Automatic LPI Radar Signal Sensing Method Using Visibility Graphs

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ABSTRACT The issue of the low probability of intercept (LPI) radar signal sensing has received considerable attention. Furthermore, the development of military technology further increased demand for it in future electronic warfare (EW). Meanwhile, so far, a systematic understanding of how radar signal detection and recognition technology contributes to EW is still lacking. Therefore, this study aims to contribute to this growing area of research by exploring an automatic method for detecting and identifying radar signals based on visibility graphs (VG), which can extract more network and feature information in the two-dimensional space of VG. In this article, the signal to be measured is subjected to wavelet noise reduction. Secondly, auto-correlation processing is performed on the signal, we subsequently convert the signal into a VG complex network. Then the average degree and weighted operation of the network are used for signal detection and recognition, respectively. Experiments in the last part indicate that the proposed method provides excellent performance, such as the robustness to noise and the probability of classification, over several state-of-the-art algorithms.

INDEX TERMS Visibility graphs, LPI radar, signal detection, signal recognition, machine learning.

I. INTRODUCTION

Low probability of intercept (LPI) [1] radar signal detection and recognition technology has been a research focus in recent years. Signal detection is the premise of radar reconnaissance system, which is also the basic content of electronic warfare (EW) and electronic intelligence (ELINT). Meanwhile, LPI radar waveform recognition has been an important direction of EW research since 1980 [2]. LPI radar waveform recognition based on the detected signal will provide a presence of threat and radar working mode such that necessary measures or countermeasures against enemy radars can be taken by electronic support (ES) and electronic attack (EA) systems. LPI radars have very low peak power, wide spectrum, high duty cycle, and low SNR, which makes it difficult for EW receivers to detect and identify them [3]. Therefore, the research on LPI radar signal detection and recognition is of great significance for future EW.

Signal detection is an enduring topic in the study of the presence or absence of LPI radar signals embedded within the random noise. To address this issue, reference [4] introduced a two-dimensional representation of signals using Wigner-Ville distribution (WVD), noise reduction, feature extraction, and LPI radar signal detection is performed using radial basis neural networks. When the signal-to-noise ratio (SNR) is -5dB and the false alarm probability is 10%, the successful detection probability is 60%. Different filters and signal detection design strategies for LPI radar signals are introduced in [5], and the traditional FFT algorithm is used for comparison. The performance gain is above 8dB, and the average detection probability is 80% at an SNR of -8dB. In [6], a semi-blind method based on principal component analysis and sequence synchronization is proposed. Simulation experiments verify that the detection method performs better than energy detection under low SNRs. In [7], a periodic Wigner-Ville Hough transform method is proposed, which has higher detection and estimation probability than Wigner-Ville Hough transform in low SNRs, but only for linear frequency modulation continuous wave (LFMCW). Reference [8] propose a time-frequency representation method based on Choi-Williams distribution (CWD). The radial integration method based on integral rotation factor was used to detect LPI radar signals in time-frequency images, the correct detection probability is 86% with a SNR of -6dB.

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An approach for detecting polyphase code signals based on fractional Fourier transform (FrFT) is illustrated in [9], specifically, searching for the main peak energy of the FrFT spectrum, and verifying the correctness of the algorithm based on the existing prior energy. When the SNR is -5dB, the average detection probability is 60%, but the algorithm exists only for chirp signals. A frequency-domain detection method based on fast Fourier transform and piecewise autocorrelation function is addressed in [10]. The results show that when the SNR is -8dB, the average detection probability of eight signals is above 90%.

In recent years, there has been an increasing amount of literature on recognition methods of LPI radar signals. Most of the methods use time-frequency analysis to preprocess the signals. The purpose is to be able to recognize two-dimensional images obtained by time-frequency transformation with machine learning algorithms [11]. A radar waveform recognition technology with WVD and CWD transformations on LPI radar signals is stated in [12], the authors extract corresponding features from the images, and perform supervised classification with neural networks. When the SNR is 0dB, the average recognition probability is 80%. In [13], an LPI radar signal recognition technology based on Elman neural network (ENN) is proposed. By extracting features from CWD images and using principal component analysis (PCA) algorithm to extract main features, the recognition probability of 8 kinds of LPI radar signals 90% at -2dB. In [14], a classification of LPI radar signals is carried out by using a convolutional neural network (CNN). Similarly, CWD technology was used to perform time-frequency analysis on 8 types of LPI radar signals, and a series of image processing operations were performed. By using CNN, the overall recognition probability is 92% at an SNR of -6dB. In [15], automatic LPI radar signal recognition technology based on spectral correlation density (SCD) is elaborated. The obtained signal SCD is performed to classify the signal features using a support vector machine (SVM), and the classification performance is 78% at a SNR of -2dB. In [16], LPI radar waveforms recognition technology based on a single-shot multi-box detector (SSD) and a supplementary classifier is proposed, which also effectively improved the recognition probability. Reference [17] propose a method to estimate the signal in noise using WVD and then use time and frequency domain technology to extract features and classify the obtained parameters by FrFT, which is also improved the classification performance of LPI radar signals.

