A SIMPLE MODEL FOR r-PROCESS SCATTER AND HALO EVOLUTION

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ABSTRACT

Recent halo-star observations of heavy elements produced by rapid neutron capture (r-process) show a striking behavior: within a single star, the relative abundances of r-process elements heavier than Ba are the same as those of solar system matter, while across stars with similar metallicity Fe/H, the r/Fe ratio varies by more than 2 orders of magnitude. In this paper we present a simple analytic model that describes a star’s abundances in terms of its “ancestry,” i.e., the number of nucleosynthesis events (e.g., supernova explosions) that contributed to the star’s composition. This model leads to a very simple analytic expression for the abundance scatter versus Fe/H, which is in good agreement with the data and with more sophisticated numerical models. We investigate two classes of scenarios for r-process nucleosynthesis, one in which r-process events occur in only ~4% of supernovae but iron synthesis is ubiquitous, and one in which iron nucleosynthesis occurs in only about 9% of supernovae (the Wasserburg-Qian model). We find that the predictions in these scenarios are similar for [Fe/H] ≥ −2.5, but that these models can be readily distinguished observationally by measuring the dispersion in r/Fe at [Fe/H] ≤ −3.

Subject headings: Galaxy: abundances — nuclear reactions, nucleosynthesis, abundances — stars: abundances

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1. INTRODUCTION

Neutron capture processes dominate the nucleosynthesis of elements beyond the iron peak. The physics of two neutron capture mechanisms has long been understood, and the astrophysical site for slow neutron capture nucleosynthesis (the s-process) has been shown to be intermediate-mass asymptotic giant branch stars (e.g., Busso, Gallino, & Wasserburg 1999). However, the site for rapid neutron capture nucleosynthesis (the r-process) has not yet been unambiguously identified, although it is very likely connected to massive stars. The production site itself might be found in some or all Type II supernova events (e.g., Woosley et al. 1994) or in binary neutron star mergers (e.g., Eichler et al. 1989; Rosswog et al. 2000). Truran (1981) showed that halo stars are a particularly useful laboratory for the study of the r-process, as the observed neutron capture abundance patterns in these stars indicate that the s-process component drops out and the r-process dominates, as one looks at stars with [Fe/H] ≤ −2.2 (Burris et al. 2000). Recent years have shown that halo stars indeed give unique insight into the r-process. With the advent of high-dispersion, high signal-to-noise ratio (S/N) measurements, the abundance observations within and among halo stars has led to surprising new discoveries with important consequences for theories of the r-process as well as halo formation.

The halo star with perhaps the most striking r-process composition is CS 22982-052. With [Fe/H] = −3.1, this is an ultra–metal poor star; its abundance ratios through the iron peak are typical for a halo star. However, this star has also been observed in 20 r-process elements from barium upward (the “heavy” r-process), with abundances (Sneden et al. 1996, 2000)

$$\left[ \frac{r}{Fe} \right] \sim 1.7 \Rightarrow \frac{r}{Fe} \sim 50 \left( \frac{r}{Fe} \right)_\odot \tag{1}$$

The element-to-element scatter in this ratio is consistent with the observational errors. This remarkable trend does not appear to hold for the “light” r-process, 40 < Z < 56 (Sneden et al. 2000); this might be related to meteoritic evidence for two r-process components (Wasserburg, Busso, & Gallino 1996; Wasserburg & Qian 2000b). Nevertheless, the fact that the heavy r-process elements agree so well with each other is stunning, given that this star could well have abundances that reflect the nucleosynthesis of a single supernova, while the solar abundances average over many generations of supernovae. The strong implication is that there is a unique astrophysical site that dominates heavy r-process nucleosynthesis.

1 In the standard notation, \([A/B] = \log(A/B) - \log(A/B)_\odot\); we take ultra–metal poor stars to be those with [Fe/H] < −2.5.
This result has profound implications. First, the mere presence of the r-process in very metal poor stars points to origins associated with massive stars. Moreover, given the apparent universality of the heavy r-process, one might expect all ratios among heavy r-process nuclides to always be constant (with pollution by the s-process increasing with [Fe/H]). This leads to the prediction that at the lowest metallicities, when the s-process has not turned on, ratios among heavy r-process elements should be fixed, and the same across all stars. So far, this appears to be true in the other ultra-metal poor star for which there are similarly good data, HD 115444 (Westin et al. 2000), which has \([\text{Fe}/\text{H}] = -3.0\) and \([r/\text{Fe}] = 0.96\). Very recently, Cayrel et al. (2001) have shown that CS 31082-001 (with \([\text{Fe}/\text{H}] = -2.91\)) has Os/Fe and Ir/Fe abundances that give \([r/\text{Fe}] = 1.98\) and thus \(r/\text{Fe} = 96\). This would be the highest r-process overabundance yet observed. Clearly, a systematic study of r-process abundance patterns in this star is of the highest priority (particularly since this is the first star in which uranium has been detected).

CS 22892-052 and HD 115444 have iron abundances that are identical within errors, and both show a remarkable constancy in their \(r/\text{Fe}\) ratios for different r-elements within each star. However, these stars show a significant difference in their mean \(r/\text{Fe}\) ratios (see Fig. 1). This is an example of the large scatter in \(r/\text{Fe}\) ratios that has been observed in halo stars. As seen in Figure 1, the dispersion in \(r/\text{Fe}\) ratios is small at \([\text{Fe}/\text{H}] \lesssim -1\) but grows with decreasing metallicity, finally spanning 2 orders of magnitude for ultra-metal poor stars. The observed large scatter in \(r/\text{Fe}\) in Population II (Pop II) stars, and the very high \(r/\text{Fe}\) in a few, demands that not all stars make both the heavy r-process elements and iron.

Models for heavy r-process nucleosynthesis and for Pop II chemical evolution must account for the remarkable constancy of \(r/\text{Fe}\) among different elements within individual stars, as well as for the variability of \(r/\text{Fe}\) between stars. These facts together suggest the following simple picture for Pop II \(r/\text{Fe}\). We assume that the observed Pop II r-process abundances in each star reflect heavy r-process contributions of a few supernovae. The basic ideas are the following:

1. Different r-process-to-iron ratios arise from different mixing between the dominant heavy r-process sources and the dominant Fe sources.
2. The scatter in r-process-to-iron then reflects the amount of mixing between the two sources.

The key is the inhomogeneity of the halo, which this scheme in fact quantifies.

