A predictive model for spiral wound reverse osmosis membrane modules: The effect of winding geometry and accurate geometric details

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**A B S T R A C T**

A new one-dimensional predictive model for spiral wound modules (SWMs) applied to reverse osmosis membrane systems is developed by incorporating a detailed description of the geometric features of SWMs and considering flow in two directions. The proposed model is found to capture existing experimental data well, with similar accuracy to the widely-used plate model in which the SWM is assumed to consist of multiple thin rectangular channels. However, physical parameters that should in principle be model-independent, such as membrane permeability, are found to differ significantly depending on which model is used, when the same data sets are used for parameter estimation. Conversely, when using the same physical parameter values in both models, the water recovery predicted by the plate-like model is 12–20% higher than that predicted by the spiral model. This discrepancy is due to differences in the description of geometric features, in particular the active membrane area and the variable channel heights through the module, which impact on predicted performance and energy consumption. A number of design variables – the number of membrane leaves, membrane dimensions, centre pipe radius and the height of feed and permeate channels – are varied and their effects on performance, energy consumption and calculated module size are analysed. The proposed spiral model provides valuable insights into the effects of complex geometry on the performance of the SWM as well as of the overall system, at a lower computational cost.

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**1. Introduction**

Reverse osmosis (RO) processes have been widely used in many applications, especially for producing nearly pure water from seawater in desalination plants, and have seen a dramatic increase in their market share in recent years (Elimelech and Phillip, 2011; Fritzmann et al., 2007; Ghaffour et al., 2013; Greenlee et al., 2009; Kim et al., 2009; Malaeb and Ayyoub, 2011; Semiat, 2008). In an RO process external hydraulic pressure that exceeds the osmotic pressure difference between two solutions is applied to the side where the more concentrated solution is placed. As a result, water passes through a semi-permeable RO membrane at a rate that is proportional to the difference between the external pressure gradient and the osmotic pressure gradient, while salts dissolved in concentrated solution are rejected. Using RO membrane systems, seawater can be separated into pure water and concentrated brine containing the rejected salts.

RO membrane modules are commercially manufactured for their use in large scale plants and spiral wound modules (SWMs) are most widely adopted among commercially available RO membrane modules (Schwinge et al., 2004). In SWMs, several sheets of RO membranes and feed and permeate spacers are alternately stacked and wrapped around a perforated centre pipe, forming separate feed and permeate channels. Due to the wrapping of membranes and spacers, the module has a complex geometry that can be difficult to capture in mathematical models.

Models of SWMs have been reported by many authors in order to predict performance and energy consumption, which have a direct influence on the overall RO process (Avlonitis et al., 1991, 1993; Boudinar et al., 1992; Dickson et al., 1992; Evangelista and Jonsson, 1988; Gerald et al., 2005; Riverol and Pilipovic, 2005; Schock and Miquel, 1987; Senthilmurugan et al., 2005; Taniguchi, 1978;...
**Nomenclature**

**Symbols**

| Symbol | Definition |
|--------|------------|
| A      | Water permeability constant [m³/m² s Pa] |
| A_c    | Cross-sectional area of feed channel [m²] |
| A_c,p  | Cross-sectional area of permeate channel [m²] |
| A_c,SWM| Total cross-sectional area of feed channel at entrance [m²] |
| A_t    | Trans-membrane area [m²] |
| ATot   | Total membrane area [m²] |
| B      | Salt permeability constant [m/s] |
| c      | Concentration [kg/m³] |
| d_1    | Constant used in parametric equations |
| d_2k   | Constant used in parametric equations |
| d_3j   | Constant used in parametric equations |
| d_h    | Hydraulic diameter [m] |
| E_sp   | Specific energy consumption [kWh/m³] |
| F      | Objective function |
| H      | Spacer thickness [m] |
| H_k    | Parameter for curve k [m] |
| h      | Height [m] |
| j      | Index of membrane leaves |
| J_w    | Water flux [m³/m² s] |
| J_w_eff| Effective water flux [m³/m² s] |
| J_s    | Salt flux [kg/m² s] |
| k_sp   | Frictional coefficient [-] |
| L      | Active length of membrane leaves [m] |
| L_g    | Glue line thickness in length direction [m] |
| L_i    | Initial length of membrane leaves [m] |
| m      | Number of variables |
| N_leaf | Number of membrane leaves |
| n      | Number of data |
| P      | Pressure [Pa] |
| P_atm  | Atmospheric pressure [Pa] |
| P_drop | Pressure drop in feed channel [Pa] |
| P_p,in | Permeate pressure at the closed end [Pa] |
| Q      | Volumetric flowrate [m³/s] |
| R      | Gas constant [m³ Pa/K kmol] |
| R_w    | Water recovery [%] |
| r_c    | Radius of centre pipe [m] |
| r_k    | Radial position of curve k [m] |
| r_oc   | Radius of outer cover [m] |
| s      | Arc length [m] |
| s_flow| Flow path length of permeate stream |
| u      | Flow velocity [m/s] |
| V      | Control volume [m³] |
| W      | Active width of membrane leaves [m] |
| W_g    | Glue line thickness in width direction [m] |
| W_i    | Initial width of membrane leaves [m] |
| w_j    | Weighting factor of j-th variable |
| x      | Permeate flow direction in cartesian coordinate system for unwound membrane [m] |
| x_kj   | x-coordinate for spiral curve kj in cartesian coordinate system [m] |
| x_c    | x-coordinate for centre pipe curve in cartesian coordinate system [m] |
| x_oc   | x-coordinate for outer cover curve in cartesian coordinate system [m] |
| y      | Channel height direction in cartesian coordinate system for unwound membrane [m] |
| y_kj   | y-coordinate for spiral curve kj in cartesian coordinate system [m] |
| y_c    | y-coordinate for centre pipe curve in cartesian coordinate system [m] |
| y_oc   | y-coordinate for outer cover curve in cartesian coordinate system [m] |
| y_i,j  | i-th measured values of j-variable |
| y*_i,j | i-th calculated values of j-variable |
| z      | Feed flow direction in cartesian coordinate system for unwound membrane [m] |

**Greek letters**

| Symbol | Definition |
|--------|------------|
| μ      | Dynamic viscosity [Pa s] |
| ε      | Small gap between outermost layer and outer cover [m] |
Sirkar et al., 1982; van der Meer and van Dijk, 1997; Zhou et al., 2006). Dickson et al. developed a model for a spiral wound RO module by assuming it behaves as a stack of thin channels (Dickson et al., 1992). A number of other studies were also conducted under the assumption of thin channels to simplify the complex geometry of SWMs (Avlonitis et al., 1991, 1993; Boudinar et al., 1992; Dickson et al., 1992; Geraldes et al., 2005; Senthilmurugan et al., 2005; Schock and Miquel, 1987). A commonly adopted model introduced by Senthilmurugan et al. is based on the assumption that the wound membranes in SWMs can be treated as unwound so that the configuration is similar to a plate-and-frame module (Senthilmurugan et al., 2005). Boudinar et al. (1992) investigated the effect of the membrane width-to-length ratio, the number of membrane leaves and the width of glue lines, by means of a computational model in which radial in-homogeneities were considered. Although there is a large body of literature on distributed models for SWMs in RO processes, the effect of winding membranes and spacers has not been considered explicitly. In other words, the plate-and-frame modelling approach cannot differentiate SWMs from plate-and-frame modules in terms of module design due to the assumptions used to simplify the complex geometry of SWMs. For instance, issues such as the asymmetric nature of wound membranes, where two folded halves do not overlap exactly when wound around the central tube, or the effect of spacer geometry, have not been represented. To the best of our knowledge, the only model that incorporates the asymmetric nature of wound membranes was developed for a modified SWM used in a forward osmosis process (Gu et al., 2011).

Since predictive models are intended to be used not only for the prediction of membrane performance and energy consumption but also for optimisation and control of an overall RO system, even minor errors introduced in a single SWM model may accumulate and affect the reliability of optimisation and control studies and real time control of RO plants (Alnouri and Linke, 2012; Bartman et al., 2009a, 2009b, 2010; Emad et al., 2012; Geraldes et al., 2005; Li, 2012; Li and Noh, 2012; Zhu et al., 2009a, 2009b).

In this study, a new model for SWM is developed by incorporating the design variables required in order to take into account the unique geometric features of SWMs. Such a detailed model of geometry is expected to make it possible not only to predict the performance of a spiral wound module with a specific design but also to optimise the design of SWMs to accomplish a certain objective, which might necessitate enhanced mass transfer and a reduced pressure drop inside the membrane module. In developing this more detailed approach, a model of the spiral geometry for a cross-section of the module is proposed in Section 2, based on parametric expressions with respect to a winding angle. These mathematical expressions can be used to obtain accurate geometric characteristics such as channel height, cross-sectional and trans-membrane area and flow path length. This geometric description is linked directly to the prediction of the performance and energy consumption in a single SWM, via mass and energy balances. In Section 3, the performance of the proposed model is investigated. Comparisons are made with a plate-like model for SWM in order to evaluate the differences between the spiral and plate-like models. The spiral model is then validated through comparisons with experimental data in the literature. Finally, the effects of varying the geometry of a SWM are investigated using the spiral model and an initial attempt at module optimisation is made.

