Structures in $^{9,10}$Be and $^{10}$B studied with the tensor-optimized shell model

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We study the structures of $^{9,10}$Be and $^{10}$B with the tensor-optimized shell model (TOSM) using an effective interaction based on the bare nucleon–nucleon interaction AV8′. The tensor correlation is treated in TOSM using the full space of 2p2h configurations including high-momentum components. The short-range correlation is treated with the unitary correlation operator method. It is found that the level orders of the low-lying states of $^{9,10}$Be and $^{10}$B are entirely reproduced. For $^{9}$Be, ground band states are located at relatively higher energies than the experiments, which indicates missing $\alpha$ clustering correlation in these states as seen in the case of $^{8}$Be. In addition, the tensor force gives a larger attraction for $T = 1/2$ states than for $T = 3/2$ ones for $^{9}$Be. The level order of the three nuclei is found to be sensitive to the presence of the tensor force in comparison with the results using the Minnesota effective interaction without the tensor force.

1. Introduction

It is an important problem in nuclear physics to realize how the nucleon–nucleon (NN) interaction explains the nuclear structure. Recently, the Green’s function Monte Carlo method (GFMC) has made it possible to calculate nuclei up to a mass of around $A \sim 12$ using bare NN interaction [1,2]. It is desirable to develop a new method to calculate nuclear structure with large nucleon numbers by taking care of the characteristics of the NN interaction.

One of the characteristics of the NN interaction is the presence of the strong tensor force which is mainly caused by the pion exchange and explains the large nuclear binding energy [1,3]. The NN tensor force produces the strong $sd$ coupling in energy, which makes the deuteron bound. The $d$-wave component in the wave function induced by the tensor force is spatially compact as compared with the $s$-wave one due to high momentum brought by the tensor force [4]. Experimentally, possible evidence of the high-momentum component of the nucleon coming from the tensor correlation in the nucleus has been developed using the $(p, d)$ reaction [5,6].

It is known that the tensor correlation in the $\alpha$ particle is generally strong [7,8], although its amount depends on the choice of the NN interactions. The large binding energy of the $\alpha$ particle can be related to the presence of the tensor force. In light nuclei, the $\alpha$ particles are often developed as
a cluster, e.g., in $^8$Be and the Hoyle state of $^{12}$C \cite{9,10}. For $^8$Be, the $0^+_1$, $2^+_1$, and $4^+_1$ states are regarded as consisting of two weakly interacting $\alpha$ particles. On the other hand, the excited states above $4^+_1$ can be regarded as shell-like states, because the $\alpha$ decay is not always favored. For $^{12}$C, the ground state is rather a shell-like state and some of the excited states including the Hoyle state are recognized as triple $\alpha$ cluster states. It is worth discussing the coexistence of the $\alpha$ cluster state and the shell-like state in light nuclei. In particular, we focus on the role of the tensor force on this phenomenon.

In the bare $NN$ interaction, the tensor force is of intermediate range and we can treat the contribution of tensor force in a reasonable shell model space \cite{11–14}. We name this method the tensor-optimized shell model (TOSM). In the TOSM wave function, the shell model basis states are employed with full optimization of two particle–two hole (2p2h) states. There is no truncation of the particle states in TOSM, where spatial shrinkage of the particle states is essential to obtain the convergence of the tensor contribution involving high-momentum components \cite{15–17}. In addition, we use the unitary correlation operator method (UCOM) to describe the short-range correlation in the $NN$ interaction \cite{18,19}. We describe the nuclei with TOSM+UCOM using a bare $NN$ interaction.

So far, we have obtained successful results of TOSM for He and Li isotopes and $^8$Be \cite{13,14,20,21}. It is found that the $(p_{1/2})^2 (s_{1/2})^{-2}$ type of the 2p2h configuration of the $pn$ pair is selectively and strongly mixed in $^4$He by the tensor force like a deuteron-type correlation \cite{12,22}. In the neutron-rich side, owing to the specific 2p2h excitations by the tensor force above, the $p_{1/2}$ occupation of extra neutrons suppresses the tensor correlation of nuclei. This is due to the Pauli blocking between the 2p2h configurations and the motions of the extra $p_{1/2}$ neutrons. This configuration dependence of the tensor correlation affects the splitting energy between the $p_{3/2}$- and $p_{1/2}$-dominant states in the neutron-rich He and Li isotopes. In $^{11}$Li, this tensor-type blocking effect naturally explains the halo formation with a large mixing of the $s_{1/2}$ component \cite{23}.

In the previous study \cite{24}, we have applied TOSM to $^8$Be and investigated how the TOSM describes two kinds of structures of the shell-model and $\alpha$ clustering type states in $^8$Be. The $^8$Be energy spectrum shows two groups. One is the ground-band three states of $0^+_1$, $2^+_1$, and $4^+_1$. These states are understood as the two-$\alpha$ cluster states in the intrinsic structure \cite{9,25}. The other group is the highly excited states starting from the $2^+_2$ state at $E_x = 16.6$ MeV experimentally. Above this state, many spin-states are observed with relatively small decay widths, less than about 1 MeV, and their decay processes are not always $\alpha$ emission. In the highly excited states, experimentally the $T = 1$ states are degenerated with the $T = 0$ states and are considered as isobaric analog states of $^8$Li and $^8$B. In TOSM, we have nicely described these properties of $^8$Be including the level order of $T = 0$ and $T = 1$ states in the highly excited states. One of the failures in TOSM is the small energy distance between the ground band head ($0^+_1$) and the highly excited state of $2^+_2$ ($T = 0$), which is 10.2 MeV in TOSM while 16.6 MeV in the experiment. The relative energy between the ground band $4^+_1$ state and the $2^+_2$ state is 2 MeV, which is smaller than the experimental value of 6 MeV.

