Lagrangian Evolution of the Weyl Tensor

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ABSTRACT

We derive the evolution equations for the electric and magnetic parts of the Weyl tensor for cold dust from both general relativity and Newtonian gravity. In a locally inertial frame at rest in the fluid frame, the Newtonian equations agree with those of general relativity. We give explicit expressions for the electric and magnetic parts of the Weyl tensor in the Newtonian limit. In general, the magnetic part does not vanish, implying that the Lagrangian evolution of the fluid is not purely local.

Subject headings: cosmology: theory — large-scale structure of the universe — gravitation

1. Introduction

Gravity is a long-ranged force. According to this Newtonian perspective, the motion of a mass element is affected by distant elements. In general relativity, however, the motion of a freely-falling body is determined by the local curvature of the spacetime manifold: in a locally inertial frame, the motion of nearby mass elements is governed by the Riemann curvature tensor. The Newtonian and relativistic viewpoints are made consistent by the fact that the Riemann tensor incorporates Newtonian gravitational tidal fields.
In the Newtonian approach, tidal fields are obtained from the gradient of the gravity vector, whose determination requires a sum over all mass elements. Thus, it came as a surprise to us that, in general relativity, the tidal field evaluated at the position of a freely-falling mass element might evolve according to a purely local equation — provided that certain stringent conditions are met.

Matarrese, Pantano, & Saez (1993) and Croudace et al. (1994), following earlier work of Barnes & Rowlingson (1989), showed that if a quantity known in general relativity as the magnetic part of the Weyl tensor vanishes in the Newtonian limit, the Newtonian tidal field obeys a local Lagrangian evolution equation until trajectories intersect. In other words, the tidal field following a mass element changes with time in a way depending only on local fluid variables — the density and velocity gradient. Because these variables themselves obey Lagrangian equations (local aside from the tidal field) — it seemed that the one might be able to evolve the fluid variables and tidal field independently for all mass elements until trajectories intersect. In this situation, all nonlocal information is incorporated into the initial value of the tidal field.

Bertschinger & Jain (1994) used this fact to study the nonlinear evolution of density perturbations in the expanding universe. They noted (as did Matarrese et al. 1994) that the magnetic part of the Weyl tensor did not necessarily vanish in the Newtonian limit. However, they assumed that it could be approximately neglected, and solved the coupled nonlinear fluid and tidal evolution equations for general irrotational motion starting from the growing mode of cosmic density perturbations. They found that nonlinear coupling of the fluid shear and tidal field would favor filamentary gravitational collapse as opposed to the sheetlike pancake collapse predicted on the basis of kinematical theory (Zel’dovich 1970). This surprising conclusion rests on an important unchecked assumption.

Until now there has been no Newtonian derivation of the tidal evolution equation. The equations of motion for the Riemann tensor in general relativity have, instead, been projected into the local fluid frame. This work was pioneered by Kundt & Trümper (1961), Hawking (1966), and Ellis (1971, 1973). It has resulted in a powerful covariant Lagrangian fluid description of matter and gravitational fields. This method has been applied to the evolution of cosmic density fluctuations by Hawking (1966), Ellis & Bruni (1989), Hwang & Vishniac (1990), and many later workers. In linear perturbation theory the magnetic part of the Weyl tensor vanishes for irrotational perturbations so that the Lagrangian fluid equations reduce to local equations. In cosmological perturbation theory the Lagrangian fluid approach, while elegant and free of gauge ambiguities, offers no compelling advantage compared with traditional Eulerian methods.

Traditionally, two approaches have been used to study gravitational dynamics in
cosmology. The first is Eulerian: the fluid and gravitational variables (mass density, velocity, etc.) are defined on a grid and evolved according to partial differential equations. This method works well for a collisional fluid in which pressure forces prevent the intersection of trajectories, but it breaks down for cold dust (pressureless collisionless matter, e.g., cold dark matter) after mass elements intersect. The second method is that of Lagrangian trajectories, exemplified by N-body simulations: the mass is discretized into particles whose positions and velocities are integrated using the appropriate equations of motion.

The Lagrangian fluid approach offers a third way: the density and velocity gradients, but not necessarily the positions and velocities, are integrated for individual mass elements. Clearly, this procedure is incomplete without computing the trajectories of mass elements. One can state the density of a given mass element (provided that it is not superposed on another element) but, without integrating the trajectories, one cannot say where it is. While the trajectories can be integrated here as in N-body simulations, this presents an extra complication. For some purposes, we would be happy to know the density and velocity gradient of every mass element even if their positions are unknown.

This explains the attraction of local Lagrangian evolution. However, the applicability of the method remains unclear as long as questions remain about the magnetic part of the Weyl tensor.

In this paper we address three questions: Does the magnetic part of the Weyl tensor vanish in the Newtonian limit? Can the Lagrangian evolution equations for the Weyl tensor be derived in the Newtonian limit? Are they local in general? In the following sections we show that the answers are, respectively, not necessarily, yes, and no.

2. Evolution of the Weyl tensor in General Relativity

In this section we present the general relativistic derivation of the evolution equations. Although this material largely repeats the work of Ellis (1971, 1973), it is a necessary prelude to the Newtonian derivation. Besides defining the Weyl tensor and other quantities, we establish some notation that is used throughout this paper and we clarify the conditions required to obtain the Newtonian limit of general relativity. We adopt the conventions and notations of Misner, Thorne, & Wheeler (1973), including metric signature $+2$.

The Weyl tensor is the traceless part of the Riemann curvature tensor (Misner et al. 1973):

$$C_{\mu \nu \kappa \lambda} \equiv R_{\mu \nu \kappa \lambda} - \frac{1}{2} \left( g_{\mu \nu \kappa \sigma} R^\sigma_{\lambda} + g_{\mu \nu \sigma \kappa} R^\sigma_{\lambda} \right) + \frac{R}{6} g_{\mu \nu \kappa \lambda} ,$$

(1)
where, for convenience in what follows, we have defined

\[ g_{\mu\nu\kappa\lambda} \equiv g_{\mu\kappa}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\kappa}. \]  

(2)

The full Riemann tensor follows from the Weyl tensor and the Ricci tensor \( R_{\mu\nu} \equiv R^\kappa_{\mu\kappa\nu} \) and its trace \( R \equiv R^\mu_\mu \). Through the Einstein field equations,

\[ R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 8\pi G T_{\mu\nu}, \]  

(3)

the Ricci tensor gives the contribution to the spacetime curvature from local sources with energy-momentum tensor \( T_{\mu\nu} \). The Weyl tensor gives the contribution due to nonlocal sources. Therefore, Newtonian tidal forces will be represented in the Weyl tensor.

