A Weighted Control Scheme for Topology Optimization

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Abstract. The SIMP and RAMP approaches are widely used to solve the discretized topology optimization problems with continuous design variables. Based on these two methods, this paper proposes a weighted control scheme for the purpose of taking advantage of the high efficiency of SIMP and the high stability of RAMP. The scheme is established by introducing a weighted factor and, it allows the designer to switch between SIMP and RAMP optionally. The negative feedback control technique is introduced to the proposed scheme to determine the proper value of weighted factor. With the proposed method, a desired target is firstly set by the designer to indicate the final goal. The topology optimization problem is solved by the weighted scheme to obtain process variable. The process variables and the desired target value together constitute the input matrix of the proposed scheme. Next, the error is estimated by subtract the process variable from desired target value. A correction is then applied by error-based regulator according to the error information. Finally, the desired value of weighted factor is achieved by eliminating the error to a permissible range. The weighted control scheme is verified by the heat conduction topology optimization problem.

1. Introduction

The design of geometry and topology of structural layout has great impact on the performance of structures[1]. How to place the material within a prescribed design domain to obtain the best structural performance becomes a noteworthy problem. Topology optimization gives ways to solve this problem, which involves the determination of features such as the number and location and shape of holes and the connectivity of the domain[2].

Mathematically, the topology optimization problem can be described as: find the subdomain \( \Omega_s \) with volume constraint \( G_0 \leq 0 \) and possibly \( M \) other constraints \( G_j \leq 0 \), \( j = 1...M \), included in a predetermined design domain \( \Omega \), that optimizes a given objective function \( F \) (e.g. compliance). Considering the density approach, the subdomain \( \Omega_s \) is described by introducing a density variable \( \rho \), the variables attain properties 1 in this subdomain \( \Omega_s \) and 0 elsewhere. The topology optimization problem is treated by discretizing the design domain \( \Omega \) into \( N \) finite elements and letting the material distribution be described by the \( N \) element[3]. The SIMP scheme[4,5] is widely used due to its ease of implementation and high efficiency. SIMP penalizes the intermediate densities (densities between 0 and 1) via the proportional material stiffness

\[
E_{\rho}(\rho_i) = \rho_i^p E_0, \quad g(\rho_i) = \rho_i^p, \quad f_0(\mathbf{u}) = E_0
\]

where \( p \) is the penalty factor, \( E_0 \) is the Young modulus of solid material. The penalization is achieved without the use of any explicit penalization scheme.
Quite similar to SIMP interpolation is the RAMP scheme[6], which penalizes the intermediate densities via the rational material stiffness

\[ E_q(\rho_i) = \frac{\rho_i}{1 + q(1 - \rho_i)} E_0, \quad g(\rho_i) = \frac{\rho_i}{1 + q(1 - \rho_i)}, \quad f_0(\mathbf{u}) = E_0 \quad (2) \]

where \( q \) is the penalty factor. The reason for introducing RAMP scheme is to alleviate non-concavity of the original SIMP interpolation scheme[7]. That is, RAMP scheme ensures compliance is a convex function when \( q \) is zero and a concave function for a finite and a priori known value on \( q \). The main difference between SIMP and RAMP is rather that the latter has a non-zero gradient for \( \rho_i = 0 \) which has an influence on convergence properties [8]. This makes RAMP more stable than SIMP, but not as efficient as SIMP.

As for SIMP and RAMP schemes, although the characteristics of the two are not the same, the optimization processes are coincident with each other, so it is capable of taking a unified interpolation of these two penalty schemes. The unified interpolation is usually expressed in a weighted format and is used for multiple material structures. For two material structures without void, one can use an interpolation that works with a weighted average of the Hashin-Shtrikman upper and lower bounds for each material property independently[9]; for two material structures with void, one can apply a hybrid of the SIMP and the Hashin-Shtrikman interpolation scheme to make use of the best features of both[10]. Other forms of weighted format are realized as additional constrains, that is, use the explicit penalization as extra constraints or as weighted penalty terms to the objective function.

In this article, we aim to propose a weighted interpolation scheme that takes advantage of the high efficiency of SIMP and the high stability of RAMP. To achieve this goal, the primary problem is to determine the appropriate value of weighted factor. Usually, the continuation method is suggested for this type of problem[11]. However, the parameter tuning process of the continuation method is a long manual process, and automatic parameter tuning method will be of great help in implementing the weighted scheme.

