Pre-Big-Bang Requires the Universe to be Exponentially Large From the Very Beginning

Nemanja Kaloper, Andrei Linde, and Raphael Bousso

Department of Physics, Stanford University, Stanford, CA 94305-4060, USA

(March 28, 2022)

We show that in a generic case of the pre-big-bang scenario, inflation will solve cosmological problems only if the universe at the onset of inflation is extremely large and homogeneous from the very beginning. The size of a homogeneous part of the universe at the beginning of the stage of pre-big-bang (PBB) inflation must be greater than $10^{19} l_s$, where $l_s$ is the stringy length, which is the only natural length scale in PBB cosmology. The total mass of an inflationary domain must be greater than $10^{72} M_s$, where $M_s \sim l_s^{-1}$. If the universe is initially radiation dominated, then its total entropy at that time must be greater than $10^{68}$. If the universe is closed, then at the moment of its formation it must be uniform over $10^{24}$ causally disconnected domains. The natural duration of the PBB stage in this scenario is $M_p^{-1}$. If the universe is open, then its initial state should have been very homogeneous over an infinitely large distance, in order to account for the homogeneity of our part of the universe. We argue that the initial state of the open PBB universe could not be homogeneous because of quantum fluctuations. Independently of the issue of homogeneity, one must introduce two large dimensionless parameters, $g_0 > 10^{-3} M_p^{-1}$, and $B > 10^{48} g_0^{-2} > 10^{31}$, in order to solve the flatness problem in the PBB cosmology. A regime of eternal inflation does not occur in the PBB scenario. This should be compared with the simplest versions of the chaotic inflation scenario, where the regime of eternal inflation may begin in a universe of size $O(M_p^{-1})$ with vanishing initial radiation entropy, mass $O(M_p)$, and geometric entropy $O(1)$. We conclude that the current version of the PBB scenario cannot replace usual inflation even if one solves the graceful exit problem in this scenario.

PACS: 98.80.Cq SU-ITP-97/46

I. INTRODUCTION

After 15 years of development of the theory of inflationary universe, cosmologists believe that something like inflation is indeed necessary in order to construct an internally consistent cosmological theory. The best hopes for the theory of all fundamental interactions are related to superstring theory. Unfortunately it has been known for at least a decade that it is extremely hard to construct a working mechanism for inflation in string theory. The familiar inflationary scenarios, which have been developed in the context of general relativity or even its scalar-tensor extensions, cannot be derived from the string effective action on a generic background. The reason is that in the low energy effective action the string moduli, and in particular the dilaton field, couple nonminimally to other degrees of freedom, and hence run during cosmological evolution. The running of the dilaton slows down the expansion of the horizon, and as a result, the horizon and flatness problems cannot be solved.

Hopefully, this problem is only temporary: it may be related to our limited understanding of string theory, which rapidly changes every year. It might be possible also, that inflation can be implemented in the simplest versions of string theory, but in a rather nontrivial way. One of the most interesting suggestions in this respect is the pre-big-bang (PBB) scenario developed in [1].

This scenario assumes that inflation occurred in a stringy phase prior to the big bang. It relies on the running of the dilaton, which at some time starts to dominate the evolution, and causes the scale factor to diverge in the future. Further, the scenario assumes that there exists a mechanism which eventually becomes important and overturns the dilaton-dominated expansion into a simple power-law, thus avoiding the big bang singularity and solving the horizon problem at the same time. The exit, however, is difficult to attain. For example, it is known, thanks to several exact no-go theorems [2–4], that it cannot be obtained in the effective potential approximation.

As a result, the current version of PBB holds that the exit should occur in the strong coupling/large curvature sector of string theory, where higher order quantum corrections become important.

The PBB scenario has some other problems. It is not easy to obtain density perturbations with a nearly-flat spectrum, it is not quite clear how to solve the primordial monopole problem, etc. Despite these difficulties, the possibility that PBB cosmology may provide a realization of inflationary cosmology in the context of some string-inspired models is certainly very interesting. However, recently it has been argued by Turner and Weinberg [3] that there also exists a fine-tuning problem in the PBB scenario. If the dilaton-dominated era in PBB is preceded by a non-inflating phase, then in order for...
inflation to solve the horizon and flatness problems, the initial universe at the onset of the dilaton-dominated era should be very large. The authors of Ref. [3] concluded that PBB is less robust, and therefore less attractive as an implementation of the inflationary paradigm. On the other hand, from Ref. [4] it was not quite obvious how strong this fine-tuning should be, and whether one can avoid this problem altogether if the universe is open [5], or if one somewhat modifies the PBB scenario [6]. In fact, the largeness of the size of the PBB universe was noticed in the very first papers on the PBB model [7], but it was not considered a real problem.

More recently, another attempt has been made [9] to argue that PBB is unnatural essentially because at the time of the exit the universe should be very large. However, in our opinion, the author of Ref. [9] somewhat misinterpreted the PBB theory and ignored the possibility that this problem can be solved by PBB inflation.

In this paper we will discuss the fine-tuning problem in PBB, using only the most general and widely accepted premises: (1) generic initial conditions which allow that the universe has not been dilaton-dominated throughout its past, (2) the existence of an inflationary phase, which metamorphoses into a post-inflationary universe like our own via a successful process of branch-changing, and (3) the validity of the description of the model by string theory with higher order corrections, implying that \( g_s \leq 1 \). Using this, we will show that if our part of the universe appeared as a result of pre-big-bang inflation, then it should have originated from a homogeneous domain of exponentially large initial size \( L_i > 10^{19} l_s \), where \( l_s = M_s^{-1} \) is the stringy scale. Such a domain should have enormously large initial mass and initial entropy at the onset of PBB inflation.

Immediate consequences of this result look different for a closed universe and for an open one, but the main conclusions are very similar. A closed universe appears from a singularity, and its description in terms of the effective action used in the PBB theory becomes possible only at a stringy time \( t_s = t_s = M_s^{-1} \) after the singularity. We will show that at that time the PBB universe must consist of at least \( 10^{24} \) causally disconnected regions with nearly equal density. The emergence of such a huge homogeneous universe is extremely improbable: this is the well-known horizon problem which inflationary theory is supposed to solve. To solve this problem in the context of the PBB cosmology one would need an additional stage of inflation, not related to the PBB scenario.

If the PBB universe is spatially open, it does not have an initial singularity. Instead, it starts as an infinitely large patch of Minkowski space (or, more precisely, the Milne universe) in the very far past, with almost vanishing matter content except for an infinitesimally small, spatially homogeneous dilaton kinetic energy density. Such a universe should shrink for an infinitely long time, until the dilaton density grows sufficiently large to cause the scale factor to bounce and undergo superinflation.

The initial homogeneity of an open PBB universe filled by a dilaton field \( \Phi \) with an infinitesimally small energy density can be completely destroyed even by extremely small but finite classical perturbations of the dilaton field. Thus, in order to explain why our universe is large, one would need to assume that it is infinite, and in order to explain homogeneity of our local part of the universe, one would need to assume that the early universe was relatively homogeneous on an infinitely large scale. Moreover, we will argue that the initial density of the open universe was so small that its homogeneity could be easily destroyed by quantum fluctuations which were present in the very early universe.

Of course, if the universe is infinite, one can always find a sufficiently large and homogeneous patch of spacetime which can undergo PBB inflation [6]. Parts of the universe similar to the one where we live now can appear as a result of PBB inflation of homogeneous domains with an initial size greater than \( 10^{19} l_s \). However, one may argue that in a contracting universe with chaotic initial conditions, the only natural size of a homogeneous region is \( l_s \). Therefore, the spontaneous formation of a homogeneous domain of size \( 10^{19} l_s \) seems extremely improbable.

We will show also that even if it were possible to solve the homogeneity problem in an open PBB universe, the possibility to solve the flatness problem relies on the existence of two large dimensionless parameters, \( g_0^{-2} > 10^{53} \), and \( B > 10^{38} g_0^{-2} > 10^{91} \). Thus, in order to explain why our universe is flat, one must first explain the origin of these two large dimensionless parameters.

