Decay Constants of Heavy Vector Mesons in Relativistic Bethe-Salpeter Method

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ABSTRACT

In a previous letter, we computed the decay constants of heavy pseudoscalar mesons in the framework of relativistic (instantaneous) Bethe-Salpeter method (full $0^-$ Salpeter equation), in this letter, we solve the full $1^-$ Salpeter equation and compute the leptonic decay constants of heavy-heavy and heavy-light vector mesons. The theoretical estimate of mass spectra of these heavy-heavy and heavy-light vector mesons are also presented. Our results for the decay constants and mass spectra include the complete relativistic contributions. We find

\[ F_{D^*_s} \approx 375 \pm 24, \quad F_{D^*_s} \approx 340 \pm 23 (D^{*0}, D^{*\pm}), \quad F_{B^*_s} \approx 272 \pm 20, \quad F_{B^*_s} \approx 238 \pm 18 (B^{*0}, B^{*\pm}), \]

\[ F_{B_{cs}} \approx 418 \pm 24, \quad F_{J/\Psi} \approx 459 \pm 28, \quad F_{\Psi(2S)} \approx 364 \pm 24, \quad F_{\Upsilon} \approx 498 \pm 20 \quad \text{and} \]

\[ F_{\Upsilon(2S)} \approx 366 \pm 27 \text{ MeV}. \]
1 Introduction

The decay constants of mesons are very important quantities [1, 2, 3, 4] and the study of them has become an interesting topic in recent years [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. These constants play important roles in many aspects, such as in the determination of Cabibbo-Kobayashi-Maskawa matrix elements, in the leptonic or nonleptonic weak decays of mesons, in the neutral $D - \bar{D}$ or $B - \bar{B}$ mixing process, etc.

In a previous letter [19], the decay constants of heavy-heavy and heavy-light pseudoscalar mesons are calculated in the framework of relativistic instantaneous Bethe-Salpeter method [20] (also called Salpeter method [21]), good agreement of our predictions with recent lattice, QCD sum rule, other relativistic model calculations as well as available experimental data is found.

In this letter, we extend our previous analysis to include vector mesons, present the relativistic calculation of heavy-heavy and heavy-light vector decay constants in the framework of full Salpeter equation. The instantaneous Bethe-Salpeter equation, which is also called full Salpeter equation, is a relativistic equation describing a bound state. Since this method has a very solid basis in quantum field theory, it is very good in describing a bound state which is a relativistic system. In a earlier paper [22], we solved the full $0^-$ Salpeter equation of pseudoscalar mesons; in another paper [23], we solved the full $1^{--}$ Salpeter equation of equal-mass vector mesons. The predictions of these relativistic methods agree very well with other theoretical calculation and recent experimental data. In this letter, we extend the analysis of equal-mass vector system to non-equal mass system by solving the full $1^-$ Salpeter equation of vector mesons, and use this method to predict the values of decay constants for heavy-heavy and heavy-light vector mesons. There are some input parameters in our method, we need to fix them when we solve the full $1^-$ Salpeter equation. In our calculation, we fix the input parameters by fitting the experimental data of mass spectra, so we also present the mass spectra for heavy vector mesons.

This letter is organized as following, in section 2, we introduce the relativistic Bethe-Salpeter equation and Salpeter equation. In section 3, we give the formula of relativistic wave function and decay constants of vector meson. We solve the full Salpeter equation, obtain the mass spectra and wave function of vector mesons. Finally, we use these relativistic wave function to calculate the decay constants of heavy vector mesons and show the numerical results and conclusion in section 4.
2 Instantaneous Bethe-Salpeter Equation

In this section, we briefly review the instantaneous Bethe-Salpeter equation and introduce our notations, interested reader can find the details in Ref. [22].

