Energy-momentum Prescriptions in General Spherically Symmetric Space-times

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Einstein, Landau-Lifshitz, Papapetrou, Weinberg, and Möller energy-momentum prescriptions in general spherically symmetric space-times are investigated. It is shown that for two special but not unusual classes of general spherically symmetric space-times several energy-momentum prescriptions in Schwarzschild Cartesian coordinates lead to some coincidences in energy distribution. It is also obtained that for a special class of spherically symmetric metrics Möller and Einstein energy-momentum prescriptions give the same result for energy distribution if and only if it has a specific dependence on radial coordinate.

1. INTRODUCTION

One of the old and basic problems in General Relativity (GR) which is still unsolved is the localization of energy. In Special Relativity one can define a symmetric tensor, $T^b_{a,b}$, as energy-momentum tensor which satisfies the conservation laws, i.e.

$$T^{b}_{a,b} = 0, \quad (1)$$

that means the energy-momentum tensor is a conserved and localized quantity in special relativity (and classical mechanics). In fact, in any local point of manifold no contribution of this quantity is produced or eliminated. But, in GR Eq. (1) is not acceptable. Because, it is not a tensor equation and is not valid in all reference frames. Using covariant derivative instead of partial one, we reach to the following equation which is invariant in all reference frames [1].

$$T^{b}_{a,b} = \frac{1}{\sqrt{-g}}(\sqrt{-g} T^{b}_{a,b} - \Gamma^{b}_{ac} T^{c}_{b} = 0 \quad (2)$$

where $\Gamma^{b}_{ac}$ are the connection coefficients. Considering Eq. (2), it is obvious that Eq. (1) is no longer satisfied. So, in this situation this energy-momentum tensor is not a localized quantity and the problem of energy-momentum localization in GR arises. If we want to keep the localization characteristics of energy in GR, we must look for a new quantity such as $\varepsilon_{a} T^{b}_{a,b}$, instead of $T^{b}_{a,b}$, which its partial derivative vanishes, i.e.

$$\varepsilon_{a} T^{b}_{a,b} = 0. \quad (3)$$

Considering the relation between partial and covariant derivatives Eq. (2), the following would be a suitable candidate [2]

$$\varepsilon_{a} T^{b}_{a,b} = (-g)^{\frac{1}{2}} (T^{b}_{a} + t^{b}_{a}). \quad (4)$$

where $g = \det(g_{ab})$ and $n$ is a positive integer that shows the weight. In fact, in this way we consider gravitational fields effects in energy distribution as an additional term in energy-momentum tensor. In other words, $\varepsilon_{a} T^{b}_{a}$ is an energy-momentum complex of matter plus gravitational fields where $t^{b}_{a}$ is not a true tensor, but rather is a pseudo-tensor that describes the localization of gravitational energy-momentum.

It should be noted that $\varepsilon_{a} T^{b}_{a}$ can be written as the divergence of some “super-potential” $H^[bc]^{[a]}$ that is anti-symmetric in its two upper indices [3] as

$$\varepsilon_{a} T^{b}_{a} = H^{[bc]}_{[a]}. \quad (5)$$

In addition, a new quantity like $U^{bc}_{a}$ can also play the role of $H^{[bc]}_{[a]}$ if

$$U^{bc}_{a} = H^{[bc]}_{[a]} + \Psi^{bc}_{a} \quad (6)$$

and divergence or double divergence of $\Psi^{bc}_{a}$ is identically zero, i.e.

$$\Psi^{bc}_{a,c} \equiv 0 \quad or \quad \Psi^{bc}_{a,c,b} \equiv 0. \quad (7)$$

So, the quantity $\Theta^{b}_{a}$ which is defined by this new super-potential remains conserved locally as

$$\Theta^{b}_{a} = U^{bc}_{a,c} \Rightarrow \Theta^{b}_{a,c} = 0. \quad (8)$$

Considering this freedom on the choice of superpotentials, many different energy-momentum prescriptions (EMPs) have been proposed by different authors, for example Einstein [4], Landau and Lifshitz [5], Møller [4], Bergmann [6], Weinberg [1], Papapetrou [7], Tolman [8], Komar [9], Penrose [10] and Qadir and Sharif [11] prescriptions.

