3} = \frac{\pi}{4} + \frac{\psi}{2} = 45^\circ - \frac{\psi}{2}$ |
|                    | Using an angle defined as the average between the angles $\varphi$ and $\psi$ [9] | $\alpha_{sh1} = \frac{\pi}{4} + \frac{\varphi + \psi}{2} = \frac{\pi + \varphi + \psi}{4}$ |
|                    |                    | $\alpha_{sh3} = \frac{\pi}{4} - \frac{\varphi + \psi}{2} = \frac{\pi - \varphi - \psi}{4}$ |
|                    | Using angle of inclination $\nu$ [10], varying in range $0 \leq \nu \leq \varphi$ | $\alpha_{sh1} = \frac{\pi}{4} + \frac{\nu}{2}$, $0 \leq \nu \leq \varphi$ |
|                    |                    | $\alpha_{sh3} = \frac{\pi}{4} - \frac{\nu}{2}$, $0 \leq \nu \leq \varphi$ |

The third way to determine the angle of inclination of the sliding surface is to find the angle that is the average between the angles of internal friction and dilatancy [9]. Half the magnitude of this angle characterizes the angle of inclination of the sliding surface.

The fourth method postulates some arbitrary value of the angle of inclination of the sliding surface to the main platforms. The value of this angle can take any value from zero to the critical value of the angle of internal friction [10]. But even in this case, the inclination of the sliding surface is characterized by only one angle value.

Given our analysis, we can conclude that the regulatory documents of Russia governing the verification of road structures for shear in the ground contain an error. The essence of such an error is to compare the shear stresses acting along one surface with their limiting value on another surface.

At the heart of calculation of shear stresses from the transport load is based on the Mohr–Coulomb criterion, in which the soil parameters are cohesion and the angle of internal friction. The limiting surface of this criterion is the Mohr hexagon. In the experimental determination of the cohesion and the angle of internal friction by the methods of triaxial compression, circles of limit stresses are drawn to which the limit tangent line is drawn. This straight line is the projection onto the $\tau-\sigma$ plan of the Mohr pyramid. When constructing circles of limit stresses, the value of the minimum main stress $\sigma_3$ is randomly set in the experiment, for example, $\sigma_3=50$ kPa for the first stress circle, $\sigma_3=100$ kPa for the second stress circle, and so on. The limit value of the maximum main stress $\sigma_{1\text{limit}}$ for each given $\sigma_3$ is determined as a result of the experiment under the condition that the sample is destroyed or deformed by a limit value. The values of the limiting deformation $\varepsilon_{1\text{limit}}$ take quite large (15% in Russia and 20% abroad). Of course, such a deformation is elastoplastic, with the largest part being plastic deformation. The essence of the limiting surface of any plasticity condition is that inside this surface there are all possible combinations of stress tensor components that cause elastic deformation. Thus, the hypothesis underlying the conclusion of the equation of the limit surface, suggests that inside the surface located the area elastic state of the ground and outside the surface the area of the plastic ground state. During deformation $\varepsilon_{1\text{limit}}=15$% under the limit line of Coulomb and inside the Mohr pyramid, there is an area of an elastoplastic state, rather than an elastic one, as
required by the initial idea. Therefore, when a limiting state occurs according to the Mohr – Coulomb criterion, the most dangerous point of the soil base experiences a deformation of 15%. Wherein subgrade draft may exceed the permissible values of the depths of the irregularities, regulated for coatings, and due by residual deformations of both the subgrade and layers of pavement.

This implies the need to either justify the limit deformation in determining the cohesion and the angle of internal friction in the triaxial compression experiment, or to modify the Mohr – Coulomb criterion or replace it with another criterion.

From the analysis that follows, the relevance of work designed at improving the calculation of pavement in the ground, by means of a reasonable replacement of the Mohr – Coulomb criterion with another plasticity condition.

2. Materials and methods
The Mohr – Coulomb traditional criterion can be represented in various recording forms [11–13]. In the table 3 shows the equations of limiting state according to this criterion [11–13], as well as other analytical conditions of soil plasticity [14–20], as well as the modified Mohr – Coulomb criterion with three parameters proposed by us in [21].

From the analysis of the first equation of the limiting state of the criterion Mohr – Coulomb, presented in table 3, it follows that it is part of the criterion of soil resistance of the subgrade to shear, presented in table 1. Any equation of the limiting state according to the Mohr – Coulomb criterion, from those given in table 3, contains only one value of the angle of internal friction. Therefore, the traditional Mohr – Coulomb criterion does not contain the first remark made by us to the calculation of the table 1. This criterion has been successfully used in calculating the first critical load on soils [22].

