Physical insight into electromagnetic kinetic energy transducers and appropriate energy conditioning for enhanced micro energy harvesting

Joachim Leicht¹, Thorsten Hehn², Dominic Maurath¹, Christian Moranz¹ and Yiannos Manoli¹ ²
¹ Fritz Huettinger Chair of Microelectronics, Department of Microsystems Engineering - IMTEK, University of Freiburg, Germany
² HSG-IMIT - Institute of Micromachining and Information Technology, Villingen-Schwenningen, Germany
E-mail: joachim.leicht@imtek.uni-freiburg.de

Abstract. This paper proposes a new method for modeling electromagnetic kinetic energy transducers and gives analytical expressions that enable the design of efficient energy conditioning circuitry. The introduced transducer modeling approach achieves high accuracy without requiring a large set of parameters. The presented transducer characterization allows physical insight into fully assembled and packaged transducers in order to extract the required transducer model parameters without knowledge of the individual components. Moreover, the electromagnetic coupling, the parasitic damping, and the optimal load can be modeled with a dependence on the external excitation. Precise co-simulation with CMOS integrated energy conditioning circuitry is possible implementing this model in a circuit simulator.

1. Introduction
Kinetic micro energy harvesting enables the use of ambient vibrations in order to supply low power electronic systems such as wireless sensor nodes [1, 2]. In order to guarantee autonomous and reliable system operation, the ambient energy extraction has to be optimized. Therefore, a well-designed kinetic energy harvesting system composed of a kinetic micro power transducer and an efficient energy conditioning such as active rectification is needed (Figure 1).

Efficient system design requires an accurate transducer model and precise definitions of the optimal operation conditions of the entire system. In order to design an efficient energy harvester highly accurate transducer modeling is mandatory and a large set of model parameters is needed [3, 4]. These parameters are difficult to extract from a fabricated harvester without deep transducer design knowledge which usually applies to circuit designers. However, for the design of efficient energy conditioning circuits a reduced parameter set is sufficient that describes the optimum transducer operation precisely. Harvester disassembly allows to extract this set [5].

A fully packaged small-scaled electromagnetic kinetic energy transducer prototyped by the HSG-IMIT (Figure 1) is characterized and modeled accurately in this work without disassembly. Moreover, definitions are proposed that detail the optimum operation of energy conditioning based on ultra-low power CMOS integrated active rectification with low voltage drop [6].
2. Electromagnetic kinetic energy transducer modeling and characterization

Figure 2 details the applied equivalent circuit model comprising the needed model parameters [7]. Maximum power \( P_{\text{max}} \) is harvested at resonance when the transducer load resistance \( R_{\text{load}} \) matches its optimum \( R_{\text{opt}} \) [3–6, 8]. By means of a defined transducer excitation ensured by a controlled vibration table [6] the maximum power, the optimal load resistance, and the resonance frequency \( f_{\text{res}} \) can be measured by performing sweeps over frequency and over the load resistance.

The characterized electromagnetic transducer is a low damped electromagnetic spring-mass-damper system and so the resonance frequency almost equals the natural frequency \( f_0 \) [8]. Thus, for all presented expressions \( f_{\text{res}} \) is assumed to be \( f_0 \) and the spring stiffness \( k \) can be given as

\[
k = (2\pi f_0)^2 m = (2\pi f_{\text{res}})^2 m,
\]

where \( m \) is the seismic mass.