The difference in the relative energy is considered to come from the insufficient description of two-$\alpha$ clustering structure in the ground band states using the shell model basis. It was also pointed out that the tensor force contribution is stronger in the ground band states than the highly excited states, which can also be related to the $\alpha$ clustering in the ground band states. In the highly excited states, the $T = 0$ states has a rather larger tensor force contribution than the values of the $T = 1$ states. This trend can be related to the isospin dependence of the tensor force, namely the one-pion exchange nature of the nuclear force. It is interesting to perform a similar analysis for other nuclei neighboring $^8$Be, focusing on the tensor correlation.
In this study, on the basis of the successful results of $^8\text{Be}$, we proceed with the analysis of the states of $^9\text{Be}, ^{10}\text{Be}$ and $^{10}\text{B}$ using TOSM and clarify the different structures of these nuclei as functions of the excitation energy. We investigate the structures of each state of the three nuclei from the viewpoint of the tensor force in analogy with $^8\text{Be}$. These nuclei are the systems in which one or two nucleons are added to $^8\text{Be}$, and then we perform a similar analysis of $^8\text{Be}$ for these nuclei. In this study, we use the same effective interaction as the one for $^8\text{Be}$, based on the bare $NN$ interaction $A V^8'$. This interaction is defined to simulate almost exactly the few-body results of $^4\text{He}$ using the TOSM wave function, retaining the characteristics of the bare $NN$ interaction as much as possible. We also investigate how the tensor force determines the energy spectrum of $^9\text{Be}, ^{10}\text{Be}$ and $^{10}\text{B}$ in comparison with the results using the effective Minnesota interaction without the tensor force.

In Sect. 2, we explain the methods of TOSM and UCOM. In Sect. 3, we show the results for $^9\text{Be}, ^{10}\text{Be}$ and $^{10}\text{B}$ and discuss the characteristics of each energy level in relation to the tensor force. A summary is given in Sect. 4.

2. Theoretical method

We explain the tensor-optimized shell model (TOSM). We define a standard shell-model state for $p$-shell nuclei with $A$ nucleons in order to introduce the TOSM. The standard shell-model state $\Psi_S$ is dominated by the low-momentum component and is given as

$$\Psi_S = \sum_{k_S} A_{k_S} |(0s)^4(0p)^{A-4}; k_S\rangle. \quad (1)$$

Here, the $s$ shell is closed and the $p$ shell is open. The index $k_S$ is the label to distinguish various shell-model configurations with the amplitude $A_{k_S}$. In the concept of the TOSM, the tensor force can excite two nucleons to various two-particle states with high-momentum components in each shell-model state. Hence, we limit TOSM configurations up to the two particle–two hole (2p2h) excitations connected with the standard shell-model configurations.

The high-momentum components brought by the strong tensor force are included in the TOSM via the excitations of two nucleons from the $s$ and $p$ shells in the standard shell-model state $\Psi_S$ to higher shells as the 2p2h states. We have 2p2h configurations in TOSM as

$$|2p2h; k_2\rangle = |(0s)^{n_s}(0p)^{n_p}(\text{higher})^2; k_2\rangle. \quad (2)$$

We put the constraints $n_s + n_p = A - 2$ and $2 \leq n_s \leq 4$. The index $k_2$ is the label to distinguish various 2p2h states. Here, “higher” means higher shells above the $s$ and $p$ shells and the particle states in TOSM.

We also allow the 1p1h excitations in addition to the 2p2h ones for shell model consistency:

$$|1p1h; k_1\rangle = |(0s)^{n_s}(0p)^{n_p}(\text{higher})^1; k_1\rangle, \quad (3)$$

with the constraint $n_s + n_p = A - 1$ and $2 \leq n_s \leq 4$. These 1p1h states can bring the high-momentum components and also improve the standard shell-model state $\Psi_S$ in the radial components.

In addition to the 1p1h and 2p2h states with high-momentum component, we further extend the standard shell-model states $\Psi_S$ in Eq. (1) by allowing excitations of the two particles in the $s$ to $p$ shells. This excitation is to take into account the part of the tensor correlation within the $s$ and $p$ shells. We denote these extended shell-model states as $|0p0h; k_0\rangle$ where no particle is excited to the particle states above the $s$ and $p$ shells. We include these basis states in the TOSM configurations.
instead of $\Psi_S$. We explicitly express the extended shell-model states as

$$\ket{0p0h; k_0} = \ket{(0s)^{n_s} (0p)^{n_p}; k_0}. \quad (4)$$

Here, the constraints $n_s + n_p = A$ and $2 \leq n_s \leq 4$. The index $k_0$ is the label to distinguish the configurations.

Superposing the $0p0h$, $1p1h$, and $2p2h$ configurations, we give the total wave function $\Psi$ of TOSM as

$$\Psi = \sum_{k_0} A_{k_0} \ket{0p0h; k_0} + \sum_{k_1} A_{k_1} \ket{1p1h; k_1} + \sum_{k_2} A_{k_2} \ket{2p2h; k_2}. \quad (5)$$

Here, three kinds of amplitude $\{A_{k_0}, A_{k_1}, A_{k_2}\}$ are variationally determined by using the minimization of the total energy.

We explain how to construct the radial wave functions of a single nucleon in each configuration. The $0p0h$ states are expressed using the harmonic oscillator wave functions as the ordinary shell model states. The $0s$ and $0p$ basis wave functions are involved and their length parameters are determined independently as variational parameters for the total energy. The $0p0h$ states using the $s$ and $p$ shells become the dominant part of the wave function for $p$-shell nuclei. For intruder states such as the positive parity states in $^9$Be, some nucleons occupy the $sd$-shell as the dominant configurations, and the $2p2h$ excitations from each dominant configuration can produce the strong tensor correlation. However, we do not include the $sd$-shell in the $0p0h$ configurations because the model space for the $sd$-shell becomes huge.

The $1p1h$ and $2p2h$ states have the particle states above the $s$ and $p$ shells. For particle states, we employ the Gaussian basis functions to express the single-particle states which can have high-momentum properties owing to the tensor force [26,27]. The Gaussian basis functions are sufficient to effectively express the high-momentum components of a single nucleon by adjusting the length parameters [12]. This technique has been used in the previous studies of TOSM and also the cluster models [28].

We superpose a sufficient number of Gaussian basis functions with various length parameters and the radial components of the particle states can be fully expressed in each configuration of TOSM. The particle states are orthogonalized to the hole states of $0s$ and $0p$ shells and among themselves [11,12]. We construct the orthonormalized single-particle basis function for the particle states using a linear combination of non-orthogonal Gaussian bases. The particle wave functions can have high-momentum components caused by the tensor force with all the possible angular momenta until the total energy converges. In the numerical calculation, the partial waves of the basis states are taken up to $L_{\text{max}}$. We take $L_{\text{max}}$ as 10 to get convergence of the energy and Hamiltonian components in the present analysis. For the number of Gaussian basis functions, typically at most 10 basis functions are used with various range parameters.

It is noted that when we give the probabilities and occupation numbers of each orbit in the states of nuclei, these numbers are given by the summation of all the orbits with the same spin having different radial behaviors.

