Optical Flow Fields: Dense Correspondence Fields for Highly Accurate Large Displacement Optical Flow Estimation

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Abstract—Modern large displacement optical flow algorithms usually use an initialization by either sparse descriptor matching techniques or dense approximate nearest neighbor fields. While the latter have the advantage of being dense, they have the major disadvantage of being very outlier-prone as they are not designed to find the optical flow, but the visually most similar correspondence. In this article we present a dense correspondence field approach that is much less outlier-prone and thus much better suited for optical flow estimation than approximate nearest neighbor fields. Our approach does not require explicit regularization, smoothing (like median filtering) or a new data term. Instead we solely rely on patch matching techniques and a novel multi-scale matching strategy. We also present enhancements for outlier filtering. We show that our approach is better suited for large displacement optical flow estimation than modern descriptor matching techniques. We do so by initializing EpicFlow with our approach instead of their originally used state-of-the-art descriptor matching technique. We significantly outperform the original EpicFlow on MPI-Sintel, KITTI 2012, KITTI 2015 and Middlebury. In this extended article of our former conference publication we further improve our approach in matching accuracy as well as runtime and present more experiments and insights.

Index Terms—optical flow, dense matching, correspondence fields.

1 INTRODUCTION

Finding the correct dense optical flow between images or video frames is a challenging problem. While the visual similarity between two image regions is the most important clue for finding the optical flow, it is often unreliable due to illumination changes, deformations, repetitive patterns, low texture, occlusions or blur. Hence, basically all dense optical flow methods add prior knowledge about the properties of the flow, like local smoothness assumptions [1], structure and motion adaptive assumptions [2], the assumption that motion discontinuities are more likely at image edges [3], or the assumption that the optical flow can be approximated by a few motion patterns [4]. The most popular of these assumptions is the local smoothness assumption. It is usually incorporated into a joint energy based regularization that rates data consistency together with the smoothness in a variational setting of the flow [1]. One major drawback of this setting is that fast minimization techniques usually rely on local linearization of the data term and thus can adapt the motion field only very locally. Hence, these methods have to use image pyramids to deal with fast motions (large displacements) [5]. In practice, this fails in cases where the determined motion on a coarser scale is not very close to the correct motion of a finer scale.

In contrast, for purely data based techniques like approximate nearest neighbor fields [6] (ANNF) and sparse descriptor matches [7] there are fast approaches that can efficiently perform a global search for the best match on the full image resolution. However, as there is no regularization, (approximate) nearest neighbor fields (NNF) usually contain many outliers that are difficult to identify. Furthermore, even if outliers can be identified they leave gaps in the motion field that must be filled. Sparse descriptor matches usually contain fewer outliers as matches are only determined for carefully selected points with high confidence. However, due to their sparsity the gaps between matches are usually even larger than in outlier filtered ANNF. Gaps are problematic, since a motion for which no match is found cannot be considered. Despite these difficulties, ANNF and sparse descriptor matches gained a lot of popularity as initial step of large displacement optical flow algorithms.
Nowadays, most top-performing methods on challenging datasets like MPI-Sintel [8] rely on such techniques. However, although most pixel-dense approaches use powerful patch matching [9] techniques like propagation and random search, conventional patch matching approaches are tailored to find the ANNF. This is suboptimal for optical flow estimation. The intention behind ANNF is to find the visually closest match (NNF), which is often not identical to the optical flow. An important difference is that NNF are known to be very noisy regarding the offset of neighboring pixels, while optical flow is usually locally smooth and occasionally abrupt (see Figure 1).

In this article we show that it is possible to create dense correspondence fields that contain significantly fewer outliers than ANNF regarding optical flow estimation – not because of explicit regularization, smoothing (like median filtering) or a different data term, but by using patch matching techniques in a novel way that is more favorable for optical flow estimation and by combining it with our novel multi-scale matching strategy. We call our correspondence fields Optical Flow Fields (short: Flow Fields) as they are tailored for optical flow estimation, while they are at the same time dense and purely data term based like ANNF.

Our main contributions are:

• A novel multi-scale correspondence field matching strategy that features powerful non-locality in the image space (see Figure 7 a), but locality in the flow space (for smoothness) and can utilize scales as effective outlier sieves. It allows to obtain better results with scales than without, even for tiny objects and other details.

• We extend the common forward backward consistency check by a novel two way consistency check as well as region and density based outlier filtering.

• We show the effectiveness of our approach by clearly outperforming ANNF and by obtaining competitive results on MPI-Sintel [8], KITTI 2012 [10] and 2015 [11].

• Several experiments to analyze our approach.

In this extended article we also present improved versions of our conference approach [12], that are much more accurate (Flow Fields+) or more accurate and at the same time much faster (Flow Fields+ Fast) than our conference version. We also present additional experiments and insights.

2 Related Work

Dense optical flow research started more than 30 years ago with the work of Horn and Schunck [1]. We refer to publications like [13], [14], [15] for a detailed overview of optical flow methods and the general principles behind it.

One of the first works that integrated sparse descriptor matching for improved large displacement performance was Brox and Malik [16]. Since then, several works followed the idea of using (sparse) descriptors [3], [17], [18], [19], [20], while few works used dense ANNF instead [4], [21]. Chen et al. [4] showed that remarkable results can be achieved on the Middlebury evaluation portal by extracting the dominant motion patterns from ANNF. Revaud et al. [3] compared ANNF to Deep Matching [18] for the initialization of their approach, called EpicFlow. They found that Deep Matching clearly outperforms ANNF. We will use their approach for optical flow estimation and show that this is not the case for our approach. Deep Matching is a semi-dense descriptor matching technique tailored for optical flow that does not use patch matching techniques like our approach.

