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Eigenvalue inequalities for the buckling problem of the drifting Laplacian of arbitrary order.

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Summary: In this paper, we investigate the buckling problem of the drifting Laplacian of arbitrary order on a bounded connected domain in complete smooth metric measure spaces (SMMSs) supporting a special function, and successfully obtain a general inequality for its eigenvalues. By applying this general inequality, if the complete SMMSs considered satisfy some curvature constraints, we can obtain a universal inequalities for eigenvalues of this buckling problem.

MSC:
35P15 Estimates of eigenvalues in context of PDEs
35J40 Boundary value problems for higher-order elliptic equations
53C20 Global Riemannian geometry, including pinching
53C42 Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)
74K20 Plates

Keywords:
eigenvalues; universal inequalities; poly-drifting Laplacian; weighted Ricci curvature

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