Blind Two-Dimensional Super Resolution in Multiple Input Single Output Linear Systems

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Abstract—In this paper, we consider a multiple-input single-output (MISO) linear time-varying system whose output is a superposition of scaled and time-frequency shifted versions of inputs. The goal of this paper is to determine system characteristics and input signals from the single output signal. More precisely, we want to recover the continuous time-frequency shift pairs, the corresponding (complex-valued) amplitudes and the input signals from only one output vector. This problem arises in a variety of applications such as radar imaging, microscopy, channel estimation and localization problems. While this problem is naturally ill-posed, by constraining the unknown input waveforms to lie in separate known low-dimensional subspaces, it becomes tractable. More explicitly, we propose a semidefinite program which exactly recovers time-frequency shift pairs and input signals. We prove uniqueness and optimality of the solution to this problem. Moreover, we provide a grid-based approach which can significantly reduce computational complexity in exchange for adding a small gridding error. Numerical results confirm the ability of our proposed method to exactly recover the unknowns.

Index Terms—Super-Resolution, Semidefinite Programming, Convex optimization.

I. INTRODUCTION

Super-resolution is the problem of recovering high-resolution information from low-resolution data. In this letter, we assume a linear time-varying (LTV) system in which the inputs are continuous-time band-limited arbitrary signals \(x_j(t)\) and the output vector \(y(t)\) is a weighted superposition of time and frequency shifted versions of the inputs:

\[
y(t) = \sum_{j=0}^{N_1-1} \sum_{k=1}^{S} b_{j,k} x_j(t - \tilde{\tau}_k) e^{j2\pi \tilde{\nu}_k}.
\]  

Here, \(b_{j,k} \in \mathbb{C}\) are some unknown weights and \(\tilde{\tau}_k, \tilde{\nu}_k \in [-T/2, T/2]\) represent continuous time and frequency shifts, respectively. \(\tilde{\tau}_k\) and \(\tilde{\nu}_k\) are not constrained to lie on a predefined domain of grids. The number of input signals \(N_1\) is known and we want to recover the unknowns \((b_{j,k}, \tilde{\tau}_k, \tilde{\nu}_k, x_j(t), S)\). Many applications in communication and signal processing match this model, including radar imaging [1], channel estimation [2], microscopy [3], astronomy [4] and localization problems [5], [6]. In channel estimation, a wireless channel can be modeled as a LTV system with delay-Doppler shifts [7]. A challenging problem in channel estimation is pilot contamination caused by sharing the non-orthogonal pilots among users. One way to deal with this problem is applying techniques that do not need any pilot signal, named blind methods [2]. As another example, we can mention spying radar where an enemy collocated transmitters send unknown signals to some objects. The goal is to detect the intended objects and the transmitted signals. Since the transmitters are collocated, the delay and Doppler introduced depend only on the object and not the transmitter signal.

In recent years, super-resolution methods based on convex optimization has attracted much attention [8]–[13] due to their superior performance. This approach was first proposed by Candès and Fernandez-Granda in [8]. They used total variation norm for exact recovery of 1D spikes under a minimum separation condition with known system function and full measurements. Then, [10] provided an atomic norm framework to estimate locations and amplitudes of a spike train in the frequency domain. In [11], the authors apply atomic norm minimization to recover time and frequency shifts in radar application. They adapted super-resolution techniques of [8] to a single-input single-output (SISO) system with known band-limited input signal. The authors in [16] investigate a similar problem, yet intend to estimate the time-frequency shifts in a blind way where the low-pass point spread function applied to the transmitter signal is unknown. In this paper, we study the systems with multiple inputs and single output (MISO) as in [1]. Besides its generality, this model matches the “collocated” transmitter scenario with “many” targets in MISO radar systems [17] and differs from the previous models used in prior works. Here, shifts are independent from inputs and only depends on the system function. Moreover, unlike [16], we assume the input signals belong to “different” subspaces with disparate dimensions. While our model in [1] matches the well-studied model in MISO radar systems [17], the strategy used in [1], [16] is not directly applicable in this setting. Generally, our goal in this paper is to find a strategy to detect the time-frequency shifts \((\tilde{\tau}_k, \tilde{\nu}_k)\) \(k = 1, ..., S\) as well as the transmitters’ signals \(x_j(t)\)’s in the MISO model [1]. To this end, we first take samples of [1] to reach an under-determined system of linear equations in matrix-vector form. We assume that input signals are of band-width \(W\) and periodic with \(T\).  Also, \(y(t)\) is observed over a time interval of \(T\). Then, by assuming that the input signals lie in different known low-dimensional subspaces, we apply the lifting trick [18] and formulate the problem as an atomic norm minimization [10]. Next, we propose a semidefinite programming (SDP) as a relaxation to the dual problem and find a strategy to detect the time-frequency shifts. The input signals are then estimated by finding the least square solution to the resulting over-determined problem (containing more equations than unknowns) with known time-frequency shifts. Due to the high

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computational complexity of the proposed SDP in practice, we also provide a grid-based approach where the time-frequency shifts can be recovered on a fine domain of grids via nuclear norm minimization.