The Bethe-Salpeter (BS) equation is read as [20]:

\[
(\not{p}_1 - m_1) \chi(q)(\not{p}_2 + m_2) = i \int \frac{d^4k}{(2\pi)^4} V(P, k, q) \chi(k),
\]

where \(\chi(q)\) is the BS wave function with the total momentum \(P\) and relative momentum \(q\) of the bound state, and \(V(P, k, q)\) is the kernel between the quarks in the bound state. \(p_1, p_2\) are the momenta of the constituent quark 1 (heavy or light) and anti-quark 2 (heavy or light), respectively. The total momentum \(P\) and the relative momentum \(q\) are defined as:

\[
p_1 = \alpha_1 P + q, \quad \alpha_1 = \frac{m_1}{m_1 + m_2},
\]

\[
p_2 = \alpha_2 P - q, \quad \alpha_2 = \frac{m_2}{m_1 + m_2}.
\]

The BS wave function \(\chi(q)\) satisfy the following normalization condition:

\[
\int \frac{d^4k d^4q}{(2\pi)^4} Tr \left[ \chi(k) \frac{\partial}{\partial P_0} \left[ S_1^{-1}(p_1) S_2^{-1}(p_2) \delta^4(k - q) + V(P, k, q) \right] \chi(q) \right] = 2iP_0,
\]

where \(S_1(p_1)\) and \(S_2(p_2)\) are the propagators of the two constituents. In many applications, the kernel of BS equation is "instantaneous", i.e., in the center mass frame of the concerned bound state \((P = 0)\), the kernel \(V(P, k, q)\) takes the simple form:

\[
V(P, k, q) \Rightarrow V(k, q) = V(|\vec{k}|, |\vec{q}|, \cos \theta),
\]

where \(\theta\) is the angle between the vectors \(\vec{k}\) and \(\vec{q}\). Then the BS equation reduced to the Salpeter equation.

It is convenient to divide the relative momentum \(q\) into two parts, \(q_\parallel\) and \(q_\perp\), a parallel part and an orthogonal one to the total momentum of the bound state, respectively.

\[
q_\mu = q_\parallel^\mu + q_\perp^\mu,
\]

\[
q_\parallel = (P \cdot q/M^2) P^\mu, \quad q_\perp^\mu = q^\mu - q_\parallel^\mu.
\]

Correspondingly, we have two Lorentz invariant variables:
\[ q_p = \frac{(P - q)}{M}, \quad q_T = \sqrt{q_p^2 - q^2} = \sqrt{-q_T^2}. \]

In the center of mass frame \( \vec{P} = 0 \), they turn out to the usual component \( q_0 \) and \( | \vec{q} | \), respectively. Now the volume element of the relative momentum \( k \) can be written in an invariant form:

\[ d^4k = dk_p k_T^2 dk_T ds d\phi, \tag{3} \]

where \( \phi \) is the azimuthal angle, \( s = (k_p q_p - k \cdot q) / (k_T q_T) \). The instantaneous interaction kernel can be rewritten as:

\[ V(| \vec{k} - \vec{q} |) = V(k_\perp, q_\perp, s). \tag{4} \]

Let us introduce the notations \( \varphi_p(q_\perp^\mu) \) and \( \eta(q_\perp^\mu) \) for three dimensional wave function as follows:

\[ \varphi_p(q_\perp^\mu) \equiv i \int \frac{dq_\parallel}{2\pi} \chi(q_\parallel^\mu, q_\perp^\mu), \]
\[ \eta(q_\perp^\mu) \equiv \int \frac{k_T^2 dk_T ds}{(2\pi)^2} V(k_\perp, q_\perp, s) \varphi_p(k_\perp^\mu). \tag{5} \]

Then the BS equation can be rewritten as:

\[ \chi(q_\parallel, q_\perp) = S_1(p_1) \eta(q_\perp) S_2(p_2). \tag{6} \]

