SPIN-DEPENDENT, INTERFERENCE AND T-ODD
FRAGMENTATION AND FRACTURE FUNCTIONS*

O.V. Teryaev

Joint Institute for Nuclear Research, Dubna, 141980 Russia

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Fracture functions, originally suggested to describe the production of
diffractive and leading hadrons in semi-inclusive DIS, may be also applied
at fixed target energies. They may also include interference and final state
interaction, providing a source for azimuthal asymmetries at HERMES and
(expecially) A polarization at NOMAD. The recent papers by Brodsky,
Hwang and Schmidt, and by Gluck and Reya, may be understood in terms
of fracture functions.

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1. Introduction

QCD factorization allows one to express the cross-sections and polariza-
tion observables of hard processes in terms of convolutions of partonic sub-
process and non-perturbative functions, describing the hadron-parton and
parton-hadron transitions. The studies of various spin effects results in the
extension of their possible types. As usual, the case of Single Spin Asymme-
tries (SSA) is especially difficult, requiring the interference and final state in-
teractions, producing the imaginary phase. The most widely known objects
are parton distributions, describing the fragmentation of hadrons to partons
and related to the forward matrix elements \( \sum_X \langle P|A(0)|X\rangle\langle X|A(x)|P\rangle = \langle P|A(0)A(x)|P\rangle \) of renormalized non-local light-cone quark and gluon op-
\[\text{operators. As they do not contain any variable, providing the cut and cor-
responding imaginary phase (to put it in the dramatic manner, the proton is}
\text{stable), the } T\text{-odd distribution functions can not appear in the framework}
\text{of the standard factorization scheme. At the same time, they may appear}
\text{effectively, when the imaginary phase is provided by the cut from the hard}
\text{process, but may be formally attributed to the distribution [1]. Another}

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well-known object is fragmentation function, describing the fragmentation of partons to hadrons and constructed from the time-like cutvertices of the similar operators \( \sum A(0) A(x) \). Now, they may contain the cut with respect to the time-like parton momentum squared \( k^2 \) (which was space-like in the case of distributions), corresponding, at the hadronic level, to the jet mass. This may give rise to the number of \( T \)-odd fragmentation functions, including jet handedness \( [2] \), Collins function \( [3] \) and interference fragmentation functions \( [4] \).

The FRACTURE Function (FF) \( [5] \), whose particular example is represented by the Diffractive Distribution (DD) \( [6] \), is related to the object \( \sum_i \langle P_1 A(0) \rangle [P_2, X] [P_2, X A(x)] [P_1] \), combining the properties of FRAgmentation and struTURE functions. They describe the correlated fragmentation of hadrons to partons and vice versa. Originally this term was applied to describe the quantities integrated over the variable \( t = (P_1 - P_2)^2 \), while the fixed \( t \) case is described by the so-called extended fracture functions.

2. **Interference and \( T \)-odd fracture functions**

They may be also extended \( [7] \) to describe SSA in such processes. Namely, such functions can easily get the imaginary phase from the cut produced by the variable \( (P_1 + k)^2 \). Due to the extra momentum of produced hadron \( P_2 \), the number of possible \( P \)-odd combinations increases. Therefore, they may naturally allow for the \( T \)-odd counterparts.

The \( T \)-odd part of (inclusive) DIS was studied long ago, when the non-local analysis of twist 3 terms was presented for the first time \( [8] \). As soon as DIS does not contain any cuts, these effects require the real \( T \)-violation and are of a pure academic interest for the foreseen future of spin experiments. At the same time, the similar effects for the crossing related process of semi-inclusive annihilation correspond to the distributions substituted by fragmentation functions. As the latter may contain the imaginary cuts, simulating the \( T \)-violation, the performed calculation is starting to be more related with physics. Namely, it describes the production of transverse polarized baryon (one should typically think about \( A_t \), whose polarization is easily revealed in its weak decay) in the annihilation of the unpolarized leptons \( [9] \). The consideration of TOFF is actually completely similar. One should just substitute the transverse polarization of the baryon by the product of the transverse component of produced particle momentum and the longitudinal polarization of the initial particle \( s_T \rightarrow P_{2T} s_L / M \). Such a simultaneous appearance of the momentum and polarization of the different particles is the natural consequence of the correlated fragmentation of hadrons to partons and vice versa, described by FF. The resulting expression for the hadronic tensor, combining the contributions of quark and quark–gluon TOFF’s is
the straightforward counterpart of that for annihilation of unpolarized leptons (see (18) of [9]), up to the mentioned substitution and the change of fragmentation function $c_{
u}$ to the TOFF $F(x, \xi, t)$. The longitudinal proton polarization ($s_L$)-dependent part is taking the following form:

$$W^{\mu\nu} = \frac{s_L}{Q^2} \sum_{i=q,\bar{q}} e_i^2 x_B F_i(x_B, \xi, t) \left[ (2x_B P_i^\nu + q^\nu)\epsilon^\nu P_f P_q + (2x_B P_i^\nu + q^\nu)\epsilon^\nu P_f P_q \right].$$

(1)

The case of polarized partons, rather than hadrons, corresponds to the matrix elements of axial, rather than vector operators. Another generalization may be provided by the case of the multihadron fragmentation. It is this latter case, considered by Collins as a “polarized beam jets” [10], which is the first description of TOFF. In the case of the produced baryons, rather than pions, the number of possible TOFF substantially increases. In the case of unpolarized target, the direction of transverse polarization of produced $A$ may be defined by both lepton and hadron scattering planes. It is the former case, which may be described by the same expression (1), with pion momentum substituted by $A$ transverse polarization, which results in return to the mentioned formula of [9]. Note that full set of $T$-odd fracture functions may be studied along the line discussed here [11] by dropping the requirement of $T$-invariance (as $T$-violation may be simulated by imaginary phases from FSI).

Let us now discuss the possible experimental manifestations of these effects.

3. $T$-odd Fracture Functions at HERMES and NOMAD

First point, which should be mentioned in this connection, is the necessity for minor generalization of FF. Namely, one should consider the possibility of the hadron 2 being different from the hadron 1 (pion for HERMES and $A$ for NOMAD. This generalization is in fact straightforward and do not require any changes in the proof of factorization.

One may also worry, why the correlated fragmentation could be important for the hadrons, which are produced in the current, rather than target, fragmentation region, studied by HERMES. This generalization is more serious. It is based on the fact, that the invariant measure of such a correlation is provided by the squared momentum transfer $t = -Q^2 z/x$, which can be rather small for HERMES and NOMAD kinematics. Of course part of that smallness comes from the smallness of $Q^2$, but it is well known, that because of “handbag dominance” the scaling in $Q^2$ happens much earlier than in $t$. Consequently, the corrections to the factorized distribution and fragmentation functions, provided by fracture functions, may be important, especially at lower $z$. 


One should mention in this connection the successful application of handbag dominance in the area of GPD, having, as it was mentioned above, much in common with FF. Namely, it is a description of large-angle (real) Compton scattering by the convolution of a handbag diagram and GPD [12].

The confirmation of the importance of FF at NOMAD comes from the Monte-Carlo simulation reported at this conference [13]. The substantial contribution of $A_s$ happens to result from the target remnants even in the current fragmentation region. Moreover, the qualitative reason for that is the insufficient energy of $A$ to break the string, modeling the fragmentation process [14], which corresponds to small $t$ argument discussed above.

As soon as the FF gives the important contribution to cross section, TOFF should be equally important for $T$-odd SSA. In this sense, NOMAD provides the first evidence for TOFF.

As to TOFF role for HERMES, the observed angular distributions of produced pions do not contain, within the experimental errors, the term $\sin 2\phi$, which is allowed by the general kinematic analysis, but happens to be compatible with zero. The expression (1) produces only $\sin \phi$, term, providing the natural explanation of this fact.

