Deep reinforcement learning for scheduling in large-scale networked control systems

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Abstract: This work considers the problem of control and resource scheduling in networked systems. We present DIRA, a Deep reinforcement learning based Iterative Resource Allocation algorithm, which is scalable and control-aware. Our algorithm is tailored towards large-scale problems where control and scheduling need to act jointly to optimize performance. DIRA can be used to schedule general time-domain optimization based controllers. In the present work, we focus on control designs based on suitably adapted linear quadratic regulators. We apply our algorithm to networked systems with correlated fading communication channels. Our simulations show that DIRA scales well to large scheduling problems.

Keywords: Networked control systems, deep reinforcement learning, large-scale systems, resource scheduling, stochastic control.

1. INTRODUCTION

Sequential decision making in uncertain environments is a fundamental problem in the current data-driven society. Typical problems occur in cyber-physical systems such as smart-grids, vehicular traffic networks, Internet of Things or networked control systems (NCS), see, e.g., Stojmenovic (2014). Many of these problems are characterized by decision making under resource constraints. NCS consist of many inter-connected heterogeneous entities (controllers, sensors, actuators, etc.) that share resources such as communication & computation. The availability of these resources do not typically scale well with system size; hence effective resource allocation (scheduling) is necessary to optimize system performance.

A central problem in NCS is to schedule data transmissions to available communication links. Traditionally, this is tackled by periodic scheduling or event-triggered control algorithms, see Heemels et al. (2012) and Park et al. (2018). To solve the sensor scheduling problem, Ramesh et al. (2013) proposed a suboptimal approach, where scheduler and control designs are decoupled. They also show that for linear single system problems with perfect communication it is computationally difficult to find optimal solutions. It must be noted that scheduling controller-actuator signals results in non-convex optimization problems, see Peters et al. (2016).

Networked controllers often use existing resource allocation schemes, however, such schemes typically reduce waiting times, and/or maximize throughputs, see Sharma et al. (2006). However, such approaches are not context aware, i.e. they do not take into account consequences of scheduling decisions on the system to be controlled.

An additional challenge for scheduling problems stems from the inherent uncertainty in many NCS. Specifically, an accurate communication dynamics model is usually unknown, see also Eisen et al. (2018). To this end, a combined scheduling and control design, which can use system state information and performance feedback, where optimal control and resource allocation go hand in hand to jointly optimize performance is highly desirable. In the present work, we tackle this problem by combining deep reinforcement learning and control theory.

Deep reinforcement learning (RL) is a combination of RL techniques with deep function approximators to break Bellmans curse of dimensionality, see Bellman (1957). Advances in deep RL has shown to be a promising toolbox to deal with uncertain large-scale problems. The most prominent algorithm is deep Q-learning, which achieved (super) human level performance in playing Atari games, see Mnih et al. (2015). Deep RL has been applied successfully to various control applications. Baumann et al. (2018) applied the recent success of deep actor-critic algorithms in an event-triggered control scenario. In Lenz et al. (2015), the authors combined system identification based on deep learning with model predictive control.

The potential of deep RL has also been applied to resource allocation problems, see for example Mao et al. (2016). Furthermore, the work of Demirel et al. (2018) and Leong et al. (2018) has shown the potential of deep RL for control and scheduling in NCS. However, these solutions do not extend well to large-scale problems, since the combinatorial complexity of the proposed algorithms grows rapidly with the number of systems and available resources.

The main contribution of this work is DIRA a Deep RL based Iterative Resource Allocation algorithm. This algorithm is tailored to large-scale control-aware resource
2. SCHEDULING AND CONTROL

2.1 Networked control system architecture

We consider a spatially distributed large control system of \( N \) discrete-time linear subsystems with state vectors \( x_k \in \mathbb{R}^{n_i} \) and control inputs \( u_k \in \mathbb{R}^m \). We write \( x_k \in \mathbb{R}^n \) and \( u_k \in \mathbb{R}^{m_N} \) for the concatenated state- and control-vector, respectively, where \( n := \sum_{i=1}^{N} n_i \). We denote by \( w_k \) the concatenated additive noise vector of i.i.d. noise processes \( w_k \sim \mathcal{N}(0, \Sigma_w) \). The system dynamics are considered to be coupled, while the input (actuator) dynamics are assumed to be independent; hence the overall linear system dynamic is given by:

\[
x_{k+1} = Ax_k + \begin{bmatrix} B^1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & B^N \end{bmatrix} u_k + w_k.
\]

We assume that the pair \([A, B]\) is controllable and that the pair \([A, Q^{1/2}]\) is observable. An illustration of the system model is shown in figure 1.

