Could Dark Energy be a Manifestation of Gravity?

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ABSTRACT

It is shown that so-called dark energy could possibly be a manifestation of the gravitational vortex producing the ‘gravitomagnetic’ (GM) force field: associated with cosmic matter rotation and inertial spacetime frame dragging. The general relativistic Gödel-Obukhov spacetime metric which incorporates expansion and rotation of the Universe is used to evaluate this force. This metric is expressed here in spherical comoving coordinates. Through a cosmic time-scale evolution, it is shown that cosmic acceleration is expected when the magnitude of the radial repulsive GM force exceeds that of the usual attractive gravitational ‘gravitoelectric’ (GE) force: associated with just cosmic matter and spacetime warping. In general, this phenomenon of cosmic accelerated expansion appears to have occurred twice in the history of the Universe: the inflationary phase and the present-day acceleration phase. It is suggested in this model that the two phases may or may not be related. The cosmological model presented here is described in the context of Einstein’s Theory of General Relativity in Riemann-Cartan spacetime, which includes cosmic rotation, its effects, and it being considered as an intrinsic part of spacetime. Also, a derived analytical expression for the cosmic primordial magnetic field is presented; and how it might relate to the GM field through the spin density of the cosmic matter is discussed.

Key words: cosmology: miscellaneous – cosmology: theory – dark energy – gravitation – methods: analytical – methods: numerical.

1 INTRODUCTION

Einstein’s Theory of General Relativity together with ordinary matter, described by the standard model of particle physics, cannot fully explain the observational data from Type Ia supernovae (Perlmutter et al. 1998; Perlmutter et al. 1999; Riess et al. 1998; Riess et al. 2004), the matter power spectrum of large scale structure (Tegmark et al. 2004), and the anisotropy spectrum of the cosmic microwave background radiation (Spergel et al. 2007), with all these data suggesting the presence of ‘dark energy.’ From general relativity, assuming homogeneity and isotropy, the standard cosmological model is commonly described by the Friedmann, Lemaître, Robertson, Walker (FLRW) 1920s and 1930s solutions to the Einstein field equations for an expanding universe (equation 32). According to this standard cosmological model, the expansion of the Universe, if it contains only non-negative mass-energy density $\rho$ and pressure $p$, decelerates, as expected on the grounds that gravity is attractive and the cosmological constant $\Lambda$ is zero. The recent observations of cosmic acceleration, first discovered and confirmed by Perlmutter et al. (1998) and Riess et al. (1998) from type Ia supernovae, can only be explained by considering repulsive gravity. In the standard cosmological model, this is achieved by models introducing matter with negative pressure and/or $\Lambda \neq 0$ [see Perivolaropoulos (2006), and references therein]. Observations suggest that the alleged cosmic acceleration can only be a very recent phenomenon and must have set in during the late stages of the matter dominated expansion of the Universe. Recent observations by Riess et al. (2004) identify the transition from a decelerating to an accelerating universe to be at $z = 0.46 \pm 0.13$. Now, among suffering from the coincidence problem (e.g., Why is the energy density of matter and radiation nearly equal to the dark energy density today?) and the cosmological constant problem (e.g., How could $\Lambda$ have been so large during inflation but so incredibly small today?), the standard model does not explain why the acceleration has started in the recent past.
To avoid resorting to anthropic principle arguments to gain acceptance of the above mentioned models, of $p < 0$ and/or $\Lambda \neq 0$, which are constrained by the so-called standard cosmological model, perhaps we should seek a wider understanding of a general relativistic cosmology, not constrained by non-rotation, a spacetime being an extension of the standard cosmological model. This ‘new’ standard cosmological model then should take into account cosmic rotation as well as cosmic expansion: two degrees of freedom. When this is done, we find that a repulsive force of gravity is a natural occurrence and could possibly provide an explanation for the recent acceleration phase of the present-day universe, and possibly shed light on our understanding of the physics of so-called inflation in the early universe.

In this paper, the nature of so-called dark energy is investigated. The aim is to answer the question, Could dark energy be a manifestation of gravity? i.e., Could it be that component of gravity, the so-called gravitomagnetic (GM) force field, associated with cosmic rotation and inertial spacetime frame dragging? Inertial is used here in the general dynamical sense. The Gödel-Obukhov metric (Obukhov 2000; Jain, Modgil, & Ralston 2007) which incorporates expansion and rotation of the universe, derived from general relativity, is used to define spacetime separation (or distance), in this quest to answer the above question. Importantly, the Gödel-Obukhov metric or geodesic line element can ensure the absence of closed timelike curves, making it completely causal, different from the originally proposed Gödel metric (Gödel 1949). The Gödel-Obukhov cosmological model (Obukhov 2000) contains parameters which smoothly interpolate between this cosmology and the standard FLRW cosmology (which describes an isotropic and homogeneous universe filled with matter: commonly represented by an ideal fluid). Note, the independent nature of vorticity as associated with shear of a fluid and pure rotation does not allow limits on cosmic rotation to be placed by limits on vorticity (Obukhov 2000; Obukhov, Chrobok, & Scherfner 2002; Jain et al. 2007). Namely, the Gödel-Obukhov spacetime metric is shear-free but the vorticity and expansion are nontrivial. It is not vorticity of pure cosmic rotation that would lead to anisotropy of the microwave background radiation temperature distribution, but effects of vorticity associated with a shearing force.

The Gödel-Obukhov model does not conflict with any known cosmological observations. Among distinctive predictions of the model are effects on the propagation of light (Jain et al. 2007). Cosmic rotation affects a polarization of radiation, which propagates in this curved spacetime, resulting in some observable anisotropy (Obukhov 2000). This appears to have been confirmed (Birch 1982, 1983; Kendall & Young 1984; Nodland & Ralston 1997a,b; Jain & Saralo 2006). Observational tests have been done, that do not require redshift information, by Jain & Ralston (1999). They found significant signals of anisotropy in a large sample of data. Several other observations of radiation propagating on cosmological scales have been found to indicate a preferred direction, all of which are aligned along the same axis (e.g., Ralston & Jain 2004; Hutsemékers et al. 2005). The origin of these effects, however, may be independent of gravitation and restricted to modifications of the electromagnetic sector in which polarization observations are exquisitely sensitive (Jain et al. 2007). Perhaps, the model presented in this present manuscript will lend support to the possibility that such effects may indeed be gravity related.

Dark energy is the popular motivation to consider models beyond the standard Friedmann-Robertson-Walker spacetime metric. Current data (Jarosik et al. 2011; Tegmark et al. 2004) appear to fit a flat cosmology with $\Omega_{m0} \sim 0.27$ and $\Omega_{\Lambda} \sim 0.73$ for matter and dark energy density parameters, respectively, in the popular lambda cold dark matter (ΛCDM) cosmological model, which assume negative pressure in a FLRW cosmology. These fits assume an isotropic universe, while, at face value, the data used in the fits substantially contradicts isotropy (Jain et al. 2007), at least it appears so from the above observational tests. Jain et al. (2007) used large redshift type Ia Supernova data (see Jain et al. 2007, and references therein) and related magnitudes, to place constraints upon parameters appearing in the Gödel-Obukhov metric (which does not have the restriction of an isotropic universe). This is done by obtaining bounds on an anisotropic redshift versus magnitude relationship and on accompanying parameters of the Gödel-Obukhov metric. They found that the outcome depends on what are used for the host galaxy extinctions. The most reasonable fits do not show any signals requiring anisotropy. Yet, the existence of some small anisotropy cannot be ruled out. It appears that their findings are consistent with present-day observations, and it might be reasonable to investigate models that perhaps yield some anisotropy, particularly the Gödel-Obukhov model.

The Gödel-Obukhov metric, with exact general relativistic solutions as expressed by the cosmic scale factor $R = R(t)$ and its derivatives (Obukhov 2000), describing the evolution of $R(t)$, just as commonly done for the Friedmann-Robertson-Walker metric, avoids the principal difficulties of old cosmological models with rotation, where $R(t)$ describes the expansion of physical spatial distances. For example, the Gödel-Obukhov metric is consistent with isotropy of the microwave background radiation, like the standard cosmology, and it produces no parallax effects. The final state according to this metric depends on the values of two cosmological coupling constants (discussed below) in which torsion can cause the universe to either accelerate ($\frac{\dot{R}}{R} > 0$) or prevent cosmological collapse ($\frac{\dot{R}}{R} \equiv 0$), with these constants playing a similar role to that played by the cosmological constant, $\Lambda$, in the FLRW spacetime cosmology.

Using the Gödel-Obukhov metric to define separation of spacetime events, we find that cosmic acceleration is expected when the radial repulsive GM force (associated with rotational energy) exceeds the attractive radial ‘gravitoelectric’ (GE) force (associated with rest mass energy or mass-energy). This appears to be somewhat the idea behind Einstein’s introduction

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1 The terms gravitomagnetic and gravitoelectric defined in this manuscript are not the same as those defined in so-called gravitoelectro-
of the cosmological constant $\Lambda$, when he introduced it in his Theory of General Relativity to explain how the Universe could resist collapse under the inward force of gravity. However it was later thrown out by Einstein as his greatest blunder. It seems that he did not see any use for $\Lambda$ in a universe already expanding according to the Hubble law, before discarding it. In any case, maybe $\Lambda$, in a general sense, is like the Hubble constant that changes over time with the age of the Universe. That is, just as the Hubble constant relates to cosmic expansion of the Universe, and, as we shall see, cosmic rotation, perhaps $\Lambda$ relates only to the effects of cosmic rotation. In the Gödel-Obukhov spacetime cosmology, presented here, two cosmological coupling constants $|\lambda_1$ and $\lambda_3$ (Obukhov 2000)] due to rotation and torsion [as related to general relativistic frame dragging or the so-called Lense-Thirring (1918) effect] in curved spacetime take on the role of $\Lambda$. Moreover, we find that the Gödel-Obukhov spacetime cosmology appears not only to explain recent observations of the accelerated expansion, but suggests a dynamical description of the Universe over time that is a natural general relativistic extension of the standard FLRW cosmology. In this sense the answer to the question posed in the title appears to be yes.

The organization of this paper is as follows. In Section 2 a detailed description is presented of the model used here to explain the present epoch acceleration of the Universe: as being a gravitational-rotational-inertial phenomenon. A formalism containing the astrophysical and mathematical descriptions of the components to validate the model’s claims is presented in Section 3. It includes analytical derivations of the Gödel-Obukhov metric in spherical comoving coordinates, the cosmic radial GM and GE force fields, and the density of the Universe, where the GE and GM fields are the gravitational analogues of electric and magnetic force fields, respectively. In general, the GE and GM fields relate directly to the total mass and rotation, respectively, of a gravitating system (see Williams 2002). Also, included are the cosmological parameters used, which includes the cosmic rotational (or angular) velocity, the scale factor, and the Hubble constant. The numerical results from evolving the analytical expressions for the GE and GM accelerations over time are presented in Section 4 and a discussion is presented in Section 5. Included in Section 5 an analytical expression for the cosmic magnetic field (equation 74) given by the Gödel-Obukhov metric in terms of the spin density is evolved over time and compared with observations and theory. Surprisingly, this expression suggests how the magnetic field and the GM field might be related through the spin density. Conclusions are presented in Section 6.

2 MODEL DESCRIPTION

It seems reasonable to assume that at $t \approx 0$ the Universe had, at least, two degrees of freedom: translational, associated with the expansion and collapse (or infall) in the $\hat{e}_r$-direction and rotational, associated with cosmic rotation in the $\hat{e}_\phi$-direction, where $\hat{e}_r$ and $\hat{e}_\phi$ are global unit vectors connected to the cosmological spacetime metric. Rotation at $t \approx 0$ is consistent with the Gödel-Obukhov spacetime metric (Obukhov 2000), which allows for rotation and expansion. For the initial conditions of the Universe we will assume that a gravitationally unbound ‘hot’ energetic rotating plasma existed at $t \sim 10^{-43}$ s, the Planck time, corresponding to the Planck length, $l_p \sim 1.6 \times 10^{-33}$ cm, wherein beyond it is believed that quantum-gravity must be used, since both general relativity and quantum mechanics simultaneously become important on such small scales and high density. We assume that this rotating matter is ‘embedded’ in an inertially expanding spacetime coordinated frame, inertia as in Newton’s first law of motion. This inertially expanding frame, however, can be associated with an inertial force that wants to drag (or torque) the cosmic spacetime matter out of rotation, i.e., $\omega_{\text{FD}} < 0$, where $\omega_{\text{FD}}$ is the inertial frame dragging angular velocity in the $\hat{e}_\phi$-direction. It seems that the cosmic rotation is coupled with gravity and the inertial expansion is associated with the initial force that ‘ignited’ the Big Bang.

Now, let us go further to assume that the Big Bang was perhaps due to this inertial force ‘stretching’ the cosmic matter apart as spacetime expands (like splitting the nucleus of an atom) with a cosmic cataclysmic quantum-gravitational, $E_U = Mc^2$, type explosion that caused infinitely dense matter to expand relativistically outward due to a force with a strength similar to that of the singularity that gives rise to the event horizon of a black hole [which draws matter inward, even light, due to warped spacetime according to Einstein], where $M_U$ is the total mass of the Universe at $t = 0$, and $E_U$ is the total energy of the Universe. It seems that this inertial force wants to ‘flatten’ spacetime wherein world lines will have straight instead of curved geodesics. This inertial force field appears to play an intrinsic part in the process from whence our Universe is expanded. Perhaps its initial force was the force, as mentioned above, needed to release the infinitely large potential energy trapped in pre-existing Big Bang conditions in which, at least, the physical forces of nature were unified.

The existence of this inertial field appears to be similar to the idea that Einstein had in proposing, in his Theory of General Relativity (see de Sitter 1916), that ‘world-matter’ (the matter he postulated to be the origin of the energy of inertia) was located at the boundary of the Universe, and controlled and provided energy for the whole Universe, from so-called supernatural masses. Einstein, it seems, was using his idea of conversion of matter to energy. It appears that de Sitter (1917) later persuaded Einstein to adopt a new hypothesis with the world-matter not at the boundary of the Universe, but distributed...
over the whole Universe, which is finite (a sphere or an ellipse), though unlimited (i.e., no boundary). In this new hypothesis inertia is produced by the whole of world-matter, and gravitation is produced by local deviations from homogeneity.

Now if we constitute Einstein’s new hypothesis with properties of the proposed inertial field, then, based on this constitution, the global gravitational field of the rotating cosmic matter is a deviation from uniformity of the cosmic inertial expansion. If the energy of inertia is to keep the Universe expanding, then deviations from this due to gravity and rotation would have negligible effects locally, i.e., on small scales, but on large scales, say globally, any significant deviation may have an effect on the expansion rate. This has been realized in the standard cosmological Big Bang model described by the Friedmann-Robertson-Walker metric, and incorporated in the deceleration parameter:

\[ q = q(t) \equiv q_r = -\frac{\ddot{R}R}{R^2} \]  

(1)

which depends on the Hubble constant \( H(t) \), as related to the mass (or mass-energy) density \( \rho(t) \) of the Universe (see equation (3)), and the scale factor \( R \) and its derivatives, with \( \Lambda = \rho = 0 \), where, when the subscript is 0, it means \( t = t_0 \), the present epoch.

A further example of affecting the expansion is the recently observed cosmic acceleration. Now, gravitating systems are associated with mass and, it seems safe to say, rotation (through conservation of angular momentum). Since the mass density has an effect on the spacetime expansion, thus determining the geometry in the standard cosmological model, then cosmic rotation (appearing to a property of gravity through conserved angular momentum) may also have an effect on the expansion. The apparent tendency of the inertial spacetime force (or world-matter) to stay expanding in a flat spacetime and the dragging of this spacetime on the rotating frame associated with gravity (the mass-energy curving spacetime) are the physical mechanisms proposed here to be the origin of recently observed acceleration of the cosmic expansion.

