Non-Gaussian distribution of nearest-neighbour Coulomb peak spacings in metallic single electron transistors

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The distribution of nearest-neighbour spacings of Coulomb blockade oscillation peaks in normal conducting aluminum single electron transistors is found to be non-Gaussian. A pronounced tail to reduced spacings is observed, which we attribute to impurity-specific parametric charge rearrangements close to the transistor. Our observation may explain the absence of a Wigner-Dyson distribution in the experimental nearest-neighbour spacing distributions in semiconductor quantum dots.

I. INTRODUCTION

A single electron transistor (SET) consists of a small conductive island, coupled to two leads via tunnel barriers, and a nearby gate used to tune the electrochemical potential of the island. The Coulomb blockade, characterized by the charging energy $E_C$ needed to add a single electron to the island, governs the electronic properties of such devices. This leads to the observation of pronounced conductance oscillations, commonly denoted as Coulomb blockade (CB) oscillations, as a function of the gate voltage $V_g$. These effects are of electrostatic origin and can be analyzed in a purely classical picture. However, a variety of additional effects can be studied in SETs, depending on the material they are made of. In superconducting islands, for example, Cooper pair formation leads to significant modifications of the device characteristics. SETs can also be realized in two-dimensional electron gases residing in semiconductor hosts such as Si MOSFETs or Ga(Al)As heterostructures. Discrete energy levels and phase coherence effects superimposed on the Coulomb blockade can be observed. Such devices, also known as `quantum dots', have therefore become model systems to investigate numerous distinct effects. Broad attention has recently been paid to experiments measuring the distribution of nearest-neighbour spacings (NNS) of the CB oscillation peaks in quantum dots. From random-matrix theory calculations, the NNS distribution is expected to obey Wigner-Dyson statistics. However, the experimentally observed distributions differ significantly from the random-matrix theory predictions. In order to separate classical charging effects from quantum mechanics, it is generally accepted to use a constant-interaction (CI) model, which assumes that the electrostatics of the system is invariant under a change of the charge on the island by integer multiples of the elementary charge $e$.

In this paper, we report on NNSs of CB peaks in metallic, i.e. purely electrostatic or ‘classical’ SETs. In a simple picture appropriate for metallic devices, one would expect to observe constant peak spacings $\Delta V_g = e/C_g$ (with a distribution broadened by thermal fluctuations only), where $C_g$ is the capacitance between island and gate. However, we observe a strongly asymmetric NNS distribution with a pronounced tail to small peak separations.

II. EXPERIMENTS AND RESULTS

We have measured high quality Al/AlO$_x$/Al SETs written by electron beam lithography and fabricated by standard two angle evaporation technique\textsuperscript{7} with intermediate room temperature oxidation of the first layer (oxygen pressure of 2.5 mbar for 20 min) to develop the tunnel barriers. The substrate was silicon covered by 600 nm thermally grown SiO$_x$. The design of the SETs is drawn schematically in fig. 1\textsuperscript{a}). The typical parameters of the devices were $R_t = 1 \ldots 10$ M$\Omega$ for the tunnel resistances, $C_j = 40 \ldots 200$ aF and $C_g \approx 50$ aF for the junction and gate capacitances, respectively. The measurements were performed in a dilution refrigerator at temperatures down to 5 mK, whereas the effective electron temperature of the devices was determined to be 45 mK (as deduced from the thermal smearing of the charge occupation number in an electron box\textsuperscript{8}). Measurements were performed over periods of several days. The devices were highly stable for constant voltages applied, showing no drifts or spontaneous jumps for days. We attribute this stability to the very slow device cooling of about 1 day, allowing the impurities to be frozen in their lowest, most stable state.
The measured 1/f noise was identified as dominant SET input noise due to background charge fluctuation, being of magnitude comparable with typical noise figures (≈ 10^{-4} \sqrt{\text{Hz}} at 10 Hz) reported for other metallic SETs. A magnetic field of \( B = 1...4 \) T was applied to suppress superconductivity.

FIG. 1. a) Schematic layout of the investigated SET devices. The tunnel barriers are formed between the overlap of island and electrodes. b) Typical measured conductance oscillations (dots). Perfect correspondence between experimental data and theoretical fit (see main text) is shown for the middle peak (solid line). The distributions of CB peak amplitudes (together with a Gaussian fit) and NNSs are shown in c) and d), respectively. The inset in d) shows \( \Delta V_g \) as a function of the peak position.

