Quantum Phase Transitions in Dissipative Tunnel Junctions

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The Ueda-Guinea model of a dissipative tunnel junction is investigated. This model accounts for final state effects associated with single-electron tunneling. A quantum phase transition emerges, marking a boundary between insulating (Coulomb blockade) and conducting phases. The system is analyzed by large-N techniques, self-consistent harmonic approximation, and Monte Carlo methods.

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I. INTRODUCTION

Over 30 years ago, Mahan and Nozieres and deDominicis predicted the existence of power law behavior in the absorption edges for X-ray transitions in metals. This phenomenon is due to the influence of a suddenly switched-on potential, due to the (screened) core hole, on the electrons. There are two aspects to this physics. One is the orthogonality catastrophe due to the change of electronic wavefunctions in the presence of the core hole. The other is formation of an excitonic resonance between the liberated electron and the core hole. Recently, several authors have noted the relevance of such nonequilibrium effects to mesoscopic systems such as tunnel junctions and quantum dots.

The model we will study has been described in the recent work of Bascones et al., based on the original work of Ueda and Guinea. Briefly, we consider a junction consisting of two banks, left (L) and right (R), each described by a noninteracting Hamiltonian of the form

\[ H_\alpha = \sum_k \varepsilon_k c_k^{\dagger} c_k, \]

where \( \alpha = L, R \). The tunneling between these banks is described by \( H_{\text{tunnel}} = -t e^{i\phi} \sum_{k,k',\alpha} c_k^{\dagger} c_{k'} \). The phase variable \( \phi \) is a collective coordinate conjugate to the charge transfer \( Q \) between left and right banks: \( \{ Q, \phi \} = e^{i\phi} \). Associated with this charge transfer is a Coulomb energy, \( \varepsilon_Q = (Q - Q_{\text{offset}})^2/2C \), where \( C \) is a capacitance and \( Q_{\text{offset}} \equiv \nu e \) accounts for the charge environment of the junction. Finally, the interaction between electrons and the global charge is written as:

\[ H_{\text{int}} = (Q - Q_{\text{offset}}) \sum_{k,k',\alpha} U_{kk'}^{\alpha} c_{k}^{\dagger} c_{k'} c_{k'}^{\dagger} c_k. \]

The fermionic degrees of freedom are quadratic in the Hamiltonian \( H = H_L + H_R + H_{\text{tunnel}} + H_Q + H_{\text{int}} \), and can be integrated out, leaving an effective model whose only dynamical degree of freedom is the phase field \( \phi \). When \( U_{kk'}^{\alpha} = 0 \), the resultant effective action is that obtained by Ben-Jacob, Mottola, and Schönh (BMS). This approach implicitly assumes that the energy level spacing in each of the banks is small on the scale of the charging energy \( E_C \equiv e^2/2C \) and temperature \( k_B T \). When the effects of \( H_{\text{int}} \) are accounted for, one obtains a modified Euclidean BMS action of the form:

\[ S[\phi(s)] = \frac{1}{4} \int_0^L ds \left( \frac{d\phi}{ds} \right)^2 + \alpha \int_0^L ds \int_0^L ds' K(s - s') \times \left\{ 1 - \cos \left( \phi(s) - \phi(s') \right) \right\}. \]

The kernel \( K(s - s') \) is given by

\[ K(s) = \left| \frac{\pi}{L} \csc \left( \frac{\pi |s|}{L} \right) \right|^{2-\epsilon}. \]

Here, \( L = E_C/k_B T \) is the dimensionless inverse temperature, and \( \alpha = g_{\infty}/4\pi^2 \), where \( g_{\infty} \) is the high temperature conductance of the junction in units of \( e^2/h \). The BMS model is recovered for \( \epsilon = 0 \). The parameter \( \epsilon \) is a sum over contributions from the two banks \( j = L, R \), with \( \epsilon_j = -\left( \delta_j/\pi \right)^2 \) due to the orthogonality catastrophe and \( \epsilon_j = (2\delta/\pi) - (\delta_j/\pi)^2 \) if excitonic effects are relevant.

