VALIS: A direct Vlasov solver for modelling laser plasma interaction

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Abstract. An accurate treatment of the relativistic Vlasov-Maxwell system is of fundamental importance to a broad range of plasma physics topics of direct relevance to inertial confinement fusion and fast ignition. With recent advances in high performance computing, the direct solution of Vlasov’s equation is becoming increasingly viable as an additional modelling tool, alongside established particle in cell methods. We introduce VALIS, an algorithm for the numerical solution of the Vlasov-Maxwell system in two spatial dimensions with two momentum dimensions, and demonstrate its application to the problem of modelling short-pulse laser absorption in short-scale length, over-dense targets with system parameters comparable to those achieved on the HELEN laser system at AWE.

1. Introduction
With high intensity, short pulse, laser systems we are able to heat matter to extreme temperatures creating exciting opportunities for high energy density physics experiments and exotic inertial fusion schemes, such as Fast Ignition\cite{1}. The mechanisms by which the short-pulse beam delivers its energy, how this energy is distributed and how it is transported into the target are complex and interdependent. A detailed understanding of these problems is essential for designing and interpreting experiments on future laser systems, such as ORION and VULCAN 10PW, and will be of paramount importance to the success of the Fast Ignition path to Inertial Confinement Fusion (ICF).
Here we consider the absorption of incident laser energy in a short scale-length density ramp using a recently developed direct Vlasov solver, VALIS\cite{2}. Kinetic models, such as VALIS, derive from a statistical treatment of the plasma retaining information about the distribution of particle momenta absent from hydro models. In addition to the conventional spatial dimensions, a kinetic model may have up to three velocity (or equivalently momentum) dimensions. Together, these describe a phase space of up to six dimensions. The complexity of such models allows a detailed consideration of non-linear kinetic phenomena, but renders problems analytically and computationally challenging. These models can be solved, together with the appropriate equations of electrodynamics, either directly or via particle in cell (PIC) methods. The PIC approach is widely adopted owing to its flexibility, stability and underlying simplicity but the noise free solution offered by direct Vlasov solvers is often desirable\cite{3}.

2. The relativistic 2D Vlasov Maxwell system
We adopt a relativistic Vlasov-Maxwell system with two spatial (x,y) and two momentum (ux, uy) dimensions – a ‘2D2P’ system. Vlasov’s equation for the electron distribution function f is given by:

$$ \frac{\partial f}{\partial t} + \frac{\mathbf{u}}{\gamma} \cdot \nabla f - \frac{e}{m_e} \left( \mathbf{E} + \frac{\mathbf{u}}{\gamma} \times \mathbf{B} \right) \cdot \nabla_x f = 0 $$

(1)

where: $ f = f(x, u) = f(x, y, u_x, u_y)$; $ \mathbf{u} = \gamma \mathbf{v} = \gamma(v_x, v_y, 0)$; $\gamma = \sqrt{1 + \frac{u^2}{c^2}}$; $\mathbf{E} = (E_x, E_y, 0)$ and $\mathbf{B} = (0, 0, B_z)$.

Maxwell’s equations are updated on a staggered grid via a second order finite difference scheme. In these simulations, the ions are treated as an immobile, initially neutralising, background but VALIS has recently been enhanced to allow for the consideration of an arbitrary number of species, on different momentum grids with arbitrary charge and mass.

