Vector Optical Beam with Controllable Variation of Polarization during Propagation in Free Space: A Review

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Abstract: The vector optical beam with longitudinally varying polarization during propagation in free space has attracted significant attention in recent years. Compared with traditional vector optical beams with inhomogeneous distribution of polarization in the transverse plane, manipulating the longitudinal distribution of polarization provides a new dimension for the expansion of the applications of vector optical beams in volume laser machining, longitudinal detection, and in vivo micromanipulation. Two theoretical strategies for achieving this unique optical beam are presented in the way of constructing the longitudinally varying phase difference and amplitude difference. Relevant generation methods are reviewed which can be divided into the modulation of complex amplitude in real space and the filtering of the spatial spectrum. In addition, current problems and prospects for vector optical beams with longitudinally varying polarization are discussed.

Keywords: vector optical beam; structured light; Bessel beam; Poincaré sphere

1. Introduction

Unlike the scalar optical beam with a homogeneous state of polarization (SoP), the vector optical beam [1], with spatially inhomogeneous distributions of polarization, has attracted increasing interest in recent years due to the unique spatial structure of polarization which has a significant influence on the spatio-temporal evolution of light and the interactions between light and matter. It has been demonstrated that the vector beams can shape the focal field [2,3], control optical nonlinear processes [4,5], enhance the longitudinal component [6], and induce strong magnetic impulses [7]. These features raise the prospect of using vector beams in quantum information [8], single molecule imaging [9], particle acceleration [10], nonlinear optics [11], optical microfabrication [12,13], optical micromanipulation, and optical tweezers [14,15].

As one type of structured light [16], the vector beam is supported by spatial manipulation technology of the optical beam. Polarization is an important vector property of light and provides a degree of freedom for customization of the optical beam. Compared with the manipulation of the amplitude and phase distribution in one optical field, the modulation of polarization is more complex. The flexibility of vector beams has been a problem for a long time. Over the past few years, the passive generation method, based on electro-optical modulation outside the resonator [17,18], which effectively promotes the flexibility of polarization manipulation in free space and the diversity of vector optical beams, has developed rapidly [19–24].

Previous studies of vector beams have mostly focused on the polarization distribution in the transverse plane. In addition to the well-known azimuthally and radially varying vector optical field (VOFs) [25], many special transverse distributions, such as bipolar- [26],
2. The Principle of Generating Vector Optical Beams with Polarization Varying along a Longitudinal Direction in Free Space

An arbitrary vector optical beam can be divided into two components with different polarization and complex amplitude [18]. Using VOFs with transverse distribution of polarization as examples, the VOF can be generally expressed as

\[ E(r, \phi) = E_1(r, \phi) \exp[i\delta_1(r, \phi)]e_1 + E_2(r, \phi) \exp[i\delta_2(r, \phi)]e_2 \]  

where \( r \) and \( \phi \) are the radial and azimuthal coordinates, \( E_1 \) and \( E_2 \) are the amplitude distributions of these two beams, \( \delta_1 \) and \( \delta_2 \) are the phase distributions, and \( e_1 \) and \( e_2 \) represent orthogonal polarization base vectors. The complex spatial distribution of polarization in VOF, from another perspective, originates from the superposition of two orthogonally polarized beams with different complex amplitude distributions. The difference may occur in the amplitude, phase, or both. For example, assuming two superposed beams are carrying left- and right-hand circular polarizations and their amplitudes are the same, but the phase distributions are different \( \delta_1 = -\phi_2 = \phi \). In this configuration, the superposition of these two beams leads to the generation of localized linear-polarized VOF. If the phase distributions are the same but the amplitudes of the two beams are \( E_1 = \cos\phi \) and \( E_2 = \sin\phi \), localized linear-polarized VOF can also be generated when the polarization of the two beams are orthogonally linear-polarized. Therefore, to generate the vector optical beam

\[ E(r, \phi) = E_1(r, \phi) \exp[i\delta_1(r, \phi)]e_1 + E_2(r, \phi) \exp[i\delta_2(r, \phi)]e_2 \]

[Figure 1. Schematic diagram of the longitudinal variation of polarization in free space.]

[2. The Principle of Generating Vector Optical Beams with Polarization Varying along a Longitudinal Direction in Free Space]

