Modified Bonnor-Ebert spheres with ambipolar diffusion heating

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ABSTRACT

Magnetic fluctuations through the molecular cloud cores can produce ambipolar diffusion (AD) heating, which consequently can produce temperature gradients through the core. The aim of this paper is to investigate the effects of these produced temperature gradients on the radius and mass of the non-isothermal modified Bonnor-Ebert spheres (MBES). Here, we use the parameter $\kappa$ to represent the magnetic fluctuations through the molecular cloud cores. This parameter introduces the change of magnetic filed strength in the length-scale. The results show that increasing of $\kappa$ leads to an increase of the radius and mass of MBES. The most important result is existence of the gravitationally stable high-mass prestellar cores at the low-density molecular medium with great magnetic fluctuations.

Subject headings: ISM: clouds – stars: formation – hydrodynamics – instabilities

1. Introduction

The gravitational-unstable and collapsing subset of the molecular cloud cores are known as prestellar cores (Ward-Thompson et al. 1994), which will convert to an individual stellar system with its universal initial mass function (Offner et al. 2014). Thus, investigation of the prestellar cores is essential to understand the initial stages of star (and stellar system) formation. Many authors use the usual isothermal Bonnor-Ebert sphere (e.g. Stahler & Palla 2004, Section 9.1) to approximate the density profiles of observed prestellar cores (e.g, Marsh et al. 2016). However, in general, the assumption of isothermal case is not accurately valid because real dense cores are subjected to heating and cooling processes (Goldsmith & Langer 1978). For this purpose, some authors assumed non-isothermal cases to approximate
self-consistently the temperature and density profiles through the thermally balanced hydrostatic spherical prestellar cores (e.g., Evans II et al. 2001, Juvela & Ysard 2011, Spilä, Harju & Juvela 2015). In these works for non-isothermal modified Bonnor-Ebert spheres (MBES), the ambipolar diffusion (AD) heating was not considered, while in the magnetic fluctuating medium, this heating mechanism is very important (e.g., Li, Myers & McKee 2012).

Observations of prestellar cores show the effects of magnetic turbulence through them. For example, Hildebrand et al. (2009) developed a method based on a dispersion function about local mean magnetic fields. Besides providing a measure for the turbulent dispersion, their method also gives an accurate estimate of the turbulent to the mean magnetic field strength ratio. Koch, Tang & Ho (2010) applied and extended the method developed in Hildebrand et al. (2009) across a range of scales in the same star formation regions. They found the magnetic field turbulent dispersion, its turbulent-to-mean field strength ratio, and the large-scale polarization angle correlation length as a function of the physical scale at the star formation sites. Girart et al. (2013) also found evidence of magnetic field diffusion at the core scales, far beyond the expected value for AD. They concluded that it may be possible that the diffusion arises from fast magnetic reconnection in the presence of turbulence. Similarly, we can also refer to the recent observational evidences of Pillai et al. (2015) and Houde et al. (2016) for high-mass star-forming regions.

Magnetic turbulence can produce magnetic fluctuations through prestellar molecular cloud cores. Thus, prestellar cores are a magnetically fluctuating medium, and the AD heating is important and must be considered. For this purpose, we consider the physical situation of a spherically symmetric, gravitationally bound, thermally supported prestellar core, which is also heated by internal magnetic slippage of particles with AD mechanism in the magnetic fluctuating medium. Section 2 gives the formulation of the quasi-static thermally equilibrium prestellar cores, and in section 3 some remarkable conclusions are presented.

2. Hydrostatic thermally equilibrium cores

In spherical polar coordinates, the usual hydrodynamic equations for spherically symmetric thermally dominated cases are

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{GM}{r^2}, \tag{2}
\]
\[
\frac{\partial M}{\partial t} + u \frac{\partial M}{\partial r} = 0, \quad \frac{\partial M}{\partial r} = 4\pi r^2 \rho,
\]
(3)

\[
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \gamma p \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) = -(\gamma - 1) \rho \Omega(\rho, T),
\]
(4)

\[
p = \frac{k_B}{\mu m_H} \rho T,
\]
(5)

where the mass density \(\rho\), the enclosed mass \(M\), the radial flow velocity \(u\), the thermal gas pressure \(p\), and the temperature \(T\), in general, depend on the radius \(r\) and time \(t\); \(G\) is the gravitational constant, \(\gamma\) is the polytropic index, and \(k_B, \mu \approx 2.3\) and \(m_H\) are Boltzmann constant, the mean molecular weight and the hydrogen mass, respectively. The net cooling function is represented by \(\Omega(\rho, T)\).

