Expectation values of flavor-neutrino currents in field theoretical approach to oscillation problem —— formulation

Kanji Fujii and Takashi Shimomura
Department of Physics, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan
(Dated: March 26, 2022)

As a possible approach to the neutrino oscillation on the basis of quantum field theory, the expectation values of the flavor-neutrino currents are investigated by employing the finite-time transition matrix in the interaction representation. Such expectation values give us in the simplest form a possible way of treating the neutrino oscillation without recourse to any one flavor-neutrino states. The present paper is devoted to presenting the formulation and the main structures of the relevant expectation values.

PACS numbers: 14.60.Pq

I. INTRODUCTION

Many papers have been published concerning the theoretical basis of the standard formulas of the neutrino oscillations. As to the works appeared till the middle of the year 2001, we can find the list of references in the review article written by Buete[1]; in this article he gave his view by arranging the controvercial points in the neutrino- and meson-oscillations.

A. Proposal of problems

The analyses of the neutrino experimental data have been usually done on the basis of the standard formula of the neutrino oscillation. In the present work we concentrate on the following two problems included in the ordinary derivation of the standard formula of neutrino oscillations in vacuum [2, 3].

First we summarize the ordinary derivation of the standard formula in vacuum [2, 3]. Let’s consider the neutrino with momentum \( k \) and helicity \( r = -1 \), produced through the charged-current weak interaction. Such a flavor-neutrino state is assumed to be a superposition of the mass eigenstates, expressed as

\[
\langle \nu_\sigma (k) \rangle = \sum_j \langle \nu_\sigma (k) | \nu_j (k) \rangle \langle \nu_j (k) |, \quad \sigma = (e, \mu, \cdots), j = (1, 2, \cdots); \quad (1.1)
\]

the matrix \( U = [U_{\sigma j}] \) with \( U_{\sigma j} = \langle \nu_\sigma (k) | \nu_j (k) \rangle \) is unitary.

By employing the time evolution of \( \langle \nu_j (k) | \),

\[
\langle \nu_j (k) | e^{-i \omega_j (k) t} | \nu_j (k) \rangle = \sqrt{k^2 + m^2_j}, \quad (1.2)
\]

the flavor-neutrino state is seen to evolve in time as

\[
\langle \nu_\rho (k) | t \rangle = \sum_{\lambda, j} U_{\rho j} e^{-i \omega_j (k) t} U_{j \lambda}^* \langle \nu_\lambda (k) | \quad (1.3)
\]

where

\[
A_{\sigma \rho} (k; t) = \left( U e^{-i \omega_{\text{diag}} (k) t} U^\dagger \right)_{\rho \sigma}, \quad \omega_{\text{diag}} = \begin{pmatrix} \omega_1 (k) \\
\omega_2 (k) \\
\vdots \end{pmatrix}. \quad (1.4)
\]
Then, the transition probability is
\[ P_{\sigma \rightarrow \rho}(t) = |A_{\sigma\rho}(k;t)|^2 \]
\[ = \sum_{j,l} U_{\sigma j}^* U_{\rho j} U_{\sigma l} U_{\rho l} e^{i(\omega_j(k) - \omega_l(k))t}. \]  \hspace{1cm} (1.5)

Under the condition of the neutrino velocities
\[ v_j(k) = \frac{|k|}{\omega_j(k)} \simeq 1 \]  \hspace{1cm} (the light velocity in vacuum),

the time \( t \) which is regarded as the time of neutrino propagation is replaced by the distance \( L \) from source to detector approximately. Then we have
\[ P_{\sigma \rightarrow \rho}(L) = \sum_{j,l} |U_{\sigma j}|^2 |U_{\rho j}|^2 + 2 \text{Re} \sum_{j > l} U_{\sigma j} U_{\rho j}^* U_{\sigma l} U_{\rho l} e^{i \frac{\Delta m^2_{jl}}{2|k|} L}. \]  \hspace{1cm} (1.7)

here we used \( \omega_j(k) - \omega_l(k) \simeq \frac{(m_j^2 - m_l^2)}{2|k|} = \frac{\Delta m^2_{jl}}{2|k|} \) which holds for extremely relativistic neutrinos.

Now we mention the two problems included in the derivation described above.

**Problem I:** The matrix \( U = |U_{\sigma\rho}| \) is defined as the matrix which diagonalizes the mass matrix in the flavor bases \( M ; \)
\[ M = \begin{pmatrix} m_{ee} & m_{e\mu} & \cdots \\ m_{\mu e} & m_{\mu\mu} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad M = M^\dagger \]  \hspace{1cm} (1.8)

\[ U^\dagger M U = \begin{pmatrix} m_1 \\ \vdots \\ m_2 \end{pmatrix} = M_{\text{diag}} \]  \hspace{1cm} (1.9)

(1.9) leads to
\[ U^\dagger \left( |k| + \frac{MM^\dagger}{2|k|} \right) U = |k| + \frac{m^2_{\text{diag}}}{2|k|} = \omega_{\text{diag}}(k), \]  \hspace{1cm} (1.10)

in the extremely relativistic case.

The important point is that the matrix \( U \) is set equal to the mixing matrix \( Z^{1/2} = \left[ Z_{\mu\rho}^{1/2} \right] \) between the two kinds of neutrino fields, i.e.
\[ \nu_F(x) = Z^{1/2} \nu_M(x), \quad Z^{1/2 \dagger} Z^{1/2} = 1 \]  \hspace{1cm} (1.11)

for
\[ \nu_F(x) = \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \vdots \\ \nu_\tau(x) \end{pmatrix}, \quad \nu_M(x) = \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \\ \vdots \\ \nu_3(x) \end{pmatrix}, \]  \hspace{1cm} (1.12)

where \( Z^{1/2} \) is defined by
\[ Z^{1/2 \dagger} M Z^{1/2} = \text{M}_{\text{diag}}; \]  \hspace{1cm} (1.13)

\( M \) is the mass matrix for the "free part" \( \mathcal{L}_0(x) = -\bar{\nu}_F(x)(\bar{\nu} + M)\nu_F(x) \) in the total Lagrangin
\[ \mathcal{L}(x) = \mathcal{L}_0(x) + \mathcal{L}_{\text{int}}(x). \]  \hspace{1cm} (1.14)

Here, \( \mathcal{L}_{\text{int}}(x) \) is assumed to have no bilinear term and no derivative of the neutrino fields. The Hamiltonian \( H_0(x) \) obtained from \( \mathcal{L}_0(x) \) is
\[ H_0(x) = \bar{\nu}_F(x)(\bar{\gamma} \cdot \partial + M)\nu_F(x) \]
\[ = \sum_j \bar{\nu}_j(x)(\bar{\gamma} \cdot \partial + m_j)\nu_j(x), \]  \hspace{1cm} (1.15)
which means the energy is diagonal with respect to $\nu_j(x)'s$, irrespectively of the momentum. Thus, in spite of the situation that, apart from a trivial phase factor, we can take

$$Z^{1/2} = U,$$

there is a certain inconsistency. It is often asserted [4] that such an inconsistency disappears in extremely relativistic neutrinos as in existing experiments; therefore, the state (1.14) with $U = Z^{1/2}$ reflects correctly the present experimental situation. It is, however, not so simple conceptually for us to solve such an inconsistency from the field theoretical viewpoint as noted in Refs. [5]-[8]. The problem with which we are now confronted is what relation the state has with the field-operator relation; in other words, what definition should be given to the one-particle state of flavor neutrino properly under the condition $\nu_F(x) = Z^{1/2}\nu_M(x)$.

In connection with this problem, various authors have proposed [9]-[11] that the neutrino oscillation should be investigated on the basis of the quantum field theory by examining transition amplitudes, in which flavor neutrinos appear only intermediate states between source- and detection-interactions, as schematically shown in FIG. 1. In such approaches, it is unnecessary for us to prepare any one-particle state of flavor neutrino from the beginning; therefore, on the contrary to the standard formulation, we are free from the trouble concerning Problem I.

**FIG. 1: Real experimental situation**

**Problem II**: The replacement of ct by the source-detector distance $L$ for obtaining (1.7) reflects such a situation that $P_{\sigma\rightarrow\rho}(L)$ mimics the usual experimental circumstances more closely than $P_{\sigma\rightarrow\rho}(t)$, because the times of reactions are not known and we measure distances rather than times. This simple replacement causes some doubts concerning the consistence of this procedure with the approximation $\omega_j(k) - \omega_l(k) \simeq \Delta m^2_{jl}/(2|k|)$. Concerning a proper definition of space-dependent probability $P_{\sigma\rightarrow\rho}(L)$, an interesting approach called "the current-density approach" has been proposed by Ancochea et al. [12]; some modification is found in Ref. [13]. In this quantum mechanical approach, the oscillation formulas $P_{\sigma\rightarrow\rho}(L)$, as a function of the source-detector distance $L$, are obtained after integrating the probability current density over a surface $\partial A$ around a detector as well as the time duration of measurements;

$$P_{\sigma\rightarrow\rho}(L) = \int_{t_f}^{t_i} dt \int_{\partial A} dS \cdot j(x, t).$$

(1.17)

By employing the appropriate probability current densities of $K^0$ and $\bar{K}^0$ constructed with the use of wave-packet functions of $K_s$ and $K_L$, the usual formulas have been shown to be derived.

Blasone et al. [14] considered the neutrino oscillation along the line of the current density approach. The current density $j_\rho(x, t)$ is replaced by the expectation values

$$\langle \nu_\sigma(x_t, t_I) | j_\rho(x, t) | \nu_\sigma(x_t, t_I) \rangle,$$

(1.18)

where $| \nu_\sigma(x_t, t_I) \rangle$ is an initial wave-packet state of the flavor constructed on the flavor vacuum in accordance with the formalism given by Blasone and Vitiello [15]. It is pointed out that in low-energy region of neutrinos certain significant deviations from the standard formula may appear.

**B. Purpose and starting formula**

The purpose of the present work is to investigate the neutrino oscillation by taking into consideration the two kinds of approaches explained in connection with Problems I and II. We consider the formula (1.17) with $j_\rho(x, x^I)$
replaced by a certain kind of expectation value in the quantum field theory. In order to do this, we notice that in the interaction representation the expectation value of a physical observable \( F(x) \) at a space-time point \( x = (x, x^0) \) with respect to a state \( \ket{\Psi(x^0)} \) is given, in accordance with e.g. Umezawa’s textbook [15], by
\[
\langle \Psi(x^0) \vert F(x) \vert \Psi(x^0) \rangle = \langle \Psi(x^0) \vert S^{-1}(x^0, x_1^0) F(x) S(x^0, x_1^0) \vert \Psi(x^0) \rangle,
\]
where
\[
S(x^0, x_1^0) = \sum_{m=0} (-i)^m \int_{x_1^0}^{x_2^0} d^4y_1 \int_{x_1^0}^{y_2^0} d^4y_2 \cdots \int_{x_1^0}^{y_{m-1}^0} d^4y_m H_{\text{int}}(y_1) H_{\text{int}}(y_2) \cdots H_{\text{int}}(y_m).
\]

We consider the simplest case, where one \( \pi^+ \) decays into \( \nu_\mu \) and \( \bar{\nu}_\mu \); for convenience we write \( \pi^+ \to \nu_\sigma + \bar{l}_\sigma \). As the initial state \( \ket{\Psi_\pi(x^0)} \), we adopt a wave-packet state which is to be explained later in Sec.III. So we examine the expectation value of the flavor-neutrino current \( j^{\nu_\sigma}_\rho(x) \), which is to be defined below, with respect to the state \( \ket{\Psi_\pi(x^0)} \).

![FIG. 2:](image_url)

The content of the first-step investigation is schematically shown in FIG.2. Thus this is seen to give the simplest case, in which the oscillation probabilities are calculated in accordance with the quantum field theory as well as without recourse to any flavor neutrino state. After a small modification, it will be seen that we can take into account the observation of \( l^+_\sigma \) as schematically shown in FIG.3.

![FIG. 3:](image_url)

The relevant weak interaction in (1.14) is written as
\[
\mathcal{L}_{\text{int}}(x) = \sum_\rho [\bar{\nu}_\rho(x) J_\rho(x) + \bar{J}_\rho(x) \nu_\rho(x)]; \quad \text{(1.21)}
\]
\[
J_\rho(x) = \delta / \delta \bar{\nu}(x) \cdot \mathcal{L}_{\text{int}}^\text{eff} = i f_{\pi\rho} v^a l_\rho(x) \partial_a \phi(x), \quad \text{(1.22)}
\]
\[
\bar{J}_\rho(x) = \delta \mathcal{L}_{\text{int}} / \delta \nu(x) \text{ eff} = i f^{\ast}_{\rho} \bar{l}_\rho(x) v^a \partial_a \phi^\dagger(x), \quad \text{(1.23)}
\]
with \( v^a = \gamma^a(1 + \gamma_5) \). (We use the Kramers representation \( \gamma \)-matrices as explained in Refs. [6, 7].) Following the natural definition of "currents" [17], we consider the infinitesimal phase transformation

\[
\nu_F(x) \rightarrow \nu'_F(x) = e^{i\varphi(x)}\nu_F(x),
\]

(1.24)

with

\[
\varphi(x) = \begin{pmatrix} \varphi_e(x) \\ \varphi_\mu(x) \\ \cdots \end{pmatrix},
\]

and obtain

\[
\mathcal{L}(x) \rightarrow \mathcal{L}'(x) = \mathcal{L}(x) + \delta\mathcal{L}(x),
\]

\[
\delta\mathcal{L}(x) = -i\bar{\nu}_F(x) (\gamma^a \partial_a \varphi(x) + [M, \varphi(x)]) \nu_F(x) + \delta\mathcal{L}_{\text{int}}.
\]

(1.25)

By remembering \( \mathcal{L}_{\text{int}} \) has no derivative of \( \nu_F(x) \)-field, we obtain the flavor neutrino current as

\[
j_{(\rho)}(x) = \frac{\partial\delta\mathcal{L}(x)}{\partial(\partial_a \varphi^\rho(x))}
\]

\[
= -i\bar{\nu}_\rho(x)\gamma^a \nu_\rho(x)
\]

\[
= -i \sum_{j,l} Z_{\rho j}^{1/2} Z_{\rho l}^{1/2} \bar{\nu}_j(x)\gamma^a \nu_l(x);
\]

(1.26)

its 4-divergence is given by

\[
\partial_a j_{(\rho)}^a(x) = \frac{\partial\delta\mathcal{L}(x)}{\partial(\varphi^\rho(x))}
\]

\[
= -i \left[ (\bar{\nu}_F(x)M)_\rho \nu_\rho(x) - \bar{\nu}_\rho(x)(M\nu_F(x))_\rho \right] + \frac{\partial\mathcal{L}_{\text{int}}(x)}{\partial\varphi^\rho(x)}.
\]

(1.27)

The right-hand is called the "diffusion term" [13, 14]. In the interaction representation, in which the \( \nu_M(x) \)-field satisfies the free equation

\[
(\partial + M_{\text{diag}})\nu_M(x) = 0,
\]

(1.28)

the total flavor current of neutrinos is seen to be conserved;

\[
\sum_\rho \partial_a j_{(\rho)}^a = 0.
\]

(1.29)

The remaining part of the present paper is organized as follows. In Sec.II, we perform some model calculations to see the relation of the current-operator expectation values (1.19) to corresponding transition amplitudes. In Sec.III, we investigate the structures of the expectation values for \( j_{(\rho)}^a(x) \) with respect to the wave-packet state, decaying into \( \nu_\sigma \) and \( \bar{l}_\sigma \). In Sec.IV, we discuss the structures of the same kind of the expectation values in the boson model for simplicity, in which the observation of a charged boson in decay products is taken into consideration, and in Sec.V summarize results and future tasks. The detailed analyses including numerical calculations are to be left to the subsequent paper [18].