Using EMPs has some problems that are mentioned in the following. Except a few of them including Möller, Penrose, and Komar prescriptions, for other prescriptions

[1] Although the Tolman and Einstein prescriptions are different in their forms, but in fact they are equal [21].
all calculations must be done in Cartesian coordinate system. Moreover, some of them are non-symmetric in exchanging of their indices. So, conserved angular momentum can not be defined for that ones which are non-symmetric. Another drawback of using EMPs is that they may give different results for the same space-time. Finally, physical concept of these non-tensorial quantities has been unclear for a long time. However, Cheng, Nester, and Chen [12] showed that they can be considered as the boundary term of Hamiltonian and therefore are quasi-local.

The problems associated with the concept of energy-momentum complexes resulted in some researchers even doubting the concept of energy-momentum localization. Misner et al. [2] argued that to look for a local energy-momentum is looking for the right answer to the wrong question. He showed that the energy can be localized only in systems which have spherical symmetry. They also expressed that pseudo-tensor approach could conflict with the equivalence principle. Cooperstock and Saracino [13] argued that if energy is localizable for spherical systems, then it can be localized in any system. In 1990, Bondi [14] argued that a non-localizable form of energy is not allowed in GR. Recently, besides EMPs, it was suggested another viewpoint for energy problem in GR that is in agreement with EMPs theory about localization of energy, i.e. EMPs in “Tele-Parallel Gravity”, (see [13]). On the other hand, some people do not believe in localization of energy and momentum in GR. In addition, some physicists propose a new concept in this regard: “quasi-localization”. A large number of definitions of quasi-local masses have been proposed [10]. Unlike EMPs theory, quasi-localization theory does not restrict one to use particular coordinate system, but this theory have also its drawbacks (see Bergqvist [17]). In general, there is no generally accepted definition for energy and momentum in GR till now.

Despite of mentioned drawbacks of using EMPs, many authors have been interested in this topic and have reached interesting results [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Especially Virbhadra and colleagues [21, 22] showed that for a specific class of space-times, i.e. Kerr-Schild class, and even more general space-times, Einstein, Landau-Lifshitz, Papapetrou, and Weinberg prescriptions (ELLPW) lead to the same results in Kerr-Schild Cartesian coordinates. Moreover, Virbhadra [21] and Xulu [23] used ELLPW prescriptions and Møller prescription in general spherically symmetric (GSS) space-time, respectively, and found different expressions for the energy in a sphere with radius \( r \) in Schwarzschild Cartesian coordinates. But, in Kerr-Schild Cartesian coordinates ELLPW prescriptions lead to the same results in GSS space-time. It is not clear why different EMPs “coincide” in the Kerr-Schild Cartesian coordinates, but not in the Schwarzschild Cartesian coordinates. Considering this confusion, in this paper we focus on EMPs in GSS space-times in Schwarzschild Cartesian coordinates.

There is no proved performance between different EMPs. However, Palmer [24] and Virbhadra [21] discussed the importance of Einstein EMP while Lessner [25] believed that Møller prescription is a powerful tool for calculating the energy-momentum in GR. Recently, some authors have presented their interest to study Møller and Einstein prescriptions and to find a relation between them [23, 26, 27]. They have tried to compare these prescriptions in different space-times.

The remainder of the paper is organized as follows. In section 2 we introduce several EMPs which are used in this paper and give previous obtained results for these prescriptions in GSS space-time in Schwarzschild Cartesian coordinates. In section 3 we consider two special but not unusual spherically symmetric space-times and calculate their energy distribution by using different EMPs. In section 4, considering mentioned classes of GSS metrics, we find a unique form of this class in which Einstein and Møller prescriptions lead to the same result. Finally, we summarize and conclude in section 5.

Conventions: We use geometrized units in which \( c = G = \hbar = 1 \) and the metric has signature \((+−−−)\). Latin indices take values 0...3. The comma and semicolon, respectively, stand for the partial and covariant derivatives.

