The Axial Current in Electromagnetic Interaction

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Abstract

We discussed the possibility that the charged axial currents of matter fields could be non-conserved in electromagnetic interaction at $O(e)$ order. It means that chiral symmetry is broken explicitly by electromagnetic interaction. This explicit symmetry breaking of chiral symmetry is shown to lead the mass differences between the charged and neutral particles of matter fields.

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Axial current is Noether current of chiral symmetry, which is a good symmetry in 2-flavour, i.e., SU(2) symmetric world, but broken more or less in SU(3) symmetric world. The global nature of the chiral symmetry generates the massless Nambu Goldstone (NG) boson through its spontaneous symmetry breaking (SSB) [1]. The pseudo-scalar (PS) bosons are the explicit manifestation of the NG bosons.

In the intermediate energy nuclear physics, however, the PS bosons have finite masses by explicit symmetry breaking (ESB) of chiral symmetry due to non-vanishing current quark masses, for the instance of pion, \[ m_\pi^2 = \frac{1}{2}(m_u + m_d) \langle \bar{q}q \rangle , \]
where \( \langle \bar{q}q \rangle = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle \) is the vacuum expectation value (VEV) of quark pairs. Therefore the conservation of axial current is not maintained any more, so that one presumes the partial conservation of the axial current (PCAC), \( \partial_\mu A_\mu^a = -f_\pi m_\pi^2 \pi^a \). But the SSB property of the vacuum is still preserved even in the ESB because the ESB is included perturbatively in order to retain the SSB property of vacuum.

On the other hand, electro-magnetic (EM) interaction with matter fields are usually introduced as an external U(1) local gauge field. The constraint of gauge invariance gives well defined interaction Lagrangian of the matter fields with EM fields.

Naturally one expects that the EM interaction Lagrangian influences on the axial current of the system, for instance, \( \pi - N \) system in this letter. We show that i) the axial current is not conserved any more in EM interaction. Therefore, the Lagrangian modified by external EM interaction causes another ESB mechanism of chiral symmetry at \( O(\epsilon) \) order besides the ESB due to finite pion mass. ii) This ESB gives rise to the mass differences among the matter fields.

Before going to further discussions, we note two following facts : i) One has the anomalous non-conservation of the axial current beyond the classical field theory, known as the Adler-Bell-Jakiew (ABJ) anomaly [2]. But this anomaly appears at \( O(\epsilon^2) \) order although it has fruitful physical consequences. ii) One is used to the concept of “minimal coupling scheme” in the EM coupling to the matter fields, which is widely used in the nuclear physics relevant to the photon reaction. But this scheme is not justified in the following sense. Given the
Lagrangian of matter fields, one derives (axial) current and includes the EM interaction by the minimal coupling, i.e., replaces \( \partial_\mu \) in the axial current by \( \partial_\mu - ieB_\mu \) where \( B_\mu \) is photon field and \( e \) is the EM coupling constant. But one could not be sure that the current modified this way equals to the current obtained directly from the Lagrangian including the EM interaction in gauge invariant way.

Since the above arguments are well reflected in the linear \( \sigma \) model, we start from the Lagrangian of the following linear \( \sigma \) model

\[
L_\sigma = L_N + L_{\pi N} + L_\pi
\]

with

\[
L_N = \overline{\Psi} i\gamma^\mu \partial_\mu \Psi , \quad L_{\pi N} = \overline{\Psi} g(\sigma + i\vec{T} \cdot \vec{\pi}) \gamma_5 \Psi , \quad L_\pi = \frac{1}{2} \left[ (\partial_\mu \vec{\pi})^2 + (\partial_\mu \sigma)^2 \right] - \frac{1}{2} \mu^2 (\vec{\pi}^2 + \sigma^2) - \frac{\lambda}{4} (\vec{\pi}^2 + \sigma^2)^2 .
\]

Although this Lagrangian is invariant under SU(2) chiral transformations, \( \Psi \rightarrow \Psi' = exp(i\gamma_5 \vec{\eta} \cdot \vec{\tau} \gamma_5) \Psi, \overline{\Psi} \rightarrow \overline{\Psi}' = \overline{\Psi} exp(i\gamma_5 \vec{\eta} \cdot \vec{\tau} \gamma_5), \vec{\pi} \rightarrow \vec{\pi}' = \vec{\pi} - \sigma \vec{\eta}, \) and, \( \sigma \rightarrow \sigma' = \sigma + \vec{\pi} \cdot \vec{\eta}, \) it describes only massless fermion and massless bosons. Later on we will show the mass generation of the matter fields and its modification by the EM interaction.

