Chapter 1

Wolfgang Kummer and the Vienna School of Dilaton (Super-)Gravity

Luzi Bergamin* and René Meyer†

* ESA Advanced Concepts Team, ESTEC, DG-PI, Keplerlaan 1, 2201 AZ Noordwijk, The Netherlands.
† Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany.

Wolfgang Kummer was well known for his passion for axial gauges and for the formulation of gravity in terms of Cartan variables. The combination of the two applied to two-dimensional dilaton gravity is the basis of the “Vienna School”, which provided numerous significant results over the last seventeen years. In this review we trace the history of this success with particular emphasis on dilaton supergravity. We also present some previously unpublished results on the structure of non-local vertices in quantum dilaton supergravity with non-minimally coupled matter.

1.1. Historical Introduction

1.1.1. Early Attempts to Non-Einsteinian Gravity in 2D

The earliest works by Wolfgang Kummer connected to gravity in two dimensions1–4 date back to the year 1991, where he realized together with Dominik Schwarz that the Katanaev-Volovich model,5–7a

\[ S = \int d^2 x \sqrt{-g} \left[ \frac{\mu^2}{2} R_{abcd}^2 + \frac{\gamma^2}{2} T_{abcd}^2 + \lambda \right], \]  

(1.1)

\[ = \int d^2 x \sqrt{-g} \left[ \frac{\mu^2}{2} R_{\mu\nu}^2 - \gamma^2 T_{\mu\nu}^a + \lambda \right], \]  

(1.2)

*Email: bergamin@tph.tuwien.ac.at
†Email: meyer@mppmu.mpg.de

Here \( R_{abcd} \) are the components of the curvature two-form written with tangent space indices only, \( T_{abcd} \) are the components of the torsion form, and \( \mu, \gamma, \lambda \) are constants. In the second line, we expressed everything in terms of the Ricci scalar \( R = R_{\mu\nu} \mu\nu \) and the Hodge dual of the torsion form, \( T_{\mu\nu}^a = \epsilon_{\mu\nu} \tau^a \).
easily can be solved in an axial gauge using light-cone variables and applying the first order formulation of gravity theories, since the theory exhibits a (nonlinear) Yang-Mills like gauge structure. In that work, the global structure of the solutions, in particular the classification according to their singularities, was discussed as well. Certainly, this insight was based on Wolfgang Kummer’s long-standing experience with noncovariant gauges in non-Abelian gauge theories. Furthermore they observed the existence of two branches of classical solutions: A constant curvature branch with vanishing torsion, yielding de Sitter space in the case of (1.1), and a nontrivial branch of solutions with both curvature and torsion, which is however labeled by a conserved quantity relating the torsion scalar and the Ricci scalar in a gauge invariant manner. As we will see, this conserved quantity exists in a much larger class of two-dimensional gravity theories and has the interpretation of a quasi-local mass.

The noncovariant gauge was then put to good use to show the renormalizability of the Katanaev-Volovich model (1.1) in a fixed background quantization around flat space, albeit its nonpolynomial interactions. Though these interactions lead to an infinite set of ultraviolet divergent one-loop graphs, only a single quantity is renormalized, such that the renormalized Greens functions reduce to the tree-level graphs. Infrared divergences were shown to be treatable by introducing a small mass regulator, and it was realized that, although no physical S-matrix exists in the model because of the lack of propagating on-shell degrees of freedom, correlators of physical observables such as the Ricci curvature scalar can still yield interesting information.

Based on the idea that the integrability of (1.1) should be due to an additional dynamical symmetry, it was then realized by Wolfgang Kummer and collaborators that the Katanaev-Volovich model can be reformulated as a \( sl(2,\mathbb{R}) \) gauge theory. They went on to analyze the constraint structure of the theory, and found that the secondary first class constraints form a deformed \( iso(2,1) \) algebra, i.e. a deformation of the Poincaré algebra of \( (2 + 1) \)-dimensional Minkowski space. The deformation parameter turned out to be the constant \( \gamma \), and sending \( \gamma \to \infty \) restored an undeformed

\[ R^2 \text{ gravity without torsion}^{13} \]

as can be seen from the first order formulation (1.3). For this reason it was named “Einstein branch”. Different limits of the parameters \( \mu, \gamma, \lambda \) in (1.3) yield further 2d gravity models: Taking \( (\mu, \gamma) \to \infty \) and integrating out \( X_a \) afterwards yields 2D BH Gravity. The Jackiw-Teitelboim model is obtained in the same limit when rescaling the cosmological constant such
iso(2, 1) symmetry. They found that the above-mentioned conserved quantity was one generator of the center of the deformed symmetry algebra, while a second generator vanished on the constraint surface. Wolfgang Kummer investigated the Katanaev-Volovich model minimally coupled to real scalars or fermions as well, and found the same dynamical symmetry, albeit the system in general is no longer integrable. Integrability was found to be preserved for chiral solutions, i.e. if the fermion has definite handedness, whereby the conserved quantity is just conserved in time but no longer in space, as its spatial divergence is connected to the spatial parts of the chiral current, \( \chi_L \frac{1}{\sqrt{-g}} \partial_X L \). These currents provide a central extension of the algebra of constraints. This result already hinted towards a generalized conservation law including the matter contributions. In Ref. 12 it was also mentioned that this dynamical symmetry and the classical integrability might hint towards nonperturbative quantum integrability of the Katanaev-Volovich model, a statement which is in fact correct for a large class of generalized two-dimensional dilaton gravities.

After so many interesting facets of the Katanaev-Volovich model being found, it might seem that it would already have revealed all of its mysteries. This was, however, not the case. Ikeda and Izawa wrote (1.1) in first order form,

\[
S = \int_{M_2} [\phi d\omega + X_a (De^a)] + \int_{M_2} d^2 x \sqrt{-g} \left[ \lambda - \frac{\phi^2}{8\mu^2} + \frac{X_a X^a}{4\gamma^2} \right], \tag{1.3}
\]

a fact which was then used by Wolfgang Kummer and Peter Widerin to further analyze the symmetry structure of the theory: They found a field-dependent off-shell global symmetry of (1.1) whose conserved charge is the conserved quantity mentioned above. Furthermore, they showed, driven by the desire to reinterpret the algebra of constraints in a framework not involving the Hamiltonian phase space, that the deformed iso(2, 1) symmetry could be re-formulated as a current algebra. The currents were readily identified with the components of the energy-momentum tensor in lightcone gauge, which were shown to fulfill (on compactified space) a Virasoro algebra, similar to the situation in string theory. As will become clear to the reader later, this first order reformulation of a much wider class of gravity
theories, together with the choice of lightcone gauge, lies at the heart of
the results outlined in this article.

Also the case with minimally coupled bosonic\textsuperscript{23} and fermionic\textsuperscript{24} matter
offered some new surprises: A different lightcone gauge, defined in terms of
‘lapse’ and ‘shift’ variables, together with the first order formulation (1.3)
of the gravity sector, allowed a counting of free functions in the general
solution of the constraints and the equations of motion. It was found that
the inclusion of scalars and fermions does spoil integrability of (1.1), but
only in a mild way, since only one non-trivial first order PDE needs to be
solved.

The combination of Eddington-Finkelstein gauge and first order formu-
lation (1.3) also played an important role in the nonperturbative quantum
treatment of the Katanaev-Volovich model:\textsuperscript{25} In that work Wolfgang Kummer
and Florian Haider found the correct path integral measure that al-
lowed to integrate out both the Zweibeine as well as the spin connection.
This measure differs from the one used in the perturbative approach\textsuperscript{4} by
a factor of \((-g)^{-3/4}\), arising from the Gaussian path integration over the
fields $\phi, X^a$ in lightcone gauge. As different path integral measures cor-
respond in a renormalizable theory to different renormalization schemes
with different counterterms, the nonperturbatively useful measure differed
from the perturbative one only by a shift in the counterterm present in
the quantum effective action.\textsuperscript{4} Besides reading off the correct measure just
from the symplectic structure which can be found directly from the first
order form (1.3), they also derived the same result from a canonical BRST
analysis. We will see in section 1.1.3 that this method reveals best the
underlying symmetry structure of the quantum theory, being in essence
a nonlinear Yang-Mills theory. Imposing again lightcone gauge the quan-
tum effective action may be calculated exactly and they found that all
local quantum effects disappear right away, i.e. the counterterms vanish.
They also discuss that the same result can be obtained by careful renor-
malization in the fixed-background quantization of Ref. 4, as expected for
a gauge-invariant quantum effective action. The Katanaev-Volovich model
thus shows no local quantum effects, and all Green functions are deter-
mined by their tree-level contributions. Nonetheless, the theory is neither
classically nor quantum mechanically trivial since there exists a global de-
gree of freedom,\textsuperscript{26} namely the quasilocal mass (conserved quantity). Its role
becomes even more important when considering spacetimes with boundary
(see Sect. 1.3).
1.1.2. First Order Formulation of Generalized 2D Dilaton Gravity

Ref. 25 was the last in a series of papers\textsuperscript{1–4,9–12,22,24,25} devoted to “Non-Einsteinian Gravity in $d = 2$”, i.e. the Katanaev-Volovich model (1.1). In the subsequent works Wolfgang Kummer and his collaborators\textsuperscript{27–32} realized that a much larger class of gravity theories in two dimensions can be treated along similar lines. This step was possible thanks to a generalization of the first order action (1.3) to

$$S_{\text{FOG}} = \int_M \left( \phi d\omega + X^a De_a + \epsilon V(\phi, Y) \right),$$

(1.4)

with $Y = X^a X_a / 2$, which first appeared in Ref. 33. These models, called First Order gravities (FOGs), in general do not permit a formulation exclusively in terms of the geometric quantities (curvature and torsion), since the possibility to eliminate the fields $X$ and $X^a$ (which actually represents a Legendre transformation\textsuperscript{34}) is not guaranteed. However, already in Ref. 30 it was then realized that it is preferable to eliminate the torsion dependent part of the spin-connection instead of the dilaton. Remarkably this procedure only involves linear and algebraic equations for any potential $V$, which thus may be reinserted into the action without restrictions.\textsuperscript{35} In general, this yields higher derivative theories of gravity. Still, if one restricts to models of type

$$V(\phi, Y) = V(\phi) + YU(\phi)$$

(1.5)

the ensuing second order action are Generalized Dilaton Theories (GDTs)

$$S_{\text{GDT}} = \frac{1}{2} \int d^2 x \sqrt{-g} \left[ \phi R - U(\phi)(\nabla \phi)^2 + 2V(\phi) \right].$$

(1.6)

The classical equivalence also holds on the quantum level,\textsuperscript{36} as long as possible additional matter fields do not couple to torsion, which is the case for scalars and fermions in two dimensions, but for example not for four-dimensional spherically reduced fermions.\textsuperscript{37}

Analogous to the Katanaev-Volovich model, all matterless FOGs with potential (1.5) are classically integrable. Their equations of motion\textsuperscript{3}$^\dagger$

$$0 = d\phi + X^- e^+ - X^+ e^-, \quad 0 = (d \pm \omega)X^\pm \mp Ve^\pm + W^\pm, \quad (1.7)$$

$$0 = d\omega + \left( \frac{\partial V}{\partial \phi} \right)^\pm + W, \quad 0 = (\partial \pm \omega)e^\pm + \left( \frac{\partial V}{\partial X^\pm} \right)^\pm, \quad (1.8)$$

