Spatial correlation of atmospheric wind at scales relevant for large scale wind turbines

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Abstract. Wind measurements a short distance upstream of a wind turbine can provide input for a feedforward wind turbine controller. Since the turbulent wind field will be different at the point/plane of measurement and the rotor plane the degree of correlation between wind speed at two points in space both in the longitudinal and lateral direction should be evaluated. This study uses a 2D array of mast mounted anemometers to evaluate cross-correlation of longitudinal wind speed. The degree of correlation is found to increase with height and decrease with atmospheric stability. The correlation is furthermore considerably larger for longitudinal separation than for lateral separation. The integral length scale of turbulence is also considered.

1. Introduction
An inherent property of the atmospheric wind challenging wind turbine engineers is gustiness. Atmospheric turbulence will cause fatigue loading of the wind turbine structure and reduce its lifetime [1]. Recently turbine control strategies involving short term predictions of the hub height or rotor equivalent wind speed have been introduced with the goal of mitigating fatigue loads and increasing power output [2, 3]. The approach is based on nacelle lidar wind speed measurements of the incoming flow ~1 diameter upstream of the rotor plane, which gives a ~10-20 second prediction of rotor wind speed for a feedforward controller. The ideal preview distance will be a tradeoff between prediction time and coherence and will also be influenced by lidar measurement errors[4]. An important consideration for choosing the preview distance will be the relevant turbulent length scales and required coherence.

Taylor’s hypothesis states that for short separation distances and large turbulent length scales the turbulent wind field can be considered “frozen” and advected by the mean wind. This assumption is however not valid under all circumstances and for all turbulent scales, and in reality the correlation between the wind speed time series measured at two points is always smaller than one. The validity of Taylor’s hypothesis on scales relevant for large wind turbines was evaluated by Schlipf et al. who found the assumption 90% accurate over a stream wise distance of 0.2 rotor diameters and up to scales on the order of two rotor diameters, measured for a 5 MW turbine [5]. It must however also be considered that the wind speed varies across the rotor area, both due to mean wind shear and large scale turbulent fluctuations. The rotational sampling of the wind turbine blades in a turbulent wind field could increase fatigue loading. Some previous investigations of spatial correlation of atmospheric wind speed at low heights can be found in the literature [6, 7], but results are harder to find for heights
~100 meters. There is however a large number of studies made on spatial and temporal correlation of turbulent flows in general, see e.g. review by Wallace [8]. In this study we will evaluate the longitudinal, lateral and vertical correlation of horizontal wind speed at scales relevant for large scale wind turbines based on point measurements from mast mounted ultrasonic anemometers.

2. Methods
A description of the spatial correlation of wind speed requires simultaneous measurements of wind speed at multiple points in space. A pulsed lidar is capable of measuring the axial wind speed upstream of a wind turbine at several distances simultaneously, but the spatial averaging makes it less ideal for correlation measurements and it can not measure the wind across the rotor plane. A scanning lidar system can measure the wind speed at any point in space, but not simultaneously. Multiple point measurements using sonic anemometers are therefore preferred, but require an array of masts, which is rarely available. In this study a 2D array of Gill WindObserver II ultrasonic anemometers was used for data collection. Horizontal wind speed and wind direction was sampled at a rate of 1Hz. Temperature was measured with calibrated pt100 probes.

Data were filtered by wind direction inside a sector of ± 5° relative to the axis between the masts for the longitudinal correlations and a sector ± 5° relative to the normal of this axis for the transversal correlation. A stationarity filter ensuring wind speed variations within 10%, wind direction variations within 10 degrees, and temperature variations within 0.5 degrees between consecutive 10-min mean values was applied. Furthermore linear detrending was applied to each timeseries.

