Supplementary Materials

Multivariate Behavioral Research

Assessing Omitted Confounder Bias in Multilevel Mediation Models

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Centering in $1 \rightarrow 1 \rightarrow 1$ Mediation Models

One important decision in specifying a $1 \rightarrow 1 \rightarrow 1$ model is the scaling (centering) of the Level–1 predictors. Researchers recommend centering the Level–1 predictors (Enders & Tofighi, 2007). The choice of centering depends on the particular research question (Enders, 2013; Enders & Tofighi, 2007). When a researcher is interested in both Between and Within relationships for a specific predictor, there are two choices of centering strategies: centering within cluster 2 (CWC2) and centering at the grand mean 2 (CGM2) (Kreft, de Leeuw, & Aiken, 1995). CWC2 centers variables within cluster (school) and adds the cluster (school) means as covariates into the model at Level 2; CGM2 centers a Level–1 predictor at the grand mean (e.g., across all schools) while also including the cluster (school) means as covariates into the model at Level 2. If one is interested in estimating differential Between and Within effects, one may use either CWC2 or CGM2. Both CWC2 and CGM2 are “equivalent” in that the coefficient estimates in CGM2 are linear transformation of the coefficients in CWC2 and vice versa (Kreft et al., 1995). In CGM2, the coefficient associated with the cluster mean predictor is the contextual effect, an estimate of the differential effect of Between and Within effects (Blalock, 1984); However, in CWC2 the coefficient is an estimate of the Between effect. In both CWC2 and CGM2, the coefficient associated with the centered Level–1 predictor is an estimate of the Within effect.

Another issue in applying CWC2 or CGM2 is the choice between the observed instead of latent cluster means to measure a Level–2 construct and to estimate Between (e.g., school–level) effects (Muthén & Muthén, 1998-2012). The reliability of using the observed instead of a latent cluster mean to represent a Level–2 construct plays an important role in estimating a Between effect. Each Level–1 observation might be considered an “item” that represents a Level–2 construct (Snijders & Bosker, 2012). Using the observed mean to represent a Level–2 construct when the number of observations (e.g., items) at Level–1 is “small” might result in less than perfect reliability (e.g., coefficient $\alpha < .8$, Cronbach,
1951). In this case, using the observed cluster means could also underestimate Between effects. On the other hand, Lüdtke et al. showed that using observed means could result in smaller standard errors of the Between coefficients. Finally, both methods yielded similar results for significance tests of Between coefficients. In sum, using latent cluster mean would yield less biased and efficient Between estimates than would the observed cluster mean. The choice between observed and latent cluster mean depends on the research question and design specifications of a particular multilevel study (Lüdtke et al., 2008).

**Estimating A $1 \rightarrow 1 \rightarrow 1$ Model in Mplus**

Mplus is one of the few programs that can estimate multilevel structural equation modeling (MSEM) with random slopes and latent cluster means. We highlight nuances in Mplus syntax to estimate an MSEM. Preacher, Zyphur, and Zhang (2010) provide a detailed explanation of estimating different types of multilevel model using observed and latent cluster means. We discuss two models and highlight subtle differences in Mplus syntax that might be confusing: (a) a $1 \rightarrow 1 \rightarrow 1$ model with random intercepts only and (b) a $1 \rightarrow 1 \rightarrow 1$ model with random slopes and intercepts (our focus).

**A $1 \rightarrow 1 \rightarrow 1$ model with random intercepts only.** Suppose that we want to estimate this model with latent cluster mean. The default and only option in Mplus is CWC2. Mplus does latent mean clustering, i.e., decomposing a Level–1 variable into Between and Within when the observed variable is (a) measured at Level–1 and (b) is not declared as Within in the VARIABLE command section. Also, for the random intercept model, one needs to specify the following command ANALYSIS: TYPE = TWOLEVEL. Example 9.1 from Mplus Manual Version 7 (Muthén & Muthén, 1998-2012, pp. 262–263) explains this example:

The difference between this part of the example and the first part is that the covariate $x$ is decomposed into two latent variable parts instead of being treated as an observed variable as in conventional multilevel regression modeling. The
decomposition occurs when the covariate x is not mentioned on the WITHIN statement and is therefore modeled on both the within and between levels. When a covariate is not mentioned on the WITHIN statement, it is decomposed into two uncorrelated latent variables,

\[ x_{ij} = x_{wij} + x_{bj} \]

where \( i \) represents individual, \( j \) represents cluster, \( x_{wij} \) is the latent variable covariate used on the within level, and \( x_{bj} \) is the latent variable covariate used on the between level. This model is described in Muthén (1989, 1990, 1994). The latent variable covariate \( x_b \) is not used in conventional multilevel analysis. Using a latent covariate may, however, be advantageous when the observed cluster–mean covariate \( x_m \) does not have sufficient reliability resulting in biased estimation of the between–level slope (Asparouhov & Muthén, 2006b; Lüdtke et al., 2008).

For analytical results, we used CWC2 with latent mean cantering to derive analytical results because of research questions and convenience. For estimating the \( 1 \rightarrow 1 \rightarrow 1 \) model in sensitivity analysis section, however, we used CGM2 because it was the only choice in Mplus; CGM2 coefficients were linearly transformed to obtain CWC2 coefficients.

We can use either CWC2 or CGM2 to estimate a \( 1 \rightarrow 1 \rightarrow 1 \) model in the sensitivity analysis. We used CGM2 in Mplus. While for analytical derivations we used CWC2, for estimating the \( 1 \rightarrow 1 \rightarrow 1 \) model in sensitivity analysis, one could use either CWC2 and CGM2 because these two methods are equivalent for the \( 1 \rightarrow 1 \rightarrow 1 \) model. That is, the coefficient estimates in CGM2 are linear transformation of the coefficients in CWC2 and vice versa. For a \( 1 \rightarrow 1 \rightarrow 1 \) model with random slopes and intercepts and latent mean centering, Mplus only allows CGM2 strategy. We used this option along with additional command in Mplus to compute the Between effect (See MODEL CONSTRAINT command). Because of the equivalency, the results of the proposed methodology remains the same for the reasons explained in the manuscript.
A 1 \rightarrow 1 \rightarrow 1 with random slopes and intercepts. Suppose that we want to estimate this model with latent cluster mean. The default and only option in Mplus is CGM2 with latent mean clustering. Mplus uses the latent mean as a Level–2 predictor in the model when the observed variable is (a) measured at Level-1 and (b) is not declared as Within in the VARIABLE command section. It is important to note that for this model, the following command, which is different from the random intercepts model, should be specified: TYPE = TWOLEVEL RANDOM; An extension of Example 9.2 from Mplus Manual Version 7 (Muthén & Muthén, 1998-2012, p. 267) explains this example:

This [using latent mean] is achieved when the individual-level observed covariate is modeled in both the within and between parts of the model. This is requested by not mentioning the observed covariate x on the WITHIN statement in the VARIABLE command. When a random slope is estimated, the observed covariate x is used on the within level and the latent variable covariate xbj is used on the between level.

