$D_{s3}^{*}(2860)$ and $D_{s1}^{*}(2860)$ as the 1D $c\bar{s}$ states

Zhi-Gang Wang

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we take the $D_{s3}^{*}(2860)$ and $D_{s1}^{*}(2860)$ as the $1^3D_3$ and $1^3D_1$ $c\bar{s}$ states, respectively, study their strong decays with the heavy meson effective theory by including the chiral symmetry breaking corrections. We can reproduce the experimental data $\text{Br}(D_{sJ}^{*}(2860) \to D^*K)/\text{Br}(D_{sJ}^{*}(2860) \to DK) = 1.10 \pm 0.15 \pm 0.19$ with suitable hadronic coupling constants, the assignment of the $D_{sJ}^{*}(2860)$ as the $D_{s3}^{*}(2860)$ is favored, the chiral symmetry breaking corrections are large. Furthermore, we obtain the analytical expressions of the decay widths, which can be confronted with the experimental data in the future to fit the unknown coupling constants. The predictions of the ratios among the decay widths can be used to study the decay properties of the $D_{s3}^{*}(2860)$ and $D_{s1}^{*}(2860)$ so as to identify them unambiguously. On the other hand, if the chiral symmetry breaking corrections are small, the large ratio $R = 1.10 \pm 0.15 \pm 0.19$ requires that the $D_{sJ}^{*}(2860)$ consists of at least four resonances $D_{s1}^{*}(2860)$, $D_{s2}^{*}(2860)$, $D_{s3}^{*}(2860)$, $D_{s3}^{**}(2860)$.

PACS numbers: 13.25.Ft; 14.40.Lb

Key Words: Charmed mesons, Strong decays

1 Introduction

In 2006, the BaBar collaboration observed the $D_{sJ}^{*}(2860)$ meson with mass $2856.6 \pm 1.5 \pm 5.0$ MeV and width $(48 \pm 7 \pm 10)$ MeV in decays to the final states $D^0K^+$ and $D^+K_S^0$ \cite{1}. There have been several possible assignments. Beveren and Rupp assign the $D_{sJ}^{*}(2860)$ to be the first radial excitation of the $D_{s0}^{*}(2317)$ based on a coupled-channel model \cite{2}. Colangelo, Fazio and Nicotri assign the $D_{sJ}^{*}(2860)$ to be the $1^3D_3$ $c\bar{s}$ state using the heavy meson effective theory \cite{3}. Close et al assign the $D_{sJ}^{*}(2860)$ to be the $2^3P_0$ state in a constituent quark model with novel spin-dependent interactions \cite{4}. Zhang et al assign the $D_{sJ}^{*}(2860)$ to be the $2^3P_0$ or $1^3D_3$ state based on the $3^3P_0$ model \cite{5}. Li, Ma and Liu share the same interpretation based on the Regge phenomenology \cite{6}. However, Ebert, Faustov and Galkin observe that the $D_{sJ}^{*}(2860)$ does not fit well to the Regge trajectory $D_{sJ}^{*}(2112), D_{sJ}^{*}(2573), D_{sJ}^{*}(2860), \cdots$ \cite{7}. Later, Li and Ma assign the $D_{s1}^{*}(2700)$ to be the $1^3D_1 - 2^3S_1$ mixing state and the $D_{sJ}^{*}(2860)$ to be its orthogonal partner, or the $D_{sJ}^{*}(2860)$ to be the $1^3D_3$ state based on the $3^3P_0$ model \cite{8}. Zhong and Zhao assign the $D_{sJ}^{*}(2860)$ to be the $1^3D_3$ state with some $1^3D_2 - 1^3D_2$ mixing component using the chiral quark model, i.e. they assume that the $D_{sJ}^{*}(2860)$ arises from two overlapping resonances $9, 10$. Vijande, Valcarce and Fernandez assign the $D_{sJ}^{*}(2860)$ to be the $c\bar{s} - c\bar{s}\bar{n}\bar{n}$ mixing state \cite{11}. Chen, Wang and Zhang assign the $D_{sJ}^{*}(2860)$ to be the $1^3D_3$ state based on a semi-classic flux tube model \cite{12}. Badalian and Bakker assign the $D_{sJ}^{*}(2860)$ to be the $1^3D_3$ state based on the QCD string model \cite{13}. Guo and Meissner take the $D_{sJ}^{*}(2860)$ as the dynamically generated $D_1(2420)K$ bound state \cite{14}.

