Loss distribution approach for company operational risk analysis

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Abstract. The measurement of potential operational losses as an assessment of capital adequacy is needed by the company. One value that can be used to measure the potential loss of a company is Value at Risk (VaR). This paper aims to measure VaR based on the loss distribution using the Bayesian method where the loss frequency is assumed to be Poisson distribution and the loss severity is assumed to be the distribution Lognormal. VaR with Monte Carlo (MC) and Fast Fourier Transformation (FFT) simulations are compute using the values of the parameter of distributions that have been obtained by using the Bayesian method. In this paper, a numerical example of VaR estimation of a bank company is demonstrated to find the estimate VaR from several sources i.e. internal fraud, external fraud, business practices, employment practices, process failure, system failure and damage to assets. The results showed that the estimate VaR value equal to $8.2336044 with Monte Carlo and $8.522028 with FFT simulation at 99% confidence level. These means that the company has the opportunity to experience operational risk losses which exceeds $8.2336044 and $8.522028 in the coming year by 1%.

1. Introduction

Risks arise because of the uncertainty in the future that results in losses and even destruction. Bank also faces risks from potentially anticipated or unanticipated events that have a negative impact on income and capital. Based on Bank Indonesia Regulation Number 5/8/PBI/2003 [1], one type of risks faced by the bank is operational risk. Operational risk is the risk which among others is caused by inadequate or malfunctioning of internal processes, human error, system failure, or the presence of external problems that affect the company's operations, neglect of labour and safety regulations, failure to meet professional obligations to customers, and loss of bank assets physically due to natural disasters [2]. Most bank companies allocate capital for operational risk ranging from 15-25% [3].

Proper measurement of operational risk must be carried out in order to determine the appropriate capital allocation for operational risk and meet the needs of capital adequacy ratio. Advance Measurement Approach (AMA) is a guideline of operational risk measurement as outlined in OJK Regulation No.11 / POJK.03 / 2016 and OJK Circular No. 24 / SEOJK.03 / 2016 [4], [5]. This guideline is more sensitive to risk because it requires the calculation of operational losses that have occurred at least within a period of 3 years [6]. There are several types of approach models in the AMA method, namely Loss Distribution Approach (LDA), Bootstrapping Approach, Bayesian Approach, and Extreme Value Theory (EVT) [7], [8]. LDA use historical data of operational risk to construct loss frequency.
and loss severity distribution which later use to construct a new distribution function known as multiple distribution or compound distribution through convolution process. Since representing convolution process in closed form is not feasible, approximation or numerical techniques usually used such as Monte Carlo (MC), Fast Fourier Transformation (FFT), Panjer algorithm, and single loss approximation [9]. The discussion of the LDA method can be found in several references, see e.g. [10], [11], [12], [13], and [14]. According to Esterhuyen et al [15] dan Angela et. al. [16], LDA is the most accurate approach in measuring operational risk. The LDA model obtained is then used to calculate the maximum loss that a company can experience at a certain confidence level or known as Value at Risk (VaR).

The formal application of LDA using only historical data can result in VaR values that are too high or too low [17]. A good measure of operational risk uses several components namely historical data, current market conditions, current and planned controls in the bank. The last three components are expert opinion and are subjective. Bayesian method is statistical method that can be used to include expert opinions in data analysis [17], [18], [19]. This paper discusses the parameter estimation in the loss frequency and loss severity distribution of the LDA model for quantification of operational risk by involving historical data and expert opinion through the Bayesian method. And then use the distribution obtained to conduct Monte Carlo (MC) and Fast Fourier Transformation (FFT) simulation to calculate the VaR value. Numerical examples of the process are discussed using Vanderloo Bank data discussed as an illustration [20].

