Trade and Development between Similar Economies: Theoretical Modeling and Simulation

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Abstract
This paper attempts to model the link between trade and development between countries with similar factor endowments and proposes a new kind of stable equilibrium in the intermediary sector of the economies. The data were elicited through simulation using Microsoft Excel 2010. The results arrived at were archived empirically using theoretical modeling and simulation. The results derived show that when a country has monopoly power in the production of intermediate goods, there is a possibility of gain in trade and exchange, and therefore, it is a win-win situation for all countries with similar factor endowments, other things being equal. The results also showed that the profits that emerged in the intermediate goods sector were positive. The results indicated that profits are generated from both domestic and foreign demand, especially from the country’s monopoly power in the production of intermediate goods. Lastly, the study also demonstrated that capital formation and the injection of liquidity into the regulated economy are efficient, and by assuming that households’ consumption at each point in time is efficient, the control of money supply and prices were found to be a necessary condition for countries with similar characteristics to converge towards stationary equilibrium. Based on the findings, it is recommended that intensification of trade in intermediate goods is a necessary policy for development policies for CEMAC and ECOWAS, where monopoly power and the control of money supply and prices are essential for equilibrated, stable growth.

Keywords
Development, Trade, Stationary Equilibrium, Modeling and Simulation
1. Introduction

The traditional theory of international trade, inspired by the theories of absolute and relative advantages of Adam Smith and David Ricardo, favoured the specialisation of countries according to their natural endowments. The differences in factor endowments between countries were to stimulate and increase trade between them. Thus, one of the explanations generally put forward for the low levels of trade between African countries is the similarity of their factor endowments and their production of agricultural products, mining, or energy. Raw materials lead to beneficiary trade between countries. However, since the end of the Second World War, the aspirations of all nations for industrialisation and development have given a new direction to international trade, tending to overturn traditional theses (Hellier, 1993). Rather, there is an increase in trade between countries with factor endowments, technologies, and comparable or even similar products. According to UNIDO (2013), the largest share of world exports of manufactured goods is traded between industrialized countries (48.7% in 2011). Calculations made using data from the World Bank (2012) show that, from 1960 to 2010, US exports and imports to and from high-income countries represented 71% and 69% of their foreign trade, respectively. Despite the tremendous growth in trade, the population of the poor, living on less than one dollar per day increased from 474.4 million in 1987 to 552 million in 2000 (Thalut & Kelese, 2019). As a result, most countries tended not only to rely on the exploitation of natural resources to make ends meet but also to make up for what they could not produce domestically through international trade.

Moreover, we can observe intensification and, consequently, an increase in trade between countries with similar economies. Thus, from 1960 to 1983, exports from the European Union (EU) to industrialised countries, which averaged 78% of its total exports, rose to 85% over the period from 1984 to 2010. Its imports went from 79% to 82%. The development of trade within sub-Saharan Africa follows the same increasing trends. Exports, which averaged 6% of the region’s total over the period 1960 to 1992, doubled from 1993 to 2010, while imports increased from 8% to 13%. Thus, in attenuation of the classic theses of specialization translated into the Heckscher-Ohlin-Samuelson (HOS) model, the similarity of comparative advantages tends to reinforce and increase economic exchanges and the well-being of populations (Krugman, 1979). This rule is valid in the presence of similarities in the internal environments of countries, their technological level, and the structure of their demand, according to Krugman (1979) and Lancaster (1980).

How to describe the exchanges between two economies that use similar technologies to produce intermediate and final consumer goods? This is the type of trade that one might wish to see established between developing countries, in particular those in Africa, which, for the most part, are endowed with significant natural resources: raw materials, mining resources, and energy. These countries, with similar economic potential and grouped together in sub-regional commu-
nities (ECOWAS, CEMAC), have the possibility of rapidly developing their economies, their industries, and the standard of living of their populations by trading with similar countries that have complementary resources. The contribution of the paper is to show how trade is linked to the development of countries with similar factor endowments and how control of money supply can lead to the prosperity of each economy. The study is divided into two sections. Section 1 describes the production of final and intermediate consumption goods as well as the money supply. Section 2 analyses how household consumption demands and can result in stationary equilibrium.

2. Description of Production

In this section, the study tries not to index variables by time. Most of the variables are considered to be static. Only the entries that describe the formation of capital and real cash are dynamic. Assume that each economy has two production sectors: an intermediate good and a final good production sector. The unique consumer good (final good) is produced in pure and perfect competition, using mainly intermediate consumption goods as inputs. The final consumer good is a perishable and non-storable good. In each economy, the production sector of the intermediate good is a national monopoly. The intermediate good is produced using capital and labor. The only tradable goods between economies are intermediate goods. We are not interested in demographics. We assume that the size of the population in each country is constant and normalized to 1. This assumption is however restrictive, especially in underdeveloped countries where the population growth rate is generally positive. It is only used to simplify the writings of the model and to tackle the aspects that seem essential to us.

In the description given, the sectors of the economy are well connected; the demand for final consumer goods is closely linked to the demand for intermediate goods, and vice versa. The national economy is well connected to the external economy. In the equilibrium of international trade, absolute parity of purchasing power is guaranteed. A devaluation of the national currency has a double effect. On one hand, it increases local and foreign demand for the domestic intermediate consumer’s goods. On the other hand, it increases the cost of living in the national economy and reduces it in foreign country. International trade is a game where each economy wins, by making a profit on its monopoly production of intermediate goods. Currency is issued by the government to finance the economy and achieve full employment of productive capacities. Issuing money is similar to issuing a right to create goods and services. Seigniorage anticipates the creation of profit in the intermediate goods sector.