In addition, the study of visibility graphs (VG) is also one of the areas of interest in recent years. The purpose of VG and horizontal visibility graphs (HVG) methods is to convert time series into complex networks [18]–[19]. A method to quickly convert time series to VG is proposed in [20]. Reference [21] introduce VG theory into the analysis of wall turbulence time series, which provided strong support for accurate time series analysis of uneven turbulence. In [22], a weighted complex network method based on VG is addressed, which assigns a weight to each neighboring edge, and then performs signal detection and recognition processing on the obtained weights.

There are already many previous works in LPI radar signal detection and recognition, however, few EW users have been able to draw on any systematic research into VG. This article proposes an LPI radar signal processing method based on VG theory. First, the signal intercepted by the electronic warfare receiver is subjected to wavelet noise reduction preprocessing, and its autocorrelation is calculated. Then convert the signal to be tested into a VG complex network. Then, Use the average degree of the network, the detection threshold is calculated by setting the false alarm probability value of the pure AWGN noise signal, and the real signal is detected. Furthermore, the VG network is weighted and the network is feature extracted. Support vector machine (SVM) and k-NearestNeighbor (kNN) [27] algorithms are used to classify the extracted features, including 18 types of LPI radar signal intra-pulse modulation types, which are MP, LFM, BPSK, QPSK, FSK, V, LFM-BPSK, LFM-QPSK, Barker, Frank, P1, P2, P3, P4.

The main contribution of this article is to provide a high-precision and adaptive automatic LPI radar signal detection method. When the SNR is -10dB, the signal detection probability can be close to 100%. Also, the algorithm is simple to implement and can be used in real-time signal processing. Moreover, an LPI radar signal recognition method using statistical pattern recognition introduced in this article, which has a high recognition probability. Also, the algorithm is easier to implement than the state-of-the-art deep learning algorithms such as CNN.

The rest of this article is organized as follows. Section II describes the related work of LPI radar waveform, including wavelet preprocessing, VG, and the VG/HVG representation of their autocorrelation analysis. The LPI radar signal detection and recognition are discussed in Section III. Section IV briefly describes the validate of this article, including the simulation experiment setup, results, and discussion. Section V finalizes the paper with the concluding remarks with notes on the direction of future research.

II. RELATED WORK

A. LPI RADAR SIGNALS

In this section, a brief introduction of the 18 LPI radar signals is provided, which are summarized into a table for explanation. Then we briefly introduce VG and HVG, including the representation of LPI radar signals with respect to them.

Assume that the radar pulse signal observation equation is

$$y[k] = x[k] + n[k] = A \exp \left[ j \left( 2\pi f_c[k] (kT_s) + \phi[k] \right) \right] + n[k]$$

(1)

where $y[k]$ represents the discrete-time signal intercepted by the EW receiver, $x[k]$ is the discrete-time complex radar signal, $n[k]$ is the complex additive white Gaussian noise (AWGN), $A$ is the complex amplitude which is constant within a pulse width, $f_c[k]$ is the carrier frequency, $k$ is the
TABLE 1. Radar signal modulation type.

| Modulation type | \( f_c[k] \) [Hz] | \( \varphi[k] \) [rad] |
|-----------------|--------------------|---------------------|
| MF2             | constant           | constant1           |
| LFM             | \( f_0 + \frac{Mf_c}{\tau_{pw}} (kT_s) \) | constant |
| BPSK            | constant           | \((0, \pi)\)        |
| QPSK            | constant           | \((0, \frac{\pi}{2}, \frac{3\pi}{2})\) |
| FSK             | \( \{ f_1, f_2, \ldots, f_{\text{NF}} \} \) | constant |
| V               | \( \left\{ \begin{array}{ll} f_0 + \frac{Mf_c}{\tau_{pw}} (kT_s), & 0 \leq k \leq \frac{\tau_{pw}}{2} \\ f_0 + \frac{Mf_c}{\tau_{pw}} (kT_s), & \frac{\tau_{pw}}{2} \leq k \leq \tau_{pw} \end{array} \right. \) | \((0, \pi)\) |
| LFM-BPSK        | \( f_0 + \frac{Mf_c}{\tau_{pw}} (kT_s) \) | \((0, \pi)\) |
| LFM-QPSK        | \( f_0 + \frac{Mf_c}{\tau_{pw}} (kT_s) \) | \((0, \frac{\pi}{2}, \frac{3\pi}{2})\) |
| Barker          | constant           | \(+1, +j, -1, +j, +i\) |
| Frank           | constant           | \( \frac{2M}{N_c} (i-1)(j-1) \) |
| PL              | constant           | \(- \frac{2M}{N_c} (2j - 1)(2i - 1)M + (i-1)j \) |
| P2              | constant           | \(- \frac{2M}{N_c} (2i - 1)(2j - 1)M \) |
| P3              | constant           | \(- \pi(i-1)^2/N_c \) |
| P4              | constant           | \( \pi(i-1)^2/N_c - \pi(i-1) \) |
| T1              | constant           | \( \text{mod} \left\{ \frac{2\pi m}{n} \right\} \) |
| T2              | constant           | \( \text{mod} \left\{ \frac{2\pi m}{n} \right\} \) |
| T3              | constant           | \( \text{mod} \left\{ \frac{2\pi m}{n} \right\} \) |
| T4              | constant           | \( \text{mod} \left\{ \frac{2\pi m}{n} \right\} \) |