For this end, the error-controlled regulation technique is introduced in this paper to determine the suitable value of weighted factor automatically. The weighted interpolation scheme and error-controlled regulator constitute the negative feedback control system. The feedback system estimates the operational error and realize the parameter tuning by eliminating the operational errors to a certain range.

2. The Weighted Control Scheme

2.1. The Weighted Density Interpolation Function

The penalty methods are introduced to steer 0-1 solutions. A key part of these methods is the introduction of an interpolation function that expresses the physical quantities. In SIMP and RAMP schemes, the solid material is assumed to be material 1, the void is assumed to be material 2, the densities interpolates between the material void and solid can be expressed as

\[ E(0) = 0, \quad E(1) = E_0 \quad (3) \]

meaning that if a final design has density zero or one in all points, this design is a black-and-white design for which the performance has been evaluated with a correct physical model. Accordingly, we propose a material stiffness that takes a weighted form of SIMP and RAMP and satisfies (3)

\[ E_r(\rho_i) = \left[ (1 - \theta) \frac{\rho_i}{1 + r(1 - \rho_i)} + \theta \rho'_i \right] E_0, \quad f_0(\mathbf{u}) = E_0 \quad (4) \]

where \( \theta \) is the weighted factor and \( r \) is the penalty factor. The density interpolation function can be expressed as
The weighted factor determines the weights between SIMP and RAMP, one can switch between SIMP and RAMP optionally by adjusting $\theta$. The weighted control scheme turns to SIMP when $\theta = 1$, and transforms to RAMP when $\theta = 0$. Fig 1(a) depicts the penalty curves of SIMP and RAMP schemes, Fig 1(b) shows one of the penalty surfaces of the weighted control scheme, the penalty factor is set to 5 and weighted factor varies from 0 to 1. From the Fig 1 we can find that the penalty surface varies continuously from RAMP to SIMP when $\theta$ increase from 0 to 1.

2.2. The Tuning Method of the Weighted Factor

For the determination of $\theta$, we establish an negative feedback control system to search the desired $\theta$ value automatically. The feedback system can be interpreted as an error-based control system whose control flow can be summarized as follows: a measurement of the process variable is subtracted from a desired target to estimate the operational error in system status, which is then used by a regulator to reduce the gap between the measurement and the desired value. The regulator modifies the input of the system according to its interpretation of the error information.

The regulator applied is the error-controlled regulator, which is typically carried out using a Proportional-Integral-Derivative Controller (PID controller). The regulator signal is derived from a weighted sum of the error signal, integral of the error signal, and derivative of the error signal

$$u(t)=K_pe(t)+K_i\int_0^te(t)dt+K_d\frac{de(t)}{dt}$$

where $u(t)$ is the output control variable of PID controller, the error $e(t)$ is defined by $e(t)=r(t)-y(t)$, $r(t)$ is the desired target value and $y(t)$ is the measured process variable, respectively. $K_p$, $K_i$ and $K_d$ denote the coefficients for the proportional, integral and derivative terms, respectively. In this paper, we use the Ziegler-Nichols tuning method to determine the three coefficients.

The feedback control structure is displayed in Fig 2 with a block diagram representation. In this scheme, the process is the object to be controlled. The purpose of control is to make the process variable $y(t)$ follow the desired target value $r(t)$. To this end, the manipulated variable $u(t)$ is changed at the command of the PID controller.
Figure 2. The feedback control structure of the weighted control scheme

Considering the topology optimization control with the weighted interpolation (4), the iterative number of which is controlled to a desired number by adjusting $\theta$. The process variable $\gamma(t)$ is the iterative number, the desired target $r(t)$ is the final number. The weighted factor $\theta$ starts from 0, the iterative number is measured by a counter. The operational error is the departure of the iterative number as measured by the counter from the desired loop number. This measured error is interpreted by the PID controller to command the value of $u(t)$. The weighted factor $\theta$ is updated according to the control variable $u(t)$. The iterative number is then routed back as a cause-and-effect input to the control system to calculate the error again. The control system stops until the desired loop number is reached to a permissible value. Summing up the above control process, the workflow of tuning method of $\theta$ can be described as:

