Ranking Support Vector Machine with Kernel Approximation

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Learning to rank algorithm has become important in recent years due to its successful application in information retrieval, recommender system, and computational biology, and so forth. Ranking support vector machine (RankSVM) is one of the state-of-the-art ranking models and has been favorably used. Nonlinear RankSVM (RankSVM with nonlinear kernels) can give higher accuracy than linear RankSVM (RankSVM with a linear kernel) for complex nonlinear ranking problem. However, the learning methods for nonlinear RankSVM are still time-consuming because of the calculation of kernel matrix. In this paper, we propose a fast ranking algorithm based on kernel approximation to avoid computing the kernel matrix. We explore two types of kernel approximation methods, namely, the Nyström method and random Fourier features. Primal truncated Newton method is used to optimize the pairwise L2-loss (squared Hinge-loss) objective function of the ranking model after the nonlinear kernel approximation. Experimental results demonstrate that our proposed method gets a much faster training speed than kernel RankSVM and achieves comparable or better performance over state-of-the-art ranking algorithms.

1. Introduction

Learning to rank is an important research area in machine learning. It has attracted the interests of many researchers because of its growing application in areas like information retrieval systems [1], recommender systems [2, 3], and computational biology [4]. For example, in document retrieval domain, a ranking model is trained based on the training data of some queries. Each query contains a group of corresponding retrieved documents and their relevance levels labeled by humans. When a new query arrives for prediction, the trained model is used to rank the corresponding retrieved documents for the query.

Many types of machine learning algorithms have been proposed for the ranking problem. Among them, RankSVM [5], which is extended from the basic support vector machine (SVM) [6], is one of the commonly used methods. The basic idea of RankSVM is transforming the ranking problem into pairwise classification problem. The early implementation of RankSVM [7] was slow because the explicit pairwise transformation led a large number of the training samples. In order to accelerate the training process, [8] proposed a primal Newton method algorithm to solve the linear RankSVM problem without the need of explicit pairwise transformation. And [9] proposed the RankSVM based on the structured output learning framework.

As with the SVM, kernel trick can be used to generalize the linear ranking problem to nonlinear case for RankSVM [7, 9]. Kernel RankSVM can give higher accuracy than the linear RankSVM for complex nonlinear ranking problem [10]. The nonlinear kernel can map the original features into some high-dimensional space where the nonlinear problem can be ranked linearly. However, the training time of kernel RankSVM dramatically grows as the training data set increases in size. The computational complexity is at least quadratic in the number of training examples because of the calculation of kernel matrix. Kernel approximation is an efficient way to solve the above problem. It can avoid computing kernel matrix by explicitly generating a vector representation of data that approximates the kernel similarity between any two data points.

The approximation methods can be classified into two categories: the Nyström method [11, 12] and random Fourier
2. Background and Related Works

In this section, we present the background and related works of learning to rank algorithm and RankSVM.

2.1. Learning to Rank Algorithms. Learning to rank algorithms can be classified into three categories: pointwise approach, pairwise approach, and list-wise approach.

(i) Pointwise: it transforms the ranking problem into regression or classification on single objects. Then existing regression or classification algorithms are directly applied to model the labels of single objects. This approach includes McRank [16] and OC SVM [17].

(ii) Pairwise: it transforms the ranking problem into regression or classification on object pairs. It can model the preferences within the object pairs. This approach includes RankSVM [5] and RankBoost [18].

(iii) List-wise: it takes ranking lists as instances in both learning and prediction and can optimize the list-wise loss function directly. This approach includes ListNet [19], AdaRank [20], BoltzRank [21], and SVM MAP [22].

In this paper, we focus on the pairwise ranking algorithm based on SVM.