We use a Hamiltonian with a bare $NN$ interaction for mass number $A$:

$$H = \sum_i A T_i - T_{\text{c.m.}} + \sum_{i<j} A V_{ij}, \quad (6)$$

$$V_{ij} = v_{ij}^C + v_{ij}^T + v_{ij}^{LS} + v_{ij}^{\text{Cmb}}. \quad (7)$$
Here, \( T_i \) and \( T_{\text{c.m.}} \) are the kinetic energies of each nucleon and a center-of-mass part. We take a bare interaction \( V_{ij} \) such as AV8' [2] consisting of central \( v_{ij}^C \), tensor \( v_{ij}^T \), and spin-orbit \( v_{ij}^{LS} \) terms, and the Coulomb term \( v_{ij}^{\text{Coul}} \).

We take care of the center-of-mass excitations considering the Hamiltonian of center-of-mass motion known as the Lawson method [29]. In this study, we take the value of \( \hbar \omega \) for the center-of-mass motion as the averaged one for the \( 0s \) and \( 0p \) orbits in the \( 0p0h \) states considering the weight of the occupation numbers in each orbit [13]. Adding this center-of-mass Hamiltonian as the Lagrange multiplier to the original Hamiltonian in Eq. (6), we can effectively project out the excitation of the center-of-mass motion.

The energy variation for the TOSM wave function \( \Psi \) defined in Eq. (5) is performed with respect to the two kinds of variational parameters; one is the length parameters of the Gaussian basis functions and the other is the amplitudes of the TOSM configurations, \( A_{k_0}, A_{k_2}, \) and \( A_{k_2} \) given in Eq. (5).

We employ UCOM to take into account the short-range correlation coming from the \( NN \) interaction [18,19]. We express the correlated wave function \( \Phi \) in terms of the TOSM wave function \( \Psi \) as \( \Phi = C \Psi \), and the unitary operator \( C \) is defined as

\[
C = \exp \left( -i \sum_{i<j} g_{ij} \right) .
\]  

(8)

We truncate the transformed Hamiltonian \( C^\dagger HC \) using UCOM at the two-body level [18] by considering the nature of short-range correlation.

The two-body Hermite operator \( g \) in Eq. (8) is given as

\[
g = \frac{1}{2} \left\{ p_r s(r) + s(r) p_r \right\} .
\]  

(9)

where \( p_r \) is the operator for the radial part of the relative momentum and is conjugate to the relative coordinate \( r \). The function \( s(r) \) is determined to minimize the total energy of nuclei. In the present analysis, we use the same \( s(r) \) functions determined in \(^4\text{He} \) [13,14]. To simplify the calculation, we adopt the ordinary UCOM in this analysis, instead of the \( S \)-UCOM. The \( S \)-UCOM extensively introduces the partial-wave dependence in \( s(r) \), in particular between the \( s \)-wave and other partial waves [12]. It has been obtained that \( S \)-UCOM improves the short-range part of the relative \( d \)-wave components of the nucleon pair, and the \( sd \) coupling and the total energy are increased in \( S \)-UCOM over the UCOM case.

2.1. \( NN \) interaction

In this paper, we use two kinds of \( NN \) interactions for comparison; one is the bare AV8' interaction, which consists of central, \( LS \), and tensor terms. This interaction is used in the benchmark calculation of \(^4\text{He} \) given by Kamada et al. without the Coulomb term [8]. The other is the effective Minnesota (MN) \( NN \) interaction, which does not have the tensor force. For MN interaction, we choose the \( u \) parameter as 0.95 for the central part and use set III of the \( LS \) part [30]. In the case of MN interaction with Coulomb interaction and without the \( LS \) force, the binding energy of \(^4\text{He} \) with TOSM is 29.72 MeV, which is very close to the rigorous calculation of 29.94 MeV [31].

In TOSM with AV8' interaction, as shown in Table 1, it is found that the tensor and \( LS \) contributions for \(^4\text{He} \) give smaller values than those in the stochastic variational method (SVM) using correlated Gaussian basis functions [31], which is one of the rigorous calculations [8]. One of the reasons for the shortage in TOSM is the contributions of the higher configurations beyond the \( 2p2h \) space.
Table 1. Hamiltonian components in TOFM, SVM, and TOSM for $^4\text{He}$ in MeV. In TOSM, AV8$'_{\text{eff}}$ is used [24].

|       | Energy | Kinetic | Central | Tensor | LS   |
|-------|--------|---------|---------|--------|------|
| TOSM  | -22.30 | 90.50   | -55.71  | -54.55 | -2.53|
| SVM   | -25.92 | 102.35  | -55.23  | -68.32 | -4.71|
| TOFM  | -24.18 | 95.50   | -54.67  | -61.32 | -4.09|
| TOSM with AV8$'_{\text{eff}}$ | -26.16 | 95.45   | -56.17  | -62.43 | -3.02|

The other reason is the two-body truncation of the transformed Hamiltonian with UCOM. There remains a small contribution in the short-range part of the tensor force and also of the $LS$ force, which can couple with the short-range UCOM and results in the many-body term [18].

Horii et al. estimate the amount of this coupling by using the few-body SVM [22] without UCOM. In Ref. [22], they propose the one-pair type coupling by the tensor force introducing the single $Y_2$ function in the global vector for angular momentum, which produces the $d$-wave component. This model is called the tensor-optimized few-body model (TOFM). The physical concept of TOFM is the same as that of TOSM except for the UCOM part to describe the short-range correlation. In TOFM, the short-range correlation is directly treated in the wave function. On the other hand, this correlation is approximately taken into account in the TOSM using UCOM. It is shown that TOFM gives a good binding energy of $^4\text{He}$ with AV8’, as shown in Table 1, as compared with the benchmark calculation (SVM). The energy of TOFM is lower than the value of TOSM by about 2 MeV [22] and this difference comes from the use of UCOM in TOSM. Considering the difference between TOSM and TOFM, the three-body UCOM term is one of the possibilities to overcome the lack of energy from UCOM [12,18].

In the previous analysis of $^8\text{Be}$ [24], we have considered the shortages of the tensor and $LS$ contributions in TOSM and introduce the effective interaction based on the AV8’, which retains the characteristics of the $NN$ interaction as much as possible. We include these effects in TOSM to simulate phenomenologically the TOFM results of $^4\text{He}$ as closely as possible by increasing the corresponding matrix elements. We enhance the tensor matrix elements by 10%, namely, the enhancement factor $X_T = 1.1$, and the $LS$ matrix elements by 40%, $X_{LS} = 1.4$. It is found that the total energy is good and more than 90% of the tensor component and kinetic energy of the SVM calculation shown in Table 1 is reproduced. It is also found that the present results of $^4\text{He}$ almost reproduce the TOFM solutions. This indicates that the missing effects of TOSM are effectively recovered by using the phenomenological enhancements of the matrix elements. In the present analysis, we use this parameter set for TOSM and call this interaction “AV8’ eff”, which retains the important characteristics of the bare $NN$ interaction. This AV8’ eff interaction successfully describes the level order of $^8\text{Be}$ [24], but it is not obvious whether this interaction is applicable to the systematic description of the $p$-shell nuclei. Hence, we examine the applicability of this interaction using the present analysis of the $p$-shell nuclei with TOSM.