This scenario makes quantitative predictions:

1. There is a maximum to \(r/\text{Fe}\), namely, the production ratio in those stars that produce heavy r-process elements, undiluted by any (r-poor) Fe events
2. The minimum to the \(r/\text{Fe}\) ratio is that produced by the r-poor events. This minimum may be as small as \(r/\text{Fe} = 0\).
3. The placement of a star between these extremes quantifies an admixture of nucleosynthesis sources recorded in the star. Thus, (a) at the very earliest times, when single supernova events are really all that contaminate a given star, the \(r/\text{Fe}\) scatter only populates the extremes, (b) at later times, but before mixing is efficient, the regions between the extremes will be filled in, and (c) at still later times, mixing becomes efficient, and then the scatter decreases toward a “universal” mix of the two sources.

If this line of reasoning is correct, it has the very significant implication that \(r/\text{Fe}\) becomes a tracer of the inhomogeneity of the halo. One can see the transition from single to multiple events and also the transition from multiple sources to a well-mixed Galaxy. The details of these transitions allow one to correlate metallicity and mixing, and we can at least grossly confirm the time sequence of the nucleosynthesis events.

Several groups have used similar arguments to motivate detailed models that explain the observed \(r/\text{Fe}\) (specifically, Eu/Fe) scatter in terms of r-process nucleosynthesis and an inhomogeneous chemical evolution of the Galactic halo. Ishimaru & Wanjaro (1999) construct a one-zone model for the halo but introduce abundance dispersion through a Monte Carlo realization of individual supernova events with yields of Fe and Eu that are strong functions of mass. They obtain good fits to the observed scatter for models in which the heavy r-process is produced by a small fraction of supernovae, either in a very narrow range \((8–10 M_\odot)\) at the low-mass end or in a larger range at the high-mass end. Travaglio, Galli, & Burkert (2001a) create a stochastic model and use this to generate Monte Carlo realizations of a number of elements, to include several r-process species. In their model, r-progenitors in the \(8–10 M_\odot\) range give a good fit to the data, but progenitors in the \(15–30 M_\odot\) range overproduce Eu/Fe at low metallicity. Argast et al. (2000) use the Eu yields of Tsujimoto & Shigeyama (1998), which favor lower r-process progenitor masses, but drop the assumption of one-zone evolution. They model the spatial inhomogeneity and incomplete mixing of expanding supernova remnants in the halo and again find good agreement.
with the observed Eu/Fe dispersion. We follow these groups in using Eu as a tracer of the r-process.2

The goal of the present paper is to aid in the understanding of these detailed models and to help focus attention on the key physics relevant for this problem. We show (§ 2) that the r/Fe scatter follows quite generally from the stochastic behavior of the halo chemical evolution and that one can derive simple, explicit, analytical expressions for the abundance scatter as a function of metallicity. We do this for two scenarios, one in which the heavy r-process occurs in only a small fraction of supernovae but Fe is ubiquitous (§ 2.1), and the scenario of Wasserburg & Qian (2000a, hereafter WQ00), in which the converse holds (§ 2.2). Fortunately, these scenarios can be distinguished by future measurements of r/Fe scatter in ultra-metal poor stars, and we present observational strategies to do this (§ 3). Discussion and conclusions appear in § 4.

2. TWO SCENARIOS FOR THE HEAVY r-PROCESS ORIGIN

The observed scatter in heavy r/Fe ratios demands that heavy r-process elements and iron not be coproduced in the same ratios in the same events. Rather, there must be r-rich and r-poor events. Also, very high and very low r/Fe ratios are observed. The highest r/Fe event observed places a lower limit on the r/Fe yields of the r-rich events. Furthermore, the fact that the highest r/Fe is much larger than the Pop II mean implies that the r-rich, high-r/Fe events occur at a very different rate from low-r/Fe events. One can envision either “iron-dominated” schemes in which high-r/Fe events are rare, but iron-producing events are not, or alternatively, the possibility that heavy r-process events are the more common, with iron-producing events more rare. We now consider each of these scenarios in turn.

2.1. Iron-dominated Models

Our model has two basic simplifying assumptions. First, we assume that the supernova heavy r-process yields are bimodal, i.e., that some supernovae have a high heavy r-process yield and others a low one (possibly zero). For each supernova, we have r-ejecta of mass $m_{r}(m)$. We denote the objects with the larger r/Fe ratio as class A and those with the lower ratio as class B. Thus, we have, averaging over the portion of the initial mass function (IMF) governing each population, r-process yields $m_{r,a}$ and $m_{r,b}$.

Second, we assume that all supernovae make iron, and furthermore, that the Fe yield is the same constant value for all supernovae.3 Thus, we have an ejected iron mass for one event of $m_{Fe}(m) = m_{Fe} = \text{const}$. While this is not precisely true, the calculated yields (see, e.g., Woosley & Weaver 1995) for progenitor masses in the range $10–30 \, M_{\odot}$ (more massive stars are both fewer in number and more likely to yield a black hole than neutron star remnants) typically vary over a narrow mass range of the order of $0.1–0.2 \, M_{\odot}$ per event (while the oxygen yields can increase significantly with progenitor mass). The relative constancy of the Fe yields can be understood on the basis of the fact that the postshock conditions of temperature and density immediately above the mass cut (beyond the outer regions of the degenerate iron core) are themselves relatively insensitive to the mass of the progenitor. The implied factor of the order of 2 scatter in the iron yields with mass would neither alter the general character of our results for r/Fe nor change the conclusions we draw from this study.

Moreover, it is important to note that in general, a strong variation of Fe with progenitor mass would lead to a larger variance in r/Fe than we calculate. In this sense, our assumptions yield a lower limit on the predicted scatter. Thus, while it is clearly an idealization to assume a strictly constant iron yield, it has the virtues of being simple and conservative, while not departing far from detailed model calculations.

With these assumptions, supernova ejecta can take one of two values of r/Fe. For type A events,

$$\left(\frac{r}{Fe}\right) = \frac{m_{r,a}/A_{r}}{m_{Fe}/A_{Fe}},$$

(2)

where $A_{r}$ is the mean mass number of the $r$-process species and $A_{Fe} \approx 56$ is that for iron. A similar expression holds for type B events. To simplify notation, we define the scaled r/Fe ratio to be

$$R = \frac{r/Fe}{(r/Fe)}_{\odot},$$

(3)

which implies that $[r/Fe] = \log R$.

Thus, a supernova will produce a fixed amount of iron and will yield r-elements at either the high value $R_{a}$ or the low value $R_{b}$. Denote the fraction of class A supernovae (averaged over the halo IMF) to be $f_{a}$; then the fraction of class B supernovae is of course $f_{b} = 1 - f_{a}$.

The heavy r/Fe observations at a metallicity [Fe/H] $\sim -1$ (Edvardsson et al. 1993; Woolf, Tompkin, & Lambert 1995; Jehin et al. 1999) converge to a value $[r/Fe] \sim 0.3$, i.e., $(R) \sim 2$.4 We take this value as the mean $R$ attained when averaging over a large ensemble of progenitors, populated to reflect the underlying $f_{a}$ and $f_{b}$ dictated by the IMF.

