QCD SUM RULES AND B DECAYS (\textasteriskcentered)

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Abstract

We review some recent applications of QCD sum rules to $B$-decays. These include the determination of the leptonic constant $f_B$ and of the semileptonic transition amplitudes of the $B$-meson into negative and positive parity charmed states.

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1. Introduction

Considerable interest has been given in the past years to applications of QCD sum rules to $B$ (and $D$) physics, resulting in a large number of estimates for the $B$ and $D$ transition matrix elements.

The formalism consists in relating via dispersion relations low energy parameters, such as hadronic masses and couplings, to the QCD dominated short-distance Operator Product Expansion (OPE) of current correlators [1,2]. Such an expansion is assumed to hold in the presence of non-perturbative QCD effects, which are parametrized by a set of fundamental vacuum condensates, describing the breaking of asymptotic freedom, and inducing power corrections in the correlators. These condensates are fundamental constants which cannot be computed from perturbation theory: however, they are “universal” and, once fitted from channels where the physics is known, their values can be used to make predictions for the other channels. Thus, being fully field-theoretic and relativistic, and incorporating the fundamental features of QCD, this method aims to be a means to interface from first principles hadronic observables to the basic quark and gluon QCD Lagrangian. From this point of view, it should represent a convenient framework to study the properties of heavy-light quark systems and of quarkonia, which avoids the notion of constituent quark bound state wavefunction and should thus reduce the model dependence to a minimum.

Most recently, the attention has been focussed on the applications of QCD sum rules to the semileptonic and the purely leptonic $B$- and $D$-meson decays, motivated on one side by the observed features of the data [3, 4] (also in the perspectives of increased precision and of rare decays measurements, as allowed by planned charm and beauty factories), and on the other side by the theoretical achievements offered in principle by the heavy quark effective theory (HQET) [5]. In this regard, it turns out that QCD sum rules allow to study the behaviour of amplitudes for $qQ$ mesons for $m_Q \to \infty$ ($Q$ represents the heavy quark, $q$ the light one). Therefore they can be used to verify the predictions of spin-flavor heavy quark symmetry, and to test quantitatively the resulting scaling laws relating the different amplitudes as well as the validity of approximations based on the $1/m_Q$ expansions. Accordingly, the sequel will be devoted to applications of QCD sum rules to several aspects of $B$ physics. Specifically, we shall briefly review the predictions for the leptonic constant $f_B$, analyzing in particular the determination of the beauty quark mass $m_b$, and of the semileptonic $B$-decays to negative and to positive parity charmed mesons.
2. The leptonic decay constant $f_B$

Leptonic decay constants, governing the decays $D \rightarrow \mu \nu$, $B \rightarrow \mu \nu$, etc., are defined as the vacuum-to-meson matrix elements:

$$<0|\bar{Q}\gamma_\mu \gamma_5 q|P(p)> = i f_P \ p_\mu,$$

with $Q = c, b$ the heavy quarks and $q = u, d, s$ the light ones, so that $P = B, D$. The theoretical interest stems from their significant role as normalizations of numerous predictions for heavy meson transitions. In particular, by dominating the description of $B - \bar{B}$ mixing, $f_B$ strongly affects the determination of the CKM matrix, with important implications on CP violation in the beauty sector [6]. Furthermore, of great interest is the scaling behaviour for very large (infinite) $m_Q$:

$$f_P = \frac{\text{const}}{\sqrt{m_P}},$$

modulo logs, which reflects the connection with the constituent quark wavefunction, but in fact results from the heavy quark symmetry and accordingly is a general consequence of QCD.

The present indications are $f_D < 310 \text{ MeV}$ [7] and $f_{D_s} = (232\pm45\pm20\pm48) \text{ MeV}$ [8], whereas no information is available on $f_B$. To get a feeling on the sensitivity required to measure $f_B$, for $|V_{ub}/V_{cb}| = 0.1$ with $|V_{cb}| = 0.045$ and $f_B = 1.5 f_\pi$ ($f_\pi = 132 \text{ MeV}$) one finds $BR(B \rightarrow \mu \nu) = 5.1 \times 10^{-7}$ and $BR(B \rightarrow \tau \nu) = 1.1 \times 10^{-4}$.

Let us now briefly describe the method of QCD sum rules as applied to $f_B$. One starts from the two-point correlator at euclidean $Q^2 = -q^2$:

$$\Pi_5(q^2) = i \int d^4x \exp(iqx) <0|T(J_5(x)J_5(0)\dagger)|0>,$$

where $J_5(x) =: \bar{Q}(x)i\gamma_5 q(x) :$ is the quark bilinear operator which interpolates the pseudoscalar meson field, and one can neglect the light quark mass $m_q$ with respect to $m_Q$. For other mesonic states, the appropriate interpolating fields must be used in (3), e.g. $V_\mu =: \bar{Q}\gamma_\mu q :$ for the vector particle $B^*$, etc. In QCD $\Pi_5(q^2)$ satisfies a twice-subtracted dispersion relation. Actually, to improve the convergence, emphasize the contribution of the hadronic ground state one is interested in, and get rid of unknown subtraction constants, suitable weight functions are used in the dispersive integrals. One is thus led to consider “moments”, each corresponding to a different version of QCD sum rules, such
as the Hilbert moments at $Q^2 = 0$, leading, for $n = 1, 2, \ldots$, to the finite energy sum rules (FESR):

$$M_n(0) \equiv \frac{(-1)^n}{(n+1)!} \left( \frac{d}{dQ^2} \right)^{n+1} \Pi_5(Q^2)|_{Q^2=0} = \int_0^\infty ds \frac{1}{s^{n+2}} \frac{1}{\pi} Im\Pi_5(s); \quad (4)$$

and the exponential moments

$$\tilde{\Pi}_5(\sigma) = \int_0^\infty ds \exp(-\sigma s) \frac{1}{\pi} Im\Pi_5(s), \quad (5)$$

where $1/\sqrt{\sigma}$ is a mass scale. A review of other versions can be found in [2].

The hadronic spectral function on the right sides of (4) and (5) is usually parametrized in terms of the ground state meson pole plus a “continuum” of higher states, starting at some threshold $s_0$ and modelled by the asymptotic freedom (AF) expression:

$$\frac{1}{\pi} Im\Pi_5(s)|_{HAD} = \frac{f_P^2 m_P^4}{m_Q^2} \delta(s - m_P^2) + \frac{1}{\pi} Im\Pi_5(s)|_{AF} \theta(s - s_0). \quad (6)$$

In (6), to two loops [9]:

$$\frac{1}{\pi} Im\Pi_5(s)|_{AF} = \frac{3}{8\pi^2} m_Q^2 \left( \frac{1-x)^2}{x} \right) \left\{ 1 + \frac{4\alpha_s}{3\pi} \left[ \frac{18}{8} - 2Li_2 \left( \frac{-x}{1-x} \right) 
- \ln \left( \frac{x}{1-x} \right) \ln \left( \frac{1-x}{1-x} \right) + \left( \frac{3}{2} - \frac{x}{1-x} \right) \ln \left( \frac{x}{1-x} \right) 
+ \frac{1}{1-x} \ln \left( \frac{1}{1-x} \right) \right] \right\}, \quad (7)$$

where $x \equiv m_Q^2/s$.

The left sides of (4) and (5) can be evaluated by means of the OPE, in terms of short-distance perturbative QCD, determined by the integral of (7) from $m_Q^2$ up to infinity, plus non-perturbative power corrections, determined by quark and gluon operator vacuum condensates, ordered for increasing dimension. These vacuum expectation values essentially allow the extrapolation of the asymptotic freedom contribution, completely known in terms of $\alpha_s$ and $m_Q$, down to “moderate” mass scales, relevant to hadrons.