We consider that the system is to be controlled by a central control unit (CCU), which has access to all state vectors \( x_k \). The key feature of the problem at hand is that the control inputs \( u_k \) are transmitted over fading channels prone to dropouts and are coupled through a scheduling constraint. Ideally at time \( k \) the CCU wishes to transmit control signals to all \( N \) subsystems, but due to limited communication resources, it is only possible to transmit to \( M \) of them \((M < N)\). Thus, the CCU is allowed to only select a subset \( \{(i^*_k, j_k) \mid 1 \leq j \leq M\} \), where \( i^*_k \in \{1, \ldots, N\} \) and \( i_k^* \) is a candidate control value for subsystem \( i \).

As shown in figure 1, control value are sent over a network. With regards to communication, we use correlated fading channel models as described in Wang and Moayeri (1995). More precisely, we consider \( M \) channels with state-space fading results into a communication drop out with a probability according to the \( d \)-th component of the crossover probability vector \( e_j \). We define the NCS inputs as:

\[
u_k := \begin{cases} \hat{u}_k, & \text{if } \hat{u}_k \text{ is successfully received}, \\
0, & \text{otherwise}. \end{cases}
\]

This corresponds to a zero-input strategy in case of no available control data. We refer the reader to Schenato (2009) for a comparison between zero-input and hold-input strategies over lossy networks.

We consider that the parameters defining the communication network (2) are unknown. Estimating unknown parameters for a possibly time varying environment in an online manner is a difficult problem, Eisen et al. (2018). Often, the estimation relies on repeated test signals, with a large enough sample size, which could be expensive. In this work, we assume that the CCU has to act solely based on information gathered when transmitting over the network.

2.2 Joint scheduling and control problem

In our current NCS setup, the CCU has to schedule each of the \( M \) channels to one of the \( N \) subsystem actuators. For each channel \( j \) we define decision variables \( a_{i,k} \in \{1, \ldots, N\} \) such that \( a_{i,k} = i \) if channel \( j \) is scheduled to subsystem \( i \). At any time \( k \) the action space of the scheduling problem is given by:

\[ \mathcal{A}_k = \{a_k = (a_{1,k}, \ldots, a_{M,k}) \mid 1 \leq a_{j,k} \leq N, 1 \leq j \leq M \} . \]

Hence, the action space has a size of \( |\mathcal{A}_k| = N^M \). (We allow the CCU to use multiple distinguishable resources to close the controller actuator links.) At time \( k \), the CCU selects \((u_k, a_k) \in \mathbb{R}^{m_M} \times \mathcal{A}_k \), where \( u_k \) is the control action and \( a_k \) is the scheduling action.

We consider that the CCU wishes to find a stationary joint control-scheduling policy \( \pi \) mapping states to admissible

\[ \pi = \{(u_k, a_k) \mid u_k \text{ is a candidate control value for subsystem } i, 1 \leq j \leq M \} . \]

Note that the decisions space generalize to transmitting a candidate control value \( \hat{u}_j \) using parallel multiple channels to enhance the probability of successful reception, such that also scenarios where \( M \geq N \) can be considered. The CCU is therefore allowed to schedule more freely to improve control performance.
control signals, i.e. \( \pi : \mathbb{R}^n \to \mathbb{R}^{mM \times A} \). We can represent the joint control-scheduling policy by a pair \((\pi_c, \pi_s)\), where

\[
\pi_c : \mathbb{R}^n \to \mathbb{R}^{|\mathcal{A}|}, \quad \pi_s : \mathbb{R}^n \to \mathcal{A}_s.
\]

Observe that \( \pi(x_k) = (\pi_c(x_k), \pi_s(x_k)) \in \mathbb{R}^{mM} \times \mathcal{A}_s \), while \( \pi_c \) maps to candidate control signals for all actuators. Thus the CCU selects the elements of \( \pi_c(x_k) \) and the scheduling action \( \pi_s(x_k) \) to evaluate an admissible pair \((a_k, a_s)\) to \( \mathbb{R}^{mM} \times \mathcal{A}_s \).

