Brief review on the limit state function of dynamic scour protections

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Abstract. Offshore foundations, namely for offshore wind, wave and tidal applications, often require the use of scour protections. Rip-rap scour protections are an important element of the foundation to ensure that the natural frequency stays within the design limits. Scour protection design still presents a remarkable empirical nature, which typically leads to uncertainty on their behaviour under extreme met-ocean conditions. Therefore, reliability assessment of scour protections has been seen as a possibility to account for design uncertainty and to optimise the scour protections. However, the definition of a suitable limit state function is still a matter of research focus, namely, regarding the proper definition of the acceptable damage level for dynamic scour protections. This research provides a brief review on the recent studies related to both the limit state function and the calculation of damage numbers through bathymetric data. A discussion is raised on how the methodologies for calculating the damage number may influence the limit state function and a theoretical example is given to assess the effects on the probability of failure. Results have shown that the acceptable damage number requires a clearer definition, which should be based on the number of layers of rock material and the area of filter exposure. In addition, this research highlights the need for alternative ways to assess damage.

1. Introduction

Offshore foundations, namely for offshore wind, wave and tidal applications, often use rubble-mound material as the main component of the scour protection. Rip-rap scour protections are an important element of the foundation to ensure that the natural frequency stays within the design limits. However, scour protection design still presents a remarkable empirical nature, which typically leads to uncertainty under extreme met-ocean conditions [1].

Aiming at uncertainty reduction and a better perception of the protection’s failure, recent research has been developed on the topic of reliability based design of scour protections, e.g. [1], [2] and [3]. These works follow the trend of reliability and risk analysis applied to other maritime and coastal structures, commonly made of rock material, e.g. breakwaters [4]. The application of these techniques
to rubble-mound structures is still far from the mature development registered in other types of known structures, *e.g.* structural design of tubular elements for an offshore platform or mooring lines for floating foundations. In [5], a review of scientific and technical challenges of reliability analysis of scour protections, for offshore wind foundations, is extensively given. A key aspect identified in [5], which requires further insight and discussion for a proper application of reliability techniques, is the definition of an accurate limit state function, which might be suitable to define the protections’ failure. While [3] extensively addresses the failure mode known as the “erosion of the top layer” for both static scour protections, without movement of rock material, and dynamic scour protections, where movement is allowed, there is still a lack of studies considering other failure modes (*e.g.* [6]).

In order to understand the failure mechanism of the rock armour material at these protections, more knowledge is required in terms of the physical scour pattern and damage severity at the top layer. Therefore, physical modelling studies have been extensively dedicated to the analysis of damage in scour protections, namely, under waves and current combined (*e.g.* [7, 8]). In [8], a physical model is used to discuss the design limits of the damage number (S3D), introduced by [9], to describe the failure of dynamic scour protections. The results showed that further research on the damage definition and range is required. It was latter seen in [5] that the methodology used to derive the damage number from bathymetric profiles influences the damage numbers obtained, thus ultimately influencing the reliability assessment of a scour protection, by means of its limit state function.

In addition, currently existing methods to define the protection’s failure in physical models are often based on visual criteria (*e.g.* [9, 10]). Despite the importance of visual damage criteria, they may contribute to cases where damage is sometimes undetected or miss-evaluated. Even though several works have been performed on alternative ways of evaluating damage (*e.g.* [11, 12]), the damage assessment based on the damage number combined with visual observation remains as the most used criteria for rip-rap scour protections at offshore wind foundations.

The present research aims at highlighting the importance of the acceptable damage number and its calculation for a proper reliability assessment. The effects of this parameter in the limit state function are shown and discussed, by means of a physical model and the computation of damage according to different methodologies. It is proven that the sub-area arrangement used to analyse the bathymetric profiles influences the damage number significantly. An example is used to identify the potential effects that damage assessment methodologies may have on the limit state function of a scour protection.

