From Pebbles and Planetesimals to Planets and Dust: The Protoplanetary Disk–Debris Disk Connection

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Abstract

The similar orbital distances and detection rates of debris disks and the prominent rings observed in protoplanetary disks suggest a potential connection between these structures. We explore this connection with new calculations that follow the evolution of rings of pebbles and planetesimals as they grow into planets and generate dusty debris. Depending on the initial solid mass and planetesimal formation efficiency, the calculations predict diverse outcomes for the resulting planet masses and accompanying debris signature. When compared with debris disk incidence rates as a function of luminosity and time, the model results indicate that the known population of bright cold debris disks can be explained by rings of solids with the (high) initial masses inferred for protoplanetary disk rings and modest planetesimal formation efficiencies that are consistent with current theories of planetesimal formation. These results support the possibility that large protoplanetary disk rings evolve into the known cold debris disks. The inferred strong evolutionary connection between protoplanetary disks with large rings and mature stars with cold debris disks implies that the remaining majority population of low-mass stars with compact protoplanetary disks leaves behind only modest masses of residual solids at large radii and evolves primarily into mature stars without detectable debris beyond 30 au. The approach outlined here illustrates how combining observations with detailed evolutionary models of solids strongly constrains the global evolution of disk solids and underlying physical parameters such as the efficiency of planetesimal formation and the possible existence of invisible reservoirs of solids in protoplanetary disks.

Unified Astronomy Thesaurus concepts: Protoplanetary disks (1300); Debris disks (363); Planet formation (1241); Planetesimals (1259); Circumstellar matter (241)

1. Introduction

Stars form surrounded by disks, the material from which planets form. Over the first million years (or so) in the life of a disk, its solids are aggregated into planetesimals and protoplanets, and eventually into planets. The first step in this transformation may be hastened as disk solids collect in inhomogeneities—ring-like pressure bumps and other features—triggering processes such as the streaming instability, which create planetesimals. The planetesimals eventually grow into planets, which shape the disk gas and dust, producing rings, gaps, inner holes, and other structures. The concentration of solids in rings may spur further planetesimal (and planet) formation. After the disk gas dissipates, residual planetesimals “left behind” in the planet formation process may eventually reveal themselves, as they collide and produce disks of debris that glow in the reprocessed light from the central star.

Some of the most dramatic evidence in support of this picture comes from the millimeter continuum morphologies of Class II (protoplanetary) disks, many of which show the substructure that forming planets are expected to induce. When imaged at millimeter wavelengths and in scattered light, large disks (radii $\gtrsim$25–30 au) show rings, gaps, central cavities, and other features at distances of 20–200 au from the star (e.g., Avenhaus et al. 2018; Huang et al. 2018; Cieza et al. 2019; Long et al. 2019, and references therein). While central cavities have been imaged for more than a decade (e.g., Andrews et al. 2010; Andrews 2015), the realization that narrow rings and gaps are common features of disks at large radii is a discovery of the Atacama Large Millimeter/submillimeter Array (ALMA) era (Partnership et al. 2015). Whereas disk central cavities are thought to be created by a high-mass giant planet that orbits within the cavity, the narrow rings and gaps can be created by lower-mass (approximately Neptune-mass) planets (e.g., Bae et al. 2018; Lodato et al. 2019).

Emission associated with orbiting gas giant planets has also been detected in disks with central cavities through direct imaging techniques (Sallum et al. 2015; Kepller et al. 2018; Haffert et al. 2019; Zurlo et al. 2021) and spectro-astrometry (e.g., Brittain et al. 2019). However, the lower-mass ice giants thought to be responsible for narrow rings and gaps at large radial distances ($\gtrsim$25–30 au) remain beyond our ability to detect directly. Because gas and ice giants at such large orbital radii have no counterpart in the solar system, these protoplanetary disk structures appear to point to an even greater diversity of planet formation outcomes than previously contemplated.

Another valuable clue in support of this picture, that planetesimals—the hypothesized building blocks of planets—commonly form in protoplanetary disks, comes from debris disks, the dusty debris that is found around some post-T Tauri and main-sequence stars. Best explained as the result of collisions between large parent bodies, and identified by their infrared and millimeter excesses, debris disks accompany stars over a wide range of ages. At ages 10 Myr to beyond 1 Gyr, approximately 20% to 25% of FGK stars have detected cold
excesses at \( \sim 100 \mu m \) (e.g., Bryden et al. 2009; Carpenter et al. 2009; Eiroa et al. 2013; Sibthorpe et al. 2018), corresponding to debris at distances of 40 au. When imaged at high angular resolution, the debris also commonly shows substructure such as rings and gaps (e.g., Hughes et al. 2018; Marino et al. 2018, 2020; Nederland et al. 2021).

Here we explore the possible evolutionary connection between the rings and gaps observed in protoplanetary disks and those in debris disks. Given that both protoplanetary disks and debris disks show structured continuum emission in the form of rings and gaps over similar radial distances (20–200 au) around approximately solar-mass stars, it seems plausible that large protoplanetary disks evolve into the known cold debris disk population. We complement related work on this topic (e.g., Michel et al. 2021) using new models of the evolution of rings of solids on million-year to billion-year timescales. The new ring models contrast with earlier generations of models that explored extended disks of solids as the origin of debris disk emission (e.g., Kenyon & Bromley 2008, 2010).

Using the new models, we examine how the efficiency of planetesimal formation affects not only the outcome of planet formation, but also its associated debris production. By comparing the model results to observations of debris disk populations, we explore the disk conditions (total initial mass of solids, planetesimal formation efficiency) that can reproduce the properties of debris disks (their incidence rate and luminosity) as a function of stellar age. We also use the comparison to explore questions such as whether cold debris disks feature in the evolutionary histories of all disks or only a special subset.

In Section 2, we review observations that link protostellar disks to debris disks. After a brief review of planetesimal formation in Section 3.1, we set up a suite of numerical calculations in rings with populations of small and large solids (Section 3.2) and describe the formation of planets (Section 3.3), debris disks (Section 3.4), and gaps (Section 3.5). After discussing the calculations in the context of observations and other models (Section 4), we conclude with a brief summary (Section 5).

2. Properties of Protoplanetary Disks and Debris Disks

2.1. Protoplanetary Disks

The millimeter continuum emission from large, bright protoplanetary disks (continuum sizes \( \gtrsim 50 \) au) is often highly structured. In deep, high angular resolution ALMA images of the brightest disks in nearby star-forming regions, the continuum emission typically arises from multiple concentric rings with radii of \( \sim 10–150 \) au, widths of a few au to tens of au, and dust masses of \( 10–70 \, M_\odot \) (Partnership et al. 2015; Huang et al. 2018; Long et al. 2018; Dullemond et al. 2018). Within larger samples of protoplanetary disks that span a wider range in disk properties, including lower millimeter continuum flux, large resolved structures are less common. Of 147 disks in Ophiuchus studied by the ODISEA survey, the great majority have millimeter continuum emission restricted to radial distances \( <15 \) au. Only \( \sim 20\% \) of the sources show continuum emission larger than 20 au in radius (see Figure 12 of Cieza et al. 2019). In a sample of 32 disks in Taurus–Auriga that cover a broad range of continuum brightness, only a modest fraction of disks shows large rings (Long et al. 2019). Nine of the 27 sources studied with stellar masses \(<1.6 \, M_\odot \) have rings with effective millimeter continuum sizes \( >40 \) au; the remainder have more compact emission.

To extrapolate the results of Long et al. (2018, 2019) and estimate the fraction of Taurus sources with large rings, we consider what is known about the entire Taurus disk population (e.g., Luhman et al. 2010; Akesson et al. 2019). We follow the rough selection criteria of Long et al. (2019) and select the Taurus Class II sources with spectral types M3 or earlier and no companions at angular separations of 0″14–1″. We also impose an upper stellar mass limit of 1.6 \( M_\odot \) to mirror the upper stellar mass limit of the debris disk samples. With these criteria, there is a parent sample of 77 sources. The fraction with large rings in this sample is at least 9/77 = 12%.

The actual fraction is likely to be larger. Beyond the 32 disks studied by Long et al. (2019), 17 additional Taurus T Tauri stars were known to have ALMA observations at the time the Long et al. paper was written. Several of these sources have rings on large scales, e.g., AA Tau (Loomis et al. 2017), LkCa 15 (Facchini et al. 2020), and DM Tau (Hashimoto et al. 2021). Others show smooth emission without rings, e.g., CW Tau, CY Tau, and DG Tau (Simon et al. 2017; Bacciotti et al. 2018). At least one source is compact without large rings (CX Tau; Facchini et al. 2019), and five other sources have binary companions within 1″ and would be excluded from our sample. The morphology of the remaining five sources is unknown (unpublished).

As a result, of the 17 additional sources, three definitely have large rings; the remaining five sources of unknown morphology may also have large rings. If all eight of these sources do in fact have large rings, the fraction of Taurus Class II sources with large rings could be as high as 17/77 or \( \sim 22\% \). The true fraction of disks with large rings is larger if other (as yet unobserved) sources among the parent sample of 77 sources also have rings.

At the same time, a robust estimate of the fraction of young stars with large rings requires that we also account for the lack of rings among the population of (diskless) Class III sources with similar ages, stellar masses, and companion properties as the Class II sources. As a group, the Class III sources represent 25% of all T Tauri stars in Taurus (e.g., Luhman et al. 2010; Luhman 2018; Esplin & Luhman 2019). Accounting for these diskless sources reduces the fraction of young stars with large rings by a factor of \( \sim 3/4 \). In summary, the fraction of young stars in Taurus with large rings is plausibly \( \sim 17\% \), although with significant uncertainty (J. Bae & A. Isella, 2021, private communication). Further ALMA imaging of a larger sample is needed for a robust estimate. Taking a similar approach, van der Marel & Mulders (2021) estimated a fraction of \( \sim 16\% \) of structured disks in a sample study of almost 700 disks in nearby star-forming regions.

Figure 1 illustrates the range of sizes and widths of continuum emission rings observed in T Tauri disks (primarily from the DSHARP sample; Table 1 of Huang et al. 2018). In the left side of the diagram, the purple bars indicate the radial extent of bright rings; the blue bars indicate the rough radial extent of the dust continuum emission from the disk. The high angular resolution of the DSHARP observations (30–60 mas; equivalent to 5–8 au at the distance of the targets) probed structures at much smaller angular scales than was possible with the lower-resolution, snapshot ALMA observations of Long et al. (2019; \( \sim 120 \) mas or \( \sim 16 \) au).
2.2. Debris Disks

Spatially resolved images of debris disks also show rings in scattered light and thermal emission (e.g., Hughes et al. 2018). Figure 1 shows the radial extents of debris disks (light-colored bars) and their rings (dark-colored bars), as measured by direct imaging (Hughes et al. 2018). As shown in Figure 1, much of the emission from debris arises from within \( a = 10–150 \, \text{au} \) of the star, similar to the orbital distances from which the millimeter continuum arises from protoplanetary disks. The fractional widths of the rings are typically \( \Delta a/a = 0.1–0.6 \), with \( \Delta a \) typically ranging from a few au to 30 au, overlapping the range of widths of protoplanetary disk rings.

While fine substructure (i.e., narrow rings) is reported more commonly for protoplanetary disks than debris disks, only a few debris disks have been imaged with sufficient sensitivity and angular resolution to detect such substructure (see discussion in Marino et al. 2020; Nederlander et al. 2021). Of the six sources studied to date at high sensitivity and angular resolution, four show gaps in their continuum emission, suggesting that finer substructure may be common among large debris disks, as in protoplanetary disks.\(^4\) Future ALMA imaging is needed to explore this possibility.

Surveys of varying sensitivities to dust temperature and fractional luminosity have reported detection rates of debris disk emission. As described in Hillenbrand et al. (2008), the FEPS Survey used the Spitzer Space Telescope to carry out a census of cool dust surrounding 328 solar-type stars in the age range 3 Myr to 3 Gyr located at distances of 10–200 pc. Over the age range 30 Myr to 3 Gyr, \( \sim10\% \) of solar-type stars show evidence for cold debris, with their rising spectral energy distributions toward 70 \( \mu \text{m} \) indicating dust temperatures of \( \sim45–85 \, \text{K} \) for dust in equilibrium with the stellar radiation field. The detected excesses are bright at young ages, with fractional excess luminosities of \( L_d/L_\ast \sim 10^{-3}–10^{-4} \) at 30 Myr, declining with increasing stellar age to \( L_d/L_\ast \sim 10^{-4} \) at 3 Gyr.

While FEPS focused on the excess properties of FGK stars younger than 3 Gyr, other Spitzer surveys investigated the excess properties of older stars. Bryden et al. (2009) reported the excess properties of planet-bearing stars with spectral types F5–K5 that are known from radial velocity studies to harbor one or more planets; the majority of the stars are 4–10 Gyr old where, as in the case of most of the FEPS sources, the ages are based on chromospheric activity and the calibration of Mamajek & Hillenbrand (2008). The excess properties of the sample are statistically indistinguishable from those of a comparison sample of comparable nearby stars (i.e., similar in spectral type and age) without known planetary companions.

To illustrate the limits placed by these Spitzer results on possible evolutionary paths for disk solids, the upper panel of Figure 2 shows detections (purple dots) and upper limits (smaller gray dots; Bryden et al. 2009; Carpenter et al. 2009) as a function of stellar age. Protoplanetary disk sources in the FEPS sample are excluded. Only the FEPS sources within 80 pc are shown to highlight the value of the constraints placed by the upper limits on the nearby sample. The full FEPS sample spans a large range in distance (out to \( \sim150 \) pc), and upper limits on the distant sources are weak. For the few sources in common between the FEPS and Bryden et al. (2009) samples (HD38529 and HD150706), we adopted the excess properties reported by FEPS. A few sources with unusually high flux uncertainties and upper limits were removed from the Bryden et al. sample (HD 4203, HD 46375, HD168746, and HD330075).

To obtain the FEPS upper limits shown, we converted the 70 \( \mu \text{m} \) flux upper limits (typically \( \sim10 \, \text{mJy, } 1\sigma \)) to upper limits on \( L_d/L_\ast \) assuming a typical ratio of \( r = (L_d/L_\ast)/(F_d/F_\ast) = 10^{-5} \) a factor appropriate for the typical temperature of detected far-IR excesses (60 K; Hillenbrand et al. 2008). The conversion factor is insensitive to temperature in the range 40–80 K (e.g., Figure 9 of Hillenbrand et al. 2008). Following Carpenter et al. (2009) for both the FEPS sources discussed here and the Herschel sources discussed below, the plotted upper limits in Figure 2 of \( L_d/L_\ast = r(F_d/F_\ast + 3\sigma) \) use the reported value of \( F_d/F_\ast \) when its value is \( >0 \) and 0 otherwise. The upper limits shown for the Bryden et al. (2009) survey are from their paper.

Following the Spitzer studies, the DEBRIS and DUNES surveys searched for infrared excess emission at 100 \( \mu \text{m} \) and 160 \( \mu \text{m} \) with the Herschel Space Observatory (Eiroa et al. 2013; Sibthorpe et al. 2018). Both programs observed several sources at 70 \( \mu \text{m} \); DUNES acquired additional data at 250–500 \( \mu \text{m} \). The surveys targeted 275 (DEBRIS) and 133 (DUNES) unique FGK stars with distances within 25 pc (DEBRIS; Sibthorpe et al. 2018) and 20–25 pc (DUNES; Eiroa et al. 2013). Stellar activity ages range from 1 Myr to 11 Gyr (100 Myr to 10 Gyr) for DEBRIS (DUNES), with median ages of \( \sim3 \, \text{Gyr} \).

The DEBRIS (DUNES) survey detected excess emission from debris disks around 47 (31) stars for a nominal detection rate of 17% (23%). In DEBRIS, the detection rate is similar across the age bins 0.1–1 Gyr, 1–3 Gyr, and 3–10 Gyr. Corrected for incompleteness, the incidence rate for FGK stars
\[ \text{Figure 1. Comparison of the observed positions of bright rings in the continuum emission from protoplanetary disks around T Tauri stars (left side; Huang et al. 2018) to bright rings within extended dusty debris disks surrounding FGK main-sequence stars (right side; Hughes et al. 2018). For each vertical bar, light (dark) regions indicate the extent of the disk (rings). Disks around main-sequence stars are ordered by age, from 12 Myr for star 24 (HD 146897) to 8.2 Gyr for star 48 (\upsilon \text{Cen}). Colors denote the spectral type—F (green), G (gold), or K (orange)—of the central star. The vertical gray dashed line separates pre-main-sequence (PMS) from main-sequence (MS) stars. The horizontal gray bands represent the two grids used in the numerical calculations described in Section 3.} \]
is \( \sim 28\% \). For DUNES, the volume-limited detection rate is \( \sim 20\% \) and is independent of spectral type for FGK stars (see also Montesinos et al. 2016).

