Observational constraints on soft dark energy and soft dark matter: challenging ΛCDM cosmology

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Soft cosmology is an extension of standard cosmology allowing for a scale-dependent equation-of-state (EoS) parameter in the dark sectors, which is one of the properties of soft materials in condensed-matter physics, that may arise either intrinsically or effectively. We use data from Cosmic Microwave Background (CMB), Baryonic Acoustic Oscillations (BAO), Supernovae Type Ia (SNIa), and Redshift space distortion (RSD) probes, in order to impose observational constraints on soft dark energy and soft dark matter. We examine three simple models, corresponding to the minimum extensions of ΛCDM scenario, namely we consider that at large scales the dark sectors have the EoS's of ΛCDM model (dust dark matter and cosmological constant respectively), while at intermediate scales either dark energy or dark matter or both, may have a different EoS according to constant “softness” parameters s_{de} and s_{dm}. The observational confrontation shows that for almost all datasets the softness parameters deviate from their ΛCDM values, in a prominent way for soft dark energy and mildly for soft dark matter, and thus the data favor soft cosmology. Finally, performing a Bayesian evidence analysis we find that the examined models are certainly preferred over ΛCDM cosmology.

I. INTRODUCTION

The concordance paradigm of cosmology has been proven very efficient, both qualitatively and quantitatively, in describing the Universe features at early and late times, as well as at large and small scales. However, the appearance of a huge and increasing amount of observational data of constantly improving precision, places the former under a lasting testing. In this procedure, even slight deviations and tensions between theory and observations, as well as purely theoretical motivations, has lead to a large variety of extensions and modifications of the standard model of cosmology [1–3].

In the usual avenues of modification one may introduce various new sectors, fields, fluids, alongside the usual particles [4, 5], or one may construct new gravitational theories with richer structure and phenomenology [6, 7]. Nevertheless, there is a rather strong assumption in all of these classes of theories, scenarios and models, namely that the Universe sectors can be described by the physics, the thermodynamics and hydrodynamics of usual matter, namely of “hard” matter. In particular, the underlying assumption is that the behavior of the Universe at large scales can be determined by the interactions between its individual constituents, and thus one can introduce the individual sectors’ energy densities and pressures corresponding to “particles” flowing collectively in a simple way.

Recently the possibility of soft cosmology appeared in the literature [8]. In this framework one introduces small deviations from standard cosmology due to the effective appearance of soft properties in the Universe sectors. Since soft matter, due to scale-dependent effective interactions that are not present at fundamental scales [9, 10], is characterized by complexity and simultaneous co-existence of phases, one feature of soft cosmology is the consideration of a scale-dependent equation-of-state (EoS) parameter for the dark sectors. Thus, one can consider that, intrinsically or effectively, dark energy and/or dark matter may have a different EoS at large scales, i.e., at scales entering the Friedmann equations, and a different one at intermediate scales, i.e., at scales entering the perturbation equations.

We mention here that there has been extensive work in the literature in which the authors consider by hand various parametrizations of the dark-energy EoS parameter, where it evolves in time (e.g., the Chevallier-Polarski-Linder (CPL) parametrization [11, 12]). Hence, one has equal right to consider EoS parameters that change with scale instead of time, which is a very developed and well-studied case in condensed-matter physics. Namely, since the physics of dark energy and dark matter is not known at the fundamental level, it is justifiable to consider that at an effective level they exhibit soft properties. Eventually, since all these cosmological models are phenomenological, the confrontation with observational data will be the crucial test for their viability.

The above possible deviation of the large-scale (ls) and
intermediate-scale (is) EoS can be quantified by introducing the softness function $s$. Although in general $s$ can (and should) be scale-dependent, the simplest case is when it is just a constant. Hence, in this simplest scenario one introduces the softness parameters for the dark energy $s_{de}$ and dark matter $s_{dm}$ sector as [8]

$$w_{de-is} = s_{de} \cdot w_{de-is}$$

$$w_{dm-is} + 1 = s_{dm} \cdot (w_{dm-is} + 1),$$

where $w_{de-is}, w_{dm-is}$ are the large-scale EoS for dark energy and dark matter respectively, while $w_{de-is}, w_{dm-is}$ the intermediate-scale ones (mind the difference in the two parametrizations in order to handle the fact that dust dark matter EoS at large scales is zero). Obviously, standard cosmology is recovered for $s_{de} = s_{dm} = 1$, in which case the large-scale and intermediate-scale EoS for each sector coincide. We mention here that the above consideration is independent of the gravitational theory, namely it can be applied both to the framework of general relativity, as well as to modified gravity.