An important milestone regarding fast ANNF estimation was PatchMatch [9]. Nowadays, there are even faster ANNF approaches [6], [22]. There are also approaches that try to obtain correspondence fields tailored to optical flow. Lu et al. [23] used superpixels to gain edge aware correspondence fields. Bao et al. [24] used an edge aware bilateral data term instead. While the edge aware data term helps them to obtain good results – especially at motion boundaries, their approach is still based on the ANNF strategy to determine correspondences, although it is unfavorable for optical flow. HaCohen et al. [25] presented a multi-scale correspondence field approach for image enhancement. While it does well in removing outliers, it also removes inliers that are not supported by a large neighborhood (in each scale). Such inliers are especially important for optical flow as they cannot be determined by the classical coarse to fine strategy. Our approach cannot only preserve such isolated inliers, but can also spread them if needed (Figure 7 a)).

A technique that shares the idea of preferring locality (to avoid outliers) with our approach is region growing in 3D reconstruction [26], [27]. It is usually computationally expensive. A faster GPU parallelizable alternative for region growing based on PatchMatch [9] was presented in our previous work [28]. It shares some ideas with our basic approach in Section 3.1, but was not designed for optical flow estimation and lacks many important aspects of our approach in this paper.

Recently, Hu et al. [29] improved the runtime of our multi-scale matching strategy [12] by not performing bi-linear interpolation and by not considering every pixel in propagation. This improves runtime speed at the cost of accuracy. Furthermore, we recently created a CNN based data term [30] for our Flow Fields approach.

3 Our Approach

In this section we detail our Flow Fields approach, our extended outlier filter and the data terms used in the tests of our paper. The idea of our approach is described in two steps. First we introduce a basic (single-scale) Flow Fields approach in Section 3.1. Then we build our full multi-scale Flow Fields approach on top of it in Section 3.2. This approach we also call conference approach, as it was already presented in the conference version of this article [12]. In addition, we present in this extended article improved versions of our approach called Flow Fields+ in Section 3.3 and a faster version of Flow Fields+ called Flow Fields+ Fast in Section 3.4.

Given two images $I_1, I_2 \subset \mathbb{R}^2$ we use the following notation: $P_r(p_i)$ is an image patch with patch radius $r$ centered at a pixel position $p_i = (x, y), i \in I_i i = 1, 2$. The total size of our rectangular patch is $(2r + 1) \times (2r + 1)$ pixels. Our goal is to determine the optical flow field of $I_1$ with respect to $I_2$ i.e. the displacement field for all pixels $p_1 \in I_1$, denoted by $F(p_1) = M(p_1) - p_1 \in \mathbb{R}^2$ for each pixel
The first step of our basic approach is similar to the kd-tree based initialization step of the ANNF approach of He and Sun [6]. We do not use any other step of [6] as we have found them to be harmful for optical flow estimation, since they introduce resistant outliers, whose matching errors are below those of the ground truth. Once introduced, a purely data based approach without regularization cannot remove them anymore. Hence, the secret is to avoid finding them. ANNF approaches try to reproduce the NNF that contains all resistant outliers, but due to their approximate nature they fail doing so – which is beneficial for optical flow estimation. In our (basic) approach we want to reinforce this property even more to find even less resistant outliers, while still keeping track of inliers.

Our approach, outlined in Figure 2, works as follows: First we calculate the Walsh-Hadamard Transform (WHT) [31] for all patches \( P_r(p_2) \) centered at all pixel positions \( p_2 \) in image \( I_2 \) similar to [6]. In contrast to them we use the first 9 bases for all three color channels in the CIELab color space. The resulting 27 dimensional vectors for each pixel are then sorted into a kd-tree with leaf size \( l \). We also split the tree in the dimension of the maximal spread by the median value. After building the kd-tree we create WHT vectors for all patches \( P_r(p_1) \) at all pixel positions in image \( I_1 \) as well and search the corresponding leaf within the kd-tree (where it would belong to if we would add it to the tree). All \( l \) entries \( L \) in the leaf found by the vector of the patch \( P_r(p_1) \) are considered as candidates for the initial flow field \( F(p_1) \). To determine which of them is the best we calculate their matching errors \( E_d \) with a robust data term \( d \) (see Section 3.5), and only keep the candidate with the lowest matching error in the initial Flow Field, i.e.

\[
F(p_1) = \arg \min_{p_2 \in L} (E_d(P_r(p_1), P_r(p_2))) - p_1.
\]

This is similar to reranking in [6]. We call points in the initial flow field arising directly from the kd-tree seeds. Larger \( l \) increase the probability that both correct seeds and resistant outliers are found. However, if both are found at a position the resistant outlier prevails. Thus, it is advisable to keep \( l \) small and to utilize the local smoothness of optical flow to propagate rare correct seeds in the initial flow field into many surrounding pixels – outliers usually fail in this regard as their surrounding does not form a smooth surface. The propagation of our initial flow values works similar to the propagation step in the PatchMatch approach [9] i.e. flow values are propagated from position \((x, y - 1)_1\) and \((x - 1, y)_1\) to position \( p_1 = (x, y)_1 \) as follows:

\[
F(p_1) = \arg \min_{p_2 \in G_1} (E_d(P_r(p_1), P_r(p_2))) - p_1
\]

\[
G_1 = \{F((x, y - 1)_1), F((x - 1, y)_1)\} + p_1
\]

(2)

\( G_1 \) are the considered flows for our first propagation step. It is important to process positions \((x, y - 1)_1\) and \((x - 1, y)_1\) with Equation 2 before position \((x, y)_1\) is processed. This allows the propagation approach to propagate into arbitrary directions within a 90 degree angle (see Figure 3 a)). As optical flow varies between neighboring pixels, but propagation can only propagate existing flow values our next step is a random search step. Here, we modify the flow of each pixel \( p_1 \) by a random uniformly distributed offset \( O_{rnd} \) of at most \( R \) pixels. If the matching error \( E \) decreases we replace the flow \( F \) by the new flow \( F + O_{rnd} \). \( O_{rnd} \) is a subpixel accurate offset which leads to subpixel accurate positions \( M(p_1) \). The pixel colors of \( M(p_1) \) and \( P_r(M(p_1)) \) are determined by bilinear interpolation. Early subpixel accuracy not only improves overall accuracy, but also helps to avoid outliers as subpixel accurate matches have a smaller matching error.