To fix the values of the model parameters $f_{a}$ and $R_{b}$, consider the bulk composition of material that has been supernova enriched. For now, we are interested in the average (well-mixed) composition of the matter; in the next section,

2 It is worth noting that studies of Ba/Fe scatter have been made as well (e.g., Raiteri et al. 1999; Tsujimoto, Shigeyama, & Yoshii 1999). These analyses are complicated by the fact that solar Ba is dominated by s-process production. Of course, at early times, one expects to see only the small, residual r-process component. Indeed, this is apparent in the data of the burst of Burris et al. (2000), but these data also show that the s-process component for Ba already begins to dominate at [Fe/H] $\sim -2.5$. Also, due to hyperfine and isotopic splitting, Ba abundances in very metal poor stars have large uncertainties and are very dependent on assumed isotopic abundance fractions and atmospheric models (e.g., the microturbulent velocity; see Sneden, Cowan, & Truran 2001). In contrast, europium is always produced predominantly by the r-process. Furthermore, there are significant observational difficulties associated with using barium to map Galactic chemical evolution (see discussion in Burris et al. 2000). For these reasons, in this paper we take Eu as our tracer of the r-process.

3 It is possible that some supernovae make neither heavy r nor Fe. Even if such events exist, they are irrelevant to the r/Fe ratios but do in principle affect the r/Fe and Fe/H ratios via dilution. In practice, however, supernovae account for very little of the dilution of matter, and thus r-process and Fe-free events should not have a significant impact on the model.

4 Above this metallicity, r/Fe decreases, but this is presumably due to the addition of the Fe yields of Type Ia supernovae. We do not include these events (and implicitly assume their heavy r-process yield to be zero); thus, we do not attempt to model r/Fe at [Fe/H] $> -1$. 
we compute deviations from the mean. Let the total number of supernova events be \( N = N_a + N_b \). Of these, on average there are \( N_a = f_a N \) events of type A, and \( N_b = f_b N \) of type B. Thus, we have that

\[
\langle R \rangle = \frac{M_r / A_r}{M_{Fe} / A_{Fe}} = \frac{(f_a m_{r,a} + f_b m_{r,b}) / A_r}{m_{Fe} / A_{Fe}} \tag{4}
\]

\[
f_a R_a + f_b R_b = R_0 + f_a (R_a - R_b) . \tag{5}
\]

We can use the observed r-process data to estimate the values of \( R_a \) and \( R_b \). We take the highest observed relative admixture of class A and B supernovae is now fixed. Equation (5) gives

\[
f_a = \frac{\langle R \rangle - R_b}{R_0 - R_b} \approx \frac{\langle R \rangle}{R_0} , \tag{7}
\]

where the last expression uses the observed fact that \( R_0/\langle R \rangle \ll 1 \). For \( \langle R \rangle = 2 \) and \( R_0 = 50 \), equation (7) gives \( f_a = 3.5\% \) for \( R_b = 0.25 \), and \( f_a = 4.0\% \) for \( R_b = 0 \). Thus, we see that the observed fact of the very large \( r/Fe \) variations immediately implies that class A events are required to be uncommon, regardless of their physical nature. That is, the events that produce the heavy r-process elements originate from a small fraction, \( \sim 4\% \), of all massive stars.

2.1.1. Basic Model

With the model parameters fixed by observations, we are now in the position to model the scatter of \( r/Fe \). To do this, we create a set of halo stars and deduce the history (the nucleosynthetic ancestry) of each. We assume that each of the halo stars we create incorporates gas that has been enriched by some number \( N_{SN} \) of supernova of either type: this is the number of supernova ancestors for the star. We create stars by allowing \( N_{SN} \) to run from 1 to \( N_{max} \) and sample equal intervals in \( N_{SN} \). Note that we are free to chose the number of halo stars that we create and that this number is unrelated to the number of supernova ancestors assigned to a given halo star.

Note also that this approach is rather different from the traditional chemical-evolution modeling (e.g., Pagel 1997) that follows the time evolution of matter and its composition, so that a star’s composition reflects the gas composition at the time of its birth and thus is connected explicitly to the time history of star formation. Here, a star’s composition is set by its ancestry, which is determined from a procedure that does not explicitly make reference to time. We now describe this procedure.

The metallicity of a star follows from the assumption that all stars have the same iron yield, so that \( M_{Fe} = m_{Fe} N_{SN} \). We take the observed relative abundance \( [r/Fe] \sim 1.7 \) (eq. [1]) as a measure of \( R_0 \). On the other hand, the lower limit of the \( r/Fe \) scatter is not as well determined. The available data suggests that the minimum, if any, differs depending on the elemental \( r \)-tracer used. At most, we have \( [r/Fe]_{min} \sim -0.6 \), giving \( R_0 \approx 0.25 \); it is also possible that \( [r/Fe]_{min} \sim 1.7 \), giving \( R_0 = 0.02 \) at upper limits to \( r/Fe \) have not been reported, but this may be a selection effect. A key issue is whether there could have been lower \( r/Fe \) values than the ones reported; such information places useful constraints on heavy r-process production. For our purposes, as long as \( R_b \ll \langle R \rangle \), then we can take \( R_0 = 0 \), as we see below.

Note that in our model, the Eu yield is

\[
m_{Eu,a} = 152 R_a \left( \frac{Eu}{Fe} \right) _\odot m_{Fe} \approx 5 \times 10^{-7} M_\odot \tag{6}
\]

using the SN 1987A iron yield \( m_{Fe} = 0.07 M_\odot \) (e.g., Woosley 1988). This Eu yield is in good agreement with, e.g., the value adopted by Travaglio et al. (2001a).

Once one establishes the observed values for \( \langle R \rangle, R_a, \) and \( R_b \), the relative admixture of class A and B supernovae is now fixed. Equation (5) gives

\[
f_a = \frac{\langle R \rangle - R_b}{R_0 - R_b} \approx \frac{\langle R \rangle}{R_0} , \tag{7}
\]

where the last expression uses the observed fact that \( R_0/\langle R \rangle \ll 1 \). For \( \langle R \rangle = 2 \) and \( R_0 = 50 \), equation (7) gives \( f_a = 3.5\% \) for \( R_b = 0.25 \), and \( f_a = 4.0\% \) for \( R_b = 0 \). Thus, we see that the observed fact of the very large \( r/Fe \) variations immediately implies that class A events are required to be uncommon, regardless of their physical nature. That is, the events that produce the heavy r-process elements originate from a small fraction, \( \sim 4\% \), of all massive stars.

