Decay of Z into Three Pseudoscalar Bosons

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Abstract

We consider the decay of the $Z$ boson into three pseudoscalar bosons in a general two-Higgs-doublet model. Assuming $m_A$ to be very small, and that of the two physical neutral scalar bosons $h_1$ and $h_2$, $A$ only couples to $Z$ through $h_1$, we find the $Z \to AAA$ branching fraction to be negligible for moderate values of $\tan \beta \equiv v_2/v_1$, if there is no $\lambda_5(\Phi_1^\dagger \Phi_2)^2 + h.c.$ term in the Higgs potential; otherwise there is no absolute bound but very large quartic couplings (beyond the validity of perturbation theory) are needed for it to be observable.
If the standard $SU(2) \times U(1)$ electroweak gauge model is extended to include two scalar doublets, there will be a neutral pseudoscalar boson $A$ whose mass may be small. In that case, the decay of the $Z$ boson into $3\, A$’s may not be negligible. This process was first studied in a specific model. It was then discussed in a more general context. More recently, it has been shown that there is a lower bound on $m_A$ of about 60 GeV in the Minimal Supersymmetric Standard Model (MSSM), hence the decay $Z \rightarrow AAA$ is only of interest for models with two scalar doublets of a more general structure. Even in the context of supersymmetry, this is possible if there exists an additional $U(1)$ gauge factor at the TeV scale.

In this paper we consider a general two-Higgs-doublet model and identify the conditions for which the decay $Z \rightarrow AAA$ may be enhanced, despite the nonobservation of $e^+e^- \rightarrow h + A$, where $h$ is either one of the two neutral scalar bosons of the model. We will show that in principle this decay is limited only by the scalar coupling $\lambda_1 - \lambda_2$ as defined below. However, if $\lambda_5 = 0$, which is true in a large class of models, then it may be bounded as discussed below.

Let the Higgs potential $V$ for two $SU(2) \times U(1)$ scalar doublets $\Phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)$ be given by

$$V = m_1^2 \Phi_1^+ \Phi_1 + m_2^2 \Phi_2^+ \Phi_2 + m_{12}^2 (\Phi_1^+ \Phi_2 + \Phi_2^+ \Phi_1) + \frac{1}{2} \lambda_1 (\Phi_1^+ \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^+ \Phi_2)^2 + \frac{1}{2} \lambda_3 (\Phi_1^+ \Phi_1)(\Phi_2^+ \Phi_2) + \frac{1}{2} \lambda_4 (\Phi_1^+ \Phi_2)(\Phi_2^+ \Phi_1) + \frac{1}{2} \lambda_5 (\Phi_1^+ \Phi_2)^2 + \frac{1}{2} \lambda_5^* (\Phi_2^+ \Phi_1)^2,$$

where the discrete symmetry $\Phi_1 \rightarrow \Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$ is only broken softly by the $m_{12}^2$ term. Assume $\lambda_5$ to be real for simplicity. Define $\tan \beta \equiv v_2/v_1$ as is customary, where $v_{1,2} = \langle \phi_{1,2}^0 \rangle$ are the usual two nonzero vacuum expectation values. The pseudoscalar neutral Higgs boson is then

$$A = \sqrt{2} (\sin \beta \text{Im} \phi_1^0 - \cos \beta \text{Im} \phi_2^0),$$

(2)
with mass given by

\[ m_A^2 = -m_{12}^2(\tan \beta + \cot \beta) - 2\lambda_5 v^2, \]

(3)

where \( v^2 \equiv v_1^2 + v_2^2 \), and the charged Higgs boson is

\[ h^\pm = \sin \beta \phi_1^\pm - \cos \beta \phi_2^\pm, \]

(4)

with

\[ m_{h^\pm}^2 = m_A^2 + (\lambda_5 - \lambda_4)v^2. \]

(5)

To get the maximum \( Z \to AAA \) rate, we let \( m_A = 0 \), i.e.

\[ m_{12}^2 = -2\lambda_5 v^2 \sin \beta \cos \beta. \]

(6)