TABLE 2. Sequential VG/HVG motifs of order 3.

| Motif | M0 | M1 |
|-------|----|----|
| VG    | \( (y_k, y_i, y_j) \) | \( (y_k, y_i, y_j) \) |
| HVG   | \( (y_k, y_i, y_j) \) | \( (y_k, y_i, y_j) \) |
| M0    | \( \text{Modulo} \left\{ \frac{2\pi m}{n} \right\} \) | \( \text{Modulo} \left\{ \frac{2\pi m}{n} \right\} \) |
| M1    | \( \text{Modulo} \left\{ \frac{2\pi m}{n} \right\} \) | \( \text{Modulo} \left\{ \frac{2\pi m}{n} \right\} \) |

In practice, LPI radar signals are generally classified into three types: frequency modulation, phase modulation, and combined modulation. When the radar signal type is frequency modulation \( f_c[k] \), the phase portion \( \varphi[k] \) is constant; when the radar signal type is phase modulation \( \varphi[k] \), the frequency portion \( f_c[k] \) is constant [5], and the third one is the combination of frequency modulation and phase modulation. The 18 modulation types are shown in Table 1, including simple pulses, frequency modulation, phase modulation, and combined modulation.

In the table, \( f_s \) is the sampling frequency, \( f_0 \) is the signal frequency, \( f_0 \) is the code rate, \( N \) is the number of symbols, \( \tau_{pw} \) is the duration, \( B \) is the signal width of the LFM, and \( k \) is the frequency modulation slope. In polyphase codes, \( M \) is the number of steps and \( N_c = M^2 \) is the sequence length or compression ratio. In multi-time codes, \( m \) is the number of segments of the symbol sequence, \( j = 0, 1, 2, \ldots, k - 1 \) is the number of segments of the step frequency waveform, \( n \) is the number of phase states of the symbol sequence, \( t_m \) is the modulation period, and \( \Delta F \) is the modulation bandwidth.

B. VG AND HVG

A VG is an undirected graph for a time-ordered sequence \( S = \{y_1, y_2, \ldots, y_n\} \). Assume that three arbitrary nodes \( i, j, k \) are labeled corresponding to data \( y_i, y_j \) and \( y_k \), respectively. The two nodes \( i \) and \( j \) (assuming \( i < k < j \) without loss of generality) are connected, if and only if one straight line can connect \( y_i \) and \( y_j \), without any intersecting intermediate data \( y_k \). Then there is

\[
y_k < y_i + \frac{k-i}{j-i} (y_j - y_i), \quad \forall k : i < k < j
\]  

When adjacent edges are connected, there are only two connection situations: M0 and M1 between any three points. VG and HVG meet three criteria: 1) each vertex connecting to at least the adjacent vertex, 2) all vertices are undirected, it can be summarized as shown in Table 2. Consider the case of three vertices, and so on for multiple vertices, and 3) change the horizontal or vertical scale of all vertices at the same time, and the connection mode is unchanged [24].

The traditional power spectrum algorithm, that is, performing fast Fourier transform (FFT) on the autocorrelation signal, this method cannot be used for phase modulation signals, because it cannot distinguish the bandwidth itself. However, VG and HVG are noticeably different, they have the advantage of being able to describe subtle features.
After converting the auto-correlated signal into VG or HVG, the VG representation of the autocorrelation analysis is shown in Fig. 1, for the 18 LPI radar waveforms considered in this article.

It is found that all VG images have no values on the main diagonal and have values on the second diagonal. That is the result of a constant connection between adjacent nodes in the VG algorithm, but without connecting with itself. Furthermore, we can see that different LPI radar waveforms have different VG representations, which can be utilized for signal recognition.

1) MONOPULSE (MP)
MP signal is relatively simple, whose frequency and phase portion are constant, rarely applied to the radar application. The VG representation of the autocorrelation analysis for MP is shown in Fig. 1 (a).

2) LFM & V
The instantaneous frequency of the LFM signal can be expressed as \( f_0 + st \), where, \( s = B/\tau_{pw} \) is the chirp rate. If \( s > 0 \), the instantaneous frequency \( f_c \) increases linearly, otherwise the instantaneous frequency \( f_c \) decreases. Thus, for a given time-width of the LFM signal, the distance resolution increases with bandwidth. That is, the LFM signal has greater improvement in distance resolution than the MP signal. The VG representation of the autocorrelation analysis for LFM is shown in Fig. 1 (b). V is a combination of two LFM signals. In \( (0, \frac{\tau_{pw}}{2}) \), the chirp rate is \( s \), and the chirp rate is \( -s \) in \( \left[ \frac{\tau_{pw}}{2}, \tau_{pw} \right] \). The VG representation of the autocorrelation analysis for V is shown in Fig. 1 (f).