2. Model development

In order to describe the actual features of an SWM more realistically and accurately, it is useful to understand how an SWM is assembled. As shown in Fig. 1, the procedure involves attaching several sheets of permeate spacers around a perforated centre pipe. A sheet of feed channel spacer is inserted inside a folded flat sheet membrane (also referred to as a membrane leaf). Glue is applied on the outer surface of the folded membrane, along edges as shown in Fig. 1. A folded membrane containing a feed spacer is inserted between each permeate spacer, which results in the membrane and the two types of spacers being arranged in an alternating fashion. Due to the glue lines on the outer membrane edges, membrane “envelopes” are formed by sealing the permeate spacers, which ensures the feed and permeate streams...
flow independently without being mixed with each other. The sets of membranes and spacers are rolled around the centre pipe and casos with an outer cover. As a result, the permeate stream flows in a spiral direction from the sealed end of the envelope and to the perforated centre pipe, where it is collected. The feed or retentate stream flows in the axial direction as shown in Fig. 1. Each half of a folded membrane sheet is referred to as a membrane half-sheet. Thus, if there is more than one membrane leaf in the module, the permeate stream flows between two half-sheets that belong to different membrane leaves, while the feed stream flows between two half-sheets that belong to the same membrane leaf.

A number of design variables must be specified to fully describe a SWM. These include the number of membranes leaves, \( N_{\text{leaf}} \); the dimensions of the membrane half-sheets,\(^{1}\) namely the length \( L_s \), the width \( W'_l \); the thickness of the spacers in the feed and permeate channels, \( H_f \) and \( H_p \), respectively; the width of glue lines in the length and width directions, \( L_g \) and \( W_g \), respectively; and the radius of the centre pipe, \( r_c \). These design variables are incorporated in mathematical models of the SWM in different ways, depending on the geometrical assumptions introduced in each model.

### 2.1. Models of membrane geometry

In this study, two types of mathematical models are investigated to represent membrane geometry: a plate-like model and a spiral model. The former has been adopted in most studies (Avlonitis et al., 1991; Boudinar et al., 1992; Dickson et al., 1992; Evangelista and Jonsson, 1988; Geraldes et al., 2005; Riverol and Pilipovic; 2005; Schock and Miquel, 1987; Senthilmurugan et al., 2005; Taniguchi, 1978; Sirkar et al., 1982; van der Meer and van Dijk, 1997; Zhou et al., 2006), while the latter is introduced here to allow for a more accurate geometrical description. A brief description of the plate-like model is presented in the Supporting information (Section A); for a more detailed discussion, the reader is referred to references (Avlonitis et al., 1991; Boudinar et al., 1992; Dickson et al., 1992; Senthilmurugan et al., 2005; van der Meer and van Dijk, 1997; Zhou et al., 2006). The details of the proposed spiral model are presented in this section.

The development of a detailed spiral model aims to take more realistic geometric features into consideration so as to facilitate a more accurate prediction of the performance of SWMs as well as the potential use of the model for module design and optimisation of the overall process. The most distinctive feature of an SWM is the winding arrangement of membrane sheets around the centre pipe, which results in a spiral-shaped cross section of the membranes in the module.

We describe the spiral module with a set of equations that provide accurate values of geometric parameters such as the flow path length, the variation of the channel heights within the module and the module cross-sectional area as a function of the module design variables. The two halves of a folded membrane sheet are differentiated by referring to the half-sheet closest to the core of the module as the inner membrane, and that furthest from the core as the outer membrane. Consider, as shown in Fig. 2 (a), a module consisting of a single membrane sheet folded in half and wound around the centre pipe. With two equally-sized half sheets being wound, the inner and outer membranes end at different angular positions, denoted in the figure by points F and G respectively. This leads to a reduction in the active membrane area, which can be of the order 1–19% depending on the size of the centre pipe and the number of leaves. The Cartesian coordinates of the centre pipe exterior, the inner and outer membranes, the centre of the feed channel and the outer cover can be described as a function of a winding angle \( \theta \). Fig. 2 (b) illustrate an SWM cross section with two membrane leaves. The leaf 1 starts from the angle \( \theta \), which is denoted by \( A \) and \( B \) for the inner and outer membrane, respectively, while the starting angle of the leaf 2 is positioned at \( \theta = 180^\circ \),

\(^{1}\) The dimensions of a membrane sheet, rather than a half-sheet, can of course be used equivalently, but the dimensions typically reported in plate-like models correspond to a half-sheet so we adopt this convention here.
which is determined by equidistant arrangement of leaves around the centre pipe. Key angular positions are also noted in Fig. 2, which are discussed later to elaborate the nature of the spiral model.

The Cartesian coordinates of the boundary of the centre pipe, $x_c$ and $y_c$, are expressed as:

$$x_c(\theta) = r_c \cos \theta$$

$$y_c(\theta) = r_c \sin \theta$$

where the angle $\theta$ ranges from 0 to $2\pi$. When there are multiple membrane leaves, it is assumed that they are equally distributed around the centre pipe. For each membrane leaf $j$, three spiral curves, denoted by subscript $k$, are defined to yield the coordinates of the inner membrane ($k = \text{in}$), of the outer membrane ($k = \text{out}$) and of the centre of the feed channel (the half-feed channel curve, $k = \text{hf}$). The folding lines of membranes are neglected in the reconstructed spiral geometry as they are trivial compared to the total membrane area. The Cartesian coordinates $x_{kj}$ and $y_{kj}$ of a point at angle $\theta$ on membrane leaf $j$ and curve $k$ are expressed via the following equations:

$$x_{kj}(\theta) = r_k(\theta) \cos (\theta + d_{3j}) = d_1(\theta + d_{2k}) \cos (\theta + d_{3j})$$

$$y_{kj}(\theta) = r_k(\theta) \sin (\theta + d_{3j}) = d_1(\theta + d_{2k}) \sin (\theta + d_{3j})$$

$$r_k(\theta) = d_1(\theta + d_{2k})$$

$$d_1 = \frac{N_{\text{leaf}}(H_f + H_p)}{2\pi}$$

$$d_{2k} = \frac{H_k + r_c}{d_1}$$

$$d_{3j} = \frac{2\pi(j - 1)}{N_{\text{leaf}}}$$



![Fig. 2. Schematic illustration of (a) a cross-section of an SWM with a single leaf and (b) a cross-section of an SWM with two leaves.](image-url)
where \( r_k(\theta) \) is the shortest distance between curve \( k \) at angle \( \theta \) and the centre of the module. In Eq. (7), \( H_k \) is a constant that indicates the distance between curve \( k \) at the folded end of the membrane leaf and that is defined differently for an inner or outer membrane and a half feed channel curve, but that is independent of the specific leaf being considered:

\[
H_k = \begin{cases} 
H_p & \text{for } k = \text{in} \\
H_p + H_f & \text{for } k = \text{out} \\
H_p + 0.5H_f & \text{for } k = hf,
\end{cases}
\]

(9)

The angle \( \theta_{k,j} \) ranges from an initial value \( \theta_{i,k,j} \) to a final value \( \theta_{f,k,j} \) which depend on the leaf number, the number of times curve \( kj \) winds around the centre pipe as well as some of the module design variables. The initial angular position is given by:

\[
\theta_{i,k,j} = d_{3j},
\]

(10)

and the final angular position \( \theta_{f,k,j} \) is given by the following equation:

\[
W_k = \int_{\theta_{i,k,j}}^{\theta_{f,k,j}} \left( \frac{dy_{kj}}{d\theta} \right)^2 + \left( \frac{dx_{kj}}{d\theta} \right)^2 d\theta
\]

(11)

Note that the initial angular position of curve \( kj \) is the same for the inner and outer membranes and the half-feed curve, and that the angle \( \theta_{f,k,j} \) can be expected to be larger than \( 2\pi \). The radius of the outer cover is found based on the final angular position of a wound membrane:

\[
r_{oc} = r_{kj}(\theta = \theta_{f,\text{out},j}) + \varepsilon
\]

(12)

where a small gap, \( \varepsilon \), is added between the outermost layer of membranes and the outer cover in order to maintain numerical stability. Note that since an outer membrane is placed further from the centre of the module than the corresponding inner membrane, the outer membrane is closest to the outer cover of the module. Thus, the radius of the cover must be at least as large as the final radius of any outer membrane. In the case of multiple membrane leaves, any outer membrane could be used because of the assumption of even distribution of the leaves, so leaf 1 has been chosen arbitrarily in Eq. (12).

Since the outer cover is perfectly circular, Eqs. (1) and (2) can be used to obtain the Cartesian coordinates of the outer cover, \( x_{oc} \) and \( y_{oc} \), by replacing \( r_k \) with the outer cover radius \( r_{oc} \):

\[
x_{oc}(\theta) = r_{oc} \cos \theta
\]

(13)

\[
y_{oc}(\theta) = r_{oc} \sin \theta
\]

(14)

where the angle \( \theta \) ranges from 0 to \( 2\pi \).

