Short-range correlations in \((e,e'p)\) and \((e,e'pp)\) reactions on complex nuclei

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Abstract

The influence of short-range correlations (SRC) on the triple-coincidence \((e,e'pp)\) reactions is studied. The non-relativistic model uses a mean-field potential to account for the distortions that the escaping particles undergo. Apart from the SRC, that are implemented through a Jastrow ansatz with a realistic correlation function, we incorporate the contribution from pion exchange and intermediate \(\Delta_{33}\) currents. The \((e,e'pp)\) cross sections are predicted to exhibit a sizeable sensitivity to the SRC. The contribution from the two-nucleon breakup channel to the semi-exclusive \(^{12}\text{C}(e,e'p)\) cross section is calculated in the kinematics of a recent NIKHEF-K experiment. In the semi-exclusive channel, a selective sensitivity in terms of the missing energy and momentum to the SRC is found.

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The \((e,e')p\) reaction has been explored extensively to study the single-particle properties of complex nuclei. The optimum kinematical conditions for such investigations are those in which the residual \((A-1)\) nucleons are behaving as spectators in the reaction of the target nucleus with the external electromagnetic field. The opposite situation in which a breakup of the \((A-1)\) system is observed is not only a direct indication for the existence of correlations in nuclei but can be explored to improve on our understanding of those correlations. As the most important correlations in the nucleus are believed to be of two-body nature, two-nucleon knockout is expected to be a major source of multi-particle breakup.

Correlations of the short-range type have been the subject of extensive theoretical studies for many years. In reactions with external electromagnetic probes, short range correlations (SRC) are competing with correlations of longer range. Therefore, in order to reach a maximum sensitivity to the SRC, special care must be taken in the choice of the kinematics. In two-nucleon knockout reactions induced by real photons, correlations related to pion exchange were shown to play a predominant role [1]. In the non-relativistic limit, however, pion exchange is not affecting the charge operator and as such two-nucleon knockout reactions with virtual photons, which are also probing longitudinal degrees of freedom, are expected to open better perspectives to study SRC. On the other hand, it should be realized that beyond the quasi-elastic peak the longitudinal part of the inclusive \((e,e')\) cross section was experimentally verified to be small. This means that in predominantly longitudinal kinematics, \((e,e'p...)\) cross sections are anticipated to be small. Another property that can be exploited to maximize the effect of the SRC is the fact that pion exchange currents are of charge-exchange nature. Consequently, the two-proton emission channel is expected to open better perspectives in the study of SRC than proton-neutron emission.

In this paper we aim at estimating the effect of SRC for the \((e,e'pp)\) reaction on complex nuclei. Further we investigate in how far the semi-exclusive \((e,e'p)\) channel exhibits a sensitivity to the SRC and can be used to discriminate between the different prescriptions for the correlation functions. In all calculations presented here we coherently add the contribution from the short-range effects in the nuclear wave functions to the contribution from the pion exchange currents.

Using standard rules, the \((e,e'N_aN_b)\) cross section can be cast in the form :

\[
\frac{d^8\sigma}{dE_d\Omega_d d\Omega_a d\Omega_p} (e,e'N_aN_b) = \frac{1}{4(2\pi)^8} k_a k_b E_a E_b f_{rec} \sigma_M \times \left[ v_L W_L + v_T W_T + v_S W_{TT} + v_I W_{LT} \right] ,
\]

where \(\sigma_M\) is the Mott cross section, \(f_{rec}\) a recoil factor and the coefficients \(v\) contain the electron kinematics. The structure functions \(W\) are defined in terms of the transition matrix elements :

\[
m_{fi}(\lambda) = \langle \Psi_f | J_{\lambda}^{[1]} + J_{\lambda}^{[2]} | \Psi_i \rangle \quad (\lambda = 0, \pm 1) ,
\]

where the virtual photon is assumed to probe both one-body \(J_{\lambda}^{[1]}\) and two-body currents \(J_{\lambda}^{[2]}\) in the target nucleus. In the absence of correlations beyond those implemented in the mean-field, the wave functions \(\Psi\) reduce to Slater determinants and only two-body currents will produce non-zero matrix elements. Here, we consider \(A\)-body wave functions in the context of the Correlated Basis Function (CBF) theory [2,3] :

