Thirty Years of Precision Electroweak Physics

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J.J. Sakurai Prize Talk
APS Meeting
Albuquerque, N.M., April 2002.

Abstract

We discuss the development of the theory of electroweak radiative corrections and its role in testing the Standard Model, predicting the top quark mass, constraining the Higgs boson mass, and searching for deviations that may signal the presence of new physics.

Dedicated to my unforgettable friends and collaborators: M.A.B. Bég, R.E. Behrends, J.J. Giambiagi, and J.J. Sakurai, in memoriam.

\(^1\)to appear in a future issue of Journal of Physics G.
1 Brief Historical Perspective

The title of my talk, “Thirty Years of Precision Electroweak Theory”, refers to the fact that the Standard Model of Particle Physics (SM), proposed originally by Weinberg, Salam, and Glashow [1], emerged, with very important contribution by other physicists, in the period 1967-1974.

This theory has a basic property, shown by ’t-Hooft, Veltman, B.W. Lee, Zinn-Justin, Becchi, Rouet and Stora, and others, namely it is a renormalizable theory [2]. This implies that it can be studied at the level of its quantum corrections by perturbative field theoretic methods, since the ultraviolet infinities encountered in the calculations can be absorbed as unobservable contributions to the masses and couplings of the theory.

At roughly the same time, a very ingenious and useful method to regulate ultraviolet divergences, namely dimensional regularization, was proposed almost simultaneously in three different parts of the world, by ’t-Hooft and Veltman in the Netherlands, by Bollini and Giambiagi in Argentina, and by Ashmore in Italy [3]. I first learned of dimensional regularization in a memorable conversation with Bollini and Giambiagi that took place in Buenos Aires in January 1972. Soon afterward, we found out that ’t-Hooft and Veltman implemented this method in the very important context of gauge theories. The application of dimensional regularization to infrared divergences was proposed soon afterward, by Gastmans and Meuldermans, and by William (Bill) Marciano and me, while Bill was my research student at NYU [4]. A bit later, Bill wrote a paper on his own extending the analysis to the regularization of mass singularities [5]. Dimensional regularization of infrared and mass singularities is widely employed at present, particularly in QCD calculations.

Once the renormalizability of the SM was recognized, it became natural to explore this theory at the level of its quantum corrections. Already in the seventies there were a number of interesting developments:

a) the one-loop electroweak corrections (EWC) to \( g - 2 \) date from that period.

b) Weinberg showed that there are no violations of \( O(\alpha) \) to parity and strangeness conservation in strong interaction amplitudes [6].

c) Gaillard and Lee studied processes which are forbidden at the tree level, but occur via loop effects, and showed that the GIM mechanism
generally suppresses neutral current amplitudes of $\mathcal{O}(G_F \alpha)$ \[7\].

d) Veltman, and Chanowitz, Furman, and Hinchliffe discovered that heavy particles do not generally decouple in the EWC of the SM, and that a heavy top quark gives contributions of $\mathcal{O}(G_F M_t^2)$ to the $\rho$ parameter \[8\].

e) Bollini, Giambiagi, and I studied the cancellation of ultraviolet divergences in several fundamental natural relations of the SM \[9\].

My own main objective since the 70’s has been the study of the EWC to allowed processes, with the aim of bringing the theory into close contact with precise experiments. The desiderata of these studies are:

i) To verify the SM at the level of its quantum corrections.

ii) To search for discrepancies or inferences that may signal the presence of new physics beyond the SM.

These are essentially the objectives of what is now called Precision Electroweak Physics.

At the time I felt that there was a problem that required urgent attention in order to test the tenability of the SM, namely the issue of Cabibbo universality or, in modern language, the test of the unitarity of the CKM matrix. From studies in the framework of the Fermi theory that preceded the SM, it was known that, in order to test Cabibbo universality, it is necessary to evaluate the radiative corrections to muon decay and the Superallowed Fermi transitions in $\beta$-decay. Nearly forty years ago it was shown that, to first order in $G_F$, but all orders in $\alpha$, the photonic corrections to $\mu$-decay are convergent in the Fermi V-A theory, after mass and charge renormalization \[10\]. However, there was a big practical and conceptual problem: in the Fermi V-A theory the corrections to $\beta$-decay were known to be logarithmically divergent!

