A New Model for the Collective Behavior of Animals

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We propose a new model in order to study behaviors of self-organized system such as a group of animals. We assume that the individuals have two degrees of freedom corresponding one to their internal state and the other to their external state. The external state is characterized by its moving orientation. The rule of the interaction between the individuals is determined by the internal state which can be either in the non-excited state or in the excited state. The system is put under a source of external perturbation called “noise”. To study the behavior of the model with varying noise, we use the Monte-Carlo simulation technique. The result clearly shows two first-order transitions separating the system into three phases: with increasing noise, the system undergoes a phase transition from a frozen dilute phase to an ordered compact phase and then to the disordered dispersed phase. These phases correspond to behaviors of animals: uncollected state at low noise, flocking at medium noise and runaway at high noise, respectively.

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I. INTRODUCTION

The collective behavior of animals is a widely observed phenomenon in various biological systems. It is one of the main topics which have been extensively investigated during the last two decades using methods borrowed from many different areas of science including physics, applied mathematics and engineering.

The flocking is a behavior of some animal species where they stay together in a group for social reasons. They derive many benefits from this behavior including defence against predators, easier collective moving, enhanced foraging success and higher success in finding a mate. When they are faced with a danger such as predators, their natural instinct is to flee not to fight. They use their natural herding instinct to bind together in a group for safety. All individuals of the group will move away from the predator in the same direction and then stampede as fast as they can when being under the predator’s attack. If there is no danger, then they are spread to find foods instead of staying in the flocking state. Well-known examples are found in populations such as large schools of fish\textsuperscript{1} or gatherings of birds\textsuperscript{2}. Biologically, it is known that the flocking behavior is advantageous for survival of a population\textsuperscript{3, 5}: reducing the risk of capture by predators, increasing higher mating efficiency, easier search for food, efficient learning of external stimuli, and reducing overall aggression\textsuperscript{6, 9}.

In 1987, Reynolds first suggested a simple model consisting of three rules: separation, alignment, and cohesion rules\textsuperscript{10}. These rules describe the behavior of each individual in interaction with other neighboring individuals. All or some of the three rules were mathematically expressed and then analyzed by Vicsek and his coworkers\textsuperscript{11, 14}. They mainly focused on the transition between coherently moving and runaway in a stampede.

The flocking behavior has been conventionally studied through simulation in two frameworks: population (Eulerian or continuum models) and individuals (agents or particle-based models)\textsuperscript{8, 15, 16}. In the population framework, the flock was collectively addressed while flock-density was used as a key variable to present spatial and temporal dynamics of aggregation frequently with partial differential equations of advection-diffusion reaction\textsuperscript{17, 18}. In the individual framework, the flock of agents has been simulated by using ordinary and stochastic equations of motion to describe interactions among agents\textsuperscript{11, 13, 21}. This approach attempted to replicate naturally observed phenomena from not only animal groups but also other self-propelled characteristics\textsuperscript{22} and to compare the evolved characteristics with those of actual animal flocking\textsuperscript{23} in order to better understand possible mechanisms by which these characteristics may have evolved.

The previous models have predicted with success flocking behavior of animal groups at high noise. They have, however, many difficulties in dealing with uncollected states at very low noise. In addition, the transition from the ordered phase to the disordered phase at high noise shows a second-order character with these models in contradiction with the sudden character observed by experiments where all individuals immediately stampede in different directions at the transition. This behavior is more likely a first-order transition.

Section II is devoted to the description of the model. Section III shows the Monte Carlo (MC) simulation re-
sults for testing our model. Conclusions are given in Sec. IV.

II. THE MODEL

In biology, all the members of an animal group are spread to find foods if there is no danger. In this situation, they are distributed in the space with a small concentration and out of alignment. So, we say the group of animals is in a “uncollected” behavior. When the animals are faced with danger such as predators, they bind together in a small area for safety with the same orientation and high concentration. This state is called “flocking” state. Facing a danger, animals will move away from the predator in the same direction and then stampede as fast as they can when being under the predator’s attack. At the final stage, they are in a “runaway” state.

