Nonzero neutrino masses are the first definitive need to extend the standard model. After reviewing the basic framework, I describe the status of some of the major issues, including tests of the basic framework of neutrino masses and mixings; the question of Majorana vs. Dirac; the spectrum, mixings, and number of neutrinos; models, with special emphasis on constraints from typical superstring constructions (which are not consistent with popular bottom-up assumptions); and other implications.

1 Neutrino Preliminaries

1.1 Definitions

A Weyl fermion is the minimal fermionic field. It has two degrees of freedom of opposite chirality, related by CPT or Hermitian conjugation, such as $\psi_L \leftrightarrow \psi_R$. (Which is called the particle and which the antiparticle is a matter of convenience.) An active (a.k.a. ordinary or doublet) neutrino $\nu_L$ is in an $SU(2)$ doublet with a charged lepton partner and therefore has normal weak interactions. Its CPT conjugate $\nu^c_R$ is a right-handed antineutrino. A sterile (a.k.a. singlet or right-handed) neutrino $N_R \leftrightarrow N^c_L$ is an $SU(2)$ singlet. It has no interactions except by mixing, Higgs couplings, or beyond the standard model (BSM) interactions. Sterile neutrinos are present in almost all extensions of the standard model. The only questions are whether they are light and whether they mix with the active neutrinos, as suggested by the LSND experiment.

Fermion mass terms convert a spinor of one chirality into the other. A Dirac mass connects two distinct Weyl spinors (usually active to sterile), such as $m_D \bar{\nu}_L N_R + h.c.$ There are four components ($\nu_L, N_R$ and their conjugates), and one can define a conserved lepton number $L$. An active-sterile Dirac mass violates weak isospin by 1/2 unit, $\Delta I = 1/2$, and can be generated by a Yukawa coupling to a Higgs doublet. This is analogous to quark and charged lepton masses, but raises the question of why $m_D$ (i.e., the Yukawa coupling) is so small. There are variant types of Dirac masses in which an active $\nu_L$ is coupled to a different flavor of active $\nu^c_R$, e.g., $m_D \bar{\nu}_e L \nu^c_\mu R$, in which $L_e - L_\mu$ conserved. This has $\Delta I = 1$ and may emerge as a limit of a model with Majorana masses.

A Majorana Mass connects a Weyl spinor with its own CPT conjugate. There are only two components, and lepton number is necessarily violated by two units, $\Delta L = \pm 2$. An active neutrino Majorana mass term $\frac{1}{2}(m_T \bar{\nu}_L \nu^c_R + h.c.)$ has $\Delta I = 1$, requiring a Higgs triplet or a higher dimensional operator with two Higgs doublets. A sterile Majorana mass term $\frac{1}{2}(m_S \bar{N}^c_L N_R + h.c.)$ has $\Delta I = 0$. It could in principle be a bare mass, but most concrete models have additional constraints and require the expectation value of a Higgs singlet.

One can also consider mixed models involving both Majorana and Dirac mass terms. For example, the case $m_S$ (or $m_T$)
\( \ll m_D \) involves two almost degenerate Majorana neutrinos (pseudo-Dirac\(^a\)). Another well-known example is the seesaw model, in which \( m_S \gg m_D \) and \( m_T = 0 \).

For three families the most general \((6 \times 6)\) mass matrix is

\[
L = \frac{1}{2} \left( \bar{\nu}_L \, N_L^c \right) \left( \begin{array}{cc} m_T & m_D \\ m_D^T & m_S \end{array} \right) \left( \begin{array}{c} \nu_R^c \\ N_R \end{array} \right) + \text{hc},
\]

where \( \nu_L \) (\( N_R \)) represent 3 flavors of active (sterile) neutrinos, and the \( 3 \times 3 \) submatrices are (a) the active Majorana mass matrix\(^b\) \( m_T = m_T^T \), generated by a Higgs triplet; (b) the Dirac mass matrix \( m_D \), generated by a Higgs doublet; and (c) the sterile Majorana mass matrix \( m_S = m_S^T \), generated by a SM singlet.

### 1.2 Neutrino Mass Patterns

The Solar neutrino oscillation parameters\(^1\) are now confirmed by SNO and Kamland to fall in the large mixing angle (LMA) region, with \( \Delta m^2_{\odot} \sim 8 \times 10^{-5} \text{ eV}^2 \) and large but non-maximal mixing angle \( \theta_\odot \). The atmospheric neutrinos are characterized by \( |\Delta m^2_{\text{Atm}}| \sim 2 \times 10^{-3} \text{ eV}^2 \), with large mixing consistent with maximal. Reactor experiments establish that the third angle, \( U_{e3} \) is small. For maximal atmospheric mixing and neglecting \( U_{e3} \) this would imply mass eigenstates

\[
\begin{align*}
\nu_3 &\sim \nu_+, \\
\nu_2 &\sim \cos \theta_\odot \, \nu_- - \sin \theta_\odot \, \nu_e, \\
\nu_1 &\sim \sin \theta_\odot \, \nu_- + \cos \theta_\odot \, \nu_e,
\end{align*}
\]

where \( \nu_\pm \equiv \frac{1}{\sqrt{2}} (\nu_\mu \pm \nu_\tau) \). Depending on the sign of \( \Delta m^2_{\text{Atm}} \) one can have either the normal hierarchy, which is analogous to the quarks and charged leptons, or the inverted one. Degenerate patterns refer to the possibility that the overall mass scale, which is

