GRAPE – Dilepton
(Version 1.1)
A Generator for Dilepton Production
in ep Collisions

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Abstract: GRAPE-Dilepton is a Monte Carlo event generator for dilepton production in ep collisions. The cross-section calculation is based on the exact matrix elements in the electroweak theory at tree level. The dilepton productions via $\gamma\gamma$, $\gamma Z^0$, $Z^0 Z^0$ collisions and via photon internal conversion are taken into account. In addition, the effects of the $Z^0$ on/off-shell production are also included. The relevant Feynman amplitudes are generated by the automatic calculation system GRACE. The calculation of the proton vertex covers the whole kinematical region. This generator has an interface to PYTHIA and SOPHIA to obtain complete hadronic final states.
PROGRAM SUMMARY

Title of program: GRAPE-Dilepton (v1.1)

Program obtainable from: CPC Program Library, Queen’s University of Belfast, N. Ireland and from http://www-zeus.desy.de/~abe/grape/.

Operating system under which the program has been tested: UNIX

Programming language used: Fortran77

Memory required to execute with typical data: 7 Mwords for integrations, 9 Mwords for event generations

Keywords: dilepton, lepton-pair, ep collision, Bethe-Heitler, Z boson, dipole form factor, hadron tensor, lepton tensor, structure function, parton density, GRACE

Nature of physical problem: A precise estimation of the cross section of the electroweak dilepton production in ep collisions is required in various physics analyses, where 8~48 Feynman diagrams can contribute.

Method of solution: The automatic calculation system GRACE is used to obtain all of the relevant helicity amplitudes. The phase space is divided into the 3 regions according to the kinematics at the proton vertex, and the 3 different calculation methods are applied. The radiative corrections are included using the structure function and the parton shower methods.

Restrictions on the complexity of the problem: Higgs, the proton-Z⁰ coupling and lepton pair production through photon radiation from the proton are not included. The contribution from the resolved photon, i.e. Drell-Yan process in ep collisions is not included.

Typical running time: 1 hour for a cross-section integration and 1 msec per 1 event for an event generation
1 Introduction

In the study of electron/positron-proton (ep) collisions, a precise estimation of the dilepton\textsuperscript{1} production cross-sections in the electroweak (EW) interaction is important since it could become a significant background for various physics analyses such as, for example, exclusive $J/\psi$ or $\Upsilon$ production and new physics searches. So far only the generator $\text{LPAIR}$\textsuperscript{1} has been used in experimental analyses to estimate the dilepton background\textsuperscript{2}. The calculation of $\text{LPAIR}$ is based on the diagrams of the photon-photon collision process\textsuperscript{3}, so-called two-photon Bethe-Heitler (2-$\gamma$ BH), corresponding to the diagrams of Fig. 1-(a) or Fig. 2-(a) with the photon contribution only. This process is dominant in most of the phase space. It is, however, expected that in the region of low invariant masses of the dilepton system, QED-Compton type (CO) diagrams (i.e. photon internal conversion process) as seen in Fig. 1-(b) and Fig. 2-(b) become dominant. In the high mass region, there is an additional interesting process, i.e. $Z^0$ production, which is implemented in the MC event generator $\text{EPVEC}$\textsuperscript{4}. $\text{EPVEC}$, however, does not include 2-$\gamma$ BH nor CO diagrams. In the di-\(e\) channel, interference effects of the final state $e^-e^-$ or $e^+e^+$ should also be taken into account, which are included neither in $\text{LPAIR}$ nor in $\text{EPVEC}$.

In this paper, a new MC event generator $\text{GRAPE-Dilepton}$ for the dilepton production in ep collisions is presented. The FORTRAN code to calculate the Feynman amplitudes is generated by $\text{GRACE}$\textsuperscript{5} which is an automatic calculation system. $\text{GRACE}$ has been used mainly for $e^+e^-$ interactions so far. This is the first time for $\text{GRACE}$ to be applied to the case where there is a composite particle (i.e. proton) in the initial state. $\text{GRAPE}$ stands for a GRAce-based generator for Proton-Electron collisions.

This generator has the following features.

- The cross-section calculation is based on the exact matrix elements in the electroweak theory at tree level. Not only 2-$\gamma$ BH but also the dilepton productions via $\gamma Z^0$ and $Z^0Z^0$ collisions are taken into account. CO and $Z^0$ on/off-shell production are also included. Interference effects of the final state $e^{\pm}e^{\pm}$ are taken into account in the di-\(e\) channel. It is possible to select any sub-set of diagrams in the calculation.

- All fermion masses are kept non-zero both in the matrix elements and in the kinematics, which makes it possible to use this program with arbitrary small

\textsuperscript{1}The word \textit{dilepton} represents di-electron(di-\(e\)), di-muon(di-\(\mu\)) and di-tau(di-\(\tau\)) in this paper.
scattering angles of $e^\pm$ and/or small invariant masses of dilepton down to the kinematical limits.

- The calculation of the proton vertex covers the whole kinematical region by dividing it into 3 categories of elastic, quasi-elastic and DIS (Deep Inelastic eq Scattering) processes, as described in the next section in detail.
- Both of Initial State Radiation (ISR) and Final State Radiation (FSR) can be included.

## 2 Physics aspects

This generator simulates the $ep$ interaction: $e_{(in)}^\pm p_{(in)} \rightarrow e^\pm l^+l^- X$ where $e_{(in)}^\pm$ and $p_{(in)}$ indicate the electron/positron and the proton in the initial state respectively; $e^\pm$ and $l^+l^-$ are the scattered electron/positron and the produced dilepton respectively. The relevant processes are classified into 3 categories using the negative momentum transfer squared at the proton vertex ($Q^2_p$) and the invariant mass of the hadronic system ($M_{had}$);

\begin{align}
Q^2_p & \equiv - \left\{ p_{e^\pm (in)} - (p_{e^\pm} + p_{l^+} + p_{l^-}) \right\}^2, \\
M^2_{had} & \equiv \left\{ (p_{e^\pm (in)} + p_{p(in)}) - (p_{e^\pm} + p_{l^+} + p_{l^-}) \right\}^2,
\end{align}

where $p_{e^\pm (in)}$ and $p_{p(in)}$ are the 4-momenta of the incoming lepton and the proton after ISR, respectively. $p_{e^\pm}$ and $p_{l^\pm}$ are those of the scattered lepton and the produced leptons before FSR, respectively. The 3 categories are

- $M_{had} = M_p$ (elastic),
- $Q^2_p < Q^2_{min}$ OR $M_p + M_{\pi^0} < M_{had} < M_{cut}$ (quasi-elastic),
- $Q^2_p > Q^2_{min}$ AND $M_{had} > M_{cut}$ (DIS),

where $M_p$ and $M_{\pi^0}$ are the masses of the proton and the neutral pion, respectively. $Q_{min}$ is set to around 1 GeV depending on the Parton Density Function (PDF) used in the DIS process. The recommended value for $M_{cut}$ is 5 GeV.

