Quantum state preparation using multi-level-atom optics

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Abstract. One of the most important characteristics for controlling processes on the quantum scale is the fidelity or robustness of the techniques being used. In the case of single atoms localized in micro-traps, it was recently shown that the use of time-dependent tunnelling interactions in a multi-trap setup can be viewed as analogous to the area of multi-level optics. The atom’s centre-of-mass can then be controlled with a high fidelity, using a STIRAP-type process. Here, we review previous work that led to the development of multi-level atom optics and present two examples of our most recent work on quantum state preparation.

1. Introduction

The ability to explore the single-particle quantum nature of matter has become feasible in the past two decades due to advances that have been made in laser cooling and trapping techniques [1]. Neutral atoms, molecules and ions have been cooled to temperatures such that their de Broglie wavelength is of the order of optical wavelengths, thereby facilitating the development of analogous experiments and theory for both particles and optical fields. One particular area that has emerged is that of atom optics, whereby atoms are used to probe the quantum world in a manner similar to quantum optics. The textbook example of quantum optics experiments involves the interaction of a two-level atom with a coherent radiation field giving rise to Rabi oscillations in the probability amplitudes of the atomic states. The extension of such relatively simple systems to more complex three-level systems yields a wealth of stunning and surprising results, whereby the effect of atomic coherence and quantum interference plays a crucial role in controlling the optical properties of an atomic medium. Such effects include electromagnetically induced transparency (EIT) [2], coherent population trapping (CPT) [3], and stimulated Raman adiabatic passage (STIRAP) [4, 5], in which two counter-intuitively pulsed light fields are used to coherently transfer the population of an atom between two internal states through an intermediate level. These phenomena have been used extensively to explore slow light [6], highly efficient resonant non-linear optics, the control of spontaneous emission [7] and the coherent storage/release of light [8] amongst many other topics [9].

In recent years, there has been significant interest focused on the control and manipulation of single, neutral atoms due to the potential applications that can stem from this research in areas as diverse as understanding the basics of quantum mechanics to potential applications for quantum information processing techniques [10]. In 2004, Eckert et al. considered the atom optics analogue to three-level quantum optics and they coined the term three-level atom optics (TLAO) for this new area of research [11]. They proposed using three, spatially separated, atom traps in a linear array to replace the three internal quantum states in a Λ configuration. Using this analogy, they demonstrated the coherent manipulation and transfer of neutral atoms from one trap to another by exploiting a time-dependent tunnelling interaction between neighbouring traps.
Figure 1. Left: Schematic setup for three level atom optics. All traps have the same trapping frequency $\omega$. The time dependence of the tunnelling strength is realized by decreasing and increasing the distance between the traps. Right: Timing sequence for the distance between the traps during the STIRAP process.

The three in-line traps were modelled as three separate harmonic potentials with identical frequencies, $\omega$, under the assumption that, initially, a single atom existed in the ground vibrational state of the leftmost trap (see Fig. 1). A counter-intuitive approach-reproach sequence was used for the position of the three traps in order to achieve complete population transfer from the leftmost trap to the rightmost trap, while keeping the middle trap stationary. This counter-intuitive sequence is analogous to the pulsing and timing of the pump and Stokes coherent laser fields in conventional STIRAP processes [4].

Eckert et al. used the fact that one of the eigenstates of the three-level system is a dark state that involves the ground states of the left and right traps, with no contribution from the middle trap:

$$|ψ⟩ = \cos \theta |0⟩_L - \sin \theta |0⟩_R ,$$

(1)

where $\theta$ is a mixing angle defined by the tunnelling (or effective Rabi) frequencies between the left and middle traps, $Ω_{LM}$, and the right and middle traps, $Ω_{MR}$, with $\tan θ = Ω_{LM}/Ω_{RM}$. The tunnelling frequencies are a function of the trap separations and, for harmonic potentials, they are given by [11]

$$\frac{Ω_R(d)}{ω_R} = \frac{-1 + e^{2d} [1 + d(1 - erf(d))]}{\sqrt{\pi} (e^{2d^2} - 1)/2d} .$$

(2)

One can now easily see that if $\theta$ is changed adiabatically from 0 to $\pi/2$ it is possible to obtain a complete population transfer of the atom from the left trap ($|0⟩_L$) to the right trap ($|0⟩_R$). Physically, this corresponds to approaching the rightmost trap to the middle and, after a suitable time delay, approaching the leftmost trap to the middle.

