Exactly solvable charged dilaton gravity theories in two dimensions

Youngjai Kiem\(^{(a)}\), Chang-Yeong Lee\(^{(b)}\) and Dahl Park\(^{(c)}\) *

\(^{(a)}\) School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea

\(^{(b)}\) Department of Physics, Sejong University, Seoul 143-747, Korea

\(^{(c)}\) Department of Physics, KAIST, Taejon 305-701, Korea

Abstract

We find exactly solvable dilaton gravity theories containing a $U(1)$ gauge field in two dimensional space-time. The classical general solutions for the gravity sector (the metric plus the dilaton field) of the theories coupled to a massless complex scalar field are obtained in terms of the stress-energy tensor and the $U(1)$ current of the scalar field. We discuss issues that arise when we attempt to use these models for the study of the gravitational back-reaction. 04.60.Kz, 04.20.Jb, 04.40.Nr
In black hole physics, the exactly solvable two dimensional model of Callan, Giddings, Harvey and Strominger (CGHS) played a significant role \[1\]. One of the main virtues of this model is its exact solvability and this aspect inspired many further developments. Using the exact classical solutions of the model, Schoutens, Verlinde and Verlinde concretely realized the black hole complementarity idea by mapping the dilaton gravity theory to a critical string theory \[2\]. Balasubramanian and Verlinde performed a detailed analysis of the gravitationally dressed mass shell dynamics using the CGHS model \[3\], based on the approach of Kraus and Wilczek \[4\]. The CGHS model captures the essential physics of the s-wave Einstein gravity, while being a vastly more tractable model for analytical investigations.

The main focus here is to find a class of two dimensional dilaton gravity theories that include \(U(1)\) gauge fields and, at the same time, are simple enough to allow some analytic treatment of their dynamics. By studying these kinds of models one hopes to further understand the black hole physics in extremal and near-extremal regimes. The dynamics of charged black holes shows many of the intricate and interesting features. In this regard, we mention that the recent advent of the D-brane technology has provided us with a good motivation to better understand the (near) extremal black holes \[5\]. In D-brane approaches, by including a number of gauge fields, it is possible to consider the extremal limit where the Hawking temperature vanishes. It is near this limit where most of the analysis were performed \[5\]. In contrast, the Hawking temperature of the CGHS model is strictly a constant that is related to the cosmological constant. In this note, we find one parameter class of exactly solvable two dimensional dilaton gravity models that contain a \(U(1)\) gauge field and a complex charged scalar field \[6\]. These models have the extremal limit with the vanishing Hawking temperature and, at the same time, retain the virtue of the CGHS model, i.e., the exact solvability of the gravity-dilaton sector.

We consider the dilaton gravity theories given by the following action \[7\].

\[
I = \int d^2x \sqrt{-g} e^{-2\phi} (R + \gamma g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + \mu e^{2\lambda \phi} - \frac{1}{4} e^{\epsilon \phi} F^2 - \frac{1}{2} e^{-2(\delta - 1)\phi} g^{\alpha\beta} (\partial_\alpha - ieA_\alpha) X (\partial_\beta + ieA_\beta) X^*)
\]
The fields in our consideration include a metric tensor $g_{\alpha\beta}$, the dilaton field $\phi$, a $U(1)$ gauge field $A_\alpha$, and a massless complex scalar field $X$. The scalar field $X$ has the $U(1)$ charge $e$.

The signature choice for our metric tensor is $(-+)$. The two dimensional scalar curvature is denoted as $R$ and the $U(1)$ field strength is $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. We use a convention $F^2 \equiv g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu}$. A specific choice for real parameters $\gamma$, $\mu$, $\lambda$, $\delta$ and $\epsilon$ corresponds to a particular dilaton gravity theory. The CGHS model [1], which has been under intensive investigation partly due to their exact solvability at the classical level, corresponds to the case when $\gamma = 4$, $\mu > 0$, $\lambda = 0$, and $\delta = 0$. The s-wave four dimensional Einstein gravity has the value $\gamma = 2$, $\mu = 2$, $\lambda = 1$ and $\delta = 1$, and when the electro-magnetic fields are included, $\epsilon = 0$ [8]. Although its importance is significant, the four dimensional Einstein gravity has not been solved exactly even at the s-wave level, which provides some motivation for devising a simple and exactly solvable model theory of gravity that mimics its physical properties.