Using observed $X_{ij}$ at the within level and latent $X_{bj}$ at the between level means CGM2 with latent mean clustering. Although not explicitly emphasized, Mplus switches to CGM2 when the latent cluster mean is used. That is, in specifying a random slopes model, if one uses observed cluster means, Mplus estimates CWC2. However, if one uses latent cluster mean, Mplus estimates CGM2. This point, not widely known, is mentioned in supplemental materials by Preacher et al. (2010) available at http://dx.doi.org/10.1037/a0020141.supp

Calculate Regression Effects

In this section, we describe steps to calculate the coefficients, $ds$, $ls$, $ss$, and $qs$ necessary to compute adjusted results.

1. Obtain Between and Within estimate of sample variance-covariance matrices, $S_B$
and $S_W$, respectively. For example, Mplus (Muthén & Muthén, 1998-2012) can produce these estimates.

2. Use the Between and Within sample estimates to calculate the regression coefficients.

For Step 1, we provided sample Mplus syntax for the empirical example to specify $1 \rightarrow 1 \rightarrow 1$ model (See the section “Mplus Input” in this document). Note that we assume this model to be potentially “misspecified”. The estimates for this model are potentially “biased” as a result of the omitted confounders. The Mplus output (See the section “Mplus Output” in this document) contains the estimates of the Between and Within sample variance–covariance matrices for the variables $X$, $M$, and $Y$. From these matrices one can also calculate the Between and Within correlation matrices for $X$, $M$, and $Y$.

In Step 2, the correlation matrices are augmented with correlation coefficients between $Z$ and $X$, $M$, and $Y$. These correlation coefficients are plausible values set by the researcher. As a result, the “augmented” Between and Within sample correlation matrices can be calculated for each set of plausible correlation values. The correlation coefficients are then used to calculate standardized regression coefficients for the Between and Within regression coefficients using computational formulas in Cohen, Cohen, West, and Aiken (2003, Chapter 3) and Mauro (1990). Unstandardized coefficients can also be calculated using standard deviations obtained from the diagonal elements of the Between and Within sample covariances. Note that for identification purposes, Between and Within parts of the latent proxy variables are scaled to have a mean of zero and $SD$ of one. To illustrate, we show the calculation for $l_{ZX}$ in Equation B4 and $l_{YZ}$ in Equation B5. The calculations for the coefficients $ds$, $ss$, and $qs$ follow a similar computational approach.

$$l_{ZX} = \frac{\rho_{ZX}^{(W)}}{\sigma_{\eta_{Xij}}}$$

In the formula above, $\rho_{ZX}^{(W)}$ and $\sigma_{\eta_{Xij}}$ denote the population within–cluster correlation between $Z_{ij}$ ($\eta_{Zij}$) and $X_{ij}$ ($\eta_{Xij}$) and within–cluster population standard deviation of
$X_{ij}(\eta_{X_{ij}})$; superscript “W” denotes Within. To compute an estimate, $\rho_{ZX}^{(W)}$ is replaced by plausible values set by the researcher and $\sigma_{\eta_{X_{ij}}}^{2}$ is replaced with a within–cluster estimate obtained from software output such as Mplus. For example, $l_{YZ}$ in Equation B5 is calculated as follows:

$$l_{YZ} = \sigma_{\eta_{Y_{ij}}} \frac{\rho_{YZ}^{(W)}(1 - \rho_{XM}^{(W)}) + \rho_{YX}^{(W)}(\rho_{ZX}^{(W)} - \rho_{YZ}^{(W)}) + \rho_{YM}^{(W)}(\rho_{ZM}^{(W)}\rho_{XM}^{(W)} - \rho_{ZX}^{(W)})}{1 + 2\rho_{XM}^{(W)}\rho_{ZM}^{(W)}\rho_{ZX}^{(W)} - \rho_{XM}^{(W)} - \rho_{ZM}^{(W)} - \rho_{ZX}^{(W)}}$$

All the population $\rho$ values are replaced by the sample estimates from the “augmented” Within correlation matrix that includes plausible values for the correlations between $Z$ and $X$, $M$, and $Y$ as well as the sample estimates of correlations between $X$, $M$, and $Y$; $\sigma_{\eta_{Y_{ij}}}^{2}$ is the population within–cluster standard deviation of $Y_{ij}(\eta_{Y_{ij}})$, which can be estimated from Mplus output. It should be noted that plausible values for the correlations between $Z$ and $X$, $M$, and $Y$ are chosen so that the resulting augmented matrices are positive definite (Rousseeuw & Molenberghs, 1994).

**R Code**

In this section, we explain R functions used to conduct a sensitivity analysis for a two–level $1 \rightarrow 1 \rightarrow 1$ model with the random intercepts and slopes. We also explain specific R commands used to generate the results for the empirical example in the current manuscript. To follow the materials presented below readers need to have basic familiarity with the statistical software environment R (R Development Core Team, 2014).

Two main functions to conduct sensitivity analysis for the $1 \rightarrow 1 \rightarrow 1$ model are `SensitivityB` and `SensitivityW`. These functions are located in a file called `sensitivity.R`. To load these functions into your R session, run the following command:

```r
source("sensitivity.R")
```
**Input.** Below is the list of the input arguments for both functions `SensitivityB` and `SensitivityW`:

- **R**: A $3 \times 3$ Between or Within correlation matrix. $R$ needs to be a matrix with row and column dimension names "x", "m", and "y". For example, dimension names for $R$ can be set as follows:

  ```r
dimnames(R) <- list(c("m", "x", "y"), c("m", "x", "y"))
  ```

- **est**: A vector of estimates for coefficients $a$ and $b$. The vector elements must have names for $a$ and $b$ specified as "a" and "b".

- **sd**: A vector of Between or Within estimates of SDs for $X$, $M$, and $Y$. These values are the square root of diagonal of the Between or Within variance–covariance matrices. The vector elements must have the names in the following format: "x", "m", and "y".

- **rZXRange**: A vector of plausible values for correlation between $X$ and $Z$.

- **rZMRange**: A vector of plausible values for correlation between $M$ and $Z$.

- **rZYRange**: A vector of plausible values for correlation between $Y$ and $Z$.

- **covajbj**: Covariance between random coefficients $a_j$ and $b_j$. This argument is only used in `SensitivityW` function.

- **ncpus**: Number of processors to run computations in parallel.

