An Approximation Solution to Estimate the Ultimate Bearing Capacity of Smooth Strip Footings on Ponderable Soil

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Abstract: The procedure for calculating the bearing capacity of smooth footings is employed to calculate the bearing capacity of strip foundations on general $c\cdot\phi\cdot\gamma$ soil by the method of characteristics. The numerical results are in good agreement with other known computations. The surcharge ratio, $\lambda$, is found to be another good parameter for evaluation of the bearing capacity without superposition. The approximate formula for estimating the bearing capacity factor $N_\gamma$ is then proposed considering the influence of $\lambda$ in addition to $\phi$ with an error of no more than $\pm 3\%$ compared to the numerical results. The error caused by the superposition method is systematically analyzed and the influences of $\lambda$ and $\phi$ on the superposition error are discussed.

1. Introduction

There are many types of soils encountered in marine engineering throughout the world; the physical and mechanical properties of which differ greatly [1-4]. Among the mechanical properties of these soils, the cohesion of soil and the internal friction angle of soil are the two main parameters used to estimate the strength of the soil. The current bearing capacity formulas for different types of foundations originate from the formula for strip footings considering the shape factors, load inclination factors, depth factors, etc [5-10]. Therefore, the bearing capacity of strip footings is regarded as one of the classic issues in geotechnical engineering. The bearing capacity of a strip footing subjected to a vertical load is currently expressed as

$$q_u = qN_q + cN_c + \frac{1}{2}\frac{\gamma}{B}N_\gamma$$

(1)

where $q_u$ is the ultimate bearing capacity; $q$, $c$, $\gamma$, and B represent the equivalent surcharge load at the footing base, the soil cohesion, the soil unit weight, and the footing width, respectively; $N_q$, $N_c$, and $N_\gamma$ are the bearing capacity factors related to $q$, $c$ and $\gamma$, respectively. Terzaghi approximately calculated the three parts of contribution by superposition: $N_q$ and $N_c$ are obtained on weightless soil (i.e., $q\neq0, c\neq0, \gamma=0$) and $N_\gamma$ is deduced by a surface footing on ponderable granular soil (i.e., $q=0, c=0, \gamma\neq0$). The expressions of $N_q$ and $N_c$ in Equation (1) are widely accepted in the following closed form [11]

$$N_q = (N_q - 1)\cot \phi$$

(2)

$$N_c = e^{\tan \phi} \tan \left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

(3)

Where, $\phi$ represents the internal friction angle of the soil. The well-know expressions above have been proven to be exact for weightless soils [12,13].
However, the bearing capacity factor $N_γ$ of a strip footing on a surface granular soil has been discussed for decades by many researchers and the expressions for $N_γ$ proposed by different investigators vary widely \cite{13-17}. Furthermore, the bearing capacity factor $N_γ$ is found to be influenced by other parameters such as $q$, $c$, $γ$, and $B$ \cite{13,18-22}, and at present, a satisfactory expression of $N_γ$ has not been acquired. For the bearing capacity of strip foundations on $c$-$ϕ$-$γ$ soil, the solutions acquired by the superposition method are conservative compared to the results calculated in one failure mechanism \cite{23-26}. But the errors caused by the superposition method have not been systematically estimated, and lack the exact bearing capacity solution on general $c$-$ϕ$-$γ$ soil.

In this paper, the procedure for calculating the bearing capacity of smooth footings, called BCSF, is written to compute the bearing capacity of strip footings by the method of characteristics. The method of characteristics is one of the main methods used for bearing capacity problems, the solution of which is proved to be exact \cite{27,28}. As the method of characteristics has been introduced by many researchers \cite{14,27-30}, it is unnecessary to introduce detailed computations of the process in this work, as it is available in many references. The present computations are compared to other known results. The surcharge ratio $\lambda$ ($\lambda=(q+c\tan \phi)/\gamma B$) first proposed by Zhu et al. \cite{21} is adopted as another parameter to evaluate the bearing capacity without using superposition. The bearing capacity factor $N_γ$ is then discussed, and an approximate formula is proposed considering the influence of $\lambda$ in addition to $ϕ$. The error caused by the superposition method is systematically analyzed and the influences of $\lambda$ and $ϕ$ on the superposition error are discussed.