A distinguishing feature of many inflationary models is the existence of a regime of self-reproduction of inflationary domains. This regime leads to especially profound consequences in the chaotic inflation scenario [10,11], but it occurs in many other inflationary models as well [2]. The self-reproduction of inflationary domains may alleviate the initial condition problem even for those inflationary models where the initial conditions required for inflation are unnatural [13]. We have investigated the possibility that a similar mechanism might alleviate the problem of initial conditions in the PBB inflationary cosmology, but we have found that the dynamics of PBB precludes the possibility of self-reproduction.

II. PRE-BIG-BANG

We begin here by reviewing the PBB scenario, to the extent needed for our investigation. The dynamics of the model is given by the four-dimensional effective action of string theory, which contains the metric, dilaton and possibly other matter fields. In the string frame, to the lowest order, the action can be written as

\[
S = \int d^4 x \sqrt{|g_{\mu\nu}|} \frac{e^{-\sigma}}{l_s^2} \left[ \frac{1}{2} R + \frac{1}{2} (\nabla \sigma)^2 + L_m(Y, g_{\mu\nu}, \sigma) \right].
\]

(1)
where $\sigma$ is the dimensionless dilaton field, which determines the string coupling constant $g = \exp(\sigma/2)$, $g_{\mu\nu}$ is the string-frame metric and $Y$ denotes any additional matter degrees of freedom. The dimensional parameter $l_s$ is the string scale, given by $l_s \sim \sqrt{\alpha'}$, and is close to today’s value of the Planck scale. We set $\hbar = c = 1$, so that mass, time and length all have the same units, given by $l_s$. The simplest variant of PBB has been given for spatially flat FRW backgrounds, and has since been investigated in more complicated situations. The crux of the scenario is that any solution starting in the weak coupling regime $g_s \ll 1$ always evolves such that the dilaton grows (i.e., the coupling increases), until it dominates the evolution. Then the universe begins the accelerated expansion, since the scale factor grows superexponentially towards a singularity in the future. The scenario further requires details of the exit scenario, we will not discuss the prospects for actually constructing it. We assume a simple equation of state for the matter, and which might require more complicated equations of state. This will not restrict the generality of our arguments, since we will be studying the conditions at or before the onset of inflationary expansion. There, the validity of the effective action implies that matter behaves ordinarily.

After the onset of the inflationary phase, where all other contributions (matter, spatial curvature etc.) to $1/\ell^2$ except the dilaton are negligible, one can easily find the pure metric-dilaton solutions \[ \text{(1)}. \] The expanding solutions are divided into two classes separated by the curvature singularity. For $t < 0$, one has

\[ a_+ = a_0 (t - t_0)^{-\frac{1}{\sigma}}, \quad H_+ = \frac{1}{\sqrt{3(t - t_0)}}, \quad e^{-\sigma} = e^{-\sigma_0} (t - t_0)^{1 + \sqrt{3}}, \]

whereas for $t > 0$, one has

\[ a_- = a_0 t^{-\frac{1}{\sigma}}, \quad H_- = \frac{1}{\sqrt{3t}}, \quad e^{-\sigma} = e^{-\sigma_0} t^{1 - \sqrt{3}}, \]

The solutions for $t < 0$ are by now widely referred as to the $(+)$, or superinflationary, branch, and those for $t > 0$ as the $(-)$, or post-inflationary, branch. For later purposes it will be useful to re-express the $(+)$-branch solution in terms of the effective Planck mass squared,

\[ \Phi = M_p^2(t) = \frac{1}{l_p^2(t)} = l_s^{-2} e^{-\sigma}, \]

as follows:

\[ a_+ = |t_0|^\frac{1}{\sqrt{3}}, \quad \Phi = \frac{1}{l_p^2(0)} |t|^{\sqrt{3} + 1}, \]  

where the initial time $t_0$ is the aforementioned time of the onset of the pole-dominated era.

Note that the hypersurface $t = 0$ is singular, lying in the future of the $(+)$ (inverse power-law) branch solutions and in the past of the $(-)$ (power-law) branch solutions. The role of the branch-changing mechanism is to connect the two branches, $(+)$ chronologically preceding the $(-)$, in such a way as to remove the singularity, and to allow the superexponential growth of the $(+)$ branch to play the role of inflation. For simplicity, in this article we will assume that such a matching of solutions is possible, without delving into a detailed analysis of string physics. However, we need to review carefully the phenomenological aspects of branch-changing. The current lore of PBB is to assume that the exit must occur roughly when the description given by the effective action \[ \text{(3)} \] breaks down, but that the dynamics can be described by a sum of \[ \text{(4)} \] and higher order corrections, both in $g_s$ and in $\alpha'$. Since the lowest-order string effective action \[ \text{(1)} \] is a truncation of a double expansion in both the inverse string tension $\alpha'$ and the string coupling $g_s = \exp(\sigma/2)$, the description based on it breaks down either when $R \sim 1/\alpha'$ or

\[ \text{The proper definition of branches is derived from solving the quadratic constraint equation in $1/\ell^2$ for $\dot{\sigma}$, and the sign of each branch is determined by the sign of the square root which arises in the solution.}
when $g_s \sim 1$, whichever happens first. But the higher
genus terms alone cannot saturate the growth of the
curvature
, and so even if the exit is precipitated by $g_s \sim 1$, the curvature
would continue to grow until again $R \sim 1/\alpha'$. Similarly, the higher derivative terms cannot
complete the exit by themselves, and thus if the break-
down of the action (4) is caused by $R \sim 1/\alpha'$ effects, the exit does not occur until the coupling reaches $g_s \sim 1$
, while the curvature remains approximately constant. If this happens, the dilaton-driven inflation is sup-
planted at a late time by an exponential phase [14,15,8]. Since
the size of the observable part of the universe ex-
trapolated back to the Planck time is greater than the
order of
, the time of the exit the Hubble length must be of the or-
planted at a late time by an exponential phase [14,15,8]. In any case, all exit scenarios in PBB postulate that at
the time of the exit the Hubble length must be of the or-
der of string scale, $H^{-1} \sim l_s$, while the string coupling is
always assumed to be at most of order unity[9]. Therefore,
using the subscripts $i$ and $f$ to denote the beginning and
the end of the inflationary era, we see that when the exit
occurs it must be

$$H_f^{-1} \sim |t_f| \sim l_s , \quad \Phi_f \leq l_s^{-2} . \quad (7)$$

This illustrates how the conclusion in fact hinges on the
assumption that $g_s \leq 1$. Given this, the numerical values
of any bound which we will obtain using (6) will be very
robust.

After the exit has been completed, the solution continues as a ($-$) branch. Hence, from our vantage point, the
Big Bang would really correspond to the emergence of
the universe from the string phase, at the time roughly
the beginning of PBB inflation, we consider the energy
density of any bound which we will obtain using (6) will be very
robust. In order to determine if PBB is a viable inflationary model, we need to check if the
dilaton-dominated expansion solves cosmological problems.

The resolution of the horizon problem requires that inflation naturally produce the “initial” condition
that the size of the observable part of the universe ex-
trapolated back to the Planck time is greater than the

\[ \frac{\Phi_i}{l_{pl}} \geq \sqrt{3} \times 10^{44} l_s \]. \tag{8}

It is important that these conditions should hold for any
version of the PBB universe, open, flat or closed. Moreover,
one can show that the same conditions follow from the
requirement of flatness of the universe, even if the
homogeneity of the universe is ensured by some other
mechanism. To be a successful model of inflation, the
pre-big-bang model must satisfy all of these inequalities
together with (6).

### III. INITIAL CONDITIONS AT THE ONSET OF INFLATION

As we have mentioned in the previous section, the
initial size of the homogeneous inflationary domain $L_i$
should be greater than $t_i$. Using Eq. (8), one finds the
following lower bound on $L_i$:

$$L_i \geq e^{44} l_s \sim 2 \times 10^{19} l_s . \tag{9}$$

This is not much better than the situation in the non-
inflationary big bang cosmology, where it was necessary
to assume that the initial size of the homogeneous part
of our universe was greater than $10^{30} l_{pl}$.

The measure of the curvature of the spacetime (the
square root of the square of the Riemann tensor) at this
time is given by $R \sim t_i^{-2} \lesssim 10^{-38} l_s^{-2}$. Hence, in
order to explain the flatness of the universe in the PBB
scenario one should start with a universe which is flat
from the very beginning.