The algebraic properties of the curvatures are simple in two dimensions. For example, we have $R_{\alpha\beta} - g_{\alpha\beta}R/2 = 0$ and $g_{\alpha\beta}F^2 = 2g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu}$ as algebraic identities. These identities simplify the actual calculations. Additionally, the behavior of the gravity sector, which includes the graviton and the dilaton, and the $U(1)$ gauge field sector is particularly simple in two dimensional space-time. The metric tensor and the dilaton field have the total of four independent components. Recalling that we have two reparameterization invariance $x^\alpha \rightarrow x^\alpha + \epsilon^\alpha(x^\alpha)$, the physical propagating degrees of freedom for the gravity sector is absent. Likewise, the gauge field has two components and the existence of the $U(1)$ gauge symmetry suggests that there are also no physical propagating degrees of freedom for this sector.

The equations of motion from the action, Eq. (1), are given by

$$-D_\alpha D_\beta \Omega + g_{\alpha\beta}D\cdot D\Omega - \frac{\gamma}{8}(g_{\alpha\beta} \frac{(D\Omega)^2}{\Omega} - 2 \frac{D_\alpha \Omega D_\beta \Omega}{\Omega})$$

$$-\frac{\mu}{2} g_{\alpha\beta} \Omega^{1-\lambda} - \frac{1}{8} g_{\alpha\beta} F^2 \Omega^{1-\epsilon/2} = T^X_{\alpha\beta},$$

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$$-\frac{\mu}{2} g_{\alpha\beta} \Omega^{1-\lambda} - \frac{1}{8} g_{\alpha\beta} F^2 \Omega^{1-\epsilon/2} = T^X_{\alpha\beta},$$
\[ R + \frac{\gamma}{4} \left( \frac{(D\Omega)^2}{\Omega^2} - 2 \frac{D \cdot D\Omega}{\Omega} \right) + (1 - \lambda) \mu \Omega^{-\lambda} - \frac{1}{4} (1 - \frac{\epsilon}{2}) \Omega^{-\epsilon/2} F^2 = T^\chi_\Omega, \quad (3) \]

\[ g^{\mu\beta} \left( (1 - \epsilon) \frac{D\Omega}{\Omega} F_{\alpha\beta} + D_\mu F_{\alpha\beta} \right) = j^X_\alpha, \quad (4) \]

and

\[ (D_\alpha + ie A_\alpha)(g^{\alpha\beta} \Omega^\delta (D_\beta + ie A_\beta) X^*) = 0 \quad (5) \]

along with its complex conjugate. We introduce \( \Omega = e^{-2\phi} \) and \( D_\alpha \) denotes a covariant derivative, along with \( D \cdot D = g^{\alpha\beta} D_\alpha D_\beta \), the two dimensional Laplacian. The stress-energy tensor of the scalar field \( X \) is given by

\[ T^X_{\alpha\beta} = \frac{\Omega^\delta}{2} \left( (\partial_\alpha - ie A_\alpha) X (\partial_\beta + ie A_\beta) X^* - \frac{1}{2} g_{\alpha\beta} g^{\mu\nu} (\partial_\mu - ie A_\mu) X (\partial_\nu + ie A_\nu) X^* \right), \quad (6) \]

and the dilaton charge and the \( U(1) \) current of the scalar field are

\[ T^X_{\alpha i} = \frac{\delta}{2} \Omega^\delta - 1 g^{\mu\nu} (\partial_\mu - ie A_\mu) X (\partial_\nu + ie A_\nu) X^* \quad (7) \]

\[ j^X_\alpha = \frac{1}{2} \Omega^{\delta - 1 + \epsilon/2} (ie (X \partial_\alpha X^* - X^* \partial_\alpha X) - 2e^2 A_\alpha XX^*). \quad (8) \]

We note that, due to the absence of the mass term for the scalar field, the stress-energy tensor \( T^X_{\alpha\beta} \) is traceless, i.e., \( g^{\alpha\beta} T^X_{\alpha\beta} = 0 \).

The key simplification of the equations of motion occurs when we rescale the metric via

\[ g_{\alpha\beta} \rightarrow \Omega^{-\gamma/4} g_{\alpha\beta}. \quad (9) \]

This transformation cancels the kinetic term for the dilaton field in the action, Eq. (1), up to total derivative terms. The remaining changes in Eq. (1) can be summarized formally by

\[ \epsilon \rightarrow \epsilon - \frac{\gamma}{2}, \quad \lambda \rightarrow \lambda + \frac{\gamma}{4}. \]