Once the renormalizability of the SM was recognized, it was apparent that the old conundrum could be solved in the new framework. I argued with myself: if the theory is renormalizable and I compute something physical, I should get a finite result! Around 1974 I found the answer in a simplified version of the SM, neglecting the strong interactions \[11\]. However, a realistic evaluation of the EWC to $\beta$-decay is particularly challenging, since one is dealing with a very low-energy-transfer process affected by the strong interactions. Fortunately, and almost miraculously, their effect can be controlled
to a large extent using current algebra techniques and associated Ward identities. The final result was simple and encouraging: i) aside from some small, short-distance QCD corrections, the result coincided with the regularized answer in the Fermi V-A theory, with the cutoff replaced by $M_Z$! ii) The corrections turned out to be sizable. They are dominated by a large logarithmic term

$$3\left(\frac{\alpha}{\pi}\right) \ln \left(\frac{M_Z}{2E_m}\right) \approx 3.4\%,$$

where $E_m = \mathcal{O}(MeV)$ is the end-point of the positron spectrum in β-decay. Furthermore, such large corrections are phenomenologically necessary to ensure, to good approximation, the tenability of the SM in the analysis of universality. For me, this was the smoking gun of the SM at the level of its quantum corrections!

Towards the end of the 70’s Bill and I thought that experimentalists would probably search for the $W$ and $Z$ bosons and hopefully measure their masses. It seemed a good idea to study at the loop level the relationship between $M_W$, $M_Z$ and $G_F$, $\alpha$, as well as the other parameters of the SM, such as $M_H$, $M_f$.

How to do it?

At the time we had precise knowledge of $G_F$ (which in my papers I had defined via the muon lifetime evaluated in the Fermi V-A theory) and $\alpha$, and a less accurate knowledge of the electroweak mixing parameter $\sin^2 \theta_W$ from $\nu - N$ deep inelastic scattering via the neutral and charged currents. So it became clear that it was necessary to evaluate the EWC to the last two processes to establish the connection with $\sin^2 \theta_W$, and to muon decay to obtain the relation with $G_F$ and $\alpha$.

Since this required dealing with a number of processes involving neutral and charged currents, I strongly felt that the first step in the strategy should be to develop a simple method to renormalize the Electroweak Sector of the SM. I proposed this in a paper with a related title: “Radiative Corrections in the $SU(2)_L \times U(1)_Y$ theory: a Simple Renormalization Framework” [14]. This approach, with important subsequent contributions by other physicists, is currently known as the on-shell scheme (OS). In the same paper, I applied the OS scheme to $\mu$ decay in the SM and introduced the EWC $\Delta r$, whose significance I will briefly discuss later. In two subsequent papers with Bill, the OS scheme was applied to study the EWC to $\nu - N$ deep inelastic scattering via the neutral and charged currents [15, 16]. This trilogy of papers achieved
our aim to establish contact with the expected measurements of \( M_W \) and \( M_Z \) (which were carried out later). In fact, the papers led to more accurate predictions of \( M_W, M_Z \) using the OS relations [14, 17]

\[
s^2 c^2 = \frac{A^2}{M_Z^2 (1 - \Delta r)}, \tag{1}
\]

\[
A^2 = \frac{\pi \alpha}{\sqrt{2} G_F}, \tag{2}
\]

\[
s^2 = \sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}, \tag{3}
\]

and the information from \( \nu - N \) scattering.

During the 80’s Bill and I also employed a hybrid \( \overline{MS} \) scheme where couplings are defined by \( \overline{MS} \) subtractions but masses are still the physical ones. It plays an important rôle in GUT predictions, which we also studied [18].

The two schemes, OS and \( \overline{MS} \), were applied systematically to additional important processes, such as \( \nu \)-lepton scattering [19] and atomic parity violation [20].

As experiments improved, the rôle of the EWC became more important and Bill and I became part of a large collaboration, led by Paul Langacker, whose aim was to elucidate the comparison between theory and experiment. This culminated in a detailed review paper [21]. Some of the estimates of this analysis were \( M_W = 80.2 \pm 1.1 \text{ GeV}, M_Z = 91.6 \pm 0.9 \text{ GeV} \), with central values within 0.2 GeV and 0.4 GeV from the current ones, respectively. We also obtained \( M_t < 180 \text{ GeV} @ 90\% \text{ CL} \) for \( M_H < 100 \text{ GeV} \). Over the years, Paul remained an invaluable and very close collaborator.

Meanwhile, in the mid-eighties, a serious problem arose in the analysis of the Superallowed Fermi transitions. Experiments on eight transitions reached great accuracy and showed a significant departure from the expectations of the conserved-vector-current hypothesis (CVC), which is an integral part of the SM. Simple theoretical arguments convinced me that the problem was related to the evaluation of the two-loop corrections of \( O(Z \alpha^2) \), which had been carried out numerically many years before. My student Roberto Zucchini and I studied this correction analytically, reviewed the analysis of the eight transitions in the light of our calculation, and found very good agreement with CVC [22], a result that was confirmed by new numerical evaluations by Jaus and Rasche.

