BCS Instability and Finite Temperature Corrections to Tachyon Mass in intersecting $D$-branes

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S.P.C., B. Sathiapalan, S. Sarkar, JHEP, 2014
S.P.C, S. Sarkar, ongoing

ISM, 2014, Puri.

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• **MOTIVATION:** Sakai Sugimoto model...BCS instability and tachyonic instability

• **INTERSECTING D1-BRANES:** A simpler problem with relevant features

• **TACHYONIC INSTABILITY:** the zero temperature spectrum

• **FINITE TEMPERATURE CORRECTIONS:** two-point functions, UV, IR divergences

• **CONCLUSION**
MOTIVATION: Sakai-Sugimtoto

- Sakai-Sugimoto model: A string theoretic model for Quantum Chromodynamics (QCD) giving a holographic description.

- Construction:
  - A background consisting of $N_c$ number of overlapping $D4$-branes compactified on an $S^1$ (Witten,'98).
  - Imposing anti-periodic boundary conditions on the $S^1$ breaks Supersymmetry. Scalars and fermions are massive at 1-loop and low energy theory is $SU(N_c)$ pure YM. (boundary)
  - Insertion of $N_f$ number of probe $D8$ and $\bar{D}8$-branes on the background (transverse to the $S^1$) (bulk).
• The SS-model demonstrates Chiral Symmetry Breaking geometrically.
• Holography: $U(N_f)_L \times U(N_f)_R$ symmetry of QCD = gauge symmetry of the $N_f$ number of $D8 - \bar{D}8$ pairs in the bulk.
• There is an upper cutoff for the radial direction (SUGRA) to the $S^1$ transverse to the $D8 - \bar{D}8$. As the radial coordinate approaches this cutoff the size of the $S^1$ shrinks.
• $D8 - \bar{D}8$-pair merges into $D8$-brane,
• $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_L$. 

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MOTIVATION

• In conventional QCD The Nambu, Jona-Lasinio-model of chiral symmetry breaking elucidates certain apparent similarities between chiral symmetry breaking and the BCS instability in superconductors.

• Inspired by this similarity, a holographic model of BCS superconductivity has been proposed within the broken chiral symmetric scenario in the Sakai Sugimoto model. (N. Sarkar, S. Sarkar, B. Sathiapalan, K. Rama)

• proposal: BCS instability (Cooper pairing between Baryons) in the boundary ($D_4$ wrapped on $S^1$) corresponds to tachyonic instability in the bulk ($D_8$).
The formation of Cooper pairs in the boundary: introduce a finite Baryon number density on the boundary theory i.e. a Chemical Potential for Baryon number.

How?: A point source of Baryon number in the bulk which creates a cusp singularity in the bulk. For two $D_8$-branes, $SU(2)$ is broken and the branes intersect at one angle between them. (Bergman, Lifschytz, Lippert)

In the SS-model a configuration of two intersecting $D_8$-branes were found to have a tachyonic instability in the bulk spectrum which is proposed to correspond to Cooper pairing instability in the boundary theory. (B. Sathiapalan, et.al.)
The tachyon mode is identified as the lowest mode in the open string excitation between the intersecting branes. (B Sathiapalan et. al., K. Hashimoto & Nagaoka, A. Hashimoto & Taylor)

There is a stable minimum in the presence of electric field. (B. Sathiapalan et.al.)

Another way of stabilizing: Finite temperature field theory.

Computation: Finite temperature one-loop mass-squared corrections to the tree-level tachyon.

Finite temperature effects: Existence of $T_c$ at which the effective mass-squared of the tachyon vanish. Our main goal is to calculate the $T_c$. 
However this problem is difficult to handle in the case of $D8$-branes on a curved $D4$-background. But many of the technical features are captured by a much simpler set-up consisting of two intersecting $D1$-branes on a flat background.