Using Eqs. (1)–(14), the cross section of an SWM with \( N_{\text{leaf}} \) membrane leaves can be reconstructed. By tracking the permeate stream starting from the glued end of the outermost layer, the contribution of the inner and outer membranes to the permeate flow can be analysed. To do so, the feed channel height must be partitioned between the two membrane half-sheets. Where both half-sheets are active (e.g., between points A and C of the inner membrane and between B and D of the outer membrane in Fig. 2 (a)), it is assumed that permeation is distributed equally between them so that each half-sheet treats a flow corresponding to half the feed channel height. When only one half-sheet is active (e.g., between points D and G in Fig. 2 (a)), the entire feed channel height is considered for the active membrane. To capture this behaviour, it is convenient to divide the inner membrane for each leaf into three regions, as shown in Fig. 2, in order to calculate the feed channel height:

Region 1 consists of a fully functional inner membrane and no outer membrane. The feed channel height corresponding to the inner membrane of leaf \( j \), \( h_{f,\text{in},j} \), is equal to the radius of the outer cover minus the distance between the centre of the module and the inner membrane.

\[
h_{f,\text{in},j}(\theta) = r_{oc}(\theta) - r_{in,j}(\theta) \quad \text{for} \quad \theta_{f,\text{out},j} \leq \theta < \theta_{f,\text{in},j}
\]

(15)

Region 2 consists of a fully functional inner membrane and an inactive outer membrane. Both sides of the outer membrane are in contact with the feed stream, so there is no permeation through the outer membrane. The feed channel height corresponding to the inner membrane of leaf \( j \) is equal to the thickness of feed spacers and remains constant throughout Region 2.

\[
h_{f,\text{in},j}(\theta) = r_{in,j}(\theta) - r_{out,j}(\theta) \quad \text{for} \quad \theta_{f,\text{in},j} - \frac{2\pi}{N_{\text{leaf}}} \leq \theta < \theta_{f,\text{out},j}
\]

(16)

Region 3 consists of fully functional inner and outer membranes. The outer membrane is now in contact with the permeate stream and water flux through an outer membrane is affected by the feed channel shared with the inner membrane. Thus the half feed channel height is allocated to each half-sheet membrane in this region. The heights of the inner and outer membranes, \( h_{f,k,j} \), \( k = \text{in, out} \), are given by:

\[
h_{f,k,j}(\theta) = \begin{cases} 
|k_{j}(\theta) - r_{kj}(\theta)| & \text{for} \ 0 \leq \theta < \theta_{f,\text{in},j} - \frac{2\pi}{N_{\text{leaf}}}
\end{cases}
\]

(17)

The outer membrane only experiences two regimes: it is active with a constant channel height equal to half the feed spacer thickness in Region 3 and it is inactive in Regions 1 and 2. Unlike the feed channel, the permeate channel does not need to be partitioned between the
inner and outer membranes. Nevertheless, the permeate channel height for membrane leaf \( j \), \( h_{p,j} \) varies, with two distinct regions and is given by:

\[
\begin{align*}
h_{p,j}(\theta) &= r_{in,j}(\theta) - r_{out,j}(\theta - 2\pi) \quad \text{at} \quad 2\pi < \theta < \theta_{f,\text{in},j} \\
h_{p,j}(\theta) &= r_{in,j}(\theta) - r_c \quad \text{at} \quad 0 < \theta < 2\pi
\end{align*}
\]  

(18)

2.2. Transport equations

There are three main modelling approaches to describe transport phenomena through membranes: pore-flow (Soltanieh and Gill, 1981; Wijmans and Baker, 1995) solution-diffusion (Paul, 2004; Soltanieh and Gill, 1981; Wijmans and Baker, 1995) and irreversible thermodynamic models (Kedem and Katchalsky, 1958; Spiegler and Kedem, 1966). It is widely recognised that the solution-diffusion model is most suitable for an RO process since RO membranes are assumed to be nonporous so that diffusion is a predominant mass transport mechanism (Soltanieh and Gill, 1981; Wijmans and Baker, 1995). The solution-diffusion model is therefore employed in this study. In addition, the concentration polariisation caused by the selectivity of RO membranes is taken into account. Details of model equations used to describe membrane permeation and concentration polariisation are presented in the Supporting information (Section B). Although the presence of spacers in the feed channel has a significant impact on mass and momentum transfer, it is not accounted for in this study since the focus of this study is to develop a model that can capture the winding geometry. Nonetheless, pressure drop affected by spacers is taken into consideration by introducing a frictional coefficient, which is shown in the latter part of the current section.

Mass transfer along the feed and permeate channels should also be considered in order to obtain the local flow velocity, concentration and pressure that are used to calculate the water and salt permeate fluxes at the corresponding locations. Gravitational effects on mass and momentum transfer are assumed to be negligible (Fletcher and Wiley, 2004) since spiral wound modules are placed horizontally in large-scale operations. Based on the mass transfer present in a differential control volume illustrated in Fig. 3, the overall and component balance equations are formulated under the assumption of incompressible flow to yield the volumetric flowrate and concentration in each feed and permeate channels. It is worth noting that the model developed is two-directional while flow in each channel is assumed to be one-dimensional as is standard practice in plate-like models (Avlonitis et al., 1991; Boudinar et al., 1992; Evangelista and Jonsson, 1988; Senthilmurugan et al., 2005).

Before deriving the transport equations, key geometric parameters are obtained. By applying Eqs. (1)–(18), the flow path length of curve \( k \) for leaf \( j \), \( s_{kj} \), is given by:

\[
\frac{ds_{kj}(\theta)}{d\theta} = \sqrt{\left(\frac{dx_{kj}(\theta)}{d\theta}\right)^2 + \left(\frac{dy_{kj}(\theta)}{d\theta}\right)^2},
\]

(19)

the trans-membrane area of membrane \( kj \), \( A_{t,kj} \), is:

\[
A_{t,kj}(\theta, z) = s_{kj}(\theta) z,
\]

(20)
the cross-sectional area of the feed channel \( A_{c,f,k,j} \), where \( k=in, out \) as appropriate for each region and \( j \) is the membrane leaf:

\[
\frac{dA_{c,f,in,j}(\theta)}{d\theta} = \frac{1}{2}r_{in,j}(\theta)^2 - r_{oc}(\theta)^2 \quad \text{for Region 1}
\]

\[
\frac{dA_{c,f,in,j}(\theta)}{d\theta} = \frac{1}{2}r_{in,j}(\theta)^2 - r_{out,j}(\theta)^2 \quad \text{for Region 2}
\]

\[
\frac{dA_{c,f,k,j}(\theta)}{d\theta} = \frac{1}{2}r_{k,j}(\theta)^2 - r_{h,j}(\theta)^2 \quad (k = in \text{ and } out) \quad \text{for Region 3},
\]

and the volume of the feed channel, \( V_f \), is given by:

\[
V_{f,k,j}(\theta, z) = A_{c,f,k,j}(\theta) z
\]

where \( z \) is the position along the longitudinal direction.

The geometric parameters for the permeate channel are calculated in the same manner but using the permeate flow direction. The cross sectional area of the permeate channel of leaf \( j \), \( A_{c,p,j} \), is given by:

\[
A_{c,p,j}(\theta, z) = h_{p,j}(\theta) z,
\]

the flow path length for the permeate flow, \( \tilde{z}_j \), is approximated by averaging the arc length of the inner membrane and either the outer membrane or the centre pipe:

\[
\tilde{z}_j(\theta) = \frac{1}{2} [s_{in,j}(\theta) + s_{out,j}(\theta)] \quad \text{at } 2\pi < \theta < \theta_{f,in,j}
\]

\[
\tilde{z}_j(\theta) = \frac{1}{2} s_{in,j}(\theta) + \int_0^\theta \sqrt{\left(\frac{dx_c(\theta)}{d\theta}\right)^2 + \left(\frac{dy_c(\theta)}{d\theta}\right)^2} \, d\theta \quad \text{at } 0 < \theta < 2\pi,
\]

and the volume of the permeate channel, \( V_{p,j} \), is given by:

\[
V_{p,j}(\theta, z) = A_{c,p,j}(\theta, z) \tilde{z}_j(\theta)
\]

The transport equations are derived from the overall and component mass balance equations. Variations along the channel height are assumed negligible compared to cross-flow streams in the \( z \) and \( \theta \) direction although the variation of concentration at membrane surfaces is considered using the concentration polarisation model. Within each channel, the flow is assumed to be one-directional, that is, only the \( z \) and \( \theta \) velocity component are nonzero for the feed and permeate streams, respectively. The diffusion terms can be neglected in the component balance equation since convective mass transfer dominates here. As a result, simplified equations are expressed as in Eqs. (26) and (27).

\[
- \frac{dJ_{w,kj}(\theta, z)}{dz} \frac{dA_{c,f,kj}(\theta)}{d\theta} - J_{w,kj}(\theta, z) \sqrt{\left(\frac{dx_{kj}(\theta)}{d\theta}\right)^2 + \left(\frac{dy_{kj}(\theta)}{d\theta}\right)^2} = 0
\]

\[
- \frac{dJ_{w,kj}(\theta, z) c_{f,kj}(\theta, z)}{dz} A_{c,f,kj}(\theta) - J_{w,kj}(\theta, z) c_{f,kj}(\theta, z) \sqrt{\left(\frac{dx_{kj}(\theta)}{d\theta}\right)^2 + \left(\frac{dy_{kj}(\theta)}{d\theta}\right)^2} = 0
\]

where \( u_{f,kj} \) is the flow velocity in the feed channel for leaf \( j \) and \( c_{f,kj} \) the feed concentration, which contribute to the performance of membrane \( kj \). \( J_{w,kj} \) and \( J_{f,kj} \) represent the water and salt fluxes through membrane \( kj \), which are obtained individually using Eqs. (B.5) and (B.6) in the Supporting information. Using the aforementioned relations for the key geometric parameters presented in Eqs. (19)–(25), the following differential equations are obtained with respect to \( z \) and \( \theta \).

