A SIMPLE CALCULATION IN SERVICE OF CONSTRAINING THE RATE OF FU ORIONIS OUTBURST EVENTS FROM PHOTOMETRIC MONITORING SURVEYS

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ABSTRACT

The FU Orionis class of young stellar objects is enigmatic and rare. The members are interpreted to be “outbursting,” that is, they are currently in a state of enhanced accretion by several orders of magnitude relative to the more modest disk-to-star accretion rates measured in typical T Tauri stars. They are key to our understanding of the history of stellar mass assembly and pre-main-sequence evolution, and are also critical when considering the chemical and physical evolution of the circumstellar environment—where planets form. A common supposition is that all T Tauri stars repeatedly undergo such outbursts, more frequently in their earlier evolutionary stages when the disks are more massive, so as to build up the requisite amount of stellar mass on the required timescale. However, the actual data supporting this traditional picture of episodically enhanced disk accretion are limited, and the observational properties of the known sample of FU Ori objects are quite diverse. To improve our understanding of these rare objects, we outline the logic used to meaningfully constrain the rate of FU Ori outbursts and present numbers to guide parameter choices in the analysis of time domain surveys.

Key words: circumstellar matter – stars: formation – stars: pre-main sequence – stars: variables: T Tauri, Herbig Ae/Be

1. INTRODUCTION

Pre-main-sequence stars are highly variable due to mechanisms operating in the stellar photosphere, in the magnetosphere and innermost regions of the circumstellar disk, or in the disk atmosphere. While the phenomenology has received attention since the work of Joy (1945) and was pioneered in the CCD era by Herbst et al. (1994), it is only in the last several years that large-scale, quality, multi-year-duration and moderate-cadence (e.g., Rice et al. 2012; Parks et al. 2014; Findeisen 2015), or multi-week and high-cadence (e.g., Cody & Hillenbrand 2010, 2011; Cody et al. 2014) sampling of the photometric time series phase space has become commonplace. These recently available data sets have enabled more complete characterization of the diversity of young star variability than was previously possible, and they serve to further our understanding of the physical processes associated with young stars.

A number of identified or hypothesized variability mechanisms are illustrated in Figure 1. Both periodic and aperiodic phenomena are represented with short (minutes and hours) to long (years and decades) duration processes causing a wide range of brightness changes. Recent monitoring programs have contributed to the quantification of variability types, amplitudes, and timescales. However, depending on survey parameters such as cadence, duration, and photometric precision, only some of the plausibly operating physical phenomena are identifiable in any given time series data set. Thus, the relative distribution of young stars in the plane of amplitude versus timescale shown in Figure 1 is unknown.

1.1. Typical Young Star Variability Patterns

At lower amplitudes and intermediate timescales, significant progress has been made toward understanding observed young star variability phenomena. Simple periodic modulation is attributed to the rotation of photospheric inhomogeneities across the pre-main-sequence stellar disk. For “cool starspots” the analogy can be made to similar phenomena on the much older Sun, but with significantly enhanced amplitudes (up to ~0.1 mag). In addition, there may be rotationally modulated “hot starspots” attributed to accretion stream footprints. Periodic variability has been well studied over the past several decades, mostly in connection with attempts to measure angular momentum evolution and its relation to disks and accretion.

By contrast, aperiodic variability of a stochastic nature, or perhaps only a briefly coherent nature, is less well understood. This is in large part due to the lack of an obvious correlation between specific physics and specific variability patterns, although aperiodic behavior is generally associated with circumstellar rather than stellar phenomena. The presence of a disk, accretion, and outflow are implicated. Variability amplitudes can exceed 1 mag but the typical rms is approximately 0.1 mag (e.g., Grankin et al. 2007, 2008 and Venuti et al. 2015 for optical; Carpenter et al. 2001, 2002 for near-infrared; and Morales-Calderón et al. 2011 for mid-infrared studies). Findeisen et al. (2013) identified moderate-amplitude, intermediate timescale fading and brightening categories in the optical data, with fades lasting from weeks to years and likely being caused by obscuring material in the few tenths to few AU range, and bursts lasting from days to months and likely arising in disk processes that also occur outside of the magnetospheric region.

Recently, Cody et al. (2014), Stauffer et al. (2014, 2015), and McGinnis et al. (2015) published high-quality light curves of young stellar objects with disks in the young cluster NGC 2264. The combination of precision, cadence, continuous duration, and wavelength coverage revealed exquisitely detailed photometric fluctuations that were used to identify the following broad categories of variability.
1. Flux dips of a periodic or quasi-periodic nature are interpreted as circumstellar dust passing through our line of sight as it orbits, probably near the inner disk. The dust is hypothesized in different scenarios as being either entrained in accretion or wind flows between the inner disk radius and the stellar photosphere (explaining “narrow dip” stars) or located in warps in the inner disk regions (explaining “broad dip” stars a.k.a. AA Tau analogs). Flux dips that are aperiodic but repeating are also likely related to dust in the accretion flow.

2. Brightening events relative to a more steady (but still variable) flux level are interpreted as unsteady mass transfer from the inner disks to stars. The accretion flow has either variable flux along or variable penetration of a roughly dipolar magnetosphere. These events are aperiodic and there is no corresponding periodic category.

3. Phenomena rooted in the stellar photosphere include periodic modulation due to temperature inhomogeneities (cool/hot spots) that rotate with the photosphere. Aperiodic events include discrete flares due to coronal-like magnetic activity.

