An experiment-based module for building quantum mechanics concepts using Mach-Zehnder interferometry

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Abstract. To introduce the fundamental concepts and formalism of Quantum Mechanics at the base of Quantum Calculus and Quantum Cryptography we developed an experiment-based module centered on the use of a Mach-Zehnder laser interferometer equipped with linear polarizers. By analyzing the constructive and destructive interference, students are guided to understand the meaning of mutual exclusivity and incompatibility of polarization states (H, V) and (45°, 135°). A MATLAB simulation allows the understanding of the experimental results in terms of single photons. The module was tested as a pilot intervention on a group of Computer Science PhD students at the University of Verona.

1. Introduction
Teaching Quantum Mechanics at graduate and post-graduate levels is becoming increasingly important in non-physics specializations such as Mathematics and Computer Science. In particular, working in the advanced fields of Quantum Computing and Quantum Cryptography requires a deep comprehension of basic quantum mechanics concepts and formalism also in an operational way. A Qubit is a quantum mechanical superposition of mutually exclusive physical quantities; Quantum Key Distribution consists in the generation (e.g. with a laser) of a train of quantum particles (e.g. single photons or two correlated photons) and on the measurement of incompatible observables (e.g. polarizations in the rectilinear vertical/horizontal and diagonal 45°/135° bases). This kind of professional formation thus requires learning how to manage concepts and formal structures concerning photon polarization and requires building a functional understanding of the superposition principle, the mutual exclusive and incompatible properties, and of the new concept of measurement in quantum mechanics.
In this work, we present an experiment-based module for teaching quantum mechanics to Computer Science post-graduate students. Starting from the works of Ghirardi and Michelini [1,2], we developed and experimented in a pilot intervention a didactic approach centred on the superposition principle in the space of photon polarization states aimed at building the basic quantum mechanics concepts and related formal structures by means of experiments based on laser interferometry and computer simulations, with a focus on quantum information applications.

2. The didactic module
The didactic module was tested as a pilot intervention, also followed by a Quantum Computing expert, with five Computer Science PhD students at the University of Verona (Italy) and was organized into two phases: a first interactive lecture (three hours) devoted to the exploration of the interaction of light with birefringent crystals and polarizers addressing the concepts of intensity and polarization in terms of electromagnetic wave and single-photon models; and a second interactive laboratory session (four hours) based on two notable experiments of modern physics, namely the Which-Way and the Quantum
Eraser, using a Mach Zehnder laser interferometer, with the aim of understanding, through the classical optics analogy, the quantum mechanical concepts underlying quantum interference.

2.1. First phase (3h): inquiry-based lecture
The first introductory session starts from a qualitative exploration of the interaction of light using a calcite birefringent crystal and Polaroid filters to address the concepts of intensity, polarization, and the Malus law and their interpretation in terms of electromagnetic waves [2]. Then, the photon model is introduced leading to the interpretation of the intensity of a photon beam as related to their number, of the Malus law in probabilistic terms, and leading to the recognition of polarization as a property of photons.

This arises the question of the meaning of polarization as a property of a single photon and of the superposition principle for a single photon in the space of polarization states up to the difference between a statistical mixture of photons and a beam of photons each one in a superposition state. The Dirac formalism that allows describing the polarization in the circular left/right, rectilinear vertical/horizontal or diagonal 45°/135° bases is explained, thus highlighting the distinction between mutual exclusive (orthogonal) states and incompatible observables, and the concept of measurement in quantum mechanics.

2.2. Second phase (4h): interactive experiment-based session
The second interactive experiment-based session makes use of the Thorlabs optics kit for the Quantum Eraser experiment [3]. The experimental set-up is a Mach-Zehnder laser interferometer, equipped with linear polarizers, described in detail in the next section, which allows to perform measurements on separated arms at the cost of some difficulty in the alignment. With this set-up students are guided to understand through constructive and destructive interference the meaning of vertical and horizontal (as well as 45° and 135°) polarization states being mutual exclusive (orthogonal) and of the two rectilinear (vertical/horizontal) and diagonal (45°/135°) bases being mutual incompatible.

