Neutrinos and Primordial Nucleosynthesis

G. Mangano\textsuperscript{a} and P.D. Serpico\textsuperscript{b}

\textsuperscript{a}Dipartimento di Scienze Fisiche, Università di Napoli \textit{Federico II} and INFN, Sezione di Napoli, Complesso Universitario di Monte Sant’Angelo, Via Cintia, I-80126 Napoli, Italy
mangano@na.infn.it

\textsuperscript{b}Max Planck Institut für Physik, Werner Heisenberg Institut, Föhringer Ring 6, 80805, München, Germany
serpico@mppmu.mpg.de

The importance of the Big Bang Nucleosynthesis (BBN) as a unique tool for studying neutrino properties is discussed, and the recent steps towards a self-consistent and robust handling of the weak reaction decoupling from the thermal bath as well as of the neutrino reheating following the $e^+ - e^-$ annihilation are summarized. We also emphasize the important role of the Cosmic Microwave Background (CMB) anisotropy in providing an accurate and independent determination of the baryon density parameter $\omega_b$. The BBN is presently a powerful parameter-free theory that can test the standard scenario of the neutrino decoupling in the early Universe. Moreover it can constrain new physics in the neutrino sector. The perspectives for improvements in the next years are outlined.

1. Introduction

The Standard Cosmological Model predicts the existence of a neutrino background (C\nu B) filling our Universe with densities of the order $n_\nu \approx 100 \text{ cm}^{-3}$ per flavor and thermal energies of $\mathcal{O}(1 \text{ K})$, which the data for the mass splitting coming from the neutrino oscillation experiments put nowadays in the non-relativistic regime. As for the much better studied photon microwave background (CMB), a detailed analysis of the C\nu B properties would provide a unique window on the physical conditions in the early stages of the life of the Universe. Moreover, the peculiar environment of a thermalized neutrino medium, impossible to reproduce in laboratory experiments, may give in principle some insights on exotic properties of neutrino physics, as the existence of sterile degrees of freedom that may be excited in such extreme conditions.

The incredibly weak interaction of the neutrinos, especially at such low energies, makes hopeless at present any perspective of direct detection of C\nu B. Nevertheless, given their extremely low interaction rate, the natural out-of-equilibrium driving force of the expansion of the Universe pushed them to decouple from the thermal bath much earlier than the CMB, when the temperature was $\mathcal{O}(1 \text{ MeV})$. This temperature is close to the electron mass $m_e$, setting the scale of the electron/positron annihilation, and both are close to the $\mathcal{O}(0.1 \text{ MeV})$ scale of the synthesis of the light nuclei via thermonuclear fusion. This suggests that interesting phenomena involving the C\nu B indeed can affect the pattern of nuclides coming from the cosmic cauldron.

BBN is a privileged laboratory for the C\nu B studies with respect to other cosmological probes, such as CMB anisotropies or the Large Scale Structure (LSS), since it is sensitive to the $\nu$ (weak) interactions as well as to the shape of the $\nu_e - \bar{\nu}_e$ phase space distributions entering the $n \leftrightarrow p$ inter-conversion rates

\begin{align}
\nu_e + n & \leftrightarrow e^- + p , \\
\bar{\nu}_e + p & \leftrightarrow e^+ + n , \\
n & \leftrightarrow e^- + \bar{\nu}_e + p .
\end{align}

Apart from the energy density due to the extra (\textit{i.e.} non electromagnetic) relativistic degrees of freedom, typically parameterized via an effective number of neutrinos $N_{\text{eff}}$, the BBN tests the dy-
namical properties of the neutrinos in a thermalized (almost) CP-symmetric medium.

Other cosmological observables are instead sensitive only to the CMB gravitational interaction. It follows that CMB mainly probes $N_{\text{eff}}$ at a much later epoch and the LSS (mainly) the neutrino mass scale $m_n$, since neutrinos turn into non-relativistic species just in time to influence the structure formation dynamics.