The recent plethora of monitoring data is enabling more detailed and quantitative studies of light-curve shapes and variability timescales (e.g., Findeisen et al. 2015) and amplitudes. The current overall direction of work in the field is to link a mathematical description of typical variability patterns among young stars more directly to the driving physics.

### 1.2. Rare Burst and Rarer Outburst Behavior

Young star bursts and outbursts are indicated in Figure 1 with amplitudes above ~1 mag and timescales longer than ~1 week. The statistics regarding these phenomena currently are quite poorly constrained.

Young stars that undergo repeated outbursts with ~2–4 mag amplitudes and characteristic burst timescales of weeks to months are called EX Lup-type variables (Herbig et al. 2001; Herbig 2008; Lorenzetti et al. 2009). V1647 Ori-type outbursts are also repeating, with larger ~4–5 mag amplitudes and similar or perhaps longer characteristic timescales of up to a year or so (Aspin et al. 2006, 2009). Finally, the FU Ori stars undergo ~4–6 mag brightness increases on timescales of a few months to years (Herbig 1989) and then decay over at least decades to centuries (empirically) or millenia (theoretically). Because of the long timescales, repeated bursts have not yet been observed among individual members of the FU Ori class.

To explain the moderate-amplitude bursts, there are a number of types of instabilities in circumstellar disks that have been suggested to lead to variable, and possibly cyclic, inward mass flow. Figure 1 also shows the relevant ranges in amplitude and timescale for various models. Magnetic reconnection instabilities are modeled by Lovelace et al. (1995) and Romanova et al. (2002), Rayleigh–Taylor instabilities are investigated by Romanova et al. (2004), diffuse instabilities are proposed by Goodson & Winglee (1999) and Romanova et al. (2005), and viscous instabilities are advocated by D’Angelo & Spruit (2010, 2012). The largest-amplitude and longest-timescale events are thought to be driven by inner disk instabilities and associated rapid mass accretion, as described in detail below.

The FU Ori outbursts are the most extreme and sustained, while the other burst categories are less extreme but more frequent. Even the stochastically variable burst behavior mentioned above as “typical” is also probably driven by nonsteady or unstable accretion, perhaps within the magnetospheric region. Thus, there is likely a broad continuum of burst behavior. Understanding the frequency of occurrence of different types of accretion burst events is important for improving our understanding of stellar mass assembly. However, how each of these different burst categories influence pre-main-sequence and disk evolution is currently unknown. Rates are currently unstudied for the lower-amplitude and more frequent events. For the higher-amplitude and less frequent FU Ori stars, the event rate has been estimated in various ways in previous literature as discussed below, although the numbers range over several orders of magnitude.

Here, we focus on the infrequent, well-defined, large-amplitude burst events exhibited by FU Ori stars. The difference between modeling these events and modeling the related EX Lup stars (shorter duty cycle and lower amplitude) and V1647 Ori-type objects (shorter duty cycle but similar amplitude, with evidence of contributions from both accretion enhancement and extinction reduction in their brightening) is that the FU Ori stars can be identified for decades after their bursts, whereas the other types of objects must be identified during the weeks to months to years that they are bursting. We leave such an effort to possible future contributions.

### 2. THE FU ORIONIS CLASS OF OBJECTS

#### 2.1. Phenomenology

The FU Ori stars are a very rare group of young pre-main-sequence objects. For the prototype FU Ori and the two other defining members of the class, V1057 Cyg and V1515 Cyg,
their outbursts were summarized observationally by Herbig (1977), while Larson (1980), Herbig (1989), and Hartmann & Kenyon (1985, 1996) discussed more general issues. Vittone & Errico (2005) and Reipurth & Aspin (2010) present recent review material. FU Ori itself, the prototype of the class, was known as an irregular variable star associated with the λ Ori region; it underwent a ~6 mag outburst in 1936 and has decayed only 0.5–1 mag (where the range indicates the uncertainty in the peak magnitude) since this time. V1057 Cyg was discovered as the emission line object LkHα 190 by Herbig (1958) and was presumed to be a normal T Tauri star until it underwent a 5.5 mag outburst in 1969, thus becoming the second known FU Ori star; its decay is much more rapid, and it is now only 2 mag above quiescence. V1515 Cyg, also a known variable before its outburst, experienced a much slower rise taking a dozen years to reach peak flux around 1980, 4 mag above quiescence, and similar to FU Ori decayed only very slowly. In each of the above cases, with the source luminosity increasing by factors of hundreds to thousands, the FU Ori outburst illuminated a reflection nebula.

In addition to their outbursts, the FU Ori objects are further distinguished among young stars by the fact that their optical spectra show unique signatures of early spectral type and low photospheric density, similar to F or G supergiants (Herbig 1977), while near-infrared spectra suggest M supergiants (Mould et al. 1978; Sato et al. 1992; Greene et al. 2008). The spectra are broadened by several tens to roughly 100 km s−1, which is much wider than the typical absorption line widths of accreting T Tauri stars. FU Ori stars also show spectral features and profiles associated with inflow and outflow processes, such as classical P Cygni line profiles in Hα and purely blueshifted absorption in higher Balmer lines, as well as similar signatures in He, Na D, Ca II H&K, and K I, attributed to winds. X-ray emission is also detected (Skinner et al. 2006). The broadband spectral energy distributions (SEDs) of FU Ori stars (e.g., Gramajo et al. 2014) exhibit large infrared excesses due to circumstellar material that is both thermally emitting and viscously dissipating energy from rapid accretion. Significant for the disk instability interpretation is the finding by Gramajo et al. (2014) that 80% of FU Ori disks have \( M_{\text{disk}} > 0.1 M_\odot \), more massive than any of the disks among the well-studied sample of T Tauri stars in Taurus, and that 90% have accretion rates \( \dot{M} = 10^{-6} M_\odot \text{yr}^{-1} \), higher than 95% of T Tauri stars; for any individual source, of course, one must be cautious of reported model parameters.

