Does confined turbulent convection ever attain the ‘asymptotic scaling’ with 1/2 power?

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Abstract. We examine turbulent thermal convection at very high Rayleigh numbers using helium gas in a cylindrical container of diameter-to-height aspect ratios of 1 and 4, and confirm that the Nusselt number, $Nu$, follows an approximately 1/3 power of the Rayleigh number, $Ra$, up to some value of $Ra$ that depends on various experimental conditions, such as the nearness to the critical point of helium as well as the aspect ratio. An enhancement of heat transport occurs for higher $Ra$, coinciding with a substantial increase in Prandtl number as well as in various measures of Boussinesq conditions, and marking the transition from the region in which $Nu \approx 0.064Ra^{1/3}$ to another in which $Nu \approx 0.078Ra^{1/3}$. By necessity, the Nusselt number in the transition region increases more steeply than 1/3. The transitional slope, which happens to be close to 1/2, does not occur at unique values of $Ra$ for given $Pr$ and so should not be mistaken for the ‘ultimate regime’ of Kraichnan. By comparing various experiments performed under different conditions (but in the apparatus with the same horizontal surfaces), we empirically find that substantially negative values of a non-dimensional parameter related to fluid conductivity and viscosity correlate well with the observed enhancement of heat transport.
1. Introduction

In a previous study of turbulent convection [1] in a cylindrical container of diameter-to-height aspect ratio unity, we noted that the Nusselt number–Rayleigh number law attained an approximately 1/3 power at sufficiently high Rayleigh numbers. This result was valid in the range of $Ra$ for which all known experimental corrections were negligible and the Boussinesq approximation was well satisfied (for instance, the Boussinesq parameter $\alpha\Delta T$ had remained smaller than 0.05). The definitions are standard: the Rayleigh number $Ra \equiv \alpha\Delta T g H^3/\nu\kappa$, where $\alpha$ is the isobaric thermal expansion coefficient of the fluid, $\Delta T$ is the temperature difference between the bottom and top walls, $g$ the acceleration due to gravity, $H$ the vertical dimension of the convection cell, and $\nu$ and $\kappa$, respectively, the kinematic viscosity and the thermal diffusivity of the fluid. The Nusselt number, $Nu$, is the ratio of the measured heat flux to that calculated for molecular conduction alone under the same conditions. The use of cold helium gas, pioneered by Threlfall [2] and Castaing et al [3], has made it possible to achieve not only these high $Ra$ but also a wide range of $Ra$ in a single apparatus [4]. The latter is accomplished by suitably adjusting the experimental operating points in the pressure–temperature phase space of the gas.

For $Ra > 2 \times 10^{14}$, the Nusselt number began to exceed the 1/3 power of $Ra$. We had not analyzed these data in detail for two important reasons. Firstly, the Boussinesq parameter $\alpha\Delta T$ had started to increase to something substantial like 0.2 around $Ra = 2 \times 10^{15}$. Although some authors seem to believe that Boussinesq approximation holds even if $\alpha\Delta T$ is as high as 0.2, we chose to be conservative. Secondly, while the bulk of the attainable $Ra$ was achieved well away from the critical point of helium gas, using only the dependence on $\rho^2$ to increase $Ra$ ($\rho$ being the density of the fluid), the high end of the $Ra$ range was attained by taking advantage of the diverging specific heat (as well as the thermal expansion coefficient to which it is thermodynamically related) close to the critical point. These divergences are noticeable quite ‘far’ from the critical point, as measured by the reduced temperature. The increasing specific heat is associated with increasing Prandtl number, $Pr = \nu/\kappa$, which is another dynamical parameter for the flow, although the Prandtl number dependence is presumed to be weak (e.g. [5, 6]). Furthermore, the uncertainty in the $Pr$ dependence associated with the laminar–turbulent transition of the thermal and viscous boundary layers has not been subjected to adequate testing. These reasons prompted us to take a rather cautious approach in [1]; in particular, we refrained from attributing the increase in the $Nu$–$Ra$ slope to the attainment of Kraichnan’s ‘ultimate regime’ [7]. We were partly dissuaded by the fact that increasing the

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**References**

1. [Threlfall](http://www.njp.org/) [2]
2. [Castaing et al](http://www.njp.org/) [3]
3. [5, 6]
4. [7]
values of Pr pushed the experimental system well away from conditions for which one might have expected the Kraichnan regime to occur.