Limiting to operators of dimensionality up to six, combining (4), (6) and (7) one obtains the FESR, in the form ($m_q \to 0$):

$$\frac{f_P^2}{m_P^{2n}} = \frac{m_Q^2}{m_P^2} \int_{m_Q^2}^{s_0} ds \frac{1}{s^{n+2}} \frac{1}{\pi} Im\Pi_5|_{AF} + M_n(0)|_{NP}, \quad (8)$$
where the non-perturbative part is given by:

\[
M_n(0)_{NP} = \frac{-m_Q \langle \bar{q}q \rangle}{m_Q^{2n+2}} \left[ 1 - \frac{\langle \alpha_s G^2 \rangle}{12\pi m_Q \langle \bar{q}q \rangle} - \frac{1}{4} (n+2)(n+1) \frac{M_0^2}{m_Q^2} \right]
- \frac{4}{81} (n+2)(n^2 + 10n + 9) \pi \alpha_s \rho \frac{\langle \bar{q}q \rangle}{m_Q^2}.
\]  

(9)

Alternatively, combining (5), (6) and (7) one obtains the exponential (also called Laplace or Borel) version of QCD sum rules for \( f_P \) [10]:

\[
\frac{f_P^2 m_P^4}{m_Q^2} \exp (-\sigma (m_P^2 - m_Q^2)) = \int_{m_Q^2}^{s_0} ds \exp (-\sigma (s - m_Q^2)) \frac{1}{\pi} |m \Pi_5|_{AF} + \bar{\Pi}_5(\sigma)|_{NP}
\]

(10)

where

\[
\bar{\Pi}_5(\sigma)|_{NP} = -m_Q \langle \bar{q}q \rangle \left[ 1 - \frac{\langle \alpha_s G^2 \rangle}{12\pi m_Q \langle \bar{q}q \rangle} + \frac{M_0^2 \sigma}{2} \left( 1 - \frac{m_Q^2 \sigma}{2} \right) \right]
+ \frac{8 \pi \alpha_s \rho \langle \bar{q}q \rangle}{27 m_Q} \left( 2 - \frac{m_Q^2 \sigma}{2} - \frac{m_Q^4 \sigma^2}{6} \right).
\]

(11)

Sum rules for the meson mass \( m_P \) can be obtained from ratios of FESR with different \( n \), or from the logarithmic derivative of (10) with respect to \( \sigma \). Applications of other versions of QCD sum rules to the estimate of \( f_P \) are presented e.g. in [11] and [2].

In (9) and (11), \( M_0^2 \) parametrizes the dimension 5 quark-gluon condensate \( \langle g_s \bar{q}q \sigma_{\mu\nu} G^{\mu\nu} q \rangle = M_0^2 \langle \bar{q}q \rangle \), and \( \rho \neq 1 \) represents the deviation of the dimension 6 four-quark condensate from pure factorization. Typical values, obtained from PCAC and from applications of the method to other channels, are e.g. \( \langle \bar{q}q \rangle (1 \text{ GeV}) \simeq -0.016 \text{ GeV}^3 \), \( \langle \alpha_s G^2 \rangle \simeq 0.02 \div 0.06 \text{ GeV}^4 \), \( M_0^2 \simeq 0.5 \div 0.8 \text{ GeV}^2 \) and \( \rho \simeq 1 \div 3 \). In practice, results for \( f_B \) are not sensitive to the value of the gluon condensate and \( \rho \). Conversely, more important is their dependence on the input values of quark masses, typical values being \( m_b (Q^2 = m_b^2) \simeq 4.6 \div 4.8 \text{ GeV} \) and \( m_c (Q^2 = m_c^2) \simeq 1.3 \div 1.4 \text{ GeV} \).

Regarding the practical use of QCD sum rules, we remark that the non-perturbative expansions in (9) and (11) have been truncated to the first few significant terms (of lowest dimension), neglecting higher condensates. The validity of such an approximation clearly depends on the values of \( n \) or \( \sigma \). Indeed, larger \( n \) and \( \sigma \) emphasize the ground state in (8) and (10) and minimize the
role of the continuum, but correspondingly also increase the size of higher power corrections. Thus, in practical applications one considers as a compromise the first few values of \( n \), or a suitable range of \( \sigma \), such that the truncation of the OPE is justified. Another point concerns the role of the continuum threshold \( s_0 \), which is not determined \textit{a priori}. Clearly, the predictions for the hadronic observables must not be sensitive to the values of this parameter. Thus, the procedure to exploit the FESR (8) is to search for a “duality window”, \textit{i.e.} a range in which the predicted \( m_B \) (and \( m_D \)) agrees with the measured one and is stable against \( s_0 \) (and \( n \)). In this window one then solves for \( f_B \), and this estimate should be reasonably reliable. Similarly, in the case of the exponential sum rules (10) one imposes stability of the predicted \( m_P \) and \( f_P \) in the range of \( \sigma \) and \( s_0 \) mentioned above.

In Table 1 we list the results for \( f_B \) (and for completeness also for \( f_D \)) previously obtained from FESR and from exponential sum rules [12,13,14], using \( \alpha_s \) and \( \langle \bar{q}q \rangle \) evaluated at a scale \( m_Q \). The authors of [10] use \( m_b = 4.8 \text{ GeV} \), which is somewhat larger than the values of \( m_b \) used by the other groups. This shows that the method is sensitive to the input value of the heavy quark mass. The “±” in Table 1 accounts for reasonable variations of the input parameters. Within \( SU(3) \) breaking effects, the results for \( f_D \simeq f_{D_s} \) are compatible with the experimental data [7].

| Method            | \( f_B/f_\pi \) | \( f_D/f_\pi \) | Ref.     |
|-------------------|-----------------|-----------------|----------|
| Laplace           | 1.02 ± 0.11     | 1.33 ± 0.19     | [10]     |
| Hilbert moments   | 1.35 ± 0.25     | 1.70 ± 0.20     | [12]     |
| Laplace & Hilbert | 1.38 ± 0.14     | 1.31 ± 0.12     | [13]     |
| Laplace           | 1.29 ± 0.15     |                 | [14]     |
| Laplace (HQET)    | 1.48 ± 0.39     | 1.29 ± 0.23     | [17]     |
| Laplace (HQET)    | 1.7 ± 0.2       |                 | [19]     |

\textbf{Table 1} QCD sum rules predictions for \( f_P \)

Another important point is that the size of the \( O(\alpha_s) \) corrections in the sum rule for \( f_B^2 \) turn out to be rather large, of the order of 40%. Moreover, at
this two loop level there is an ambiguity in the definition of the argument of $\alpha_s$, i.e. whether it should be $m_b$ or the much smaller confinement scale of the order of $1 \text{ GeV}$, which can make a substantial numerical difference.