For the elastic process, the diagrams in Fig.[I] are calculated with the following dipole form factor for the proton-proton-photon vertex ($\Gamma^{\mu}_{pp\gamma}$) with the on-shell proton. The general form of the elastic proton vertex can be written as

\begin{equation}
\Gamma^{\mu}_{pp\gamma} = e_p \left( F_1(Q^2_p)\gamma^\mu + \frac{K_p}{2M_p} F_2(Q^2_p) i\sigma^{\mu\nu} q_\nu \right)
\end{equation}
where $e_p$ indicates the electric charge of the proton, $q$ is the 4-momentum transfer at the proton vertex ($q^2 = -Q_p^2$), $F_1(Q_p^2)$ and $F_2(Q_p^2)$ are the 2 independent form factors, and $\kappa_p$ is the anomalous magnetic moment of the proton (see, for example, [6]). The electric and magnetic form factors $G_E^p(Q_p^2)$ and $G_M^p(Q_p^2)$, respectively are defined as follows,

$$
\begin{pmatrix}
G_E^p(Q_p^2) \\
G_M^p(Q_p^2)
\end{pmatrix} = \begin{pmatrix}
F_1(Q_p^2) - \frac{\kappa_p Q_p^2}{4 M_p} F_2(Q_p^2) \\
F_1(Q_p^2) + \kappa_p F_2(Q_p^2)
\end{pmatrix}.
$$

Using the Gordon decomposition and the scaling law of the form factor,

$$G_E^p(Q_p^2) = G_M^p(Q_p^2)/|\mu_p|,$$

the following formula which is used in this program is obtained,

$$
\Gamma_{\mu p\gamma} = e_p \left( \mu_p G_E^p(Q_p^2) \gamma^\mu - \frac{(p_{\mu p(\text{in})}^\mu + p_{\mu p(\text{out})}^\mu)}{2 M_p} \frac{\kappa_p}{1 + \frac{Q_p^2}{4 M_p^2}} G_E^p(Q_p^2) \right)
$$

where $\mu_p = (1 + \kappa_p) \mu_B$, $\mu_B$ is the Bohr magneton, and $p_{\mu p(\text{out})}$ indicates the 4-momentum of the scattered proton. $G_E^p(Q_p^2)$ is calculated according to the formula of the dipole fit,

$$G_E^p(Q_p^2) = \left( 1 + \frac{Q_p^2}{0.71 \text{ GeV}^2} \right)^{-2}.$$

The only difference between the elastic and the quasi-elastic processes is the treatment of the proton vertex and the simulation of the hadronic final state. The quasi-elastic proton vertex can be described using the hadron tensor in the following form assuming parity and current conservation (for example, see [6]),

$$W^{\mu \nu} = W_1 \left(-g^{\mu \nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2 \frac{1}{M_p^2} \left(p_{\mu p(\text{in})}^\mu - \frac{p_{\mu p(\text{in})} \cdot q}{q^2} q^\mu\right) \left(p_{\nu p(\text{in})}^\nu - \frac{p_{\nu p(\text{in})} \cdot q}{q^2} q^\nu\right).$$

$W_1(Q_p^2, M_{\text{had}})$ and $W_2(Q_p^2, M_{\text{had}})$ are the electromagnetic proton structure functions. The hadron tensor is contracted with the lepton tensor $L^{\mu \nu}$ numerically to obtain the cross section,

$$d\sigma \sim L^{\mu \nu} W^{\mu \nu}.$$
In this version, $W_1$ and $W_2$ are parameterized with Brasse et al. \cite{7} for $M_{\text{had}} < 2 \text{ GeV}$ (the proton resonance region), and with ALLM97 \cite{8} for $M_{\text{had}} > 2 \text{ GeV}$. These two parameterizations are based on fits to the experimental data on the measurement of the total $\gamma^* p$ cross-sections. The exclusive hadronic final state is generated using the MC event generator SOPHIA \cite{9} in the event generation step.

For the DIS process with the Quark Parton Model, the diagrams in Fig. 2 are calculated. PDFLIB \cite{10} is linked to obtain parton densities with $Q^2_p$ as a QCD scale. The simulation of the proton remnant and the hadronization are performed by PYTHIA \cite{11}. It should be noted that the lowest order calculation in this process is valid only for the region of

$$ u \overset{\text{def}}{=} |\{p_{q(i)} - (p_{l+} + p_{l-})\}| \gtrsim 25 \text{ GeV}^2, \quad (10) $$

where $p_{q(i)}$ is the 4-momentum of the incoming quark. The value of $u$ corresponds to the virtuality of the $u$-channel quark in the diagrams in Fig. 2-(b),(c). When it is nearly or smaller than 25 GeV$^2$, the lowest order calculation is not correct as explained in \cite{4} since QCD corrections become large. In this case, the dilepton production should be treated as Drell-Yan process between the proton and the resolved photon from the beam lepton, which is not implemented in this program. The cut: $u > 25 \text{ GeV}^2$ is explicitly applied in this program if the diagrams other than BH are included.

The effect of ISR is included in the cross-section calculation using the structure function method described in \cite{12}, where the momentum transfer squared on the beam lepton, \textit{i.e.} $\{p_{e\pm(i)} - p_{e\pm}\}^2$ is used as a QED scale. When ISR turns on, the correction for the photon self energy, \textit{i.e.} the vacuum polarization, is included according to the parameterization in \cite{13} by modifying photon propagators. FSR is performed by PYTHIA using the parton shower method when the event is generated.
Fig. 1: Feynman diagrams included in the (quasi-)elastic process. $e=\{e^+, e^-\}$, $l^\pm=\{e^\pm, \mu^\pm, \tau^\pm\}$. N means a (dissociated) proton or a nucleon resonance.
Fig. 2: Feynman diagrams included in the DIS process. \( e = \{ e^+, e^- \} \), \( l^\pm = \{ e^\pm, \mu^\pm, \tau^\pm \} \) and \( q = \{ \bar{u}, d, s, c, b, t \} \).
3 Program structure

Physics events are generated with the 2 steps; the MC integration step by the executable: `integ` and the event generation step by the executable: `spring`, as illustrated in Fig. 3. In both steps, the program is controlled by an ASCII file: `grape.cards`. The file is read by the executables with help of `FFREAD` [14]. The contents of `grape.cards` are explained in the next section.