To generate the EM interactions (external U(1) gauge field) with the fermion fields we firstly consider \( L_N \) and introduce photon field \( B_\mu \) via the covariant derivative \( D_\mu \Psi = (\partial_\mu + iV_\mu) \Psi, \) where \( V_\mu = -eT_3B_\mu \) with \( T_i = \tau_i/2. \) It has the following transformation rule in order to be invariant under a local transformation \( h(x), \) i.e., \( D'_\mu \Psi' = hD_\mu \Psi, \)

\[
\delta V_\mu = i(\partial_\mu h)h^{-1} - [V_\mu , h]h^{-1} .
\]

Since the Gell-Mann Levy method is a very convenient tool to derive the current from the given Lagrangian, we assume temporarily the local chiral transformation \( \eta(x) \) to extract axial current [3]. The change of \( L_N \) in the local SU(2) chiral transformation, \( \delta L_N = \overline{\Psi}' i\gamma^\mu D_\mu \Psi' - \overline{\Psi} i\gamma^\mu D_\mu \Psi, \) is given by

\[
\delta L_N = i\overline{\Psi}[i\gamma^\mu(\partial_\mu \vec{T} \cdot \vec{\eta})\gamma_5 - i\gamma_\mu(eB^\mu \epsilon_{5le}T_i\eta_c)\gamma_5] \Psi .
\]

Using the Gell-Mann Levy equations we get the following axial current and its divergence
\[ A_\mu^c(N) = - \frac{\partial \delta L_N}{\partial (\partial_\mu \eta_c)} = \bar{\Psi} \gamma^\mu T_c \gamma_5 \Psi \quad (4) \]
\[ \partial_\mu A_\mu^c(N) = \frac{\partial \delta L_N}{\partial \eta_c} = e \epsilon_{3le} \bar{\Psi} B T_1 \gamma_5 \Psi . \]

The EM interaction does not affect the axial current itself, but breaks its conservation apart from the case of neutral axial current, where the ABJ anomaly plays a role in breaking the conservation of the axial current at \(O(e^2)\) order.

The transformation rule of the mesons comes from retaining the interaction Lagrangian \(L_{\pi N}\) chiral invariant, so that \(\phi \rightarrow \phi' = \phi - f \bar{\eta}\) \(\phi\) is related to \(\sigma\) and \(\pi\) fields through chiral field \(U = \exp(i \phi \cdot \vec{T} / f) = \frac{1}{2f}(\sigma + i \vec{\pi} \cdot \vec{\pi})\) which transforms as \(U \rightarrow U' = U g\) with \(g = \exp(-i \vec{T} \cdot \bar{\eta}(x))\).

For the meson sector, \(L_\pi = \frac{1}{2}[\partial_\mu \sigma]^2 + (\partial_\mu \vec{\pi})^2 = f^2 Tr(\partial_\mu \sigma \partial^\mu \sigma^+)\),

we make the similar procedures to take the EM interaction with \(\phi\) fields into account. Exploiting the covariant derivative for \(U\) field, \(D_\mu U = \partial_\mu U + i [U, V_\mu]\), we find the following transformation rule of \(V_\mu\) to be invariant under a local transformation \(h\), i.e. \(D'_\mu U' D'^\mu U' = D_\mu U D^\mu U\),

\[ \delta V_\mu = -i(\partial_\mu h^+) h - [V_\mu, h^+] h . \quad (6) \]

The change of \(L_\pi\) under transformation \(g\), \(\delta L_\pi = f^2 Tr(D_\mu U' D'^\mu U') - f^2 Tr(D_\mu U D^\mu U)\), is represented as

\[ \delta L_\pi = f^2 Tr(\alpha_\beta \beta^\mu + \beta_\mu \alpha^\mu + i \beta_\mu V^\mu + i V_\mu \beta^\mu + i(\alpha_\mu - V_\mu)[\frac{\vec{T} \cdot \bar{\phi}}{f}, \beta^\mu] + i V_\mu O^\mu(\bar{\phi}^2) - \alpha_\mu [V_\mu, \vec{T} \cdot \bar{\eta}]) \quad (7) \]

where \(\alpha_\mu = \frac{1}{f} \partial_\mu (\vec{T} \cdot \bar{\phi})\), \(\beta_\mu = -i \partial_\mu (\vec{T} \cdot \bar{\eta})\) and local chiral transformation \(\eta = \eta(x)\) is also assumed. Upto \(O(V_\mu, \phi, \eta, e) = O(1)\) order, we obtain

\[ A_\mu^c(\pi) = - \frac{\partial \delta L_\pi}{\partial (\partial_\mu \eta_c)} = f \partial_\mu \phi_c + f e B^\mu \epsilon_{3le} \phi_t + O(2) \quad (8) \]
\[ \partial_\mu A_\mu^c(\pi) = fe \epsilon_{3le} B^\mu \partial_\mu \phi_t + O(2) . \]
Likewise to the nucleon case, the conservation of the axial current is broken at $O(e)$ order. But in the axial current the EM effects showed up directly.

Finally, total axial current and its divergence are changed by the EM interaction in the following way

$$A_\mu^c = \bar{\Psi}\gamma^\mu T_c\gamma_5 \Psi + f\partial^\mu\phi + feB^\mu\epsilon_{3lc}\phi_l + O(2)$$  \hspace{2cm} (9)

$$\partial_\mu A_\mu^c = e\epsilon_{3lc}(\bar{\Psi}BT_c\gamma_5 \Psi + fB^\mu\partial_\mu\phi_l) + O(2)$$

$$= e\epsilon_{3lc} B_\mu A_\mu^c + O(2) .$$

This form can be reduced to the normal form if $e$ goes to zero.