In this work we desire to confront soft dark energy and soft dark matter with data from Supernovae Type Ia (SNIa), Baryonic Acoustic Oscillations (BAO) and Cosmic Microwave Background (CMB) observations. In particular, we want to examine whether non-trivial values for the softness parameters, namely values different than one, are allowed by the data, and if yes whether the scenario of soft cosmology is favored comparing to ΛCDM paradigm. Interestingly enough we find that this is indeed the case: Soft dark energy and soft dark matter are favored over ΛCDM, although they have one more parameter.

II. SOFT COSMOLOGY

In this section we briefly review soft cosmology. We consider a flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry with metric

$$ds^2 = dt^2 - a^2(t) \gamma_{ij} dx^i dx^j,$$

while extension to non-flat case is straightforward. Additionally, we introduce the usual baryonic matter and usual radiation, the dark matter sector, as well as the dark energy sector. Since the cosmological scales are suitably large that allow us to neglect the microphysics of the universe ingredients and describe them effectively, we introduce their energy momentum tensors as

$$T^{(i)}_{\mu\nu} = (\rho_i + p_i) u_\mu u_\nu + p_i g_{\mu\nu},$$

where $\rho_i$ and $p_i$ are the energy density and pressure of the fluid corresponding to the $i$-th sector (with $i$ being $b$, $r$, $dm$, $de$ denoting baryonic matter, radiation, cold dark matter, and dark energy respectively), and with $u_\mu$ the 4-velocity vector field. Hence, the dynamics of the universe at the background level is determined by the two Friedmann equations

$$H^2 = \frac{k^2}{3}(\rho_b + \rho_r + \rho_{dm} + \rho_{de}),$$

$$2\dot{H} + 3H^2 = -k^2(\rho_b + \rho_r + \rho_{dm} + \rho_{de}),$$

with $H \equiv \dot{a}/a$ the Hubble parameter (dots denote time derivatives), and $k^2 = 8\pi G$. Moreover, the conservation equation $\nabla_{\mu} T^{(tot)}_{\mu\nu} = \nabla^\alpha [\sum_i T^{(i)}_{\mu\nu}] = 0$ for non-interacting fluids results to the separate conservation equations

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0,$$

while the extension to interacting cosmology is straightforward. Finally, the equations close by assuming that the fluids are barotropic, and thus the pressure is a function of the energy density, the simplest case of which is

$$p_i = w_{i-is} \rho_i,$$

with $w_{i-is}$ the equation-of-state parameter of the $i$-th sector. Note that we have added the subscript “is”, denoting “large scales”, since for the moment we describe the background, i.e. the large-scale, evolution of the universe. Lastly, we mention that the above equations are of general validity, holding for every model of dark matter and dark energy, while the concordance ΛCDM scenario is recovered for $\rho_{de} = -p_{de} = \Lambda/κ^2$, with $\Lambda$ the cosmological constant.

Having described the evolution at the background level, we can proceed to the description of small perturbations around the FLRW background. Focusing to linear scalar isentropic perturbations in the Newtonian gauge we write

$$ds^2 = -(1 + 2\Psi) dt^2 + a^2(t)(1 - 2\Phi) \gamma_{ij} dx^i dx^j,$$

and thus we extract the perturbation equations [8]

$$\ddot{\delta}_i + (1 + w_{i-is})(\frac{\dot{\theta}_i}{a} - 3\dot{\Phi}) + 3H(\gamma^{(i)2}_{\text{eff}} - w_{i-is})\dot{\delta}_i = 0,$$

$$\ddot{\theta}_i + H \left[ 1 - 3w_{i-is} + \frac{w_{i-is}}{H(1 + w_{i-is})} \right] \theta_i - \frac{k^2 \gamma^{(i)2}_{\text{eff}} \delta_i}{(1 + w_{i-is})a} - \frac{k^2 \dot{\Phi}}{a} = 0.$$

In these expressions $\delta_i \equiv \delta \rho_i/\rho_i$ are the density perturbations while $\theta_i$ is the divergence of the fluid velocity, and $k$ is the wavenumber of the Fourier modes (in the case of ΛCDM paradigm the corresponding dark-energy perturbations are not considered). Additionally, we have defined the effective sound speed square for the $i$-th sector as

$$c^{(i)2}_{\text{eff}} = \frac{\delta p_i}{\delta \rho_i}.$$
which determines the clustering properties, being zero for maximal clustering and 1 for no clustering.