Using differently crafted approach and reproach sequences, the authors in [11] were able to create spatial superposition states between the left and right traps and show that these states are the atom optics analogy to the dark state in CPT. They also showed that the STIRAP process is robust against noise and found that the population of the dark state was 99% robust, for 5% noise on the position of the traps, therefore proving useful for realistic atom interferometry experiments.

Finally, Eckert et al. demonstrated an atom optics equivalent to EIT, by inhibiting tunnelling between the leftmost and middle traps. In quantum optics, EIT modifies the properties of a medium by converting an opaque medium into a transparent medium by the addition of a driving field [2]. They proposed that this atom optics approach could be used for the generation of phase gates for quantum logic.

It should be noted that Eckert et al. also pointed out that TLAO is not limited to particles in the ground state. The only requirement is for the translational mode to be a zero mode, i.e. energy must be
Figure 2. Schematic of the setup for the four level technique to create a superposition state. The asymptotic ground states of the individual traps are indicated in red and the symmetric and the anti-symmetric eigenstates of the double-well trap are shown in blue. Note that to aid visualisation, and in contrast to the requirement outlined in the text, in this schematic, $\omega_R$ is chosen to have the same value as $\omega$.

conserved during the process. This relaxed the often technically difficult requirement of ground state cooling. An additional advantage is that the entire process can evolve in time scales approaching the millisecond range, in stark contrast to the quantum optics case, which has strict requirements concerning the duration for the processes (pulses of 50 ns are typical) and restrictions resulting from dipole selection rules and predefinition of the quantum states involved in the process. Eckert et al. have recently extended these ideas to wave-guides, realistic trapping potentials and non-linear settings [12].

2. State Preparation

Expanding the ideas explored above, we now show how to exploit dark states using more than three levels. In particular we will demonstrate how a coherent, centre-of-mass superposition state of a single atom between two traps can be created, in which the symmetry is purely determined by external experimental parameters [13]. This process is similar to a proposal for optical systems using a $\Lambda$ system with a manifold of levels [14]. The setup we study is shown in Fig. 2, and consists of a linear array of four traps. The two leftmost traps are simple harmonic potentials, $V(x) = \frac{1}{2}m\omega^2x^2$, with identical trapping frequencies, $\omega$. The ground states are denoted by $|0\rangle_L$ and $|0\rangle_M$. The trap on the right hand side of the setup is a double-well potential made up of two harmonic potentials of trapping frequency $\omega_R$, and its asymptotic ground states are denoted by $|0\rangle_{RL}$ and $|0\rangle_{RR}$. Ground state splitting occurs, however, within the double-well potential, and the asymptotic eigenstates are combined to yield a symmetric and anti-symmetric state in the trap. While this results in a lowering of the energy of the symmetric eigenstate, the energy of the anti-symmetric eigenstate is raised. This effect is the basis for the process outlined in the following.

For a transition process between quantum states to be successful, it is essential that resonance conditions be satisfied for the states involved. If we consider the case for the three single traps as discussed above, this translates into the requirement that all three traps must have the same trapping frequency. In the example shown in Fig. 2 the equi-frequency constraint will, however, lead to a failure of the transfer process for an atom initially located in the left trap to end up in the double-well trap on the right. This arises from the fact that neither the symmetric nor the antisymmetric ground state of the double-well trap are in resonance with the ground states of the single harmonic traps.

While, at a first glance, this may appear to be a strong disadvantage for such a scheme, it does, in fact, hold the key to the realization of many possible and interesting scenarios. By adjusting the trapping frequencies of the traps that form the double-well potential, we can bring either the symmetric or the anti-symmetric eigenstate in resonance with the energies of the middle and the left trap. This can easily
be achieved by either increasing or decreasing $\omega_R$ with respect to $\omega$. To find the necessary condition for this to occur, let us consider the Hamiltonian of the system, given by

\[
H = \hbar \begin{pmatrix}
0 & -\Omega_{LM}(t) & 0 & 0 \\
-\Omega_{LM}(t) & 0 & -\Omega_{MR}(t) & 0 \\
0 & -\Omega_{MR}(t) & \omega - \omega_R & -\Omega_R \\
0 & 0 & -\Omega_R & \omega - \omega_R
\end{pmatrix}.
\]  

(3)