\(^a\) A Dirac neutrino can be thought of as the limiting case, with two degenerate Majorana neutrinos with maximal mixing and opposite CP parity.

\(^b\) The off diagonal terms in \( m_{T,S} \) are still considered Majorana as long as there is no way to define a conserved \( L \).

not determined by oscillation experiments, is large compared to the mass differences. This was once strongly motivated by hot dark matter (HDM) scenarios, but now HDM serves only as an upper limit. The inverted and degenerate patterns may be radiatively unstable\(^2\).

### 2 The Basic Framework

Many ideas\(^1\) have previously been put forward as alternatives to oscillations amongst the 3 active neutrinos for the Solar and atmospheric neutrinos. These include oscillations into sterile neutrinos; neutrino decay; decoherence between the quantum components of the wave function; new flavor changing interactions; Lorentz, CPT, or equivalence principle violation\(^3\); and (for the Solar neutrinos) large magnetic moments and resonant spin flavor precession (although there are strong constraints from stellar cooling)\(^4\). These could typically describe the contained (lower energy) atmospheric events, but most schemes were excluded by (higher energy) upward throughgoing events, e.g., because they depend on \( LE \) or \( L \) rather than \( L/E \). This conclusion has been further strengthened by observation of a dip in the \( L/E \) spectrum by SuperKamiokande, a clear indication of oscillations. Similarly, the KamLAND reactor results eliminated alternatives to the LMA oscillation picture for the Solar neutrinos; this has also been strengthened by a suggestion of a dip in the KamLAND spectrum.
data is also now sufficiently good to indirectly
demonstrate the need for MSW (matter) effects in the Sun\(^5\), although direct confirma-
tion would require observation of the transition between the low (high) energy vacuum
(MSW) regimes in a future \(pp\) neutrino ex-
periment.

Thus, these alternatives are now ex-
cluded as the dominant mechanism. Em-
phasis has shifted to precision theoretical\(^6\)
and experimental studies to search for or con-
strain such effects as small perturbations\(^c\)
and to further test the Standard Solar Model.

3 Majorana or Dirac

One of the most important questions for un-
derstanding the origin of neutrino masses is
whether they are Majorana or Dirac. Many theorists are convinced that they
must be Majorana, because (a) no stan-
dard model gauge symmetry forbids Majo-
rana masses; (b) nonperturbative electroweak
processes (sphalerons) and black holes vi-
olate \(L\) (in practice, such effects are negligi-
ably small); and (c) standard grand unified
theories (GUTS) violate \(L\). However, these
arguments are not compelling because there
could be additional symmetries to forbid or
strongly suppress \(L\) violation, analogous to
the strong suppression of proton decay. For
example, there could well be new gauge sym-
metries (e.g., a \(Z'^9\)) at the TeV-scale which
could forbid Majorana masses. Similarly,
constraints in superstring constructions are
extremely restrictive and could forbid or sup-
press them\(^10\). Therefore, Dirac or pseudo-
Dirac masses are serious possibilities.

The only practical way to distinguish
Majorana and Dirac masses experimentally
is neutrinoless double beta decay \((\beta\beta_{0\nu})\)\(^11\).
If observed, \(\beta\beta_{0\nu}\) would imply Majorana

\(^{c}\)For example, small flavor changing operators\(^7\) could
shift the Solar parameters slightly, to coincide with a
different KamLAND oscillation minimum\(^8\).

masses\(^d\). As discussed below, the converse
is not true.

4 The Spectrum

Another key uncertainty (and constraint on
models) is whether there is a normal, in-
verted, or degenerate spectrum. It should
eventually be possible to distinguish the nor-
mal and inverted hierarchies using long base-
line oscillation effects, because the MSW
matter effects associated with \(\Delta m^2_{\text{Atm}}\) change
sign. It may also be possible to distinguish
from the observed energy spectrum in a fu-
ture supernova because of matter effects in
the supernova and in the Earth\(^12\). Planned
and proposed \(\beta\beta_{0\nu}\) experiments would be
sensitive to Majorana masses predicted by
the inverted and degenerate spectra. Unfor-
nately, nonobservation could be due either
to a normal hierarchy or to Dirac masses.

