Position Measurement-Induced Collapse:
A Unified Quantum Description of Fraunhofer
and Fresnel Diffractions

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Abstract

Position measurement-induced collapse states are shown to provide a unified quantum description of diffraction of particles passing through a single slit. These states, which we here call ‘quantum location states’, are represented by the conventional rectangular wave function at the initial moment of position measurement. We expand this state in terms of the position eigenstates, which in turn can be represented as a linear combination of energy eigenfunctions of the problem, using the closure property. The time-evolution of the location states in the case of free particles is shown to have position probability density patterns closely resembling diffraction patterns in the Fresnel region for small times and the same in Fraunhofer region for large times. Using the quantum trajectory representations in the de Broglie-Bohm, modified de Broglie-Bohm and Floyd-Faraggi-Matone formalisms, we show that Fresnel and Fraunhofer diffractions can be described using a single expression. We also discuss how to obtain the probability density of location states for the case of particles moving in a general potential, detected at some arbitrary point. In the case of the harmonic oscillator potential, we find that they have oscillatory properties similar to that of coherent states.

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1 Introduction

An early attempt to describe the Fraunhofer diffraction in optics using quantum theory was made by Epstein and Ehrenfest [1], by combining the concept of a light quantum with Bohr’s correspondence principle. However, a consistent quantum treatment of the diffraction of particles passing through a single slit is not yet at hand and it is pointed out (see, for example, [2]) that the usual textbook discussions of such phenomena are fraught with ambiguities and omissions. This observation is based mainly on the fact that such treatises directly use the classical wave optics itself, instead of making a quantum mechanical approach, to explain diffraction, interference etc. This fundamental issue is addressed in some recent works [3, 4, 5], and the authors have proceeded further to explore the possibility that the diffraction through a single slit can be regarded as a prototype experiment to all position measurements. That is, they considered the slit in the diaphragm as a device for measuring the position of the incident particle, hoping that such an analysis shall be helpful for a general understanding of the quantum theory of position measurement. It is also noted that in the literature, the only calculations which treat diffraction through single slit as ‘position measurement’ are those cited above.

Among these works, the earliest one due to Marcella [3] proposes an expression to calculate the intensity of the diffracted beam using a reduced state ket of the particle. The collapse of state due to the process of measurement leads to this wave function, which is equivalently the projection of an initial state ket into the final one. The Fourier transform of the resulting function, which is simply the momentum space wave function in the same case, is squared to give an expression that is comparable with the standard result in Fraunhofer diffraction. Rothman and Boughn [4] have shown that this procedure in [3] is not compatible with standard quantum mechanics. Recently, Fabbro [5] has come up with an extension of the formalism in [3], in an attempt to rectify its drawbacks. Here it is inferred that these drawbacks come from the way in which the wave function reduction has been applied in it. The model in [5] claims to predict the intensity of diffracted wave at large angles, focusing on the case of diffraction at infinity (Fraunhofer diffraction) only.

In the present work, we intend to study this issue and to suggest an alternative approach to that in [3]. In the first part, the general case of performing a measurement of position on a particle in one dimension is taken up. Initially, we consider a free particle and later a particle under an arbitrary potential. When a
position measurement takes place at a point, it is assumed to lead to a collapse of the original wave function. Following the conventional approach in orthogonal measurements [6, 7], we assume the collapse to be in such a way that the resulting function is a nonzero constant inside the region of a one-dimensional box centred about the point and zero outside it. Though this collapsed state is not a position eigenstate, we can consider it as a superposition of position eigenstates whose eigenvalues lie inside the box. Here use is made of the fact that the position eigenstates have the general form of the Dirac delta function. The crucial steps to follow are the expansion of the above collapsed wave function by substituting the delta function with the left hand side of the closure relation for energy eigenstates in this case, and the introduction of the unitary time evolution of the collapsed state. The resulting states fall into a category of its own, which we here call ‘position measurement-induced collapse state’ or ‘location state’ for short. After developing such a method for the position measurement of a free particle in one-dimension and later for a particle in a harmonic oscillator potential, we apply it to the phenomenon of diffraction of particles passing through a single slit. Our numerical calculations demonstrate that the location states can play an important role in explaining phenomena such as quantum diffraction and interference.

