Near horizon symmetries, emergence of Goldstone modes and thermality

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Quantum mechanically horizons are believed to be thermal in nature for a long time. The microscopic origin of this thermality is the main question behind our present investigation, which reveals possible importance of near horizon symmetry. It is this symmetry which is assumed to be spontaneously broken by the background spacetime, generates the associated Goldstone modes. In this paper we construct a suitable classical action for those Goldstone modes, and show that all the momentum modes experience nearly the same inverted harmonic potential, leading to an instability. Thanks to the recent conjectures on the chaos and thermal quantum system, particularly in the context of an inverted harmonic oscillator system. Going into the quantum regime, the system of large number of Goldstone modes with the aforementioned instability is shown to be inherently thermal. Interestingly the temperature of the system also turns out to be proportional to that of the well known horizon temperature. Therefore, we hope our present study can illuminate an intimate connection between the horizon symmetries and the associated Goldstone modes as a possible mechanism of the microscopic origin of the horizon thermality.

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I. INTRODUCTION: BMS SYMMETRY AND GOLDSTONE MODES

Symmetry breaking phenomena is ubiquitous in nature. Across a large span of physical problems in particle physics, cosmology and condensed matter physics, it is not only the symmetry, but its spontaneous breaking also plays very crucial role in understanding the low energy properties. Symmetries in nature are broadly classified into two categories. A symmetry which acts globally on the physical fields are called global symmetry. Most importantly for each global continuous symmetry there exists associated conserved charge which encodes important properties of the system under consideration. Another class of symmetry which acts locally on the fields, generally known as gauge symmetry, makes the description of the system redundant. Unlike global continuous symmetries, gauge symmetry does not have associated non-trivial conserved charge \cite{1} \cite{2}. However, as compared to the global symmetry the most striking property of a global continuous symmetry lies in its spontaneous breaking phenomena which plays very important role in understanding the low energy behaviour of the system under consideration. If a global continuous symmetry of a system breaks spontaneously, associated Goldstone boson mode emerges, whose dynamics will characterise the underlying states and their properties of the system \cite{3}. On the other hand, breaking of gauge symmetry is inherently inconsistent with the theory under consideration.

In the present paper, we will be trying to understand the dynamics of the Goldstone boson modes associated with a special class of global symmetry arising at the boundary of a spacetime with nontrivial gravitational background. The generic underlying symmetry of a gravitational theory is spacetime diffeomorphism which is a set of local general coordinate transformation. Therefore, diffeomorphism can be thought of as a gauge symmetry of the gravitational theory. However, it is well known that a gauge symmetry in the bulk acts as a non-trivial global symmetry at the boundary. Therefore, even if gravitational theory can be formulated as a gauge theory, theory of Goldstone modes can still be applicable and information about the microscopic gravitational states may be extracted from the boundary global symmetry. Symmetries near the boundary of a spacetime has been the subject of interest for a long time \cite{4}–\cite{14}.

One of the popular and important examples of such a bulk-boundary correspondence is the well known global Bondi-Metzner-Sachs (BMS) group \cite{4}–\cite{6} of transformation. BMS group is an infinite dimensional global symmetry transformation which acts non-trivially on the asymptotic null boundary of an asymptotically flat spacetime. The original study \cite{5} was done on the asymptotic null boundary of an asymptotically flat spacetime. Subsequently the analysis on another null boundary namely the event horizon of a black hole spacetime has been studied \cite{15}–\cite{17}. The basic idea is to find out the generators which preserve the boundary structure of a spacetime of our interest under diffeomorphism. Usually, one encounters two types of generators, one is super-translation associated with time reparametrization and other one is super-rotation associated with angular rotation. Over the years it is observed that these generators can play a crucial role in understanding the horizon entropy of a black hole \cite{18}–\cite{28}. Since then there is a constant effort to understand these symmetries and its role to uncover the microscopic structure of the horizon thermodynamics. Although there is no significant progress till now, but the motivation is still there which led to some of the recent attempts \cite{29}–\cite{35}.

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Moreover in the series of remarkable papers [36]-[42] a deep connection between ward identities associated with the aforementioned BMS supertranslation symmetries and Weinberg’s soft graviton theorem has been unraveled. It is argued that the soft photons are the Goldstone boson modes arising due to spontaneous breaking of the asymptotic symmetries. Hence an equivalence has been established between Weinberg’s soft photon theorem and BMS symmetries [41]. More interestingly the same BMS transformation is shown to be closely related with the gravitational memory effect [38][43]. Subsequently same effect has been shown to arise near the black hole horizon as well [44].

In the present paper our focus will be on the Killing horizon specifically in Rindler and Schwarzschild background. Those horizons behave like another null boundary where bulk diffeomorphism acts non-trivially in terms of BMS-like global symmetry [45][46]. Associated with those global symmetry on the horizon, black hole microstates have been conjectured to be played by the soft hairs which are essentially the Goldstone boson modes associated with the symmetry broken by the macroscopic black hole state [47]-[53]. Although the appearance of Goldstone modes in the context of BMS symmetry exits, its dynamical behavior has not been studied in a concrete way. It is believed that the dynamics of those modes should play crucial role in understanding the microscopic nature of the black holes. Having set this motivation, in the present paper we will study the dynamics of those Goldstone modes following the standard procedure.