To find the lower bound for the mass of the universe at
the beginning of PBB inflation, we consider the energy
stored in the dilaton field inside the horizon volume. Using
equations (6), (3), (3), (3), and $L_i \sim |t_i|$, we find

$$M_i \sim \alpha_i^2 L_i^3 l_{pl}^{-2} \sim |t_i| l_{pl}^{-2} \geq e^{165} M_s \sim 10^{72} M_s . \tag{10}$$

One of the manifestations of the flatness problem in the
usual big bang theory is that the universe today is $10^{40}$
times heavier than the natural mass scale $M_p$. In order
to resolve this problem in the context of PBB cosmology,
one should begin with a universe which has the mass $M >
10^{72} M_s$, where $M_s$ is the only natural mass scale at the
PBB stage (and is approximately the same as the Planck
mass after the Big Bang, as we have discussed above). Thus, PBB does not seem to give us any real advantage in trying to explain the large mass of the universe.

The best way to express the size of the universe in dimensionless units is to calculate its entropy. For the universe filled with ultrarelativistic matter the total entropy coincides, up to a factor $O(1)$, with the total number of particles in the universe. In spacetimes possessing an event horizon, however, one has entropy even in the absence of ordinary matter. For example, the entropy of a horizon-size domain of de Sitter space dominated by a scalar field with a potential energy density $V(\phi)$ is proportional to the area of the event horizon, \[ S = 2\pi A_l^{-2} = 8\pi^2 t_l^{-2} H^{-2} \] This gives 
\[ S \sim \frac{3M_p^4}{8V(\phi)}. \] (11)

Similarly, we can estimate the entropy associated with the event horizon in PBB inflation, \[ S = 2\pi A_t l^{-2}, \] (12)
where $A_t$ is the area of the cosmological horizon at the beginning of PBB inflation, and the parameter $l$ is the characteristic scale for the problem. We leave the option open that $l$ can be either $l_s$ or $l_p$. When $l = l_p$, we see that since $A_t \sim 4\pi L^2 \sim 4\pi |t_i|^2$, Eq. (12) gives (using the conditions (2) and (8))
\[ S \sim 8\pi^2 |t_i|^2 t_p^{-2} \geq 8\pi^2 e^{210} \geq 10^{93}. \] (13)

Hence the initial horizon entropy is exponentially large. In fact, it is not very different from the total entropy of the observable part of the present universe $S \gtrsim 10^{88}$.

One may argue, however, that perhaps in the stringy phase one should calculate the area of the horizon in stringy units. If we take this point of view, our estimate of the entropy becomes smaller, but still remains extremely large: 
\[ S \sim 8\pi^2 |t_i|^2 t_s^{-2} \geq 8\pi^2 e^{89} \geq 2 \times 10^{38}. \] (14)

Hence even this much weaker estimate gives an extremely large quantity.

So far we have taken into account only the gravitational entropy. Including any matter entropy would increase the total entropy. To confirm this we can estimate the entropy of a universe which is initially dominated by radiation with energy density $\rho_t$. The total entropy $S_r$ within the horizon of initial size $|t_i|$ is given by $S_r \sim (\rho_t t_i^4)^{3/4}$. From the constraint equation in (4) we find that $\rho_t t_i^4 \sim t_i^2 \Phi_i$. Using the solution for radiation dominated PBB inflation obtained in Ref. [3] (case $b \gg 1$ in their notation), one can easily find $t_i$ and $\Phi_i$, just as we did before for the case of dilaton-dominated PBB inflation. This leads to the following constraint on the total entropy of radiation in this scenario: 
\[ S_r \gtrsim 10^{88}. \] (15)

The initial entropy can be used to evaluate the probability of spontaneous formation of a homogeneous domain of an inflationary universe. Indeed, one may argue that the entropy is the measure of complexity of the universe, and the probability of creation of a universe with a huge entropy should be exponentially suppressed, 
\[ P \sim e^{-S} = \exp \left( - \frac{3M_p^4}{8V(\phi)} \right). \] (16)

This result exactly coincides with the result obtained in Ref. [8] by a different method. In inflationary cosmology it implies that the probability of inflation starting in a state with $V(\phi) \sim M_p^4$ is not exponentially suppressed. In other words, one does not need to fine-tune initial conditions in the simplest models of chaotic inflation, where inflation may start at $V(\phi) \sim M_p^4$.

Meanwhile, the estimate $P \sim e^{-S}$ for the probability of quantum creation of an inflationary universe leads to a vanishingly small number in the context of the PBB scenario. For the dilaton dominated regime we get, in the very best case, 
\[ P \sim e^{-S} \lesssim \exp \left( -10^{38} \right), \] (17)
whereas for the radiation dominated universe the probability is even much smaller,
\[ P \sim e^{-S} \lesssim \exp \left( -10^{68} \right). \] (18)

These arguments show just how serious the fine-tuning problem is in PBB inflation. In order to be able to solve the horizon and flatness problems, PBB inflation has to start from a universe which is already very large, very dense and very homogeneous and isotropic. Its initial entropy must also be exponentially large, suggesting that this initial state is very improbable.

One can look at this issue from another point of view. Suppose we give up the desire to solve all cosmological problems by a PBB stage, and simply ask what is a natural duration of this stage. One can immediately deduce the answer from Eq. (13). In order to avoid exponential suppression of the probability of creation of a PBB universe, it should be created at $|t_i| \lesssim l_p \sim M_p^{-1}$. Thus, one may expect that the typical duration of PBB inflation is given by the Planck time.

However, in our evaluation of the probability of formation of a homogeneous inflationary domain of size $\sim 10^{19}$ $l_s$ we did not take into account the possibility that such a state may naturally appear as the result of some pre-inflationary dynamics. We will investigate this possibility in the next two sections.
IV. CLOSED PRE-BIG-BANG UNIVERSE

In the usual big bang cosmology quantum fluctuations of the metric are extremely large, the terms $\sim R^2$ in the effective action are greater than $M_p^2 R$, and the effective action approach breaks down at the time $t \lesssim t_p \sim M_p^{-1}$. Therefore one may say that the universe (i.e., classical space-time) “materializes” at the Planck time $t_p \sim M_p^{-1}$. One could try to argue that in the PBB theory the effective action approach breaks down near the big bang, but not in the beginning of the PBB expansion. This seems to be the case for the (+)-branch PBB solutions given in (1). They are described by the string effective action (4) in the far past, with ever weaker coupling and smaller curvature. However, if the universe is inhomogeneous, which is clearly a more generic situation, the answer to this question can be completely different. We should expect that, prior to inflation, the universe on a large scale must be entirely inhomogeneous - after all, one of the roles of inflation is to take one such universe and smooth it out by rapid expansion. If such a solution had a curvature singularity in the past, the action should be extended by higher derivative, or higher order $\alpha'$, corrections, regardless of the magnitude of the string coupling $\alpha'$.

As a result, such a universe would be fully shaped by string physics at the beginning, controlled by the scale $l_s = \sqrt{\alpha'}$, and not by classical phenomena. The question in this case is how big such a universe would have to be initially, in order to survive until the beginning of inflation, and inflate to give the post-big-bang.

The best way to investigate the naturalness of initial conditions in inflationary cosmology is to study a closed universe, which at the same time provides us with the understanding of the behavior of those parts of the universe which are locally very dense. In the big bang theory the total lifetime of a closed universe filled with radiation is proportional to $M/M_p^2$, so a universe of the natural mass $\sim M_p$ immediately collapses unless it begins inflating soon after its formation. That is why the simplest models of inflationary theory based on chaotic inflation are so much better than the new inflation models from the point of view of initial conditions: In the chaotic inflation models the process of exponential expansion may begin immediately after the big bang (4).

Here we would like to analyze the same problem in the context of the PBB theory. The exact spatially closed dilaton-metric solutions of (3) are

$$a = \sqrt{B_{l_p}(0)} \frac{(\cos \eta)^{(1+\sqrt{3})/2}}{3^{1/4} (-\sin \eta)^{(\sqrt{3}-1)/2}},$$

$$\Phi = \frac{1}{l_p^2(0)} \frac{(-\sin \eta)}{\cos \eta} \sqrt{3},$$

(19)

where $\eta$ is the conformal time, defined by $dt = d\eta$. Here $B = -\alpha'^3 \Phi$ is a parameter which does not change during the PBB evolution; $l_p(0)$ is the value of the effective Planck scale at the onset of inflation.