Measurement results of \( P_{\text{max}} \) and \( R_{\text{opt}} \) at resonance are presented in Figure 3 for different transducer peak excitation accelerations \( a_{\text{ex}} \). By means of these results and the easily measurable coil series resistance \( R_{\text{coil}} \) the expressions

\[
d_m = \frac{d}{m^2} = \frac{a_{\text{ex}}^2 (R_{\text{opt}} - R_{\text{coil}})}{8 R_{\text{opt}} P_{\text{max}}},
\]

\[
c_m = \frac{c}{m} = \frac{a_{\text{ex}} (R_{\text{opt}} - R_{\text{coil}})}{(8 R_{\text{opt}} P_{\text{max}})^{1/2}} = \sqrt{d(R_{\text{opt}} - R_{\text{coil}})}
\]

can be derived, where \( d_m \) embodies the parasitic damping \( d \) normalized to the square of the seismic mass \( m^2 \) and \( c_m \) the electromagnetic coupling normalized to \( m \).
Figure 3. Measured maximum power $P_{\text{max}}$ and measured optimal load resistance $R_{\text{opt}}$ of the electromagnetic kinetic energy transducer.

Figure 4. Normalized parasitic damping $d_m$ and corresponding normalized electromagnetic coupling $c_m$.

Figure 5. Normalized equivalent circuit model of the electromagnetic kinetic energy transducer connected to a load resistance.

Figure 6. Normalized equivalent circuit model of the transducer connected to a resistive load at resonance.

Figure 4 illustrates calculation results of $d_m$ and $c_m$ as well as the corresponding linear fits which are used in circuit simulations. This allows accurate simulations even though the optimal load resistance, the electromagnetic coupling as well as the parasitic damping depend on the external excitation as demonstrated in Figure 3 and Figure 4 \[4, 9\].

The normalized parameters $d_m$ and $c_m$ result in the normalized equivalent circuit model of the electromagnetic kinetic energy harvester shown in Figure 5. In Figure 6 the corresponding reduced normalized model valid at resonance is given. It is demonstrated that the seismic mass is completely incorporated in the normalized parameters at resonance for this linear load scenario.

3. Guidelines for optimal energy conditioning circuitry design

In order to harvest the highest possible energy by means of active rectification the transducer must be excited at resonance and the active rectifier (Figure 1) must be operated at the maximum power point (MPP) (Figure 7) \[6\]. Maximum power point operation can be accomplished via a MPP-harvesting interface \[10\]. The design of such an interface requires knowledge about the optimum transducer load and the optimum rectifier load that enables MPP operation.

The normalized model given in Figure 6 allow to express the optimal load resistance $R_{\text{opt}}$ and the maximum load power $P_{\text{max}}$ of the electromagnetic kinetic energy transducer at resonance using the original model parameters \[3–5, 8\] as well as the extractable normalized parameters:

$$R_{\text{opt}} = \frac{c_m^2}{d_m} + R_{\text{coil}} = \frac{c_m}{d_m}^2 + R_{\text{coil}},$$

$$P_{\text{max}} = \frac{\dot{V}_{\text{opt}}^2}{2R_{\text{opt}}} = \frac{(m\ddot{a}_\text{exc})^2}{8d(c^2 + R_{\text{coil}}d)} = \frac{\dot{a}_\text{exc}c_m^2}{8d_m(c_m^2 + R_{\text{coil}}d_m)} = \frac{\dot{a}_\text{exc}^2 R_{\text{opt}} - R_{\text{coil}}}{8d_m R_{\text{opt}}},$$
where \( \hat{V}_{\text{opt}} \) is the optimal peak voltage across \( R_{\text{opt}} \) delivered by the transducer.

Assuming an ideal full wave bridge rectifier composed of ideal diodes allows to give approximate MPP-harvesting interface design expressions by means of \( R_{\text{opt}} \), \( P_{\text{max}} \), and \( R_{\text{coil}} \):

\[
P_{\text{MPP}} \approx P_{\text{max}} \frac{R_{\text{opt}} - R_{\text{coil}}}{R_{\text{opt}}},
\]

\[
V_{\text{MPP}} \approx \sqrt{2P_{\text{max}} R_{\text{opt}} - R_{\text{coil}}} = \hat{V}_{\text{opt}} \frac{R_{\text{opt}} - R_{\text{coil}}}{R_{\text{opt}}},
\]

\[
R_{\text{MPP}} \approx 2(R_{\text{opt}} - R_{\text{coil}}),
\]