Thus, the equations of motion are simplified to be

\[ - D_\alpha D_\beta \Omega + g_{\alpha\beta} D \cdot D\Omega - \frac{\mu}{2} g_{\alpha\beta} \Omega^{1-\lambda-\gamma/4} - \frac{1}{8} g_{\alpha\beta} F^2 \Omega^{1-\epsilon/2+\gamma/4} = T^X_{\alpha\beta}, \quad (10) \]
\[ R + (1 - \lambda - \frac{\gamma}{4})\mu\Omega^{-\lambda-\gamma/4} - \frac{1}{4}(1 - \frac{\epsilon}{2} + \frac{\gamma}{4})\Omega^{-\epsilon/2+\gamma/4}F^2 = T^X_\Omega, \]  
(11)

\[ g^{\mu\beta}[(1 - \frac{\epsilon}{2} + \frac{\gamma}{4})\frac{D_{\mu}\Omega}{\Omega}F_{\alpha\beta} + D_{\nu}F_{\alpha\beta}] = j^X_\alpha. \]  
(12)

The stress-energy tensor \( T^X_{\alpha\beta} \) and \( T^X_\Omega \) do not change, while we have a modified expression for \( j^X_\alpha \) as;

\[ j^X_\alpha = \frac{1}{2}\Omega^{\delta - 1 + \epsilon/2 - \gamma/4}(ie(XX^*X^* - X^*X) - 2e^2A_{\alpha}XX^*). \]  
(13)

The equations of motion for the scalar field \( X \), Eq. (5) and its complex conjugate, do not change under the rescaling of the metric.

With equations of motion written in the above fashion, we observe that imposing the conditions for the parameters

\[ 1 - \lambda - \frac{\gamma}{4} = 0, \quad 1 - \frac{\epsilon}{2} + \frac{\gamma}{4} = 0, \quad \delta = 0 \]  
(14)

renders exactly solvable theories. We note that in the absence of the \( U(1) \) gauge field, the CGHS model satisfies the condition \( 1 - \lambda - \gamma/4 = 0 \) and \( \delta = 0 \). For the choice of parameter values for the CGHS model, the above conditions determine \( \epsilon = 4 \). The consequences of the conditions are manifold. In Eq. (10), the non-linear terms for \( \Omega \) field are absent on the left hand side and \( T^X_{\alpha\beta} \) does not depend on the \( \Omega \) field. Additionally, \( j^X_\alpha \) also does not depend on the \( \Omega \) field, and \( T^X_\Omega \) vanishes identically. We have

\[-D_\alpha D_\beta \Omega + g_{\alpha\beta}D \cdot D\Omega - (\frac{\mu}{2} + \frac{1}{8}F^2)g_{\alpha\beta} = T^X_{\alpha\beta} \]  
(15)

\[ R = 0 \]  
(16)

\[ g^{\mu\nu}D_\mu F_{\alpha\nu} = j^X_\alpha. \]  
(17)

Eq. (14) implies that the metric \( g_{\alpha\beta} \) is flat. We can thus choose to use the flat conformal coordinates \( ds^2 = -dx^+dx^- \) for the description of the space-time geometry. Under this gauge choice for the space-time coordinates, we can rewrite Eq. (13) as
\[ \partial^2_{+} \Omega = -T^{X}_{++} \quad , \quad \partial^2_{-} \Omega = -T^{X}_{--} \quad , \quad \partial_{+} \partial_{-} \Omega = -\frac{\mu}{4} + \frac{f^2}{2}, \quad (18) \]

where we introduce \( F_{++} = f = -F_{--} \) and we use the fact that the trace of \( T^{X}_{\alpha \beta} \) vanishes.

Eq. (17) becomes

\[ 2\partial_{+} f = -j^{X}_{+} = -ie(X\partial_{+} X^{*} - X^{*}\partial_{+} X) + 2e^{2}A_{+} XX^{*} \quad (19) \]

\[ 2\partial_{-} f = j^{X}_{-} = ie(X\partial_{-} X^{*} - X^{*}\partial_{-} X) - 2e^{2}A_{-} XX^{*}. \]

The scalar field \( X \) satisfies

\[ (\partial_{+} + ieA_{+})((\partial_{+} + ieA_{-})X^{*}) + (\partial_{-} + ieA_{-})((\partial_{+} + ieA_{+})X^{*}) = 0, \quad (20) \]

from Eq. (5), and its complex conjugate. The crucial point is that Eqs. (19) and (20) are identical to the ones in the flat space-time scalar electrodynamics, and they can be solved to determine \( A_{\pm} \) and \( X \) with proper initial and boundary conditions, up to gauge transformations. This will in turn determine \( T^{X}_{\pm \pm} \) via

\[ T^{X}_{\pm \pm} = \frac{1}{2}(\partial_{\pm} - ieA_{\pm})X(\partial_{\pm} + ieA_{\pm})X^{*}. \quad (21) \]

