Workspace Design for 6 Degree of Freedom Machining Bed

M F Shah¹, Z Kausar²*, and S S Farooq¹

¹Department of Mechanical Engineering, Khwaja Fareed University of Engineering and Information Technology, Abu Dhabi Road, Rahim Yar Khan, Pakistan
²Department of Mechatronics Engineering, Air University, E-9, Islamabad, Pakistan

*Email: zkau001@aucklanduni.ac.nz

Abstract. Machining is a process in which raw material is cut into desired shape and size using different material removal processes. The accuracy and precision of workpiece is effected by present day high speed spindle technologies. To deal with such a problem it is proposed to replace current machining techniques with a 6 DOF machining bed which gives motion to workpiece in six degrees whereas the tool remains static. An issue in using such machining beds is workspace due to limitation of its joints movement and this should be known. This paper presents an algorithm for the determination of workspace for such machining beds. The algorithm is implemented in simulation which verifies the suitability of bed for machining in 6 degree of freedom.

1. Introduction

The process of removal of material from a workpiece in order to obtain the desired shape and dimensions can be termed as machining. Considered to be a manufacturing part for metals machining can also be done on ceramics, composites and wood. Shaping, sawing, broaching, turning and drilling all can be categorized as types of machining [1]. The process of machining comes into play where high accuracy and precision are required. Conventional machining technologies use high speed tools which allow the material to remove very quickly. But these high speed tools are vulnerable to internal and external vibrations which may result in poor precision and low accuracy [2]. Current CNC technologies have multiple tool axis, but the accuracy of these machines is influenced by various types of errors and tool wear [3,4]. Such problem can be solved by using a mechanism in which the tool remains static and the workpiece moves accordingly. A parallel machining bed able to give rotation to workpiece in 6 DOF was proposed by Shah et al [5] and Kausar et al [2] in their research works. These types of parallel manipulators are widely known as Stewart Platform. In such manipulators shown in figure 1 the top plate gives motion in 6 degrees [6,7]. These parallel manipulators give high stiffness and have high load bearing capacities [8] hence have gained attention from researchers and industries as well.

A challenge in the proposed system is the limitation of size of the workpiece. It is very important before starting this process of machining that the volume within which the machining bed can reach for given shape of the workpiece. Workspace of a parallel manipulator is of much concern while designing it. From both theoretical and industrial point of view the optimum design of such parallel manipulators can make them of more use [9]. The volume within which the end effector of the parallel manipulators can reach is the basic requirement for their practical usage [10]. Collection of all points \((x, y, z)\) where the
robot can reach in at least one of its orientation is known as its reachable workspace [3]. The workspace of parallel manipulators is limited [11] because of limitations on its joints, link interface and limited range of link lengths. Keeping in view above mentioned issues workspace for each length of parallel manipulators is calculated separately using inverse kinematics [10] but this research proposes an algorithm to calculate the cumulative workspace for the machining bed. The paper analyzes the workspace of a 6 DOF parallel manipulator used for the machining purpose by first solving the inverse kinematics presented in section 2 and later simulating the workspace using proposed algorithm with and without constraints.

2. Kinematic Modeling
The problem is stated as a position of the end effector of the manipulator is known to us depending upon the coordinates of the workpiece, respective leg lengths are required to be determined. For this purpose inverse kinematics of the parallel machining bed was used.

From figure 1 it can be seen that two plates of machining bed are connected through six legs with variable lengths. The base plate of the machining bed was considered as the reference frame[12]. There are many orientations of legs in which these platforms are proposed[13–17], we here propose that the legs along with the ball and socket joint are attached at an equal angle of 60 degrees with the spherical top plate whereas at the bottom these lengths along with the universal joints are place in form of pairs at equidistance from each other and other pair of legs as well.

A schematic view of the ith leg of parallel machining bed is shown in figure 2. L_i is the length of ith leg of machining bed, T is the translational vector, P_i is the point which defines the upper anchor point from the center of the movable top plate and B_i is the point which defines the anchor point from the center of the reference bottom plate. G_i is the resultant vector obtained as a result of head to tail rule addition [3]. Equation obtained for ith leg of the machining bed by using geometrical relations and vector addition is (1):

\[ \vec{L}_i = \vec{T} + \vec{p}_R R \vec{P}_i - \vec{B}_i \]  

Where \( \vec{p}_R R \) is rotation matrix from \( O_B \) to \( O_P \). Rotation matrix was designed using (2):

\[ \vec{p}_R R = R_Z(\psi) + R_Y(\theta) + R_X(\varphi) \]  

\[ = \begin{pmatrix} c(\psi) c(\theta) & -s(\psi) c(\varphi) + c(\psi) s(\theta) s(\varphi) & s(\psi) s(\varphi) + c(\psi) s(\theta) c(\varphi) \\ s(\psi) c(\theta) & c(\psi) c(\varphi) + s(\psi) s(\theta) s(\varphi) & -c(\psi) s(\varphi) + s(\psi) s(\theta) c(\varphi) \\ -s(\theta) & c(\theta) s(\varphi) & c(\theta) c(\varphi) \end{pmatrix} \]  

