On Type II Superstrings in Less Than Four Dimensions

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Abstract

We study gauge theories which are associated with classical vacua of perturbative Type II string theory that allows for a conformal field theory description. We show that even if we compactify seven spatial dimensions (allowing for one macroscopic dimension to arise non-perturbatively) the Standard Model cannot be obtained from the perturbative sector of Type II superstrings. Therefore, the construction of the Standard Model (or extensions thereof) from M theory must involve fields that are non-perturbative from the Type II perspective. We also address the case of eight compact dimensions.
One of the outstanding questions in string theory is how does it describe nature. At present, we are unable to make definitive statements about the underlying dynamics that selects our string vacuum. Nevertheless, one can use phenomenological constraints as guidelines to explore realistic string models in various calculable regimes. The primary purpose of such explorations is not to find the model which fully describes our world, but to examine the possibilities within the string theoretical framework.

In this respect, a perturbative limit of M theory which was largely unexplored is Type II string theory (for earlier work in this direction, see, e.g., References [1-3]). This is due to the no-go theorem in Ref. [1], where it was shown on general grounds that Type II string theory does not contain the Standard Model. In this short note, we revisit this no-go theorem by compactifying this theory down to three dimensions. We also address compactifications to two dimensions.

Of course, our spacetime appears to be manifestly four-dimensional (rather than three or lower dimensional), but as the string coupling becomes strong, a new dimension can open up. In this limit, one may hope to recover an (approximately) Lorentz invariant four-dimensional spacetime. In fact, this transition from three to four dimensions was used in the proposal of Ref. [6] as a possible solution to the cosmological constant problem. Another motivation for studying this setup is that its M theory lift should correspond to compactifications on seven-dimensional spaces which may not even have a geometrical interpretation.

We will show that there is enough room to accommodate the Standard Model gauge group \(SU(3) \times SU(2) \times U(1)\) and matter fields transforming in appropriate representations under the non-Abelian gauge groups. However, the \(U(1)\) quantum numbers carried by the candidate quarks and leptons cannot be identified with hypercharge. Therefore, the Standard Model, if obtained from M theory, must contain sectors that are non-perturbative

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1The conformal field theory description of the heterotic string compactified on manifolds with exceptional holonomy to three and two dimensions was considered in Ref. [4]; that of Type II strings on non-compact manifolds with \(G_2\) holonomy was recently studied in Ref. [5].

2A possible realization of this idea as M theory compactified on a \(Spin(7)\) manifold which asymptotes to a \(G_2\) manifold times \(S^1\) was recently discussed in Reference [6].

3For geometrical compactifications on seven-manifolds with \(G_2\) holonomy and their Type II orientifold cousins, see References [8,9] and [10], respectively. M-theory models on Calabi-Yau threefolds times \(S^1/Z_2\) [11] and asymmetric orbifolds [12] have also been considered [13].
from the Type II point of view.

The argument is similar in spirit to that in Ref. [1]. The only requirements are superconformal invariance and the decomposition of the super-stress tensor $T$ into a sum of contributions from the Minkowski space-time and the internal conformal field theory. There is no need to assume any particular compactification scheme, orbifold or otherwise, space-time supersymmetric or not. The internal conformal field theory is not required to have any geometrical interpretation.

Our starting point is a classical background of Type II string theory which is defined by a conformal field theory with local $(1,1)$ world-sheet supersymmetry. The matter (primary) superfields are defined through their operator product expansions (OPEs) with the (left-moving) holomorphic super-stress tensor $T(z, \theta) = T_F(z) + \theta T_B(z)$ and likewise with its anti-holomorphic counterpart $\overline{T}(\overline{z}, \overline{\theta}) = \overline{T}_F(\overline{z}) + \overline{\theta} \overline{T}_B(\overline{z})$ where $(z, \theta)$ and $(\overline{z}, \overline{\theta})$ are the holomorphic and anti-holomorphic coordinates parametrizing the two-dimensional superspace. The OPE of $T(z, \theta)$ with itself is given by

$$T(z_1, \theta_1) \cdot T(z_2, \theta_2) = \frac{i \hat{c}}{z_{12}^3} + \frac{3\theta_{12}}{2z_{12}^2} T(z_2, \theta_2) + \frac{i}{2} D_2 T + \frac{\theta_{12}}{z_{12}} \partial_2 T + \ldots$$

(1)

where $z_{12} = z_1 - z_2 - \theta_1 \theta_2$ and $\theta_{12} = \theta_1 - \theta_2$. Since the ghosts for the local super-reparametrization invariance contribute a central charge $\hat{c}_{ghost} = -10$, the total superconformal anomaly can cancel only if the central charge of the superconformal field theory (SCFT) describing the matter superfields has $\hat{c} = 10$.