Still few years ago, the BBN theory together with the observations of the abundances of pristine nuclides were used to determine the baryon to photon ratio $\eta \equiv n_B/n_\gamma$, or, equivalently, the baryon fraction of the universe $\omega_b = \Omega_bh^2$. Nowadays $\omega_b \approx 0.023$ is fixed to better than 5% accuracy by detailed CMB anisotropies analysis [1], thus leaving the BBN as an over-constrained (and thus, very predictive) theory. Once $\omega_b = 0.023 \pm 0.001$ is plugged into the BBN theory, the prediction for the deuterium, which is the nuclide most sensitive to $\omega_b$, nicely fits the range of the observed values in high redshift, damped Ly-α systems [2], thus offering a remarkable example of internal consistency of the current cosmological scenario. Moreover, the predictions of other light nuclei which at least qualitatively agree with the observed values are likely to put constraints on the Galactic chemical evolution ($^4\text{He}$) or to the temperature scale calibration or depletion mechanisms in PopII halo stars ($^7\text{Li}$).

Apart from the uncertainty on $\omega_b$, the $^2\text{H}$, $^3\text{He}$ and $^7\text{Li}$ abundance predictions are mainly affected by the nuclear reaction uncertainties. An updated and critical review of the nuclear network and a new protocol to perform the nuclear data regression has been presented in [3] and widely discussed in [4], to which we address for details.

On the other hand, the predicted value of the $^4\text{He}$ mass abundance, $Y_p$, is poorly sensitive to the nuclear network details and has only a weak, logarithmic dependence on $\omega_b$, being fixed essentially by the ratio of neutron to proton number density at the onset of nucleosynthesis. Its crucial dependency on the weak rates [1] and on the (standard or exotic) neutrino properties will be briefly discussed in the following section.

2. Weak Rates, $^4\text{He}$, and relic neutrinos

As a first approximation, the neutrino decoupling can be described as an instantaneous process taking place around 2-4 MeV, without any overlap in time with $e^+ - e^-$ annihilations. All $\nu$ species would then keep perfect Fermi-Dirac distributions, with temperature $T_\nu$ smaller than the photon one $T$ since they do not benefit in this instantaneous decoupling scheme of the entropy release from $e^+ - e^-$ annihilations. The asymptotic ratio $T/T_\nu$ for $T << m_\nu$ can be evaluated in an analytic way, and turns out to be $(11/4)^{1/3} \approx 1.401$.

More accurate calculations by solving the kinetic equations have been performed, and they show a partial entropy transfer to the neutrino plasma. As a consequence, the neutrino distributions get distorted. In [5,6] it was shown that with a very good approximation the distortion in the $\alpha$-th flavor can be described as

$$f_{\nu_\alpha}(x, y) \simeq \frac{1}{e^{xy} + 1} \left( 1 + \sum_{i=0}^{3} c_i^\alpha(x) y^i \right)$$

(2)

where $x \equiv m_\nu/T_\nu$, and $y \equiv p/T_\nu$. The evolution of the $c_i^\alpha$ and $c_i^\sigma$ with $x = \mu/T$ are shown in Figure 1 and 2 versus $z = m_\nu/T$, respectively. Notice that the electron neutrinos get a larger entropy transfer than the $\mu$ and $\tau$ since they also interact via charged currents with the $e^\pm$ plasma. By fully consistently including order $\alpha$ QED corrections to the photon and $e^\pm$ equation of state, in [3] the energy density in the neutrino fluid is found to be enhanced by 0.935% (for $\nu_\mu$) and 0.390% (for $\nu_\tau$ and $\nu_\tau$) and the effective ratio $T/T_\nu \approx 1.3984$ is slightly lower than the previous instantaneous decoupling estimate. In Figure 3 we show the evolution of this ratio versus $z$. Put in terms of $N_{\text{eff}}$, the standard prediction is then 3.04 instead of 3.

How much the $^4\text{He}$ prediction is influenced by these phenomena? In several papers (see [7,8] and references therein) the value of $Y_p$ has been computed by improving the evaluation of the rates [1] including electromagnetic radiative corrections, finite mass corrections and weak magnetism effects, as well as the plasma and thermal radiative effects. In particular in [4] it has been also consid-
Figure 1. The evolution of the electron neutrino distortion coefficients $c_{e0}$ (solid), $c_{e1}$ (dashed), $c_{e2}$ (dotted) and $c_{e3}$ (dot-dashed) versus $z = m_e/T$.

Figure 2. The evolution of the $\mu$ and $\tau$ (collectively denoted by $x$) neutrino distortion coefficients $c_{x0}$ (solid), $c_{x1}$ (dashed), $c_{x2}$ (dotted) and $c_{x3}$ (dot-dashed) versus $z = m_e/T$.

