A fluctuating energy-momentum may produce an unstable world

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Abstract

Quantum gravitational effects may induce stochastic fluctuations in the structure of space-time, to produce a characteristic foamy structure. It has been known for some time now that these fluctuations may have observable consequences for the propagation of cosmic ray particles over cosmological distances. We note here that the same fluctuations, if they exist, imply that some decay reactions normally forbidden by elementary conservation laws, become kinematically allowed, inducing the decay of particles that are seen to be stable in our universe. Due to the strength of the prediction, we are led to consider this finding as the most severe constraint on the classes of models that may describe the effects of gravity on the structure of space-time. We also propose and discuss several potential loopholes of our approach, that may affect our conclusions. In particular, we try to identify the situations in which despite a fluctuating energy-momentum of the particles, the reactions mentioned above may not take place.

1 Introduction

In the last few years the hunt for possible minuscule violations of the fundamental Lorentz invariance (LI) has been object of renewed interest, in particular because it has been understood that cosmic ray physics has an unprecedented potential for investigation in this field [1–6]. Some authors [3,4,7] have even invoked possible violations of LI as a plausible explanation to some puzzling observations related to the detection of ultra high energy cosmic rays (UHECRs) with energy above the so-called GZK feature [8], and to the unexpected shape of the spectrum of photons with super-TeV energy from sources...
at cosmological distances. Both types of observations have in fact many uncertainties, either coming from limited statistics of very rare events, or from accuracy issues in the energy determination of the detected particles, and most likely the solution to the alleged puzzles will come from more accurate observations rather than by a violation of fundamental symmetries. For this reason, from the very beginning we proposed [5] that cosmic ray observations should be used as an ideal tool to constrain the minuscule violations of LI, rather than as evidence for the need to violate LI. The reason why the cases of UHECRs and TeV gamma rays represent such good test sites for LI is that both are related to physical processes with a kinematical energy threshold, which is in turn very sensitive to the smallest violations of LI. UHECRs are expected to suffer severe energy losses due to photopion production off the photons of the cosmic microwave background (CMB), and this should suppress the flux of particles at the Earth at energies above $\sim 10^{20}$ eV, the so called GZK feature. Present operating experiments are AGASA and HiRes, and they do not provide strong evidence either in favor or against the detection of the GZK feature [9]. A substantial increase in the statistics of events, as expected with the Auger project and with EUSO should dramatically change the situation and allow to detect the presence or lack of the GZK feature in the spectrum of UHECRs. These are the observations that will provide the right ground for imposing a strong limit on violations of LI. For the case of TeV sources, the process involved is pair production [10] of high energy gamma rays on the photons of the infrared background. In both cases, a small violation of LI can move the threshold to energies which are smaller than the classical ones, or move them to infinity, making the reactions impossible. The detection of the GZK suppression or the cutoff in the gamma ray spectra of gamma ray sources at cosmological distances will prove that LI is preserved to correspondingly high accuracy [5].

The recipes for the violations of LI generally consist of requiring an explicit modification of the dispersion relation of high energy particles, due to their propagation in the “vacuum”, now affected by quantum gravity (QG). This effect is generally parametrized by introducing a typical mass, expected to be of the order of the Plank mass ($M_P$), that sets the scale for QG to become effective.

However, explicit modifications of the dispersion relation are not really necessary in order to produce detectable effects, as was recently pointed out in Refs. [11–14] for the case of propagation of UHECRs. It is in fact generally believed that coordinate measurements cannot be performed with precision better than the Planck distance (time) $\delta x \geq l_P$, namely the distance where the metric of space-time must feature quantum fluctuations. A similar line of thought implies that an uncertainty in the measurement of energy and momentum of particles can be expected, according with the relation $\delta p \simeq \delta E \simeq p^2/M_P$. 

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The consequences of these uncertainty relations on the propagation of high energy cosmic rays have been investigated by many authors. In a previous paper [15] (see also [16]) we have actually argued that the effects of fluctuating dispersion relations may induce observable consequences on cosmic ray propagation even at energies as low as \( \approx 10^{15} \text{ eV} \).

Very recently it has been argued ([17], see also [18]) that generic fluctuations may already be inconsistent with detection of interference features from very distant sources; this result however has been criticized in [19].