The propagators of the two constituents can be decomposed as:

\[ S_i(p_i) = \frac{\Lambda^+_i(q_\perp)}{J(i)q_p + \alpha_i M - \omega_i + i\epsilon} + \frac{\Lambda^-_i(q_\perp)}{J(i)q_p + \alpha_i M + \omega_i - i\epsilon}, \tag{7} \]

with

\[ \omega_i = \sqrt{m_i^2 + q_T^2}, \quad \Lambda^\pm_i(q_\perp) = \frac{1}{2\omega_i} \left[ \frac{P}{M} \omega_i \pm J(i)(m_i + q_\perp) \right], \tag{8} \]

where \( i = 1, 2 \) for quark and anti-quark, respectively, and \( J(i) = (-1)^{i+1} \). Here \( \Lambda^\pm_i(q_\perp) \) satisfy the relations:

\[ \Lambda^+_i(q_\perp) + \Lambda^-_i(q_\perp) = \frac{P}{M}, \quad \Lambda^\pm_i(q_\perp) \frac{P}{M} \Lambda^\pm_i(q_\perp) = \Lambda^\pm_i(q_\perp), \quad \Lambda^\pm_i(q_\perp) \frac{P}{M} \Lambda^\mp_i(q_\perp) = 0. \tag{9} \]

Due to these equations, \( \Lambda^\pm \) may be considered as \( P- \)projection operators, since in the rest frame \( \vec{P} = 0 \) they turn to the energy projection operator.

Introducing the notations \( \varphi^{\pm \pm}_p(q_\perp) \) as:

\[ \varphi^{\pm \pm}_p(q_\perp) = \Lambda^\pm_i(q_\perp) \frac{P}{M} \varphi_p(q_\perp) \frac{P}{M} \Lambda^\pm_i(q_\perp), \tag{10} \]
and taking into account with $\frac{P_q}{M} = 1$, we have
\[ \varphi_p(q) = \varphi^{++}_p(q) + \varphi^{+-}_p(q) + \varphi^{-+}_p(q) + \varphi^{--}_p(q) \]

With contour integration over $q_p$ on both sides of Eq.(6), we obtain:
\[ \varphi_p(q) = \frac{\Lambda^+_p(q) \gamma_p(q) \Lambda^+_p(q)}{(M - \omega_1 - \omega_2)} - \frac{\Lambda^-_p(q) \gamma_p(q) \Lambda^-_p(q)}{(M + \omega_1 + \omega_2)} \]
and we may decompose it further into four equations as follows:
\[ (M - \omega_1 - \omega_2) \varphi^{++}_p(q) = \Lambda^+_p(q) \gamma_p(q) \Lambda^+_p(q) \]
\[ (M + \omega_1 + \omega_2) \varphi^{-+}_p(q) = -\Lambda^-_p(q) \gamma_p(q) \Lambda^-_p(q) \]
\[ \varphi^{+-}_p(q) = \varphi^{-+}_p(q) = 0 . \quad (11) \]

The complete normalization condition (keep all of the four components appearing in Eq.(11)) for BS equation turns out to be:
\[ \int \frac{q^2 dq}{2\pi^2} Tr \left[ \frac{P}{M} \varphi^{++} - \frac{P}{M} \varphi^{-+} - \frac{P}{M} \varphi^{-+} - \frac{P}{M} \varphi^{--} \right] = 2P_0 . \quad (12) \]

### 3 Relativistic Wave Function and Decay Constant of Vector Meson

The general form for the relativistic wave function of vector state $J^P = 1^-$ can be written as 16 terms constructed by $P$, $q$, $\epsilon$ and gamma matrix. Because of the approximation of instantaneous, the 8 terms with $P \cdot q$ become zero, so the general form for the relativistic Salpeter wave function of vector state $J^P = 1^-$ can be written as [23]:
\[ \varphi^\lambda_{-1}(q) = q_{\perp} \cdot \epsilon^\lambda_{\perp} \left[ f_1(q) + \frac{P}{M} f_2(q) + \frac{g_{\perp}}{M} f_3(q) + \frac{P g_{\perp}}{M^2} f_4(q) \right] + M \varphi^\lambda f_5(q) 
+ g^\lambda_{\perp} P f_6(q) + (\epsilon^\lambda_{\perp} - q_{\perp} \cdot \epsilon^\lambda_{\perp}) f_7(q) + \left( P \varphi^\lambda f_8(q) + P q_{\perp} \cdot \epsilon^\lambda_{\perp} f_9(q) \right) \quad (13) \]