The DC current through the devices was measured as a function of bias and gate voltages \( V_b \) and \( V_g \), respectively. Due to the DC measurement technique we took large sets of sufficiently dense points by variation of the voltages in the ranges \(| V_b | \leq \frac{1}{4} E_C/e \) and \(| V_g | \leq 1 \) V. The latter corresponds to a difference of some hundred electrons on the island. The data was analyzed by fitting the conductance peaks with \( G(V_g) = G_0(\delta V_g/w)/\sinh(\delta V_g/w) \), where \( \delta V_g = | V_g^{n+1} - V_g^n | \), yielding amplitude \( G_0 \), width \( w = (k_B T/2E_C)(e/C_g) \) and position \( V_g^0 \). A partial trace of typical CB oscillations is shown in fig. 1b) together with a theoretical curve fitting. As expected for our devices, the amplitudes are found to be constant over the entire \( V_g \) range, with a standard deviation of typically 1.5%, as shown in fig. 1c). However, the distribution \( P(\Delta V_g) \) of NNS values \( \Delta V_g(n) = V_g^{n+1} - V_g^n \), where \( n \) is the peak index, is not Gaussian but shows a significant number of events with reduced values, cf. fig. 1d). The NNS distributions were quantitatively independent of \( V_b \) variations. The 8 samples investigated all showed similar behaviour. The main peak in \( P(\Delta V_g) \), containing the majority of the events (≈ 60...80 % for different samples), fits well to a Gaussian, whose width scales linearly with temperature.

We should mention that samples cooled at a much faster rate typically show strongly enhanced noise levels. Consequently, measurements of CB peak statistics with such devices yielded significantly broadened NNS distributions (not shown).

In order to investigate reproducibility of the reduced NNS events, we have performed measurements on a smaller \( V_g \) range, where only very few NNSs with reduced values are detected. Figure 2 shows traces with 16 conductance peaks each, taken from two consecutive \( V_g \) scans in the same direction. Both traces show two shifts in \( \Delta V_g \) at the same positions. It has been found in general that the position range where NNSs significantly smaller than the mean value \( \langle \Delta V_g \rangle \) occur, is well reproduced as a function of \( V_g \). In addition, a clear difference in low \( \Delta V_g \) positions between up and down scans was observed, suggesting a hysteretic behaviour. More details on the reproducibility and the hysteresis effect will be published elsewhere.
FIG. 2. Two traces of measured CB oscillations taken from consecutive $V_g$ scans swept in one direction (the second trace is offset by 50 nS). Two reduced NNSs in each trace are observed (denoted by arrows), occurring at the same $V_g$ positions for both scans. The vertical dashed lines correspond to a perfectly constant CB peak spacing of $\Delta V_g = e/C_g$.

The CB peak position fluctuations do not show significant correlation as a function of $V_g$. The standard deviation of the $n$-th neighbour peak spacing is very closely proportional to $\sqrt{n}$, which is expected for uncorrelated events. We could not find any specific periodicity from a Fourier analysis of the CB peak position spectrum either.

The measured noise was essentially proportional to the SET gain $dI/dV_g$, i.e., dominated by device input noise. In very few cases a significantly increased noise level was observed, with a non-zero correlation with events of reduced CB peak width. This is attributed to the well-known dynamic switching of background charges (‘random telegraph noise’, RTN) for certain $V_g$ values close to the fluctuator threshold. Considering the rare occurrence of correlated excess noise with a non-average NNS, we conclude that the fluctuators producing dynamic noise are not primarily responsible for the observed reduction of NNSs.

In addition, we should emphasize that the measured fluctuation distributions did not depend on the absolute $V_g$ range considered (cf. also inset in fig. 1d).

III. DISCUSSION

Based on the theoretically expected behaviour of our SET transistors and the experimental results discussed above, we explain the observations with discontinuous switching of two-level tunnelling systems (TLTS) where the displacement of a single charge modifies the transistor island potential. Figure 3(a) shows schematically how such switching events can explain the systematic occurrence of reduced NNSs. Consider a TLTS in a metastable state, located in between the gate and the SET island. Exceeding a particular $V_g$ threshold, a charge rearrangement can be induced in the TLTS. In response to this, the electrochemical potential $\mu_{SET}$ of the SET island, which is usually tuned continuously by $V_g$, experiences a sudden jump in the same direction as the $V_g$ variation, independent of the scan direction. Consequently, a smaller $\Delta V_g$ is needed in order to change the island occupation number by one. Within this picture, the tail in the NNS distribution reflects the spatial and energetical distribution of TLTSs in some region between the island and the gate electrode. The adjustment of a dipole following the variation of an electric field is equivalent to the picture of introducing a medium with increased dielectric constant, increasing $C_g$ and decreasing $\Delta V_g$.

