Cosmological constraints from supernova data set with corrected redshift

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Abstract. Observations of distant type Ia supernovae (SNe Ia), used as standard candles, support the notion that the Cosmos is filled with a mysterious form of energy, the dark energy. The constraints on cosmological parameters derived from data of SNe Ia and the measurements of the cosmic microwave background anisotropies indicate that the dark energy amounts to \( \frac{1}{70\%} \) of all the energy contained in the Universe. In the hypothesis of a \( \Omega_m + \Omega \Lambda = 1 \), we investigate if the dark energy is really required in order to explain the SNe Ia experimental data, and, in this case, how much of such unknown energy is actually deduced from the analysis of these data and must be introduced in the \( \Lambda \)CDM model of cosmology. In particular we are interested in verifying if the Einstein–de Sitter model of the expanding Universe is really to be ruled out. By using a fitting procedure based on the Newton method search for a minimum, we reanalyzed the “Union compilation” reported by Kowalski et al. (2008) formed by 307 SNe, obtaining a very different estimate of the dark energy, that is \( \frac{1}{60\%} \). Furthermore, in order to balance the correction of the apparent magnitude of SNe Ia, due to the dilation or stretching of the corresponding light curve width, we introduce a suitable modified redshift. Taking into account this correction, we refitted the Union compilation dataset after a selection cut. The main result that emerges from our analysis is that the values of \( \Omega_m \) and \( \Omega \Lambda \) strongly depend on the fitting procedure and the selected sample. In particular, the constraint we obtain on the mass density, normalized by the critical mass density, is \( \Omega_m = 0.7 \) for a sample of 252, and \( \Omega_m = 1 \) for a sample of 242 SNe Ia respectively. The latter case does not imply the existence of any additional form of dark energy.

1. Introduction

It is well known that the supernovae (SNe) Ia have a significant intrinsic brightness due to their very homogenous initial conditions, i.e. thermonuclear explosions of white dwarfs. For this reason they are among the best standard candles providing cosmological distance determination, with a precision of approximately 8\%, and thus determination of cosmological parameters [1]. The detection of SNe Ia thus turns out to be essential in order to probe the cosmological expansion (for a review, see ref. [2]).

After the first systematic efforts to detect low–redshift (\( z < 0.1 \)) SNe Ia, carried out by the Calán/Tololo Supernova Survey [3, 4, 5], several international teams, the Supernova Cosmological Project (SCP) and the High-Z SN Search (HZSNS) performed extensive searches for SNe Ia at high redshift (out to \( z \leq 0.8 \)) [6, 7, 8, 9, 10]. In the wake of these successful

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experiments, a fit of the magnitude-redshift data of 42 SNe Ia (at redshifts between 0.18 and 0.83), discovered by the Supernova Cosmology Project (SCP), jointly with a set of 18 supernovae from the Calán/Tololo Supernova Survey, was performed at redshifts below 0.1 [11]. Practically, this study compared the distance modulus of the measured SNe Ia with that which would be expected for an Einstein–de Sitter flat Universe, leading to the truly surprising discovery of a positive vacuum energy density ($\Omega_A > 0$), i.e. the dark energy, which opposes the self–attraction of matter and causes the expansion of the Universe to accelerate (for a review, see refs. [12, 13]). This result is consistent with the full sky temperature maps based on seven years of data from WMAP, which are well fit by a minimal six-parameter flat $\Lambda$CDM model. In fact, by using the WMAP data in conjunction with baryon acoustic oscillation data from the Sloan Digital Sky Survey and priors on the Hubble constant $H_0$ from Hubble Space Telescope observations, it was possible to estimate that $\Omega_A = 0.728^{+0.015}_{-0.016}$ [14].

In more recent years, several large-scale ground-based projects have been started [15, 16, 17, 18, 19], the Hubble Space Telescope playing a key role in high-precision optical and infrared follow-up of SNe discovered from the ground. It was therefore possible to draw up new SN Ia compilations, combining high-redshift ($z \sim 0.5$) with low-redshift ($z \sim 0.05$) samples, collected in different projects [20, 21, 22, 23, 24, 25, 26]. However, the corresponding cosmological constraints were consistent with a very wide range of dark energy models. This fact is basically caused by statistical errors but, most importantly, by systematic uncertainties as well. As a matter of fact, the statistical uncertainties in measurements of dark energy parameters are on the same order as systematic uncertainties. Moreover, the different results significantly depend on which light curve fitter has been used. This fact clearly emerges from subsequent works [27, 28, 29, 30].