$^\dagger$The ±-indices are lightcone indices in tangent space, cf. App. A.
in the matter free case, $W = \frac{\delta L^{(m)}}{\delta \phi} = 0$, $W^{\pm} = \frac{\delta L^{(m)}}{\delta \epsilon^\pm} = 0$, can be solved just by form manipulations. In a patch with $X^+ \neq 0$, the solution is

\[ ds^2 = 2e^+ \otimes e^- = 2df d\phi + \xi(\phi) df^2, \]

(1.9)

\[ Q(\phi) = \int U(y) dy, \quad w(\phi) = \int e^{Q(y)} V(y) dy, \]

(1.10)

\[ \xi(\phi) = 2e^{Q(C - w)|_{\phi=\phi(\phi)}} \quad d\phi = d\phi e^Q, \]

(1.11)

\[ C = e^{Q(\phi)} Y + w(\phi). \]

(1.12)

Here $f$ is a free function on two-dimensional space-time. By using $f$ and $\tilde{\phi}$ directly as coordinates the line element (1.9) is found naturally in Eddington-Finkelstein gauge, although no diffeomorphism gauge fixing was employed. The conserved quantity (1.12), which enters the Killing norm (1.11), is exactly the nontrivial integral of motion found in the Katanaev-Volovich model (1.1). The solutions (1.9) can have many Killing horizons, and in fact can be extended globally.\textsuperscript{21}\textsuperscript{,29,30,38,39} As for the Katanaev-Volovich model the integrability extends to cases with special “chiral” matter.\textsuperscript{21}

FOG also allows for a convenient reformulation as a so-called Poisson-Sigma-Model (PSM),\textsuperscript{40–42} illuminating some of its structure more clearly: Grouping together the target-space coordinates $X^I = (\phi, X^a)$ and the gauge fields $A_I = (\omega, e^a)$, FOG (1.4) can be written as

\[ S_{PSM} = \int_{M^2} \left[ dX^I \wedge A_I + \frac{1}{2} P^{IJ} A_J \wedge A_I \right], \]

(1.13)

where $P^{IJ} = \{ X^I, X^J \}$ is a Poisson tensor related to a Schouten-Nijenhuis bracket defined on the manifold. Since the Poisson tensor must have a vanishing Nijenhuis tensor with respect to this bracket,

\[ P^{IL} \partial_L P^{JK} + \text{perm} (IJK) = 0, \]

(1.14)

the PSM action is invariant under the symmetries

\[ \delta X^I = P^{IJ} \varepsilon_J, \quad \delta A_I = -d\varepsilon_I - (\partial_I P^{JK}) \varepsilon_K A_J. \]

(1.15)

For $P^{IJ}$ linear in $X^I$ the symmetries constitute a linear Lie algebra and (1.14) reduces to the Jacobi identity for the structure constants of a Lie group. Finally, we mention that the variation of $A_I$ and $X^I$ in (1.13) yields the PSM field equations

\[ dX^I + P^{IJ} A_J = 0, \quad dA_I + \frac{1}{2} (\partial_I P^{JK}) A_K A_J = 0. \]

(1.16)
All PSMs are in essence topological theories of gauge fields on a by itself dynamical target space. They also allow for the straightforward inclusion of other topological degrees of freedom (such as gauge fields in two dimensions). The central object in (1.13) is the Poisson tensor $P^{IJ}$, which for bosonic FOG (1.4) reads

$$P^{\phi \pm} = \pm X^\pm, \quad P^{+-} = X^+ X^- U(\phi) + V(\phi), \quad P^{IJ} = -P^{JI}. \quad (1.17)$$

The PSM formulation of FOG is very useful in the context of supergravity, see Sect. 1.2. Since the Poisson tensor (1.17) has odd dimension it cannot have full rank. Therefore there appears at least one “Casimir function” defined by the condition

$$\{X^I, C\} = P^{IJ} \frac{\partial C}{\partial X^J} = 0. \quad (1.18)$$

The conserved quantities are thus central elements of the algebra of target space coordinates under the Schouten-Nijenhuis bracket. It is straightforward to show the conservedness by using the equations of motion (1.16), as well as to reproduce the form of (1.12).

### 1.1.3. Exact Path Integral Quantization

In the years 1997-1999 Wolfgang Kummer, Herbert Liebl and Dimitri Vassilevich \(^{36,43,44}\) found that an exact path integral quantization is possible for the first order formulation (1.4), (1.6). Even more, if (1.4) is coupled to matter, \(^{36,43-51}\) the geometric sector $(\omega, e^a, X^a, \phi)$ can still be integrated out exactly, yielding a nonlocal and nonpolynomial effective action for the matter fields, which then can be treated perturbatively. In this section, we will shortly review the most important steps in the path integral quantization, more details are explained in the context of supergravity in Sect. 1.2 or can be found e.g. in the review \(^21\).

The goal is to evaluate the path integral

$$Z = \int \mathcal{D}(\Phi, \phi, X^a, \omega_\mu, e^a_\mu) e^{iS_{\text{FOG}}[\phi, X^a, \omega, e^a]} + iS_{\text{max}}[\phi, e^a, \Phi] \quad (1.19)$$

where $\Phi$ denote all kinds of matter fields, which should not couple directly to the spin connection if the equivalence with generalized dilaton theories (1.6) shall not be lost. Since the model can be formulated as a nonlinear gauge theory with diffeomorphisms and local Lorentz invariance as symmetries, the first step is to construct an appropriate gauge fixed action. To this end, one analyzes the constraints in the theory, by first noting that (1.4)
furnishes a natural symplectic structure with canonical variables\(^6\) \(q^I = (\phi, X^a)\), \(p_I = (\omega_1, e^a_1)\), and \(\bar{p}_I = (\omega_0, e^a_0)\). The second set of momenta \(\bar{p}_I\) does not appear with time derivatives in the Lagrangian and thus its conjugate coordinates are constrained to zero, \(\bar{q}_I \approx 0\), which constitutes three primary first class constraints. These constraints give rise to three secondary first class constraints \(G_I = \{\bar{p}_I, \mathcal{H}\}\). If the matter extension yields additional second class constraints, as e.g. in the case of fermions,\(^{48}\) the Poisson bracket should be replaced by a the Dirac bracket. The three constraints \(G_I\) form a closed nonlinear algebra

\[
\{G_I, G_J\} = f_{K IJ} \delta(x - x'),
\]

(1.20)

with structure functions \(f_{K IJ}\). A Virasoro-like algebra closing on derivatives of \(\delta\)-functions typical for 2D gravity models can be recovered from linear combinations of the \(G^I\).\(^{22,23}\) As expected for a theory of gravity the Hamiltonian density vanishes on the constraint surface,

\[
\mathcal{H} = \dot{q}^I p_I - L = -G^I \bar{p}_I.
\]

(1.21)

From this knowledge it is now straightforward to construct the gauge fixed action with ghosts, using the BVF formalism.\(^{54-56}\) Introducing one pair of ghosts and antighosts for each secondary first class constraint, \((c_I, p^c_I)\),\(^f\) one finds the nilpotent BRST charge

\[
\Omega = G^I c_I + \frac{1}{2} p^c_k f_{K IJ} c_I c_J, \quad \{\Omega, \Omega\} = 0.
\]

(1.22)

having the same structure as in ordinary Yang-Mills theory, albeit the field-dependence in \(f_{K IJ}\). At this point it is important to follow Wolfgang Kummer’s concept of temporal gauges and to fix e.g. \((\omega_0, e^a_0) = (0, 1, 0)\), which again establishes Eddington-Finkelstein gauge for the metric. This can be implemented by a simple multiplier gauge \(\Psi = p^c_2\).\(^{57}\) another possibility was used in Ref. 36. Now the gauge fixed Hamiltonian \(\mathcal{H}_{gf} = \{\Omega, \Psi\}\), via Legendre transformation\(^8\) yields the gauge fixed Lagrangian,

\[
\mathcal{L}_{gf} = \dot{q}^I \bar{p}_I + \dot{q}^I p_I + G^+ + p^c_k (\delta_0 e^c_k + f_{K IJ} c_I c_J).
\]

(1.23)

\(^{6}\)Note that these conventions accord with most literature on supergravity\(^{52,53}\) but differ from those in Ref. 21, where the components of the gauge fields were chosen as momenta rather than canonical coordinates.

\(^{f}\)Strictly speaking it would also be necessary to introduce (anti)ghosts for the primary constraints. The procedure is straightforward, as described in chapter 7 of Ref. 21, and does not yield much new insight.

\(^{8}\)Of course the Legendre transform also has to be done w.r.t. possible matter canonical variables present. We tacitly ignored this here, as we do not want to specify the matter content yet, but describe the overall structure.
The path integral (1.19) can now be evaluated as follows: First all (anti)ghosts are integrated out yielding the Faddeev-Popov determinant $\text{Det} \Delta_{FP} = \text{Det}(\partial_0 \delta^L_K + f_K^{+L})$. Now an important observation is made: Eq. (1.23) only depends linearly on $p_I$, while $\text{Det} \Delta_{FP}$ is independent thereof. After eventual integration of matter momenta the geometric fields $p_I$ thus can be integrated. This generates three “functional $\delta$ functions”, whose arguments contain parts of the equations of motion and imply in the matterless case ($j^I$ are sources for the $p_I$)

$$0 = \partial_0 \phi - X^+ - j^\phi, \quad 0 = \partial_0 X^+ - j^+, \quad (1.24)$$

$$0 = (\partial_0 + X^+ U(\phi))X^- + V(\phi) - j^-, \quad (1.25)$$

The final integration of $X^I = q^I$ sets these fields to the formal solution of the above equations given in terms of Green functions for $\partial_0$ acting on the sources. Note that these solutions depend nonlocally on the sources, and in particular the solution of (1.25) is also nonpolynomial. In defining the Green functions the asymptotic values of $X^I$ are fixed, which also fixes the quasilocal mass. For this reason, the path integral as it is yields local physics. It is a question worth investigating whether in the path integral formalism an “integration over masses” is possible — see Sect. 1.3 for some further comments on this topic. Finally, it should be mentioned that the integration over $q^I$ cancels exactly the Faddeev-Popov determinant.

In the absence of matter fields the story ends here since the path integral is fully evaluated. If an integration over matter fields is left, the so-far obtained effective action is a complicated, nonlocal and nonpolynomial expression in the matter fields. Still, it is possible to derive in a systematic way non-local vertices of the interaction of matter with the quantized gravitational background. Therefrom, higher loops\textsuperscript{43} and scattering processes\textsuperscript{45} can be calculated.\textsuperscript{45} For a real scalar field unitarity, i.e. absence of information loss, and CPT invariance of the tree-level four-particle S-matrix element was found. The specific heat of the Witten black hole was also shown to be positive\textsuperscript{58} once loop corrections are taken into account.

A phenomenon worth mentioning is the “virtual black hole (VBH)”;\textsuperscript{46,58–60} Some of the nonlocal interaction geometries resemble black holes, although being off-shell entities. The curvature scalar of a VBH has a $\delta$-peak at some point and is discontinuous up to that peak (see the Penrose diagram in Fig. 2 of ref. 61, an effective line element can also be found there). Although this interpretation is a feature of the chosen gauge, in the calculation of (gauge invariant) S-matrix elements one has to integrate over all such VBHs, which leads to the idea that nonperturbative and
nonlocal excitations such as VBHs might also play an important role in higher-dimensional quantum gravity.