In total we perform alternately 4 propagation and 3 random search steps (all with the same \( R \)) as shown in Figure 2. While the first propagation step is performed to the right and bottom, the subsequent three propagation steps are performed into the directions shown in Figure 3 c). Many approaches that perform propagation (e.g. [6]) do not consider different propagation directions. Even the original

1. For WHTs patches must be split in the middle. We found that the matching quality does not suffer from splitting uneven patches with size \((2r + 1)\) into patches of size \( r \) and \( r + 1 \).
PatchMatch approach only considers the first two directions. While these already include all 4 main directions, we have to consider that propagation actually can propagate into all directions within a quadrant (see Figure 3) and that there are 4 quadrants in the full 360 degree range.

Extensive propagation with random search (which we call spreading) is important to distribute rare correct seeds into the whole Flow Field. The locality of spreading (with small $R$) prevents the flow field from introducing new outliers not existing in the initial flow field (see Figure 4).

### 3.2 Flow Fields

Our basic Flow Fields still contain many resistant outliers arising from kd-tree initialization. We can further reduce their amount (and the amount of initial inliers) by not determining an initial flow value for each pixel. This helps as inliers usually propagate much further than outliers (optical flow is smooth, outliers are usually not). However, to cover the larger flow variations between fewer inliers (that are further apart from each other) the random search distance $R$ must be increased, which raises the danger of adding close by resistant outliers. A way to avoid this is to increase the patch influence area as well, either by raising $r$ or by determining the optical flow on a downsampled image. This helps for instance in the presence of repetitive patterns or poorly textured regions, but creates new failure cases e.g. close to motion discontinuities and for small objects. Furthermore, a larger influence area and larger $R$ leads to less accurate matches.

Our solution (outlined in Figure 5) avoids most of the disadvantages of large influence areas while being even more robust: First we define that $P^p_r(p_i)$ is a subsampled patch at pixel position $p_i$ with patch radius $r+n$ that consists of only each $n$th pixel within its radius including the center pixel, i.e. (see Figure 3) for an illustration):

$$\begin{align*}
(x^*, y^*) \in P^p_r((x, y)) & \Rightarrow \frac{|x^* - x|}{n} \mod n = 0 \\
\frac{|y^* - y|}{n} \mod n = 0
\end{align*}$$

The pixel colors for $P^p_r(p_i)$ are not determined from image $I_i$, but from a low-pass filtered version of $I_i$ that we call $I^p_r$, i.e. we use scale-spaces [32]. While scale-spaces are similar to using image pyramids and using $P_r$ on a $n$ times downsampled image, scale-spaces have the advantage that we can perform high-quality interpolation at low computational cost up to pixel accuracy in the full image resolution. Furthermore, $p_i$ is an actual pixel position on the full resolution, which prevents upsampling errors. Our low-pass filtering approach to obtain $I^p_r$ is described in Section 3.6.

We always start with $n = 2^k$. Our full Flow Fields approach first initializes only each $n$th pixels $p^i_1 = (x_n, y_n)_1$ with $x_n \mod n = 0$ and $y_n \mod n = 0$ (see Figure 5).

Initialization is performed similar to the basic approach: $$F(p^i_1) = \arg \min_{p_2 \in L} \left( E_d(P^p_r(p^i_1), P^p_r(p_2)) \right) - p^i_1 \quad (4)$$

Note that the kd-tree samples $L$ are identical to those of the basic approach. We still use non-subsampled patches $P_r(p_i)$ for the WHT vectors for an accurate initialization.

After initialization we perform propagation and random search similar to the basic approach. Except that we only propagate between points $p^i_m$ i.e. $(x_n - n, y_n)_1, (x_n, y_n - n)_1 \rightarrow (x_n, y_n)_1$ etc. (see Figure 5) and that we use $R = R \times n$ as maximum random search distance. After determining $F(p^i_1)$ using patches $P^m_r$, we determine $F(p^i_m), m = 2^k - 1$ in the same way using patches $P^m_r$. Hereby, the samples $F(p^i_m)$ are used as seeds instead of kd-tree samples. Positions $p^i_m$ that are not part of $p^i_1$ receive an initial flow value in the first propagation step of the scale $k - 1$. This approach is repeated up to the full resolution $F(p^i_1) = F(p_1)$ (see Figure 2 and 5).

As demonstrated in Figure 4 our spreading (propagation + random search) is usually too local to introduce new (resistant) outliers. On the other hand, spreading of finer scales is likely to remove outliers persisting in coarser scales, since resistant outliers are often not resistant on all scales. This is due to the fact that matching error minimas are different on different scales. Formally: If $G_n$ = $\arg \min_{p_2} E_d(P^p_r(p_1), P^p_r(p_2))$ is the global minimum match at scale $n$ then we cannot imply that it is the minimum for a different scale as well i.e. $G_n1 = p_2 \Rightarrow G_n2 = p_2$. As a result, scales serve as a kind of outlier sieve. The outlier sieve effect can be seen in Figure 6.

