Characteristics of manipulative in mathematics laboratory

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Abstract. A manipulative is a teaching aid designed such that students could understand mathematical concepts by manipulating it. This article aims to provide an insight to the characteristics of manipulatives produced in the mathematics laboratory of Universitas Ahmad Dahlan, Indonesia. A case study was conducted to observe the existing manipulatives produced during the latest three years and classified the manipulatives based on the characteristics found. There are four kinds of manipulatives: constructivism manipulative, virtual manipulative, informative manipulative, and game-based manipulative. Each kinds of manipulative has different characteristics and impact towards the mathematics learning.

1. Introduction
The definition of a good mathematics teaching practice is oftenly related to the achievement of the learning goals. If the students could achieve the learning goals through a particular teaching approach, then the approach can be regarded as good [1]. One of the mathematics leaning goals is to create what the students learn become meaningful [2, 3]. In this case, representation becomes important since mathematics teachers cannot bring the mathematics object in front of their students [4, 5]. Furthermore, the idea of reducing the abstract sense in mathematics learning has been massively promoted to help students understand the mathematics concepts better [1, 2, 6, 5, 7–9]. Therefore, they need manipulatives to show what the objects look like and what properties attached to the objects.

Manipulative is a kind of teaching aid designed to represent abstract mathematical ideas concretely. It can be in a form of visual and physical appearance which can be manipulated by students through hands-on-activities [8]. Manipulative is believed to play important role in encouraging students to learn mathematics. Some studies report that manipulatives, both concrete and virtual, have significant impact in students’ achievement [10–12]. Manipulative provides opportunity for the students, especially in the thinking stage of operational concrete to understand a concept by supporting the process of assimilation and accommodation [6].

On the other hand, simply using manipulative does not guarantee that the learning would be effective [10]. There are many factors which must be considered in using the manipulatives. Teacher needs to consider the relevancy with material or concept [13], the goal of using the manipulative (game for fun or finding a concept) [5], the learning approach, and the students’ characteristics [10]. In some cases, one or two considerations above are failed to be well understood by teacher such that the learning goals could not be achieved optimally.

Despite on the challenge of the effectiveness, the use of manipulative is still promising to help students construct their knowledge through meaningful activities. That is why mathematics teachers
and pre-service teachers need to develop the appropriate manipulatives to support their professional skill.

The mathematics education department of Universitas Ahmad Dahlan, Indonesia, is one that have a huge attention in developing manipulatives to support mathematics learning. During the last three years, there are many manipulatives produced in its laboratory based on the need analysis of the material and the current curriculum. This article aims to provide an insight to the characteristics of manipulatives produced in the mathematics laboratory which is then can be used to track the potential effectiveness and challenge for the use of the manipulatives at school.

2. Method
This research was an exploratory case study towards the characteristics of manipulatives produced by the mathematics education laboratory in Universitas Ahmad Dahlan. As the nature of exploratory case study which allows data collection conducted prior to the definition the case [14, 15], we explored the manipulatives before we get down to the characteristics as the focus topic.

All manipulatives produced in the latest three years were documented, classified, and then their characteristics were observed upon the appropriate use for mathematics learning. The documentation result is also confirmed by a secondary data taken from the laboratory. It was a record of manipulative register in which all manipulatives produced should be registered before displayed and used in the future. The data was then classified based on the characteristics found. The classification also considered what kind of learning approach is compatible with the use of the manipulatives. Furthermore, interviews to the corresponding lecturers who advised the production of the manipulatives were also conducted to gain more explanation towards the findings.

3. Results and discussion
The laboratory record gave an early information that there are 127 manipulative products registered during the latest three years. Further documentation was conducted and successfully found all of the products with various conditions. It was only 74.8% of them which can be accessed and used by students or lecturers in appropriate condition, while the rest are damaged. Most of the damaged products are the ones produced three years ago, having paperboard or styrofoam as its material. It leads to the first finding that the manipulative products with such materials only last no more than three years for appropriate use. An interview addressed to the lecturer who advised for the production, we refer to as Subject 1, confirming that the durability of the products is an important issue they concern currently.

