Summary and Overview of Working Group VI: $V_{us}$ and $V_{ud}$

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We briefly review the current status of the determination of $|V_{us}|$ and $|V_{ud}|$, with particular attention to the latest experimental and theoretical developments on $|V_{us}|$ since the first CKM Workshop [1].

1 Introduction

Despite the great experimental and theoretical progress in semileptonic $b$ decays, at present the most precise constraints on the size of CKM matrix elements are still extracted from the low-energy $s \to u$ and $d \to u$ semileptonic transitions. In a few cases these can be described with excellent theoretical accuracy, and combining the constraints on $|V_{ud}|$ and $|V_{us}|$ we can perform the most stringent test of CKM unitarity. In particular, the best determination of $|V_{us}|$ is obtained from $K \to \pi\ell\nu$ decays ($K_{\ell3}$), whereas the two most stringent constraints on $|V_{ud}|$ are obtained from superallowed Fermi transitions (SFT), i.e. beta transitions among members of a $J^P = 0^+$ isorotip of nuclei, and from the neutron beta decay. In addition to these key modes, a promising and complementary information on $|V_{ud}|$ is extracted from the pion beta decay ($\pi_{e3}$), while significant constraints on $|V_{us}|$ are obtained also from Hyperon and $\tau$ decays.

In all cases the key observation which allow a precise extraction of the CKM factors is the non-renormalization of the vector current at zero momentum transfer in the $SU(N)$ limit (or the conservation of the vector current) and the Ademollo Gatto theorem [2]. The latter implies that the relevant hadronic form factors are completely determined up to tiny isospin-breaking corrections (in the $d \to u$ case) or $SU(3)$-breaking corrections (in the $s \to u$ case) of second order. As a result of this fortunate situation, the accuracy on $|V_{us}|$ is approaching the 1% level and the one on $|V_{ud}|$ the 0.05% level. If we make use of the unitarity relation

$$U_{uu} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ab}|^2 = 1,$$  \hspace{1cm} (1)

the present accuracy on $|V_{ud}|$ and $|V_{us}|$ is such that the contribution of $|V_{ab}|$ to Eq. (1) can safely be neglected, and the uncertainty of the first two terms is comparable. In other words, $|V_{ud}|$ and $|V_{us}|$ lead to two independent determinations of the Cabibbo angle both around the 1% level, and the unitarity relation $U_{uu} = 1$ can be tested at the 0.1% level.

A detailed discussion about the extraction of $|V_{us}|$ and $|V_{ud}|$ from the key observables mentioned above can be found in Ref. [11] and will not be repeated here. However, we stress that a few significant developments have been achieved since the publication of Ref. [11]:

- The new measurement of $\text{BR}(K_{\ell3}^+)$ by BNL-E865, at the 2% level of accuracy, has been confirmed and finalized [3].
- KLOE has announced new preliminary measurements of $K_{\ell3}^0$ and $K_{\ell3}^0$ branching ratios both at the 2% level of accuracy [4].
- Performing a complete analysis of $K_{\ell3}$ decays in CHPT at $O(p^6)$, Bijnens and Talavera have shown that, at this level of accuracy, the amount of $SU(3)$ breaking in $f_+(0)$ could be extracted in a model-independent way from the measurement of slope and curvature of the scalar form factor $f_0(t)$ [5].
- Cabibbo, Swallow and Winston have reanalyzed Hyperon semileptonic decays, showing that theses modes can lead to an independent extraction $|V_{us}|$ with a precision which is not far from the one presently obtained from $K_{\ell3}$ decays [6].

As we shall discuss in the following, these new results do not change substantially the overall picture presented in Ref. [11], but provide a good starting point to reach, within a few years, a determination of the Cabibbo angle well below the 1% level.

2 Status of $|V_{us}|$

The steps necessary to extract $|V_{us}|$ from each of the $K_{\ell3}$ decay mode can be summarized as follows:

1. experimental determination of the photon-inclusive decay rate $\Gamma(K \to \pi\ell\nu + n\gamma)$;
2. experimental determination (or, if not available, theoretical evaluation) of the momentum dependence of the two form factor, $f_+(t)$ and $f_0(t)$ (the latter being relevant only for $K_{\ell3}^0$ modes);
Table 1. Summary of isospin-breaking factors from Ref. [7], in units of $10^{-2}$; the entries with [*] are from Ref. [9].

