Quantum Geometry as a Relational Construct

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Abstract

The problem of constructing a quantum theory of gravity is considered from a novel viewpoint. It is argued that any consistent theory of gravity should incorporate a relational character between the matter constituents of the theory. In particular, the traditional approach of quantizing a space-time metric is criticized and two possible avenues for constructing a satisfactory theory are put forward.
I. INTRODUCTION.

The search for a theory of quantum gravity has been one of the most excruciating tasks of modern theoretical physics. The reasons for this are multiple: To begin with, we have excellent theories to describe all other interactions in the standard model of particle physics, gravity being the missing item in this approach. However, and perhaps even more important is the fact that the realm of quantum gravity seems to confront us with deep conceptual problems. Among many such problems we have diffeomorphism invariance and its connection with observables, the issue of initial conditions for the wave function of the universe, and the problem of time in a quantum theory of the space-time, etc. [1]. Another example is the so-called information loss paradox associated with the quantum evaporation of a black hole.

The problem of finding a quantum theory of gravity has been attacked from several perspectives, the most notable being the canonical quantization approach, à la Wheeler-DeWitt [2]; Loop Quantum Gravity [3], in which there is a natural place for the notion of a “wave function of the universe;” and the String/M Theory program [4]. The last two have recently enjoyed a certain degree of success, in particular by their achievements on one front that has been long considered as a “first test” that must be passed by a candidate theory of Quantum Gravity: The identification of the fundamental degrees of freedom and the evaluation through statistical mechanical methods of the entropy associated with a Black Hole [5].

However, despite these recent achievements there is a lingering sense of frustration in some parts of the gravitational physics community related to the fact that not much progress seems to have emerged from these theories in addressing many of the outstanding conceptual problems mentioned above.

We usually feel that there will be a need for a quantum description of gravity only when the energy scales of the interactions reaches the Planck scale. This seems to be a very simplistic view, first because the technology available is reaching the point where certain type of experiments once thought to belong only in the Gedanken realm of Einstein, are now becoming possible: Can we test the equivalence principle with an object in a state of quantum superposition of different energy eigenstates? Yes [6]. Can we hope to see quantum fluctuations in the space-time metric? Yes [7]. The list of such experiments is increasing and it seems reasonable to expect that in the near future we might be able to perform experiments with quantum mechanical sources of the gravitational field (such as a SQUID carrying a superposition of currents in opposite directions [9]). Moreover, we would like to have, at least in principle, a way to envision a manner to describe space-time at, say, the atomic scale, whose curvature is associated with a nucleus which is in a state for which the position is not well defined. There is certainly no Planck scale involved here! Note that this last issue could be accessible to experiment in setups analogous to those considered in [10].

The purpose of this article is to suggest a new approach to this issue that centers on the conceptual problems underlying our notions of space-time in view of the quantum nature of the matter that inhabits it. The hope is that by doing so we might be guided towards a description of gravity at the quantum level that will be able from the outset to deal with the conceptual problems on which little light seems to be being shed by the most popular approaches to the subject.

In this paper we will take the view that gravity is NOT LIKE the other forces in nature,
basically because gravity describes the stage where the rest of the physics take place while the other forces are just actors on this stage. The previous statement is meant to appeal to our intuition, and should not be taken in the literal sense, among other reasons because, as we will later argue on this paper, the stage is in a sense just a set of relations between the actors. A more precise way to argue that gravitation is not just another force is to note that gravity alone is responsible for determining the causal structure and thus the commutation properties of other fields.