In (3) and in rest of this paper c is used as abbreviation for cosine and s for sine, whereas \( \psi, \theta \) and \( \varphi \) are angels in respective axis. Distribution of angels between six legs of the machining bed at the top plate...
as well as the bottom plate are shown in figure 3. Initially it was assumed that legs translate \( a \) units in \( z \) axis only. Keeping in view this assumption the \( \mathbf{T} \) vector was written as shown in (4):

\[
\mathbf{T} = \begin{pmatrix}
0 \\
0 \\
a
\end{pmatrix}
\quad (4)
\]

Using the data obtained in (4) and interpreting the values of \( P_i \) and \( B_i \) from figure 3 and figure 4, (1) was solved for each of the six legs of the machining bed. Resulting expressions are listed in (5) to (10).

\[ L_1 = \begin{pmatrix}
rc\psi c\theta - Rc12.5 \\
rs\psi c\theta - Rc12.5 \\
a - rs\theta
\end{pmatrix}
\quad (5) \\
L_2 = \begin{pmatrix}
rc60c\psi c\theta - rs60s\psi s\phi + rs60c\psi s\theta s\phi - Rc47.5 \\
rc60s\psi c\theta + rs60c\psi c\phi + rs60s\phi s\psi s\theta - Rs47.5 \\
a - rc60s\theta + rs60c\theta s\phi
\end{pmatrix}
\quad (6) \\
L_3 = \begin{pmatrix}
-rc60c\psi c\theta - rs60s\psi s\phi + rs60c\psi s\theta s\phi + Rc47.5 \\
-rc60s\psi c\theta + rs60c\psi c\phi - rs60s\phi s\psi s\theta - Rs47.5 \\
a + rc60s\theta + rs60c\theta s\phi
\end{pmatrix}
\quad (7) \\
L_4 = \begin{pmatrix}
-rc\psi c\theta + Rc12.5 \\
-rc\psi c\theta - Rs12.5 \\
a + rs\theta
\end{pmatrix}
\quad (8) \\
L_5 = \begin{pmatrix}
-rc60c\psi c\theta + rs60s\psi c\phi + rs60c\psi s\theta s\phi + Rc17.5 \\
-rc60s\psi c\theta - rs60c\psi c60 + rs60s\phi s\psi s\theta + Rs17.5 \\
a + rc60s\theta - rs60c\theta s\phi
\end{pmatrix}
\quad (9) \\
L_6 = \begin{pmatrix}
rc60c\psi c\theta + rs60s\psi c\phi - rs60c\psi s\theta s\phi - Rc17.5 \\
rc60s\psi c\theta - rs60c\psi c60 - rs60s\phi s\psi s\theta + Rs17.5 \\
a - rc60s\theta - rs60c\theta s\phi
\end{pmatrix}
\quad (10)

Leg lengths are determinable knowing the orientation of the center of the top plate of the machining bed where the workpiece is to be moved, \( T \) depending upon the shape of the workpiece, rotation matrix (3) and \( P_i \) and \( B_i \).

3. Workspace
3.1. Algorithm
A number of techniques are proposed to determine workspace margins for a manipulator. These techniques include analytical methods, iterative methods and numerical methods. To design workspace for the machining bed a MATLAB® code is developed by using leg lengths obtained using (5)-(10). The algorithm of the workspace design is as given in figure 4.

![Figure 4. Workspace design algorithm for the 6 DoF machining bed.](image)

3.2. Simulation results
The algorithm proposed in section 3.1 is simulated for the machining bed in Matlab. For each leg and each angle a step size of 0.4 is given from the interval -1 to 1 radians. For each sliding movement, a, the program is run for the step size of 20 from the interval of 0 to 800mm. Then the mapping of leg lengths of the machining bed obtained through inverse kinematics is obtained against the angles to demonstrate the movement of the machining bed. Finally graphics was added for separate mapping and enhanced understanding of each leg as shown in figure 4. In second test all the parameters were kept as in first case but a geometric constraint was added to restrict the machining bed kinematics (Leg Lengths) between 400 to 750mm. The resulting dexterous workspace obtained for the machining bed is shown in figure 5. The calculated work space is for the whole platform, including legs and movable top plate. Any change in the parameters will result in change of the work space. The work space obtained is of zig-zag shape. So, machining bed will work for any work piece falling within limits of constraints applied.

4. Conclusions
An algorithm is proposed for design of workspace for a six degree of freedom machining bed. The workspace requirement may change with the change in shape and size of the workpiece. The proposed algorithm draws a cylinder (boundary conditions) knowing maximum size of leg actuators and then draws trajectory of the workpiece and confirms whether the workpiece shape lies within boundary condition or not. The same algorithm may be applied to design parameters of legs knowing the maximum requirement of the workpiece machining. Future work may include the control of workspace and position of the workpiece.
**Figure 5.** Workspace of a 6 D.O.F Machining Bed  
**Figure 6.** Dexterous Workspace for 6 D.O.F Machining Bed

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