An intelligent computing and control model of topological relation between spatial objects based on fuzzy theory

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Abstract. The technological progress in the field of artificial intelligence (AI) has brought new opportunities and challenges to the intelligent development and innovative research in fuzzy spatial relations, fuzzy theory is the basic theory of intelligent information processing, and the paper gives a brief analysis of fuzzy calculation method in spatial analysis and spatial relation calculation. In particular, the spatial relations of fuzzy objects are more difficult to express and describe accurately. This paper proposes a method to calculate the topological relations between fuzzy objects with 4-Intersection model and 9-Intersection model, in the analysis of spatial relations, the extension from deterministic entity to uncertain entity is realized with fuzzy set theory, the fuzzy space matrix is constructed, then fuzzy topological relations and fuzzy membership are calculated and output, the model simplifies many complex relations, and it can describe eight topological relations including disjoint, meet, overlap, covers, contains, covered-by, inside and equal. In practical application, the position of spatial objects can be controlled and adjusted according to topological relations. Through analysis of algorithm and fuzzy membership calculation, the model is effective in theory.

1. Overview of spatial direction relation
For the description of topological relations between fuzzy objects, Cohn proposed “the Egg-Yolk model”,[1] the fuzzy entity is represented by egg yolk and albumen respectively. Among them, egg yolk represents the internal of fuzzy entity, and albumen represents the boundary of fuzzy entity. The paper focuses on modeling the propagation of attribute uncertainties in logical and arithmetical operations in GIS analysis using fuzzy set and statistical techniques, and the fuzzy entity is partitioned by using the cut set of the fuzzy set.[2] Research on fuzzy topological relations,[3][4][5] they are not very mature at present, and most of them are still on the description of fuzzy topological relationship, which is less in practical application.

This paper is based on the fuzzy division of uncertain objects, and it proposes a method to calculate the topological relationship between fuzzy objects with 4-Intersection model and 9-Intersection model. The topological relation of entity is taken as a reference space relation vector, and the concept of topological relation vector difference is proposed, then the membership of fuzzy topological relation is calculated after adjusting the reference space relation vector by clustering method.

2. Topological relation description and calculation of fuzzy entity
There are 4-Intersection model and 9-Intersection model to descript topological spatial relations of fuzzy objects. [6] Topological relations between objects are presented by judging intersection of interior, external and boundary is empty or not in 4-Intersection model and 9-Intersection model, there
are described with formula (1) and formula (2) respectively. Among them, \( A^0 \), \( A^- \) and \( \partial A \) denote internal, external and boundary of \( A \) respectively.

\[
T(A_i, A_j) =
\begin{bmatrix}
A^0_i \cap A^0_j & A^0_i \cap \partial A_j \\
\partial A_i \cap A^0_j & \partial A_i \cap \partial A_j
\end{bmatrix}
\]

(1)

\[
T(A_i, A_k) =
\begin{bmatrix}
A^0_i \cap A^0_k & A^0_i \cap \partial A_k \\
\partial A_i \cap A^0_k & \partial A_i \cap \partial A_k
\end{bmatrix}
\]

(2)

According to the above calculation methods of 4-Intersection model and 9-Intersection model, it can be extended to the calculation model of fuzzy objects. There are described with formula (3) and formula (4) respectively.

\[
\mu_{T4} =
\begin{bmatrix}
\begin{array}{c}
\begin{bmatrix}
f(\tilde{A}^0_1 \cap \tilde{A}^0_2), & f(\tilde{A}_1^0 \cap \tilde{\partial A}_2) \\
\tilde{f}(\partial \tilde{A}_1 \cap \tilde{A}_2), & \tilde{f}(\partial \tilde{A}_1 \cap \tilde{\partial A}_2)
\end{bmatrix}
\end{array}
\end{bmatrix}
\]