Assuming a simple system of an electron switching locally between two sites, the measured CB peak spacing statistics $P(\Delta V_g)$ can be analyzed considering spacial distribution and type of such TLTSs. According to electrostatic dipole calculations, we derive the position of the electron and its displacement which allow a variation of island charge on the order of 10% (as in our experiments): an electron located very close to the island ($\leq 1$ nm) and facing the gate electrode requires a displacement (radially away from the island) by $2 \ldots 4$ nm. A process of a charge displacement by a few nanometres is very well consistent with other studies on charge trapping. However, by slightly increasing the electron’s distance from the island, the necessary displacement quickly grows to length scales for which the observed reproducibility of TLTS switching becomes very unlikely. The largest electric fields are found between island and gate electrode, whereas the field is shielded or strongly reduced elsewhere, thereby reducing the trap switching effect to a negligible level. This explains the asymmetry of $P(\Delta V_g)$ with a tendency to lower values. Hence, the shape of the $P(\Delta V_g)$ distribution is determined by geometry and materials of the device and the surroundings.
SET transistors are known to be the best electrometers to date, with a sensitivity to charge variations by a small fraction of the electron charge $e$. In contrast to the random dynamic background charge fluctuations, the TLTSs in our case are highly stable in time, depending (in first approximation) on electrical potential variations only. The thermal activation energy is typically much larger than our temperature range investigated. The reproducibility of the effects suggests a well-defined system of individual charge traps with negligible interaction. Defects, acting as charge traps, may particularly reside in oxides, at semiconductor heterointerfaces, or generally at any disordered interface or lattice. The low-frequency noise in metallic SET transistors is commonly attributed to background charge fluctuations mainly in the substrate, with a small probability of traps in the tunnel junctions.\footnote{A few studies on RTN have also been reported for semiconducting nanostructure devices.} Switching of background charges can be detected directly with the SET provided the device is in a sensitive state of non-zero gain, i.e., within a conductance peak. In our case, the switchings predominantly reduce the width of the peak. Under low bias conditions, most of these events occur in between the CB peaks and are not seen in the peak width. However, we can increase the detection range of the peaks for TLTS switching by making them wider, e.g., by increasing the bias voltage. The CB peak widths $w$ show a distribution $P(w)$ qualitatively similar to $P(\Delta V_g)$. The peak width in $P(w)$ reflects the temperature of the system. As plotted in fig. 3b, the position of the main $P(\Delta V_g)$ peak shows no variation with $V_b$, and the fraction of the tail events (FTE) in $P(\Delta V_g)$ also remains constant within experimental errors. On the other hand, the $P(w)$ main peak position has a thermally broadened minimum at $V_g = 0$ and increases almost linearly with increasing $|V_b|$, cf., fig. 3c. The FTE of the $P(w)$ distribution behaves proportionally to $w$, which is indicated by their ratio in fig. 3d). This confirms that the discontinuous jumps in $V_g$ are uniformly distributed along the $V_g$ axis, independent of the state of the SET transistor, i.e., whether it is in the CB regime or not. Apparently, the positions of the jumps depend on the gate potential only. Our arguments are further supported by correlated low tail events between the $P(\Delta V_g)$ and $P(w)$ distributions.

On one hand, our experimental results reveal important information for the understanding of charge fluctuation mechanisms in nanostructures, hopefully leading to an improvement of reliable and stable devices. This is particularly crucial for developments like quantum computing, ultra-low noise electrometers or metrological applications. We have demonstrated the possibility to detect directly discrete fluctuations of the background charge configuration, allowing a quantitative characterization of substrates or other dielectrics of interest.

On the other hand, we wish to emphasize in particular the impact of our results on the lively discussion on CB peak statistics in semiconductor quantum dots. Our experiments show that even in the absence of single particle energy levels on the SET island, the NNS distribution can deviate significantly from a Gaussian, since it is an intrinsic feature of the island to react with high sensitivity to background charge rearrangements. So far, all experiments studying the NNS distribution of quantum dots have been performed by measuring CB oscillations as a function of a gate electrode.\footnote{The observed peak spacings in $V_g$ have been corrected using the CI model. However, the remaining NNS distribution does contain the modification of level spacings as a consequence of rearrangements in the random background charge configuration and cannot solely be attributed to the energy spectrum of the quantum dot. The charge and potential distribution in a two-dimensional electron gas (2DEG) is known to be fairly inhomogeneous and sensitive to even small perturbations of electromagnetic field.\footnote{Furthermore, single electron charging effects among isolated regions due to non-uniform potential distribution in a 2DEG have recently been observed.\footnote{Consequently,}}
charge sensitive nanodevices made of semiconducting structures may reveal a significantly modified behaviour due to charging effects, of the origin described above. In terms of our model explanations, it can be easily understood, e.g., why Simmel et al. observe much broader NNS distributions in quantum dots defined in Si MOSFETs than those distributions observed in Ga(Al)As heterostructures, since there are more traps in SiO$_2$ than in heterostructures grown by molecular beam epitaxy.

In order to go beyond the CI model, we therefore suggest that the charge rearrangements in the vicinity of the quantum dot should be measured independently. In detail, one could define a metallic 'control' SET on top of a quantum dot, which is used to correct each individual peak spacing of the quantum dot for the charge fluctuations in the environment.

In summary, we have measured non-Gaussian distributions of nearest-neighbour spacings in normal conducting aluminum single electron transistors. A significant part of the peak spacings is reduced to lower values. We interpret this effect in terms of reproducible background charge rearrangements, which take place in close vicinity to the SET island, and are predominantly induced by gate voltage changes.

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