When the tunnel junction is placed in series with a capacitor, forming a single electron box, the external charge environment is accounted for by a topological term in the action, \( \Delta S_{\text{top}} = -2\pi i \nu W[\phi] \), where \( W[\phi] = [\phi(L) - \phi(0)]/2\pi \) is the winding number of the phase field \( \phi \).

The model is a version of the ubiquitous dissipative quantum rotor. The \( \epsilon = 0 \) case has been studied by several authors. Physical quantities are smoothly dependent on \( \alpha \) and there is no phase transition at any finite \( \alpha^2 \). For \( \epsilon > 0 \), however, a quantum \( (T = 0) \) phase transition is present at a critical value \( \alpha_c(\epsilon) \), as first noted by Kosterlitz. Here, we investigate this phase transition using a large-\( N \) expansion, the self-consistent harmonic approximation, and finally Monte Carlo simulations.

II. LARGE-\( N \) THEORY

The large-\( N \) generalization of this problem was first discussed in unpublished work by Renn. Consider the
The saddle point theory is thus a Gaussian theory, with

$$F = C \alpha \left[ K(0) - \dot{K}(\alpha) \right] \dot{\bar{n}}(\omega) \right|^2$$

where $\omega_n = 2\pi n/L$ is a bosonic Matsubara frequency, and

$$\dot{K}(\omega_n) = \int_0^L ds K(s) e^{i\omega_n s}.$$  

The saddle point theory is thus a Gaussian theory, with correlation functions

$$\langle n_i(s) n_j(s') \rangle = G(s) \delta_{ij}$$

$$G(s) = \frac{1}{2L} \sum_{\omega_n} \frac{1}{4\omega_n^2 + \lambda + \alpha[K(0) - \dot{K}(\omega_n)]} e^{-i\omega_n s}.$$  

Extremizing the free energy $F = -L^{-1} \text{Tr}_n \exp(-S[\bar{n}])$ with respect to $\lambda$ yields the equation $G(0) = q$, which is to be solved for $\lambda(\alpha, L)$.

In the zero temperature ($L \to \infty$) limit, $\omega_n \to \omega$ becomes a continuous quantity, and $\dot{K}(0) - \dot{K}(\omega) = C_\epsilon |\omega|^{1-\epsilon}$, with

$$C_\epsilon = \frac{\pi}{\Gamma(2-\epsilon) \cos\left(\frac{\pi}{2}\epsilon\right)}.$$  

Straightforward analysis of the integral shows that $\lambda$ vanishes for $\alpha > \alpha_c(\epsilon)$, where

$$\alpha_c(\epsilon) = \frac{1}{4C_\epsilon} \left( \frac{2A_c}{\pi q\epsilon} \right)^{1+\epsilon}$$

$$A_c = \int_0^\infty \frac{dt}{1 + t^{1+\epsilon}}.$$  

FIG. 1: Solution to the large-$N$ model at finite $L$ with $\epsilon = 0.25$ (top) and $\epsilon = 0.75$ (bottom). $\lambda$ versus $\alpha$ for $L = 8, 32, 128, 512$ (thin lines) and $L = \infty$ (thick line). $\alpha_c(\epsilon = 0.25) = 0.5619$, and $\alpha_c(\epsilon = 0.75) = 0.1226$.

From $\lim_{\epsilon \to 0} A_\epsilon = 1$ and $C_{\epsilon = 0} = \pi$, we find, restoring $q = 1/N$, that for small $\epsilon$ the critical point occurs at $\alpha_c(\epsilon) = N/2\pi^2\epsilon$. This is to be contrasted with the renormalization group result of Kosterlitz, $\alpha^{RG}_{\epsilon = 0}(\epsilon) = (N-1)/2\pi^2\epsilon$. Note that these results agree to leading order in $1/N$.

Further analysis reveals the critical behavior of $\lambda(\alpha)$ in the vicinity of the critical point:

$$\lambda(\alpha) \approx D_{\epsilon}(\alpha_c(\epsilon) - \alpha)^\nu$$

$$D_{\epsilon} = \frac{[2^{1-\epsilon}C_{\epsilon}(\epsilon)]^{-\epsilon}}{(1+\epsilon) \alpha_c(\epsilon)}$$

$$B_{\epsilon} = \frac{\epsilon}{2\epsilon - 1} \int_0^\infty \frac{dt}{1 + t^{1+\epsilon-1}}$$

where $\nu = \max(1, \epsilon^{-1} - 1)$.