Considered the effect of the neutrino spectra distortions and of the process

$$\gamma + p \leftrightarrow \nu_e + e^+ + n,$$

which is kinematically forbidden in vacuum, but allowed in the thermal bath. The latter is shown to give a negligible contribution, while the neutrino distortion has a significant influence on the rates only at relatively late times (see Figure 1), when however the neutron to proton density ratio is already frozen. Nonetheless, one would expect effects up to $\mathcal{O}(1\%)$ due to the neutrino reheating. However, the spectral distortion and the changes in the energy density and $T_\nu(T)$ conspire to almost cancel each other, so that $Y_p$ is changed by a sub-leading $\mathcal{O}(0.1\%)$. This effect is of the same order of the predicted uncertainty coming from the error on the measured neutron lifetime, $\tau_n = 885.7 \pm 0.8$ s \cite{tau_n}, and has to be included in quoting the theoretical prediction, $Y_p = 0.2481 \pm 0.0004$ ($1\sigma$, for $\omega_b = 0.023$).

Unfortunately, there is no hope at present to single out such a small effect, given the observational situation plagued by systematics. The value of $Y_p$ is usually derived by extrapolating to zero metallicity the measurements done in dwarf irregular and blue compact galaxies, that are among the least chemically evolved galaxies. The typical statistical errors are of the order of 0.002 (i.e., at the 1% level), but the systematics are such that in the recent reanalysis \cite{Y_p} the authors argue for the conservative range $0.232 \leq Y_p \leq 0.258$, i.e. a $1\sigma$ error of $\mathcal{O}(5\%)$. On the other hand, it is worth stressing that BBN provides the best limit on some $C_{\nu B}$ properties. For example, the range $0.232 \leq Y_p \leq 0.258$ gives the limit $-1.2 \leq \Delta N_{\text{eff}} \leq 0.7$, only slightly dependent on $\omega_b$, and much stronger than the typical actual CMB constraint on this parameter.

Similarly, a possible $C_{\nu B}$ asymmetry due to non-vanishing neutrino chemical potentials is much better constrained by the BBN, which limits (see e.g. \cite{xi_e})

$$|\xi_e| \equiv |\mu_{\nu_e}/T_\nu| \leq \mathcal{O}(0.1),$$

while both CMB or LSS probes are only sensitive to $\xi \sim \mathcal{O}(1)$. The former is indeed mainly fixed by the effect of the (slightly degenerate) $\nu_e - \bar{\nu}_e$ distributions in the weak rates \cite{xi_e}, while the lat-
The evolution of $T/T_\nu$ versus $z = m_e/T$. The asymptotic value at small temperatures is 1.3984.

...are only sensitive to the extra energy density present in the $\xi \neq 0$ case. It was recently realized [12] that because of flavor oscillation in the primordial plasma, using present determination of mass differences and mixing angles from atmospheric and solar neutrinos, the three asymmetry parameters $\xi_\alpha$ should be very close each other, so the bound of Equation (4) can be extended to all neutrino flavors.

In conclusion, the precision cosmology era has opened a new opportunity for the BBN theory to probe the very early universe, and in particular neutrino physics in such a unique environment. The theoretical predictions are in this respect quite robust, and will be further sharpened by new nuclear reactions measurements, as well as by refining the CMB determination of $\omega_b$, especially for the $^2\text{H}$, $^3\text{He}$ and $^7\text{Li}$ nuclides. For the $^4\text{He}$, which is the most sensitive probe of new physics, only a significant improvement in understanding the observational systematics could offer deeper insights on physics of the early Universe.

REFERENCES

1. D. N. Spergel et al., Astrophys. J. Suppl. 148 (2003) 175
2. D. Kirkman et al., Astrophys. J. Suppl. 149 (2003) 1
3. P. D. Serpico, nucl-th/0407051
4. P. D. Serpico et al., astro-ph/0408076
5. G. Mangano et al., Phys. Lett. B534 (2002) 8
6. S. Esposito et al., Nucl. Phys. B590 (2000) 539
7. R. E. Lopez and M. S. Turner, Phys. Rev. D59 (1999) 103502
8. S. Esposito et al., Nucl. Phys. B568 (2000) 421
9. S. Eidelman et al., Phys. Lett. B592 (2004) 1
10. K. A. Olive and E. D. Skillman, astro-ph/0405588
11. A. Cuoco et al., Int. J. Mod. Phys. A19 (2004) 4431
12. A. D. Dolgov et al., Nucl. Phys. B632 (2002) 363