Dynamic Modelling and Pendulation Laws of Cable Hoisting System

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Abstract. The dynamic characteristics of the cable hoisting system (CHS) are studied by combining theoretical modelling and visual simulation. The dynamic model is derived using D’Alembert’s principle. It contains the effect of horizontal motion and lifting motion, and also takes into account the quality of the cable to improve the accuracy of model. The Runge-Kutta method is used to solve model numerically, and simulation is conducted within Adams environment to verify the dynamic model. Based on the dynamic model and numerical calculation, the effect of horizontal motion, lifting motion and system parameters on swing was studied. The results show that the dynamic model and numerical calculations are accurate; during the movement, the payload oscillation is mainly affected by the initial acceleration, sudden change of acceleration and lifting speed; the length of steel cable and the mass of hoisting payload determine the frequency of CHS, which does not affect the payload oscillation; the oscillation and frequency are affected by lifting laws; and the study of acceleration laws can be used to guide the design and control of CHS.

1. Introduction
High-altitude cable hoisting system (CHS) is to transfer variety of material from one location to another location by the steel cable in order to load and unload huge materials where heavy loads must be moved with extraordinary precision [1]. The CHS acceleration is required for motion and induces undesirable load swing, which poses safety hazards, affects precise positioning, and restricts work efficiency [2, 3]. In order to study the swing of the CHS, it is necessary to establish a dynamic model, plan a more reasonable movement and explore the swing control technology and method. However, due to its underactuated and nonlinear characteristics [4], the system is vulnerable to external interference, indirect control and poor control robustness of the hoisting payload.

Thus, many scholars have done a lot of research on cable crane system, and established two-dimensional two degrees of freedom model of nonlinear system in the early stage. Jaafar, Mohamed, Jamian, Abidin, Kassim and Ab Ghani [5] studied the effect of parameters such as the length of the cable, the mass of the crane payload and the weight of the trolley on the trolley displacement and swing angle of the crane under different voltage inputs. The references [6, 7] only studied the horizontal motion and ignored the lifting movement to control the swing of the lifting load, which has great limitations. Kral, Kreuzer and Wilmers [8] modelled the boom crane as a planar pendulum and the ship as a two-dimensional rigid body undergoing heave, surge, and pitch. The model was used to study the influence of cable hoisting on cargo pendulations. Gao, Chen and Zhang [9] established the
nonlinear model of the crane system by using the Lagrange equation method, and cancelled one degree of freedom of the three-dimensional model to obtain two-dimensional crane system dynamic model.

Additionally, some scholars have studied the helicopter suspension system. Lucassen and Sterk [10] were the first to study the helicopter suspension model. The relatively simple model studied the influence of helicopters and lifting systems. This model is limited to three degrees of freedom in the vertical plane, ignoring the aerodynamic force and moment of the lifting weight, so as to analyse the stability of the suspension flight. Finally, the dynamic equation of the suspension was linearized and the data were analysed.

Scholars have studied many researches on overhead crane about the swing characteristics and anti-swing control strategy [11, 12]. However, there are few studies on the movement planner of the CHS [13, 14]. Analysing the effect of the hoisting point movement laws on the hoisting payload swing is conducive to better movement planner, so as to ensure the operating efficiency of lifting and reduce energy consumption.

In this paper, the dynamic model of CHS is derived using D’Alembert’s principle, the factors such as cable mass, horizontal motion and lifting motion are considered into the model and the effects of main parameters on pendulation by using the dynamic model are studied.

2. Kinematics of the Cable Hosting System
The dynamic model is of great significance to the exploration of motion laws, the method of swing control and the formulation of operation strategy. Because Adams simulation process is cumbersome and inconvenient to study laws and methods, it is used to verify dynamic models and numerical algorithms. Before establishing the CHS model, the model processing and system motion analysis should be carried out. In order to facilitate the analysis of the dynamic characteristics of the system and the design of effective control methods, the hoisting payload is modeled as a point mass, the steel cable is modeled as an inextensible link.

