Non-probabilistic models for in-plane elastic properties of cellular hexagonal honeycomb cores involving imprecise parameters

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Abstract
Inherent imprecisions present in the basic parameters of cellular honeycomb cores, such as the cell angle, the material properties, and the geometric parameters, need to be considered in the analysis and design to meet the high-performance requirements. In this paper, imprecisions associated with the basic parameters of honeycomb cores are considered. Non-probabilistic models for the in-plane elastic properties of hexagonal honeycomb cores are developed in which the imprecisely defined input and response parameters are represented by only their mean values and variations without the requirement of knowing the probability density distributions of the imprecise parameters as is required for probabilistic methods. Thus, the proposed models predict not only the nominal values of the in-plane elastic properties but also their variations from the respective mean values. The applicability of the proposed models is demonstrated by considering the analysis of the in-plane elastic properties of a honeycomb core made of aluminum 5052-H-32 in which the core material properties are defined by their mean values and variations. The results show that realistic variations of the in-plane elastic properties are obtained using the proposed non-probabilistic models. The sensitivity of the in-plane elastic properties to the imprecisions present in each basic parameter is also investigated.

Keywords
Elastic properties, honeycomb cores, non-probabilistic, imprecisions, universal grey

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Introduction
Cellular materials are widely used as cores in sandwich structures due to their distinct characteristics such as having an extra lightweight and high stiffness and strength values. These characteristics have led designers to use cellular materials in many engineering applications, such as aerospace and automotive structural applications, that require high performance and reliability. Hexagonal honeycomb, balsa, foam, and corrugated cores are typical cellular materials that can be used as cores in sandwich structures. Hexagonal honeycombs, which epitomize two-dimensional cellular materials, are commonly used as cores because of their low weight, high bending strength, and flexural rigidity. In recent decades, many closed-form deterministic analytical models have been developed to determine the mechanical properties of hexagonal honeycomb cores. The dynamic and failure of honeycomb core sandwich panels

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with high-velocity impact were studied by Sun et al.\textsuperscript{4} using experimental and numerical methods. Zhang et al.\textsuperscript{5} conducted an experimental study to investigate the mechanical behavior of hexagonal honeycombs under quasi-static combined compression-shear loading. Ha et al.\textsuperscript{6} proposed a new honeycomb, known as bio-inspired honeycomb, and studied the geometrical configurations on the energy absorption of bio-inspired honeycomb sandwich panels. He et al.\textsuperscript{7} presented a comprehensive study using numerical, experimental, and theoretical method on the residual flexural properties honeycomb core sandwich panels that are subjected to low-velocity impact. Sun et al.\textsuperscript{8} investigated the influences of some structural parameters, such as face sheet thickness, wall and cell geometries, on the behavior of honeycomb cores sandwich panels subjected to low-velocity impact. However, hexagonal honeycomb cores exhibit disorders in the geometries and properties of materials from which the cores are made. For example, the material properties of the core material, cell angle, and geometry are inherently subjected to irregularities and imprecisions during the formation and manufacturing of cores. Thus, using the available deterministic models for mechanical properties cannot assure the high performance and reliability of hexagonal honeycombs because the inherent scatter in the basic parameters is neglected. The variabilities associated with engineering system responses with stringent design requirements need to be known in order to achieve a desirable performance. Hence, variations must be considered when determining the response of hexagonal honeycomb cores so that improved and more realistic response representations can be obtained.

Several studies have considered the variabilities associated with cellular materials; however, these have often been in terms of their stochastic/probabilistic nature. Zhu et al.\textsuperscript{9} studied the effects of irregularities in the cell size and shape of foams on their elastic properties by developing models for the finite element analysis of random Voronoi foams. Hohe and Becker\textsuperscript{10} and Hohe and Bechmann\textsuperscript{11} presented a numerical analysis using stochastic procedures to investigate the effective mechanical properties of two-dimensional cellular materials taking into account the presence of microstructural irregularities. Flores et al.\textsuperscript{12} developed a multi-scale finite element method using the Gaussian process emulator to study the stochastic mechanical response of structures with honeycomb cores. The probabilistic distribution of the residual strength of impacted sandwich structures with honeycomb cores was estimated by Kim et al.\textsuperscript{13} in which the mean and variance values of the original properties were used in the probabilistic analysis. Parsi et al.\textsuperscript{14} developed a probabilistic model for predicting the dynamic contact force of honeycomb structures subjected to impacts at low velocities. Probabilistic methods have also been used in analysis and design in many engineering fields, such as vibrations,\textsuperscript{15–18} fracture mechanics,\textsuperscript{19–22} computational fluid mechanics,\textsuperscript{23,24} and structural stability.\textsuperscript{25,26}