1.2. Two-Dimensional Dilaton Supergravity

In the following we will leave the field of bosonic dilaton gravity, since many results thereof have been summarized in various reviews.\textsuperscript{21,61} Instead we will concentrate on dilaton supergravity in two dimensions, a field in which Wolfgang Kummer made substantial contributions as well but which is not covered exhaustively by the existing reviews.

Of course, dilaton supergravity in two dimensions existed long before Wolfgang Kummer entered the field. Many early attempts were based on superspace techniques,\textsuperscript{62} which led to a generalized dilaton supergravity action of the form\textsuperscript{63}

\[ S = \int d^2x d^2\theta E\left( S - \frac{1}{4} U(\Phi) D^\alpha \Phi D^\beta \Phi + \frac{1}{2} u(\Phi) \right) , \]  

(1.26)

where \( S \) and \( \Phi \) are the supergravity and dilaton superfields, resp, and \( U(\Phi) \) and \( u(\Phi) \) two dilaton dependent functions (potentials) defining the model.\textsuperscript{63} Despite its successes and advantages,\textsuperscript{63,65,66} in particular the straightforward way to couple additional gauge or matter fields, the superspace formulation shares its drawbacks with the purely bosonic second order action: It does not include bosonic torsion, exact solutions are not easy to obtain although the matterless model is integrable, and a nonperturbative quantization is cumbersome. Some attempts to relax the standard condition of vanishing bosonic torsion led to a mathematically complex formalism\textsuperscript{67,68} and thus were not pursued further.

1.2.1. 2D Dilaton Supergravity from Graded PSMs

In this situation it appeared promising to extend FOG (1.4) to dilaton supergravity. Already in Refs. 69,70 gauge theoretic methods were used to find supersymmetric extensions of some dilaton gravity models. However, this approach is limited to models representable as a linear gauge theory, in particular the Jackiw-Teitelboim model.\textsuperscript{17,18} Its extension to nonlinear gauge theories showed that generalized dilaton supergravity can be treated...
in this framework.\textsuperscript{40,71} This led to a first order action for dilaton supergravity using free differential algebras,\textsuperscript{72} which however is restricted to vanishing bosonic torsion. This model, which is classically equivalent to the action (1.26) for $U = 0$, then was shown to be a special case of a graded PSM.\textsuperscript{71}

Wolfgang Kummer, Martin Ertl and Thomas Strobl then showed\textsuperscript{35} that the use of graded PSMs (gPSMs) provides a simpler and more systematic tool to find supergravity extensions of FOG. To this end one replaces the target space (Poisson manifold) of the action (1.13) by a graded Poisson manifold.\textsuperscript{1} On this manifold we use the coordinates (scalar fields) $X^I(x) = (X^i(x), X^\alpha(x))$, where $X^i$ refers to bosonic (commuting) coordinates, while $X^\alpha$ are fermions (anti-commuting coordinates.) All equations (1.13)--(1.18) have been displayed in such a way that they hold for gPSMs as well, if one replaces the permutations in (1.14) by graded permutations and keeps in mind that $P^{ij}$ is now graded anti-symmetric.

As in the non-supersymmetric case, not every gPSM describes a supergravity model, but certain additional structures are needed. Firstly this concerns the choice of target space and gauge fields. In the application to two-dimensional $N = (1, 1)$ supergravity, the bosonic fields in (1.4) are complemented by two Majorana spinors, $\psi_\alpha$ ("gravitino") and $\chi_\alpha$ ("dilatino"):

\begin{equation}
A_I = (\omega, e_a, \psi_\alpha) \quad X^I = (\phi, X^a, \chi_\alpha) \quad (1.27)
\end{equation}

Secondly, local Lorentz invariance determines the $\phi$-components of the Poisson tensor

\begin{equation}
P^{\alpha\phi} = X^b \epsilon_a \quad P^{\alpha\phi} = -\frac{1}{2} \chi^3 \epsilon_a \quad (1.28)
\end{equation}

Finally it should be noted that the existence of a target space metric $\eta_{ab}$ is assumed as an extra structure.\textsuperscript{1} Remarkably it was possible to solve the nonlinear Jacobi identity (1.14) explicitly with just these three assumptions.\textsuperscript{35,68} Having defined the fermionic extension, all components of the graded Poisson tensor may be written as a systematic expansion in these fields, restricted by local Lorentz invariance encoded in the $\epsilon_\phi$ component in (1.15). As an example $P^{ab}$ must be of the form $P^{ab} = V(\phi, Y) + \chi^2 \epsilon_3(\phi, Y)$, where $V(\phi, Y)$ is the bosonic potential (1.5) of the model. Then it is a straightforward but tedious calculation to solve the condition (1.14) order by order in the fermionic fields. Though a purely algebraic solution

\textsuperscript{1}Instead of a graded target space, supersymmetric PSMs can be obtained by grading the world-sheet manifold. While such models attracted interest in the context of string theory,\textsuperscript{74,75} they are less useful to derive supergravity actions.

\textsuperscript{2}Attempts to relax some of these conditions have been presented in Refs. 76,77.
L. Bergamin and R. Meyer

was found,\textsuperscript{35,68} this result was not yet satisfactory, as it depends besides the bosonic potential \( \mathcal{V}(\phi, Y) \) on five arbitrary Lorentz covariant functions. However, many choices of these five functions impose new singularities and obstructions in the variables \( \phi \) and \( Y \) in points where the bosonic potential remains regular. In extreme cases this prohibits any supersymmetrization at all.

From this result it is obvious that not all Lorentz covariant gPSMs permit an interpretation as dilaton supergravity, but a suitable implementation of supersymmetry transformations is needed.\textsuperscript{k} This can be achieved by restricting the fermionic extension \( P^{\alpha \beta} \) to the form

\[
P^{\alpha \beta} = -2iX^a \gamma_a^{\alpha \beta} + Z(\phi, Y, \chi^2)^3, \tag{1.29}
\]

where the first term generates supersymmetry transformations of the form \( \{ Q^\alpha, Q^\beta \} \) in the commutator of two symmetry transformations (1.15). The solution of the Jacobi identities now proved a unique class of \( N = (1, 1) \) dilaton supergravity models with Poisson tensor\textsuperscript{l}

\[
P^{ab} = \left( V + U \right) - \frac{1}{2} \lambda^2 \left( \frac{VU + V'}{2} + \frac{2V^2}{u^3} \right), \tag{1.30}
\]

\[
P^{ab} = \frac{U}{4} X^a \gamma_a^{b} \gamma \gamma^\alpha \gamma^\beta + \frac{iV}{u} (\chi^b \gamma^\alpha), \tag{1.31}
\]

\[
P^{\alpha \beta} = -2iX^c \gamma_c^{\alpha \beta} + \left( u + \frac{U}{8} \chi^2 \right) \gamma^3, \tag{1.32}
\]

whereby \( u(\phi) \) acts as a prepotential for the bosonic potential \( V(\phi) \)

\[
V(\phi) = -\frac{1}{8} \left( (u^2)' + u^2 U(\phi) \right). \tag{1.33}
\]

First, this result was established from symmetry arguments\textsuperscript{52} by constructing a deformed superconformal algebra as previously done for FOG.\textsuperscript{23} Then it turned out that the supergravity action from (1.28) and (1.30)–(1.32),

\[
S = \int_{\mathcal{M}} \left( \phi \phi + X^a D e_a + \chi^\alpha D \psi_\alpha + e \left( V + U \right) - \frac{1}{2} \lambda^2 \left( \frac{VU + V'}{2} + \frac{2V^2}{u^3} \right) \right)
+ \frac{U}{4} X^a \left( \chi \gamma^b \gamma^\beta \psi + \frac{iV}{u} (\chi \gamma^a e_a \psi) \right)
+ iX^a (\psi \gamma_a \psi) - \frac{1}{2} \left( u + \frac{U}{8} \chi^2 \right) (\psi \gamma_a \psi), \tag{1.34}
\]

\textsuperscript{k}This procedure is similar to the choice of “consistent” supergravity in the standard formulation.\textsuperscript{78–81}

\textsuperscript{l}Notice that the potential \( U(\phi) \), which determines the torsion part of the spin connection, in most of the literature on supergravity is denoted by \( Z(\phi) \).
after elimination of the auxiliary fields $X^a$ and $\omega$ is equivalent to the action (1.26) after integrating out superspace.\textsuperscript{82} As in FOG there exists at least one Casimir function, which receives additional fermionic contributions:

$$C = e^Q (Y - \frac{1}{8} u^2 + \frac{1}{16} \chi^2 (u' + \frac{1}{2} u U))$$ \hspace{1cm} (1.35)$$

The Casimir function plays a crucial role in the discussion of BPS states.\textsuperscript{83} From the symmetry transformation of the dilatino in Eq. (1.15) it follows that the fermionic part of the Poisson tensor for BPS solutions does not have full rank. For purely bosonic states this implies $\text{Det} P^{\alpha \beta} = 8Y - u^2 = 0$ and thus $C = 0$. Furthermore, an additional fermionic Casimir function exists and it can be checked\textsuperscript{35,82} that the quantity

$$\tilde{c} = e^{\frac{1}{2}Q} \sqrt{\left| X^{++} \right|} \left( \chi^- - \frac{\chi^+}{2\sqrt{2}X^{++}} \right)$$ \hspace{1cm} (1.36)$$

for $C = 0$ is a Casimir function as defined in (1.18), which corresponds to the unbroken supersymmetry of the BPS state.\textsuperscript{83} Besides the purely bosonic states there also exist BPS states with non-vanishing fermionic fields, which however must have a trivial bosonic background.\textsuperscript{83} Nonetheless, some of these states exhibit the interesting feature that $\chi^2(u' + \frac{1}{2} u U) \neq 0$ and thus the bosonic part of the Casimir vanishes, while its fermionic extension (its soul) is non-zero.