1. Build the optimization problem using (4), initialize the optimization parameter $r$ and $\theta$
2. Build the negative feedback control system with PID controller and the optimization problem built in step 1, initialize the PID coefficients $K_p$, $T_i$ and $T_d$, set the maximum permissible error and set-point
3. Solve the optimization problem and measure the process variable
4. Calculate the error between the measured process variable and set-point
5. If the error is in the scope of maximum permissible error, outputs $\theta$; otherwise, calculate the PID control variable using (6)
6. Update $\theta$ based on the PID control variable
7. Reassign the updated $\theta$ to the weighted interpolation function and calculate the error again
8. If the error is within the range of maximum permissible error, outputs $\theta$; otherwise, return back to step 5

3. Illustrative Examples

In this section, we implement heat conduction topology optimization problem to test the application effects of the weighted control scheme. In the following, we first define the optimization problem and then display the experimental results. For the examples below, design domains are discretized by square bi-linear 4-node finite elements, the Young modulus of solid material is $E_0 = 1$ and the Poisson ratio $\nu = 1/3$, the proportional coefficient $K_p = 2.5$, integral coefficient $K_i = 0.2$, and differential coefficient $K_d = 0.005$.

The heat conduction problem can be regarded as a multi-physics problem. The phrase “multi-physics” means that optimization problems require modeling in elastic and thermal areas (temperature field gives rise to a thermal expansion that may influence the elastic field). Figure 3 shows the typical heat conduction design problem, the square plate is evenly heated (constant source term in all nodes) and the center of the left edge is a heat sink, i.e. the temperature is set to zero. The design task of topology optimization is to find out such an optimal configuration of the structure that produces the least heat for a prescribed volume of available solid materials.
In the experimental test, we set penalty factors to fixed value, and pick two different parameters as the desired target values of control system, which are loop number and $M_{nd}$. The experiments are divided into two sub tests. In sub test 1, the control system is not limited by the maximum permissible error, which means the control system will stop the tuning once the process variable $y(t)$ is lower than the desired target $r(t)$. In sub test 2, the control system is limited by the maximum permissible error, which means the control system will stop the adjustment only when $y(t)$ is in the range of maximum permissible error. Fig 4 displays the final designs of SIMP, RAMP and the two sub tests.

Based on the test results, SIMP gets a relatively low loop number than that of RAMP. Since the purpose of this experiment is to test the control effects of negative feedback control system, we pick a number that less than that of SIMP as the desired target, which is 130. The test result of the weighted control scheme is $loop = 87$, which is much lower than those of SIMP and RAMP. The $M_{nd}$ value is 24.5097, which is also significantly lower than the value of SIMP. The changing curves of loop and $\theta$ are depicted in the Fig 5(a).

As seen in Fig 5(a), $\theta$ begins iteration from 0, where the weighted scheme is equal to SIMP at this moment. The loop number here is the same with that of SIMP, which is 155. After three iterations, the
control system stops since the loop number is less than 130. The value of $\theta$ is updated to 0.4375, the end design of this value is shown in Fig 4(c). We can see from the figure that the end design is clear-cut, without checkerboards and mesh-dependence.

For sub test 2, we set a number that the same with the desired target of test 1. The maximum permissible error is set to 20, which means the control system will stop only when $y(t)$ is between 110 and 150. The control process is depicted in Fig 5(b). The first three iterations are the same as test 1, $\theta$ value becomes negative at the fourth iteration, the control system reallocates $\theta$ to 0 as the value range is 0 to 1. As a result of this case, the loop number returns back to 155. Then after another iteration, the loop number reaches to 111, the control system stops the tuning and outputs $\theta$. The $\theta$ value is 0.4025, $M_{ad}$ is 24.8457, and corresponding end design is displayed in Fig 4(d).

4. Conclusions
In this paper, a weighted density interpolation function is proposed for topology optimization problem, the purpose of which is to take advantage of both the benefits of SIMP and RAMP. The penalty effects of the weighted interpolation function are tested. For the determination of weighted factor, we propose a negative feedback control system to find the desired values automatically based on the specific requirements. The control effects of the negative feedback control system are tested through the heat conduction problem. The experimental results show that the final designs are clear and cut, and the weighted factor can be well tuned, which verified the effectiveness of the proposed method.

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6. References
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