5 We assume this hold whenever both classes A and B exist; below we consider the effect when there is a time delay before the onset of one class. The constancy of \( r/Fe \) can be determined observationally. Current data samples the low metallicities too sparsely to make a strong statement (Burris et al. 2000), but with more observations, one should be able to test for a metallicity dependence of \( r/Fe \).
observationally determined constant (eq. [7]). For an individual star, the fraction of class A (high-$r$) supernovae $f_{A,*} = N_{A,*}/N_{SN}$ is also a random number, as is its $r$/Fe value:

$$R_* = f_{A,*} R_a + f_{b,*} R_b = R_0 + (R_a - R_b) \frac{N_{A,*}}{N_{SN}}. \tag{9}$$

The dispersion in $N_{A,*}$ naturally induces a dispersion in $R_*$:

$$\sigma(R_*) \approx (R_a - R_b) \frac{\sqrt{N_{A,*}}}{N_{SN}} \approx \frac{\langle R \rangle}{\sqrt{N_a}}\left(\frac{f_a}{f_{Fe \text{min}}} \right)^{-1/2} \langle R \rangle, \tag{10}$$

where we have successively assumed that $f_a \approx 1$ and $R_b \ll \langle R \rangle$. Thus, we see that the fractional dispersion in $R$ is just given by the counting statistics of $N_{A,*}$: $\sigma(R_*)/(R_*) \approx 1/N_a^{1/2}$. We now can put each star on the ($R$, [Fe/H]-plane, and the model is complete.

This model thus has three free parameters that can be fixed by observations: $\langle R \rangle$, $R_a$, and $R_b$; there is one parameter $f_{Fe \text{min}}$ that is (for now) determined by theory. We can rewrite equation (10) explicitly in terms of these parameters,

$$\sigma_R = \left(\frac{f_a}{f_{Fe \text{min}}} \right)^{-1/2} \langle R \rangle, \tag{11}$$

and thus

$$\sigma_{r/Fe} = \frac{1}{f_{Fe \text{min}}^{1/2} \ln 10} 10^{-(|\text{Fe/H}| - |\text{Fe/H}|_{\text{min}})/2} = 10^{-|(\text{Fe/H}) + 3.3)/2}. \tag{12}$$

The model thus predicts that the intrinsic scatter in $r$/Fe falls as $(\text{Fe/H})^{-1/2}$. This is indeed consistent with the observed scatter in halo-star data, as seen in Figure 1. Note that the envelope includes both the observational errors as well as the intrinsic scatter given by equation (11).

Of course, the data also contain observational errors, both in $r$/Fe and [Fe/H]. Detailed abundance analyses of individual stars with high-resolution spectra (e.g., CS 22892-052; Sneden et al. 2000; HD 115444: Westin et al. 2000) indicate observational uncertainties for $\sigma(r$/Fe) and $\sigma([\text{Fe/H}])$ of less than 0.10. Lower resolution surveys of groups of stars (e.g., Burris et al. 2000) suggest abundance uncertainties for $\sigma(r$/Fe) more typically of $\approx 0.2$. We include these in the model, adopting representative values of $\sigma(r$/Fe)$_{\text{obs}} = 0.10$ dex and $\sigma([\text{Fe/H}])_{\text{obs}} = 0.05$ dex, with the supposition that the errors are uncorrelated with the intrinsic dispersion.

In the binomial model just described, there are quantization effects due to the integral values of $N_{A,*}$ and $N_{b,*}$. These are particularly noticeable when $\langle N_{A,*} \rangle < 1$, which occurs when $N_{SN} < 1/f_a \approx 25$ or $|\text{Fe/H}| < -2.6$. In this regime, most points will cluster at $R_0$, with only a few at higher values. Furthermore, $R_* = \langle R \rangle N_{A,*}/N_{SN}$ is also quantized. The lower envelope of the nonzero values is given by the case in which $N_{a} = 1$, so that $R_{\text{env}} = \langle R \rangle/N_{SN}$ and $|r$/Fe$|_{\text{env}} = -|\text{Fe/H}| + \log(R)$.

It is not necessary that a star’s heavy-element ancestry be quantized in this way. For example, one can plausibly imagine that the mixing of successive generations of supernova ejecta is not an “all or nothing” prospect, but rather that a given parcel of ISM gas and dust can be enriched to different degrees by the ancestors it had. This is indeed very likely to be the case, which would mean that the quantization effects of the binomial model are spurious (and thus the binomial results are only to be trusted in the regime in which $(N_{A,*} > 1$ or $|\text{Fe/H}| < -2.6$).

Thus, it is of interest to consider making the number of ancestors of each class a continuous random variable. This can be done in a way that naturally generalizes from the binomial distribution; in that case, the total number of supernova progenitors $N_{SN}$ is fixed, and the fraction $f_{A,*} = N_{A,*}/N_{SN}$ of progenitors of class A is a random variable that has a mean $\langle f_{A,*} \rangle = f_a$ but can take only discrete rational values because of the integral nature of $N_{A,*}$. We continue to take $N_{SN}$ to be fixed, but we now assume that the fraction $f_{A,*}$ of progenitors of class A is now a continuous random variable, with mean $\langle f_{A,*} \rangle = f_a$. We thus want to draw $f_{A,*}$ from a continuous distribution with values in the interval $[0, 1]$ and with a mean we are free to choose. These requirements are met by the beta distribution, which has a distribution function

$$
\xi(f) = \frac{1}{B(a, b)} f^{a-1}(1 - f)^{b-1}
$$

for $f \in [0,1]$, and where $B(a, b)$ is the beta function, which can be expressed in terms of the gamma function via $B(a, b) = \Gamma(a)\Gamma(b)/\Gamma(a + b)$. We fix the parameters $a$ and $b$ of the distribution by simultaneously fitting the mean $\langle f \rangle = a/(a + b)$ to the observed value $f_a$, and forcing the variance $\sigma^2(f) = ab(a + b)^{-2}(a + b + 1)$ to the same value as the binomial case, $\sigma^2(f) = f_a(1 - f_a)/N_{SN}$. These conditions are satisfied if $a = f_a(N_{SN} - 1)$ and $b = (1 - f_a)(N_{SN} - 1)$.

We again turn to a halo star of fixed metallicity $[\text{Fe/H}] = [\text{Fe/H}]_{\text{min}} + \log N_{SN}$. Our procedure is to draw $f_{A,*}$ according to the beta distribution $\xi(f)$. We then find $N_{A,*} = f_a N_{SN}$ and $N_{b,*} = (1 - f_a) N_{SN} = N_{SN} - N_a$. Since $f_a$ is a continuous variable, so are $N_{A,*}$ and $N_{b,*}$. Our choices of the parameters $a$ and $b$ guarantee that $\langle R_* \rangle = \langle R \rangle$ and that $\sigma(R)$ is identical to the value found for the binomial case, so that equations (10) and (11) still hold. Results for a Monte Carlo simulation of a stellar population appear in Figure 2 (top). The general trend is quite similar to that for the binomial case, but the continuous nature of the parent distribution guarantees that quantization effects are now absent. We see that at the very lowest metallicities $[\text{Fe/H}] \leq -3.5$, the points cluster at $r$/Fe$_{\text{min}}$ and have a mean value below that at higher metallicity. This feature has a similar origin to that of its counterpart in Figure 2 (top). At the lowest metallicities, the small-number statistics for the supernova parents is a dominant effect, and the $r$-rich events are too rare to be expected in a sample of the size presented here.

