Gravitational complementary principle:

A new approach to quantum gravity

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Abstract

A new idea of quantum gravity is developed based on Gravitational Complementary Principle. This principle states that gravity has dual complement features: The quantum and classical aspects of gravity are complement and absolutely separated by the planck length into planckian and over-planckian domains, respectively. The classical Einstein equations are correct at the fundamental level at over-planckian domain and general relativity is not a low energy limit of a more fundamental theory. The quantum gravity is totally confined to the planckian domain with a new kind of ultra-short range interaction, mediated by massive (Planck mass) particles, through the virtual microscopic wormholes of the Planck scale with action $\hbar$. There is no room for gravitons or extra dimensions in

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this scenario. It is shown that the hierarchy problem can solve the cosmological constant problem via this new quantum gravity.
1 Introduction

In the past few decades remarkable progress has been put forward in constructing a unified theory of the forces of nature. Weinberg-Salam theory of electroweak interaction has unified the electromagnetic and weak interactions [1], while the so-called grand unified theories incorporates the strong interaction into a wider gauge theory $SU(5)$. However, gravity as the odd one out of four forces of nature resists against a consistent formulation in the quantum framework. Combining general relativity with quantum mechanics is the last hurdle to be overcome in the "quantum revolution" [2]. Many different and powerful (perturbative and non-perturbative) approaches have been pursued to quantize gravity, but as yet a completely satisfactory quantum gravity theory remains elusive [3]. String and superstring theories, on the other hand, as the best candidate to formulate a full quantum theory of gravity, have not yet proceed successfully in this direction [3]. The quantum formulation of gravity involves some fundamental conceptual and technical difficulties. The dual role played by the field $g_{\mu\nu}$ as both the background spacetime structure and the quantity which describes the dynamical aspect of quantum gravity causes the entire notion of causality to be ill-defined. On the other hand, the quantum theory of gravity is non-renormalizable due to the dimensionality of the gravitational coupling constant.

From other points of view, combining general relativity with quantum mechanics is inconsistent. General relativity demands four-dimensional covariance while quantum mechanics is a theory of measurement on 3-dimensional space-like hypersurfaces.

In spite of these technical difficulties and conceptual complications it is generally believed that Einstein’s general relativity is a low energy limit of a more fundamental background
independent quantum theory of gravity [3], [4]. It is discussed that the spacetime metric is coupled to matter sources through Einstein equations and quantum theory applies to these matter sources. Thus, any attempt to treat gravity classically leads to serious problems and the only way to avoid these difficulties is to treat the spacetime metric in a probabilistic fashion which means quantum gravity.

In what follows, we will try to develop a new idea of quantum gravity which may shed light on the current problems in quantizing gravity, cosmological constant and hierarchy problems. We emphasize that this idea is still far from being complete, but it is hoped that it can stimulate further investigations in this direction.

2 Gravitational complementary principle

The so-called complementary principle, introduced by Bohr, is one of the most important basis of quantum mechanics. This fundamental principle in mathematical language is equal to the Heisenberg uncertainty principle which leads to the basic commutation relations on which the standard quantum mechanics stands. In quantum mechanics, position and momentum, just like particle behavior and wave aspects of a system respectively, are complement properties of the system and the theory *does not admit the possibility of an experiment in which both could be established simultaneously*. In this way, Bohr complementary principle BCP resolved some serious difficulties in the advent of quantum mechanics. The two slit experiment was one of confusing behaviors of nature which was satisfactory described by this principle. The prescription was to describe this experiment completely by the wave aspect and avoid any particle behavior. In fact, any attempt to describe it by the particle behavior would destroy
the whole interference pattern.

Inspired by Bohr complementary principle BCP and its key role in the foundation of quantum mechanics we introduce *Gravitational Complementary Principle* GCP:

Classical and quantum aspects of a gravitational system are complement properties of the system and the gravity theory *does not admit the possibility of a gravitational interaction in which both classical and quantum aspects could be established simultaneously*. A quantum mechanical experiment demands *wave* or *particle* description according to the nature of that experiment. For example, the two slit experiment needs wave description whereas the photoelectric experiment demands particle description. A gravitational interaction, however, demands *classical* or *quantum* description according to the size (or energy scale) of the domain in which that interaction takes place\(^1\). We then introduce a characteristic length scale for demarcation between the classical and quantum features of gravitational interactions. Since GCP is absolutely strict on the classical and quantum domains, then this length scale must be a fundamental one to strictly forbid any intrusion between these disjoint domains.