The expected average cost following a stationary policy \( \pi \) with an initial state \( x \) reads

\[
J^\pi(x) = \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}_{x_k, u_k \sim \xi \sim \mathcal{E}} \left\{ \sum_{k=0}^{T} g(x_k, u_k) \mid x_0 = x \right\},
\]

where \( \mathbb{E} \{ \cdot \} \) denotes the expected value where \( \xi \) is distributed according to \( \mathcal{E} \). Here, \( \mathcal{E} \) denotes the system environment represented by the stochastic processes \((W_k, Z_k^1, \ldots, Z_k^M, k \geq 0)\). We define the single state costs \( g(x, u) \) as quadratic costs, i.e. \( g(x, u) := x^T Q x + u^T Ru \), where \( Q \) is positive semi-definite and \( R \) is positive definite.

The direct minimization of \( J^\pi(x) \) over all admissible policies for all states \( x \in \mathbb{R}^n \) is a difficult problem. This stems from the fact that the set of admissible control signals is clearly non-convex. Additionally, the expectation has to be carried out with respect to the Markov process dynamics of each channel, which are assumed to be unknown a priori. These challenges motivate the use of model-free learning techniques in combination with optimal control methods to find a suboptimal solution for the joint scheduling and control problem.

3. DEEP RL FOR CONTROL AWARE RESOURCE SCHEDULING

To obtain a tractable solution, we shall decompose the joint scheduling and control problem into the following parts:

(i) A deep reinforcement learning based scheduling agent DIRA, which iteratively picks actions \( a_k \in \mathcal{A}_s \) at every \( k \).

(ii) A (suboptimal) time-varying linear quadratic controller, which computes candidate control signals \( \hat{a}_k \) based on the expected success rate of each controller-actuator link.

In Section 3.3 we will detail on (i). Section 4 describes our controller design (ii) and shows how to combine our decomposition to an algorithmic approach for the control communication co-design problem. Before proceeding, we give some background on deep reinforcement learning.

3.1 Background on deep RL

In RL an agent seeks to find a solution to a Markov decision process (MDP) with state space \( \mathcal{S} \), action space \( \mathcal{A} \), transition dynamics \( P \), reward function \( r(s, a) \) and discount factor \( \gamma \in (0, 1] \), by interacting with an environment \( \mathcal{E} \), via a policy \( \pi : \mathcal{S} \to \mathcal{A} \). The aim is to solve the Bellman equation

\[
Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left\{ r(s, a) + \gamma \max_{a' \in \mathcal{A}} Q^*(s', a') \mid s, a \right\},
\]

see, e.g., Bertsekas and Tsitsiklis (1996). Deep RL approaches such as deep Q-Learning seek to estimate \( Q^* \) (also known as Q-factors) for every state-action pair. The resulting optimal policy is given by

\[
a^* = \arg \max_{a \in \mathcal{A}} Q^*(s, a), \quad \forall s \in \mathcal{S},
\]

and is popularly referred to as a “greedy policy with respect to \( Q^* \)”. A neural network based approximation of the Q-factors, given by \( Q(s, a; \theta) \approx Q^*(s, a) \) with weights \( \theta \), is called a Deep Q-Network (DQN), see Mnih et al. (2015). Deep Q-Learning seeks to identify optimal neural network weights \( \theta^* \). This is done by performing mini-batch gradient descent steps to minimize the squared Bellman loss \( (y_k - Q(s, a; \theta))^2 \), at every time \( k \), with the target values

\[
y_k = r(s, a) + \gamma \max_{a' \in \mathcal{A}} Q(s', a'; \theta).
\]

These mini-batches are sampled from an experience replay \( R \) with capacity \( G \) to decorrelate training from interaction, i.e. to reduce the bias towards recent interactions. Finally, the targets \( y_k \) are usually computed using a separate neural network with weights \( \theta_{target} \), see Section 5.3.

