Two-Graviton Interaction in PP-Wave Background in Matrix Theory and Supergravity

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Abstract

We compute the two-body one-loop effective action for the matrix theory in the pp-wave background, and compare it to the effective action on the supergravity side in the same background. Agreement is found for the effective actions on both sides. This points to the existence of a supersymmetric nonrenormalization theorem in the pp-wave background.
1 Introduction

The Matrix model of M-theory proposed by [1, 2] is believed to give a non-perturbative description of quantum gravity. In the original paper [1], the authors computed graviton scattering in flat space using the Matrix model and found exact agreement with 11-dimensional Supergravity, which is believed to be the low energy limit of M-theory. The result also suggested the existence of a nonrenormalization theorem for the $v^4$ term in the effective action, which was subsequently confirmed by the study of supersymmetry in [5, 6, 7, 8, 9, 10, 11]. Since then, more detailed investigations have been performed in flat space [12, 13, 14, 15, 16, 17, 18, 19]. The Matrix model in a weakly curved background was proposed by Taylor and Van Raamsdonk in [20]. The case of a space weakly curved in the transverse directions was checked explicitly in [21].

The Matrix Theory action in the pp-wave background was proposed in [24]. Since this new action is exact in this curved background, it could provide further tests of the Matrix conjecture beyond the proposal on weakly curved backgrounds in [20]. In addition it is different from the case in [21], because the background metric now has a nontrivial $g_{++}$ component.

The goal of this paper is to compare two-graviton scattering in the pp-wave background in Matrix Theory and Supergravity. An agreement will provide evidence for: (1) the existence of a supersymmetric nonrenormalization theorem in pp-wave background for the terms compared; (2) Matrix model as a description of M-theory in the pp-wave background. As pointed out by [22, 23], Matrix Theory in a generic curved background is not expected to agree with Supergravity. In the pp-wave case, however, we do find precise agreement as will be shown in this paper. This is likely to be a result of the large number of supersymmetries of the pp-wave background.

The paper is organized as follows. In Section 2 we briefly review the previously known results in flat space and in weakly curved backgrounds. In Section 3 the approximations and limits of our computation are discussed. In Section 4 through Section 7 we present the computation on the Matrix Theory side. In Section 8 we present the computation on the Supergravity side. In Section 9 we discuss some possible future directions. Some of the technical details are given in the Appendices.
2 A Brief Review of Known Results

In [13], the one-loop effective potential for two gravitons in flat spacetime background was computed in Matrix Theory to be:

\[ V_{\text{eff}}^{1-\text{loop}} = \frac{15N_pN_s v^4}{16M^9 R^3 r^7} \]  

(1)

where \( N_p \) and \( N_s \) are the numbers of D0-branes making up the probe graviton and source graviton, respectively, \( v \) and \( r \) are the transverse relative velocity and distance between them, \( M \) is the 11-dimensional Planck mass, and \( R \) is the radius of compactification in DLCQ. This effective potential agrees precisely with the Supergravity result [16].

In [21], the effective potential for a weakly curved background with nontrivial transverse metric components was computed. Again agreement was found. In fact, the only modification needed was the replacement of \( r \) by \( d \), the geodesic distance between the two gravitons.

3 The Effective Potential, \( V_{\text{eff}} \)

In this paper, the main object for comparison on both sides is the effective potential \( V_{\text{eff}} \). The computation is carried out in the DLCQ formalism, which was proposed in Susskind’s finite N conjecture [2], and further elucidated by [3, 4]. In this formalism \( x^- \) and \( x^- + 2\pi R \) are identified. \( p_- \) is therefore quantized in units of \( 1/R \).

The implications of such a lightlike compactification, however, are far from trivial [25]. One such complication arises from the longitudinal zero modes, which appear to cause perturbative amplitudes to diverge. In addition, there are concerns that the DLCQ of M-theory in the low energy limit is not necessarily the DLCQ of 11-dimensional Supergravity because some exotic degrees of freedom such as membranes wrapped around the lightlike direction may contribute.

Here we are going to take the viewpoint in [26]. Essentially, the presence of a source exerts a pressure that decompactifies the region surrounding it, rendering \( x^- \) effectively spacelike by providing a nonzero \( g_- \) component in the metric. In the limit of large \( N \), this bubble of 11-dimensional space expands, and the approximation of Supergravity as a low energy description is thus justified. This view is further elucidated in [27], and we do not expect new issues to arise in the pp-wave background.

It was also argued in [27] that the perturbative calculations in Matrix Theory and Supergravity have different regions of validity \( (E/M > N^{1/3} \) and \( E/M < N^{-1}) \). Therefore, in general there is no reasons why they should match as each effective action is valid only within its own energy scale. Thus a mismatch does not immediately invalidate the Matrix
Conjecture. An exact match, however, will point to the existence of a nonrenormalization theorem, which protects the terms evaluated from gaining higher loops corrections. If such a nonrenormalization theorem does exist, then the agreement of both sides can be viewed as positive evidence for the Matrix Conjecture. It is with these points in mind that the comparison of the effective action is made here.

On the Matrix Theory side, the effective potential is computed up to 1-loop. As in flat space, it should correspond to to terms of order $\kappa_{11}^2$ on the Supergravity side. The relation $\kappa_{11}^2 = 16\pi^5/M^9$ means only terms of order $1/M^9$ are relevant on the Matrix Theory side for the purpose of such comparison.

A natural length scale that arises on the Matrix Theory side is $1/(M^3 R)^{1/2}$, which for convenience we will denote as $(\alpha)^{1/2}$. In addition to the low velocity and large $r$ approximation necessary to facilitate comparison in flat space, we will assume also:

$$\frac{\alpha^2 \mu^2}{r^2} << 1$$

where $\mu$ is the 123+ component of the four-form field strength.

This dimensionless number, as we will see in eqn(3), is simply the relative strength of the new terms in the action arising from the pp-wave background to the quartic terms already present in flat space. In the opposite limit, $\frac{r^2}{\alpha^2 \mu^2} << 1$, the effective potential on the Matrix Theory side resums to give $1/\mu$ dependence, which does not appear possible to be reproduced on the Supergravity side. In fact, this is nothing new. A similar issue arises already in flat space, where the effective potential only matches when we take the small $v$ and large $r$ limit, or more precisely, by expanding on the small parameter $v \alpha/r^2$. In other words, even with the existence of a nonrenormalization theorem, the results on both sides should only be compared at very large $r$, where Supergravity is applicable.

4 Background Field Method

We will follow the Background Field Method as reviewed in [29]. $X$ is expanded into a background field $B$ and a fluctuating field $Y$, i.e. $X = B + Y$. Only the part of the action that is quadratic in $Y$ will be of interest below.

The Matrix Model action in the DLCQ of M-theory in the maximally supersymmetric

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1 This $\alpha$ should not be confused with the string scale $\alpha'$.
2 This can be seen in eqn(14), a typical term in the effective potential.
pp-wave background is \[ S = \int dt \text{Tr} \left\{ \sum_{I=1}^{9} \frac{1}{2R} (D_0 X^I)^2 + \frac{(M^3 R)^2}{4R} \sum_{I,J=1}^{9} [X^I, X^J]^2 + (M^3 R) \sum_{J=1}^{9} \psi^T \gamma^J [\psi, X^J] \right. \\
\left. + \frac{1}{2R} \left[ -\frac{\mu}{3} \sum_{i=1}^{9} (X^i)^2 - \frac{\mu}{6} \sum_{a=1}^{9} (X^a)^2 \right] - \frac{\mu}{4} \psi^T \gamma_{123} \psi \\
- \frac{(M^3 R)}{R} \sum_{i,j,k=1}^{3} \epsilon_{ijk} \text{Tr} (X^i X^j X^k) \right\} \tag{3} \]

where \( D_0 X = \partial_t X^I - i[X_0, X^I] \)

Taking the ratios of any of the \( \mu \)-dependent terms to the \( \mu \)-independent non-derivative terms gives the parameter in eqn(2). In other words, the assumption stated in the previous section is identical to treating the new terms arising from the pp-wave background as a perturbation to flat space. Note that this is exactly the opposite of the approximation made in \[28\], where the \( \mu \)-independent terms are treated as perturbations to the \( \mu \)-dependent terms. While the computation of the 1-loop effective potential is possible in both limits on the Matrix Theory side, an agreement with Supergravity is possible only in the large \( r \) limit given in eqn(2).

In what follows, unless stated otherwise, we will always assume the indices \( i \) goes from 1 to 3, \( a \) goes from 4 to 9, and \( I \) goes from 1 to 9.