For the combined set of DEBRIS and DUNES detections, the median \( L_d/L_* \) is roughly an order of magnitude smaller than the median dust luminosity of the Spitzer detections. For both Herschel programs, blackbody dust temperatures have a broad range, 16–300 K for DEBRIS and 20–100 K for DUNES; the median dust temperatures are 48 K (DUNES) and 63 K (DEBRIS). Inferred radii for the dust are 1–300 au with a median \( \sim 20 \) au for DEBRIS and 7–200 au with a median of \( \sim 30 \) au for DUNES. Assuming realistic dust properties would place the emission at larger radii.

The lower panel of Figure 2 shows the Herschel detections (green dots) and upper limits (smaller gray dots) for the 43 sources with FGK spectral types (effective temperatures of 4000–7200 K) and ages as estimated from stellar activity (e.g., Vican 2012). To translate flux detection upper limits to \( L_d/L_* \), the reported 3\( \sigma \) upper limit on \( F_d/F_* \) at 100 \( \mu \)m was converted to \( L_d/L_* \) assuming a ratio \( r = (L_d/L_\star)/(F_d/F_\star) \approx 10^{-5.4} \), the value appropriate for an excess at 100 \( \mu \)m that has a temperature of \( \sim 55 \) K, the median temperature of detected excesses. The actual adopted value of \( r \) is appropriate for the stellar temperature of each source.

To illustrate a larger range of excess properties observed among debris disks, Figure 3 supplements the Spitzer and Herschel detections (purple and green dots, respectively) with those from Matrà et al. (2018), which compiles properties of (bright) debris disks that have been spatially resolved at millimeter wavelengths. Sources with FGK spectral types not included in the Spitzer (Carpenter et al. 2009; Bryden et al. 2009) and Herschel (Eiroa et al. 2013; Sibthorpe et al. 2018) samples are shown. The gray arrows show the median upper limits from FEPS at ages \( \leq 100 \) Myr and from Herschel at ages \( \geq 100 \) Myr in 0.1 intervals of log stellar age.

Limits on \( L_d/L_* \) for younger stars with ages \( \leq 4–5 \) Myr are rare. Lovell et al. (2021) detected one source with cold debris in an ALMA survey of six class III T Tauri stars that are likely members of the 1–3 Myr Lupus association; the other sources in the Lovell et al. (2021) study are likely to be members of the Sco-Cen Association (Michel et al. 2021). Future observations of class III T Tauri stars in other star-forming regions would improve links between class II sources with \( L_d/L_* \approx 3 \times 10^{-2} \) (e.g., Michel et al. 2021) and debris disks with \( L_d/L_* \approx 10^{-2} \).

Finally, parallax data from the Gaia satellite have revolutionized our knowledge of nearby moving groups of young stars (e.g., Faherty et al. 2018; Gagné et al. 2018, 2020; Ujjwal et al. 2020), leading to new membership catalogs that allow for better probes of the frequency of cold debris disks in the 20–150 Myr age range where the Spitzer and Herschel surveys have poor statistics. Among the \( \sim 10 \) F stars within the 20–25 Myr old \( \beta \) Pic moving group, 50% (75%) have dusty material at \( a \geq 40 \) au (\( a \geq 1 \) au; Pawellek et al. 2021). For F stars in older moving groups (20 stars in the Tucana/Horologium association and the Columba and Carina groups), the cold debris disk frequency declines to \( \sim 30\% \) at 45 Myr and \( \sim 15\% \) at 150 Myr.5 However, few of the new 45–150 Myr old debris disks have \( a \geq 30 \) au.

With \( L_d/L_* \approx 2 \times 10^{-4} \) to \( 2 \times 10^{-3} \), the cold debris disks in the \( \beta \) Pic moving group have dust luminosities similar to the \( L_d/L_* \) of stars with ages of 10–40 Myr in Figure 3. The new systems with ages of 40–50 Myr from other moving groups have lower dust luminosities, \( L_d/L_* \approx 2 \times 10^{-5} \) to \( 10^{-4} \), which

\[5\] At 45 Myr (150 Myr), six of 20 (one of seven) stars in these samples have blackbody radii larger than 10 au.
These data thus follow the observed trend of decreasing L_d/s with stellar age. Continued analysis of cold debris in 30–100 Myr old stars would provide essential connections between the younger more luminous debris disks and those much older than 100 Myr.

To summarize, debris disk detections fall along a broad swath in L_d/L_\ast and decline with time (Figure 3). The most stringent upper limits in L_d/L_\ast are \lesssim 10^{-5} at ages of 0.1–10 Gyr (from the Herschel surveys), and \sim 10^{-4} at 10–100 Myr (from FEPS). Current data suggest the frequency of cold debris disks is roughly constant at \sim 25% for stellar ages of \sim 50 Myr to 10 Gyr. Within the much younger \beta Pic moving group (\sim 20–25 Myr), the frequency may be higher, \sim 50%, based on the study of a small sample of stars. As discussed below, future observations that lead to greater certainty in the debris disk frequency and L_d/L_\ast for stars with ages of 20–150 Myr will bear on the variety of ways in which protoplanetary disks evolve into debris disks (Section 4).

3. Evolution of Rings of Solids

To understand whether the rings of solids observed at a \approx 20–200 au in young stars could plausibly evolve into the rings of debris detected at similar a in much older stars, we perform a suite of multi-annulus coagulation calculations. For the geometry of the rings, we rely on the observed properties outlined in Section 2. In previous calculations, Kenyon & Bromley (2008, 2010) considered the evolution of swarms of 1–100 km planetesimals in disks extending from 30–150 au. These calculations matched the time evolution of available data for L_d/L_\ast rather well. However, for disks with the largest L_d/L_\ast, the models predict that dust is produced at increasingly large distances from the central star at late times. This trend is not observed among known debris disks (e.g., Najita & Williams 2005; Kennedy & Wyatt 2010; Matthews et al. 2014; Hughes et al. 2018; see also Figure 1). Moreover, theory currently favors scenarios where planets grow in seas of small and large solids (see below). Together, these observational and theoretical developments motivate an updated set of models of debris production. To support our choices for the initial mix of pebbles and planetesimals, we briefly review recent theoretical results. Following this summary, we outline the numerical procedures and then describe results of new calculations.

3.1. Background

In the core accretion model, planet formation is a three-step process. Within a circumstellar disk of gas and dust, micron-sized dust grains grow into centimeter-sized pebbles, then kilometer-sized or larger planetesimals, then planets. Within each step, various uncertainties in the initial conditions, the physical properties of the gas and solids, and the important chemical and physical processes prevent a robust understanding of the path from grains to planets. For this discussion, we summarize the current picture of planet formation, highlighting several areas of significant uncertainty that we explore in our models.

Initially, the gas and dust are well mixed (e.g., Chiang & Youdin 2010; Youdin 2010; Youdin & Kenyon 2013, pp. 1–51; Liu & Ji 2020). As the disk evolves, small grains collide slowly, stick together, and grow into larger and larger aggregates (e.g., Dominik & Tielens 1997; Wurm & Blum 1998; Blum & Wurm 2008; Birnstiel et al. 2016; Nimmo et al. 2018). As growth proceeds, particles compact significantly (e.g., Weidling et al. 2009). Particles with larger filling factors are less well-coupled to the gas. When grains uncouple from the gas, they settle to the midplane and collide at higher velocities. Interactions between particles become more elastic, which limits additional growth (e.g., Zsom et al. 2010; Kelling et al. 2014; Kruss et al. 2017). Detailed studies suggest that particles experience a “bouncing barrier” at sizes \sim 1–10 cm beyond which agglomeration effectively ceases (see also, Brauer et al. 2008; Windmark et al. 2012; Gandlach & Blum 2015; Kruss & Wurm 2020; Teiser et al. 2021).

Although the presence of charged or organic grains may circumvent the bouncing barrier (e.g., Homma et al. 2019; Steinpilz et al. 2019), recent analyses have concentrated on the “streaming instability” as a way to generate kilometer-sized or larger planetesimals from ensembles of centimeter-sized ‘pebbles’ (Youdin & Goodman 2005). In this mechanism, aerodynamic drag concentrates pebbles into clumps with large over-densities compared to the typical solid-to-gas ratio throughout the disk (e.g., Johansen & Youdin 2007; Johansen et al. 2007, 2009). Continued concentration of pebbles within the clumps enables the formation of planetesimals with radii r \approx 100–1000 km (e.g., Birnstiel et al. 2016; Simon et al. 2016; Schäfer et al. 2017; Yang et al. 2017; Li et al. 2018; Sekiya & Onishi 2018; Lenz et al. 2019; Liu et al. 2019; Li et al. 2019; Chen & Lin 2020; Umurhan et al. 2020; Pan & Yu 2020; Gerbig et al. 2020; Squire & Hopkins 2020).

In recent numerical studies of the streaming instability, the size distribution of the largest planetesimals and the efficiency of planetesimal formation depend on the physical conditions of the gaseous disk and the size distribution of pebbles (e.g., Simon et al. 2016; Li et al. 2018; Abod et al. 2019; Carrera et al. 2020; Klähre & Schreiber 2020; Gole et al. 2020; Rucska & Wadsley 2021). Large solid-to-gas ratios generated by radial drift and low turbulence (\alpha \approx 10^{-4}) favor efficient concentration of mono-disperse (i.e., single-sized) sets of pebbles into much larger solids. High-turbulence (\alpha \gtrsim 10^{-3}), smaller solid-to-gas ratios, and broader size distributions of pebbles appear to limit the ability of the streaming instability to form large planetesimals (however, see also McNally et al. 2021). Among calculations with identical starting conditions, local fluctuations in these and other physical conditions within the disk lead to variations in the maximum size r_{\text{max}} of a planetesimal and the fraction f of the initial solid mass in pebbles that is concentrated into massive planetesimals.

Among other options for planetesimal formation, such as turbulent clustering (e.g., Cuzzi et al. 2008; Pan et al. 2011; Hartlep & Cuzzi 2020) and the settling instability (Squire & Hopkins 2018), outcomes for f and r_{\text{max}} are also uncertain. For any instability mechanism, local chemistry, radial diffusion, and sublimation modify the growth of pebbles and the concentration of pebbles into planetesimals (e.g., Ida & Guillot 2016; Hyodo et al. 2019). Prior to the onset of instability, the porosity and compactness of pebbles are also uncertain (e.g., Okuzumi et al. 2012; Kataoka et al. 2013).

Once planetesimals form, the path to protoplanets is more certain. In systems with f \approx 1 (completely efficient planetesimal formation), the growth of massive protoplanets may be too slow to form super-Earth-mass and larger planets during the likely lifetime of the gaseous disk (e.g., Kenyon & Bromley 2008; Lissauer et al. 2009; Kenyon & Bromley 2009, 2010;
Kobayashi et al. 2010; Levison et al. 2010; D’Angelo et al. 2014; Mordasini et al. 2015; Bodenheimer et al. 2018; D’Angelo et al. 2021). From analytical considerations, Goldreich et al. (2004) and Rafikov (2005) demonstrated that a few large planetesimals in a sea of pebbles grow rapidly due to the small scale height of the pebbles. Subsequent numerical calculations of “pebble accretion” yield $10M_\oplus$ and larger ice giants on timescales of a few million years (Kenyon & Bromley 2009; Bromley & Kenyon 2011a; Lambrechts & Johansen 2012). More recent studies with $r_{\text{max}} \gtrsim 100$ km and $f \lesssim 10^{-2}$ illustrate the ability of pebble accretion to form ice and gas giants in many circumstances on short timescales (e.g., Matsumura et al. 2017; Alibert et al. 2018; Lin et al. 2018; Bitsch et al. 2019; Johansen & Bitsch 2019; Lambrechts et al. 2019; Klahr & Schreiber 2020; Morbidelli 2020; Voelkel et al. 2020; Chambers 2021; Kobayashi & Tanaka 2021).

In contrast to the many studies of giant planet formation via pebble accretion, there have been few attempts to investigate the long-term evolution of the debris signatures produced by growing protoplanets in a sea of pebbles. Formation scenarios for a planet nine at $a > 200$ au in the solar system illustrate how systems with an initial $r_{\text{max}} = 100$ km and various $f$ generate super-Earth-mass planets and very luminous debris disks at 200–750 au around solar-type stars on timescales of 0.1–1 Gyr (Kenyon & Bromley 2015, 2016a). The debris disks in some model systems have properties similar to those observed in the bright debris disks orbiting HD 107146, HD 20628, and HD 207129 (Corder et al. 2009; Krist et al. 2010, 2012; Marshall et al. 2011; Ricci et al. 2015; Marino et al. 2018). In the next sections, we consider whether swarms of pebbles and planetesimals can produce debris disks similar to those in the Herschel and Spitzer samples described in Section 2.

### 3.2. Initial Conditions

To follow the evolution of pebbles and planetesimals in a ring, we use the multi-annulus coagulation routine within Orchestra, an ensemble of codes for planet formation. As outlined in The ORCHESTRA Code the Appendix A, the code uses a particle-in-a-box algorithm for collision rates and energy scaling for collision outcomes. Particles evolve dynamically with Fokker–Planck routines. To avoid the extra free parameters associated with the gaseous component of the ring, we ignore radial drift and condensation of solids by the gas. Kenyon & Bromley (2008, 2010, 2012) describe the formulation and procedures in more detail.

For this study, we perform calculations in two separate grids, each with 28 concentric annuli. With an inner radius of 36 au (60 au) and an outer radius of 54 au (90 au), the grids cover a reasonable subset of the rings observed in protoplanetary and debris disks (Figure 1). Within each annulus, particles occupy distinct mass bins with sizes ranging from 1 $\mu$m to 10$^3$ km, orbital eccentricity $e$, and inclination $i$. Initially, solids have sizes of 1 cm (residual pebbles) and 100 km (planetesimals produced by the streaming instability), eccentricity $e_0 = 10^{-3}$, and inclination $i_0 = e/2$. These parameters are appropriate for solids recently liberated from a protostellar disk with turbulence parameter $\alpha \sim 10^{-3}$.

Solids initially have a total mass $M_0$, a surface density distribution $\Sigma(a)$, and a fraction $f$ of mass in large planetesimals (see Section 3.1). To select these parameters, we rely on the observational constraints described above and shown schematically in Figure 4. Among class II sources with ages $\sim 1$ Myr, roughly a quarter have a compact disk with an outer radius of $\sim 30$ au and bright rings of solids at larger distances (Figure 4, top left; see also Figure 1). The rings are well fit with Gaussian distributions of pebbles having total masses $\sim 10–60 M_\oplus$ and dispersions $\sigma \approx 4–8$ au (Dullemond et al. 2018). To span this range, we adopt upper limits of $M_0 = 10 M_\oplus$ at 45 au and $M_0 = 45 M_\oplus$ at 75 au. For $\Sigma(a)$, we adopt a fixed ratio $\sigma/a = 1/15$; solids then have a Gaussian $\Sigma(a)$ centered at 45 au (75 au) with $\sigma = 3$ au (5 au). Defining $P$ as the orbital period around the central star, the maximum $M_0$ and the Gaussian $\Sigma(a)$ yield a similar ratio for $P/\Sigma$ at 45 au and 75 au and therefore similar timescales for the growth of planets (e.g., Lissauer 1987).

In the majority of class II sources with compact disks and no bright rings (Figure 4, bottom left), the lack of millimeter emission at large radii is consistent with the solids outside the compact disk having either (i) low mass if composed primarily of pebbles or (ii) higher mass if composed primarily of large planetesimals. These possibilities motivate a broad range in $M_0$ (from 0.01 $M_\oplus$ to the adopted upper limits) and $f$ (0–1). A plausible third option for the lack of millimeter emission in these sources is a small vertical scale height, which prevents solids from intercepting a detectable fraction of light from the central star. For the calculations reported here, we do not consider this possibility but return to it in the discussion.

