Cluster decay calculations based on mean-field potentials

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Abstract. Based on mean-field model, an effective cluster potential has been suggested and applied successfully to the calculations of various cluster decays including diproton, $^6$He and heavier cluster decays. One of the advantages of the new potential is no new parameter introduced. All the parameters keep the same values as in mean-field model, which leads to self-consistent descriptions for both single-particle and cluster motions.

1. Introduction
Cluster decays have been observed widely from light to heavy nuclei. In light nuclei, cluster structures are particularly favored when nuclei are excited to the vicinities of the cluster decay thresholds [1]. In heavy nuclei, cluster decays can happen in their ground states. Cluster decays have long played an important role in the studies of nuclear structures. For example, fine $\alpha$-decay measurements in neutron-deficient Pb-Hg region can provide unique shell structure information [2, 3] and evidence for shape coexistence [4]. Especially in the syntheses of superheavy nuclei, the $\alpha$-decay measurement is the unique method to identify new elements at present.

More exotic is molecular-type cluster structures in which there are two or more clusters with similar masses. Early as 1960s, the resonant states observed in the $^{12}$C+$^{12}$C scattering were explained for molecular structures of the compound nucleus $^{24}$Mg [5, 6]. Recent experiment [7] and theory [8] gave interesting molecular structure in $^{12}$Be with two $\alpha$-particles and four covalent neutrons. The famous $^{12}$C Hoyle state which is at an excitation energy of 7.65 MeV is considered to have three $\alpha$ molecular structure. However, it is still unclear that the Hoyle state has three $\alpha$-particles in chain or in triangle.

In theory, cluster decay is considered a process of quantum tunneling of a cluster particle through a potential barrier, which is called the Gamow model [9]. A right barrier is crucial for the calculation of decay width or lifetime. Experimentally, $\alpha$-cluster potentials can be extracted from $\alpha$ scatterings but are generally energy- and method-dependent. Theoretically, Phenomenological $\alpha$-cluster potentials have been suggested for the calculations of $\alpha$-decay half-lives and spectroscopic properties [10, 11, 12]. Microscopic cluster potentials obtained from the double folding method have also been successfully applied to the investigations of $\alpha$-decays and $\alpha$-scatterings [13, 14, 15, 16, 17].
2. Model

In our previous works [18, 19, 20, 21], we suggested cluster potentials based on mean-field models. One of the advantages of such mean-field cluster potentials is that all the potential parameters can keep the same values as in the mean-field potentials [18, 19, 20, 21], which gives self-consistent descriptions for both single-particle and cluster motions and also makes predictions more reliable. Our mean-field cluster potentials have been successfully applied to the calculations of various cluster decays including diproton, α, heavier cluster decays [18, 19] and molecular structures as well [19, 21].

In the spherical case, the cluster potential can be simply written as (e.g., [11])

\[
V(r) = V_N(r) + V_C(r) + \frac{\hbar^2}{2\mu r^2}(L + \frac{1}{2})^2,
\]

that contains the nuclear potential \(V_N(r)\), the Coulomb potential \(V_C(r)\) and the Langer modified centrifugal potential with \(L\) and \(\mu\) for the angular momentum carried by the cluster and the reduced mass of the cluster-core system, respectively [18]. We construct the nuclear potential between the cluster and the remaining core as follows [18, 19],

\[
V_N(r) = \lambda[N_c v_n(r) + Z_c v_p(r)],
\]

where \(\lambda\) is a folding factor which scales the nuclear potential to give right decay energy [18, 19]. \(N_c\) and \(Z_c\) are the neutron and proton numbers of the cluster, respectively. \(v_n(r)\) and \(v_p(r)\) are the single-neutron and -proton potentials (excluding the Coulomb potential) respectively, generated by the core. The single-particle potentials can be obtained by mean-field models, such as the Skyrme-Hartee-Fock (SHF) [18, 19, 20] or the simpler Woods-Saxon potential [21]. The Coulomb potential \(V_C(r)\) is well defined in physics and should not be folded. We approximated the Coulomb potential by \(V_C(r) = Z_c v_n(r)\) in the SHF potential calculation (where \(v_n(r)\) is the single-proton Coulomb potential obtained with the density of the protons of the core) [18, 19, 20]. Such a cluster potential actually assumes that the cluster is structureless.

The folding factor \(\lambda\) can be determined using the Bohr-Sommerfeld quantization condition,

\[
\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}} |Q_L^* - V(r)| = (2n + 1)\frac{\pi}{2} = (G - L + 1)\frac{\pi}{2},
\]

where \(r_1, r_2\) (and \(r_3\)) are classical turning points obtained by \(V(r) = Q_L^*\) (the decay energy). The global quantum number \(G = 2n + L\) (\(n\) is the node number in the radial wave function of the cluster tunneling motion) is determined by the Wildermuth rule [22], giving \(G = \sum_i^A g_i\) where \(A_c\) is the nucleon number of the cluster and \(g_i\) is the oscillator quantum number of a cluster nucleon orbiting the core. The \(g_i\) numbers can be easily obtained from the Nilsson diagram. For the lowest-energy cluster decay, for example, cluster nucleons should occupy single-particle orbits immediately above the Fermi levels of the daughter nucleus. For a cluster rotational spectrum with a given intrinsic structure, the \(G\) number is fixed.