3) BINARY PHASE SHIFT KEYING (BPSK) AND QUADRATURE PHASE SHIFT KEYING (QPSK)
Phase encoding signals are different from frequency-modulated signals, which divides wide pulses into several short pulses connected. Each short pulse has the same width and the same frequency. The only difference is the phase of the carrier frequency, and there is a certain relationship between the phases. The phase \( \phi[k] \) of BPSK is 0 or \( \pi \), and the number of 0 or \( \pi \) is stochastic within a symbol width. Similarly, the QPSK phase \( \phi[k] \) is \( \{ -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, -\frac{3\pi}{4} \} \), and there is stochastic in number of the phase within a symbol width. The VG representation of the autocorrelation analysis for BPSK / QPSK is shown in Fig. 1 (c) (d), respectively.

4) LFM & BPSK/QPSK
The combined modulation radar signal discussed in this article is a combination of LFM and BPSK / QPSK modulation technology. This type of signal has higher processing gain, increased LPI characteristics, and lower interception probability for non-collaborator. The frequency term of these signals meets the conditions of LFM, and the phase also meets the conditions of BPSK / QPSK. Not accidentally, there are FSK / PSK hybrid technologies, and the VG representations of the autocorrelation analysis for LFM / BPSK and LFM / QPSK are shown in Fig. 1 (g) and (h), respectively.

5) FSK
The FSK signal frequency \( f_i \) of the LPI radar transmitter is selected from the FM series \( \{ f_1, f_2, \ldots, f_N \} \). Its advantages are simple structure and distance resolution that does not depend on the FM rate, which is suitable for tracking and large wideband signal processing. The performance of FM radar depends on precoding, which greatly increases the intercept probability, for example, Costas coded signal [1]. The VG the autocorrelation analysis for of FSK is shown in Fig. 1 (e).

6) POLYPHASE CODE
A multivariate pseudo-random sequence is called a polyphase coded signal, which is obtained by phase modulating a
continuous carrier. The number of symbols in a sequence is determined by the designer. As the number of symbols increases, the interception receiver has a lower probability of interception. The multi-code signals considered in this article include Barker, Frank, P1, P2, P3, and P4. Their VG the autocorrelation analysis for correspond to (i)-(n) in Fig. 1. Because the non-cooperative interception receiver does not know the number of Barker codes, this situation demonstrates how collaborators used innovative military strategies in Barker code design. Frank coding can be seen as an approximation to the LFM waveform, using step frequencies. The P1 code can be also approximated by the LFM waveform ladder. The P2 code changes the initial phase based on the P1 code. The P3 code is a synchronous oscillator at the end of the frequency sweep. The imaginary and real parts are sampled at the Nyquist sampling rate. The P4 code consists of discrete phases of the LFM waveform at specific time intervals.

7) MULTI-TIME CODE
Unequally to the multi-code, the phase state of the multi-time code can be adjusted by the user, that is, the time occupied by each phase state changes during the duration of the entire waveform. The multi-time codes involved in this article include T1, T2, T3, T4, and their VG the autocorrelation analysis for correspond to (o)-(r) in Fig. 1. Multi-time code is similar to the step frequency and chirp signal, but it does not have the same characteristics. The multi-time code will improve the sidelobe performance with the increase of the number of phase states, and different phase states can generate arbitrary time-bandwidth waveforms.

The overall LPI radar signal detection and identification technology proposed in this article is illustrated in Algorithm I. Where the SDP is the successful detection probability, and the SRP is the successful recognition probability.

Algorithm I LPI Radar Signal Detection and Recognition Algorithm

Input: Signal to be tested through the AWGN channel
Output: SDP, SRP
1: Given LPI radar parameters including sampling frequency \( f_s \), signal frequency \( f_0 \), code rate \( f_b \), the number of symbols \( N \), \( \{f_1, f_2, \ldots, f_N\} \) of FSK, various parameter settings including polyphase code and multi-time code
2: Wavelet denoising for the signal to be tested through the AWGN channel
3: Perform Autocorrelation on the obtained signal to be tested
4: Set the threshold based on Neyman-Pearson Criterion and perform the detection operation
5: Converted the detected real signal into VG or HVG
6: Weighted for VG or HVG network
7: Perform feature extraction on weighted complex networks
8: Classify by features using SVM and KNN algorithms

C. WAVELET PREPROCESSING
The advantage of wavelet transform is that the original signal to be measured is refined by transforming, and the signal is characterized. It fulfills localized and multi-scale analysis of the signal in the wavelet domain. The wavelet transform method shows good performance in denoising processing [25]. LPI radar signals generally have low SNR characteristics, so reasonable noise reduction techniques are one of the key technical points for signal detection and recognition. This article proposes a signal detection and recognition technology based on wavelet denoising. The function \( f(n) \) in any space is expanded under the wavelet basis, which is called wavelet transform, and the discrete wavelet transform is defined as [28]

\[
W_f(j, k) = 2^{\frac{j}{2}} \sum_{n=0}^{N-1} f(n) \varphi \left( 2^{-j} n - k \right)
\]