Equations (10), (11) are just the standard ones of the literature [13, 14], with the only change being the replacement of the EoS’s of the various sectors by $w_{i}^{-ls}$, which as we described is the essence of the soft-matter properties. Specifically, we consider that at intermediate scales, namely at the scales dominating the perturbation equations, the dark matter and dark energy fluids may have a different EoS than the one they have at large scales, namely at scales entering the Friedmann equations. Actually this is a quite reasonable consideration, since there is no fundamental reason of why the EoS should be the same at all scales, since the behavior of a sector at perturbation level is in general independent of its behavior at large scales (for instance the effective sound speed square is an independent input from the large-scale EoS in standard cosmology, and in the same lines the intermediate-scale EoS is an independent input from the large-scale EoS in soft cosmology). However, we mention here that one should be careful with the above simplified formulation not to spoil the total energy-momentum tensor conservation, otherwise a more rigid formulation of soft cosmology would be needed, in the lines of condensed-matter literature [9, 10].

As explained in [8], the reason for the appearance of soft properties in the dark sectors may be intrinsic or effective. In the first case, the unknown microphysics of dark-energy and/or dark matter may induce complexity at intermediate scales which results to scale-dependent EoS, in a similar way that intermediate-scale interactions bring about complexity and alter the intermediate-scale EoS in soft matter materials such as polymers, colloids, surfactants, liquid crystals, etc [9, 10]. In the second case, softness may arise effectively due to the dark-energy clustering, since the development of cluster structure in the dark-energy sector creates intermediate-scale effective interactions (for instance the interaction between two dark-matter clusters below the dark-energy clustering scale, namely two dark-matter clusters with sparse dark energy between them, will be different than the interaction between two dark-matter clusters with a dark-energy cluster between them). Finally, once again we mention that the consideration of softness is independent from the underlying gravitational theory, and can be applied also in the case where dark energy is of effective gravitational origin.

Apart from the above change in the EoS, one applies all the steps of standard cosmology. For instance, considering the Poisson equation at sub-horizon scales:

$$-\frac{k^2}{a^2} \Psi = \frac{3}{2} H^2 \sum_i \left[ \left( 1 + 3c_{\text{eff}}^{(2)} \right) \Omega_i \delta_i \right],$$

(13)

with $\Omega_i \equiv \kappa^2 \rho_i/(3H^2)$ the density parameters for the various sectors, we can eliminate the fluid velocities and extract equations for the density perturbations. Lastly, the whole analysis can be extended in various ways, including viscosity, heat flux, interactions between dark energy and dark matter, etc.

In summary, in this first approach on soft cosmology we apply all techniques and equations of standard cosmology, with the only change being to allow for a different EoS at the background and perturbation levels, namely the consideration of relations (1) and (2). Note that we start with a simple model in which the scale-dependent EoS are determined by a simple, constant softness parameter for each sector, since we focus on sub-horizon scales $k \gg aH$ and thus to perturbation modes affected only by the intermediate-scale dark-energy EoS (the full analysis, in which different perturbation modes are affected by different EoS according to their scale, and where $s_i = s_i(k)$, will be presented elsewhere). Finally, standard cosmology is recovered for $s_{de} = s_{dm} = 1$.

In the following we will confront the scenario with observational data, and we are interested in extracting the constraints on the softness parameters $s_{de}$ and $s_{dm}$. Since dark energy and dark matter may be soft independently, we will examine all combinations, namely soft dark energy with usual dark matter, soft dark matter with usual dark energy, and soft dark energy and dark matter simultaneously.

The last step is to consider specific models of standard cosmology and construct their soft extension. In the present work we focus on the simplest soft cosmological models, namely the soft extensions of ΛCDM paradigm.

1. Model 1: Soft dark energy

As the first soft extension of ΛCDM scenario we consider a model where dark matter is the usual, non-soft, dust one at all scales, while dark energy is the soft component with large-scale behavior that of a cosmological constant. Hence, we impose fixed $s_{dm} = 1$, namely

$$w_{dm-1s} = w_{dm-is} = 0$$

(14)

as in the standard dust dark matter case, while we set

$$w_{de-1s} = -1$$
$$w_{de-is} = s_{de}w_{de-1s} = -s_{de}.$$  

(15)

Thus, $s_{de}$ is the only extra free parameter comparing to ΛCDM cosmology, and the latter is recovered for the value $s_{de} = 1$.