2.2. Linear RankSVM. Linear RankSVM is a commonly used pairwise ranking algorithm [5]. For the web search problem with \(n\) queries and a set of documents of each query, features \(x_i \in \mathbb{R}^d\) are extracted from the query-document pair \((q_i, \text{doc}_j)\) and label \(y_{ij} \in \mathbb{Z}\) is the relevance level of the \text{doc}_j to the query \(q_i\). Thus, the training data is a set of label-query-instance tuples \((y_{ij}, q_i, x_i)\). Let \(\mathcal{P}\) denote the set of preference pairs. If \((i, j) \in \mathcal{P}\), \text{doc}_i and \text{doc}_j are in the same query \((q_i = q_j)\) and \text{doc}_i is preferred over \text{doc}_j \((y_i > y_j)\). The goal of linear RankSVM is to get a ranking function

\[
f(x) = w^T x
\]

such that \(\forall (i, j) \in \mathcal{P}, f(x_i) > f(x_j) = w^T x_i > w^T x_j\), and \(w \in \mathbb{R}^d\).

RankSVM has a good generalization due to the margin-maximization property. According to [27], the margin is defined as the closest distance between two data points when the data points project to the ranking vector \(w\):

\[
d = \min \frac{w^T (x_i - x_j)}{\|w\|}, \quad \forall (i, j) \in \mathcal{P}. \tag{2}
\]

Maximizing the margin is good because data point pairs with small margins represent very uncertain ranking decisions. RankSVM can guarantee to find a ranking vector \(w\) with the maximum margin [27]. Figure 1 shows the margin-maximization of four data points for linear RankSVM. The weights of two linear ranking, namely, \(w_1\) and \(w_2\), can both rank the four data correctly. But \(w_1\) generalizes better than \(w_2\) because the margin \(d_1\) of \(w_1\) is larger than the margin \(d_2\) of \(w_2\).

For L1-loss (Hinge-loss) linear RankSVM [5], the objective loss function is

\[
\frac{1}{2} \|w\|^2 + C \sum_{(i, j) \in \mathcal{P}} \max \left(0, 1 - w^T (x_i - x_j)\right), \tag{3}
\]

where \(C\) is the regularization parameter. Equation (3) can be solved by standard SVM classification on pairwise difference vectors \((x_i - x_j)\). But this method is very slow because of the large size of \(\mathcal{P}\).

In [8], an efficient algorithm was proposed to solve the L2-loss (squared Hinge-loss) linear RankSVM problem

\[
\frac{1}{2} \|w\|^2 + C \sum_{(i, j) \in \mathcal{P}} \max \left(0, 1 - w^T (x_i - x_j)\right)^2. \tag{4}
\]

They used a \(p \times n\) sparse matrix \(A\) to obtain the pairwise difference training sample \((x_i - x_j)\) implicitly (\(p = |\mathcal{P}|\)). If \((i, j) \in \mathcal{P}\), there exists a number \(k\) such that \(A_{ki} = 1\) and...
A_{jk} = -1 and the rest is 0. Let X = [x_1, \ldots, x_n]^T. Equation (4) can be written as
\[
\frac{1}{2} \|w\|^2 + C \sum_{(i,j) \in P} \max(0, 1 - w^T (\phi(x_i) - \phi(x_j))) + \sum_{(i,j) \in P} \alpha_{ij}^* Q_{ij}\nu.
\]
where D is a p \times p diagonal matrix with D_{(i,j)} = 1 if 1 - w^T (x_i - x_j) > 0 and 0 otherwise. Then, (5) is optimized by primal truncated Newton method in O(nd + p).

2.3. Kernel RankSVM. The key of kernel method is that if kernel function \( \kappa \) is positive definite, there exists a mapping \( \phi \) into the reproducing kernel Hilbert spaces (RKHS), such that
\[
\kappa(x, x') = \langle \phi(x), \phi(x') \rangle,
\]
where \( \langle \cdot , \cdot \rangle \) denotes the inner product. The advantage of the kernel method is that the mapping \( \phi \) never has to be calculated explicitly.

