Uplink CoMP under a Constrained Backhaul and Imperfect Channel Knowledge

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Abstract

Coordinated Multi-Point (CoMP) is known to be a key technology for next generation mobile communications systems, as it allows to overcome the burden of inter-cell interference. Especially in the uplink, it is likely that interference exploitation schemes will be used in the near future, as they can be used with legacy terminals and require no or little changes in standardization. Major drawbacks, however, are the extent of additional backhaul infrastructure needed, and the sensitivity to imperfect channel knowledge. This paper jointly addresses both issues in a new framework incorporating a multitude of proposed theoretical uplink CoMP concepts, which are then put into perspective with practical CoMP algorithms. This comprehensive analysis provides new insight into the potential usage of uplink CoMP in next generation wireless communications systems.

Index Terms

CoMP, network MIMO, constrained backhaul, imperfect CSI, joint detection, interference cancellation, multiple access channel, interference channel

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I. INTRODUCTION

A. Motivation

Mobile network operators are experiencing an exponentially growing demand for mobile data rates at a stagnating average revenue per user (ARPU), driving the need for larger spectral efficiency. It is known, however, that especially urban cellular systems are mainly limited through inter-cell interference [2]. To overcome this limitation, coordinated multi-point (CoMP) was proposed in [3], [4], and has been selected as a key technology for long term evolution (LTE)-Advanced [5]. In the uplink, for instance, multi-cell joint signal processing enables the exploitation of interference [6], [7], rather than treating it as noise, promising vast gains in spectral efficiency and fairness [8], [9]. Beside key challenges, such as synchronization in time and frequency, a major concern of uplink CoMP is its demand for additional backhaul [10], and its sensitivity to imperfect channel state information (CSI) [1].

This paper performs an analysis of various uplink CoMP concepts under a constrained out-of-band backhaul and imperfect CSI. The joint observation of these two major issues from a theoretical and practical perspective sheds a new light on the value of particular CoMP schemes in next generation wireless communication systems.

B. Related Work

Considering the aspect of a constrained backhaul, an uplink CoMP scenario is related to the CEO-problem [11], where a number of agents make noisy, but correlated observations on the same random source, and use capacity-constrained links to a central estimation officer (CEO), who aims at reconstructing the source with minimum distortion. For a Gaussian source and noise, and a quadratic distortion measure, the rate-distortion trade-off was found in [12], respectively.

In [13], transmission from a two-antenna user equipment (UE) to two base stations (BSs) linked to a central processing unit was considered as a particular CEO problem setup. The work was based on distributed Wyner-Ziv compression [14], though its optimality could not yet be proved. The work was extended in [15], [16] to the case of multiple UEs, pointing out that compression can trade-off one UE’s rate versus the other, and to an arbitrary number of BSs with symmetric inter-cell interference in a circular Wyner model in [17]–[20].

While previous citations considered the exchange of quantized receive signals and centralized decoding, it can be beneficial under strongly constrained backhaul to use decentralized decoding.
where the BSs exchange decoded data bits [21]–[23], or quantizations of transmit sequences [23], [24] for (partial) interference subtraction. The benefit of adapting between different cooperation strategies depending on the channel realization has been pointed out in [21], [25].

Concrete CoMP algorithms have been proposed using centralized [26], [27] or decentralized decoding [28], [29], where the latter schemes involve an iterative exchange of likelihood information on transmitted bits. Considering the overall rate/backhaul trade-off, however, iterative schemes are only marginally superior to single-shot cooperation [27], [30]. In general, each BS may only exchange information connected to its own UEs [31], or also that on interfering UEs [32], and the rate/backhaul trade-off strongly depends on the quantization approach [33].

A different perspective on backhaul-constrained uplink CoMP is to see the setting as an interference channel (IC) with partial receiver-side cooperation [34], distinguishing between gains in degrees of freedom [35] and in power. However, the cited work also considers scenarios of strong interference (acc. to [35]) that are unlikely to occur in the context of cellular systems, as the assignment of UEs to BSs would simply be swapped on a reasonable time basis, and excludes the important option of centralized multi-user decoding.

Considering the aspect of imperfect CSI, first information theoretical steps concerning the impact on single-input single-output (SISO) links were taken in [36], and extended to point-to-point multiple-input multiple-output (MIMO) links in [37]. The impact on uplink CoMP was studied from a signal processing perspective in [38], [39] and in information theory in [1].

C. Main Contribution of this Work

This work yields new conclusions on uplink CoMP in practical systems, providing

- a framework incorporating a multitude of information theoretic concepts provided by various authors, putting these in perspective with a variety of proposed signal processing schemes.
- numerical results considering both information theoretic bounds as well as practical constraints, and hence yielding an insight into the value of sophisticated signal processing.
- reasonably complex models reflecting interference scenarios likely to occur in practical cellular systems. While these models do not enable closed-form analysis, they yield more relevant conclusions than overly simplified models as, e.g., in [17]–[19].
D. Terminology

In this work, the terms CoMP and BS cooperation refer to schemes where BSs exchange received signals or information connected to the data bits of certain UEs in order to improve data rates. Schemes that only make use of coordination between BSs, for example joint scheduling or interference rejection combining (IRC), are considered non-cooperative. The term backhaul infrastructure refers to the overall connectivity of BSs and the network, while any backhaul quantity always refers to the backhaul capacity required by a cooperative scheme in addition to that of a non-cooperative system.

E. Outline

In Section II, the transmission model and basic BS cooperation schemes are introduced, inner bounds on capacity regions under imperfect CSI for infinite, no, or partial BS cooperation are derived, and performance regions are introduced. In Section III, the overall CoMP gain is quantified for different scenarios, and the introduced BS cooperation schemes are evaluated w.r.t. the achievable rate/backhaul trade-off. The value of BS cooperation in conjunction with source coding or superposition coding is discussed, before Monte Carlo simulations using a slightly larger setup emphasize the gain of adaptation between different BS cooperation strategies. In Section IV, parallels are drawn between the analyzed theoretical concepts and proposed practical algorithms, and the value of iterative BS cooperation and other practical aspects are discussed. The work is concluded in Section V.