(3)

\[
\mu_{T9} =
\begin{bmatrix}
\begin{array}{c}
\begin{bmatrix}
f(\tilde{A}^0_3 \cap \tilde{A}^0_4), & f(\tilde{A}_3^0 \cap \tilde{\partial A}_4), & f(\tilde{A}_3^0 \cap \tilde{\partial A}_4^-) \\
\tilde{f}(\partial \tilde{A}_3 \cap \tilde{A}_4), & \tilde{f}(\partial \tilde{A}_3 \cap \tilde{\partial A}_4), & \tilde{f}(\partial \tilde{A}_3 \cap \tilde{\partial A}_4^-)
\end{bmatrix}
\end{array}
\end{bmatrix}
\]

(4)

Among them, \( \tilde{A}^0 \), \( \tilde{\partial A} \) and \( \tilde{A}^- \) represent fuzzy internal, fuzzy external and fuzzy boundary of \( A \) respectively. The calculation methods of each fuzzy value in 4-Intersection model matrix are described from formula (5) to formula (8), the model can be distinguishable 16 relations in theory.

\[
f(\tilde{A}^0_1 \cap \tilde{A}^0_2) = \max \left\{ \frac{\text{area}(\tilde{A}^0_1 \cap \tilde{A}^0_2)}{\text{area}(\tilde{A}^0_1)}, \frac{\text{area}(\tilde{A}^0_1 \cap \tilde{A}^0_2)}{\text{area}(\tilde{A}^0_2)} \right\}
\]

(5)

\[
f(\tilde{A}^0_1 \cap \tilde{\partial A}_2) = \max \left\{ \frac{\text{area}(\tilde{A}^0_1 \cap \tilde{\partial A}_2)}{\text{area}(\tilde{A}^0_1)}, \frac{\text{area}(\tilde{A}^0_1 \cap \tilde{\partial A}_2)}{\text{area}(\tilde{\partial A}_2)} \right\}
\]

(6)

\[
f(\tilde{\partial A}_1 \cap \tilde{A}^0_2) = \max \left\{ \frac{\text{area}(\tilde{\partial A}_1 \cap \tilde{A}^0_2)}{\text{area}(\tilde{\partial A}_1)}, \frac{\text{area}(\tilde{\partial A}_1 \cap \tilde{A}^0_2)}{\text{area}(\tilde{A}^0_2)} \right\}
\]

(7)

\[
f(\tilde{\partial A}_1 \cap \tilde{\partial A}_2) = \max \left\{ \frac{\text{area}(\tilde{\partial A}_1 \cap \tilde{\partial A}_2)}{\text{area}(\tilde{\partial A}_1)}, \frac{\text{area}(\tilde{\partial A}_1 \cap \tilde{\partial A}_2)}{\text{area}(\tilde{\partial A}_2)} \right\}
\]

(8)

The calculation methods of each fuzzy value in 9-Intersection model matrix are described from formula (9) to formula (17), the model can be distinguishable 512 spatial relations in theory such as disjoint, meet, overlap, covers, contains, covered-by, inside and equal.

\[
f(\tilde{A}^0_3 \cap \tilde{A}^0_4) = \max \left\{ \frac{\text{area}(\tilde{A}^0_3 \cap \tilde{A}^0_4)}{\text{area}(\tilde{A}^0_3)}, \frac{\text{area}(\tilde{A}^0_3 \cap \tilde{A}^0_4)}{\text{area}(\tilde{A}^0_4)} \right\}
\]

(9)

\[
f(\tilde{A}^0_3 \cap \tilde{\partial A}_4) = \max \left\{ \frac{\text{area}(\tilde{A}^0_3 \cap \tilde{\partial A}_4)}{\text{area}(\tilde{A}^0_3)}, \frac{\text{area}(\tilde{A}^0_3 \cap \tilde{\partial A}_4)}{\text{area}(\tilde{\partial A}_4)} \right\}
\]

(10)
Among them, $f$ denotes fuzzy membership function between fuzzy objects, and each membership is represented by a maximum, area represents region of intersection between fuzzy objects.