2. Loss distribution

2.1. Loss distribution

Loss distribution method for quantification of operational risk constructs compound distribution of operational loss. This method is based on modeling N frequencies of loss and severities $X_1, X_2, ..., X_N$ of operational loss events [21]. Then the loss distribution in the time period $t$ can be formulated as follows:

$$Z_t = \sum_{i=1}^{N_t} X_i$$  \hspace{1cm} (1)

where $N_t$ as the frequency of operational loss events in the time period $t$ is a discrete random variable, whereas $X_i, i = 1, 2, ..., N_t$, as an operational loss severity for each event over a period of time $t$ is a continuous random variable. Frequency $N$ and severity $X$ are assumed to be mutually independent random variables.

2.2. Frequency loss distribution

There are many types of discrete random variable distribution which can be used to explain the frequency of operational loss events. In this article, the frequency of operational loss events $N$ is assumed followed Poisson distribution with $f(N|\lambda) = \frac{\lambda^N e^{-\lambda}}{N!}, \lambda \geq 0$. If $N_1, N_2, ..., N_T$ independent and identically distributes with density, then the likelihood function is $(N|\lambda) = \prod_{i=1}^{T} e^{-\lambda} \frac{\lambda^{N_i}}{N_i!}$. Using Pure Bayesian approach, it is assumed that the prior distribution for $\lambda$ is $\Gamma(\alpha, \beta)$ with the density function equal to

$$\pi(\lambda|\alpha, \beta) = \frac{\lambda^{\alpha-1} e^{-\frac{\lambda}{\beta}}}{\Gamma(\alpha) \beta^\alpha}, \lambda > 0; \alpha > 0; \beta > 0$$  \hspace{1cm} (2)

and marginal distribution of random variable $N$ is equal to

$$l(N) = \int_0^\infty l(N|\lambda) \pi(\lambda|\alpha, \beta) \, d\lambda = \frac{\Gamma(\sum_{i=1}^{T} N_i + \alpha)}{\prod_{i=1}^{T} \Gamma(\alpha) \beta^\alpha (\alpha + \beta)} \sum_{i=1}^{T} \frac{N_i}{\alpha + \beta}$$  \hspace{1cm} (3)

Since $l(N|\lambda) \pi(\lambda|\alpha, \beta) = \frac{\lambda^{(\sum_{i=1}^{T} N_i + \alpha) - 1} e^{-\left(\frac{\lambda}{\beta}\right)}}{\prod_{i=1}^{T} \Gamma(\alpha) \beta^\alpha}$, then the posterior distribution of loss frequency $\hat{\pi}(\lambda|N)$ is as follows,
\[ \hat{f}(\lambda|N) = \frac{l(N|\lambda)\pi(\lambda|\alpha,\beta)}{l(N)} = \frac{\lambda^{(\sum_{i=1}^{T}N_{i}+\alpha)-1}e^{-\left(\frac{T+1}{\beta}\right)\lambda}}{\prod_{i=1}^{T}N_{i}!\Gamma(\alpha)\beta^{\alpha}} \times \left( \frac{\Gamma\left(\sum_{i=1}^{T}N_{i}+\alpha\right)}{\Gamma(\sum_{i=1}^{T}N_{i}+\alpha)} \right) \]

It can be seen that the posterior distribution is Gamma distribution with

\[ \alpha_1 = \sum_{i=1}^{T}N_{i} + \alpha, \beta_1 = \frac{\beta}{\beta+T+1} \]

and \( T \) equal to number of periods. Then, the Bayes estimator for \( \lambda \) is as follows,

\[ E(\lambda|N) = \alpha_1\beta_1 \]

2.3. Severity loss distribution

The severity of operational loss events \( X \) is assumed followed Lognormal distribution \( LN(\mu, \sigma^2) \) then \( Y = \ln X \) followed Normal distribution \( N(\mu, \sigma^2) \) with \( f(Y|\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(Y-\mu)^2}{2\sigma^2}} \), \( 0 < \mu < \infty ; 0 < \sigma^2 < \infty \) and the expected value of severity losses is

\[ E(Y|\mu, \sigma) = \mu \]

If \( Y_1, Y_2, \ldots, Y_T \) independent and identically distributes with density, then the likelihood function is

\[ l(Y|\mu, \sigma^2) = \prod_{i=1}^{T} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(Y_i-\mu)^2}{2\sigma^2}} \]

