Analysis of the directional pattern of a non-equidistant antenna array with a random arrangement of radiating elements in three-dimensional space

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A three-dimensional antenna array with a spherical surface was investigated. The channels were randomly placed at array points with a vertical and horizontal step equal to 0.45 wavelengths. For uniform filling of the sphere, the minimum allowable distance between the channels was introduced, depending on the number of channels and the sphere size. The maximum side lobe level depends on the number of channels. To calculate the path difference in a given direction, the coordinate system was rotated so that one of the coordinate axes coincided with the selected direction. To find the radiation pattern, the scalar product of the amplitude-phase distribution vector corresponding to the signal emission direction was calculated by the complex-conjugate vector of the amplitude-phase distribution corresponding to all possible directions of signal arrival from an external source.

Introduction

Currently, two-dimensional (flat) equidistant antenna arrays are widely used in aircraft and ship radars, as well as in missile attack warning radars. Such arrays make it possible to perform electronic scanning in two planes: in azimuth and elevation, to form the radiation pattern zeros for active jammers, thereby suppressing interference, to obtain a small level of the radiation pattern side lobes using the weight processing of the receiving channels with the Hamming window, and provide high directional action and small angular resolution element.

Unfortunately, all these wonderful possibilities are limited in the direction of scanning to some sectors and cannot be realized at full 360 degrees in azimuth. In a surveillance radar station, such an array has to be rotated in azimuth, which is very difficult due to the need to transmit information from a large number of channels through a rotating joint and a large array mass for the decimeter and meter ranges. Therefore, a flat array immediately turns into a linear (one-dimensional) vertical array, in which, to further reduce the number of information transmission channels, the directional patterns of 3 or 4 receiving channels in elevation angle are formed directly at ultrahigh frequency using digitally controlled phase shifters.

A radical solution to the problem of transmitting information from the antenna array to the processing device is the use of a three-dimensional (volumetric) non-equidistant antenna array, which electronically scans the entire space by rotating the coordinate system in azimuth and elevation. Such an array is called the "Crow's Nest" [1, 2, 3]. The antenna's name was derived from the Crow's Nest designation for a platform at the top of a sailing ship's mast, which is used as an observation deck for observation in all
directions. The non-equidistance of the three-dimensional antenna array is ensured by the random placement of the receiving-transmitting channels within the given shape volume.

An example of a non-equidistant three-dimensional antenna array is the one developed by the Fraunhofer Institute for High Frequency Radar Physics and Technology (FHR), shown in figure 1. This antenna is implemented in the centimeter wavelength range and has a sphere diameter of 1.83 m, containing about 2000 single radiating elements.

1. Amplitude-phase signal distribution and the directional pattern of a three-dimensional antenna array

The amplitude-phase signal distribution over the antenna array elements is determined by the path difference, that is, the distance between the array elements in the direction of signal arrival. Distance is a dimensionless quantity and is specified in wavelengths.

To calculate the path difference in a given direction, it is necessary to rotate the coordinate system so that one of the coordinate axes coincides with the selected direction. The difference in the path of each element is calculated relative to the array center and is its coordinate along this axis.

To calculate the path difference, we will graphically depict the signal arrival direction and one array element. The results are shown in figure 2.

In this figure $A$ is the wave front arrival direction, $E$ is the point of the signal receiver location, $P_E = R_E$. To find out the path difference from the $A$ direction, we will rotate the coordinate system so that the $AP$ line becomes the vertical axis of the new coordinate system.

For this purpose, the Cartesian coordinates of the $E$ point are calculated as follows:

\[ x_E = R_T \cos \alpha_E \cos \varphi_E, \]
\[ y_E = R_T \cos \alpha_E \sin \varphi_E, \]
\[ z_E = R_T \sin \alpha_E \] (1)

First, we multiply the coordinate system rotation matrix relative to the $Z$ axis by the $\varphi_A$ angle [4, 5]. As a result, we get:

\[ \begin{pmatrix} \cos \varphi_A & \sin \varphi_A & 0 \\ -\sin \varphi_A & \cos \varphi_A & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_E \\ y_E \\ z_E \end{pmatrix} = \begin{pmatrix} x_E \cos \varphi_A + y_E \sin \varphi_A \\ -x_E \sin \varphi_A + y_E \cos \varphi_A \\ z_E \end{pmatrix}. \]