The normalized cross-correlation coefficient for horizontal separation was calculated from

\[
R_{xy} = \frac{1}{N} \sum_{i=1}^{N} \frac{(u_1(t) - \bar{u}_1)(u_2(t + \tau) - \bar{u}_2)}{\sigma_{u_1} \sigma_{u_2}}
\]

where \(u_1\) and \(u_2\) is the time series of horizontal wind speed measured at two points in space and \(\tau\) is the time lag. In the results for the longitudinal separation the time lag is converted to a distance lag using the mean wind speed in order to make the x-axis independent of wind speed. The vertical cross-correlation is calculated as a function of vertical separation distance \(\Delta z\):

\[
R_{xy} = \frac{1}{N} \sum_{i=1}^{N} \frac{(u_1(t, z_{ref}) - \bar{u}_1)(u_2(t, z_{ref} - \Delta z) - \bar{u}_2)}{\sigma_{u_1} \sigma_{u_2}}
\]

Atmospheric stability is calculated from the Richardson number [9]

\[
Ri \approx Ri(z_m) = \frac{\partial \Delta \theta z_m}{(\Delta U)^2} \ln \left(\frac{z_2}{z_1}\right)
\]

given at height \(z_m = (z_1z_2)^{1/3}\) where \(\Delta \theta\) and \(\Delta U\) are the virtual potential temperature difference and the horizontal wind speed difference between \(z_1 = 10m\) and \(z_2 = 100m\). The Richardson number is used to estimate the Monin-Obukhov length, \(L\).

\[
\zeta = \frac{z_m}{L} = Ri
\]

\[
\zeta = \frac{z_m}{L} = \frac{Ri}{1 - 5Ri}
\]

\(Ri < 0\)

\(0 < Ri < 0.2\)

The atmospheric stability is then divided into the following stability classes:
Very stable: $0 < L < 100$
Stable: $100 < L < 500$
Neutral: $|L| > 500$
Unstable: $-500 < L < -100$
Very unstable: $-100 < L < 0$

2.1. The Frøya site

The measurements for this study were made at the Skipheia site at the western tip of the island Frøya on the coast of Mid-Norway. The terrain surrounding the site consists of a combination of low vegetation, bare rock and open water from the onshore sectors, and open sea from the south-western direction. The average roughness length determined from the mean wind profile is 0.001 m. Wind directions from south-east and south-west will be used for the longitudinal and lateral correlation estimates respectively as seen in Figure 1. The Skipheia measurement station has three masts positioned in a triangular formation as shown in Figure 1a. Only data from the two 100 m masts “Mast 2” and “Mast 4” are used in this study. “Mast 2” is instrumented with 12 Gill WindObserver 2D ultrasonic anemometers mounted in pairs at 6 levels between 10 m and 100 m above ground level while “Mast 4” is instrumented with the same anemometers at the corresponding 40 m and 100 m heights. The separation distance between the masts is 79 meters while the effective separation between the upstream anemometers varies somewhat with wind direction because of the boom orientation.

![Figure 1](image)

**Figure 1.** (a) The layout and orientation of the mast array at Skipheia. (b) An aerial view of the terrain surrounding the site with the station located by the red marker.

In Figure 2 the distribution of wind speed, stability and wind direction for the site can be seen. Since the axis between the two masts used for horizontal correlation measurements is offset from the prevailing wind directions there is a limited amount of data for some of the bins. The availability of coinciding data for correlated anemometers also varies for different atmospheric conditions. The number of averaged 10-min periods will therefore vary, but is generally on the order of 100 for the horizontal separation correlations, 1000 for the vertical separation correlations and the auto correlations.
3. Results

3.1. Autocorrelation and integral scale of turbulence

The autocorrelation function of the wind speed time series provides a time scale over which the wind speed is correlated to itself, i.e. the duration of the largest fluctuations. This can be used to estimate the integral scale of turbulence which approximates the size of the largest turbulent eddies. The integral of the autocorrelation function from zero lag to the first x-axis crossing is used to estimate the integral time scale $T$ for wind speeds between 10 and 15 m/s. The alternative method using the lag at $1/e$ crossing was also investigated, giving slightly smaller values as also found by Flay and Stevenson [7]. The autocorrelation coefficients increase with height (Figure 3a) and accordingly also the calculated time scales (Figure 3b).

3.2. Horizontal correlation

Cross correlation coefficients for the longitudinal and lateral separations based on two-point wind speed measurements are presented in Figure 4. The time lag is here, for each cross-correlation, converted to a distance lag using the mean wind speed. All stability conditions are included here.

Figure 2. (a) The distribution of stability classes at Skipheia (b) Wind rose at 100 m height.

Figure 3. (a) Autocorrelation for wind speeds in the range 10-15 m/s and neutral stability (b) Integral time scale of turbulence calculated from the autocorrelation function.