Another important element in the present work is the use of quantum trajectories for the description of diffraction. We agree with the observation made by Rothman and Boughn that Ref. [3] adopts a hidden variable point of view, though it is in a very crude form. On the other hand, in this paper, we use some standard nonlocal hidden variable theories for the quantum description of diffraction. Historically, hidden variables were viewed by quantum physicists as simply part of attempts to make quantum mechanics a causal and local theory. More specifically, according to earlier conventional viewpoint, hidden variable theories cannot violate the well-known Bell’s inequalities [8]. However, Bell himself has identified and publicised that one cannot rule out the existence of nonlocal hidden variable theories, such as the de Broglie-Bohm (dBH) theory [9, 10]. Recently, modified de Broglie-Bohm (MdBB) [11, 12, 13] and Floyd-Faraggi-Matone (FFM) [14, 15] quantum trajectories, which are also nonlocal hidden variable theories, are widely discussed. These trajectory formalisms give the same predictions as those in standard quantum theory (except that of the trajectories, which are not directly measurable). Hence generally they agree with the latter on all predictions of experimental results. In this paper, we have shown that quantum trajectories such as those in the dBH, MdBB and FFM formalisms can be of valuable help in explaining diffraction. By this approach, a unified description of the phenomenon can be obtained, applicable both in the Fresnel and Fraunhofer regions.

The paper is organized as follows. We start from the current studies [3, 4, 5] on the above problem of quantum diffraction in Sec. 2 giving a careful account of the procedure in [3]. In Sec. 3 the perspective on the measurement of position...
adopted in this paper and the location state wave function for a free particle in one dimension are presented. The corresponding problem when the particle is moving in an arbitrary potential is discussed in Sec. 4. How to describe diffraction of particles with the help of both location states and quantum trajectories is explained in Sec. 5. The last section comprises a discussion of our results.

2 Diffraction as a measurement process

![Figure 1: Experimental set up](image)

Here we briefly review the work in [3] and its modifications suggested in [5]. A single-slit diffraction experiment with an apparatus as shown in Fig. 1 is considered. On a diaphragm kept in the plane \( x = 0 \), a slit of width \( \Delta y = a \) is made with center at \( y = 0 \). Let the slit be of infinite depth along the \( z \)-axis. It is assumed that in the product wave function of the incident free particle, only the part along the \( y \)-axis is affected by diffraction. This part of the initial state gets reduced to \( |\psi_y^{y=0,\Delta y=a}\rangle \) at the moment of collapse. We denote this collapsed ket as \( |\psi_0^{0,a}\rangle \) for short. The corresponding quantum wave function of the particle in the position space is \( \psi_0^{0,a}(y) \equiv \langle y|\psi_0^{0,a}\rangle \). In the momentum space, this state is described by \( \langle p_y|\psi_0^{0,a}\rangle \), which may be denoted as \( \phi_0^{0,a}(p_y) \). The probability that the particle is scattered with its \( y \)-momentum between \( p_y \) and \( p_y + dp_y \) is then given by

\[
P(p_y)dp_y = |\phi_0^{0,a}(p_y)|^2 dp_y.
\]  

(1)
Here, the wave function in the momentum space is related to the corresponding wave function in position space by

\[
\phi_{y}^{0,a}(p_y) \equiv \langle p_y | \psi_{y}^{0,a} \rangle = \int dy \langle p_y | y \rangle \langle y | \psi_{y}^{0,a} \rangle = \frac{1}{\sqrt{2\pi}} \int dy \exp\left(-\frac{ip_y y}{\hbar}\right) \psi_{y}^{0,a}(y). \tag{2}
\]

