This paper summarizes current results for rare non-hadronic $b$ decay processes. The world average $B(b \to s\gamma) = (3.14 \pm 0.48) \times 10^{-4}$ is in agreement with the standard model prediction. Upper limits on $b \to s\ell^+\ell^-$ and $b \to \ell^+\ell^-$ are also given. Finally $B^+ \to \ell^+\nu$ upper limits are presented as well as the world average value of $(255 \pm 21 \pm 28)$ MeV for $f_{D_s}$ from $D_s^+ \to \ell^+\nu$ decay rate measurements.
1 Introduction

This paper will describe current experimental results on both inclusive and exclusive $B$ meson decays to charmless final states containing one photon or two leptons. Specifically the topics include $b \rightarrow s\gamma$, $b \rightarrow s\ell^+\ell^-$, $b \rightarrow s\nu\overline{\nu}$, the associated exclusive reactions for these final states, and decays to dileptons and diphotons. I will also discuss the annihilation processes $B^- \rightarrow \ell^-\overline{\nu}$ and $D^+_s \rightarrow \mu^+\nu$.

These processes, with the exception of the lepton-neutrino final state, proceed through higher order weak interactions involving loops, which are often called “Penguin” processes, for unscientific reasons. A Feynman loop diagram is shown in Fig. 1 that describes the transition of a $b$ quark into a charged -1/3 $s$ or $d$ quark, which is effectively a neutral current transition. The dominant charged current decays change the $b$ quark into a charged +2/3 quark, either $c$ or $u$.

![Figure 1: Loop or “Penguin” diagram for a $b \rightarrow s$ or $b \rightarrow d$ transition.](image)

The intermediate quark inside the loop can be any charge +2/3 quark. The relative size of the different contributions arises from different quark masses and CKM elements. In terms of the Cabibbo angle ($\lambda=0.22$), we have for $t,c,u$: $\lambda^2,\lambda^2,\lambda^4$. The mass dependence favors the $t$ loop, but the amplitude for $c$ processes can be quite large $\approx 30\%$. Moreover, as pointed out by Bander, Silverman and Soni, interference can occur between $t$, $c$ and $u$ diagrams and lead to CP violation. In the standard model it is not expected to occur when $b \rightarrow s$, due to the lack of a CKM phase difference, but could occur when $b \rightarrow d$. In any case, it is always worth looking for this effect; all that needs to be done, for example, is to compare the number of $K^{*-}\gamma$ events with the number of $K^{*-}\gamma$ events.

There are other possibilities for physics beyond the standard model to appear. For example, the $W^-$ in the loop can be replaced by some other charged object such as a Higgs; it is also possible for a new object to replace the $t$. 
2 Standard Model Theory

In the Standard Model the effective Hamiltonian for the intermediate $t$ quark is given by

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) .$$  \hspace{1cm} (1)

Some of the operators are

$$O_1 = \pi^+_L \gamma_{\mu} b^-_L \gamma^\mu c^+_L, \quad O_7 = \frac{e}{16\pi^2} m_b \pi_L F_{\mu\nu} b^X_{\mu} F^{\nu\mu} .$$  \hspace{1cm} (2)

The matrix elements are evaluated at the scale $\mu = M_W$ and then evolved to the $b$ mass scale using renormalization group equations, which mixes the operators:

$$C_i(\mu) = \sum_j U_{ij}(\mu, M_W) C_j(M_W) .$$  \hspace{1cm} (3)

3 $b \to s\gamma$

This process occurs when any of the charged particles in Fig. 1 emits a photon. The only operator which enters into the calculation is $C_7(\mu)$. CLEO first measured the inclusive rate, as well as the exclusive rate into $K^*(890)\gamma$. I will report here on an updated CLEO measurement using 1.5 times the original data sample and a new measurement from ALEPH.