II. System Model and Basics

A. Transmission Model

We consider an uplink transmission from $K$ UEs to $M$ BSs, as shown in Fig. I and denote the sets of UEs and BSs as $\mathcal{K} = \{1..K\}$ and $\mathcal{M} = \{1..M\}$, respectively. We assume that each UE has $N_{ue} = 1$ transmit antenna, as this is the configuration in the recently completed standard LTE Release 8 [40]. The BSs can be equipped with any number $N_{bs}$ of receive antennas each. We assume that transmission takes place over a frequency-flat channel, where all entities are perfectly synchronized in time and frequency. Each UE $k \in \mathcal{K}$ has a set $\mathcal{F}_k$ of discrete messages which it maps onto a set $\mathcal{X}_k$ of Gaussian unit power transmit sequences of length $N_{sym}$ symbols,
using an encoding function $e(\cdot)$. We denote all messages of all UEs as $\mathcal{F}_{\text{all}} = \mathcal{F}_1 \cup \mathcal{F}_2 \cup \cdots \cup \mathcal{F}_K$, and all transmit sequences as $\mathcal{X}_{\text{all}} = \mathcal{X}_1 \cup \mathcal{X}_2 \cup \cdots \cup \mathcal{X}_K$. The overall transmission from all UEs to the BSs in one single channel access $1 \leq t \leq N_{\text{sym}}$ is given as

$$\mathbf{y}[t] = \mathbf{Hs}[t] + \mathbf{n}[t],$$

(1)

where $\mathbf{y}[t] = [y_{1,t}, y_{1,t}, \ldots, y_{2,t}, \ldots, y_{M,1} \cdot y_{M,N_{m}}]^T \in \mathbb{C}^{N_{bs} \times 1}$ are the signals received at the BSs, and the channel matrix is given as $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K] \in \mathbb{C}^{N_{bs} \times K}$, where each column $\mathbf{h}_k$ is connected to UE $k$. The channel is assumed to be block-fading, where each element is taken from an independent, zero-mean Gaussian distribution $h_{i,j} \sim \mathcal{N}(0, E \{ |h_{i,j}|^2 \})$. $s[t] \in \mathbb{C}^{K \times 1}$ are the symbols transmitted from the UEs, which are given as

$$\forall k \in \mathcal{K} : s_k[t] = \sum_{F \in \mathcal{F}_k} \sqrt{\rho_F} [e(F)]_t,$$

(2)

where $\rho_F \in \mathbb{R}_0^+$ is the transmit power assigned to message $F$. We use $\mathcal{P} = \{ \rho_F : F \in \mathcal{F}_{\text{all}} \}$ to capture the overall power allocation. According to (2), each UE $k$ transmits a weighted superposition of sequences in set $\mathcal{X}_k$. $\mathbf{n}[t] = [n_{1,1}, n_{1,1}, n_{2,1}, \ldots, n_{M,1} \cdot n_{M,N_{m}}]^T \in \mathbb{C}^{N_{bs} \times 1}$ is additive Gaussian noise at the receiver side with covariance $E_t\{\mathbf{n}[t](\mathbf{n}[t])^H\} = \sigma^2 \mathbf{I}$. We state the covariance of the transmitted signals as $E_t\{\mathbf{s}[t](\mathbf{s}[t])^H\} = \mathbf{P} = \text{diag}(\mathbf{p})$ with $\mathbf{p} \in \mathbb{R}_0^+ [K \times 1]$, where each element $p_k$ corresponds to the overall transmit power (over all transmitted sequences) of UE $k$, and assume that the transmit powers are subject to the power constraint $\mathbf{P}^\text{max} - \mathbf{P} \succeq 0$.

Hence, each UE has an individual power constraint defined by the entries of the diagonal matrix $\mathbf{P}^\text{max} = \text{diag}(\mathbf{p}^\text{max})$ with $\mathbf{p}^\text{max} \in \mathbb{R}_0^+[K \times 1]$. The transmit covariance connected to only a subset of messages $\mathcal{F} \subseteq \mathcal{F}_{\text{all}}$ is denoted as $\mathbf{P}(\mathcal{F})$, where the diagonal elements are given as

$$[\mathbf{P}(\mathcal{F})]_{k,k} = \sum_{\forall F \in (\mathcal{F}_k \cap \mathcal{F})} \rho_F.$$

(3)

We also use $\forall k \in \mathcal{K} : \mathcal{S}_k = \{ s_k^{[1]} \cdots s_k^{[N_{\text{sym}}]} \}$ as the superposition of all sequences transmitted by UE $k$, $\forall m \in \mathcal{M} : Y_m$ as the sequence of all symbols received at all antennas of BS $m$, and $\forall m \in \mathcal{M}, 1 \leq a \leq N_{bs} : N_{m,a}$ as the noise sequence received by BS $m$ at antenna $a$. As indicated in Fig. [1] the BSs are assumed to be connected through a mesh of error-free out-of-band backhaul links, where we denote as $\beta \in \mathbb{R}_0^+$ the sum backhaul capacity required in addition to that of a non-cooperative system. Note that in our setup, it is sufficient if a UE can be decoded by any involved BS, which then forwards the decoded bits to the network, circumventing cases of strong interference [35]. The symbol index $t$ is omitted in the sequel for brevity.
B. Modeling of Imperfect Channel Knowledge

To incorporate the impact of imperfect (receiver-side) CSI into our model, we assume that all BSs have the same knowledge of the compound channel estimate

\[ \hat{H} = H + E, \]  

(4)

with \( \hat{H} \in \mathbb{C}^{[N_{\text{BS}} \times K]} \), where the error term \( E \in \mathbb{C}^{[N_{\text{BS}} \times K]} \) is a random variable of covariance

\[ E \left\{ \text{vec}(E)\text{vec}(E)^H \right\} = \frac{\sigma_{\text{pilots}}^2}{N_p \cdot p_{\text{pilots}}} \cdot I = \sigma^2_E \cdot I. \]  

(5)

The estimated channel \( \hat{H} \) and error \( E \) are assumed to have multiple independent realizations per block of \( N_{\text{sym}} \) symbols (due to multiple pilots per block). Equation (5) is based on the Kramer-Rao lower bound \[41\], yielding the absolute estimation error variance, if optimal channel estimation has been performed based on \( N_p \) pilots of power \( p_{\text{pilots}} \), subject to Gaussian noise with variance \( \sigma^2_{\text{pilots}} \). Note that \( \sigma^2_{\text{pilots}} \) can differ from \( \sigma^2 \) if multi-cell (quasi-)orthogonal pilot sequences are employed. In the sequel, we assume unit-power pilots \((p_{\text{pilots}} = 1)\), and choose \( N_p = 2 \), which has been motivated through the observation of a concrete channel estimation scheme in a frequency-selective OFDMA system for a channel of average coherence time and bandwidth in \[42\]. Let us now state the following theorem:

**Theorem 1 (Modified transmission equation under imperfect CSI):** An inner bound for the capacity region (considering average rates over many estimation errors) of the transmission in (1) under imperfect CSI can be found by observing the capacity region connected to the transmission

\[ y = H^e s + v + n, \]  

(6)

which involves a power-reduced effective channel \( H^e \in \mathbb{C}^{[N_{\text{BS}} \times K]} \) with elements

\[ \forall i, j : \ h^e_{i,j} = \frac{h_{i,j}}{\sqrt{1 + \sigma^2_E / E \left\{ |h_{i,j}|^2 \right\}}}, \]  

(7)

and is subject to an additional Gaussian noise term \( v \in \mathbb{C}^{[N_{\text{BS}} \times 1]} \) with diagonal covariance

\[ E \left\{ vv^H \right\} = \Phi^v = \Delta \left( \bar{E}^e P (F_{\text{all}}) \left( \bar{E}^e \right)^H \right), \]  

where \( \forall i, j : \bar{e}^e_{i,j} = \sqrt{\frac{E \left\{ |h_{i,j}|^2 \right\} \cdot \sigma^2_E}{E \left\{ |h_{i,j}|^2 \right\} + \sigma^2_E}} \),

(8)

and \( \Delta(\cdot) \) sets all off-diagonal values of the operand to zero.