| $K_{e3}$ | $\delta_{SU(2)}$ | $\delta_{\rho\rho^*}$ | $\Delta I(\partial f_3, \partial f_0)$ |
|---------|----------------|----------------|-----------------|
| $K_{e3}^+$ | 2.4 ± 0.2 | 0.32 ± 0.16 | -1.27 |
| $K_{e3}^0$ | 0 | 0.46 ± 0.08 | -0.32 |
| $K_{\mu3}^+$ | 2.4 ± 0.2 | 0.006 ± 0.16 | 0.0 ± 1.0 [*] |
| $K_{\mu3}^0$ | 0 | 0.15 ± 0.08 | 1.7 ± 1.0 [*] |

Thanks to the complete $O(p^4, \epsilon p^2)$ analysis of isospin breaking corrections in the framework of CHPT ($\epsilon$ stands for both $\epsilon^2$ and $m_u - m_d$) by Cirigliano et al. [7], the theoretical error due to the step n. 3 is around 0.3%. This means that if we combine the first three steps in this list for the four different $K_{\ell3}$ decay modes, we should obtain four independent determination of the product $f_3(0)|V_{us}|$ affected by a very small theoretical error.

The master formula for a combined analysis of this type is:

$$|V_{us}| \cdot f_3^{K^{\ell3}}(0) = \left[ \frac{192 \pi^3 T_i}{G_F^2 M_K^2 C_i S_{\text{ew}} P_0(\partial f_3, \partial f_0)} \right]^{1/2} \times \frac{1}{1 + \delta_{SU(2)}^i + \delta_{\rho\rho^*}^i + \frac{1}{2} \Delta I(\partial f_3, \partial f_0)},$$

(2)

where $C_i = 1$ ($2^{-1/2}$) for neutral (charged) modes, $S_{\text{ew}} = 1.0232$ denotes the universal short-distance electroweak correction factor [8] and $P_0(\partial f_3, \partial f_0)$ the non-radiative phase space integral. The small correction terms in the second line of Eq. (2) denote the isospin-breaking effects computed in [7] and reported in Table 1.

Applying this master formula to the presently available data leads to the plot in Fig. 1 which provides an update of similar analyses presented in Ref. [11]. The first point to be noted is that the new generation of experiments already produced single measurements which compete with the PDG values. Performing a naive average of all the points in Fig. 1 leads to an error of $|V_{us}| \cdot f_3^{K^{\ell3}}(0)$ around 0.3%, which would be negligible with respect to the theoretical uncertainty in $f_3^{K^{\ell3}}(0)$ (point n. 4 in the list). However, we cannot perform a naive average of all the points in Fig. 1 given their internal consistency: if we include a scale factor following the usual PDG procedure [10], the error goes up to about 0.8%, which is worse than what is obtained without the new results. The central value of the average moves by less than 0.3% with the inclusion of the new points, this is why we stated in the previous section that the overall picture is essentially unchanged with respect to Ref. [11].

As can be seen from Fig. 1 the only point which is badly consistent with the others is the new $B(K_{e3}^+)$ measurement by BNL–E865. This new result differ by 2.3$\sigma$ with the average of older measurement of the same channel, and by more than 3$\sigma$ with the average of the neutral modes (once theoretical isospin-breaking corrections are applied). Given this situation, it is clear that new independent measurements of $B(K_{e3}^+)$ — soon expected by KLOE and NA48 [12] — are particularly interesting. If the BNL-E865 result is confirmed, it means that isospin-breaking corrections have been badly underestimated in Ref. [7] and the extraction of $|V_{us}|$ from $K_{\ell3}$ decays is more complicated than expected. If charged and neutral modes turn out to be compatible, we would become more confident about the theoretical treatment of $K_{\ell3}$ decays and we could hope to reach, in a short time, an overall error on $|V_{us}|$ substantially below 1%.

As far as the theoretical estimate of $SU(3)$-breaking is concerned, an interesting new development is provided by the work of Bijnens and Talavera [15]. They have pointed out that, within CHPT, the local $SU(3)$-breaking contribution to $f_3(0)$ of $O(p^6)$ (i.e. the leading local contribution), can be unambiguously predicted in terms of the the first two derivatives of $f_0(t)$ (which in principle are experimentally accessible). In particular, the slopes $\lambda_0$ and $\lambda_0'$, defined by

$$f_0(t) = f_3(0) \left[ 1 + \lambda_0 \frac{t}{m_3^2} + \lambda_0' \frac{t^2}{m_3^4} \right], \quad t = (p_K - p_\pi)^2,$$