In the integration step by the executable: `integ`, an effective total cross-section (in unit of pb) and probability distributions are calculated by `BASES` [15]. The results are stored in a file: `bases.rz` which has the Ntuple format provided by the `HBOOK` package [16]. At the same time, the information related to the convergency status of the integration is output into an ASCII file: `bases.result`.

In the event generation step by the executable: `spring`, unweighted events are generated. This is done by a routine: `SPRING` [15] according to the probability distributions in `bases.rz`. The results of the event generation are stored in the `PYTHIA` common block `/PYJETS/`. After filling `/PYJETS/`, `spring` calls a routine: `USRSTR` in which user specific procedures are put. Its template is found in the appendix. The event information in `/PYJETS/` is also available in a Ntuple file: `grp.rz`.

The calculated cross-section is found in `bases.result` or at the end of the standard output from `spring`. The status of the event generation is output into an ASCII file: `spring.result`. Looking at the file, users should find a reasonable agreement between generated distributions by `spring` and calculated ones by `integ`. The procedure to make the executables is described in the `README` file.

4 Input data cards

The input data in `grape.cards` are explained in this section. All of the items are optional and are set to default values if not specified. Default values are written in the brackets starting with `D=`. The items are not explicitly displayed in case that they are the only one for their cards.

- **KFLBEAM**
  
  KF code of the lepton beam (INTEGER, D=-11); 11:electron, -11:positron.

- **EPOL** \( P \theta \phi \)
  
  Polarization of the lepton beam (REAL);
Fig. 3: Flowchart for the program structure

\[ P = \text{degree of the polarization in the range } [-1, 1] \ (D=0.) \]
\[ \theta = \text{polar angle of the polarization vector in degree } (D=0.) \]
\[ \phi = \text{azimuthal angle of the polarization vector in degree } (D=0.) \]

The positive direction of the z-axis on the polarization vector is in the direction of the lepton beam.

- **EBEAM**
  Lepton beam momentum in MeV/c (REAL, D=27520.).

- **PBEAM**
  Proton beam momentum in MeV/c (REAL, D=820000.).

- **PROCESS**
  Process type of the proton vertex (INTEGER, D=1);
  1:elastic, 2:quasi-elastic, 3:DIS.

- **LPAIR**
  Dilepton channel (INTEGER, D=2); 1:di-\(\epsilon\), 2:di-\(\mu\), 3:di-\(\tau\).

- **ISR**
  Initial state radiation flag for the beam lepton (INTEGER, D=1); 0:off, 1:on.
Table 1: Possible combinations of QFLV and MERGE

- **QFLV**
  Scattered quark in the DIS process (INTEGER, D=1);
  1:u, 2: ¯u, 3:d, 4: ¯d, 5:s, 6: ¯s, 7:c, 8: ¯c, 9:b, 10: ¯b, 11:t, 12: ¯t.

- **MERGE**
  Merging mode in the DIS process (INTEGER, D=0); 0:off.
  In some cases, contributions from different quarks can be included in the cross-section calculation adding the parton densities if the mass difference is negligible. The possible combinations of QFLV and MERGE are written in Table 1. The mass of the quark specified with QFLV is used in the amplitude and the kinematics calculations.

- **NGROUP**
  Author group described in the PDFLIB manual (INTEGER, D=5).

- **NSET**
  PDF set described in the PDFLIB manual (INTEGER, D=5).
  The default is GRV94(LO).

- **GRASEL**
  Feynman diagram selection (INTEGER, D=3);
  1: 2-γ Bethe-Heitler (without \(e^\pm e^\pm\) interference in case of di-\(\mu\)e),
  2: 2-γ Bethe-Heitler (including \(e^\pm e^\pm\) interference in case of di-\(\mu\)e),
  3: QED diagrams (i.e. all the diagrams except for the \(Z^0\) contribution),
  4: EW diagrams (i.e. all the diagrams),
13: QED-Compton type diagrams only,
14: $Z^0$ production diagrams only.

In case of di-$\mu$, $\tau$, the first and the second selections give the same result.

- **ITMX1**
  Number of iterations in the grid optimization step of BASES. (INTEGER, D=4).
  This should be larger than 2.

- **ITMX2**
  Number of iterations in the integration step of BASES (INTEGER, D=10).
  This should be larger than 5.

- **NCALL**
  Number of sampling points in each iteration of BASES (INTEGER, D=1000000).
  This should be large so that any accuracy of each iteration in the integration step of BASES is better than 0.5%.

- **NGEN**
  Number of events to be generated by *spring* (INTEGER, D=100).

- **NMOD** $N_{mod}$
  Printing a message per $N_{mod}$ events in the event generation (INTEGER, D=1000).

- **PSISR**
  Switch for the initial state parton shower by PYTHIA (INTEGER, D=1); 0:off, 1:on.
  This has an effect only on event generations of the DIS process. No effect on elastic and quasi-elastic events. This item is copied to MSTP(61) in the PYTHIA common block /PYPARS/.

- **PSFSR**
  Switch for the final state parton shower by PYTHIA (INTEGER, D=1); 0:off, 1:on.
  This item is copied to MSTP(71) in the PYTHIA common block /PYPARS/.

- **PSBRA**
  Parton shower branchings in PYTHIA (INTEGER, D=2);
  1:QCD, 2:QCD+QED.
  This item is copied to MSTJ(41) in the PYTHIA common block /PYDAT1/.

- **PSSUP**
  Suppression of the PYTHIA parton shower (INTEGER, D=0); 0:off, $\geq$1:on.
  This item is copied to MSTJ(40) in the PYTHIA common block /PYDAT1/.

- **PYDECAY**
  Switch for fragmentation and decay in PYTHIA (INTEGER, D=1); 0:off, 1:on.
No effect on elastic and quasi-elastic events.
This item is copied to MSTP(111) in the PYTHIA common block /PYPARS/.

- **PRIPT**
  Primordial $k_t$ distribution in the proton (INTEGER, D=1);
  0:off, 1:gaussian, 2:exponential.
No effect on elastic and quasi-elastic events. This item is copied to MSTP(91) in the PYTHIA common block /PYPARS/.

- **PYLIST**
  Printing the contents of /PYJETS/ (LOGICAL, D=TRUE).

