Quantum Zeno dynamics of a matter-wave bright soliton

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The quantum measurement problem, namely how the deterministic quantum evolution leads to probabilistic measurement outcomes, remains a profound question to be answered. In the present work, we propose a spectacular demonstration and test of the subtle and peculiar character of the quantum measurement process. We show that a bright soliton supported by a Bose-Einstein condensate can be reflected as a whole by an electron beam, with neither attraction nor repulsion between the condensate’s neutral atoms and the beam’s electrons. This macroscopic reflection is purely due to the quantum Zeno dynamics induced by the frequent position measurement of the condensate’s atoms by the electron beam. As an example of application, just as a soccer player would stop a coming ball, an electron beam moving backward with half the velocity of the bright soliton can precisely stop the soliton. This offers an entirely new and useful tool for manipulating bright solitons.
If we assume—according to our current best understanding—that the evolution of the world is governed by the quantum theory, how then can this underlying deterministic evolution enable the quantum measurement process to give probabilistic outcomes? This quantum measurement problem remains a profound question to be answered \[1, 2\]. In particular, due to the peculiar laws of quantum mechanics, frequently ascertaining whether a system is inside a region can effectively set up an impenetrable wall around this region. This is called the quantum Zeno dynamics (QZD) \[3, 4\]. It is deeply related to the quantum measurement process \[3\] and thus studies of it can help unveil the secrets behind the hitherto mysterious measurement process. QZD has been realized in various physical systems \[3, 10\]. Besides its fundamental significance, it has been realized to be a useful tool for quantum state engineering \[10–17\]. Interestingly however, so far experimental realizations of the QZD have been mainly focused on systems composed of several to a handful of discrete quantum levels, while the QZD of the continuous spatial distribution of a quantum mechanical system has been much less explored \[18–20\]. Recently, such spatial QZD has been observed in a Bose-Einstein condensate depleted by an electron beam \[18\]. The electron beam can be seen as a continuous position measurement constantly ascertaining whether any atom is in the region irradiated by the beam. It was observed that beyond some threshold, a more intense electron beam led to less depletion of the condensate, evidencing that the atoms in the condensate are being repelled by the stronger measurement, i.e., the QZD.

FIG. 1. Concept and core messages. (a) An electron beam can reflect a matter-wave bright soliton through QZD. The bright soliton is formed from a Bose-Einstein condensate with attractive interaction. Normally, an electron beam simply knocks atoms present in its beam path out of the condensate. This can also be viewed as ascertaining whether any atom is along the beam path. When the electron beam is intense enough, this frequent ascertaining induces a QZD that repels atoms from entering the beam path. We show that the matter-wave bright soliton will also be reflected by this QZD repulsion, and that it can remain a soliton after being reflected. (b) Calculated time evolution of the density distribution of an incoming bright soliton. The horizontal axis is the position, while the vertical axis is the evolution time. The gray rectangular region (denoted “EB”) represents the electron beam. The soliton is reflected by the electron beam. After being reflected, the solitonic character is not preserved and the wave packet spreads out. (c) Same as (b) but for a stronger dissipation strength $\gamma$. The reflected fraction $P_{\text{refl}}$ rises from 0.54 to 0.62. The wave packet after reflection spreads out slower. (d) Same as (c) but for a sharper beam edge, $w = 0.1$ compared to $w = 0.8$ in (c). The reflected fraction further rises from 0.62 to 0.91. Most importantly, the solitonic character is well preserved by the reflection in that the wave packet after reflection propagates with a constant shape. (e) The surviving fraction of the atoms as a function of time. The three curves correspond to subfigures (b)-(d) respectively.

