The anisotropic coupling of gravity and electromagnetism in 
Hořava-Lifshitz theory

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Abstract

We analyze the electromagnetic-gravity interaction in a pure Hořava-Lifshitz framework. To do so we formulate the Hořava-Lifshitz gravity in 4+1 dimensions and perform a Kaluza-Klein reduction to 3+1 dimensions. The complete $z = 4$ theory is, in principle, power-counting renormalizable. The critical value of the dimensionless coupling constant in the kinetic term of the action is $\lambda = 1/4$. The relativistic symmetry of the kinetic term for $\lambda = 1$ is broken by the potential terms in the Hořava-Lifshitz formulation leaving the only critical value of the action to be $\lambda = 1/4$. It is the kinetic conformal point for the non-relativistic electromagnetic-gravity interaction. In distinction, the corresponding kinetic conformal value for pure Hořava-Lifshitz gravity in 3+1 dimensions is $\lambda = 1/3$. The critical value is protected from quantum corrections by the presence of a second class constraint. We analyze the geometrical structure of the critical and noncritical cases, they correspond to different theories. The physical degrees of freedom propagated by the noncritical theory are the transverse traceless graviton, the transverse gauge vector and two scalar fields. In the critical theory one of the scalars is absent, only the dilaton scalar field is present. The gravity and vector excitations propagate with the same speed, which at low energy can be taken to be the speed of light. The field equations for the gauge vector in the non-relativistic theory have exactly the same form as the relativistic electromagnetic field equations arising from the Kaluza-Klein reduction of General Relativity, and are equal to them for a particular value of one of the coupling constants. The gauge vector is now coupled to the anisotropic Hořava-Lifshitz gravity. The potential in the Hamiltonian is a polynomial of finite grade in the gauge vector and its covariant derivatives.

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I. INTRODUCTION

Hořava-Lifshitz gravity \([1]\) is a candidate for a perturbatively renormalizable theory of quantum gravity \([2–15]\). The main idea is to break the Lorentz symmetry \([16]\) in order to allow higher order derivative terms in the potential which improve the quantum behaviour of the theory without the introduction of ghost fields \([17]\). There is no space-time geometry in the formulation since there is an anisotropic scaling of the time and space coordinates. The geometrical framework is a foliation with Riemannian leaves parametrized by the time variable. The theory is manifestly invariant under the diffeomorphisms preserving the foliation. Hence only reparametrizations of the time variable are allowed. Besides this Riemannian geometry the theory introduces the lapse function and shift vector as fields defined on the leaves parametrized by the time coordinate. This formulation allows the lapse function to have different dimensions than in a space-time formulation. Moreover, the overall coupling constant of the action becomes dimensionless. The Hořava-Lifshitz gravity may also arise from the gauging of the Newton-Cartan geometry \([18]\).

In this work we consider a non-projectable version of the Hořava-Lifshitz in 4 + 1 dimensions, and perform a Kaluza-Klein (KK hereinafter) reduction to 3 + 1 dimensions. The main goal will be the analysis of the gravity-gauge vector interaction. In order to have a power counting renormalizable theory in 4 + 1 dimensions it is necessary to introduce \(z = 4\) interaction terms in the potential. However, in this work we only consider the \(z = 1\) terms since we want to contrast with the relativistic electromagnetic vector field obtained from Kaluza-Klein compactifications in General Relativity (GR from now on). In order to preserve the invariance under diffeomorphisms on the leaves of the foliation these terms in the potential have to be constructed in terms of the 4-dimensional Riemannian tensor and the lapse function and covariant derivatives of them \([2]\). Once this requirement is satisfied one may dimensionally reduce to 3 + 1 dimensions \(a\ la\ KK\). The same approach for GR in 4 + 1 dimensions gives rise to the relativistic coupling of gravity and electromagnetism in 3 + 1 dimensions. Hence our approach will determine how far the anisotropic coupling in Hořava-Lifshitz theory is from the relativistic one. This is a relevant point since electrodynamics is a very well established theory.

One important aspect in our analysis will be to compare our approach following a reduction from 4 + 1 dimensions, to the usual one consisting in coupling the 3 + 1 Hořava-Lifshitz
gravity to the relativistic energy momentum tensor of the electromagnetism.

In section II we present the Hořava-Lifshitz theory in $4+1$ dimensions, we distinguish the $\lambda \neq 1/4$ and $\lambda = 1/4$ theories. In section III we analyze the $3+1$ theory following a KK reduction. In section IV we discuss the gauge vector field equations and compare to the Maxwell equations. In section V we consider in particular the $\lambda = 1/4$ theory. Finally in section VI we give our conclusions.

II. HOŘAVA-LIFSHITZ IN FIVE DIMENSIONS

We consider a five dimensional manifold foliated by four dimensional Riemannian leaves with metric $G_{\mu\nu}dx^\mu \otimes dx^\nu$, $\mu, \nu = 1, 2, 3, 4$, and parametrized by a time variable. Besides a four vector $N_\mu$ and a scalar $N$ under diffeomorphims on the Riemannian foliation are introduced, the shift and the lapse respectively. Both are scalar densities under time reparametrizations. The Hořava-Lifshitz action on this geometrical framework is given by

$$S(G_{\mu\nu}, N_\rho, N) = \int dtdx^4 N \sqrt{G} \left[ K_{\mu\nu}K^{\mu\nu} - \lambda K^2 + \beta^{(4)} R + \alpha a_\mu a^\mu + V(G_{\mu\nu}, N) \right] ,$$

where $a_\mu = \partial_\mu LnN$ and $K_{\mu\nu}K^{\mu\nu} - \lambda K^2$ is the kinetic term and the remaining terms describe the potential of the theory. We have written explicitly the lowest order terms in spatial derivatives, the higher order ones are contained in $V(G_{\mu\nu}, N)$, and are constructed from powers of the four dimensional Riemannian tensor and the covariant derivatives of the lapse $N$. The highest order derivative terms of the physical degrees of freedom should be at least of order eight and with an elliptic symbol, in order to obtain a power counting renormalizable theory. We will perform a low energy analysis and consequently we will not consider the term of the potential denoted by $V(G_{\mu\nu}, N)$, only the potential terms which are explicitly written will be considered. This second order truncation in $3+1$ dimensions is related to the Einstein-aether theory \[19, 22\. $K_{\mu\nu}$ is the extrinsic curvature of the Riemannian leaves,

$$K_{\mu\nu} = \frac{1}{2N}(\dot{g}_{\mu\nu} - \nabla_\mu N_\nu - \nabla_\nu N_\mu) ,$$

and $K$ its trace

$$K = G^{\mu\nu}K_{\mu\nu}.$$
λ is a dimensionless coupling constant, while α and β are the coupling constants at low energy in the potential of the theory. In this section we analyze the above Hořava-Lifshitz theory in 4 + 1 dimensions. In the next section we consider its KK reduction to 3 + 1 dimensions.

