Preferred Basis in a Measurement Process

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Abstract

The effect of decoherence is analysed for a free particle, interacting with an environment via a dissipative coupling. The interaction between the particle and the environment occurs by a coupling of the position operator of the particle with the environmental degrees of freedom. By examining the exact solution of the density matrix equation one finds that the density matrix becomes completely diagonal in momentum with time while the position space density matrix remains nonlocal. This establishes the momentum basis as the emergent ‘preferred basis’ selected by the environment which is contrary to the general expectation that position should emerge as the preferred basis since the coupling with the environment is via the position coordinate.

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The act of measurement for quantum systems has defied understanding as this process involves a collapse of the state vector to one of the eigenstates of the dynamical operator which is being measured. The process of collapse is nonunitary \[1\] and cannot be described quantum mechanically. In recent years, the ‘decoherence’ approach \[2–6\] to the quantum measurement problem has successfully tackled many conflicts between the predictions of conventional quantum theory and classical perceptions. This approach seems to provide convincing explanations for the emergence of ‘classicality’ from an underlying quantum substrate. In this approach, the measuring apparatus, which is often macroscopic, is never isolated and is constantly interacting with a large environment. The physical system, which comprises of the quantum system, the measuring apparatus, and the environment, has a large number of independent parameters or degrees of freedom. However, we are often interested in only a small number of these degrees of freedom to describe the outcome of a measurement. Decoherence is a consequence of ‘ignoring’ large numbers of degrees of freedom. More technically, if we are describing the interaction between the apparatus and the environment by a density matrix \[6\], and trace over the environment degrees of freedom, the reduced density matrix of the apparatus is driven diagonal as a consequence of the environmental influence. The pure state density matrix thus gets reduced to a mixture. This amounts to saying that superpositions vanish and the density matrix can be interpreted in terms of classical probabilities. However, even in this approach the question of the basis in which the density matrix becomes diagonal, i.e., the ‘preferred basis’ \[6\] in which superpositions vanish, is not quite understood. It seems that regardless of the initial condition, the environment always selects a special set of states in which the density matrix is diagonal and hence classically interpretable \[6\]. It is important to know what these preferred bases are for specific systems since they are directly related to the emergent ‘classicality’ of macroscopic systems as a consequence of decoherence. It seems plausible that such a preferred basis would be decided by the system operator which is coupled to the environmental degrees of freedom. For example, for a free particle if the position operator is involved in coupling with the environmental degrees of freedom, one expects that the density matrix would be
driven diagonal in the position space. It has been shown by Zurek [6] that the coherence between two Gaussian wave packets separated in space by $\Delta x$ is lost on a time scale which is typically

$$\theta = \tau \left( \frac{\hbar}{\Delta x \sqrt{4mk_BT}} \right)^2$$  \hspace{1cm} (1)$$

where $m$ is the mass of the particle, $k_B$ is Boltzmann’s constant, $T$ is the temperature of the heat bath and $\gamma^{-1}$ is the characteristic relaxation time of the system. For classical systems and standard macroscopic separations $\Delta x$, the ratio $\theta/\tau$ can be as small as $10^{-40}$ [6], suggesting that the density matrix becomes diagonal in position space almost instantaneously, making position the ‘preferred basis’. Here, however, we show that for a free particle such is not the case. This implies, therefore, that to observe a dynamical variable, the system-apparatus coupling requires a careful consideration.

I. THE MASTER EQUATION

The problem of a quantum system interacting with an environment has been studied in great detail by many authors in the context of quantum dissipative systems [7–12] and the quantum measurement problem [2–6,13,14]. Here we employ the method studied extensively by Caldeira and Leggett [9] and others [10] to study a free particle coupled to a collection of harmonic oscillators, which constitutes the environment. The interaction between the free particle and the environment is linear via a coordinate-coordinate coupling [8,10]. The total Hamiltonian for the composite system can be written as

$$H = \frac{P^2}{2m} + \sum_j \left( \frac{p^2_j}{2m_j} + \frac{m_j\omega^2_j}{2} \left[ x_j - \frac{c_jQ}{m_j\omega^2_j} \right]^2 \right).$$  \hspace{1cm} (2)$$