In the stationary \((\partial/\partial t = 0)\) quasi-static \((u \rightarrow 0)\) thermally equilibrium state, the net cooling function \(\Omega(\rho, T)\) must be zero at each radius \(r\) (i.e., locally thermal balance). To calculate the thermal balance within the molecular cloud cores, we need to consider heating and cooling processes affecting the gas and the dust. For cooling processes, we use the Table 2 of Goldsmith (2001) to parameterize the cooling function as \(\Lambda_0 \left( T/10^K \right)^\beta \) where \(\Lambda_0\) and \(\beta\) are generally functions of density (see, Fig. 1 of Nejad-Asghar 2011). There are several different heating mechanisms in models of interstellar matter. Here, we consider heating processes of cosmic rays (e.g., Glassgold & Langer 1973) and dissipation of magnetic energy via AD (e.g., Li, Myers & McKee 2012).

The heating due to cosmic rays with sufficient energies (\(\sim 100\text{MeV}\)) to penetrate dense clouds is commonly about \(\Gamma_{CR} = 2.5 \times 10^{-4}\text{erg g}^{-1}\text{s}^{-1}\), with assumption of an ionization rate per \(H_2\) molecule of \(2 \times 10^{-17}\text{s}^{-1}\) and a mean energy gains per ionization of \(19\text{eV}\) (Glassgold & Langer 1973). The heating due to AD can be obtained by low ionization fraction approximation. In the limit of low ionization fraction (i.e., \(\rho = \rho_n + \rho_i \approx \rho_n\), where \(\rho_n\) and \(\rho_i\) are neutral and ion densities, respectively), the inertia of charged particles being negligible so that the Lorentz force will be balanced by the equally important drag force. The drag force per unit volume exerted on the neutrals by ions is \(f_d = \gamma_{AD} \rho_i \rho v_d = \gamma_{AD} \epsilon \rho^{3/2} v_d\), where \(\gamma_{AD} \sim 3.5 \times 10^{13}\text{cm}^3\text{g}^{-1}\text{s}^{-1}\) is the collisional drag coefficient in the molecular clouds, and we used the relation \(\rho_i = \epsilon \rho^{1/2}\) between ion and neutral densities in the local ionization equilibrium state with \(\epsilon \sim 3 \times 10^{-16}\text{g}^{1/2}\text{cm}^{-3/2}\) (Shu 1992). In this way, the drift velocity of ions is

\[
v_d = v_i - v_n \approx \frac{1}{4\pi \gamma_{AD} \epsilon \rho^{3/2}} (\nabla \times B) \times B.
\]
(6)

The magnitude of drift velocity, \(v_d\), is inversely proportional to the power of density and directly proportional to the magnetic field strength and its gradient. Shu (1992, Equation 27.9) used typical values of \(B \Delta B/\Delta x \sim (30\mu G)^2/0.1\text{pc}\) to estimate the typical drift speeds through the molecular clouds. Li, Myers & McKee (2012) divided the magnetic field into a steady...
component and a fluctuating one, and used the mean squared method to estimate upper limits on the drift speed. Here, we use the parametric relation $v_d = B^2 \kappa^2 / 16 \pi^2 \gamma_{AD} \rho^{-5/2}$, where the parameter $\kappa \equiv \Delta B/\Delta x$ is the change of magnetic field strength in the length-scale $\Delta x$. In this way, the heating due to ambipolar diffusion can be represented as

$$\Gamma_{AD} = \frac{f_d v_d}{\rho} = \frac{B^2 \kappa^2}{16 \pi^2 \gamma_{AD} \rho^{-5/2}}.$$  \hspace{1cm} (7)