It was originally pointed out by Planck that combining the universal constants \(G, \hbar\) and \(c\) gives a new fundamental unit of length, namely the Planck length

\[
L_{pl} = \left( \frac{G\hbar}{c^3} \right)^{1/2} = 1.616 \times 10^{-33} \text{cm}.
\]

The Planck length should be thought of as the *zero point length* of the spacetime and any correct theory of quantum gravity must incorporate this feature in suitable form [5]. This

\(^1\) Similar ideas have already been proposed. For example, in [5], the author speaks of a thermodynamic spacetime and it is possible to separate the domain of validity of classical and quantum gravitational effects while connecting them through thermodynamic identities.
fundamental length scale is generic to the gravitational interactions, due to the presence of $G$, and there is no analogous typical length scale associated with other three interactions. This is a unique feature of gravitational interaction so as to be distinguished from other interactions. This tells us that gravity is different in nature from other forces. The existence of this length scale in the framework of gravitational interactions is completely consistent with GCP provided we absolutely divide the classical and quantum features of the gravitational interactions into disjoint over-planckian and planckian domains, respectively. While other three interactions obey the rules of BCP (quantum mechanics) at over-planckian domain, it seems gravity at this domain does not. Rather, it obeys the rules of quantum mechanics at planckian domain! At over-planckian domain it is just a pure classical field theory.

GCP states that: unlike other three forces where a typical interaction can be described both classical and quantum mechanically by the same action principle, the gravitational interaction with dual features at over-planckian and planckian domains is described by two different kinds of interactions and action principles in these domains. In other words, unlike for example the electromagnetic interactions where $A_\mu$ plays same (dynamical) role in classical and quantum descriptions, in the case of gravitational interactions $g_{\mu\nu}$ does not play the same role in classical and quantum descriptions. In fact, $g_{\mu\nu}$ does not play a dynamical role in quantum description. It merely can be described by the classical field theory of gravity (Einstein-Hilbert action) at over-planckian domain and has nothing to do with quantum features of gravity which are trapped at planckian domain. Another interaction and action principle is responsible for the quantum features of gravitational interaction at planckian domain! The principles of quantum theory do not apply to gravity at over-planckian domain and the classical general relativity is
correct at the fundamental level. The classical Einstein’s general relativity at over-planckian domain is no longer a low energy limit of a more fundamental theory of quantum gravity. One can not use $g_{\mu\nu}$ as a quantum field to describe the quantum features of gravity at over-planckian domain. The quantum features of gravity are totally confined and trapped in the planckian domain and are described by a field other than $g_{\mu\nu}$. Therefore, any attempt to study the quantum gravity is strictly limited into the planckian domain and the resultant theory of quantum gravity will be $g_{\mu\nu}$ independent. In this way, we will hope to have a new quantum gravity without technical and conceptual difficulties, discussed so far, arising due to dual roles of $g_{\mu\nu}$ as classical background and quantum fields. In fact, all above problems arise because we violate GCP by assuming dual roles for $g_{\mu\nu}$, simultaneously.

Just like the two slit experiment where the assumption of particle behavior (photon) for radiation violates BCP and destroys the whole interference pattern, the assumption of quantum behavior (graviton) for gravitation $g_{\mu\nu}$ at over-planckian domain violates GCP and destroys the whole quantization pattern, for example by the appearance of non-renormalizable divergences!

As a rule, a wrong question from nature yields a nonsense answer. Most of the current approaches to quantum gravity obey this rule, concerning GCP. In these approaches, usually a simultaneous combination of classical and quantum features of gravity is used to quantize gravity which violates GCP. For example, we know the quantum version of Einstein equation

$$G_{\mu\nu} = 8\pi G_N < T_{\mu\nu} >,$$  

leads to a serious problem that the gravitational field behaves in a discontinuous acausal way