1This situation is different from that encountered in underdeveloped countries, which very often have highly developed countries as their main economic partner, which have great monopoly power. These Third World countries for the most part produce goods that they do not consume, and mainly consume goods that they do not produce. The productive sectors of their economy are totally disconnected. In these economies, capital is generally owned by companies in developed countries, and is used only for the exploitation of natural resources. Profits made with the help of foreign capital are repatriated, and poor countries become poorer.
2.1. Production of the Final Consumer Good

Consider two economies of similar size, which use identical technologies to produce intermediate and final consumer goods. Each economy has the monopoly on the intermediate good that it produces and which is specific to it\(^2\). The two countries exchange only the intermediate consumer good. There are no customs duties between the countries and the transport costs are assumed to be negligible. Mobility of labour is assumed to be immobile. The index \(N\) stands for domestic economy while index \(F\) stands for the foreign economy.

Each country is an open economy with two sectors of production; production of the intermediate good and final good. The final good \((Y)\) is a pure and perfect competition product, which is produced using as inputs, intermediate consumption goods \(x_N\) and \(x_F\) following technology\(^3\) below:

\[
Y = \left( x_N^\theta + x_F^\theta \right)^{\frac{1}{\theta}} \tag{1}
\]

\(x_N\) is the quantity of the local intermediate consumption good used in the production of the final consumer good; the final consumer good \(Y\); the final consumer good is a perishable good, which cannot be preserved from one period to another; \(x_F\) is the quantity of imported intermediate consumption good (produced by the foreign country), used to produce the final consumption good; \(\theta, 0 < \theta < 1\), reflects monopoly power\(^4\) that each country owns on the intermediate consumer good that it produces. When \(\theta = 1\), intermediate consumer goods are perfect substitutes. When \(\theta\) is close to 0, the monopoly power \(1 - \theta\) is high; when \(\theta\) is close to 1, the monopoly power is weak. When: \(0 < \theta < 1\), intermediate goods are no longer perfect substitutes; they complementary to each other in the sense that it is always more profitable to combine the two intermediate goods\(^5\) \(x_N\) and \(x_F\) in the process of producing the final good (rather than using a single intermediate good to produce).

The curve below is obtained when \(\theta = 0.5\). Figure 1 shows the variations in the production of the final good, when there are 10 units of intermediate goods, which are divided between \(x_N\) and \(x_F = 10 - x_N\).

Figure 1 shows that maximum output is obtained when \(X_N\) is equal to \(X_F\). The graph also revealed that labour used as an input face a diminishing return.

\(^2\)Everything happens, as if each country has its own raw materials which allowed it to manufacture a single intermediate consumer good.

\(^3\)This technology is borrowed from the monopolistic competition models of (Dixit & Stiglitz, 1977).

\(^4\)Intermediate consumer goods—local and foreign are substitutable, but they complement each other in the production process of the final good.

\(^5\)This description relatively plausible for countries of the same size, monopoly power may seem identical. But, in general a small developing country does not have the same monopoly power as a large developed country. To simplify the entries, notations, and to avoid estimation problems, we assume that is the same for both economies.

\(^6\)In the context of trade between a developed country and a developing country, we could look at a basket of raw materials: coffee, cocoa, cotton, wood, uranium, oil, gas, cobalt, coltan... and like a basket imported intermediate consumer goods from the new technologies sector: these are computers, cell phones, trucks, tractors, processing plants, new technologies, etc.
When \( 0 < \theta < 1 \), Figure 1, is concave, symmetrical, it reaches its maximum when \( x_F = x_N \). This production function is quite revealing of the reality that would occur within the framework of North-South, or South-South exchanges. In the context of North-South trade, it is more efficient in the production process to combine raw materials and high-tech products in order to obtain the best production of the final good. In the context of South-South trade \( x_F \) and \( x_N \), baskets of raw materials would be added to each other to produce final consumer goods. But, in reality the optimal production is not necessarily obtained when \( x_F = x_N \).

Figure 2(a) and Figure 2(b), shows the iso-production curves of the final consumption good of level 10, obtained respectively for \( \theta = 0.5 \) and for \( \theta = 0.3 \). In effect, to produce 10 units of the final consumption good \( Y \), we can use 10 units of intermediate good \( X_N \) and 10 units of intermediate good \( X_F \). When \( \theta = 0.5 \), it suffices to use 2.5 units of locally produced intermediate goods and 2.5 units of imported intermediate goods to produce 10 units of final

A more realistic production function would be of the form: 

\[
Y = a(x_F)^\theta + b(x_N)^\theta
\]

where \(a\) and \(b\) are positive real numbers, which take into account the weighting of each intermediate good in the production function of the final good. The disadvantage of this formulation, is the addition of additional parameters, which weigh down the writings. For the sake of simplicity, we retain the production function formulated in expression (1).
consumer goods. When \( \theta = 0.3 \), it suffices to use 0.99 quantity of \( X_N \) and 0.99 quantity of \( X_F \) to produce 10 units of the final consumption goods. The variations of \( X_N \) and \( X_F \) on the iso-production curve of good \( Y \) depend on \( \theta \). In general, to produce 10 units of final consumer goods \( Y \), it suffices to combine quantities of \( X_N \) and \( X_F \) in the proportions \( X_F = X_N = X_F = 10 \times \left( \frac{1}{\theta} \right) \).