II. SYSTEM MODEL

We sample $y(t)$ at rate $\frac{1}{T}$ based on 2WT-theorem \[19\] to collect $L := WT$ samples, assumed to be an odd number. By applying the discrete Fourier transform (DFT) and inverse DFT (IDFT), and defining normalized parameters $\tau_k = \frac{t_k}{T}$ and $\nu_k = \frac{\nu_k}{T}$ we obtain:

$$y(p) := y(p/W) = \frac{1}{T} \sum_{k=1}^{S} \sum_{j=0}^{N-1} x_j(l)e^{i2\pi(m(p-1)/T - \tau_k)}e^{i2\pi(p\nu_k - m\tau_k)}$$

(2)

This is an under-determined linear system with $L$ equations and $N_1L + 3S^2 + 1$ unknowns. To achieve a unique solution, we impose a subspace constraint \[16, 18, 20, 22\], which is common in a wide range of applications \[23\]. We assume that the input signals $x_j := [x_j(-N), \ldots, x_j(N)]^T$ belong to known subspaces with dimensions $K_j \ll L$.

$$x_j = D_j h_j, \quad D_j := [d_j^1, \ldots, d_j^N]^H \in \mathbb{C}^{L \times K_j}, \quad \|h_j\|_2 = 1.$$  

(3)

It is convenient to rewrite (2) in matrix form using Dirichlet kernel:

$$D_N(t) := \frac{1}{T} \sum_{m=-N}^{N} e^{i2\pi tm}.$$  

(4)

Applying this definition to (2) and replacing $x_j(l) = d_j^H h_j$, yields (See Appendix \[A\] for detailed derivation):

$$y(p) = \sum_{k=1}^{S} \sum_{j=0}^{N-1} D_N(\frac{p}{T} - \tau_k)D_N(\frac{m}{T} - \nu_k) d_j^H h_j e^{i2\pi mp}.$$  

(5)

Define vector $s := [\tau, \nu]^T$ and atoms $a_j(s_k) \in \mathbb{C}^{L \times 2}$ as:

$$[a_j(s_k)]_{(m,l),1} = D_N(\frac{m}{T} - \tau_k)D_N(\frac{l}{T} - \nu_k).$$  

(6)

The dictionary part $\tilde{D}_j := D_j^H \in \mathbb{C}^{L \times K_j}$ is defined as:

$$[\tilde{D}_j]_{(m,l),1} := \frac{1}{T} \sum_{p=1}^{T} d_j^H(p-l) e^{i2\pi mp}, \quad p, l, m = -N, \ldots, N.$$  

(7)

Substituting (6) and (7) in (5) and using the lifting trick \[18\], we obtain:

$$y(p) = \sum_{k=1}^{S} \sum_{j=0}^{N-1} b_k a_j^H(s_k) \tilde{D}_j h_j$$

$$= \text{Tr} \left( \sum_{j=0}^{N-1} \tilde{D}_j^H \sum_{k=1}^{S} b_k h_j a_j(s_k)^H \right) = \sum_{j=0}^{N-1} \langle B_j, \tilde{D}_j^H \rangle,$$  

(8)

where $B_j := \sum_{k=1}^{S} b_k h_j a_j^H(s_k)$. We define a linear operator $\chi : \mathbb{C}^{N_1-1} \mathbb{C}^{K_j \times L^2} \rightarrow \mathbb{C}^L$ that maps a matrix tuple to a vector and the input matrix tuple $\mathcal{B} := (B_j)_{j=0}^{N_1-1} \in \oplus_{j=0}^{N_1-1} \mathbb{C}^{K_j \times L^2}$. The observation vector $y := [y(-N), \ldots, y(N)]^T$ can be represented as:

$$y = \chi(\mathcal{B}).$$  

(9)