\footnote{This property of quantum gravity is similar to the weak interactions in which the massive gauge bosons are mediated in a small range.}
and this difficulty seems to be avoided only by treating spacetime metric in a probabilistic fashion [4]. Using GCP, we show that this is not necessarily the case. In fact, Eq.(1) is generally a wrong question from the system and so leads to the above problem. According to GCP, the field $g_{\mu\nu}$ is a pure classical object at over-planckian domain and has nothing to do with any effect in which the observer may play the key role by the act of observation. Therefore, $g_{\mu\nu}$ can in no way couple to a matter source with some quantum probabilistic (observer dependent) features. One can resolve this confliction by assuming that any source of matter which is quantized over 3-hypersurfaces, behaves in a fully deterministic way against the 4-dimensional classical object $g_{\mu\nu}$. In fact, for the given excited states $|\alpha>$, $|\beta>$ the object $<\alpha|T_{\mu\nu}|\beta>$ implies the ignorance of 3-D observer about the full behavior of the system under consideration, and impelling $g_{\mu\nu}$ as a 4-D object to obey this ignorance is not really justified. In other words, every quantized non-gravitational system at over-planckian domain which behaves in a probabilistic way for 3-D observers, may behave against 4-D classical gravity in a fully deterministic and observer independent way\(^3\). Eq.(1) is a simultaneous combination of classical ($g_{\mu\nu}$) and quantum probabilistic ($<\alpha|T_{\mu\nu}|\beta>$) notions in quantizing gravity at over-planckian domain and so violates GCP. According to GCP, since this is not a correctly written equation then it yields nonsense answer, namely a discontinuous acausal behavior of the gravity. By this (wrong) equation and its nonsense answer one may not conclude that $g_{\mu\nu}$ should be quantized at over-planckian domain. However, it is important to note that the vacuum expectation value $<0|T_{\mu\nu}|0>$ is understood as a classical (observer independent) concept and so can couple to the geometry through equation (1). For example we know the

\(^3\)This is in spirit of Einstein’s believe: *God does not play dice on $g_{\mu\nu}$ background!*
mass of fundamental particles are proportional to the Higgs field vacuum expectation value \( <0|\phi|0> \). On the other hand, these particles can trivially couple to the geometry through classical Einstein equation. Therefore, one concludes that \( <0|\phi|0> \) must couple to the geometry as well, through equation (1).

In the covariant perturbation method also one writes the spacetime metric \( g_{\mu \nu} \) as

\[
g_{\mu \nu} = \eta_{\mu \nu} + h_{\mu \nu}, \tag{2}
\]

where \( \eta_{\mu \nu} \) is a background spacetime and \( h_{\mu \nu} \) represents a self-interacting spin-2 quantum field propagating on this background. This approach leads to a non-renormalizable perturbation theory \([4]\). As is easily seen, in this approach a simultaneous combination of classical and quantum field is presented which apparently violates GCP. According to GCP, the object \( g_{\mu \nu} \) is intrinsically a classical field at over-planckian domain and so can not be a simultaneous combination of classical and quantum parts. Indeed, the non-renormalizability of quantum gravity in this approach is a direct consequence of GCP violation which leads the pattern for quantizing gravity to be completely lost, in the same way as a wave interference pattern is lost when one violates BCP by introducing the particle behavior in the two slit experiment. Eq.(2) is just correct at over-planckian domain provided one does not interpret \( h_{\mu \nu} \) as a quantum field (graviton) in this domain. This equation just describes a classical gravitational wave \( h_{\mu \nu} \) propagating on the classical background \( \eta_{\mu \nu} \). Since gravitational interaction does not obey BCP (wave-particle duality) at over-planckian domain, then there is no quantum particle (graviton) associated with the gravitational wave, \( h_{\mu \nu} \).

In the canonical approach one attempts to construct a quantum theory in which the Hilbert space carries a representation of the operators corresponding to the full metric, or some func-
tions of the metric, without background metric to be fixed. This approach also uses simultaneously the classical and quantum notions by associating $g_{\mu\nu}$ with operator representation.

It seems in all other current approaches to quantum gravity one can find simultaneous use of classical and quantum features of gravity.

3 Ultra-short range quantum gravity

Up to know, we have introduced GCP which tells us that a gravitational interaction at over-planckian domain is completely described by classical general relativity with no notions of quantum gravity, in principle. GCP learns us that at over-planckian domain general relativity is not the low energy limit of a fully quantized gravity. There is no quantum gravity (graviton) at over-planckian $g_{\mu\nu}$ domain, at all. Quantum gravity is absolutely confined and trapped at planckian domain where $g_{\mu\nu}$ is not defined. If so, the important question is: what is the real nature and physics of quantum gravitational interactions at planckian domain?

