Design and analysis of a new bicontact cam mechanism

S Alaci¹, F-C Ciornei¹ and F Buium²

¹Mechanics and Technologies Department, „Stefan cel Mare” University of Suceava, Suceava, Romania
² „Mechanical Engineering, Mechatronics and Robotics Department, “Gheorghe Asachi” Technical University of Iasi, Iasi, Romania

E-mail: stelian.alaci@usm.ro

Abstract. Based on an observation from a previous work, the authors use the degree of passive freedom of a mechanism with roller follower to propose a new type of mechanism. In the new mechanism, the roller is replaced by a jointed element that forms two contacts with the cam. The obtaining of the cam profile is the main disadvantage of the cam mechanisms but in the present work, by using a cam formed by two connected discs, it is aimed to be avoided. The method is illustrated by designing the mechanism in CAD software followed by the simulation of its movement. To optimize the movement of the final element, the equivalent mechanism with lower pairs is used which has the advantage that the lengths of the elements are constant. The kinematics of the final element is particularly sensitive to the variation of the dimensions of the mechanism, fact illustrated by an example.

1. Introduction

In applied engineering is seldom met the requirement that a stipulated motion must be ensured to a certain element component of a structure. The mechanical solutions of these problems are amid the linkages and cam mechanisms. The linkages are advantageous as robust and technologically realistic but have the main drawback the improbability of verifying the stipulated motion. Frequently the motion attained at the final element only approximates the theoretical motion of the element. Responsible of this aspect is the fact that the mechanisms has a reduced number of elements and implicitly a reduced number of parameters to be adjusted for diminishing the discrepancy between the actual motion and the theoretical motion. Obviously, with increased number of elements of the mechanism, the possibility of better approximating the theoretical motion required for the driven element also raises.

Freudenstein [2] offers a series of solutions of basic linkages as function generators. But all the disadvantages mentioned above are removed by employing cam mechanisms [3-4]. The cam mechanisms have as chief benefit the reduced number of elements (cam, follower and on occasion, roller) and the possibility of – by adequate cam design, ensuring the exact motion for the considered element [5]. New transmission solutions transmission solution between an actuating element [6-7] and a last element are permanently asked for [8] and the cam mechanisms can be regarded as such an original solution. But the main inconvenience of cam mechanisms is the technologically difficult shape of the cam, and therefore it must be obtained by CNC machining. The significance of the subject is highlighted by Flores [8] who presents the manner of synthesis of a mechanism with rotating cam and translating roller follower and also a review of the most significant papers regarding this subject.
2. Structural considerations

In a recent paper [10] it is underlined the fact that mobility of a mechanism with translating roller follower is \( M = 2 \) that is, after the cam performs the specified motion, the roller has a passive motion about its axis. The fork or snake-tongue element jointed to the ground makes two contacts with the \((\gamma)\) profile of the cam. The solution presented in figure 1 is significant only by theoretical reasons since for a conscientious analysis it is observed that during the motion of the mechanism, the point \( D \) describes a curve \((\gamma')\) in the plane of the cam, curve that can be used as profile of a new cam. Positioning the tip \( D \) of the translating follower 3 on the \((\gamma')\) profile, a new mechanism with fewer elements which ensures the same motion to the follower as the initial one is obtained.

\[
\begin{align*}
E & \quad B_1 \quad D \\
(\gamma') & \quad 2 \quad (\gamma)
\end{align*}
\]

**Figure 1.** The bicontact mechanism.

3. Proposal for the new mechanism

The structural scheme of the new mechanism is based on the mechanisms from figure 1.

\[
\begin{align*}
E & \quad B_1 \quad D \\
(\gamma') & \quad 2 \quad (\gamma)
\end{align*}
\]

**Figure 2.** The new proposed mechanism.
The novelty in this case consists in the fact that the two contacts $B_1$ and $B_2$ are formed in two different parallel planes. The driving element of the mechanism is made by two cams bonded to each other. The imagined mechanism is presented in figure 2, modelled using a CAD software.

The two cams are two discs assembled eccentrically and with phase shift, on the input shaft. For a set of chosen parameters, the kinematics simulation for the velocity of the follower is presented in figure 3 which attests the functionality of the mechanism.