For L1-loss RankSVM, the objective loss function with the kernel mapping \( \phi \) has the form [7]
\[
\frac{1}{2} \|w\|^2 + C \sum_{(i,j) \in P} \max(0, 1 - w^T (\phi(x_i) - \phi(x_j))) + \sum_{(i,j) \in P} \alpha_{ij}^* Q_{ij}\nu.
\]
The primal problem of (7) can be transformed to the dual problem using the Lagrange multipliers.
\[
\begin{align*}
\max_a: & \sum_{ij} \alpha_{ij} - C \sum_{(i,j) \in P} \sum_{(u,v) \in P} \alpha_{ij}^* \alpha_{uv}^* Q_{ij}\nu, \nu
\end{align*}
\]
subject to: \( 0 \leq \alpha_{ij} \leq C, \ \forall (i, j) \in P, \)

where each Langrange multiplier \( \alpha_{ij} \) corresponds to the pair index \((i, j)\) in \( P \) and
\[
Q_{ij}\nu = (\phi(x_i) - \phi(x_j))(\phi(x_i) - \phi(x_j)) = \kappa(x_i, x_u) + \kappa(x_j, x_u) - \kappa(x_i, x_u) - \kappa(x_j, x_u).
\]
Solving the kernel RankSVM is a large quadratic programming problem. Instead of directly computing the matrix \( Q \), we can save the cost by \( A \) in (5).
\[
Q = AK^T, \quad \text{where} \ K_{ij} = \kappa(x_i, x_j).
\]
The ranking function of the kernel RankSVM has the form
\[
f(x) = \sum_{(i,j) \in P} \alpha_{ij}^* (\kappa(x_i, x) - \kappa(x_j, x)) + \sum_{(i,j) \in P} \alpha_{ij}^* Q_{ij}\nu.
\]
The computation of \( Q \) requires \( O(n^2) \) kernel evaluations. It is difficult to scale to large kernel RankSVM by solving (8).

Several works have been proposed to accelerate the training speed of kernel RankSVM, such as 1-slack structural method [9], representer theorem reformulation [27], and pairwise problem reformulation [10]. However, these methods are still slow for large-scale ranking problem because the computational cost is at least quadratic in the number of training examples.

3. RankSVM with Kernel Approximation

3.1. A Unified Model. The drawback of kernel RankSVM is that it needs to store many kernel values \( \kappa(x_i, x_j) \) during optimization. Moreover, \( \kappa(x_i, x_j) \) needs to be computed for new data \( x \) during the prediction, possibly for many vector \( x \). This problem can be solved by approximating the kernel mapping explicitly:
\[
\kappa(x, x') = \langle \tilde{\phi}(x), \tilde{\phi}(x') \rangle,
\]
where \( \tilde{\phi} \) is the mapping of kernel approximation. The original feature \( x \) can be mapped into the approximated Hilbert space by \( \tilde{\phi} \). The objective function of RankSVM with the kernel approximation can be written as
\[
\frac{1}{2} \|w\|^2 + C \sum_{(i,j) \in P} \ell(w^T \tilde{\phi}(x_i) - w^T \tilde{\phi}(x_j)),
\]
where \( \ell \) is a loss function for SVM, such as \( \ell(t) = \max(0, 1 - t) \) for L1-loss SVM and \( \ell(t) = \max(0, 1 - t) \) for L2-loss SVM. The problems of (13) can be solved using linear RankSVM after the approximation mapping. The kernel never needs to be calculated during the training process. Moreover, the weights \( w \) can be computed directly without the need of storing any training sample. For new data \( x \), the ranking function is
\[
f(x) = \tilde{\phi}(x)^T w.
\]

Our proposed method mainly includes mapping process and ranking process.

(i) Mapping process: the kernel approximation is used to map the original data into high dimensional space. We use two kinds of kernel approximation methods, namely, the Nyström method and random Fourier features, which will be discussed in Section 3.2.

(ii) Ranking process: the linear RankSVM is used to train a ranking model. We use the L2-loss RankSVM because of its high accuracy and fast training speed. The optimization procedure will be described in Section 3.3.

