Cosmological Constant Implementing Mach Principle in General Relativity

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Abstract
We consider the fact that noticing on the operational meaning of the physical concepts played an impetus role in the appearance of general relativity (GR). Thus, we have paid more attention to the operational definition of the gravitational coupling constant in this theory as a dimensional constant which is gained through an experiment. However, as all available experiments just provide the value of this constant locally, this coupling constant can operationally be meaningful only in a local area. Regarding this point, to obtain an extension of GR for the large scale, we replace it by a conformal invariant model and then, reduce this model to a theory for the cosmological scale via breaking down the conformal symmetry through singling out a specific conformal frame which is characterized by the large scale characteristics of the universe. Finally, we come to the same field equations that historically were proposed by Einstein for the cosmological scale (GR plus the cosmological constant) as the result of his endeavor for making GR consistent with the Mach principle. However, we declare that the obtained field equations in this alternative approach do not carry the problem of the field equations proposed by Einstein for being consistent with Mach’s principle (i.e., the existence of de Sitter solution), and can also be considered compatible with this principle in the Sciama view.

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1 Introduction
In the one hundredth anniversary of the advent of GR, it would be instructive to investigate the intellectual roots of this theory. Actually, we propose to scrutinize the well-known fact

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that Einstein was influenced by Mach’s thoughts and positivism, particularly, the idea that one
does not know the meaning of a concept unless one has a method for measuring it. Applying this
idea in physics leads to the belief that one is not allowed to enter any concept into physics unless
an operational definition is proposed for it. In this respect, Einstein’s view, and also Bridgman’s
effort [1], gave origin to a method of philosophical investigation on the meaningfulness of scientific
theories that nowadays goes under the name of operationism, however, that a theory calls for
experiments to prove or disprove its assertions is certainly not new in physics. In this regard, the
footprint of this belief can be traced to the appearance of special relativity [2]. Indeed, Einstein’s
attempt for making a consistency between the Newtonian mechanics and the Maxwell theory led
him to provide operational definitions for the concepts of simultaneity, time interval and spatial
interval, that were finally manifested in the theory of special relativity. However, although
special relativity can be considered as a good substitution for the Newtonian mechanics, that is
also consistent with the Maxwell equations, but still it suffers from the lack of an operational
definition for the inertial observer.

In this context, the existed operational definition for the inertial observer, that was proposed
by Newton, was unacceptable for Einstein because it introduces the concept of absolute space
which itself has no operational meaning and has mostly been considered as a metaphysical one
that, usually believed, there is no room for it in physics. In this respect, Mach’s idea about the
inertia could have provided Einstein with an acceptable operational definition for the inertial
observer as an observer who moves at a constant velocity with respect to the far stars. In
fact, Mach was one of those who were against the absolute space and absolute motion as pure
mental constructs that cannot be produced in experience [3], and instead, he presumed that the
appearance of the inertial force can be related to the relative motion with respect to the far
massive bodies [3]–[5]. As, in the earliest expression [4] of Einstein’s realization of Mach’s idea
(that can be traced in his paper in 1912 [6]), he mentioned that Mach’s idea suggests that the
whole inertia of a material point can be considered as the result of the presence of all other masses
depended on a kind of interaction with them. Such a realization led Einstein to conclude two
principles, namely, the general covariance and the equivalence principles. Then, his endeavors,
in employing these two principles and implementing the Mach idea through determination of the
metric tensor, $g_{\mu\nu}$, via a tensor that describes matter and energy, resulted in the manifestation
of the theory of GR [7, 8]. He also hoped that this theory would not have any solution in
the absence of matter as the consequence of Mach’s principle[1] Nevertheless, even the trivial
Minkowski spacetime exists as a vacuum solution of it, and this fact caused Einstein to be
unsatisfied with the theory.

