Electromagnetic and gravitational radiation from the coherent oscillation of electron-positron pairs and fields

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Integrating equations of particle-number and energy-momentum conservation and Maxwell field equations, we study the oscillation and drift of electron and positron pairs coherently with fields after these pairs are produced in external electromagnetic fields. From the electric current of oscillating pairs, we obtain the energy spectrum of electromagnetic dipole radiation. This narrow spectrum is so peculiar that the detection of such radiation can identify pair production and oscillation in strong laser fields. We also obtain the energy spectrum of gravitational quadrupole radiation from the energy-momentum tensor of oscillating pairs and fields. Thus, we discuss the generation of gravitational waves on the basis of rapid development of strong laser fields.

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Introduction. Positron and electron pairs are produced from the vacuum in a constant electromagnetic field and the production rate is sizable when the field strength reaches the critical value ($E_c = 1.3 \times 10^{16} \text{ V/cm}$); see Refs. [1, 2]. To reach this critical value in laboratory, based on recent advanced laser technologies, there are many ongoing experiments: x-ray free-electron laser (XFEL) facilities [3], optical high-intensity laser facilities such as Vulcan or ELI [4], and SLAC E144 using nonlinear Compton scattering [5] for details, see Refs. [6–8]. This leads to the physics of ultrahigh intensity laser-matter interactions in the critical field [10].

We focus on the backreaction and screening effects of electron and positron pairs on external electric fields that lead to the phenomenon of plasma oscillations: electrons and positrons moving back and forth coherently with alternating electric fields. In a constant electric field $E_{\text{ext}}$, the phenomenon of plasma oscillations is studied in two frameworks: (1) the semiclassical QED with a quantized Dirac field and classical electric field [11, 12]; and (2) the kinetic description using the Boltzmann-Vlasov equation (or equations of particle-number and energy-momentum conservation) and the Maxwell equations [13, 14]. In Ref. [11], two frameworks are discussed. The first framework is semiclassical, where quantized fermion fields $\psi$ satisfy the Dirac equation in an external classical potential $A_\mu$, which satisfies the Maxwell equation coupling to the mean value of charged fermion current. These equations are numerically integrated in (1+1)-dimensional case. The second framework is classical — the description of particle distribution or density is adopted, and the kinetic equation of the Boltzmann-Vlasov for particle density and the Maxwell equation for fields are numerically integrated. The results obtained in two frameworks are in good quantitative agreement [11], for details, see Refs. [2]. In this paper, we adopt the second framework to investigate the plasma oscillations of electron and positron pairs in the $(2+1)$ space-time with the presence of both electric and magnetic fields. We obtain not only the frequencies of plasma oscillations, but also the oscillating pattern of electron and positron pairs in the $(2+1)$ space-time. In addition, we obtain the energy spectra of electromagnetic and gravitational radiation from plasma oscillations.

PLASMA OSCILLATION.

In 1931 Sauter [15] and four years later Heisenberg and Euler [16] provided a first description of the vacuum properties in constant electromagnetic fields. They identified a characteristic scale of strong field $E_c = m_e^2 c^3/\hbar e$, at which the field energy is sufficient to create electron-positron pairs from the vacuum. In 1951, Schwinger [17] gave an elegant quantum-field theoretic reformulation of their result in the spinor and scalar QED framework (see also [18]). The special attention was given for the presence of magnetic fields [19]. In the configuration of constant electromagnetic fields, the pair-production rate per unit volume is given by

$$\Gamma \propto \frac{\alpha \varepsilon^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \frac{n \pi \beta/\varepsilon}{\tanh n \pi \beta/\varepsilon} \exp \left(-\frac{n \pi E_c}{\varepsilon}\right),$$

where the two Lorentz invariants $\varepsilon$ and $\beta$ are

$$\varepsilon \equiv \sqrt{(S^2 + \mathcal{P}^2)^{1/2} + S}, \quad \beta \equiv \sqrt{(S^2 + \mathcal{P}^2)^{1/2} - S}. \quad (2)$$

