Disentangling the role of geometry and interaction in many-body system dynamics: the emergence of anomalous dynamics from the underlying singular continuous spectrum

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We study the dynamical properties of an interacting many-body systems with an underlying non-trivial energy potential landscape which can induce a singular continuous single-particle energy spectrum. Our aim is to show that there is a non-trivial competition between the quasi-periodicity of the lattice - i.e. the geometry of the system- and many-body interaction, which give rise to anomalous propagation properties.

Introduction - The discovery of quasicrystals in 1982[1] and of protocols to produce large and stable samples[2] has triggered the theoretical studies aiming at understanding the origin of their unusual physical properties. Among others, their peculiar transport features, such as the resistivity increase with both decreasing temperature or increasing the sample purity [3], have attracted an increasing attention [4]. It was soon realized that this behavior is strictly linked to the singular continuous (SC) nature of the single-particle spectrum, with the accompanying critical eigenfunctions [5], whose scaling properties can account for anomalous transport and diffusion, and can partially explain the unusual behavior of these materials [4]. Before the discovery of quasicrystalline materials, the mathematical concept of SC spectrum [6] was thought to be a purely mathematical elucidation with no physical counterpart [7]. The SC part is not easily accessible, and, often, its presence is inferred after removing the absolutely continuous (AC) and pure-point (PP) parts from the whole spectrum, provided a set of non-zero measure is left over.

The role of SC spectra in the dynamics of non-interacting systems has been investigated in Ref. [9] and its link to anomalous propagation of correlations and in the spreading of an initially localized wave-packet has been investigated in Refs. [8,10]. A particularly interesting, exemplary physical model where the nature of the spectrum plays a crucial role is the Aubry-André model (AAM), which describes particles hopping in a one-dimensional “quasi-periodic” lattice. It has been proven that displays a metal-to-insulator transition [11,12] with the spectrum being AC and PP in the metal and insulating phases, respectively, whereas it is purely SC at the transition point. The model has been realized with ultracold atoms loaded in a bichromatic optical lattice [13–15]. Due to the presence of interactions, such system displays a non-trivial phase diagram [16–18], with the appearance of a mobility edge [19,20] and of a many-body-localized phase separating an ergodic from a localized one [14–15].

Inspired by recent experiments [14–15] reporting the observation of the dynamical slowing down of an interacting gas loaded in an incommensurate bichromatic lattice, we aim at providing an explanation for these observations based on the nature of the single-particle energy spectrum of the AAM model. Our results, valid for small interactions, point towards the phase diagram sketched in Fig.[1] in terms of the interaction strength, \(U\), and of the onsite potential of the AAM, \(\lambda\). We have found that a threshold value \(\lambda_0\) exists, slowly decreasing with increasing \(U\), such that for \(\lambda > \lambda_0\) the single-particle spectrum has a PP nature, with a frozen dynamics and a localized system; on the other hand, for \(\lambda < \lambda_0\), the dynamics is ergodic with typical time scales of the order of that of a single particle with an AC spectrum. These two extreme behaviors are separated by an intermediate region where the dynamics is still ergodic but on time scales much larger than the typical single-particle time. This is where the slowing down of the dynamics, reported in Ref. [14], occurs. Our findings imply that such a behavior is due to a non-trivial competition between the underlying order induced by the potential energy landscape, and the many-body interactions.

The model and physical quantities - We consider a gas of spin-1/2 particles in one dimension, described by the Fermi-Hubbard model:

\[
\hat{H} = \sum_{n,\sigma} \epsilon_n \hat{c}_{n,\sigma}^\dagger \hat{c}_{n,\sigma} - \frac{J}{2} \sum_{n,\sigma} (\hat{c}_{n+1,\sigma}^\dagger \hat{c}_{n,\sigma} + \text{h.c.}) + \frac{U}{2} \hat{n}_{n,\uparrow} \hat{n}_{n,\downarrow},
\]  

where \(\epsilon_n\) is the onsite energy, \(U\) the on-site interaction between particles with different spin in the s-wave approximation, \(\hat{c}_{n,\sigma}^\dagger\) (\(\hat{c}_{n,\sigma}\)) are fermion creation (annihilation) operators at site \(n\) with spin \(\sigma\) and \(\hat{n}_{n,\sigma} = \hat{c}_{n,\sigma}^\dagger \hat{c}_{n,\sigma}\) the corresponding num-

![Figure 1.](image-url)
ber operator. We work with open boundary conditions not to enforce any artificial periodicity. The AAM is obtained by setting $\epsilon_n = J\lambda \cos(2\pi n)\frac{21}{23}$ with $\tau = \left(\sqrt{5} + 1\right)/2$.

