Branes and anti-de Sitter spacetimes

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Abstract: We consider a series of duality transformations that leads to a constant shift in the harmonic functions appearing in the description of a configuration of branes. This way, for several intersections of branes, we can relate the original brane configuration which is asymptotically flat to a geometry which is locally isometric to $\text{adS}_k \times E^l \times S^m$. These results imply that certain branes are dual to supersingleton field theories. We also discuss the implications of our results for supersymmetry enhancement and for supergravity theories in diverse dimensions.

Duality symmetries have played a prominent rôle during the last few years by providing a handle on non-perturbative physics. Using a web of duality symmetries one can connect all known string theories and eleven-dimensional supergravity, leading to a conjectural master theory, the $M$-theory\cite{1}, that contains all others as special limits. The $U$-duality group of $M$-theory compactified on a torus is usually assumed to be the (discretized) version of the global symmetry group of the various maximal supergravity theories obtained from eleven-dimensional supergravity by toroidal compactification in dimensions $d \leq 10$. There are, however, also duality transformations\cite{2,3} outside this group that change the asymptotic geometry of spacetime and connect Poincaré supergravities and gauged anti-de Sitter supergravities. These new duality symmetries were instrumental in the recent microscopic derivation of the Bekenstein–Hawking entropy formula for non-extremal black holes\cite{4}, and they will be the topic of this contribution.

We shall show that using these duality transformations one can map supersymmetric configurations of $M$-branes and $D$-branes that are asymptotically flat to configurations that are locally isometric to $\text{adS}_k \times E^l \times S^m$, where $\text{adS}_k$ is the $k$-dimensional anti-de Sitter space, $E^l$ is the $l$-dimensional Euclidean space and $S^m$ is the $m$-dimensional sphere. We shall further argue that the worldvolume theories of the membrane ($M2$), the fivebrane ($M5$) of $M$-theory, as well as the worldvolume theory of the self-dual threebrane ($D3$) of IIB string theory, are dual to the supersingleton field theories of $\text{adS}_4$, $\text{adS}_7$ and $\text{adS}_5$, respectively.

The basic mechanism that converts an asymptotically flat space to an anti-de Sitter space is a dualization w.r.t. an isometry that is spacelike everywhere, except at spatial infinity, where it becomes null (cf. \cite{5}). This isometry is a combination of spacelike isometries and a timelike isometry. The latter is present in any stationary solution. Since the isometry involves time its orbits are non-compact. We shall therefore consider the following limiting procedure. We shall first compactify the time coordinate and then at the end let the radius of the time coordinate go to infinity. Notice that the time coordinate is naturally compact in the anti-de Sitter space that we obtain, after all dualities have been performed. Thus, taking the infinite radius limit corresponds to considering the covering space.

The effect of the new duality symmetries is to remove the constant part from the harmonic functions appearing in the brane solutions. We shall first discuss the case of a single extremal brane. The case of multiple intersections of branes will be dealt with afterwards. The results hold also for non-extremal...
branes and non-extremal intersections of extremal branes. We shall consider the case of type II string theory, and we shall also lift our results to M-theory branes. In all cases, the brane under consideration is wrapped on a torus, i.e. all the worldvolume coordinates are taken to be periodic. Hence these solutions can be dualized, using an appropriate chain of (standard) duality transformations, to the plane wave solution

\[ ds^2 = -H^{-1}dt^2 + H(dx_1 + H^{-1}dt)^2 + (dx_2^2 + \cdots + dx_9^2), \]
\[ B_{01} = 0; \quad e^{-2\phi} = 1 \]  

(1)

where

\[ H(r) = 1 + \frac{Q}{r^6}; \quad r^2 = x_2^2 + \cdots + x_9^2. \]  

(2)

The solution (1) is dual to the fundamental string solution. The off-diagonal part in (1) was chosen in such a way that the antisymmetric tensor of the dual solution is regular at the horizon.\(^6\)

The solution (1) is a solution in the same coordinate system (of course, (3) is a solution in the transformed coordinate system since (1) is a solution).