Also the complete classical solution now is straightforward to obtain.\textsuperscript{35,68,82} The bosonic line element again may be written in the form (1.9). From this result the extremality of BPS Killing horizons\textsuperscript{84,85} is immediate since by virtue of the constraint (1.33) the conformally invariant potential acquires the form $w(\phi) = -e^Q u^2/8$. Thus, the Killing norm $\xi$ in Eq. (1.11) for BPS solutions is positive semi-definite, an eventual horizon must be extremal and represents a ground state.\textsuperscript{83}

Coupling of additional fields (e.g. gauge fields or matter fields) is straightforward for FOG, but in supergravity this task becomes considerably more complicated since a correct supersymmetry transformation of the new fields must be ensured. Since it is known that all genuine gPSM supergravity models are classically equivalent to superspace models, the coupling of matter fields can be inferred therefrom. As an example, it has been demonstrated in Ref. 83 that conformal matter fields can be coupled non-minimally to the general action (1.34). The matter action for a chiral multiplet with real scalar field $f$ and Majorana fermion $\lambda_\alpha$ and with coupling function $K(\phi)$ for second order supergravity is obtained from standard
superspace techniques as

\[
S_{\text{matter}} = \int d^2 x e \left[ K \left( \frac{1}{2} (\partial^m f \partial_m f + \imath \lambda \gamma^m \partial_m \lambda) + \imath (\psi_n \gamma^m \gamma^n \lambda) \partial_m f \right) 
+ \frac{1}{4} (\psi_n \gamma_m \gamma_n \psi^m) \lambda^2 \right] + \frac{u}{8} K' \lambda^2 - \frac{1}{4} K' (\chi \gamma^3 \gamma^m \lambda) \partial_m f 
- \frac{1}{32} \left( K'' - \frac{1}{2} \left[ K' \right]^2 \right) \chi^2 \lambda^2 \right]. \tag{1.37}
\]

Without further changes this action provides the correct extension of genuine gPSM supergravity by a conformal matter field. Still, the supersymmetry transformations of some of the gPSM fields acquire additional terms from the matter action.\textsuperscript{83}

The gPSM approach to two-dimensional dilaton supergravity is not restricted to the above example of \( N = (1, 1) \) supergravity. As an example the gPSM version of the model of Ref. \textsuperscript{86} has been presented in Refs. \textsuperscript{87,88}. This model deals with chiral or twisted-chiral \( N = (2, 2) \) supergravity. Thus besides a pair of complex Dirac dilatini/gravitini an additional graviphoton gauge field is needed in the supergravity multiplet, while the (twisted-)chiral dilaton multiplet is complemented by an additional Lorentz scalar \( \pi \):

\[
X_I = (\phi, \pi, X^a, \chi^\alpha, \bar{\chi}^\alpha), \quad A_I = (\omega, B, e_a, \psi_\alpha, \bar{\psi}_\alpha). \tag{1.38}
\]

Again symmetry constraints fix certain components of the Poisson tensor. Local Lorentz invariance and supersymmetry are implemented analogously to (1.28) and (1.29) and supersymmetry is encoded as \( P^{\alpha \beta} = -2i \chi^\beta \gamma^\alpha \chi^\beta + \) terms \( \propto \gamma^3 \). The additional \( B \) gauge symmetry imposes (here for chiral supergravity)

\[
P^{a \pi} = 0, \quad P^{a \pi} = -i 2 \chi^\beta \gamma^3 \beta \alpha, \quad P^{a \pi} = -i 2 \chi^\beta \gamma^3 \beta \alpha. \tag{1.39}
\]

Starting with these restrictions it is again possible to solve the non-linear Jacobi identity (1.14) and to derive the complete action of extended supergravity.\textsuperscript{87} Again, once reformulated as a gPSM the model can be solved explicitly, whereby some subtleties arise as a consequence of the grading of the Poisson manifold.\textsuperscript{88} The model is found to have at least two Casimir functions, one being the \( N = (2, 2) \) extension of (1.12), the second one is the charge with respect to the graviphoton field. This new conserved quantity diverges in the limit \( C \rightarrow 0 \) unless all fermionic contributions vanish. Nevertheless, after integrating all equations of motion it is found that there exist well-defined solutions at \( C = 0 \) with non-vanishing fermion
fields. However, the remaining conserved quantity (the charge with respect to the graviphoton) can no longer be expressed entirely in terms of the target space variables, i.e. for those special solutions this conserved quantity is not a Casimir function of the graded Poisson manifold. This is a unique feature of graded PSMs, since for purely bosonic PSMs the existence of Casimir-Darboux coordinates excludes such a behavior.

1.2.2. Quantization of 2D Supergravity

Having formulated dilaton supergravity as a gPSM it is self-evident to try to extend the nonperturbative quantization outlined in Sect. 1.1.3 to dilaton supergravity.\textsuperscript{47,53}

1.2.2.1. Quantization Without Matter

Let us first consider the dilaton supergravity action (1.34) without matter couplings. As in Sect. 1.1.3 the canonical coordinates are chosen as $q^I = \tilde{X}^I$, $p_J = A_{1J}$, $\bar{p}_J = A_{0J}$ and $\bar{q}^J \approx 0$ are the primary first class constraints. In supergravity these coordinates obey the graded Poisson bracket\textsuperscript{m}

$$\{q^I, p_J\} = (-1)^{I+J+1} \{p^J, q^I\} = (-1)^I \delta^I_J \delta(x - x') .$$  \hspace{1cm} (1.40)

The secondary constraints

$$G^I = \partial_1 q^I + P^{IJ} p_J ,$$  \hspace{1cm} (1.41)

despite the new fermionic constraints formally obey the same constraint algebra (1.20) as in the purely bosonic case. The structure functions in the matterless case are simply $f_K^{IJ} = -\partial_K P^{IJ}$ with $P^{IJ}$ as given in (1.28) and (1.30)–(1.32), and with fermionic derivatives being left-derivatives. The ensuing Hamiltonian (1.21) again is quantized by introducing ghosts, which now obey the graded commutation rules $[c_I, p_J^c] = -(-1)^{(I+1)(J+1)} [p^c_J, c_I] = \delta^I_J$. This yields an additional sign in the BRST charge\textsuperscript{53}

$$\Omega = G^I c_I + \frac{1}{2} (-1)^I p^c_K f_K^{IJ} c_J c_I .$$  \hspace{1cm} (1.42)

The BRST charge (1.42) is found to be nilpotent as a consequence of the graded Jacobi identity of $P^{IJ}$ (1.14). This characteristic does not depend on the specific form of the Poisson tensor as found in (1.30)–(1.32) and thus also applies to graded dilaton models not related to the supergravity action.

\textsuperscript{m}This bracket should not be confused with the Schouten bracket, associated to the Poisson tensor $P^{IJ}$ in (1.13).
(1.26). Also, any gPSM gravity model without matter is free of ordering problems although the constraints are nonlinear in the fields.33

Imposing again a multiplier gauge $\bar{p}_I = a_I$ with $a_I = -i \delta_I^{++}$ one arrives at the gauge fixed Lagrangian

$$L^0_{\text{g.f.}} = \dot{q}^I p_I + p^I \dot{c}_I - i P^{+++} p_J .$$

(1.43)

The evaluation of the path integral now follows exactly the same steps as in the purely bosonic case. Having integrated out all ghosts the effective Lagrangian including sources for $p_I$ and $q^I$ becomes

$$L^0_{\text{eff}} = \dot{q}^I p_I - i P^{+++} p_J + q^I j_{qI} + j_p^I p_I$$

(1.44)

which again is linear in all $p_I$. Integration of the momenta thus yields five functional $\delta$ functions, whose arguments imply (for the Poisson tensor (1.28), (1.30)-(1.32))

$$\dot{q}^i = -i q^{++} - j_p^i, \quad \dot{q}^{++} = -j_p^{++},$$

$$\dot{q}^{--} = i \left( e^{-Q} w' + q^{++} q^{-} U + \frac{1}{2 \sqrt{2}} q^- e^{-Q/2} \left( \sqrt{-w} \right)^{''} \right) - j_p^{--},$$

$$\dot{q}^+ = -j_p^+, \quad \dot{q}^- = -i \left( e^{-Q/2} \left( \sqrt{-w} \right) q^+ - \frac{1}{2} U q^{++} q^- \right) - j_p^- .$$

(1.45)  

(1.46)  

(1.47)

These functional $\delta$ functions may be used to evaluate the remaining $q^I$ integration. As expected, this exactly cancels the super-determinant from the integration of the ghosts. The final generating functional becomes

$$W[j^I, j_{qI}] = \exp i L^0_{\text{eff}}, \quad L^0_{\text{eff}} = \int d^2 x \left( B^I j_{qI} + \bar{L}^0_{\text{amb}}(j^I p, B^I) \right),$$

(1.48)

where $B^I$ are the solutions of Eqs. (1.45)-(1.47) and $\bar{L}^0$ are the so-called “ambiguous terms” 21, 25, 36. An expression of this type is generated by the integration constants $G_I(x^1)$ from the term $\int dx^0 \int dq^0 (\delta_0^{-1} A^I) j_{qI}.$

From this result it is straightforward to calculate the quantum effective action

$$\Gamma \left( \langle q^I \rangle, \langle p_I \rangle \right) := L^0_{\text{eff}} (j_{qI}, j^I p) - \int d^2 x \left( \langle q^I \rangle j_{qI} + j^I p \langle p_I \rangle \right),$$

(1.49)

in terms of the mean fields

$$\langle p_I \rangle := \left. \frac{\delta L^0_{\text{eff}}}{\delta j_p^I} \right|_{j_{qI} = 0}, \quad \langle q^I \rangle := \left. \frac{\delta L^0_{\text{eff}}}{\delta j_{qI}} \right|_{j_p^I = 0} = B^I \bigg|_{j_p^I = 0}. $$

\(^{21}\)Notice that the elements of the spin-tensor decomposition are related to lightcone indices as $v^{++} = i v^{\ominus}$, $v^{--} = -i v^{\ominus}$, cf. Eq. (A.8). Thus $\beta_{++}$ is imaginary.
By re-expressing all sources in terms of the (classical) target space coordinates it is found\textsuperscript{53} that—up to boundary terms—the quantum effective action is nothing but the gauge fixed classical action. Thus similar to FOG\textsuperscript{36} two-dimensional dilaton supergravity exhibits local quantum triviality. Still, the quantization procedure is not completely trivial as the generating functional (1.48) is essentially non-local.

1.2.2.2. Quantization Including Matter Fields

Local quantum triviality provides an important consistency check, the main purpose of the nonperturbative quantization procedure is the straightforward way to couple matter fields perturbatively to the fully quantized geometric background.\textsuperscript{21,43,44,57} While an immediate extension to supergravity could be expected for the matterless theory on general grounds, this came as a surprise for the theory including matter fields.\textsuperscript{47,53} The matter action (1.37) is obtained from the superspace action by a non-trivial integration of auxiliary fields and exactly this step, in the second order formalism, generates quartic ghost terms, which would spoil the nonperturbative quantization procedure. In the first order formulation this does not happen, which makes the full integration over geometry possible in the first place.

From the matter action (1.37) together with the matter fields and momenta, $q = f$, $p = \partial L_{(m)}/\partial \dot{q}$ and $q^a = \lambda^a$, $p_a = \partial L_{(m)}/\partial \dot{q}^a$, the Hamiltonian density follows as $H_{(m)} = \dot{q} p + \dot{q}^+ p_+ + \dot{q}^- p_- - L_{(m)}$. The total Hamiltonian density is the sum of this contribution and (1.21). For the Poisson bracket of two matter field monomials one finds \{ $q$, $p'$ \} $= \delta(x-x')$ and \{ $q^a$, $p'_b$ \} $= -\delta_{ab} \delta(x-x')$. We do not provide the explicit form of the matter Hamiltonian, as it can again be written in terms of secondary constraints, $H = G_I^T \tilde{p}_I$. The constraints itself divide into matter and geometry, $G_I^T = G_{(q)}^T + G_{(m)}^T$, where $G_{(q)}^T$ has been derived in (1.41) and the matter part reads ($\partial = \partial_1$):

\begin{equation}
G_{(m)}^{++} = -\frac{K}{4p_{++}}(\partial q - \frac{1}{K} p)^2 + i(\partial q - \frac{1}{K} p)(\frac{K}{p_{++}} p_+ q^+ - \frac{K'}{4\sqrt{2} p_{++}} q^- q^-) \\
+ \frac{iK'}{4\sqrt{2}}(\partial q + \frac{1}{K} p)q^+ q^+ + \frac{K}{\sqrt{2}} q^+ \partial q^+ - \frac{K'}{2\sqrt{2} p_{++}} p_+ q^- q^- q^+ \\
- p_{--} \left( \frac{u K'}{4} - \frac{1}{8} (K'' - \frac{K'^2}{K}) q^- q^+ \right) q^- q^+ 
\end{equation} 

(1.51)
temporal gauge as used there the gauge fixed Lagrangian becomes path integral along the same lines as for the matterless case. In the same Thanks to this unexpectedly simple result we can pursue to formulate a different non-linearities the theory does not exhibit ordering problems.

charge as given in Eq. (1.42). Furthermore it is again found that despite the logical perturbation theory again stops at first order, most important observation a lengthy calculation unravels that the homo-

expressions it can be shown that the secondary constraints with respect to the structure functions arise, explicit expression can be found in Ref. 47.