4. Establishing the law of motion of the mechanism

The proposed mechanism covers a number of parameters large enough to be modified in order to obtain the set of values which best approximate the law of motion. These parameters are: the radius of the discs $AB_1 = R_1$, $AB_2 = R_2$; the assemblage eccentricities $AC_1 = \delta_1$, $AC_2 = \delta_2$; the angular phase shift $C_1AC_2 = \alpha$; the radius of the bolts $r_1$, $r_2$; the length of intermediate elements $B_1D = l_1'$, $B_2D = l_2''$; the angle between the intermediate elements $B_1DB_2 = \gamma$; the eccentricity of the follower $ex$ defined as the oriented distance from the origin $A$ to the axis of motion of the final element. For optimization purpose, taking into account the circular profiles of the two cams, that is constant radius, the equivalent mechanism form figure 4 is convenient.

The equivalent mechanism form figure 4 is a mechanism of family 3 for which the analytical kinematical analysis is difficult to carry out. The vector contour method [11] is applied for completing the positional analysis of the mechanism. To this end, for identifying the independent contours, the graph of the mechanism, presented in figure 5, is utilized. By studying the graph of the mechanism, figure 5, two independent loops are found in the structure of the mechanism, that is: $0-1-2-4-5-0$ and $0-1-3-4-5-0$. The closing equations of the two contours are:

\[
\begin{align*}
AC_1 + C_1B_1 + B_1D + DA &= 0 \quad (1) \\
AC_2 + C_2B_2 + B_2D + DA &= 0 \quad (2)
\end{align*}
\]

By considering that the angle of the driving element $\varphi_1$ is the angle between the angle bisector of the $C_1AC_2$ angle with the positive half axis $Ox$, the projection equation of the two contours are:
\[
\begin{align*}
\delta_1 \cos (\varphi_1 - \alpha / 2) + l_2 \cos \varphi_2 + l'_4 \cos \varphi'_4 - x_D &= 0 \\
\delta_1 \sin (\varphi_1 - \alpha / 2) + l_2 \sin \varphi_2 + l'_4 \sin \varphi'_4 - y_D &= 0
\end{align*}
\] (3)

and

\[
\begin{align*}
\delta_2 \cos (\varphi_1 + \alpha / 2) + l_3 \cos \varphi_3 + l''_4 \cos \varphi''_4 - x_D &= 0 \\
\delta_2 \sin (\varphi_1 + \alpha / 2) + l_3 \sin \varphi_3 + l''_4 \sin \varphi''_4 - y_D &= 0
\end{align*}
\] (4)

Figure 4. The equivalent mechanism.

Figure 5. The graph of the mechanism.

In the relations (3) and (4), \(l_2\) and \(l_3\) represent the lengths of the replacing elements for cam-follower higher pairs and \(\varphi_2, \varphi_3\) are the angles made by these with the positive half axis \(Ox\). In a
similar manner the angles $\phi'_4$ and $\phi''_4$ formed by the vectors $\overline{B_1D}$ and $\overline{B_2D}$ with the positive half axis $Ox$ are defined. The unknowns from the systems of equations (3) and (4) are the angles $\phi_2, \phi_3, \phi'_4, \phi''_4$ and the ordinate $y_D$ of the point $D$. Since five unknowns are identified and four equations are available, a supplementary equation is needed. This equation representing a relationship between the angles $\phi'_4$ and $\phi''_4$ is obtained based on the representation form figure 6.

![Figure 6. The relationship between the angles $\phi'_4$ and $\phi''_4$.](image)

The equation (5) can be rewritten under the firm:

$$\phi'_4 - \phi''_4 = \delta$$  

(5)

From the two members of the equation (6), the common value is denoted $\phi_4$, and the angles $\phi'_4, \phi''_4$ can now be expressed as:

$$\phi'_4 = \phi_4 + \delta / 2$$

$$\phi''_4 = \phi_4 - \delta / 2$$  

(7)

And thus the symmetry is kept for the systems of equations (3) and (4).

$$\begin{align*}
\delta_1 \cos(\phi_1 - \alpha / 2) + l_1 \cos\phi_2 + l'_4 \cos(\phi_4 + \delta / 2) - x_D &= 0 \\
\delta_1 \sin(\phi_1 - \alpha / 2) + l_1 \sin\phi_2 + l'_4 \sin(\phi_4 + \delta / 2) - y_D &= 0
\end{align*}$$  

