Antisymmetric tensor matter fields: an abelian model

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Abstract

We present a simple renormalizable abelian gauge model which includes antisymmetric second-rank tensor fields as matter fields rather than gauge fields known for a long time. The free action is conformally rather than gauge invariant. The quantization of the free fields is analyzed and the one-loop renormalization-group functions are evaluated. Transverse free waves are found to convey no energy. The coupling constant of the axial-vector abelian gauge interaction exhibits asymptotically free ultraviolet behavior, while the self-couplings of the tensor fields do not asymptotically diminish.

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1 The lagrangian

The structure of the kinetic term and interactions for antisymmetric tensor matter fields can be obtained by dynamical generation \[1\] from fundamental spinors like in the Nambu–Jona–Lasinio model. We shall not stop at the details of that and proceed to studying the resulting lagrangian which could as well be guessed in some other way.

It should be understood from the very beginning that the fields under consideration are \textit{not} the antisymmetric tensor gauge fields introduced a long time ago \[2\] by analogy with the electromagnetic vector field. We are going to deal with the matter fields, the free action for which is conformally rather than gauge invariant. Such fields appeared in extended conformal supergravity theories \[3\], but they have not specifically been studied there in detail.

We present a simple abelian gauge model that includes this new type of matter fields, analyze their quantization, and compute the one-loop renormalizations in the model.

The lagrangian density in four dimensions is of the form
\[
\mathcal{L} = \frac{1}{2} (\partial \lambda T_{\mu\nu})^2 - 2 (\partial \mu T_{\mu\nu})^2 - \frac{1}{4} (\partial \mu A_\nu - \partial \nu A_\mu)^2 + i \bar{\psi} \gamma_\mu A_\mu \psi + 4h \gamma_5 A_\mu \psi + 4h \bar{\psi} \gamma_\mu A_\mu \psi + \frac{1}{4} h \left[ \frac{1}{2} (A_\mu T_{\mu\nu})^2 - 2 (A_\mu T_{\nu\mu})^2 \right] + 2 T_{\mu\nu} T_{\nu\mu} T_\lambda T_{\lambda\mu} ,
\]
(1)

where \(A_\mu\) is a real axial-vector gauge field, \(T_{\mu\nu} = - T_{\nu\mu}\) is a real tensor matter field, and \(\psi\) is a Dirac spinor field.

We denote \(\tilde{T}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} T_{\alpha\beta}\), \(\sigma_{\mu\nu} = \frac{1}{2} i [\gamma_\mu, \gamma_\nu]\), use the \((+---)\) Minkowski metric, and always imply covariant contractions of repeated Lorentz greek indices without distinguishing their positions. The \(\epsilon\) tensor and \(\gamma_5\) satisfy
\[
\epsilon_{\mu_1...\mu_4} \epsilon_{\nu_1...\nu_4} = - g_{\mu_1} [\nu_1 ... g_{\mu_4} \nu_4], \quad \tilde{T}_{\mu\nu} = - T_{\mu\nu},
\]
\[
\text{tr} (\gamma_5 \gamma_{\mu_1} ... \gamma_{\mu_4}) = - 4i \epsilon_{\mu_1...\mu_4}, \quad \gamma_5^2 = 1, \quad \frac{1}{2} i \epsilon_{\mu\nu\alpha\beta} \sigma_{\alpha\beta} = \gamma_5 \sigma_{\mu\nu} .
\]

There are two useful identities which follow from antisymmetry of \(T_{\mu\nu}\) and the definition of the \(\epsilon\) tensor:
\[
\tilde{T}_{\mu\lambda} \tilde{T}_{\lambda\nu} = T_{\mu\lambda} T_{\lambda\nu} + \frac{1}{2} g_{\mu\nu} T_{\alpha\beta}^2, \quad T_{\mu\lambda} \tilde{T}_{\lambda\nu} = \frac{1}{4} g_{\mu\nu} T_{\alpha\beta} \tilde{T}_{\beta\alpha} .
\]
(2)
It can directly be checked that our lagrangian (1) is invariant, up to a total derivative, under the infinitesimal abelian gauge transformations
\[
\delta A_\mu = \partial_\mu \omega, \quad \delta \psi = -i \hbar \gamma_5 \psi, \quad \delta \bar{\psi} = -i \hbar \bar{\psi} \gamma_5,
\]
\[
\delta T_{\mu\nu} = -2 \hbar \omega \tilde{T}_{\mu\nu}, \quad \delta \tilde{T}_{\mu\nu} = 2 \hbar \omega T_{\mu\nu}.
\]
The ratio of the charges of the spinor and tensor fields is fixed by requiring the invariance of the Yukawa-like $y$ interaction term.