In the above,

\[
\langle p_y | y \rangle = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{ip_y y}{\hbar}\right), \tag{3}
\]

is the position eigenfunction in the momentum space. Marcella assumes the collapsed wave function in position space, in a position measurement such as the above, to be of the normalised form

\[
\psi_{y}^{0,a}(y) = \begin{cases} 
\frac{1}{\sqrt{a}} & -a/2 \leq y \leq a/2 \\
0 & |y| > a/2 \end{cases}, \tag{4}
\]

at the instant of measurement (passage through the slit). The momentum space wave function corresponding to this may be evaluated using Eq. (2) as

\[
\phi_{y}^{0,a}(p_y) = \frac{2\hbar}{p_y \sqrt{2\pi a}} \sin\left(\frac{ap_y}{2\hbar}\right), \tag{5}
\]

which is the Fourier transform of (4). The probability density for this particle to have momentum around \( p_y \) can be found using Eq. (1) as

\[
P(p_y) = \frac{a}{2\pi} \left[ \sin\left(\frac{ap_y}{2\hbar}\right) \right]^2. \tag{6}
\]

This is valid at the moment of passage of the particle through the slit. Marcella substitutes \( p_y = psin\theta \) in the above expression, where \( \theta \) represents the scattering angle from the centre of the slit. Putting \( \alpha = ap_y/2\hbar \), one writes the above equation as

\[
P(\alpha) = \frac{a}{2\pi} \left( \frac{\sin\alpha}{\alpha} \right)^2. \tag{7}
\]

This then appears to give the standard result for Fraunhofer diffraction. But as noted by Rothman and Boughn [4], Marcella simply describes the state of the
particle at the slit and is not concerned with what takes place at the distant observation screen. Moreover, these authors have pointed out that the method used in [3] implicitly makes the same approximations as in the treatment of interference, etc., in classical optics. Eq. (5) gives the probability amplitude for the momentum $p_y$ at $t = 0$, which is the Fourier transform of the reduced wave function (4), and both of them describe the same state at the moment of detection. It is pointed out in [4] that Eq. (7) is not the same as the probability amplitude for the angle $\theta$ in a diffraction experiment without some form of a ‘hidden variable’ approach.

In the recent work in [5], Marcella’s formalism is extended to obtain a quantum mechanical description of diffraction through a single slit, in the Fraunhofer region and at all angular ranges. By considering the wave function (4) as a projection of an initial state $|\psi_{y_{in}}\rangle$ into the final one, an analysis in the light of quantum measurement theory is made. The following modifications are made in the approach in [3] to obtain a quantum mechanical diffraction formula: (1) The position filtering corresponds to a measurement of the three spatial coordinates, instead of one, and (2) it must be completed by an “energy-momentum filtering”. These, it is argued, are necessary to obtain a final state compatible with a ‘kinematic constraint’ and also with the constraint that the presence of the diffracted wave is only beyond the diaphragm. The modified theory attempts to provide a formula for the intensity of the diffracted wave over the whole range of diffraction angle, for the case of diffraction at infinity (Fraunhofer region). However, the conceptual difficulties raised by [4] are not addressed any further in [5].

3 Location state: free particle

Before going into the theory of diffraction, we consider the measurement of position of a particle in one dimension. In the standard framework of quantum mechanics, if the system consists of a single particle in one-dimension (whose position coordinate is denoted here as $y$), and if the measurement is of this observable $y$ with infinite precision at some point $y'$, the initial state gets reduced to a position eigenstate $|y'\rangle$. In the position representation, this eigenket can be written as [7, 16]

$$\langle y|y'\rangle = \delta(y-y'),$$

which is the Dirac $\delta$-function. One can write this as

$$\langle y|y'\rangle = \delta(y-y') = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk_y e^{ik_y(y-y')}.$$  