An important observation is that no similar enhancement of heat transport occurred in our aspect ratio 1/2 experiments [4], and a recent repetition of the measurements [8, 9] verified this conclusion. For aspect ratios of 1 and 1/2, the bottom and top plates were the same; we simply changed the vertical distance between the two plates to attain different aspect ratios. Thus, we still need to explain how this enhancement of heat transport could occur in some experiments and not in others. Plausible corrections, for example for finite plate conductivity [10], would only change the data by an order of a per cent. A significant difference between the experiments may be changes in the boundary layer structure, according to estimates in [1] and [12].

In an effort to explain the enhancement of heat transport increases for aspect ratio unity, we have now pushed the Rayleigh number higher than before. We have collected additional data for aspect ratio unity using operating points in the pressure–temperature phase space that are somewhat different from those reported in [1]. With the aid of these new data, we find that the Nusselt number simply changes from one region of 1/3 power-law dependence on Ra to another that is roughly 20% higher in numerical value (see figure 1). The ‘transition’ region between the two 1/3-regions has, by necessity, a larger slope, which happens to be close to 1/2, but this ‘special’ rational number does not necessarily have any intrinsic significance in this instance. By all available estimates [1], the transition in aspect ratio unity is not related specifically to the laminar–turbulent transition of the boundary layers. That the increased slope in the transition

Figure 1. A subset of heat transport data for aspect ratio unity showing a power-law region with an exponent of about 1/3 followed by a ‘transition’ to another region with the same scaling but 20% higher in value. In the transition region, the effective $\log Nu - \log Ra$ slope is approximately 1/2 (the dotted line has a slope of 1/6, which, when added to 1/3, yields 1/2). In this and subsequent plots, open symbols represent data beyond the onset of the transition defined here, and solid symbols refer to the same data before transition. This convention does not apply to the data of Chavanne et al [17] for the aspect ratio 1/2 (open stars), which are included here for comparison. Regarding the transition to enhanced heat transport evident in our data (triangles), we do not find unique correspondence to traditional non-Boussinesq parameters such as $\alpha \Delta T$. For data on the upper dashed line, the operating points were within 10% of the critical values of temperature and density, as defined in the text.
region is followed by a second region of 1/3 slope would indicate that it is not the ultimate regime of Kraichnan’s. This argument by itself is insufficient to dismiss Kraichnan’s regime. For instance, it could be said that this ultimate scaling regime could be affected by some non-Boussinesq process which simply reduces the effective slope. What provides compelling and sufficient evidence against the interpretation of the ultimate regime is the data obtained [11] in a container of aspect ratio 4 along similar paths in the phase space. In this case, the highest Rayleigh number attained was smaller roughly by a factor of 4\(^3\) (due to the \(H^3\) factor in the definition of \(Ra\)). We found a similar transition in these experiments as well, at an \(Ra\) that was roughly a factor of 4\(^3\) lower than for the \(\Gamma = 1\) experiments (with similar \(Pr\)).

This increase in heat transport does not correlate well with the usual non-Boussinesq parameter \(\alpha \Delta T\), as will be discussed in section 3. We should particularly note that the relatively small increases of \(\alpha \Delta T\) in the region of transition do not allow us to quantitatively explain our observations \textit{a la} Ahlers \textit{et al} [13, 14], who describe the behavior of ethane gas near its critical point. Another recent work [15] shows that the heat transport near the critical point of \(SF_6\) may be enhanced or diminished depending on the operating point in the pressure–temperature diagram. We will discuss these points in section 5, particularly in the context of the numerical work [16]. Our finding is that the onset of the transitional behavior correlates well with a sufficiently negative value of a parameter that characterizes the temperature dependence of viscosity and conductivity. None of the above references specifically addressed the \textit{transition} between different convection regimes, which is the focus of the present work.