Eqs. (26) and (27) are modified in order to use them for the permeate channel since water and salt fluxes through both the inner and outer membrane contribute to the change of the permeate flowrate and concentration, respectively.

\[
- \frac{dJ_{w,ijn}(\theta, z)}{d\theta} \frac{h_{p,j}(\theta)}{h_{p,j}(\theta)} - J_{w,ijn}(\theta, z) \sqrt{\left(\frac{dx_{ijn}(\theta)}{d\theta}\right)^2 + \left(\frac{dy_{ijn}(\theta)}{d\theta}\right)^2} +
\]

\[
\frac{J_{w,out,j}(\theta - 2\pi, z)}{h_{p,j}(\theta)} \sqrt{\left(\frac{dx_{out,j}(\theta - 2\pi)}{d\theta}\right)^2 + \left(\frac{dy_{out,j}(\theta - 2\pi)}{d\theta}\right)^2} = 0 \quad \text{at } 2\pi < \theta < \theta_{f,in,j}
\]

\[
- \frac{dJ_{w,ijn}(\theta, z)}{d\theta} \frac{h_{p,j}(\theta)}{h_{p,j}(\theta)} - J_{w,ijn}(\theta, z) \sqrt{\left(\frac{dx_{ijn}(\theta)}{d\theta}\right)^2 + \left(\frac{dy_{ijn}(\theta)}{d\theta}\right)^2} = 0 \quad \text{at } 0 < \theta < 2\pi
\]
where \( u_{p,j} \) is the flow velocity and \( c_{p,j} \) the feed concentration in the permeate channel which is formed by leaf \( j \). As shown in Eqs. (26) to (31), the contributions of the inner and outer membranes to the permeate flowrate and concentration depend on the angular range.

Since pressure provides the driving force for water permeation, the local pressure in each channel needs to be evaluated. The presence of spacers between the membranes induces changes in the permeate pressure, \( P_{f,kj} \), that are modelled with an appropriate correlation for pressure drop, given by Zhou et al. (2006):

\[
\frac{dP_{f,kj}(\theta, z)}{dz} = \frac{12k_{sp,f} \mu u_{f,kj}(\theta, z)}{[h_{f,kj}(\theta)]^2}
\]  

(32)

where \( k_{sp,f} \) is the friction coefficient in the feed channel, \( \mu \) the dynamic viscosity, assumed to be constant and equal to that of water. For the permeate channel pressure, \( P_{p,j} \), Eq. (32) is modified in order to express the pressure drop with respect to the winding angle \( \theta \) using the equations for the spiral curves:

\[
\frac{dP_{p,j}(\theta, z)}{d\theta} = \frac{12k_{sp,p} \mu u_{p,j}(\theta, z) dS_j(\theta)}{[h_{p,j}(\theta)]^2}
\]  

(33)

where \( k_{sp,p} \) is the friction coefficient in the permeate channel, and \( u_{p,j} \) the flow velocity in the permeate channel for leaf \( j \).

Boundary conditions for Eqs. (26) to (33) are given as follows:

\[
u_{f,k}(\theta, z) = \frac{Q_{f,0}}{A_{c,SWM}} \quad \text{at} \quad z = 0 \quad \text{and for all} \quad \theta \in [0, \theta_{f,kj}] \quad (k = \text{in, out})
\]  

(34)

\[
c_{f,k}(\theta, z) = c_{f,0} \quad \text{at} \quad z = 0 \quad \text{and for all} \quad \theta \in [0, \theta_{f,kj}] \quad (k = \text{in, out})
\]  

(35)

\[
P_{f,k}(\theta, z) = P_{f,0} \quad \text{at} \quad z = 0 \quad \text{and for all} \quad \theta \in [0, \theta_{f,kj}] \quad (k = \text{in, out})
\]  

(36)

\[
P_{p,j}(\theta, z) = P_{am} \quad \text{at} \quad \theta = 0 \quad \text{and for all} \quad z \in [0, L]
\]  

(37)

\[
u_{p,j}(\theta, z) = j_{w,in,j}(\theta, z) \quad \text{at} \quad \theta = \theta_{f,in,j} \quad \text{and for all} \quad z \in [0, L]
\]  

(38)

\[
c_{p,j}(\theta, z) = j_{w,in,j}(\theta, z) \quad \text{at} \quad \theta = \theta_{f,in,j} \quad \text{and for all} \quad z \in [0, L]
\]  

(39)

where \( A_{c,SWM} \) is the total cross-sectional area of the feed channel at the entrance of the SWM and \( Q_{f,0} \), \( c_{f,0} \) and \( P_{f,0} \) are operating feed flowrate, concentration and pressure, respectively. \( L \) indicates the \( z \)-coordinate of the exit in the feed channel. Permeate pressure at the centre pipe is assumed to be atmospheric pressure. Permeate flowrate and concentration at the closed end can be determined based on water and salt fluxes at the corresponding locations.

The model equations presented so far cannot be solved analytically. The differential equations in Eqs. (26)–(33) are transformed to algebraic equations using the first-order backward finite difference method and an iterative algorithm is employed to solve the equations for flux, volumetric flowrate, concentration and pressure. For detailed information on the numerical procedure, refer to Section D of the Supporting information.

3. Results and discussions

The proposed model is applied to single SWMs and to RO processes with several SWMs. The results are organised as follows. First of all, simulation results for a single SWM, using the plate-like and spiral models are presented, using an aqueous sodium chloride solution as feed. The plate-like and spiral models are compared under various operating conditions with respect to the spatial variations obtained from each model as well as overall performance and specific energy consumption. Secondly, the spiral model is validated against an extensive set of experimental data. It is then investigated further by studying the effects of module design parameters: the number of leaves, membrane dimensions, centre pipe radius and feed and permeate channel heights.

The model parameters, water and salt permeability constants and friction coefficients in the feed and permeate channel, are either extracted directly from the literature or estimated using published experimental data. All model parameters and module specifications used in the present study are listed in Table 1, along with the sources of data. The length and width of the membrane indicate the initial dimensions of the membrane without taking into account the glued areas on membranes to spacers. The glue line width in each direction is found based on the values of initial and active membrane dimensions presented in the literature (Avlonitis et al., 1991, 1993). The friction coefficients used in this study are chosen to produce a pressure drop in each channel of a similar magnitude to the measured values reported in the literature (Avlonitis et al., 1991, 1993). The centre pipe radius is arbitrarily chosen for an initial simulation and this
Table 1
List of model parameters and module specifications used in simulations.

| Symbol | Name | Value | Source |
|--------|------|-------|--------|
| $N_{0,\text{av}}$ | Number of leaves | 1 | Avlonitis et al. (1991, 1993) |
| $l_0$ [m] | Membrane length | 0.946 | Avlonitis et al. (1991, 1993) |
| $W_0$ [m] | Membrane width | 1.34 | Avlonitis et al. (1991, 1993) |
| $H_0$ [m] | Feed spacer thickness | $7.7 \times 10^{-4}$ | Avlonitis et al. (1991, 1993) |
| $H_0$ [m] | Permeate spacer thickness | $4.3 \times 10^{-4}$ | Avlonitis et al. (1991, 1993) |
| $l_p$ [m] | Glue line width in length direction | $4 \times 10^{-2}$ | Avlonitis et al. (1991, 1993) |
| $W_p$ [m] | Glue line width in direction | $17 \times 10^{-2}$ | Avlonitis et al. (1991, 1993) |
| $r_c$ [m] | Centre pipe radius | $3 \times 10^{-2}$ | Arbitrarily chosen |
| Salt type | Aqueous NaCl solution | | |
| $A$ [m$^3$/m$^2$s Pa] | Pure water permeability constant | $2.74 \times 10^{-12}$ | Patrakloulou et al. (2013) |
| $B$ [m$^3$/m$^2$s] | Salt permeability constant | $1.74 \times 10^{-8}$ | Patrakloulou et al. (2013) |
| $k_{w,j}$ [-] | Frictional coefficient in feed channel | 2.8 | Avlonitis et al. (1991, 1993) |
| $k_{p,j}$ [-] | Frictional coefficient in permeate channel | 30 | Avlonitis et al. (1991, 1993) |
| $\varepsilon_{\text{pump}}$ [-] | Efficiency of high pressure pump | 0.85 | Greenlee et al. (2009), Semiat (2008), and Kim et al. (2009). |
| $\varepsilon_{\text{exchanger}}$ [-] | Efficiency of pressure exchanger | 0.98 | Greenlee et al. (2009), Semiat (2008), and Kim et al. (2009). |
| $\alpha$ | Coefficient for Eq. (8.8) | 1.85 | McCutcheon and Elimelech (2006) |
| $\beta$ | Exponent for Eq. (8.8) | 0.33 | McCutcheon and Elimelech (2006) |
| $\gamma$ | Exponent for Eq. (8.8) | 0.33 | McCutcheon and Elimelech (2006) |
| $\delta$ | Exponent for Eq. (8.8) | 0.33 | McCutcheon and Elimelech (2006) |
| $r$ [m] | Small gap in Eq. (12) | $0.05 \times 10^{-3}$ | Arbitrarily chosen |

is followed by an analysis of performance and energy consumption with varying radii. In order to predict the net energy consumption, efficiency parameters for a high pressure pump and pressure exchanger are introduced and selected based on recent studies (Greenlee et al. 2009; Semiat, 2008; Kim et al. 2009). The permeability constants are obtained from the literature (Patrakloulou et al., 2013). Constants for the Sherwood correlation in Eq. (8.8) (in the Supporting information) are found in the literature (McCutcheon and Elimelech, 2006), based on one of simplest correlations to describe a rectangular channel.