### A. Kubo Formula

When $N = 2$ the symmetry group $O(2)$ has a single generator, hence there is one vector potential. For the $O(N)$-symmetric case, a local gauge is effected by $n(s) \to R(s)n(s)$, where $R(s) \in O(N)$ is given by

$$R(s) = \exp[iA_n(s)T^a]$$
where the $T^a$ generate the Lie algebra $\mathfrak{o}(N)$. These $\frac{1}{2}N(N-1)$ generators are $N \times N$ Hermitian antisymmetric tensors; they may be chosen to satisfy the normalization $\text{Tr}(T^a T^b) = N \delta^{ab}$. (For $N = 2$ the sole generator is $T = \sigma^h$.) There are therefore $\frac{1}{2}N(N-1)$ vector potentials $A_a(s)$, variation with respect to which defines the $O(N)$ currents,

$$I_a(s) = \frac{\delta S_{\text{int}}[\mathbf{A}(s)]}{\delta A_a(s)}|_{\mathbf{A}=0} = 2i\alpha n_b(s) T^a_{kl} \int_0^L ds K(u-s) n_l(s)$$

and the noise current-current correlation function

$$\langle T^a_I(s) T^b_I(0) \rangle = \left< \frac{\delta^2 S_{\text{int}}}{\delta A_a(s) \delta A_b(0)} \right>_{\mathbf{A}=0} = 2N\alpha \delta^{ab} \left[ \delta(s) \int_0^L du K(u) G(u) - K(s) G(s) \right].$$

The conductance (in units of $e^2/h$) is computed according to the Kubo formula,\n
$$g_{ab}(i\omega_n) = \frac{2\pi}{\omega_n} \int_0^L ds e^{i\omega_n s} \langle T^a_I(s) T^b_I(0) \rangle .$$

We define $g(i\omega_n)$ by $g_{ab}(i\omega_n) \equiv N g(i\omega_n) \delta_{ab}$. We have no expression for the analytic continuation of the conductance to real frequencies. As a diagnostic of any phase transition, we will examine the quantity $g_{\alpha} \equiv g(i\omega_n=1)$. We find

$$g_{\alpha} = \frac{4\pi^2\alpha}{L} \left( \frac{L}{\pi} \right)^\epsilon \int_0^L ds \sin^\epsilon (\pi s/L) \left< \cos \left( \phi(s) - \phi(0) \right) \right> .$$

A similar expression was proposed by Bascones et al.\n
$$g_\alpha = \frac{4\pi^2\alpha}{L} \left( \frac{L}{\pi} \right)^\epsilon \left< \cos \left( \phi \left( \frac{1}{2} L \right) - \phi(0) \right) \right> .$$

At the quantum critical point, we can evaluate DC ($\omega \rightarrow 0$) limit of $g(i\omega_n, \alpha_c, T = 0)$. We obtain

$$g_c = \pi (1 - \epsilon) \cot \frac{1}{2} \pi \epsilon .$$

Note that while the critical coupling $\alpha_c$ is $q$-dependent, the critical conductance $g_c(\epsilon)$ is universal and independent of $q$. The approximations $g_{\alpha}$ and $g_\alpha$ also take on universal values at criticality:

$$g_{\alpha}^c = \frac{1}{\pi} \int_0^\pi d\theta \left( \frac{\sin \theta}{\theta} \right)^\epsilon \times g_c(\epsilon)$$

$$g_c^c = \left( \frac{2}{\pi} \right)^{1+\epsilon} \times g_c(\epsilon) .$$

The fully nonlinear theory with action

$$S[\phi(s)] = \int_0^L ds \frac{1}{4} \left( \frac{\partial \phi}{\partial s} \right)^2$$

$$+ \alpha \int_0^L ds \int_0^L ds' K(s-s') \left[ 1 - \cos \left( \phi(s) - \phi(s') \right) \right]$$

is replaced with a trial Gaussian theory described by the quadratic action

$$S_0[\phi(s)] = \frac{1}{2} \int_0^L ds \int_0^L ds' V(s-s') \left[ \phi(s) - \phi(s') \right]^2$$

$$= \frac{1}{2} \sum_{\omega_n} \hat{G}^{-1}(\omega_n) \left| \hat{\phi}(\omega_n) \right|^2 ,$$

where

$$\hat{G}(\omega_n) = \frac{1}{2L} \left[ \hat{V}(0) - \hat{V}(\omega_n) \right] .$$