The time scale found is considerably larger in unstable conditions, which indicates a higher spatial correlation in the stream wise direction. Using Taylor’s hypothesis and the mean wind speed (12.5 m/s) to convert between a time scale and a length scale we compare the measurements with the model from ESDU 85020. The model fits the data quite well for neutral/unstable conditions.
because of limited data availability. A peak can be observed as expected around the separation distance between the masts (79m). If Taylor’s hypothesis was perfectly valid the correlation at this point should be one. Considering a wind speed preview distance on the order of one rotor diameter with a hub height of 100 m, the correlation coefficient of hub height wind speed is on average 0.8. The actual peak correlation value depends on the height and is lower at 40 m, which might be related to the increase of both wind speed and integral scale of turbulence with height.

The cross-correlation curves at both 100 m and 40 m height appear to level off to a zero correlation at a distance lag of around 500 meters, which also gives an indication of the longitudinal extent of the largest gusts. The lateral correlation of the streamwise wind speed component shown in Figure 4b also increases with height, but is as low as 0.28 at 100 m. Therefore a single hub height wind speed might not be very representative for the wind speed fluctuation over the rotor plane and some kind of spatial average of the wind speed over the rotor area should rather be used.

Comparing the zero lag correlations in Figure 4a and b confirms that the longitudinal scale of turbulence is considerably larger than the lateral scale as also shown by Kader et al.[6]. This indicates that the shape of a wind gust is oval both in the horizontal plane, and in the vertical plane shown in Figure 5. The use of a rotor averaged upstream wind preview will likely filter out much of the hub height turbulent fluctuation and only the largest gust will be used by the wind turbine controller. A small positive shift of the lateral correlation peak can be seen for reasons unknown. Two separate data loggers were used for the two masts, and a small time shift might explain part of the error. Also a systematic offset in the wind direction relative to the perpendicular mast axis could also introduce a bias. The size of the dataset is however limited, and the scatter is quite large as indicated by the 10th and 90th percentile.

3.3. Vertical correlation

Vertical cross correlations are calculated for all measurement levels using both 10 m and 100 m as reference heights. In general the spatial correlations calculated from 100 m are much higher than those from 10 m. This is supported by the increase of integral length scale with height shown in Figure 3b, and is especially apparent for high wind speeds. The correlation also increases substantially with mean wind speed which also increases with height. Therefor it is important to consider the hub height, mean wind speed and stability conditions when assessing the spatial correlations functions.
Figure 5. Vertical cross correlation as a function of vertical separation (a) downwards from 100 m and (b) and upwards from 10 m for neutral stability conditions.

For low wind speeds the correlation across the rotor span is very low and it is doubtful whether a feed-forward control would significantly increase power production. As is seen for the auto correlation, atmospheric stability also influences the vertical correlation of horizontal wind speed. The highest correlation is found for unstable conditions which are mainly associated with a convective atmosphere at low to moderate wind speeds. For extreme wind load conditions on large structures, during high wind speeds where neutral stability could be expected, the spatial correlation is lower.

Figure 6. Vertical cross correlation as a function of vertical separation (a) downwards from 100 m and (b) and upwards from 10 m for mean wind speed of 10-15 m/s.

4. Conclusion
Correlation coefficients of longitudinal wind speeds at separation distances relevant for large scale wind turbines have been evaluated. The longitudinal cross-correlation of horizontal wind speed shows a sharp peak of 0.7-0.8 around the downstream separation distance indicating that the larger turbulent structures are sustained to some degree when advected downwind as predicted by Taylor’s hypothesis. Furthermore the correlation coefficient for horizontal wind speed increases with height for both longitudinal and lateral separation. This could be related to the integral scale of turbulence which increases both with height and atmospheric instability. The longitudinal correlation is however substantially higher than the lateral and vertical correlation which indicates that the turbulent structures are oval in shape. Since the lateral and vertical correlation is low compared to the longitudinal correlation a rotor averaged wind speed preview should be preferred over a hub height preview in order to filter out turbulent structures which are small compared to the rotor area. The
vertical spatial correlation decreases more rapidly at a height of 10 m than at a typical wind turbine hub height of 100 m.

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