In contrast to ordinary multi-scale approaches, our approach is non-local in the image space. Figure 7 a) demonstrates how powerful this non-locality is. The flow field is only initialized by two flow values with a flow offset of 52 pixels to each other (Figure 7 b)). This is more than the random search step of all scales together can traverse. Thus, the orange flow is a propagation barrier for the violet flow (Like gray pixels in Figure 3 a)). Anyhow, our approach manages to distribute the violet flow and similar flows determined by random search throughout the whole image. We originally performed the experiment to prove that the

Fig. 5. Illustration of our multi-scale Flow Fields approach. Flow offsets saved in pixels are propagated in all arrow directions.

Fig. 6. Outlier sieve effect. Outliers disappear through propagations on different scales. For visualization purposes the valid gray pixels of the scales in Figure 5 are enlarged to fill the whole pixel space. Scales for the numbers are: 1: $n=8$ after KD-tree initialization, 2:$n=8$ after propagation, 3:$n=4$ after propagation, 4:$n=1$ after propagation (we skipped $n=2$). The full images can be found in our supplementary material.
flow can be propagated into the arms starting from the body, but our approach even can obtain the flow for nearly the whole image with such poor initialization.

Figure 7 c) shows that we can even find tiny objects with our multi-scale approach: The 3 marked objects are well persevered in c) due to their presence in the coarsest scale d). Remarkably, these objects are only preserved when using multi-scale matching. Our basic approach without scale-scales only preserves parts of the upper object (a butterfly) riddled with outliers, although its seeds are a superset of the seed of the multi-scale approach – but it fails in avoiding resistant outliers. Our multi-scale approach preserves tiny objects due to unscaled WHTs (initialization) and since the image gradients around tiny objects create local minima in $E_{ds}$ even for huge patches $P_{r}^{n}$. This is sufficient as lower minima (resistant outliers) are successfully avoided by our search strategy. Our visual tests showed that our approach with $k = 3$ in general preserves tiny objects and other details better than our basic approach. With too large $k$ (> 3) tiny objects are, due to lack of seeds, not that well preserved.

3.3 Flow Fields+

Our original approach uses 4 propagation iterations containing 3 random search iterations with a fixed random search distance $R$. In our improved approach first presented in this article, we instead use two different random search distances. First we perform 4 propagation iterations (containing 3 random search iterations) with $R^+ = 2R$ and then 8 propagation iterations (containing 7 random search iterations) with $R$. For the different scales this means that we use $R^+_n = R^+ * n$ and $R_n = R * n$. Our four search directions are hereby repeated every 4 propagation iterations. The larger $R^+$ helps to further distribute sparse matches in difficult situations like large flow variations with only few correct seeds, while the smaller $R$ is required for accurate convergence. Large random search distances increase the risk of finding resistant outliers, but we found that the positive effect prevails if $R/R^+$ is chosen reasonably.

| $n$ | 8 | 4 | 4 | 2 | 2 | 1 |
|-----|---|---|---|---|---|---|
| $n^*$ | 8 | 6 | 4 | 3 | 2 | 1 |

### TABLE 1

Scales and sub-scales used for our improved approach Flow Fields+.

3.3.1 Sub-Scales

Besides different random search distances our improved approach also uses sub-scales. While our ordinary scales are limited to scaling factors of $n \in \{2^k, k \in \mathbb{N}\}$, sub-scales $n^* \in \mathbb{N}$ can additionally contain values that are not a multiple of two. In our improved approach we use sub-scales for the patch size $P_{r}^{n^*}(p_i)$, image blur $I_{r}^{n^*}$ and random search distances $R_{n^*}^+ = R^+ * n^*$ and $R_{n^*} = R * n^*$, but not for propagation and random search positions of the scales (i.e. everything shown in Figure 2). Here we only use valid $n$. Table 1 shows the $n$ and $n^*$ used for the different scales in our tests of our improved approach with sub-scales.

3.4 Flow Fields+ Fast

The Flow Fields+ Fast approach aims to be much faster and still more accurate than the original Flow Fields approach. Compared to Flow Fields+ we omit sub-scales. Furthermore, we use only 4 propagations with $R$, similar to the original Flow Fields approach for the finest and thus computationally most expensive scale. Coarser scales are still executed with $4 \times R^+$ and $8 \times R$ like in the Flow Fields+ approach.

3.4.1 Flow Fields+ Fast x2

Flow Fields+ Fast x2 is an even faster version that does not execute the finest scale at all and only uses 4 propagations with $R$ on the 2. finest scale. Starting from the 3. finest scale this approach also uses $4 \times R^+$ and $8 \times R$. Furthermore, we only add one pixel in a 2x2 region to the KD-Tree as KD-Tree creation would otherwise be a significant time factor. As the approach does not process the finest scale, it creates only one match in each 2x2 region. This is not an issue since we sparsify matches before computing the final optical flow (See Section 3.8).

3.5 Data Terms

In this article we consider the following data terms:

1) Census transform [33]. It is computationally cheap, illumination robust and to some extend edge aware. We use the sum of census transform errors over all color channels in the CIELab color space for $E_d$.

