Sivers Differential Cross Section for $\pi + X$ Production via $p^\uparrow + p$ Collisions, and the Quark Sivers Function

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Abstract

We recently estimated pion+$X$ production via proton collisions with a polarized proton target to enable the Collins fragmentation function to be determined via future experiments. In the present article we first estimate the Silvers differential cross section obtained from experiments with the target proton having both positive and negative polarizations, and then estimate the Sivers Function using quark distribution functions.

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1 Introduction

Recently we estimated the differential cross section for a proton colliding with a polarized proton target to produce a pion and a hadron $X$, $\frac{d\sigma}{dy}$, with $y$ the rapidity[1], with a factor $D_{q\rightarrow q\bar{q}}$, the fragmentation probability for a polarized quark to fragment into a $q\bar{q}$[2]. From this cross section, and the fragmentation probability to produce a hadron from a transversely polarized quark, one can extract the Collins fragmentation function[3].

The present article is motivated in part by the Letter of Intent by the E1039 collaboration[4], with spokespersons from the Los Alamos National Laboratory, to measure the Sivers Function[5] via Drell-Yan experiments. A Drell-Yan process[6] occurs in hadron-hadron scattering with a quark from one hadron and an antiquark from the other hadron annihilating to create a boson. A number of Deep Inelastic Scattering experiments[7, 8, 9, 10] have measured non-zero values for the Sivers Function, but the E1039 Collaboration plan to make the first measurement of the Sivers Function of sea quarks.

In the following section the Sivers Function obtained from proton collision experiments with a polarized proton target is discussed and the Sivers differential cross section is estimated. In the next section the Sivers Function in terms of the quark distribution function in polarized protons is defined and used for our estimate of the Sivers Function.
The Sivers and Collins Functions in terms of experimental cross sections are defined, e.g., in Refs[7, 10]. The Sivers function was defined in terms of quark distribution functions with polarized protons in Ref.[11].

2.1 Sivers and Collins Functions in terms of experiments

For proton collisions with a polarized proton target producing a pion and a hadron (X), which is similar to the differential cross section estimated in Ref[1] for the Collins Fragmentation Function, except that both up and down proton polarizations are needed. The Sivers and Collins Functions are given in terms of the asymmetry

\[ A(\phi_h, \phi_s) = \frac{1}{S^T} \frac{\sigma_{p^+ p \rightarrow \pi X}(\phi_h, \phi_s) - \sigma_{p^- p \rightarrow \pi X}(\phi_h, \phi_s)}{\sigma_{p^+ p \rightarrow \pi X}(\phi_h, \phi_s) + \sigma_{p^- p \rightarrow \pi X}(\phi_h, \phi_s)} \approx A_C(\phi_h + \phi_s) \sin(\phi_h + \phi_s) + A_S(\phi_h - \phi_s) \sin(\phi_h - \phi_s), \]

where \((\phi_h, \phi_s)\) are the azimuthal angles of the (hadron momentum, target spin) relative to the scattering plane, \(S^T\) is the component of the target proton spin orthogonal to the scattering plane, \(A_C\) is the Collins Function, and \(A_S\) is the Sivers Function. Since the cross sections \(\sigma_{p^+ p \rightarrow \pi X}(\phi_h, \phi_s)\), \(\sigma_{p^- p \rightarrow \pi X}(\phi_h, \phi_s)\) are very difficult for experiments to measure we estimate the Sivers differential cross section in the next subsection and the Sivers Function in the following section.

2.2 Sivers Differential Cross Section

From Eq[1] we define the Sivers Differential Cross Section \(\frac{d\sigma_{Sivers}}{dy}\) as

\[ \frac{d\sigma_{Sivers}}{dy} = \frac{d\sigma_{p^+ p \rightarrow \pi X}}{dy} - \frac{d\sigma_{p^- p \rightarrow \pi X}}{dy}. \] (2)

From Ref[1] Eq(11) the differential cross section used to possibly evaluate the Collins Fragmentation function is with \(y\) the rapidity

\[ \frac{d\sigma_{p^+ p \rightarrow \pi X}}{dy} = Aqq[\Delta f_g(x(y), 2m)\Delta f_g(a/x(y), 2m) + \Delta f_d(x(y), 2m)\Delta f_d(a/x(y), 2m) \Delta f_d(x(y), 2m)\Delta f_d(a/x(y), 2m) + \Delta f_u(x(y), 2m)\Delta f_u(a/x(y), 2m) + \Delta f_u(x(y), 2m)\Delta f_u(a/x(y), 2m)] \frac{dx(y)}{dy} \left[ D_g^{*} \rightarrow q\bar{q} \right], \]

with \(Aqq\) a known constant and[12]
\[ \Delta f_g(x) = 15.99 - 700.34x + 13885.4x^2 - 97888.x^3 \]
\[ \Delta f_d(x) = -5.378 + 205.6x - 4032.77x^2 + 28371.x^3 \]
\[ \Delta f_u(x) = 8.44 - 292.16x + 5675.16x^2 - 39722.x^3 \]
\[ \Delta f_{\overline{u}}(x) = -1.447 + 64.67x - 1268.24x^2 + 8878.32x^3 \]
\[ \Delta f_{\overline{d}}(x) = \Delta f_u(x), \quad (4) \]

with \( \Delta f_g \) the gluon distribution function and \( \Delta f_q, \Delta f_{\overline{u}} \) the quark and anti-quark distribution functions.

