Spinodal instability of baryon-rich quark matter

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Abstract. The spinodal instabilities of both confined and expanding baryon-rich quark matter are studied in a transport model derived from the Nambu-Jona-Lasino model. Appreciable higher-order density moments are seen as a result of the first-order phase transition in both cases. The skewness of the quark number event-by-event distribution in a small sub-volume of the system becomes appreciable for the confined quark matter. For the expanding quark matter, the density fluctuations lead to enhanced the dilepton yield.

1. Introduction

The beam energy scan (BES) experiments [1, 2, 3] at the Relativistic Heavy Ion Collider (RHIC) are designed for the search of the critical end point (CEP) in the baryon-rich region of the QCD phase diagram. At CEP, the first-order phase transition between the hadronic matter and the quark-gluon-plasma (QGP) changes to a second-order one. To understand the properties of this phase transition, both schematic models and hydrodynamic approach [4, 5, 6] have been used to study how the spinodal instability of a QGP leads to the self-amplified deviation from its equilibrium state during the phase transition. Using the test particle method, the spinodal instabilities of both the confined and expanding quark matter have been studied in a transport model that is derived from the Nambu-Jona-Lasino (NJL) model [7]. By varying the coupling strength of the vector interaction $G_V$ in the NJL model, different equations of state are obtained for a baryon-rich quark matter. A first-order phase transition occurs if $G_V = 0$ with the CEP at about $T_c \simeq 70$ MeV and $n_c \simeq 0.9$ fm$^{-3}$ [8], and it is absent if $G_V = G_S$. The density fluctuations induced by the spinodal instability can be quantified by the scaled density moments $\langle \rho^N \rangle / \langle \rho \rangle^N$ [6], where $\langle \rho^N \rangle \equiv \int d^3r \rho(r)^{N+1} / \int d^3r \rho(r)$. They are all equal to one for a uniform density distribution but become larger if the density is nonuniform. Also, density fluctuations can affect the dilepton yield in heavy ion collisions. Results on these observables for the two cases of with and without a first-order phase transition are compared in the following for both the confined and the expanding quark matter.

2. Quark matter in a box

Shown in the left window of Fig. 1 by the dotted, dashed, and solid lines are the time evolution of scaled density moments for $N = 2, 4$ and $6$, respectively, during the first-order phase transition of a quark matter that is confined in a box with an initially uniform net quark density $n_q = 0.5$ fm$^{-3}$ and temperature $T = 20$ MeV. They are obtained by averaging over 1,000 events. The scaled moments are seen to increase during the phase separation and reach their saturated values at...
Figure 1. Time evolution of scaled density moments during a first-order phase transition for a quark matter confined in a box with an initially uniform net quark density $n_q = 0.5 \text{ fm}^{-3}$ and temperature $T = 20 \text{ MeV}$.

about $t = 40 \text{ fm/c}$, when the phase separation almost ends. Also, moments with larger $N$ increase faster and saturate at larger values.

Figure 2. Time evolution of the event-by-event distribution of the number of quarks in a sub-volume of size 0.6 fm$^3$ (left window) and 30 fm$^3$ (right window) for a quark matter of temperature $T = 20 \text{ MeV}$ and average net quark density $n_q = 0.5 \text{ fm}^{-3}$ inside the spinodal region. The total number of events is 1000.

The number of quarks inside a sub-volume also varies drastically from event to event during a first-order phase transition as shown in Fig. 2 for the central cell from 1000 events at $t = 0$, 20, and 40 fm/c by the solid, dashed and dotted lines, respectively, for the two cases of sub-volume of size 0.6 fm$^3$ (left window) and 30 fm$^3$ (right window). The left panel of Fig. 2 clearly shows
that the distribution for the small sub-volume becomes asymmetric as time increases, starting with an initial skewness of 0.11 and increasing to 0.60 at 20 fm/c and 0.75 at 40 fm/c, agreeing with the prediction in Ref. [9]. This feature is absent in the right panel of Fig. 2 for the larger sub-volume, where the distribution remains essentially symmetric with increasing time.

