In this paper we investigate the spatial and temporal variability of $N$ over the city of Milan, using the historical record of soundings in the period 1991–2007, using 00GMT and 12GMT soundings and performing evaluations at intervals 300–700 m, 800–1800 m, 1500–2500 m, (00GMT only) and 2500–3500 m, 3000–4000 m, 4000–5000 m, 7000–8000 m (both 00GMT and 12GMT).

The values obtained reveal that the Brunt-Väisälä frequency is subject to a moderate change with height over the entire observation period, and once the height is fixed shows only weak seasonal changes. At the height interval 1500–2500 m, the maximal values of $N$ are observed between December and January, whereas from April to September smaller values of $N$ represent a flat plateau. These variations generally decrease with increasing height. They are still recognizable in the interval 2500–3500 m, and fully diminish at 7000–8000 m.

**KEY WORDS:** Brunt-Väisälä frequency, free atmosphere, static stability

**ABSTRACT**

This paper investigates the time variation of the Brunt-Väisälä frequency over the area of Milan on period 1991–2007, for which easily accessible low-resolution soundings are available.

The types of time variation addressed are the average seasonal change referred to the whole period considered, and changes in yearly average.

Provisions have been taken to prevent and avoid, in the limits possible, perturbations due to PBL dynamics.

**DATA AND METHOD**

Data used in this work are low resolution soundings taken at the Linate airport (16080 LIML, lat. 45°27’N, lon. 9°16’E, 4 km E of Milan), as available from the Wyoming University download facility.

The time period considered spans 1991 to 2007. In this interval, sounding are available at standard hours 00GMT, 06GMT, 12GMT and 18GMT. Of these, profiles 06GMT and 18GMT are not always present and have then not been considered. At site longitude, 00GMT is always nighttime, and 12GMT daytime.

Direct use of sounding data to estimate the Brunt-Väisälä frequency is not possible due to measurement errors and other perturbations. Then, individual profiles have been fitted by monotonic cubic splines whose value has been sampled at the ends.
of a family of height intervals (300–700 m, 800–1800 m, 1500–2500 m, 2500–3500 m, 3000–4000 m, 4000–5000 m, 7000–8000 m). Interval Brunt-Väisälä frequency has then been estimated from these samples using the first order difference approximation

$$N^2 = \frac{g}{\nu} \frac{\Delta \theta_v(z)}{\Delta z}.$$ 

On 12GMT, estimates of the Brunt-Väisälä from height intervals 300–700 m, 800–1800 m and 1500–2500 m have been excluded to prevent contamination from PBL dynamics. On 00GMT data from intervals 300–700 m and 800–1800 m have been used for checking purposes and not used to draw conclusions, as their value is likely to reflect PBL dynamic properties inherited through the residual layer.

The actual cubic spline fitting has been done using the routine CUBGCV [Hutchinson, 1986], which per se does not guarantee the monotonicity of the fitting function. Individual fitted profiles have then been evaluated in the useful range (300 m to 8000 m) and any non-monotonic item has been removed. This approach is actually more severe than imposing monotonicity constraint during spline identification, and has been used to reduce the opportunity of ill-behaved profiles to contaminate the statistics. Of the 20135 total profiles considered, 5049 yielded non-monotonic splines and have been consequently discarded.

The surviving profiles have then been evaluated at experimental height levels, and the absolute difference between experimental and estimated temperature values have been inspected.

At end of these computing, a data set containing values of $N$ relative to the height intervals mentioned has been constituted, with contamination from PBL dynamics reduced as possible. Data in set are time stamped with date and time (00 and 12GMT), allowing separation of nocturnal and diurnal estimates.

Various time averages have then been formed starting from the data set, namely:

- Whole data set and yearly average and standard deviation of $N$ data with same reference hour (00 or 12GMT)
- Bootstrap estimate of the 95% confidence limits on period and yearly averages of $N$ data with same reference hour (00 or 12GMT)
- “Typical year” of $N$ data with same reference hour (00 or 12GMT) from the whole data set.

RESULTS

The averages of mean and maximum absolute difference between experimental and estimated temperature have been found respectively equal to 0.59 and 1.76 K for profiles at 00GMT and 0.56 and 1.70 K at 12GMT. The corresponding empirical density functions at 00GMT are shown in figure 1.

