Effects of Hob Tip Radius and Addendum Modification Coefficients on Tooth Bending Stress of Spur Gears under Dynamic Conditions

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Abstract. The paper presents a bending stress analysis of spur gear pairs in order to enhance the effects of the tooth root influence factors in relation to the hob tip geometry and addendum modification coefficients. These factors are evaluated in accordance with ISO 6336 for the basic rack profiles. In order to improve the accuracy of this analysis, a dynamic model is considered to analyse the shared dynamic load for a meshing cycle. All these influence factors are included in a global factor \( Y_d \). A comparative study has been performed to investigate the effects of the hob tip radius, addendum modification coefficients and pinion speed on the variation of the factor \( Y_d \) on the meshing cycle. The approach makes for improved in predicting the tooth bending stress at the design stage.

1. Introduction
The large bending stress in the root fillets of loaded gear teeth represents one of the primary cause of gear tooth failure. There are a large number of parameters are involved in the design of a gear. Between these, the tooth root fillet radius, addendum modification coefficients and dynamic working condition play an important role on the performance of gear transmissions [1-5].

The generation of tooth root fillet radius depends on the hob tip radius and addendum modification coefficient. In the design of spur and helical gears, the gear teeth with involute profile are geometrically defined by the basic rack with the tooth geometry recommended in ISO 53 [6].

Gear standards such as ISO [6] or DIN [7], which are used widely for gear design, included different values for the tooth radius \( r_0 \). Thus, ISO 53 recommends \( r_0 = (0.25/0.3/0.38)m_n \), while DIN 867 recommends a tooth radius \( r_0 = (0.25/0.3)m_n \).

The gear teeth are generated from a rack-type tool (hob or rack cutter) which is conjugated to the standard rack. The hob tool form is presented in figure 1. For an addendum of \( h_{ma} = 1.25m_n \) and a pressure angle of \( \alpha_0 = 20^\circ \), the maximum radius is \( r_0 = 0.472m_n \), where \( m_n \) represents tooth module. When the tip radius is greater than 0.38 \( m_n \), the straight portion of the cutter addendum is less than the standard value of 1.0 \( m_n \).

A large variation exists in specification of the hob tip radius. Thus, in [3] the radius \( r_0 = 0.25m_n \) is considered, while in [8] \( r_0 = (0.1-0.5)m_n \) and \( r_0 = 0.16m_n \) [9].
Therefore, it would be desirable to obtain pertinent information of the influence of the hob tip radius on the tooth influence factors used in the design stage of gear transmissions.

There are few studies about the dynamic effect on tooth bending stress bending stress. Between these, Dai et.all [10] investigates the static and dynamic tooth root strains in spur gear pairs using a finite element approach, while in [5] an analytical study on the effect of hob offset on the dynamic tooth strength of spur gears is presented.

In this paper, we investigate the effects of the hob tip radius on the dynamic bending stress of spur gears by including the dynamic sharing load on the meshing cycle. The effects of the tooth profile modifications are included in this investigation.

2. Factors of tooth influence bending stress

According to ISO 6336-3 [11], the tooth root stress \( F_{r} \) is expressed as

\[
F_{r} = \frac{F_{t}}{b \cdot m_{n}} Y_{F} Y_{S} Y_{B} Y_{DT} K_{A} K_{V} K_{F_{T}} K_{F_{m}} K_{F_{a}}
\]  

(1)

where

- \( F_{t} \) is the nominal tangential load, \( b \) is the facewidth, \( m_{n} \) is the normal module; \( Y_{F} \) is the form factor;
- \( Y_{S} \) is the stress correction factor which takes into account the influence on nominal tooth root stress;
- \( K_{F_{T}} \) is the transverse load factor for tooth root stress; \( K_{V} \) is the dynamic factor; \( K_{F_{m}} \) is the face load factor for tooth root stress; \( K_{A} \) and \( K_{V} \) are respectively application and dynamic factors;
- \( Y_{B}, Y_{G}, Y_{DT} \) are respectively helix angle, rim thickness and deep tooth factors.