In the present work, we show a spectacular demonstration and test of the subtle and peculiar character of the quantum measurement process can be realized using this new experimental capability and a bright soliton via QZD. Solitons is one of the most fascinating phenomena in Bose-Einstein condensates. Unlike ordinary waves that constantly change their shape and often very soon spreads out and disappears, a soliton is a solitary wave packet that retains its shape as it propagates, due to a cancellation between dispersion and nonlinearity. Studies of solitons have
been widely carried out in the context of Bose-Einstein condensates \cite{21,31} and helps to reveal the properties of this exotic state of matter. In particular, a Bose-Einstein condensate with attractive atom-atom interactions can support a bright soliton with a localized density distribution \cite{29}, which holds great promise for application in precision interferometry \cite{32,33,34,35}. It is known that the behavior of a bright soliton is akin to a classical particle. In particular, just as a classical particle, it can be repelled by a potential barrier and preserve its solitonic character \cite{36}.

In the present work we show that a bright soliton can be repelled as a whole by the electron beam. Since there is neither attraction nor repulsion between the condensate’s atoms and the beam’s electrons, this spectacular reflection at the macroscopic scale is purely due to the QZD induced by the frequent position measurement of the condensate’s atoms by the electron beam, and thus purely attributable to the peculiar character of the quantum measurement process.

We show that the key for the QZD induced by an electron beam to repel a bright soliton and preserve its solitonic character, and at the same time cause as small atom loss as possible, is to have a sharp beam edge, apart from having a high enough intensity as one would probably naturally expect for QZ D. That is, the intensity of the electron beam should rise from zero to a high value in a short enough distance. Also, if one has the freedom to choose to work with a slower soliton, the performance would also improve dramatically.

As an example of application of the proposed phenomenon, we show that just as a soccer player would stop a coming ball, a backward moving electron beam with half the velocity of the coming bright soliton can precisely stop the translational motion of the soliton. This offers an entirely new and useful tool for manipulating bright solitons.

**Results**

**Physical system.** We consider a one-dimensional system: A bright soliton supported by an attractive Bose-Einstein condensate moves toward an electron beam (cf. Fig. 1a). We follow the time evolution of the system by solving the time-dependent Gross-Pitaevskii equation with a dissipation term \cite{18,20}:

\[
i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} - g |\psi(x,t)|^2 \psi(x,t) - i\gamma \Gamma(x) \psi(x,t).
\]

(1)

Here \( \psi(x,t) \) is the wave function of the condensate. \( g > 0 \) is the nonlinearity parameter arising from attractive atom-atom interaction. The electron beam knocks atoms present in its beam path out of the condensate. This dissipative effect is characterized by the strength parameter \( \gamma \) and the beam profile

\[
\Gamma(x) = e^{-\frac{(x-x_b)^2}{2w^2}},
\]

(2)

where \( w \) characterizes the sharpness as well as the width of the electron beam while \( x_b \) is the position of the center of the beam. Without loss of generality, we set \( m = \hbar = g = 1 \), and normalize the initial \( \psi(x,0) \) such that

\[
\int_{-\infty}^{\infty} dx |\psi(x,0)|^2 = 1.
\]

(3)

**Quantum Zeno dynamics of a matter-wave bright soliton.** In spatial regions with negligible dissipation, equation (1) supports the bright soliton solution:

\[
\psi_{\text{soliton}}(x,t) = Ae^{ix} \text{Sech} \left[ A \sqrt{g}(x - x_0 - vt) \right] e^{-\frac{1}{2}it(v^2-A^2\gamma)},
\]

(4)

which describes a solitary wave that at \( t = 0 \) is localized around \( x_0 \) and moves with velocity \( v \) while maintaining its shape unchanged. \( A \) is the amplitude of the soliton. For our initial condition, we set \( t = 0 \) in the above equation and normalize according to equation (3). The initial condition reads:

\[
\psi_{\text{ini}}(x) = \frac{\sqrt{g}}{2} e^{ix} \text{Sech} \left[ \frac{g}{2}(x - x_0 - vt) \right].
\]

(5)

We start the bright soliton in a region with negligible dissipation toward the electron beam (cf. Fig. 1b) and follow the time evolution. A calculated time evolution of the density distribution is shown in Fig. 1b. The soliton is indeed reflected by the electron beam. However, the solitonic character is not preserved in that the wave packet quickly spreads out after reflection. A fraction of 0.54 of the initial condensate is reflected, other atoms are lost due to dissipation from the beam. In Fig. 1c, a stronger dissipation strength parameter \( \gamma \) is used while all other parameters are kept the same. The reflection fraction rises from 0.54 to 0.62. The wave packets spreads out slower after being reflected.

