A picture of the Yang-Mills deconfinement transition and its lattice verification

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Abstract

In the framework of the center vortex picture of confinement, the nature of the deconfining phase transition is studied. Using recently developed techniques which allow to associate a center vortex configuration with any given lattice gauge configuration, it is demonstrated that the confining phase is a phase in which vortices percolate, whereas the deconfined phase is a phase in which vortices cease to percolate if one considers an appropriate slice of space-time.

Heuristics of the center vortex picture

A discussion of the deconfinement transition in Yang-Mills theory presupposes a picture of the phenomenon of confinement. Conversely, any picture of confinement should be able to accommodate the deconfinement phase transition. The work presented here is concerned specifically with the so-called center vortex picture of confinement; this picture is based on the conjectured presence of center vortices in typical Yang-Mills gauge configurations. These vortices represent closed magnetic flux lines in three space dimensions, describing closed two-dimensional world-sheets in four space-time dimensions. Space-time in the following will always be considered Euclidean. The magnetic flux represented by the vortices is furthermore quantized such that a Wilson loop linking vortex flux takes a value corresponding to a nontrivial center element of the gauge group. In the case of $SU(2)$ color discussed here, the only such element is $(-1)$. For $N$ colors, there are

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$N - 1$ different possible vortex fluxes corresponding to the $N - 1$ nontrivial center elements of $SU(N)$.

Consider an ensemble of center vortex configurations in which the vortices are distributed randomly, specifically such that intersection points of vortices with a given two-dimensional plane in space-time are found at random, uncorrelated locations. In such an ensemble, confinement results in a very simple manner. Let the universe be a cube of length $L$, and consider a two-dimensional slice of this universe of area $L^2$, with a Wilson loop embedded into it, circumscribing an area $A$. On this plane, distribute $N$ vortex intersection points at random, cf. Fig. 1 (left). According to the specification above, each of these points contributes a factor $(-1)$ to the value of the Wilson loop if it falls within the area $A$ spanned by the loop; the probability for this to occur for any given point is $A/L^2$.

The expectation value of the Wilson loop is readily evaluated in this simple model. The probability that $n$ of the $N$ vortex intersection points fall within the area $A$ is binomial, and, since the Wilson loop takes the value $(-1)^n$ in the presence of $n$

Figure 1: Simple models for confining (left) and deconfining (right) vortex ensembles.
intersection points within the area $A$, its expectation value is

$$
\langle W \rangle = \sum_{n=0}^{N} (-1)^n \binom{N}{n} \left( \frac{A}{L^2} \right)^n \left( 1 - \frac{A}{L^2} \right)^{N-n} = \left( 1 - \frac{2\rho A}{N} \right)^N \xrightarrow{N \to \infty} \exp(-2\rho A)
$$

where in the last step, the size of the universe $L$ has been sent to infinity while leaving the planar density $\rho = N/L^2$ of vortex intersection points constant. Thus, one obtains an area law for the Wilson loop, with the string tension $\sigma = 2\rho$.

This simple mechanism lies at the core of the center vortex picture of confinement. After having been proposed already in [1, 2], evidence that the Yang-Mills dynamics actually favors the formation of magnetic flux tubes arose in the framework of the Copenhagen vacuum [3]. Also lattice studies were initiated with the aim to study vortices [4, 5]. These studies in essence defined vortices via their effect on Wilson loops, as discussed above. While this definition has the advantage of being gauge invariant, it does not allow to easily localize vortices, i.e. associate a collection of vortex world-surfaces with any given lattice gauge configuration.