There are three complementary future
probes of the absolute mass scale:

- Tritium beta decay experiments measure
the quantity, \(m_\beta \equiv \Sigma_i |U_{ei}|^2 |m_i|\), where \(U\)
is the leptonic mixing matrix. The KA-
TRIN experiment should be sensitive to
\(m_\beta \sim 0.2\ \text{eV}\), compared to the present up-
per limit of \(2\ \text{eV}\). It could only see a signal
for the degenerate cases.

- Cosmological (large scale structure) obser-
vations are sensitive to \(\Sigma \equiv \Sigma_i |m_i|\). The
most stringent claimed limit is \(\Sigma \lesssim 0.42\)
eV\(^13\). Using future Planck data, it may be
possible to extend the sensitivity down to
\(0.05 - 0.1\ \text{eV}\), close to the minimum value
\(0.05\ \text{eV} \sim \sqrt{|\Delta m^2_{\text{Atm}}|}\) allowed by the oscil-
lation data. However, there are significant
theoretical uncertainties.

- The \(\beta\beta_{0\nu}\) amplitude is proportional to the
effective mass \(m_{\beta\beta} \equiv \Sigma_i U^2_{ei} m_i\), where there

\(^{d}\)\(\beta\beta_{0\nu}\) could also be driven by new interactions, such
as \(R\)-parity violation in supersymmetry. These would
also lead to Majorana masses at some level.
Figure 3. Cosmological and oscillation constraints on $\Sigma$ and $m_{\beta\beta}$ for the normal (N.H.) and inverted (I.H.) hierarchies.

...can be cancellations due to signs or (Majorana) phases in $U_{ei}$ or in $m_i$ (depending on conventions). Proposed experiments would be sensitive to $m_{\beta\beta} \sim 0.02 \text{ eV}$, corresponding to Majorana masses predicted by the inverted or degenerate spectra. If observed there would be a significant uncertainty in the actual value of $m_{\beta\beta}$ due to the nuclear matrix elements.

There is a claimed observation of $\beta\beta_0\nu$, corresponding to $0.17 < m_{\beta\beta} < 2.0 \text{ eV}$. This would be extremely important if confirmed, but would not be easy to reconcile with the current expectations from cosmology and oscillations, as shown in Figure 3 (from 14).

5 Neutrino Mixings

The leptonic mixing (Pontecorvo, Maki, Nakagawa, Sakata 17) matrix $U_{PMNS}$ is due to the mismatch of the charged lepton and neutrino mixings, $U_{PMNS} = U^*_e U_\nu \equiv U$. It is very different from the quark mixing matrix $U_{CKM}$. Whereas the latter has small mixing angles, two of the leptonic mixings are large.

The atmospheric angle $\theta_{23}$ is consistent with maximal, while the Solar angle is large but not maximal, $\tan^2 \theta_{12} = 0.40^{+0.09}_{-0.07}$. On the other hand, the third angle $\sin^2 2\theta_{13} < 0.03$ (90%). A better knowledge is important because the angles may be a critical test of models, especially the deviation of $\theta_{12}$ from maximal and the value of $\theta_{13}$. (The latter is also of some urgency because leptonic CP violation in oscillations vanishes for $\theta_{13} = 0$.)

The observed large mixings came as something of a surprise, especially in frameworks such as grand unification, in which the simplest models would yield small mixings similar to those in $U_{CKM}$. The mixings can be associated with either the charged leptons ($U_e$) or the neutrinos ($U_\nu$), or both. One can always choose a basis in the space of lepton families in which one or the other is the identity matrix, but that basis might not be the one in which the family or other symmetries or constraints are most apparent. Until recently, most models assumed either $U_e \sim I$ or $U_\nu \sim I$, with the large mixings due to the other sector. In this case it is easier to achieve bimaximal mixing, $\theta_{23} = \theta_{12} = \frac{\pi}{4}$ than the observed $\theta_{12}$. Recently, several authors have pointed out that the observations are consistent with a bimaximal $U_\nu$ and small (Cabibbo-like) deviations due to $U_e \neq I$. (In fact, the central value of $\theta_{12}$ is $\sim \frac{\pi}{4} - \theta_{\text{Cabibbo}}$ 18.)

6 The Number of Neutrinos

There are two major constraints on the number of neutrino types. The invisible Z width implies $N_\nu = 2.9841(83)$, where $N_\nu$ is the number of active neutrinos with $m_\nu < M_Z/2$. Clearly, there is room for only three. Other unobserved new particles from Z decay would also give a positive contribution to $N_\nu$.