A very effective way to further increase the reflected fraction and minimize the atom loss is using an electron beam with a sharper edge. This is because while a strong enough dissipation will reflect atoms through QZD and cause very small atom loss, a weaker dissipation is less effective in reflecting and the soliton would go through such weakly dissipative region, losing atoms. However, before reaching the region where QZD reflection happens effectively, the
soliton always has to go through a region with weaker dissipation which causes atom loss. By using a sharper beam edge, this lossy region is greatly shortened, and as a result the atom loss can be greatly reduced.

Fig. 1 shows the results for an electron beam with a sharper edge than with \( w = 0.1 \) compared to \( w = 0.8 \) in Fig. 1, while all other parameters are kept the same. The reflected fraction rises greatly from 0.62 to 0.91. Most importantly, now the solitonic character is indeed preserved by the reflection in that the wave packet maintains a constant shape as it propagates after the reflection. This indicates the balance between dispersion and nonlinearity required for shape stability is preserved by the QZD reflection, even though a small fraction of the atoms are lost due to dissipation. In calculating Fig. 1 the initial soliton velocity \( v = -0.45553 \), the initial position of the soliton \( x_0 = 10 \), and the center position of the beam \( x_b = -5 \).

The principle quantity of interest here is the successfully reflected fraction \( P_{\text{refl}} \equiv \int_{x_b}^{x_0} |\psi(x, t_{\text{final}})|^2 dx \), where \( t_{\text{final}} \) is the end time of the evolution chosen such that the reflection has been completed and no further atom loss happens. On the one hand, a larger \( P_{\text{refl}} \) means smaller atom loss according to its definition. On the other hand, according to our experience, the higher the \( P_{\text{refl}} \) value, the better the solitonic character is preserved after reflection.

Apart from the sharpness of the beam edge, other factors can also influence the value of \( P_{\text{refl}} \). These include the strength parameter of the dissipation \( \gamma \), and the velocity \( v \) of the incoming soliton. As one would naturally expect for QZD, for stronger dissipation, due to QZD the boundary of the monitored region will be more similar to an impenetrable wall. As a result the atom loss during reflection will be reduced. For the velocity dependence, the essence of QZD is that the coherences between the monitored and unmonitored regions are repeatedly set to zero by the measurements with a high enough repetition frequency that coherent transmission between the regions cannot build up. Whether the repetition frequency is high enough is affected by the kinetic energy. If the kinetic energy is very high, the time scale of the coherent evolutions will be very short so that the requirement for the repetition frequency will be very high. From another point of view, for lower kinetic energies, the same repetition frequency, or equivalently the same intensity of a continuous monitoring such as by the electron beam, can induce a more pronounced QZD than for higher kinetic energies. So for a slower incoming soliton, due to its lower kinetic energy the QZD reflection will be more effective and the atom loss will be reduced.

In Figs. 2a-f we show the results of a systematic investigation on the effects of varying \( w, v \) and \( \gamma \). As can be seen, \( P_{\text{refl}} \) is higher when the sharpness parameter \( w \) decreases, or when the incoming velocity \( v \) decreases, or when the strength parameter of dissipation \( \gamma \) increases. In the overwhelming majority of cases, only reflection is visible. For high incoming velocity, small dissipation and a narrow beam, a faint transmission is also visible, as shown in Fig. 2g.