In 2009, the BaBar collaboration confirmed the $D_{sJ}^{*}(2860)$ in the $D^*K$ channel, and measured the ratio $R$ among the branching fractions \cite{15},

$$R = \frac{\text{Br}(D_{sJ}^{*}(2860) \to D^*K)}{\text{Br}(D_{sJ}^{*}(2860) \to DK)} = 1.10 \pm 0.15 \pm 0.19.$$

The observation of the decays $D_{sJ}^{*}(2860) \to D^*K$ rules out the $J^P = 0^+$ assignment \cite{2, 4, 5, 6}. On the other hand, if we take the $D_{sJ}^{*}(2860)$ as the $1^3D_3$ state, Colangelo, Fazio and Nicotri obtain the value $R = 0.39$ based on the heavy meson effective theory \cite{3}, while in the $3^3P_0$ model, Zhang et al obtain the value $0.59$ \cite{5}, Li and Ma obtain the value $0.75$ \cite{8}, Song et al obtain the value

\footnotetext{1}{E-mail:zgwang@aliyun.com.}
The two-body strong decays

In Ref. [3], Colangelo, Fazio and Nicotri take the leading order heavy meson effective Lagrangian. Recently, Wu and Huang study the strong decays of the $D^*_s(2860)$ with the heavy meson effective theory in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

|     | Expt [18, 19] | [17] | [21] | [22] |
|-----|--------------|------|------|------|
| $1^3D_1$ | 2859         | 2913 | 2899 | 2913 |
| $1^3D_2$ | –            | 2931 | 2900 | 2900 |
| $1^3D_3$ | –            | 2961 | 2926 | 2953 |
| $1^3D_4$ | 2860         | 2871 | 2917 | 2925 |

Table 1: The masses of the 1D $c\bar{s}$ mesons from the potential models compared to the experimental data.

0.55 $\sim$ 0.80 [15]. Recently, Godfrey and Jardine obtain the value 0.43 based on the relativized quark model and the pseudoscalar emission decay model [17]. The theoretical values differ from the experimental value greatly.

Recently, the LHCb collaboration observed a structure at 2.86 GeV with significance of more than 10 standard deviations in the $\overline{D}^0 K^-$ mass spectrum in the Dalitz plot analysis of the decays $B^0_d \rightarrow \overline{D}^0 K^- \pi^+$, the structure contains both spin-1 and spin-3 components (i.e. the $D^{*+}_{s3}(2860)$ and the $D^{*+}_{s3}(2860)$, respectively), which supports an interpretation of these states being the $J^{P} = 1^-$ and $3^-$ members of the 1D family [18, 19]. The measured masses and widths are $M_{D^{*+}_{s3}} = (2860.5 \pm 2.6 \pm 2.5 \pm 6.0)$ MeV, $M_{D^{*+}_{s3}} = (2859 \pm 12 \pm 6 \pm 23)$ MeV, $\Gamma_{D^{*+}_{s3}} = (53 \pm 7 \pm 4 \pm 6)$ MeV, and $\Gamma_{D^{*+}_{s3}} = (159 \pm 23 \pm 27 \pm 72)$ MeV, respectively. Furthermore, the LHCb collaboration obtained the conclusion that the $D^{*+}_{s3}(2860)$ observed by the BaBar collaboration in the inclusive $e^+e^- \rightarrow \overline{D}^0 K^- X$ production and by the LHCb collaboration in the $pp \rightarrow \overline{D}^0 K^- X$ processes consists of at least these two resonances [15, 20].

According to the predictions of the potential models [7, 21, 22], see Table 1, the masses of the 1D $c\bar{s}$ states is about 2.9 GeV. It is reasonable to assign the $D^{*+}_{s1}(2860)$ and $D^{*+}_{s3}(2860)$ to be the $1^3D_1$ and $1^3D_3$ $c\bar{s}$ states, respectively [18, 19]. However, the theoretical values $R$ differ from the experimental value greatly in the case of the $D^{*+}_{s3}(2860)$ or the $1^3D_3$ assignment of the $D^{*+}_{s3}(2860)$. In Ref.[5], Colangelo, Fazio and Nicotri take the leading order heavy meson effective Lagrangian. The two-body strong decays $D^{*+}_{s3}(2860)$ to $D^+K$, $DK$ take place through the relative F-wave, the final $K$ mesons have the three momenta $p_K = 584$ MeV and 705 MeV, respectively. The decay widths

$$\Gamma(D^{*+}_{s3}(2860) \rightarrow D^+K, DK) \propto p_K^7, \quad (2)$$

where $p_K^7 = 2.3 \times 10^{19}$ MeV$^7$ and $8.6 \times 10^{19}$ MeV$^7$ in the decays to $D^+K$ and $DK$, respectively. Small difference in $p_K$ can lead to large difference in $p_K^7$, so we have to take into account the heavy quark symmetry breaking corrections and chiral symmetry breaking corrections so as to make robust predictions.

In this article, we take into account the chiral symmetry-breaking corrections, and study the two-body strong decays of the $D^{*+}_{s1}(2860)$ and $D^{*+}_{s3}(2860)$ with the heavy meson effective Lagrangian, and try to reproduce the experimental value $R = 1.10 \pm 0.15 \pm 0.19$ by assigning the $D^{*+}_{s3}(2860)$ to be the $D^{*+}_{s3}(2860)$ and $D^{*+}_{s3}(2860)$, respectively. Recently, Wu and Huang study the strong decays of the $D^{*+}_{s0}(2317)$ and $D^{*+}_{s1}(2460)$ by including the chiral symmetry breaking corrections [23]. The heavy meson effective theory have been applied to identify the charmed mesons and bottom mesons [3, 24, 25, 26, 27], and to calculate the radiative, vector-meson, two-pion decays of the heavy quarkonium states [28].