Let us decompose the superconformal field theory into a direct product of a D-dimensional space-time and an internal superconformal field theory. The super-stress tensor can be written as $T(z, \theta) \equiv T^D(z, \theta) + T^{int}(z, \theta)$ where $T^D$ and $T^{int}$ anti-commute with each other. The central charge of the space-time SCFT is $\hat{c} = D$, and hence $\hat{c}_{int} = 10 - D$. For $D = 4$, $\hat{c}_{int} = 6$, it was shown in Ref. [1] that the central charge of the internal conformal field theory is not large enough to accommodate the Standard Model gauge group, if there are massless fermions in the appropriate representations of $SU(3) \times SU(2)$ playing the role of quarks and leptons. It was also shown there [1] that chiral fermions transform only under gauge symmetries from one side of the superstring, so that one can focus, say, on the left-movers.

In three dimensions, the situation seems better. The central charge of the internal conformal field theory is $\hat{c}_{int} = 7$. As in References [1,2], one can give a complete list of 3D gauge groups allowed in perturbative Type II string theory by considering all possible semi-simple Lie algebras with central charge $\hat{c} \leq 7$. Let $J^a$ be a holomorphic supercurrent of the Kac-Moody algebra. The OPEs of $J(z, \theta)$ with $T(z, \theta)$ and itself are given by,
\[ T(z_1, \theta_1) \cdot J^a(z_2, \theta_2) = \frac{\theta_{12}}{z_{12}} J^a(z_2, \theta_2) + \frac{1}{z_{12}} D_2 J^a(z_2, \theta_2) + \frac{\theta_{12}}{z_{12}} \partial_2 J^a(z_2, \theta_2) + \ldots, \]
\[ J^a(z_1, \theta_1) \cdot J^b(z_2, \theta_2) = \frac{1}{2z_{12}} \cdot k\delta^{ab} + i f^{abc} J^c(z_2, \theta_2) + \ldots, \tag{2} \]

where \( f^{abc} \) are the structure constants of the semi-simple Lie group. Unitarity requires the Kac-Moody level, \( k_i \), and the super-Kac-Moody level, \( \hat{k}_i = k_i - C_A \), of each group factor, \( G_i \), to be non-negative integers. (\( C_A \) denotes the eigenvalue of the quadratic Casimir operator in the adjoint representation.) In terms of these and the dimension of \( G_i \), the central charge is given by \([14]\):
\[ \hat{c}(G_i) = \frac{d(G_i)}{3} + \frac{2d(G_i) k_i - C_A}{k_i}. \tag{3} \]

Descendant (secondary) fields do not give rise to massless states. Thus, Standard Model fields must be described by primary fields. However, primary fields transforming under representations of \( G_i \) (other than the identity representation) can only be present if \( \hat{k}_i > 0 \).

Applying this to the Standard Model we consider the super-Kac-Moody algebra corresponding to \( SU(3)_{k_3} \times SU(2)_{k_2} \times U(1)_{k_1} \). The central charges \( \hat{c}_3 \) and \( \hat{c}_2 \) of \( SU(3)_{k_3} \) and \( SU(2)_{k_2} \) are given by
\[ \hat{c}_3 = \frac{8}{3} + \frac{16}{3} \frac{k_3 - 3}{k_3}, \tag{4} \]
\[ \hat{c}_2 = 1 + 2 \frac{k_2 - 2}{k_2}, \tag{5} \]
while \( U(1)_{k_1} \) contributes central charge \( \hat{c}_1 = 1 \) for any \( k_1 \). It was the observation that \([1]\)
\[ \hat{c}_3 + \hat{c}_2 + \hat{c}_1 \geq 20/3, \tag{6} \]
while \( \hat{c}^{\text{int}} = 6 \) that excluded the perturbative Type II superstring from further phenomenological considerations. One possible way out, as noted in \([1]\), is to give up one spatial dimension. We will now show that the situation is actually significantly worse than that.