In addition to the action above, there are terms arising from the ghosts and gauge fixing, which we simply state below:

\[
S_{gf} = \int dt \text{Tr} \left[ -\frac{1}{2R} (\partial_t X_0 + i[B_i, X_i])^2 \right] \tag{4}
\]

\[
S_{ghost} = \int dt \text{Tr} \left[ \overline{\sigma} \partial_t c - \partial_t \overline{\sigma} [X_0, c] + \overline{\sigma} [B_i, [X_i, c]] \right] \tag{5}
\]

Thus, the complete Matrix Theory action is:

\[
S_M = S + S_{gf} + S_{ghost} \tag{6}
\]

To simplify the notation, we will put \( M^3 R = 1/\alpha = 1 \). This factor can be restored by dimensional analysis. It is also convenient to define \( g^2 \equiv R \), which corresponds to a loop counting parameter in the Matrix Theory.
4.1 Expansion About The Background

The fields $X$, $\psi$, and $c$ are expanded in the following way, with a purely bosonic background. Here we set $N_p = N_s = 1$, i.e. we deal with $2 \times 2$ matrices (much interesting work has been done on matrix quantum mechanics in flat space, e.g. [33, 34]). We will later restore $N_p$ and $N_s$:

$$X_\mu = B_\mu + gY_\mu \; ; \; \mu = 0, 1, 2, ..., 9$$

$$B_I = \begin{pmatrix} x_I & 0 \\ 0 & 0 \end{pmatrix} \; ; \; Y_I = \begin{pmatrix} \zeta_I & z_I \\ \bar{z}_I & \bar{\zeta}_I \end{pmatrix}$$

$$B_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \; ; \; Y_0 = \begin{pmatrix} \zeta_0 & z_0 \\ \bar{z}_0 & \bar{\zeta}_0 \end{pmatrix}$$

$$\psi = \begin{pmatrix} \eta \\ \theta \end{pmatrix} \; ; \; c = \begin{pmatrix} \epsilon \\ c_1 \\ c_2 \end{pmatrix}$$

The above background has the interpretation of one graviton (the source) sitting at the origin, while another graviton (the probe) approaches from the position given by $x^I$ in the matrix $B$. We will use the shorthand $r^2 = \sum_{I=1}^{9} (x^I)^2$.

After a Wick rotation, where we define $S = iS^{(E)}$ and $\tau = it$, and at the same time rotating $X_0$ to $iX_0^{(E)}$, the quadratic part of the action is:

$$S^{(E)}_{boson} = \int d\tau \left\{ -\frac{1}{2} \zeta_0 \partial^2 \zeta_0 - \frac{1}{2} \bar{\zeta}_0 \partial^2 \bar{\zeta}_0 + \frac{1}{2} \bar{\zeta}_i (\partial^2 - (\mu/3)^2) \bar{\zeta}_i + \frac{1}{2} \zeta_a (\partial^2 + (\mu/6)^2) \zeta_a \\
+ \frac{1}{2} \bar{\zeta}_i (\partial^2 + (\mu/3)^2) \bar{\zeta}_i + \bar{\zeta}_a (\partial^2 + (\mu/6)^2) \bar{\zeta}_a \\
+ \bar{z}_0 (\partial^2 + r^2) z_0 - 2i\partial_\tau x_I (\bar{\zeta}_I z_0 - \bar{z}_0 z_I) \\
+ \bar{z}_i (\partial^2 + r^2 + (\mu/3)^2) z_i + \bar{z}_a (\partial^2 + r^2 + (\mu/6)^2) z_a - i\mu \epsilon_{ijk} x_i \bar{z}_j z_k \right\} (7)$$

$$S^{(E)}_{fermion} = \int d\tau \left\{ \eta (i\partial_\tau - i\frac{\mu}{4} \gamma_{123}) \eta + \bar{\eta} (i\partial_\tau - i\frac{\mu}{4} \gamma_{123}) \bar{\eta} + 2\bar{\theta} (i\partial_\tau + x_I \gamma_I - i\frac{\mu}{4} \gamma_{123}) \theta \right\} (8)$$

$$S^{(E)}_{ghost} = \int d\tau \left\{ \bar{\epsilon} \partial^2 \epsilon + \bar{\bar{\epsilon}} \partial^2 \bar{\epsilon} + \bar{c}_1 (\partial^2 - r^2) c_2 + \bar{c}_2 (\partial^2 - r^2) c_1 \right\} (9)$$

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3Another possible interpretation is a transverse five brane at the origin [32].

4For simplicity, all subsequent superscripts of $(E)$ on the Euclideanized fluctuation fields will be omitted.
4.2 The Sum Over Mass

The partition function, \( Z \) of the above action can be computed as a product of functional determinants. The 1-loop effective action \( \Gamma \) is then simply related to \( Z \) via:

\[
\exp(-\Gamma) = Z
\]

The 1-loop effective potential is defined as:

\[
\Gamma = -\int d\tau V_{\text{eff}}
\]

To first approximation, however, it is not necessary to compute the functional determinants. As was suggested by Talfjord and Periwal [31] and Talyor [30], one could deduce the effective potential by simply evaluating the mass spectrum of the fluctuating fields. From the masses, the 1-loop contribution to \( V_{\text{eff}} \) could be easily deduced using the formula:

\[
V_{\text{eff}}^{1-\text{loop}} = -\frac{1}{2} \left( \sum_{\text{real bosons}} m_b - \sum_{\text{real fermions}} m_f - \sum_{\text{real ghosts}} m_g \right)
\]

The physical reasons for this is that at large distances, i.e. the limit where Supergravity is valid, all the string stretching between the D0-branes can be assumed to lie in their ground state. This result can also be verified using the complete expression for \( V_{\text{eff}} \) in terms of functional determinants. We provide an argument for this in Appendix A. In what follows, we will omit the superscript \( 1-\text{loop} \), assuming this is understood. The contribution from tree level, which does not concern us here, is simply the Lagrangian with \( X \) replaced by \( B \). Both contributions will be put back together at the end in eqn(19).

One important point to note is that this method is valid only up to the lowest powers of \( v \), as is already known in the flat space case. In flat space, the above formula reproduces every term predicted by a Supergravity computation with the right coefficients, but the Matrix Theory corrections to Supergravity, i.e. terms with even higher powers of \( v \) and \( 1/r \) which would not be found in Supergravity, will not come out with the correct coefficients. In fact, the parameter \( \alpha \) can be treated as the counting parameter for this purpose. All terms of order \( \alpha^3 \), which is basically \( \kappa^2_{11} \) in the Supergravity language, will be found on the Supergravity side, but terms on the Matrix Theory side with higher powers of \( \alpha \), which represent short distance effects, should be treated as corrections. To compute them correctly, one needs to make use of the complete expression in terms of functional determinants.

For our purpose, however, the above approach is sufficient. We are not interested in computing the correction to Supergravity, rather we would like to check whether the terms already predicted by Supergravity in the pp-wave background can be reproduced by a Matrix
Theory calculation.

5 A Simple Case

In the next section we will work out a more efficient method to compute $V_{eff}$ without explicitly diagonalizing the mass matrix. Nevertheless, it is instructive to work out the simplest case in a direct approach to get the basic idea of the computation.

In this simple case, we put $x^8 = b$ and $x^9 = v\tau$, while all the other $x^I$ are set to zero\(^5\). Here $b$ is a constant, which can be interpreted as the impact parameter of the approaching probe graviton towards the source sitting at the origin. In this case, the mass matrix constructed from eqn(7), (8) and (9) is easily diagonalized to give the mass spectrum listed in Table 1.

It should be noted that the velocity in the table above is measured in Euclidean time $\tau$, i.e. $v = \frac{\partial x}{\partial \tau}$. In a comparison with Supergravity, a Wick rotation back into Minkowski time $t = -i\tau$ is required, which introduces extra minus signs in $V_{eff}$.

With the mass spectrum at hand, $V_{eff}$ can be evaluated using eqn(12):

$$V_{eff} = -\frac{1}{2}(2\mu^2/3 + 6\mu/6 - 8\mu/4) - \frac{1}{2}\left\{6\sqrt{r^2 + \mu^2/3^2} + 10\sqrt{r^2 + \mu^2/6^2} + 2\sqrt{r^2 + \eta_+}ight.$$ 
$$+ 2\sqrt{r^2 + \eta_-} - 8\sqrt{r^2 + \mu^2/4^2 + v} - 8\sqrt{r^2 + \mu^2/4^2 - v} - 4r\right\}$$

(13)

At this point it is useful to restore the factors of $M^3R$, which we denote as $1/\alpha$. For instance, the first square root term in the above equation becomes:

$$\sqrt{\frac{r^2}{\alpha^2} + \frac{\mu^2}{3^2}}$$

(14)

This can in turn be written as:

$$\frac{r}{\alpha}\sqrt{1 + \frac{1}{3^2}(\frac{\alpha}{r^2})(\alpha\mu^2)}$$

The expression for $V_{eff}$ given above, being a Matrix Theory result, is only expected to match with Supergravity in the large $r$ limit (if it does at all!). Defining the large $r$ limit

