Production of $q q \bar{q} \bar{q}$ final states in $e^- e^-$ collisions in the left-right symmetric model

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Abstract

We consider the reaction $e^- e^- \rightarrow q q \bar{q} \bar{q}$ as a test of lepton number non-conservation in the framework of the left-right-symmetric electroweak model. The main contributions to this process are due to Majorana neutrino exchange in $t$-channel and doubly charged Higgs ($\Delta^{--}$) exchange in $s$-channel with a pair of right-handed weak bosons ($W_R$) as intermediate state. We show that in a linear $e^- e^-$ collider with the collision energy of 1 TeV (1.5 TeV) the cross section of this process is 0.01 fb (1 fb), and it will, for the anticipated luminosity of $10^{35}$ cm$^{-2}$, be detectable below the $W_R$ threshold. We study the sensitivity of the reaction on the masses of the heavy neutrino, $W_R$ and $\Delta^{--}$. 
1 Introduction

The electroweak model with the left-right (LR) gauge symmetry $SU(3) \otimes SU(2) \otimes SU(2)_R \otimes U(1)_{B-L}$, proposed in [1], is one of the most popular extensions of the Standard Model (SM). It gives a better understanding of parity violation than SM and it maintains the lepton-quark symmetry in weak interactions. Parity is in it broken spontaneously, and embedding of the model into the SO(10) grand unified scheme [2] can be implemented consistently when the scale of the discrete LR-symmetry breaking is more than 1 TeV or so.

Perhaps the most important property of the LR-model is its ability to provide, in terms of the seesaw mechanism [3], a simple and natural explanation to the smallness of the masses of the ordinary neutrinos. This results from the mixing of the ordinary left-handed neutrinos with right-handed neutrinos, which quite naturally achieve a Majorana mass of the order of $2-3 W_R$-masses [4]. The ordinary neutrinos are predicted by this model to be very light, but – in contrast with the SM – not exactly massless, Majorana particles. The recent observation by the SuperKamiokande experiment of the atmospheric neutrino oscillations [5] confirmed that at least some of the neutrino species do have a mass, giving an additional argument in favour of the LR-symmetric model.

An essential ingredient of the LR-model are the triplet scalars. They are needed to break the LR-symmetry in a consistent way so that at low energies the model reproduces the SM interactions and at the same time give rise to the seesaw mass mechanism of neutrinos. Their interactions with fermions break the lepton number by two units, $|\Delta L| = 2$, as do the Majorana mass terms of neutrinos they give rise to. The $e^-e^-$-collisions give the most pure environment to study the $|\Delta L| = 2$ interactions, because the corresponding SM background is suppressed as the lepton number is conserved in the SM. In the literature different observable lepton number violating processes, including doubly charged Higgs production [3], vector-boson pair and triple production for electron-positron and electro-electron colliders [4, 5], have been investigated.

In the present paper we will study the lepton-number violating process

$$e^-e^- \rightarrow q \bar{q} \bar{q} \bar{q}$$

(1.1)

with various quark flavour combinations. This process, as it breaks the lepton number, is forbidden in the SM. One would expect to obtain indirect evidence of the LR-model
via this process well below the threshold of $W_R^\pm$, the gauge boson of the right-handed interactions, and other new particles predicted by the model.

According to the existing plans the Next Linear Collider (NLC) will operate at energies up to $\sqrt{s} \approx 1 - 2$ TeV, and it is assumed to have a luminosity of the order of $10^{35}$ cm$^{-2}$ [9]. We will show in this paper that with this kind of equipment it will be possible to detect the reaction (1.1) for a reasonable choice of the parameters of the LR-symmetric model and obtain quite strong mass constraints for the new gauge and Higgs bosons of the model.

The organization of this article is as follows: in Section 2 we give the description of particle content, lagrangian and general properties of the LR-model; in Section 3 we derive the amplitudes of the reaction (1.1) and discuss the corresponding reactions with a leptonic final state; in Section 4 we present the numerical results of our calculations; in Section 5 we discuss the SM background; and Section 6 is devoted to conclusions.

