Phototube non-linearity correction technique

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Abstract. Scintillation light is often detected by photo-multiplier tube (PMT) technology. PMTs are however intrinsically non linear devices, especially when operated with high light yield scintillators and high input photon flux. Many physical effects (e.g. inter-dynode field variation, photocathode resistivity, etc.) can spoil the ideal PMT behavior in terms of gain, ending up in what are addressed as the under-linearity and over-linearity effects.

Established techniques implemented in the PMT base (e.g. increasing bleeding current, active voltage divider, etc.) can mitigate these effects, but given the unavoidable spread in manufacturing and materials, it turns out that, with respect to linearity at the percent level, every PMT sample is a story of its own.

The residual non linearity is usually accounted for with polynomial correction of the spectrum energy scale, starting from the position of a few known energy peaks of calibration sources, but uncertainly remains in between of calibration peaks. We propose to retrieve the calibration information from the entire energy spectrum and not only the position of full energy peaks (FEP), by means of an automatic procedure that also takes into account the quality (signal/noise ratio) of the information about the non-linearity extracted from the various regions of the spectrum.

1. Introduction
Radiation detectors based on recently developed scintillators with very high light yield and fast light emission (e.g. LaBr\textsubscript{3}:Ce, with 63 photons/keV and 18 ns decay time [1]) may easily suffer from non-linearity effects of the associated photo-multiplier tube (PMT) devices, especially in case of high energy $\gamma$-ray/particle detection. In principle, also the non-linearity of the scintillator itself should be taken into account, but, particularly for LaBr\textsubscript{3}:Ce and the energy range considered in this paper, that is not the dominant factor. Non negligible PMT under-linearity and over-linearity effects often emerge as the result of many interacting physical effects (e.g. inter-dynode field variation, photocathode resistivity etc.), which often can be mitigated, but not completely eliminated, by established techniques implemented in the PMT base (e.g. increased bleeding current, active voltage divider, etc.).

The residual non-linearity (e.g. of the order of a few percents and different from sample to sample of PMT) is usually accounted for with polynomial calibration of the spectrum energy scale based on the expected position of the energy peaks of a few calibration sources, but uncertainly still remains in between of the calibration peaks.

With this in mind, we start to introduce in section 2.1 an automatic algorithm that improves the signal-to-noise (S/N) ratio of the experimental energy spectra and increases the effectiveness of the new technique, proposed in section 2.2, that retrieves the calibration information from the entire
energy spectrum and not only from the position of the known full energy peaks (FEP). In section 3 the effectiveness of the proposed methods is demonstrated using experimentally acquired energy spectra.

2. Methods

The experimental set-up used for acquisition of all the energy spectra is schematically represented in Fig. 1: a 3"x3" LaBr3:Ce doped scintillator (by Saint Gobain) is coupled to a Hamamatsu R10233-100-SEL PMT equipped with the LaBrVD PMT base developed in Milano, while a Lecroy HRO 12 bits oscilloscope sends the digitized PMT waveforms to a PC running the MATLAB software. PMT power supply is adjustable, in order to easily characterize the non-linearity as a function of the applied high voltage level.

Thanks to the new proposed method for non-linearity correction, which takes into account the entire shape of the spectrum, only a few calibration sources were required, as shown in Fig. 2, i.e. a $^{137}\text{Cs}$ source, an AmBe-Ni and the internal radioactivity of the LaBr scintillator.

2.1. Algorithm for signal-to-noise improvement

In order to improve the signal ("true counts") vs noise ("statistical fluctuations") ratio of the energy spectra, we propose a method based on a few steps: i) the energy axis $E$ is rescaled to get uniform peaks FWHM (e.g. according to the square root of energy); ii) the counts axis $C$ is rescaled to get comparable peak height (e.g. through to the logarithm function); iii) the resulting function $C(E)$ is decomposed by Fourier techniques and synthesized back, neglecting the high-order harmonics, so that the intrinsic absolute energy resolution of the detector at low energy is preserved and at the same time the S/N ratio for high energy peaks for only few distributed counts is improved (see Fig. 3, 4, 5).

2.2. Algorithm for non-linearity correction

While ideal systems should have a perfectly proportional relationship between the physical energy of the detected events and the amplitude of the correspondent digitized waveforms, this is almost never the case with real detectors, especially if they are based on PMT technology.

Indeed, also in case of LaBr3:Ce detectors the predominant source of non proportionality can be identified with the associated PMT device, apart from the tens-of-keV range of energy.

The non-linearity characteristic of a given combination of PMT and HV power supply level can be easily estimated by comparing two energy spectra obtained in the very same experimental condition apart from the two HV levels (see Fig. 6). If one energy spectrum is acquired with low-enough PMT gain so that all non-linearity effects are practically negligible, although resolution at low energy may be clearly compromised, it is always possible to characterize the PMT non-linearity at any given high voltage level by simply comparing the associated non-linear spectrum with the linear one.