In terms of the two Lorentz invariants, the scalar $S \equiv (E^2 - B^2)/2 = (\varepsilon^2 - \beta^2)/2$, and the pseudoscalar $\mathcal{P} = E \cdot B = \varepsilon \beta$. In order to focus on studying the phenomenon of plasma oscillations, as a model for quantitative calculations, we postulate an initial configuration of constant electromagnetic fields: (i) The electric and magnetic fields are perpendicular to each other ($E \perp B$).
(ii) Their amplitudes are different (|E| > |B| ≠ 0) in the laboratory frame, i.e., the rest frame of electron-positron pair production. For this electromagnetic configuration \( \mathcal{P} = 0 \) and leading term \( (n = 1) \), Eq. (4) yields

\[
S = \frac{m_e^2}{4\pi^3} \left( \frac{2S_e}{E_z^c} \right) \exp \left[ -\pi E_z^c (2S)^{1/2} \right],
\]

where the critical field \( E_z^c \equiv m_e^2/e \) and \( m_e (-e) \) is the electron mass (charge). Note that Eq. (3) is valid only for \( S > 0 \), i.e., \( |E| > |B| \) and \( \mathbf{E} \perp \mathbf{B} \). In this case \( \beta = 0 \) and \( \varepsilon^2 = 2S \). Eq. (3) is equivalent to the case for a purely electric field \( E = 2S \). Equation (3) approaches zero as \| \mathbf{B} \| \) approaches \| \mathbf{E} \| \). As shown below, we have chosen an electric field strength \( \mathbf{E} \) that is significantly larger than the magnetic one \( \mathbf{B} \); otherwise, Eq. (3) would approximately vanish for \( S ≈ 0 \) and \( \mathcal{P} = 0 \), analogously to the field configuration of a monochromatic laser beam (plane wave \( S = \mathcal{P} = 0 \)). We will also discuss the situation in which electromagnetic fields are parallel. It is an important issue for future investigations how initial configurations are dynamically generated from the outset.

We adopt \( \hbar = c = 1 \) and Compton units of length \( \lambda_C = h/m_e \), time \( \tau_C = h/m_e \varepsilon^2 \), energy scale \( m_e \varepsilon^2 \) and critical field strength \( E_z^c \).

In the kinetic description for plasma fluids of positrons (+) and electrons (−), with single-particle spectra \( p_{\pm}^0 = (p_{\pm}^c + m_e^2)^{1/2} \), we define the number densities \( n_{\pm}(t, x) \) and “averaged” velocities \( \mathbf{v}_{\pm}(t, x) \) of the fluids:

\[
n_{\pm} \equiv \int \frac{d^3p_\pm}{(2\pi)^3} f_{\pm}, \quad \mathbf{v}_{\pm} \equiv \frac{1}{n_{\pm}} \int \frac{d^3p_\pm}{(2\pi)^3} \left( \frac{p_{\pm}}{p_{\pm}^0} \right) f_{\pm},
\]

where \( f_{\pm} = f_{\pm}(t, p_{\pm}, x) \) is the distribution function in phase space. The four-velocities of the electron and positron fluids are \( U_{\pm}^\mu = (1, \mathbf{v}_{\pm}) \), the Lorentz factor \( \gamma_{\pm} = (1 - \mathbf{v}_{\pm}^2)^{-1/2} \), and the comoving number densities \( \bar{n}_{\pm} = n_{\pm} \gamma_{\pm}^{-1} \). The collisionless plasma fluid of electrons and positrons coupling to electromagnetic fields is governed by the equations of particle-number and energy-momentum conservation and the Maxwell equations:

\[
\frac{\partial (\bar{n}_\pm U_{\pm}^\mu)}{\partial x^\mu} = S; \quad \frac{\partial T_{\mu\nu}^{\text{em}}}{\partial x^\nu} = -F_{\mu\nu}^\text{pol}(J_{\pm}^\mu + J_{\pm}^\text{pola}),
\]

\[
\frac{\partial F_{\mu\nu}}{\partial x^\mu} = -4\pi (J_{\text{cond}}^\mu + J_{\text{pola}}^\mu + J_{\text{ext}}^\mu),
\]

where we have an external electric current \( J_{\text{ext}}^\mu = (\rho_{\text{ext}}, \mathbf{J}_{\text{ext}}) \), electron and positron fluid currents \( J_{\pm}^\mu \equiv \pm e\bar{n}_\pm U_{\pm}^\mu \), and energy-momentum tensors:

\[
T_{\mu\nu} = \bar{p}_{\pm} g_{\mu\nu} + (\bar{p}_{\pm} + \bar{\varepsilon}_{\pm}) U_{\pm}^\mu U_{\pm}^\nu, \quad T_m^\mu = \sum_{\pm} T_{\mu\nu}^\text{pola}.
\]