- parabolic-[27] and elliptic-[28] symmetry, double-mode [18], and fractal VOFs, [29] have been generated. However, with a degree of spatial freedom, the propagation direction is also a significant dimension to construct inhomogeneous distributions of polarization. The unique beam, with customized polarization distribution in the longitudinal direction, was first generated in 2015 [30] and has attracted increasing attention in recent years. Beyond enriching the forms of vector beam, the variation of polarization along the direction of propagation provides increased scope for the interaction between light and matter, especially in nonlinear effect and spin–orbit coupling. Moreover, it also offers advantages in the applications of material deep processing, remote polarimetry, and three-dimensional micromanipulation. In order to realize the variation of polarization in the longitudinal direction, a direct method is to modulate the propagation environment of the polarized beam, including by artificially constructing the anisotropic media with a spatially modulated optic axis [31–34], or by utilizing the self-induced nonlinear effects of optical beams in a nonlinear medium [35–38]. However, for situations in which the propagation environment cannot be manipulated, such as in free space, the direct methods are unsuitable and the possibility of modulating the beam at the initial plane to indirectly control the longitudinal distribution of polarization should be taken into consideration. In recent years, many indirect methods have been proposed to generate vector optical beams with longitudinally varying polarization in free space based on different theoretical and experimental strategies. The schematic diagram of vector optical beams with longitudinally varying polarization in free space is shown in Figure 1. Here, we focus on the most recent research into the unique vector beam with controllable polarization distribution along the longitudinal direction in free space generated by indirect methods. Different theories and experimental methods for generating this vector optical beam are reviewed in detail. Based on experimental results, the variation of polarization and its rate during propagation in free space are discussed with respect to existing problems and possible development directions of the vector optical beam with longitudinally varying polarization.
with varying polarization along the longitudinal direction in free space, there are two strategies available: constructing the longitudinally varying phase difference and amplitude difference, respectively.

2.1. Construction of Varying Phase Difference in the Longitudinal Direction

With reference to the generation of VOF based on phase modulation, vector beams with varying polarization during propagation in free space can be achieved by constructing a longitudinally varying phase difference between two superposed beams. To achieve this configuration, Bessel beams based on linear phase modulation are commonly used.

An ideal Bessel beam can be described by

$$E(r, \phi, z) = A_0 \exp(ik_z z) J_n(k_r r) \exp(\pm il\phi)$$

where $J_n$ is an nth-order Bessel function, $k_r$ and $k_z$ are the radial and longitudinal components of the wave vector. The wave number $k = \sqrt{k_r^2 + k_\perp^2}$, where $k_\perp$ denotes the transverse component of wave vector, including the radial and azimuthal components. Generally, the Bessel beam can be generated with the help of an axicon [39–41], or using computer holographic techniques [42–45], to add a linear phase modulation at the initial optical field. The phase modulation can be described as

$$\phi(r) = \exp(-ik_r r) \exp(\pm il\phi)$$

This phase modulation leads to a gradual convergence and produces an approximately nondiffracting region of $z_{\text{max}} = r_{\text{max}}k/k_r$, where $r_{\text{max}}$ is the maximum radius of the phase mask. If $l = 0$, an approximately zeroth-order Bessel beam is generated at the finite nondiffracting region. As the wave number can be deemed to be a constant, the longitudinal component of the wave vector $k_z$ varies with the transverse component. If two orthogonal-polarized Bessel beams, $E_1$ and $E_2$, are generated by the initial phase modulations with different radial wave vectors, a dynamic phase difference will appear during propagation in free space which can be expressed as

$$\delta(z) = (k_{z2} - k_{z1})z$$

Therefore, the vector Bessel beam (VBB) with spatially varying polarization during propagation is generated

$$E = E_1 + E_2 = E_1 e_1 + E_2 \exp[i\delta(z)] e_2$$

2.2. Construction of Varying Amplitude Difference in the Longitudinal Direction

In addition to the phase modulation, some attention has been devoted to the modulation of amplitude along the direction of propagation, which is a reliable way to generate vector optical beams with longitudinally varying polarization in free space. Considering the superposition of two zeroth-order Bessel–Gaussian beams with orthogonal polarizations, the electric field $E$ can be expressed as [46]

$$E(r, z) = \exp\left(-\frac{r^2}{w_0^2}\right) J_0(k_r r) \exp(ik_z z) \left[ E_1(z)e^{i\delta_1(z)} e_1 + E_2(z)e^{i\delta_2(z)} e_2 \right]$$

where $E_{1,2}(z)$ are axial amplitude distributions corresponding to these two polarization eigenstates; $\delta_{1,2}(z)$ are phase retardations independent of dynamic phase. When $\delta_1 - \delta_2$ is fixed, it is the difference of axial amplitude that determines the variation of polarization during propagation. This amplitude modulation can be achieved by spatial spectrum
engineering [47,48]. According to the modified Fourier transform, the quasi-Bessel beams with reshaped axial envelopes can be expressed in cylindrical coordinates as [46]

$$E_{1,2}(z) = \int_0^k U_{1,2}\left(\sqrt{k^2-k_z^2}, z = 0\right)e^{ik_z z}dz$$

(7)

where $U_{1,2}$ represents the spatial spectra. The customization of amplitude distribution in the longitudinal direction can be transferred to the manipulation of the spatial spectrum. By illuminating a tailored mask placed at the Fourier plane as a spatial spectrum filter, the desired amplitude distribution will be obtained. Based on inverse Fourier transform, the spatial spectra determined by the desired amplitude distribution can be described as [46]

$$U_{1,2}\left(\sqrt{k^2-k_z^2}, z = 0\right) = \frac{1}{2\pi k_z}F\left[E_{1,2}(z)\exp(ik_0 z)\right]$$

(8)

Different spatial spectrum filters enable customized amplitude relations between two orthogonally polarized components, which ensures the accurate control of polarization during propagation in free space. Moreover, this method can enable uniform axial intensity distribution.

