Time-Dependent Transport in Mesoscopic Structures

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A discussion of recent work on time-dependent transport in mesoscopic structures is presented. The discussion emphasizes the use of time-dependent transport to gain information on the charge distribution and its collective dynamics. We discuss the RC-time of mesoscopic capacitors, the dynamic conductance of quantum point contacts and dynamic weak localization effects in chaotic cavities. We review work on adiabatic quantum pumping and photon-assisted transport, and conclude with a list which demonstrates the wide range of problems which are of interest.

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1. Introduction

Time-dependent transport in mesoscopic physics is a fascinating subject which yields a wealth of information which cannot be obtained otherwise. Electric fields, induced through oscillating voltages at the contacts or gates defining the sample, or induced by time-dependent magnetic fluxes, couple to the charge distribution of the mesoscopic structure. Time-dependent transport is thus an investigation of the charge distribution and its dynamics. In addition to the purely scientific interest, the investigation of time-dependent transport in small structures is important if applications are envisioned, such as in quantum computing, or in more practical problems such as capacitance and current standards [1]. In all possible applications we are interested not in the stationary-time independent behavior of small structures, but in driving them from one state to another and in doing this as fast as possible.

This work emphasizes a number of basic issues, provides some comments on the existing literature and attempts to make a few suggestions for further
research. The paper is not a scholarly review of the field. Nevertheless, due to the variety of topics addressed, we touch on a large portion of the recent literature and it is hoped that the article can thus also serve as a useful guide to the more recent work in this field.

Time-dependent phenomena can be roughly classified depending on whether they are a consequence of external forcing (externally applied voltages, time-dependent fluxes) or whether they occur spontaneously (thermal frequency frequency-dependent fluctuations, frequency-dependent shot noise). We can further distinguish, whether we deal with a linear process and analyze dynamic susceptibilities (dynamic conductance, magnetic susceptibilities) or whether we deal with a non-linear process (rectification, photon-assisted tunneling). Furthermore it is useful to distinguish between low frequency phenomena where we follow adiabatically a sequence of equilibrium states and a high-frequency non-adiabatic regime where the system is driven far from the ground state. These are already sixteen categories and it is clear that here we cannot address all of them. In addition we can combine different categories such as the investigation of shot noise in the presence of photon-assisted tunneling which is an example of an externally forced system in which we are in the non-linear, non-adiabatic regime, but are interested in a spontaneous process (the fluctuation spectrum). It is clear that we cannot provide a reasonable discussion of all these diverse phenomena. To limit the scope, we consider only externally forced phenomena: spontaneous dynamic processes (fluctuations) in mesoscopic systems are reviewed in Ref. 2.

2. Basic considerations

2.1. Formulation of the problem

It is useful to consider first some basic aspects of the problem at hand. We ask: are there some general principles which we can or even must use when formulating a description of a time-dependent process? Already, when faced with the task to determine the equilibrium electrostatic potential of a mesoscopic structure, we become aware of the fact that we must look beyond the conductor whose state is of immediate interest to us. In a typical mesoscopic structure the equilibrium electrostatic potential depends not only on the conductor itself but also on the other nearby electric charges provided by donors or acceptors, by gates and by contacts. To find the equilibrium electrostatic potential such additional nearby conductors must necessarily be part of the consideration. This fact is of particular importance in time-dependent transport, since what counts is not the externally applied field (presumed to be known) but the total electric field generated by all the relevant charges, whether they are within the conductor or away from it,
Time-Dependent Transport

on a gate or on the surface of a reservoir. Unlike in dc-transport where we get away with investigating particle currents, the total current \( j(r) \) in time-dependent transport is the sum of the displacement current \( (\varepsilon_L/4\pi)\partial E(r)/\partial t \) and the particle current \( j_p(r) \),

\[
j(r) = (\varepsilon_L/4\pi)\partial E(r)/\partial t + j_p(r).
\] (1)

Here \( \varepsilon_L \) is the dielectric constant (for simplicity taken to be space and time-independent). The total current is conserved,

\[
div j(r) = 0. \quad (2)
\]

Eqs. (1) and (2) are a consequence of the continuity equation and the Poisson equation. Eq. (2) states that along a line that is tangential to the current vector \( j \), the length of this vector is an invariant. Like the conservation law of energy permits the transformation of kinetic energy into potential energy, so similarly here, we are permitted to transform particle current into displacement current and vice versa. Very importantly, while the particle current exists only inside electric conductors, the displacement current is not limited to the conductor, but exists wherever we have a time-dependent electric field.

It is the total current which counts experimentally, not the particle current. This is particularly clear, if we can assume that all electromagnetic fields are localized. This assumption underlies the electrical engineering networks composed of \( R, C, L \) elements and possibly more complicated nonlinear elements. As a consequence of the localization of the electromagnetic fields, the currents at the terminals of such a network add up to zero (there is overall current conservation) and the sum of all charges in the network is also conserved. The localization of the electric field means that any field line which emanates from the conductor terminates a) again on the conductor, b) at a nearby gate or capacitor which is included in our consideration or c) at a reservoir (electrical contact) which must also be included into our consideration. The localization of electric fields means that we can find a volume, denoted \( V_\Omega \), large enough, such that the electric flux through the surface of this Gauss volume vanishes. Naturally, this implies that the total charge \( Q_\Omega \) within this Gauss volume vanishes and implies that the sum of all currents flowing in and out of this volume must add up to zero. It is thus reasonable to demand that we should provide a description of time-dependent transport such that the overall charge vanishes

\[
Q_\Omega(t) = 0 \quad (3)
\]

and such the sum of all currents at all the contacts (labeled \( \alpha = 1, 2, 3, \)) of
Fig. 1. Mesoscopic capacitor connected via a single lead to an electron reservoir and capacitively coupled to a gate. $V_1$ and $V_2$ are the potentials applied to the contacts, $U$ is the electrostatic potential of the cavity.

the contacts of the conductor and nearby capacitors adds up to zero,

$$\sum_{\alpha} I_\alpha(t) = 0. \quad (4)$$

Only if these conditions are fulfilled do we get answers which depend only on potential differences (answers which are gauge invariant).

In the simplest case the Gauss volume also coincides with the mesoscopic region in which phase-coherent electron motion is relevant. The Gauss volume separates then the mesoscopic region from the exterior macroscopic circuit to which we can apply the usual engineering description in terms of $R$, $C$, $L$ elements, current and voltage sources, noise etc. However, it should be noticed that this is not the only point of view: We might insist on treating the mesoscopic system and the external circuit on the same footing. For instance in the circuit considered in Ref. [3] the potential distribution depends on the location of the battery vis-a-vis the conductor. On the quantum level, such an approach would demand that we write down Hamiltonians for current and voltage sources, microwave generators, etc.