In order to clarify and better understand the methodology of our calculation, in the first half of our paper we consider Rindler space time with flat spatial section. In the later half we consider the asymptotically flat Schwarzschild black hole. Once we have a gravitational background, we first identify the global symmetry associated with the null boundary surface [15]-[17][54] imposing the appropriate boundary conditions. Boundary conditions are such that the near horizon form of the metric remains invariant after the symmetry transformation. However, macroscopic quantities such as mass, charge and angular momentum characterizing the physical states of a black hole under consideration will change under those symmetry transformation. Such phenomena can be understood as a spontaneous breaking of the aforementioned boundary global symmetry by the black hole background. We, therefore, expects the associated dynamical Goldstone boson modes. As mentioned earlier in this paper we will study the dynamics of those Goldstone boson modes which may shed some light on the possible microscopic states of the black holes.

II. RINDLER BACKGROUND

In this section we will consider the simplest background and try to understand the symmetry breaking phenomena as described in the introduction. The Rindler metric, in the Gaussian null coordinate is expressed as

$$ds^2 = -2 r o dv^2 + 2 dv dr + \delta_{AB} dx^A dx^B .$$  (1)

The Rindler horizon is located at $r = 0$. $\alpha$ is the acceleration parameter which characterizes the macroscopic state of the background spacetime. Symmetry properties of the horizon can be extracted from the following fall off and gauge conditions,

$$\mathcal{L}_\zeta g_{rr} = 0 , \quad \mathcal{L}_\zeta g_{vr} = 0 , \quad \mathcal{L}_\zeta g_{Ar} = 0 ; \quad (2)$$

$$\mathcal{L}_\zeta g_{rv} \approx O(r) ; \quad \mathcal{L}_\zeta g_{vA} \approx O(r) ; \quad \mathcal{L}_\zeta g_{AB} \approx O(1) . \quad (3)$$

Here, $\mathcal{L}_\zeta$ corresponds to the Lie variation for the diffeomorphism $x^a \rightarrow x^a + \zeta^a$. The above conditions are satisfied for the following form of the diffeomorphism vector,

$$\zeta^a \partial_a = F(v, y, z) \partial_v - r \partial_v F(v, y, z) \partial_r$$

$$- r \partial^A F(v, y, z) \partial_A . \quad (4)$$

Note that in this case we have only one diffeomorphism parameter $F$ which characterizes the symmetry of the Rindler horizon. Since for constant $F$, it essentially gives the time translation, the general form of this time diffeomorphism which acts non-trivially on the $r = 0$ hypersurface, is called supertranslation. For details of this analysis, we refer to [16, 17, 54].

In order to study the dynamics, let us first find the modified metric which are consistent with the aforementioned gauge (24) and fall-off (25) conditions. Important point to remember that the Lie variation of the metric component in our analysis is defined up to the linear order in $\zeta^a$ and hence we express the form of $\zeta^a$ (4) valid up to linear order in $F$. Under this diffeomorphism vector (4), the modified metric takes the following form:

$$g_{ab} = g_{ab}^{(0)} + h_{ab}$$

$$= -2 r o dv^2 + 2 dv dr + \delta_{AB} dx^A dx^B$$

$$+ \left[ - r \partial_v (\alpha \partial_A F + \partial^2_v F) \right] dv^2$$

$$+ \left[ - 2 r \alpha \partial_A F + \partial_A \partial_v F \right] dv dx^A$$

$$+ \left[ - 2 r \partial_A \partial_B F \right] dx^A dx^B . \quad (5)$$

In the above, $g_{ab}^{(0)}$ is the original unperturbed metric (1), whereas all linear in $F$ terms are incorporated in $h_{ab}$. Under the following supertranslation symmetry transformation,

$$v' = v + F(v, x^A) , \quad x'^A = x^A - r \partial^A F(v, x^A) , \quad (6)$$

we can clearly see the macroscopic state parameter $\alpha$ of the original Rindler background transforms into

$$\alpha \rightarrow \alpha + \left( \alpha \partial_v F + \partial^2_v F. \right) . \quad (7)$$

Therefore, this change of macroscopic state by the symmetry transformation can be understood as a breaking of
the boundary symmetry of the Rindler spacetime [52]. As $F$ is the parameter associated with the broken symmetry generator, following the standard procedure of Goldstone mode analysis, we promote $F$ as a Goldstone boson field. However, all the measure will be done with respect to the usual unprimed coordinate, and dynamics of the mode is defined on the $r = 0$ hyper-surface.