3. The Experimental Generation of Vector Optical Beams with Longitudinally Varying Polarization in Free Space

Unlike the traditional VOF with transverse polarization distribution, controlling the longitudinal distribution of polarization in free space requires an indirect modulation method. The controllable and flexible variation of polarization in the longitudinal direction has always been sought and much effort has been spent on the generation method of this special vector beam in recent years. The controllability reflects the accuracy and predictability of polarization at each propagation position, while the flexibility implies the diverse and adjustable distribution of polarization in the longitudinal direction. The generation methods in free space can be broadly divided into the modulation of complex amplitude in real space and the filtering of the spatial spectrum.

3.1. Modulation on the Complex Amplitude in Real Space

In order to generate the beam whose polarization varies with propagation distance in free space, one approach is to modulate the initial phase distribution. The phase modulation determines the transverse component of the wave vector and will further influence the longitudinal component $k_z$. If two collinear orthogonally polarized optical beams have different $k_z$, a dynamic phase difference as a function of propagation distance will appear, which drives the variation of polarization during propagation. It is found that the initial phase modulation of two collinear polarized beams can be implemented based on the phase masks and the holographic grating.

3.1.1. Single-Path Generation Method Based on Phase Mask

In 2015, Moreno et al. proposed a type of VBB whose polarization varies with propagation distance, by adding an extra radial phase retardation profile between two orthogonal linear-polarized components [30]. The schematic diagram is shown as Figure 2a. In theory, a collimated light enters a diffractive axicon which acts as a radial diffraction grating to produce an approximate Bessel beam. The modulation of radial diffraction grating can be expressed as

$$g(r) = \exp\left(-i\frac{2\pi r}{d}\right)$$

(9)

where $d$ denotes the grating period. If a birefringent axicon is placed in front of the diffraction grating, a spatially varying phase shift will be introduced between two orthogonal linear polarizations as a function of radius, which further influences the polarizations at different longitudinal positions.
The polarization of the generated zeroth-order Bessel beam at different propagation distances are measured. It changes from linearly polarized at 45° to LCP to linearly polarized at 135° to RCP within the propagation region of 75 cm. It is demonstrated that higher order Bessel beams with varying polarization during propagation in free space can also be achieved by adding a spiral phase to the radial diffractive grating [30] which can be expressed as

$$g(r, \phi) = \exp \left[ -i \left( \frac{2\pi r}{d} + l\phi \right) \right]$$  \hspace{1cm} (10)

where $l$ denotes the topological charge of the spiral phase. Unlike for the zeroth-order Bessel beam, the resulting nondiffracting beam shows a dark center and a surrounding ring corresponding to the $l$th-order Bessel function, but the variation of polarization at the central ring is the same with those in zeroth-order Bessel beam.

Moreno et al. further generated higher order VBBs with the distribution of polarization varying during propagation in free space [49]. By means of adding different spiral and radial phases onto two polarization components, they achieved VBBs with variable charge and polarization in the longitudinal direction. In addition, by tailoring the additional phase shift between the two orthogonal linear polarizations as a function of the distance, the rotation rate of localized linear polarization of the VBBs can be controlled. The experimental setup is shown in Figure 3, which is similar to Figure 2b. The phase masks encoded on SLM have additional spiral phases with opposite topological charges, and extra wave plates are inserted in the optical path.
The polarization distribution of the generated 1st-order VBB where \( l_1 = -l_2 = 1 \) are analyzed by the intensity profiles after passing through polarizers in which two lobes are displayed. The orientation of the lobes rotates with propagation distance, indicating the rotations of localized linear polarization during propagation. After a propagation distance of \( Z/2 \), the radially polarized VOF turns to an azimuthally polarized VOF with the same charge. The rotating rate of linear polarization in the VBB is approximately uniform at 0.18 deg/mm. Moreno et al. also attempted to change the topological charge of the vector beam along the propagation axis by using multi-mode phase modulation. Each phase mask with fixed radial phase distribution is divided into different radial sectors where different topological charges of the spiral phase are attached. Experiments were carried out by dividing the phase mask into five radial sectors, with different charges \( l_1 = -l_2 = 1, 2, 3, 2, 1 \) corresponding to different propagation distances. Two, four, and six lobes can be observed after an analyzer, corresponding to the charges of VOF of 1, 2, 3, which means the charge of localized linear-polarized VOF changes along the propagation axis.