The current census of FU Ori objects numbers only two dozen at most, with some still considered debatable.

2.2. Physical Interpretation

To explain the high-amplitude outbursting FU Ori stars, theoretical models invoking thermal, gravitational,4 or magneto-rotational disk instabilities, and combinations thereof, have been proposed (Bonnell & Bastien 1992; Bell & Lin 1994; Clarke & Syer 1996; Kley & Lin 1999; Armitage et al. 2001; Vorobyov & Basu 2005; Boley et al. 2006; Zhu et al. 2009; Martin & Lubow 2014). There are also suggestions of instability triggering through dense cluster or close binary or planet interactions (e.g., Lodato & Clarke 2004; Reipurth & Aspin 2004; Pfalzner 2008).

The FU Ori broadband SEDs are well-reproduced by the disk models of Zhu et al. (2007, 2008) in which an outburst having an accretion rate of \( \sim 10^{-5} M_\odot \text{yr}^{-1} \) is driven by gravitational instability and associated magnetorotational instability that extends out to several AU (Zhu et al. 2010). For the thermal instability mechanism (Bell & Lin 1994), by contrast, the outbursting zone is confined to \( < 0.1 \text{AU} \) at \( 10^{-5} M_\odot \text{yr}^{-1} \), but could reach 1 AU at \( 10^{-3} M_\odot \text{yr}^{-1} \); the SED fits of this model are not as good (Bell et al. 1995), although the light curves are well matched. The enhanced mid-plane disk accretion rates can be compared to those typically inferred for classical T Tauri stars, which are more like \( 10^{-8} M_\odot \text{yr}^{-1} \) (Gullbring et al. 1998).

In the standard picture (e.g., Hartmann & Kenyon 1996), accretion outbursts are expected to decline in frequency, and perhaps amplitude, with increasing source age. More recent studies of protostellar accretion history (Vorobyov & Basu 2010; Stamataklos et al. 2012; Bae et al. 2014; Dunham et al. 2014) include disk instability physics in various ways in simulations, which results in the promotion of fragmentation. Accretion burst behavior can be produced at ages typically much less than \( \sim 1 \text{Myr} \) in the form of short-lived, 1.5–2 orders of magnitude enhancement in the disk accretion rates. Padoan et al. (2014), conversely, advocate for the importance of variable infall (rather than disk accretion) rates, which are prolonged in their turbulent cloud fragmentation model relative to the more isolated core/envelope/disk models which rely on disk instabilities to build up the stellar mass.

It is widely assumed that FU Ori stars are typical solar-type young stellar objects with central star masses \( \sim 0.3–1.5 M_\odot \) and ages \( \sim 1–2 \text{Myr} \). The predominant view is that they are Class II-type sources with a significant amount of the luminosity in their quiescent stages coming from envelope infall. Some FU Ori stars, however, are demonstrated to be Class II-type sources, such as PTF 10qpf (Miller et al. 2011), which was a routine M3e classical T Tauri star with little or no evidence based on the SED for envelope infall.5 Empirically, therefore, the episodic accretion behavior appears to continue into the more optically revealed, envelope-free phase of star formation and pre-main-sequence contraction, and so possibly beyond 1 Myr of age.

3. ESTIMATING OUTBURST RATES

The frequency of an astrophysical phenomenon is often derived by considering the number of detections of a particular category of object, the biases for or against detection as well as the total number of sources in a parent sample from which the categories could be established, and information about the relevant timescale. With these values in hand, one can arrive at a rate of occurrence. A well-understood numerator and a similarly well-defined denominator are required for rigorous statistical assessment.

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4 Though not widely recognized, the thermal instability model for FU Ori stars was first proposed by Paczynski in 1975, as reported by Paczynski (1978) and Trimble (1976).

5 The Gramajo et al. (2014) study identifies one-third of known FU Ori stars as Class II, but detailed examination shows likely errors in both directions. Most of their Class II designations (BBW 76 excepted) have flat or rising SEDs, as expected for Class I and so-called flat-spectrum sources, and some of their Class I designations (e.g., HBC 722 = PTF 10qpf) include far-infrared photometry that is significantly confused by other sources in the beam.
3.1. Previous Approaches to FU Ori Outburst Rates

Based on their relative number of FU Ori stars and regular T Tauri stars known at that time, Herbig (1977) indicated that the FU Ori outburst rate was \(10^{-4}\ \text{year}^{-1}\) star\(^{-1}\), and that the phenomenon was recurrent.

Hartmann & Kenyon (1996) estimated the FU Ori outburst frequency assuming the Miller & Scalo (1979) star formation rate within 1 kpc of the Sun, i.e., 0.01–0.02 \(M_{\odot}\) yr\(^{-1}\), and also seem to assume an average mass close to solar, giving \(\sim 0.01\) new low-mass stars per year. For the census at the time of 5 known outbursts (FU Ori, V1057 Cyg, V1515 Cyg, V1735 Cyg, and V346 Nor) in 60 years within 1 kpc, an integrated outburst rate of 0.08 year\(^{-1}\) is calculated. Dividing one rate by the other, a typical low-mass star thus has \(\sim 10\) outbursts. Dividing the T Tauri disk lifetime of \((1-10) \times 10^6\) years by 10 outbursts provides one outburst every \(10^{5-6}\) years.