Here the pressure \( \bar{p}_{\pm} \) and energy density \( \bar{\varepsilon}_{\pm} \) are related by the equation of state \( \bar{p}_{\pm} = \rho_{\pm}(\varepsilon_{\pm}) \) in the fluid comoving frame. In the laboratory frame, the electron and positron energy density \( p_{\pm}^0 = T_{\pm}^{00} \) and momentum density \( \rho_{\pm} \equiv T_{\pm}^{\mu\nu} \) are given by \( p_{\pm}^0 = (\bar{\varepsilon}_{\pm} + \bar{p}_{\pm} \gamma_{\pm}^2) \), and \( \rho_{\pm} = (\bar{\varepsilon}_{\pm} + \bar{p}_{\pm}) \gamma_{\pm}^2 \). The conducting four-current density

\[
J_{\text{cond}}^\mu = e(\bar{n}_+ U_+^\mu - \bar{n}_- U_-^\mu), \quad \partial_{\nu} J_{\text{cond}}^\mu = 0,
\]

and the polarized four-current density \( J_{\text{pola}}^\mu = \sum_{\pm} J_{\text{pola}}^\mu \) with \( J_{\text{pola}}^\mu = (\rho_{\text{pola}}^\mu, \mathbf{J}_{\text{pola}}) \) defined by (4)

\[
F_{\mu\nu}^\text{pola} = \sum_{\pm} F_{\mu\nu}^\text{pola}, \quad \sum_{\pm} = \int \frac{d^3p_{\pm}}{(2\pi)^3} p_{\pm}^\mu A_{\nu},
\]

where \( A \) is related to Eq. (3) by \( S = \int d^3p_{\pm}/[(2\pi)^3 p_{\pm}^0] A \). \( F_{\mu\nu} \) and \( T_{\mu\nu}^\text{pola} \) are the field strength and the energy-momentum tensor of electromagnetic fields.

We now assume external electromagnetic fields \( \mathbf{E}_{\text{ext}} = E_{\text{ext}} \hat{\mathbf{z}} \) and \( \mathbf{B}_{\text{ext}} = B_{\text{ext}} \hat{\mathbf{x}} \), where \( E_{\text{ext}} \) and \( B_{\text{ext}} \) are constant fields in space and time. As will be shown below, in this system, the electron-positron fluid velocities (Eqs. (8) and (9)) and \( B_{\text{z}}(t, y, z) \) are the electromagnetic fields created by the motion of electron and positron pairs.

We adopt the approximations \( \bar{p}_{\pm} = 0, \bar{\varepsilon}_{\pm} \approx m_e \bar{n}_{\pm}, \) and \( \varepsilon_{\pm} = \bar{\varepsilon}_{\pm} \gamma_{\pm}^2 \) when the pair number density is not very large for \( E \approx E_z^c \). Using Eqs. (8) and (9), we obtain

\[
J_{\text{pola}}^\mu \approx \frac{e z_{\pm}}{E^2} (m_e \gamma_{\pm} \Sigma S), \quad J_{\text{pola}}^0 \approx \frac{v_{z}^e E_z + v_{y}^e E_y}{E^2} m_e \gamma_{\pm} \Sigma S,
\]

where \( E^2 = E_z^2 + E_y^2 \). The total electric current and charge densities of the electron-positron fluid are composed by Eqs. (3) and (9) as

\[
J_z = e_+ n_+ v_{z}^e + e_- n_- v_{z}^e + J_{\text{pola}}^z + J_{\text{ext}}^z, \quad J_y = J_y(z \rightarrow y) \quad \text{and} \quad \rho = \sum_{\pm} (e_{\pm} n_{\pm} + J_{0,\text{pola}}), \quad \text{where the positron and electron charge} \quad e_{\pm} = \pm e.
\]