2.2. Mesoscopic capacitors

One of the elementary distinctions between dc-transport and ac-transport is that we can drive current not only through particle transport but also through a displacement and thus can correlate particle currents in conductors which are not connected by a dc-conductance path. This aspect of dynamic conductance is perhaps most simply illustrated by considering a mesoscopic capacitor. Fig. 1 shows a mesoscopic cavity connected only via one lead to an electron reservoir and separated from a back gate by an insulating layer. Note that there is no dc-transport possible in this structure.
**Time-Dependent Transport**

We are interested in the dynamic conductance \( G_{\alpha\beta}(\omega) = \frac{dI_\alpha(\omega)}{dV_\beta(\omega)} \) which gives the current at contact \( \alpha \) in response to an oscillating potential at contact \( \beta \). We present the solution obtained in Ref. [4]. For simplicity, we assume that we can consider the electrostatic potential \( U(\omega) \) in the cavity as uniform. Furthermore, we follow the literature on the Coulomb blockade and instead of the Poisson equation describe the relation between the charge on the cavity and the potential with the help of a geometrical capacitance \( C \). The gate is described as macroscopic conductor. In response to an oscillating potential \( dU(\omega) \) on the cavity and an oscillating gate voltage \( dV_2(\omega) \) we have an oscillating charge on the cavity given by \( dQ(\omega) = C(dU(\omega) - dV_2(\omega)). \) We have to find \( dU(\omega) \). Ref. [4] now considers first the response of non-interacting carriers to an oscillation of the potential at contact 1 assuming that the potential \( U \) in the cavity is held fixed. This response is

\[
G^0(\omega) = \frac{e^2}{\hbar} \int dE \text{Tr}[1 - \mathbf{s}^\dagger(E)\mathbf{s}(E + \hbar\omega)] \frac{f(E) - f(E + \hbar\omega)}{\hbar\omega}.
\]  

(5)

where \( \mathbf{s} \) is the scattering matrix which relates the incident current amplitudes in reservoir 1 to the outgoing current amplitudes in reservoir 1. \( \mathbf{1} \) is the unit matrix with dimension equal to the number of scattering channels. The trace is just the sum over all scattering channels. \( f \) is the Fermi function in reservoir 1. Note that \( G^0(0) = 0 \) since the scattering matrix is unitary. The total current at contact 1 is the result not only of the oscillating reservoir voltage \( dV_1 \) but also depends on the oscillating electric potential in the cavity. We write thus for the current at contact 1,

\[
dI_1(\omega) = G^0(\omega)dV_1(\omega) - i\omega\Pi(\omega)dU(\omega)
\]

with a response function \( \Pi(\omega) \) which we now determine. For the charge conserving answer which we seek, an overall potential shift cannot have an effect on the system. Thus if we subtract from all oscillating potentials \( dU(\omega) \) we must find the same current as given above. This is the case only if \( \Pi(\omega) = (-i/\omega)G^0(\omega) \). Using this and observing that the current at contact 1 is also the time-derivative of the charge on the cavity gives

\[
dI_1(\omega) = G^0(\omega)dV_1(\omega) - G^0(\omega)dU(\omega) = -i\omega C(dU(\omega) - dV_2(\omega)).
\]  

(6)

This equation now determines the potential

\[
dU(\omega) = \frac{dV_1 - i\omega C/G^0(\omega)dV_2}{1 - i\omega C/G^0(\omega)}.
\]  

(7)

Inserting this potential back into Eq. (5) gives the conductance \( G \equiv G_{11} = G_{22} = -G_{12} = -G_{21} \) of the interacting system,

\[
G(\omega) = \frac{-i\omega C}{1 - i\omega C/G^0(\omega)}.
\]  

(8)
We have now achieved a current conserving answer: Whether we measure at contact 1 or 2 we have to find the same current. We next would like to find the \( RC \)-time of the mesoscopic capacitor. To this end we consider \( 1 - i\omega C/G^0(\omega) \) and expand it to first order in frequency. (This requires an expansion to second order in frequency of \( G^0(\omega) \)). This gives us a dynamic conductance of the form

\[
G(\omega) = \frac{-i\omega C}{1 - i\omega R_q C}\mu. \tag{9}
\]

The dynamic conductance of the mesoscopic capacitor is, like that of a macroscopic capacitor, determined by an \( RC \)-time. But instead of only purely classical quantities, we obtain now expressions which contain quantum corrections due to the phase-coherent electron motion in the cavity. It turns out that the \( RC \)-time can be expressed with the help of the Wigner-Smith time-delay matrix

\[
N = \frac{1}{2\pi i} \frac{ds}{dE}. \tag{10}
\]

The sum of the diagonal elements of this matrix determines the density of states

\[
N = TrN = \frac{1}{2\pi i} Tr[\hat{s}\hat{s}^\dagger \frac{ds}{dE}] \tag{11}
\]

and gives rise to a "quantum capacitance" \( e^2 N \) which in series with the geometrical capacitance determines the electrochemical capacitance

\[
C^{-1}_\mu = C^{-1} + (e^2 N)^{-1}. \tag{12}
\]

The resistance which counts is the charge relaxation resistance (index \( q \))

\[
R_q = \frac{e^2}{2\hbar} \frac{Tr[\hat{N}[\hat{N}]}{[Tr\hat{N}]^2}. \tag{13}
\]

For simplicity we have given these results, Eqs. (10 - 13) only in the zero temperature limit. It is instructive to consider a basis in which the scattering matrix is diagonal. Since we have only reflection all eigenvalues of the scattering matrix are of the form \( exp(i\phi_n) \) where \( \phi_n \) is the phase which a carrier accumulates from the entrance to the cavity through multiple scattering inside the cavity until it finally exits the cavity. Thus the density of states can also be expressed as

\[
N = \frac{1}{2\pi} \sum_n (d\phi_n/dE) \tag{14}
\]

and is seen to be proportional to the total Wigner time delay carriers experience in the cavity. The time delay for channel \( n \) is \( \tau_n = \hbar d\phi_n/dE \).
Similarly we can express the charge relaxation resistance in terms of the energy derivatives of phases and we obtain in the zero-temperature limit,

\[ R_q = \frac{e^2}{2h} \left( \sum_n \frac{d\phi_n}{dE} \right)^2 \]

\[ \left( \sum_n \frac{d\phi_n}{dE} \right)^2 \]

(15)

\( R_q \) is thus determined by the sum of the squares of the delay times divided by the square of the sum of the delay times. We now briefly discuss these results. First, our Eq. (12) for the electrochemical capacitance predicts that it is not a purely geometrical quantity but that it depends on the density of states of the cavity. This effect is well known from investigations of the capacitance of the quantized Hall effect. More recent work investigates the mesoscopic capacitance of quantum dots and wires and is often termed capacitance spectroscopy. In addition, to the average behavior our results can also be used to investigate the fluctuations in the capacitance. Similar to the universal conductance fluctuations there are capacitance fluctuations in mesoscopic samples due to the fluctuation of the density of states. Such effects can be expected to be most pronounced if the contact permits just the transmission of a single channel. Then it is necessary not only to investigate the fluctuations of the mean square fluctuations but the entire distribution function. Such an investigation was carried out by Gopar et al.\(^5\) in the single channel limit and by Brouwer and the author\(^6\) and Brouwer et al.\(^7\) for chaotic cavities with quantum point contacts which are wide open (many channel limit). Since the Coulomb energy \( e^2/C \) is typically much larger than the level separation \( \Delta \) these fluctuations are small and possibly hard to observe.