We proceed to reformulate the action using the Hamiltonian formalism. To do so we have to distinguish the \( \lambda \neq 1/4 \) and \( \lambda = 1/4 \) cases. At \( \lambda = 1/4 \) the kinetic term is conformal invariant. This is analogous to the \( \lambda = 1/3 \) theory in the Hořava-Lifshitz gravity in 3 + 1 dimensions which propagates the same degree of freedom as GR and has the same quadrupole radiative behaviour as GR \[13, 27, 28\]. We begin our analysis with the \( \lambda \neq 1/4 \) case. The conjugate momentum to \( G_{\mu\nu} \) is given by

\[
\pi^{\mu\nu} = \frac{\partial L}{\partial \dot{G}_{\mu\nu}} = \sqrt{G} \left( K^{\mu\nu} - \lambda G^{\mu\nu} K \right), \tag{4}
\]

and its trace by

\[
\pi = G_{\mu\nu} \pi^{\mu\nu} = \sqrt{G} \left( 1 - 4\lambda \right) K. \tag{5}
\]

The Hamiltonian density obtained from the Legendre transformation is

\[
\mathcal{H} = \sqrt{G} N \left[ \frac{\pi^{\mu\nu} \pi_{\mu\nu}}{G} + \frac{\lambda}{(1 - 4\lambda)} \frac{\pi^2}{G} - \beta^{(4)} R - \alpha a^\mu a^\mu \right] + 2\pi^{\mu\nu} \nabla_\mu N_\nu + \sigma P_N, \tag{6}
\]

we have added the last term with the Lagrange multiplier \( \sigma \) because the theory is subject to a primary constraint given by the conjugate momentum to the lapse \( N \) equal to zero, since no time derivatives of \( N \) appears in Lagrangian density.

The field equations arising from the canonical Lagrangian are the following. The conservation of the primary constraint, or equivalently variation with respect to \( N \) yields the Hamiltonian constraint

\[
\frac{\pi^{\mu\nu} \pi_{\mu\nu}}{G} + \frac{\lambda}{(1 - 4\lambda)} \frac{\pi^2}{G} - \beta^{(4)} R - \alpha a^\mu a^\mu + 2\alpha \nabla_\mu a^\mu = 0, \tag{7}
\]

which ends up being a second class constraint. Variations with respect to \( \pi^{\mu\nu} \) gives

\[
\dot{G}_{\mu\nu} = \frac{2N \pi^{\mu\nu}}{\sqrt{G}} + \frac{2\lambda}{(1 - 4\lambda)} G_{\mu\nu} N \frac{\pi^\nu}{\sqrt{G}} + \nabla_\mu N_\nu + \nabla_\nu N_\mu, \tag{8}
\]

while variations with respect to \( G_{\mu\nu} \) yields

\[
-\dot{\pi}^{\mu\nu} = -\frac{1}{2} N G^{\rho\mu} \left[ \frac{\pi^{\lambda\rho} \pi_{\lambda\nu}}{\sqrt{G}} + \frac{\lambda}{(1 - 4\lambda)} \frac{\pi^2}{\sqrt{G}} \right] + 2N \left[ \frac{\pi^{\mu\lambda} \pi_{\nu\lambda}}{\sqrt{G}} + \frac{\lambda}{(1 - 4\lambda)} \frac{\pi^{\mu\nu} \pi}{\sqrt{G}} \right]
+ \beta \sqrt{G} N \left[ \frac{\beta^{(4)} R^{\mu\nu}}{2} - \frac{1}{2} R G^{\mu\nu} \right] - \beta \sqrt{G} \left[ \nabla^{(\mu} N^{\nu)} - G^{\mu\nu} \nabla_\lambda \nabla^\lambda N \right] \tag{9}
\]

\[
-\frac{1}{2} \alpha \sqrt{G} N G^{\mu\nu} a_\rho a^\rho + \alpha \sqrt{G} N a^\mu a^\nu + 2\nabla_\rho \left[ \pi^{\rho(\mu} N^{\nu)} \right] - \nabla_\rho \left[ \pi^{\mu\nu} N^\rho \right].
\]
where
\[ A^{(\mu B^\nu)} = \frac{1}{2} (A^\mu B^\nu + A^\nu B^\mu). \] (10)

Finally, variations with respect to \( N_\mu \) determine the first class constraints of the theory
\[ \nabla_\mu \pi^{\mu \nu} = 0. \] (11)

Variations with respect to \( P_N \) yields \( \dot{N} = \sigma \). This equation merely fixes the Lagrange multiplier \( \sigma \). The physical degrees of freedom propagated by these non-linear evolution equations on an Euclidean four dimensional background given by
\[ G_{\mu \nu} = \delta_{\mu \nu}, \quad \pi^{\mu \nu} = 0, \quad \text{together with } N_\mu = 0, \quad N = 1, \] (12)
may be obtained by considering a first order perturbation
\[ G_{\mu \nu} = \delta_{\mu \nu} + k_{\mu \nu}, \]
\[ \pi^{\mu \nu} = \Omega^{\mu \nu}, \]
\[ N_\mu = n_\mu, \]
\[ N = 1 + n, \]
and then determining the propagation of \( k_{\mu \nu}, n_\mu, n \) from the evolution equations up to first order in perturbations. We obtain to first order
\[ (4) R_{\mu \nu} = \frac{1}{2} \partial_\lambda \partial_\nu k_{\mu \lambda} + \frac{1}{2} \partial_\mu \partial_\lambda k_{\nu \lambda} - \frac{1}{2} \partial_\mu \partial_\nu k_{\lambda \lambda} - \frac{1}{2} \Delta k_{\mu \nu}, \] (14)
\[ (4) R = \partial_\mu \partial_\lambda k_{\mu \lambda} - \Delta k_{\mu \mu}. \] (15)