Multiplying the resulting vector by the coordinate system rotation matrix relative to the $Y$ axis by the $\alpha_A$ angle, we find the coordinate vector of the $E$ point:

\[ \begin{pmatrix} \cos \alpha_A & 0 & \sin \alpha_A \\ 0 & 1 & 0 \\ -\sin \alpha_A & 0 & \cos \alpha_A \end{pmatrix} \begin{pmatrix} x_E \cos \varphi_A + y_E \sin \varphi_A \\ -x_E \sin \varphi_A + y_E \cos \varphi_A \\ z_E \end{pmatrix} = \begin{pmatrix} \cos \alpha_A \left( x_E \cos \varphi_A + y_E \sin \varphi_A \right) + z_E \sin \alpha_A \\ -x_E \sin \varphi_A + y_E \cos \varphi_A \\ -\sin \alpha_A \left( x_E \cos \varphi_A + y_E \sin \varphi_A \right) + z_E \cos \alpha_A \end{pmatrix}. \]

The first coordinate in the resulting vector is $x$, the second is $y$, and the third is $z$. Therefore, the travel difference relative to the origin in the $A$ direction is calculated as follows:

\[ x = \cos \alpha_A \left( x_E \cos \varphi_A + y_E \sin \varphi_A \right) + z_E \sin \alpha_A. \] (2)
We transform the obtained path difference into the signal phase, with the help of which we calculate the complex amplitude of the signal arriving from the \( A \) direction at the \( E \) point after the coordinate system rotation. The result is a vector of the phase distribution over the antenna array elements, which depends on the chosen direction:

\[
B = \left( e^{-j2\pi x_1}, \ldots, e^{-j2\pi x_n} \right).
\]  (3)

Here \( X = (x_1, \ldots, x_n) \) is a vector formed from the path differences for each element of the antenna array, where \( n \) is the number of array elements.

To find the directional pattern, it is necessary to calculate the scalar product of the amplitude-phase distribution vector corresponding to the directional pattern main lobe (signal emission direction) \( B_g \) by the complex-conjugate amplitude-phase distribution vector corresponding to each direction of the received signal arrival from an external source \( B_s \):

\[
d = B_s \times B_g^*.
\]  (4)

If the received signal is the radiated signal reflection result, that is, the vectors being multiplied differ only by complex conjugation, we obtain the maximum directional pattern value:

\[
d_{\text{max}} = B_s \times B_g^*.
\]  (5)

To calculate the directional pattern, we use simulation modeling.

2. Influence of a three-dimensional antenna array parameters on the directional pattern

The width of the main lobe of the directional pattern (DP) emanating from the array center depends on its size, that is, on the diameter of the sphere in which the array transmitting and receiving elements are randomly placed. Since the sphere is symmetrical, that is, its vertical size is equal to the horizontal size, then after the coordinate system rotation, the DP main lobe width in two mutually perpendicular planes, the intersection line of which coincides with the DP maximum direction, will be the same. The DP main...
lobe width is measured at the half power level and is equal to 2 degrees with a sphere radius of 12.5 wavelengths [6, 7]. In the original coordinate system, the DP main lobe azimuth width depends on the elevation angle.

When calculating the DP, we change the azimuth from 0 to 359 degrees, and the elevation angle from -90° to + 90°. As a result, with an angle sampling step of 0.5 degrees, the directional pattern will be an array of 361 rows and 720 columns for each elevation and azimuth, respectively. When assessing the array step effect on the directional diagram, it was assumed that the same number of array elements is randomly located inside the sphere, equal to 128. When placing, the minimum distance between the elements was chosen as possible, but such that the specified number of elements could fit inside the selected volume sphere. For 128 array elements with a sphere radius of 12.5, this distance will be 3.3. The array step will be changed from 0.4 to 0.55 at 0.05 wavelengths.

The normalized DP, obtained as a result of experiments under the indicated conditions, are shown in figures 3, 4, 5 and 6, where the target azimuth is chosen equal to 180°, and the elevation angle is 0°. In the foreground of each of the figures there is a line corresponding to an elevation angle of + 90°, in the back -90°.

From the analysis of the above figures, it follows that changing the array step does not affect the directional pattern width, which turned out to be equal to 2° for the chosen size of the array aperture of 12.5 wavelengths. An exception is the array step equal to 0.5 wavelengths, which is used as the maximum possible in linear and flat arrays. With such an array spacing, shown in figure 5, the radiation pattern has two identical main lobes, one is in the selected direction, the other is in the opposite azimuth, the latter being cut into two parts for 0° and 359°. It should be noted that the width of the side lobes in azimuth increases with an increase in the elevation angle magnitude, however, the height of the side lobes remains, on average, the same as at small elevation angles.