The article is arranged as follows: we derive the strong decay widths of the charmed mesons $D^{*+}_{s1}(2860)$ and $D^{*+}_{s3}(2860)$ with the heavy meson effective theory in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.
The strong decays with the heavy meson effective theory

In the heavy quark limit, the heavy-light mesons $Qar{q}$ can be classified in doublets according to the total angular momentum of the light antiquark $\bar{s}_t$, $\bar{s}_t = \bar{s}_q + \bar{L}$, where the $\bar{s}_q$ and $\bar{L}$ are the spin and orbital angular momentum of the light antiquark, respectively [29]. In this article, the relevant doublets are the $L = 0$ (S-wave) doublet $(P, P^*)$ with $J_{P_t}^{P} = (0^-, 1^-)_2$, and the $L = 2$ (D-wave) doublets $(P_1^*, P_2)$ and $(P_2, P_3^*)$ with $J_{P_t}^{P} = (1^-, 2^-)_2$ and $(2^-, 3^-)_2$, respectively. In the heavy meson effective theory, those doublets can be described by the effective super-fields $H_a$, $X_a$, and $Y_a$, respectively [30].

\[
H_a = \frac{1 + \gamma_5}{2} \{ P^{\mu \nu} \gamma_{\mu} - P_a^{\gamma 5} \},
\]
\[
X_a^\mu = \frac{1 + \gamma_5}{2} \left\{ P^{\mu \nu} \gamma_5 \gamma_\nu - P_{a \nu}^{\gamma} \gamma_5 \left( \frac{g^{\mu \nu} - \frac{\bar{X}^a}{3} (\gamma^\mu + \gamma^\nu)}{3} \right) \right\},
\]
\[
Y_a^{\mu \nu} = \frac{1 + \gamma_5}{2} \left\{ P^{\mu \nu \gamma} \gamma_\sigma - P_{a \nu}^{\gamma} \gamma_5 \left[ g^{\mu \gamma} g^{\sigma \nu} - \frac{g^{\mu \gamma} (\gamma^\mu - \nu^\nu)}{5} - \frac{g^{\sigma \nu} (\gamma^\nu - \nu^\nu)}{5} \right] \right\},
\]

where the heavy meson fields $P^{(\alpha)}$ contain a factor $\sqrt{M_{P^{(\alpha)}}}$ and have dimension of mass $\frac{3}{2}$. The super-fields $H_a$ contain the S-wave mesons $(P, P^*)$; $X_a$, $Y_a$ contain the D-wave mesons $(P_1^*, P_2)$, $(P_2, P_3^*)$, respectively.

The light pseudoscalar mesons are described by the fields $\xi = e^{iM_\eta}$, where

\[
\mathcal{M} = \lambda^j \mathcal{P}^j = \begin{pmatrix}
\sqrt{\frac{2}{\pi}} \pi^0 + \sqrt{\frac{1}{\eta}} \eta & \pi^+ & K^+ \\
\pi^- & -\sqrt{\frac{2}{\pi}} \pi^0 + \sqrt{\frac{1}{\eta}} \eta & K^0 \\
K^- & \bar{K}^0 & -\sqrt{\frac{2}{\pi}} \eta
\end{pmatrix},
\]

and the decay constant $f_\pi = 130$ MeV.

At the leading order approximation, the heavy meson chiral Lagrangians $\mathcal{L}_X$ and $\mathcal{L}_Y$ for the strong decays to the light pseudoscalar mesons can be written as:

\[
\mathcal{L}_X = \frac{g_X}{\Lambda} \text{Tr} \left\{ \bar{H}_a X_a^{\mu} (iD_{\mu} A + iD_\mu A_a) \gamma_5 \right\} + h.c.,
\]
\[
\mathcal{L}_Y = \frac{1}{\Lambda^2} \text{Tr} \left\{ \bar{H}_a Y_a^{\mu \nu} [g_Y (iD_{\mu} A_a) A_\lambda + \bar{g}_Y (iD_\nu A_\lambda A_a + iD_{\nu} iD_\mu A_a) \gamma_5] \right\} + h.c.,
\]

where

\[
D_\mu = \partial_\mu + V_\mu,
\]
\[
V_\mu = \frac{1}{2} \left( \xi \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right),
\]
\[
A_\mu = \frac{1}{2} \left( \xi \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right),
\]
\[
\{D_\mu, D_\nu\} = D_\mu D_\nu + D_\nu D_\mu.
\]

the hadronic coupling constants $g_X$, $g_Y$ and $\bar{g}_Y$ are parameters and can be fitted to the experimental data [31, 32], $\Lambda$ is the chiral symmetry breaking scale and chosen as $\Lambda = 1$ GeV [27].
We construct the chiral symmetry breaking Lagrangians $\mathcal{L}^X$, $\mathcal{L}^Y$ accordingly to Ref.333.