**Proof:** Briefly, the theorem is based on the fact that (6) overestimates the detrimental impact of imperfect CSI by assuming \( v \) to be a Gaussian random variable with a different realization in each channel use. The proof is stated in the Appendix.
Note that our model of the channel estimate in (4) deviates from that in, e.g., [37], where the authors start with the assumption of an unbiased minimum mean square error (MMSE) channel estimate which is uncorrelated from its estimation error. Both models, however, lead to Theorem 1 while the model considered in this work has the advantage that $\sigma_E^2$ is given as an absolute channel estimation noise term, where the different impact on weak or strong links becomes evident in (8). In general, the model implies that $\sigma_E^2$, as well as the average gain of all links $E\{|h_{i,j}|^2\}$, are known to the receiver side.

C. Capacity Region Under Infinite BS Cooperation

If an infinite backhaul infrastructure enables full cooperation between all BSs, we are observing a multiple access channel (MAC). In this context, there is no benefit of superimposed messages [43], hence we can constrain the used messages to

$$\forall k \in K : \mathcal{F}_k := \{F_k\}, \quad \mathcal{F}_{\text{all}} := \{F_1, F_2, \ldots, F_K\} \quad \text{and} \quad \mathcal{P} := \{\rho_{F_1}, \rho_{F_2}, \ldots, \rho_{F_K}\} \quad (9)$$

and state the following theorem:

Theorem 2 (Capacity region under infinite BS cooperation): An inner bound for the capacity region of the uplink transmission in (1) under infinite BS cooperation is given as

$$\mathcal{R}_\infty = \bigcup_{\mathcal{P} : P_{\text{max}} - P(\mathcal{F}_{\text{all}}) \geq 0} \mathcal{R}_\infty(\mathcal{P}) \quad (10)$$

where $\bigcup$ denotes a convex hull operation, and all rate tuples $r \in \mathcal{R}_\infty(\mathcal{P})$ fulfill $\forall k \in K : 0 \leq r_k \leq \nu_{F_k}$ and $\forall \mathcal{F} \subseteq \mathcal{F}_{\text{all}}$:}

$$\sum_{F \in \mathcal{F}} \nu_F \leq \log_2 \left| \mathbf{I} + (\sigma^2 \mathbf{I} + \mathbf{\Phi}_v\mathbf{v})^{-1} \mathbf{H}^\mathsf{c} \mathbf{P} (\mathcal{F}) (\mathbf{H}^\mathsf{c})^H \right|,$$

(11)

where $\nu_F$ is the rate connected to message $F$.

Proof: The proof is a straightforward application of [44] to (6) and given in [45].

Equation (11) states that the sum rate of any subset of UEs is limited by the sum capacity of the channel, assuming that all other UEs have already been decoded and their signals subtracted from the system. Note that under imperfect CSI, a certain extent of noise covariance $\mathbf{\Phi}_v\mathbf{v}$ remains, having a detrimental impact on any cooperation strategy that will be explored later. If the sum rate is to be maximized and all links have equal average power, (11) simplifies to the expression derived for point-to-point MIMO transmission under imperfect CSI in [37].
D. Capacity Region without BS Cooperation

Without BS cooperation, our scenario is similar to a Gaussian IC, where the capacity is known only for certain interference cases. The tightest known inner bound [46] is based on superposition coding (SPC), where common messages are decoded individually by multiple receivers. Our setup differs in the way that we can swap the assignment of UEs to BSs, or let BSs decode multiple UEs, such that scenarios of strong interference [35] are avoided. This reduces the range of scenarios for which common message concepts are known to be beneficial, and the increased background noise level due to imperfect CSI renders these even less attractive. As SPC in general suffers from signal-to-noise ratio (SNR) gaps inherent in practical coding schemes, and requires UE modifications in conjunction with more complex signaling, we again constrain ourselves to one message per UE as in (9). Term $a \in \{1..M\}$ captures BS-UE assignment, i.e. denotes the BS where each UE is decoded at, and we introduce
\[
\forall m \in M : \mathcal{F}^{[m]}(a) = \{ F_k \in \mathcal{F}_{all} : a_k = m \} \quad \text{and} \quad \mathcal{F}^{[m]}(a) = \{ F_k \in \mathcal{F}_{all} : a_k \neq m \} \quad (12)
\]
as the sets of messages decoded or not decoded by BS $m$, respectively. We now state:

**Theorem 3 (Inner bound on the capacity region without BS cooperation):** An inner bound of the capacity region of the transmission in (1) without BS cooperation is given as
\[
R_0 = \bigcup_{a, \mathcal{P}} \mathcal{R}_0 (a, \mathcal{P}), \quad (13)
\]
where all rate tuples $r \in \mathcal{R}_0 (a, \mathcal{P})$ fulfill $\forall k \in \mathcal{K} : 0 \leq r_k \leq \nu_{F_k}$ and $\forall m \in M :$
\[
\forall \mathcal{F}' \subseteq \mathcal{F}^{[m]}(a) : \sum_{F \in \mathcal{F}'} \nu_F \leq \log_2 \left| I + \left( \Phi^{ii}_m \right)^{-1} H^e_m P (\mathcal{F}) (H^e_m)^H \right|
\]
with $\Phi^{ii}_m = \sigma^2 I + \Phi^{vw}$ Impact of imp. CSI + $H^e_m P (\mathcal{F}^{[m]}(a)) (H^e_m)^H$, Interference
\[
\text{Impact of imp. CSI} + \text{Interference}, \quad (14)
\]
where $H^e_m$ and $\tilde{E}_m^e$ denote effective channel and part of $E^e$ from (8), resp., connected to BS $m$.

**Proof:** The theorem is a straightforward extension of the work in [46] to the transmission in (6) with an arbitrary number of communication paths but a single message per UE. 

Note that the non-cooperative capacity region from Theorem 3 implicitly makes use of IRC, as (14) exploits the spatial structure of interference. We will later also observe the performance of frequency division multiplex (FDM), where the UEs focus their transmit power on orthogonal resources, and hence interference is avoided. As such schemes play a minor role in the context of CoMP, however, corresponding capacity expressions are omitted.
E. Base Station Cooperation Schemes For Finite Backhaul

We investigate four BS cooperation schemes, constrained to scenarios with \( M = K = 2 \) for clarity. The schemes are initially considered with only one phase of information exchange between BSs, but the benefit of iterative BS cooperation will be discussed in Section [IV-A].