\[
\Psi = \mathcal{G} | \Psi > ,
\]
where \( \mathcal{G} \) projects on the Slater determinants the correlations that are absent in an independent-particle model (IPM) description. The dominant terms in the operator \( \mathcal{G} \) are the central, or Jastrow, component \((\prod_{i<j} f_C(r_{ij}))\) and the tensor term \((\prod_{i<j} f_{tr}(r_{ij})S_{ij}\vec{r}_i\vec{r}_j)\) \([3]\). Here, we retain only a state independent Jastrow correlation term. For an initial calculation, the tensor terms can be neglected given the smallness of the function \( f_{tr} \) relative to \( f_C \) \([3]\). For the particular case of two-nucleon breakup in the final state, the correlated initial and final wave function read then:

\[
\Psi_i = \prod_{i<j=1}^{A} f_C(r_{ij}) \Psi_i
\]

\[
\Psi_f = | \vec{k}_a \frac{1}{2}m_{sa}, \vec{k}_b \frac{1}{2}m_{sb} > \prod_{i<j=1}^{A-2} f_C(r_{ij}) | J_{RM_R} > ,
\]

where \( | J_{RM_R} > \) is the state in which the residual A-2 system is created, \( r_{ij} = | \vec{r}_i - \vec{r}_j | \) and \( | \vec{k}_a \frac{1}{2}m_{sa}, \vec{k}_b \frac{1}{2}m_{sb} > \) determines the momentum and spin orientation of the ejected nucleon pair. Inserting the above wave functions into the matrix elements \([2]\) leads to an expression which cannot be calculated exactly with the presently available computers. Various cluster expansions, however, have been developed to calculate the matrix elements to a high degree of accuracy \([5]\). Here, we rely on a technique developed by Weise, Huber and Danos \([6]\). Essentially it reduces the calculation of expectation values between correlated wave functions to evaluating matrix elements between Slater determinants with an effective operator that is constructed from the original transition operator and the correlation function \( f_C \). It can be readily verified that the matrix elements of Eq. \([2]\) can be rewritten as:

\[
\langle \vec{k}_a \frac{1}{2}m_{sa}, \vec{k}_b \frac{1}{2}m_{sb}; J_{RM_R} | \mathcal{H}^{eff}_\lambda | \Psi_i \rangle ,
\]

with

\[
\mathcal{H}^{eff}_\lambda = \prod_{i<j=1}^{A-2} \left( 1 - g^i(r_{ij}) \right) \left( \sum_{k=1}^{A} J^{[1]}_\lambda (k) + \sum_{k<l=1}^{A} J^{[2]}_\lambda (k, l) \right) \prod_{m<n=1}^{A} (1 - g(r_{mn})) .
\]

In the above equation we have introduced the correlation function \( g(r_{12}) \equiv 1 - f_C(r_{12}) \). As outlined in Ref. \([8]\), the effective transition hamiltonian \( \mathcal{H}^{eff} \) can be expanded into different orders in \( g \). For the present purposes, the expansion in \( g \) is expected to converge rapidly. First, chances are relatively small that virtual photons will interact with heavily correlated three, four, ...-nucleon clusters at normal nuclear densities. Further, two-body photoabsorption is anticipated to represent the dominant absorption mechanism in two-nucleon knockout processes, thus giving a natural constraint on the expansion into the different orders of \( g \). The convergence of the cluster expansion in the different orders of \( g(r_{12}) \) has recently been investigated for the \( ^3\text{He}(e,e'\text{d}) \) case by Leidemann et al. \([2]\). They concluded that in the \( (e,e'\text{d}) \) case a first order calculation would be sufficient to incorporate the main effects of the SRC. The deviation between the first order and full calculation was found to grow with increasing missing momentum \( p_m \) but remained reasonable over the whole \( p_m \) range.