In the limit $\eta \ll 1$, the solution (19) coincides with the flat-space superinflating solution (3), with $|t| \sim |\eta| \sqrt{3}/(\sqrt{3}-1)$.

The onset of inflation corresponds roughly to $\eta \sim -\pi/4$, when $\sin \eta \sim \cos \eta$, and $a \sim -l_s \sim \sqrt{B_{l_p}(0)}$, $\Phi_i \sim l_p^{-2}(0)$. If we want the closed universe to be large enough to incorporate a homogeneous inflationary domain of size $L_* \sim 2 \times 10^{19} l_s$ (3), one should have $B \gtrsim 4 \times 10^{38} l_p^2/l_p^2(0) = 4 \times 10^{38} g_0^{-2}$. Taking into account that, according to Eq. (3), $g_0^{-2} = l_p^2/l_p^2(0) \sim \Phi_i/\Phi_f > \exp(70\sqrt{3})$, one finds the following constraint on the string coupling constant $g_0^2 \sim 10^{-53}$ we get the constraint

$$B \gtrsim 4 \times 10^{91}.$$  (22)

For smaller $g_0$, the constraint on $B$ becomes even stronger.

The beginning of the evolution of the closed PBB universe is described by Eq. (19) in the limit $\eta \rightarrow -\pi/2$. In this case the solution (19) can be approximated by the solution (3), where $t \sim (\pi/2 - |\eta|) \sqrt{3}/(\sqrt{3}-1)$. Thus, $\eta \rightarrow -\pi/2$ again corresponds to $t \rightarrow 0$, but now the scale factor $a$ vanishes, and $R$ and $\Phi$ diverge (which implies that the string coupling $g_s$ vanishes). Here $t$ corresponds to the time after the initial singularity in the closed PBB universe. In what follows we will concentrate on the investigation of conditions near this singularity.

As we have discussed earlier, the description based on the effective action (4) breaks down when $R \sim l_s^{-2} \sim M_p^{-2}$, since the higher order $\alpha'$ corrections must be added regardless of the value of the string coupling $g_s$. Further, it does not make much sense to consider universes of size smaller than $l_s$ in the effective action description, since in string theory we cannot fit any low energy mode in such a universe. Hence, in this limit the solution is valid until $a \sim l_s$ or $R \sim t^{-2}$, whichever comes first when one goes toward the singularity at $t = 0$. From Eq. (4) we see that $a \sim t^{-1/\sqrt{3}}$ and $H^{-1} \sim t$ at small $t$. Thus $H$ grows much faster than $a^{-1}$ in the limit $t \rightarrow 0$. As a result, near the initial singularity the curvature $R = 6H + 12H^2 + 6k/a^2$ is dominated by the kinetic terms, $R \sim H^2 \sim t^{-2} \gg a^{-2}$. This means that the effective action description breaks down not at $a \sim l_s$, but at the stringy time $t \sim t_s \sim l_s \sim M_s^{-1}$. Thus, the $k = 1$ universe emerges out of the initial stringy phase at the stringy time $M_s^{-1}$ after the singularity.

We would like to point out that this is a general conclusion for all stringy models where the universe experiences
a power-law expansion $a \sim t^\beta$ near the singularity, with $\beta = O(1)$. Indeed, in this regime $H \sim t^{-1}$, and $R$ receives a contribution $12H^2 \sim t^{-2}$, which becomes greater than $M_p^2$ for $t < M_p^{-1}$. This means that the stringy time $t_s \sim M_p^{-1}$ plays the same role in stringy cosmology as the Planck time in the standard big bang theory.

If we take a $k = 1$ PBB inflationary solution which satisfies all our constraints (11), (12), (13), (14) and (15), and extrapolate it backwards towards the initial singularity, by the time the curvature again reaches the string scale the universe will still be very large. Indeed, as we have already mentioned, at $t \ll t_i \sim -\sqrt{\mathcal{B} p}(0)$ one has $a \sim t^{1/\sqrt{3}}$. Thus at $t \sim l_s$ one has

$$a \sim l_s (\sqrt{\mathcal{B} p}(0)/l_s)^{1-1/\sqrt{3}} \gtrsim 10^8 l_s.$$  \hfill (23)

Note that if at the initial stage the size of the universe is $10^8$ times greater than the horizon $t_s \sim l_s$, then one should require that the universe is homogenous in $10^{24}$ causally disconnected domains of size $\sim l_s$. One can estimate the probability of this event as

$$P \sim \exp(-10^{24}).$$  \hfill (24)

This clearly demonstrates that the condition (23) requires the universe to be unnaturally large from the very beginning.

If one wants to avoid this probability suppression, one needs to consider a closed universe which consists of just one causally connected domain of size $l_s$ at the stringy time $t_s \sim l_s$. Such a universe would be immediately dilatonic-dominated, would start inflating - and would need to exit right away, after having been inflating for an amount of time merely of order $l_s$. In this case one would have $l_s \simeq l_s$, so the total duration of the PBB stage would be $\sim M_p^{-1}$. Such a universe would never inflate enough to solve the horizon and flatness problems.

Now let us estimate the total mass of the universe at this moment (at the stringy time $t_s \sim M_p^{-1}$ after the initial singularity in the PBB scenario). The calculation is essentially the same as the one which leads to the estimate of the mass at the onset of inflation (10). We again have

$$M_b \sim \sigma_b^2 L_\theta^3 l_p^{-2}(b),$$  \hfill (25)

where the index $b$ denotes quantities at the moment of birth of the universe, at $t \sim t_s$. A good estimate of the size of a closed universe is $L_b \sim a$, where $a$ is given in Eq. (23). Using $\sigma_b^2 \sim R_b \sim l_s^{-2}$ and (23), and recalling that $B > 10^{91}$, we find that

$$M_b \sim \sigma_b^2 L_\theta^3 l_p^{-2}(b) \sim M_b B \gtrsim 10^{91} M_b.$$  \hfill (26)

Thus the mass of the PBB universe at the moment of its creation is even much greater than its mass at the moment when inflation begins (10).

Similarly, one can obtain a constraint on the initial value of the string coupling constant $g^2 = (\Phi l_p^2)^{-1}$ at the moment $t_s$ after the initial singularity:

$$g_b^2 < 10^{-67}.$$  \hfill (27)

This constraint may be viewed as very unnatural from the point of view of M-theory. Indeed, weak coupling limits of consistent string theories are obtained as dimensional reductions of the 11D M-theory, which admits 11D supergravity as its low energy limit. In the process of dimensional reduction of M-theory to a string theory, the string coupling (or equivalently the dilaton field) is identified with the size of the eleventh direction. In particular, if $L_{11}$ is the eleven-dimensional Planck length, we have the following relationship between the size of the 11th direction and the 10D dilaton field: $R_{11} = L_{11} \exp (\Sigma/3)$, where $R_{11}$ is the size of the 11th direction and $\Sigma$ the 10D dilaton. In order to make contact with four-dimensional physics, on which we focus in this article, we must further reduce the 10D string theory to a 4D one, which for our purposes has been defined by the truncation of the action given in (1). This reduction in the context of the PBB cosmology is usually performed on some internal rigid Calabi-Yau three-fold, and if the volume of the Calabi-Yau three-fold is $V_6$, we get the following equation:

$$l_s^2 = \frac{L_s^8}{V_6}.$$  \hfill (28)