Let \( P \) be the price of the final good; \( P_N \) and \( P_F \) are the respective prices of intermediate goods \( X_N \) and \( X_F \); \( e \) is the exchange rate quoted to the uncertain \( e^7 \) for the domestic economy; is the quantity of national currency that should be given in exchange for one unit of foreign currency; is the quantity of local currency that should be given in exchange for a unit of intermediate good; is the amount of local currency that should be given in exchange for one unit of foreign intermediate good. The profit in the sector of production of the final good is written:

\[
P \cdot Y - P_N \cdot X_N - e \cdot P_F \cdot X_F
\]

\(^7\)We will introduce the currency later.
The demand for intermediate goods $X_N$ and $X_F$ results from the maximization of profit in the final good sector. Under the assumption of pure and perfect competition (free entry and zero profit) in the final good sector, we deduce:

\[
x_N = \left( e \cdot \frac{P_F}{P_N} \right)^{\frac{1}{\theta}} \cdot Y
\]

\[
x_F = \left( \frac{P_N}{e \cdot P_F} \right)^{\frac{1}{\theta}} \cdot Y
\]

\[
P = \left[ \left( \frac{P_N}{e \cdot P_F} \right)^{\frac{\theta}{\theta-1}} + \left( e \cdot P_F \right)^{\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}}
\]

The first part of Equations (2a) and (2b) shows that intermediate goods $X_N$ and $X_F$ can be substitutable, complement each other in the production process of the final good. The increase in demand for one of these two goods leads to an increase in demand for the other. Equation (2a) also shows that when the exchange rate $e$ increases, the demand for the locally produced intermediate good $x_N$ increases. Similarly, when $P_F$ increases, $x_N$ also increases. Thus, a devaluation of the local currency (increase in the exchange rate) produces the same effects, on the demand for the local intermediate good $x_N$, as an increase in the foreign price $P_F$. It is important to note, that the demand for the local intermediate good $x_N$ also increases as its price decreases $P_N$ or if the demand for the final good increases $Y$. These results are close to what is observed in reality or international trade. Economies of similar size, which have the same monopoly power over the intermediate goods that they produce, devaluation of the local currency lead to an increase in demand of the locally produced intermediate good and a reduction in local demand for the imported intermediate good.

In Equation (2c), the price of the final consumers’ goods is a kind of harmonic average of the prices of foreign and local intermediate goods. Devaluation would lead to an increase in the price of the final consumer goods in the domestic economy; everything being equal, this would result in an increase in the cost of living. Similarly, any increase in the price of intermediate consumer good $P_N$ and $P_F$ would also lead to an increase in the price of the final consumer good $P$.

In addition to Equations (2a) and (2b), at equilibrium demand of final good is deduced as

\[
Y = \left[ \left( \frac{P}{P_N} \right)^{\frac{1}{\theta}} + \left( \frac{P}{e \cdot P_F} \right)^{\frac{1}{\theta}} \right]^{-1} \cdot (x_N + x_F)
\]
intermediate goods and prices. Assuming that the sectors of the domestic economy are connected\(^9\) with that of the foreign economy, from Equation (2d) it is shown that (other things being equal), a devaluation of the domestic currency would lead to an increase in the production of the final good. In an economy where welfare of households depends on final consumption, devaluation of the local currency would have a negative effect on the well-being of households in the local economy, though it is assumed to peak in the intermediate run following the J curve. Symmetrically, the same token of analysing can quickly be done to verify what happens in the foreign market for the final good as observed subsequently. The profit on the final good\(^10\) \(Y^*\) is given as follows.

\[
P^* \cdot Y^* = \frac{1}{e} \cdot P_N \cdot x_N^* - P_F \cdot x_F^*
\]

where, \(Y^* = \left[ \left( x_N^* \right)^\theta + \left( x_F^* \right)^\theta \right]^{\frac{1}{\theta}}\) is the quantity of final consumer good produced in the foreign country. Note that the exchange rate \(\frac{1}{e} \cdot P_N\) is in terms of foreign currency, that is, it is per the price of a unit of local intermediate good. By using again the hypothesis of pure and perfect competition (free entry and zero profit) in the final good sector in the foreign country, the following can be deduced:

\[
x_N^* = \left( \frac{e \cdot P_F}{P_N} \right)^{\frac{1}{\theta}} \cdot x_F^* = \left( \frac{e \cdot P^*}{P_N} \right)^{\frac{1}{\theta}} \cdot Y^*
\]  \hspace{1cm} (2a')

\[
x_F^* = \left( \frac{P_N}{e \cdot P_F} \right)^{\frac{1}{\theta}} \cdot x_N^* = \left( \frac{P^*}{P_F} \right)^{\frac{1}{\theta}} \cdot Y^*
\]  \hspace{1cm} (2b')

\[
P^* = \left[ \left( \frac{1}{e} \cdot P_N \right)^{\frac{\theta}{\theta-1}} + \left( P_F \right)^{\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}}
\]  \hspace{1cm} (2c')

\[
Y^* = \left[ \left( \frac{P^*}{P_F} \right)^{\frac{1}{\theta}} + \left( \frac{e \cdot P^*}{P_N} \right)^{\frac{1}{\theta}} \right]^{\frac{1}{\theta}} \cdot (x_N^* + x_F^*)
\]  \hspace{1cm} (2d')

From Equation (2a') it is clearly shown that devaluation of the local currency would lead to an increase in foreign demand for a local intermediate good, everything being equal. On the other hand, devaluation, reduces the price of the final consumer goods in the foreign country, and therefore improved on quality of life through reduction in the cost of living in the foreign country (Equation

\(^9\)In developed countries, sectors of the economy are fully connected to each other. However, in developing countries, production sectors are generally disconnected. Some Third World economies produce predominantly what they do not consume, and consume what they do not. There is no link between sectors of the economy. Devaluation in this case can have perverse effects on the economies of underdeveloped countries.

\(^10\)To designate the foreign country, quantities and prices are marked with a star. For example, is the foreign demand for a local intermediate good \(x_F^*\).
By combining Equations (2c) and (2c'), an expression for the relative price between domestic and foreign goods can be established:

\[ P = e \cdot P' \]  

Equation (3) reflects the absolute purchasing power parity between the domestic and foreign countries. At international trade equilibrium, by relaxing or holding customs duties and transport costs constant, the same quantity of currency is used to buy the same quantity of goods and services in both countries. Devaluation of the local currency has two effects: 1) it increases foreign demand for the local intermediate consumption good on one hand; 2) it also increases the cost of living in the local economy and reduces it in the foreign country in the long run. Based on the aforementioned arguments, the proposition of the effect of devaluation can be verified simply as follow. When, for example, the exchange rate changes from \( e = 1 \), to \( e = 2 \); if the local economy is willing to offer, for example, 1 bushel of coffee in exchange for a laptop, he would now have to offer 2 bushels of coffee to get the same laptop. It is observed that with devaluation, the foreign country obtains the local intermediate goods at a very low price, which directly contributed to the low cost of production and thus low price. The low price overseas indeed contributes to reduction in the cost of living in the foreign country. Meanwhile, in the local economy, the exchange rate doubles, resulting in local prices doubling, and hence the cost of living becomes unbearable. Although the devaluation has the effect of stimulating both local and foreign demand for domestic intermediate goods, it equally leads to an increase in the cost of living in the local economy.