1) Distributed Interference Subtraction (DIS) [27]: This concept is often paralleled to decode-and-forward in relaying: One BS decodes (part of) one UE’s transmission and forwards the decoded data to the other BS, in the CoMP case for (partial) interference cancellation. For a particular BS-UE assignment \( a = [1, 2]^T \) and cooperation direction \( b = 1 \) as shown in Fig. 3(a), let us assume UE 1 transmits messages \( F_1^1 \) and \( F_1^{1\rightarrow 2} \), mapped onto sequences \( X_1^1 \) and \( X_1^{1\rightarrow 2} \). Both messages are decoded by BS 1, after which message \( F_1^{1\rightarrow 2} \) is forwarded to BS 2. As the signals received by both BSs are correlated, we consider compressing the decoded bits via Slepian-Wolf source coding [47] at BS 1, before forwarding \( w(F_1^{1\rightarrow 2}) \). At BS 2, message \( F_1^{1\rightarrow 2} \) is reconstructed with \( Y_2 \) as side-information (if source coding was applied), and message \( F_2^2 \) is decoded based on the interference-reduced receive signals \( \hat{Y}_2 = Y_2 - \hat{H}_2^e \) \( \rho F_1^{1\rightarrow 2} e(F_1^{1\rightarrow 2}) \).

Theorem 4 (Inner bound on DIS capacity region): An inner bound on the capacity region of a DIS setup (with any assignment \( a \) and cooperation direction \( b \)) under backhaul \( \beta \) is given as

\[
\mathcal{R}^\text{dis} (\beta) = \bigcup_{a, b, P} \{ P_{\text{max}} = P(R_{\text{all}}) \geq 0 \} \mathcal{R}^\text{dis} (\beta, a, b, P),
\]

where all \( r \in \mathcal{R}^\text{dis}(\beta, a = [1, 2]^T, b = 1, P) \) fulfill \( \forall k \in \{1, 2\} : r_k \geq 0, r_1 \leq \nu_{F_1^1} + \nu_{F_1^{1\rightarrow 2}} \) and

\[
\nu_{F_1^1} \leq \log_2 \left| I + \left( \sigma^2 I + \Phi_1^{\text{yw}} + H_1^e P (F_2^2) (H_1^e)^H \right)^{-1} H_1^e P (F_1^1) (H_1^e)^H \right| \leq 0 \text{ without Slepian-Wolf source coding}
\]

\[
\nu_{F_1^{1\rightarrow 2}} \leq \log_2 \left| I + \left( \sigma^2 I + \Phi_2^{\text{yw}} + H_2^e P \left( \{ F_1^1, F_2^2 \} \right) (H_1^e)^H \right)^{-1} H_1^e P (F_1^{1\rightarrow 2}) (H_1^e)^H \right| \leq \beta + \log_2 \left| I + \left( \sigma^2 I + \Phi_2^{\text{yw}} + H_2^e P \left( \{ F_1^1, F_2^2 \} \right) (H_2^e)^H \right)^{-1} H_2^e P (F_2^2) (H_2^e)^H \right|,
\]

where \( \Phi_1^{\text{yw}} \) and \( \Phi_2^{\text{yw}} \) are the submatrices of \( \Phi^{\text{yw}} \) in (8) connected to BSs 1 and 2, respectively.

Proof: The rate of of message \( F_1^{1\rightarrow 2} \) is constrained on one hand in (17) as it has to be decoded by BS 1 (interfered by messages \( F_1^1 \) and \( F_2^2 \)), and on the other hand in (18) by the rate of the backhaul plus the rate at which it could be decoded by BS 2 without cooperation. Message \( F_2^2 \) can then be decoded free of interference from message \( F_1^{1\rightarrow 2} \) (see Eq. (19)).
Note that the decoding order at the forwarding BS is important, i.e. the forwarded message has to be decoded first such that its rate is low w.r.t. the level of interference it represents.

2) Compressed Interference Forwarding (CIF) [23], [24]: This scheme is similar to DIS in the way that both BSs decode their UE individually, while one BS offers the other a certain extent of interference subtraction. Here, however, the BSs exchange quantized transmit sequences. Using CIF, the rate/backhaul operation point can be adjusted through choosing an appropriate degree of quantization, rather than using SPC. Let us again fix the assignment \( a = [1, 2]^T \) and cooperation direction \( b = 1 \) as in Fig. 3(b) and observe the case where BS 1 decodes message \( F_1^1 \), calculates the originally transmitted sequence \( X_1^1 = e(F_1^1) \) and forwards a quantized version \( q(X_1^1) \) to BS 2. We optionally consider that a source-encoded version \( w(q(X_1^1)) \) is forwarded, exploiting side-information at BS 2. The latter BS then reconstructs \( q(X_1^1) \) and computes an interference-reduced version of its received signals \( \hat{Y}_2 = Y_2 - \hat{H}^e_2\sqrt{\rho_{F_1^1} q(X_1^1)} \), from which message \( F_2^2 \) can be decoded.

**Theorem 5 (Inner bound on CIF capacity region):** An inner bound on the capacity region of a CIF setup (with any assignment \( a \) and cooperation direction \( b \)) under backhaul \( \beta \) is given as

\[
\mathcal{R}^{\text{cif}}(\beta) = \bigcup_{a, b, \mathcal{P}} \mathcal{R}^{\text{cif}}(\beta, a, b, \mathcal{P}),
\]

where all rates \( r \in \mathcal{R}^{\text{cif}}(\beta, a = [1, 2]^T, b = 1, \mathcal{P}) \) fulfill \( \forall k \in \{1, 2\} : r_k \geq 0 \) and

\[
r_1 \leq \log_2 \left| 1 + \left( \sigma^2 I + \Phi_1^{\text{vy}} + H_1^e P \left( F_2^2 \right) (H_1^e)^H \right)^{-1} H_1^e P \left( F_1^1 \right) (H_1^e)^H \right| (21)
\]

\[
r_2 \leq \log_2 \left| 1 + \left( \sigma^2 I + \Phi_2^{\text{vy}} + h_{2,1}^e h_{1,1}^e \right)^{-1} H_2^e P \left( F_2^2 \right) (H_2^e)^H \right| (22)
\]

\[
\xi_{F_1^1} \geq \frac{\rho_{F_1^1}}{\max(2\beta-2, 1)} \quad \text{or} \quad \xi_{F_1^1} \geq \frac{\rho_{F_1^1}}{2\beta} \quad \text{or} \quad \xi_{F_1^1} \geq \frac{\rho_{F_1^1} \cdot \kappa}{2\beta - 1 + \kappa} (23)
\]

with \( \kappa = E \left\{ x_1^1 (x_1^1)^H \right\} Y_2 = 1 + \rho_{F_1^1} \left( h_{2,1}^e \right)^H \left( \sigma^2 I + \Phi_2^{\text{vy}} + h_{2,1}^e P \left( F_2^2 \right) (h_2^e)^H \right)^{-1} h_{2,1}^e \right\}^{-1}(24)

and where \( h_{m,k}^e \in \mathbb{C}^{[N_m \times 1]} \) is the effective channel between BS \( m \) and UE \( k \).