We choose to study the finite temperature effects in this simpler set-up. We are able to do so because the tachyon dynamics is a local phenomenon and not influenced significantly by curvature effects.
Validity

- The low energy theory on the brane can be described by the DBI action for the massless fields on the brane. This is valid as long as only energies $<< \frac{1}{\alpha'}$ are being probed.
- We can study this as a quantum theory with a cutoff $\Lambda < \frac{1}{\sqrt{\alpha'}}$ and proceed to study the corrections due to the massless mode quantum and thermal fluctuations.
- The Yang-Mills action (in $D \leq 3 + 1$) is finite.
- Thermal corrections should be unambiguously finite.
- Supersymmetry ensures finiteness.
INTERSECTING $D_p$-BRANES

- Consider two $D_p$-branes: world-volume:
  \[ S_{p+1} = \frac{1}{g_{YM}^2} \text{tr} \int d^p x \left[ -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi_I D^\mu \Phi_I + \frac{1}{2} [\Phi_I, \Phi_J]^2 \right] + \text{Fermions} \]

- $a = \{1, 2, 3\} = SU(2)$ gauge index,
- $I = 1, \cdots, 8 = \text{transverse directions}.$
- Background: $\langle \Phi_1^3 \rangle = qx$, separation between branes;
- $x$: world-volume. (Hashimoto, Nagaoka, D.Lust et al, etc.)
- Slope: $q = \left( \frac{1}{\pi \alpha'} \right) \tan(\frac{\theta}{2}).$
- $q = 0$: Coincident branes.
- Putting $A_0^a = 0$ removes ghosts.
The background fields: \( \{(\Phi_1^{1B}, A_2^{xB}); (\Phi_2^{1B}, A_1^{xB})\} \). The Lagrangian for the background fields decouple into two pieces, one for each of these doublets.

Define bosonic doublets \((\tau = it)\).

\[
\zeta(x, \tau) = \begin{pmatrix} A_2^{xB}(x, \tau) \\ \Phi_1^{1B}(x, \tau) \end{pmatrix}, \quad \zeta'(x, \tau) = \begin{pmatrix} A_1^{xB}(x, \tau) \\ \Phi_2^{1B}(x, \tau) \end{pmatrix}
\]

In each doublet the fields satisfy a set of coupled differential equations.

There are two sectors of solutions: \( m_n^2 = (2n - 1) \frac{g}{g_{YM}} \), \( m_n^2 = 0 \). Two different sets of normalized eigenfunctions for each of these doublet fields.
(\(A_x^2, \Phi_1^1\)), (\(A_x^1, \Phi_1^2\)):

Normalized Eigenfunctions:

\[
m_n^2 = \frac{(2n-1)q}{g_{YM}^2}
\]

\[
A_n(x) = \pm \mathcal{N} e^{-\frac{qx^2}{2}} \left( H_n(\sqrt{q}x) + 2nH_{n-2}(\sqrt{q}x) \right) e^{ik_\alpha x^\alpha},
\]

\[
\Phi_n(x) = \mathcal{N} e^{-\frac{qx^2}{2}} \left( H_n(\sqrt{q}x) - 2nH_{n-2}(\sqrt{q}x) \right) e^{ik_\alpha x^\alpha},
\]

\(n \neq 1\).

Normalized Eigenfunctions:

\[
m_n^2 = 0
\]

\[
\tilde{A}_n(x) = \pm \mathcal{N}' e^{-\frac{qx^2}{2}} \left( H_n(\sqrt{q}x) - 2(n-1)H_{n-2}(\sqrt{q}x) \right) e^{ik_\alpha x^\alpha},
\]

\[
\tilde{\Phi}_n(x) = \mathcal{N}' e^{-\frac{qx^2}{2}} \left( H_n(\sqrt{q}x) + 2(n-1)H_{n-2}(\sqrt{q}x) \right) e^{ik_\alpha x^\alpha},
\]

\(n \neq 0\).
• We turn on all the other fields as fluctuations. For the other bosonic fields:

\[ \Phi_{1n}^{1,2}(x) = \mathcal{N}^{1,2} e^{-\frac{qx^2}{2}} H_n(\sqrt{qx}) e^{ik_\alpha x^\alpha}, \quad m_n^2 = (2n + 1) \frac{q}{g_{YM}^2} \]

where \( I \neq 1 \)

• The third gauge components of all fields are massless

• \( \{ A^3_x, A^3_\alpha, \Phi^3_I \} \sim e^{ik.x} \)
The fermions in the picture play a crucial role in ensuring the UV finiteness of one-loop computations.

We shall restrict our discussion to only $D1$-branes now. We have a complete calculation for this case. For $D2$ and $D3$-branes the work is still in progress.