where the $\epsilon^\lambda_{\perp}$ is the polarization vector of the vector meson. The equations
\[ \varphi^{+-}_{-1}(q) = \varphi^{-+}_{1-}(q) = 0 \quad (14) \]
give the constraints on the components of the wave function, so we have the relations
\[ f_1(q) = \frac{g_{\perp} f_3(q) + M^2 f_5(q)}{M(m_1 + m_2)q^2_{\perp}} \]
\[ f_7(q) = \frac{f_5(q) M(m_1 m_2 + \omega_1 \omega_2 + q^2_{\perp})}{(m_1 - m_2)q^2_{\perp}} \]
\[ f_2(q_\perp) = \frac{-q_\perp^2 f_4(q_\perp) + M^2 f_6(q_\perp)}{M(\omega_1 + \omega_2)q_\perp^2} \left( m_1 \omega_2 - m_2 \omega_1 \right), \quad f_8(q_\perp) = \frac{f_6(q_\perp)(m_1 \omega_2 - m_2 \omega_1)}{(\omega_1 - \omega_2)q_\perp^2}. \]

Then there are only four independent wave functions \( f_3(q_\perp), f_4(q_\perp), f_5(q_\perp) \) and \( f_6(q_\perp) \) been left in the Eq.(13). Following the Ref.[22], put Eq.(13) into Eq.(11) and take trace, we obtain four coupled integral equations, by solving them we obtain the numerical results of mass spectra and wave functions for the corresponding bound states.

In our calculation, we choose the center-of-mass system of the heavy meson, so \( q_\parallel \) and \( q_\perp \) turn out to be the usual components \((q_0, \vec{0})\) and \((0, \vec{q})\), \( \omega_1 = (m_1^2 + \vec{q}^2)^{1/2} \) and \( \omega_2 = (m_2^2 + \vec{q}^2)^{1/2} \). Wave functions \( f_3(\vec{q}), f_4(\vec{q}), f_5(\vec{q}) \) and \( f_6(\vec{q}) \) will fulfill the normalization condition:

\[
\int \frac{d\vec{q}}{(2\pi)^3} \frac{16\omega_1 \omega_2}{3} \left\{ 3f_5f_6 \frac{M^2}{m_1 \omega_2 + m_2 \omega_1} + \frac{\omega_1 \omega_2 - m_1 m_2 + \vec{q}^2}{(m_1 + m_2)(\omega_1 + \omega_2)} \left[ f_4f_5 - f_3 \left( f_4 \frac{\vec{q}^2}{M^2} + f_6 \right) \right] \right\} = 2M. \quad (15)
\]

In our model, Cornell potential, a linear scalar interaction plus a vector interaction is chosen as the instantaneous interaction kernel \( V \) [22]:

\[
V(\vec{q}) = V_s(\vec{q}) + \gamma_0 \otimes \gamma^0 V_v(\vec{q}),
\]

\[
V_s(\vec{q}) = -\frac{(\lambda + V_0) \delta^3(\vec{q})}{\pi^2 (\vec{q}^2 + \alpha^2)^2}, \quad V_v(\vec{q}) = -\frac{2}{3\pi^2} \frac{\alpha_s(\vec{q})}{(\vec{q}^2 + \alpha^2)},
\]

where the coupling constant \( \alpha_s(\vec{q}) \) is running:

\[
\alpha_s(\vec{q}) = \frac{12\pi}{27} \frac{1}{\log(a + \frac{\vec{q}^2}{\Lambda_{QCD}^2})},
\]

and the constants \( \lambda, \alpha, a, V_0 \) and \( \Lambda_{QCD} \) are the parameters that characterize the potential.