It may be noted that this is the same as the closure relation in this case.
On the other hand, in an actual position measurement such as the one described in the above section, the collapsed wave function is postulated to take the form (4). But this can be written as a superposition of position eigenkets,

\[
\psi_{y}^{0,a}(y) = \frac{1}{\sqrt{a}} \int_{-a/2}^{a/2} dy' \delta(y - y').
\]  

(10)

Using equation (9), this expression can be seen to be

\[
\psi_{y}^{0,a}(y) = \frac{1}{2\sqrt{a}} \int_{-a/2}^{a/2} dy' \int_{-k_m}^{k_m} dk_y e^{ik_y(y-y')},
\]  

(11)

where the limit \( k_m \to \infty \) has to be taken. One can rewrite this equation as

\[
\psi_{y}^{0,a}(y) = \frac{1}{2\sqrt{a}} \int_{-k_m}^{k_m} dk_y \left[ \int_{-a/2}^{a/2} dy' e^{-ik_y y'} \right] e^{ik_y y},
\]  

(12)

which is a superposition of eigenstates \( e^{ik_y y} \) of the Hamiltonian operator in this case, with the coefficients contained in the square bracket. Let this wave function be denoted as \( \Psi_{y}(y,0) \equiv \psi_{y}^{0,a}(y) \). We can introduce the unitary time evolution of the wave function for \( t > 0 \) as

\[
\Psi_{y}(y,t) = \frac{1}{2\sqrt{a}} \int_{-k_m}^{k_m} dk_y \left[ \int_{-a/2}^{a/2} dy' e^{-ik_y y'} \right] e^{ik_y y} e^{-iE_y t/\hbar},
\]  

(13)

under the limit \( k_m \to \infty \). Here \( E_y = \hbar^2 k_y^2 / 2m \). In this form, we call it the ‘position measurement-induced collapse state’ or ‘location state’. It is important to note that such time-evolution of the wave function is not considered in [3].

We have plotted this location state wave function in Eq. (13) for various values of \( t \), putting \( \hbar/m = 1 \), \( a = 0.1 \) and by fixing \( k_m = 10^8 \), in Fig. 2. This value of \( k_m \) appears to be reasonably large, for any further increase in it does not lead to significant change in the plot of \( |\Psi_{y}(y,t)|^2 \) versus \( y \), for all values of \( t \). For \( t = 0 \), we are able to regain the real rectangular wave function given in Eq. (4) with very good accuracy. Thus without the factor \( e^{-iE_y t/\hbar} \) in the integral, this gives nothing new, except the original reduced wave function. It is notable that this commonly conceived wave function in Eq. (4) has a spreading for \( t > 0 \). As can be seen from the figures, it is not simply a spreading of the rectangular wave function in (4) as such; for small values of \( t \), the modulus square of the wave function in position space acquires the shape of the intensity distribution for Fresnel diffraction and for large \( t \), it approaches the pattern for Fraunhofer diffraction.

In summary, our method consists of first rewriting the rectangular wave function into the form (10), then substituting for the \( \delta \)-function in the integrand with the right hand side of Eq. (9) (which is the closure relation in this case), and finally postulating the unitary time-evolution as per Eq. (13). Note that in this
section, we have only described the spreading of a one-dimensional rectangular wave function with time and have plotted the probability density, with the results resembling the diffraction patterns. For an actual treatment of diffraction, we must specify the value of time corresponding to the distance to the screen, which we shall do in Sec. 5.

To our knowledge, the above method of obtaining the patterns similar to Fresnel and Fraunhofer from the spreading of the same rectangular wave function is not discussed elsewhere in the literature. This formalism to describe the time-evolution of a rectangular wave function shall be useful not only for diffraction, but also for other cases of position measurement of particles moving in general potentials, as demonstrated in the next section.