Indeed, in this regard it is instructive to consider the “static approximation” of (10), where $m_Q$ becomes infinitely heavy. This can formally be obtained by defining the “non-relativistic” variables:

$$\sqrt{s} = m_Q + \omega; \quad m_P = m_Q + \omega_R; \quad \tau = 2m_Q\sigma,$$

(12)

where $\tau$ is a euclidean time and the “binding energy” $\omega_R$ should be flavor-independent in the heavy quark limit. Expanding up to the first order $1/m_Q$, and retaining the condensates up to dimension 5, Eq.(10) becomes [15,16,17]:

$$\left(f_P\sqrt{m_P}\right)^2 \left(\frac{m_P}{m_Q}\right)^3 \exp\left(-\tau \omega_R\right) =$$

$$\frac{3}{\pi^2} \int_0^{\omega_0} d\omega \omega^2 \left(1 - \frac{2\omega}{m_Q}\right) \exp\left(-\tau \omega\right) \left\{1 + \frac{\alpha_s}{\pi} \left[2 \ln\left(\frac{m_Q}{2\omega}\right) + \frac{13}{3} + \frac{4\pi^2}{9} - \frac{4\omega}{m_Q}\right]\right\}$$

$$- \langle \bar{q}q \rangle \left[1 - \frac{M_0^2 \tau^2}{16} \left(1 - \frac{4\tau}{m_Q}\right) + \frac{\langle \alpha_s G^2 \rangle}{12\pi m_Q \langle \bar{q}q \rangle}\right].$$

(13)

Analogously to the relativistic version (10), a sum rule for $\omega_R$ can be obtained from the logarithmic derivative of Eq.(13) with respect to $\tau$.

By construction, this non-relativistic form should reproduce the large $m_Q$ dimensional behaviour implied by (2), plus $1/m_Q$ corrections. In addition, it should be noticed that, for $m_Q \to \infty$, the right hand side of (13) has no explicit reference to $m_Q$, and actually now involves mass scales of the order of the confinement scale $\sim 1 \text{ GeV}$ (the same is expectedly true for $\omega_R$). This fact naively suggests to take a low energy, confinement scale as the argument of the running $\alpha_s$ and $\langle \bar{q}q \rangle$.* However, Eq.(13) clearly shows that there is a difficulty with the $O(\alpha_s)$ correction, associated with the appearance of a large (diverging) $\ln m_Q$, so that the limit $m_Q \to \infty$ is only defined after a resummation of these terms to all orders. Also, one can see that the finite coefficient in the $\alpha_s$ correction is of the order of 10, giving rise to a large contribution, regardless of the choice of the argument of the strong coupling constant. The separation of the $\ln m_Q$ dependence from the sum rule and the definition of the appropriate scale require the

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* This suggestion was also made in [18], discussing the application of QCD sum rules to quarkonium states.
renormalization group improved summations of the leading \((\alpha_s \ln m_Q)^n\) and the next-to-leading \(\alpha_s (\alpha_s \ln m_Q)^n\) terms to all orders in perturbation theory. This has been recently done by formulating the QCD sum rules in the framework of the HQET \([17,19]\).

Basically, in this theory one has to investigate the two-point correlator

\[
\Pi_5(w) = i \int d^4x \exp(ikx) \langle 0 | \hat{J}_5(x) \hat{J}_5(0) \rangle | 0 >, \tag{14}
\]

where \(\hat{J}_5 = \bar{q}\gamma_5 h_Q(v)\) is the effective pseudoscalar current, with \(h_Q(v)\) the velocity-dependent heavy quark field. The ground state contribution is essentially determined by \(\langle 0 | \hat{J}_5 | P(v) \rangle = \sqrt{m_Q} F(\mu)\), where \(F(\mu)\) is an effective theory low energy constant, independent of \(m_Q\) but depending on a renormalization scale \(\mu\). The relation between the physical and the HEQT \(f_P\) is:

\[
f_P \sqrt{m_P} = C_F(\frac{m_Q}{\mu}) F(\mu) + O(\frac{\Lambda_{QCD}}{m_Q}), \tag{15}\]

where \(C_F\) is a QCD short-distance coefficient, which is computed up to the next-to-leading order \([20,21,17]\). The left side of (15) must be \(\mu\)-independent; to the considered order the \(m_Q\) and the \(\mu\)-dependences in \(C_F(\frac{m_Q}{\mu})\) factorize, so that (15) can be suitably rewritten as:

\[
f_P \sqrt{m_P} = \hat{C}_F(m_Q) \hat{F}, \tag{16}\]

where

\[
\hat{C}_F(m_Q) = (\alpha_s(m_Q))^{d/2} \left[ 1 + \frac{\alpha_s(m_Q)}{\pi} Z \right], \tag{17a}\]

\[
\hat{F} = (\alpha_s(\mu))^{-d/2} \left[ 1 - \frac{\alpha_s(\mu)}{\pi} (Z + \delta) \right] F(\mu). \tag{17b}\]

In (17) \(d = \gamma_0/\beta_0\) with \(\gamma_0\) and \(\beta_0\) the leading order anomalous dimension and QCD \(\beta\)-function, \(Z\) is a scheme-independent constant and \(\delta\) is scheme-dependent (for the explicit values we refer e.g. to [17]). According to (17b), \(\hat{F}\) is a scale-independent, universal constant of QCD.

By the same procedure leading to (13) one obtains a QCD sum rule of the form \((m_Q = \infty)\):

\[
F^2(\mu) \exp(-\tau \omega_R) = \frac{3}{\pi^2} \int_{0}^{\omega_0} d\omega \omega^2 \exp(-\tau \omega) \left\{ 1 + \frac{\alpha}{\pi} \left[ 2 \ln \left( \frac{\mu \tau}{2} \right) + \frac{13}{3} + \frac{4\pi^2}{9} - 2 \ln (\tau \omega) + \delta \right] \right\} - \langle \bar{q}q \rangle(\mu) \left[ 1 - \frac{M_0^2 \tau^2}{16} + \cdots \right], \tag{18}\]

8
where the ellipses denote contributions of order $\alpha_s$ and of higher dimensional condensates. Eq. (18) corresponds to (13) at $m_Q = \infty$, except from the replacement of $m_Q$ by $\mu$ in the log and the appearance of $\delta$. Both the $\mu$-dependence and $\delta$ must cancel when considering the sum rule for the renormalization-group invariant $\hat{F}$. Indeed, the summation of the $\ln \mu$ to all orders produces the factor $(\alpha_s(2/\tau)/\alpha_s(\mu))^{-d}$ and, taking into account (17b), the running $\alpha_s$ must be evaluated at a low-energy scale of the effective theory. The structure of the QCD sum rule for $\hat{F}$ is finally [17]:

$$
(\alpha_s(2/\tau))^d \left(1 + 2\frac{\alpha_s}{\pi} Z\right) \hat{F}^2 \exp(-\tau \omega_R)
$$

$$
= \frac{3}{\pi^2} \int_0^{\omega_0} d\omega \, \omega^2 \exp(-\tau \omega) \left[1 + \frac{\alpha_s}{\pi} \left(\frac{13}{3} + \frac{4\pi^2}{9} - 2 \ln(\tau \omega)\right)\right]
$$

$$
- \langle \bar{q}q \rangle |_{2/\tau} \left[1 - \frac{M_0^2 \tau^2}{16} + \cdots\right],
$$

(19)

where $\alpha_s$ is evaluated at a confinement scale.

The analysis then proceeds by looking for ranges in $\tau$ and in $\omega_0$ where the prediction for $\hat{F}$ is stable. The results exhibit a dependence on the value of $\omega_R$, which is typically of the order of $0.5 \div 0.8$ GeV, depending on the value of the heavy quark mass $m_b$ (or $m_c$). Such a dependence somehow limits the accuracy of the predictions. Also, there is an uncertainty associated to the actual choice for the low energy scale in the running strong coupling constant. Although the different choices formally have an impact at the $\alpha_s^2$ level, in practice they make a substantial difference because the $\alpha_s$ corrections are large by themselves.

Combining the various uncertainties, the final result for the “static limit” of $f_B$, defined as

$$
f_{B}^{\text{stat}} = \frac{\hat{C}(m_b) \hat{F}}{\sqrt{m_B}},
$$

(20)

is $f_{B}^{\text{stat}} = 200 \div 300$ MeV, in good agreement with the value recently found from lattice calculations [22]. However, the price is represented by a 100% order $\alpha_s$ correction, which questions the validity of the perturbative expansion in this case.