For very small incoming velocity, the reflected fraction \( P_{\text{refl}} \) can be very high. In Fig. 2h such a case is shown. The incoming velocity \( v = -0.015625 \). \( P_{\text{refl}} \) reaches a very high value of 0.996 and the solitonic character is very well preserved. Curiously, as can be seen from the figure, for such small incoming velocity of the soliton, even at the closest encounter there is a pronounced gap between the beam and the soliton. The electron beam seemingly can reflect the soliton without any contact, an impossible feat. In Fig. 2 we plot the electron beam profile \( \Gamma(x) \) together with the density distribution \( |\psi(x)|^2 \) and the amplitude distribution \( |\psi(x)| \) of the soliton at the lowest encounter at \( t = 640 \). As shown, although the density distribution seems quite cleanly separated from the electron beam, there is a sizable presence of the amplitude near the position of the beam. This close encounter effects the reflection.

With the numerical evidences presented so far for the beneficiary effect of having a small \( w \) for promoting \( P_{\text{refl}} \), one may still ask the question, is a small \( w \) advantageous because it describes a sharper beam edge or a narrower beam width? Considering numeric evidences presented so far these two views are indistinguishable, because a small \( w \) means both a sharper edge and a narrower beam width, due to the Gaussian form we have chosen for the beam (cf. equation (4)). To have a numeric resolution of this question, we investigate a beam profile given by

\[
\Gamma_{\text{new}}(x) = \begin{cases} 
  e^{-\left(\frac{x-x_1}{w}\right)^2}, & x < x_1 \\
  1, & x_1 \leq x < x_r \\
  e^{-\left(\frac{x-x_r}{w}\right)^2}, & x \geq x_r.
\end{cases}
\]  

(6)

This profile is shown by the black solid line in Fig. 2. This new beam has the same sharp edge as a Gaussian beam with the same \( w = 0.1 \) (red dashed curve in Fig. 2), but is much wider. These two beam profile both give \( P_{\text{refl}} = 0.94 \), while a Gaussian beam with similar width to the new beam (blue dotted curve in Fig. 2 with \( w = 0.8 \)) gives \( P_{\text{refl}} = 0.76 \). Since the new beam has a sharp edge but not a narrow width, this evidences that it is the sharpness of the beam edge instead of a narrow beam width that is beneficiary in promoting \( P_{\text{refl}} \).

Other parameters used in calculating Fig. 2 are: \( x_0 = 10, x_b = -5 \) except for Figs. 2h and i where \( x_b = -7 \). Fig. 2k, \( \gamma = 400, v = -0.25 \). Fig. 2j, \( w = 0.1, v = -0.25 \). Fig. 2l, \( w = 0.1, \gamma = 400 \). Fig. 2m, \( v = -0.25 \). Fig. 2n, \( w = 0.1 \). Fig. 2o, \( w = 0.1, v = -1.51241, \gamma = 25 \). Figs. 2h and i, \( \gamma = 100, w = 0.1 \). Fig. 2p, \( x_1 = -6, x_r = -5, v = -0.25, \gamma = 200 \).

**An example of application.** As an example of application for the above discussed QZD reflection of a bright soliton by an electron beam, we show that the translational motion of a bright soliton can be precisely stopped using this mechanism. As seen in Figs. 1p and 1q, and Figs. 2p and 2q, the reflected soliton always has the same speed as...
that of the incoming soliton. This inspires us to learn from a soccer player to use the mechanism in the following way in order to stop the soliton: we employ a moving electron beam that is moving in the same direction of the soliton and with half the soliton’s velocity, cf. Fig. 3a. The beam profile now is given by the time-dependent function:

\[ \Gamma_{\text{mov}}(x, t) = e^{-\left[\frac{x-(x_0-vt/2)}{w}\right]^2}. \]  

(7)

As can be seen from the calculated time-evolution of the density distribution shown in Fig. 3b, after the beam-soliton encounter the soliton is precisely stopped. The parameter used in calculating Fig. 3b are \( v = -0.25, \gamma = 100, w = 0.1, x_d = -5, x_0 = 10 \). The same method can also be employed to engineer the final velocity of the soliton to be other chosen values.