### 2. Methodology

#### 2.1. Damage number based design

In [9], the design of dynamic scour protections for monopile foundations is based on a predicted damage number (S3D), whose predictive equation was derived from an extensive set of physical model tests. The predictive equation (1) is a regression based equation obtained from a set of damage numbers calculated directly from the bathymetric profiles taken after a certain number of waves (N).

\[
\frac{S_{3D}}{N^{0.62}} = a_0 \frac{U_{m}^2 T_{m,1.0}^2}{g d (s-1) T_{m,1.0}^2 D_{50}^2} + a_1 \left(\frac{U_{m}}{w_s}\right)^2 \left(\frac{U_{s} + a_4 U_{m}}{a_3}ight)^2 \left(\frac{\sqrt{d}}{g D_{50}^2}\right)^{0.5} \tag{1}
\]

where \(U_m\) is the average current velocity, \(s\) is the ratio between sediment’s density \(\rho_s\) and water density \(\rho_w\), \(g\) is the gravitational acceleration, \(d\) is the water depth, \(U_m\) is the orbital bottom velocity and \(w_s\) is the sediments’ fall velocity. \(T_{m,1.0}\) is the energy spectral wave period, which for a JONSWAP spectrum, with \(\gamma = 3.3\) can be obtained from the peak period \(T_p\) as \(T_{m,1.0} = m_{1.0}/m_0 = 1.107 T_p\). In (1) \(b_0, a_0, a_2\) and \(a_3\) are equal to 0.243, 0.00076, -0.022 and 0.0079, respectively. The constants \(a_1\) and \(a_4\) depend on the wave-current relative direction, the current velocity and the Ursell number. A detailed calculation of these parameters is given in [9].

The damage numbers derived from the bathymetric profiles, *e.g.* those which are not predicted but, instead, obtained through physical modelling of prototype conditions, are obtained as a function of the
eroded volume ($V_e$) over a specified sub-area ($A_{sub}$) of the scour protection itself, (2). In the original formulation, [9] defines the sub-area as being equal to the cross-section of the monopile foundation, with $D_p$ being the pile diameter (Figure 1). Then the representative damage number of the protection is equal to the maximum value for all the $S_{3D}$ calculated from Eq. (2), i.e. $S_{3D\text{max}}$.

\[ S_{3D} = \frac{V_e}{D_{n50} A_{sub}} = \frac{V_e}{D_{n50} \pi \frac{D_p^2}{4}} \]  

(2)

**Figure 1.** Scheme of a scour protection divided in sub-areas equal to the pile cross-section.

De Vos et al. [9] proposes a design procedure for dynamic scour protections by means of Eq. (1), where local sea-state conditions, the rock material characteristics and a predefined damage level (e.g. $S_{3D} = 0.5$) serve as input. Then the equation can be solved in order to calculate $D_{n50}$, thus providing the nominal median diameter of the armour layer. For further details and discussion of this methodology, [3] and [9] are recommended. As defined in [9], the damage number provides a measure on the number of layers of rock material, which have been removed over the specified sub-area. [9] also defines failure visually as in [10], i.e. the scour protection fails when the filter layer is exposed over an area equal or larger than $4D_{50}^2$.

The underlying design assumption of this methodology is the proper definition of an acceptable damage level. In [9], a value of $S_{3D}$ smaller or equal to 0.25 is suggested for scour protections without movement of the armour material (static scour protection), whereas a value of $S_{3D}$ between 0.25 and 1.00 is suggested for dynamic scour protections. $S_{3D}$ values above 1.00 were identified through physical modelling as being within the failure domain of the scour protection.

The reference values presented by [9] were identified as conservative since dynamically stable scour protections were found for larger values of $S_{3D}$. Research performed by [3, 8] concluded that the division of the physical model into sub-areas has an important effect on the maximum damage number. Alternative divisions are suggested in [5], which concludes that further discussion on the definition of the acceptable damage is required. However, this implies a deeper study on the definition of the sub-areas used in (2), which ultimately affect the regression leading to (1), thus having an effect on the design methodology of dynamic scour protections.