We can consistently integrate both Eq. (18) and Eq. (19), once we determine \( j^{X}_{\pm} \) and \( T^{X}_{\pm \pm} \). Via the equation of motion for the \( X \) field, Eq. (20), we see that the integrability condition for Eqs. (19),

\[ \partial_{+} j^{X}_{-} + \partial_{-} j^{X}_{+} = 0, \quad (22) \]

which represents the \( U(1) \) current conservation (all other fields in our consideration being neutral), holds. Similarly, upon using Eqs. (19) and (20), we verify that the integrability conditions

\[ \partial_{-} T^{X}_{++} = -f \partial_{+} f \quad , \quad \partial_{+} T^{X}_{--} = -f \partial_{-} f \quad (23) \]

for Eqs. (18) are satisfied. Eqs. (23) represent the stress-energy conservation for the scalar field \( X \) and the \( U(1) \) gauge field. We note that in the presence of non-vanishing charge currents, \( T^{X}_{\pm \pm} \) are not chirally conserved.
The general solution for Eq. (19) is solved to be

\[ f(x^+, x^-) = f(x^+_0, x^-_0) - \frac{1}{2} \int_{x^+_0}^{x^+} j^X_+(x^+, x^-_0) dx^+ - \frac{1}{2} \int_{x^-_0}^{x^-} j^X_-(x^-, x^+_0) dx^- + \frac{1}{2} \int_{x^+_0}^{x^+} j^X_+(x^+, x^-) dx^+ \]  

(24)

where \( x^+_0 \) are arbitrary constants. This solution determines the \( U(1) \) field strength \( f \) in terms of the incoming \( (j^X_+) \) and the outgoing \( (j^X_-) \) charge currents. The general solution for Eq. (18) is

\[ \Omega(x^+, x^-) = \Omega(x^+_1, x^-_1) - \mu/4 (x^+ - x^+_1)(x^- - x^-_1) \]  

(25)

\[ + \frac{1}{2} \int_{x^+_2}^{x^+} \int_{x^-_2}^{x^-} f^2(x'^+, x'^-) dx'^- dx'^+ + \frac{1}{2} \int_{x^+_3}^{x^+} \int_{x^-_3}^{x^-} f^2(x'^+, x'^-) dx'^- dx'^+ \]

\[ - \int_{x^+_4}^{x^+} \int_{x^-_4}^{x^-} T^X_{++}(x'^+, x^+_2) dx'^+ dx^+ - \int_{x^-_5}^{x^-} \int_{x^+_5}^{x^-} T^X_{--}(x^-_2, x^-) dx^- dx^- \]

where \( x^+_1, x^+_2 \) and \( x^+_3 \) are arbitrary constants. The dilaton field is determined in terms of the scalar field stress-energy tensor \( T^X_{\pm\pm} \) and the \( U(1) \) field strength, which, in turn, is given in Eq. (24) via the charge currents of the scalar field. These are the main results of this note. Now that we solved the dilaton and the metric in terms of matter stress-energy tensor and the \( U(1) \) current, we reduced our problem to that of the two dimensional scalar electrodynamics in flat space-time.

Static solutions of the theories in our consideration have been discussed in the literature [9]. The particularly simple case is when we have the gravity-dilaton sector of the CGHS model. Thus, in our further consideration we set \( \gamma = 4 \). The static solutions in this case are given by

\[ \Omega = e^{-2\phi} = \Omega_0 - (\mu/4 - f_0^2/2)x^+x^- \]  

(26)

for the dilaton field and

\[ ds^2 = -dx^+dx^- / \Omega \]

for the metric. Here \( f_0 \) is the charge of the \( U(1) \) gauge field. This solution exhibits the general feature of the higher dimensional stringy charged black holes. If the magnitude of the charge
$f_0$ is less than the critical value $\sqrt{\mu/2}$, we have a black hole geometry with one horizon, which is at $x^- = 0$. Hidden inside this horizon, we have the curvature singularity at $x^+x^- = 2\Omega_0/(\mu/2 - f_0^2)$. Unlike the Reissner-Nordstrom black holes, the stringy charged black hole has a curvature singularity at the would-be inner horizon, and this is the property that our solutions share [10]. If $f_0^2 = \mu/2$, we have space-time geometries that have flat incoming region and flat outgoing region. This behavior is also similar to the higher dimensional stringy charged black holes where extremal solutions have a long throat that connects two flat space-time regions [10]. If $f_0^2 > \mu/2$, we have naked singularities. This consideration suggests that we can regard $f_0^2 = \mu/2$ case as extremal. Indeed, by computing the period of the Euclideanized time coordinate, we determine the Hawking temperature to be