Concerning the $O(1/m_Q)$ corrections due to the breaking of the heavy quark symmetry, a systematic analysis is possible by combining (19) with (13), and reintroducing the terms which vanish in the infinite mass limit [17,19]. One source of corrections, which can be immediately assessed, is related to the factor $(m_P/m_Q)^3$ on the left hand side of (13). This shows that, although naively of
order \((\Lambda_{QCD}/m_Q)\), the expansion parameter is actually a factor \(\times (\omega_R/m_Q)\).

For current values of quark masses, \(1/m_Q\) corrections are then expected to be reasonably small for \(f_B\), but very large for \(f_D\).

The results of the complete analysis of the sum rules including \(1/m_Q\) corrections are reported in Table 1, and one can compare them with the values previously found from their relativistic counterparts. Within the uncertainties, the overall picture indicates a leptonic constant of the \(B\) comparable (or even larger) to that of the \(D\), and in the range of possible measurements at a beauty factory.

3. A determination of the beauty quark mass

In the previous section we mentioned the dependence of the QCD sum rule estimates of \(f_B\) on the value of \(m_b\). This is true also for other transition matrix elements. Thus, a precise determination of \(m_b\) is highly desirable in order to reduce the uncertainty in the theoretical predictions. The ideal source of information on \(m_b\) should be represented by the application of QCD sum rules to the \(\Upsilon\) system, where the data are extremely precise. A typical result of such analysis, using exponential moments in the static approximation, is \(m_b = 4.65 \pm 0.05\) GeV [23]. It turns out that estimates of \(m_b\) are affected by the approximations used to confront theory and experiment, and in particular by the role of the \(\alpha_s\) corrections [2,23].

In [18] the convenience of studying ratios of the exponential moments \((5)\) was pointed out. In fact, in ratios the importance of higher order radiative corrections could be reduced considerably, in particular in the non-relativistic limit. The latter corresponds to infinite quark mass, and only the leading mass term was considered in [18]. Moreover, since the ratios are found to depend sensitively on the quark mass, they should be suitable for the extraction of \(m_b\). As an attempt to pursue improvements over the past analyses, it should be interesting to reconsider the QCD sum rules proposed in [18], including the next-to-leading mass corrections to the non-relativistic limit [24].

To this aim, following the familiar procedure, we start from the correlator:

\[
\Pi_{\mu\nu}(q) = i \int d^4x \exp(\mathrm{i}q x) <0|T \left(V_\mu(x) V_\nu(0)\right)^\dagger|0>
= (-g_{\mu\nu}q^2 + q_\mu q_\nu) \Pi(q^2), \tag{21}
\]

with \(V_\mu(x) = \bar{b}(x)\gamma_\mu b(x)\). From the exponential moment

\[
\Pi(\sigma) = \frac{1}{\pi} \int_0^\infty ds \exp(-\sigma s) \mathrm{Im}\Pi(s), \tag{22}
\]
we define the ratio, sensitive to the mass of the ground state:

\[
R(\sigma) = -\frac{d}{d\sigma} [\ln \Pi(\sigma)] = \frac{\Pi'(\sigma)}{\Pi(\sigma)},
\]

which tends to \(m_\Upsilon^2\) for \(\sigma \to \infty\).

In the non-relativistic limit, \(\sqrt{s} = 2m_b + \omega\), Eqs.(22) and (23) become respectively:

\[
\Pi(\tau) = \frac{1}{\pi} \int_0^\infty d\omega \exp (-\tau \omega) IIm(\omega),
\]

where \(\sigma = \tau/4m_b\), and

\[
R(\sigma = \tau/4m_b) = 2m_b - \frac{d}{d\tau} [\ln \Pi(\tau)].
\]

At the two-loop level in perturbative QCD [25]:

\[
\frac{1}{\pi} IIm(\Pi(q^2)_{AF}) = \frac{1}{8\pi^2} v(3 - v^2) \times \left\{ 1 + \frac{4\alpha_s}{3} \left[ \frac{\pi}{2v} - \frac{(v + 3)}{4} \left( \frac{\pi}{2} - \frac{3}{4\pi} \right) \right] \right\} \times \Theta(s - 4m_b^2),
\]

where \(v = \sqrt{1 - 4m_b^2/s}\) (\(m_b\) is here the b-quark pole mass). The leading non-perturbative contribution is given by the gluon condensate [1]:

\[
\Pi(q^2)_{NP} = \frac{1}{48q^4} \left[ \frac{3(v^2 + 1)(1 - v^2)^2}{2v^5} \ln \frac{1 + v}{1 - v} - \frac{3v^4 - 2v^2 + 3}{v^4} \right] \frac{\alpha_s}{\pi} G^2.
\]

Expanding (26) to leading plus next-to leading orders in \(m_b \to \infty\), we finally obtain the following QCD expression for \(R(\tau)\) defined in Eq.(25):

\[
R(\tau) = 2m_b \left\{ 1 + \frac{3}{4m_b\tau} \left( 1 - \frac{5}{6m_b\tau} \right) - \frac{1}{3} \left( \frac{\pi}{m_b\tau} \right)^{1/2} \alpha_s(\tau) \right\} \times \left\{ 1 - \frac{2}{\pi m_b\tau} \left( \frac{19}{2} \pi + \frac{3}{16\pi} - \frac{5}{8} \right) \right\} + \frac{\pi^2}{48} \left( \frac{\tau}{m_b} \right)^2 \frac{\alpha_s}{\pi} G^2,
\]

where

\[
\alpha_s(\tau) = \frac{12\pi}{23 \ln (4m_b/\tau A^2)}.
\]

The ratio (28) must be compared with a corresponding ratio involving the experimental data. We parametrize the latter by the sum of narrow \(\Upsilon\) resonances
followed by a hadronic continuum starting at a threshold $s_0$, modelled by perturbative QCD as given in (26):

$$
\Pi(\sigma)_{EXP} = \frac{27}{4\pi \alpha_{EM}^2} \sum_V \Gamma_V^\gamma m_V \exp(-\sigma m_V^2)
+ \frac{1}{\pi} \int_{s_0}^{\infty} ds \exp(-\sigma s) Im\Pi_{QCD}.
$$

(30)

Analogously to (13), one can notice from (28) that in practice the sum rule determines the difference between the hadronic ground state mass $m_\Upsilon$ and $2m_b$, which is related to scales relevant to the confinement dynamics.

The values of $m_b$ are then determined by matching the theoretical expression (28) to the square root of the experimental ratio

$$
R(\sigma)_{EXP} = -\frac{d}{d\sigma} \ln \Pi(\sigma)_{EXP}.
$$

(31)

Stability requires the matching to occur in a range of $\sigma$ and $s_0$ (actually the sum rule is extremely well saturated by the $\Upsilon$ family, so that the continuum has very little role), with corrections terms in (28) at a safe, few percent level. Varying $\Lambda_{QCD}$ and of $\langle \alpha_s G^2 \rangle$ in the currently allowed range one finds: $m_b = 4.71 \pm 0.05$ GeV.

As a comment to this determination, we notice that the subleading quark mass corrections can be important. For instance, the $O(\alpha_s/m_b \sqrt{m_b})$ term in (28) is of the same size and sign as the non-perturbative term, for the relevant values of $\tau$. Hence, it would not be fully justified to keep the latter and ignore the former. Thus, at this level, with all correction terms safely small, the accuracy on $m_b$ is essentially limited by the uncertainties in $\Lambda_{QCD}$, known to a factor of 2, and in the gluon condensate, known to a factor of 3.