3. Expanding quark matter

For an expanding quark matter as in relativistic heavy ion collisions, the initial partons are obtained from a multiphase transport (AMPT) model with vanishing parton scattering cross sections in Zhang’s parton cascade (ZPC)[10], with the hadronic afterburner based on a relativistic transport (ART) [11, 12] turned off and with string melting [13] that uses the heavy ion jet interaction generator (HIJING) [14, 15, 16] as the input. In the AMPT, these partons are assumed to be produced on the light cone, i.e., on a thin disk perpendicular to the longitudinal axis or beam direction, which is reasonable in collisions at the top energy of RHIC. For collisions at lower energies such as in the BES experiments, the initial parton distribution is expected to be more extended in the longitudinal direction, and we take this into account by increasing the thickness of the initial disk to 2dN/square root of N for head-on collisions, where d is the diameter of the colliding nuclei, and assuming that the initial partons are uniformly distributed in this disk. Using 1000 test partons for each physical partons produced from Au+Au collisions at zero impact parameter and a center-of-mass energy square root of sNN = 2.5 GeV and smearing the distribution of initial partons from the AMPT with a longitudinal length of d/3.5 to mimic the geometry of the initial quark configuration for collisions at square root of sNN = 7 GeV, we have found that some parts of the system go through the spinodal region when the SU(3) NJL model with GV = 0.

![Figure 3](image_url)

**Figure 3.** The phase trajectories of the central part of an expanding quark matter obtained by averaging over 100 events for the cases with (solid line) and without (dashed line) a first-order phase transition using the initial parton distribution from the AMPT model that is smeared in the longitudinal direction as described in the text. The spinodal region in the case of GV = 0 is shown by the gray color, and it disappears for GV = GS. The region where quark matter can be bound in the case of GV = 0 is shown by red color.

As shown by the solid line in Fig. 3, the phase trajectory of the central part of the system,
obtained by averaging over 100 events for the case with a first-order phase transition, goes into
the spinodal instability region, which is colored in gray, at about 6 fm/c after expansion, and
moves out of this region at about 8.5 fm/c. For the case without a first-order phase transition,
the trajectory is shown by the dashed line in Fig. 3. The temperatures are obtained by solving
the equations of state using both the energy density and the baryon number density as the inputs.
It’s shown in Fig. 3 that the expansion is slowed down by the first-order phase transition. The
simulation also shows that in the case with a first-order phase transition, i.e. \(G_V\) is set to zero,
a disk of high density appears due to the lower pressure as a result of the spinodal instability,
which later transforms into several disjointed clusters on a ring at \(z = 0\). These cold and dense
clusters stay in the spinodal instability region even at \(t = 14 \text{ fm/c}\).

![Figure 4](image-url)

Figure 4. Scaled density moments averaged over 100 events as functions of time for the cases
with (black lines) and without (red lines) a first-order phase transition.

These clusters can again be quantified by the event-averaged scaled density moments as
shown in Fig. 4 for the 2nd (dotted curve), 4th (dashed curve), and 6th (solid curve) order
scaled moments with the black and red colors denoting results obtained with and without a
first-order phase transition, respectively. It is seen that the density moments keep on increasing
during the spinodal decomposition. Such a feature is more prominent for the higher-order scaled
density moments.

Because the rate of the dilepton production through quark-antiquark annihilation is
proportional to the product of their densities, it will also be enhanced as the density fluctuations
become large. The dilepton invariant mass spectra obtained by averaging over 100 events are
plotted in Fig. 5 by the solid and dashed lines for the cases with and without a first-order phase
transition, respectively. As expected, more dileptions are produced from a quark matter with a
first-order phase transition.

4. Summary
The spinodal instability of a baryon-rich quark matter has been studied in the transport model
that is based on the NJL model. Dense clumps are observed in both confined and expanding
quark matter, and they lead to the enhancement of higher-order scaled density moments. The
skewness of the quark number event-by-event distribution in a small sub-volume also increases
Figure 5. Dilepton yield as a function of the invariant mass $M_{e^+e^-}$ for the cases with (solid line) and without (dashed line) a first-order phase transition in an expanding quark matter with the blast wave initial conditions.

during the first-order phase transition in the confined quark matter, but this feature disappears if the sub-volume is large. The density fluctuations further enhance the dilepton production through quark-antiquark annihilation in the expanding quark matter.

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