A complementary constraint comes from big bang nucleosynthesis (BBN) 19, in which the predicted $^4\text{He}$ abundance depends sensitively on the competition between weak and
expansion rates, and therefore on the number of relativistic particles present at $T \sim 1$ MeV. This implies $N'_\nu < 3.1 - 3.3$, where $N'_\nu$ counts the active $\nu$'s with $m_\nu \lesssim 1$ MeV. It also includes sterile $\nu$'s, which could be produced by oscillation effects, for a wide range of masses and mixings with the active neutrinos. It does not include $N_R$ for light Dirac neutrinos unless they could be produced by new BSM interactions.

6.1 LSND

LSND has claimed evidence for oscillations, especially $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, with $|\Delta m^2_{\text{LSND}}| \gtrsim 1$ eV$^2$. If this is confirmed by the Fermilab Mini-BooNE experiment, it would strongly suggest the existence of one or more light sterile neutrinos which mix with the active $\nu$'s of the same chirality (since the $Z$ width does not allow a fourth active neutrino)$^c$. Such sterile neutrinos are present in most models, but the active-sterile mixing would require Dirac and Majorana mass terms which are both tiny and either comparable or $m_S \ll m_D$ (pseudo-Dirac), which are difficult to achieve theoretically.$^f$ It is also difficult to accomodate experimentally. It is well established that neither the Solar nor the atmospheric oscillations are predominantly into sterile states. Furthermore, a combination of Solar, atmospheric, Kamland, and reactor and accelerator disappearance limits exclude both the $2 + 2$ and $3 + 1$ schemes$^g$. These refer respectively to the possibilities that the Solar and atmospheric pairs (which contain admixtures of sterile) are separated by a gap of $\sim 1$ eV, and to the possibility of three closely spaced (mainly active) states separated from the fourth (mainly sterile) state by $\sim 1$ eV. However, some $5 \nu$ (i.e., $3 + 2$) patterns involving mass splittings around 1 eV$^2$ and 20 eV$^2$ are more successful$^h$.

The constraints on sterile neutrinos from BBN and large scale structure are also severe. The $2 + 2$ and $3 + 1$ patterns$^g$ are again apparently excluded$^{20,21}$, although there are some (highly speculative/creative) loopholes. These include: (a) Large $\nu$ asymmetries to suppress or compensate the steriles$^k$. (b) Late time phase transitions to suppress $\nu$ masses and sterile mixings until after neutrino decoupling ($T \lesssim 1$ MeV)$^{29}$. (c) Time varying $\nu$ masses (due to coupling to special scalar fields) so that the steriles were too massive to produce cosmologically (with possible implications for matter effects and for dark energy)$^{30}$. (d) A low reheating temperature after inflation$^{31}$. On the other hand, sterile neutrinos could play a useful role in r-process nucleosynthesis$^{32}$ and in understanding pulsar kicks$^{33}$.

Instead of mixing with sterile neutrinos, one can invoke CPT violation, which could manifest itself as a difference in the neutrino and antineutrino masses, allowing three mass differences for active neutrinos and antineutrinos. The original proposal$^{34}$ was excluded by Kamland, which observed $\bar{\nu}_e$ disappearance for the Solar parameters. An alternative, which mainly affects the atmospheric oscillations$^{35}$, has been shown to be excluded by global data analyses$^{20}$. Still surviving is

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$^c$An alternative, nonstandard neutrino interactions$^{23}$, is strongly disfavored by the KARMEN data.

$^f$Some possible mechanisms and described in$^{24,25}$. For a review, see$^{21}$

$^g$The $3 + 2$ schemes have not been studied in detail, but are also likely to be problematic.

Figure 4. $2 + 2$ and $3 + 1$ patterns.
a hybrid scheme which also invokes a sterile neutrino. Another possibility is that CPT violation manifests itself in quantum decoherence rather than in the masses.

7 Models of Neutrino Mass

There are an enormous number of models of neutrino mass. Models constructed to yield small Majorana masses include: the ordinary (type I) seesaw; models with heavy Higgs triplets (type II seesaw); TeV (extended) seesaws, with $m_\nu \sim m_{5+1}/M$, e.g., with $M$ in the TeV range; radiative masses (i.e., generated by loops); supersymmetry with $R$-parity violation; mass generation by terms in the Kähler potential; anarchy (random entries in the mass matrices); large extra dimensions (LED), possibly combined with one of the above.

Small Dirac masses may be due to: higher dimensional operators (HDO) in intermediate scale models (e.g., associated with a $U(1)'$ or supersymmetry breaking); large intersection areas in intersecting brane models; or large extra dimensions, from volume suppression if $N_R$ propagates in the bulk.

I will only describe a few of these in more detail, as well as comment on the possibilities in superstring constructions, which may lead to variant forms.