2 Description of the model

In the LR-model quarks and leptons are assigned to the following $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ representations [11]:

$$Q_{iL} = \begin{bmatrix} u \\ d \end{bmatrix}_{iL} = (2, 1, \frac{1}{3}); \quad Q_{iR} = \begin{bmatrix} u \\ d \end{bmatrix}_{iR} = (1, 2, \frac{1}{3})$$

(2.1)

$$\Psi_{iL} = \begin{bmatrix} \nu \\ e \end{bmatrix}_{iL} = (2, 1, -1); \quad \Psi_{iR} = \begin{bmatrix} \nu \\ e \end{bmatrix}_{iR} = (1, 2, -1),$$

(2.2)

where $i$ is the flavour index. In addition to the SM particles, each family contains a right-handed neutrino. The gauge sector differs from the SM due to presence of right-handed gauge bosons $W_R$ and $Z_R$. The scalar sector should contain essentially more particles than in the SM. In order to generate fermion masses one needs the Higgs bidoublet with the following quantum numbers:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} = (2, 2^*, 0),$$

and with the following vacuum expectation value (VEV)

$$< \Phi > = \frac{1}{\sqrt{2}} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$$
This is, however, not enough to accomplish the spontaneous symmetry breaking of the
gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ into the SM symmetry, but some other Higgs
field with non-vanishing $B-L$ is needed. There are several alternatives for the additional Higgs multiplet [12], but if one wants to generate neutrino masses through the
seesaw mechanism, the triplet Higgs field $\Delta_R$, sometimes also called a Higgs-Majoron, is needed:

\[ \Delta_R = \begin{pmatrix} \Delta^+_{R}/\sqrt{2} & \Delta^0_{R} & -\Delta^+_{R}/\sqrt{2} \\ \Delta^0_{R} & \Delta^+_{R}/\sqrt{2} & \Delta^-_{R}/\sqrt{2} \\ -\Delta^+_{R}/\sqrt{2} & \Delta^-_{R}/\sqrt{2} & \Delta^0_{R} \end{pmatrix} = (1, 3, 2) \]  

with the vacuum expectation value:

\[ \langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_R \end{pmatrix} \]  

If one imposes an explicit $L \leftrightarrow R$ symmetry, the corresponding left-handed Higgs-
Majoron field should also be introduced:

\[ \Delta_L = \begin{pmatrix} \Delta^+_{L}/\sqrt{2} & \Delta^0_{L} & -\Delta^+_{L}/\sqrt{2} \\ \Delta^0_{L} & \Delta^+_{L}/\sqrt{2} & \Delta^-_{L}/\sqrt{2} \\ -\Delta^+_{L}/\sqrt{2} & \Delta^-_{L}/\sqrt{2} & \Delta^0_{L} \end{pmatrix} = (3, 1, 2) \]  

with the vacuum expectation value:

\[ \langle \Delta_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_L \end{pmatrix} \]  

As far as masses of neutrinos and gauge bosons are concerned, the presence of the
left-handed Higgs-Majoron is not, however, essential.

The most general potential describing self-interactions of the scalar fields introduced
above can be found, e.g., in [12]. There exist severe phenomenological bounds on the
parameters of this potential, particularly from the limitations on the flavour changing
neutral currents (FCNC). Since the choice $v_L = k_2 = 0$ satisfies these bounds [12] we
will restrict ourselves in this paper to this case. This choice means in particular that
we do not allow any mixing between charged vector boson fields $W_L$ and $W_R$. Then
the masses of the charge vector bosons are given by the expression

\[ M^2_{W_L} = \frac{1}{4}g^2_Lk_1^2, \]  

\[ M^2_{W_R} = \frac{1}{4}g^2_R(2v_R^2 + k_1^2). \]
In the case of explicit left-right symmetry gauge couplings of both \( SU(2) \) groups should be equal \((g_R = g_L \simeq 0.64)\). Without this symmetry the internal consistency within the model requires nevertheless \( g_R \geq 0.55g_L \) \([3]\). The experimental value of the left-handed charged boson mass is \( M_{W_L} = 81 \) GeV, while the lower bound from Tevatron is \( M_{W_R} > 650 \) GeV \([7]\).

As for the fermion masses, they come from the Yukawa interactions of quarks and leptons:

\[
-\mathcal{L}_{Yuk} = \bar{\Psi}_i^L(f_{ij} \Phi + g_{ij} \tilde{\Phi}) \Psi_j^R + h.c. + Q_i^L(f_{ij}^Q \Phi + g_{ij}^Q \tilde{\Phi}) Q_j^R + h.c. + \\
+ h_{R;ij} \Psi_{iR}^T C \sigma_2 \Delta_R \Psi_{jR} + h_{L;ij} \Psi_{iL}^T C \sigma_2 \Delta_L \Psi_{jL} + h.c.,
\]

where \( \tilde{\Phi} = \sigma_2 \Phi^* \sigma_2 \) and \( i, j \) are flavour indices. This yields the usual quark \( 3 \times 3 \) mass matrix and charged lepton masses, while for the neutrino one obtains the seesaw mass matrix:

\[
M = \begin{pmatrix}
m_L & m_D \\
m_D^T & m_R
\end{pmatrix}.
\]