A related McKee & Offner (2010) estimation assumes the Fuchs et al. (2009) star formation rate within 1 kpc of the Sun, \(8 \times 10^{-8}\) \(M_{\odot}\) yr\(^{-1}\) and an average mass of 0.5 \(M_{\odot}\) to obtain 0.016 low-mass stars per year capable of outburst. These authors also suggest that 25% of the mass of a star is accreted through such enhanced accretion episodes.

An updated census of outbursting objects by Greene et al. (2008) using the catalog of Abrahám et al. (2004) tabulates 18 spectroscopically identified FU Ori stars within 1 kpc; Reipurth & Aspin (2010) provide a more recent compilation. Dividing the number of presently known FU Ori objects by the star formation rate above, a typical low-mass star spends \(\sim 1250\) years of its life in an FU Ori phase. Although consistent with the Hartmann & Kenyon figure for a one century outburst length, the coincidence probably is not meaningful since the increase in the FU Ori census in the last 15 years is offset by the higher star formation rate assumed.

The above statements are equivalent to saying that the rate of FU Ori outbursts in active star-forming regions is approximately \(10^{-5}\) year\(^{-1}\) star\(^{-1}\), independent of whether all T Tauri stars go through such a phase or whether only a subset is prone to outbursts. However, since the population of FU Ori stars is incomplete, this rate is likely an underestimate.

Another line of reasoning is that a typical T Tauri star must have a lifetime average accretion rate of \(\sim 10^{-8}\) \(M_{\odot}\) yr\(^{-1}\) to account for its estimated mass and age. This high average accretion rate combined with the consequently lower-than-expected luminosity of “protostars” led Kenyon et al. (1990) to argue that FU Ori stars were an important mechanism in which much of the time could be spent accumulating mass at low rates and relatively little time at higher rates. If, for example, the star alternates between a quiescent phase of accreting \(10^{-7}\) \(M_{\odot}\) yr\(^{-1}\) (higher than typical) and an outburst phase of accreting \(10^{-4}\) \(M_{\odot}\) yr\(^{-1}\) (also higher than what is thought typical), then one can account for all of the star’s accretion if it spends 1% of its lifetime in outburst. Given an assumed typical outburst lifetime of \(\sim 100\) years, this implies that an outburst rate closer to \(10^{-4}\) year\(^{-1}\) star\(^{-1}\) is needed, or an even higher rate of \(10^{-3}\) year\(^{-1}\) star\(^{-1}\) if the more typical quiescent and outburst accretion rate values of \(10^{-8}\) \(M_{\odot}\) yr\(^{-1}\) and \(\sim 10^{-5}\) \(M_{\odot}\) yr\(^{-1}\), respectively, are adopted. Note that BBW 76 has been at maximum light since at least 1900 (Reipurth et al. 2002), and V883 Ori since at least 1888 (Cederblad 1946); similar to FU Ori, these stars are not rapidly fading. Thus, the actual duration of FU Ori outbursts may be a factor of several longer than what is typically assumed.

Based on the above logic, the outburst rate needed to reconcile the T Tauri accretion rate versus age problem using the episodic FU Ori outburst model is inversely proportional to the assumed outburst lifetime. Longer duration bursts mean lower rates and, conversely, if we allow for some fraction of shorter-duration FU Ori outbursts with lifetimes of only \(\sim 30\) years, then the needed outburst rate is higher by factors of several than the numbers above.

Bae et al. (2014) recently produced a distribution of accretion rates during the infall and early disk accretion phases up to 1 Myr, based on two-dimensional (2D) hydrodynamic models that include heating and cooling. Roughly 26% of the time is spent at an infall rate of \(10^{-5.75}\) \(M_{\odot}\) yr\(^{-1}\), 64% of the time is spent at the quiescent disk rate of \(10^{-7.75}\) \(M_{\odot}\) yr\(^{-1}\), and 10% of the time is spent in outburst at \(10^{-5}\) \(M_{\odot}\) yr\(^{-1}\) (their Figure 16). This results in an average accretion rate of \(1.5 \times 10^{-6}\) \(M_{\odot}\) yr\(^{-1}\) over the first \(\sim 1\) Myr, which is more than enough to accrete the stellar mass using the simple logic above.

The burst rate in these new Bae et al. (2014) models is significantly higher than in previous gravitational instability scenarios where, based on their Figure 3, approximately 24 outbursts in the first 0.3 Myr and only 2 in the next 0.7 Myr are suggested from the more standard models, whereas from their Figure 10 the number of outbursts is higher by at least a factor of two. Converting to an estimated FU Ori rate results in values around \(8 \times 10^{-4}\) year\(^{-1}\) star\(^{-1}\) in the Class I stage when the envelope dominates the disk extending to about 0.25 Myr, by which point the disk mass dominates, then dropping to about \(3 \times 10^{-5}\) year\(^{-1}\) star\(^{-1}\) into the Class II stage.

Considering the results of the numerous lines of reasoning above, it is not an exaggeration to say that there is currently a factor of at least 30 range among existing estimates of the FU Ori outburst rate.