3. Results

3.1. $^9\text{Be}$

We discuss the structures of $^9\text{Be}$ with negative parity states in TOSM using AV8’ eff, which is used in the analysis of $^8\text{Be}$. The total binding energy of the ground state of $^9\text{Be}$ is obtained as 27.71 MeV in TOSM, which is considerably smaller than the experimental value of 58.17 MeV. This difference
mainly comes from the missing higher configurations beyond the 2p2h excitations in TOSM and also the three-nucleon interaction. The $\alpha$ clustering correlation in $^9\text{Be}$ is also considered to explain the energy difference [24].

We show the excitation energy spectra of $^9\text{Be}$ in Fig. 1 for $T = 1/2$ and 3/2. In the experimental spectrum, the spins of most of the highly excited states are not confirmed yet for $T = 1/2$. It is found that there can be two groups of states in $^9\text{Be}$ for the experimental spectrum; one is up to the excitation energy of 12 MeV and the other is the highly excited states starting from 14 MeV, which are degenerated with the $T = 3/2$ states. The relative energy between $5/2^-$ and the state with $E_x = 13.8$ MeV in $^9\text{Be}$ is about 2 MeV in the experiments, while these state are overlapped in some regions in the calculated spectrum. In the TOSM results, for $T = 1/2$, we almost reproduce the experimental level order of the low-lying states starting from the ground $^3/2^-$ states. For the $T = 3/2$ states the excitation energies are lower than the experimental values by about 6 MeV.

Here we discuss the analogy of level structures between $^9\text{Be}$ and $^8\text{Be}$. The binding energy of $^8\text{Be}$ with AV8' eff is obtained as 30.19 MeV, which is smaller than the experimental value 56.50 MeV. In the previous analysis of $^8\text{Be}$ [24], we have similarly obtained the two groups in the spectrum; one is the three ground band states of $0^+$, $2^+$, and $4^+$, and the other is the highly excited states starting from the $2^+_2$ state at 16.6 MeV of the excitation energy, as shown in Fig. 2. In the figure the spectrum is normalized at the $2^+$ ($T = 1$) state of $^8\text{Be}$, which is regarded as the shell-like state. In TOSM, the relative energy between $4_1^+$ and $2_2^+$ is about 2 MeV and smaller than the experimental value of 6 MeV. The main reason or difference is considered to be the missing energy coming from the $\alpha$ clustering component in the ground band states of $^8\text{Be}$. We consider the same effect on the $^9\text{Be}$ spectrum. For this purpose, we renormalize the spectrum to the $T = 3/2$, $J^{\pi} = 3/2^-$ state as shown in Fig. 3. It is found that there is a good correspondence in the $T = 3/2$ states between TOSM and the conditions.

![Excitation energy spectrum of $^9\text{Be}$ using AV8' eff](image-url)
Fig. 2. Excitation energy spectrum of $^{8}\text{Be}$ using AV8$'_{\text{eff}}$ normalized to the $2^+(T = 1)$ state.

Fig. 3. Excitation energy spectrum of $^{9}\text{Be}$ using AV8$'_{\text{eff}}$ normalized to the $3^-_{3/2}(T = 3/2)$ state.

experiments. This indicates that the TOSM nicely described the $T = 3/2$ states and the states are considered to be mainly the shell-like states, as was obtained for $^{9}\text{Li}$ [14].

For $T = 1/2$ states, it is found that the energy of the ground state is missed by about 6 MeV in comparison with the experiments in Fig. 3. This value is close to the $^{8}\text{Be}$ case, as shown in Fig. 2. In the $^{8}\text{Be}$ case, we have discussed the $\alpha$ clustering effect using the two-$\alpha$ cluster model and the possible energy gain from the $\alpha$ clustering is estimated as 5 MeV [24]. For the $^{9}\text{Be}$ case, it is expected that the
Table 2. RMS radii of $^9$Be and $^{10}$B in the units of fm.

|        | TOSM | Experiment [32] |
|--------|------|-----------------|
| $^8$Be | 2.21 | —               |
| $^9$Be | 2.32 | 2.38(1)         |
| $^{10}$Be | 2.31 | 2.30(2)       |
| $^{10}$B | 2.20 | —               |

inclusion of the $\alpha + \alpha + n$ three-body component may help the lack of energy of $^9$Be in TOSM for low-lying states. These results imply that the mixture of the $\alpha$ cluster component in the TOSM basis states is desirable to improve the energy spacing of two groups in $^9$Be. It is interesting to develop the TOSM to include the $\alpha$ clustering correlation explicitly and to express the tensor contribution in each $\alpha$ particle.

For the ground state of $^9$Be, the matter radius is obtained as 2.32 fm in TOSM, as shown in Table 2, which is slightly smaller than the experimental value of 2.38(1) fm [32]. This trend of radius occurs in the case of $^8$Be (2.21 fm) [24] in comparison with the two-$\alpha$ cluster model (2.48 fm). The small radii of $^8$Be and $^9$Be in TOSM indicate that the $\alpha$ clustering correlation is not fully included in the present solution of TOSM. In the shell model, it is generally difficult to express the asymptotic form of the spatially developed $\alpha$ clustering states. Naively, many particle–many hole excitations in the shell model bases might be necessary to assist the formation of well-separated two-$\alpha$ clusters in space. In TOSM, the 2p2h excitation is used to incorporate the tensor correlation in the single $\alpha$ particle with high-momentum components by about 10% [12,13]. This indicates that when two $\alpha$ particles are established in $^8$Be and $^9$Be, each $\alpha$ particle independently needs the 2p2h components to express the tensor correlation, although the probability of this situation is expected to be not so high. In the TOSM, the approximation of the 2p2h excitations might restrict the spatial cluster formation in $^8$Be and $^9$Be.