As the kinetic term of the matter fermion \( \lambda \) is first order only, this part of the action leads to constraints as well. From \( p_+ = -K p_+ + q^+ / \sqrt{2} \) and \( p_- = K p_- - q^- / \sqrt{2} \), the usual primary second-class constraints are deduced:

\[
\Psi_+ = p_+ + K \sqrt{2} p_{++} q^+ \approx 0, \quad \Psi_- = p_- - K \sqrt{2} p_{--} q^- \approx 0.
\]

These second class constraints are treated by substituting the Poisson bracket by the “Dirac bracket” \({ f, g }^* = \{ f, g \} - \{ f, \Psi_\alpha \} C^{\alpha \beta} \{ \Psi_\beta, g \}, \) where \( C^{\alpha \beta} C_{\beta \gamma} = \delta^\alpha_\gamma \) and \( C_{\alpha \beta} = \{ \Psi_\alpha, \Psi_\beta \}. \) Despite the complexity of these expressions it can be shown that the secondary constraints with respect to the Dirac bracket obey an algebra of the type (1.20). In the case of minimal coupling, \( K(\phi) = 1 \), the structure functions of the matterless theory (1.20) are reproduced, while in the generic case additional matter contributions to the structure functions arise, explicit expression can be found in Ref. 47.

The quantization follows the same steps as in the matterless case. As most important observation a lengthy calculation unravels that the homological perturbation theory again stops at first order, with the BRST charge as given in Eq. (1.42). Furthermore it is again found that despite the different non-linearities the theory does not exhibit ordering problems. Thanks to this unexpectedly simple result we can pursue to formulate a path integral along the same lines as for the matterless case. In the same temporal gauge as used there the gauge fixed Lagrangian becomes

\[
L_{\text{gt}} = \dot{q}^I p^I + \dot{q}^a p_a + \dot{p}_c c^I - i p^{++} p_+ - i (-1)^K p^*_c C^{I \alpha \beta} c^K
\]

\[
+ \frac{i K}{4} (\dot{q}^I p^I - \dot{q}^a p_a - \dot{p}_c c^I) + (\dot{q}^I p^I - \dot{q}^a p_a - \dot{p}_c c^I) \frac{K^I}{4 \sqrt{2} p_{++} q^{-} q^{+}}
\]

\[
+ \frac{i K'}{4 \sqrt{2} p_{++} q^{-} q^{+}} - \frac{i}{\sqrt{2}} K q^+ \dot{q}^+ + \frac{i}{2 \sqrt{2}} K' \frac{p_{--}}{p_{++}} q^{-} q^{+} + \frac{i K'}{4 \sqrt{2} p_{++} q^{-} q^{+}}
\]
\[ \bar{\psi}_I^I, \bar{\psi}_I^\alpha \text{ are trivial and the ghosts just yield the super-determinant } s\text{det} M_{IJ} = s\text{det} \left( \delta_{IJ} \partial_0 + i f_I^{++} \right). \]

The fermionic momenta \( p_\alpha \) by means of the constraint (1.55) are integrated trivially as well, while this is possible for \( p \) after a quadratic completion. The ensuing determinant can be absorbed by the re-definition of the path-integral measure of \( \bar{\psi} \) and \( \bar{\psi}_\alpha \) with correct superconformal properties.

This yields the effective matter Lagrangian

\[ L_m = iK p_{++} \bar{\psi}^2 - \frac{K}{2\sqrt{2}} p_{++} \bar{\psi}^+ \bar{\psi}^+ + \frac{K}{2\sqrt{2}} p_{--} \bar{\psi}^- \bar{\psi}^-. \]

Since this expression is again linear in \( p_I \) all geometric variables can be integrated out and one is left with the integration of the matter variables, which must be treated perturbatively. It should be noted that the above Lagrangian explicitly depends on the prepotential \( u \) as a consequence of the elimination of (superspace) auxiliary fields. This is different than in all bosonic models, where a strict separation of the potentials appearing in the geometric part \( (V \text{ and } U) \) and the one of the matter extension, \( K \), occurs.

Having performed the remaining integrals of all geometric quantities one is left with a path integral in the matter fields \( \psi \) and \( \psi_\alpha \):

\[ \mathcal{W} = \int \mathcal{D}(\psi, \psi_\alpha) \exp \left( i \int d^2x \left( K \bar{\psi} \partial \psi - i \sqrt{2} \bar{\psi}^+ \partial \psi^+ \right) + \frac{K'}{2\sqrt{2}} \partial \psi^+ \partial \psi^+ - B^I j_{qI} + \bar{L}(j^{I}_p, B^I) + \bar{q} j + q^\alpha J_\alpha \right). \]

Here, the \( B^I \) are solutions to the functional \( \delta \) functions

\[ \bar{q}^{\delta} = q_{(g)}^{\delta}, \quad \bar{q}^{++} = q_{(g)}^{++} - iK \bar{q}^2 - \frac{K}{2\sqrt{2}} \bar{q}^+ \bar{q}^+ - i \frac{K'}{2\sqrt{2}} \bar{q}^+ \bar{q}^+ , \] (1.59)
\[ \dot{q}^- = q^{(g)} - \frac{K}{\sqrt{2}} q^- q^- - \frac{i K'}{2 \sqrt{2}} q^- q^- \]

\[ - i \left( \frac{u K'}{4} - \frac{1}{8} \left( K'' - \frac{1}{2} \frac{K^2}{K} \right) q^- q^+ \right) q^- q^+ \]

\[ q^+ = q^{(g)} + 2i K q q^+ , \quad \dot{q}^- = q^{(g)} , \]

with the \( q^{(g)} \) being the right hand sides of Eqs. (1.45)-(1.47).

1.2.2.3. Four-Point Vertices

From the matter Lagrangian (1.57) and the differential equations (1.59)–(1.61) it is possible to derive the non-local vertices of matter to lowest order (tree level.) These results were presented in Ref. 53 for minimal coupling, \( K(\phi) = 1 \), here we derive the more general result for non-minimal couplings. In this calculation the concept of localized matter\(^{44,58,59}\) is used, here we follow the notation of Ref. 53 and define

\[ \Phi_i(x) = \frac{1}{2} \dot{q}(x) \partial_y q(x) \Rightarrow a_i \delta^2 (x - y) , \]

\[ \Psi_i^{\pm \pm}(x) = \frac{1}{2} \dot{q}^\pm (x) q^{\mp \pm}(x) \Rightarrow b_i^{\pm \pm} \delta^2 (x - y) , \]

\[ \Pi_i^\pm (x) = \partial_y q(x) q^{\mp}(x) \Rightarrow c_i^\pm \delta^2 (x - y) , \]

\[ \Lambda(x) = q^- q^+ (x) \Rightarrow e \delta^2 (x - y) . \]

Notice that with our choice of gauge only \( \Psi_i^{++} \) and \( \Pi_i^+ \), but no terms in \( \Psi_i^{--} \) or \( \Pi_i^- \) appear in the interaction. Furthermore, due to supersymmetry \( a_0 \) and \( b_0^{++} \) only appear in the linear combination

\[ A_0 = 2i a_0 - \sqrt{2} b_0^{++} , \]

which thus will be used as abbreviation in the following. Thanks to the local quantum triviality of the matterless theory, the lowest order vertices can be determined from the matter interaction terms in the gauge-fixed Lagrangian (1.57) by solving to first order in localized matter the classical equations of motion of the geometrical variables involved.\(^{21,44,58}\) To this end the asymptotic integration constants must be chosen in a convenient way. Following the calculations of the purely bosonic case\(^{44,58}\) \( q^0 (x^0 \to \infty) = x^0 \), which implies \( q^{++} (x^0 \to \infty) = i \). In addition \( p_--(x^0 \to \infty) = i e q^0 \) may now be imposed. Finally we have to fix the asymptotic value of the Casimir function (1.35), \( C(x^0 \to \infty) = C_\infty \). Due to the matter interactions \( dC \neq 0 \), but the conservation law receives contributions from the matter fields as
well.\textsuperscript{83} Finally, considering the asymptotic values of the fermions $q^+$ and $q^-$ we follow Ref. 53 and set $q^+_{\infty} = q^-_{\infty} = 0$. This considerably reduces the complexity of the equations of motion since all contributions quadratic in the fermions vanish as they are second order in localized matter.

It turns out that the gauge fixed equations with the above choice of the asymptotic values can be solved explicitly to first order in localized matter. Therefore all non-local four-point vertices can be evaluated exactly to lowest order. To economize writing of the subsequent non-local quantities we introduce the notations

\begin{equation}
[f g]_{x^0} = f(x^0)g(x^0), \quad [f g]_{x^0 \pm y^0} = [f g]_{x^0} \pm [f g]_{y^0}.
\end{equation}

Furthermore, the abbreviation $h_{xy} = \theta(y^0 - x^0)\delta(x^1 - y^1)$ is used. We do not repeat the complete solution here, as an example the Casimir function gets the new contributions

\begin{equation}
C = -m_{\infty} + \left[iA_0(m_{\infty}[K]_y + [Ku]_y) + \sqrt{2}[e^QK]_y b_{0}^{--} - \frac{1}{4}[e^QuK]_y e^h_{xy}\right] \quad (1.68)
\end{equation}

$m_{\infty}$ is the integration constant of $q^{-} = ie^{-Q}m_{\infty} + \ldots$, which, however, turns out to be equivalent to the asymptotic value $-C_{\infty}$. Notice that all contributions except this integration constant are proportional to $h$ and thus to first order in localized matter all geometric variables may be replaced by their asymptotic values in that equation. Of course, this is equivalent to the statement that $C$ is constant in the absence of matter fields.