\(^5\)Note that by putting all $x^i$ to zero for $i = 1, 2, 3$, we made sure that in this case the Myers term will not contribute to the mass matrix.
| $m^2$          | Fields            |
|---------------|-------------------|
| 0             | $\zeta^0$         |
| $\mu^2/3^2$   | $\zeta^i$; $i = 1, 2, 3$ |
| $\mu^2/6^2$   | $\zeta^a$; $a = 4, ..., 9$ |
| 0             | $\tilde{\zeta}^0$ |
| $\mu^2/3^2$   | $\tilde{\zeta}^i$; $i = 1, 2, 3$ |
| $\mu^2/6^2$   | $\tilde{\zeta}^a$; $a = 4, ..., 9$ |
| $r^2 + \mu^2/3^2$ | $\bar{z}^i, z^i$; $i = 1, 2, 3$ |
| $r^2 + \mu^2/6^2$ | $\bar{z}^a, z^a$; $a = 4, ..., 8$ |
| $r^2 + \eta_+$ | $\bar{z}^0 + \bar{z}^0, z^0 + z^9$ |
| $r^2 + \eta_-$ | $\bar{z}^0 - \bar{z}^0, z^0 - z^9$ |
| $\mu^2/4^2$   | $\eta$ (8)        |
| $\mu^2/4^2$   | $\tilde{\eta}$ (8) |
| $r^2 + \mu^2/4^2 + v$ | $\theta$ (8) |
| $r^2 + \mu^2/4^2 - v$ | $\tilde{\theta}$ (8) |
| 0             | $\bar{\epsilon}, \epsilon$ |
| 0             | $\bar{\epsilon}, \epsilon$ |
| $r^2$         | $\bar{c}_I, c_I$; $I = 1, 2$ |

Table 1: The Mass Spectrum for a Simple Case. The numbers inside the round brackets indicate the number of physical degrees of freedom of the fermions with the given mass. $\eta_\pm$ is given by $\pm [\frac{\mu^2}{6\sigma^2} \pm \sqrt{(\frac{\mu^2}{6\sigma^2})^2 + 16v^2}]$. 
by eqn(2), we can then expand the 1-loop effective potential in powers of $\alpha^2 \mu^2 / r^2$. Thus, expanding $V_{\text{eff}}$ gives:

$$V_{\text{eff}} = \alpha^3 \left( \frac{15}{16} \frac{v^4}{r^7} + \frac{7}{96} \frac{\mu^2 v^2}{r^5} + \frac{1}{768} \frac{\mu^4}{r^3} \right) + O[\alpha^5] \quad (15)$$

Wick rotating $v$, and restoring $N_p, N_s$ gives:

$$V_{\text{eff}} = \frac{N_p N_s}{M^9 \mathcal{R}^3} \left( \frac{15}{16} \frac{v^4}{r^7} - \frac{7}{96} \frac{\mu^2 v^2}{r^5} + \frac{1}{768} \frac{\mu^4}{r^3} \right) + O[\alpha^5] \quad (16)$$

The $\alpha^3$ terms give the factor $1/M^9$, which translates into $\kappa_{11}^2$ in the Supergravity language. This is the order we are interested in. We throw away the higher powers of $\alpha$ (which are always accompanied by powers of $1/r$) because they correspond to short distance corrections to Supergravity, just as in flat space.

Here the first term is just the flat space result. The second and the third term are the interesting ones, with new $\mu^2 v^2$ and $\mu^4$ dependence created by the pp-wave background. A comparison of their coefficients with Supergravity will show exact agreement.

## 6 Mass Matrix Computation

In the more general cases, when the velocity and the impact parameter point in arbitrary directions, calculating the effective potential $V_{\text{eff}}$ by finding the entire $m^2$ spectrum, then taking their square roots and expanding them in powers of $\mu$ and $v$ becomes inefficient, since in the most general case this involves finding the eigenvalues of mass matrices of very high dimension.

Instead, it is possible to make use of the sum over mass formula in eqn(12) without explicitly diagonalizing the mass matrix. Let us denote the square of the mass matrix as $W = M^2$. Since there is never any mixing between the bosons, the fermions and the ghosts, we can study their mass matrices separately.

In terms of $W$, the sum over mass formula becomes:

$$V_{\text{eff}}^{1-\text{loop}} = -\frac{1}{2} \text{tr}(\sqrt{W_b} - \sqrt{W_f} - \sqrt{W_g}) \quad (17)$$

The square root of $W$ can be defined unambiguously by its expansion in powers of $\alpha/r^2$ in the Supergravity limit, as was discussed in Section 5. Note that $M_b$ is defined to be the mass matrix for real bosons. If it is taken to be the mass matrix for the complex bosons, then there will be an extra factor of two in front of $\sqrt{W_b}$. 


6.1 Simple Recipe for Mass Matrix

In this subsection we will give a simple recipe for writing out $M^2$ for both the bosons and the fermions. The mass for the ghosts is exactly the same as in the simple case of Section 5.

First of all, we should note that the mass of $\zeta^i$ and $\zeta^a$ are always $\mu/3$ and $\mu/6$ respectively for $i = 1, 2, 3$ and $a = 4, ..., 9$. The mass of all eight physical degrees in $\eta$ is always $\mu/4$. These are independent of the background $B$. Mixing occurs only among the $z^i$ and among the $\theta$ and $\bar{\theta}$. Hence in what follows, we will denote the component arising from say $z^I z^J$ in the bosonic Lagrangian simply as $(M^2)_{IJ}$ without mentioning $z$ explicitly. Note also that $M^2$ is symmetric.

6.1.1 Rules for Bosons

1. $(M^2)_{00} = r^2$; $(M^2)_{ii} = r^2 + \mu^2/3^2$; $(M^2)_{aa} = r^2 + \mu^2/6^2$;
2. $\dot{x}^I = v_I$ mixes $z^0$ and $z^I \Rightarrow (M^2)_{0I} = -2v_I$
3. $x^1 = b_1$ mixes $z^2$ and $z^3$... etc $\Rightarrow (M^2)_{jk} = i\mu\epsilon_{ijk}b_i$

Note that Rule 3 applies only to $z^i$ but not $z^a$. Such mixing is the effect of the Myers term in the Matrix Theory action.

6.1.2 Rules for Fermions

The mass matrix for the fermions can be written in a closed form:

$$M^2 = r^2 + \mu^2/4^2 + \sum_{I=1}^{9} v_I \gamma_I + \sum_{i=1}^{3} \frac{i\mu x^i}{4} \{\gamma_i, \gamma_{123}\}$$

7 The General Case

Once the mass matrix squared $W = M^2$ is known, eqn\ref{17} can then be used to compute the 1-loop effective potential explicitly. In accordance with our earlier discussions, only terms up to order $\alpha^3 \sim 16\pi^5/M^9 = \kappa_{11}^2$ are kept. After restoring all factors of $M^3 R$, $N_p$, and $N_s$, the 0 and 1-loop effective potential is given by:
\[ V_{\text{eff}}^{0,1-\text{loop}} = \frac{N_p}{2R} \left( \sum_{i=1}^{9} v_i^2 + g_{++} \right) + \frac{N_p N_s}{M^9 R^3} \left\{ \frac{15(\sum_{i=1}^{9} v_i^2)^2}{16 r^7} - \frac{\mu^2 \sum_{i=1}^{9} v_i^2}{96 r^5} - \frac{7 \mu^2 \sum_{a=4}^{9} v_a^2}{96 r^5} \right. \\
+ \left. \frac{15 \mu^2}{32 r^7} \left[ \sum_{i=1}^{3} x_i^2 \left( - 3 \sum_{i=1}^{3} v_i^2 + \sum_{a=4}^{9} v_a^2 \right) + 2(\sum_{i=1}^{3} x_i v_i)^2 \right] \right\} \]

\[ + \frac{\mu^4 N_p N_s}{R^3 M^9} \frac{1}{768 r^7} \left\{ 32 \left[ \sum_{i=1}^{3} (x_i')^2 \right]^2 + \left[ \sum_{a=4}^{9} (x_a')^2 \right]^2 - 12 \sum_{i=1}^{3} (x_i')^2 \cdot \sum_{a=4}^{9} (x_a')^2 \right\} \]

(19)

This is the equation to be compared with the Supergravity result. Notice the effective potential has manifest \( SO(3) \times SO(6) \) symmetry, as should be expected from the symmetry of the original Matrix Theory action. Just as in flat space \cite{20}, one should be able to recast this 1-loop effective potential in the form \( T^\mu_\nu G_{\mu\nu} \). A comparison with the Supergravity side will indeed confirm this, as this is precisely the form of the effective potential on the Supergravity side as derived in Appendix B.

Having computed the effective potential on the Matrix Theory side, the next step will be to compare it with the result from a Supergravity calculation. Before this could be done, the issue of gauge choice has to be addressed.

It is necessary to make a gauge choice when solving the Einstein equations. A gauge choice corresponds to a choice of the coordinate system one uses to describe the physics. On the Matrix Theory side, such a choice of coordinates was made right from the very beginning: The action in eqn\cite{3} was written in coordinates that made the \( SO(3) \times SO(6) \) symmetry manifest. Before a comparison is possible, a corresponding choice of coordinates, i.e. a choice of gauge has to be made on the Supergravity side.

