Excited hadrons from improved interpolating fields

Tommy Burch\textsuperscript{a}, Christof Gattringer\textsuperscript{a}, Leonid Ya. Glozman\textsuperscript{b}, Reinhard Kleindl\textsuperscript{b}, C. B. Lang\textsuperscript{b}, and Andreas Schäfer\textsuperscript{a} (for the BGR [Bern-Graz-Regensburg] Collaboration)

\textsuperscript{a} Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany.
\textsuperscript{b} Institut für Physik / Theoretische Physik, Universität Graz, A-8010 Graz, Austria.

The calculation of quark propagators for Ginsparg-Wilson-type Dirac operators is costly and thus limited to a few different sources. We present a new approach for determining spatially optimized operators for lattice spectroscopy of excited hadrons. Jacobi smeared quark sources with different widths are combined to construct hadron operators with different spatial wave functions. We study the Roper state and excited $\rho$ and $\pi$ mesons.

1. Optimized quark sources

The low-lying hadron spectrum shows a few features which are fingerprints of QCD. In the meson sector the pion occurs as an almost-Goldstone boson with its squared mass vanishing proportional to the quark mass in contrast to all other mesons. The observed ordering of the lowest positive, $1/2^+$, $N(1440)$, and negative parity excitations of the nucleon, $1/2^-$, $N(1535)$ is 'unnatural'. A physical picture based on linear confinement, Coulomb and color-magnetic terms, always arranges the first radial excitation above the first orbital excitation, i.e. the excited states have alternating parities.

Whereas ground state spectroscopy on the lattice is by now a well understood physical problem with impressive agreement with experiment, the lattice study of excited states is not so far advanced. In a lattice calculation the masses of excited states show up in the sub-leading exponentials of Euclidean two point functions. A direct fit of a single correlator is cumbersome since the signal is strongly dominated by the ground state. Also with methods such as constrained fits \cite{1} or the maximum entropy method \cite{2} one still needs very high statistics for reliable results.\cite{3} An alternative method is the variational method \cite{4} where one diagonalizes a matrix containing all cross correlations of a set of several operators with the correct quantum numbers. For a large enough and properly chosen set of basis operators each eigenmode is then dominated by a different physical state. After normalization the largest eigenvalue gives the correlator of the ground state, the second-largest eigenvalue corresponds to the first excited state, and so on.

It is important to optimize the spatial properties of the interpolating operators. An example for this fact is the Roper state where the variational method,\cite{4} based on nucleon operators that differ only in their diquark content but have the same spatial wave function, did not lead to success.\cite{5} It can be argued that a node in the radial wave function is necessary to capture reliably the Roper state or other radially excited hadrons. Recently we demonstrated that an elegant solution is to combine Jacobi smeared quark sources with different widths to build the hadron operators and compute the cross-correlations in the variational method. We find good effective mass plateaus for the first and partly the second radially excited states. The propagators are then fitted using standard techniques.

Already in \cite{6} Jacobi smeared sources were combined with point sources and cross-correlations studied in similar spirit (see also \cite{7}).

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The technique of Jacobi smearing is well known [3,10]. The smeared source lives in the timeslice \( t = 0 \) and is constructed by iterated multiplication with a smearing operator \( H \) on a point-like source. The operator \( H \) is the spatial hopping part of the Wilson term at timeslice 0; it is trivial in Dirac space and acts only on the color indices. This construction has two free parameters: The number of smearing steps \( N \) and the hopping parameter \( \kappa \). These can be used to adjust the profile of the source. Here we work with two different sources, a narrow source \( n \) and a wide source \( w \) with parameters given by

\[
\begin{align*}
n &: N = 18, \quad \kappa = 0.210, \quad \sigma/2 \approx 0.27 \text{ fm}, \\
w &: N = 41, \quad \kappa = 0.191, \quad \sigma/2 \approx 0.41 \text{ fm}.
\end{align*}
\]  

(1)

\( N \) and \( \kappa \) were chosen such that the profiles approximate Gaussian distributions with the indicated half-widths [7]. We remark that the two sources allow the system to build up radial wave functions with and without a node. The parameters were chosen such that simple linear combinations \( c_n n + c_w w \) of the narrow and wide profile approximate the first and second radial wave functions of the spherical harmonic oscillator: The coefficients \( c_n \sim 0.6, c_w \sim 0.4 \) approximate a Gaussian with a half-width of \( \sigma/2 \sim 0.33 \text{ fm} \), while \( c_n \sim 2.2, c_w \sim -1.2 \) approximate the corresponding excited wave function with one node.