Since the usual PBB scenario assumes that the Calabi-Yau is rigid (i.e., has constant size), we can set $\sigma = \Sigma$. The natural volume of the Calabi-Yau three-fold is the string volume, $V_6 \sim l_s^6$. We can see this if we recall that strings cannot probe distances shorter than $l_s$, and hence it does not make sense to set the volume of the Calabi-Yau to be less than $l_s^6$. Also, since today the Planck scale is roughly equal to string scale, $l_p \sim l_s$, if the volume were considerably larger than the string volume, there would be Kaluza-Klein modes with masses much smaller than the Planck mass today. To prevent this, for simplicity we can assume that the Calabi-Yau manifold is compactified at the string scale. In any case, this will give us a rough order-of-magnitude estimate, and any deviation from it could be absorbed in redefining $a$. A possible subtlety involving D-branes and the fact they can probe distances shorter than $l_s$ can be ignored since the PBB scenario is defined in the perturbative sector of string theory, where D-branes are super-heavy. All this leads to the conclusion that $l_s \sim L_{11}$. Therefore, we find that when a successful PBB universe has emerged from the initial string phase, it must have satisfied

$$\frac{R_{11}(b)}{a(b)} \sim l_s g_b^{2/3} \sim g_b^2 \frac{(\sqrt{3}+1)/3}{g_0^{2/3}} \leq e^{-10(\sqrt{3}+1)} g_0^{2/\sqrt{7}} \gtrsim e^{-98} \sim 10^{-42}. $$  \hfill (29)

which clearly indicates that if viewed as an M-theory configuration, the initial universe must be an extremely asymmetric one in order to inflate.
V. OPEN PRE-BIG-BANG UNIVERSE

The problems described in the previous section arise if the universe is closed. If the universe is initially flat or open, there is no initial singularity at the stringy time $t_s \sim t_\ast$. Here we briefly review the issue of naturalness of initial conditions for the spatially open cosmologies. Instead of carrying out a comprehensive analysis, we merely outline generic kinematic conditions and indicate the potential dangers.

The exact spatially open dilaton-metric solutions of (2) are

$$
a = \frac{\sqrt{B l_p(0)}}{3^{1/4}} \left( \cosh \eta \right)^{(1+\sqrt{3})/2} (-\sinh \eta)^{(\sqrt{3}-1)/2},
$$

$$
\Phi = \frac{e^{-\sigma}}{l_s^2} = \frac{1}{l_s^2(0)} \left( -\sinh \eta \right)^{\sqrt{3}}.
$$

The line element is

$$
ds^2 = a^2(\eta) \left( -d\eta^2 + \frac{dr^2}{1 + r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right),
$$

where $\eta$ is the conformal time, defined by $dt = ad\eta$. As earlier, we have $B = -a^6 \Phi / l_s(0)$ is defined as (approximately) the value of the effective Planck scale at the onset of dilaton domination.

In the limit $\eta \ll 1$, the solution (30) is again approximated by the superinflating flat-space solution (3), with $|t| \approx |\eta|^{\sqrt{3}/(\sqrt{3}+1)}$.

The onset of PBB inflation occurs at the time $t_i \sim -\sqrt{B l_p(0)}(0)$. This corresponds to $\sinh \eta \sim -1/\sqrt{3} + 1$. At this time, the scalar field $\Phi$ is roughly $\Phi \sim 0.32 l_p^{-2}(0)$, and hence the string coupling is

$$
g = \frac{l_p(t)}{l_s} \sim 1.77g_0.
$$

Let us now compare this to the conditions in the limit when the universe is “infinitely young”, i.e., $\eta \to -\infty$. In this limit, $-\tanh \eta \to 1$, and so $\Phi = 1/l_p^2(0)$, giving for the initial string coupling $g_0 = l_p(0)/l_s \ll 1$. Hence the string coupling $g$ is minimized by its initial value, in contrast to what happens in the spatially closed and flat cases.

Since the inflationary stage of the open universe PBB theory does not differ much from the inflationary stage of the closed universe scenario, the constraints on the parameters $B$ and $g_0$ in the open PBB universe remain practically the same as in the closed universe case:

$$
B \gtrsim 4 \times 10^{38} g_0^{-2},
$$

and

$$
g_0^2 \lesssim 10^{-53},
$$

so that

$$
\Phi_0 > g_0^{-2} l_s^{-2} > 10^{53} M_s^2.
$$

For $g_0^2 = 10^{-53}$, the parameter $B$ should be greater than $10^{91}$.

In the limit $\eta \to -\infty$ we can approximate $-\sinh \eta \sim \cosh \eta \sim e^{-\eta}/2$, which gives

$$
a = \frac{\sqrt{B l_p(0)}}{2 \cdot 3^{1/4}} e^{-\eta}.
$$

Using $t = \int a(\eta)d\eta$, we find that up to an additive constant,

$$
t = -\frac{\sqrt{B l_p(0)}}{2 \cdot 3^{1/4}} e^{-\eta},
$$

which yields $a = |t|$ and $H = -|t|^{-1}$ when $t \to -\infty$. From the relation $B = a^6 \Phi = -|t|^3 \Phi$ it follows that

$$
\Phi(t) = \Phi_0 - \frac{B}{2t^2}.
$$

Let us discuss the homogeneity problem in this scenario. Eq. (38) shows also that the difference between $\Phi_0$ and $\Phi(t)$ in the early universe was very small:

$$
\Delta \Phi = \Phi_0 - \Phi(t) = \frac{B}{2t^2}.
$$

In the limit when $t \to -\infty$ (going backwards in time), the spatial curvature $a^{-2}$ vanishes as $t^{-2}$, whereas the energy density is falling even faster: $\rho = \frac{8\pi}{3} \sim \frac{B}{2t^2} t^{-6}$. The same is true for the Riemann tensor, $R_{\mu\nu\alpha\beta}$. It is completely determined by the energy density, so it vanishes equally fast: $R_{\mu\nu\alpha\beta} \sim \frac{8\pi}{3} \sim \frac{B}{2t^2} t^{-6}$. Thus, the younger the universe, the flatter the space-time.

As we have already mentioned, in order for the $k = -1$ PBB universe to solve the homogeneity, flatness and horizon problems, at the onset of inflation the linear dimension of a homogeneous and isotropic patch which starts to inflate must be at least as big as $L_0 \sim \sqrt{B l_p(0)}$. This happens at $t_i \sim -\sqrt{B l_p(0)}$. In the past this domain should be much greater, of size $L(t) \sim |t|$, because of the growth of the scale factor $a \sim |t|$.

Thus, the initial size of a homogeneous domain must be finite. One may wonder whether it is a good idea to explain the large size of our universe by assuming that it is infinite, and to explain its homogeneity by assuming that the universe was homogeneous on an infinitely large scale from the very beginning. The only consistent version of an open universe theory which explains how a universe may become homogeneous on an infinitely large scale is given by inflationary cosmology [2]. In the models proposed in [3], the homogeneity of an open universe is explained in a very nontrivial way by a preceding stage of indefinitely long false vacuum inflation. However, in our case we do not have any pre-inflation...
before the pre-big-bang. Therefore we do not know why
the universe should be even approximately homogeneous
from the very beginning, so that $\frac{2\Phi}{p} \lesssim 1$ over an infinitely
large scale.

Indeed, as we have already mentioned, in the limit $t \to -\infty$ the energy density of the matter and the
curvature of the spacetime in an open PBB universe were infinitesimally small. This means that the universe
was practically indistinguishable from empty Minkowski
space. It looks as an open universe only due to the pres-
ence of an infinitesimally small amount of matter mov-
ing in a coherent way. Thus, to produce a contracting
open PBB universe one should take an empty Minkowski
space, add an infinitesimally small amount of nearly ho-
logenously distributed dilaton field $\Phi$, and make this
field move in a coherent manner all over an infinite uni-
verse. This seems to be much more complicated than to
take a Planck-size domain filled with a scalar field $\phi$ in
a chaotic inflation scenario, and let this domain inflate
and self-reproduce, and create an infinitely large amount
of exponentially large homogeneous domains $[1]$.

The possibility that the universe was homogeneous at
$t \to -\infty$ may become even more complicated when one
takes into account quantum fluctuations of the field $\Phi$.
Typically, one expects quantum fluctuations to be im-
portant only near the cosmological singularity. However,
this is not the case in the open PBB universe. In the
limit $t \to -\infty$, the classical value of energy of the scalar
field $\Phi$ was vanishingly small, and quantum fluctuations
could play a dominant role.