The laser interferometry and the polarization measurements allow also to conduct the Which-Way Feynman experiment of the double-slit to demonstrate the complementarity of wave and particle nature of light [4,5]. The limit down to the single photon interpretation is obtained through a computer MATLAB simulation, shown below, which allows to understand the experimental results in terms of single photons. On the simulation, the laser source is modelled as a train of discrete particles and the interference figure obtained in the Mach-Zehnder set-up is built from the (slow) arriving of separate hits by sampling the wave-intensity as probability function of the photon position in space.

3. Materials and methods

3.1. Optical instrumentation: the Mach-Zehnder interferometer
The Mach-Zehnder interferometer is an amplitude division interference system based on two beam splitters and two reflecting mirrors (figure 1). A collimated laser (532nm) entering the first beam splitter (BS1) has its amplitude equally divided between the reflected and transmitted beams on two different paths, the lower and upper arm of the system. The mirrors (M) swap directions to redirect the beams to the second beam splitter (BS2), which recombines the beams leading to an interference phenomenon, with observation of constructive or destructive interference in the two screen (S0 and S1), according to the relative phase difference. A perfect balanced Mach-Zehnder interferometer leads to the phase difference of $\Delta \Phi = \pi$ and of $\Delta \Phi = 0$ in the two screens, according to the orientation of the beam splitters (see figure 1c). In our configuration, the beam splitter are positioned opposite and we observe a constructive interference in the beam exiting the system with the same direction of the entering.

Generally, in a Mach Zehnder interferometer a difference in the length paths is introduced and an interference pattern is observed in the longitudinal axis. In our configuration, with balanced arms, we observe a circular symmetrical fringe pattern that is due to the divergence of the beam passing the lens.
The pattern, clearly with opposite annular rings starting by the destructive or constructive central spot, is then observed at a fixed time on both screens. Fringe visibility is very high, thus facilitating the demonstration of experiments to the students (figure 2b). The interferometer is equipped with a set of three linear polarization rotators (mounted in figure 1a); two of them are positioned in the upper and lower arms, the third is positioned at the exit of the system before one detector (S1) during Which-Way and Quantum Eraser experiments.

Figure 1. Mach-Zehnder interferometer: a) mounted device with the three polarization rotators; b) basic optical scheme; c) phase-shift at beam splitter (from Thorlabs manual [6]).

Figure 2. Circular interference pattern: a) scheme (from Thorlabs manual [6]); b) our observation.

3.2. Experiments

3.2.1. Experiment 1 – Superposition of states of spatially separated states. In the experiment (fig. 3), the two polarization rotators are first placed in the two separated upper and lower arm inside the interferometer, after the BS1. With one polarizer fixed and moving the other, the student is asked to explore different relative orientations in order to discuss mutual exclusive (orthogonal) polarization states by observing the pattern on the screen. The sequence includes: step a) parallel polarizers oriented 0°/0° with interference on the two screen; step b) orthogonal polarizers 0°/90° causing the interference
patterns disappear. The student is also asked to make a prevision on the intermediate orientation. Then, a third polarizer is added at the exit of the system, in one arm direction, in order to discuss incompatible polarization basis by observing and comparing the pattern on the two screens. In step c) the diagonal orientation 45° in the upper arm restores the interference pattern in that arm.

3.2.2. Experiment 2 – Which-way and Quantum Eraser. The same setup (figure 3) allows to perform the traditional “Which-Way” Feynman experiment of the double-slit in order to discuss the complementarity of wave and corpuscular nature of light. This approach was shown in previous works by Dimitrova and Weiss [5]. Here the same sequence of the first experiment is re-interpreted: the polarizers with different 0°/90° orientation placed in the two arms inside the system provide a which-way measurement switching from wave (interference) to corpuscular (no interference) behaviour; adding the third polarizer the information is erased and the interference restored.

![Figure 3](image)

**Figure 3.** Experiment sequence (sketch). Two polarization rotators positioned in separated arms at different orientation: a) parallel 0°/0° (interference observed); b) orthogonal 0°/90° (interference disappears); c) adding the third polarizer in diagonal orientation 45° at the exit of the upper arm restores the interference in the upper screen.