The cluster decay energy from an excited state is obtained by

\[
Q_L^* = Q_0 + E_J^*,
\]

where \(Q_0\) is the cluster \(Q\) value for the ground state, and \(E_J^*\) is the excitation energy of the given state with the spin \(J\). The decay process can occur only if the state has a positive \(Q_L^*\) value (i.e., above the decay threshold). The spin \(L\) carried by the cluster is determined by the coupling rule of the spins of mother and daughter nuclei. One can also assume a same folding factor for a given cluster structure in the excited and ground states of the mother nucleus, i.e.,
Experiments have well established the decays, such as [18]. The calculated half-lives agree with experimental data within a factor of 2 [18].

3. Calculations and discussions

Before the calculations of cluster decays, we test the validity of the proposed cluster potential by investigating the volume integral $J_R$ per interacting nucleon pair for $\alpha$-cluster decays or $\alpha$-cluster scatterings [13]

$$J_R = \frac{4\pi}{A_\alpha A_d} \int_0^\infty V_N(r)r^2 dr,$$

where $A_\alpha$ and $A_d$ are the mass numbers of the $\alpha$ and daughter, respectively. In the potential calculations, we take the SkI4 force which has been developed with a good isospin property and can well reproduce the bulk properties of nuclei. For the $\alpha$ decay from the ground state of $^{212}$Po, for example, we obtained a folding factor $\lambda = 0.595$ by using the Bohr-Sommerfeld condition, which leads to a volume integral, $J_R^{\text{cal.}} = 325$ MeV fm$^3$ [18]. Actually, obtained volume integrals are quite stable for a given mass region: $J_R^{\text{cal.}} = 330$ MeV fm$^3$ for trans-tin nuclei and $J_R^{\text{cal.}} = 290$ MeV fm$^3$ for trans-lead nuclei, agreeing well with the experimental values of $J_R^{\text{exp.}} \approx 300 - 350$ MeV fm$^3$ for a wide range of nuclei, obtained by $\alpha$ scattering experiments (see e.g., Ref. [13] for experimental data).

The $\alpha$ decays from the ground states of even-even trans-tin and trans-lead nuclei have been well studied experimentally. We have performed systematical calculations for the $\alpha$ decays of these two mass regions. The calculated results are quite encouraging [18]. We obtained quite stable folding factor which is $\lambda \sim 0.5$ for trans-tin mass region and $\lambda \sim 0.6$ for trans-lead region [18]. The calculated half-lives agree with experimental data within a factor of 2 [18].

We have also investigated heavier cluster decays from the ground states of trans-lead nuclei. Experiments have well established the decays, such as $^{14}$C, $^{20}$O, $^{24}$Ne, $^{28}$Mg and $^{32}$Si. The present parameter-free calculations for the partial half-lives of the heavy cluster decays agree with experimental data within one order of magnitude [18]. As an exotic example, we have investigated the diproton radioactivity which has been observed experimentally. In our model, we assume the diproton as a cluster with a folding potential of $V_{2p} = \lambda \times 2\nu_p(r) + 2\nu_c(r)$. The typical examples of diproton decays from the ground states of $^{16}$Ne and $^{38}$Ti have been calculated. The folding factors determined with using experimental decay energies are $\lambda = 0.66$ and 0.75 for $^{16}$Ne and $^{38}$Ti, respectively [18]. The calculations give quite reasonable half-lives of the diproton emissions of the two nuclei, within one order of magnitude compared with data [18].

In Ref. [20], we gave the detailed and systematical calculations of $\alpha$ decays for nuclei from $Z = 78$ to $Z = 118$, compared with available data. Our calculations reproduce systematically the observed decay properties, giving evidence to testify the validity of the mean-field-based cluster potential. In the paper [20], we also studied the deformation effect on the cluster decay by calculating an averaged decay width, $\Gamma = \int_{0}^{\pi/2} \Gamma(\theta)\sin\theta d\theta$ ($\theta$ is the angle between

the folding factor $\lambda$ is determined in the case of the ground state and used for the calculations of cluster decays from excited states. Since the decay calculation is very sensitive to the $Q'_L$ value, the experimental $Q_0$ and $E'_J$ have been used in numerical calculations.

The partial cluster decay width is calculated by [11, 19]

$$\Gamma = \frac{\hbar^2}{4\mu} \exp\left[-2 \int_{r_1}^{r_2} k(r)dr \right], \quad \text{(5)}$$

where $k(r) = \sqrt{\frac{2\mu}{E}}[Q'_L - V(r)]$ is the wave number, and $P$ is the preformation factor of the cluster being formed in the mother. For even-even nuclei, it has been testified that the $P = 1$ assumption under the use of the Bohr-Sommerfeld condition can well reproduce the experimental half-lives of various cluster decays [11, 18]. The decay half-life is obtained by $T_{1/2} = \hbar ln2/\Gamma$. 