A stochastic/probabilistic analysis represents each imprecise parameter as a random variable that follows a specific probability distribution. In real-life applications, the full probabilistic information of imprecise quantities is not readily available, which leads to unknown probability density distributions for the imprecisely defined quantities. Thus, an exact stochastic/probabilistic analysis becomes unpractical in the absence of exact probability density distributions of each imprecisely defined parameter, as in most engineering applications. However, due to readily available maximum and minimum values, mean values, and variations are commonly used parameters to represent the imprecisions associated with quantities that are subjected to imprecision. Several non-probabilistic methods have been proposed to model imprecisions associated with engineering systems where the probability density distributions of imprecise parameters are not needed\textsuperscript{27–33} Recently, universal grey mathematics has been used in many studies to quantify imprecisions associated with engineering systems.\textsuperscript{34–38} These studies have shown that universal grey mathematics has the ability to realistically quantify imprecisions with less computational effort and without overestimating variations in response parameters. Universal grey mathematics only requires the mean values and the corresponding variations of imprecisely defined parameters to model imprecisions in imprecise systems. The superiority of this method lies in the satisfaction of the distributive property, which leads to the realistic quantification of imprecisions in the response parameters without overestimation.

This work presents the modeling and analysis of the in-plane elastic properties of hexagonal honeycomb cores when the basic quantities are defined in terms of mean values and variations. Non-probabilistic models are presented based on universal grey mathematics for the modeling of imprecisions in the response quantities. Unlike probabilistic models, which require the probability distribution density of imprecise parameters to be exactly known, the proposed models can express the in-plane elastic properties of a hexagonal honeycomb core in terms of their mean values and variations with the knowledge of only the mean values and variations of the basic input quantities. Additionally, a sensitivity analysis is performed in which the influence of the variation of each basic variable on the variations of the response quantities is investigated.

**In-plane elastic properties of cellular hexagonal honeycomb cores**

Several approaches are available in the literature to estimate the elastic properties of cellular honeycomb...
Imprecise models for in-plane elastic properties of a honeycomb core

Universal grey mathematics is used to predict the in-plane elastic properties of a honeycomb core in the presence of imprecise basic parameters. The arithmetic operations associated with the universal grey number theory are presented by Rao and Liu\cite{Rao} where each imprecise quantity $X$ is represented by its extreme values; the lower bound $\underline{X}$ and upper bound $\overline{X}$. Using $m_X = \frac{\underline{X} + \overline{X}}{2}$ and $\delta_X = \frac{\overline{X} - \underline{X}}{m_X} \times 100$, the extreme values can be represented by the mean value and the percent variation of the imprecise quantity as $X = m_X(1 - 0.01\delta_X)$ and $\overline{X} = m_X(1 + 0.01\delta_X)$. Thus, in this paper, the arithmetic operations are remodeled in which each imprecise quantity $X$ is represented by its mean value $m_X$ and the percent variation $\delta_X$.

The basic input quantities are defined in terms of their mean values and the percent variations. Consequently, the response quantities of a honeycomb core are also represented by their mean values and percent variations. In this work, the imprecise parameter $X$ is represented in the form

$$X = (m_X, \delta_X)$$

where $m_X$ and $\delta_X$ represent the mean value and the respective percent variation of the imprecise parameter $X$, respectively. Then, the analytical expressions defined in equations (1)–(5) in conjunction with universal grey mathematics are used to predict the response quantities, namely, the in-plane Young’s moduli ($E_1^*$ and $E_2^*$), the in-plane shear modulus ($G_{12}^*$), and the Poisson’s ratios ($\nu_{12}^*$ and $\nu_{21}^*$), in terms of their mean values and respective percent variations.

However, the aforementioned expressions do not consider the inherent imprecisions in the basic quantities, namely, the wall length $l$, wall height $h$, wall thickness $t$, cell angle $\theta$, and cell wall material properties. In real-life applications, during the manufacturing processes, environmental effects and other sources of imprecisions, the basic quantities are subjected to imprecisions which lead to imprecise elastic properties of honeycomb cores. To achieve high performance and reliability, the imprecisions in the basic input quantities need to be considered in the analysis. In the next section, the imprecisions associated with the basic parameters are considered by proposing analytical expressions for the in-plane elastic properties of a honeycomb core using the universal grey mathematics.

$$\begin{align*}
E_1^* &= E_l \left(\frac{l}{h} + \frac{\sin \theta}{\cos \theta}\right) \\
E_2^* &= E_s \left(\frac{t}{h} + \frac{\sin \theta}{\cos \theta}\right) \\
G_{12}^* &= G_{12} \left(\frac{h}{t} + \frac{\sin \theta}{\cos \theta}\right) \\
\nu_{12}^* &= \frac{\cos^2 \theta}{\left(\frac{h}{t} + \frac{\sin \theta}{\cos \theta}\right) \sin \theta} \\
\nu_{21}^* &= \frac{\left(\frac{h}{t} + \frac{\sin \theta}{\cos \theta}\right) \sin \theta}{\cos^2 \theta}
\end{align*}$$

where $m_X$ and $\delta_X$ represent the mean value and the respective percent variation of the imprecise parameter $X$, respectively. Then, the analytical expressions defined in equations (1)–(5) in conjunction with universal grey mathematics are used to predict the response quantities, namely, the in-plane Young’s moduli ($E_1^*$ and $E_2^*$), the in-plane shear modulus ($G_{12}^*$), and the Poisson’s ratios ($\nu_{12}^*$ and $\nu_{21}^*$), in terms of their mean values and respective percent variations.