### II. RELATION OF CURRENT EXPECTATION VALUE TO TRANSITION MATRIX ELEMENT

#### A. The case with no mixing

We perform the simplest model calculation with the aim of seeing the relation of a current expectation value to the absolute square of the relevant transition matrix element. First we examine the case of no mixing interaction.
Let’s consider the simple situation where (pseud-)scalar particles $A, B, C$ have the weak interaction

$$L_{\text{int}} = f_{ABC} \phi_C^\dagger(x) \phi_B(x) \phi_A(x) + H.c.$$  
$$= - \mathcal{H}_{\text{int}}(x). \quad (2.1)$$

An $A$-particle described by the complex field $\phi_A(x)$ decays into $\bar{B}$ and $C$ particles. We write the current of the $C$-field as

$$j_b^{(C)}(x) = i \left( \phi_C^\dagger(x) \partial_\mu \phi_C(x) - \partial_\mu \phi_C^\dagger(x) \phi_C(x) \right)$$
$$\equiv i \phi_C^\dagger \partial_\mu \phi_C(x). \quad (2.2)$$

We execute, in the interaction representation, perturbative calculations in the lowest order of $\mathcal{H}_{\text{int}}$ by employing the plane wave expansion

$$\phi_f(x) = \int d^4p \frac{1}{(2\pi)^{3/2}2E_f(p)} \left[ \alpha_f(p)e^{ipx} + \beta_f^\dagger(p)e^{-ipx} \right], \quad f = A, B, C; \quad (2.3)$$

where $px = p \cdot x - E_f(p)x^0$, $E_f(p) = \sqrt{p^2 + m^2}$; the commutation relations among the expansion-coefficient operators are

$$[\alpha_f(p), \alpha_g(q)] = [\beta_f(p), \beta_g^\dagger(q)] = 2E_f(p)\delta_{fg}\delta(p - q), \quad \text{others} = 0; \quad (2.4)$$

the vacuum $|0\rangle$ is defined by

$$\alpha_g(p)|0\rangle = \beta_g(p)|0\rangle = 0 \quad \text{for} \quad g = A, B, C. \quad (2.5)$$

the number operator of $\phi_C$-field is given by

$$N_C(x^0) = i: \int d^3x j_b^{(C)}(x):$$
$$= \int d^3k \frac{1}{2E_C(k)}[\alpha_C^\dagger(k)\alpha_C(k) - \beta_C^\dagger(k)\beta_C(k)]. \quad (2.6)$$

(The symbol $:\cdot:$ means the normal ordering.)

In the following we examine the forms of expectation values of the current and the number operator with respect to one $A$-particle state

$$|A(p; x^0)\rangle = S(x^0, x_f^0)^{-1} : j_b^{(C)}(x) : S(x^0, x_f^0)\alpha_A^\dagger(p)|0\rangle. \quad (2.7)$$

We define the expectation value of the current with respect to the state $|A(p; x_f^0)\rangle$ as

$$\mathcal{E}(A(x_f^0); \text{C-current}(x))_b \equiv \langle 0| \alpha_A(p)S(x_f^0, x_f^0)^{-1} : j_b^{(c)}(x) : S(x_f^0, x_f^0)\alpha_A^\dagger(p)|0\rangle_{\text{con}}, \quad (2.8)$$

and also that of the number operator as

$$\langle N_C(x^0) \rangle_{A(p; x_f^0)} = i \int d^3x \mathcal{E}(A(x_f^0); \text{C-current}(x))_4. \quad (2.9)$$

We take out only the connected part in the expectation values, as designated in R.H.S. of (2.8). In the lowest order (i.e. the second order) of the weak interaction, we have

$$\langle N_C(x^0) \rangle_{A(p; x_f^0)} = \langle 0| \alpha_A(p) \int_{x_f^0}^{x^0} dz H_{\text{int}}(z)N_C(x^0) \int_{x_f^0}^{x^0} dy H_{\text{int}}(y)\alpha_A^\dagger(p)|0\rangle,$$  
$$\quad (2.10)$$

which consists of two parts;

$$\langle N_C(x^0) \rangle_{A(p; x_f^0)} = \langle n_C(x^0) \rangle_{A(p; x_f^0)} + \langle \tilde{n}_C(x^0) \rangle_{A(p; x_f^0)},$$  
$$\quad (2.11)$$
where

\[ n_C = \int d^3k \frac{1}{2E_C(k)} \alpha_C^\dagger(k) \alpha_C(k), \quad \bar{n}_C \equiv -\int d^3k \frac{1}{2E_C(k)} \beta_C^\dagger(k) \beta_C(k). \tag{2.12} \]

The first part, represented as FIG. 4 (a), is given by

\[ \langle n_C(x^0) \rangle_{A(p,x^0)} = \int_{x_1^0}^{x_0^0} d^4y |f_{ABC}|^2 (2\pi)^3 \int_{x_1^0}^{x_0^0} d^3y e^{-ipz + i\phi^\dagger_B(0)|\phi_B(y)|0} |\phi_C(z)n_C\phi_C(y)|0 \]

\[ = \int_0^t d\tau d^3x e^{i\tau(E_A(p) - E_B(q) - E_C(k)(z^0 - y^0))} \bar{V}_{AB} f_{ABC} |f_{ABC}|^2 \frac{1}{(2\pi)^6 E_B(q)2E_C(k)} \bigg|_{q=-p-k}, \tag{2.13} \]

here, \( t \equiv x^0 - x_1^0, \ V = \int d^3x. \) Similarly, the second part in R.H.S. of (2.11), represented as FIG. 4 (b), is given by

\[ \langle \bar{n}_C(x^0) \rangle_{A(p,x^0)} = \int_{x_1^0}^{x_0^0} d^4y |f_{ABC}|^2 (2\pi)^3 \int_{x_1^0}^{x_0^0} d^3y e^{-ipz + i\phi^\dagger_B(0)|\phi_B(y)|0} |\phi_A(z)\phi_A(y)|0 \]

\[ \times (\phi_C(z)\bar{n}_C\phi_C(y)|0) \alpha_A(p)\phi_A(z)\bar{\alpha}_A(p) |0 \]

\[ = -\int_0^t d\tau d^3x e^{i\tau(E_A(p) + E_B(q) + E_C(k)(z^0 - y^0))} \bar{V}_{AB} f_{ABC} |f_{ABC}|^2 \frac{1}{(2\pi)^6 E_B(q)2E_C(k)} \bigg|_{q=-p-k}, \tag{2.14} \]

By adding the damping factor \( e^{-|z^0 - y^0|}, \epsilon > 0, \) to the integrands of (2.11) and (2.14), we obtain

\[ \langle n_C(x^0) \rangle_{A(p,x^0)} = V |f_{ABC}|^2 \int d^3k (2\pi)^6 E_B(q)2E_C(k) \]

\[ \times \frac{e^{i(E_A(p) - E_B(q) - E_C(k) + i\epsilon)t} - 1}{E_A(p) - E_B(q) - E_C(k) + i\epsilon} \frac{e^{-i(E_A(p) - E_B(q) - E_C(k) - i\epsilon)t} - 1}{E_A(p) - E_B(q) - E_C(k) - i\epsilon} \bigg|_{q=-p-k}, \tag{2.15} \]

\[ \langle \bar{n}_C(x^0) \rangle_{A(p,x^0)} = -V |f_{ABC}|^2 \int d^3k (2\pi)^6 E_B(q)2E_C(k) \]

\[ \times \frac{e^{-i(E_A(p) + E_B(q) + E_C(k) - i\epsilon)t} - 1}{E_A(p) + E_B(q) + E_C(k) - i\epsilon} \frac{e^{i(E_A(p) + E_B(q) + E_C(k) + i\epsilon)t} - 1}{E_A(p) + E_B(q) + E_C(k) + i\epsilon} \bigg|_{q=-p-k}. \tag{2.16} \]

Thus we see that due to the higher configuration in the intermediate state of FIG. 4 (b) in comparison with FIG. 4 (a), the main contribution to \( \langle N_C(x^0) \rangle_{A(p,x^0)} \) comes from \( \langle n_C(x^0) \rangle_{A(p,x^0)} \); i.e.

\[ \langle N_C(x^0) \rangle_{A(p,x^0)} \simeq \langle n_C(x^0) \rangle_{A(p,x^0)}. \tag{2.17} \]

On the other hand, the transition amplitude from the initial A state at \( x_1^0 \) to \( \bar{B}(q) \) and \( C(k) \) at \( x^0 \) is, in the first order of the weak interaction, given by

\[ M(A(p,x^0)_1 \rightarrow \bar{B}(q) + C(x^0) \equiv \langle 0 | \beta_B(q)\alpha_C(k)i \int_{x_1^0}^{x^0} d^4y |f_{ABC}|^2 \frac{1}{2E_C(k)} \phi_C(z)\phi_A(z)\alpha_A^\dagger(p) |0 \rangle \]

\[ = if_{ABC} \int_{x_1^0}^{x^0} d^4y \frac{e^{iz(p-q-k)}}{(2\pi)^{3/2}}, \tag{2.18} \]
from which we obtain

\[ \int \frac{d^3q}{2E_B(q)} \int \frac{d^3k}{2E_C(k)} |M|^2 = |f_{ABC}|^2 V \int d^3q \int d^3k \frac{\delta(p - q - k)}{(2\pi)^6 2E_B(q) 2E_C(k)} \]
\[ \times \int_{x_0}^{x_0} \int_{x_0}^{x_0} dz^0 dy^0 e^{i(z^0 - y^0)(E_A(p) - E_B(q) - E_C(k))} \]
\[ = \langle n_C(x^0) \rangle_{A(p,x^0)} \simeq \langle N_C(x^0) \rangle_{A(p,x^0)}. \]  

(2.19)

Diagramatically we may express the above equality as FIG. 5.

\[ \int x_0^0 \int x_0^0 d^4 z d^4 y \frac{A(p,x)}{A(p,x)} \]
\[ \simeq \int \int \frac{d^3q d^3k}{2E_B(q) 2E_C(k)} \left| \int x_0^0 d^4 z \right| \frac{B(q)}{C(k)} \]

FIG. 5: Relation of the expectation value of number operator to the transition probability.

A derivation which is somewhat more general than that explained above is given when we assume the $\tilde{n}_C$-contribution to the expectation values (2.8) is smaller than $n_C$-contribution, because the former arises from higher configurations; i.e.

\[ \langle N_C(x^0) \rangle_{A(p,x^0)} = \langle 0 | \alpha_A(p) S^{-1}(x^0, x^0) [n_C + \tilde{n}_C] S(x^0, x^0) \alpha_A^\dagger(p) | 0 \rangle^\text{con}, \]
\[ \simeq \int \frac{d^3k}{2E_C(k)} \langle 0 | \alpha_A(p) S^{-1}(x^0, x^0) \alpha_C^\dagger(k) | 0 \rangle^\text{con} \langle 0 | \alpha_C(k) S(x^0, x^0) \alpha_A^\dagger(p) | 0 \rangle^\text{con} \]
\[ = \int \frac{d^3k}{2E_C(k)} \sum_{\text{phase vol}} |\langle C(k); X | S(x^0, x^0) | \alpha_A(p) \rangle|^2. \]  

(2.20)

A similar consideration applies also to the weak interaction process, in which a source and a target are included and the particle C scattered from the target is observed, as shown in FIG. 6:

\[ \sum \text{phase vol} \]
\[ B \rightarrow X \rightarrow A \]
\[ N_C(x^0) \]
\[ \simeq \sum \text{phase vol} \]
\[ (x^0) \]
\[ B \rightarrow C \rightarrow A \]

FIG. 6: Relation of the expectation value of number operator to the transition probability. Both include production and detection processes.

In the case where X is one D-particle state, RHS of FIG. 6 corresponds to the process examined by Yabuki and Ishikawa [11] when the mixing exists in the intermediate state.
B. The case with mixing

Along the aim of the present paper, it is important for us to consider the case where there exists a "flavor" degree of freedom with mixing among (pseudo-)scalar fields \( \phi_{C\rho}(x) \), \( \rho = 1, 2, \cdots \). The relevant Lagrangian is

\[
\mathcal{L}(x) = \mathcal{L}_0(\phi_A - \phi_B - \phi_C) - (\partial^\mu \phi_C^\dagger(x) \cdot \partial_\mu \phi_C(x) - \phi_C^\dagger(x) M^2 \phi_C(x)) + \sum_\rho \left\{ f_{ABC\rho} \phi_{C\rho}(x) \phi_B(x) \phi_A(x) + H.c \right\},
\]

\[
\phi_C(x) \equiv \left( \begin{array}{c} \phi_\mu(x) \\ \phi_\mu(x) \\ \vdots \end{array} \right), \quad M^2 = \left( \begin{array}{ccc} m_{ee}^2 & m_{e\mu}^2 & \cdots \\ m_{e\mu}^2 & m_{\mu\mu}^2 & \cdots \\ \vdots & \vdots & \ddots \end{array} \right), \quad (M^2)^\dagger = M^2,
\]

Similarly to (1.11)-(1.13), we have

\[
Z^{1/2} M^2 Z^{1/2} = M^2_{\text{diag}} = \left( \begin{array}{ccc} m_1^2 & 0 & \cdots \\ 0 & m_2^2 & \cdots \\ \vdots & \vdots & \ddots \end{array} \right),
\]

\[
\phi_C(x) = Z^{1/2} \phi_C(x)^{(M)}, \quad \phi_C^{(M)}(x) = \left( \begin{array}{c} \phi_{C1}(x) \\ \phi_{C2}(x) \\ \vdots \end{array} \right), \quad Z^{1/2} Z^{1/2} = I.
\]

We assume \( m_j^2 > 0 \) for all \( j = 1, 2, \cdots \), and now consider the interaction representation, where \( \phi_q \)-fields, \( q = A, B, C_j \)’s, satisfy the free equations of motion with masses \( m_q \)'s, and Fock space is constructed on the vacuum \( |0\rangle \), defined in the same way as (2.24).

We examine the structures of the expectation value of \( N_{C\rho}(x^0) \) defined by

\[
N_{C\rho}(x^0) \equiv i \int d^3 x j_b^{(C\rho)}(x) : \quad (2.25)
\]

with

\[
j_b^{(C\rho)}(x) = i \phi_{C\rho}^\dagger(x) \overleftrightarrow{\partial}_b \phi_{C\rho}(x), \quad \rho = e, \mu, \cdots \quad (2.26)
\]

here, needless to explain, the symbol : : represents the normal ordering with respect to \( \{ \alpha_{CJ}(k), \beta_{CJ}(k), \quad j = 1, 2, \cdots \} \) and their Hermitean conjugates. Concretely we have

\[
N_{C\rho}(x^0) = \sum_{j,l} Z_{ij} Z_{jl}^{1/2} \int d^3 k \frac{1}{4} \left[ \frac{1}{E_{CJ}(k)} \left( \frac{1}{E_{CJ}(k)} + \frac{1}{E_{CJ}(k)} e^{i(E_{CJ}(k) - E_{CJ}(k)) x^0} \beta_{CJ}^\dagger(-k) e^{i(E_{CJ}(k) + E_{CJ}(k)) x^0} \right] \right.
\]

\[
= \left. + \left( - \frac{1}{E_{CJ}(k)} + \frac{1}{E_{CJ}(k)} \right) \alpha_{CJ}^\dagger(k) \beta_{CJ}^\dagger(k) e^{i(E_{CJ}(k) - E_{CJ}(k)) x^0} \right]
\]

\[
= \left. + \left( - \frac{1}{E_{CJ}(k)} - \frac{1}{E_{CJ}(k)} \right) \beta_{CJ}(-k) \alpha_{CJ}(k) e^{-i(E_{CJ}(k) + E_{CJ}(k)) x^0} \right]
\]

(2.27) here, \( \beta_{CJ}(q) \) with \( \{ q = -k, q_0 = E_{CJ}(k) \} \) is written simply as \( \beta_{CJ}(-k) \).