If not otherwise stated, we consider the same initial state as in the experiment [14], with two particles with opposite spin on even sites; the subsequent time evolution is then generated by the Hamiltonian in Eq.(1). In the case of ultracold gases, this corresponds to a quench where at the initial time $t = 0$ both nearest-neighbour tunneling and onsite interaction are brought to finite values on a time scale much shorter than the tunneling time $J^{-1}$, but large enough not to excite transitions to higher bands [22].

At the single particle level, the dynamics of the system is encoded in the lesser and greater Green’s functions defined as $G^<_{\sigma}(t';t) = i\langle \hat{c}_\sigma(t')\hat{c}_\sigma(t) \rangle_0$ and $G^>_{\sigma}(t';t) = -i\langle \hat{c}_\sigma(t)\hat{c}_\sigma(t') \rangle_0$ respectively, where the average is over the initial state, and we use the notation $s = n, \sigma$. Information on the spectral properties, instead, are encoded in the spectral function $A(T, k, \omega) = iN^{-1}\sum_{n,m} e^{i(\epsilon_n - \epsilon_m)k} \int \frac{d\tau}{2\pi} \delta^{\omega + \epsilon_n - \epsilon_m - 2\tau} \Omega(T + \frac{\tau}{2}, T - \frac{\tau}{2})$ where $\Omega = Tr_c\{G^r - G^\lambda\}$ and the trace is over the spin degrees of freedom.

To compute these quantities we resort to the non-equilibrium Green’s functions technique, by solving numerically the Dyson equation for the single-particle Green’s function. Our approach closely follows the one of Refs. [22] and is an extension of the self-consistent approach presented in Ref. [23] for bosonic systems. The self-energy entering the Dyson equation is calculated in the second-Born approximation [26]; more details on the numerical implementation will be presented elsewhere [27].

Geometry-induced anomalous diffusion - We start by looking at the role of the geometry of the potential energy landscape, which affects the spreading of the correlations due to its influence on the nature of the single-particle energy spectrum. The spreading of correlations in a non-interacting system with a continuous energy spectrum (corresponding to extended eigenstates) is ballistic with a maximum velocity determined by both the energy spectrum and the initial state but it is always finite and bounded from above by the Lieb-Robinson bound [28]. In the case of a discrete energy spectrum (with exponentially localized eigenstates), the spreading is suppressed and correlations develop only in a finite region whose size is proportional to the localization length, thus going to zero in the thermodynamics limit.

In order to quantify the spreading of the correlations we use the variance of the probability distribution defined as $P_s(t) = \langle |G^<_{\sigma}(0; t)|^2 \rangle$ similarly to what has been done in Ref. [29]. Due to the absence of interaction, the spin degree of freedom factorizes is irrelevant; therefore, we can consider spinless fermions when $U = 0$.

In Fig. 2 panel a), we show the probability distribution $P_s(t)$ for the AAM with $L = 200$ sites in the metallic (extended) phase ($\lambda = 0.8$) and at the transition point ($\lambda = 1$). It can be clearly seen that, below the transition point, the spreading is ballistic, whereas, at the transition point, it acquires a (possibly anomalous) diffusive behavior. By focusing on the variance $\sigma(t)$ of $P_s(t)$, and assuming a power law behavior, $\sigma(t) \propto t^\alpha$ for $Jt \gg 1$, we looked at the behavior of the exponent $\alpha$ for different system sizes and different values of $\lambda$. The results are shown in Fig. 2 panel b), where we can see that for the AAM, the expansion tends to be ballistic in the thermodynamic limit as $\alpha = 1$ for $\lambda < 1$. On the other hand, at $\lambda = 1$ the exponent drops to $\alpha \approx 1/2$, thus signaling the presence of a diffusive behavior. For $\lambda > 1$, we observe a shrinking of the region where the power-law behavior is in order as the system size increases. The residual, observed expansion, therefore, can be attributed to the tails of the exponentially localized eigenstates (due to the finite size).