The TOSM with AV$^8'_{\text{eff}}$ is found to reproduce entirely well the energy level order of $^9$Be. We examine the roles of the tensor matrix elements in determining the energy spectrum of $^9$Be. For this purpose, we try to see the structures of $^9$Be by changing the strengths of the tensor forces around AV$^8'_{\text{eff}}$. The same analysis was performed for $^8$Be [24].

We start from the effective interaction AV$^8'_{\text{eff}}$, namely, $X_T = 1.1$ and $X_{LS} = 1.4$, and change the tensor strength. The result is shown in Fig. 4, normalized to the $\frac{3}{2}^-$ ($T = 3/2$) state. We have confirmed that the band structures, such as the energy spacings, do not depend on the changes of $X_T$ in this parameter range. It is found that the relative energy between the low-lying $T = 1/2$ states and the three $T = 3/2$ states increases as the tensor force is strengthened. This result indicates that the $T = 1/2$ states are more sensitive to the tensor correlation than the $T = 3/2$ states. This is related to the attractive nature of the $T = 0$ channel of the tensor force. Hence, as the tensor correlation becomes stronger, the smaller isospin $T = 1/2$ states gain more energy than the larger isospin $T = 3/2$ states. Essentially, the level orders in each isospin state do not change so much. This is considered as follows. The tensor force strongly couples the 0p0h and 2p2h configurations, in which the particle states involve high-momentum components. This coupling induced by the tensor force contributes to the total energy of every state commonly within the same isospin state. Hence, the low-lying relative spectra of $^9$Be does not show the strong dependence of the tensor force strength.

The mixing of particle states with high momentum in the wave function contributes to the energies of every state commonly within the same isospin state.
Fig. 4. Energy spectrum of $^9$Be with TOSM by changing the tensor strength about $\pm 5\%$ around $A V8'_\text{eff}$ keeping $X_{LS} = 1.4$, where the black lines denote the $T = 1/2$ states and the blue lines the $T = 3/2$ states.

The Hamiltonian components of each state are discussed later. The same effect of tensor force on the isospin of nuclei is confirmed in $^8$Be [24], in which the $T = 0$ and $T = 1$ states are compared and the $T = 0$ states tend to gain energy more as the tensor correlation becomes stronger. From this result of $^9$Be, it is found that the tensor force affects the relative energies between the $T = 1/2$ and $T = 3/2$ states. This is just the same conclusion as obtained for $^8$Be [24].

We also show the energy spectrum of $^9$Be using the effective MN interaction to see the effect of the tensor force on the spectrum. In this study, we reduce the strength of the $LS$ force in the MN interaction by 30% to give the same $LS$ splitting energy of 1.5 MeV in $^5\text{He}$ using $A V8'_\text{eff}$ in TOSM [13,24]. The binding energy of the $^9$Be ground state is obtained as 66.50 MeV, which is close to the experimental value of 58.17 MeV. This interaction gives a radius of 1.96 fm for $^9\text{Be}$, much smaller than the $A V8'_\text{eff}$ case of 2.32 fm. This means that MN cannot reproduce the nuclear saturation properties, which is related to the tensor force.

We discuss the excitation energy spectrum with MN as shown in Fig. 5. The spectrum reproduces the overall trend of the experiment including the level density. However, the level order is different from the experiment in some parts. At lower excitation energies of the $T = 1/2$ states, it is found that the $\frac{1}{2}^-$ is higher than the experimental value. For the $T = 3/2$ states, the location of the $\frac{3}{2}^-$ state is much different from the experiment, while the other two states of $\frac{1}{2}^-$ and $\frac{3}{2}^-$ are close to the experiments. From these results obtained using MN interaction, there are differences in the excitation energies of some specific states. As compared with the MN results, the $A V8'_\text{eff}$ interaction gives the nice energy spectrum of $^9\text{Be}$ for two isospin states.

We discuss the Hamiltonian components in each state of $^9\text{Be}$ using $A V8'_\text{eff}$ in TOSM to understand the contributions of the tensor force explicitly. In Table 3, we show the Hamiltonian components of the $^9\text{Be}$ states.
Fig. 5. Excitation energy spectrum of $^9$Be with TOSM using the Minnesota interaction (MN).

Table 3. Hamiltonian components in MeV for $^9$Be for $T = 1/2$ and 3/2 states.

| State ($T = 1/2$) | Energy  | Kinetic | Central | Tensor   | $LS$  |
|------------------|---------|---------|---------|----------|-------|
| $\frac{1}{2}^1$  | -23.92  | 214.75  | -126.17 | -101.09  | -11.40|
| $\frac{1}{2}^-$   | -19.92  | 212.31  | -114.46 | -101.06  | -16.72|
| $\frac{1}{2}^-$   | -17.51  | 210.56  | -113.92 | -97.77   | -16.38|
| $\frac{3}{2}^-$   | -27.71  | 217.49  | -127.10 | -102.07  | -16.03|
| $\frac{3}{2}^-$   | -21.98  | 213.46  | -123.70 | -101.25  | -10.47|
| $\frac{3}{2}^-$   | -20.28  | 211.81  | -115.56 | -101.83  | -14.69|
| $\frac{5}{2}^-$   | -25.64  | 216.57  | -124.50 | -101.49  | -16.23|
| $\frac{5}{2}^-$   | -21.74  | 213.69  | -122.08 | -100.36  | -12.99|
| $\frac{5}{2}^-$   | -19.07  | 211.14  | -113.16 | -100.95  | -16.10|
| $\frac{7}{2}^-$   | -22.73  | 214.93  | -121.75 | -99.08   | -16.82|
| $\frac{7}{2}^-$   | -20.71  | 212.86  | -114.29 | -99.93   | -19.37|
| $\frac{7}{2}^-$   | -16.72  | 210.57  | -115.92 | -98.45   | -12.92|
| $\frac{9}{2}^-$   | -18.91  | 212.67  | -117.4589 | -98.31  | -15.81|
| $\frac{9}{2}^-$   | -13.78  | 208.87  | -113.1296 | -96.90  | -12.62|

| State ($T = 3/2$) | Energy  | Kinetic | Central | Tensor   | $LS$  |
|------------------|---------|---------|---------|----------|-------|
| $\frac{1}{2}^-$  | -17.59  | 187.78  | -106.57 | -85.95   | -12.85|
| $\frac{1}{2}^-$  | -18.50  | 189.10  | -107.86 | -84.25   | -15.49|
| $\frac{5}{2}^-$  | -14.81  | 187.48  | -106.51 | -83.83   | -11.94|
It is interesting to discuss the differences between the $T = 1/2$ and $T = 3/2$ states of $^9\text{Be}$. In Table 3, it is shown that the $T = 3/2$ states possess smaller tensor contributions than the $T = 1/2$ case by about 15 MeV. The kinetic energies also show a similar trend due to the high-momentum component brought by the tensor force. This result for the $T = 3/2$ states can be related to the isospin dependence of the tensor force, in which the $T = 1/2$ states can easily contain the $T = 0$ nucleon pair states which induce the stronger tensor correlation than the $T = 1$ nucleon pair case. This result is consistent with the dependence of the energy spectrum on the tensor strength, as shown in Fig. 4. We have confirmed the same feature of the isospin dependence of the tensor and kinetic contributions in the case of $^8\text{Be}$ [24]. The direct relation between the state and the interaction for the isospin property should be carefully examined elsewhere.