Similarly to (2.28), we write the expectation value of \( j_b^{(C\rho)}(x) \) as

\[
\mathcal{E}(A(x^0); C_\rho\text{-cur}(x))_b \equiv \langle 0 | \alpha_{A}(p) S(x^0, x^0) \rangle : j_b^{(C\rho)}(x) : S(x^0, x^0) \alpha_{A}^\dagger(p) | 0 \rangle \quad \text{as con,}
\]

(2.28) which leads to

\[
\langle N_{C\rho}(x^0), A(p, x^0) \rangle_4 \equiv i \int d^3 x \mathcal{E}(A(x^0); C_\rho\text{-cur}(x))_4.
\]

(2.29)
As we see from (2.27), \(N_{\rho}(x^0)\) consists of 4 terms, which we write \(N_{\rho}(x^0)^{(n)}\), \(n = 1, 2, 3, 4\), in order;

\[
N_{\rho}(x^0)^{(1)} = \sum_{j,l} Z_{p,j}^{1/2} Z_{l}^{1/2} \int \frac{d^3 k}{(2\pi)^3} \left[ \frac{1}{4 E_C(k)} + \frac{1}{2 E_C(l)} \right] \times \alpha_{Cj}(k) \alpha_{Cl}(k) e^{i(E_C(k) - E_C(l)) x^0},
\]

(2.30)

\[
N_{\rho}(x^0)^{(3)} = \left[ N_{\rho}(x^0)^{(4)} \right]^\dagger.
\]

(2.31)

In the lowest (i.e. the second ) order contribution with respect to the weak interaction in (2.21), we have for

\[
\langle N_{\rho}(x^0)^{(n)} \rangle_{A(p,x^0)} = \langle 0 | \alpha_A(p) \int_{x_I^0}^{x_f} dz H_{\text{int}}(z) N_{\rho}(x^0)^{(n)} \int_{x_f}^{x_0} dy H_{\text{int}}(y) \alpha_A^\dagger(p) | 0 \rangle_{\text{con}},
\]

(2.32)

\[
\langle N_{\rho}(x^0)^{(3+4)} \rangle_{A(p,x^0)} = i^2 \langle 0 | \alpha_A(p) \int_{x_I^0}^{x_f} dz \int_{x_I^0}^{x_f} dy [H_{\text{int}}(z) H_{\text{int}}(y) N_{\rho}(x^0)^{(3)} + N_{\rho}(x^0)^{(4)}] \alpha_A^\dagger(p) | 0 \rangle_{\text{con}}.
\]

(2.33)

In the following, we assume only one flavor \(\sigma\) is dominant in \(H_{\text{int}}(x)\), similarly to \(\pi^+\) decay (i.e. \(\pi^+ \to \mu^+ + \nu_\mu\)), so that we take

\[
H_{\text{int}}(x) = -f_{ABC\sigma} \phi_C^\dagger(x) \phi_B(x) \phi_A(x) + H.c.
\]

(2.34)

Then we obtain, as the respective contributing diagrams from the diagrams FIG. 7(1) and (2),

\[
\langle N_{\rho}(x^0)^{(1)} \rangle_{A(p,x^0)} = V |f_{ABC\sigma}|^2 \sum_{j,l} Z_{p,j}^{1/2} Z_{l}^{1/2} Z_{\sigma j}^{1/2} Z_{\sigma l}^{1/2} \int \frac{d^3 k}{(2\pi)^3} \times e^{i(E_C(k) - E_C(l))} \frac{1}{4 E_C(k)} + \frac{1}{2 E_C(l)} \times e^{i(E_A(p) - E_B(q) - E_C(j)) z^0} e^{i(E_A(p) + E_B(q) + E_C(l)) z_0} |_{q = p - k},
\]

(2.35)

\[
\langle N_{\rho}(x^0)^{(2)} \rangle_{A(p,x^0)} = -V |f_{ABC\sigma}|^2 \sum_{j,l} Z_{p,j}^{1/2} Z_{l}^{1/2} Z_{\sigma j}^{1/2} Z_{\sigma l}^{1/2} \int \frac{d^3 k}{(2\pi)^3} \times e^{i(E_C(k) - E_C(l))} \frac{1}{4 E_C(k)} + \frac{1}{2 E_C(l)} \times e^{-i(E_A(p) + E_B(q) + E_C(j)) z^0} e^{i(E_A(p) + E_B(q) + E_C(l)) z_0} |_{q = -p - k}.
\]

(2.36)

here, \(t = x^0 - x_I^0\).

---

FIG. 7: Diagram representing \(A(p,x^0)_1\) and \(A(p,x^0)_2\), respectively

Due to the same reason as explained in the preceding subsection A, \(\langle N_{\rho}(x^0)^{(1)} \rangle_{A(p,x^0)}\) contributes dominantly over \(\langle N_{\rho}(x^0)^{(2)} \rangle_{A(p,x^0)}\).
As to \( \langle N_{CP}(x^0)^{(3)} \rangle_{A(p,x^0)} \), the diagrams represented by FIG. 8 contribute to it, the form of which is concretely given as

\[
\langle N_{CP}(x^0)^{(3)} \rangle_{A(p,x^0)} = V|f_{ABC\sigma}|^2 \sum_{j,l} Z_{\rho_{pl}}^{1/2} Z_{\sigma_{pl}}^{1/2} Z_{\sigma_{lj}}^{1/2} Z_{\sigma_{lj}}^{1/2} \int_0^t d\tau_0 \int_0^{\pi} dy_0 \\
\times \left[ \frac{1}{|E_{CJ}(k)|} - \frac{1}{E_{CJ}(k)} \right] \cdot \frac{1}{2E_B(q)} e^{i(E_{CJ}(k) + E_{CJ}(k))t} \\
\times \left[ e^{i(E_A(p) - E_B(q) - E_{CJ}(k))z_0} e^{i(E_A(p) + E_B(q) - E_{CJ}(k))y_0} \right]_{q=p-k} \\
+ e^{i(-E_A(p) - E_B(q) - E_{CJ}(k))z_0} e^{i(E_A(p) + E_B(q) - E_{CJ}(k))y_0} \right]_{q=-p+k} \\
= \left[ \langle N_{CP}(x^0)^{(4)} \rangle_{A(p,x^0)} \right]^*. \tag{2.37}
\]

FIG. 8: Diagrams representing \( \langle N_{CP}(x^0)^{(3)} \rangle_{A(p,x^0)} \)

Due to the factor \( 1/E_{CJ}(k) - 1/E_{CJ}(k) \) in the integrand, \( \langle N_{CP}(x^0)^{(3+4)} \rangle_{A(p,x^0)} \) has its magnitude much smaller than that of \( \langle N_{CP}(x^0)^{(1)} \rangle_{A(p,x^0)} \).

From the model calculation explained above, though more detailed numerical analyses are necessary, we are possible to regard the magnitude of \( \langle N_{CP}(x^0)^{(1)} \rangle_{A(p,x^0)} \) as dominant over other \( \langle N_{CP}(x^0)^{(n)} \rangle_{A(p,x^0)} \)'s; i.e.

\[
\langle N_{CP}(x^0) \rangle_{A(p,x^0)} \simeq \langle N_{CP}(x^0)^{(1)} \rangle_{A(p,x^0)}. \tag{2.38}
\]

If we add by hand the term due to the lifetime of A-particle in the way as

\[
e^{iE_A(p)z_0} \rightarrow e^{i(E_A(p) - \Gamma_A(p)/2)z_0} \quad \text{and} \quad e^{-iE_A(p)y_0} \rightarrow e^{(-iE_A(p) - \Gamma_A(p)/2)y_0}. \tag{2.39}
\]

in \( \langle N_{CP}(x^0)^{(1)} \rangle_{A(p,x^0)} \), we obtain

\[
\langle N_{CP}(x^0)^{(1)} \rangle_{A(p,x^0)} = V|f_{ABC\sigma}|^2 \sum_{j,l} Z_{\rho_{pl}}^{1/2} Z_{\sigma_{pl}}^{1/2} Z_{\sigma_{lj}}^{1/2} Z_{\sigma_{lj}}^{1/2} \\
\times \left[ E_A(p) - E_B(q) - i\Gamma_A(p)/2 \right] \left[ E_A(p) - E_B(q) - i\Gamma_A(p)/2 \right]_{q=p-k} \tag{2.40}
\]
Under the condition $\Gamma_A(p)t \gg 1$, we have

$$\langle N_{C\rho}(x^0) \rangle_A(p,x_0) \approx V_{FAB}\sigma \sum_{j,l} Z_{\rho j}^{1/2} Z_{\rho l}^{1/2} Z_{\sigma j}^{1/2} Z_{\sigma l}^{1/2} \int \frac{d^3k}{(2\pi)^6} e^{i(E_{Cj}(k)-E_{Ci}(k))k} G_{ABjl}(p,q = p-k,k),$$

(2.41)

$$G_{ABjl}(p,q,k) = \frac{1}{4} \left[ \frac{1}{E_{Cj}(k)} + \frac{1}{E_{Ci}(k)} \right] \frac{1}{2E_B(q)} \left[ \frac{1}{E_A(p) - E_B(q) - E_{Cj}(k) + i\Gamma(p)/2} \right]$$

$$\times \left[ \frac{1}{E_A(p) - E_B(q) - E_{Ci}(k) - i\Gamma(p)/2} \right].$$

(2.42)

E. q. (2.40) should be compared with E. q. (2.45). It will be meaningful to examine whether $G_{ABjl}$ depends weakly on $\{j,l\}$, and then, whether (2.41) depends on $t$, characteristically different from the standard formula (1.5).

III. EXPECTATION VALUES OF FLAVOR-NEUTRINO CURRENTS WITH RESPECT TO ONE-PION WAVE PACKET STATE

A. Starting Relations

We examine the case where flavor neutrinos are produced through a boson decay such as $\pi^+ \rightarrow \mu^+ + \nu_\mu$; for convenience, neutrinos with only one flavor $\sigma$ are produced. As the initial $\pi^+$-state $\left| \Psi((p); X, x^0) \right\rangle$, we adopt, in accordance with Giunti et al. [10],

$$\left| \Psi((p); X, x^0) \right\rangle = \int d^3p A^*_\pi(p, (p); X, x^0) \frac{1}{\sqrt{2E_\pi(p)}} \alpha_\pi(p) |0\rangle,$$

(3.1)

$$A^*_\pi(p, (p); X, x^0) = A^*_\pi(p, (p); X, x^0) e^{-ip \cdot X + iE_\pi(p)x^0}$$

$$= \frac{1}{(\sqrt{2\pi} \sigma_\pi)^{3/2}} \exp \left[ \frac{-\left( \left( \frac{p - \langle p \rangle}{4\sigma_\pi^2} \right) \cdot \left( \frac{X - \langle X \rangle}{4\sigma_\pi^2} \right) - i\frac{1}{8\sigma_\pi^2} \right)}{2} \right].$$

(3.2)

Here, the operator $\alpha_\pi(p)$ appears in the plane-wave expansion of the $\pi^+$-field, as already given by (2.40). The state normalization is given by

$$\langle \Psi((p); X, x^0) | \Psi((p'); X', x^0) \rangle$$

$$= \exp \left[ \frac{1}{8\sigma_\pi^2} \right] \left( \langle p \rangle - \langle p' \rangle \right)^2 \right] \left( \langle p \rangle = \langle p' \rangle \right) \rightarrow 1.$$  

(3.3)

We have the following relations:

(i)

$$\int d^3p A^*_\pi(p, (p); X, x^0) (-i \frac{\partial}{\partial X}) A_\pi(p, (p); X, x^0)$$

$$= \int d^3p |A_\pi(p, (p); X, x^0)|^2 = \langle p \rangle,$$

(3.4)

(ii)

$$\int d^3p A^*_\pi(p, (p); X, x^0) (-i \frac{\partial}{\partial x^0}) A_\pi(p, (p); X, x^0)$$

$$= \int d^3p E_\pi(p) |A_\pi(p, (p); X, x^0)|^2$$

$$\simeq \int d^3p E_\pi(p) \langle \nu_\pi \rangle (p - \langle p \rangle) |A_\pi(p, (p); X, x^0)|^2 = E_\pi(p),$$

(3.5)
where
\[ \langle v_\pi \rangle \equiv \left[ \frac{\partial E_\pi}{\partial p} \right]_{p=\langle p \rangle} = \frac{\langle p \rangle}{E_\pi(\langle p \rangle)}; \] (3.6)

hereafter \( E_\pi(\langle p \rangle) \) is written as \( \langle E_\pi \rangle \) for simplicity.

(iii) By employing the above relations (3.3) - (3.5) and the energy and momentum operators of \( \pi \)-field
\[ \left( \hat{H} \right) \equiv \int \frac{d^3p}{2E_\pi(p)} \left( E_\pi(p) \right) [\alpha_\pi^\dagger(p)\alpha_\pi(p) + \beta_\pi^\dagger(-p)\beta_\pi(-p)], \] (3.7)
we easily obtain
\[ \langle \Psi(\langle p \rangle; X, x_f^0) | \left( \hat{H} \right) | \Psi(\langle p \rangle; X, x_f^0) \rangle \approx \langle E_\pi \rangle = \langle p \rangle. \] (3.8)

(iv) The wave packet in the coordinate space is defined by
\[ A_\pi^X(x, x_f^0; X, x_f^0, \langle p \rangle) \equiv \int \frac{d^3p}{(2\pi)^{3/2}} A_\pi^X(p, \langle p \rangle; X, x_f^0) e^{i(p \cdot x - E_\pi(p) x_f^0)}. \] (3.9)

By using the approximate form of \( E_\pi(p) \) in (3.3), we obtain
\[ A_\pi^X(x, x_f^0; X, x_f^0, \langle p \rangle) \approx \frac{1}{\sqrt{2\pi} \sigma_{\pi\pi}} \exp \left[ i \langle p \rangle \cdot (X - x) - i \langle \pi \rangle (x_0 - x_f^0) - \left( \frac{(X - x) - \langle \pi \rangle (x_0 - x_f^0)}{4\sigma_{\pi\pi}^2} \right)^2 \right], \] (3.10)
where \( \sigma_{\pi\pi} = 1/2 \). The normalization is
\[ \int d^3xA_\pi^X(x, x_f^0; X, x_f^0, \langle p \rangle)A_\pi(x, x_f^0; X', x_f^0, \langle p \rangle') = \int d^3pA_\pi^X(p, \langle p \rangle; X, x_f^0)A_\pi(p, \langle p \rangle'; X', x_f^0), \] (3.11)
which is the same as (3.3).

(v)
\[ \int d^3x |A_\pi(x, x_f^0; X, x_f^0, \langle p \rangle)|^2 \simeq X + \langle \pi \rangle (x_0 - x_f^0). \] (3.12)

Next, the relevant effective weak interaction, causing \( \pi^+ \rightarrow \bar{\nu}_\sigma + \nu_\sigma \) decay is written as
\[ \mathcal{H}_W(x) = -\mathcal{L}_{\text{mat}}(x) = -[\bar{\nu}_\sigma(x)J_\sigma(x) + \bar{J}_\sigma(x)\nu_\sigma(x)]; \] (3.13)
the effective forms of \( J_\sigma(x) \) and \( \bar{J}_\sigma(x) \) have been given by (1.22) and (1.28). The spin 1/2-field \( \psi(x) \) is expanded in the interaction representation as
\[ \psi(x) = \sum_{\text{helicity}} \int \frac{d^3k}{\sqrt{(2\pi)^3 2E(k)}} \left[ \alpha(kr)e^{ikx}u(kr) + \beta^\dagger(kr)e^{-ikx}v(kr) \right]. \] (3.14)

The concrete forms of \( u(kr) \) and \( v(kr) \) in Kramers representation [10] as well as other details are given in Refs. [2] and [11].

