Quenched Chiral Behavior of Hadrons with Overlap Fermions

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We study the quenched chiral behavior of hadrons with the pseudoscalar mass as low as \( \approx 280 \, \text{MeV} \). We look for quenched chiral logs in the pion mass, determine the renormalized quark mass, and observe quenched artifacts in the \( a_0 \) and \( N^* \) propagators. The calculation is done on a quenched lattice of size 20\( \times \)4 and \( a = 0.148(2) \, \text{fm} \) using overlap fermions and an improved gauge action.

1. Simulation Details

Using a \( \beta = 7.60 \) (tree-level tadpole-improved) Lüscher–Weisz\(^\text{[1,2]} \) gauge action, we study the chiral properties of hadrons on a 20\( \times \)4 lattice with the overlap fermion\(^\text{[3,4]} \). The massive Dirac operator\(^\text{[5–7]} \) is defined so that the tree-level renormalization of mass and wavefunction is unity.

\[
D(m_0) = (1 - \frac{m_0a}{2\rho})D(\rho) + m_0a \tag{1}
\]

\( \epsilon(H) = H/\sqrt{H^2}, H = \gamma_5D_w, \) and \( D_w \) is the usual Wilson fermion operator, except with a negative mass parameter \( -\rho = 1/2\kappa - 4 \) in which \( \kappa_c < \kappa < 0.25 \); we take \( \kappa = 0.19 \) in our calculation which corresponds to \( \rho = 1.368 \). See\(^\text{[3]} \) for more details of the simulation.

2. Pion Mass and Chiral Logs

To search for chiral logs, we fit \( m_\pi^2a^2 \) to \( \text{Eq. (2)} \)

\[
m_\pi^2a^2 = Am_0a\{1 - \delta[\ln(Am_0a/\Lambda_\chi^2a^2 + 1)] + Bm_0^3a^2
\]

The best fits which give stable values of \( A \) and \( B \) and with errors less than half of the fitted values of \( \delta \) for a range of \( \Lambda_\chi = 0.6 \, \text{GeV} \) to 1.4 \( \text{GeV} \) are listed in the table.

| \( \Lambda_\chi \) | A      | B      | \( \delta \) | \( \chi^2/\text{NDF} \) |
|------------------|--------|--------|-------------|------------------|
| 0.6              | 1.72(7)| 3.0(8) | 0.23(7)     | 0.18             |
| 0.8              | 1.42(11)| 3.0(8) | 0.28(11)    | 0.18             |
| 1.0              | 1.17(24)| 3.0(8) | 0.34(17)    | 0.18             |

We plot the fit with \( \Lambda_\chi = 0.8 \, \text{GeV} \) as a solid line in Fig. 1. To check if there is indeed a quenched chiral log, we fit \( m_\pi^2a^2 \) alternatively without the chiral log term and find

\[
m_\pi^2a^2 = 1.88(10)m_0a - 2.3(11)m_0^2a^2 + 7.8(30)m_0^3a^3
\]

with \( \chi^2/\text{NDF} = 0.46 \). A quadratic fit (shown in Fig. 1 as the dotted line) has \( \chi^2/\text{NDF} = 0.90 \). Thus the presence of a chiral log term is favored.

3. Quark Masses

To renormalize our estimates of the quark mass, we follow\(^\text{[6]} \) to determine the renormalization constants \( Z_{M}^{-1} = Z_{S} = Z_{P} \) and thus...
Figure 1. $m^2 a^2$ versus $m_0 a$ calculated from $G_{A_1} p (p = 0, t)$ with linear plus quadratic fit (dotted line) and chiral log fit (solid line) from Eq. (3) with $\Lambda_\chi = 0.8 \text{ GeV}$.

$$m^{\text{RGI}} = Z_M (g_0) m_0 (g_0): \text{ calculate}$$

$$Z_M (g_0) = U_M \cdot \frac{1}{(r_0 m_\pi)^2} |(r_0 m_\pi)^2 = x_{\text{ref}}$$

using their tabulated values

$$U_M = \begin{cases} 
0.181(6), & x_{\text{ref}} = 1.5736 \\
0.349(9), & x_{\text{ref}} = 3.0 \\
0.580(12), & x_{\text{ref}} = 5.0
\end{cases}$$

where $U_M$ is a universal factor which they estimated from published results of another regularization, namely, $\mathcal{O}(a)$-improved Wilson.