Since the randomness of the array elements’ arrangement should affect the DP maximum side lobe magnitude, it is of great interest to calculate its mean value and the standard deviation value (SDV). Knowing the SDV will make it possible to assess the possibility of reducing the maximum side lobe by 3 levels by enumerating all possible transceiver locations and finding the best one.

The average maximum side lobe value (MSL) of the normalized directional pattern and its standard deviation for different array steps, except for 0.5, are shown in table 1.

For further research, we will choose an array step equal to 0.45.

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**Figure 3.** 3D DP of 128 elements’ array with an array step of 0.4λ and an elevation angle of 0°

**Figure 4.** 3D DP of 128 elements’ array with array step of 0.45λ and an elevation angle of 0°
Table 1

| Results          | Array step |
|------------------|------------|
|                  | 0.4        | 0.45       | 0.55       |
| Average DP MSL   | 0.295      | 0.295      | 0.295      |
| DP MSL SDV       | 0.018      | 0.019      | 0.015      |

Without changing the DP width in azimuth and elevation at the half power level and the array step, we will restrict ourselves to changing the number of transceivers from 64 to 1024, doubling this number each time.

Directional patterns obtained as a result of experiments with a different number of transceivers located at random with given placement parameters are shown in figures 7, 8, 9, 10 and 11.

The average value of the normalized directional pattern maximum side lobe and its standard deviation for a different number of transceivers are given in table 2.

It follows from the table that for each double increase in the number of transceivers placed randomly inside a sphere of constant radius equal to $12.5\lambda$, which provides a beam width of $2^\circ$, the level of the directional pattern side lobes decreases by 1.4 times. The value of the standard deviation of the directional pattern maximum side lobe changes differently with each increase in the number of array elements by a factor of 2, in contrast to its mean value, for which all changes are the same. So, with an increase in the number of elements from 64 to 128, the maximum side lobe SDV decreases by 1.2 times, from 128 to 256 and from 256 to 512 - by 1.45 times, and from 512 to 1024, the SDV decreases by 1.5 times. ...

It follows that, starting with 256 array elements, the SDV decreases by about 1.5 times and, therefore, the maximum side lobe average level for 2048 transceivers will be 0.076, and the SDV is 0.004.
Comparing the DP of arrays with the same parameters, shown in figures 4 and 8, we note that the existing difference in the shape and size of the DP side lobes is due to the difference in the placement of the transceivers inside the sphere.

Let us now consider the effect of changes in the elevation level on the directional pattern. Since the sphere is symmetric, and the directional pattern width is in two planes, the direction of the intersection line of which coincides with the direction to the DP maximum, is the same, then for any angular position of the beam in the initial coordinate system, depending on the azimuth and elevation angle, the DP width in both planes in the local the coordinate system, one axis of which coincides with the signal arrival direction and, accordingly, with the directional pattern, will not change, since the spherical antenna aperture in these planes is the same.
Table 2

| Results          | Number of elements |
|------------------|--------------------|
|                  | 64     | 128    | 256    | 512    | 1024   |
| Average DP MSL   | 0.413  | 0.295  | 0.21   | 0.149  | 0.106  |
| DP MSL SDV       | 0.023  | 0.019  | 0.013  | 0.009  | 0.006  |

In the original coordinate system, when the elevation angle changes, the end of a vector of the same length will be on a circle with a smaller radius than when the elevation angle is 0. So, with an elevation angle of 60 degrees, the circle radius will be half of the maximum. In the limit with an elevation angle of 90 degrees, the circle radius is 0 and the azimuth is not defined, since the vector projection onto the horizontal plane is a point. When the circle radius is halved, the angle at which a part of the perimeter of the same length circle is visible will double. Hence it follows that the directional pattern width in azimuth at an elevation angle of 60 degrees should be twice as wide as the directional pattern at an elevation angle equal to zero. These considerations are illustrated in figure 12.

![Figure 11](image)

Figure 11. 3D DP of 1024 elements’ array with an array step of $0.45\lambda$, and an elevation angle of 0°

Since the circle radius described by the end of the fixed length vector when the elevation angle changes remains constant, the width of the directional pattern in elevation does not change.

The study of the directional pattern dependence on elevation angle changes will be carried out with the DP width in azimuth and elevation at a half power level equal to 2 degrees. The array step will take the previous value, equal to 0.45 wavelengths, and the number of transmit-receive elements will remain equal to 128. The azimuth will be fixed at a value equal to 180 degrees. The elevation angle will take on different values of 0, 60, 85, 89 degrees. The results of the DP calculation are shown in figures 13, 14, 15, 16.