It is interesting to compare this behavior with that of a model which does not show any transition but has a pure SC energy spectrum. We have choose the on-site Fibonacci model (OFM) obtained by setting $\epsilon_n = J\lambda(n + 1) - |n|$ in Eq.(1) whose spectrum has been proved to be SC [30]. The results are shown in Fig. 2 panel c). We see that increasing $\lambda$ results in a stronger deviation from the ballistic
behavior which on its own might seem surprising due to the non-interacting nature of the system we are considering. On the other hand, this behavior can be traced back to the critical nature of the eigenfunctions together with the SC nature of the spectrum \[4, 10, 30\], shared by other aperiodic structures \[31, 33\].

We can draw two main conclusions from the above observations. The AAM for \(\lambda < 1 (\lambda > 1)\) behaves as any non-interacting system with a continuous (discrete) energy spectrum, inducing ballistic (suppression of) propagation of correlations. At the transition point, on the other hand, the spreading turns diffusive, a behavior usually arising in the presence of interactions and/or phase boundaries (as is the case for the AAM at \(\lambda = 1\)). In the OFM, the system shows anomalous diffusion despite the absence of any phase transition and \(\partial\) or crossover between different phases. On the other hand, the AAM and the OFM share a common feature: the nature of the single-particle spectrum at the transition point for the AAM and that of the OFM for any finite value of \(\lambda\) is singular continuous with critical eigenstates, which manifest as an anomalous diffusive behavior.

**Interplay between interaction and geometry** - The presence of interaction can alter the transport properties in a substantial way; for example, when the non-interacting single particle eigenfunctions are extended states, the spreading turns from ballistic to diffusive as the interaction strength is increased \[22, 25\]. On the other hand, we have just shown that an anomalous diffusion can arise in a non-interacting system solely due to the properties of the underlying geometry of the potential energy landscape. A natural question to ask is then how the dynamical properties due to a non-trivial underlying geometry are affected by the interactions. To answer this question, we look at the dynamics of a many-body interacting system described by the Hamiltonian in Eq. (1) both for the AAM and the OFM.

In order to investigate the properties of such a system, we resort to experimentally accessible physical quantities \[14, 15, 22\] so that our results can be benchmarked with experimental results. Let us introduce the particle imbalance, defined as: \(\Delta N(t) = (N_e(t) - N_o(t))/N_{tot}\), where \(N_{e(o)(t)}\) is the number of particles at the even (odd) sites at time \(t\) and \(N_{tot}\) is the total number of particles in the system. This quantity is a good witness of the diffusion properties: for a system in a delocalized (ergodic) phase, we expect that, regardless of the initial state, \(\Delta N(t) \rightarrow 0\) at long enough times \((Jt \gg 1)\) as all particles will be equally redistributed among different sites. On the other hand, in a localized phase \(\Delta N(t) \rightarrow C(\lambda, U) \neq 0\) at long times.

Refs. \[14, 15\] have reported that, in the AAM, this is indeed true away from the non-interacting critical point. Close to \(\lambda = 1\), instead, \(\Delta N \rightarrow 0\) with a power law behavior. The latter is a signature of a non-trivial interplay between the effect of interaction and geometry that we want to investigate here in more detail.