Let $X$ and $Y$ be imprecise parameters that are defined in terms of their mean value $m$ and percent variation $\delta$ as $X = (m_X, \delta_X)$ and $Y = (m_Y, \delta_Y)$, respectively. Applying universal grey mathematics, the arithmetic operations that are required between any imprecise parameters, $X$ and $Y$, are given by:

$$X + Y = (m_X + m_Y, \delta_X + \delta_Y)$$

$$X \times Y = (m_X + m_Y, m_X m_Y)$$

$$\frac{X}{Y} = (m_X + m_Y, \delta_X + \delta_Y)$$

which lead to imprecise elastic properties of honeycomb cores. To achieve high performance and reliability, the imprecisions in the basic input quantities need to be considered in the analysis. In the next section, the imprecisions associated with the basic parameters are considered by proposing analytical expressions for the in-plane elastic properties of a honeycomb core using the universal grey mathematics.
The imprecise forms of the models associated with the in-plane elastic properties of a hexagonal honeycomb core can be derived using the basic arithmetic operations defined in equations (7)–(10) in conjunction with the analytical deterministic expressions given in equations (1)–(5). The nature of the arithmetic operations (equations (7)–(10)) and the conditions associated with the subtraction and division operations (equations (8) and (10)) lead to several and long expressions depending on the nature of the desired in-plane elastic model. For example, depending on the numerical values of the mean values and the respective percent variations of the basic quantities, the derivation of the imprecise form of the Young’s modulus of a honeycomb core in direction 1 $E_{1}^{1}$ leads to eight imprecise models that can be used because of the presence of three division operations in equation (1) which are $\frac{\delta}{\cos\theta}$, $\frac{1}{\cos\theta}$, and $\frac{\sin\theta}{(\frac{\delta}{\cos\theta})\sin\theta}$ in which each division operation leads to two possible formulas due to the two conditions associated with equation (10). Thus, a computer program is developed using MATLAB to consider all possible non-probabilistic models for predicting the mean values and percent variations of the in-plane elastic properties of a honeycomb core where the process of the MATLAB code can be described as:

1. Enter the mean values and percent variations of all basic parameters.
2. Call (function E1), (function E2), (function G12), (function v12), or (function v21) to calculate the desired property.
3. Use (function addition), (function subtraction), (function multiplication), and (function division) to perform each arithmetic operation present in each of the formulas defined in equations (1)–(5).

To illustrate the process, the computations of the mean value and the percent variation of the Young’s modulus in direction 2, $E_{2}= (m_{E_{2}}, \bar{\delta_{E_{2}}})$, of a hexagonal honeycomb core that is made of aluminum 5052-H-32 is considered. By applying the imprecise parameters of the basic quantities of the honeycomb core, $E_{s} = (70.3 \times 10^{3} \text{MPa}, \pm 1.5\%)$, $l = (3.8 \text{mm}, \pm 0.5\%)$, $h = (5.7 \text{mm}, \pm 1\%)$, $t = (0.08 \text{mm}, \pm 2\%)$, and $\theta = (30^\circ, \pm 1.5\%)$, the parameters, the mean value $m_{E_{2}}$ and the percent variation $\bar{\delta_{E_{2}}}$, which describe the imprecision associated with the Young’s modulus in direction 2, $E_{2}$, can be computed as follows

\[
m_{E_{2}} = \frac{m_{E_{1}}(\frac{\delta}{\cos\theta})}{m_{\cos\theta}(1 - 0.01^2 \delta_{\cos\theta})}
\]

\[
\left[1 - 0.01^2 \delta_{(\frac{\delta}{\cos\theta})}\delta_{\cos\theta} + 0.01^2 \delta_{E_{2}}(\delta_{\cos\theta} - \delta_{(\frac{\delta}{\cos\theta})})\right]
\]

\[
= 2.0231 \text{ MPa}
\]

\[
\bar{\delta}_{E_{2}} = \frac{\delta_{E_{2}}(\frac{\delta}{\cos\theta}) + \delta_{\cos\theta}(1 - 0.01^2 \delta_{E_{2}}(\frac{\delta}{\cos\theta})\delta_{\cos\theta} + 0.01^2 \delta_{E_{2}}(\delta_{\cos\theta} - \delta_{(\frac{\delta}{\cos\theta})}))}{1 - 0.01^2 \delta_{(\frac{\delta}{\cos\theta})}\delta_{\cos\theta} + 0.01^2 \delta_{E_{2}}(\delta_{\cos\theta} - \delta_{(\frac{\delta}{\cos\theta})})}
\]