4. Semileptonic B decays into charmed final states in the limit of infinitely heavy quarks

The hadronic matrix elements that describe the decays into negative-parity states:

$$
B \to D \ell \nu
$$

(32)

and

$$
B \to D^* \ell \nu
$$

(33)

can be written in terms of form factors as follows:

$$
< D(p') | \bar{c} \gamma_{\mu} b | B(p) > = G_+ (p + p')_\mu + G_- q_\mu
$$

(34)
\[
\frac{1}{i} \left< D^*(p', \epsilon) | \bar{c} \gamma_\mu b | B(p) \right> = i F \epsilon_{\mu \nu \alpha \beta} \epsilon^{* \nu} (p + p')^\alpha q^\beta \quad (35)
\]
\[
\frac{1}{i} \left< D^*(p', \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | B(p) \right> = F_0 \epsilon^*_\mu + F_+ (\epsilon^* \cdot p)(p + p')^\mu + F_- (\epsilon^* \cdot p) q^\mu \quad (36)
\]
where \( G_\pm, F, F_{0, \pm} \) are functions of \( q^2 = (p - p')^2 \).

These form factors will be discussed for finite values of the heavy quark masses in Section 6. Here we consider the results obtained in the limit \( m_Q \to \infty \). In the infinite heavy quark mass limit one can build up an effective field theory (HQET) \([5]\) and obtain useful predictions for the above form factors. More precisely, one assumes that \( m_Q \) is much larger than the average momentum of the light degrees of freedom \( q^\mu : m_Q >> |q^\mu| \). If we write \( \frac{p^\mu}{m_Q} = m_Q v^\mu + k^\mu = m_Q \nu_Q^\mu \), where \( v^\mu \) is the hadron velocity, \( \nu_Q^\mu \) is the quark velocity and \( k^\mu \) is a residual small momentum, in the limit

\[
m_Q \to \infty \, , \quad \frac{p^\mu}{m_Q} \text{ fixed} \quad (37)
\]
we get

\[
v_Q^\mu = v^\mu \quad (38)
\]
i.e. the heavy quark velocity is equal to the hadron velocity: QCD interactions do not change \( \nu_Q^\mu \) that is therefore conserved. Eq.(38) is called velocity superselection rule \([5]\).

One can show that in the limit (37) the heavy quark propagator becomes:

\[
\frac{\hat{p} + 1}{2} i \frac{1}{vk}, \quad (39)
\]
and the heavy quark gluon vertex assumes the form:

\[-ig T^a v_\mu, \quad (40)\]

because, as a consequence of the equations of the motion, the effective heavy quark field operator \( h_v(x) = \frac{\hat{p} + 1}{2} e^{im_Q v x} Q(x) \) satisfies the equation

\[
\hat{\nu} h_v = h_v. \quad (41)
\]

These results follow from a strong interaction effective Lagrangian which is the starting point of HQET:

\[
\mathcal{L} = \sum_v \mathcal{L}_{h_v} = \sum_v i \bar{h}_v v^\mu D_\mu h_v \quad . \quad (42)
\]
The effective Lagrangian (42) has two symmetries that are not present in the original QCD lagrangian and are a consequence of the heavy quark mass limit:

1) $SU(2)$ spin symmetry, which follows from the absence of $\gamma$ matrices in (42).

2) $SU(N)$ ($N = 2$ if only charm and beauty are considered) flavour symmetry, which follows from the independence of the lagrangian from the heavy quark flavour.

Similar to Eq.(2) relating $f_B$ to $f_D$, because of these symmetries one can relate several physical quantities that are in principle independent. In particular, all the form factors in Eqs.(34-36) can be expressed in terms of one universal form factor $\xi$ (Isgur-Wise function). $\xi$ is a function of $w = v \cdot v'$, where $v$ and $v'$ are the $B$ and $D$ four velocities:

$$\xi = \xi(w)$$ (43)

with

$$w = v \cdot v' = \frac{m_B^2 + m_D^2 - q^2}{2m_Bm_D}.$$ (44)

Notice that $\xi(1) = 1$ since the vector charge is the generator of the flavour symmetry.

The matrix elements in (34-36) are expressed in terms of $\xi$ (neglecting $1/m_Q$ corrections) as follows:

$$< D(v')|V^\mu|B(v) > = \sqrt{m_Bm_D} \xi(w) (v + v')^\mu$$ (45)

$$< D^*(v', \epsilon)|V^\mu - A^\mu|B(v) > = \sqrt{m_Bm_D} \xi(w) \left[ ie^{\mu\nu\alpha\beta} \epsilon_\nu v_\alpha v_\beta \right. - \left. (1 + w)\epsilon^{\mu*} + (\epsilon^* \cdot v)v'^\mu \right].$$ (46)

The relations among the form factors in Eqs.(34-36) and the universal function $\xi$ can be obtained by a straightforward calculation.

We shall also consider the semileptonic transitions

$$B \to D^{**}\ell\nu ,$$ (47)

where $D^{**}$ are positive parity $p$-wave charmed mesons. According to the quark model one expects four states: $^{2S+1}L_J = ^3P_2, ^3P_1, ^3P_0$ (corresponding to total
spin $S = 1$) and $^1P_1$ (corresponding to total spin $S = 0$). In the HQET the heavy quark spin $s_Q$ is decoupled from the light quark angular momentum $\vec{s}_\ell = \vec{J} - \vec{s}_Q$, so that it is convenient to work with the two degenerate multiplets $J^P = (0^+_1, 1^+_1)$ and $J^P = (1^+_3, 2^+_3)$ which differ by the value $s_\ell = 1/2$ and 3/2 respectively (the relative orbital momentum is $L = 1$). The 1$^+_1$ states defined above are given by the following linear combinations [26,28]:

$$|1^+_{3/2} > = \sqrt{\frac{2}{3}} |^1P_1 > + \sqrt{\frac{1}{3}} |^3P_1 >$$

$$|1^+_{1/2} > = \sqrt{\frac{1}{3}} |^1P_1 > - \sqrt{\frac{2}{3}} |^3P_1 >$$

while $|0^+_{1/2} > = |^3P_0 >$ and $|2^+_{3/2} > = |^3P_2 >$. The 2$^+_3/2$ state has been observed experimentally with a mass of 2460 MeV: it is denoted by $D^*_2(2460)$. The 1$^+_3/2$ meson can be identified with $D_1(2420)$ even though a 1$^+_1/2$ component is likely to be contained in this state [26,29]. They are both narrow ($\Gamma \leq 20$ MeV) since their strong decays occur in $d$-wave, in contrast with the states $D_0$ (the 0$^+_1/2$ state) and $D'_1$ (the 1$^+_1/2$ state) that can also decay in $s$-wave.