Using Pure Bayesian approach, it is assumed that parameter \( \sigma \) is known and the prior distribution for \( \mu \) is \( N(\mu_0, \sigma_0^2) \) with the density function equal to

\[ \pi(\mu|\mu_0, \sigma_0^2) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0^2}} \]

and marginal distribution of random variable \( Y \) is equal to

\[ l(Y) = \int_{-\infty}^{\infty} l(Y|\mu, \sigma^2)\pi(\mu|\mu_0, \sigma_0^2) \, d\mu = \frac{B\sqrt{\pi}}{\sqrt{\rho}} \]

with

\[ B = \left( \prod_{i=1}^{T} \frac{1}{\sigma_0\sqrt{2\pi}} \right) \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{1}{2\rho} \left( \frac{\sum_{i=1}^{T}Y_i}{\sigma^2} + \frac{\mu_0}{\sigma_0^2} \right)^2} \exp \left( -\frac{1}{2} \frac{\sum_{i=1}^{T}Y_i^2}{\sigma^2} + \frac{\mu_0^2}{\sigma_0^2} \right) \]

(10)

Since \( l(Y|\mu, \sigma^2)\pi(\mu|\mu_0, \sigma_0^2) \) is as follows,

\[ \hat{f}(\mu|Y) = \frac{l(Y|\mu, \sigma^2)\pi(\mu|\mu_0, \sigma_0^2)}{l(Y)} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\rho} \left( \mu - \frac{\sum_{i=1}^{T}Y_i}{\sigma^2} + \frac{\mu_0}{\sigma_0^2} \right)^2} \]

(11)

It can be seen that the posterior distribution is Normal distribution with

\[ \mu_1 = \frac{1}{\rho} \left( \frac{\sum_{i=1}^{T}Y_i}{\sigma^2} + \frac{\mu_0}{\sigma_0^2} \right) \]

and \( \sigma_1^2 = 1/\rho \) with \( \rho = \frac{\tau_0^2 + \sigma^2}{\sigma^2 \rho_0^2} \)

(12)

Then, the Bayes estimator for \( \mu \) is as follows,

\[ E(\mu|Y) = \mu_1 \]

3. The research method

This paper will discuss the estimation of compound loss distribution with Bayesian method and the simulation of VaR with Monte Carlo simulation and Fast Fourier Transformation. An example of operational loss data from a bank is used as an example of the computation [20]. VaR estimation of a bank company is demonstrated to find the estimate VaR from each sources i.e. Internal Fraud, External Fraud, Client, Product, and Business, Business Distortion and System Failure, and Execution, Delivery, and Process Management.
The Bayesian method step to find the estimate of loss frequency distribution for each risk source based on the theoretical explanation above are as follows: (1) set the highest and lowest value of $\lambda$ as an expert opinion based on historical data, current market conditions, current and planned controls in the bank. The mean of the loss frequency data can be used as the first assumption to set the value of $\lambda$, (2) run the simulation to generate 1000 random $\lambda$ values with the highest and lowest $\lambda$ values from step (1) and use it to compute $E(\lambda)$, (3) use the simulation result in step (2) to choose the value of $a$ and $b$ which satisfied the equation (14) below and $\rho = 0.667$ a recommended estimate of the probability of prior distribution [17] by trial and error, (4) run the gamma distribution simulation developed using matching a given functional form method to find $\alpha, \beta$ the parameter of prior loss frequency distribution with the information of $a, b$ from step (3) previously, (5) compute $\alpha_1, \beta_1$ the parameter of posterior loss frequency distribution where $\alpha_1 = \sum_{t=1}^{T} n_t + a, \beta_1 = \frac{\beta}{\beta + T}$ and $T$ equal to number of periods, (6) compute $\lambda = \alpha_1 \beta_1$.

$$P(a \leq \lambda \leq b) = \rho = F(b) - F(a)$$

(14)

where

$$F(y) = \int_{0}^{y} \frac{\Gamma(a+\frac{y}{\mu})}{\Gamma(a)\mu^a} dx$$

(15)

