DEVELOPMENT OF TOOL FOR THE IDENTIFICATION OF STIFFNESS AND DAMPING COEFFICIENTS OF JOURNAL BEARING

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Abstract- Liquid Lubricated bearings are generally utilized in rotating machines due to its long existence with minimal effort and intrinsic damping. The stiffness and damping of the journal bearing impacts the rotor dynamics. The present work is to evaluate the stiffness and damping coefficients of the cylindrical journal bearing. Reynolds lubrication equations for determining the stiffness and damping coefficients of a journal bearing were introduced. These governed equations were formulated utilizing finite difference method (FDM). A 'C' program is generated for discretization for FDM and solving the governed equations to estimate pressure distribution, followed by stiffness and damping coefficients of journal bearing. The evaluated values through the created "C" program are validated against the values produced by rotor dynamic analysis package MADYN 2000.

Keywords- Journal bearing, stiffness, damping, finite difference, discretization, dynamics

I. INTRODUCTION

Fluid lubricated journal bearings are most widely used in turbines because of its long life with low cost and inherent damping. They support the rotating shaft by a thin fluid film. This fluid film is generated by rotating action of journal inside the sleeve. Lubricant film formed thus prevents metal to metal contact and assures smooth running and continuous operation without undergoing failure. The schematic of journal bearing is shown in Fig.1.

Lund and Thompson [1] developed a finite perturbation method to figure the linear stiffness and damping coefficients of oil lubricated bearings. After differentiating the Reynolds equation with respect to displacements and velocities, four new perturbed Reynolds equations were gotten. The four stiffness and damping coefficient were founded for the elliptical bearing.
Booker, J.F [2] exhibited that for dynamically loaded journal bearings the exemplary analysis issue is predicting the motion of a journal center under discretionary loading. Numerous strategies for arrangement are accessible most give agreeable outcomes however contrast in their generality and computational efficiency. Goenka [3] demonstrated the adaptability of the analysis by investigating 17 different cases of a connecting-rod bearing. The calculation time is high to the point that sometimes the analysis of dynamically loaded bearings utilizing the finite element method isn't justified.

Klit, P. and Lund, J.W. [4] portrays how the dynamic bearing coefficients are obtained from a solution to the variational equivalent of Reynolds equation. A perturbation method is applied to find the individual dynamic coefficients. The Finite Element Method is utilized as a part of the numerical assessment of the equations. The flow is assumed to be laminar, the lubricant is Newtonian. Allowance is made for viscosity-temperature dependency in circumferential and axial directions. Solution of these equations is extremely sensitive to the precision of the pressure gradients, as there is a great variation in the pressure in a hydrodynamic bearing.

Pafelias, T. A. and Broniarek, C. A. [5] discussed about Reynolds’ equation for an partial journal bearing with both transitional and tilting motions. He has tackled these equations by methods of finite difference technique. An arrangement of sixteen spring and sixteen damping coefficients is utilized to build up a “point matrix” for a bearing of finite dimensions with misalignment, in compatible with the current transfer matrix techniques.

Oscar DE Satiago[6] has showed that rotordynamic analysis relies on fluid-film bearing force coefficients for prediction of rotor response to unbalance and rotor bearing system stability maps. Dynamic force coefficients, namely stiffness and damping, are derived from infinitesimal journal motions about an equilibrium position (Lund [1]). For plain cylindrical bearings, the bearing dynamic parameters are functions of the Somerfield number, relating static load, shaft speed, lubricant viscosity, and the bearing clearance, diameter, and length.

Omidreza Ebrat et al. [7] showed a point by point journal bearing analysis for exact evaluation of film dynamic characteristics. The new formulation depends on a local perturbation of the oil film at each computational node that catches the vital impacts of journal misalignment and bearing structural deformation in rotor dynamics .He computed the coefficients by utilizing finite difference method and infinitesimal perturbation method.

Press William. H., Teukolsky [8] has prescribed solving static pressure and remaining perturbed pressures by gauss seidal method, which used to bring the relation for solving the pressures.

