Implementing black hole as efficient power plant

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Treating the black hole molecules as working substance and considering its phase structure, we study the black hole heat engine by a charged anti-de Sitter black hole. In the reduced temperature-entropy chart, it is found that the work, heat, and efficiency of the engine are independent of the black hole charge. Applying the Rankine cycle with or without a back pressure mechanism to the black hole heat engine, the efficiency is numerically solved. The result shows that the black hole engine working along the Rankine cycle with a back pressure mechanism has a higher efficiency. This provides a novel and efficient mechanism to produce the useful mechanical work with black hole, and such heat engine may act as a possible energy source for the high energy astrophysical phenomena near the black hole.

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Introduction.—Black hole has been a fascinating object since general relativity predicted its existence. Compared with other stars, a black hole has sufficient density and a huge amount of energy. Near a black hole, there are many high energy astrophysical phenomena related to the energy flow, such as the power jet and black holes merging. Thus extracting energy from or using black hole is an interesting and outstanding challenge subject.

As early as 1969, Penrose [1] proposed that a particle, at rest at infinity, arrives the ergosphere of a Kerr black hole and then decays into two photons, one of which with negative energy crosses the event horizon, while other one with positive energy escapes to infinity. This process provides us a mechanism to extract the rotational energy of the black hole, and the maximum efficiency for a single particle via this process is 121% [2].

Several years ago, Banados, Silk, and West [3] showed that Kerr black holes can act as high energy particle accelerators and the center of mass energies of two colliding particles could reach the Planck scale when some fine tunings are satisfied. This mechanism was also extended to the collisional Penrose process that two particles collide inside the ergosphere of a rotating black hole and produce two particles, one of which falls into the black hole while other one escapes to infinity. The net efficiency of the process can approach to 130% [3, 4] for a single particle, or to a higher efficiency [5, 6]. Replacing the two collided particles with two black holes, there will be the black holes merging, which is a violent astrophysical phenomenon in our Universe. During the collision, a large amount of energy is released through the gravitational waves, which were recently directly observed by the LIGO scientific Collaboration [7]. Black hole bomb [8, 9] is also another interesting mechanism to extract energy from black holes due to the superradiant instability.

Moreover, energy extraction can also be achieved with a thermodynamic method. Dolan [10] pointed out that, in an isentropic and isobaric process, the anti-de Sitter (AdS) black hole can yield mechanical work by decreasing its angular momentum. And an extremal electrically neutral or charged rotating black hole can have an efficiency of up to 52% or 75%.

Most recently, Johnson [11] operated a simple holographic black hole heat engine along a thermodynamic cycle to extract the useful mechanical work. The exact efficiency formula was also obtained entirely in terms of the black hole mass [12]. This provides a novel thermodynamic way to extract energy using the black hole. However, importantly, we argue that the thermodynamic property and phase transition of the working substance should be included in for two reasons: i) a cycle is crucially built by utilizing the thermodynamic property of the working substance. For example, water working in a steam power plant will undergo liquid-vapour phase transition during each cycle. ii) Raising the engine efficiency through reducing the low-temperature heat sources will unavoidably encounter the phase transition of the substance. For a black hole engine, we here suggest that the working substance is the fluid constituted with the virtual black hole molecules [13], which carry the degrees of freedom of black hole entropy. Thus, the aim of this letter is to investigate the black hole heat engine implemented with a practical thermodynamic cycle when the thermodynamic property and the phase transition of the black hole molecules is included in.

Phase diagram.—Let us first proceed with the thermodynamic property of the working substance. In the extended phase space, the cosmological constant is treated as a pressure, \( P = -\Lambda/8\pi \) [14], and the small-large black hole phase transition of van der Waals type was studied [15] for the four-dimensional charged AdS black hole. The critical point is \( (P_c, T_c, S_c) = (1/96\pi Q^2, \sqrt{6}/18\pi Q, 6\pi Q^2) \). The state equation describing this phase transition in the reduced parameter space can be expressed in the following form

\[
\tilde{T} = \frac{1}{8S^{3/2}} \left( 3\tilde{S}^2 + 6\tilde{S} - 1 \right) ,
\]

(1)

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The heat change is \( \Delta U \) with the path and \( \Delta V \) steady-state flow system. Thus one arrives at the first law from A to B. For simplicity, we can cast the process into the system follows a path transferring state A to state B, the work and heat can be obtained with integrating areas by casting the process into the system, while \( dU \) is the amount of energy stored in the system, \( dH \) measures the energy stored by the working substance rather than the system.