For the explicit expressions of the vertices one derives the relevant interaction terms from (1.57) as

\begin{equation}
L_{(m)} = (2i\Phi_0 - \sqrt{2}\Psi_0^{++})Kp_{++} + \sqrt{2}Kp_{--}\Psi_0^{-} + 2K\Phi_1 + \frac{i}{2\sqrt{2}}K'p_{++}q^{+} - 2iKp_{+}\Psi_0^{+} + \frac{i}{2\sqrt{2}}K'p_{--}q^{-}\Psi_0^{--} + \frac{K'}{2\sqrt{2}}q^{+}\Pi_{1}^{+} + \sqrt{2i}K\Psi_{1}^{++} + \frac{i}{4}uK'p_{--}\Lambda. \quad (1.69)
\end{equation}

With the solution obtained one now finds that the vertices depend on seven
different functions:

\[ V_1(x, y) = -K(x^0)K(y^0) \left( 2[w]_{x^0, y^0} - (x^0 - y^0) \right) \left( [w']_{x^0, y^0} \right) \]

\[ + \left[ \frac{K'}{K}(w + m_\infty) \right]_{x^0, y^0} h_{xy}, \]

\[ V_2(x, y) = -\sqrt{2}iK(x^0)K(y^0) \left( \sqrt{-w} \right)_{x^0, y^0} \left( \sqrt{-w} \right)_{x^0, y^0} \]

\[ - \frac{1}{2} \left[ \frac{K'}{K}(w + m_\infty) \right]_{x^0, y^0} h_{xy}, \]

\[ V_3(x, y) = 2iK(x^0)K(y^0)(x^0 - y^0)\delta(x^1 - y^1), \]

\[ V_4(x, y) = -\frac{i}{4}K(x^0) \left[ e^{\Phi} K' \right]_{y^0} (x^0 - y^0)\delta(x^1 - y^1), \]

\[ V_5(x, y) = \sqrt{2}K(x^0) \left[ e^{\Phi} K' \right]_{y^0} (x^0 - y^0)\delta(x^1 - y^1), \]

\[ V_6(x, y) = -\frac{1}{\sqrt{2}}K(x^0) \left[ \sqrt{-w} \right]_{x^0, y^0} \left[ e^{\Phi} K' \right]_{y^0} (h_{xy} - h_{yz}), \]

\[ V_7(x, y) = -\frac{i}{\sqrt{2}}K(x^0)K(y^0)(h_{xy} - h_{yz}). \]

The full interaction vertices from these functions are obtained as\(^{21,53,58}\)

\[ V = \int_x \int_y \Xi_i(x)\Xi_j(y)V_{ij}(x, y), \]

where \( \Xi_i \) is one of the contributions from localized matter according to (1.62)–(1.65) and \( V_{ij} \) is—up to eventual constants—the vertex function from (1.70)–(1.76) that describes the interaction between \( \Xi_i \) and \( \Xi_j \). \( V_1 \) determines the interaction of

\[ \hat{q}\hat{q}(x) \to \hat{q}\hat{q}(y) = -4V_1(x, y), \]

\[ \hat{q}^+\hat{q}^+(x) \to \hat{q}^+\hat{q}^+(y) = 2V_1(x, y), \]

\[ \hat{q}\hat{q}(x) \to \hat{q}^+\hat{q}^+(y) = -2\sqrt{2}iV_1(x, y), \]

while \( V_2 \) yields

\[ \hat{q}\hat{q}^+(x) \to \hat{q}\hat{q}^+(y) = V_2(x, y). \]

These are the only vertices that do not vanish for minimal coupling.\(^{54}\)

\( K = 1 \). Both of them are conformally invariant, but while \( V_1 \) vanishes at \( x^0 = y^0 \), \( V_2 \) does not unless \( K = 1 \), which thus yields a local four-point interaction for non-minimal coupling. The vertex function \( V_1 \) with
minimal coupling vanishes for models with \( w \propto \phi \), in particular for the CGHS model. Since \( V_1 \) is the only vertex function of bosonic models with minimal couplings, the CGHS model exhibits scattering triviality to this order. This does not apply to the supersymmetric extension of the CGHS model since \( V_2 \) does not vanish here.

\( V_3 \) appears in two different interaction terms in (1.69) leading to the four vertices

\[
\dot{q}q(x) \rightarrow \partial q\dot{q}(y) = 2iV_3(x,y) , \quad \dot{q}^+q^+(x) \rightarrow \partial q\dot{q}(y) = -\sqrt{2}V_3(x,y) , \tag{1.82}
\]

\[
\dot{q}q(x) \rightarrow \partial q^+q^+ = 2\sqrt{2}V_3(x,y) , \quad \dot{q}^+q^+(x) \rightarrow \partial q^+q^+ = 2V_3(x,y) . \tag{1.83}
\]

The remaining two interactions with \( (2i\Phi_0 - \sqrt{2}\Psi_0^+) \) as initial or final state are

\[
\dot{q}q(x) \rightarrow q^-q^+(y) = 2iV_4(x,y) , \quad \dot{q}^+q^+(x) \rightarrow q^-q^+(y) = -\sqrt{2}V_4(x,y) , \tag{1.84}
\]

\[
\dot{q}q(x) \rightarrow \dot{q}^-q^-(y) = 2iV_5(x,y) , \quad \dot{q}^+q^+(x) \rightarrow \dot{q}^-q^-(y) = -\sqrt{2}V_5(x,y) . \tag{1.85}
\]

Notice that these vertices are not conformally invariant as they cannot be written exclusively in terms of the conformally invariant potential \( w \). Finally, for non-minimal coupling besides (1.81) there exist two additional vertices with mixed bosonic/fermionic initial and final states,

\[
\dot{q}q^+(x) \rightarrow \dot{q}q^-(y) = V_6(x,y) , \tag{1.86}
\]

\[
\dot{q}^+(x) \rightarrow \partial q\dot{q}^- = V_7(x,y) . \tag{1.87}
\]

\( V_6 \) is not conformally invariant but zero for \( x^0 = y^0 \), while \( V_7 \) exactly behaves the other way around.

### 1.3. Two-Dimensional Dilaton Gravity with Boundaries

In all calculations of the previous sections boundary terms were assumed to vanish and asymptotically the values of the target space variables \( \phi \) and \( X^I \) were fixed, which removes eventual boundary degrees of freedom. However, in many applications this setup is not suitable and a careful treatment of

---

\(^{92}\)It might come as a surprise that a conformally invariant model generates vertices which are not invariant under this transformation. Nonetheless, it should be remembered that conformal invariance applies to the complete Lagrangian, here (1.69), including the (asymptotic) matter states.\(^{48,61}\)
boundary terms is necessary. Already in Ref. 32 it was found by Wolfgang Kummer and Stephen Lau that the action (1.4) should be complemented by the boundary term

\[ S_{\text{boundary}} = \int_{\partial M} \left( X \omega + \frac{1}{2} X \partial \ln \left| \frac{e^+}{e^-} \right| \right) \quad (1.88) \]

in order to make the theory globally equivalent to the model (1.6) complemented with the standard York-Gibbons-Hawking boundary term.\(^93,94\) If Dirichlet boundary conditions are chosen for \( X, e^+, \) and \( e^- \) the second term in Eq. (1.88) is formulated exclusively in terms of fields fixed at the boundary. However, this term is essential to restore invariance under unrestricted Lorentz transformations.\(^p\)

In one of his last publications\(^97\) Wolfgang Kummer resumed the discussion of boundary terms in FOG from a quite different point of view. This work was motivated by results from Refs. 98,99 where it was argued that black hole entropy should emerge from Goldstone-like degrees of freedom that emerge from a symmetry breaking in the presence of a horizon.\(^q\) In these works a stretched horizon was imposed as a boundary, implemented by suitable boundary constraints. It was then found that these constraints break parts of the symmetry, which allowed to deduce the correct entropy by means of the Cardy formula.\(^100\) Since the formalism of Refs. 98,99 does not allow to impose sharp horizon constraints, it however remained open in which sense the stretched horizon really is a special choice of a boundary. In FOG the Eddington-Finkelstein type solutions are not singular at the horizon and thus the first order formulation provides the possibility to replace the stretched horizon in Refs. 98,99 by a true horizon. This led to the idea to study FOG with boundaries, once chosen as a generic boundary and once chosen as a horizon, and to compare these two situations.

In Ref. 97 the boundary was considered at a fixed value of \( x^1 \), whereby \( x^0 \) represents Hamiltonian time. This choice allowed to implement the specific values of the fields fixed at the boundary as boundary constraints, which then were introduced in the Hamiltonian analysis. Since the dilaton

\(^p\)In order to ensure a well-definedness of the semiclassical approximation and thus of thermodynamics of black hole spacetimes, further boundary counterterms, solely depending on the boundary values of the fields held fixed there, can be important. In the Euclidean approach these counterterms have been discussed in Refs. 95,96.

\(^q\)This is thought to be a spontaneous symmetry breaking happening in the full dynamical theory. However, because of the inability to treat the fully dynamical picture, the investigation of Carlip’s idea is done by implementing the symmetry breaking explicitly through boundary constraints.
is constant at the horizon, the first boundary constraint was chosen as
\( B_1[\eta] = (p_1 - \hat{p}_1)\eta|_{\partial M}. \) A generic boundary is determined by the two
additional constraints (\( \eta \) is a smearing function)
\[
B_2[\eta] = (\bar{q}_2 - E_0^{-}(x^0))\eta|_{\partial M}, \quad B_3[\eta] = (\bar{q}_3 - E_0^{+}(x^0))\eta|_{\partial M}, \quad (1.89)
\]
which turns all secondary constraints \( G^I \) into second class constraints. On-shell (1.89) with the choice \( E_0^{-}(x^0) \equiv 0 \) could be used to fix the boundary to be a horizon. However, off-shell this choice is problematic since the Killing
norm expressed in terms of target space variables, Eq. (1.11), need not
vanish and not surprisingly it was found that the Hamiltonian treatment
of (1.89) becomes singular at the horizon. Still, the first order formulation
offers a different set of horizon constraints, namely
\[
B_2[\eta] = \bar{q}_2\eta|_{\partial M}, \quad B_3[\eta] = p_3\eta|_{\partial M}, \quad (1.90)
\]
which removes all the problems encountered with (1.89). As an important
difference to the generic boundary it is now found that two of the three
secondary constraints, namely the Lorentz constraints and diffeomorphisms
along the boundary, remain first class. This picture was confirmed by constructing the reduced phase space. Both situations have zero physical
degrees of freedom in the bulk, but while a generic boundary exhibits one
pair of boundary degrees of freedom (which could be related to mass and proper time as previously found by Kuchař\[101\]), no boundary degrees of freedom are left at the horizon. This suggests a quite different picture
of black hole entropy.\[97,102\] The physical degrees of freedom present on a
generic boundary are converted into gauge degrees of freedom on a horizon
and entropy arises because approaching the black hole horizon does not
commute with constructing the physical phase space.

Already during the preparation of Ref. 97 it was realized by Wolfgang
Kummer and his collaborators that it could be advantageous to choose the
boundary at constant value of Hamiltonian time. Indeed, the extremely
complex constraint algebras emerging from the calculations above destroyed
any hopes to quantize the model along the lines of Sect. 1.1.3, not to mention
the impossibility to couple matter fields. Nonetheless, if in the Hamil-
tonian picture the boundary is rather chosen as initial or final values, the
canonical formalism is not affected at all. However, since for a spacelike
boundary the boundary values are no longer fixed via boundary constraints,
the necessary restrictions should be obtained from the “lost constraints,”\[v

\[v\text{Integrating out the } p_I \text{ reproduces only the equations of motion (1.24)-(1.25). There is another set of these equations with spatial rather than time-derivatives, which should} \]
which turned out to be difficult to tackle. Thus this line of investigations
was given up in favor of the picture presented in Ref. 97. Only recently, this
unfinished work was continued103 and it was shown that for the matterless
theory the result expected from previous works,97,101 namely the existence
of one boundary degree of freedom – the mass – was obtained. In particular,
fixing $e_{\pm}^k$ at the boundary instead of $X^k$ implies for the arguments of the
functional $\delta$ functions (1.24)-(1.25) not to evaluate the path integral com-
pletely, but rather leaving an integration over field boundary values uneval-
uated. Furthermore, additional contributions from the Gibbons-Hawking
boundary term made the evaluation the of quantum equations of motion
and the identification of the additional boundary degree of freedom possible.
The formalism presented in Ref. 103 thus may provide a way to finally
do a path integration over the remaining boundary degree of freedom – a
real “sum over boundary conditions” labeled by the mass of the spacetime.
Though relevant questions regarding the path integral remained open this
work, and will hopefully addressed in the future, it shows that Wolfgang
Kummer’s philosophy of the “Vienna School of dilaton gravity” remains a
powerful formalism which also in the future will provide deeper insight into
important questions of classical and quantum gravity.