- **NLIST**
  Number of events whose /PYJETS/ is printed out (INTEGER, D=10).

- **NTPYT**
  Output of generated events into a Ntuple file: grp.rz
  from the PYTHIA common block /PYJETS/ (LOGICAL, D=FALSE).
  The meanings of the Ntuple variables are in the following.
  
  \[
  \begin{align*}
  \text{npy} & : \text{Number of particles (integer)} \\
  \text{px}(1:\text{npy}), \text{py}(1:\text{npy}), \text{pz}(1:\text{npy}) & : x,y,z\text{-component of momentum in GeV/c (real*4)} \\
  \text{pe}(1:\text{npy}) & : \text{Energy in GeV (real*4)} \\
  \text{pm}(1:\text{npy}) & : \text{Mass in GeV (real*4)} \\
  \text{kf}(1:\text{npy}) & : \text{KF code (integer)} \\
  \text{sta}(1:\text{npy}) & : \text{Status code (integer)} \\
  \text{mot}(1:\text{npy}) & : \text{Line number of the mother particle (integer)}
  \end{align*}
  \]

- **Q2RNGME**  
  \text{Min} \text{ Min} \text{ Max}
  Range for the negative momentum transfer squared at the electron vertex $Q_e^2$ without ISR (REAL), i.e. $Q_e^2 = -\{(p_{e\pm(in)} - p_{e\pm})\}^2$ where $p_{e\pm(in)}$ is a 4-momentum of the incoming lepton after ISR.
  
  \[
  \begin{align*}
  \text{Min} & = \text{the minimum in GeV}^2 (D=0.), \\
  \text{Max} & = \text{the maximum in GeV}^2 (D=1.E20). 
  \end{align*}
  \]
  In case of di-$e$ with $e^\pm e^\pm$ interference, smaller one of the two $Q_e^2$ values is used.

- **Q2RNGOB**  
  \text{Min} \text{ Max}
  Range for the negative momentum transfer squared at the electron vertex $Q_e^2$ including ISR (REAL), i.e. $Q_e^2 = -\{(p_{e\pm(in)} - p_{e\pm})\}^2$ where $p_{e\pm(in)}$ is a 4-momentum of the incoming lepton before ISR.
Min = the minimum in GeV^2 (D=0.),
Max = the maximum in GeV^2 (D=1.E20).

In case of di-\(e\) with \(e^\pm e^\pm\) interference, smaller one of the two \(Q^2\) values is used.

- **MHAD** **Min** **Max**
  Range for the mass of the hadronic system \(M_{had}\) (REAL);
  - \(Min = \) the minimum in GeV (D=1.08),
  - \(Max = \) the maximum in GeV (D=1.E20).
  No effect on elastic events.

- **Q2P** **Min** **Max**
  Range for the negative momentum transfer squared at the proton vertex \(Q^2_p\) (REAL);
  - \(Min = \) the minimum in GeV^2 (D=0.),
  - \(Max = \) the maximum in GeV^2 (D=1.E20).
  In case of the DIS process, \(Q^2_p\) is used as a QCD scale for PDF.

- **THMIN** \(\theta_{\min}^{(1)} \theta_{\min}^{(2)} \theta_{\min}^{(3)} \theta_{\min}^{(4)}\)
- **THMAX** \(\theta_{\max}^{(1)} \theta_{\max}^{(2)} \theta_{\max}^{(3)} \theta_{\max}^{(4)}\)
- **EMIN** \(E_{\min}^{(1)} E_{\min}^{(2)} E_{\min}^{(3)} E_{\min}^{(4)}\)
- **EMAX** \(E_{\max}^{(1)} E_{\max}^{(2)} E_{\max}^{(3)} E_{\max}^{(4)}\)
- **PMIN** \(P_{\min}^{(1)} P_{\min}^{(2)} P_{\min}^{(3)} P_{\min}^{(4)}\)
- **PMAX** \(P_{\max}^{(1)} P_{\max}^{(2)} P_{\max}^{(3)} P_{\max}^{(4)}\)
- **PTMIN** \(P_{t\min}^{(1)} P_{t\min}^{(2)} P_{t\min}^{(3)} P_{t\min}^{(4)}\)
- **PTMAX** \(P_{t\max}^{(1)} P_{t\max}^{(2)} P_{t\max}^{(3)} P_{t\max}^{(4)}\)

- (1) : for scattered proton or quark,
- (2) : for scattered \(e^\pm\),
- (3) : for produced \(l^\pm\),
- (4) : for produced \(l^\pm\).

The above 8 data cards are used for describing the detector cut in the laboratory frame (REAL). Each final state particle is required to satisfy the following,

\[
\theta_{\min}^{(i)} < \theta < \theta_{\max}^{(i)} \quad \text{AND} \quad E_{\min}^{(i)} < E < E_{\max}^{(i)}
\]

\[
\text{AND} \quad P_{\min}^{(i)} < P < P_{\max}^{(i)} \quad \text{AND} \quad P_{t\min}^{(i)} < P_t < P_{t\max}^{(i)}
\]

where \(\theta\) (degree), \(E\) (GeV), \(P\) (GeV/c) and \(P_t\) (GeV/c) indicate polar angle, energy, momentum and transverse momentum, respectively. The default values correspond to not applying this cut.

- **THPTMCT** \(\theta_{\min} \theta_{\max}\)
• **PTMXCT**  \( P_{t_{\text{min}}} \ P_{t_{\text{max}}} \)

Using the above 2 data cards, final state leptons are required to satisfy the following (REAL),

\[
\theta_{\text{min}} < \theta^M < \theta_{\text{max}} \quad \text{AND} \quad P_{t_{\text{min}}} < P_{t^M} < P_{t_{\text{max}}}
\]

where \( P_{t^M} \text{(GeV/c)} \) indicates the maximum transverse momentum among the 3 final state leptons \((e^\pm, l^\mp, l^\pm)\), and \( \theta \text{(degree)} \) is the polar angle of the lepton with \( P_{t^M} \). The default values correspond to not applying this cut.

• **MASSLL**  \( \text{Min1 Max1 Min2 Max2} \)

Range for the mass of the produced dilepton system (REAL);

\( \text{Min1} = \text{the minimum in GeV } (D=0.), \)
\( \text{Max1} = \text{the maximum in GeV } (D=1.E20). \)

In case of di-\( \mu, \tau \), \( \text{Min2} \) and \( \text{Max2} \) are not used.