Figure 2 shows a sub-area arrangement alternative to [9], which was applied in [13]. In this arrangement, the sub-areas vary between each other and are divided between concentric circular rings. These sub-areas vary depending on the ring and sector on which the sub-area is located. The areas of the outer rings are larger than the ones near the pile, but the number of sectors remains the same. Since the eroded volume is being evaluated in different sub-areas, differences are expected when (2) is applied,
thus invalidating a direct comparison with the limits for the acceptable damage discussed in [9]. In [8], a comparison between the data presented in [9] and [13] is made and it is concluded that the damage numbers obtained for dynamic scour protections can largely exceed the value of $S_{3D}$ equal to 1.00 (see [3, 5]). [5] discusses the use of an arrangement of overlapped sub-areas that may potentially provide a more accurate knowledge on the damage pattern occurring at the protection. However, the literature shows that such concept is yet to be systematically applied to a large set of bathymetric data, as in [9] and in a shorter extent as in [13]. The present application demonstrates that the use of different sub-areas not only impacts directly the assessment of the damage number, but it also affects the assessment of the reliability of a scour protection (Section 3.1).

![Figure 2. Scheme of sub-areas arranged in concentric circular rings.](image)

### 2.2. Reliability of dynamic scour protections

The definition of the ultimate limit state of a dynamic scour protection, for the erosion of the top layer failure mode, can be defined as in [1], (3) where the reference level of $S_{3D} = 1.00$ is used as the resistance parcel of the scour protection ($R$) and the expected damage for the local loading conditions is calculated from (1), which defines the limit state function $g(X)$. A full description on how to perform reliability analysis for dynamic scour protections is given in [1, 2 and 3].

$$g(X) = R - S_{3D} = 1.00 - S_{3D}$$

(3)

where $X$ represents the vector of random variables included in the calculation of $S_{3D}$, e.g. $U_m, U_c, T_p$. While (3) seems to be linear, the design procedure is dependent on relation given in (1), which is highly non-linear, in particular, when considering the design variable $D_{n50}$. Therefore, and considering the relatively low computation effort required to simulate (3), it was shown by [3] that Monte-Carlo simulation method can be applied to assess the domain of $g(X)$. In order to obtain a measure of the protection’s reliability, the probability of failure ($P_f$) can be calculated, as in (4).

$$P_f = \frac{\#(g(X) \leq 0)}{n} = \frac{\sum I(g(X))}{n}$$

(4)

where $n$ is the number of simulations, i.e. the number of times $g(X)$ is calculated for random hydrodynamic conditions and remaining variables of (1). $I(g(X))$ is an indicator function equal to 1 if $g(X) \leq 0$ or 0 if $g(X) > 0$. If $g(X) \leq 0$, the scour protection fails, i.e. the top layer is eroded, because the acceptable level of damage, i.e. the $S_{3D} = 1.00$, has been exceeded for certain random conditions.

Here the definition of failure is linked to the non-compliance with the design criteria. In practical situations, the actual physical failure (exposure of the filter) may not have occurred yet. However, as the design criterion is not being met the protection is considered to be in eminent failure. Taking this
definition into consideration, it is noted that if $R$ changes from 1.00 to any other value, then the probability of failure will also change.

Therefore, it is important to understand, which value of $R$ should be used. The changes in the sub-areas definition may lead to changes in the acceptable damage limits as defined in [9]. Moreover, the calculation of the $S_{3D}$ from the bathymetric data has a direct impact on the regression fit used to develop the predictive design formula of the $S_{3D}$, also included in $g(X)$.

2.3. Physical model

The aforementioned changes, influence the probability of failure. In this research, a minor example of a physical model test is provided to show that the damage number may vary depending on the sub-area arrangement and to provide a prospective discussion on the potential effects on the probability of failure.

The physical model consists in a protected monopile foundation, with rip-rap scour protection with an extent of $5D_p$ and an outer slope of 1:3. The model had a geometrical scale of 1:50 with Froude’s similitude. The test included the effect of waves and current combined, with a $D_{50}$ of 4.2 mm and a protection thickness of $3D_{50}$, which is within the tested range in [9]. The rock material density was 2630 kg/m$^3$. A ratio of 0.84 between the median diameter ($D_{50}$) and the nominal median diameter ($D_{n50}$) was considered.

A total of 9000 waves were performed and no visual exposure of the filter was noticed. The wave-current flume had a total length of 32 m with a cross-section, 1 m wide by 1 m height. Full details of the experimental setup, equipment, measurements accuracy and testing conditions are given in [13]. The scour protection was subjected to a mean significant wave height of 0.11 m (5 m in prototype) and a peak period of 1.60 s (11 s in prototype). A depth averaged current velocity of 0.15 m/s was measured at 40% of the mean water level, counting from the sand bed to surface. Figure 3 shows the drained tested model before testing and after 9000 waves. The scour protection showed very minor movements of the armour material.