The momentum spectrum of the $\gamma$ peaks close to its maximum value at half the $B$ mass. If we had data with only $B$ mesons, it would be easy to pick out $b \to s\gamma$. We have, however, a large background from other processes. At the $\Upsilon(4S)$, the $\gamma$ spectrum from the different background processes is shown. The largest is $\pi^0$ production from continuum $e^+e^-$ collisions, but another large source is initial state radiation (ISR), where one of the beam electrons radiates a hard photon before annihilation. The backgrounds and the expected signal are illustrated in Fig. 2. Similar backgrounds exist at LEP.

To remove background CLEO used two techniques originally, one based on “event shapes” and the other on summing exclusively reconstructed $B$ samples. Examples of idealized events are shown in Fig. 3. CLEO uses eight different shape variables described in Ref. [3], and defines a variable $r$ using a neural network to distinguish signal from background. The idea of the $B$ reconstruction analysis is to find the inclusive branching ratio by summing over exclusive modes. The allowed hadronic system is comprised of either a $K_s \to \pi^+\pi^-$ candidate or a $K^+\pi^-$ combined with 1-4 pions, only one of which can be neutral. The restriction on the number and kind of pions maximizes efficiency while
minimizing background. It does however lead to a model dependent error. For all combinations CLEO evaluates
\[ \chi^2_B = \left( \frac{M_B - 5.279}{\sigma_M} \right)^2 + \left( \frac{E_B - E_{beam}}{\sigma_E} \right)^2, \]
(4)
where \( M_B \) is the measured \( B \) mass for that hypothesis and \( E_B \) is its energy. \( \chi^2_B \) is required to be \( < 20 \). If any particular event has more than one hypothesis, the solution which minimizes \( \chi^2_B \) is chosen. For events with a reconstructed \( B \) candidate CLEO also considers the angle between the thrust axis of the \( B \) and the thrust axis of event with the \( B \) candidate removed, \( \cos(\theta_t) \). This is highly effective in removing continuum background.

Another neural network is used to combine \( r, \chi^2_B, \cos(\theta_t) \) into a new
variable $r_c$ and events are then weighted according to their value of $r_c$. This method maximizes the statistical potential of the data. Fig. 3 shows the photon energy spectrum of the inclusive signal, compared with the model of Ali and Greub. A fit to the model over the photon energy range from 2.1 to 2.7 GeV/c gives the branching ratio result shown in Table 1, where the first error is statistical and the second systematic.

![Figure 4: The background subtracted photon energy spectrum from CLEO. The dashed curve is a spectator model prediction from Ali and Greub.](image)

Table 1: Experimental results for $b \rightarrow s\gamma$

| Sample   | Br (GeV/c) |
|----------|------------|
| CLEO     | $(3.15 \pm 0.35 \pm 0.41) \times 10^{-4}$ |
| ALEPH    | $(3.11 \pm 0.80 \pm 0.72) \times 10^{-4}$ |
| Average  | $(3.14 \pm 0.48) \times 10^{-4}$ |
| Theory   | $(3.28 \pm 0.30) \times 10^{-4}$ |

ALEPH reduces the backgrounds by weighting candidate decay tracks in a $b \rightarrow s\gamma$ event by a combination of their momentum, impact parameter with respect to the main vertex and rapidity with respect to the $b$-hadron direction. The result is shown in Table 1 and the resulting photon energy spectrum in Fig. 3. The world average value experimental value is also given, as well as the theoretical prediction.

The standard model prediction is in good agreement with the data. It is evaluated by including $u$, $c$ and $t$ loops. The $t$ loop has the CKM factors $V_{tb}V_{ts}^*$. The $c$ and $u$ can be combined, since by unitarity

$$V_{cb}V_{cs}^* + V_{ub}V_{us}^* = -V_{tb}V_{ts}^* .$$

(5)
Thus, comparing the model prediction with the data in Table II gives a measurement of $|\frac{V_{ts}}{V_{tb}}| = 0.98 \pm 0.28$, consistent with the expectation of unity.

The consistency with standard model expectation has ruled out many models. Hewett has given a good review of the many minimal supergravity models which are excluded by the data.