The above form can be expressed into the more familiar representation known as "the minimal coupling scheme" in current level, if we introduce the generalized covariant derivative $D_\mu(\pm) = \partial_\mu \mp ieB_\mu$ in the total axial current $A^{(\pm)\mu} = A^{(\pm)\mu}(N) + A^{(\pm)\mu}(\pi)$,

$$D_\mu(\pm) A^{(\pm)\mu} = 0 ,$$  \hspace{2cm} (10)

$$A^{(\pm)\mu}(N) = \bar{\Psi}\gamma^\mu \tau_\pm \gamma_5 \Psi \hspace{0.5cm} , \hspace{0.5cm} A^{(\pm)\mu}(\pi) = f(\partial^\mu \mp ieB^\mu)\phi_\pm ,$$

where $\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$ and $\phi_\pm = \frac{1}{\sqrt{2}}(\phi_1 \pm i\phi_2)$. This result can be also derived by exploiting the generalized Euler equation \[4\] and applied to the quenching problem of induced pseudo-scalar coupling constant in the radiative muon capture \[5\].

Non-conservation of the axial current as shown above means just that the chiral symmetry is broken explicitly by EM interaction at $O(e)$ order. Now let’s discuss the physical meaning of the ESB by the EM interaction. Actually the chiral symmetry is a global, so that we have to switch off the locality of chiral symmetry. Therefore the change of $L_\sigma$ due to EM interaction is finally given at $O(e)$ order,

$$\delta L_\sigma = L_\sigma^{e.m.} - L_\sigma = \bar{\Psi}\gamma_\mu B^\mu \gamma_5 T_l \Psi + eB^\mu\epsilon_{3lc}\phi_l\partial_\mu\phi + O(e^2) .$$  \hspace{2cm} (11)

Usually, in order to create the pion mass, the chiral ESB term $\zeta_\sigma$ should be included into $L_\sigma$. By exploiting the Gell-Mann Levy method, similarly to the above, one sees $\partial_\mu A_\mu^c = -\zeta_{\pi c}$.

The $\zeta_\sigma$ term is to generate the $\pi^0$ mass, $m^2_{\pi^0} = \zeta / \langle \sigma \rangle$. Here $\langle \sigma \rangle$ is the mean field taken.
in the degenerate vacuum, as a result the vacuum is spontaneously broken, to introduce new field \( \sigma' = \sigma + \langle \sigma \rangle \) which is vanishing at the ground state. On the other hand, the fermion matter field acquires a mass \( M_N = -g \langle \sigma \rangle \). Therefore \( \zeta \) is finally given as \( -m_{\pi^0}^2 M_N / g \).

Now let us switch on the EM interaction. Then \( \zeta \) is changed as \( \zeta' \) which includes effectively the additional ESB term (see eqs. (8-9) and (11)). Here we would like to remind that \( \zeta \) is introduced perturbatively in order not to change the SSB property of vacuum. In this context, \( \zeta' = \zeta + \delta \zeta(e) \) can be also safely introduced perturbatively because \( \delta \zeta(e) \) is much smaller than \( \zeta \). Similarly, \( \langle \sigma \rangle \rightarrow \langle \sigma' \rangle \). Finally, since we can conjecture that the masses of charged particles are obtained from those of neutral particles with additional inclusion of EM interaction we get

\[
\zeta = -m_{\pi^0}^2 M_N / g, \quad \zeta' = -m_{\pi^\pm}^2 M_P / g, \quad (12)
\]

where \( M_N(M_P) \) and \( m_{\pi^0}(m_{\pi^\pm}) \) mean the masses of neutron(proton) and neutral(charged) pions, respectively. If we assume \( \zeta \simeq \zeta' \), we obtain a relation of mass ratio

\[
\left( \frac{m_{\pi^0}}{m_{\pi^\pm}} \right)^2 = \frac{M_P}{M_N}. \quad (13)
\]

This relation shows that the charged particle mass is heavier than neutral particle in case of meson, while in case of fermion neutral particle is heavier than charged particle. But, actually this relation is deviated about 7 \% if we use the measured values. The mass difference in pion can be explained by the virtual photon propagation, which comes from the interaction of the 2nd term in eq.(11), but the mass difference in nucleon cannot be fully explained only by EM interaction. Therefore the 7 \% deviation should be attributed to the effects at quark level of hadrons.

Finally we make a brief summary. If we introduce the EM interaction to the matter fields in U(1) gauge invariant way, the chiral symmetry is broken explicitly in the total Lagrangian, so that the axial current is not conserved any more. This ESB of chiral symmetry just gives the mass differences between the charged and neutral particles in the matter fields. In specific, such mass difference in the pion sector is just reversed in case of the nucleon,
namely, \( m_{\pi^\pm} > m_{\pi^0} \) while \( M_N > M_P \). We showed a mass relation among the particles. The small deviation in the nucleon side should be understood at quark level.

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