**Output.** Each function generates a data frame with the variables described below. For example, function `SensitivityB` generates a data frame with the following variables:

- **rzx**: Correlation between $X$ and $Z$
• **rzm**: Correlation between \( M \) and \( Z \)

• **ryz**: Correlation between \( Y \) and \( Z \)

• **abBetBiased**: Unadjusted ("biased") indirect effect estimate

• **abBetTrue**: Adjusted ("unbiased") indirect effect estimate

Then, one can use the resulting data frames to produce sensitivity contour plots. Below, we show R commands used to generate the results in the empirical example:

```r
estW <- c(0.086, 0.433)
estB <- c(0.024, 0.784)

names(estW) <- names(estB) <- c("a", "b") # names the vector elements

## Read the estimates of between and within covariance matrices
## produced by Mplus. See the Mplus output section.
sw_vec <- scan("spw.txt") # Within estimate
sb_vec <- scan("estsigb.txt") # Between estimate

## vechr.reverse function from lavaan package converts a vector to
## a matrix
library(lavaan)

SW <- vechr.reverse(sw_vec) # within covariance matrix
SB <- vechr.reverse(sb_vec) # between covariance matrix

nam <- c("m", "y", "x") # dim names

dimnames(SW) <- dimnames(SB) <- list(nam, nam) # assigning dim names

RW <- cov2cor(SW) # correlation matrix
RB <- cov2cor(SB) # correlation matrix

sdW <- sqrt(diag(SW)) # compute the SDs from diagonal values
sdB <- sqrt(diag(SB)) # compute the SDs from diagonal values

## Covariance between aj and bj
```
covajbj <- 0

## Generating plausible values
stp <- 0.01  # the amount of step
rZXRange <- c(0, 0.1, 0.3, 0.5)
rZMRange <- seq(-0.5, 0.5, stp)
rZYRange <- seq(-0.5, 0.5, stp)

library(parallel)  # load parallel library for parallel computing
resB <- SensitivityB(RB, est = estB, sd = sdB, rZXRange = rZXRange,
    rZMRange = rZMRange, rZYRange = rZYRange, ncpus = ncpus)
resW <- SensitivityW(RW, est = estW, sd = sDW, covajbj = covajbj,
    rZXRange = rZXRange, rZMRange = rZMRange, rZYRange = rZYRange,
    ncpus = ncpus)

library(lattice)  # To generate sensitivity contour plots using lattice library
library(grid)
library(gridExtra)

## Between plot
trellis.par.set(theme = col.whitebg())  # Set the coloring scheme
przx0 <- contourplot(abBetTrue ~ rzm * ryz, resB, main = expression(r[ZX] ==
    0), xlab = expression(r[ZM]), ylab = expression(r[ZY]), region = TRUE,
    subset = rzx == "rZX=0", par.settings = list(fontsize = list(text = 8,
    points = 10)), cuts = 6)
przx.1 <- contourplot(abBetTrue ~ rzm * ryz, resB, main = expression(r[ZX] ==
    0.1), xlab = expression(r[ZM]), ylab = expression(r[ZY]), region = TRUE,
    subset = rzx == "rZX=0.1", par.settings = list(fontsize = list(text = 8,
    points = 10)))
przx.3 <- contourplot(abBetTrue ~ rzm * ryz, resB, main = expression(r[ZX] ==
0.3), xlab = expression(r[ZM]), ylab = expression(r[ZY]), region = TRUE,
subset = rzx == "rZX=0.3", par.settings = list(fontsize = list(text = 8,
points = 10)))
grid.arrange(przx0, przx.1, przx.3, ncol = 2)

## Within plot
trellis.par.set(theme = col.whitebg())
przxw0 <- contourplot(abWithTrue ~ rzm * ryz, resW, main = expression(r[ZX] ==
0), xlab = expression(r[ZM]), ylab = expression(r[ZY]), region = TRUE,
subset = rzx == "rZX=0", par.settings = list(fontsize = list(text = 8,
points = 10)), cuts = 6)
przxw.1 <- contourplot(abWithTrue ~ rzm * ryz, resW, main = expression(r[ZX] ==
0.1), xlab = expression(r[ZM]), ylab = expression(r[ZY]), region = TRUE,
subset = rzx == "rZX=0.1", par.settings = list(fontsize = list(text = 8,
points = 10)))
przxw.3 <- contourplot(abWithTrue ~ rzm * ryz, resW, main = expression(r[ZX] ==
0.3), xlab = expression(r[ZM]), ylab = expression(r[ZY]), region = TRUE,
subset = rzx == "rZX=0.3", par.settings = list(fontsize = list(text = 8,
points = 10)))
grid.arrange(przxw0, przxw.1, przxw.3, ncol = 2)

Sensitivity.R Function

## There are two main functions to conduct sensitivity analysis for a
## 1-1-1 model called SensitivityB and SensitivityW located in a file
## 'sensitivity.R'. To load these functions into your R session, run
## the following command:
## source('sensitivity.R')
## Input and output values for each function is listed below.

## R: A 3 by 3 Between or Within correlation matrix. R needs to be a matrix with row and column dimension names '{x}', '{m}', and '{y}'. For example, dimension names for R can be set as follows: `dimnames(R) <- list(c('m', 'x', 'y'), c('m', 'x', 'y'))`

## est: A vector of estimates for coefficients a and b. The vector elements must have names for a and b as '{a}' and '{b}'.

## sd: A vector of between and estimates SDs of X, M, and Y. These values are the square root of diagonal of the Between or Within variance-covariance matrices. The vector elements must have the names of the following format: '{x}', '{m}', and '{y}'.

## rZXRange: A vector of plausible values for correlation between X and Z.

## rZMRange: A vector of plausible values for correlation between M and Z.

## rZYRange: A vector of plausible values for correlation between Y and Z.

## covajbj: Covariance between random coefficients aj and bj. This argument is only used in SensitivityW function.

## ncpus: Number of processors to run computations in parallel.

### Extra Utility Functions

```r
b_1p <- function(ryx, sx, sy) {
  b <- ryx * sy/sx
  return(as.vector(b))
}

b1_2p <- function(ry1, ry2, r12, sy, s1) {
  # x1 is the primary variable
```
\[
\beta_1 \leftarrow (r_{y_1} - r_{y_2} \times r_{12})/(1 - r_{12}^2)
\]

\[
b_1 \leftarrow \beta_1 \times s_y/s_1
\]

\[
\text{return}(\text{as.vector}(b_1))
\]

corCompute <- \text{function}(x) \{
  ## accepts list of parameters and return a cor matrix
  x <- x[1:6]
  corM <- \text{lavaan::vech.reverse}(x, FALSE)
  corM <- corM + \text{diag}(1, 4)
  corM <- corM + \text{diag}(1, 4)
  \text{return}(corM)
\}

bz_3p <- \text{function}(x) \{
  rzx <- x[1]
  rzm <- x[2]
  ryz <- x[3]
  rxm <- x[4]
  ryx <- x[5]
  rym <- x[6]
  sx <- x[7]
  sm <- x[8]
  sy <- x[9]
  sz <- x[10]
  num <- ryz \times (1 - r_{x_m}^2) + ryx \times (rzm \times rxm - rzx) + rym \times (rzx \times rxm - rzm)
  den <- 1 + 2 \times rxm \times rzm \times rzx - rxm^2 - rzm^2 - rzx^2
  b <- \text{as.vector}(\text{num}/\text{den} \times s_y)
\}
return(b)
}