The absence of techniques allowing to carry out such an identification for a long time posed a considerable obstacle to the study of center vortex physics, especially the study of their global properties. These properties, however, constitute a crucial aspect for many applications, as a closer examination of the above heuristic picture shows. Namely, for vortex intersection points to be distributed in a sufficiently random manner on a space-time plane to induce an area law for the Wilson loop, the vortices must form networks which percolate throughout space-time. To see this, consider the converse, namely that vortices can be separated into clusters of bounded extension. This implies that any vortex intersection point on a plane comes with a partner a finite distance (smaller than the bound on the cluster extension) away, because vortices are closed. For simplicity, assume the pairs of intersection points to occur with a fixed mutual distance $d$, and distribute $N$ pairs on a space-time plane containing a Wilson loop of area $A$, cf. Fig. 1 (right), where the lines between the points in the figure are merely to guide the eye in identifying pairs of points. Now, the probability that any given pair contributes a factor $(-1)$ to the Wilson loop is $pPd/L^2$, where $P$ denotes the perimeter of the loop, since only pairs whose midpoints lie within a strip of width $d$ around the Wilson loop are able to contribute a factor $(-1)$, and they do this with a probability $p$ related to the angular distribution of the pairs. Note that $p$ is independent of the dimensions of the Wilson loop. The probability that $n$ pairs contribute a factor $(-1)$ is again binominal, in complete analogy to above, and one consequently obtains a perimeter
law for the expectation value of the Wilson loop in the limit of an infinite universe,

$$\langle W \rangle = \left(1 - \frac{2pPd}{L^2}\right)^N \xrightarrow{N \to \infty} \exp(-\rho pPd)$$

(2)

where $\rho = 2N/L^2$ again denotes the density of points. Thus, in the absence of percolation, confinement disappears. This leads to the conjecture that the deconfinement phase transition in the vortex picture may take the guise of a percolation transition. However, as already indicated above, to test such global properties of vortices in lattice experiments, new techniques are needed which allow to associate a vortex world-sheet configuration with any given lattice gauge configuration. These techniques have only been furnished quite recently, sparking renewed interest in the vortex picture. The present work is one contribution to these efforts.

**Locating vortices on the lattice**

The abovementioned techniques, introduced in [6, 7, 8], employ a two-step procedure familiar from the dual superconductor picture of confinement. First, one uses the gauge freedom to bring a given gauge configuration as close as possible to the collective degrees of freedom under consideration; in the case of the dual superconductor, that is the Abelian degrees of freedom, in particular, the monopoles. The second step consists of projecting onto these degrees of freedom, i.e. neglecting residual deviations away from, say, Abelian configurations in the case of the dual superconductor. This second step clearly constitutes a truncation of the theory. This idea was adapted to the case of vortex degrees of freedom as follows [6, 7, 8]. One fixes gauge configurations to the maximal center gauge,

$$\max \sum_i |\text{tr } U_i|^2$$

(3)

where the $U_i$ are the link variables on a space-time lattice. This procedure biases links towards elements of the center of the gauge group. Next, one performs a truncation of the configurations, namely center projection,

$$U \rightarrow \text{sign } \text{tr } U$$

(4)

i.e. one replaces each $SU(2)$ link variable by the center element closest to it in the group. Thus, one remains with a lattice of center elements. Such a lattice can be
associated in the standard fashion with a vortex configuration. One examines all plaquettes on the lattice, and if a plaquette takes the value \((-1)\), a vortex is said to pierce that plaquette. Thus, vortices in the lattice formulation are defined on the dual lattice, i.e. the lattice shifted by the vector \((a/2, a/2, a/2, a/2)\) w.r.t. the original one, \(a\) denoting the lattice spacing. One can easily convince oneself that the vortices defined in this way have all the properties postulated further above.

Having isolated vortices on the lattice, the first question to answer is whether these degrees of freedom do indeed determine the physics of confinement, i.e. whether they furnish the full string tension found in exact calculations without any truncations. Without this basis, more detailed considerations of vortex properties run the risk of being academical. One carries out two lattice experiments, both times using the full Yang-Mills action as a weight, but in one experiment, one calculates the observable in question, such as the Wilson loop, using the full configurations; in the other experiment, one uses the center projected configurations. If the results agree, the observable is said to display center dominance. Center dominance for the string tension has indeed been verified in \(SU(2)\) lattice gauge theory both at zero temperature [6, 7, 8] and at finite temperatures [9, 10], including the so-called “spatial string tension” all the way into the deconfined regime. Furthermore, the vortex density obeys the proper scaling law as dictated by the renormalization group for physical quantities, cf. [11] (note erratum in [9]) and [8].