Consistent with what is allowed by the Gödel-Obukhov spacetime metric, the frame dragging velocity, \( \omega_{FD} \), can be \(< 0 \) or \( > 0 \), with the \(< 0 \) solution naturally being chosen if we express \( \omega_{FD} \) in the usually sense, as we shall see in the following section. To understand physically what is meant by \( \omega_{FD} < 0 \), we use the analogy of a rotating black hole. As mentioned above we assume that the Universe has two major degrees of freedom, rotation and expansion (which includes infall). Now, in the case of a massive rotating black hole, the gravitational force of the black hole drags inertial frames into rotation, with \( \omega_{FD} > 0 \). But in the case of the matter of the Universe, undergoing what one might call an ‘anti-gravitational’ expansion (i.e., a reversed gravitational collapse), the inertial force of the expansion appears to drag (or torque) the spacetime cosmic matter of the Universe out of rotation, with \( \omega_{FD} < 0 \). In both cases, a force is produced, which we refer to as the GM force, that acts on any moving matter or particle in the dragged frame. It is like say the Coriolis force acting on moving matter in a rotating frame, analogous to the Lorentz force acting on a charged particle in a magnetic field: hence came the word GM (see e.g., Thorne, Price, & Macdonald 1986). However, importantly, in the case of the Universe, with \( \omega_{FD} < 0 \), this GM force is of a repulsive nature, and could very well be associated with the present-day observed cosmic acceleration. Note, in the case of the black hole, with \( \omega_{FD} > 0 \), the GM force is also of a repulsive nature (see Williams 2004, 2002).

The mass density \( \rho(t) \) regulates the warping of spacetime, while supplying the inhomogeneity to the original homogeneously expanding Universe. It appears that the cosmic inertial expansion is a flat spacetime phenomenon like light (or electromagnetic radiation) traveling in an inertial frame, i.e., without the influence of gravity. The expansion, it seems, behaves like light, with general relativistic gravitation properties as well. In other words, the expansion appears to have the inertial properties of electricity and spacetime properties of gravitation. This would be expected, it seems, of a large scale, once unified force field, of, at least, gravity and electromagnetism.

Now, consistent with the degrees of freedom stated above, we will assume that at \( t \approx 10^{-43} \) s, at least three large-scale forces, producing the following magnitudes of acceleration, were present to act on the mass-energy of the Universe: (1) the GE acceleration of gravity \( g_{GE} \) associated with \( \rho(t) \), (2) the GM acceleration of gravity \( g_{GM} \) associated with cosmic rotational velocity \( \omega_{rot} \), and (3) the initial acceleration of the expansion \( a_1 \) associated with the inertial spacetime expanding coordinate frame (indicated by subscript I). Next, we consider the following scenario: If we assume that the energy, or the work done by the force, of the Big Bang at \( t = 0 \) was enough to overcome the singularity binding force associated with \( g_{GE} \) (due to the mass-energy that initially warped spacetime closed), then we will have expansion with \( a_1 \geq g_{GE} \), implying a flat or open universe. We assume that at the event of the Big Bang the Universe became gravitationally unbound, with matter transforming to relativistic particle expansional and rotational energy. The Universe, in general, will expand with the expansion velocity given to it by the force of expansion, \( F_I = F_I(t) \), which appears to be expressed by \( F_I \sim E_{UC}e^{-2H^2r} \), where, again, \( H = H(t) \) is the Hubble constant, and \( r = r(t) \) is the spacetime separation between events (equation 34). Since in the Big Bang ‘explosion’ matter was converted entirely into energy, then the expansion velocity \( v_I, \) at \( t \approx 0 \), was at least \( \approx c, \) the speed of light. The acceleration of the expansion, \( a_1 \sim H^2r, \) will decrease over time. If the scenario ended here, add inflation, and exclude rotation, this would be a universe explained by the standard FLRW cosmology. But with the existence of \( g_{GM} \), of a fictitious-like force, associated with the non-inertial rotating frame, the expanding mass-energy of the Universe will experience an additional acceleration, perhaps like that of the recently observed cosmic acceleration. Now, this is where
the FLRW cosmology develops the well known problems pointed out in Section 1, i.e., when attempting to explain the physics of the accelerated expansion (or cosmic acceleration) we observe to exist in the present-day Universe.

It appears that the inconsistencies in the standard cosmological model of the Friedmann-Robertson-Walker spacetime metric are due to our lack of considering the effects of cosmic rotation, which requires the use of a rotating and expanding cosmological spacetime metric, like that employed in this present paper. In the following sections the physics we need to further discuss the model described above, and to test its validity with observations, is devised.

3 FORMALISM

3.1 The Gödel-Obukhov Spacetime Metric in Spherical Coordinates

The Gödel-Obukhov (Jain et al. 2007; Obukhov 2000; Obukhov, Korotky, & Hehl 1997) shear free and spatially homogeneous spacetime metric, defining separations of events in Cartesian comoving coordinates, is given by (Jain et al. 2007; Obukhov 2000)

\[ ds^2 = dt^2 - 2\sqrt{\sigma(t)}R(t)e^{\sigma z}dt - R^2(t)(dx^2 + k\sigma^{2m\sigma}dy^2 + dz^2) , \tag{2} \]

where \( \sigma(t) \), \( m \), and \( k \) are related geometrical parameters; \( R = R(t) \) is a time dependent scale factor, and \( k > 0 \) ensures absence of closed timelike curves (note, \( k \) is not the curvature index); with \( c = 1 \) unless noted otherwise. Clearly, \( \sigma(t) \) must be \( > 0 \), and for definiteness, we choose \( m > 0 \) (Obukhov 2000). According to equation (2) the Universe is spatially homogeneous, rotating, and expanding. Note, the usual Gödel (1949) metric that suffers from the presence of closed timelike curves is obtained by setting

\[ R(t) = 1, \quad \sigma(t) = 1, \quad m = 1, \quad k = -\frac{1}{2} \]

in equation (2). The magnitude of the global cosmic rotational velocity \( \omega_{\text{rot}} \) oriented along the \( z \)-axis is (Obukhov 2000; Jain et al. 2007)

\[ \omega_{\text{rot}} = \sqrt{\omega_{\mu\nu}\omega^{\mu\nu}} = \frac{m}{2R} \sqrt{\frac{\sigma}{k + \sigma}} \geq 0 \tag{3} \]

or

\[ m = 2R\omega_{\text{rot}} \sqrt{\frac{k + \sigma}{\sigma}} \tag{4} \]

(see equation 2), where, recall, \( R = R(t) \) and \( \sigma = \sigma(t) \). Thus, we see that, vanishing of \( m \) and/or \( \sigma(t) \) yields zero vorticity.

Upon assuming a spinning fluid of proper angular momentum along the \( \hat{z} \)-axis with electromagnetic dynamical characteristics in a Riemann-Cartan spacetime (Obukhov 2000; Minkevich 2006), Obukhov (2000) gives an exact solution to Einstein’s field equations: an equation of motion describing the evolution of the scale factor \( R \). From Obukhov (2000) we can show that

\[ \frac{\dot{R}}{R} = -H^2 + \frac{\omega_{\text{rot}}^2(k + \sigma)(3\sigma + 4k)}{3\sigma} + \frac{1}{\omega_{\text{rot}}^2} \left( \frac{k + \sigma}{144k} \right) (4\lambda_3^2 - \lambda_1^2) \frac{R^4}{R^8} + \frac{8\pi G}{3c^2} \left( \frac{k + \sigma}{k} \right) (c^2\rho - p - \frac{B^2}{R^2}) , \tag{5} \]

where, the variables \( H \), \( \omega_{\text{rot}} \), \( B \) (the cosmic magnetic field strength), \( \rho \), and \( p \), are all functions of time; \( \lambda_1 \) and \( \lambda_3 \) are cosmological coupling constants of the curvature, spin, and torsion tensors (Obukhov 2000) mentioned in Section 1. It appears that the parameters \( \sigma \) and \( k \), in a sense, determine the magnitude of acceleration of a fluid element due to rotation of the Universe; we shall see more evidence of this in Section 3.2. Note, \( B \) is related to the spin density (angular momentum per unit volume), as we shall see in Section 5.3. Thus, one can see the repulsive nature of the above equation of motion, for the scale factor \( R \), in a Riemann-Cartan spacetime, as found, it appears, independently by Obukhov (2000) and Minkevich (2005). Minkevich, Garkun, & Kudin (2007) have found this repulsive characteristic not only in the extreme conditions of the early Universe, but also at sufficiently small energy densities of later times. Minkevich et al. (2007) conclude that the effect of the accelerated cosmological expansion, even of today, is geometrical in nature and is connected with the geometrical structure of spacetime. Indeed this might be the case: it appears that these authors are finding the effect that frame dragging, producing the GM field, has on the geometry of spacetime. In this present paper, with the author’s model proposed independently and unaware of concluding remark by Minkevich et al. (2007), it is shown that the recently observed cosmic acceleration may be the effect of the frame dragging nature of cosmic rotation, interacting with an inertially expanding spacetime geometry. Since the GM field is an inherent gravitational property of a rotating spacetime, it seems, we can safely say that the repulsive nature found by at least one of these authors (Obukhov 2000), as in the spacetime evolution of equation (5), is due to the presence of the GM field, more or less. Note, in deriving equation (4), for a specific epoch time \( t \),

\[ H = H(t) \equiv \frac{\dot{R}}{R} , \tag{6} \]
the Hubble constant, was used. In addition, one cannot help but notice that the first term on the right-hand side of equation (5) is the same as that in the standard FLRW model: wherein the equation of motion of the scale factor reduces exactly to this term when $\Lambda = p = K$ (curvature) = 0 and $q = 1$ (see equations [32] and [33]). We will return to this and similar comparisons in Section 5.4.

For the G"odel-type universe of equation (2), the evolution of the scale factor reveals several possible stages of the Universe as pointed out by Obukhov (2000), and elaborated on here, in this present paper, based on equation (5). The first stage is short and occurs in the vicinity $t = 0$. There is no initial cosmological singularity due to the dominating spin contribution, a characteristic of Einstein’s gravitational theory in Riemann-Cartan spacetime (Minkevich 2006), in which $R(t = 0) \neq 0$ implies a regular, as opposed to a singular, spacetime metric in the transition from compression to cosmological expansion. The duration of this first stage is $\ll 1 \text{ s}$, since the spin term quickly decreases with the growth of the scale factor (Obukhov 2000), as can be seen somewhat in the third term on the right-hand side of equation (4). But, importantly, notice that $\omega_{\text{rot}}$ in the denominator will cause this accelerating term to increase over time in some degree as $\omega_{\text{rot}} \rightarrow 0$. Now, at this stage ($\ll 1 \text{ s}$) the fluid source describing the material of the Universe is characterized by the stiff matter equation of state: $p = c^2 \rho$, where $p$ is the pressure and $\rho$ is the mass-energy density of matter and radiation. But before this stage, at very small $R(t)$, perhaps at $t \lesssim 10^{-35} \text{ s}$, with the equation of state possibly being that of a gravitating false vacuum ($p = -c^2 \rho$), however not a necessity, the right-hand side of equation (5) shows that the fourth term (which appears to be related to the inertial spacetime expansion) together with the second and third terms (which appear to be related to cosmic rotation) will produce perhaps a large acceleration, one that might be associated with the inflationary stage of the standard cosmological model. During inflation it is commonly accepted that $R(t) \sim e^{Ht}$, assuming $H$ to be a constant during inflation (i.e., since it is approximately constant over the very short timescale of inflation). Next comes the stage when the scale factor increases like $R(t) \propto t^{1/2}$, while the equation of state is of the radiation type, $p \approx c^2 \rho/3$. This 'hot universe' expansion lasts until the Universe becomes matter dominated. After this the 'modern' stage starts with the effective dust equation of state $p \approx 0$. The scale factor still increases, now like $R(t) \propto t^{2/3}$, but the deceleration of the expansion takes place. The final stage depends on the value of the the third term referred to as the cosmological term by Obukhov (2000), containing $\lambda_1$ and $\lambda_3$, which specifically are made up of coupling constants relating torsion and curvature, where torsion can either accelerate the expansion or prevent cosmological collapse. That is, either the future evolution enters the eternal de Sitter type accelerated expansion, or expansion ends and a contraction phase starts. Notice the striking similarity of this cosmological term and accelerated expansion to the popular view of the cosmological constant $\Lambda$ as the source of the present-day observed accelerated expansion.

It appears that torsion of spacetime might be directly related to inertial frame dragging, and, thus the GM force field. In support of this, the characteristics of torsion given by Mao et al. (2007), that a rotating body also generates torsion through its rotational angular momentum, and the torsion in turn affects the motion of spinning objects such as gyroscopes, are exactly those of the GM field (see e.g., Williams 2002; Thorne et al. 1986). Therefore, it seems reasonable to associate the dominant repulsive nature of equation (5) with that of the GM field. In this paper, we derive the GM field associated with cosmic rotation to see what role it may have in the recently observed cosmological accelerated expansion. Further analysis of equation (5) will allow us to identify, as we shall see in Section 5.4, the suspected GM acceleration and other terms one would expect to be measured by a rotating and expanding comoving frame observer.

The above stages are consistent with the model description proposed in Section 2 which includes being consistent with a Big Bang cosmology: first proposed by Lemaitre (1931a). This could mean that if expansion is part of the conditions occurring around $t = 10^{-43} \text{ s}$, then rotation, which appears to be a natural phenomenon associated with gravitation, could very well be a part also. So avoiding the initial singularity that exists at $R(t=0)=0$ for the standard FLRW cosmology, suggests that the G"odel-Obukhov metric allows us to get somewhat closer to conditions existing at $t = 0$. That is, perhaps cosmic rotation and cosmic expansion are intrinsic parts left over from the earlier quantum-gravitational spacetime makeup of the primordial matter of the Universe at $t \simeq 0$. Image that cosmic expansion and deceleration of cosmic rotation of the Universe are a reversed process of gravitational contraction (or collapse) and conservation of angular momentum. This helps one to conceive the strong possibility of how the two: rotation and expansion, cannot, it appears, be separated in the physics to describe the Universe, as commonly done by assuming $\omega_{\text{rot}} = 0$.

The G"odel-Obukhov metric of equation (2) converted into spherical coordinates for non-stationary and axisymmetrical characteristics of spacetime, which require that the metric coefficients be independent of the azimuthal $\phi$ coordinate: $g_{\mu \nu} \equiv g_{\mu \nu}(t, r, \theta)$, is given by

$$
\begin{align*}
\text{ds}^2 &= dt^2 - 2\sqrt{\sigma(t)}R(t)e^{mr \sin \theta}r \sin \theta dt d\phi - R^2(t)(dr^2 + r^2 d\theta^2 + ke^{2mr \sin \theta}r^2 \sin^2 \theta d\phi^2),
\end{align*}
$$

(7)

where the usual transformation equations from Cartesian to spherical polar coordinates have been used. We identify the following metric coefficients for our convenience:

$$
\begin{align*}
ge_{tt} &= 1, \\
ge_{t\phi} &= -\sqrt{\sigma(t)}R(t)e^{mr \sin \theta}r \sin \theta = g_{\phi t}, \\
ge_{rr} &= -R^2(t),
\end{align*}
$$
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\[ g_{\theta \theta} = -R^2(t)r^2, \]
\[ g_{\phi \phi} = -R^2(t)k e^{2mr \sin \theta} r^2 \sin^2 \theta. \]  

Further, for our convenience, we find the corresponding inverse metric components:

\[ g^{tt} = \frac{k}{k + \sigma(t)}, \]
\[ g^{t\phi} = -\frac{\sqrt{\sigma(t)}}{R(t)e^{mr \sin \theta} r \sin \theta [k + \sigma(t)]} = g^{\phi t}, \]
\[ g^{rr} = \frac{1}{R^2(t)}, \]
\[ g^{\theta \theta} = -\frac{1}{R^2(t)r^2}, \]
\[ g^{\phi \phi} = \frac{1}{R^2(t)e^{2mr \sin \theta} r^2 \sin^2 \theta [k + \sigma(t)]}, \]  

where, in general, from inverse matrix operations,

\[ g^{tt} = g^{\phi \phi} g^{tt} g^{\phi \phi} - g^{2t\phi}, \]
\[ g^{t\phi} = -\frac{g^{t\phi}}{g^{tt} g^{\phi \phi} - g^{2t\phi}} = g^{\phi t}, \]
\[ g^{rr} = \frac{1}{g_{rr}}, \]
\[ g^{\theta \theta} = \frac{1}{g_{\theta \theta}}, \]
\[ g^{\phi \phi} = \frac{g^{tt}}{g_{tt} g_{\phi \phi} - g_{t\phi} g_{t\phi}}. \]  

The frame dragging velocity for the Gödel-Obukhov metric of equation (7), with diagonal component signature (+, -, -, -, -), is given by

\[ \omega_{FD} = \frac{-g^{tt}}{g_{\phi \phi}} \sqrt{\sigma(t)} \]
\[ = -\frac{g^{tt}}{R(t)e^{mr \sin \theta} r \sin \theta}. \]  

where we are being consistent with the geometrical expression given by Bardeen, Press, & Teukolsky (1972) for the frame dragging velocity. In equation (11), we see that frame dragging velocity in the \( \hat{e}_\phi \) direction can be either positive or negative because of the square root, but \( \omega_{FD} < 0 \) occurs naturally it seems, producing, as we shall see in Section 3.2, a repulsive gravitational force.