### Main Functions

SensitivityB <- function(R, est, sd, rZXRange = c(-0.3, -0.1, 0, 0.1, 0.3), rZMRange = seq(-0.99, 0.99, 0.02), rZYRange = seq(-0.99, 0.99, 0.02), ncpus = 3L) {
  RYX_B = R["x", "y"] # Observed correlation between the independent and dependent variables.
  RMX_B = R["x", "m"] # Observed correlation between the independent variable and the mediator.
  RYM_B = R["m", "y"] # Observed correlation between the mediator and the dependent variable.

  aBiased_B = est["a"]
  bBiased_B = est["b"]
  sx_B = sd["x"]
  sm_B = sd["m"]
  sy_B = sd["y"]
  rVecM = rZMRange # seq(-.99,.99,.02)
  rVecY <- rZYRange  # seq(-.5,.5,.01)
  rVecX <- rZXRange  # c(-.3,-.1,0,.1,.3)

  input <- expand.grid(rzx = rVecX, rzm = rVecM, ryz = rVecY, rxm = RMX_B, ryx = RYX_B, rym = RYM_B, sx = sx_B, sm = sm_B, sy = sy_B, sz = 1,
                       aBiased = aBiased_B, bBiased = bBiased_B)

  input <- data.frame(input)

  inputList <- split(input, 1:nrow(input))

  ## We would like to acknowledge that following code is adopted from
  ## from the lavaan package

  have_mc <- have_snow <- FALSE

  if (.Platform$OS.type != "windows" && ncpus > 1L)
have.mc <- TRUE else if (.Platform$OS.type == "windows" && ncpus > 1L)

have_snow <- TRUE

res1 <- if (ncpus > 1L && (have.mc || have_snow)) {
    if (have.mc) {
        parallel::mclapply(inputList, trueCoefB, mc.cores = ncpus)
    } else if (have_snow) {
        cl <- parallel::makePSOCKcluster(rep("localhost", ncpus)) #
        if (RNGkind()[1L] == "L'Ecuyer-CMRG")
            parallel::clusterSetRNGStream(cl) #
        parallel::clusterExport(cl, list("corCompute", "trueCoefB",
                        "b_1p", "b1_2p", "bz_3p"))
        res <- parallel::parLapply(cl, inputList, trueCoefB) #
        parallel::stopCluster(cl) #
        res
    }
} else lapply(inputList, trueCoefB)

res1 <- do.call("rbind", res1)
res1 <- data.frame(res1)
ord1 <- with(res1, order(rzm, ryz))
resN <- res1[ord1, ]

resN$rzx <- factor(resN$rzx, levels = sort(unique(rVecX)), labels = paste("rZX=",
                        sort(unique(rVecX)), sep = ""))

return(resN)

SensitivityW <- function(R, est, sd, covajbj = 0, rZXRange = c(-0.3, -0.1, 0, 0.1, 0.3),
 rZMRange = seq(-0.99, 0.99, 0.02), rZYRange = seq(-0.99, 0.99, 0.02), ncpus = 3L) {

ryx = R["x", "y"]  ##Observed correlation between the independent and dependent variables.
rm = R["x", "m"]   ##Observed correlation between the independent variable and the mediator.
ry = R["m", "y"]  ##Observed correlation between the mediator and the dependent variable.

aBiased = est["a"]
bBiased = est["b"]
sx = sd["x"]
sm = sd["m"]
sy = sd["y"]

rVecM <- rZMRange
rVecY <- rZYRange
rVecX <- rZXRange

input <- expand.grid(rzx = rVecX, rzm = rVecM, ryz = rVecY, rxm = rm,
                     ryx = ryx, rym = ry, sx = sx, sm = sm, sy = sy, sz = 1, aBiased = aBiased,
                     bBiased = bBiased, covajbj)

input <- data.frame(input)

inputList <- split(input, 1:nrow(input))

## We would like to acknowledge that following code is adopted from
## from the lavaan package

have_mc <- have_snow <- FALSE

if (.Platform$OS.type != "windows" && ncpus > 1L)
  have_mc <- TRUE else if (.Platform$OS.type == "windows" && ncpus > 1L)
  have_snow <- TRUE

have_mc

have_snow

res1 <- if (ncpus > 1L && (have_mc || have_snow)) {
  if (have_mc) {
    parallel::mclapply(inputList, trueCoefW, mc.cores = ncpus)
} else if (have_snow) {
    cl <- parallel::makePSOCKcluster(rep("localhost", ncpus))  
    if (RNGkind()[[1L]] == "L'Ecuyer-CMRG")
        parallel::clusterSetRNGStream(cl)  
    parallel::clusterExport(cl, list("corCompute", "trueCoefW", 
        "b_1p", "b1_2p", "bz_3p"))
    res <- parallel::parLapply(cl, inputList, trueCoefW)  
    parallel::stopCluster(cl)  
    res
}
} else lapply(inputList, trueCoefW)

res1 <- do.call("rbind", res1)
res1 <- data.frame(res1)
ord1 <- with(res1, order(rzm, ryz))
resN <- res1[ord1, ]
resN$rzx <- factor(resN$rzx, levels = sort(unique(rVecX)), labels = paste("rZX=",
    sort(unique(rVecX)), sep = ""))

return(resN)

### Additional Utility Functions within coefficients adjusted values

trueCoefW <- function(x) {
    x <- unlist(x)
    xMat <- corCompute(x)

    if (matrixcalc::is.positive.definite(xMat)) {
        rzx <- x[1]
        rzm <- x[2]
        ryz <- x[3]
    } else {
        cl <- parallel::makePSOCKcluster(rep("localhost", ncpus))  
        if (RNGkind()[[1L]] == "L'Ecuyer-CMRG")
            parallel::clusterSetRNGStream(cl)  
        parallel::clusterExport(cl, list("corCompute", "trueCoefW", 
            "b_1p", "b1_2p", "bz_3p"))
        res <- parallel::parLapply(cl, inputList, trueCoefW)  
        parallel::stopCluster(cl)  
        res
    }
}

res1 <- do.call("rbind", res1)
res1 <- data.frame(res1)
ord1 <- with(res1, order(rzm, ryz))
resN <- res1[ord1, ]
resN$rzx <- factor(resN$rzx, levels = sort(unique(rVecX)), labels = paste("rZX=",
    sort(unique(rVecX)), sep = ""))

