Abstract
We investigate the $Q^2$ evolution of parton distributions at small $x$ values, obtained in the case of flat initial conditions. The contributions of twist-two and (renormalon-type) higher-twist operators of the Wilson operator product expansion are taken into account. The results are in excellent agreement with deep inelastic scattering experimental data from HERA.

Key-words: Quantum Chromodynamics, the deep-inelastic scattering, structure function, parton distribution, twist.

1 Introduction
The measurements of the deep-inelastic scattering structure function $F_2$ in HERA \cite{1,2} have permitted the access to a very interesting kinematical range for testing the theoretical ideas on the behavior of quarks and gluons carrying a very low fraction of momentum of the proton, the so-called small $x$ region. In this limit one expects that non-perturbative effects may give essential contributions. However, the reasonable agreement between HERA data and the next-to-leading order (NLO) approximation of perturbative QCD that has been observed for $Q^2 > 1\text{GeV}^2$ (see the recent review in \cite{3}) indicates that perturbative QCD could describe the evolution of structure functions up to very low $Q^2$ values, traditionally explained by soft processes. It is of fundamental importance to find out the kinematical region where the well-established perturbative QCD formalism can be safely applied at small $x$.

The standard program to study the small $x$ behavior of quarks and gluons is carried out by comparison of data with the numerical solution of the DGLAP equations by fitting the parameters of the $x$ profile of partons at some initial $Q_0^2$ and the QCD energy scale $\Lambda$ (see, for example, \cite{4,5}). However, if one is interested in analyzing exclusively the small $x$ region ($x \leq 0.01$), there is the alternative of doing a simpler analysis by using some of the existing analytical solutions of DGLAP in the small $x$ limit (see \cite{3} for review). This was done so in Ref. \cite{10-19} where it was pointed out that the HERA small $x$ data can be interpreted in terms of the so called doubled asymptotic scaling phenomenon related to the asymptotic behavior of the DGLAP evolution discovered in \cite{10} many years ago.

Here we illustrate results obtained recently in \cite{7} and demonstrate some (preliminary) results of \cite{9}, where the contributions of higher-twist operators (i.e. twist-four ones and...
twist-six ones) of the Wilson operator product expansion are taken into account. The importance of the contributions of higher-twist operators at small-$x$ has been done in many studies (see [11]).

We would like to note that the results of [7] are the extension to the NLO QCD approximation of previous leading order (LO) studies [10, 6]. The main ingredients are:

1. Both, the gluon and quark singlet densities are presented in terms of two components ($'+'$ and $'−'$) which are obtained from the analytical $Q^2$ dependent expressions of the corresponding ($'+'$ and $'−'$) parton distributions moments.

2. The '$−'$ component is constant at small $x$, whereas the '$+$' component grows at $Q^2 \geq Q^2_0$ as $\sim \exp (\sigma_{NLO})$, where

$$
\sigma_{NLO} = 2\sqrt{(\hat{d}_+ s + \hat{D}_+ p)\ln x},
$$

and the LO term $\hat{d}_+ = -12/\beta_0$ and the NLO one $\hat{D}_+ = \hat{d}_+ + \hat{d}_+ \beta_1/\beta_0$ with $\hat{d}_+ = 412 f/(27 \beta_0)$. Here the coupling constant $a_s = \alpha_s/(4\pi)$, $s = \ln(\alpha(Q^2_0)/\alpha(Q^2))$ and $p = \alpha(Q^2_0) - \alpha(Q^2)$, $\beta_0$ and $\beta_1$ are the first two coefficients of QCD $\beta$-function and $f$ is the number of active flavors.

2 Basic formulae

Our purpose is to show the small $x$ asymptotic form of parton distributions in the framework of the DGLAP equation starting at some $Q^2_0$ with the flat function:

$$
f^r_{a^2}(Q^2_0) = A_a \quad (\text{hereafter } a = q, g),
$$

where $f^r_{a^2}$ are the leading-twist parts of parton (quark and gluon) distributions multiplied by $x$ and $A_a$ are unknown parameters that have to be determined from data. Through this work at small $x$ we neglect the non-singlet quark component.