2. Model 2: Soft dark matter

As another soft extension of ΛCDM scenario we consider a model where dark energy is the usual cosmological constant, with

$$w_{de-1s} = w_{de-is} = -1,$$  

(16)
however dark matter is soft with
\[ w_{dm-ls} = 0 \]
\[ w_{dm-is} = s_{dm} - 1, \tag{17} \]
according to (2). Thus, \( s_{dm} \) is the only extra free parameter comparing to ΛCDM cosmology, and the latter is recovered for the value \( s_{dm} = 1 \).

3. Model 3: Soft dark energy and soft dark matter

In this more advanced extension of ΛCDM scenario we consider a model in which both dark energy and dark matter have soft properties, namely we set \[ w_{de-ls} = -1 \]
\[ w_{dm-ls} = 0, \tag{18} \]
while at intermediate scales we consider (1) and (2). In this case there are two extra free parameters, namely \( s_{de} \) and \( s_{dm} \), and ΛCDM paradigm is recovered for \( s_{de} = s_{dm} = 1 \).

III. OBSERVATIONAL DATASETS AND STATISTICAL METHODOLOGY

In this section we describe the observational datasets and the methodology to constrain the cosmological models under investigation. The cosmological probes that we use are the following.

- **Cosmic Microwave Background (CMB) Observations**: We use the CMB measurements from the Planck 2018 final release. Specifically, we use the CMB temperature and polarization angular power spectra plikTTTEEE+lowl+lowE [15, 16].

- **Baryon Acoustic Oscillations (BAO)**: We consider several measurements of the BAO data from different galaxy surveys, namely 6dFGS [17], SDSS-MGS [18], and BOSS DR12 [19], as used by the Planck 2018 team [16].

- **Pantheon sample of Supernovae Type Ia (SNIa) data**: We consider the Pantheon sample of the SNIa consisting of 1048 data points which are distributed in the redshift interval \( z \in [0.01, 2.3] \) [20].

- **Redshift Space Distortion (RSD)**: We consider 22 data points of \( f\sigma_8 \) from Table I of [21], presented in Table I below.

In order to constrain the parameter space of each cosmological model of the previous section, we have modified the publicly available Markov Chain Monte Carlo (MCMC) package CosmoMC [37] which supports the Planck 2018 likelihood [15], and additionally it is well equipped with the Gelman-Rubin convergence statistics, quantified through \( R - 1 \) [38]. We mention that we continue the running of the chains until their convergences achieve \( R - 1 < 0.02 \).

| \( z \) | \( f\sigma_8 \) | \( \sigma_{f\sigma_8} \) | \( \Omega_{m0, fid} \) | Ref. |
|---|---|---|---|---|
| 0.02 | 0.428 | 0.0465 | 0.3 | [22] |
| 0.02 | 0.398 | 0.065 | 0.3 | [23], [24] |
| 0.02 | 0.314 | 0.048 | 0.266 | [24], [25] |
| 0.1 | 0.37 | 0.13 | 0.3 | [26] |
| 0.15 | 0.49 | 0.145 | 0.31 | [27] |
| 0.17 | 0.51 | 0.06 | 0.3 | [28] |
| 0.18 | 0.36 | 0.09 | 0.27 | [29] |
| 0.38 | 0.44 | 0.06 | 0.27 | [29] |
| 0.25 | 0.3512 | 0.0583 | 0.25 | [30] |
| 0.37 | 0.4602 | 0.0378 | 0.25 | [30] |
| 0.32 | 0.384 | 0.095 | 0.274 | [31] |
| 0.59 | 0.488 | 0.06 | 0.307 | [32] |
| 0.44 | 0.413 | 0.08 | 0.27 | [33] |
| 0.6 | 0.39 | 0.063 | 0.27 | [33] |
| 0.73 | 0.437 | 0.072 | 0.27 | [33] |
| 0.6 | 0.55 | 0.12 | 0.3 | [34] |
| 0.86 | 0.4 | 0.11 | 0.3 | [34] |
| 1.4 | 0.482 | 0.116 | 0.27 | [35] |
| 0.978 | 0.379 | 0.176 | 0.31 | [35] |
| 1.23 | 0.385 | 0.099 | 0.31 | [36] |
| 1.526 | 0.342 | 0.07 | 0.31 | [36] |
| 1.944 | 0.364 | 0.106 | 0.31 | [36] |

TABLE I: Observational data of the redshift space distortion (RSD). The first column contains the redshift, the second the observed value, the third the corresponding error and the fourth the \( \Omega_{m0} \) of the fiducial ΛCDM cosmology used to extract the measurements from the LSS power spectrum. This dataset was compiled by [21].