It is also remarkable to note that the constraints on the value of $\Omega_A$, which is the main goal of this research, depend strongly on redshift of the SNe Ia. In fact, the data at $z > 1$ are not able to provide robust constraints on the existence of the dark energy. Lastly, the physical nature of the dark energy is at the moment lost in the meanders of theoretical speculations [31, 32, 33, 34, 35, 36, 37, 38].

In the present paper, we challenge the current standard model of cosmology, the $\Lambda$CDM model, which includes the cosmological constant as an essential feature. In particular we debate on the use of the stretch factor that intervenes in the SN Ia light curves to modify the time axis around the peak of the luminosity. Actually, without this empirical transformation, the light curves of different SNe Ia are not congruent with each other, but have a shape and a maximum that differ substantially among the samples. However, we believe that any correction applied on the apparent magnitude of each SNe Ia implies a correction on its corresponding redshift, as we discuss in section 2. From this perspective we have reanalysed the so-called “Union compilation”, which is a compilation of 414 SNe Ia (which reduces to 307 after selection cuts) provided by ref. [27] together with a framework to analyze datasets in a homogeneous manner 1.

The paper is structured as follows. In §2 we introduce the correction that we carried on the redshift of SNe. In §3 we explain the fitting procedure and the selection cuts on the original sample that have led to the compilation of two subsets. The resulting parameters of the fits are reported together with the Hubble diagrams. Finally, in §4 we summarize and discuss the results.

2. Apparent magnitudes and Redshift corrections

The apparent magnitudes of SNe measured by different groups at different redshifts must be corrected, before putting them together in the Hubble diagram, in such way that the absolute magnitude can be considered the same constant for all the SNe forming the sample. The photometric images are corrected for galactic extinction, $K$–correction, etc., and finally undergo

1 Recently this analysis chain has been further refined in ref. [39], the “Union2 sample”.
a light-curve width-luminosity correction [6]. The light curve of each SN is compared with a template and the peak apparent magnitude is corrected assuming a functional form [11]

\[ m_{\text{eff}} = m_{\text{peak}} + \alpha(s - 1), \]

where \( \alpha \) is a coefficient that will be determined by a fitting algorithm, simultaneously with the other cosmological parameters; \( s \) is the so called “stretch factor” that suitably stretches or contracts the time axis of each light curve around the date of maximum light, affecting both the rising and declining part of the light curve of each SN.

The stretch factor was introduced to reduce the intrinsic dispersion in the absolute brightness of supernovae, but it really acts as a physical mechanism and it was first considered due to a local astrophysical effect (for example the temperature - dependent variation in the opacity) by the authors in ref. [6]. Then, four years later, they same authors admitted that: “a single stretch factor varying the timescale of the SNe Ia accounts very well for the restframe B band light curve both before and after maximum light” but “It is not understood from the current status of the theory of SNe Ia whether this is a fortuitous coincidence or a reflection of some physics timescale” [40].

The behavior of the SNe Ia light curves is usually interpreted as a strong support for time dilation due to an expanding Universe. However, non-standard cosmological models have been suggested as alternative explanations for acceleration without dark energy. As a matter of fact, if we use a model-independent method to compare quantitatively the ability of some dark energy models and non-standard types of cosmology (braneworld, \( f(R) \), kinematic models, etc.) to represent the Union2 sample data [39], none of the considered cosmological models can be rigorously rejected on the basis of the current SN Ia data [41].

Here we propose an alternative view of the stretch factor that we interpret as a virtual displacement of the supernova from the measured redshift to a new effective redshift. Actually the width of the light curves is larger for SNe Ia at higher redshift \( (z > 0.1) \) than it is for local ones. In fact, due to redshift, the observed width evolves by a time-dilatator factor \( (1 + z) \). Still the same authors [40] adjusted the SCP and Calan/Tololo photometric data to the maximum intensity \( I_{\text{max}} \) and scaled the time axis by a width factor \( w = s(1 + z) \). Remarkably, with the application of this single stretch timescale parameter, the SNe Ia data fell all on a common curve at the level of the measurement uncertainty.