The observed properties of debris disks also motivate a range in $M_0$ and $f$. Among solar-mass stars with ages of $\sim$1 Gyr, roughly 25% have bright rings of cold debris at $a > 40$ au (Figure 4, top right). In the remaining 75%, cold debris is absent or undetectable (Figure 4, bottom right). The goal of the calculations is to identify the initial conditions and evolutionary paths that connect the protoplanetary disks (left) to the debris disks (right), while satisfying the constraints on the incidence rates and dust luminosities of known debris disks as a function of stellar age (e.g., Figures 1 and 3). One simple hypothesis we explore is whether the massive rings of protoplanetary solids (upper-left panel of Figure 4) evolve into rings of debris at similar distances at billion-year ages (upper-right panel of Figure 4).

Other evolutionary scenarios are also plausible. If some of the 75% of class II sources that appear as compact disks at millimeter wavelengths possess substantial rings of solids that are invisible at millimeter wavelengths as a result of their lower mass or lack of pebbles, these may evolve into bright debris disks at late times (e.g., Wyatt 2008; Kenyon & Bromley 2010; Matthews et al. 2014; Krivov & Wyatt 2021, and references therein). To explore the evolution of these systems, we also follow the evolution of low-mass rings of pebbles and more massive rings composed of large planetesimals.

Based on these considerations, we consider initial masses $M_0 = 0.01–45 M_\oplus$ for eight values of $f$ between 0 and 1. As summarized in Tables 1 and 2 in Appendix C, some combinations of $M_0$ and $f$ are not physically realizable in a ring with pebbles and large planetesimals. For example, with $M_0 = 0.01 M_\oplus$ and $f = 10^{-3}$, the mass in a single 100 km planetesimal exceeds $M_0 \times f$, the total mass allocated for all larger bodies.

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*6 The lower mass limit is somewhat smaller than current mass estimates for the Kuiper Belt (0.02–0.06 $M_\oplus$; Pijewka & Pijewski 2018; Di Ruscio et al. 2020), which has a dust luminosity well below Herschel sensitivity limits (e.g., Backman et al. 1995; Vitense et al. 2012).*
In identifying the relevant evolutionary path (or combination of paths) for protoplanetary and debris disks, we aim to ensure continuity between the starting and ending points outlined in Figure 4. That is, if young stars simply move horizontally in Figure 4, from bright rings at 1 Myr to bright rings at 1 Gyr and from compact disks into diskless main-sequence stars, we are guaranteed to match the observed fraction of debris disks at every epoch from ∼50 Myr to 10 Gyr. However, if some sources with compact disks and invisible rings of solids at 1 Myr evolve into observable debris disks at 1 Gyr, then a similar fraction of class II sources with bright rings must evolve into diskless main-sequence stars. In this case, we need to ensure that the fraction of class II sources that evolve diagonally upward in the figure matches the fraction that evolve diagonally downward.

If the high fraction of cold debris disks in the β Pic moving group is typical of all 20–25 Myr old stars (50%; Pawellek et al. 2021), then a large fraction of class II sources without bright rings need to evolve into bright debris disks at 20–25 Myr and then fade below current detection limits for stellar ages of 50–100 Myr. An additional goal of this study is to understand the physical properties of protoplanetary disks that lead to this type of evolution.

The following sections describe the growth of the largest objects (Section 3.3), the evolution of the dust luminosity from collisional debris (Section 3.4), and the impact of the largest objects on the radial distribution of the debris and gap formation (Section 3.5).

3.3. Growth of Planets

As each calculation proceeds, pebbles and planetesimals collide and merge into larger objects. Along with mergers, debris from collisions deposits mass into other mass bins. At first, the low initial e and i limit collisions between particles within different annuli. Evolution of e and i allows for more interactions among all particles; these interactions disperse collision debris throughout the grid. Most collisions yield some debris with particle sizes smaller than 1 μm. This material is assumed to be ejected by radiation pressure from the central star. Over 10 Gyr of evolution, the amount of lost material ranges from less than 1% to more than 90% (see Tables 1 and 2 in Appendix C). Ejected solids do not interact with other solids in the grid.

To set some expectations for the calculations, we estimate the timescale for the growth of planets in rings at 45 and 75 au. In Kenyon & Bromley (2010), 100 km planetesimals grow into Pluto-mass planets on a timescale $t_{1k} \sim 10$ Gyr at 80 au in a disk with the solid surface density distribution of a minimum mass solar nebula. In these calculations, $t_{1k} \propto P/\Sigma$, where P is the orbital period, and \Sigma is the initial surface density (see also Lissauer 1987; Goldreich et al. 2004; Rafikov 2005; Kenyon & Bromley 2008). Scaling these results to 45 au and 75 au,

$$t_{1k} \approx 2\text{ Gyr} \left( \frac{M_{\text{ref}}}{M_0} \right),$$

where $f = 1$ and $M_{\text{ref}} = 10 M_\oplus$ (45 $M_\oplus$) is the mass contained in rings at 45 au (75 au). When $f < 1$, analytical estimates suggest $t_{1k} \propto f^n$, with $n \approx 0.5$–1.0 (Goldreich et al. 2004; Rafikov 2005). The calculations here provide good tests of these estimates, as discussed in the Appendix.

In the full set of calculations, outcomes are sensitive to $M_0$ and $f$. Systems with more mass in solids tend to evolve more rapidly and form the largest planets and the brightest debris disks. For fixed initial mass, rings with more of their initial mass in planetesimals evolve more slowly. When $f$ is large, planetesimals compete for pebbles. As the initial population of pebbles declines, growth depends on infrequent planetesimal–planetesimal collisions, and the evolution stalls. When $f$ is small, planetesimals do not compete for pebbles; growth is then rapid.

Figure 5 illustrates the growth of the largest object in systems with $f = 0$. At the start of each calculation, gravitational focusing is negligible; pebbles grow roughly linearly with time. Slow pebble growth allows collisional damping to reduce e and i by an order of magnitude. As the largest particles approach kilometer sizes, collision velocities remain low despite increased dynamical friction and viscous stirring by the largest objects. Larger gravitational focusing factors initiate an explosive phase of runaway growth, where planetesimals with maximum radii $r_{\text{max}} \sim 1$ km rapidly reach much larger sizes, $r_{\text{max}} \gtrsim 1000$ km. After the runaway, a few protoplanets continue to accrete more and more material from the smaller objects and to stir up the velocities of the leftovers. At first, collisional damping among the pebbles counters stirring by protoplanets. Eventually, stirring overcomes damping and initiates a collisional cascade, where kilometer-sized and smaller objects are ground into smaller and smaller particles. The cascade robs protoplanets of remaining small solids; growth ceases.

Aside from the timescale and the final mass, the evolution of the mass of the largest protoplanet is amazingly independent of $a$ and $M_0$. All of the curves in Figure 5 have the same linear phase for $M \lesssim 10^{12}$ g followed by a nearly vertical rise to a maximum planet mass that scales with $M_0$. During runaway growth, planets accrete nearly all of the mass in their annuli (and sometimes in adjacent annuli). With more mass in protoplanets than pebbles, stirring reduces gravitational focusing factors. The evolution is then oligarchic, where the largest
protoplanets slowly accumulate leftover pebbles (e.g., Kokubo & Ida 1998; Ormel et al. 2010). Once pebbles are exhausted, growth ceases. In the most massive rings, Earth-mass planets form in 3–10 Myr. When \( M_0 \) is smaller, growth takes longer and results in lower-mass protoplanets. In the lowest-mass rings we consider, Pluto-mass planets form on billion-year or longer timescales. The progression of longer timescales to produce lower-mass planets when \( M_0 \) is small is characteristic of coagulation calculations (Equation (1)).

When \( f > 0 \), the final planet mass correlates well with \( M_0 \) and \( f \). At 75 au (Figure 6, upper panel), rings with \( f \approx 10^{-2} - 10^{-5} \) have a similar explosive growth phase as systems with \( f = 0 \). Initially, 100 km planetesimals slowly accrete pebbles and the occasional planetesimal. Once their masses exceed \( 10^{23} \text{ g} \), the runaway quickly carries them to super-Earth (\( M_0 = 45 \, M_{\oplus} \), thick orange curve) to super-Mars (\( M_0 = 15 \, M_{\oplus} \), thin orange curve) masses. Unlike the systems in Figure 5, the lack of solids with masses intermediate between planetesimals and pebbles allows protoplanets to accrete a large fraction of the mass in their respective annuli. Oligarchic growth is limited. Protoplanets maintain roughly constant masses for the rest of the calculation.

As the initial mass in pebbles drops, protoplanets grow more slowly and just manage to reach the mass of Mars (Figure 6, green and purple curves in the upper panel). Systems with \( f \approx 0.5 \) produce many Pluto-mass planets in 10–100 Myr. Continued accretion of pebbles and leftover planetesimals during an extended oligarchic growth phase enables protoplanets to match the mass of Mars. When \( f \approx 1 \), it takes \( 1–3 \) Gyr for several protoplanets to exceed the mass of Pluto. This timescale is close to the estimate derived from scaling previous calculations in 30–150 au disks of solids (Equation (1)). Oligarchic growth eventually carries some of these to the mass of Mars.

At 45 au, the behavior of rings with \( 3–10 \, M_{\oplus} \) and \( f = 10^{-5} \) to \( f = 1 \) closely follows the evolution at 75 au (Figure 6, lower panel). Although timescales are similar, the lower-mass rings at 45 au prevent planets from reaching the same masses as the higher-mass rings at 75 au. Thus, Mars-mass (Earth-mass) planets form in 3–10 \( M_{\oplus} \) (15–45 \( M_{\oplus} \)) rings at 45 au (75 au) in 30–100 Myr when \( f \approx 10^{-4} – 0.1 \). For \( f = 0.5 \), timescales to reach Pluto-mass planets at 45 au are somewhat longer than at 75 au; Mars-mass planets take much longer, \( \approx 3–10 \) Gyr (green curves). Calculations with \( f = 1 \) (purple curves) yield a few Mars-mass planets only at the highest mass considered; lower-mass systems manage to form planets with \( 3–5 \, M_P \) (Pluto masses).

When rings have less mass than the examples in Figure 6, lower-mass planets form on longer timescales (Figure 7). At 45 au (lower panel), rings with \( M_0 = 1 \, M_{\oplus} \) (0.3 \( M_{\oplus} \)) and \( 0 < f < 1 \) produce Pluto-mass planets on timescales of 100 Myr to 1 Gyr (green and orange curves). After 10 Gyr, the planets in these systems have masses of \( 2–10 \, M_P \). Growth is negligible when \( f = 1 \) (purple curves); maximum masses are \( \approx 0.1 \, M_P \). The more massive systems at 75 au easily generate Mars-mass planets in 300 Myr to 1 Gyr when \( f \) is very small (Figure 7, upper panel, orange curves). In each of these systems, planetesimals accrete nearly all of the available pebbles during runaway growth and then maintain a roughly constant mass for the rest of the calculation. As \( f \) increases, growth slows; the mass of the largest planets drops by several orders of magnitude (green and purple curves). When \( f = 0.5 \), Pluto-mass planets form on \( 1–10 \) Gyr timescales. Very efficient planetesimal formation (\( f = 1 \)) leads to the growth of Pluto-mass planets at late times when \( M_0 = 5 \, M_{\oplus} \); in lower-mass rings, large planets fail to form after 10 Gyr.

Very-low-mass rings produce much smaller planets. Mixes of solids and pebbles with \( M_0 = 0.01–0.1 \, M_{\oplus} \) in rings at 45 au struggle to make a single Pluto. For any \( f \neq 1 \), the \( M_0 = 0.1 \, M_{\oplus} \) models have a modest runaway that converts 200 km
and the timescale to reach this mass scale inversely with the mass of Pluto Charon-mass objects. As Pluto. When set of dwarf planets with masses 5 to 10 times the mass of three planetesimals into 900 km dwarf planets. Rings with a factor of Figure 7. Growth of the largest objects in medium-mass rings of solids at 45 au (lower panel) and at 75 au (upper panel). The legend associates $M_0$ and $f$ for each curve. Numbers in parentheses indicate initial mass in $M_0$ for thick (first number) and thin (second number) lines. The horizontal dashed lines indicate the mass of Pluto (lower) and Mars (upper). The final mass of the largest object and the timescale to reach this mass scale inversely with $M_0$: lower-mass rings produce lower-mass planets.

planetesimals into 900 km dwarf planets. Rings with a factor of three (10) less mass produce several Charon-mass objects (Kuiper Belt objects) with radii of 500 km (250 km). Rings with no pebbles barely evolve over 10 Gyr.

At 75 au, the lowest-mass rings in our study show a little more activity. Models with $M_0 = 0.5 M_⊕$ and $f ≤ 0.1$ begin a robust runaway at 1 Gyr. After 3 Gyr, these rings have a modest set of dwarf planets with masses 5 to 10 times the mass of Pluto. When $f ≈ 0.5$, the delay in the runaway to 3–6 Gyr fails to produce a Pluto-mass dwarf planet, but generates a few Charon-mass objects. As $M_0$ drops to 0.01–0.1 $M_⊕$, growth is modest, yielding a few 200–300 km objects in 0.1 $M_⊕$ rings and many 150 km planetesimals in 0.01 $M_⊕$ rings. Once again, systems with $f = 1$ barely evolve over the age of the universe.

To collate some results from the calculations in a convenient form, Tables 1–2 list the timescale to form at least one Pluto-mass planet, $t_{1P}$, the maximum radius $r_{\text{max}}$ of the largest planet at 10 Gyr, the final mass in solids $M_f$ at the end of each calculation, and other useful parameters. In addition to examining the relation between $t_{1P}$, $M_f$, and $f$, the discussion in the Appendix includes several examples of collisional damping and some comparisons with previous calculations.

3.4. Evolution of Dust Luminosity

Throughout each calculation, the dust luminosity depends on the vertical scale height and the surface area of small solids. Initially, pebbles and planetesimals have the same typical orbital inclination, $i_0 = 5 × 10^{-4}$. At large distances $a = 30–90$ au, the upper and lower surfaces of the swarm intercept a negligible fraction of stellar flux (e.g., Kenyon & Hartmann 1987; Chiang & Goldreich 1997). The maximum stellar luminosity intercepted by pebbles is then the fraction of solid angle subtended by a ring with vertical scale height $H$ above and below the midplane at distance $a$ from the central star: $L_{d,\text{max}} = H/\alpha = 5 × 10^{-4}$ for the adopted $i_0 = 5 × 10^{-4}$. Ignoring the negligible contribution from large planetesimals for rings with any $f$, the initial dust luminosity is

$$L_{d,0}/L_* ≈ \begin{cases} 7.3 \times 10^{-4} (1 - f) \left( \frac{M_0}{M_\oplus} \right) & a = 45 \text{ au}, M_0 \lesssim 0.7 M_\oplus \\ 1.9 \times 10^{-4} (1 - f) \left( \frac{M_0}{M_\oplus} \right) & a = 75 \text{ au}, M_0 \lesssim 2.6 M_\oplus \end{cases}$$

When $M_0$ exceeds the limits quoted in Equation (2), $L_{d,0}/L_* = 5 \times 10^{-4}$.

These expressions for $L_{d}/L_*$ establish clear constraints on plausible evolutionary paths for rings of pebbles and planetesimals. To exceed the FEPS upper limits of $L_{d}/L_*$ ≈ $10^{-4}$ for young stars with ages $≤ 100$ Myr, rings at 45 au (75 au) must have initial masses $M_0 \gtrsim 0.15 M_\oplus$ (0.5 $M_\oplus$). Lower-mass disks require additional dust production from a collisional cascade. Among older stars, exceeding the Herschel upper limits of $L_{d}/L_*$ ≈ $3 \times 10^{-6}$ requires much less mass, $M_0 \gtrsim 0.005 M_\oplus$ (0.02 $M_\oplus$) at 45 au (75 au). If the time evolution of $L_{d}/L_*$ is slow, the small Herschel upper limits allow a broader set of rings to match observed systems.

As each calculation proceeds, the dust luminosity responds to changes in $H$ and the surface area of small solids. When $f < 10^{-2}$, collisional damping initially reduces $H$ by a factor of 2 to 5. Debris production is minimal; $L_{d}/L_*$ drops as $H$ falls. As planetesimals accrete more and more pebbles, stirring (damping) becomes more (less) efficient. Debris production and $H$ begin to grow; $L_{d}/L_*$, rings. Rings of solids with larger $f$ have more stirring from growing planetesimals and less damping. The dust luminosity then rises at the start of each calculation. In all systems, $H$ eventually reaches a maximum; debris generated from collisions of protoplanets and planetesimals falls below the loss of small particles from the collisional cascade. The dust luminosity then fades with time.