Figure 1. $|V_{us}| \cdot f_3^{K^{\ell3}}(0)$ from the four $K_{\ell3}$ modes, including the very recent result from BNL–E865 [12] and preliminary results from KLOE [1]: the (black) points without labels correspond to the old published results averaged by PDG [10]. The full (dashed) horizontal line denotes the average without (with) the new data.
should be measured with absolute errors of 10^{-3} (\lambda_0) and 10^{-4} (\lambda_0') in order to reach a prediction of \f_s(0) at the 1\% level. Although very challenging, this goal is not impossible for high-statistics experiments such as KLOE and NA48.\footnote{Note that this measurements could be performed either in K_{\ell}^0 or in K_{\rho}^0 channels, since the relative isospin-breaking corrections are known.} An interesting complementary approach to estimate the amount of S U(3)-breaking in \f_s(0) is provided by Lattice QCD. Unfortunately, at present none of this new techniques can lead to a numerical prediction, and the most reliable figure is still represented by the Leutwyler-Roos result: f_0^\text{K}\pi(0) = 0.961 \pm 0.008 \text{[13]}. Combining it with |V_{ud}| : f_s^\text{K}\pi(0) = 0.2115 \pm 0.0015, obtained from the published data on B(K_{\ell}^0) and B(K_{\rho}^0) only, one finds [14]

\[ |V_{ud}|_{K_{\ell}^0} = 0.2201 \pm 0.0016_{\text{exp}} \pm 0.0018_{\text{the}(f_s)} \]
\[ = 0.2201 \pm 0.0024 . \]

### Other determinations of |V_{ud}|

Alternative strategies to determine |V_{ud}| are offered by tau-lepton and Hyperon decays

**Tau decays.** The novel strategy to determine \( V_{ud} \) via \( \tau \) decays, proposed in Ref. [15] and illustrated in this workshop by Jamin [16], relies on the fact that, using the OPE, we can express theoretically the hadronic width of the \( \tau \) lepton and the appropriate moments — for both Cabibbo-allowed \( \langle R^d_{\ell} \rangle \text{ and Cabibbo-suppressed } \langle R^U_{\ell} \rangle \text{ transitions — in terms of strange-quark mass and CKM matrix elements. Originally, this feature has been exploited to determine } m_s \text{ using } |V_{ud}| \text{ as input. The authors of Ref. [15] have inverted this reasoning: they have employed the range } m_t(2 \text{ GeV}) = 105 \pm 20 \text{ MeV, derived from other observables, to determine } |V_{ud}| \text{ from hadronic } \tau \text{ decays. Using the lower moments only (} k = 0 \text{) they obtained}

\[ |V_{ud}|_{\tau} = 0.2173 \pm 0.0044_{\text{exp}} \pm 0.0009_{\text{th}} \pm 0.0006_{V_{ud}} \]
\[ = 0.2173 \pm 0.0045 , \]

where the theoretical error reflects the uncertainty in \( m_t \), the dependence on \( V_{ud} \) correspond to the safe range \( |V_{ud}| = 0.9739 \pm 0.0025 \), and the dominant experimental error reflects the inputs \( R_{\ell}\Sigma = 0.1625 \pm 0.0066 \text{ and } R_{\ell\Sigma} = 3.480 \pm 0.014 \text{[13].} \text{ A reduction in the uncertainty of } R_{\ell}\Sigma \text{ by a factor of two, which should easily be reached at } B \text{ factories, would make this extraction of } |V_{ud}| \text{ competitive with the one based on } K_{\ell}^0 \text{ decays. As already stressed in Ref. [11], in this perspective it would be highly desirable also to estimate the systematic uncertainty of the method (e.g. extracting } V_{ud} \text{ from higher } R^d_{\ell} \text{ moments, and obtaining additional constraints on the } m_t \text{ range). Future precise measurements of } \tau \text{ hadronic moments, with a good flavor tag, could allow to reach this goal.}