It is found that the $\frac{3}{2}^-$ ground state with $T = 1/2$ possesses the largest tensor contribution of $-102$ MeV, and also the largest kinetic energy and central contributions. As the excitation energy goes up, the states tend to reduce the matrix elements of each Hamiltonian component. The $LS$ contribution depends on the states, because the $LS$ matrix elements generally depend on the single-particle configurations, such as the occupation of the $p_{1/2}$ and $p_{3/2}$ orbits.

Dominant configurations of several states in $^9\text{Be}$ are shown in Table 4 for $T = 1/2$ and Table 5 for $T = 3/2$. It is found that the $T = 3/2$ states tend to have single configuration properties, and the configuration mixing occurs more strongly in the $T = 1/2$ states than the $T = 3/2$ states.

For reference, the $\frac{1}{2}^+$ state of $^9\text{Be}$ is obtained at the excitation energy of 8.4 MeV in TOSM, which is located higher than the experimental value by 6.7 MeV. The $\frac{1}{2}^+$ state is observed very close to the $\alpha + \alpha + n$ threshold energy. For this intruder state, the last neutron can occupy the $1s$ orbit coupled with the $^8\text{Be} (0^+)$ state weakly. In the present TOSM, the description of the dominant configuration for the intruder states is insufficient, as explained in Sect. 2, because the $1s$ shell is treated as the higher shells. In the dominant $0p0h$ configurations for $^9\text{Be}(\frac{1}{2}^+)$, one nucleon is excited from the $0s$
Table 5. Dominant configurations of $^9$Be in $T = 3/2$ with their squared amplitudes $(A_k^l)^2$ using the AV8'$_{\text{eff}}$ interaction.

| Configuration | $\frac{1}{2}^-$ | $\frac{3}{2}^-$ |
|---------------|----------------|----------------|
| $(0s)^4(0p_{3/2})_{02}^2(0p_{1/2})^3$ | 0.82 | 0.06 |
| $(0s)^4(0p_{3/2})_{02}^2(0p_{1/2})^3$ | 0.53 | 0.11 |
| $(0s)^4(0p_{1/2})_{02}^2(0p_{1/2})^3$ | 0.59 | 0.77 |
| $(0s)^4(0p_{1/2})_{02}^2(0p_{1/2})^3$ | 0.10 | |

Fig. 6. Excitation energy spectrum of $^{10}$Be with $T = 1$ using AV8'$_{\text{eff}}$ and MN interactions. The left-hand side shows experimental data.

shell to the $0p$ shell in TOSM. The result of TOSM indicates that the inclusion of the $sd$-shell in the $0p0h$ configurations in Eq. (4) could improve the description of $^9$Be($\frac{1}{2}^+$).

3.2. $^{10}$Be

We discuss the structures of $^{10}$Be for their positive parity states with $T = 1$ in TOSM. The binding energy of $^{10}$Be with AV8'$_{\text{eff}}$ is obtained as 29.91 MeV, which is smaller than the experimental value 64.98 MeV. The energy spectrum of $^{10}$Be is shown in Fig. 6. Experimentally, the spins of the highly excited states are not assigned yet, a similar situation to the $^9$Be case.

From the comparison between TOSM and experiments, in the low-lying states, the $0^+_2$ state is located at about 8 MeV of the excitation energy, which is 2 MeV different from the experiment. We will focus on the structure of the $0^+_2$ states later. The excitation energies of the $2^+_2$ states are well reproduced in TOSM from $2^+_1$ to $2^+_4$. The $4^+_1$ state, which is considered to form the band structure with the ground and $2^+_1$ states, is located at a lower excitation energy than the experiment. This trend
of the small band energy can also be seen in the case of $^8\text{Be}$ in TOSM in Fig. 2 [24]. We also predict the low-lying $1^+$ state at the excitation energy of about 5.8 MeV, which is not confirmed experimentally. The matter radius of the ground state of $^{10}\text{Be}$ is 2.31 fm in TOSM, as shown in Table 2, which is close to the experimental value of 2.30(2) fm [32].

For the $0^+$ series, several theories suggest the two-$\alpha$ clustering state in the $0^+_2$ with a large mixing of the $sd$ shell of valence two neutrons [33–37]. On the other hand, in TOSM, the mixings of the $sd$-shell are not large in all $0^+$ states, as shown in Table 6. This is related to the large excitation energy of the $1^+_2$ state in $^9\text{Be}$, as was explained. The $sd$-shell corresponds to the intruder orbit, and the lowering of the $sd$-shell in the low-excitation energy can often be seen in the neutron-rich $p$-shell nuclei, such as in $^{11}\text{Be}$ and $^{12}\text{Be}$ [38,39]. In TOSM, it is interesting to examine this phenomenon by extending the standard shell-model state to include the $sd$-shell in Eq. (1), although this extension needs a huge model space and computational effort at present.

In Fig. 6, we show the energy spectrum of $^{10}\text{Be}$ using MN interaction, which does not have the tensor force. The whole trend of the spectrum agrees with experiment for low-lying states except for the $0^+_2$, which is highly located at $E_x = 16$ MeV in TOSM with MN interaction. In the $0^+_2$ state, two neutrons are excited from $0p_{3/2}$ to $0p_{1/2}$, which makes the large splitting energy. This is related to the large splitting energy of $^9\text{Be}$ between the $\frac{3}{2}^-$ and $\frac{1}{2}^-$, as shown in Fig. 5.

In Table 7, we show the Hamiltonian component using $AV8'_{\text{eff}}$. Among the $0^+$ states, the ground $0^+$ state has the largest contribution of the kinetic part and also of the central and $LS$ force. On the other hand, the $0^+_2$ state has the largest tensor contribution. If the $0^+_2$ state has a relation to the developed $\alpha$ cluster state, a strong tensor contribution might appear from the $\alpha$ clusters in the nucleus. The results of TOSM might suggest the $\alpha$ clustering in the $0^+_2$ of $^{10}\text{Be}$.