The motivation of this work is the following question: How would we have arrived at a description of gravitation if our normal physical experiences came from the quantum world, and the classical limit had not been discovered. More precisely, what is the analogue of the conceptual path that took us from special to general relativity, if the starting point was a quantum rather than a classical theory? Much has been said about the possibility that both quantum theory and general relativity might need modifications when we want to bring them together into a quantum theory of space-time. The ideas that will be presented here constitute an example of one such approach, one which falls in the category of those based on relational ideas, several of which have been considered previously but as far as the authors know, none of them considered the issue in the light we suggest in this work. We will elaborate on this view in the rest of the paper. In Section II we will describe the general considerations that we take as guiding our search for a framework with which to start the construction of a theory of Quantum Gravity. This is the core of this paper and the remainder is presented to help to envision the type of construct we expect to be appropriate. In Section III we will sketch a proposal based on the generalization of the algebraic approach to quantum field theory. In Section IV we will suggest a line based on the relativization of the notion of quantum state as proposed by C. Rovelli and others. We will end in Section V with a brief discussion centered on the main objections that could be raised to such a program.

II. GEOMETRY AND PHYSICS

Here we want to analyze the notion of Physical Geometry, that is the geometry that we associate to the physical world in a way that implicitly shows the contrast with idealized mathematical geometry.

Let us start by considering what is the role of geometry in classical (as opposed to quantum) physics. More precisely we want to analyze the meaning of such assignment of geometry to our physical world. We have learned from Einstein that we measure intervals between events. In order to do this we must first identify these events, by for example singling out one point on the world line of a point-like object, which can be done for example by considering the intersection of two such world lines (We can be thinking of the “passing of an object in front of a particular observer”, or the emission of a light pulse from a certain source), and second we must use a physical device to actually measure the interval. We would use a “clock” if the interval is time-like, a ruler if the interval is space-like, or define it

1Ideas in this direction have been explored within the string theory perspective in [11].
as null if the two events can be connected by a light signal (to be precise we must ensure that the measurement is done along the prescribed interval). In all these cases we need physical objects, clocks, rulers, or light pulses, in order to be able to define what the geometry is. This is not just saying the obvious, that we need such objects to measure the geometry. What we want to stress is that we need to specify these physical objects in order to define what we mean by “the geometry of our physical world”. We note for example that in principle it is not a priori certain that the geometry could not depend on the objects we choose to employ to define it. For instance we could choose to measure spatial intervals with a ceramic rods or with metallic rods, and imagine a situation in which the temperature trough the region being examined is not uniform. Under such circumstances the straightforward determination of the geometry of the region would give different results. For instance, the geometry determined with the ceramic rods could be flat while the one determined with the metallic rods could have nonzero curvature. This is all obvious, and everybody would tell us that the use of the ceramic rods is the correct choice in this case because of the large thermal dilation coefficient of the metal. Actually we would be told that even if we use ceramic rods we need to correct for such thermal effects, and only then we would obtain a determination of the “true geometry” which would be equivalent to that which is determined by means of ideal “length preserving” rods. One would normally expect to see the quotation marks of the previous phrase on the word ideal, however, what we want to point out is that, there is in principle a problem of establishing what is a “length preserving” object. How would we know, even in principle when an object does not change its length? By measuring its length in different circumstances? Yes! However, in order to measure its length we need to use a length preserving object! We must emphasize that the issue is not entirely technical. We could consider —and in practice we do— replacing standard physical objects by generalized objects: actual physical objects complemented by well defined prescriptions of how to correct for the changes in their assigned lengths under specific circumstances (such as taking into account the corresponding thermal expansion coefficients and accompanying every length determination with a simultaneous temperature determination). We would still be faced with the issue of having to define the operational procedure to determine the length preserving assignment to a physical object or a generalized object. We could envision the consideration of more modern methods, as for example, choosing to measure an object’s length with light signals and clocks, while taking the speed of light as 1 by definition. This only converts our problem into that of choosing clocks that run at a “constant” rate. It is evident that this faces us with complete analogous conundrums. How would we measure the rate of ticking of a clock to determine whether it is constant without the use of another clock?