A comparison of the above equation with the general expression for \( V_{\text{eff}} \) in eqn\cite{56} will in the end determine the correct gauge choice for the Supergravity computation. There will be a further discussion about gauge choice in the Supergravity section.

8 The Supergravity Effective Potential

To find the two-body effective action, one only needs to solve for the metric perturbation caused by the source graviton at the linear order (\( \sim \kappa_1^2 \)).

The action is given by:

\[ S = S_G + S_A + S_P \]

(20)

\( S_G \) is the Einstein action for the metric:

\[ S_G = \frac{1}{2} \kappa_1^2 \int d^{11} x \sqrt{|g|} R \]

(21)
$S_A$ is the action for the three-form:

$$S_A = -\frac{2}{\kappa_{11}^2} \int d^{11}x \left\{ \sqrt{|g|} F_{\mu \nu \lambda \xi} F_{\mu \nu \lambda \xi} + \frac{1}{12} \frac{1}{3!(4!)^2} \epsilon^{\mu_1 \ldots \mu_{11}} A_{\mu_1 \mu_2 \mu_3} F_{\mu_4 \ldots \mu_7} F_{\mu_8 \ldots \mu_{11}} \right\}$$

(22)

$S_P$ is the action for the source graviton (the subscript $P$ means "particle"):

$$S_P = C_P \frac{1}{2} \int_{-\infty}^{+\infty} d\xi \left( \frac{1}{\beta(\xi)} g_{\mu \nu}(y) \frac{dy^\mu}{d\xi} \frac{dy^\nu}{d\xi} - \beta(\xi) m^2 \right)$$

(23)

with $C_P$ being some constant.

The above action gives the equations of motion for the metric, the 3-form field, and the source graviton, all listed below.

The Einstein equation:

$$R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} = \kappa_{11}^2 \left[ [T_{\mu \nu}]_A + [T_{\mu \nu}]_P \right]$$

(24)

The Maxwell equation:

$$\partial_\mu \left( \sqrt{|g|} F^{\mu \nu \lambda \xi} \right) - \frac{1}{1152} \epsilon^{\nu \lambda \xi \rho_1 \ldots \rho_8} F_{\rho_1 \ldots \rho_4} F_{\rho_5 \ldots \rho_8} = 0$$

(25)

The geodesic equation:

$$\frac{d^2 y^\mu}{d\xi^2} + \Gamma^{\mu}_{\rho \nu}(y) \frac{dy^\rho}{d\xi} \frac{dy^\nu}{d\xi} = 0$$

(26)

$[T_{\mu \nu}]_A$ and $[T_{\mu \nu}]_P$ are the stress tensors obtained by varying $S_A$ and $S_P$ w.r.t. the metric, given below:

$$[T_{\mu \nu}]_A = \frac{1}{12\kappa_{11}^2} \left( F_{\mu \lambda \xi \rho} F^{\lambda \xi \rho} - \frac{1}{8} g_{\mu \nu} F^{\rho \sigma \lambda \xi} F_{\rho \sigma \lambda \xi} \right)$$

(27)

$$[T_{\mu \nu}]_P (x) = \frac{C_P}{2} \frac{1}{\sqrt{|g(x)|}} g_{\mu \rho}(x) g_{\nu \lambda}(x) \int_{-\infty}^{+\infty} d\xi \frac{1}{\beta(\xi)} \frac{dy^\rho(\xi)}{d\xi} \frac{dy^\lambda(\xi)}{d\xi} \delta^{(11)}(x - y(\xi))$$

(28)

Setting $C_P$ to zero means the absence of the source graviton. In this case, a solution to the above equations of motion is the pp-wave background. The metric $g_{\mu \nu}$ and the 4-form field strength are given by:

$$g_{++} = 1, \quad g_{+\nu} = -\mu^2 \left[ \frac{1}{9} \sum_{i=1}^{3} (x^i)^2 + \frac{1}{36} \sum_{a=4}^{9} (x^a)^2 \right], \quad g_{AB} = \delta_{AB}$$

(29)

$$F_{123+} = \mu$$

(30)

In our convention, $\mu, \nu, \rho, \ldots$ take the values $+, -, 1, \ldots, 9$; $A, B, C, \ldots$ take the values $1, \ldots, 9$; $i, j, k, \ldots$ take the values $1, \ldots, 3$; and $a, b, c, \ldots$ take the values $4, \ldots, 9$.
The introduction of a source graviton, i.e. a non-zero $C_P$, perturbs the above pp-wave solution:

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu} \equiv G_{\mu\nu}; \quad F_{\mu\nu\rho\sigma} \rightarrow F_{\mu\nu\rho\sigma} + f_{\mu\nu\rho\sigma}$$

It suffices to solve the geodesic equation at the zeroth order of $C_P$, which gives a solution

$$x^+ = \xi, \quad x^- = 0, \quad x^A = 0$$

and the corresponding stress tensor of the source graviton is then:

$$[T_{\mu\nu}]_P(x) = p^+ g_{\mu+} g_{\nu+} \delta(x^-) \prod_{A=1}^{9} \delta(x^A)$$

(31)

where $p^+ \equiv \frac{C_P}{2\beta_0}$ is a constant (note that $\beta(\xi)$ is a constant $\beta_0$ for a geodesic) and in what follows we will use $p^+$ instead of $C_P$. Note that the order of $\kappa_1^2$ is the same as the order of $p^+$. Also note that the only non-vanishing component of $[T_{\mu\nu}]_P$ is $[T_{--}]_P = p^+ \delta(x^-) \prod_{A=1}^{9} \delta(x^A)$.

In what follows we will integrate everything over the $x^-$ direction, thus getting rid of $\delta(x^-)$ and derivatives w.r.t $x^-$. On the Matrix Theory side, the effective potential was only computed up to 1-loop. In Supergravity language, that means we are only looking at order $\kappa_{11}^2$. To find the effective potential on the Supergravity side up to this order, we need only the linearized (i.e. to the linear order of $p^+$) Einstein equation and Maxwell equation.

We consider static solutions which has no $x^+$ dependence. Also we restrict our attention to metric and gauge field perturbations that go to zero at infinity. The linearized Einstein equation in 11 dimension is:

$$\delta R_{\mu\nu} = \kappa_{11}^2 \left[ \delta T_{\mu\nu} + \frac{1}{g_{\mu\nu}} \left(T^{\alpha\beta} h_{\alpha\beta} - g^{\alpha\beta} \delta T_{\alpha\beta} \right) \right] \equiv T_{\mu\nu}$$

(32)

where the perturbation to the total stress tensor is given by

$$\delta T_{\alpha\beta} = [\delta T_{\alpha\beta}]_A + [T_{\alpha\beta}]_P$$

(33)

$[\delta T_{\alpha\beta}]_A$ is the perturbation to the stress tensor of the gauge field, which is to be expressed in terms of the perturbation to the field strength.

First look at the $(- -)$ component of the Einstein equation:

$$\delta R_{--} = -\frac{1}{2} \sum_{A=1}^{9} \frac{\partial^2 h_{--}}{\partial x^A \partial x^A}$$

(34)

and

$$T_{--} = \kappa_{11}^2 \delta T_{--} = \kappa_{11}^2 [T_{--}]_P = \kappa_{11}^2 p^+ \prod_{A=1}^{9} \delta(x^A)$$

(35)
where $[\delta T_-]_A = 0$ (as can be readily verified) has been used.

This gives

$$h_- = \frac{\kappa_{11}^2 p^+ 15}{\pi^4 16 |\vec{x}|^7}$$

where we use $\vec{x}$ to denote the 9-dimensional vector in the transverse directions.