FIG. 2: Large-$N$ results for $\alpha_c(\epsilon)$ (solid) compared with the Kosterlitz RG value $\alpha_c^{\text{RG}} = 1/2\pi^2 \epsilon$ (dashed). Inset: critical conductance $g_c(\epsilon)$ within the large-$N$ approximation, with $N = 2$.

### III. SELF-CONSISTENT HARMONIC APPROXIMATION

The nonlinearity of the system is replaced with a trial Gaussian theory described by the quadratic action

$$S_0[\phi(s)] = \frac{1}{2} \sum_{\omega_n} \hat{G}^{-1}(\omega_n) \left| \hat{\phi}(\omega_n) \right|^2 ,$$

where

$$\hat{G}(\omega_n) = \frac{1}{2L} \left[ \hat{V}(0) - \hat{V}(\omega_n) \right] .$$
\[ V(s), \text{ or equivalently } \hat{G}(\omega_n), \text{ is treated variationally, so we extremize by setting} \]
\[ \frac{\partial}{\partial \hat{G}(\omega_n)} \left\{ F_0 + \frac{1}{L} \langle S - S_0 \rangle_0 \right\} \tag{26} \]

where \( F_0 = -L^{-1} \text{Tr}_\phi \exp(-S_0[\phi]) \). This leads to the following self-consistent equation:
\[ \frac{1}{\hat{G}(\omega_n)} = \frac{1}{2} \omega_n^2 + 2\alpha \int_0^L ds \left( 1 - \cos(\omega_n s) \right) K(s) \Gamma(s) \tag{27} \]

where
\[ \Gamma(s) = \exp \left( -\frac{1}{2} \left\langle [\phi(s) - \phi(0)]^2 \right\rangle_0 \right) \]
\[ = \exp \left( -\frac{2}{L} \sum_{n=1}^{\infty} \left( 1 - \cos(\omega_n s) \right) \hat{G}(\omega_n) \right). \tag{28} \]

We iterate these equations to self-consistency.

The dimensionless conductance \( g_A(\alpha, L) \) is plotted for \( \epsilon = 0.1 \) and \( \epsilon = 0.2 \) in fig. 3. As the dimensionless inverse temperature \( L \) is increased from \( L = 16 \) to \( L = 256 \), the curves apparently cross at a critical point, \( \alpha_c(\epsilon) \). For \( \alpha > \alpha_c \), the conductance increases as the temperature is lowered, indicating a conducting phase. For \( \alpha < \alpha_c \) the conductance vanishes as the temperature is lowered, i.e., the Coulomb gap survives. For larger values of \( \epsilon \), however, a spurious first order transition preempts this critical behavior, as shown in fig. 4. The solution to the SCHA is hysteretic, and discontinuous, provided \( L \) is large enough.

**IV. COULOMB GAS REPRESENTATION AND MONTE CARLO**

Including the offset charge, the Euclidean action for our system is \( S = S_0 + S_{\text{int}} + S_{\text{top}}, \) where
\[ S_0[\phi] = \frac{1}{4} \int_0^L ds \left( \frac{\partial \phi}{\partial s} \right)^2 \tag{29} \]
\[ S_{\text{int}}[\phi] = -\alpha \int_0^L ds \int_0^L ds' K(s - s') \cos[\phi(s) - \phi(s')] \tag{30} \]
\[ S_{\text{top}} = -i\nu [\phi(L) - \phi(0)] = -2\pi i\nu W[\phi], \tag{31} \]

where \( W[\phi] = [\phi(L) - \phi(0)]/2\pi \) is the winding number of the phase field. In the phase representation, the topological term represents a purely imaginary contribution to the action. However, in the Coulomb gas representation, the action remains purely real.