The entries are \( 3 \times 3 \) matrices given by \( m_D = (f k_1 + g k_2)/\sqrt{2} \), \( m_L = \sqrt{2} h_L v_L \) and \( m_R = \sqrt{2} h_R v_R \). We will also ignore possible mixing between the lepton families, so that these matrices are assumed diagonal. Natural seesaw condition implies \( m_{Di} \approx m_{li} \), where \( m_{li} \) is the charged lepton mass, while the evident phenomenological left-right hierarchy implies \( v_R >> k_1 \) and hence \( m_{Ri} >> m_{Di} \). The ensuing neutrino masses are \( m_{\nu_1} \simeq m_{D_i}^2/m_{Ri} \) and \( m_{\nu_2} \simeq m_{Ri} \). The mixing angle \( \eta \) between left-handed and right-handed neutrino states is given by

\[
\tan 2\eta_i = \frac{2m_{Di}}{m_{Ri}},
\]

Since it is natural that scale of right-handed neutrino masses is of order \( 1-3 \) \( M_{W_R} \) \([1, 3]\), the following values of the mixing angle \( \eta \) are reasonable:

\[
\eta_1 \approx \frac{m_e}{m_R} = 0.5 \cdot 10^{-6}, \\
\eta_2 \approx \frac{m_\mu}{m_R} = 10^{-4}, \\
\eta_3 \approx \frac{m_\tau}{m_R} = 2 \cdot 10^{-3}.
\]

We will use these values in the following calculations.
3 Feynman amplitudes

Let us first give the arguments that make us to consider the reaction (1.1) particularly suitable for testing the LR-model. First of all, the final state particles are all light, so that there is no kinematical suppression for the process, in contrast with, e.g., the $W_R$ pair production. Consequently, one may expect to detect evidence of the LR-model through this reaction well below the $W_R$ threshold. The same is true, of course, for the leptonic final states, for example for the reaction $e^- e^- \rightarrow \mu^- \mu^- \mu^- \mu^+$. Reactions with ordinary neutrinos in the final state are not very useful as invisibility of neutrinos makes them not easy to distinguish from the background processes. Also, reactions with final state electrons are not that good because of the possible mix-up of the initial and final state particles.

In Fig. 1. and Fig. 2 we show Feynman diagrams for the reactions $e^- e^- \rightarrow \mu^- \mu^- \mu^- \mu^+$ and $e^- e^- \rightarrow b \bar{b} \ell \bar{\ell}$, respectively. The reason for our studying the four-quark final states instead of the four-lepton final states becomes evident from these figures. One can see that the reaction with leptons in the final state does not involve charged vector bosons as intermediate states but is quite sensitive to the structure of the neutral current sector, while the reaction with quarks in the final state involves charged vector bosons, particularly the right-handed boson $W_R$, but not the neutral ones. There is a variety of extensions of the Standard Model where one has extra neutral gauge boson(s), such as the superstring-inspired E(6) models [18], but no new charged gauge bosons, in contrast with the LR-model. Hence the reactions like $e^- e^- \rightarrow b \bar{b} \ell \bar{\ell}$ that involve charged currents but not neutral currents offer a more unambiguous test of the LR-symmetric model than the leptonic processes.

Consequently, we have chosen the processes $e^- e^- \rightarrow q \bar{q} \bar{q} \bar{q}$ for our further investigation. We prefer final states with $b$-quarks as the $b$-jets are relatively easy to identify in experiment (the same should be expected for $t$-jets) [19]. From this point of view, the best process for a study would be $e^- e^- \rightarrow b \bar{b} \ell \bar{\ell}$. However, as will be seen from our numerical results, it will possible to measure the cross section also for the 4-jet reactions with no $b$-jets, as well as for the reactions with a single $b$-jet.

The Feynmann graphs for the process $e^- e^- \rightarrow b \bar{b} \ell \bar{\ell}$ are presented in Fig.2. Some of these diagrams may be safely neglected without any substantial effect on our numerical results. First of all, since the left-handed electron neutrino is very light ($m_{\nu_1} < 1$ eV) compared with the right-handed one (for which the seesaw mechanism in its simplest
form implies $m_{\nu_2} \simeq 1 - 2 \text{ TeV}$) and with the collision energy, the amplitudes 2, 3, 6 and 7 in Fig.2 have, in comparison with, say, diagram 9, an extra overall factor of $m_{\nu_1}/m_{\nu_2}$ or $m_{\nu_1}/\sqrt{s}$ due to the lepton-number-violating neutrino propagator and they may be therefore ignored. Diagrams 4, 5 and 8 contain, due to neutrino mixing, a small parameter $\sin \eta_1$ in the $e\nu_2W_L^+$ vertex:

$$L_{e\nu_2W_L} \simeq \sin \eta_1 \cdot \frac{g_R}{\sqrt{2}} W_{\mu L}^+ \bar{\Psi}_e \gamma^\mu \Psi_{\nu_2L} + h.c., \quad (3.1)$$

and also their contribution can be neglected. Hence there are only two amplitudes, corresponding to the diagrams 1 and 9, that are relevant.