The value of the bare quark mass $m_0 a$ that reproduces a given pseudoscalar mass (through $x_{\text{ref}} = (r_0 m_\pi)^2$, with $r_0 / a = 4.05(2)$ for our lattice) is obtained by interpolation of our data for the pion mass. From this we obtain $Z_M^{-1} = Z_S$ at three values of quark mass (the lowest two quark masses are roughly half-strange and strange) as shown in Fig. 2.

The operators and action are $\mathcal{O}(a)$ improved. For the axial vector renormalization constant, $Z_A$, we find small $(m_0 a)^2$ dependence as well.

Figure 2. Renormalization constant $Z_M^{-1} = Z_S$.

$$Z_S (m_1 = 0) = 1.333 \pm 0.034 \chi^2 / \text{NDF} = 0.86$$

Thus we fit $Z_M^{-1} = Z_S$ to a constant.

$$Z_M^{-1} = Z_S = 1.233(34) \quad \chi^2 / \text{NDF} = 0.86 \quad (6)$$

From the $(m_\pi a)^2$ versus $m_0 a$ figure, we obtain $m_0 a$ at the physical pion mass. Using $m^{\text{RGI}} = m_0 / Z_S$ and, from the 4-loop calculation with $N_f = 0$, $Z_S^{MS} = Z_S / 0.72076$, we obtain (very preliminary results) for $(m_u + m_d) / 2$.

$$m^{\text{RGI}} = 7.3(7)(10) \text{ MeV} \quad (7)$$

$$m^{\text{MS}} (\mu = 2 \text{ GeV}) = 5.3(6)(7) \text{ MeV} \quad (8)$$

4. Chiral Loops in Quenched Propagators

Bardeen et al. [12] present evidence of a strong “quenched chiral loop” (QCL) effect, attributable to an $\eta' - \pi$ intermediate state, in the valence propagation of the scalar, isovector meson ($a_0$). The effect is manifested in a negative-metric contribution to the two-point correlation function which increases for decreasing quark mass. They were able to uncover the effect only after using their pole-shifting procedure, which mitigates the poor chiral properties of the Wilson fermions. We see this effect clearly with overlap fermions without the need for any such procedure.
Figure 3. 2pt corr. func. for $a_0$ with bare quark masses $m_{qa} = 0.0150, 0.0164, 0.0192, \text{and } 0.0274$.

Fig. 3 shows the two-point scalar-isovector local-local correlators for quark masses $m_{qa} = 0.0150, 0.0164, 0.0192, \text{and } 0.0274$; they are plotted on a linear scale to show the correlator becoming negative at early times ($t/a \approx 1 - 2$, the source is at $t_0 = 3$) and then asymptotically approaching zero from below.

We also see the effect for $N^*(\frac{1}{2}^-)$, Fig. 4 shows the two-point $N^*$ local-local correlator for quark masses $m_{qa} = 0.0150, 0.0547, \text{and } 0.1642$. The correlation function becomes negative at successively earlier times for decreasing mass. The effect becomes very pronounced at the lighter masses. See [13] for more details.

5. Summary

We found evidence for chiral logs in the dependence of the pion mass on quark mass. (See [3] for more details.) We also presented, for the first time, our very preliminary results for the renormalized light quark mass and the effect of chiral loops on the $a_0$ and $N^*$ propagators. In a companion proceeding [10], we study zero modes and their effect on the pion propagator, and show our results for the axial renormalization constant $Z_A$ and pion axial decay constant $f_\pi$, and the presence of chiral logs in the pseudoscalar decay constant, $f_P$. See [3] for complete details.

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