We have discarded a formally divergent constant from our action,
\[ \Delta S = L \int_0^L ds K(s), \tag{32} \]

**FIG. 3:** Results of the self-consistent harmonic approximation for \( \epsilon = 0.10 \) and \( \epsilon = 0.20 \), with dimensionless inverse temperature \( L = 16, 32, 64, 128, 256 \). The crossing of the curves at a single point indicates a second order phase transition.

**FIG. 4:** Results of the self-consistent harmonic approximation for \( \epsilon = 0.30 \) and \( \epsilon = 0.40 \), with dimensionless inverse temperature \( L = 16, 32, 64, 128, 256 \). For high temperatures, the curves seem to cross at a single point. However, a spurious first order transition intervenes at low \( T \) (large \( L \)).
which may be rendered finite through an ultraviolet regularization of $K(s)$, \textit{viz.,}

$$K_b(s) = \left\{ \frac{\pi}{L} \csc \left( \frac{\pi |s|}{L} \right) \cdot \left[ 1 - e^{-L\sin(\pi |s|/L)/\pi b} \right] \right\}^{2-\epsilon},$$

so that $K_b(0) = b^{-(1-\epsilon)}$

The partition function for our problem is given by

$$\Xi(\nu) = \Xi_0 \left( e^{i\nu(\phi(L)-\phi(0))} \right) \sum_{N=0}^{\infty} \frac{1}{N!} \left( -S_{\text{int}} \right)^N$$

$$= \Xi_0 \sum_{N=0}^{\infty} \frac{\alpha^N}{N!} \left( \frac{\pi L}{\nu} \right)^{\frac{N}{2}} \sum_{s_n} \left\langle e^{i\nu\phi(L)} e^{-i\nu\phi(0)} e^{i\sum_{k=1}^{N} [\phi(s_k^+) - \phi(s_k^-)]} \right\rangle_0$$

where the average is with respect to the bare action $S_0$. This allows us to transform the complex action (due to the topological term) in the phase representation to a purely real action in a dipole gas representation\textsuperscript{28}. The coordinates $\{s_k^\pm\}$ are interpreted as locations of positive and negative charges. Each factor of $(-S_{\text{int}})$ introduces another pair of such charges, \textit{i.e.} a dipole. We now write the field $\phi(s)$ as a sum of winding plus a periodic pieces:

$$\phi(s) = \frac{2\pi W s}{L} + \eta + \varphi(s)$$

$$\varphi(s) = \frac{1}{\sqrt{L}} \sum_{\omega_n} \varphi(\omega_n) e^{-i\omega_n s},$$

where $\eta$ is a constant and $W$ is the winding number. The prime on the sum denotes exclusion of the $n = 0$ term, which is accounted for by $\eta$. The bare action is then

$$S_0 = \frac{2\pi W^2}{L} + \frac{1}{4} \sum_{\omega_n} |\varphi(\omega_n)|^2$$

Thus, $\left\langle |\varphi(\omega_n)|^2 \right\rangle_0 = 2/\omega_n^2$, and

$$C(s) = \frac{1}{2} \left\langle |\varphi(s) - \varphi(0)|^2 \right\rangle$$

$$= \frac{L}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos(2\pi ns/L)$$

$$= \left\{ s \left( 1 - \frac{s}{L} \right) \right\}_{\text{per}},$$

where the subscript indicates that the expression is to be periodically extended from its value on the interval $s \in [0, L]$. Summing over the winding number $W$ and averaging over $\phi(s)$, we obtain the $N$-dipole pair Boltzmann weight,

$$\vartheta_N(s_1^+, \ldots, s_N^-) = \frac{\alpha^N}{N!} \prod_{j=1}^{N} K(s_j^+ - s_j^-) \times$$

![FIG. 5: Single pair potentials $V_C(s)$ and $V_C(s) + V_d(s)$ for $\nu = 0$ (solid), $\nu = 0.25$ (dashed), and $\nu = 0.5$ (dot-dashed). Upper panels have $L = 5$; lower panels have $L = 25$. In all cases $b = 0.2$.](image-url)
and where the location of each charge is restricted to the interval $s_0 \in [0, L]$, so that $P \in [-NL, NL]$.