The decay constant \( F_V \) of vector meson is defined as

\[
\langle 0 | \bar{q}_1 \gamma_\mu q_2 | V, \epsilon \rangle \equiv F_V M \epsilon_\mu^V,
\]

which can be written in the language of the Salpeter wave functions as:

\[
\langle 0 | \bar{q}_1 \gamma_\mu q_2 | V, \epsilon \rangle = \sqrt{N_c} \int Tr \left[ \gamma_\mu \varphi_1^{-}(\vec{q}) \frac{d\vec{q}}{(2\pi)^3} \right] = 4M \sqrt{N_c} \epsilon_\mu^V \int \frac{d\vec{q}}{(2\pi)^3} f_5(\vec{q}).
\]

Therefore, we have

\[
F_V = 4 \sqrt{N_c} \int \frac{d\vec{q}}{(2\pi)^3} f_5(\vec{q}),
\]

(19)
Table 1: Mass spectra in unit of MeV for heavy vector meson. ‘Ex’ means the data from experiments [24]. ‘Th’ means the predictions from our theoretical estimate.

|     | $B^*_c$ | $B^*_s$ | $B^*_d$ | $D^*_s$ | $D^*_d$ |
|-----|--------|--------|--------|--------|--------|
| Ex(1S) | 5416.6 | 5325.0 | 5325.0 | 2112.1 | 2010.0 |
| Th(1S) | 6336.9 | 5416.6 | 5326.2 | 5322.9 | 2112.0 |
| Th(2S) | 6918.5 | 5957.6 | 5842.3 | 5837.7 | 2673.0 |

4 Numerical Results and Conclusion

In our method, there are some parameters that have to be fixed when performing the calculations. In Ref. [22], we fixed the values of the input parameters by fitting the mass spectra for heavy pseudoscalar mesons of $0^-$ states, and we hope to choose the same parameters for pseudoscalar and vector mesons. But we find, when we solve the full Salpeter equation Eq.(11) of heavy vector mesons with same parameter set as in Ref. [22] for pseudoscalar mesons, our predictions of mass spectra of vector mesons do not agree well with experimental data, and can not explain the vector-pseudoscalar mass splitting. We argue that we choose a very simple interaction kernel Eq.(16), while the forms of pseudoscalar and vector wave functions are very different (see Eq.(13) in this letter and Eq.(20) in Ref.[22]). The latter decrease the connection between the pseudoscalar and vector states, so we like to choose different parameters to fit experimental data and to present the mass splitting between the vector and pseudoscalar mesons. We find the following parameters can fit data very well,

\[ a = e = 2.7183, \quad \alpha = 0.06 \text{ GeV}, \quad V_0 = -0.49 \text{ GeV}, \quad \lambda = 0.21 \text{ GeV}^2, \quad \Lambda_{QCD} = 0.27 \text{ GeV} \quad \text{and} \]

\[ m_b = 5.158 \text{ GeV}, \quad m_c = 1.7551 \text{ GeV}, \quad m_s = 0.535 \text{ GeV}, m_d = 0.377 \text{ GeV}, \quad m_u = 0.371 \text{ GeV}, \quad (20) \]

and we show our predictions of mass spectra for heavy vector mesons as well as the experimental data in Table 1 and Table 2.

In table 1, we show the results for ground state (1S) and first radial excitation state (2S), one can see that our mass predictions for ground states of heavy-light mesons can fit the experimental data [24] very well. In table 2, we show the mass spectra of the first eight states for vector $cc$ system. As can be seen from the table, our mass results below $2D$ state agree well with experimental data, while the masses for $2D$ and $4S$ states are about $50 \sim 60$ MeV lower than the experimental data.
Table 2: Mass spectra in unit of MeV for $c\bar{c}$ vector system.