It turns out to be a \( (1 + 2) \)-dimensional problem in space-time coordinates \((t, y, z)\). Equations (8) and (9) are reduced to (i) the particle-number and energy conservation,

\[
\frac{\partial \bar{n}_+}{\partial t} + \frac{\partial n_+ v_{z}^e}{\partial z} + \frac{\partial n_+ v_{y}^e}{\partial y} = S,
\]

\[
\frac{\partial \bar{n}_-}{\partial t} + \frac{\partial n_- v_{z}^e}{\partial z} + \frac{\partial n_- v_{y}^e}{\partial y} = \varepsilon_{\pm} n_{\pm} v_{z}^e E_z + \varepsilon_{\pm} n_{\pm} v_{y}^e E_y + m_e \gamma_{\pm} S;
\]

(ii) the momentum conservation,

\[
\frac{\partial \bar{p}_+}{\partial t} + \frac{\partial \bar{p}_+ v_{z}^e}{\partial z} + \frac{\partial \bar{p}_+ v_{y}^e}{\partial y} = e_+ n_+ E_z - e_- n_- v_{z}^e B_z + E_z J_{0,\text{pola}}^0.
\]
with \((z \leftrightarrow y, B_x \rightarrow -B_x)\); (iii) Maxwell equations \(\nabla \cdot \mathbf{E} = 4\pi \rho, \nabla \cdot \mathbf{B} = 0\),
\[
\frac{\partial \mathbf{E}_x}{\partial t} + \frac{\partial \mathbf{B}_y}{\partial y} = -4\pi J_z, \quad \frac{\partial \mathbf{E}_y}{\partial y} - \frac{\partial \mathbf{E}_z}{\partial z} = -\frac{\partial \mathbf{B}_x}{\partial t},
\]
with \((z \leftrightarrow y, B_x \rightarrow -B_x)\). The pair-production rate [Eq. 3] can be approximately used for varying electromagnetic fields [Eq. 10], provided \(E(t, y, z)\) and \(B(t, y, z)\) created by electron-positron pair oscillations vary very slowly compared with the rate of electron-positron pair productions \(O(m_e c^2 / \hbar)\). This is justified if the inverse adiabaticity parameter \(\eta = m_e E / c \gg 1\), where \(\omega_p\) is the frequency of plasma oscillations.

We are in the position of numerically integrating the basic equations (12) and (13). The initial conditions \((t = 0)\) are given by the constant electromagnetic fields \(E_z = E_{ext} = E_{ext}\) and \(B_x = B_{ext}\). To simplify numerical integrations, we assume the \((z - y)\) homogeneity that the electron-positron fluid quantities and electromagnetic fields are independent of \(y\) and \(z\). As a result, Eqs. (12) and (13) are reduced to ordinary differential equations, and Eq. (13) leads to \(B_x = 0\), i.e., the magnetic field \(B_x\) of Eq. (10) is a constant in space and time. The initial condition \(E_y = 0\) leads to the solution \(E_y = 0\) and \(J_y = 0\) for \(t \neq 0\), because \(v_x^+ = -v_y^+\) and \(v_y^0 = v_y^0 > 0\). This is verified in the following numerical calculations.

To illustrate the plasma oscillations of pairs and fields, we consider two cases: (i) \(E_{ext} = E_c\) and \(B_{ext} = 0.1 E_c\); (ii) \(E_{ext} = E_c\) and \(B_{ext} = 0.3 E_c\). Due to the presence of the magnetic field \(B_x\), pairs are not only oscillating up and down in the \(\hat{z}\) direction, as first shown in Ref. [11], but they also move in the \(\hat{y}\) direction. In Fig. 1 we show the trajectory and velocity of pairs produced at \(z = y = 0\) and \(t = 0\). When \(B_x \neq 0\) and \(dv_y \sim ev_x B_x dt\), \(v_y\) increases for \(v_y > 0\) and decreases for \(v_y < 0\). In the case of \(B_x\) being small enough compared with \(E_x\), \(v_y\) does not change its sign (see Fig. 1). \(B_x = 0.1 E_c\) in the period of one circle oscillation in the \(\hat{z}\) direction; therefore pairs move forward in the \(\hat{y}\) direction. When \(B_x = 0.3 E_c\), \(v_y\) changes its sign (see Fig. 1). \(B_x = 0.3 E_c\); therefore pairs also oscillate back and forth, while they are moving in the \(\hat{y}\) direction. In contrast to the case \(B_x = 0\), the negative \(E_z\) amplitude is smaller than the positive \(E_z\) amplitude (see Fig. 1). The reason is that \(v_y\) increases in the phase of positive decreasing \(v_z\) when \(E_z < 0\); i.e., the electric energy goes to the kinetic energy of the motion in the \(\hat{y}\) direction.