A. Dynamical equation for $F$

As we have already pointed out, in order to compute the action for $F$ the natural and straightforward method will be to expand the standard Einstein-Hilbert action along with Gibbons-Hawking York (GHY) surface term for the new perturbed metric (5) near the $r = 0$ surface. The full action is defined as,

$$S_F = \frac{1}{16\pi G} \int d^4x \delta (ar) \sqrt{-g'} R' + \int d^3x \frac{2}{\sqrt{h}} K$$ \hspace{1cm} (8)

Here $R'$ is the Ricci scalar calculated for the newly constructed metric $g'_{ab}$ (5) and $g'$ is the corresponding determinant. The second term is the GHY boundary term defined on an arbitrary timelike hypersurface. $K$ is the trace of extrinsic curvature near boundary and $h$ is the induced metric on a timelike hypersurface. To understand the dynamics of the modes associated with the underlying horizon symmetry it would be natural to project the action on the $r = 0$ hyper-surface. First we will compute the action at arbitrary $r$, and then project the action on the hyper-surface by using Dirac-delta function in the first part of the proposed action. Whereas the second term, the Gibbons-Hawking-York (GHY) term, is first calculated on a $r = constant$ surface and then the horizon limit ($r \to 0$) has been taken. In the Dirac-delta function $\delta (> 0)$ has been included in the argument to make it dimensionless.

Now we are in a position to expand our proposed action (30) in terms of the transformed metric (5). If the background metric is $g_{ab} = g_{ab}^{(0)} + h_{ab}$, with $h_{ab}$ as a small fluctuation, in general the Taylor series expansion of the action around background metric $g_{ab} = g_{ab}^{(0)}$ can be written as

$$S = S(g_{ab}^{(0)}) + h_{ab} \left( \frac{\delta S}{\delta g_{ab}} \right)^{(0)} + h_{ab} h_{cd} \left( \frac{\delta^2 S}{\delta g_{ab} \delta g_{cd}} \right)^{(0)} + \ldots$$ \hspace{1cm} (9)

The first term of the above equation obviously does not contribute to the dynamics. Given the background metric to be a solution of equation of motion, the second term vanishes as it is essentially proportional to the Einstein’s equation of motion. Third term introduces the quadratic action for the Goldstone field $F$. For our purpose of the present paper, we will restrict only to free action for the Goldstone mode which is at the quadratic order. All the higher order in $F$-terms we left for our future discussions. The first part of the action (30) takes the following form:

$$S_1 = \frac{1}{16\pi G} \frac{1}{\alpha} \int d^3x \left( \sqrt{-g'} R' \right)_{r=0}$$

$$= \frac{1}{16\pi G} \frac{1}{\alpha} \int dv dy dz \left[ -6\alpha \partial_y F \partial_y F - 6\alpha \partial_z F \partial_z F + 4\alpha \partial_y F \partial_y F - 12\alpha \partial_y F \partial_z F - 6(\partial_y \partial_z F)^2 + 4\partial_y^2 F \partial_z F \right]$$

$$+ 4\partial_y^2 F (\alpha \partial_y F + \partial_y^2 F)$$ \hspace{1cm} (10)

As it is well known fact that the total derivative terms in Lagrangian keep the dynamics unchanged, we can ignore those terms here. Then one finds the reduced action as,

$$S_1 = \frac{1}{8\pi G} \frac{1}{\alpha} \int dv dy dz \left[ -6\alpha \partial_y F \partial_y F - 6\alpha \partial_z F \partial_z F - 2(\partial_y \partial_z F)^2 - 2(\partial_z \partial_y F)^2 \right]$$ \hspace{1cm} (11)

Next we concentrate on the second part of the action (30) which is GHY boundary term. Here the trace of the extrinsic curvature of this boundary surface $K$ is given by $K = -\nabla_a N^a$, where $N^a$ is considered as the unit normal to the $r = constant$ hypersurface. For metric (5), its lower component is given by $N_a = (0, 1/ \sqrt{2r}(\alpha + \alpha \partial_r F + \partial_r^2 F), 0, 0)$. Therefore in the near horizon limit ($r \to 0$), one gets the following form of the action coming from the GHY term:

$$S_2 = \frac{1}{8\pi G} \int d^3x \left[ \alpha + \left( \alpha \partial_r F + \frac{1}{2} \partial_r^2 F + \frac{1}{2\alpha} \partial_r^3 F \right) 

+ \frac{1}{2\alpha^2} \left( \alpha^2 \partial_r F \partial_r^2 F + \alpha (\partial_r^2 F)^2 + \alpha \partial_r F \partial_r^3 F + \partial_r^2 F \partial_r^3 F \right) \right]$$ \hspace{1cm} (12)

The first term in the above expression is a constant, independent of $F$ and the other terms in the first line are purely total derivative in nature. Hence, they can not contribute to the dynamics. For the rest of the terms, one can easily show that those can be expressed as total derivative in $v$ which also have no effect in the required dynamics. Keeping all these in mind we can discard the above part of the action. Hence the final form of action will be (11).