Once we have reached the plane wave solution we dualize along the isometry generated by \( \partial/\partial x_1' = \partial/\partial x_1 - (1/2)\partial/\partial t \). The norm squared of this vector is \( Q/r^6 \). So the isometry is spacelike everywhere except at spatial infinity where it becomes null. The result is a fundamental string solution but with zero constant part in the harmonic function, i.e.

\[ ds^2 = \tilde{H}(r)^{-1}(-dt^2 + dx_1'^2) + (dx_2^2 + \cdots + dx_9^2), \]
\[ B_{01} = \tilde{H}(r)^{-1}; \quad e^{-2\tilde{\phi}} = \tilde{H}(r); \quad \tilde{H}(r) = \frac{Q}{r^6}. \]  

(3)

where we have dropped the primes. From here one can dualize back to the original brane configuration.

It is instructive to decompose the new duality transformation into two steps. First, do a coordinate transformation to reach adapted coordinates and then perform a standard T-duality transformation. The coordinate transformation, which we shall call the shift transformation, is given by\(^2\)

\[ x_1' = x_1; \quad t' = t + \frac{1}{2}x_1 \]  

(4)

This transformation yields (with the primes dropped)

\[ ds^2 = -\tilde{H}^{-1}dt^2 + \tilde{H}(dx_1 + \tilde{H}^{-1}dt)^2 + (dx_2^2 + \cdots + dx_9^2) \]  

(5)

i.e. (1) but with the harmonic function \( H \) replaced by \( \tilde{H} \). Subsequent T-duality along \( x_1 \) yields (3). Observe that (1) and (3) are both solutions in the same coordinate system (of course, (3) is a solution in the transformed coordinate system since (1) is a solution).

The shift transformation can be applied to more general configurations of fundamental strings, solitonic fivebranes, D-branes, waves and Kaluza-Klein monopoles. We will make the following assumptions: (i) the harmonic functions only depend on the overall transverse coordinates, of which there are at least three, and (ii) the configurations are built according to the intersection rules based on the ‘no-force’ condition (5) and references therein). Together, these requirements imply that there are at most four independent charges. Moreover, the fraction of supersymmetry preserved is \( 1/2^n \) for \( n \leq 3 \) and \( 1/8 \) for \( n = 4 \) if \( n \) is the number of charges. Then any harmonic function appearing in the description of such a configuration can be shifted by a constant. This can be achieved by mapping the ‘brane’ corresponding to this harmonic function to a wave, using S and T-dualities. Then we can apply the same shift transformation (3) as before to change the harmonic function. Finally we can map back to the original configuration using the same chain of duality transformations.

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1. More generally, one may dualize w.r.t. \( \partial/\partial x_1' = (1/\alpha)(\partial/\partial x_1 - (1/2)\partial/\partial t) \), where \( \alpha \) is arbitrary. This duality yields also (1) but with \( Q \to Q/a^2 \), and the radius \( R_1 \) of the \( x_1 \) coordinate becomes \( R_1 \alpha \).

2. For the more general dualization mentioned in the previous footnote, the transformation is \( x_1' = ax_1; \quad t' = (1/\alpha)(t + (1/2)x_1) \).
The shift transformations in the harmonic functions are particularly interesting for the case of brane configurations with some simple near horizon geometry. Such branes interpolate between this geometry and Minkowski spacetime at infinity. In eleven dimensions, the membrane interpolates between $M_{11}$ and $adS_4 \times S^7$, whereas the fivebrane interpolates between $M_{11}$ and $adS_7 \times S^4$. We may dimensionally reduce the $M2$-brane to a fundamental (type IIA) string, do the shift procedure as explained before, and lift this configuration up to eleven dimensions again. This will give a configuration which is locally isometric to the $adS_4 \times S^7$ solution of eleven-dimensional supergravity, that is just the $M2$-brane solution with the harmonic function not containing the constant term. Notice that the solution is only locally isometric to the $adS_4 \times S^7$ solution since the original brane was wrapped on $T^2$, so in the final configuration the various coordinates carry appropriate global identifications. The compact spacetime isometries associated with the torus are needed in order to be able to dualize the original $M2$ to a wave. Similar remarks apply to all cases that we discuss. The $M5$-brane is similarly connected to the $adS_7 \times S^4$ solution. In fact, $adS_4 \times S^7$ and $adS_7 \times S^4$ are known to be maximally supersymmetric vacua of $d=11$ supergravity. There is one ten-dimensional $p$-brane with similar properties: the self-dual threebrane. Its near horizon geometry, $adS_5 \times S^3$, is itself a maximally supersymmetric vacuum of type IIB supergravity.