\[
\alpha_{\text{MPP}} \approx 2 \arccos \left( \frac{R_{\text{opt}} - R_{\text{coil}}}{R_{\text{opt}} + R_{\text{coil}}} \right),
\]

where \( P_{\text{MPP}} \) is the maximum rectifier load power enabled by the optimal rectifier load resistance \( R_{\text{MPP}} \). At the MPP the rectified voltage \( V_{\text{rect}} \) (Figure 1) is driven to \( V_{\text{MPP}} \). This voltage level induces the conduction angle \( \alpha_{\text{MPP}} \) of the ideal diodes. The given expressions are also applicable for a low voltage drop active rectifier with low power loss as verified in Section 4.

Excitations below the mechanical seismic mass displacement limit of the packaged harvester shown in Figure 1 when loaded with the optimal resistance at resonance are considered [3].

### 4. Results and verifications

Figure 7 shows simulation results (dashed) and measurement results (solid) of \( P_{\text{max}} \) for an excitation of \( \hat{a}_{\text{ex}} = 1 \text{ m/s}^2 \) at resonance. Moreover, measured (solid) and simulated (dashed) characteristics of the active rectifier load power \( P_{\text{rect}} \) are shown. These simulation results are achieved with model parameters valid for the actual excitation (\( \hat{a}_{\text{ex}} = 1 \text{ m/s}^2 \)). Simulation and measurement results correspond almost perfectly. The dotted characteristics show simulation results achieved with model parameters determined at another excitation (\( \hat{a}_{\text{ex}} = 3 \text{ m/s}^2 \), Figure 4) different from the actual excitation (\( \hat{a}_{\text{ex}} = 1 \text{ m/s}^2 \)). The mismatch of these dotted characteristics compared to the measurement results demonstrates that it is insufficient to use only a single set of constant parameters. The presented simulations are performed with the original model shown in Figure 2 implemented in the circuit simulator applying the real weight of the seismic mass.

In Figure 8 the seismic mass \( m \) is swept up to the weight of the packaged transducer which is the possible maximum weight by applying the normalized model shown in Figure 5 with the
linear fits of the calculated parameters (Figure 4) implemented in the circuit simulator. Figure 8 shows the maximum transducer load power $P_{\text{max}}$, the optimal transducer peak voltage $V_{\text{opt}}$, the maximum active rectifier load power $P_{\text{MPP}}$, and the optimal rectified voltage $V_{\text{MPP}}$ for an excitation of $\hat{a}_{\text{ex}} = 1 \text{ m/s}^2$ at resonance. These simulation results show almost independence of the seismic mass $m$, demonstrating that $m$ needs not to be known exactly in order to accurately simulate the optimum operation of the kinetic energy harvesting system shown in Figure 1. This optimum operation simulation accuracy allowed by the normalized model is of major importance for the design of an efficient MPP-harvesting interface. Hence, there is no need to know $m$ exactly or to disassemble the harvester [5] in order to extract the weight of the seismic mass.

Figure 9 presents the measured maximum active rectifier load power $P_{\text{MPP}}$ and the corresponding voltage $V_{\text{MPP}}$ at resonance. Furthermore, corresponding predicted values achieved by means of (6) and (7) are given. In Figure 10 simulation results of the optimal rectifier load resistance $R_{\text{MPP}}$ and the optimal conduction angle $\alpha_{\text{MPP}}$ as well as the predicted results calculated with (8) and (9) are shown. The MPP prediction precision demonstrated in Figure 9 and Figure 10 verify the utility of the energy conditioning design guidelines given in Section 3.

5. Conclusion
The proposed electromagnetic kinetic energy harvester model allows precise co-simulations with pure resistive and nonlinear loads such as active rectifiers also for variable excitations. Fully packaged harvesters can be parameterized by means of the presented transducer characterization without harvester disassembly. The proposed optimal energy conditioning design methodology allow appropriate interface circuitry in order to increase the harvested electrical energy output.

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