To remedy this issue, at first he thought the problem as the consequence of boundary con-
ditions at infinity, and thus, attempted to remove the Minkowski solution by imposing suitable
boundary conditions which do not impart any absolute structure to spacetime [4]. However, he
did not succeed. After a while, he abandoned his boundary condition formulation of Mach’s
principle, and tried to implement Mach’s idea through proposing a model for the universe [9];
wherein, nowadays, it is believed that a cosmological model depends more on symmetries than
on initial conditions. In this context, Einstein claimed that, in a finite and closed universe with

[1]More specific, the *strong* version of the Mach principle (namely, if there is no matter then there is no
geometry). However, the resulted theory only satisfies its *weak* version (namely, the matter distribution determines
the geometry).
no boundary region, the local metric is not determined in part by the boundary conditions, but rather only by the matter distribution in the universe. In other words, his impression was that by having no boundary region, there would be no room for non–Machian determination of the metric. In this scenario, he was forced to enter an additional constant term (which the theory also allows its inclusion) into the theory, and presented the field equations in the form

\[ G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]  \hspace{1cm} \text{(1)}

in four dimensions, where \( G_{\mu\nu} \) is the Einstein tensor as a function of the metric and its derivatives, \( T_{\mu\nu} \) is the energy–momentum tensor, and \( \Lambda \) is the cosmological constant. Now, Einstein expected that these amended equations would not admit any matter–free solution in the model of a closed universe. However, in the early 1917, a vacuum solution was found by de Sitter [10]. Thus, despite his later attempts for showing the theory to be consistent with Mach’s principle [11], he finally gave up the struggle and, his enthusiasm for Mach’s principle began to decrease [4, 12]. Nevertheless, the cosmological constant still survives in the theory and has obtained some other applications.

By this brief consideration, we have endeavored to emphasize on the fact that focusing on the operational meaning of physical concepts could have played an impetus role (at the beginning) in the appearance of GR. Hence, still insisting on this point of view, let us also consider the operational definition for the factor \( G \) in the Einstein equation. In this regard, it is well–known that this factor in GR is a constant which enters into the theory by making it consistent with the Newtonian theory of gravitation in the weak field limit. Actually, it is the same gravitational constant that appears in the Newtonian law of gravitation, and hence, the operational definition for it is a dimensional constant that can be gained through the measurement in an appropriate experiment, such as the Cavendish experiment [14]. For a historical review, and the list of attempts that have been performed for measuring this constant, see Ref. [15].

As all these experiments specify the value of this coupling constant locally (at most in the scale of solar system), one can say that \( G \), and hence the Einstein equation, is operationally meaningful only in a local area. Regarding this point, and insisting on the viewpoint that any concept in a theory should be operationally meaningful, pose a question on how one can apply such a theory to the large scale of the universe as a whole. In this respect, as the conformal invariant theories do not include any dimensional “constant” and also are free from belonging to any specific scale, it seems that it would be instructive to consider GR as a conformal invariant model and then, reduce the model to a theory for the cosmological scale by breaking down the conformal symmetry. This is a task that we propose to perform in this work. Hence, in the next section, we take a glance on the conformal invariance while emphasizing on our intended points. Then, in Sect. 3, we introduce the desired model for our purpose, and conclude the outcomes. Some final remarks are left to the last section.

Throughout this work, the signature is \((- , + , + , +)\), the lower case Greek indices run from zero to three, and we use units in which \( \hbar = 1 = c \).

Although, Einstein gave convincing argument against the absolute time by use of the operational method, but Bridgman [13] declares his failure in carrying it over into GR.
2 Conformal Invariance

It is evident that the values of the units of mass, length and time employed in any theory should be arbitrary. Regarding this point, the principle of conformal invariance has been developed as a principle that requires all fundamental equations in physics to be invariant under local changes of the unit systems used [16]–[18]. In principle, these changes correspond to conformal transformations of the metric which, loosely speaking, stretch/shrink all lengths and durations via spacetime–dependent conversion factors. This requirement poses a fundamental symmetry, namely, the conformal symmetry in physics which is of ever–increasing attention in modern physical theories. As in this regard, many interesting results have been obtained by applying this symmetry, see, e.g., Refs. [19]–[25].

In addition, in the conformal invariant theories, it is well–known that in the absence of dimensional parameters, the conformal invariance requires the vanishing of the trace of the energy–momentum tensor of the matter conformally coupled to gravitation, see, e.g., Ref. [26]. Also, in such theories, in the presence of dimensional parameters, the conformal invariance can be established if such parameters are conformally transformed according to their dimensions as well [17]. Indeed, in the conformal invariant material theories, the dimensional parameters are replaced by scalar fields to allow for the local changes of the unit systems, and thus, a general feature of such theories is the presence of varying dimensional coupling constants. It is also well–known that breaking down the conformal symmetry can be performed by introducing a dimensional constant into the theory. Regarding these points, it is possible to acquire gravitational coupling through conformal symmetry breaking by looking at the massless scalar matter field conformally coupled to gravity as a variant of GR, see Ref. [27]. Inspired by such an approach, and noticing that conformal transformations are also considered as transformations between metrics belonged to different scales, we propose a method for applying GR to cosmological scale.

