Structural health monitoring: Frequency domain analysis of beam with breathing crack

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Abstract. Structural Health Monitoring (SHM) system can be considered as an Internet-of-Things (IoT) system implemented to engineering structure to provide real-time information on the integrity of the structure. In doing so, the system acquires data from various sensors distributed across the structure and infers damage sensitive quantities. In the research, we proposed a damage index derived from the governing dynamics of the structure in the frequency domain. The proposal was empirically verified by applying the index to analyze the case of a cracked cantilever beam subjected to a harmonic load. The value of the damage index was computed at some points surrounding the crack to understand the effect of the distance of the monitoring point to the crack to the index sensitivity. The results suggest that the index is sensitive to the crack particularly when the observation point is near the crack.

Keywords: structural health monitoring, Euler-Bernoulli beam, internet of things, finite element method, damage indicator

Introduction

Structural Health Monitoring (SHM) is a system integrating an engineering subsystem, a subsystem of sensors and data acquisition, a subsystem of information and communication technology, a database subsystem, and a structural management subsystem. The system is essential to provide a guarantee of operating engineering systems safely and reliably. Many engineering systems have deployed SHM including bridges [1–7] and aircraft [8–11]. Those subsystems are interrelated to set up a complete SHM system as illustrated in Fig. 1 [12]. The component that labeled `On-bridge' can be considered to be for any engineering structure of interest.
Figure 1. The Architecture of Structural Health Monitoring System [12].

The flow of information and assessment of the structural integrity occurs with the following sequence. It is all started from the sensor subsystem, labeled with SS. Generally, the subsystem consists of sensors for structural responses such as strain, acceleration, or displacement. The information is acquired and recorded temporarily by the Data Acquisition and Transmission Subsystem (DATS). An additional system called Portable Inspection and Maintenance Subsystem (PIMS) may also be incorporated.

The second major component is labeled 'Central Control Office', consisting of subsystems of Data Processing and Control Subsystem (DPCS), Data Management Subsystem (DMS), and Structural Evaluation Subsystem (SES). The latter subsystem is one of the most critical subsystems that translates the structural response data to structural integrity level, and damage location and level. The subsystem reduces the response data into features that are sensitive to structural damages for both location and level.

As for the damage sensitive features, many research articles have considered the structural natural frequencies and mode shapes [13–19]. Many recent publications proposed machine learning techniques for SHM [9, 10, 20–27]. Besides, the structural curvature, elastic energy, and power spectral density during vibration also widely used. Although each feature has its benefits and limitations, it is generally accepted that the detection is only possible when the damage has reached a certain level [28].

In the research, we propose a new damage sensitive index in the frequency domain. We derive the feature from the structural governing dynamics and is expected to be more sensitive to damage than the existing technique. Traditionally, in the design phase of engineering structures, the structural governing dynamics are only be utilized to estimate deformation and stress, making sure structures can be operated safely. In operation, the governing dynamics are being used to estimate the loads subjected to the structures, making sure that structures are not subjected beyond their critical loads. Thus, this paper offers a new application of the structural governing dynamics that is for monitoring of structural integrity. We demonstrate the index by analyzing a cracked beam specimen and compare the results with that by using the natural frequency method. The index was evaluated on an Euler-Bernoulli beam with damage smeared uniformly in the structure in [29] and on an idealized system in [30].

Research methods
In this section, we describe our proposal of the damage sensitive index or damage indicator. We present the index in two domains: time and frequency. We also describe the data for the evaluation of the proposal. We also explain all relevant aspects of the computation of the index. We begin with our proposal of the damage index.
We consider a beam structure which is a single-dimensional basic structure. For this structural type, Euler-Bernoulli beam theory was established to relate the applied load $q(x)$ with the beam lateral displacement $w(t, x)$. The relation is expressed in the form of the differential equation:

$$EI \frac{\partial^4 w(t, x)}{\partial x^4} = -\rho \frac{\partial^2 w(t, x)}{\partial t^2} + q(x).$$  \hspace{1cm} (1)$$

We note that the relationship is held if some conditions are fulfilled: the displacement is small such that the surface of the beam cross-section remains flat before and after the deformation and the section is also assumed to be perpendicular to the beam neutral plane upon deformation.

In Eq. (1), the symbol $E$ denotes the beam elastic modulus, $I$ denotes the second moment of area of the beam cross-section, and $\rho$ is the material density. The dynamic equilibrium equation is applicable for any point on the beam at any time. We hypothesize that a deviation from the condition may signify a change in either the material properties or the beam geometry or both. Corrosion or crack may reduce the beam cross-section. Thus, the change may reflect the deterioration in material integrity.