A comparison with the observed data in Figure 2 (top) shows that the beta distribution model gives a good fit to the available data. We thus conclude that the observed scatter in $r$/Fe can be understood by our simple scenario in which halo stars stochastically sample the yields of two different populations of heavy-element producing events. In this
interpretation, the degree of scatter increases with decreasing metallicity because of counting statistics, so that the lowest metallicity events record the nucleosynthesis of a few \((N_{SN}/Fe/Fe_{min})\) events. Furthermore, since \(\sigma(R)/\langle R \rangle \approx f_a / H_{SN}\), a good measure of the scatter in \(r/Fe\) allows one to find \(N_{SN}\), and thus one can fix the parameter \((Fe/H)_{min}\). In this way, one can empirically measure the single-event iron yield and also determine the number of nucleosynthesis events recorded at a particular metallicity. The fact that the observed distribution is reasonably enclosed by the theory curves that use \((Fe/H)_{min} = -4\) (e.g., Audouze & Silk 1995) indicates that this value is a reasonable first approximation.

### 2.1.2. Introducing Stellar Lifetimes

Having achieved a reasonable fit to the data, we are emboldened to consider refinements to this model. Specifically, we turn to the question of timescales. Note that we are only concerned here with the differences in the timescales between the two classes of supernovae; we are not able to constrain the absolute timescales characteristic of either class. This limitation is overcome with more detailed models that take advantage of the high quality of solar system iso-topic data to derive quantitative limits on the rates of different \(r\)-process components (Cameron 1993; Wasserburg et al. 1996). Such detailed models are ultimately required to fully understand halo-star data but are beyond the scope of the semianalytic approach taken here.

Thus far, we have implicitly assumed that there is always the same relative probability of \(r\)-rich and \(r\)-poor nucleosynthesis events. We note that since the \(r\)-rich events are much more rare than the \(r\)-poor events, \(f_a \sim 4\%\), this means that the rates of the two types are dissimilar by the same factor and further implies that the mean timescale for each event to occur is correspondingly different: \(A\) events are separated by timescales on average \(\sim 24\) times longer than \(B\) events. This effect is purely statistical and is independent of the lifetimes of the progenitor stars.

However, the assumption of equal progenitor lifetimes may break down at very early times. One expects that the two classes \(A\) and \(B\) of nucleosynthesis events stem from different physics. For example, the \(r\)-rich and \(r\)-poor classes could correspond to progenitor stars with different masses and thus different lifetimes, or heavy \(r\)-process production could arise from binary neutron star mergers and thus require some time delay for in-spiral. In either case, one expects a time lag between the first events of one class versus the first events of the other. During this initial period, the \(r/Fe\) ratio of the ISM and of any new stars will only be that of the allowed class of nucleosynthesis events. There will thus be no dispersion in \(r/Fe\), or rather, there will be a smaller intrinsic dispersion that arises within the allowed class. Once events of the other class can occur, a large \(r/Fe\) dispersion will set in, and the \(r/Fe\) will scatter around its mean value.

We can crudely simulate this time delay between the onset of the two classes as follows. We note that the iron abundance increases with time. Thus, a time delay can be encoded as a delay in \(Fe/H\). We choose a value \((Fe/H)_{cut}\) as the cutoff that marks the onset of the lagged class of nucleosynthesis events. To allow for dispersion in the birth times of the first stars, we introduce some randomness in this cutoff by making the cutoff number \(N_{cut}\) of events a Poisson random variable, with mean \(N_{cut} = Fe_{cut}/Fe_{min}\). We then require that if \(N_{SN} < N_{cut}\), only events of the allowed class \(i\) can occur, and thus \(R = R_i\). If \(N_{SN} > N_{cut}\), then we follow the standard procedure described above.

Results appear in Figure 2 (middle), using \(Fe_{cut} = 10Fe_{min}\), which roughly corresponds to timescales \(\tau_{long} = 10\tau_{short}\), where the longer lifetime is that of the species cutoff. As we have required, at \(Fe/H < 10Fe_{cut}\), the dispersion vanishes and the points lie at the level of the allowed class. While this is clearly an oversimplified description, it nevertheless gives a qualitative sense of the kind of behavior one expects if there is indeed a significant disparity between the timescales for the production of \(r\)-rich and \(r\)-poor events. A discrepancy in timescales is required if the heavy \(r\)-process production is determined by the progenitor star’s mass. As we suspect the mass to be the controlling parameter, we predict that one of the two bottom panels in Figure 2 represents the trend that will be observed in ultralow-metal poor stars.

Thus, a discovery of the sort of behavior seen in Figure 2 (middle) would indicate which class has the shorter production timescale, and would thus provide a key clue as to the astrophysical origin of the dominant \(r\)-process site. Furthermore, a determination of \(Fe_{cut}/Fe_{min}\) would help to
quantify (in a model-dependent way) the time delay itself. If class A stars have the longer timescale, \( \tau_a > \tau_b \), then physically this means that (1) the heavy \( r \)-process producers evolve more slowly, and thus (2) these stars should show up later, so that there should be a limiting low metallicity below which all stars have \( R_a = R_b \). The appearance of the first stars with \( R_a > R_b \) marks the time of the first heavy \( r \)-process stars and thus \( t \geq \tau_a \).

We can be more specific by using the fact that heavy \( r \)-process stars must be a small fraction of all supernovae. This means that the progenitors must represent a limited range in the massive star IMF: either \( 8-10 M_\odot \) or \( \geq 25 M_\odot \). If \( \tau_a \) is long, this would select the \( 8-10 M_\odot \) range, for which specific supernova models have been proposed (Meyer & Brown 1997; Wheeler, Cowan, & Hillebrandt 1998; Freiburghaus et al. 1999) and chemical-evolution models discussed (Travaglio et al. 1999, 2001a, 2001b; Raiteri et al. 1999; Ishimaru & Wanajo 1999). This would in turn mean that \( \tau_a \sim \tau(10 M_\odot) \sim 3 \times 10^7 \) yr. Now consider the opposite case. If A stars have the shorter timescale, then \( \tau_a < \tau_b \), and the heavy \( r \)-process stars should appear first. Consequently, there should be a limiting low metallicity below which all stars have \( R_a = R_b \), and the appearance of the first stars with \( R_a < R_b \) marks the time of the first non-\( r \)-process stars and thus \( t \geq \tau_b \). The short \( \tau_b \) implies the high-mass \( (m \geq 25 M_\odot) \) progenitor mass range, and thus \( \tau_b \gtrsim \tau(25 M_\odot) \sim 10^7 \) yr.