Compactification down to three dimensions implies \( \hat{c}^{\text{int}} = 7 \), and therefore
\[ \frac{16}{3} \frac{k_3 - 3}{k_3} + 2 \frac{k_2 - 2}{k_2} \leq \frac{7}{3}. \tag{7} \]

\(^4\)At the level of the superconformal algebra, the level of a \( U(1) \) Kac-Moody algebra is not well defined. However, in many cases, it can be read off from the anomaly polynomial \([13]\). Here, we simply mean the normalization of the \( U(1) \) charge.
The only allowed values are \((k_2, k_3) = (3, 4)\) or \((4, 4)\). While the latter case saturates the bound, in the former there must be an additional superconformal field theory with \(\hat{c} = 1/3\). However, the central charge for unitary superconformal field theories with \(\hat{c} < 1\) is highly restricted. The only allowed values are \([14]\):

\[
\hat{c} = 1 - \frac{8}{m(m + 2)}, \quad m = 2, 3, 4, \ldots,
\]

and combinations thereof. Since \(\hat{c} = 1/3\) is not allowed, the case \((k_2, k_3) = (3, 4)\) must be rejected. We are left with \((k_2, k_3) = (4, 4)\), with no room for an extra conformal field theory. Note that the gauge couplings for the higher level models are given in terms of the string coupling

\[
g_i^2 k_i = g_s^2.
\]

Since \(SU(3)\) and \(SU(2)\) are realized at the same Kac-Moody level, they have the same gauge coupling at the string scale — indicating the unification of the \(SU(3)\) and \(SU(2)\) gauge couplings slightly below the string scale consistent with observation. This is in contrast with the perturbative heterotic string case where the Kac-Moody levels are generally unconstrained.

The conformal dimension of a primary field corresponding to the highest weight representation \(r\) is

\[
h_r = \frac{C_r}{2k + C_A},
\]

where \(C_r\) is the quadratic Casimir of \(r\). Hence, for the fundamental representation \(N\) of \(SU(N)_k\),

\[
h_N = \frac{N^2 - 1}{2N(k + N)}.
\]

The conformal dimension of a primary field carrying charge \(Q\) with respect to an Abelian Kac-Moody factor realized at level \(k_1\) is given by

\[
h_Q = \frac{1}{2k_1} Q^2,
\]

where \(k_1\) serves as a normalization constant.

Identifying the \(U(1)\) with hypercharge the conformal weights of various Standard Model fields are:

| Field | \(SU(3)\times SU(2)\times U(1)_Y\) | Conformal weight \(h\) |
|-------|----------------------------------|------------------------|
| \(Q_L\) | \((3, 2)_{\frac{1}{6}}\) | \(\frac{53}{168} + \frac{1}{72k_1}\) |
The conformal weight of a massless field is $h = \frac{1}{2}$. It is easy to see that there is no choice of the normalization factor $k_1$ such that all Standard Model fields are massless.

The reason that $\hat{c}_{\text{int}} = 7$ conformal field theory fails to describe the Standard Model is that it barely contains its gauge groups plus quarks and weak doublets. Therefore, particles with the same non-Abelian gauge quantum numbers must have the same hypercharge (up to a sign) to be simultaneously massless. This implies that the up and down quark singlets are predicted to carry the same amount of $U(1)$ charge and hence this $U(1)$ gauge field cannot be hypercharge.

It is interesting to note that if we compactify Type II string theory down to two dimensions, we have $\hat{c}_{\text{int}} = 8$ which can accommodate an extra $U(1)'$. The complete Standard Model spectrum can be massless as long as it is supplemented by the appropriate $U(1)'$ charges. It turns out, however, that these charges are incompatible with the Yukawa structure of the Standard Model. One can categorize the possible gauge groups for $\hat{c}_{\text{int}} = 8$:

- (i) $SU(3)_4 \times SU(2)_4 \times U(1) \times U(1)'$
- (ii) $SU(3)_4 \times SU(2)_4 \times U(1) \times (\hat{c} = 1 \text{ SCFT})$
- (iii) $SU(3)_4 \times SU(2)_3 \times U(1) \times (\hat{c} = \frac{2}{3} \text{ SCFT}) \times (\hat{c} = \frac{2}{3} \text{ SCFT})$
- (iv) $SU(3)_4 \times SU(2)_6 \times U(1) \times (\hat{c} = \frac{2}{3} \text{ SCFT})$
- (v) $SU(3)_5 \times SU(2)_5 \times U(1)$
- (vi) $SU(3)_6 \times SU(2)_3 \times U(1)$