The magnetic field strength, $B$, is evaluated in the Troland & Crutcher (2008) for a set of 34 molecular cloud cores. Their evaluations show that the magnetic field strengths are in the range of 0.5 to 50 $\mu$G. The authors use a scaling relation of the field strength with density, which is usually parameterized as a power law, $B \propto \rho^n$ (Crutcher 2012). In the strong-field models, $\eta \lesssim 0.5$ is predicted (e.g., Mouschovias & Ciolek 1999), while for weak magnetic fields, $\eta \approx 0.66$ is predicted (Mestel 1966). Here, we assume that the magnetic field is so strong that as ambipolar diffusion increases the mass-to-flux ratio, the density $\rho$ increases faster than $B$. Thus, we are in strong-field models with $\eta \lesssim 0.5$. According to the Figure 6 of Crutcher (2012), we choose an approximate value of $B \approx 12$ $\mu$G for number density $n = \rho/\mu m_H \approx 10^3$ cm$^{-3}$, and $B \approx 100$ $\mu$G for number density $n = \rho/\mu m_H \approx 10^6$ cm$^{-3}$. By choosing these values for the magnetic field strengths, we have a power-law approximation as $B \approx 100$ $\mu$G $(\frac{n}{10^6 \text{cm}^{-3}})^{0.3}$. In this way, the condition for domination of thermal pressure over the magnetic pressure is $\log(n/\text{cm}^{-3}) \gtrsim 7.25 - 2.5 \log(T/\text{K})$. According to the Table 1 of Koch, Tang & Ho (2010), the maximum value of $\Delta B$ is $\sim 0.9 B$, which corresponds to cloud core with dimension 60 mpc at W51 e2/e8, observed with SMA interferometer. On the other hand, like the work of Li, Myers & McKee (2012), we assume that the AD heating, which is produced by magnetic fluctuations, is wave dissipation with its maximum wavelength equal to the physical scale $l_0$. In this way, gradients cannot exist on scales larger than $l_0/2\pi$. Here, we choose the minimum value of $l_0$ equal to 23 mpc, which corresponds to the cloud core at Orion BN/KL, observed with SMA interferometer (Koch, Tang & Ho 2010, Table 1), so that we have $\Delta x \sim 3$ mpc. Thus, we expect the maximum value of the parameter $\kappa$ in the prestellar cores be equal to $\sim 3 \mu$G/1 mpc. Since there is a lot of uncertainties in the values of the magnetic field fluctuations and dissipation scales, we assume a parametric value for $\kappa$, which includes roughly all of these uncertainties. We express $\kappa$ in the unit of $0.3 \mu$G/1 mpc and perform the calculations for $0 \lesssim \kappa \lesssim 10 \times 0.3 \mu$G/1 mpc. Thus, the AD heating (7) can be rewritten as

$$\Gamma_{AD} = 2.3 \times 10^{-9} \left( \frac{\kappa}{0.3 \mu \text{G/1mpc}} \right)^2 \left( \frac{n}{10^6 \text{cm}^{-3}} \right)^{-1.9} \text{erg g}^{-1} \text{s}^{-1}. \hspace{1cm} (8)$$

The thermal balance (i.e., $\Omega(\rho, T) = \Lambda_0(n) (T/10 \text{K})^{\beta(n)} - \Gamma_{CR} - \Gamma_{AD} = 0$) at the
molecular clouds leads to a relation between the temperature and density as follows

\[ T = 10 \text{ K} \times \left[ \frac{\Gamma_{CR}}{\Lambda_0(n)} + \frac{2.3 \times 10^{-9}}{\Lambda_0(n)} \left( \frac{\kappa}{0.3 \mu \text{G}/1 \text{ mpc}} \right)^2 \left( \frac{n}{10^6 \text{ cm}^{-3}} \right)^{-1.9} \right]^{-\frac{1}{\beta(n)}}. \] (9)

The results of (9) are plotted in the Fig. 1 for different values of \( \tilde{\kappa} \equiv \kappa/\left(0.3 \mu \text{G}/1 \text{ mpc}\right) \). The dash-line in this figure, \( \log(n/\text{cm}^{-3}) = 7.25 - 2.5 \log(T/\text{K}) \), shows that our assumption of thermally dominated prestellar core is approximately correct, especially for greater values of \( \kappa \). Departure from the thermally dominated assumption leads to consideration of the pressures from both MHD turbulence and the mean field gradient; the resulting equilibrium configuration will not be spherical. Here, we use the spherical assumption for prestellar cores as a first approximation, and the effects of magnetic pressures will be considered in the subsequent work.