Triple gauge boson couplings are of great interest in checking the standard model. If there were an anomalous $WW\gamma$ coupling it would serve to change the standard model rate. $p\bar{p}$ collider experiments have also published results limiting such couplings. In a two-dimensional space defined by $\Delta \kappa$ and $\lambda$, the D0 constraint appears as a tilted ellipse and the $b\rightarrow s\gamma$ as nearly vertical bands. In the standard model both parameters are zero.

4 The Exclusive Decays $K^{\ast}\gamma$ and $\rho\gamma$

The CLEO measurements have not as yet been updated. The exclusive branching ratio is far more difficult to predict than the inclusive. CLEO measures $\mathcal{B}(B \rightarrow K^{\ast}(890)\gamma) = (4.2 \pm 0.8 \pm 0.6) \times 10^{-5}$, with this exclusive final state comprising $(18 \pm 7)\%$ of the total rate.

CLEO also limits $\mathcal{B}(B \rightarrow \rho\gamma) < 1.2 \times 10^{-5}$ at 90% confidence level. This leads to a model dependent limit on $|V_{td}/V_{ts}|^2 < 0.45 - 0.56$, which is not very significant. It may be possible that improved measurements can find a meaningful limit, although that has been disputed.
5 \( b \rightarrow s\ell^+\ell^- \) and \( b \rightarrow s\nu\bar{\nu} \)

The diagrams that contribute to \( b \rightarrow s\ell^+\ell^- \), where \( \ell \) refers to either an electron or muon are shown in Fig. 6. The diagrams for \( b \rightarrow s\nu\bar{\nu} \) are similar, with \( \nu \) replacing \( \ell \), except that only the \( Z^0 \) contributes in the left-hand diagram.

![Diagram](image)

Figure 6: Loop or “Penguin” diagram for a \( b \rightarrow s\ell^+\ell^- \) transition.

The operator structure is more complicated as \( C_9 \) and \( C_{10} \) contribute along with \( C_7 \). CP violation can be looked at in both the branching ratios and the polarization of the lepton pair. No signals have been seen as yet in any inclusive or exclusive modes. Current upper limits are given in Table 2 and Table 3.

| Sample | \( b \rightarrow s\mu^+\mu^- \) | \( b \rightarrow se^+e^- \) | \( b \rightarrow s\nu\bar{\nu} \) |
|--------|-----------------|-----------------|-----------------|
| SM theory | \((0.8 \pm 0.2) \times 10^{-5}\) | \((0.6 \pm 0.1) \times 10^{-5}\) | \((3.8 \pm 0.8) \times 10^{-5}\) |
| CLEO | \(< 5.7 \times 10^{-5}\) | \(< 5.7 \times 10^{-5}\) | \(< 7.7 \times 10^{-4}\) |
| ALEPH | \(< 5.7 \times 10^{-5}\) | \(< 5.7 \times 10^{-5}\) | \(< 7.7 \times 10^{-4}\) |