### 2.2. The Production of the Intermediate Consumer Goods

Let us now turn to the production of intermediate goods. Intermediate consumer goods are produced by combining capital and labor, following technology:

\[ x_N + x_N^* = (K)^\alpha \cdot (N)^{1-\alpha} \]  
\[ x_F + x_F^* = (K^*)^\alpha \cdot (N^*)^{1-\alpha} \]  

where, \( x_N \) is the local demand for the local intermediate good; \( x_N^* \) is the foreign demand for the local intermediate good; \( (K)^\alpha \cdot (N)^{1-\alpha} \) is the supply of the local intermediate good; \( K \) is the amount of capital used in the production of the local intermediate good; \( N \) is the amount of labor used in the production of the local intermediate good. \( x_N + x_N^* \) is the global demand (local and foreign) for a local and foreign-produced intermediate good. Equation (4b) describes the production of the foreign intermediate good. Assumed that local and foreign technologies are the same, and the economies have similar characteristics.

The production of intermediate goods is a source of national monopolies. In the local intermediate good sector, national firm seeks to maximise profit by choosing the price \( P_N \), the quantity of labor \( N \) and capital \( K \) which permits to
maximize its profit:
\[
\pi = \frac{P_N}{P} (x_N + x_N^*) - w \cdot N - r \cdot K
\]
where \( w \) is the real wage rate, it is the quantity of final good that should be offered in exchange for a unit of labor; \( r \) is the real rental price of capital, it is the number of units of final good that should be offered in exchange for one unit of capital. Recalled \( \gamma \) is the Lagrange multiplier and form the Lagrangian\(^{11}\) for the maximisation problem of the local firm which produces the intermediate consumer goods:
\[
L = \frac{P_N}{P} \left( x_N + x_N^* \right) - w \cdot N - r \cdot K + \gamma \left[ (K)^{\alpha} \cdot (N)^{1-\alpha} - x_N - x_N^* \right]
\]
The first-order conditions with respect to \( P_N, K \) and \( \gamma \), we have
\[
\frac{\gamma}{\theta} = \frac{P_N}{P} \tag{5a}
\]
\[
w = \gamma \cdot (1-\alpha) \cdot \frac{(K)^{\alpha} \cdot (N)^{1-\alpha}}{N} \tag{5b}
\]
\[
r = \gamma \cdot \alpha \cdot \frac{(K)^{\alpha} \cdot (N)^{1-\alpha}}{K} \tag{5c}
\]
\[
x_N + x_N^* = (K)^{\alpha} \cdot (N)^{1-\alpha} \tag{5d}
\]
Thus, the profit in the local intermediate good sector is given as:
\[
\pi = \frac{P_N}{P} \left( x_N + x_N^* \right) \cdot (1-\theta) \tag{6a}
\]
Profit is derived from the global demand (local and foreign) for an intermediate good and monopoly power \( 1-\theta \) of the country over its production of an intermediate good. Profit is considered as a real good of the same nature as the intermediate consumer good. Following the same token of analysis, the lines that profit participates in the formation of capital. Real household income is:
\[
w \cdot N + r \cdot K = \frac{P_N}{P} \left( x_N + x_N^* \right) \cdot \theta \tag{6b}
\]
Household income depends on local and foreign demand and monopoly power. When monopoly power is larger, household income is low and profit is high.

So far, the analysis has being focus on comparative static, that is, we have not to introduce time lag into our equations. For dynamic exposition, it is necessary to introduce time lag/lead, to describe the evolution of capital, from one period to another. Time lead was also used to describe the evolution of the money supply. Assume that the profits are fully reinvested in capital, so that the capital evolves according to the process:
\[
^{11} x_N = \left( \frac{P}{P_N} \right)^{\frac{1}{\alpha}} \cdot Y \quad \text{and} \quad x_N^* = \left( \frac{e \cdot P^*}{P_N} \right)^{\frac{1}{\alpha}} \cdot Y^* .
\]
The index indicates the time; \( K_{t+1} \) is the desired level of capital for the future period \( t+1 \); \( \delta \), \( 0 < \delta < 1 \) is the capital depreciation rate; \( K_t \) is the capital level at the time \( t \); \( \pi_t \) is the profit made in the period \( t \); \( K_0 > 0 \) is given as is the level of capital that is existing in the economy at the initial date \( t = 0 \). Equation (6c) describes the rule of capital formation.

\[
K_{t+1} = (1-\delta) \cdot K_t + \pi_t \tag{6c}
\]

Equation (6d) describes the capital formation equation described by Equations (6c) and (6d) is unusual in most studies and therefore, one of the major contributions of this article.