In classical physics we are in effect “solving” these dilemmas by the use of judicious definitions which make use of the following empirical fact, F1: There exist objects and clocks (usually generalized objects and clocks) that when used to define the lengths of intervals, result in empirical laws of physics that are particularly simple (as for example, the law of inertia) [14]. In this way we select the objects that give an empirical content to our assignment of geometry to the classical level of description of our world. What are the objects, actually the class of objects \( \mathcal{O} \), that give a meaning to the geometry at the quantum level? Does such class of objects exist at all? Are there different classes of objects, say \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \), each leading to equally simple laws of nature, which however are different for different selections
of the class of objects? In this case we would confront head on the problem of having two different geometries for the same physical situation, one associated with the class $O_1$ and the other with the class $O_2$! We will not attempt to give definite answers to these questions here, but will finish this illustrative sequence by a question for which we expect the answer to be negative: If a class of objects can be so chosen, would we expect it to contain enough objects to give meaning to geometry at all scales? Note that the limiting scale could be coming from the physics of the objects and not necessarily from the gravity sector alone, that is, the limiting scale need not be the Planck scale (See, for instance, the discussion in [15] and references therein).

Recognizing and keeping in mind these points could turn out to be important (and we will argue that they are indeed essential) for considerations about the nature of a Quantum Theory of Gravitation. In this paper we will take the stand that the standard views on these issues rely on idealizations that are not unlike the idealizations of pre-relativistic physics about the existence of absolute space and absolute time, or those in pre-quantum mechanics about the existence of well defined particle trajectories or the existence of a fully deterministic physical world. That is, we will take the view that, in the same way that those idealizations turned out to be impediments for the advancements towards Special Relativity and Quantum Mechanics, the notion of a physical geometry existing independently of the physical objects with which to determine it (in the sense of defining it), is an impediment towards the construction of a quantum theory of gravitation.

There are several issues associated with the previous considerations. First, we note that, while at the classical level we must consider the objects which are used to define the geometry (i.e the objects satisfying $F_1$) as classical objects, at the quantum level the natural objects that should be considered as defining the geometry should be themselves “quantum objects”. Thus, given that the meaning of the physical geometry arises solely from statements which must be expressible in terms of the objects selected to define it, such meaning should be taken to be a certain codification of correlations between physical “events” (this word is used here in an imprecise sense because at this point the discussion refers to both the classical and the quantum cases) associated with such objects. At the classical level we could be talking about the number of ticks of a clock along the world line segment joining two events with which the clock eventually coincides. At the quantum level we need to consider very different sorts of things. One must then expect such correlations and their codification to depend on the type of objects one is considering, and therefore the kind of objects representing this information (the kind of objects that represent geometry) should be expected to be very different in the classical and the quantum cases. In particular, we don’t expect the quantum objects to be described by the same constructs as ordinary quantum matter. These differences should be as significant, say, as a space-time vector (considered here as describing the state of a classical particle) is from a wave function (considered here as describing a quantum state of a particle). Note that up to this point we have not specified what we mean by “quantum objects”. Our considerations have so far been very general. In Sections IV and V descriptions of a more specific character will be given in the context of the corresponding approaches. In considering these issues we must keep in mind that there are in principle two roles played by the physical geometry in a geometric theory of gravitation: 1) It is an entity codifying relationships between “events” defined in terms of the “matter” content of the theory, and 2) it is a dynamical entity whose behavior is part of the subject
of the theory.

Let us try to be a bit more specific at this point by saying not what should be done, but what should not be done and why. The (most widely accepted) classical description of gravitation involves a space-time (i.e. a manifold with a pseudo-Riemannian metric) $(M, g_{ab})$, which carries information about the behavior of geodesics, their points of intersection, which would define events, the length of the interval between two such events along a given geodesic, etc. It is natural that such an object (i.e. a tensor field) should be used to describe the geometry which is defined in terms of point particles whose state can be described in terms of a four vector $u^a$ and a point $x$ in space-time. On the other hand, when the geometry is defined in terms of quantum fields [at this point the use of these words should not be taken to indicate the standard mathematical description of such objects, but the underlying physical entities they are thought to represent], the physical counterpart of a geodesic (viewed as the world line of a classical test particle) or the physical counterpart of a four vector tangent to a space-time point (viewed as a perfectly localized particle with a well defined four momentum), cease to exist! That is, the objects that would give meaning to $g_{ab}$ are not part of the realm of physics we are attempting to describe. Therefore we should not try to construct a quantum counterpart of $g_{ab}$, i.e. a quantized gravitational field $\hat{g}_{ab}$ or any equivalent object, because such a construct would be a quantum object carrying information about classical correlations! An unholy mixture indeed. In fact, doing this, would have meant that we changed aspect 2) of the geometry from classical to quantum, but did not change aspect 1)\footnote{A remark is in order. The previous discussion suggests that even if one already has been successful in constructing a quantum theory of gravity, and one tries to define the equivalent of $\hat{g}_{ab}$, the task will be non-trivial or even impossible.}.