3. The calculation method of topological relations between fuzzy objects

3.1. The basic principle of the algorithm

According to algorithm of spatial topological relationship,\cite{7} it can complete the judgment of eight basic topological relations, including disjoint, meet, overlaps, covers, contains, equal, covered-by and inside.\cite{8} Among them, contains, inside, covers and covered-by are symmetrical in the practical application of the topological relations between areas, overlaps, covers and covered-by can be further simplified as an intersection topological relation. Eventually, they will be simplified as five topological relations including disjoint, meet, covered-by, equal and intersection. Whether the interior, exterior and boundary of two objects intersect or not can be represented by 0 or 1. If there is intersection, it can be represented by 1, otherwise it can be represented by 0.

Each relation matrix is represented by four values in the 4-intersection model,\cite{9} the corresponding space relation vector of the simplified five topological relations is shown in formula (18).

$$ST^4 = \{TR^4_d, TR^4_m, TR^4_c, TR^4_i, TR^4_e\}$$

Among them, $TR^4_d=(0,0,0,0)$ denotes disjoint, $TR^4_m=(0,0,0,1)$ denotes meet, $TR^4_c=(1,1,0,0)$ denotes covered-by, $TR^4_i=(1,1,1,1)$ denotes intersection, $TR^4_e=(1,0,0,1)$ denotes equal.

Similarly, Each relation matrix is represented by nine values in the 9-intersection model,\cite{10} the corresponding space relation vector of the simplified five topological relations is shown in formula (19).

$$ST^9 = \{TR^9_d, TR^9_m, TR^9_c, TR^9_i, TR^9_e\}$$

$$f(\tilde{A}_3^0 \cap \tilde{A}_4^0) = \max \left\{ \frac{\text{area}(\tilde{A}_3^0 \cap \tilde{A}_4^-)}{\text{area}(\tilde{A}_3^0)}, \frac{\text{area}(\tilde{A}_3^0 \cap \tilde{A}_4^-)}{\text{area}(\tilde{A}_4^0)} \right\}$$

$$f(\partial\tilde{A}_3 \cap \tilde{A}_4^0) = \max \left\{ \frac{\text{area}(\partial\tilde{A}_3 \cap \tilde{A}_4^-)}{\text{area}(\partial\tilde{A}_3)}, \frac{\text{area}(\partial\tilde{A}_3 \cap \tilde{A}_4^-)}{\text{area}(\tilde{A}_4^0)} \right\}$$

$$f(\partial\tilde{A}_3 \cap \partial\tilde{A}_4) = \max \left\{ \frac{\text{area}(\partial\tilde{A}_3 \cap \partial\tilde{A}_4^-)}{\text{area}(\partial\tilde{A}_3)}, \frac{\text{area}(\partial\tilde{A}_3 \cap \partial\tilde{A}_4^-)}{\text{area}(\partial\tilde{A}_4)} \right\}$$

$$f(\tilde{A}_3^- \cap \tilde{A}_4^-) = \max \left\{ \frac{\text{area}(\tilde{A}_3^- \cap \tilde{A}_4^-)}{\text{area}(\tilde{A}_3^-)}, \frac{\text{area}(\tilde{A}_3^- \cap \tilde{A}_4^-)}{\text{area}(\tilde{A}_4^-)} \right\}$$

$$f(\tilde{A}_3^- \cap \tilde{A}_4^0) = \max \left\{ \frac{\text{area}(\tilde{A}_3^- \cap \tilde{A}_4^0)}{\text{area}(\tilde{A}_3^-)}, \frac{\text{area}(\tilde{A}_3^- \cap \tilde{A}_4^0)}{\text{area}(\tilde{A}_4^0)} \right\}$$

$$f(\tilde{A}_3^- \cap \partial\tilde{A}_4) = \max \left\{ \frac{\text{area}(\tilde{A}_3^- \cap \partial\tilde{A}_4^-)}{\text{area}(\tilde{A}_3^-)}, \frac{\text{area}(\tilde{A}_3^- \cap \partial\tilde{A}_4^-)}{\text{area}(\partial\tilde{A}_4)} \right\}$$