4 Location state: general potential

Let us now consider the general case of position measurement of a particle in a potential $V(y)$, where the measurement is made at some arbitrary point $y_0$. Here we assume that the collapsed wave function at $t = 0$ is nonzero only between $y_0 - a/2$ and $y_0 + a/2$. In the simpler case, this state is described by a constant wave function similar to that in Eq. (4), but centred at $y_0$, at the moment of measurement.
Let the eigenkets of the Hamiltonian operator in this case be denoted by \(|u_n\rangle\), so that the corresponding position space energy eigenfunctions are \(u_n(y) \equiv \langle y | u_n \rangle\). If we perform an idealised measurement at the point \(y'\) with infinite precision, the resulting reduced wave function would be a \(\delta\)-function. One can use the closure relation

\[
\sum_n u^*_n(y')u_n(y) = \delta(y - y'),
\]

(14)
to represent this position eigenstate while it is in this potential. This can be used to expand the reduced wave function \(\psi(y, 0)\) at \(t = 0\), as in Eq. (10),

\[
\psi_{y^0, a}(y) = \frac{1}{2\pi \sqrt{a}} \int_{y_0-a/2}^{y_0+a/2} dy' \sum_n u^*_n(y')u_n(y),
\]

(15)
where the limit \(n \to \infty\) must be taken. We denote this wave function as \(\Psi(y, 0) \equiv \psi_{y^0, a}(y)\). As in the previous case, we rewrite the above equation and introduce the time-evolution of the wave function of the particle as

\[
\Psi(y, t) = \frac{1}{2\pi \sqrt{a}} \sum_n \left[ \int_{y_0-a/2}^{y_0+a/2} dy' u^*_n(y') \right] u_n(y)e^{-iE_n t/\hbar},
\]

(16)
where again the limit \(n \to \infty\) may be taken. Here \(E_n\) are the energy eigenvalues of the particle when it is in this potential.

We have plotted the above wave function for a harmonic oscillator with potential \(V(y) = \frac{1}{2} m \omega^2 y^2\). Here we use

\[
u_n(y) = (2^n n! \sqrt{\pi} \sigma)^{-1/2} \exp \left[ -\frac{1}{2} \left( \frac{y}{\sigma} \right)^2 \right] H_n \left( \frac{y}{\sigma} \right)
\]

(17)
and the corresponding energy eigenvalues \(E_n = \hbar \omega (n + \frac{1}{2})\), with \(\sigma = \sqrt{\hbar/(m \omega)}\) in the above equation (16). The oscillator is assumed to be detected at \(y_0 = 10\) by a slit of width \(a = 2\), with \(\hbar/m = 1\). The plots given in Fig. 3 are for different values of \(t\), with an upper limit for \(n\) as \(n_m = 250\). It can be seen that the pattern gradually changes from that of Fresnel diffraction to Fraunhofer diffraction.

For the above values of parameters, the period of oscillation of the harmonic oscillator is \(2\pi\). We have also plotted the patterns for a full period, at values of \(t = 0.0, 1.0, 2.0, 3.0, \pi, 4.0, 5.0, 6.0, 2\pi, 7.0, 8.0\) and \(9.0\) in Fig. 4. It can be clearly seen that the probability density of location state is periodic and maintains the diffraction pattern as it moves along, changing from the Fresnel to Fraunhofer and vice-versa, in this case. The oscillatory behaviour has some similarity with that of coherent states, and it indicates that the characteristics of location states deserves to be studied in detail.
5 Quantum treatment of diffraction

Now we turn to the actual quantum treatment of diffraction of free particles when they pass through a single slit of width \(a\) made on a diaphragm placed in a region where there is no other potential. The experimental set-up is as described in Sec. 2. Let the particle be incident on the diaphragm from left, and the incident wave function be of the form \(\Psi_x(x,t)\Psi_y(y,t)\Psi_z(z,t) = Ne^{i(k_xx - E_xt/\hbar)}\). This plane wave is assumed to have a constant wave vector \(\vec{k}\), whose components are \(k_x > 0, k_y = 0, k_z = 0\), so that \(\Psi_y\) and \(\Psi_z\) are constants and the initial wave function corresponds to a wave progressing along the positive \(x\)-axis. We assume the collapse to occur at \(t = 0\), when the particle passes through the slit. The part \(\Psi_x(x,t)\) of the collapsed wave function is not affected. For \(t \geq 0\), the part \(\Psi_y(y,t)\) is assumed to be given by equation (13), which is the location state along the \(y\)-axis. The product wave function that describes the particle is now