In the most common formulation of general relativity, the Einstein equations are regarded as field equations for the metric tensor components, since the Ricci tensor can be obtained from the metric tensor and its first and second derivatives. Because the Riemann tensor also follows from the metric and its derivatives, the Weyl tensor can be calculated from the solution to the field equations.

However, we are interested in an alternative formulation of general relativity in which the Weyl tensor is treated as the fundamental geometrical quantity and the Ricci tensor follows algebraically from the Einstein equations for a given distribution of energy and momentum. This formulation requires equations of motion for the Weyl tensor independently of the Einstein equations. They follow from the contracted Bianchi identities (Kundt & Trümper 1961). In terms of the Weyl and Ricci tensors, these are

\[ \nabla^\kappa C_{\mu\nu\kappa\lambda} = \nabla_{[\mu} R_{\nu]\lambda} + \frac{1}{6} g_{\lambda[\mu} \nabla_{\nu]} R^\kappa_\kappa \]  

(4)

Gradient symbols denote the covariant derivative with respect to \( g_{\mu\nu} \). Square brackets around a pair of indices denote antisymmetrization, e.g., \( A_{[\mu\nu]} \equiv \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu}) \). Symmetrized indices are surrounded by parentheses: \( A_{(\mu\nu)} \equiv \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu}) \). Substituting the Einstein equations (3) into the Bianchi identities (4) now provides field equations for the Weyl tensor:

\[ \nabla^\kappa C_{\mu\nu\kappa\lambda} = 8\pi G \left( \nabla_{[\mu} T_{\nu]\lambda} + \frac{1}{3} g_{\lambda[\mu} \nabla_{\nu]} T^\kappa_\kappa \right). \]  

(5)

Although the Weyl tensor has 256 components, only 10 of them are independent in four dimensions. It is convenient to incorporate these 10 components into two symmetric second rank tensors using the 4-velocity field \( u^\mu(x) \) to split the Weyl tensor as follows:

\[ E_{\mu\nu}(u) \equiv u^\kappa u^\lambda C_{\mu\kappa\nu\lambda}, \quad H_{\mu\nu}(u) \equiv \frac{1}{2} u^\kappa u^\lambda \epsilon_{\alpha\beta\kappa(\mu} C^{\alpha\beta}_{\nu)\lambda}. \]  

(6)
These tensors, called the electric and magnetic parts of the Weyl tensor, respectively, fully determine the Weyl tensor for any non-null normalized \( u^\mu \) (not just the 4-velocity of the matter). Indeed, the full Weyl tensor can be reconstructed from its electric and magnetic parts (Ellis 1971):

\[
C_{\mu\nu\kappa\lambda} = (g_{\mu\alpha\beta} g_{\kappa\lambda\delta} - \epsilon_{\mu\alpha\beta\kappa} \epsilon_{\lambda\gamma\delta}) u^\alpha u^\gamma E^{\beta\delta}(u) + (\epsilon_{\mu\alpha\beta\kappa} g_{\kappa\lambda\gamma\delta} + g_{\mu\alpha\beta\kappa} \epsilon_{\lambda\gamma\delta}) u^\alpha u^\gamma H^{\beta\delta}(u). \tag{7}
\]

Equation (7) is the exact inverse of equations (8) provided \( g_{\mu\nu} u^\mu u^\nu = \pm 1 \) and the same \( u^\mu \) is used in both equations. Ellis (1971) has a sign error in the first term of his version of equation (7) at the end of his section 4.3.3. Note that we have used the fully antisymmetric tensor \( \epsilon_{\mu\nu\kappa\lambda} = (-g)^{1/2} [\mu\nu\kappa\lambda] \), where \( g \) is the determinant of \( g_{\mu\nu} \) and \( [\mu\nu\kappa\lambda] \) is the completely antisymmetric Levi-Civita symbol defined by three conditions: (1) \([0123] = +1\), (2) \([123\mu\nu] = +1\) changes sign if any two indices are exchanged, and (3) \([\mu\nu\kappa\lambda] = 0\) if any two indices are equal. (Ellis uses the tensor \( \eta_{\mu\nu\kappa\lambda} = -\epsilon_{\mu\nu\kappa\lambda} \). We have compensated for the sign change in defining \( H_{\mu\nu} \).

The Weyl tensor is completely independent of the 4-velocity \( u^\mu \). Only the electric and magnetic parts depend on \( u^\mu \) — in particular, they are orthogonal to \( u^\mu \) where \( E_{\mu\nu} u^\nu = H_{\mu\nu} u^\nu = 0 \). Thus, in the local rest frame defined by \( u^\mu \), only the spatial components of \( E_{\mu\nu} \) and \( H_{\mu\nu} \) are non-vanishing. Moreover, these tensors are trace-free: \( E^\mu_\mu = H^\mu_\mu = 0 \). Each therefore has 5 independent components, accounting for the 10 independent components of the Weyl tensor.

Equation (7) yield equations of motion for the electric and magnetic parts of the Weyl tensor. The results depend, of course, on the energy-momentum tensor \( T_{\mu\nu} \). We assume that the matter is a perfect fluid for which \( T_{\mu\nu} = (\rho + p) u_\mu u_\nu + pg_{\mu\nu} \) with \( u^\mu \) being the fluid 4-velocity and \( \rho \) and \( p \) being the proper mass density and pressure, respectively, in the fluid frame. Using the same 4-velocity to split the Weyl tensor into its electric and magnetic parts (Ellis 1971):

\[
(\text{div}-E) : \quad P^\alpha_\mu P^\beta_\nu \nabla_\nu E^{\alpha\beta} + \epsilon^{\mu\nu\alpha\beta} u_\nu \sigma_{\alpha\gamma} H^{\gamma\beta} - 3H^\nu_\nu \omega^\nu = \frac{8\pi}{3} GP^{\mu\nu} \nabla_\nu \rho, \tag{8}
\]