It can be assumed that the maximum load during gear tooth meshing is applied to the Highest Point of Single Tooth Contact (HPSTC). According to the standard ISO 6336, the nominal stress is calculated by using the assumption that the critical section lies at the point where a line with an inclination of 30 deg to the tooth centerline touches with the fillet internally (Figure 2).

The tooth influence factors \( Y_{F} \) and \( Y_{S} \) are determined by gear geometry and are influenced by the tooth root fillet. Thus, the factor \( Y_{F} \) is expressed as [12]:

\[
Y_{F} = \frac{6(h_{F_{c}}/m_{n}) \cos \alpha_{F_{n}}}{(s_{F_{n}}/m_{n})^{2} \cos \alpha_{n}}
\]

(2)

where \( h_{F_{c}} \) and \( s_{F_{n}} \) are presented in Figure 2 and are depended of hob tip radius coefficient \( r_{o}^{*} \) [12].
For the influence factor $Y_F$ the following expression is used:

$$Y_S = (1.2 + 0.13 L_a) q_s^p$$  \hspace{1cm} (3)

$$p = \frac{1}{1.21 + 2.3 L_a}$$  \hspace{1cm} (4)

$$L_a = s_{Fn} / h_{Fe}$$  \hspace{1cm} (5)

$$L_a = s_{Fn} / h_{Fe}$$  \hspace{1cm} (6)

$$q_s = s_{Fn} / (2 \rho_F)$$  \hspace{1cm} (7)

$$\rho_F = r_o^* + \frac{2G^2}{\cos (\pi \cdot \cos^2 \varphi - 2G)}$$  \hspace{1cm} (8)

$$G = r_o^* - h_{ao} + x$$  \hspace{1cm} (9)

where $\rho_F = \rho_F / m_n$, $r_o = r_o / m_n$, $h_{ao} = h_{ao} / m_n$.

In order to improve the evaluation of the load distribution in the transverse direction and the dynamic condition, the effect of the combined factors $K_V \cdot K_{Fa}$ from (1) is substituted by the dynamic factor $c_{dl}$ which is considered in this analysis. The dynamic factor $c_{dl}$ of the single tooth pair is defined as the ratio of the single dynamic load $F_{dl}$ to the static load $F_n$ and permits to underline the effect of load distribution over meshing teeth under dynamic condition.

In order to consider the effects of hob tip geometry and addendum modification coefficients on bending stress of spur gears under dynamic conditions, the effects of the factors combined $Y_F Y_S K_V K_{Fa}$ from (1) on the tooth bending stress are analyzed by considering the global factor $Y_D = Y_F Y_S C_{dl}$.

3. The dynamic model of a gear pair

The typical dynamic model for a gear pair in mesh is shown in figure 3. In this model, the teeth are considered as springs and the gear blanks as inertia masses. The gear mesh interface is represented by the time-varying mesh stiffness $k_i(t)$, the viscous damper $c$ and the composite tooth profile error $e_i(t)$. The motion transfer along the line of action between the two gears represents the dynamic transmission error (DTE) and can be expressed as $\delta = r_{bl} \theta_1 - r_{b2} \theta_2$. The differential equations of motion can be expressed as

$$J_1 \ddot{\theta}_1 + c \dot{\theta}_1 r_{bl} + \sum_{i=1}^{N} r_{bi} F_{di} = T_1$$  \hspace{1cm} (10)

$$J_2 \ddot{\theta}_2 - c \dot{\theta}_2 r_{b2} - \sum_{i=1}^{N} r_{bi} F_{di} = -T_2,$$  \hspace{1cm} (11)

where $\theta_1, \theta_2$ are the rotation angle of the pinion and the driven gear, respectively, $J_1$ and $J_2$ are the mass moments of inertia of the gears, $T_1$ and $T_2$ denote the external torques applied on the gear system, $r_{bl}, r_{b2}$ are the base circle radii of the gears.
The damping coefficient is calculated by

\[ c = 2\xi \sqrt{m_c k_m}, \]  

(12)

where \( \xi \) represents the damping ratio factor, \( m_c \) represents the equivalent inertia mass and \( k_m \) is the average mesh stiffness of the gear pair.