$$f(\tilde{A}_3^- \cap \partial\tilde{A}_4^-) = \max \left\{ \frac{\text{area}(\tilde{A}_3^- \cap \partial\tilde{A}_4^-)}{\text{area}(\tilde{A}_3^-)}, \frac{\text{area}(\tilde{A}_3^- \cap \partial\tilde{A}_4^-)}{\text{area}(\partial\tilde{A}_4^-)} \right\}$$

$$f(\tilde{A}_3^- \cap \tilde{A}_4^-) = \max \left\{ \frac{\text{area}(\tilde{A}_3^- \cap \tilde{A}_4^-)}{\text{area}(\tilde{A}_3^-)}, \frac{\text{area}(\tilde{A}_3^- \cap \tilde{A}_4^-)}{\text{area}(\tilde{A}_4^-)} \right\}$$
Among them, \( TR^{d} = (0,0,1,0,0,1,1,1,1) \) denotes disjoint, \( TR^{m} = (0,0,1,0,1,1,1,1,1) \) denotes meet, \( TR^{c} = (1,1,0,0,1,0,0,1,1) \) denotes covered-by, \( TR^{i} = (1,1,1,1,1,1,1,1,1) \) denotes intersection, \( TR^{e} = (1,0,0,0,1,0,0,0,1) \) denotes equal.

In the calculation of topological relations of fuzzy objects, the result of \( \mu_{T4} \) or \( \mu_{T9} \) is a fuzzy space topological relation vector composed of fuzzy values from 0 to 1 rather than a value like 0 or 1. However, these fuzzy relation vectors, they are difficult to intuitively let people know the topological relations between fuzzy objects, which must be transformed into a certain spatial topological relations.

Definition 1: Difference degree of spatial relation vector refers to the sum of absolute value of difference each component of two spatial relation vectors. Let’s assume that \( ST(k) \) and \( \mu_{Ti} \) denote any two \( i \)-dimensional spatial vectors, difference degree is shown in formula (20).

\[
R_k (\mu_{Ti} , ST(k)^{i}) = \frac{1}{i} \sum_{j=0}^{i-1} | Y(\mu_{Ti}) - Y(ST(k)^{i})|, \quad 1 \leq k \leq 5, i \in \{4,9\}
\]  

Among them, \( Y(\mu_{Ti}) \) and \( Y(ST(k)^{i}) \) denote the \( j \)-th sub-vector of vector \( \mu_{Ti} \) and \( ST(k)^{i} \), \( | Y(\mu_{Ti}) - Y(ST(k)^{i})| \) denotes the absolute value of difference between of the \( j \)-th sub-vector of vector \( \mu_{Ti} \) and \( ST(k)^{i} \), \( k \) denotes the \( k \)-th kind of spatial topological relation of \( ST^{9} \) or \( ST^{4} \). The difference degree is smaller, it denotes more relevant between two vectors, and the spatial topological relations are more similar.

Fuzzy membership of topological relation is shown in formula (21).

\[
\mu_{ST(k)^{i}} = 1 - R_k (\mu_{Ti} , ST(k)^{i}), 1 \leq k \leq 5
\]  

The fuzzy membership of the current fuzzy topological relation vector \( \mu_{Ti} \) affiliated to the reference space relation \( ST(k)^{i} \) can be obtained by formula (21). The membership is bigger, the fuzzy topological relation is affiliated to it. The specific implementation steps are as follows.

(1) Let’s assume that there are \( M \) pairs fuzzy regions, the topological relation vector \( \mu_{Ti} \) between each pair of fuzzy regions is calculated, thus it can be obtained \( M \) fuzzy topological relation vectors.

(2) The differences \( R_k (\mu_{Ti}, TR(k)^{i}) \) between these quantitative topological relation vectors and reference spatial relation vectors are calculated by formula (20) respectively.

(3) According to the value of difference, the fuzzy membership \( \mu_{ST(k)^{i}} \) of fuzzy spatial topological relation vector is affiliated to the reference space is calculated by formula (21).

(4) According to the value of fuzzy membership, the fuzzy topological relation affiliated to what kind of topological relation in reference space is determined.