\[
(\dot{H}) : \quad P^\mu_\alpha P^\nu_\beta \frac{DH^{\alpha\beta}}{d\lambda} - P^{\alpha(\mu} E^{\nu)\beta\gamma\delta} u_\beta \nabla_\gamma E_{\alpha\delta} - 2u_\alpha a_\beta E_\gamma (u^\nu)\alpha\beta\gamma + \Theta H^{\mu\nu}
+ P^{\mu\nu} (\sigma_{\alpha\beta} H_{\alpha\beta}) - 3H^{\alpha(\mu} \sigma^{\nu)} + H^{\alpha(\mu} \omega^{\nu)} = 0, \tag{9}
\]

\[
(\text{div}-H) : \quad P^\mu_\alpha P^\nu_\beta \nabla_\nu H^{\alpha\beta} - \epsilon^{\mu\nu\alpha\beta} u_\nu \sigma_{\alpha\gamma} E^{\gamma\beta} + 3E^\mu_\nu \omega^\nu = -8\pi p G(\rho + p) \omega^\mu, \tag{10}
\]

\[
(\dot{E}) : \quad P^\mu_\alpha P^\nu_\beta \frac{DE^{\alpha\beta}}{d\lambda} + P^{\alpha(\mu} E^{\nu)\beta\gamma\delta} u_\beta \nabla_\gamma H_{\alpha\delta} + 2u_\alpha a_\beta H_\gamma (u^\nu)\alpha\beta\gamma + \Theta E^{\mu\nu}
+ P^{\mu\nu} (\sigma_{\alpha\beta} E_{\alpha\beta}) - 3E^{\alpha(\mu} \sigma^{\nu)} + E^{\alpha(\mu} \omega^{\nu)} = -4\pi G(\rho + p) \sigma^{\mu\nu}. \tag{11}
\]

We have adopted Ellis’ names for these equations, inspired by their similarity to the Maxwell equations (Ellis 1971, 1973).
In writing equations (8)–(11) we have defined several new quantities. The derivatives of $E_{\alpha \beta}$ and $H_{\alpha \beta}$ are projected into the rest space of $u_{\nu}$ using the projection tensor $P_{\mu \nu} \equiv g_{\mu \nu} + u_{\mu}u_{\nu}$. The covariant derivative along the fluid worldline is $D/d\lambda \equiv u^\nu \nabla_\nu$. The velocity gradient has been decomposed into the acceleration 4-vector $a_{\nu}$, expansion scalar $\Theta$, shear tensor $\sigma_{\mu \nu}$, and vorticity tensor $\omega_{\mu \nu}$ or its dual $\omega^\mu_{[\mu \nu]}$:

\[
\nabla_\mu u_\nu = -u_\mu \frac{Du_\nu}{d\lambda} + P^\alpha_{\mu} P^\beta_{\nu} \nabla_\alpha u_\beta = -u_\mu a_\nu + \frac{1}{3} \Theta P_{\mu \nu} + \sigma_{\mu \nu} + \omega_{\mu \nu} ;
\]
\[
\Theta = \nabla_\mu u^\mu , \quad \sigma_{\mu \nu} = \sigma_{(\mu \nu)} , \quad \omega_{\mu \nu} = \omega_{[\mu \nu]} = \epsilon_{\mu \nu \alpha \beta} u^\alpha \omega^\beta .
\]

In the fluid rest frame, $\omega_i$ is half the usual three-dimensional vorticity $(\vec{\nabla} \times \vec{v})_i$. Ellis (1971) defines $\omega_{\mu \nu}$ and $\omega^\mu$ with the opposite sign.

Equations (9) and (11) represent Lagrangian evolution equations for the Weyl tensor: the time derivatives are taken along the fluid worldlines. Equations (8) and (10) are constraint equations. However, only the divergences of $E_{\mu \nu}$ and $H_{\mu \nu}$ are constrained; the “curl” parts (the gradient terms in eqs. [9] and [11]) may be specified arbitrarily on some initial hypersurface. This is in contrast with Newtonian gravity, where the gravitational field is fully determined from the matter distribution through the static Poisson equation. It is similar to electromagnetism in that the gravitational field may contain a source-free part corresponding (in the weak-field limit) to gravitational radiation.

Equations (8)–(11) are fully covariant tensor equations and can be applied in any coordinate system. Because of the key role played by the fluid 4-velocity, however, it is most convenient to evaluate them in the fluid rest frame. We shall assume that the matter is cold (pressureless) dust, in which case the 4-acceleration $a^\mu$ vanishes, so that the fluid rest frame is a locally inertial frame. Using locally flat coordinates in this frame, the equations of motion for the electric and magnetic parts of the Weyl tensor become

\[
\nabla_j E^j_i - \epsilon_{ijk} \sigma^{jl} H^k_l - 3 H_{ij} \omega^j = \frac{8\pi}{3} G \nabla_i \rho ,
\]
\[
\frac{dH_{ij}}{dt} + \nabla_k \epsilon^{kl} (i H)_{jl} + \Theta H_{ij} + \delta_{ij} \sigma^{kl} H_{kl} - 3 \sigma^k_{(i} H_{j)k} - \omega^k_{(i} H_{j)k} = 0 ,
\]
\[
\nabla_j H^j_i + \epsilon_{ijk} \sigma^{jl} E^k_l + 3 E_{ij} \omega^j = -8\pi G \rho \omega_j ,
\]
\[
\frac{dE_{ij}}{dt} - \nabla_k \epsilon^{kl} (i E)_{jl} + \Theta E_{ij} + \delta_{ij} \sigma^{kl} E_{kl} - 3 \sigma^k_{(i} E_{j)k} - \omega^k_{(i} E_{j)k} = -4\pi G \rho \sigma_{ij} .
\]

Note that these equations assume $u^\mu = (1, \vec{0})$ at the point where they are being applied because we are using locally flat coordinates in the fluid rest frame, but the gradient of the 3-velocity does not necessarily vanish. In section 4 we shall furnish a Newtonian derivation of these equations.
3. Weyl Tensor for a Perturbed Robertson-Walker Spacetime

As we saw in the preceding section, the electric and magnetic parts of the Weyl tensor are only partly constrained by the matter distribution. They are fixed, however, when the metric is specified. In this section we shall obtain the Weyl tensor components and its electric and magnetic parts for a perturbed Robertson-Walker spacetime.

