Computer Simulation for Air-coupled Ultrasonic Testing

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Abstract. Air-coupled ultrasound is used as non-contact ultrasonic testing method. For wider application of air-coupled ultrasonic technique, it is required to know situation of ultrasonic propagation between air and solid. Transmittance of the ultrasonic waves from air to solids is extremely small with $10^{-5}$, however it was revealed that, by using computer simulation methods based on the two-stage elastic wave equation in which two independent variables of stress and particle velocity are used, visualization calculation of ultrasonic propagation between air and solid was possible. In this report, the calculation of air-coupled ultrasound using the new Improved-FDM for computer simulation of ultrasonic propagation in solids is shown. Waveforms obtained by 1-dimensional calculation are discussed for principle and performance of the calculation. Visualization of ultrasonic incidence to cylindrical steel pipe is demonstrated as an example to show availability for ultrasonic testing.

1. Introduction

Air-coupled ultrasonic technique[1,2] is utilized for non-destructive testing for advanced materials and structures. It requires no couplant other than air for ultrasonic transmission to the object of the testing. However, ultrasonic propagation between air and solid is extremely inefficient because of large difference in acoustic impedances of them. Moreover, their ultrasonic velocities differ extremely, therefore the propagation behaviour is complicated. Although the computer visualization is effective for analysis of ultrasonic behaviour, traditional method based on FDM(finite difference method) using displacement as independent variable could not treat directly ultrasonic propagation between air and solid. For the computer visualization, author was developed Improved-FDM which uses displacement as the variable[3], and recently the method was renewed in which particle velocity and stress are used as the variables[4]. Using former and new Improved-FDMs for visualization of ultrasonic propagation between air and steel, it was revealed that the new Improved-FDM could calculate ultrasound of air and steel in unified calculation model. In this report, principle of the calculation is explained and ultrasonic visualization for ultrasonic propagation between air and steel pipe is demonstrated.

2. Principle of FDM calculation for air-coupled ultrasound.

In comparison between ultrasonic physical quantities, ultrasonic transmittance between air and solid are attractive as shown in table 1 in which relations between air and steel are described. Between the group of stress and pressure and the group of particle velocity and displacement, they have complementary relationship. For example, in the case of propagation from air to steel, particle velocity decreases to $10^{-5}$, whereas Stress increases to 2. This complementary characteristic plays important role in the calculation for ultrasonic propagation between materials with huge difference of acoustic impedance.
Table 1. Ultrasonic transmittance between air and steel, in the case of normal incidence at room temperature. $Z_1$ and $Z_2$ is acoustic impedance of materials.

|                | Equations for transmittance from medium-1 to medium-2. | Transmittance from air to steel. | Transmittance from steel to air. |
|----------------|--------------------------------------------------------|----------------------------------|----------------------------------|
| Stress, Pressure | $T_p = 2Z_2 / (Z_2 + Z_1)$                             | 1.999982                         | 1.76x10^{-5}                     |
| Particle velocity, Displacement | $T_v = 2Z_1 / (Z_2 + Z_1)$                             | 1.76x10^{-5}                     | 1.999982                         |

For the computer visualization, volume of air and solid is discretized and elastic wave equation is transformed into finite difference formula. Figure 1 shows calculation model and elastic wave equations for Improved-FDMs. In former Improved-FDM, only displacement $u$ is used as independent variable as shown in equation (3). In this case, subtraction in the formula can cause numerical error of “loss of trailing digits” or “loss of information”. So, in the case of the propagation from air to solid, calculation can’t reproduce extremely small displacements at solid surface, and in actual calculation, the transmittance was calculated to be 0. On the other hand, in new Improved-FDM, the two-stage elastic wave equation shown in equations (1) and (2) is used, and independent variables of particle velocity $v$ and stress $T$ are used. On each time-step of calculation, particle velocity and stress are calculated and updated by using each other. That is, each variable plays complementary role, and this process enables calculation of both transmissions between air and solid without loss of trailing digits. For example, in the case from air to solid, information of ultrasonic displacement is carried by mediation of the stress, without loss of information, as supposed on table 1.

Figure 1. Discretized solid model and wave equations used in Improved-FDMs[3][4]. Equations are of 1-dimension for simplicity. Corner points of unit cubes are calculation nodes for particle velocity $v$ and displacement $u$, and the center points of unit cubes represent stress $T$, density $\rho$ and elastic constant $C$.