|   | 1S     | 2S     | 1D    | 3S    | 2D    | 4S    | 3D    | 5S    |
|---|--------|--------|-------|-------|-------|-------|-------|-------|
| Ex($c\bar{c}$) | 3096.916 | 3686.093 | 3770.0 | 4040  | 4159  | 4415  |       |       |
| Th($c\bar{c}$)  | 3096.8   | 3690.9  | 3759.8 | 4065.2| 4108.2| 4344.2| 4371.6| 4567.2|

Table 3: Mass spectra in unit of MeV for $b\bar{b}$ vector system.

|   | 1S     | 2S     | 1D    | 3S    | 2D    | 4S    | 3D    | 5S    |
|---|--------|--------|-------|-------|-------|-------|-------|-------|
| Ex($b\bar{b}$) | 9460.30 | 10023.26 | 10355.2 |       | 10580.0 |       | 10865 |       |
| Th($b\bar{b}$)  | 9460.3  | 10029  | 10130  | 10379  | 10438  | 10648  | 10690 | 10868 |

We also calculate the mass spectra for vector $b\bar{b}$ system, we find our prediction with upper parameter set Eq.(20) can not fit experimental data. The reason is due to that in $b\bar{b}$ system, there are double heavy $b$ quarks, and the flavor $N_f = 4$, so we have to choose a new set of parameters as well as smaller value of coupling constant [25]. We change the previous scale parameters to $\Lambda_{QCD} = 0.21$ GeV, $m_b = 5.1242$ GeV, and other parameters are not changed. With this set of parameters, the coupling constant at the scale of bottom quark mass is $\alpha_s(m_b) = 0.23$, and obtained the mass spectra of $b\bar{b}$ systems. We show the numerical results and experimental data in Table 3. One can see that our predictions can fit the experimental data very well, even for higher states.

By fitting the mass spectra of heavy mesons, we fixed the parameters and obtained the relativistic wave functions for heavy mesons. Put the obtained wave functions into Eq.(19), we calculated the decay constants for heavy-heavy and heavy-light vector mesons. In Table 4, we show our estimates of decay constants for heavy-light ground state (1S) and first radial excitation state (2S) as well as the $B_{c}^{*}$ vector mesons. In Table 5, we show our estimates of decay constants for $c\bar{c}$ and $b\bar{b}$ systems. We also show the theoretical uncertainties of our results for decay constants in Table 4 and Table 5. These uncertainties are obtained by varying all the input parameters simultaneously within ±10% of the central values, and taking the largest variation of the decay constant.

For comparison, in Table 6, we show our predictions for decay constants and recent theoretical predictions as obtained by other methods. For example, we show the results from Ref. [30], which is also in BS method, but they used different interaction kernel, different form of wave function, especially,
Table 4: Decay constants of heavy vector meson in unit of MeV.

|       | $F_{B^*_c}$ | $F_{B^*_s}$ | $F_{B^*_d}$ | $F_{B^*_u}$ | $F_{D^*_s}$ | $F_{D^*_d}$ | $F_{D^*_u}$ |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1S    | 418±24      | 272±20      | 239±18      | 238±18      | 375±24      | 341±23      | 339±22      |
| 2S    | 331±21      | 246±13      | 222±15      | 221±14      | 312±17      | 290±16      | 289±16      |

Table 5: Decay constants in unit of MeV for $c\bar{c}$ and $b\bar{b}$ vector systems.

|       | 1S | 2S | 1D | 3S | 2D | 4S | 3D | 5S |
|-------|----|----|----|----|----|----|----|----|
| $F_V(c\bar{c})$ | 459±28 | 364±24 | 243±17 | 319±22 | 157±11 | 288±18 | 174±12 | 265±16 |
| $F_V(b\bar{b})$ | 498±20 | 366±27 | 261±21 | 304±27 | 155±11 | 259±22 | 176±10 | 228±16 |

different reduction method from ours, they chose Thompson equation to reduce the full BS equation, while we choose instantaneous approach to reduce the full BS equation. We also show the ratios of decay constant $F_{B^*_c}/F_{B^*}$ and $F_{D^*_c}/F_{D^*}$ in Table 6. Not like the pseudoscalar case, where we find good agreement for pseudoscalar decay constants between different models, from Table 6, we find rough agreement between the values of vector decay constants estimated by different methods, this means we need more effort for the knowledge of vector decay constants.