### 3.3 $^{10}\text{B}$

We analyze the level structure of $^{10}\text{B}$ with $T = 0$ states as shown in Fig. 7. The binding energy of $^{10}\text{Be}$ with $AV8'_{\text{eff}}$ is obtained as 29.13 MeV, which is smaller than the experimental value of 64.75 MeV. In the figure, we employ two kinds of $NN$ interaction; one is $AV8'_{\text{eff}}$ which is used for $^8\text{Be}$, $^9\text{Be}$, and $^{10}\text{Be}$. For reference, the other is the original $AV8'$ without the modification of the tensor and $LS$ forces.

It is found that the spin of the ground state is obtained as the $1^+$ state for $AV8'$, which is different from the experimental situation. This result is commonly obtained for other calculations using the bare $NN$ interaction without three-nucleon interaction [1]. On the other hand, in the case of $AV8'_{\text{eff}}$, we can reproduce the ground state spin and also the low-lying spectra. In particular, the numbers of levels for each spin are almost reproduced in TOSM. These results indicate that the effective treatment of the tensor and $LS$ forces gives the proper state dependence to explain the level order of $^{10}\text{B}$, although this treatment is not related to the three-nucleon interaction. The matter radius of the ground state is obtained as 2.20 fm, as shown in Table 2.

### Table 6. Nucleon occupation numbers of $^{10}\text{Be}$ ($0^+$).

|             | $0s_{1/2}$ | $0p_{1/2}$ | $0p_{3/2}$ | $1s_{1/2}$ | $d_{3/2}$ | $d_{5/2}$ | $1p_{1/2}$ | $1p_{3/2}$ |
|-------------|------------|------------|------------|------------|-----------|-----------|------------|------------|
| $0^+_1$     | 3.77       | 0.92       | 4.90       | 0.04       | 0.06      | 0.05      | 0.03       | 0.05       |
| $0^+_1'$    | 3.78       | 1.71       | 4.09       | 0.04       | 0.06      | 0.05      | 0.03       | 0.05       |
| $0^+_2$     | 3.78       | 1.83       | 3.97       | 0.04       | 0.06      | 0.05      | 0.03       | 0.04       |
| $0^+_3$     | 3.78       | 1.40       | 4.41       | 0.05       | 0.06      | 0.05      | 0.03       | 0.05       |
Table 7. Hamiltonian components in MeV for $^{10}$Be with $T = 1$.

| State $^+$ | Energy | Kinetic | Central | Tensor | $LS$ |
|------------|--------|---------|---------|--------|------|
| $0^+_1$ | -29.91 | 245.17 | -146.18 | -108.78 | -20.13 |
| $0^+_2$ | -21.98 | 239.92 | -136.77 | -109.86 | -15.27 |
| $0^+_3$ | -18.00 | 238.28 | -138.19 | -105.00 | -13.09 |
| $0^+_4$ | -14.93 | 235.56 | -126.81 | -106.01 | -17.66 |
| $1^+_1$ | -24.09 | 240.83 | -136.22 | -110.68 | -18.03 |
| $1^+_2$ | -22.08 | 240.24 | -137.64 | -107.53 | -17.14 |
| $1^+_3$ | -17.06 | 237.27 | -128.07 | -107.73 | -18.53 |
| $1^+_4$ | -14.32 | 234.37 | -127.18 | -107.43 | -14.07 |
| $2^+_1$ | -27.30 | 244.03 | -142.77 | -108.07 | -20.48 |
| $2^+_2$ | -25.15 | 242.71 | -142.04 | -108.49 | -17.33 |
| $2^+_3$ | -23.46 | 241.13 | -138.09 | -107.58 | -18.92 |
| $2^+_4$ | -20.89 | 239.81 | -136.68 | -107.90 | -16.12 |
| $3^+_1$ | -22.61 | 241.53 | -139.81 | -107.50 | -16.82 |
| $3^+_2$ | -18.90 | 238.09 | -131.79 | -106.81 | -18.38 |
| $3^+_3$ | -17.29 | 236.83 | -130.74 | -107.98 | -15.41 |
| $3^+_4$ | -16.55 | 237.11 | -132.14 | -106.35 | -15.17 |
| $4^+_1$ | -21.40 | 240.82 | -136.36 | -105.72 | -20.14 |
| $4^+_2$ | -17.55 | 238.17 | -134.68 | -105.47 | -15.58 |
| $4^+_3$ | -15.61 | 236.28 | -131.51 | -106.35 | -14.03 |

Fig. 7. Comparison of the excitation energy spectrum of $^{10}$B with $T = 0$ using AV8'$_{eff}$, AV8', and MN interactions in TOSM.
Table 8. Hamiltonian components in MeV for $^{10}$B with $T = 0$.

| State | Energy | Kinetic | Central | Tensor | LS  |
|-------|--------|---------|---------|--------|-----|
| $0^+_1$ | -17.95 | 237.52  | -124.77 | -110.81 | -19.89 |
| $0^+_2$ | -6.34  | 230.69  | -122.20 | -106.50 | -8.33 |
| $1^+_1$ | -28.30 | 224.92  | -142.09 | -113.98 | -17.15 |
| $1^+_2$ | -27.90 | 244.67  | -140.88 | -112.57 | -19.17 |
| $1^+_3$ | -20.92 | 240.35  | -138.94 | -108.90 | -13.42 |
| $1^+_4$ | -19.61 | 239.07  | -134.93 | -108.84 | -14.90 |
| $2^+_1$ | -27.20 | 243.81  | -141.27 | -113.08 | -16.66 |
| $2^+_2$ | -26.01 | 243.19  | -140.16 | -113.27 | -15.77 |
| $2^+_3$ | -20.01 | 239.29  | -137.94 | -110.68 | -10.68 |
| $2^+_4$ | -16.29 | 236.51  | -126.45 | -109.25 | -17.11 |
| $3^+_1$ | -29.13 | 245.40  | -141.60 | -111.81 | -21.12 |
| $3^+_2$ | -24.96 | 242.99  | -139.21 | -111.33 | -17.47 |
| $3^+_3$ | -23.44 | 241.30  | -136.57 | -111.69 | -16.47 |
| $3^+_4$ | -18.18 | 238.73  | -134.21 | -107.38 | -15.34 |
| $4^+_1$ | -25.93 | 243.42  | -138.77 | -110.92 | -19.65 |
| $4^+_2$ | -17.42 | 237.81  | -133.97 | -109.02 | -12.23 |
| $4^+_3$ | -10.00 | 233.34  | -124.36 | -106.86 | -12.10 |
| $5^+_1$ | -19.23 | 239.95  | -135.30 | -108.49 | -15.40 |

We also calculate the $^{10}$B energy spectrum by using the MN interaction for comparison. The result is shown in Fig. 7. It is found that the level density is smaller than the experiment. One of the reasons for this result comes from the strong effect of the effective $LS$ force in MN for $^{10}$B, as similarly seen in $^9$Be and $^{10}$Be. The ground state radius of $^{10}$B is obtained as 1.88 fm. This small radius in the MN interaction affects the saturation property and provides the small level density.