### 3.1. Simulation results

The plate-like model and the spiral model are simulated under the same conditions, and comparisons are made in terms of predicted membrane performance and specific energy consumption. The same membrane leaf size is used in all cases, and it is noted that this results in different active membrane areas and module dimensions. Since the focus of this paper is to present the spiral model, simulation results from the spiral model are mainly presented in this section while the spatial variations obtained from the plate-like model are displayed in Section F of the Supporting information for comparison purposes.

#### 3.1.1. Simulation results under various conditions

As shown in Fig. 4, a general trend in the variation of predicted water flux obtained with the plate-like model and the spiral model can be observed, specifically that the values generated with the plate-like model are larger than those computed with the spiral model. The higher value of water flux from the plate-like model can be explained by noting that the effective membrane area and the actual membrane area are the same in the plate-like model, but that the effective membrane area is smaller in the spiral model. Based on the assumptions made for the plate-like model, the total amount of water and salt permeation taking place in a single leaf SWM is calculated by doubling the amount of permeation through one side of a folded membrane. As a result, the dead membrane area is not taken into account, which leads to overestimation of the membrane area and the module performance by approximately 5% and 11–20%, respectively. Due to the higher water recovery predicted by the plate-like model, the resulting specific energy consumption is lower than that calculated by the spiral model since the specific energy consumption is defined as the ratio of net energy consumption to water production rate.

Both models respond in the same way to varying inlet conditions. At high inlet flowrate and low salt concentration, larger water flux can be achieved. The specific energy consumption, however, is larger at high flowrate due to the increased energy requirements to pump a larger amount of feed water, but there is only a slight increase in produced permeate water. When feed pressure increases from 50 × 10$^5$ Pa to 70 × 10$^5$ Pa, the water flux increases almost linearly whereas a variation in the specific energy consumption is not significant.

Relative differences between the two models for a single module lie in the range of 11–20% and 1.5–4.5% for performance and specific energy consumption, respectively; in a large scale system such errors may accumulate as the feed flow passes through multiple modules arrayed in series.

#### 3.1.2. Spatial variations obtained from the spiral model

Since the spiral model takes into account the spiral direction $\theta$-axis for the permeate flow as presented in Fig. 3, the $\theta$-axis is transformed to the $x$-axis as presented in Fig. F.1 for the plate-like model (in the Supporting information), using the following relation in order to display spatial variations in the Cartesian coordinates.

$$x(\theta) = \int_{\theta_{i,k}}^{\theta_{j,k}} \sqrt{\left(\frac{dx_{ij}}{d\theta}\right)^2 + \left(\frac{dy_{ij}}{d\theta}\right)^2} d\theta - \int_{\theta_{i,k}}^{\theta_{j,k}} \sqrt{\left(\frac{dx_{ij}}{d\theta}\right)^2 + \left(\frac{dy_{ij}}{d\theta}\right)^2} d\theta \text{ at } \theta_{i,k} < \theta < \theta_{j,k} (k = \text{in, out})$$

(40)

where $x$ is the coordinate of the permeate flow direction as in the plate-like model, which is equivalent to the $\theta$-direction in the spiral model. The spatial variations obtained from the spiral model can be plotted with respect to the $z$-and $x$-axes, along which the feed and permeate streams flow. Fig. 5 shows the spatial variation of water fluxes and flow velocity and pressure in the permeate channel. Different
results are obtained for inner and outer membranes, which are distinguished by subscripts in and out. Due to the presence of dead space in the outer membrane, the active width of the outer membrane is not the same as that of the inner membrane, as presented in Fig. 5.

The spatial variations obtained from the spiral model are similar to those generated by the plate-like model except that there are unusual discontinuities in the spiral model. In Fig. 5, the water flux through the inner membrane exhibits a sharp increase near the closed end of a permeate channel where the coordinate of the x-axis is zero. At the outlet of the SWM module, a percentage change in water flux at the interface between Region 2 and 3 is found to be approximately −9%. This can be interpreted as a consequence of the sudden drop in feed channel height, which affects concentration polarisation and consequently water and salt permeation.

For the permeate channel, on the other hand, no discontinuity appears except for that in the permeate flow velocity since the permeate flow is influenced by both inner and outer membranes. The discontinuity in the permeate flow velocity profile is caused by the sudden expansion of the permeate channel near the centre pipe. An interesting observation is that the qualitative distributions of all state variables, J, Q, u, c and P, for the outer membrane are similar to those of Region 1 in the inner membrane, although the magnitudes are slightly different. For instance, both water fluxes through the inner membrane and outer membranes at the outlet of the SWM range between $5.5 - 5.6 \times 10^{-6}$ m$^3$/(m$^2$·s). In addition to differences in spatial profiles, quantitative differences exist in the characteristics of the SWM calculated using the plate-like and spiral models. In particular, the pressure at the closed permeate channel, which is an important factor in determining the driving force for water permeation, is found to be affected. The spiral model predicts a permeate pressure of approximately 1.25 bar, whereas the prediction of the plate-like model is around 2.5 bar.

3.2. Model validation

Plate-like models such as the one used here have been shown to provide reliable simulation results, in good agreement with experimental data (Avlonitis et al., 1991, 1993; Boudinar et al., 1992; Schock and Miquel, 1987; Senthilmurugan et al., 2005; Taniguchi, 1978). The spiral model developed in this study also needs to be validated against experimental data including detailed measurements of permeate flowrate, feed outlet pressure and permeate pressure at the closed end under various feed flowrates, pressures and temperatures obtained from
pure water experiments (Avlonitis et al., 1991) and from aqueous solutions of sodium chloride (Boudinar et al., 1992; Taniguchi, 1978). Due to the differences in the plate-like and spiral models, model parameters derived from the same data sets for the two models may be expected to differ. Parameter estimation is carried out by solving an optimisation problem in which the objective function is to minimise the sum of squared errors between predicted and measured output, as shown in Eq. (41).

\[ F = \min \sum_{i=1}^{n} \sum_{j=1}^{m} w_i^j (y_i^j - \hat{y}_i^j)^2 \]  

(41)

where \( F \) is the objective function, \( n \) the number of experiments, \( m \) the number of measured variables, \( w_i^j \) a weighting factor for the \( j \)-th variable, \( y_i^j \) the value of measured variable \( j \) in experiment \( i \), and the \( \hat{y}_i^j \) the value of variable \( j \) at the conditions of experiment \( i \) as calculated by the model. The friction coefficients reported in the literature cannot be used directly due to the different pressure drop model adopted in this study, but the literature value of the ratio between the friction coefficients for the permeate and feed spacers is applied (Avlonitis et al., 1991; Boudinar et al., 1992; Taniguchi, 1978), i.e., \( k_{sp,p}/k_{sp,f} = 4.1 \) for the data of Taniguchi (1978) and \( k_{sp,p}/k_{sp,f} = 48 \) for the data of Boudinar et al. (1992). A total of \( n \) sets of experimental data are divided into two categories: ‘estimation sets’ for parameter estimation and ‘validation sets’ for model validation. Estimated parameters are used in the plate-like and spiral models to predict performance under the experimental conditions of the validation sets. Predicted results are then compared to experimental measurements to assess the validity of the model by means of average absolute relative errors (AARE) as shown in Fig. 6. Further information can be found in Section G of the Supporting information.