The top panel of Figure 8 illustrates the smooth evolution of the relative dust luminosity $L_{d}/L_*$ for a set of calculations with $f = 0.5$ at 75 au. When half of the initial mass is in pebbles, the initial $L_{d}/L_*$ ranges from $L_{d}/L_* \sim 10^{-6}$ for $M_0 = 0.01 M_\oplus$ to $5 \times 10^{-4}$ for $M_0 = 2.6–45 M_\oplus$ (Equation (2)). As the most massive rings evolve (15–45 $M_\oplus$, black and purple curves), stirring overcomes collisional damping; $H$ increases. Despite the lack of debris production from planetesimal growth, $L_{d}/L_*$ grows with time. Over the next 10–30 Myr, increased debris production roughly balances losses from the collisional cascade; $L_{d}/L_*$ remains roughly constant. As protoplanet growth slows considerably at 100–200 Myr, stirring initiates a stronger cascade that removes more and more of the remaining small solids. The dust luminosity then begins a roughly linear decline. With more massive protoplanets that form earlier and stir solids more strongly, the 45 $M_\oplus$ rings begin to decline before the 15 $M_\oplus$ rings. Once the strong cascade begins, $L_{d}/L_*$ follows the same roughly linear decline in both systems.

Lower-mass models have a similar behavior. In rings with 0.5–5 $M_\oplus$ (light green, dark green, and blue curves), the dust luminosity slowly declines with time during most of the first 100 Myr to 1 Gyr of evolution. During this decline, protoplanets gradually reach the mass of Pluto. Stirling from these large solids increases $H$. Larger relative velocities enhance debris production, but mass removal from the cascade keeps $L_{d}/L_*$ falling. Eventually, stirring becomes more effective; the cascade strengthens and removes more and more small solids.
from the ring. The dust luminosity then declines more rapidly, joining the roughly linear decline of the higher-mass models.

In the lowest-mass models (0.01–0.1 $M_\odot$, beige and orange curves), the evolution is extremely slow. Over 10 Gyr, planetesimal masses grow by a factor of 3 (0.01 $M_\odot$) to 30 (0.1 $M_\odot$). The collisional cascade is equally weak; the dust luminosity changes little over 10 Gyr. Early in the evolution (1–10 Myr), these systems show a small rise in $L_{d}/L_*$ due to the extra production of small dust grains relative to the loss of pebbles. The systems then begin a protracted decline. Were the calculations extended to 20–30 Gyr, the decline in $L_{d}/L_*$ would speed up and eventually join the other models on a roughly linear decline of $L_{d}/L_*$ with time.

Compared to the models with $f=0.5$, systems with small $f$ undergo a more erratic evolution (Figure 8, lower panel). When massive rings have $f \approx 10^{-4}$–$10^{-2}$, runaway growth is explosive; protoplanets rapidly reach super-Earth masses (Figures 5–6). The dust luminosity echoes this behavior (black, purple, and blue curves). Initially, $L_{d}/L_*$ drops by a factor of $\sim 2$–5 as collisional damping among pebbles reduces the scale height of the smallest solids. After the runaway begins, protoplanets stir up smaller solids; the cascade starts to produce copious amounts of debris. With more debris and a larger scale height, the dust luminosity rises dramatically to $\sim 10^{-2}$ and then drops precipitously as the cascade depletes the rings of pebbles and smaller solids. During the steep decline, protoplanet–protoplanet collisions replenish the debris and produce additional brief rises in $L_{d}/L_*$.

After a nearly billion-year-long decline, a few large collisions create enough debris to fuel a final, modest rise in the dust luminosity, $L_{d}/L_*$ $\approx 10^{-6}$–$10^{-5}$ that lasts until $\sim$10 Gyr.

Calculations with $f \lesssim 10^{-4}$ behave in a similar fashion. Early on, damping is more important; $L_{d}/L_*$ declines by a factor of 10–20. As kilometer-sized planetesimals evolve into super-Earths, rapid growth of $H$ allows $L_{d}/L_*$ to reach $\sim 10^{-2}$ somewhat earlier than models with $f = 10^{-4}$–$10^{-2}$. In these systems, the drop in $L_{d}/L_*$ is more dramatic, falling well below $10^{-7}$, and the recovery is smaller than displayed by the black, purple, and blue curves in the lower panel of Figure 8.

Rings with a larger initial mass in planetesimals, $f=0.1$–0.5, evolve more smoothly (light green and orange curves). In these systems, $L_{d}/L_*$ rises by a factor of four during the first 10–20 Myr when planetesimals grow into Pluto-mass planets (Figure 6). While protoplanets continue to accrete pebbles, the dust luminosity levels off and begins to decline. The systems then enter an extended oligarchic growth phase from $\sim 100$ Myr to 10 Gyr, where the dust luminosity declines roughly linearly with time, from $L_{d}/L_*=10^{-3}$ to $L_{d}/L_*=10^{-5}$. Near the end of this epoch, both calculations exhibit a small rise in $L_{d}/L_*$ from several collisions among the remaining planetesimals and protoplanets.

When $f=1$, the ponderous evolution of $L_{d}/L_*$ closely parallels the slow growth of planetesimals into Mars-mass planets (Figure 8, beige curve). During the first 30 Myr, $L_{d}/L_*$ rises slowly from $\sim 10^{-9}$ to $\sim 10^{-6}$. Growth of planetesimals and a rise in debris production powers a more rapid rise to $L_{d}/L_*=10^{-3}$ at 200–300 Myr. The system then enters oligarchic growth, where protoplanets continue to grow, and the cascade grinds leftovers to dust. Unlike other systems with smaller $f$, these rings maintain a roughly constant luminosity from 300 Myr to 10 Gyr. Eventually, the dust luminosity will decline; however, the decline will occur after the central star leaves the main sequence.

When the initial mass in solids is smaller than 45 $M_\odot$, the evolution of the dust luminosity in models with $f \lesssim 10^{-2}$ is slower and less dramatic. The overall shape in the $L_{d}(t)/L_*$ curve follows the examples in the top panel of Figure 8, with a fairly constant $L_{d}/L_*$ at the start of the calculation followed by a nearly linear decline. Superimposed on this generic evolution is a series of spikes in $L_{d}/L_*$ generated by debris from occasional giant collisions between protoplanets and planetesimals. The amplitudes of these spikes decline with decreasing initial mass in solids. Rings with $M_0 = 10 M_\oplus$ at 45 au and $M_0 = 15 M_\oplus$ at 75 au have large spikes as in the black, purple, and blue curves in the lower panel of Figure 8, while rings with 0.3–1 $M_\oplus$ at 45 au and 0.5–1.5 $M_\oplus$ at 75 au have modest spikes. Lower-mass systems have insignificant spikes in $L_{d}/L_*$ during the overall decline.

In systems with $f=1$, the evolution in $L_{d}/L_*$ becomes less and less interesting with decreasing $M_0$. Rings with $M_0 \gtrsim 1 M_\oplus$ (5 $M_\oplus$) at 45 au (75 au) have $L_{d}/L_*=10^{-6}$ at 5–10 Gyr. As $M_0$ decreases from these limits, the maximum dust luminosity also drops. At 45 au, the maximum dust luminosity falls to $3 \times 10^{-7}$ for 0.3 $M_\oplus$ rings to less than $10^{-7}$ for 0.1 $M_\oplus$ rings. Low-mass rings at 75 au are equally invisible with current technology; all systems with $M_0 \lesssim 0.5 M_\oplus$ have a maximum $L_{d}/L_*$ smaller than the limit of current technology.
than $10^{-7}$. The tables in the Appendix include the maximum $L_d/L_*$ for all calculations and allow for a more extensive comparison among the calculations.

To connect these results to previous studies, we compare with published analytical models of collisional cascades (e.g., Wyatt & Dent 2002; Dominik & Decin 2003; Krivov et al. 2008; Löhne et al. 2008). In a cascade at 75 au where the radius $r_c$ of the largest object participating in the cascade does not change, the dust luminosity is constant at early times and then falls linearly with time, $L_d/L_*=L_0/(1+t/\tau_0)$. However, numerical models demonstrate that $r_c$ also declines with time (e.g., Kenyon & Bromley 2017). Including this behavior in the analytical model yields a steeper decline in the dust luminosity, $L_d/L_*=L_0/(1+t/\tau_0)^{1.12}$, where $\tau_0=1.12\alpha_0$ and $t_0=r_cP/(12\pi\Sigma)$ is the collision time (Kenyon & Bromley 2017). The $\alpha$ term is a function of the ratio of the collision energy to the binding energy of planetesimals.

The dotted line in each panel of Figure 8 shows the luminosity evolution for $L_0=4\times10^{-3}$ and $t_0=30$ Myr. The dust luminosity is nearly constant for $5–10$ Myr and then declines. Although the analytical model has a somewhat larger $L_d/L_*$ at early times, it matches numerical models for massive disks with $f=0.1–0.5$ at late times. Tracking the behavior of the numerical models for less-massive disks requires smaller $L_0$ and larger $t_0$.

In the Kenyon & Bromley (2017) analytical model, the reference luminosity $L_0$ and the timescale $\tau_0$ depend primarily on $M_0$ and $r_c$; $L_0 \propto M_0 r_c^{1/2}$ and $\tau_0 \propto r_c M_0^{-1}$. For the cascade in Figure 8, the collision velocity is just large enough to shatter objects with $r \approx r_c$; then, $r_c \approx 0.2–0.5$ km and $M_0 \approx 50–55 M_\oplus$. For fixed $r_c$, reducing $L_0$ by a factor of 100 yields $M_0 \approx 0.5 M_\oplus$ and $t_0 \approx 3$ Gyr. This model provides a reasonable match to numerical calculations with $M_0 \approx 0.1–0.5 M_\oplus$.

Analytic cascade models with the larger planetesimals expected from streaming instability models, e.g., $r_c \approx 100–200$ km, require unreasonably large $M_0$ to approximate the evolution in Figure 8 (see also Shannon & Wu 2011; Krivov & Wyatt 2021). Adopting $r_c \approx 100$ km instead of $r_c \approx 0.3$ km requires $\sim 17$ times more mass to achieve $L_0 \approx 4 \times 10^{-3}$. To match the short collision time, $\tau_0 \approx 30$ Myr, collision velocities must be $\sim 3–4$ times larger than the minimum required to shatter $100$ km objects. While this model provides a reasonable match to the simulation, the high mass in solids, $M_0 \approx 800–900 M_\oplus$, makes this solution unattractive compared to the analytical model with small planetesimals or the numerical model with a mix of pebbles and large planetesimals.

Although not shown in Figure 8, analytic models also match the results of numerical models at 45 au. For $M_0=10 M_\oplus$ and $r_c=0.2$ km, an analytical model with $L_0 \approx 2 \times 10^{-3}$ and $\tau_0 \approx 25$ Myr tracks the numerical models of disks with $f=0.1–0.3$ for evolution times exceeding 30–40 Myr. Lower-mass disks with larger $\tau_0$ also match the simulations. Compared to calculations at 75 au, the analytic model prefers a somewhat smaller $r_c$ and shorter $\tau_0$ to track the numerical results adequately at 45 au. Once again, much larger $r_c$ requires unreasonably large $M_0$.

### 3.5. Gap Formation

In addition to the dust luminosity, the coagulation calculations provide a quantitative measure of the radial distribution of dust as a function of time. For low-mass rings that generate little dust luminosity, $L_d/L_* \lesssim 10^{-4}$, final protoplanet masses are typically less than the mass of Mars. These planets tend to follow the initial surface density distribution and cluster near the center of the ring. With little mass in planets, the dust also has a Gaussian distribution in surface density with a peak in the center of the ring, e.g., at 45 au and at 75 au.

More massive rings have more obvious features in the radial surface density of the dust. The planets in these calculations do not follow the initial surface density distribution and are more evenly distributed among the 28 annuli in each ring. Super-Earth-mass planets tend to eliminate all solids from their annuli and sometimes from adjacent annuli. These systems thus have 1–3 au gaps in the surface density of dust along the orbits of the super-Earths. Rings with several super-Earths have multiple gaps.

Predicting the structure of these gaps requires a parallel set of coagulation and $n$-body calculations to allow the gravity of planets to open gaps with sizes that depend on the mass of the planet and the remaining mass in smaller solids within the ring (e.g., Kokubo & Ida 1995; Rafikov 2001; Bromley & Kenyon 2011b, 2013, and references therein). These calculations are computationally expensive (e.g., Bromley & Kenyon 2020; Kenyon & Bromley 2021); we defer them to a later study. Here, we use previous results to infer the likely structure of gaps in the dust distribution of rings with massive planets.

For this initial exploration of gap formation, we compare the relative separation of protoplanet orbits to the Hill radius, $r_H=a(M/M_*)^{1/3}$. From analytic and numerical calculations, planets clear out a “ring of influence” with a radial extent $\delta a \approx 2\sqrt{3} r_H$ on either side of their orbits (e.g., Gladman 1993; Kokubo & Ida 1995; Rafikov 2001). Among pairs of protoplanets, those separated by more than $4r_H$ in semimajor axis do not interact dynamically (e.g., Kokubo & Ida 1995; Chambers et al. 1996; Weidenschilling et al. 1997; Chambers 2001; Bromley & Kenyon 2006; Kenyon & Bromley 2006); each carves out its own gap with an extent comparable to $\delta a$.

Protoplanets on closer orbits interact dynamically and produce large gaps in an “interaction region” defined by the extent of their chaotic orbits prior to a merger or ejection event. This region has a width of $2–3 \delta a$.

Figure 9 illustrates the evolution for a calculation with $M_0=45 M_\oplus$ and $f=0.01$. With a surface density maximum in the middle of the ring, planets grow fastest (slowest) at 75 au (60–65 au and 85–90 au). Once 500–600 km protoplanets form at 30 Myr, runaway growth begins. Over the next 70 Myr, the five fastest-growing protoplanets each surpass the mass of Mars. Smaller protoplanets have much smaller masses, $\sim 1–3 M_\oplus$. The horizontal lines in the figure illustrate the extent of the rings of influence for each of the five largest protoplanets. Rings of influence for the two most massive protoplanets at 73–75 au overlap and contain a lower-mass protoplanet at 72 au. Rings of influence for the other two protoplanets at 67 au and at 81 au contain several much smaller planetesimals but no other massive protoplanet.

The close proximity of the two largest protoplanets in Figure 9 has two observable outcomes. Initially, the two central protoplanets interact dynamically, scattering smaller solids out of their orbits and trying to move to a larger separation. This interaction involves the protoplanet at 72 au and begins a period of chaotic growth, where the three protoplanets at 72–75 au move chaotically through the grid and sweep up smaller objects along their orbits. Eventually, this process
involves the outer two protoplanets at 67 au and 81 au. Subsequent collisions and mergers among the five largest protoplanets, the smaller protoplanets with radii \( \sim 1000\text{–}4000\text{ km} \), and the smaller planetesimals are likely to leave behind one or two super-Earth-mass planets (e.g., Goldreich et al. 2004; Kenyon & Bromley 2006). Throughout chaotic growth, the ring of smaller solids expands radially inward and outward. Thus, at \( \gtrsim 200\text{ Myr} \), ALMA observations would reveal thermal emission from a larger ring of small solids with a central depression where the remaining super-Earths orbit.

In this example, it seems likely that the size of the gap will be larger than the standard \( 4 \sqrt{3} r_H \) expected for a single massive planet in a ring of small solids. The five protoplanets at 67–82 au would experience chaotic growth within an “interaction region” extending from \( \sim 65\text{ au} \) to \( \sim 83\text{ au} \). Once chaotic growth ends, this region would have few small solids. Instead of the \( \sim 50 M_\oplus \) single planet required to create such a large gap, this gap would contain 1–2 super-Earths with a total mass \( \sim 5 M_\oplus \).

Among the suite of calculations with \( M_0 = 10 M_\oplus \) at 45 au and \( M_0 = 15–45 M_\oplus \) at 75 au, many have protoplanets with overlapping rings of influence (Figure 10). At 45 au (left panel), two systems have as many as a half dozen protoplanets within an interaction region (top left and middle left panels). Another system has \( \sim 20 \) large objects in an interaction region that takes up most of the ring. Calculations at 75 au yield similar systems where a few or many massive objects will interact chaotically. Some may yield 1–2 super-Earths; others may produce many Mars-mass planets.

In each of these calculations, we expect that chaotic evolution will produce gaps and perhaps narrow rings of small solids within broader, more diffuse rings. With many massive protoplanets evolving within the interaction region, the final sizes of dark gaps may be much larger than expected from the final masses of protoplanets.