In the seventies and eighties I developed a fruitful collaboration with M. A. B. Bég. Together, and often with other physicists, we wrote several
papers and two extensive and, to some extent, pedagogical reviews on Gauge Theories of Weak Interactions [23].

In the seventies I participated in an ambitious project, led by T. D. Lee, to study non-topological solitons in quantum field theories [24]. Since my post-doctoral years at Columbia, T. D. Lee, one of the great masters of our discipline, has been for me a constant source of motivation and learning.

During the seventies, eighties, and nineties, I continued my close collaboration with Bill, with B. A. Kniehl, and several of my students, former students, and post-doctoral associates: S. Sarantakos, S. Bertolini, R. Zucchini, G. Degrassi, S. Fanchiotti, P. Gambino, J. Papavassiliou, K. Philippides, M. Passera, P. A. Grassi, A. Ferroglia, and G. Ossola.

Around 1989, LEP and SLC started operations, and FNL began the accurate measurement of $M_W$. LEP soon determined $M_Z$ with great precision. This prompted a change in strategy: $\alpha$, $G_F$, and $M_Z$ were adopted as the basic input parameters, a great effort was made to study the observables at the $Z$ peak, namely the line shape and the various asymmetries and widths measured at LEP and SLC, and there was a major improvement in the comparison between theory and experiment.

## 2 Input Parameters

As I mentioned before, there are three very accurately determined quantities that play a special role as input parameters:

i) $\alpha = 1/137.03599959(38)(13)$, $\delta\alpha = \pm 0.0037$ ppm, derived most precisely from $g(e) - 2$.

ii) $G_F \equiv G_\mu = (1.16637 \pm 0.00001) \times 10^{-5}$ GeV$^{-2}$, $\delta G_F = \pm 9$ ppm, where I defined $G_\mu$ from the muon lifetime using the finite photonic corrections of the V-A Fermi theory:

$$
\delta = 1 + \frac{\alpha}{2\pi} \left(\frac{25}{4} - \pi^2\right) \left[1 + \frac{2\alpha}{3\pi} \ln\left(\frac{m_\mu}{m_e}\right)\right] + 6.701 \left(\frac{\alpha}{\pi}\right)^2 + \cdots
$$

The $\mathcal{O}(\alpha)$ term has been known for a long time [25], the logarithmic term of $\mathcal{O}(\alpha^2)$ was derived several years later [26], while the last term was evaluated very recently [27]. It very nearly cancels the logarithmic
term of $O(\alpha^2)$. Including very small $O(\alpha m_e^2/m_\mu^2)$ contributions one has:

$$\delta = 1 - 4.1995 \times 10^{-3} + 1.5 \times 10^{-6} + \cdots,$$

(5)

where the second and third terms stand for the one and two-loop contributions. This reveals two interesting points: i) when the corrections are expressed in terms of $\alpha$, as in Eq.(4), by a fortuitous cancellation the $O(\alpha^2)$ effects are very small, and the original $O(\alpha)$ calculation turns out to be very accurate. ii) It took about four decades to evolve from the one-loop to the complete two-loop calculation! This is a sobering indication of how difficult it may be to achieve the new frontier of complete two-loop calculations in the SM!

iii) $M_Z = 91.1875 \pm 0.0021$ GeV, $\delta M_Z = \pm 23$ ppm

3 Basic Electroweak Corrections

There are a number of basic electroweak corrections that play an important role in the analysis of the SM.

The EWC $\Delta r$ that appears in Eq.(4) depends on the various physical parameters of the SM such as $\alpha$, $M_W$, $M_Z$, $M_H$, $M_f$, etc., where $M_f$ stands for a generic fermion mass. From a theoretical point of view, the significance of Eqs.(1-3) is that they provide the relation between the physical parameters of the Fermi theory (low-energy effective theory), namely $G_F$ and $\alpha$, with those of the SM (underlying theory), namely $\alpha$, $M_W$, $M_Z$, $M_H$, $M_f$, . . . , at the level of the quantum corrections. Eqs.(1-3) are currently used to calculate $M_W = M_W(M_H)$ leading to very sharp constraints on $M_H$. As is clear from Eqs.(1-3), $\Delta r$ is a physical observable.