Apart from dividing the phase mask and modulating the optical beam multiple times based on the polarization selectivity of SLM, vector optical beams with longitudinally varying polarization can also be generated by cascading SLMs in a single-path system. Gao et al. demonstrated a Bessel-type vector beam, named oscillating polarized (OP) vector beam, with a spatially oscillating polarization along the optical axis based on the cascade of SLMs [50]. To obtain the variable phase shift along the propagation axis, they used the combination of SLM1 and SLM2 displaying the different phase patterns shown in Figure 4a. When a plane wavefront illuminates an axicon-like phase mask uploaded on SLM1, it will be converted to a conical wave. Then, the beam passes through the SLM2, therefore achieving different additional phase modulation. The schematic diagram is shown at the top of Figure 4a. The oscillating phase shift \( \delta(z) \) along the optical path will be obtained in the overlap region.

Based on the oscillating phase shift in the longitudinal direction, Gao et al. constructed an experimental setup to achieve the variation of polarization along the propagation direction in free space, which is shown in Figure 4b. The horizontal linearly polarized fundamental Gaussian beam illuminates the phase mask on the SLM1 which performs as the combination of an axicon and a \( l \)th-order spiral phase modulation. An approximate Bessel beam with an azimuthal topologic charge of \(-l\) will be generated because of the reflective SLM1. The half wave plate (HWP) is placed at an angle of 22.5° to the horizontal direction in order to rotate the polarization orientation of the generated Bessel beam at 45°. After reflection by SLM2, only the horizontal polarization component will be modulated. Except for the difference in radial phase modulating with SLM1, the SLM2 will provide another spiral phase modulation with a topologic charge of 2\( l \) by the uploaded phase mask. A 45° QWP is used to transform the two linearly polarized Bessel beams into left and right circularly polarized beams. The experimental results of generated 1st- and 2nd-order OP vector beams are shown in Figure 4c. It can be observed that the orientation of polarization
in the localized linearly polarized VOF rotates with the propagation distance. The rotation angle has a relation with the oscillating phase shift and azimuthal topologic charge as

$$\theta = \frac{\delta(z)}{2l}$$

(11)

which is coincident with the measured rotation angle at different propagation distances, as shown in Figure 4d. The highest rotating rate achieved by this method is ~0.15 deg/mm.

Figure 4. Generation of an OP beam with spatial oscillating polarization. (a) Schematic diagram of generating oscillating phase shift along the propagation direction by using two SLMs. (b) Experimental setup for generating OP vector beams. (c) Experimental results of generated OP vector beams. (d) The angle of rotation as a function of propagation distance. Adapted with permission from [50].

The above approaches to generate vector optical beams with longitudinally varying polarization in free space all rely on digital phase mask [51], enabled by SLM, and require a particular SoP at the input plane. Recently, a form-birefringent was demonstrated by Capasso et al. to impart the arbitrary polarization along the propagation direction without a priori knowledge of the incident polarization [52]. The metasurface is composed of dielectric nanopillars with structured birefringence in the way of subwavelength-spaced arrays. By controlling the constituent and arrangement, the metasurface can manipulate the incident waveform converting to any desired output [53] based on the Jones calculus [54] and dual phase holography [55]. Optical metasurfaces have become a current subject of intense research due to their unprecedented capability in manipulating the polarization at subwavelength resolution.

The desired polarization distribution along the propagation direction can be achieved by the superposition of two (or more) monochromatic waveforms of the same wavelength but with different SoP and spatial frequencies, which can be expressed in the Jones vector as

$$E(z) = E_1(z) + E_2(z) = e^{i k_z z} \begin{bmatrix} E_{1x} e^{i \frac{\Delta \delta z}{2}} + E_{2x} e^{i \frac{\Delta \delta z}{2}} \\ E_{1y} e^{i \frac{\Delta \delta z}{2}} + E_{2y} e^{i \frac{\Delta \delta z}{2}} \end{bmatrix}$$

(12)

where $\Delta k_z = k_{z2} - k_{z1}$ and $k_z$ is the average spatial frequency $(k_{z2} + k_{z1})/2$. Metasurfaces are a particularly convenient platform for obtaining the desired polarization response. The metasurfaces proposed by Capasso et al. can act as z-dependent polarizing devices, as shown in Figure 5a, which can also be treated as a z-dependent $2 \times 2$ Jones matrix that
implements a transformation from an incident scalar light into a vector beam with varying polarization along the optical axis. The transformation function can be expressed as

$$\tilde{U}(r, z) = e^{-i\omega t} \sum_{m=-N}^{N} A^{(m)} \Phi (r) e^{i k m z}$$

(13)

where $A^{(m)}$ are $2 \times 2$ matrix-valued coefficients and can be obtained from the equation exhibiting similarity with a Fourier integral that

$$\begin{bmatrix} A_{11}^{(m)} & A_{12}^{(m)} \\ A_{21}^{(m)} & A_{22}^{(m)} \end{bmatrix} = \frac{1}{L} \int_{0}^{L} \begin{bmatrix} F_{11}(z) & F_{12}(z) \\ F_{21}(z) & F_{22}(z) \end{bmatrix} e^{-i k m z} dz$$