In case of di-\( e \) with \( e^\pm e^\pm \) interference, there are two masses; \( M_{e^+e^-}^{(1)}, M_{e^+e^-}^{(2)} \) \((M_{e^+e^-}^{(1)} < M_{e^+e^-}^{(2)}), \) and they are required to satisfy the following,

\[
\text{Min1} < M_{ee}^{(1)} < \text{Max1} \quad \text{AND} \quad \text{Min2} < M_{ee}^{(2)} < \text{Max2}.
\]

• **MASSLL**  \( \text{Min Max} \)

Range for the mass of the final state lepton system of \( e^\pm l^\mp l^\pm \) (REAL);

\( \text{Min} = \text{the minimum in GeV } (D=1. ), \)
\( \text{Max} = \text{the maximum in GeV } (D=1.E20). \)

• **MASSQLL**  \( \text{Min Max} \)

Range for the mass of the scattered quark and produced dilepton system of \( q l^+l^- \) (REAL);

\( \text{Min} = \text{the minimum in GeV } (D=5. ), \)
\( \text{Max} = \text{the maximum in GeV } (D=1.E20). \)

This cut has an effect only on the DIS process.

In case of di-\( e \) with \( e^\pm e^\pm \) interference, smaller one of the 2 values is used.

• **IVISI**  \( N_{visi} \)

• **THEVMIN**  \( \theta_{\text{min}}^{(1)} \ \theta_{\text{min}}^{(2)} \ \theta_{\text{min}}^{(3)} \ \theta_{\text{min}}^{(4)} \)

• **THEVMAX**  \( \theta_{\text{max}}^{(1)} \ \theta_{\text{max}}^{(2)} \ \theta_{\text{max}}^{(3)} \ \theta_{\text{max}}^{(4)} \)

• **EVMIN**  \( E_{\text{min}}^{(1)} \ E_{\text{min}}^{(2)} \ E_{\text{min}}^{(3)} \ E_{\text{min}}^{(4)} \)

• **EVMAX**  \( E_{\text{max}}^{(1)} \ E_{\text{max}}^{(2)} \ E_{\text{max}}^{(3)} \ E_{\text{max}}^{(4)} \)

• **PTVMIN**  \( P_{t_{\text{min}}}^{(1)} \ P_{t_{\text{min}}}^{(2)} \ P_{t_{\text{min}}}^{(3)} \ P_{t_{\text{min}}}^{(4)} \)

• **PTVMAX**  \( P_{t_{\text{max}}}^{(1)} \ P_{t_{\text{max}}}^{(2)} \ P_{t_{\text{max}}}^{(3)} \ P_{t_{\text{max}}}^{(4)} \)

(1) : for scattered proton or quark,
(2) : for scattered $e^\pm$,
(3) : for produced $l\bar{l}$,
(4) : for produced $l^\pm$.

The above 6 data cards are used for describing the detector cut in the laboratory frame (REAL except for $N_{visi}$: INTEGER). $N_{visi}$ particle(s) are required to satisfy the following,

$$\theta^{(i)}_{\text{min}} < \theta < \theta^{(i)}_{\text{max}} \text{ AND } E^{(i)}_{\text{min}} < E < E^{(i)}_{\text{max}} \text{ AND } P_{\text{t}}^{(i)}_{\text{min}} < P_{\text{t}} < P_{\text{t}}^{(i)}_{\text{max}}$$

where $\theta$ (degree), $E$ (GeV) and $P_{\text{t}}$ (GeV/c) indicate polar angle, energy and transverse momentum, respectively. As for $N_{visi}$, $D=-1$, which corresponds to not applying this cut. The test run at the end of this paper is instructive for understanding this cut.

5 Summary

A new Monte Carlo generator for dilepton production in the framework of the electroweak theory was presented. The whole kinematical region on the proton vertex is covered. This generator can be used for quantitative and precise estimations of processes which come in addition to the two-photon Bethe-Heitler contributions.

Acknowledgements

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Appendix A. Event store in /PYJETS/

Each line in the PYTHIA event store has the following meaning according to the PYTHIA convention.

| Line number | Meaning |
|-------------|---------|
| 1,2         | Beam particles (1:p, 2:e±) |
| 3,4         | Partons from the beam particles before ISR (3:from p, 4:from e±) |
| 5,6         | Partons from the beam particles after ISR; the initial state in the matrix element calculation (5:from p, 6:from e±) |
| 7,8,9,10    | Final state particles before FSR; the final state in the matrix element calculation (7:p, 8:e±, 9:l∓, 10:l±) |
| 11~         | Final state particles after FSR, fragmentation and decay |

In case of the (quasi-)elastic process,

- a parton from the beam proton is the proton itself, \( i.e. \) the 1st and the 3rd lines are the same,
- the beam proton always makes no ISR, so that the 1st, 3rd and 5th lines are the same,
- the 2nd and the 4th lines are also the same.

In case of the DIS process with both ISR and FSR, all of the lines have different contents in general.

As for di-\( e \) events, there are 2 identical particles in the final state. In GRAPE, those 2 particles are distinguished in the following way;

- in case of \( e^±e^± \) interference off, a particle stored in the 8th line is a scattered lepton, and one in the 10th line comes from the 2-\( \gamma \) collision (\( i.e. \) a produced lepton),
- in case of \( e^±e^± \) interference on, a lepton stored in the 8th line has smaller transverse momentum than that of a lepton in the 10th line.
Appendix B. Routines/function related to users

- **Function** `DRN(ISEED)`
  provides uniform random numbers. All other random number routines are linked to this one. This routine is stored in the `BASES` library (`libbases.a`).
- **Subroutine** `DRNSET(ISEED)`
  performs an initialization for `DRN(ISEED)`. This is also stored in `libbases.a`.
- **Subroutine** `READ_CARDS(LUN,filename)`
  reads input data cards from `grape.cards`.
- **Subroutine** `SETMAS`
  provides masses/widths of particles and the QED coupling constant.
- **Subroutine** `USRSTR(Ievt,Ngen)`
  can be modified by users to access the information on generated events. User initialization/termination procedures are also put in this routine.
Appendix C. User event storing routine: USRSTR