3.2 DQN for resource allocation

Suppose now that a control policy \( \pi_s \) is given. In principle, we can find a scheduling policy \( \pi_c \) in an control aware manner using the DQN paradigm as described in Section 3.1. For that purpose, we define an MDP \( M_1 \), with state space \( \mathcal{S}_1 = \mathbb{R}^n \), action space

\[
\mathcal{A}_1 = \{(a_{1,k}, \ldots, a_{M,k}) \mid 1 \leq a_{i,k} \leq N, 1 \leq i \leq M \},
\]

reward signal \( r_k = -g(x_k, u_k) \) and discount factor \( \gamma \in (0, 1] \). By assigning the negative one-stage cost as a reward, a solution to \( M_1 \) minimizes the discounted cost

\[
\limsup_{T \to \infty} \mathbb{E}_{x_k, u_k \sim \mathcal{E}} \left\{ \sum_{k=0}^{T-1} \gamma^k g(x_k, u_k) \mid \pi_c \right\}.
\]

Unfortunately, the direct application of Deep Q-Learning to solve this MDP is infeasible when \( N \) and \( M \) are large, since the algorithm is usually divergent for large action spaces of size \( |\mathcal{A}| = N^M \). The next subsection presents a reformulation of \( M_1 \), which goes beyond DQN to address this scalability issue. In section 4 we present a control policy design \( \pi_c \), which adapts to the learned schedule \( \pi_s \).

3.3 An MDP for iterative resource allocation

The intractability of the scheduling problem for large action spaces, is addressed next by exploiting the inherent iterative structure of the resource allocation problem. Let us say that the system state is \( x_k \) at time \( k \). The scheduler (DIRA) iteratively picks component-actions \( a_{1,k}, \ldots, a_{M,k} \) to obtain scheduling action \( a_k \). Recall that \( a_{j,k} = i \) when subsystem \( i \) is allocated to channel \( j \). Once \( a_k \) is picked, the CCU transmits the associated control signals, see Fig. 1. After that, the scheduler receives acknowledgment for successful transmissions and a reward \( r_k \) (performance feedback).

To implement the above described iterative procedure, we introduce intermediate states. For this, we construct an \( M \)-dimensional representation vector

\[
h_k := (h_{1,k}, \ldots, h_{M,k})
\]
where $h_{1,k}$ is associated with component-action $a_{j,k}$. The intermediate states are defined as $(x_k, h_k)$, where each element of $h_k$ is assigned once at a fast rate as illustrated in Fig. 2 with $N = M = 3$. Let the system state at time $k$ be $x_k$. Before picking $a_{j,k}$’s, $h_k$ is initialized to the zero vector. Now, action $a_{1,k}$ is picked as a function of $(x_k, h_k)$. Let us say $a_{1,k} = 2$, then $h_k$ is updated to $(10, 00, 00)$, i.e. $h_{1,k} := 10$ (where 10 is the binary representation for 2) and all other $h_{j,k}$’s remain unchanged. Now, if $a_{2,k} = 1$ and $a_{3,k} = 3$, then $h_k$ is updated to $(10, 01, 00)$ followed by an update to $(10, 01, 11)$. Thus $a_k = (2, 1, 3)$ is selected.

To embed such an iterative procedure in an MDP, it is convenient to proceed as follows: Let $x, x' \in S_1$ and $h, h' \in H_1$, where $H_1$ denote the space of all possible representation vectors $h$. We say that $(x, h)$ and $(x', h')$ are “equivalent” iff $x = x'$. We define an equivalence class by $[x] := \{(x', h) \mid (x', h') \in S_1 \times H_1, x' = x\}$. Between times $k$ and $k+1$, the system state $x_k$ is frozen, while the representation vector $h_k$ changes.² Hence all the intermediate states are equivalent. The idea of freezing a portion of the state space is inspired by Mao et al. (2016). We defined this equivalence, since we assign the same reward $r_k$ to all intermediate actions $a_{j,k}$. This is because the $a_{j,k}$’s are combined to obtain the action $a_k$, which in turn results in one single stage cost.

We therefore have a natural MDP reformulation $M_2$: $S_2$ : contains all state equivalence classes as defined above. $A_2$ : is given by $\{1, \ldots, M\}$. $r$ : is given by $-g(x_k, u_k)$. $\gamma$ : is the discount factor such that $\gamma \in (0, 1]$. In the next section we will solve $M_2$ using the DQN paradigm. The important consequence of the reformulation is that $M_2$ has an action space of size $|A_2| = N$, opposed to $|A_1| = N^M$.