2. Apparatus

The \(\Gamma = 1\) measurements were made in the apparatus discussed in [1]. The diameter and height of the cell were 50 cm. The top and bottom plates, 3.8 cm in thickness, were made of copper annealed under oxygen-free conditions and were heated uniformly using a specially designed distributed heater. While a constant heat flux occurred at the bottom plate, measurements were initiated only after a constant temperature was reached in the steady state. The top plate was connected to a helium reservoir through a distributed and adjustable thermal link, and its temperature was kept constant by means of a resistance bridge and servo. The stainless steel sidewall had a thickness of 0.267 cm. The inner convection cell was insulated by three thermal shields at various graded temperatures, residing in a common vacuum space. Due to the relatively large height of the apparatus, corrections were made for the small adiabatic temperature gradient across the fluid layer (see [18], appendix to chapter 14).

The \(\Gamma = 4\) results [11] were obtained in the same apparatus by reducing the vertical distance between the two plates by a factor of 4. The aspect ratio of 1/2, to which a brief reference has been made, was achieved by doubling the vertical distance corresponding to \(\Gamma = 1\).

3. High \(Ra\) heat transfer data for \(\Gamma = 1\) and 4

In figure 1, we plot the new data for \(\Gamma = 1\) cell. For convenience, the key experimental parameters are listed in table 1; the parameter \(\xi\) in the last column will be defined at the beginning of section 4. For \(Ra < 10^{14}\), we observe a slope of 1/3 in log \(Nu\)–log\(Ra\) plots (the lower dashed line with \(NuRa^{-1/3} = 0.064\)). The data for \(Ra > 4 \times 10^{14}\) settle to another regime of 1/3 slope in the log\(Nu\)–log\(Ra\) relation (figure 1). This regime is represented by the upper

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dashed line corresponding to $Nu Ra^{-1/3} = 0.078$, about 20% higher than the lower dashed line. The jump from $Nu Ra^{-1/3} = 0.064$ to 0.078 requires a transitional region in which the $Nu-Ra$ slope has to be necessarily larger than 1/3; this slope happens to be roughly 1/2, as indicated by the dotted line. Depending primarily on the reduced density, this enhanced slope begins at different values of $Ra$: for the data of [1], slightly smaller densities than here were used, and the rise with a slope of about 1/2 occurred at a slightly higher $Ra$, as we shall see below. In further discussion, the heat transport data exceeding the lower horizontal line of figure 1 are identified by open symbols. Included for comparison are the data of Chavanee et al [17]. The latter data, in which the ITS-90 temperature scale was adopted, are more similar to ours than they are different.

Table 1. Experimental parameters for the present data with $\Gamma = 1$. Tabulations of all other previously published data can be found in [1, 11].