This routine is called \( (N_{gen} + 2) \) times where \( N_{gen} \) is the number of generated events. The 1st call is for the user initialization and the last one for the user termination phase. The following is the template file prepared in the GRAPE package (usrstr.f).

```fortran
subroutine USRSTR(Ievt, Ngen)
  implicit NONE
  *------------------- Arguments -------------------
  integer Ievt, Ngen
  * Ngen : # of events to be generated
  * Ievt : Counter --- < 1 ===> Initialization phase
  * 1 - Ngen ===> Event generation phase
  * > Ngen ===> Termination phase
  *-------------------------------------------------------------------
  *-------- PYTHIA common --------
  integer N,NPAD, K(4000,5)
  double precision P(4000,5), V(4000,5)
  common /PYJETS/ N,NPAD,K,P,V !!! Event Record !!!
  integer MINT(400)
  double precision VINT(400)
  common /PYINT1/ MINT,VINT
  * (See PYTHIA manual for details.)
  *-------------------------------------------------------------------
  *-------- Local variables --------
  integer LUN1, LUN2, LUN3
  parameter (LUN1=41, LUN2=42, LUN3=43)
  * (You can use the above logical unit numbers.)
  *-------------------------------------------------------------------

  ******** Initialization of USER Event Storing ********
  if (Ievt .LT. 1) then
    endif
  *****************************************************

  *********** <<< USER Event Storing >>> ***********
  if ((Ievt .GE. 1).and.(Ievt .LE. Ngen)) then
    endif
  ****************************************************

  ********** Termination of USER Event Storing ********
  if (Ievt .GT. Ngen) then
    endif
  *****************************************************

  return
end
```

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References

[1] S. P. Baranov, O. Dünger, H. Shooshtari and J.A.M. Vermaseren, 
LPAIR — A Generator for Lepton Pair Production, 
Proc. of the workshop on Physics at HERA, Vol.3, p.1478, 
Eds: W. Buchmüller and G. Ingelman, DESY(1992)

[2] D. Hoffmann, L. Favart, Lepton Pair Monte Carlo Generators for HERA Physics, 
Proc. of the workshop on Monte Carlo Generators for HERA Physics, 
Eds: A.T. Doyle, G. Grindhammer, G. Ingelman and H. Jung, DESY(1999)

[3] J.A.M. Vermaseren, Nucl. Phys. B229 (1983) 347

[4] U. Baur, J.A.M. Vermaseren and D. Zeppenfeld, Nucl. Phys. B375 (1992) 3

[5] T. Ishikawa, T. Kaneko, K. Kato, S. Kawabata, Y. Shimizu and H. Tanaka, 
KEK Report 92-19 (1993)

[6] F. Halzen and A. D. Martin, Quarks and Leptons: An Introductory Course in Modern Particle Physics, John Wiley & Sons, 1984

[7] F. W. Brasse et al., Nucl. Phys. B110 (1976) 413

[8] H. Abramowicz and A. Levy, DESY-97-251 (hep-ph/9712415)

[9] A. Mücke et al., Comput. Phys. Commun. 124 (2000) 290

[10] H. Plothow-Besch, Comput. Phys. Commun. 75 (1993) 396

[11] T. Sjöstrand, Comp. Phys. Comm. 82 (1994) 74

[12] J. Fujimoto, M. Igarashi, N. Nakazawa, Y. Shimizu and K. Tobimatsu, 
Prog. Theor. Phys. Suppl. 100 (1990) 285

[13] H. Burkhardt and B. Pietrzyk Phys. Lett., B356 (1995) 398

[14] R. Brun, et al., FFREAD User Guide and Reference Manual, CERN DD/US/71, CERN Program Library I302, February 1987

[15] S. Kawabata, Comput. Phys. Commun. 88 (1995) 309
[16] R. Brun and D. Lienart, "HBOOK User Guide – Version 4", CERN Program Library Y250, 1988

[17] M. Glück, E. Reya and A. Vogt, Z. Phys. C67 (1995) 433
TEST RUN INPUT AND OUTPUT

An example of the input data cards

One example is presented with the following condition:

- process: $e^+ q \rightarrow e^+ q \mu^+ \mu^- \ (q=\{\bar{u}, \bar{d}, \bar{s}\})$ with GRV94(LO) [17],
- BH diagrams only,
- 70% polarization of the $e^+$ beam in the direction of the proton beam,
- cuts: (1) & (2) & (3) & (4),
  (1) $Q^2_p > 1 \text{GeV}^2$ & $M_{\text{had}} > 5 \text{GeV}$,
  (2) for scattered $q$, $\theta > 10^{\circ}$ & $P_t > 15 \text{GeV}/c$,
  (3) (invariant mass of $\mu^+ \mu^-$) > $4 \text{GeV}$,
  (4) 2 of the following 3 requirements for the final state leptons are satisfied,
  - for $e^+$: $5^{\circ} < \theta < 175^{\circ}$ & $P_t > 5 \text{GeV}$,
  - for $\mu^+$: $20^{\circ} < \theta < 160^{\circ}$ & $P_t > 3 \text{GeV}$,
  - for $\mu^-$: $20^{\circ} < \theta < 160^{\circ}$ & $P_t > 3 \text{GeV}$.

Following is the corresponding grape.cards. The same file is put in the directory sample.

```
LIST
NCALL  1400000
C ==================================================
C << Polarization of the Lepton Beam >>
C   (1)  (2)  (3)
EPOL   -0.7  0.  0.
C ==============================================================
C << Process in the Proton Vertex >>  (1:elastic, 2:quasi-elastic, 3:DIS)
PROCESS  3
C ==============================================================
C << Produced Lepton-pair >>  (1:di-e, 2:di-mu, 3:di-tau)
LPAIR   2
C ==============================================================
C << Scattered Quark in DIS >>
C   (1:u, 2:u-bar, 3:d, 4:d-bar, 5:s, 6:s-bar, 7:c, ..., 12:t-bar)
QFLV    1
MERGE  123456
C ==============================================================
C << PDF set in DIS >>  (See PDFLIB manual.)
NGROUP  5
```
NSET 5
C =========================================================================
C ######################################################################
C =========================================================================
C << Electroweak Dilepton Production >>
GRASEL 2
C =========================================================================
C ######################################################################
C =========================================================================
C << Mass Range for the Hadronic System >> (only for quasi-elastic and DIS)
MHAD 5. 300.
C ------------
C << Q2 Range for the Proton Vertex >>
Q2P 1. 1.E20
C ------------
C << Cuts for each Final-state Particle >>
C <p/q> <e+-> <l-+> <l+->
THMIN 10. 0. 0. 0.
THMAX 180. 180. 180. 180.
EMIN 0. 0. 0. 0.
EMAX 1.E20 1.E20 1.E20 1.E20
PMIN 0. 0. 0. 0.
PMAX 1.E20 1.E20 1.E20 1.E20
PTMIN 15. 0. 0. 0.
PTMAX 1.E20 1.E20 1.E20 1.E20
C ------------
C << Mass cuts >>
MASSLL 4. 1.E20
0. 1.E20
C ------------
C << Cuts for One or Some of the Final-state Particles >>
IVISI 2
C <p/q> <e+-> <l-+> <l+->
THEVMIN 0. 5. 20. 20.
THEVMAX 0. 175. 160. 160.
EVMIN 0. 0. 0. 0.
EVMAX 0. 1.E20 1.E20 1.E20
PTVMIN 0. 5. 3. 3.
PTVMAX 0. 1.E20 1.E20 1.E20
C =========================================================================
STOP