return(resN)
rxm <- x[4]
ryx <- x[5]
rym <- x[6]
 sx <- x[7]
 sm <- x[8]
 sy <- x[9]
 sz <- x[10]
 aBiased <- x[11]
 bBiased <- x[12]
 covajbj <- x[13]
lzx <- b_1p(rzx, sx, sz)
 lyz <- bz_3p(x)
lmz <- b1_2p(ry1 = rzm, ry2 = rxm, r12 = rzx, sy = sm, s1 = sz)  # M is y and z is x
qzx <- b1_2p(ry1 = rzx, ry2 = rzm, r12 = rxm, sy = sz, s1 = sx)
qzm <- b1_2p(ry1 = rzm, ry2 = rzx, r12 = rxm, sy = sz, s1 = sm)
 aBias <- lmz * lzx
 bBias <- lyz * qzm
 cpBias <- lyz * qzm
abBiased <- as.vector(aBiased * bBiased) + covajbj
 a <- aBiased - aBias  # unbiased (adjusted) estimate
 b <- bBiased - bBias  # unbiased (adjusted)
 ab <- as.vector(a * b)  # unbiased (adjusted) estimate of indirect effect
 grp = ifelse(ryz < 0, 1, 0)  # This is used to distinguish two curves (if exists)
 res <- c(rzx, rzm, ryz, abWithBiased = abBiased, abWithTrue = ab,
        grp = as.vector(grp))
} else res <- c(rzx, rzm, ryz, abWithBiased = NA, abWithTrue = NA,
        grp = NA)
```r
c return(res)
}

### between coefficients adjusted values

trueCoefB <- function(x) {
  x <- unlist(x)
  xMat <- corCompute(x)
  if (matrixcalc::is.positive.definite(xMat)) {
    rzx <- x[1]
    rzm <- x[2]
    ryz <- x[3]
    rym <- x[4]
    rxy <- x[5]
    rym <- x[6]
    sx <- x[7]
    sm <- x[8]
    sy <- x[9]
    sz <- x[10]
    aBiased <- x[11]
    bBiased <- x[12]
    dzx <- b_1p(rzx, sx, sz)
    dyz <- b_2p()
    dmz <- b_1_2p(ry1 = rzm, ry2 = rym, r12 = rzx, sy = sm, s1 = sz)  
    szx <- b_1_2p(ry1 = rzx, ry2 = rzm, r12 = rym, sy = sz, s1 = sx)
    szm <- b_1_2p(ry1 = rzm, ry2 = rzx, r12 = rym, sy = sz, s1 = sm)
    abias <- dmz * dzx
    bbias <- dyz * szm
    cpBias <- dyz * szm
  }
}
```
OMITTED CONFOUNDER BIAS

abBiased <- as.vector(aBiased * bBiased)
a <- aBiased - aBias  # unbiased (adjusted) estimate
b <- bBiased - bBias  # unbiased (adjusted) estimate
ab <- as.vector(a * b)  # unbiased (adjusted) estimate of indirect effect
grp = ifelse(ryz < 0, 1, 0)  # This is used to distinguish two curves (if exists)
res <- c(rzx, rzm, ryz, abBetBiased = abBiased, abBetTrue = ab,
         grp = as.vector(grp))
} else res <- c(rzx, rzm, ryz, abBetBiased = NA, abBetTrue = NA, grp = NA)
return(res)

Mplus Input

TITLE: MSEM for mediation analysis
random intercepts and slopes;
DATA: FILE = mplusdata.txt;
VARIABLE:
NAMES = id x m y;
USEVARIABLES= id x m y;
MISSING=.;
CLUSTER=id;
DEFINE:
standardize x m y;
ANALYSIS: TYPE IS TWOLEVEL RANDOM;
MODEL:  ! model specification follows
%WITHIN%  ! Model for Within effects follows
sa | m ON x;  ! regress m on x, call the random slope sa
sb | y ON m;  ! regress y on m, call the random slope sb
sc | y ON x; ! regress y on x, call the random slope sc
%BETWEEN% ! Model for Between effects follows
sa sb sc x m y; ! estimate Level-2 (residual) variances for sa, sb, sc, x, m, and y
sa WITH sc; ! estimate Level-2 covariances of sa with sc, x, m, and y
sa WITH sb (covab); ! estimate Level-2 covariance of sa and sb, call it cab
sb WITH sc; ! estimate Level-2 covariances of sb with sc, x, m, and y
m ON x (acon); ! regress m on x, call the slope acon, acon = contextual effect
!, not the Between slope
y ON m (bcon); ! regress y on m, call the slope bcon, bcon = contextual effect
!, not the Between slope
y ON x (cpcon); ! regress y on x
[sa] (aw); ! estimate the mean of sa, call it aw
[sb] (bw); ! estimate the mean of sb, call it bw
[sc] (cpw); ! estimate the mean of sc, call it cpw
MODEL CONSTRAINT: ! section for computing indirect effects
NEW(ab bb cpb indx indxWRONG); ! name the indirect effects
ab=aw+acon; ! compute Between a path
bb=bw+bcon; ! compute Between b path
cpb=cpw+cpcon; ! compute Between b path
indx=aw*bw; ! compute the Within indirect effect
indxWRONG=aw*bw+covab; ! compute the Within indirect effect--not correct
indx=ab*bb; ! compute the Between indirect effect
OUTPUT: samp TECH1 TECH8; ! request parameter specifications, starting values,!
optimization history, and confidence intervals for all effects
Savedata:
sample = spw.txt; ! Saves Within variance-covariance estimate
sigb = estsigb.txt; ! Saves Between variance-covariance estimate

Mplus Output

Mplus VERSION 7.3 (Mac)
MUTHEN & MUTHEN
11/15/2014 12:21 PM

INPUT INSTRUCTIONS

TITLE: MSEM for mediation analysis
random intercepts and slopes;
DATA: FILE = mplusdata.txt;
VARIABLE:
NAMES = id x m y;
USEVARIABLES = id x m y;
MISSING = .;
CLUSTER = id;
DEFINE:
standardize x m y;
ANALYSIS: TYPE IS TWOLEVEL RANDOM;
MODEL: ! model specification follows
%WITHIN% ! Model for Within effects follows
sa | m ON x; ! regress m on x, call the random slope sa
sb | y ON m;  ! regress y on m, call the random slope sb
sc | y ON x;  ! regress y on x, call the random slope sc
%BETWEEN% ! Model for Between effects follows
sa sb sc x m y;  ! estimate Level-2 (residual) variances for sa, sb, sc, x, m, and y
sa WITH sc;  ! estimate Level-2 covariances of sa with sc, x, m, and y
sa WITH sb (covab);  ! estimate Level-2 covariance of sa and sb, call it cab
sb WITH sc;  ! estimate Level-2 covariances of sb with sc, x, m, and y
m ON x (acon);  ! regress m on x, call the slope acon, acon = contextual effect
!, not the Between slope
y ON m (bcon);  ! regress y on m, call the slope bcon, bcon = contextual effect
!, not the Between slope
y ON x (cpcon);  ! regress y on x
[sa] (aw);  ! estimate the mean of sa, call it aw
[sb] (bw);  ! estimate the mean of sb, call it bw
[sc] (cpw);  ! estimate the mean of sc, call it cpw
MODEL CONSTRAINT: ! section for computing indirect effects
NEW(ab bb cpb indw indwWRONG);  ! name the indirect effects
ab=aw+acon;  ! compute Between a path
bb=bw+bcon;  ! compute Between b path
cpb=cpw+cpcon;  ! compute Between b path
indw=aw*bw;  ! compute the Within indirect effect
indwWRONG=aw*bw+covab;  ! compute the Within indirect effect--not correct
indb=ab*bb; ! compute the Between indirect effect