7.1 Textures

Neutrino textures are specific guesses about form of the $3 \times 3$ neutrino mass matrix or the Dirac and Majorana matrices entering seesaw models. These are often studied in connection with models also involving quark and charged lepton mass matrices, such as grand unification (GUTs), family symmetries, or left-right symmetry. These textures are not unique, even given perfect information about the quark and lepton masses and mixings, because the forms are changed when one rotates to different bases for the left and right-handed fermions. However, it is hoped that there is some basis in which the underlying symmetries of the theory are especially simple, and that finding a successful texture (typically involving hierarchies of large and small elements) will provide a clue. Some examples, which lead to each of the possible neutrino hierarchies, are:

$$\text{Normal: } m_\nu = m \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, \ \epsilon \ll 1$$

$$\text{Inverted: } m_\nu = m \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \text{small}$$

$$\text{Degenerate: } m_\nu = mI + \text{small.} \ (3)$$

Another complication is that most models predict the form of the textures at the Planck or GUT scales, so that one must run the mass and mixing parameters to low energy to compare with experiment. For the neutrinos this is especially important for degenerate or inverted cases $m_i \sim m_j$ (with the same sign), in which case there may either be instabilities or the radiative generation of large mixings.

7.2 Dirac Masses

One promising mechanism for small Dirac neutrino (or other) masses is that elementary Yukawa couplings $LN_LH_2$ may be forbidden by new symmetries (e.g., $U(1)'$) of the low energy theory or by string constraints, but that very small effective couplings are generated by higher dimensional operators, such as

$$L_\nu \sim \left( \frac{S}{M_{Pl}} \right)^p LN_LH_2, \ \langle S \rangle \ll M_{Pl}$$

$$\Rightarrow m_D \sim \left( \frac{\langle S \rangle}{M_{Pl}} \right)^p \langle H_2 \rangle. \ (4)$$

$S$ is a standard model singlet field assumed to acquire a TeV. For large $p$ one could have $\langle S \rangle$ close to $M_{Pl}$, as can occur in heterotic string models with an anomalous $U(1)'$. For smaller $p$, $\langle S \rangle$ could be at an intermediate scale $\ll M_{Pl}$, e.g., $\langle S \rangle \sim 10^8$ GeV for $p = 1$. The scale of $\langle S \rangle$ could be associated
with the breaking of a non-anomalous $U(1)'$ along a $D$ and (almost) $F$ flat direction, or with supersymmetry breaking. A variant on such models could have additional operators that naturally yield ordinary-sterile mixing should it be needed. Such mechanisms are compatible with the general features of string constructions, but there have been no detailed models.

7.3 The (Ordinary) Seesaw Model

The ordinary (type I) seesaw is the most popular model for small neutrino masses. It assumes that $m_T = 0$ in (1) and that the eigenvalues of $m_S$ are $\gg m_D$ (e.g., $10^{12}$ GeV), yielding an effective Majorana mass matrix

$$m_{\nu}^{\text{eff}} = -m_D m_S^{-1} m_D^T$$

(5)

for the 3 light active $\nu$'s.

The ordinary seesaw is usually implemented in connection with grand unification. For example, in $SO(10)$ the $N_R$ occurs naturally, although large Higgs multiplets such as the 126 (or higher dimensional operators) must be invoked to generate $m_S$. Most of the explicit $SO(10)$ models yield the normal hierarchy.

The grand unified theory seesaw model is an elegant mechanism for generating small Majorana neutrino masses, which leads fairly easily to masses in the correct range. It also provides a simple framework for leptogenesis, in which the decays of heavy Majorana neutrinos produce a lepton asymmetry, which is later partially converted to a baryon asymmetry by electroweak sphaleron effects.

However, the expectation of the simplest grand unified theories is that the quark and lepton mixings should be comparable and that the neutrino mixings should be small, rather than the large mixings that are observed. This can be evaded in more complicated GUTs, e.g., (a) those involving highly non-symmetric (lopsided) mass matrices, in which there are large mixings in the right-handed charge $-1/3$ quarks (where it is unobservable) and in the left-handed charged leptons $U_e$ (this is harder to achieve in $SO(10)$ than $SU(5)$); or (b) in those with complicated textures for the heavy Majorana neutrino mass matrix, i.e., involving an $m_D, m_S$ conspiracy to give large $U_{\nu}$ mixings. However, the need to do so makes the GUT seesaw concept less compelling. Furthermore, a number of promising extensions of the standard model or MSSM do not allow the canonical GUT seesaw. For example, the large Majorana masses needed are often forbidden, e.g., by extra $U(1)'$ symmetries predicted in many string constructions. Similarly, it is difficult to accommodate traditional grand unification (especially the needed adjoint and high dimension Higgs multiplets needed for GUT breaking and the seesaw) in simple string constructions. Such constructions also tend to forbid direct Majorana mass terms and large scales. Finally, the active-sterile neutrino mixing required in the schemes motivated by the LSND experiment is difficult to implement in canonical seesaw schemes.

