Numerical simulation of focused ultrasonic waves in soft biological tissues with subsequent generation of shear waves

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Abstract. Numerical simulation of the propagation of ultrasonic waves is carried out based on the numerical solution of the Westervelt equation by the pseudospectral method with visualization. The modeling conditions are selected in accordance with the physical model of a soft biological environment. A comparison of physical and numerical modeling of the evolution of shear waves is carried out, and the elastic characteristics of polymer phantoms are determined.

1. Introduction
Numerical modeling of a physical experiment makes it possible to predict the measurement result, therefore, to solve the problem of computer simulation of environments with given parameters, a large number of algorithms, software modules and independent packages have been developed. Modern computer facilities allow high-precision calculations, but they place high demands on the hardware of the machine, therefore, it becomes important to choose an algorithm that efficiently uses the power of computing modules with the lowest possible load, while ensuring sufficient accuracy.

2. Numerical modeling
2.1 Evolutionary Westervelt Equation
It is convenient to simulate the propagation of waves in elastic media in the MATLAB package, which is optimized for working with large matrices, and for the convenience of working with the space, the k-Wave toolbox, created to calculate the propagation of waves in elastic media.

The numerical model of a set of k-wave scripts implies working with space as an initialization of a matrix of a given size, each element of which is assigned the necessary physical parameters necessary to calculate the field at a point. In k-wave space, this matrix is called the kgrid.

Numerical solution of the problem of acoustic wave propagation in soft biological tissues is reduced to solving the Westervelt equation in partial derivatives [1-2].

\[
\frac{\partial^2 p}{\partial \tau \partial z} = \frac{c_0}{2} \Delta p + \frac{\beta}{2 \rho_0 c_0^3} \frac{\partial^2 p^2}{\partial \tau^2}, \quad \tau = t - z / c_0,
\]

z - direction along the beam axis, \( \beta \) - nonlinearity coefficient
The equation is solved using the pseudospectral k-space method, where spatial gradients are computed using an FFT and temporal gradients are computed using a corrected k-space difference scheme. Each of these schemes is implemented using the finite element method.

The essence of the method lies in its name. The area in which the solution of differential equations is sought is divided into a finite number of subareas (elements). In each of the elements, the form of the approximating function is arbitrarily chosen. In the simplest case, this is a first degree polynomial. Outside of its element, the approximating function is equal to zero. The values of the functions at the boundaries of the elements (at the nodes) are the solution to the problem and are not known in advance. The coefficients of the approximating functions are usually sought from the condition of equality of the values of neighboring functions at the boundaries between elements (at nodes). These coefficients are then expressed in terms of the values of the functions at the nodes of the elements. A system of linear algebraic equations is compiled. The number of equations is equal to the number of unknown values at the nodes at which the solution of the original system is sought, in direct proportion to the number of elements and is limited only by the capabilities of the computer. Since each of the elements is associated with a limited number of neighboring ones, the system of linear algebraic equations has a sparse form, which greatly simplifies its solution [2].

In order to solve the Westervelt equation, it must be reduced to a weak formulation, for which we introduce the initial and boundary conditions:

\[
\begin{aligned}
p(0, t) &= f(t) \\
p(x, 0) &= 0 \\
p(L, t) &= 0 \\
\frac{\partial p}{\partial t}(t = 0) &= 0 
\end{aligned}
\]

\(f(t)\) is the source function at \(x = 0\), \(v(t)\) is the test function.

\[
\int_0^L \frac{\partial^2 p}{\partial x^2} V(x) dx - \frac{1}{C^2} \int_0^L \frac{\partial^2 p}{\partial t^2} V(x) dx = \frac{-\beta}{\rho_0 c_0^4} \int_0^L \frac{\partial^2 p}{\partial x^2} V(x) dx
\]

This is the first weak wording. We require \(v\) to be sufficiently smooth and satisfying \(v(0) = v(L) = 0\), since \(p\) is essential. We solve by integration by parts:

\[
\int_0^L \frac{\partial p}{\partial x} V(x) dx \left[ \frac{\partial}{\partial x} V(x) \right]_0^L - \int_0^L \frac{\partial p}{\partial x} V(x) dx = -\int_0^L \frac{\partial p}{\partial x} V(x) dx
\]

where \(V\) becomes 0 at \(x = 0\) and \(x = L\).

\[
\int_0^L \frac{\partial p}{\partial x} V(x) dx + \frac{1}{C^2} \int_0^L \frac{\partial^2 p}{\partial t^2} V(x) dx = \frac{-\beta}{\rho_0 c_0^4} \int_0^L \frac{\partial^2 p}{\partial x^2} V(x) dx
\]

which is now true only for smooth \(v\) satisfying \(v(0) = v(L) = 0\).