\[
\mathcal{L}^X = \frac{k_X^2}{\Lambda^2} \text{Tr} \left\{ \bar{H}_a X_\mu^a (iD_\mu A + i\mathcal{P} A_\mu) / (m_\xi)_{ca} \gamma_5 \right\} \\
+ \frac{k_X^2}{\Lambda^2} \text{Tr} \left\{ \bar{H}_a X_\nu^a (m_\eta)_{cb} (iD_\mu A + i\mathcal{P} A_\mu) \bar{A} \gamma_5 \right\} \\
+ \frac{k_Y^2}{\Lambda^2} \text{Tr} \left\{ \bar{H}_a X_\mu^a (iD_\mu A + i\mathcal{P} A_\mu) / (m_\xi)_{cc} \gamma_5 \right\} \\
+ \frac{k_Y^2}{\Lambda^2} \text{Tr} \left\{ \bar{H}_a X_\nu^a (iD_\mu A + i\mathcal{P} A_\mu) / (m_\xi)_{ch} \gamma_5 \right\} \\
+ \frac{1}{\Lambda^2} \text{Tr} \left\{ \bar{H}_a X_\mu^a \left[ k_X^2 \{ iD_\mu, iD_\nu \} A_\lambda + k_X^2 \{ iD_\mu, iD_\nu \} v \cdot A \right] \gamma^\lambda \gamma_5 \right\} + h.c.,
\]

where

\[
\{D_\mu, D_\nu, D_\rho\} = D_\mu D_\nu D_\rho + D_\mu D_\rho D_\nu + D_\nu D_\mu D_\rho + D_\nu D_\rho D_\mu + D_\mu D_\rho D_\nu + D_\mu D_\nu D_\rho, \tag{8}
\]

\[
m_\eta = \text{diag}(m_u, m_d, m_s), m_\xi = \xi_0 m_\eta, \xi^1\text{diag}(1,0,0), \text{the hadronic coupling constants} k_X^{1/4}, k_X^2, k_X^3, \bar{k}_X^5 \text{ with } j = 1, 5 \text{ can be fitted to the experimental data.} \]

From the heavy meson chiral Lagrangians $\mathcal{L}^X$, $\mathcal{L}^Y$, $\mathcal{L}^X$ and $\mathcal{L}^Y$, we can obtain the decay widths $\Gamma$ of the strong decays to the light pseudoscalar mesons,

- $\Gamma(1^- \to 2^- + \mathcal{P}_j) = \frac{M_f \left( p_j^2 + m_{P_j}^2 \right)}{6\pi M_f} F_j^2$, \tag{9}
- $\Gamma(1^- \to 0^- + \mathcal{P}_j) = \frac{M_f \left( p_j^2 + m_{P_j}^2 \right)}{9\pi M_f} F_j^2$, \tag{11}
where

\[
F_j = \frac{2g_X}{f_\pi\Lambda}\lambda^j_{ba} + \frac{4k_X^j}{f_\pi\Lambda^2}\lambda^j_{bc}(m_q)_{ca} + \frac{4k_X^j}{f_\pi\Lambda^2}(m_q)_{bc}\lambda^j_{ca} + \frac{4k_X^j}{f_\pi\Lambda^2}(m_q)_{ec} + \frac{4k_X^j}{f_\pi\Lambda^2}\delta_{ba}\lambda^j_{cd}(m_q)_{dc}
- \frac{2(k_X^j + \bar{k}_X^j + \bar{k}_X^j)}{f_\pi\Lambda^2}\sqrt{p_j^2 + m_f^2}\lambda^j_{ba},
\]

\[
p_f = \frac{\sqrt{(M^2_i - (M_f + m_P)^2)(M^2_i - (M_f - m_P)^2)}}{2M_i},
\]

the \(i\) (or \(b\)) and \(f\) (or \(a\)) denote the initial and final state heavy mesons, respectively.

\* \((2^-, 3^-)_\pi \to (0^-, 1^-)_\pi + P_j,\)

\[
\Gamma(3^- \to 1^- + P_j) = \frac{4M_f p_f^7}{105\pi M_i} F_j^2, \quad (13)
\]

\[
\Gamma(3^- \to 0^- + P_j) = \frac{M_f p_f^7}{35\pi M_i} F_j^2, \quad (14)
\]

\[
\Gamma(2^- \to 1^- + P_j) = \frac{M_f p_f^7}{15\pi M_i} F_j^2, \quad (15)
\]

\[3\text{ Numerical Results and Discussions}\]

The input parameters are taken as \(M_{K^+} = 493.677\text{ MeV}, M_{K^0} = 497.614\text{ MeV}, M_\eta = 547.862\text{ MeV}, M_{D^+} = 1869.5\text{ MeV}, M_{D^{*+}} = 1864.91\text{ MeV}, M_{D^0} = 1969\text{ MeV}, M_{D^{*0}} = 2010.29\text{ MeV}, M_{D_{s1}} = 2006.99\text{ MeV}, M_{D_{s2}} = 2112.3\text{ MeV}\) from the Particle Data Group [44].