3.2 The Cosmic ‘Gravitomagnetic’ (GM) Force

We now derive an expression for the GM force, \( F_{GM} \), exerted on a test particle (or an object such as a galaxy) of cosmic space momentum \( P \). Using the analogy of expressions found in Williams (2002, 2005), for a rotating general relativistic system such as our Universe, the GM force measured by a comoving observer can be given by

\[ \left( \frac{dP}{d\tau} \right)_{GM} = H_{ij} P^j, \quad \text{i.e.,} \quad \left( \frac{dP}{d\tau} \right)_{GM} = \mathbf{H} \cdot \mathbf{P} \]  

(d\( \tau \) is the proper time interval), with

\[ H_{ij} = e^{-\nu} (\beta_{GM})_j, \]

(the vertical line indicates the covariant derivative in 3-dimensional absolute space), where, like in the case of a rotating black hole (Thorne et al. 1986),

\[ (\beta_{GM})^r = (\beta_{GM})^\theta = 0, \quad (\beta_{GM})^\phi = -\omega_{FD}; \]  

\( \omega_{FD} \) is given by equation (11). The field \( \mathbf{H} \) is called the GM tensor field, and \( \beta_{GM} \) is sometimes called the GM potential. We perform the metric component operations in eq. (12) to give the following expression for the GM force exerted, \( F_{GM} \):

\[ \left( \frac{dP}{d\tau} \right)_{GM} = F_{GM} = [(F_{GM})_r, (F_{GM})_\theta, (F_{GM})_\phi] \]
Here we are only interested in the radial component of eq. (14), where we are assuming that the other components are not important in explaining cosmological acceleration along the line-of-sight of the observer. Therefore, we need to determine the force in the radial direction:

\[
(F_{\text{GM}})_r = H_{r\theta} P^\theta + H_{r\phi} P^\phi \\
= H_{r\theta} g^{\theta\theta} P_\theta + H_{r\phi} g^{\phi\phi} P_\phi,
\]

where we have used \( F^\mu = g^{\mu\nu} P_\nu \). (Note, equation 14 is a general expression, existing for any gravitating and rotating system.)

In equation (12) we can identify the so-called blueshift factor \( e^{-\nu} \equiv \sqrt{g^{tt}} \) for a metric with a signature of diagonal components of the type as in equation (7). Thus, from equation (9),

\[
e^{-\nu} \equiv \sqrt{g^{tt}} = \sqrt{\frac{k}{k + \sigma(t)}}.
\]

Next we determine the relevant GM tensor components: \( H_{r\theta} \) and \( H_{r\phi} \), to be substituted into equation (14). These components are given in equation (12):

\[
H_{ij} = \sqrt{g^{tt}} (\beta_{\text{GM}})_{j|i}
\]

(repeated indices of \( i, j, k \) sum over \( r, \theta, \phi \)), where we have used equation (16). In general the covariant derivative in 3-dimensional absolute space is given by

\[
\beta_{j|i} = \beta_{j,i} - \Gamma^k_{ji} \beta_k;
\]

so that

\[
H_{r\theta} = \sqrt{g^{tt}} (\beta_{\text{GM}})_{\theta|r} = -\sqrt{g^{tt}} \Gamma^\theta_{r\theta} (\beta_{\text{GM}})_{\phi}.
\]

since, as given by equation (13), \((\beta_{\text{GM}})_{\theta} = (\beta_{\text{GM}})_\theta = 0 \) and \((\beta_{\text{GM}})_{\phi} \neq 0\), where

\[
(\beta_{\text{GM}})_{\phi} = g_{\phi\phi}(\beta_{\text{GM}})^\phi_{\phi} = g_{\phi\phi}(-\omega_{FD}) = \mp \sqrt{\sigma(t)} R(t)e^{nr\sin \theta} \sin \theta,
\]

upon substitutions from equations (8) and (11). Note, \((\beta_{\text{GM}})_{\phi}\) is measured in units of length and \((\beta_{\text{GM}})^\phi_{\phi}\) in units of length. Note also that for \( \omega_{FD} > 0 \), \((\beta_{\text{GM}})^\phi_{\phi} < 0 \) (\( \text{GM potential} \)), and \((\beta_{GM})_{\phi} > 0 \); for \( \omega_{FD} < 0 \), \((\beta_{GM})^\phi_{\phi} > 0 \), and \((\beta_{GM})_{\phi} < 0 \). Now similarly,

\[
H_{r\phi} = \sqrt{g^{tt}} (\beta_{\text{GM}})_{\phi|r} = \sqrt{g^{tt}} [\beta_{\text{GM}}]_{\phi,r} - \Gamma^\phi_{\phi,r} (\beta_{\text{GM}})_{\phi} = \sqrt{g^{tt}} \left[ \frac{\partial (\beta_{\text{GM}})_{\phi}}{\partial r} - \Gamma^\phi_{\phi,r} (\beta_{\text{GM}})_{\phi} \right].
\]

It can be shown that \( H_{r\theta} = 0 \), from equations (8), (9), (19), and (24); therefore, the GM force in the radial direction relative to an arbitrary comoving observer, given by equation (15), reduces to

\[
(F_{\text{GM}})_r = H_{r\phi} g^{\phi\phi} P_\phi.
\]

We want to simplify and analyze the above vector component to see under what conditions it may contribute to a repulsive accelerating force, i.e., we want the GM radial force component to be repulsive \((>0)\). We first evaluate the partial derivative of equation (21). The negative value of equation (20) is chosen, consistent with the note stated above following equation (20) concerning \( \omega_{FD} < 0 \), and, importantly, consistent with the description of the model presented in Section 2 concerning the frame dragging, \( \omega_{FD} < 0 \), which tends to drag the Universe out of rotation. Thus, from equation (20),

\[
\frac{\partial (\beta_{\text{GM}})_{\phi}}{\partial r} = -\sqrt{\sigma(t)} R(t) \sin \theta \frac{\partial}{\partial r} (r e^{mr\sin \theta}) = -\sqrt{\sigma(t)} R(t) \sin \theta e^{mr\sin \theta} (mr \sin \theta + 1).
\]

We next evaluate \( \Gamma^\phi_{\nu\rho} \) of equation (21). In general the affine connection is given by

\[
\Gamma^\phi_{\nu\rho} = \frac{1}{2} g^\lambda_{\nu\rho} \left( \frac{\partial g_{\phi\lambda}}{\partial x^\mu} + \frac{\partial g_{\phi\mu}}{\partial x^\lambda} - \frac{\partial g_{\mu\lambda}}{\partial x^\phi} \right).
\]
upon substitution of nonzero metric components from equations (8) and (9) into equation (24). Now we substitute from equations (9), (23), (25), and (20) into equation (21) yielding
\omega_{\rho \text{ matter}} equation (30) can be set equal to the mass density, of the energy-momentum tensor, \( T_{\mu \nu} \), where, again,
\( T_{\mu \nu} \) is the energy-momentum tensor. Further, this result will allow us to test the validity of this assumption once the exact GE term is identified.

3.3 The Cosmic ‘Gravitoelectric’ (GE) Force

We now calculate the cosmic gravitational force throughout an assumed spherical axisymmetric universe (of infinite extent to an arbitrary comoving observer). We will assume that the equation of state and scale factor relations, derived from the Einstein field equations, by way of a FLRW cosmological model, are still at least approximately valid in the Godel-Obukhov cosmology. Support of this assumption is that the rotating and expanding Godel-Obukhov metric (equation (7)), in the limit of large times and nearby distances, reduces to the open metric of Friedmann (Carneiro 2002). Moreover, we will also assume that the derivation of the GE force for the spherical axisymmetric case is not much different from that of the spherical symmetric (FLRW) case. The result will allow us to test the validity of this assumption once the exact GE term is identified in equation (4).

Note, the Godel-Obukhov metric is shear free, spatially homogeneous, and isotropic in the cosmic microwave background (CMB) radiation (like in the standard FLRW cosmology) for any moment of cosmological time \( t \) (Obukhov 2000). Further, and in summary, the cosmological model of equation (7), with rotation and expansion, does not suffer from the three major problems associated in the past with cosmic rotation. This cosmological model is causal, isotropic in the CMB radiation, and parallax free; and thus, the limits on the cosmic rotation, obtained earlier from the study of CMB radiation and of the parallaxes in a rotating world, are not true for the class of cosmologies in which the Godel-Obukhov metric of equation (7) is a member (Obukhov 2000).

The gravitational potential inside the Universe is assumed to be given by the post-Newtonian approximation
\[ \Phi(r) = -G \int d^3r' \frac{T^{00}(r')}{|\mathbf{r} - \mathbf{r}'|} \]
where the integral is over the distance from the arbitrary observer located at \( r' = r_{\text{obs}} \equiv 0 \), along the line-of-sight, out to an arbitrary distance \( r \); and where
\[ T^{00} = \sum m_n \delta^3(\mathbf{r} - \mathbf{r}') \]
for a gravitational bound system of masses \( m_n \). The component \( T^{00} \) is the rest-mass density, or commonly referred to as the mass density, of the energy-momentum tensor, \( T^{\mu \nu} \), which serves as the source of the gravitational field. For nonrelativistic matter equation (30) can be set equal to the mass density \( \rho(r') \). Again, we are assuming spatial homogeneity, and isotropy in CMB radiation only, which we will refer to as the ‘quasi-Cosmological Principle.’ In the FLRW model, the Cosmological Principle of spatial homogeneity and spatial isotropy is assumed, which is consistent with CMB temperature measurements.
critical mass density is given by

\[ \rho \approx 4 \pi G \rho r M, \]

where by deductive reasoning \( r \) is a measure of spacetime separation, identified as the same \( r \) as in the spacetime metric of equation (7). Dividing equation (51) by \( M \) gives the universal gravitational acceleration for a given epoch at a distance \( r \) from an arbitrary comoving observer. Equation (51) satisfies the above requirement that the gravitational acceleration goes to zero as \( r \to 0 \), as measured by the comoving local inertial spacetime observer. The validation of the above reasoning used in deriving equation (51) will be given in the following section. Importantly, we shall see that \( \rho \) is just the mass density in the equation of motion of the scale factor in the standard FLRW cosmology when \( \Lambda = k = p = 0 \) (see equation 52). Notice that the GE force given by equation (31) is negative and opposite the sign of the radial component of the GM force given by equation (23). This repulsive nature of the GM force is consistent with it acting to accelerate the cosmic expansion of the Universe. To test this claim of consistency, in Sections 4 and 5, we will compare the magnitudes of the accelerations produced by the GM and GE forces at redshift \( z \sim 0.5 \), to see which is dominant.

To summarize, we are assuming that the Universe can be represented by a spherical axisymmetric cosmology (though infinite in extent to a comoving observer). The force \( F_{GE} \) of equation (51) expresses the gravitational force, due to the average mass density \( \rho \) (being a constant throughout the Universe for any given epoch according to the quasi-Cosmological Principle), acting on say a galaxy of mass \( M \) at a distant \( r \), as measured by an arbitrary observer, where, for this observer, \( r \to 0 \), which means that \( F_{GE} \to 0 \), as it should locally, in accordance with the Equivalence Principle, assuming any arbitrary comoving observer in the Universe is in free fall. Notice, however, the same is not true for \( (F_{GM})_r \) of equations (27) and (28), i.e., \( F_{GM} \) does not go to zero at the observer, where \( r \to 0 \), because \( F_{GM} \) exerts a force on locally moving inertial frames; then only if \( \omega_\text{rot} \to 0 \) will \( F_{GM} \to 0 \). In other words, the GM force in general acts on the momentum of a test body or particle [with effects being readily seen in a strong gravitational potential such as that of a black hole (e.g., see Williams 2004)]. The findings in this present paper suggest that the GM acceleration has a significant dynamical effects on spacetime comoving objects in a rotating frame, and, thus, on the expansion rate of the Universe.

### 3.4 The Density of the Universe

We now derive an expression for the mass density \( \rho(t) \) of the Universe, which includes any contribution from radiation. We assume that the standard FLRW cosmological model is approximately correct. The Friedmann-Lemaître’s solutions to Einstein’s gravitational field equations yield the following acceleration equation:

\[ \frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4 \pi G}{3} \left[ \rho + \frac{3p}{c^2} \right], \]

where the Robertson-Walker metric was used and \( K \) (≡ curvature) = 0. Then the general expression for the time-dependent critical mass density is given by

\[ \rho_c(t) = \frac{3H^2(t)}{4\pi G}, \]

with \( \Lambda = p = 0 \), implying specifically a Friedmann cosmology, where we have used equations (1) and (2); \( \rho_c \) is the density needed to make the Universe flat. Note, the way in which \( \rho_c \) of equation (33) was derived, from the standard FLRW cosmology (equation 32), does not rule out contribution from dark energy, but only sets \( \Lambda = 0 \), for an isotropic universe: no pressure gradients (\( \nabla p = 0 \)) and nonrelativistic matter or matter dominated (\( p = 0 \)) universe. This does, however, suggests that dark
for a qualitative and somewhat quantitative analysis of the model described in this paper we choose the following parameters

\[ \rho(t = t_0)c \equiv \rho_c(t_0) = \frac{3q_0H_0^2}{4\pi G^2}, \]  

where we find that

\[ \rho_c(t_0) \approx 9.5 \times 10^{-30} \text{ g cm}^{-3} \]  

for the currently suggested values of \( H_0 \approx 71 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( q_0 \approx \frac{1}{2} \). This value of deceleration parameter \( q_0 \) indicates a flat universe, which implies that \( \Omega \equiv \rho(t_0)/\rho_c(t_0) = 1 \), consistent with observational data finding that \( \Omega \approx 1 \) (Jarosik et al. 2011; Tegmark et al. 2004), if we assume that \( \rho \equiv \rho_{\text{mat}} + \rho_{\text{rot}} \); then division by \( \rho_c \) yields \( \Omega = \Omega_{\text{mat}} + \Omega_{\text{rot}} \approx 1 \), where observations suggest that \( \Omega_{\text{rot}} \approx 0.27 \) and \( \Omega_{\text{rot}} \approx 0.73 \) requiring \( \Lambda \neq 0 \) and \( p < 0 \) in the standard FLRW cosmology, specifically defining the \( \Lambda \text{CDM} \) cosmological model. This model, however, fails to tell the true nature of so-called dark energy, leaving the subject open to speculation. Nevertheless, equation (35) can be identified as the source of the universal gravitational field of attraction and will be used in the GE force given by equation (31), which gives the GE force for different epochs, where evaluated using equation (34) gives the present strength.

Now we return to give validity to the expression for the universal gravitational force of attraction (equation (31), and, thus, to the reasoning that led to its derivation. Upon substitution of the critical density of the Universe (equation 33) into equation (31) and dividing by the mass \( M \), we obtain the GE acceleration [i.e., the gravitational force per unit mass \((g_{\text{GE}})_r = \ddot{r}\)] which is, as would be expected, the same as that of the standard FLRW cosmology, when \( \Lambda = p = 0 \) and equation (33) is substituted into equation (32), where

\[ r(t) = R(t)\chi, \]

relating the physical distance \( r \) to the comoving coordinate distance \( \chi \), and its derivatives, have been used. The vector \( \chi \) comoves with the cosmic expansion. One can think of equation (37) as a coordinate grid which expands with time. Galaxies remain at fixed locations in the \( \chi \) coordinate system. The scale factor \( R(t) \) then tells how physical separations are growing with time, since the coordinate distances \( \chi \) are by definition fixed. Further, solving for \( q \), we can identify equation (36) as that of equation (11), with \( H \) given by equation (6), i.e., we identity the deceleration parameter as defined in the standard FLRW model. Again, this is what one would expect for the behavior of the GE force of equation (31), as it relates to the standard model, and, thus, this can serve to validate the reasoning behind assumptions made in its derivation. Importantly, note, equation (32) is exactly equal to the first term on the right-hand side of equation (5) with \( q = 1 \). Therefore, this term can be identified as the GE deceleration of the scale factor in the Gödel-Obukhov spacetime (we will return to this discussion in Section 5). So, in summary, the validity of the derivation leading to equation (31), which can be used to express the GE acceleration approximately in both the Gödel-Obukhov and FLRW cosmologies, has been established. The assumption that the derivation of the GE force for the spherical axisymmetric case is not much different from that of the spherical symmetric (FLRW) case has been validated, at least qualitatively; and it appears from equations (5) and (30) that the strengths will differ quantitatively by a factor of \( q \).