![Dynamic model of a gear pair.](image)

Figure 3. Dynamic model of a gear pair.

The dynamic normal load between two meshing gear teeth is expressed as

\[ F_{di} = k_i(t)(\delta(t) - \varepsilon_i(t)). \]  

(13)

For a pair of contacting teeth \( i \), the time-varying mesh stiffness \( k_i(t) \) acts as a parameter excitation. The gear tooth is modeled to be a nonuniform cantilever beam supported by a flexible fillet region and foundation [13]. The effect of bending, shear and Hertzian contact deformation is taken into account in the analytical method when calculating the tooth deformation. The analytical modelling considers the real tooth profile and the geometrical parameters which are specific for spur gear pairs.

The teeth pairs in contact act like parallel springs. Therefore, the total mesh stiffness during each engagement cycle can be written as a function of the position of contact point on the action line.

\[ k_i = k_{i}^1 + k_{i}^{II}, \quad \text{for double-tooth contact} \]
\[ k_i = k_{i}^1, \quad \text{for simple-tooth contact} \]

where I and II are the mating points of the teeth pairs.

The shared dynamic load is analyzed by using the dynamic factor of the single tooth pair defined as \( \frac{c_{dl}}{F_n} \).

4. Numerical results and discussions

The effects of hob tip geometry and addendum modification coefficients on bending stress of spur gears under dynamic condition are analyzed by using the factor \( Y_D \), where

\[ \sigma_F = \sigma_{FK} \cdot Y_D. \]  

(14)

\[ \sigma_{FK} = \frac{F_i}{b \cdot m_n} Y_{\beta} K_A K_y K_{F\beta}. \]  

(15)

\[ Y_D = Y_F Y_S C_{dl}. \]  

(16)
Specifications of the pertinent geometrical and kinematics parameters of the analyzed gear pairs are shown in table 1, where $z_1, z_2$ represent the tooth number of a gear pair, $x_{1,2}$ are the addendum modification coefficients and $\varepsilon_\alpha$ is the transverse contact ratio. These parameters are for spur gear pairs having: face-width of gears, $b=20\ [\text{mm}]$ and center distance, $a = 90\ [\text{mm}]$. Additionally, the damping ratio $\xi=0.12$ is considered in the dynamic analysis. A numerical value $F_n/b = 120\ \text{N/mm}$ corresponding for transmitting load is considered in the numerical analysis.

| $z_1$ | $z_2$ | $m_n$ [mm] | $x_{1\min}/r_0^*$ | $x_1$ | $x_2$ | $\varepsilon_\alpha$ | AB [mm] | AC [mm] | BD [mm] | DE [mm] |
|-------|-------|------------|-------------------|-------|-------|-------------------|---------|---------|---------|---------|
| 14    | 31    | 4          | 0.36/0.10         | 0.36  | -0.36 | 1.495 5.84       | 6.56    | 5.96    | 5.84    |
|       |       |            | 0.26/0.25         | 0.26  | -0.26 | 1.517 6.10       | 7.46    | 5.70    | 6.10    |
|       |       |            | 0.18/0.38         | 0.18  | -0.18 | 1.534 6.30       | 8.23    | 5.50    | 6.30    |
| GA2   |       |            | 4                 | 0.8   | 0.8    | 1.301 3.55       | 2.23    | 8.25    | 3.55    |
| GB1   |       |            | -0.22/0.10        | -0.22 | 0.22   | 1.65 5.78        | 8.91    | 3.07    | 5.78    |
|       |       |            | -0.32/0.25        | -0.32 | 0.32   | 1.65 5.76        | 9.52    | 3.09    | 5.76    |
|       |       |            | -0.40/0.38        | -0.40 | 0.40   | 1.647 5.72       | 10.05   | 3.13    | 5.72    |
| GB2   |       |            | 3                 | 0.0   | 0.0    | 1.647 5.73       | 7.491   | 3.12    | 5.73    |
| GB3   |       |            | 3                 | 0.5   | -0.5   | 1.577 5.11       | 4.01    | 3.74    | 5.11    |
| GB4   |       |            | 3                 | 0.8   | -0.8   | 1.260 2.29       | -1.83   | 6.55    | 2.29    |