$$T_H = \frac{1}{2\pi}\sqrt{\frac{\mu}{4} - \frac{f_0^2}{2}}$$

and the mass of the black hole is given by

$$M = \Omega_0\sqrt{\frac{\mu}{4} - \frac{f_0^2}{2}}.$$  

Under the suggested condition of the extremality, we find the Hawking temperature vanishes. This is a generic behavior of charged extremal black holes.

The usual way to compute the mass is to integrate the perturbed metric over a space-like hypersurface. However, we can demonstrate the validity of the mass expression given above by considering the injection of charge-neutral stress-energy flux. For this purpose, we rewrite the dilaton expression as

$$\Omega = (\Omega_0 + \int^{x^+} dx^+ T^X_{++}) - k^2 x^+ (x^- + \frac{1}{k^2} \int^{x^+} T^X_{++} dx^+)$$

where we set $T^X_{--} = 0$, use an integration by parts and introduce $k = \sqrt{\mu/4 - f_0^2/2}$. Since we consider the neutral scalar field, the black hole charge $f_0$ does not change. The asymptotically flat coordinates $v$ and $u$ are related to the Kruskal-like coordinates $x^\pm$ via

$$v = \frac{1}{k} \ln x^+ , \quad u = -\frac{1}{k} \ln (-x^-)$$

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outside the black hole, and we see that
\[
\int_{x^-}^{x^+} dx^{++} T_{++}^X = \frac{1}{k} \int v T_{vv}^X.
\]
Since the mass is in fact \( \Omega_0 = M/k \), we find that the first part of Eq. (24) denotes the dynamical increase of black hole mass by the injection of the matter stress-energy. The second part of Eq. (29) shows the shift interaction considered by 't Hooft [11].

Given our results on the classical back-reaction on the space-time caused by the propagating charged scalar fields as shown in Eqs. (24) and (25), the natural next step is to use the gravity theories in this note for the investigation of the quantum gravitational back-reaction. The crucial issue then is the introduction of a boundary. The same situation happens in the quantization of the CGHS model by Schoutens, Verlinde and Verlinde [2]. To mimic the dynamics of the four dimensional \( s \)-wave Einstein gravity, they introduce a reflecting boundary that plays the role of \( r = 0 \) in the \( s \)-wave Einstein gravity. We note that this also allows one to avoid the strong coupling region to be asymptotic in- or out-regions. Once this boundary is introduced, the properties of the back-reaction in the CGHS model become qualitatively similar to those of the \( s \)-wave Einstein gravity [12]. Ultimately, one hopes to relate the behavior of the models considered here to the \( s \)-wave reduction of the five dimensional (or four dimensional) supergravity theory [13]. We note that similar to the \( s \)-wave reduction of the Einstein gravity, we wrote down the two dimensional \( s \)-wave action of the type IIB supergravity on \( T^5 \) in Ref [14]. In other words, our models may share the same qualitative properties as those of the \( s \)-wave supergravity theories when an interacting boundary is introduced (including the gauge interactions). In the context of the CGHS model, the dynamical moving mirror of Ref. [15] provides one such example. In view of the recent developments involving D-brane technology, the following possibility comes out. The important dynamics of the D-brane approach takes place at the level of \( s \)-wave. It is a tempting idea, then, to have a description in terms of an effectively two dimensional field theory with non-trivial boundary interactions, rather similar to the Callan-Rubakov effect. In this picture, it may be possible that we can regard the whole D-brane configurations at
weak coupling as a boundary point where we impose some form of the reflecting boundary condition. As the loop correction makes the D-branes massive, we expect quite non-trivial dynamics of the boundary point itself. Although our models are different from the low energy effective theory of the D-brane systems, we expect that they may provide an insight along this direction in a simpler setting. We plan to discuss the (semi-classical) quantization of our model theories in the future.

ACKNOWLEDGMENTS

Y. K. and C.Y. L. were supported in part by the Basic Science Research Institute Program, Ministry of Education, BSRI-96-2442. D. P. would like to thank Prof. Jae Kwan Kim for helpful guidance and discussions. We also thank W.T. Kim and H. Verlinde for useful discussions.
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