2. Computations of bearing capacity

Cox (1962) discussed the bearing capacity of strip footings on $c$-$ϕ$-$γ$ soil with variation of the dimensionless ratio $G$, which is defined as

$$G = \frac{\gamma B}{2c^*}$$

where $c^*$ is the relative cohesion, equal to $c+qtan \phi$.

It can be inferred that the denominator of $G$ is related to $\lambda$ using the formula below

$$\lambda = \frac{\cot \phi}{2G}$$

Martin (2003) confirmed that Cox’s results\cite{18} are in good agreement with computations by ABC (Analysis of Bearing Capacity). When $q=0$, $c=1$ kPa, $B=2$ m, and $\gamma$ is equal to the value determined by equation (4), the bearing capacity $q_u$ can be calculated by the present method and $q_u/c^*$ is then obtained as shown in table 1. The numerical results in table 1 indicate the results of the present method’s calculations are very close to those by Cox (1962) and Martin (2003).

| $ϕ/°$ | $G$ | $\lambda$ | $q_u/c^*$ |
|-------|-----|-----------|-----------|
|       |     |           | present method | Cox (1962) | Martin (2003) |
| 10    | 0   | $\infty$ | 8.3449     | 8.34       | 8.345        |
|       | 0.01| 283.56    | 8.3522     | 8.35       | 8.352        |
|       | 0.1 | 28.36     | 8.4167     | 8.42       | 8.417        |
|       | 1   | 2.84      | 9.0200     | 9.02       | 9.02         |
|       | 10  | 0.28      | 13.5581    | 13.6       | 13.56        |
| 20    | 0   | $\infty$ | 14.8348    | 14.83      | 14.83        |
|       | 0.01| 137.37    | 14.8692    | 14.87      | 14.87        |
|       | 0.1 | 13.74     | 15.1743    | 15.2       | 15.17        |
|       | 1   | 1.37      | 17.8929    | 17.9       | 17.89        |
|       | 10  | 0.14      | 37.7627    | 37.8       | 37.76        |
| 30    | 0   | $\infty$ | 30.1400    | 30.14      | 30.14        |
|       | 0.01| 86.60     | 30.2914    | 30.29      | 30.29        |
|       | 0.1 | 8.66      | 31.6146    | 31.6       | 31.61        |
|       | 1   | 0.87      | 42.8703    | 42.9       | 42.87        |
For the bearing capacity of rough footings, Zhu et al. (2003) found that the normalized bearing capacity, $p_u = \frac{q_u + ccot\phi}{\gamma B}$, is only related to the surcharge ratio $\lambda$ at a determined $\phi$. The same conclusion can be verified by numerical calculations of smooth footings. When $\phi$ is equal to 25°, the computations of normalized $p_u$ under a series of $q$, $c$, $\gamma$, and $B$ values are shown in Table 2. As can be seen from the table, although the bearing capacity, $q_u$, differs greatly with the variations in $q$, $c$, $\gamma$, and $B$, $p_u$ is a constant when $\lambda$ is fixed. A large number of numerical examples at other $\phi$ also support the same conclusion.

### Table 2. Normalized $p_u$ under a series of $q$, $c$, $\gamma$, and $B$ values ($\phi=25^\circ$).

| $q$ (kPa) | $c$ (kPa) | $\gamma$ (kN/m$^3$) | $B$ (m) | $\lambda$ | $q_u$ (kPa) | $p_u$ |
|-----------|-----------|----------------------|---------|-----------|-------------|-------|
| 10        | 0         | 20                   | 5       | 0.1       | 336.96      | 3.37  |
| 2         | 0         | 10                   | 2       | 0.1       | 67.39       | 3.37  |
| 1         | 2.145     | 20                   | 2.8     | 0.5       | 184.10      | 3.37  |
| 4         | 0.933     | 20                   | 3       |           | 200.18      | 3.37  |
| 10        | 0         | 20                   | 1       |           | 167.99      | 8.40  |
| 10        | 4.663     | 10                   | 4       | 0.5       | 325.99      | 8.40  |
| 5         | 2.332     | 20                   | 1       |           | 162.99      | 8.40  |
| 0         | 11.658    | 20                   | 2.5     |           | 394.99      | 8.40  |
| 20        | 0         | 20                   | 1       |           | 283.19      | 14.16 |
| 18        | 0         | 10                   | 1.8     | 1         | 254.8       | 14.16 |
| 10        | 13.989    | 10                   | 4       |           | 536.39      | 14.16 |
| 0         | 23.315    | 20                   | 2.5     |           | 657.99      | 14.16 |