Considering a reversible thermodynamic process, i.e., the system follows a path transferring state A to state B, the work and heat can be obtained with integrating the first law from A to B. For simplicity, we can cast the process into the \( P-V \) chart, in which the area constructed with the path and \( V \)-axis is equal to the negative work for the closed system, and the area constructed with the path and \( P \)-axis is equal to the work for the steady-state flow system. In addition, the \( T-S \) chart is complementary to the \( P-V \) chart. For the two systems, the heat can be both measured with the area under the path. Therefore we can obtain the work or heat through measuring the areas by casting the process into the \( P-V \) chart or \( T-S \) chart.

If a device operates in a cyclical manner along a closed thermodynamic process and exchanges only heat and work with its surroundings, we call it a heat engine. And we require that no working substance is lost or gained during the cycle. For a simple heat engine, it shares two characteristics: a heat source at high temperature and a heat sink at lower temperature. At high temperature, the engine absorbs the heat \( Q_1 \) (positive) from the high temperature source, and \( Q_2 \) (negative) from the heat sink. During one cycle, we have \( \Delta U = 0 \) or \( \Delta H = 0 \), which leads to

\[
Q_1 + Q_2 + W = 0, \tag{5}
\]

where \( W \) (negative) measures the work done by the surroundings to the engine during one cycle. The closed and steady-state flow systems share the same expression, so we will not distinguish them. Another important characteristic parameter of a heat engine is its efficiency, which is defined as the ratio of work output from the machine and heat input at high temperature,

\[
\eta = \frac{W}{Q_1} = 1 + \frac{Q_2}{Q_1}. \tag{6}
\]

Since \( Q_2 < 0 \), we have \( \eta < 1 \). Explicitly, in order to obtain the efficiency, one needs to calculate \( Q_1 \) and \( Q_2 \). Thus we can cast the cycle into the \( T-S \) chart only, and then the efficiency can be obtained by measuring the areas under the paths.

Next, we will turn to the Carnot engine, which is a particularly simple design conceived by Carnot in 1824. The Carnot cycle contains four reversible steps, i.e., two isothermal steps and two adiabatic steps. In general, the Carnot cycle is described in the \( P-V \) chart, which appears as an irregular loop. However, when casting this four steps in the \( T-S \) chart, one will find that this cycle is a regular rectangle, and its area can be quite easily calculated. We sketch the cycle in the \( T-S \) chart in Fig. 2(a).

Steps A-B and C-D described by the vertical lines are adiabatic. And steps B-C and D-A described by the horizontal lines are two isothermal steps. For a steady-state flow Carnot engine, steps D-A and A-B are implemented with compressor, and steps B-C and C-D with expansion turbine. Then the efficiency for the Carnot engine will be

\[
\eta = 1 + \frac{Q_2}{Q_1} = 1 - \frac{\text{area}(ADEF A)}{\text{area}(BCEFB)} = 1 - \frac{T_2}{T_1}. \tag{7}
\]
The useful work done by the engine in one cycle is just the negative of the area enclosed by the cycle.

Before applying this cycle to a black hole engine, one needs to be familiar with the thermodynamic property of the working substance. As stated above, the black hole system exhibits a small-large black hole phase transition. And in the reduced parameter space, the phase structure is found to be charge-independent. So here we would like to extend the study to the reduced $T$-$S$ chart. The reduced heat and work have maximal values, i.e., $Q$ and $W$ similarly decrease, and approach to 3.68 and 524. When increasing $T_1$ from its minimum, $Q$ and $W$ decrease, and approach to 3.68 and 5.32 at $T_1$=1.

The black hole molecules, which act as the working fluid, start at state A and approach to state B in an adiabatic process. Next they follow one isobaric line from state B to state E. During this process, these molecules undergo a small-large black hole phase transition from state C to state D with a constant temperature. Then the molecules perform a useful work and reduce the temperature from state E to state F such that the molecules change from the large black hole phase to a coexistence phase of small and large black holes. Finally, these molecules return to state A with shrinking their specific volume.

