Obtaining a light-like planar gauge.

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ABSTRACT

In the usual and current understanding of planar gauge choices for Abelian and non Abelian gauge fields, the external defining vector $n^\mu$ can either be space-like ($n^2 < 0$) or time-like ($n^2 > 0$) but not light-like ($n^2 = 0$). In this work we propose a light-like planar gauge that consists in defining a modified gauge-fixing term, $\mathcal{L}_{GF}$, whose main characteristic is a two-degree violation of Lorentz covariance arising from the fact that four-dimensional space-time spanned entirely by null vectors as basis necessitates two light-like vectors, namely $n_\mu$ and its dual $m_\mu$, with $n^2 = m^2 = 0$, $n \cdot m \neq 0$, say, e.g. normalized to $n \cdot m = 2$.

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1 Introduction.

Gauge fields regardless of their Abelian or non Abelian character are by definition fields that display some sort of gauge invariance. Fields which are connected by this invariance must be accounted only once in the process of quantization, otherwise all sort of pathologies arise. The standard and pragmatic way in which one proceeds in dealing with this issue is the introduction of a gauge-breaking term in the Lagrangian density which fixes the gauge choice. This choice is obviously a matter of taste and convenience of one’s preference, although practice has taught us that some choices may bring more technically demanding difficulties than others.

In this respect, algebraic non-covariant gauges have posed many challenges concerning not only the question of their usefulness but also what concerns the inherent technical difficulties and subleties associated with them. For example, an open problem in the non-homogeneous axial gauge, also known as planar gauge [1,2], has to do with the fact that “it is not possible” to define it in the light-like case [3]. Let us briefly review the reason why this is considered so.
The non-homogeneous axial case is defined by the condition:

\[ n^\mu A_\mu^a \equiv n \cdot A^a = \Phi^a \]

where \( n_\mu \) is the arbitrary external and constant vector, \( A_\mu^a \) are the gauge fields (for definiteness we deal with general non-Abelian fields) and \( \Phi^a \) are a set of scalar fields belonging to the adjoint representation of \( SU(N) \) Lie algebra \([3]\). Then the gauge fixing Lagrangian density term is chosen to be:

\[
\mathcal{L}_{GF} = \frac{1}{2n^2} \partial_\mu \Phi^a \partial^\mu \Phi^a + \lambda^a (\Phi^a - n \cdot A^a). \tag{1}
\]

where \( \lambda^a \) are the Lagrange multipliers. It is easy to see that these last equations are in fact equal to:

\[
\mathcal{L}_{GF} = \frac{1}{2n^2} \partial_\mu (n \cdot A^a) \partial^\mu (n \cdot A^a). \tag{2}
\]

Depending on the type of external vector \( n_\mu \) that we consider the following cases are defined:

- \( n^2 < 0 \): space-like planar gauge,

- \( n^2 > 0 \): time-like planar gauge.

However, the light-like planar gauge is not defined, since in this case we meet a singularity at \( n^2 = 0 \).
But in the next section we will see that there is a way out to this inconvenience.

2 Our proposal.

We begin this section by reviewing some important concepts related to null vectors as basis for four dimensional space-time. The “light-likeness” condition $n^2 = 0$ does not uniquely define the necessary external vector $n_\mu$ to implement the gauge condition. The reason for this is most easily seen considering a particular case where $n_\mu = (n_0, 0, 0, n_3)$, in which case the condition $n^2 = 0$ gives as solutions either $n_0 = +n_3$, or $n_0 = -n_3$ with $n_0 > 0$. Therefore those components of the light-like vector are not linearly independent; hence the two possibilities are $n_\mu = (n_0, 0, 0, +n_3)$ and $m_\mu = (n_0, 0, 0, -n_3)$.

These peculiar light-cone vector properties have been demonstrated by Leibbrandt [4] to be connected to the Newman-Penrose [5] tetrad formalism in the context of gravitation and cosmology, where a four-dimensional basis is spanned entirely by null vectors. In his work, Leibbrandt demonstrated that the two-dimensional vector sub-space $(n_0, n_3)$ cannot be spanned solely by the vector $n_\mu$, because this vector possesses linearly dependent components.