In the research, we use the Euler-Bernoulli theory not to predict a beam deformation, nor to predict the exerted force, but to estimate the beam integrity. For the purpose, we propose:

$$d(t, x) = \left| EI \frac{\partial^4 w(t, x)}{\partial x^4} + \rho \frac{\partial^2 w(t, x)}{\partial t^2} - q(x) \right|. \hspace{1cm} (2)$$

We hypothesize when the beam is intact, and $E$, $I$, and $\rho$ are assigned with values on the intact condition, the damage indicator $d(t, x)$ should be zero or very small. The indicator may shift from zero when the beam contains damages. We assume that damages alter the beam deformation $w(t, x)$. For SHM purposes, we have the freedom to choose the observation or measurement point. We may select the point where the external load is absent. Thus, it simplifies the computation of the damage indicator.

We also evaluate the index in the frequency domain. For the purpose, we apply the Fourier transform and obtain:

$$\hat{d}(\omega, x) = EI \frac{\partial^4 \hat{w}(\omega, x)}{\partial x^4} - \rho \omega^2 \hat{w}(\omega, x). \hspace{1cm} (3)$$

Equations (2) and (3) suggest the fourth-order differentiation of the displacement is required to compute the indexes. We compute the term $\partial^4 w/ \partial x^4$ by the finite difference approximation of:

$$\frac{\partial^4 w(x, t)}{\partial x^4} \approx \frac{w(x - 2h, t) - 4w(x - h, t) + 6w(x, t) - 4w(x + h, t) + w(x + 2h, t)}{h^4}, \hspace{1cm} (4)$$

where $h$ is taken as the element length.

As for the term $\partial^2 w/ \partial t^2$, we firstly fit the displacement time-history data with a cubic spline function and then take the first and the second derivatives of the function to provide the acceleration. The derivation of the cubic spline function is as the following.

We consider $n$ data points: $\{(t_1, s(t_1)), (t_2, s(t_2)), \ldots, (t_n, s(t_n))\}$. Strictly, we have the condition: $t_1 < t_2 < \cdots < t_n$. We fit the segment in $[t_j, t_{j+1}]$ with a third-order polynomial where $j$ is an index from 1 up to $n$. The polynomial can be written in the Newton form as:

$$p_j(t) = a_{1,j} + c_{2,j}(t - t_j) + c_{3,j}(t - t_j)^2 + c_{4,j}(t - t_j)^3. \hspace{1cm} (5)$$
To complete the polynomial, we should determine the values of the four coefficients: $c_{1,j}$, $c_{2,j}$, $c_{3,j}$, and $c_{4,j}$. They can be computed by:

\begin{align}
  c_{1,j} &= s(t_j), \\
  c_{2,j} &= m_j, \\
  c_{3,j} &= \left[\frac{t_{j+1} - t_j}{\Delta t_j}\right]s - c_{4,j}\Delta t_j, \\
  c_{4,j} &= \frac{m_j + m_{j+1} - 2[t_{j+1} - t_j]}{\Delta t_j^3}. \tag{9}
\end{align}

The term $m_j$ denotes the gradient at the point $t_j$. The terms $[t_i, ..., t_{i+k}]s$ denotes the $k$th divided difference of $s$ at the points $t_1, ..., t_{i+k}$. It can be computed by:

\[ [t_i, ..., t_{i+k}]s = \frac{[t_{i+1}, ..., t_{i+k}]s - [t_i, ..., t_{i+k-1}]s}{t_{i+k} - t_i} \]

Given the slopes $m_1$ at the first point $t_1$ and $m_n$ at the last point $t_n$, we can compute the gradients at the points $t_2, t_3, ..., t_{n-1}$ by solving a set of linear equations:

\[ \Delta t_j \cdot m_{j-1} + 2(\Delta t_{j-1} + \Delta t_j) \cdot m_j + \Delta t_{j+1} \cdot m_{j+1} = 3(\Delta t_j [t_{j-1}, t_j]s + \Delta t_{j-1} [t_j, t_{j+1}]s), \tag{10} \]

leading to the gradients $m_2, m_3, ..., m_{n-1}$. With these results, we establish the cubic splines for interpolation of the data $w(x, t)$ across the time domain.

We have discussed our proposal for the damage index in which the structural integrity can be evaluated. Now, we discuss our approach the obtain data to empirically evaluate our proposal.

For the current evaluation, we study the case of a breathing crack on a 2D beam. this model is adopted from Ref. [29]. The beam has a length of 300 mm, a width of 20 mm and a thickness of 20 mm. At its midsection, the beam has a crack with a length of 6 mm. The beam is made of steel material. As the reference did not provide the material data, we assume Young’s modulus of 207 GPa, Poisson ratio of 0.3, and density of 8050 kg/m$^3$. Later, we find our results of the natural frequencies are slightly different than the reference. We speculate it may be due to a difference in the material data. Thus, the data are generated from a more complex but realistic condition than that where the damage indicator is derived.