3.2. How to Measure the FU Ori Rate Empirically

One can consider it to be a reasonable goal to determine the frequency of FU Ori outbursts to within a factor of two. In the calculations that follow, we evaluate an experimental design intended to constrain the rate to a factor of two or better at confidence C, with a chance of success (i.e., a measure of the risk in carrying out the experiment) of R. Both C and R have values in the range \([0, 1]\). We assume a region of sky containing a number \(N_{\text{YSO}}\) of young stars with disks (massive disks are required in the standard instability scenario for the outbursts) that is monitored over a duration \(\Delta t\). We further assume that outbursts occur at some average rate of \(P_0 = 10^{-4}\) year\(^{-1}\) star\(^{-1}\), that outbursts do not repeat on observable timescales (i.e., less than a century, essentially a constraint on \(P_0 \Delta t < 1\) star\(^{-1}\)), and that any outburst occurring in the monitored interval is detected.

The number of observed FU Ori outbursts is then well modeled by a binomial distribution,

\[ P(k \mid N, p) = \binom{N}{k} p^k (1 - p)^{N-k}, \]

which, in our case with probability \(p = P_0 \Delta t\), becomes

\[ P(N_{\text{event}} \mid N_{\text{YSO}}, P_0 \Delta t) = \binom{N_{\text{YSO}}}{N_{\text{event}}} \times (P_0 \Delta t)^{N_{\text{event}}} (1 - P_0 \Delta t)^{N_{\text{YSO}} - N_{\text{event}}}. \]
The number of FU Ori events that are expected to occur is $N_{\text{event}} = N_{\text{YSO}} r_0 \Delta t$ and the number detected would be $N_{\text{obs}} = cN_{\text{event}}$, where we assume $c = 1$ in the following. The probability of $n_{\text{obs}}$ or more detections $P(N_{\text{obs}} \geq n_{\text{obs}})$ is given by the appropriate number of binomial terms, $\sum_{k=n_{\text{obs}}}^{\infty} P(k | N_{\text{YSO}}, r_0 \Delta t)$.

We assume that, in practice, the estimated outburst rate $r$, measured in outbursts per star per unit time, will be inferred from the observations $N_{\text{obs}}$, for example, using $r = N_{\text{obs}}/\epsilon N_{\text{YSO}} \Delta t$. We assume that the uncertainty associated with the measurement of $r$ will be evaluated using the confidence interval formalism. We then ask how large a survey must be to have a high probability of detecting enough outbursts (given fluctuations around the mean true outburst rate $r_0$) to produce a tight confidence interval around the measurement $r$.

Given an observed number of events $N_{\text{obs}}$ and an inferred outburst rate $r$, an observer can define a confidence interval of $(r_{\text{lower}}, r_{\text{upper}})$, where the lower and upper limits on the outburst rate $r$ are set by the desired confidence level $C$ and by $N_{\text{obs}}$. For a symmetric confidence interval, these limits satisfy the following equalities:

$$P(n_{\text{obs}} \leq N_{\text{obs}}) = B(N_{\text{obs}}, N_{\text{YSO}}, r_{\text{upper}} \Delta t) = \frac{1 - C}{2}$$
$$P(n_{\text{obs}} \leq N_{\text{obs}}) = B(N_{\text{obs}}, N_{\text{YSO}}, r_{\text{lower}} \Delta t) = \frac{1 + C}{2},$$

where, in general, $B(n_{\text{obs}}, n, p)$ is the cumulative distribution function for a binomial distribution with $n$ trials of probability $p$ that realizes $n_{\text{obs}}$ events. $B$ is monotonic with respect to all three arguments: holding the other two arguments fixed, $B$ increases as $n_{\text{obs}}$ increases, decreases as $n$ increases, and decreases as $p$ increases.

For the experiment to provide a factor of two constraint, a symmetric confidence interval around the nominal measurement $r$ must lie entirely inside the interval $[r/2, 2r]$. Thus, we need to consider the case where $r_{\text{lower}} \geq 1/2 r$ and $r_{\text{upper}} \leq 2 r$. Substituting into the two equations above and expressing $r$ in terms of $N_{\text{obs}}$, we want to design the experiment so that it satisfies, with probability $R$, the inequalities

$$B\left(N_{\text{obs}}, N_{\text{YSO}}, \frac{2 N_{\text{obs}}}{c N_{\text{YSO}}} \right) \leq \frac{1 - C}{2}$$
$$B\left(N_{\text{obs}}, N_{\text{YSO}}, \frac{1 N_{\text{obs}}}{2 c N_{\text{YSO}}} \right) \geq \frac{1 + C}{2}.$$

The adjustable parameters in this experimental setup are the sample size $N_{\text{YSO}}$ and the monitoring interval $\Delta t$.

For $c = 1$, $B(N_{\text{obs}}, N_{\text{YSO}}, 2 N_{\text{obs}}/N_{\text{YSO}})$ is a decreasing function of $N_{\text{obs}}$, while $B(N_{\text{obs}}, N_{\text{YSO}}, N_{\text{obs}}/2 c N_{\text{YSO}})$ is an increasing function. Therefore, for reasonable values$^6$ of $N_{\text{YSO}}$ and $C$, both of these criteria will be satisfied for sufficiently high $N_{\text{obs}}$. For a factor of two constraint at $C = 0.90$, we typically need $N_{\text{obs}} \sim 8$ for a broad range of $N_{\text{YSO}}$.