We start with the following line element:

\[
ds^2 = a^2(\tau) \left\{ -(1 + 2\psi) d\tau^2 + 2w_i d\tau dx^i + [(1 - 2\phi)\gamma_{ij} + 2h_{ij}] dx^i dx^j \right\}, \quad \gamma^{ij} h_{ij} = 0. \quad (17)
\]

The perturbations \(\psi, \phi, w_i,\) and \(h_{ij}\) vanish for a Robertson-Walker spacetime with expansion scale factor \(a(\tau)\) (\(\tau\) is conformal time) and 3-metric \(\gamma_{ij}\) (the metric of a constant curvature space). We treat the perturbations as being small quantities.

As written, our metric is completely general. To reduce the gauge freedom we impose the following transversality constraints on \(w_i\) and \(h_{ij}\):

\[
\gamma^{ij} \nabla_i w_j = 0, \quad \gamma^{jk} \nabla_k h_{ij} = 0, \quad (18)
\]

where \(\nabla_i\) is the spatial covariant derivative relative to \(\gamma_{ij}\), whose inverse is \(\gamma^{ij}\). It can be shown that, with these conditions, \(\psi\) and \(\phi\) are identical to the gauge-invariant scalar mode variables \(\Phi_A\) and \(-\Phi_H\), respectively, of Bardeen (1980). Moreover, \(w_i\) corresponds to the “vector mode” (gravitomagnetism) and \(h_{ij}\) to the “tensor mode” (gravitational radiation). As discussed in detail by Bertschinger (1994), the gauge choice implied by equation (18) has several advantages over other choices such as the synchronous gauge. First, it is essentially unique — there is no residual gauge freedom associated with spatially inhomogeneous redefinitions of the coordinates. Second, by eliminating scalar mode contributions from \(w_i\) and \(h_{ij}\), and vector mode contributions from \(h_{ij}\), the physics of the modes is simplified. Finally, the perturbation variables are small provided that physical curvature perturbations are small, so that the coordinates \((\tau, x^i)\) are nearly identical to locally flat coordinates scaled by a homogeneous expansion factor. These last two advantages facilitate relating calculations in this gauge to the Newtonian limit (i.e., locally flat spacetime with slow source speeds relative to the comoving frame).

Before giving the Weyl tensor components, we first present the equations of motion for the metric perturbation variables obtained from the Einstein equations. These will provide intuition about the physics of the different types of perturbations (scalar, vector, tensor) and will facilitate comparison with the Newtonian limit.

The scalar and vector metric perturbation fields obey the following equations (Bertschinger 1993) derived from the Einstein equations in the limit of distance scales small
compared with the curvature and Hubble distances and with nonrelativistic shear stresses (the latter condition implying $\psi = \phi$):

$$\vec{\nabla} \cdot \vec{g} = -4\pi Ga^2 (\rho - \bar{\rho}) \ , \ \vec{\nabla} \times \vec{g} + \partial_\tau \bar{H} = 0 \ , \ \vec{\nabla} \cdot \bar{H} = 0 \ , \ \vec{\nabla} \times \bar{H} = -16\pi Ga^2 \vec{f}_\perp \ ,$$

where

$$\vec{g} \equiv -\vec{\nabla} \psi - \partial_\tau \vec{w} \ , \ \vec{H} \equiv \vec{\nabla} \times \vec{w} \ .$$

We use vector notation for three-dimensional vectors in the spatial hypersurfaces with metric $\gamma_{ij}$. Note that $\bar{\rho}(\tau)$ is the background density and $\vec{f}_\perp = (\rho \vec{v})_\perp$ is the transverse energy current (transversality implying $\vec{\nabla} \cdot \vec{f}_\perp = 0$) evaluated in the comoving frame (so that $\vec{v}$ is the peculiar velocity, $\vec{v} = d\vec{x}/d\tau$ to lowest order in the metric perturbations).

If one neglects $\vec{w}$, then equations (19)–(20) reduce to Newtonian gravity in comoving coordinates. Because the source for the gravitomagnetic field $\vec{H}$ is smaller by $O(v/c)$ than the source for Newtonian gravity, $\vec{H}$ usually is unimportant. Note that equations (19) are nearly identical with the Maxwell equations; the important difference is the absence of longitudinal current and a displacement current ($\partial_\tau \vec{g}$) term in the “Ampère” law for $\vec{\nabla} \times \vec{H}$. This difference implies that both $\vec{g}$ and $\vec{H}$ are essentially static fields without radiation.

The absence of gravitational radiation from $\vec{g}$ and $\vec{H}$ is what one expects for a spin-2 field (the graviton): radiation must be present only in the spatial tensor mode. Indeed, from the Einstein equations one can show that the tensor mode obeys the wave equation

$$\left(\partial_\tau^2 + 2\eta \partial_\tau - \nabla^2 + 2K\right) h_{ij} = 8\pi Ga^2 \Sigma_{ij,T} \ ,$$

where $\eta \equiv d\log a/d\log \tau$, $\nabla^2 = \gamma^{ij} \nabla_i \nabla_j$ is the spatial Laplacian, $K$ is the spatial curvature constant, and $\Sigma_{ij,T}$ is the proper transverse-traceless stress.

The field equations (19)–(21) show that for a fluid with velocity $\vec{v}$, $w_i = \psi \times O(v/c)$ and $h_{ij} = \psi \times O(v/c)^2$. These relations will be important when we compare with the Newtonian limit.