\[
= \pm 6.6364\%
\]


\[
E_{c}(i) = \left( m_{E}, \delta_{E}(i) \right) = \left( m_{E}, \delta_{E}(i) \right)
\]

\[
= \left( m_{E}, m_{E} \left( 1 + 0.01^2 \times 3 \delta_{E}^2 + 0.01^4 \delta_{E} \delta_{E} + 0.01^2 \times 3 \delta_{E} \delta_{E} \right), \right)
\]

\[
\frac{1 + 0.01^2 \times 3 \delta_{E}^2 + \delta_{E} \delta_{E} + 0.01^2 \times 3 \delta_{E} \delta_{E} + 0.01^2 \times 3 \delta_{E} \delta_{E}}{1 + 0.01^2 \times 3 \delta_{E}^2 + \delta_{E} \delta_{E} + 0.01^2 \times 3 \delta_{E} \delta_{E} + 0.01^2 \times 3 \delta_{E} \delta_{E}}
\]

\[= (0.6567, \pm 5.9937\%)
\]

\[
h \sin \theta = \left( m_{\cos \theta} \cos \theta, \delta_{\cos \theta} \cos \theta \right)
\]

\[
= \left( m_{\cos \theta} \cos \theta, \frac{m_{h \cos \theta} \cos \theta \delta_{\cos \theta} \cos \theta}{m_{h \cos \theta} \cos \theta + m_{\sin \theta} \delta_{\sin \theta}} \right)
\]

\[= (2, \pm 0.7151\%)
\]

\[
\cos^2 \theta = \left( m_{\cos \theta} \cos \theta, \delta_{\cos \theta} \cos \theta \right)
\]

\[
= \left( m_{\cos \theta} \cos \theta \right)^2 \frac{0.01^2 \delta_{\cos \theta} \cos \theta}{1 + 0.01^2 \times 3 \delta_{\cos \theta} \cos \theta}
\]

\[= (0.6495, \pm 1.3603\%)
\]

\[
t \sin \theta = \left( m_{\sin \theta} \sin \theta, \delta_{\sin \theta} \sin \theta \right)
\]

\[
= \left( m_{\sin \theta} \sin \theta, \frac{m_{h \sin \theta} \cos \theta \sin \theta}{m_{h \sin \theta} \cos \theta + m_{\sin \theta} \delta_{\sin \theta}} \right)
\]

\[= (0.0211, \pm 1.5002\%)
\]

\[
h \sin \theta = \left( m_{h \sin \theta} \sin \theta, \delta_{h \sin \theta} \sin \theta \right)
\]

\[
= \left( m_{h \sin \theta} \sin \theta, \frac{m_{h \sin \theta} \cos \theta \sin \theta}{m_{h \sin \theta} \cos \theta + m_{\sin \theta} \delta_{\sin \theta}} \right)
\]

\[= (1.5, \pm 0.5\%)
\]

\[
\cos \theta = \left( m_{\cos \theta} \cos \theta, \delta_{\cos \theta} \cos \theta \right)
\]

\[
= \left( \cos \left( m_{0} \right), \frac{\cos \left( m_{0} \right) \cos \left( 1 - 0.01 \delta_{\theta} \right) - \cos \left( 1 - 0.01 \delta_{\theta} \right)}{0.02 \cos \left( m_{0} \right)} \right)
\]

\[= (0.8660, \pm 0.4535\%)
\]

\[
\sin \theta = \left( m_{\sin \theta} \sin \theta, \delta_{\sin \theta} \sin \theta \right)
\]

\[
= \left( \sin \left( m_{0} \right), \frac{\sin \left( m_{0} \right) \cos \left( 1 - 0.01 \delta_{\theta} \right) - \sin \left( 1 - 0.01 \delta_{\theta} \right)}{0.02 \sin \left( m_{0} \right)} \right)
\]

\[= (0.5, \pm 1.3604\%)
\]

Similarly, the mean values and the percent variations of the other in-plane elastic properties of a hexagonal honeycomb core, $E_{1}$, $G_{12}$, $\nu_{12}$, and $\nu_{21}$, can be computed by using the basic arithmetic operations defined in equations (7)–(10) in conjunction with their respective deterministic expressions. Also, for comparison, the combinatorial method is used to find the mean values and percent variations in the in-plane elastic properties of honeycomb cores in the presence of imprecise input parameters. The combinatorial method determines the extreme values of the response parameters by, first, considering all combinations of the extreme values of the basic parameter and determining the response parameters for all combinations, then the minimum and maximum of the obtained possible response values are taken as the lower bound $X$ and upper bound $X$, respectively. However, the combinatorial method is not practical due the large number of analyses needed to find the extreme values of the response parameters. In the following section, the imprecise behavior of the in-plane elastic properties of a hexagonal honeycomb core made of aluminum 5052-H-32 is investigated, where the basic parameters of wall length $l$, wall height $h$, wall thickness $t$, cell angle $\theta$, and the cell wall material properties are considered to be in the form of their mean values and percent variations.