We will use the \( T + L \) decomposition of the four dimensional tensors,
\[ k_{\mu \nu} = k_{\mu \nu}^{TT} + \frac{1}{3} \left( \delta_{\mu \nu} - \frac{\partial_\mu \partial_\nu}{\Delta} \right) k^T + \partial_\nu k_{\mu}^{L} + \partial_\mu k_{\nu}^{L}. \] (16)

We obtain,
\[ (4) R_{\mu \nu} - \frac{1}{2} G_{\mu \nu} (4) R = -\frac{1}{2} \Delta k_{\mu \nu}^{TT} + \frac{1}{3} \left( \delta_{\mu \nu} - \frac{\partial_\mu \partial_\nu}{\Delta} \right) \Delta k^T. \] (17)

From (11) we obtain
\[ \Omega^L = 0 \]
\[ \Omega_{\mu \nu} = \Omega_{\mu \nu}^{TT} + \frac{1}{3} \left( \delta_{\mu \nu} - \frac{\partial_\mu \partial_\nu}{\Delta} \right) \Omega^T. \] (18)
From (8) we get

\[ \dot{k}_{\mu \nu}^{TT} = 2\Omega_{\mu \nu}^{TT} \]  
\[ \dot{k}^T = \frac{2}{(1 - 4\lambda)(1 - \lambda)} \Omega^T \]  
\[ \dot{k}_\mu^L = N_\mu + \frac{\lambda}{(1 - 4\lambda)} \frac{\partial_\mu}{\Delta} \Omega^T, \]  

from (9)

\[ \dot{\Omega}_{\mu \nu}^{TT} = \frac{\beta}{2} \Delta k_{\mu \nu}^{TT} \]  
\[ \dot{\Omega}^T = -\beta \Delta k^T - 3\beta \Delta n, \]  

and from (7)

\[ \beta \Delta k^T + 2\alpha \Delta n = 0. \]  

Finally, we may combine the above equations to obtain

\[ \ddot{k}_{\mu \nu}^{TT} = \beta \Delta k_{\mu \nu}^{TT} \]  
\[ \ddot{k}^T = \beta \frac{(1 - \lambda)}{(1 - 4\lambda)} \frac{3\beta - 2\alpha}{\alpha} \Delta k^T, \]

we remark that these equations are gauge independent. They describe the propagation of the physical degrees of freedom of the theory. (25) describes the evolution of 5 degrees of freedom while (26) describes the propagation of an additional one. It follows that when reducing a la KK these field equations to a 3 + 1 dimensions, the 5 degrees of freedom decompose into 2 + 2 + 1 corresponding to the two physical degrees of freedom of the graviton, two degrees of freedom of a vector gauge field and one degree of freedom of a scalar field, all of them propagating with the same velocity $\sqrt{\beta}$. In order to show this decomposition of the physical degrees of freedom we assume the fields are independent of one spatial coordinate. We first invert the $T + L$ decomposition, to obtain

\[ k_L^\nu = \frac{1}{\Delta} \partial_\rho k_{\rho \nu} - \frac{1}{2} \frac{\partial_\nu}{\Delta} \left( \partial_\chi \partial_\rho k_{\chi \rho} \right) \]  
\[ k^T = k_{\mu \mu} - \frac{\partial_\rho}{\Delta} h_{\rho \lambda} \]  
\[ k_{\mu \nu}^{TT} = k_{\mu \nu} - \frac{1}{3} \left( \delta_{\mu \nu} - \frac{\partial_\mu \partial_\nu}{\Delta} \right) k^T - \partial_\mu k^L_\nu - \partial_\nu k^L_\mu. \]
We now decompose $k_{\mu \nu}$ into

\begin{align*}
  k_{ij} &= h_{ij} \\
  k_{4i} &= A_i \\
  k_{44} &= \phi,
\end{align*}

and perform a three dimensional $T + L$ decomposition of $h_{ij}$ and $A_i$

\begin{align*}
  h_{ij} &= h^T_{ij} + \frac{1}{2} \left( \delta_{ij} - \frac{\partial_i \partial_j}{\Delta} \right) h^T + \partial_i h^L_j - \partial_j h^L_i \\
  A_i &= A^T_i + \partial_i A^L.
\end{align*}

By replacing (31) and (32) into (27), (28) and (29), we obtain

\begin{align*}
  k_{4i}^L &= h_i^L \\
  k_{44}^L &= A^L \\
  k^T &= h^T + \phi \\
  k_{4i}^{TT} &= A^T_i \\
  k_{44}^{TT} &= \frac{1}{3} (2\phi - h^T) \\
  k_{ij}^{TT} &= h_{ij}^{TT} - \frac{1}{6} \left( \delta_{ij} - \frac{\partial_i \partial_j}{\Delta} \right) (2\phi - h^T).
\end{align*}

Finally, from (25) we have

\begin{align*}
  \ddot{h}_{ij}^{TT} &= \beta \Delta h_{ij}^{TT} \\
  \ddot{A}^T &= \beta \Delta A^T \\
  2\ddot{\phi} - \dddot{h}^T &= \beta \Delta (2\phi - h^T).
\end{align*}

(34) describes the propagation of the two degrees of freedom of the graviton, (35) of the two degrees of freedom of the gauge vector and (36) of a scalar field, all of them propagating with velocity $\sqrt{\beta}$. In addition, we have one propagating scalar degree of freedom described by (26).