An essential difference of the antisymmetric tensor matter fields from their gauge counterparts of ref. [2] is the possibility of introducing a renormalizable self-interaction in an abelian model. Its form, which is presented by the $q$ term in eq. (1), is uniquely determined by the gauge invariance.

It is also important to point out that no explicit mass terms can be introduced because neither $M^2 T_{\mu\nu}^2, M^2 T_{\mu\nu} \tilde{T}_{\nu\mu}, m \bar{\psi} \psi$, nor $m \bar{\psi} \gamma_5 \psi$ would be gauge-invariant. The fields may acquire masses owing to a spontaneous symmetry breaking, for example, after adding scalar fields in the model. We do not tackle this problem for the time being, although it is of importance for physical applications [4].

2 Quantization of free fields

The canonical hamiltonian $\mathcal{H} = \dot{\varphi} \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \mathcal{L}$ for the free part of the tensor-field lagrangian (1) can be reduced to the following form:

\[
\mathcal{H}_T = (\partial_0 T_{0j})^2 - \left(\bar{\partial} T_{0j}\right)^2 + 2 (\partial_j T_{0j})^2
\]
\[
+ (\partial_0 T_j)^2 - \left(\bar{\partial} T_j\right)^2 + 2 (\partial_j T_j)^2,
\]
where $\bar{\partial}$ and latin indices refer to space dimensions with the Euclidean summation implied, the mixed time-space tensor components $T_{0j}$ form a 3-dimensional vector, and the purely space components $T_{jk} = \epsilon_{jkl} T_l$ are parameterized by an axial 3-vector $T_j$.

The classical equations of motion

\[
\partial_\lambda^2 T_{\mu\nu} + 2 \partial_\lambda (\partial_\mu T_{\nu\lambda} - \partial_\nu T_{\mu\lambda}) = 0
\]
can explicitly be rewritten in components as
\[
\left[ \delta_{jk} \left( \partial^2_0 + \vec{\partial}^2 \right) - 2 \partial_j \partial_k \right] T_{0k} = -2 \epsilon_{jkl} \partial_0 \partial_k T_l ,
\]
\[
\left[ \delta_{jk} \left( \partial^2_0 + \vec{\partial}^2 \right) - 2 \partial_j \partial_k \right] T_k = 2 \epsilon_{jkl} \partial_0 \partial_k T_{0l} .
\]
All their solutions satisfy at the same time the D’Alembert equation, therefore the standard decomposition in plain waves can be used with subsequently extracting the positive- and negative-frequency components \[5\].

In momentum representation the basis of the solutions is described as follows. Since the hamiltonian does not mix longitudinal and transverse field configurations, they stay independent.

There are similar longitudinal massless excitations of scalar and pseudoscalar type, with \( T_{0j} (p) \) or \( T_j (p) \) parallel to \( p_j \); \( p_0 = \sqrt{\vec{p}^2} \). The corresponding secondarily quantized field can be written through the creation and annihilation operators as
\[
T_{0j} (x) = \int \frac{d^3 \vec{p}}{2 (2 \pi p_0)^{3/2}} \left\{ a^\dagger (-\vec{p}) \exp [i (p_0 x_0 - \vec{p} \cdot \vec{x})] - a (\vec{p}) \exp [-i (p_0 x_0 + \vec{p} \cdot \vec{x})] \right\},
\]
which reduces the hamiltonian \([\text{3}]\) to \( \int d^3 \vec{p} \, p_0 \ a^\dagger (\vec{p}) \ a (\vec{p}) \) after the normal ordering.