Thus, in the next section, first we generalize GR to a conformal invariant theory via replacement of the gravitational constant by a scalar field (to let the unit systems being arbitrary at every point) and choosing suitable coefficients. As conformal transformations can also be assumed to be transformations between metrics belonged to different scales, hence, the conformal symmetry makes the new theory free from belonging to any scale. Then, we break the conformal symmetry to single out a preferred conformal frame corresponding to a specific scale. As breaking down the conformal symmetry also amounts to considering a constant value for the scalar field (i.e., the varying gravitational coupling), we perform the symmetry breaking by fixing the value of the scalar field in a way that the theory is reduced to a conformal frame which is characterized by the large scale characteristics of the universe as a whole. Thus, in this way, we apply GR to cosmological scale.

3 The Model

Following the mentioned explanations, in a 4–dimensional spacetime, we replace the Einstein–
Hilbert action by the well–known conformal invariant action functional

\[ S[\varphi, g_{\mu\nu}] = \frac{1}{2} \int d^4x \sqrt{-g} \left( \frac{1}{6} R \varphi^2 + \partial_{\mu} \varphi \partial^{\mu} \varphi + \frac{1}{2} \lambda \varphi^4 \right), \]  

(2)

where \( R \) is the Ricci scalar, \( \varphi \) is a real scalar field non–minimally coupled to the gravity (that as explained, it is a substitution for the gravitational constant in GR), and \( \lambda \) is a dimensionless coupling constant. Action (2) is invariant under the conformal transformations

\[ \varphi \rightarrow \tilde{\varphi} = \Omega^{-1}(x) \varphi \quad \text{and} \quad g_{\mu\nu}(x) \rightarrow \tilde{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x), \]  

(3)

apart from a complete divergent term that gives no contribution to the variation of the relevant action, where the conformal factor \( \Omega(x) \) is a non–vanishing arbitrary smooth function of the spacetime coordinates.

Before adding an action for the matter, let us remind that the establishment of the conformal symmetry in the vacuum sector of a gravitational model confronts one with a problem concerning the incorporation of matter to the gravity. Indeed, as all the conformal frames are dynamically equivalent, it raises the question that to which frame the matter should be coupled. As a choice, we trust the weak equivalence principle (although, it is not reliable at quantum level), and according to this principle, the metric appears in the matter part should be the same one that describes the gravitational field. Thus, we consider the total action by adding a matter source \( S_m \) with the same metric and independent of \( \varphi \) to action (2), i.e. \( S = S[\varphi, g_{\mu\nu}] + S_m \). Now, varying the total action with respect to the scalar and metric fields yields

\[ \Box \varphi - \frac{1}{6} R \varphi - \lambda \varphi^3 = 0 \]  

(4)

and, with \( \varphi \neq 0 \),

\[ G_{\mu\nu} - \frac{3}{2} \lambda \varphi^2 g_{\mu\nu} = 6\varphi^{-2} \left( T_{\mu\nu} + T_{\mu\nu}^{[\varphi]} \right), \]  

(5)

where

\[ T_{\mu\nu}^{[\varphi]} \equiv -\partial_{\mu} \varphi \partial_{\nu} \varphi + \frac{1}{2} g_{\mu\nu} \partial_{\rho} \varphi \partial^{\rho} \varphi - \frac{1}{6} (g_{\mu\nu} \Box - \nabla_{\mu} \nabla_{\nu}) \varphi^2 \]  

(6)

That is usually used as a model for a matter field conformally coupled to gravity. This action may also be considered corresponding to the action of Brans–Dicke theory in the special case of \( \omega = -\frac{3}{2} \), where under transformation \( \tilde{g}_{\mu\nu} = \varphi^2 g_{\mu\nu} \), one can get it as \( S[\tilde{g}_{\mu\nu}] = \frac{1}{2} \int d^4x \sqrt{-\tilde{g}} \left( \frac{1}{6} R \tilde{g} - \frac{1}{2} \lambda \tilde{g} \right) \) without the scalar field [28]. Nevertheless, it should be noted that the scalar field in the Brans–Dicke theory has a different meaning than the one in our approach. In this theory, the value of the gravitational constant varies with position as the consequence of Mach’s principle, i.e. the effect of matter distribution in the universe on the value of this constant at every point. However, in action (2), \( G \) has been replaced by a scalar field as the consequence of the conformal invariance and the arbitrariness of unit systems at every point, actually, the same as the scalar field proposed by Dirac in the Weyl–Dirac action [29].