**Numerical example**

The analysis of a hexagonal honeycomb core made of aluminum 5052-H-32 is considered to demonstrate the applicability of the proposed imprecise models of the in-plane elastic properties. The analysis is performed using the basic input quantities shown in Table 1 in which three different $\alpha = h/l$ ratios are considered, namely, $\alpha = 1$, $\alpha = 1.5$, and $\alpha = 2$. To investigate the behavior of the response quantities in the presence of imprecisions, five cases are considered in which different variations are introduced in the basic input quantities in each case. The sensitivity of the in-plane elastic properties to each imprecise input parameter is studied over a variation range of 0\% to ±5\% from the respective mean value. In addition, the influence of the cell angle on the response quantities in the presence of imprecisions is investigated.

Tables 2 to 5 present the results obtained for the in-plane elastic properties of a hexagonal honeycomb core considering five cases of imprecisely defined input parameters. The results show that the Young's modulus in direction 2, $E_{2}$, the Poisson's ratio, $\nu_{12}$, and the shear modulus, $G_{12}$, have acceptable degrees of variation with respect to the variations introduced in the basic input parameters in all of the cases. However, the Young's modulus in direction 1, $E_{1}$, exhibits a realistic variation but approximately doubled the introduced variations in
Table 1. Basic input parameters in terms of mean values and variations.

| Parameter | Mean value $m$ | Variation, $\delta$ |
|-----------|----------------|---------------------|
|           | $\text{Case 1}$ | $\text{Case 2 (%)}$ | $\text{Case 3 (%)}$ | $\text{Case 4 (%)}$ | $\text{Case 5 (%)}$ |
| $E$       | 70,300         | $0 \pm 1$          | $\pm 2$          | $\pm 3$          | $\pm 1.5$          |
| $l$       | 3.8            | $0 \pm 1$          | $\pm 2$          | $\pm 3$          | $\pm 0.5$          |
| $h$       | $\text{ad}$   | $0 \pm 1$          | $\pm 2$          | $\pm 3$          | $\pm 1$           |
| $t$       | 0.08           | $0 \pm 1$          | $\pm 2$          | $\pm 3$          | $\pm 2$           |
| $\theta$  | 30             | $0 \pm 1$          | $\pm 2$          | $\pm 3$          | $\pm 1.5$         |

Table 2. In-plane elastic properties in terms of mean values and variations for $\alpha = h/l = 1$.

| Case   | Response parameter | $E_1$ | $E_2$ | $\nu_{12}$ | $\nu_{21}$ | $G_{12}$ |
|--------|--------------------|-------|-------|------------|------------|----------|
| Case 1 | Mean, $m$          | 1.5149| 1.5149| 1.0000     | 1.0000     | 0.3787   |
|        | Variation, $\delta$ | $\pm 0$| $\pm 0$| $\pm 0$     | $\pm 0$     | $\pm 0$   |
| Case 2 | Mean, $m$          | 1.5155| 1.5151| 1.0000     | 1.0000     | 0.3787   |
|        | Variation, $\delta$ | $\pm 2.8132$| $\pm 1.6045$| $\pm 0.6046$| $\pm 0.6046$| $\pm 1$  |
| Case 3 | Mean, $m$          | 1.5176| 1.5166| 1.0002     | 1.0000     | 0.3787   |
|        | Variation, $\delta$ | $\pm 5.6224$| $\pm 3.2084$| $\pm 1.2094$| $\pm 1.2094$| $\pm 2$  |
| Case 4 | Mean, $m$          | 1.5210| 1.5166| 1.0004     | 0.9999     | 0.3787   |
|        | Variation, $\delta$ | $\pm 8.4240$| $\pm 4.8113$| $\pm 1.8143$| $\pm 1.8143$| $\pm 3.0001$  |
| Case 5 | Mean, $m$          | 1.5199| 1.5168| 1.0002     | 1.0000     | 0.3793   |
|        | Variation, $\delta$ | $\pm 8.3343$| $\pm 6.0669$| $\pm 1.2403$| $\pm 1.2403$| $\pm 6.4917$  |

Table 3. In-plane elastic properties using combinatorial method in terms of mean values and variations for $\alpha = h/l = 1$.