In Table 8, we list the Hamiltonian components for each state using $AV^8_{\text{eff}}$. In the ground state region, the $1^+_1$ state shows the largest tensor contribution and also the largest kinetic energy, which are correlated by the tensor force. The $3^+_1$ shows the largest $LS$ contribution. The $2^+_1, 2^+_2$ states also show the rather large tensor contribution.

Comparing $^{10}$B and $^{10}$Be in the Hamiltonian components, it is found that the tensor contributions are rather larger in $^{10}$B than those of $^{10}$Be. This trend is natural from the viewpoint of the attractive effect of the $T = 0$ channel of the tensor force. On the other hand, the $LS$ contributions do not entirely show the large difference in the two nuclei.

4. Summary

The nucleon–nucleon ($NN$) interaction has two specific characters, the tensor force originated from the pion exchange and the short-range repulsion. We have treated these two characters of the $NN$ interaction in nuclei on the basis of the tensor-optimized shell model (TOSM) with the unitary correlation operator method (UCOM), TOSM + UCOM. The TOSM basis states optimize the two particle–two hole (2p2h) states fully by using the Gaussian expansion method. The 2p2h states in TOSM play an important role in the description of the strong tensor correlation with the high momentum of nucleon motion in nuclei. Using TOSM + UCOM, we have investigated the structures of three
nuclei, $^{9,10}\text{Be}$ and $^{10}\text{B}$, as an extension of the previous analysis of $^{8}\text{Be}$. In this paper, we mainly focus on the different structures appearing in the low-lying and the excited states of $^{9}\text{Be}$ with two isospin $T = 1/2$ and $T = 3/2$ states. We used the effective $NN$ interaction for TOSM based on the $AV8'$ interaction, which retains the characteristics of the bare $NN$ interaction and simulates the few-body calculation of $^{4}\text{He}$ as a reference nucleus.

For $^{9}\text{Be}$, it is found that TOSM reproduces fairly well the excitation energy spectrum of $^{9}\text{Be}$ for two isospin states, except for the energy spacing between the low-lying states and the highly excited states; the latter group is close to the $T = 3/2$ states in excitation energy. The small energy spacing is considered to come from the missing $\alpha$ clustering component in the low-lying states in TOSM. We have obtained the same situation for $^{8}\text{Be}$ as the small energy distance between the states of $T = 0$ and $T = 1$. The common result between $^{8}\text{Be}$ and $^{9}\text{Be}$ indicates the necessity of the explicit component of $\alpha$ clustering in the TOSM basis states for two nuclei, in particular, in the low-lying energy region, in which the $\alpha$ cluster correlation is considered to exist strongly.

For highly excited states, we normalize the energy spectrum of $^{9}\text{Be}$ to the $\frac{3}{2}^-$ ($T = 3/2$) state, because this state is the isobaric analog state of the $^{9}\text{Li}$ ground state and TOSM has successfully described the structures of Li isotopes. This normalization of the $^{9}\text{Be}$ spectrum is useful to understand the energy locations of the ground band states and the highly excited states in TOSM in comparison with the experiment. It is found that TOSM gives almost the same level order as the experiments for both the $T = 1/2$ and $T = 3/2$ states, although the spins of highly excited states are not experimentally confirmed yet. This result indicates that the state dependence of the $NN$ interaction is correctly treated in TOSM. On the other hand, when we employ the effective Minnesota interaction without the tensor force, the results show a different energy level order. This difference means that the state dependence of the $NN$ tensor force is important in explaining the level order of $^{9}\text{Be}$, which is correctly treated in TOSM. It is also found that the $T = 1/2$ states in $^{9}\text{Be}$ show a stronger tensor contribution than the $T = 3/2$ states. This is naturally understood from the $T = 0$ attractive channel of the tensor force, originated from the one-pion exchange phenomenon.

To understand the roles of the non-central forces explicitly, we have examined the dependencies of the tensor matrix elements on the $^{9}\text{Be}$ structures. The tensor force gives a larger attraction for $T = 1/2$ states than for $T = 3/2$ ones for $^{9}\text{Be}$, which makes the energy difference of two isospin states large. This result is also confirmed in the $^{8}\text{Be}$ analysis between the $T = 0$ and $T = 1$ states.

For $^{10}\text{Be}$, we have obtained a nice energy spectrum for low-lying states, while the highly excited states are not confirmed for spins experimentally. Among the $0^+$ states, the $0^+_2$ state possesses the largest tensor contribution and is dominated by the $p$-shell configuration, and the mixing of the $sd$-orbit is small. This situation is different from the recent theoretical analysis of $^{10}\text{Be}$ ($0^+_2$) by using the $\alpha$ cluster model. The $\alpha$ cluster model suggests the large mixing of the $sd$-orbit of valence-two neutrons with the developed two-$\alpha$ clustering. In TOSM, the small mixing of $sd$-orbit is also related to the higher energy of the $1^+_2$ state of $^{9}\text{Be}$. It is also found that the tensor contribution of $0^+_2$ is the largest value among the $0^+$ states. This might be related to the $\alpha$ clustering in this state.

For $^{10}\text{B}$, we have reproduced the correct spin of the ground state using the effective $NN$ interaction. The tensor contributions of each state of $^{10}\text{B}$ are generally larger than those of $^{10}\text{Be}$. This is because the number of $pn$ pairs is larger in $^{10}\text{B}$ than $^{10}\text{Be}$, which plays an important role in the tensor correlation in nuclei.

For the interaction, we phenomenologically introduce the effective $NN$ interaction based on the bare interaction for TOSM, which entirely describes the level order of $^{9,10}\text{Be}$ and $^{10}\text{B}$ in addition to
the results for $^8$Be. It is interesting to examine the applicability of this interaction to the systematic description of the light nuclei in the future.

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