The addendum modification coefficient $x_{\min}$ to avoid undercutting results in the following form [14]:

$$x_{\min} = h_{ao}^* - \frac{z \cdot \sin \alpha_\alpha^2}{2} - r_0^* (1 - \sin \alpha_\alpha).$$

A computer program was developed for simulating the dynamic characteristics of spur gear pairs. The equations of motion are solved by the fourth-order Runge-Kutta method. The dynamic loads are calculated using detailed contact analysis at each time step of meshing cycle. Computer analysis of dynamic characteristics includes different gear pairs with combination of addendum modifications and gear ratio.

The variation of the the global factor $Y_D$ for the engagement cycle is presented in figures 4 and 5 in relation to the amount of hob radius factor $r_0^*$, pinion speed and addendum modification coefficients. It can be observed that the fluctuation of the global factor $Y_D$ is affected by the amount of the tip hob radius factor $r_0^*$.

The effects of addendum modification coefficients on the variation of the dynamic global factor $Y_D$ are presented in figure 6 in relation to the amount of the hob tip radius factor $r_0^*$. In these figures the following colours are used: red – for $r_0^* = 0.1$; blue – for $r_0^* = 0.25$; black - for $r_0^* = 0.38$.

Referring to figures 4, 5 and 6, the following mesh points were used to represent the successive positions of contact point of a tooth as it passes through the zone of loading: the initial point of engagement, A; the lowest point of single-tooth contact, B; the highest point of single-tooth contact, D; and the final point of engagement, E. Section AB and DE are double - tooth contact zone and section BD is the single tooth - contact zone.
Figure 4. Variation of global factor $Y_D$ as a function of pinion speed with various hob tip radius

Figure 5. Variation of global factor $Y_D$ as a function of pinion speed with various hob tip radius
In order to underline the dynamic effect in this analysis, in table 2 are presented the ratio $Y_{FD}/Y_{D_{max}}$ for the pinion speed $\omega_1 = 300\text{s}^{-1}$, where $Y_{FD} = Y_F Y_S$. The favorable effect of the larger addendum modification coefficient on the global factor $Y_D$ is more evident with increase the hob radius factor $r_0^*$. 

### 5. Conclusions

An analytical procedure for analysis the dynamic bending stress of spur gear pairs is presented. The global factor $Y_D = Y_F Y_S C_{dl}$ is proposed in this analysis, where the tooth root factors $Y_F$ and $Y_S$ are
calculated according to the standard ISO 6336-3, while the dynamic aspects are based on the dynamic analytical model. This procedure predicts the combined effects of the hob tip radius and addendum modification coefficients on the variation of the bending stress during the meshing cycle for different pinion speeds. In order to improve the accuracy of gear stress analysis, a dynamic model for spur gears is used. The time-varying mesh stiffness along the path of contact is included in the procedure by using an exact analytical model.

The results show that a larger value for the hob tip radius permits to reduce the tooth root stress concentration. At the same time, the effect of the tooth tip radius of the hob on the tooth undercutting limit of gears is included in the analysis.

The method described in the present paper aims to complete the ISO Standard calculation procedure for improve of the tooth dynamic stress analysis.

6. References

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