Note that the aforesaid action (11) contains higher derivative terms of $F$. Therefore, the theory of Goldstone boson modes emerging on the boundary of a gravitational theory turns out to be higher derivative in nature. However, if we want to trace back the origin of this higher derivative action, it is from the diffeomorphically transformed metric components which already contains the derivative term. However, we will see those higher derivative terms will be crucial for our subsequent discussions on the horizon properties. This connection could be an interesting topic to investigate further. However, one of the important point here is that at the background label system is not Lorentz invariant. The generalized
Euler-Lagrangian equation, defined for higher derivative theory, is:

$$\frac{\partial L}{\partial F} - \partial_\mu \left( \frac{\partial L}{\partial (\partial_\mu F)} \right) + \partial_\mu \partial_\nu \left( \frac{\partial L}{\partial (\partial_\mu \partial_\nu F)} \right) = 0. \quad (13)$$

With this the equation of motion is found to be

$$3\alpha^2 \partial_y^2 F + 3\alpha^2 \partial_z^2 F - 2\partial_y \partial_z F - 2\partial_z \partial_y F = 0. \quad (14)$$

Since induced horizon geometry has flat spacial section, we take the following solution ansatz for the above equation,

$$F_{mn} = f_{mn}(v) \frac{1}{\alpha} \exp \left[ i(\alpha y + \alpha z) \right]. \quad (15)$$

Hence the general solution for Goldstone mode would be,

$$F(v, y, z) = \sum_{m,n} C_{mn} F_{mn}, \quad (16)$$

Substitution of (15) in (14) yields

$$\partial_v^2 f_{mn}(v) - \frac{3\alpha^2}{2} f_{mn}(v) = 0. \quad (17)$$

Important point to note that every individual mode \((m, n)\), will follow the same equation of motion of a simple oscillator in an inverted harmonic potential. The solution will be,

$$f_{mn}(v) = A \exp \left[ i(\sqrt{3/2})\alpha v \right] + B \exp \left[ -i(\sqrt{3/2})\alpha v \right] \quad (18)$$

for all \(m, n\). In the above, \(A\) and \(B\) are arbitrary constants to be determined. So far we talked about the classical dynamics of the Goldstone mode. It is apparent that the system is unstable because of the inverted harmonic potential at least at the tree level Lagrangian. We already know that the horizon is a special place in the entire spacetime region, as any two hypothetical observers spatially separated by the horizon can never communicated to each other. Therefore, it would have been unusual, had there been just simple stable free field like Lagrangian for the Goldstone modes. The connection between these special nature of the horizon and emergence of instability is the subject of study for a long time. Our goal of this paper would be to shed some light on this issue. Does the emergence of the inverted harmonic potential has anything to do with the thermal nature of the black hole horizon? Of course in order to understand this, we need to go to beyond the classical regime. In the next section we will try to make this connection considering a recent proposal \cite{55,56}.

### B. Thermal behaviour of the field solution

In this section we consider the quantum mechanical treatment of the Goldstone boson mode discussed so far. It has recently been conjectured that Lyapunov exponent \(\lambda\) of a thermal quantum system, in presence of quantum chaos, is bounded by the temperature \(T\) of the system as \(\lambda \leq 2\pi T/\hbar\) \cite{57}. Based on this result further conjecture has been made in the reference \cite{56,58} which says that a chaotic system with a definite Lyapunov exponent could be fundamentally thermal by reversing the above inequality. To justify the argument, one of the interesting example the author has studied is the semi-classical dynamics of a particle in an inverted harmonic potential\footnote{The choice of the inverted harmonic oscillator stems from the fact that the particle motion is unstable under this potential and hence, at the classical level, any small perturbation can lead induction of chaos in the motion (for example, see \cite{59}).}, and showed that the quantum correction induces an energy emission by the particle under study obeying thermal probability distribution. Therefore, the connection between the semi-classical chaotic system and the thermal nature is emerged. Interestingly, for our present system, each individual Goldstone boson mode behaves like an inverted harmonic oscillator. Hence, the aforementioned connection between the thermal emission and the semi-classical chaotic dynamics could be a potential reason for the thermal nature of the black hole horizon. Even more interestingly, every individual Goldstone boson mode parametrized by \((m, n)\) see the same inverted potential, which may also indicate the universality of the thermal nature of the horizon. Our present claim is ambitious and exciting which needs detailed future exploration. Hereafter we will quote the main result of the reference \cite{55,56}. The argument will be based on simply quantizing the harmonic oscillator in the inverted harmonic potential. From the mode equation (17), the associated Hamiltonian \(H\) can be expressed as,

$$H = -\hbar^2 \frac{\partial^2}{\partial^2 f_{mn}} - 3\alpha^2 f_{mn}^2. \quad (19)$$

Hence, the Schrödinger equation for the wave function \(\Phi(f)\) becomes,

$$-\hbar^2 \frac{\partial^2 \Phi}{\partial^2 f_{mn}} - 3\alpha^2 f_{mn}^2 \Phi = E \Phi. \quad (20)$$