The beam is discretized into 6000 square-shape elements. Each element size is 1 mm. The model is built in Ansys, a finite element package, with the Plane183 element type. The type has 8 nodes in rectangular element shape. It has quadratic displacement behavior. Each node has two degrees of freedom: translations in the nodal $x$ and $y$ directions. The finite element model of the beam is shown in Fig. 2.

**Figure 2.** The Mesh of the Finite Element Model of the 2D Beam with a Crack. The beam is clamped on its left and its right side is subjected to a harmonic load $f(t)$. 

\[ \text{Fig. 2. The Mesh of the Finite Element Model of the 2D Beam with a Crack.} \]
As for the dynamic analysis, from which the model responses are recorded and used for the damage index analysis, the nonlinearity due to the crack is taken into account. On the overlapping nodes on the crack faces, gaps elements are build to allow the crack faces interacting in compression only. The element type for the gap elements is Conta178. The model is subjected to a sinusoidal force on the nodes along with the model right end, see Fig. 1. The forcing function is at the frequency of 18.153 Hz, about one-tenth of the natural frequency of the first mode. The analysis is performed for 128 ms. The structural displacement on nodes across the specimen surface is sampled at a rate of 0.5 ms.

Results
The main idea we wish to demonstrate in the research is a proposal for a new damage index. The index is proposed in the frequency domain as for the time domain, it has been discussed in our previous publication (see [29]). To achieve the objective, we apply the index to a case of a cracked beam and compare the present damage index with the most popular damage index based on the change of the natural frequency.

Firstly, we present the values of the proposed damage index across a range of frequencies. Second, we present the results of a modal analysis where we show the natural frequency of the beam on the conditions with and without crack.

By comparing those results, one should obtain an understanding of the performance of the present proposal concerning one of the best and most widely damage sensitive features.

The values of the current damage index for the cracked-beam are shown in Fig. 3. We compute the damage index for cracked condition as well for the healthy condition. We compute it on various points of observation such that we may assess how the distance contributes to the sensitivity of the index to crack.

![Figure 3. The Computed Frequency-Domain Damage Index at a Number of Locations for the Beam with and without Crack.](image)

From the eleven observation points, distributed evenly from $x = -50$ mm to $x = 50$ mm with uniform spacing between points of 10 mm, we witness that the values of the index are very small when the beam contains no crack. As for the condition with crack, the index values are much higher except for the measurement points far from the crack at $x = \pm 50$ mm. The sensitivity of the index to crack
diminishes with distance from the crack where the most sensitive response is obtained at the measurement point above the crack.

The sensitivity of the index to crack is also remarkably high at two frequencies: 181.5 Hz and 18.2 Hz. The first frequency is associated with the first mode. The second is the loading frequency.

The second results are the changes in the natural frequencies due to the crack. We present the natural frequency data in Table 1 for the first five frequencies. The mode shapes associated with those frequencies are presented in Fig. 4. We note that the natural frequency and the mode shape are obtained from a modal analysis with the assumption that the system is linear and the interaction between the crack faces is ignored. The changes in the natural frequencies, presented in the last column of the table, are very small. The largest is around 5%, associated with the second mode shape. This mode is associated with the condition of a large bending deformation on the crack position.

As the final note, we conclude that the present damage index is far more sensitive to crack as a large crack extension that reduced the beam cross-section area by 30% is only to change the natural frequency by a small amount, which may hardly detectable in actual measurement.

### Table 1. The Natural Frequencies in Hz of the Beam with and without crack

| Modes | Healthy          | Damaged          | Current Change (%) |
|-------|------------------|------------------|--------------------|
|       | Current | FE. [29] | Exp. [29] | Current | Exp. [29] | Change |
| 1     | 181.5    | 185.1    | 185.2    | 179.0    | 174.7    | 1.4    |
| 2     | 1115.0   | 1159.9   | 1160.0   | 1052.9   | 1155.3   | 5.6    |
| 3     | 3028.9   | 3247.6   | 3245.0   | 3027.9   | 3134.8   | 3.4    |
| 4     | 4229.1   |          |          | 4156.5   |          | 1.7    |
| 5     | 5700.4   |          |          | 5453.9   |          | 4.3    |

### Figure 4. The Mode Shapes of the First Five Frequencies with and without Crack
Conclusion
In the research, we have discussed a new damage index derived from the governing dynamics of a beam structural member. The index is verified in the sensitivity to a crack by using empirical data collected by a finite element simulation of a cracked beam. Insight about the sensitivity and the influence of the position of the observation points are discussed in a great length. As for a future research topic, we suggest comparing the index sensitivity with the case of the time domain.

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