In a spacetime with arbitrary dimension \( n \), the conformal invariance can be achieved if it is accompanied by the appropriate rescaling of the scalar field as \( \varphi \rightarrow \Omega^{w}(x) \varphi \), where the conformal weight \( w \) depends on the dimension of the scalar field of the model and is \( w = 1 - n/2 \), see, e.g., Ref. [30]. However, the resulted two new fields still remain independent.

However, in general, one can assume that the metrics in the gravitational and the matter parts are different (although conformally related), and achieves interesting results, see, e.g., Ref. [25].
and as usual the symmetric energy–momentum tensor of matter is defined as $T_{\mu\nu} \equiv -(2/\sqrt{-g}) \times \delta S_m/\delta g^{\mu\nu}$. By taking the trace of equation (5), while using equation (4), one obtains

$$T_{\mu\mu} = 0,$$

which is similar to the well–known condition on the matter field conformally coupled to gravitation.

At this stage, as explained before, we break the symmetry and fix the value of the scalar field by introducing a dimensional constant, say $\Lambda$, into the theory. For this purpose, we add

$$S_\alpha(x) = \frac{1}{2} \int d^4x \sqrt{-g} \alpha(x) \left( \frac{1}{2} \lambda \varphi^4 - \frac{1}{3} \alpha \Lambda \varphi^2 \right)$$

(8)

to the total action. In this case, the corresponding equations (4) and (5) are

$$\Box \varphi - \frac{1}{6} R \varphi - (1 + \alpha) \lambda \varphi^3 + \frac{1}{3} \alpha \Lambda \varphi = 0$$

(9)

and, with $\varphi \neq 0$,

$$G_{\mu\nu} - 3 \left[ \frac{1}{2} (1 + \alpha) \lambda \varphi^2 - \frac{1}{3} \alpha \Lambda \right] g_{\mu\nu} = 6 \varphi^{-2} \left( T_{\mu\nu} + T_{[\mu]}^{[\nu]} \right),$$

(10)

while the variation with respect to $\alpha$ gives a constant value to the scalar field, i.e. $\varphi^2 = 2\Lambda/(3\lambda)$, although its value is not definite yet. However, by substituting this constant value of $\varphi$ into the trace of equation (10), one also gets

$$\varphi^2 = -\frac{T_{\mu\mu}^\mu}{R/6 + 2\Lambda/3},$$

(11)

i.e., as expected, the conformal symmetry breaking is equivalent to coupling a matter with non–zero trace energy–momentum tensor to gravity, while singles out a specific conformal frame corresponding to the constant value of the scalar field.

Now, to consider the symmetry breaking as a cosmological effect, we plausibly take $\Lambda$ of the order of observational bound on the cosmological constant, i.e. $R_0^{-2}$, where $R_0$ is the radius of the universe at the present epoch (i.e., roughly the Hubble distance). We also approximate $T^{\mu\mu}_\mu$ by its average value, that is the mass density of the universe in central mass frame, i.e. $\langle T^{\mu\mu}_\mu \rangle \sim -M_0 R_0^{-3}$ (where the negative sign is due to the signature), while neglecting the (positive) energy content of the universe (like photons), as these are identically traceless. By considering these approximations in relation (11), the Ricci scalar should also have a constant value (just as in GR when the trace of the energy–momentum tensor is a constant). Further, following the symmetry breaking as the cosmological effect, and as the dimension of this constant is one over length square, we take it to be of the order of $R_0^{-2}$ too, for $R_0$ being a length that characterizes the

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7 Note that, in four dimensions, with the conformal transformations (3) not only the field equation (4) is conformally invariant, but if, in addition to (3), the traceless symmetric energy–momentum tensor of the matter transforms as $\tilde{T}_{\mu\nu} = \Omega^{-2}(x) T_{\mu\nu}$, then the field equation (5) and the conservation equation $\nabla_{\mu} T^{\mu\nu} = 0$ are also conformally invariant, i.e. $\nabla_{\mu} \tilde{T}^{\mu\nu} = 0$ [25].