The following lagrangian vertices give rise to the diagrams 1 and 9:

$$h_{R,11} \cdot \Delta^{--} \Psi^T_{eL} C \Psi_{eL} + h.c., \quad (3.2)$$

where $h_{R,11}$ is defined in (2.9),

$$- \frac{g^2_R}{\sqrt{2}} v_R \cdot \Delta^{--} W_R^+ W_R^- + h.c. \quad (3.3)$$

which originates in the kinetic term of the Higgs-Majoron field, and

$$- \frac{g_R}{\sqrt{2}} V_{tb} W_{\mu R}^+ \bar{f}_R \gamma_\mu b_R. \quad (3.4)$$

The total amplitude is then:

$$M_{ss'tt'qq'} = e^T_s(p_1) T_{\mu \mu'} e^T_{s'}(p_2) \cdot \left( \frac{i g_R}{\sqrt{2}} V_{tb} \right)^2 \cdot$$

$$\frac{1}{\sqrt{2}} \left[ \bar{f}_s \gamma_\mu \gamma_R b_q \cdot \bar{f}_{s'} \gamma_\mu' \gamma_R b_{q'} + \bar{f}_s \gamma_\mu \gamma_R b_{q'} \cdot \bar{f}_{s'} \gamma_\mu' \gamma_R b_q \right] \quad (3.5)$$

Here $e_s, b_q, \bar{f}_s$ denote the electron, $b$ and $\bar{f}$ 4-spinors with the corresponding spin indices, $\gamma_R \equiv (1 + \gamma_5)/2$, $V_{tb}$ is the element of the right-handed Kobayasi-Maskawa matrix. $T_{\mu \mu'}$ contains the contributions from different channels:

$$T_{\mu \mu'} = T^s_{\mu \mu'} + T^t_{\mu \mu'} + T^u_{\mu \mu'}. \quad (3.6)$$

The $s, t, u$ indices correspond to the Mandelstam variables if one treats charged bosons involved in the considered Feynman diagrams as a final state particles (in other words, each of the two $b, \bar{f}$ clusters is treated as a single particle). Then, in correspondence with [3], we have:

$$T^s_{\mu \mu'} = h_{R11} (1 + \gamma_5) \cdot \frac{iC \cdot i}{k^2 - M^2_{\Delta^{--}}} (-2i) \frac{g^2_R}{\sqrt{2}} g^{\nu \nu'} \Pi_{\mu \nu}(k_1) \Pi_{\mu' \nu'}(k_2) \quad (3.7)$$
\[ T_{\mu\nu}^t = \left( \frac{i g_R}{\sqrt{2}} \right)^2 \gamma^\nu \gamma_R \cdot i C^{-1} \frac{H^+ + m}{p^2 - m^2} \gamma^\nu \gamma_R \Pi_{\mu\nu}(k_1) \Pi_{\mu'\nu'}(k_2) \]  

\[ T_{\mu\nu}^u = \left( \frac{i g_R}{\sqrt{2}} \right)^2 \gamma^\nu \gamma_R \cdot i C^{-1} \frac{H^+ + m}{p^2 - m^2} \gamma^\nu \gamma_R \Pi_{\mu\nu}(k_2) \Pi_{\mu'\nu'}(k_1) \]  

Here \( k \) is the 4-momentum of the doubly charged Higgs, \( p \) is the 4-momentum of the Majorana neutrino, \( k_{1,2} \) are the 4-momentum of the charged bosons, and

\[ \Pi_{\rho\lambda}(q) = \frac{-i}{q^2 - M_{W_R}^2 + \frac{i}{2} \Gamma_{W_R} M_{W_R}} \left( g_{\rho\lambda} - \frac{q_{\rho} q_{\lambda}}{M_{W_R}^2} \right) \]

is the Breit-Wigner form of massive vector boson propagator (this is the form the propagators are used by CompHEP [10]). For the “t-channel” amplitude one has \( p = p_1 - k_1 \), \( p + p_2 = k_2 \), while for the “u-channel” amplitude the momentum conservation law implies \( p = p_1 - k_2 \), \( p + p_2 = k_1 \).

We estimate the width of the right-handed boson to be \( \Gamma_{W_R} \approx \Gamma_{W_L} \cdot M_{W_R}/M_{W_L} \), and that of the doubly-charged Higgs-Majoron \( \Gamma_{\Delta^{--}} \approx 0.053 M_{\Delta^{--}} \). For the right-handed neutrino we assume \( \Gamma_{\nu_R} \approx 7 - 70 \text{ GeV} \) for \( m_{\nu_2} \sim 1-2 \text{ TeV} \), but the width has not much effect on our results since the right-handed neutrinos are far from their pole in our case.