Here \(E\) is equivalent to energy per mode. Even more interestingly what is emerged from our present calculation that all the modes with quantum number \((n, m)\) are degenerate with respect to \(E\). This observation seems to suggest that the horizon under study can carry entropy because of those degenerate quantum states. However, in order to have finite entropy, we need to have an upper limit on the value of \((m, n)\), which must be proportional to the only scale available in the theory namely Planck scale. Once we considers quantum mechanical correction, the classical trajectories of the mode under consideration will have finite probability of taking non-classical trajectories. Such as for \(E < 0\), and \(E > 0\), a particular mode
will have finite probability of transmission (T) through and reflection (R) off the potential respectively. It is easy to prove that the probability of this transition takes the following Boltzmann form,

\[ P_{T/R} = \frac{1}{\sqrt{\frac{2\pi}{\beta E}}} + 1 \]

For details of this derivation in the case of inverted harmonic oscillator, see [55]. It is clear from the above expression that for large absolute value of the energy \( E \), probability amplitude from classical path to quantum transmission or reflection will be \( e^{\beta |E|} \). This probability amplitude can be easily attributed to a two level system with temperature \( T \), whose ground state is represented as the classical trajectories and excited state is quantum one. The temperature of the system, in our present case, can be easily identified as

\[ T = \frac{\hbar}{2\pi} \sqrt{\frac{3\alpha^2}{2}} \]

Our naive analysis based on [55], shows that semiclassical Goldstone boson dynamics can capture the well known thermal behaviour of the horizon. Moreover, the temperature turned out to be proportional to the acceleration of the Rindler frame. This is an important observation as we know that the Rindler horizon is thermal with respect to its own origin. In this case also the temperature is proportional to \( \alpha \), known as Unruh temperature [60]. However, the proportionality constant appeared to be different. Another important outcome of our analysis is the emergence of infinite number of degenerate states which can be associated with the entropy on this horizon. We will take up this issue in our future publication. The microscopic origin of the horizon thermodynamics is a subject of intensive research for a long time. Our present analysis hints towards an important fact that the BMS symmetry near the horizon could play important role in understanding the thermal nature and possible origin of the underlying microscopic states of a black hole. Motivated by our analysis, in the subsequent section we will discuss about the Schwarzschild black hole.

### III. SCHWARZSCHILD BLACK HOLE

So far we have discussed about the dynamics of Goldstone boson mode in Rindler background. To this end we perform similar analysis considering Schwarzschild black hole background. The near horizon geometry of the Schwarzschild black is again Rindler, however, with two dimensional sphere at each point. Therefore, we expect similar behavior of the Goldstone mode for this case as well. As we go along we also notice the main differences with flat Rindler case.

The Schwarzschild metric in Eddington-Finkelstein coordinate \((v, r, \theta, \phi)\) is expressed as,

\[ ds^2 = -(1 - 2M/r)dv^2 + 2dvdr + r^2g_{AB}dx^A dx^B \]

The event horizon is located at \( r = 2M \). \( M \) is the mass of the black hole which characterizes the macroscopic state of the background spacetime. Asymptotic symmetry properties of the horizon can be extracted from similar fall off and gauge conditions for the metric components,

\[ L_{\zeta}g_{rr} = 0, \quad L_{\zeta}g_{vr} = 0, \quad L_{\zeta}g_{Ar} = 0 ; \quad L_{\zeta}g_{AB} \approx O(1) \]

Here, \( L_{\zeta} \) corresponds to the Lie variation for the diffeomorphism \( x^a \to x^a + \zeta^a \). The primary motivation to consider the aforementioned conditions is essentially to preserve the form of the metric under the diffeomorphism. As has already been observed in our previous case, those diffeomorphisms in turn renormalizes the state of the black hole parameter such as mass \( M \) of the Schwarzschild black hole. Similar to our previous analysis after solving the above gauge fixing conditions with the imposed fall-off conditions, the diffeomorphism vectors turned out to be,

\[ \zeta^a \partial_a = F(v, x^A)\partial_v - (r - 2M)\partial_r F(1/r - 1/2M)\gamma^{AB}\partial_B F \partial_A. \]

Again we have one unknown function \( F \) which is identified as supertranslation generator. Under this transformation the background metric takes of following form [49],

\[ g'_{ab} = g_{ab} + \gamma_{ab} \]

\[ = -(1 - 2M/r)dv^2 + 2dvdr + r^2\gamma_{AB}dx^A dx^B \]

\[ + [2M/r(1 - 2M/r)\partial_v F - 2(1 - 2M/r)\partial_r F - 2(r - 2M)\partial_r^2 F] dv^2 + \left[ -(1 - 2M/r)\partial_A F - (r - 2M)\partial_A F + r^2\partial_A F (1/r - 1/2M) \right] dv dx^A \]

\[ + \left[ -2(2M - r)r\gamma_{AB}\partial_r F \right. \]

\[ - (1/r - 1/2M)(\partial_E F\gamma^{DE}\partial_D\gamma_{AB}) \]

\[ + \left. \gamma_{AD} \partial_B (\partial_E F\gamma^{DE}) \right] dx^A dB \]