Then the mass-squared matrix spanning the two neutral scalar Higgs bosons \( \sqrt{2} \text{Re} \phi_{1,2}^0 \) is given by

\[ \mathcal{M}^2 = 2v^2 \begin{bmatrix} \lambda_1 \cos^2 \beta + \lambda_5 \sin^2 \beta & (\lambda_3 + \lambda_4) \sin \beta \cos \beta \\ (\lambda_3 + \lambda_4) \sin \beta \cos \beta & \lambda_2 \sin^2 \beta + \lambda_5 \cos^2 \beta \end{bmatrix}. \]

(7)

Consider now the following two linear combinations:

\[ h_1 = \sqrt{2}(\sin \beta \text{Re} \phi_1^0 - \cos \beta \text{Re} \phi_2^0), \]

(8)

\[ h_2 = \sqrt{2}(\cos \beta \text{Re} \phi_1^0 + \sin \beta \text{Re} \phi_2^0). \]

(9)

It is well-known that \( h_1 \) couples to \( AZ \) but not \( ZZ \), whereas \( h_2 \) couples to \( ZZ \) but not \( AZ \). However, the process \( e^+e^- \to h + A \) is in general possible because \( h \) will normally have a \( h_1 \) component, thereby putting a constraint on \( m_A \) if kinematically allowed. For our purpose, we will require \( h_1 \) and \( h_2 \) to be mass eigenstates, in which case \( m_A \) is unconstrained by the nonobservation of \( e^+e^- \to h + A \) even if \( m_2 \) is small, as long as \( m_1 \) is larger than the \( e^+e^- \) center-of-mass energy. This allows us to have the maximum effective coupling of \( Z \) to \( AAA \) as shown below.

The requirement that \( h_1 \) and \( h_2 \) be mass eigenstates leads to the condition

\[ \lambda_2 \sin^2 \beta - \lambda_1 \cos^2 \beta + (\lambda_3 + \lambda_4 + \lambda_5)(\cos^2 \beta - \sin^2 \beta) = 0. \]

(10)
As a result, the masses of \( h_{1,2} \) are given by

\[
m_1^2 = [\lambda_1 \cos^2 \beta + \lambda_2 \sin^2 \beta + \lambda_5 - \lambda_3 - \lambda_4] v^2, \tag{11}
\]

\[
m_2^2 = [\lambda_1 \cos^2 \beta + \lambda_2 \sin^2 \beta + \lambda_5 + \lambda_3 + \lambda_4] v^2. \tag{12}
\]

Note that in the MSSM, Eq. (10) cannot be satisfied in the presence of radiative corrections.

We now extract the \( h_1 AA \) coupling from Eq. (1), using Eqs. (2) and (8). We find it to be given by

\[
\sin 2\beta \left( \frac{2}{\sqrt{2}} (\lambda_1 - \lambda_2) \right) v, \tag{13}
\]

where Eq. (10) has been used. As a function of \( \beta \), this expression is obviously maximized at \( \sin 2\beta = \pm 1 \). On the other hand, our conditions so far do not limit the combination \( \lambda_1 - \lambda_2 \), hence there is no absolute bound on \( Z \to AAA \) in this general case.

Let us consider the case \( \lambda_5 = 0 \). This is natural in a large class of models where the two Higgs doublets are remnants\(^3\) of a gauge model larger than the standard model such that they are distinguishable under the larger symmetry. In that case, we have

\[
m_1^2 = 2(\lambda_1 - \lambda_3 - \lambda_4) v^2 \cos^2 \beta = 2(\lambda_2 - \lambda_3 - \lambda_4) v^2 \sin^2 \beta, \tag{14}
\]

and we can rewrite (13) as

\[- \frac{m_1^2}{v} \cot 2\beta. \tag{15}\]

The above expression appears to be unbounded as \( \sin 2\beta \to 0 \). However, that would require very large quartic scalar couplings. This can be seen two ways. First, since (15) is equal to (13), we need an extremely large value of \( \lambda_1 - \lambda_2 \). Second, from Eq. (14), we see also that if \( \sin \beta \) is small, then \( \lambda_2 - \lambda_3 - \lambda_4 \) has to be big, and if \( \cos \beta \) is small, then \( \lambda_1 - \lambda_3 - \lambda_4 \) has to be big. Thus we will choose moderate values of \( \tan \beta \) in (15) for the following discussion.

