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A Mathematical Model Development for the Lateral Collapse of Octagonal Tubes

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Abstract. Many researches has been done on the lateral collapse of tube. However, the previous researches only focus on cylindrical and square tubes. Then a research has been done discovering the collapse behaviour of hexagonal tube and the mathematic model of the deformation behaviour had been developed [8]. The purpose of this research is to study the lateral collapse behaviour of symmetric octagonal tubes and hence to develop a mathematical model of the collapse behaviour of these tubes. For that, a predictive mathematic model was developed and a finite element analysis procedure was conducted for the lateral collapse behaviour of symmetric octagonal tubes. Lastly, the mathematical model was verified by using the finite element analysis simulation results. It was discovered that these tubes performed different deformation behaviour than the cylindrical tube. Symmetric octagonal tubes perform 2 phases of elastic-plastic deformation behaviour patterns. The mathematical model had managed to show the fundamental of the deformation behaviour of octagonal tubes. However, further studies need to be conducted in order to further improve on the proposed mathematical model.

1. Introduction
Several studies had been performed with regards to the lateral collapse of tubes compressed between two flat rigid plates which basically focus on the cylindrical tube. The earliest mathematical model was developed based on rigid perfectly plastic material model where the load-deformation prediction was based on the energy balance method and it took into account the geometrical components of stiffening phenomenon [1, 2, 10, 11]. Those mathematical models were improved then by introducing a rigid linear strain hardening material model [3]. Lastly, the mathematical model was improved further by relating the contribution of strain hardening over a variable length of plastic hinges [4]. This vastly improved mathematical model and still being applied until now.

During the compression process, the cylindrical tube deformed by undergoing three major phases which resulted in three stages in the force-deformation curve. The first stage is the linear increasing force, followed by a flat force and lastly an unbounded increasing force. The first stage is the elastic collapse, the second stage is the plastic collapse and the last stage is the densification of the cylindrical tube [5]. During the plastic collapse, three types of behaviour can take place which are the strain hardening, strain softening and perfectly-plastic [6]. The illustration of these behaviours is shown in figure 1.
However, all of these mathematical models focused on cylindrical tube. Based on these previous works, a mathematical model to predict the collapse behaviour of square and symmetric hexagonal tubes had been developed [7]. It was discovered that, unlike cylindrical tubes, symmetrical hexagonal tubes were more suitable with rigid, perfectly plastic mathematical model.

In this study, the aim was to investigate and hence produce a more suitable mathematical model for the deformation behaviour pattern of symmetric octagonal tubes. In this study, the deformation theories as discussed in early research work [5-7] were still taken into consideration. Furthermore, another type of deformation behaviour will be taken into consideration. It was stated that, there were two type force-displacement curves under the compression loading which were named as Type-I and Type-II [8]. Type I execute flat force whereas Type-II performs an initial peak force and then followed with an immediately reduced force. Figure 2 shows the illustration of these deformation types.

This paper is organized as follows. In section 2, the mathematical model are discussed. In section 3, the simulation procedure is presented. The comparison of mathematical model and simulation results are presented in section 4. The discussion of the results is made in section 5. Finally, our work of this paper is summarized in the last section.

2. Mathematical Model

The deformation of the symmetric octagonal tubes is assumed to have two phases of deformation. As shown in figure 3, the phase 1 collapse is where the hinge occurred at all the vertices which make all the oblique sides collapse but all the horizontal and vertical sides remain unbothered. The collapse model can be taken as the collapse of a symmetric hexagonal tube [7]. Thus,

\[
F_1 = \frac{ybh^2 \cos \theta_1}{H_1 \left[1-\left(\frac{4\cos \theta_1}{2H_1}\right)^2\right]^2}; \quad 0 < \delta \leq 2H_1
\]  

(1)

and the maximum deformation of phase 1 is
For the second phase, after all the oblique sides have fully deformed the shape will be transformed to rectangular shape. Then the deformation of the transformed rectangular tubes will take place which the vertical sides will collapse until the whole shape become flat. The deformation model can be taken similar with the deformation model of square tube and with some modification for phase 2; it can be define as shown below

\[ F_2 = \frac{Ybh^2 \cos \theta_1}{H_1[1 - \cos \theta_1]^2} + \frac{Ybh^2}{H_2[1 - \left(\frac{\delta - 2H_1}{H_2}\right)^2]} \]  \text{ for } 2H_1 < \delta \leq 2H_1 + H_2 \quad (3)

Hence the fully model of the collapse of a symmetric octagonal tube is given by,

\[
F = \begin{cases} 
\frac{Ybh^2 \cos \theta_1}{H_1[1 - \cos \theta_1]^2} & \text{for } 0 < \delta \leq 2H_1; \\
\frac{Ybh^2 \cos \theta_1}{H_1[1 - \cos \theta_1]^2} + \frac{Ybh^2}{H_2[1 - \left(\frac{\delta - 2H_1}{H_2}\right)^2]} & \text{for } 2H_1 < \delta \leq 2H_1 + H_2; \\
\infty & \text{for } \delta > 2(R_1 \cos \theta_1 + R_2).
\end{cases}
\]  \quad (4)