As has already been discussed for the Rindler metric with flat spatial section, for the present case the modification \( h_{ab} \) due to following super-translation,

\[ v' = v + F; \; \; x'^A = x^A + (1/r - 1/2M)\gamma^{AB}\partial_B F \]

the macroscopic black hole state parameter \( M \) renormalizes to,

\[ \frac{1}{M} \to \frac{1}{M} + \frac{1}{M} \left( \partial_v F + 4M\partial_r^2 F \right) \]

Therefore, this change of macroscopic state by the symmetry transformation can similarly be understood as a breaking of the boundary super-transformation symmetry.
with $F$ as the broken symmetry generator. In a similar manner, we promote $F$ as a Goldstone boson field with the following action,

$$S_F = \frac{1}{16\pi G} \left[ \int d^4x \delta(r - 2M) \sqrt{-g'} R' + \int d^3x 2\sqrt{h} K \right].$$

(30)

Here $R'$ is the Ricci scalar calculated for the newly constructed metric $g'_{ab}$ (27) and $g'$ is the corresponding determinant. Hence neglecting the total derivative term from the Lagrangian, Einstein-Hilbert action $S_1$ comes out to be,

$$S_1 = \frac{M}{8\pi G} \int dv \ d\theta d\phi \left[ \frac{-3}{2(2M)^2} \csc \theta \partial_\phi F \partial_\phi F - \frac{3}{2(2M)^2} \sin \theta \partial_\theta F \partial_\theta F + 4 \sin \theta \partial_\theta F \partial_\phi F - 2 \csc \theta \partial_\theta (\partial_\phi F)^2 - 2 \sin \theta (\partial_\phi F)^2 \right].$$

(31)

Here the nonvanishing lower components of $N^a$ is given by

$$N_r = \frac{1}{\sqrt{f(r) - (2M/r)f(r)\partial_r F + 2f(r)\partial_r F + 2r f(r)\partial_r^2 F}}$$

(32)

where $f(r) = 1 - 2M/r$. Hence for GHY boundary term the action can be expressed as,

$$S_2 = -\frac{M}{8\pi G} \int d^3x \sin \theta \left[ 1 + (\partial_\theta F + 2M \partial_\phi^2 F) + (2M \partial_\theta F \partial_\phi^2 F + 8M^2 (\partial_\phi^2 F)^2 + 8M^2 \partial_\theta F \partial_\phi^2 F + 32M^3 \partial_\phi^2 F \partial_\phi^3 F) \right].$$

(33)

which is again either total derivative term as was the case for Rindler space. The dynamics of the Goldstone mode will be governed by $S_1$, and equation of motion is given by,

$$-8 \sin \theta \partial_\phi^2 F + \frac{3}{(2M)^2} \cos \theta \partial_\phi F + \frac{3}{(2M)^2} \sin \theta \partial_\phi^2 F + \frac{3}{(2M)^2} \csc \theta \partial_\phi^2 F - 4 \sin \theta \partial_\phi^2 \partial_\theta F - 4 \cos \theta \partial_\phi^2 \partial_\theta F - 4 \csc \theta \partial_\phi^2 \partial_\phi^2 F = 0.$$

(34)

In this analysis full metric has been considered. Since we are interested in the near horizon symmetries, the near horizon metric could be enough to obtain the same result. For completeness, we explicitly demonstrated this in Appendix A.

Since the action has the rotational symmetry, we can take the following solution ansatz for Goldstone boson modes in terms of spherical harmonic,

$$F(v, \theta, \phi) = \frac{1}{k} \sum_{lm} c_{lm} f_{lm}(v) Y_{lm}(\theta, \phi),$$

(35)

with $c_{lm}$ are constant coefficients and $f_{lm}$ are the time dependent mode function. This is consistent with the spherically symmetric Schwarzchild geometry. The factor $1/k = 4M$ is introduced for dimensional reason. Substituting the form of $F$ (35) in (34) we get following equation of motion for $f_{lm}(v)$

$$[l(l + 1) - 2] \partial_v^2 f_{lm} - \frac{3}{16M^2} l(l + 1) f_{lm} = 0. \quad (36)$$

Since the near horizon geometry of the Schwarchild black hole is Rindler with sphere as spatial section, one notices some significant differences in the mode dynamics governed by eq. (36) and that of the previous case in eq.(17). Most importantly, for spatial spherical geometry the effective potential perceived by every individual mode parametrized by $(l, m)$ is no longer universal but dependent upon the angular momentum $l$. Before we discuss the implications of this dependence, let us take a look at the behaviour of individual modes.

- For $l = 0$ mode, the equation reduces to,

$$\partial_v^2 f_{00}(v) = 0 . \quad (37)$$

The solution of the above equation is $f_{00} = c_1 v + c_2$. By choosing $c_1 = 0$, this s-wave mode becomes trivially constant.

- For $l = 1$, the associated mode becomes identically vanishing, $f_{1m} = 0$. This seems to suggest that at the linear level, the p-wave dipole excitation can not be a microscopic state of a black hole. This may have some intimate connection with the nonexistence of gravitational dipole radiation, which needs further exploration.