Here, $P$, $Q$ are the momentum and position coordinates of the free particle, and $p_j$ and $x_j$, those of the jth harmonic oscillator. $c_j$s are coupling strengths and $\omega_j$s are the frequencies of the oscillators. The Hamiltonian of [2] is known as the independent oscillator model. More frequently seen in the literature is the ‘linear coupling’ model where the Hamiltonian is
without the ‘counter term’ (last term in the summation) and the coupling is represented
by adding a term of the form $Q \sum_j c_j x$. As pointed out by Ford et al [8], the ‘linear coupling’
Hamiltonian is unphysical and corresponds to a ‘passive’ heat bath and is not invariant under
translations. All information about the harmonic oscillator heat bath which is required for
the description of the particle via a reduced density matrix is contained in the spectral
density function [8,9] and the initial temperature of the bath. Using the Feynman- Vernon
influence functional technique [11], Caldeira and Leggett have shown that for a white (ohmic)
noise spectrum in the high temperature limit, one can write an equation of motion for the
reduced density matrix $\rho$ of the free particle. In the position representation this equation
can be can be written [6,12] as

$$
\frac{\partial \rho_R(x, y, t)}{\partial t} = \left[ -\frac{\hbar}{2im} \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) - \gamma (x - y) \left\{ \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right\} - \frac{D}{4\hbar^2}(x - y)^2 \right] \rho_R(x, y, t),
$$

(3)

where $m$ is the mass of the particle, $\hbar$ is Planck’s constant, $\gamma$ is the Langevin friction coefficient and $D$ has the usual interpretation of the diffusion constant. $\gamma$ and $D$ are related to
the parameters of the Hamiltonian 2. For a high temperature thermal bath, $D = 2m\gamma k_B T$.

[3] has been used to study the dynamics of systems like the free particle and the harmonic
oscillator in interaction with a heat bath [3,14]. In the following we will be looking at the
exact solution of [3] in the position and momentum representations.

II. PREFERRED BASIS

Consider the exact solutions of [3] derived earlier by Kumar [12] for an initial Gaussian
wave packet

$$
\psi(x, 0) = \frac{1}{(\sigma \sqrt{\pi})^{1/2}} \exp(-x^2/2\sigma^2)
$$

(4)

where $\sigma$ is the width of the wave packet. The solutions [12,16] in wave vector and position
representations in terms of the changed coordinates $Q = u - v; q = (u + v)/2$ in wave vector
space and $R = (x + y)/2; r = (x - y)$ in position space, are
\[ \rho_d(Q, q, t) = 2 \sqrt{\frac{\pi}{N(\tau)}} \exp \left[ -\frac{1}{N(\tau)} \left[ q + \frac{i\hbar Q}{2\sigma^2 m \gamma} e^{-\tau} (1 - e^{-\gamma}) - \frac{iQ D}{4\hbar \gamma^2 m} (1 - e^{-\gamma})^2 \right]^2 \right. 
- \left. \frac{\hbar^2}{4\sigma^2 m^2 \gamma^2} (1 - e^{-\gamma})^2 + \frac{\sigma^2}{4} + \frac{D}{2m^2 \gamma^3} (2\tau - 3 + 4e^{-\tau} - e^{-2\tau}) \right] Q^2 \] (5)

where \( \tau = \gamma t \) and

\[ N(\tau) \equiv \left( D/2\hbar^2 \gamma \right) (1 - e^{-2\tau}) + \left( 1/\sigma^2 \right) e^{-2\tau}, \] (6)

and

\[ \rho_d(R, r, t) = 2 \sqrt{\frac{\pi}{N(\tau)}} \exp \left[ -\left[ \frac{1}{4\sigma^2} e^{-2\tau} + \frac{D}{8\hbar^2 \gamma} (1 - e^{-2\tau}) \right] - \frac{1}{M(\tau)} \left[ R \right. 
- \left. \frac{ihr}{2\sigma^2 m \gamma} e^{-\tau} (1 - e^{-\gamma}) - \frac{iDr}{4\hbar \gamma^2 m} (1 - e^{-\gamma})^2 \right]^2 \right] \] (7)