In Figure 1 we show the diagram for the decay \( Z \to AAA \) with an intermediate virtual \( h_1 \). To maximize this rate, we minimize \( m_1 \) to be just above the maximum experimental
\(e^+e^\text{−}\) center-of-mass energy, which is 172 GeV up to now but will soon be 183 GeV. As for \(h_2\), it interacts exactly as the one Higgs boson of the standard-model, from which we have the experimental limit[^7] of \(m_2 > 65\) GeV. However, \(m_2\) is not directly involved in the \(h_1AA\) coupling here. Note also that \(\lambda_4\) by itself must be large and negative so that \(m_{h^\pm}\) of Eq. (5) can be greater than \(m_t - m_b\) for \(m_A = 0\), so as to prevent the decay \(t \to b + h^+\). This condition is not satisfied in the MSSM where \(\lambda_4 = -g_2^2 / 2\), hence \(m_A = 0\) is not allowed there[^4].

Assuming \(\lambda_5 = 0\) and using Eq. (15) with \(m_1 = 180\) GeV and \(|\cot 2\beta| = 1\) (i.e. \(\tan \beta = 0.4\) or 2.4), we now calculate the \(Z \to AAA\) decay rate, following Ref. [1]. The amplitude is given by

\[
\mathcal{M} = g_Z \frac{m_1^2 \sqrt{2}}{v} \left[ \frac{\epsilon \cdot k_1}{(p - k_1)^2 - m_1^2} + \frac{\epsilon \cdot k_2}{(p - k_2)^2 - m_1^2} + \frac{\epsilon \cdot k_3}{(p - k_3)^2 - m_1^2} \right],
\]

where \(g_Z = e / \sin \theta_W \cos \theta_W\), \(p\) is the four-momentum of the \(Z\) boson, and \(k_{1,2,3}\) are those of the \(A\)'s. The effective coupling used in Ref. [1] is now determined to be

\[
\lambda_{\text{eff}} = \frac{m_1^2 \sqrt{2}}{v^2} \simeq 1.5.
\]

Using the estimate of Ref. [1], this \(Z \to AAA\) rate is then about \(1.0 \times 10^{-7}\) GeV. Hence its branching fraction is about \(4 \times 10^{-8}\) which is clearly negligible. To obtain a branching fraction of \(10^{-6}\), we need \(\cot 2\beta = 5\) (i.e. \(\tan \beta = 0.1\) or 10). In this case, either \(\lambda_1 - \lambda_3 - \lambda_4\) or \(\lambda_2 - \lambda_3 - \lambda_4\) in Eq. (14) has to be about 53.5. If \(\lambda_5 \neq 0\), then we cannot use Eqs. (14) and (15), but Eq. (13) is still valid. To obtain a branching fraction of \(10^{-6}\), we will then need \(|\lambda_1 - \lambda_2|\) to be about 53.5. Thus in both scenarios, one or more quartic scalar couplings have to be very large and beyond the validity of perturbation theory.

If \(h_1\) and \(h_2\) are not exact mass eigenstates, then there is an additional contribution from \(h_1 - h_2\) mixing which is necessarily very small from the constraint of experimental data if
$m_2$ is below 172 GeV. The $h_2AA$ coupling is given by

$$\frac{v}{\sqrt{2}} \left( \frac{m_2^2}{2v^2} - 2\lambda_5 [1 - \sin^2 \beta \cos^2 \beta] \right).$$  

(18)

If $\lambda_5 = 0$, this expression is bounded independent of $\tan \beta$ and the overall contribution (including the small $h_1 - h_2$ mixing) is negligible. If $\lambda_5 \neq 0$, then its value has to be huge for the process to be observable.

The reason that $\Gamma(Z \to AAA)$ is so small is twofold. One is that with the higher energy reached by LEP2, the nonobservation of $Z \to h + A$ forces $m_1$ to be much greater than $M_Z$. The other is that for $m_1 >> M_Z$, the leading term in $\mathcal{M}$ vanishes because $\epsilon \cdot (k_1+k_2+k_3) = 0$, resulting in a very severe suppression factor[1]. Our conclusion is that the decay $Z \to AAA$ is not likely to be observable in a general two-Higgs-doublet model with parameters in the perturbative regime.

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Figure Caption

Fig. 1. One of 3 diagrams for the decay $Z \rightarrow AAA$. The other 2 are obvious permutations.