Two other important relations are [28, 29]

$$\hat{s}^2 \hat{c}^2 = \frac{A^2}{M_Z^2(1 - \Delta \hat{r})},$$

(6)

$$\hat{s}^2 = \frac{A^2}{M_W^2(1 - \Delta \hat{r}_W)},$$

(7)

where $\hat{s}^2 \equiv \sin^2 \hat{\theta}_W(M_Z)$ is the electroweak mixing parameter defined by modified minimal subtraction, and evaluated at the scale $\mu = M_Z$. It is employed in the $\overline{MS}$ scheme and plays a crucial rôle in GUT studies. $\Delta \hat{r}$
and $\Delta \hat{r}_W$ are the relevant EWC. I introduced Eq. (3) and evaluated $\Delta \hat{r}$ while visiting CERN in August of 1989, at the time LEP was starting operations. By the end of Aug., LEP had measured $M_Z$ within 160 MeV. Using $\Delta \hat{r}$ and the new $M_Z$ measurement, $\hat{s}^2$ could be determined with significantly greater precision. Further, the improved determination of $\hat{s}^2$ was consistent with supersymmetric Grand Unification [28]!

The $\overline{MS}$ and OS versions of the electroweak mixing parameter, namely $\hat{s}^2$ and $s^2$, are related by [30]

$$\hat{s}^2 = s^2 \left(1 + \frac{e^2}{s^2} \Delta \hat{\rho}\right),$$

$$\Delta \hat{\rho} = \text{Re} \left[\frac{A_{WW}(M_W^2)}{M_W^2} - \frac{A_{ZZ}(M_Z^2)}{M_Z^2 \hat{\rho}}\right]_{\overline{MS}},$$

where $A_{WW}(q^2)$ and $A_{ZZ}(q^2)$ are the $W - W$ and $Z - Z$ transverse self-energies, $\hat{\rho} = (1 - \Delta \hat{\rho})^{-1}$, and $\overline{MS}$ denotes the $\overline{MS}$ renormalization and the choice $\mu = M_Z$.

Another important version of the electroweak mixing parameter is $s_{eff}^2 = \sin^2 \theta_{eff}$, used by the Electroweak Working Group (EWWG) to analyze the data at the $Z$ resonance.

The relations between $s_{eff}^2$ and $\hat{s}^2$ and $s^2$ are given by [31]

$$s_{eff}^2 = \text{Re} k_l(M_Z^2) \hat{s}^2 = \text{Re} k_l(M_Z^2) s^2,$$

where $k_l(q^2)$ and $k_l(q^2)$ are electroweak form factors. In particular, because of a fortuitous cancellation of effects, $\text{Re} \dot{k}_l(M_Z^2)$ is very close to unity and $s_{eff}^2 - \hat{s}^2 \approx 10^{-4}$.

It is also very convenient to employ the expression [32, 33]

$$s_{eff}^2 c_{eff}^2 = \frac{A^2}{M_Z^2 (1 - \Delta r_{eff})},$$

$$\Delta r_{eff} = \Delta \hat{r} + \frac{e^2}{s_{eff}^2 c_{eff}^2} \Delta \dot{k} \left(1 - s_{eff}^2 c_{eff}^2\right) (1 + x_t) + \cdots,$$

where $x_t = 3G_\mu M_t^2 / 8\sqrt{2}\pi^2$ is the leading contribution to $\Delta \hat{\rho}$.

The neutral current vertex of the $Z$ boson into an $f - \bar{f}$ pair has the form

$$< f \bar{f} | J^Z | 0 > = V_{f\bar{f}}(q^2) \bar{u}_f \gamma_\mu \left[\frac{I_{3f}}{2} (1 - \gamma_5) - \dot{k}_f(q^2) s^2 Q_f\right] v_f,$$
where $V_f(q^2)$, $k_f(q^2)$, and its OS counterpart $k_f(q^2)$ are electroweak form factors, and $I_{3f}$ and $Q_f$ denote the third component of weak isospin and the charge of fermion $f$.

The incorporation of QCD effects of $O(\alpha \bar{\alpha} s, \alpha \bar{\alpha}^2 s)$ in the basic EWC was studied with B.A. Kniehl and my student Sergio Fanchiotti [34].

4 Asymptotic Behaviors

The basic corrections $\Delta r$, $\Delta \hat{r}$, $\Delta \hat{r}_W$, $\Delta r_{eff}$, $\Delta \hat{\rho}$, $\hat{k}_f$, ... have been studied in great detail by several groups. Here I can only point out the asymptotic behaviors for large $M_t$, $M_H$ at the one loop level:

$$\Delta r \sim -\frac{3\alpha}{16\pi s^4} \frac{M_t^2}{M_Z^2} + \frac{11\alpha}{24\pi s^2} \ln \left( \frac{M_H}{M_Z} \right) + \cdots ,$$

$$\Delta r_{eff} \approx \Delta \hat{r} \sim -\frac{3\alpha}{16\pi \bar{s}^2 \bar{c}^2} \frac{M_t^2}{M_Z^2} + \frac{\alpha}{2\pi \bar{s}^2 \bar{c}^2} \left( \frac{5}{4} - \frac{3}{4} \bar{c}^2 \right) \ln \left( \frac{M_H}{M_Z} \right) + \cdots .$$

Eqs. (14, 15) reveal a quadratic dependence on $M_t$, a logarithmic dependence on $M_H$. Also, the asymptotic behaviors in $M_t$ and $M_H$ have opposite signs, which explains the well-known $M_t - M_H$ correlation.