However, in our opinion, the correction of the apparent magnitude should be balanced by a correspondent correction of the redshift. In fact, since the transformation from the observer frame to the SN rest frame is obtained by dividing the time interval by the factor \( w = s(1 + z) \), this is equivalent to consider a time-dilatator factor \( (1 + z_{\text{corr}}) = s(1 + z) \), and SNe shall not be considered at redshift \( z \) anymore, but at somewhat corrected redshift \( z_{\text{corr}} \), such that

\[ z_{\text{corr}} = s + sz - 1. \]

In the light of our interpretation of the stretch factor, we think that the SN Ia data have to be refitted by using the above corrected redshift, which takes into account nothing else than the correction of the apparent magnitude of SNe adducted by the scientists of the SCP on the photometric data.

3. Hubble diagram construction and cosmological parameter fitting

We considered the data reported in ref. [27], even if these authors did not strictly preserved the original meaning of \( s \). Actually, in the recent update of the SNe sample [39] the meaning of the stretch factor was completely lost because the new fitting algorithm SALT2 returns a parameter \( x_1 \) that must be converted to the old \( s \) using a correlation formula that would become a source of other statistical errors.
The effective magnitude of the $i$–th of the $N$ SNe forming our modified sample is obtained from the formula:

$$m_{i}^{\text{eff}} = m_{i}^{\text{peak}} + \alpha(s_{i} - 1) - \beta c_{i}$$  \hspace{1cm} (3)$$

where $c$ is the rest frame color at maximum as defined in ref. [27] and $\beta$ is an additional parameter, which has been determined by the fitting procedure too.

The $\chi^{2}$ corresponding to that of eq.(3) is given as

$$\chi^{2} = \sum_{i=1}^{N} \frac{(m_{i}^{\text{eff}} - 5 \log_{10} D_{i} - \tilde{M})^{2}}{(\sigma_{m_{i}}^{2} + \alpha^{2} \sigma_{s_{i}}^{2} - \beta^{2} \sigma_{c_{i}}^{2}) + \sigma_{\text{int}}^{2} + \sigma_{v_{i}}^{2}},$$  \hspace{1cm} (4)$$

where $D$ is the Hubble free luminosity distance

$$D = (1 + z) \int_{0}^{z} \frac{dz'}{\sqrt{\Omega_{m}(1 + z')^{3} + \Omega_{\Lambda}}},$$  \hspace{1cm} (5)$$

$\tilde{M}$ is the magnitude zero point offset

$$\tilde{M} = M + 5 \log_{10} \frac{C}{H_{0}} + 25,$$  \hspace{1cm} (6)$$

$C$ is the speed of light, and $M$ is the absolute magnitude which is assumed to be constant after that the above mentioned corrections of apparent magnitudes have been implemented. $\sigma_{m_{i}}$, $\sigma_{s_{i}}$, $\sigma_{c_{i}}$ are the errors related to the magnitude, stretch factor, and color factor, respectively. We neglected the errors on $z$ and $z_{\text{corr}}$. As in ref. [42], we considered an error due to a peculiar velocity dispersion of 300 km sec$^{-1}$, which can be written:

$$\sigma_{v_{i}} = 0.005 \ln 10 \left( \frac{1}{1 + z_{i}} + \frac{1}{\sqrt{\Omega_{m}(1 + z_{i})^{3} + \Omega_{\Lambda}}} \int_{0}^{z_{i}} \frac{dz}{\sqrt{\Omega_{m}(1 + z')^{3} + \Omega_{\Lambda}}} \right).$$  \hspace{1cm} (7)$$

Furthermore, we supposed that our Universe is flat, so that $\Omega_{m} + \Omega_{\Lambda} = 1$.