To answer this question, one may look for quantum gravitational interactions of ultra-short range at planckian domain. In so doing, one may get suspicious to the following relations

\[ M_{pl}^2 = \hbar \frac{c}{G_N}, \quad \text{(3)} \]

\[ L_{pl}^2 = \hbar \frac{G_N}{c^3}. \quad \text{(4)} \]

These relations are reminiscent of the well-known relations in quantum mechanics

\[ E = \hbar \omega, \quad \text{(5)} \]

\[ P = \hbar K, \quad \text{(6)} \]
which describe quantitatively the wave-particle duality or in a sense BCP. Here, the Planck constant $\hbar$ relates the dual features of nature, namely the wave aspects and particle behaviors of a quantum mechanical system. Inspired by this key role of $\hbar$ in BCP, one may think that it may play a similar role in relating other dual features of nature, namely the classical aspects and quantum behaviors of a gravitational system. In other words, Eqs.(3), (4) play the same role in GCP as Eqs.(5), (6) does in BCP. Therefore, since the factors $\frac{c}{G_N}$ and $\frac{G_N}{c^3}$ in the r.h.s of Eqs.(3), (4) are pure classical quantities one may think that $M_{pl}$ and $L_{pl}$ in the l.h.s must be quantities representing the quantum features of the gravitational system. Since $c$ and $G$ describe a classical long range ($c$), gravitational ($G$) interaction, respectively, then $M_{pl}$ and $L_{pl}$ may be interpreted respectively as the particle’s mass and length scale of the quantum gravity interaction$^4$. Since we assume the quantum rules are just valid in this domain then according to Heisenberg uncertainty relation the huge mass of $M_{pl}$ implies that quantum gravitational interactions have ultra-short range $L_{pl}$, as desired in consistent with the assumption that they are confined to the planckian domain.

$^4$One may wish as well to interpret the Planck time $T_{pl}$ as the duration of quantum gravitational interaction. This leads to the result that this interaction is mediated by the light velocity; a result which is not consistent with the huge mass $M_{pl}$ of mediating particle. But, it is easily seen that $T_{pl}$ plays no fundamental role in this game. This is because, $T_{pl}$ is related to $L_{pl}$ merely by the classical gravity factor $c$ which plays no role in the quantum gravity sector. This is not the case for $M_{pl}$ and $L_{pl}$, because they are related not only by $c$ but also by $\hbar$ which connects the classical and quantum domains. $M_{pl}$ and $L_{pl}$ are independent factors of quantum gravity, just like momentum and position in quantum mechanics. $T_{pl}$ in this game is a redundant factor which plays no important role.
The Schwarzschild radius of the planck mass is obtained as

\[ R_s = 2L_{pl}, \] (7)

which means the range of quantum gravitational interaction \( L_{pl} \), is inside the Schwarzschild radius. In fact, the quantum particle with mass \( M_{pl} \) fits into the Schwarzschild radius because its Compton wavelength \( L_{pl} \) is smaller than \( R_s \). Therefore, the quantum gravitational interactions are always hidden from the domain of classical gravity. The appearance of Planck size Schwarzschild radius is in complete consistency with GCP, provided we assume the existence of microscopic wormholes with Planck size throat to mediate Planck mass particles as the quantum gravitational interaction. The wormholes with Planck size throat can hide the Planck range quantum gravitational interaction from outside the horizon. In fact, the throat of these wormholes may define the zero point length of \( g_{\mu\nu} \), namely the Planck length, below which \( g_{\mu\nu} \) is not defined. This throat is the boundary of the government of the classical gravity!

In conclusion, it seems a quantum gravitational interaction takes place as follows: whenever two particles are accelerated toward huge energies to collide each other at a ultra-short distance of quantum gravity regime, a virtual microscopic wormhole of Planck size is created and the particles interact with each other quantum gravitationally, through this wormhole, by exchanging virtual Planck mass particles. \(^5\)

The most important point is the emergence of a new definition for the mass. If gravity has two different features at over-planckian and planckian domain, then there are two different definition of mass in these domains: \( \text{classical gravitational mass CGM} \) and \( \text{quantum gravitational mass QGM} \).