The average absolute residual is close to the 0.5 K error selected when performing the data interpolation, even after the non-monotonic profiles have been excluded from the data set. If per-radiosounding maxima of absolute residuals are considered, the value of their average does not exceed 2 K. Both mean and maximum absolute deviations are small respect to the typical temperature variation along the height range 300-8000m, and suggest a good visual adaptation of the smooth reconstructed profiles to the experimental data.

In figure 2, two specific examples of reconstructed profiles are shown along with the corresponding experimental radio sounding points. The good adaptation suggested by residuals is confirmed by visual inspection; the problem with low resolution radio soundings available for public download is also evident, the number and position of experimental data points being remarkably non-uniform from sounding to sounding, and widely spaced
along the vertical even close to ground in some instances. This may make difficult using directly these data for further more detailed analyses, namely detecting by visual means the height of the PBL to exclude boundary-layer affected evaluation in a more sophisticated manner.

Table 1 presents the period average of the Brunt-Väisälä frequency at different height intervals. The error values represent the maximum deviation of the average from lower and upper limits of the respective 95% confidence interval, obtained using the bootstrap method. Individual standard deviations are also shown, to present the overall variability of the data set.

Table 2 shows the yearly average and standard deviation of Brunt-Väisälä frequency for the height intervals 1500–2500, 2500–3500 and 7000–8000 meters. All averages and standard deviations are computed using only 00GMT soundings. Averages are given with error values defined as the maximum shifts from the lower and upper limits of 95% confidence limits obtained using the bootstrap method.

Figures 3 and 4 illustrate the seasonal changes in the Brunt-Väisälä frequency, using the 00GMT soundings over the entire observational period. Figure 3 presents the 1500–2500 m data via a box-plot summarizing the monthly empirical distributions; and
Figure 4 compares the monthly means from the two height intervals 1500–2500 m and 7000–8000 m; the variation bars in figure 4 represent the monthly standard deviations over the whole data period (1991–2007).

DISCUSSION

The data presented show a modest tendency of the period-averaged $N$ to decrease with increasing height (for convenience, the height is attributed to the lower limit of each interval). The pattern observed is

| Height above Geoid (m) | $N$ at 00GMT | Standard deviation | $N$ at 12GMT | Standard deviation |
|------------------------|--------------|--------------------|--------------|--------------------|
| 300–700 m              | 0.0136 ± 0.0002 | 0.0041             | n/a          | n/a                |
| 800–1800 m             | 0.0105 ± 0.0001 | 0.0036             | n/a          | n/a                |
| 1500–2500 m            | 0.0110 ± 0.0001 | 0.0035             | n/a          | n/a                |
| 2500–3500 m            | 0.0109 ± 0.0001 | 0.0031             | 0.0116 ± 0.0001 | 0.0029            |
| 3000–4000 m            | 0.0111 ± 0.0001 | 0.0030             | 0.0115 ± 0.0001 | 0.0026            |
| 4000–5000 m            | 0.0111 ± 0.0001 | 0.0028             | 0.0108 ± 0.0001 | 0.0028            |
| 7000–8000 m            | 0.0089 ± 0.0002 | 0.0043             | 0.0087 ± 0.0002 | 0.0056            |
independent on the choice of the sounding hour (00GMT or 12GMT).

By analyzing the yearly averages of $N$ it can be noticed that the changes are small. At 1500 m the minimum value occurred in the year 2003, characterized in Northern Italy by the warmer summer in the whole period considered. However, the relation between $N$ and the yearly average surface temperature in Linate has not been investigated, and may be the subject of future work.

Comparing the yearly variation of $N$ at 1500–2500 m to other altitudes, it can be noticed the latter's present a smoother behavior, with fluctuations on a longer time scale. The minimum Brunt-Väisälä frequency at 2500 m (respectively 7000 m) occurred on 2005 (respectively 2007).

The monthly variation in the Brunt-Väisälä frequency at 00GMT exhibit a seasonal dynamics (figure 3), characterized by a maximum in December and January waning out to a lower plateau from April to August.