We would like to note that new HERA data [2] show a rise of $F_2$ structure function at low $Q^2$ values ($Q^2 < 1\text{GeV}^2$) when $x \to 0$ (see Fig.2, for example). The rise can be explained in a natural way by incorporation of higher-twist terms in our analysis (see the part 2.2).

We shortly compile below the main results found in [7, 9] at the LO approximation (the leading-twist results at the NLO approximation may be found in [11]). The full small $x$ asymptotic results for parton distributions and $F_2$ structure function at LO of perturbation theory is:

$$
F_2(x, Q^2) = e \cdot f_q(x, Q^2),
$$

$$
f_a(x, Q^2) = f^+_a(x, Q^2) + f_a^−(x, Q^2),
$$

where the '$+$' and '$−'$ components $f^\pm_a(x, Q^2)$ are given by the sum

$$
f^\pm_a(x, Q^2) = f^{r^2, \pm}_a(x, Q^2) + f^{hr, \pm}_a(x, Q^2)
$$

of the leading-twist parts $f^{r^2, \pm}_a(x, Q^2)$ and the higher-twist parts $f^{hr, \pm}_a(x, Q^2)$, respectively.
2.1 The contribution of twist-two operators

The small $x$ asymptotic results for PD, $f_{a}^{T2}$:

$$f_{g}^{T2,+}(x, Q^2) = \left( A_g + \frac{4}{9} A_q \right) I_0(\sigma) e^{-\overline{d}_+(1)s} + O(\rho),$$  \hspace{1cm} (5) \\
$$f_{q}^{T2,+}(x, Q^2) = \frac{f}{9} \left( A_g + \frac{4}{9} A_q \right) \rho I_1(\sigma) e^{-\overline{d}_+(1)s} + O(\rho),$$  \hspace{1cm} (6) \\
$$f_{g}^{T2,-}(x, Q^2) = -\frac{4}{9} A_q e^{-d_-(1)s} + O(x),$$  \hspace{1cm} (7) \\
$$f_{q}^{T2,-}(x, Q^2) = A_q e^{-d_-(1)s} + O(x),$$  \hspace{1cm} (8)

where $\overline{d}_+(1) = 1 + 20 f / (27 \beta_0)$ and $d_-(1) = 16 f / (27 \beta_0)$ are the regular parts of $d_+$ and $d_-$ anomalous dimensions, respectively, in the limit $n \to 1$. The functions $\hat{I}_\nu (\nu = 0, 1)$ are related to the modified Bessel function $I_\nu$ and to the Bessel function $J_\nu$ by:

$$\hat{I}_\nu(\sigma) = \begin{cases} I_\nu(\sigma), & \text{if } s \geq 0 \\ J_\nu(\sigma), & \text{if } s < 0 \end{cases}.$$  \hspace{1cm} (9)

The variables $\sigma$ and $\rho$ are given by

$$\sigma = 2 \sqrt{|\overline{d}_+ s ln(x)|}, \quad \rho = \sqrt{\frac{|\overline{d}_+ s|}{ln(1/x)}} = \frac{\sigma}{2 ln(1/x)}.$$  \hspace{1cm} (10)

2.2 The higher-twist contributions

Using the results in [9] (which are based on calculations [12, 13]), we show the effect of higher-twist corrections in the renormalon case (see recent review of renormalon models in [14]). We present the results below making the following substitutions in the corresponding twist-two results presented in Eqs. (3)-8:

$f_{g}^{T2,+}(x, Q^2)$ (see Eq. (5)) $\to f_{g}^{hr,+}(x, Q^2)$ by

$$A_a I_0(\sigma) \to A_a \cdot \frac{16 f}{15 \beta_0} \left\{ \frac{\Lambda_{2,a}^2}{Q^2} \left( \frac{2}{\rho} I_1(\sigma) + \left[ K_{ga}(f) - \ln \left( \frac{\Lambda_{1,a}^2}{Q^2} \right) \right] I_0(\sigma) \right) \right.$$
$$- \frac{8}{7} \Lambda_{2,a}^4 \left( \frac{2}{\rho} I_1(\sigma) + \left[ K_{qa}(f) - \frac{11}{112} \ln \left( \frac{\Lambda_{2,a}^2}{Q^2} \right) \right] I_0(\sigma) \right) \right\},$$  \hspace{1cm} (11)

where $\Lambda_{1,a}^2$ and $\Lambda_{2,a}^4$ are magnitudes of twist-four and twist-six corrections and

$$K_{ga}(f) = \frac{101}{60} - \frac{8 f}{81}, \quad K_{qa}(f) = \frac{121}{60} - \frac{7 f}{81};$$

$f_{q}^{T2,+}(x, Q^2)$ (see Eq. (6)) $\to f_{q}^{hr,+}(x, Q^2)$ by

$$A_a \rho I_1(\sigma) \to A_a \cdot \frac{128 f}{45 \beta_0} \left\{ \frac{\Lambda_{2,a}^2}{Q^2} \left( \frac{2}{\rho} I_1(\sigma) + \left[ K_{ga}(f) - \ln \left( \frac{\Lambda_{1,a}^2}{Q^2} \right) \right] I_0(\sigma) \right)
$$
$$- \frac{8}{7} \Lambda_{2,a}^4 \left( \frac{2}{\rho} I_1(\sigma) + \left[ K_{qa}(f) - \frac{11}{112} \ln \left( \frac{\Lambda_{2,a}^2}{Q^2} \right) \right] I_0(\sigma) \right) \right\},$$  \hspace{1cm} (12)

\footnote{From now on, for a quantity $k(n)$ we use the notation $k(n)$ for the singular part when $n \to 1$ and $\overline{k}(n)$ for the corresponding regular part.}
Figure 1: The structure function $F_2$ as a function of $x$ for different $Q^2$ bins. The experimental points are from H1 [1]. The inner error bars are statistic while the outer bars represent statistic and systematic errors added in quadrature. The dashed and dot-dashed curves are obtained from fits (based on leading-twist formulae) at LO and NLO respectively with fixed $Q^2_0 = 1 \text{ GeV}^2$. The solid line is from the fit at NLO giving $Q^2_0 = 0.55 \text{ GeV}^2$.

where

$$K_{qq}(f) = \frac{11}{60} - \frac{2f}{27}, \quad K_{qg}(f) = -\frac{3}{20} - \frac{7f}{81};$$

$$f_{g}^{r2-}(x, Q^2) \text{ (see Eq.}(8)) \rightarrow f_{g}^{chr-}(x, Q^2) \text{ by}$$

$$A_q \rightarrow A_q \cdot \frac{16f}{15\beta_0^2 Q^2} \left( \ln \left( \frac{Q^2}{x^2 \Lambda_{1,q}^2} \right) + \frac{5}{2} \ln \left( \frac{1}{x} \right) - \frac{359}{120} + \frac{4f}{81} \right) \ln \left( \frac{Q^2}{x^2 \Lambda_{1,q}^2} \right)$$

$$+ \frac{143}{1680} - \frac{7f}{81} \right) \right) - A_g \cdot \frac{128f^2}{1215\beta_0^2} \left\{ \frac{\Lambda_{1,g}^2}{Q^2} - \frac{8\Lambda_{2,g}^4}{7 Q^4} \right\}; \quad (13)$$

$$f_{g}^{r2-}(x, Q^2) \text{ (see Eq.}(8)) \rightarrow f_{g}^{chr-}(x, Q^2) \text{ by}$$