(5) The number of pairs of fuzzy regions contained in each type of the new topological relation set is calculated, it denotes \( M = \sum_{i=1}^{N} N_{i} \). The reference space relation vector \( ST(m)^{i} \) of \( m \) topological relations can be recalculated according to formula (22) respectively.

\[
ST(m)^{i} = (1/N_{i}) \sum_{i=0}^{N} Y(\mu T^{i})_{m}, 1 \leq m \leq 5, i \in \{4,9\}
\]  

(6) Substituting \( ST(m)^{i} \) into formula (20) to replace \( ST(k)^{i} \), difference of new reference space vector in new topological relation is calculated. The fuzzy membership of each fuzzy topological relation vector affiliated to the new topological relation is calculated, and the new topological relation set is obtained. Go back to step (5), iteration \( n \) times, and compared with the previous calculation results, until each component changes very little in reference spatial relation vector, so as to determine the final reference space relation vector.

3.2. Algorithm for calculating fuzzy topological relations

Input: fuzzy entity \( A_{3} \) (vector data), fuzzy entity \( A_{4} \) (vector data) and thresholds (\( \lambda \))

Output: fuzzy topological relations and fuzzy membership
Algorithm execution steps:

Step1: according to the dividing method of boundary, interior and exterior of the fuzzy entity,[14] the closed regions composed of the boundary, internal and external finite discrete point sets of fuzzy entity $A_3$ and $A_4$ are obtained respectively.

Step2: the inner set region and the boundary set region are substituted into formula (5) to formula (8) to obtain the topological relation vector of $\{f_{11}, f_{12}, f_{21}, f_{22}\}$ with 4-Intersection model.

Step3: the differences $R_1(\mu_{T4}, TR^4_d)$, $RA_{T4}, TR^4_m)$, $R_3(\mu_{T4}, TR^4_i)$, $R_4(\mu_{T4}, TR^4_e)$ and $R_5(\mu_{T4}, TR^4_c)$ between fuzzy topological relation vectors and reference spatial relation vectors are calculated by formula (20) respectively.

Step4: the fuzzy membership $\mu_{ST(1)} = 1 - R_1(\mu_{T4}, TR^4_d)$, $\mu_{ST(2)} = 1 - R_2(\mu_{T4}, TR^4_m)$, $\mu_{ST(3)} = 1 - R_3(\mu_{T4}, TR^4_i)$, $\mu_{ST(4)} = 1 - R_4(\mu_{T4}, TR^4_e)$ and $\mu_{ST(5)} = 1 - R_5(\mu_{T4}, TR^4_c)$ are calculated by formula (21) respectively.

Step5: maximum $\text{Max} \{\mu_{ST(1)}^4, \mu_{ST(2)}^4, \mu_{ST(3)}^4, \mu_{ST(4)}^4, \mu_{ST(5)}^4\}$ is used to determine the fuzzy vector affiliated to the fuzzy topological relation and calculate the number of fuzzy topological relation vectors in new topological relation.

Step6: the new reference space relation vector is calculated by formula (22), and the change of previous reference space relation vector is calculated. If the change is greater than $\lambda$, then with reference space relation vector of new topological relation substitute the previous reference space relation vector. If fuzzy topological relation vector is invariant, then it return step3.

Step7: iteration n times, when the change of reference space relation vector is smaller than $\lambda$, then fuzzy topological relation and membership of the previous calculation are output.

4. Conclusion

Fuzziness is one of the basic characteristics of spatial entity. With the wide application of artificial intelligence technology in spatial relationship analysis, it needs more appropriate theories and methods to analyze spatial fuzzy objects. Fuzzy theory has unique predominance in dealing with the entity with fuzzy and uncertainties. In this paper, the topological relationship between spatial objects is analyzed by fuzzy computing method, fuzzy 4-Intersection model and fuzzy 9-Intersection model are designed, and then the fuzzy measure of the whole process is obtained. Fuzzy 4-Intersection model and fuzzy 9-Intersection model are computable model. It is helpful to describe the topological relations from accurate entity to fuzzy entity, it can overcome the disadvantage that the results of existing model are inconsistent due to the separation of accurate entity and fuzzy entity.

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