Discussion

In summary, we have shown that the QZD induced by an electron beam can reflect a bright soliton supported by an attractive Bose-Einstein condensate and the solitonic character can be well preserved. This reflection at the macroscopic scale constitute a demonstration and test of the subtle and peculiar character of the quantum measurement process. According to our best knowledge, this idea of reflecting a bright soliton using pure dissipation has not been put forward before. Technically, the key to preserve the solitonic character and minimize atom loss is to have a sharp beam edge, apart from the perhaps more naturally expected high dissipation strength. Also, if one can choose to work with a slow incoming soliton, the performance will also improve dramatically. Finally, we have shown that this phenomenon can be employed as an entirely new and useful tool for manipulating the motional state of the bright soliton. Compared to the more familiar optical potentials generated by laser beams, an electron beam can be focused much more sharply and thus can explore regimes difficult for an optical potential to reach. Also, if used in combination with optical potentials, it is readily conceivable that much more interesting phenomena and techniques await to be discovered. The phenomena studied in the present work can be readily realized using the recent experimental capability [18] combining an electron beam and a Bose-Einstein condensate. Our work shows this new experimental capability offers a great opportunity to study the spatial QZD, and more generally a many-body system out of equilibrium.
FIG. 2. Detailed analysis of the QZD reflection of a bright soliton. (a) The reflected fraction $P_{\text{refl}}$ as a function of the sharpness parameter $w$ of the electron beam. As can be seen, $P_{\text{refl}}$ increases as $w$ decreases, namely, for a sharper beam edge. (b) $P_{\text{refl}}$ increases with the strength parameter $\gamma$ of the dissipation. (c) $P_{\text{refl}}$ increases as the incoming velocity $v$ of the bright soliton decreases. (d) Contour plot of $P_{\text{refl}}$ as a function of both $w$ and $v$. $P_{\text{refl}}$ increases as $w$ or $v$ decreases. (e) $P_{\text{refl}}$ increases as $w$ decreases or $\gamma$ increases. (f) $P_{\text{refl}}$ increases as $v$ decreases or $\gamma$ increases. (g) In a small minority of cases, if the incoming velocity is high, the dissipation strength is small and the beam is narrow, a faint transmission is visible. (h) For very small incident velocity, the reflection $P_{\text{refl}}$ can be very high. Curiously, in such cases it is very pronounced that the soliton appears to be repelled long before it even hits the beam. This may seem impossible. However, as shown in (i), at the closest encounter ($t=640$), though the density $|\psi(x)|^2$ seems quite separated from the beam, the amplitude $|\psi(x)|$ has a sizable presence near the beam and is able to interact with the beam and effect the reflection. (j) The key for effective reflection is a sharp edge but not narrow beam width. A wide beam with a sharp edge (black solid) reflects as efficiently as a narrow sharp beam (red dashed), both achieving $P_{\text{refl}} = 0.94$. 
FIG. 3. (a) An example of application: an electron beam moving backward with half the velocity of the incoming soliton can precisely stop the soliton. (b) Calculated time evolution of the density distribution. After encounter with the electron beam at around $t \sim 100$ the soliton’s translational motion is precisely stopped.
ACKNOWLEDGMENTS

The work is supported by the National Natural Science Foundation of China (Grant No. 11575082, No. 11761161001, No. 11535004, No. 11375086, and No. 11120101005, No. 11235001) and by the International Science & Technology Cooperation Program of China (Grant No. 2016YFE0129300).

AUTHOR CONTRIBUTIONS

X.Z. performed the calculations, X.Z., X.F., C.X., Z.R. and J.P. discussed the results and wrote the paper.

Competing interests: The authors declare no competing interests.

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