| $T$ (K) | $\rho$ (g cm$^{-3}$) | $\Delta T$ (K) | $Ra$ | $Nu$ | $Pr$ | $\xi$ |
|--------|-----------------|---------------|-----|------|-----|-----|
| 5.333  | 0.0248          | 0.0663        | $1.3 \times 10^{13}$ | 1473.5 | 1.3 | 0.948 |
| 5.311  | 0.0242          | 0.0728        | $1.34 \times 10^{13}$ | 1495.4 | 1.28 | 1.029 |
| 5.308  | 0.0263          | 0.0662        | $1.67 \times 10^{13}$ | 1617.6 | 1.38 | 0.944 |
| 5.399  | 0.0262          | 0.0676        | $1.75 \times 10^{13}$ | 1654.4 | 1.34 | 0.915 |
| 5.434  | 0.0277          | 0.077         | $1.82 \times 10^{13}$ | 1690.6 | 1.4  | 0.916 |
| 5.356  | 0.0296          | 0.0572        | $2.24 \times 10^{13}$ | 1805.8 | 1.55 | 0.882 |
| 5.369  | 0.0274          | 0.0875        | $2.46 \times 10^{13}$ | 1862.3 | 1.41 | 0.909 |
| 5.366  | 0.0295          | 0.0761        | $2.94 \times 10^{13}$ | 1938  | 1.54 | 0.836 |
| 5.371  | 0.0295          | 0.0861        | $3.3 \times 10^{13}$ | 2003.7 | 1.53 | 0.945 |
| 5.376  | 0.0295          | 0.0969        | $3.69 \times 10^{13}$ | 2060.3 | 1.53 | 0.883 |
| 5.449  | 0.0317          | 0.089         | $4.21 \times 10^{13}$ | 2174.5 | 1.62 | 0.837 |
| 5.408  | 0.0335          | 0.0813        | $5.32 \times 10^{13}$ | 2354.5 | 1.78 | 0.783 |
| 5.519  | 0.0379          | 0.0691        | $6.96 \times 10^{13}$ | 2620.9 | 2.02 | 0.572 |
| 5.612  | 0.0397          | 0.0742        | $7.91 \times 10^{13}$ | 2726.1 | 2.05 | 0.474 |
| 5.607  | 0.0444          | 0.0646        | $1.23 \times 10^{14}$ | 3208.5 | 2.46 | 0.237 |
| 5.376  | 0.0447          | 0.0522        | $2.04 \times 10^{14}$ | 3758.9 | 3.23 | 0.095 |
| 5.884  | 0.0660          | 0.0657        | $2.36 \times 10^{14}$ | 4465.6 | 2.81 | 0.3  |
| 5.883  | 0.0674          | 0.067         | $2.47 \times 10^{14}$ | 4351.8 | 2.82 | 0.3  |
| 5.253  | 0.0444          | 0.0464        | $2.94 \times 10^{14}$ | 4260.8 | 3.93 | 0.202 |
| 5.287  | 0.0443          | 0.0634        | $3.39 \times 10^{14}$ | 4447.8 | 3.66 | 0.072 |
| 5.604  | 0.0591          | 0.047         | $3.43 \times 10^{14}$ | 5279.6 | 3.85 | 0.81  |
| 5.621  | 0.0671          | 0.0415        | $3.82 \times 10^{14}$ | 5687.4 | 4.05 | 1.102 |
| 5.624  | 0.0671          | 0.0472        | $4.3 \times 10^{14}$ | 5840.9 | 4.02 | 1.213 |
| 5.632  | 0.0660          | 0.0532        | $4.57 \times 10^{14}$ | 6021.3 | 3.94 | 0.971 |
| 5.267  | 0.0498          | 0.0336        | $5.85 \times 10^{14}$ | 5518.7 | 5.88 | 1.67  |
| 5.551  | 0.0660          | 0.0471        | $6.31 \times 10^{14}$ | 6754.1 | 4.75 | 1.462 |
| 5.271  | 0.0495          | 0.0427        | $6.68 \times 10^{14}$ | 5874  | 5.6  | 1.48  |
| 5.371  | 0.055           | 0.0421        | $8.41 \times 10^{14}$ | 6629.6 | 6.03 | 1.991 |
| 5.432  | 0.066           | 0.0337        | $1.11 \times 10^{15}$ | 8029.6 | 7.04 | 3.023 |
| 5.401  | 0.066           | 0.0262        | $1.17 \times 10^{15}$ | 8356.9 | 8.1  | 3.822 |
| 5.333  | 0.066           | 0.0194        | $2.06 \times 10^{15}$ | 9881.9 | 12.08 | 7.398 |
| 5.298  | 0.066           | 0.0165        | $3.33 \times 10^{15}$ | 11397 | 16.25 | 11.88 |
| 5.287  | 0.066           | 0.0135        | $3.34 \times 10^{15}$ | 11678.1 | 18.03 | 11.435 |
Figure 2. A subset of the aspect ratio 4 data from [11], showing a power-law region of approximate exponent 1/3 followed by an enhancement at higher $Ra$. The open symbols have about the same value of the parameter $\ell_c$ as the data in the transition region of figure 1 (see the text). We thus do not necessarily expect to see a saturation to the new 1/3 power for these data.

The present data on the upper dashed line were obtained within 10% of the critical point, i.e. $\ell_c < 0.1$, where

$$\ell_c = \left[ \left( \frac{\rho - \rho_c}{\rho_c} \right)^2 + \left( \frac{T - T_c}{T_c} \right)^2 \right]^{1/2}. \quad (1)$$

Those on the lower dashed line correspond to $\ell_c > 0.3$. We included those data from [1] with $\ell_c < 0.1$ here as well, but do not identify them separately since they agree well with the present data (although we shall comment on them separately below). We emphasize that the transition from one region of the 1/3 power to the other occurs at different Rayleigh numbers depending on the particular path taken by the experimental protocol in the pressure–density phase space—occurring at lower $Ra$ for larger densities (closer to the critical value). The data in transition are distinguished by having intermediate values $0.1 < \ell_c < 0.3$. Plotting the data from different experiments with many different experimental paths in the phase plane will therefore appear as scatter. While we do not consider the enhancement effect to be a ‘critical point effect’ (we are, in any case, dealing with non-equilibrium phenomena), the critical point is indeed important in so far as it produces large variations of fluid properties for small temperature differences.

