Vibrational dynamics of concentrated-mass cantilevers in atomic force acoustic microscopy: Presence of modes with selective enhancement of vertical or lateral tip motion

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Abstract. Concentrated-mass (CM) cantilevers previously proposed by the author have features significantly effective for atomic force acoustic microscopy (AFAM), in which sample stiffness can be detected at nano scale by a vibrating tip. CM cantilevers improve the sensitivity of the detection and simplify the dynamics then lead to success in evaluations of the elastic modulus. The present study proposed a new type of CM cantilevers based on analyses of the vibrational dynamics taking account of previously ignored factors including the lateral contact stiffness and the inertia moment of a particle attached as a CM. A rod-like particle was attached on a tip in the new type to enhance the inertia moment in addition to the translational inertia. The first two modes behaved like one-freedom models, namely translational (vertical) and rotational (lateral) motions of the attached mass and the tip. Experiments on a sapphire wafer verified that the vertical and lateral stiffness can be simultaneously evaluated without mutual interference.

1. Introduction
A dynamic operation of atomic force microscopy (AFM), in which a tip mounted on a cantilever is vibrated in contact with a sample, has allowed the direct imaging of force gradients, namely contact stiffness, with a lateral resolution of about 10 nm or less. The implementation of AFM, known as atomic force acoustic microscopy (AFAM) [1], relies on the dependence of the resonance frequency $f$ of the cantilever on the contact stiffness $k'$. The present author previously proposed a concentrated-mass (CM) cantilever as a way of enhancing the sensitivity in $k'$-detection [2]. The vibration of a CM cantilever obeys a one-freedom model consisting of a point-mass and a spring because of sufficient inertia of the CM attached on a tip site. The sensitivity reaches to the maximum $df/dk'(k'/f) = 0.5$. The dynamics was discussed by using the continuum theory of cantilever vibration [2], where effects of the inertia moment of a CM, the lateral stiffness, the tip height and the oblique sample surface were ignored. The present study presents an analytical model of the CM cantilever vibration including these effects.

For normal cantilevers without CM the dynamics have been studied [1, 3-5]. It has been shown that the lateral component of the contact stiffness produces a moment at the tip site of a cantilever due to the presence of a tip height and then the vertical and lateral motions of a tip intensely couples to affect the resonant frequency. The present analyses for CM cantilevers were similar to those for normal cantilevers except for the boundary conditions related with the inertia and the inertia moment of a CM.
However fruitful results were drawn, namely the existence of vibration modes selectively enhancing a vertical or lateral tip motion. From experiments on sapphire wafer, we proposed a new type of CM cantilevers, by which the vertical and lateral stiffness can be simultaneously evaluated without mutual interference.

2. Theory and experiment

2.1. Theoretical model

Figure 1 shows an example of CM cantilevers, where the gold rod as a CM was adhered on the silicon cantilever. The CM cantilever was designed as a new type of enhancing inertia moment. It is assumed in the analytical model that the CM is a thin rod of mass \( m_{ad} \) and length \( l_m \). A tip of length (height) \( h \) is attached at a distance \( l_t \) from the end of the CM rod (see figure 2). The rod is connected to the end of the flexural cantilever of mass \( m_{can} \) and length \( l \). The gravitational center of the rod is at a distance \( l_{g} \) from the connected end (\( l_{g} = \frac{l_m}{2} \) for uniform rods). The sample is represented by the vertical stiffness \( k_v^{*} \) and the lateral stiffness \( k_l^{*} \). The CM cantilever is inclined with an angle of \( \theta \) and has a spring constant \( k_c \) relating the deflection at the tip site to the force acting in the tip direction.

Natural flexural vibrations of the cantilever are considered under small-amplitude. Referring to time \( t \) and \( x \) axis originating at the clamped end (\( x = 0 \)) along the cantilever, the equation of motion for the flexural part (\( 0 < x < l \)) of the cantilever is written by

\[
\frac{d^2 w}{dx^2} + \left( \frac{t^4}{3} \right) (k_v^{*} / m_{can}) \left( \frac{d^4 w}{dx^4} \right) = 0,
\]

where \( w = w(x, t) \) is the deflection and \( k_v^{*} \) is the spring constant of the flexural part, related to \( k_v \) by \( k_v^{*} = k_v \left[ 1 + 3(l_t/l) + 3(l_t/l)^2 \right] \). The boundary conditions are given by \( w = \frac{\partial w}{\partial x} = 0 \) at \( x = 0 \). The conditions for the shear force \( F = (k_v^{*} \frac{l}{3}(\frac{d^3 w}{dx^3}) \) at \( x = l \) and for the bending moment \( M = (k_v^{*} \frac{l^3}{3}(\frac{d^2 w}{dx^2}) \) at \( x = l \) are derived from the equations of translational motion and rotational motion on the gravitational center of the CM rod:

\[
F = \frac{\partial^2 w}{\partial t^2} x = \frac{l_t}{l} \text{ and } M = -[\frac{\partial^2 w}{\partial t^2} (x = l)] + J_c \frac{\partial^2 w}{\partial t^2} (x = l) \cdot \]

where \( w_n \) is the deflection at the tip site (\( x = l + l_t \)) and \( J_c \) is the moment of inertia on the gravitational center of the CM rod. The effective contact stiffness \( k_v^{*} \), etc., is defined by \( \hat{k}_v = k_v^{*} \cos^2 \theta + k_l^{*} \sin^2 \theta \), \( \hat{k}_l = k_l^{*} \cos^2 \theta + k_v^{*} \sin^2 \theta \), and \( \hat{k}_b = (k_v^{*} - k_l^{*}) \cos \theta \sin \theta \).

Substituting \( w = W(x) \sin \omega t \) into equations (1) to (3) and the other boundary conditions, we can determine the vibration mode \( W(x) \) and also obtain the following frequency equation for eigenvalue \( \lambda = (3 r^2 m_{can}/k_v^{*})^{1/4} \):

\[
C_0 + C_1 \cos \lambda \cosh \lambda + C_2 \sin \lambda \cosh \lambda + C_3 \cosh \lambda + C_4 \sin \lambda \sinh \lambda = 0,
\]
where

\[
C_0 = 1 + PQ - D_1 D_2 - P \alpha \lambda V,
C_1 = -1 + PQ + D_1 D_2 + P \alpha \lambda V
\]

\[
C_2 = P + Q - PQ \alpha \lambda \left(1 + \frac{\lambda^2}{(l_g / l)^2} + J_C \right. (m_{ad} l^2)) \right) + 2PD_1 (l_i / l) + Q \lambda^2 (l_i / l)^2
\]

\[
C_3 = P - Q + PQ \alpha \lambda \left(1 - \frac{\lambda^2}{(l_g / l)^2} + J_C \right. (m_{ad} l^2)) \right) + 2PD_1 (l_i / l) + Q \lambda^2 (l_i / l)^2
\]

\[
C_4 = 2\sqrt{PD_1 D_2} - 2PQ \lambda^2 (l_g / l) + 2Q \lambda^2 (l_i / l)
\]

\[
V = 1 - Q \alpha \lambda^3 J_C \left/(m_{ad} l^2) \right) + 2D_1 (l_i - l_g) / l + (D_1 / D_2) [J_C \right/(m_{ad} l^2) + (l_g - l_i) / l^2]
\]

\[
P = \lambda^3 / [3(k_c^* / k_{c0}) \cos^2 \theta + r \sin^2 \theta], \quad Q = \lambda^3 / [3(h/l)(k_c^* / k_{c0}) (r \cos^2 \theta + \sin^2 \theta)]
\]

\[
D_1 = (r - 1) \tan \theta / [(h/l)(r + \tan^2 \theta)], \quad D_2 = (h/l)(r - 1) \tan \theta / (1 + r \tan^2 \theta)
\]

\[
r = k_c^* / k_c^*, \quad \alpha = m_{ad} / m_{can}, \quad J_C = \xi (m_{ad} l^2 / 12)
\]

The coefficient \(\xi\) in the equation for \(J_C\) is an adjustment factor counting for effects of a non-zero diameter of the CM rod.

2.2. Experimental procedure

Experimental setup was the same as that in reference [6]. An atomic force microscope (SII Co, Ltd.) was utilized together with a lock-in amplifier and a function generator. For measurements of contact resonance, the cantilever was oscillated by means of a piezo device beneath a sample. The cantilever was also similar to that in reference [6] except for the CM, made by cutting a gold wire of \(I_{30}\) \(P_{m}\) into a length of \(l_m = 100\) \(P_{m}\). The other data of the cantilever were as follows: \(k_c = 0.86\) N/m, \(l = 187\) \(\mu m\), \(l_t = 0.55 l_m\), \(h = 24\) \(\mu m\), \(m_{ad} = 1.38\) \(\mu g\), \(\alpha = 44.9\), \(l_g = l_m / 2\), \(\theta = 14\) degree, the first resonant frequency \(f_0 = 4.9885\) kHz, and the second resonant frequency \(f_0 I = 61.5\) kHz. A laser spot for detection of the deflection angle was set at \(x = 75\) \(P_{m}\). The apex radius of the Pt/Ti-coated silicon tip was less than 35 nm. A contact force of 472 nN was applied to make the tip flat-ended [6]. The sample surface was a polished sapphire wafer [C-plane (0001)].