21
A part of the standard output from integ

---

--- DIS process
(Scattering of e and u quark)
--- Muon-pair production

grace 2.1(5)

(c)Copyright 1990-1998 Minami-Tateya Group (Japan)

>>> Graph selection
jselg =
  1 : 0
  2 : 0
  3 : 0
  4 : 0
  5 : 0
  6 : 0
  7 : 0
  8 : 0
  9 : 1
 10 : 0
 11 : 0
 12 : 0
 13 : 1
 14 : 0
 15 : 0
 16 : 0
 17 : 0
 18 : 0
 19 : 0
 20 : 0
 21 : 0
 22 : 0
 23 : 0
 24 : 0
 25 : 0
Kinematics Initialization

Information (in Lab. frame)

- P of electrons = 27.5200 GeV
- P of protons = 820.000 GeV
- Mass of electron = 5.10999E-04 GeV
- Mass of proton = 0.938272 GeV
- \sqrt{S} = 300.444 GeV
- P of CMS = 792.480 GeV
- E of CMS = 847.521 GeV
- Gamma of CMS = 2.82089
- Beta Gamma of CMS = 2.63770

Mass range for the hadronic system
- Min. = 5.00000 GeV
- Max. = 300.000 GeV

PDFLIB Initialization started

PDFLIB Version: 7.09 Released on 970702 at 16.05
in the CERN Computer Program Library W5051
Library compiled on 970702 at 16.05

Nptype = 1 Ngroup = 5 Nset = 5 Name = "GRV94-LO" CrMode = -1
NFl = -5, LO = 1, Tmas = 180.00 GeV/c**2
QCDL4 = 0.2000 GeV, QCDL5 = 0.1530 GeV
Xmin = 0.10E-05, Xmax = 0.99999E+00,
Q2min = 0.400 (GeV/c)**2, Q2max = 0.10E+07 (GeV/c)**2

PDFLIB Initialization finished

ISR for incoming lepton using Structure Function method

End of Kinematics Initialization

BASES Version 5.1
coded by S.Kawabata KEK, March 1994

Parameters for BASES
(1) Dimensions of integration etc.

- No of dimensions: \( N_{\text{dim}} = 9 \) (50 at max.)
- No of Wilds: \( N_{\text{wild}} = 7 \) (15 at max.)
- No of sample points: \( N_{\text{call}} = 1399680 \) (real) 1400000 (given)
- No of subregions: \( N_{\text{g}} = 48 \) / variable
- No of regions: \( N_{\text{region}} = 6 \) / variable
- No of Hypercubes: \( N_{\text{cube}} = 279936 \)

(2) About the integration variables

| i  | \( X_L(i) \)    | \( X_U(i) \)    | \( I_G(i) \) | Wild |
|----|----------------|----------------|-------------|------|
| 1  | 0.000000E+00   | 1.000000E+00   | 1           | yes  |
| 2  | 0.000000E+00   | 1.000000E+00   | 1           | yes  |
| 3  | 0.000000E+00   | 1.000000E+00   | 1           | yes  |
| 4  | 0.000000E+00   | 1.000000E+00   | 1           | yes  |
| 5  | 0.000000E+00   | 1.000000E+00   | 1           | yes  |
| 6  | 0.000000E+00   | 1.000000E+00   | 1           | yes  |
| 7  | 0.000000E+00   | 1.000000E+00   | 0           | no   |
| 8  | 0.000000E+00   | 1.000000E+00   | 0           | no   |

(3) Parameters for the grid optimization step

- Max. # of iterations: \( I_{\text{TMX1}} = 4 \)
- Expected accuracy: \( A_{\text{cc1}} = 0.2000 \% \)

(4) Parameters for the integration step

- Max. # of iterations: \( I_{\text{TMX2}} = 10 \)
- Expected accuracy: \( A_{\text{cc2}} = 0.0100 \% \)

Date: 0/ 4/12 21:29

Convergency Behavior for the Grid Optimization Step

| IT | Eff | R_Neg | Estimate | Acc % | Estimate(+ Error) | order | Acc % ( H: M: Sec ) |
|----|-----|-------|----------|-------|-------------------|-------|---------------------|
| 1  | 0.00| 4.586E-02 | 6.435 | 4.586406(+0.295122)E-02 | 6.435 | 0 : 1:37.57 |
| 2  | 0.00| 5.205E-02 | 0.295 | 5.203249(+0.015359)E-02 | 0.295 | 0 : 7:23.86 |
| 3  | 0.00| 5.196E-02 | 0.194 | 5.196162(+0.008625)E-02 | 0.166 | 0 :13:52.62 |

Date: 0/ 4/12 21:29

Convergency Behavior for the Integration Step

| IT | Eff | R_Neg | Estimate | Acc % | Estimate(+ Error) | order | Acc % ( H: M: Sec ) |
|----|-----|-------|----------|-------|-------------------|-------|---------------------|
| 1  | 0.00| 5.196E-02 | 0.194 | 5.196162(+0.010077)E-02 | 0.194 | 0: 20:27.03 |
| 2  | 0.00| 5.201E-02 | 0.295 | 5.203249(+0.015359)E-02 | 0.295 | 0: 7:23.86 |
| 3  | 0.00| 5.196E-02 | 0.194 | 5.196162(+0.008625)E-02 | 0.166 | 0:13:52.62 |

****** END OF BASES ********
<< Computing Time Information >>

(1) For BASES
   H: M: Sec
   Overhead   : 0: 0: 0.12
   Grid Optim. Step : 0:13:52.62
   Integration Step : 1: 5:42.20
   Go time for all : 1:19:34.94

(2) Expected event generation time
   Expected time for 1000 events : 0.40 Sec

Making bases.rz...
   ---> BASE1: finished
   ---> BASE3: finished
   ---> BASE4: finished
   ---> BASE5: finished
   ---> RANDM: finished
   ---> PLOTH: finished
   ---> PLOTB: finished
   ---> BSRSLT: finished

