The galactocentric radius dependent upper mass limit of young star clusters: stochastic star formation ruled out

Jan Pflamm-Altenburg\textsuperscript{1*}, Rosa A. Gonzalez-Lópezlira\textsuperscript{2*} and Pavel Kroupa\textsuperscript{1*}

\textsuperscript{1} Helmholtz-Institut für Strahlen- und Kernphysik (HISKP), University of Bonn, Nussallee 14-16, D-53115 Bonn, Germany
\textsuperscript{2} Centro de Radioastronomía y Astrofísica, UNAM, Campus Morelia, Michoacán, México, C.P. 58089

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ABSTRACT

It is widely accepted that the distribution function of the masses of young star clusters is universal and can be purely interpreted as a probability density distribution function with a constant upper mass limit. As a result of this picture the masses of the most-massive objects are exclusively determined by the size of the sample. Here we show, with very high confidence, that the masses of the most-massive young star clusters in M33 decrease with increasing galactocentric radius in contradiction to the expectations from a model of a randomly sampled constant cluster mass function with a constant upper mass limit. Pure stochastic star formation is thereby ruled out. We use this example to elucidate how naive analysis of data can lead to unphysical conclusions.

Key words: stars: formation – galaxies: individual: M33 – galaxies: star clusters: general

1 INTRODUCTION

Two of the most fundamental distribution functions in astronomy are the initial stellar mass function (IMF) in star clusters and the initial mass function of star clusters (ICMF). Observations have shown that the form of both distribution functions, the IMF in star clusters (Kroupa 2001, 2002; Kroupa et al. 2011) and the ICMF (e.g. de Grijs et al. 2003) seem to be universal.

There is less agreement about how the high-mass regime of both functions is populated. On the one hand the upper mass limits are treated to be independent of the environment (e.g. Gieles et al. 2006; Gieles 2009; Parker & Goodwin 2007; Lamb et al. 2010) and therefore the most-massive objects are determined by a pure size-of-sample effect. That is, the typical mass of the most-massive object that occurs in a sample increases with the size of the sample. On the other hand the formation of the most-massive objects could naturally require appropriate physical conditions. In the case of the IMF there is a growing amount of evidence that massive stars are not forming in low-mass star clusters (Weidner & Kroupa 2006; Weidner et al. 2010; Hen et al. 2012). This is incompatible with the idea that the high-mass regime of the IMF is populated entirely randomly. In the case of the ICMF this is less clear. But recently, Larsen (2009) found that if the mass distribution of young star clusters is fitted by a Schechter-type function, then the critical mass, $M_{\text{crit}}$, where the ICMF turns down at the high-mass end, is larger for star-burst galaxies than for Milky Way type galaxies. I.e., the most-massive star clusters in star-burst galaxies are more massive than those in normal disk galaxies. Larsen (2009) suggested that this may be due to the high-pressure environment in star-burst galaxies.

Consequently, if the gas density determines the physical upper mass limit for star cluster formation in whole galaxies, then the same effect is expected to be present within disk galaxies on smaller scales. As gas densities generally increase with decreasing galactocentric distance, the most-massive star clusters should form predominantly in the inner regions. One might be tempted to leap to the conclusion that this is the result of a size-of-sample effect because the star formation rate density and therefore the cluster formation rate density in the central regions is higher than in the outer ones.

In Sec. 2 we show that the most-massive young star clusters in M33 seem to be in agreement at first and naive sight with a pure size-of-sample effect. In Sec. 3 we demonstrate by using bins with equal number of star clusters that the masses of the most-massive star clusters decrease with increasing galactocentric radius therewith ruling out a purely randomly sampled constant ICMF.

We proceed strictly logically: we assume that sampling cluster masses from the constant ICMF is stochastic and then find extremely significant disagreement with the ob-
The stochasticity hypothesis is therewith falsified with extremely high confidence.