The most common numerical approach to Vlasov’s equation is to adopt some form of time-splitting scheme[4,5]. Conservative[6,7] and semi-Lagrangian[8] approaches have been most popular, more recently some attempts have been made at adaptive Vlasov codes[3]. In a comparison of conservative schemes[7], the Piecewise Parabolic Method (PPM)[9] has been shown to be both robust and accurate across a range of tests. The PPM method is conservative and formally third order away from strong limiters. PPM also ensures monotonicity so that false oscillations, potential sources of kinetic instabilities, cannot be introduced. While for some tests the semi-Lagrangian scheme may be more accurate than high-order conservative schemes (such as PPM), these differences are small[10]. In addition semi-Lagrangian schemes require extra steps to enforce positivity, these additional steps have to date only been implemented in dimensionally split form. Unfortunately the semi-Lagrangian technique must be applied in full multidimensional, i.e. unsplit, form for relativistic problems to ensure global conservation[11]. VALIS is parallelised via domain decomposition - this is a necessity given the need to advance quantities on a 4D grid. The simulations discussed here were run on grids of at least (nx, ny, nux, nuy) = (640, 192, 210, 210) distributed over between 256 and 640 processing elements (PEs). VALIS has been tested on up to 1024 PEs on AWE’s CRAY XT3 system and demonstrates near-ideal scaling (a speed up of ~1.994 for each doubling in the number of PEs).

3. Short pulse absorption in short scale-length plasmas

We consider the interaction of a short laser pulse with a steep, exponential density ramp. VALIS adopts a system of normalised, dimensionless, units in which velocities are normalised to the speed of light in vacuum and time is normalised to the inverse of the electron plasma frequency. Key system parameters for the simulations discussed here are given in Table 1. We consider three density scale lengths: ‘short’, ‘medium’ and ‘long’ - although all are effectively short when compared with long-pulse laser-plasma interactions. The density ramp is angled at between 0 and 30 degrees to the P-polarised incident beam.

Figure 1 shows the transverse electric field and electron density profiles for one case. The interaction of the incident beam with the density ramp is clearly visible as is the reflection of the beam at the critical surface. In all cases, populations of hot electrons are generated which move into the target (see Fig. 2). It is common for this hot population to be characterised by a single temperature $T_{\text{hot}}$. However, it is often inadequate to describe the complex phase space structures generated in the absorption region with a single ‘temperature’. Often the distribution function contains beams and plateaux, this makes estimating $T_{\text{hot}}$ a problematic and subjective undertaking. We define $T_{\text{hot}}$ in terms of the normalised directed energy calculated from the electron distribution function

$$ T_{\text{hot}} \approx \frac{\int E_x(u)K(u)f_u du}{\int K(u)f_u du} $$

(4)

where $E_x$ is the directed energy in x and $K$ is the kinetic energy, both as a function of momentum. Since the aim, ultimately, is to model how the deposited energy is absorbed and transported into the dense material, it is also useful to compare the change in the energy of the electron population at the top of the ramp - in the dense material but away from the absorption region.
Figure 1: Density profile and transverse field for the medium scale length short-pulse laser plasma interaction simulation, angled at 30 degrees, at \( t = 740 \), equivalent to 105fs.

| Parameter       | Simulation Parameters | VALIS units |
|-----------------|-----------------------|-------------|
| Temperature     | 5keV                  | 0.02        |
| Number density  | 1.58e28 m\(^{-3}\)    | 1.0         |
| Scale length, ‘short’ | 0.05 \( \mu \)m | 1.25       |
| ‘medium’        | 0.5 \( \mu \)m       | 12.5        |
| ‘long’          | 3.4 \( \mu \)m       | 80.0        |
| Target angles   | 0\(^{\circ}\), 10\(^{\circ}\), 30\(^{\circ}\) | 0, 10, 30  |
| Laser wavelength, \( \lambda \) | 1.06 \( \mu \)m | 8\( \pi \) |
| Laser pulse length | 78.9 fs     | 560         |
| Laser spot size | 5 \( \mu \)m        | 120         |
| \( \Omega \lambda^2 \) | 5.5e18 Wcm\(^{-2}\)\( \mu \)m\(^{2}\) | 157.9      |
| Distance        | 0.042 \( \mu \)m     | 1           |

Table 1: System parameters for short-pulse LPI demonstration.