In the HQET the matrix elements for the decays (47) can be written in terms of two universal functions $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ [27]. The form factor $\tau_{3/2}(w)$ describes the transitions into the 2$^+_3/2$ and 1$^+_3/2$ states, and $\tau_{1/2}(w)$ those to the 1$^+_1/2$ and 0$^+_1/2$ states. The hadronic matrix elements for the decays (47) in the $m_Q \to \infty$ limit can be written as [27]:

$$< D_0(v')|A_\mu|B(v) > = -2\sqrt{m_Bm_D} \tau_{1/2}(w) (v - v')_\mu$$

$$< D'_1(v', \epsilon)|V_\mu - A_\mu|B(v) > = \sqrt{m_Bm_D} \tau_{1/2}(w) \times$$

$$\times \left[ 2(w - 1) \epsilon^*_\mu + 2(\epsilon^* \cdot v)v'_\mu - i\epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu}(v + v')^\alpha(v - v')^\beta \right]$$

$$< D_1(v', \epsilon)|V_\mu - A_\mu|B(v) > = \sqrt{m_Bm_D} \tau_{3/2}(w) \times$$

$$\times \left\{ \left[ (1 - w^2) \frac{\epsilon^*_\mu + (\epsilon^* \cdot v)}{\sqrt{2}} [3v_\mu + (w - 2)v'_\mu] ight. ight.$$ 

$$\left. + i \frac{w - 1}{2\sqrt{2}} \epsilon_{\mu\nu\alpha\beta} \epsilon^* \epsilon^\nu(v + v')^\alpha(v - v')^\beta \right\}$$

$$< D^*_2(v', \epsilon)|V_\mu - A_\mu|B(v) > = \sqrt{m_Bm_D} \tau_{3/2}(w)$$

$$\left[ i \frac{\sqrt{3}}{2} \epsilon_{\mu\alpha\beta} \epsilon^* \epsilon^\nu v_\nu(v + v')^\beta(v - v')^\gamma - \sqrt{3}(w + 1)\epsilon^*_\mu v^\alpha + \right.$$

$$\left. \sqrt{3} \epsilon^*_\alpha v^\alpha v^\beta v'_\mu \right]$$
where now $V^\mu = \bar{c}_v \gamma^\mu b_v$, $A^\mu = \bar{c}_v \gamma^\mu \gamma_5 b_v$, $c_v$ and $b_v$ are charm and beauty quark operators in HQET, and $m_D^{\ast \ast}$ is the mass of the positive parity charmed resonances. In the following Section we shall review the QCD sum rules calculation of $\xi(w), \tau_{1/2}(w)$ and $\tau_{3/2}(w)$.

5. Three point functions

To obtain QCD sum rules for the form factors one starts from the three-point correlators [30]:

$$\Pi_{\Gamma,\mu}(p, p', q) = i^2 \int dx dy e^{i(p' \cdot x - p \cdot y)} < 0|T(\bar{J}_\Gamma(x)J^V, A^\mu(0)J_5(y))|0> . \tag{53}$$

In (53) $J_{\Gamma}$ denotes the currents interpolating the final charmed meson with $\Gamma$ the appropriate set of Lorentz indices (e.g. $J_{\Gamma} = \bar{q}c, \bar{q}i\gamma_5 c$ and $\bar{q}(\gamma_\nu \overleftrightarrow{\partial_\mu} + \gamma_\mu \overleftrightarrow{\partial_\nu} - \frac{1}{2} g_{\mu\nu} \overleftrightarrow{\partial_\rho})c$ for 0± and 2+ respectively), and $J_5$ is the analogous operator for the initial $B$-meson, see Eq.(3). The form factors are extracted by expanding, in each case, Eq.(53) in independent Lorentz structures.

Similar to the analysis of two-point functions, we write a sum rule by computing (53) in two different ways: by including hadronic states plus a continuum of states modelled by perturbative QCD on one side, and by performing an OPE in the framework of QCD on the other side, accounting for the asymptotic freedom contribution plus higher dimensional non-perturbative condensates. Finally, these alternative representations are matched to one another, and the convergence is improved by one of the methods discussed in Sect.2, e.g. by taking the Borel transform of both sides of the sum rule. The double Borel transform, as applied to any of the Lorentz invariant functions $\Pi_i(p^2, p'^2, q^2)$ appearing on the left hand side of (53), looks as follows:

$$\Pi_i(\sigma, \sigma', q^2) = -\frac{1}{4\pi^2} \int \rho_i(s, s', q^2) \exp(-s\sigma) \exp(-s'\sigma') \, ds \, ds' , \tag{54}$$

where $\Pi_i(\sigma, \sigma', q^2)$ is obtained applying the Borel transform to the OPE of $\Pi_i(p^2, p'^2, q^2)$, and the double spectral function $\rho_i(s, s', q^2)$ is saturated by the lowest hadronic states, involving the form factors of interest, plus the QCD continuum starting at some thresholds $s_0, s'_0$.

In order to perform the heavy-quark mass limit and then estimate the dependence of the the form factors on $w$, we estimate the sum rules at $q^2 = 0$, 

and take the limit $m_Q \equiv m_b \to \infty$ with $m_c = m_b / \sqrt{Z}$, $Z$ fixed [31]. The form factors then become functions of $w$ via Eq.(44) at $q^2 = 0$, which reads:

$$\sqrt{Z} = w + \sqrt{w^2 - 1}.$$  \hspace{1cm} (55)

In the case of $0^- \to (0^-, 1^-)$ semileptonic transitions, this procedure reproduces the main features of the results obtained by directly working in the infinite heavy-quark limit [17,32]. As for the Borel parameters $\sigma$ and $\sigma'$, in the heavy-quark limit, in analogy with (12), we write $\tau_1 = 2m_b \sigma$ and $\tau_2 = 2m_c \sigma'$. For the continuum thresholds we put $s_0' = m_b^2 / Z + 2m_b \mu_+ / \sqrt{Z}$, $s_0 = m_b^2 + 2m_b \mu_-$ and for the meson masses we introduce the "binding energies" as follows: $m_B^2 = m_b^2 + 2m_b \omega_R$, $m_P^2 = m_b^2 / Z + 2m_b \omega_R' / \sqrt{Z}$ where $P = D, D^*, D^{**}$.

The resulting sum rules for the universal form factors are [33]:

$$\xi(w) = \frac{e^{(\omega_R \tau)}}{F^2} \left\{ \frac{3}{4\pi^2} \frac{Z}{(\sqrt{Z}+1)^3} H(w) - <\bar{q}q> \left[ 1 - \frac{\lambda^2 \tau^2 (w - 1)}{48} \right] e^{-\lambda^2 \tau^2 (w+1)/16} \right\} \hspace{1cm} (56)$$

$$\tau_{1/2}(w) = \frac{e^{(\omega_R \tau_1 + \omega_R' \tau_2)}}{f^{(+)}} \left\{ \frac{3}{8\pi^2} \frac{Z}{\sqrt{Z}+1} J(w) - <\bar{q}q> \left[ 1 - \frac{\lambda^2 \tau_1 \tau_2 (w + 1)}{12} \right] e^{-\left( \lambda^2 (\tau_1^2/8 + \tau_2^2/8 + 2w \tau_1 \tau_2/8) \right)} \right\} \hspace{1cm} (57)$$

$$\tau_{3/2}(w) = \frac{e^{(\omega_R \tau_1 + \omega_R' \tau_2)}}{f_T F^2} \frac{\sqrt{3}}{\pi^2} \frac{Z^{3/2}}{(\sqrt{Z}+1)^5} I(w), \hspace{1cm} (58)$$

where $\lambda^2 = M_0^2/2$ arises from the contribution of the dimension 5 operator introduced with regard to Eqs.(9) and (11). In Eq.(56) we have set $\tau_1 = \tau_2 = \tau/2$ and $\omega_R = \omega_R'$, which ensures $\xi(1) = 1$ for any value of $\tau$. Actually, once $\hat{F}$ is replaced by the sum rule (19), the dependence on $\omega_R$ drops from the ratio, and this minimizes the sensitivity of $\xi(w)$ on the values of this quantity.

Explicit expressions for $O(\alpha_s)$ corrections to (56) have been presented in [34]. The complete $\alpha_s$ corrections for the case of (57) and (58) are not available yet.