The $(-A)$ component of the Einstein equation is,

$$\delta R_{-A} = -\frac{1}{2} \sum_{B=1}^{9} \frac{\partial^2 h_{-A}}{\partial x^B \partial x^B} + \frac{1}{2} \sum_{B=1}^{9} \frac{\partial^2 h_{-B}}{\partial x^A \partial x^B}$$

and

$$T_{-A} = 0$$

which gives

$$h_{-A} = 0$$

Now we look at the linearized Maxwell equation, in terms of the gauge potential perturbation $a_{\mu\nu}$ (note $f_{\lambda\mu\nu} = \partial_\lambda a_{\mu\nu} - \partial_\mu a_{\nu\lambda} + \partial_\nu a_{\rho\lambda} - \partial_\rho a_{\lambda\mu}$). We choose to work in the “Lorentz gauge” where $\sum_{D=1}^{9} \partial_D a_{\mu\nu} = 0$. The upper $(AB+)$ component of the Maxwell equation gives:

$$\sum_{D=1}^{9} \partial^2_{D} a_{AB} - \sum_{D=1}^{9} \partial_D [h_- F_{DAB+}] = 0$$

Using the expression for $h_-$ that we just found, we have:

$$a_{ij} = \frac{\mu \kappa_{11}^2 p^+ 15}{\pi^4 32} \sum_{k=1}^{3} \varepsilon_{ijk} \frac{x^k}{|\vec{x}|^7}$$

while all other $a_{AB-}$’s vanish. This gives the field strength:

$$f_{-ijk} = \frac{\mu \kappa_{11}^2 p^+ 15}{\pi^4 32} \sum_{k=1}^{3} \varepsilon_{ijk} \left[7 \sum_{i=1}^{3} (x^i)^2/|\vec{x}|^9 - 3 \frac{1}{|\vec{x}|^7}\right]$$

$$f_{-ijb} = \frac{\mu \kappa_{11}^2 p^+ 15}{\pi^4 32} \sum_{k=1}^{3} \varepsilon_{ijk} \left[7 \frac{x^k x^b}{|\vec{x}|^9}\right]$$

Next consider the upper $(ABC)$ component of the Maxwell equation. Using the fact that $h_{-A} = 0$ and $a_{AB-} = 0$ except for $a_{ij-}$, we have:

$$\sum_{D=1}^{9} \partial^2_{D} a_{ABC} = 0$$
hence, all \( a_{ABC} = 0 \). Now the \((A+\) component. Using \( h_{-A} = 0 \) we get
\[
\sum_{D=1}^{9} \partial_D^2 a_{A-+} = 0 \tag{44}
\]
thus \( a_{A-+} = 0 \). Now we go back to look at the \((+A)\) component of the Einstein equation. Using \( h_{-A} = 0 \), we get
\[
\delta R_{+A} = -\frac{1}{2} \sum_{B=1}^{9} \frac{\partial^2 h_{+A}}{\partial x^B \partial x^B} + \frac{1}{2} \sum_{B=1}^{9} \frac{\partial^2 h_{+B}}{\partial x^A \partial x^B} \tag{45}
\]
Using \( a_{A-+} = 0, a_{ABC} = 0, \) and \( h_{-A} = 0 \), we get
\[
T_{+A} = 0 \tag{46}
\]
So we conclude that
\[
h_{+A} = 0 \tag{47}
\]
Now consider the \((+-)\) component of the Einstein equation
\[
\delta R_{+-} = -\frac{1}{2} \sum_{A=1}^{9} \frac{\partial^2 h_{+-}}{\partial x^A \partial x^A} + \frac{1}{2} \sum_{A=1}^{9} \frac{\partial g_{++}}{\partial x^A} \frac{\partial h_{+-}}{\partial x^A} \tag{48}
\]
and
\[
T_{+-} = \frac{1}{6} (\mu^2 h_{-} - \mu f_{-23}) \tag{49}
\]
In writing \( T_{+-} \), we made use of the following equations:
\[
[\delta T_{+-}]_A = \frac{\mu^2}{4 \kappa_{11}^2} h_{-} \]
\[
[\delta T_{ij}]_A = \frac{1}{4 \kappa_{11}^2} \delta_{ij} (-2 \mu f_{-23} - \mu^2 h_{-})
\]
\[
[\delta T_{bc}]_A = \frac{1}{4 \kappa_{11}^2} \delta_{bc} (2 \mu f_{-23} + \mu^2 h_{-})
\]
\[
[\delta T_{ib}]_A = -\frac{\mu}{4 \kappa_{11}^2} \sum_{j,k=1}^{3} \epsilon_{ijk} f_{-jkb} \tag{50}
\]
Solving this Einstein equation we have:
\[
h_{+-} = -\frac{\mu^2 \kappa_{11}^2 \rho_+}{\pi^4} \left[ \frac{5}{64} \sum_{i=1}^{3} (x^i)^2 \frac{1}{|\vec{x}|^7} + \frac{1}{192} \frac{1}{|\vec{x}|^5} \right] \tag{51}
\]
The \((AB)\) component of the Einstein equation reads:

\[
\delta R_{AB} = -\frac{1}{2} \left[ \sum_{C=1}^{9} \frac{\partial^2 h_{AB}}{\partial x^C \partial x^C} - \sum_{C=1}^{9} \frac{\partial^2 h_{AC}}{\partial x^B \partial x^C} - \sum_{C=1}^{9} \frac{\partial^2 h_{BC}}{\partial x^A \partial x^C} + \sum_{C=1}^{9} \frac{\partial^2 h_{CC}}{\partial x^A \partial x^B} + 2 \frac{\partial^2 h_{+-}}{\partial x^A \partial x^B} \right] + \frac{1}{4} \left[ 2h_{--} \frac{\partial^2 g_{++}}{\partial x^A \partial x^B} + 2g_{++} \frac{\partial^2 h_{--}}{\partial x^A \partial x^B} + \frac{\partial g_{++}}{\partial x^A} \frac{\partial h_{--}}{\partial x^B} + \frac{\partial g_{++}}{\partial x^B} \frac{\partial h_{--}}{\partial x^A} \right]
\]

(52)

and

\[
\begin{align*}
\mathcal{T}_{ij} &= -\frac{1}{3} \delta_{ij} \left( 2\mu f_{-123} + \mu^2 h_{--} \right) \\
\mathcal{T}_{bc} &= \frac{1}{6} \delta_{bc} \left( 2\mu f_{-123} + \mu^2 h_{--} \right) \\
\mathcal{T}_{ib} &= -\frac{\mu}{4} \sum_{j,k=1}^{3} \epsilon_{ijk} f_{-jkb}
\end{align*}
\]

(53)

So far the need to make a gauge choice for the metric has not arisen. Now to solve for \(h_{AB}\) we must make a gauge choice for the metric. Let \(G^{\rho\sigma}\) and \(\Gamma_{\rho\sigma}^{\mu}\) denote the complete inverse metric and Christoffel symbol respectively (by "complete", we mean they include both the unperturbed and perturbed part). We shall fix the gauge by specifying \(G^{\rho\sigma}\Gamma_{\rho\sigma}^{\mu}\).

As can be easily verified,

\[
G^{\rho\sigma}\Gamma_{\rho\sigma}^{A} = \sum_{C=1}^{9} \partial_C h_{AC} - \frac{1}{2} \partial_A \left( \sum_{C=1}^{9} h_{CC} + 2h_{+-} - g_{++}h_{--} \right)
\]

(54)

so we need to specify \(G^{\rho\sigma}\Gamma_{\rho\sigma}^{A}\) to fix the gauge.

Using the above expressions for \(G^{\rho\sigma}\Gamma_{\rho\sigma}^{A}\), we can rewrite \(\delta R_{AB}\) as

\[
\delta R_{AB} = -\frac{1}{2} \left[ \sum_{C=1}^{9} \frac{\partial^2 h_{AB}}{\partial x^C \partial x^C} - \partial \left( \frac{G^{\rho\sigma}\Gamma_{\rho\sigma}^{A}}{\partial x^A} \right) - \partial \left( \frac{G^{\rho\sigma}\Gamma_{\rho\sigma}^{B}}{\partial x^B} \right) + \frac{1}{2} \left( \frac{\partial g_{++}}{\partial x^A} \frac{\partial h_{--}}{\partial x^B} + \frac{\partial g_{++}}{\partial x^B} \frac{\partial h_{--}}{\partial x^A} \right) \right]
\]

(55)

In general relativity we often use the “harmonic gauge” where we set \(G^{\rho\sigma}\Gamma_{\rho\sigma}^{A} = 0\) (which is satisfied by the unperturbed pp-wave background). Here, however, we shall opt for a different gauge.
As derived in the Appendix B, the effective potential is given by:

\[
V_{\text{eff}} = \frac{N_p}{R} \left\{ \frac{1}{2} \left[ v^2 + g_{++} + h_{++} + g_{--} \left( \frac{1}{4} g_{++} h_{--} - h_{++} \right) \right] + \sum_A [2h_{+,A} - h_{-,A}(v^2 + g_{++})]v^A + \sum_{A,B} h_{AB}v^Av^B \right\} + \frac{1}{8} h_{--}v^4 - \frac{1}{2} v^2 \left( h_{++} - \frac{1}{2} g_{++} h_{--} \right) \} \tag{56}
\]

where \( N_p \) is the number of D0-branes forming the probe graviton, and \( v^A \equiv \dot{x}^A, \ v^2 \equiv \sum_{A=1}^9 (v^A)^2 \). As \( h_{+,A}, h_{-,A} \) all vanish, they simply drop out of the effective potential.