| Case   | Response parameter | $E_1$ | $E_2$ | $\nu_{12}$ | $\nu_{21}$ | $G_{12}$ |
|--------|--------------------|-------|-------|------------|------------|----------|
| Case 1 | Mean, $m$          | 1.5149| 1.5149| 1.0000     | 1.0000     | 0.3787   |
|        | Variation, $\delta$ | $\pm 0$| $\pm 0$| $\pm 0$     | $\pm 0$     | $\pm 0$   |
| Case 2 | Mean, $m$          | 1.5217| 1.5218| 1.0005     | 1.0005     | 0.3798   |
|        | Variation, $\delta$ | $\pm 9.3910$| $\pm 9.5140$| $\pm 3.1462$| $\pm 3.1462$| $\pm 7.5902$  |
| Case 3 | Mean, $m$          | 1.5421| 1.5425| 1.0021     | 1.0019     | 0.3830   |
|        | Variation, $\delta$ | $\pm 12.890$| $\pm 11.760$| $\pm 3.7192$| $\pm 3.7192$| $\pm 10.864$  |
| Case 4 | Mean, $m$          | 1.5765| 1.5774| 1.0047     | 1.0043     | 0.3885   |
|        | Variation, $\delta$ | $\pm 27.534$| $\pm 27.878$| $\pm 9.4163$| $\pm 9.4163$| $\pm 22.432$  |
| Case 5 | Mean, $m$          | 1.5270| 1.5245| 1.0008     | 1.0006     | 0.3808   |
|        | Variation, $\delta$ | $\pm 12.890$| $\pm 11.760$| $\pm 3.7192$| $\pm 3.7192$| $\pm 10.864$  |

The basic parameters. For example, as shown in Table 2, in the case of $h/l = 1$, when variations of $\pm 2\%$ are introduced in all the basic parameters (Case 3), the extreme values of $E_2$, $\nu_{12}$, and $G_{12}$ deviate from the respective mean values by $\pm 3.2084\%$, $\pm 1.2094\%$, and $\pm 2\%$, respectively. However, the variation corresponding to $E_1$ is found to be $\pm 5.6224\%$. The results presented in Table 2 are compared with the ones predicted by the combinatorial method which are given in Table 3. Clearly, as shown in Tables 2 and 3, the variations predicted by the present models secure sharper variations than the variations predicted the combinatorial method. Similar behavior is observed for $h/l = 1.5$ (Table 4) and $h/l = 2$ (Table 5) in all response quantities. The reason for this behavior is that the expression used to compute the Young’s modulus (equation (1)) involved a greater number of imprecisely defined parameters than were used in the computations of the other response quantities (equations (2)–(5)). Additionally, as the ratio of $h/l$ increases, the variations corresponding to $E_2$ and $G_{12}$ also increase. However, the variations predicted for $E_1$ and $\nu_{12}$ decrease as the ratio of $h/l$ increases. For example, in the case of $h/l = 1$ (Table 2), when $\pm 3\%$ is introduced in all the basic quantities (Case 4), the variations corresponding to $E_1$, $E_2$, $\nu_{12}$, and $G_{12}$ are $\pm 8.4240\%$, $\pm 4.8113\%$, $\pm 1.8143\%$, and $\pm 3\%$, respectively. When $h/l = 1.5$ (Table 4), $E_1$, $E_2$, $\nu_{12}$, and $G_{12}$ deviate from their
respective mean values by $\pm 8.1988\%$, $\pm 5.0375\%$, $\pm 1.5876\%$, and $\pm 3.2266\%$, respectively.

Figure 2 shows the sensitivities of the in-plane elastic properties, where $h/l = 1$, to each input parameter, namely, wall length $l$, wall height $h$, wall thickness $t$, cell angle $\theta$, and the cell wall elastic modulus $E_w$, when variations of $\pm 0\%$ to $\pm 5\%$ are introduced in each input quantity. Compared to the other input quantities, the imprecisions associated with the wall length $l$ and thickness $t$ of the cell highly influence the variations in the elastic moduli $E_1$ and $E_2$, and the shear modulus $G_{12}$, of the honeycomb core. This is because the parameters $l$ and $t$ appear more often in the expressions of $E_1$, $E_2$, and $G_{12}$ compared to the other input parameters. Furthermore, the nonlinearity associated with the parameters $l$ and $t$ in the expressions of the response quantities is a cause of the high sensitivity behavior. For example, when a variation of $\pm 2\%$ is present only in the wall length $l$ of the cell, the response quantity $E_1$ varies by $\pm 7.3213\%$. By contrast, the elastic modulus $E_1$ exhibits a variation of $\pm 2\%$ when only the cell wall elastic modulus $E_w$ is imprecisely defined. Figure 2 also shows that the variations associated with the Poisson’s ratio $\nu_{21}$ are influenced by only the variations presented in the wall height $h$ and cell angle $\theta$. However, the Poisson’s ratio is not affected by the variations presented in the other input quantities, namely, wall length $l$, wall thickness $t$, and the cell wall elastic modulus $E_w$.