In this paper we derive another important consequence of the fluctuating dispersion relations introduced in [15]: a particle propagating in vacuum acquires an energy dependent fluctuating effective mass, which may be responsible for kinematically forbidden decay reactions to become kinematically allowed. If this happens, particles that are known to be stable would decay, provided no other fundamental conservation law is violated (e.g.: baryon number conservation, charge conservation). A representative example is that of the reaction \( p \rightarrow p + \pi^0 \), that is prevented from taking place only due to energy conservation. With a fluctuating metric, we find that if the initial proton has energy above a few \( 10^{15} \text{ eV} \), the reaction above can take place with a cross section typical of hadronic interactions, so that the proton would rapidly lose its energy. Similar conclusions hold for the electromagnetic process \( p \rightarrow p + \gamma \).

The fact that particles that would be otherwise stable could decay has been known for some time now [20,21] and in fact it rules out a class of non-fluctuating modifications of the dispersion relations for some choices of the sign of the modification: the new point here is that it does not appear to be possible to fix the sign of the fluctuations, so that the conclusions illustrated above seem unavoidable.

The plan of the paper is the following: in §1 we discuss our main results, putting strong emphasis on the underlying assumptions. In §2, we set the framework for kinematical computations of the thresholds for the processes presented in §3. Finally in the last section we argue that the comparison of our predictions with experimental data indicates a strong inconsistency, implying that the framework of quantum fluctuations currently discussed in most literature is in fact ruled out. The strength of this conclusion leads us to try to identify possible loopholes in our working assumptions. The ways to avoid the dramatic effects of the fluctuating energy-momentum of a particle should be mainly searched in the dynamics of Quantum Gravity. These effects, in which our knowledge is poor to say the least, might forbid processes even when these processes are kinematically allowed due to the fluctuations in the energy and momentum.
2 The effect of Space-Time fluctuations on the propagation of high energy particles.

In this section we summarize the formalism introduced in [15] following [12]. The basic points can be listed as follows:

- the values of energy (momentum), fluctuate around their average values (assumed to be the result that in principle could be recovered with an infinite number of measurements of the same observable):

  \[ E \approx \bar{E} + \alpha \frac{\bar{E}^2}{M_P} \]  
  \( E \approx \bar{E} + \alpha \frac{\bar{E}^2}{M_P} \) (1)

  \[ p \approx \bar{p} + \beta \frac{\bar{p}^2}{M_P} \]  
  \( p \approx \bar{p} + \beta \frac{\bar{p}^2}{M_P} \) (2)

  with \( \alpha \) and \( \beta \) normally distributed variables (with \( O(1) \) variance) and \( p \) the modulus of the 3-momentum (for simplicity we assume rotationally invariant fluctuations);

- the dispersion relation fluctuates as follows:

  \[ P_\mu g^{\mu\nu} P_\nu = E^2 - p^2 + \gamma \frac{p^3}{M_P} = m^2 \]  
  \( P_\mu g^{\mu\nu} P_\nu = E^2 - p^2 + \gamma \frac{p^3}{M_P} = m^2 \) (3)

  and \( \gamma \) is again a normally distributed variable. Eqs. (1-3) are assumed to hold for \( E, p << M_P \).

In principle, a theory of Quantum Gravity (QG) should be able to predict the specific properties of space-time and the dispersion relations written above, but this approach is clearly out of reach as of today. From this the need of assuming a gaussian form for the fluctuations introduced above. It is however worth stressing once more (see also [15]) that the results that one gets are not very sensitive to this assumption, namely any symmetric distribution with variance \( \approx 1 \), within a large factor, would give essentially the same results. Besides assuming gaussianity, we also assume that \( \alpha, \beta \) and \( \gamma \) are uncorrelated random variables; again, this assumption reflects our ignorance in the dynamics of QG.

The form chosen for the fluctuations can be guessed in a variety of ways, as will be discussed in the last section; notice that we have chosen a very general form in order to derive our results in a way that we consider as unbiased as possible. We also assume energy momentum conservation relations on the fluctuating quantities, \( i.e. \)

\[ \sum \left[ \bar{E}_i + \alpha_i \frac{\bar{E}_i^2}{M_P} \right] = \sum \left[ \bar{E}_f + \alpha_f \frac{\bar{E}_f^2}{M_P} \right] \]  
\( \sum \left[ \bar{E}_i + \alpha_i \frac{\bar{E}_i^2}{M_P} \right] = \sum \left[ \bar{E}_f + \alpha_f \frac{\bar{E}_f^2}{M_P} \right] \) (4)
\[
\sum \left[ \bar{p}_i + \beta_i \frac{\bar{p}_i^2}{M_p} \right] = \sum \left[ \bar{p}_f + \beta_f \frac{\bar{p}_f^2}{M_p} \right]
\]  
(5)

where the index \( i \) (\( f \)) denotes values of energy and momentum in the initial (final) state of a reaction.