IV. RESULTS

In this section we present the observational constraints on the three soft cosmological scenarios presented in section II, using the datasets and methodology described in the previous section. We mention that apart from the free parameters of the models, namely \( s_{de} \) and \( s_{dm} \), some of the key derived parameters are as usual \( H_0 \), \( \sigma_8 \), and \( r_{\text{drag}} \), where \( H_0 \) is the Hubble constant at present time (in units Km/s/Mpc), \( \sigma_8 \) is the matter-power spectrum normalization on scales of \( 8h^{-1} \) Mpc, and \( r_{\text{drag}} \) is the sound horizon at the epoch of baryon decoupling. Moreover, we set \( c_{\text{eff}}^{(dm)} = 0 \) as usual however we do handle \( c_{\text{eff}}^{(de)} \) as a free parameter in [0, 1] in order to be quite general on the dark-energy clustering properties, since as described above the dark-energy clustering can lead to the effective appearance of softness even if softness is intrinsically absent.
A. Observational constraints on soft cosmology

1. Model 1: Soft dark energy

The observational constraints for this model are summarized in Table II for CMB, CMB+BAO and CMB+BAO+Pantheon, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD datasets. Additionally, in Fig. 1 we provide the one-dimensional marginalized posterior distributions for some selected parameters, and the two-dimensional likelihood contours.

Focusing on the key parameter, \( s_{de} \), we find that for CMB alone, \( s_{de} = 0.573^{+0.345}_{-0.326} \) (at 68% CL), which does not coincide with \( s_{de} = 1 \) at more than 68% CL (recall that \( s_{de} = 1 \) corresponds to the ΛCDM model). However, within 95% CL, \( s_{de} \) is consistent to 1 which indicates the ΛCDM cosmology. When the BAO data are added to CMB, we again find that \( s_{de} = 0.561^{+0.357}_{-0.354} \) (at 68% CL for CMB+BAO) which clearly shows that at more than 68% CL, we have a signal for soft DE but indeed similar to the CMB alone case, the 95% CL constraint on \( s_{de} \) is consistent to its corresponding value for ACeDM. The conclusion on the \( s_{de} \) does not change for the remaining datasets, such as CMB+BAO+Pantheon, CMB+BAO+Pantheon, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD. It is quite interesting to mention that CMB alone and all the combined observational datasets clearly indicate the preference for soft dark energy and hence a deviation from the ΛCDM scenario.

We would like to mention here that the model at hand behaves as ΛCDM at large scales, and eventually as wCDM with \( w = -s_{de} \) at intermediate scales, however it is a new model, different from both. Hence, the fact that we find \( w = -s_{de} = -0.573^{+0.345}_{-0.326} \) is not in contradiction with the fact that in wCDM ones finds \( w = -1.58^{+0.52}_{-0.41} \) [16], since every cosmological scenario is a new scenario and thus its confrontation with the data can give quite independent and different results.

Concerning the key derived parameter \( H_0 \), we find that the obtained constraints are almost similar to what we have observed from Planck 2018 [16], and thus within this scenario the existing discrepancy of the Hubble constant between the Planck (within ΛCDM) [16] and SH0ES collaboration [39] is not alleviated, which was of course expected since the background evolution is identical to ΛCDM model. Similarly, if we also concentrate on the estimated values of the \( S_8 \) parameter from CMB alone and other combined datasets, we do not find any evidence for a lower value of \( S_8 \) which thus means that the tension on this parameter is not alleviated within this scenario.

2. Model 2: Soft dark matter

The observational constraints for this model are summarized in Table III, while in Fig. 2 we present the one-dimensional marginalized posterior distributions for some selected parameters, as well as the two-dimensional likelihood contours, for several cosmological datasets, namely, CMB, CMB+BAO, CMB+BAO+Pantheon, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD.