In the next section we analyze the exact KK reduction of the Hořava-Lifshitz in 4 + 1 dimensions. Among other properties we will determine the coupling of Hořava-Lifshitz gravity to the Maxwell gauge potential and two scalar fields.
III. NON-PERTURBATIVE KALUZA-KLEIN REDUCTION TO 3+1 DIMENSIONS

In this section we perform the KK reduction of the $4 + 1$ Hořava theory in a non-pertubative approach. We decompose the 4-dimensional Riemannian metric $G_{\mu\nu}$ in the following form

$$\begin{pmatrix} \gamma_{ij} + \phi A_i A_j & \phi A_j \\ \phi A_i & \phi \end{pmatrix},$$

(37)

where $\gamma_{ij}$ is a 3-dimensional Riemannian metric. We denote $\det (\gamma_{ij}) \equiv \gamma$, thus we have $G \equiv \det (G_{\mu\nu}) = \gamma \phi > 0$, hence $\phi > 0$. The inverse metric is then given by

$$\begin{pmatrix} \gamma_{ij} & -A^j \\ -A^i & 1/\phi + A_k A^k \end{pmatrix},$$

(38)

where $\gamma^{ij}$ are the components of the inverse of $\gamma_{ij}$ and $A^i = \gamma^{ij} A_j$. The decomposition (37) is invertible

$$\begin{align*}
\gamma_{ij} &= G_{ij} - \frac{G_{4i} G_{j4}}{G_{44}} \\
A_j &= \frac{G_{4j}}{G_{44}} \\
\phi &= G_{44}.
\end{align*}$$

(39-41)

We then have

$$\pi^{\mu\nu} \dot{G}_{\mu\nu} = \pi^{ij} \dot{\gamma}_{ij} + p^i \dot{A}_i + p \dot{\phi},$$

(42)

where

$$\begin{align*}
p^{ij} &= \pi^{ij} \\
p^i &= 2\phi A_j \pi^{ij} + 2\pi^{i4} \phi \\
p &= \pi^{ij} A_i A_j + 2\pi^{i4} A_i + \pi^{44}.
\end{align*}$$

(43-45)

Equations (39)-(45) define a canonical transformation. In fact,

$$\{ G_{\mu\nu} (x), \pi^{\rho\lambda} (\bar{x}) \}_{PB} = \frac{1}{2} \left( \delta^\rho_\mu \delta^\lambda_\nu + \delta^\rho_\nu \delta^\lambda_\mu \right) \delta (x - \bar{x}),$$

(46)
\begin{align*}
\{\gamma_{ij}(x), p^{kl}(\tilde{x})\}_P &= \frac{1}{2} \left( \delta_i^k \delta_j^l + \delta_i^k \delta_j^l \right) \delta(x - \tilde{x}), \\
\{A_i(x), p^j(\tilde{x})\}_P &= \delta_i^j \delta(x - \tilde{x}), \\
\{\phi(x), p(\tilde{x})\}_P &= \delta(x - \tilde{x}),
\end{align*}

and all other Poisson brackets being zero. The canonical Lagrangian in terms of the new variables assuming \( \partial_4 = 0 \), is then given by

\[ \mathcal{L} = p^{ij} \dot{\gamma}_{ij} + p^i \dot{A}_i + p \phi + P_N \dot{N} - \mathcal{H}, \]

where the Hamiltonian density is given by

\[ \mathcal{H} = \frac{N}{\sqrt{\gamma \phi}} \left[ \phi^2 p^2 + p^{ij} p_{ij} + \frac{p^i p_i}{2\phi} + \frac{\lambda}{(1 - 4\lambda)} \left( p^{ij} \gamma_{ij} + p \phi \right)^2 - \gamma \phi \beta^{(4)} R - \gamma \phi \alpha a_i a^i \right] - \Lambda \partial_i p^i - \Lambda_j \left( \nabla_i p^{ij} - \frac{1}{2} p^j \gamma^{jk} F_{ik} - \frac{1}{2} p \gamma^{ij} \partial_j \phi \right) - \sigma P_N, \]

where

\[ (4) R = R - \frac{\phi}{4} F_{ij} F^{ij} - \frac{2}{\sqrt{\phi}} \nabla_i \nabla^i \sqrt{\phi}, \]

\( R \) is the curvature and \( \nabla_i \) the covariant derivative associated to the 3-dimensional metric \( \gamma_{ij} \). Indices are raised and lowered using \( \gamma_{ij} \) and its inverse \( \gamma^{ij} \). \( \Lambda \) and \( \Lambda_j \) are the Lagrange multipliers associated to a combination of the constraints (53) while \( \sigma \) is the Lagrange multiplier associated to the constraint

\[ P_N = 0 \]

that is, the conjugate momentum to \( N \) equal zero. These are the primary constraints of the formulation.

The KK reduction of the momentum constraint (53) yields

\[ H^i \equiv \partial_i p^i = 0 \]
\[ H^j \equiv \nabla_i p^{ij} - \frac{1}{2} p^j \gamma^{jk} F_{ik} - \frac{1}{2} p \gamma^{ij} \partial_j \phi = 0 \]
The conservation of these primary constraints is satisfied and the conservation of (53) yields the Hamiltonian constraint

\[
H_N \equiv \frac{1}{\sqrt{\gamma \phi}} \left[ \phi^2 p^2 + p^{ij} p_{ij} + \frac{p^i p_i}{2 \phi} + \frac{\lambda}{(1 - 4 \lambda)} (p^{ij} \gamma_{ij} + p \phi)^2 - \beta \gamma \phi R + \frac{\beta}{4} \gamma \phi^2 F_{ij} F^{ij} \right. \\
+ 2 \beta \gamma \sqrt{\phi} \nabla_i \nabla^i \sqrt{\phi} + \alpha \sqrt{\gamma \phi} a_i a^i + 2 \alpha \sqrt{\gamma} \nabla_i \left( \sqrt{\phi} a^i \right) = 0. \tag{56}
\]

The Dirac’s procedure to determine the constraints of the theory ends at this step. It turns out that (54) and (55) are first class constraints while (53) and (56) are second class constraints. The first class constraints, once they are satisfied initially, they are preserved by the evolution equations obtained by taking variations of the action with respect to the independent fields. The second class ones have to be imposed at any time.