The Nyström method is data dependent and the random Fourier features method is data independent [28]. The Nyström method can usually get a better approximation than random Fourier features, whereas the Nyström method is slightly slower than the random Fourier features. Additionally, in the ranking process, we can replace the L2-loss RankSVM with any other linear ranking algorithms, such as ListNet [19] and FRank [23].

3.2. Kernel Approximation

3.2.1. Nyström Method. Nyström method gets a low-rank approximation of kernel matrix \( K = [\kappa(x_i, x_j)]_{i,j} \) by uniformly sampling \( m \ll n \) examples from \( X \), denoted by
Random Fourier features is 

\[ \tilde{\phi}(\omega) = \sqrt{\frac{2}{m}} \left[ \cos(\omega^T \mathbf{x} + b_1), \ldots, \cos(\omega^T \mathbf{x} + b_m) \right]^T, \]  

(21)

where \( \omega \) is sampled from the distribution \( p(\omega) \). Since \( p(\omega) \) and \( \kappa(\mathbf{x}, \mathbf{y}) \) are real, \( \tilde{\phi}(\omega)(\mathbf{x}) = \sqrt{\frac{2}{m}} \mathbf{c}(\mathbf{x}, \mathbf{y})^T \mathbf{y} \)

where \( \mathbf{c}(\mathbf{x}, \mathbf{y}) \) is defined in Algorithm 2. Random Fourier features. 

**Algorithm 2: Random Fourier features.**

Ensure: \( \tilde{\phi}(\omega) \)

(1) Compute the Fourier transform \( \mathbf{p}(\omega) \). Since \( p(\omega) \) and \( \kappa(\mathbf{x}, \mathbf{y}) \) are real, \( \tilde{\phi}(\omega)(\mathbf{x}) = \sqrt{\frac{2}{m}} \mathbf{c}(\mathbf{x}, \mathbf{y})^T \mathbf{y} \)

where \( \mathbf{c}(\mathbf{x}, \mathbf{y}) \) is defined in Algorithm 2. Random Fourier features.

3.3. Ranking Optimization. In this section, we solve the L2-loss (squared Hinge-loss) ranking problem of (13) after the kernel approximation mapping of training data

\[ \frac{1}{2} \| \mathbf{w} \|^2 + C \sum_{(i,j) \in \mathcal{P}} \max(0, 1 - w^T (\tilde{\phi}(\mathbf{x}_i) - \tilde{\phi}(\mathbf{x}_j)))^2. \]  

(22)

Similar as (5), the loss function can be rewritten as

\[ \frac{1}{2} \| \mathbf{w} \|^2 + C \left( 1 - \mathbf{A} \tilde{\Phi} \mathbf{w} \right)^T \mathbf{D} \left( 1 - \mathbf{A} \tilde{\Phi} \mathbf{w} \right), \]  

(23)

where \( \tilde{\Phi} = [\tilde{\phi}(\mathbf{x}_1), \ldots, \tilde{\phi}(\mathbf{x}_n)]^T \). The gradient and the generalized Hessian matrices of (23) are

\[ \mathbf{g} = \mathbf{w} + 2C \tilde{\Phi}^T \mathbf{A}^T \mathbf{D} \left( \mathbf{A} \tilde{\Phi} \mathbf{w} - 1 \right), \]  

(24)

\[ \mathbf{H} = \mathbf{I} + 2C \tilde{\Phi}^T \mathbf{A}^T \mathbf{D} \mathbf{A} \tilde{\Phi}, \]

where \( \mathbf{I} \) is the identity matrix. The Hessian matrix does not need to be computed explicitly using truncated Newton method [8]. The Newton step \( \mathbf{H}^{-1} \mathbf{g} \) can be approximately computed using linear conjugate gradient (CG). The main computation of linear CG method is the Hessian-vector multiplication \( \mathbf{Hs} \) for some vector \( \mathbf{s} \)

\[ \mathbf{Hs} = \mathbf{s} + 2C \tilde{\Phi}^T \mathbf{A}^T \mathbf{D} \mathbf{A} \tilde{\Phi} \mathbf{s}. \]  