Since after the duality the asymptotic geometry has changed, the degrees of freedom should organize themselves into representations of the appropriate anti-de Sitter group. The latter has some representations, the so-called singleton representations, that have no Poincaré analogue. They have appeared in studies of spontaneous compactifications of eleven-dimensional supergravity on spheres. In particular, the fields of the supersingleton representation appear as coefficients in the harmonic expansion of the eleven-dimensional fields on the corresponding sphere. A crucial property is that the singleton multiplets can be gauged away everywhere, except on the boundary of the anti-de Sitter space $\mathcal{M}_5$. In particular, it has been argued in the past that the singleton field theories of $adS_4$, $adS_7$, $adS_5$ and $adS_3$ correspond to membranes $[11]$, fivebranes $[11,12]$, self-dual threebranes $[11,12]$ and strings $[3]$, respectively. It has actually been shown that, in all cases, the world-volume fields of the corresponding $p$-brane form a supersingleton multiplet. Notice also that the anti-de Sitter group $SO(d-1,2)$ coincides with the conformal group in one dimension lower. Therefore, one ends up with a conformal field theory on the boundary. Since we are considering extremal branes, the theory on the boundary is also supersymmetric.

It is well-known that the worldvolume theory (in the limit that gravity decouples) of the membrane $M2$, of the fivebrane $M5$, of the self-dual threebrane $D3$, as well as of the string, is a superconformal field theory. We therefore conclude that these branes are $U$-dual to supersingletons (see also $[4]$).

Following these considerations there should exist topological field theories in anti-de Sitter spaces that when restricted to the boundary yield superconformal field theories. For the case of $adS_5$, this topological theory is simply a Chern-Simons theory. It is indeed well-known that a Chern-Simons theory on a manifold with boundary induces a WZW model on this boundary. Further evidence for this duality for the case of the $D3$-brane was recently given in $[15]$. In this work, the boundary of the anti-de Sitter space was considered at spatial infinity with topology $S^1 \times S^3$ and only a $U(1)$ multiplet was considered. In our case, the boundary should carry the appropriate global identifications, i.e. it should be $S^1 \times T^3$ since the original brane was wrapped on a torus. It will be interesting to analyze whether the $N=4$ $SU(k)$ SYM theory on $T^4$ can be extended to a topological theory on $adS_5$. Similar remarks apply for the case of $M2$ and $M5$ (for a recent work on the worldvolume theory of $M5$, see $[11]$).

Next let us consider the effect of our duality transformations on orthogonal intersections of $M$-branes. The cases of interest are the ones with a simple near horizon geometry which is a product containing an anti-de Sitter spacetime and a sphere. There is only one intersection of two $M$-branes which falls into this class, namely the $M2 \perp M5$ solution,

$$
\begin{align*}
ds^2 &= H_2^{-\frac{2}{3}}H_5^{-\frac{4}{3}}(-dt^2 + dx_1^2) + H_2^{-\frac{2}{3}}H_5^\frac{2}{3}(dx_2^2) + H_2^\frac{1}{3}H_5^{-\frac{4}{3}}(dx_3^2 + \cdots + dx_6^2) \\
&\quad + H_2^\frac{2}{3}H_5^\frac{2}{3}(dx_7^2 + \cdots + dx_{10}^2),
\end{align*}
$$

\begin{align}
F_{012} &= \pm \partial_1 H_2^{-1}, \\
F_{2\alpha\beta\gamma} &= \pm \epsilon_{\alpha\beta\gamma}\partial_{\alpha} H_5,
\end{align}