For the steam power plant, A-B process is implemented with a feed-pump, B-E process with a boiler of the steam generator and a superheater, E-F process with a turbine, and F-A process with a condenser. For the black hole engine, we may imagine that there exist four effective mechanisms to implement the four processes. The efficiency for this Rankine cycle is

$$\eta = 1 - \frac{\text{area(AFHJA)}}{\text{area(JBCDEHJ)}}. \quad (10)$$

We show the reduced heat and work in Fig. 3. The low temperature is set to $T_2 = 0.1$, while $T_1$ freely varies. One thing worth to note is that during the cycle, state F should fall into the coexistence range, thus $T_1$ has a minimum value larger than $T_2$. $Q$ and $W$ are plotted with $T_E=1.1$. At the minimum temperature $T_1 = 0.37$, the reduced heat and work have maximal values, i.e., $Q = 602$ and $W = 524$. When increasing $T_1$ from its minimum, $Q$ and $W$ decrease, and approach to 3.68 and 3.32 at $T_1$=1.

The efficiency is described in Fig. 4 with thick red lines for $T_E=1.1$, 1.2, and 1.5, respectively, from bottom to top. It monotonously increases with $T_1$. And the higher the $T_E$, the higher the efficiency. Moreover, from this double $y$-axis figure, we can clearly obtain two results: i) at low reduced temperature $T_1$, we have more useful work while low efficiency. ii) at high $T_1$, we get higher efficiency but less useful work.
keep in mind that the low temperature is also set to $T_E = 0.1$. From the figure, one can clearly see that the behavior of this BPR cycle is very different from the Rankine cycle without the back pressure mechanism. Both more work and higher efficiency can be achieved at the same time. Thus, the left thing is to raise the reduced temperature $T_1$ for a high efficiency. However, we should keep in mind that $T_1$ is not the highest temperature for the engine to operate, while $T_E$ is. Since the normal working of the heat engine is limited with the temperature, $T_E$ should not be too high. We list some numerical data in Table I which shows that in order to achieve the efficiency $\eta$, how high the temperatures $T_1$ and $T_E$ should be. For small $\eta$, the highest temperature $T_E$ of the heat engine is small. However, if one wonder an efficiency larger than 90%, $T_E$ will rapidly increase. For example, $\eta = 96\%$ requires $T_E = 3.70$, and $\eta = 98\%$ requires $T_E = 7.45$. Nevertheless, the BPR cycle still has a higher efficiency than the standard Rankine cycle.

| $\eta$ | 20\% 40\% 60\% 80\% 90\% 93\% 96\% 98\% |
|--------|-------------------|
| $T_1$  | 0.12 0.15 0.20 0.29 0.42 0.50 0.64 0.86 |
| $T_E$  | 0.14 0.20 0.32 0.70 1.45 2.09 3.70 7.45 |

TABLE I: Efficiency and temperatures for the BPR cycle with $T_2 = 0.1$.

Summary.—In this letter, we applied the Rankine cycle to the black hole heat engine by analogy with the steam power plant. We first examined the phase structure for the black hole molecules working as substance in the engine. Then after making a comparison between the small-large black hole phase transition and the liquid-vapour phase transition of water, we operated the black hole engine along the Rankine cycle with which the steam power plant operates. In the reduced $T$-$S$ chart, the work, heat, and efficiency of the engine were found to be charge-independent. Furthermore, our result shows that, when a back pressure mechanism is introduced for the black hole heat engine, the engine efficiency will approach its maximum. For example, the efficiency for $T_2 = 0.1$ reaches 98.62\%, which is only slightly smaller than the bounded efficiency 99.07\% of the corresponding Carnot engine. More importantly, the engine also produces the maximal work at the same time. Thus this result may provide us a novel and efficient mechanism to produce the useful mechanical work with the black hole.

At last, we would like to make a few comments. Here we dealt with the charged AdS black hole heat engine operating along the Rankine cycle. During the cycle, the engine produces useful work to its surroundings. Of course, one can consider a reverse cycle, the refrigeration cycle for the black hole engine if the work is provided by its surroundings. Moreover, one can consider the heat engine by the rotating AdS black holes or black holes in higher-derivative gravity, where the phase diagram is similar to the water with a triple point being found. These may provide a novel way to transfer heat energy to mechanical work, and black hole heat engine may act as a possible energy source for the high energy astrophysical phenomena near the black holes. On the other hand, according to the AdS/CFT correspondence, finding out the holographic dual description of such thermodynamic cycle in the large $N$ field theory is also an interesting and important subject.
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