Next let us discuss briefly the charge relaxation resistance \( R_q \). First we note that the resistance unit is not the von Klitzing \( h/e^2 \) but \( h/2e^2 \). The factor two arises since the cavity is coupled to one reservoir only. Thus only half the energy is dissipated as compared to dc-transport through a two terminal conductor. Second, we note that in the single channel limit, Eq. (15) is universal and given just by \( h/2e^2 \). This is astonishing since if we imagine that a barrier is inserted into the lead connecting the cavity to the reservoir one would expect a charge relaxation resistance that increases as the transparency of the barrier is lowered. In the large channel limit, Eq. (15) is proportional to \( 1/N \), where \( N \) is the number of scattering channels, and it can be shown that its ensemble averaged value is indeed proportional to \( 1/T \), if each channel is connected with transmission probability \( T \) to the reservoir.\(^8\) Thus in the large channel limit Eq. (15) behaves as expected.

Using the fluctuation dissipation theorem we also obtain the fluctuations of the current, the charge on the cavity, and the potential. Ref.\(^4\) gives a direct derivation of these fluctuation spectra without invoking the fluctuation dissipation theorem.
Markus Büttiker

The above discussion emphasizes the role of interaction in the investigation of time-dependent problems. The discussion also highlights that capacitances and charge relaxation resistance, or taken together, the RC-time, are fundamental for our understanding of ac-transport. Clearly, the simple view taken here which neglects exchange correlations and for poor contacts neglects effects which arise from the discreteness of the charge, leaves much room for improvement. For a discussion of capacitance fluctuations, taking into account the discreteness of charge we refer the reader to Kaminski et al.\textsuperscript{9} which builds on earlier work by Flensberg\textsuperscript{10} and Matveev.\textsuperscript{11}

3. The dynamic conductance matrix

Consider now an arbitrary geometry consisting of a mesoscopic conductor with \( M \) contacts, \( \alpha = 1, 2, 3, ..., M \) and \( N - M \) gates, \( \alpha = M + 1, ..., N \). We can use a similar approach as outlined above\textsuperscript{12} or in fact an approach which uses the full potential landscape\textsuperscript{13,14} to determine the dynamic conductance matrix \( G_{\alpha\beta}(\omega) = dI_{\alpha}(\omega)/dV_{\beta}(\omega) \). For simplicity, we assume that the external circuit connecting the various contacts exhibits zero impedance in all branches. An expansion of the dynamic conductance to second order in frequency is,

\[
G_{\alpha\beta}(\omega) = G_{\alpha\beta}(0) - i\omega E_{\alpha\beta} + \omega^2 K_{\alpha\beta} + O(\omega^3). \tag{16}
\]

Here the first term is the dc-conductance. This matrix has non-vanishing elements only for \( \alpha < M \) and \( \beta < M \), i.e. between contacts that permit carrier transmission. The second term is called the emittance matrix. If either \( \alpha > M \) or \( \beta > M \) and if both \( \alpha > M \) and \( \beta > M \) the elements of this matrix are purely capacitive. They are determined by the conductor to gate capacitances and by gate-gate capacitances. Even if \( \alpha < M \) or \( \beta < M \) the elements of this matrix might have the same sign as expected from a capacitance matrix. But for ballistic structures, or other structures with high transmission, the coefficients of this matrix might be dominated by kinetic effects and have a sign that we would expect if the system also contains inductive elements. Typically only coupling to the Poisson equation is considered and not to the full Maxwell equations. In this case we call a coefficient of the emittance matrix with a sign opposite to what is expected for a capacitance, kinetic-inductive. Below, we consider an example of such a matrix.

The term second order in frequency is dissipative and of the type \( C_\mu^2 R_q \), but again with a sign that depends on the kinetics of the transport.
3.1. The emittance matrix of a quantum point contact

As an example, we consider here the emittance matrix of a quantum point contact (QPC). A QPC is a small constriction in a two-dimensional electron gas which allows the transmission of only a few conducting channels. We consider a symmetric QPC with two gates as shown in Fig. 2a. and ask for its capacitance and low-frequency admittance. Again we greatly simplify the electrostatic problem by assuming that there are only two regions $\Omega_1$ and $\Omega_2$ to the left and to the right of the constriction with sizes of the order of the screening length (see Fig. 2a.). We are interested in the charge variation in these two regions. The theory now deals with two potentials $\delta U_1$ and $\delta U_2$ which describe the departure away from equilibrium of the electrostatic potentials in these regions. We only present the result that describes the opening of the first quantum channel. Thus the QPC has a transmission probability $T$ and a reflection probability $R$. Furthermore, we assume that at equilibrium the QPC has a right-left symmetry. The two gates are taken to be at the same voltage $V_3$. Thus in effect, the two gates act like a single gate. As in the previous section, the gate will be treated as a macroscopic conductor. Charge conservation is taken into account by requiring that the sum of the charges in $\Omega_1$, $\Omega_2$ and at the gates vanishes, $dq_1 + dq_2 + dq_3 = 0$.

The geometrical capacitance is now also a matrix $dq_i = C_{ij}dU_j$ and due
to the symmetry of the problem, the geometric capacitance matrix can be written in the form

$$C = \begin{pmatrix} C_0 + C & -C_0 & -C \\ -C_0 & C_0 + C & -C \\ -C & -C & 2C \end{pmatrix},$$

(17)

where $C_0$ is the geometric capacitance between $\Omega_1$ and $\Omega_2$ and where $C$ is the geometric capacitance between these regions and the gates. The emittance matrix can be expressed with the help of two electrochemical capacitances $C_g$ and $C_\mu$. Here $C_g$ is the electrochemical capacitance of the QPC vis-a-vis the gate and $C_\mu$ is the capacitance which determines to which extend we can charge the regions $\Omega_1$ and $\Omega_2$ differently, i.e. build up a dipole across the QPC. Denoting the quantum contribution to the capacitance of the two regions by $D = \frac{e^2 N_{\Omega_1 + \Omega_2}}{2}$ Ref. [15] finds

$$C_g = \frac{1}{C^{-1} + (D/2)^{-1}};$$

(18)

$$C_\mu = \frac{RC_0 + (C/2)(1 + R) + 2C_0 C_g D^{-1}}{1 + 2(2C_0 + C)D^{-1}}.$$

(19)

Note that $C_g$ is of the same form as Eq. (12), whereas the $C_\mu$ now depends on the reflection probability of the QPC. For the emittance matrix Ref. [15] finds

$$E_{11} = E_{22} = RC_\mu - DT^2/4 + C_g T/2,$$

$$E_{12} = E_{21} = C_g - E_{11},$$

$$E_{13} = E_{31} = E_{23} = E_{32} = -E_{33}/2 = -C_g.$$  

(20)

To elucidate the content of these equations consider the limiting case in which $C_0$ tends to zero. In this limit both the charge of the QPC and on the gate are fixed. We have $C_g = 0$ and all elements of the emittance matrix vanish except four elements which are equal in magnitude $E_{11} = E_{22} = -E_{21} = -E_{21} \equiv E$ with $E = \frac{E_C\mu - DT^2/4}{E_F/3}$. For a small transmission probability the first term in $E$ dominates: We have a very weakly leaking capacitor. On the other hand as the first channel becomes transparent and $T$ tends to one, we have a ballistic conductor. The emittance is negative and has the sign characteristic not of a capacitive response but of a (kinetic) inductive response. The full curve in Fig. 2b, which stays positive, is the capacitance $C_\mu$ as a function of the value $eU_0$ at the saddle point of the QPC potential and the curve which departs from this line and reaches negative values is $E$. The potential range shown covers the opening of three successive quantum channels which are separated by $E_F/3 = 7/3meV$. The dashed
Time-Dependent Transport

lines in Fig. 2b. show the behavior of the capacitance $C_\mu$ and the emittance element $E_{11}$ for $C = C_0$. The features in the capacitance become smaller and the transition from capacitive to kinetic inductive behavior extends over a voltage region corresponding to the opening of several channels.