Variations with respect to \(p^{ij}, p^i, \) and \(p\) give the field equations

\[
\dot{\gamma}_{ij} = \frac{N}{\sqrt{\gamma \phi}} \left[ 2 p_{ij} + \frac{2 \gamma_{ij} \lambda}{(1 - 4 \lambda)} (p^{lm} \gamma_{lm} + p \phi) \right] + \nabla (i \Lambda_j), \tag{57}
\]

\[
\dot{A}_i = \frac{N p_i}{\sqrt{\gamma \phi^3}} + \partial_i \Lambda + \frac{1}{2} \Lambda_j \gamma^{jk} F_{ik}, \tag{58}
\]

\[
\dot{\phi} = \frac{N}{\sqrt{\gamma \phi}} \left[ 2 \phi p^2 + \frac{2 \lambda}{(1 - 4 \lambda)} (p^{lm} \gamma_{lm} + p \phi) \phi \right] + \frac{1}{2} \Lambda^i \partial_i \phi. \tag{59}
\]

Variations with respect to \(\gamma_{ij}, A_i, \) and \(\phi\) yield the field equations

\[
\dot{p}^{ij} = \frac{N}{2 \sqrt{\gamma \phi}} \left[ \phi^2 p^2 + p^{lk} p_{lk} + \frac{1}{\phi} p^i p_i + \frac{\lambda}{(1 - 4 \lambda)} (p^{lm} \gamma_{lm} + p \phi)^2 \right] - \frac{N}{\sqrt{\gamma \phi}} \left[ 2 p^{jk} p^j_k + \frac{1}{2 \phi} p^i p^j + \frac{2 \lambda}{(1 - 4 \lambda)} (p^{lm} \gamma_{lm} + p \phi) p^{ij} \right] + N \sqrt{\gamma \phi} \beta \left[ \frac{R}{2} \gamma^{ij} \right. \\
- R^{ij} \bigg] + \beta \sqrt{\gamma} \left[ \nabla (i \nabla^j) \left( N \sqrt{\phi} \right) - \gamma^{ij} \nabla_k \nabla^k \left( N \sqrt{\phi} \right) \right] + \frac{\beta}{2} N \sqrt{\gamma \phi^3} \left[ F^{in} F_{in} - \frac{\gamma^{ij}}{4} F_{mn} F^{mn} \right]\tag{60}
\]

\[
+ \alpha N \sqrt{\gamma \phi} \left[ \frac{\gamma^{ij}}{2} a_k a^k - a^i a^i \right] - \nabla_k \left[ p^{(i} \Lambda^{j)} - \frac{p^{ij}}{2} \Lambda^k \right] + \frac{1}{2} \Lambda^i p^j \gamma^{jm} F_{lm} + \frac{1}{2} p \Lambda^i \partial^i \phi,
\]

\[
\dot{p}^i = \beta \partial_j \left( N \sqrt{\gamma \phi^3} F^{ij} \right) - \frac{1}{2} \partial_k \left( \Lambda^k p^j - \Lambda^i p^k \right), \tag{61}
\]
\[ p = -\frac{N}{\sqrt{\gamma}} \left[ \frac{3}{2} \sqrt{\phi} p^2 - \frac{1}{2\sqrt{\phi}} pp^i p_{ij} - \frac{3}{4\sqrt{\phi}} p^i p_i + \frac{\lambda}{(1 - 4\lambda)} \left( \frac{3}{2} \sqrt{\phi} p^2 \right) \right. \\
\left. + \frac{pp^i \gamma_{ij}}{\sqrt{\phi}} - \frac{1}{2} \left( \frac{p^i \gamma_{ij}}{\sqrt{\phi}} \right)^2 \right) - \gamma \beta \left( \frac{1}{2} \sqrt{\phi} R - \frac{3}{8} \sqrt{\phi} F_{ij} F_{ij} \right) \left( \frac{3}{2} \sqrt{\phi} p^2 + pp^i \gamma_{ij} \right) \right] - \beta \frac{\sqrt{\gamma}}{\sqrt{\phi}} \nabla_j \nabla^i N \left[ + \frac{1}{2} \partial_i (p\Lambda) \right] . \]

(62)

(54)-(62) are the complete set of field equations of the 3+1-dimensional theory, it describes the gauge vector-gravity interaction together with two additional scalar fields as already mentioned in the 4+1-dimension formulation of the previous section. We may obtain the perturbative equations directly from this 3+1-formulation.

So, the perturbations around Euclidean space are defined by introducing the variables \( h_{ij}, \Omega_{ij}, n, n_4 \) and \( n_4 \) in the following way

\[
\gamma_{ij} = \delta_{ij} + \epsilon h_{ij}, \quad p^i = \epsilon \Omega_{ij}, \quad N_i = \epsilon n, \quad N_4 = \epsilon n_4, \quad N = 1 + \epsilon n. \quad (63)
\]

While for the scalar \( \phi \) and the vector \( A_i \) fields we have

\[
A_i = \epsilon \xi_i, \quad p^i = \epsilon \zeta_i, \quad \phi = 1 + \epsilon \tau, \quad p = \epsilon \chi. \quad (64)
\]

So, at linearized level the evolution field equations become

\[
\dot{\tau} = 2\chi + \frac{2\lambda}{(1 - 4\lambda)} (\chi + \Omega), \quad (65)
\]
\[
\dot{\chi} = -\frac{\beta}{2} \Delta h - \beta \Delta n, \quad (66)
\]
\[
\dot{\xi}_i = \zeta_i - \partial_i \tau, \quad (67)
\]
\[
\dot{\zeta}_i = \beta \partial_j (\partial_i \xi_j - \partial_j \xi_i), \quad (68)
\]
\[
\dot{h}_{ij} = 2\Omega_{ij} + \frac{2\delta_{ij} \lambda}{(1 - 4\lambda)} (\Omega + \xi) + 2\partial_i n_{ij}, \quad (69)
\]
\[
\dot{\Omega}_{ij} = -\frac{\beta}{2} \left( \delta_{ij} - \partial_i \partial_j \right) \Delta h + \frac{\beta}{2} \Delta h_{ij} - \frac{\beta}{2} \left( \delta_{ij} - \partial_i \partial_j \right) \Delta \left( n + \frac{\gamma}{2} \right), \quad (70)
\]