2 THE SIZE-OF-SAMPLE EFFECT

The initial cluster mass function (ICMF) of young star clusters, \( \xi_{cl}(M_{cl}) \), determines the number of young star clusters, \( dN \), in the mass interval \([M_{cl}, M_{cl}+dM_{cl}]\). Observations have shown that the ICMF can be described by a power-law,

\[
\xi_{cl}(M_{cl}) = \frac{dN}{dM_{cl}} \propto M_{cl}^{-\beta}.
\]  

(1)

In M33 [Sharma et al. (2011)] determine a slope of \( \beta = 2 \) for the complete young star cluster population in M33. This is in agreement with the slope of lower-mass young embedded star clusters in the solar neighbourhood (Lada & Lada 2003) as well as with extragalactic studies (e.g. Zhang & Fall 1998; de Grijs et al. 2003). Throughout the whole paper, young refers to the ages of the star clusters in the sample in Sharma et al. (2011) which we have used in our analysis. The majority of these star clusters have ages \( \lesssim 10 \) Myr. A minor fraction of the star clusters have ages up to a few tens of Myr.

The ICMF is widely interpreted as a simple universal probability density distribution function. As a consequence the masses of the most-massive star clusters are determined entirely by the size-of-sample effect (e.g. Gieles et al. 2006; Gieles 2009): If only a few star clusters are randomly drawn from the ICMF, then it is unlikely that a very massive star cluster is among this small sample. Contrary, if a very large number of star clusters are drawn from the same ICMF then it is very likely that a massive star cluster is part of this set.

Before we proceed analysing the star cluster data by Sharma et al. (2011) it should be mentioned that, due to stochastic effects, the determination of star cluster masses based on integrated photometry might be questionable for low-mass star clusters (e.g. Maíz Apellániz 2000; Fouesneau & Lançon 2010). For our analysis we use the masses of the most massive star clusters, which are not expected to be influenced strongly by stochastic IMF effects. The low-mass star clusters only contribute to the size-of-sample of the considered sub-set of star clusters. As these subsets are defined by the position of the star clusters in M33 and not by their masses, stochastic IMF effects on the mass determination are unimportant.

Figure 1 shows the masses versus the galactocentric radius (black points) of the young star cluster sample in M33 from Sharma et al. (2011). In order to allow a statistical analysis only young star clusters above the completeness limit of \( \approx 600 \, M_\odot \) are considered. The sample is divided into radial bins with a constant width of 2 kpc. Within each bin the 1st, 2nd, 3rd, 4th, and 5th most-massive young star cluster is determined. The i-th most-massive young star cluster in a particular bin is represented by its mass and a radial position which is the average of the radial positions of all star clusters in the particular bin. The black solid lines connect the points of the i-th most massive star cluster in radial direction.

It can be seen in Fig. 1 that there is the general tendency that the i-th most-massive star cluster becomes less massive with increasing galactocentric radius. At the same time, the number of young star clusters in the bins (number below the star cluster sample) decreases with increasing galactocentric radius, too. Such a behaviour is expected if the most-massive star clusters are determined by the size-of-sample effect. Theoretically, the probability density distribution of the i-th most-massive star cluster for a size-of-sample, \( N \), is given by (e.g. Pflamm-Altenburg & Kroupa 2008)

\[
p_{i,N}(M_{cl}) = N^{(N-1)} \left( \int_{M_{l}}^{M_{u}} \xi_{cl}(M_{cl}) \, dM_{cl} \right)^{N-1} \xi_{cl}(M_{cl}) \left( \int_{M_{l}}^{M_{u}} \xi_{cl}(M_{cl}) \, dM_{cl} \right)^{-1-1},
\]  

(2)

where \( M_{l} \) is the lower mass limit and \( M_{u} \) is the upper mass limit of the cluster mass function and \( \xi_{cl}(M_{cl}) \) needs to be normalised,

\[
\int_{M_{l}}^{M_{u}} \xi_{cl}(M_{cl}) \, dM_{cl} = 1.
\]  

(3)

Figure 2 shows the theoretical distribution function of the 2nd most-massive star cluster, derived from eq. 2 with \( M_{l} = 600 \, M_\odot \) (the completeness limit), \( M_{u} = 10^7 \, M_\odot \), and \( \beta = 2 \) for three different sizes-of-sample (\( N = 136, 79, 23 \), corresponding to the three radial bins 2–4 kpc, 4–6 kpc, and 6–8 kpc). The filled black circles mark the mass of the 2nd most-massive star cluster in the particular bin. At first sight, the masses of the most-massive young star clusters seem to be in agreement with the size-of-sample effect, suggesting that there is no need for a deviation from the picture of a purely randomly sampled universal and therefore environment independent cluster mass function.