An alternative discussion of the admittance of a QPC is presented by Aronov et al., who attempt to find the entire potential landscape. Strangely, however, they find regions in which the electric field points against the overall voltage drop. Of course that is not forbidden by any general principle, but for a QPC we expect the potential landscape to be a smoothly varying function both in the equilibrium state and in the presence of slowly oscillating external potentials.

3.2. Negative capacitance?

Before continuing the discussion, it is worthwhile to discuss briefly the rather entrenched practice to speak about negative capacitance of conductors which exhibit a (kinetic) inductive response $E_{11} < 0$ rather than as expected a capacitive response $E_{11} > 0$. As an example we cite here only two items. We emphasize that in a dynamical conductance measurement it is the emittance $E$ which is measured and not really the capacitance $C$. The example we have discussed shows that the capacitances $C_\mu$ and $C_g$ stay positive, independently of whether the emittance is positive or negative. There are examples for which the compressibility is negative and in such a case the term negative capacitance might be appropriate.

3.3. Magnetic field symmetry of dynamic conductance

Geometrical capacitances are independent of magnetic field. Through the density of states, however, the electrochemical capacitance becomes magnetic field dependent. As long as it is only the total (global or local) density of states which counts, capacitance coefficients are even functions of magnetic field. For a conductor with $M \geq 2$ contacts, the emittance matrix of Eq. (16) obeys the Onsager reciprocity symmetry $E_{\alpha\beta}(B) = E_{\beta\alpha}(-B)$. It can be shown that this symmetry relation applies also to the purely capacitive elements in the emittance matrix and that therefore capacitance elements exist which are not even functions of magnetic field. Experiments demonstrating this for a geometry where a small gate overlaps the edge of a two-dimensional conductor with two contacts have been carried out by Chen et al. in the integer regime and by Moon et al. in the fractional quantized Hall regime. In contrast, if we consider an arrangement of conductors each of which is connected by only one lead to a reservoir, the emittance matrix (which in this case is a pure capacitance matrix) is an even function of magnetic field. Under the same condition $K_{\alpha\beta}$ is also an
even function of magnetic field but only as long as inelastic scattering can be neglected. This later point, the change of symmetry depending on inelastic scattering is clearly interesting and deserves further work. An experiment on such a geometry is reported in Ref. [22]. A classification of the magnetic field symmetry is given in Ref. [23].

### 3.4. Frequency-dependent weak localization

In a pioneering experiment Pieper and Price [24] investigated the dynamic conductance of a mesoscopic Aharonov-Bohm ring with frequencies up to the GHz range. Denoting the time for diffusion across the metallic ring by \( \tau_d = L^2 / D \), the frequency is large enough to measure the real and imaginary part for \( \omega \tau_d < 1 \) and \( \omega \tau_d > 1 \). A drawback of the experiment is that the temperature \( kT > \hbar \omega \) at all accessible frequencies. As a consequence, as discussed in Refs. [25,26], the amplitude of the Aharonov-Bohm oscillations is nearly frequency independent. Below, I discuss briefly the frequency dependence of the weak localization in quantum chaotic cavities discussed in the charge neutral limit by Aleiner and Larkin [27] and for a chaotic cavity in proximity to a gate by Brouwer [6] and the author.

Ref. [6] proceeds by first evaluating the conductance matrix, Eq. (5), in the absence of screening. The currents are evaluated in response to an oscillating voltage in the contacts under the condition that the potential in the cavity is held fixed. For a quantum chaotic cavity coupled to reservoirs via two large contacts with \( N_1 \) and \( N_2 \) channels, and using random matrix theory [28] to perform the ensemble averages, Ref. [6] finds in an expansion up to order \( 1 \) in \( N^{-1} \), where \( N = N_1 + N_2 \), for the ensemble averaged conductance \( \langle G_{\mu\nu}(\omega) \rangle \) the following results. To leading order (order \( N \)) there is a classical contribution

\[
\langle G^{cl,\mu\nu}(\omega) \rangle = \delta_{\mu\nu} N_\mu - \frac{N_\mu N_\nu}{N(1 - i\omega \tau_d)}.
\]

and to order \( 1 \) there is a weak localization correction

\[
\langle \Delta G^{\mu\nu}(\omega) \rangle = \frac{(2 - \beta)N_\mu}{\beta N(1 - i\omega \tau_d)} \left( \frac{N_\nu(1 - 2i\omega \tau_d)}{N(1 - i\omega \tau_d)^2} - \delta_{\mu\nu} \right),
\]

where \( \tau_d = (h/N)\langle dn/d\varepsilon \rangle = (h/N \Delta) \) is the dwell time and \( \Delta \) is the mean level spacing. The index \( u \) indicates that we deal with an unscreened conductance. The symmetry index \( \beta = 1 \) (2) in the absence (presence) of a time-reversal-symmetry breaking magnetic field; \( \beta = 4 \) in zero magnetic field with strong spin-orbit scattering. The matrix \( \langle G_{\mu\nu}(\omega) \rangle = \langle G^{cl,\mu\nu}(\omega) \rangle + \langle \Delta G^{\mu\nu}(\omega) \rangle \) is not current conserving. Next let us compare this result with the case when screening is taken into account. Coupling to a nearby gate is again described.
Time-Dependent Transport

by a geometrical capacitance. Ref. [9] finds for the screened admittance
\[ \langle G_{\mu\nu}(\omega) \rangle = \delta_{\mu\nu} N_{\mu} - \frac{N_{\mu} N_{\nu}}{N(1 - i\omega\tau)}, \]
and the weak localization contribution
\[ \langle \Delta G_{\mu\nu}(\omega) \rangle = \frac{(2 - \beta)N_{\mu}}{\beta N(1 - i\omega\tau_{d})} \left( \frac{N_{\nu}(1 - 2i\omega\tau)}{N(1 - i\omega\tau)^2} - \delta_{\mu\nu} \right), \]
where \( \tau^{-1} = \tau_{d}^{-1} + e^2 N/hC \) is the \( R_{q}C_{\mu} \) time. The electrochemical capacitance of the cavity is
\[ C_{\mu}^{-1} = C^{-1} + (e^2(dn/d\varepsilon))^{-1} = C^{-1} + (Ne^2\tau_{d}/h)^{-1} \]
and the charge relaxation resistance is
\[ R_{q} = \frac{h}{e^2 N} = \frac{h}{e^2 N_{1} + N_{2}} \]
For the product we have \( R_{q}C_{\mu} = \tau \). It is interesting to compare the charge relaxation resistance \( R_{q} \) with the dc-resistance \( R = (h/e^2)(1/N_{1} + 1/N_{2}) \) which is the series addition of the contact resistances and is thus dominated by the smaller of the two contacts. In contrast, the inverse of \( R_{q} \) is the parallel addition of the contact conductances, and \( R_{q} \) is thus dominated by the larger of the two contacts.