**OUTPUT:** samp TECH1 TECH8; ! request parameter specifications, starting values, ! optimization history, and confidence intervals for all effects

**Savedata:**

sample = spw.txt; ! Saves Within variance-covariance estimate

sigb = estsigb.txt; ! Saves Between variance-covariance estimate

INPUT READING TERMINATED NORMALLY

MSEM for mediation analysis
random intercepts and slopes;

**SUMMARY OF ANALYSIS**

| Description                          | Value   |
|--------------------------------------|---------|
| Number of groups                     | 1       |
| Number of observations               | 12000   |
| Number of dependent variables        | 2       |
| Number of independent variables      | 1       |
| Number of continuous latent variables| 3       |

Observed dependent variables
Continuous
M Y

Observed independent variables
X

Continuous latent variables
SA SB SC

Variables with special functions

Cluster variable ID

Estimator MLR
Information matrix OBSERVED
Maximum number of iterations 100
Convergence criterion 0.100D-05
Maximum number of EM iterations 500
Convergence criteria for the EM algorithm
Loglikelihood change 0.100D-02
Relative loglikelihood change 0.100D-05
Derivative 0.100D-03
Minimum variance 0.100D-03
Maximum number of steepest descent iterations 20
Maximum number of iterations for H1 2000
Convergence criterion for H1 0.100D-03
Optimization algorithm EMA

Input data file(s)
mplusdata.txt
Input data format  FREE

SUMMARY OF DATA

Number of missing data patterns  1
Number of clusters  300
Average cluster size  40.000

Estimated Intraclass Correlations for the Y Variables

| Intraclass | Intraclass |
|------------|------------|
| Variable   | Correlation| Variable   | Correlation | Variable   | Correlation |
| M          | 0.649      | Y          | 0.526      | X          | 0.207       |

COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value  0.100

PROPORTION OF DATA PRESENT
### Covariance Coverage

|     | M   | Y   | X   |
|-----|-----|-----|-----|
| M   | 1.000 |     |     |
| Y   | 1.000 | 1.000 |     |
| X   | 1.000 | 1.000 | 1.000 |

### SAMPLE STATISTICS

**NOTE:** The sample statistics for within and between refer to the maximum-likelihood estimated within and between covariance matrices, respectively.

### ESTIMATED SAMPLE STATISTICS FOR WITHIN

**Means**

|     | M   | Y   | X   |
|-----|-----|-----|-----|
| 1   | 0.000 | 0.000 | 0.000 |

**Covariances**

|     | M   | Y   | X   |
|-----|-----|-----|-----|
|     |     |     |     |
OMITTED CONFOUNDER BIAS

\[
\begin{array}{lll}
M & 0.351 \\
Y & 0.167 & 0.474 \\
X & 0.062 & 0.096 & 0.793 \\
\end{array}
\]

Correlations

\[
\begin{array}{ccc}
M & Y & X \\
\hline
M & 1.000 \\
Y & 0.408 & 1.000 \\
X & 0.117 & 0.156 & 1.000 \\
\end{array}
\]

ESTIMATED SAMPLE STATISTICS FOR BETWEEN

Means

\[
\begin{array}{llll}
M & Y & X \\
\hline
1 & 0.000 & 0.000 & 0.000 \\
\end{array}
\]

Covariances

\[
\begin{array}{lll}
M & Y & X \\
\hline
M & 0.649 \\
Y & 0.475 & 0.526 \\
X & -0.001 & -0.024 & 0.207 \\
\end{array}
\]
### Correlations

|     | M       | Y       | X       |
|-----|---------|---------|---------|
| M   | 1.000   |         |         |
| Y   | 0.813   | 1.000   |         |
| X   | -0.002  | -0.072  | 1.000   |

**Maximum Log-Likelihood Value for the Unrestricted (H1) Model is**

-39127.241

### Univariate Higher-Order Moment Descriptive Statistics

**Univariate Higher-Order Moment Descriptive Statistics**

| Variable | Mean | Skewness | Minimum | % with 20%/60% | Percentiles | 40%/80% | Median |
|----------|------|----------|---------|----------------|-------------|---------|--------|
|          |      |          |         |                | Sample Size |         |        |
|          |      |          |         |                | Variance    |         |        |
|          |      |          |         |                | Kurtosis    |         |        |
|          |      |          |         |                | Maximum     |         |        |
|          |      |          |         |                | Min/Max     |         |        |
| M        | 0.000| -0.083   | -4.254  | 0.01%          | 12000.000   | 0.271  | 0.820  |
| Y        | 0.000| 0.002    | -4.677  | 0.01%          | -0.794      | -0.229 | 0.018  |
### Model Fit Information

**Number of Free Parameters**  
21

**Loglikelihood**

- **H0 Value**: -29160.280  
- **H0 Scaling Correction Factor**: 1.1811 for MLR

**Information Criteria**

- **Akaike (AIC)**: 58362.559  
- **Bayesian (BIC)**: 58517.805  
- **Sample-Size Adjusted BIC**: 58451.069  

\[(n^* = (n + 2) / 24)\]
### Model Results

|                | Two-Tailed | Estimate | S.E. | Est./S.E. | P-Value |
|----------------|------------|----------|------|-----------|---------|
| **Within Level** |            |          |      |           |         |
| **Variances**   |            |          |      |           |         |
| \( X \)         | 0.793      | 0.010    | 77.038 | 0.000     |
| **Residual Variances** | |          |      |           |         |
| \( M \)         | 0.177      | 0.002    | 76.502 | 0.000     |
| \( Y \)         | 0.102      | 0.001    | 70.322 | 0.000     |
| **Between Level** |            |          |      |           |         |
| \( M \) ON \( X \) | -0.062     | 0.112    | -0.556 | 0.579     |
| \( Y \) ON \( M \) | 0.350      | 0.082    | 4.293  | 0.000     |
| \( Y \) ON \( X \) | -0.063     | 0.092    | -0.685 | 0.493     |
| **SA WITH**     |            |          |      |           |         |
| \( SC \)        | 0.011      | 0.010    | 1.062  | 0.288     |
| \( SB \)        | 0.004      | 0.021    | 0.206  | 0.837     |
| Variable | Parameter 1 | Parameter 2 | Parameter 3 | Parameter 4 |
|----------|-------------|-------------|-------------|-------------|
| SB WITH SC | -0.005 | 0.016 | -0.320 | 0.749 |
| Means | | | | |
| X | 0.000 | 0.028 | 0.000 | 1.000 |
| SA | 0.086 | 0.027 | 3.216 | 0.001 |
| SB | 0.433 | 0.043 | 10.162 | 0.000 |
| SC | 0.078 | 0.021 | 3.713 | 0.000 |
| Intercepts | | | | |
| M | 0.006 | 0.048 | 0.131 | 0.895 |
| Y | -0.067 | 0.042 | -1.593 | 0.111 |
| Variances | | | | |
| X | 0.207 | 0.017 | 12.345 | 0.000 |
| SA | 0.208 | 0.018 | 11.874 | 0.000 |
| SB | 0.510 | 0.045 | 11.284 | 0.000 |
| SC | 0.125 | 0.011 | 11.316 | 0.000 |
| Residual Variances | | | | |
| M | 0.684 | 0.061 | 11.301 | 0.000 |
| Y | 0.501 | 0.073 | 6.874 | 0.000 |
| New/Additional Parameters | | | | |
| AB | 0.024 | 0.109 | 0.217 | 0.828 |
| BB | 0.784 | 0.071 | 10.994 | 0.000 |
| CPB | 0.015 | 0.090 | 0.165 | 0.869 |
| INDB | 0.019 | 0.086 | 0.216 | 0.829 |
| INDW | 0.037 | 0.012 | 3.056 | 0.002 |
INDWWRON 0.042 0.024 1.746 0.081