We found a minimum of our $\chi^{2}$ in eq.(4) starting from an intrinsic dispersion of the relation $\sigma_{\text{int}} = 0.15$, and we updated this value refitting until we have obtained a reduced $\chi_{\text{red}}^{2} = \chi^{2}/\text{DOF} \approx 1$. To this aim we used the “Newton method search for a minimum” developed in Mathematica in a way very similar to the algorithm adopted in ref. [42]. The same results can be equivalently obtained using iteratively the “NonlinearRegress” functionality of Mathematica.

We applied this procedure to the standard sample of 307 SNe Ia reported in ref. [27]. The resulting four parameters ($\Omega_{m}$, $\tilde{M}$, $\alpha$, $\beta$) of the fitting procedure are listed in Table 1, which clearly shows, in the first row, a difference of more than 10% in the estimate of $\Omega_{m}$ between our procedure and the well known standard result.

Next, we corrected the measured redshift $z$ by eq.(2), and after this correction, we neglected the 35 following SNe because their $z_{\text{corr}}$ are now negative: 1993ag, 1993o, 1993h, 1992br, 1992bp, 1992bo, 1992bl, 1992aq, 1990af, 2001cn, 2000bh, 1993ac, 1994m, 1994t, 1995ak, 1996bo, 2000fa, 2000dk, 2000cn, 2000cf, 1999gd, 1999ek, 1999cc, 1998eg, 1998ef, 1998dx, 1998co, 1998ab, 1998v, 1997dg, 1997y, 1999bm, 1995ap, h283, n326.

Then, we considered the remaining 272 SNe, and we made a fitting test without taking into account the measurement errors, and imposing $\Omega_{m} = 1$. Actually, we did not want a really blind analysis of the data, but we wanted to test if the Einstein-de Sitter model, without any cosmological constant, should really be excluded from the cosmological theory. Considering the
Table 1. Fitting procedure results.

| $z$  | $N$    | $\Omega_m$ | $\bar{M}$ | $\alpha$ | $\beta$  | $\sigma_{\text{int}}$ | $\chi^2_{\text{red}}$ |
|------|--------|-------------|------------|----------|----------|------------------------|-----------------------|
| normal | 307 | 0.393 ± 0.046 | 24.015 ± 0.035 | 0.81 ± 0.12 | 1.15 ± 0.10 | 0.25 | 1.056 |
| corrected | 252 | 0.70 ± 0.22 | 24.24 ± 0.11 | 11.01 ± 0.41 | 1.25 ± 0.28 | 0.32 | 1.005 |
| corrected | 242 | 1 | 24.400 ± 0.034 | 10.57 ± 0.33 | 1.23 ± 0.24 | 0.20 | 1.075 |

parameters resulting from the test fit, 20 SNe were removed from the 272 SN sample having the highest residuals with respect to the Einstein-de Sitter model: they are: 1992ag, 1992ae, 1992p, 1990o, 1999gp, 1994s, 1999aa, 1999aw, 1996j, 1996cl, 2001jn, 2001jf, 040mb, 04Sas, 2003dy, e138, g142, k396, p534. After this selection cut, we applied our fitting procedure considering also the measurement errors and the corresponding results are listed in the second line of Table 1. With respect to the 307 SN data set, we obtained a completely different estimation of the mass density, i.e. $\Omega_m = 0.7$.

In order to verify the stability of our result, we removed other 10 SNe from our sample (1992bc, 2001cz, 1995ac, 1995ao, 2002w, 04D3df, 03D1cm, bo10, g055, m138). This time, the value of $\Omega_m$ changed again and became equal to the unity, causing the vanishing of any form of dark energy. Note that the minimum of $\chi^2$ in the last case falls outside the range of the allowed values $0 \leq \Omega_m \leq 1$. So we have computed some three parameters fits for fixed values of $\Omega_m$ in the right range. The result is that $\chi^2$ is a decreasing function of $\Omega_m$ that has the smallest acceptable value just when we put $\Omega_m = 1$. The values of the remaining three parameters are reported in Table 1. The behavior of the Hubble diagrams are shown in figures 1 and 2 for the 252 and the 242 SN subset, respectively.