\[
\Psi_x(x,t)\Psi_y(y,t) = N \frac{e^{i(k_xx - E_xt/\hbar)}}{2\pi\sqrt{a}} \int_{-k_m}^{k_m} dk_y \left[ \int_{-a/2}^{a/2} dy' e^{-ik_y y'} \right] e^{i(k_y y - E_y t/\hbar)},
\]  
(18)

where \(E_y = \hbar^2 k_y^2 / 2m\). To obtain the diffraction pattern on the screen placed at \(x = D\), one has to plot the modulus square of this product function on the screen.
Figure 4: Probability distribution for location states of a harmonic oscillator for $T = (a) 0.0, (b) 1.0, (c) 2.0, (d) 3.0, (e) \pi, (f) 4.0, (g) 5.0, (h) 6.0, (i) 2\pi, (j) 7.0 (k) 8.0 (l) 9.0$.

We do not consider a factor $\Psi_z(z,t)$ in the above product since the free incoming particle has $k_z = 0$ and also since the slit is of infinite depth along the $z$-axis. Thus $\Psi_z(z,t)$ will always remain a constant.

It is easily seen that $|\Psi_x(x,t)\Psi_y(y,t)|^2$ does not depend on $x$. The only variation for this probability density is along the $y$-direction and for different values of $t$, the patterns change as in Fig. 2. Hence the probability density on a screen evaluated using the above wave function shall be independent of the position $x = D$ of the screen, but will depend on time $t$ for a fixed $D$. On the other hand, the patterns obtained in experiments depend on $D$, but for a fixed $D$, they are time-independent. Thus unless we specify some time $T$ in the above expression corresponding to a given value of $D$, there is no definite theoretical prediction of the pattern on the screen to compare with experiment. It is clear that if we adhere to standard quantum mechanics alone, some sort of discrepancy arises in this case.

The way out of this puzzle is to assume that there are point particles moving...
along trajectories, as in nonlocal hidden variable theories. The dBB, MdBB and the FFM formulations are such hidden variable theories. In the above case of single slit diffraction, the particles move with a constant \( x \)-component of velocity \( v_x = \hbar k_x/m \) in all the three formalisms. (Though the FFM case differ with the former two cases with regard to time parametrization \[18\], \[19\], the free particle motion in it is identical with that in the former ones.) Therefore the time with which a particle from the slit reaches the screen placed at \( x = D \) is \( T = D/v_x \). We shall now attempt to plot the probability distribution versus \( y \), on this screen at \( D \) for a time evaluated according to this formula. The plots shown in Fig. 5 are for a slit of width \( a = 0.1 \).

![Diffraction patterns predicted according to the location state formalism (blue lines) when compared to that in \[3\] (dotted red line) for values of \( T = (a) 0.0005, (b) 0.00075, (c) 0.001, (d) 0.01 \)

Figure 5: Diffraction patterns predicted according to the location state formalism (blue lines) when compared to that in \[3\] (dotted red line) for values of \( T = (a) 0.0005, (b) 0.00075, (c) 0.001, (d) 0.01 \)