\textbf{Figure 3.} A cross-section of a symmetric octagonal tube.
3. Simulation Procedures
In this simulation, the shapes were compressed in between two flat rigid plates (see figure 4). ABAQUS finite element analysis software was used to perform the simulation procedure. The deformation behaviour was simulated using quasi static analyses. Arbitrary Lagrange Euler solver (ALE) and "Static-General Implicit" analysis procedure was used to develop the three dimensional (3D) model. 3D modelling space and deformable behaviour with a solid extrude base feature was used to replicate the shape of the symmetric octagonal tubes. 3D modelling space and the ‘discrete rigid’ type behaviour with a shell extrude base feature was used to replicate the rigid plates. The top and bottom flat plates were set rigid and assigned with 100 R3D4 (quadrilateral rigid elements) and meshed by using 15000 to 30000 ‘C3D8R’ type elements or 8 node brick elements. The material properties were set based on the stainless properties, i.e. Young's modulus, \(E=200\) GPa, Poisson’s ratio, \(\nu=0.35\). yield strength, \(\sigma_Y=207\) MPa and ultimate strength, \(\sigma_U=517\) MPa.

![Figure 4](image-url)  
**Figure 4.** The diagram of symmetrical tube placed in between two rigid plates.

4. Results

![Figure 5](image-url)  
**Figure 5.** Comparing mathematical and simulation results of force-deformation/total height relationship of symmetric octagonal tubes, (a) \(\theta_1=45^\circ\) and (b) \(\theta_1=60^\circ\).

Figure 5 shows the comparison of the force vs. deformation/total height based from the simulation method and the mathematical method for the symmetric octagonal tubes of various angle, \(\theta_1\). Figure 5
(a) shows the comparison of the force-deformation/total height between the simulation result and mathematical result for symmetric octagonal tubes with $\theta_1 = 45^\circ$. The results for both methods were quite closed. Clearly, both mathematical and simulation results show two phase of curves. For mathematical method, the first phase has a slightly increasing force before the force jumped suddenly. The second phase force is slightly flat curve and lastly the force became unbounded. For the simulation method, the first phase force is flat and then followed with a jump force. The second phase force is a slightly flat curve and then there is a small increase force and after that the force increased very fast and became unbounded.

Figure 5 (b) is the comparison of the force-deformation/total height between the simulation result and mathematical result for symmetric octagonal tubes with $\theta_1 = 60^\circ$. Both mathematical and simulation results also show two phase of curves. For the first phase, both methods show pretty close result with a flat force. Then, for both methods it was followed with a jump force. However, the jump force under the simulation method was a bit higher than under the mathematical method. The second phase force of the mathematical method is flat and then the force became unbounded. Meanwhile, the second phase force of the simulation method begins with a slightly increasing curve. Then, the force became a flat and lastly the force increased unboundedly.

5. Discussion
All the simulation of the force-deformation/total height of symmetric octagonal tubes of different $\theta_1$ also has shown two phases of deformation curve. For phase 1, it was began with an initial immediate linear increasing force and then followed with a long flat force. The long flat force for both sample were found to be static. This shows that the symmetric octagonal tubes were experiencing perfectly plastic during phase 1 deformation. After that, phase 2 plastic deformations took place with a jump force and followed with a curve force. The jump and the curve were more obvious for $\theta_1 = 60^\circ$. The highly jump and the curve effect was happened due to the strain hardening of the straight vertical sides of the transformed rectangular shape. The effect was higher for $\theta_1 = 60^\circ$ because a high force was needed to collapse the vertical sides of the symmetric octagonal tubes during the second phase. Then after the curve force, there was an immediate increasing force which was due to the densification of the tubes where the vertical sides were almost to collapse completely and the top and bottom wall of the symmetric octagonal tubes were nearly to contact and the symmetric octagonal tubes was almost flat.
Comparing the simulation and the mathematical results for symmetric octagonal tubes, it was found out that both methods had a very good agreement during the first phase. During the second phase, the symmetric octagonal tube with $\theta_1 = 45^\circ$ shown a better agreement than the symmetric octagonal tube with $\theta_1 = 60^\circ$. When $\theta_1 = 60^\circ$, the mathematical model under estimated the simulation method during the second phase of the deformation which was caused by strain hardening effect that is not included in the mathematical method.

6. Conclusions
The rigid, perfectly plastic with 2 phases of elastic-plastic mathematical model for collapse behaviour pattern which were used in this study had managed to show the fundamental collapse pattern of symmetric octagonal tubes. This has been verified by the finite element analysis using ABAQUS software which showed a good agreement for symmetric octagonal tube with $\theta_1 = 45^\circ$. However, for the symmetric octagonal tube with $\theta_1 = 60^\circ$, during phase 2 the plastic deformation undergoing strain hardening phenomenon, and was not shown by the perfectly plastic model. Thus, further studies need to be conducted to get a better/improved mathematical model for symmetric octagonal tubes with $\theta_1 = 60^\circ$.

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