Basically, the CHS is made up of the hoisting point moving along a horizontal axis with a payload hung from a steel cable, as shown in Figure 1, where A is the hoisting point, B is the hoisting payload. This work studied the movement of the hoisting payload when the hoisting point moves horizontally and the steel cable moves up or down. Therefore, the coordinate system is established with the hoisting point as the origin of the coordinate system, the direction of hoisting point movement as X axis, and the direction of hoisting payload movement vertically as Z axis. Selecting the position vector of the hoisting point \( \mathbf{s} \), the position vector of the steel cable \( \mathbf{l} \), and the position vector of the hoisting payload \( \mathbf{x} \) as the system degrees of freedom (DOF). It is also convenient for the subsequent simulation verification of Adams. In Figure 1, the hoisting point is electrically driven by a motor force \( \mathbf{F}_1 \), and the steel cable is electrically driven by a motor force \( \mathbf{F}_2 \).

![Figure 1. A simple model for two-dimensional cable hoisting system.](image-url)
The motion of the CHS consists of horizontal motion and rotational motion of the steel cable. As shown in Figure 1, the position of the hoisting point in coordinate system is given by

$$\begin{align*}
x_A &= s \\
z_A &= 0
\end{align*}$$

(1)

The position of the hoisting payload is given by

$$\begin{align*}
x_B &= x \\
z_B &= \sqrt{l^2 - (s - x)^2}
\end{align*}$$

(2)

The movements of the hoisting cable and rolling cable are different and should be calculated separately. The position of the hoisting cable is given by

$$\begin{align*}
x_A &= (s + x) / 2 \\
z_A &= \sqrt{l^2 - (s - x)^2} / 2
\end{align*}$$

(3)

The position of the rolling cable is considered to be the same as the hoisting point.

The acceleration of the hoisting point, the hoisting payload, and the hoisting cable can be expressed as:

$$\begin{align*}
a_{As} &= \ddot{s} \\
a_{Ac} &= 0 \\
a_{Bs} &= \dot{x}
\end{align*}$$

$$\begin{align*}
a_{Br} &= -(s - x)(l^2 - (s - x)^2)\ddot{s} + (s - x)(l^2 - (s - x)^2)\dot{x}
\end{align*}$$

$$\begin{align*}
\alpha &= l(\ddot{x} - (s - x)^2)\ddot{\theta} - ((x - s)\ddot{l} + l(\dot{s} - \dot{x}))\dot{l}/(2\sqrt{l^2 - (s - x)^2})
\end{align*}$$

(4)

The angle between the cable and the positive direction of $Z$ axis is given by

$$\theta \approx \tan \theta = \frac{s - x}{z_B}$$

(5)

The angular acceleration of the steel cable of the center of mass can be expressed as:

$$\alpha = -(l(s - x)(l^2 - (s - x)^2)\ddot{x} + l^2(l^2 - (s - x)^2)\ddot{s} - l^2(s - x)^2)\dot{x} - 2((l^2 + (s - x)^2 / 2)\ddot{l} - 3l(\dot{s} - \dot{x})(s - x) / 2)(x - s)\ddot{x} + l(\dot{s} - \dot{x}) / (l^2 - (s - x)^2)^{5/2}$$

(6)

According to the above analysis, $s$, $l$, and $x$ are the system degrees of freedom, $F_1$ and $F_2$ are independent external forces, so the system is an underactuated mechanical system with three degrees of freedom. The relationship between the five factors can be established by the dynamic model. We can observe the movement of the hoisting payload, and analyze the pendulation laws.

3. Dynamic Model of the Cable Hosting System

Since the cable hoisting system is a typical multivariable dynamic system and the upper end of the steel cable as a boundary condition has nonholonomic constraint, the equations of motion of the CHS are derived using D’Alembert’s principle, which is a general method to solve the dynamics problems.
of the system of constrained particle. Its characteristic is that solving the dynamic problems by using
the method of studying equilibrium problems of statics. Its equation is given as:

\[
\sum F_i + \sum F_{Ni} + \sum F_u = 0 \\
\sum M_o(F_i) + \sum M_o(F_{Ni}) + \sum M_o(F_u) = 0
\]  

(7)

Where, \( i = 1, 2, \ldots, n \). According to the D'Alembert's principle, we obtain the inertial force and the
dynamics of the particle system, so as to establish the dynamic balance equation. The equations of
dynamic equilibrium of the CHS can be written as:

\[
\sum F = 0, F_i = F_i(s-x)l + ma_{\alpha} - l\rho\alpha_{\alpha} - (L-l)\rho\alpha_{\alpha} = 0 \\
\sum F = 0, F_z = l\rho g + ma_{\beta} - l\rho\alpha_{\beta} = 0 \\
\sum M = 0, -F_i(l/2)(z_{gb} /l) + (l/2)(s-x) / l\rho m - l\rho\alpha / 12 = 0
\]  

(8)

Where \( m \) is the payload mass, \( L \) is the total length of the steel cable, \( \rho \) is the linear density of
cable.