In order to study the phase transition behavior, we have to map the group of animals into a physical system. We consider the animal as a particle with two degrees of freedom: one is an external parameter \( \sigma_i \) characterizing the animal orientation, and the other one is the internal parameter \( S_i \) indicating either it is in the non-excited or in the excited individual state. The internal individual state is defined with the help of two Ising spins for each animal: the animal is in the internal excited state if its spins are parallel. There are thus \( n = 2N \) spins with \( N \) being the number of animals in the system. In the absence of an external noise (or the temperature in statistical physics), the system of internal 2N Ising parameters \( S_i \) is in the disordered phase if these spins are randomly anti-parallel, then we say the internal state is excited. Otherwise, the internal state is non-excited with two parallel spins in each animal. The total number of “up” spins in Ising model could be evaluated by \( \{2\} \):

\[
n_{\uparrow} = \frac{n}{1 + \exp(-2\epsilon/\eta)},
\]

where \( \epsilon > 0 \) is the energy of a spin and \( \eta \) the external noise. Hence, the number of down spins is \( n_{\downarrow} = n - n_{\uparrow} \). We denote by \( N_0 \) the number of non-excited individuals:

\[
N_0 = n_{\uparrow} - n_{\downarrow} = \frac{2n_{\uparrow} - n}{2} = n_{\uparrow} - N.
\]

Thus the number of excited individual is \( N_e = N - N_0 \). Note that when \( \eta \to 0 \), \( n_{\downarrow} \to n \) or \( N_0 \to N \), namely all animals are non-excited. At high noise, \( n_{\uparrow} \to n/2 \) or \( N_e \to N \), so that all animals are excited.

Now, we put this system of individuals on the lattice where each individual can move on 2D triangular lattice of linear size \( L \). The number of lattice sites should be greater than that of individuals, i.e., \( L^2 \gg N \). We denote by \( \sigma_i \) the orientation of individual \( S_i \): \( \sigma_i \) is defined as in a \( q \)-state Potts model, i.e. \( \sigma_i = 1, 2, \ldots, q \). For simplicity, we consider \( q = 6 \), so the orientations \( \sigma_i = 1, 2, \ldots, 6 \) of individual \( S_i \) can be defined by the vectors which connect a site to its nearest neighbors (NN) with the following angles measured from the \( x \) axis: \( \varphi = 0, \pi/3, \ldots, 5\pi/3 \).

The interaction between nearest-neighboring animals is given by the Hamiltonian

\[
\mathcal{H} = - \sum_{\langle i,j \rangle} J_{i,j} \cos[\pi(\sigma_i - \sigma_j)/3],
\]

where the sum \( \sum_{\langle i,j \rangle} \) is made over the nearest neighboring individuals \( S_i \) and \( S_j \). \( J_{i,j} \) is the exchange interaction between two individuals which depends on their internal state: \( J_{i,j} = 0 \) if both individuals \( S_i \) and \( S_j \) are non-excited, and \( J_{i,j} = J > 0 \) if otherwise. Let us note that the above Hamiltonian is the clock Potts model which has been solved for \( q = 2, 3 \) and 4 \( \{25\} \). The case of very large \( q \) has been solved by Fröhlich and Spencer \( \{26\} \). However, for \( q = 5, 6, \ldots \) there are not (yet) exact results.

III. THE PHASE-TRANSITION BEHAVIOR

We use the MC simulation technique to study the above model. The main physical quantities such as the order parameter \( Q \) and the concentration \( \rho \) are defined by

\[
Q = \frac{q}{N(q-1)} \sum_{i=1}^{N}(\sigma_i^{\text{max}} - N/q),
\]

\[
\rho = \frac{1}{N} \sum_{i=1}^{N} n_i,
\]

with \( q = 6 \) and \( \sigma^{\text{max}} = \max(\sum_{i} \sigma_i) \). In Eq. 5 \( n_i \) is the number of NN individuals around \( S_i \).

Before showing the results let us adopt the following notations. The ordering of the system is quantified by the order parameter \( Q \). When the individuals have different orientations, the order parameter \( Q \approx 0 \), then the system is in the orientationally disordered phase. Whereas, when the system is in the ordered phase, the order parameter reaches to the maximum value \( Q = 1 \), namely all the individuals have the same orientation. On the other hand, the quantity \( \rho \) in Eq. \( \{2\} \) characterizes the spatial distribution of the individuals. Hence, the behavior of animals can be adequately described by the two parameters \( Q \) and \( \rho \).

In the simulations, we use \( \epsilon = 0.04, J = 1.0 \) (taken as the unit of energy) and \( N = 100, 400 \) and 900 with the lattice size \( L = 4 \times \sqrt{N} \). At each \( \eta \), the equilibration time lies around \( 4 \times 10^6 \) MC steps per individual and we compute statistical averages over \( 8 \times 10^6 \) MC steps per individual. Periodic boundary conditions are used in the \( xy \) plane.

We plot in figures \( \{1\} \) and \( \{2\} \) the order parameter and the concentration as a function of external noise for several system sizes. It clearly shows the existence of three phases which are separated by the two transitions at very low noise and high noise. In phase I at low noise, the
FIG. 1: Order parameter versus noise with the system sizes $N = 100$ (circle), 400 (square) and 900 (diamond). The insets show the enlarged scale at low (a) and high (b) noise.