Within the adopted assumptions the effective interaction hamiltonian \([6]\) reads:

\[
\mathcal{H}^{eff}_\lambda \approx \sum_{i=1}^{A} J^{[1]}_\lambda (i) - \sum_{i<j=1}^{A} \left( J^{[1]}_\lambda (i) + J^{[1]}_\lambda (j) \right) g(r_{ij}) + \sum_{i<j=1}^{A} J^{[2]}_\lambda (i, j) (1 - g(r_{ij})) .
\]
Strictly speaking, this effective interaction Hamiltonian also contains terms in $g^\dagger J_\lambda$ and $g^\dagger J_\lambda g$. All of those terms, however, involve final-state correlations that according to Eq. (5) refer solely to the coordinates of the A-2 spectator nucleons. As such they do not contribute to the matrix elements of Eq. (5). In deriving the expression (7) we have further neglected all three- and four-body operators. The latter require respectively three and four active nucleons in the virtual photon absorption process. We expect these mechanisms to produce small contributions in the two-nucleon breakup channel. It remains to be investigated whether these multi-body operators ($N \geq 3$) give rise to sizeable three and more nucleon knockout strength. The first term in the effective transition Hamiltonian (7) does not contribute to the two-nucleon knockout channel. The second term is a typical SRC effect and will be referred to as the “SRC current” in the remainder of this paper.

For the longitudinal one-body operator we consider the charge density (neglecting the Darwin-Foldy term) $J_0^{[1]} = \sum_{i=1}^A e_i \delta(\vec{r} - \vec{r}_i)$. The transverse one-body current $J_{\pm 1}^{[1]}$ which is part of the SRC current is constructed from the common convection and magnetization term. The two-body current $J_\lambda^{[2]}$ is determined along the lines explained in Ref. [1]. In short, we have considered all diagrams with one exchanged pion including those with an intermediate $\Delta_{33}$ excitation. In the non-relativistic limit, the transverse current operator that originates from this procedure has several components including the seagull, pion-in-flight and $\Delta$-isobar term. The correction for the finite extension of the interacting hadrons to $J_\lambda^{[2]}$ is handled in the standard manner by introducing a monopole $\pi NN$ form factor with a cutoff mass $\Lambda_\pi = 1250$ MeV/c$^2$, which is consistent with the Bonn boson exchange model for the nucleon-nucleon interaction. In the context of $(\gamma, NN)$ reactions, the correlation function $g(r_{ij})$ in the last term of the interaction Hamiltonian (7) was shown to produce an overall reduction of the two-body current contribution $J_\lambda^{[2]}$ of less than 10%.
For the calculations presented below this effect has been neglected.

In the choice of the central correlation function \( g \) we have been led by the apparent success of the recent Variational Monte Carlo (VMC) \([5,4]\) and Fermi Hypernetted Chain (FHNC) calculations \([9]\) in treating complex nuclei. In the calculations of Ref. \([9]\) a Gaussian correlation function \( g(r_{12}) = \alpha e^{-\beta r_{12}^2} \) was put forward. For \(^{16}\text{O}\) the following values for the two parameters were obtained: \( \alpha = 0.51 \) and \( \beta = 1.52 \text{ fm}^{-2} \). In Fig. we compare this FHNC result with the correlation function obtained with VMC techniques. It is noted that the two independent calculations produce central correlations that bear a strong resemblance with each other. All results presented below are obtained with the Gaussian correlation function. The VMC correlation function was checked to produce very similar results.

In the calculation of the two-nucleon knockout cross sections we do not attempt to include the full complexity of the final state interaction and adopt a direct knockout reaction model. This means that within our model assumptions photoabsorption on a correlated nucleon pair does imply that the active nucleons are ejected from the target system and become asymptotically free particles. The distorting effect of the residual A-2 system on the wave functions for the escaping particles is implemented. This is accomplished by performing a partial wave expansion for both of the escaping particles in terms of the eigenfunctions of a mean-field potential \([10]\). In this procedure, the initial and final state are guaranteed to remain orthogonal, thus avoiding spurious contributions entering the matrix elements. This is particularly of importance for the semi-exclusive calculations, where integrations over a large fraction of phase space are carried out. The main effect of the distortions on the calculated \((e,e'NN)\) cross sections is a reduction relative to the results obtained in a plane wave approach. As such the effect of the distortions is similar to what was observed for \((\gamma,NN)\) reactions \([10]\). Also in the optical potential calculations of Ref. \([11]\) the final state interaction was reported to bring about a moderate reduction of the two-nucleon knockout cross sections.