In Ref. [20], we gave the detailed and systematical calculations of $\alpha$ decays for nuclei from $Z = 78$ to $Z = 118$, compared with available data. Our calculations reproduce systematically the observed decay properties, giving evidence to testify the validity of the mean-field-based cluster potential. In the paper [20], we also studied the deformation effect on the cluster decay by calculating an averaged decay width, $\Gamma = \int_{0}^{\pi/2} \Gamma(\theta)\sin\theta d\theta$ ($\theta$ is the angle between
the cluster-emission direction and the deformation symmetry axis of the nucleus [18]. The \( \alpha \)-decay calculations also reflect well the shell closure around \( N = 126 \).

The suggested mean-field-based cluster potential has also been successfully applied to the calculations of cluster decays from excited states, studying so-called molecular structures [19]. We calculated the \( \alpha \)-decay widths of excited states which belong to the sequence \( 0^+, 2^+, 4^+ \cdots \) in \( ^8\text{Be} \), well reproducing the measured decay widths [19]. \(^{20}\text{Ne} \) is an interesting nucleus in which excited states at excitation energies higher than \( \approx 4.7 \) MeV (i.e., the \( \alpha \)-decay threshold) are considered to have the molecular structure of \( \alpha + ^{16}\text{O} \). There have already been good experimental measurements on the resonant widths of the molecular states with positive or negative parities. Our calculations can well reproduce the experimental widths within a factor of about two for 7 observed resonant states at excitation energies ranging from 5.8 MeV to 22.9 MeV [19].

Compared with beryllium and neon nuclei discussed above, carbon isotopes have more complicated geometric structures with three \( \alpha \)-particles which can form chain or triangular cluster structure. Our calculations show these two structures lead to different decay widths or lifetimes [19]. In the numerical calculations of the \( \alpha \)-decay widths of excited states in carbon isotopes, we assumed that the remaining cores (i.e., two \( \alpha \)-particles + possible covalent neutrons) are well deformed with an approximate axis ratio of 2:1. The core produces a cluster potential which can be obtained also from a mean-field model. We have made detailed calculations for \(^{12,13,14}\text{C} \), giving reasonable results compared with data.

In the carbon isotopes, \(^{12}\text{C} \) is an especially interesting nucleus. The large abundance of \(^{12}\text{C} \) in the universe is attributed to the second \( 0^+ \) state which accelerate the fusion process from helium to carbon. The 7.6 MeV \( 0^+_2 \) excited state (just above the threshold of the \( \alpha + ^8\text{Be} \) fusion) is called the Hoyle state predicted by Hoyle in 1950s. Though 60 years passed, its structure which is chain or triangular is still unclear. For the decay to \( \alpha + ^8\text{Be} \), our calculation with a 3\( \alpha \) chain gave a decay width of 5.9 eV [19] which is close to the experimental data of 8.5 \(+ 1.0 \) eV. The calculation with assuming a 3\( \alpha \) triangular configuration decaying to \( \alpha + ^8\text{Be} \) shows a smaller width of 1.6 eV. From the comparison between calculated and experimental decay widths, the chain structure seems to be more likely. However, we cannot exclude the triangular structure. It is also possible that the 3\( \alpha \)-chain and -triangular configurations might coexist. Indeed, our preliminary calculations based on a 3-body Hamiltonian with a Gaussian force shows that these two configurations coexist in the Hoyle state, while the \( 2^+_2 \) state built on the Hoyle state is predicted to be more triangular [23]. The \( 2^+_2 \) excited Hoyle state was observed recently with an excitation energy of \( E_x \sim 10 \) MeV [24].

4. Summary
We have suggested a cluster potential which is based on single-particle mean field. The advantage of the potential is no new free parameter introduced, which makes predictions more reliable. This also leads to self-consistent descriptions for both single-particle and cluster motions. The potential includes automatically the isospin dependence. In most of our numerical calculations, the microscopic Skyrme force has been adopted. With such a mean-field-based cluster potential, we have successfully investigated various cluster decays including diproton, \( \alpha \) and heavier cluster decays, with calculated decay widths or lifetimes agreeing well with available experimental data. Some useful structure information has been obtained. For example, the calculations of the \(^{12}\text{C} \) Hoyle state gave a large component of the 3\( \alpha \)-chain structure. As a verification of the validity of the cluster potential, we have calculated the \( \alpha \)-decay volume integral of interaction, giving right values agreeing well with experimental data obtained by \( \alpha \) scatterings.

Acknowledgments
This work has been supported by the National Key Basic Research Program of China under Grant 2013CB834400, and the National Natural Science Foundation of China under Grant
Nos. 11235001 and 10975006.

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