8 This condition is not a holonomic constraint, and we have introduced an auxiliary field $\alpha(x)$ to have such a condition, wherein with the obtained constant value of $\varphi$, the field equations also yield $\alpha = 9\lambda T_{\mu\mu}^\mu/(2\Lambda^2)$. 

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size of the universe. Then, replacing back these approximations into relation (11) leads to the estimation of constant value of \( \phi \) as

\[
\phi^2 \sim \frac{6 M_0}{5 R_0} \sim G^{-1},
\]

where, in the second proportionality, we have used the well–known empirical cosmological coincidence dimensionless relation \( G M_0/R_0 \sim 1 \), \[31\]–\[33\], that is the Schwarzschild radius of the universe agrees with its size. At last, by inserting the obtained fixed value of \( \phi \) into the field equations (10), one obtains

\[
G_{\mu\nu} - \Lambda g_{\mu\nu} \sim 5 G T_{\mu\nu}.
\]

(13)

Therefore, in this alternative approach, wherein the attempt has been performed to achieve an extension of GR for the cosmological scale, we have also reached to the same value for \( G \) as is obtained locally, and to the same field equations as Einstein proposed for this scale through his approach of making GR consistent with Mach’s principle. However, in the proposed model, the field equations (13) have been achieved as the result of breaking down the conformal symmetry by fixing the value of the scalar field through introducing the cosmological constant. Moreover, in the absence of matter, the field equations (9) and (10) yield \( \Lambda = 0 \), however, see also the following properties.

Also, almost the same result can be obtained via this model with no potential term \( (\lambda \phi^4) \) in action \[2\], but through a different way of symmetry breaking as has been considered in Ref. \[27\]. Actually, the variation of this new action (used in that paper) gives the same field equations (4) and (5) with \( \lambda = 0 \) terms, and yet with the same traceless energy–momentum tensor, i.e. equation (7). However, to break down the conformal symmetry in this potential–free action, a dimensional mass term for the scalar field, i.e. \( M^2 \phi^2/2 \), has been added to its related Lagrangian, which leads to the corresponding field equations

\[
\partial_\mu \partial^\mu \varphi - \frac{1}{6} R \varphi - M^2 \varphi = 0
\]

(14)

and

\[
G_{\mu\nu} - 3 M^2 g_{\mu\nu} = 6 \varphi^{-2} \left( T_{\mu\nu} + T^{(\varphi)}_{\mu\nu} \right),
\]

(15)

with \( T^\mu_{\mu} = -M^2 \varphi^2 \). Thus, by the same approximation for the energy–momentum tensor and assuming the mass \( M \) of the order of \( R_0^{-1} \), the scalar field is fixed. Then, inserting it into

\[9\]

It also infers that the inertial energy of a particle (with mass \( m \)) is due to the gravitational potential energy of the matter of the universe upon it, i.e. \( mc^2 - GM_0 m / R_0 \sim 0 \), more or less a mathematical formulation of the Machian point of view \[34\]–\[35\].

\[10\]

The constant value of \( \phi \) and those approximations also give \( \lambda = 5/(9 M_0 R_0) \) and \( \alpha = -5/2 \). In addition, the proportionality factor 5 in equation (13) can be improved. That is, if one considers all the employed approximations as \( \Lambda = a R_0^{-2} \), \( R = b R_0^{-2} \), \( T^\mu_{\mu} = -c M_0 R_0^{-3} \) and \( R_0 / M_0 = d G \), hence, instead of the proportionality factor 5, one will get the equality factor \( d (b + 4a) \). Furthermore, one can usually consider \( d \sim 1 \sim b \) up to the first order, and also uses the relation \( \Lambda = 3 \Omega_\Lambda H^2 \) while assumes \( R_0 = \beta H_0^{-1} \) (i.e., \( R_0 = \beta d_H \) at the present epoch). Thus, with the Planck data \[36\] for \( \Omega_\Lambda \) at the present epoch, if the resulted factor is supposed to be \( 8 \pi \), it will yield \( \beta \simeq 2 \).

\[11\]

Note that, there is no explanation in this paper about the meaning one can attribute to the scalar field; and however, as mentioned, the approach of this work inspired us in figuring out the way of applying conformal symmetry, and then, breaking it as a method for acquiring a generalization of a theory operationally valid in some scale, for some other scales.
equation (15), leads to the same field equations (13) (where the factor 5 has been replaced by 6) with an effective cosmological constant, of the right sign, i.e. $3M^2 \sim R_0^{-2}$, that in turn, is proportional to $T_{\mu \nu} \left| \varphi_{\text{fixed}} \right.$.