The first critical load is the pressure on the soil half-space at which a limit state on Mohr – Coulomb plasticity condition arises at the most dangerous point in the soil mass according. The conclusion of such a formula is to substitute into the equation of the limit state of formulas that allow us to calculate \( \sigma_1 \) and \( \sigma_3 \) at the most dangerous point. The formulas for calculating \( \sigma_1 \) and \( \sigma_3 \) can be represented as the product of pressure and depth functions. Next, the resulting equation is solved with respect to pressure. When solving the equation, the depth is taken equal to the distance from the surface of the half-space to the most dangerous point at which the shear stress is of greatest value.

In addition, the Mohr – Coulomb criterion is widely used in solving problems by the method of limiting equilibrium of the soil, which is used to determine the limit pressure on the soil from various loads and types of impact [25–28].
Table 3. The equations of the limiting state of the plasticity criterion.

| Name of criterion                                      | Limit state equation                                                                                     |
|--------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| 1. Criterion Mohr–Coulomb [11–13]                      | \[
\frac{1}{\cos \varphi} \cdot \sigma_1 - \sigma_3 - \tan \varphi \cdot \frac{\sigma_1 + \sigma_3}{2} = c \\
\frac{\sigma_1 - \sigma_3}{2 \cdot c \cdot \tan \varphi + \sigma_1 + \sigma_3} = \sin \varphi \\
\sigma_1 \cdot (1 - \sin \varphi) - \sigma_3 \cdot (1 + \sin \varphi) = 2 \cdot c \cdot \cos \varphi
\] |
| 2. Criterion Mohr–Coulomb in invariant form [13], inclusive Lode angle \( \Theta \) | \[
\sqrt{J_2} = \left( \frac{I_1 \cdot \sin \varphi}{3} + c \cdot \cos \varphi \right) \left( \cos \Theta - \frac{\sin \Theta \cdot \sin \varphi}{\sqrt{3}} \right)
\] |
| 3. Criterion Tresca [14], inclusive resistance to undrained shear | \[
\frac{\sigma_1 - \sigma_3}{2} = c_u
\] |
| 4. Criterion Drucker–Prager [15, 16]                    | \[
\sqrt{J_2} - a \cdot I_1 = k; \ a = \frac{2 \cdot \sin \varphi}{\sqrt{3 \cdot (3 \pm \sin \varphi)}}, \ k = \frac{6 \cdot c \cdot \cos \varphi}{\sqrt{3 \cdot (3 \pm \sin \varphi)}}
\] |
| 5. Criterion Matsuoka–Nakai [17, 18]                    | \[
\frac{I_1 \cdot I_2}{I_3} = k_{M-N}; \ k_{M-N} = \frac{9 - \sin^2 \varphi}{1 - \sin^2 \varphi}
\] |
| 6. Criterion Lade–Duncan [18–20]                        | \[
\frac{I_1^3}{I_3} = k_{L-D}; \ k_{L-D} = \frac{(3 - \sin \varphi)^3}{(1 - \sin \varphi) \cdot \cos^2 \varphi}
\] |
| 7. Modified criterion Mohr–Coulomb with three parameters [21] | \[
\frac{1}{2} \left( \sigma_1 \cdot \left( 1 - \sin \varphi_{cr} \right)^d \cdot \left( 1 + \sin \varphi_{cr} \right)^d \cdot \sigma_3 \right) = c
\] |

where \( k_{M-N} \) soil strength parameter related to the angle of internal friction

where \( k_{L-D} \) soil strength parameter related to the angle of internal friction

where \( \varphi_{cr} \) – angle of internal friction established by triaxial tests at limit deformation \( \varepsilon_{\text{lim}} = 15\% \); 

\( d \) – the third parameter, the value of which varies in the range 0 ≤ d ≤ 0.5 and depends on the vertical deformation of the sample, taken at triaxial compression as the limiting value 8% < \( \varepsilon_{\text{lim}} < 12\% \)

Nevertheless, the Mohr – Coulomb criterion has a second drawback associated with a large value of limit deformation at which its material parameters are determined. In figure 1 shows the limiting surfaces of the Mohr – Coulomb criterion and the Hardening Soil model.
Figure 1. Limit surfaces: a Mohr-Coulomb criterion; b Hardening Soil model; $f_s$ and $f_c$ – functions for deviator and isotropic loading.