\[ D_{q^+ \rightarrow q\overline{q}} = \int_0^1 dz D_{q^+ \rightarrow q\overline{q}}(z) \simeq 2.87 \times 10^{-3}, \quad (5) \]

With \( D_{q^+ \rightarrow q\overline{q}}(z) \) evaluated from Ref.[2]

From Eqs(2,3) the differential cross section
\[ \frac{d\sigma^{Sivers}}{dy}, \quad D^{Sivers} = D_{q^+ \rightarrow q\overline{q}} - D_{q^- \rightarrow q\overline{q}}, \]

is
\[ \frac{d\sigma^{Sivers}}{dy} \propto \frac{dx(y)}{dy} \frac{1}{x(y)} D^{Sivers}. \quad (6) \]

\[ D^{Sivers} = \int_0^1 dz (D_{q^+ \rightarrow q\overline{q}}(z) - D_{q^- \rightarrow q\overline{q}}(z)) \simeq 7.44 \times 10^{-3} \simeq 2.6 \times D_{q^+ \rightarrow q\overline{q}}. \quad (7) \]

Note also that for \(-1 \leq y \leq 1\)
\[ (1/\Delta f_g(x)\Delta f_g(a/x)) \times [\Delta f_g(x)\Delta f_g(a/x) + \Delta f_d(x)\Delta f_d(a/x) + \Delta f_d(x)\Delta f_d(a/x) + \Delta f_u(x)\Delta f_u(a/x), 2m] + \Delta f_{\overline{u}}(x)\Delta f_{\overline{u}}(a/x)] \simeq 1.4 \quad (8) \]

Therefore, from Eqs(7,8)
\[ \frac{d\sigma^{Sivers}}{dy} \simeq 3.54 \times \frac{d\sigma_{p^+ p^{-} \rightarrow \pi X}}{dy}. \quad (9) \]

Therefore from Figure 2 in Ref[1] one finds \( \frac{d\sigma^{Sivers}}{dy} \) by simply multiplying the differential cross section in Figure 2 in Ref[1] by 3.54
$d\sigma^{Sivers}/dy$ for $E=200$ GeV is shown in figure 1

![Figure 1: $d\sigma^{Sivers}/dy$ (µb) for $E=200$ GeV via $p^+p - p^+p$ collisions](image)

### 3 Sivers Function in terms of Quark Distribution Functions

The Sivers Function was defined in terms of quark distribution functions in Refs\cite{11,13}.

The Sivers Function $f_S(z, k_T^2)$ is given by\cite{13}

\[
f_{qp^\uparrow}(z, k_T^2) - f_{qp^\uparrow}(z, -k_T^2) = f_{qp^\uparrow}(z, k_T^2) - f_{qp^\downarrow}(z, k_T^2) = -f_S(z, k_T^2) \left( \vec{P} \times \vec{k}_T \cdot \vec{S} \right) \frac{1}{2M}, \tag{10}
\]

where $\vec{P}$ is the proton momentum, $\vec{k}_T$ is quark momentum perpendicular to $\vec{P}$, $\vec{S}$ is the normalized proton spin, with $S^2 = -1.0$, and $M$ is the proton mass.
With the choice of frame the target rest frame, the Sivers Function can be derived from the quark distribution function with polarized protons:

\[ f_S(z) \simeq (f_{qp}^\uparrow(z) - f_{qp}^\downarrow(z))(P/2M) \]

\[ \simeq (f_{qp}^\uparrow(z) - f_{qp}^\uparrow(-z))(P/2M). \]  

(11)

Therefore, an approximate estimate of the Sivers Function can be obtained from the quark fragmentation function \( D_{q^+ \rightarrow q\bar{q}}(z) \) from Ref[1], which was obtained from Ref[2].

3.1 Estimate of the Sivers Function in terms of Quark Distribution Functions

We make an approximate estimate of the Sivers function \( f_S(z) \) using \( f_{qp}^\uparrow(z) \simeq D_{q^+ \rightarrow q\bar{q}}(z) \) The estimate of \( D_{q^+ \rightarrow q\bar{q}}(z) \) from the Eq(31) in Ref[2] was discussed in Ref[1].

\[ f_{qp}^\uparrow(z) \simeq D_{q^+ \rightarrow q\bar{q}}(z) \simeq 2.125 \times 10^{-3}(6z + 6z^2 - 15z^3 - 12z^4 + 15z^5), \]  

(12)

with \( 0 \leq z \leq 1 \). Therefore from Eqs(11,12)

\[ f_S(z) \simeq 2.125 \times 10^{-3} \frac{P}{2M}(12z - 30z^3 + 30z^5). \]  

(13)

For \( E = \sqrt{P^2 + M^2} = 200 \text{ GeV} \) and the proton mass \( M \simeq 1 \text{ GeV} \), \( P/2M \simeq 100 \), giving

\[ f_S(z) \simeq 0.2125(12z - 30z^3 + 30z^5). \]  

(14)

The Sivers function, \( f_S(z) \), is shown in Figure 2.
4 Conclusions

We have estimated the Sivers Differential Cross Section. As shown in Figure 1 it should be possible to measure $\frac{d\sigma^{Sivers}}{dy}$. We have also estimated the Sivers Function, shown in Figure 2.

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