The Bayesian method step to find the estimate of loss severity distribution for each risk source based on the theoretical explanation above are as follows: (1) set the minimum and maximum severity loss of $\mu$ as an expert opinion, (2) run the simulation to generate 1000 random $\mu$ values with the highest and lowest $\mu$ values from step (1) and use it to compute $E(\mu)$ (3) use the simulation result in step (2) to choose the value of $a$ and $b$ which satisfied the equation (16) below by trial and error using information from step (2), and $\rho = 0.667$ a recommended estimate of the probability of distribution prior [17]. It is assumed that $\sigma$ is known, the standard deviation of the loss severity data can be used as the first assumption to set the value of $\sigma$, (4) run the simulation developed using matching a given functional form method to find $\mu_0, \sigma_0$ the parameter of prior loss severity distribution with the information of $a, b$ and $\rho = 0.667$ from the previous step, (5) compute $\mu_1, \sigma_1^2$ the parameter of posterior loss severity distribution as follows, $\mu_1 = \frac{1}{\rho} (\frac{\sum_{i=1}^{K} y_i}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}), \sigma_1^2 = 1/\rho$ with $\rho = \frac{k^{2} + \sigma_0^2}{\sigma_0^2 \sigma_0^2}$, (6) compute $\mu = \mu_1$.

$$P(a \leq \mu \leq b) = \rho = \Phi \left( \frac{(\ln b - \frac{1}{2}\sigma^2 - \mu_0)}{\sigma_0} \right) - \Phi \left( \frac{(\ln a - \frac{1}{2}\sigma^2 - \mu_0)}{\sigma_0} \right)$$

(16)

The simulation of VaR using Monte Carlo simulation or Fast Fourier Transformation are the following steps: (1) for parameter $\lambda$ which is obtained $E(\lambda|n)$, simulate the number of operational events $N$ from the frequency distribution $f(n|\lambda)$, (2) for parameter $\mu$ obtained at $E(\mu|y)$, simulate severity $X_i, i = 1, 2, ..., N$. From severity distribution $f(y|\mu)$, (3) determine losses in the time period $t$ at $Z_t$, (4) repeat steps 1-3 as much $K$ times to build a sample of loss distribution $Z_t$, (4) find the VaR value at the confidence level $(1 - \alpha)$ as the $(1 - \alpha)^{th}$ quartile of the sample of loss distribution $Z_t$ in step (3).

4. Research, analysis and discussions

The following Table 1 is a summary of operational loss frequency and severity data from 2009–2011. From this information, the prior and posterior distribution of loss frequency and loss distribution is computed and described in Table 2 and 3 below. While the estimate VaR value with Monte Carlo (MC) and Fast Fourier Transform (FFT) is described in Table 3.

| Statistic          | Risk Sources |
|--------------------|--------------|
| Internal fraud     | 15           |
| External fraud     | 13           |
| Business practices | 9            |
| Employment practices | 12        |
| Process failure    | 19           |
| System failure     | 21           |
| damage to assets   | 9            |

Table 1. Summary of data on yearly operational loss severity 2009-2011 (in $)
Table 2. The prior and posterior distribution of loss frequency

| Risk source          | Prior          | Posterior       |
|----------------------|----------------|-----------------|
| Internal Fraud       | $\Gamma(1.73, 0.162968)$ | $\Gamma(16.73, 0.030178)$ |
| External Fraud       | $\Gamma(1.896, 0.118746)$ | $\Gamma(14.896, 0.028232)$ |
| Business Practices   | $\Gamma(1.872, 0.090472)$ | $\Gamma(10.872, 0.026279)$ |
| Employment Practices | $\Gamma(1.896, 0.118746)$ | $\Gamma(13.896, 0.028232)$ |
| Process Failure      | $\Gamma(1.641, 0.217447)$ | $\Gamma(20.641, 0.031647)$ |
| System Failure       | $\Gamma(1.754, 0.224917)$ | $\Gamma(22.754, 0.0318)$ |
| Damage to Assets     | $\Gamma(1.896, 0.118746)$ | $\Gamma(10.896, 0.028232)$ |