where

\[ M(\tau) \equiv \sigma^2 + \frac{\hbar^2}{\sigma^2 m^2 \gamma^2} (1 - e^{-\gamma})^2 + \frac{D}{2m^2 \gamma^3} (2\tau - 3 + 4e^{-\tau} - e^{-2\tau}). \] (8)

The time dependence of the above two solutions can be studied for two regimes: \( \tau \gg 1 \), i.e., for times much larger than the characteristic relaxation time \( \gamma^{-1} \), and \( \tau \ll 1 \), i.e., for times much smaller than \( \gamma^{-1} \). The first regime \( (\tau \gg 1) \) is of greater importance in all real life systems.

A. \( \tau \gg 1 \)

It is clear from the form of the solution \[ \] that as \( \tau \to \infty \), the off-diagonal elements of the density matrix in the momentum representation vanish. This decay is exponential and occurs in a time

\[ t_d = \frac{m^2 \gamma^2}{DQ^2}. \] (9)

The density matrix \[ \] becomes completely diagonal at long times and assumes the form

\[ \rho(0, u, t) = 2 \sqrt{\frac{\pi}{N(\tau)}} \exp \left\{ -\frac{u^2}{N(\tau)} \right\}, \] (10)
making momentum the obvious choice for the preferred basis. \( \rho \) is the classical Maxwell distribution that one would obtain from a classical Fokker-Planck equation. We would like to point out here that the expression \( \tau > > 1 \) for \( \tau > > 1 \) which implies a time scale much greater than the relaxation time of the system \( \gamma^{-1} \). Moreover \( \rho \) is valid for a high temperature heat bath, where \( \hbar \gamma / k_B T \ll 1 \). From the expression \( \rho \) one can see that the decoherence time \( \tau_d \) decreases with increasing \( T \) and \( \gamma \). On the other hand the distribution function \( f \) in the position representation does not become completely diagonal as \( \tau \to \infty \), but assumes the form

\[
\rho_d(R, r, t) = 2 \sqrt{\frac{\pi}{M(\tau)}} \exp \left[ -\frac{D}{8h^2\gamma} r^2 - \frac{1}{M(\tau)} \left[ R + \frac{iDr}{4h^2\gamma m} (1 - e^{-\tau})^2 \right]^2 \right], \tag{11}
\]

with

\[
M(\tau) \simeq \frac{D\tau}{m^2\gamma^3}. \tag{12}
\]

The density matrix is obviously non-diagonal in the position space representation which is a consequence of the fact that \( \rho \) and \( \rho_d \) are related by Fourier transforms. The asymptotic width for the distribution in variable \( r \) is \( D/8h^2\gamma \), which for a thermal bath is \( \pi/2\lambda_d^2 \), since

\[
h^2\gamma/D = \lambda_d^2/4\pi, \tag{13}
\]

where \( \lambda_d \) is the thermal de Broglie wavelength of the particle \( (h/\sqrt{2m\pi k_B T}) \). One can see that if the extent of 'off-diagonality' is much greater than \( \lambda_d \), the magnitude of the off-diagonal elements, which is weighted by \( e^{-r^2\pi/2\lambda_d^2} \), is very small and the density matrix in position space can be considered nearly diagonal. In principle, however, it remains nonlocal to the extent of the deBroglie wavelength, \( \lambda_d \). It is interesting to see that for the initial condition considered by Zurek \( \rho \), where the initial position-space density matrix contains four well-separated peaks \( (\Delta x > > \sigma) \), an exact solution of \( \rho \) shows that for the peaks which are along the diagonal for which \( R \simeq \pm \Delta x; r \simeq 0 \), the major contribution to the density matrix is from the diagonal elements, which are peaked around \( \pm \Delta x \). However, for the peaks along the off-diagonal, for which \( R \simeq 0; r \simeq \pm \Delta x \) the factor \( \exp(-r^2\pi/2\lambda_d^2) \) (see \( \rho \))
is now exp\((-\Delta x^2\pi/2\lambda_d^2)\). One can see that for \(\Delta x >> \lambda_d\), this factor is very small and hence the elements of the density matrix corresponding to these two off-diagonal peaks are negligible in magnitude. Thus, position seems to emerge as an approximate preferred basis, which works well only when one is probing length scales which are much larger than \(\lambda_d\).