While massive stars seem required to explain the heavy \( r \)-process simply because of their evolutionary timescale, it is possible that the heavy \( r \)-process is not made in the core collapse and explosion, but rather through the merger of supernova remnants, via neutron star–neutron star mergers (e.g., Eichler et al. 1989; Rosswog et al. 2000). As pointed out recently by Qian (2000), an important constraint on this scenario comes from the magnitude of the halo-star \( r/F \) scatter, which appears to be too small (!) to reconcile with the low merger rate and thus high yields of the coalescence events. An additional issue for neutron star coalescence is the timescale of the merger, which is an extra delay in the appearance of the heavy \( r \)-process compared to the other supernova products (such as iron). This timescale is a strong function of the orbital semimajor axis, and particularly the eccentricity (Peters 1964). As shown in population synthesis calculations (e.g., Fryer, Woosley, & Hartmann 1999), this leads to a wide distribution of in-spiral timescales, spanning a few Myr to \( \sim 1 \) Gyr. Thus, if the heavy \( r \)-process does come from binary neutron star mergers, then the first events had to have the correct orbital parameters to allow a very rapid evolution.

Finally, even if observations establish the lack of a signature of the kind seen in Figure 2 (middle, bottom), this would provide useful information about the heavy \( r \)-process as well. We would learn that there is no significant difference in the timescales for production of the two classes of events. This would rule out any models that require such a delay and would challenge the presumption that the heavy \( r \)-process yields are a function of the mass of the progenitor stars. Alternatively, it may suggest a very different scenario entirely, as we see below.

### 2.2. \( r \)-Process–dominated Models

In the previous section, we assumed that heavy \( r \)-process nucleosynthesis events are rare, while iron-production events are ubiquitous (for massive stars). We now turn to the reverse case, which has been developed in some detail by WQ00. The essential ingredients are the following, in chronological order:

1. **Prompt enrichment.** —The first, zero-metallicity (Population III) stars are assumed to be very massive and short lived, producing an uneven Galactic floor of \([Fe/H] \sim -4\) to \(-3\), but no \( r \)-process. The inhomogeneity of this population will lead to \( r/F \) scatter in this metallicity range.

2. **Production of the \( r \)-process in high-frequency (H) events.** —The first of the Pop II supernovae produce the heavy \( r \)-process without any Fe. These are WQ00’s high-frequency H events.

3. **Iron production in low-frequency (L) events.** —Iron-producing supernovae also exist, which cause the light \( r \)-process (i.e., sub-barium, which we do not consider here), but none of the heavy \( r \)-process. They occur at a much lower rate, \( \sim 1/10 \) of the H rate, and thus will on average first commence after \( \sim 10 \) H lifetimes.

We model the WQ00 scenario as follows; our approach and some details are very similar to the model discussed in the previous section. WQ00 note that the Pop III events need not occur in a homogeneous or well-mixed way, which will lead to global \([Fe/H]\) variations between \(-4\) and about \(-3\). Thus, we assume that a single Pop III parent would have ejecta that mix with its surroundings, leading to an enrichment of \([Fe/H] = [Fe/H]_{\text{min,III}} = -4\); this is the Pop III iron yield. This sets the Fe floor. These events do not make any \( r \)-process, which raises the possibility that an observable abundance record of this stage remains, as we discuss shortly. It is possible, indeed it is likely, that the first stars of the next stage of Galactic evolution form prior to the onset of the first H events that make the first \( r \)-process elements. Any such stars will have only Pop III abundances and thus will be void of any \( r \)-process. The number of such \( r \)-free stars depends on the details of star formation immediately after Pop III and thus is difficult to estimate. We thus make the simple assumption that the first H events are not significantly delayed after the formation of the first halo stars, so that there is not a significant population of Pop III, \( r \)-free stars.

In the next stage, the Pop II stars commence. The pure heavy-\( r \) H events occur first, as they have rates 10 times higher and thus recurrence times 10 times shorter than those of the Fe-only L events. Each H event is taken to produce the same heavy \( r \)-process yield, and thus the total heavy \( r \)-production is just the number \( N_h \) of parents times the yield of a single event. Once the L events commence, they produce only Fe (and the light \( r \)-process, which we do not consider). WQ00 put the Fe yield of a single event to be about \([Fe/H]_l = -2.5\), i.e., 30 times more than that of the Pop III events. We assume that all L events have this yield.

We thus have two epochs to model. The first is the period after the Pop III events but before the first L event, and the second is the rest of the Pop II phase, up to \([Fe/H] = -1\). We begin modeling each halo star’s ancestry by determining which of these epochs it samples. To do this, we choose a metallicity in the range \(-4 \leq [Fe/H] \leq -1\).\(^6\) We note that whenever \([Fe/H] < [Fe/H]_l = -2.5\), then the star must

\(^6\) Again, we are free to sample any metallicity in this range, as we are not modeling the massive stars but only the present-day halo stars that record the nucleosynthetic past.
have arisen in the first, H-only epoch and have no L-event progenitors. In this case, the number of Pop III progenitors is then fixed to be $N_{\text{III}} = \frac{\text{Fe}}{\text{Fe}_{\text{min,III}}}$.

We then determine the number of H events in our halo star’s lineage by choosing $N_h$. As in the previous section, we note that the specific distribution of $N_h$ is determined by the detailed physics of star formation and gas dynamics in the earliest epoch in our Galaxy. The distribution is constrained by the H event-to-L event ratio of 10, which demands that on average there be 10 H events before the first L event occurs. Independent of the specific distribution, a crucial issue is the expected correlation between $N_h$ and $N_{\text{III}}$ (and thus Fe). Since these events occur in two different phases by two different populations, we make the simple assumption that the two are uncorrelated. We then have $R_s = r_h N_h / \text{Fe}$, with Fe already fixed and with $r_h$ the heavy $r$-process yield of a single H event (determined below). Note that since $N_h$ is uncorrelated with Fe, then the mean trend $\langle R \rangle = \langle N_h \rangle r_h / \text{Fe}$, which scales as Fe$^{-1}$. That is, the mean $r/Fe$ trend for $-4 \leq [\text{Fe/H}] \leq -2.5$ should not be constant, but should decrease with logarithmic slope $-1$. Scatter about the $r/Fe$ trend is created by the variance in $N_h$; $\sigma_R = \sigma(N_h) r_h / \text{Fe} = \sigma(N_h) / \langle N_h \rangle \langle R \rangle$. For illustration, we choose $N_h$ from a uniform distribution in (0, 10), which gives $\langle R \rangle = 5 r_h / \text{Fe}$ and $\sigma_R = \langle R \rangle / \sqrt{3}$.