We redefine the hadronic coupling constants \(\bar{g}_Y = g_Y^\prime + \tilde{g}_Y, \tilde{k}_Y^j = \left(k_Y^j + \bar{k}_Y^j\right)/\bar{g}_Y, j = 1 - 5\), and write down the decay widths of the \(D^*_{s3}(2860)\) explicitly from Eqs.(13-14).

\[
\Gamma(D_{s3}^{*+} \to D^{*0} + K^+) = \frac{16\bar{g}_Y^2 M_D p_K^7}{105\pi f_\pi^2\Lambda^4 M_{D_{s3}}} \left(1 + \frac{2m_u k_Y^1}{\Lambda} + \frac{2m_s k_Y^2}{\Lambda} + \frac{2(m_u + m_d + m_s)k_Y^3}{\Lambda} \right)^2,
\]

\[
\Gamma(D_{s3}^{*+} \to D^0 + K^+) = \frac{4\bar{g}_Y^2 M_D p_K^7}{35\pi f_\pi^2\Lambda^4 M_{D_{s3}}} \left(1 + \frac{2m_u k_Y^1}{\Lambda} + \frac{2m_s k_Y^2}{\Lambda} + \frac{2(m_u + m_d + m_s)k_Y^3}{\Lambda} \right)^2,
\]

\[5\]
\[ \Gamma(D_{s3}^{*+} \to D^{*+} + K^0) = \frac{16\eta^2 M_D p_K^7}{105 \pi f_s^4 M_{D_{s3}}} \left(1 + \frac{2m_d k_Y^1}{\Lambda} + \frac{2m_s k_Y^2}{\Lambda} + \frac{2(m_d + m_s) k_Y^3}{\Lambda} \right) \]
\[ - \left(3\sqrt{p_K^2 + m_Y^2 k_Y^2}\right)^2, \]
\[ \Gamma(D_{s3}^{*+} \to D^+ + K^0) = \frac{4\eta^2 M_D p_K^7}{35 \pi f_s^4 M_{D_{s3}}} \left(1 + \frac{2m_d k_Y^1}{\Lambda} + \frac{2m_s k_Y^2}{\Lambda} + \frac{2(m_d + m_s) k_Y^3}{\Lambda} \right) \]
\[ - \left(\frac{m_d + m_s - 2m_s k_Y^3}{\Lambda}\right) \left(3\sqrt{p_K^2 + m_Y^2 k_Y^2}\right)^2, \]
\[ \Gamma(D_{s3}^{*+} \to D_{s3}^+ + \eta) = \frac{8\eta^2 M_D p_\eta^7}{105 \pi f_s^4 M_{D_{s3}}} \left(1 + \frac{2m_d k_Y^1}{\Lambda} + \frac{2m_s k_Y^2}{\Lambda} + \frac{2(m_d + m_s) k_Y^3}{\Lambda} \right) \]
\[ - \left(\frac{m_d + m_s - 2m_s k_Y^3}{\Lambda}\right) \left(3\sqrt{p_\eta^2 + m_Y^2 k_Y^2}\right)^2. \] (18)

We define the ratios \( R_{0+}, R_{+0}, R_s \) among the decay widths,
\[ R_{0+} = \frac{\Gamma(D_{s3}^{*+} \to D^{*0} + K^+)}{\Gamma(D_{s3}^{*+} \to D^{0} + K^+)}, \]
\[ R_{+0} = \frac{\Gamma(D_{s3}^{*+} \to D^{*+} + K^0)}{\Gamma(D_{s3}^{*+} \to D^{+} + K^0)}, \]
\[ R_s = \frac{\Gamma(D_{s3}^{*+} \to D_{s3}^+ + \eta)}{\Gamma(D_{s3}^{*+} \to D_{s3}^+ + \eta)}. \] (20)

The ratios \( R_{0+}, R_{+0}, R_s \) are independent on the hadronic coupling constants \( \eta_Y, k_Y^1, k_Y^2, k_Y^3, k_Y^4 \), we can absorb the coupling constants \( k_Y^1, k_Y^2, k_Y^3, k_Y^4 \) into the effective coupling \( \eta_Y \) or set \( k_Y^1 = k_Y^2 = k_Y^3 = k_Y^4 = 0 \).

Firstly, let us assign the \( D_{s3}^*(2860) \) to be the \( D_{s3}^*(2860) \), then we can obtain the value,
\[ k_Y^1 = 0.33223 \pm 0.01248, \] (21)

by setting the \( \frac{R_{0+} + R_{+0}}{2} \) to be the experimental data, \( \frac{R_{0+} + R_{+0}}{2} = R = 1.10 \pm 0.15 \pm 0.19 \). On the other hand, if we retain only the leading order coupling constant \( \eta_Y \), then \( \frac{R_{0+} + R_{+0}}{2} = 0.3866 \), which is consistent with the value 0.39 obtained by Colangelo, Fuzio and Nicotri. The value of the hadronic coupling constant \( k_Y^1 + k_Y^2 \) in the chiral symmetry breaking Lagrangian is about \( \frac{1}{4} \) of that of the hadronic coupling constant \( q_Y + \bar{q}_Y \) in the leading-order Lagrangian according to the relation \( \frac{k_Y^1 + k_Y^2}{(q_Y + \bar{q}_Y)} \). However, taking into account such chiral symmetry breaking term can enlarge the ratio \( R \) about 2.8 times.

Then we write down the prediction of the ratio \( R_s \),
\[ R_s = 0.42 \pm 0.06 \ (0.18), \] (22)
the value 0.18 in the bracket comes from the leading order heavy meson effective Lagrangian \( \mathcal{L}_Y \), i.e. only the \( \tilde{g}_Y \) is retained. The chiral symmetry breaking corrections are rather large, the present predictions can be confronted with the experimental data in the futures to study the chiral symmetry breaking corrections.