(14)

where $\tilde{F}$ may take the form of an arbitrary $z$-dependent Jones matrix with a periodicity of $L$ along the propagation direction. Based on the structure of dielectric nanopillars with structured birefringence, the desired Jones matrix function can be implemented and the corresponding metasurfaces can be locally represented by a spatial arrangement of linearly birefringent wave-plate-like elements as

$$\tilde{f}(x, y) = R(-\phi(x, y)) \begin{bmatrix} e^{i \delta_x(x, y)} & 0 \\ 0 & e^{i \delta_y(x, y)} \end{bmatrix} R(\phi(x, y))$$

(15)

where $R(\phi)$ is the $2 \times 2$ rotation matrix. Due to anisotropy of dielectric nanofins, unit cells of this topology provide two propagating modes that experience different phase delays, $\delta_x$ and $\delta_y$. By changing the dimensions of the nanopillars and the pillar’s angular orientation in the transverse plane, the phase retardation and rotational angle can be adjusted. Furthermore, to break the constraints that limit the possible polarization behaviors, a multidimensional matrix reconstruction is implemented based on dual matrix holography.

Figure 5. (a) The schematic of a $z$-dependent polarizing device based on the metasurface. (b) The experimental setup of generating optical beam with polarization varying with propagation by metasurface [52].

The experimental setup based on a metasurface combined with a 4f system is depicted in Figure 5b. The collimated polarized Gaussian beam illuminates the metasurface. The desired spectrum is filtered at the Fourier plane of L1 and then transformed back to the real space via L2 before being recorded by a CCD camera. The polarization is measured.
by Stokes polarimetry at different propagation distances using a polarizer and a QWP. When the metasurface serves as an analyser whose transmission axis varies as a function of propagation distance, the polarization varies continuously from 0$^\circ$ to 90$^\circ$ within the region $z = 2.5$ mm to $z = 5.5$ mm. The intensity decays gradually along the propagation direction under $x$-polarized incident illumination, while the intensity increases over the same propagation region under $y$-polarized incident illumination. Aside from serving as the polarizer, the metasurface can also act as an HWP with the direction of fast axis varying along the optical axis, which will change the orientation polarization of an incident linearly polarized beam during propagation. The generated central linear polarization rotates $\sim 180^\circ$ within a propagation region of 3 mm, corresponding to a rotation rate of $\sim 60^\circ$/mm, while the intensity distribution is approximately unchanged as a Bessel function profile during propagation. Finally, a QWP-like metasurface is fabricated with a rotation of its fast axis along the propagation direction, thereby modifying the ellipticity of an incident polarization. The generated polarization of the output beam evolves from linear polarization to elliptical polarization, the LCP then reverts to the original linear polarization, and repeats in the northern hemisphere, forming a closed-looped trajectory on the Poincaré sphere. The direction of the fast axis of the wave-plate-like element can be adjusted by rotating the metasurface and the desired conversion of polarization can be achieved at arbitrary propagation positions without a priori knowledge of the incident polarization.

3.1.2. Double-Path Generation Method Based on Holographic Gratings

The single-path generation method is simple and highly effective. However, when using SLM as the phase modulation device, the phase mask is limited by the modulation depth of the SLM which constrains the frequency of the longitudinal polarization oscillation. Although the metasurface may solve the problem of phase modulation depth, this manufactured element lacks flexibility compared to the digital modulation method based on SLM. The generation method based on digital holographic gratings is a valid way to promote the modulation depth without losing the flexibility of the longitudinal manipulation of polarization.

Li and Zhao et al. demonstrated an experimental setup using a Sagnac interferometer combining with the 4f system [56]. In theory, the Pancharatnam–Berry (PB) phase [57–60] is introduced to account for the variation of polarization in the longitudinal direction. To achieve the longitudinally varying PB phases, the modified Sagnac interferometer is used [19,61,62], as depicted in Figure 6a. The system comprises two main parts. One is the interferometer section which is made up of two mirrors (M1 and M2) and a reflective phase SLM used to prepare transverse PB phases, while the other part is the transverse-to-longitudinal structuring based on an axicon. An input expanded beam is divided into two orthogonally polarized components by a polarization beam splitter (PBS), and then enters the Sagnac interferometer. The designed holographic grating placed at the front focal plane of L3 is imprinted on the SLM. The $\pm 1$st-order diffraction beams are filtered through the same open aperture A placed on the Fourier plane of L3. After the collimating lens L4, two collinear beams with orthogonal polarizations and different phase modulations pass through an axicon to achieve the transverse-to-longitudinal transformation of the PB phase, and the VBB with spatially varying polarization in the propagation direction is generated.
When the orthogonal circular polarizations are chosen as bases and a QWP is inserted after the PBS, the variation of polarization along the equator of the Poincaré sphere can be achieved, as shown in Figure 6b. It has been demonstrated that the generated central linear polarization distribution changes from horizontal linearly polarized to linearly polarized at 45° to vertical linearly polarized to linearly polarized at 135° and back to horizontal linearly polarized within the propagation region of 11.2 cm. The rotation rate of the orientation of linear polarization can be estimated as ~1.6 °/mm. When choosing the horizontal and vertical polarization as bases, the variation of polarization along a meridian of the Poincaré sphere can be achieved. In this case, an HWP is inserted after the PBS. The generated central polarization experiences a variation from 45° linearly polarized to right circularly polarized, to 135° linearly polarized, to left circularly polarized, and back to 45° linearly polarized within a region of 112 mm. Based on this experimental setup, a higher-order VBB with varying polarization during propagation can also be generated. As shown in Figure 6c, the orientation of polarization in the localized linear-polarized VOF rotates with propagation distance at a uniform rate.