![Figure 1](image-url) The experimental set-up used for the acquisition of the energy spectra. The adjustable gain of the DAQ F/E signal allows to fit the various amplitude of the PMT output signal, for different HV levels, to the fixed ADC dynamic range.
Figure 2. Reference energy spectrum (10 millions counts, acquired at 500 cps rate) ranging from 100 keV up to 9 MeV, with a few recognizable peaks from two calibration sources: $^{137}$Cs (662 keV) and AmBe-Ni, plus LaBr$_3$ intrinsic radioactivity. As high energy events are detected with lower efficiency, the S/N ratio in the right side is usually considerably lower.

Figure 3. The energy spectrum of Fig. 1 (blue line) superimposed with a new version of the same spectrum with improved S/N ratio (red line), obtained by applying the procedure introduced in Sec. 2.2. Magnified regions of the two spectra are shown in Fig. 3 and Fig. 4. The new energy spectrum, if required, can be easily re-sampled according also to different bin scales.

Figure 4. $^{137}$Cs energy peak (662 keV) and surrounding region of the Fig. 3 spectra, showing total coincidence between the original and the improved S/N ratio spectra.

Figure 5. AmBe(Ni) full energy peak (8.9 MeV) with first escape peak and surrounding region of the Fig. 3 spectra, clearly showing the effectiveness of S/N improvement.
The final, slowly varying non-linearity correction function is determined by applying a Least Mean Squares (LMS) technique that takes into account: i) the non-linear spectrum shape; ii) the "shift" information, e.g. determined as in Fig. 7 and iii) the S/N quality of the shift information, i.e. higher in correspondence of narrow peaks (like the one in Fig. 4) and lower around nearly flat portions of the spectrum (like the 2300-3000 bin region of Fig. 3). An example of such a function is shown in Fig. 9, in the form of: bin(e(bin)+1).

Practically speaking, the entire procedure can be thought of as the search for a correction function that locally "stretches or shrinks" the ADC bin scale of the non-linear energy spectrum, so to make its shape as much as possible identical to the shape of the reference linear spectrum.

A more detailed description of the two methods will be readily published in a forthcoming technical paper.

Figure 6. These 4 energy spectra have all been acquired using Fig. 1 set-up but different HV power supply level. While the blue spectrum (710 V, 15 mV@662 keV) is practically linear, significant over-linearity appears for higher HV values (e.g. 810 V for the red spectrum, with 2x signal amplitude compared to the blue one; 900 V and 4x amplitude for the green one; 975 V and 6x amplitude for the black one). Monotonic increase of non-linearity is evident for this PMT.

Figure 7. As an example of part of the procedure described in Sect. 2.2, the black peak from the non-linear spectrum (left panel) is left shifted and aligned with the blue one from the linear spectrum (as in right panel), thus maximizing the local cross-correlation function of the two spectra.

3. Experimental validation
As an applicative example of the proposed method, the red energy spectrum of Fig. 8 (acquired at 810 V) has been compared against the blue energy spectrum (acquired at 710 V and practically linear, but with degraded energy resolution at low energy). After estimating the non-linearity function (e.g. in the form of bin=bin*(1+e(bin) as the one represented in Fig. 9), the non-linearity of the red spectrum has been corrected and the two energy spectra have been superimposed in Fig. 10. Because the non-linearity correction introduces non-uniform binning in the energy scale, the resulting spectrum has been also uniformly re-sampled at 1 keV/bin, obtaining the final black energy spectrum in Fig. 10. Once the detector resolution as a function of energy is supplied to the algorithm, the entire procedure is fast and runs in a completely unattended way.
Figure 8. The two energy spectra have been acquired in the same experimental conditions, apart from the HV level. The blue spectrum at 710 V is practically linear, while the red spectrum at 810 V shows significant over-linearity.

Figure 9. The ideal bin=F(bin) function of a perfectly linear spectrum (blue) and the equivalent function (red), estimated using the proposed method, for a spectrum with significant 8% over-linearity at maximum energy.

Figure 10. The effectiveness of the proposed method for non-linearity correction is shown here: the blue (linear) and red (linearized) spectra of Fig. 8 are now superimposed, while the black spectrum, derived from the red one after 1 keV uniform binning using the algorithm for S/N improvement.

4. Conclusions
A new method for PMT non-linearity estimation and correction has been developed, that takes into account the entire shape of the energy spectrum and not only the few peak positions of the calibration sources. Especially in case of LaBr₃ scintillator, this method can thus take advantage also of the detector significant internal radioactivity, to provide a quick and effective procedure for non-linearity correction that can also run unattended. Improvement of S/N ratio of the high energy regions of the spectrum is also obtained by a new proposed method based on Fourier signal decomposition. A more detailed analysis of the two proposed algorithms will be given in a forthcoming technical paper.

References
[1] Giaz A. et al., 2013, Nucl. Instr. and Met. A, 729, 910-921.