We can also define the ratios \( R_s \) and \( \tilde{R}_s \),

\[
R_s^0 = \frac{\Gamma(D_{s1}^{*+} \rightarrow D^{*0} + K^+)}{\Gamma(D_{s1}^{-+} \rightarrow D^{-+} + K^0)},
\]
\[
\tilde{R}_s^0 = \frac{\Gamma(D_{s1}^{*+} \rightarrow D^0 + K^+)}{\Gamma(D_{s1}^{-+} \rightarrow D^+ + K^0)},
\]

which are sensitive to the chiral symmetry breaking corrections associate with \( \tilde{k}_Y \). We can estimate the \( \tilde{k}_Y \) by confronting the ratios \( R_s^0 \) and \( \tilde{R}_s^0 \) with the experimental data in the future.

Now we assign the \( D_{s1}^{*+}(2860) \) to be the \( D_{s1}^{*+}(2860) \) and study the strong decays of the \( D_{s1}^{*+}(2860) \) as the \( 1^3D_1 \) state. Firstly, let us redefine the hadronic coupling constants \( k_X^j = \frac{k_X^j}{g_X}, j = 1-4 \), \( k_X^j = (k_X^j + \tilde{k}_X^j + \tilde{k}_X^j)/g_X \), and write down the decay widths explicitly from Eqs.(10-11),

\[
\Gamma(D_{s1}^{*+} \rightarrow D^{*0} + K^+) = \frac{2g^2_XM_{D^*}}{9\pi f_{\pi}^2\Lambda^2 M_{D_s^{*+}}^2} \left( \frac{1}{\Lambda} + \frac{2m_u \tilde{k}_X^1}{\Lambda} + \frac{2m_s \tilde{k}_X^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\tilde{k}_X^3}{\Lambda} \right)^2,
\]

\[
\Gamma(D_{s1}^{*+} \rightarrow D^0 + K^+) = \frac{4g^2_XM_{D^*}}{9\pi f_{\pi}^2\Lambda^2 M_{D_s^{*+}}^2} \left( \frac{1}{\Lambda} + \frac{2m_u \tilde{k}_X^1}{\Lambda} + \frac{2m_s \tilde{k}_X^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\tilde{k}_X^3}{\Lambda} \right)^2,
\]

\[
\Gamma(D_{s1}^{*+} \rightarrow D^+ + K^0) = \frac{2g^2_XM_{D^*}}{9\pi f_{\pi}^2\Lambda^2 M_{D_s^{*+}}^2} \left( \frac{1}{\Lambda} + \frac{2m_u \tilde{k}_X^1}{\Lambda} + \frac{2m_u \tilde{k}_X^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\tilde{k}_X^3}{\Lambda} \right)^2,
\]

\[
\Gamma(D_{s1}^{*+} \rightarrow D^+ + \eta) = \frac{4g^2_XM_{D^*}}{27\pi f_{\pi}^2\Lambda^2 M_{D_s^{*+}}^2} \left( \frac{1}{\Lambda} + \frac{2m_u \tilde{k}_X^1}{\Lambda} + \frac{2m_u \tilde{k}_X^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\tilde{k}_X^3}{\Lambda} \right)^2 - \frac{(m_u + m_d - 2m_s)\tilde{k}_X^4}{\Lambda} - \sqrt{\frac{\Lambda^2}{\Lambda^2} + \frac{m_u^2 + m_d^2 + m_s^2 + \tilde{k}_X^2}{\Lambda^2}},
\]

\[
\Gamma(D_{s1}^{*+} \rightarrow D^+ + \eta) = \frac{8g^2_XM_{D^*}}{27\pi f_{\pi}^2\Lambda^2 M_{D_s^{*+}}^2} \left( \frac{1}{\Lambda} + \frac{2m_u \tilde{k}_X^1}{\Lambda} + \frac{2m_s \tilde{k}_X^2}{\Lambda} + \frac{2(m_u + m_d + m_s)\tilde{k}_X^3}{\Lambda} \right)^2 - \frac{(m_u + m_d - 2m_s)\tilde{k}_X^4}{\Lambda} - \sqrt{\frac{\Lambda^2}{\Lambda^2} + \frac{m_u^2 + m_d^2 + m_s^2 + \tilde{k}_X^2}{\Lambda^2}}.
\]
Then we define the ratio $\mathcal{R}$,

$$\mathcal{R} = \frac{1}{2} \left\{ \frac{\Gamma(D_{s1}^{*+} \to D^{*0} + K^+)}{\Gamma(D_{s1}^{*+} \to D^0 + K^+)} + \frac{\Gamma(D_{s1}^{*+} \to D^{*+} + K^0)}{\Gamma(D_{s1}^{*+} \to D^{+} + K^0)} \right\},$$