Here, we have reached a situation that requires careful attention. The (naive) conclusion that this aspect of the observations is in agreement with the picture of a simple universal probability density distribution function of the ICMF does not imply that a different model where the
formation of the most-massive star clusters is limited by the local physical conditions, as for example the local gas density (Pflamm-Altenburg & Kroupa 2008), is ruled out. These lax dealings have lead to serious misconclusions. E.g., Maschberger & Clarke (2008) analyse whether the most-massive star in young low-mass star clusters is determined by a size-of-sample effect while drawing from a constant IMF with a constant upper mass limit and they state: “Our conclusion (in support of the random drawing hypothiss) remains provisional.” But they have not tested the alternative of a varying upper mass limit and how well such a model agrees with the observations. Thus, careless reading of Maschberger & Clarke (2008) then leads to the acceptance of a useless statement. E.g., Eldridge (2012) refers to this work writing: “Maschberger & Clarke (2008) also made a detailed study of all available information and also favour PSS” (pure-stochastic-sampling).

3 RADIAL DEPENDENCE

In order to exclude the influence of a possible size-of-sample effect in the analysis of the star cluster masses, we divide the sample in bins such that they contain an equal number of star clusters. If the ICMF is a simple universal and environment independent probability density distribution function, then no radial dependence of the i-th most-massive star cluster is expected.

The sample of young star clusters in M33 from Sharma et al. (2011) contains 591 clusters. For the analysis we only consider all 358 more massive than 600 $M_\odot$ which is approximately the completeness limit. In order to construct as many as possible bins containing the same number of star clusters we choose a size-of-sample of $N = 17$. This leads to only one redundant star cluster, which is the outer-most one. This has of course no effect on the statistical analysis.

In each bin we determine the 1st to 5th most-massive star cluster. These are shown in Fig. 3 and are connected by solid lines. The radial position of the i-th most-massive star cluster is the average of the radial distances of all star clusters in the particular bin. It can be seen that the trend of a decrease of the mass is preserved, whereas no trend is expected if the upper-mass regime were purely stochastically populated.

The radial distance dependence of the i-th most massive star cluster can be described by a linear fit,

$$\log_{10} \left( \frac{M_i}{M_\odot} \right) = a \frac{r}{\text{kpc}} + b. \quad (4)$$

The coefficients $a$ and $b$ are listed in Table 1. Two cases are considered: i) all star clusters, ii) only star clusters with a galactocentric distance less than 4.5 kpc. Except for the most massive star cluster the 2nd to 5th most massive star cluster show a very pronounced radial dependence.

If the ICMF were universal then on average no radial trend is expected. However, it could be possible that this snapshot of the most-massive star cluster sample in M33 is just the result of stochasticity of a universal ICMF. In order to determine how likely such a radial distribution is under the assumption of a constant and radially independent upper mass limit is, we perform a Monte-Carlo experiment: for each bin we draw $N = 17$ star clusters from an ICMF with $\beta = 2$, $M_1 = 600 M_\odot$, and $M_{\text{up}} = 10^7 M_\odot$. The 1st to 5th most massive star clusters are identified. Their radial positions are the radial average of the star clusters in the particular bin. The sets are fitted by eq. (4) identical to the treatment of the observational data. Figure 3 shows the distribution of the slope $a$ after $10^6$ repetitions for case (ii), i.e. only considering star clusters with a galactocentric distance less than 4.5 kpc.