From the definitions of $p_u$ and $\lambda$, the bearing capacity $N_\gamma$ in Equation (1) can be written as

$$N_\gamma = 2(p_u - \lambda N_q)$$

(6)

If the bearing capacity $N_q$ is calculated from Equation (3), it is can be inferred that the bearing capacity is only influenced by $\lambda$ when $\phi$ is constant. The surcharge ratio, $\lambda$, is thus regarded as another parameter that has impact on $N_\gamma$ in addition to $\phi$.

3. Analysis of the bearing capacity $N_\gamma$

3.1. $N_\gamma$ in the case of $\lambda=0$

The bearing capacity $N_\gamma$ have been discussed by many researchers for the case of $q=0$, $c=0$, and $\gamma\neq0$. The surcharge ratio, $\lambda$, equals 0 under this condition and $N_\gamma$ can be obtained by the following equation:

$$N_\gamma = \frac{2q_u}{\gamma B}$$

(7)

The $N_\gamma$ values calculated by the present method at different angles are given in table 3. The present values for the case of $\lambda=0$ are in good agreement with those by the method of characteristics and are within the range of the lower and upper bounds defined by the limit analysis. The numerical results of Frydman and Burd [31] using FLAC and Woodward and Griffiths [32] using FEM are also consistent with the values obtained by the present method. The upper-bound solutions in the table are relatively
large compared to the present results. The present results can be taken as exact solutions as they are between the lower and upper bounds given by Hjaj et al. [16], which are very close to each other.

| $\phi$ ($^\circ$) | Present method | Chen (1975) | Bolton and Lau (1993) | Michalowski (1997) | Frydman and Bard (1997) | Woodward and Griffiths (1998) | Ukritchon et al. (2003) | Smith (2005) | Hjaj et al. (2005) | Kumat (2009) |
|------------------|----------------|------------|--------------------|------------------|------------------------|-----------------------------|------------------------|------------|-----------------|-------------|
| 5                | 0.0845         | 0.248     | 0.10               | 0.09             | 0.127                  | --                          | 0.08-0.09              | --         | 0.0862-0.0914   | 0.087       |
| 10               | 0.2810         | 0.723     | 0.50               | 0.29             | 0.423                  | --                          | 0.3                    | 0.27-0.30 | 0.283-0.299     | 0.282       |
| 15               | 0.6987         | 1.641     | 1.20               | 0.71             | 1.050                  | --                          | 0.7                    | 0.68-0.75 | 0.701-0.737     | 0.699       |
| 20               | 1.5787         | 3.452     | 2.70               | 1.60             | 2.332                  | --                          | 1.5                    | 1.52-1.73 | 1.578-1.665     | 1.577       |
| 25               | 3.4614         | 7.163     | 5.90               | 3.51             | 5.020                  | --                          | 3.4                    | 3.33-3.94 | 3.454-3.653     | 3.457       |
| 30               | 7.6530         | 15.19     | 12.7               | 7.74             | 10.92                  | 7.9                         | 7.6                    | 7.18-8.54 | 7.623-8.078     | 7.644       |
| 35               | 17.5897        | 33.87     | 28.6               | 17.8             | 24.75                  | 18.9                        | --                     | 15.7-21.2 | 17.46-18.51     | 17.55       |
| 40               | 43.19          | 81.75     | 71.6               | 44               | 60.22                  | 42                          | --                     | 38.5-54.2 | 42.77-45.42     | 43.08       |