Interpreting (13) for $\nu = 0$, we have that the system consists of a gas of $N$ dipole pairs. Each of the $2N$ charges interacts with every other charge according to a one-dimensional Coulomb potential $-\sigma \sigma' |s - s'|$. In addition, each charge is logarithmically attracted to its unique mate - the other member of its dipole pair - via a potential $V_d(s) = -\ln K(s) \simeq (2 - \epsilon) \ln |s|$. For $\nu \neq 0$ there is an additional electric field of strength $2\nu$ present. $\alpha$ is the dipole fugacity; $\alpha \to 0$ will suppress the appearance of dipoles. Hence, in the absence of any short-time cutoff $b$, short-distance dipole pairs will proliferate without limit. A simple estimation of the dipole density, ignoring the Coulomb interactions, is $n_{\text{dip}} \sim b^{-(1-\epsilon)}/(1-\epsilon)$. In our Monte Carlo simulations we have used $b = 1$; the essential physics is rather weakly dependent on $b$, though. Grand canonical averages are to be computed in the usual way, i.e., $\langle A \rangle = \text{Tr}(\rho A)/\text{Tr} \rho$, where the trace entails a sum over all possible numbers $N$ of dipole pairs and integration over their $2N$ charge coordinates.

In figure 5 we plot the Coulomb energy for a single dipole pair,

$$V_C(s) = |s| + \ln \frac{H_L(s + \nu L)}{H_L(0)} + 2\nu s$$

as well as the total dipole energy $V_C(s) + V_d(s)$ for $\nu = 0$, $\nu = \frac{1}{2}$, and $\nu = \frac{1}{2}$ for $L = 5$ and $L = 25$. Note how the interaction becomes asymmetric (yet still properly periodic) when $\nu$ is neither integer nor half odd integer. Furthermore, the Coulomb interaction is effectively cancelled when $\nu = \frac{1}{2}$.

The XY phase correlation function,

$$G(s) = \langle e^{i\phi(s)} e^{-i\phi(0)} \rangle,$$

is simply related to the dipole separation correlation function,

$$h(s) = \langle \sum_i \delta(s_i^+ - s_i^- - s) \rangle.$$

To see this, define the quantity

$$Q_N(\tau^+, \tau^-) = (N + 1) \int_0^L ds_1^+ \int_0^L ds_1^- \cdots \int_0^L ds_N^+ \int_0^L ds_N^- \varrho_{N+1}(s_1^+, s_1^-, \ldots, s_N^+, \tau^+, \tau^-).$$

One then has

$$\alpha K(g) G(s) = \frac{1}{L^2} \sum_{N=0}^\infty \int_0^L d\tau Q_N(\tau + s, \tau) = h(s).$$

What is computed in the Coulomb gas Monte Carlo calculation is the dipole separation correlation function $h(s)$, from which the XY phase correlator is obtained via $G(s) = h(s)/\alpha K(g)$. The conductance is then obtained using (15).

The renormalized charging energy, $E_c^*$, is given by

$$\frac{E_c^*}{E_c} = \frac{1}{2} \frac{\partial}{\partial \nu^2} \bigg|_{\nu=0} = 1 - \frac{L^2}{2} \left( \frac{H''_L(P)}{H_L(P)} \right) + 2 \left( \frac{H'_L(P)}{H_L(P)} \right) - \frac{L}{2} (P^2).$$

Note that when the dipole fugacity $\alpha$ vanishes, there are no pairs at all and one obtains

$$\frac{E_c^*}{E_c} \bigg|_{\alpha=0} = 1 - \frac{L}{2} \frac{H''_L(0)}{H_L(0)}.$$

which also follows from an analysis of the noninteracting Hamiltonian $H_{\alpha=0} = E_c (\hat{n} + \nu)^2$. In this case the renormalized charging energy $E_c^*$ interpolates between its low temperature value of $E_c^* = E_c$ and its high temperature limit of $E_c^* = 0$.

A. Results

Typical raw data for $g_\alpha(\alpha, L)$ are shown in fig. 6. Two phases are identified for every positive value of $\epsilon$: a small
\( \alpha \) insulating phase in which the Coulomb gap persists and \( g_\alpha \) decreases as \( T \to 0 \) \((L \to \infty)\), and a large \( \alpha \) conducting phase in which \( g_\alpha \) diverges as \( T \to 0 \), indicating a nonlinear \( I-V \) relation at \( T=0 \). The boundary between these phases is marked by a critical value \( \alpha_c(\epsilon) \), at which there is a quantum phase transition. At \( \alpha = \alpha_c \), the conductance becomes temperature-independent (provided \( T \) is low enough that the system is in the scaling regime). Monte Carlo results for \( \alpha_c(\epsilon) \) and \( g_\alpha^*(\epsilon) \) are presented in fig. 7 along with comparisons to large-\( N \) results.