The expectation value of the flavor neutrino current \( j_\nu^\rho(x) \), given by (1.26), when the initial state is \( |\Psi_\pi(\langle p \rangle; X, x_f^0)\rangle \), given by (3.1), is expressed as, in accordance with (1.19),
\[ \langle \Psi_\pi(\langle p \rangle; X, x_f^0) | S^{-1}(x^0, x_f^0) : j_\nu^\rho(x) : S(x^0, x_f^0) | \Psi_\pi(\langle p \rangle; X, x_f^0) \rangle. \] (3.15)
This consists of a sum of two parts in the lowest order of $\mathcal{H}_{W}$:

\[
\mathcal{E}(x^0; \langle p \rangle, X, x_1^0)^{\alpha}_{\rho}(1) \rho \equiv \langle \Psi_\pi((p); x, x_1^0) | \int_{x_1^0}^{x^0} d^4z d^4y \mathcal{H}_W(z) : j_{(\rho)}^a(x) : \mathcal{H}_W(y) \mid \Psi_\pi((p); x, x_1^0) \rangle^{\text{con}}, \quad (3.16)
\]

\[
\mathcal{E}(x^0; \langle p \rangle, X, x_1^0)^{\alpha}_{\rho}(1) \rho \equiv \langle \Psi_\pi((p); x, x_1^0) | i^2 \int_{x_1^0}^{x^0} d^4z d^4y \\
\times \left[ \mathcal{H}_W(y) \mathcal{H}_W(z) : j_{(\rho)}^a(x) : + j_{(\rho)}^a(x) : \mathcal{H}_W(z) \mathcal{H}_W(y) \right] \mid \Psi_\pi((p); x, x_1^0) \rangle^{\text{con}}. \quad (3.17)
\]

In the following we examine (3.16), because this includes a dominant contribution.

### B. Form of $E_{(1)}^{\alpha}_{\rho}$

As shown in FIG. 4, $E_{(1)}^{\alpha}_{\rho}$ consists of two parts expressed as

\[
\begin{align*}
\left[ \mathcal{E}(x, x^0; \langle p \rangle, X, x_1^0)^{\alpha}_{\rho}(1) \rho \right] \\
\left[ \mathcal{E}(x, x^0; \langle p \rangle, X, x_1^0)^{\alpha}_{\rho}(2) \rho \right] \\
= \langle \Psi_\pi((p); x, x_1^0) | \int_{x_1^0}^{x^0} d^4z d^4y \left[ \bar{j}_\sigma(z) \nu_\sigma(z) : j_{(\rho)}^a(x) : \bar{\nu}_\sigma(y) J_{\sigma}(y) \right] \mid \Psi_\pi((p); x, x_1^0) \rangle^{\text{con}}. \quad (3.18)
\end{align*}
\]

![Diagram](image-url)

**FIG. 9:** Diagrams representing $E_{(1)}^{\alpha}_{\rho}$

We obtain

\[
\begin{align*}
\mathcal{E}(x, x^0; \langle p \rangle, X, x_1^0)^{\alpha}_{\rho}(1) \rho &= \int_{x_1^0}^{x^0} d^4z d^4y \left[ \bar{j}_\sigma(z) \nu_\sigma(z) : j_{(\rho)}^a(x) : \bar{\nu}_\sigma(y) J_{\sigma}(y) \right] \\
&\times \langle 0 | i f_{(\rho)}^a(x) | 0 \rangle \ \langle 0 | i f_{(\rho)}^a(x) | 0 \rangle \ \langle 0 | i f_{(\rho)}^a(x) | 0 \rangle \ \langle 0 | i f_{(\rho)}^a(x) | 0 \rangle \\
&= \int_{x_1^0}^{x^0} d^4z d^4y \int d^4p' \int d^4p \frac{-\langle f_{(\rho)}^a \rangle^2}{(2\pi)^4 \sqrt{2 E_\pi(p') E_\pi(p)}} A_\pi(p', \langle p \rangle) A_\pi^*(p, \langle p \rangle) \ \\
&\times \int d^4k \int d^3k(2\pi)^{-9} \sum_{j,l} Z_{\rho j}^{1/2} L_{\rho j}^{1/2} Z_{\sigma j}^{1/2} L_{\sigma j}^{1/2} B_{\sigma j}^{1/2} \left( q, p', p, k', k \right) \\
&\times e^{-i\mathbf{p}'(x-X)} + iE_\pi(p')(x^0 - x_1^0) e^{-i\mathbf{p}(y - X)} - iE_\pi(p)(y^0 - x_1^0) \ \\
&\times e^{i(q+k')z - i(k_1 + q)y} e^{i(k_1 - k)z}, \quad (3.19)
\end{align*}
\]
where \( k_l = (k, \omega_l(k)) \), \( \omega_l(k) = \sqrt{k^2 + m^2} \):

\[
B^{(1)}_{\sigma, jl} (q; p', p; k', k) = \sum_{s, r, r'} \tilde{u}_s (qs) \tilde{p} (1 + \gamma_5) u_j (k' r') (-i) \tilde{u}_j (k' r') \gamma^a u_l (kr) \cdot \tilde{u}_l (kr) \tilde{p} (1 + \gamma_5) v_\sigma (qs). \tag{3.20}
\]

By taking the new parameters

\[
t = x^0 - x^0_j, \quad Z^0 = z^0 - x^0_j, \quad Y^0 = y^0 - x^0_j,
\]

and adding by hand the factor of \( \pi \) lifetime in the same way as, we obtain

\[
E(x^0; \langle p \rangle, X, x^0_j)_{(1)} \rho = \int_0^t \int_0^t \int_0^t dZ^0 dY^0 \int d^3p' \int d^3p \frac{|f_{\pi\sigma}|^2}{(2\pi)^3 \sqrt{2E_\pi(p')} 2E_\pi(p)} A_\pi(p', \langle p \rangle) A_\pi^\dagger(p, \langle p \rangle) \\
\times \int d^3q \int d^3k' \int d^3kg(2\pi)^{-9} \sum_{j,l} Z^{1/2}_\sigma Z_{\sigma j}^{1/2} Z_{\sigma l}^{1/2} Z_{\sigma l}^{1/2} Z_{\sigma j}^{1/2} \tilde{B}^{(1)}_{\sigma, jl} (q; p', p; k', k) \\
\times \int \int d^3z d^3y e^{ix(-p'+q+k') + iy(p-q-k)} e^{ix(-p' - p)} + d^3z d^3y e^{ix(-k' + k)} \\
\times e^{it(\omega_j(k') - \omega_l(k))}.
\tag{3.22}
\]

The factors depending on \( Z^0 \) and \( Y^0 \), when we perform the \( Z^0 \)- and \( Y^0 \)-integrations and require the conditions

\[
t \Gamma_\pi (p') \gg 1, \quad t \Gamma_\pi (p) \gg 1,
\]

lead to the factor

\[
\left\{ \begin{array}{l}
E_\pi(p') - E_\pi(q) - \omega_j(k') + i \frac{\Gamma_\pi(p')}{2} \\
E_\pi(p) - E_\pi(q) - \omega_l(k) - i \frac{\Gamma_\pi(p)}{2}
\end{array} \right\}^{-1} \tag{3.24}
\]

Then we see that possible candidates having a connection with the oscillation in R.H.S. of \( E(x^0)_{(1)} \rho \) are

\[
i) \quad e^{i(\omega_j(k') - \omega_l(k))t} \quad \text{and} \quad ii) \quad e^{i(p' - p) X + i(-k' + k) \cdot x} \rightarrow e^{i(-k' + k) \cdot L} \quad \text{with} \quad L = x - X.
\tag{3.25}
\]

We will examine the situation in the next section.

Similar to \( E(x^0)_{(1)} \rho \), we obtain

\[
E(x^0_j; \langle p \rangle, X, x^0_j)_{(2)} \rho = \int_0^t \int_0^t \int_0^t dZ^0 dY^0 \int d^3p' \int d^3p \frac{|f_{\pi\sigma}|^2}{(2\pi)^3 \sqrt{2E_\pi(p')} 2E_\pi(p)} A_\pi(p, \langle p \rangle) A_\pi^\dagger(p', \langle p \rangle) \\
\times \sum_{j,l} Z^{1/2}_\sigma Z_{\sigma j}^{1/2} Z_{\sigma l}^{1/2} Z_{\sigma l}^{1/2} Z_{\sigma j}^{1/2} \tilde{B}^{(2)}_{\sigma, jl} (q; p, p'; k, k') \\
\times e^{iZ^0(-E_\pi(p') - E_\pi(q) - \omega_j(k') + i \Gamma_\pi(p')/2)} e^{iY^0(E_\pi(p) + E_\pi(q) + \omega_l(k) + i \Gamma_\pi(p)/2)} \tag{3.26}
\]

here,

\[
\tilde{B}^{(2)}_{\sigma, jl} (q; p, p'; k, k') = \sum_{s, r, r'} \tilde{u}_s (qs) \tilde{p} (1 + \gamma_5) u_j (k' r') (-i) \tilde{u}_j (k' r') \gamma^a v_\sigma (qs) \cdot \tilde{v}_l (kr) \cdot \tilde{u}_l (kr) \tilde{p} (1 + \gamma_5) u_\sigma (qs). \tag{3.27}
\]
The factor corresponding to (3.24) is

\[
\left\{ \left[ E_\pi(p') + E_\sigma(q) + \omega_j(k') - i \frac{\Gamma_\pi(p')}{2} \right] \left[ E_\pi(p) + E_\sigma(q) + \omega_l(k) + i \frac{\Gamma_\sigma(p)}{2} \right] \right\}^{-1} \quad (3.28)
\]

Though detailed numerical analyses are necessary, we can expect the main contribution in \( E_{(1)\rho}^{\alpha} \) comes from \( E_{(1)\rho}^{\sigma} \).

In the next subsection, we perform further evaluations of \( E_{(1)\rho}^{\alpha} \).

### C. Evaluation of \( E_{(1)\rho}^{\alpha} \)

With the aim of examining the effect due to the wave packet of \( \pi^+ \), we perform the integrations in R.H.S. of (3.22) with respect to \( p' \) and \( p \) firstly; then the \( z \) and \( y \) integrations are carried out.

#### 1. \( p' \)- and \( p \)-integrations

We consider the \( p \) integration appearing in (3.22),

\[
\int d^3p A_\pi^\dagger(p, \langle p \rangle) e^{iY^0(-E_\pi(p) + i\Gamma_\pi(p)/2 + i\langle y - X \rangle_p F(p)},
\]

and take the approximation, similar to (3.31),

\[
e^{iY^0(-E_\pi(p) + i\Gamma_\pi(p)/2) \approx e^{iY^0(-\langle E_\pi \rangle - \langle \Gamma \rangle_p)} e^{i\langle y - X \rangle_p} + i\langle \Gamma \rangle_p/2},
\]

here, \( \langle \Gamma \rangle_p \equiv \Gamma_\pi(p) \). Also \( F(p) \) in the integrand is assumed to be replaced approximately by \( F(\langle p \rangle) \); then we have

\[
\approx F(\langle p \rangle) \int \frac{d^3p}{[\sqrt{2\pi\sigma_\pi}]^3/2} \exp \left[ -\frac{(p - \langle p \rangle)^2}{4\sigma_\pi^2} + i(p - \langle p \rangle) \cdot (y - X - Y^0 \langle v_\pi \rangle) \right]
\]

\[
+ i(p) \cdot (y - X) + iY^0(-\langle E_\pi \rangle + i\langle \Gamma \rangle_p/2)
\]

\[
= F(\langle p \rangle) \int \frac{d^3p}{[\sqrt{2\pi\sigma_\pi}]^3/2} \exp \left[ -\frac{1}{4\sigma_\pi^2}(p - 2i\sigma_\pi^2(y - X - Y^0 \langle v_\pi \rangle))^2 \right]
\]

\[
\times \exp \left[ -\sigma_\pi^2(y - X - Y^0 \langle v_\pi \rangle)^2 + i\langle p \rangle \cdot (y - X) + iY^0(-\langle E_\pi \rangle + i\langle \Gamma \rangle_p/2) \right]
\]

\[
\approx F(\langle p \rangle)(2\sigma_\pi\sqrt{2\pi})^{3/2} \exp \left[ -\frac{1}{2}\sigma_\pi^2(y - X - Y^0 \langle v_\pi \rangle)^2 + i\langle p \rangle \cdot (y - X) + iY^0(-\langle E_\pi \rangle + i\langle \Gamma \rangle_p/2) \right].
\]

Thus, (3.22) is written as

\[
\mathcal{E}(\mathbf{x}, x^0; \langle p \rangle, \mathbf{X}, x^0_\rho(1)_{\rho}) \sim \int_0^t dZ^0 dY^0 \int d^3z \int d^3y \frac{|f_{\pi\sigma}|^2}{(2\pi)^5/2} \langle E_\sigma \rangle \int d^3q \int d^3k' \int d^3k
\]

\[
\times \sum_{j,l} Z_{j,l}^{1/2} Z_{j,l}^{1/2} Z^{1/2} Z^{1/2} B_{\sigma,j,l}(q; \langle p \rangle, \langle p \rangle; k', k) [2\sigma_\sigma \sqrt{2\pi}]^3
\]

\[
\times e^{iz_\pi(-\langle p \rangle + q + k') + iy \cdot \langle (p) - q - k \rangle}
\]

\[
\times e^{iz_\pi((E_\pi) - E_\sigma(q) - \omega_j(k') + i\langle \Gamma \rangle_p)/2} e^{iY^0(-\langle E_\sigma \rangle + E_\sigma(q) + \omega_l(k) + i\langle \Gamma \rangle_p)/2}
\]

\[
\times e^{iz_\pi(-\langle k' \rangle + k + t_i \omega_j(k') - \omega_l(k))}
\]

\[
\times \exp \left[ -\sigma_\pi^2(z - X - Z^0 \langle v_\pi \rangle)^2 - \sigma_\sigma^2(y - X - Y^0 \langle v_\pi \rangle)^2 \right].
\]

We see

\[
y - X - Y^0 \langle v_\pi \rangle = 0, \quad z - X - Z^0 \langle v_\pi \rangle = 0
\]

(3.33)
correspond to the classical relations as shown in Fig.11.
2. z- and y-integrations

By employing

$$-\sigma^2 (y - X - Y^0 \langle v_\pi \rangle)^2 + iy \cdot ((p) - q - k)$$

we obtain from

$$-\sigma^2 ((p) - q - k)^2 + i(Y^0 \langle v_\pi \rangle + X)((p) - q - k),$$

we get

$$\mathcal{E}(x, x^0; \langle p \rangle, X, x^0)_{(1-1)_\rho}$$

$$\simeq \int_0^t dZ dY^0 \frac{|f_{\pi\sigma}|^2}{(2\pi)^3} \langle E_{\pi} \rangle \int d^3q \int d^3k' \int d^3k \sum_{j,l} Z_{\sigma j}^{1/2} Z_{\rho j}^{1/2} Z_{pl}^{1/2} Z_{\sigma l}^{1/2}$$

$$\times B^{(1)\sigma}_{\sigma,ji}(q; \langle p \rangle, \langle p \rangle, k', k) [2\sigma_\pi \sqrt{2\pi}]^3 \left[ \frac{\sqrt{\pi}}{\sigma_\pi} \right]^6$$

$$\times \exp \left[ -\frac{1}{4\sigma^2_\pi} ((p) - q - k)^2 - \frac{1}{4\sigma^2_\pi} ((p) - q - k')^2 \right]$$

$$\times e^{iZ^0 ((E_{\pi} - E_\sigma(q) - \omega_j(k') + i(\Gamma_\pi/2 + \langle v_\pi \rangle) \cdot (p) + q + k')) e^{iY^0 ((E_{\pi} + E_\sigma(q) + \omega_l(k) + i(\Gamma_\pi/2 + \langle v_\pi \rangle) \cdot (p) - q - k))}$$

$$\times e^{i(x - X) \cdot (-k' + k) + it(\omega_j(k') - \omega_l(k))}. \quad (3.35)$$

3. k- and k'-integrations

Similarly we can perform k- and k'-integrations approximately. For a fixed K, which is defined by

$$K \equiv \langle p \rangle - q \quad (3.36)$$

the main contributions to the k and k'-integrations come from the regions

$$k \simeq K, \quad k' \simeq K. \quad (3.37)$$

Then, we are allowed to employ the relation

$$\omega_l(k) \simeq \omega_l(K) + \frac{\partial \omega_l}{\partial k} \bigg|_K \cdot (k - K) = \omega_l(K) + v_l(K) \cdot (k - K), \quad (3.38)$$