In figure 1a, the limiting surface of the Mohr – Coulomb criterion is given, which limits the region of permissible stress states or the zone of elastic deformation. But the value of the limit deformation in experiments on triaxial compression is assumed to be 15 or 20%. Such deformations are elastoplastic. This means that the parameters of strength of the Mohr – Coulomb criterion are determined not at elastic deformation, but with elastoplastic.

The Hardening Soil model is based on the hyperbolic dependence of the deformation $\varepsilon_1$ on the stress deviator $q=\sigma_1-\sigma_3$ during the primary triaxial application of the load and linear dependences during the unloading and its repeated application. Compared to the Mohr – Coulomb criterion, the Hardening Soil model contains more parameters. The yield surfaces of the Hardening Soil model, bounding the elastic region, do not have a fixed position in the space of principal stresses. The elastic area bounded by these yield surfaces can expand upward, approaching the Mohr – Coulomb limit surface, under shear hardening. With isotropic hardening, the elastic region increases, but not up, but to the right. Note that compared to the Mohr – Coulomb criterion, the Hardening Soil model is more accurate and applicable to more tasks. For our analysis, it is important that this model demonstrates the need for shear hardening in order for the elastic area to reach sizes limited by the limiting surface of the Mohr – Coulomb criterion.

Therefore the conclusion that the improvement of the methodology for calculating the soil of subgrade can be carried out in one of two directions:

– in substantiating the value of the limiting deformation in triaxial compression experiments, at which the cohesion and the angle of internal friction are determined;

– replacing the Mohr – Coulomb criterion with another plasticity condition with higher shear stresses, which will require an increase in pavement thickness, which will lead to a decrease in the pressure on the subgrade and the deformations it experiences.

In [21], the possibility of using other analytical dependencies such as the Drucker – Prager, Lade – Duncan, and Matsuoka – Nakai criteria instead of the Mohr – Coulomb criterion was considered. The limiting surfaces of these criteria are considered on the deviatorial plane. As a result, it was found that all intersect with the Mohr hexagon in compression angles and / or tensile angles. Thus, all the compared criteria give the same result under stress state, characterized by stresses $\sigma_1>\sigma_2=\sigma_3$. Such a stress state occurs along the axis of symmetry of the load distributed over a circular platform. This section is taken as the calculated one when assessing the resistance of soils to shear. Therefore, for this section, replacing the Mohr – Coulomb criterion with another analytical condition of plasticity is impractical, because all these criteria give the same result.

This gives rise to the idea of the possibility of replacing the Mohr – Coulomb criterion with an empirical condition in which the shear stresses are greater than the shear stress determined by the first formula in table 1.
In the table 4 shows the work of R.F. Craig [29] and G.K. Arnold [30]. Both authors give formulas for determining the limit value of the minimum main stress $\sigma_3$, at the occurrence of which the material experiences a limit state at a given value of the maximum main stress $\sigma_1$. G.K. Arnold [30], analyzing the experimental data, found that in most cases the destruction of the sample or its deformation to a limiting value occurs at a minimum main stress $\sigma_3$, the value of which is greater than the limiting value calculated from the traditional Mohr – Coulomb criterion. Therefore G.K. Arnold introduced in the fundamental formulas recommended by R.F. Craig, edits that increase the value of stress $\sigma_3$ in the zone of soil undergoing an active Rankine state. Dependencies R.F. Craig [29] and G.K. Arnold [30] are shown in table 4.

| The author of approach | Ground mass area                  | Formula for determining the value of minimum holding stress |
|------------------------|----------------------------------|----------------------------------------------------------|
| R.F. Craig [29]        | Zone of active Rankine state     | $\sigma_3 = \sigma_1 \cdot K_{act} - 2 \cdot c \cdot \sqrt{K_{act}}$; $K_{act} = \frac{1 - \sin \varphi}{1 + \sin \varphi}$ |
|                        | Zone of passive Rankine state    | $\sigma_3 = \sigma_1 \cdot K_p - 2 \cdot c \cdot \sqrt{K_p}$; $K_p = \frac{1 + \sin \varphi}{1 - \sin \varphi}$ |
|                        | Zone of elastic state            | $\sigma_3 = \sigma_1 \cdot K_0$; $K_0 = 1 - \sin \varphi$ |
| G.K. Arnold [30]       | Zone of active Rankine state     | $\sigma_3 = \sigma_1 \cdot K_{act} - 2 \cdot c \cdot K_{act}$; $K_{act} = \frac{1 - \sin \varphi}{1 + \sin \varphi}$ |
|                        | Zone of passive Rankine state    | $\sigma_3 = \sigma_1 \cdot K_p - 2 \cdot c \cdot K_p$; $K_p = \frac{1 + \sin \varphi}{1 - \sin \varphi}$ |
|                        | Zone of elastic state            | $\sigma_3 = \sigma_1 \cdot K_0$; $K_0 = 1 - \sin \varphi$ |