It is straightforward, though algebraically tedious, to compute the components of the Weyl tensor for the metric given above. The result is:

$$C^0_{\alpha i} = -\frac{1}{2} D_{ij}(\psi + \phi) - \frac{1}{2} \dot{W}_{ij} + \frac{1}{2}(\partial_\tau^2 + \nabla^2 - 2K) h_{ij} \ ,$$

$$C^0_{ijk} = (\nabla_k W_{ij} - \nabla_j W_{ik}) + \frac{1}{4}(\nabla^2 + 2K)(\gamma_{ij} w_k - \gamma_{ik} w_j) + (\nabla_j \dot{h}_{ik} - \nabla_k \dot{h}_{ij}) \ ,$$

$$C^i_{jkl} = \gamma^i_{jmn} \gamma^m_{kp} \left[ C^m_{0p0} + (\nabla^2 - 3K) h^m_p \right] + \gamma^i_{jmn} \gamma^p_{kl} \nabla_p \nabla^m h^m_q \ .$$
where $\gamma^{ij}$ is used to raise the components of $\nabla_i$ and $h_{ij}$ and we have defined several auxiliary quantities:

$$D_{ij} \equiv \nabla_i \nabla_j - \frac{1}{3} \gamma_{ij} \nabla^2, \quad W_{ij} \equiv \nabla (w_j), \quad \gamma_{ijkl} \equiv \delta^i_k \gamma_{jl} - \delta^i_l \gamma_{jk}.$$  

(25)

All other components of the Weyl tensor follow from the symmetry relations

$$C_{\mu\nu\kappa\lambda} = C_{[\mu\nu][\kappa\lambda]} = C_{\kappa\lambda\mu\nu}.$$  

(26)

Note that because we are retaining only first-order terms in the metric perturbations, the Weyl tensor components may be raised and lowered using the unperturbed metric. Thus, up to powers of the expansion factor, we may regard equations (22)–(24) as giving us the Weyl tensor components in locally flat coordinates.

The electric and magnetic parts of the Weyl tensor follow from equations (6). We first give them using the 4-velocity of comoving observers, $u^\mu = u_c^\mu = (a^{-1}, \vec{0})$ for the metric of equation (17). Note that because the Weyl tensor is already first-order in the metric perturbations, we need the 4-velocity only to zeroth-order. We obtain (cf. Bruni, Dunsby, & Ellis 1992, eqs. [113] and [115])

$$E_{ij}(u_c) = \frac{1}{2} D_{ij}(\psi + \phi) + \frac{1}{2} \dot{W}_{ij} - \frac{1}{2} \left( \partial^2 + \nabla^2 - 2K \right) h_{ij},$$

(27)

$$H_{ij}(u_c) = \frac{1}{2} \nabla (i) H_{j} + \nabla k \epsilon^{kl} (i) \dot{h}_{jl}.$$  

(28)

These components are given in the comoving coordinates of equation (17). To obtain the components in a locally flat coordinate system at rest relative to the comoving frame (to first order in the metric perturbations) one simply multiplies $E_{ij}$ and $H_{ij}$ by $a^{-2}$.

In the Newtonian limit, $\psi = \phi$ is the Newtonian potential and $E_{ij}$ is simply the gravitational tidal field (the traceless double gradient of the potential, with a factor of $a^{-2}$ required to convert the gradients from comoving to proper coordinates). The magnetic part of the Weyl tensor appears to have no Newtonian counterpart. It depends on gravitomagnetism and gravitational radiation, as do the corrections to the electric part.

Next we must obtain the electric and magnetic parts of the Weyl tensor in the fluid frame moving with 4-velocity $u^\mu$. Two steps are required for this computation. First we must evaluate $E_{\mu\nu}$ and $H_{\mu\nu}$ using $u^\mu = a^{-1}(\gamma, \vec{v})$ rather than $u_c^\mu$ in equation (6), where $\vec{v}$ is the peculiar velocity and $\gamma \equiv (1 - v^2)^{-1/2}$. Then we must transform from comoving coordinates to locally flat coordinates in the fluid rest frame. The result is

$$E'_{ij}(u) = a^{-2} \Lambda_{i}(\vec{v}) \Lambda_{j}(\vec{v}) u^k u^\lambda C_{\mu\nu\kappa\lambda}.$$  

(29)
and similarly for $H_{ij}$. The Weyl tensor may be computed from $E_{ij}(u_c)$ and $H_{ij}(u_c)$ using equation (9). A prime is used to indicate that the components are given in the transformed frame moving with velocity $\vec{v}$ relative to the unprimed (comoving) frame, and $\Lambda^k_i(\vec{v})$ is the Lorentz transformation corresponding to a boost $\vec{v}$: $\Lambda^0_0 = \gamma$, $\Lambda^0_i = \Lambda^i_0 = \gamma v^i$, $\Lambda^i_j = \delta^i_j + (\gamma - 1) v^i v_j/v^2$. We are allowed to use special relativity here because we are working in locally flat coordinates. [The factor $a^{-2}$ converts $E_{\mu\nu}(u)$ from comoving to locally flat coordinates as described in the previous paragraph.]

The two-stage transformation described above gives the following results:

$$E'_{ij} = E_{ij} + 2v_k\epsilon^{kl}(iH_j)_{l} + O(v/c)^2, \quad H'_{ij} = H_{ij} - 2v_k\epsilon^{kl}(iE_j)_{l} + O(v/c)^2,$$

where, in each frame (primed and unprimed) the components are evaluated using locally Minkowski coordinates. (To reduce the clutter we have dropped the arguments $u$ and $u_c$ from the primed and unprimed tensors, respectively.) Note that, although we began with a covariant definition of $E_{\mu\nu}$ and $H_{\mu\nu}$, here we are evaluating them in two different locally inertial frames such that in each frame the tensors are purely spacelike. Our results are reminiscent of the Lorentz transformation of the ordinary electric and magnetic fields. Ellis (1973) shows how the electric and magnetic fields may be defined as spacelike 4-vectors using the 4-velocity field to split the electromagnetic field strength tensor in a way similar to what we have done here for the Weyl tensor.

### 4. Newtonian Evolution of the Weyl Tensor

In the preceding sections we have derived the equations of motion for the electric and magnetic parts of the Weyl tensor using general relativity, and we have related these fields to the Newtonian gravitational potential and other metric perturbation fields of a perturbed Robertson-Walker spacetime. In this section we provide a Newtonian derivation of these results for cold dust. We will work in a perturbed Robertson-Walker spacetime, but a similar derivation can be carried through in a non-cosmological setting.

We begin with the continuity and Poisson equations in comoving coordinates (Bertschinger 1992):

$$\frac{\partial \delta}{\partial \tau} + \vec{\nabla} \cdot [(1 + \delta)\vec{v}] = 0, \quad \nabla^2 \phi = 4\pi G a^2 (\rho - \bar{\rho}).$$

The mass density is written $\rho = \bar{\rho}(\tau)(1 + \delta)$ and the peculiar velocity is $\vec{v} = d\vec{x}/d\tau$. Gradients are taken with respect to comoving coordinates. The cosmological Poisson
equation differs from its flat spacetime counterpart in that the mean background density is subtracted from the source. Equations (31) are valid to first order in $v/c$ on scales much smaller than the Hubble distance $cH^{-1}$. Note that these equations are valid even after orbits intersect provided that $\delta$ and $\vec{v}$ are taken to be the total density fluctuation and the average fluid velocity.