The wavelet transform is an integral transform, \( W_f(j, k) \) is called the wavelet transform coefficient, \( N \) is the number of sampling points, \( j \) is the number of decomposition layers, and the wavelet base includes two parameters of the decomposition layer \( j \) and translation \( k \), which is also very different from the Fourier transform. The construction of wavelet base \( \varphi(n) \) needs to satisfy: 1) it contains only local non-zero domains, and the value of the function outside the window is 0; 2) it does not contain a direct current trend component, that is, \( \varphi(0) = \frac{1}{N} \sum_{n=-\infty}^{+\infty} f(n) = 0 \); and 3) there is a band-pass property, that is, as the parameter \( j \) of the decomposition layer increases, the time window extends, the bandwidth shrinks, the bandwidth narrows, the center frequency decreases, and the frequency resolution becomes higher. If \( j \) decreases, the opposite is true.

D. WEIGHTED COMPLEX NETWORK
According to [30], the weighting method for defining undirected complex networks needs to satisfy the following properties. 1) Vertices cannot connect to themselves, 2) There is at most one edge between two vertices, 3) Each edge is undirected. When there is a connection relationship between two vertices, the weighting algorithm uses the following calculation method

\[
w_{ij} = \arctan \frac{n_j - n_i}{t_j - t_i}, \quad j > i
\]  

where \( w_{ij} \) is the weighted value of the joint node \( n_i \) and the node \( n_j \), and the weighted complex network algorithm is illustrated by taking three nodes as an example. Considering that the three nodes 1, 2, and 3 respectively correspond to values of 10, 15, and 25, then \( w_{12} = \arctan \frac{15-10}{2-1} = 1.3734 \), \( w_{13} = \arctan \frac{25-10}{3-1} = 1.4382 \), \( w_{23} = \arctan \frac{25-15}{2-1} = 1.4711 \). The weighting algorithm is simple to implement, and the size of the weight is only related to the distance between the two points and the vertex value. The discriminativeness of the weights obtained in the interval \([-\pi/2, \pi/2]\) is better than other weighting algorithms [22], [31].
III. LPI RADAR SIGNAL DETECTION AND RECOGNITION

A. LPI RADAR SIGNAL DETECTION

Signal detection is to detect the existence of the signal, the aim of automatic signal detection is to analyze the modulated LPI radar signals embedded within the AWGN [23]. A signal detection method based on VG is proposed in this article, which converts the signal to be measured after preprocessing technology into VG, and then performs the optimal decision of binary detection based on the statistical characteristics of VG. Assume that the discrete-time LPI radar signal can be expressed as shown in formula (1), in the case of binary detection, \( H_0 \) and \( H_1 \) can be expressed as [29]

\[
H_0 : r[n] = s_{0k}, \quad k = 1, 2, \ldots, N
\]

\[
H_1 : r[n] = s_{1k}, \quad k = 1, 2, \ldots, N
\]

(6)

In the algorithm design process of this article, the average degree of VG is used as the detection basis. Among them, \( s_{0k} \) is the average degree of VG of AWGN, and the decision \( H_0 \) is valid. The \( n_k \) noise itself obeys the \( N(0, \sigma^2_n) \) distribution, \( s_k \) is the general expression form of the radar signal, \( s_{1k} \) represents the VG average degree of the signal \( s_k \) after passing the AWGN channel, and the decision \( H_1 \) is valid.

When the signal length is \( N \), the expression of the average degree can be written as

\[
A_{d} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij}
\]

(7)

Among them, \( d_{ij} \) is the VG degree of the \( i \)th row and the \( j \)th column, and the value is 0 or 1. Then \( H_0 \) and \( H_1 \) can be rewritten as

\[
H_0 : r[n] = ad(w[k]), \quad k = 1, 2, \ldots, N
\]

\[
H_1 : r[n] = ad(s_k + w[k]), \quad k = 1, 2, \ldots, N
\]

(8)

Likelihood ratio function is

\[
\Lambda(r; s_k) = \frac{p(r; s_k | H_1)}{p(r | H_0)}
\]

(9)

where \( p(r; s_k | H_1) \) is the probability density function of \( r[n] \) under \( H_1 \), which depends on the unknown parameter \( s_k \), and \( p(r; s_k) \) is the probability density function of \( r[n] \) under \( H_1 \).

The generalized likelihood ratio test can be expressed as

\[
\Lambda(r; \tilde{s}_k) = \frac{p(r; \tilde{s}_k | H_1)}{p(r | H_0)} > \eta
\]

(10)

where \( \eta \) is the detection threshold. In multiple stochastic trials, it is well-known that the average value of the VG average degree of the pure AWGN signal does not change with the SNR, therefore, the detection threshold is set based on the size of the false alarm probability \( p_{fa} \).

B. LPI RADAR SIGNAL RECOGNITION

Since the modulation recognition of radar signals gives information about the type of the pulse compression technique used by the LPI radar and radar working mode, such as tracking, strike, etc. Hence, it plays a significant role in the design of the EW receiver.