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.165E-05
(ratio of smallest to largest eigenvalue)

TECHNICAL 1 OUTPUT

PARAMETER SPECIFICATION FOR WITHIN

NU

M Y X

------- ------- -------
1 0 0 0

LAMBDAB

SA SB SC M

Y

------- ------- ------- -------
------- ------- ------- -------
M 0 0 0 0 0
0
### LAMBDA

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| Y | 0 | 0 | 0 | 0 | 0 |
| X | 0 | 0 | 0 | 0 | 0 |

### THETA

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| M | 0 |   |   |   |   |
| Y | 0 | 0 | 0 |   |   |
| X | 0 | 0 | 0 | 0 | 0 |

### ALPHA

|   | SA | SB | SC | M |
|---|----|----|----|---|
| Y |    |    |    |   |

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| Y | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
OMITTED CONFOUNDER BIAS

ALPHA

X

---------
1 0

BETA

| SA | SB | SC | M |
|----|----|----|----|
| Y  | 0  | 0  | 0  | 0  |

| SA | 0  | 0  | 0  | 0  | 0 |
|----|----|----|----|----|
| SB | 0  | 0  | 0  | 0  | 0 |
|----|----|----|----|----|
| SC | 0  | 0  | 0  | 0  | 0 |
|----|----|----|----|----|
| M  | 0  | 0  | 0  | 0  | 0 |
|----|----|----|----|----|
| Y  | 0  | 0  | 0  | 0  | 0 |
|----|----|----|----|----|
| X  | 0  | 0  | 0  | 0  | 0 |

BETA

X
|       | SA | SB | SC | M | Y | X |
|-------|----|----|----|---|---|---|
|       | 0  | 0  | 0  | 0 | 0 | 0 |

\[
\begin{array}{cccc}
SA & SB & SC & M \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

|       | X |
|-------|---|
|       | 0 |

\[
\begin{array}{c}
X \\
3 \\
\end{array}
\]
PARAMETER SPECIFICATION FOR BETWEEN

NU

\[
\begin{array}{ccc}
M & Y & X \\
\hline
1 & 0 & 0 & 0 \\
\end{array}
\]

LAMBDA

\[
\begin{array}{cccc}
SA & SB & SC & M \\
\hline
Y & & & \\
\hline
M & 0 & 0 & 0 & 0 \\
0 & & & \\
Y & 0 & 0 & 0 & 0 \\
0 & & & \\
X & 0 & 0 & 0 & 0 \\
0 & & & \\
\end{array}
\]

LAMBDA

\[
\begin{array}{c}
X \\
\hline
M & 0 \\
Y & 0 \\
X & 0 \\
\end{array}
\]
|    | M  | Y  | X  |
|----|----|----|----|
|    | 0  | 0  | 0  |
|    | 0  | 0  | 0  |

|    | S A | S B | S C | M  |
|----|-----|-----|-----|----|
|    | 0   | 0   | 0   | 7  |
|    | 1   | 4   | 5   | 6  |
|    | 8   |     |     |    |

|    | S A | S B | S C | M  |
|----|-----|-----|-----|----|
|    | 0   | 0   | 0   | 9  |
|    | 1   |     |     |    |
|   | SA   | SB   | SC   | M   | Y   | X   |
|---|------|------|------|-----|-----|-----|
| SA| 0    | 0    | 0    | 0   | 0   | 0   |
| SB| 0    | 0    | 0    | 0   | 0   | 0   |
| SC| 0    | 0    | 0    | 0   | 0   | 0   |
| M | 0    | 0    | 0    | 0   | 0   | 0   |
| Y | 0    | 0    | 0    | 0   | 11  | 0   |
| X | 0    | 0    | 0    | 0   | 0   | 0   |

\[ \text{BETA} \]
\[ X \]

\[ \text{-----} \]
|   | SA   | SB   | SC   | M   | Y   | X   |
|---|------|------|------|-----|-----|-----|
| SA| 0    | 0    | 0    | 0   | 0   | 0   |
| SB| 0    | 0    | 0    | 0   | 0   | 0   |
| SC| 0    | 0    | 0    | 0   | 0   | 0   |
| Y | 10   | 12   | 0    |     |     |     |
| X | 0    | 0    | 0    | 0   | 0   | 0   |

\[ \text{PSI} \]
\[ \text{SA} \quad \text{SB} \quad \text{SC} \quad \text{M} \quad \text{Y} \]
OMITTED CONFOUNDER BIAS

---

---

SA 13
SB 14 15
SC 16 17 18
M 0 0 0 19
Y 0 0 0 0
X 0 0 0 0

---

PSI
X

---

X 21

PARAMETER SPECIFICATION FOR THE ADDITIONAL PARAMETERS

NEW/ADDITIONAL PARAMETERS

AB BB CPB INDB

INDW

---

---

1 22 23 24 25

26
NEW/ADDITIONAL PARAMETERS

INDWWRON

--------

1 27

STARTING VALUES FOR WITHIN

| NU | M    | Y    | X    |
|----|------|------|------|
|    | 0.000| 0.000| 0.000|

| LAMBDA | SA | SB | SC | M |
|--------|----|----|----|---|
| Y      |    |    |    |   |
|--------|----|----|----|---|
|        |    |    |    |   |
|--------|----|----|----|---|
|        |    |    |    |   |

| M | 0.000 | 0.000 | 0.000 | 1.000 |
|   | 0.000 |
| Y | 0.000 | 0.000 | 0.000 | 0.000 |
|   | 1.000 |
| X | 0.000 | 0.000 | 0.000 | 0.000 |
|   | 0.000 |
### LAMBDA

|   |   |   |
|---|---|---|
| M | 0.000 |
| Y | 0.000 |
| X | 1.000 |

### THETA

|   |   |   |
|---|---|---|
| M | 0.000 |
| Y | 0.000 | 0.000 |
| X | 0.000 | 0.000 | 0.000 |

### ALPHA

|   |   |   |   |   |
|---|---|---|---|---|
| Y |   |   |   |   |
| X |   |   |   |   |
|   |   |   |   |   |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 |