2) Patch based SIFT flow [34] (for experiments with our original conference approach) and Pixel-wise SIFT features [7] (for experiments with our improved approaches). Reasoning for the decision to switch to SIFT is provided in Section 3.6.

a) Pixel-wise SIFT features: the error between SIFT features is determined with the $L_2$ distance. Due to the large feature vector of $S = 128$ dimensions only $r = 0$ is affordable in our approach ($r = 1$ has already 9 times more operations).
3.7 Outlier Filtering

A common approach of outlier filtering is to perform a forward backward consistency check. We found that the robustness of the consistency check can be further improved by calculating the backward flow two instead of only one time. This helps as our approach is randomized. Hence, two backward flows with different pseudo-random numbers are not identical which is why outliers often diverge into different directions. This property can be further reinforced by using different patch radii \( r \) and \( r_2 \) for both backward flows. We delete a pixel if it is not consistent to both backward flows i.e.

\[
|F(p_1) + F_b^1(p_1 + F(p_1))| < \epsilon, j \in 1, 2
\]

is not fulfilled for one of the two backward flows \( F_b^j \). For a 3-way check an additional forward flow could be added, but for a 2-way check an extra backward flow performs better (see supplementary material for an explanation).

After the consistency check many of the remaining outliers form small regions that were originally connected to removed outliers. Thus, we remove these regions as follows: First, we segment the partly outlier filtered flow field into regions. Neighboring pixels belong to the same region if the difference between their flow is below 3 pixels. Then, we test for regions with less than \( s \) pixels if it is possible for that region to add at least one outlier that was removed by the consistency check with the same rule. If this is possible, we found a small region that was originally connected to an outlier and we remove all points in that region.

3.8 Sparsification and Dense Optical Flow

To fill the gaps created by outlier filtering we use the edge preserving interpolation approach proposed by Revaud et al. [3] (EpicFlow). We found that EpicFlow does not work very well with too dense samples. Thus, we select only one sample in each \( q \times q \) region in the outlier filtered flow field if the region still contains at least \( e \) samples. \( q \) is set to 3, except for \( \text{Flow Fields+ Fast } x2 \) where \( q = 3 \) does not fit (a sampling size of \( 2 \times 2 \) cannot be assigned to \( 3 \times 3 \) patches). Here, we use \( q = 4 \) (and we also test \( q = 8 \) as a faster alternative). This is our last consistency check. We found that even after region based filtering most remaining outliers are in sparse regions where most flow values were removed. The sample that is selected is the sample for which the sum of both forward backward consistency check errors is the smallest.
Most approaches can obtain an endpoint error (EPE) in the subpixel range.

- KITTI 2012 [10]: It was created from a platform on a driving car and contains images of city streets. The motions can become large when the car is driving.
- KITTI 2015 [11]: An improved version of KITTI 2012, where other cars actually drive (in KITTI 2012 other cars are just standing in the street).

The remainder of this section is structured as follows: In Section 4.1 we detail parameter selection. In Section 4.2 – 4.5 we analyze our approach with various kinds of experiments. In Section 4.6 we evaluate our different approaches on the MPI-Sintel and KITTI 2015 training set, while we evaluate our best approach on the test sets of all four major evaluation portals in Section 4.7. Finally, in Section 4.8 we present visual results of our approach.

### 4.1 Parameter Selection

Here we detail parameter selection for our approach. In our experiments we use \( l = 8 \) equivalent to [6] and \( k = 3 \) as it showed to perform best for the tested optical flow benchmarks. In general, visual tests with large images showed that \( k_{\text{good}} \approx \log_4(\text{NumImagePixels}/6000) \) seems to be a reasonable approximation for a good \( k \). Note that this is only based on few visual observations and might vary depending on other parameters and the dataset.

We set \( R = 1 \) for experiments on MPI-Sintel and \( R = 1.5 \) for experiments on KITTI. These values are based on the experiments in Figure 9 right. The results of our conference approach Flow Fields are created with a fixed \( R = 1 \). The parameters \( \epsilon, e, s \) are tuned coherently for our results in Section 4.7 on the corresponding training set with \( \epsilon \pm 0.5, e \pm 1, s \) and \( \pm 50 \). Determined parameters for our public results can be found in our supplementary material.

In our experiments we use the census transform data term for the MPI-Sintel and Middlebury datasets with \( r = 8 \) and \( r_2 = 6 \) for our conference approach Flow Fields and \( r = 4 \) and \( r_2 = 3 \) for our improved approaches Flow Fields+

### 4.1.1 Influence of parameters

The influence of our parameters can be seen in Figure 9. (Fast). While the values of the conference approach are based on a few incoherent tests with the Flow Fields approach the values of our improved approaches are based on tests of different \( r \) on the whole MPI-Sintel training set.

For our experiments on KITTI 2012 and 2015 we use data terms based on the deformation and scale robust SIFT features instead (improved approach: SIFT, conference approach: SIFT flow with \( r = 3, S = 12, r_2 = 2, S_2 = 12, S_2 \) is \( S \) for 2. backward flow). We use SIFT here as the KITTI dataset contains image patches of walls and the streets that are undergoing extreme scale changes and deformations (due to large viewing angles). Thus, patch-based approaches perform poorly here [4]. By using a different more appropriate data term for KITTI we also demonstrate that in our approach the data term can easily be adapted to the problem.

For EpicFlow [3] applied on our approach we use their standard parameters which are tuned for Deep Matching features [18]. As there are no standard parameters for KITTI 2015 we use slightly modified KITTI 2012 parameters. For a fair comparison we use the same parameters (tuning \( \epsilon, e, s \) for ANNF does not affect our results), data term and WHTs in CIELab space for our tests with the ANNF approach [6] (the original approach performs even worse). This includes ANNF results in Section 4.2 and in Figure 1 and 15.
means 4 propagation iterations, “12” means 12 iterations. “4+8” means 4 iterations with \( R^+ \) and 8 iterations with \( R \) as described in Section 3.3. As can be seen, our approach of using 4+8 iterations performs the best if \( R \) is chosen reasonably. For too large \( R \) the error increases faster than with 12 fixed \( R \) iterations, as the 4+8 approach also uses \( R^+ = 2R \). While the difference between 4 and 12 iterations is larger than between 12 and 4+4, 8+4 has the benefit that it has the same runtime as 12. In our conference approach we simply used 4 iteration with \( R = 1 \), which is suboptimal.