We define the Newtonian gravity vector in comoving coordinates,

$$\vec{g} \equiv -\vec{\nabla} \phi , \quad \vec{\nabla} \cdot \vec{g} = -4\pi G a^2 \bar{\rho} \delta . \quad (32)$$

This differs from equation (20) in that we neglect $\partial_\tau \vec{w}$. For slowly moving sources we expect $\partial_\tau \sim \vec{v} \cdot \vec{\nabla}$ so that the neglected contribution is $O(v/c)^2$ compared with the Newtonian gravitational field.

For convenience we will decompose the velocity gradient into trace and traceless symmetric and antisymmetric parts as in equation (12):

$$\nabla_i v_j = \frac{1}{3} \theta \delta_{ij} + \sigma_{ij} + \omega_{ij} . \quad (33)$$

This equation defines $E_{ij}$ in the Newtonian case; note that here, as in equation (27), we use the comoving components (Cartesian so that $\gamma_{ij}$ becomes $\delta_{ij}$), which differ by a factor $a^2(\tau)$ from the proper components. Equations (27) and (33) agree to $O(v/c)$.

Returning now to our derivation, we substitute $\vec{g}$ into the continuity equation to obtain

$$\vec{\nabla} \cdot \left( \frac{\partial a \vec{g}}{\partial \tau} \right) = 4\pi G a^3 \vec{f} , \quad \vec{f} \equiv \rho \vec{v} = \vec{f}_\parallel + \vec{f}_\perp . \quad (34)$$

We have decomposed the mass current in the comoving frame into longitudinal and transverse parts obeying $\vec{\nabla} \times \vec{f}_\parallel = 0$ and $\vec{\nabla} \cdot \vec{f}_\perp = 0$. Integrating equation (34) yields

$$\frac{\partial \vec{g}}{\partial \tau} + \frac{\dot{a}}{a} \vec{g} = 4\pi G a^2 \vec{f}_\parallel . \quad (35)$$

The source involves only the longitudinal mass current because the Newtonian gravity vector is longitudinal. We rewrite equation (35) using the Lagrangian derivative following a fluid element, $d/d\tau = \partial/\partial \tau + \vec{v} \cdot \vec{\nabla}$:

$$\frac{d\vec{g}}{d\tau} + \frac{\dot{a}}{a} \vec{g} = \vec{v} \cdot \vec{\nabla} \vec{g} + 4\pi G a^2 \vec{f}_\parallel . \quad (36)$$
Before proceeding further we replace the transverse mass current by a transverse vector field $\vec{H}$ obeying the field equations
\[ \nabla \times \vec{H} = -16\pi G a^2 \vec{f}_\perp , \quad \nabla \cdot \vec{H} = 0 . \tag{37} \]
We recognize this as the gravitomagnetic field of equations (19) and (20). However, in the Newtonian case we imply no relation between $\vec{H}$ and spacetime metric perturbations; we simply regard $\vec{H}$ as a dynamical field related to $\vec{f}_\perp$.

We further define a traceless tensor
\[ H_{ij} \equiv -\frac{1}{2} \nabla_j H^j_i + 2v_k \epsilon^{kl} \nabla_j g_{kl} = H_{ij} + \epsilon_{ijk} A^k , \tag{38} \]
which we have decomposed into a symmetric part
\[ H_{ij} = -\frac{1}{2} \nabla (i H_j) - 2 v_k \epsilon^{kl} (i E_j)_l \tag{39} \]
and an antisymmetric part with dual
\[ A_i = \frac{2}{3} v_i \nabla^2 \phi - E_{ij} v^j + \frac{1}{4} \left( \nabla \times \vec{H} \right)_i . \tag{40} \]

To first order in $v/c$, equation (39) agrees exactly with the magnetic part of the Weyl tensor in the fluid frame (eqs. [28] and [30]).

For reference we provide some useful identities following from the definitions of equations (32)–(33) and (37)–(40). First are two differential identities for the divergence and curl of $A$:
\[ \nabla_i A^i = \frac{2}{3} \theta \nabla^2 \phi - \sigma^{ij} E_{ij} , \tag{41} \]
\[ \epsilon^{kl} \nabla_k A_l = -8\pi G a^2 \bar{\rho} \left( 1 + \frac{1}{3} \delta \right) \omega_i - \epsilon_{ijk} \sigma^{jl} E_{ij}^k + E_{ij} \omega^j . \tag{42} \]

Next are two differential identities for $H_{ij}$. The first is
\[ \nabla_j H^j_i + \epsilon_{ijk} \sigma^{jl} E_{ij}^k + 3 E_{ij} \omega^j = -8\pi G a^2 \bar{\rho} \omega_j . \tag{43} \]

Aside from the factor $a^2$ present on the right-hand side because we are using comoving rather than proper coordinates, this result agrees exactly with equation (15). The last identity will be useful in deriving equation (16):
\[ \epsilon^{kl} \nabla_k H_{ij} = -\nabla_j A_i + \frac{2}{3} \left( \nabla^2 \phi \right) \nabla_j v_i + \frac{2}{3} \theta E_{ij} + \delta_{ij} \sigma^{kl} E_{kl} - 4 \sigma_{(i} E_{j)k} - 4 \omega_{[i} E_{j]k} . \tag{44} \]
To derive equation (16) we first substitute equation (40) into equation (37) and use equations (32) and (36) to get
\[
\frac{d\vec{g}}{d\tau} + \frac{\dot{a}}{a} \vec{g} = 4\pi G a^2 \bar{\rho} \vec{v} + \vec{A}.
\] (45)
Taking the gradient gives
\[
\frac{d}{d\tau} \nabla_i g_{j} + \frac{\dot{a}}{a} \nabla_i g_{j} + (\nabla_i v^k)(\nabla_k g_{j}) = 4\pi G a^2 \bar{\rho} \nabla_i v_j + \nabla_i A_j.
\] (46)
The traceless symmetric part of this equation is
\[
\frac{dE_{ij}}{d\tau} + \frac{\dot{a}}{a} E_{ij} + \frac{1}{3} \theta E_{ij} + \frac{2}{3} \sigma_{ij} \nabla^2 \phi + \sigma^{k}(i E_{j})_k - \frac{1}{3} \delta_{ij} \sigma^{kl} E_{kl} - \omega^{k}(i E_{j})_k =
\]
\[
- 4\pi G a^2 \bar{\rho} \sigma_{ij} - \nabla_{(i} A_{j)} + \frac{1}{3} \delta_{ij} \nabla_k \Lambda^k.
\] (47)
The trace and traceless antisymmetric parts of equation (46) give no further information because they simply reproduce the divergence and curl of $\vec{A}$ that are given above.