- Upper-bound method
- Method of characteristics
- Fast lagrangian analysis of continua (FLAC)
- Finite element method (FEM)
- Limit analysis (lower-bound method and upper-bound method)

### 3.2. Approximation of $N_f$

If $\lambda$ is not equivalent to 0, the numerical value of $N_f$ obtained form Equation (6) is found to increase with increasing $\lambda$. The maximum $N_f$ is available when $\lambda$ approaches $\infty$, which means the unit weight of the soil, $\gamma$, decreases to 0. The values of maximum $N_f$ are also listed in Table 3, when $\lambda$ is $\infty$. It should be noted that the numerical results of $N_f$ are calculated for the case of $\lambda=104$ because $N_f$ will approach the upper-bound solution given by Chen (1975) in the Hill mechanism, the theoretical formula of which is expressed as

$$N_f = \frac{1}{4} \tan u ((\tan u e^{3\phi} - 1) + \frac{3\sin \phi}{1 + 8\sin^2 \phi} [(\tan u - \frac{\cot \phi}{3}) e^{3\phi} + \tan u \frac{\cot \phi}{3}] + 1)$$

where $u = \pi/4 + \phi/2$ and $f = \tan \phi$.

To distinguish the $N_f$ value in different surcharge ratios, $N_f$ is noted as $N_{f,\min}$ and $N_{f,\max}$ in the case of $\lambda=0$ and $\lambda=\infty$, respectively. The $N_{f,\min}$ and $N_{f,\max}$ results by the present method can be approximately written in the form of the fitting formula suggested by Diaz-Segura (2013) as

$$N_{f,\min} = (0.5 N_q - 0.08) \tan(1.34 \phi)$$

$$N_{f,\max} = (N_q + 0.6) \tan(1.3 \phi)$$

The approximate formula for $N_f$ considering the influence of $\lambda$ is proposed as

$$N_f = \frac{N_{f,\min}}{1 + (A_0 \lambda)^{0.75}} + \frac{N_{f,\max}}{1 + (A_0 \lambda)^{0.75}}$$

where $A_0$ is a fitting coefficient related to $\phi$ and the proposed value is determined as $A_0 = 3.774 \phi - 6.774 \phi^2 - 1.552 \phi + 5.202$.

The approximate curves as well as numerical values are plotted in figure 1. The $N_f$ values from the computations of Cox(1962) and Martin(2003) in table 1, can be acquired using
The inferred results from Equation (13) are also marked in Figure 1. As can be seen from Figure 1, the approximation curves are in good agreement with the numerical results. Further estimation shows that the error between the approximation and the exact numerical result is no more than ±3% for any case if $\phi$ is within 40°. The $N_\gamma$ values deduced by Martin (2003) perfectly support the present results. The outcomes by Cox (1962) outcomes are also close to the present calculations on the whole, except that relatively larger errors occur in two cases ($G=0.01$ when $\phi=10^\circ$ and $20^\circ$) because of poor accuracy for $q_u/c^*$.

Figure 1. The variation of $N_\gamma$ versus $\lambda$ at different friction angles.
4. Error evaluation caused by the superposition method

Calculation of the bearing capacity factor, $N_γ$, by the traditional superposition method is obtained for the circumstance of $λ=0$ ($q=0$, $c=0$, $γ≠0$), the value of which is the minimum named as $N_{γ, min}$ for a determined $ϕ$. If the bearing capacity obtained by the superposition method is noted as $s_u$, the equation of $s_{umin}$ can be written as

$$s_{umin} = 0.5(\frac{γBN_{γ, min}}{\lambda N_q - c cot / γB + 0.5N_γ})$$

The error, $ε$, caused by the superposition method is estimated by the formula as follows:

$$ε = \frac{q_u - q_u}{q_u} = \frac{0.5(N_{γ, min} - N_γ)}{\lambda N_q - c cot / γB + 0.5N_γ}$$