In Fig. 3 we plot (i) the electric current density of pairs \(J_x\) as a function of the time, which is the source of electromagnetic radiation, and (ii) the total energy-momentum tensor of pairs and fields \(T_{\mu \nu} = T_{\mu \nu}^m + T_{\mu \nu}^e\) as functions of the time, which are the sources of gravitational radiation.

Before ending this section, we would like to present some discussions on the role of magnetic fields. In the particular initial configuration of fields \(\mathbf{E} \perp \mathbf{B}\) and \(|\mathbf{E}| > |\mathbf{B}|\) considered in this paper, by integrating Eqs. (3), (4) and (6), we show the oscillating electric field strength (see Fig. 1), and the number and current densities of pairs (see Fig. 2) are suppressed by magnetic fields, compared with their counterparts in the absence of magnetic fields. However, we cannot conclude that such magnetic suppression is generally true. For example, when electromagnetic fields are parallel \((\mathbf{E} \times \mathbf{B} = 0\) and \(|\mathbf{E}| > |\mathbf{B}|\)), Eq. (1) yields (see Ref. 13)
\[
\frac{\Gamma}{V} \simeq \frac{\alpha |\mathbf{B}| |\mathbf{E}|}{\pi} \coth\left(\frac{\pi |\mathbf{B}|}{|\mathbf{E}|}\right) \exp\left(-\frac{\pi E_c}{|\mathbf{E}|}\right),
\]
indicating that the pair-production rate receives an enhancement \((\pi |\mathbf{B}| / |\mathbf{E}|) \coth(\pi |\mathbf{B}| / |\mathbf{E}|)\) to the prefactor, compared with the rate in the absence of magnetic fields [19, 20] (see also [1, 2]). It is worthwhile to study the phenomenon of plasma oscillations by numerically integrating Eqs. (5), (10) and (14) consistently with the initial configuration of parallel electromagnetic fields [21].

**Electromagnetic and gravitational radiation.** We attempt to study electromagnetic and gravitational radiation generated, respectively, by the electric current and energy-momentum tensor of pairs and fields. Suppose that we observe this radiation in the wave zone; that is, at distances much larger than the dimension \(\mathcal{R}\) of the plasma oscillations, and also much larger than \(\omega \mathcal{R}^2\) and
of Eq. (16), we set
the electromagnetic radiation in Eq. (15), and for quadrapole
the nonvanishing components are

The gravitational energy radiated per unit solid angle per
frequency interval is then given by

\[ \frac{d^2E^{em}}{d\omega d\Omega} = 2 \int_{\mathcal{V}} d^3x' \int_{\mathcal{T}} dt' e^{i\omega t' - ikx'} \left[ \frac{\partial J_z(x', t')}{\partial t'} \right] ^2. \tag{15} \]

The gravitational energy radiated per unit solid angle per
frequency interval is then given by

\[ \frac{d^2E^{grav}}{d\omega d\Omega} = 2\omega \left[ T^{\mu\nu}(k, \omega)T_{\mu\nu}(k, \omega) - \frac{1}{2} |T_{\nu\nu}(k, \omega)|^2 \right], \]

\[ T_{\mu\nu}(k, \omega) = \int_{\mathcal{V}} d^3x' \int_{\mathcal{T}} dt' T_{\mu\nu}(x', t') e^{i\omega t' - ikx'}. \tag{16} \]

where \(|k| = \omega \) and \( T^{\mu\nu}(x', t') = T^m_{\mu\nu}(x', t') + T^e_{\mu\nu}(x', t') \).