The final form of the wave function is determined through the variational method [5]. In this approach one computes a complete correlation matrix of operators \( O_i, i = 1, 2, \ldots R \) that create from the vacuum the state which one wants to analyze. The eigenvalues \( \lambda^{(k)}(t) \) of the correlation matrix behave as \( \lambda^{(k)}(t) \propto e^{-t M_k} [1 + O(e^{-t \Delta M_k})] \), where \( \Delta M_k \) is the distance of \( M_k \) to nearby energy levels. The hadron sources we use for the correlation matrix are constructed from the narrow and wide quark sources.

### 2. Excited nucleon signals

For our quenched calculation we use the chirally improved Dirac operator [14]. It is an approximation of a solution of the Ginsparg-Wilson equation which governs chiral symmetry on the lattice. This operator is well tested in quenched ground state spectroscopy [12] where pion masses down to 250 MeV can be reached at a considerably smaller numerical cost than needed for exact Ginsparg-Wilson fermions. For ground states the chirally improved action shows very good scaling behavior. The gauge configurations were generated on a \( 12^3 \times 24 \) lattice with the Lüscher-Weisz action [13]. The inverse gauge coupling is \( \beta = 7.9 \), giving rise to a lattice spacing of \( a = 0.148(2) \text{ fm} \) as determined from the Sommer parameter [14]. The statistics of our ensemble is 100 configurations. We use 10 different quark masses \( m \) ranging from \( am = 0.02 \) to \( am = 0.20 \).

Our analysis is based on the interpolator \( \varepsilon_{abc}(u_{\rho} C \gamma_5 d_{\rho}) u_{\gamma} \). Each of the three quarks can be smeared either narrow (\( n \)) or wide (\( w \)). This gives 8 possible combinations (\( nnn, nww \), etc.). From a subset of 4 of these operators (after projection to definite parity) we calculate the correlation matrix \( C(t) \) which we then use in the variational method. The exponential decay of three eigenvalues is clearly identified. We identify these signals with the nucleon, the Roper state and the next positive parity resonance \( N(1710) \). A detailed discussion of further checks on the correct identification of the Roper state can be found in [7] (see also [3] concerning the problem of nucleon-\( \eta' \) ghost contributions).

### 3. Excited meson signals

As another test of our approach we discuss the \( \pi^- \) and \( \rho \) mesons and their radial excitations. For the \( \rho \) we use the interpolators \( \Pi(x) \gamma_i d(x) \) and \( \Pi(x) \gamma_i \gamma_5 d(x) \), for the pion \( \Pi(x) \gamma_5 d(x) \) and \( \Pi(x) \gamma_5 \gamma_5 d(x) \). Again we use wide and narrow quark sources for both interpolating fields, corresponding to 3 operators each (the combinations \( nw \) and \( wn \) give identical correlators and one of them can be omitted). When diagonalizing the \( 3 \times 3 \) matrix with either interpolator we see a pronounced exponential decay only for the two larger (in magnitude) eigenvalues, \( \lambda^{(1)}(t) \) and \( \lambda^{(2)}(t) \). The smallest eigenvalue \( \lambda^{(3)}(t) \) does not show a clear effective mass plateau. This is an indication that this eigenvalue couples to an unphysical quenched ghost state [15,16,3]. The final results for the masses as a function of the quark mass are
Figure 1. Masses of $\rho(770)$, $\rho(1450)$, $\pi(1400)$ and $\pi(1300)$ as a function of the quark mass (the experimental data were converted to lattice units with the Sommer parameter scale).

A crucial test of our method is to check whether indeed the ground state is built from a nodeless combination of our sources and the excited states do show nodes. This question can be addressed by analyzing the eigenvectors of the correlation matrix. This has been done in [7] and indeed confirms the expectation.

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