FIG. 10: Classical Relation.
and that for $\omega_j(k')$. Then we rewrite the exponent of (3.35) as

$$-\frac{1}{4\sigma^2}(k - K)^2 - i\omega_l(k)(t - Y^0) - iY^0 \langle v_\pi \rangle \cdot (k - K) + ik \cdot (x - X)$$

$$\simeq -\frac{1}{4\sigma^2}(k - K)^2 + i(k - K) \cdot [L - v_l(t - Y^0) - Y^0 \langle v_\pi \rangle]$$

$$- i\omega_l(k)(t - Y^0) + iK \cdot L$$

$$= -\frac{1}{4\sigma^2}(k - K)^2 - 2i\sigma^2 F_l(K; L, t, Y^0) - \sigma^2 F_l^2(0, t, Y^0)$$

$$- i\omega_l(k)(t - Y^0) + iK \cdot L,$$  (3.39)

where $L = x - X$ and

$$F_l(K; L, t, Y^0) = L - v_l(K)(t - Y^0) - Y^0 \langle v_\pi \rangle.$$  (3.40)

By employing (3.40), we obtain from (3.35)

$$\mathcal{E}(x, x^0, (p), X, X^0)_{(t=0)}$$

$$\simeq \int_0^t \int_0^t dZ^0 dY^0 \left[ \frac{|\pi^0|^2}{(2\pi)^{12/2}} \right] \int d^3q \sum_{j,l} Z_{\rho j}^{1/2} Z_{pl}^{1/2} Z_{sk}^{1/2} Z_{sl}^{1/2}$$

$$\times B_{x,j}^{(1, 0)}(q; (p), (K), (L)) 2\sigma_\pi \sqrt{2\pi^3} \left[ \frac{\pi^0}{\sigma_\pi} \right]^6 (2\sigma_\pi \sqrt{\pi})^6$$

$$\times e^{|Z^0|^2(\langle E_\pi \rangle - E_\pi (q) + i(\Gamma/2))} e^{Y^0 (\langle E_\pi \rangle + E_\pi (q) + i(\Gamma/2))}$$

$$\times \exp \left[ -\sigma^2 F_l(K; L, t, Z^0)^2 - \sigma^2 F_l^2(K; L, t, Y^0)^2 \right]$$

$$\times e^{i\omega_j(K)(t - Z^0) - i\omega_l(K)(t - Y^0)} \bigg|_{K=(p) - q}.$$  (3.41)

In R.H.S. of (3.41), the $L$-dependent oscillation factor in (3.35), $\exp[i(k - k') \cdot L]$, disappears. The $L$-dependence is expected to be recovered due to $F^2_j$- and $F^2_l$-terms through the time integration in (3.41). In order to see this possibility qualitatively, we evaluate roughly the $Y^0$- and $Z^0$- integration in (3.41) under the classical relations (as shown in FIG 11)

$$F_l(K; L, t, Y^0) = F_l(K; L, t, Z^0) = 0 ;$$  (3.42)

where $L = x - X$ and $Z^0_c$ of (3.42) are given by

$$Y^0_c = \frac{v_l t - L}{v_l - \langle v_\pi \rangle} ; \quad Z^0_c = Y^0_c (v_l \rightarrow v_j),$$  (3.43)

or

$$t - Y^0_c = \frac{L - \langle v_\pi \rangle t}{v_l - \langle v_\pi \rangle} = \frac{L - Y^0_c \langle v_\pi \rangle}{v_l}, \quad t - Z^0_c = t - Y^0_c (v_l \rightarrow v_j).$$  (3.44)
By assuming $0 < \{Y_{c}, Z_{c}^{0}\} < t$, and by taking the rough evaluation of integrations
\[
\int_{0}^{t} dZ^{0} \int_{0}^{t} dY^{0} \cdots \simeq \int_{0}^{t} dZ^{0} \int_{0}^{t} dY^{0} \delta (Z^{0} - Z_{c}^{0}) \delta (Y^{0} - Y_{c}^{0}),
\]
we obtain
\[
(3.41) \simeq \frac{-|f_{\pi\sigma}|^{2}}{\sqrt{2\pi}^{9}} (2\sigma_{\pi})^{3} \sum_{j,l} Z_{\pi j}^{1/2} Z_{\pi j}^{1/2*} Z_{\pi l}^{1/2} Z_{\pi l}^{1/2*} \int d^{4}q B_{\sigma,ji}(q; \langle p \rangle, \langle p \rangle ; K, K)
\times \exp [(P_{ji})_{\text{clas}}]\bigg|_{K=(p)-q}, \tag{3.45}
\]
where
\[
(P_{ji})_{\text{clas}} \equiv i(\omega_{i}(K) - \omega_{j}(K))t - Z_{c}^{0}(i\delta E_{j} + \frac{\langle \Gamma_{\pi} \rangle}{2}) + Y_{c}^{0}(i\delta E_{l} - \frac{\langle \Gamma_{\pi} \rangle}{2}),
\]
with $\delta E_{j} = E_{\sigma}(q) + \omega_{j}(K) - (E_{\pi})$. We obtain easily
\[
\text{Im}(P_{ji})_{\text{clas}} = (L - \langle v_{\pi} \rangle t) \frac{v_{j} - v_{l}}{(v_{j} - \langle v_{\pi} \rangle)(v_{l} - \langle v_{\pi} \rangle)} \left[ \frac{v_{j} + v_{l} - \langle v_{\pi} \rangle}{v_{j}v_{l}} K + \langle E_{\pi} \rangle - E_{\sigma} \right]. \tag{3.47}
\]
This unfamiliar form suggests a more precise evaluation of the integrations in (3.41) is necessary, which we will consider as follows

4. $Z^{0}$- and $Y^{0}$-integrations

For simplicity, we write $F_{j}(K; L, t, Z^{0})$ as $F_{j}(Z^{0})$, and define $Z_{c}^{0}$ by
\[
\left[ \frac{d}{dZ_{c}^{0}} (F_{j}(Z^{0})) \right]^{2} = 0. \tag{3.48}
\]
Then we obtain
\[
Z_{c}^{0} = \frac{1}{(v_{j} - \langle v_{\pi} \rangle)^{2}} (v_{j} - \langle v_{\pi} \rangle) \cdot (v_{j}t - L)
= 2\sigma_{\pi}^{2} \sigma_{\pi}^{2} (v_{j} - \langle v_{\pi} \rangle) \cdot (v_{j}t - L), \tag{3.49}
\]
\[
\sigma_{\pi}^{2}[F_{j}(Z^{0})]^{2} = \frac{1}{2\sigma_{\pi}^{2}} (Z^{0} - Z_{c}^{0})^{2} + \sigma_{\pi}^{2}[F_{j}(Z^{0})]^{2}, \tag{3.50}
\]
where
\[
\sigma_{\pi}^{2}(v_{j} - \langle v_{\pi} \rangle)^{2} = \frac{1}{2\sigma_{\pi}^{2}}. \tag{3.51}
\]

(3.41) is rewritten as
\[
\mathcal{E}(L, t; \langle p \rangle )_{(1-1)}^{(a)} \equiv \mathcal{E}(\chi, x, \sigma, \langle p \rangle, \chi, x, x_{l} ; (1-1))
\simeq \frac{-|f_{\pi\sigma}|^{2} (2\sigma_{\pi})^{3}}{2 \langle E_{\sigma} \rangle} \int d^{4}q \sum_{j,l} Z_{\pi j}^{1/2} Z_{\pi j}^{1/2*} Z_{\pi l}^{1/2} Z_{\pi l}^{1/2*} B_{\sigma,ji}(q; \langle p \rangle, \langle p \rangle ; K, K)
\times \exp \left[ i(\omega_{j}(K) - \omega_{l}(K))t - \sigma_{\pi}^{2} F_{j}(K; L, t, Z_{c}^{0})^{2} - \sigma_{\pi}^{2} F_{l}(K; L, t, Z_{c}^{0})^{2}
- Z^{0}(i\delta E_{j} + \frac{\langle \Gamma_{\pi} \rangle}{2}) + Y^{0}(i\delta E_{l} - \frac{\langle \Gamma_{\pi} \rangle}{2}) \right]
\times M_{j}(t; Z^{0}) \cdot M_{l}(t; Z_{c}^{0}). \tag{3.52}
\]
where
\[
M_j(t; Z^0) = \int_0^t dZ^0 \exp \left[ -\frac{1}{2\sigma^2_{\pi j}} (Z^0 - Z^0_0)^2 - (Z^0_0 - Z^0)(i\delta E_j + \frac{\Gamma_j}{2}) \right].
\] (3.53)

The integration form of (3.53) is expressed as
\[
M_j(t, Z^0) = \sqrt{2\sigma^2_{\pi j}} \exp \left[ \frac{1}{2} \sigma^2_{\pi j} (i\delta E_j + \frac{\Gamma_j}{2})^2 \right] J_j(u_j(t); \alpha_j, \beta_j),
\] (3.54)

where
\[
\alpha_j = \frac{\sigma^2_{\pi j} \delta E_j}{\sqrt{2}}, \quad \beta_j = \frac{1}{\sqrt{2\sigma^2_{\pi j}}} (-Z^0 + \frac{\sigma^2_{\pi j}}{2} \Gamma_j),
\]
\[
J_j(u_j(t); \alpha_j, \beta_j) = \int_0^{u_j(t)} du_j \exp[-(u_j + i\alpha + \beta_j)^2],
\] (3.55)

with \( u_j = Z^0 / (\sqrt{2}\sigma_{\pi j}) \) and \( u_j(t) = t / (\sqrt{2}\sigma_{\pi j}) \). We define the two functions as
\[
f_c(y; \alpha, \beta) = \int_0^y du \exp[\alpha^2 - (u + \beta)^2] \cos(2\alpha(u + \beta)),
\]
\[
f_s(y; \alpha, \beta) = \int_0^y du \exp[\alpha^2 - (u + \beta)^2] \sin(2\alpha(u + \beta));
\] (3.56)

then, (3.54) is expressed as
\[
M_j(t, Z^0) = \sqrt{2\sigma^2_{\pi j}} \exp \left[ \frac{1}{2} \sigma^2_{\pi j} (i\delta E_j + \frac{\Gamma_j}{2})^2 \right] |J_j(u_j(t); \alpha_j, \beta_j)| \exp[-i\Theta(u_j(t); \alpha_j, \beta_j)],
\] (3.57)

where
\[
\Theta(y; \alpha, \beta) = \arctan \left[ \frac{f_s(y; \alpha, \beta)}{f_c(y; \alpha, \beta)} \right].
\] (3.58)

Thus, (3.52) is expressed as
\[
\mathcal{E}(L, t; \langle p \rangle) = -\frac{|f_{\pi \pi}|}{2} \frac{(2\sigma_{\pi})^3}{\langle E_{\pi} \rangle} \int dq^3 \sum_{j=i} Z_{\pi j}^{1/2} Z_{\pi j}^{1/2*} Z_{\pi i}^{1/2*} Z_{\pi i}^{1/2} \times B_{\pi j}(q; \langle p \rangle; \langle K \rangle) (2\sigma_{\pi j} \sigma_{\pi i}) \exp \left[ P_{ji} + \frac{1}{2} \sigma^2_{\pi j} (i\delta E_j + \frac{\Gamma_j}{2})^2 + \frac{1}{2} \sigma^2_{\pi i} (-i\delta E_i + \frac{\Gamma_i}{2})^2 \right] \times |J_j(u_j(t); \alpha_j, \beta_j)||J_i(u_i(t); \alpha_i, \beta_i)| \exp \left[ -i(\Theta(u_j(t; \alpha_j, \beta_j)) - \Theta(j \rightarrow i)) \right] |_{K=\langle p \rangle - q},
\] (3.59)

where
\[
P_{ji} \equiv i(\omega_j(K) - \omega_i(K))t - \sigma^2_{\pi} [F_j(Z^0)]^2 - \sigma^2_{\pi} [F_i(Y^0)]^2
\]
\[
- Z^0 (i\delta E_j + \frac{\Gamma_j}{2}) - Y^0 (-i\delta E_i + \frac{\Gamma_i}{2}),
\] (3.60)
\[
[F_j(Z^0)]^2 = \frac{1}{(v_j - \langle v_{\pi} \rangle)^2} [(v_j - \langle v_{\pi} \rangle)^2(v_j t - L)^2 - (v_j - \langle v_{\pi} \rangle) \cdot (v_j t - L)]^2
\] (3.61)

with \( v_j = K / \sqrt{K^2 + m^2_j}, K = \langle p \rangle - q \).

Though the \( t \) - and \( L \)-dependent part (3.59) is complicated, it becomes a little simpler when the parallel condition is assumed. From \([F_j(Z^0)]^2 + F_i(Y^0)] \rangle_{\text{para}} = 0\), which leads to the classical relation
\[
[Z^0]_{\text{para}} = \frac{(u_j t - L)}{v_j - \langle v_{\pi} \rangle},
\] (3.62)
we obtain \( \text{Im}[\mathcal{P}_{ji}]_{\text{para}} \),

\[
\text{Im}[\mathcal{P}_{ji}]_{\text{para}} = (L - \langle v_\tau \rangle t) \frac{v_j - v_i}{(v_j - \langle v_\tau \rangle)(v_i - \langle v_\tau \rangle)} \left[ - \frac{v_j + v_i - \langle v_\tau \rangle}{v_j v_i} K + \langle E_\tau \rangle - E_\sigma \right],
\]

(3.63)

which is the same as (3.47).

We can examine the qualitative structure of (3.59) under the condition of neutrino high energies in comparison with \( m_j \)'s. This is to be explained in the last section Sec.V.

IV. EXPECTATION VALUE OF NEUTRINO CURRENT WITH AN ADDITIONAL OBSERVED PARTICLE

As pointed out in Sec.III, it is worthwhile for us to examine the structure of the expectation value of the flavor current, where the charged lepton observation is taken into consideration.