| Mode | CLEO | CDF | DELPHI | Theory |
|------|------|-----|--------|--------|
| \( K^-\mu^+\mu^- \) | \(< 1.0 \times 10^{-5}\) | \(< 1.0 \times 10^{-5}\) | \(< 1.0 \times 10^{-5}\) | 0.2-1.0\times 10^{-6} |
| \( K^-e^+e^- \) | \(< 1.0 \times 10^{-5}\) | \(< 1.0 \times 10^{-5}\) | \(< 1.0 \times 10^{-5}\) | 0.2-1.0\times 10^{-6} |
| \( K^0\mu^+\mu^- \) | \(< 2.1 \times 10^{-5}\) | \(< 2.1 \times 10^{-5}\) | \(< 2.1 \times 10^{-5}\) | 0.2-1.0\times 10^{-6} |
| \( K^0e^+e^- \) | \(< 1.7 \times 10^{-5}\) | \(< 1.7 \times 10^{-5}\) | \(< 1.7 \times 10^{-5}\) | 0.2-1.0\times 10^{-6} |
| \( K^{*0}\mu^+\mu^- \) | \(< 1.1 \times 10^{-5}\) | \(< 1.1 \times 10^{-5}\) | \(< 1.1 \times 10^{-5}\) | 0.8-4.2\times 10^{-6} |
| \( K^{*0}e^+e^- \) | \(< 1.4 \times 10^{-5}\) | \(< 1.4 \times 10^{-5}\) | \(< 1.4 \times 10^{-5}\) | 1.1-6.0\times 10^{-6} |
| \( K^{*0}\nu\bar{\nu} \) | \(< 10^{-3}\) | \(< 10^{-3}\) | \(< 10^{-3}\) | \(1-5\times 10^{-5}\) |
| \( B_s \rightarrow \phi\nu\bar{\nu} \) | \(< 5.4 \times 10^{-3}\) | \(< 5.4 \times 10^{-3}\) | \(< 5.4 \times 10^{-3}\) | \(< 5.4 \times 10^{-3}\) |
6  \( b \rightarrow \ell^+\ell^- \) or \( b \rightarrow \gamma\gamma \)

The standard model diagrams for neutral \( b \) decays into two leptons are shown in Fig. 7. (I have not shown the \( \nu\bar{\nu} \) final state.) Larger rates are expected for \( B_s \) decays than \( B_d \) decays since \( V_{ts} \) is larger than \( V_{td} \).

![Figure 7: Diagrams for a \( b \rightarrow \ell^+\ell^- \) and \( b \rightarrow \gamma\gamma \).](image)

Only upper limits have been determined in these modes. The limits are orders of magnitude higher than the predictions. The best limit in each mode is given in Table 4.

| \( B_d \) mode | \( \gamma\gamma \) | \( e^+e^- \) | \( \mu^+\mu^- \) | \( \tau^+\tau^- \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Exp.            | \( L3 \)        | \( <38 \)      | \( <5.9 \)       | \( <0.86 \)      |
| \( B \times 10^6 \) |                | \( 90\% \)    | \( 90\% \)       | \( 95\% \)       |
| Confidence      |                |                |                |                    |
| Theory          |                |                |                | \( 10^{-8} \)    |

| \( B_s \) mode | \( \gamma\gamma \) | \( e^+e^- \) | \( \mu^+\mu^- \) | \( \tau^+\tau^- \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Exp.            | \( L3 \)        | \( <148 \)      | \( <54 \)       | \( <2.6 \)       |
| \( B \times 10^6 \) |                | \( 90\% \)    | \( 10^{-14} \)  | \( 95\% \)       |
| Confidence      |                |                |                |                    |
| Theory          |                |                |                | \( 10^{-9} \)    |

7  The decay \( B^- \rightarrow \ell^-\bar{\nu} \)

8  \( B^- \rightarrow \ell^-\bar{\nu} \)

This reaction proceeds via the annihilation of the \( b \) quark with the \( \bar{\nu} \) into a virtual \( W^- \) which materializes as \( \ell^-\bar{\nu} \) pair as illustrated in Fig. 8. The decay rate for this process can be written as

\[
\Gamma(B^- \rightarrow \ell^-\bar{\nu}) = \frac{G_F^2}{8\pi} f_B^2 m_{\ell}^2 M_B \left( 1 - \frac{m_{\ell}^2}{M_B^2} \right)^2 |V_{ub}|^2 ,
\]

(6)
where $f_B$ is the so called “decay constant,” a parameter that can be calculated theoretically or determined by measuring the decay rate. This formula is the same for all pseudoscalar mesons using the appropriate CKM matrix element and decay constant.

![Diagram for a $B^- \to \ell^+ \bar{\nu}$ decay.](image)

Figure 8: Diagram for a $B^- \to \ell^+ \bar{\nu}$ decay.