Figures 3 to 6 show the behavior of the variations associated with the in-plane elastic properties for different values of cell angle $\theta$, considering three cases of imprecisions in all input parameters: $\pm 1\%$, $\pm 2\%$, and $\pm 3\%$. As can be seen in Figure 3, the variations associated with the elastic modulus $E_1$ decrease as the cell angle increases from $10^\circ$ to $57.47^\circ$, then a sharp increase is observed as the cell angle increases further. This behavior is due to the presence of the trigonometric terms in the respective expressions. Figure 4 shows that the variations corresponding to the elastic modulus $E_2$ increase with the cell angle for all cases $\delta_{all\ basic\ parameters} = 1, 2,$ and $3$. Regarding the variation in the Poisson’s ratio $\nu_{21}$, a behavior similar to that seen in the elastic modulus $E_1$ is noticed (shown in Figure 5) in which the variations decrease when the cell angle changes from $10^\circ$ to $39.82^\circ$. The variations significantly increase when the cell angle further increases.
As shown in Figure 6, the modulus of rigidity $G_{12}$ exhibits variations approximately consistent with those introduced in all the basic quantities up to a cell angle of $30^\circ$.

**Conclusion**

This work, for the first time, introduces the modeling and analysis of imprecisions in the in-plane elastic
properties of a hexagonal honeycomb core with imprecisely defined parameters without the requirement of knowing the exact probability distributions of the imprecise parameters as is required for probabilistic approaches. The proposed models are non-probabilistic and can express the in-plane elastic properties in terms of only mean values and percent variations when the basic parameters are available in the form of mean values and percent variations. The variations predicted by the proposed models were found to be realistic and acceptable compared to the imprecisions present in the basic variables. Also, the present results secure sharper and more realistic variations compared to the ones predicted by the combinatorial method. One of the advantages of the proposed models, which cannot be achieved when imprecisions are not considered, is the performance of a sensitivity analysis. This analysis determined the influence of the variations present in each basic variable on the variations of the in-plane elastic properties.