4. Discussion

The models described in Section 3 illustrate how the collisional evolution of rings of solids—composed of pebbles and planetesimals—follows diverse evolutionary histories depending on the efficiency of planetesimal formation \( (f) \) and the initial mass in solids \( (M_0) \). These parameters establish whether rings can grow Pluto-, Mars-, or super-Earth-mass planets in 1 Myr to 10 Gyr (Section 3.3; Figures 5–7; see also Tables 1–2). Diverse dust luminosity histories result (Section 3.4; Figure 8). The growing planets should create gaps in the radial distribution of solids that reflect their mass and growth history (Section 3.5; Figures 9–10).

Here we compare the model results from Section 3 with observations of debris disks to identify the plausible initial conditions and evolutionary paths that connect protoplanetary disks to debris disks (Figure 4). We also associate planet formation outcomes—Pluto, Mars, or (super-)Earth—with the various paths to debris disks. Figure 11 compares the fractional luminosity \( L_d/L_* \) of the ring models (colored lines) with the observed \( L_d/L_* \) values of debris disks described in Section 2. In each panel, the colored dots from Figure 3 are reproduced as black dots; gray arrows repeat the median upper limits from Figure 3. The comparison demonstrates that the evolution of massive rings of pebbles and planetesimals at 45–75 au plausibly explains the observed dust luminosities of known debris disks with stellar ages of 10 Myr to 10 Gyr.

4.1. Bright Young Disks

As shown in the two panels of Figure 11, the population of bright young disks \( (L_d/L_* \gtrsim 10^{-3}) \) at 10–100 Myr is best matched by models of high-mass rings \( (3–45 M_\oplus) \) with a wide range of planetesimal formation efficiencies \( (f = 0–0.5) \). In
models with $f \lesssim 10^{-2}$, occasional collisions among large protoplanets create copious debris and pronounced spikes in $L_{d}/L_{*}$ (Figure 11, purple curve in the lower panel). The largest spikes rival the observed $L_{d}/L_{*}$ of the brightest debris disks with ages of 10–100 Myr (upper panel; 75 au). Fainter debris systems in this age range are well matched by rings with smaller masses, smaller radii, or smaller $f$ (Figure 11, upper panel).

In contrast to these evolutionary paths, rings with similar or larger masses, but completely efficient planetesimal formation ($f = 1$) produce very little debris at early times (Figure 11, orange curve in the lower panel). If planetesimals form with such high efficiency, matching the observed fractional luminosities of the bright young disks requires very large initial masses ($\sim 1000 M_{\oplus}$; Shannon & Wu 2011; Krivov & Wyatt 2021). As shown here, such large masses are unnecessary; more modest planetesimal formation efficiencies and the typical masses of observed protoplanetary disk rings can account for the observed properties of bright young disks.

4.2. Fainter Old Disks

Debris disks trend fainter with age, with $L_{d}/L_{*} \sim 10^{-5}$ at ages beyond 1 Gyr (Figures 2, 3, and 11). The downward trend of $L_{d}/L_{*}$ with age is readily explained by the simple fading of the bright young disks of Section 4.1. Rings with $f \approx 0.01$–0.5 follow the classical evolution in dust luminosity of $L_{d}/L_{*} \propto t^{-1}$ (see also Wyatt & Dent 2002; Dominik & Decin 2003; Krivov et al. 2008; Löhr et al. 2008; Wyatt 2008; Kenyon & Bromley 2017; Kobayashi & Löhne 2014) via the evolutionary pathway illustrated by nearly all of the curves in the upper panel of Figure 11. We identify this evolution as the “bright-stalwart” pathway shown in Figure 12. This interpretation of the known debris disks—that they arise from the fading of a population of initially massive disks partially composed of pebbles and planetesimals—is consistent with the similar fraction of debris disks and protoplanetary disk rings at each evolutionary age ($\sim 25$%; Section 2). Thus, the bright-stalwart pathway accounts for both the incidence rates and luminosities of the known debris disks in the age range $\sim 50$ Myr to 10 Gyr.

The general agreement of this evolutionary path with the observed properties of debris disks potentially limits the possible role of other evolutionary histories. For example, the fainter old disks could also be explained with a population of low-mass rings ($\sim 1 M_{\oplus}$) with modest $f$ that are moderately bright at early times ($L_{d}/L_{*} \sim 10^{-5}$) and evolve more horizontally in Figure 11, reaching $L_{d}/L_{*} \sim 10^{-5}$ at 10 Gyr. This pathway—which we refer to as ‘steady glow’—is illustrated by the blue and green curves in the lower panel of Figure 11 (also the gold curve in Figure 12). Alternatively, high-mass disks ($10–40 M_{\oplus}$) made entirely of planetesimals pursue a stealthy evolutionary path that is extremely faint at early times ($L_{d}/L_{*} < 10^{-4}$) and brightens at late times to currently observable levels of $L_{d}/L_{*} \sim 10^{-3}$. The orange curve in the lower panel of Figure 11 (also in Figure 12) shows this “late-bloomer” pathway.

If we assume that $\sim 25$% of stars start out with massive protoplanetary disk rings (Section 2) and subsequently follow the bright-stalwart path, we can account for the observed luminosities and incidence rates of debris disks with age; this assignment leaves little room for significant contribution from the steady-glow and late-bloomer pathways. If these were important pathways, each populated roughly equally to the classical bright-stalwart pathway, the incidence rate of debris disks at early times (from the bright-stalwart pathway) would be three times smaller than the rate at late times when the steady-glow and late-bloomer populations become detectable with $L_{d}/L_{*} \sim 10^{-5}$ at $\sim 1$ Gyr. We can further rule out a significant steady-glow population, because it would also overproduce sources at ages of 0.1–1 Gyr with $L_{d}/L_{*} \sim 10^{-4}$. The debris disk incidence rate in this age and luminosity range is restricted by the Herschel survey upper limits (Section 2). Future debris disk surveys that reach fainter luminosity limits at ages 10–300 Myr can provide additional, direct constraints on the steady-glow ($L_{d}/L_{*} \sim 10^{-5}$) and late-bloomer ($L_{d}/L_{*} \sim 10^{-6}$) populations.

One can imagine combining the evolutionary tracks from Section 3 in other ways. For example, if the incidence of cold debris disks at 10–30 Myr is as large as the 50% rate derived for the small population of F stars in the 20–25 Myr old $\delta$ Pic moving group (Pawellek et al. 2021), a significant population of disks must rise to high dust luminosity within a few tens of millions of years before fading significantly beyond 100 Myr. This behavior, which we identify as the “early-flare” pathway, could explain the high incidence rate of cold debris disks at young ages and the much lower rate among stars with ages $\geq 50$ Myr. In the calculations, this behavior occurs in rings with...
high masses and $f \lesssim 10^{-3}$, where the combined impact of damping, rapid planet growth, and an efficient cascade results in a system where $L_d/L_a$ rises from $\lesssim 10^{-4}$ to $\sim 10^{-2}$ and declines back down to $\lesssim 10^{-4}$ on timescales of 100 Myr (Figure 8, lower panel, black, purple, and blue curves; Figure 12, green curve). The late decline in $L_d/L_a$ in these models is as fast or faster than the $L_d/L_a \propto t^{-2}$ required to eliminate the descendants of the β Pic stars from the DEBRIS and DUNES samples at ages of $\approx 1$ Gyr (Pawellek et al. 2021).

We might further imagine that in addition to the early-flare pathway producing the vast majority of the luminous debris disks at ages $\lesssim 40$–50 Myr, we also have the steady-glow and late-bloomer pathways dominating among much older stars. With this combination, nearly all of the early flares need to fall below current detection limits before rings in the steady-glow and late-bloomer pathways begin to contribute to the population. Contributions from the classical bright-stalwart pathway could “smooth over” the transition between these pathways.

Invoking three pathways—early flare, steady glow, and late bloomer—faces several hurdles. Selecting the proper mix to ensure a high incidence rate at the youngest ages and to maintain a roughly constant rate of $\sim 25\%$ for all older stars would require some fine tuning for ages where early flares are fading away and the other pathways first become detectable. In addition, the required mix of initial conditions—massive disks with $f = 0$ for early flares, massive disks with $f = 1$ for late bloomers, and intermediate-mass disks with $f \approx 0.1$–0.5 for steady-glow sources—seems unlikely, unless there are physical mechanisms that can produce such dramatically different planetesimal efficiencies in different disks. The larger parameter space of $f$ that produces the classical bright-stalwart sources seems more plausible and more in line with the predictions of simulations of the streaming instability (e.g., Rucská & Wadsley 2021).

We can also distinguish these pathways with a different approach, by quantifying the disk substructure created by any massive, embedded planets that form. Massive rings ($10$–40 $M_⊕$) that follow the classical bright-stalwart pathway ($f = 0.1$–0.5; Figure 12, blue curve) or the late-bloomer path ($f = 1$; Figure 12, orange curve) would build Mars-mass planets by 10 Gyr. In contrast, massive rings with lower $f$ that “burn bright, fade fast” (the early-flare pathway; $f \lesssim 10^{-3}$; Figure 12, green curve) would create more massive objects—super-Mars- to super-Earth-mass planets—in 20–30 Myr (Figure 6). Modest-mass rings that follow the roughly horizontal steady-glow pathway ($\sim 1$–5 $M_⊕$
f = 0.3–0.5; Figure 12, gold curve) would only build Pluto-mass objects by 10 Gyr (Figure 7).

The gaps created by the more massive planets could be resolved spatially. If a ring with a radius of 75 au creates a planet with the mass of Earth, Mars, or Pluto, the planet would open a gap with a fractional width of at least 2\(d a/a = 4\sqrt{3}\left(M_p/M_\odot\right)^{1/3}\) (Section 3), which corresponds to an angular width of 54 mas, 24 mas, and 8 mas at a distance of 140 pc. In comparison, the Next Generation Very Large Array (ngVLA) is anticipated to deliver angular resolutions of 0.5 mas to 50 mas at wavelengths of 2.6 mm to 25 cm (e.g., Matthews et al. 2018; Tobin et al. 2018; Chalmers et al. 2020, and references therein).

Thus, future observations could test the hypothesis that the classical bright-stalwart evolution is the primary pathway for debris disks, with few systems pursuing either the late-bloomer or the steady-glow paths. In other words, observations could probe whether protoplanetary disks are born with massive, initially dark rings of planetesimals (late-blooming rings) or initially modest-mass rings of pebbles and planetesimals (steady-glow rings).

4.3. The Story of Solids in Disks

If bright stalwarts dominate the evolutionary pathways for debris disks, one simple interpretation of the results described here is that extended and compact protoplanetary disks evolve differently, i.e., disks evolve horizontally in Figure 4. The 25% of T Tauri systems with large ringed protoplanetary disks produce detectable debris throughout their lives, from \(\sim\)10 Myr to 10 Gyr, and are the showy, attention-grabbing celebrities of the debris disk world. In contrast, the majority of T Tauri systems with compact protoplanetary disks live quieter, tidier lives. Few of them are born with “late-blooming” massive, initially dark rings of planetesimals or “steady” modest-mass rings of pebbles and planetesimals. That is, if rings are typically a mixture of pebbles and planetesimals at the end of the protoplanetary disk phase, then compact disks typically leave behind <1\(M_\oplus\) of solids at large radii, a small reservoir that produces little debris over the lifetime of the star. Instead of each disk evolving along the classical path of Class 0/I/II/III sources into debris disks (e.g., Cieza et al. 2007; Wahhaj et al. 2010; Williams & Cieza 2011; Hardy et al. 2015), this interpretation implies that the known debris disks are a chapter in the history of only a subset of low-mass stars.

While most current observations support this simple picture, the true story may be more complex if the high incidence rate of cold debris disks among the small sample of F stars in the \(\beta\) Pic moving group is typical of all 20–25 Myr old solar-type stars. As noted earlier, a high incidence rate of debris that persists for a short time may imply that a significant fraction of the compact protoplanetary disks actually possess massive rings of solids with very small \(f\) that are also undetectable because of their very small initial scale height (Section 3.2). These disks would evolve into bright early-flare sources that appear suddenly on the debris disk stage and quickly fade below current detection limits.

This potential complexity aside, the divergent debris-production histories of large and small protoplanetary disks connect back to the more fundamental question of what sets the initial distribution of solids in protoplanetary disks. One possibility is that some protoplanetary disks are born large and others small, a consequence of the initial angular momentum of the cloud core and the extent to which angular momentum is shed (or transported) as collapse proceeds (e.g., Terebey et al. 1984; Matsumoto et al. 1997; Basu 1998; Yorke & Bodenheimer 1999; Nakamura 2000; Krasnopolsky & Königl 2002; Tscharnuter et al. 2009; Joos et al. 2012; Tomida et al. 2015; Hennebelle et al. 2016; Zhao et al. 2020, and references therein). Alternatively, most disks may be born with similar (large) sizes, but some experience greater inward migration of solids and others do not. Disks that create planets (or other disturbances) early on, at large radii, can induce pressure bumps that trap solids and prevent inward migration (e.g., Pinilla et al. 2012; Zhu & Stone 2014; van der Marel et al. 2018). If bright stalwarts dominate the production of debris disks, one or more of these scenarios are efficient in concentrating or placing a significant solid mass (10–40 \(M_\oplus\)) in rings at large radii in about 20%–25% of disks.

Our analysis and results complement ideas discussed in the literature. In a recent examination and interpretation of the dust masses and luminosities of protoplanetary and debris disk sources, Michel et al. (2021) proposed a similar picture to the one described here: they hypothesized that debris disks are the descendants of large structured protoplanetary disks, which preserve their solids against inward radial drift through the action of dust trapping in pressure bumps. Their compact protoplanetary disk counterparts result from disks without such pressure bumps, whose solids drift inward to small disk radii. In support of their picture, they noted the similar sizes of protoplanetary disk structures (e.g., the central cavity radii of transition disks) and the blackbody radii of debris disks. However, they also offered the caveat that their protoplanetary and debris disk samples did not span the same spectral types, with the debris disks skewed to earlier-type stars. They also asked for modeling that would support their hypothesized scenarios.

Our study complements this work by assembling protoplanetary and debris disk data sets that are matched in stellar mass and by comparing orbital distances (of protoplanetary rings and debris) and their incidence rates as a function of age. Our study also provides the detailed modeling that supports the hypothesis of Michel et al. (2021). We confirm that rings of solids with the properties inferred for protoplanetary disk rings (sizes, solid masses) can indeed evolve to produce the observed luminosities of known debris disks. Moreover, the approximately constant incidence rates of protoplanetary rings and debris disks as a function of age, when combined with our models, leaves little room for nonstructured (i.e., compact) protoplanetary disks to sustain distant reservoirs of solids >1\(M_\oplus\). This result supports the underlying assumption in Michel et al. (and many other studies) that “what you see is what you get”, i.e., that protoplanetary disks with compact millimeter emission have a compact solid mass distribution.

Our results also overlap—but may be less compatible—with the picture described by van der Marel & Mulders (2021). If they are correct that giant planets create protoplanetary disk rings and eventually migrate in close to the star where they are detected as transiting and radial velocity planets, the radial distribution of the planetesimals that result may not be ring-like. That is, if a giant planet creates a pressure bump outside its orbit where small solids collect and planetesimals form, the pressure bump moves with the planet as it migrates inward, whereas the planetesimals are poorly coupled to the gas and are left behind. If small solids are continually captured by the pressure bump and planetesimals form, the radial distribution
of planetesimals is broadened as the pressure bump migrates. If the giant planets migrate from \( \sim 40 \) au to \( \sim 1 \) au and planetesimals form continuously behind the migrating planet, a very broad disk of planetesimals can result (e.g., Figure 2 of Shibaike & Alibert 2020; see also Miller et al. 2021).

Such a broad distribution of planetesimals would eventually generate debris at increasing orbital distance with age (e.g., Kenyon & Bromley 2008, 2010), a trend that is not observed in the debris disk population (e.g., Najita & Williams 2005; Kennedy & Wyatt 2010; Matthews et al. 2014; Hughes et al. 2018; see also Figure 1). In addition, if small solids migrate inward with the pressure bump, away from the planetesimals they create, the planetesimal-pebble mixture (i.e., the planetesimal mass fraction) will be altered. The scenario we explore relies on the longevity of narrow rings (of planetesimals and pebbles with an appropriate mixture) at large radii to account for the properties and demographics of the known debris disk population.