All that remains is to find the probability that $N_{\text{obs}}$ will be high enough as a function of $N_{\text{YSO}}$. Since $N_{\text{obs}}$ is drawn from a binomial distribution with $N_{\text{YSO}}$ trials and probability $r_{\text{lower}} \Delta t$ (not the observed rate $r$ which appears in the preceding paragraphs), this can be done by choosing trial values of $N_{\text{YSO}}$, finding the minimum number of $N_{\text{obs}}$ that would provide a factor of two constraint on $r$, and then checking whether the probability of actually observing $N_{\text{obs}}$ events, given the true rate $r_0$, is at least our risk tolerance $R$. In other words, we predict the results of a future statistical analysis while integrating over all possible experimental outcomes to which said analysis would be applied. The expressions above are solved simultaneously for the two parameters $N_{\text{obs}}$ and $N_{\text{YSO}}$. The smallest value of $N_{\text{YSO}}$ for which a solution exists is the minimum survey size.

The trade-offs in parameter space between event rate $r$, survey baseline $\Delta t$, and minimum survey size $N_{\text{YSO}}$ are illustrated in Figure 2. The survey size required to probe a given outburst rate is approximately inversely proportional to the available baseline (i.e., the expected event count is approximately constant). A sample of approximately one million young stars with disks is needed in order to probe outburst rates $\sim 10^{-5} \text{ year}^{-1} \text{ star}^{-1}$ within a 1 year survey. For longer time baselines, the survey samples can be smaller and still probe the same outburst rate—only $\sim 10^5$ stars are needed for a decade survey and $10^6$ for a century baseline in order to probe the same rate as above. We note right away that typical studies today report information on, at best, several thousand accreting young stars, and so cannot yet even begin to meaningfully constrain the FU Ori rate on empirical grounds.

Figure 3 demonstrates the effect of considering different time baselines and outburst rates for samples under $10^5$ stars, a reasonable upper limit to the census of “known” young stars.

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$^6$ For an example of unreasonable values, let $N_{\text{YSO}} = 2$. The first condition can be satisfied only if $N_{\text{obs}} = 1$ but $B(1, 2, 1/2) = 0.9375$, and so the second condition cannot be satisfied if $C > 0.875$. 
within a few kiloparsecs. Samples larger than this are required if we aim to measure in a short survey the true FU Ori outburst rate, if it is much lower than $10^{-4}$ year$^{-1}$ star$^{-1}$.

Figure 4 illustrates the impact of altering some of the fixed parameters in the simulations above: the factor of two sensitivity to deviations from the particular outburst rate, and the required probability of a successful experiment. As expected, a larger survey, having smaller statistical fluctuations, provides either a higher probability of meeting specific precision goals for the rate, or better precision on the rate. The bottom panel shows that relative to the factor of two constraint that we have specified as adequate, improving our knowledge of the FU Ori rate to a factor of 1.5 requires 4 times as much survey data, while knowledge to only a factor of 3 requires 3 times less survey data. The top panel illustrates the relative insensitivity of the survey size requirement to the risk of obtaining a less precise result on the rate than the experimental design. In other words, there is little to be gained by increasing the chance that a given survey size will not detect enough FU Ori outbursts to obtain a reliable constraint on the FU Ori rate.

Figure 4. For a 30 year baseline and an outburst rate of $10^{-4}$ year$^{-1}$ star$^{-1}$ (i.e., $n_0 \Delta t = 3 \times 10^{-4}$), the survey size needed to detect a factor of two deviation from the hypothetical outburst rate, at 90% confidence ($C = 0.9$), and 90% of the time ($R = 0.9$). The required survey size is plotted as a function of (top panel) the probability $R$ that the survey will achieve the confidence bound, and (bottom panel) measurement precision on the hypothetical outburst rate. Larger surveys provide either a higher probability of meeting specific precision goals on the rate, or better precision on the rate. Approximately 3500 stars are needed for a factor of two precision on the rate (bottom panel) at 90% confidence (top panel) for this time baseline and true outburst rate.

Figure 3. Cuts across Figure 2 that illustrate relationships among parameters for survey sizes consisting of less than $10^5$ young stars with disks. The top panel shows the outburst rate $n_0$ for various values of the time baseline $\Delta t$, while the bottom panel shows the time baseline $\Delta t$ for various values of the outburst rate $n_0$ probed. Both panels assume the same 90% chance of constraining the outburst rate to a factor of two.

$C = 0.9$.
now by including particularly deep historical photographic plate data. A total sample size above $10^5$ stars would systematically shift the lines to more rapid returns, as would relaxing the requirements of 90% confidence or factor of 2 knowledge of the rate.

Finally, in Figure 5, we explore the dependence of our ability to know the FU Ori rate on what the event rate actually is. As expected, the rarer the occurrence of FU Ori events, the looser the rate constraint from any given survey. The tip of the plot where the constraint is reduced from a factor of 2 to nearly a factor of 5, the probability of detecting at least one outburst for this sample size over this survey duration is exactly 90%. For higher event rates, we would detect more outbursts and would improve our knowledge of the rate.

4. APPLICATION TO MODERN SURVEY DATA

Increased interest in young star variability mechanisms has led to more, larger (both wider field and longer duration), and often multiwavelength time series data sets. The value of the earliest possible first epoch observation is considerable, as illustrated in Figure 2. Over a 100 year baseline would mean either a reduction in the number of stars needed in the survey or an improvement for the limit able to be placed on the FU Ori rate—both by the same factor of 100—relative to a 1 year baseline survey.