The computation on Matrix Theory side in section 7 tells us that in the effective potential there are no terms of the form \( v^a v^b \) for \( a \neq b \), nor are there terms of the form \( v^i v^a \). This suggests we choose the gauge such that \( h_{ab} \propto \delta_{ab} \), and \( h_{ia} = 0 \). To make \( h_{ab} \propto \delta_{ab} \), we set:

\[
G^{\rho\sigma} \Gamma^a_{\rho\sigma} = \frac{1}{2} h_{--} \partial_a g_{++} \tag{57}
\]

then, to make \( h_{ia} = 0 \), we set:

\[
\partial_b \left( G^{\rho\sigma} \Gamma^i_{\rho\sigma} \right) = \frac{1}{2} \partial_i g_{++} \partial_b h_{--} - \frac{\mu}{2} \epsilon_{ijk} f_{-jkb}
\]

which implies

\[
G^{\rho\sigma} \Gamma^i_{\rho\sigma} = \frac{35 \mu^2 \kappa_{11}^2 p^+}{96 \pi^4} \frac{x^i}{|\vec{x}|^7} \tag{58}
\]

In this gauge, the Einstein equation gives:

\[
h_{ab} = \delta_{ab} \frac{\mu^2 \kappa_{11}^2 p^+}{\pi^4} \frac{1}{96} \left[ \frac{15}{2} \sum_{k=1}^3 (x^k)^2 \left( |\vec{x}| \right)^7 - \frac{1}{|\vec{x}|^5} \right] \tag{59}
\]

\[
h_{ij} = \delta_{ij} \frac{\mu^2 \kappa_{11}^2 p^+}{\pi^4} \frac{1}{96} \left[ -15 \sum_{k=1}^3 (x^k)^2 \left( |\vec{x}| \right)^7 + \frac{1}{2} \frac{1}{|\vec{x}|^5} \right] + \frac{\mu^2 \kappa_{11}^2 p^+}{\pi^4} \frac{15}{64} \frac{x^i x^j}{|\vec{x}|^7} \tag{60}
\]

Now let us look at the upper \((AB-)\) component of the Maxwell equation. It gives the following equations:

\[
\sum_{D=1}^9 \partial_D^2 a_{ij} + g_{++} \sum_{D=1}^9 \partial_D^2 a_{ij} - \sum_{D=1}^9 \partial_D g_{++} \left( \partial_D a_{ij} + \partial_i a_{jD} - \partial_j a_{Di} - \partial_j a_{Di} \right) + \mu \sum_{k=1}^3 \epsilon_{ijk} \left\{ - \sum_{D=1}^9 \partial_D h_{Dk} + \sum_{m=1}^3 (\partial_m h_{mk} - \partial_k h_{mm}) \right\}
\]

\[
\left\{ \partial_k \left[ \frac{1}{2} \left( g_{++} h_{--} + \sum_{D=1}^9 h_{DD} \right) \right] \right\} = 0 \tag{61}
\]
\[ \sum_{D=1}^{9} \partial_D^2 a_{bc} = 0 \]  
\[ \sum_{D=1}^{9} \partial_D^2 a_{ib} = 0 \]  

Solving them gives:

\[ a_{ij} = \mu \kappa \left( \sum_{k=1}^{3} \epsilon_{ijk} x^k \right) \frac{1}{384 \ |x|^7} \left[ -29 \sum_{m=1}^{3} (x^m)^2 + 9 \sum_{a=4}^{9} (x^a)^2 \right] \quad (64) \]
\[ a_{bc} = 0 \quad (65) \]
\[ a_{ib} = 0 \quad (66) \]

They give the field strength:

\[ f_{ijk} = \frac{\mu^3 \kappa^2 \Pi^+}{\pi^4} \left( \sum_{k=1}^{3} \epsilon_{ijk} x^k \right) \frac{5}{384} \frac{x^b}{|x|^9} \left[ -41 \sum_{m=1}^{3} (x^m)^2 + 9 \sum_{a=4}^{9} (x^a)^2 \right] \quad (67) \]

As can be easily checked, all the \( a_{\mu
u\rho} \) we have found indeed satisfy the Lorentz gauge.

Finally, we consider the \((++)\) component of the Einstein equation:

\[ \delta R_{++} = -\frac{1}{2} \sum_{A=1}^{9} \partial_A^2 h_{++} + \frac{1}{2} \sum_{A,B=1}^{9} \partial_A g_{++} \partial_B h_{AB} - \frac{1}{4} \sum_{A,B=1}^{9} \partial_A g_{++} \partial_A h_{BB} \]
\[ + \frac{1}{2} \sum_{A,B=1}^{9} h_{AB} \partial_A \partial_B g_{++} + \frac{1}{2} \sum_{A=1}^{9} \partial_A g_{++} \partial_A h_{--} + \frac{1}{2} \sum_{A=1}^{9} g_{++} \partial_A g_{++} \partial_A h_{--} \]
\[ - \frac{1}{4} \sum_{A=1}^{9} h_{--} (\partial_A g_{++})^2 \quad (68) \]

and

\[ T_{++} = -\frac{\mu}{2} \left( 2 f_{123} + \mu \sum_{i=1}^{3} h_{ii} \right) + \frac{\mu}{6} g_{++} (2 f_{-123} + \mu h_{--}) \quad (69) \]

From this we find

\[ h_{++} = \frac{\mu^4 \kappa^2 \Pi^+}{\pi^4} \frac{1}{6912 \ |x|^7} \left\{ 116 \left[ \sum_{i=1}^{3} (x^i)^2 \right]^2 + 2 \left[ \sum_{a=4}^{9} (x^a)^2 \right]^2 - 17 \sum_{i=1}^{3} (x^i)^2 \cdot \sum_{a=4}^{9} (x^a)^2 \right\} \quad (70) \]

To summarize, the nonzero components of the metric perturbation are: \( h_{--} \) [eqn(36)], \( h_{++} \) [eqn(51)], \( h_{ab} \) [eqn(59)], \( h_{ij} \) [eqn(60)], \( h_{++} \) [eqn(70)]; and the nonzero components of the field strength perturbation are: \( f_{-ijk}, f_{-ijb} \) [eqn(42)] and \( f_{+ijk}, f_{+ijb} \) [eqn(67)].
Substituting the expressions for the metric into our formula for $V_{eff}$ in eqn(56), averaging $h_{\mu\nu}$ over $x$ (i.e. dividing by $2\pi R$), and noting that $\kappa^2_{11} = \frac{16\pi^5}{117}$, $p^+ = \frac{N_p}{R}$, we find

$$V_{eff} = \frac{N_p}{2R} (v^2 + g_{++}) + \frac{15 N_p N_s}{16 M^9 R^3} \frac{v^4}{|x|^7}$$

$$+ \frac{\mu^2 N_p N_s}{R^3 M^9} \left\{ \left( \frac{1}{96} \frac{1}{|x|^5} - \frac{15}{32} \frac{(x^1)^2 + (x^2)^2 + (x^3)^2}{|x|^7} \right) \sum_{i=1}^{3} (v^i)^2 + \frac{15}{16} \sum_{i,j=1}^{3} x^i x^j v^i v^j \right\}$$

$$+ \frac{\mu^4 N_p N_s}{R^3 M^9} \frac{1}{768 |x|^7} \left\{ 32 \left[ \sum_{i=1}^{3} (x^i)^2 \right]^2 + \left[ \sum_{a=4}^{9} (x^a)^2 \right]^2 - 12 \sum_{i=1}^{3} (x^i)^2 \cdot \sum_{a=4}^{9} (x^a)^2 \right\}$$

Comparison of the above formula with eqn(19) on the Matrix Theory side shows exact agreement.

We would like to emphasize the approximation involved once again. We treated the source graviton as a perturbation to the exact pp-wave background, and the calculation was performed only to first order in $p^+$. However the solution that we found for these linearized equations is exact in $\mu$.

### 9 Discussion and Future Directions

In this paper, the effective potentials of Matrix Theory and Supergravity describing graviton scattering are computed, and they are found to agree exactly at order $\kappa^2_{11}$, up to quantum corrections at short distances. This provides evidence that the Matrix Theory action proposed in [24] describes M-theory in the pp-wave background. Furthermore, it implies the existence of a nonrenormalization theorem protecting the terms studied against higher loop corrections on the Matrix Theory side. Such nonrenormalization theorems have been studied in flat space [5, 6, 7, 8, 9, 10, 11]. In these papers, the $SO(9)$ symmetry in the transverse space appears to be a crucial ingredient. In the pp-wave background, however, this is broken to $SO(3) \times SO(6)$, so it will be interesting to see how the result could be extended in this case.

Our result at order $\mu^2$ agrees with Taylor and Van Raamsdonk’s proposal in [20] for Matrix Theory in a weakly curved background up to linear terms. As mentioned in their discussion, their proposal is proven only in the case where the background is produced by well-defined Matrix Theory configurations. It is not the case for the pp-wave background, so their proposal for the Matrix Theory in this background, while convincing, is not a proven fact. Thus the result at $\mu^2$, i.e. terms linear in the background, can be treated as
additional evidence for their proposal, similar to the explicit calculation in [21], this time with a nontrivial $g_{++}$ metric component.