**Vortex percolation properties**

Given techniques allowing to locate vortex world-sheets in space-time, or vortex loops on three-dimensional slices thereof, it is possible to discriminate between different vortex clusters. In the following, three-dimensional slices of space-time, where one of the space directions is left away, will be considered, since this displays the relevant percolation properties most clearly. To define a cluster, one finds a link on the dual lattice which is part of a vortex and furthermore locates all adjacent links which are also part of the vortex. This is repeated with all new links found, until no further links exist which are connected with the cluster in question. Having detected all vortex clusters in this manner, it is possible to determine the space-time extension of each cluster, i.e. the largest distance between any pair of points on the cluster. In a percolating phase, most of the available vortex length will be organized into clusters of the maximal possible extension, whereas in a phase with no vortex percolation, most of the vortex material present in the configuration
will be concentrated in clusters much smaller than the typical extension of the universe.

To generate “vortex material distributions” which allow to read off which scenario is realized, one simply measures both the extension of each cluster as well as the

Figure 2: Vortex material distributions.
number of links contained in it, and adds the latter number to a bin corresponding to the cluster extension in question. Fig. 2 displays such distributions, obtained for $\beta = 2.4$ on $12^3 \times N_t$ lattices [10], which have been normalized such that the integral over the distributions gives unity, and where the cluster extension on the horizontal axis is in units of the maximal extension possible in the universe in question. In view of Fig. 2, one indeed obtains a transition from a confining phase, in which vortices percolate, to a deconfining phase, in which they cease to percolate. This confirms the conjecture proposed above in the introductory section. If one analyzes the small vortex clusters dominating the deconfined phase in more detail, one finds that a large part of these vortices wind in the (Euclidean) temporal direction, i.e. the space-time direction whose extension is identified with the inverse temperature. Therefore, one finds that the typical configurations in the two phases can be characterized as displayed in Fig. 3 in a three-dimensional slice of space-time, where one space direction has been left away. Note that Fig. 3 also furnishes an explanation of the spatial string tension in the deconfined phase. A spatial Wilson loop embedded into Fig. 3 (right) can exhibit an area law, since intersection points of winding vortices with the minimal area spanned by the loop can occur in an uncorrelated fashion despite those vortices having small extension. Note also the dual nature of this (magnetic) picture as compared with electric flux models [12]. In such models, electric flux percolates in the deconfined phase, while it does not percolate in the confining phase.

Outlook

While it has thus been established how vortices generate the confining and deconfining phases of Yang-Mills theory, it remains to be clarified what the essential features of the dynamics underlying their behavior are. One interesting observation in this context is that a simple model of vortices as random surfaces in four-dimensional space-time already is able to generate the vortex phenomenology described above, i.e. a percolating confining and a non-percolating deconfining phase, separated by a transition as a function of temperature. The necessary ingredients are an action per unit vortex area (i.e. a Nambu-Goto term), and an action penalty related to the curvature of the vortex surfaces. By construction, this model can be understood in terms of the entropy associated with random surfaces in a given space-time domain; it contains no further dynamics. Evaluating the partition function of such a model amounts to counting possible vortex surface
configurations given a certain vortex density (enforced by the Nambu-Goto term), and given an ultraviolet cutoff on the space-time fluctuations of the surfaces (enforced by the curvature penalty). A detailed report on a lattice investigation of this model will be given in an upcoming publication. Further issues being, or recently having been, investigated include: The Pontryagin index associated with center vortex configurations [13, 14], and the breaking of chiral symmetry [13]; the continuum meaning of the maximal center gauge [14]; generalizations to $SU(3)$ color [3, 13]; and whether a random surface model for vortices can be justified in terms of a low-energy effective theory describing infrared Yang-Mills dynamics [14].

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