The probability of a random drawing event of the radial trend is obtained from the cumulative distribution. In those cases where the cumulative distribution of the Monte-Carlo simulations is zero, because $10^6$ repetitions are too few, the left wings of the distributions of the slope $a$ are fitted by a Gaussian curve and extrapolated. The probabi-
ties are listed in Table 1. E.g., the observed slope of the 3rd most-massive star cluster if all star clusters are considered is $a = -0.146$. The probability that a slope of $a = -0.146$ or steeper is the result of a randomly sampled ICMF with a radius-independent upper mass limit of $M_u = 10^7 M_\odot$ and slope $\beta = 2$.

**Table 1.** Slopes of the radial distribution of the $i$-th massive star cluster in M33

| range | $i$ | $b$ | $a$ | $P(\leq a)$ |
|-------|----|----|----|-----------|
| tot   | 5  | 4.209 | -0.146 | $5.3 \times 10^{-19}$ |
| tot   | 4  | 4.348 | -0.158 | $2.2 \times 10^{-16}$ |
| tot   | 3  | 4.465 | -0.146 | $1.3 \times 10^{-9}$ |
| tot   | 2  | 4.672 | -0.149 | $2.5 \times 10^{-6}$ |
| tot   | 1  | 4.674 | $-1.7 \times 10^{-4}$ | 0.52 |

| $\leq 4.5$ kpc | 5  | 4.430 | -0.239 | $6.4 \times 10^{-11}$ |
| $\leq 4.5$ kpc | 4  | 4.526 | -0.233 | $6.6 \times 10^{-8}$ |
| $\leq 4.5$ kpc | 3  | 4.583 | -0.197 | $1.4 \times 10^{-4}$ |
| $\leq 4.5$ kpc | 2  | 4.827 | -0.213 | $1.4 \times 10^{-3}$ |
| $\leq 4.5$ kpc | 1  | 5.153 | -0.208 | $3.6 \times 10^{-2}$ |

Listed are the coefficients $a$ and $b$ of the linear fitting function (eq. 4) of the observed radial distribution of the 1st–5th most-massive star cluster. The last column contains the probability that the value of the slope $a$ or steeper in column 4 is the result of a randomly sampled ICMF with constant and galactocentric radius-independent upper mass limit of $M_u = 10^7 M_\odot$ and slope $\beta = 2$.

**4 PARAMETER DEPENDENCE**

In order to explore how the probabilities depend on the chosen parameters of the ICMF, we also perform Monte-Carlo simulations with different values of the slope and the upper cluster mass limit. The probabilities are obtained in the same way as described above. Figure 5 shows the probabilities for the whole set of star clusters for an upper mass limit of $M_u = 10^7 M_\odot$ (filled symbols) and $M_u = 10^9 M_\odot$ (open symbols), and for three different slopes $\beta = 1.7$ (triangles), $\beta = 2.0$ (squares), and $\beta = 2.3$ (circles). Figure 5 is the same as Fig. 3 but only for star clusters with a galactocentric radius of $\leq 4.5$ kpc. In the cases where no open symbol appears it lies at the same position as the corresponding filled symbol. It can be seen that, except for the most-massive star clusters, it is unlikely that the radial trend of the most massive star clusters is the stochastic result of a purely randomly sampled constant ICMF. In general, the probabilities decrease considerably with a steeper slope of the ICMF and the order of the star cluster.

There are several studies which conclude that the ICMF of young star clusters may be better described by a Schechter-type function than by a single-part power-law (e.g. Gieles et al. 2002; Larsen 2009). Therefore, we also test a Schechter-type function with a low-mass-end slope of $\beta = 2$ and a turn-down mass of $M_{\text{cut}} = 2.1 \times 10^5 M_\odot$ (Larsen 2009). The probabilities that the radial distribution of the $i$-th most-massive star cluster is consistent with a randomly sampled constant ICMF are shown in Fig. 7. A constant randomly sampled Schechter-type ICMF is ruled out, too.

The analysis has been performed for a size-of-sample of $N = 17$ and it might be interesting what the results are for a different size-of-sample. We now set $N = 32$. In this case all 358 clusters are binned in 11 radial bins and only 6 redundant star clusters remain. Fig. 8 shows the radial dependence of the most to the 5th-most massive star cluster. It can be seen that the radial trend of a decrease of the masses of the most-massive star clusters persists. The probabilities that these trends can be the result of a randomly sampled constant ICMF ($\beta = 2$ and $M_u = 10^7 M_\odot$) is shown in Fig. 9.