Classic Optics models the interference pattern as a variation in space, at fixed time, of the total intensity of monochromatic electromagnetic waves due to the phase shift given by different optical paths across the two arm, with the known result I \( \propto \cos^2 \Delta \Phi \).

Starting from the two-slit Young interference model students had to to face the new problem that a single photon cannot be divided, like the light beam. In the Quantum modelling (detailed below) we find an analogous expression for the probability of detecting the photon in the screen, obtained as square modulus of the output wave function. This allows us to discuss the problem of the physical meaning of the wave function and the standard probability interpretation.

3.2.3. Experiment 3 – single-photon simulation A simulation helps us in stretching the experiments to single-photon observation: like in the intensity beam analogy the measured intensities give the detection probabilities.

A MATLAB code has been designed in order to discuss with the students the probability interpretation of the wave function and the link between the classic wave and quantum particle behavior. The simulation constructs, step by step and in real-time, the probability from single arrivals in space of a high number of photons, launching random events with a given pdf. The probability function is sampled with the rejection method. A real-time plot shows how the discrete probability of finding the photon in a certain point at given time tends to the light intensity pattern (continuous) observed in the screen.

The Graphic User Interface of the MATLAB developed code is shown in figure 4. The MZ interference pattern (I \( \propto \cos^2 \)) is built from the (slow) arriving of separate hits by sampling the wave-intensity as probability function of the photon position in space. The 3D pattern is rendered using circular symmetry from the 2D sampling. A BIP sound at each launched photon is used to capture student attention to
discrete events. The GUI parameters include the event rate, the step of the space sampling of the CCD detector.

**Figure 4.** Graphic User Interface of the MATLAB script developed. It shows in the top windows the light intensity interference function ($\propto \cos^2$) used as theoretical pdf for launching the random events. The bottom windows plot in real time the photon counted at a line detector and the construction of the continuous pattern from the discrete events.

4. **Discussion**

4.1. **Connections of optical interferometry with quantum information**

At the basis of the proposed approach there is the profound connection between optical interferometry and quantum information. Indeed, our idea is to use the quantum information approach to which computer science students are more comfortable for linking the underlying physical concepts of quantum mechanics. The following issues were recalled and discussed with the students.

Quantum information is stored and transported by a two-level quantum mechanical system, e.g. the polarization of the photon. The possible physical states of the quantum bit are represented by the two states $|0\rangle$ and $|1\rangle$, with the difference respect to the classic analogous that the qubits may be prepared in an infinite number of coherent superposition of states $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$; the measurement on the state of qubit $|\psi\rangle$ gives the outcome $0$ or $1$ with the respective probability $|\alpha|^2$ and $|\beta|^2$, leaving the qubit in state $|0\rangle$ or $|1\rangle$, depending on the outcome. Qubits are thus represented by vectors in the 2-dimensional Hilbert space spanned by the computational standard basis $|0\rangle$, $|1\rangle$, where $|0\rangle$ and $|1\rangle$ are, for example, the conjugate pair of photon polarizations in the rectilinear, diagonal, or circular bases, or the reflected and transmitted path of the photon in the beam splitter.

Qbits are processed in a quantum computation through reversible logical gates that perform unitary transformations on the quantum states, therefore represented as 2x2 unitary matrix. The quantum measurement on the output state thus provides the result of the computation. It is known that small-scale quantum circuits can be simulated in linear optical setups by introducing a single-photon representation of qubits and a correspondence between traditional optics components, e.g. beam splitters, phase shifters, and polarization rotators, and single-bit quantum gates [7]. For example, a loss-less symmetric beam splitter is known to act as Hadamard gate, a polarization rotator as controlled-NOT gate.
Single-photon Mach-Zehnder interferometer in figure 1 is then shown to be a quantum circuit, the “which-way” variable containing the information of the path taken by the photon is the “location” or “dual-rail” qubit. The first beam splitter (BS1) gate acts on the entering photon transforming the input state (e.g. |0⟩) in the superposition of states |0⟩ and |1⟩, on which finally the second beam splitter gate (with opposite orientation in our setup) acts by transforming back the location qubit at its initial state |0⟩, i.e. the photon exits the circuit in the same direction (constructive interference observed at the detector S0 and destructive interference observed at detector S1).