The axicon-like radial phase modulation in the initial plane has been widely used to achieve the oscillating phase delay and to further drive the variation of polarization along the propagation direction. However, Wang et al. demonstrated that the radial wave number of axicon-like phase modulation will perform an additional amplitude modulation to the Bessel profile, which determines the overall intensity [63]. Considering the superposition of two collinear beams with different radial components of wave vector and orthogonally circularly polarized bases, the generated beam in cylindrical coordinates at initial plane can be expressed as

$$E(r, \theta, z = 0) = E_{01} e^{-ikr z} \mathbf{e}_j + E_{02} e^{-ikr z} \mathbf{e}_r$$

(16)

the propagating equation can be expressed under the paraxial approximation as

$$E(r, z) = e^{ikz} \left[ \sqrt{I_1} e^{i(k - \frac{\omega^2}{2z})z} \mathbf{e}_i + \sqrt{I_2} e^{i(k - \frac{\omega^2}{2z})z} \mathbf{e}_r \right]$$

(17)

where

$$I_q(r, z) = \frac{2\pi}{k} E_0^2 q^2 I_0^2(k_q r), q = 1, 2$$

(18)

Since the Bessel function is influenced by the radial wave vector $k_r$, a square relation between $k_r$ and the intensity $I$ can also be found. It is evident that an equal-amplitude superposition at the initial plane cannot be maintained during propagation, which may further influence the accuracy of the longitudinal distribution of polarization.

**Figure 6.** (a) Schematic representation of the experimental setup based on a Sagnac interferometer combined with the 4f system. (b) Schematic of the variation of polarization along the equator of the Poincaré sphere. (c) Schematic representation of the polarization distribution of higher-order VBB with varying polarization during propagation [56].
To solve this problem, Wang et al. proposed an amplitude–phase joint modulation method based on a two-dimensional (2-D) binary phase holographic grating (BP-HG) and 4f system. The experimental setup is shown in Figure 7a. The horizontal linearly polarized beam illuminates a phase-only SLM after expanding and collimating. The SLM is located at the input plane of the 4f system which is composed of a pair of lenses (L1 and L2). The designed 2-D BP-HG imprinted on the SLM diffracts the incident beam into different orders, and only the two +1st-orders are allowed to pass through the Fourier plane of L1 and are converted by two QWP into the left- and right-hand circularly polarized beams. Then the two +1st-order diffraction beams are recombined by a Ronchi grating (RG), which is placed at the rear focal plane of L2. The complex amplitude transmission function of the BP-HG can be expressed as

\[ t(x, y) = \begin{cases} 
  e^{i\psi_1}, & \text{when } \text{rect}\left(\frac{x}{a_1}\right)\text{rect}\left(\frac{y}{a_2}\right) \otimes \text{comb}\left(\frac{x}{d} + \frac{\psi_1}{2\pi}, \frac{y}{d} + \frac{\psi_2}{2\pi}\right) = 0, \\
  e^{i\psi_2}, & \text{otherwise.} 
\end{cases} \]  

(19)

where rect(x) and comb(x) are rectangular and comb functions, \( \otimes \) is a convolution operator, \( \psi_1 \) and \( \psi_2 \) are the binary phases of the grating, \( \psi_1 \) and \( \psi_2 \) are the phase modulations adding on the two +1st-orders, \( d \) is grating period, while \( a_1 \) and \( a_2 \) are the stripe widths in x and y direction, respectively. The two +1st-order left- and right-handed circularly polarized diffraction beams carrying the respective space-varying phases can be rewritten as

\[ E_{+1,0} \propto \frac{a_1}{d} \left( 1 - \frac{a_2}{d} \right) \text{sinc}\left( \frac{a_1}{d} \right) \exp(i\psi_1) e_l \]  

(20)

\[ E_{0,+1} \propto \frac{a_2}{d} \left( 1 - \frac{a_1}{d} \right) \text{sinc}\left( \frac{a_2}{d} \right) \exp(i\psi_2) e_r \]  

(21)

Figure 7. Generation of localized linearly polarized vector beams with longitudinally varying polarization based on the amplitude–phase joint modulation method. (a) Generation system based on two-dimensional binary phase holographic grating and 4f system. (b) Intensity profiles of generated vector beams at different propagation distance. (c) Dependence of the orientation angle of local linear polarization on propagation distance [63].
Based on the above relations, if only concentrating on the beam center and attempting to achieve an equal-amplitude superposition during propagation, the relationship between the duty ratio and the radial wave number becomes quite simple as

\[
\frac{(1 - \delta_1^2)^2 \sin^2 (\frac{\pi}{4})}{(1 - \delta_2^2)^2 \sin^2 (\frac{\pi}{4})} \cdot \frac{k_{2}^2}{k_{1}^2} = 1
\]

(22)

This means the equal-amplitude superposition can be effectively guaranteed during propagation by adjusting the duty ratio \(a_1/d\) and \(a_2/d\).