### 3.5 Cosmological Parameters

For a qualitative and somewhat quantitative analysis of the model described in this paper we choose the following parameters of equations (3), (11), and (7): \( \sigma, m, k, \) and \( \omega_{\text{rot}} \), based on observations and, of course, on theoretical insight. A possible way to express evolution of the force \( F_{\text{GM}} \) of equation (25) over time is to let

\[ \sigma = \sigma(t) \equiv e^{ct^2/t_0}, \]

and let \( k \) be defined as a function of \( \sigma(t) \) by Obukhov’s (2000) model relation

\[ k = c_2\sigma(t), \]

where, when estimated from the Gödel-Obukhov metric and general relativity, \( c_2 \approx 71 \), using \( q_0 = 0.01, (\omega_{\text{rot}})_0 = 0.1H_0, H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (again, the 0 subscripts indicate the present epoch), with \( \omega_{\text{rot}} = \omega_{\text{rot}}(t) = (\omega_{\text{rot}})_0 \). A reasonable choice for the constant \( c_1 \) appears to be \( c_1 \approx -115 \). The value of \( c_1 \) is related to the magnitude of the force \( F_{\text{GM}} \) of equation (25):
for example, upon changing from $c_1 = -105$ to $c_1 = -115$, in these model calculations, the magnitude of the force, for a typical case, increases by about two orders of magnitude. Note, with such choices, at $t = 0$, $\sigma(t) \approx 1$ and $k \approx 71$, consistent with the $k \geq 0$ requirement for causality. We will use the value $c_2 \approx 71$, since making it relatively larger of smaller appears to have little effect on the model outcome.

We will assume an analytical expression (equation 54) consistent with the more recent estimate, given below, for the ratio of the magnitude of cosmic rotation to the Hubble constant, where observations of anisotropy in electromagnetic propagation from distant radio sources, expected typically of cosmic rotation, are used to determine the estimate (Obukhov et al. 1997; Nodland & Ralston 1997a):

$$\frac{(\omega_{rot})_0}{H_0} = 6.5 \pm 0.5,$$  \hspace{1cm} (40)

with galactic coordinate direction $l = 50^\circ \pm 20^\circ$, $b = -30^\circ \pm 25^\circ$. This value is larger than the estimate (Obukhov 1992):

$$\frac{(\omega_{rot})_0}{H_0} = 1.8 \pm 0.8,$$  \hspace{1cm} (41)

with direction $l = 295^\circ \pm 25^\circ$, $b = 24^\circ \pm 20^\circ$, obtained from Birch’s (1982, 1983) data. Moreover, recent analysis of the large-scale distribution of galaxies (Broadhurst, Ellis, Koo, & Szalay 1990) has revealed an apparently periodic structure of the number of sources as a function of red shift. From this we get yet another estimate of the rotational velocity which appears necessary to produced this observed periodicity effect. This estimate gives the largest ratio of the three (Obukhov 2000; Korotky & Obukhov 1994):

$$\frac{(\omega_{rot})_0}{H_0} \approx 74$$  \hspace{1cm} (42)

(compare equations 10 and 11). It is clear from above that further careful observations and statistical analyzes will be extremely important in overcoming the inconsistencies, in establishing the true value of the cosmic rotation (or vorticity), which may result from too few empirical data.

Next, we use our defined scale factor $R(t)$ to relate the Hubble constant with time. In general, with assuming the usually power-law solution for the scale factor as a function of time ($R \propto t^n$) according to the FLRW cosmological model, based on the the assumptions of Section 3.3.

$$R(t) \equiv \left( \frac{t}{t_0} \right)^n,$$  \hspace{1cm} (43)

normalized at the present epoch $t = t_0$. Using equations 1 and 6, we get the general expressions

$$H_t = n't^{-1},$$  \hspace{1cm} (44)

and

$$q = \frac{(n - 1)}{n},$$  \hspace{1cm} (45)

where, in equation 11, $\equiv n'$, which gives for $n' = n = \frac{2}{3}$ an age of the Universe (9.22 $\times$ 10$^9$ yr) too low to be consistent with recent observational estimates of $H_0 \approx 71$ km s$^{-1}$ Mpc$^{-1}$, with $q = \frac{1}{2}$ according to equation 15. On the other hand, the expression $H_t = t^{-1}$, $n' = 1$, gives an age (13.8 $\times$ 10$^9$ yr), which is consistent with recent observational estimate with cosmic acceleration (Lineweaver 1999), and without acceleration, in the absence of deceleration (Freedman et al. 2001). So it seems reasonable to assume the following limits for the present age $t_0$:

$$\frac{2}{3H_0} < t_0 < \frac{1}{H_0},$$  \hspace{1cm} (46)

i.e., $\frac{2}{3} < n' \lesssim 1$. Note, with $n = 1$, then according to equation 15 $q = 0$, implying an open universe in the standard FLRW cosmology (compare equation 1). Specifically, for concreteness, it appears appropriate to choose $t_0 \equiv H_0^{-1}$ (n’ = 1) for the present epoch, but with $n = \frac{2}{3}$ in equation 43. Note, with observations suggesting that the age of the Universe is closer to the Hubble time ($H_0^{-1}$), instead of that given by the standard FLRW cosmological model ($\frac{2}{3}H_0^{-1}$), implies that the Universe has at least not decelerated continuously. The discrepancies leading to the limits above can possibly be attributed to the evolution of $R(t)$, i.e., how it might change as the Universe undergoes phase changes, thus reflecting how the value of n might change, where $n = \frac{2}{3}$, recall, is also the starting scale factor at $t \sim 0$ of Lemaître’s (1931b) expanding cosmological model. Moreover, for completion, during inflation (indicated by the subscript ‘inf’), it appears that

$$\frac{[R(t)]_{\text{final}}}{[R(t)]_{\text{initial}}} = e^{H \int dt} \approx e^{H_{\text{inf}}} \Delta t$$  \hspace{1cm} (47)

(i.e., $e^{H \int dt} \approx e^{H_{\text{inf}}}$), where $[R(t)]_{\text{initial}}$ and $[R(t)]_{\text{final}}$ are the initial and final scale factors before and after inflation; $H_{\text{inf}} = 1/t_{\text{inf}}$ is the Hubble parameter at the onset of inflation, which remains approximately constant during inflation; and
Could Dark Energy be a Manifestation of Gravity?

4 NUMERICAL MODEL RESULTS

For comparison and completion, plotted in Fig. 1 is the cosmic scale factor. Figure 1(a) displays a schematic plot of the scale factor given by equation (23), with \( n = \frac{1}{3} \) or \( n = \frac{2}{3} \), over a time period from when the age of the Universe was \( \sim 10^{-43} \) s to the present estimated age of \( t_0 = 13.8 \times 10^9 \) yr. The lower time limit corresponds to a Hubble radius \( \sim 3 \times 10^{-33} \) cm, of the order of the Planck scale, expanding out to a radius \( \sim 1.3 \times 10^{28} \) cm. The Hubble radius, mentioned above, is related to the causally-connected observable Universe and is given by \( r_H = c/H \). Immediately following the Planck era we believe that the Universe was at least a thermal causally connected spacetime gaseous plasma. We assume that \( n = \frac{1}{3} \) in equation (43) at \( t = 10^{-43} \) s, with this value lasting up to the beginning of inflation, as indicated in Fig. 1(a). Here also we are assuming that at the onset of inflation the Universe was at least radiation dominated (setting \( n = \frac{1}{3} \)), where at this epoch it appears a ‘false vacuum’ or the release of a type of quantized-gravity binding-like energy, resulting from symmetry braking of at least three of the fundamental forces (strong, gravitational, electromagnetic), drove inflation. The details as to what initiated inflation are yet to be understood; at present we can only speculate. After inflation, during the radiation dominated era, from when the age of the Universe was \( t \sim 10^{-34} \) s to \( t = t_{eq} \sim 1.7 \times 10^{12} \) s \( \simeq 53,968 \) yr, we continue to set \( n = \frac{1}{3} \) in equation (43), where \( t_{eq} \) is the time of matter and radiation equality (Liddle 2003). Beyond \( t_{eq} \) we set \( n = \frac{2}{3} \), indicating matter dominance, producing the step-like feature clearly seen in Fig. 1(b) at \( t = t_{eq} \sim 4 \times 10^{-6} \) \( t_0 \). Before this time relativistic particles dominated. As the Universe continues in a matter dominated phase after recombination, at \( t \sim 350,000 \) yr after the Big Bang, in equation (43) we still have \( n = \frac{2}{3} \) up to the present epoch. Note, this expression for \( R(t) \) indicates a flat, decelerating universe (i.e., \( q = \frac{1}{4} > 0 \), using equation (43)), as would be expected in a FLRW expanding cosmology. Yet, this is only somewhat consistent with observations, because recent observations appear to indicate an open, accelerating universe, at least for the present epoch, with \( q < 0 \) according to the standard FLRW cosmological model. This would cause \( R(t) \) to have a somewhat steeper incline (or slope) near the present epoch than that displayed. For example, for \( q = -0.2 \), equation (45) gives \( n = 1.25 \). Now, Fig. 1(b) displays \( R(t) \) of equation (43) over the time (138 yr \( \leq t \leq 13.8 \times 10^9 \) yr) that the gravitational accelerations, the GM and the GE, are calculated from the forces of equations (28) and (31), respectively, as we shall see below. The lower time limit is set here by the computational capacity of the computer in the units used in calculating the GM acceleration from equation (28). This limit will, however, be overcome in Section 5. Notice, the step-like feature is an ‘artifact’ resulting from normalizing \( R(t) \) at the present epoch (compare equation (43)).

Displayed in Fig. 2 are the evolutions of the magnitudes of the cosmic gravitational accelerations (force per unit mass)
Figure 2. The magnitudes of the radial accelerations \((g_{GM})_r\) (solid curve) and \((g_{GE})_r\) (short-dashed curve) produced by the gravitomagnetic (GM) and gravitoelectric (GE) forces, respectively, in cgs units, versus \(t/t_0\), representing the evolution from \(t = 138\) yr after the Big Bang up to the present time \(t = t_0\): (a) Evolution of accelerations at a distance with \(z = 0.65\) (see text). Note, the \((g_{GM})_r\) component extends down to \(\sim 2 \times 10^{-15}\) cm s\(^{-2}\), then begins to turn up. (b) Evolution of accelerations at a distances with \(z = 0.55\). (c) Evolution of acceleration at a distances with \(z = 0.5\). (Note, the \(x\)-axis extends to 1.2 \(t_0\) so that we can clearly see the values of the functions at \(t = t_0\), i.e., at the present epoch.)

\[(g_{GM})_r = \left\{ \frac{k \sigma(t)}{[k + \sigma(t)]^3} \right\}^{1/2} \left[ \frac{(m/c)r \sin \theta + 1}{R(t) \sin \theta_c \sin \theta} \right] \left\{ 1 - \frac{\sigma(t) + 2k}{2[k + \sigma(t)]} \right\} \omega_{rot} c; \tag{48}\]

and

\[(g_{GE})_r \approx -\frac{4}{3} \pi G \rho r; \tag{49}\]

with \(\theta = \frac{\pi}{3}\) in equation (48) (see discussion in Section 5.5). The above radial gravitational accelerations, \((g_{GM})_r\) and \((g_{GE})_r\), are measured at a coordinate separation distance \(r\), corresponding to a particular redshift \(z\), by an arbitrary comoving observer, as this distance expands over time, while the gravitational accelerations at that distance evolve over time, from \(t \simeq 138\) yr after the Big Bang to the present estimated age of the Universe: \(t_0 \simeq 13.8 \times 10^9\) yr.

In these calculations we step through the independent variable \(t\) by assuming the following:

\[t = f_i t_0, \tag{50}\]

where \(f_i\) is the fraction of the total time that we step through, normalized to equal one at the present epoch, i.e., \(t = t_0\); and subscript \(i\) indicates a step size. The Hubble constant then evolves as

\[H(t) = \frac{H_0}{f_i} \tag{51}\]

The evolving mass density \(\rho = \rho(t)\) of equation (49) is thus given by equation (33) for a flat universe according to the standard
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Figure 2 – continued (d) Evolution of accelerations at a distance with $z = 0.45$. (e) Evolution of accelerations at a distance with $z = 0.35$. (f) Evolution of the accelerations at $z = 0$, where $(g_{GE})_r \to 0$. Note, $(g_{GM})_r$ reaches a maximum finite magnitude at $z = r = 0$, as measured at the arbitrary comoving observer (see text).

FLRW cosmology. Similarly, the evolving distance $r = r(t)$ is assumed to be given by

$$r(t) \simeq \frac{cz}{H(t)} = \frac{cz}{H_0} f_t,$$

(52)

for recession velocities $\ll c$, which is just the nonrelativistic Hubble law. Note, for $z \geq 1$, the relativistically corrected Hubble law (equation 56) must be used for accuracy; this will be discussed further in Section 5.2.

The above evolving distance $r(t)$ is for a specific $z$, measured by a present-day observer, indicating how a specific comoving coordinate point in spacetime has evolved. Substitution of the evolving variables: $r(t)$, $\sigma(t)$ (equation 35), $R(t)$ (equation 40), and $\rho(t)$ (equation 33), into $(g_{GM})_r$ and $(g_{GE})_r$, above (equations 48 and 49), allows us to see how these cosmic gravitational accelerations have evolved at that specific comoving coordinate point, indicated by $z$, as measured by an arbitrary present epoch observer. Note, it shall be interesting to see what happens to $(g_{GM})_r$ as $t$ approaches zero and what role it may play in the cosmic inflationary era. It is suspected that it may play a significant role. We shall see if our suspicions are true in Section 5 where we will attempt to go back in time as far as theoretically possible using a valid approximation to the GM acceleration of equation (48). However, for the present, we find that for $z \to 0$ in equation (48), $(g_{GM})_r$ reaches a finite maximum value of $(g_{GM})_r \sim 3.9 \times 10^{14}$ cm s$^{-2}$ at an arbitrary comoving observer as shown in Fig. 2(f). Now, by equation (48) and conservation of angular momentum, as $t$ decreases, in Fig. 2, for $t < 0.01 t_0$ (or $< 1.38 \times 10^8$ yr after the Big Bang), $(g_{GM})_r$ increases with increasing $\omega_{rot}$ as would be expected. Yet, on the other hand, as $t$ increases, for $t > 0.01 t_0$, $(g_{GM})_r$ first decreases as would be expected, but then as $\omega_{rot}$ gets smaller and smaller, $(g_{GM})_r$ once again increases, at least for the $z$ values shown (compare Fig. 2; see also Fig. 3). The behavior of $(g_{GM})_r$ for larger values of $z$ will be discussed in Section 5.2. The above behavior of $(g_{GM})_r$ appears to be consistent with the third term on the right-hand side of equation (5); and, importantly, in Fig. 2(c), the magnitude of $(g_{GM})_r$ overtakes that of $(g_{GE})_r$, indicating a net positive acceleration or repulsive force per unit mass. We shall return to this discussion in the following section.
Figure 3. (a) Evolution of the cosmic rotational velocity ($\omega_{\text{rot}}$)$_t$ over time (see text), from $t = 138$ yr to the present $t_0 = 13.8 \times 10^9$ yr for $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$.