Equation (8) can be represented by the following matrix-vector form:

\[
\begin{bmatrix}
M_{11} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{s} \\
\dot{\theta}
\end{bmatrix}
+
\begin{bmatrix}
H_1 \\
H_2 \\
H_3
\end{bmatrix}
=
\begin{bmatrix}
Q_1 \\
Q_2 \\
Q_3
\end{bmatrix}
\]  

(9)

Where,

\[
\begin{align*}
M_{11} &= (-\rho l^2 - 2ml) / (2l) \\
M_{12} &= (-2\rho Ll + \rho l^2) / (2l) \\
M_{13} &= 0 \\
M_{21} &= -(s-x)(\rho l / 2 + m) / (l^2 - (s-x)^2)^{3/2} \\
M_{22} &= (s-x)(\rho l / 2 + m) / (l^2 - (s-x)^2)^{3/2} \\
M_{23} &= -l(\rho l / 2 + m) / (l^2 - (s-x)^2)^{3/2} \\
M_{31} &= -l^2(\rho l^3 / 6 + ml^2 - m(s-x)^2) / (2(l^2 - (s-x)^2)^{3/2}) \\
M_{32} &= (\rho l^3 / 6 + m(s-x)^2l^2 - m(s-x)^4) / (2(l^2 - (s-x)^2)^{3/2}) \\
M_{33} &= -l(s-x)(\rho l^3 / 6 + ml^2 - m(s-x)^2) / (2(l^2 - (s-x)^2)^{3/2})
\end{align*}
\]  

(10)

\[
H_1 = 0 \\
H_2 = (l^3(\rho l / 2 + m)^3 + l(s-x)^3(\rho l / 2 + m)^3 + l^3(\rho l / 2 + m) / (s-x)^3 + g(l^2 - (s-x)^2)(\rho l + m)(l^2 - (s-x)^2)^{3/2} - 2x_\theta(\rho l / 2 + m) / (l^2 - (s-x)^2)^{3/2}) \\
H_3 = gm(s-x) / l^2 + (s-x)(\rho l^3 / 3 + \rho(s-x)^2l^2 / 6 + ml(s-x)^2l^2 - m(s-x)^4) \\
\]  

(11)
\[ Q_1 = F_1(s - x) / l - F_1 \]
\[ Q_2 = F_2(l^2 - (s - x)^2)^{1/2} / l \]
\[ Q_3 = F_3(l^2 - (s - x)^2)^{1/2} / 2 \]

It can be seen from the dynamic model that \( x \) is related to \( s, l, F_1 \) and \( F_2 \). The motion of the hoisting payload can be obtained by planning the movement of the hoisting point and the steel cable, and then the effect of the horizontal movement and the lifting movement on the hoisting payload can be analysed. And the equations of motion of the CHS can be reformed as:

\[
F_1 = 2(M_{31} \ddot{x} + M_{32} \ddot{s} + M_{33} \ddot{l}) / z_B
\]
\[
F_2 = l(M_{21} \ddot{x} + M_{22} \ddot{s} + M_{23} \ddot{l}) / z_B
\]
\[
\ddot{x} = (((s - x)M_{22} - 2M_{32}) / z_B - M_{12}) \ddot{s} + ((s - x)M_{23} - 2M_{33}) \ddot{l} / z_B
\]
\[
+ (s - x)H_2 - 2H_3 / z_B - H_1 / (M_{11} - ((s - x)M_{21} - 2M_{31}) / z_B)
\]

Then the Runge-Kutta method is used to solve a multi degree of freedom vibration equation, and obtain the time domain response curve of vibration. Suppose that:

\[
x_1 = x
\]
\[
x_2 = \dot{x}
\]

where \( x_1 \) is the displacement of the horizontal motion of the hoisting payload, \( x_2 \) is the velocity of the horizontal motion of the hoisting payload. After substituting \( x_1 \) and \( x_2 \) into equation (15), now we can rewrite the equation as:

\[
\begin{pmatrix}
\ddot{x}_1 \\
\ddot{x}_2
\end{pmatrix} =
\begin{pmatrix}
x_2 \\
(((s - x)M_{22} - 2M_{32}) / z_B - M_{12}) \ddot{s} + ((s - x)M_{23} - 2M_{33}) \ddot{l} / z_B
\end{pmatrix}
\]

The values corresponding to \( x_1 \) and \( x_2 \) to each time are obtained by using ode45 module.