A. Boson model; the plane wave case

We first consider the boson model in the plane wave case with the aim of seeing the way how to take into account the observation of an additional particle. We examine the same model as that in Sec.IIB;

\[
\mathcal{H}_{\text{int}} = -J_{ABC} \phi_{C}^\dagger(x) \phi_{B}(x) \phi_{A}(x) + H.c.
\]

\[
= \mathcal{H}_1(x) + \mathcal{H}_2(x),
\]

(4.1)

\[
\mathcal{H}_1(x) = \phi_{C}^\dagger(x) J_{y}(x), \quad \mathcal{H}_2(x) = \mathcal{H}_1^\dagger(x) = J_{y}^\dagger(x) \phi_{C}(x),
\]

(4.2)

\[
j_{b}^{(C \rho)}(x) = i \phi_{C}^\rho(x) \nabla_{b} \phi_{C \rho}(x).
\]

(4.3)

From the expectation value (2.25) we pick up the part corresponding to FIG.1, i.e.

\[
\mathcal{E}(A(x)_{i}^0; C \rho - \text{cur}(x))_{b}^{(t-1)} = \int_{x_i^0}^{x_i^0} d^4 z \int_{x_i^0}^{x_i^0} d^4 y \langle 0 | \alpha_A(p) H_2(z) : j_{b}^{(C \rho)}(x) : H_1(x) \alpha_A^\dagger(p) | 0 \rangle_{\text{con}}
\]

\[
= \int_{x_i^0}^{x_i^0} d^4 z \int_{x_i^0}^{x_i^0} d^4 y \langle 0 | \alpha_A(p) J_{y}^\dagger(z) \cdot J_{y}(y) \alpha_A^\dagger(p) | 0 \rangle
\]

\[
\times \langle 0 | \phi_{C}(z) : j_{b}^{(C \rho)}(x) : \phi_{C \rho}^\dagger(y) | 0 \rangle_{\text{con}}.
\]

(4.4)

The first part in the last integrand includes the propagator \( \langle 0 | \phi_{B \sigma}^\dagger(z) \phi_{B \sigma}(y) | 0 \rangle \). This can be effectively set equal to

\[
\langle 0 | \phi_{B \sigma}^\dagger(z) \phi_{B \sigma}(y) | 0 \rangle = \langle 0 | \phi_{B \sigma}^\dagger(z) \int \frac{d^3 q}{2 E_B(q)} \beta_{B \sigma}^\dagger(q) | 0 \rangle \langle 0 | \beta_{B \sigma}(q) \phi_{B \sigma}(y) | 0 \rangle
\]

\[
= \langle 0 | \phi_{B \sigma}^\dagger(z) \left[ - \int \tilde{n}_{B \sigma}(q) d^3 q \right] \phi_{B \sigma}(y) | 0 \rangle_{\text{con}}
\]

(4.5)

with \( -\tilde{n}_{B \sigma}(q) = \beta_{B \sigma}^\dagger(q) \beta_{B \sigma}(q)/(2E_B(q)) \). As seen from

\[
\langle 0 | \beta_{B \sigma}(Q') \left[ - \int \tilde{n}_{B \sigma}(q) d^3 q \right] \beta_{B \sigma}^\dagger(Q) | 0 \rangle_{\text{con}} = \int \frac{d^3 q}{2 E_B(q)} 2 E_B(Q') \delta(Q' - q) 2 E_B(q) \delta(q - Q)
\]

\[
= \langle 0 | \beta_{B \sigma}(Q') \beta_{B \sigma}^\dagger(Q) | 0 \rangle.
\]

(4.6)

Thus we can regard the expectation value of the flavor current with the additional particle observation is given by

\[
\mathcal{E}(A(x)_{i}^0; C \rho - \text{Cur}(x), B_{x}(q) - \text{obs})_{b}^{(t-1)} = \int_{x_i^0}^{x_i^0} \int_{x_i^0}^{x_i^0} d^4 z d^4 y \langle 0 | \alpha_A(p) J_{y}^\dagger(z)(-\tilde{n}_{B \sigma}(q)) J_{y}(y) \alpha_A^\dagger(p) | 0 \rangle_{\text{con}}
\]

\[
\times \langle 0 | \phi_{C \sigma}(z) : j_{b}^{(C \rho)}(x) : \phi_{C \sigma}^\dagger(y) | 0 \rangle_{\text{con}}.
\]

(4.7)
The concrete form of (4.7) is given by
\[
E \times \int \int x \int \langle i A x | E \int x A^\dagger | x \rangle \frac{1}{(2\pi)^6 2E_B(q)} \sum_{j,l} Z^1_{\sigma j} Z^1_{\rho j} Z^1_{l \sigma j} e^{iz(-p+q)+iy(-q+p)}
\]
\times \int d^3k d^3k' \frac{i^2(k_b + k_h) e^{i(k-k')x + i(k-k')y}}{(2\pi)^6 2\omega_j(k) 2\omega_j(k')}
\]
\[
= \int_0^t dt \ |f_{ABC\sigma}|^2 \int d^3y d^3p \sum_{j,l} Z^1_{\sigma j} Z^1_{\rho j} Z^1_{l \sigma j} e^{i(\omega_j(k) - \omega_j(k')) t} \frac{i^2}{2\omega_j(k) 2\omega_j(k')}
\]
\times \left[ \frac{2k}{i(\omega_j(k) + \omega_j(k'))} \right] e^{i(E_A(p) - E_B(q) - \omega_j(k)) t - i(E_A(p) - E_B(q) - \omega_j(k)) y} |_{k=p-q},
\]
where \( t = x^0 - x^0_0 \). We see (4.8) leads certainly to
\[
\int d^3x \int d^3k iE(A(x^0_0); C_\rho \cdot \text{Cur}(x), B_\sigma(q) \cdot \text{obs})^{(I-1)} = \langle N_{C_B}(x^0_0) \rangle \langle A(p); x^0_0 \rangle,
\]
given by (4.9).

Due to the plane-wave calculation, the \( x \)-dependence disappears in (4.8); thus, there is no spatial coordinates specifying the observation. In the next subsection we examine the model calculation by employing the wave packets for the external particles (i.e. for \( \pi^+ \) and the "observed" \( B_\sigma \)-particle).

### B. Boson model; the wave-packet case

First we have to take out the part, which is suitable for the present purpose, of
\[
\langle \Psi_A(p); X_A, x^0_1 \rangle \int_{x^0_1}^{x^0} d^4z d^4y \mathcal{H}_{\text{int}}(z); j^{(C_\rho)}(x) : \mathcal{H}_{\text{int}} | \Psi_A(p); X_A, x^0_1 \rangle^{\text{con}}.
\]

Here, we choose the wave-packet state in the same way as (5.1):
\[
|\Psi_A(p); X_A, x^0_1 \rangle = \int d^3p A^+_\sigma(p, \langle p ; X_A, x^0_1 \rangle) \frac{1}{\sqrt{2E_A(p)}} \alpha^+_\sigma(p) |0\rangle,
\]
\[
A^+_\sigma(p, \langle p ; X_A, x^0_1 \rangle) = \frac{1}{(\sqrt{2\pi\sigma_A})^3/2} \exp \left[ \frac{- (p - \langle p \rangle)^2}{4\sigma_A^2} - ip \cdot X_A + iE_A(p) x^0_1 \right].
\]

This state is normalized in the way as noted by (3.3). (4.11) is regarded as the state obtained through substituting the wave-packet state for the plane-wave state \( \frac{1}{\sqrt{2E_A(p)}} \alpha^+_\sigma(p) |0\rangle \). In the same way, the wave-packet of \( B_\sigma \) incorporated in (4.12) through the substitution
\[
\frac{1}{2E_B(|q\rangle \beta^+_B(q) |0\rangle |0\rangle \beta_{B_\sigma}(q) \langle q \rangle \rightarrow |\Psi_B(q, X_B, x^0_B) \rangle \langle \Psi_B(q, X_B, x^0_B) |}
\]

(See FIG. 12)

FIG. 12: Diagram reprenting (4.7)
with
\[ |\Psi_B((q) : X_B, x_B^0)\rangle = \int d^3q A_B^*(q, \langle q) : X_B, x_B^0) \frac{\beta_B^i(q)}{\sqrt{2E_B(q)}} |0\rangle, \]
(4.14)

where \(A^*_B\) is defined similarly to (4.13).

Thus, we are led to examine
\[
\mathcal{E}[A(p) : X_A, x_A^0] ; C_p \cdot \text{Cur}(x), B_\sigma ((q) : X_B, x_B^0)_{\text{b}}^{(I-1)}
\]
\[
= \int x^0_B \int x_B^1 \int d^4z \int |f_{ABC\sigma}|^2 \langle \Psi_A(p) : X_A, x_A^0 | (4.13) \rangle \phi_A^i(z) \phi_A(\langle q) : X_A, x_A^0)\rangle_{\text{con}}
\times \langle 0| \phi_B^i(z) \langle \Psi_B((q) : X_B, x_B^0)|\Psi_B((q) : X_B, x_B^0) | \phi_B\sigma(y) | 0\rangle
\times \langle 0| \phi_C\sigma(x) \phi_C\sigma(y) | 0\rangle_{\text{con}}.
\]
(4.15)

We add a remark on the choice of (4.13). As noted above the state normalization is changed from \(\langle 0| \alpha_A(p') \alpha^i(p) | 0\rangle = 2E_A(p) \delta(p' - p)\) to
\[ \langle \Psi_A((p') : X_A, x_A^0) | \Psi_A((p) : X_A, x_A^0)\rangle = \exp \left[ -\frac{\langle (p') - (p) \rangle^2}{8\sigma_A^2} \right]. \]
(4.16)

In the same way, we adopt (4.13) which satisfies
\[
\langle 0| \beta_{\sigma}\sigma(Q') \int d^3q \langle \Psi_B((q) : X_B, x_B^0)\rangle | \Psi_B((q) : X_B, x_B^0) | \beta_{\sigma}^i(Q) | 0\rangle
= \int d^3q \sqrt{2E_B(Q')2E_B(Q)} A_B^*(Q', \langle q) A_B(Q, \langle q) \rangle
= \sqrt{2E_B(Q')2E_B(Q)} \exp \left[ -\frac{\langle (Q') - (Q) \rangle^2}{8\sigma_B^2} \right],
\]
(4.17)

instead of adopting
\[
\int d^3q A_B^*(q, \langle q) A_B(q, \langle q) \rangle \frac{1}{2E_B(q)} \beta_{\sigma}^i(q) | 0\rangle (0| \beta_{\sigma}(q).
\]
(4.18)

(4.13) satisfies
\[ \langle 0| \beta_{\sigma}(Q') \left( \int d^3q \right. \left. \right)^{I-1} \beta_{\sigma}^i(Q) | 0\rangle = 2E_B(Q) \delta(Q' - Q), \]
(4.19)

which corresponds to the plane-wave normalization.

We obtain from (4.13)
\[
\mathcal{E}[A(p) : X_A, x_A^0] ; C_p \cdot \text{Cur}(x), B_\sigma ((q) : X_B, x_B^0)_{\text{b}}^{(I-1)}
\]
\[
= \int x^0_B \int x_B^1 \int d^4z \int |f_{ABC\sigma}|^2 \int d^3p' \int d^3p \int d^3q' \int d^3q \int d^k \int d^k \frac{1}{(2\pi)^6 \sqrt{2E_A(p')2E_A(p)2E_B(q')2E_B(q)}}
\times \sum_{j,s} Z_{1/2_j} Z_{1/2_s} Z_{1/2_r} \ A_A(p', \langle p) A_A^i(p, \langle p) A_B^i(q', \langle q) A_B(q, \langle q) \rangle
\times \frac{i^2(k_j^i + k_i^j)}{(2\pi)^6 \omega_j(k') \omega_i(k)} e^{i(-p' + q' + k') \cdot z + i(p - q - k) \cdot y + i(p' - p) \cdot x_A - i(q' - q) \cdot x_B + i(-k' + k) \cdot x'}
\times \exp \left[ i(E_A(p') - E_B(q'))(z^0 - x_B^0) + i(-E_A(p) + E_B(q) + \omega_j(k))(y^0 - x_B^0)
+ i(\omega_j(k') - \omega_i(k))(x^0 - x_B^0) + i(E_B(q') - E_B(q))(x_B^0 - x_B^0) \right].
\]
(4.20)
We perform the integrations with respect to \( p', p, q' \) and \( q \) by employing the approximate relations as already used in Sec.III (e.g.(3.29)). and obtain

\[
\mathcal{E}[A(p); X_A(x^0); C_p \text{-Cur}(x), B_s((q); X_B(x^0))]_{1}^{(1-1)}
\]

\[
\simeq \int_0^\infty \int_0^\infty dZ^0 dY^0 \int d^3 z \int d^3 k \int d^3 k' \frac{(2\sqrt{2\pi \sigma A})^3 (2\sqrt{2\pi \sigma B})^3}{(2\pi)^6 2 \langle E_A \rangle 2 \langle E_B \rangle} |f_{ABC}s|^2
\]

\[
\times \sum_{j,i} Z_{\pi j}^{1/2} Z_{\rho j}^{1/2} Z_{\mu j}^{1/2} Z_{\nu j}^{1/2} \sigma_{\mu j} \sigma_{\nu j} \frac{i^2 (k'_j + k_i)_b}{(2\pi)^6 2 \omega_j (k'_j) \omega_i (k)} e^{i(-k'_j + k_i) \times + i(\omega_j (k'_j) - \omega_i (k)) t}
\]

\[
\times \exp \left[ i (p) \cdot (y - X_A) + (-i \langle E_A \rangle - \langle \Gamma_A \rangle / 2) Y^0 + i (p) \cdot (z + X_A) + (i \langle E_A \rangle - \langle \Gamma_A \rangle / 2) Z^0
\]

\[
+ i \langle q \rangle \cdot (y + X_B) + i \langle E_B \rangle \langle Y^0 - X^0_B \rangle + i \langle q \rangle \cdot (z - X_B) - i \langle E_B \rangle \langle Z^0 - X^0_B \rangle
\]

\[
- h(y, Y^0) - h(z, Z^0) + i(-\omega_j (k') Z^0 + \omega_i (k) Y^0 + i(k' \cdot z - k \cdot y)) \right],
\]

where the weak-decay width of \( \Lambda \)-particle is added by hand; \( t = x^0 - x^0_f, Y^0 = y^0 - x^0_f, Z^0 = z^0 - x^0_f, X_B = x^0_B - x^0_f \);

\[
h(y, Y^0) = \sigma^2_A (y - X_A) - \langle v_A \rangle Y^0)^2 + \sigma^2_B (y - X_B) - \langle v_B \rangle (Y^0 - X^0_B))^2,
\]

With the aim of execute the \( z \)- and \( y \)-integrations, we first examine the structure of

\[
\exp \left[ -h(z, Z^0) + i(-\langle p \rangle + \langle q \rangle + k') \cdot z \right],
\]

which is included in the integrand of (4.21). The concrete form of \( z \) defined by

\[
\frac{\partial h(z, Z^0)}{\partial z} \bigg|_z = 0,
\]

is given by

\[
z = \frac{1}{\sigma^2_A + \sigma^2_B} \left[ \sigma^2_A X_A + \langle v_A \rangle Z^0 + \sigma^2_B (X_B - \langle v_B \rangle X^0_B - Z^0) \right];
\]

then, we obtain

\[
h(z, Z^0) = \frac{\sigma^2 A \sigma^2 B}{\sigma^2_A + \sigma^2_B} \left| \mathbf{F}^{AB} (Z^0, X^0_B) \right|^2,
\]

with

\[
\mathbf{F}^{AB} (Z^0, X^0_B) \equiv \mathbf{X}_B - \langle \mathbf{v}_B \rangle (X^0_B - Z^0) - \langle \mathbf{v}_A \rangle Z^0 - \mathbf{X}_A,
\]

and

\[
h(z, Z^0) = (\sigma^2_A + \sigma^2_B) (z - z)^2 + h(z, Z^0).
\]

We see \( \exp[-h(z, Z^0)] \) is maximum at \( z = z \), and

\[
h(z, Z^0) = 0 \leftrightarrow \mathbf{F}^{AB} (Z^0, X^0_B) = 0.
\]

\( \mathbf{F}^{AB} (Z^0, X^0_B) \) leads to the relation among classical trajectories.