Calculated cross sections for the \(^{12}\text{C}(e,e'pp)\)

\(^{10}\text{Be}\) reaction leaving the residual nucleus in a \((1p_{3/2})^{-2}\) two-hole state are shown in Fig. 2. The presented cross sections are obtained in so-called coplanar and symmetrical kinematics \([12]\) \((\phi_p = 0^\circ, \phi_{p'} = 180^\circ \text{ and } \theta_p = \theta_{p'}\)\). The kinetic energy for one of the ejected protons was fixed such that \( T_p = T_{p'} \) for \( \theta_p = \theta_{p'} = 90^\circ \). For the curves of Fig. we have summed over the two possible values for the momentum \( J_R = 0^+ \text{ and } 2^+ \) in which the residual nucleus can be created. In the absence of correlations beyond the IPM, the sole contributing two-body current to the \((e,e'pp)\) channel is of isobaric origin. The kinematics for Fig. 2a) are taken from a recent NIKHEF-K \(^{12}\text{C}(e,e'pp)\) experiment \([14]\). The curves of Fig. 2b) are obtained in more longitudinal kinematics (the longitudinal polarization \( \epsilon_L = v_L/2v_T \) is 0.85 for kinematics b) and 0.30 for a)). Although the effect of the SRC is slightly bigger in more longitudinal kinematics, the predicted increase in the cross section is less than a factor of two in the peak of the cross section.

The \((e,e'pp)\) results presented in Refs. \([12,15]\) predict a much stronger sensitivity to the SRC for some particular choices of the correlation function. Particularly with the so-called ”omy” correlation function \([13]\) spectacular increases in the \((e,e'pp)\) cross sections were observed. For completeness the hard-core ”omy” correlation function has been added to Fig. 1. The ”omy” correlation function has been derived by minimizing the ground-state energy of \(^{16}\text{O}\) with a rather crude form of the nucleon-nucleon interaction that has a state-independent hard core radius \( c = 0.6 \text{ fm} \) \([13]\). It is obvious that in the ”omy” parametrization the central correlation corrections to the wave functions are much bigger than what modern theories predict. A similar type of trend is observed...
with respect to the effect of the SRC on the (e,e′pp) cross sections, confirming their sensitivity to the choice of the correlation function.

Although two-nucleon knockout reactions remain obvious choices to study nucleon-nucleon correlations, in what follows we illustrate that (e,e′p) studies can be used to test theories of SRC when performed under carefully selected kinematics. In particular, we will concentrate on the semi-exclusive (e,e′p) reaction performed at high missing momenta (\(\vec{p}_m = \vec{p}_p - \vec{q}\)) and energies (\(E_m = \omega - T_p - T_{A-1}\)). Results of calculations for the semi-exclusive \(^{12}\text{C}(e,e^{'p})\) reaction including short-range effects, pion-exchange and isobaric currents are shown in Fig. 3. The kinematics are from Ref. [16] representing the first semi-exclusive (e,e′p) measurement on a complex nucleus for a whole range of proton angles. The calculations have been performed along the lines explained in Refs. [1,17]. The model assumes that the semi-exclusive strength above the two-nucleon emission threshold arises from two-nucleon knockout. Consequently, the calculations involve an integration over phase space of the undetected particle (either a proton or a neutron) and a sum over all possible quantum states |\(J_R M_R\)\> of the residual nucleus. Accordingly, the semi-exclusive (e,e′p) cross sections are determined by two-nucleon knockout matrix elements of the type \(\langle \rangle\). Just as for the (e,e′NN) channel the relative importance of the different types of correlations can be investigated by retaining a selected number of terms in the interaction hamiltonian of Eq. (7). Looking at Fig. 3 it becomes clear that the SRC contribution (dotted line) represents a sizeable contribution of the measured strength at the forward proton angles. At backward proton angles, the SRC contribution becomes marginal and the calculated cross section is dominated by the pionic and isobaric degrees of freedom. As such, the data at large \(\theta_p\)’s are ideal to gauge the meson-exchange process.
$^{12}\text{C}(e,e'p) ; \varepsilon=478 \text{ MeV} ; \omega=212 \text{ MeV} ; q=270 \text{ MeV/c}$

FIG. 3. Calculated contribution from the two-nucleon breakup channel to the semi-exclusive $^{12}\text{C}(e,e'p)$ reaction for $\varepsilon=478 \text{ MeV}$, $\omega=212 \text{ MeV}$ and $q=270 \text{ MeV/c}$. For the dotted line only the SRC current is retained. The dot-dashed line represents the result when only accounting for the pion degrees of freedom (including intermediate $\Delta^3_{33}$ creation). The solid line is obtained when coherently adding the short-range and mesonic contributions. The data are from Ref. [16]. The proton angles are expressed relative to the direction of the momentum transfer $\vec{q}$. For the forward proton angles (27, 42 and 74°) the azimuthal angle $\phi_p = 180^o$, for the other angles $\phi_p = 0^o$. 