To implement the numerical model at the first stage, it is necessary to calculate the matrix of signals in time space. Each source emits a signal as a set of point sources,

\[
S = \{ A_{st}, t < t_s \} \quad 0, t > t_s
\]

where \(t\) is the simulation time, is the time during which the source emits, and at each point of the system the resulting signal is equal to the sum of signals from all sources

\[
S_{res} = \sum_{i=1}^{k} S_i
\]

where \(-S\) signal of a point source, \(k\) - the number of point sources.

The next step is the transition to the frequency domain using the FFT algorithm.

2.2. Focusing the ultrasonic wave. k-WAVE
In this work, the numerical simulation of the Shear Wave Elasticity Imaging SWEI method was carried out. This method was implemented on the Verasonics system with an open architecture at the Department of Acoustics in the Laboratory of Biomedical Technologies, Medical Instrumentation and Acoustic Diagnostics "MedLab", Lobachevsky state university [3-5]. This allows you to check the results obtained using numerical simulations in practice.

The calculation of the magnitude of the radiation force is based on the results of focusing [5]. On the basis of these results, the magnitude of the imaginary sources is recalculated for the last stage of modeling.

The implementation of the numerical solution of the simulation problem implies a division into stages: first, the environment in which the simulation is carried out is set (it can be either a linear medium or a nonlinear space), then a transducer is placed (in this case, it is a model of a standard linear transducer for ultrasound studies), and finally, simulation of wave propagation in the medium.

For the simulation to work, you need to create a space in which the wave propagation will be calculated. In the k-Wave module, a workspace is a collection of points that form a matrix called a k-grid, or k-space. For each of these points, the values of the speed of sound, density, and other parameters are prescribed, if required.

The radiation source in this model is a point. But if necessary, you can put several sources, or group several points into one emitter.

In the case of a linear sensor, the radiator is a phased array of 128 elements. Within the given numerical model, these are 128 point sources (Fig. 1). To obtain a shear wave, it is necessary to focus the emitters to a point. This is achieved using a quadratic phase incursion on each emitter; the center of the sensor is considered as zero. The medium in this model is assumed to be homogeneous, with such characteristic parameters as density $\rho = 1030$ kg / m$^3$ and speed of sound $C = 1540$ m / s.

![Figure 1. Visualization of the calculated focusing pressure of ultrasonic emitters, similar to the L7-4 linear medical transducer](image)

The calculation of the radiation force and the generation of a shear wave are carried out on the basis of the results of modeling the focusing of an ultrasonic wave. Here, the radiation sources are placed in the center so as to emit in the region of the focusing spot with a total power equivalent to the pressure obtained in the spot at the focusing stage.

3. **Comparison of physical and numerical simulation of shear waves in polymer phantoms**

Experimental determination of the shear wave velocity, Young's modulus, and viscosity of the medium was carried out using a Verasonics acoustic system in a calibrated CIRS Model 049 Elasticity...
QA Phantom Spherical polymer phantom with spheres 10 mm and 20 mm in diameter located at
different depths. The spheres in the phantoms were of four types with different values of Young's
modulus (Type I – IV), indicated in the accompanying documents. These elements were in a polymer
medium (matrix), the elastic characteristics of which were also known. The advantage of phantoms is
that they are made of Zerdine polymer material, the characteristics of which do not depend on changes
in external temperature and applied pressure.
To compare the results of numerical simulations with physical simulations, we used the results
obtained by measuring the shear wave velocity in a CIRS phantom. The initial conditions of the model
were set in accordance with the passport values of the phantom, as well as the settings of the
measuring sensor.
Properties of the medium: \( C = 1540 \text{ m/s}, \rho_0 = 1010 \text{ kg/m}^3 \). Source parameters: \( f = 5\text{MHz}, 128 \)
emitters (L7-4 sensor).
The simulation was run on a computer with the following specifications: Ryzen 5 3400G, 16GB
RAM, no GPU acceleration. The calculation time for focusing the sensor radiation was 1 min. 26 sec.,
and the processing time of shear wave generation 2 min. 5 sec.

4. Acknowledgement
This work was supported by a grant from the Ministry of Science and Higher Education of the Russian
Federation (project No. 0729-2020-0040).

|                  | Numerical simulation (K-wave package) Speed (m / s) | Physical modeling (Verasonics) Speed (m / s) |
|------------------|-----------------------------------------------------|---------------------------------------------|
| Matrix           | 4.17                                                | 2.68                                        |
| I                | 1.84                                                | 1.34                                        |
| II               | 2.07                                                | 1.81                                        |
| III              | 4.85                                                | 3.46                                        |
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