By noting

\[
\exp \left[ -(\sigma^2_A + \sigma^2_B) (z - z)^2 \right] = \exp \left[ \frac{1}{2(\sigma^2_A + \sigma^2_B)} (-\langle p \rangle + \langle q \rangle + k)^2 \right]
\]

\[
\times \left[ \mathbf{z} - \mathbf{z} \frac{i}{\sigma^2_A + \sigma^2_B} (-\langle p \rangle + \langle q \rangle + k)^2 + i(-\langle p \rangle + \langle q \rangle + k) \cdot \mathbf{z} \right],
\]
we perform the z and y integrations of (4.21) and obtain

\[
\mathcal{E}[A(p) ; X_A, x^0_B) ; C_{\rho A}(x), B_{\sigma}((q) ; X_B, x^0_B)]_{(I-1)}
\]

\[
\simeq \int_0^t \int_0^t \int d^3k' \int d^3k \ |f_{ABC\sigma}|^2 \left( \frac{2\sqrt{2\pi} \sigma_A}{(2\pi)^{3/2} \langle E_A \rangle} \right)^3 \left( \frac{\pi}{\sigma_A^2 + \sigma_B^2} \right)^3 
\]

\[
\times \sum_{j,i} Z_{j(i)}^{1/2} Z_{j(i)}^{1/2} Z_{j(i)}^{1/2} Z_{j(i)}^{1/2} \frac{i^2(k'_j + k_i)^6}{(2\pi)^6 2 \omega_j(k') 2 \omega_i(k)} e^{i(k' - k) \cdot x + i(\omega_j(k') - \omega_i(k)) t} 
\]

\[
\times \exp \left[ -h(z, Z^0) + i(-\langle p \rangle + \langle q \rangle + k') \cdot z - h(y, Y^0) + i\langle p \rangle - \langle q \rangle - k \cdot y 
\right. 
\]

\[
- \frac{1}{4(\sigma_A^2 + \sigma_B^2)} \left( (-\langle p \rangle + \langle q \rangle + k')^2 + (-\langle p \rangle + \langle q \rangle + k)^2 \right) 
\]

\[
+ iZ^0((E_A) - (E_B) - \omega_j(k')) + iY^0(- (E_A) + (E_B) + \omega_i(k)) - \frac{i}{2} \langle \Gamma_A \rangle (Z^0 + Y^0) \right]. \quad (4.31)
\]

Next we proceed to integrate with respect to k' and k. By employing the definitions

\[
\langle k \rangle = \langle p \rangle - \langle q \rangle, \quad \langle \omega_j \rangle = \omega_j(\langle k \rangle), \quad \langle v_j \rangle = v_j(\langle k \rangle), \quad (4.32)
\]

and

\[
\exp \left[ -h(z, Z^0) + i(-\langle k \rangle + k') \cdot z + iZ^0((E_A) - (E_B) - \langle \omega \rangle - \langle v_j \rangle \cdot (k' - \langle k \rangle)) 
\right. 
\]

\[
- ik' \cdot x + i(\langle \omega_j \rangle + \langle v_j \rangle \cdot (k' - \langle k \rangle)) t - \frac{1}{4(\sigma_A^2 + \sigma_B^2)} (-\langle k \rangle + k')^2 \right] 
\]

\[
= \exp \left[ -h(z, Z^0) - \frac{1}{4(\sigma_A^2 + \sigma_B^2)} \left( k' - k + 2i(\sigma_A^2 + \sigma_B^2)(-z + Z^0(\langle v_j \rangle + x - \langle v_j \rangle t)^2 
\right. 
\]

\[
- \sigma_A^2 + \sigma_B^2) \{ -z + Z^0(\langle v_j \rangle + x - \langle v_j \rangle t)^2 + iZ^0((E_A) - (E_B) - \langle \omega \rangle) 
\]

\[
- i\langle k \rangle \cdot x + i\langle \omega \rangle t \right], \quad (4.33)
\]

we obtain from (4.31)

\[
\mathcal{E}[A(p) ; X_A, x^0_B) ; C_{\rho A}(x), B_{\sigma}((q) ; X_B, x^0_B)]_{(I-1)}
\]

\[
\simeq \sum_{j,i} Z_{j(i)}^{1/2} Z_{j(i)}^{1/2} Z_{j(i)}^{1/2} Z_{j(i)}^{1/2} \int_0^t \int d^3k' d^3k \ |f_{ABC\sigma}|^2 \left( \frac{2\sqrt{2\pi} \sigma_A}{(2\pi)^{3/2} \langle E_A \rangle} \right)^3 \left( \frac{\pi}{\sigma_A^2 + \sigma_B^2} \right)^3 
\]

\[
\times [2\sqrt{2\pi} (\sigma_A^2 + \sigma_B^2)]^6 \times e^{i(\omega_j(\langle k \rangle) - \omega_i(\langle k \rangle)) t} 
\]

\[
\times \exp \left[ -G_j(Z^0, X_A; x^0_B, X_B; t, x) - G_i(Y^0, X_A; x^0_B, X_B; t, x) 
\right. 
\]

\[
+ iZ^0((E_A) - (E_B) - \langle \omega \rangle) + iY^0(- (E_A) + (E_B) + \langle \omega \rangle) - \frac{i}{2} \langle \Gamma_A \rangle (Z^0 + Y^0) \right] 
\]

\[
\times \frac{i^2}{(2\pi)^6 2 \omega_j(\langle k \rangle) 2 \omega_i(\langle k \rangle)} \left[ 2 \langle k \rangle \right] \left[ i(\omega_j(\langle k \rangle) + \omega_i(\langle k \rangle)) \right] \right], \quad (4.34)
\]

where

\[
G_j(Z^0, X_A; x^0_B, X_B; t, x) = h(z, Z^0) + (\sigma_A^2 + \sigma_B^2)\{x - z - \langle v_j \rangle (t - Z^0)\}. \quad (4.35)
\]

In order to see the properties of the integrand, it seems convenient for us to define

\[
F^{AC}(Z^0) = x - \langle v_A \rangle Z^0 - \langle v_j \rangle (t - Z^0) - X_A, \quad (4.36)
\]

\[
F^{BC}(Z^0, X_B^0) = F^{AC}(Z^0, X_B^0) - F^{AC}(Z^0) 
\]

\[
= X_B - \langle v_B \rangle (X_B^0 - Z^0) + \langle v_j \rangle (t - Z^0) - x, \quad (4.37)
\]
where \(\mathbf{F}^{AB}(Z^0, X_B^0)\) is given by (4.27). The factor \(x - z - \langle v_j \rangle (t - Z^0)\) included in \(G_j\), given by (4.35), is expressed as

\[
x - Z - \langle v_j \rangle (t - Z^0) = \mathbf{F}_j^{AC}(Z^0) + \langle v_A \rangle Z^0 + X_A
\]

\[
- \frac{1}{\sigma_A^2 + \sigma_B^2} \left[ \sigma_A^2 (X_A + \langle v_A \rangle Z^0) + \sigma_B^2 (X_B - \langle v_B \rangle (X_B^0 - Z^0)) \right]
\]

due to (4.26)

\[
= \mathbf{F}_j^{AC}(Z^0) - \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2} \mathbf{F}^{AB}(Z^0, X_B^0)
\]

\[
= \frac{1}{\sigma_A^2 + \sigma_B^2} \left[ \sigma_A^2 \mathbf{F}_j^{AC}(Z^0) - \sigma_B^2 \mathbf{F}_j^{BC}(Z^0, X_B^0) \right];
\]

(4.38)

thus we obtain from (4.26) and (4.35)

\[
G_j(Z^0, X_A; X_B, X_B; t, x)
\]

\[
= \frac{1}{\sigma_A^2 + \sigma_B^2} \left[ \sigma_A^2 \sigma_B^2 (\mathbf{F}^{AB}(Z^0, X_B^0))^2 + \sigma_A^2 \mathbf{F}_j^{AC}(Z^0) - \sigma_B^2 \mathbf{F}_j^{BC}(Z^0, X_B^0) \right]^2
\]

\[
= \sigma_A^2 (\mathbf{F}_j^{AC}(Z^0)) + \sigma_B^2 (\mathbf{F}_j^{BC}(Z^0, X_B^0))^2.
\]

(4.39)

Here we give a remark on the possibility of the minimum of \(G_j\) to be 0. The condition \(G_j = 0\) for \(Z^0 = Z_{\text{class}}^0\) is

\[
\mathbf{F}_j^{AC}(Z_{\text{class}}^0) = 0, \quad \text{and} \quad \mathbf{F}_j^{BC}(Z_{\text{class}}^0) = 0, \quad \text{[due to (4.39)]}
\]

leading respectively to

\[
Z_{\text{class}}^0 = \frac{1}{\langle \langle v_j \rangle - \langle v_A \rangle \rangle^2} \langle \langle v_j \rangle - \langle v_A \rangle \rangle \cdot (-x + X_A + \langle v_j \rangle t),
\]

(4.41)

\[
Z_{\text{class}}^0 = \frac{1}{\langle \langle v_j \rangle - \langle v_B \rangle \rangle^2} \langle \langle v_j \rangle - \langle v_A \rangle \rangle \cdot (-x + X_B + \langle v_B \rangle X_B^0 + \langle v_j \rangle t),
\]

(4.42)

together with the components of \(-x + X_A + \langle v_j \rangle t\) and \(-x + X_B + \langle v_B \rangle X_B^0 + \langle v_j \rangle t\) perpendicular respectively to \(\langle v_j \rangle - \langle v_A \rangle\) and \(\langle v_j \rangle - \langle v_B \rangle\) to be equal 0. Eqs. (4.41) means \(\mathbf{F}^{AB}(Z_{\text{class}}^0, X_B^0) = 0\) [due to (4.37)], which leads to

\[
Z_{\text{class}}^0 = \frac{1}{\langle \langle v_A \rangle - \langle v_B \rangle \rangle^2} \langle \langle v_A \rangle - \langle v_B \rangle \rangle \cdot (X_B - \langle v_B \rangle X_B^0 - X_A),
\]

(4.43)

together with the component of \(X_B - \langle v_B \rangle X_B^0 - X_A\) perpendicular to \(\langle v_A \rangle - \langle v_B \rangle\) to be 0. These \(Z_{\text{class}}^0\)'s give three kinds of the classical relations. We see, however, these \(Z_{\text{class}}^0\)'s should not be taken to be equal especially due to the \(j\)-independence of (4.33).

It is necessary for us to consider how to perform the \(Y^0\) - and \(Z^0\)-integrations over a finite time-interval; this is to be considered in the next subsection where the minimum of \(G_j\) is examined.

C. Time integrations

We examine in detail the structures of the integrand in (4.26).

1. The minimum point \(Z^0\) of \(G_j\)

We write for simplicity \(G_j\) as \(G_j(Z^0)\) and rewrite it in the form

\[
G_j(Z^0) = \alpha_j (Z^0 - Z^0)^2 + G_j(Z^0).
\]

(4.44)

\(Z^0\) is determined by

\[
\frac{dG_j(Z^0)}{dZ^0} \bigg|_{Z^0} = 0 = 2 \left[ \sigma_A^2 (\langle v_j \rangle - \langle v_A \rangle) \cdot \mathbf{F}_j^{AC}(Z^0) - \sigma_B^2 (\langle v_j \rangle - \langle v_B \rangle) \cdot \mathbf{F}_j^{BC}(Z^0, X_B^0) \right]
\]
due to (4.39)

\[
Z^0 = 2\sigma^2_j \left[ -\sigma^2_A(\langle v_j \rangle - \langle v_A \rangle)^2 + \sigma^2_B(\langle v_j \rangle - \langle v_B \rangle)^2 \right] + \sigma^2_A(\langle v_j \rangle - \langle v_A \rangle) \cdot F^AC_j(Z^0 = 0) - \sigma^2_B(\langle v_j \rangle - \langle v_B \rangle) \cdot F^{BC}_j(0, X^0_B) \right]
\]

which leads to

\[
Z^0 = 2\sigma^2_j \left[ -\sigma^2_A(\langle v_j \rangle - \langle v_A \rangle)^2 + \sigma^2_B(\langle v_j \rangle - \langle v_B \rangle)^2 \right] + \frac{1}{\sigma^2_{jA}} \equiv \frac{1}{\sigma^2_{jA} - \sigma^2_{jB}}.
\]

with

\[
\frac{1}{2\sigma^2_{jA}} \equiv \sigma^2_A(\langle v_j \rangle - \langle v_A \rangle)^2, \quad \frac{1}{2\sigma^2_{jB}} \equiv \sigma^2_B(\langle v_j \rangle - \langle v_B \rangle)^2.
\]

By noting \(\alpha_j\) to be given by \(\frac{1}{2}(d^2 G_j/d(Z^0)^2) = 1/(2\sigma^2_j)\), we have

\[
G_j(Z^0) = \frac{1}{2\sigma^2_j}(Z^0 - Z^0)^2 + G_j(Z^0)
\]

\[
= \frac{1}{2\sigma^2_j}(Z^0 - Z^0)^2 + \sigma^2_A(F^AC_j(Z^0))^2 + \sigma^2_B(F^{BC}_j(Z^0, X^0_B))^2.
\]

2. Another expression of \(G_j(Z^0)\)

We write the points \(Z^0\)’s giving the minima of \((F^AC_j)^2\) and \((F^{BC}_j)^2\) as \(T^AC_j\) and \(T^{BC}_j\), respectively;

\[
F^AC_j(T^AC_j) \cdot (\langle v_j \rangle - \langle v_A \rangle) = 0, \quad F^{BC}_j(T^{BC}_j, X^0_B) \cdot (\langle v_j \rangle - \langle v_B \rangle) = 0.
\]

We obtain

\[
T^AC_j = \frac{1}{((\langle v_j \rangle - \langle v_A \rangle)^2(\langle v_j \rangle - \langle v_A \rangle) \cdot (-x + X_A + \langle v_j \rangle t) = 2\sigma^2_A \sigma^2_A((\langle v_j \rangle - \langle v_A \rangle) \cdot (-F^AC_j(Z^0 = 0)),
\]

\[
T^{BC}_j = \frac{1}{((\langle v_j \rangle - \langle v_B \rangle)^2(\langle v_j \rangle - \langle v_B \rangle) \cdot (-x + \langle v_j \rangle t - \langle v_B \rangle X^0_B + X_B) = 2\sigma^2_B \sigma^2_B((\langle v_j \rangle - \langle v_B \rangle) \cdot F^{BC}_j(Z^0 = 0, X^0_B);\n\]

then (4.40) leads to

\[
Z^0 = \sigma^2_j \left[ \frac{1}{\sigma^2_{jA}} T^AC_j + \frac{1}{\sigma^2_{jB}} T^{BC}_j \right]
\]

\[
= \frac{1}{\sigma^2_{jA} + \sigma^2_{jB}} \left[ \sigma^2_{jB} T^AC_j + \sigma^2_{jA} T^{BC}_j \right].
\]

By noting

\[
\frac{1}{2} \frac{\partial^2}{\partial (Z^0)^2} \left[ (F^AC_j)^2 \right] = \left[ \frac{1}{2\sigma^2_{jA}} \right], \quad \frac{1}{2} \frac{\partial^2}{\partial (Z^0)^2} \left[ (F^{BC}_j)^2 \right] = \left[ \frac{1}{2\sigma^2_{jB}} \right],
\]
we have from (4.39)

\[ G_j(Z^0) = \frac{1}{2\sigma_j^2}(Z^0 - T_j^{AC})^2 + \sigma_j^2[F_j^{AC}(T_j^{AC})]^2 + \frac{1}{2\sigma_j^2}(Z^0 - T_j^{BC})^2 + \sigma_j^2[F_j^{BC}(T_j^{BC}, X_B^0)]^2. \] (4.54)

It is easy for us to confirm the equality

\[ \frac{1}{2\sigma_j^2}(Z^0 - Z^0)^2 + \frac{1}{2\sigma_j^2}(T_j^{AC} - T_j^{BC})^2 \]

\[ = \frac{1}{2\sigma_j^2}(Z^0 - T_j^{AC})^2 + \frac{1}{2\sigma_j^2}(Z^0 - T_j^{BC})^2; \] (4.55)

because, due to (4.52) we have

\[ \text{L.H.S. of (4.50) = 1, given by (3.54), we can write} \]

\[ \text{R.H.S. of (4.55)}. \] (4.56)

Thus we obtain

\[ G_j(Z^0) = \sigma_j^2[F_j^{AC}(T_j^{AC})]^2 + \sigma_j^2[F_j^{BC}(T_j^{BC}, X_B^0)]^2 + \frac{(T_j^{AC} - T_j^{BC})^2}{2(\sigma_j^2 + \sigma_j^2)}. \] (4.57)