4 Numerical results

By means of CompHEP [10] we have derived the squared matrix elements for \( e^- e^- \rightarrow b \bar{b} \ell^- \ell^- \) and computed the ensuing the cross sections at the collision energies \( \sqrt{s} = 1 \) TeV and \( \sqrt{s} = 1.5 \) TeV. The results depend on a number of unknown parameters of the LR-model, the most important ones being the masses of the right-handed boson \( W_R \) and doubly charged Higgs-Majoron \( \Delta^{--} \). As was discussed before, we consider theory without \( W_L - W_R \) mixing and neglect small effects of the seesaw mixing. We restrict ourselves to the manifestly left-right symmetric case, implying that the left and right-handed interactions have the same coupling strength, i.e. \( g_L = g_R \), and that the Kobayashi-Maskawa mixings of the right-handed charged currents are exactly the same as those of the left-handed ones, in particular \( V_{tb}^R = V_{tb}^L = V_{tb} \).

We evaluate the values of the gauge coupling constants at the linear collider energy scale \( \sqrt{s} \) through one-loop massless renormalization group equations of the SM without the Higgs boson contribution:

\[ g^2(s) = g^2(M_Z^2) \left( 1 + \frac{g^2(M_Z^2)}{16\pi^2} \frac{10}{3} \log \frac{s}{M_Z^2} \right)^{-1} \]
\[ g'^2(s) = g'^2(M_Z^2) \left( 1 - \frac{g'^2(M_Z^2)}{16\pi^2} \frac{20}{3} \log\frac{s}{M_Z^2} \right)^{-1}, \]

which are related to \( e \) and \( \sin \theta_W \) as:

\[ e^2 = \frac{g'^2 - g^2}{g^2 + g'^2}, \quad \sin^2 \theta_W = \frac{g'^2}{g^2 + g'^2}. \]

Here \( M_Z \) is the neutral \( Z \)-boson mass. We do not take into account in these equations the additional particles of the LR-model, making the assumption that they are effectively decoupled due to their large mass. The effects of any possible light Higgs particle are also neglected since they are anyhow relatively small. At the energy scale of order of the SM neutral \( Z \) boson mass (\( \sqrt{s} = M_Z = 91 \) GeV) we use the standard electroweak input [14, 15, 16].

In Fig. 3 we show the energy dependence of the total cross section of the process \( e^-e^- \to b\bar{b}\ell\bar{\ell} \) for various values of masses of the triplet Higgs \( \Delta^{--} \) and the right-handed neutrino \( \nu_2 \). In all the cases the right-handed boson mass is taken to be \( M_{W_R} = 700 \) GeV. We remind that the present experimental lower bound from the Tevatron measurements is \( M_{W_R} \simeq 650 \) GeV [17].

The plot Fig. 3a presents the cross section for the case of the right-handed neutrino mass \( m_{\nu_2} = 1 \) TeV and for three different values of \( M_{\Delta^{--}} \) (600, 1000, 1500 GeV) as indicated in the figure. The characteristic behaviour of these curves is that they all have a resonance at \( M_{\Delta^{--}} = \sqrt{s} \) and the cross section grows by several orders of magnitude above the \( W_R \) threshold. The value of the cross section at the resonance is determined by the \( \Delta^{--} \) width and depends on the assumption we made on it in the previous section, while the growth above the \( W_R \) threshold is easy to understand since the phase space above the threshold contains the poles of the charged right-handed boson propagator.

In the plot in Fig. 3b presents the cross section in the case \( m_{\nu_2} = 1.5 \) TeV for the triplet Higgs mass values \( M_{\Delta^{--}} = 400, 800, 1200, 2000 \) GeV. A comparison with Fig. 3a shows that the increase in the right-handed neutrino mass makes the cross section larger. The reason for this is obvious: the amplitude represented by the diagrams 1 and 9 of Fig. 2 increases with the growth of Yukawa coupling of the neutrino \( h_{R,11} \).

As we keep \( M_{W_R} \) and \( g_R \) fixed, also the the VEV \( v_R \) of the Higgs triplet is fixed, and so the increase of the right-handed neutrino mass is solely due to a corresponding increase of the Yukawa coupling \( h_{R,11} \). This makes the contribution of the diagrams 1 and 9 larger than in the case of Fig. 3a. On the whole, the intimate relation of
the neutrino mass and lepton number violating couplings in the LR model is directly reflected in the behaviour of the cross sections.