FIG. 2: Concentration versus noise with $N = 100$ (circles), 400 (squares) and 900 (diamonds).

The system is in the disordered phase with low concentration ($Q \approx 0$ and $\rho \approx 0$). This phase is equivalent to the uncollected behavior of an animal group which is dispersed over the whole space, it is called “frozen” phase. Phase III, at high noise, is spatially sparse as the phase I, but the difference between them is that animals are not frozen but very mobile in all directions in phase III. It describes the runaway behavior. At medium noise, the animals are in phase II where they are compactly moving in an ordered phase with $Q \approx 1$ and the maximum concentration $\rho \approx 6$. This phase corresponds to the flocking behavior.

The spatial distribution of the individuals is also presented in Fig. 3 with snapshots taken at several values of noise $\eta = 0.0522, 0.1367, 0.5000$ and 0.5667, at the final stage of a MC simulation. The vectors indicate their position and moving orientation. One sees that Figs. 3a,d show that the system has the same distribution where they are randomly distributed in the plane with different orientations. These phases correspond to the frozen uncollected animals and mobile runaway animals but the snapshots do not permit to see the difference. Fig. 3b) shows the flocking behavior, and Fig. 3c) shows the distribution of the system at the noise closes to the II-III transition point.

At this stage, let us examine again Fig. 1 it shows the discontinuous order parameter at both transitions I-II and II-III, indicating a signature of a first-order transition. This abrupt behavior corresponds to the fact that the animals immediately flock when they are faced with danger, and they runaway when being under a predator’s attack. The first-order character observed in our simulations are in agreement with these behaviors experimentally observed. To confirm the first-order transition we have performed the energy histogram at the transi-
Let us discuss on the phase transition at high noise. It is known that the $q$-state Potts model localized on a lattice has a first-order transition with $q \geq 4$ in two dimensions. The clock Potts model localized on a lattice was solved only for $q \leq 4$ \cite{22}, though it was solved for very large $q$ \cite{20}. The model treated in the present paper corresponds to a mobile Potts model where particles can go from one lattice site to another. This was never solved before. So we cannot compare our first-order transition with analytical results. Note however that the transition II-III corresponds not only to an orientational disordering of Potts parameter but also to the breaking of the compactness of the flocking state. This is similar to a melting transition where the solid phase melts into the liquid phase. The melting has often a first-order character in three dimensions \cite{23}. In two dimensions, it is known that long-range solid ordering does not survive at finite temperatures if the system has isotropic short-range interactions according to Nelson and Halperin \cite{28}: at a first critical temperature, bound pairs of dislocations formed at low temperatures are unbound, giving rise to a phase with no translational ordering but with orientational hexatic structure. The latter phase undergoes a “critical” phase transition to the disordered phase. It was however found by MC simulation that the second transition is not critical but it is a first-order two-dimensional disclination melting \cite{29}. Our result shown in Figs. 3 (c,d) and 4 indicate that our model undergoes a first-order transition from the hexatic ordered phase to the liquid phase, in agreement with earlier MC simulations \cite{29}.

Finally, to compare our mobile Potts model with the corresponding localized Potts model, we have simulated the same system defined by \cite{10} including the internal degree of freedom on a lattice of size $L^2 = N$, i.e. the individuals are not allowed to move on the lattice. We show in Fig. 5 results of both mobile and localized Potts models: the transition temperature is about 0.5844 for the mobile case, while it is $\eta \approx 1.1126$, much higher, for the localized Potts model.

IV. CONCLUSIONS

We proposed in this paper a simple model for studying the behavior of animal groups as a function of an external perturbation, called “noise”, such as dangers coming from predators. We showed that with increasing noise, the system has three phases I, II and III separated by two transitions, the first transition occurs at a low noise and the second one at high noise. The three phases are frozen, ordered and melted phases which correspond respectively to the following behaviors of animals: uncollected state, flocking state and runaway state. Both transitions from one phase to another are found to be of the first-order.

This model is similar to the lattice gas model developed by Csahók et al. \cite{12} but in our model the alignment rules of the individuals depend on the internal state, excited or non-excited, of its neighbors including itself. Therefore, our model can be applied not only for studying the phase transition behavior of animals at high noise as in previous models, but also for analyzing the animal behavior at low noise where animals are in a uncollected, frozen phase. The frozen phase and the melted phase are both disordered and spread in the space, but their difference lies on the animal mobility: in phase I animals are slow (frozen) and in phase III they are mobile (stampeding state). This is very similar to the difference between the frozen spin glass phase and the paramagnetic phase in statistical physics where spins are random in both phases but frozen at low temperatures and freely slipping at high temperatures.

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