which is independent on the hadronic coupling constants $g_X$, $\bar{k}_X$, $\bar{k}_X^2$, $\bar{k}_X^3$, $\bar{k}_X^4$. We can also absorb the coupling constants $\bar{k}_X$, $\bar{k}_X^2$, $\bar{k}_X^3$ into the effective coupling $g_X$ or set $\bar{k}_X = \bar{k}_X^2 = \bar{k}_X^3 = \bar{k}_X^4 = 0$. By setting $\mathcal{R} = \bar{R} = 1.10 \pm 0.15 \pm 0.19$ [15], we can obtain the value,

$$\bar{k}_X^3 = 1.0555 \pm 0.01953.$$  

The value of the hadronic coupling constant $\bar{k}_X^3$ in the chiral symmetry breaking Lagrangian is as large as that of the hadronic coupling constant $g_X$ in the leading-order Lagrangian according to the relation $\bar{k}_X^3 = (k_X^0 + k_X^2 + \bar{k}_X^4)/g_X$. It is unreasonable, and the assignment of the $D_{s1}^*(2860)$ as the $D_{s3}^*(2860)$ is not favored.

Those strong decays of the $D_{s1}^*(2860)$ take place through the relative P-wave, the decay widths are proportional to $p_f^5$, while the strong decays of the $D_{s3}^*(2860)$ take place through the relative F-wave, the decay widths are proportional to $p_f^7$. The decay widths of the $D_{s1}^*(2860)$ are much insensitive to the $p_f$ compared to that of the $D_{s3}^*(2860)$. At the present time, there is no experimental data to fit the hadronic coupling constants. In the leading order approximation, i.e. we neglect the chiral symmetry breaking corrections, the ratios $\overline{\mathcal{R}}$, $\bar{R}$ and $\overline{\mathcal{R}}_s$ among the decay widths are

$$\overline{\mathcal{R}} = 0.24 \ (0.46 \sim 0.70),$$

$$\bar{R} = \frac{\Gamma(D_{s1}^{*+} \to D_s^{*+} + \eta)}{\Gamma(D_{s1}^{*+} \to D^0 + K^+)} = 0.177 \ (0.10 \sim 0.14),$$

$$\overline{\mathcal{R}}_s = \frac{\Gamma(D_{s1}^{*+} \to D_s^{*+} + \eta)}{\Gamma(D_{s1}^{*+} \to D_s^{*+} + \eta)} = 0.17,$$

where in the bracket we present the values from the recent studies based on the $^3P_0$ model [15]. Also in the $^3P_0$ model, Zhang et al obtain the value 0.16 [5]. The present value $\overline{\mathcal{R}}$ differs greatly from that obtained in Ref. [16], while it is compatible with that obtained in Ref. [5]. In Ref. [17], Godfrey and Jardine obtain the value 0.34 based on the relativized quark model and the pseudoscalar emission decay model, which is larger than the present calculation. The present predictions can be confronted with the experimental data in the future to study the strong decays of the $D_{s1}^*(2860)$.

In the leading order approximation, we obtain the values $R = 0.39$ and $\overline{\mathcal{R}} = 0.24$ in the cases of assigning the $D_{s1}^{*+}(2860)$ to be the $D_{s3}^+(2860)$ and $D_{s1}^+(2860)$ respectively, which differ from the experimental value $1.10 \pm 0.15 \pm 0.19$ greatly [15]. If the $D_{s1}^{*+}(2860)$ observed by the BaBar collaboration in the inclusive $e^+e^- \to D^0 K^- X$ production and by the LHCb collaboration in the $pp \to D^0 K^- X$ processes consists of two resonances $D_{s1}^+(2860)$ and $D_{s1}^*(2860)$ [15,20], we expect to obtain an even smaller ratio $R$ in case of the chiral symmetry breaking corrections are small. On the other hand, if the $D_{s1}^{*+}(2860)$ consists of at least four resonances $D_{s1}^+(2860)$, $D_{s2}^+(2860)$, $D_{s3}^+(2860)$, $D_{s3}^*(2860)$, the large ratio $R = 1.10 \pm 0.15 \pm 0.19$ is easy to count for, as the $J^P = 2^-$ mesons $D_{s2}^*$(2860) and $D_{s2}^*(2860)$ only decay to the final states $D^+K$, see Eq.(9) and Eq.(15).

4 Conclusion

In this article, we take the $D_{s3}^*(2860)$ and $D_{s1}^*(2860)$ as the $1^3D_3$ and $1^3D_1$ $c\bar{s}$ states, respectively, study their strong decays with the heavy meson effective theory by including the chiral symmetry breaking corrections. We can reproduce the experimental value of the ratio $R$,
\[ R = \frac{\text{Br} (D^*_{sJ}(2860) \to D^* K)}{\text{Br} (D^*_{sJ}(2860) \to D K)} = 1.10 \pm 0.15 \pm 0.19, \] with suitable hadronic coupling constants, the assignment of the \( D^*_{sJ}(2860) \) as the \( D^*_{s3}(2860) \) is favored. The chiral symmetry breaking corrections are large, we should take them into account. Furthermore, we obtain the analytical expressions of the decay widths, which can confronted with the experimental data in the future from the LHCb, CDF, D0 and KEK-B collaborations to fit the unknown coupling constants. The present predictions of the ratios among the decay widths can be used to study the decay properties of the \( D^*_{s3}(2860) \) and \( D^*_{s1}(2860) \) so as to identify them unambiguously. On the other hand, if the chiral symmetry breaking corrections are small, the large ratio \( R = 1.10 \pm 0.15 \pm 0.19 \) requires that the \( D^*_{sJ}(2860) \) consists of at least four resonances \( D^*_{s1}(2860), D^*_{s2}(2860), D^*_{s2}'(2860), D^*_{s3}(2860) \).