In general, most studies on capital formation assumed that capital accumulates according to the equation: \( K_{t+1} = (1-\delta) \cdot K_t + I_t \), where \( I_t \) is the level of investment that one chooses at the time \( t \). This paper assumed all the profits are ploughed back or reinvested, that is, \( I_t = \pi_t \). In addition, everything happens as if the sector production of the intermediate good is managed by a national firm, which reinvests all profits in capital. At stationary state, this simple rule of capital formation can lead to optimum profits for households, companies of the economies of the countries. Moreover, solving the problem of the foreign national firm, which has the monopoly in the production of the foreign intermediate good, gives:

\[
\pi_t^* = \frac{P_{F_t}}{P_t} \cdot (x_{F_t} + x_{F_t}^*) \cdot (1-\theta) > 0
\]

It can be inferred based on the preposition on profit that trade is not a zero-sum profit for both economies. In other words, international trade is beneficial for both countries engaged in trade. When each economy has a monopoly on the production of intermediate good, international trade is a game where each of the economies wins, by constantly releasing a positive profit that participates in the formation of its capital. As in the home economy, we assume that in the foreign economy, capital accumulates according to the equation:

\[
K_{t+1}^* = (1-\delta) \cdot K_t^* + \pi_t^*
\]

By combining Equations (4) which translate the equality between supply and demand for an intermediate good and Equations (2) which describe the demand for an intermediate good, we obtain:

\[
(K_t)^\alpha \cdot (N_t)^{1-\alpha} = \left( \frac{P_t}{P_{N,t}} \right)^{\frac{1}{1-\alpha}} \cdot (Y_t + Y_t^*) \tag{7a}
\]

\[
(K_t^*)^\alpha \cdot (N_t^*)^{1-\alpha} = \left( \frac{P_t}{e \cdot P_{F,t}} \right)^{\frac{1}{1-\alpha}} \cdot (Y_t + Y_t^*) \tag{7b}
\]
In Equations (7a), (7b) and (7c), there is no relation between the present and the past, or between the present and the future. These equations could well be written without using time. Equations (7a) and (7b) show that the world demand \( Y_t + Y_t^* \) for the final good depends on the production of the intermediate good in each country. Each of the economies is linked to the global economy. When profits are fully reinvested in both economies (and initial capital stocks low in both countries), it can result in a future increase in the production of the intermediate good in each country and an increase in global demand for the final consumer good. The ratio of Equations (7a) and (7b) gives:

\[
\left( \frac{K_t}{K_t^*} \right)^{\alpha} \cdot \left( \frac{N_t}{N_t^*} \right)^{1-\alpha} = \left( \frac{e \cdot P_{E,t}}{P_{N,t}} \right)^{1-\theta}
\]  

(7c)

Equation (7c) describes the equilibrium of production of intermediate goods in the two economies. Equation (7c) is similar to Equation (2b). It shows that at international trade equilibrium, the production ratio in the local and foreign economies must at each date respect the price ratio. Devaluation should theoretically increase the production of the local intermediate good (all other things being equal). Conversely, if the production of the local intermediate good increases very sharply in relation to the production of the foreign intermediate good, it would be necessary to devalue the local currency (increase the exchange rate) so that trade between the two countries remains compatible with the external equilibrium of the trade.

By combining Equations (2a) and (2a’) and noting that in equilibrium, the supply of the final good is equal to the demand for the final good we deduce:

\[
\frac{Y_t}{Y_t^*} = \left( \frac{e \cdot P_{E,t}}{P_{N,t}} \right)^{\theta}
\]  

(8)

Equation (8) like Equation (7c) is static. Equation (8) shows that at equilibrium on the market of the final good and at the equilibrium of foreign trade, the ratio between local and foreign production of the final good must respect the price ratio of intermediate goods.

### 2.3. Money Supply

Equation (9a) describes the evolution of the money supply. Indeed, if Equations (7) and (8) are static, then the evolution of the money supply is dynamic. Let’s assume that the money supply is exogenous and it is managed by the government of each economy. In the home economy, the money supply evolves according to the equation:

\[
M_t = (1 - \varepsilon) \cdot M_{t-1} + P_t \cdot \pi_t
\]  

(9a)

where \( M_t \) is the face value of the currency at the start of the period \( t \); \( P_t \) is the price of the final consumer goods; \( \pi_t \) is the profit created in the home economy during the period \( t \); it is also in real terms, the surplus of the trade balance;
$M_{t-1}$ is the nominal value of the money supply in the previous period; $M_0 \geq 0$, given by the economy; $\varepsilon$, $0 < \varepsilon < 1$ is the rate of loss of money, it is the proportion of money existing at the start of the period $t-1$, which leaves the economic circuit before the start of the period $t$. $\varepsilon$ is a positive constant close to 0. Since there is no uncertainty in the model, everything goes as if at the start of each period $t$, the government anticipates the profit that the economy will generate, and creates a quantity of equivalent currency, so that during the period the supply of money is neither insufficient, no surplus and in a way that adequately finances its economy. The level of actual cash is defined by: $m_t = \frac{M_t}{P_t}$ is such that:

$$m_t = (1 - \varepsilon) \frac{m_{t-1}}{\mu_t} + \pi_t,$$

where $\mu_t = \frac{P_t}{P_{t-1}}$ is the price ratio (which also reflects the level of inflation) between the period: $t-1$ and the period $t$. The government’s constraint is written.

$$\tau_t = m_t - (1 - \varepsilon) \frac{m_{t-1}}{\mu_t}$$

$\tau_t$ is seigniorage; at the start of each period, the government transfers $\tau_t$ without compensation to households. $\tau_t$ is the means by which the government reorganises the rights of economic agents. Issuing money is like issuing a right to create goods and services; the financing anticipates the creation of a profit of a real value equal to $\pi_t$.