Fig. 3 reports the imbalance \(\Delta N(t)\) for the AAM and for two different interaction strengths a) \(U = 0.6J\) and b) \(U = 1.4J\). The top (bottom) row shows \(\Delta N(t)\) for values of \(\lambda < 1 (\lambda > 1)\). We see that, for \(\lambda < 1\), \(\Delta N(t) \rightarrow 0\), whereas, for \(\lambda > 1\), there are two appreciably different behaviors. There exist a critical value of the interaction \(U_c(\lambda)\) such that for \(U < U_c(\lambda)\) \(\Delta N(t) \rightarrow C \neq 0\) whereas for \(U \geq U_c(\lambda)\) \(\Delta N(t) \rightarrow 0\) with a power-law behavior. In order to characterize such a behavior we fitted the imbalance with a power-law of the form \(\Delta N(t) = at^{-\beta}\). In Fig. 4, we show two density plots reporting the exponent \(\beta\) and the constant \(a\), respectively. We see that, for \(\lambda < 1\), we have \(\beta \approx 0\) and \(a \approx 0\), thus signaling that there is no power law and that \(\Delta N(t)\) vanishes, as expected in the delocalized phase. For \(\lambda > 1\) and for small enough interactions \(U\), we have \(\beta \approx 0\) and \(a \neq 0\) thus signaling localization at long times. A non trivial power law is found for \(\lambda > 1\) and \(U > U_c(\lambda)\), witnessed by the exponent \(\beta\)
being \( \neq 0 \). It is also interesting to note that as \( U \) increases, a power law can be found also for values of \( \lambda \leq 1 \); this is easily understood in a mean field picture: the many-body interaction is responsible for an effective increase of the local energy at each site due to the presence of a particle with opposite spins.

The key point is now to understand whether or not the power-law behavior is a legacy of the transition at \( U = 0 \), or it has a deeper origin. In order to shed light onto this puzzle, we looked for comparison at the interacting OFM. In Fig. 5 we show the imbalance \( \Delta N(t) \) for two values of the interaction \( U \) and for each of them we considered several values of the parameter \( \lambda \). We see that for small values of \( \lambda \) and at long times \( \Delta N(t) = 0 \), whereas for larger \( \lambda \) a power-law behavior emerges once again, similarly to what we have seen for the AAM, and sharing similar features; namely that the power-law exponent (not shown) increases with increasing \( U \) at fixed \( \lambda \).

Another interesting aspect emerges as we look at the spectral function \( \Lambda(T, k, \omega) \) for \( Jt \gg 1 \) shown in the panels c) and d) of Figs. 6. We see that the role of the potential \( \lambda \) is to open gaps in the spectrum, whereas interaction tends to both broaden the peaks in momentum and to close gaps in the energy spectrum. Therefore, in the AAM for \( \lambda > 1 \), the two parameters \( \lambda \) and \( U \) compete in broadening the spectral function in momentum, but have contrasting effects on the energy gaps. Notice that, in the case of the OFM, we would have expected peaks located at specific values of the momentum related to the strength of \( \lambda \); these are not visible due to the small number of sites considered here.

From Fig. 5 panel c) and d), we can see that the appearance of the power-law behavior in \( \Delta N(t) \) is accompanied by the start of a broadening in momentum, which, in that case, is not due to localization, but most likely to the merge of the broadened peaks mentioned above.

We therefore conjecture that the power-law behavior arises as a non-trivial competition between the geometry of the underlying energy landscape and the two-body interaction. In the reminder of this letter, we will search for further evidences of a link of this behavior to the nature of the single particle energy spectrum.