- For all remaining modes $l \geq 2$, we get the inverted harmonic oscillator potential similar to our previous case. One important difference is the angular momentum dependence of the inverted harmonic potential. Therefore, the universality of all the modes with respect to their time dynamics is lost as opposed to our previous study in Rindler metric with spatial section. However, it can be checked that numerically the inverted potential depends very weakly on the value of $l$, which we will discuss in terms of temperature in the next subsection. Nonetheless, the mode equation looks likes,

$$\partial_v^2 f_{lm} - k^2 \Omega^2 f_{lm}(v) = 0 , \quad (38)$$

where,

$$\Omega = \sqrt{\frac{3l(l + 1)}{(l(l + 1) - 2)}. \quad (39)}$$

The general solution can be written as,

$$f_{lm}(v) = A \exp(\Omega kv) + B \exp(-\Omega kv). \quad (40)$$
Hereafter we can proceed along the same line as discussed before. Important difference would be the state dependent inverted harmonic potential

\[ V_{\text{harmonic}} = -\frac{1}{2} \Omega(l)^2 k^2 f_{lm}^2. \]  

Therefore, strictly speaking for the present case degenerate states will be only for \( m \) within \((-l, l)\). However, let us point out that if we consider numerical values into consideration, the value of \( \Omega \) is confined within a very narrow region

\[ \sqrt{\frac{6}{2}} < \Omega(l) < \sqrt{\frac{9}{2}}. \]  

Hence, one can approximately consider all the quantum states of the Goldstone boson parametrized by \((l, m)\) with \( l \geq 2 \), are quasi-degenerate. Unlike the previous case for the Rindler spacetime with flat spatial section, the emission probability for the present case would be identified with Boltzmann distribution with temperature,

\[ T_l = \frac{\hbar}{8\pi M} \Omega(l), \]  

which will weakly depend upon the value of angular momentum quantum number \( l \). We can define an average temperature

\[ T_{\text{avg}} = \frac{\hbar}{8\pi M} \left( \sum_l \frac{\Omega(l)}{\sum_l 1} \right) = \frac{\hbar}{8\pi M} \left( \sqrt{3} \right) = \sqrt{3} T_{\text{BH}}, \]  

where, \( T_{\text{BH}} \) is the usual black hole temperature, given by the Hawking expression \[61\]. Here again we observed that the Goldstone modes are inherently thermal in nature. The obtained temperature is proportional to the Hawking expression for that of the Schwarzschild horizon.

From the analysis so far what we can infer is that since the origin of the Goldstone modes are associated with the breaking of symmetries of the horizon, those modes can be a potential candidate for the microscopic states of a black hole. Quantum mechanically all those states turned out to be thermal with a specific temperature. However, origin of different expressions for the temperature compared with that of the usual Hawking temperature needs to be explored in detail. Furthermore, nature of degeneracy of those Goldstone states appears to be dependent upon the spacetime background. Such as for Rindler spacetime with plane symmetric horizon, all the modes emerged as degenerate and, therefore, each mode fills the same temperature. On the other hand for Schwarzschild black hole this is not the case as the degeneracy of states has been lifted by the less symmetric spherical horizon. Nevertheless, we hope that this thermal nature of the Goldstone modes at the quantum level can be inferred for all types of horizon. We keep this for our future project.

**IV. SUMMARY AND CONCLUSIONS**

Microscopic origin of the thermodynamic nature of the black hole is one of fundamental questions in the theory of gravity. It is obvious that within the framework of Einsteinian gravity this question can not be answered. However, the recent understanding of infrared behavior of gravity opens up a new avenue towards understanding this question. In the gravitational theory, one of the interesting infrared properties is the emergence of infinite dimensional symmetry at null infinity which leads to soft graviton theorem. Over the years it has been observed that analogous symmetry exists near the null horizon which can play important role in explaining the microscopic origin of horizon thermodynamics. Here we particularly concentrated on the BMS-like symmetry in the near horizon region. Under the diffeomorphism symmetry, appropriate boundary conditions are imposed in such a way that the near horizon form of the metric remains unchanged. It is observed that in this process the macroscopic parameters, like mass (surface gravity), get modified. This change in macroscopic parameters is argued to be the phenomena of symmetry breaking on the horizon and corresponding parameter can be viewed as the Goldstone mode.

In the present paper our main effort was to explore the dynamics of these Goldstone modes. For the purpose of our present study, we consider two simple gravitational background. One is simple Rindler spacetime with flat Killing horizon and the other one is Schwarzschild black hole. Our preliminary investigation at tree level reveals that the horizon is indeed a special place where the dynamics of the Goldstone mode in momentum space is governed by inverse harmonic potential. As mentioned earlier, in the framework of classical Einsteinian gravity it is difficult to understand this situation as those modes are simply unstable. Interestingly, at the quantum level this instability \[56\] can have a nice interpretation in terms of inherent thermality in connection with its chaotic behavior, which may provide us a first glimpse of microscopic view of the horizon thermodynamics. Interestingly, for both the gravitational backgrounds, as expected the temperature turned out to be proportional to the surface gravity which is similar to the expression (except a numerical factor) given by Unruh \[60\] and Hawking \[61\]. This led us to think that these Goldstone modes might be candidates for the microscopic description of the horizon thermality. Even more interestingly, we found out large number of degenerate states for Rindler and quasi-degenerate states for Schwarzschild black holes which may be responsible for the horizon entropy. We will take up these issues in more detail in future publication.