Eventually, the field equations (13), that are obtained from this alternative approach, possess the following properties.

- First, the coupling constant $G$, that is acquired via breaking down the conformal symmetry, is related to the mass distribution in the visible universe in the form of $GM_0/R_0 \sim 1$.

- Second, as the breaking down of the symmetry is equivalent to considering a matter field with non–zero trace, hence, the existence of matter in the universe is a presupposition for obtaining these field equations.

- Third, the presence of the cosmological constant in the field equations is actually the indication of the above presupposition.

These properties make the field equations (13) different from those proposed by Einstein.

Now, as the motivation behind the Einstein field equations was making consistency between GR and Mach’s principle, let us investigate the consistency of the field equations (13) with this principle too. In this context, according to the second and third properties, the vacuum solution for these field equations is meaningless. Although, by having such a presumption one cannot check the consistency of these field equations with Mach’s principle (through the vacuum solution) in a closed universe, but, in this alternative approach, the de Sitter solution cannot either be as an evidence against Einstein’s expected consistency between these field equations and the Mach principle. On the other hand, as the relation between $G$ and the mass distribution in the visible universe (i.e., $GM_0/R_0 \sim 1$), besides being an observational relation, is also a representation of Mach’s principle proposed by Sciama [34], the first property indicates the consistency of the field equations (13) with Mach’s principle in the Sciama view. Therefore, one can claim that in this work, the cosmological constant is playing the same role as Einstein hoped, i.e. the cosmological constant introduced into GR has implemented Mach’s principle via this alternative approach.

Also, an accepted view toward Mach’s principle is the formulation by Bondi, i.e. “local inertial frames are determined through the distributions of energy and momentum in the universe by some weighted averages of the apparent motions” [37], that has been illustrated in the cosmological perturbation theory through the existence of the gauges in the theory called “Machian gauges” in which the distribution of $\delta T^\mu_\nu$ determines uniquely and instantaneously the rotations and accelerations of local inertial frames via Einstein’s field equations [38]. Moreover, it has been shown that in such a gauge, only differences of rotation rates of inertial frames are determinable for the case of a closed universe, and hence, no absolute rotations exist [39] in accord with Mach’s ideas that all motions are relative. However, in the field equations (13), as the value of the gravitational coupling depends on the energy–momentum tensor of the matter in the universe, it may change the results, and this matter can be investigated.

On the sidelines, although significant on its own, we should emphasize that the result of the approach (that implements the cosmological constant should actually be related to the matter) somehow reminds one of the interpretation of the vacuum fluid and the vacuum energy density, see, e.g., Refs. [40, 41]. And more important, as the gravitational field equations in the
alternative approach cannot be considered without a non–zero cosmological constant term, and wherein also, in the reverse side, the matter being somehow dependent on it, such a situation reminds one of the ether issue, see, e.g., Ref. [42] and references therein. In this respect, we should mention that even Einstein himself claimed [43, 44] that, in principal, the general theory of relativity is merely an ether theory.

4 Final Remarks

By insisting on the viewpoint that any concept in a theory should possess an operational meaning, we have applied GR to the cosmological scale through its replacement with a conformal invariant model and then, have reduced this model to a conformal frame characterized by the large scale characteristics of the universe. Finally, we have reached to the same field equations (GR plus a cosmological constant) that Einstein proposed for this scale with the purpose of making consistency between GR and Mach’s principle, although he was not successful. However, in this alternative approach, the theory itself determines the constant $G$ in the cosmological scale. Also, we have declared that the field equations obtained in this approach do not carry the problem of Einstein’s field equations for being consistent with Mach’s principle and, they can be considered compatible with this principle in the Sciama view as well.

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