In Fig. 3c we present the cross section for $m_{\nu_2} = 2$ TeV for three different values of the triplet Higgs mass, $M_{\Delta^{--}} = 800, 1200, 1600, 2000$ GeV. In all cases $M_{W_R} = 700$ GeV. The value $m_{\nu_2} = 2$ TeV, when $M_{W_R} = 700$ GeV, corresponds to a value of the coupling constant $h_{R,11}$ close to unity. Going beyond this to higher neutrino masses would not yield reliable results due to the unitarity bound.

Given the cross sections, it is interesting to study what will be the sensitivity of the NLC in testing the central parameters of the LR-model through the reaction (1.1). In the following we will present the contours in the $M_{W_R} - M_{\Delta^{--}}$ plane corresponding to various event rates of $e^- e^- \to b \bar{b} t \bar{t}$. We consider the collision energies 1 TeV and 1.5 TeV and the anticipated luminosity of $10^{35}$ cm$^{-2}$ · s$^{-1}$ appropriately scaled with the collision energy (we consider the luminosity to be approximately proportional to the collision energy). At the collision energy $\sqrt{s} = 1.5$ TeV the process with the cross section $\sigma = 0.01, 0.1, 1$ fb would produce 30, 300 and 3000 events per year, correspondingly.

In deriving the contours one cannot use the computed cross sections as such but has to impose several phase space cuts to make quark jets unambiguously identified. Following the arguments of [19] we apply the following cuts:

- Each b-jet should have energy more than 10 GeV.
- Each t-jet should have energy more than 190 GeV.
- The opening angle between two detected jets should be greater than 20°.
- The angle between each detected jet and the colliding axis should be greater than 36°.
- The total energy of the event should be greater than 400 GeV.

In Fig. 4 we display the $M_{W_R} - M_{\Delta^{--}}$ histogram of the cross section of $e^- e^- \to b \bar{b} t \bar{t}$ and the contour levels corresponding to $\sigma = 0.015, 0.15, 1.5$ fb for the colliding energy $\sqrt{s} = 1$ TeV and right-handed neutrino mass $m_{\nu_2} = 1.5$ TeV. The histogram has the evident resonanse behavior in $M_{\Delta^{--}}$, when $M_{W_R}$ is kept fixed, while the increase of the charged boson mass with $M_{\Delta^{--}} = \text{const}$ causes the decrease of the cross section. This happens because the gauge coupling remains fixed and hence the increase of charged boson mass leads to the increase of the Higgs-Majoron VEV which
should be compensated (since neutrino mass is also fixed) by the decrease of the Yukawa coupling \( h_{R,11} \).

As can been seen from these contours, near the \( \Delta^{--} \) resonance the process will be sensitive to values of \( M_{WR} \) that are much above the present lower limit of 650 GeV obtained from direct searches at Tevatron, and that also exceed the collision energy, assuming that some tens of annual events is enough for the signal. Away from resonance, the bound one could obtain on \( M_{WR} \) is about 700 GeV, i.e. no improvement to the present bound. The constraint on the mass of the doubly charged Higgs \( \Delta^{--} \) is generally stronger than that on the \( M_{WR} \).

As the cross section is proportional to the mass of neutrino, the larger \( m_{\nu_2} \) the more stringent are the ensuing constraints. This is demonstrated in Fig. 5 where \( m_{\nu_2} = 2 \) TeV.

Increasing the collision energy will, of course, lead to more restrictive bounds. In Fig. 6 and 7 we present the sensitivity contours for \( \sqrt{s} = 1.5 \) TeV with the masses of the right-handed neutrinos 1 and 1.5 TeV, respectively. The achievable limit for \( M_{WR} \) is now about 1.5 TeV at the triplet Higgs resonance and outside the resonance about 1 TeV, a considerable improvement to the present bound.

Let us now consider the case when the final state quarks are light. As one would expect, the results are very similar to the heavy quark case considered above. We checked this by the CompHEP calculations and found that the relative difference in the cross section is of the order of \( m_t/\sqrt{s} \), i.e. 20–25%. If we impose for the counterparts of the top quarks, the \( c \) quarks, in the reaction \( e^- e^- \to s \bar{s} c \bar{c} \) the cut \( E_{c_1,2} > 190 \) GeV, which is very effective in diminishing the SM background (see below), the cross sections differ not more than 12 %. Accordingly, we have these approximative relationships among the heavy and light quark cases:

\[
\sigma(bb\bar{t}\bar{t}) \approx \sigma(ss\bar{c}\bar{c}) \approx \sigma(dd\bar{u}\bar{u}). \tag{4.12}
\]

Assuming that the Kobayashi-Maskawa matrix elements for the right-handed currents are the same as for the left-handed ones, the greatest non-diagonal element is \( |V_{us}| \approx 0.221 \). This will yield a suppression factor of \( 4 \cdot 10^{-2} \) to the cross section \( \sigma(ss\bar{c}\bar{u}) \) as compared with the cross sections above that contain only diagonal currents, and for the other non-diagonal processes the suppression is even stronger.