The fermions: sixteen left and sixteen right moving Majorana-Weyl fermions, grouped into two different sets of eight pairs distinguished by their e.o.m.

\[(\partial_0 + \partial_x)L \pm qxR = 0\]

\[(-\partial_0 + \partial_x)R \pm qxL = 0\]
• The Eigenfunctions for the Fermions: $m_n = \pm \sqrt{2nq}$

\[
L_n = \mathcal{N}_f e^{-\frac{q x^2}{2}} \left( -\frac{i}{\sqrt{2n}} H_n(\sqrt{q}x) + H_{n-1}(\sqrt{q}x) \right)
\]

\[
R_n = \pm \mathcal{N}_f e^{-\frac{q x^2}{2}} \left( -\frac{i}{\sqrt{2n}} H_n(\sqrt{q}x) - H_{n-1}(\sqrt{q}x) \right)
\]

and their complex conjugates.

• $L^3_i$ and $R^3_i$ are massless fermions(plane waves).
The bosonic doublets $\zeta_k$ are eigenvectors corresponding to the mass squared eigenvalue:

$$m_k^2 = \lambda_k = \frac{(2k - 1)q}{g_{YM}^2}$$

where $k = 0$ corresponds to tachyonic modes.
Main Idea: We implement the background field method
Fluctuations participate only at the level of loop.
We use perturbation theory to construct the full spectrum for the fluctuations.

Technical Difficulties:
Harmonic Oscillator basis
Bosonic amplitudes (two-point functions for tachyon): problem of IR + UV divergences
IR div. occur due to massless fields in the loops

\[ UV \, div \sim \sum_n \frac{1}{\sqrt{n}} \] arise from Quantum Corrections \((T = 0)\)

Fermionic amplitudes: problem of UV divergences
IR problem: A two-step Resolution

Step 1: Calculate the finite $T$ 1-loop mass-corrections for the massless fields namely, $\Phi_1^3$, $\Phi_I^3$, ($I \neq 1$) and $A_x^3$.

$m_n^2 = 0$: Infinitely degenerate massless modes corresponding to the zero eigenvalue sector: diagonalized mass matrices as a function of temperature (numerically).

Step 2: These temperature dependent masses modify the propagators in the tachyonic amplitudes.

The tachyon two-point functions are computed self-consistently (numerical computation).
• **UV problem**: for all fields.

• Finite $T$ 1-loop bosonic and fermionic amplitudes: Each term is UV divergent.

• Sum over discrete momentum $n$ (fields coupled to the background are massive)

• Integral over continuous momentum (massless modes).

• Compute the integrals involved in the vertices and expand the sums over $n$ about $n = \infty$: leading order $\frac{1}{\sqrt{n}}$.

• Cancellation between Bosonic and fermionic terms yeilds finite answer.
• No divergence from temp-dependent part.
• One-loop corrections to the tachyon mass term: set all external momenta in the Feynman diagrams $= 0$ and integrate/sum over the loop momenta.
One-loop diagrams:

Figure: one-loop bosons

Figure: one-loop fermions
FINITE TEMPERATURE CORRECTIONS

- **What do we expect?:**
  Tachyon: tree-level mass squared $= -\frac{q}{g_{YM}^2}$.

- **Corrections:**
  Quantum Corrections ($T = 0$) + Thermal Corrections ($T \neq 0$).

- **Expand the finite temperature integrands and summands about $\beta = 0$:**
  Leading order behaviour is given by $\frac{1}{\beta \sqrt{q}}$.

- The parameter $q$ provides a scale for supersymmetry breaking.
  The effective mass of the tree-level tachyon

$$m^2(q, T) = -\frac{q}{g_{YM}^2} + \left( m_0^2 + \frac{T}{\sqrt{q}} \left( \sum_n \frac{1}{\sqrt{\lambda_n}} + \cdots \right) + \mathcal{O}\left( \frac{g_{YM}^2}{q} \right) \right)$$

- $m_0^2$: Quantum corrections ($T = 0$). Only true for $1 + 1$-dimensions.
Sample plot for massless field: $\Phi^3_I$, $m^2_0 = 1.6 g^2_{YM}$
FINITE TEMPERATURE CORRECTIONS

- Numerical Plot: \( m^2(q, T) \) vs \( T \), \( |g^2_{YM}| = 1/100 \).