(25)

Assuming that the embedding space \( \tilde{\phi} \) has \( m \) dimensions, the total complexity of this method is \( O(n m + p) \) where \( p = |\mathcal{P}| \). The main step of our proposed algorithm is described in Algorithm 3. We calculate the approximation embedding \( \tilde{\phi} \) using the Nyström method or random Fourier features in line (1). Then \( \tilde{\phi} \) is applied to all training samples in line (2). The linear RankSVM model with primal truncated Newton method is applied in the embedding space in line (3)–(11).


4. Experiments

4.1. Experimental Settings. We use three data sets from LETOR (http://research.microsoft.com/en-us/um/beijing/projects/letor), namely, OHSUMED, MQ2007, and MQ2008, to validate our proposed ranking algorithm. The examples of the data sets are extracted from the information retrieval data collections. These data sets are often used for evaluating new learning to rank algorithms. Table 1 lists the properties of each algorithm.

The hyperparameters of the algorithms are selected by grid search. The regularization parameter $C$ of each algorithm is chosen from $[2^{-12}, 2^{-11}, \ldots, 2^2]$. For kernel RankSVM and our approximation methods, the parameter $\gamma$ of RBF kernel is chosen from $[2^{-12}, 2^{-11}, \ldots, 2^2]$. For MQ2007 dataset, the number of sampling for kernel approximation $m$ is set to 2000, whereas $m = 500$ for the other datasets. All experiments are conducted on a high performance server with 2.0 GHz 16-cores CPU and 64 GB of memory.

The remaining two lines represent RankNyström and RankRandomFourier, respectively. In the beginning, the performances of kernel approximate methods are worse than linear RankSVM. But along with the increase of $m$ (the number of sampling of approximation), both of the kernel approximate methods can outperform the linear RankSVM. We also observe that RankNyström gets better results than RankRandomFourier when $m$ is small and the two methods obtain similar results when $m = 2000$.

4.2. Comparison of the Nyström Method and Random Fourier Features. Figure 2 shows the performance comparison of RankSVM with the Nyström method and random Fourier features on MQ2007 dataset. We take the linear RankSVM algorithm, RankSVM-Primal, as the baseline method, which is plotted as dotted line. The remaining two lines represent RankNyström and RankRandomFourier, respectively. In the beginning, the performances of kernel approximate methods are worse than linear RankSVM. But along with the increase of $m$ (the number of sampling of approximation), both of the kernel approximate methods can outperform the linear RankSVM. We also observe that RankNyström gets better results than RankRandomFourier when $m$ is small and the two methods obtain similar results when $m = 2000$.

4.3. Comparison with Linear and Kernel RankSVM. In this part, we compare our proposed kernel approximation ranking algorithms to other linear and kernel RankSVM algorithms. We take $N = 2000$ for the kernel approximation. Table 2 gives the results of different RankSVM algorithms on the first fold of MQ2007 dataset. The linear RankSVM algorithms use less training time, but their MeanNDCG values are lower than the values of the kernel RankSVM algorithms. Our kernel approximation methods obtain better performance than the kernel RankSVM-TRON with much faster training speed in this dataset. The training time of our kernel approximation methods is about ten seconds, whereas the training time of the kernel RankSVM-TRON is more than 13 hours. The result of random Fourier features is slightly better than the RankNyström method. Moreover,
Figure 2: Performance comparison of RankSVM with the Nyström method and random Fourier features on MQ2007 dataset. (a) NDCG@1; (b) NDCG@3; (c) MeanNDCG; (d) MAP.