(8)

$$\begin{align*}
\delta_2 \cos(\phi_1 + \alpha / 2) + l_1 \cos\phi_3 + l''_4 \cos(\phi_4 - \delta / 2) - x_D &= 0 \\
\delta_2 \sin(\phi_1 + \alpha / 2) + l_1 \sin\phi_3 + l''_4 \sin(\phi_4 - \delta / 2) - y_D &= 0
\end{align*}$$  

(9)

The following operations are performed upon the systems of equations (8) and (9): the terms containing trigonometric functions of the angles $\phi_2$ and $\phi_3$ are brought in the right members; the equations are raised to power 2 and after that summing member by member, the unknowns $\phi_2$ and $\phi_3$ respectively are eliminated.

Subsequently, the system results:
At this stage, the equations of the system (10) are subtracted one of each other and a linear equation of is obtained, the unknown $y_D$ depending on the angles $\phi_1$ and $\phi_4$.

The equation is solved and then the obtained expression is replaced into one of the equations (10), resulting a trigonometric equation of unknown $\phi_4$. The form of this equation is intricate and a numerical method should be applied for solving it. Obviously, the angle $\phi_4$ will depend on $\phi_1$, the angle of position of the driving element.

Finally, the vertical displacement of the last element as a function of the angle of position of the driving element is achieved.

$$y_D = y_D(\phi_1, p_1, p_2, \ldots, p_k)$$

where by $p_k$ were denoted the constructive parameters of the mechanism.

5. Optimization of the constructive parameters of the mechanism

It is considered that the imposed law of motion of the driving element is stipulated under the form of table dependency:

$$[Y_{Dk}, \varphi_{1k}]$$

The parameters $p_1, \ldots, p_k$ must be found by imposing to the function:

$$\Phi(p_1, p_2, \ldots, p_k) = \sum_k [y_D(\varphi_{1k}, p_1, \ldots, p_k) - Y_{Dk}]^2$$

the condition to reach the minimum. This condition is equivalent to the system:

$$\frac{\partial \Phi(p_1, p_2, \ldots, p_k)}{\partial p_j} = 0, j = 1, \ldots, k$$

The concrete form of the system is complicated due to the fact that the equations are transcendental. To solve the system (14), a numerical procedure is requested which, at its turn, needs, for process initialization, a set of values of the constructive parameters. Furthermore, since the system is formed by transcendental equations, it is likely to present several solutions and thus, to guess the set of values for the initialization of the numerical procedure is not an easy task.

A solution for surpassing this difficulty is the parametric modelling of the mechanism by means of specialised software. For the case of planar mechanisms the SAM software is particularly useful. The SAM software is straightforward to apply but another gain is the possibility offered of continuous change of the dimensions of the analyzed mechanism with simultaneous response of the effects of these changes upon the envisaged parameters.

The modelling of the mechanism from figure 4 is presented in figure 7 together to the displacement of the last element. To be underlined that a insignificant change of the dimensions may lead to major effects in the kinematical behaviour of the mechanism. In figure 7 is proved that, for a small alteration of the position of the node 5, the existence condition of the crank isn’t satisfied any more.
Figure 7. Kinematical analysis performed using SAM software.
6. Conclusions
The paper has as starting point the remark that the structural analysis of a mechanism with cam and translating roller follower reveals the presence of a passive degree of freedom, given by the rotation motion of the roller about its axis. The obstruction of this motion does not modify the motion of the follower but only changes the rolling friction between the roller and the cam into sliding friction.

A previous work proposed a new mechanism where the roller was replaced by an intermediate element which made two contacts with the cam; in the present paper, the proposed structural solution is a mechanism where the intermediary element accomplishes two contacts but with different cams, bonded to each other and positioned into different planes.

The mechanism is modelled and animated using CAD software, for the particular case when the two cams are two circular discs eccentri cally assembled. After that, the number of independent cycles is determined, the unknown kinematical parameters are identified and the projection equations of closing equations of independent vector contours are written.

The particularities of the solving manner are presented for the obtained system of equations. The foremost problem is to find the constructive parameters that allow for accomplishing the imposed law of motion of the follower and for this the manner of employing modelling software is presented. The applied software permits finding a set of preliminary values of the parameters used in the initialization of numerical optimization procedure.

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