Comparison of the two results shows that screening leads almost everywhere to the replacement of the dwell time \( \tau_{d} \) by the \( RC \)-time \( \tau \). The dwell term survives only in the weak localization correction which depends on both time-scales. That the dwell time can survive in the weak localization term is explained by the fact that the time reversed paths which give rise to weak localization can be viewed as (charge neutral) electron-hole trajectories. Since in typical experiments the charging energy is a few times larger than the level spacing the weak localization term depends on vastly different time scales \( \tau << \tau_{d} \) and this double time-scale behavior should be observable in experiment.

The conductance matrix, Eqs. (23, 24) is current conserving if the gate contact is included. The elements of the conductance matrix relating to the gate (contact 3) can be obtained by using the sum rules \( \sum_{\mu} G_{\mu\nu} = \sum_{\nu} G_{\mu\nu} = 0 \). Of course we can also insist that the unscreened result is current conserving and apply the above sum rules to determine the remaining elements of the conductance matrix. That corresponds to a cavity which has an infinite capacitance towards the gate.

3.5. The pulsed cavity

Consider now an experiment where we apply a voltage pulse \( V_{\alpha}(t) = a_{\alpha}\delta(t) \) to one of the contacts. The Fourier transform of such a pulse is a con-
stant and the resulting current is thus proportional to the frequency integral of the conductance matrix element. The current at contact \( \mu \) in response to a pulse at contact \( \nu \) is thus proportional to the frequency integral over the conductance \( G_{\mu\nu}(\omega) \), or \( G_{\mu\nu}(t) = (1/2\pi) \int d\omega \exp(-i\omega t) G_{\mu\nu}(\omega) \). The aim of the discussion presented here is two fold. First, we would like to establish the connection between Ref. [6] and works [29] which discuss a novel time scale which is intermediate between the dwell time \( \tau_d \) and the Heisenberg time \( \tau_H \). The Heisenberg time is \( \tau_H = N\tau_d \). It is the time scale at which a quantum system starts to notice the energy level structure. This connection concerns only the non-interacting system. Our second aim is to point out that the interacting system behaves differently and the intermediate time mentioned above does not appear, at least not within the limits of the first two terms in the \( 1/N \)-expansion of the conductance.

For the classical part of the ensemble averaged conductance this gives for the non-interacting system (for \( \mu = 1, 2, \nu = 1, 2 \))

\[
\langle G_{\mu\nu}^{cl,u}(t) \rangle = \frac{N_\mu N_\nu}{N\tau_d} \exp(-t/\tau_d)
\] (26)

an exponential decay determined by the dwell time, whereas for the interacting system the decay is very much faster since it is determined by \( \tau \),

\[
\langle G_{\mu\nu}^{cl}(t) \rangle = \frac{N_\mu N_\nu}{N\tau} \exp(-t/\tau).
\] (27)

On a log scale we have \( \ln[\langle G_{\mu\nu}^{cl,u}(t) \rangle N/N_\mu N_\nu] = -t/\tau_d \) for the non-interacting system and \( \ln[\langle G_{\mu\nu}^{cl}(t) \rangle N/N_\mu N_\nu] = -t/\tau \) for the classical, interacting system.

Let us next investigate the weak localization term. For the non-interacting case (index \( u \)), we find

\[
\ln[\langle \Delta G_{\mu\nu}^{u}(t) \rangle N/N_\mu N_\nu] = -t/\tau_d + \ln\left[ \frac{2-\beta}{N_\nu} (\delta_{\mu\nu} - \frac{N_\nu}{N}(\frac{t}{\tau_d})(2 - \frac{t}{2\tau_d})) \right].
\] (28)

The time-dependence is now already somewhat complicated as a consequence of the fact that the pole determined by the dwell time is of third order in the weak localization term (see Eq. (24)). While the initial time-dependence is governed by \( \tau_d \) there exists for the orthogonal and symplectic examples, a regime when the last term in the parenthesis proportional to \( t^2 \) becomes dominant and a deviation from simple exponential behavior becomes observable. The term proportional to \( t^2 \) will be of order 1 at a time \( t^2 = t^2_\nu = N\tau_d^2 = \tau_d\tau_H \) where \( \tau_H = N\tau_d \) is the Heisenberg time. This establishes the connection between the results of Ref. [6] and the discussions in Refs. [29].
Time-Dependent Transport

For the interacting system, there is now an additional time scale $\tau$ in the weak localization term in addition to the dwell time $\tau_d$. As a consequence, the pole in the weak localization term determined by the dwell time is only first order. The result is that even if screening is not complete (finite capacitance) there are no prefactors which grow proportional to $t^2$, but only with $t$. The result is simple only in the limit of complete screening ($\tau = 0$), where the decay of the weak localization term is exponential with the dwell time. We have $G_{11}(t) = G_{22}(t) = -G_{12}(t) = -G_{21}(t) \equiv G(t)$ with

$$\langle G(t) \rangle = G^{dc}[\delta(t) + \frac{2 - \beta}{\beta} \frac{1}{N} \frac{1}{\tau_d} \exp(-t/\tau_d)]$$

where $G^{dc}$ is the dc-conductance of the cavity. Note that in this case the classical response is instantaneous (within our approximations).

4. Pumping

4.1. Adiabatic quantum pumping

Different types of electron pumps have been of interest for a number of years. Adiabatic quantum pumping investigates the current in response to two (slowly) oscillating potentials $U_1(t) = u_1 \sin(\omega t)$ and $U_2 = u_2 \sin(\omega t - \phi)$ which in practice are applied to the system by varying two gate voltages. The pumping is called adiabatic since the frequencies are sufficiently slow for the system to follow a quasi-stationary state. It is called quantum pumping since the direction and magnitude of the current that is pumped depend on the sample specific quantum nature of the electron wave functions. Different theoretical approaches have been put forth for metallic diffusive conductors and chaotic cavities with perfect contacts and with contacts which are almost transparent. Below I present some results of the work of Brouwer. The experiment by Switkes et al. investigates the current response as a function of the phase difference $\phi$.

The charge which is expelled through contact $\alpha$ in response to an oscillating potential $U_1$ is given by

$$\delta Q_\alpha(t) = e^2 N(\alpha, 1) \delta U_1(t)$$

where

$$N(\alpha, 1) = -(1/4\pi ie) \sum_\beta Tr(\delta s^\dagger_{\alpha\beta}(\delta s_{\alpha\beta}/\delta U_1) - (\delta s^\dagger_{\alpha\beta}/\delta U_1)s_{\alpha\beta})$$

is the emittance of the conductor into contact $\alpha$. For the case of interest here, in the presence of two potentials the charge emitted through contact
\[ \delta Q_\alpha(t) = e^2(N(\alpha, 1)\delta U_1(t) + N(\alpha, 2)\delta U_2(t)). \] 

(32)

The total charge expelled in a time interval from 0 to \( T \) is

\[ dQ_\alpha(t) = e^2 \int_0^T dt(N(\alpha, 1)dU_1/dt + N(\alpha, 2)dU_2/dt). \] 

(33)

For the case considered here, where \( U_1 \) and \( U_2 \) are periodic functions in time with period \( T \), the pair describes in this parameter space a closed path \( S \). With the help of Green’s theorem, this integral can be written as a surface integral of the surface enclosed by the path \( S \),

\[ dQ_\alpha(t) = e^2 \int_S \left( \frac{\partial}{\partial U_1} N(\alpha, 2) - \frac{\partial}{\partial U_2} N(\alpha, 1) \right) dU_1 dU_2. \] 