Figure: Mass-squared Vs \( \beta \)

Figure: Mass-squared Vs \( T \)
FINITE TEMPERATURE CORRECTIONS

\[ T_c = \sum_n \left( \frac{1}{\sqrt{\lambda_n}} + \cdots \right) \left( \sqrt{q} \left( \frac{q}{g_Y^2} - \tilde{m}_0^2 \right) \right) \]  \hspace{1cm} (0.1)

\[ \tilde{m}_0^2 = 1.6 \] is the dimensionless zero temperature quantum correction.

| \( q \) | \( T_c \) (leading order analytical) | \( T_c \) (numerical) |
|--------|----------------------------------|---------------------|
| 0.1    | 3.34                             | 3.38                |
| 0.2    | 9.48                             | 9.51                |
| 0.3    | 16.73                            | 16.79               |

Table : Comparing between analytical and numerical values of \( T_c \).
The finite temperature effects remove the tachyon instability in intersecting $D1$-branes and stabilize the configuration.

The effective mass-squared of the tree-level tachyon grows linearly with temperature as expected in $(1+1)$-dimensions.

The zero temperature quantum corrections are independent of the parameter $q$ $(1+1$-dim.).

At finite temperature the superconducting instability transits into a stable normal phase.

This phenomenon bears the hallmark of a phase transition.
FUTURE DIRECTIONS

• To do the full stability analysis we must compute the full finite temperature effective action for the tachyon, which calls for computing higher point functions.

• Our results can be generalized to higher dimensional branes ($D2$ and $D3$) without much difficulty. It will be interesting to study the issue of phase transition in higher dimensions. (ongoing)

• By scaling arguments (scaling the integrals by powers of $\beta$) we see that the finite temperature behaviour in $p + 1$-dims ($p > 1$) is $T^{p-1}$.

• Question of adding $\alpha'$-corrections in the loop may be interesting.

• Open string world-sheet perspective: calculating the annular amplitudes at finite $T$. 
THANK YOU!
The one-loop corrections from the bosonic diagrams with 4-pont vertex

\[ \Sigma^1(w, w', k, k', \beta, \omega) = \frac{1}{2} N \sum_{m} \left[ \sum_{n} \left( \frac{F_1(k, k', n, n)}{\omega^2_m + \lambda_n} + \frac{\tilde{F}_1(k, k', n, n)}{\omega^2_m} + \frac{7F_2(k, k', n, n)}{\omega^2_m + \gamma_n} \right) \right. 
+ \left. \int \frac{dl}{2\pi \sqrt{q}} \left( \frac{7F'_2(k, k', l, -l)}{\omega^2_m + l^2} + \frac{F'_3(k, k', l, -l)}{\omega^2_m + l^2} \right) \right] \delta_{w+w'} \]

(0.2)

where \( F' \)'s denote the four point vertices in this expression.

\[ V^A_{i} = -\frac{N}{g^2} \mathcal{F}^A_i(k, k', n/l, n'/l') \delta_{w+w'+m+m'} \]

(0.3)

\[ N = \sqrt{q}/\beta. \]
Finite Temperature Corrections

- The one-loop corrections from the bosonic diagrams with 3-pont vertex

\[ \Sigma^2(w, w', k, k', \beta, q) = -\frac{1}{2} qN \sum_{m, n} \left[ \int \frac{dl}{2\pi \sqrt{q}} \frac{F_4(k, l, n) F_4^*(k', l, n)}{(\omega_m^2 + \lambda_n)\omega_{m'}^2} ight. \]

\[ + \int \frac{dl}{2\pi \sqrt{q}} \frac{\tilde{F}_4(k, l, n) \tilde{F}_4^*(k', l, n)}{\omega_m^2 \omega_{m'}^2} \]

\[ + \int \frac{dl}{2\pi \sqrt{q}} \left( \frac{7 F_5(k, l, n) F_5^*(k', -l, n)}{(\omega_m^2 + \gamma_n)(\omega_{m'}^2 + l^2)} + \frac{F_5'(k, l, n) F_5'^*(k', -l, n)}{(\omega_m^2 + \lambda_n)(\omega_{m'}^2 + l^2)} \right) \]

\[ + \int \frac{dl}{2\pi \sqrt{q}} \frac{\tilde{F}_5(k, l, n) \tilde{F}_5^*(k', -l, n)}{\omega_m^2 (\omega_{m'}^2 + l^2)} \right] \delta_{w+w'} \]