Now, knowing the relation between temperature and density, equation (5) leads us to determine the gradient of pressure as

\[ \frac{dp}{dr} = \frac{k_B}{\mu m_H} \left( T + \rho \frac{dT}{d\rho} \right) \frac{d\rho}{dr}, \] (10)

so that the stationary \((\partial/\partial t = 0)\) quasi-static \((u \to 0)\) state of the momentum equation (2) becomes

\[ \frac{d\rho}{dr} = -\frac{\mu m_H G}{k_B} \frac{M \rho}{r^2 \left( T + \rho \frac{dT}{d\rho} \right)}. \] (11)

We use the non-dimensional quantities \( \tilde{n} \equiv n/n_r, \tilde{T} \equiv T/T_r, \tilde{\kappa} \equiv r/(k_B T_r/\mu m_H)^{1/2}, \) and \( \tilde{M} \equiv M/4\pi(k_B T_r/\mu m_H)^{1/2} \mu m_H n_r \), where \( n_r = 10^6 \text{ cm}^{-3} \) and \( T_r = 10 \text{ K} \) are the reference density and temperature, respectively. In this way, the equations (3) and (11) become

\[ \frac{d\tilde{M}}{d\tilde{r}} = \tilde{r}^2 \tilde{n}, \] (12)

\[ \frac{d\tilde{n}}{d\tilde{r}} = -\frac{\tilde{M} \tilde{n}}{\tilde{r}^2 (\tilde{T} + \tilde{n} \frac{d\tilde{T}}{d\tilde{n}})}, \] (13)

which can be integrated numerically (e.g., with Runge-Kutta method), from the origin \( \tilde{r} = 0 \) with the boundary conditions \( \tilde{n}(0) = \tilde{n}_c \) and \( \tilde{M}(0) = 0 \), where \( \tilde{n}_c \) is the non-dimensional central density.

The results for density and temperature profiles of the stationary quasi-static thermally equilibrium prestellar cores with central density \( \tilde{n}_c = 0.1, 1, \) and 10 are shown in the Fig. 2. The profiles of isothermal case \((\tilde{T} = \text{const.})\) are also plotted in this figure, for comparison.
As can be seen, consideration of the AD heating departs the profiles of the stationary quasi-static thermally equilibrium prestellar cores from the isothermal case. These departures are greater for greater magnetic fluctuating parameter $\kappa$, at the envelopes of the prestellar cores with smaller densities. This result for departure of profiles is consistent with the Fig. 1, which shows that importance of AD heating is at lower densities.

In the actual cases, the density does not fall to zero, but to some value $\tilde{n}_0$ characterizing the external medium. If we fix $\tilde{n}_0$, for each density contrast $\tilde{n}_c/\tilde{n}_0$, we can find the radius $\tilde{r}_0$ of the prestellar core and its corresponding non-dimensional mass $\tilde{M}_0$. The molecular cloud core mass, $M_0$, versus logarithm of density contrast, $n_c/n_0$, are plotted in Fig. 3 for three different values of density of external medium equal to $n_0 = 10^3$, $10^4$, and $10^5$ cm$^{-3}$. With increasing density contrast, core mass first rises to a maximum value of $M_{MBES}$, attained at the radius $r_{MBES}$. Then, the mass drops to a minimum, and eventually approaches in an oscillatory fashion to the asymptotic limit value, which represents the mass of the singular sphere (with density $\propto 1/r^2$). In the isothermal Bonnor-Ebert case, the density contrast corresponded to the first maximum is $n_c/n_0 = 14.1$ (e.g., Stahler & Palla 2004, Figure 9.2).

Prestellar cores with low density contrast are mainly confined by the external pressure, and not self-gravity. This situation changes as we progress along the curves in the Fig. 3 to the right of the first maximum (i.e., models of higher $n_c/n_0$). These prestellar cores have gravity-dominated configurations, and an arbitrarily small initial perturbation in the structure grows rapidly with time, leading ultimately to collapse. In the isothermal case, a cloud with the critical value of mass and radius corresponding to the first maximum is known as the Bonnor-Ebert sphere. For non-isothermal cases, we use the nomenclature of modified Bonnor-Ebert sphere (MBES). The critical values of mass and radius of the MBESs, versus $\kappa$, are plotted in the Fig. 4 for different values of external densities.