The asymptotic behavior of the neutral current amplitude is

$$\text{NC ampl.} \sim \frac{G_F}{1 - x_t} ,$$

where $x_t$ is defined after Eq. (12).

Additional contributions to $\Delta r$ and $\Delta r_{eff}$ lead to variations

$$\delta M_W/M_W \approx -0.205 \delta (\Delta r) ,$$

$$\delta s_{eff}^2/s_{eff}^2 \approx 1.52 \delta (\Delta r_{eff}) .$$

5 The $M_t$ Prediction

A very good example of the successful interplay between theory and experiment was provided by the $M_t$ prediction and its subsequent measurement. Before 1995, the top quark could not be produced directly, but it was possible to estimate its mass because of its virtual contributions to the EWC. In Nov. 94, a global analysis by the EWWG led to the indirect determination

$$M_t = 178 \pm 11^{+18}_{-19} \text{GeV} ,$$

where $\delta r$ is defined after Eq. (12),

$$\delta M_W/M_W \approx -0.205 \delta (\Delta r) ,$$

$$\delta s_{eff}^2/s_{eff}^2 \approx 1.52 \delta (\Delta r_{eff}) .$$
where the central value corresponds to $M_H = 300 \text{ GeV}$, the first error is experimental, and the second reflects the shift in the central value to $M_H = 65 \text{ GeV} (-19 \text{ GeV})$ or $M_H = 1 \text{ TeV} (+18 \text{ GeV})$. This may be compared with the current measurement $(M_t)_{exp} = (174.3 \pm 5.1) \text{ GeV}$.

This quite successful prediction was due to the quadratic $M_t$-dependence of the basic corrections, as illustrated in Eqs. (14, 15, 16).

6 Renormalization Schemes

As discussed in Section 4, the EWC have been carried out in a number of renormalization frameworks. Two of the most frequently employed are:

On-Shell (OS) Scheme. It is “very physical”, since it identifies renormalized couplings and masses with physical, scale-independent observables, such as $G_F, \alpha, M_Z, M_W, M_H, M_t, \ldots$

$\overline{\text{MS}}$ Scheme. It has very good convergence properties. This is related to the fact that in this scheme one essentially subtracts the pole terms and, therefore, the calculations follow closely the structure of the unrenormalized theory. As a consequence, it avoids large finite corrections frequently induced by renormalization. It employs inherently scale-dependent couplings such as $\alpha(\mu), \hat{s}^2(\mu)$, which play a crucial rôle in the analysis of Grand Unification. On the other hand, this leads to a residual scale dependence in the calculation of observables in finite orders of perturbation theory. The choice $\mu = M_Z$ is frequently made.

Very recently, a novel approach was proposed with my students Ferroglia and Ossola [33, 35]:

Effective Scheme. It shares the good convergence properties of the $\overline{\text{MS}}$ approach, but the calculation of observables in this scheme is strictly scale independent in finite orders. It employs scale-independent quantities such as $s^2_{\text{eff}}, G_F, M_Z^2$ as basic parameters. The reason that the Effective Scheme shares the good convergence properties of the $\overline{\text{MS}}$ approach is related to the fact that, as mentioned before, $\sin^2 \theta_{\text{lept}}$ and $\hat{s}^2(M_Z)$ are numerically very close (Cf. discussion after Eq. (14)).
7 The running of $\alpha$

A very important contribution to the EWC is due to the running of $\alpha$ to the $M_Z$ scale (vacuum polarization contributions):

$$\alpha(M_Z)/\alpha = 1/(1 - \Delta \alpha).$$ (20)

The light quarks’ contribution ($u$-$b$) is evaluated using dispersion relations and the experimental cross section for $e^+e^- \rightarrow$ hadrons at low $\sqrt{s}$, and perturbative QCD (PQCD) at large $\sqrt{s}$. Recently, “theory driven” calculations claim to reduce the error by using PQCD down to low $\sqrt{s}$ values.