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References
1. Gibson L, Ashby M, Schajer G, et al. The mechanics of two-dimensional cellular materials. Proc R Soc Lond A Math Phys Sci 1982; 382: 25-42.
2. Gibson L and Ashby M. Cellular solids: structures and properties, 2nd ed. Cambridge: Cambridge University Press, 1997.
3. Sather E and Krishnamurthy T. An analytical method to calculate effective elastic properties of general multifunctional honeycomb cores in sandwich composites. Technical Memorandum, NASA Langley Research Center, Hampton, V/A, 2019. NASA/TM-2019-220275.
4. Sun G, Chen D, Wang H, et al. High-velocity impact behaviour of aluminium honeycomb sandwich panels with different structural configurations. Int J Impact Eng 2018; 122: 119–136.
5. Zhang D, Lu G, Ruan D, et al. Quasi-static combined compression-shear crushing of honeycombs: an experimental study. Mater Des 2019; 167: 107632.
6. Ha NS, Lu G and Xiang X. Energy absorption of a bio-inspired honeycomb sandwich panel. J Mater Sci 2019; 54: 6286-6300.
7. He W, Lu S, Yi K, et al. Residual flexural properties of CFRP sandwich structures with aluminum honeycomb cores after low-velocity impact. Int J Mech Sci 2019; 161–162: 105026.
8. Sun G, Huo X, Wang H, et al. On the structural parameters of honeycomb-core sandwich panels against low-velocity impact. Compos B Eng 2021; 216: 108881.
9. Zhu H, Hobdell J and Windle A. Effects of cell irregularity on the elastic properties of open-cell foams. Acta Mater 2000; 48: 4893-4900.
10. Hohe J and Becker W. A probabilistic approach to the numerical homogenization of irregular solid foams in the finite strain regime. Int J Solids Struct 2005; 42: 3549–3569.
11. Hohe J and Bechmann C. Probabilistic homogenization of hexagonal honeycombs with perturbed microstructure. Mech Mater 2012; 49: 13–29.
12. Flores E, DiazDelaO F, Friswell M, et al. A multi-scale finite element approach for the random mechanical response of honeycomb-cored structures. In: 53rd AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics and materials conference, 23–26 April 2012, Honolulu, Hawaii, pp.1–14.
13. Kim J, Lee S, Jin J, et al. Estimation for probabilistic distribution of residual strength of sandwich structure with impact-induced damage. Renew Energy 2013; 54: 219-226.
14. Parsi S, Rajeev A, Uddin A, et al. Probabilistic contact force model for low velocity impact on honeycomb structure. Sustain Resilient Infrastruct 2019; 4: 51–65.
15. Cheng J and Xiao R. Probabilistic free vibration and flutter analyses of suspension bridges. Eng Struct 2005; 27: 1509–1518.
16. Cheng J and Xiao R. Probabilistic free vibration analysis of beams subjected to axial loads. Adv Eng Softw 2007; 38: 31–38.
17. Chen J, Zeng X and Peng Y. Probabilistic analysis of wind-induced vibration mitigation of structures by fluid viscous dampers. J Sound Vib 2017; 409: 287–305.
18. Huang T and Schroder K. A Bayesian probabilistic approach for damage identification in plate structures using responses at vibration nodes. Mech Syst Signal Process 2021; 146: 106998.
19. Sandvik A, Ostby E and Thaulow C. A probabilistic fracture mechanics model including 3D ductile tearing of bi-axially loaded pipes with surface cracks. Eng Fract Mech 2008; 75: 76–96.
20. Su C and Zheng C. Probabilistic fracture mechanics analysis of linear-elastic cracked structures by spline fictitious boundary element method. Eng Anal Bound Elem 2012; 36: 1828–1837.
21. Chowdhury M, Song C and Gao W. Probabilistic fracture mechanics with uncertainty in crack size and orientation using the scaled boundary finite element method. Comput Struct 2014; 137: 93–103.
22. Long X, Liu K, Jiang C, et al. Probabilistic fracture mechanics analysis of three-dimensional cracked structures considering random field fracture property. Eng Fract Mech 2019; 218: 106586.
23. Naderi M and Khamehchi E. Cutting transport efficiency prediction using probabilistic CFD and DOE techniques. J Pet Sci Eng 2018; 163: 58–66.
24. Karimi M, Salehi S, Raisee M, et al. Probabilistic CFD computations of gas turbine vane under uncertain operational conditions. Appl Therm Eng 2019; 148: 754–767.
25. Liu J, Luo Y, Wang L, et al. A probabilistic framework for stability assessment of existing spatial structures. J Constr Steel Res 2019; 156: 96–104.
26. Gao K, Do D, Li R, et al. Probabilistic stability analysis of functionally graded graphene reinforced porous beams. Aerosp Sci Technol 2020; 98: 105738.
27. Santoro R, Muscolino G and Elishakoff I. Optimization and anti-optimization solution of combined parameterized and improved interval analyses for structures with uncertainties. Comput Struct 2015; 149: 31–42.
28. Muscolino G, Softi A and Giunta F. Dynamics of structures with uncertain-but-bounded parameters via pseudo-static sensitivity analysis. Mech Syst Signal Process 2018; 111: 1–22.
29. Muscolino G and Santoro R. Dynamics of multiple cracked prismatic beams with uncertain-but-bounded depths under deterministic and stochastic loads. J Sound Vib 2019; 443: 717–731.
30. Wang C and Matthies HG. Non-probabilistic interval process model and method for uncertainty analysis of transient heat transfer problem. Int J Therm Sci 2019; 144: 147–157.
31. Jena SK, Chakraverty S and Malikan M. Implementation of non-probabilistic methods for stability analysis of nonlocal beam with structural uncertainties. Eng Comput. Epub ahead of print 22 February 2020. DOI: 10.1007/s00366-020-00987-z.
32. Popova E and Elishakoff I. Novel interval model applied to derived variables in static and structural problems. Arch Appl Mech 2020; 90: 869–881.
33. Fu C, Feng G, Ma J, et al. Predicting the dynamic response of dual-rotor system subject to interval parametric uncertainties based on the non-intrusive metamodel. Mathematics 2020; 8: 736.
34. Alazwari MA and Rao SS. Interval-based uncertainty models for micromechanical properties of composite materials. J Reinf Plast Compos 2018; 37: 1142–1162.
35. Alazwari MA and Rao SS. Modeling and analysis of composite laminates in the presence of uncertainties. Compos B Eng 2019; 161: 107–120.
36. Rao SS and Wang S. Uncertainty-based structural optimization using universal grey number theory. AIAA J 2019; 57: 5002–5013.
37. Rao SS and Alazwari MA. Failure modeling and analysis of composite laminates: interval-based approaches. J Reinf Plast Compos 2020; 39: 817–836.
38. Nejadpak A and Rao SS. Universal grey finite elements for heat transfer analysis in the presence of uncertainties. ASCE ASME J Risk Uncertain Eng Syst B Mech Eng 2020; 6: 031004.
39. Rao SS and Liu XT. Universal grey system theory for analysis of uncertain structural systems. AIAA J 2017; 55: 3966–3979.
40. Hussain M, Khan R, Badshah S, et al. Investigation of static and fatigue behavior of honeycomb sandwich structures under bending load. Tech J Univ Eng Technol Taxila Pak 2017; 22: 72–80.

Appendix

Notation

\( m \) = mean value of an imprecisely defined parameter
\( \delta \) = percent variation
\( E_s \) = Young’s modulus of a core material, (MPa)
\( l \) = wall length, (mm)
\( h \) = wall height, (mm)
\( t \) = wall thickness, (mm)
\( \theta \) = cell angle, (degree)
\( E^* \) = Young’s modulus of a honeycomb core, (MPa)
\( v^* \) = Poisson’s ratio of a honeycomb core
\( G^* \) = shear modulus of a honeycomb core, (MPa)