**B. \(\tau \ll 1\)**

In this regime, one is probing the system at time scales which are much smaller than the characteristic relaxation time \(\gamma^{-1}\). This regime is not very realistic since one is usually interested in the state of the system long after it has been left in an environment. If we retain terms only up to first order in \(\tau\) in (8) and (9), the momentum and position space density matrices assume the forms

\[
\rho_d(Q, q, t) = 2\sqrt{\frac{\pi}{N(\tau)}} \exp\left[\frac{-1}{N(\tau)} \left[q + \frac{i\hbar Q}{2\sigma^2 m \gamma} \tau (1 - \tau) - \frac{iQD}{4\hbar \gamma m} \tau^2 \right]^2 - \left[\frac{\hbar^2}{4\sigma^2 m \gamma^2} \tau^2 + \frac{\sigma^2}{4} Q^2 \right]\right],
\]

(14)

where

\[
N(\tau) \simeq (D\tau/\hbar^2 \gamma) + (1/\sigma^2)(1 - 2\tau),
\]

(15)

and

\[
\rho_d(R, r, t) = 2\sqrt{\frac{\pi}{M(\tau)}} \exp\left[-\frac{1}{4\sigma^2}(1 - 2\tau) + \frac{D\tau}{4\hbar^2 \gamma} r^2 - \frac{1}{M(\tau)} [R \left[\frac{i\hbar r}{2\sigma^2 m \gamma} \tau (1 - \tau) + \frac{iDr}{4m \gamma^2 \hbar} \tau^2 \right]^2\right],
\]

(16)

where

\[
M(\tau) \simeq \sigma^2 + \frac{\hbar^2 \tau^2}{\sigma^2 m^2 \gamma^2}.
\]

(17)

It is clear from (14) and (16) that the density matrices in both representations remain nondiagonal and neither show any exponential decay with time for the off-diagonal elements. For the position density matrix (16), the leading order decaying term is \(~\)
\[
\exp \left( -\frac{1}{2} \left\{ \frac{D}{2\hbar^2\gamma} - \frac{1}{\sigma^2} \right\} r^2 \tau \right).
\]
Note that \( Dr^2\tau \sim 4\pi r^2\tau/\lambda_d^2 \) and the 'decay' time is similar to \( \tau_d \) obtained by Zurek \( 6 \). However, since this is valid only for \( \tau << 1 \), it cannot be interpreted as a complete exponential decay of the off-diagonal elements in position space. In this time regime, thus, both the position and the momentum space density matrices remain highly nonlocal and one cannot talk in terms of any emergent preferred basis.

From the above analysis it is clear that in the \( \tau \to \infty \) regime, which is a significant regime for realistic systems, momentum emerges as the basis selected by the environment. The position space density matrix remains nonlocal, the extent of nonlocality being \( \lambda_d \).

To summarize, we have clarified the roles of position and momentum for a free particle which is dissipatively coupled to a heat bath. The nature of the emergent preferred basis is expected to depend on the form of the system-environment coupling. We use a Hamiltonian with a coordinate-coordinate coupling between the system and the environment, but it is unitarily equivalent to other types of coupling (coordinate-momentum etc.) \( 8 \). The preferred basis need not necessarily be that of the system variable which couples to the environmental degrees of freedom. For the coordinate-coordinate coupling, we have shown that the momentum basis clearly is the emergent preferred basis. This is contrary to the general expectation that position should emerge as the preferred basis.

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