Knowledge of $f_B$ is important because it is used to determine constraints on CKM matrix elements from measurements of neutral $B$ mixing. Since the decay is helicity suppressed, the heavier the lepton the larger the expected rate. Thus looking for the $\tau^- \bar{\nu}$ has its advantages. The big disadvantage is that there are least two missing neutrinos in the final state. The most stringent limit has been set by L3 of $< 5.7 \times 10^{-4}$ at 90% confidence level, using a missing energy technique. This is still one order of magnitude higher than what is expected. Other limits are poorer.

Since $f_B$ is so difficult to measure, models, especially lattice gauge models, are used. However, it is prudent to test these models. $D_s^+ \to \mu^+ \nu$ can be used; it is Cabibbo favored and the predicted branching ratio is close to 1%. Several groups have made measurements. The results are shown in Table 5. I have changed the values of $f_{Ds}$ according to the new PDG $D_s$ decay branching fractions for the normalization modes and have corrected the old CLEO result by using the new fake rates determined in their updated analysis. In addition, there are new results using the $D_s^+ \to \tau^+ \nu$ decay from the L3 collaboration of $(309 \pm 58 \pm 33 \pm 38)$ MeV, and $(330 \pm 95)$ MeV from the DELPHI collaboration. The world average value for $f_{Ds}$ is $(255 \pm 21 \pm 28)$ MeV, where the common systematic error is due the error on the absolute branching ratio for $D_s^+ \to \phi \pi^+$. These numbers are consistent with C. Bernard’s world average for lattice theories of $(221 \pm 25)$ MeV.

### 9 Conclusions

The study of rare non-hadronic $b$ decays has been very important in confirming the standard model. (Although, we may have wished for non-standard model effects to show up in these decays.) We have seen that the decay rate for $b \to s \gamma$ is consistent with the standard model within errors and many non-standard models have been excluded. Further improvements in the statistical accuracy
Table 5: Measured values of $f_{D_s}$ from experimental values of $\Gamma(D_s^+ \to \mu^+\nu)$

| Collaboration | Observed Events | Published $f_{D_s}$ value (MeV) | Corrected $f_{D_s}$ value (MeV) |
|---------------|-----------------|---------------------------------|---------------------------------|
| CLEO (old)    | 39±8            | 344 ± 37 ± 62 ± 42              | 282 ± 30 ± 43 ± 34              |
| WA75          | 6               | 232 ± 45 ± 20 ± 48              | 213 ± 41 ± 18 ± 26              |
| BES           | 3               | 430±150±40                      | Same                            |
| E653          | 23.2 ± 6.0±1.0  | 194 ± 35 ± 20 ± 14              | 200 ± 35 ± 20 ± 26              |
| CLEO          | 182±22          | -                              | 280 ± 19 ± 28 ± 34              |

of the data can be expected, but improvements in the systematic errors will be slow, as will reductions in the theoretical errors.

The next inclusive decay to be studied will be $b \to s\ell^+\ell^-$. CLEO has shown that there is a large background from events where both $b$'s decay semileptonically at the $\Upsilon(4S)$. This background can, in principle, be removed by insisting that the two leptons come from the same vertex. The upcoming higher luminosity $e^+e^-$ experiments Babar, Belle and CLEO III may be able to see this mode. If they are not successful, perhaps hadronic $b$ collider experiments (BTeV, LHC-B) will be. Though some have thought that inclusive measurements are not possible at such machines, the CLEO technique that finds the inclusive rate by summing the exclusive channels can be used here, albeit with no neutral pions rather than the one allowed by CLEO.

Decays to dileptons can only be seen at hadronic machines, because of the small predicted rates. Charged $B$ decays to $\ell^-\bar{\nu}$ are difficult but can possibly be done at $\Upsilon(4S)$ machines with large integrated luminosity.

Exclusive modes are very important for studies of CP violation. Final states such as $K^*\ell^+\ell^-$ can be studied for rate asymmetries and differences in dilepton polarization. The only exclusive mode seen thus far is $K^*\gamma$, but hopefully that will soon be augmented.

Clearly the study of rare $b$ decays has just begun. We look forward to learning much from these reactions.

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