2. ENERGY DISTRIBUTION OF THE MOST GENERAL NON-STATIC SPHERICALLY SYMMETRIC SPACE-TIME

2.1. Energy-momentum Prescriptions (EMP)

In this section we introduce several EMPs which are used in this paper, i.e. Einstein, Landau-Lifshitz, Papapetrou, Weinberg, and Møller prescriptions. Specific form of each energy-momentum pseudo-tensor, conservation laws, and energy-momentum 4-vectors are listed in Table I. Weinberg, Landau-Lifshitz, and Papapetrou pseudo-tensors are symmetric in exchanging their indices and so, using them, one can define a conserved angular momentum. Moreover, we must perform our calculations in Cartesian coordinate system in all of above-mentioned prescriptions, except Møller prescription in which all coordinate systems are acceptable. Interested readers can refer to the mentioned references for more details. As mentioned in the previous section, prescriptions’ differences are just in a curl term. This topic was discussed in reference [30] in more detail.

In the third column of Table I Gaus’ theorem is used. In the surface integrals \( n_a \) represents the components of a normal one form over an infinitesimal surface element \( ds \). In spherically symmetric space-times, suitable surface of integration would be a sphere with radius \( r \). In addition, \( ds \) would be equal to \( r^2 \sin(\theta)d\theta d\phi \). The results of calculations according to each of the individual forms in GSS space-times will be presented in the following subsections.
2.2. General Spherically Symmetric (GSS) Space-time

Most general non-static spherically symmetric space-time is described by the following line element.

\[ ds^2 = B(r,t)dt^2 - A(r,t)dr^2 - 2F(r,t)dt\,dr - D(r,t)r^2(d\theta^2 + \sin^2 \theta \,d\phi^2). \]

(9)

Transforming the line element [9] which is in spherical coordinates \((t, r, \theta, \phi)\) into Cartesian coordinates \((t, x, y, z)\), we have (according to \(x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta\))

\[ ds^2 = B(r,t)dt^2 - 2F(r,t)(\frac{x}{r}dx + \frac{y}{r}dy + \frac{z}{r}dz)dt - D(r,t)(dx^2 + dy^2 + dz^2) - (D(r,t) - A(r,t))(\frac{x^2dx + y^2dy + z^2dz}{r}). \]

(10)

Using Einstein, Landau-Lifshitz, and Weinberg prescriptions for the metric [10], the energy for each EMP inside a 2-sphere with radius \(r\) is given by [21]

\[ eE = \frac{r}{2} \frac{B(A - D - D') - F(r \dot{D} - F)}{2\sqrt{AB + F^2}}, \]

(11)

\[ \mu E = \frac{r}{2} \frac{D(A - D - D')}{2}, \]

(12)

\[ wE = \frac{r}{2} \frac{(A - D - D')}{2} \]

(13)

where prime and dot denote the partial derivatives with respect to the coordinates \(r\) and \(t\), respectively. For static and spherically symmetric space-times \((F = 0; A, B, \text{ and } D\) only depend on \(r\) coordinate\) we have [21]

Using Möller prescription for line element [9] or [10], one can find [23]
It is obvious from Eqs.\textsuperscript{[11] [12] [13] [15]} that EMPs disagree for the most general non-static spherically symmetric space-time. It should be noted that in obtaining all above expressions for the energy, Schwarzschild Cartesian coordinate \((t, r, \theta, \phi)\) have been used, while using Kerr-Schild Cartesian coordinates for a general non-static spherically symmetric space-time of the Kerr-Schild class described with the line element

\[
d s^2 = B(u, r) du^2 - 2 du dr - r^2 (d \theta^2 + \sin^2 \theta d \phi^2)
\]

leads to the same result for Einstein, Landau-Lifshitz, and Weinberg prescriptions given by \textsuperscript{[21]}

\[
E = \frac{r}{2} (1 - B(u)).
\]

In Eq.\textsuperscript{(16)}, \(u\) coordinate is related to both \(t\) and \(r\) coordinates as \(u = r + t\).

According to Virbhadra in reference \textsuperscript{[21]} “It is not clear why different definitions coincide when calculations are carried out in Kerr-Schild Cartesian coordinates, but disagree in Schwarzschild Cartesian coordinates. At this stage it is not known if this is accidental or points out something interesting.”. This viewpoint is our main motivation to study some special cases of the line element \textsuperscript{[9]} and to investigate EMPs in Schwarzschild Cartesian coordinate if there is any coincidence between them in these cases.