The two transverse waves involve both vector and axial-vector fields of equal magnitude, being orthogonal to \( \vec{p} \) and to each other. The fields are represented as follows
\[
T_{0j} (x) = \int \frac{d^3 \vec{p}}{2 (2 \pi)^{3/2} \sqrt{p_0}} \left\{ b^\dagger (-\vec{p}) \, n_j (-\vec{p}) \exp [i (p_0 x_0 - \vec{p} \cdot \vec{x})] + b (\vec{p}) \, n_j (\vec{p}) \exp [-i (p_0 x_0 + \vec{p} \cdot \vec{x})] \right\},
\]
\[
T_j (x) = \int \frac{d^3 \vec{p}}{2 (2 \pi p_0)^{3/2}} \left\{ - b^\dagger (-\vec{p}) \, n_l (-\vec{p}) \exp [i (p_0 x_0 - \vec{p} \cdot \vec{x})] + b (\vec{p}) \, n_l (\vec{p}) \exp [-i (p_0 x_0 + \vec{p} \cdot \vec{x})] \right\},
\]
where \( \vec{n} (\vec{p}) \cdot \vec{p} = 0 \). However, the hamiltonian on these solutions of the equations of motion turns into zero, that is the model possesses unusual zero-energy excitations.
It can easily be seen that the square form in the lagrangian (1) for our tensor fields is non-degenerate (as opposed to the gauge fields [2]). The causal propagator is well defined

$$\langle T_{\mu\nu}(-p) T_{\alpha\beta}(p) \rangle = \frac{i}{p^2 + i0} \Pi_{\mu\nu\alpha\beta}(p),$$

$$\Pi_{\mu\nu\alpha\beta}(p) = \frac{1}{2} \left( g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha} \right) - \frac{g_{\mu\alpha} p_{\nu} p_{\beta} + g_{\nu\beta} p_{\mu} p_{\alpha} - g_{\nu\alpha} p_{\mu} p_{\beta}}{p^2},$$

$$\Pi_{\mu\nu\rho\sigma}(p) \Pi_{\rho\sigma\alpha\beta}(p) = \frac{1}{2} \left( g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha} \right).$$

Its tensorial structure just repeats the structure of the kinetic operator in the lagrangian.

This completes the quantization of the free antisymmetric tensor matter fields. It is worth mentioning that the contribution of the quartic potential term of eq.(1) to the hamiltonian can be rewritten as (with the matrix notation in Lorentz indices used for brevity)

$$- \frac{1}{4} q \left( \frac{1}{2} \text{tr}^2 T^2 - 2 \text{tr} T^4 \right) = q \left[ \frac{1}{2} \left( T_0^2 - T_j^2 \right)^2 + 2 (T_0 T_j)^2 \right].$$

It is evidently nonnegative-definite if $q \geq 0$. Thus, the existence of the vacuum should not be violated by the self-interaction.

3 One-loop renormalizations

By the power-counting rules the lagrangian (1) seems to be renormalizable. However, it is well known that in an axial-vector gauge theory there exists the Adler – Bell–Jackiw anomaly which leads to non-conservation of the axial vector current and can destroy the gauge invariance of counterterms and renormalizability since the three-loop level. To avoid this difficulty, we have to adjust the set of the fields that contribute to the anomaly at one loop so that their contributions canceled. The simplest way to achieve this is to introduce a partner for every charged particle with the opposite axial-gauge charge.

Thus, the cancellation of the anomaly will be guaranteed if we add another spinor $\chi$ with the charge $-h$ instead of $h$ and, as a partner for $T_{\mu\nu}$, another
antisymmetric tensor field $U_{\mu\nu}$ with the charge $-2h$. Identities (2) generalize to two fields as
\[ \tilde{T}_{\mu\lambda} U_{\lambda\nu} + \tilde{U}_{\mu\lambda} T_{\lambda\nu} = \frac{1}{2} g_{\mu\nu} \text{tr} (T \tilde{U}). \]