3. Household Consumption and Stationary Equilibrium

Section 2 deals with analysis household problems assuming a static model. It also supports the proposition that households are all the same. Each household supplies labour which the national firm rents to add to its raw materials in order to produce the intermediate consumer goods. The prepositions highlighted show that controlling the money supply and prices are a necessary stationary equilibrium condition for convergence growth. When the economy begins with a low capital stock, capital, household consumption, and production increase year by year and converge to a stationary equilibrium. The size of the population is constant and normalised to 1. Households do not value leisure, the demand for work $L_t$ on each period in time is such that: $L_t = N_t = 1$. Households receive a salary $w_t$ in return for their work. The capital belongs to the households. Households in every economy rent capital for their business. There is no labour mobility. The capital owned by households is remunerated at the rate $r_t$. All markets are open on each period $t$. Households use their income from work and interest to purchase the final consumption goods $Y_t$. In the local economy, the household budget constraint at each period is written:
\[ C_t + K_{t+1} + m_t = w_t \cdot N_t + r_t \cdot K_t + (1-\delta)K_t + \pi_t + \tau_t + (1-\epsilon)\frac{m_{t-1}}{\mu_t} \quad (10a) \]

where: \( K_{t+1} = (1-\delta) \cdot K_t + \pi_t \) is the capital accumulation equation; \( m_t = \tau_t + (1-\epsilon)\frac{m_{t-1}}{\mu_t} \) describes the actual cash formation on the time \( t \);

\[ C_t = w_t \cdot N_t + r_t \cdot K_t = \delta Y_t = \delta \frac{P_{N,t}}{P_t} \cdot (K_t)^\alpha \cdot (N_t)^{1-\alpha} \quad (10b) \]

is household consumption at the time \( t \). Expression (10b) describes the actual resource constraint. The household budget constraint \( C_t \) denotes current household consumption. This model implies that households consume all production of the final consumer good, \( C_t = Y_t \); \( K_{t+1} \) is the level of capital that households anticipate for the future period \( t+1 \); \( m_t \) is the level of real cash that households want to spend during the current period; \( (1-\epsilon)\frac{m_{t-1}}{\mu_t} \) is the actual cash level existing at the end of the previous period; \( \tau_t \) funds the difference between the actual cash in existence at the end of the period \( t-1 \) and the cash desired at the period \( t \).

In the budget constraint equation, money serves to reduce transaction costs and makes it possible to avoid introducing the notion of debt (to finance the difference between the existing cash and the desired cash). But, in reality, most central banks are owned by international financiers who lend the money they freely create to governments, so that the money created becomes a debt for the economy that uses it, households must repay it with interest. We illustrate our remarks in a theoretical framework, where money is created by the government of each country to finance its economy.

In general, the problem of households is to maximize their inter-temporal utility function, under the constraint of their budget. In our model, the household objective is to attain its optimum, for any instantaneous utility function, increasing for consumption. Indeed, on each period \( t \), the instantaneous utility of households is maximum, because households consume all the available production of the final consumer good \( \theta Y_t \). In each period, the production of the final consumer good is optimal since it stems from the process of optimizing corporate profit. It follows, therefore, that the household objective is to maximize utility, for any function of increasing utility for consumption.

The level of capital for all future periods is fully determined, as soon as the initial conditions of the economy are known. Indeed, the capital accumulation rule is described in Equation (6b) \( K_{t+1} = (1-\delta) \cdot K_t + \pi_t \), where \( \pi_t = (1-\theta) \cdot \frac{P_{N,t}}{P_t} \cdot (K_t)^\alpha \cdot (N_t)^{1-\alpha} \). Recalled that at equilibrium on the labor market, \( N_t = 1 \) and therefore \( K_{t+1} = (1-\delta) \cdot K_t + (1-\theta) \cdot \frac{P_{N,t}}{P_t} (K_t)^\alpha \). Since \( K_0, P_0 \) and \( P_{N,0} \) are assumed to be known at the time \( t=0 \), we can calculate \( K_t \). Using Equation (9a), we can choose \( M_t \) and \( P_t \). A simple solution, consists in
choosing at each period, $t \geq 1$ \( P_t = P_0 \) and \( \frac{P_{t-1}}{P_t} = \bar{a} \) where \( \bar{a} > 0 \), and thus Equation (2c) would permit us to calculate the prices \( P_{N,t} \) and \( e_t P_{P,t} \). The knowledge of the report \( \frac{P_{N,t}}{P_t} = \bar{a} \), allows determining all the continuation of future capital. However, this solution is not unique, but the choice of the rule: \( \frac{P_{N,t}}{P_0} = \bar{a} \), is always feasible. We could even fix; \( P_0 = P_t = 1 \), \( \forall t \geq 0 \) in this case the final consumer good is chosen as cas.

**Proposal 1** (written for the local economy):

At equilibrium in the markets, if the ratio, \( \frac{P_{N,t}}{P_t} = \bar{a} \), \( \bar{a} > 0 \)

a) then there is a steady-state for capital determined by:

\[
K_t = \bar{K} = \left[ \frac{\bar{a}(1-\theta)}{\delta} \right]^{\frac{1}{1-\alpha}}
\]

b) Capital \( K_t \) converges to its stationary state \( \bar{K} \), whatever the initial value of. \( K_0 > 0 \).

If, \( K_0 \leq \bar{K} \) then \( (K_t)_{t>0} \) is an increasing sequence that converges to. \( \bar{K} \). If, \( K_0 \geq \bar{K} \), then \( (K_t)_{t>0} \) is a decreasing sequence that converges to \( \bar{K} \). Thus, capital converges in each of the economies, as soon as the ratio between the price of the intermediate good and the price of the final good is constant.

**Remark**: If the economy starts with a low initial capital stock \( (K_0 < \bar{K}) \), household consumption \( C_t = \theta \cdot (K_t)^{\alpha} \) and the production of the final good \( Y_t = \theta \cdot (K_t)^{\alpha} \) will increase and will converge towards \( \bar{C} = \bar{Y} = \theta \cdot \bar{K}^{\alpha} \). If, on the other hand, the economy starts with a high capital stock \( (K_0 > \bar{K}) \), then the final consumption of households and the production of the final good will decrease and will converge towards \( \theta \cdot \bar{K}^{\alpha} \).