We also reconfigured the apparatus for a larger aspect ratio, $\Gamma = 4$, and a subset of these data [11] at the highest $Ra$ obtained is shown in figure 2. As in figure 1, we plot the data normalized by the 1/3 power of $Ra$ for $Ra > 10^{10}$. There is a considerable range of 1/3 scaling, followed by a transition region in which the local exponent has a higher value than 1/3 (open symbols). Clearly, the enhancement of heat transport begins at a considerably lower value of $Ra$ compared to aspect ratio unity, reflecting that the enhancement is governed by the experiment’s
Figure 3. Upper figure (a): heat transfer data for aspect ratio unity as in figure 1. Separately identified here are data from an earlier experiment [1], represented by circles. The convention for open and solid symbols applies as in the two previous figures. The two paths traced by the present data correspond to different loci in the pressure–temperature plane, demonstrating that \( Ra \) alone does not determine the transition point. For the same data sets, the fractional deviation of the thermal conductivity \( k \) against \( Ra \) is plotted in the lower figure (b). Here the occurrence of transition is well correlated with negative values of this parameter.

operating point in the phase space, not the value of \( Ra \). For \( \Gamma = 4 \), there is no clear upper region of 1/3 power; this is reasonable if we compare the data in the transition region (open symbols) with those of the \( \Gamma = 1 \) experiment. Indeed, for the open symbols in figure 2, we have \( 0.17 < \ell_c < 0.3 \), which corresponds to the transitional data for unity aspect ratio but not to the line of the upper 1/3 power. The solid symbols in figure 2 (region of 1/3 scaling) are likewise distinguished by having \( \ell_c > 0.3 \), again in quantitative agreement with the data represented by solid symbols in figure 1.

We now add the older data of Niemela and Sreenivasan [1] in which we originally observed the enhancement effect. We also show some new data taken under similar conditions in figure 3(a). Here the data of [1] are represented by circles, where, as before, open (closed) symbols refer to data above (below) the transition, while the new data are represented by triangles. The Nusselt number follows two different paths in the pressure–temperature phase plane, where we have different dynamical behaviors occurring in precisely the same experimental setup and with the same \( Ra \), the difference being that helium has different properties in the two cases. In particular, the data are not affected by the history of experimental conditions.
4. An interpretation

In the lower figure 3(b), we plot the fractional variation of thermal conductivity \(k\) across the layer, \(\Delta k/k\), for the data of the upper figure 3(a). Here and below we define changes in any property \(X\) by \(\Delta X = X_H - X_C\), \(X_H\) being the value at the hot plate temperature and \(X_C\) that at the cold plate temperature. The data show a strong collapse in the region of enhancement where the Nusselt number itself does not show a unique relation with \(Ra\). The fractional deviation of thermal conductivity exhibits an exponential decay for high \(Ra\), although the values are small in absolute magnitude. To account for the relatively large effect observed, we take the hint from figure 3(b) and form a quantity with units of a pressure difference for a fluid of density \(\rho\),

\[
\Delta P = \left( \frac{Pr \Delta k}{\Delta v} \right) \Delta T,
\]

and normalize it by the thermal energy density of the fluid

\[
P_{th} = \rho C_P \Delta T.
\]

The resulting dimensionless pressure difference can be reduced to a parameter \(\xi\) given by

\[
\xi = \frac{\mu \Delta k}{k \Delta \mu}
\]

keeping the density constant. Here, \(\mu\) is the shear viscosity of the fluid. The quantity \(\xi\) measures the ratio of viscous variations to thermal variations across the fluid layer\(^3\).

In all the experiments, the dimensionless parameter \(\xi\) changes sign and becomes negative as the transition begins. The abscissa in figure 4 is \(Ra\Gamma^{3.2}\) for all experiments with \(\Gamma = 1\) and 4.