In the Winter 2002 analysis [36], the EWWG employs two determinations:

$$\Delta \alpha_h^{(5)} = 0.02761 \pm 0.00036,$$ (21)

and

$$\Delta \alpha_h^{(5)} = 0.02747 \pm 0.00012.$$ (22)

The leptonic contribution is

$$\Delta \alpha_l = 0.03150.$$ (23)

8 Evidence for Electroweak Corrections

A) Evidence for EWC beyond the running of $\alpha$ [37]. It can be obtained by measuring $\Delta r$. Using $(M_W)_{\text{exp}} = 80.451 \pm 0.033$ GeV [36], and Eqs.(4-3) one finds $(\Delta r)_{\text{exp}} = 0.03107 \pm 0.00200$. The contribution to $\Delta r$ from the running of $\alpha$ is $\Delta \alpha = 0.05911 \pm 0.00036$, where I used Eqs.(21,23). The EWC not associated with $\Delta \alpha$ is $(\Delta r)_{\text{exp}} - \Delta \alpha = -0.02804 \pm 0.00203$, which differs from 0 by 13.8 $\sigma$! A similar result is obtained by comparing $(s^2_{\text{eff}})_{\text{exp}} = 0.23149 \pm 0.00017$ [36] and $(s^2)_{\text{exp}} = 0.22162 \pm 0.00064$. The difference is 0.00987 $\pm$ 0.00066 or 14.9 $\sigma$! And it is due to EWC not involving $\Delta \alpha$. In fact, this difference is dominated by the correction $c^2 \Delta \rho$ in Eq.(8).

B) Evidence for Electroweak Bosonic Correction (EWBC) [38]. They include loops involving the bosonic sector, W’s, Z, H. They are subleading numerically, but very important conceptually. Evidence for these correction can be found by measuring $(\Delta r)_{\text{eff}}$. Using $(s^2_{\text{eff}})_{\text{exp}} =$
0.23149 ± 0.00017 and Eq. (11), we find \((\Delta r_{\text{eff}})_{\text{exp}} = 0.06047 ± 0.00048\).

Subtracting the contribution of the EWBC, but retaining the fermionic corrections, the theoretical value is \((\Delta r_{\text{eff}})_{\text{theor}}^{\text{subtr}} = 0.05106 ± 0.00083\). The difference is \((\Delta r_{\text{eff}})_{\text{exp}} - (\Delta r_{\text{eff}})_{\text{theor}}^{\text{subtr}} = 0.00941 ± 0.00096\), a 9.8 σ effect!

### 9 Theoretical Pursuit of the Higgs Boson

The Higgs boson is the fundamental missing piece of the SM! With \(M_t\) measured, to what extent can \(M_H\) be constrained? For large \(M_t\), the EWC contain contributions proportional to \(\ln M_H/M_Z\). We need precise calculations! Theorists distinguish two classes of errors:

1) parametric, such as \(\delta M_t, \delta \Delta \alpha_h^{(5)}, \delta s_{\text{eff}}^2, \ldots\)

2) uncertainties due to the truncation of the perturbative series (i.e. uncalculated higher order effects). What is the status of the higher order corrections? Contributions of \(\mathcal{O}(\alpha), \mathcal{O}(\alpha \log M_Z/M_f)^n\), and \(\mathcal{O}(\alpha^2 \log M_Z/M_f)\) were analyzed during the period 1979-84. Those of \(\mathcal{O}(\alpha^2(M_t/M_W)^4), \mathcal{O}(\alpha \alpha_s), \) and \(\mathcal{O}(\alpha \alpha_s^2(M_t/M_W)^2)\) were studied from the late 80’s to the middle 90’s.

Of more recent vintage are the corrections of \(\mathcal{O}(\alpha^2(M_t/M_W)^2)\). Large \(M_t\) expansions were employed to evaluate the irreducible contributions of this order to the basic corrections, which were then incorporated in the calculation of \(s_{\text{eff}}^2\) and \(M_W\), as functions of \(M_H\), in three schemes: \(\overline{MS}\), and two versions, OSI and OSII, of the OS scheme, with two different implementations of the QCD corrections [39]. A large reduction was found in the scheme and residual scale dependences. Maximal variations, among the schemes, for given \(M_H\), amounted to \(\Delta s_{\text{eff}}^2 \approx 3 \times 10^{-5}\) and \(\Delta M_W \approx 2\) MeV. Including additional QCD uncertainties: \(\Delta s_{\text{eff}}^2 \approx 6 \times 10^{-5}\) and \(\Delta M_W \approx 7\) MeV. In the case of \(M_W\), the results can be compared with important new calculations that include all two-loop contributions to \(\Delta r\) that contain a fermion loop [40]. Again one finds \(\Delta M_W \approx 7\) MeV. The study of the \(\mathcal{O}(\alpha^2 M_t^2/M_W^2)\) contributions has been extended to the partial widths \(\Gamma_f(f \neq b)\) of the \(Z\) [41] and to the Effective Scheme [8, 11]. The incorporation of the \(\mathcal{O}(\alpha^2 M_t^2/M_W^2)\) had also a felicitous consequence: the 95% CL upper bound \(M_H^{(95)}\) was reduced by \(\approx 30\%\) [13].
10 Simple Formulae for $s_{eff}^2$, $M_W$, $\Gamma_l$