1) FEATURE EXTRACTION

Average Weighted Degree

Consider the average weighted degree of a complex network as one of the best statistical attributes, cause it reasonably as the most direct and most reflective factor of network complexity. Average weighted degree can be expressed as

\[
A_w = \sum_{j \in B(i)} w_{ij}
\]

where \( B(i) \) is the field of node \( i \). The average weighted degree is the average of the total weights of all vertex connections in the network.

Average Clustering Coefficient

The average clustering coefficient reflects the clustering of the connection nodes, and its definition [26] can be written as

\[
C_i = \frac{\left( A^2 \right)_{ij} - 1}{\left( A^2 \right)_{ii} - 1}
\]

(12)

where \( C_i \) is the clustering coefficient of node \( i \), \( A \) is the VG or HVG matrix, and \( \bar{C} \) is the average clustering coefficient, respectively. This coefficient is directly processed by the network as a whole, therefore it is universal.

2) CLASSIFIER

k-NearestNeighbor (kNN)

kNN is a commonly used supervised learning algorithm [27]. The basic principle is that for a given test sample set, find the \( k \) training samples closest to it in the training set based on a certain distance metric, and then make predictions based on the information of the \( k \) nearest neighbors. kNN is generally used for pattern recognition and classification.

During the classification process, the average value of the most frequently occurring tag categories in the \( k \) training samples can be selected as the prediction result. A weighting algorithm can also be selected to determine the weight according to the distance. Generally, the closer the distance, the greater the weight.

It can be found from Fig. 2, that different \( k \) values have different discrimination results. When \( k = 1 \), it is discriminated as a square; when \( k = 3 \), it is discriminated as a star; when \( k = 5 \), it is discriminated as a square again. Therefore, for the kNN algorithm, choosing the appropriate \( k \) value has a great influence on the classification performance. If the value
of $k$ is too small, the influence of the noise component will have a relatively large impact on the classification result. For example, when the value of $k$ is 1, if the nearest point is a noise signal, a misjudgment will occur. If the value of $k$ decreases, the overall model will become more complicated and prone to overfitting. If the value of $k$ is too large, that is, the training examples in the neighborhood with large test points are used for prediction, in this way, instances that are far away from the input target point will also affect the prediction point, making the prediction result wrong. An increase in the value of $k$ means that the overall model is relatively simple. If $k$ is the total number of samples, all instances are obtained, for sake of taking the most points in a category of the instance, which has no practical significance for prediction.

Regarding the selection of the distance formula, Euclidean distance and Manhattan distance, etc. can be chosen. Suppose there are two points $(x_1, y_1)$ and $(x_2, y_2)$, the Euclidean distance is defined as $d = \sqrt{(x_1 - y_1)^2 + (x_2 + y_2)^2}$, and the Manhattan distance is defined as $d = |x_1 - x_2| + |y_1 - y_2|$.

### Algorithm 2 $k$NN Algorithm

**Input:** Training set $D = \{(x_1, y_1), (x_2, y_2), \cdots, (x_N, y_N)\}$, where $x_i \in X \subseteq \mathbb{R}^k$ is the feature vector of the instance, and $y_i \in Y = \{c_1, c_2, \cdots, c_k\}$ is the category of the instance.

**Output:** The instance $x$ of category $y$

1. According to the given distance metric, find $k$ points closest to $x$ in the training set $D$.
2. The field of $x$ covering these $k$ points is denoted as $N_k(x)$.
3. Determine the category $y$ of $x$ according to the classification decision rule in $N_k(x)$

$$
y = \arg \max_{i \in I} \sum_{x_i \in N_k(x)} I(y_i = c_j), i = 1, 2, \cdots, N; j = 1, 2, \cdots, K,$$

where $I$ is the indicator function, when $y_i = c_i$, $I$ is 1, otherwise it is 0.

![Figure 3. SVM sample distribution. (a)Linearly separable; (b)Linearly inseparable.](image)

### Support Vector Machine (SVM)

Support vector machine is a classifier with supervised learning. Its main function is a generalized linear classifier that classifies data. Its decision boundary is the maximum margin hyperplane that solves learning samples. As shown in Fig. 3 (a), if the feature space where the input data is located is used as the decision boundary, the hyper-plane separates the learning target by stars and squares, and the point-to-plane distance of any sample is greater than or equal to 1, the sample is said to be linearly separable. That is, for the input data $X = \{X_1, X_2, \cdots, X_N\}$ and the target sample $y = \{y_1, y_2, \cdots, y_N\}$, two conditions must be satisfied, 1) there is an interval such that $\frac{1}{\|w\|} + \frac{1}{\|w\|} = \frac{1}{\|w\|}$, then the boundary distance is $2d = \frac{1}{\|w\|^2}$ (2) Both $w^TX + b \geq 1, w^TX + b \leq -1$ are satisfied at the same time, and the two types of stars and squares on the interval boundary are supported vector.