In order to compare this approach to the “standard model” of this effect, which is now probably represented by the convolution of chiral-odd transversity distribution with chiral- and $T$-odd Collins fragmentation function [15], one may try to look for the dependence on the variable $x$ and $z$, which should be factorizable in that approach at leading order [16]. At the same time, there is no reason for such a factorization in the case of TOFF. The current level of accuracy, unfortunately, does not seem to allow for such a check.

4. Fracture functions as a framework for distribution and fragmentation models

FF (and in particular TOFF) provide a natural framework for understanding recent suggestions, extending the scope of SIDIS. In particular, the target spin dependent fragmentation functions, suggested by Gluck and Reya [17], perfectly fit to the definition of spin-dependent fracture function. The criticism of this paper may therefore be reformulated as a suggestion about possible role of fracture functions. Note that because fracture functions include all the information about the target, the expressions for spin-dependent cross section should not contain the spin-independent parton distributions anymore.

The model calculation of SSA by Brodsky, Hwang and Schmidt (BHS) [18] may also be related to ($T$-odd) fracture function. Indeed, their asymmetry is large only provided the pion transverse momentum is small, which signals about the possibility of correlation between distribution and fragmentation functions.
Moreover, the smallness of transverse momentum makes standard twist classification inapplicable, as the familiar twist 3 suppression factor $M/P_T$ is now not small. The elegant suggestions of Collins [19] to attribute BHS asymmetry to gluonic path ordered exponential may be considered as another manifestation of effective $T$-odd distribution [1]. The twist of the effect deserves special discussion. Although the imaginary phase induced by exponential may appear already at the leading twist level, it does not change the helicity structure of the amplitude, and cannot produce the interference and asymmetry. The latter appears at subleading (according to the standard counting rules) level and is suppressed as $M/P_T$. In BHS model this factor, however, is defined not to be small.

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REFERENCES

[1] D. Boer, P.J. Mulders, O.V. Teryaev, Phys. Rev. D57, 3057 (1998).
[2] A.V. Efremov, L. Mankiewicz, N.A. Tornqvist. Phys. Lett. B284, 394 (1992).
[3] J.C. Collins, Nucl. Phys. B396, 161 (1993).
[4] R.L. Jaffe, Xue-min Jin, Ji-an Tang, Phys. Rev. D57, 5920 (1998).
[5] L. Trentadue, G. Veneziano, Phys. Lett. B323, 201 (1994).
[6] J.C. Collins, Phys. Rev. D57, 3051 (1998); [Erratum, Phys. Rev. D61, 019902 (1999)].
[7] O.V. Teryaev, T-odd Diffractive Distributions, in Proceedings of IX Blois Workshop, ed. by V. Kudrat and P. Zavada, p. 211; $T$-odd Fracture Functions, in SPIN-01, Proceedings of 2001 Dubna Spin Meeting, p. 105.
[8] A.V. Efremov, O.V. Teryaev, Yad. Fiz. 39, 1517 (1984).
[9] O.V. Teryaev, In SPIN-96 Proceedings. Edited by C.W. de Jager, T.J. Kuetel, P.J. Mulders, J.E. Oberski, M. Oskam-Tamboezer, World Scientific, 1997, p. 594.
[10] J.C. Collins, hep-ph/9610263.
[11] J. Blümlein, talk at this conference not submitted to the proceedings.
[12] A.V. Radyushkin, Phys. Rev. D58, 114008 (1998).
[13] D. Naumov, Acta Phys. Pol. B33, 3791 (2002), these proceedings.
[14] A. Kotsinian, private communication.
[15] A.V. Efremov, Acta Phys. Pol. B33, 3755 (2002), these proceedings.
[16] E. Leader, A.V. Sidorov, D.B. Stamenov, Acta Phys. Pol. B33, 3695 (2002), these proceedings.
[17] M. Gluck, E. Reya, hep-ph/0203063.
[18] S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B530, 99 (2002).
[19] J.C. Collins, Phys. Lett. B536, 43 (2002).