Figure 3 displays how the cosmic rotational velocity decreases over time: Specifically plotted, as we shall see below, is a derived analytical expression consistent with equation (40). We shall see that these model calculations suggest that the rotational velocity $\omega_{\text{rot}}$ has a value $\sim 6.3H$, which is consistent with present-day observations. In what follows, an analytical expression is derived for the cosmic rotational velocity $\omega_{\text{rot}}$ of equation (48), whose numerical value is consistent with observations: As usual the magnitude of the angular velocity (or the angular frequency) is given by

$$\omega = \frac{d\phi}{dt} \approx \frac{2\pi}{t},$$

(53)

assuming simple harmonic-like motion. From equation (53), it seems reasonable to relate the Hubble constant, the rate of cosmic expansion, to $\omega_{\text{rot}}$, the rate of cosmic rotation, by

$$\omega_{\text{rot}} \sim 2\pi H \sim 6.3H,$$

(54)

where we have use $t \sim \frac{1}{H}$. Importantly, this expression is consistent with observations; compare equation (40). Equation (54) is plotted in Fig. 3.

5 DISCUSSION

In the following sections, we analyze further and discuss the results above. We will look at the long term behavior of the GM and GE accelerations over time. Because of the large interval of time covered, we do this in two separate epochs, to compensate for the limited capacity of the computer. However, since these calculations are computations of analytical equations, none of the physics is lost, because the values and times converge at the ‘interface,’ indicative of a specific cosmological time.

5.1 The Gravitomagnetic (GM) and the Gravitoelectric (GE) Accelerations: From 138 yr to Present

We first analyze the GM and the GE accelerations over the time for which we have the exact analytical expression for the GM acceleration (equation 48), with the GE acceleration given by equation (49). As expressed in Fig. 2, this time is $138$ yr $\leq t \leq 13.8 \times 10^9$ yr after the Big Bang. Note, equation (48) is referred to as the exact in comparison to the approximation to this equation we will use in the following section to find its value in the early universe ($t < 138$ yr).

As mentioned in Section 4, the evolution of the GM and GE accelerations over time at a distance $r(t)$ as measured by a present epoch arbitrary comoving observer for a specific $z$ (see equation 29) as related to the cosmological distance given by equation (37) is plotted in Fig. 2. These model calculations show that the GM acceleration (equation 48) goes to zero at $z \gtrsim 0.7$ as measured by an arbitrary comoving present observer. This implies that the Universe was in a decelerating phase for $z$ at least greater than $\sim 0.7$, i.e., at an earlier cosmic time; this also is consistent with observations (Riess et al. 2001). Figures 2(a) and 2(b) seem to show that the Universe starts to decelerate at a slower and then an even slower rate at $z = 0.65$ and 0.55, respectively. This is because according to Fig. 2(a), ($g_{\text{GM}}$)$_r$ begins to increase at $\sim 8 \times 10^9$ yr after the Big Bang, from a minimum value $\sim 2 \times 10^{-15}$ cm s$^{-2}$, at the spacetime coordinate point, $r$, associated with $z = 0.65$ (as measured by a present epoch observer); and according to Fig. 2(b), ($g_{\text{GM}}$)$_r$ begins to increase at $\sim 6 \times 10^9$ yr after the Big Bang, from
In Section 4, and as can be seen in Fig. 2(f), the decoupling of radiation and matter, and the hydrogen atom formed causing the Universe to become transparent while $g \sim 1$; dashed curve is for $(g_{GE})_r$, $n = 2/3$; dash-dotted curve is for $(g_{GE})_r$, $n = 1/2$. Note, $(g_{GE})_r$, $n = 2/3$ and $(g_{GE})_r$, $n = 1/2$ converge at the present epoch, $t_0 \simeq 4.3 \times 10^{17} \, s \simeq 13.8 \times 10^9 \, yr$, since $(g_{GM})_r$, depends on the scale factor, which is normalized at $t = t_0$.

There is a factor of $1/2$ difference between $(g_{CE})_r$, $n = 2/3$ (implying $g = 1/2$) and $(g_{GE})_r$, $n = 1/2$ (implying $g = 1$), where $n = 2/3$ is for matter dominance and $n = 1/2$ is for radiation dominance. (b) From $t = 10^{-9} t_0 = 138 \, yr$ to $t_0 = 13.8 \times 10^9 \, yr$. Solid curve is for $(g_{GM})_r$, short-dashed curve for $(g_{CE})_r$, evaluated at the Hubble radius, being consistent with (a). The step-like feature on the curve for $(g_{GM})_r$, indicates the change from radiation dominance to matter dominance at $t = t_{eq} \sim 54,000 \, yr$ (see text), as $n = 1/2$ goes to $n = 2/3$. Note, $(g_{GM})_r \rightarrow 0$ at $t \sim 0.6 t_0$ (not shown; see text).

A minimum value $\sim 9 \times 10^{-13} \, \text{cm s}^{-2}$, at the spacetime coordinate point associated with $z = 0.55$. Importantly, we see that Fig. 2(c) is consistent with recent observations that suggest that the Universe entered into an accelerating phase at $z \sim 0.5$, with $(g_{GM})_r > (g_{CE})_r$. Figure 2(d) shows how as $z$ gets smaller, which means that the distance from the arbitrary comoving observer is getting smaller, the GM acceleration gets larger and larger; and we find that, as $z \rightarrow 0$, the GM acceleration will continue to get larger until a maximum value of $(g_{GM})_r \sim 4 \times 10^{14} \, \text{cm s}^{-2}$ is reached at the comoving observer, as mentioned in Section 3 and as can be seen in Fig. 2(f).

Now looking at Fig. 2(c) in more detail and using what is known from particle physics, up to $t \sim 2.7 \times 10^{-5} t_0$, where, again, $t_0 = 13.8 \times 10^9 \, yr$, the Universe was filled with ionized gas. Then around this time, $t$, recombination occurred with the decoupling of radiation and matter, and the hydrogen atom formed causing the Universe to become transparent while entering into a ‘dark-age’ phase. The fundamental force of gravitation became dominant on small scales, eventually leading to large-scale structures, as the Universe continued to cool. The so-called dark ages persisted up to around $t \sim 4 \times 10^{-10} t_0$, until quasars and/or protogalaxies formed, and then the Universe entered a ‘cosmic renaissance’ phase, ending the dark ages, and entering into a reionization (of hydrogen) phase. Figure 2(c) shows that the magnitude of the GM acceleration becomes less than that of the GE acceleration during the so-called dark ages at $t \sim 1 \times 10^{-4} t_0$, indicating that the Universe goes into a decelerating phase at the evolving cosmological distance $r(t)$, associated with $z = 0.5$. As the magnitude of $(g_{GM})_r$ continues to fall below that of $(g_{CE})_r$, it reaches a minimum at $t \sim 0.3 t_0$ in the galaxy evolution phase, and then once again climbs above that of $(g_{CE})_r$ at $t \sim 0.9 t_0$ as galaxies continue to evolve, reaching a value of $\sim 1.4 \times 10^{-7} \, \text{cm s}^{-2}$, greater than that of $(g_{CE})_r \sim 1.7 \times 10^{-8} \, \text{cm s}^{-2}$, at $t = t_0$, indicating the possibility for the Universe to be in an accelerating phase for $z \gtrsim 0.5$ [compare Figs. 2(a) through 2(e)].

### 5.2 The GM and the GE Accelerations at the Hubble Radius: From Time of Planck Scale to Inflation to the Present

Upon substitution of equation (54) into equation (18), setting $r \approx 0, \theta = \frac{\pi}{2}$, and using model parameters defined in Section 3.5 (equation (33) $\sigma = \exp[-115/(t/t_0)], k = 71 \sigma, H_1 \sim 1/t$), we can derive an approximate analytical expression for the GM cosmic acceleration that appears to be valid at early times in the Universe as well as later times when the scale factor of equation (43) is normalized at the present epoch [i.e., when $f_i$ of equations (50) to (52) equals 1]. Thus, we find that

$$
(g_{GM})_r \sim 1.8 \times 10^{7} \frac{z^0}{t^0} \exp \left( \frac{57.5t}{t_0} \right) \frac{\text{cm s}^{-2}}{s}
$$

where $n$ is defined in equation (43).

For $t = t_0$ (i.e., $H = H_0$) equation (55) gives the same value as that calculated using the exact analytical expression for the GM acceleration (equation (38), measured at an arbitrary comoving present epoch observer: $(g_{GM})_r \sim 4 \times 10^{14} \, \text{cm s}^{-2}$, for $H_0 = 71 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, by letting $z \rightarrow 0$ or $r \rightarrow 0$; compare equation (52) and Fig. 2(f)).

This appears to validate our use of equation (55) at earlier times for which $r \approx 0$ and $t \neq t_0$.

Displayed in Fig. 4 are the evolutions of the magnitudes of the GM and the GE accelerations $(g_{GM})_r$ and $(g_{GE})_r$. 

![Figure 4](image-url)
respectively. Plotted in Fig. 4(a) are the magnitudes of \((g_{\text{GM}})_{r}\) (equation \(\text{(55)}\)) and \((g_{\text{GE}})_{r}\) (equation \(\text{(19)}\)), from the Planck time \((t_{P} \approx 5.3 \times 10^{-44} \text{ s})\) with length \(r = ct \equiv l_{P} \approx 1.6 \times 10^{-33} \text{ cm}\) up to the Hubble time of \(t \approx 4 \times 10^{9} \text{ s} \approx 138 \text{ yr} = 10^{-8} t_{0}\). Note, for any given epoch, the magnitudes of \((g_{\text{GM}})_{r}\) and \((g_{\text{GE}})_{r}\) are evaluated at the Hubble radius, i.e., the limit of the causally-connected observable Universe for any arbitrary observer. If we assume that \(n = \frac{2}{3}\), being consistent with a Lemaître early cosmology, then from the Planck time up to the time of inflation \((\sim 10^{-36} \text{ s})\), \((g_{\text{GM}})_{r}\) goes from \(\sim 10^{38}\) down to \(\sim 10^{33}\), respectively, factors larger than \(|(g_{\text{GE}})_{r}|\). The net acceleration of the expansion might be sufficient to inflate the Universe to outside of a causally connected region: given by the Hubble radius (i.e., \(r_{H} = ct \equiv cH^{-1}\)), consistency with what we assume to have occurred in our standard FLRW cosmological with inflation. The basic requirement is that the scale factor must increase by a factor of at least \(10^{27}\) in a very short time. This could imply that perhaps what we call inflation may have been, or at least in part, caused by \((g_{\text{GM}})_{r}\), and thus is related to frame dragging in some degree. The above statement, however, would have to be investigated. Notice, even up to age \(t \sim 1 \text{ s}\), the proposed limit for inflation to have occurred to be consistent with nucleosynthesis (Liddle 2003), \((g_{\text{GM}})_{r}\) is still \(\sim 10^{33}\) orders of magnitude larger than \(|(g_{\text{GE}})_{r}|\). Since, however, we want this present model to be consistent with the standard cosmological model, and with the assumption that \((g_{\text{GM}})_{r}\) is related to spacetime torsion, we must keep in mind that the negative terms in the last acceleration component on the right-hand side of equation \((\text{5})\) could very well come into play to keep the expansion of the Universe at a rate consistent the standard model.

In other words – making a brief deviation to clarify this cosmological distance given by equation \((\text{7})\) – the relativistic Hubble law:

\[
r = \frac{v}{H_{0}} = \left[\frac{(1+z)^{2} - 1}{(1+z)^{2} + 1}\right] \frac{c}{H_{0}},
\]

for \(H_{0} = 71 \text{ km s}^{-1}\text{Mpc}^{-1}\), where \(z\) can be solved for:

\[
z = \left(\frac{1 + rH_{0}/c}{1 - rH_{0}/c}\right)^{1/2} - 1.
\]

Note, \(r = r_{P}\) for \(z \gtrsim 30\), according to equation \((\text{56})\), appears to simply suggests that \(r \lesssim r_{H}\) is always the observable causally connected Universe for any \(z \gtrsim 30\), which is relativistically true. Equation \((\text{56})\) gives the spatial separation at a common time from an arbitrary comoving observer when the light was emitted from say a distant galaxy at \(r \leq r_{H}\).

In other words – the proper distance \(r(t)\) (equation \((\text{57})\)), measured according to a standard clock at rest with this arbitrary comoving observer, for \(r(t = t_{0}) = cH_{0}^{-1}\), is equivalent to redshift \(z \gtrsim 30\) according to the relativistic Hubble law:

\[
r_{\text{proper}} = \int_{0}^{\chi} \sqrt{|g_{rr}|} \, dr,
\]

where \(\chi\) is the comoving coordinate distance (associated with the proper physical distance), discussed in Section \((\text{4.3})\) and it can be shown from the metric component, \(g_{rr}\), of equation \((\text{8})\), upon integration, that

\[
r_{\text{proper}} = R(t)\chi = r(t),
\]

which is just equation \((\text{57})\). The proper distance measured by a comoving observer can be understood as a spacelike separation using a hypothetical ruler to measure the separation at the time of emission, from say a distant galaxy, as opposed to a lightlike (null) separation using light-travel time in an expanding universe to measure the separation. Both \(r(t)\) and \(\chi\) are spacelike separations between events: this means that they are imaginary \((\chi \equiv -\sqrt{-1})\) and cannot lie on the world line of any body or particle. From the proper distance \(r(t)\) of equation \((\text{57})\), its derivative with respect to time, and equation \((\text{61})\), the Hubble law can be derived exactly. The proper distance might be called the dynamical distance. The recession velocity for the proper (or so-called dynamical) cosmological distance is always \(\lesssim c\), at \(r \leq r_{H}\), i.e., causally connected regions. Now, since the Hubble law is derived exactly from the metric proper radial distance, and since the dynamics are what we are concerned with in this present manuscript, we appropriately use the proper distance in these calculations.

Finally, before proceeding, for the proper distance \(r\), if we consider the ratio of the approximated distance (equation \((\text{62})\)) to the more accurate relativistically corrected distance (equation \((\text{55})\)), we obtain for \(z = 0.03, 0.5,\) and \(1\) the ratios \(\sim\)
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1, 1.3, and 1.7, respectively. In these present model calculations, to give an explanation for the observed so-called dark energy, we need only consider the evolution of the GM (equation 45) and GE (equation 49) radial accelerations over spacetime points for \( z < 1 \), since equation 48 goes to zero, as measured by a present epoch comoving observer, for \( z \approx 0.7 \) using equation 52, the nonrelativistic Hubble law; and, it goes to zero for \( z \approx 1.5 \) using equation 56, the relativistic Hubble law [compare Figs. 2 and 4(b)]. Both these \( z \) values where \( (g_{GM})_{r} \rightarrow 0 \) are qualitatively consistent with observations. Thus, the results of this present model, giving an explanation for so-called dark energy, do not change qualitatively, and these results change only slightly quantitatively, when the approximate distance (equation 52), as opposed to the relativistically corrected distance (equation 56), is used for \( z \lesssim 0.68 \). Importantly, the accelerated expansion appears to set in, theoretically, according to this model, at \( z \sim 0.5 \), consistent with observations, when using either the nonrelativistic or relativistic Hubble law to determine \( r \), the physical distance. Therefore, the above, and the fact that most large-scale galaxy surveys use the nonrelativistic Hubble law, appear to justify our use of equation 52 in this present manuscript. Nevertheless, some relevant results using the more accurate relativistic Hubble law will be discussed below.

Proceeding, the early evolution \((t < 10^{-5} t_{0})\) of the GM acceleration as measured by an arbitrary comoving observer at a specific time is expected not to be qualitatively, and perhaps not much quantitatively, different from that seen in Fig. 4(a), where \((g_{GM})_{r}\) in these times is given by equation 50 for sufficiently small \( r \). Thus, in these early times \((g_{GM})_{r}\) decreases with increasing time or decreasing angular velocity, as would be expected (see also Fig. 3), independently of \( r \), recalling that \( r = r(t) \). But the evolution of \((g_{GM})_{r}\) (equation 45) from \( t = 10^{-8} t_{0} \) (\( = 138 \) yr) up to the present epoch will be quite different from that of Fig. 4(b) \((z \sim 30)\), for a sufficiently smaller value of \( z \) (associated with an \( r \)), as can be seen in Fig. 2, wherein cosmological acceleration of the expansion appears to begin at \( z \sim 0.5 \), as measured by a present epoch arbitrary comoving observer; this can be seen in Fig. 2(c). We discuss this further in the next paragraph.