1.1 Theoretical Background

The process of computing bearing dynamic coefficients begins with solving for the pressure field in the fluid film between the journal and bearing surfaces.

Considering converging oil film wedge an infinitesimally small element in the oil film at a distance x where the pressure is p and the shear stress is $\tau_x$ and $\tau_z$ as appeared in Fig.2. The pressure p varies with distance x and z only and the shear stress is a function of both x and y.

![Figure 2. Equilibrium of the element in x direction](image-url)
Similarly equilibrium of element in z direction

\[ p \times dx \times dy - \left( p + \frac{\partial p}{\partial z} \right) dx \times dy + \tau x \times dx \times dz - \left( \tau x + \frac{\partial \tau x}{\partial y} dy \right) \times dx \times dz = 0 \]  

Using equations (1), (2), (3), (4) and equations of newton's law of viscosity

\[ \frac{\partial \tau x}{\partial x} = -\frac{\partial p}{\partial x} \] \hspace{1cm} (3)

\[ \frac{\partial \tau z}{\partial z} = -\frac{\partial p}{\partial y} \] \hspace{1cm} (4)

II. REYNOLDS EQUATION FOR THE CYLINDRICAL JOURNAL BEARINGS

Consider the following cylindrical journal bearing as shown in the Fig. 3.

The circumference of the bearing is unwrapped to represent the variation of film thickness as shown in Fig. 4.
The film unwrapped in the circumferential direction \(x = R\phi\). \(\phi\) is the angle measured between the journal and bearing centers. Substituting the value of \(x\) in eq (9) yields

\[
\frac{1}{R^2} \frac{\partial}{\partial \phi} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = \frac{6U}{R} \frac{\partial h}{\partial \phi} + 12 \frac{\partial h}{\partial t} \tag{10}
\]

2.1 Perturbation of Reynolds Equation: For a journal bearing rotating at steady speed, the pressure is a function of the local film thickness \(h\) and its rate of change \(\dot{h}\) i.e. pressure relies upon the displacement of rotor in \(x\) and \(y\) direction and velocity of rotor in \(x\) and \(y\) direction.

Using Taylor expansion and neglecting second order terms, the pressure can be written as

\[
p = p_o + \frac{\partial p}{\partial x} \Delta x + \frac{\partial p}{\partial y} \Delta y + \frac{\partial p}{\partial \dot{x}} \Delta \dot{x} + \frac{\partial p}{\partial \dot{y}} \Delta \dot{y} \tag{11}
\]

\[
\Delta h = \Delta x \sin \phi + \Delta y \cos \phi \tag{12}
\]

As \(x=R\phi\) and substituting (11),(12) in eq.(10). After dropping the higher order terms and separation of variables due to the arbitrary small perturbations \(\Delta x, \Delta y, \Delta \dot{x}, \Delta \dot{y}\) yields

\[
L_o(p_o) = \frac{6U}{R} \frac{\partial h_o}{\partial \phi} + 12 \frac{\partial h_o}{\partial t} \tag{13}
\]

\[
L_o \left( \frac{\partial p}{\partial x} \Delta x \right) = \frac{6U}{R} \left( \Delta x \cos \phi \right) - L_x(p_o) \tag{14}
\]

\[
L_o \left( \frac{\partial p}{\partial y} \Delta y \right) = \frac{6U}{R} \left( \Delta y \sin \phi \right) - L_y(p_o) \tag{15}
\]

\[
L_o \left( \frac{\partial p}{\partial \dot{x}} \Delta \dot{x} \right) = 12 \left( \Delta \dot{x} \sin \phi \right) \tag{16}
\]

\[
L_o \left( \frac{\partial p}{\partial \dot{y}} \Delta \dot{y} \right) = 12 \left( \Delta \dot{y} \cos \phi \right) \tag{17}
\]