3. The expression of \( \mathcal{E}_s^{(t-1)} \)

(4.51) is expressed as

\[ \begin{align*}
\mathcal{E}[A(p); X_A, x_A^0]; C_{\rho\sigma} & \text{-Cur}(x), \bar{B}_{\sigma} & \text{-Cur}(q); \mathcal{X}_B, x_B^0]^{(t-1)} \\
\simeq & \sum_{j,i} Z_{ij}^{1/2} Z_{ij}^{1/2*} \sum_{i,j} |f_{A,B}\sigma|^2 (\sigma_j^2 \sigma_j^2)^{3/2} \left[ \frac{2}{\pi} \right]^{3} \frac{i^2}{2 \langle E_A \rangle} \frac{\langle E_B \rangle}{2 \langle \omega_j \rangle} \frac{\langle \omega_i \rangle}{2 \langle \omega_i \rangle} \left[ \langle i \rangle (\omega_j) + \langle \omega_i \rangle \right] \\
\times & \exp \left[ i (\langle \omega_j \rangle - \langle \omega_i \rangle) t - G_j(Z^0, X_A; X_B^0, X_B; t, x) - G_i(Y^0, X_A; X_B^0, X_B; t, x) \\
& - Z^0 (i \Delta E_j + \frac{\langle \Gamma_A \rangle}{2}) + Y^0 (i \Delta E_i + \frac{\langle \Gamma_A \rangle}{2}) \right] \\
\times & I_j(x, t; X_A, X_B, X_B^0) I_i^*(x, t; X_A, X_B, X_B^0),
\end{align*} \] (4.58)

\[ I_j(x, t; X_A, X_B, X_B^0) \equiv \int_0^{X_B^0} dZ^0 \exp \left[ - \frac{(Z^0 - Z^0)^2}{2\sigma_j^2} - (Z^0 - Z^0)(i \Delta E_j + \frac{\langle \Gamma_A \rangle}{2}) \right]; \] (4.59)

here, we

\[ \Delta E_j \equiv \langle E_B \rangle + \langle \omega_j \rangle - \langle E_A \rangle. \] (4.60)

and required a physical condition on the time ordering

\[ x^0 \geq x_B^0 \geq (y^0, z^0), \quad \text{i.e.} \quad t \geq X_B^0 > (Y^0, Z^0). \] (4.61)

In the same way as \( M_j(t; Z^0) \) given by (3.54), we can write \( I_j \) as

\[ I_j = \sqrt{2\sigma_j^2} \exp \left[ \frac{1}{2\sigma_j^2}(i \Delta E_j + \frac{\langle \Gamma_A \rangle}{2})^2 \right] J_j(u_j(X_B^0); \alpha_j, \beta_j), \] (4.62)
where \( \hat{\alpha}_j = \sigma_j \Delta E_j / \sqrt{2} \), \( \hat{\beta}_j = (\mathbf{Z}^0 + \frac{1}{2} \sigma_j^2 (\Gamma_A) ) / (\sqrt{2} \sigma_j) \) and \( u_j(X_B^0) = t / (\sqrt{2} \sigma_j) \). Then, (4.58) is expressed as

\[
\mathcal{E}[A((p); X_A, x_A^0); C_{\mu} \text{Cur}(x), \mathcal{B}(q) : X_B, x_B^0)]_{(I-1)} = \sum_{j,i} Z_{\rho j}^{1/2} Z_{\rho i}^{1/2} Z_{\sigma i}^{1/2} |f_{ABC}\sigma|^2 (\sigma^2_A \sigma^2_B)^{1/2} \left[ \frac{2}{\pi} \right]^3 \frac{i^2}{2(E_A) 2(E_B) 2(\omega_j) 2(\omega_i)} \times \left[ i((\omega_j) + (\omega_i)) \right] \exp \left[ Q_{ji} + \frac{1}{2} \sigma^2_{ji} (i \Delta E_j + \frac{\langle \Gamma_A \rangle}{2})^2 - \frac{1}{2} \sigma^2_i (i \Delta E_i - \frac{\langle \Gamma_A \rangle}{2})^2 \right] \times |J_j(u_j(X_B^0); \hat{\alpha}_j, \hat{\beta}_j)| \times |J_i(u_i(X_B^0); \hat{\alpha}_i, \hat{\beta}_i)| \exp \left[ i(\Theta(u_j(X_B^0); \hat{\alpha}_j, \hat{\beta}_j) - \Theta(j \to i)) \right],
\]

where

\[
Q_{ji} = i((\omega_j) - (\omega_i)) t - G_j(Z_0^0) - G_i(Y_0) - Z_0^0(i \Delta E_j + \frac{\langle \Gamma_A \rangle}{2}) + Y_0(i \Delta E_i - \frac{\langle \Gamma_A \rangle}{2}).
\]

We see (4.64) has structures similar to those of (3.59), and examine in Sec.V its qualitatives of \( t \)- and \( L \)-dependence under the condition of neutrino high energies. Here we add a remark on \( G_j(Z_0^0) \) as follows. From (4.58) and (4.57), \( G_j(Z_0^0) \) is given by

\[
G_j(Z_0^0) = \sigma^2_A (F_j^{AC}(Z_0^0))^2 + \sigma^2_B (F_j^{BC}(Z_0^0, X_B^0))^2
\]

\[
= \sigma^2_A (F_j^{AC}(T_j^{AC}))^2 + \sigma^2_B (F_j^{BC}(T_j^{BC}, X_B^0))^2 + \frac{(T_j^{AC} - T_j^{BC})^2}{2(\sigma^2_A + \sigma^2_B)}. \tag{4.65}
\]

As noted in the subsection B of Sec.IV, we can not take two \((Z_0^0)_{\text{clas}}' \)'s (given by (4.59) and (4.12)) to be equal. This means \( T_j^{AC} \) and \( T_j^{BC} \), given by (4.59) and (4.51), are not equal to each other, since the expressions of (4.50) and (4.51) are equal respectively to (4.11) and (4.12). Thus, due to (4.52), we have

\[
T_j^{AC} > Z_0^0 > T_j^{BC} \quad \text{or} \quad T_j^{AC} < Z_0^0 < T_j^{BC}. \tag{4.66}
\]

Under the parallel condition we obtain

\[
(T_j^{AC})_{\text{para}} = \frac{(v_j) - L}{(v_j) - (v_A)}, \quad L = x - X_A, \tag{4.67}
\]

\[
(T_j^{BC})_{\text{para}} = \frac{1}{(v_j) - (v_B)} ((v_j) - L_{BA} - (v_B) X_B^0), \quad L_{BA} = X_B - X_A, \tag{4.68}
\]

\[
(Z_0^0)_{\text{para}} = \frac{1}{\sigma^2_{ja} + \sigma^2_{jb}} \left[ \sigma^2_j (T_j^{AC})_{\text{para}} + \sigma^2_j (T_j^{BC})_{\text{para}} \right]; \tag{4.69}
\]

the minimum of \( G_j(Z_0^0) \) is given by

\[
G_j(Z_0^0)_{\text{para}} = \frac{1}{2(\sigma^2_{ja} + \sigma^2_{jb})} [(T_j^{AC})_{\text{para}} - (T_j^{BC})_{\text{para}}]^2. \tag{4.70}
\]

V. DISCUSSIONS AND FINAL REMARKS

We have investigated the structures of the expectation values of the flavor current with respect to an one-particle state which plays a role of the neutrino source. The present approach is certainly simple one to the neutrino oscillation without recourse to any one-particle state of flavor neutrino. In Sec.III, the expectation value of the flavor neutrino current with respect to one \( \pi^+ \) wave-packet state was given as (3.59). In Sec.IV, we have examined in the boson model the same type of the expectation value with an additional "observed" charged particle, and shown that the main part of the expectation value of (1.63) has the structures similar to those of (3.59) derived in Sec.III.

The expectation values (1.59) and (1.63) have somewhat complicated structure in comparison with the standard oscillation formula; therefore, in order to make clear characteristic features included in (3.59) and (1.03), we are
necessary to perform numerical calculations, which will be done in a subsequent paper \[18\]. Here we give some remarks in the following.

It is worthwhile for us to examine the features of contributions from high-energy neutrinos to \[3.50\] and \[4.63\].

First we considered the structures of the integration of the integrand of \[3.50\] in the case of \[m_j^2/k^2 \ll 1\] for all \(j\)’s and \(1 - \langle v_\pi \rangle \approx \mathcal{O}(1)\).

As to the phase parts, we obtain from \[5.03\]

\[
\text{Im}(\mathcal{P}_{ji})_{para} \simeq \frac{\Delta m_{ji}^2}{2k} \left( \frac{L - t \langle v_\pi \rangle}{1 - \langle v_\pi \rangle} \right) \left[ 1 + \frac{\langle \delta E \rangle}{k(1 - \langle v_\pi \rangle)} \right]; \tag{5.1}
\]

with \(\langle \delta E \rangle = E_\pi + K - \langle E_\pi \rangle\), and also

\[
-\Theta(u_j(t); \alpha_j, \beta_j) + \Theta(u_i(t); \alpha_i, \beta_i) \simeq -\frac{dv_j}{dm_j} \left[ \frac{\partial \Theta(u_j(t); \alpha_j, \beta_j)}{\partial v_j} \right]_{m_j=0} \Delta m_{ji}^2 + (j \to i). \tag{5.2}
\]

By using the \(j\)-independent quantity

\[
\left[ \frac{\partial \Theta}{\partial v} \right]_0 = \left[ \frac{\partial \Theta(u_j(t); \alpha_j, \beta_j)}{\partial u_j} \right]_{m_j=0} = -\frac{1}{|J_0(u(t); \alpha, \beta)|} e^{\alpha^2} \left[ e^{-\beta^2} \sqrt{(\alpha')^2 + (\beta')^2} \sin(2\alpha\beta + \Theta - \varphi^1) \right.
\]

\[
+ e^{-(u(t) + \beta)^2} \sqrt{(\alpha')^2 + (u(t) + \beta')^2} \sin(2\alpha(u(t) + \beta) + \Theta - \varphi^2) \bigg], \tag{5.3}
\]

where primes (‘) denote the derivative with respect to neutrino velocity, \(v\), and

\[
\varphi^1 = \arctan \left( \frac{\alpha'}{\beta'} \right), \quad \varphi^2 = \arctan \left( \frac{\alpha'}{(u(t) + \beta)'} \right),
\]

we obtain

\[
-\Theta(u_j(t); \alpha_j, \beta_j) + \Theta(u_i(t); \alpha_i, \beta_i) \simeq \frac{\Delta m_{ji}^2}{2k^2} \left[ \frac{\partial \Theta}{\partial v} \right]_0, \tag{5.4}
\]

This is the additional phase coming from the finite-time integrals in \[3.32\]. One can see from \[5.03\] that this additional phase will vanish when \(\beta\) and \(u(t) + \beta\) become much larger than 1 as well as \(\alpha\). This result can be understood because, since \(\beta\)’s represent the ratio of the distances \((Z^0, t - Z^0)\) between the peak of gaussian \((Z^0)\) and both limits of integral region \((0, t)\) to the gaussian width \((\sigma_{\pi j})\), \(\beta\)’s \(\gg 1\) means that the gaussian is so sharp that one can regard the both limits as infinite. Then the finite-time integral is reduced to the real function. However, the situation will be different when \(\beta\)’s are smaller than 1 and \(\alpha\). Numerical calculations are expected to make a behavior of this phase clearer.

As mentioned before, the condition

\[
m_j^2/k^2 \left( \frac{1}{1 - \langle v_\pi \rangle} \right) \ll 1, \tag{5.5}
\]

is necessary for deriving \[5.03\]. By taking \(\langle v_\pi \rangle \sim 0\) and \(\langle \delta E \rangle / k \sim 0\), we obtain

\[
\text{Im}(\mathcal{P}_{ji})_{para} \simeq \frac{\Delta m_{ji}^2}{2k} L, \tag{5.6}
\]

which is the same as the standard one, apart from \(K = K(q) = (p) - q\). It will be instructive to evaluate typical \(\langle v_\pi \rangle\)-values a shown in Table II(a); by the way, rough values of \(K\) by assuming \(|\delta E_j|/\langle E_\pi \rangle \lesssim \mathcal{O}(1/10)\) are given in Table II(b).

It is possible for us to extract information on the coherence length from \(|J_j(u_j(t); \alpha_j, \beta_j)| \cdot |J_i(u_i(t); \alpha_i, \beta_i)|\) as follows as \(\text{Re}\mathcal{P}_{ji}\). This is one of the remaining tasks.
Next we give a remark on the structures of (4.64). In this case, \( \langle k \rangle = (\langle p \rangle - \langle q \rangle) \) is fixed when \( \langle p \rangle \) and \( \langle q \rangle \) are given. We have the phase parts in (4.63) expressed as

\[
\text{Im}(Q_{ji})_{\text{para}} = \left[ (\omega_j) - (\omega_i) \right] t - Z^0_j \Delta E_j + Y^0_i \Delta E_i \right]_{\text{para}}, \tag{5.7}
\]

and \( (Z^0)_{\text{para}} \) given by (4.69) and \( (Z^0)_{\text{para}} \) given by (3.62). This difference arises from (4.66), i.e. from \( T^{AC}_j \neq T^{BC}_j \), which leads to nonvanishing \( G_j(Z^0_j) + G_i(Y^0_i) \) para in Re\[Q_{ji}]_{para} and also to the additional \( v_j \)-dependence of \( Z^0 \) through \( \sigma^2_{jA} \) and \( \sigma^2_{jB} \).

|   | \( \langle E \rangle \) (MeV) | 200 | 300 | 500 | 1000 |
|---|--------------------------------|-----|-----|-----|------|
| (a) | \( \langle p \rangle \) (MeV) | 142.8 | 265.3 | 480.0 | 990.1 |
| | \( \langle u_\pi \rangle /c \) | 0.71 | 0.88 | 0.96 | 0.99 |
| (b) | \( k \) (MeV) | 74 | 122 | 211 | 427 |

* Values of \( \frac{1}{2}(m_{\pi}^2 - m_{\mu}^2)/((E_\pi) - \langle p \rangle) \)

TABLE I: (a) and (b)

[1] M. Buethe, Oscillations of nutrinos and mesons in quantum field theory, hep-th/0109119, Sep, 2001; Phys. Rev. D66, 013003 (2002); Phys. Rept. 375, 105 (2003).
[2] S. M. Bilenky and B. Pontecorvo, Phys. Lett. 61B, 248(1976); Phys. Rept. 41, 225(1978). See also the recent articles,
[3] E.g., M. Fukugita and T. Yanagida, Physics and Astrophysics of Nuetrinos, edited by M. Fukugita and A. Suzuki (Springer-Verlag, Tokyo, 1994), p.1 ; S. M. Bilenky, Lectures given at the 1999 European School of High Energy Physics, Cast Papiemicka, Slovakia, Aug. 22-Sep.4, 1999
[4] E.g., C. Giunti, C. W. Kim, and U. W. Lee, Phys. Rev. D45, 2414(1992); S. M. Bilenky and C. Giunti, hep-ph/0102320, 2001
[5] M. Blasone and G. Vitiello, Annals of Phys. 244, 283(1995); 245,363(E)(1995); Phys. Rev. D60, 111302(1999).
[6] K. Fujii, C. Habe, and T. Yabuki, Phys. Rev. D59, 113003(1999); 60, 099903(E)(1999).
[7] K. Fujii, C. Habe, and T. Yabuki, Phys. Rev. D64, 013011-1(2001).
[8] K. Fujii, C. Habe, and M. Blasone, hep-ph/0212076 Dec.2002.
[9] C. Giunti, C. W. Kim, J. A. Lee, and U. W. Lee, Phys. Rev. D48, 4310(1993).
[10] W. Grimus, P. Stockinger, and S. Mohanty, Phys. Rev. D59, 013011(1999).
[11] T. Yabuki and K. Ishikawa, Prog. Theor. Phys. 108, 347(2002).
[12] B. Ancochea, A. Bramon, R. Muñoz-Tapia, and M. Nowakowski, Phys. Lett. B389, 149(1996).
[13] M. Zralek, Acto Phys. Polon. B29, 3925(1998).
[14] M. Blasone, P. Jizba, and G. Vitiello, Phys. Lett. B517,471(2001); M. Blasone, P. P. Pachêco, and H. W. C. Tseung, hep-ph/0212402 Dec.2002.
[15] H. Umezawa, Sorayushi-ron (Misuzu Shyobo, Tokyo 1953), ch.13; Quantum Field Theory (North-Holland Pub.Co, Amsterdam, 1956), ch.13.
[16] E.g. E. Cornaldesi, Nucl. Phys. 7, 305(1958).
[17] E.g. V. de. Alfaro, S. Fubini, G. Furlan, and C. Rossetti, Currents in Hadron Physics (North-Holland Pub.Co, Amsterdam.London; American Elsevier Pub.Co, Inc., New York, 1973), ch.2.
[18] T. Shimomura, in preparation.