For CMB dataset alone, the estimated value of the softness parameter is \( s_{dm} = 1.00094^{+0.00088}_{-0.00088} \) at 68% CL, which shows that \( s_{dm} \neq 1 \) (recall that \( s_{dm} = 1 \) corresponds to ΛCDM cosmology) at 68% CL and therefore we obtain a preference for the soft dark matter within 1σ (although the 95% CL bounds on \( s_{dm} \) (\( s_{dm} = 1.00094^{+0.00075}_{-0.00180} \)) makes it consistent to the value 1). Thus, a mild indication of the soft dark matter is still preferred for this case. When BAO data are added to CMB, we find that \( s_{dm} \) allows 1 within 68% CL (\( s_{dm} = 1.00080^{+0.00099}_{-0.00089} \) for CMB+BAO at 68% CL). For the remaining combinations with Pantheon and RSD, our conclusion does not change comparing to CMB+BAO. This implies that within 68% CL, even though \( s_{dm} \) is consistent with the value 1, different values are allowed too. Interestingly, contrary to the previous scenario of soft dark energy, in this case one can clearly notice that the softness parameter \( s_{dm} \) is correlated with \( S_8 \) and \( r_{drag} \) as shown in Fig. 2. Specifically, the correlation of \( s_{dm} \) with \( S_8 \) is very appealing in the context of cosmological tensions since from the 2D plot between \( (s_{dm}, S_8) \) (see Fig. 2), one can notice that the lower values of \( S_8 \) are indicated for values of \( s_{dm} \) below the value of \( s_{dm} = 1 \). We note that any deviation of \( s_{dm} \) from 1 indicates the preference of soft dark matter. Even though for the present employed datasets we do not find any such strong preference for low values of \( S_8 \), however, this certainly demands the analysis with the cosmic shear measurements [40, 41]. This will be performed in a separate analysis.

Finally, similarly to the soft dark energy scenario, the constraints on \( H_0 \) are similar to the reported values by Planck 2018 [16], which is expected since the background evolution of the present model is ΛCDM scenario. Our observation on the \( S_8 \) parameter does not also change similar to the soft dark energy scenario.

3. Model 3: Soft dark energy and soft dark matter

Let us now investigate the model where both dark energy and dark matter are allowed to be soft.
| Parameters   | CMB          | CMB+BAO       | CMB+BAO+Pantheon | CMB+BAO+RSD | CMB+BAO+Pantheon+RSD |
|-------------|--------------|---------------|------------------|-------------|----------------------|
| $\Omega_m h^2$ | 0.12636      | 0.11940       | 0.11924          | 0.11873     | 0.11865              |
| $\Omega_b h^2$ | 0.0236       | 0.02242       | 0.02243          | 0.02246     | 0.02246              |
| $100\theta_{MC}$ | 1.04990      | 1.04101       | 1.04102          | 1.04105     | 1.04106              |
| $\tau$       | 0.0548       | 0.0560        | 0.0564           | 0.0567      | 0.0572               |
| $n_s$        | 0.9641       | 0.9663        | 0.9667           | 0.9675      | 0.9681               |
| $\ln(10^9A_s)$ | 3.049        | 3.047         | 3.047            | 3.047       | 3.047               |
| $s_{de}$     | 0.8375       | 0.8104        | 0.8104           | 0.8104      | 0.8104              |
| $\Omega_m$   | 0.345        | 0.337         | 0.348            | 0.348       | 0.348               |
| $\sigma_8$   | 0.9293       | 0.8104        | 0.8104           | 0.8104      | 0.8104              |
| $H_0$        | 67.71        | 67.69         | 67.71            | 67.99       | 67.99               |
| $S_8$        | 0.8364       | 0.8260        | 0.8241           | 0.8241      | 0.8241              |
| $\sigma_{eq}$ | 3410.69      | 3389.11       | 3385.56          | 3374.06     | 3372.12             |
| $r_{drag}$   | 147.01       | 147.20        | 147.23           | 147.33      | 147.34              |

TABLE II: The 1σ and 2σ confidence level constraints on the free and derived cosmological parameters of Model 1: Soft dark energy, along with the mean values of the parameters within the 1σ area of the MCMC chain, for CMB, CMB+BAO, CMB+BAO+Pantheon, CMB+BAO+RSD, and CMB+BAO+Pantheon+RSD datasets. We mention that $S_8 = \sigma_8 \sqrt{\Omega_m h^2}/0.3$, $h = H_0/100$ Km/s/Mpc is the normalized Hubble parameter, and $\sigma_{eq}$ is the redshift at the matter-radiation equality.