Besides, from the constraints we have

\[
\partial_i \Omega_{ij} = 0 \quad (71)
\]
\[
\beta \Delta \tau + 2\alpha \Delta n + \beta \Delta h = 0. \quad (72)
\]
In order to cast the physical degrees of freedom propagated at linearized level we use the orthogonal transverse/longitudinal decomposition obtaining

\[ \dot{\xi}_i^T = \zeta_i^T, \]  
\[ \dot{\zeta}_i^T = \beta \Delta \xi_i^T, \]  

so, combining (73) and (74) we get the following wave equation for the photon

\[ \ddot{\xi}_i^T - \beta \Delta \xi_i^T = 0, \]  

spreading with velocity \( \sqrt{\beta} \). From equations (69) and (70) we obtain the following wave equation for the graviton

\[ \ddot{h}_{ij}^{TT} - \beta \Delta h_{ij}^{TT} = 0, \]  

it’s noteworthy that the graviton has the same spread velocity as the gauge vector, i.e, \( \sqrt{\beta} \). The longitudinal modes \( \xi^L \) and \( h^L_i \) are gauge modes. They are not physical excitations. The remaining terms obtained from the decomposition of the equations (69) and (70) are

\[ \dot{h}_i^T = 2 \Omega_i^T + \frac{4 \lambda}{(1 - 4 \lambda)} (\Omega^T + \chi), \]  
\[ \dot{\Omega}_i^T = -\frac{\beta}{2} \Delta h_i^T - 2 \beta \Delta n - \beta \Delta \tau, \]  

and the longitudinal terms

\[ n_i + \frac{\lambda}{(1 - 4 \lambda)} \frac{\partial}{\Delta} (\Omega^T + \chi) = 0, \]  

The above equation (79) allows to determine \( n_i \). So, solving (72) for \( \Delta n \) we get

\[ \Delta n = -\frac{\beta}{2 \alpha} (\Delta \tau + \Delta h^T), \]  

and combining it with (65), (66), (77) and (78) we obtain

\[ \ddot{h}_i^T - 2 \ddot{\tau} = \beta \Delta (h_i^T - 2 \tau) \]  
\[ \ddot{h}_i^T + \dot{\phi} = \frac{\beta (1 - \lambda) (3 \beta - 2 \alpha)}{\alpha (1 - 4 \lambda)} \Delta (h_i^T + \phi). \]

These results are in exact correspondence with the ones obtained in the previous section.
IV. THE ELECTROMAGNETIC FIELD EQUATIONS IN HOŘAVA FORMULATION

Let us analyze in more details equations (54), (58) and (61). Since (54) is a first class constraint it generates a gauge transformation on $A_i$,

$$
\delta A_i = \{\langle \lambda \partial_i p^i \rangle, A_i (x) \}_P B = \partial_i \lambda (x),
$$

where

$$
\langle \lambda \partial_i p^i \rangle = \int d^3 \tilde{x} \lambda (\tilde{x}) \partial_i p^i (\tilde{x}).
$$

(84)

The action and field equations are invariant under this gauge transformation. $\Lambda_i$ are Lagrange multipliers associated to the first class constraints (55), they can be fixed to zero in order to simplify the analysis of (58) and (61). If we denote $A_0 \equiv \Lambda$ and

$$
F^{0i} = - \frac{1}{N^2} \gamma^{ij} F_{0j},
$$

then eliminating $p^i$ in (58) and (61) we obtain the equation

$$
\partial_0 \left( F^{0j} N \sqrt{\gamma} \phi^3 \right) + \beta \partial_j \left( F^{ji} N \sqrt{\gamma} \phi^3 \right) = 0.
$$

(86)

In the ADM decomposition of a 4-dimensional space-time metric $g_{\mu\nu} dx^\mu \otimes dx^\nu$,

$$
g_{00} = -N^2 + \gamma_{ij} N^i N^j,
g_{0i} = g_{i0} = N_i, \quad N_i = \gamma_{ij} N^j,
g_{ij} = \gamma_{ij},
$$

(87)

with inverse given by

$$
g^{00} = - \frac{1}{N^2},
g^{0i} = g^{i0} = \frac{N^i}{N^2},
g^{ij} = \gamma^{ij} - \frac{N^i N^j}{N^2},
$$

(88)

if we fix $N_i = 0$ as a gauge fixing of the space-like diffeomorphisms, then we have that our definition of $F^{0i}$ is

$$
F^{0i} = g^{0\mu} g^{i\nu} F_{\mu\nu},
$$

(89)
the space-time definition of the contravariant curvature of the potential $A_\mu$, where $A_0$ is the Lagrange multiplier of the constraint (54), the generator of gauge transformation.

The non-relativistic electromagnetic equations are exactly the relativistic ones if $\beta = 1$. In fact, $N\sqrt{\gamma} = \sqrt{|g|}$ and (86) becomes the relativistic equations of the electromagnetic coupled to the dilaton field. If $\phi = 1$ we have

$$\partial_\mu \left( \sqrt{|g|} F^{\mu i} \right) = 0,$$

(90)

while (54) can be expressed, when $\phi = 1$, as

$$\partial_\mu \left( \sqrt{|g|} F^{\mu 0} \right) = 0.$$

(91)

The velocity of the propagation is in general $\sqrt{\beta}$ and it is the same for the gravity and gauge vector excitations. It is interesting that this property is a consequence of starting with a power counting renormalizable theory in 5-dimensions. We can now identified the gauge vector with the electromagnetic potential. Equations (86) and (91) are the anisotropic electromagnetic equations. Notice that (91) arising directly from (54) does not involve any coupling constant. Since the potential in the Hamiltonian of the complete theory is constructed from polynomial expressions in terms of the Riemann tensor and the lapse function and its covariant derivatives up to $2z$ derivatives, $z$ being the time scaling which is 4 in the $4+1$ dimensional theory, its reduction to $3+1$ dimensions is a polynomial (of finite grade) in the gauge vector and its covariant derivatives.