\(^3\) Since roughly half the total temperature drop occurs over each of two boundary layers, it does not change the essential argument whether we consider property variations over boundary layers or the entire cell.

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The $\Gamma$ factor arises because (i) the paths through the pressure–temperature phase space for different experiments are essentially similar, so the critical point is approached at Rayleigh numbers that are roughly $4^3 = 64$ times lower for $\Gamma = 4$ than for $\Gamma = 1$, reflecting the $H^3$ factor in $Ra$; and (ii) the exponent 3.2, not significantly different from 3, provides a slightly better collapse of the data perhaps representing slight differences in the operating paths in the two cases. From figure 4, it is clear that the enhanced heat transport coincides with sufficiently negative values of the dimensionless pressure difference $\xi$. For small negative $\xi$, we do not observe any measurable enhancement of heat transport above the 1/3 power; however, for negative magnitudes of unity and greater, the system is clearly in a state of enhanced heat transport for all experiments presented. The data of Chavanne et al. also exhibit substantially negative values of $\xi$ at high $Ra$ but the large systematic scatter due to property uncertainties (from varying mean temperatures) makes it impossible to adequately test the present correlation (the same systematic uncertainty gave rise to the substantial scatter of their data in figure 1).

In figure 4, we include for comparison a few values of $\xi$ for room temperature fluids commonly used for convection studies. For example, with a bottom-to-top-plate temperature difference of 10 K and a mean temperature of 40 K, we estimate that $\xi = -0.1$ for water (shown as the dash-dotted line) and about 1.1 (short-dashed line) for air and ethanol. To our knowledge, these values of $\xi$ are not associated with any enhancement of heat transport, nor has any enhancement been observed for similar $\xi$ in our helium experiments. However, mercury could produce values of $\xi \approx -0.5$ (dash-dot-dot line). Indeed, a curious enhancement of heat transport was reported in [19] at high $Ra$ for which $\xi$ (which becomes increasingly negative as the temperature difference is increased) appears to have reached values between $-0.5$ and $-1$ (the reported temperature difference for the highest $Ra$ data are apparently of the order of 100 K). Of course, mercury becomes nearly as conductive as the bounding plates at the highest $Ra$ in that experiment, so the interpretation of these results may be subject to additional uncertainty. The values of $\xi$ for ethane near the critical point, which was used in the studies [13, 14], do not appear to have this same dependence on temperature. These authors find that the main contributor to an enhancement effect on the Nusselt number is due to large values of the parameter $\alpha T$ (among the parameters they specifically considered). We reiterate that $\alpha T$ in our experiments is not large enough to account quantitatively for our observations of the enhancement according to their model.

In fact, it is worth looking at the variation in $\alpha T$ for the data of figure 3. As seen in figure 5, like most non-Boussinesq parameters, $\alpha T$ increases with the approach to the critical point (coinciding generally with large $Ra$) but we do not observe any correlation between the increasing magnitude of this parameter and the existence of the transition, as denoted by the change from solid to open symbols.

It remains for us to understand why the $\Gamma = 1/2$ data of [4] do not show an enhancement of heat transport. There, negative values of $\xi$ occur above $Ra = 10^{15}$, for which the viscous boundary layer has become turbulent according to various estimates summarized in [1]. Indeed, of the three experiments of [1, 4], the $\Gamma = 1/2$ flow is the only one for which the boundary layers are turbulent with any certainty and the only one in which enhanced heat transport does not occur. A point to consider is that the usual signatures of a coherent large-scale flow filling the entire apparatus—manifest either as a prominent peak in the power spectrum of temperature on the side wall or as strong periodicity in its Fourier transform—are lacking in the aspect ratio 1/2 experiments at these high $Ra$. We should further point out that this latter experiment was not conducted at constant mean temperature and that a small effect could possibly be masked.
Figure 5. The variation in $\alpha \Delta T$ with $Ra$ for the aspect ratio unity data shown in figure 3. The region in which both the occurrence and the absence of transition are observed—corresponding to the overlapping of open and closed symbols—does not correlate with the magnitude of $\alpha \Delta T$.

by systematic uncertainties in both pressure and temperature. Since the range of variation in the pressure and temperature was significantly larger in [17], even larger trends could well be masked there.