Here $\Omega_{LM}(t)$ and $\Omega_{MR}(t)$ represent the time-dependent tunnelling frequencies between the states $|0\rangle_L$ and $|0\rangle_M$ and $|0\rangle_M$ and $|0\rangle_R$, respectively. The tunnelling frequency between the two states $|0\rangle_{RL}$ and $|0\rangle_{RR}$ is given by $\Omega_R$ and is constant at all times. The condition for the Hamiltonian in eq. (3) to have an eigenstate with an eigenvalue zero is given by the following relationship between the trapping frequencies, $\omega$ and $\omega_R$, and the tunnelling frequency within the double-well trap, $\Omega_R$

\[
\omega - \omega_R = \pm \Omega_R.
\]  

(4)

Since $\Omega_R$ is positive, this condition implies the existence of two dark states, one for $\omega > \omega_R$ and one for $\omega < \omega_R$. These correspond exactly to the symmetric and the anti-symmetric eigenstates and are given by

\[
|\psi\rangle^\pm = \cos \theta |0\rangle_L - \sin \theta \left[ \frac{1}{\sqrt{2}} (|0\rangle_{RL} \pm |0\rangle_{RR}) \right],
\]  

(5)

\[
= \cos \theta |0\rangle_L - \sin \theta |0\rangle^\pm_R,
\]  

(6)

where the mixing angle, $\theta$, is defined as

\[
\tan \theta = \sqrt{2} \frac{\Omega_{LM}}{\Omega_{MR}}.
\]  

(7)

In order to create a coherent superposition for a single atom between the two components of the double-well trap, we initially place the atom in the leftmost trap. Since there is no direct coupling to the final state, indirect coupling using the STIRAP method is utilised. This involves following two stages for the process: first, the tunnelling interactions between the double-well and the middle trap are increased and decreased and, second, the same procedure is followed for the left and the middle trap, only with a time-delay $\Delta t$. As above, this increase and decrease in the tunnelling strength is achieved by varying the distances between the traps, while ensuring that no mechanism other than tunnelling is energetically allowed at any time during the process evolution. The whole process is facilitated without ever moving the middle trap and the total time for one approach and reproach process is given by $T$. While it is vital that $T + \Delta t$ be chosen such that it is large enough to ensure the whole process is adiabatic with respect to excitations within the left and the middle trap, it is also essential that the process be adiabatic with respect to the tunnel splitting in the double-well trap, leading to the condition

\[
T + \Delta t > \frac{1}{\Omega_R}.
\]  

(8)

Once the above process has been carried out, the atomic wavefunction has been transferred completely from the left hand trap to the double well trap and it possesses a symmetry that is determined by the difference between the trapping frequencies, $\omega - \omega_R$. To demonstrate this effect, we have numerically integrated the Schrödinger equation and the results are shown in Fig. 3. It is clearly seen that the symmetry of the final state is correctly chosen by the system and that it depends on the trapping frequency $\omega_R$. We have recently shown that this process is very robust, even in the presence of noise [13]

One detail worth noting from Fig. 3 is that there is never any population at $z = 0$ during the transfer process and this is in accordance with the properties of the dark state.
Figure 3. Dark state transition with a double trap on the right hand side. The graph on the left shows the anti-symmetric situation for \( \omega - \omega_R = -\Omega_R \), as clearly visible by the node between the distribution within the two halves of the double-well trap. The graph on the right shows the symmetric final state for \( \omega - \omega_R = \Omega_R \).

Figure 4. Schematic of the setup for state filtering. The trapping frequencies for the traps in the middle and on the right are three times the value of the frequency of the trap on the left. Initially a particle is in a superposition of the ground and first excited state of the left trap.

3. State Filtering

In the above example, we have shown how the resonance condition for a STIRAP-type transport process can be used for selective quantum state preparation by choosing the trapping frequencies. In this section we will demonstrate a simple example, whereby this condition can be used to create a spatial superposition state from a superposition state in energy space.