(34)

Explicitly, in terms of the scattering matrices this result becomes

\[ Q_\alpha(T) = -\left( \frac{e}{2\pi i} \right) \sum_\beta \int_S Tr[\frac{\delta s^\dagger_{\alpha\beta}}{\delta U_1} \frac{\delta s_{\alpha\beta}}{\delta U_2} - \frac{\delta s^\dagger_{\alpha\beta}}{\delta U_2} \frac{\delta s_{\alpha\beta}}{\delta U_1}] dU_1 dU_2. \] 

(35)

The resulting current at contact \( \alpha \) is

\[ I_\alpha = \frac{e\omega \sin(\phi) u_1 u_2}{2\pi} \sum_\beta \text{Im} Tr[\delta s^\dagger_{\alpha\beta}(\delta s_{\alpha\beta}/\delta U_1)(\delta s_{\alpha\beta}/\delta U_2)]. \] 

(36)

Experimentally, the potentials \( U_1 \) and \( U_2 \) are not known. What is controlled are the voltages applied to the gates. Thus the considerations above should be extended in this direction. Brouwer\cite{Brouwer} does consider screening and finds for the problems he investigates only hardly noticeable changes compared to the unscreened result.

An experiment testing these predictions was performed by Switkes et al.\cite{Switkes} for a quantum chaotic cavity which is connected to reservoirs via quantum point contacts which are completely transparent for the lowest quantum channel. Oscillating voltages are applied to two of the gates used to define the geometry of the cavity. Switkes et al. measure the voltage which is produced by the pumping in an infinite external impedance circuit. The voltage is described by the simple expression \( V_{dot} = A_0 \sin(\phi) + B_0 \) with \( A_0 \) and \( B_0 \) extracted from fits to the data. It is found that \( A_0 \) fluctuates randomly (as a function of magnetic field) with an average that is roughly forty times smaller than the fluctuation amplitude. Similarly \( B_0 \) is a very small correction. For low pumping amplitudes the experimental data agree well with theory. We conclude the description of this adiabatic pump by emphasizing its wide applicability to test phase coherent fluctuation properties in a wide range of mesoscopic systems.
Time-Dependent Transport

4.2. Mechanical pumping

Thus far we have only considered electrical degrees of freedom. It is quite interesting that during recent years, different pumping mechanisms have been investigated, which are based on mechanical pumping. The pumps based on launching surface acoustic waves through a mesoscopic sample provide one example. Another example are shuttle pumps investigated by Gorelik et al.\textsuperscript{35}: a quantum dot oscillates between the left and right reservoir and during each oscillation transfers one charge. Experiments are reported in Ref. \textsuperscript{36}. The accuracy of such a pump is the subject of work by Weiss and Zwerger.\textsuperscript{37} Such developments are important, especially in view of current and capacitance standards.\textsuperscript{1} They are scientifically interesting since the domain of electro-mechanical mesoscopic effects remains largely unexplored.

5. Photon-assisted transport

5.1. Quantum dots

A successful set of experiments has been carried out by applying microwaves in the GHz range to quantum dots. Whereas earlier experiments dealt with quantum dots for which the density of states could effectively be viewed as continuous, recently an experiment by Oosterkamp et al.\textsuperscript{38} succeeded in effect to see photon-assisted transport via the ground state and or through an excited state. Photon-assisted transport is a non-linear mechanism whereby a carrier absorbs and re-emits a photon and a dc-current is generated. In the dots investigated in Ref. \textsuperscript{38} the charging plays an important role. Theoretically such a situation was treated by Bruder and Schoeller\textsuperscript{39} extending a master equation approach to the Coulomb blockade into the dynamic regime. The experimenters compare their results with the discussion of Tien and Gordon of photon-assisted transport (PAT) which states that the essential effect of the ac-voltage drop over the tunneling barrier (forming the contact of the dot to the reservoir) is to modify the static tunneling rate $\Gamma(E)$ according to

$$\Gamma_{\text{PAT}}(E) = \sum_n J_n^2(\alpha) \Gamma(E + n\hbar\omega)$$

where $J_n$ is the n-th order Bessel function and $\alpha = eV/\hbar\omega$ with $V$ the voltage drop accross the barrier. Note that the Tien and Gordon approach predicts that the current through a barrier in the presence of a microwave field can be obtained from the static $I-V$-characteristic. In Ref. \textsuperscript{40} Eq. (37) is used to formulate a master equation for a two state charge model (corresponding to a dot with $N$ and $N+1$ electrons). Note that for $N = 2$ distributed over five
single particle levels this gives already ten equations for the probabilities of all different dot configurations. To compare with experiments voltage drops of different magnitude are permitted across the contacts. This implies that at least one reservoir potential is taken to be oscillatory in addition to the central potential in the dot. (We note that this is in contrast to most of the theoretical work, which treats this problem by assuming that it is only the dot potential or the energy levels in the dot which oscillate). By and large the model works remarkably well.

5.2. Quantum Point Contacts

It is instructive to compare the success of experiments in quantum dots with similar experiments aiming at observing photon-assisted transport in quantum point contacts (QPC). Despite a number of efforts, no clear signature of photon-assisted transport has been observed in QPC’s. Instead it seems to be possible to explain the experiments in terms of heating generated by the microwaves. The success in quantum dots is likely due to the fact that it is a problem with high tunnel barriers which help to generate localized electric fields and help to isolate the dots from the leads. There are many theoretical works on photon-assisted QPC’s and wires. It would thus be very interesting to find a way to observe photon-assisted transport in such structures to test these predictions. A recent work suggests that one should try to localize the electric field for instance with the help of superconductors placed above the QPC with only a narrow opening. We remark here only, that it is not sufficient to localize the external field: what counts is the total field. In a QPC or a quantum wire the poor screening will generate a total field over a large region even if the external field is well localized.

5.3. Role of displacement currents

In a number of works it is stated that displacement currents play no role in photon-assisted transport. This is correct in the sense that if we evaluate a dc-current the displacement part of the current \( \epsilon_L/4\pi \partial \mathbf{E}/\partial t \) does of course not contribute. However, the particle current \( j_p \) is a function of the total electric field and moreover, it is a non-linear function of the field. To investigate the role of the displacement current in photon-assisted transport, it is useful to remember, that even so we are mainly interested in the dc-current, photon-assisted transport is also associated with time-dependent currents. In the presence of sinusoidal applied voltages \( dV_\alpha(t) = dV_\alpha(\omega) \cos(\omega t) \), the resulting currents at contact \( \alpha \) have components at all harmonics of \( \omega \). In other words a theory is needed not only for \( I_\alpha(\omega = 0) \) but also for \( I_\alpha(n\omega) \). The Fourier comoments \( n = \pm 1 \) are the currents at the driving frequency. Ref. investigated this and presents a theory based
Time-Dependent Transport

on an RPA screening approach, in terms of an expansion of the currents in powers of the Fourier amplitude of the driving voltages. This theory is formulated such that the overall charge of the conductor and gates obeys Eq. (3) for all harmonics. It is shown, that the self-consistent potential within the dot is essentially determined by the currents and charges at $n = \pm 1$. The self-consistent potential in turn also determines the zero-frequency Fourier component. In fact, one can view photon-assisted transport as a down conversion of the displacement fields at frequency $\omega$. RPA might not be the proper approach to treat problems in which charge quantization is important, but the central point made in Ref. 45 is clearly independent of the self-consistent scheme that is applied.