It can be easily deduced that $ε$ is within the range below

$$ε_L ≤ ε ≤ ε_U$$

where $ε_L$ and $ε_U$ are the minimum and max defined as

$$ε_L = \frac{0.5(N_{γ, min} - N_γ)}{\lambda N_q - c cot / γB + 0.5N_γ}$$

$$ε_U = \frac{0.5(N_{γ, min} - N_γ)}{\lambda N_q - c cot / γB + 0.5N_γ}$$

The error $ε$ is equal to $ε_L$ under the condition of $q=0$ and turns into $ε_U$ when $c=0$. If $ϕ$ and $λ$ are fixed, it can be inferred that the error caused by using the superposition method is largest on cohesive soil without surcharge and is least on granular soil with surcharge. For a constant $ϕ$, $ε$ is always within the range of $ε_L$ and $ε_U$, regardless of the values of $q$, $c$, $γ$ and $B$. The error in the bearing capacity resulting from the traditional superposition method can be theoretically evaluated for $λ$ at a determined $ϕ$. The variation of $ε$ with $λ$ at different friction angles are plotted in figure 3. When $λ$ is constant, figure 4 gives the curves of $ε_L$ and $ε_U$ versus $ϕ$. In the two figures, the value of $ε$ changes between the dashed line and the solid line for a determined $ϕ$ or $λ$. 

![Graph showing error evaluation](image-url)
The figure 2 and figure 3 reveal that:

1. The curves of $\epsilon_L$ and $\epsilon_U$ versus $\lambda$ are similar to a lognormal distribution. When $\phi$ is a constant between $5^\circ$-$40^\circ$, the largest error occurs when $\lambda$ is between 0.04 and 0.3. The value of $\lambda$ corresponding to the maximum error increases with increasing $\phi$.

2. The curves of $\epsilon_L$ and $\epsilon_U$ converge as $\phi$ increases, which indicates for a fixed $\lambda$, the impacts of $q$, $c$, $\gamma$, and $B$ on $\epsilon$ get smaller with increasing $\phi$. 
(3) The \( \lambda_L \) and \( \lambda_U \) relate to the maximum values of \( \varepsilon_L \) and \( \varepsilon_U \), respectively, and are not equivalent. The former is relatively larger than the latter. But the difference between the two values becomes smaller with increasing \( \phi \).

(4) The maximum error, \( \varepsilon_L \), decreases when \( \phi \) increases to some extent. The absolute value of the maximum error is about 24% when \( \phi = 5^\circ \) and reduces to about 16% when \( \phi \) reaches 40°. But there is an obvious superposition error when \( \lambda \) is around 0.1 regardless of the value of \( \phi \).

5. Conclusions
The BCSF procedure is employed to calculate the bearing capacity of smooth strip footings based on the method of characteristics. The present computations of bearing capacity \( q_u \) and the bearing capacity factor \( N_\gamma \) are compared to other researchers’ results. An approximate formula to evaluate the numerical results of the \( N_\gamma \) is proposed and the approximations are found to be in good agreement with the numerical calculations. The error caused by the superposition method is comprehensively discussed with the variation of the surcharge ratio \( \lambda \). The following conclusions can be reached from these comparisons and analyses:

The bearing capacity and corresponding \( N_\gamma \) determined by the present method are close to the values given by other researchers. The bearing capacity factor \( N_\gamma \) can be regarded as an exact solution, as the present value is between the range of the lower and upper bounds given by Hjiaj et al. (2005) when \( \lambda \) equals 0.

The difference between the exact \( N_\gamma \) and the value obtained by the traditional superposition method on a general \( c-\phi-\gamma \) soil comes from neglecting the surcharge ratio \( \lambda \). To get a better \( N_\gamma \), it is critical to consider the contribution of \( \lambda \) in addition to the friction angle \( \phi \). The formula for \( N_\gamma \) proposed in this paper provides a good approximate value with the largest error no more than 3% and is suitable for evaluating the bearing capacity without using the superposition method.

The bearing capacity obtained by the traditional superposition method gives a visible error on the conservative side. The values of bearing capacity are underestimated by 16%-24% for smooth footings from the view of maximum error. A further discussion shows that the maximum error occurs when \( \lambda \) is between 0.04 and 0.3 at a determined \( \phi \).

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