The doubling of the fields gives rise to the appearance of new possible interactions. Along with the kinetic and the minimal gauge terms, which look the same as in eq. (1) with $h \rightarrow -h$, the most general gauge-invariant terms allowed to be added are
\[
\mathcal{L}_{\text{add.}} = z \sigma_{\mu\nu} U_{\mu\nu} \chi + \frac{1}{4} r \left( \frac{1}{2} \text{tr}^2 U^2 - 2 \text{tr} U^4 \right) + \frac{1}{2} s \text{tr}^2 (TU) + v \left[ \frac{1}{4} \text{tr} T^2 \text{tr} U^2 - \text{tr} (T^2 U^2) \right] + w \left[ \frac{1}{4} \text{tr} T^2 \text{tr} U^2 - \text{tr} (TU T U) \right],
\]
besides the mixed mass term $m^2 \text{tr} (TU)$ which we do not want to introduce since it generates tachyon states, violating the positivity of the free hamiltonian. It will not be generated in perturbation theory if we do not introduce it from the very beginning. On the other hand, in the minimal subtraction renormalization scheme, which we use, renormalizations of the dimensionless couplings do not depend on any masses. The mass term may be essential when we shall consider the spontaneous symmetry breaking in a more elaborate extended model.

Without slightly complicating the calculations of Feynman diagrams, we can provide both the spinors $\psi$ and $\chi$ with an isotopic index runnig through $n$ values. This trivial generalization does not break the cancellation of the anomaly.

In our one-loop calculations we use the standard Feynman gauge. The Feynman rules for eqs. (1) and (4) are presented in the appendix.

To unambiguously evaluate the divergent contributions in one loop, we can apply the regularization by dimensional reduction [6].

Here are our results for the renormalization-group $\beta$ functions and anomalous dimensions of the fields, obtained with the aid of the computer program FORM for analytic evaluation:

\[
\begin{align*}
16 \pi^2 \gamma_T &= \frac{4}{3} n y^2 + \frac{4}{3} h^2, \\
16 \pi^2 \gamma_U &= \frac{4}{3} n z^2 + \frac{4}{3} h^2, \\
16 \pi^2 \gamma_{\psi} &= h^2 - 6y^2, \\
16 \pi^2 \gamma_{\chi} &= h^2 - 6z^2, \\
\beta_{h^2} &= \gamma_A h^2 = (16 \pi^2)^{-1} \left( \frac{8}{3} n - 6 \right) h^4, \\
16 \pi^2 \beta_{g^2} &= \left[ \frac{10}{3} h^2 + \left( \frac{4}{3} n - 12 \right) y^2 \right] y^2, \\
\end{align*}
\]
In the Feynman gauge the ultraviolet divergencies in the Yukawa vertices happen to cancel, which leaves only the propagator contributions in eqs. (8) and (9).

The most interesting fact is that at $n=1$ and 2 the gauge charge exhibits asymptotically free behavior, eq. (7), which is due to the negative contribution of the antisymmetric tensor matter fields (that of the spinors is positive as in the usual quantum electrodynamics, while the abelian axial-vector field possesses no gauge self-interaction).

As $n<3.5$ or $n>9$, there exists a special renormalization-group solution, consistent with eqs. (7) – (9), which makes the Yukawa charges proportional to the gauge charge

$$y^2 = z^2 = \left(2 - \frac{11}{9-n}\right) h^2.$$ 

However, the explicit test of eqs. (10) – (14) with the aid of the computer program Mathematica shows that at $n=1$, 2, and 3 there are no consistent special solutions with proportional real values for all the rest of the couplings. Thus, the self-interaction of the tensor fields does not asymptotically diminish in the ultraviolet limit (as it does not diminish for ordinary scalar matter fields in the $\phi^4$ theory). This can make the asymptotic freedom of the gauge charge in our toy model unstable with respect to higher-order corrections,
when at the third loop a contribution of the quartic self-couplings to $\beta_{h^2}$ will appear. There still remains a hope, however, that supplementing other fields may generate complete one-charge special solutions, respecting asymptotic freedom to all orders of perturbation theory and possibly related to a higher symmetry or supersymmetry.

Thus, we have demonstrated the possibility of introducing a new type of matter fields into the gauge quantum field theory. Some of their properties, like zero-energy free waves and asymptotic freedom in an abelian model, are quite unusual. Ahead stay further investigations into non-abelian models, spontaneous symmetry breaking, and the physics associated with extending the standard electroweak theory to include antisymmetric tensor matter fields, which would substantially modify the Higgs sector.