In (57), (58) we use the leptonic constants $f^{(+)}$ and $f_T$ defined as follows:

$$< 0 | V_{\mu} | M(0^+_{1/2}; p) > = i \frac{f^{(+)}}{\sqrt{m_Q}} p_\mu,$$  \hspace{1cm} (59)

$$< 0 | J_{\mu\nu} | M(2^+_{3/2}; p, \epsilon) > = \epsilon_{\mu\nu} f_T \sqrt{m_Q}.$$  \hspace{1cm} (60)
The numerical values for $f_T$ and $f^{(+)}$ are obtained, in analogy with $\hat{F}$, by two-point QCD sum rules [33] and the results are as follows: $f_T = (0.18 \pm 0.02) \text{ GeV}^2$ and $f^{(+)} = (0.46 \pm 0.06) \text{ GeV}^2$.

The functions $H(w)$, $J(w)$ and $I(w)$ in (56)-(58) are given by:

$$H(w) = \frac{1}{\sqrt{Z} - 1} \int_0^{2\mu_-} dy \int_0^{2\mu_-} dy' (y + y') e^{-\tau(y+y')/4} \Theta(2wyy' - y^2 - y'^2)$$

(61)

$$J(w) = \frac{1}{(\sqrt{Z} - 1)^3} \int_0^{2\mu_+} dy \int_0^{2\mu_-} dy' (y - y') e^{-(y\tau_2/2 + y'\tau_1/2)} \times$$

$$\Theta(2wyy' - y^2 - y'^2)$$

(62)

$$I(w) = \frac{1}{(\sqrt{Z} - 1)^3} \int_0^{2\mu_+} dy \int_0^{2\mu_-} dy' ((y^2 + 2yy')(Z - \sqrt{Z} + 1) - 3\sqrt{Z}y'^2) \times$$

$$e^{-(y\tau_2/2 + y'\tau_1/2)} \Theta(2wyy' - y^2 - y'^2)$$

(63)

where $\mu_{\pm}$ are the effective thresholds in the heavy quark limit.

The stability analysis of the results for the form factors is an obvious extension of the method described in Sect.2. One observes explicitly that the non-perturbative terms turn out to be numerically negligible for $\tau_{3/2}$, and therefore have not been included in (58). Moreover, it should be noted that the optimal values of the Borel parameters $\tau_1$ and $\tau_2$ as well as the continuum thresholds $\mu_+$ and $\mu_-$ are taken at the same values obtained in the analysis of two-point functions [33], in particular $\mu_- = 0.7 \pm 0.1 \text{ GeV}$ and $\mu_+ = 1.3 \pm 0.1 \text{ GeV}$. Finally, the form factors $\tau_{1/2}$ and $\tau_{3/2}$ should include a multiplicative $O(\alpha_s)$ correction which depends on the anomalous dimension of the current operators and on $w$. Its numerical effect is small and can be included in the theoretical uncertainties of the results. Furthermore, as far as $\xi$ is concerned, we notice that the large $O(\alpha_s)$ corrections affecting $\hat{F}$ (see discussion in Section 2) should drop from the ratio in (56), which also preserves $\xi(1) = 1$.

An important role in the numerical analysis of $\xi$ is played by the continuum. In spite of the denominators $(\sqrt{Z} - 1)^n = (w - 1 + \sqrt{w^2 - 1})^n$, with $n = 1, 3$, the integrals $H(w)$, $J(w)$ and $I(w)$ are finite for $w \to 1$. On the other hand the slope of the universal function $\xi$ turns out to be divergent, a result which is unnatural. As shown in [17] this is an artifact of the approximations used in modelling the
higher mass resonances as a continuum of states. To cure this problem for the “Isgur-Wise” function $\xi(w)$ one can modify the integration region in (61) without affecting the normalization constraint at zero recoil $\xi(1) = 1$ (which is easily seen to be verified by the sum rule). One can replace, for example, the $\Theta$ function by $\Theta(2w y y' - y^2 - y'^2) \times \Theta[2\mu(1 + w - \sqrt{w^2 - 1} - y - y')]$ (case a) or, as another possibility, one can just change the upper limit of integration $2\mu \rightarrow 2\mu(1 + w + \sqrt{w^2 - 1})/(1 + w)$ (case b). Both procedures lead to a finite slope. However, they add some significant uncertainty to the determination of $\xi(w)$ for values of $w$ relevant to nonzero recoil, and in particular by varying the continuum integration domain in Eq.(56) one obtains different values for the slope $\xi'(1)$. The value that better fits the experimental data corresponds to the case a and is [17]:

$$\xi'(1) = -1.28 \pm 0.25,$$  \hspace{1cm} (64)

while in [35] it is suggested to adopt the case b, which leads to:

$$\xi'(1) = -0.70 \pm 0.10.$$  \hspace{1cm} (65)

An analogous result has been found in [34] including $O(\alpha_s)$ corrections. We observe that all these values satisfy the lower bound:

$$|\xi'(1)| \geq 0.25$$  \hspace{1cm} (66)

obtained in [36].

Regarding the numerical evaluation of $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ according to Eqs.(57)-(58), we notice that, in this case, there is no general normalization condition on the form factors at $w = 1$. In Fig.1 we depict the functions $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ for the central values of $\mu_+$ and $\mu_-$ and with $1/\tau_1 \simeq 1/\tau_2 \simeq 1.5 \text{GeV}$. We observe that the numerical values at $w = 1$: $\tau_{1/2}(1) \simeq \tau_{3/2}(1) \simeq 0.22$ are in reasonable agreement with the outcome of the constituent quark model in [27]. Moreover, they also agree with the estimate $\tau_{1/2}(1) \simeq 0.35 \pm 0.2$ given in [35]. Finally, we observe that the values in Fig.1 are inside the upper bound put by the optical sum rule [37]:

$$|\tau_{1/2}(1)|^2 + 2|\tau_{3/2}(1)|^2 < \frac{1}{2} \frac{m_D - m_e}{m_{D^{*\ast}} - m_D} \simeq 0.5.$$  \hspace{1cm} (67)

In Table 2 we list the values of the branching ratios resulting from the analysis presented in this Section.
| Channel         | Branching Ratio                          |
|-----------------|------------------------------------------|
| B → D(0−) ℓ ν  | 1.4 × 10^{-2} \left( \frac{V_{cb}}{0.04} \right)^2 |
| B → D(1−) ℓ ν  | 4.3 × 10^{-2} \left( \frac{V_{cb}}{0.04} \right)^2 |
| B → D_0 ℓ ν    | 5 × 10^{-4} \left( \frac{V_{cb}}{0.04} \right)^2 |
| B → D'_1 ℓ ν   | 7 × 10^{-4} \left( \frac{V_{cb}}{0.04} \right)^2 |
| B → D_1 ℓ ν    | 1 × 10^{-3} \left( \frac{V_{cb}}{0.04} \right)^2 |
| B → D_2^∗ ℓ ν  | 2 × 10^{-3} \left( \frac{V_{cb}}{0.04} \right)^2 |

**Table 2.** Branching ratios obtained for the various B-meson semileptonic transitions in the heavy-quark limit [33] (\(\tau_B = 1.21\) ps).