Displayed in Fig. 4(b) are the magnitudes of the accelerations, \((g_{GM})_{r}\), and \((g_{GE})_{r}\), between the interval 138 yr \( \leq t \leq t_{0}\), measured by an arbitrary comoving observer, at the Hubble distance \( r(t) = r_{H} = ct \), i.e., at \( z \sim 30 \) according to equation 55, as this spacetime coordinate distance (or point) evolves over time, with \( t_{0} \) indicating the present epoch Hubble radius (causally-connected observable Universe). In this case, \((g_{GM})_{r}\) falls below the magnitude \(|(g_{GE})_{r}|\) twice, at \( t \sim 2 \times 10^{-6} t_{0} \) and \( t \sim 5 \times 10^{-5} t_{0} \), because of the so-called artifact in changing from radiation dominance to matter dominance occurring at the equilibrium time \( t = t_{eq} \sim 4 \times 10^{-6} t_{0} \). This is where \( n \rightarrow \frac{4}{3} \) at the radiation-matter equilibrium time \( t = t_{eq} \sim 1.7 \times 10^{12} \) s \(( = 4 \times 10^{-6} t_{0})\), for \( \Omega_{mat} \simeq 0.3 \) (Liddle 2003); compare Fig 4(b). Then, although not shown on Fig. 4(b), at \( t \approx 0.6 t_{0} \) \(( \approx 8 \times 10^{9} \) yr\), \((g_{GM})_{r}\) falls to zero, while \(|(g_{GE})_{r}| \sim 5.8 \times 10^{-8} \) cm s\(^{-2}\). After this, \((g_{GM})_{r}\) remains zero up to the present epoch \((t_{0})\) at which \(|(g_{GE})_{r}| \sim 3.5 \times 10^{-8} \) cm s\(^{-2}\), consistent with a decelerating universe, as measured by a present epoch arbitrary observer. But, recall, as \( z \) becomes less than one, at \( t = t_{0} \), as measured by a present epoch observer, \((g_{GM})_{r}\) becomes larger than \((g_{GE})_{r}\), for \( z \lesssim 0.5 \), indicating perhaps the Universe enters into an accelerating phase [compare Figs. 2(a) through 2(e)], consistent with recent observations. Note, as mentioned earlier, at \( z = r = 0 \) and \( t = t_{0} \), indicating at the present epoch arbitrary comoving observer, \((g_{GM})_{r}\), \( \sim 4.9 \times 10^{15} \) cm s\(^{-2}\) [compare Fig. 2(f)].

After a quantitative comparison of the calculated results, it appears that \((g_{GM})_{r}\) is independent of \( r \) for small \( r \), i.e., \((g_{GM})_{r}\) has the same value for any value of \( r(t) \) as measured by an arbitrary comoving observer at a specific epoch up to some time \( t \equiv t_{crit} \). For example, notice in Figs. 2 and 4(b), over the range of values of \( z \) measured by a present epoch arbitrary observer, the value of \((g_{GM})_{r}\) \( \approx 41.5 \) cm s\(^{-2}\) at \( t = 10^{-5} t_{0} \) is the same, although the values of \( r(t) \) are different. Analytically, this is because the exponent of the exponential term in equation 48 goes to zero, meaning \( e^{0} = 1 \), for small \( r \); and because \((m/c)r \sin \theta \lesssim 1 \), where \( m \) is given by equation 41; thence \((g_{GM})_{r}\) appears independent of \( r \) (or \( z \)) to compare equations 48 and 55. Now, comparing different values of \( z \) in the range \( 4 \times 10^{-6} \leq z \leq 30 \), with \((g_{GM})_{r}\) evolving over time, as in Figs. 2 and 4, with the scale factor being normalized at the present epoch, it appears that \((g_{GM})_{r}\) begins showing dependence on \( r(t) \) at the critical time \( t_{crit} \sim 1.2 \times 10^{4} \) yr \(( = 9 \times 10^{-7} t_{0})\) after the Big Bang, and then showing significant dependence at times \( t \gtrsim 2.8 \times 10^{8} \) yr \(( = 0.02 t_{0})\). This means that \((g_{GM})_{r}\) begins to change significantly over time, wherein it starts to increase. Before discussing below the importance of this observation, we readily see that this validates our use of equation 55 to evaluate \((g_{GM})_{r}\) in the early Universe: it is independent of \( r \) for small \( r \) and it matches or converges to the value of \((g_{GM})_{r}\) \( \approx 41.5 \) cm s\(^{-2}\), evaluated using the exact analytical expression (equation 48), at the so-called interface: \( t = 10^{-5} t_{0} = 138 \) yr \( \approx 4.4 \times 10^{8} \) s [compare Figs. 2 and 4].

Continuing, this independence of \((g_{GM})_{r}\), for small \( r \), at early times, as the Universe evolves over time, and then becoming significantly dependent on \( r \) (or \( z \)), as measured by an arbitrary comoving observer, for \( t \gtrsim 0.02 t_{0} \), can be seen in Figs. 2 and 4. This behavior can be further understood by looking at the resulting equation 55 for very small \( r \) at early times and for very small \( r \) at later times. Equation 55 shows what happens for small spacetimes, when the exponential goes to one in equation 48, as mentioned above, and \((g_{GM})_{r}\) decreases somewhat linearly with increasing time [compare Fig. 4(a)]. But as time continues to increase, the exponential in equation 55 begins to increase, and \((g_{GM})_{r}\) reaches a finite value at \( t = t_{0} \), \( z = 0 \), like that seen in Fig. 2(f), which was evaluated, along with the other figures of Fig. 2, using the exact analytical expression for \((g_{GM})_{r}\) (equation 48). So, a present-day arbitrary comoving observer will measure the value of \((g_{GM})_{r}\) to be different at different \( z \) (or \( r \)) values; and, importantly, its values at \( z \lesssim 0.5 \) dominate over \(|(g_{GE})_{r}|\), contributing possibly to the recently observed acceleration of the cosmic expansion.
Thus, looking at Fig. 2 and what happens at \( t \geq 0.02t_0 \) enable us to see how the GM acceleration \((g_{GM})_r\) might once again dominate over the GE deceleration \((g_{GE})_r\), if it were an intrinsic part of the equation of motion of the expansion (or cosmic scale factor) in the expanding and rotating Universe of equation (5) to produce the observed present-day cosmic acceleration. This we suppose in Sections 5.3 and 5.6 where we will consider, as mentioned earlier, how the GM acceleration, and thus inertial spacetime frame dragging, might be related to the torsion term in the Gödel-Obukhov equation of motion (equation (5)).

Further, and for completion, from using the relativistic Hubble law (equation (59)) to determine \( \rho \), the physical proper distance, in equation (48), the following appears to occur. For any \( z > 10 \), \((g_{GM})_r \to 0 \) (or negligibly small) at around \( t = 0.6t_0 \approx 8.3 \times 10^9 \) yr after the Big Bang, i.e., about \( 5.5 \times 10^9 \) years ago, and remains zero up to the present epoch \( t = 13.8 \times 10^9 \) yr (compare Fig. 4). There is no turning up of the curve, like that of the curves found in Fig. 2, of \((g_{GM})_r\) versus cosmic time, for these large values of \( z \). The case of Fig. 4(b), which is evaluated at the Hubble radius \( r_H \), equivalent to \( z = 30 \) according to the relativistic Hubble law, is an example of this. Then from around \( z = 6 \), \((g_{GM})_r\) gradually changes from zero to nonzero values over time and decreasing redshifts, as measured by an arbitrary comoving observer, consistent with observations that indicate the coming presence of so-called dark energy. At \( z \approx 1 \) is where \((g_{GM})_r \approx 2.4 \times 10^{-17} \text{ cm s}^{-2}\) first becomes nonzero at \( t = t_0 \), as measured by a present epoch observer, but \((g_{GM})_r\), is still much less than \(|(g_{GE})_r| \approx 2.1 \times 10^{-8} \text{ cm s}^{-2}\) at this redshift. Then at around \( z = 0.5 \), as mentioned above, \((g_{GM})_r\), becomes greater than \(|(g_{GE})_r|\), indicating the Universe goes into an accelerating expansion phase [compare Fig. 2(c)]. Further details of these results using the relativistic Hubble law, at these large redshifts, are presented elsewhere (Williams 2011, in preparation).

So, in summary, the above appears to suggest that the GM field strength was once very large in the past as shown in Fig. 4(a); became negligibly small (\( \approx 0 \)) around \( t = 0.6t_0 \), about \( 5.5 \times 10^9 \) years ago, for \( z > 10 \), as shown somewhat in Fig. 4(b); and, then, from \( z \leq 6 \) the GM field gradually increased over time, and decreasing \( z \), from zero to a nonzero value at \( z \approx 1 \). The case of Fig. 4(b), which is evaluated at the Hubble radius \( r_H \), equivalent to \( z = 30 \) according to the relativistic Hubble law, is an example of this. Then from around \( z = 6 \), \((g_{GM})_r\) gradually changes from zero to nonzero values over time and decreasing redshifts, as measured by an arbitrary comoving observer, consistent with observations that indicate the coming presence of so-called dark energy. At \( z \approx 1 \) is where \((g_{GM})_r \approx 2.4 \times 10^{-17} \text{ cm s}^{-2}\) first becomes nonzero at \( t = t_0 \), as measured by a present epoch observer, but \((g_{GM})_r\), is still much less than \(|(g_{GE})_r| \approx 2.1 \times 10^{-8} \text{ cm s}^{-2}\) at this redshift. Then at around \( z = 0.5 \), as mentioned above, \((g_{GM})_r\), becomes greater than \(|(g_{GE})_r|\), indicating the Universe goes into an accelerating expansion phase [compare Fig. 2(c)]. Further details of these results using the relativistic Hubble law, at these large redshifts, are presented elsewhere (Williams 2011, in preparation).

5.3 The GM Acceleration, Cosmic Rotation, and the Present epoch Observer

Figure 5 displays what happens to equation (48) in a general sense when \( z \) is held constant at \( z = 0.5 \) [Fig. 5(a)] or at \( z = 0 \) [i.e., \( r = 0 \): Fig. 5(b)] and \( \delta \) is allowed to vary, where

\[
(\omega_{\text{rot}})_0 = \delta H_0 \tag{58}
\]

is the present-day angular velocity (see equations (40) and (42), with \( H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1} = 2.3 \times 10^{-18} \text{ s}^{-1} \). In Figure 5(a) \( \delta \) was allowed to vary between \( 74 \geq \delta \geq 1 \). We see that \((g_{GM})_r \to 0\) exponentially (i.e., quickly), left of the maximum, at any value \((\omega_{\text{rot}})_0 \geq 10H_0\); falling from a maximum of \( \sim 5 \times 10^{13} \text{ cm s}^{-2} \) at \( \delta = 0.1 \); and \((g_{GM})_r \to 0\) linearly (i.e., slowly), right of the maximum, as \((\omega_{\text{rot}})_0 \to 0\) at \( \delta = 10^{-28} \). Note, \((\omega_{\text{rot}})_0 = 2\pi H_0 \approx 1.5 \times 10^{-17} \text{ s}^{-1}\) is the value used in the calculations of this present manuscript (see equation (57)). The reason for this behavior is that, for a present epoch observer, with \( R(t = t_0) = 1 \), at large \( \delta \), the exponential expression in the denominator of equation (48) dominates, causing \((g_{GM})_r\) to go exponentially to zero; and at small \( \delta \), the exponential expression goes to one and \((\omega_{\text{rot}})_0\) in the numerator goes to zero in a linear-like fashion. This behavior will be important when we next analyze the equation of motion describing the expansion and deceleration of the Gödel-Obukhov spacetime cosmology. We shall see in the following section that this behavior is similar to what one would expected of the third term in equation (56), again with \( R(t_0) = 1 \). Figure 5(b), with \( \delta \) allowed to vary between \( 74 \geq \delta \geq 1 \) at \( r = z = 0 \), shows, again in a general sense, what the strength of \((g_{GM})_r\) would be as measured by a present epoch observer \( (t = t_0) \). The reason for the linear-decline in \((g_{GM})_r\), is because with \( r = 0 \) the exponential expression in the denominator of equation (48), equals one, resulting in \((g_{GM})_r\), decreasing as \((\omega_{\text{rot}})_0\) in the numerator decreases.

Although the above analysis presented in Fig. 5 may not have any real physical significance, save limiting the proportionality constant relating \( H_0 \) and \((\omega_{\text{rot}})_0\) (equation (58)), it shows the general behavior of the dependence of \((g_{GM})_r\) on \((\omega_{\text{rot}})_0\) at \( z = 0.5 \) and \( z = 0 \), as measured by a present epoch observer.

5.4 Comparing the Standard FLRW Spacetime with the Gödel-Obukhov Spacetime

In this section we will analyze the equations of motion of the scale factor that contains terms that control spacetime acceleration and deceleration of the Universe over time: equations (48) and (4), for the standard FLRW and the Gödel-Obukhov spacetimes, respectively. Substitution of equation (48) into the second term on the right-hand side of equation (48) and comparing equation (48), allows us to identify this second term as the GE acceleration that decelerates the Universe, particularly when
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Figure 5. The GM cosmic acceleration and rotation: \((g_{GM})_r\) vs. \(\omega_{rot}\). (a) At redshift \(z = 0.5\), scale factor \(R(t) = 1\), at \(t = t_0\). Note, \(\delta\) in equation (58) decreases along the curve, as \(\omega_{rot}\) decreases (see text). (b) At redshift \(z = 0\), scale factor \(R(t) = 1\), at \(t = t_0\). Note, for comparison, using \(\delta = 2\pi\) (see equation (54), as it is done throughout these calculations, at \(t = t_0\), with \(H_0 = 71\) km s\(^{-1}\)Mpc\(^{-1}\), \((\omega_{rot})_0 \sim 1.5 \times 10^{-17}\) s\(^{-1}\).

\[\Lambda = p = 0:\]
\[
\frac{\dot{R}}{R} = -qH^2; \tag{59}
\]
or
\[q \equiv q_{FLRW} = -\frac{\dot{R}}{RH^2}. \tag{60}\]

Equation (60) is just equation (1), where equation (59) has been used. Notice, we have defined \(q \equiv q_{FLRW}\) to distinguish between deceleration parameters in the two spacetimes we are considering: the FLRW and the Gödel-Obukhov, where \(q \equiv q_{GO}\) in the Gödel-Obukhov spacetime (indicated by the subscript ‘GO’). We will see below how the two may correlate. So, we find in general from equations (59) and (60) that

\[q_{FLRW} = 0 \Rightarrow \text{coasting},\]
\[q_{FLRW} > 0 \Rightarrow \text{deceleration},\]
\[q_{FLRW} < 0 \Rightarrow \text{acceleration}, \tag{61}\]

again, for \(\Lambda = p = 0\).

On the other hand, in the Gödel-Obukhov spacetime we can identify the GE deceleration as the first term on the right-hand side of equation (6), and exactly that of the standard FLRW model if we assume \(q = 1\) (see equation (59), as mentioned in Section 3). Moreover, it seems reasonable to identify the fourth term on the right-hand side of equation (6), involving the mass density \(\rho\), as that describing the initial inertial expansion over time. We define this acceleration term, which we associated with the initial cosmic expansion by

\[
\left(\frac{\dot{R}}{R}\right)_I \equiv 2\left(\frac{k + \sigma}{k}\right)qH^2, \tag{62}\]

where equation (43) has been used (see Section 3.4 for validation of its use). This term being proportional to \(H^2\) and, thus, the density, will be very large in the very early Universe. Now using equation (47) and its derivatives, we can express this inertial spacetime cosmic expansion in terms of the physical separation distance \(r\):

\[
\ddot{r} \equiv (a_I)_r = 2\left(\frac{k + \sigma}{k}\right)qH^2r, \tag{63}\]

where \(r\) is given by equation (47). So, we have identified equation (63) as the acceleration due to expansion. If we use the expressions for \(\sigma\) and \(k\) given in equations (38) and (40), respectively, with values given in Section 3.5 for constants \(c_1\) and \(c_2\), equation (63) reduces to

\[
(a_I)_r \sim 2qH^2r. \tag{64}\]

This is consistent with the of order expression given near the end of Section 2 which was derived from dimensional analysis.