4. JOINT CONTROL AND COMMUNICATION

4.1 Controller design

Until now we have considered how to schedule resources for a given control policy. We achieved “control awareness” of our scheduler by providing the negative one stage costs as rewards in the MDPs, see Section 3.3. Conversely, we would also like to achieve a form of “schedule awareness” for our control algorithm. Additionally, we would like to incorporate system knowledge, i.e. knowledge of the matrices $A$ and $B$. These points motivate a simple yet effective controller design building on linear quadratic control. Specifically, we parameterize a linear quadratic regulator by the expected rate at which the control loop is closed via each input dynamics and use an approximation of the average decision and channel success probabilities to update the controller during runtime.

Let $\delta_i^k$ be random variables such that $\delta_i^k = 1$, if a control signal $u_i^k$ is successfully received at actuator $i$. Now the system dynamics in (1) can be written in terms of $\delta_i^k$.

Define $\Delta_k = \text{diag}(\{I m \delta_i^k\}_{i=1}^N)$, where $I_n$ denotes the identity matrix of dimension $m \times m$. Then define $B_{\Delta k} = B_{\Delta k}$.

Consider the LQR problem with finite horizon $T$:

$$
\min_{\{u_k\}} \mathbb{E}_{x_k, B_k} \left\{ x_0^T Q x_T + \sum_{k=0}^{T-1} (x_k^T Q x_k + u_k^T R u_k) \right\},
$$

s.t. $x_{k+1} = A x_k + B_{\Delta k} u_k + w_k$.

Assume that $B_{\Delta k}$ are independent with finite second moments, then the dynamic programming framework yields the following optimal finite horizon solution to (3)

$$
u_k^* = - \left( R + \mathbb{E} \{ B_{\Delta k} K_{k+1} B_{\Delta k} \} \right)^{-1} \mathbb{E} \{ B_{\Delta k} K_{k+1} A x_k \} \quad \text{with} \quad K_T = Q,
$$

$$
K_k = A^T K_{k+1} A + Q - A^T K_{k+1} B_{\Delta k} \left( R + \mathbb{E} \{ B_{\Delta k} K_{k+1} B_{\Delta k} \} \right)^{-1} \mathbb{E} \{ B_{\Delta k} \} \quad \text{for} \quad k = 0, \ldots, T-1.
$$

We will use the steady state controller

$$
u_k^* = - \left( R + \mathbb{E} \{ B_{\Delta k} K_{\infty} B_{\Delta k} \} \right)^{-1} \mathbb{E} \{ B_{\Delta k} \} \quad \text{with} \quad K_\infty = A^T K_\infty A + Q - A^T K_\infty B_{\Delta k} \left( R + \mathbb{E} \{ B_{\Delta k} K_{\infty} B_{\Delta k} \} \right)^{-1} \mathbb{E} \{ B_{\Delta k} \}.
$$

It is important to point out that equation (6) does not necessarily have a solution, i.e., (4) does not need to converge to a stationary value, see Section 3.1 of Bertsekas (2017). The following lemma establishes conditions such that (6) has a steady state solution. It extends a result from Ku and Athans (1977) to the present case, where $B$ is disturbed by a multiplicative diagonal matrix.

**Lemma 1.** A steady state solution for (4) exists if

$$\lambda_{\max}(\Gamma A) < 1,
$$

where $\Gamma$ is defined by

$$\Gamma = \text{diag} \left\{ I_m \left( \sqrt{1 - \frac{\mathbb{E}(\delta_i^k)^2}{\mathbb{E}(\delta_i^k)}} \right)^N \right\},
$$

and $\lambda_{\max}(\cdot)$ denotes the largest absolute eigenvalue.