$$A_q \rightarrow A_q \cdot \frac{128f}{45\beta_0^2 Q^2} \left\{ \frac{\Lambda_{1,g}^2}{Q^2} \left( \ln \left( \frac{Q^2}{x^2 \Lambda_{1,g}^2} \right) + \frac{5}{2} \ln \left( \frac{1}{x} \right) - \frac{359}{120} + \frac{4f}{81} \right) \ln \left( \frac{Q^2}{x^2 \Lambda_{1,g}^2} \right)$$

$$+ \frac{1871}{600} + \frac{309f}{1215} + \frac{24f^2}{(81)^2} \right) - \frac{8\Lambda_{2,g}^4}{7 Q^4} \left( \ln \left( \frac{Q^2}{x^2 \Lambda_{2,g}^2} \right) + \frac{52}{21} \right) \ln \left( \frac{1}{x} \right)$$

$$- \frac{10237}{3360} + \frac{4f}{81} \right) \ln \left( \frac{Q^2}{x^2 \Lambda_{2,g}^2} \right) + \frac{33301}{11200} + \frac{3377f}{19440} + \frac{24f^2}{(81)^2} \right) \right\}$$

$$- A_g \cdot \frac{128f^2}{405\beta_0^2 Q^2} \left\{ \ln \left( \frac{Q^2}{x^2 \Lambda_{1,g}^2} \right) - \frac{259}{60} - \frac{7f}{81} - \frac{8\Lambda_{2,g}^4}{7 Q^4} \right\} \ln \left( \frac{Q^2}{x^2 \Lambda_{2,g}^2} \right) - \frac{1817}{3360} - \frac{7f}{81} \right) \right\} \quad (14)$$

From Eqs. (11)-(14) one can notice that the higher-twist terms modify the flat condition Eq.(1). They lead to a rise of parton distributions and, thus, $F_2$ structure function at low value $Q^2_0$, when $x \rightarrow 0$. This is in agreement with HERA data [2], as it is shown in next section.
3 Results of the fits

With the help of the results presented in the previous section we have analyzed $F_2$ HERA data at small $x$ from the H1 collaboration (first articles in \cite{1, 2}). In order to keep the analysis as simple as possible we have fixed the number of active flavors $n_f = 4$ and $\Lambda_{\overline{MS}}(n_f = 4) = 250$ MeV, which is a reasonable value extracted from the traditional (higher $x$) experiments. Moreover, we put $\Lambda_{1,a} = \Lambda_{2,a}$ in agreement with \cite{15}. The initial scale of the parton densities was also fixed into the fits to $Q_0^2 = 1$ GeV$^2$, although later it was released to study the sensitivity of the fit to the variation of this parameter. The analyzed data region was restricted to $x < 0.01$ to remain within the kinematical range where our results are accurate.

Fig. 1 shows $F_2$ calculated from the fit (based only on leading-twist formulae) with $Q^2 > 1$ GeV$^2$ in comparison with 1994 H1 data (first article in \cite{1}). Only the lower $Q^2$ bins are shown. One can observe that the NLO result (dot-dashed line) lies closer to the data than the LO curve (dashed line). The lack of agreement between data and lines observed at the lowest $x$ and $Q^2$ bins suggests that the flat behavior should occur at $Q^2$ lower than 1 GeV$^2$. In order to study this point we have done the analysis considering $Q_0^2$ as a free parameter. Comparing the results of the fits (see \cite{7}) one can notice the better agreement with the experiment at fitted $Q_0^2 = 0.55$ GeV$^2$ (solid curve) is apparent at the lowest kinematical bins.

Fig. 2 shows $F_2$ calculated from the fit at LO (based on leading-twist and higher-twist formulae) in comparison with 1995 H1 and ZEUS data \cite{2}. One can observe that these results (solid line) lies closer to the data than the twist-two results (dashed line). We have done the analysis considering $Q_0^2$ as a free parameter. Comparing the results of the fits (see \cite{9}) one can notice the better agreement with the experiment at fitted $Q_0^2 = 0.61$ GeV$^2$, which is close to $Q_0^2$ in the analysis of 1994 H1 data (see Fig. 1).
4 Conclusions

We have shown that the results developed recently in [7, 9] have quite simple form and reproduce many properties of parton distributions at small $x$, that have been known from global fits.