Equation (47) involves the symmetrized gradient of the antisymmetric part of $H_{ij}$ defined by equation (38), which can be replaced in favor of the antisymmetrized gradient of the symmetric part $H_{ij}$ using equations (41) and (44). The necessary link is the identity
\[
\nabla(A_j) - \frac{1}{3} \delta_{ij} \nabla^k A_k = -\nabla_k \varepsilon^{kl}(H_j)_l + \frac{2}{3} \theta E_{ij} + \frac{2}{3} \sigma_{ij} \nabla^2 \phi - 4\sigma^k(i E_{j})_k + \frac{4}{3} \delta_{ij} \sigma^{kl} E_{kl}. \] (48)
When this is substituted into equation (47) we obtain
\[
\frac{dE_{ij}}{d\tau} + \frac{\dot{a}}{a} E_{ij} - \nabla_k \varepsilon^{kl}(H_j)_l + \theta E_{ij} + \delta_{ij} \sigma^{kl} E_{kl} - 3\sigma^{k}(i E_{j})_k - \omega^{k}(i E_{j})_k = -4\pi G a^2 \bar{\rho} \sigma_{ij}. \] (49)
If the variables are converted to proper coordinates, $dt = ad\tau$, $E_{ij} \rightarrow a^2 E_{ij}$, etc., and one recalls that the comoving expansion scalar is $\theta = a\Theta - 3\dot{a}/a$, one obtains equation (16).

We have succeeded in deriving the div-$H$ and $\dot{E}$ equations from Newton’s laws; what about the other two field equations for the Weyl tensor? Care is required because these equations involve terms $O(v/c)^2$, as we can see by the following argument. From their definitions, we see that the units of $H_{ij}$ and $E_{ij}$ differ by one power of a velocity: $H_{ij} \sim v E_{ij}$. From their respective field equations, one sees that $v$ must be of order the matter peculiar velocity. Dimensional analysis of equations (13) and (14) shows that the $H_{ij}$ terms must be divided by $c^2$ and that they are $O(v/c)^2$ relative to the other terms. (The same analysis applied to eqs. (15) and (16) shows that all terms are of the same order.) In the strictly Newtonian limit one would drop terms $O(v/c)^2$; for example, equations (13) neglect relativistic corrections of this order.
Indeed, equations (13) and (14) are satisfied in the Newtonian limit, as one can see by evaluating the divergence and curl of $E_{ij}$ from equation (33):

$$\nabla_j E^j_i = \frac{2}{3} \nabla_i \nabla^2 \phi , \quad \nabla_k \epsilon^{kl} (i E_j)_{il} = 0 . \tag{50}$$

Using the Poisson equation for $\nabla^2 \phi$, one obtains

$$\nabla_j E^j_i = \frac{8\pi}{3} G a^2 \nabla_i \rho . \tag{51}$$

Aside from the use of comoving coordinates, this equation and the second of equations (50) agree with equations (13) and (14) in the Newtonian limit.

This derivation is not fully satisfactory because we have not accounted for all the terms in equations (13) and (14). Fortunately, with a little care we can derive the full equations.

First, we must use the $E_{ij}$ and $H_{ij}$ obtained using general relativity rather than Newtonian gravity, although we shall make the simplifying assumptions that gravitational radiation and the gravitational effect of shear stress, as well as terms that are explicitly $O(v/c)^2$, can be neglected. Using equations (27), (28), and (30), in comoving coordinates we obtain the results

$$E_{ij} = \left( \nabla_i \nabla_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) \phi + \frac{1}{2} \dot{W}_{ij} + 2v_k \epsilon^{kl} (i H_j)^{(0)} , \quad H_{ij} = -\frac{1}{2} \nabla_i H^j - 2v_k \epsilon^{kl} (i E_j)^{(0)} , \tag{52}$$

where superscript $(0)$ indicates the quantity is to be evaluated setting $v_k = 0$. We must include in equations (52) terms that are explicitly first-order in $v_k$, even though we will evaluate the evolution equations in the fluid rest frame for comparison with equations (13) and (14), because we will need spatial derivatives of $E_{ij}$ and $H_{ij}$. This fact shows that we can, however, safely ignore terms that are explicitly quadratic in $v_k$.

The divergence of $E_{ij}$ from equation (52) gives

$$\nabla_j E^j_i = \frac{8\pi}{3} G a^2 \nabla_i \rho + \frac{1}{4} \nabla^2 \ddot{w}_i + \epsilon_{ijk} \sigma^{jl} H^{kl} + 3H_{ij} \omega^j , \tag{53}$$

where we have retained only the terms that do not vanish when $v_k = 0$. If we can show that $\nabla^2 \ddot{w}_i = \partial_t (16\pi G a^2 f_{\perp i})$ vanishes in the fluid frame, then equation (53) implies equation (13). We can show this using the following trick. In the fluid frame, $\vec{f} = \vec{f}_{\parallel} + \vec{f}_{\perp} = 0$, by definition. Also, the fluid frame is freely-falling, so that the fluid acceleration measured at the fluid element must vanish. Equation (36) then yields $\vec{f}_{\parallel} = 0$, hence $\vec{f}_{\perp} = 0$ and therefore $\nabla^2 \ddot{w}_i = 0$. Since this result holds for all time (the fluid continues to remain at rest in this inertial frame), $\nabla^2 \ddot{w}_i = 0$. Equation (13) then follows.
To derive the $\dot{H}$ equation we take the curl of $E_{ij}$ from equation (52), again discarding terms that do not vanish when $v_k = 0$

$$\nabla_k \epsilon^{kl} (i E_{ij}) = \frac{1}{2} \nabla (i \dot{H}_{ij}) - \theta H_{ij} - \delta_{ij} \sigma^{kl} H_{kl} + 3 \sigma^{k} (i H_{jk}) + \omega^{k} (i H_{jk}) .$$  (54)