This dynamics, still evident at 2500–3500 m, is not evident at 7000–8000 m (figure 4), where an almost constant behavior manifests.

At all altitudes investigated, however, the seasonal variation of monthly values taken at 00GMT is in the same order of magnitude of monthly variation. In figure 3 the bands from 25th to 75th percentiles shows a strong variability in monthly data. The variation bars in figure 4 further confirm this. In order to clarify this point in a definitive manner, analysis of 00GMT high resolution sounding data on a longer period may be necessary.

### Table 2: Yearly $N$ at 00GMT

| Year | 1500–2500 m | 2500–3500 m | 7000–8000 m |
|------|-------------|-------------|-------------|
|      | Mean (± Std. dev) | Mean (± Std. dev) | Mean (± Std. dev) |
| 1991 | 0.0115 ± 0.0005 (0.0034) | 0.0112 ± 0.0003 (0.0030) | 0.0088 ± 0.0003 (0.0025) |
| 1992 | 0.0115 ± 0.0005 (0.0033) | 0.0111 ± 0.0003 (0.0029) | 0.0084 ± 0.0003 (0.0023) |
| 1993 | 0.0113 ± 0.0004 (0.0035) | 0.0108 ± 0.0003 (0.0030) | 0.0089 ± 0.0003 (0.0029) |
| 1994 | 0.0111 ± 0.0004 (0.0031) | 0.0106 ± 0.0004 (0.0031) | 0.0092 ± 0.0006 (0.0058) |
| 1995 | 0.0110 ± 0.0004 (0.0033) | 0.0109 ± 0.0003 (0.0031) | 0.0090 ± 0.0004 (0.0033) |
| 1996 | 0.0113 ± 0.0004 (0.0036) | 0.0113 ± 0.0003 (0.0031) | 0.0087 ± 0.0003 (0.0027) |
| 1997 | 0.0112 ± 0.0004 (0.0031) | 0.0108 ± 0.0003 (0.0027) | 0.0093 ± 0.0007 (0.0050) |
| 1998 | 0.0110 ± 0.0003 (0.0031) | 0.0112 ± 0.0003 (0.0029) | 0.0099 ± 0.0013 (0.0068) |
| 1999 | 0.0107 ± 0.0004 (0.0032) | 0.0107 ± 0.0003 (0.0028) | 0.0096 ± 0.0010 (0.0062) |
| 2000 | 0.0109 ± 0.0003 (0.0030) | 0.0110 ± 0.0003 (0.0029) | 0.0089 ± 0.0005 (0.0045) |
| 2001 | 0.0107 ± 0.0004 (0.0033) | 0.0110 ± 0.0003 (0.0028) | 0.0089 ± 0.0004 (0.0044) |
| 2002 | 0.0107 ± 0.0003 (0.0030) | 0.0109 ± 0.0003 (0.0026) | 0.0081 ± 0.0005 (0.0030) |
| 2003 | 0.0105 ± 0.0005 (0.0033) | 0.0108 ± 0.0003 (0.0031) | 0.0089 ± 0.0005 (0.0047) |
| 2004 | 0.0107 ± 0.0004 (0.0033) | 0.0106 ± 0.0004 (0.0026) | 0.0088 ± 0.0004 (0.0033) |
| 2005 | 0.0112 ± 0.0006 (0.0058) | 0.0109 ± 0.0005 (0.0057) | 0.0091 ± 0.0006 (0.0059) |
| 2006 | 0.0108 ± 0.0004 (0.0034) | 0.0107 ± 0.0003 (0.0027) | 0.0079 ± 0.0006 (0.0029) |
| 2007 | 0.0106 ± 0.0004 (0.0032) | 0.0107 ± 0.0003 (0.0028) | 0.0083 ± 0.0003 (0.0025) |
Figure 3: Typical year of Brunt–Väisälä frequency between 1500 and 2500 m (00 GMT)

Figure 4: Comparison between typical year of Brunt–Väisälä frequency at different altitudes (00 GMT)
CONCLUSIONS

The method used yielded estimates of the Brunt-Väisälä frequency made from the automatic processing of publicly available radio sounding data, and providing values compatible with typical value of 0.01.