Indeed, let us estimate a typical amplitude of quan-
tum fluctuations on a scale comparable with the initial
size of our homogeneous domain $L(t) \sim |t| \sim |H^{-1}|$. The equation of motion for the Fourier modes of dilaton per-
turbations can be written as follows:

$$
\ddot{\Phi}_q + 3H \dot{\Phi}_q + \frac{q^2}{a^2} \Phi_q = 0 ,
$$

where $q$ is the comoving wavenumber, or equivalently,
the inverse comoving wavelength. At large $|t|$, the field
$\Phi = l_p^{-2}$ approaches a constant limit $\Phi_0$, so we can ig-
no re the variation of the Planck mass. Then, one can rep-
cent $\Phi$ as $l_p^{-2}(1 + \phi l_p)$, where $l_p \approx l_p(0)$ is ap-
proximately constant. The action (1) in terms of $\phi$ is
$$
\int d^4x \sqrt{-g} \left\{ R l_p^{-2} / 2 + (\nabla \phi)^2 / 2 + \ldots \right\} .
$$
Thus the field $\phi$ has canonical normalization. Hence its quantum fluctuations on the scale $H^{-1}$ are roughly given by $\delta \phi \sim \frac{\delta \Phi}{l_p} \sim \frac{\sqrt{\Phi H}}{2 \pi} |t|^{-1}$. and therefore we can write

$$
\delta \Phi \sim \frac{\delta \phi}{l_p} \sim \frac{\sqrt{\Phi H}}{2 \pi} \sim \frac{1}{2 \pi l_p(0)} |t|^{-1} .
$$

These quantum fluctuations oscillate with a period $T \sim |t|$, so they do not actually change much during the
subsequent evolution of the universe. Therefore they can
hardly be distinguished from the classical dilaton field $\Phi$.

In order to appreciate the importance of these quantum
fluctuations, one should compare them with the differ-
ence $\Delta \Phi = \Phi_0 - \Phi(t) = \frac{T}{2} |t|^{-2}$. This difference is the only feature which distinguishes the contracting open PBB universe from Minkowski space. It is easy to show that quantum fluctuations become much greater than $\Delta \Phi$ for $|t| \gg B l_p(0)$.

One can reach a similar conclusion by comparing the energy density of the homogeneous component of the scalar field, $\rho = \frac{\Phi^2}{2} \sim \frac{B^2}{2 \pi} |t|^{-6}$, and quantum fluctuations of the energy density of this field, which includes terms such as $\frac{(\dot{\phi})^2}{2} \sim \frac{\langle (\dot{\phi})^2 \rangle}{2} \sim t^{-4}$. We see that the energy density of quantum fluctuations on the scale $|t|$ is much greater than the energy density of the homogeneous component of the scalar field for $|t| \gg B l_p(0)$.

This means that at the very beginning of the evolution of the universe in the PBB scenario, quantum fluctuations $\delta \Phi$ are always much greater than $\Delta \Phi$, and their energy is much greater than the energy of the field $\Phi$. They completely destroy the homogeneity of the open universe PBB solution on the scale $|t|$ which is important for the subsequent PBB inflation. In other words, the assumption of initial homogeneity of the open PBB universe seems to be internally inconsistent when quan-
tum fluctuations are taken into account.

We should emphasize that we are not discussing here the small density perturbations $\frac{2\delta \rho}{\rho} \lesssim 1$ and small pertur-
bations of the metric in the open universe background,
which can be studied with the standard methods of per-
turbation theory. We do not study here the Jeans insta-
bility, which may or may not exist in the PBB cosmology.
Rather we argue that the initial perturbations discussed
above completely destroy the homogeneous background,
making it look like a collection of pieces of open uni-
vesses and closed universes with completely different den-
sity, matched together to provide a chaotic distribution
of segments of space-time with different properties. It is
correct that in the limit $t \to -\infty$ the density of matter
decreases, so we are rapidly approaching Milne space, or Minkowski space $[3]$. The question, however, is whether small perturbations of density in the limit $t \to -\infty$ are greater than the density of the homogeneous component of the field. If this is the case, then the simple descrip-
tion of the universe in terms of Friedmann open or closed
universe models becomes invalid. The way to see it is to compare the part of the Riemann tensor induced by the homogeneous dilaton field, $R_{\mu\nu\rho\sigma} \sim \frac{B^2}{2 \pi} |t|^{-6}$, with the fluctuations of the Riemann tensor related to quantum perturbations, which is proportional to $\Phi_0^{-1} t^{-4}$. Obvi-
ously, quantum fluctuations completely change the ge-
ometry of space for $|t| \gg B l_p(0)$.

Moreover, in order to ensure the homeogeneity of an
inflationary domain of initial size $L_0 \sim \sqrt{B l_p(0)}$, one
would need the universe to be homogeneous not only on a
scale $\sim |t|$, but also on scales much greater than $|t|$. Indeed,
the conditions at any point of the universe at a
moment $|t_0|$ can be influenced by effects occurring at a distance equal to its particle horizon,

$$L_h = a(t) \int_0^t \frac{dt}{a(t)} \sim |t| \ln \frac{|t|}{|t_0|} \gg |t|. \quad (43)$$

As we have argued above, quantum fluctuations tend to destroy homogeneity on a scale $\sim |t|$. Similar arguments show that quantum fluctuations also destroy homogeneity on a larger scale $\sim |t| \ln |t|/|t_0|$.

In fact, perturbations which behave as $|t|^{-1}$ exist at the classical level, too [4], but their amplitude was unknown, so it was hard to evaluate their significance. One could do it only by considering them as small perturbations on the open universe background. As we believe, this is not the case for quantum fluctuations, which can completely destroy this background.

Despite the chaos created by the large density perturbations discussed above, one may expect that in an infinite universe there always are many sufficiently large domains where the universe will be sufficiently homogeneous [5]. In such domains inflation will begin, just like in the chaotic inflation scenario [4], so one may argue that one should concentrate on such domains and ignore those domains which are too inhomogeneous to inflate.

Indeed, it is quite possible that despite initial inhomogeneities, the universe always remains homogeneous on a stringy scale simply because the scale of inhomogeneities in string theory can hardly become smaller than $l_s$, just like the scale of inhomogeneity in gravity theory cannot be smaller than the Planck scale. However, as we have discussed in section III, PBB inflation can solve the cosmological problems only if the size of the initially homogeneous domain is at least 19 orders of magnitude greater than the stringy scale. At the moment we are unaware of any mechanism which would ensure homogeneity of the universe on such a large scale [4]. Thus, we are returning exactly to the same problem as in the case of the closed universe.

One could argue that in a certain sense this might not be such a great problem. The argument could go as follows: In the standard big bang theory the universe was required to be homogeneous at the Planck time on the scale $10^{30} l_p$. In the PBB cosmology the requirement is much more modest; the size of a homogeneous domain should be greater than $10^{19} l_s$. Thus, in the absence of any natural model of stringy inflation, this may still be considered a substantial progress. Indeed, it seems much more probable to achieve homogeneity in a domain of the size $10^{19} l_s$ and then increase this size by the PBB inflation up to $10^{30} l_p$, rather than to start from the very beginning with a homogeneous domain of the size $10^{30} l_p$.

However, this argument does not seem to help much. Indeed, if the initial homogeneous domains were produced by chance rather than by some additional stage of pre-inflation, then it seems much easier to produce homogeneous domains of the size $10^{18} l_s$ rather than the homogeneous domains of the size $10^{19} l_s$. Domains of the initial size $10^{18} l_s$ will inflate and produce locally homogeneous universes with stars and planets like ours, but such universes will be extremely inhomogeneous on the superlarge scale comparable to the scale of the horizon $\sim 10^{28}$ cm.

This is the main reason why we need inflation, and why inflation must somewhat “overshoot.” In the situation where a short stage of inflation is more probable than a long stage of inflation, the universe may be quite hospitable to the existence of life as we know it, but it will be very inhomogeneous on the scale of the horizon. This would contradict observational data which show that the universe on the scale of the horizon is homogeneous at the level better than $10^{-4}$.

It was mainly for this reason that all open inflationary universe models proposed prior to [19] failed. It was always possible to assume that inflation somewhat “undershoots,” and the universe remains open if it was open from the very beginning. But this “undershooting” implied that inflation could not solve the homogeneity problem. As we have already mentioned, in the models proposed in Ref. [19] this problem was solved by a prolonged stage of inflation preceding the creation of a single-bubble open universe. There is no such stage in the current version of the PBB scenario.