7.4 Triplet models

An alternative class of models involves the introduction of a Higgs triplet $T = (T^+ T^0)$ with weak hypercharge $Y = 1$. Majorana masses $m_T$ can then be generated from the Yukawa couplings $L_T = \lambda_T^i L_i T L_j$ if $(T^0) \neq 0$ but $\ll$ the electroweak scale. An early version, the Gelmini-
Roncadelli model, assumed spontaneous $L$ violation. The original model has been excluded because the decay $Z \rightarrow \text{scalar} + \text{Majoron}$ (the Goldstone boson of $L$ violation) would increase the $Z$ width by the equivalent to two extra neutrinos. This can be evaded in invisible Majoron models, in which the Majoron is mostly singlet. However, most of the more recent triplet models assume a very heavy triplet mass $M_T$, and break $L$ explicitly by including $THH$ couplings, giving large a Majoron mass. This coupling will induce a very small $vEV$ for the triplet suppressed by $M_T^{-1}$ (the type II seesaw). The type II models are often considered in the context of $SO(10)$ or left-right symmetry, with both the ordinary and triplet mechanisms competing and with related parameters. However, they can also be considered independently. A general supersymmetric version would involve the superpotential

$$
W_\nu = \lambda^T_{ij} L_i T L_j + \lambda_1 H_1 TH_1 + \lambda_2 H_2 T H_2 + M_T TT + \mu H_1 H_2,
$$

(6)

where $T$, $T$ are triplets with $Y = \pm 1$, and typically $M_T \sim 10^{12} - 10^{14}$ GeV. This induces a seesaw-type $vEV$ and triplet Majorana mass

$$
\langle T^0 \rangle \sim -\lambda \langle H^0_2 \rangle^2 \Rightarrow m_{ij} = -\lambda^T_{ij} \lambda_2 \frac{\langle S \rangle}{M_T^2} v^2.
$$

(7)

Equivalently, one can integrate out the heavy triplet, inducing a higher dimensional effective mass operator coupling two lepton doublets to two Higgs doublets,$ \frac{1}{M_T^2} \lambda^\nu_{ij} (L_i H_2) (L_j H_2)$.

8 Neutrinos in String Constructions

Some of the key ingredients of most GUT and bottom up models are either absent or different in known semi-realistic string constructions, both heterotic and intersecting brane. In particular, string constructions typically yield bifundamental, singlet, and adjoint representations of the gauge groups, not the large representations usually invoked in GUT model building. Moreover, string symmetries and constraints may forbid couplings allowed by the apparent symmetries of the four-dimensional field theory. In particular, the superpotential terms leading to Majorana masses may be absent, or if they are present they may not be diagonal (i.e., connecting the same family or same flavor), leading to nonstandard mass matrices. Furthermore, GUT Yukawa relations are typically broken. Another difference is that the nonzero superpotential terms are related to gauge couplings, so they may naturally be equal or simply related, with hierarchies in effective Yukawas typically due to higher-dimensional operators in heterotic models or due to the areas of intersection triangles in intersecting brane constructions.

8.1 The Seesaw in String Constructions

There seem to be no fundamental difficulties in generating Dirac masses in string constructions, which can be small by the mechanism described in Section 7.2. However, Majorana masses are more difficult. Several questions arise when one attempts to embed the seesaw model: (a) Can one generate a large effective $m_S$ from superpotential terms like

$$
W_\nu \sim c_{ij} \frac{S^{\gamma+1}}{M_{Pl}^\gamma} N_i N_j \Rightarrow (m_S)_{ij} \sim c_{ij} \frac{\langle S \rangle^{\gamma+1}}{M_{Pl}^\gamma},
$$

(8)
consistent with $D$ and $F$ flatness? ($S^{q+1}$ can represent a product of $q + 1$ different SM-singlet fields, or can even contain SM-nonsinglets if one allows for condensations of products of fields.) (b) Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints? If (a) and (b) are satisfied, there is a possibility of an ordinary seesaw. However, to obtain anything resembling typical bottom-up constructions, there are two more conditions. (c) Is $c_{ii} = 0$? This is motivated by the fact that diagonal terms in the superpotential are rare in string constructions. One could still have a seesaw with off diagonal terms, but it would be very non-standard. Finally (d): Are the assumptions usually made in bottom-up constructions relating the neutrino sector to the quark and charged lepton masses maintained?

There have been relatively few detailed investigations of neutrino masses in string models, but none have been consistent with all of these assumptions. Some constructions lead to a conserved $L$ and Dirac masses $49$. Only one construction $50$, based on flipped $SU(5)$, has found a possibly flat direction that can yield an ordinary seesaw, but that one is very non-GUT-like in detail. A detailed study of the $Z_3$ orbifold is in progress $10$, but so far has yielded no Majorana mass terms.