(6)

where the signs differentiate between brane and anti-brane, and $\epsilon_{\alpha\beta\gamma}$ is the volume form of the three
sphere surrounding the intersection. The harmonic functions are

\[ H_2 = 1 + \frac{Q_2}{r^2}, \quad H_5 = 1 + \frac{Q_5}{r^2}, \quad (7) \]

where \( r^2 = x_1^2 + x_5^2 + x_6^2 + x_{10}^2 \). With this choice of harmonic functions the solution is asymptotically Minkowski. The near horizon geometry is the geometry for \( r \to 0 \). In this limit the constant parts of the harmonic functions become negligible. If one takes the harmonic functions (7) without the constant terms one still has a solution, which can also be obtained by performing the shift transformation. In the latter case, however, we get the solution below plus a set of global identifications, namely the coordinates \( x_i, i = 1, \ldots, 6 \) are now all periodic. We get

\[
    ds^2 = Q_2^{-\frac{4}{5}} Q_5^{-\frac{4}{5}} r^2 (-dt^2 + dx_1^2) + Q_2^{-\frac{4}{5}} Q_5^{\frac{4}{5}} (dx_2^2) + Q_2^{\frac{4}{5}} Q_5^{-\frac{4}{5}} (dx_3^2 + \cdots + dx_6^2) + Q_2^{\frac{4}{5}} Q_5^{\frac{4}{5}} \left( \frac{1}{r^2} dr^2 + d\Omega_5^2 \right).
\]

Near the horizon the spacetime factorizes into the product of an \( adS_3 \) spacetime (with coordinates \( t, x_1, r \)), a three-sphere \( S^3 \) of radius \( Q_5^{\frac{4}{5}} \), and a flat Euclidean five-dimensional space \( E^5 \). Similarly, one finds the other possible orthogonal eleven-dimensional intersections that give rise to this kind of geometry: for three charges these are the \( M_2 \perp M_2 \perp M_2 \) and \( M_5 \perp M_5 \perp M_5 \) (with three overall transverse directions), and for four charges the \( M_2 \perp M_2 \perp M_5 \perp M_5 \) intersection. We tabulate the results of the shift transformation below. The right hand column denotes the geometry which the dual solution is locally isometric to. These are also the geometries near the horizon considered in \([17]\). Let us emphasize that the two geometries coincide only locally: in order to perform the shift transformation, we need to periodically identify some of the coordinates.

| \( M_2 \) | \( adS_4 \times S^3 \) |
| \( M_5 \) | \( adS_7 \times S^4 \) |
| \( D3 \) | \( adS_5 \times S^6 \) |
| \( M_2 \perp M_5 \) | \( adS_3 \times E^{10} \times S^5 \) |
| \( M_2 \perp M_2 \perp M_2 \) | \( adS_2 \times E^5 \times S^3 \) |
| \( M_5 \perp M_5 \perp M_5 \) | \( adS_3 \times E^5 \times S^4 \) |
| \( M_2 \perp M_2 \perp M_5 \perp M_5 \) | \( adS_2 \times E' \times S^2 \) |

Table 1

The product geometries have the form \( adS_{p+2} \times E^q \times S^{9-p-q} \), where \( p \) is the spatial dimension of the intersection, \( q \) is the number of relative transverse coordinates and \( 9-p-q \) is the number of overall transverse coordinates minus one. A wave can be added to the common string of the \( M_2 \perp M_5 \) and \( M_5 \perp M_5 \perp M_5 \) intersections (as well as to the single \( M_2 \) and \( M_5 \) branes). It modifies only the \( adS \) part of the corresponding geometry.