Our results for \( \alpha_c(\epsilon) \) differ significantly from those of Bascones et al.\(^{12} \), who obtained \( \alpha_c \) through analysis of the renormalized charging energy, assuming \( E^*_c(\alpha) \sim (\alpha_c - \alpha)^{1/\nu} \). (Our values for \( \alpha_c(\epsilon) \) are approximately four times smaller throughout the range \( 0 < \epsilon \leq \frac{1}{2} \)). In fact, we find that our raw data for \( E^*_c/E_c \) versus \( \alpha \) are in good agreement with those of Bascones et al.\(^{30} \). This agreement is noteworthy since their Monte Carlo was carried out in the phase representation whereas our is in the Coulomb gas representation. However, it is very difficult to reliably extract \( \alpha_c \) from the charging energy data, as our results shown in fig. 8 show. At a temperature corresponding to \( L = 50 \) (ref. 12) or \( L = 40 \) (this work), there is no detectable signature of the phase transition at \( \alpha = \alpha_c \). Similar behavior is found in the large-\( N \) results of fig. 11. In the large-\( N \) theory, \( \lambda(\alpha) \) plays the role of an energy gap, similar to \( E^*_c \). As is evident from fig. 11 even at relatively low temperatures of \( 1/L \approx 1/32 \), extrapolation of \( \alpha_c \) based on the zero-temperature critical (i.e., power law) behavior is problematic. Indeed, for fixed \( 1/L \), varying \( \alpha \) takes the system through renormalized classical, quantum critical, and quantum disordered regimes\(^{23} \), and the single parameter \( L = \infty \) behavior of \( \lambda(\alpha) \) is insufficient to extract \( \alpha_c \).

Finally, we plot conductance versus charge offset \( \nu \) for \( \epsilon = 0.2 \) and \( \epsilon = 0.8 \) in figs. 9 and 10, respectively. Two values of \( \alpha \) on either side of \( \alpha_c \) are chosen, corresponding to opposite temperature dependences at \( \nu = 0 \). We find a curious double peak structure in the vicinity of \( \nu = \frac{1}{2} \) at higher temperatures, but which disappears as \( T \to 0 \). For \( \nu = \frac{1}{2} \) and \( \epsilon = 0.8 \) the conducting state prevails even at small values of \( \alpha \). This feature is emphasized in fig. 11 where \( g_\lambda(\alpha, \nu = \frac{1}{2}) \) is contrasted for \( \epsilon = 0.8 \) and \( \epsilon = 0.1 \). For \( \epsilon = 0.8 \), a conducting state is observed down to values of \( \alpha \) as small as 0.01, well below the \( \nu = 0 \) critical value of \( \alpha_c \approx 0.07 \). Hence, it is possible for the junction to exhibit opposite temperature dependences in the troughs \((\nu \approx 0)\) and peaks \((\nu \approx \frac{1}{2})\) of the conductance as the gate voltage is varied. This result is to be contrasted with the behavior at \( \epsilon = 0.1 \), where the conductance at \( \nu = 0.5 \) is very weakly temperature dependent. Indeed, at \( \epsilon = 0 \) and \( \nu = \frac{1}{2} \), the inverse charging energy is known to diverge very weakly\(^{23} \). While the experiments of Joyez et al.\(^{34} \) seem to be perfectly consistent with the more familiar \( \epsilon = 0 \) behavior, anomalous temperature dependence in the troughs has been observed in the conductance of quantum dots\(^{35} \), although it is hardly clear that the nonequilibrium effects which we consider.