The enhanced heat transport observed in experiments with laminar boundary layers might occur if the plume production was enhanced. An important influence could also occur due to the so-called ‘acoustic’ modes [20, 21], often referred to as the ‘piston effect’, which plays a role in convection very close to the critical point. These modes are considered in [15] to lead to a substantial reduction in heat transport with large and increasing $Pr$ (and hence large and increasing values of isothermal compressibility). However, these same authors found that for $Pr$ of the order of 1–10 with supercritical conditions (farther from the critical point), small increases in $Pr$ resulted in a modest enhancement of heat transport. The supercritical results of [15] are not unlike those reported here, although the density in our case is not held at the critical value. These authors did not observe the present transitional change which, as we have noted, correlates reasonably well with variations in fluid conductivity and viscosity through the parameter $\xi$. Of course, since $Pr$ increases with $Ra$, it would not be surprising if there is some positive correspondence between increasing $Pr$ and the transition to enhanced heat transport, but a strong correlation does not exist with $Pr$, especially for the data obtained for the same aspect ratio. We also draw attention to [16]. These authors numerically varied each individual fluid property separately and showed that the largest changes in $Nu$ corresponded best to changes in the thermal expansion coefficient, but also noted that ‘the results will depend on the path by which one approaches the critical point. At present, we are not in a position to comment generally on all non-Boussinesq effects, especially at very high Rayleigh numbers.’ The present results are thus completely consistent on this point.

We cannot be certain, of course, that $\xi$ is the best or correct parameter to describe the observations in the absence of a plausible theory, and we have relied on an empirical approach taking into account a number of experiments having the same boundary conditions.
Since plume production occurs in boundary layers, it is conceivable that details of the heated surfaces (here kept constant) should have an effect as well, making it difficult, in principle, to quantitatively compare experiments in different apparatus. Both SF₆ experiments under supercritical conditions relatively far from the critical point (Pr of the order of unity) and the present experiments conducted in helium gas under similar conditions produce states of enhanced heat transport. Whether this is a coincidence or can be related to a common mechanism requires further study.

5. Summary and conclusions

We have observed an enhancement of the turbulent heat transport at high Ra in a number of experiments performed in the same apparatus. Comparison with other studies where non-Boussinesq effects are shown to be due mainly to increases in the thermal expansion coefficient cannot quantitatively explain these results, because $\alpha \Delta T$ does not attain large enough values in the present experiments. With respect to Pr changes, it appears that there exist qualitative similarities with [15]: when Pr is not too large, enhanced heat transport corresponds to higher Pr but a reduction in heat transport occurs for high values of Pr.

Our results show that the enhancement correlates well with negative values of the order of $-1$ of the dimensionless pressure difference $\xi$, based on molecular properties of the fluid. The fact that this enhancement of heat transport is observed only for laminar boundary layers points to enhanced plume production as a possible physical mechanism. Apparently, enhanced plume production is suppressed by the presence of turbulence in the boundary layers, as evidenced by the data in the aspect ratio 1/2 experiments. Recall that negative $\xi$ is obtained in that flow only for $Ra > 10^{15}$ for which the viscous boundary layer, by known estimates, has already undergone a transition to turbulence. In laminar flows, the actual enhancement effect may be expected to depend on the fine details of the heated surfaces, and a systematic study of this aspect is a worthwhile objective for future work.

At the level of detailed knowledge that we now possess, we cannot rule out that some competing effects, such as the changing Prandtl number and increased expansion coefficient, conspire to create the second plateau of the $-1/3$ power. It is reasonable to ask whether the second region of the $1/3$ slope might have continued to follow the nearly $1/2$ power, consistent with Kraichnan’s result, if there were no significant non-Boussinesq effects or the ‘piston effect’ or something else. The main empirical evidence against the Kraichnan argument is that the transition does not coincide with unique values of the similarity parameters Ra and Pr, as shown most clearly in flows with substantially different heights, but also in the same aspect ratio apparatus (figure 3). In any case, the transition to the enhanced state is quite sharp and can be characterized well. In this work, the $1/2$ scaling observed is simply a transition between two states of nearly $1/3$ scaling, with the upper state roughly corresponding in magnitude to the heat transport measured in [17] (see figure 1). The authors of that reference, however, have drawn a different conclusion.

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