It is well-known\([9]\) that some solutions exhibit supersymmetry enhancement at the horizon. For example, the \( M_2 \), \( M_5 \) and \( D3 \)-branes break one half of supersymmetry, whereas their near horizon geometries are maximally supersymmetric vacua of \( d=11 \) supergravity. Some other cases of supersymmetry enhancement for static \( p \)-brane solutions in different dimensions are known, and in all these cases the near horizon geometry contains a factor \( adS_k \times S^m \). We find that in fact all solutions of table 1 exhibit supersymmetry enhancement at the horizon. It turns out\([9]\) that the condition for unbroken supersymmetry, \( \delta \psi_M = 0 \) where \( \psi_M \) is the eleven-dimensional gravitino, in the background of the intersections in table 1, reduces to the geometric Killing spinor equations on the anti-de Sitter, Euclidean and spherical factors of the geometry. In the case of the \( M_2 \perp M_5 \) intersection there is one additional projection, whereas for the intersections with three and four charges there are two projections needed. As one projection reduces the supersymmetry by a factor one half and since anti-de Sitter, Euclidean and spherical geometries all admit the maximal number of Killing spinors, one concludes that the solutions in the right column of
table 1 have double the amount of supersymmetry as compared to their brane counterparts in the left column.

Furthermore we observe that, for the configurations in table 1, a dimensional reduction over one or more of the relative transverse directions will always give rise to lower dimensional solutions which also exhibit supersymmetry enhancement at the horizon. This is because reduction over such directions, corresponding to the Euclidean directions in the shifted solutions in the right column, cannot interfere with supersymmetry. Further applications of $T$-duality in the relative transverse directions or $S$-duality will also preserve the quality of supersymmetry enhancement. This way one obtains a large class of solutions exhibiting supersymmetry enhancement, including all previously known ones, such as the four and five-dimensional extremal black holes with nonzero entropy. There are some other interesting features that all these solutions have in common. They have regular (i.e. finite) dilaton at the horizon (or no dilaton in eleven dimensions), and in the shifted solutions the dilaton is a constant everywhere. Besides, the antisymmetric field strengths become covariantly constant in the shifted solutions, as is the case for the Bertotti-Robinson solution. Finally, these solutions are all non-singular.

At first sight, it seems that the shift transformation relates two solutions with different amounts of unbroken supersymmetry. However, as we mentioned, the shift transformation involves $T$-duality transformations along the worldvolume coordinates, which therefore have to be compact. Only half of the Killing spinors of anti-de Sitter spacetime survive compactification of the overall worldvolume directions as one can immediately verify by looking at the explicit expression of the Killing spinors\cite{19}. Only locally there is an enhanced supersymmetry.

We conclude by making some comments on the implications of our results for supergravity theories in various dimensions. Each configuration in 11 dimensions with effective geometry of the type $adS_k \times E^l \times S^m$ corresponds to a solution of 11d supergravity with the appropriate amount of supersymmetry. It also follows directly that, after reduction along $p \leq l$ of the flat directions, the geometry $adS_k \times E^{l-p} \times S^m$ is a solution with the same amount of supersymmetry (counting the number of spinor components) in $11-p$ dimensions. In addition we can deduce the existence of solutions with a certain amount of supersymmetry after spontaneous compactification on the sphere $S^m$. These compactifications are expected to give rise to solutions of gauged supergravities in $11-m-p$ dimensions with geometry $adS_k \times E^{l-p}$. Several of these results are well-known, such as the spontaneous compactification of 11d supergravity on $S^7$, giving gauged $N=8$ supergravity in $d=4$, and the $adS_7 \times S^4$ and $adS_5 \times S^5$ solutions of $d=11$ supergravity and type IIB supergravity. The anti-de Sitter parts of these solutions are maximally supersymmetric vacua of gauged maximal supergravities in seven and five dimensions\cite{21}.

Let us also make some remarks about the 10d case. The $M_7 \times S^3$ solution of type I supergravity corresponding to the fivebrane with shifted harmonic function must correspond to the 1/2 supersymmetric $M_7$ solution with linear dilaton of gauged $N=2$ $d=7$ supergravity\cite{21}. Also, the solution of a string in a fivebrane gives rise, after the shift, to $adS_3 \times E^4 \times S^3$ geometry, and $adS_3 \times T^4$ is a solution of $N=2$ $d=7$ gauged supergravity as well\cite{22}. These observations strongly suggest that $N=1$ $d=10$ supergravity compactified on a three-sphere yields $N=2$ $d=7$ gauged supergravity (see also\cite{23}). Similar studies can be made for the other cases.

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