The experimental results are shown in Figure 7b and c. The top row of Figure 7b shows the total intensity distribution while the remainder are the intensity distributions after a linear polarizer. The second and third rows show the situation when the amplitude is modified by adjusting the duty ratio of 2-D BP-HG where the field center can be completely extinguished after a polarizer. This means the desired linear polarization is generated accurately. However, if no amplitude modulation is implemented at the initial plane, the central amplitude of two superposed beams will be unequal during propagation in free space. An undesired elliptical polarization which cannot be completely extinguished after a polarizer, as shown in the bottom row of Figure 7b, will also occur. The orientation angle of the central linear polarization as a function of the propagation distance is shown in Figure 7c. When the radial wave number of two superposed beams are changed, the experimental results all show extremely high linear fitness and good agreement with the theoretical calculations, which means a stable rotation of central linear polarization with controllable rotation rate can be achieved. The highest rotation rate achieved based on the amplitude–phase joint modulation method is ~6.7°/mm. Relying on this amplitude-phase joint modulation method, two coherent beams with orthogonally circular polarization, different radial wave number and adjustable amplitude ratio are collinearly superposed to achieve the accurate and controllable manipulation of polarization distribution along the propagation direction in free space.

3.2. Filtering of Spatial Spectrum

Phase modulation in real space is the commonly used method to manipulate the polarization distribution both in the transverse and longitudinal directions. However, for the vector optical beam with varying polarization along propagation direction in free space, the initial phase modulation will inevitably influence the intensity distribution of the propagating field. In order to improve the axial intensity distribution of the generated beams with longitudinally varying polarization, Zhao et al. utilized the Sagnac interferometer [19,56,61,62] to construct the superposition of two beams with complementary axial intensities and orthogonal SoPs based on the spatial spectrum optimization approach proposed by Cižmár and Dholakia [47], thereby generating a quasi-Bessel beam with uniform axial intensity but varying polarization upon propagation [46]. The schematic of the theoretical configuration is shown in Figure 8.

Figure 8. Schematic of the theoretical configuration of reshaping the axial intensity distributions of quasi-Bessel beams [46].
A quasi-Bessel beam composed of two orthogonally polarized beams with linearly varying axial intensity, for which the axial fields can be expressed as

\[ E_{H,V}(z) \exp(i\delta_{H,V}) = \sqrt{I_0}(1 \pm az)/2 \exp(\pm i\pi/4) \]  

(23)

where \( a \) is a constant determining the varying period. With the axial intensities of two polarization components constantly varying, the polarization also changes correspondingly. The results at \( z = -15, -10, 0, 10, 15 \) cm are measured respectively, and the ellipticity of the polarization experiences a variation from increase to decrease, thereby the transformation of the polarization of this quasi-Bessel beam continuously moving along the meridian of the Poincaré sphere. More importantly, because of the complementary axial intensity distributions of these two polarization components, the total axial intensity has an approximately uniform profile in the non-diffractive region. Nevertheless, the variation of the ellipticity angle varies with different angles when propagating over the same distance. In order to achieve uniform variation of polarization, another axial envelope is designed as

\[ E_H(z) = \sqrt{I_0} \sin(2\pi bz) \]  

(24)

\[ E_V(z) = \sqrt{I_0} \cos(2\pi bz) \]  

(25)

where \( b \) denotes the varying period of envelopes. As the phase difference is zero, the quasi-Bessel beam is always linearly polarized. The polarization direction has a relationship with the longitudinal distance as

\[ \tan(\pi/2 - \psi) = E_V/E_H = \tan(2\pi bz) \]  

(26)

The distributions of Stokes parameters at \( z = 0, 1.9, 3.8, 5.6, \) and \( 7.5 \) cm are measured. The results demonstrate the quasi-Bessel beam retains a linear polarization upon propagation that shows a periodic variation of polarization along the equator of the Poincaré sphere.

