The possibility of quasi-bound state formation of $\eta$-meson with helium isotopes

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The necessary conditions of quasi-bound state formation of $\eta$-meson with isotopes $^3$He, $^4$He have been found within the framework of optical potential model. These conditions have been compared with the findings about helium nucleus densities and with the available information about $\eta$N-scattering length. Thus, we have conclude that within the framework of discussed model $\eta^{-3}$He quasi-bound state formation is not possible, but $\eta^{-4}$He quasi-bound state formation is possible with the great probability.

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The interaction of $\eta$-mesons with helium isotopes has been considered in the frame of optical potential for the purpose of $\eta^{-3}$He and $\eta^{-4}$He quasi-bound state formation. Let us connect the optical potential distributions with the nuclear densities of the discussed nuclei using the findings of root-mean-square radii. For the description of low energy $\eta$N-interaction let us use a well-known fact of resonance domination $S_{11}(1535)$ in the amplitude of this interaction at such energies. In that case the optical potential of $\eta$-A interaction of $U(r)$ takes the following form [1]:

$$2\mu U(r) = -4\pi(1 + \frac{m_\eta}{m_N})\rho(r)a_0,$$

where $m_\eta$, $m_N$ are the meson and nucleon masses, $\mu$ is the reduced meson-nucleus mass, $a_0$ is the $\eta$N-scattering length, $\rho(r)$ is the spherically symmetrical density of nucleon in nuclei, which has been chosen in Fermi form:

$$\rho(r) = \frac{\rho_0}{1 + \exp(\frac{r-R_c}{a})}, \quad (2)$$

Here $R_c$ is the half-density radius, $a$ is the thickness of nucleus diffusion surface layer, $\rho_0$ is the nucleon density of nucleus in the center. For the nucleus with the nucleon number $A$, two parameters in distribution (2) may be fixed by the conditions:

$$A = \int_0^\infty r^2\rho(r)dr, \quad \langle r^2 \rangle = \frac{1}{A}\int_0^\infty r^4\rho(r)dr, \quad (3)$$

where $\langle r^2 \rangle^{1/2}$ is the root-mean-square radius ($r_{rms}$) of the nucleus. The knowledge of rms radii of $^3$He, $^4$He nuclei [2] leaves only one free parameter of $U(r)$ radial distribution, which is called “diffuseness” and stand for $a/R_c$. The nucleus densities depend on the diffuseness parameters, as it may be seen in figures 1, 2.

For the formation of quasi-bound state in the complex potential with complex energy eigenvalue $E = -i(\varepsilon + \frac{\Gamma}{2})$ where $\varepsilon$ is the binding energy, and $\Gamma$ is the level width, the definite relation between absolute values of imaginary and real parts of this potential is required, at which the bound state is possible [3, 4]. The calculated formation boundaries, $\varepsilon \approx 0$, of the discussed $\eta$-nuclei in the dependence of imaginary potential part on the real one for different nucleon distributions in nuclei $^3$He and $^4$He are shown in figures 3, 4 in the complex plane of free $\eta$N-scattering length. It is evident that at the nucleus diffuseness decrease for the quasi-bound state formation the greater real potential part is required, that is, real parts of $\eta$N-scattering length. At the diffuseness increase, when $a/R_c$ is over 0.25, the formation boundaries of $\eta$-nuclei practically stop shifting to the left, that limits the dependence of quasi-bound state formation on the nuclear density distribution. In view of impossibility of complex $\eta$N-scattering length experimental determination $a_0$ is found indirectly, that is, they are model dependent and differ greatly from paper to paper [2, 6, 7, 8, 9, 10, 11, 12] (see table 1 in paper [13]). The values of $\eta$N-scattering length getting into the darkened areas in figures 3, 4 demonstrate the possibility of $\eta$-nucleus formation at the attraction potential initiated by such $a_0$. And on the contrary, if these values are left in the white parts of figures 3, 4 the formation is not possible.

The bound states spectrum simulating results in $\eta^{-3}$He system are contradictory [13, 14, 15] and ambiguous [16]. The same situation presents in calculations of bound states in $\eta^{-4}$He system [13, 17]. One may see in figure 3 that formation of quasi-bound state $\eta^{-3}$He for the known data concerning length of $\eta$N-scattering and root-mean-square radius of $^3$He nucleus is impossible for any diffuseness of $^3$He nucleus density. If we use “off-shell” length of $\eta$N-scattering in potential (1) as authors of paper [13] insist, the conclusion will be the same because the real and imaginary parts of $a_0$ should, according to the cited paper, decrease proportionally. On the contrary, the existence of quasi-bound state of $\eta^{-4}$He within the limits of used model is possible and is almost independent from the diffuseness of $^4$He nucleus density as it may be seen in figure 4 if $|Re(a_0)| \geq 0.60$ fm.

The experimental result for both reactions $d + d \rightarrow ^4$He + $\eta$, $d + p \rightarrow ^3$He + $\eta$ near the thresholds point

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FIG. 1: The nucleon density distributions of $^3$He nucleus for different diffuseness parameter values $a/R_c$ and fixed root-mean-square radii. The curve parameters are given in table 1.

| $r_{rms}$ (fm) | $N^0$ | $a/R_c$ | $R_c$ (fm) | $\rho_0$ (nucleon/fm$^3$) |
|----------------|-------|----------|------------|-----------------------------|
| 1.9            | 1     | 0.10     | 2.210      | 0.090                       |
|                | 2     | 0.15     | 1.991      | 0.074                       |
|                | 3     | 0.20     | 1.770      | 0.093                       |
|                | 4     | 0.25     | 1.571      | 0.114                       |

FIG. 2: The nucleon density distributions of $^4$He nucleus for different diffuseness parameter values $a/R_c$ and fixed root-mean-square radii. The curve parameters are given in table 2.

| $r_{rms}$ (fm) | $N^0$ | $a/R_c$ | $R_c$ (fm) | $\rho_0$ (nucleon/fm$^3$) |
|----------------|-------|----------|------------|-----------------------------|
| 1.6            | 1     | 0.05     | 2.021      | 0.113                       |
|                | 2     | 0.10     | 1.874      | 0.132                       |
|                | 3     | 0.15     | 1.687      | 0.163                       |
|                | 4     | 0.20     | 1.499      | 0.203                       |
|                | 5     | 0.25     | 1.331      | 0.250                       |
FIG. 3: Curves are the boundaries of quasi-bound states in system $\eta - {^3}\text{He}$. The darkened areas are the areas of quasi-bound state formation of $\eta$-meson with $^3\text{He}$ nucleus in the complex plane of $\eta N$-scattering length for different diffuseness $a/R_c$ parameters: $1 - 0.25; 2 - 0.15; 3 - 0.1$. $\eta N$-scattering lengths have been taken from works: $\blacksquare - [5]; \bullet - [6]; \blacktriangle - [7]; \blacktriangledown - [8]; \blacklozenge - [9]; \square - [10]; \blacktriangledown - [11]; \bigcirc - [12]$.

FIG. 4: Curves are the boundaries of quasi-bound states in system $\eta - {^4}\text{He}$. The darkened areas are the areas of quasi-bound state formation of $\eta$-meson with $^4\text{He}$ nucleus in the complex plane of $\eta N$-scattering length for different diffuseness $a/R_c$ parameters: $1 - 0.25; 2 - 0.15; 3 - 0.05$. $\eta N$-scattering lengths have been taken from works: $\blacksquare - [5]; \bullet - [6]; \blacktriangle - [7]; \blacktriangledown - [8]; \blacklozenge - [9]; \square - [10]; \blacktriangledown - [11]; \bigcirc - [12]$.

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