We analyse here only a radial dependence. This means that the observed environmental dependency of the upper mass limit of the ICMF must be given by the spatial location of the star clusters. The underlying physical reason for the derived result might be that the surface density of the gas in M33 tendentially decreases with increasing galactocentric radius (Heyer et al. 2004) and that the formation of very massive star clusters require a sufficiently large local gas reservoir. Furthermore, it is found that the masses of the most-massive young star clusters in M33 scale with the surface gas density (González-Lópezlira, Pflamm-Altenburg & Kroupa 2012). In this context, the existence of the very massive star cluster in the outer region should not be an exception: the local gas density must be high enough in order to form it.

The high-probability in the case of the most-massive star cluster does not mean that a non-universal ICMF is ruled out but only that the consistency with a universal ICMF is much higher than for the star clusters with a lower order. This is not surprising. Figure 10 shows the distribution of the 1st to 5th-most massive star cluster for a size-of-sample of $N = 17$ star clusters and an ICMF with slope $\beta = 2$ between $M_l = 600 M_\odot$ and $M_h = 10^7 M_\odot$. The distribution of the most-massive star cluster is much broader than for the 5th-most massive star cluster. Specifically, the FWHM for the most-massive star cluster is 1.0 dex, whereas the FWHM for the 5th-most massive star cluster is only 0.35 dex. If there is a radial dependence of the upper mass limit of the ICMF (being higher in the central region, i.e. in regions of higher gas densities) then the broad distribution of the most-massive young star cluster smears out the radial distribution.
trend. This effect has also been mentioned in Larsen (2009) stating that the error is smaller for the 5th brightest star cluster than for the brightest. Therefore, it is more reliable to use the fifth-most massive star cluster and thus a randomly sampled constant and environmentally independent ICMF is ruled out.

A final remark should be made if disruption processes of young star clusters can cause the observed trend. Disruption due to mass loss by stellar evolution and gas expulsion are internal processes and should occur in the same way at each position in the galaxy and can therefore not create the observed trend. Disruption due to tidal effects should depend on the position of a star cluster in a galaxy. However, tidal effects mainly influence on the evolution of lower-mass star clusters and we assume that high-mass star cluster are not disrupted on the short time scale of \( \lesssim 10-20 \) Myr.

### 5 CONCLUSION

We have found that the formation of very massive star clusters is increasingly suppressed with increasing galactocentric radius in M33. We have ruled out with extremely high significance that this is the result of a size-of-sample effect, where a constant and environment independent ICMF is populated entirely randomly and environmental effects can be neglected.

The straightforward conclusion is that very massive star clusters require special physical conditions in order to form, as for example high gas surface densities. This is indeed a very plausible notion, because gas surface densities decrease tendentially with increasing galactocentric radius.

If the ICMF is not independent of the environment, why should the stellar IMF be? Indeed, direct analysis of Galactic star forming regions shows that the formation of high-mass stars takes place in high-mass star clusters which can not be a size-of-sample effect. Weidner & Kroupa
Figure 9. The probabilities for a constant power-law ICMF with $\beta = 2$ and $M_u = 10^7 M_\odot$ but with a size-of-sample of $N = 32$. The filled symbols show the probabilities for all star clusters, the open symbols show the probabilities only for star clusters with a galactocentric distance of $\leq 4.5$ kpc.

Figure 10. The distribution of the 1st to 5th most-massive star cluster for a size-of-sample of $N = 17$ star clusters and an ICMF with slope $\beta = 2$ between $M_l = 600 M_\odot$ and $M_u = 10^7 M_\odot$. The arrows mark the FWHM of the distribution of the most-massive star cluster (1.0 dex) and of the fifth most-massive star cluster (0.35 dex).

In this context the work presented here fundamentally supports the IGIMF theory and its consequences.

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