**SC spectrum in interacting systems** - By looking at the spreading of particles, we have observed three markedly different behaviors for the AAM: i) a complete delocalization over the time scale of the single particle hopping, which occurs for small interaction and for \( \lambda < \lambda_c \); ii) a localized region, where spreading is suppressed even in the presence of interactions in the parameter region \( \lambda > \lambda_c \) and \( U > U_c(\lambda) > 0 \); iii) a power law decay of \( \Delta N(t) \) at long times in a region \( \lambda \geq \lambda_c \) and \( U \leq U_c(\lambda) > 0 \). A similar behavior has been observed in the OFM, where depending on the strength of the on-site potential (and, therefore, of the singularity nature of the spectrum), there exists a region in the \((U, \lambda)\) plane where \( \Delta N(t) \) displays a power-law decay, and which separates two regions where the imbalance either shows long-living oscillations, or it decays to zero. We now want to link this behavior to the nature of the single particle spectrum by introducing two quantities: the time averaged imbalance correlation, \( C(\tau) = \langle \Delta N^2(t) \rangle^{-1} \langle \Delta N(t) \Delta N(t + \tau) \rangle \), (where the brackets denote a time average over \( t \)) and its Cesáro (time) average \( \langle C^2(\tau) \rangle = T^{-1} \int_0^T ds \ C^2(s) \). By following the discussions in Refs. 9, 23, 22, 37, we can use them to investigate the nature of the spectrum of the signal in \( \Delta N(t) \), using the Ruelle-Amrein-Georgescu-Enss (RAGE) theorem and Wiener lemma which state that if \( \lim_{t \to \infty} C(t) \neq 0 \), then there can be no AC component in the spectrum; if \( \lim_{t \to \infty} \langle C^2(\tau) \rangle = 0 \), then there can be no PP component in the spectrum. If both conditions hold, then the spectrum must be purely SC in nature due to Lesbegue classification theorem of positive measures.
From Fig. 6 we can see that for $\lambda < 1$ $C(t) \to 0$ at long times, and at the same time $\langle C^2(t) \rangle_T \to 0$ for large $T$, thus signaling the presence of an AC component (due to the extended eigenstates) but a lack of a PP one in the spectrum of the signal $\Delta N(t)$, as expected. On the other hand, for large enough $\lambda$, we have the opposite behavior; namely, $C(t) \neq 0$ and $\langle C^2(t) \rangle_T \neq 0$ for large $T$, witnessing the presence of a PP component in the spectrum, reminiscent of the discrete spectrum for $\lambda > 1$ in the non-interacting case. For intermediate values of $\lambda$ and $U$, we observe that there are pairs of values $(U, \lambda)$ for which $C(t) \to 0$ as a power law and at the same time $\langle C^2(t) \rangle_T \to 0$, therefore signaling the absence of both AC and PP components in the spectrum; due to the Lesbegue decomposition of positive measures, we therefore conclude that in that region the spectrum is SC in nature. These regions correspond to the ones we observed a power-law behavior in the decay of $\Delta N(t)$ in the previous discussion.

To rule out the role of the transition at $\lambda = 1$ and $U = 0$, we refer again to the OFM which has no transition at all in the non-interacting case. In Fig. 7 we recover the same qualitative behavior we just discussed for the AAM; namely, that there are regions where both $C(t) \to 0$ and $\langle C^2(t) \rangle_T \to 0$, signaling the presence of a SC spectrum. This is particularly interesting because, as a side result, we obtained that SC spectra are robust when many-body interactions are added, thus leaving hope of observing the unusual properties of quasicrystalline materials also in moderately interacting systems.

As a side but important remark, we want to stress here that, whereas in the AAM in order to observe these features in a clear way one has to resort to large enough system sizes, due to the reduced region of existence of the SC spectrum in the non-interacting case (which is only one point in the whole parameter space), in other quasicrystals, such as the Fibonacci-like ones, this region is much wider and more robust against finite size effects. Indeed, whereas for the AAM we had to consider system of $L = 40$ sites, for the OFM we were able to observe them already for $L = 20$ sites.

### Conclusions

To summarize, we have investigated the dynamics of the interacting Aubry-André model, focusing on the diffusion of particles initially held in an uncorrelated state with an inhomogeneous spatial distribution. We have found that, in the AAM, the anomalous behavior, which occurs only at the transition point in the non-interacting case, is found to occur in a finite region of the plane $(U, \lambda)$ in the presence of interaction. Aiming at showing that this behavior is not a legacy of the non-interacting transition point, we looked for a comparison at the interacting on-site Fibonacci model, which lacks transitions of any type in the non-interacting case. We have found that an anomalous redistribution of particles occurs for this model as well. In order to explain this behavior, we resorted to the Lesbegue decomposition of positive measures to show that the power-law behavior is a consequence of the singular continuous nature of the single particle spectrum of the system.

We conjecture that, in the interacting case, the power-law behavior is linked to anomalous diffusion similarly to what happens in non-interacting systems with quasicrystalline geometries [4]. In analogy to the non-interacting case, we expect that the microscopic mechanism behind such a behavior has to be sought in the critical nature of the single particle natural orbi tensors of the reduced density matrix of the system at stationarity.

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