At this point we will use the analogy of the standard FLRW cosmology, where we set \(\Lambda = p = K\) (curvature) = 0 in equation (32) to define the deceleration parameter \((q_{FLRW})_r\) and its relation to the mass density \(\rho_1\); here we will set \(\omega_{rot} = B = p = 0\) in equation (5) to define the deceleration parameter \((q_{GO})_r\) and its relation to the mass density. Equation (5)
reduces to
\[
\frac{\ddot{R}}{R} = -H^2 + \frac{8\pi G}{3} \left( \frac{k + \sigma}{k} \right) \rho
\]  
(65)

(compare with equation 32 using equation 33). Using the model parameters of Section 3.5 as was done in equation 64, substituting in the mass density now given by
\[
\rho = \frac{3q_{GO} H^2}{4\pi G},
\]  
(66)

and dividing through by \(-H^2\), equation (65) yields
\[
q_{GO} \simeq \frac{1}{2} \left( \frac{\ddot{R}}{RH^2} + 1 \right);
\]  
(67)

or
\[
q_{GO} \simeq \frac{1}{2} (1 - q_{FLRW})
\]  
(68)

(compare equation 66). So by equation 68 we now have a relationship between the deceleration parameters of the standard FLRW and Gödel-Obukhov cosmologies, which appears to be consistent with observations, as can be seen in the following:

For
\[
q_{GO} \simeq \frac{1}{2} q_{FLRW} = 0 \Rightarrow \text{coasting};
\]
\[
q_{GO} < \frac{1}{2} q_{FLRW} > 0 \Rightarrow \text{deceleration};
\]
\[
q_{GO} > \frac{1}{2} q_{FLRW} < 0 \Rightarrow \text{acceleration};
\]  
(69)

compare equation 61.

Next, looking at the second and third terms on the right-hand side of equation (5), it appears that these are related to the fictitious-like forces associated with cosmic rotation, and at least one to general relativistic inertial frame dragging. These fictitious forces, analogous to Newtonian centrifugal and Coriolis forces, appear in the equation of motion of an object in a rotating frame. It is called a fictitious force because it does not appear when the motion is expressed in a Newtonian inertial frame of reference (i.e., a frame that is not rotating nor dragged into rotation). The second term can be easily identified as a centrifugal-like acceleration, associated with the vorticity (i.e., the rotation), that decreases over time, being proportional to \(\omega_{rot}^2\), and largest in the early Universe. This second term appears to be associated with the initial cosmic rotational energy, similar to the fourth term on the right-hand side of equation (5) in which we identified above (equation 62) as that being associated with the initial inertial cosmic expansion energy. The third term, as mentioned earlier in Section 3.1, is related to the torsion, coupled with spin (or rotation) and curvature, of spacetime, and appears to be directly related to general relativistic inertial spacetime frame dragging, where we would expect it to be some sort of Coriolis-like force. This third term behaves similar to that seen in Fig. 5(a), i.e., it goes to zero for large \(\omega_{rot}\), which would be at early times; increases as \(\omega_{rot}\) decreases over time; and, then, at later times, decreases as, perhaps, the magnitude of the cosmic magnetic field \(B\) (Obukhov 2000) decreases over time. Note, we shall consider this \(B\) and its relation to \(\omega_{rot}\) further in the following section. Now, since in the case of a moving test particle in the gravitational potential well of a compact rotating object, the GM force exerted on the particle has been related to a Coriolis-like force, one would expect the GM general relativistic so-called fictitious force (exerted on a moving ‘test galaxy’) here, and thus the third term on the right-hand side of equation (5), to be related to a Coriolis-like force. So, supposing this to be the case, based on

(i) the evolution of \((g_{GM})_r\), presented in Fig. 2 and its consistency with recent observations of cosmic accelerated expansion as discussed in Sections 5.1 and 5.2,
(ii) that \((g_{GM})_r\) is a Coriolis-like force derived using the Gödel-Obukhov metric (equation 7), and
(iii) support by the behavior of the third term: being somewhat similar to that of Fig. 5(a),

it seems reasonable that the acceleration produced by this third term should be at least approximately equal to that of the GM force given by equation 15. But, in order to set these accelerations equal, and solve for the cosmological coupling constants, \(\lambda_1\) and \(\lambda_3\), whose derivations are otherwise very complex (see Obukhov 2000), we must first express equation 48 in the form of a component of the cosmic scale factor equation of motion (equation 5). This is done in the following section.

5.5 The GM Field, Spin, and the Electromagnetic Field

In this section we will analyze the terms in equation 5 involving the magnetic field, \(B\), and the meaning of this electromagnetic field. As a vector representation this field is the \(F_{12} = B_z\) component of the electromagnetic field tensor due to
the electrodynamic characteristics of the spacetime matter. Here we consider the Universe to be a spinning Einstein-Cartan (source of spacetime curvature) fluid of charge density rotating about the \( \hat{e}_z \) axis, i.e., \( \omega_{\text{rot}} = \omega_{\text{rot}} \hat{e}_z \), like that stated in Section 3 (Obukhov 2000; Obukhov & Korotky 1987). We assume that torsion is generated by the spin tensor of such a fluid. Importantly, and somewhat surprisingly, it appears that \( B \) is related to the GM field, as we shall see below.

Using equation (37), its derivatives, and dividing equation (13) through by \( r \), we have the GM acceleration expressed in desired form – referring to the above section, in units of equation (5), i.e., \( s^{-2} \), expressed as an acceleration of the cosmic scale factor:

\[
\left( \frac{\dot{R}}{R \text{GM}} \right)^\text{GM} = \left\{ \frac{k \sigma(t)}{[k + \sigma(t)]^3} \right\}^{1/2} \left\{ \frac{(m/c) r \sin \theta + 1}{R(t) \sin \theta c^{(m/c) r \sin \theta}} \right\} \left\{ 1 - \frac{\sigma(t) + 2k}{2[k + \sigma(t)]} \right\} \frac{\dot{\omega}_{\text{rot}} c}{r}, \tag{70}
\]

where, \( m \), being proportional to \( \dot{\omega}_{\text{rot}} \), is again given by equation (4). Notice that the terms in the above equation are consistent with the GM force producing a Coriolis-like acceleration.

Note, in these calculations (as mentioned in Section 4) in reference to equation (13), we set \( \theta = \frac{\pi}{2} \), measured locally by an arbitrary comoving observer. This appears to be consistent with the quasi-Cosmological Principle in a rotating universe, where, for this local observer at the centre of the metric (equation 4), we propose four-dimensional spacetime to be described (or at least understood) by two-dimensional spacetime spherical axisymmetrical concentric hypersurfaces of spacelike separations. We assume each surface, surrounding the local observer, to be embedded with the constituents of the Universe at any given time, and rotating at the corresponding angular velocity, \( \omega_{\text{rot}} = \omega_{\text{rot}}(t) \equiv \dot{\omega}_{\text{rot}}(t) \) (compare equations (71) and (73)). Each surface of infinitesimal thickness represents a specific epoch of time, with time increasing (locally) toward the centre of these concentric surfaces of constant time \( t \) as well as constant \( \omega_{\text{rot}} \), for observers whose world lines coincide at any point on a particular surface. An arbitrary comoving observer of the metric of equation (4), with \( \theta = \frac{\pi}{2} \), would be at the centre of a cross section of such axisymmetrical and homogeneous surfaces of constant time. It might be easier to visualize this local arbitrary observer to be at the global ‘centre’ of the Universe (although not physically nor a necessity) such that the local coordinates coincide with the global system. So this arbitrary comoving observer’s line-of-sight at any given time and location will be from the centre of a disc plane of infinitesimal thickness, intersecting surrounding concentric hypersurfaces, whose radial distances from the observer are measured by \( r(t) \), the proper or physical distance (see equations 37 and 57). Such a geometrical configuration for a local arbitrary comoving observer with \( \theta = \frac{\pi}{2} \) appears to be unique to local observers in a rotating universe, and appears to be consistent with the quasi-Cosmological Principle of homogeneity and isotropy in the CMB radiation, but now including spatial isotropy in the direction of \( r \) (for axial symmetry), i.e., pseudo-spatial isotropy in \( r \) resembling that for the nonrotating FLRW spherical symmetric case. This proposed geometry is for a present-day comoving arbitrary observer, where the scale factor \( R(t) \) (equation 23) is normalized at the present time \( t = t_0 \), at \( r = 0 \), corresponding to redshift \( z = 0 \), with \( (GM)_r \approx 4 \times 10^{14} \text{ cm s}^{-2} \) at \( t_0 \), as can be seen in Fig. 2(f). As \( r \), the proper or physical distance, increases, this observer is looking back in spacetime to an earlier time, indicated by concentric spacetime hypersurfaces. This is the arbitrary comoving observer that measures the parameters displayed in Figs. 2 through 6, described in Sections 4 and 5.

Now with the note above in mind, we set the third term on the right-hand side of equation (3) that is associated with torsion, approximately equal to that of equation (70) above, associated with frame dragging, and solve for \( 4\lambda_1^2 - \lambda_2^2 \), the coupling constants of torsion, curvature, and spin, with \( 4\lambda_1^2 \gg \lambda_2^2 \) to prevent cosmological collapse (Obukhov 2000), as suggested to be the case in recent observations:

\[
4\lambda_1^2 - \lambda_2^2 \approx \left( \frac{144k}{k + \sigma} \right) \left\{ \frac{k \sigma(t)}{[k + \sigma(t)]^3} \right\}^{1/2} \left\{ \frac{(m/c) r \sin \theta + 1}{\sin \theta c^{(m/c) r \sin \theta}} \right\} \left\{ 1 - \frac{\sigma(t) + 2k}{2[k + \sigma(t)]} \right\} \frac{R^2 \dot{\omega}_{\text{rot}}^2 c}{B r}, \tag{71}
\]

where in cgs units, \( \lambda_1 \) and \( \lambda_2 \) have units of \( \text{cm g}^{-1} \), and \( B \) has units of gauß (\( \approx \text{g}^{1/2} \text{cm}^{-1/2} \text{s}^{-1} \)). The torsion, as related to torque and rotation, appears to be caused by the inertial spacetime expansion as it attempts to drag (or torque) the cosmic spacetime matter out of rotation (as described in Section 2). I refer to the cosmic expansion frame as an inertial (or ‘flat’) spacetime frame in a general relativistic dynamical sense because it has inertial force properties as well as inertial motion properties, appearing to be, in straight lines away from an arbitrary comoving observer (i.e., along the observer’s line-of-sight) as expected of an inertially flat expanding spacetime frame. In other words, in a general relativistic sense, the inertial frame of linear expansion is torquing or dragging the non-inertial frame of rotation, of cosmic spacetime mass density \( \rho_c \), out of rotation, in a sense, producing the GM force (per unit of moving mass) that accelerates the cosmic expansion. So, the torsion it appears is caused by the expanding frame’s inertia to rotation, producing the negative frame dragging angular velocity (see equation 11). This behavior is analogous to how a sufficiently large mass density can warp (or curve) spacetime causing the inertial cosmic expansion to decelerate, clearly seen in the standard FLRW cosmological model (equation 32), thus producing the so-called GE force (equation 31) compare also equation 5.

Moreover, in general relativity, the effect that frame dragging has on moving objects in spacetime is described by the GM force field. Similar to that experienced by moving objects (i.e., test particles) in the gravitational potential well of a rotating black hole [see fig. 2 in Williams (2002)], where the frame dragging angular velocity is in the positive azimuthal direction and
produces a positive radial force, this cosmological spacetime frame dragging is, however, in the negative azimuthal direction but produces a positive radial force as well. The difference in the sign of the frame dragging velocity appears to be due to the involvement of both the translational inertial and rotational non-inertial frames in an expanding and rotating universe, acting on freely falling or comoving observers, such as galaxies; whereas, with the black hole only the rotational frame produced by the black hole is involved, acting on freely falling local inertial frame observers, such as particles of plasma, dragging these observers into rotation in the direction that the black hole is rotating. Now, in general, both the translational cosmic spacetime inertial and cosmic rotational non-inertial frames are undergoing a form of acceleration: a change in the velocity for freely falling or comoving observers, such as galaxies; whereas, with the black hole only the rotational frame produced by the black hole is involved, acting on freely falling local inertial frame observers, such as particles of plasma, dragging these observers into rotation in the direction that the black hole is rotating. Next, the derivation of equation (71) suggests that there might be a relationship between the GM and the electromagnetic fields; we will look further at this somewhat surprising discovery below. The magnetic field given by Obukhov (2000) is from application of an ideal fluid plasma of spin and energy-momentum, to a cosmological model with rotation (shear-free) and expansion, in the framework of the Einstein-Cartan theory of gravity. Note, the Einstein-Cartan theory of gravity is just Einstein’s theory including rotation (and its effect on spacetime), a natural extension to describe a universe with expansion and rotation. This cosmic magnetic field, seen in equations (5) and (71), and mentioned above, is the $F_{12} = B_z$ component of the electromagnetic field tensor describing the electrodynamic characteristics of matter in the Universe. It appears that this field can be expressed over time by (Obukhov 2000)

$$B_t = [-2R(t)\omega_{rot}\tau]^{1/2},$$

where $\tau_t = \tau(t) < 0$ is the spin density, in units of $g \text{cm}^{-1}s^{-1}$; again, $R(t)$ and $\omega_{rot} = (\omega_{rot})_t$ are given by equations (49) and (71), respectively. The spin density by definition is the angular momentum per unit volume. It then seems reasonable to express the spin density as

$$\tau_t \sim -\rho \omega_{rot} t^2,$$

where, also, $r = r(t)$. This means that $B$ could be a primordial cosmic global magnetic field intrinsic to the rotating and expanding spacetime matter in the Universe. Since $B$ of equation (72) depends on the spin density, $\tau_t$, and this spin density can be expressed in terms of the average mass density $\rho$, by equation (73), with great surprise and excitement, equations (72) and (73) appear to show or at least lead us to the long sought relationship, by Einstein (as well as by the author after being inspired by the work of Einstein), between the electromagnetic field (with the electric field canceling assuming equal number of positive and negative charges) and the gravitational field, namely, between the magnetic field and the GM field. It appears that they can be related by the spin density. Further details of this relationship and of its importance in other astrophysical phenomena are beyond the scope of this paper and will be discussed elsewhere in a forthcoming paper (Williams, in preparation). But important here is that these findings, in the context of the model considered in this paper, reveal how the GM field might be associated with the magnetic field through the spin density $\tau_t$, which can be expressed in terms of the average mass density $\rho$. In the following, we shall compare values given by equations (72) and (73) in the early and late Universe with other theoretical estimated values, as well as with observations. First, for comparison, we used the author’s derived spin density $\tau_t$ of equation (73) and the cosmological parameters used [$q_0 = 0.01$ and $(\omega_{rot})_0 = 0.1H_0$] by Obukhov (2000), with $H_0 = 71 \text{km} \text{s}^{-1}\text{Mpc}^{-1}$, to see if the values of the parameter and Obukhov (2000) agree for spin density, $\tau_0$, at the present epoch. The density from equation (73) is calculated to give $\rho_c = 1.89 \times 10^{-31} \text{g cm}^{-3}$; then, evaluation of equation (73) at the Hubble radius ($r_H \approx 1.3 \times 10^{26} \text{cm}$) yields $\tau_0 \sim -7.4 \times 10^6 \text{g cm}^{-1}s^{-1}$. The magnitude of this value is smaller than the magnitude of Obukhov’s (2000) estimated value ($\tau_0 \sim -5 \times 10^6 \text{g cm}^{-1}s^{-1}$) when using the same $q_0$ and $(\omega_{rot})_0$ as used by Obukhov (2000). However, using the same $q_0 = 0.01$, but letting $(\omega_{rot})_0 = 2\pi H_0$ (equation (54), as used by the author, equation (73)), then yields $\tau_0 \sim -4.7 \times 10^6 \text{g cm}^{-1}s^{-1}$. As we can see, this value is approximately equal to the average value estimated by Obukhov (2000), given above. Since Obukhov (2000) does not give the derived analytical expression used to calculate $\tau_0$, it cannot be commented on here as to why our values agree for the same $q_0$, but with a different relationship between $H_0$ and $(\omega_{rot})_0$, except that perhaps this is the reason Obukhov (2000) calculates the modern-day magnetic field strength to be $\sim 10^3$ times larger than the established upper limit from astrophysical observations. Nevertheless, from the above, it seems safe to conclude that the deduced analytical expression for $\tau_t$ (equation (73) is at least of order true. Next, the cosmic magnetic field of equation (72) can be expressed as

$$B_t = [2R(t)\omega_{rot}^2\rho_t^2]^{1/2},$$

where we have used equation (73). It appears that $B_t$ above is a frozen-in cosmic primordial magnetic field modified only
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1e-04
0.001
0.01
0.1
1
10
100
1000
1e-08
1e-07
1e-06
1e-05
1e-04
0.001
0.01
0.1
1
1e-08
1e-07
1e-06
1e-05
1e-04
0.001
0.01
0.1
1

B(t) (gauss)
t/to

(a)

(b)

(c)

Figure 6. Evolution of the cosmic primordial universal magnetic field: (a) \(B_t\) versus \(t\) at the \(z \approx 30\) (at the Hubble radius, \(cH_0^{-1} \approx ct_0\)). (b) \(B_t\) versus \(t\) at the \(z = 0.5\) (\(\approx 2 \times 10^3\) Mpc). (c) \(B_t\) versus \(t\) at the \(z = 4 \times 10^{-6}\) (\(\approx 17\) kpc).

through the expansion process, such that the magnetic flux is conserved (Asseo & Sol 1987). Then \(B_t\) would be that measured by a comoving arbitrary observer at the centre of the metric of equation (7). As the physical distance \(r\) from this observer is increased, the observer is looking back in time, as usual, because of the finite speed of light. So with this in mind one would expect \(B_t\) to be larger at large \(r\), as measured by a present-day observer (i.e., where the scale factor of equation 43 is normalized to one), and smaller as \(r\) gets smaller, until it reaches the value measured locally, between galaxies. This \(B_t = B_0\) would be the present-day value of the cosmic magnetic field that has been somewhat dissipated, due to the spacetime expansion of the Universe. Below we will calculate values measured by this arbitrary comoving observer.