![FIG. 7: Critical coupling \( \alpha_c(\epsilon) \) (top) and critical conductance \( g_\alpha^*(\epsilon) \) (bottom), comparing data from Monte Carlo (points) and large-\( N \) (smooth curve) calculations. The dashed line in the top figure is the Kosterlitz RG result \( \alpha^*_{RG} = 1/2\pi^2\epsilon \).](image1)

![FIG. 8: Dimensionless renormalized charging energy \( E^*_c/E_c \) versus \( \alpha \) at \( L = 10 \) (filled triangles), \( L = 20 \) (open squares), and \( L = 40 \) (filled circles) for \( \epsilon = 0.50 \) (top panel) and \( \epsilon = 0.80 \) (bottom panel). Arrows indicate the location of the phase transition as obtained from crossing of conductance curves.](image2)
are uniquely responsible for this phenomenon.

V. CONCLUSIONS

Nonequilibrium shakeup processes have the potential to drastically affect the physics of tunnel junction behavior, by allowing for a conducting phase in which the Coulomb blockade is completely suppressed. The phase transition is made manifest within a large-$N$ approach, although it was first predicted over 25 years ago by Kosterlitz\cite{24} in his renormalization group studies of $O(N)$ classical spin chains with long-ranged (power law) interactions.

The phase transition has two principal signals. First, the renormalized charging energy $E^*_c$ vanishes for $\alpha \geq \alpha_c$. Second, while the conductance vanishes for $\alpha < \alpha_c$ and diverges for $\alpha > \alpha_c$, precisely at the transition $g_c = g(\alpha_c)$ is finite and universal (although $\epsilon$-dependent). Extracting the critical value $\alpha_c$ from numerical data at finite temperature can be tricky; we have found. In particular, the renormalized charging energy shows little signal at $\alpha_c$ even at dimensionless inverse temperatures on the order of $L = E_c/k_B T \sim 40$; this behavior is borne out explicitly in our large-$N$ studies. It is more reliable to obtain $\alpha_c$ from the crossing of the conductance curves $g(\alpha, L)$.

An extension of the model investigated here to the case of granular systems has been recently considered.\cite{36}

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VII. APPENDIX: ABSENCE OF LONG-RANGED ORDER FOR $\epsilon = 0$

Applying Mermin’s classic Bogoliubov inequality arguments, Simánek\cite{38} has proven the absence of long-ranged order in the $1/n^2$ XY chain. Here we (trivially) extend Simánek’s work to the continuum, and show how the $\epsilon = 0$ case is marginal, i.e. absence of order cannot be so proven for $\epsilon > 0$.

We begin with an action

$$S[P(s), \phi(s)] = \int_0^L ds \left[ \frac{1}{2} P^2(s) + \frac{1}{4} (\partial_s \phi)^2 - h \cos \phi(s) \right] + \alpha \int_0^L ds \int_0^L ds' K(s - s') (1 - \cos[\phi(s) - \phi(s')]) .(53)$$
Note that when $A$ is a functional of $\phi(s)$ alone, the functional integral over the momentum field $P(s)$ cancels between numerator and denominator, i.e. we recover the modified BMS model.

The Bogoliubov inequality guarantees
\[
\langle A^* A \rangle \geq \frac{|\langle \{C, A^* \} \rangle |^2}{\langle \{C, \{C^*, S \} \} \rangle},
\]
for an arbitrary functional $C[P, \phi]$. Following Šimánek, we take
\[
A[P, \phi] = \int_0^L ds \sin(\phi(s)) e^{-i\omega_n s}
\]
and
\[
C[P, \phi] = \int_0^L ds P(s) e^{-i\omega_n s},
\]
where $\omega_n = 2\pi n/L$ is a Matsubara frequency. In the $L \to \infty$ limit, the Bogoliubov inequality then establishes the following identity:
\[
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{m^2}{\hbar^2 \omega^2 + 2\alpha C_n |\omega|^{1-\epsilon}} \leq 1,
\]
where $m = \langle \cos(\phi(0)) \rangle$ is the average magnetization density. When $\epsilon = 0$, the integral diverges as $-m^2 \ln(hm)$, hence the inequality demands $m = 0$ when $h = 0$, i.e. no long-ranged order. However, the case $\epsilon = 0$ is marginal, and for any $\epsilon > 0$ the integral is convergent when $h = 0$. While this does not rigorously prove the existence of an ordered phase for $\epsilon > 0$, it is at least consistent with our findings.

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