3.2.1. Parameter estimation from pure water experiments

Parameters are first derived using data from experiments with pure water. Three measured variables \((m = 3)\) are used, \( Q_p \) in \( m^3/s \), \( P_{f,out} \) in Pa and \( P_{p,in} \) in Pa, with respective weighting factors \( w^1 = 10^{12}, w^2 = 10^{-9} \) and \( w^3 = 10^{-9} \), to reflect the differences in scaling between the volumetric output and pressures. The operating conditions and measured output at \( 20 \degree C \) are used in order to find a constant water permeability parameter at isothermal conditions and a total of 16 sets of data \((n = 16)\) are extracted from the literature (Avlonitis et al., 1991).

| Model          | Estimated parameters | \( k_{sp,f} \) |
|----------------|----------------------|---------------|
| Spiral model   | 3.43                 | 2.40          |
| Plate-like model | 3.29                 | 0.54          |

Table 2 Parameters estimated using the spiral and plate-like models.
The parameters obtained for each model are listed in Table 2. The water permeability obtained with the spiral model is approximately 4% larger than that with the plate-like model, which is encouraging as this parameter is intrinsic to the membrane. The values of the friction coefficients for the feed channel, on the other hand, are very different; the friction coefficient estimated with the spiral model is approximately 4 times larger than that with the plate-like model. The smaller magnitude of the friction coefficient in the plate-like model can be explained by the larger pressure drops that are predicted with the plate-like model when the same parameter values are used in both models.

AARE for the plate-like model are comparable with those for the spiral-model despite the differences in the parameters. When using the spiral model, the maximum relative errors for permeate flowrate, outlet feed pressure and permeate inlet pressure are approximately 8%, 2% and 10%, respectively.

3.2.2. Parameter estimation from aqueous salt solution experiments

Further validation of the spiral model is undertaken using measured data on permeate flowrate and concentration found in existing studies and comparisons are also made between results calculated by the spiral model developed in this study and by other models for SWMs (Boudinar et al., 1992; Senthilmurugan et al., 2005; Taniguchi, 1978). Boudinar et al. (1992) reported their experimental and numerical simulation results, as well as comparisons with Taniguchi’s experimental data (Taniguchi, 1978) for a different type of SWM. Senthilmurugan et al. (2005) also developed a model for SWM and used experimental data from Boudinar et al. (1992) and Taniguchi (1978)’s work in order to estimate model parameters. Therefore in this study, two sets of experimental data – one from Taniguchi (1978) Taniguchi (1978)’s work (a total of 10 sets) and the other from Boudinar et al. (1992) (a total of 15 sets) – are used. Subsequently the results obtained with the spiral model are compared with the simulation results of Boudinar et al. (1992) and Senthilmurugan et al. (2005).

An additional parameter, salt permeability constant, B, is also estimated since the feed solutions used in the experiments are aqueous sodium chloride solutions with different concentrations. The weighting factors for Q_p and c_p are chosen to be 10^10 and 10^-4, respectively. Module specifications for each SWM and the corresponding experimental data can be found in the literature (Boudinar et al., 1992; Senthilmurugan et al., 2005; Taniguchi, 1978). The parameters estimated for each set of experiments are listed in Table 3.

AARE of the spiral model in permeate flowrates and concentrations are presented in Fig. 7 along with those of two other models. The results calculated by all three models are comparable. AARE for the spiral model are about 1.6% and 3.4% for permeate flowrate and concentration, respectively.

Similarly, calculations with the spiral model and experimental data by Boudinar et al. (1992) are compared and the AAREs of three model calculations are depicted in Fig. 8. The spiral model results in AAREs of 6.8% and 5.8% for permeate flowrate and concentration, respectively.

It can be seen that the AAREs for the three calculations are comparable. Given that relatively simple correlations for mass transfer coefficient and pressure drop are employed in the spiral model together with the assumption of constant water permeability, the spiral model provides a good representation of the performance of the SWM and its dependence on key operating conditions. It is expected that more accurate correlations for the mass transfer coefficients and pressure drop would lead to an increased reliability in the analysis of the performance of SWMs.

| Experiment data     | Estimated parameters |
|---------------------|----------------------|
|                     | A [10^{-12} m^2/(Pa s)] | k [10^{-8} m/s] | k_{op} [/] |
| Taniguchi [17]      | 2.61                  | 15.0           | 42.8        |
| Boudinar et al. [13] | 4.47                  | 3.86           | 2.40        |
3.3. Effect of design parameters

A unique characteristic of the proposed spiral model is that it enables the effect of a broader range of decisions in membrane module design to be investigated. Here, the effects of the number of membrane leaves and the length or width of each leaf are first analysed using both the plate-like and spiral models, for a fixed total membrane area. This is followed by an investigation of the effects of the centre pipe radius and channel height using the validated spiral model, as it captures the impact of these parameters in detail.

3.3.1. Number of leaves and membrane dimensions

A total of 17 cases are studied by varying the number of leaves and dimensions of the membranes as specified in Table 4, where case numbers starting with \( W \) or \( L \) indicate that the width or length of the membrane is varied (\( W_i = 0.45–4.5 \text{ m} \) and \( L_i = 0.3–3 \text{ m} \)), respectively, while keeping a constant total membrane area of 4.5 \( \text{ m}^2 \). Other module parameters such as channel heights and glue line width are kept the same as those in Table 1. The feed inlet conditions used for the simulations are \( 6 \times 10^{-6} \text{ m}^3/\text{s}, 35 \text{ kg/m}^3 \) and \( 60 \times 10^3 \text{ Pa} \) for feed flowrate, salt concentration and pressure, respectively.

Water flux, permeate flowrate and specific energy consumption for the 17 cases are predicted by the plate-like model and by the spiral model and the results are depicted in Fig. 9. The water flux and permeate flowrate predicted by the plate-like model are higher than those predicted by the spiral model in most cases. However, for the largest and the second largest width (Case W1 and W2), an opposite behaviour to this general trend is exhibited as can be seen in Fig. 9 (a) and (b). Due to the presence of the glue line width, the permeate flowrate

![Fig. 7](image_url)

Fig. 7. Average absolute relative errors (AARE) of model calculations (by spiral model, Boudinar et al., 1992 and Senthilmurugan et al., 2005) against experimental data (Taniguchi, 1978). (a) Permeate flowrate, (b) permeate concentration. Bars are the AARE and the lines on each bar indicate the range of absolute relative errors.

![Fig. 8](image_url)

Fig. 8. Average absolute relative errors (AARE) of model calculations (by spiral model, Boudinar et al., 1992 and Senthilmurugan et al., 2005) against experimental data (Boudinar et al., 1992). (a) Permeate flowrate, (b) permeate concentration. Bars are the AARE and the lines on each bar indicate the range of absolute relative errors.

| Case No. | W1 | W2 | W3 | W4 | W5 | W6 | W7 | W8 | W9 |
|----------|----|----|----|----|----|----|----|----|----|
| \( N_{\text{ref}} \) [\( \cdot \)] | 1  | 2  | 3  | 4  | 5  | 6  | 8  | 9  | 10 |
| \( W_i \) [\( \text{m} \)]  | 4.5| 2.25| 1.5| 1.125| 0.9| 0.75| 0.5| 0.5| 0.45|
| Case No. | L1 | L2 | L3 | L4 | L5 | L6 | L7 | L8 | –  |
| \( N_{\text{ref}} \) [\( \cdot \)] | 1  | 2  | 3  | 4  | 5  | 6  | 8  | 10 | –  |
| \( L_i \) [\( \text{m} \)]  | 3  | 1.5| 1  | 0.75| 0.6| 0.5| 0.375| 0.3| –  |

Table 4

List of cases with varying number of leaves and width of the membrane, with a fixed length of 1 \( \text{ m} \) and a total membrane \( A_{\text{mem}} \) of 4.5 \( \text{ m}^2 \) (W1–W9) and with varying number of leaves and length of the membrane, with a fixed width of 1.5 \( \text{ m} \) and a total membrane \( A_{\text{mem}} \) of 4.5 \( \text{ m}^2 \) (L1 to L8).
declines as the number of leaves increases, that is, the effective membrane area becomes smaller even if the total area of membranes used is constant.

The differences in the two models have implications for the optimal design of SWMs, where it may be desirable to maximise permeate flowrate or minimise specific energy consumption. The results obtained in our work indicate that the method by which geometric parameters are represented can influence the optimal module design. Based on the results shown in Fig. 9 (b) and (c), the largest permeate flowrate and smallest specific energy consumption can be obtained when an SWM is designed to have three leaves (at \( W_t = 1.5 \) m) according to the plate-like model while the spiral model predicts an optimal design to be a 2-leaf module (at \( W_t = 2.25 \) m). When the number of leaves and the length of membrane leaves are varied (Fig. 9 (e) and (f)), from L1 to L8, both the plate-like and spiral models predict Case L2 (with \( L_t = 1.5 \) m) to be the best design with respect to permeate flowrate while the designs that exhibit the smallest specific energy consumption are also different: L4 (\( L_t = 0.75 \) m) for the plate-like model and L3 (\( L_t = 1 \) m) for the spiral model.

As explained earlier, the pressure drop along a feed channel and the permeate pressure at the closed end are predicted differently by the plate-like model and the spiral model (Table 5). An interesting observation can be made based on the pressure drops predicted for cases W1 to W9; as the width decreases, the pressure drop predicted by the plate-like model tends to increase (from \( 0.35 \times 10^5 \) Pa to \( 0.54 \times 10^5 \) Pa) whereas the pressure drop predicted by the spiral model decreases slightly (from \( 0.29 \times 10^5 \) Pa to \( 0.26 \times 10^5 \) Pa). This may be caused by the differences in the cross-sectional area of flow in the feed channel, which affects flow velocity and consequently pressure drop.