In addition to the relationships (4.12) one can immediately write down the following
approximative relations:

\[ 2\sigma(bb\bar{t}\bar{t}) \approx \sigma(bs\bar{c}) \approx \sigma(bd\bar{u}) \approx \sigma(sd\bar{c}). \]

The factor of two in front of the cross section of \( e^-e^- \rightarrow bb\bar{t}\bar{t} \) originates in the identity of the final state quarks and antiquarks. We have checked also these relations by CompHEP.

In conclusion, we have the following relations between the cross sections of the reactions with no, one and two \( b \)-jets in the final state:

\[ \sigma(0b) \approx \sigma(1b) \approx 4 \cdot \sigma(2b); \quad (4.13) \]

This relation may be very useful as a test of the LR-model. We show the sensitivity contours for the processes with one final state \( b \)-jet and with no final state \( b \)-jets in Fig. 8 and Fig. 9, respectively. In correspondence with (4.13) they will yield more severe restrictions for the \( M_{W_R} \) mass than the heavy quark final state and would give an essential improvement to the present bound.

5 The SM background

The main SM background of the reaction \( e^-e^- \rightarrow bb\bar{t}\bar{t} \) is due to the process \( e^-e^- \rightarrow \nu_e\nu_e b\bar{b}t\bar{t}, \) which has the same visible particles in the final state. (In the case of light quarks in the final state one has the analogous background process.) Furthermore, since it is not possible to distinguish \( b \) and \( \bar{b} \) jets from each other, the SM process \( e^-e^- \rightarrow e^-e^- b t \bar{b} \bar{t} \) (and analogously for the other quark combinations) is another important source of background.

To analyse these processes we first note that quark-antiquark pairs can be produced only from \( W, Z \) or Higgs lines. Therefore one can start with considering the processes \( e^-e^- \rightarrow e^-e^-W^+W^-; \) \( e^-e^- \rightarrow e^-e^-ZZ; \) \( e^-e^- \rightarrow e^-e^-ZH; \) \( e^-e^- \rightarrow e^-e^-HH; \) \( e^-e^- \rightarrow \nu_e\nu_e W^-W^- \), which were analysed in [20]. The first reaction has the largest cross section: at \( \sqrt{s} = 1 \text{ TeV} \) about 800 fb and at \( \sqrt{s} = 1.5 \text{ TeV} \) about 1100 fb. All the other processes are at least 50 times smaller and may be neglected here. If we use for a conservative estimation the so-called \textit{product } \times \textit{decay} (or narrow width) approximation [19], which assumes that the intermediate \( W_L^\pm \)-bosons are mostly on shell, we will get for the background values of order 50–70 fb, which is inconveniently high in comparison
with our signal. However, if we impose the cut of 50 GeV on the energies of the final state electrons (whose energy we assume to be possible to determine as missing energy although particles themselves may not be distinguishable from the beam particles), the cross section $\sigma(e^- e^- \rightarrow e^- e^- W^+ W^-)$ diminishes by 3 orders of magnitude and yields the background at 1 TeV on the 0.1 fb level and at 1.5 TeV on the 0.03 fb. There is a further suppression in the case of the $bb\bar{t}\bar{t}$ due to the fact the intermediate $W_L$ bosons should actually be away from the pole as the invariant mass of its decay products $b, \bar{t}$ should be greater than $m_t$. This yields altogether 8 orders of magnitude suppression of the background, making it fully harmless.

The situation will the the same for the light quarks if one applies the corresponding cut on the 2-jet invariant mass, i.e. $s > m_t^2$. Thus by means of the cut combined with the measurements of the missing energy it is possible to make the SM background about 7 orders of magnitude smaller than the investigated process.

Even without measuring the missing energy associated with the electrons, imposing just the cut on the invariant mass, it will be possible to make the SM background 4 orders of magnitude smaller than the investigated signal.

Thus we can conclude that the $e^- e^- \rightarrow q \bar{q} \bar{q} \bar{q}$ processes in the LR-symmetric model may be well observed above the SM background.