“I myself admit that three years ago I did not concern in limiting the specification of material. Thus, some students used fragile material and thus it was not durable. For now and then, I have set the specification of material which could maintain the durability of the products, such as plastic coated paperboard instead of paperboard only” (Subject 1 testimony).

The manipulatives in the laboratory are products of two subjects. They are the Mathematics Learning Media Development and the Learning Multimedia. The former produces concrete products while the later produces virtual manipulatives (CDs containing Macromedia Flash files). The documentation of the data can be seen in Table 1.

| Table 1. Manipulative products based on its type |
|-------------------------------------------------|
| Type                                    | Percentage Number of Products |
| Concrete product | 36.2% |
| Virtual product   | 63.8% |

Based on the Table 1, most of manipulatives produced were the virtual ones, as the product of the Learning Multimedia subject. Furthermore, the observation towards the products’ characteristics found
that the products can be categorized into three types of manipulatives, (1) the constructivist manipulative, (2) the informative teaching aid/model, and (3) the mathematics fun games. The result of classification can be seen in Table 2.

**Table 2.** Manipulative products based on its characteristics

| Type                     | Percentage | Number of Products |
|--------------------------|------------|--------------------|
| Constructivist manipulative | 10.2%      |                    |
| Informative teaching aid/model | 81.9%      |                    |
| Mathematics fun games    | 7.9%       |                    |

Based on the Table 2, most of the manipulative products can be categorized as informative teaching aid, while the other two categories share the rest percentage. Each type of manipulative has its own characteristics. The characteristics also determine the compatibility of its use in the mathematics learning.

### 3.1. Constructivist manipulative

Constructivist manipulative, or usually just called manipulative, refers to manipulative definition in general. Manipulative material is defined as concrete object representing mathematical ideas which is designed such that students could understand mathematical concepts by manipulating it [8]. It can be touched, moved around, and appeals to several senses including the socio-cultural needs of the students [7]. By using the keywords of definition, we can understand the important attribution that the object shall be able to be manipulated by the students. On the contrary, if the students cannot do any manipulation towards the model, then it is not categorized in this classification.

In the mathematics education laboratory, there is only 10.2% of the manipulatives produced. We tried to find out whether this number is a “by design” phenomena by interviewing the lecturers. Two lecturers were interviewed, the Subject 1 and Subject 2, and they gave quite similar answer. They said that they let the students analyze the material and design the manipulative based on their creativity.

> “First, I instructed the students (mathematics education department) to work in group, analyzing the material, and making assumption if particular material is considered difficult by students (primary or secondary school). Then, they designed the relevant manipulative and my role is controlling that they will not mislead the mathematical concept” (Subject 1).

> “Sometimes I ask them to pay attention on some materials which are usually considered problematic for secondary school students. Then, they work in group designing what manipulative would be useful for learning” (Subject 2).

From the interview above, there is no control on what kind of manipulative the students will produce. This phenomena will lead to further question about what makes the manipulative effective for learning, and do they aware of the students’ specific behavior while learning by utilizing manipulative.

A simple example of the manipulative in this category is presented in Figure 1.
If we look at the Figure 1, the logic pipe can be categorized as constructivist manipulative since the student could manipulate the model in some designed conditions. The knot of the pipe represents the true or false of statement. If it is “ON”, the water flows, which means “TRUE” statement, while “OFF” means “FALSE” statement. The compound sentence represented by Figure 1 is disjunction. The combination of “ON” and “OFF” conditions will lead to the truth value of the disjunction. If the water flows, then the disjunction is “TRUE”, and vice versa. Using this logic pipe, students can find the table of truth value of a disjunction.

The use of this manipulative indeed requires attention from teachers to aware of the goals and approach of learning. Many studies refers the use of manipulative to the idea of constructivism addressed by Piaget and Vigotsky, who suggest that students need to construct knowledge by themselves through questioning activities and the teacher provides assistance which the students need in a form of scaffolding [5]. In here, manipulative is suitable with constructivism learning and contextual learning where the students could find the concepts through discussion and interaction with the manipulative [3]. Furthermore, manipulative also support the heuristic of problem solving which is also problematic in the context of Indonesian secondary school students as reported by some studies [16]. The other issue is that it is essential for teachers to make the students realize the manipulatives as tools instead of as toys. If manipulatives are regarded as "toys", students will think that all they can do is playing with the manipulative rather than learning something which the manipulatives bring to them.