**Evidence**: To prove part a) of Proposition 1, it suffices to replace \( K_{t+1} \) and \( K_t \) by \( \bar{K} \) and by in the equation:

\[
K_{t+1} = (1-\delta) \cdot K_t + \bar{a} \cdot (1-\theta) \cdot (K_t)^{\alpha} \cdot (N_t)^{-\alpha}
\]

and knowing that at equilibrium in the labor market, \( N_t = 1 \), it becomes

\[
\bar{K} = \left[ \bar{a}(1-\theta) \right]^{\frac{1}{1-\alpha}}. \text{ Let us prove that the sequel} \ (K_t)_{t>0} \text{ is monotonous. Suppose that} \ K_0 \leq \bar{K} \text{ and we first show that the sequence is bounded by} \ \bar{K}. \text{ Let’s do it by induction on the time} \ t, \ t \geq 0, \ K_t \leq \bar{K}. \text{ Since:} \ 0 < \delta, \theta, \alpha < 1 \text{ and} \ \bar{a} > 0, \text{ then,}
\]

\[
(1-\delta) \cdot K_t + \bar{a} \cdot (1-\theta) \cdot (K_t)^{\alpha} \leq (1-\delta) \cdot \bar{K} + \bar{a} \cdot (1-\theta) \cdot (\bar{K})^{\alpha} = \bar{K}
\]

So \( K_{t+1} \leq \bar{K} \). This proves by induction on that the sequence \( (K_t)_{t>0} \) is bounded by \( \bar{K} \). It is therefore necessary to study the monotony of what follows.
Let’s study the variations of:

\[ K_{t+1} - K_t = -\delta \cdot K_t + \bar{\pi} \cdot (1-\theta) \cdot (K_t)^\alpha \]

as a function of \( K_t \). Table 1 below summarizes the variations of the function \( K_{t+1} - K_t \) in terms of \( K_t \), when \( K_t \) vary from 0 to \(+\infty\). Table 1 shows that \( \forall K_t \leq \bar{K}, \ K_{t+1} - K_t \geq 0 \).

Thus, if \( K_0 \leq \bar{K}, \ (K_t)_{t \geq 0} \) is an increasing sequence and bounded by \( \bar{K} \). We have shown symmetrically that if \( K_0 \geq \bar{K}, \ (K_t)_{t \geq 0} \) is a decreasing sequence undermined by \( \bar{K} \), which completes the proof.

Equation (9a) gives a degree of freedom on the choice of the price \( P_t \) and the money supply \( M_t \). It is possible to fix \( P_t = 1 \) at each period or even to choose the constant report \( \frac{P_{t+1}}{P_t} = \bar{\mu} \). Propositions 1 and 2 can be written symmetrically for the foreign economy. It revealed that controlling the money supply and prices allows economies to converge towards a stationary equilibrium. Starting from a low capital stock, household consumption, the level of real cash and the production of the intermediate good and the final good, converge towards a stationary equilibrium.

**Proposal 2** (Case of Domestic Economy):

a) If the ratio \( \frac{P_{t+1}}{P_t} = \mu_{t+1} = \bar{\mu} > 1 \), and if the ratio \( \frac{P_{N+1}}{P_t} = \bar{\alpha} \) is constant (\( \bar{\alpha} > 0 \)), then the real cash level converges to its steady-state \( \bar{m} = \frac{\bar{\mu} \cdot \delta \cdot \bar{K}}{\bar{\mu} + \epsilon - 1} \) whatever the initial value of \( m_0 > 0 \). This convergence takes place if \( \epsilon = 0 \), in which case, the actual cash flow stream converges to \( \bar{m} = \frac{\bar{\mu} \cdot \delta \cdot \bar{K}}{\bar{\mu} - 1} \).

b) If the ratio \( \frac{P_{t+1}}{P_t} = \bar{\mu} = 1 \), \( \forall t \geq 0 \); and if the ratio \( \frac{P_{N+t}}{P_t} = \bar{\alpha}, \ \forall t \geq 0 \); the level of actual cash converges to steady-state \( \bar{m} = \frac{\delta \cdot \bar{K}}{\epsilon} \).

**Table 1. Variations of change in capital stocks over time.**

| \( K_t \) | \( K_{t} - K_{t-1} \) | \( \frac{d(K_{t+1} - K_t)}{dK_t} \) | \( K_{t+1} - K_t \) |
| --- | --- | --- | --- |
| 0 | \( \frac{\alpha \cdot \bar{\alpha} \cdot (1-\theta) \cdot (K_t)^\alpha}{\delta} \) | \( -\delta + \alpha \cdot \bar{\alpha} \cdot (1-\theta) \cdot (K_t)^\alpha \) | \( \frac{\bar{\alpha} \cdot (1-\theta) \cdot (K_t)^\alpha}{\delta} \) |

Source: Computed by Author, 2021.
**Evidence:** It suffices, to consider the stationary state of the capital, then to replace $m_t$ and $m_{t-1}$ by $m_t$, $K_t$ by $\overline{K}$, $\mu_t$ by $\overline{\mu}$, in the equation:

$$m_t = (1 - \varepsilon) \frac{m_{t-1}}{\mu_t} + \pi_t,$$

where $\pi_t = (1 - \theta) (K_t)^\alpha$.

Recall that the monotony of the actual cash stock is not guaranteed. Indeed, under the conditions of proposition 2,

$$m_t - m_{t-1} = \frac{1 - \varepsilon - \overline{\mu}}{\overline{\mu}} \cdot m_{t-1} + (1 - \theta) \left[ (1 - \delta) \cdot K_{t-1} + (1 - \theta) \cdot (K_t)^\alpha \right].$$

The first member to the right of this equality is always negative, and the second member is always positive. The sign of $m_t - m_{t-1}$ therefore depends on the parameters of the model and the initial values $m_0, K_0$.