Table 3. The prior and posterior distribution of loss severity

| Risk source          | Prior          | Posterior       |
|----------------------|----------------|-----------------|
| Internal Fraud       | $N(7.10775, 0.007413)$ | $N(3.736711, 0.00365)$ |
| External Fraud       | $N(7.2693, 0.024007)$ | $N(3.284559, 0.009797)$ |
| Business Practices   | $N(7.2922, 0.064135)$ | $N(2.713307, 0.019175)$ |
| Employment Practices | $N(7.1092, 0.2652812)$ | $N(2.652812, 0.010369)$ |
| Process Failure      | $N(7.5577, 0.095301)$ | $N(1.197459, 0.11064)$ |
| System Failure       | $N(6.9521, 0.043635)$ | $N(1.26644, 0.061849, 0)$ |
| Damage to Assets     | $N(7.1558, 0.049523)$ | $N(2.32707, 0.01207)$ |

Table 4. VaR value with Monte Carlo (MC)

| Risk source          | 90%         | 95%         | 99%         |
|----------------------|-------------|-------------|-------------|
| Internal Fraud       | $1,961334$  | $2,514062$  | $3,272709$  |
| External Fraud       | $0,1070221$ | $0,1415731$ | $0,2173632$ |
| Business Practices   | $0,1837849$ | $0,2455571$ | $0,4037028$ |
| Employment Practices | $0,1916615$ | $0,2544923$ | $0,3866242$ |
| Process Failure      | $1,045881$  | $1,310917$  | $1,981496$  |
| System Failure       | $0,7798336$ | $0,9795639$ | $1,433006$  |
| Damage to Assets     | $0,2650816$ | $0,3424482$ | $0,5387025$ |
| Total                | $4,534599$  | $5,788614$  | $8,233604$  |

Table 5. VaR value with Fast Fourier Transform (FFT)

| Risk source          | 90%         | 95%         | 99%         |
|----------------------|-------------|-------------|-------------|
| Internal Fraud       | $1,844581$  | $2,456247$  | $3,630029$  |
| External Fraud       | $0,1014164$ | $0,1384211$ | $0,2258336$ |
| Business Practices   | $0,1806444$ | $0,2385438$ | $0,3922412$ |
| Employment Practices | $0,184$     | $0,2372543$ | $0,3935903$ |
### Process Failure

|                | Estimate (Monte Carlo) | Estimate (FFT) |
|----------------|------------------------|----------------|
| $1,043211$     | $1,317593$             | $2,055948$     |

### System Failure

|                | Estimate (Monte Carlo) | Estimate (FFT) |
|----------------|------------------------|----------------|
| $0,8042771$    | $0,9989833$            | $1,356059$     |

### Damage to Assets

|                | Estimate (Monte Carlo) | Estimate (FFT) |
|----------------|------------------------|----------------|
| $0,2606379$    | $0,3297777$            | $0,4683265$    |

### Total

|                | Estimate (Monte Carlo) | Estimate (FFT) |
|----------------|------------------------|----------------|
| $4,418768$     | $5,71682$              | $8,522028$     |

Based on the VaR validity test, the best estimate VaR value is resulted from MC simulation at a 90%, 95%, and 99% confidence level for $4.534599, $5.788614 and $8.2336044 respectively. Whereas, the estimate VaR value resulted from FFT simulation are $4.418768, $5.71682, and $8.522028 at confidence level 90%, 95%, and 99%, and respectively.

### 5. Conclusion

The steps in quantifying operational risk using the Bayesian method are estimating the parameters of the prior distribution of the loss distribution consisting of the distribution of the loss frequency and the distribution of the loss severity by matching a given functional form. The distribution of loss frequency is Poisson ~ Gamma while the distribution of loss severity is Lognormal ~ Normal. The estimate VaR value equal to $8.2336044 with Monte Carlo and $8.522028 with FFT simulation at 99% confidence level. These means that the company has the opportunity to experience operational risk losses which exceeds $8.2336044 and $8.522028 in the coming year by 1%.

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