In practice, the number of shifts $S$ is much smaller than the number of samples $L$. Therefore, in the atomic set

$$\mathcal{A}_j = \{h_j a_j^H(s) : s \in [0,1]^2, \|h_j\|_2 = 1, h_j \in \mathbb{C}^{K_j}\},$$

(10)

a small number of atoms are active. To promote sparsity, we use the atomic norm:

$$\|B_j\|_{\mathcal{A}_j} = \inf \{t > 0 : B_j \in t \text{ conv}(\mathcal{A}_j)\}$$

$$= \inf_{b_k \in \mathbb{C}, s \in [0,1]^2, \|h_j\|_2 = 1} \sum_{k=1}^{S} |b_k| : B_j = \sum_{k} b_k h_j a_j^H(s_k),$$

(11)

and propose the following optimization problem to recover the time-frequency shift pairs:

$$\mathcal{P}_1: \min_{B_j \in \mathbb{C}^{K_j \times L^2}} \|B_j\|_{\mathcal{A}_j} \quad \text{s.t.} \quad y(p) = \sum_{j=0}^{N_1-1} \langle \tilde{B}_j, \tilde{D}_j^H \rangle.$$  

(12)

The optimization to calculate $\|B_j\|_{\mathcal{A}_j}$ is over infinite dimensional variables and thus computationally intractable. To cope with this issue, we consider the dual problem and find a SDP relaxation for it. Aside from this, we propose a grid-based approach (dividing the region $[0,1]^2$ into grids) to solve (12) directly, leading to a reduced computational burden. In general, we take the following steps:

1) Solve (12) to recover $(\tau_k, \nu_k)$ and $S$.

2) Find the least squares solutions of the following linear equation to estimate $b_k h_j$ (in grid-based approach, $s_k$ are replaced with grid points): \[13\]

$$\sum_{j=0}^{N_1-1} \left[ a_j^H(s_1) \tilde{D}_j^{1,N} \ldots a_j^H(s_{N_1}) \tilde{D}_j^{N_1,N} \right] b_{k} h_j = \begin{bmatrix} y(-N) \\ \vdots \\ y(+N) \end{bmatrix}.$$  

(13)

A. Dual Approach

The dual problem of (12) is given by (the detailed derivation is included in Appendix \[B\]):

$$\max_{q \in \mathbb{C}^L} \langle q, y \rangle \quad \text{s.t.} \quad \|\chi^*(q)\|_1 \leq 1,$$  

(14)

where $\chi^* : \mathbb{C}^L \rightarrow \oplus_{j=0}^{N_1-1} \mathbb{C}^{K_j \times L^2}$ denotes the adjoint operator of $\chi$ such that $\chi^*(q)_j = \sum_{p=-N}^{N} q_p \tilde{D}_j^H(p-l)$ (see Appendix \[C\] for details). The dual norm in (14) is obtained as:

$$\|\chi^*(q)\|_1 \leq 1.$$  

(15)

Replacing (13) in (14) yields:

$$\mathcal{P}_1^*: \max_{q \in \mathbb{C}^L} \langle q, y \rangle \quad \text{s.t.} \quad \|\chi^*(q)\|_1 \leq 1, \quad s \in [0,1]^2.$$  

(16)

The primal convex problem (12) has only equality constraint. Therefore, strong duality holds and in the optimal points
we can claim \( \sum_{j=0}^{N-1} \| B_j \|_{\alpha_j} = \langle \hat{q}, \hat{y} \rangle_R \). The following theorem states optimality and uniqueness of the solution to this problem. Define the dual polynomial function as:

\[
  f_j(s) := [x^*(q)]_j a_j(s) = \sum_{p=-N}^{N} q_p D_p^H a_j(s) \in \mathbb{C}^{K_j}. \tag{17}
\]

**Theorem 1.** For the true support \( \mathcal{J} = \{ s_k \}_{k=0}^{S} \) and the observation vector according to (8), the matrix tuple \( \mathcal{B} = \mathcal{B} \) is the unique optimal solution of (12) provided that the following conditions hold:

1. There exist 2D trigonometric vector polynomials [17] with complex coefficients \( q = [q(-N), \ldots, q(N)]^T \) such that:

\[
  f_j(s_k) = \text{sign}(b_k) h_j, \forall s_k \in \mathcal{J} \tag{18}
\]