### 4.4. Connection to Planetesimal Formation

To place the results of Figure 12 in the context of planetesimal formation theories, we classify each calculation in terms of the four evolutionary pathways. Figure 13 summarizes the results. Rings with (i) \( M_0 \lesssim 0.03 \, M_\oplus \) and any \( f \) or (ii) \( M_0 = 0.1\text{--}0.3 \, M_\oplus \) and \( f = 1 \) never generate dust in amounts detectable with Herschel (gray symbols). More massive rings (\( M_0 \gtrsim 1 \, M_\oplus \)) with \( f = 1 \) follow the late-bloomer pathway (orange symbols). Although the maximum dust luminosity of late bloomers grows with \( M_0 \), \( L_{\text{d}} / L_* \) never reaches detectable levels before \( \sim 1 \) Gyr.

Among the \((M_0, f)\) combinations considered here, steady-glow and bright-stalwart outcomes are the most numerous. Rings with \( M_0 = 0.1\text{--}1 \, M_\oplus \) and any \( f = 1 \) usually follow the steady-glow pathway (gold symbols). More massive systems (\( M_0 \gtrsim 3 \, M_\oplus \)) with \( f = 0.01\text{--}0.5 \) are bright stalwarts (blue symbols). The boundary between these two outcomes depends on \( a_0 \). When the ring is closer to the host star (e.g., \( a_0 \sim 45 \) au), dust intercepts a larger fraction of stellar radiation and brightens earlier in the evolutionary sequence. The dust in more distant rings (e.g., \( a_0 \sim 75 \) au) intercepts less stellar radiation; this lower dust luminosity falls below Spitzer detection limits for stellar ages of \( 10\text{--}100 \) Myr. Somewhat more massive rings, \( \sim 3 \, M_\oplus \) at \( 75 \) au, are more luminous at \( 10\text{--}100 \) Myr and remain detectable throughout their evolution.

For rings with \( M_0 \gtrsim 3\text{--}5 \, M_\oplus \) and \( f \lesssim 10^{-2} \), the early evolution of massive solids is stochastic. Sometimes, ensembles of \( 1000\text{--}2000 \) km protoplanets undergo a series of mergers that produce several super-Earths and generate a rapid rise in \( L_{\text{d}} / L_* \) on timescales of \( 10\text{--}30 \) Myr. However, this early-flare evolution (green symbols) depletes the system of the intermediate-mass solids that fuel the collisional cascade; \( L_{\text{d}} / L_* \) then drops quickly. When mergers of protoplanets are uncommon, the system maintains a plentiful supply of intermediate-mass solids, which power a more slowly evolving cascade where the dust luminosity rises and then declines more slowly. On timescales \( \lesssim 50\text{--}100 \) Myr, this bright-stalwart evolution generates super-Mars-mass planets but not super-Earths.

A preference for the bright-stalwart pathway instead of the early-flare and late-bloomer tracks is consistent with the current understanding of planetesimal formation. In recent numerical studies of the streaming instability (e.g., Simon et al. 2016; Li et al. 2018; Abod et al. 2019; Carrera et al. 2020; Klahr & Schreiber 2020; Gole et al. 2020; Carrera et al. 2020; Klahr & Schreiber 2020; Gole et al. 2020; Schreiber 2020; Gole et al. 2020; Rucska & Wadsley 2021), the planetesimal formation efficiencies required for bright stalwarts (\( f \approx 0.1\text{--}0.5 \)) are more common than either \( f \lesssim 10^{-2} \) (early flare) or \( f \approx 1 \) (late bloomer). Simulations that provide stronger constraints on \( f \) would enable better estimates of the importance of each of the four pathways outlined in Figure 12.

The results derived here suggest a way to test numerical simulations observationally. Although rings with a broad range of \( f \) can match a specific measurement of \( L_{\text{d}} / L_* \), robust detection of substructure within a ring may provide useful limits on \( f \). For example, if bright debris disks (\( L_{\text{d}} / L_* \sim 10^{-3} \)) at an age of \( 10\text{--}30 \) Myr commonly have a gap that is wide enough to require a several-Earth-mass planet, the numerical calculations discussed here require \( f \lesssim 0.01 \) in order to form such a massive object. For example, \( f \approx 0.01 \) and the bright-stalwart pathway (orange symbols) indicate models that failed to produce an observable \( L_{\text{d}} / L_* \) with Herschel (see also Tables 1\text{--}2). To avoid overlap, some points have been displaced horizontally.

### 4.5. Caveats and Open Questions

The calculations considered here employ standard well-tested techniques and begin with starting conditions that are consistent with observations \((M_0)\) and theory \((f)\). In a Gaussian ring with a constant gas-to-dust ratio, neglect of radial drift from gas drag has little impact as drift velocities are small. The initial orbital parameters, \( e = 10^{-3} \) and \( i = 5 \times 10^{-4} \), are plausible; however, other options are possible. If centimeter-sized pebbles are well-coupled to the gas, they would have a large vertical scale height and a larger initial inclination than considered here (e.g., Chiang & Youdin 2010; Riols & Lesur 2018; Krapp et al. 2020). As the gas dissipates over
several million years, pebbles decouple from the gas; damping from collisions and residual gas would then act rapidly and reduce \( \epsilon \) and \( \iota \) to values similar to the starting conditions considered here. The growth of larger solids and the evolution of \( L_d/L_e \) in the four pathways would differ little from our description.

If interactions with the gas or other physical processes should produce smaller \( \epsilon \) and \( \iota \) than considered here, outcomes could change dramatically. Except for models with \( f = 1 \), all sequences would begin with smaller \( L_d/L_e \). Solids in rings following the late-bloomer and steady-glow pathways would adjust on 10–100 Myr timescales and then follow the evolution described above. Within the bright-stalwart and early-flare pathways, planetesimals would have larger gravitational focusing factors and grow more rapidly. While it would take a little extra time for \( L_d/L_e \) to begin to rise, these systems might produce larger planets and brighter debris disks. The early-flare tracks might be more peaked and fade more rapidly; the bright-stalwart tracks might be somewhat brighter but would fade on timescales similar to those described above. In both pathways, it might be easier for high-resolution observations of the debris to identify dark gaps and bright rings due to the more energetic early stages of planet formation.

Aside from including the \( n \)-body component of Orchestra, other changes to the approach (e.g., fragmentation parameters, number of annuli per ring, or number of mass bins per annulus) are unlikely to change the outcomes significantly. As discussed in Appendix B, modifying the fragmentation parameters yields changes of order \( \sim 2 \) in \( L_d/L_e \). Previous tests of the coagulation code demonstrate that improving the mass and spatial resolution of a calculation produces similarly small changes in outcomes as a function of evolution time (e.g., Kenyon & Bromley 2016b, 2017).

Finally, the calculations presented here do not explore whether and how the widths of the rings change with time. The ring widths may evolve depending on the mass of the planets that form in the ring and other dynamical processes. Future calculations are needed to explore this issue.

5. Summary

Rings of solids at \( a \gtrsim 30–40 \) au offer a way to resolve a long-standing disconnect between detailed evolutionary models of debris disks and their observed properties. The lack of a strong trend in the orbital distance of debris with age (e.g., Najita & Williams 2005; Kennedy & Wyatt 2010; Matthews et al. 2014; Hughes et al. 2018; see also Figure 1), which is predicted by earlier generations of planet formation models, is readily explained if the parent bodies that produce the debris are initially distributed in discrete rings rather than over a broad range of orbital radii. The prominent rings that are commonly observed in T Tauri (protoplanetary) disks, which have an incidence rate similar to that of cold debris disks around low-mass stars (\( \sim 20\%–25\% \)), suggest a compelling starting point for debris disk evolution (Section 2).

We have explored the potential connection between protoplanetary rings and debris disks, using a new set of evolutionary calculations that follow the evolution of rings of solids spanning a range of initial properties. The results show that diverse evolutionary histories are possible as a function of planetesimal formation efficiency \( f \) and initial solid mass \( M_0 \) (Section 3). Depending on these parameters, rings of solids can grow Pluto-, Mars-, or super-Earth planets in 0.01–10 Gyr (Section 3.3). The resulting dust luminosity histories are also diverse and fall into four main pathways (Figure 12): an always-bright classical evolutionary pathway that encompasses the known debris disks (“bright stalwart”); tracks that burn bright and fade fast (“early flare”); those that maintain a relatively constant, lower luminosity from 10 Myr to 10 Gyr (“steady glow”); and those that brighten dramatically at late times to a detectable luminosity (“late bloomer”). The largest objects that form via these pathways are expected to clear detectable gaps in the radial distribution of the accompanying debris (Figures 9–10; Section 3.5).

When compared with the model tracks, the known population of bright young debris disks (\( L_d/L_e \gtrsim 10^{-3} \) at 50–100 Myr) is well matched by rings that start out with high initial mass (\( M_0 = 5–40 M_{\oplus} \)). As they evolve, these systems pass through the \( L_d/L_e \), distribution of known debris disks as a function of age, an outcome that is consistent with a large range in planetesimal formation efficiency (\( f \lesssim 0.5 \)). Thus, if \( \sim 25\% \) of stars start out with massive protoplanetary rings and follow this classical “bright-stalwart” evolutionary pathway, we can readily account for the observed luminosities and incidence rates of debris disks over time (Section 4.2).

Although most current observations are consistent with this simple picture, the debris disk incidence rate in the 10–50 Myr age range is not well known. The true story may be more complex if the high incidence rate of cold debris disks among the small sample of F stars in the \( \beta \) Pic moving group is typical of 10–50 Myr old stars. A large population of such sources may indicate that a significant fraction of disks follow the “early-flare” pathway, in which massive rings of solids with very small \( f \) that are also undetectable because of their very small initial scale height brighten dramatically into observable debris disks, then quickly fade below current detection limits. Future observations that constrain the debris disk frequency and \( L_d/L_e \) of stars in the 10–50 Myr age range are needed to understand how often protoplanetary disks pursue this evolutionary pathway.

Constraints from an even earlier stage of evolution, from the Class III phase, are also important to understand the evolutionary pathways of solids in disks. The recent work of Lovell et al. (2021), which studies a small sample of Class III sources in Lupus, is a good start in this direction. Larger samples and deeper observations are needed to understand whether most Class III sources follow the approximately constant luminosity evolution of the “bright-stalwart” disks at early times, and/or if disks populate the fainter “steady-glow” evolutionary tracks.

This uncertainty aside, the ability of the classical “bright-stalwart” pathway to explain the known debris disks appears to limit the role of the other generic pathways in producing known debris disks. The inferred strong evolutionary connection between the \( \sim 20\%–25\% \) of protoplanetary disks with large rings and the \( \sim 20\%–25\% \) of mature stars with cold debris disks implies that the majority population of compact protoplanetary disks (\( \sim 75\%–80\% \) of all disks) leave behind only modest masses of residual solids at large radii (\( \lesssim 1 M_{\oplus} \)) and evolve primarily into mature low-mass stars without detected debris at \( a \gtrsim 30–40 \) au (Figures 4 and 12). In other words, the cold debris disks studied to date are a chapter in the history of a minority of low-mass stars (Section 4.3).

We can test this interpretation by looking for dynamical evidence of the planets predicted to form under these.
conditions. Planets should produce gaps in the accompanying debris, which may be resolvable spatially with facilities such as the ngVLA (Section 4.2). Resolving gaps in young disks can also place constraints on the efficiency of planetesimal formation (Section 4.3). Improved debris disk demographics can also test this picture. More sensitive surveys that probe \( L_d/L_* \) down to \( 10^{-5} \)–\( 10^{-6} \) in the 10–300 Myr age range can directly constrain the extent to which disks follow the “steady-glow” and “late-bloomer” paths. It is also important to study larger samples of young stars (<100 Myr) and to characterize the incidence rate of bright cold debris disks \( (L_d/L_* \sim 10^{-4} \rightarrow 10^{-2}) \) to infer whether they represent ~20% of low-mass stars or a much larger fraction. A much larger fraction (Pawellek et al. 2021) would indicate a more complex situation and a possibly significant role for the “early-flare” pathway.

Although the picture we have described motivates and awaits new tests, our analysis illustrates how models of the evolution of rings of solids, when combined with observational constraints on the demographics of debris disks (e.g., their incidence rate as a function of luminosity and age), can strongly constrain the global evolutionary pathways of debris disks and place constraints on current important unknowns, such as the efficiency of planetesimal formation and the masses of possible dark reservoirs of solids in young disks.

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Appendix A
The ORCHESTRA Code

To derive the evolution of solid particles within a disk or a ring, we perform sets of numerical calculations with ORCHESTRA, an ensemble of computer codes designed to track the accretion, fragmentation, and orbital evolution of solid particles ranging in size from a few microns to thousands of kilometers (Kenyon 2002; Bromley & Kenyon 2006; Kenyon & Bromley 2008; Bromley & Kenyon 2011a, 2013; Kenyon & Bromley 2016b, 2016a). For the calculations described here, we establish a radial grid of 28 or 56 concentric annuli distributed in equal intervals of \( a^{1/2} \) between \( a_{\text{in}} \) and \( a_{\text{out}} \). Each annulus has 140 mass bins; the interval between adjacent bins is \( \delta = m_{i+1}/m_i = 2 \). The minimum particle radius is \( r_{\text{min}} = 1 \mu m \); the largest possible object in the grid has a mass of roughly 50 \( M_\oplus \).

Within each calculation, solids have initial mass density \( \rho \), orbital eccentricity \( e_0 \), orbital inclination \( i_0 \), and total mass \( M_0 \). Each annulus in the grid has initial surface density of solids \( \Sigma_0 \propto a^{-3/2} \) (disks) or \( \Sigma_0 \propto e^{-(a-a_0)/\Delta a} \) (rings). As the calculations proceed, collisions between particles modify the number and masses of objects in each mass bin. Collisional damping and gravitational interactions between solids change the orbital parameters \( e \) and \( i \).

To evolve the size and velocity distributions of solids in time, ORCHESTRA derives collision rates and outcomes with standard particle-in-a-box algorithms (Kenyon & Bromley 2012). Systems start with an initial size distribution \( n(r) \). When a pair of solids collides, the mass of the merged object is

\[
m = m_1 + m_2 - m_{\text{esc}},
\]

where \( m_1 \) and \( m_2 \) are the masses of the colliding particles. The mass of debris ejected in a collision is

\[
m_{\text{esc}} = 0.5 (m_1 + m_2) \left( \frac{Q_s}{Q_D} \right)^{\frac{1}{2}},
\]

where \( Q_s = m_1 m_2 v^2/(m_1 + m_2)^2 \) is the center-of-mass collision energy, \( v \) is the collision velocity, and the exponent \( b_2 \) is a constant of the order of unity (e.g., Davis et al. 1985; Wetherill & Stewart 1993; Kenyon & Luu 1999; Benz & Asphaug 1999; O’Brien & Greenberg 2003; Kobayashi & Tanaka 2010; Leinhardt & Stewart 2012). The binding energy of solids, \( Q_D \), is the energy required to disperse half of the combined mass, \( m_1 + m_2 \), to infinity:

\[
Q_D^* = Q_s r_v^2 + Q_g r_v^2,
\]

where \( \rho = 1.5 \) g cm\(^{-3} \) is the mass density, and \( (Q_s, Q_g, e_s, e_g) \) are model parameters (e.g., Benz & Asphaug 1999; Leinhardt & Stewart 2012). In this expression, the first (second) term is the strength (gravity) component of the binding energy. We choose parameters for normal ice: \( Q_s = 4 \times 10^6 \) erg cm\(^{-1/2} \) g\(^{-1} \), \( e_s = -0.4, Q_g = 0.3 \) erg cm\(^{1.65} \) g\(^{-1} \), and \( e_g = 1.35 \) (Schlichting et al. 2013; Kenyon & Bromley 2020).

To place the debris in the grid of mass bins, we set the mass of the largest collision fragment as

\[
m_{\text{max},d} = m_{10} \left( \frac{Q_s}{Q_D^*} \right)^{-b_2} m_{\text{esc}},
\]

where \( m_{10} \approx 0.01–0.5 \) and \( b_2 \approx 0–1.25 \) (Wetherill & Stewart 1993; Kenyon & Bromley 2008; Kobayashi & Tanaka 2010; Weidenschilling 2010). When \( b_2 \) is large, catastrophic (cratering) collisions with \( Q_s \gtrsim Q_D^* \) (or \( Q_s \gtrsim Q_g^* \)) crush solids into smaller fragments. Lower-mass objects have a differential size distribution \( N(r) \propto r^{-\delta} \) with \( \delta \approx 3–4 \). After placing a single object with mass \( m_{\text{max},d} \) in an appropriate bin, we place material in successively smaller mass bins until (i) the mass is exhausted or (ii) the mass is placed in the smallest mass bin. Any material left over is removed from the grid.