\[ V_i^3 = -\frac{N_3}{2g^2} F_i^3(k, k', n/l, n'/l') \delta_{w+w'+m+m'} \]
After performing the Matsubara sums, the mass correction for the four-point vertices become

\[
\Sigma^1(k, k', \beta, q) = \frac{1}{2} \left[ \sum_n \frac{F_1(k, k', n, n)}{\sqrt{2n-1}} \left( \frac{1}{2} + \frac{1}{e^{\beta \sqrt{(2n-1)q}} - 1} \right) \right. \\
+ \sum_m \left( \frac{\tilde{F}_1(k, k', n, n)}{\omega_m^2} + \int \frac{dl}{2\pi \sqrt{q}} \frac{F_3(k, k', l-l)}{\omega_m^2} \right) \\
+ \sum_n \left( \frac{7F_2(k, k', n, n)}{\sqrt{2n+1}} \left( \frac{1}{2} + \frac{1}{e^{\beta \sqrt{(2n+1)q}} - 1} \right) \right. \\
+ \left. \left. \left( \int \frac{dl}{2\pi \sqrt{q}} \frac{15N}{2l^2} \left( (\beta l/2) \coth(\beta l/2) - 1 \right) \right) \right] \\
\] (0.6)
The mass correction for the three-point vertices after the Matsubara sum assumes the form

\[
\Sigma^2(k, k', \beta, q) = -\frac{1}{2} \sum_n \left[ \int \frac{dl}{2\pi \sqrt{q}} \frac{F_4(k, l, n) F_4^*(k', l, n)}{2n - 1} \left[ \left( \frac{\sqrt{q}}{\beta \omega_m^2} - \frac{1}{\sqrt{2n - 1}} \left( \frac{1}{2} + \frac{1}{e \sqrt{(2n - 1) q \beta} - 1} \right) \right) \right] \\
+ \int \frac{dl}{2\pi \sqrt{q}} \sum_m \tilde{F}_4(k, l, n) \tilde{F}_4^*(k', l, n) \omega_m^4 \right] \\
+ \int \frac{dl}{2\pi \sqrt{q}} \left[ \frac{7F_5(k, l, n) F_5^*(k', -l, n)}{l^2 - (2n + 1)q} \left( \frac{1}{\sqrt{2n + 1}} \left( \frac{1}{2} + \frac{1}{e \sqrt{(2n + 1) q \beta} - 1} \right) \right) \\
- \frac{1}{l} \left( \frac{1}{2} + \frac{1}{e l \beta - 1} \right) \right] \\
+ \int \frac{dl}{2\pi \sqrt{q}} \left[ \frac{F_5'(k, l, n) F_5'^*(k', -l, n)}{l^2 - (2n - 1)q} \left( \frac{1}{\sqrt{2n - 1}} \left( \frac{1}{2} + \frac{1}{e \sqrt{(2n - 1) q \beta} - 1} \right) \right) \\
- \frac{1}{l} \left( \frac{1}{2} + \frac{1}{e l \beta - 1} \right) \right] \\
+ \int \frac{dl}{2\pi \sqrt{q}} \frac{\tilde{F}_5'(k, l, n) \tilde{F}_5'^*(k', -l, n)}{l^2} \left( \frac{\sqrt{q}}{\beta \omega_m'^2} - \frac{1}{l} \left( \frac{1}{2} + \frac{1}{e l \beta - 1} \right) \right) \delta_{w+w'} \right]
\]

(0.7)
The fermionic corrections are accompanied with diagrams with only 3-point vertices.

\[
\Sigma^3(w, w', k, k', \beta, q) = (8N) \sum_{n, m, m'} \int \frac{dl}{2\pi \sqrt{q}} \frac{1}{(i\omega_m + \sqrt{\lambda'_n})} \times \left[ \frac{F^R_6(k, n, l)F^R_6^*(k', n, l)}{(i\omega_m + l)} + \frac{F^L_6(k, n, l)F^L_6^*(k', n, l)}{(i\omega_m - l)} \right. \\
+ \left. \frac{F^L_7(k, n, l)F^L_7^*(k', n, l)}{(i\omega_m + l)} + \frac{F^R_7(k, n, l)F^R_7^*(k', n, l)}{(i\omega_m - l)} \right] \delta_{w + w'}
\]

where \( \omega_m = \frac{(2m+1)\pi}{\beta} \)
The fermionic corrections are accompanied with diagrams with only 3-point vertices.