Where the following differential operators from [7] are used:

\[
L_o(z) = \frac{1}{R^2} \frac{\partial}{\partial \phi} \left( h_o^3 \frac{\partial}{\partial \phi} \left( \frac{\partial}{\partial z} \frac{\partial}{\partial \phi} \left( \frac{\partial}{\partial z} \right) \right) \right)
\]

\[
\{L_x(z)\} = \frac{\partial}{\partial z} \left( 3h_o^2 \frac{\partial}{\partial z} \left( \Delta x \sin \phi \right) \right) + \frac{1}{R^2} \frac{\partial}{\partial \phi} \left( 3h_o^2 \frac{\partial}{\partial \phi} \left( \Delta y \cos \phi \right) \right)
\]

To non dimensionlise equation, Let

\[
H = \frac{h}{c}, H = 1 + \varepsilon \cos(\phi - \phi_o), \quad U = R\Omega, \quad P = \frac{p}{6\mu\Omega \left( \frac{c}{R} \right)^2}, \quad Z = \frac{z}{R} \tag{18}
\]

Where \(\Omega\) is the rotational speed of the rotor.

\[
\frac{\partial h}{\partial t} = (\dot{x} \sin \phi + \dot{y} \cos \phi), X = \frac{x}{c}, Y = \frac{y}{c}, \dot{X} = \frac{\dot{x}}{c\Omega}, \dot{Y} = \frac{\dot{y}}{c\Omega}
\]

From substitution of equation (18) in (10)
\[ \frac{\partial}{\partial \Phi} \left( H^2 \frac{\partial P}{\partial \Phi} \right) + \frac{\partial}{\partial Z} \left( H^2 \frac{\partial P}{\partial Z} \right) = \frac{\partial H}{\partial \Phi} + 2(\dot{X} \sin \Phi + \dot{Y} \cos \Phi) \quad \square = c(1 + \varepsilon \cos(\Phi - \Phi_0)) \]

\[ \varepsilon = \frac{e}{c} \text{ eccentricity ratio} \]

### 2.2 Computational By Finite Difference Method-Infinitesimal Perturbation (FDM-IFP)

The IFP method uses the partial derivatives of the bearing forces with respect to displacements and velocities to calculate the stiffness and damping coefficients.

\[ F_x = k_{xx}x + k_{xy}y + c_{xx}\dot{x} + c_{xy}\dot{y} \]
\[ F_y = k_{yx}x + k_{yy}y + c_{yx}\dot{x} + c_{yy}\dot{y} \]

Non dimensionalised equation

\[ k_{xx} = -C_f \int_0^{2\pi} \int_0^L \frac{\partial P}{\partial x} \sin \Phi d\Phi dZ, \quad k_{xx} = \frac{\partial F_x}{\partial x} \]

\[ C_f = \frac{C_8}{c} \text{ for } k_{xx}, k_{xy}, k_{yx}, k_{yy}, C_f = \frac{C_8}{c\Omega^2} \text{ for } c_{xx}, c_{xy}, c_{yx}, c_{yy} \]

\[ C_8 = 6\mu R^2 \left( \frac{R}{c} \right) \]

\[ P_x, P_y, P_\dot{x}, P_\dot{y} \text{ are perturbing pressures and } c \text{ is the clearance of the bearing.} \]

\[ P_x = \frac{\partial P}{\partial x}, P_y = \frac{\partial P}{\partial y}, P_\dot{x} = \frac{\partial P}{\partial \dot{x}}, P_\dot{y} = \frac{\partial P}{\partial \dot{y}} \]

Similarly the other stiffness and damping coefficients \( k_{yy}, k_{xy}, \ldots, c_{yy} \) are obtained. FDM chosen to solve Reynolds equation. The finite difference method which uses the infinitesimal perturbation to calculate eight dynamic coefficients. From Fig.5. using forward difference

[Figure 5. Nodes for Half step difference method[7].]

