NUMERICAL STUDY OF NATURAL CONVECTION IN SQUARE CAVITY USING LATTICE BOLTZMANN METHOD: OBSTACLES EFFECT

A. BENDARAA\textsuperscript{1,}\textsuperscript{*}, M. NOUNI\textsuperscript{1}, M. OUHROUM\textsuperscript{1}, M. M. CHARAFI\textsuperscript{1}, A. HASNAOUI\textsuperscript{1}

\textsuperscript{1}LS3M polydisciplinary faculty of Khouribga, Sultan Mly Slimane University 25000, Morocco.

*Corresponding author: bendaraa.anass@usms.ac.ma

Abstract: In this work, the natural convection is studied in a closed square cavity differentially heated, using the Lattice Boltzmann numerical Method. Completely filled with a newtonian fluid, the cavity is managed by two rigid walls in right and left sides, and is provided with two rectangular adiabatic obstacles interposed on the mid-horizontal walls. The height of the two obstacles is varied from 10\% to 40\% of the side (L). The calculation code was performed for a laminar flow and an incompressible fluid with a Prandtl number $Pr = 0.71$ (the case of air). The computation code thus developed was used to analyse the effect of the length of obstacles in the mid-square cavity. The obtained results were validated by comparing with those of a previously published works. It has been deduced that the addition of the obstacles influences the thermal and hydrodynamic profiles of the flow inside the cavity and its thermal performances.

Keywords: Natural convection, Numerical methods, Lattice Boltzmann Method, Finite difference method (FD), obstacles.

1. Introduction

The heat transfer phenomenon by natural convection in confined spaces have been and continue to be the subject of numerous research activities in the fundamental field as well as in the experimental fields. Many works and applications studied will be able to satisfy the requirements of many industrial applications, in particular in building, solar collectors, cooling of electronic components, nuclear engineering, heating and ventilation, Etc. Natural convection is fundamental interest to mathematicians, physicists, chemists and biologists, and many others scientists. As a result, it has become essential for any engineer, whatever the field in which he will be called, to have good knowledge of the fundamental laws of thermal science.
To solve this type of problems, we must resort to numerical methods [1] if we cannot solve them analytically. Among these most used methods, we can cite, finite volumes (FVM), finite elements (FEM), finite differences (FDM) methods. In this work, we have used the Lattice Boltzmann Method to solve the problem of convection [2], [3]. We will present a contribution in the use of this new method which is based on the thermal mesoscopic approach of the Boltzmann equation on a Lattice, applied to the problem of heat transfer by natural convection within a closed square cavity and differentially heated. The cavity is equipped with two adiabatic obstacles. It is considered that the two vertical walls of the cavity are isothermal and the horizontal ones are adiabatic. The obstacles effect has become the subject of significant investigation by researchers in the field giving example of [4] who have studied the effect of fins on convective flow.

The Lattice Boltzmann method, commonly called LBM [5], is a recent new alternative for the numerical simulation of physical phenomena which emerged in the last decade. The method which is based on the resolution of the Boltzmann equation with the hydrodynamic approach, it describes the movement of a fluid at a mesoscopic level (speed, pressure, density) by the distribution of these different particles [6]. This method is well suited to simulation because it represents a very interesting new approach relative to classical methods, such as finite volumes, finite differences and finite elements which are based on the discretization of the Navier-Stokes equations. The Lattice Boltzmann method has proven advantages by its efficiency in the solution of fluid mechanics problems lies mainly in the simplicity of the modelling and the simulation of complex flow phenomena, it is very suitable in the implementation which allows an easy and efficient parallel calculation. The study presented in this work aims to analyse the phenomenon of heat transfer by natural convection in laminar mode, in a square cavity with two horizontal adiabatic walls and two vertical isothermal and differentially heated walls [7]. The cavity completely filled with Newtonian fluid is provided with two adiabatic obstacles of rectangular shape, inserted on the horizontal walls. The study is carried out for Rayleigh numbers from $10^4$ to $10^6$ and a Prandtl number Pr = 0.71. The influence of the height of the obstacles (H) will also be treated for values of 10% to 40% of (L) Height of the cavity.

## 2. Mathematical formulation

Let us consider a square cavity of height L (Fig. 1). This cavity is filled with a fluid having a Prandtl number (Pr = 0.71) initially at the temperature $T_c$. The horizontal walls of the cavity are assumed to be adiabatic. The vertical walls are maintained at constant temperatures $T_f$ and $T_c$ (hot and cold temperatures respectively), so a horizontal thermal gradient is established. The non-slip condition is applied to all walls of the cavity [8]. The fluid filling the cavity is subjected to the only action of the gravity field (g) directed downwards in a Cartesian coordinate system ($o, x, y$). The fluid is supposed to be Newtonian and incompressible. The governing equations follow the equation of continuity, the equations of conservation of momentum and energy represented as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (1)

$$x \rightarrow U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)$$  \hspace{1cm} (2)
y \rightarrow U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = \frac{\partial P}{\partial Y} + Pr \left( \frac{\partial V}{\partial X}^2 + \frac{\partial V}{\partial Y}^2 \right) + Pr Ra \theta \quad (3)

u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (4)

\begin{align*}
\text{u = v = 0, } \frac{\partial T}{\partial y} = 0
\end{align*}

**Fig. 1: Schematic representation for the physical model.**

- **Boundary conditions of the problem**
  - Hot wall: \( x = 0 \text{ and } 0 \leq y \leq 1 \) \( U = 0; V = 0; T = T_s \)
  - Cold wall: \( x = 1 \text{ and } 0 \leq y \leq 1 \) \( U = 0; V = 0; T = T_c \)
  - Bottom and top walls: \( y = 0; \ y = 1 \text{ and } 0 \leq x \leq 1 \) \( U = 0; V = 0; \frac{\partial T}{\partial y} = 0 \)