**Proof:** Equation (21) bounds the achievable rates of message \( F_1^1 \) interfered by message \( F_2^2 \), while (22) bounds the rate of message \( F_2^2 \) that is subject to a residual extent of interference from message \( F_1^1 \), depending on the quantization noise power \( \xi_{F_1^1} \in \mathbb{R}_0^+ \). Equation (23) gives exactly this quantity as a function of backhaul \( \beta \), where we distinguish between the cases of a practical quantizer as given in [41] or operation on the rate-distortion bound [43], without or with source.
coding [49]. For the latter case, (24) denotes the variance of the symbols in \( X_1^1 \) conditioned on the signals \( Y_2 \) received by BS 2.

3) Distributed Antenna System - Decentralized Decoding (DAS-D) [50]: In a third cooperation scheme based on decentralized decoding, the BSs exchange quantized receive signals rather than decoded bits or transmit sequences. For a particular assignment \( a = [1, 2]^T \) as shown in Fig. 3(c), we assume the UEs transmit messages \( F_1^1 \) and \( F_2^2 \), respectively, mapped onto sequences \( X_1^1 \) and \( X_2^2 \). Both BSs now create quantized versions \( q(Y_1), q(Y_2) \) of their received signals, and forward these over the backhaul. Optionally, source coding can be applied, such that \( w(q(Y_1)), w(q(Y_2)) \) are exchanged. Both BSs then use this information and their received signals to reconstruct \( q(Y_1), q(Y_2) \), and then decode messages \( F_1^1, F_2^2 \), respectively.

Theorem 6 (Inner bound on DAS-D capacity region): An inner bound on the capacity region of DAS-D (for any BS-UE assignment \( a \)) under sum backhaul \( \beta \) is given as

\[
R_{\text{dasd}}(\beta) = \bigcup_{a, P : P_{\max} - P(\mathcal{F}_{\text{all}}) \geq 0} R_{\text{dasd}}(\beta, a, P),
\]

where all rates \( r \in R_{\text{dasd}}(\beta, a = [1, 2]^T, P) \) fulfill \( \forall k \in \{1, 2\} : r_k \geq 0 \) and

\[
r_1 \leq \log_2 \left| \mathbf{I} + \left( \mathbf{H}^c \mathbf{P} (F_2^2) (\mathbf{H}^c)^H + \begin{bmatrix} 0 & 0 \\ 0 & \Phi_2^{q1} \end{bmatrix} + \Phi^{vv} + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{H}^c \mathbf{P} (F_1^1) (\mathbf{H}^c)^H \right|,
\]

\[
r_2 \leq \log_2 \left| \mathbf{I} + \left( \mathbf{H}^c \mathbf{P} (F_1^1) (\mathbf{H}^c)^H + \begin{bmatrix} \Phi_1^{q1} & 0 \\ 0 & 0 \end{bmatrix} + \Phi^{vv} + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{H}^c \mathbf{P} (F_2^2) (\mathbf{H}^c)^H \right|,
\]

with

\[
\sum_{m=1}^{2} \log_2 \left| \mathbf{I} + (\Phi_m^{q1})^{-1} \Delta (\Psi_m^{yy}) \right| + 2MN_{\text{bs}} \leq \beta \text{ or } \sum_{m=1}^{2} \log_2 \left| \mathbf{I} + (\Phi_m^{q1})^{-1} \Psi_m \right| \leq \beta
\]

(28)

Performance of a practical quantizer

Rate-dist. th. (opt. source coding)

where \( \Psi_m \) is either the receive signal covariance at BS \( m \), i.e. \( \Psi_m := \Phi^{yy}_m = \mathbf{H}^c_m \mathbf{P}(\mathcal{F}_{\text{all}}) (\mathbf{H}^c_m)^H + \Phi^{vv}_m + \sigma^2 \mathbf{I} \), or the receive signal covariance conditioned on the signals received by the other BS (if we consider source coding), i.e. \( \Psi_m := \Phi^{yy}_{m|m'} \) with \( \forall m \in \{1, 2\}, m' \neq m : [15], [16] \)

\[
\Phi^{yy}_{m|m'} = \mathbf{H}^c_m \left( \mathbf{I} + \mathbf{P}(\mathcal{F}_{\text{all}}) (\mathbf{H}^c_{m'})^H (\Phi^{vv}_{m'} + \sigma^2 \mathbf{I})^{-1} \mathbf{H}^c_{m'} \right)^{-1} \mathbf{P}(\mathcal{F}_{\text{all}}) (\mathbf{H}^c_m)^H + \Phi^{vv}_m + \sigma^2 \mathbf{I}
\]

(29)

Proof: The message rates are constrained in (26) and (27) due to interference and quantization noise on the antennas of the corresponding remote BS. The quantization noise covariances are limited through (28), where we again consider a practical scheme quantizing each dimension separately (losing one bit to the rate-distortion bound in each real dimension) [48], operation on the rate-distortion bound [43], or the latter including source coding [15], [16].
Investing different portions of backhaul into the two cooperation directions allows trading the rate of one UE against the other. The calculation of (weighted sum-rate) optimal quantization noise covariances for (28) has been studied in detail in [15], [16].

4) Distributed Antenna System - Centralized Decoding (DAS-C): We finally consider the case that both UEs are decoded jointly by one of the BSs, and the other BS is degraded to a remote radio head (RRH) that quantizes and forwards received signals, possibly being oblivious to transmitted codewords [20]. To incorporate various proposed concepts, let us assume for a fixed local, non-cooperative decoding [17], [20], [51]. BS 1 decodes messages $F_1$ and $F_2$, respectively, which are individually decoded by both BSs, and messages $F_1^1$ and $F_2^2$, respectively, which are jointly decoded by BS 2. This model (see Fig. 3(d)), hence reflects the concept of common messages, known to be beneficial in the context of an IC [46] and for centralized detection [25], and also the concept of local, non-cooperative decoding [17], [20], [51]. BS 1 decodes messages $F_1^1$, $F_1^{1,2}$ and $F_2^{1,2}$, and subtracts the corresponding transmit sequences from the received signals to construct

$$\hat{Y}_1 = Y_1 - \hat{h}_{1,1}^* \left( \sqrt{P_{F_1^{1,2}}} X_1^{1,2} + \sqrt{P_{F_1^1}} X_1^1 \right) - \hat{h}_{1,2}^* \cdot \sqrt{\rho_{F_2^{1,2}}} \cdot X_2^{1,2}. \quad (30)$$

This is then quantized to $q(\hat{Y}_1)$ (and optionally source-encoded to $w(q(\hat{Y}_1))$ and forwarded to BS 2. The latter BS also decodes messages $F_1^{1,2}$ and $F_2^{1,2}$, subtracts their impact on $Y_2$, and uses $Y_2$ plus the information provided by BS 1 to finally decode messages $F_1^2$ and $F_2^2$.