The estimates permitted to constitute a data set spanning a 17 years time period and various reference altitudes, from which a climatology of the Brunt-Väisälä frequency has been obtained. Key elements of this climatology are:

- A relative constancy of yearly averages, with minor variations possibly correlated to other climatological indicators;
- The existence of a seasonal variation, more evident at low altitudes, superposed to a strong variability.

All findings described in this paper are by their very nature local, and their extension to other sites is not automatic. In particular, a relation between the kind and amplitude of seasonal variation with latitude is likely to occur, and an extension of this study to other sites is currently planned by the authors.

The climatology of the Brunt-Väisälä frequency described in this work is suitable of improvement, at the cost of using more advanced data and processing options. In particular:

- Use of high resolution radio soundings would allow the visual detection of anomalies, such as residual layers, allowing the precise removal of the part of profiles directly affected by PBL dynamics;
- High resolution soundings would also allow the direct detection of convective PBL, allowing its removal and so improving the value of data at 12GMT, currently underused;
- Explicit imposition of monotonicity on the temperature fitting cubic splines, although less conservative than the non-monotonic profile removal adopted, would allow a more efficient use of the data available – this change would be of even higher value when the more "nervous" high resolution data are used.

Addressing these items would allow to explore the behavior of the Brunt-Väisälä frequency at altitudes lower than the ones taken into account in this work, with possible practical implications. These extensions are currently under evaluation by the authors, and may constitute the subject of a future work.

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**Giovanni Grandoni** has received his degree of Doctor of Physics (specialization in Geophysics) at the University “La Sapienza” of Rome.
Starting his carrier in Enea (Italian National Agency for New Technologies, Energy and Environment) at 1980, he was engaged in investigations on the air quality by developing and implementing a multi-sources original deterministic model about the pollutant dispersion and transport into the atmosphere and the deposition to the ground level.
Being a scientific leader of the Project “ATMOSFERA®®” he has carried out activities and resources to develop neural network stochastic models to predict air quality in some Italian big cities (Rome, Milan, Naples).
He coordinates the scientific activities and collaborates with Italian (Rome, Salento) and foreign universities (ASU, FMI, RSHU, Helsinki) to characterize the micro-meteorological state of the atmosphere into the PBL and develop new technologies and scientific tools oriented to the air quality control.
Author and co-author of numerous publications, he has been also awarded for achieved outcomes.
Maria Cristina Mammarella graduated from the “La Sapienza” University (specialization in Mathematics). She joined the ENEA in 1983 to carry out weather studies and research on meteodiffusivity and air quality and she made a decisive contribution to the development of an automatic intelligent station based on neural network, able to forecast air pollution level 72 hours in advance (A.T.M.O.S.F.E.R.A.©®). Besides A.T.M.O.S.F.E.R.A.©® projects, carried out in Rome, Milan and Naples, M.C. Mammarella made, or coordinated, several ground applied research projects for air quality control, such as A.R.T.E.M.I.S.I.A, in Udine district and A.R.T.E.M.I.S.I.A 2 in Sicily. Recently M.C. Mammarella focused her research on linking forecast systems to neural network using the studying of “Atmospheric Boundary Layer”, which has fundamental effects on spreading pollution agents in the air. She is involved with international research groups (such as Arizona State University, the Finnish Meteorological Institute and Helsinki University) for developing new instruments and technologies. Credits for the results obtained were given to her by OCSE from Paris, by SAS Institute, by CIVR from Department for Scientific Research. She was proposed as a candidate from Accademia dei Lincei for the Italgas Award and in 2008 she received the Eunomia Award. The results of her studies were published in scientific journals and conference proceedings.

Dr. Maurizio Favaron has obtained his degree in applied mathematics at the University of Milan in 1987. His research interests are ultrasonic anemometry, physics of the Planetary Boundary Layer, and pollutant dispersion, with works published in some peer-reviewed journals. As project leader and software engineer he contributed to the development of the Meteoflux® real-time eddy covariance self-standing automatic station, and currently participates in the development of data acquisition systems and models for micro-meteorological and hydrological applications. Since 2004 he is member of the Geological Society of America (GSA) and since 2009 member of the Association for Computing Machinery (ACM).