Very simple formulae, that reproduce accurately the numerical results of the detailed codes in the range $20 \text{ GeV} \leq M_H \leq 300 \text{ GeV}$, have been recently presented [42]. They are of the form

$$s_{eff}^2 = (s_{eff}^2)_0 + c_1 A_1 + c_2 A_2 - c_3 A_3 + c_4 A_4,$$

$$M_W = M_W^0 - d_1 A_1 - d_2 A_2 - d_3 A_3 - d_4 A_4,$$

$$\Gamma_l = \Gamma_l^0 - g_1 A_1 - g_2 A_2 + g_3 A_3 - g_4 A_4,$$

where $\Gamma_l$ is the leptonic partial width of the $Z$, $c_i, d_i, g_i (i = 1 - 5)$ are constants given in Ref. [42], and

$$A_1 \equiv \ln (M_H/100 \text{ GeV}) , \quad A_2 \equiv [\Delta \alpha_h^{(5)}/0.02761] - 1,$$

$$A_3 \equiv (M_t/174.3 \text{ GeV})^2 - 1 , \quad A_4 \equiv [\alpha_s(M_Z)/0.118] - 1. \quad (27)$$

In constructing these expressions, the input values $M_t = 174.3 \pm 5.1 \text{ GeV}$, $\Delta \alpha_h^{(5)} = 0.02761 \pm 0.00036$, $\alpha_s(M_Z) = 0.118 \pm 0.002$, were employed [36]. A very useful feature is that Eqs. (24-27) retain their accuracy over the rather large range $0.0272 \leq \Delta \alpha_h^{(5)} \leq 0.0283$ that encompasses the recent calculations.

We now discuss some instructive physical applications of Eqs. (24-27).

i) Using only Eq. (24) with $(s_{eff}^2)_{exp} = 0.23149 \pm 0.00017$ [36], one finds

$$M_H = 124^{+82}_{-52} \text{ GeV} ; \quad M_H^{95} = 280 \text{ GeV},$$

where $M_H^{95}$ stands for the 95% CL upper bound.

ii) Using only Eq. (25) with $(M_W)_{exp} = 80.451 \pm 0.033 \text{ GeV}$ [36], one obtains

$$M_H = 23^{+49}_{-23} \text{ GeV} ; \quad M_H^{95} = 122 \text{ GeV}.$$

Thus, at present, $(M_W)_{exp}$ constrains $M_H$ much more sharply than $s_{eff}^2$!

In fact, the above $M_H$ value is well below the direct exclusion bound $M_H > 114 \text{ GeV}$ (95% CL).

iii) Use $A_1$, derived from $(s_{eff}^2)_{exp}$ and Eq. (24), to predict $M_W$ via Eq. (25). This leads to:

$$(M_W)_{indir.} = (80.374 \pm 0.025) \text{ GeV},$$
which differs from \((M_W)_{\text{exp}}\) by 1.86\(\sigma\). The above value is close to \((M_W)_{\text{indir.}} = 80.379 \pm 0.023\) GeV, obtained in the global analysis \[30\].

to be compared with \(M_H = 85_{-34}^{+54} \text{GeV} ;\ M_H^{95} = 196 \text{GeV}\) in the recent EWWG fit \[38\].

The current \((s_{\text{eff}}^2)_{\text{exp}}\) determination by the EWWG has \(\chi^2/\text{d.o.f.} = 10.6/5\), which corresponds to a CL of only 6%. There is an intriguing dichotomy: from the leptonic observables \((A_l(SLD), A_l(P_{\tau}), A_{f_b}^{(0,l)})\) one finds \((s_{\text{eff}}^2)_l = 0.23113 \pm 0.00021\), while the hadronic measurements \((A_{f_b}^{(0,b)}, A_{f_b}^{(0,c)}, < Q_{f_b}>)\) lead to \((s_{\text{eff}}^2)_h = 0.23220 \pm 0.00029\). Thus, there is a 3 \(\sigma\) difference between the determinations of \(s_{\text{eff}}^2\) from the leptonic and hadronic sectors! It is also interesting to note that \((s_{\text{eff}}^2)_l\) leads to

\[M_H = 59_{-29}^{+50} \text{GeV} ;\ M_H^{95} = 158 \text{GeV}\]

which are closer to the \(M_H\) values derived from \((M_W)_{\text{exp}}\).