What is needed is to construct an object $\hat{G}$ (the double hat is meant to indicate that this is a quantum object but not necessarily of the same sort as a quantum field $\hat{\phi}(x)$) which carries information about correlations of quantum objects. The analogy we should use as a guide should be of the following type: The metric is an object that encompasses information about the way we transport (specifically, parallel transport) the description of the state of say a classical particle (four vector) from one point to another, and thus the quantum geometry should be described by an object that itself encompasses information about how to (in some yet undefined sense) “parallel transport” the description of a quantum state of an object from one region to another (or from one “observer” to another). Needless to say, the previous statement should be taken merely as a philosophical guiding principle, and any attempt at a specific realization should start by stating exactly what the objects are that will be used in defining the quantum geometry, what exactly is a quantum state of such an object is, and exactly what these correlations that the geometry is meant to codify. Given that an ordinary quantum mechanical state of a system has a much richer structure than its classical counterpart (The Hilbert space of a one particle system is infinite dimensional while the classical phase space is six-dimensional) there is a much larger set of possible aspects the correlations describing the geometry could refer to. Note also that we are envisioning the existence of some notion of localization which is in principle absent in the notion of a
state in quantum field theory, either in flat or curved space-time.

An alternative way of indicating the type of object that we expect would describe the quantum geometry (i.e. the kind of object that \( \hat{G} \) should be), is the following: We can think of the metric in general relativity as a codification of how to join local inertial frames to cover extended regions of space-time. Actually, we know that given the metric and a space-time point there is a well defined procedure to construct locally inertial coordinates (Riemann normal coordinates), and thus by carrying out such constructions at several points we can end up with an approximate description of the global space-time which would be covered by local inertial frames, whose global properties would be encoded in the rules that allow us to translate from one frame to another, and the description of vectors in the intersections of the regions covered by two such frames. By analogy we expect that the quantum geometry would codify the way to translate the description of the states of a quantum systems from one “local frame” to another.

The next issue we are immediately confronted with is that of the notion of physical geometry in the COMPLETE absence of objects. That is, we imagine having selected the type of objects that, in principle, give meaning to the physical geometry but would want to consider the possibility of associating a geometry to a physical situation in which these objects are absent. In the classical realm what we mean in these cases is the following: We consider the empirical content of the description of, say, an empty space-time by a certain metric to be associated to the way that “test” objects (particles, light pulses, clocks etc) “would behave if placed” in that space-time. This presents no particular problem in this case because of two very special features of classical physics: First, there is no limit to how closely we can approximate, say, an ideal test particle (which would not have an effect on the geometry) by a real physical particle. We can make its four momentum as small as we wish without any effect on our ability to localize it as much as we want. Second, in classical physics the object is unaffected by its observation. It is clear that these two nice features do not survive the passage from classical to quantum physics: The first feature is denied to us by the Uncertainty Principle, and the second feature, which had allowed us to identify (in the classical realm) the observed and unobserved object, must be removed from consideration in the quantum realm, because failure to do so often results in internal contradictions. That is, we cannot identify, say, a double slit experiment in which one of the holes is being monitored for the passage of a particle, with a double slit experiment in which this monitoring in not being done. We know that at the quantum-mechanical level the description of a joint state of two systems is not the same as the sum of the descriptions of the two systems. Thus we do not expect a viable identification of “a geometry with” and “a geometry without” the objects that serve to define it. For an earlier discussion of this point see [8].