0.000

|   |   |
|---|---|
| X |   |
|   | 1 |
|   | 0.000 |
|     | SA  | SB  | SC  | M   |
|-----|-----|-----|-----|-----|
| Y   | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |

|     | SA  | SB  | SC  | M   |
|-----|-----|-----|-----|-----|
| X   | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
|     | 0.000 | 0.000 | 0.000 | 0.000 |
### PSI

|     | SA       | SB       | SC       | M       |
|-----|----------|----------|----------|---------|
| Y   | ---------| ---------| ---------|---------|

|     | SA       | SB       | SC       | M       |
|-----|----------|----------|----------|---------|
| Y   | 0.000    | 0.000    | 0.000    | 0.500   |
| X   | 0.000    | 0.000    | 0.000    | 0.000   |

### STARTING VALUES FOR BETWEEN

|     | M       | Y       | X       |
|-----|---------|---------|---------|
| X   | 0.500   |         |         |
|   | SA | SB | SC | M  |
|---|----|----|----|----|
| 1 | 0.000 | 0.000 | 0.000 |

**LAMBDA**

|   |   |   |   |   |
|---|---|---|---|---|
| M | 0.000 | 0.000 | 0.000 | 1.000 |
| Y | 0.000 | 0.000 | 0.000 | 0.000 |
| X | 0.000 | 0.000 | 0.000 | 0.000 |

**LAMBDA**

|   |   |
|---|---|
| M | 0.000 |
| Y | 0.000 |
| X | 1.000 |

**THETA**

|   | M | Y | X |
|---|---|---|---|
| M | 0.000 |
| Y | 0.000 | 0.000 |
|---|-------|-------|
| X | 0.000 | 0.000 | 0.000 |

**ALPHA**

| SA | SB | SC | M |
|----|----|----|---|
| Y  |     |    |   |
| 1  | 0.000 | 0.000 | 0.000 | 0.000 |

0.000

**ALPHA**

| X  |     |     |     |
|----|-----|-----|-----|
| 1  | 0.000 |

**BETA**

| SA | SB | SC | M |
|----|----|----|---|
| Y  |     |    |   |
| 1  | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 |
| 0.000 |
| 0.000 |
| 0.000 |
| 0.000 |
|       | A    | B    | C    | D    | E    |
|-------|------|------|------|------|------|
|       | 0.000| 0.000| 0.000| 0.000| 0.000|
| M     | 0.000| 0.000| 0.000| 0.000| 0.000|
|       | 0.000| 0.000| 0.000| 0.000| 0.000|
| Y     | 0.000| 0.000| 0.000| 0.000| 0.000|
|       | 0.000| 0.000| 0.000| 0.000| 0.000|
| X     | 0.000| 0.000| 0.000| 0.000| 0.000|

**BETA**

|       | A    | B    | C    | D    | E    |
|-------|------|------|------|------|------|
|       | 0.000| 0.000| 0.000| 0.000| 0.000|
| SA    | 0.000|      |      |      |      |
| SB    | 0.000|      |      |      |      |
| SC    | 0.000|      |      |      |      |
| M     | 0.000|      |      |      |      |
| Y     | 0.000|      |      |      |      |
| X     | 0.000|      |      |      |      |

**PSI**

|       | A    | B    | C    | D    | E    |
|-------|------|------|------|------|------|
|       |      |      |      |      |      |
| SA    | 1.000|      |      |      |      |
| SB    | 0.000| 1.000|      |      |      |
| SC    | 0.000| 0.000| 1.000|      |      |
| M     | 0.000| 0.000| 0.000| 0.500|      |
OMITTED CONFOUNDER BIAS

\[
\begin{aligned}
Y & \quad 0.000 & 0.000 & 0.000 & 0.000 \\
& \quad 0.500 \\
X & \quad 0.000 & 0.000 & 0.000 & 0.000 \\
& \quad 0.000 \\
\hline \\
\text{PSI} & \ & \ & \ & \\
\hline \\
\text{X} & \ & \ & \ & \\
\hline \\
\text{X} & \ & \ & \ & 0.500 \\
\hline \\
\end{aligned}
\]

STARTING VALUES FOR THE ADDITIONAL PARAMETERS

NEW/ADDITIONAL PARAMETERS

\[
\begin{aligned}
\text{AB} & \quad 0.500 & 0.500 & 0.500 & 0.500 \\
\text{BB} & \quad 0.500 & 0.500 & 0.500 & 0.500 \\
\text{CPB} & \quad 0.500 & 0.500 & 0.500 & 0.500 \\
\text{INDB} & \quad 0.500 & 0.500 & 0.500 & 0.500 \\
\text{INDW} & \quad 0.500 & 0.500 & 0.500 & 0.500 \\
\end{aligned}
\]

NEW/ADDITIONAL PARAMETERS

\[
\begin{aligned}
\text{INDWWRON} & \quad 0.500 \\
\end{aligned}
\]
TECHNICAL 8 OUTPUT

| E STEP | ITER | LOGLIKELIHOOD | ABS CHANGE | REL CHANGE | ALGORITHM |
|--------|------|-------------|------------|------------|-----------|
| 1      |      | -0.36926775D+05 | 0.00000000 | 0.00000000 | EM        |
| 2      |      | -0.29361604D+05 | 7565.1706780 | 0.2048695 | EM        |
| 3      |      | -0.29162226D+05 | 199.3780926  | 0.0067904 | EM        |
| 4      |      | -0.29160301D+05 | 1.9252962   | 0.0000660 | EM        |
| 5      |      | -0.29160280D+05 | 0.0209272   | 0.0000007 | EM        |
| 6      |      | -0.29160280D+05 | 0.0002962   | 0.0000000 | EM        |

TECHNICAL 8 OUTPUT FOR THE H1 MODEL

| E STEP | ITER | LOGLIKELIHOOD | ABS CHANGE | REL CHANGE | ALGORITHM |
|--------|------|-------------|------------|------------|-----------|
| 1      |      | -0.39133672D+05 | 0.00000000 | 0.00000000 | EM        |
| 2      |      | -0.39127270D+05 | 6.4021543  | 0.0001636 | EM        |
| 3      |      | -0.39127242D+05 | 0.0278645  | 0.0000007 | EM        |
| 4      |      | -0.39127241D+05 | 0.0007304  | 0.0000000 | EM        |
| 5      |      | -0.39127241D+05 | 0.0000229  | 0.0000000 | EM        |

SAVEDATA INFORMATION

Sample/H1/Pooled-Within Matrix
OMITTED CONFOUNDER BIAS

Save file
spw.txt
Save type COVARIANCE
Save format Free

Estimated Sigma Between Matrix

Save file
estsigb.txt
Save type COVARIANCE
Save format Free

Beginning Time: 12:21:10
Ending Time: 12:21:10
Elapsed Time: 00:00:00

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