==> Directory : //bn
   3 (N) BASES data(real*8)
   1 (N) BASES data(integer*4)
   2 (N) BASES data(real*4)
A part of the standard output from spring

Suppressing decay of the following particles in PYTHIA,

K_S0  eta  Lambda0  Sigma+  Sigma0  Sigma-  Xi0
Xi-  Omega-  D+  D0  D_s+  Lambda_c+  mu-
tau-  pi+  K+  K_L0  p0

Loading bases.rz...
---› BASE1 : finished
---› BASE3 : finished
---› BASE4 : finished
---› BASE5 : finished
---› RANDM : finished
---› PLOTH : finished
---› PLOTB : finished
---› BSRSLT: finished

****************************************************************************
****************************************************************************
**  **
**  **
** *......* Welcome to the Lund Monte Carlo! **
** *:::!!:::::::::::* 
** *::::::!!::::::::::::::* PPP Y Y TTTTT H H III A 
** *::::::::!!::::::::::::::::* P P Y Y T H H I A A **
** *:::::::::!!:::::::::::::::::* PPP Y T HHHHH I AAAAA **
** *::::::::!!:::::::::::::::::*! P Y T H H I A A **
** *::::::!!::::::::::::::* !!**
** !! *:::!!:::::::::::* !! This is PYTHIA version 6.136 **
** !! pp !! Last date of change: 30 Nov 1999 **
**!! ep !! Disclaimer: this program comes **
**!! pp !! with no guarantees. Beware **
**!! e+e- !! of errors and use common sense **
**!! !! when interpreting results. **
**!! ep !! Copyright T. Sjostrand (1999) **
**!! ep !! An archive of program versions and documentation is found on the web: **
**!! ep !! http://www.thep.lu.se/~torbjorn/Pythia.html **
**!! pp !! When you cite this program, currently the official reference is **
**!! pp !! T. Sjostrand, Computer Physics Commun. 82 (1994) 74. **
**!! pp !! The supersymmetry extensions are described in **
**!! ep !! S. Mrenna, Computer Physics Commun. 101 (1997) 232 **
**!! pp !! Also remember that the program, to a large extent, represents original **
**!! pp !! physics research. Other publications of special relevance to your **
**!! pp !! studies may therefore deserve separate mention. **
**!! pp !! Main author: Torbjorn Sjostrand; Department of Theoretical Physics 2, **
**!! pp !! Lund University, Solvvegan 14A, S-223 62 Lund, Sweden; **
**!! pp !! phone: + 46 - 46 - 222 48 16; e-mail: torbjorn@thep.lu.se **
**!! pp !! SUSY author: Stephen Mrenna, Physics Department, UC Davis, **
**!! pp !! One Shields Avenue, Davis, CA 95616, USA; **
**!! pp !! phone: + 1 - 530 - 752 - 2661; e-mail: mrenna@physics.ucdavis.edu **
**
PYINIT: initialization of PYTHIA routines

PYTHIA will be initialized for p+ on e+ user configuration

px (GeV/c) py (GeV/c) pz (GeV/c) E (GeV)
p+ 0.000 0.000 820.000 820.001
e+ 0.000 0.000 -27.520 27.520

I corresponding to 300.444 GeV center-of-mass energy

PYMAXI: summary of differential cross-section maximum search

ISUB Subprocess name Maximum value
308 e+ uu^dd^ss^ -> e+ q m+ m- 5.1962D-02

------------------ START of SPRING at 1:21(13/ 4/ 0)
Number of generated events = 100

Event listing (summary)

| particle/jet | particle type | KS | KF | orig | p_x | p_y | p_z | E  | m  |
|--------------|---------------|----|----|------|-----|-----|-----|-----|-----|
| 1            | p+            | 21 | 2212 | 0    | 0.000 | 0.000 | 820.000 | 820.001 | 0.938 |
| 2            | e+            | 21 | -11 | 0    | 0.000 | 0.000 | -27.520 | 27.520 | 0.001 |
| 3            | d            | 21 | 1    | 1    | -0.343 | -0.058 | 168.409 | 168.409 | 0.000 |
| 4            | e+            | 21 | -11 | 2    | 0.000 | 0.000 | -27.520 | 27.520 | 0.000 |
| 5            | d            | 21 | 1    | 3    | 1.167 | -0.401 | 138.481 | 138.487 | 0.000 |
| 6            | e+            | 21 | -11 | 4    | 0.000 | 0.000 | -27.497 | 27.497 | 0.000 |
| 7            | d            | 21 | 1    | 0    | -10.149 | 16.228 | 96.119 | 96.007 | 0.004 |
| 8            | e+            | 21 | -11 | 0    | 0.009 | 0.020 | -6.958 | 6.958 | 0.001 |
| 9            | mu-          | 21 | 13   | 9    | 19.804 | -7.523 | -12.433 | 24.564 | 0.106 |
| 10           | mu+          | 21 | -13 | 0    | -8.498 | -9.126 | 34.256 | 36.455 | 0.106 |
|              |              |    |     |      |       |       |     |     |     |
| 11           | e+           | 1  | -11 | 8    | 0.009 | 0.020 | -6.958 | 6.958 | 0.001 |
| 12           | mu-          | 1  | 13   | 9    | 19.804 | -7.523 | -12.433 | 24.564 | 0.106 |
| 13           | gamma        | 1  | 22   | 4    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 14           | mu+          | 1  | -13  | 10   | -7.882 | -8.464 | 31.770 | 33.810 | 0.106 |
| 15           | gamma        | 1  | 22   | 2    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 16           | (d) A        | 12 | 1    | 7    | -9.379 | 15.009 | 77.561 | 79.565 | 0.004 |
| 17           | (g) I       | 12 | 21   | 7    | -1.386 | 0.556 | 21.044 | 21.097 | 0.000 |
| 18           | (g) I       | 12 | 21   | 3    | -1.510 | 0.343 | 29.905 | 29.945 | 0.000 |
| 19           | (uu_1) V    | 11 | 2203 | 1    | 0.343 | 0.058 | 651.591 | 651.591 | 0.771 |
| Subprocess | Generated | Tried    | Sigma    | Cross-section |
|------------|-----------|----------|----------|---------------|
| 0 All included subprocesses | 100 | 100 | 5.196D-02 | 1.00E00 |
| 008 e+ uu\`dd\`ss\` -> e+ q m+ m- | 100 | 100 | 5.196D-02 | 1.00E00 |

Fraction of events that fail fragmentation cuts = 0.00000