3. **Operated method**

In this work we performed the Lattice Boltzmann Method to solve the natural convection problem. This method is based on kinetic of gazes. The governing equation will be discretized as follow: The kinetic equation for the distribution function (temperature distribution, species distributions. \( f_k(x, t) \) can be written as:

\[
\frac{\partial f_k(x, t)}{\partial t} + c_k \frac{\partial f_k(x, t)}{\partial x} = \Omega_k
\]

(5)

The left-hand side terms represent the streaming process, where the distribution of stream function along with the lattice link with velocity: \( c_k = \frac{\Delta x}{\Delta t} \)

The right-hand term \( \Omega_k \) represents the rate of change of distribution function \( f_k \) in the collision process. BGK approximation for the collision operator can be approximated as [9]–[11].
The term $\tau$ represents a relaxation time toward the equilibrium distribution ($f_k^{eq}$), which is related to the diffusion coefficient on the macroscopic scale. The relation will be discussed later on.

As usual, the LBM consists of two steps, collisions and streaming.

The collisions step without forcing function is:

$$f_k \left( x, y, t + \Delta t \right) = f_k \left( x, y, t \right) \left[ 1 - \omega \right] + \omega f_k^{eq} \left( x, y, t \right)$$

(7)

$k = 0; \ldots; 8$.

The streaming step is:

$$f_k \left( x + \Delta x, y + \Delta y, t + \Delta t \right) = f_k \left( x, y, t + \Delta t \right)$$

(8)

For the temperature field:

$$g_k \left( x + \Delta x, t + \Delta t \right) - g_k \left( x, t \right) = -\frac{1}{\tau_\alpha} \left[ g_k \left( x, t \right) - g_k^{eq} \left( x, t \right) \right]$$

(9)

Where $\tau_\alpha$ is the relaxation time. $f_k^{eq}$ and $g_k^{eq}$ are the local equilibrium distribution function.

$g_k^{eq}$ is defined as follow:

$$g_k^{eq} = \omega_\phi \phi \left( x, t \right) \left( 1 + \frac{c_k \vec{u}}{c_s^2} \right)$$

(10)

where $\vec{u}$ is advection velocity vector, and $c_k$ is unit vector along the streaming direction. The weight factors are $\omega_\phi = 4/9$, $\omega_2 = \omega_5 = \omega_6 = \omega_7 = \omega_8 = 1/9$, and $\omega_2 = \omega_5 = \omega_6 = \omega_7 = \omega_8 = 1/36$.

Where: $c_s = c_s^2 / \sqrt{3}$ is the lattice speed of sound.

The dependent variable $\phi \left( x, t \right)$ in Eq. 10, can be related to the distribution function $g_k^\phi$, as $\phi \left( x, t \right) = \sum_{i=1}^{8} g_k \left( x, t \right)$.

4. Validation of calculation code

In the first, to validate our calculation code made with LBM, we studied natural convection in a differentially heated closed square cavity, the two horizontal walls of the top and bottom of which are considered adiabatic. This type of configuration is associated with a classic problem intensively studied and used to check the accuracy of numerical codes (Fig. 1). It is emphasized that the right-side wall of the cavity is subjected to a cold temperature and that of the left to a hot temperature, which produces between the two faces, a horizontal temperature gradient ($\Delta T = T_c - T_f$). We compare the results of the streamlines and isotherms deduced with those of the literature. A 100x100 network is used. The number of Prandtl $Pr$ is fixed at 0.71 (air), and the number of Rayleigh $Ra$, defined by the preceding Eq varies from $10^2$ to $10^4$. 
This section is reserved for a comparative study between the results of the LBM and the finite difference method to see the concordance rate between the results of the two methods and to try to validate the present calculation code. To do this, we have studied the problem of natural convection in a closed square cavity filled with air, differentially heated, whose two horizontal top and bottom walls are considered adiabatic. This type of configuration is associated with a classical problem that has been intensively studied and used for the verification of the accuracy of calculation codes. It is pointed out that the right and left side walls are maintained at fixed cold and hot temperatures respectively. The validation of the calculation code was performed for the test case (unobstructed square cavity), with a Prandtl number (Pr) set to 0.71 (air case), and the Rayleigh number (Ra) between $10^4$ and $10^6$. Figures 2 and 3 clearly show that the results of the stream and isothermal lines obtained are similar to those found by the finite difference method[12], [13], which leads us to admit the present computation code to study other future applications.