### 3 Decay of stable particles

In this section we consider three specific decay channels, that illustrate well, in our opinion, the consequences of the quantum fluctuations introduced above. We start with the reaction

\[ p \rightarrow p + \pi^0 \]

and we denote with \( p \) (\( p' \)) the momentum of the initial (final) proton, and with \( k \) the momentum of the pion. Clearly this reaction cannot take place in the reality as we know it, due to energy conservation. However, since fluctuations have the effect of emulating an effective mass of the particles, it may happen that for some realizations, the effective mass induced to the final proton is smaller than the mass of the proton in the initial state, therefore allowing the decay from the kinematical point of view. Since no conservation law or discrete symmetry is violated in this reaction, it may potentially take place.

For the sake of clarity, it may be useful to invoke as an example the decay of the \( \Delta^+ \) resonance, which is structurally identical to a proton, but may decay to a proton and a pion according to the reaction \( \Delta^+ \rightarrow p + \pi^0 \), since its mass is larger than that of a proton. From the physical point of view, the effect of the quantum fluctuations may be imagined as that of exciting the proton, inducing a mass slightly larger than its own (average) physical mass.

Following [15] we expect to find that for momenta above a given threshold, depending on the value of the random variables, the decay may become kinematically allowed. In general, the probability for this to happen has to be calculated numerically from the conservation equations supplemented by the dispersion relations.

Although a full calculation is possible, it is probably more instructive to proceed in a simplified way, in which only the fluctuations in the dispersion relation of the particle in the initial state are taken into account. Neglecting the corresponding fluctuations in the final state should not affect the conclusions in any appreciable way, unless the fluctuations in the initial and final states are correlated.

In this approximation, the threshold for the process of proton decay to a proton and a neutral pion can be written as follows (neglecting corrections to
order higher than $p/M_P$):

$$\gamma \frac{2p_{th}^3}{M_P} - 2m_\pi m_p - m_\pi^2 = 0,$$

(6)

with solution

$$p_{th} = \left(\frac{(2m_p m_\pi + m_\pi^2)M_P}{2\gamma}\right)^{\frac{1}{3}}.$$

(7)

For negative values of $\gamma$, the above equation has no positive root; this happens in 50% of the cases. Since the gaussian distribution is essentially flat in a small interval around zero, the distribution of thresholds for positive $\gamma$ (i.e. in the remaining 50% of the cases) peaks around the value for $\gamma \approx 1$, meaning that the threshold moves almost always down to a value of $\approx 10^{15}$ eV [5,15]; essentially the same result holds for generic fluctuations (i.e. not confined to the dispersion relations) affecting only the incident particle, namely the one with the highest energy.

The reason why the effects of fluctuations are expected to occur at such low energies is that in that energy region the fluctuation term becomes comparable with the rest mass of the particle. In fact the same concept of rest mass of a particle may lose its traditional meaning at sufficiently high energies [14].

It can be numerically confirmed that independent fluctuations of momenta (and/or of the dispersion relations) of the decay products are more likely to make the decay easier rather than more difficult, due to the nonlinear dependence of the threshold on the strength of fluctuations: the probability that the decay does not take place is in fact $\approx 30\%$. In the remaining cases, the decay will occur if the momentum of the initial proton is larger than $p_{th}$. The distribution of $p_{th}$ is essentially identical to the one reported in [15] for other reactions.

All the discussion reported so far remains basically unchanged if similar reactions are considered. For instance the reaction $p \rightarrow \pi^+ n$ is kinematically identical to the one discussed above. For all these reactions, we expect that once they become kinematically allowed, the energy loss of the parent baryon is fast. For the case of nuclei, all the decays that do not change the nature of the nucleon leave $(A,Z)$ unchanged, so we do not expect any substantial blocking effect in nuclei.

Another reaction that may be instructive to investigate is the spontaneous pair production from a single photon, namely

$$\gamma \rightarrow e^+ e^-.$$
In this case, following the calculations described above, we obtain the following expression for the threshold:

\[ p'_{th} = \left( \frac{4m_e^2 M_P}{2\gamma'} \right)^{\frac{1}{3}} \]  

(8)

and \( p'_{th} \) is of the order of \( 10^{13} \) eV. Again, if the reaction becomes kinematically allowed, there does not seem to be any reason why the reaction should not take place with a rate dictated by the typical cross section of electromagnetic interactions.