Table 2: Results of different RankSVM algorithms on the first fold of MQ2007 dataset. We take \( m = 2000 \) for the kernel approximation method.

| Algorithm          | Type   | Loss | \( C \)   | \( g \)   | Mean-NDCG | Time (s) |
|--------------------|--------|------|-----------|-----------|-----------|----------|
| RankSVM-TRON       | linear | L1    | \( 2^{-5} \) | \(-\)      | 0.5265    | 1.9      |
| RankSVM-Struct     | linear | L1    | \( 2^{-1} \) | \(-\)      | 0.5268    | 2.2      |
| RankSVM-Primal     | linear | L2    | \( 2^{-10} \) | \(-\)     | 0.5270    | 1.2      |
| RankSVM-TRON       | RBF    | L1    | \( 2^{-2} \) | \( 2^{-5} \) | 0.5310    | 47463.5  |
| RankNystöm         | RBF    | L2    | \( 2^{-2} \) | \( 2^{-5} \) | 0.5330    | 10.9     |
| RankRandomFourier  | RBF    | L2    | \( 2^{-2} \) | \( 2^{-5} \) | **0.5336** | 16.1     |
Table 3: Performance comparison on TD2004 dataset.

| Algorithm          | NDCG@1 | NDCG@3 | NDCG@5 | P@1   | P@3   | MAP   |
|--------------------|--------|--------|--------|-------|-------|-------|
| AdaRank-MAP [20]   | 0.4133 | 0.4017 | 0.3932 | 0.4133| 0.3422| 0.3308|
| AdaRank-NDCG [20]  | 0.3600 | 0.3838 | 0.3769 | 0.3600| 0.3289| 0.2986|
| FRank [23]         | 0.4400 | 0.479   | 0.4362 | 0.4400| 0.3867| 0.3809|
| ListNet [19]       | 0.4400 | 0.4371 | 0.4209 | 0.4400| 0.4000| 0.3721|
| RankBoost [18]     | 0.4800 | 0.4640 | 0.4368 | 0.4800| 0.4044| 0.3835|
| RankSVM-Struct [9] | 0.4400 | 0.492   | 0.3935 | 0.4400| 0.3511| 0.3505|
| RankSVM-Primal [8] | 0.4666 | 0.4468 | 0.4277 | 0.4666| 0.4000| 0.3793|
| RankNystöm         | 0.4933 | 0.4348 | 0.4254 | 0.4933| 0.3911| 0.3899|
| RankRandomFourier  | 0.4933 | 0.4422 | 0.4265 | 0.4933| 0.4000| 0.3924|

Table 4: Performance comparison on OHSUMED dataset.

| Algorithm          | NDCG@1 | NDCG@3 | NDCG@5 | P@1   | P@3   | MAP   |
|--------------------|--------|--------|--------|-------|-------|-------|
| RankSVM-Struct [9] | 0.5515 | 0.4850 | 0.4729 | 0.6338| 0.5898| 0.4478|
| ListNet [19]       | 0.5326 | 0.4732 | 0.4613 | 0.6338| 0.5895| 0.4487|
| AdaRank-MAP [20]   | 0.5388 | 0.4682 | 0.4613 | 0.6338| 0.5895| 0.4487|
| AdaRank-NDCG [20]  | 0.5330 | 0.4790 | 0.4673 | 0.6719| 0.5984| 0.4498|
| RankBoost [18]     | 0.4632 | 0.4555 | 0.4494 | 0.5576| 0.5609| 0.4411|
| RankRLS [24]       | 0.5490 | 0.4770 | 0.4530 | 0.6440| 0.5860| 0.4470|
| RankSVM-Primal [8] | 0.5645 | 0.5004 | 0.4782 | 0.6710| 0.5983| 0.4473|
| RankNystöm         | 0.5730 | 0.4874 | 0.4780 | 0.6801| 0.5890| 0.4473|
| RankRandomFourier  | 0.5728 | 0.4965 | 0.4804 | 0.6801| 0.5983| 0.4472|

Table 5: Performance comparison on MQ2007 dataset.