Figure 2: Left: Electron \((x, u_x, u_y)\) phase space showing hot electrons produced by absorption of laser pulse into an overdense \((16n_c)\) plasma ramp with a scale-length of \( \sim 0.05 \) laser wavelengths. Centre and right: Electron \((u_x, u_y)\) phase space spatially integrated over the top of the plasma ramp; short scale length angled at 30 degrees (centre); long scale length angled at 0 degrees (right).

Table 2 summarises the results for various target conditions. The long scale length case absorbs much more of the incident energy but the result is to heat the foot of the ramp. This generates a hot, but not particularly numerous, population with an effective hot electron temperature in excess of 400keV (see Fig. 4), this population is not particularly well suited to carrying energy into the dense material, as can be seen in Table 2. In the short scale length case the laser interacts directly with the dense material, and despite utilising a smaller fraction of the available EM energy, the laser is able to heat the dense material more effectively. The medium scale length case, at normal incidence, is broadly similar to the short case. The average hot electron energy and absorption fraction are almost identical, however with the absorption region farther away from the top of the density ramp, we find that the heating in the dense material is reduced. The medium scale length case is improved significantly when the target is angled. The mean energy of the hot population in this case is significant and the absorption climbs to over 50\%, although the total energy change at the top of the ramp remains smaller than all three short scale length cases.

Changing the angle of the short scale length target also produces an increase in the mean hot electron energy (as we would expect since the laser’s electric field is now has a component into the density ramp) and improves absorption slightly. However, it proves less effective at raising the temperature at the top of the ramp.
The long scale length case was generally unaffected by a change in target angle from 0 to 10 degrees. At higher angles the refraction of the beam should be clearly apparent.

![Table 2: Absorption fraction, $T_{\text{hot}}$ and average kinetic energy change of electrons at the top of the ramp, in each case.](image)

| Run          | $f_{\text{abs}}$ | $T_{\text{hot}}$ (keV) | $\Delta E_{\text{dense}}$ (keV) |
|--------------|------------------|-------------------------|---------------------------------|
| Short, 0°    | 0.30             | 100                     | 35                              |
| Short, 10°   | 0.31             | 110                     | 34                              |
| Short, 30°   | 0.37             | 200                     | 22                              |
| Med., 0°     | 0.28             | 145                     | 12                              |
| Med., 30°    | 0.52             | 300                     | 20                              |
| Long, 0°     | 0.88             | 400                     | 8                               |
| Long, 10°    | 0.90             | 440                     | 10                              |

Table 2: Absorption fraction, $T_{\text{hot}}$ and average kinetic energy change of electrons at the top of the ramp, in each case.

4. Conclusions

Collimated, energetic beams will be able to heat material to higher temperatures and at greater depths, so the properties of the hot electron population produced during short pulse laser-matter interaction is an important consideration for high energy density physics experiments[12] and Fast Ignition schemes. These results indicate that the longer scale-length system absorbs more incident energy and produces the hottest population of electrons in the absorption. However there are other considerations, such as how much of this is delivered to the dense material of the target — in these simulations the average energy of the electrons, at the end of the simulation, is higher in the medium scale length case angled at 30 degrees to the laser normal. But the temperature change of the dense material is greatest in the shortest scale lengths, where the laser is interacting with the material directly. The impact of return current heating and collisional effects will dominate transport beyond the absorption region, so a self consistent model for absorption and transport is needed to fully assess these issues. Also, it is not possible, in practice, to control the scale length of the pre-plasma so tightly. Even if measures are taken to mitigate the pre-pulse (as was the case for experiments on the HELEN laser at similar intensities[12]) there will still be shot-to-shot variation in the plasma scale length. Future work will study the properties of the hot electrons produced in similar, but more extensive systems, with mobile ions and over a wider range of initial parameters — particularly the irradiance. This will allow us to better understand the mechanisms which produce hot electrons and how these, along with the properties of the hot electron population, are influenced by the laser and plasma conditions.

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5. References

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