The previous discussion very strongly suggests that we must abandon the notion of assigning a geometry to a physical situation in the COMPLETE absence of objects. Moreover, we believe that the fundamental objects describing the matter in our world are not particles, but rather fields, and that therefore vacuum is not in any sense “the absence of everything”, but rather merely a state of the matter in the universe, and that, moreover, there is in general (in particular in non-stationary space-times) no state that can be uniquely described by the word “vacuum”.
Finally we point out a conclusion that seems to emerge from the points we made above: There should be no place for a counterpart of Minkowski space-time, and moreover there should be a fundamental limit to the degree to which such space-time can be approached. Suppose we want to have a region with a geometry as close to Minkowski as possible, that is a geometry that is as well defined as possible and as flat as possible at any scale. In order for it to be as well defined as possible it should be probed with quantum objects in as much detail as possible, but in order for it to be as flat as possible we must, among other things (here we are guiding ourselves by the correspondence principle which in this case indicates that matter will be associated with curvature), place as little matter as possible in that region. The two requirements are certainly incompatible. Thus there should be bounds on the flatness of the physical geometry.

Several remarks are in order: 1) It might seem that the discussion above leads to a theory in which Lorentz invariance is violated. Certainly this is a possible implication of our arguments. This is particularly intriguing, since (local) Lorentz invariance is at the heart of our present formulation of physics, needed implicitly in the mere definition of spinor fields on space-time. One possibility is that Poincare invariance is fundamental at the level of defining unitary representations (and thus having the different types of possible "particles"), but that this symmetry is not explicitly present in the formulation of quantum geometry (just as Poincare invariance is meaningless in general relativity). One would expect to recover approximate Poincare invariance at some scale. 2) We have not touched upon the validity of the equivalence principle. It is our understanding that, as presently formulated, the equivalence principle refers to classical objects, and thus possible violations can be expected when quantum objects come into play. On the other hand, we expect that there should be some local validity of the equivalence principle, which in ordinary terms is achieved when the quantum system is localized enough as to be in an idealized (local) inertial frame. Finally, let us remark that it would be desirable to have a quantum replacement for the equivalence principle that would serve us as a guiding principle in the search for a quantum theory of gravity, just as the classical equivalence principle led Einstein to general relativity.

III. AN APPROACH BASED ON THE ALGEBRAIC FORMULATION OF Q.F.T.

The proposal we are going to present here is very sketchy and should only been seen as a possible starting point to develop a framework with the characteristics discussed above. Its main virtue is that allows us to present in a relatively concrete way the type of construct we have in mind. The motivation of this particular proposal is to remove from the start one of the conceptual problems alluded to at the beginning of this paper: That of the meaning of the wave function for the universe, for which there is by definition nobody that can make the measurements that normally give it content. The present proposal will consider the usual type of quantum states to have in general only local validity and can, but need not, 3

3One view on this issue that has gained popularity is associated with the decoherence functional etc., but lingering problems exist associated with the need to divide the universe into systems playing different roles, one that remains the object of the description and a second that plays the
lead to global states. Thus, at least the need for an object like the wave function of the universe disappears from the start. A further motivation for introducing this feature is the analysis by Sorkin [20] of the applicability of the standard Q.M. interpretation of the measurement problem to quantum field theory, which indicates, that if it is taken literally, leads to unacceptable violations of causality (it allows EPR-type experiments to actually transmit information). One basic feature of this analysis is the assumption that we can associate a quantum state with arbitrarily large regions in all circumstances, and thus it seems appropriate to call this assumption into question.