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Appendix. Feynman rules for antisymmetric tensor matter fields

\[
\begin{align*}
T_{\mu\nu} &\rightarrow T_{\alpha\beta} \Rightarrow U_{\mu\nu} \rightarrow U_{\alpha\beta} \Rightarrow \frac{i}{p^2 + i0} \Pi_{\mu\nu\alpha\beta}(p), \\
A_\mu \rightarrow A_\nu &\Rightarrow -\frac{i}{p^2 + i0} g_{\mu\nu}, \\
\psi \rightarrow \psi &\Rightarrow \chi \rightarrow \chi \Rightarrow \frac{i}{p} \gamma_\mu, \\
T_{\mu\nu} &\rightarrow T_{\alpha\beta} \Rightarrow \frac{1}{2} h \left( p_\mu \epsilon_{\lambda\nu\alpha\beta} - p_\nu \epsilon_{\lambda\mu\alpha\beta} - g_{\lambda\alpha} p_\rho \epsilon_{\rho\beta\mu\nu} + g_{\lambda\beta} p_\rho \epsilon_{\rho\alpha\mu\nu} + q_\alpha \epsilon_{\lambda\delta\mu\nu} - q_\beta \epsilon_{\lambda\alpha\mu\nu} - g_{\lambda\mu} q_\rho \epsilon_{\rho\alpha\nu\beta} + g_{\lambda\nu} q_\rho \epsilon_{\rho\mu\alpha\beta} \right), \\
U_{\mu\nu} &\rightarrow U_{\alpha\beta} \Rightarrow -\frac{1}{2} h \left( p_\mu \epsilon_{\lambda\nu\alpha\beta} - p_\nu \epsilon_{\lambda\mu\alpha\beta} - g_{\lambda\alpha} p_\rho \epsilon_{\rho\beta\mu\nu} + g_{\lambda\beta} p_\rho \epsilon_{\rho\alpha\mu\nu} + q_\alpha \epsilon_{\lambda\delta\mu\nu} - q_\beta \epsilon_{\lambda\alpha\mu\nu} - g_{\lambda\mu} q_\rho \epsilon_{\rho\alpha\nu\beta} + g_{\lambda\nu} q_\rho \epsilon_{\rho\mu\alpha\beta} \right), \\
\psi &\rightarrow \psi \Rightarrow i h \gamma_5 \gamma_\mu, \\
\chi &\rightarrow \chi \Rightarrow -i h \gamma_5 \gamma_\mu,
\end{align*}
\]
\( T_{\mu \nu} A_{\rho} A_{\sigma} \Rightarrow U_{\mu \nu} U_{\rho \sigma} \Rightarrow 4 i \hbar^2 \left( (g_{\mu \alpha} g_{\nu \beta} - g_{\mu \beta} g_{\nu \alpha}) g_{\lambda \rho} - g_{\alpha \beta} (g_{\nu \lambda} g_{\beta \rho} + g_{\nu \rho} g_{\beta \lambda}) + g_{\mu \beta} (g_{\nu \lambda} g_{\alpha \rho} + g_{\nu \rho} g_{\alpha \lambda}) + g_{\nu \alpha} (g_{\mu \lambda} g_{\beta \rho} + g_{\mu \rho} g_{\beta \lambda}) - g_{\nu \beta} (g_{\mu \lambda} g_{\alpha \rho} + g_{\mu \rho} g_{\alpha \lambda}) \right); \)

\( \psi \gamma \gamma \gamma \Rightarrow i y \sigma_{\mu \nu} \),

\( \chi \gamma \gamma \gamma \Rightarrow i z \sigma_{\mu \nu} \),

\( T_{\mu \nu} T_{\mu \nu} T_{\mu \nu} T_{\mu \nu} \Rightarrow 4 i \prod_{j=1}^{4} \left[ \frac{1}{2} \left( g_{\mu_j \alpha_j} g_{\nu_j \beta_j} - g_{\mu_j \beta_j} g_{\nu_j \alpha_j} \right) \right] \times \)