### 3. Summary and conclusions

The fluctuations of the magnetic field strength through the molecular clouds can produce AD heating. In this research, the effects of AD heating to produce temperature gradient through thermally supported, dynamically balanced prestellar spherical cores are investigated. This temperature gradient changes the maximum allowed mass and radius of the gravitationally stable cores (i.e., MBES). The issue of MBES is important in the star formation theory.

Thermal balance of the molecular clouds demand forcefully a relation between density and temperature as outlined in Fig. 1 for different values of magnetic fluctuation parameter
κ. From thermal balancing state (Fig. 1), we know the relation between temperature and density, so that we can find both temperature $T(r)$ and density $\rho(r)$ in which the stationary hydrostatic equilibrium of prestellar cores are satisfied. The results for temperature and density profiles are shown in the Fig. 2. The inclusion of AD heating leads to the departure of the density and temperature profiles from the isothermal case, especially in the outer regions of envelope with lower densities. This effect leads to the formation of the gravitationally stable high-mass prestellar cores in the low density medium.

The Fig. 3 depicts the core mass as function of density contrast. For fixed external density $n_0$, increasing of central density leads to rise of core mass to a maximum value of $M_{MBES}$ attained at the critical radius $r_{MBES}$. The prestellar cores with masses lower than $M_{MBES}$ are gravitationally stable, while if the core mass reaches to the critical value $M_{MBES}$ or a larger value, the core will be gravitationally unstable and collapse occurs. The Fig. 3 shows that in the high-density medium, ambipolar diffusion is not important, while in the low-density medium, it is important so that we can have high-mass cores that are gravitationally stable.

Importance of magnetic fluctuation parameter $\kappa$ to produce high-mass prestellar cores at low-density molecular clouds are shown in Fig. 4. In isothermal case, the critical density contrast for first maximum is attained at $n_c/n_0 = 14.1$, while in the non-isothermal cases without AD heating ($\kappa = 0$), this value increases so that its corresponding critical radius $r_{MBES}$ lies below the isothermal value, as outlined by Sipilä, Harju & Juvela (2015). Considering of magnetic fluctuation and AD heating ($\kappa \neq 0$) increases the $r_{MBES}$, and leads to existence of gravitationally stable high-mass prestellar cores at the low-density molecular medium.

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Fig. 1.— The temperature-density relation for thermal balancing state of the molecular clouds. The parameter $\tilde{\kappa} \equiv \Delta B / \Delta x$ represents the change of magnetic field strength at length-scale $\Delta x$, in unit of $0.3 \, \mu G / 1 \, \text{mpc}$. The dash-line is $\log(n/cm^{-3}) = 7.25 - 2.5 \log(T/K)$ in which for densities greater than this value, the assumption of thermally dominated prestellar core is approximately correct.
Fig. 2.— The density and temperature profiles of dynamically balanced, spherical molecular cloud cores in the isothermal case (solid), and in the non-isothermal cases with $\kappa = 0$ (dash), 5 (dot), and 10 (dash-dot), for three values of central density equal to $10^5\text{ cm}^{-3}$ (upper-panels), $10^6\text{ cm}^{-3}$ (middle-panels), and $10^7\text{ cm}^{-3}$ (lower-panels).
Fig. 3.— The core mass versus density contrast for fixed external densities $n_0 = 10^3 \text{ cm}^{-3}$ (upper-panel), $10^4 \text{ cm}^{-3}$ (middle-panel), and $10^5 \text{ cm}^{-3}$ (lower-panel), for isothermal case (solid), and non-isothermal cases with $\tilde{\kappa} = 0$ (dash), 5 (dot), and 10 (dash-dot). In the lower-panel, the non-isothermal cases with different $\tilde{\kappa}$ overlap to each other.
The mass and radius of MBES versus magnetic fluctuation parameter $\kappa$, for fixed external density $n_0 = 10^3 \text{cm}^{-3}$ (solid), $10^4 \text{cm}^{-3}$ (dot), and $10^5 \text{cm}^{-3}$ (dash). The corresponded isothermal results are depicted by arrows on the vertical axes.