The result at order $\mu^4$ is beyond linear order in the background, and hence is a completely new result. In fact, our calculation explicitly shows that there are no higher powers of $\mu$ in the effective action of the Supergravity side at the order $\kappa_{11}^2$. On the Matrix Theory side, higher powers of $\mu$ are also not expected for long distances. When they do appear, they are always accompanied by higher powers of $\alpha/r^2$, which indicates that they are corrections to Supergravity at short distances. However, as it stands, the corrections for velocity dependent terms are unreliable because they are computed using the sum over mass formula, which is exact only for terms independent of velocity or terms proportional to $\kappa_{11}^2$, as is shown in the Appendix A. Evaluating these corrections exactly requires going beyond the sum over mass formula, and an efficient way of handling the mass matrix will be of use.

An obvious future direction is to push our Matrix Theory computation to 2-loop. This can test whether a nonrenormalization theorem exists for the terms compared. A 2-loop computation is also necessary if we are to extend our result to three-body interactions.

In this paper, a very special bosonic background is chosen, with the source graviton sitting at the origin. It may be of interests to generalise to the case where neither of the gravitons are fixed at the origin.

A very interesting prospect is to calculate scattering from a transverse fivebrane, recently constructed in [32]. However, the effect of a transverse fivebrane in this background may only be visible through a higher loop computation on the Matrix Theory side.

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**Appendix A**

**Sum Over Mass Formula**

In this appendix we will prove the sum over mass formula in eqn(12).

The effective action $\Gamma$ of a theory expanded upon a background $B$ in the background
field method is given by:

\[ \Gamma = \frac{1}{2} [ Tr_{\text{boson}} \ln(-\partial^2_\tau + W_b(\tau)) - Tr_{\text{fermion}} \ln(-\partial^2_\tau + W_f(\tau)) - Tr_{\text{ghost}} \ln(-\partial^2_\tau + W_{gh}(\tau))] \] (72)

Here \( W = M^2 \) is the mass matrix squared for the fluctuating fields, and the trace \( Tr \) is over both the functional space and the field component indices (which are, besides the \( U(2) \) indices, the spacetime indices \( 0, 1, \ldots, 9 \) for the bosons, and the 16 Dirac spinor indices for the fermions).

Take the trace of the boson, for example:

\[
\Gamma_{\text{boson}} = \frac{1}{2} Tr_{\text{boson}} \ln(-\partial^2_\tau + W(\tau)) = -\frac{1}{2} Tr \int_0^\infty ds \exp[-s(-\partial^2_\tau + W(\tau))] \] (73)

The trace over functional space can be computed using the “plane-wave” basis wavefunctions \( |\omega> = \frac{1}{\sqrt{2\pi}} e^{-i\omega\tau} \),

\[
\Gamma_{\text{boson}} = -\frac{1}{2} tr \int d\tau \int \frac{d\omega}{2\pi} \int_0^\infty ds \frac{1}{s} e^{i\omega \tau} \exp[-s(-\partial^2_\tau + W(\tau))] e^{-i\omega \tau} \] (74)

The trace \( tr \) is now over only the field component indices. If we define \( V_{\text{eff}} \) by:

\[
\Gamma = -\int d\tau V_{\text{eff}} \] (75)

Then, the bosonic part of \( V_{\text{eff}} \) becomes:

\[
V_{\text{eff}}(\text{boson}) = \frac{1}{2} tr \int \frac{d\omega}{2\pi} \int_0^\infty ds \frac{1}{s} e^{i\omega \tau} \exp[-s(-\partial^2_\tau + W(\tau))] e^{-i\omega \tau} \] (76)

The operator in the middle can be rewritten in the following way:

\[
\exp[-s(-\partial^2_\tau + W(\tau))] = X e^{-sW(\tau)} e^{s\partial^2_\tau} \] (77)

Where \( X \) is defined as:

\[
X \equiv \exp[-s(-\partial^2_\tau + W(\tau))] e^{-s\partial^2_\tau} e^{sW(\tau)} = 1 + \text{commutator terms} \] (78)

The commutator terms give corrections to Supergravity, so for the purpose of this paper, which is to see whether Matrix model can reproduce Supergravity results, we can ignore
them. This claim will be proven shortly, after the result from approximating $X = 1$ is examined. In this approximation, we have:

$$V_{\text{eff}}(\text{boson}) = \frac{1}{2} \text{tr} \int \frac{d\omega}{2\pi} \int_{0}^{\infty} \frac{ds}{s} \exp[-s(\omega^2 + W(\tau))]$$

$$\Rightarrow V_{\text{eff}} = -\frac{1}{2} \text{tr} M$$

(79)

Note that $M$, the square root of $W$, can be defined through its expansion in powers of $1/r$. Putting everything together, and minding the minus signs for the fermions and the ghosts, we get the sum over mass formula in eqn(12). Now, we return to the claim made above, that the commutator terms in $X$ will not contribute to terms in the Supergravity limit. To show this, we first write $X$ in a most general form:

$$X = \sum_{n,m} K_{[m]}^{[n]}(W) s^n \partial^m$$

(80)

Here $K_{[m]}^{[n]}(W)$ is a general function of $W$ and its derivatives, and is defined by the above equation. Looking back at the definition of $X$ in eqn(78), we see that $n$ counts the number of terms involved in forming the commutator, and $m$ is the number of derivatives not acting on $W$. For example, when $n = 0$, it implies $m = 0$, and $K_{[0]}^{[0]} = 1$, corresponding to the approximation we made above. All the other values of $n$ correspond to commutator terms in $X$, and in particular, $K = 0$ when $n = 1$, because a commutator takes at least two terms.

Putting $X$ in terms of $K$ into $V_{\text{eff}}$, we will encounter the following factor inside the integrand:

$$e^{i\omega \tau} \partial^m e^{-i\omega \tau} = \sum_{l=0}^{m} (m)_l (-i\omega)^l \partial^{m-l} = \sum_{l=0}^{[m/2]} (m)_l (-\omega^2)^l \partial^{m-2l}$$

(81)

$[m/2]$ is the biggest integer no larger than $m/2$. In the last line, we made use of the fact that $\omega$ will be integrated from $-\infty$ to $+\infty$ so that any odd functions in the integrand will give zero. As a result, only terms with even powers of $\omega$ are kept. Therefore, the effective potential becomes:
\[ V_{\text{eff}}(\text{boson}) = \frac{1}{4\pi} \sum_{n,m} \text{tr} \int d\omega \int_0^\infty ds \frac{K_{[m]}^n e^{i\omega \tau}}{s} \partial^n e^{-i\omega \tau} e^{-sW} e^{-s\omega^2} \]
\[ = \frac{1}{4\pi} \sum_{n,m} \sum_{l=0}^{[m/2]} \text{tr} \int d\omega \int_0^\infty ds \frac{K_{[m]}^n e^{-sW}}{s} \partial^m e^{-sW} e^{-s\omega^2} \]
\[ = \frac{1}{4\pi} \sum_{n,m} \sum_{l=0}^{[m/2]} \text{tr} \int_0^\infty ds \frac{1}{s} \left( \frac{\partial^l}{\partial s} \sqrt{s} \right) K_{[m]}^n \partial^m e^{-sW} \]
\[ = \text{tr} \sum_{n,m} \sum_{l=0}^{[m/2]} \frac{1}{4} \frac{\Gamma(n-l-1/2)}{\Gamma(1/2-l)} K_{[m]}^n \partial^m e^{-sW} \]
\[ \text{Thus, the effective potential can be recasted in the following form:} \]
\[ V_{\text{eff}} = \text{tr} \sum_{n,m} \sum_{l=0}^{[m/2]} \frac{1}{4} \frac{\Gamma(n-l-1/2)}{\Gamma(1/2-l)} \alpha^{2(n-l)-1} K_{[m]}^n (W) \partial^m e^{-sW} \]
\[ \frac{1}{(\alpha^2 W)^{n-l-1/2}} \] (83)

As before, \( \alpha = 1/(M^3 R) \). The reason these factors of \( \alpha \) are inserted will be clear shortly.

In a comparison of 1-loop Matrix Theory with Supergravity, the relevant terms on the Supergravity side are proportional to \( \kappa_{11}^2 \), which is of order \( \alpha^3 \) on the Matrix Theory side. This means any terms of higher powers of \( \alpha \) are irrelevant for such a comparison as they represent only Matrix Theory corrections to Supergravity, and finding them is not the purpose of this paper. In other words, to examine whether the Matrix Theory can reproduce Supergravity in the appropriate limit, only terms up to \( \alpha^3 \) need to be kept.