In practice, an automated comparison of two fields at widely separated epochs would select stars with measured magnitudes that differed by some threshold $\Delta m$ as candidate FU Ori objects, to be confirmed or rejected by follow-up observations. The actual magnitude increase $\Delta m$ to which such a strategy is complete is always larger than $\Delta m_{\text{true}}$ because of measurement errors in the photometry at each epoch. Allow a star to have a true magnitude $M_1$ in epoch 1 and a true magnitude $M_2$ in epoch 2. Let the star have observed magnitudes $m_1$ and $m_2$ that can be modeled by Gaussian random variables with respective means $M_1$ and $M_2$ and variances $\sigma_1^2$ and $\sigma_2^2$. The magnitude difference is then also a Gaussian random variable with mean $M_2 - M_1$ and variance $\sigma_1^2 + \sigma_2^2$. In this model, $m_2 - m_1 = (M_2 - M_1) - 1.282 \sqrt{\sigma_1^2 + \sigma_2^2}$ with 90% probability. A survey with a cutoff $\Delta m$ for identifying an outburst detection will be 90% complete to outbursts with amplitude larger than $\Delta m + 1.282 \sqrt{\sigma_1^2 + \sigma_2^2}$.

In an actual data analysis, the value of $\Delta m$ can be chosen arbitrarily based on the expected outburst amplitude and the quality of the available photometry. Statistical outliers based on photometric errors are a minor contaminant even if one targets only the weaker outbursts. For $\Delta m$ of 2 mag and poor photographic photometry, such as $\sigma_1 = 0.3$ mag, $\Delta m = 1.46$ mag with a statistical false positive rate of $3 \times 10^{-4} \text{star}^{-1}$. Raising the magnitude threshold, for example, to exclude astrophysical false positives drawn from the large variety of young stars and other types of variables showing shorter-term variations at the roughly 2 mag level, would effectively increase the sample size needed to ensure good statistics.

4.1. Recent Results on High-amplitude Outbursts

Attempting to address the FU Ori rate problem, Scholz (2012) used the near-infrared 2MASS-UKIDSS ∼8 years baseline to investigate several hundred stars, and Scholz et al. (2013) used the mid-infrared Spitzer-WISE 5 year baseline to compare the photometry of several thousand stars. Consequently, as clearly illustrated in Figure 2, they could only constrain the FU Ori rate to much higher values (by several orders of magnitude) than the historical estimates of the FU Ori rate, discussed in Section 3.1, established based on alternate considerations.

From a survey with PTF consisting of over 1000 observations over the past 5.5 years toward the North America and Pelican Nebulae, two large-amplitude brightening events were discovered: PTF 10nvq (V2492 Cyg), which is likely a combination of accretion burst behavior and variable obscuration (Hillenbrand et al. 2013) perhaps analogous to V1647 Ori; and PTF 10qpf (V2493 Cyg), which is a bona fide FU Ori star (Semkov et al. 2010; Miller et al. 2011) previously known as the Class II classical T Tauri star LkHα 188/G4 or HBC 722. Both were discovered within the first season of PTF monitoring and—given the only a few thousand star sample of known young stars in the region—would suggest an FU Ori rate significantly higher than the canonical value.

Considering a parent sample of ∼2000 stars in the region with evidence of circumstellar material, and assuming perfect detection efficiency, the implied event rate is then $9 \times 10^{-5}$ outbursts per star per year. From the simulations presented here, the 90% confidence interval for this rate is $[3 \times 10^{-5}, 4 \times 10^{-4}]$ outbursts per star per year. Of course, statistical estimates based on only a single detection are not adequate predictors. At the implied event rate, the chance of not seeing another FU Ori outburst for another 20 years is only 3%, while at the lower bound, there is a 30% chance of no additional outbursts in the next 20 years.
4.2. Recent Results on Low-amplitude Bursts

In addition to the bona fide FU Ori star outbursts, there are also lower-amplitude bursts that have been identified, including the EX Lup group illustrated in Figure 1, as well as even shorter-time-scale and lower-amplitude bursts that are more routinely found in modern survey data. These less extreme but more common types may also play an important role in the accretion history of a star.

Among a sample of young stars selected based on their variability and infrared excess, Findeisen et al. (2013) found in the North America and Pelican Nebulae region that 12% ± 3% exhibited bursting behavior on timescales from 0.1 to 100 days, with amplitudes from 0.2 to 1.5 mag. If confirmed as accretion-driven events, then the cases of repeated bursts reported there may be the first empirical examples of accretion-driven cyclic behavior caused by processes in the inner disk, such as those predicted on timescales of a few tens of days in the disk instability mechanisms described in Section 2.2.

The simulations presented above are not applicable to these shorter-duration burst events since we have not included either the survey cadence (which needs to be short enough to catch and time-resolve the burst) or the burst duration (which needs to be long enough to be detected) as model parameters in our simple framework.

4.3. The Numerator

As stated earlier, estimating an event rate requires measuring a number of detections, with considerations to bias, from among the number of objects for which detections could be made. The numerator in the rate equation is the number of FU Ori outbursts observed. There are, in fact, two sides of the FU Ori outburst phenomenon which may be amenable to detection: one is the rise side or the outburst itself, occurring over weeks or months; and the other is the decay side, occurring over decades and centuries.