Table 6: Decay constants and ratios of decay constants estimated by different methods. (NRQM: Nonrelativistic Constituent Quark Model, RQM: Relativistic Quark Model, QL: Quenched Lattice QCD, BS: Bethe-Salpeter method, RM: Relativistic Mock Meson Model)

|       | $F_{B^*_c}$ | $F_{B^*_s}$ | $F_{B^*_d}$ | $F_{B^*_u}$ | $F_{D^*_s}$ | $F_{D^*_d}$ | $F_{D^*_u}$ | $F_{B^*_c}/F_{B^*}$ | $F_{B^*_s}/F_{B^*}$ | $F_{B^*_d}/F_{B^*}$ | $F_{B^*_u}/F_{B^*}$ |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|---------------------|---------------------|---------------------|---------------------|
| ours  | 272±20      | 238±18      | 1.14±0.08   | 375±24      | 340±22      | 339±22      | 1.10±0.06    |                     |                     |                     |                     |
| NRQM[26] | 236±14      | 151±15      | 1.55±0.06   | 326±21      | 223±23      | 223±19      | 1.41±0.06    |                     |                     |                     |                     |
| RQM[27] | 214         | 195         | 1.1         | 335         | 315         | 1.06        |                     |                     |                     |                     |                     |
| QL[28] | 217         | 190         | 1.10(2)±2   | 254         | 234         | 1.04(1)±2   |                     |                     |                     |                     |                     |
| QL[29] | 229±20      | 196±24      | 1.17(4)±3   | 272±16      | 245±20      | 223±19      | 1.11(3)      |                     |                     |                     |                     |
| BS[30] | 164         | 194±8       | 1.16±0.09   | 298±11      | 262±10      | 228±16      | 1.14±0.09    |                     |                     |                     |                     |
| RM[31] | 225±9       | 194±8       | 1.16±0.09   | 298±11      | 262±10      | 228±16      | 1.14±0.09    |                     |                     |                     |                     |

There are other interesting quantities, such as the ratios of vector to pseudoscalar decay constant $F_V/F_P$, which are sensitive to the difference between the vector and pseudoscalar wave functions. In Table 7 we show our estimates of these ratios.

In conclusion, we calculated the decay constants of heavy vector mesons in the framework of the rela-
Table 7: Ratios of vector to pseudoscalar decay constant.

| Ratio   | Value       |
|---------|-------------|
| $\frac{F_{\Upsilon}}{F_{\eta_b}}$ | 1.30±0.24   |
| $\frac{F_{B_s^*}}{F_{B_s}}$     | 1.26±0.28   |
| $\frac{F_{B_c^*}}{F_{B_c}}$     | 1.21±0.27   |
| $\frac{F_{J/\Psi}}{F_{\eta_c}}$ | 1.57±0.23   |
| $\frac{F_{D_s^*}}{F_{D_s}}$     | 1.51±0.26   |
| $\frac{F_{D_c^*}}{F_{D_c}}$     | 1.48±0.26   |

 relativistic Salpeter method. Our relativistic estimate results are $F_{D_s^*} \approx 375 \pm 24$, $F_{D_c^*} \approx 340 \pm 23$ ($D^{*0}, D^{*\pm}$), $F_{B_s^*} \approx 272 \pm 20$, $F_{B_c^*} \approx 238 \pm 18$ ($B^{*0}, B^{*\pm}$), $F_{B_s} \approx 418 \pm 24$, $F_{J/\Psi} \approx 459 \pm 28$, $F_{\Upsilon(2S)} \approx 364 \pm 24$, $F_{\Upsilon} \approx 498 \pm 20$ and $F_{\Upsilon(2S)} \approx 366 \pm 27$ MeV.

This work was supported by the National Natural Science Foundation of China (NSFC).

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