Appendix A. Notations and conventions

Most of the notation follows the one used in Refs. 35,68, which should be
consulted for further explanations.

For indices of target-space coordinates and gauge fields the notation

\[ X^I = (X^i, X^\alpha) = (X^{\phi}, X^a, X^\alpha) = (\phi, X^a, \chi^\alpha), \]
\[ A_I = (A_i, A_\alpha) = (A_{\phi}, A_a, A_\alpha) = (\omega, e_a, \psi_\alpha), \]

\[ (A.1) \]
\[ (A.2) \]

i.e. capital Latin indices are generic, $i,j,k \ldots$ are bosonic, $a,b,c\ldots$ denote
the anholonomic coordinates and Greek indices are fermionic. The summa-
convention is always $NW \to SE$, e.g. for a fermion $\chi$: $\chi^2 = \chi^\alpha \chi_\alpha$. Our
conventions are arranged in such a way that almost every bosonic expres-
sion is transformed trivially to the graded case when using this summation
convention and replacing commuting indices by general ones. This is possi-
ble together with exterior derivatives acting from the right, only. Thus the

fix the remaining freedom in the choice of certain integration functions (depending on
$x^1$). These are the “lost constraints”, which play the role of Ward identities for the
diffeomorphism and local Lorentz invariance. See also Ref. 57 and references therein.
graded Leibniz rule is given by
\[ d(AB) = AdB + (-1)^B (dA)B . \]  
(A.3)

In terms of anholonomic indices the metric and the symplectic \(2 \times 2\) tensor are defined as
\[ \eta_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \epsilon_{ab} = -\epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \]  
(A.4)

The metric in terms of holonomic indices is obtained by
\[ g_{\alpha\beta} = \epsilon_a^b \epsilon_m^a \eta_{ab} \]  
and for the determinant the standard expression
\[ \epsilon = \det \epsilon_m^a = \sqrt{-\det g_{\alpha\beta}} \]  
is used. The volume form reads
\[ \epsilon = \frac{1}{2} \epsilon_{ab} \epsilon^b \wedge \epsilon^a ; \]  
by definition \(\ast \epsilon = 1\).

Covariant derivatives of anholonomic indices with respect to the geometric variables \(e_a = d_x^m e_{am}\) and \(\psi_\alpha = d_x^m \psi_{am}\) include the two-dimensional spin-connection one form \(\omega^{ab} = \omega^{a\beta} b\). When acting on lower indices the explicit expressions read
\[ (De)_a = d e_a + \omega^{ab} e_b \]  
\[ (D\psi)_\alpha = d \psi_\alpha - \frac{1}{2} \omega^{\alpha\beta} \psi_\beta \]  
(A.5)

For Majorana spinors in chiral representation,
\[ \chi^\alpha = (\chi^+, \chi^-) , \quad \chi_\alpha = \begin{pmatrix} \chi^+ \\ \chi^- \end{pmatrix} , \]  
(A.7)

upper and lower chiral components are related by \(\chi^+ = \chi_-, \chi^- = -\chi_+\), \(\chi^2 = \chi^\alpha \chi_\alpha = 2\chi_- \chi^+\). Vectors conveniently are used in the spin tensor decomposition \(v^{\alpha\beta} = \frac{1}{\sqrt{2}} \epsilon^{\alpha\beta} \gamma^3\). Due to the additional factor \(i\) the spin tensor components are related to standard light cone components as
\[ v^{++} = iv^{\oplus} , \quad v^{--} = -iv^{\ominus} , \]  
(A.8)

in particular the spin tensor components of a real vector are imaginary. This notation implies that \(\eta^{++} = 1, \epsilon^{--} = -\epsilon^{++} = -\epsilon^{++} = 1\) and for the \(\gamma\) matrices one finds
\[ (\gamma^{++})_\alpha^\beta = \sqrt{2i} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} , \quad (\gamma^{--})_\alpha^\beta = -\sqrt{2i} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} . \]  
(A.9)
References