**Hyperon semileptonic decays.** A new analysis of Hyperon semileptonic decays has recently been presented in Ref. [6]. On general grounds, these processes are not so clean as \( K_{\ell}^0 \text{ decays since: i) the hadronic matrix elements of the axial current (not protected by the Ademollo-Gatto theorem) are also involved, ii) the convergence of the chiral expansion is slower and the corresponding coefficients are known with less accuracy. As shown in Ref. [6], the first problem can be circumvented by fitting the ratio of axial over vector current at zero momentum transfer (\( g_1/f_1 \)) from data (similarly to what is done for the extraction of } V_{ud} \text{ from the neutron beta decay). By doing so, and neglecting possible S U(3) and isospin-breaking breaking terms in } f_1(0) \text{ due to quark masses, the authors of Ref. [6] obtain}

\[ |V_{ud}|_{\text{Hyp}} = 0.2250 \pm 0.0027_{\text{exp}} , \]

where the average is dominated by the two values from A \( (0.2224 \pm 0.0034) \text{ and } \Sigma \ (0.2282 \pm 0.0049) \text{ semileptonic decays. The fact that the error in [6] is very close to the final error in [3] and it is in better agreement with CKM unitarity is rather stimulating. However, we stress that the comparison between [5] and the final error in [3] is not appropriate, since the latter does include an estimate of the theoretical uncertainty due to light-quark masses. The calculation of S U(3)-breaking effects in the matrix elements of the vector current at zero momentum transfer is more difficult in the baryonic sector than in meson one, and indeed the existing estimates are affected by sizable uncertainties (see e.g. Ref. [19]).\footnote{Note that in the case of the } f_1(0) \text{ the leading non-local term of } O(\rho^4) \text{ is known to excellent accuracy and the first ambiguities arises at the two-loop level, while in the baryonic sector there are sizable ambiguities already at the one-loop level [19].}}

### 3 Status of |V_{ud}| and CKM Unitarity

The situation of |V_{ud}| has not changed since the publication Ref. [11]. As stressed by Abele [23] at this workshop, the nine independent measurement of SFT in different nuclei show a remarkable internal consistency once the appropriate universal and structure-dependent radiative corrections are included. The latter have been recently re-analyzed in Ref. [20], confirming (and thus strengthening the confidence in) the older analyses, which leads to the global average [21]

\[ |V_{ud}|_{\text{SFT}} = 0.9740 \pm 0.0005 . \]
attention to recent experiments with a high degree of polarization to measure $g_A/g_V$ (which represent the dominant source of uncertainty). Their average leads to [11]

$$|V_{ud}| = 0.9731 \pm 0.0015 ,$$

which is expected to improve substantially in the near future thanks to the upgrade of the PERKEO experiment in Heidelberg [22].

The third and completely independent approach to $|V_{ud}|$, namely the determination via the $\beta$-decay of the charged pion, appears to be very promising in the long term due to the excellent theoretical accuracy of the corresponding decay amplitude [23]. The present experimental precision for the tiny branching ratio of this transition does not allow yet to compete with SFT and $\eta$ determinations; however, the situation is improving thanks to the PIBETA experiment at PSI [24]. The preliminary result of the PIBETA Collaboration [25],

$$B(\pi^+ \rightarrow \pi^0 e^+\nu) = (1.044 \pm 0.007_{\text{stat}} \pm 0.009_{\text{syst}}) \times 10^{-8},$$

combined with the theoretical analysis of Ref. [23], leads to

$$|V_{ud}| = 0.9765 \pm 0.0056_{\text{exp}} \pm 0.0005_{\text{th}}$$

$$= 0.9765 \pm 0.0056 ,$$

(8)

where the error should be reduced by about a factor of 3 at the end of the experiment.

**CKM unitarity**

The two measurements of $|V_{ud}|$ from SFT and nuclear beta decay, reported in Eqs. (6) and (7) respectively, are perfectly compatible. Combining them in quadrature one obtains

$$|V_{ud}| = 0.9739 \pm 0.0005 ,$$

(9)

a result which is not modified by the inclusion in the average of the present $\eta$ data. The compatibility of SFT and nuclear beta decay results is clearly an important consistency check of Eq. (9). However, it should also be stressed that the theoretical uncertainty of inner radiative corrections (which contribute at the level of $\pm 0.04\%$) can be considered to a good extent a common systematic error for both determinations. Thus the uncertainty quoted in Eq. (9) is mainly of theoretical nature and should be taken with some care. Using the unitarity relation (11) we can translate Eq. (9) into a prediction for $|V_{us}|$:

$$|V_{us}| = 0.2269 \pm 0.0021 ,$$

(10)

to be compared with the direct determination in Eq. (5).