The experiments are now modelled using Dirac formalism, introduced in the first lesson, in order to link the qubit quantum processing to the optical chain, as follows

\[
|\psi_{\text{in}}\rangle = |\psi_{\text{out}}\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \rightarrow |\psi_{\text{out}}\rangle = \cos \frac{\Delta \phi}{2} + i \sin \frac{\Delta \phi}{2} |\psi_{\text{in}}\rangle
\]

with \(\Delta \phi = \phi_0 - \phi_1\) the phase shifting collected in the two arms, e.g. according to the beam splitters orientation. The probabilities of measuring the photon entering the system state |0⟩ at the screens S0 and S1 are then

\[
P_{S0} = |\langle 0|\psi_{\text{out}}\rangle|^2 = \cos^2 \frac{\Delta \phi}{2} \quad P_{S1} = |\langle 1|\psi_{\text{out}}\rangle|^2 = \sin^2 \frac{\Delta \phi}{2}
\]

Turning now to the set of polarizers in the interferometer, we show that the information of the polarization of the photon (e.g. ||⟩, |⟩ in the horizontal-vertical basis) is stored in the “polarization” qubit in product state with the “location” mode, forming a 2-qubits system. We have that the polarization rotators may act on the entering photon, conditionally on its location, by changing the state to different polarizations ||⟩ and |⟩, thus labelling the two paths as distinguishable and causing the interference to disappear (Feynman which-way experiment, the location and polarization qubits are entangled).

5. Final remarks and conclusion

Starting from previous approaches [1,2,5] we designed a new didactic experiment-based module for building quantum mechanics concepts using Mach-Zehnder interferometry and linking optics with quantum information. The potential of the Mach-Zehnder interferometer in quantum teaching is well recognized in the context of virtual laboratories [8]. Here, we focus a double approach based on a real experiment, mounted on a portable optical bench, and on simulations.

The Mach-Zehnder interferometer equipped with polarizers has shown to be an efficient setup to lead students to reflect on the difference between the polarization and the dual-rail qubits, through a deeper comprehension of the physical meaning of the superposition principle, the mutual exclusive and incompatible properties, and of the new concept of measurement in quantum mechanics.

Clearly, naturally helped by vector composition rules in the plane, students visualize that a diagonal polarization of 45° is an equal superposition of horizontal and vertical polarizations. But what about a superposition of the single-photon locations in the two arms of the interferometer? The quantum superposition of states \(\sum_{i=0}^{1} \frac{|i\rangle}{\sqrt{2}} (|0\rangle + |1\rangle)\) represents a photon that simultaneously stays in the upper and lower arms of the system and not a probabilistic mixture of the states |upper⟩ and |lower⟩. Single photon is indivisible.

This illustrates to the student that superposition and probability mixture are different concepts: when measuring the superimposed state of the single photon after the first beam splitter port, we get an equal probability of 1/2 of finding it in the upper or lower path; but, after the photon travelled across the entire system, subjected to a phase transformation, we have that in the second beam splitter the two paths interfere leading to the state |ψ_{out}\rangle, with the probabilities \(P_{S0}\) and \(P_{S1}\) of finding it at the two screens S0 (lower) and S1 (upper). A phase shift of \(\Delta \phi = 0\) gives a unitary probability of finding the photon in the lower screen, clearly in contradiction with the misconception of considering the photon travelling the arms with probability of 1/2 and then, again, passing through the last beam splitter with a probability of 1/2 to exit the two directions.

Quantum interference and the simultaneous travelling of the single photon in both arms may be then underlined.
The module, proposed as a first pilot intervention to a small group of Computer Science PhD students and followed by a Quantum Computing expert as well, proved its feasibility as part of an integrated course on Quantum Information based on the use of optical interferometry and quantum circuits that is planned for future work.

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