At the present epoch, as measured by this comoving observer, with \(q = \frac{1}{2}\), \(n = \frac{2}{3}\) using equation (43), normalized at \(t = t_0\), \(\rho_0 = \rho_c(t_0)\), for \(H_0 = 71\) km s\(^{-1}\) Mpc\(^{-1}\) = \(2.3 \times 10^{-18}\) s\(^{-1}\), \(\omega_0 \approx 1.5 \times 10^{-17}\) s\(^{-1}\) (equation 54), \(t_0 = H_0^{-1}\), and \(r = r_H = cH_0^{-1}\), we find that \(B_0 \sim 8 \times 10^{-4}\) gauss at the Hubble radius (\(\approx 1.3 \times 10^{28}\) cm). This can be seen in Fig. 6 where the magnetic field of equation (74) is evolved over time from when the Universe was 138 years-old to the present. Figures 6(a) and 6(b) display the evolution of the field strength \(B_t\) measured at the Hubble radius (\(r_H\), at \(z \sim 30\) according to equation 56) and at \(z = 0.5\), respectively, by an arbitrary comoving observer. These figures are to be compared with those of Figs. 4(b) and 2(c), respectively, which plot the GM acceleration over time.

For the very early Universe, this comoving observer measures the following as spacetime evolves. At the Planck scale (indicated by subscript \(P\)), \(t = t_P = \sqrt{\frac{\hbar G}{2\pi c^3}} \approx 5.4 \times 10^{-44}\) s, with \(\rho_P \sim 1.8 \times 10^{92}\) g cm\(^{-3}\) (equation 34), \(B_P \sim 1.8 \times 10^{37}\) gauss, according to equation (72), at the Hubble radius (\(r = r_P = ct_P \approx 1.6 \times 10^{-31}\) cm or the so-called Planck length), where \(\hbar\) is the Planck constant. The best way to express \(B_t\), it appears, at least during inflation, and to see clearly how the magnetic field falls off over time, is to use

\[
\rho_c = \frac{\rho_0}{R^3(t)}.
\]
then equation (74) can be expressed as

$$B_t = \left[ \frac{2\omega_{\text{rot}}\rho t}{R^2(t)} \right]^{1/2}.$$  

(76)

Now, we assume that inflation occurs at the characteristic times between $10^{-36}$ s $\leq t_{\text{inf}} \leq 10^{-34}$ s, where during inflation $H_t \approx$ constant, since $\Delta t \approx 9.9 \times 10^{-35}$ s $\ll 1$ (see Section 5.5). The Hubble radii ($r = r_{\text{inf}} \sim c t_{\text{inf}}$) corresponding to the time interval above are $3 \times 10^{-26}$ cm $\leq r_{\text{inf}} \leq 3 \times 10^{-24}$ cm. From equation (74) or equation (76) we calculate the magnetic field at the beginning of inflation to be $B_t \sim 4.7 \times 10^{32}$ gauss. Since we assumed that $H_t$ is approximately constant during inflation, it seems reasonable to assume that $\omega_{\text{rot}}$ ($\sim 2\pi H$) is also approximately constant. We found in Section 5.5 that the initial and final scale factor at the beginning and after inflation are related by $R_{\text{final}} = 10^{43} R_{\text{initial}}$. It can be shown from equation (77) that the physical radius after inflation is given by $r_{\text{final}} = 10^{43} r_{\text{initial}}$. Thus we can see that upon substitution into equation (76), the $10^{43}$ factors in the numerator and denominator will cancel. Thus, it appears that $B_t$ is also approximately constant during inflation, such that $B_t$ will have approximately the same value before and after inflation. Therefore, after inflation one can continue to use equation (74) or (76) to express the cosmic primordial magnetic field. Then, after inflation, at the Hubble radius $\sim 10^{-24}$ cm, for $t = 10^{-34}$ s, we find that $B_t \sim 2 \times 10^{31}$ gauss.

Using equation (74) or equation (76) we now calculate the cosmic magnetic field at recombination/decoupling and at the present epoch ($B_0$). We will assume as suggested by particle physics that recombination/decoupling occurs $\sim 350,000$ years after the Big Bang (Liddle 2003). We can find values for this primordial frozen-in intergalactic (as related to spaces between galaxies) universal magnetic field $B_t$, as measured by an arbitrary comoving observer, at the present-day, normalized (see equations [77] and [43]), spacetime coordinate distance $r(t = t_0) \sim 17$ kpc [or $z(t = t_0) \sim 4.6 \times 10^{-6}$]. Recall that this observer is located at the centre of the Gödel-Obukhov metric (equation 77), which could be centred on an arbitrary comoving galaxy. Figure 6(c), shows how this cosmic $B_t$ has evolved over time, at the above coordinate distance, from the spacetime separation $r(t = 138 \text{ yr})$ to the present $r(t_0)$. From this figure we see that $B_t = B_{\text{sec}} \sim 4 \times 10^{-6}$ gauss and $B_t = B_0 \sim 3 \times 10^{-9}$ gauss at recombination and at the present epoch, respectively. Importantly, the value for $B_0 \sim 3 \times 10^{-9}$ gauss is consistent with the upper limit constraint placed on the present strength of any primordial homogeneous magnetic field, which is $B_0 \lesssim 4 \times 10^{-9}$ gauss, for $\Omega_0 = 1$ and $H_0 = 71 \text{ km s}^{-1} \text{Mpc}^{-1}$ (Barrow, Ferreira, & Silk 1997). The Cosmic Background Explorer (COBE) measurements provide this constraint: set by how the amplitude of the magnetic field is related to amplitude of the microwave background anisotropies on large scales. Pasquale, Scott, & Olimto (1999) study the effect of inhomogeneity on the Faraday rotation of light from distant quasi-stellar objects to find a consistent limit of $B_0 \lesssim 10^{-9}$ gauss.

The above present model calculated values of the cosmic magnetic field appear to be consistent with those that would allow the Universe to evolve into its present state, particularly like the cosmological model of Zeldovich (see Asseo & Sol 1987, and references therein). Such cosmological models depend on the choice of the metric, of the equation of state, and of the cosmological constant $\Lambda_0$, where most often taken to be $\Lambda_0 = 0$, until recently with the advent of dark energy. Such models are characterized by the expansion factors, the density of constituents, and the magnetic field, which evolve according to the Einstein-Maxwell field equations. The magnetic field energy is assumed to be modified only through the expansion process, and not through an exchange of energy with other constituents: matter or radiation, which means that the magnetic flux is conserved and a uniform magnetic field evolves like $1/R_t$. It can be shown from the possibility for a uniform magnetic field to exist even in the absence of an electric current ($\nabla \times B = 0$). This can be interpreted as the evolution of the magnetic field compatible with the Einstein equations, as characteristic of a magnetic field geometrically frozen-in independently of the conductivity of the matter.

Notice that the present epoch magnetic field of equation (74) or (76) goes to zero at the arbitrary comoving observer, just as does the gravitational acceleration ($g_{\text{GE}}$) of equation (49), because $r$ goes to zero. However, as we saw earlier in section 4 this is not the case for the GM acceleration ($g_{\text{GM}}$) of equation (48), which goes not go to zero, because the angular velocity $\omega_{\text{rot}}$ of equation (51) goes not to zero. The former statement appears to be consistent with the relationship pointed out above between mass density and magnetic field through the spin density. On the other hand, the latter statement is consistent with $\omega_{\text{rot}}$ being related to both: ($g_{\text{GM}}$) and $B_t$, with $B_t$ being related directly to the spin density (equation 73) as well.

5.6 The Gödel-Obukhov Spacetime Equation of Motion Extension

Now, if we assume that the theoretical model presented above is valid to describe processes in the Universe concerning the so-called dark energy, then the equation of motion of the scale factor of equation (63) can now be expressed as

$$\frac{\dot{R}}{R} \approx -H^2 + \frac{\omega_{\text{rot}}^2}{k} \left( \frac{k}{k + \sigma} \right) (k + \sigma)(3\sigma + 4k) + \left( \frac{3k}{k + \sigma} \right)^{1/2} \left[ \frac{1}{Re^{m/c}} \right] \left( \frac{e^2 - p - B^2}{R^2} \right),$$

(77)

This equation will be discussed in more detail in the next section.
in units of $s^{-2}$, where, the GM acceleration of equation (70), due to frame dragging, has been substituted in for the torsion term, which is the same as substituting equation (71) into equation (5). Note, $θ$, originally found in the metric of equation (7), has been set equal to $\frac{\pi}{2}$, as done in these model calculations (see equation 35). Equations (11) and (25) or (35) suggest that $θ = \frac{\pi}{2}$ is the only well-behaved solution, i.e., for example, for $z = 0.5$, as measured by a present-day comoving observer, $θ = \frac{\pi}{2}$ gives the minimum value for $(g_{GM})$, of equation (35), with this value increasing quickly to infinity at $θ ≈ ±\frac{\pi}{2}$. It appears that $θ = \frac{\pi}{2}$ naturally sets up the geometry (in a rotating universe) for a local arbitrary comoving observer surrounded by concentric spacetime axisymmetrical hypersurfaces that look the same in radial directions along the line-of-sight of this observer, as discussed in Section 5.5. This is consistent with the quasi-Cosmological Principle of homogeneity and isotropy for such an observer who measures distances governed by, and from the centre of, the metric of equation (4). (Perhaps the use of cylindrical spacetime coordinates would clarify why $θ = \frac{\pi}{2}$ is a ‘special’ value.) Again, model parameters for $k$ and $σ$ are defined in Section 3.5; $B$, as related to the spin density, is given above (equation 22 or 26); and $m$ is given by equation (4). Note, the GM-inertial frame dragging term of equation (77), believed, as suggested from these present model calculations, to be responsible for the recent accelerated cosmic expansion, and identified to be related directly to torsion of spacetime, is the only term dependent on the coordinate or spacetime separation $r$. This dependence on $r$ is consistent with the accelerated cosmic expansion being observed to be a function of $r$ (or $z$) as measured by a present-day arbitrary comoving observer, i.e., consistent with the apparently coming into being of the recent accelerated cosmic expansion at $z ≈ 0.5$. As described by the GM acceleration of equation (48), this GM-inertial frame dragging (torsion) term decreases over time from large $z$ to smaller $z$, but at some point in its later stage of evolution, it starts to increase with decreasing $z$, as can be seen in Fig. 2, and discussed in details in Sections 5.1 and 5.2. Thus, equation (77) appears to adequately explain the overall expansion of the Universe, wherein we consider cosmic rotation to be that of spacetime matter and is an inherent property of the geometrical and dynamical makeup of the Universe.

Importantly, we see that the pressure $p$ need not be negative to explain the recently observed cosmic acceleration, and Einstein’s cosmological constant can remain his ‘greatest blunder’; compare equation (32). We have found instead that this acceleration can be explained by considering the effects of the GM field and inertial spacetime frame dragging in a rotating and inertially expanding universe. So, in entirety, it appears that equation (5), equation (77), a possible extension of equation (5), and the metric of equation (4) constitute a complete description of the spacetime dynamics of our expanding and apparently rotating Universe, past and present.

6 CONCLUSIONS

With the recent discovery of so-called dark energy (appearing to comprise $≈ 70$ per cent of the Universe), it seems we know little about the Universe we live in, save the mass-energy we can see and somewhat the effects of gravity this mass-energy displays. Now our understanding is limited even more in addition to our already lack of knowledge of what composes so-called dark matter (appearing to comprise $≈ 30$ per cent of the Universe). The above percentages are based on observations and the standard FLRW cosmology; and, importantly, they suggest that we should perhaps seek further an understanding of the underlining force: gravity. Such understanding could possible solve the problem of our recently, and somewhat embarrassingly, increased lack of understanding. In this paper, I have adhered to the above suggestion by seeking an understanding of dark energy by considering it to be a component of gravity, a by product, arising in a general relativistically expanding and rotating cosmological spacetime.

In this manuscript is presented a general relativistic model to describe the dynamical evolution of the Universe. This model appears to answered the question, ‘Could dark energy be a manifestation gravity?’ and the answer given is yes. In this model, the recently observed cosmic acceleration can be explained by considering the effects of the GM field due to inertial spacetime frame dragging in an expanding and rotating universe. These model calculations seem to show that application of the Gödel-Obukhov spacetime metric of equation (2), or equation (7), in an Einstein-Cartan general relativistic spacetime, leading to the derivation of equation (5), the equation of motion of the scale factor $R$, and, perhaps, subsequently in the form of equation (77), yield a complete description of how our Universe has evolved over time. Nonvanishing torsion of the Riemann-Cartan spacetime (Mao et al. 2007) and the GM field of inertial frame dragging appear to be one in the same in this description. Importantly, we see that the pressure $p$ need not be negative, nor do we have to resurrect the cosmological constant $\Lambda$ to explain so-called dark energy, as proposed by previous models constrained by the standard FLRW cosmology.

Not only does the model presented here appear to describe the dynamical evolution of the Universe, but it appears to show a surprising relationship between the cosmic magnetic field and the GM field, through the spin density, which could possibly be a link between electromagnetism and gravity. This apparent relationship needs to be investigated further, not only on the astronomical level but the atomic level as well. However, this relationship should not be of a total surprise, since we have already made the analogy of the magnetic field of a rotating charge and the GM field of a rotating mass, based on their similarities, and, particularly, how the Lorentz force on a moving charged particle in a magnetic force field is analogous to the Coriolis-like force on a moving particle in a GM force field (see e.g., Braginsky, Caves, Thorne 1977; Thorne et al. 1986).
As a future calculation, one might use the results of the cosmological model presented in this manuscript to evaluate the Friedmann-like equation, describing the evolution of the scale factor, derived from the gravitational field equations by Obukhov (2000), used in the derivation of equations 5, 71, and 77. It appears that equation (5.42) of Obukhov (2000) can be used, applying the Equivalence Principle, to see what fraction of the critical density the GM acceleration, due to frame dragging (or torsion), contributes to making the observed density parameter $\Omega \approx 1$. This might also shed light on the true nature of dark matter.

Finally, whether all the concepts presented in this manuscript are fully valid, as they appear, or not, particularly those leading directly to equation 77, the fact remains that the Gödel-Obukhov cosmology of equation 5 exhibits cosmic acceleration in the equation of motion of the scale factor, which could possibly explain the recently observed cosmic accelerated expansion. If the extended form (equation 77) is indeed valid it shows that this cosmic acceleration became important in recent times.

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