| Algorithm          | NDCG@1 | NDCG@3 | MeanNDCG | P@1   | P@3   | MAP   |
|--------------------|--------|--------|----------|-------|-------|-------|
| RankSVM-Struct [9] | 0.4096 | 0.4063 | 0.4966   | 0.4746| 0.4315| 0.4645|
| ListNet [19]       | 0.4002 | 0.4091 | 0.4988   | 0.4640| 0.4334| 0.4652|
| AdaRank-MAP [20]   | 0.3821 | 0.3984 | 0.4891   | 0.4392| 0.4230| 0.4577|
| AdaRank-NDCG [20]  | 0.3876 | 0.4044 | 0.4914   | 0.4475| 0.4305| 0.4602|
| RankBoost [18]     | 0.4134 | 0.4072 | 0.5003   | 0.4823| 0.4348| 0.4662|
| LambdaMART [25]    | 0.4147 | 0.4119 | 0.5011   | —     | —     | 0.4660|
| BL-MART [25]       | 0.4200 | 0.4224 | 0.5093   | —     | —     | 0.4730|
| CRR [26]           | —      | —      | 0.5000   | —     | —     | 0.4660|
| RankSVM-Primal [8] | 0.4109 | 0.4063 | 0.4973   | 0.4747| 0.4317| 0.4655|
| RankNystöm         | 0.4242 | 0.4138 | 0.5036   | 0.4888| 0.4394| 0.4695|
| RankRandomFourier  | 0.4224 | 0.4136 | 0.5036   | 0.4871| 0.4386| 0.4698|

the L2-loss RankSVM can get better performance than the L1-loss RankSVM on this dataset. The MeanNDCG of RankSVM-Primal (linear) is slightly higher than RankSVM-TRON (linear). The kernel approximation methods get better MeanNDCG than RankSVM-TRON with RBF kernel.

4.4. Comparison with State-of-the-Art. In this part, we compare our proposed algorithm with the state-of-the-art ranking algorithms. Most of the results of the comparison algorithms come from the baselines of LETOR. The remaining results come from the papers of the algorithms. The hyperparameters C and γ of our proposed kernel approximation RankSVM are selected by grid search as in Section 4.1.

Table 3 provides the comparison of testing NDCG and MAP results of different ranking algorithms on the TD2004 dataset. The number of sampling for kernel approximation m is set to 500. We can observe that the kernel approximation ranking methods can achieve the best performances on 3 terms of all the 6 metrics. Also, the results of RankNystöm and RankRandomFourier are similar.

Table 4 provides the performance comparison on the OHSUMED dataset. m is set to 500. We once observe that RankRandomFourier achieves the best performances on 3 metrics of all the 6 metrics. RankNystöm gets the best results on 2 metrics.

Table 5 provides the comparison of results on the MQ2007 dataset. m is set to 2000. We observe that RankNystöm...
obtains the best scores on 3 metrics on MQ2007 dataset. BL-MART also achieves the best scores on 3 metrics. However, BL-MART trains 10,000 LambdaMART and creates bagged model by randomly selecting a subset of these models, whereas our proposed RankNyström algorithm only trains one model.

5. Conclusions

In this paper, we propose a fast RankSVM algorithm with kernel approximation to solve the problem of lengthy training time of kernel RankSVM. First, we proposed a unified model for kernel approximation RankSVM. Approximation method is used to avoid computing kernel matrix by explicitly approximating the kernel similarity between any two data points. Then, two types of methods, namely, the Nyström method and random Fourier features, are explored to approximate the kernel matrix. Also, the primal truncated Newton method is used to optimize the L2-loss (squared Hinge-loss) objective function of the ranking model. Experimental results indicate that our proposed method requires much less computational cost than kernel RankSVM and achieves comparable or better performance over state-of-the-art ranking algorithms. In the future, we plan to use more efficient kernel approximation and ranking models for large-scale ranking problems.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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