2. The sets \( \left\{ a_j^H(s_k) D_j^{-N}, \ldots, a_j^H(s_k) D_j^{N} \right\}_{k=1}^{S} \) are linearly independent.

**Proof.** If \( q \) satisfies (18), then, it will be in the feasible set of (16) which is observed from (15) and (17). Conversely, if \( q \) satisfies (18) and (19), then \( (\mathcal{B}, q) \) is a primal-dual optimal solution pair. To show this, by the definition of atomic norm in (11), we have:

\[
  \langle q, y \rangle_R = \langle x^*(q), D_j \rangle_R = \sum_{j=0}^{N-1} \langle x^*(q)_j, B_j \rangle_R \tag{19}
\]

On the other hand,

\[
  \langle q, y \rangle_R = \langle x^*(q), \mathcal{B} \rangle_R = \sum_{j=0}^{N-1} \langle x^*(q)_j, B_j \rangle_R \tag{19}
\]

We conclude that \( \sum_{j=0}^{N-1} \| B_j \|_{\alpha_j} = \langle q, y \rangle_R \). Hence, \( \mathcal{B} \) is the primal optimal and \( q \) is the dual optimal solution. To check the uniqueness, assume that \( \mathcal{B} \) is another solution supported on \( \mathcal{J} \neq \mathcal{J} \) with \( B_j := \sum_{s_k \in \mathcal{J}} b_k h_j a_j^H(\bar{s}_k) \), then:

\[
  \langle q, y \rangle_R = \langle x^*(q), \mathcal{B} \rangle_R = \sum_{j=0}^{N-1} \| a_j^H(\bar{s}_k) \|_{\alpha_j} \| B_j \|_{\alpha_j} \tag{19}
\]

Since the set of atoms and their shifts in \( \mathcal{J} \) are linearly independent, having the same support means \( \mathcal{B} = \mathcal{B} \). This result violates strong duality and therefore, \( \mathcal{B} \) is the unique solution of (12).

The infinite number of constraints in the dual problem (19) makes it computationally intractable. To overcome this difficulty, we use a semidefinite programming and extend (16) to multiple constraints based on (24). The dual polynomial functions in (17) can be written as (see (16) Appendix C):

\[
  f_j(s) = \sum_{p, m=-N}^{N} \left( \frac{1}{t} [q_p \sum_{l=-N}^{N} b^*_l e^{2\pi \frac{m p - l}{T}}] \right) e^{-2\pi \left( m r + p \tau \right)} \tag{20}
\]

and we define \( \tilde{Q}_j \) is \( \mathbb{C}^{K_j \times L^2} \) such that:

\[
  [\tilde{Q}_j]_{(i, (p,m),)} = \frac{1}{t} [q_p \sum_{l=-N}^{N} b^*_l e^{2\pi \frac{m p - l}{T}}] \tag{21}
\]

Finally, the SDP relaxation can be written as:

\[
  P_1^\prime : \max_{q, Q_j} \langle q, y \rangle_R \tag{22}
\]

s.t. \( \left( Q_j, \tilde{Q}_j^H \right) \geq 0 \), \( Tr((\theta_m \otimes \theta_r) Q_j) = \delta_{m, r} \)

where \( \theta_r \) is a Toeplitz matrix with ones on its i-th diagonal and zeros elsewhere. (22) can be solved by standard solvers.

**B. Grid-Based Approach**

Program \( P_1^\prime \) in (22) is a high-precision method with high computational complexity of order \( O(L^2) \). Here, we provide a grid-based approach with reduced computational complexity in exchange for higher estimation error.

Suppose the time-frequency shifts \( (\tau_k, h_k) \) lie on a \( (1/G, 1/G) \)-grid. So, the observation vector is expressed as:

\[
  y(l) = \frac{1}{G} \sum_{r,s=0}^{G-1} u_{r,s} e^{2\pi i r p} \sum_{j=0}^{N-1} \sum_{m, l=-N}^{N} x_j(l)e^{-2\pi \frac{m p - l}{T}} e^{2\pi \left( m r + p \tau \right)}, \tag{23}
\]

where \( x_j(l) = d_j^H h_j \). Using the discrete version of the variables defined in the previous section, we get:

\[
  y(l) = \sum_{r,s=0}^{G-1} \sum_{j=0}^{N-1} b'_{r,s} a_j(g)^H \tilde{D}_j^H h_j = \sum_{j=0}^{N-1} \langle B_j', \tilde{D}_j^H \rangle, \tag{24}
\]

where \( g = [r/G, s/G], b' \in \mathbb{C}^{G^2} \) is a sparse vector such that \( b'_{(r,s),1} := b'_{r,s} \), and \( B_j' := \sum_{r,s=0}^{G-1} b'_{r,s} a_j(g)^H \).