If \((s_{\text{eff}}^2)_h - (s_{\text{eff}}^2)_l\) is due to a statistical fluctuation, one possibility is to increase the error by \(\chi^2/\text{d.o.f.}^{1/2}\), according to the PDG prescription. This results in \(s_{\text{eff}}^2 = 0.23149 \pm 0.00025\). Interestingly, increasing the error in \(s_{\text{eff}}^2\) leads to a smaller \(M_H^{95}\) in the combined \(s_{\text{eff}}^2-M_W-\Gamma_l\) analysis: \((223 \text{GeV} \to 201 \text{GeV})!\) The reason is that increasing the \(s_{\text{eff}}^2\) error gives enhanced weight to the \(M_W\) input, which prefers a smaller \(M_H\) value.

If \((s_{\text{eff}}^2)_h - (s_{\text{eff}}^2)_l\) is due to new physics involving the \((t,b)\) generation, a substantial, tree-level change in the \(Zb_R\bar{b}_R\) coupling would be required \[44, 45\]. Very recently, it has been pointed out that if the \((s_{\text{eff}}^2)_h\) and \((s_{\text{eff}}^2)_l\) discrepancy were to settle on the leptonic value, a scenario with light \(\tilde{\nu}, \tilde{l}\), and \(\tilde{g}\) would improve the agreement with the electroweak data and the direct lower bound on \(M_H\) \[46\].
11 Global Fit

The SM describes rather well a large number of observables. Recent fits lead to:

\[ M_H = 85^{+54}_{-34} \text{ GeV} ; \quad M_H^{95} = 196 \text{ GeV} \quad [36], \]
\[ M_H = 90^{+50}_{-33} \text{ GeV} ; \quad M_H^{95} = 197 \text{ GeV} \quad [47] . \]

There are no major deviations from the SM fit. For instance, in the EWWG group analysis, the largest differences are 3 $\sigma$ for $\sin^2 \theta_W(\nu N)$, $-2.64 \sigma$ for $A_f^{(0, b)}$, 1.73 $\sigma$ for $M_W$, 1.63 $\sigma$ for $\sigma^0_{had}$, and 1.50 $\sigma$ for $A_f(SLD)$.

Thus, there are no major disagreements or compelling signals for new physics.

Nonetheless, it is very important to explore for new physics. For example, if the central values of $M_t$ and $M_W$ remain as they are now, but the errors shrink sharply as expected at Tevatron/LHC or even much better at LC+GigaZ, a discrepancy would be established with the SM, that can be accommodated in the MSSM. It is also very important to remember that $M_H < 135 \text{ GeV}$ in the MSSM. As emphasized by Bill in his talk, the measurement and analysis of the muon anomaly $g - 2$ is of particular interest at present. If a conclusive deviation from the SM prediction were established, an intriguing possibility would be the presence of supersymmetric contributions!

12 Precision Studies, Quantum Field Theory and Fundamental Physical Concepts

The foundations of the SM are firmly rooted in major developments in Quantum Field Theory: Yang-Mills Theories, their quantization and renormalizability; BRS symmetry; spontaneous symmetry breaking; renormalization schemes and their implementation; new techniques of computation, etc. . .

Precision studies have also led to unexpected byproducts. I mention two in which I was involved:

i) The discovery of the cancellation of mass-singularities in integrated transition probabilities (first paper in Ref. [25]). This was an important motivation for the Kinoshita-Lee-Nauenberg (KNL) Theorem.

ii) The elucidation of the concepts of mass and width of unstable particles [48] and Partial Widths [49]. In 1991 I realized that, in the context of
gauge theories, the conventional on-shell definitions of mass and width are gauge dependent in next-to-next-to-leading order, and proposed to solve this severe conceptual and practical problem in terms of definitions based on the complex-valued position of the propagator’s pole.

**Concluding Remarks**

With improving experimental precision, the study of electroweak and QCD corrections plays an increasingly important role.

The modern era of these studies, in the framework of the Fermi theory, started in the mid-fifties, in collaboration with R. E. Behrends and R. J. Finkelstein, who was our mentor 50. Since that time until the emergence of the SM, the significance of the problem of universality attracted the attention of several first rate theorists.

However, at any given moment, the number of physicists engaged in these studies was very limited: you could count us with the fingers of one hand!

The emergence of the SM created a new theoretical framework where these studies can be carried out in a theoretically consistent manner. At the same time we got lucky: experimental physics moved in the direction of precision electroweak physics and a rich phenomenology emerged! In fact, it is very likely that precision electroweak physics will continue to be an important component in the future development of our science.

For me, a particularly rewarding experience is to walk into a room at a Conference or a Workshop and see dozens of talented young theorists (some of them my own students and collaborators) working in this frontier area of Physics!

**Acknowledgments**

The author is indebted to J. Erler for illuminating communications. This work was supported in part by NSF Grant No. PHY-0070787.
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