Let us start by reminding the reader that in the algebraic approach to quantum field theory, which seems to be the appropriate setting to consider quantum fields in arbitrary space-times [19], one forgoes the notion of a Hilbert space of states, and takes as starting point an algebra $\mathcal{A}$ (more precisely a $C^*$ algebra) of abstract operators (usually thought of as corresponding to the smearing of polynomials in the field operators by suitable smooth functions) which are identified with the observables of the theory, and the states are then defined to be linear mappings $\omega : \mathcal{A} \to \mathbb{C}$ satisfying a positivity condition $\omega(A^*A) \geq 0, \forall A \in \mathcal{A}$ (and a normalization condition $\omega(I) = 1$ ), and where the meaning of $\omega(A)$ is that of the expectation value of the observable $A$ in the state $\omega$.

The idea of the approach is to consider localized states in such a way as to reproduce locally the setup of quantum field theory, but deny in general the existence of global states, and instead introduce rules to perform “parallel transportation” of these states. The complete collection of such rules being identified as the quantum geometry.

Let us now present the sketch: Let $M$ be a manifold, and let $\mathcal{A}$ be a $C^*$ algebra of abstract operators, each associated with a particular open set $U$ of $M$. For every open set $U$ we define the subalgebra $\mathcal{A}_U$ of local operators associated with $U$ to be the collection of all elements of $\mathcal{A}$ associated with open sets contained in $U$. A local state at $U$ is a mapping $\omega_U : \mathcal{A}_U \rightarrow \mathbb{C}$ with similar properties as before.

Two comments are in order: 1) Normally we would have expected that once the state at $U$ is specified it would determine the state for all of the domain of dependence $D(U)$ of $U$. However stating something like this would presuppose that the causal structure is determined. Instead, we would hope to define the domain of dependence, and thus the causal structure, in terms of the “correlations” of the states associated with the different regions. We would not expect, for example, that the causal structure would be always well defined! 2) Note that even though we might have the local states defined everywhere in the sense that we might have an open covering $\{U_\alpha\}, \alpha \in J$ of $M$ and an assignment of a local state $\omega_{U_\alpha}$ for every element of the covering, we might still not have a well defined global state! We might lack information about correlations.

Next we would need an axiom assuring that the local laws of physics are “everywhere” the same. We can try something like this: For all $p, q \in M$ there exist neighborhoods $U_p$, of $p$ and $U_q$ of $q$ and a mapping $\phi : \mathcal{A}_{U_p} \rightarrow \mathcal{A}_{U_q}$ which is an algebra isomorphism. This mapping need not be unique.

Next we would describe the geometry. The idea is to generalize the notion of connec-
tion. We do not want however to consider parallel transport along a line, nor do we have objects associated to points that we might wish to transport along such a line. The natural generalization seems to be the assignment of the notion of parallel transport to every one-parameter family of diffeomorphisms that takes an open set into another continuously. A quantum geometry for the manifold $M$ will be an assignment of a mapping $\Psi_f : \mathcal{A}_U \to \mathcal{A}_U$, to every one-parameter family of diffeomorphisms $f(\lambda)$ taking an open set $U$ to an open set $V$. This assignment would then be a realization of $\hat{G}$.

This setup has touched so far only kinematical aspects. We would next need to introduce dynamical aspects as well as self consistency restrictions. The fact that we must at some point be able to recover GR, and the appropriate quantum field theory in flat space-time in the corresponding limits, indicate that the dynamics of $\hat{G}$, must be associated with the generalized state of the matter fields, and the dynamics of the matter fields must be compatible with the causal structure that in the appropriate circumstances emerges from $\hat{G}$. Note that some steps in the direction of defining an algebraic QFT in a diffeomorphism invariant context were given in [16].