1. W. Kummer and D. J. Schwarz, Classical and quantum theory of non-Einsteinian 2d-gravity, *Czech. J. Phys.* **41**, 13–22, (1991).
2. W. Kummer and D. J. Schwarz. NonEinsteinian gravity with torsion at d = 2. In *Strings and Symmetries, 1991: SUNY Stony Brook, 20-25 May 1991*, pp. 168–169. World Scientific, (1992).
3. W. Kummer and D. J. Schwarz, General analytic solution of R**2** gravity with dynamical torsion in two-dimensions, *Phys. Rev.* **D45**, 3628–3635, (1992).
4. W. Kummer and D. J. Schwarz, Renormalization of R**2** gravity with dynamical torsion in d = 2, *Nucl. Phys.* **B382**, 171–186, (1992).
5. M. O. Katanaev and I. V. Volovich, String model with dynamical geometry and torsion, *Phys. Lett.* **B175**, 413–416, (1986).
6. M. O. Katanaev and I. V. Volovich, Two-dimensional gravity with dynamical torsion and strings, *Ann. Phys.* **197**, 1, (1990).
7. M. O. Katanaev, Complete integrability of two-dimensional gravity with dynamical torsion, *J. Math. Phys.* **31**, 882, (1990).
8. W. Kummer, Ghost Free Nonabelian Gauge Theory, *Acta Phys. Austriaca.* **41**, 315–334, (1975).
9. H. Grosse, W. Kummer, P. Presnajder, and D. J. Schwarz, Nonlinear gauge symmetry in R**2** gravity in two-dimensions, *Czech. J. Phys.* **42**, 1325–1329, (1992).
10. W. Kummer and D. J. Schwarz, Two-dimensional R**2** gravity with torsion, *Class. Quant. Grav.* **10**, S235–S238, (1993).
11. H. Grosse, W. Kummer, P. Presnajder, and D. J. Schwarz, Novel symmetry of nonEinsteinian gravity in two-dimensions, *J. Math. Phys.* **33**, 3892–3900, (1992).
12. W. Kummer. Deformed ISO(2,1) symmetry and non-Einsteinian 2d-gravity with matter. In eds. D. Bruncko and J. Urban, *Hadron Structure '92*, (1992). Stara Lesna, Czechoslovakia.
13. H. Kawai and R. Nakayama, Quantum R**2** gravity in two-dimensions, *Phys. Lett.* **B306**, 224–232, (1993).
14. T. Strobl, Quantization and the issue of time for various two-dimensional models of gravity, *Int. J. Mod. Phys.* **D3**, 281–284, (1994).
15. D. Cangemi and R. Jackiw, Gauge invariant formulations of lineal gravity, *Phys. Rev. Lett.* **69**, 233–236, (1992).
16. H. Verlinde. Black holes and strings in two dimensions. In *Trieste Spring School on Strings and Quantum Gravity*, pp. 178–207 (April, 1991).
17. C. Teitelboim, Gravitation and Hamiltonian structure in two space-time dimensions, *Phys. Lett.* **B126**, 41, (1983).
18. R. Jackiw. Liouville field theory: a two-dimensional model for gravity. In ed. S. Christensen, *Quantum theory of gravity : essays in honor of the 60th birthday of Bryce S.DeWitt*, pp. 327–344, Bristol, (1984). Hilger.
19. N. Ikeda and K. I. Izawa, Quantum gravity with dynamical torsion in two-dimensions, *Prog. Theor. Phys.* **89**, 223–230, (1993).
20. N. Ikeda and K. I. Izawa, Gauge theory based on quadratic Lie algebras and 2-d gravity with dynamical torsion, Prog. Theor. Phys. **89**, 1077–1086, (1993).
21. D. Grumiller, W. Kummer, and D. V. Vassilevich, Dilaton gravity in two dimensions, Phys. Rept. **369**, 327, (2002).
22. W. Kummer and P. Widerin, NonEinsteinian gravity in d=2: Symmetry and current algebra, Mod. Phys. Lett. **A9**, 1407–1414, (1994).
23. M. O. Katanaev, Canonical quantization of the string with dynamical geometry and anomaly free nontrivial string in two-dimensions, Nucl. Phys. **B416**, 563–605, (1994).
24. W. Kummer. Exact classical and quantum integrability of R**2 + T**2 gravity in (1+1) dimensions. In eds. J. Carr and M. Perrottet, *International Europhysics Conference On High-Energy Physics (HEP 93)*. Editions Frontieres, (1993).
25. F. Haider and W. Kummer, Quantum functional integration of nonEinsteinian gravity in d = 2, Int. J. Mod. Phys. **A9**, 207–220, (1994).
26. P. Schaller and T. Strobl, Canonical quantization of non-Einsteinian gravity and the problem of time, Class. Quant. Grav. **11**, 331–346, (1994).
27. W. Kummer. Unified treatment of all 1+1 dimensional gravitation models. In eds. J. Lemonne, C. Vander Velde, and F. Verbeure, *International Europhysics Conference On High Energy Physics (HEP 95)*. World Scientific, (1995).
28. W. Kummer and P. Widerin, Conserved quasilocal quantities and general covariant theories in two-dimensions, Phys. Rev. **D52**, 6965–6975, (1995).
29. W. Kummer. General treatment of all 2d covariant models. In ed. S. Moskalik, *12th Hutsalian Workshop On Methods Of Mathematical Physics*. Hadronic Press, (1995).
30. M. O. Katanaev, W. Kummer, and H. Liebl, Geometric Interpretation and Classification of Global Solutions in Generalized Dilaton Gravity, Phys. Rev. **D53**, 5609–5618, (1996).
31. M. O. Katanaev, W. Kummer, and H. Liebl, On the completeness of the black hole singularity in 2d dilaton theories, Nucl. Phys. **B486**, 353–370, (1997).
32. W. Kummer and S. R. Lau, Boundary conditions and quasilocal energy in the canonical formulation of all 1 + 1 models of gravity, Annals Phys. **258**, 37–80, (1997).
33. T. Strobl. *Poisson structure induced field theories and models of 1+1 dimensional gravity*. PhD thesis, Technische Universität Wien, (1994).
34. T. Strobl. Gravity in two spacetime dimensions. Habilitation thesis, (1999).
35. M. Ertl, W. Kummer, and T. Strobl, General two-dimensional supergravity from Poisson superalgebras, JHEP. **01**, 042, (2001).
36. W. Kummer, H. Liebl, and D. V. Vassilevich, Exact path integral quantization of generic 2-d dilaton gravity, Nucl. Phys. **B493**, 491–502, (1997).
37. H. Balasin, C. G. Boehmer, and D. Grumiller, The spherically symmetric standard model with gravity, Gen. Rel. Grav. **37**, 1435–1482, (2005).
38. T. Klösch and T. Strobl, Classical and quantum gravity in (1+1)-
dimensions. Part 1: A unifying approach, *Class. Quant. Grav.* **13**, 965–984, (1996).
39. T. Klösch and T. Strobl, Classical and quantum gravity in 1+1 dimensions, part II: The universal coverings, *Class. Quant. Grav.* **13**, 2395–2422, (1996).
40. N. Ikeda, Two-dimensional gravity and nonlinear gauge theory, *Ann. Phys.* **235**, 435–464, (1994).
41. P. Schaller and T. Strobl, Poisson sigma models: A generalization of 2-d gravity Yang- Mills systems. In *Finite dimensional integrable systems*, pp. 181–190, (1994). Dubna.
42. P. Schaller and T. Strobl, Poisson structure induced (topological) field theories, *Mod. Phys. Lett.* **A9**, 3129–3136, (1994).
43. W. Kummer, H. Liebl, and D. V. Vassilevich, Non-perturbative path integral of 2d dilaton gravity and two-loop effects from scalar matter, *Nucl. Phys.* **B513**, 723–734, (1998).
44. W. Kummer, H. Liebl, and D. V. Vassilevich, Integrating geometry in general 2d dilaton gravity with matter, *Nucl. Phys.* **B544**, 403–431, (1999).
45. P. Fischer, D. Grumiller, W. Kummer, and D. V. Vassilevich, S-matrix for s-wave gravitational scattering, *Phys. Lett.* **B521**, 357–363, (2001). Erratum ibid. **B532** (2002) 373.
46. D. Grumiller, Virtual black hole phenomenology from 2d dilaton theories, *Class. Quant. Grav.* **19**, 997–1009, (2002).
47. L. Bergamin. Quantum dilaton supergravity in 2d with non-minimally coupled matter. In eds. P. Fiziev and M. Todorov, *Gravity, Astrophysics, and Strings @ the Black Sea*, pp. 17–28, Sofia, (2005). St.Kliment Ohridski University Press.
48. D. Grumiller and R. Meyer, Quantum dilaton gravity in two dimensions with fermionic matter, *Class. Quant. Grav.* **23**, 6435–6458, (2006).
49. R. Meyer. Constraints in two-dimensional dilaton gravity with fermions. To appear in the proceedings of International V.A. Fock School of Advances of Physics (IFSAP 2005), St. Petersburg, Russia, 21–27 Nov 2005., (2005).
50. R. Meyer. Classical and quantum dilaton gravity in two dimensions with fermions. Master’s thesis, (2006).
51. R. Meyer. Quantizing two-dimensional dilaton gravity with fermions: The Vienna way. In eds. H. Kleinert and R. Jantzen, *The eleventh Marcel Grossman meeting*. World Scientific, (2008).
52. L. Bergamin and W. Kummer, Graded Poisson sigma models and dilaton-deformed 2d supergravity algebra, *JHEP* **05**, 074, (2003).
53. L. Bergamin, D. Grumiller, and W. Kummer, Quantization of 2d dilaton supergravity with matter, *JHEP* **05**, 060, (2004).
54. E. S. Fradkin and G. A. Vilkovisky, Quantization of relativistic systems with constraints, *Phys. Lett.* **B55**, 224, (1975).
55. E. S. Fradkin and T. E. Fradkina, Quantization of relativistic systems with boson and fermion first and second class constraints, *Phys. Lett.* **B72**, 343, (1978).
56. I. A. Batalin and G. A. Vilkovisky, Relativistic S matrix of dynamical systems with boson and fermion constraints, *Phys. Lett.* **B69**, 309–312, (1977).
57. D. Grumiller. Quantum dilaton gravity in two dimensions with matter. PhD thesis, Technische Universität Wien, (2001).
58. D. Grumiller, W. Kummer, and D. V. Vassilevich, Virtual black holes in generalized dilaton theories and their special role in string gravity, Eur. Phys. J. C30, 135–143, (2003).
59. D. Grumiller, W. Kummer, and D. V. Vassilevich, The virtual black hole in 2d quantum gravity, Nucl. Phys. B580, 438–456, (2000).
60. D. Grumiller, Virtual black holes and the S-matrix, Int. J. Mod. Phys. D13, 1973–2002, (2004).
61. D. Grumiller and R. Meyer, Ramifications of lineland, Turk. J. Phys. 30, 349–378, (2006).
62. P. S. Howe, Super Weyl transformations in two-dimensions, J. Phys. A12, 393–402, (1979).
63. Y.-C. Park and A. Strominger, Supersymmetry and positive energy in classical and quantum two-dimensional dilaton gravity, Phys. Rev. D47, 1569–1575, (1993).
64. D. Grumiller. Three functions in dilaton gravity: The good, the bad and the muggy. Lectures given at 14th International Hutsulian Workshop on Mathematical Theories and their Physical and Technical Applications (Timpani - Mathyphys 2002), Chernivtsi, Ukraine, (2002).
65. A. Bilal, Positive energy theorem and supersymmetry in exactly solvable quantum corrected 2-d dilaton gravity, Phys. Rev. D48, 1665–1678, (1993).
66. S. Nojiri and I. Oda, Dilatonic supergravity in two-dimensions and the disappearance of quantum black hole, Mod. Phys. Lett. A8, 53–62, (1993).
67. M. F. Ertl, M. O. Katanaev, and W. Kummer, Generalized supergravity in two dimensions, Nucl. Phys. B530, 457–486, (1998).
68. M. Ertl. Supergravity in two spacetime dimensions. PhD thesis, Technische Universität Wien, (2001).
69. V. O. Rivelles, Topological two-dimensional dilaton supergravity, Phys. Lett. B321, 189–192, (1994).
70. D. Cangemi and M. Leblanc, Two-dimensional gauge theoretic supergravities, Nucl. Phys. B420, 363–378, (1994).
71. N. Ikeda, Gauge theory based on nonlinear Lie superalgebras and structure of 2-d dilaton supergravity, Int. J. Mod. Phys. A9, 1137–1152, (1994).
72. J. M. Izquierdo, Free differential algebras and generic 2d dilatonic (super)gravities, Phys. Rev. D59, 084017, (1999).
73. T. Strobl, Target-superspace in 2d dilatonic supergravity, Phys. Lett. B460, 87–93, (1999).
74. L. Bergamin, Generalized complex geometry and the Poisson sigma model, Mod. Phys. Lett. A20, 985–996, (2005).
75. I. Calvo, Supersymmetric WZ-Poisson sigma model and twisted generalized complex geometry, Lett. Math. Phys. 77, 53–62, (2006).
76. T. Strobl, Gravity from lie algebroid morphisms, Commun. Math. Phys. 246, 475–502, (2004).
77. M. Adak and D. Grumiller, Poisson-sigma model for 2D gravity with non-metricity, Class. Quant. Grav. 24, F65, (2007).
78. D. Z. Freedman, P. van Nieuwenhuizen, and S. Ferrara, Progress toward a
theory of supergravity, Phys. Rev. D13, 3214–3218, (1976).
79. D. Z. Freedman and P. van Nieuwenhuizen, Properties of supergravity the-
ory, Phys. Rev. D14, 912, (1976).
80. S. Deser and B. Zumino, Consistent supergravity, Phys. Lett. B62, 335,
(1976).
81. R. Grimm, J. Wess, and B. Zumino, Consistency checks on the superspace
formulation of supergravity, Phys. Lett. B73, 415, (1978).
82. L. Bergamin and W. Kummer, The complete solution of 2d superfield su-
pergravity from graded Poisson-sigma models and the super pointparticle,
Phys. Rev. D68, 104005, (2003).
83. L. Bergamin, D. Grumiller, and W. Kummer, Supersymmetric black holes
in 2-d dilaton supergravity: baldness and extremality, J. Phys. A37, 3881–
3901, (2004).
84. G. W. Gibbons and C. M. Hull, A Bogomolny bound for general relativity
and solitons in N=2 supergravity, Phys. Lett. B109, 190, (1982).
85. K. P. Tod, All metrics admitting supercovariantly constant spinors, Phys.
Lett. B121, 241–244, (1983).
86. W. M. Nelson and Y. Park, N=2 supersymmetry in two-dimensional dilaton
gravity, Phys. Rev. D48, 4708–4712, (1993).
87. L. Bergamin and W. Kummer, Two-dimensional N=(2,2) dilaton super-
gravity from graded Poisson-sigma models I: Complete actions and their
symmetries., Eur. Phys. J. C39, S41–S52, (2005).
88. L. Bergamin and W. Kummer, Two-dimensional N = (2,2) dilaton super-
gravity from graded Poisson-sigma models. II: Analytic solution and BPS
states, Eur. Phys. J. C39, S53–S63, (2005).
89. P. A. M. Dirac, Lectures on Quantum Mechanics. (Belfer Graduate School
of Science, Yeshiva University, New York, 1996).
90. M. Rocek, P. van Nieuwenhuizen, and S. C. Zhang, Superspace path integral
measure of the n=1 spinning string, Ann. Phys. 172, 348, (1986).
91. U. Lindstrom, N. K. Nielsen, M. Rocek, and P. van Nieuwenhuizen, The
supersymmetric regularized path integral measure in x spacce, Phys. Rev.
D37, 3588, (1988).
92. J. C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Evanscent
black holes, Phys. Rev. D45, 1005–1009, (1992).
93. J. W. J. York, Role of conformal three geometry in the dynamics of gravita-
tion, Phys. Rev. Lett. 28, 1082–1085, (1972).
94. G. W. Gibbons and S. W. Hawking, Action integrals and partition functions
in quantum gravity, Phys. Rev. D15, 2752–2756, (1977).
95. D. Grumiller and R. McNees, Thermodynamics of black holes in two (and
higher) dimensions, JHEP. 04, 074, (2007).
96. L. Bergamin, D. Grumiller, R. McNees, and R. Meyer, Black Hole Thermo-
dynamics and Hamilton-Jacobi Counterterm, J. Phys. A41, 164068, (2008).
97. L. Bergamin, D. Grumiller, W. Kummer, and D. V. Vassilevich, Physics-to-
gauge conversion at black hole horizons, Class. Quant. Grav. 23, 3075–3101,
(2006).
98. S. Carlip, Horizon constraints and black hole entropy, *Class. Quant. Grav.* **22**, 1303–1312, (2005).
99. S. Carlip, Horizon constraints and black hole entropy, (2005).
100. J. L. Cardy, Operator Content of Two-Dimensional Conformally Invariant Theories, *Nucl. Phys.* **B270**, 186–204, (1986).
101. K. V. Kuchař, Geometrodynamics of Schwarzschild black holes, *Phys. Rev.* **D50**, 3961–3981, (1994).
102. L. Bergamin and D. Grumiller, Killing horizons kill horizon degrees, *Int. J. Mod. Phys.* **D15**, 2279–2284, (2006).
103. L. Bergamin and R. Meyer. Two-dimensional quantum gravity with boundary. In eds. P. Fiziev and M. Todorov, *Gravity, Astrophysics, and Strings @ the Black Sea*. St.Kliment Ohridski University Press, (2008).