The oil film is divided into a grid of desired size.

\[ \left( \frac{\partial P}{\partial \Phi} \right)_{i,j} = P_{i+1/2,j} - P_{i-1/2,j} \]
\[ \left( \frac{\partial H}{\partial \Phi} \right)_{i,j} = H_{i+1/2,j} - H_{i-1/2,j} \]
\[ \left( \frac{\partial P}{\partial Z} \right)_{i,j} = P_{i,j+1/2} - P_{i,j-1/2} \]

After non dimensionalising terms in equations(13),(14),(15),(16),(17) and applying forward difference approximation for the terms.

\[ \frac{\partial}{\partial \Phi} \left( H^2 \frac{\partial P}{\partial \Phi} \right)_{i,j} = \frac{(H^3)_{i+1/2,j}P_{i+1,j} + (H^3)_{i-1/2,j}P_{i-1,j} - ((H^3)_{i+1/2,j} + (H^3)_{i-1/2,j})P_{i,j}}{\Delta \Phi^2} \]

Similarly
\[ \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P}{\partial Z} \right)_{i,j} = \frac{(H^3)_{i+1/2,j} P_{i+1,j} + (H^3)_{i-1/2,j} P_{i-1,j} - ((H^3)_{i+1/2,j} + (H^3)_{i-1/2,j}) P_{i,j}}{\Delta Z^2} \]

\[ \frac{\partial}{\partial \phi} \left( H^2 \sin \phi \frac{\partial P}{\partial \phi} \right) = \frac{S_{i,j}}{\Delta \phi^2} \]

\[ \frac{\partial}{\partial Z} \left( H^2 \cos \phi \frac{\partial P}{\partial Z} \right) = \frac{T_{i,j}}{\Delta Z^2} \]

Where

\[ S_{i,j} = (H^2)_{i+1/2,j} \sin \phi_{i+1/2,j} P_{i+1,j} + (H^2)_{i-1/2,j} \sin \phi_{i-1/2,j} P_{i-1,j} - ((H^2)_{i+1/2,j} \sin \phi_{i+1/2,j} + (H^2)_{i-1/2,j} \sin \phi_{i-1/2,j}) P_{i,j} \]

\[ T_{i,j} = (H^2)_{i+1/2,j} P_{i+1,j} + (H^2)_{i-1/2,j} P_{i-1,j} - ((H^2)_{i+1/2,j} + (H^2)_{i-1/2,j}) P_{i,j} \]

Now the Reynolds equation is of the form

\[ A_{i,j} P_{*i+1,j} + B_{i,j} P_{*i-1,j} + C_{i,j} P_{*i,j+1} + D_{i,j} P_{*i,j-1} - E_{i,j} P_{*i,j} = f_{i,j} \]

Where

\[ A_{i,j} = (H^3)_{i+1/2,j} \]

\[ B_{i,j} = (H^3)_{i-1/2,j} \]

\[ C_{i,j} = \left( \frac{\Delta \phi}{\Delta Z} \right)^2 (H^3)_{i,j+1/2} \]

\[ D_{i,j} = \left( \frac{\Delta \phi}{\Delta Z} \right)^2 (H^3)_{i,j-1/2} \]

\[ E_{i,j} = A_{i,j} + B_{i,j} + C_{i,j} + D_{i,j} \]

\[ f_{i,j} = \Delta \phi (H_{i+1/2,j} - H_{i-1/2,j}) \text{ for } P_* = P_x \]

\[ f_{i,j} = \Delta \phi^2 \cos \phi_{i} - 3 S_{i,j} - 3 \left( \frac{\Delta \phi}{\Delta Z} \right)^2 \sin \phi_{i} T_{i,j} \text{ for } P_* = P_y \]

After rewriting

\[ P_{*i,j} = \frac{A_{i,j} P_{*i+1,j} + B_{i,j} P_{*i-1,j} + C_{i,j} P_{*i,j+1} + D_{i,j} P_{*i,j-1} - f_{i,j}}{E_{i,j}} \]

Pressure can be calculated by iterating using over relaxation method given by [8].

\[ P_{*i,j}^k = P_{*i,j}^{k-1} + \alpha \left( \frac{A_{i,j} P_{*i+1,j}^{k-1} + B_{i,j} P_{*i-1,j}^{k-1} + C_{i,j} P_{*i,j+1}^{k-1} + D_{i,j} P_{*i,j-1}^{k-1} - f_{i,j}}{E_{i,j}} - P_{*i,j}^{k-1} \right) \]

\[ \alpha \text{ is over relaxation factor, } \quad 1.5 < \alpha < 1.9 \]
For $P_0$, during iteration if $P_0 < 0$ then $P_0$ is set to zero.
After $P_0$ is calculated $f_{i,j}$ for $P_x, P_y, P_x, P_y$ is calculated and boundary nodes are noted and then the same equations are solved iteratively to get $P_x, P_y, P_x, P_y$. After that they are integrated to get the dynamic coefficients.