It makes sense, therefore, to examine each factor in \( V_{\text{eff}} \) and count the powers of \( \alpha \) it contains. We begin with the mass matrix squared. By inspection of the explicit expressions given in section 6, one sees that \( W \) can always be written in the following form:
\[ W \sim \frac{1}{\alpha^2 r^2 + \frac{1}{\alpha} N} \]
\[ \Rightarrow \alpha^2 W \sim r^2 + \alpha N \]
\[ \Rightarrow (\alpha^2 W)^k \sim 1 + \alpha N + (\alpha N)^2 + \cdots \] (84)

For example, using eqn(18), we have:
\[ \alpha N_f = \alpha \left( \sum_{l=1}^9 v_l \gamma_l + \sum_{i=1}^3 \frac{i \mu \omega^i}{4} \{ \gamma_i, \gamma_{12} \} \right) + \alpha^2 \mu^2 / 4^2 \] (85)
Similarly, \( \alpha N_b \) can be constructed using the rules given in section 6, while \( \alpha N_{gh} = 0 \). From the explicit expressions of \( N \), it can be shown easily that \( \text{tr } \alpha N = 0, \text{tr } [\alpha N]^2 = 0 + O[\alpha^4] \), and \( \text{tr } [(\alpha \partial N)^2] = 0 + O[\alpha^4] \). These facts are related to the large number of supersymmetries of our system and will be of use shortly. The last line in eqn (84) is a symbolic statement that for any \( k \), whether positive or negative, \( (\alpha^2 W)^k \) will only give non-negative powers of \( \alpha \).

Another important point to note is that every \( \alpha \) arising from \( (\alpha^2 W)^k \) is accompanied by a factor of \( N \). Now look at \( K[n] \): Let \( K[n] = \sum_{p,q} K[p,n,q] \) where \( p \) is the number of \( \tau \)-derivatives acting on \( W \) inside \( K \), and \( q \) is the number of \( W \) inside \( K \). By definition, we have:

\[
\frac{p + m}{2} + q = n
\]  

(86)

For \( n = 0 \) \( \Rightarrow K = 1; \)
For \( n = 1 \) \( \Rightarrow K = 0; \)
For \( n \geq 2 \) \( \Rightarrow K \) consists of commutators. In this case, we have:

\[
\left\{ \begin{array}{l}
q < n \\
p + m < 2n
\end{array} \right.
\]  

(87)

For fixed \( q \) and \( n \geq 2 \), \( m \) and \( p \) have the following extremal values:

\[
m_{\text{min}} = 0 \Rightarrow p_{\text{max}} = 2(n - q) \\
p_{\text{min}} = 1 \Rightarrow m_{\text{max}} = 2(n - q) - 1
\]

The reason \( p_{\text{min}} = 1 \) is that \( \partial^2_\tau \) must act at least once on \( W \) to give non-vanishing commutators like \([W, [W, [W, \dot{W}]]]\) in \( K \). Look at:

\[
\alpha^{2(n-l)-1} K[n] W \partial^{m-2l} \frac{1}{(\alpha^2 W)^{n-l-1/2}} = \sum_{p,q} \alpha^{2(n-l)-1} K[p,n,q] W \partial^{m-2l} \frac{1}{(\alpha^2 W)^{n-l-1/2}}
\]

\[
= \sum_{p,q} \alpha^{2(n-q)-1-2l} K[p,n,q] (\alpha^2 W) \partial^{m-2l} \frac{1}{(\alpha^2 W)^{n-l-1/2}}
\]

\[
= \sum_{p,q} \alpha^a K[p,n,q] (\alpha^2 W) \partial^{m-2l} \frac{1}{(\alpha^2 W)^{n-l-1/2}}
\]  

(88)

where \( a = 2(n - q) - 1 - 2l \). Noting \( l \leq [m/2] \leq [m_{\text{max}}/2] = [n - q - 1/2] = n - q - 1 \), we must have:

\[
l_{\text{max}} = n - q - 1
\]

\[
\Rightarrow a \geq 1
\]  

(89)

From this derivation of the lower bound of \( a \), we see that the equality holds only when \( m = m_{\text{max}} = 2(n - q) - 1 \) and \( l = l_{\text{max}} = n - q - 1 \). Then, eqn (86) gives:

\[
m - 2l = 1
\]

\[
p = p_{\text{min}} = 1
\]  

(90)
A comparison with Supergravity with one-loop Matrix Theory means keeping terms only up to $\alpha^3 \sim \kappa_{11}^2$. Therefore, we need only consider the range of $a$ to be:

$$1 \leq a \leq 3$$ (91)

For $a = 3$: There can be no factors of $\alpha N$ from $\alpha^2 W$ because they increase the powers of $\alpha$ beyond 3, hence taking us beyond the limit of Supergravity. Without any factors of $\alpha N$, the effective potential is simply:

$$V_{\text{eff}} = f(r) tr 1$$

$$\Rightarrow V_{\text{eff}} \sim tr 1 = 0$$ (92)

Here, $tr$ is again the trace over the boson minus the trace over the fermions and the ghosts.

For $a = 2$: There can be at most one $\alpha N$, either from $K(\alpha^2 W)$ or $\partial^{m-2l} \frac{1}{(\alpha^2 W)^{n-1/2}}$. But for only one $N$, we have:

$$V_{\text{eff}} \sim tr \partial^k N$$

$$= \partial^k tr N$$

$$= 0$$ (93)

For $a = 1$: Now it is possible to have $(\alpha N)^2$ coming from one of the following three cases:

(i) Both $(\alpha N)^2$ come from $\partial^{m-2l} \frac{1}{(\alpha^2 W)^{n-1/2}}$:

$$V_{\text{eff}} \sim \alpha tr \partial^{m-2l} (\alpha N)^2$$

$$\sim \alpha \partial^{m-2l} tr (\alpha N)^2$$

$$\sim \alpha O[\alpha^4]$$

$$\sim O[\alpha^5]$$ (94)

(ii) Both $(\alpha N)^2$ come from $K(\alpha^2 W)$: Since we showed that $p = 1$ when $a = 1$, there is only one $\tau$-derivative acting on $W$ in $K$, we must have either: (a) $V_{\text{eff}} \sim tr N^2$ or (b) $V_{\text{eff}} \sim tr (N \partial_\tau N) \sim \frac{1}{2} \partial_\tau tr (N^2)$. In either case, $V_{\text{eff}} = 0 + O[\alpha^5]$.

(iii) One $(\alpha N)$ comes from $K(\alpha^2 W)$ and one from $\partial^{m-2l} \frac{1}{(\alpha^2 W)^{n-1/2}}$: We already showed that $m - 2l = 1$ when $a = 1$, so we have:

$$V_{\text{eff}} \sim K \partial \frac{1}{(\alpha^2 W)^{n-1/2}}$$ (95)

This implies either: (a) $V_{\text{eff}} \sim tr (N \partial N)$ or (b) $V_{\text{eff}} \sim tr (\partial N \partial N)$

(a) is identical to case (iii) above. (b) is of order $\alpha^5$ using the fact mentioned before. This exhausts all cases contributing to terms up to order $\alpha^3 \sim \kappa_{11}^2$ in $V_{\text{eff}}$. In particular, we have shown that none of the commutator terms in $X$, corresponding to $K[m]_{in}$ with $n \geq 2$, contributes to terms relevant to Supergravity. This completes the proof of the claim made under equation (78).
Appendix B

Deriving $V_{eff}$

The Lagrangian of the probe graviton moving in a curved spacetime with metric $G_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$ is given by\(^6\):

$$
\mathcal{L} = -m\sqrt{-G_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}} = -m\sqrt{-(g_{\mu\nu} + h_{\mu\nu})\dot{x}^{\mu}\dot{x}^{\nu}}
$$

(96)

We make a Legendre transformation:

$$
\mathcal{L}' = \mathcal{L} - P^{+}_p \dot{x}^-
$$

(97)

where

$$
P^{+}_p = \frac{\delta \mathcal{L}}{\delta \dot{x}^-} = m\frac{1 + h_{-\nu}\dot{x}^{\nu}}{\sqrt{-G_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}}
$$

(98)

When we let $m \to 0$, this gives

$$
G_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = 2\ddot{x}^- + g_{++} + v^2 + h_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = 0
$$

(99)

This is a quadratic equation for $\ddot{x}^-$, which we will solve for $\ddot{x}^-$, keeping only terms of lowest order of $p^+$. Recalling that all $h$ are at least of order $p^+$, we have:

$$
\ddot{x}^- = -\left\{ \frac{1}{2} \left[ v^2 + g_{++} + h_{++} + g_{++} \left( \frac{1}{4} g_{++} h_{--} - h_{+-} \right) \right]
+ \sum_{A} \left[ 2 h_{++} - h_{-A} (v^2 + g_{++}) \right] v^A + \sum_{A,B} h_{AB} v^A v^B \right\} + \frac{1}{8} h_{--} v^4 - \frac{1}{2} v^2 \left( h_{+-} - \frac{1}{2} g_{++} h_{--} \right)
$$

(100)

Taking the limit $m \to 0$, the $\mathcal{L}'$ in eqn(97) is simply $-P^{+}_p \dot{x}^-$, which is the effective potential $V_{eff}$ that we need. It contains the interaction between the probe and the source up to terms linear in $h_{\mu\nu}$. An alternative way of writing such interaction is:

$$
\frac{\delta \mathcal{L}}{\delta G_{\mu\nu}} h_{\mu\nu} = T^{\mu\nu} h_{\mu\nu}
$$

(101)

This structure was used by Taylor and Van Raamsdonk\[^{20}\] to identify the effective potentials on both sides.

\[^6\]This approach is borrowed from\[^{16}\].
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