In practical, there may not be a hyperplane in the original sample space that can correctly divide the sample into two types. As shown in Fig. 3 (b), given the limited dimension of linear partitioning, more attributes need to be added for feature partitioning. Let $\phi(x)$ be the feature vector after the input data sample $X = \{X_1, X_2, \cdots, X_N\}$ is mapped, and the hyperplane in the feature space can be expressed as

$$
f(x) = w^T \phi(x) + b$$

To obtain the maximum boundary, the following conditions need be met

$$
\min_{w, b} \frac{1}{2} \|w\|^2, \quad \text{s.t.} y_i \left(w^T \phi(x_i) + b \right) \geq 1
$$

(14)

The dual problem can be expressed as

$$
\max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j \phi(x_i)^T \phi(x_j)
$$

s.t. $\sum_{i=1}^{N} \alpha_i y_i = 0, \alpha_i \geq 0$ (15)

It is very difficult to calculate $\phi(x_i)^T \phi(x_j)$ directly, let

$$
k(x_i, x_j) = \phi(x_i)^T \phi(x_j)
$$

(16)

Therefore, formula (15) can be rewritten as

$$
\max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j k(x_i, x_j)
$$

s.t. $\sum_{i=1}^{N} \alpha_i y_i = 0, \alpha_i \geq 0$ (17)

Then we can get

$$
f(x) = w^T \phi(x) + b = \sum_{i=1}^{N} \alpha_i y_i \phi(x_i)^T \phi(x_j) + b
$$

$$
= \sum_{i=1}^{N} \alpha_i y_i k(x_i, x_j) + b
$$

(18)

where $k(x_i, x_j)$ is called the kernel function. In the actual classification task, we don’t know which kernel function performs well. However, choosing a suitable kernel function has a great impact on the classification result. If the kernel function is not selected properly, the sample space will be
mapped to an inappropriate feature space, and the classification performance will not be very well. The commonly used kernel functions have the following forms.

1) Polynomial kernel

\[ k(x_i, x_j) = (x_i^T x_j)^n \]  

2) Radial basis kernel (RBF) or Gaussian kernel

\[ k(x_i, x_j) = \exp \left(-\frac{||x_i - x_j||^2}{2\sigma^2}\right) \]

3) Linear kernel

\[ k(x_i, x_j) = x_i^T x_j \]

Standard SVM is an algorithm for solving two-class classification problems, and cannot directly deal with multi-classification problems. Through the direct and indirect methods, the standard SVM algorithm is used to construct multiple decision boundaries to realize the multi-classification of samples. Commonly used are the direct method “one-against-one” and indirect method “one-against-all”. The One-against-one SVM directly modifies the objective function, combines multiple classification parameter solutions into one optimization problem, and solves the optimization problem to achieve multi-classification of goals. This method has high computational complexity and is difficult to implement. It is suitable for classification problems with few categories. One-against-all training sequentially classifies the samples of one category into one category, and the remaining samples into another category, so that the samples of N categories construct N SVMs. The category of the sample is selected according to the category with the highest score among all the decision boundaries, there are a total of \( \frac{N(N-1)}{2} \) decision boundaries. The One-against-all SVM algorithm can calculate all decision boundaries in one iteration.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A. EXPERIMENTAL SETUP

The experimental parameters include all parameter settings for LPI radar detection and recognition. They are the sampling frequency \( f_s = 5 \text{GHz} \), the signal frequency \( f_0 = 1 \text{GHz} \), the code rate \( f_b = 2 \text{MHz} \), the number of symbols \( N = 6 \), and the duration \( \tau_{pw} = N/f_b \). The signal width \( B = \tau_{pw} \), the frequency modulation slope \( k \) is 1, and the FSK \( \{f_1, f_2, \ldots, f_6\} \) is \{1\text{GHz}, 2\text{GHz}, 1\text{GHz}, 2\text{GHz}, 1\text{GHz}, 2\text{GHz}\}. When the LPI radar is frequency modulated, the phase information is 0; when the LPI radar is phase modulated, the carrier frequency is the signal frequency \( f_0 \). In a polyphase code, the number of steps \( M \) is 8, the sequence length or compression ratio \( N_c = M^2 = 64 \), \( i \) of P1 and P2 is \( 1, 2, 3, \ldots, M, j \) is \( 1, 2, 3, \ldots, M \), and \( i \) of P3 and P4 is \( 1, 2, 3, \ldots, N_c - 1 \). In the multi-time code, the number of segments \( m \) of the symbol sequence is 4, the number of phase states \( n \) of the symbol sequence is 2, the modulation period \( \tau_m = 16 \text{ns} \), and the modulation bandwidth \( \Delta F = 1 \text{GHz} \).

B. DETECTION

For the signals disturbed by noise in the observation samples, the presence or absence of the signals is determined according to the VG average degree of the received measurement value samples. The measured signal processed by wavelet noise reduction is windowed to find its autocorrelation, and then the signal to be measured after autocorrelation is converted into a VG complex network, and the observation sample is divided by using the concept of VG average degree, 1000 times Monte Carlo independent experiments, the results are shown in Fig. 4.

![Simulation analysis of signal detection. (a) overall threshold comparison, (b) overall success detection probability.](image-url)
In the figure, when the SNR is -12dB, the detection probability is 80.3%, and as the SNR increases, the detection probability also increases, especially when the SNR> -10dB, the detection probability is basically 100%. Therefore, it is reasonable to believe that the detection algorithm is very robust to noise. In addition, the algorithm is currently only for LPI radar signals. If it is extended to acoustic signal processing or other application scenarios, it is still applicable.