Finally, we propose a third reaction that in its simplicity may represent the clearest example of reactions that should occur in a world in which quantum fluctuations behave in the way described above. Let us consider a proton that moves in the vacuum with constant velocity, and let us consider the elementary reaction of spontaneous photon emission. In the Lorentz invariant world the process of photon emission is known to happen only in the presence of an external field that may provide the conditions for energy and momentum conservation. However, in the presence of quantum fluctuations, one can think of the gravitational fluctuating field as such an external field, so that the particle can in fact radiate a photon without being in the presence of a nucleus or some other external recognizable field. The threshold for this process, calculated following the usual procedure, is

\[ p''_{th} \approx \left( \frac{m^2 M_P \omega}{\gamma''} \right)^{\frac{1}{4}}, \]  

(9)

where \( \omega \) is the energy of the photon. This threshold approaches zero when \( \omega \to 0 \): for instance, if \( \omega = 1 \) eV, then \( p_{th} \approx 300 \) GeV for protons and \( p_{th} \approx 45 \) GeV for electrons. In other words there should be a sizable energy loss of a particle in terms of soft photons. This process can be viewed as a sort of bremsstrahlung emission of a charged particle in the presence of the (fluctuating) vacuum gravitational potential.

Based on the arguments provided in this section, it appears that all particles that we do know are stable in our world, should instead be unstable at sufficiently high energy, due to the quantum fluctuations described above. In the next section we will take a closer look at the implications of the existence of these quantum fluctuations, and possibly propose some plausible avenues to avoid these dramatic conclusions.
4 Discussion and outlook

If the decays discussed in the previous section could take place, our universe, at energies above a few PeV or even at much lower energies might be unstable, nothing like what we actually see. The decays $\text{nucleon} \rightarrow \text{nucleon} + \pi$ would start to be kinematically allowed at energies that are of typical concern for cosmic ray physics, while the spontaneous emission of photons in vacuum might even start playing a role at much lower energies, testable in laboratory experiments. Without detailed calculations of energy loss rates it is difficult to assess the experimental consequences of this process. We are carrying out these calculations, that will be presented in a forthcoming publication [22].

For the nucleon decay, the situation is slightly simpler if we assume that the quantum fluctuations affect only the kinematics but not the dynamics, an assumption also used in [15]. In this case one would expect the proton to suffer the decay to a proton and a pion on a time scale of the same order of magnitude of typical decays mediated by strong interactions. This would basically cause no cosmic ray with energy above $\sim 10^{15}$ eV to be around, something that appears to be in evident contradiction with observations

In the following we will try to provide a plausible answer to these three very delicate questions:

(1) If the particles were kinematically allowed to decay, and there were no fundamental symmetries able to prevent the decay, would it take place?
(2) Is the form adopted for the quantum fluctuations correct and if so, how general is it?
(3) If in fact the form adopted for the fluctuations is correct, how general and unavoidable is the consequence that (experimentally) unobserved decays should take place?

Although the result that particles are kinematically allowed to decay is fairly general, the (approximate) lack of relativistic invariance forbids the computation of life-times. Two comments are in order: first, the phase space for the decays described above, as calculated in the laboratory frame, is non zero and in fact it increases with the momentum of the parent particle. The effect of fluctuations can be seen as the generation of an effective (mass) $\propto p^3/M_P$. A similar effect, although in a slightly different context, was noted in [4]. Second,

1 From a phenomenological point of view, consistency with experiments would require either that the variance of the fluctuations considered above is ridicolously small ($< 10^{-24}$) or, allowing more generic fluctuations $\Delta l \propto l_P(l_P/l)^a$, that a fairly large value for $\alpha$ should be adopted [15].

2 In fact life-times can be in principle estimated in approaches in which it is possible to make transformations between frames [14,23,24], despite the lack of LI.
we do not expect dynamics to forbid the reactions: one must keep in mind that we are considering very small effects, at momenta much smaller than the Planck scale. For instance the gravitational potential of the vacuum fluctuations is expected to move quarks in a proton to excited levels, not to change its content, nor the properties of strong interactions.

There is a subtler possibility, which must be taken very seriously in our opinion, since it might invalidate completely the line of thought illustrated above, namely that the quantum fluctuations of the momenta of the particles involved in a reaction occur on time scales that are enormously smaller than the typical interaction/decay times. This situation might resemble the so called Quantum Zeno paradox, where continuously checking for the decay of an unstable particle effectively impedes its decay. This possibility is certainly worth a detailed study, that would however force one to handle the intricacies of matter in a Quantum Gravity regime. We regard this possibility as the most serious threat to the validity of the arguments in favor of quantum fluctuations discussed in this paper and in many others before it.