To follow the orbital evolution of solids, we derive collisional damping from inelastic collisions and elastic (gravitational) interactions. For inelastic and elastic collisions, we follow the statistical, Fokker–Planck approaches of Ohtsuki (1992) and Ohtsuki et al. (2002), which treat pairwise interactions (e.g., dynamical friction and viscous stirring) between all objects. We also compute long-range stirring from distant oligarchs (Weidenschilling 1989).

We assume the surface density of gas is zero throughout the grid and ignore interactions between solids and gas. Previously
published calculations with Orchestra (e.g., Kenyon & Bromley 2008, 2009, 2010) demonstrate that gas drag tends to circularize the orbits of particles with radii \( r \lesssim 1 \text{ km} \) on timescales of several million years at 30–150 au (see Adachi et al. 1976; Weidenschilling 1977; Rafikov 2004). In parallel, dynamical friction between these particles and much larger solids gradually reduces the \( e \) and \( i \) of the larger solids. Once runaway growth begins, the largest solids then have much larger gravitational focusing factors and grow much more rapidly than ensembles of solids where gas drag is neglected (see also Youdin & Kenyon 2013, pp. 1–51). Because our interest is in the evolution of solids on billion-year timescales, the lack of gas drag probably has little impact on our results. We consider several comparisons below.

Aside from circularization of orbits, the gas causes intermediate-sized solids to drift radially relative to the gas (Adachi et al. 1976; Weidenschilling 1977; Rafikov 2004). The smallest solids drift with the gas; the largest solids are not affected by the gas. Here, we avoid the complications of evolving the surface density of the gas (e.g., Alexander & Armitage 2009; Oka et al. 2011; Bromley & Kenyon 2011a; Martin & Livio 2012, 2014; Bitsch et al. 2015; Zhang & Jin 2015; Xiao et al. 2017; Shadmehri & Ghoreyshi 2019) and ignore radial drift of the solids. For our focus on long-term evolution, this assumption is a reasonable starting point.

Our solutions to the evolution equations conserve mass and energy to machine accuracy. Typical calculations require several 12 hr runs on a system with 56 cpus; over the \( 10^7–10^8 \) time steps in a typical 2–4 Gyr run, calculations conserve mass and energy to better than one part in \( 10^{10} \).

Appendix B

Binding Energy Parameters

In previous studies, Kenyon & Bromley (2008, 2010) demonstrated that outcomes of coagulation calculations are remarkably independent of initial conditions. Although the growth time for large objects scales inversely with mass, collisions rapidly erase the initial eccentricity and inclination of the solids (see also Kenyon & Bromley 2004, 2012, 2016b). The pace of growth also depends on the initial size distribution and the maximum size of the solids at the start of a calculation. However, the final radius of the largest object in an annulus is rather insensitive to these starting conditions.

Growth of the largest objects is much more sensitive to the bulk properties of solids. Solids with larger binding energy \( Q_D^s \) generate less debris during a collision and therefore grow larger with time than solids with smaller \( Q_D^s \). In a suite of coagulation calculations at 15–150 au, weak solids reach typical sizes of 3000–7000 km in 10 Gyr (Kenyon & Bromley 2010, 2012). Stronger solids achieve radii somewhat larger than 100 km on similar timescales. Despite these differences in final planet radii, large changes in the binding energy produce factor of \( \lesssim 2 \) variations in the maximum surface area of small particles (Kenyon & Bromley 2010).

As an illustration of the sensitivity of the stellar luminosity reprocessed by small particles, \( L_s \), we consider calculations in a single annulus at 30–60 au from the Sun (Kenyon & Bromley 2020). Calculations begin with \( 45 M_\oplus \) in solids. Particles have a range of sizes \( r \approx 100–500 \text{ km} \); 100 km particles initially contain most of the mass. To initiate a collisional cascade at the start of the calculation, particles have initial collision velocities of 1 km s\(^{-1}\), which guarantees that collisions among 100 km particles are destructive. Although there are no particles smaller than 100 km at the start of the calculation, destructive collisions rapidly create them.

In addition to the parameters appropriate for normal ice listed above, we perform calculations with “weak” \((Q_s = 2 \times 10^7 \text{ erg cm}^{2.19} \text{ g}^{-1})\) and “strong” \((Q_s = 7 \times 10^7 \text{ erg cm}^{2.19} \text{ g}^{-1})\) ice. For the gravitational component of \( Q_s^D \), we examine results for the Benz & Asphaug (1999) approach \((BA; Q_s = 2.1 \text{ erg cm}^{1.81} \text{ g}^{-1})\) and the Leinhardt & Stewart (2012) approach \((LS; Q_s = 0.3 \text{ erg cm}^{1.81} \text{ g}^{-1})\). To have a sense of how coagulation calculations react to these choices, we briefly outline how collision outcomes depend on \( Q_s^D \). When solids are weak (strong), they are easier (harder) to fragment during a high-velocity collision. As the largest objects grow, more (fewer) collision fragments generate a faster (slower) rise in the dust luminosity. If collisional damping is important, a larger mass in small fragments may accelerate the growth of the largest objects (e.g., Kenyon & Bromley 2015, 2016a). Over time, systems of weaker solids lose somewhat more mass and contain somewhat less mass in small solids than those with stronger solids (see also Kenyon & Bromley 2016b).

Figure 14 shows the evolution of the dust luminosity for the single-annulus calculations (Kenyon & Bromley 2020). Starting with most of the mass in 100 km objects, solids have initial collision velocities of 1 km s\(^{-1}\) and no dynamical evolution. Starting conditions are set to initiate a cascade of collisions that gradually grind 100 km objects into fine dust grains that are ejected by radiation pressure from the central star (see also Wyatt 2007). For the chosen parameters, the binding energy of 100 km objects is fairly independent of \( Q_s^D \), \( Q_s^G \), and \( e_s \). Collisions between pairs of these objects produce the same amount of mass in fragments; fragments also have the same size distribution. Across the five different calculations, large objects generate fragments at the same rate throughout 4.5 Gyr of evolution. Thus, the mass-loss rate of the system is the same.
in all five calculations. After 4.5 Gyr, all systems have the same final mass in solids.

Differences arise when the 1–10 km fragments collide. At the smallest sizes in these calculations (1–10 μm), the dust luminosity depends on the rate mass flows from the fragments to smaller and smaller sizes. For systems with the same binding energy at the largest sizes, the mass flow is largest for “weak” ice systems and is then progressively smaller as the bulk strength grows from weak to normal to strong (see also Wyatt et al. 2011; Kenyon & Bromley 2016b, 2017). With a constant production rate of 1–10 km fragments, strong ice systems retain a larger fraction of this mass than weak ice systems and therefore have a larger dust luminosity. In addition to the bulk strength, the mass flow down the cascade depends on the gravitational component of the binding energy. For particle radii \( r \gtrsim 10–100 \text{ m} \), self-gravity dominates bulk strength. In the BA formalism, 0.1–2 km fragments are stronger than in the LS formalism. Independent of the bulk strength, the mass flow rate in a system of BA solids is then smaller than in a system of LS solids. BA systems retain more small solids and have larger \( L_d \) than LS systems.

In Figure 14, all five systems follow the same evolution. During the first 1–10 Myr, the dust luminosity rises rapidly as large object collisions produce the first fragments, which in turn produce the first small dust grains. Because weaker fragments make more dust, the luminosity rises faster in systems of weak particles. At 10–100 Myr, the rapid rise in \( L_d \) from the initial set of collisions slows. Systems enter a plateau phase, where the dust luminosity slowly rises. After 3–4 Gyr, collision rates among leftover 100 km objects begin to drop, dust production slows, and the dust luminosity falls.

At late times, the evolution of the dust luminosity falls into three groups with only a factor of \( \sim 3 \) range in \( L_d \). Systems with the LS gravitational component of the binding energy have the largest mass flow down the cascade, the smallest dust mass, and the lowest dust luminosity. Although LS systems with “weak” ice initially evolve more rapidly than those with normal ice, they converge on nearly the same \( L_d \) at late times. In these two examples, collisions between 0.1 and 10 km objects set the mass flow rate down the cascade. For a collision velocity of 1 km s\(^{-1}\), the bulk strength of 1 cm and smaller particles has little impact on the population of small particles. Thus, weak ice collisions with the LS gravitational component yield the same dust luminosity at late times.

In cascades with smaller collision velocities, the difference between weak and normal bulk strength might lead to different dust luminosities at late times. However, we expect these differences to be smaller than those from adopting different relations for the gravity component of \( Q_b \).

Systems with strong ice and the BA gravity component have the smallest mass flow rate down the cascade and the largest \( L_d \). At intermediate \( L_d \) systems with the BA gravity component and either weak or normal ice have roughly the same \( L_d \) at late times. In these systems, the rate of mass flow down the cascade is the same as the strong BA model from 100 km to 0.1 km. Below 0.1 km, the mass flow rate grows substantially compared to the strong BA model and reaches roughly the same level in both calculations. Thus, these models have roughly the same \( L_d \) at late times.

The small variation in maximum \( L_d \) among these calculations suggests that the binding energy parameters are not critical components for predictions of the luminosity (see also Kenyon & Bromley 2010, 2012). To facilitate comparisons with previous studies, we perform calculations with the normal ice and LS binding energy parameters. From Figure 14, this choice produces a smaller dust luminosity than the BA binding energy parameters.

The maximum sizes of the largest objects are also somewhat smaller. Expressed another way, the normal ice bulk strength with the LS gravity component of the binding energy requires the largest mass to generate an observed maximum \( L_d \) and a maximum size for the largest objects. If the suite of calculations discussed in the main text explains observations without violating mass budget constraints (e.g., Najita & Kenyon 2014), then simulations with other choices for the binding energy parameters will match observations with lower initial masses in solids.

Appendix C
Supplemental Results

In Sections 3.3–3.5, we described the time evolution of (i) the largest objects with a broad range in \( M_0 \) and \( f \) (Figures 5–7), (ii) \( L_d/L_\star \) for selected values of \( M_0 \) and \( f \) (Figure 8), and (iii) the radial distribution of the largest objects within the rings (Figures 9–10). To illustrate several other aspects of the calculations, we describe the importance of collisional damping on runaway growth and show the relation between the growth time, the initial mass in solids, and \( f \). For completeness, this section also includes Tables of results for the full set of calculations.

Eccentricity evolution. Collisional damping, dynamical friction, and viscous stirring are central features of all calculations with \( f < 1 \). In a system with a mix of pebbles and planetesimals, collisions among pebbles are inelastic and circularize the orbits of pebbles (see also Goldreich et al. 2004, and references therein). Gravitational interactions between pebbles and planetesimals try to equalize the kinetic energy of both species. With the large mass ratio between pebbles and planetesimals, this process excites the eccentricities of pebbles and damps the eccentricities of planetesimals. Viscous stirring transfers angular momentum from planetesimals to pebbles, raising \( e \) for the pebbles.

Figure 15 illustrates the impact of these processes for a calculation with \( m_0 = 45 M_\oplus \) and \( f = 0 \) at 75 au. Initially, all pebbles have \( e = 10^{-3} \). During the first 1000 yr of evolution, collisions among pebbles create 10 cm rocks and debris with \( r \lesssim 1 \text{ mm} \). Damping reduces the eccentricity of 1–100 μm (3–10 cm) particles to \( \sim 5\times10^{-4} \). As pebbles continue to grow, modest stirring by meter-sized to kilometertized planetesimals tries to raise \( e \), but pebble damping continues to keep \( e \) low.

This balance between the smallest and largest solids in the ring continues until the runaway produces large protoplanets. At 10 Myr, several 0.25 \( M_\oplus \) planets try to stir all of the lower-mass solids. Solids with \( R \approx 100 \text{ m} \) have little mass; protoplanets effectively stir them to larger \( e \). Among smaller solids, damping maintains low \( e \). At 100 Myr, the smallest solids continue to resist viscous stirring by the largest objects. Although 100 m and larger objects now have \( e \sim 0.01–0.1 \), small solids maintain \( e \sim 2–4 \times 10^{-3} \). Small fluctuations in \( e \) for 0.1–1 m solids result from variations of the mass contained in each mass bin; bins with smaller \( e \) have more mass.

As the evolution proceeds to 1–10 Gyr, the cascade gradually reduces the population of solids with \( r \lesssim 1 \text{ m} \). With
less mass among pebbles, damping is less effective. For nearly all solids with $r \gtrsim 4 \text{ m}$, $e \approx 0.1–0.2$. Among the 1–4 m solids, damping maintains a small $e$. The linear rise in the $e$ distribution from 1 m to 1 $\mu$m results from the smaller mass in these objects: damping maintains small $e$ for 1–4 m objects but damping is less and less effective at smaller and smaller sizes. By 10 Gyr, nearly all solids have roughly the same eccentricity, $e \approx 0.3$; the largest protoplanets have $e \approx 0.1$.

Figure 16 shows snapshots of the eccentricity evolution for a massive ring with planetesimals and pebbles. During the first million years, growth is slow: mergers produce a few objects with $r \approx 200–300 \text{ km}$ and a swarm of smaller solids with $r \lesssim 10 \text{ km}$. Collisional damping reduces $e$ by 10%–20% for the smaller solids; dynamical friction lowers $e$ by almost a factor of 100 for the largest solids.

Over the next 30 Myr, the largest solids grow dramatically to nearly 10,000 km. With most of the mass, protoplanets stir themselves to larger and larger $e$. Solids with $R \approx 0.1–10 \text{ km}$ have little mass and are stirred to even larger $e$. Despite having less mass, pebbles maintain smaller $e$ through collisional damping. The combined effects of damping and stirring yield an eccentricity distribution where the largest and the smallest solids have $e \approx 2 \times 10^{-3}$. Dynamical friction between pebbles and 100–300 km solids keeps the larger solids at very small $e \approx 10^{-5}$.

As the collisional cascade continues, pebbles contain less and less mass. Damping and dynamical friction are less effective; viscous stirring dominates. Eccentricities for all solids grow from $10^{-3}$ to 0.1–0.3. At either end of the distribution, $e$ drops by a factor of 2–3. For the large solids, dynamical friction with the rest of the solids reduces $e$. The smallest solids have just enough mass for damping to reduce and maintain the smaller $e$.

These two examples illustrate how damping among pebbles fuels runaway growth of the largest planetesimals. Without damping, pebbles have a factor of 10–100 larger eccentricity. Larger eccentricity lowers gravitational focusing factors and reduces the growth rate of the largest planetesimals. Runaway growth is delayed and is much weaker. Instead of reaching super-Earth masses on 1–10 Myr timescales, systems with no damping would produce super-Earths on 100 Myr to 1 Gyr timescales. Delaying the runaway allows smaller planetesimals to accrete more pebbles, reducing the dust luminosity to levels below observed systems at 10–100 Myr. Once the runaway begins, a delayed collisional cascade would probably raise the dust luminosity to levels well above those observed at 1–10 Gyr. In this way, the lack of collisional damping would challenge our ability to match observations with an evolving swarm of pebbles and planetesimals.

**Tables of results.** To facilitate comparisons between these calculations and those in other studies, Tables 1–2 summarize results for each calculation. The first two columns list $M_0$ and $f$. In the third column, the ratio of the final mass $M_f$ to $M_0$ provides a measure of the mass ejected in the form of 1 $\mu$m and smaller particles. In energetic (weak) cascades, $M_f/M_0$ is small (large). Rings with small $f$ lose much more material than those with large $f$. The next columns measure the ability of collisional growth to concentrate solids into massive planets. The variable $t_{1k}$ in column 4 quantifies the timescale for the growth of Pluto-mass planets; $r_{\text{max}}$ in column 5 lists the radius of the largest protoplanet at 10 Gyr. Rings that generate super-Earths (Plutos) have large (small) $r_{\text{max}}$ and small (large) $t_{1k}$. The next two columns quantify the amount of mass in the largest objects at the end of each calculation: $N_t (M_f)$ is the number (mass) of objects with masses no less than 10% of the mass of the largest object. Sometimes, the calculation produces 1–3 large protoplanets; others with similar amounts of mass in large objects generate many much smaller protoplanets. These two quantities allow for a comparison of systems with similar initial masses but very different $r_{\text{max}}$. Finally, the last column quantifies the maximum dust luminosity throughout the calculation. As noted in the main text, more massive rings have more luminous debris disks. This column quantifies that statement.