So far we have considered the black hole spacetime which are static and hence generating only one Goldstone field. However for a gravitational background having intrinsic rotation such as Kerr spacetime, corresponding analysis of the Goldstone mode dynamics will be more effective. This is because in this case there will be more
than one symmetry generator. This topic is now under investigation. Finally, we want to mention that since the Goldstone modes are thermal in nature, it might be interesting to look at BMS symmetry in this way hoping that such an analysis will be able to shed some light towards the microscopic description of horizon thermodynamics.

Appendix A: Near horizon analysis of Schwarzschild black hole

As mentioned in the main text, in this section we will argue that same dynamical equations and solution for the Goldstone modes can be obtained starting from the near horizon metric of the Schwarzschild black hole. The near horizon form of the Schwarzschild metric can be obtained by Taylor expansion of all the metric components around \( (r - 2M) = \tilde{r} = 0 \). Therefore, the form of the metric is given by (in Gaussian null coordinates);

\[
ds^2 = -2\tilde{r}k \, dv^2 + 2 \, dv d\tilde{r} + (4M^2 + 4\tilde{r}M) \, d\Omega^2 , \tag{A1}
\]

where the parameter \( k = 1/4M \) is essentially the surface gravity. The diffeomorphism vector components take following form in the near horizon limit,

\[
\zeta^a \partial_a = F(v, y, z) \partial_v - \tilde{r} \partial_v F(v, y, z) \partial_v - \tilde{r} \partial_A F(v, y, z) \partial_A ; \tag{A2}
\]

\( F \) is the only unknown function of coordinates, called as super-translation parameter. To construct the modified metric we follow the same procedure as described in our main text, and it takes the following form:

\[
\delta'_{ab} = h_{ab} + \delta_{ab} = -2\tilde{r}k \, dv^2 + 2 \, dv d\tilde{r} + (4M^2 + 4\tilde{r}M) \gamma_{AB} dx^A dx^B
\]

\[
+ \left[ -2\tilde{r}(k \partial_v F + \partial_v^2 F) \right] dv^2 + \left[ -2\tilde{r}(k \partial_A F + k \partial_A \partial_v F + \partial_A \partial_v F) \right] dv dx^A + \left[ -4M^2 \gamma_{AB} \partial_v F \right]
\]

\[
- 2\tilde{r}(\partial_E F \gamma^{DE} \partial_D \gamma_{AB} + \gamma_{AD} \partial_B (\partial_E F \gamma^{DE})) \right] dx^A dx^B . \tag{A3}
\]

Following the same prescription described in the main text we obtain the action for the Goldstone mode as,

\[
A = \frac{1}{8\pi G} M \int d^3x \left[ -2 \sin \theta + 8 \csc \theta \partial_v \partial_v^2 F \right. 
\]

\[
+ \frac{-3}{2(2M)^2} \csc \theta \partial_v F \partial_v F - \frac{3}{2(2M)^2} \sin \theta \partial_v F \partial_v F 
\]

\[
+ 4 \sin \theta \partial_v F \partial_v F - \frac{3}{M} \csc \theta \partial_v F \partial_v \partial_v F 
\]

\[
+ 1 \frac{M}{M} \cos \theta \partial_v F \partial_v F - \frac{3}{M} \sin \theta \partial_v F \partial_v \partial_v F 
\]

\[
+ 4 \cos \theta \partial_v F \partial_v^2 F + \frac{1}{M} \csc \theta \partial_v^2 F \partial_v F + 4 \csc \theta \partial_v^2 F \partial_v^2 F 
\]

\[
+ \frac{1}{M} \sin \theta \partial_v^2 F \partial_v F + 4 \sin \theta \partial_v^2 F \partial_v^2 F - 6 \csc \theta \left( \partial_v \partial_v F \right)^2 
\]

\[
- 6 \sin \theta \left( \partial_v \partial_v F \right)^2 + 8 \sin \theta \partial_v F \partial_v^2 F \right] . \tag{A4}
\]

Now we can see that the action (A4) so constructed from the near horizon metric contains three types of terms. The ones which are independent and linear in \( F \), can be traced back from their origin which can be transmitted to the fact that the near horizon geometry of the Schwarzschild black hole is Rindler times a sphere, and it does not satisfy the background Einstein’s equation. We, therefore, ignore those terms as they can also be made total derivative. Non-trivial dynamics of the Goldstone modes are attributed to second order term in \( F \) in the action, and it can be easily checked that those terms are exactly the same as in (31) up to a total derivative. As a result with a proper prescription, full spacetime geometry as well as near horizon geometry of the Schwarzschild background are giving rise to the same Goldstone mode dynamics.

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