4. Implementation, Results and Discussion
The experience of manual operation controls the swing angle within a reasonable range to achieve the hoisting payload positioning and pendulation reduction. Thus, through control of the horizontal movement and the lifting movement, the pendulation can be reduced without affecting the fast and accurate positioning of the CHS. The nonlinear coupling between hoisting traction movement and its swing makes it important to study the effect of the hoisting point movement on hoisting. The pendulation of CHS is related to the acceleration of the hoisting point, the acceleration laws of the hoisting point, the lifting laws of the steel cable, as well as the length of the steel cable and the mass of the hoisting payload, etc. In view of the above aspects, this paper analysed the dynamic model of the CHS and explores its motion laws.

Before analysing the laws of motion, calculating the natural frequency of the CHS, which is similar to a simple pendulum. Since the mass of the steel cable is also taken into account in this system, the frequency is derived on the basis of the system, and the calculation results can be obtained as:

\[
L_\alpha = \frac{ml + \frac{1}{2} l \cdot \rho l}{m + \rho l}
\]
which is the centre of mass of the CHS. Then the natural frequency of the CHS can be obtained as:

\[ f = \frac{1}{2\pi} \sqrt{\frac{(m + \rho l)g}{ml + \frac{1}{2} \rho l^2}} \]  

(19)

If \( m = 2000\, \text{kg}, l = 15000\, \text{mm} \), then natural frequency of the CHS \( f = 0.129 \). We can judge the vibration of the CHS by analysing the natural frequency and velocity oscillation of the hoisting payload. Since the solution of the dynamic model can obtain two results of the displacement and speed of the hoisting payload, if the speed of the hoisting payload is basically the same as the hoisting point, the system can be considered as stable motion, so the following only analyses the speed of the hoisting payload. In this study, taking a certain garbage grab crane as an example to discuss the pendulation laws of the CHS.

**4.1. The Effect of the Acceleration of Hosting Point on Pendulation of the CHS**

The movement of the hoisting payload is affected by three factors: horizontal movement, lifting movement and itself swing. In order to express the relationship between the acceleration of the hoisting point and the pendulation more intuitively, the system only performs horizontal motion, and when the speed is a uniform motion, the speed vibration of the hoisting payload is the pendulation of the CHS. The following system parameters were fixed as: \( m = 2000\, \text{kg}, l = 15000\, \text{mm} \). The acceleration at the starting and stopping moment of the system are respectively 28.125 \( \text{mm/} \text{s}^2 \), 56.25 \( \text{mm/} \text{s}^2 \) and \( \infty \), and the speed of the uniform motion in the middle is 225 \( \text{mm/} \text{s} \).

The speed relationship between the hoisting point and the hoisting payload in Figure. 2 shows that velocity oscillation of payload is affected by the acceleration. The lower acceleration of the hoisting point, the lower velocity oscillation of payload, and when the acceleration is zero, the velocity oscillation of payload remains unchanged. The specific numerical relationship between them in Table 1 shows that when the acceleration is lower to a certain extent, the velocity oscillation of payload is not much affected. For example, when the acceleration is 28.125 \( \text{mm/} \text{s}^2 \), the velocity oscillation is not obvious. Since the length of the steel cable remains 15000 \( \text{mm} \), the frequency is 0.129, which is consistent with the natural frequency of the system.

| Acceleration \( (\text{mm/s}^2) \) | Velocity Oscillation \( (\text{mm/s}) \) | Period \( (\text{s}) \) | Frequency |
|----------------------------------|-----------------------------------|----------------|----------|
| 28.125                           | 6.868                             | 7.76           | 0.129    |
| 37.5                             | 60.673                            | 7.76           | 0.129    |
| 56.25                            | 139.221                           | 7.76           | 0.129    |
| 112.5                            | 201.926                           | 7.76           | 0.129    |
| \( \infty \)                     | 225.000                           | 7.76           | 0.129    |