3. SPECIAL CASES OF GSS

Considering Eq.\textsuperscript{(12)} and Eq.\textsuperscript{(13)}, it is obvious that Landau-Lifshitz and Weinberg prescriptions give the same result if and only if \(D = 1\). Moreover, if \(D = 1\), \(F = 0\), and \(B = A^{-1}\) we have \(\varepsilon E = \frac{2}{3} (1 - B)\). In addition, if \(F = 0\) and \(D = 1\) and we want to reach to the same results for Einstein, Landau-Lifshitz, and Weinberg prescriptions we should have \(A + B = 2\). In addition to \(F = 0\) and \(D = 1\) if \(B = A^{-1}\), then for coincidence of Einstein, Landau-Lifshitz, and Weinberg prescriptions we should have \(B = A = 1\) (flat space-time).

In the following subsections we consider two special but not unusual spherically symmetric space-times which lead to interesting results for energy distribution. In the first case we assume \(D = 1\), \(F = 0\), \(B = A^{-1} = (1 - f)^\mu\) and in the next one it is supposed that \(D = (1 - f)^{1 - \mu}\), \(F = 0\), and \(B = A^{-1} = (1 - f)^\mu\) where \(f\) is only a function of \(r\), i.e. \(f(r)\), and \(\mu\) is a constant number. Some examples of the first case are listed in Table\textsuperscript{[11]} As one can see in this Table, many important and well-known metrics belong to the above mentioned special case.

| Metric | \(f(r)\) |
|--------|---------|
| Schwarzschild | \(\frac{2m}{r}\) |
| Reissner-Nordström | \(\frac{2m}{r} - \frac{q^2}{r^2}\) |
| de-Sitter | \(\frac{4}{3} r^2\) |
| Schwarzschild de-Sitter | \(\frac{2m}{r} + \frac{4}{3} r^2\) |
| RN (anti)de-Sitter | \(\frac{2m}{r} - \frac{q^2}{r^2} + \frac{|\Delta|}{\mu^2}\) |
| Dymnikova \textsuperscript{[28]} | \(\frac{R_g(r)}{r}\) |
| ABG Black Hole \textsuperscript{[29]} | \(\frac{2m}{r} (1 - \tanh \frac{q^2}{2mr})\) |
| Conformal Scalar Dyon Black Hole \textsuperscript{[30]} | \(\frac{Q_m}{r}\) |
| De Lorenci \textsuperscript{[31]} | \(\frac{2m}{r} - \frac{q^2}{r^2} + \frac{4Q_m^2}{\mu^2}\) |
| Charged Topological Black Hole \textsuperscript{[32]} | \(1 + \frac{\Delta}{3} r^2 + (1 - \frac{Q_m}{r^2})^2\) |
| Bardeen’s Regular Black Hole \textsuperscript{[33]} | \(\frac{2m \xi^2}{(r^2 + q^2)^2}\) |

TABLE II: Some examples of a special case of GSS, i.e. \(D = 1\), \(F = 0\), \(B = A^{-1} = (1 - f)^\mu\) in Eq.\textsuperscript{(15)}

the two following subsections, using different EMPs we calculate the energy expressions and compare them with each other.

3.1. Case I, \(D = 1\)

Substituting \(D = 1\), \(F = 0\), \(B = A^{-1} = (1 - f)^\mu\) in the line element \textsuperscript{[10]} and considering Eqs.\textsuperscript{[11] [12] [13] [14] [15]} we find
The equivalence between the results remain and next terms disappear. So, we obtain Lorenci, Charged Topological Black Hole, and Bardeen’s ABG Black Hole, Conformal Scalar Dyon Black Hole, De Ber. Recent condition means that this metric belongs to reduce to $f = 0$. Substituting mentioned conditions in case I $(D = 1, F = 0, B = A^{-1} = (1 - f)^2)$ in Eq. it is not difficult to find that $f$ indicates a perturbation term from flat space-time. For small values of $f$ $(f \ll 1)$, Eqs. reduce to

$$eE = wE = L E = \frac{r}{2} \mu f$$

$$\nu E = \frac{r}{2} \mu f - \frac{r^2}{2} \mu^2 f f'$$

$$\mu E = -\frac{r^2}{2} (1 - (\mu - 1)f) \mu f'$$

The aforementioned equations are in full agreement with previous results about energy distribution of some specific spherically symmetric space-times with described conditions (see 18, 19, 21, 23, 27, 34). It is interesting to note that when $r \to \infty$ all of above expressions will be equal if