Let us turn our attention toward the question about the correctness and generality of the form adopted for the momentum fluctuations. It is generally accepted that the geometry of space-time suffers profound modifications at length (time) scales of the order of the Planck length (time), and that this leads to the emergence of a minimum measurable length. This may be reflected in a non commutativity of space-time and in a generalized form of the uncertainty principle. The transition from uncertainty in the length or time scales to uncertainty in momenta of particles is undoubtedly more contrived and deserves some attention. The expressions in Eqs. 1, 2 and 3 have been motivated in various ways [11,12,15,14,25] in previous papers. For instance, the condition \( \Delta l \geq l_P \) implies the following constraint on wavelengths \( \Delta \lambda \geq l_P \), otherwise it would be possible to design an experimental set-up capable of measuring distances with precision higher than \( l_P \). Therefore \( \Delta p \propto \Delta (\lambda^{-1}) \propto l_P \lambda^2 \). Similar arguments have been proposed, all based to some extent on the de Broglie relation \( p \propto \lambda^{-1} \).

There is certainly no guarantee that the de Broglie relation continues to keep its meaning in the extreme conditions we are discussing, in particular in models in which the coordinates and coordinate-momentum commutators are modified with respect to standard quantum mechanics and the representation of momentum in terms of coordinate derivatives generally fails. For instance in a specific (although non-relativistic) example [26] the existence of a minimum length is shown to imply that

\[
p = \frac{2}{\pi l_P} \tan \left( \frac{\pi l_P}{2 \lambda} \right).
\]
In other words, the de Broglie relation may be modified in such a way that a minimum wavelength corresponds to an unbound momentum. Notice, however, that we are considering here the effects of these modifications at length scales much larger than the Planck scale, where the correction is likely to be negligible. In general, if \( p \propto \lambda^{-1} g(l_P/\lambda) \) then \( \Delta p \propto l_P p^2 + p O(l_P^2 p^2) \). Hence, we do not expect that the result shown in the previous Section is appreciably modified.

Last but not least we notice that the fluctuations in the dispersion relations can be easily derived from fluctuations of the (vacuum) metric in the form given in [25]:

\[
ds^2 = (1 + \phi)dt^2 - (1 + \psi)dr^2
\]  

(11)

where \( \phi, \psi \) are functions of the position in space-time.

The fluctuations of the dispersion relation, Eq. 3, follow if \( \phi \neq \psi \) (i.e. non conformal fluctuations), assuming at least approximate validity of the de Broglie relation; if \( \phi = \psi \) a much milder modification \( O(pm^2/M_P) \) follows.

Having given plausibility arguments in favor of the form adopted for the fluctuations, at least for the case of non conformal fluctuations, we are left with the goal of proving an answer to the last question listed above, namely does a decay actually occur once it is kinematically allowed? Certainly the answer is positive if one continues to assume momentum and energy conservation, and modifications of these conservation laws with random terms of order \( O(p^2/M_P) \) do not change this conclusion. The question then is whether we are justified in assuming energy and momentum conservation in the form used above. For instance, in the so-called Doubly Special Relativity (DSR, [23]) theories and in general in models with deformed Poincare’ invariance, the conservation relations may be modified in a non trivial, non additive and non abelian way. For instance, in the case of proton decay considered above, momentum conservation may read as [23,24]

\[
p_p \approx p'_p + (1 + l_p E'_p) p_\pi \quad \text{or} \quad p_p \approx p'_\pi + (1 + l_p E'_\pi) p_p.
\]  

(12)

This certainly makes the probability of being above threshold smaller. However in order to qualitatively modify our results this probability should be in fact vanishingly small. For the case of low energy cosmic rays, this probability should be of the order of a typical decay time divided by the residence time of cosmic rays (mostly galactic at these energies) in our Galaxy.

We are led to conclude that allowing for modifications of the conservation relations does not appear to improve the situation to the point that the strong
conclusions derived in the previous section can be avoided. In the same
perspective, cancellation between fixed modifications of the dispersion relation
and fluctuations (of the same order of magnitude) does not seem a viable way
to proceed.

It is important however to notice that we have considered the above fluc-
tuations as independent. In a full theory (exemplified by DSR models, for
instance) one should take into account the correlations (and possible cancel-
lations $^3$) between them. In our opinion this analysis is mandatory if we want
to have a clearer idea of the extent of the implications of these theories. At the
level these calculations can be carried out at present, we think that there are
solid arguments that suggest that the fluctuations in the form assumed above
imply observational consequences which seem to be in serious contradiction
with reality.

**Note added in proof:** After submission of the present paper, analogous
considerations appeared in [27], where essentially identical conclusions are
reached.

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$^3$ In fact in DSR theories the processes described here should be absent, given the
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