By inspection of Table 1, it is evident that the absolute magnitude and the parameter $\beta$ are stable for the three samples. Instead, the cosmological parameter $\Omega_m$ changes drastically, and also the parameter $\alpha$ derived using $z_{\text{corr}}$ is very different from the one obtained with the standard $z$. The reason of the latter difference can be explained expanding $D(z_{\text{corr}})$ in the equation (5) around $s = 1$. At the first order in $(s - 1)$ we have

$$5 \log_{10} D(z_{\text{corr}}, \Omega_m) \simeq 5 \log_{10} D(z, \Omega_m) + \delta(z, \Omega_m)(s - 1),$$

where

$$\delta = \frac{5}{\ln 10} \left(1 + \frac{1 + z}{\Omega_m (1 + z)^3 + \Omega_\Lambda \int_0^z \frac{dz'}{(1+z')^3+\Omega_\Lambda}}\right).$$

If we insert this result in the equation (4) we can write $\chi^2(z_{\text{corr}})$ in terms of the normal $z$. Comparing it with the corresponding standard $\chi^2(z)$ we obtain $\alpha_{\text{norm}} = \alpha_{\text{corr}} - \delta(z_i)$ in the sum. So, considering $\delta = \sum_{i=1}^N \delta(z_i)/N$, which is the average value of $\delta$ in our sample, the estimates of $\alpha$ listed in Table 1, when we use the corrected redshift, are in effect $\alpha_{\text{corr}} \simeq \alpha_{\text{norm}} + \delta$.

4. Conclusions

We introduced a correction on the redshift of the SNe Ia that takes into account the correction of the corresponding luminosity, which is commonly performed in order to get all the SN light curves to match. In this framework, we reanalyzed the Union compilation [27] finding the following results:
Figure 1. Top: Hubble diagram for 252 type Ia SNe selected (see text) from the “Union compilation” reported in [27]. Bottom: Magnitude residuals from the best-fit flat cosmology, ($\Omega_m = 0.70$, $\Omega_\Lambda = 0.30$).
Figure 2. Top: Hubble diagram for 242 type Ia SNe selected (see text) from the “Union compilation” reported in [27]. Bottom: Magnitude residuals from the best-fit flat cosmology, ($\Omega_m = 1$, $\Omega_A = 0$).
We refitted the set of 307 type Ia SNe by a fitting procedure based on the Newton method search for a minimum. The constraint we obtained from supernovae on the dark energy density is $\Omega_{\Lambda} = 0.607 \pm 0.046$, which is quite different from the result of $\Omega_{\Lambda} = 0.713 \pm 0.028$ found in ref. [27]. This outcome reflects the fact that the fit of the SNe is very sensible to the fitting procedure and to the systematic and intrinsic errors inserted in the $\chi^2$.

- We corrected the redshift of the SNe according to eq. (2), and neglected the SNe whose redshift turned out to be negative. After a selection cut based on the residuals, we reduced the original compilation to two subsets formed by 252 and 242 SNe respectively. Fitting the first subset, we found that the values of the cosmological parameters were practically reversed ($\Omega_m = 0.70$, $\Omega_{\Lambda} = 0.30$). On the other hand, from the fit of the second subset we find even $\Omega_m = 1$, which corresponds to a Universe without dark energy.

- The fits of the SNe, both with $z$ and $z_{corr}$, depend on the number of SNe used too. The difference of few SNe between two samples changes dramatically the estimates of the cosmological parameters. The above mentioned case of the two subsets formed by 252 and 242 SNe respectively is emblematic.

In conclusion, we found that the constraint on the cosmological parameters based on the analysis of SNe is contingent on both fitting procedure and the number of SNe considered in the sample. On the contrary, the absolute magnitudes of SNe are not affected by variations in the sample.

We emphasize that it is well known that the hypothesis according to which the Universe is accelerating comes not only from the results of SNe Ia, but also from the analysis of CMB and galaxy clusters. The latter two methods converge on a result of $\Omega_{\Lambda} \simeq 0.70$, $\Omega_m \simeq 0.30$. The present work is not intended to replace the current paradigm of the cosmology (even if the debate is still alive [43]), but only to point out that the use of SNe Ia in the estimate of cosmological parameters can introduce, through the stretch factor, some instability in the final result, in addition to biases and discrepancies in the SN Ia data samples. So, all the results that come from the SNe Ia must be taken with more caution, especially when they are used to discriminate between different cosmological models.

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