Numerous projects dedicated to other (usually extragalactic) science areas are also finding young star outbursts, some of which are even FU Ori like. The currently operating ASAS, CSS, and PTF will be succeeded by next generation surveys such as ATLAS, ZTF, and LSST, all of which are capable of making discoveries in this area. The ATLAS project, for example, is predicting 5 FU Ori stars in its first year of operations.

Survey yield will depend on parameters such as sensitivity, cadence, and galactic latitude coverage, as well as on astrophysical factors such as the range in burst amplitudes and rise times, and the frequency of visibility at optical versus infrared-only versus submillimeter-only wavelengths (e.g., Johnstone et al. 2013). None of these considerations are included in our simple simulations, where they were wrapped up in the factor $\epsilon$ which was set to unity. However, the results may be generalized to the case of $\epsilon = 1$ by considering that, even if not all FU Ori sources are detected, a Poisson/binomial process will still be observed, albeit at an effectively reduced rate. A best estimate rate and a confidence interval could still be calculated, then multiplied by $1/\epsilon$.

In addition to catching the FU Ori rise phase, a number of these imminent time domain sky survey projects could also identify FU Ori objects during the decay phase by selecting for follow-up spectroscopy sources with consistent fading over their 5–10 years of operations. However, there is significant diversity in the post-outburst light curves of known FU Ori stars (see, e.g., Clarke et al. 2005, their Figure 1). For example, FU Ori itself has decayed only slightly in 50 years, while V1057 Cyg had a much more rapid decay; V1515 Cyg was slower to rise, but has had a relatively flat post-outburst light curve.

4.4. The Denominator: Census of Selected Star-forming Regions

The denominator in the rate equation is the number of young stars with the possibility to undergo an FU Ori outburst in a given survey. The true survey size is $N_{YSO}$ young stars with disks which, given a certain survey area, is generally far less than the number of objects photometrically detected in the field. Young stars are identified via a variety of mechanisms associated with either stellar or circumstellar processes. Taken in isolation, none are 100% reliable indicators of membership in <1–3 Myr old star-forming regions, but in combination they can lead to secure knowledge of the total size and identity of the targeted young stellar population.

The typically considered diagnostics of young T Tauri stars are photometric variability, low excitation emission lines, presence of quick-burning lithium, infrared excess due to circumstellar dust, ultraviolet excess due to re-radiated accretion shocks, and X-rays from both accretion and stellar coronal activity. As the FU Ori phenomenon is driven by massive disks, only so-called Class I and Class II stars should be included in assessing survey statistics. This is a much easier scenario than other statistical problems, such as the lifetimes of disks where both the disked and non-disked populations must be thoroughly known, as well as the absolute source ages. There has been great progress over the last decade in terms of census building in star-forming regions. However, the job is not complete—even for regions within only a few hundred parsecs of the Sun. Spitzer-identified (infrared excess) and Chandra-identified (X-ray) samples are incomplete, even in combination, in part because these were pointed missions. The recent availability of wide-field survey data from WISE (mid-infrared), 2MASS, and UKIDSS (near-infrared), as well as iPHAS/UVEH (optical) and GALEX (ultraviolet) can help fill in these census gaps near and in the galactic plane. Non-outburst photometric variability is also—after some decades of hiatus—again being used to identify young stars. GAIA results on kinematics will help define cluster and moving group membership for optically visible young stars.

5. DISCUSSION AND CONCLUSIONS

Currently, there are reasonable empirical constraints on the rate of occurrence of phenomena such as M dwarf flares, novae, supernovae, and gamma-ray bursts. Each can be quantified in terms of the number per year per square degree per magnitude interval. The young star FU Ori outbursts, however, which have luminosity increases between those of M dwarf flares and novae, are rare enough that only crude estimates exist for the number that occur within each young star disk lifetime. While the rate is a significant number, knowledge of the distribution of time separation between FU Ori events during the first one to few Myr when the star first becomes visible as a self-luminous hydrostatically contracting object would be more meaningful.

In this paper, we have provided a simple framework for considering how well we might be able to constrain the rate of
FU Ori outburst events. The method employs binomial statistics which are generally applicable to any phenomenon that can be treated as either occurring (e.g., an outburst is detected) or not occurring (no outburst is detected). In our presentation, we have provided numbers based on a 90% measurement confidence requirement (a parameter of the statistical analysis that depends on the data actually obtained) with a 90% probability (a parameter that refers to having a representative sampling of the astrophysics), under the assumption that all outbursts that occur are detected. Figures 2–5 illustrate the simulation results.

With the rigor specified above, empirically constraining the FU Ori outburst rate to a level of $<10^{-4} M_\odot$ yr$^{-1}$ requires $>10^5$ young stars with disks to be monitored for >1 year, or $>10^4$ such stars for >10 years. Similarly, constraining the FU Ori outburst rate to $<10^{-5} M_\odot$ yr$^{-1}$ requires $>10^5$ young stars with disks to be monitored for >10 years, or $>10^4$ such stars for >100 years. Finally, outburst rates $<10^{-6} M_\odot$ yr$^{-1}$ could be probed if $>10^6$ young stars with disks were monitored for >10 years, or $>10^5$ such stars for >100 years. In any case, more than several $10^4$ suitably disked young stars are needed in order to be on the linear parts of these scaling relations, given the likely range of true outburst rates. An extension of our calculation to shorter-duration event types such as EX Lup/V1647 Ori or lower level bursts would require consideration of the survey cadence relative to the burst duration as additional model parameters.

Finally, we note that in order to estimate the true outburst rate, attention should be paid not only to the numerator that also to careful de...