We found very good agreement between our approach based on QCD and HERA data, as it has been observed earlier with other approaches (see the review [3]). The (renormalon-type) higher-twist terms lead to the natural explanation of the rise of $F_2$ structure function at low values of $Q^2$ and $x$. The rise has been discovered in recent HERA experiments [2].

As next step of our investigations, we plan to study contributions of higher-twist operators to relations between parton distributions and deep inelastic structure functions, observed, for example, in [15, 17].

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References

[1] H1 Collab.: S. Aid et al., Nucl.Phys. B470 (1996) 3; ZEUS Collab.: M. Derrick et al., Zeit.Phys. C72 (1996) 399;

[2] H1 Collab.: I. Abt et al., Nucl.Phys. B497 (1997) 3; ZEUS Collab.: M. Derrick et al., Eur.Phys.J. C7 (1999) 609.

[3] A. M. Cooper-Sarkar et al., Int.J.Mod.Phys. A13 (1998) 3385.

[4] A.D. Martin et al., Eur.Phys.J. C14 (2000) 133; M. Gluck, E. Reya and A. Vogt, Eur. Phys. J. C5 (1998) 461; H.L. Lai et al., Eur. Phys. J. C12 (2000) 375.

[5] G. Parente et al., Phys.Lett. B333 (1994) 190; A.L. Kataev et al., Phys.Lett. B388 (1996) 179; Phys. Lett. B417 (1998) 374; Nucl. Phys. Proc. Suppl. 64 (1998) 138; Nucl. Phys. B573 (2000) 405.

[6] R.D. Ball and S. Forte, Phys.Lett. B336 (1994) 77; L. Mankiewicz et al., Phys.Lett. B393 (1997) 175.

[7] A.V. Kotikov and G. Parente, Nucl.Phys. B549 (1999) 242.

[8] A.V. Kotikov and G. Parente, [hep-ph/9810223], in Proceeding of International Conference PQFT98 (1998), Dubna; [hep-ph/0006197], in Proceeding of International Workshop on Deep Inelastic Scattering and Related Phenomena (2000), Liverpool; [hep-ph/0010352] in Proceeding of International Workshop the Diffraction 2000, Cetraro.

[9] J. Bartels, A.V. Kotikov, and G. Parente, work in progress

[10] A. De Rújula et al., Phys.Rev. D10 (1974) 1649.
[11] J. Bartels, Preprint DESY-91 074; Phys.Lett. B298 (1993) 204; Z.Phys. C60 (1993) 471; E.M. Levin, M.G. Ryskin and A.G. Shuvaev, Nucl.Phys. B387 (1992) 589; A.D. Martin and M.G. Ryskin, Phys. Lett. B431 (1998) 395; J. Bartels and C. Bontus, Phys.Rev. D61 (2000) 034009.

[12] E. Stein, M. Maul, L. Mankiewicz, and A. Schäfer, Nucl.Phys. B536 (1998) 318.

[13] A.V. Kotikov, Phys.Atom.Nucl. 57 (1994) 133; Phys.Rev. D49 (1994) 5746.

[14] M. Beneke, Phys. Report 317 (1991) 1; M. Beneke and V.M. Braun, Preprint PITHA-00-25, TPR-00-19 (hep-ph/0010288).

[15] M. Dasgupta and B.R. Webber, Phys. Lett. B382 (1996) 273.

[16] A.V. Kotikov, JETP Lett. 59 (1994) 667; A.V. Kotikov and G. Parente, Phys.Lett. B379 (1996) 195.

[17] A.V. Kotikov, JETP 80 (1995) 979; A. V. Kotikov and G. Parente, hep-ph/9609237, in Proceeding of International Workshop on Deep Inelastic Scattering and Related Phenomena (1996), Rome, p.237; Mod.Phys.Lett. A12 (1997) 963; JETP 85 (1997) 17.