$$f = \sum_{n} C_n r^{-n}$$

where $C_n$ are constant and $n$ is a positive integer number. Recent condition means that this metric belongs to asymptotically flat space-times. Considering Table one can find that Schwarzschild, Reissner-Nordström, ABG Black Hole, Conformal Scalar Dyon Black Hole, De Lorenci, Charged Topological Black Hole, and Bardeen’s Regular Black Hole space-times in this table satisfy Eq. In this situation only first terms of Eqs. remain and next terms disappear. So, we obtain the following equivalence between the results

$$eE = L E = \nu E = M E = \frac{r}{2} \mu f.$$ (26)

3.2. Case II, $D = (1 - f)^{1-\mu}$

Assuming $D = (1 - f)^{1-\mu}$, $F = 0, B = A^{-1} = (1 - f)^2$, considering Eqs. 11, 12, 13, 14, 15, and using them in the line element 10, we find

$$eE = \frac{r}{2} (-f - rf' + \mu f')$$

and for $f \ll 1$ we have

$$eE = wE = \nu E = \frac{r}{2} (-f - rf' + \mu f').$$ (27)

It should be noted that for well-known Janis-Newman-Winicour (JNW) space-time in which $f = \frac{2M_s}{r}$, $\mu = \frac{M}{M_s}$ in far away points $(r \to \infty)$ we obtain a strong coincidence between different prescriptions as

$$eE = L E = \nu E = M E = M.$$ (28)

4. EINSTEIN AND MÖLLER PRESCRIPTIONS’ COINCIDENCE

From Eq. 11 and Eq. 15 it is obvious that Einstein and Møller expressions for the energy in GSS space-time are different, generally. In this section, supposing some conditions, we look for a suitable form for GSS metric in which Einstein and Møller prescriptions give the same result. For other forms of GSS metric Einstein and Møller EMPS give different expressions for energy.

Supposing $B(r) = A(r)^{-1}, F = 0$ in 10 for coincidence of energy expressions of Einstein 11 and Møller 15 we must have

$$r(1 - BD - D'Br) = r^2 DB'.$$ (29)

For a special but not unusual case, i.e. $D = f(r)^{1-\mu}$, $B = f(r)^\mu$, Eq. transforms to the following differential equation

$$1 - f - rf' = 0.$$ (30)

Solving above differential equation, we obtain

$$F(r) = 1 + \frac{C}{r}$$ (31)

where $C$ is the integration Constant. The above equation means that with the aforementioned conditions on GSS space-time, energy expressions in Einstein and Møller prescriptions are equal only if $B$ coefficient in line element 10 has a specific form, i.e.
\[ ds^2 = (1 + \frac{C}{r})^\mu dt^2 - (1 + \frac{C}{r})^{-\nu} dr^2 (37) - r^2(d\theta^2 + \sin^2(\theta)d\phi^2) \]

where for \( \mu = 1 \) (\( D = 1 \)) we obtain \( B = 1 + \frac{C}{r} \) (Schwarzschild like), and any other choices for \( B \) do not fulfill our purpose of Einstein and Møller prescriptions’ coincidence (see [23]).

5. SUMMARY AND CONCLUSION

Considering Einstein, Landau-Lifshitz, Papapetrou, Weinberg, and Møller EMPS, we reviewed the previous results about a 2-sphere with radius \( r \) in GSS space-time. It is obtained that these energy expressions are different in Schwarzschild Cartesian coordinates and coincide in Kerr-Schild Cartesian coordinates. We used Schwarzschild Cartesian coordinates and restricted ourselves to two special but not unusual GSS space-times and obtained interesting results for Einstein, Landau-Lifshitz, Papapetrou, Weinberg, and Møller EMPS. Our results can be considered as an extension of Virbhadra’s viewpoint to Schwarzschild Cartesian coordinates which say that different EMPS may provide same bases to define a unique quantity. Finally, we find a unique form for a special GSS metric in which Einstein and Møller prescriptions lead to the same result. For other forms of GSS space-time Einstein and Møller EMPS give different expressions for the energy.

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