3. Results and discussion

3.1. Deriving the damage number from different sub-areas arrangements

Based on the physical model described, the bathymetric data was analysed under the sub-areas arrangement from [9] and [13] (Figure 4 and Figure 5). Despite the reasonable agreement in the damage pattern, where the majority of the scour occurs near the pile and at the edge region, the difference in the maximum damage number is significant and varies roughly 68% from the methodology presented in Figure 4 to Figure 5. The methodology [13] provides a larger value near the pile. This was expected due to the smaller area size considered. It can be noted that the method presented in [13] is concentrating the damage at the pile centre more than at the edges. The scour pattern is different from the typical shapes.
reported in [3, 8, 9]. At the upstream region more edge scour should be expected. The present model shows a larger edge scour at the lee-wake region. The remaining damage at the upstream side of the pile is consistent with typical damage under waves and current combined. Nevertheless, the physical model enables to prove the main assumption, i.e. the damage number is influenced by the sub-area size and the analysis applied to the bathymetric data. Results show that the damage number varies considerably depending on the methodology used, thus the reliability assessment is affected in the sense that the limits for static and dynamic stability, formerly introduced by [9], may require a re-evaluation if different sub-areas are used. This is particularly important as it can affect the limit state function, g(X), particularly if the failure of the protection (failure criterion) is defined in alternative ways, e.g. different areas of filter exposure.

Figure 4. (a) Bathymetry differences between 0 and 9000 waves, (b) sub-areas analysis according to [9], $S_{3D_{max}}=0.936$ for 24 sub-areas equal to the pile cross-section (waves from bottom to top).

Figure 5. (a) Bathymetry differences between 0 and 9000 waves, (b) sub-areas analysis according to [13], $S_{3D_{max}}=1.37$ for 48 sub-areas with radial division (waves from bottom to top).
3.2. Theoretical example of the influence in the probability of failure

Considering different values of acceptable damage leads to a direct influence on the probability of failure in the sense that the design criterion is being more or less restrictive depending on the chosen R (Eq. 3). As discussed previously these may change depending on the methodology used to calculate the S3D value from bathymetric data, whereas such methodology may also lead to alternatives to Eq. (1).

In this section, the data from Horns Rev 3 offshore wind farm, fully reported in [1] and [3], is used to show how the probability of failure is affected by the S3D value chosen for Eq. (3). Monte-Carlo method was used to evaluate the limit state function g(X) based on a bi-variate version of the kernel density estimation method for n = 300,000 simulations, which has been shown in [1] to be a sufficient number to obtain a stabilized value of the probability. The input of the random variables that form vector X is also the same as defined in [1]. In brief, the significant wave height ranged from 0.14 m to 6.11 m and the peak periods from 1.60 s to 22 s (see e.g. [1, 3]).

For the present case three curves are presented in Figure 6 for different values of D50 (0.84Dn50). The first curve shows the probability of failure for R = 0.25, the limit for static stability, the second curve for R=1.00, the limit for dynamic stability (both as defined in [9]). The third curve is calculated for R=2.00. The use of larger values of R has a physical meaning, which corresponds to considering a less restrictive design criterion. The meaning is that the designer is assuming that more layers of stones, i.e. rock armour material, can be removed from the scour protection before the filter is exposed (i.e. before failure).

Figure 6 shows that increasing the acceptable damage in the design criteria leads to smaller probabilities of failure, thus meaning that the protection for the same random loading conditions fulfills the design criterion more often. As the scour protection is designed to be dynamic, applying an acceptable damage of R = 0.25, compatible with static stability, is leading to large probabilities of failure, thus meaning that, for the same random loading conditions, the protection is often having an estimated S3D larger than 0.25. The effects on the probability of failure shown in Figure 6 are in agreement with those given in [2], where a set of copula-based models is used. As the D50 increases there is no damage on the protection for the considered conditions.

As the definition of a proper acceptable damage is a key aspect to estimate the reliability of the scour protection, the underlying question, is which value of R should be used. The answer to this relies on the failure definition, the method used to analyse the bathymetry and finally on the predictive equation used.