3.2. Informative teaching aid/model

This type of manipulative often create misunderstanding among educators. By definition in Section 3.1, this type cannot be categorized as manipulative. Therefore, we can use the term of informative teaching aid. The idea of the teaching aid is simply giving information to the students. It is a ready-made object which the students learn the concept as it is delivered.

However, some studies still refers this teaching aid, especially the virtual one, as manipulative though there is no significant manipulation conducted by students [17]. A virtual manipulative is an interactive visual representation of a mathematical object which allow students to construct knowledge [7]. If students just click to see the next material in a smartphone or giving a response to get a “Yes” or “No” feedback, then the media cannot be categorized as virtual manipulative. These media mainly adopt the expository method and provide a virtual presentation of material which then could substitute
conventional book. This media invites many sceptical idea that it does not accommodate the idea of constructivism attributed in manipulative [7].

This phenomena is also found in the multimedia produced by mathematics laboratory of Universitas Ahmad Dahlan, most of the multimedia are categorized as informative virtual teaching aid. They used Flash application to just deliver the material and the information is given by clicking some buttons provided. In fact, some applications can be utilized to develop appropriate virtual manipulative, such as Geogebra and Cabri 3D which allow students to manipulate figures [17].

An example of informative teaching aid is presented in Figure 2.

![Figure 2. Magic angle as an informative teaching aid](image)

The teaching aid presented in Figure 2 is used to give ready-made information to the students. If the students turn on the button, then various LED lamp in some indicated angles will light up. A legend information of the light meaning is provided, such as green light represent supplementary angles, or blue light means alternate interior angles, and so on. There is no possibility for students to manipulate the teaching aid.

The reasons why manipulative use is suggested by many educators is that it is promising to engage the students with constructivism which make sure the retention of information. However, inapropriate design or implementation of manipulative will cause the use of manipulative become ineffective since the students do not really understand that they construct a concept [9]. They are just being told and are risky to forget the material.

3.3. Mathematics fun games
The third type of manipulative produced in mathematics laboratory of Universitas Ahmad Dahlan is mathematics fun games. Yet, according to the definition, this type cannot be categorized as manipulative, either. It is because the students only play and follow certain rule without doing any manipulation.

This fun games is useful for drilling and promoting joyful learning. However, it has challenge for the concept retention. Students should not lost focus that they learn something (concept) [11, 18, 19]. Teachers who believe manipulatives are just used for fun are not going to genuinely incorporate manipulatives and the concepts they were meant to convey into their instructional lessons [2]. Thus, the use of mathematics fun games should also consider the learning goals.
An example of mathematics fun games in mathematics laboratory is presented in Figure 3.

Figure 3. Roulette games for quadratic equation material

The Figure 3 shows a roulette games for quadratic equation material. Students turn the roulette and get topic pointed by the arrow. They have to do problems or challenge related to the topic and then confirmed by a key answer provided in an envelope. This is a drilling method by using a fun game media.

4. Conclusion
Manipulative shall be used properly according to the goal and the approach of learning. The success of manipulative use depends on the appropriate use in the mathematics learning. Therefore, we have to consider the characteristics of the manipulative which will be developed. In the mathematics education laboratory of Universitas Ahmad Dahlan, there are various manipulatives produced every year. We found that there are types found, namely: (1) the constructivist manipulative, (2) the informative teaching aid, and (3) the mathematics fun games. The constructivist manipulative allows the students to do manipulation on the dynamic model to construct knowledge by themselves. The informative teaching aid simply gives a ready-made information to the students. It is suitable for expository learning. The mathematics fun games allow the students to play and have a drilling upon certain exercise or problems. For future research, it is potential to test the effectiveness of the three categories above to confirm the findings and also the state of the art of this research among the other relevant studies.

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