In evaluating the time derivative term we must be careful to divide the comoving components $H_j$ used here by $a$ to obtain the proper components. Then, using the fact that in the fluid frame $v_k = dv_k/d\tau = 0$, we obtain $dH_{ij}/d\tau = \partial_\tau H_{ij} = -(\partial_\tau - \dot{a}/a) \frac{1}{2} \nabla (i H_j)$. Equation (54) then reduces to

$$\frac{dH_{ij}}{d\tau} + \frac{\dot{a}}{a} H_{ij} + \nabla_k \epsilon^{kl} (i E_{ij}) + \theta H_{ij} + \delta_{ij} \sigma^{kl} H_{kl} - 3 \sigma^{k} (i H_{jk}) - \omega^{k} (i H_{jk}) = 0 .$$  (55)

Converting to proper coordinates and using the relation $\theta = a\Theta - 3 \dot{a}/a$, we recover equation (14).

It is interesting to note that in the Newtonian derivation of the $H$-dot equation (14) — but not the $E$-dot equation (16) — we had to assume vanishing acceleration in the fluid frame. The reason is clear from the relativistic equations (9) and (11) retaining the 4-acceleration. By dimensional analysis, the 4-acceleration term in equation (11) must be divided by $c^2$ compared with the term in equation (9). Thus, acceleration effects are unimportant in the Newtonian tidal evolution equation but are important in the evolution of $H_{ij}$.

5. Conclusions

We have shown the magnetic part of the Weyl tensor $H_{ij}$ does not necessarily vanish in the Newtonian limit by deriving an explicit expression for it — equation (39). As a consequence, the Lagrangian evolution of the tidal field (the electric part of the Weyl tensor, $E_{ij}$) is not purely local: it involves the gradient of $H_{ij}$ (eq. [16]). This term is of the same order in powers of $v/c$ as the other terms affecting the tidal evolution. No ambiguity remains because we have obtained identical results using both general relativity and Newtonian gravity.

It may be surprising that magnetic-like effects are significant for the evolution of the tidal field. After all, the Newtonian limit neglects the gravitomagnetic force $m \vec{v} \times \vec{H}$ relative to the gravitoelectric force $m \vec{g}$. However, changes in the tidal field are dependent on the motion of the fluid. As in the case of electromagnetism, electric fields in one frame
transform into magnetic fields in another frame. Moreover, even if we choose a frame so that $H_{ij} = 0$ (not necessarily the fluid frame), this condition is insufficient to restore locality because the evolution of $E_{ij}$ depends on the gradient of $H_{ij}$. Given the complexity of the equations, at this time we cannot provide more physical insight into our results. However, it is clear that they are implied by mass conservation and Newtonian gravity rather than some subtle relativistic effects.

Although we have shown that the tidal evolution is, in general, nonlocal, the Lagrangian fluid method still can be applied to study the nonlinear evolution of self-gravitating mass. One will simply have to evolve both $E_{ij}$ and $H_{ij}$ and compute their gradients. Since this requires knowing the positions of mass elements, the trajectories will have to be integrated simultaneously. Although this greatly complicates matters, there may be advantages in being able to compute the density and velocity gradients of individual mass elements treated as a fluid rather than by summing over particles weighted with a smoothing kernel. After trajectories intersect, however, the Lagrangian fluid method becomes substantially more complicated.

Another possibility is that an alternative approximation may prove useful in restoring locality. Since we have obtained an explicit Newtonian expression for $H_{ij}$, one can test alternative local approximations for its gradient. Of course, we still have not shown how bad an approximation it is to neglect $H_{ij}$ altogether. This can only be done by comparing with other approximations or N-body simulations. This work is currently underway.

Our results do not resolve the question of whether nonlinear gravitational collapse favors the formation of prolate filaments as opposed to oblate pancakes. This question hinges critically on the behavior of $H_{ij}$ during gravitational collapse. We leave this subject for future work.

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REFERENCES

Bardeen, J. M. 1980, Phys. Rev., D22, 1882
Barnes, A., & Rowlandson, R. R. 1989, Class. Quantum Grav., 6, 949
Bertschinger, E. 1992, in New Insights into the Universe, ed. V. J. Martinez, M. Portilla and D. Saez (New York: Springer-Verlag), 65
Bertschinger, E. 1993, in Statistical Description of Transport in Plasma, Astro-, and Nuclear Physics, ed. J. Misguich, G. Pelletier & P. Schuck, (Editions Frontières: Gif sur Yvette), in press
Bertschinger, E. 1994, Les Houches lecture notes
Bertschinger, E., & Jain, B. 1994, ApJ, 431, in press
Bruni, M., Dunsby, P. K. S., & Ellis, G. F. R. 1992, ApJ, 395, 34
Croudace, K. M., Parry, J., Salopek, D. S., & Stewart, J. M. 1994, ApJ, 423, 22
Ellis, G. F. R. 1971, in General Relativity and Cosmology, ed. R. K. Sachs (New York: Academic Press), 104
Ellis, G. F. R. 1973, in Cargèse Lectures in Physics, vol. 6, ed. E. Schatzman (New York: Gordon and Breach), 1
Ellis, G. F. R., & Bruni, M. 1989, Phys. Rev., D40, 1804
Hawking, S. W. 1966, ApJ, 145, 544
Hwang, J.-C., & Vishniac, E. T. 1990, ApJ, 353, 1
Kundt, W., & Trümper, M. 1961, Akad. Wiss. Lit. Mainz Abh. Math.-Nat. Kl., 12
Matarrese, S., Pantano, O., & Saez, D. 1993, Phys. Rev., D47, 1311
Matarrese, S., Pantano, O., & Saez, D. 1994, Phys. Rev. Lett., 72, 320
Misner, C. W., Thorne, K. S. & Wheeler, J. A. 1973, Gravitation (San Francisco: W. H. Freeman)
Zel’dovich, Ya. B. 1970, A&A, 5, 84

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