Recently, a theoretical model to obtain anomalous VBBs with varying polarization order during propagation was demonstrated by Liu et al. [65]. Compared with changing the charge of VOF by dividing the axicon into various radial sectors [49], the method introduced a continuous phase delay by designed spiral slits [66,67] is more flexible such that arbitrary polarization orders, including integers and fractions, can be generated along the propagation axis. This approach was inspired by the idea that a zeroth-order Bessel beam can be thought of as the Fourier transform of an annular slit [68]. The diffraction intensity distributions and phase profiles of anomalous Bessel beams at different propagation distances are shown in Figure 9a. A right-/left-handed circularly polarized plane wave illuminates the spiral slit corresponding to \( l = 3 \) and \(-3\), respectively. It can be observed that the topological charge of the anomalous Bessel vortex beam decreases with the propagation distance. When these two beams are collinearly superposed, the generated anomalous VBB is shown in Figure 9b. With the gradual increase of the propagation distance, the polarization order of the anomalous VBB will gradually tend to zero accompanied by the variation of polarization distribution. This characteristic may provide more possibilities and expand the applications in optical trapping, quantum communications, and optical microscopy.
4. Discussion

Methods to generate vector optical beams with polarization longitudinally varying along propagation direction have been rapidly developed in recent years. Although an increasing number of approaches to achieve the longitudinal manipulation of polarization in free space have been demonstrated, there are still some problems to be solved.

For practical purposes, a highly effective experimental method with accurate and flexible control of the longitudinal distribution of polarization in free space has always been sought. Although the single-path generation method has relatively high efficiency, the flexibility of this method is limited by the modulation depth of the phase mask. For the applications of microfabrication or micromanipulation, the variation of polarization with high oscillation frequency in a small region is desired, which requires a large value for the radial wave number and much greater modulation depth. This problem may be solved by the double-path generation method based on digital holographic grating, because the phase modulation of diffraction beams is only determined by the structure of the grating. However, the existing double-path method is confronted with the problem of low efficiency, which is mainly caused by the high loss of digital holographic grating and may be solved by optimizing the design of phase grating. A generation method which balances efficiency and flexibility will contribute to practical applications of vector optical beams with polarization longitudinally varying along the propagation direction.

The accurate and flexible customization of the longitudinal distribution of polarization is a promising direction for the development of this unique vector optical beam. Most of the reported vector optical beams with longitudinal polarization distribution exhibit uniform variations of polarization along trajectories corresponding to the equator or meridian of the Poincaré sphere, as shown in Figure 10a and b, respectively. For customizing the longitudinal distribution of polarization, the variation of polarization is predicted to trace arbitrary trajectories on the Poincaré sphere, as shown in Figure 10c, and the rate of the variation of polarization in longitudinal direction is predicted to be flexibly adjusted. The arbitrary variation of polarization in ellipticities and the long axis orientation will bring additional degrees of freedom to engineer the optical beam which may provide further possibilities for the interaction between light and matter, as well as broadening the application region of vector beams. In fact, the variation of polarization along arbitrary circular trajectories of the Poincaré sphere can be realized in experiments by controlling the parameters of superposed beams, including the polarized bases and the amplitude ratio [63].

Figure 9. (a) The diffraction intensity distributions and phase profiles of anomalous Bessel beams under different propagation distances. (b) The intensity and polarization distribution of the generated anomalous VBBs at different propagation distances. Adapted with permission from [65], copyrighter Elsevier, 2021.
Benefiting from in-depth research into the characteristics of VOFs, the characteristics of inhomogeneous polarization distributions in the transverse plane have been widely explored in many applications including nonlinear optics [4,5,69,70], microfabrication [12,13], and quantum information [8]. To date, the research into the vector optical beam with longitudinal distribution of polarization in free space has focused on the generation method of this special beam. However, effective methods to achieve the longitudinal manipulation of polarization in free space provide the technical basis for constructing diverse and desired longitudinal distributions of polarization, which may induce special responses in various mediums. The propagation-dependent polarized feature may extend the form of spin-orbit coupling in bulk material or cavities [71] as well as bringing novel physical effects related to the z-dependent nonlinear [72,73] and topological effects [74]. Combining consideration of the application prospects of the vector optical beam with the longitudinal distribution of polarization in volume laser machining [49], longitudinal detection [30], and in vivo manipulation of multiple polarizing elements [52,75], we believe that increased attention paid to the characteristics of this beam when it interacts with matter will facilitate progress in the application of structured light.

5. Summary

In conclusion, the theoretical foundations and experimental approaches to generate vector beams with longitudinal distribution of polarization in free space have been reviewed. The longitudinal variation of polarization originates from the variable difference between the complex amplitudes of two orthogonally polarized beams with propagation. Thus, there are two theoretical strategies to achieve this special vector beam, which are constructing the longitudinally varying phase difference and amplitude difference. In experiments, the generation methods can be summarized as the modulation of complex amplitude in real space and the filtering of the spatial spectrum. By tailoring the spatial phase distribution in the initial plane, a z-dependent phase difference between two orthogonally polarized beams can be achieved, and the VBB with varying polarization during propagation can be generated based on modulation in real space. In addition, based on spatial spectrum engineering, the axial intensity of two orthogonal polarization components can be modulated to realize the Bessel beam with uniform axial intensity and longitudinally varying polarization. The successful generation of vector beams with controllable variation of polarization along the propagation direction in free space enriches the structured light with a new degree of freedom and will open new paths in light’s interaction with matter. The customization of the longitudinal distribution of polarization and corresponding optimized generation methods with high efficiency and flexibility are anticipated to be achieved by further applied research into this special beam.

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