C. RECOGNITION

Identify the type of Intra-pulse modulation in the detected signal. Inverse Fourier transform of the signal to obtain the autocorrelation function, convert it to VG or HVG, then weight the VG complex network, then perform feature extraction on the weighted network. Finally, machine learning algorithms are used to identify features, including SVM and $k$NN. The indexes for judging classification performance mainly include successful recognition probability (SRP) and calculation time (CT). Let $x_i, x_j$ be the $n$ dimensional feature vector of the SVM, then the CT of $k$ ($x_i, x_j$) = $\phi(x_i)^T \phi(x_j)$ is $O(n^2)$, and the calculation cost is large, but the CT after using the kernel function ($x_i^T x_j$) is only $O(n)$, which is greatly reduced. If $N$ is the number of training examples, $n$ is the number of data dimensions, and $k$ is the number of neighbors, then the CT of $k$NN is $O(Nnk)$.

As shown in Fig. 6, 18 types of LPI radar signals are characterized. Their characteristics are significantly different at low SNRs, especially when the SNR is bigger than -5dB. The two features of the average weighted degree and average clustering coefficient are average values under 1,000 Monte Carlo independent experiments. It is easy to see from Fig. 6 (a) that the combined modulation LFM-BPSK and LFM-QPSK are easy to distinguish with the naked eye, also as the polyphase code. In Fig. 6 (b), we can know that there are several types of LFM and P2 signals that can be directly filtered. In addition, all LPI radar signals are basically concentrated near 0. Therefore, when the average clustering coefficient changes greatly, it can be concluded that the received signal is not an LPI radar signal.

Given the different kernel functions of the SVM classifier have different performance in classification. Therefore, for sake of evaluating the classification performance of these feature sets, we apply additional SVMs with RBF kernel functions and SVMs with polynomial kernel functions. Similarly, considering $k$NN experiments on different values of $k$ on combined feature vector sets, $k = 3$ and $k = 10$, respectively. Their classification performance for 18 types of LPI radar signals is shown in Fig. 7 below.

Fig. 7 shows that our proposed method has higher accuracy than some previous research results when the SNR is $-5dB$ to $5dB$. Among them, we just compare the overall experimental results with those using other existing methods and properly evaluate the SRP function.

The overall recognition probability of the algorithm proposed in this article is 97.4%, which is a great improvement compared to the LPI radar waveform recognition technology proposed in the reference. Compared with image processing algorithms, statistical signal recognition methods need to manually find and extract features, which is not as superior as deep learning algorithms (DLA) to directly extract features from images. Therefore, in the experimental comparison,
TABLE 3. Comparison between proposed technique and related studies.

| Authors               | Dataset                      | Method                              | Overall SKP |
|-----------------------|------------------------------|-------------------------------------|-------------|
| Lundan J et al. [11]  | 9 kinds of LPI radar waveforms| Time-Frequency analysis + multilayer perceptron | 76.5%       |
| Zhang M et al. [12]   | 9 kinds of LPI radar waveforms| Time-Frequency analysis + Elman neural network | 93.8%       |
| Qu, Z et al. [13]     | 12 kinds of LPI radar waveforms| Time-frequency analysis + Convolutional neural network | 98.9%       |
| Vanhuy G et al. [14]  | 12 kinds of LPI radar waveforms| Spectral correlation + SVM           | 88.7%       |
| Hoang, L. M. et al. [15]| 12 kinds of LPI radar waveforms| Single shot multi-box detector + Supplementary classifier | 98.5%       |
| Proposed technique    | 15 kinds of LPI radar waveforms| VG+SVM+4NN                          | 97.4%       |

FIGURE 6. LPI radar signal characteristic characterization map. (a) average weighted degree, (b) average clustering coefficient.

Learning require a lot of work and time, while the latter method requires very little engineering work and adds minimal computational costs. Combining these two methods can achieve the best performance. Introducing VG ideas into radar signal recognition is not only an attempt, but it also provides a new idea for the signal processing field. However, VG still has certain limitations. For example, considering it is an undirected graph, the exact same signal, the amplitude of which increases or decreases linearly, and the converted VG is the same, such a signal cannot be identified.

V. CONCLUSION

This article proposes an LPI radar detection and recognition algorithm based on VG theory. The algorithm first performs signal noise reduction processing, then wavelet noise reduction is executed and use VG average degree for signal detection, converts the detected signal into a weighted complex network, and carry out feature extraction on the network. When the SNR is greater than -9dB, the detection probability is basically 100%; and when the SNR is greater than -2dB, it is almost 100%. The experimental results also show that when considering the weighted complex network theory for analysis, the average weighting degree and average clustering coefficient are the most promising features to reveal the hidden information of the LPI radar waveform.

In future research, we will extend from VG theory to the application of radar signal sorting and radar operating mode recognition. If possible, detection and recognition schemes will be considered from the VG itself, for example, performing fast Fourier transform in the graph domain.
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