We consider \( \omega R \ll 1 \) and \( e^{-i\omega t'} \approx 1 \) for dipole
electromagnetic radiation in Eq. (15), and for quadrapole
gravitational radiation in Eq. (16). In the calculations
of Eq. (16), we set \( B_{ext} = 0 \) and \( E_{ext} = E_c \), and then
the nonvanishing components are \( T^00 = T^m_{00} + T^e_{00} \) and
\( T^{zz} = T^m_{zz} + T^e_{zz} \). Using the approximation of spatial ho-

mogeneity in Eqs. (15) and (16), we can factorize out the
volume \( \mathcal{V} = \int d^3\mathbf{r'} \), in which the total energy density
\( T_{00} = T^m_{00} + T^e_{00} = E_{ext}^2/(8\pi) \) is conserved (see Fig. 3).

Let \( \mathcal{T} \) and \( \mathcal{V} \) also be the time and volume of strong
fields \( E_{ext} \gtrsim E_c \) created by coherent laser beams. Se-

lecting different \( \mathcal{T} \) values, in Fig. 4 we plot the
electromagnetic and gravitational radiation spectra \cite{15} and
\cite{10} with \( \mathcal{V}^2 \) factored out. These two energy spectra are
narrow, and the locations (\( \omega_{peak} \)) of their peaks are
related to the coherent oscillation frequency (\( \omega_c \)) of pairs
and fields, which depend on \( \mathcal{T} \) and \( E_{ext} \) (see Ref. \cite{22}).

The peculiar energy spectrum of electromagnetic radiation
is clearly distinguishable from the energy spectra of
the bremsstrahlung radiation, electron-positron annihil-
ation and other possible background events. Therefore, it
is sensible and distinctive to detect such peculiar radi-
tive signatures to identify the production and oscillation of
electron-positron pairs in strong laser fields. As shown in
Fig. 4, gravitational radiation is much smaller than the
electromagnetic one for the reason that the gravita-
tional coupling \( Gm_c^2 = 2.5 \times 10^{-45} \) is much smaller
than the electromagnetic coupling \( e^2 = 1/137 \). In order to
achieve a sizable radiation intensity from the plasma
oscillation, the volume \( \mathcal{V} \) of oscillating pairs and strong
electric fields should be large enough and/or the strength
of strong fields should be enhanced (\( E_{ext} \gtrsim E_c \)) to in-
crease the pair density. It is worthwhile to point out that
Fig. 4 shows the numerical results of Eqs. (15) and (16)
being consistent with the approximate relation between
Eqs. (15) and (16) in the ultrarelativistic limit of charged
particles moving in external electromagnetic fields \cite{26}.

Gravitational waves from an inflationary cosmos \cite{29}
are in the high frequency band \((10^8 - 10^{13}) \) Hz. Gravi-
tational waves originating from some sources in ground
laboratories are also in this frequency band, and several
proposals have been made to detect high-frequency grav-
itational waves up to 5 GHz \cite{30}. Gravitational waves
generated from the high-energy particle beam \cite{26,31}
in the ground experiments of the Stanford Linear Col-
lider and LHC have much higher frequencies of \( O(10^{23}) \) Hz.
The frequency of gravitational wave discussed here
is \( O(10^{19-20}) \) Hz, i.e., \( O(10^{20}) \) KeV, or sub nanometer
\( O(10^{-9-10}) \) cm. It is not clear whether such gravita-
tional waves could ever be detected, or have observable
effects. One would have to build an atom-sized gravita-
tional wave detector to response incoming gravitational
wave with such high frequencies (for some more details, see Ref. [22]).

To end this paper, we remark again that the intensity of electromagnetic radiation emitted by the plasma oscillations is tens of orders of magnitude larger than their gravitational radiation; therefore any detectable signal is enormously more likely to result from the electromagnetic interaction. The prospect of detecting gravitational radiation of such ultrahigh frequencies looks dim. Nevertheless, our theoretical investigation of the gravitational radiation from the electron-positron plasma oscillation would be useful for the study of gravitational radiation emitted from particles and antiparticles in the very early Universe.

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