![Graph showing the influence of the parameter k on the approach.](image)

**Figure 10. The influence of the parameter \( k \) on our approach. We plot the main measures for each dataset.**

Figure 10 shows the influence of \( k \). While \( k = 3 \) is optimal for both MPI-Sintel as well as KITTI 2015 the error only increases slightly for larger \( k \) on KITTI but significantly on MPI-Sintel. This is likely caused by the fact that MPI-Sintel contains much more small independently moving objects than KITTI. These cannot be determined anymore if \( k \) is too large.

### 4.2 Comparison to ANNF

In the introduction we claimed that our Flow Fields are better suited for optical flow estimation than ANNF and contain significantly fewer outliers. To prove our statement quantitatively we compare our Flow Fields with different number of scales \( k \) to the state-of-the-art ANNF approach presented in [6]. We also compare to the real NNF calculated in several days on the GPU. The comparison (to our Flow Fields approach) is performed in Table 2 with 4 different measures:

- The percentage of flows with an EPE below 3 pixels.
- The EPE bounded to a maximum of 10 pixels for each flow value (EPE10). Outliers in correspondence fields can have arbitrary offsets, but the difficulty to remove them does not scale with their EPE. Local outliers can even be more harmful since they are more likely to pass the consistency check. The EPE10 considers this.
- The real endpoint error (EPE) of the raw correspondence fields. It has to be taken with care (see EPE10).
- The EPE after outlier filtering (like in Section 3.7) and utilizing EpicFlow to fill the gaps (Epic).

All 4 measures are determined in non-occluded areas only, as it is impossible to determine data based correspondences in occluded areas. As can be seen, we can determine nearly 90% of the pixels on the challenging MPI-Sintel training set with an EPE below 3 pixels, relying on a purely data based search strategy which considers each position in the image as a possible correspondence. With weighted median filtering (weighted by matching error) this number can even be improved further, but the distribution is unfavorable for EpicFlow (it probably removes important details similar to some regularization methods). In contrast, more scales up to the tested \( k = 3 \) have a positive effect on the EPE as they successfully can provide the required details.

#### 4.2.1 Differences to scaled matching of Bao et al. [24]

Bao et al. [24] also used multi-scale matching in their approach to speed it up. However, despite joined bilateral upsampling combined with local patch matching in a 3x3 window they found that the accuracy on Middlebury drops clearly due to multi-scale matching. As can be seen in Table 3, this is not the case for our approach. As expected from the experiment in Figure 7 the accuracy even rises. Note that the Epic result does biased to the value in the first row.

| Method                  | \( \leq 3 \) pixel EPE10 | EPE  | Epic |
|------------------------|--------------------------|------|------|
| Ground truth           | 100%                     | 0.0  | 0.0  |
| \( k = 3 \times + median \) | 89.2%                    | 0.91 | 4.41 |
| \( k = 3 \)            | 88.79%                   | 1.36 | 8.84 |
| \( k = 2 \)            | 86.88%                   | 1.57 | 14.65|
| \( k = 1 \)            | 79.13%                   | 2.29 | 32.51|
| ANNF [6]               | 68.05%                   | 3.38 | 59.11|
| Original EpicFlow      | -                        | -    | -    |

**Table 2**

Comparison of different correspondence fields on a representative subset (2x every 10th frame) on non-occluded regions of the MPI-Sintel training set (clean and final). Results are based on our conference approach Flow Fields. See text for details.

| Method                  | \( \leq 1 \) pixel EPE3  | EPE  | Epic |
|------------------------|--------------------------|------|------|
| Ground truth           | 100%                     | 0.0  | 0.0  |
| \( k = 3 \)            | 87.08%                   | 0.499| 1.16 |
| \( k = 2 \)            | 86.81%                   | 0.508| 2.32 |
| \( k = 0 \)            | 81.93%                   | 0.670| 12.33|
| Original EpicFlow      | -                        | -    | -    |

**Table 3**

Comparison of our conference approach Flow Fields with different scales on the Middlebury training dataset to demonstrate that the quality does not suffer from multi-scale matching like in [24]. Note that the Epic result is biased to the value in the first row.

### 4.3 Analysis of Outlier Sieve Effect

In this subsection we analyze the outlier sieve effect of our approach (Figure 6) based on resistant outlier probabilities \( P_s(d_f) \) for different setups \( s \) and distances \( d_f \) to the ground truth \( p_2^* \). With the matching error abbreviation

\[
E^p_s(p_2) = E_d(P^p_s(p_1), P^p_s(p_2)),
\]

3. No backward flow calculated
Fig. 11. We determined the probably that a point is a resistant outlier depending on the distance of the point to the ground truth match. Probabilities are plotted relative to the blue plot "1". See text for details.

we can define the following configurations for $s$:

$$C_{s=x} = E_x^2(p_2) < E_x^2(p_2)$$

$$C_{s=x+y} = C_x \land C_y$$

$$C_{s=x+y} = E_x^2(p_2) + E_y^2(p_2) < E_x^2(p_2) + E_y^2(p_2)$$

Then, the probability $P_s(d_f)$ can be written as:

$$P_s(d_f) = P(C_s \mid d_f = ||p_2 - p_2||_2)$$

$||p_2 - p_2||_2$ is the Euclidean distance to the ground truth match. Since the raw probabilities $P_s(d_f)$ are difficult to read in a plot and as we are mainly interested in the relation between probabilities we plot the relation of probabilities $P_s^{rel}(d_f)$ in Figure 11, instead. $P_s^{rel}(d_f)$ is defined as:

$$P_s^{rel}(d_f) = \frac{P_s(d_f)}{P_1(d_f)}$$

$P_1(d_f)$ is the resistant outlier probability for patches on the finest scale. As can be expected the figure shows that finer scales (like 1,2) are better for matching close-by to the ground truth while coarser scales (like 4,8) are better for matching far from the ground truth. Even better seems to be to match all scales at once (1+2+4+8). However, this is computationally expensive.