IV. APPROACH BASED ON THE PROGRAM OF RELATIVE STATES.

In a recent article C. Rovelli [13], in considering anew the measurement problem in Quantum Mechanics, in particular the collapse of the wave function in situations of the Wigner’s Friend type, comes to the conclusion that the wave function of a system should not be considered as an attribute of the system in the sense of describing the absolute physical state of the system, but rather as an attribute of the relation of one system (the described system) to another (the describing system), in the sense of encoding the information that the second system has about the first. In this way the problem of collapse of the wave function which is instantaneous in a given reference frame, can in principle be disassociated from the collapse of the wave function in other frames. The scheme has been considered as dealing with interpretational issues and was in no way intended to change the standard predictions of Quantum Mechanics. We will nevertheless adopt this idea and consider it as possible starting point for a relational scheme of the sort we have been advocating. Again, the following description is merely schematic.

Let $S$ be the collection of all subsystems in the universe and consider assigning to every ordered pair of them $(A, B)$ a density matrix in an appropriate Hilbert space, representing the information that $A$ has about $B$. When the corresponding information is non-existent the assignment will be a multiple of the identity matrix. In general we would expect that such a scheme would leave open the possibility of neglecting certain important correlations, for instance $A$ might assign a density matrix to $B$ and a density matrix to $C$, but be unable to assign more than the identity to $A$ and $C$ together because of lack of information about the correlations. We will not presuppose that it is always meaningful to talk about a system preserving its identity through “time” and thus our systems should be considered in general as localized in space and in time. We would of course need to make the previous statement meaningful in a way that is internal to the theory (i.e., in terms of considerations about correlations, etc.) as we would want space-time to be an emergent property of such a setup.
Now we can imagine another set of objects, assigned say to every pair of pairs of the form \((A, B), (A, C)\) the correlations needed for \(A\) to construct a density matrix associated to \(B\) and \(C\) together. Let us call these object “Quantum Correlators”. According to our discussions it should be possible to consider a (large enough ) set of Quantum Correlators as describing the structure of space-time. It seems that this is indeed the case, for if we are given a “complete” collection of wave-functions and quantum correlators we should be able to construct the global (space-time) states of all the quantum fields and from these, at least in favorable circumstances, one should be able to deduce the Space-time geometry.

Again, we have touched only on kinematical aspects, while clearly a dynamical content that ensures self consistency in the assignment of states to systems and subsystems, and that allows us to recover General Relativity and Quantum Mechanics in the appropriate limits needs to be considered.

V. DISCUSSION.

In this note, we have argued that the physical significance of the geometry of space-time can not be separated from the description of the matter fields that probe such geometry, and that it is therefore natural that a quantum theory of gravity should reflect this fact. We are well aware of possible dangers of adopting a purely operational approach to the construction of physical theories, where one might encounter technical difficulties and make little concrete progress. Furthermore, our position should not be seen as the same position adopted, for instance, by Rovelli and Smolin who have argued that loop quantum gravity (or its close relative, the Spin-Foam Models \[21\]) are already examples of relational quantum theories of gravity \[12,18\], in the vacuum sector.

We should also point out that we have left an important issue almost untouched. Namely, one expects that there is a scale in which the quantum description of the geometry becomes necessary. So far, we have not made a concrete proposal about where this scale can be found. In the standard approaches, one assumes that this scale will be the Planck scale (in the canonical approach) or the stringy scale (lately, theories formulated with “large extra dimensions” put this scales at lower energies). From the dimensional point of view, these seem to be the most natural choices. A new proposal for a relevant scale should, in our view, arise from the dynamics of the quantum objects involved in the formulation (See \[14\] and references therein).

Our point here is that the development of the physics of the last century presented us with several instances in which our attachment to certain idealizations based in our previous physical experience turned out to be an impediment to progress. We have made the case that the notion of a geometry that exists independently of the objects used to define it might be one such impediment. We have also pointed out that there are several outstanding problems associated with the standard interpretation of quantum theories that are particularly exacerbated when considered in a general relativistic setting, and have thus tried to formulate approaches that give some hope of helping in the resolution of those issues. After all, if quantum gravity is finally conquered and it does not help us with these interpretational problems of Quantum Mechanics, then it is very difficult to envision from whence clues for their resolution might come. We have thus advocated a relational point
of view and have suggested a few directions that could be tried for the implementation of these ideas.

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