The performance and energy consumption are affected by the number of membrane leaves as is the size of the outer module cover. The outer cover radius tends to increase as the number of leaves increases. Since the width of the membrane leaves for Cases W1 to W9 is adjusted to make the total area constant, the range of the resulting radii is narrow, between 51 mm and 52 mm. For Cases L1 to L8, however, as the number of leaves increases, the width remains unchanged and consequently the radius of the outer cover expands more rapidly between 39 mm for Case L1 and 82 mm for Case L8. Thus, using the spiral model developed in this study, the volume of the SWM relative to the effective membrane area required to meet a desired production rate can also be investigated.

| Case | Width [m] | Length [m] |
|------|-----------|------------|
| W1   | 1         | 3          |
| W2   | 2         | 3          |
| W3   | 3         | 3          |
| W4   | 4         | 3          |
| W5   | 5         | 3          |
| W6   | 6         | 3          |
| W7   | 7         | 3          |
| W8   | 8         | 3          |
| W9   | 9         | 3          |
| L1   | 1         | 3          |
| L2   | 2         | 3          |
| L3   | 3         | 3          |
| L4   | 4         | 3          |
| L5   | 5         | 3          |
| L6   | 6         | 3          |
| L7   | 7         | 3          |
| L8   | 8         | 3          |

**Table 5**

Pressure drop in the feed channel \( P_{\text{in}} \) and permeate pressure at the closed end \( P_{\text{in}} \) predicted by the plate-like (P) and spiral (S) models and calculated outer cover radius \( r_{oc} \) for varying numbers of leaves and dimensions, with the total membrane area fixed.

| Case | \( P_{\text{in}} \) [10^5 Pa] | \( P_{\text{in}} \) [10^5 Pa] | \( r_{oc} \) [mm] |
|------|-------------------------------|-------------------------------|------------------|
| W1   | 0.352                         | 0.290                         | 51.9             |
| W2   | 0.363                         | 0.274                         | 51.9             |
| W3   | 0.378                         | 0.261                         | 51.9             |
| W4   | 0.396                         | 0.248                         | 51.9             |
| W5   | 0.545                         | 0.194                         | 51.9             |
| W6   | 3.59                          | 2.29                          | 51.9             |
| W7   | 0.874                         | 0.591                         | 51.9             |
| W8   | 0.378                         | 0.261                         | 51.9             |
| W9   | 0.027                         | 0.191                         | 51.9             |
| L1   | 5.45                          | 2.12                          | 51.9             |
| L2   | 1.46                          | 1.24                          | 51.9             |
| L3   | 1.02                          | 1.39                          | 51.9             |
| L4   | 3.05                          | 3.10                          | 51.9             |
| L5   | 3.07                          | 3.07                          | 51.9             |
| L6   | 2.89                          | 2.89                          | 51.9             |
| L7   | 1.46                          | 1.46                          | 51.9             |
| L8   | 1.43                          | 1.43                          | 51.9             |

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Fig. 9. Predicted water flux, permeate flowrate and specific energy consumption (left to right). (a), (b) and (c): results for Cases W1 to W9. (d), (e) and (f): for Cases L1 to L8.
3.3.2. Centre pipe radius

The selection of the centre pipe radius for an SWM can influence module performance, energy consumption and required footprint. In order to study the effect of centre pipe radius, a single leaf SWM is simulated with varying centre pipe sizes. Only the spiral model is used here since the centre pipe radius is not taken into account in the plate-like model. The centre pipe radius is varied across a wide range between 1 mm and 65 mm, even though excessively small or large sizes may not be commercially available. Feed inlet conditions are $6 \times 10^{-4}$ m$^3$/s, 35 kg/m$^3$ and $60 \times 10^5$ Pa for feed flowrate, concentration and pressure, respectively, with all geometric parameters other than the centre pipe radius kept identical to those shown in Table 1.

The predicted water flux, permeate flowrate and specific energy consumption for various centre pipe radii are illustrated in Fig. 10. As the centre pipe radius increases, the total permeate flowrate decreases and accordingly the specific energy consumption increases. This is because a larger centre pipe causes a larger dead membrane area where water permeation does not occur in the outermost layer of wrapped membranes.

Although variations in the results with respect to the centre pipe radius is not significant, it can be concluded that an SWM assembled with a smaller centre pipe can achieve lower specific energy consumption and higher production rate. However there is a limitation to using a small radius. There should be a sufficient number of holes in the centre pipe to allow permeate to flow into it and to be collected. In addition, the number of leaves that can be attached around a centre pipe is limited by its surface area. Therefore, a minimum centre pipe radius should be specified as a constraint in order to use the spiral model for module design optimisation.

In summary, the effect of centre pipe radius on performance and specific energy consumption is not as significant as the number of leaves and membrane dimensions as presented in the previous section. However, it cannot be entirely neglected since minor differences can accumulate when multiple modules are used in a large scale RO plant.

3.3.3. Thickness of feed and permeate spacers

Spacers play an important role in an SWM since they determine the height of the feed and permeate channels. In order to study the effect of the heights of the channels, maps of effective water flux, permeate flowrate and specific energy consumption are generated by carrying calculations across a range feed and permeate spacer thicknesses, as depicted in Fig. 11. The other geometric parameters used in the simulations are kept the same as listed in Table 1 and feed inlet conditions are $1.2 \times 10^{-4}$ m$^3$/s, 35 kg/m$^3$ and $60 \times 10^5$ Pa for feed flowrate, concentration and pressure, respectively.

An optimal SWM design with respect to spacer thickness can then be obtained based on these different key performance indicators. Values for the maximum effective water flux, maximum permeate flowrate and minimum specific energy consumption are indicated in each panel in Fig. 11, along with the corresponding spacer thicknesses. It is obvious that for the two channels the pressure drop is larger at
the smaller spacer thicknesses hence larger energy consumption as shown in Fig. 11(c). Since the permeate flowrate affects the pressure drop in both feed and permeate channels and the pressure difference between the two sides is the main driving force for water permeation, the relation of the pressure driving force with both spacer thicknesses is not straightforward. Finally, a general trend is also found that thicker spacers result in larger SWM volume; at \( H_f = 0.2 \) mm and \( H_p = 0.15 \), \( r_{oc} = 32.7 \) while \( H_f = 1.0 \) mm and \( H_p = 0.75 \), \( r_{oc} = 41.2 \) mm.

It is worth noting that conditions at which either the maximum permeate flowrate or the minimum specific energy consumption is achieved cannot be applied universally due to uncertainty in the effect of spacers on the frictional coefficients. Thus, the results presented here can only be treated as indicative and it is important for accurate parameters associated with spacers to be obtained and their effects on performance and energy consumption to be scrutinised in future studies.

3.4. Large-scale RO process

Although differences in the predicted performance and energy consumption by the plate-like and spiral models are not large for a single SWM, real RO systems involve multiple stages and multiple SWM elements. In order to assess the impact of using the spiral model on a large-scale RO process, a 2-stage configuration is selected where concentrated brine from the first stage is fed into the second stage. The numbers of pressure vessels in the first and second stage are chosen to be 40 and 20, respectively, arrayed in parallel. It is assumed that six elements of SWM are loaded into each pressure vessel. Operating conditions are \( 7.0 \times 10^{-2} \) m\(^3\)/s, 35 kg/m\(^3\) and \( 60 \times 10^5 \) Pa for feed flowrate, concentration and pressure, respectively. The same model parameters and module specifications are used as listed in Table 1.

A summary of predicted overall performance and specific energy consumption by the plate-like and spiral models is given in Table 6. Similar to the trend observed with a single SWM, the plate-like model predicts higher permeate flowrates than the spiral model. It is clear that the total water recovery predicted by the plate-like model is about 20% higher than that predicted by the spiral model. The large discrepancy in predicted total water recovery might lead to substantial difference in an RO desalination process design which aims to achieve 50% seawater recovery, particularly in terms of the number of modules required and the configuration. It is also worth noting that there is hardly any difference in the specific energy consumption predicted by the two models (Table 6).

4. Conclusions

A detailed model for the spiral wound modules used in reverse osmosis (RO) processes has been developed and used to study the effects on module performance of operating conditions and, more importantly, of the geometric parameters of relevance to SWM design, such as the number of membrane leaves, their dimensions, the radius of the centre pipe, and the thickness of permeate and feed spacers. Equations to describe the module geometry and transport through the module have been developed.

The spiral model was successfully validated against experimental data from the literature and was demonstrated to be able to capture observed behaviour reliably. It was found that the friction coefficients estimated for the plate-like and spiral models from the same data sets were noticeably different, although the same pressure drop model was used in both models. Due to a lack of adequate experimental data, it is not possible to discriminate between the plate-like and spiral models and to assess the extent to which the spiral model captures relevant geometric features. Nonetheless, the relative errors between the experimental and calculated performances for the spiral model and plate-like models were found to be comparable, providing support for the validity of the spiral model.