**Fig.2: Comparison between the results of stream function from LBM and FDM for different Rayleigh numbers.**
5. Results and discussion

After validation of the calculation code by the comparative approach, this sub-section will be devoted to the study of the influence of the two adiabatic obstacles height, which varies between 0.1L and 0.4L on the convective flow within the cavity. This study is made for a Rayleigh range between $10^4$ and $10^6$. In what follows, we will show the results obtained for the two thermal and dynamic fields, these results will be accompanied by a discussion on the effect of the adiabatic obstacles on these two fields.

- Dynamic field

Figures 4-6, respectively, show the stream functions for different obstacle heights at different Rayleigh numbers between $10^4$ and $10^6$. For obstacles with a height $H=0.1L$, their presence does not affect changes in the flow structure, and the resulting configuration tends to be an obstacle-free case. This means that the phenomenon of heat transfer for this first case can remain without apparent influence.
For a height $H=0.2L$, there is a slight compression towards the centre of the cavity. This slightly influences the structure of the flow for a Rayleigh number equal to $Ra = 10^4$. As the Rayleigh number increases, the influence of the compression on the flow structure increases, which becomes more and more apparent, illustrated by a break in the central stream line.

The increase in the height of the obstacles from $H=0.3L$ to $H=0.4L$ induces a break in the unicellular structure of the flow to form two cells. The shape of the stream lines shows that they are stretched towards the upper part on the left side and the other towards the lower part on the right side. Furthermore, it can be concluded from these findings that for higher heights $H=0.2L$, we are faced with a construction of two zones of stream lines of different intensity, from which the stream lines on the left side are of a much higher intensity than the contours on the right side. This influence on mechanical profiling, from the case of convection in the presence of obstacles, becomes more and more intense by increasing the number of Rayleigh.

Fig. 4: Stream functions for different obstacle heights at $Ra=10^4$. 
Fig. 5: Stream functions for different obstacle heights at $Ra=10^5$.

Fig. 6: Stream functions for different obstacle heights at $Ra=10^6$.
- **Thermal field**

The thermal field, described by the isothermal lines, is represented respectively on figures 7 to 9, and this for different obstacle heights from $H=0.1L$ to $H=0.4L$ and at different Rayleigh numbers between $10^4$ and $10^6$. The shape of the isotherms for a height $H=0.1L$ is similar to those obtained for a square cavity without obstacles, the only difference being the adiabatic obstacles, in which the isothermal lines join these obstacles forming a mass of parallel lines along with its thickness. The presence of obstacles with a height of $H=0.2L$ slightly changes the shape of the isotherms which stratify in the centre of the cavity near the obstacles. This stratification becomes more preponderant with increasing obstacle heights ($H=0.3L$ and $H=0.4L$) and makes these layers more twisted away from these isothermal blocks, this effect becomes even more apparent with increasing Rayleigh numbers. It is also noticed that the increase in the height of the adiabatic obstacles can divide our cavity into two zones, a high-temperature zone near the warm wall and a cold temperature zone near the cooler wall, this remark is valid for all Rayleigh numbers, which may explain the use of such a process in thermal insulation applications.

*Fig.7: Isotherms for different obstacle heights at $Ra=10^4$.***
Fig. 8: Isotherms for different obstacle heights at $Ra=10^5$.

Fig. 9: Isotherms for different obstacle heights at $Ra=10^6$. 
- **Heat transfer Rate**

To concretize this result, and to try to quantify the rate of heat transfer for the case of a cavity with obstacles, as well as the effect of their presence on the diffusion of heat, it was possible to evaluate the average Nusselt number for all the previously reported cases. The results thus found are shown in Fig.10, which shows the evolution of mean Nusselt for each height of obstacle and different Rayleigh numbers. It is found that the increase in obstacle height leads to a decrease in the Nusselt number, and consequently to degradation of heat transfer for our configuration. This decrease becomes more and more important for heights higher than \( H=0.2L \), and this, by increasing the number of Rayleigh. Quantifying the rate of heat transfer employing the average Nusselt number made it possible to conclude the harmfulness of this obstacle arrangement in our study configuration.

![Fig.10: The evolution of the average Nusselt number for different Rayleigh numbers and different obstacle heights.](image)

6. **Conclusion**

The numerical study of the effect of two obstacles attached to the mid-adiabatic walls of a differentially heated square cavity on the fluid flow and heat transfer by natural convection was examined. A qualitative comparison of the results obtained by the LBM with the results of the finite difference method led us to validate our computational code, in order to conduct several numerical simulations based on the said method, to explain and solve this problem. The results thus obtained, allowed us to conclude that the increase in the height of the obstacles induces a decrease in convective heat transfer and to divide our configuration into two zones, one heated and the other cooled, which showed the feasibility of the application in thermal insulation processes. A second conclusion, concerning the effect of the variation in Rayleigh number on convective heat transfer, it was deduced that the increase in \( Ra \), led to an increase in the average Nusselt number, subsequently, a gain in terms of heat transfer rate will apply by this growth.
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