In all the figures in Fig. 5 we have plotted \( |\Psi_{\chi}(x,t)\Psi_{\gamma}(y,t)|^2 \), which is the
probability distribution corresponding to Eq. (18) (blue lines), together with the expression (7) used by Marcella [3] (dotted red lines), both normalised and evaluated for fixed values of \( D \). The different values of \( D \), along with that of \( a \) and \( \lambda \), give Fresnel numbers \( N_F \equiv \frac{a^2}{4\lambda D} = \frac{a^2 m}{8\pi \hbar T} = 0.796, 0.531, 0.398, 0.039 \), respectively. This shows that except for the last case, the diffraction is in the Fresnel region. Our plots using Eq. (18) demonstrate that at small values of time \( T \) (closer distances \( D \)), the location state pattern approaches that of Fresnel diffraction, contrary to what happens in [3]. For the last plot, we have \( N_F << 1 \) and hence the Fraunhofer regime results. The agreement in the last case can be seen to be excellent. Thus the comparison between patterns of the present location state formalism and that in [3] tells that the present one has better agreement with experiment in all regions.

We thus see that the standard results of Fresnel and Fraunhofer diffraction can be connected to the time-evolution of location states only with the help of quantum trajectory formalisms. Our success in this endeavour supports the existence of particle trajectories. In this case, dB, MdBB and FFM trajectories give the same results, for they have the same value for \( v_x = \frac{\hbar k_x}{m} \). Therefore, it is not possible to discriminate them using this experiment.

6 Discussion

The present attempt to describe quantum mechanically the phenomenon of diffraction is strongly founded on the collapse of a quantum state due to position measurement and the subsequent time-evolution of this state. In dealing with this time-evolution, we have followed the standard axioms of quantum mechanics. That these patterns (for instance, those in section 3) closely resemble the diffraction patterns obtained in experiments is reassuring and shows the connection of diffraction to the quantum location states, which we have defined in this paper. Since it is here shown possible that a single quantum mechanical expression can explain both the Fresnel and Fraunhofer diffractions, the present formulation deserves serious attention.

To obtain the diffraction pattern using the above formalism, we must take the limit \( k_m \rightarrow \infty \) or \( n \rightarrow \infty \), in the respective integrations or summations done to evaluate \( \Psi_y(y,t) \) in equations (13) and (16). We have drawn the patterns with finite values of these parameters, but it is observed that they do not get modified appreciably by increasing these values beyond that used by us. However, one must realise that in such limits, the mean value of energy in these states also must tend to infinity. It is only natural in the quantum regime that a position eigenstate, which is described by a Dirac \( \delta \)-function, involves infinite energy, for the uncertainty in momentum tends to infinity in those cases. The location
states are written as an integral over such position eigenstates, with eigenvalues lying between $-a/2$ to $a/2$. We have evaluated the mean value of energy of these location states for various $k_m$ only to find that the mean keeps on increasing. Since the variance of momentum in the state (4) is infinite, an infinite mean value of energy is unavoidable. This peculiarity is a consequence of the discontinuity of this wave function and is characteristic of all orthogonal measurements [6]. But in actual experiments, such sharp discontinuities may not exist, so that one can have a finite upper limit for $k_m$ or $n_m$, as we have taken. In any case, since the patterns do not change appreciably with their values beyond those used by us, our explanation of diffraction remains satisfactory. However, more sophisticated experiments are needed to settle the issue of energy.

It is true that both formalisms, i.e., the one in Ref. [3] and the present one, use some kind of hidden variable approaches. In the former case, while using $p_y = p \sin \theta$, it is assumed that the particles are emitted with definite values of $p_x$ and $p_y$, and that they proceed to the screen as classical particles with precisely these momenta. In fact, no time-evolution of the wave packet for $t > 0$ is considered here. Hence [3] cannot be considered as relying on a proper hidden variable theory. One must also note that the formalisms in [3] and [5] do not focus on Fresnel diffraction. The present one, on the other hand, considers the evolution of wave function and makes use of standard nonlocal hidden variable theories such as those in dBB, MdBB and FFM formalisms. Taking into account the evolution of the state makes the formalism capable of providing a satisfactory explanation even to the Fresnel diffraction, at small distances from the slit. While showing that the location states play a major role in providing this unified description of Fraunhofer and Fresnel diffractions, we find this result also as suggestive of the existence of quantum trajectories.

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