Another possibility is an extended (TeV-scale) seesaw $51$, in which the light neutrino masses are of order $m_\nu \sim m_P^{p+1}/m_S^p$, with $p > 1$ (e.g., $m \sim 100$ MeV, $m_S \sim 1$ TeV for $p = 2$). This could come about, for example, by the mass matrix

$$
\frac{1}{2} \begin{pmatrix}
\nu_L & \bar{N}_L^c & \bar{N}_L^c
\end{pmatrix}
\begin{pmatrix}
0 & m_D & 0 \\
m_D^T & 0 & m_{SS'} \\
0 & m_{SS'}^T & m_S
\end{pmatrix}
\begin{pmatrix}
\nu_R \\
N_R \\
N_R'
\end{pmatrix}
$$

(9)

where $\nu_L$, $N_R$, and $N_R'$ each represent three flavors, $m_{SS'}$ has TeV-scale eigenvalues, and $m_D$ and $m_{SS'}$ are much smaller. This may occur in certain heterotic constructions (depending on dynamical assumptions) $52$.

### 8.2 Triplets in String Constructions

If the triplet model in (6) were embedded in a string construction then one expects $\lambda_{ij}^T \sim 0$ for $i = j$, i.e., that the diagonal terms vanish at the renormalizable level, implying that $m_{ii} = 0$ to leading order $h$. That is because the existence of an $SU(2)$ triplet with $Y \neq 0$ would require a higher level embedding of $SU(2)$, e.g. $SU(2) \subset SU(2) \times SU(2)$, with the $T$ and $\bar{T}$ transforming as $(2,2)$ and the lepton (and Higgs) doublets as $(2,1)$ or $(1,2)^1$. The underlying $SU(2) \times SU(2)$ would only allow off-diagonal trilinear couplings such as

$$W \sim \lambda_{ij}^T L_i (2,1) T (2,2) L_j (1,2), \ j = 2,3,$$

yielding

$$m_\nu = \begin{pmatrix}
0 & a & b \\
a & 0 & c \\
b & c & 0
\end{pmatrix}.$$  

(11)

c = 0 for the example in (10), but it could be present for the embedding $SU(2) \subset SU(2) \times SU(2) \times SU(2)$. Alternatively, a non-zero but suppressed $c$ (or the diagonal elements) could be generated by higher dimensional operators (HDO).

This reasoning provides a stringy motivation to study the neutrino mass matrix in (11). In string constructions it is also plausible (but not necessary) to assume $|a| = |b| = |c|$ or $|a| = |b| \gg |c|$. There are enough zeroes in (11) so that one can take $a, b, c$ real w.l.o.g. by a redefinition of fields. Then $m_\nu = m_L^i$ with $\text{Tr} \ m_\nu = 0$, which implies $m_1 + m_2 + m_3 = 0$, where the eigenvalues $m_i$ are real but can be either positive or negative. This simple constraint, combined with the observed values $|\Delta m^2_{\text{Atm}}| \sim 2 \times 10^{-3}$ eV$^2$ and $\Delta m^2_{\odot} \sim 8 \times 10^{-5}$ eV$^2$, leads

$^1$One would also need either to have multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal or to generate the $\lambda_{1,2}$ by HDO. It would only be necessary for one pair to survive to low energies.

$^1$Explicit $Z_3$ constructions have been found with some but not all of these features $10$. 

to the prediction\(^7\) of an inverted hierarchy, 
m_1 = 0.046, -0.045, -0.001 eV. This corresponds to |a| ≈ |b| ≫ |c|, and approximate bi-maximal mixing, i.e., θ_⊙ ≈ θ_{Atm} ≈ π/4 for 
\[ U_e = I (|a| ≠ |b|) \text{ would lead to maximal Solar mixing and non-maximal atmospheric}. \]
It is convenient to choose the phases of the fields so that a ≈ -b.

The limiting case $a = -b, c = 0$ of (11) has actually been studied previously by many authors \(^5\), motivated by bottom-up or other theoretical considerations. There is a conserved nonstandard lepton number $L_e - L_\mu - L_\tau$, bimaximal mixing, and an inverted hierarchy with the degenerate pair forming the variant form of a Dirac neutrino involving only active states (section 1.1). The small Solar mass splitting can be induced by turning on a small |c| or diagonal element, yielding a pseudo Dirac $\nu$.