![Figure 6. Example of the influence of the acceptable damage number as introduced by [9] in the probability of failure.](image-url)
Thinking that the S3D can represent the number of layers removed over the analysed sub-area, then the probabilities derived with R = 0.25 could be analysed under a static stability criterion, e.g. [14] which is based on the threshold of motion and the critical shear-stress. A comparison between probabilities derived from statically stable and dynamically stable criteria can be seen in [1]. Although the present calculation serves its purpose of showing the influence of the acceptable damage, for statically stable scour protections, the use of g(X) based on [14] is recommended by [3].

Considering, for example, a scour protection with a thickness of rock material equal to 3D50. It seems appealing to state that depending on the sub-area being considered the exposure of the filter occurs for R = S3D = 3. A theoretical reliability analysis for the physical model of section 3.1, i.e. D50 = 0.21 m and R = 3, yields a probability of failure of 1.9×10\(^{-4}\) (see Table 1), which is lower than the ones presented for the remaining values of R (Table 1). In this sense, the values shown in Figure 6 may be seen as conservative assessments of the protection’s reliability.

Nevertheless, an important problem is that the limit state function from (3) does not directly consider the area of filter layer exposure, namely the area of 4D_{50}^2 used to identify the failure visually, as in [9] and [10]. Therefore, present research highlights the importance of developing new methodologies to calculate the damage number that might be directly related to a meaningful area of filter exposure. Thus contributing to improving the definition of g(X), both to define a proper parcel R and, eventually, a more accurate way of estimating the S3D.

| R (acceptable S3D) | Pf     |
|-------------------|--------|
| 0.25              | 2.0×10^{-2} |
| 1.00              | 4.6×10^{-3} |
| 2.00              | 1.3×10^{-3} |
| 3.00              | 1.9×10^{-4} |

4. Conclusions
Reliability analysis remains as a rather novel but important topic to scour protections. Despite the recent research aiming at understanding the failure frequency at scour protections for offshore wind foundations, there are still aspects to be clarified.

One of the most important relies on the accuracy of the limit state function, which largely depends on the definition of an acceptable damage level (R). The definition of this parameter is yet to be fully understood and is currently progressing based on the knowledge derived from physical model studies and different methodologies to calculate the damage at the top layer.

The present research addressed the influence of the acceptable damage number on the reliability assessment of a scour protection by means of a theoretical example. It was concluded that varying this parameter leads to considerable changes in the probability failure, which agrees with recent findings from [1, 2 and 3]. Moreover, an example is used to show that a value of S3D=1 may lead to a conservative estimation of the protection’s failure, in scour protections with thicknesses that exceed one times the median stone size (1D50). However, regardless of this influence’s magnitude, it was discussed that defining an acceptable damage number largely depends on the methodology employed to calculate the S3D. A brief analysis of the physical model was performed with the different areas, based on [9] and [13], being concluded that the methodology applied influences the maximum damage number. This fact formerly introduced by [5] is hereby proved. This implies that the predictive formula, Eq. (1), and the associated acceptable damage is re-evaluated if new methodologies are used. The model showed a variation in the maximum damage number close to 68%. In fact, while 24 sub-areas are used in [9], 48 are used in [13]. Thus, not only the size and shape of each sub-area differ, but the sample of damage

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[14]: Reference to the cited article.
numbers, from which the maximum is chosen, is also different. This may lead to an influence when addressing the damage distribution in the scour protection. Such aspect requires further research in order to understand how the number of sub-areas contributes to a better insight on the damage pattern and the definition of new threshold parameters for the limit state function.

Different arrangements of the sub-areas influence the limit state function, not only in terms of the failure threshold, but also on the potential alternatives for damage prediction, i.e. alternatives to Eq. 1. These findings highlight the need for further research on bathymetric analysis and damage estimation as a way to provide more accurate reliability assessments. In addition, it was seen that a clear relationship between the damage limits proposed in [9] and discussed in [8] and the visual area of exposed filter [9, 10] is yet to be determined. The use of more flexible sub-areas arrangements, as formerly suggested in [5] may provide a better insight on how to define the acceptable damage for common scour protections, allowing for limit state functions that simulate the actual exposure of the filter.

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