The setup consists of three harmonic traps in a linear configuration as shown in Fig. 4. The trapping frequencies are chosen such that \( \omega_L = \frac{1}{3} \omega_M = \frac{1}{3} \omega_R \), i.e. the middle and right traps have a trapping frequency that is three times larger than that for the trap on the left and can, therefore, be viewed as tight traps. Due to the harmonic nature of the traps, this implies that the ground states of the tight traps are in resonance with the first excited state of the trap on the left, i.e.

\[
E^1_L = \frac{3}{2} \hbar \omega_L = \frac{1}{2} \hbar \omega_{R,M} = E^0_{R,M},
\]

where \( E^m_m \) represents the energy of the \( n \)-th state for the trap \( m = \{L, M, R\} \). The Hamiltonian of the
Figure 5. Left: Atomic density during the STIRAP process. The transition process is clearly visible. Right: Final state after the filtering process (solid line, blue) and trapping potential (dashed line, red). It is clear that both states involved in the superposition are ground states. All lengths are in units of the ground state size of the left trap and time is in units of $1/\omega_L$.

The system can then be written as

$$H = \hbar \begin{pmatrix} \frac{1}{2}\omega_L & 0 & 0 & 0 \\ 0 & \frac{3}{2}\omega_L & -\Omega_{LM}(t) & 0 \\ 0 & -\Omega_{LM}(t) & \frac{1}{2}\omega_M & -\Omega_{MR}(t) \\ 0 & 0 & -\Omega_{MR}(t) & \frac{1}{2}\omega_R \end{pmatrix}. \tag{10}$$

In principle, there is a small probability for tunnelling from the ground state of the left trap into the ground state of the middle trap to occur; however, the matrix element for this process is much smaller than all other tunnelling matrix elements and we can, as a result, safely neglect it. It is, therefore, immediately clear that the ground state of the trap on the left decouples from all the other states in the system and we need only consider an effective three-level system for the dynamics, defined by $|1\rangle_L$, $|0\rangle_M$ and $|0\rangle_R$. As before, we can show that the system possesses a dark state, given by

$$|\psi_D\rangle = \cos \theta |1\rangle_L - \sin \theta |0\rangle_R. \tag{11}$$

Let us, now, consider an atom in a superposition state $|\psi_i\rangle = \frac{1}{\sqrt{2}}(|0\rangle_L + |1\rangle_L)$ and apply a STIRAP-type sequence (as detailed above) to the system. We can, immediately, deduce the final state by using the dark state above and it is given by

$$|\psi_f\rangle = \frac{1}{\sqrt{2}}(|0\rangle_L + |0\rangle_R). \tag{12}$$

Closer examination of this state shows that, while the initial state was a superposition in energy space only, the final state is a superposition in energy space and in real space. This arises since translation is a zero mode of the system and a distinct advantage of such a scheme is that it allows for relatively easy state measurement of the superposition state. While measuring vibrational quantum numbers is not always straightforward, measuring positions in space is a far easier task.

To demonstrate this process we have performed a straightforward integration of the Schrödinger equation using $|\psi_i\rangle$ as the initial state. As can be clearly seen from Fig. 5, the wavefunction is initially located entirely in the left trap. After the approaching sequence, the wavefunction is split equally between the ground states of the left and the right trap. The maximum of the density in the trap on the right hand side is larger than the maximum in the left hand side trap, due to the larger value of $\omega_R$. 

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4. Conclusion

Three level atom optics is a relatively new area of research that holds many promises for manipulating single atoms. Due to the robustness and high fidelity of the STIRAP technique, it is particularly useful for quantum state preparations. For electronic systems, one of the main advantages of this technique is its insensitivity to spontaneous emissions from the excited state - an aspect that is not an issue in atomic systems. In fact, the usefulness of STIRAP-type processes in atomic systems comes from the high degree of freedom achievable in the creation of a microtrap. The only condition that must be fulfilled is that of energy conservation. Therefore, the shape of the relevant potentials is the only restriction imposed.

In this paper, we have presented two examples in which we have exploited this freedom in order to study STIRAP-type processes for single atoms. In the first example, the final state of a system was either the symmetric or the antisymmetric ground state of a double-well trap, depending on the choice of trapping frequencies. This allows for the creation of spatial superposition states in which one has perfect control over the population distribution and also the phase relation.

In the second example, we have shown how a state initially in a superposition of two kinetic energy states can be transformed using the translational zero mode into a spatial superposition state. This process is useful since it makes it easier to measure the state of a system, due to the fact that spatial measurements are generally far less difficult to perform compared to energy measurements.

Finally, we would like to point out that single atom trapping and dynamic control over trap centres is a very active field in experimental research and significant progress has been made using strongly focused dipole traps and holographic techniques [15–17].

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