Theorem 7 (Inner bound on DAS-C capacity region): An inner bound on the capacity region of DAS-C (for any assignment $a$ and cooperation direction $b$) under backhaul $\beta$ is given as

$$R_{\text{dasc}}(\beta) = \bigcup_{P : P_{\text{max}} - P(F_{\text{all}}) \geq 0} R_{\text{dasc}}(\beta, a, b, P), \quad (31)$$

where all rate tuples $r \in R_{\text{dasc}}(\beta, a = [1, 2]^T, b = 1, P)$ fulfill $\forall k \in \{1, 2\} : r_k \geq 0$ and

$$r_1 = \nu_{F_1^1} + \nu_{F_1^{1,2}} + \nu_{F_1^2} \quad \text{and} \quad r_2 = \nu_{F_2^{1,2}} + \nu_{F_2^2} \quad (32)$$

$$\forall F \subseteq \{F_1^1, F_1^{1,2}, F_2^{1,2}\} : \sum_{F \in F} \nu_F \leq \log_2 |I + \left( H_{1}^2 \mathbf{P} \{ \{F_1^1, F_2^{1,2}\} \} (H_{1}^2)^H + \Phi_{1}^{\text{in}} \right)^{-1} \Psi_1| \quad (33)$$

$$\forall F \subseteq \{F_1^{1,2}, F_2^{1,2}\} : \sum_{F \in F} \nu_F \leq \log_2 |I + \left( H_{2}^2 \mathbf{P} \{ \{F_2^{1,2}\} \} (H_{2}^2)^H + \Phi_{2}^{\text{in}} \right)^{-1} \Psi_2| \quad (34)$$

$$\forall F \subseteq \{F_1^2, F_2^2\} : \sum_{F \in F} \nu_F \leq \log_2 |I + \left( \begin{bmatrix} \Phi_{qq} & 0 \\ 0 & \rho_{F_1^2} h_{2,1}^* (h_{2,1}^*)^H \end{bmatrix} + \Phi_{\text{in}} \right)^{-1} \Psi|, \quad (35)$$
where \( \Phi_{nn} = \Phi_{vv} + \sigma^2 I \) and \( \Psi = H^c\mathbf{P}(P)(H^c)^H \), and we have the backhaul constraint

\[
\log_2 \left| I + (\Phi_{qq})^{-1} \Delta (\Phi_{yy}) \right| + 2N_{bs} \leq \beta, \quad \log_2 \left| I + (\Phi_{qq})^{-1} \Phi_{yy} \right| \leq \beta \quad \text{or} \quad \log_2 \left| I + (\Phi_{qq})^{-1} \Phi_{yy} \right|_{1/2} \leq \beta,
\]

where \( \Phi_{yy} \) is the signal covariance at BS 1 after the subtraction of decoded messages, and \( \Phi_{yy} \) is the same quantity, but conditioned on the signals at BS 2 after message subtraction.

**Proof:** Eq. (32) states that the overall UE rates are the sum of the rates of the superimposed messages. Eqs. (33) and (34) state the sum rates of any tuples of messages decoded without BS cooperation by BSs 1 and 2, respectively, and (35) states the sum rate bound on the two messages \( F_2^1 \) and \( F_2^2 \) that are jointly decoded by BS 2. In the latter equation, we have to consider not only quantization noise (making the same differentiations w.r.t. quantization as before), but also the fact that the signals received at BS 2 are still subject to interference from message \( F_1^1 \), as this message is not known to BS 2. The backhaul constraint in (36) is based on [15], [16].

In this work, we also consider FDM scenarios where the UEs are served on orthogonal resources, but enhanced through the exchange of received signals over the backhaul. We will see that these schemes play a minor role, and hence omit equations for brevity.

### F. Performance Regions

The concept of **performance regions** was introduced in [21] to jointly capture achievable rate tuples and the corresponding backhaul requirement. A **performance point** is defined as

\[
Z = \langle r, \beta \rangle, \quad (37)
\]

and a **performance region** connected to an arbitrary BS cooperation scheme \( yz \) is defined as

\[
Z_{yz} = \bigcup \{ \langle r, \beta \rangle : r \in R_{yz}(\beta) \}. \quad (38)
\]

Note that the convex hull operation \( \bigcup \) in (38) implies the option of time-sharing along the backhaul dimension, while each region \( R_{yz} \) already incorporates time-sharing between different BS-UE assignments and cooperation directions. An example performance region is shown in Fig. 2 for DIS, CIF, DAS-D, DAS-C or FDM (all assuming practical quantization and no SPC) for \( M = K = 2, N_{bs} = 1 \) and \( H = [1, \sqrt{0.25}; \sqrt{0.5}, 1] \). We observe imperfect CSI with \( N_p = 2 \), and set \( \sigma^2 = 0.1 \) (SISO SNR of 10 dB on the main links). We plot the achievable UE rates on the
x- and y-axis, and the required backhaul on the z-axis. The top surface of the performance region hence reflects the capacity region in the non-cooperative case, while its intersection with the x-y plane inner bounds the capacity region for infinite BS cooperation. Note that the latter deviates from a pentagon shape due to imperfect CSI. For the example channel, FDM schemes are beneficial in the regime of no or very limited backhaul, DIS concepts are interesting for moderate backhaul, whereas DAS-C is the only scheme approaching MAC performance for large backhaul. DAS-D and CIF are inferior for all extents of backhaul and hence not visible.

III. Analysis of Cooperation Concepts

A. Scenarios and Channels Considered

In the sequel, we are interested in the sum-rate achievable in a small scenario with \( M = K = 2 \) and \( N_{bs} = 2 \), for infinite or no BS cooperation, and for the CoMP schemes from Section II-E. We observe different scenarios characterized by the location of the UEs, where for each UE \( k \) a normalized distance \( d_k \in [0, 1] \) denotes whether it is close to its assigned BS (small \( d_k \)), at the cell-edge (\( d_k = 0.5 \)), or closer to the other BS (\( d_k > 0.5 \)). We use exemplary channel matrices

\[
H = \begin{bmatrix}
\sqrt{\lambda_{1,1}} & \sqrt{\lambda_{1,2}e^{j(-\varphi_1/2-\varphi_{12}/2)}} \\
\sqrt{\lambda_{1,1}} & \sqrt{\lambda_{1,2}e^{j(+\varphi_1/2-\varphi_{12}/2)}} \\
\sqrt{\lambda_{2,1}e^{j(-\varphi_2/2-\varphi_{12}/2)}} & \sqrt{\lambda_{2,2}} \\
\sqrt{\lambda_{2,1}e^{j(+\varphi_2/2-\varphi_{12}/2)}} & \sqrt{\lambda_{2,2}}
\end{bmatrix}
\]

with

\[
\forall k \in \{1, 2\}, m \neq k : \lambda_{k,k} = \frac{d_k^{-\theta}}{d_k^{-\theta} + (1 - d_k)^{-\theta}}, \quad \lambda_{m,k} = \frac{(1 - d_k)^{-\theta}}{d_k^{-\theta} + (1 - d_k)^{-\theta}},
\]

where \( \lambda_{m,k} \) is the linear path gain from UE \( k \) to BS \( m \), based on a flat-plane pathloss model with pathloss exponent \( \theta = 3.5 \), is normalized by the transmit power of the UEs. We here assume multi-cell power control, where the average power by which a UE is received by both BSs is normalized to 1. We can then use \( P^{\text{max}} = I \) regardless of UE location, and the dominant links are normalized to unit gain in the case of \( d_k = 0 \). Terms \( \varphi_1, \varphi_2 \) and \( \varphi_{12} \) in (39) are phases connected to the orthogonality of the channels seen by BS 1 or 2, respectively, or the additional orthogonality of the compound channel. Unless stated otherwise, we observe channels of average orthogonality, i.e. \( \varphi_1 = \varphi_2 = \varphi_{12} = \pi/2 \), and choose \( \sigma^2 = 0.1 \), leading with (40) to a cell-center SISO SNR of 10 dB, which is motivated through system level simulations in [45].
B. Overall CoMP Gain under Imperfect CSI