V. THE KINETIC CONFORMAL POINT IN 4+1 DIMENSIONS

We discuss in this section the Hořava-Lifshitz gravity in $4+1$ dimensions for $\lambda = 1/4$. It is a different theory with respect to the $\lambda \neq 1/4$ formulation we have already considered. The physical propagating degrees of freedom are different in the two cases. The corresponding relation in Hořava-Lifshitz gravity in $3+1$ dimensions is between the $\lambda = 1/3$ and $\lambda \neq 1/3$ theories [13, 27]. In the canonical analysis in section II, in the case $\lambda = 1/4$, we obtain an additional primary constraint

$$\pi = G_{\mu \nu} \pi^{\mu \nu} = 0, \quad \mu, \nu = 1, 2, 3, 4.$$

(92)

The Hamiltonian density in this case is given by

$$\mathcal{H} = N\sqrt{G} \left[ \pi^{\mu \nu} \pi_{\mu \nu} - \beta^{(4)} R - \alpha a_\mu a^\mu \right] + 2\pi^{\mu \nu} \nabla_\mu N_\nu + \sigma P + \mu \pi,$$

(93)
where $\sigma$ and $\mu$ are Lagrange multipliers associated to two primary constraints, as before $P_N$ is the conjugate momentum to $N$. Additionally
\[ \nabla_\mu \pi^{\mu\nu} = 0 \] (94)
is the primary constraint associated to the invariance under the diffeomorphisms on the Riemannian leaves of the $4+1$ foliation. As usual it is preserved under the evolution determined by the Hamiltonian. It ends up being first class constraint.

The conservation of the primary constraints (92) and $P_N = 0$ implies two new constraints, the Hamiltonian constraint
\[ \frac{\pi^{\mu\nu}\pi^{\mu\nu}}{G} - \beta^{(4)} R + \alpha a_\mu a^\mu + 2\alpha \nabla_\mu a^\mu = 0, \] (95)
and
\[ \frac{2\pi^{\mu\nu}\pi^{\mu\nu}}{G} + \beta^{(4)} R + (\alpha - 3\beta) a_\mu a^\mu - 3\beta \nabla_\mu a^\mu = 0. \] (96)
The conservation of (95) and (96) determine Lagrange multipliers in the formulation. The Dirac’s procedure to determine the complete set of constraint of the theory ends at this stage: (92), $P_N = 0$, (95) and (96) are second class constraints, (94) are first class constraints.

We first obtain the propagating physical degrees of freedom on an Euclidean background. We consider perturbations around this backgrounds as in section II and III. We obtain from (95) and (96) using a $T + L$ ADM decomposition, at first order in perturbations,
\[ \Delta k^T = 0 \] (97)
\[ \Delta n = 0, \] (98)
which assuming, as usual, an asymptotic zero value of the fields on the leaves of the foliation imply
\[ k^T = 0 \] (99)
\[ n = 0. \] (100)

From (92) and (94) we obtain
\[ \Omega^{\mu\nu} = \Omega^{\mu\nu TT}. \] (101)

It turns out that only the $TT$ degrees of freedom remain in the action which up to second order in perturbations becomes
\[ S = \frac{1}{4} \int dt dx \left[ k^{TT}_{\mu\nu} \Delta k^{TT}_{\mu\nu} - k^{TT}_{\mu\nu} k^{TT}_{\mu\nu} \right], \] (102)
which describes the propagation of 5 physical degrees of freedom. Following the analysis in section II and section III when reducing to 3 + 1 dimensions we obtain (using the same notation)

\[ \ddot{h}_{ij} = \beta \Delta h_{ij} \tag{103} \]
\[ \ddot{A}_i = \beta \Delta A_i \tag{104} \]
\[ \ddot{\phi} = \beta \Delta \phi \tag{105} \]
\[ h^T = \phi, \tag{106} \]

that is, comparing with the $\lambda \neq 1/4$ theory there is only one propagating scalar field. All physical modes propagate with the same speed $\sqrt{\beta}$. The longitudinal components $h^T_l, A^T$ are gauge modes.

We notice that the propagating degrees of freedom are the same to the ones in GR in interaction with the electromagnetic and the dilaton fields arising from a KK reduction of GR in 4 + 1 dimensions. Although the Hořava’s theory at the kinetic conformal point we are considering breaks the relativistic symmetry, it propagates the same physical degrees of freedom as the corresponding one in GR. We now consider the complete set of field equations of the theory. In order to derive them, we assume an asymptotic decay to zero of the Lagrange multipliers of the second class constraints. It is then correct to use the Hamiltonian (93) without the explicit introduction of the whole set of second class constraints via Lagrange multipliers [29].

The field equations are, together with (92), (94), (95) and (96), the following ones,

\[ \dot{G}_{\mu\nu} = \frac{2N\pi_{\mu\nu}}{\sqrt{G}} + \mu G_{\mu\nu} + \nabla_\mu N_\nu + \nabla_\nu N_\mu, \tag{107} \]

\[ -\dot{\pi}_{\mu\nu} = -\frac{1}{2} NG\mu\nu \pi^{\lambda\rho} \pi_{\lambda\rho} \sqrt{G} + 2N \frac{\pi^{\mu\lambda} \pi^{\nu}_\lambda}{\sqrt{G}} + \beta \sqrt{G} N \left( R^{\mu\nu} - \frac{1}{2} R G^{\mu\nu} \right) \]

\[ -\beta \sqrt{G} \left[ \nabla^\mu \nabla^\nu N - G^{\mu\nu} \nabla_\lambda \nabla^\lambda N \right] - \frac{1}{2} \alpha \sqrt{G} NG^{\mu\nu} a_\rho a^\rho + \alpha \sqrt{G} \times N a^\mu a^\nu + 2\nabla^\rho \left[ \pi^{\rho(\mu} N^{\nu)} \right] - \nabla_\rho \left[ \pi^{\mu\nu} N^\rho \right] + \mu \pi^{\mu\nu}. \tag{108} \]

We can also perform the canonical transformation defined in section III and impose $\partial_4 = 0$ on the fields to obtain a 3 + 1 formulation. The Hamiltonian in this formulation becomes
\[
\mathcal{H} = \frac{N}{\sqrt{G}} \left[ \phi^2 p^2 + p^{ij} p_{ij} + \frac{p^i p_i}{2\phi} - \beta G^{(4)} R - \alpha G a_{\mu} a^{\mu} \right] - 2 \left[ \nabla_{\mu} \pi^{\mu\nu} \right] N_{\nu} \\
+ \sigma P_N + \mu \left[ p^{ij} \gamma_{ij} + p \phi \right],
\]

(109)

where \((4) R\) has the expression (52).