\[d^6\sigma/d\varepsilon'd\varepsilon d\Omega_p dT_p \left(10^{-9}\text{fm}^2/\text{sr}^2\text{MeV}^2\right)\]
part of the cross section. As the direction of proton detection moves closer to \( \vec{q} \), it is not but after coherently adding the short- and long-range terms in the effective interaction hamiltonian that a reasonable overall description of the data can be obtained.

It is worth investigating the driving mechanism behind the proton angle dependence in the relative importance of the different photoabsorption mechanisms. As will become clear in the course of this paragraph, a natural explanation of the qualitative features of the SRC cross sections in Fig. 3 is provided by the two-nucleon correlation model (TNC) as developed by Ciofi degli Atti et al. [18,19]. In the most naive version of the TNC model it is assumed that the short-range correlations in finite nuclei are governed by those configurations in which the high momentum of a bound nucleon \( \vec{p}_m \) is balanced by a momentum \( -\vec{p}_m \) of a correlated nucleon and that the remaining \((A-2)\) nucleons act as spectators in this process. Discarding all effects related to the FSI this picture predicts the following relation between missing energy and momentum in case of emission of a correlated nucleon [19,20]:

\[
E_m \approx S_{NN} + \langle E_x^{hh'} \rangle + \frac{(A-2)p_m^2}{2(A-1)M_N},
\]

(8)

where \( S_{NN} \) is the separation energy for two-nucleon knockout and \( \langle E_x^{hh'} \rangle \) the average excitation energy of the two-hole state \(|h^{-1}h^{-1}\rangle\) in which the A-2 system is created (in the \(^{12}\text{C}\) case reasonable values are \( \langle E_x^{(1p)^2} \rangle = 0 \text{ MeV}, \langle E_x^{(1p1s)} \rangle = 25 \text{ MeV} \) and \( \langle E_x^{(1s)^2} \rangle = 50 \text{ MeV} \)).

It is worth mentioning that the above relation (8) is based on purely kinematical arguments. When applied to the kinematics of Fig. 3 the above relation predicts a maximized likelihood to eject correlated protons for \( E_m \approx 85 \text{ MeV} (\theta_p = 27^\circ), E_m \approx 95 \text{ MeV} (\theta_p = 42^\circ), E_m \approx 130 \text{ MeV} (\theta_p = 74^\circ), E_m \approx 155 \text{ MeV} (\theta_p = 107^\circ), E_m \approx 165 \text{ MeV} (\theta_p = 131^\circ) \) and \( E_m \approx 170 \text{ MeV} (\theta_p = 162^\circ) \). This observation explains why the peaks of the dotted curves shift to higher \( E_m \) with increasing proton angle. Another striking feature of the dotted curves in Fig. 3 is that SRC effects loose in importance as the proton angle becomes larger. This can be explained by considering that for the backward proton angles very large missing momenta are probed and that the probability to find a nucleon is a steadily decreasing function with growing momentum.

From the above discussion it is clear that our dynamic and microscopic model to deal with the SRC effects seems to reproduce the trends predicted by the TNC model that is based on purely kinematical grounds.

Summarizing, we have developed a microscopic model that aims at estimating the quantitative effect of SRC on \((\text{e,e}')^p\) and \((\text{e,e}')^n\) cross sections in complex nuclei. In our approach, both the effects of the SRC in the nuclear wave functions and the pion-exchange currents in the photoabsorption mechanism are treated. The latter are shown to be highly competitive with the SRC and necessary to obtain a reasonable agreement with the semi-exclusive \(^{12}\text{C}(\text{e,e}')^p\) data of a recent NIKHEF-K experiment. Even for the \((\text{e,e}')^p\) cross sections the effect of the SRC was found to be of similar magnitude than the strength created by intermediate \(\Delta_{33}\) creation. Further, our microscopic approach confirms that semi-exclusive \((\text{e,e}')^p\) reactions can be very useful tools to test different theories of SRC, provided that they are performed under suitable kinematics.

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