Let us first observe the gain from no to infinite BS cooperation according to the bounds from Sections II-C and II-D. Fig. 4 shows the achievable sum-rate of both UEs, while these are simultaneously moved from the cell-center ($d_k = 0.2$) to the cell-edge and slightly beyond ($d_k = 0.6$). We observe different extents of CSI, with $N_p \in \{1, 2, \infty\}$. As intuitive, the CoMP gain is largest at the cell-edge (for average channel orthogonality), and diminishes towards the cell-center. In conjunction with multi-cell power control, CoMP can provide fairly homogeneous performance throughout all scenarios, hence improve fairness. Interestingly, the relative gain of CoMP at the cell-edge increases with decreasing CSI, while the opposite is the case towards the cell-center, as shown in Fig. 5. The former is due to array gain from which channel estimation can benefit, while the latter is the case as the weak interference links become difficult to estimate, and hence cannot be exploited for CoMP. Fig. 5 also shows results for $M = K = 3$, where the CoMP gains are larger, as (for $N_{bs} = 2$) each BS by itself cannot spatially separate all 3 UEs.

C. Performance of CoMP Schemes for Specific Channels

Let us now analyze the rate/backhaul trade-off achievable with the CoMP schemes from Section II-E. We observe a scenario with $d_1 = d_2 = 0.5$, hence a symmetric cell-edge case, in Fig. 6 and a scenario with $d_1 = 0.4$ and $d_2 = 0.2$, hence asymmetric and weaker interference, in Fig. 7. For all schemes, we show (from right to left, i.e. from less to more efficient) the performance based on a practical quantizer [48] (not applicable to DIS), that given by the rate-distortion bound [43], or through additional source coding. The dotted line shows the cut-set bound [43], resembling the case where each backhaul bit leads to an equal sum-rate increase of one bit, until MAC performance is reached. In the cell-edge case, DAS-C is superior for any extent of backhaul, and source coding is highly beneficial, as the correlation of received signals is strong. The gap to the cut-set bound is due to the fact that backhaul is inevitably wasted into the quantization of noise [15], [16]. Dashed lines indicate the (marginal) benefit of SPC, which here can be attributed to the fact that common messages can be decoded by the BSs without cooperation, reducing the extent of signal power that is quantized for cooperation [50]. In this symmetric cell-edge case, DIS and CIF yield no gain, as each BS can decode both UEs without cooperation. DAS-D provides array gain, but is inferior to DAS-C due to its inability to perform interference cancellation. In the asymmetrical scenario in Fig. 7, DIS and CIF are superior in
regimes of low backhaul. Here, the cell-edge UE 1 is decoded first (under little interference), and then the decoded bits or a quantized transmit sequence are provided from BS 1 to BS 2. It can be shown that DIS with SPC is always superior to compressed interference forwarding (CIF) [45], while the latter has practical advantages. While gains from source coding have decreased due to less signal correlation, it is beneficial to use DAS-C with SPC, more specifically with the option of local, non-cooperative decoding, as pointed out in [17], [18], [20]. However, the performance is inferior to that of a simple time-share between a decentralized and centralized approach [45].

Fig. 8 shows the best cooperation scheme as a function of $d_1$ and $d_2$, for a fixed backhaul of $\beta = 4$ bits per channel use, summarizing and extending previous observations. DAS-C is clearly superior in regimes of strong, possibly asymmetric interference, DIS in regimes of weaker, asymmetric interference, CIF in regimes of even weaker interference, while DAS-D is only interesting for very weak and highly symmetric interference. Considering that the CoMP gain in the latter regimes is marginal, it appears sufficient to adapt between DAS-C and DIS. The hashed areas in Fig. 8 indicate where such adaptation yields more than 10% sum-rate benefit.

D. Benefit of Source Coding and Superposition Coding

Fig. 9 shows the sum-rate gain (in %) of using source coding and/or SPC for DIS and distributed antenna system - centralized (DAS-C) (taking the maximum gain over all regimes of backhaul). As noted before, the gain of source coding techniques can be substantial for cases of strong interference, but these lead to a significantly increased complexity [52]. A main problem as that such schemes require the interference covariance to remain constant over a reasonable extent of time, which is questionable in a cellular uplink that is typically subject to a flashlight effect, i.e. to quickly changing background interference due to scheduling. Investing effort into SPC is clearly not attractive, as the rate increase or improvement of quantization efficiency through common messages is marginal. Further, the gain of partial local decoding is marginal for distributed interference subtraction (DIS), or can be superceded by simple time-sharing between different cooperation strategies for DAS-C.

E. Monte Carlo Simulation Results for $M = K = 3$

We now provide Monte Carlo simulation results for a scenario with $M = K = 3$. Here, average path gains are generated from (40) for $d_1 = d_2 = d_3 = 0.5$ and $d_1 = d_2 = d_3 = 0.3$, and then many
Rayleigh fading realizations are generated that fulfill \( E\{|h_{2(m-1)+1,k}|^2\} = E\{|h_{2m,k}|^2\} = \lambda_{m,k} \), providing different channel orthogonalities. We compare the following schemes:

- Non-cooperative detection employing maximum ratio combining (MRC)
- Non-cooperative detection based on IRC
- Non-coop. detection with IRC and arbitrary BS-UE assignment (see Section II-D)
- Only DIS concepts, only DAS-C concepts, or hybrid combinations, as modeled in [21]
- All BSs quantize and forward to a central network entity [18], [19], denoted DAS-N
- Backhaul-enhanced FDM, as mentioned at the end of Section II-E

We again consider both information theoretical limits with or without source coding, and performance based on practical quantization. In Fig. 10, for the cell-edge case, we can see that IRC is already substantially beneficial over MRC, and an instantaneous BS-UE assignment further improves non-cooperative performance. For cooperation, pure DAS-C strategies appear best, even under practical quantization. FDM is strongly inferior, as the avoidance of interference is inefficient when backhaul is available, and DAS-N has the disadvantage of performing quantization over one more link than schemes based on centralized decoding by a BS. In Fig. 11, in the cell-center case, we can see a significant benefit of adapting between DAS-C and DIS, especially for practical quantization schemes. Under such adaptation, about 50% of CoMP gain can be achieved with about 1.5 bits of backhaul per bit of sum-rate.