The spiral model has been compared to the more standard plate-like model, which is often used to investigate SWM performance. Our results demonstrate the advantages of using a model that describes the spiral geometry accurately. In particular, the two models were found to lead to significant differences in the predicted permeate pressures. The detailed geometric representation enables usual performance metrics such as specific energy consumption and water recovery to be calculated as well as quantities such as the outer cover size, which is required to determine the footprint of an RO process. A more detailed distributions of fluxes, flowrates, concentrations and pressures is also obtained from the model, as the effect of different regions in the module, where the outer membrane may or may not be active, is recognised. Geometric parameters such as channel height, cross-sectional area and flow path length, are found to have a strong influence on predicted performance and energy consumption.

Finally the performance and specific energy consumption of a large-scale RO process were predicted by both the plate-like and spiral models, resulting in significant differences particularly in predicting water recoveries at the same conditions. This indicates that the geometric details of single SWMs have an important impact on overall RO process performance and should be taken into account if models are to be used to support the design of SWMs.

In summary, the spiral model developed in this study can serve as a useful tool to analyse the performance and energy consumption under various operating conditions and configurations. It offers the following advantages over existing models for a spiral wound module:

- The reconstruction of cross-sectional spiral curves provides an understanding of the complex spiral geometry.
- More realistic geometric parameters – channel height, cross-sectional area, trans-membrane area and flow path length – can be obtained.
- The model is computationally tractable, making it suitable for the initial design of single modules.
- System optimisation can be performed with respect to the footprints of RO plants, which are correlated with cost as well as performance and energy.

A limitation of the present study is the reliance on mass transfer correlations which have been derived in the context of a specific geometric design. It is expected that key design decisions such as spacer thickness and design (e.g., woven vs non-woven) can have a

| Table 6 | Summary of simulation results for a 2-stage RO process using the plate-like and spiral models. |
|---------|-------------------------------------------------------------------------------------------------|
| Model   | Average water flux \( f_{w,ave} \) [10\(^{-6}\) m\(^3\)/m\(^2\) s] | Recovery \( R_w \) [%] | Specific energy consumption \( E_w \) [kWh/m\(^3\)] |
| Plate-like | 4.93 | 51.3 | 2.09 |
| Spiral | 4.12 | 42.9 | 2.10 |
significant effect on mass transfer coefficients. Future studies will be focussed on obtaining accurate correlations of model parameters and applying the proposed model to the optimisation of module design and system configuration.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.compchemeng.2016.07.029.

References

Alnouri, S.Y., Linke, P., 2012. A systematic approach to optimal membrane network synthesis for seawater desalination. J. Membr. Sci. 417–418, 96–112.
Avlonitis, S., Hanbury, W.T., Boudnar, M.B., 1991. Spiral wound modules performance. An analytical solution, Part I. Desalination 81, 191–208.
Avlonitis, S., Hanbury, W.T., Boudnar, M.B., 1993. Spiral wound modules performance. An analytical solution, Part II. Desalination 89, 227–246.
Bartman, A., Christofides, P.D., Cohen, Y., 2009a. Non-linear model-based control of an experimental reverse-osmosis water desalination system. Ind. Eng. Chem. Res. 48, 6126–6136.
Bartman, A.R., McFall, C.W., Christofides, P.D., Cohen, Y., 2009b. Model-predictive control of feed flow reversal in a reverse osmosis desalination process. J. Process Control 19, 433–442.
Bartman, A., Zhu, A., Christofides, P.D., Cohen, Y., 2010. Minimizing energy consumption in reverse osmosis membrane desalination using optimization-based control. J. Process Control 20, 1261–1269.
Boudinar, M.B., Hanbury, W.T., Avlonitis, S., 1992. Numerical simulation and optimization of spiral-wound modules. Desalination 66, 273–290.
Dickson, J.M., Spencer, J., Costa, M.L., 1992. Dilute single and mixed solute systems in a spiral wound reserve osmosis module. Part I: theoretical model development. Desalination 89, 63–88.
Elimelech, M., Phillip, W.A., 2011. The future of seawater desalination: energy, technology, and the environment. Science 333, 712–717.
Emad, A., Aljabar, A., Almutaz, A., 2012. Periodic control of a reverse osmosis desalination process. J. Process Control 22, 218–227.
Evangelou, F., Jonsson, G., 1988. Optimal design and performance of spiral wound modules I: numerical method. Chem. Eng. Commun. 72, 69–81.
Fletcher, D.F., Wiley, D.E., 2004. A computational fluids dynamics study of buoyancy effects in reverse osmosis. J. Membr. Sci. 245, 175–181.
Fritzmann, C., Iwenberg, J., Wintgens, T., Melin, T., 2007. State-of-the-art of reverse osmosis desalination. Desalination 216, 1–76.
Gárdalé, V., Pereira, N.E., de Pinho, M.N., 2005. Simulation and optimization of medium–sized seawater reverse osmosis processes with spiral–wound modules. Ind. Eng. Chem. Res. 44, 1897–1905.
Ghafoor, N., Mismimer, T.M., Amy, G.L., 2013. Technical review and evaluation of the economics of water desalination: current and future challenges for better water supply sustainability. Desalination 309, 197–207.
Greenlee, L.F., Lawler, D.F., Freeman, B.D., Marrot, B., Moulin, P., 2009. Reverse osmosis desalination: water sources, technology, and today’s challenges. Water Res. 43, 2317–2348.
Gu, B., Kim, D.Y., Kim, J.H., Yang, D.R., 2011. Mathematical model of flat sheet membrane modules for FO process: plate-and-frame module and spiral-wound module. J. Membr. Sci. 370, 403–415.
Kedem, O., Katchalsky, A., 1958. Thermodynamic analysis of the permeability of biological membrane to non-electrolytes. Biochim. Biophys. Acta 27, 229–246.
Kim, Y.M., Kim, S.J., Kim, Y.S., Lee, S., Kim, I.S., Kim, J.H., 2009. Overview of systems engineering approaches for a large–scale seawater desalination plant with a reverse osmosis network. Desalination 238, 312–322.
Li, M.H., Noh, B., 2012. Validation of model-based optimization of brackish water reverse osmosis (BWRO) plant operation. Desalination 304, 20–24.
Li, M.H., 2012. Optimal plant operation of brackish water reverse osmosis (BWRO) desalination. Desalination 293, 61–68.
Malaeb, L., Ayoub, G.M., 2011. Reverse osmosis technology for water treatment: state of the art review. Desalination 267, 1–8.
McCUTCHEON, J.R., ELIMELECH, M., 2006. Influence of concentrative and dilutive internal concentration polarization on flux behaviour in forward osmosis. J. Membr. Sci. 284, 237–247.
Patriokou, G., Sassi, K.M., Mutjaba, I.M., 2011. Simulation of boron rejection by seawater reverse osmosis desalination. Chem. Eng. Trans. 32, 1873–1878.
Paul, D.R., 2004. Reformulation of the solution-diffusion theory of reverse osmosis. J. Membr. Sci. 241, 371–386.
Riverol, C., Plišopović, V., 2005. Mathematical modeling of perfect decoupled control system and its application: a reverse osmosis desalination industrial-scale unit. J. Autom. Methods Manage. Chem. 2005, 50–54.
Schock, G., Miqael, A., 1987. Mass transfer and pressure loss in spiral wound modules. Desalination 64, 339–352.
Schwing, J., Neal, P.R., Wiley, D.E., Fletcher, D.F., Fane, A.G., 2004. Spiral wound modules and spacers – review and analysis. J. Membr. Sci. 242, 129–153.
Semia, R., 2008. Energy issues in desalination process. Environ. Sci. Technol. 42, 8153–8201.
Senthilmurugan, S., Ahiuluvalia, A., Gupta, S.K., 2005. Modeling of a spiral-wound module and estimation of model parameters using numerical techniques. Desalination 173, 269–286.
Sirkar, K.K., Dang, P.T., Rao, G.H., 1982. Approximate design equations for reverse osmosis desalination by spiral–wound modules. Ind. Eng. Chem. Process Des. Dev. 21, 517–527.
Solomo, M., Gill, W.N., 1981. Review of reverse–osmosis membranes and transport models. Chem. Eng. Commun. 12, 279–363.
Spiegel, K.S., Kedem, O., 1966. Thermodynamics of hyperfiltration (reverse osmosis): criteria for efficient membranes. Desalination 1, 311–326.
Taniguchi, Y., 1978. An analysis of reverse osmosis characteristics of ROGA spiral–wound modules. Desalination 25, 71–88.
Wijmans, J.G., Baker, R.W., 1995. The solution–diffusion model: a review. J. Membr. Sci. 107, 1–21.
Zhou, W.W., Song, L.F., Guan, T.K., 2006. A numerical study on concentration polarization and system performance of spiral wound RO membrane modules. J. Membr. Sci. 271, 38–46.
Zhu, A., Christofides, P.D., Cohen, Y., 2009a. Effect of thermodynamic restriction on energy cost optimization of RO membrane water desalination. Ind. Eng. Chem. Res. 48, 6010–6021.
Zhu, A., Christofides, P.D., Cohen, Y., 2009b. Energy consumption optimization of reverse osmosis membrane water desalination subject to feed salinity fluctuation. Ind. Eng. Chem. Res. 48, 5851–5859.
van der Meer, W.G.J., van Dijk, J.C., 1997. Theoretical optimization of spiral–wound and capillary nanofiltration modules. Desalination 113, 129–146.