Numerical simulation research on the optimization for blade shape of rotational supercavitating evaporator

Q Li¹, J P Cheng¹, Z Y Zheng¹, F C Li¹,³ and V A Kulagin²
¹ School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, China
² Department of Heat Technology and Fluid Dynamics, Siberian Federal University, Krasnoyarsk 660041, Russia
E-mail: lifch@hit.edu.cn

Abstract. Rotational Supercavitating Evaporator (RSCE) has been proposed as a new technology for seawater desalination. However, it lacks systematic researches on the blade shape of RSCE. In this paper, numerical simulations were conducted on the supercavities formed behind two-dimensional (2D) wedge-shaped cavitators of the RSCE. The cavitating flows around the 2D wedge-shaped cavitators with several certain wedge angles varied from 30 to 150 degrees under different cavitation numbers were simulated, and the empirical formulae of supercavity dimensions about cavitation number at corresponding wedge angles were obtained.

1. Introduction
Water is the source of our lives and the material basis of the human beings' survival. There exists a severe shortage of fresh water around the world, which has led to development of seawater desalination methods. The most widely used technologies are thermal methods and membrane methods, which make up about 95% of worldwide fresh water production. However, the former is limited by the scale formation and thermal transfer coefficient, and the later is limited by the membrane fouling and the recovery factor. There is a technological bottleneck in the thermal methods to evaporate water into steam free from scale formation and possess a dramatic increase in thermal transfer coefficient. Water evaporates into steam on the surface of supercavity in the same way with boiling of water. Moreover, the heat transfer coefficient of evaporation from the surface of supercavity depends on the heat-flux density and there is no scale formation. So a device named Rotational Supercavitating Evaporator (RSCE) was put forward for desalination based on supercavitation effect. It also has a good application foreground in sewage disposal, such as nuclear waste water and polluted water containing heavy metals.

Cavitation can be defined as the breakdown of a liquid medium under very low pressures and applied to cases in which the liquid is either static or in motion. It was first discovered on the propeller in the warship of British Royal Navy in 1897. The study on cavitation primarily focused on the prevention of negative influence of cavitation phenomenon such as noise, vibration and surface

³ Corresponding author.
erosion. Subsequently the great drag reduction effect of supercavitation on an underwater body was found, and it came an upsurge of research on the drag reduction of underwater vehicle based on supercavitation effect. 30 years ago, Machinski [1] introduced the stationary supercavitating cone evaporator, which can be connected to vacuum system for steam extraction for desalination. Recently Likhachev and Li [2-4] put forward RSCE and performed preliminary design and research.

In order to obtain systematic characteristics of cavity formed by the rotational supercavitating blades, numerical simulations are conducted on 2D wedge-shaped cavitators, which is a simplification of rotational supercavitating blades. The cavitating flows around the 2D wedge-shaped cavitator with certain dimension are simulated under different cavitation numbers to obtain the characteristics of cavity. Different wedge angles varied from 30 to 150 degrees of 2D wedge-shaped cavitators are selected to obtain the characteristics of cavity. As a conclusion, characteristics of planar symmetric cavity are analyzed and empirical formulae of supercavity dimensions as a function of cavitation number at corresponding wedge angles are obtained.

2. Numerical method

2.1. Governing equations
Numerical simulations are conducted on the two-dimensional steady supercavitating flows in this paper. In the numerical simulation, supercavitating flow is a multiphase and turbulent flow and mixture model based on the homogeneous equilibrium multiphase flow theory is selected as the multiphase flow model. The mixture of the gas and liquid phases is considered as homogeneous single-phase fluid and the equations in this model are shown below.

Continuity equation of the mixture phase:
\[ \nabla \cdot \left( \rho_m \vec{v}_m \right) = 0 \]

Momentum equation of the mixture phase:
\[ \frac{\partial}{\partial t} \left( \rho_m \vec{v}_m \right) + \nabla \cdot \left( \rho_m \vec{v}_m \vec{v}_m \right) = -\nabla p + \nabla \cdot \left[ \mu_m \left( \nabla \vec{v}_m + \nabla \vec{v}_m^T \right) \right] \]

Transportation equation of the vapour fraction of the gas phase:
\[ \nabla \cdot \left( \alpha_v \rho_v \vec{v}_m \right) = R_e - R_c \]

where \( \rho_m \) is the density of the mixture phase, \( \vec{v}_m \) is velocity, \( \alpha_v \) is the vapour fraction of the gaseous phase, \( \rho_v \) is the density of the gaseous phase, \( R_e \) are \( R_c \) the production rate and condensation rate of the gaseous phase, respectively. In order to depict \( R_e \) and \( R_c \), Schnerr-Sauer model [5] is adopted as the cavitation model.