We now turn to the case of stars for which we have picked a higher Fe/H $> \text{Fe}/H$. For these stars, some contribution by iron-producing L events is necessary. As before, we generate a number $N_{\text{SN}}$ of supernova parents for each halo star. Thus, each star generated has $N_h$ H parents and $N_L$ L parents, but the star’s iron abundance constrains our choice of $N_l$ and thus $N_{\text{SN}}$. Namely, the star has a metallicity

$$\left(\frac{\text{Fe}}{\text{H}}\right)_* = N_{\text{III}} \left(\frac{\text{Fe}}{\text{H}}\right)_{\text{min,III}} + N_l \left(\frac{\text{Fe}}{\text{H}}\right)_{\text{L}} = \left(\frac{\text{Fe}}{\text{H}}\right)_{\text{III,eff}} + N_l \left(\frac{\text{Fe}}{\text{H}}\right)_{\text{L}},$$

(14)

with $N_{\text{III,eff}} = (\text{Fe}_{\text{min,III}} / \text{Fe}) N_{\text{III}}$. We choose $N_{\text{III}}$ from a distribution between 0 and Fe/H$_{\text{min,III}}$ $\approx 30$; for illustration, we use the uniform distribution. Once given $N_{\text{III}}$, then $N_l$ is fixed by equation (14).

Turning to $N_h$, we must choose this in a way that is consistent with our choice of $N_l$. Note that since the L events are 10 times more rare, the variance in $N_l$ is much larger than that in $N_h$, and thus we should expect a large span in the allowed $N_{\text{SN}}$ choice at fixed $N_l$. We proceed by first estimating the total number of supernova events to be $N_{\text{SN}} = N_l / f_l$, where $f_l = 1/11 = 1 - f_h$ is the fraction of supernova of type L. Using a beta distribution, we choose $f_{h,*}$, with $a = f_h (N_{\text{SN}} - 1)$ and $b = f_l (N_{\text{SN}} - 1)$. Then we have $f_{h,*} = 1 - f_{l,*}$ and can determine $N_{\text{SN,*}} = N_l / f_{l,*}$. From these, we find $N_h = f_{h,*} N_{\text{SN,*}} = (f_{h,*} / f_l) N_l$, and the star’s $r/Fe$ ratio,

$$R_s = \frac{N_h}{N_{\text{III,eff}} + N_l} R_{\text{yl,d}},$$

(15)

where $R_{\text{yl,d}} = r_h / \text{Fe}_l$ is the ratio of the (number) yields from the two event types. Note that at high metallicity, we have $N_h, N_l \gg N_{\text{III,eff}}$, and thus,

$$R_s \rightarrow \frac{N_h}{N_l} R_{\text{yl,d}} \rightarrow \langle R \rangle.$$  

(16)

However, since WQ00 specify that H events occur 10 times more frequently than L events, we have $N_h / N_l = 10$, and from equation (16), we can infer the proper $R_{\text{yl,d}} = r_h / \text{Fe}_l$ parameter to give the correct $\langle R \rangle$. Namely, we have $R_{\text{yl,d}} = (\text{Fe} / \text{H}) / (\text{Fe}_l / \text{H}_l) = 6.3 \times 10^{-4}$.

We now turn to the $r/Fe$ dispersion at Fe/H $> \text{Fe}_l$. Note that simple analytic formulae are not easily derived for the $r/Fe$ scatter in the WQ00 scenario, owing to the presence of random variables in both the numerator and denominator of equation (15). Nevertheless, one can roughly estimate that $\sigma(R) / R \sim 1 / \langle N_{\text{III}} \rangle^{1/2}$. For Fe $> \text{Fe}_l$, this gives an intrinsic dispersion of $\sigma(R) / R \sim 1 / (\langle Fe/Fe_l \rangle)^{1/2}$, or

$$\sigma_{r/Fe} = \frac{1}{\ln 10} \left(\frac{\langle \text{Fe}/\text{H} \rangle - \langle \text{Fe}/\text{H}_l \rangle}{\langle \text{Fe}/\text{H} \rangle + \langle \text{Fe}/\text{H}_l \rangle}\right)^{1/2} = \frac{1}{\ln 10} \left(\frac{\langle \text{Fe}/\text{H} \rangle}{\langle \text{Fe}/\text{H}_l \rangle} - 1\right)^{1/2},$$

(17)

the same Fe$^{-1/2}$ scaling, with essentially the same amplitude, as that in the Fe-dominated case (eq. [12]). Thus, the WQ00 model also provides an excellent fit to the scatter in the available data, as seen in Figure 1.

Results from a Monte Carlo realization appear in Figure 3. The dashed curve shows the 2 $\sigma$ envelope that includes observational errors and the intrinsic scatter predicted by $\sigma(R) / R = 1 / (\langle Fe/Fe_l \rangle)^{1/2}$ and plotted over its range of validity. We see that the scatter at $\langle \text{Fe}/\text{H} \rangle > -2.5$ is similar to that in the Fe-dominated case and is in reasonable agreement with the present data.

However, the low-metallicity convergence behavior seen in Figure 2 (middle, bottom) does not appear in the WQ00 scenario. Rather, at $\langle \text{Fe}/\text{H} \rangle < -2.5$, we see a scatter around the $R \sim 1 / \langle \text{Fe}_l \rangle$ trend discussed above. Thus, in the WQ00 scenario, the $r/Fe$ values remain closer to the higher metallicity mean and retain more scatter than in the models of the last section. Both of these trends essentially reflect the essential difference between the very low metallicity behavior in the two scenarios: here, when $-4 \leq \langle \text{Fe}/\text{H} \rangle \leq -3$, the sources for the $r$-process and iron are in different, uncorrelated populations (very massive Pop III stars and H supernovae). On the other hand, in the “time-delay” models of the previous...
section, the $-4 \leq [\text{Fe}/\text{H}] \leq -3$ regime is due to stars where the $r$-process and iron correlation is total (i.e., the $r$-process is either always or never produced). We thus conclude that observations of the $r$/Fe scatter in this regime can thus provide important information about $r$-process astrophysics.

### 3. OBSERVATIONAL TESTS OF $r$-PROCESS ASTROPHYSICS

The models we have presented for $r$/Fe scatter all show similar trends from $[\text{Fe}/\text{H}] = -1$ down to about $-2.5$. However, below this metallicity, the differences become increasingly stark, as seen in the figures. The predicted behaviors are for points to cluster above, below, or around the mean value $[r/\text{Fe}]_{\text{avg}}$.

1. If the points all cluster around $[r/\text{Fe}] \gg [r/\text{Fe}]_{\text{avg}}$, this points to a very early population that, regardless of the small number of ancestors, maintained high $r$/Fe. This would confirm that the earliest stars produced both iron and heavy $r$-elements, and thus that the sources of the heavy $r$-process are short-lived, and thus high-mass, stars.