The non-relativistic electromagnetic field equations are the same as in the previous section, however the other field equations are different, in particular a new second class constraint appears in this formulation compared to the \(\lambda \neq 1/4\) theory.

It is important to discuss the field equations obtained from the Hamiltonian in this section, equation (109), and in section III in comparison to the field equations arising from an action in which it is assumed that the scalar field \(\phi\) is in its ground state, which we take to be \(\phi = 1, p = 0\). In fact, variations of an action subject to the restriction \(\phi = 1, p = 0\) give rise to field equations which are not equivalent to the ones in this section or in section III on which one imposes the \(\phi = 1, p = 0\) restriction.

It is straightforward to obtain the equations from the action restricted by \(\phi = 1\) and \(p = 0\). In fact, variations with respect to \(\gamma_{ij}, p^{ij}, A_i, p^i\) determine the same field equations obtained by taking variations of the canonical Lagrangian associated to (109), in the \(\lambda = 1/4\) case, and imposing afterwards \(\phi = 1, p = 0\). The main difference being that in the restricted case there are not field equations corresponding to variations on \(\phi\) and its conjugate momenta.

The analysis of the field equations in the restricted case show that the only physical degrees of freedom in the theory correspond to the \(h_{ij}^{TT}\) tensorial modes and the \(A_i^T\) vectorial modes. The corresponding perturbation equations are (103) and (104).

VI. CONCLUSIONS

We analyzed the Hořava-Lifshitz gravity theory in 4 + 1 dimensions and its KK reduction to 3 + 1 dimensions. These are in principle power counting renormalizable theories provided all \(z = 4\) interaction terms are included in the potential. We consider only the low energy potential terms in the Lagrangian and distinguish the \(\lambda \neq 1/4\) and \(\lambda = 1/4\) theories. They have a different constraint structure and consequently the physical degrees of freedom in the theories are different. The \(\lambda = 1/4\) corresponds to the kinetic conformal point of the theory.
The kinetic term of the theory is conformal invariant at that point, nevertheless the theory is not conformal invariant due to the interaction terms in the Lagrangian.

We obtain in both cases the Hamiltonian, field equations of the theories and determine the propagating degrees of freedom. In the $\lambda = 1/4$ theory, they are exactly the same as the ones in the gravity+electromagnetic+dilaton interaction described in GR. This is an extension of same result in [13, 27] for the pure Hořava-Lifshitz gravity at the kinetic conformal point, $\lambda = 1/3$ in that case. It is interesting that the introduction of the interaction of gravity with the gauge vector and dilaton fields shifts the kinetic conformal point from $\lambda = 1/3$ to $\lambda = 1/4$, although it is natural since the $\lambda = 1/4$ theory arises from a 4 + 1 formulation. This is an important point. We can start from 3 + 1 Hořava-Lifshitz gravity and coupled it to the relativistic energy momentum tensor of the electromagnetic theory. In that case if we want to have only the gravity and the electromagnetic propagation avoiding the scalar field, then we have to take the $\lambda = 1/3$ theory. In distinction, the KK reduction approach we have followed here is natural in the context of the Hořava-Lifshitz theory because we start from a foliation preserving-diffeomorphims invariant theory in 4 + 1 dimensions. The reduction process preserve the foliation. Therefore, the power-counting renormalizability of the resulting 3+1 theory is ensured in the KK approach since it comes from a power counting renormalizable theory (once the $z = 4$ terms have been incorporated). In some cases the coupling to matter fields damage the behaviour of the divergences of the vacuum theory. For example it is known that pure GR is finite at one loop but it is badly divergent at one loop when a scalar field is coupled to it. Since the final 4 + 1 power-counting renormalizable theory needs the higher order terms (at list up to $z=4$ order), our results seem to indicate that the higher order terms in the vectorial sector are needed in order to get a renormalizable theory. With respect to the speed of propagation of the graviton and the gauge vector field it is the same velocity $\sqrt{\beta}$ for both excitations, in the $\lambda \neq 1/4$ and $\lambda = 1/4$ theories.

The gauge symmetry of the gauge vector is generated by the same first class constraint as in the electromagnetic-gravity theory in GR. Moreover, the field equations for the gauge vector have the same structure as in the relativistic case. In particular if we take $\beta = 1$, the Hořava-Lifshitz theory is still an anisotropic formulation of the gravity-electromagnetic interaction in which the field equations for the gauge vector are exactly the Maxwell equations.

An interesting point to be considered is to include higher order terms in the potential and
to analyze the non-linear, non-relativistic extension of the Maxwell equations. A relevant point in this context is that the new non-linear interaction terms are necessarily polynomial of finite grade in the gauge vector field and its covariant derivatives, since the metric and its inverse are polynomial in the gauge vector field.

We also considered the theory in which the dilaton field is in its ground state. That is, we assume $\phi$ to be constant in the Lagrangian. The theory in that case propagates only the gravity and electromagnetic excitations, the transverse traceless components of the metric and the transverse gauge vector components just as in GR or Maxwell equations.

Acknowledgements

A. Restuccia is partially supported by grant Fondecyt No. 1161192, Chile. F. Tello-Ortiz thanks the financial support of the project ANT1756 and the Phd program at the Universidad de Antofagasta.

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