Therefore, for our purpose and without loss of generality, we introduce
two light-like vectors, namely \( n_\mu \) and \( m_\mu \), so that (note that here we choose a normalization factor 2 for convenience)

\[
\begin{align*}
  n^2 & = m^2 = 0, \\
  n \cdot m & = 2.
\end{align*}
\]

Our proposal is to consider the following gauge fixing term

\[
\mathcal{L}_{GF} = \frac{1}{4n \cdot m} \partial_\mu \Phi^{2a} \Sigma \partial^\mu \Phi^{2a} + \lambda^{2a}(\Phi^{2a} - \mathcal{N} \cdot A^{2a}).
\]

where

\[
\Sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]

and

\[
\mathcal{N}_\mu = \begin{pmatrix} 1 \otimes n_\mu & 0 \\ 0 & 1 \otimes m_\mu \end{pmatrix}
\]

Also we extend the dimension \( N \) of the Lie algebra to be \( 2N \) or it is extended to the complex case, so that we can split the \( \Phi^{2a} \) \((2a = 1 \cdots 2N)\) fields as:

\[
\Phi^{2a} = \begin{pmatrix} \phi^a \\ \varphi^b \end{pmatrix}
\]
where \(a = 1 \cdots N\) and \(b = N + 1 \cdots 2N\) in the Lie algebra space. It is not too hard to check that the gauge fixing term described in equation (3) will take the form

\[
L_{GF} = \frac{1}{2n \cdot m} \partial_{\mu}(n \cdot A^{a})\partial^{\mu}(m \cdot A^{a}),
\]

just working it through as it was done when going from (1) to (2) (where we have made the ansatz \(A^{a} = A^{b}\)).

This last equation is in fact free from the singularity in the light-like case since now we have \(n \cdot m\) in the denominator and this quantity differs from zero. So it is possible to define (3) to be the gauge fixing Lagrangian density term for the planar gauge in the light-like case.

Note that there is a kind of discrete symmetry in (4) when we exchange \(n \leftrightarrow m\). Making such an exchange (3) results in the “breaking” of the matrix \(\mathcal{N}\), so (3) is not invariant under this kind of symmetry, but if we use this broken gauge fixing Lagrangian, and then proceed to eliminate the \(\Phi^{a}\) fields as it was done when going from (3) to (4) then we will discover that our answer coincides with (4).
With this new gauge fixing term we can find the boson propagator,

\[
G_{\mu\nu}(q) = -\frac{i}{2(2\pi)^4q^2} \left[ 2g_{\mu\nu} - q^-(q_\mu n_\nu + q_\nu n_\mu) - q^+(q_\mu m_\nu + q_\nu m_\mu) + \frac{q_\mu q_\nu}{q^+q^-} \right. \\
+ \frac{(q^-)^2}{q^+q^-}n_\mu n_\nu + \left. \frac{(q^+)^2}{q^+q^-}m_\mu m_\nu - q^+q^- \frac{(n_\mu m_\nu + m_\mu n_\nu)}{q^+q^-} \right]
\]

(5) \equiv -\frac{i}{2(2\pi)^4q^2} \left[ G^{\text{cov}}_{\mu\nu}(q) + G^{\text{planar}}_{\mu\nu}(q, n, m) \right],

Observe that the vector propagator now is much more complex than the usual one which is obtained using only one external vector \( n_\mu \) to fix the gauge choice (one-degree violation of Lorentz covariance). On the other hand, it is clear from this new propagator that the light-cone variables \( q^+ \) and \( q^- \) do form a bilinear term \( q^+q^- \) that cannot be separated without losing important physical information in the process. In fact, separating them in the denominators means violating causality for the field operators. So, although one may be tempted to simplify the above expression into

\[
G^{\text{planar}}_{\mu\nu}(q, n, m) = -\frac{(q_\mu n_\nu + q_\nu n_\mu)}{q^+} - \frac{(q_\mu m_\nu + q_\nu m_\mu)}{q^-} + \frac{q_\mu q_\nu}{q^+q^-} \left[ \frac{q^-n_\mu n_\nu + q^+ m_\mu m_\nu - n_\mu m_\nu - m_\mu n_\nu}{q^+q^-} \right],
\]

this simplification may pose unsuspected mixing of positive- and negative-frequency modes for the quantum fields.
3 Conclusions.

In this work we have considered the algebraic noncovariant gauge choice of the inhomogeneous type — known as planar gauge — introducing a two-degree violation of Lorentz covariance on the gauge breaking term in the Lagrangean density, supported by the fact that four-dimensional space-time spanned solely by null vectors as basis cannot be accomplished without the concurrence of both pairs of dual light-like vectors $n_\mu$ and $m_\mu$. Doing this, we were able to define the planar gauge in the light-like case, however, the new boson vector propagator becomes almost prohibitively complicated for calculating Feynman integrals in the diagrams of quantum corrections to any physical processes involving them.

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