But what if, despite all our arguments, we were able somehow to solve the homogeneity problem in the open PBB scenario? Would it mean that we no longer need to have the stage of PBB inflation at all, and our constraints on $g_0$ and $B$, given by Eqs. (33) and (34), disappear?

The answer is no. As we have mentioned in Sect. II, we would still need to have inflation of the same duration as before. If we are going to solve the flatness problem, we need the scale factor of the open universe to become greater than $10^{30} M_p^{-1}$ at the Planck time. Since this scale factor at the onset of inflation is of the same order as $|t_i| \sim \sqrt{B_p(0)}$ or greater, the constraints on $B$, $g_0^2$ and $\Phi_0$ which follow from the condition of flatness of the universe coincide with the constraints, Eqs. (33), (34), and (35), obtained from the condition of homogeneity of the universe on a scale $|t_i|$.

Thus, even if one finds a way to ensure the homogeneity of an open PBB universe on a scale much greater than $|t|$ without any use of inflation, Eqs. (33) and (35),

\[\text{**In fact, a priori the scale } 10^{19} l_s \text{ (unlike the scale } l_s) \text{ does not look special in any respect. If there were any non-inflationary mechanism which would naturally produce homogeneous domains of size } 10^{19} l_s, \text{ then one would imagine that the same mechanism could possibly explain homogeneity of the universe at a much greater scale such as } \sim 10^{30} l_s. \text{ It is very hard to see how it could be otherwise - why would such a mechanism act selectively on inhomogeneities, smoothing ones at scales } \leq 10^{19} l_s \text{ and failing to smooth the ones that occur at scales } \geq 10^{10} l_s. \text{ Then one would not need a prolonged stage of PBB inflation at all.} \]
obtained from the condition of flatness of the universe, will imply that the solution \( \Phi(t) = \Phi_0 - \frac{a_0}{a} \) describing initial conditions in an open universe must contain extremely large parameters \( \Phi_0 = g_0^{-2} M_s^2 > 10^{53} M_s^2 \) and \( B > 4 \times 10^{38} g_0^{-2} > 10^{91} \).

One might try to argue that the large values of these parameters are somehow related to the postulated weakness of the string coupling in the early universe. There is a big difference, however, between a simple assumption that \( g^2 \) was small in the PBB universe, and the requirement that it must be smaller than \( 10^{-53} \). We are unaware of any natural explanation of this number in the PBB theory. Moreover, even the extraordinary smallness of the string coupling does not help to solve the flatness problem. Indeed, if one keeps \( B = \text{const} \), then in the limit \( g_0 \to 0 \) the scale factor of the open universe at the beginning of the PBB inflation vanishes, \( a \sim \sqrt{B} g_0 l_s \to 0 \). Meanwhile we need it to be greater than \( 10^{19l} \), to solve the flatness problem. That is why we get the constraint \( B \gtrsim 4 \times 10^{38} g_0^{-2} \), which implies that even for \( g_0 \sim 1 \) we would need to have \( B \gtrsim 4 \times 10^{38} \), and in the weak coupling limit \( g_0 \to 0 \) we would need to have an infinitely large \( B \).

Obviously, the requirement that \( g_0^2 \) should be smaller than \( 10^{-53} \) and \( B \) should be greater than \( 4 \times 10^{38} g_0^{-2} \) is not a solution of the flatness problem, but its reformulation, where instead of one problem we have two.

After having investigated closed and open PBB universes, we will briefly discuss the spatially flat universe. In models such as chaotic inflation, quantum fluctuations can (over)compensate the decrease of the effective “vacuum” energy due to the slow roll of the order parameter, by pushing the order parameter back towards the region of large effective mass faster than it rolls down the potential ridge. In PBB, however, the energy which drives the expansion of the universe is provided by the kinetic energy of the rolling string coupling, and the duration of inflation is limited by requiring that the coupling must be of large order unity. Hence, the self-regeneration of PBB would require that quantum fluctuations can decrease string coupling \( g_s \), or equivalently, increase the effective Planck mass squared \( \Phi = l_s^{-2} g_s^{-2} \) faster than it rolls down during the expansion.

To give a qualitative description of the conditions for self-regeneration, we should look at the dynamics of coupling inhomogeneities. As we have indicated earlier, during the dilaton-dominated epoch, the coupling obeys a simple Klein-Gordon differential equation: \( \nabla^4 \Phi = 0 \). The individual modes \( \Phi_q(t) \exp(iq \cdot \vec{x}) \) propagate according to Eq. (11). The physical wavelength \( \lambda \) of the mode \( \Phi_q \) and the comoving wavelength \( 1/q \) are related by \( \lambda = a/q \). Now, in PBB, \( H \sim 1/|t| \) and \( 1/a^2 \sim |t|^{-2/3} \). Thus, all waves with the comoving wavelength \( 1/q \geq (l_s/|t_0|)^{1+1/\sqrt{3}} \) will exit the Hubble volume of the universe at some time. Recall that the time and the background value of the scalar field \( \Phi \) are related by (1).
The metric and the Hubble parameter are uniquely determined by \( \Phi \), as can be seen from (3) and (4). We can therefore parameterize the instant when the wave exits by the magnitude of the field \( \Phi \). For a wave with the comoving wavelength \( 1/\bar{q} \) this happens when the field \( \Phi(q) \) is such that the wave vector and the physical wavelength are

\[
q = a(\Phi(q))H(\Phi(q)) , \\
\lambda = \frac{a(\Phi(q))}{q} = \frac{1}{H(\Phi(q))} .
\]

(44)

After this moment, as we see from equation (44), these modes freeze out: since the physical frequency \( q^2/a^2 \) is subleading to the damping term \( \sim H \), inside the Hubble volume the waves with wavelength greater than \( 1/H \) do not oscillate any more. Now, the self-regeneration of inflation requires that at some later instant, when \( \Phi(t) = \bar{\Phi} \), there must be a region inside the original domain which looks exactly the same as the original domain did when \( \Phi(t) = \Phi \). Modeling the self-regeneration processes by a simple doubling of domains, we see that \( \bar{\Phi} \) is related to \( \Phi \) as the instant when the wave with the comoving wavelength \( 1/\bar{q} = 1/(2q) \) froze out. Using this and the definitions (44), we see that \( \bar{\Phi} \) is determined by

\[
\bar{\Phi} = \frac{\Phi}{2\sqrt{3}} .
\]

(46)

By this time, the square of the effective Planck mass decreases by an amount

\[
\Delta \Phi = \Phi - \bar{\Phi} = \Phi \left( 1 - \frac{1}{2\sqrt{3}} \right) .
\]

(47)

This corresponds to the classical rollover and must be compensated by quantum fluctuations in order for the self-reproduction to proceed. The amplitude of quantum fluctuations was estimated in the previous section:

\[
\delta \Phi \sim \frac{\delta \phi}{l_p} \sim \sqrt{\Phi H} \frac{1}{2\pi} .
\]

(48)

If the magnitude of the quantum fluctuation \( \delta \Phi \) is greater than the magnitude of the rollover \( \Delta \Phi \) for any background value of \( \Phi \) and \( H \), the self-regeneration can occur. However, using (15) and (16), we find

\[
\frac{\delta \Phi}{\Delta \Phi} \sim \frac{2\sqrt{3}-1}{\pi(2\sqrt{3}-1)} \frac{H}{\sqrt{3} \pi (2\sqrt{3}-1) |\dot{\Phi}|} .
\]

(49)

This ratio remains exponentially small until the very end of the PBB stage, i.e.,

\[
\delta \Phi \ll \Delta \Phi .
\]

(50)

We see that quantum fluctuations at the stage of PBB inflation are never large enough to overtake the rolling of the field \( \Phi \). Therefore eternal inflation is impossible in the PBB scenario, so it cannot alleviate the problem of initial conditions in the PBB cosmology.