For new Improved-FDM:

$$\rho \frac{\partial v}{\partial t} = \frac{\partial T}{\partial x}$$  \hspace{1cm} (1)

$$\frac{\partial T}{\partial t} = C \frac{\partial^2 v}{\partial x^2}$$  \hspace{1cm} (2)

For previous method:

$$\rho \frac{\partial^2 u}{\partial t^2} = C \frac{\partial^2 u}{\partial x^2}$$  \hspace{1cm} (3)

Figure 2 shows waveforms obtained by 1-dimensional calculation of ultrasonic propagation from air to steel. In new Improved-FDM, particle velocity and stress are used and calculated. Displacement is obtained by time integration of particle velocity. The calculation was performed using the 3-dimensional program with cyclic boundary conditions. In the calculation, 2-cycles sinusoidal stress of 100kHz is loaded on one of particle velocity nodes of air. The excited displacement waveform is shown in the figure as the incident wave. Amplitude of waveform transmitted into steel is reduced to 1.75x10^{-5}, which is good agreement with table 1. Surprisingly, degradation of transmitted waveform is not observed in spite of huge reduction of its amplitude, although increase of ringing trail is observed slightly. The ringing is the common phenomenon in FDMs and relating to propagation distance. The reflected waveform from steel surface shows that the steel acts as rigid surface where phase inversion and no amplitude reduction are occurred.
Calculation conditions

Time step: $\Delta t = 20\text{ns}$
Spatial step: $\Delta x = 0.2\text{mm}$
Input wave: $10\mu\text{s} \times 2$ cycles

Steel: $\rho = 7860\text{kg/m}^3$, $C_{11} = 2.74 \times 10^{11}\text{Pa}$
Air: $\rho = 1.2\text{kg/m}^3$, $C_{11} = 1.39 \times 10^5\text{Pa}$

Figure 2. 1-dimensional calculation of ultrasonic propagation from air to steel. Vertical axis is displacement (m), and horizontal axis is time (s).

3. Visualizing calculation of ultrasonic propagation from air to steel pipe.

Important application of air-coupled ultrasonic is non-destructive testing for solid-fuel rocket[5]. Solid fuel with free from cracks and voids, and adhesion between the fuel and outer shell are important to assure normal combustion of the fuel. Requirement of no couplant and fire safety is remarkable advantage for the testing. However situation of ultrasonic propagation through rocket body was investigated only by the experiment. Here shows a simple calculation as an example for ultrasonic propagation into cylindrical body of the rocket.

Figure 3 shows 2-dimensional calculation for visualizing ultrasonic propagation from air to a cylindrical steel pipe. The calculation is performed using the 3-dimensional program with cyclic boundary condition. Two cycles of sinusoidal wave shown in figure 2 enters the steel pipe. The stiffness matrix of air used in the calculation is shown in equation (4). It is induced from Pascal’s principle as well as for water. In the figure, displacement distribution of waves in the steel pipe can not observed on the images showing outer air (Air-o) and steel pipe because of amplitude reduction with $10^{-5}$. On the other hand, on the images showing the steel pipe and inner air (Air-i), the distribution is observed clearly, and it is obvious that no reduction of displacement occurs while the wave propagates from steel to inner air. In the inner air, beam width of the longitudinal wave is observed wider in comparison with initial beam width. It is because that, in the case of the incidence from air to steel, mode conversion between longitudinal and shear wave occur with small angled incidence (~7deg.) and direct longitudinal wave and mode-converted waves are overlapped in the inner air.

$$
C = \begin{bmatrix}
1.39e05 & 1.39e05 & 1.39e05 & 0 & 0 & 0 \\
1.39e05 & 1.39e05 & 1.39e05 & 0 & 0 & 0 \\
1.39e05 & 1.39e05 & 1.39e05 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \text{Pa} \quad (4)
$$
Figure 3. 2-D. calculation of angled-incidence of ultrasound into cylindrical steel pipe. Colors (red / green) in the images indicate directions of displacements (horizontal / vertical). Outside (upper images) and inside (lower images) of the pipe are visualized separately, because of huge difference of displacement amplitude. Steel pipe is 200mm outer diameter and 20mm thickness. Calculation condition is same as figure 2. Total area size is 700x1000 steps.

4. Conclusion
It was revealed that ultrasonic propagation between air and solid is calculated precisely using new Improved-FDM, because 2 independent variables of particle velocity and stress shows complementally relationship to prevent loss of trailing digit in the calculation. This feature is brought by the two-stage wave equation, therefore it is expected that the feature is also for FDTD[6] and EFIT[7].

References
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