One cannot simultaneously obtain the observed $\Delta m^2_{\odot}$ and the observed deviation of $\theta_\odot$ from maximal in this way except by a fine-tuned cancellation of two rather large corrections. As discussed in Section 5, however, the deviation from maximal could be due to small deviations of $U_e$ from the identity. In the string context, in particular, there is no reason to assume that there are no mixings in $U_e$, so the model in (11) is viable. Assuming, for example, that

\[ U_e \sim \left( \begin{array}{ccc} 1 & -s_{12} & 0 \\ s_{12} & 1 & 0 \\ 0 & 0 & 1 \end{array} \right), \]

(12)

with $s_{12}^2 \text{ chosen so that } \theta_\odot \sim \frac{\pi}{4} - \frac{s_{12}^2}{\sqrt{2}} = 0.56 \pm 0.05$, one predicts $|U_{ee}|^2 \sim \frac{\sin^2 2 \theta_\odot}{2} \sim (0.023 - 0.081) \text{ at 90% cl}$, close to the present upper limit of 0.03; an observable $\beta \beta_\nu$, mass $m_{\beta \beta} \sim m_2 (\cos^2 \theta_\odot - \sin^2 \theta_\odot) \sim 0.020 \text{ eV};$ and a total cosmological mass $\sum |m_i| = 0.092 \text{ eV}.$

\(^7\)There is a second solution corresponding to a partially degenerate normal hierarchy for |a| ≈ |b| ≈ |c|, but this does not lead to realistic mixings.

9 Other Implications

Let me briefly mention a number of other implications of neutrino mass.

- Lepton flavor nonconservation (LFV) \(^5\), e.g., $\mu \rightarrow e \gamma$, $\mu N \rightarrow e N$, $\mu \rightarrow 3e$. Lepton and hadron FCNC are expected at some level in most BSM theories. In principle, nonzero neutrino mass and mixings violates $L$ flavor, but the effects are negligible except for neutrino oscillations. However, significant LFV is often generated along with $m_\nu$ in specific models, e.g. by $\bar{\nu}$ exchange in supersymmetry.

- Large magnetic moments are possible \(^4\), though the simplest neutrino mass models yield very small moments. There are rather stringent astrophysical limits.

- Massive neutrinos may decay, with implications for high energy \(^5\), supernova \(^5\), solar \(^5\), and cosmological $\nu$'s \(^5\).

- In addition to the possible CP violating phase in the PMNS matrix, which may be observable in long baseline experiments, Majorana neutrinos allow two additional phases. These are in principle observable in $\beta \beta_\nu$, but in practice this is difficult due to nuclear uncertainties \(^5\).

- Oscillation effects will likely lead to an equilibration of lepton asymmetries between lepton flavors. This greatly strengthens the limits on asymmetries from BBN unless there was a compensating contribution to the energy density in the early universe \(^2\).

- High energy $\nu$'s are expected from violent astrophysical events, such as active galactic nuclei and gamma ray bursts. Measurements of the $\nu_e/\nu_\mu/\nu_\tau$ ratio would be very sensitive to oscillations and decays \(^6\). $Z$-bursts (the annihilation of an ultra high energy $\nu$ with a relic $\nu$ into a $Z$) could allow
the detection of relic $\nu$’s, but only for an unexpectedly high flux \(^{62}\).

- The most popular model for baryogenesis is leptogenesis \(^{46}\), in which the asymmetric decays of the heavy Majorana neutrino in a seesaw model, $N\to lH \neq N\to \bar{l}\bar{H}$ can generate a lepton asymmetry. This $L$ asymmetry is then partially converted to a $B$ asymmetry $(n_B/n_\gamma \sim 6\times10^{-10})$ by electroweak $B + L$-violating thermal fluctuations (sphalerons) prior to the electroweak phase transition. There are severe constraints on this mechanism in supersymmetric models because of difficulties for BBN due to the decays of gravitinos produced after inflation \(^{63}\). However, these can be avoided for some versions of supersymmetry or if the heavy neutrinos are produced nonthermally. The relevant CP phases are unfortunately not directly measurable at low energies, although there may be model-dependent relations to LFV in supersymmetry \(^{64}\). There are alternative forms of leptogenesis associated with heavy triplet models \(^{48,65}\). There are also viable mechanisms for baryogenesis \(^{66}\) not related to neutrinos, such as electroweak baryogenesis (especially in extensions of the MSSM \(^{67}\)), the Affleck-Dine mechanism, etc.

### 10 Conclusions

- Nonzero neutrino masses are the first necessary extension of the standard model.
- The experimental program has been spectacularly successful.
- $m_\nu$ may well be due to GUT or Planck scale physics.
- There are many possibilities for neutrino mass, both Dirac and Majorana. In particular, string constructions are unlikely to yield the standard GUT or left/right motivated seesaw models. One should allow for the possibilities of small Dirac masses, nonstandard or extended seesaw models, and triplet models, perhaps with an inverted hierarchy.

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