2. If, conversely, the points all cluster around $[r/\text{Fe}] \ll [r/\text{Fe}]_{\text{avg}}$ (or have only upper limits for heavy $r$-process elements), this indicates that the earliest stars produced iron but failed to produce significant heavy $r$ elements. This is thus evidence for a delay until the first heavy $r$-producing stars and thus points to longer lived progenitors (lower mass supernovae or NS-NS binaries).

3. Finally, the points could continue to cluster near (but perhaps somewhat above) $[r/\text{Fe}]_{\text{avg}}$. This would indicate that the earliest stars could collectively produce both the heavy $r$-process or iron, but the persistent scatter would demand that individual stars could only produce one of the two. In other words, unlike the previous possibilities, in this case, the $r$-process and iron nucleosynthetic sources are uncorrelated. This points to a picture of the kind suggested by WQ00, wherein an early (and nonuniform) Pop III production of iron is quickly followed by rapid production of iron-free heavy $r$-process ejecta.

Thus we see that ultra–metal poor stars can provide “smoking-gun” evidence for heavy $r$-process astrophysics.

### 4. CONCLUSIONS

We have presented an analytical approach to understanding the scatter in heavy $r$-process–to-iron ratios in metal-poor halo stars. The observed $r$/Fe scatter demands that the bulk of the heavy $r$-process elements and of iron cannot be produced in the same stars. We have used a simple stochastic description of the different populations of nucleosynthetic events that contribute to the heavy $r$-process and iron abundances in each halo star. The random star-to-star variations in nucleosynthetic “ancestry” lead to scatter in $r$/Fe. The models we present are all successful in reproducing the scatter in the available data, which go down to about $[\text{Fe}/\text{H}] = -3$. We see that the abundance scatter can be understood simply in terms of the variance due to counting statistics of the ancestry of each star. We present simple and convenient analytic expressions for the scatter in this regime.

We model two basic scenarios for heavy $r$-process production; one in which the heavy $r$-process is rare, occurring in about $\sim 4\%$ of all massive stars. The other scenario, that of WQ00, takes the reverse approach, in which heavy $r$-process production is ubiquitous and iron production rare in massive stars. We find that these scenarios give very similar $r$/Fe scatter at $[\text{Fe}/\text{H}] \gtrsim -2.5$ but show divergent behavior at lower metallicity.

In closing, we remind the reader that the results here have the virtue of being model independent, in that they do not refer to detailed supernova yield calculations nor to detailed hydrodynamic mixing models. Of course, a full solution of the chemical evolution of the $r$-process will demand such calculations; our hope is that the results here will provide a useful guide in interpreting those more complete models. We have also made important simplifications, notably in our assumption of a constant, mass-independent yield of iron from type II supernovae. We look forward to a large data set on $r$-process abundances in very low metallicity stars, which will demand improvements on the simple picture presented here.

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**REFERENCES**

Argast, D., Samland, M., Gerhard, O. E., & Thielemann, F.-K. 2000, A&A, 356, 873

Audouze, J. & Silk, J. 1995, ApJ, 451, L49

Barris, D. L., et al. 2000, ApJ, 544, 302

Busso, M., Gallino, R., & Wasserburg, G. J. 1999, ARA&A, 37, 239

Cameron, A. G. W. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 47

Cayrel, R., et al. 2001, Nature, 409, 691

Edvardsson, B., et al. 1993, A&A, 275, 101

Eichler, D., Livio, M., Piran, T., & Schramm, D. N. 1989, Nature, 340, 126

Freiburghaus, C., et al. 1999, ApJ, 516, 381

Fryer, C. L., Woosley, S. E., & Hartmann, D. H. 1999, ApJ, 526, 152

Ishimaru, Y., & Wannajo, S. 1999, ApJ, 511, L33

Jehin, E., et al. 1999, A&A, 341, 241

McWilliam, A. 1998, AJ, 115, 1640

McWilliam, A., Preston, G. W., Sneden, C., & Schectman, S. 1995a, AJ, 109, 2736

McWilliam, A., Preston, G. W., Sneden, C., & Searle, L. 1995b, AJ, 109, 2757

Meyer, B. S., & Brown, J. S. 1997, ApJS, 112, 199

Pagel, B. E. J. 1997, Nucleosynthesis and Chemical Evolution of Galaxies (Cambridge: Cambridge Univ. Press)

Peters, P. C. 1964, Phys. Rev., 136, 1224

Qian, Y.-Z. 2000, ApJ, 534, L67

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1. In the final stages of completion of this paper, we became aware of the work of Qian (2001), which also considers $r$/Fe scatter in the WQ00 model. His methods and conclusions are very similar to ours.
Qian, Y.-Z. 2001, ApJ, 552, L117
Raiteri, C. M., Villata, M., Gallino, R., Busso, M., & Cravanzola, A. 1999, ApJ, 518, L91
Rosswog, S., Davies, M. B., Thielemann, F.-K., & Piran, T. 2000, A&A, 360, 171
Sneden, C., Cowan, J. J., & Truran, J. W. 2001, in Cosmic Evolution, ed. E. Vangioni-Flam, R. Ferlet, & M. Lemoine (Singapore: World Scientific), 179
Sneden, C., et al. 1996, ApJ, 467, 819
———. 2000, ApJ, 533, L139
Travaglio, C., Galli, D., & Burkert, A. 2001a, ApJ, 547, 217
Travaglio, C., Gallino, R., Busso, M., & Gratton, R. 2001b, ApJ, 549, 346
Travaglio, C., et al. 1999, ApJ, 521, 691

Truran, J. W. 1981, A&A, 97, 391
Tsujimoto, T., & Shigeyama, T. 1998, ApJ, 508, L151
Tsujimoto, T., Shigeyama, T., & Yoshii, Y. 1999, ApJ, 519, L63
Wasserburg, G. J., Busso, M., & Gallino, R. 1996, ApJ, 466, L109
Wasserburg, G. J., & Qian, Y. Z. 2000a, ApJ, 529, L21 (WQ00)
———. 2000b, Phys. Rep., 333, 77
Westin, J., Sneden, C., Gustafsson, B., & Cowan, J. J. 2000, ApJ, 530, 783
Wheeler, J. C., Cowan, J. J., & Hillebrandt, W. 1998, ApJ, 493, L101
Woolf, V. M., Tomkin, J., & Lambert, D. L. 1995, ApJ, 453, 660
Woosley, S. E. 1988, ApJ, 330, 218
Woosley, S. E., & Weaver, T. A. 1995, ApJS, 101, 181
Woosley, S. E., Wilson, J. R., Mathews, G. J., Hoffman, R. D., & Meyer, B. S. 1994, ApJ, 433, 229