\[
\Sigma^3 (w, w', k, k', \beta, q) = \sum_{n} \left[ \int \frac{dl}{2\pi \sqrt{q}} \left( \frac{-\beta \tanh \left( \frac{\beta l}{2} \right) + \beta \tanh \left( \frac{1}{2} \sqrt{2nq} \right)}{2 \left( l - \sqrt{2nq} \right)} \right) \right. \\
+ \left. \left[ (F^R_6 (k, n, l) F^R_6 (k', n, l) + F^L_7 (k, n, l) F^L_7 (k', n, l) \right. \\
\left. + F^R_6 (k, n, l) F^L_7 (k', n, l) + F^R_6 (k, n, l) F^L_7 (k', n, l)) \right] \right] \delta_{w + w'} \tag{0.8}
\]
UV FINITENESS:leading order terms

\[
\frac{1}{2} \sum_{m,n} \frac{1}{2\pi \sqrt{2n}} \frac{1}{\omega_m^2 + \lambda_n} + \frac{1}{2} \sum_{m,n} \frac{7 \times 2}{2\pi \sqrt{2n}} \frac{1}{\omega_m^2 + \gamma_n} + \sum_{m} \frac{1}{2\omega_m^2}
\]

amplitudes for \( F_1(0, 0, n, n) + F_2(0, 0, n, n) \)

\[
- \int \frac{dl}{2\pi \sqrt{q}} \left( \sum_{m} \frac{1}{2\omega_m^2} + \frac{1}{2} \sum_{m,n} \frac{1}{2\pi \sqrt{2n}} \frac{1}{\omega_m^2 + \lambda_n} \right) + \int \frac{dl}{2\pi \sqrt{q}} \sum_{m} \frac{1}{2\omega_m^2}
\]

from \( \tilde{F}_1(0, 0, n, n) \)

\[
+ \left( \frac{1}{2} (7) \int \frac{dl}{2\pi \sqrt{q}} \sum_{m} \frac{1}{\omega_m^2 + l^2} + \frac{1}{2} \times \frac{1}{2} \int \frac{dl}{2\pi \sqrt{q}} \sum_{m} \frac{1}{\omega_m^2 + l^2} \right)
\]

from \( F_4(0, l, n) \)

amplitudes for \( F'_2(0, 0, n, n) + F'_3(0, 0, n, n) \)

\[
+ \frac{1}{2} \times \frac{1}{2} \int \frac{dl}{2\pi \sqrt{q}} \sum_{m} \frac{1}{\omega_m^2 + l^2} - \sum_{m} \frac{1}{2\omega_m^2}
\]

from \( \tilde{F}'_5(0, l, n) \)

\[
- \left( \frac{1}{2} (7) \sum_{m,n} \frac{4}{2\pi \sqrt{2n}} \frac{1}{\omega_m^2 + \gamma_n} + \frac{1}{2} \sum_{m,n} \frac{4}{2\pi \sqrt{2n}} \frac{1}{\omega_m^2 + \lambda_n} \right)
\]

amplitudes for \( F_5(0, l, n) + F'_5(0, l, n) \)
FINITE TEMPERATURE CORRECTIONS

\[ - \frac{1}{2} \left( \frac{1}{16} \sum_{m,n} \frac{1}{2\pi\sqrt{2n}} \frac{1}{\omega_m^2 + \lambda_n'} \right) \]

1st term in \( \Sigma^3(w, w, 0, 0, \beta, q) \)

\[ - \frac{1}{2} \left( \frac{1}{8} \int \frac{dl}{2\pi\sqrt{q}} \sum_{m} \frac{1}{\omega_m^2 + l^2} \right) \]

2nd term in \( \Sigma^3(w, w, 0, 0, \beta, q) \)

\[ + \frac{1}{2} \left( \frac{1}{8} \sum_{m,n} \frac{4}{2\pi\sqrt{2n}} \frac{1}{\omega_m^2 + \lambda_n'} \right) \]

3rd term in \( \Sigma^3(w, w, 0, 0, \beta, q) \)

\[ (0.10) \]

The asymptotic expansions of the one-loop corrections about \( n = \infty \) gives the above terms. There is cancellation between the leading order Bosonic and Fermionic contributions.