FIG. 1: The 1σ and 2σ two-dimensional iso-likelihood contours, alongside the one-dimensional posterior distributions, for Model 1: Soft dark energy, for CMB, CMB+BAO, CMB+BAO+Pantheon, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD datasets. The combined analysis indicates a deviation from $\Lambda$CDM scenario and favors soft dark energy.
TABLE III: The 1σ and 2σ confidence level constraints on the free and derived cosmological parameters of Model 2: Soft dark matter, alongside the mean values of the parameters within the 1σ area of the MCMC chain, for CMB, CMB+BAO, CMB+BAO+Pantheon, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD datasets. We mention that $S_8 = \sigma_8 \sqrt{\Omega_{m0}/0.3}$, $h = H_0/100$ Km/s/Mpc is the normalized Hubble parameter, and $z_{eq}$ is the redshift at the matter-radiation equality.

FIG. 2: The 1σ and 2σ two-dimensional iso-likelihood contours, alongside the one-dimensional posterior distributions, for Model 2: Soft dark matter, for CMB, CMB+BAO, CMB+BAO+Pantheon, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD datasets. The analysis indicates a deviation from $\Lambda$CDM scenario and favors soft dark matter.
The analysis indicates a deviation from the mean values of the parameters within the 1σ area of the MCMC chain, for CMB, CMB+BAO, CMB+BAO+Pantheon, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD datasets. We mention that $S_b = \sigma_s \sqrt{\Omega_{mb}/0.3}$, $h = H_0/100$ Km/s/Mpc is the normalized Hubble parameter, and $z_{eq}$ is the redshift at the matter-radiation equality.
The observational constraints are summarized in Table IV, and in Fig. 3 we show the one-dimensional marginalized posterior distributions for some selected parameters, as well as the two-dimensional likelihood contours, for various cosmological datasets, namely CMB, CMB+BAO, CMB+BAO+Pantheon, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD.

For CMB dataset alone, the 68% CL constraints on the soft parameters are, \( s_{dm} = 0.0008836^{-0.0008835} \) and \( s_{de} = 0.551^{+0.321}_{-0.382} \) which clearly indicate that within 68% CL, soft cosmology is preferred and a deviation from the \( \Lambda \)CDM cosmology is perfectly suggested within 1σ. While the 95% CL bounds on the soft parameters for CMB alone allow for \( \Lambda \)CDM cosmology, we cannot exclude the possibility of soft dark matter and dark energy. When BAOs are added to CMB, the preference for soft dark energy within 68% CL still persists \( s_{de} = 0.565^{+0.350}_{-0.371} \) at 68% CL for CMB+BAO) while within 95% CL, \( s_{de} \) \( \Lambda \)CDM cosmology is allowed (i.e. \( s_{de} = 1 \)). Concerning the remaining softness parameter, i.e. \( s_{dm} \), we can see that \( s_{dm} = 1 \) is allowed within this statistical level: \( s_{de} = 1.00070^{+0.00980}_{-0.00985} \) at 68% CL for CMB+BAO.

Even though one of the softness parameters does not exhibit any strong deviation from 1 for these combined datasets, the joint picture remains in favor of a soft cosmological model and hence a deviation from \( \Lambda \)CDM cosmological scenario is suggested for CMB+BAO. The inclusion of Pantheon data to this combined dataset, i.e. CMB+BAO, does not offer any new results and we find again a preference for soft cosmology, mainly driven by the soft dark energy. The analyses in presence of RSD data, i.e. for the combined datasets, CMB+BAO+RSD and CMB+BAO+Pantheon+RSD, also prefer a deviation from \( \Lambda \)CDM cosmology within 68% CL, and thus indicating the evidence of soft dark energy in the joint picture. Finally, similarly to Model 1, we mention here that although the present model seems to be similar to \( w \)CDM at intermediate scales, overall it is a new model and thus the above fitting results are not in contradiction with the constraints of \( w \)CDM, i.e. \( w = -1.58^{+0.32}_{-0.41} \) [16].

In summary, even this combined scenario of soft dark energy and soft dark matter is favored by the data, and the analysis indicates a deviation from \( \Lambda \)CDM cosmology. Similarly to Model 2 (soft dark matter), the correlation of \( s_{dm} \) with \( S_8 \) exists in this case too (see Fig. 3). Finally, for \( H_0 \) and \( S_8 \) we find similar results with the previous two scenarios, compared to the \( \Lambda \)CDM-based Planck results [16].