We solve the dependencies presented in table 4 for the zone of active Rankine state relative to cohesion. As a result, we get the limit state equations. Solution of the formula R.F. Craig [29] will lead to one of the accepted forms of the equation of the limit state of the generally accepted criterion Mohr – Coulomb, it has the form

$$\frac{1}{2} \left( \sigma_1 \cdot \frac{1 - \sin \varphi}{1 + \sin \varphi} - \sigma_3 \cdot \frac{1 + \sin \varphi}{1 - \sin \varphi} \right) = \mathcal{C} \tag{1}$$

By performing a similar solution to the equation G.K. Arnold, we obtain a new equation of the limiting state of an empirical criterion

$$\frac{1}{2} \left( \sigma_1 \cdot \frac{1 + \sin \varphi}{1 - \sin \varphi} \cdot \sigma_3 \right) = \mathcal{C} \tag{2}$$

The left sides of equations (1) and (2) allow us to calculate the shear stresses using these plasticity conditions. Moreover, the calculation results on the left side of formula (1) coincide with the results obtained by the first dependence of the table 1. Therefore, the results of calculations by the left side of formula (1) are shear stresses according to the traditional Mohr – Coulomb criterion.

### 3. Results and discussion

A visual representation about the differences in the value of shear stresses, calculated by the formulas (1) and (2) provide graphs of their changes in depth. To construct these graphs and calculate the shear stresses, it is necessary to calculate the maximum and minimum main stresses at points belonging to the axis of symmetry of the load distributed over the circular area. To calculate the main stresses, the traditional solution was used [31, 32]. All calculations were performed for the axis of symmetry of the
load, the abscissas of all points of which are equal to zero, i.e. $x = 0$. The depth of the half-space for which the calculation of the main stresses is performed is limited by the relative value $Z/D = 3$. Here $Z$ is the absolute value of the depth, and $D$ is the diameter of the circular platform, over the area of which a load of $p$ is distributed. The results of the calculation of shear stresses are presented in the form of graphs in figure 2.

Figure 2. Diagrams of the relatively shear stresses $\tau/p$ at $\mu=0.35$: $a$ – from the Mohr – Coulomb criterion by the left side of the formula (1); $b$ – from the empirical criterion by the left part of the formula (2); 1 – maximum shear stress; 2–5 shear stresses according to the empirical criterion at the angle of internal friction $10^\circ$, $20^\circ$, $30^\circ$ and $40^\circ$; 6 – line of locations of the most dangerous points.

Analyzing the results of calculating the shear stresses according to the empirical criterion, we can conclude.

Comparing the shear stresses at the most dangerous points of the diagrams presented in figure 2, a, and figure 2b, we note that with an increase in the angle of internal friction, the difference between the shear stresses increases. So at $\phi=40^\circ$, the shear stress according to empirical condition (2) is more than 2 times higher than the similar stress according to the Mohr – Coulomb criterion, and at $\phi=10^\circ$ the shear stress according to the Mohr – Coulomb criterion is exceeded by about 1.2 times.

We can give a more detailed analysis of the difference in values according to the criteria (1) and (2). To do this, we divide the left side of the empirical criterion (2) to the left side of the traditional Mohr – Coulomb condition (1). Then we get some function $k$, which shows how many times (for the
same values of $\sigma_1$, $\sigma_3$, $\phi$ and c) the shear stress by empirical criterion (2) is greater than the shear stress by the standard Mohr – Coulomb condition. This function has the form

$$k = \frac{1 + \sin \phi}{1 - \sin \phi}.$$  \hspace{1cm} (3)

From the analysis of function (3), it follows that under the condition $\phi > 0$, the value of the shear stress in the empirical criterion is greater than the shear stresses of the original Mohr – Coulomb condition. For $\phi = 0$ the function $k = 1$ means that the shear stresses calculated by (1) and (2) are equal. Substitution of $\phi = 0$ into the equations of limit state written in formulas (1) and (2) allows us to conclude that in this case both criteria are transformed into the third strength theory, in which the shear stress is equal to the maximum shear stress.

The calculation results by formula (3) are shown in figure 3.

![Figure 3. Dependence of the value of the function k on the angle of internal friction.](image)

In figure 3 the numbers near the corresponding points indicate the value of the function $k$ at the corresponding value of the angle of internal friction.

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