Under the conditions of proposition 2 written for the domestic and foreign economy, a steady-state exists for each of the economies; the steady-state in the domestic economy is such that:

a) The equilibrium labor supply. $N_t = \overline{N} = 1, \forall t \geq 0$.

b) The steady-state equilibrium capital level is $\overline{K} = \left[ \frac{\overline{\mu} (1 - \theta)}{\delta} \right]^{1/\alpha}$.

c) The equilibrium consumption at the stationary state is $\overline{C} = \theta \cdot \overline{K}^\alpha$, the re-invested profit compensates for the depreciation of the capital: $\overline{\pi} = (1 - \theta) \cdot \overline{\mu} \cdot \overline{K}^\alpha = \delta \cdot \overline{K}$.

d) The level of real cash at steady state is $\overline{m} = \frac{\overline{\mu} \cdot \delta \cdot \overline{K}}{\overline{\mu} + \varepsilon - 1}$ and the level of inflation is $\overline{\mu} \geq 1$.

Under the conditions of Proposition 2, the economy converges to a stationary equilibrium.

Simulation of the stationary state condition, arbitrarily the values of the parameters and the initial values of the model were chosen as observed in Table 2. These values were calibrated for specific savings.

As observed in Figure 3, when the economy starts with a low capital stock, household final consumption, capital stock and output of the final good grow and converge towards their stationary equilibrium. In this model, time is discrete. You can choose the length of a period of time equal to one quarter. This means that every three months, the government intervenes to adjust, if necessary, the quantity of currency in circulation. For most of the parameter values and initial values, the economy approaches very noticeably towards its stationary equilibrium after about 100 periods. The convergence is quite fast, when the

| Table 2. Simulated Parameters. |
|--------------------------------|
| $K_0$ | $m_0$ | $\alpha$ | $\theta$ | $\overline{\pi}$ | $\overline{\mu}$ | $\varepsilon$ | $\delta$ |
|-------|-------|----------|----------|----------------|----------------|-------------|----------|
| 0.05  | 1     | 0.65     | 0.8      | 0.5           | 1.05           | 0.03        | 0.1      |

Source: Simulated by Author using Excel 2010.
Figure 3. Convergence of variables towards a stationary.

initial values, are close to the values of the stationary equilibrium.

Note that rules similar to those of the domestic economy can be proposed to
the foreign economy. The capital formation rule:

\[ K_{t+1}^* = \left(1 - \delta^*\right) \cdot K_t^* + \pi_t^* , \quad 0 < \delta^* < 1, \]

The rule of money formation:

\[ m_t^* = \left(1 - \varepsilon^*\right) \cdot \frac{m_t^*}{\mu_t} + \pi_t^* , \quad 0 < \varepsilon^* < 1, \]

The choice of price level:

\[ \frac{P_{t+1}^*}{P_t^*} = \bar{\mu} \geq 1, \quad \text{et} \quad 0 < \frac{P_{t+1}^*}{P_t^*} = \bar{\pi}, \quad \forall t \geq 0. \]

Under these rules, the foreign economy converges to a stationary equilibrium,

where the equilibrium capital is equal to: \( K^* = \left[ \frac{\pi^* (1 - \theta)}{\delta^*} \right]^{\frac{1}{1 - \alpha}} \), the equilibrium

consumption \( \bar{C}^* = \theta \left( K^* \right)^{\alpha} \), and the equilibrium real cash level \( \bar{m}^* = \frac{\bar{\mu} \cdot \delta^* \cdot K^*}{\bar{\mu} + \varepsilon^* - 1} \).
When the parameters which characterize each country are the same \((\delta = \delta^*, \varepsilon = \varepsilon^*, \alpha = \alpha^*, \mu = \mu^*)\), then the economies converge towards the same stationary equilibrium, whatever their initial state. This balance is different from that resulting from a dynamic optimization process. Indeed, dynamic optimization allows households to choose the future capital (through investment or savings) and the level of real cash (through their utility function, which could depend on the holding of actual cash). Propositions 1 and 2 assume that the money supply and future capital formation follow rules. Households in the model we have presented do not need to save. Profits are invested; these investments readjust the level of future household income. Seigniorage adjusts the level of actual cash.

4. Conclusion

International trade between countries with similar factor endowments is necessary for different sectors of the national economy to be well connected and integrated into the world market. The theoretical model developed in this paper shows that both countries can benefit from international trade and therefore international trade between two or more countries is not a zero-sum game. In addition, the model revealed that a country can still make profits by producing and consuming its raw materials whilst exporting its surplus it can import only intermediate goods necessary for its development. The knowledge gap in this article is particularly useful for most developing countries, which have developed countries as their main trading partners, with very strong monopoly power. The paper also highlights the interest of every government through its central bank to control its currency, finance its economic activities and achieve full employment of production capacities. The stabilization of exchange rates and prices, in the long run, permits the local and foreign economies to converge towards a stationary equilibrium. This work could be used to regulate economic zones in which the powers of monopolies are similar (such as CEMAC, ECOWAS, etc.) and provide the countries that constitute them with identical rules for capital formation and issuance of capital change.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Dixit, A. K., & Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *The American Economic Review, 67*, 297-308.

Hellier, J. (1993). La similitude dans l’échange international: une revue critique des approches. *Revue française d’économie, 8*, 89-134. https://doi.org/10.3406/rfeco.1993.924

Krugman, P. R. (1979). Increasing Returns, Monopolistic Competition, and International
Lancaster, K. (1980). Intra-Industry Trade under Perfect Monopolistic Competition. *Journal of International Economics, 10*, 151-175. [https://doi.org/10.1016/0022-1996(80)90052-5](https://doi.org/10.1016/0022-1996(80)90052-5)

Thalut, N., & Kelese, M. T. (2019). Trade Liberalization, Energy Consumption and Pollution: An Empirical Investigation of Kuznets’ Hypothesis in 60 Developing Countries. *SSRG International Journal of Economics and Management Studies, 6*, 10-20. [https://doi.org/10.14445/23939125/IJEMS-V6I1P102](https://doi.org/10.14445/23939125/IJEMS-V6I1P102)

UNIDO (2013). United Nations Industrial Development Organization.

World Bank. Information, Communication Technologies, & infoDev (Program) (2012). *Information and Communications for Development 2012: Maximizing Mobile*. World Bank Publications.