As already pointed out in Ref. [11], the 2.2$\sigma$ discrepancy between these two determinations could be attributed to: i) an underestimate of theoretical and, more general, systematic errors; ii) an unlikely statistical fluctuation; iii) the existence of new degrees of freedoms which spoil the unitarity of the CKM matrix. Since theoretical errors provide a large fraction of the total uncertainty in both cases, at present the solution i), or at least a combination of i) and ii), appears to be the most likely scenario. As discussed in Section 2, the situation of $K_{e3}$ decays is in rapid evolution: with help of new data and new theoretical estimates of $SU(3)$-breaking effects in these channels, we should be able to shed new light on these three scenarios in the near future. For the time being, if we are interested in a conservative estimate of the Cabibbo angle to be used in different frameworks (e.g. global CKM fits), the best we can do is to treat the two determinations in (11) and (10) on equal footing and to introduce an appropriate scale factor. Following this procedure, we confirm the estimate of $|V_{us}|$ presented in [11], namely

$$|V_{us}|_{\text{limit-}K_{e3}} = 0.2240 \pm 0.0034 .$$

(11)

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**References**

1. M. Battaglia et al., *The CKM matrix and the unitarity triangle*, hep-ph/0304132.
2. M. Ademollo and R. Gatto, Phys. Rev. Lett. 13 (1964) 264.
3. A. Shers [E865 Collaboration], hep-ex/0305042.
4. J. A. Thompson, D. E. Kraus and A. Sher, *these proceedings*, hep-ex/0307053.
5. A. Aloisio et al. [KLOE Collaboration], hep-ex/0307016. Phys. Lett. B 535 (2002) 37 [hep-ph/0203232].
6. J. Bijnens and P. Talavera, hep-ph/0303103.
7. J. Bijnens, hep-ph/0307082.
8. N. Cabibbo, E. C. Swallow and R. Winston, hep-ph/0307124; hep-ph/0307298.
9. V. Cirigliano, M. Knecht, H. Neufeld, H. Rupertberger and P. Talavera, Eur. Phys. J. C 23 (2002) 121 [hep-ph/0110133].
10. W.J. Marciano, A. Sirlin, Phys. Rev. Lett. 71 (1993) 3629; A. Sirlin, Rev. Mod. Phys. 50 (1978) 579.
11. E. S. Ginsberg, Phys. Rev. 162 (1967) 1570 [Erratum, ibid. 187 (1969) 2280]; Phys. Rev. 171 (1968) 1675.
[Erratum, ibid. 174 (1968) 2169; Phys. Rev. D 1 (1970) 229.]

10. K. Hagiwara et al., Particle Data Group Phys. Rev. D 66 (2002) 010001.

11. G. Calderon and G. Lopez Castro, Phys. Rev. D 65 (2002) 073032 [hep-ph/0111272].

12. D. Madigozhin, these proceedings [hep-ex/0307076].

13. H. Leutwyler and M. Roos, Z. Phys. C 25 (1984) 91.

14. V. Cirigliano, these proceedings and [hep-ph/0305154].

15. E. Gamiz, M. Jamin, A. Pich, J. Prades and F. Schwab, [hep-ph/0212230].

16. M. Jamin, these proceedings.

17. F. Le Diberder and A. Pich, Phys. Lett. B 289 (1992) 165.

18. M. Davier and C. Yuan, [hep-ex/0211057].

19. J. Anderson and M. A. Luty, Phys. Rev. D 47 (1993) 4975 [hep-ph/9301219]; R. Flores-Mendieta, A. Garcia and G. Sanchez-Colon, Phys. Rev. D 54 (1996) 6855 [hep-ph/9503230]; R. Flores-Mendieta, E. Jenkins and A. V. Manohar, Phys. Rev. D 58 (1998) 094028 [hep-ph/9805416].

20. I. S. Towner and J. C. Hardy, Phys. Rev. C 66 (2002) 035201 [nucl-th/0209014].

21. I. S. Towner and J. C. Hardy, [nucl-th/9809087].

22. H. Abele, these proceedings and [hep-ex/0208048].

23. V. Cirigliano, M. Knecht, H. Neufeld and H. Pichl, Eur. Phys. J. C 27 (2003) 255 [hep-ph/0209226]. V. Cirigliano, M. Knecht, H. Neufeld, H. Pichl, [hep-ph/0209226].

24. PIBETA Collaboration, M. Bychkov et al., PSI Scientific Report 2001, Vol.1, p. 8, eds. J. Gobrecht et al., Villigen PSI (2002); [http://pibeta.web.psi.ch].

25. D. Pocanic, these proceedings, [hep-ph/0307258].