### V. CONCLUSIONS

In this work we used data from Cosmic Microwave Background (CMB), Baryonic Acoustic Oscillations (BAO), Supernovae Type Ia (SNIa), and Redshift space distortion (RSD) probes, in order to impose observational constraints on soft dark energy and soft dark matter. Soft cosmology is an extension of standard cosmology obtained through the relaxation of the rather strong assumption that the dark sectors behave like simple, i.e. hard matter. Since soft matter is characterized by complexity and simultaneous co-existence of phases, in soft cosmology one allows for a scale-dependent equation-of-state parameter in the dark energy and/or dark matter, with the simplest case being a given EoS at large scales, i.e at scales entering the background evolution, and a different EoS at intermediate scales, i.e at scales entering the perturbation evolution. Such a property may arise intrinsically, due to the unknown microphysics of dark energy and dark matter, or it may arise effectively due to the dark-energy clustering which may in-

| \( \ln B_{ij} \) | Strength of evidence for model \( M_i \) |
|----------------|----------------------------------|
| \( 0 \leq \ln B_{ij} < 1 \) | Weak |
| \( 1 \leq \ln B_{ij} < 3 \) | Definite/Positive |
| \( 3 \leq \ln B_{ij} < 5 \) | Strong |
| \( \ln B_{ij} \geq 5 \) | Very strong |

TABLE V: The revised Jeffreys scale [45] used to compare the statistical efficiency of model \( M_i \) with respect to the reference model \( M_j \) (typically \( \Lambda \)CDM).
TABLE VI: The calculated values of $\ln B_{ij}$, where $i$ refers to the three soft cosmological models and $j$ stands for the reference $\Lambda$CDM scenario, for the various datasets. Positive values indicate that the examined models are favored over the reference $\Lambda$CDM scenario, while negative values indicate that the reference model is preferred. DE and DM denote dark energy and dark matter, respectively.

| Model                                      | Data                        | $\ln B_{ij}$ |
|--------------------------------------------|-----------------------------|--------------|
| Model 1: Soft DE                           | CMB                         | 3.1          |
| Model 1: Soft DE                           | CMB+BAO                     | 2.8          |
| Model 1: Soft DE                           | CMB+BAO+Pantheon            | 1.1          |
| Model 1: Soft DE                           | CMB+BAO+RSD                | 2.2          |
| Model 1: Soft DE                           | CMB+BAO+Pantheon+RSD       | 1.2          |
| Model 2: Soft DM                           | CMB                         | 2.1          |
| Model 2: Soft DM                           | CMB+BAO                     | 1.5          |
| Model 2: Soft DM                           | CMB+BAO+Pantheon            | 0.3          |
| Model 2: Soft DM                           | CMB+BAO+RSD                | 0.3          |
| Model 2: Soft DM                           | CMB+BAO+Pantheon+RSD       | 0.7          |
| Model 3: Soft DE and soft DM               | CMB                         | 2.9          |
| Model 3: Soft DE and soft DM               | CMB+BAO                     | 1.9          |
| Model 3: Soft DE and soft DM               | CMB+BAO+Pantheon            | 1.6          |
| Model 3: Soft DE and soft DM               | CMB+BAO+RSD                | 1.9          |
| Model 3: Soft DE and soft DM               | CMB+BAO+Pantheon+RSD       | 1.8          |

The calculated values of $\ln B_{ij}$, where $i$ refers to the three soft cosmological models and $j$ stands for the reference $\Lambda$CDM scenario, for the various datasets. Positive values indicate that the examined models are favored over the reference $\Lambda$CDM scenario, while negative values indicate that the reference model is preferred. DE and DM denote dark energy and dark matter, respectively.

The fact that soft dark energy and soft dark matter seem to challenge $\Lambda$CDM scenario makes it both interesting and necessary to investigate in detail many possible soft extensions of standard cosmological scenarios. For instance instead of the minimum extension of $\Lambda$CDM scenario analyzed in the present work, one could consider the soft extensions of dynamical dark-energy models such as the CPL dark-energy parametrization [11, 12], in which the large-scale dark-energy $\omega$ will be $\omega_{de,ls} = \omega_0 + \omega_a(1 - a)$, on top of which we will apply the dark-energy softness parameter $s_{de}$ according to (1). Similarly one could examine the soft extensions of various interacting models, which are known to solve the $H_0$ tension [49, 50], in which case one expects to have the advantages of both the interaction and softness. Finally, one could consider more realistic cases, where the softness parameter depends on the scale, leading to a smooth transition between large-scale and intermediate-scale equation-of-state parameters. More importantly, since soft cosmology seems to pass the basic confrontation with observational data, one could investigate the theoretical framework for its appearance, and one good starting point might be modified gravity, in which case the softness, i.e. the scale-dependent $\omega$, of the effective dark energy sector may arise naturally due to the richer structure of the gravitational theory. These analyses will be studied in future projects.

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