Acknowledgment

This work is supported by National Natural Science Foundation, Grant Number 11375063, and Natural Science Foundation of Hebei province, Grant Number A2014502017.

References

[1] B. Aubert et al, Phys. Rev. Lett. 97 (2006) 222001.
[2] E. v. Beveren and G. Rupp, Phys. Rev. Lett. 97 (2006) 202001.
[3] P. Colangelo, F. De Fazio and S. Nicotri, Phys. Lett. B642 (2006) 48.
[4] F. E. Close, C. E. Thomas, O. Lakhina and E. S. Swanson, Phys. Lett. B647 (2007) 159.
[5] B. Zhang, X. Liu, W. Z. Deng and S. L. Zhu, Eur. Phys. J. C50 (2007) 617.
[6] D. M. Li, B. Ma and Y. H. Liu, Eur. Phys. J. C51 (2007) 359.
[7] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C66 (2010) 197.
[8] D. M. Li and B. Ma, Phys. Rev. D81 (2010) 014021.
[9] X. h. Zhong and Q. Zhao, Phys. Rev. D78 (2008) 014029.
[10] X. H. Zhong and Q. Zhao, Phys. Rev. D81 (2010) 014031.
[11] J. Vijande, A. Valcarce and F. Fernandez, Phys. Rev. D79 (2009) 037501.
[12] B. Chen, D. X. Wang and A. Zhang, Phys. Rev. D80 (2009) 071502.
[13] A. M. Badalian and B. L. G. Bakker, Phys. Rev. D84 (2011) 034006.
[14] F. K. Guo and U. G. Meissner, Phys. Rev. D84 (2011) 014013.
[15] B. Aubert et al, Phys. Rev. D80 (2009) 092003.
[16] Q. T. Song, D. Y. Chen, X. Liu and T. Matsuki, arXiv:1408.0471.
[17] S. Godfrey and I. T. Jardine, Phys. Rev. D89 (2014) 074023.
[18] R. Aaij et al, arXiv:1407.7574.
[19] R. Aaij et al, arXiv:1407.7712.
[20] R Aaij et al, JHEP 1210 (2012) 151.
[21] S. Godfrey and N. Isgur, Phys. Rev. D32 (1985) 189.
[22] M. Di Pierro and E. Eichten, Phys. Rev. D64 (2001) 114004.
[23] J. Y. Wu and M. Q. Huang, arXiv:1406.5804
[24] Z. G. Wang, Phys. Rev. D83 (2011) 014009.
[25] Z. G. Wang, Phys. Rev. D88 (2013) 114003; Z. G. Wang, Eur. Phys. J. Plus 129 (2014) 186.
[26] P. Colangelo, F. De Fazio, F. Giannuzzi and S. Nicotri, Phys. Rev. D86 (2012) 054024.
[27] P. Colangelo, F. De Fazio and R. Ferrandes, Phys. Lett. B634 (2006) 235; P. Colangelo, F. De Fazio, S. Nicotri and M. Rizzi, Phys. Rev. D77 (2008) 014012; P. Colangelo and F. De Fazio, Phys. Rev. D81 (2010) 094001.
[28] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B302 (1993) 95; F. De Fazio, Phys. Rev. D79 (2009) 054015; Z. G. Wang, Eur. Phys. J. A47 (2011) 94; Z. G. He, X. R. Lu, J. Soto and Y. Zheng, Phys. Rev. D83 (2011) 054028; Z. G. Wang, Int. J. Theor. Phys. 51 (2012) 1518; Z. G. Wang, Mod. Phys. Lett. A27 (2012) 1250197; Z. G. Wang, Commun. Theor. Phys. 57 (2012) 93.
[29] A. V. Manohar and M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 10 (2000) 1; M. Neubert, Phys. Rept. 245 (1994) 259.
[30] A. F. Falk, Nucl. Phys. B378 (1992) 79; A. F. Falk and M. E. Luke, Phys. Lett. B292 (1992) 119.
[31] M. B. Wise, Phys. Rev. D45 (1992) 2188; G. Burdman and J. F. Donoghue, Phys. Lett. B280 (1992) 287; P. Cho, Phys. Lett. B285 (1992) 145; U. Kilian, J. G. Korner and D. Pirjol, Phys. Lett. B288 (1992) 360.
[32] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, F. Feruglio, R. Gatto and G. Nardulli, Phys. Rept. 281 (1997) 145.
[33] C. G. Boyd and B. Grinstein, Nucl. Phys. B442 (1995) 205; I. W. Stewart, Nucl. Phys. B529 (1998) 62; T. Mehen and R. P. Springer, Phys. Rev. D72 (2005) 034006; S. Fajfer and J. F. Kamenik, Phys. Rev. D74 (2006) 074023.
[34] K. A. Olive et al, Chin. Phys. C38 (2014) 090001.