Among the published numerical calculations of pebble accretion, only Shannon et al. (2015) consider the 1–10 Gyr evolution of a swarm of pebbles and planetesimals. Starting with $M_0 = 0.01 M_\odot$ in a ring at 42–48 au with $f = 10^{-3}$ for pebbles and kilometer-sized planetesimals, they follow the growth of larger solids with a single-annulus coagulation calculation. With a much smaller initial eccentricity $e_0 = 10^{-6}$ and binding energy for solids ($Q_s^*$, Equation (A3)), large objects in these models grow more slowly and fail to reach $r_{\text{max}} \gtrsim 1000 \text{ km}$ in 10 Gyr. In contrast, For the initial conditions...
considered here, results are similar: \( r_{\text{max}} \approx 1000 \text{ km} \) at 1–2 Gyr and \( \sim 2000 \text{ km} \) at 10 Gyr.

**Growth time as a function of initial mass.** To conclude this subsection, Figure 17 plots values for \( t_{\text{f}\ell} \) in Tables 1–2 as a function of \( M_0 \) for rings at 45 au (lower panel) and 75 au (upper panel). The data show clear relations between \( t_{\text{f}\ell} \) and \( M_0 \). For calculations with the same \( f \), the timescale to form Pluto-mass protoplanets scales inversely with the initial mass of solids, \( t_{\text{f}\ell} \propto M_0^{-1} \). Among the full ensemble, there are three trends. Systems with \( f = 1 \) (\( f \leq 10^{-5} \)) have the longest (shortest) formation times, with a difference of three orders of magnitude independent of \( M_0 \). Midway between, systems with \( 10^{-4} \leq f \leq 0.5 \) have a factor of \( \sim 30 \) shorter (longer) formation time than those with \( f = 1 \) (\( f \leq 10^{-5} \)). The short formation times with small \( f \) are a hallmark of pebble accretion (Goldreich et al. 2004; Rafikov 2005).

The relation for the slow growth rates for systems with \( f = 1 \) is straightforward to derive. In any swarm of solids, the growth time for a solid of radius \( r \) is \( t_c \propto (rP/\Sigma)[1 + (v_{\text{esc}}/v)^2]^{-1} \), where \( P \) is the orbital period, \( \Sigma \) is the surface density of solids, \( v_{\text{esc}} \) is the escape velocity of the pair of colliding solids, and \( v \) is their collision velocity (e.g., Safronov 1969; Lissauer 1987; Wetherill & Stewart 1993; Goldreich et al. 2004; Rafikov 2005; Kenyon & Bromley 2008). When \( f = 1 \), \( v_{\text{esc}} \approx v \), \( t_c \propto rP/\Sigma \).

Compared to the prediction in Equation (1) derived from scaling results in Kenyon & Bromley (2008), the growth timescales listed in Tables 1–2 are a factor of two smaller. Analysis of each calculation suggests a simple explanation: in the ring-like geometries considered here, planetesimals are concentrated in the middle of the ring. Collisions with other planetesimals outside the central annuli tend to concentrate additional material in these annuli. This additional material increases the surface density and lowers the growth time. In the full suite of calculations, the typical increase in \( \Sigma \) is a factor of two, accounting for the factor-of-two smaller growth time.

Although the smaller growth times for systems with \( f < 1 \) are easy to understand, deriving a quantitative relation is a challenge. In a system of pebbles and planetesimals, damping reduces collision velocities. With \( v \ll v_{\text{esc}} \), \( t_c \propto (rP/\Sigma)(v/v_{\text{esc}})^2 \). To evaluate \( (v/v_{\text{esc}})^2 \), there are two regimes (e.g., Goldreich et al. 2004; Rafikov 2005). In the “dispersion regime”, collision velocities exceed the Hill velocity, \( v_H = \Omega R_H \), where \( \Omega \) is the angular velocity. Analytic results then yield \( (v/v_{\text{esc}})^2 \propto f \). In the “shear regime”, \( v < v_H \), then \( (v/v_{\text{esc}})^2 \propto f^{1/2} \).

These theories predict the sense of the results in Tables 1–2, but not the clustering of \( t_{\text{f}\ell} \) for \( 10^{-4} \lesssim f \lesssim 0.5 \) and the second cluster of results for smaller \( f \). When \( f \) is roughly zero, growth is in the shear regime and very rapid as summarized in the main text. Larger \( f \) places growth more in the dispersion regime, where growth is somewhat slower. Our results suggest that the boundary between rapid and extremely rapid growth is \( f \approx 10^{-4} \). However, current theory is insufficient to isolate this boundary. At the same time, theory suggests that the growth time should scale with \( f \) for intermediate \( f \) between 0 and 1 rather than a common growth time for a range of \( f \). Because relating \( t_{\text{f}\ell} \) to the initial properties of rings of solids is not a central goal of this project, we leave a detailed investigation of this issue to a separate study.
Table 1
Results for Coagulation Calculations at 36–54 au.*

| $M_0$ ($M_\odot$) | $f$  | $M_f/M_0$ | $t_{1k}$ (Myr) | $r_{\text{max}}$ (km) | $N_{\text{f}}$ | $M_\ell$ ($M_\odot$) | $L_{d,\text{max}}/L_\odot$ |
|-------------|-----|-----------|---------------|----------------|-------------|----------------|-----------------|
| 0.01        | 1.0 | 1.000     | ...           | 167.5          | 8017        | 0.01           | <0.01           |
| 0.01        | 0.3 | 0.979     | ...           | 257.0          | 98          | <0.01          | 0.50            |
| 0.01        | 0.1 | 0.986     | ...           | 192.8          | 869         | <0.01          | 0.58            |
| 0.01        | $10^{-2}$ | 0.993 | ... | 295.1 | 75 | <0.01 | 0.61 |
| 0.01        | 0.0 | 0.993     | ...           | 453.9          | 5           | <0.01          | 0.62            |
| 0.03        | 1.0 | 1.000     | ...           | 199.5          | 23714       | 0.03           | 0.01            |
| 0.03        | 0.3 | 0.933     | ...           | 639.7          | 9           | <0.01          | 1.02            |
| 0.03        | 0.1 | 0.940     | ...           | 638.3          | 15          | <0.01          | 1.04            |
| 0.03        | $10^{-2}$ | 0.885 | ... | 540.8 | 178 | 0.01 | 1.04 |
| 0.03        | $10^{-3}$ | 0.545 | 7067.8 | 1049.5 | 17 | 0.01 | 1.05 |
| 0.03        | 0.0 | 0.962     | ...           | 736.2          | 7           | <0.01          | 1.05            |
| 0.10        | 1.0 | 1.000     | ...           | 206.1          | 78705       | 0.10           | 0.01            |
| 0.10        | 0.3 | 0.828     | 8851.2        | 1047.1         | 8           | <0.01          | 2.15            |
| 0.10        | 0.1 | 0.625     | 3143.6        | 1538.2         | 2           | 0.01           | 2.01            |
| 0.10        | $10^{-2}$ | 0.495 | 5601.9 | 1039.9 | 37 | 0.01 | 1.63 |
| 0.10        | $10^{-3}$ | 0.312 | 2293.1 | 1207.8 | 40 | 0.02 | 2.37 |
| 0.10        | 0.0 | 0.468     | 1402.8        | 1367.7         | 17          | 0.02           | 2.05            |
| 0.30        | 1.0 | 1.000     | ...           | 434.5          | 1683        | 0.02           | 0.03            |
| 0.30        | 0.3 | 0.774     | 1453.5        | 1552.4         | 10          | 0.01           | 4.95            |
| 0.30        | 0.1 | 0.587     | 820.4         | 2113.5         | 9           | 0.03           | 4.30            |
| 0.30        | $10^{-2}$ | 0.319 | 659.9 | 2387.8 | 3 | 0.02 | 5.16 |
| 0.30        | $10^{-3}$ | 0.254 | 550.4 | 1836.5 | 12 | 0.03 | 6.34 |
| 0.30        | $10^{-4}$ | 0.341 | 95.4 | 2138.0 | 15 | 0.04 | 8.32 |
| 0.30        | 0.0 | 0.752     | 317.4         | 1625.5         | 5           | 0.01           | 2.51            |
| 1.00        | 1.0 | 0.998     | ...           | 517.6          | 6295        | 0.17           | 0.20            |
| 1.00        | 0.3 | 0.750     | 427.6         | 2128.1         | 11          | 0.05           | 13.30           |
| 1.00        | 0.1 | 0.547     | 229.0         | 3076.1         | 9           | 0.09           | 10.57           |
| 1.00        | $10^{-2}$ | 0.272 | 149.8 | 3639.2 | 5 | 0.12 | 17.22 |
| 1.00        | $10^{-3}$ | 0.217 | 129.5 | 2393.3 | 6 | 0.04 | 20.04 |
| 1.00        | $10^{-4}$ | 0.186 | 30.6 | 3191.5 | 2 | 0.04 | 26.61 |
| 1.00        | 0.0 | 0.267     | 39.4          | 2055.9         | 10          | 0.04           | 7.57            |
| 3.00        | 1.0 | 0.993     | 4818.2        | 2157.7         | 49          | 0.11           | 0.66            |
| 3.00        | 0.3 | 0.676     | 98.7          | 3243.4         | 3           | 0.06           | 32.21           |
| 3.00        | 0.1 | 0.520     | 87.5          | 4385.3         | 4           | 0.16           | 29.65           |
| 3.00        | $10^{-2}$ | 0.234 | 66.6 | 5407.5 | 5 | 0.28 | 65.61 |
| 3.00        | $10^{-3}$ | 0.168 | 51.9 | 5211.9 | 2 | 0.16 | 65.92 |
| 3.00        | $10^{-4}$ | 0.136 | 19.6 | 2747.9 | 46 | 0.22 | 86.50 |
| 3.00        | $10^{-5}$ | 0.094 | 0.6 | 3396.3 | 17 | 0.27 | 110.41 |
| 3.00        | 0.0 | 0.118     | 3.1           | 4335.1         | 6           | 0.15           | 44.16           |
| 10.00       | 1.0 | 0.962     | 716.9         | 4375.2         | 22          | 0.66           | 2.18            |
| 10.00       | 0.3 | 0.561     | 19.8          | 4168.7         | 20          | 0.50           | 119.40          |
| 10.00       | 0.1 | 0.468     | 24.9          | 4405.5         | 13          | 0.48           | 132.74          |
| 10.00       | $10^{-2}$ | 0.211 | 14.5 | 6280.6 | 5 | 0.58 | 154.17 |
| 10.00       | $10^{-3}$ | 0.143 | 15.8 | 6683.4 | 6 | 0.85 | 350.75 |
| 10.00       | $10^{-4}$ | 0.140 | 6.2 | 3784.4 | 22 | 0.28 | 239.88 |
| 10.00       | $10^{-5}$ | 0.091 | 0.3 | 3097.4 | 75 | 0.81 | 340.41 |
| 10.00       | 0.0 | 0.047     | 1.0           | 3810.7         | 4           | 0.23           | 127.94          |

**Note.**

* The columns list the initial mass $M_0$ for each calculation, the initial fraction $f$ of mass in large planetesimals, the ratio of the final mass in the grid $M_f$ to the initial mass, the timescale $t_{1k}$ to produce the first protoplanet with $r \geq 1000$ km, the final radius $r_{\text{max}}$ of the largest planet, the final number $N_{\text{f}}$ of protoplanets with masses at least as large as 10% of the mass of the largest planet, the total mass $M_\ell$ in these large protoplanets, and the maximum relative dust luminosity $L_{d,\text{max}}/L_\odot$ in units of $10^{-5}$.
To illustrate the impact of differences between the calculations in this paper and those in Kenyon & Bromley (2010), we consider several examples. In Kenyon & Bromley (2010), solids with radii $r \gtrsim 1$ m evolve within 64 annuli extending from an inner radius of 30 au to an outer radius of 150 au. Solids interact with a gaseous disk; gas drag circularizes orbits.
while radial drift removes smaller particles from the grid. Initially, the gas-to-solid ratio is roughly 1:100; during the evolutionary sequence, the gas mass declines exponentially with an e-folding timescale of 10 Myr. Although Kenyon & Bromley (2010) consider a broad range of initial conditions, here we focus on calculations starting with a mono-disperse set of solids and a total mass roughly comparable with the minimum mass solar nebula (solid and gas mass).

In the calculations for this paper, the minimum size of solids is $1 \mu m$ instead of 1 m. Interactions with gas are ignored. The smaller size allows collisional damping to play a role during the collisional cascade (e.g., Kenyon & Bromley 2015, 2016b, 2016a; see also Figures 15–16). When damping is important, the mass in 0.1 mm to 10 cm particles may grow with time and allow a second phase of runaway growth for the largest particles. Thus, some solids reach larger masses than in calculations without the small particles. The lack of interactions with gas tends to slow the growth of the largest particles when their initial sizes are 1–10 km.

The lower panel of Figure 18 compares results for calculations starting with only 1 km, 10 km, or 100 km objects. At the start of each calculation, planetesimals grow slowly. Over 1–3 Myr of evolution, gas drag gradually results in larger gravitational focusing factors for the largest objects. Despite the loss of collisional debris from radial drift, 1–10 km objects reach Pluto masses in 30–100 Myr. After this spurt, gravitational interactions between the swarm of Plutos and smaller planetesimals initiate a collisional cascade, which grinds the leftovers into smaller and smaller objects. The loss of material slows growth considerably. The largest objects approach the mass of Mars but rarely grow larger.

When gas drag is not included, 1–10 km objects grow more slowly. It takes them about three times longer to reach the mass of Pluto. Once they initiate the collisional cascade, a modest amount of mass collects in millimeter- and centimeter-sized objects, which are stronger and more resistant to the cascade than meter-sized and larger solids. This reservoir allows Pluto-mass objects to grow beyond the mass of Mars and reach the mass of the Earth. The timescale to reach Earth-mass is $\sim 1$–2 Gyr.

Gaseous disks have little impact on the collisional evolution of 100 km planetesimals. On timescales of 100–300 Myr, these objects reach masses comparable to Charon, Pluto’s binary partner. At this point, collisions initiate a modest collisional cascade. In calculations with no submeter-sized particles, the cascade effectively removes debris from the grid. Although growth continues, the largest objects reach roughly the mass of Mars on 5–10 Gyr timescales. When the small particles are included, mass removal is less efficient. Mars (Earth)–mass planetes then form on 1–2 Gyr (5–7 Gyr) timescales.

Despite differences in approach, the evolution of the dust luminosity is similar in the two sets of calculations (Figure 18, upper panel). In Kenyon & Bromley (2010), a second calculation uses the loss rate of submeter-sized particles from the main coagulation calculation to derive the time evolution of 1 $\mu m$ to 1 m particles; the surface area (and thus reprocessed stellar luminosity) of these particles is therefore completely distinct from the evolution of the largest particles in the grid. In the calculations for this paper, the smallest particles evolve together with the largest particles. This approach should provide a better measure of the evolution of the dust luminosity.

Figure 18 illustrates that the dust luminosity follows the growth of the largest objects in each set of calculations. Early on, the largest particles grow slowly; collisions generate little debris. In turn, the dust luminosity is fairly small. As planetesimals grow more rapidly, collisions produce copious amounts of dust; the dust luminosity rises in step with the growth of the largest planetesimals. As the collisional cascade proceeds, there is less and less solid mass in the grid. Growth slows, debris production declines, and the dust luminosity begins to fade.

For each pair of calculations, the maximum dust luminosity correlates well with the initial sizes of planetesimals; $L_d/L_s \approx 10^{-3}$ for 1 km planetesimals, $L_d/L_s \approx 3 \times 10^{-4}$ for 10 km planetesimals, and $L_d/L_s \approx 10^{-4}$ for 100 km planetesimals. The timing of this peak is earlier for systems with gas drag and 1–10 km planetesimals and is independent of gas drag for 100 km planetesimals. Independent of the approach, all of the calculations have the same dust luminosity at 10 Gyr, $L_d/L_s \sim 10^{-4}$.

Although we do not compare these evolutionary tracks with the data in Figure 3 for clarity, a simple comparison shows that the evolution of full disks provides a poor match to the data. At 100 Myr, these models have $L_d/L_s \sim 10^{-5}$–$10^{-3}$, as in observed systems. With $L_d/L_s \approx 10^{-4}$ at 10 Gyr, all disk models decline much more slowly than the observations where $L_d/L_s \sim 10^{-5}$ for most sources. As discussed in the main text, rings of solids evolve in a similar way as the observed systems.

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