3. Results

3.1. Predicted relation of the resonant frequency versus the contact stiffness

Figure 3 shows variation in the \(f - k_v^*\) relation when increasing the length \((l_m)\) of an attached Au rod from \(l_m = 0\) with keeping the original length of the cantilever \(l + l_t = 242\) \(P_{m}\) and the diameter of rod. The other data were the same as those in subsection 2.2. The stiffness ratio \(r\) was assumed 0.8. The values of \([k_c 0 / m_{can}]^2 / (2\pi)\) for \(l_m = 18\) \(\mu m\) and \(\xi\) were set 48.713 kHz and 1.582, respectively, so that predicted values of \(f_0\) and \(f_0 I\) for \(l_m = 100\) \(\mu m\) agree with the experimental results. With an increase in \(l_m\) a steep curve appears like ABC in figure 3 (b), which coincides with the mass-spring model, namely

![Figure 3](image-url)
in spite of presence of the lateral stiffness. It is also interesting that another steep curve appears like DEF in figure 3 (c) with a further increase in $l_m$. Figure 4 shows the vibration modes at several points on the curves in figure 3 (c). It should be noted that the mode has a node at the tip site, at any points on the curve connecting the points A2-B2-C1-D1-E1-H2 [the curve DEF in figure 3 (c)]. Then the tip motion becomes purely rotational and interacts only with the lateral stiffness. The rotational vibration of a tip can be expressed by the one-freedom model in figure 5:

$$f = f_0 \left( 1 + \frac{k_v}{k_c} \right)^{1/2},$$  

(6)

Actually the broken curves in figure 6, predicted by equation (7) for several values of $r$, coincide with the results from the continuum theory (solid curves).

We found that the point-mass and spring model [equation (6)] is still valid for $k_v$-detection if $\alpha$ is larger than about 4 and $\theta$ is less than about 15 degree. Moreover the rod and spiral-spring model [equation (7)] is also applicable if the ratio of inertia moment $\beta = (m_c l_m^2)/(m_c^2 l^2) = \alpha (l_m/l)^2$ is larger than about 4. This finding is useful for simultaneous detection of the vertical and lateral stiffness.

3.2. Measurements of the contact resonant frequency and evaluation of the contact stiffness

Figure 7 shows spectra observed for a sapphire wafer with polished C-plane (0001), where [1120] is the $a_3$-axis in which the cantilever was oriented. The first peak can be identified as a contact resonance...
of the vertical motion since the frequency ($f_1 = 56.2$ kHz) was lower than $f_{0i}$. The resonance curve did not depend on the contact force. It resulted from the use of a flat tip, in which the radius $a$ of the contact area was kept constant. In contrast the second peak showed nonlinear resonance at low contact forces. The second one (the resonant frequency $f_2 = 86.9$ kHz) was the rotational modes, which caused the tip to slide at low forces. All the measurements were stable and reproducible. The effects of the orientation of the cantilever were also investigated (figure 8), where the variation in the second mode $f_2$ was small since elastic anisotropy in the C-plane is weak.

The stiffness ratio $r = 3.1$ and the vertical stiffness $k_v = 100$ N/m were obtained by using equations (6), (7) and the values of $f_1$ and $f_2$. The results are shown by circles in the figure 6. On the other hands the contact mechanics [7] provides $k_v = 2aE$, $k_l = 8aG$, and then $r = 4G/E$, where $E = [(1–\nu_t^2)/E_t + (1–\nu_s^2)/E_s]^{-1}$ and $G = [(2–\nu_t)/G_t + (2–\nu_s)/G_s]^{-1}$. The subscripts t and s indicate a tip and a sample. $E$, $G$ and $\nu$ are Young’s modulus, shear modulus and Poisson’s ratio, respectively. We obtained $r = 0.75$, assuming the moduli of bulk platinum $E_t = 168$GPa, $\nu_t = 0.377$ for the tip elasticity, and using calculated moduli for the $\text{Al}_2\text{O}_3$ sample: $E_s = E_{(0001)} = 451$GPa with $\nu_s = \nu_{[0001]} = 0.172$, $G_s = G_{[1\underline{T}2\underline{O}]} = 142$GPa with $\nu_s = \nu_{[1\underline{T}2\underline{O}]} = 0.311$, where the calculation based on crystal constants were similar to the appendix in reference [6]. The disagreement in $r$ results from the discrepancy between the experiment and the theory in a position of rotational center of the tip motion. The gravitational center of the CM rod was offset out of the cantilever axis (see figure 1) and would become the rotational center when the tip interacts with the lateral stiffness. Introducing an equivalent tip height $h_e = h + (\text{the rod radius plus half of the cantilever thickness: } 16 \text{ nm})$ instead of $h$ in the theory, we obtained $r = 1.05$, closer to the calculated value ($r = 0.75$), together with $k_v = 106$ N/m, as shown in figure 9. The contact size was estimated $2a = 0.8$ nm. While further understanding is needed on a position of the rotational center and its effects on the stiffness evaluations, the present type of CM cantilevers enable us to simultaneously evaluate the vertical and lateral stiffness without mutual interference.

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