IV. PRACTICAL CONSIDERATIONS

A. Parallels between Theory and Practice, and the Value of Iterative BS Cooperation

The previous section has revealed a central trade-off inherent to uplink CoMP:

- If BSs operate code-aware, hence perform (partial) decoding prior to cooperation, any backhaul-usage is more efficient (see results for DIS and CIF), but the schemes fail to achieve MAC performance in regimes of large backhaul.
- If BSs are oblivious to the used codeword, backhaul is wasted into the quantization of noise, but the schemes (i.e. DAS-C) asymptotically obtain the complete CoMP gain.

Proposed practical algorithms typically perform a combination of both strategies. In, e.g., [31]–[33], each BS (partially) decodes both the strongest interferer and its own UE, and forwards soft-bits to the other BS. Hence, code-awareness is used to exploit the structure in signals and
interference for efficient backhaul usage, while the soft-bits inherit information on uncertainty, which yields array and diversity gain. The fact that terminal rates are strongly constrained through the first (partial) decoding process can be alleviated by using iterative BS cooperation [28], [29], [34], hence starting with coarse decoding and refining this in each iteration. It has been shown in [30], [45], however, that for the case of iterative DIS and even under very theoretical considerations, the rate/backhaul trade-off is only marginally improved over one-shot cooperation (though the asymptotic sum-rate is improved). In practice, every backhaul usage will always inherit additional redundancy (and introduce latency), hence rendering iterative schemes even more questionable, as also observed in [27].

B. CSI Distribution and Complexity Issues

Table I summarizes key aspects of the CoMP concepts treated in this work, and adds considerations connected to the required distribution of CSI and complexity. DIS and CIF, for example, have the advantage that each BS only requires local CSI connected to their sub-part of the channel, while DAS-C requires knowledge on the compound channel at the decoding BS, hence requiring the distribution of CSI over the backhaul. In terms of complexity, CIF offers the benefit that it does not require re-modulation by a BS that performs (partial) interference subtraction. Complexity increases drastically if source coding (Wyner-Ziv, Slepian-Wolf) is performed.

V. CONCLUSIONS

Different theoretical uplink CoMP concepts have been analyzed with a special focus on a constrained backhaul infrastructure and imperfect CSI. The work has shown that strongest CoMP gains can be expected at the cell-edge, and in fact increase for diminishing CSI, whereas gains quickly vanish towards the cell-center. This reduces the set of attractive CoMP concepts to DAS-C, interesting in regimes of strong interference and based on oblivious BSs, and DIS, based on local decoding and an exchange of decoded bits, where adaptation has shown to be beneficial. Various proposed concepts based on SPC have shown to be of minor interest, while source coding appears attractive, but has to be put in perspective to major implementation challenges. A comparison of these theoretical concepts to proposed practical algorithms has shown the fundamental trade-off between efficient backhaul usage and maximum CoMP gain that has to be made, and put the practical usage of iterative BS cooperation into question.
We here sketch the proof of Theorem 1, providing details in [45]. Eqs. (1) and (4) yield
\[ y = Hs + n = (\hat{H} - E) s + n = (\hat{H}^e - E^e) s + n, \]
where \( \hat{H}^e \) is an unbiased channel estimate, and \( E^e \) is an uncorrelated estimation error with
\[ \forall i, j : \hat{h}_{i,j}^e = \frac{\hat{h}_{i,j}}{\sqrt{1+\sigma^2_E/E}} \quad \text{and} \quad E \{ |e_{i,j}^e|^2 \} = E \{ |e_{i,j}|^2 \} = \frac{E \{ |h_{i,j}|^2 \} \cdot \sigma^2_E}{E \{ |h_{i,j}|^2 \} + \sigma^2_E}. \]

Treating product \( E^e \)'s in (41) as a Gaussian random variable with a different realization in each channel access leads to an overestimation of the impact of imp. CSI [37], i.e. we can state
\[ E \{ I(S; Y|\hat{H}^e) \} \geq \log_2 \left| I + (\Phi^v + \sigma^2 I)^{-1} \hat{H}^e P (\hat{H}^e)^H \right|. \]

In this work, we are interested in observing the rates achievable with a fixed channel \( H \), averaged over many channel estimation realizations \( \hat{H}^e \), which we can approximate by
\[ I(S; Y) \geq E_{\hat{H}^e} \left\{ I(S; Y|\hat{H}^e) \right\} \approx \log_2 \left| I + (\Phi^v + \sigma^2 I)^{-1} \hat{H}^e P (\hat{H}^e)^H \right|. \]

with \( \hat{H}^e \) given in (7). Clearly, the RHS of (44) is larger or equal to (43) due to Jensen’s inequality, but numerical evaluation has shown that this aspect is negligible unless noise power and channel power are of the same order, especially in consideration of the noise overestimation in (43).

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Figure 1. Uplink transmission considered in this work.

Figure 2. Illustration of a performance region for an example channel with $M = K = 2$ and $N_{bs} = 1$. 
Figure 3. Uplink CoMP schemes for $M = K = 2$ analyzed in this work.
| UPLINK | DIS | CIF | DAS-D | DAS-C |
|--------|-----|-----|-------|-------|
| Decoding | decentralized | | centralized |
| Exchanged signals | decoded messages | quantized sequences | quantized receive signals |
| Achievable gains | SIC gain (interference cancellation) | partial interference cancellation | array + spat. mult. gain | array + spat. mult. + SIC gain |
| Suitable in scenarios | weak, asymm. interference | very weak, asymm. interf. | very weak, symm. interf. | strong interference |
| | low backh. | low backh. | low backh. | large backh. |
| Source coding concepts | provide little gain, but possible if interference is also decoded | provide little gain, and are highly questionable from implementation point of view | major potential gains, but highly questionable |
| Channel knowl. req. | local knowledge from each BS to all UEs sufficient | global CSI at all BSs | global CSI at one BS |
| Complexity | moderate, if interf. only needs re-encoding / SIC (w/o src. coding), high if dec. of mult. UEs / SIC (w/ src. coding) | low, due to simple signal subtraction | low, as only one UE is decoded | high, as all UEs are successively or jointly decoded + SIC |

Table I

Comparison of uplink BS cooperation schemes, considering practical aspects.
Figure 4. Overall gain through BS cooperation for $M = K = 2$, $N_{bs} = 2$, and channels of average orthogonality.

Figure 5. Gain of BS cooperation as a function of CSI accuracy.
Figure 6. Sum-rate as a function of backhaul for a symmetric cell-edge scenario. For each scheme, an area indicates the range between the theoretical performance limit (employing source coding) and the performance of a practical quantizer [48].

Figure 7. Sum-rate as a function of backhaul for a channel of moderate, asymmetric interference.
Figure 8. Best cooperation scheme as a function of UE locations, for a scenario with $M = K = 2$, $N_{bs} = 2$ and $\beta = 4$ bpcu.

Figure 9. Gain through source coding (sc.) or superposition coding (spc.), maximized over all extents of backhaul.
Figure 10. Monte Carlo Results for $M = K = 3$ and $N_{bs} = 2$ (cell-edge scenarios).

Figure 11. Monte Carlo Results for $M = K = 3$ and $N_{bs} = 2$ (cell-center scenarios).