**4.2. The Effect of the Acceleration Laws of Hosting Point on Pendulation of the CHS**

The motion state of the CHS is "stop-start-stop", so the movement mode is to accelerate first and then decelerate. According to the actual situation, three kinds of motion laws that meet the requirements are
listed, namely cycloid motion, quadratic motion and simple harmonic motion. They represent the three types of motion laws cycloid, polynomial and trigonometric functions, respectively. Since it has been confirmed that the pendulation of hoisting payload is affected by the acceleration, for the average speed of $225 \text{ mm/s}$, the motion distance is different, and the movement laws of the hoisting point has different influences on the pendulation of hoisting payload. Assuming that the system only moves horizontally, the following system parameters were fixed as: $m = 2000 \text{ kg}$, $l = 15000 \text{ mm}$. The motion distances of the hoisting point are 3600 mm, 7200 mm and 10800 mm, and the corresponding acceleration time is 16 s, 32 s and 48 s, respectively.

The results are shown in Figure 3 (a), (b) and (c), respectively. In these figures, when the distance is shorter, the pendulation of hoisting payload affected by the quadratic motion law is the least, but the effect will be amplified in the process of uniform acceleration and deceleration by the increasing distance. Additionally, there is a sudden change of acceleration at the midpoint of the quadratic motion law, and the pendulation phenomenon of the hoisting payload is the most obvious; Compared with the other two motion laws, the cycloidal motion law requires greater acceleration and deceleration in the same time. Thus, with the increasing of the motion distance of the hoisting point, the effect of the pendulation of the hoisting payload is smaller. When the driving distance is 3600 mm, the maximum acceleration can reach $45 \text{ mm/s}^2$, and the speed changes too fast, leading to obvious pendulation of the hoisting payload. Since the acceleration at the start and stopping moment of the simple harmonic motion law is higher and the acceleration changes suddenly, the hoisting payload is the most obvious pendulation among the three laws, whether it is during the movement or after the stopping movement.

Figure 3. The effect of acceleration laws of the hoisting point on pendulation.
4.3. The Effect of the Length of Steel Cable and Mass of Hoisting Payload on Pendulation of the CHS

According to (19), the natural frequency is related to the length of the steel cable and the mass of the hoisting payload when the linear density of the cable is constant. Assuming that the hoisting point only moves horizontally, the motion of the hoisting point is uniform motion, which speed is $225 \text{ mm/s}$. The mass of the hoisting payload is fixed as: $m = 2000 \text{ kg}$, and the lengths of the steel cables are set as: $l_1 = 1000 \text{ mm}$, $l_2 = 8000 \text{ mm}$ and $l_3 = 15000 \text{ mm}$, respectively. The result of the effect of the length of the hoisting cable on the pendulation of the hoisting payload is shown in Figure 4. The length of the steel cable is fixed as: $l = 15000 \text{ mm}$, and the mass of the hoisting payload are set as: $m_1 = 20 \text{ kg}$, $m_2 = 200 \text{ kg}$ and $m_3 = 2000 \text{ kg}$, respectively. The result of the effect of the mass of the hoisting payload on the pendulation of the hoisting payload is shown in Figure 5. The results show that the shorter length of the hoisting cable and the lighter mass of the hoisting payload, the greater frequency. However, the velocity oscillation of payload is not affected by them.

![Figure 4. The effect of the length of the hoisting cable on pendulation.](image1)

![Figure 5. The effect of the mass of the hoisting payload on pendulation.](image2)

The specific numerical relationships are shown in Table 2 and Table 3. In these Tables, the frequency is consistent with the natural frequency when the length of the cable is changed, but the frequency is slightly smaller than the natural frequency when the lifting weight is changed. Since the natural frequency is calculated by the centroid of the steel cable, it can be roughly considered that the frequency is consistent with the natural frequency. Since the natural frequency is calculated based on the centre of mass of the CHS, and the mass of the hoisting payload is too small to be close to the mass of the steel cable, resulting in errors. It can be considered that the frequency is consistent with the natural frequency.