It is easy to see that (v) and (vi) are ruled out by the same reasoning that the $SU(3)_4 \times SU(2)_4 \times U(1)$ case was ruled out for $\hat{c}_{\text{int}} = 7$. The conformal weights of primary fields in $\hat{c} < 1$ SCFT are highly restricted. One can show that for (iii) and (iv), there is no normalization constant $k_1$ such that all Standard Model fields are massless. A large class of

| Field  | Representation | Conformal Weight |
|--------|----------------|------------------|
| $U$    | $(\mathbf{3}, 1)_{\frac{1}{3}}$ | $\frac{4}{21} + \frac{1}{18k_1}$ |
| $D$    | $(\mathbf{3}, 1)_{-\frac{2}{3}}$ | $\frac{4}{21} + \frac{2}{9k_1}$ |
| $L$    | $(1, 2)_{-\frac{1}{2}}$ | $\frac{1}{8} + \frac{1}{8k_1}$ |
| $H_U, H_D$ | $(1, 2)_{\frac{1}{2}}$ | $\frac{1}{8} + \frac{1}{8k_1}$ |
| $E$    | $(1, 1)_1$ | $\frac{1}{2k_1}$ |
\( \hat{c} = 1 \) SCFTs have been studied in Reference [16], but it is not known whether this represents the complete classification. We do not consider case (ii) in detail; however, it seems very unlikely that the Standard Model can be constructed.

Let us note that it is even more difficult to embed a Grand Unified Theory (GUT) or a model with partial unification (e.g., the Pati-Salam model) within perturbative Type II string theory. The GUT gauge group with the smallest dimension \( d(G) \) is \( SU(5) \) which has \( d(G) = 24 \), hence \( \hat{c}_{\text{int}} \geq 8 \). In the case of the Pati-Salam model, the central charge of the \( SU(4) \) factor is at least 7 if we require the fundamental representation of \( SU(4) \) to be unitary (so that there are quarks upon breaking the Pati-Salam gauge group down to the Standard Model). There is not enough room even to accommodate the weak \( SU(2) \). Similarly, the left-right model cannot be obtained as well.

As we go from three to four dimensions, we are in the strong coupling regime so that strictly speaking, we cannot trust a perturbative Type II calculations. However, with supersymmetry, it is likely that the perturbative massless spectrum would stay massless even at strong coupling. Certainly, there are additional massless states, including the D0-branes whose masses go as \( 1/R_{11} \) where \( R_{11} \) is the size of the eleventh dimensions.

A possible way of evading the no-go theorem presented here is that it may not always be possible to decompose the super-stress tensor \( T \) into a sum of \( T^D \) and \( T^{\text{int}} \) which commute with each other. For example, this may be the case in a warped compactification where the \( D \)-dimensional spacetime metric depends on the coordinates in the \( (10-D) \) compactified dimensions. It would be interesting to see if this may help in relaxing the constraints in embedding the Standard Model in perturbative Type II string theory.

So far we restricted our attention to perturbative spectra of Type II string theory described by a CFT. However, these spectra may be supplemented by non-perturbative (solitonic) states even at arbitrarily weak string coupling, when the compactification manifold is singular [17]. These correspond to points in the moduli space where the associated conformal field theories are badly behaved [18]. On the Type IIA (IIB) side, the non-perturbative states are obtained from even (odd) D-branes wrapped around even (odd) collapsing cycles [1]. In this case, the gauge coupling is independent of the string coupling, but depends on the geometric moduli. It is interesting to note, that if the \( U(1)_Y \) is a linear combination of a perturbative and a non-perturbative \( U(1) \) gauge field, then our argument based

\(^5\)It is not clear, however, how this way of obtaining non-Abelian gauge symmetries from singularities can be understood in a non-geometrical setup, such as asymmetric orbifolds.
on the conformal weights does not apply. More generally, (parts of) the Standard Model
gauge group can be a diagonal subgroup of a perturbative and a non-perturbative gauge
group. This may lead to a field theoretical realization [19] of the generation-changing phase
transitions [20,21,10].

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