For \( p_v \geq p \),
\[ R_e = \frac{\rho_v \rho_i}{\rho_m} \alpha_v (1 - \alpha_v) \frac{3}{9 R_b} \sqrt{\frac{2}{3} \frac{(p_v - p)}{\rho_i}} \]

For \( p_v \leq p \),
\[ R_c = \frac{\rho_v \rho_i}{\rho_m} \alpha_v (1 - \alpha_v) \frac{3}{9 R_b} \sqrt{\frac{2}{3} \frac{(p - p_i)}{\rho_i}} \]
where \( \rho_l \) is the density of the liquid phase, \( p_v \) is the saturation vapor pressure, \( R_b \) is the radius of the bubble and defined as follows.

\[
R_b = \left( \frac{\alpha_v 3 1}{1 - \alpha_v 4\pi n} \right)^{1/3}
\]  

Herein, \( n \) is the number of the microbubbles within the liquid of unit volume and set as \( 1 \times 10^{13} \).

Turbulent flow in the numerical simulation is described by Realizable \( k - \varepsilon \) model and the Scalable wall function is adopted to solve the flow near the wall.

2.2. Boundary conditions and mesh

Because of the 2D wedge-shaped cavitator model is symmetric, half of the model can be used as the computational domain, which is shown in Figure 1. The half height of the base of the wedge-shaped cavitator is \( h_c = 5 \)mm and the wedge angles \( \gamma \) have several certain values ranged from 30 to 150 degrees. The computational domain is 5m in height and 20.5m in length, which is much bigger than the typical computational domain of an axisymmetric cavitator in case of that the cavity formed by a planar cavitator is much larger than that formed by an axisymmetric cavitator. The cavitator is placed 0.5m downstream from the inlet. Both the top and the inlet of the computational domain are set as velocity inlet with the velocity of certain values ranged from 25 to 60 m/s in axial direction. The axis is set as symmetry and the outlet is set as pressure-outlet with the pressure of 1 bar, while the other boundaries are set as fixed no-slip wall. The thermodynamic properties of water and steam are defined according to international standard for the thermodynamic properties of water and steam IAPWS95 [6] at 25°C. Density of water liquid and vapor are 997kg/m\(^3\) and 0.023075kg/m\(^3\), respectively. Viscosity of water liquid and vapor are \( 8.9011 \times 10^{-4} \)Pa·s and \( 9.8669 \times 10^{-6} \)Pa·s. The computational domain is meshed by the structured grids, and the meshes in the boundary layer near the blades surface are arranged to be denser, which are shown in Figure 2.

![Figure 1. Schematic diagram of the computational domain.](image)

![Figure 2. Schematic diagram of the mesh with: (a) enlarged part around cavitator, (b) meshes in the boundary layer.](image)

3. Results and discussions

In order to obtain the characteristics of cavities under different cavitation numbers, the cavitating flows around each cavitator with certain wedge angles are simulated under different cavitation
numbers. The chosen wedge angles are 30, 45, 60, 90, 120 and 150 degrees. Cavitation number is defined as:

\[ \sigma = \frac{p_0 - p_c}{\frac{1}{2} \rho v^2} \]  

(7)

where \( p_0 \) is the ambient pressure, \( p_c \) is the pressure inside the cavity and equals to the saturation vapor pressure, \( \rho \) is the density of liquid, and \( v \) is the velocity. Different velocities, including 25, 30, 35, 40, 50, 60m/s, are selected to get different cavitation numbers.

3.1. Mesh independence verification

In order to guarantee the accuracy and economy of the calculations, the mesh independence is verified for the first step. Meshes with different grid number were generated with different grid sizes in the downstream of the cavitator for numerical simulation. Dimensionless parameters \( \frac{L}{h_c} \) and \( \frac{H}{h_c} \) are used to measure the shape of cavity, herein \( L \) is the length of cavity and \( H \) is the maximum thickness of cavity. The results of different meshes are shown in Figure 3. As the grid number up to about 0.45 million, the length of cavity remain unchanged. So the mesh of about 0.45 million grids is selected in the further research.

![Figure 3. The results of mesh independence.](image)

3.2. Results for different cavitation numbers

In order to obtain the dependences of cavity dimensions and drag coefficient on the cavitation number, the results for the cavitator with the wedge angle of 45 degrees are analyzed. In many cases, it is possible to model the experimental dependence of the cavity length, for low values of that parameter, by a power law:

\[ \frac{L}{h_c} = A \sigma^{-n} \]  

(8)

where the exponent \( n \) is found to equal to 2 if the body is located in an infinite medium [7]. The length and maximum thickness of the cavity formed by the cavitator with the wedge angle of 45 degrees are respectively fitted, as shown in Figures 4. The fitting functions are

\[ \frac{L}{h_c} = 2.329 \sigma^{-1.83} \]  

(9)

\[ \frac{H}{h_c} = 1.469 \sigma^{-0.975} \]  

(10)

The drag coefficient is fitted in parabola very well, as shown in Figures 5. The fitting function is:

\[ C_x = 0.439 + 0.166 \sigma + 0.743 \sigma^2 \]  