**Table 2.** The effect of the length of the hoisting cable on pendulation.

| The Cable Length (mm) | Velocity Oscillation (mm/s) | Period (s) | Frequency |
|-----------------------|-----------------------------|------------|-----------|
| 1000                  | 225                         | 2          | 0.5       |
| 8000                  | 225                         | 5.67       | 0.176     |
| 15000                 | 225                         | 7.76       | 0.129     |

**Table 3.** The effect of the mass of the hoisting payload on pendulation.

| The Payload Mass (kg) | Velocity Oscillation (mm/s) | Period (s) | Frequency |
|-----------------------|-----------------------------|------------|-----------|
| 20                    | 225                         | 7.06       | 0.142     |
| 200                   | 225                         | 7.64       | 0.131     |
| 2000                  | 225                         | 7.76       | 0.129     |
4.4. The Effect of the Lifting Laws of the Steel Cable on Pendulation of the CHS

The mass of the hoisting payload is fixed as: \( m = 2000 \text{kg} \), and the motion of the hoisting point is uniform motion, which speed is \( 225 \text{ mm/s} \). The steel cable does lifting motion with the initial length of \( 15000 \text{ mm} \), which are the motion with speed of \( 0 \text{ mm/s} \), upward uniform motion with speed of \( 225 \text{ mm/s} \) and sinusoid motion, respectively. Corresponding to three motion states of no lifting motion, uniform rising motion and reciprocating motion. They correspond to three motion states: no lifting motion, uniform lifting motion and reciprocating motion.

The effect of the movement of the CHS on pendulation of the hoisting payload in Figure 6 shows that the frequency and velocity oscillation of the hoisting payload are different with different lifting motion modes. Since the frequency is affected by the length of the hoisting cable, the frequency of the uniform lifting motion is the fastest, and the frequency of the no lifting movement is the lowest. The velocity oscillation of the uniform lifting motion increases gradually, while that of the sinusoidal motion increases first and then decreases. Since the velocity oscillation is not affected by the length of steel and the acceleration of uniform lifting motion is zero, the velocity of lifting motion will affect the velocity oscillation of the hoisting payload.

In order to further study the relationship between the velocity of lifting motion and the velocity oscillation of the hoisting payload, the lifting motion is set to do the uniform lifting motion with the speed of \( 0 \text{ mm/s} \), \( 112.5 \text{ mm/s} \), \( 225 \text{ mm/s} \) and \( 450 \text{ mm/s} \), respectively, and the result in Figure 7 shows that the larger the velocity of lifting motion, the greater the velocity oscillation of the hoisting payload, and the velocity oscillation increases exponentially with time.

![Figure 6](image1.png)

**Figure 6.** The effect of the lifting laws of the steel cable on pendulation.

![Figure 7](image2.png)

**Figure 7.** The effect of the velocity of lifting motion of the steel on pendulation.

5. Conclusion

In this paper, the dynamic characteristics of cable crane system are studied by means of theoretical modelling and visual simulation. The dynamic model of two-dimensional cable hoisting system is derived using D’Alembert’s principle. In order to lay a foundation for motion planning and reduce pendulation of the CHS, the paper analyses the effect of acceleration laws of the hoisting point, the lifting laws of the steel cable and system parameters on pendulation of the model. Important points of the results have been summarized as follow:

i) The pendulation of the hoisting payload is much affected by the acceleration of the hoisting point. The lower acceleration, the lower pendulation of hoisting payload.

ii) The pendulation of the hoisting payload is affected by the acceleration laws of the hoisting point. Since the pendulation is affected by the acceleration, the quadratic motion can be selected for the short stroke, and the cycloid is the best for the general and long-distance travel.
iii) The frequency of the hoisting payload is affected by the length of the steel cable and the mass of the hoisting payload. The shorter length of the hoisting cable and the lighter mass of the hoisting payload, the greater frequency, which is consistent with the natural frequency.

iv) The pendulation of the hoisting payload is affected by the lifting laws of the steel cable. The frequency is affected by the length of the steel cable and the oscillation is affected by the velocity of lifting motion. The velocity goes up, the pendulation will increase, and the velocity goes down, the pendulation will decrease.

v) After the hoisting point stops moving, the hoisting payload vibrates at the stopping position. The pendulation of the hoisting payload is affected by the velocity, acceleration and frequency at the stopping time.

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