From the analysis of the directional pattern cross-sections shown in figures 17 and 18, which correspond to the 3D diagrams shown in figures 13 and 14, respectively, it follows that with an increase in the elevation angle from 0 to 60 degrees, when the length of the projection of the vector of the directional pattern maximum direction onto the horizontal plane decreases by 2 times, the DP width in azimuth increases from 2.5 to 5.5 degrees, which practically confirms the situation shown in figure 12.

The diagram width is calculated at the 0.707 voltage level, which corresponds to the 0.5 power level. The slight expansion of the experimental diagram in comparison with the theoretical one is due to the fact that the real size of the non-equidistant lobe is somewhat less than 12.5 due to the random
arrangement of the lobe elements and insufficient sphere filling density. With an increase in the number of elements, the discrepancy between the directional pattern theoretical and experimental width will decrease.

![Figure 12](image1.png)

**Figure 12.** Demonstration of the directional pattern width change in azimuth with an elevation angle increase from 0 to 60°

![Figure 13](image2.png)

**Figure 13.** 3D DP of 128 elements' array with an array step of 0.45 and an elevation angle of 0°

![Figure 14](image3.png)

**Figure 14.** 3D DP of 128 elements' array with an array step of 0.45λ and an elevation angle of 0°

![Figure 15](image4.png)

**Figure 15.** 3D DP of 128 elements' array with an array step of 0.45λ and an elevation angle of 85°

As for the further change in the elevation angle, leading to the next expected two-fold increase in the projection of the maximum directional pattern direction vector on the horizontal plane, it corresponds to an elevation angle equal to 85 degrees. The DP corresponding to this elevation angle is shown in figure 18, and its azimuth cross-section in the main lobe area is shown in figure. 19. From the last figure it follows that the directional pattern width with the indicated parameters at an elevation angle of 85°
degrees is 33 degrees, i.e. when the elevation angle is changed from 60 to 85 degrees, the DP width increases from 5.5 to 33 degrees, which is 3 times more than expected.

From this it follows that the considerations for the DP broadening with a change in the elevation angle shown in Figure 12 are valid only for elevation angles less than 60 degrees. At large angles, the great-circle arc from Figure 12 becomes a chord for the small-circle arc. The greater the arc length is greater than the chord length, the greater the DP azimuth expansion will be greater than expected. At an elevation angle of 89°, two intersecting circles of the same radius are formed on the sphere surface, which are visible from the sphere center at the same angle of 2°. The circle center formed by the main lobe of the pattern relative to the center of the array is located on the azimuthal circle and vice versa. The points of intersection of these circles cut off the arc from the azimuthal circle, which corresponds to the DP main lobe width in azimuth, equal to 120°. Starting from an elevation angle of 89.5°, the...
 azimuth circle is completely inside the circle formed by the DP main lobe, which width in azimuth in the original coordinate system becomes 360 °.

The average value of the normalized DP maximum side lobe and its standard deviation for different elevation angle values are given in table 3.

| Elevation angle | Results                   |
|-----------------|---------------------------|
| 0               | Average DP MSL | 0.295   |
| 60              | 0.297           |
| 85              | 0.296           |
| 89              | 0.582           |
| DP MSL SDV      | 0.018           |
|                 | 0.017           |
|                 | 0.017           |
|                 | 0.097           |

With an increase in the elevation angle of more than 60 °, the DP main lobe begins to expand sharply in azimuth, however, the average level of the maximum side lobe and its SDV, as shown in table 3, do not change up to an angle of 89 °. As shown in figure 16, the DP at this elevation angle has a regular side lobe with a width of 360 ° in azimuth, the level of which is significantly higher than the level of the other random side lobes. With an increase in the elevation angle to 90 °, this side lobe grows and turns into the main one.

Conclusion

The presence of sufficiently large side lobes in the directional pattern of a three-dimensional non-equidistant antenna array requires the obligatory use of adaptive compensators for active interference [8, 9, 10]. The absence of movement of the three-dimensional array elements' movement facilitates the formation of the directional pattern zeros in the direction of the jammers.

Unfortunately, bringing the level of the side lobes of the three-dimensional array DP to acceptable values requires a sufficiently large number of transmitting and receiving array elements and a long search for their best placement. This disadvantage is compensated to a certain extent by the possibility of circular electronic scanning of space, which can be either sequential (single-beam) scanning in elevation and azimuth, and parallel (multi-beam) in azimuth at fixed elevation angles.

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