### III. VALIDATION RESULTS

The linear dynamic coefficients of the cylindrical journal bearing are obtained using C-Programme by solving the governing equations that have been formulated.

For the following geometry and viscosity:
l=0.1m, r=0.05m, clearance=0.000075m, viscosity=0.007569 N-s/m²

| Eccentricity(e) | Attitude Angle(°) | $K_{xx}$ | $K_{xy}$ | $K_{yx}$ | $K_{yy}$ | $C_{xx}$ | $C_{xy}$ | $C_{yx}$ | $C_{yy}$ |
|-----------------|-------------------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.1             | 75.05             | 1.61     | -3.27    | 10.52    | 1.42     | 6.66     | 1.741    | 1.722    | 20.92    |
| 0.2             | 65.89             | 1.55     | -1.76    | 5.77     | 1.59     | 3.77     | 1.65     | 1.64     | 11.55    |
| 0.3             | 57.86             | 1.61     | -1.33    | 4.4      | 1.68     | 3.04     | 1.86     | 1.86     | 8.91     |
| 0.4             | 52.76             | 1.62     | -1.05    | 3.78     | 1.9      | 2.63     | 1.99     | 1.99     | 7.76     |
| 0.5             | 47.61             | 1.52     | -0.66    | 3.44     | 2.4      | 2.003    | 1.82     | 1.83     | 7.06     |
| 0.6             | 42.45             | 1.46     | -0.35    | 3.31     | 3.01     | 1.59     | 1.72     | 1.73     | 6.81     |
| 0.7             | 37.18             | 1.43     | -0.06    | 3.4      | 4        | 1.28     | 1.68     | 1.68     | 6.9      |

Kxy, Kyx…… is a nondimensional less number in the Table 1.
The stiffness in N/m and damping in Ns/m are calculated from non-dimensional stiffness and damping from Table 1.

\[
K_{xx} = \frac{k_{xx}}{W} \times C_k, \quad k_{xx} = 2.39e^7
\]
\[
c_{xx} = \frac{C_{xx}}{C_k \Omega} = 3.12e^5
\]

Where $W$ is load acting on bearing, $C$ is clearance and $\Omega$ is angular speed.

The stiffness and damping coefficients for the same cylindrical journal bearing estimated by bearing module of rotor dynamic analysis package MADYN 2000.

These values are estimated for the eccentricity ratio of 0.1. The comparison of developed C programme values, MADYN 2000 values and % error are given in Table 2.

|                      | $K_{xx}$ (N/m) | $K_{xy}$ (N/m) | $K_{yx}$ (N/m) | $K_{yy}$ (N/m) | $C_{xx}$ (Ns/m) | $C_{xy}$ (Ns/m) | $C_{yx}$ (Ns/m) | $C_{yy}$ (Ns/m) |
|----------------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|
| From the developed C-program in this project | 2.39e7         | -4.86e7        | 1.56e8         | 2.1e7          | 3.14e5          | 8.22e4          | 8.22e4          | 9.97e6          |
| % of Error           | 10             | 11.7           | 1.9            | 9              | 10.2            | 16.9            | 15.6            | 1.12            |
IV. CONCLUSIONS

The governing equations for calculation of dynamic coefficients of cylindrical journal bearing have been formulated. Linear stiffness and damping coefficients have been estimated by solving the above formulations. The solution methodology has been programmed in “C” for solving the formulations. The estimated values are validated with the values obtained from Madyn 2000 (commercially available rotor dynamic package). Estimated values obtained from present work in comparison to Madyn 2000 values are having a max percentage error of 16.

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