6. Semileptonic form factors for finite heavy quark masses

The method of QCD Sum Rules can be applied to the evaluation of the form factors in Eqs.(34-36) without performing the limit \(m_Q \to \infty\). This offers the possibility to evaluate systematically \(O(1/m_Q)\) corrections for the transitions \(B \to D, D^∗\). We shall consider also the form factors for decays into the positive parity \(0^+_{1/2}\) and \(1^+_{1/2}\) states (the other form factors have not been computed yet). The form factors are given by:

\[
\frac{1}{i} < D_0^{**}(p')|\bar{c}\gamma_\mu\gamma_5 b| B(p) >= g_+ (p' + p)_\mu + g_- q_\mu
\]

(68)

\[
\frac{1}{i} < D'_1(p', \epsilon)|\bar{c}\gamma_\mu b| B(p) >= i f \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(p' + p)^\alpha q^\beta
\]

(69)

\[
\frac{1}{i} < D'_1(p', \epsilon)|\bar{c}\gamma_\mu b| B(p) >= f_0 \epsilon^*_\mu + f_+ (\epsilon^* \cdot p)(p' + p)_\mu + f_- (\epsilon^* \cdot p)q_\mu
\]

(70)

where \(\epsilon^\mu\) is the \(D'_1\) polarization vector and the form factors \(g_\pm, f, f_0, f_\pm\) are functions of \(q^2\). For this application it is also necessary to evaluate via QCD sum rules the vacuum-to-meson constants:

\[
< 0|\bar{q}c|D_0 >= \frac{f_{D_0} m_{D_0}^2}{m_c}
\]

(71)
and
\[< 0|\bar{q}\gamma_{\mu}\gamma_5c|D'_1(p, \epsilon)> = \frac{m^2_{D'_1}}{g_{D'_1}}\epsilon_\mu(p).\] (72)

The result for the mass is \(m_{D_0} = m_{D'_1} = 2.5 \pm 0.1 \text{ GeV},\) and the leptonic constants are given in Table 3 [38].

| Leptonic constant | Theoretical result |
|-------------------|---------------------|
| \(f_{D_0}\)       | 170 ± 20 MeV        |
| \(g_{D'_1}\)      | 9.8 ± 1.5           |

**Table 3** Leptonic constants, see Eqs. (71), (72).

Our results for the values of the various form factors at \(Q^2 = 0\) are reported in Table 4. As for the \(Q^2\)-dependence of the form factors, it could be derived by QCD sum rules; however, for simplicity, we assume a simple pole behaviour \(F_i(Q^2) = F_i(0)/(1 + \frac{Q^2}{m^2}),\) which is adequate in the present case. This is also suggested by dispersion relations, where the pole masses are those of \(b\bar{c}\) mesons with appropriate quantum numbers.

| \(B \rightarrow D(0^-)\) | 0.67 ± 0.19 |
| \(B \rightarrow D^*(1^-)\) | 0.12 ± 0.02  | -0.05 ± 0.02 | 4.35 ± 0.15 |
| \(B \rightarrow D_0^{**}(0^+)\) | 0.30 ± 0.03 |
| \(B \rightarrow D^{**}(1^+)\) | 0.10 ± 0.03  | -0.02 ± 0.02 | 0.9 ± 0.3 |

**Table 4** Form factors for B decays for finite \(m_Q\)
The results for the widths are rather insensitive to the precise value of these masses; we take \( m = 6.34 \text{ GeV} \) for the \( 1^- \) pole (for \( G_+, F, f_0 \) and \( f_+ \)) and \( m = 6.73 \text{ GeV} \) for the \( 1^+ \) pole (for \( g_+, f, F_0 \) and \( F_+ \)).

From the values in Table 4 we can compute the branching ratios for the decays \( B \to D, D^*, B \to D_0, D'_1 \). The results are collected in Table 5.

| Decay mode   | BR (Theory ) | BR (Exp. [39]) |
|--------------|--------------|----------------|
| \( B \to D\ell\nu \) | \( 1.5 \left( \frac{V_{cb}}{0.04} \right)^2 \times 10^{-2} \) | \( 1.75 \pm 0.42 \pm 0.35 \times 10^{-2} \) |
| \( B \to D^*\ell\nu \) | \( 4.6 \left( \frac{V_{cb}}{0.04} \right)^2 \times 10^{-2} \) | \( 4.8 \pm 0.4 \pm 0.7 \times 10^{-2} \) |
| \( B \to D_0\ell\nu \) | \( 0.15 \left( \frac{V_{cb}}{0.04} \right)^2 \times 10^{-2} \) | \( \text{N.D.} \) |
| \( B \to D'_1\ell\nu \) | \( 0.15 \left( \frac{V_{cb}}{0.04} \right)^2 \times 10^{-2} \) | \( \text{N.D.} \) |

**Table 5** Branching ratios for finite \( m_Q \)

Comparing these results with those in Table 2, we observe that the \( 1/m_Q \) corrections affect much more strongly the decays into positive parity charmed states. This can be qualitatively understood by recalling that in the case of higher charmed resonances the expansion parameter can be expected to be \( \propto \frac{(m_{D^{**}} - m_c)}{m_c} \), which is a number of order one. For the transitions into \( D, D^* \) the corrections linear in \( 1/m_Q \) are forbidden in the most relevant form factors at \( w = 1 \) [40] and this seems to protect the leading order result. Actually, the systematic analysis of \( B \) decays into negative parity states at the next-to-leading order in \( 1/m_Q \) implies the introduction of four new functions, in addition to \( \xi \) [40]. Some of these functions are also subject to normalization conditions at the zero recoil point \( w = 1 \). They have been recently analyzed in the framework of QCD sum rules in [41].

**7. Concluding remarks**

Summarizing the results presented in the previous sections, QCD sum rules
provide a conceptually consistent (and numerically simple) method to estimate from first principles semileptonic and purely leptonic transitions of the $B$ meson.

For the case of $f_B$, predicted values are in the potential of a $B$ factory, and indeed such a measurement would be really welcome for theoretical reasons.

For $B \to D, D^*$ semileptonic transitions, the method can provide a systematic analysis of the relevant form factors, both for finite and for infinite $m_b$. In the last regard, the results (and the simplicity) of the HQET can be easily incorporated in this framework, and the Isgur-Wise function $\xi(w)$ can be predicted, naturally accommodating the zero-recoil normalization $\xi(1) = 1$. Order $\alpha_s$ effects as well as the form factors relevant to the general next-to-leading $1/m_Q$ structure can be also accounted for. Thus the situation with the transitions to negative-parity charmed states looks well settled from the theoretical point of view, waiting for more accurate experimental data, which would allow a precise and model independent determination of $V_{cb}$.

Also the $B \to D^{**}$ semileptonic transitions can be evaluated in the framework of QCD sum rules, where $D^{**}$ represents the orbitally excited charmed resonances. Indeed, the analysis of these decays is theoretically related to the previous ones in this framework, which therefore provides a unified treatment of all form factors in terms of the same fundamental parameters. Moreover, the case of $D^{**}$ final states can be discussed in the view of the heavy quark mass limit and the two universal functions $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ can be predicted. The current analysis, more difficult as it involves operators with derivatives of the quark fields, is not complete as it is in the case of $\xi(w)$, in particular the order $\alpha_s$ effects are not taken into account yet. In addition, the $\tau$’s are not normalized at zero recoil so that, in general, uncertainties are larger and predictions can be considered as more qualitative. Nevertheless, it is encouraging that the values of $\tau_i(1)$ thus found are in reasonable agreement with the independent quark model estimates, and comply with the theoretical upper bound from the optical sum rule.

Concerning the branching ratio of $B \to D^{**}$, the results indicate values of the order of $a \ few \times 10^{-3}$ for the sum of the various p-wave resonances, which is not enough to fill the observed gap between the inclusive semileptonic rate and the sum of the exclusive rates $B \to D, D^*\ell\nu$. Clearly, this issue deserves further theoretical attention and represents an interesting point for the physics at $B$ factories.
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