(11)
3.3. Results for different wedge angles

$L/h_c$ and $H/h_c$ of cavitator with different wedge angles are fitted respectively and the exponents of the fitting functions of $L/h_c$ and $H/h_c$ for different wedge angles are close to the values of -1.83 and -0.975. Thus the exponents of the fitting functions of $L/h_c$ and $H/h_c$ are fixed as -1.83 and -0.975 in the following research. The fitting results of $L/h_c$ and $H/h_c$ for the certain cavitation numbers under different wedge angles are shown in Figures 6 and 7. The cavity dimension increases rapidly as the wedge angle increases at a certain cavitation number, which means larger cavity can be formed by the cavitator with larger wedge angle under the same condition. The coefficient $A$ of each fitting function are shown in Table 1, and it can be fitted in parabola as:

$$A_{L/h_c} = 0.496 + 2.602\gamma - 0.361\gamma^2$$

$$A_{H/h_c} = 0.344 + 1.611\gamma - 0.241\gamma^2$$

The drag coefficients of cavitator with different wedge angles are fitted in parabola:

$$C_x = a + b\sigma + c\sigma^2$$

The fitting results are shown in Figures 8 and the coefficients, $a$, $b$ and $c$, are shown Table 1. It can be seen that cavitators with larger wedge angle have larger drag coefficients. So more power is needed to obtain larger cavity. Finally, empirical formulae of supercavity dimensions as a function of cavitation number and wedge angles are obtained as:

$$L/h_c = (0.496 + 2.602\gamma - 0.361\gamma^2)\sigma^{-1.83}$$

$$H/h_c = (0.344 + 1.611\gamma - 0.241\gamma^2)\sigma^{-0.975}$$

Supercavity dimensions increase with the decrease of cavitation number in power law. What's more, the exponents of empirical formulae are constant for different wedge angles, which is consistent with the work of Franc and Michel [7] and the accuracy of the numerical simulation results is verified.

4. Conclusions

Numerical simulations are conducted on the cavitating flows around the 2D wedge-shaped cavitators with wedge angles of 30, 45, 60, 90, 120 and 150 degrees under different cavitation numbers, and the empirical formulae of supercavity dimensions are obtained.
Dimensions of supercavities formed by planar symmetric cavitators with different wedge angles are fitted in power law very well under different cavitation numbers, and the variation with wedge angles is parabolic. The empirical formulae will give the guidance to the optimization for blade shape of RSCE.

Figure 6. Fitting results for the length of cavities formed by cavitators with different wedge angles.

Figure 7. Fitting results for maximum thickness of cavities formed by cavitators of different wedge angles.

Figure 8. The fitting result of drag coefficients of cavitator with different wedge angles.

Table 1. Coefficients of each fitting function for $L/h_c$, $H/h_c$ and $C_x$.

| Angle (degree) | 30  | 45  | 60  | 90  | 120 | 150 |
|---------------|-----|-----|-----|-----|-----|-----|
| Coefficient $A$ for $L/h_c$ | 1.748 | 2.329 | 2.835 | 3.684 | 4.364 | 4.843 |
| Coefficient $A$ for $H/h_c$ | 1.109 | 1.469 | 1.778 | 2.275 | 2.647 | 2.915 |
| Coefficient $a$ for $C_x$ | 0.320 | 0.439 | 0.547 | 0.730 | 0.868 | 0.967 |
| Coefficient $b$ for $C_x$ | 0.176 | 0.166 | 0.171 | 0.131 | 0.0951 | 0.0634 |
| Coefficient $c$ for $C_x$ | 0.741 | 0.743 | 0.789 | 0.966 | 1.129 | 1.273 |

Acknowledgments
This work is supported by National Natural Science Foundation of China (51276046). The authors would like to appreciate the valuable discussions with the members of Complex Flow and Heat Transfer Lab.
References

[1] Machinski A S. 1984 Hydrodynamics and thermal transfer characteristics of supercavitating evaporators for water desalination (Moscow: Kiev Order of Lenin Polytechnic Institute PhD thesis) 285 (in Russian)

[2] Likhachev D S and Li F C 2013 Desalin. Water Treat. 51 2836–49

[3] Likhachev D S and Li F C 2013 Sci. China Phys. Mech. 56 1–14

[4] Likhachev D S 2013 Study on the hydrodynamic characteristics of rotational supercavitating evaporator (Harbin: Harbin Institute of Technology PhD thesis)

[5] Schnerr G H and Sauer J 2001 Physical and numerical modeling of unsteady cavitation dynamics 4th International Conference on Multiphase Flow (New Orleans, USA, 27 May-1 June 2001)

[6] Wagner W and Pruß A 2002 J. Phys. Chem. Ref. Data 31 387–535

[7] Franc J P and Michel J M. 2006 Fundamentals of Cavitation (Berlin: Springer)