5.4. Pumping with a single localized potential?

Another remark seems here appropriate: consider a quantum dot at zero external bias. Switkes et al. 33 write: "A periodic deformation (of the potential) that depends on a single parameter cannot result in net transport; any charge that flows during the first half-period will flow back during the second". Parts of the literature are, however, in contradiction to this very plausible statement. The statement is obviously true, for the adiabatic quantum pump described above where two potentials are essential and in addition their mutual phase is of importance. It is also true for the analysis of Ref. 45, where a single spatially localized potential does not lead to a dc-current in the absence of a bias due to the unitarity of the scattering matrix. However, neither of these discussions can be viewed as a general proof. A proof could start along the following lines: potential oscillations, even in the equilibrium state, occur spontaneously, due to thermal (or zero point quantum) noise. Such spontaneous fluctuations, clearly, cannot lead to a dc-current since that would be tantamount to say that an equilibrium state does not exist. Until some very special conditions are fulfilled which do not correspond to what is possible in an equilibrium ensemble, there can be no resulting dc-current. Of course the fluctuations which invoke two potentials, as in the adiabatic quantum pump, also occur spontaneously, but with a phase that is random.

5.5. Double Dots

Coupling two quantum dots hybridizes the states of each dot and gives rise to coupled states which represent a covalent bonding and anit-bonding state. Two states with energies $E_l$ and $E_r$ on the left and the right dot in the absence of coupling are separated by an energy $\Delta E^* = E_{anti-bond} - E_{bond} = \sqrt{(\Delta E)^2 + 4|t|^2}$ where $\Delta E = E_l - E_r$ and $|t|$ is a coupling energy. A theoretical investigation of transport through double quantum dots is
Markus Büttiker

provided by Stoof and Nazarov and by Stafford and Wingreen. Electron transport is possible when an electron in the bonding state absorbs a photon and is promoted to the anti-bonding state. The condition for this process is

\[ \hbar \omega = \Delta E^* \] or

\[ \Delta E = \sqrt{(\hbar \omega)^2 - 4|t|^2}. \] (38)

An experiment by Oosterkamp et al., taking the coupling energy $|t|$ as a fit parameter, shows a remarkably good agreement with Eq. (38) over a wide range of frequencies. In view of the discussion given above, oscillations of the charge in the leads, displacement currents, etc., it is an even more astonishing result. In part this can be explained by the fact that Eq. (38) is a resonance condition, and that all interesting effects are thus buried in the coupling energy $|t|$. Clearly, it would be interesting to compare theoretical predictions for the width of the resonance with the measurements. Further, it would be interesting to see real time-oscillations of the excited state, as this was done recently in a Josephson junction circuit by Nakamura et al.

6. Breadth of the field

In the previous paragraphs we have been able to touch on a few problems related to dynamical transport in mesoscopic structures. However, a number of important topics have been omitted. Following is a list of a few topics which give an impression of the wide range of questions addressed in this field. The references given are in no way complete but are presented here only as an initial guide to a citation trail which the interested reader has to follow on his own initiative.

6.1. Closed Systems

Mesoscopic systems can be enclosed in the dielectric medium of a capacitor or a transmission line. This permits a contactless investigation of a number of susceptibilities of closed mesoscopic systems. We mention here in particular the work of Noat et al. where the orbital magnetic susceptibilities of closed squares are investigated and the work of Reulet et al. where the conductance of isolated rings is measured in a frequency range between 330MHz and 1065 MHz. Nonlinear dynamic effects in rings, such as transport in presence of a linearly increasing flux continue to be a subject of theoretical interest.

6.2. Aharonov-Bohm effect in capacitance

We have pointed out that the capacitance of mesoscopic structures is not a purely geometrical quantity but via the density of states depends on the
Time-Dependent Transport

properties of the system. For a small ring threaded by an Aharonov-Bohm
flux, the charge distribution will in general depend on the flux, and as a con-
sequence a capacitance can like the conductance of a ring exhibit Aharonov-
Bohm oscillations. Systems for which the charge quantization is important
are predicted to exhibit especially pronounced oscillations.

6.3. Weakly non-linear ac-response

An interesting regime for ac-transport is the onset of non-linearity, i.e.
the initial departure away from Ohm’s law. In this regime one might still
hope to find answers of considerable generality. We refer here only to one
recent theoretical work by Ma et al.

6.4. Resonant Double Barriers

The dynamics of resonant double barrier structures has long been a
subject of interest. We refer to Ref. and the work of Anantram for a
self-consistent discussion of ac-conductance.

6.5. ac-conductance of wires

Perfect wires provide, like quantum point contacts and resonant double
barriers another elementary system which can be investigated to test essen-
tial ideas. While the dc-conductance depends only on the equilibrium elec-
trostatic potential and for an adiabatic connection to reservoirs is quantized,
the ac-conductance is very sensitive to interactions. The single channel wire
is very often regarded as an example of a Luttinger liquid and addressed with
the help of bosonization techniques, but a discussion within RPA leads
to the same results and is instructive. The determination of the coupling
constants of a wire in proximity to a gate with the help of ac-measurements
is the subject of Ref. Wires connected to reservoirs only (no gates) but
with a realistic Coulomb interaction are the subject of Ref.

6.6. Superlattices

In the dynamic regime superlattices exhibit a wide variety of effects:

dynamical localization of carriers, absolute negative conductance, current
harmonics generation, Shapiro steps, continue to be of interest. Superlat-
tices are of discussed also as THz-photon detectors.

6.7. Dynamics of edge states

An essential aspect of the dynamics of electrical conductors are plas-
mons. Plasmons are especially interesting in high magnetic fields where they
propagate along the edges of the sample. We mention here only the experi-
Markus Büttiker

mental works by Zhitenev, et al.\[68\] who measure the time-delay of a voltage pulse, and the work by Talyanskii, et al.\[66\] who investigate the scattering of edge plasmons at a barrier.

7. Conclusion

The investigation of dynamic transport permits us to probe the inner energy scales of a conductor, especially those associated with the charge distribution and its (collective) dynamics. The ratio of experiments to theoretical works and proposals is still very small. It is hoped that this article can contribute to change this.