6 Summary

The main results of this paper can be summarized as follows. It is shown that the reaction $e^- e^- \rightarrow q \bar{q} \bar{q} \bar{q}$ may be observed at NLC for a wide range of reasonable parameter values of the left-right symmetric model and already below the $W_R$ threshold. For the collision energy $\sqrt{s} = 1.5$ TeV and luminosity $10^{35}$ cm$^{-2}$ cm$^{-2}$·s$^{-1}$ the lower limit for the mass of the right-handed gauge boson one could reach is $M_{W_R} \gtrsim 1000$ GeV. Near the doubly charged Higgs ($\Delta^{--}$) resonance the lower bound on $M_{W_R}$ may reach, and even exceed, the value of the collision energy. As the lepton number violation and neutrino masses are intimately connected through the Maojaran mass terms, the strength of the $e^- e^- \rightarrow q \bar{q} \bar{q} \bar{q}$ process increases with the growth of the mass of the right-handed neutrino. The ”non-diagonal” processes, i.e. the reactions where the $\bar{q}q$ pair or pairs in the final state mix with fermion families, are essentially suppressed, while all the ”diagonal” processes have approximately the same probability. Process $e^- e^- \rightarrow b \bar{b} \bar{t} \bar{t}$
can be identified as $b$-tagging is possible. For the processes involving only light quarks or containing just one $b$-jet are approximately related to this cross section by eq. (4.6).

The SM background can be suppressed to the level 4 orders of magnitude below the process rate if the proper cuts in the phase space are applied, and it can be made even 7 orders of magnitude below the signal level if the full energy of the event can be reconstructed with the accuracy of 50 GeV.

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Figure 1: Feynman diagramms for $e^- e^- \rightarrow \mu^- \mu^- \mu^- \mu^+$ in the LR-model.

Figure 2: Feynman diagramms for $e^- e^- \rightarrow b \bar{b} \bar{t} \bar{t}$ in the LR-model.
Figure 3: Energy dependence of the full cross section for the process $e^- e^- \rightarrow b \bar{b} \bar{t} \bar{t}$ for different values of $\Delta^{--}$ mass ($M \equiv M_{\Delta^{--}}$) and right-handed neutrino masses: $m_{\nu_2} = 1 \text{ TeV}$ (left upper picture), $m_{\nu_2} = 1.5 \text{ TeV}$ (right upper picture), $m_{\nu_2} = 2 \text{ TeV}$ (lower picture), (see comments in the text).
Figure 4: Cross section for the process $e^-e^- \rightarrow b \bar{b} \bar{t} t$ and and contours corresponding to the sensitivity levels $\sigma = 0.01\, fb$ (30 events per year), $\sigma = 0.1\, fb$ (300 events per year), $\sigma = 1\, fb$ (3000 events per year), for the energy $E = 1\, TeV$, and right-handed neutrino mass $m_{\nu_2} = 1.5\, TeV$. 
Figure 5: Cross section for the process $e^- e^- \rightarrow b \bar{b} \ell \bar{\ell}$ and contours corresponding to the sensitivity levels $\sigma = 0.01$ fb (30 events per year), $\sigma = 0.1$ fb (300 events per year), $\sigma = 1$ fb (3000 events per year), for the energy $E = 1$ TeV, and right-handed neutrino mass $m_{\nu_2} = 2$ TeV.
Figure 6: Cross section for the $e^-e^- \rightarrow b, b, \bar{t}, \bar{t}$ and its contour levels at $\sigma = 0.015$ fb (30 events per year), $\sigma = 0.15$ fb (300 events per year), at $\sigma = 1.5$ fb (3000 events per year) for the energy $E = 1.5 TeV$, and the right-handed neutrino mass $m_{\nu_2} = 1$ TeV.
Figure 7: Cross section for the $e^-e^- \rightarrow b, b, \bar{t}, \bar{t}$ and it's contour levels at $\sigma = 0.01$ fb (30 events per year), $\sigma = 0.1$ fb (300 events per year), $\sigma = 1$ fb (3000 events per year) for the energy $E = 1.5 \, TeV$, and the right-handed neutrino mass $m_{\nu_2} = 1.5 \, TeV$. 
Figure 8: Contour levels at $\sigma = 0.015$ fb (30 events per year), $\sigma = 0.15$ fb (300 events per year), $\sigma = 1.5$ fb (3000 events per year) for the processes with 1 $b$-jet or with light-quarks only in the final state (see comments in the text) for the energy $E = 1$ TeV, and the right-handed neutrino masses: $m_{\nu_2} = 1$ TeV (on the top), $m_{\nu_2} = 1.5$ TeV (in the middle) and $m_{\nu_2} = 2$ TeV (in the bottom).
Figure 9: Contour levels at $\sigma = 0.01$ fb (30 events per year), $\sigma = 0.1$ fb (300 events per year), $\sigma = 1$ fb (3000 events per year) for the processes with 1 $b$-jet or with light-quarks only in the final state (see comments in the text) for the energy $E = 1.5$ TeV, and the right-handed neutrino masses: $m_{\nu_2} = 1.5$ TeV (on the top) and $m_{\nu_2} = 1$ TeV (in the bottom).