As stated in Section 3.2 the scales of our approach serve as a kind of outlier sieve. Outliers can only survive if matching fails on all scales. This can be approximated as an & operation between the matching terms of the scales (like 1&2&4&8).

However, in our approach the random search can undo the & operation of coarser scales in its search range as it can redo already sieved outliers here (these outliers are actually just inaccurate matches due to their proximity to the ground truth). Considering this we get the black curve as approximation for the resistant outlier probability of our outlier sieve. This curve still keeps the extremely low failure rate of 1&2&4&8 for large distances, which is only 4.3% of scale 1 for a distance of 200 pixels. This is much lower the failure rate of scale 8 which is 23.2% or 1+2+4+8 (which is 18.6%). For smaller distances the benefit is lower and for distances < $R$ (or $R^2$) there is no benefit anymore. This means that our approach is very likely to find a match that is not too far from the ground truth. However regarding subpixel accuracy there is no benefit with our approach compared to ordinary approaches (for pixels where these manage to avoid outliers far from the ground truth).

4.4 Texture effect tests

Due to the small random search distance $R$ we can expect from our approach that it can flawlessly match repetitive patterns as long as the influence area of the coarsest scale is larger than the ambiguous repetitive pattern. That this is actually the case for our approach is demonstrated in the experiment in Figure 12. While we get a perfect match only with $k = 3$ the matching error strongly decreases already for fewer scales. We think that this is among other things due to the outlier sieve effect: as corner pixels can be
matched they can overwrite matches of non-corner pixel, but not the other way around. In fact this effect is also required for $k = 3$ as the images are 95x95 pixels in size. However, with the used $r = 4$ the matching patch size is only $(2r + 1)2^k = 72$ pixels. The Figure also shows that we can expect a similar effect for texture-less objects.

In natural images repetitive and texture-less objects are usually not completely ambiguous. Thus, our approach should be able to match them even if the repetitive structure exceeds the influence area of the coarsest scale. Figure 13 shows that our multi-scale approach is also noise and blur resistant. Blur resistance is also confirmed by Figure 15 c).

Fig. 14. Percentage of removed outliers versus percentage of removed inliers, for an outlier threshold of 5 pixels (We vary $\epsilon$).

4.5 Outlier Filtering

Figure 14 shows the percentage of outliers that are removed versus the percentage of inliers that are removed by different consistency checks on the MPI-Sintel training set. Both the 2x consistency check as well as the region filter increase the amount of removed outliers for a fixed inlier ratio. We also considered using the matching error $E_d$ for outlier filtering, but there is no big gain to achieve (see supplementary material).

4.6 Evaluation of our approaches

Here we compare the performance and runtime of our different approaches on the MPI-Sintel and KITTI 2015 training sets (on the test sets only the best version of an approach shall be submitted). As can be seen in the first two results in Table 4 and 5, sub-scales improve the matching accuracy, with a reasonable increase in runtime. If speed matters the \textit{Flow Fields+ Fast} approach can provide results much faster, with relatively small accuracy trade-off. \textit{Flow Fields+ Fast x2} is again much faster, while even this approach still outperforms our original conference approach in accuracy (if we do not use $q=8$). The tables also show that while the 2. consistency check improves the result, it also requires extra runtime which is why it is not recommended for our fast approaches. On MPI-Sintel the runtime of EpicFlow exceeds the runtime of \textit{Flow Fields+ Fast x2} approach. We can decrease it by increasing $q$. However, this has a clear impact on the accuracy. Our results on KITTI show that our fast feature approach F2F is comparable to the F2 approach regarding matching accuracy. In the shown test it even performs slightly better which we consider as noise as in another test it performed slightly worse.

4.7 Public Results

In this subsection we present the public results of our approach on different public evaluation portals. We consider our conference approach \textit{Flow Fields} [12], our improved approach \textit{Flow Fields+} and for completeness also our very recent CNN based publication [30] that uses CNN based features as \textit{Flow Fields+} and \textit{Flow Fields+ Fast} is not considered here as the evaluation portals request to submit only the best approach of a publication and to test variations of an approach on the training set. As results in the evaluation portals change regularly we only compare to similar approaches. For a full overview of approaches we refer to the corresponding evaluation portals [8], [10], [11], [13] (links in reference section).