Now, we propose the following optimization program to recover the time-frequency shifts:

\[
  P_2 : \min_{B_j' \in \mathbb{C}^{K_j \times 2}} \| B_j' \|_* \text{ s.t. } y(l) = \sum_{j=0}^{N-1} \langle B_j', \tilde{D}_j^H \rangle \tag{25}
\]

where \( \| \cdot \|_* \) is the nuclear norm of a matrix which is regarded as an alternative for the atomic norm in the discrete setting.

**III. Simulation Results**

In this section, we provide some experiments to confirm the accuracy of our proposed methods using CVX toolbox and SDPT3 package. In the first experiment, we have two input signals and a system with the time-frequency shift pair \( (0.24, 0.52), \) generated uniformly in the interval [0, 1] according to a minimum separation condition based on (11). We set \( N = 7, K_1 = K_2 = 1, S = 1 \) and \( N_f = 2 \). The entries of \( D_j, h_j \) and \( b_k \) are generated from complex standard normal distribution subject to \( \| h_j \|_2 = 1 \) and \( \| b_k \| = 1 \). Fig 1.
SRF shows good matching of the true and estimated signals when error is inversely related to SRF. The bottom image of Fig.2 illustrates the dual polynomials in (17) which achieve 1 in the locations of true shift pairs. The top image of Fig.2 shows the dual problem can be written as (14).

We obtain the dual function by minimizing over \(B\): 

\[
\mathcal{L}(\mathcal{B}, \mathbf{q}) = \sum_j \|B_j\|_{\mathcal{H}_j} + \langle \mathbf{q}, \mathbf{y} - \chi(\mathcal{B}) \rangle
\]

in which: 

\[
\langle \mathbf{q}, \chi(\mathcal{B}) \rangle = \langle \chi^*(\mathbf{q}), \mathcal{B} \rangle = \sum_j \langle \chi^*(\mathbf{q})_j, B_j \rangle
\]

\[
\leq \sum_j \|B_j\|_{\mathcal{H}_j} \|\chi^*(\mathbf{q})_j\|_{\mathcal{H}_j}^d
\]

and Therefore, 

\[
\mathcal{L}(\mathcal{B}, \mathbf{q}) \geq \sum_j \|B_j\|_{\mathcal{H}_j} (1 - \|\chi^*(\mathbf{q})_j\|_{\mathcal{H}_j}^d) + \langle \mathbf{q}, \mathbf{y} \rangle
\]

We obtain the dual function by minimizing over \(\mathcal{B}\) as: 

\[
H(\mathbf{q}) = \inf_{\mathcal{B}\in\mathcal{B}, \mathbf{q} \in \mathbb{C}_N^J} \mathcal{L}(\mathcal{B}, \mathbf{q}) = \begin{cases} 
\langle \mathbf{q}, \mathbf{y} \rangle 
& \|\chi^*(\mathbf{q})\|_{\mathcal{H}_j}^d \leq 1 \\
+\infty & \text{otherwise}
\end{cases}
\]

So, the dual problem can be written as (14).

\[\text{APPENDIX C}\]

**ADJOINT OPERATOR } \chi^*\**

Beginning from (16), we have: 

\[
\langle \mathbf{q}, \mathbf{y} \rangle_{\mathcal{H}} = \langle \mathbf{q}, \chi(\mathcal{B}) \rangle_{\mathcal{H}} = \sum_{p,j} \langle B_j, \tilde{D}_p^{\mathbf{h}_j} \rangle q_p \quad (26)
\]

We also have: 

\[
\langle \mathbf{q}, \mathbf{y} \rangle_{\mathcal{H}} = \langle \chi^*(\mathbf{q}), \mathcal{B} \rangle_{\mathcal{H}} = \sum_j \langle B_j, [\chi^*(\mathbf{q})]_j \rangle_{\mathcal{H}} \quad (27)
\]

From (26) and (27) we can deduce: 

\[
\sum_j \langle B_j, \tilde{D}_p^{\mathbf{h}_j} \rangle = \sum_j \langle \mathbf{q}, [\chi^*(\mathbf{q})]_j \rangle_{\mathcal{H}}
\]

Therefore: 

\[
[\chi^*(\mathbf{q})]_j = \sum_p q_p \tilde{D}_p^{\mathbf{h}_j}
\]
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