**Proof.** Consider the recursive Riccati equation (4). Define $\alpha_1 = \mathbb{E} \{ \Delta_k \}$ and $\alpha_2 = \mathbb{E} \{ \Delta_k^2 \}$. Observe that, since $B_{\Delta k} K_{k+1} B_{\Delta k}$ is symmetric, it commutes with $\Delta_k$. Thus we have $\mathbb{E} \{ B_{\Delta k} K_{k+1} B_{\Delta k} \} = \alpha_2 B_{\Delta k} K_{k+1} B_{\Delta k}$. Similarly, we can rewrite $\mathbb{E} \{ B_{\Delta k} K_{k+1} B_{\Delta k} \} = \alpha_2 B_{\Delta k} K_{k+1} B_{\Delta k}$. Now observe that

$$\alpha_2 \alpha_2^{-1} - \beta B_{\Delta k}, \text{with } \beta = \text{diag} \left\{ I_m \left( \sqrt{1 - \frac{\mathbb{E}(\delta_i^k)^2}{\mathbb{E}(\delta_i^k)}} \right)^N \right\},
$$

with $M_k := K_{k+1} - K_{k+1} B_{\Delta k} \left( \alpha_2^{-1} R + B_{\Delta k} K_{k+1} B_{\Delta k} \right)^{-1} B_{\Delta k} K_{k+1}$.
since $\beta$ commutes with $K_{k+1}$. Now following similar lines as in Ku and Athans (1977) we have a bound $M_k \leq L \forall k$, since $[A, B]$ is controllable and $[A, Q^{1/2}]$ is observable. Define $Q' := Q - A^T(1 - \beta)LA$. Then we have

$$K_k \leq A^T(1 - \beta)^{1/2}K_{k+1}(1 - \beta)^{1/2}A + Q'. \quad (8)$$

Now the recursion associated to (8) converges by Lyapunov stability theory, if the eigenvalues of $(1 - \beta)^{1/2}A = \Gamma$ lie in the unit circle and so does (7).

Under the conditions of Lemma 1 we will use (5) as a (suboptimal) time-varying control solution in combination with our iterative scheduling algorithm. More specifically, we calculate $K_\infty$ using sample based approximations of the expected values $E\{\delta_k^1\}$ and $E\{\{\delta_j\}^2\}$. In doing this, the controller varies according to the expected rate at which the controller-actuator links are closed and therefore adapts to the scheduler behavior. After a scheduling action $a_{k} \in A_k$ is chosen by $\pi_s$, we evaluate

$$u_k = -(R + E\{B_{\delta_{k}}K_\infty B_{\delta_{k}}|a_k\})^{-1}E\{B_{\delta_{k}}|a_k\} K_\infty Ax_k,$$

where the expectations are evaluated with respect to the actual scheduling action $a_k$ and the approximated channel success rates. After that, these control signals, which correspond to a one-step look-ahead controller using $K_\infty$ as terminal costs, are transmitted.

\subsection{DIRA}

The combination of the scheduler design of Section 3.3 and controller design of Section 4.1 results in the following Deep Q-Learning based Iterative Resource Allocation (DIRA) algorithm with time-varying linear quadratic regulation (LQR). For DIRA we define $s_k$ as $(x_k, h_k)$.

\textbf{Algorithm 1} DIRA with time-varying LQR

1: Initialize the Q-network weights $\theta$ and $\theta_{\text{target}}$.
2: Initialize replay memory $\mathcal{R}$ to size $G$ and fix $c_1$ & $c_2$.
3: for the entire duration do
4: Select action $a_k$ as described in section 3.3 with exploration parameter $\epsilon$.
5: Execute $a_k$ to obtain reward $r_k$ and state $s_{k+1}$.
6: Store selection history in $\mathcal{R}$, by associating $r_k$ and $s_{k+1}$ to each intermediate state of step 5.
7: for each intermediate state do
8: Sample a random minibatch of $N$ transitions $(s_k, a_k, r_k, s_{k+1})$ from $\mathcal{R}$.
9: Set $y_j = r_j + \gamma \max_{a'} Q(s_{j+1}, a'; \theta_{\text{target}})$.
10: Gradient descent step on $(y_j - Q(s_j, a_j; \theta))^2$.
11: end for
12: Every $c_1$ steps set $\theta_{\text{target}} = \theta$.
13: Every $c_2$ steps approximate $K_\infty$.
14: end for

At each time-step $k$ an action is selected by the procedure introduced in Section 3.3 in an $\epsilon$-greedy manner. Specifically, for all intermediate time-steps we pick a random action with probability $\epsilon$, and we pick a greedy action as in (3.1) with probability $1 - \epsilon$. During training, the exploration parameter $\epsilon$ is decreased to transition from exploration to exploitation. In step 8, the targets for the Q-Network are computed using a target network, with weights $\theta_{\text{target}}$, which are updated to $\theta$ every $c_1$ steps in step 12. This technique, also introduced by the authors of DQN, results in less variation of the target values, which improves learning. In step 13, we approximate $K_\infty$ using collected samples. Updating the control policy at every time-step increases the computational effort since the Riccati equation has to be solved accurately at every time-step. Additionally, updating the control policy frequently induces non-stationary in the environment, which makes learning difficult. Therefore, we update the control policy only every $c_2$ steps.