\(^5\)The virtual wormholes or particles mediating quantum gravitational interactions are created according to Heisenberg uncertainty principle \( cM_{pl}L_{pl} \sim \hbar \), which governs at the throat domain.
tational mass QGM. Since classical gravity is a long range interaction, then CGM is defined in large scale. But, quantum gravity of the type discussed in this paper is a ultra-short range interaction. Then, QGM have to be defined at ultra-short distance of planck length. When two particles are at a distance larger than the Planck length \( r \gg L_{pl} \) in \( g_{\mu\nu} \) background, no microscopic wormhole and hence no quantum gravitational interaction takes place and the CGM of particles interact classically through the smooth geometry \( g_{\mu\nu} \). However, when they are very close at Planck length a virtual wormhole with the same size is created so that there is no notion of distance between the particles at the throat. This is because, the appearance of this wormhole makes the 3-geometry to be non-simply connected. In this case the QGM of particles interact quantum gravitationally through the throat and there is no classical gravitational interaction between CGM of particles because there is no meaningful notion of distance \( r \) at the throat. This clearly explains why GCP forbids the simultaneous classical and quantum gravity descriptions.

4 Hierarchy problem can solve the cosmological constant problem

Cosmological constant and hierarchy problems are the most outstanding problems in the high energy physics. There is a rich literature about different solutions for both problems. Recently proposed brane world gravity with extra dimensions is understood to successfully solve the hierarchy problem [6]. As yet, however, it could not solve the cosmological constant problem in a successful way. This rises a very important new problem: How a fundamental theory can
solve the hierarchy problem without solving the cosmological constant problem? Conventional wisdom states that if a theory (e.g. brane gravity) is capable of solving the hierarchy problem in a very subtle way, it has to be of the same capability to solve the cosmological constant problem. Therefore, it seems a fundamental quantum gravity theory is the one that in which both problems can be solved at once. We shall present a mechanism, based on the present idea of quantum gravity, by which both problems are properly addressed, at once. In explicit expression: we show that the solution for the cosmological constant problem is nothing but the hierarchy problem!

The so-called cosmological constant problem arises in the following way

$$\Lambda_{\text{eff}} = \Lambda_0 + \mathcal{O}(M_{\text{pl}}^4),$$

where $\Lambda_{\text{eff}}$ is a combination of bare cosmological constant $\Lambda_0$ and quantum field theory contributions of the order of $M_{\text{pl}}^4$. These contributions arise due to the vacuum fluctuations of quantum fields. They can be interpreted as classical effects because they are associated with the properties of the vacuum. Therefore, they couple to classical gravity $g_{\mu\nu}$, through Einstein equation (1). Due to huge values of these contributions $\sim M_{\text{pl}}^4$ we face with the so-called cosmological constant problem, namely a large difference between the observational bound and theoretical predictions on the value of cosmological constant. On the other hand, the hierarchy problem is to explain why Planck scale is enormously larger than electroweak scale.

A wise insight to both problems reveals that they have a same feature in common, namely the Planck mass. In other words, $M_{\text{pl}}$ joins the cosmological constant and hierarchy problems. Therefore, the key solution of both problems certainly lies in the physics of Planck scale. The Planck scale physics in this paper is nothing but the ultra-short range quantum gravity. This
means the present idea of quantum gravity and the corresponding wormhole structure is the key to resolve the enigmatic situation concerning the cosmological constant and hierarchy problems. To show this, we notice that the quantum gravity features $M_{pl}$ and $L_{pl}$ in the present idea provides us with an energy density of the order of $M_{pl}^{-3} L_{pl}^3 \sim M_{pl}^4$ in the throat domain. But, this is surprisingly the same vacuum energy density associated with the quantum field contributions to the effective cosmological constant in (8). Therefore, the microscopic wormholes can act as drainpipes to evacuate the large contributions $M_{pl}^4$ from classical gravity domain to elsewhere\(^6\). What is then left is the bare cosmological constant which can in principle be adjusted to satisfy the present observations. In this way, the main purpose of the existence of ultra-short range quantum gravity is to set the effective cosmological constant to zero by its wormhole structure. In quantum field theory language, these wormholes play the role of counter terms to exactly cancel out the contributions $M_{pl}^4$ from the $g_{\mu\nu}$ universe.

However, it is very important to note that the essential condition for the occurrence of these microscopic wormholes is the existence of Planck scale energy $M_{pl}$, not electroweak energy scale $M_{EW}$. This means that the hierarchy between $M_{EW}$ and $M_{pl}$ is a natural (perhaps anthropic) set-up to solve the cosmological constant problem. The electroweak energy scale is responsible for symmetry breaking via Higgs field, whereas the Planck scale energy is responsible for vanishing the effective cosmological constant via microscopic wormholes.

\(^6\)A similar idea has already been proposed to solve the cosmological constant problem [7]. In this idea our hot universe is assumed to be in contact with other large and cool universes by Planck size wormholes to vanish the effective cosmological constant.
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