\[ \frac{1}{3} (g_{\alpha_1 \alpha_2} g_{\beta_1 \beta_2} g_{\alpha_3 \alpha_4} g_{\beta_3 \beta_4} + g_{\alpha_1 \alpha_3} g_{\beta_1 \beta_3} g_{\alpha_2 \alpha_4} g_{\beta_2 \beta_4} + g_{\alpha_1 \alpha_4} g_{\beta_1 \beta_4} g_{\alpha_2 \alpha_3} g_{\beta_2 \beta_3} - \frac{4}{3} (g_{\beta_1 \beta_2} g_{\beta_3 \beta_4} g_{\alpha_2 \alpha_3} g_{\alpha_4 \beta_1} + g_{\beta_1 \alpha_4} g_{\beta_3 \alpha_2} g_{\beta_4 \alpha_3} g_{\beta_2 \alpha_1} + g_{\beta_1 \alpha_3} g_{\beta_2 \alpha_2} g_{\beta_4 \alpha_3} g_{\beta_3 \alpha_1} + g_{\beta_1 \alpha_2} g_{\beta_2 \alpha_4} g_{\beta_3 \alpha_2} g_{\beta_4 \alpha_1} + g_{\beta_1 \alpha_4} g_{\beta_2 \alpha_2} g_{\beta_3 \alpha_3} g_{\beta_4 \alpha_1} + g_{\beta_1 \alpha_2} g_{\beta_3 \alpha_4} g_{\beta_4 \alpha_3} g_{\beta_2 \alpha_1} + g_{\beta_1 \alpha_3} g_{\beta_2 \alpha_2} g_{\beta_4 \alpha_4} g_{\beta_3 \alpha_1} + g_{\beta_1 \alpha_4} g_{\beta_2 \alpha_3} g_{\beta_4 \alpha_2} g_{\beta_3 \alpha_1} + g_{\beta_1 \alpha_3} g_{\beta_2 \alpha_4} g_{\beta_4 \alpha_2} g_{\beta_3 \alpha_1} + g_{\beta_1 \alpha_2} g_{\beta_3 \alpha_4} g_{\beta_4 \alpha_1} g_{\beta_2 \alpha_3} g_{\beta_3 \alpha_1}) \].

\[ T_{\mu \nu} T_{\mu \nu} T_{\mu \nu} T_{\mu \nu} \Rightarrow 4 i \prod_{j=1}^{4} \left[ \frac{1}{2} (g_{\mu_j \alpha_j} g_{\nu_j \beta_j} - g_{\mu_j \beta_j} g_{\nu_j \alpha_j}) \right] \times \]

\[ \frac{1}{3} (g_{\alpha_1 \alpha_2} g_{\beta_1 \beta_2} g_{\alpha_3 \alpha_4} g_{\beta_3 \beta_4} + g_{\alpha_1 \alpha_3} g_{\beta_1 \beta_3} g_{\alpha_2 \alpha_4} g_{\beta_2 \beta_4} + g_{\alpha_1 \alpha_4} g_{\beta_1 \beta_4} g_{\alpha_2 \alpha_3} g_{\beta_2 \beta_3} - \frac{4}{3} (g_{\beta_1 \beta_2} g_{\beta_3 \beta_4} g_{\alpha_2 \alpha_3} g_{\alpha_4 \beta_1} + g_{\beta_1 \alpha_4} g_{\beta_3 \alpha_2} g_{\beta_4 \alpha_3} g_{\beta_2 \alpha_1} + g_{\beta_1 \alpha_3} g_{\beta_2 \alpha_2} g_{\beta_4 \alpha_3} g_{\beta_3 \alpha_1} + g_{\beta_1 \alpha_2} g_{\beta_2 \alpha_4} g_{\beta_3 \alpha_2} g_{\beta_4 \alpha_1} + g_{\beta_1 \alpha_4} g_{\beta_2 \alpha_2} g_{\beta_3 \alpha_3} g_{\beta_4 \alpha_1} + g_{\beta_1 \alpha_2} g_{\beta_3 \alpha_4} g_{\beta_4 \alpha_2} g_{\beta_3 \alpha_1} + g_{\beta_1 \alpha_3} g_{\beta_2 \alpha_4} g_{\beta_4 \alpha_1} g_{\beta_2 \alpha_3} g_{\beta_3 \alpha_1} + g_{\beta_1 \alpha_4} g_{\beta_2 \alpha_3} g_{\beta_4 \alpha_2} g_{\beta_3 \alpha_1} + g_{\beta_1 \alpha_3} g_{\beta_2 \alpha_4} g_{\beta_4 \alpha_2} g_{\beta_3 \alpha_1}) \].
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