\textbf{Remark 2.} Acknowledgment of successful transmissions are only necessary in the learning phase. Thus, a converged policy can be used without any communication overhead.

5. NUMERICAL RESULTS

\textbf{Experimental set-up:} We conducted three sets of experiments for a varying number of subsystems and resources. Specifically, we evaluate the scaling of our algorithm by considering the pairs $(N = 4, M = 3)$, $(N = 8, M = 6)$, $(N = 12, M = 9)$. The systems are generated using random second order subsystems, which are then coupled weakly according to a random graph. Regarding stability, we generated 50% of the subsystems as stable and 50% as unstable, with at least one eigenvalue in range $(1,1.5)$. Additionally, the systems are selected such that the optimal loss per subsystem equals approximately 10. For communication, we consider two state Markov models known as Gilbert-Elliot models. We consider two channel types (“good” & “bad”) with parameter tuples

$$\left( \begin{array}{ccc} 0.9 & 0.1 & 0.9 \\ 0.1 & 0.9 & 0.1 \\ 0.9 & 0.1 & 0.9 \end{array} \right), \left( \begin{array}{ccc} 0.8 & 0.2 & 0.8 \\ 0.2 & 0.8 & 0.2 \\ 0.8 & 0.2 & 0.8 \end{array} \right), \left( \begin{array}{ccc} 0.4 & 0.5 & 0.4 \\ 0.5 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.4 \end{array} \right)$$

as introduced in section 2.1. In every experiment 1/3 of the channels are “good” and 2/3 are “bad”.

\textbf{Algorithm hyper-parameters:} In all experiments, the Q-Network is parameterized by a single hidden layer neural network, which is trained over epochs using the optimizer ADAM, see Kingma and Ba (2015). For the three sets of experiments we vary the following parameters: We used $(512, 1024, 1536)$ for the number of rectifier units in the hidden layer of the Q-Network, we trained over $(50,100,150)$ epochs, $\epsilon$ is initialized to 1 and attenuated to 0.001 at attenuation rates $(0.9999, 0.99995, 0.99999)$, and we use learning rates $(e^{-3.5}, e^{-4.5}, e^{-5.5})$. In all experiments, we use the following parameters: $\gamma = 0.95$, $G = 30000$, $T = 500$, minibatch size $32$, $c_1 = 100$, $c_2$ is set such that $K_\infty$ is updated once every epoch.

In all our experiments a uniformly random scheduling policy results in an unstable system. We observed empirically, that the use of a slightly more “clever” random policy, used to initialize the Q-Network and to explore during training, can improve learning convergence. Specifically, the random agent assigns channels randomly according to the degree of stability of each subsystem and is able to stabilize the system in each experiment, though with poor quality.

Figure 3 shows the learning progress of our iterative agent DIFA for $(N=4, M=3)$ averaged over 15 Monte Carlo runs. For comparison, also the quality of the “clever” random agent is included. For DIFA, we also display three standard deviations as a shaded blue area around the mean loss. DIFA finds a policy, which achieves a control

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3 This is similar to the reason why target networks are used.
Our simulations showed that our co-design solution is scalable to large decision spaces. In the future we plan to present DIRA, an iterative deep RL based resource allocation algorithm for control-aware scheduling in NCS. We observed that DIRA is able to achieve good performance for the training results for all three experiments. We observe that the decision space has a size of \(N^M\) which equals (64, 262144, 5159780352), respectively. DIRA is able to find good policies in these large decision spaces in a control aware manner.

6. CONCLUSION

We presented DIRA, an iterative deep RL based resource allocation algorithm for control-aware scheduling in NCS. Our simulations showed that our co-design solution is scalable to large decision spaces. In the future we plan to consider state estimation, scheduling of sensor-controller links as well as the controller-actuator side and time-varying resources. Finally, we are also working towards a stability result.

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