VII. CONCLUSIONS

In this article, we have presented arguments clarifying the problem of initial conditions in the PBB scenario of inflation. Our results suggest that the current version of the PBB scenario, described by an effective action with higher order corrections within a single string theory, cannot solve the homogeneity, isotropy, flatness and horizon problems in the way these are solved by the usual inflationary scenario. The PBB universe must be huge and homogeneous from the very beginning. Our calculations show that in order to solve the horizon, flatness and homogeneity problems, the initial size of the inflationary domain in the PBB scenario should be at least 19 orders of magnitude greater than the string length \( l_s \), which is the only natural length scale in the theory. If the universe is closed, and is to inflate enough to solve the horizon and flatness problems at the Planck time, its initial size prior to PBB inflation has to be at least \( 10^{38} \) times greater than the horizon size at the time of the universe creation. The initial mass of such a universe has to be \( 10^{91} \) times greater than the string mass. We have estimated the probability of one such event to be \( P \sim \exp(-10^{24}) \), i.e., exponentially small. If one wants to avoid this strong suppression, then the natural duration of PBB inflation appears to be as short as \( M_p^{-1} \).

If the universe is open, it must begin in a state with a vanishingly small density of the dilaton field, which should be nearly homogeneously distributed over an infinitely large length scale. We do not know whether the assumption of initial homogeneity in an infinitely large volume is a good way to explain the homogeneity of our part of the universe. Also, the initial homogeneity can be destroyed by any finite density perturbations. Moreover, we argued that the open universe solution is unstable with respect to quantum fluctuations which can make spacetime completely inhomogeneous, so its description in terms of an open universe may become inadequate. Parts of the universe similar to the one we inhabit can appear as a result of PBB inflation in homogeneous domains with an initial size greater than \( 10^{19} l_s \). Such domains may appear by chance in an infinite homogeneous universe. It seems much easier, however, to produce homogeneous domains of a smaller size. This means that typical domains produced by PBB inflation may be homogeneous on a small scale, but should be very inhomogeneous on the scale comparable to the size of the observable part of our universe.

Even if it were possible to solve somehow the problem of initial homogeneity of an open PBB universe, there is one more problem. The open universe solution is characterized by two dimensionless parameters, \( g_0 \) and \( B \). If the flatness problem is to be solved by PBB inflation, i.e., if we want to explain why the present value of \( \Omega \) is not vanishingly small, one should have \( g_0^{-2} \gtrsim 10^{53} \) and \( B \gtrsim 10^{38} g_0^{-2} > 10^{91} \). Note that to solve the flatness problem one should explain why the scale factor of the
universe at the Planck time was $10^{30}$ times greater than the Planck length. Now in order to explain the origin of the large number $10^{30}$ in the context of the PBB cosmology we must introduce two other numbers, $g_0^{-2}$ and $B$, which should be greater than $10^{53}$ and $10^{91}$ respectively. We do not see any natural explanation for appearance of these two different large dimensionless parameters in the theory.

The problems mentioned above are further exacerbated by the fact that a regime of eternal inflation is impossible in the PBB scenario. Hence, even if a region of the universe began to inflate in a PBB phase, it would remain solitary and isolated, and hence just as unlikely to produce the universe we live in.

In closing, we note that the origin of the fine-tuning seems to be twofold. First, PBB is borderline inflation, which does not satisfy the standard inflationary condition $H \ll H^2$. Second, the scenario is defined to be entirely stringy, i.e., the coupling is at most of order unity and so the full evolution of the universe from its birth through PBB to the present day is entirely within the phase space of a single string theory. The first property is a generic feature of any superinflationary model, and thus cannot be atoned. In contrast, one could attempt to relax the second condition and allow the coupling to get greater than unity by the time the exit occurs. While this might have sounded heretical prior to the development of the “web of dualities” [20], one could imagine that this region of very large coupling $g_0 \gg 1$, should be described by a weak coupling region of an $S$-dual of the original string theory in which PBB has begun, or alternatively, by a phase of the parent M-theory. Even if this were permitted, however, the original coupling would have to grow to about $10^{46}$ in order to remove the naturalness bounds we have found. This can be seen, for example, from the mass bound [14], which can be rewritten as $M \gtrsim 10^{101} M_s g_f^{-2}$ where $g_f$ is the coupling of the original theory at the time of the exit. Moreover, even if this were allowed, the current initial conditions require that the cosmological evolution must begin when at least one of the compact dimensions is about $10^{12}$ times smaller than the length scale of the space-time.

We must admit that independently of the prospects of the current version of the pre-big-bang scenario, its investigation has shown us several intriguing possibilities. The dynamics of the PBB model with an account taken of inhomogeneities produced by quantum fluctuations deserves a more detailed investigation. There is always a chance that we could have missed something important, or that our understanding of inhomogeneous PBB models is incomplete. One should remember that in the very beginning of the development of inflationary cosmology some of its authors claimed that it does not work and cannot be improved [21]. It is not inconceivable that some properties of the PBB models which look unnatural from our point of view can become natural when seen from another perspective. It would be very desirable to find a way to overcome the problems which we have found, because it could be an important step towards a realization of inflationary cosmology in the context of string theory.

VIII. ACKNOWLEDGEMENTS

The authors are grateful to Ram Brustein and Gabriele Veneziano for stimulating discussions and useful comments. This work was supported by NSF grant PHY-9219345.

[1] G. Veneziano, Phys. Lett. B265, 287 (1991); M. Gasperini and G. Veneziano, Astropart. Phys. 1, 317 (1993).
[2] R. Brustein and G. Veneziano, Phys. Lett. B329, 429 (1994).
[3] N. Kaloper, R. Madden, and K.A. Olive, Nucl. Phys. B452, 677 (1995).
[4] E.J. Copeland, A. Lahiri, and D. Wands, Phys. Rev. D50 4868 (1994); N. Kaloper, R. Madden, and K.A. Olive, Phys. Lett. B371 34 (1996); R. Easther, K. Maeda, and D. Wands, Phys. Rev. D53 4247 (1996).
[5] M.S. Turner and E. Weinberg, Phys. Rev. D56, 4604 (1997).
[6] G. Veneziano, Phys. Lett. B406, 297 (1997).
[7] A. Buonanno, K. Meissner, C. Ungarelli, and G. Veneziano, e-print hep-th/9706226.
[8] M. Maggiore and R. Stirani, e-print gr-qc/9706053.
[9] D. Coule, e-print gr-qc/9712067.
[10] A.D. Linde, Phys.Lett. B175, 395 (1986).
[11] A.D. Linde, Particle Physics and Inflationary Cosmology (Harwood Academic Publishers, Chur, Switzerland 1990).
[12] P.J. Steinhardt, in: “The Very Early Universe”, G.W. Gibbons, S.W. Hawking, S. Siklos, eds., Cambridge U.P. Cambridge, England (1982), p. 251; A.D. Linde, “Non-singular Regenerating Inflationary Universe,” Cambridge University preprint (1982); A. Vilenkin, Phys. Rev. D27, 2848 (1983); A. Linde, Phys. Lett. B238, 160 (1990).
[13] A.D. Linde, D.A. Linde, and A. Mezhlumian, Phys. Rev. D 49, 1783 (1994).
[14] M. Gasperini, M. Maggiore and G. Veneziano, Nucl. Phys. B494, 315 (1997).
[15] R. Brustein and R. Madden, Phys. Lett. B 410, 110 (1997); Phys. Rev. D 57, 712 (1998).
[16] B.A. Campbell and K.A. Olive, Phys. Lett. B345, 429 (1995).
[17] G.W. Gibbons and S.W. Hawking, Phys. Rev. D15 2752 (1977).
[18] A.D. Linde, JETP 60, 211 (1984); Lett. Nuovo Cimento 39, 401 (1984); Ya.B. Zeldovich and A.A. Starobinsky, Sov. Astron. Lett. 10, 135 (1984); V.A. Rubakov, Phys.
Lett. B148, 280 (1984); A. Vilenkin, Phys. Rev. D30, 549 (1984).

[19] J. R. Gott, Nature 295, 304 (1982); M. Bucher, A. S. Goldhaber and N. Turok, Phys. Rev. D 52, 3314 (1995); K. Yamamoto, M. Sasaki and T. Tanaka, Astrophys. J. 455, 412 (1995); A. D. Linde, Phys. Lett. B 351, 99 (1995); A. D. Linde and A. Mezhlumian, Phys. Rev. D 52, 6789 (1995).

[20] E. Witten, Nucl. Phys. B471, 135 (1996).

[21] A. H. Guth, E. J. Weinberg, Nucl. Phys. B212, 321 (1983).