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REFERENCES

1. For review see K. Flensberg, A. A. Odintsov, F. Liefrink and P. Teunissen, (unpublished).
2. Ya. M. Blanter and M. Büttiker, Phys. Rep. (unpublished).
3. J. D. Jackson, Am. J. of Physics \textbf{64}, 855 (1996).
4. M. Büttiker, H. Thomas, and A. Pretre, Phys. Lett. A\textbf{180}, 364 - 369, (1993).
5. V. A. Gopar, P. A. Mello, and M. Büttiker, Phys. Rev. Lett. \textbf{77}, 3005 (1996)
6. P. W. Brouwer and M. Büttiker, Europhys. Lett. \textbf{37}, 441-446 (1997).
7. P. W. Brouwer, K. M. Frahm, and C. W. J. Beenakker, Phys. Rev. Lett. \textbf{78}, 4737 (1997).
8. C. W. J. Beenakker, unpublished 1999 communication.
9. A. Kaminski, I. L. Aleiner, and L. I. Glazman, Phys. Rev. Lett. \textbf{81}, 685 (1998);
   A. Kaminski et al., Phys. Rev. B \textbf{59}, 9798 (1999).
10. K. Flensberg, Phys. Rev. B \textbf{48}, 11156 (1993).
11. K. A. Matveev, Phys. Rev. B \textbf{51}, 1743 (1995).
12. M. Büttiker, A. Prêtre and H. Thomas, Phys. Rev. Lett. \textbf{70}, 4114 (1993).
13. M. Büttiker, J. Phys. Condensed Matter \textbf{5}, 9361 (1993).
14. M. Büttiker, J. Math. Phys. \textbf{37}, 4793 (1996).
15. M. Büttiker and T. Christen, in "Mesoscopic Electron Transport", NATO Advanced Study Institute, Series E: Applied Science, edited by L. L. Sohn, L. P. Kouwenhoven and G. Schoen, (Kluwer Academic Publishers, Dordrecht, 1997).
   Vol. 345. p. 259.
16. T. Christen and M. Büttiker, Phys. Rev. Lett. \textbf{77}, 143 (1996).
17. I. E. Aronov, G. P. Berman, D. K. Campbell, S. V. Dudiy J. Phys.: Condens Matter \textbf{9}, 5089 (1997); I. E. Aronov, N. N. Beletskii, G. P. Berman, D. K. Campbell, G. D. Doolen and S. V. Dudiy, Phys. Rev. B \textbf{58}, 9894 (1998).
Time-Dependent Transport

18. M. Ershov et al, Appl. Phys. Lett. 70, 1828 (1997); N. A. Penin, Semiconductors, 30, 340 (1996).
19. M. Büttiker, J. Phys. Condensed Matter 5, 9361 (1993).
20. W. Chen, T. P. Smith, M. Büttiker and M. Shayegan, Phys. Rev. Lett. 73, 146 (1994).
21. J. S. Moon, J. A. Simmons, J. L. Reno, and B. L. Johnson, Phys. Rev. Lett. 79, 4457 (1997).
22. P. K. H. Sommerfeld, R. W. van der Heijden, and F. M. Peeters, Phys. Rev. B 53, 13250 (1996).
23. I. E. Aleiner and A. I. Larkin, Phys. Rev. B 54, 14423 (1996).
24. P. W. Brouwer and C. W. J. Beenakker, Phys. Rev. Lett. 82, 608 (1999).
25. M. Switkes, C. Marcus, K. Capman, and A. C. Gossard, Science 283, 1905 (1999).
26. M. Büttiker, H. Thomas and A. Prêtre, Z. Phys. B 94, 133 (1994).
27. C. Weiss and W. Zwerger, Europhys. Lett. 47, 97 (1999).
28. T. H. Oosterkamp, L. P. Kouwenhoven, A. E. A. Koolen, N. C. van der Vaart, and C. J. P. M. Harmans, Phys. Rev. Lett. 78, 1536 (1997).
29. C. Bruder and H. Schoeller, Phys. Rev. Lett. 72, 1076 (1994).
30. T. H. Oosterkamp, W. G van der Wiel, S. De Franceschi, C. J. P. M. Harmans and L. P. Kouwenhoven, cond-mat/9904359
31. J. A. Del Almo, C. C. Eugster, Q. Hu, M. R. Melloch, M. J. Rooks, Superlattices and Microstructures, 23, 122 (1998).
32. K. Yakubo, S. Feng and Q. Hu, Phys. Rev. B 54, 7987 (1996).
33. A. Grinzaig, L. Y. Groelik, V. Z. Kleiner, R. I. Shekhter, Phys. Rev. B 52, 12168 (1995).
34. C. S. Tang and C. S. Chu, Phys. Rev. B 60, 1830 (1999).
35. M. Pedersen and M. Büttiker, Phys. Rev. B 58, 12993 (1998).
36. T. H. Stoof and Yu. V. Nazarov, Phys. Rev. B53, 1050 (1996).
37. C. A. Stafford and N. S. Wingreen, Phys. Rev. Lett. 76, 1916 (1996).
38. T. H. Oosterkamp, T. Fujisawa, W. G van der Wiel, K. Ishibashi, R. V. Hijman,
Markus Büttiker

S. Tarucha and L. P. Kouwenhoven, Nature 395, 873 (1998).
49. Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, cond-mat/9904003
50. Y. Noat, H. Bouchiat, B. Reulet and D. Mailly, Phys. Rev. Lett. 80, 4955 (1998).
51. B. Reulet, M. Ramin, H. Bouchiat, and D. Mailly, Phys. Rev. Lett. 75, 124 (1995).
52. L. Gorelik, S. Kulinich, Yu. Galperin, R. I. Shekhter, and M. Jonson, Phys. Rev. Lett. 78, 2196 (1997).
53. M. Büttiker, Physica Scripta, T54, 104 (1994).
54. M. Büttiker and C. A. Stafford, Phys. Rev. Lett. 76, 495 (1996); P. Cedraschi and M. Büttiker, J. Phys. Condens. Matter 10, 3985 (1998).
55. I. V. Krive, P. Sandstroem, R. I. Shekther, and M. Jonson, Europhysics Lett. 38, 213 (1997); P. Sandstroem and I. V. Krive, Annals of Physics 257, 44 (1997).
56. E. Simanek, Phys. Rev. B 60, 4410 (1999).
57. M. V. Moskalets, Physica E 4, 111 (1999).
58. Z.-s. Ma, J. Wang, and H. Guo, Phys. Rev. B 59, 7575 (1999).
59. M. P. Anantram, J. Phys. Condens. Matter 10, 9015 (1998).
60. I. Saï and H. J. Schulz, Phys. Rev. B 52, R17040 (1995); I. Saï, Ann. Phys. (Paris) 22, 463 (1997).
61. V. V. Ponomarenko, Phys. Rev. B 54, 10328 (1996).
62. Ya. M. Blanter, F.W.J. Hekking, and M. Büttiker, Phys. Rev. Lett. 81, 1925 (1998).
63. V. A. Sablikov and B. S. Shchamkhalova, Phys. Rev. B 58, 13847 (1998).
64. B. J. Keay, S. Zeuner, S. J. Allen, Jr., K. D. Maranowski, A. C. Gossard, U. Bhattacharya, and M. J. W. Rodwell, Phys. Rev. Lett. 75, 4102 (1995).
65. K. Hofbeck, J. Genzer, E. Schomburg, A. A. Ignatov, K. F. Renk, D. G. Pavel’ev, Yu. Koschurinov, B. Melzer, S. Ivanov, S. Schaposchnikov, P. S. Kop’ev, Phys. Lett. A218, 349 (1996).
66. R. Aguado and G. Platero, Phys. Rev. Lett. 81, 4971 (1998).
67. A. Ignatov and A.-P. Jauho, J. Appl. Phys. 85, 3643 (1999).
68. N. B. Zhitenev, et al., Phys. Rev. Lett. 71, 2292 (1993).
69. V. I. Talyanskii, et al., J. Phys.: Condens. Matter 7, L435 (1995).