4.7.1 MPI-Sintel

Our results on MPI-Sintel are shown Table 6. Our conference approach \textit{Flow Fields} already clearly outperforms the original EpicFlow that is based on Deep Matching features [18]. Most of this advance is obtained in the non-occluded area but EpicFlow also rewards our better input in the occluded areas. Our improved approach \textit{Flow Fields+} again performs clearly better than our conference approach – especially on the clean set. Here it is at the moment of writing this article the best submission on the non-occluded area with an EPE of only 0.820, while the 2. best recent submission (MR-Flow, yet unpublished) has an EPE of 0.983. Still, our approach is

4. Single core runtime, in conference paper we reported multicore runtime.

### Table 4

| Method                   | parameter | EPE   | time$^*$ | time Epic |
|-------------------------|-----------|-------|----------|-----------|
| Flow Fields+             | c×2, q=3  | 2.410 | 14.0s    | 3.1s      |
| Flow Fields+ no sub-scales | c×2, q=3  | 2.438 | 11.4s    | 3.1s      |
| Flow Fields+ Fast        | c×2, q=3  | 2.448 | 6.7s     | 3.1s      |
| Flow Fields+ Fast x2     | c×2, q=4  | 2.641 | 4.5s     | 3.1s      |
| Flow Fields+ Fast x2     | c×1, q=4  | 2.526 | 1.8s     | 1.8s      |
| Flow Fields+ Fast x2     | c×1, q=8  | 2.593 | 1.2s     | 1.8s      |
| Flow Fields+ Fast x2     | c×1, q=8  | 2.693 | 1.2s     | 1.8s      |
| Original Flow Fields     | c×2, q=3  | 2.587 | 14.2s    | 3.2s      |

### Table 5

| Method                   | parameter | > 3px EPE failure rate | time$^*$ | time Epic |
|-------------------------|-----------|------------------------|----------|-----------|
| Flow Fields+             | c×2, F2   | 21.22%                 | 25.8s    | 1.8 s     |
| Flow Fields+ no sub-scales | c×2, F2   | 21.36%                 | 21.1s    | 1.8 s     |
| Flow Fields+ Fast        | c×2, F2   | 21.82%                 | 10.8s    | 1.9 s     |
| Flow Fields+ Fast        | c×1, F2   | 21.98%                 | 8.4s     | 1.9 s     |
| Flow Fields+ Fast x2     | c×2, F2F  | 25.34%                 | 4.0s     | 1.8 s     |
| Flow Fields+ Fast x2     | c×1, F2F  | 25.34%                 | 3.1s     | 1.2s      |
| Original Flow Fields     | c×2, F1   | 24.74%                 | 39.4s*   | 1.8s      |

$^*$ Single core runtime without EpicFlow.
only the 2. best for the overall error (EPE all) as MR-Flow seems to have a better interpolation into the occluded area (for which we still use EpicFlow). With better interpolation on top of our approach it might perform better here, as well. Our approach with CNN-based features [30] performs best on the final set. We think that learned features benefit from the motion blur that is only in the final set while on the clean set there is not such a big improvement possible.

### 4.7.2 Middlebury

On Middlebury our conference approach (Flow Fields) obtains an average rank of 38.0 (EpicFlow: 52.2) and an average EPE of 0.33 (EpicFlow: 0.39). Our rank is either exactly the same as EpicFlow (e.g. 69 on Army) or better (e.g. 4 instead of 53 on Urban). As already discussed in Section 4.2 the EPE rank that can be obtained with EpicFlow on Middlebury is limited, as EpicFlow is not designed for such datasets. Nevertheless, we can improve the result on some datasets. Due to the limitations with EpicFlow on Middlebury we do not create Flow Fields+ results.

### 4.7.3 KITTI 2012 and 2015

Our results on KITTI 2012 and 2015 can be seen in Table 7 and 8, respectively. As can be seen, our conference approach Flow Fields already clearly outperforms the original EpicFlow with Deep Matching features on KITTI 2012. Our improved approach Flow Fields+ performs even better. To the best of our knowledge our Flow Fields+ approach is so far the best approach both on KITTI 2012 and 2015 that does not use CNNs like [30], [36] or object segmentation and rigidity assumptions for the segmented objects like [37], [38], [39]. Thus, in contrast to all better performing approaches ours also works for non-rigid scenes or scenes where object segmentation fails and does not require to train a neural network, for which proper training data is required. Our CNN-based approach [30] performs even better, but does require proper training data.

### 4.8 Visual Results

Visual results of our approach are shown in Figure 15. EpicFlow can preserve considerably more details with our Flow Fields than with the original Deep Matching features. Even in failure cases like in Figure 15 a) (right column), our approach often still achieves a smaller EPE thanks to more preserved details. Note that the shown failure cases also happen to the original EpicFlow. Despite more details our approach in general does not incorporate more outliers. The occasional removal of important details like the one marked in Figure 15 b) remains an issue – even for our improved outlier filtering approach. The marked detail is important as the flow of the very fast moving object is different on the left (brighter green). Still, we can in general preserve more details than the original EpicFlow. Figure 15 c) shows that our approach also performs well in the presence of motion and defocus blur.

### 5 Conclusion

In this article we presented a novel correspondence field approach for optical flow estimation. We showed that our Flow Fields are clearly superior to ANNF and better suited than state-of-the-art descriptor matching techniques, regarding optical flow estimation. We also presented extended outlier filtering and demonstrated that we can obtain promising optical flow results, utilizing a modern optical flow algorithm like EpicFlow. Compared to the conference version we further improved our approach both in accuracy and runtime efficiency. We also gave a deeper insight into our approach. With our results, we hope to inspire the research of dense correspondence field estimation for optical flow.

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Fig. 15. The left 4 columns show example results. Images is the average of both input images. For ANNF we use [6] in a fair way (see text). FF means Flow Fields. OM means that the ground truth occlusion map is added (black pixels, it is incomplete at image boundaries). Filtered FF is after outlier filtering (deleted pixels in black). FF+Epic is EpicFlow applied on our Flow Fields. EpicFlow is the original EpicFlow. Right column: a) Our approach fails in the face of the right person (outlier) and at its back (blue samples too far right). Still our EPE is smaller due to more preserved details. b) The marked bright green flow is not considered due to too strong outlier filtering. This makes a huge difference here. c) We show that our Flow Fields (bottom left) perform much better in the presence of blur than ANNF (top left).

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