Influence of Stark shift and detuning on atomic entanglement induced by a thermal field of one-mode cavity

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Abstract. We considered a quantum model consisting of two effective two-level atoms not resonantly interacting with a single-mode thermal cavity field through two-photon transitions. We explored the entanglement dynamics between two atoms, and studied the effect of the Stark shift and detuning on the entanglement. The results showed that for a separable initial atomic state the Stark shift and detuning enhance the amount of atom-atom entanglement induced by the thermal field. For entangled atomic state these reduce the atom-atom entanglement oscillations.

1. Introduction

Entanglement is recognized nowadays as basic feature in the fundamentals of the quantum physics and as a source of diverse applications in quantum information and computation [1]. Among the most successful and rapidly developing ways of realizing quantum computations and communications are those using natural atoms (such as neutral atoms or ions) and artificial atoms (such as superconducting circuits, spins in solids, quantum dots and hybrid systems) [2]. Nowadays applications like quantum computations, cryptography and quantum communication are already commercially available. The progresses in these fields will allow a deeper understanding of the more fundamental physical aspects of generation, operation and control entanglement between qubits and transfer of information in communication systems. Cavity quantum electrodynamics (CQED) has been a favorite tool to test the entanglement between qubits. Many experiments have been carried out, and in recent years, qubit-qubit entangled states have been created and verified for neutral atoms, trapped ions, superconducting circuits, quantum dots and spins [2]. A lot of schemes are proposed for controlling and protecting the entanglement of qubits in a cavity. The theoretical investigations of such schemes are based on a Jaynes-Cummings model (JCM) and its generalizations [3]. Various generalizations of the JCM were proposed via using multi-levels, multi-atoms, multi-mode fields, dipole-dipole interaction, detuning, Kerr nonlinearity and so on. Two-photon processes have also been studied in cavity QED [4,5]. The Jaynes-Cummings model describing the interaction between a quantized mode of a cavity and a two-level atom is one of the simplest and most important models of light-matter interactions. JCM and its generalizations make it possible to describe all the well-known quantum effects of the interaction of natural or artificial atoms with a field:
photon antibunching and sub-Poisson statistics, vacuum Rabi oscillation, collapses and revivals of Rabi oscillations, superradiance and subradiance, qubit-qubit and field-qubit entanglement, sudden death of entanglement etc [3]. All these effects were observed experimentally for qubits of various physical nature in CQED systems [6-15]. Early CQED experiments were largely confined to the weak qubit-field coupling regime. However, improvements in cavity design and laser cooling techniques shifted the boundary of experimental research toward the strong coupling regime, leading to observations of novel phenomena such as photon-pair production [11] and two-photon absorption at intensities far below levels at which nonlinear transitions normally occur [12]. Recently, there has been novel trends in CQED research including, use of solid state photonic cavities with artificial atoms, spins in micro-cavities, circuit QED, which uses a superconducting cavity coupled to charge and flux qubits, transmons, fluxoniums, quantum dots and other atom-like entities [1, 2]. The novel systems offer much stronger coupling than traditional CQED as well as tunability and promise a whole range of new physics, including low power nonlinear optics, become a real possibility in CQED systems [13]. A lot of CQED experiments showed that JCM and its generalizations give a good predictions for qubit-field dynamics [6-16]. Moreover, there is agreement between the theoretical and experimental results not only for dynamics of the population of qubit energy levels or mean photon numbers [6-15], but for dynamics of the entanglement parameters [16]. The JCM can be extended to nonlinear versions (nondipolar light-matter interaction) [3] known as multiphoton JCM. Such type of nonlinear Hamiltonian can be engineered in trapped ion domain [17,18], quantum dots [19], neutral atoms [20] or in superconducting circuits [21, 22]. When the light-matter interaction strength becomes comparable with the bare frequencies of the system, the ultrastrong coupling regime (USR) is reached. For a qubit-cavity system in the USC regime, the JCM must be replaced with the quantum Rabi model (QRM) [23]. Felicetti and co-authors [24] proposed a superconducting circuit to implement a two-photon QRM in a solid-state device, where a qubit and a resonator are coupled by a two-photon interaction and found that fundamental quantum-optical phenomena are qualitatively modified.

In order to function optimally various quantum computations and quantum communications applications require maximally entangled states. Because of decoherence, which is generally related to noise, there is great difficulty in generating and keeping the integrity of a pure entangled states. Although the interaction between the environment and quantum systems can lead to decoherence, it may also be associated with the formation of non-classical effects such as entanglement [25]. Thus, understanding and investigating entanglement of mixed states becomes one of the actual problem of quantum information. Recently, Bose et al. [26] have shown that entanglement can always arise in the interaction of an arbitrary large system in any mixed state with a single qubit in a pure state, and illustrated this using the Jaynes-Cummings interaction of a two-level atom in a pure state with a field in a thermal state at an arbitrary high temperature. Kim et al. [27] have investigated the atom-atom entanglement in the system of two identical two-level atoms with one-photon transition induced by a single-mode thermal field. They showed that a chaotic field with minimal information can entangled atoms which were prepared initially in a separable state. Zhang directly generalizes Kim’s study to the case when the atoms are slightly detuned from the thermal field, and study how the detuning would affect atom-atom entanglement [28]. He showed that a slight detuning between the atomic transition frequency and the field frequency might cause high entanglement between the atoms and that atoms can entangled even when these are initially prepared in the excited stats. The entanglement between two identical two-level atoms through nonlinear two-photon interaction with one-mode thermal field has been studied by Zhou et al [29]. They showed that atom-atom entanglement induced by nonlinear interaction is larger than that induced by linear interaction. In [30-32] has discovered that two atoms can be entangled also through nonlinear nondegenerate two-photon interaction with two-mode thermal field. But Zhou and co-author [29] did not
take into account the dynamical Stark shifts for energy levels and possible detuning between atom and field frequency. But these mechanisms may greatly enhance the amount of atom-atom entanglement induced by cavity field [28,33-35]. For instance, the problem of controlling entanglement by dynamic Stark effect has attracted much attention [36-40]. In [41] the authors performed spectroscopy of a superconducting qubit coupled not-resonantly to a single mode of an on-chip resonator and showed that strong coupling induces a large ac Stark shift in the energy levels of the qubit.

It is of interest to investigate the dynamics of atom-atom entanglement for two-atom degenerate two-photon Jaynes-Cummings model taking into account the atom-field detuning and Stark shift. Therefore, in the present paper the investigate the influence of the detuning and ac Stark shift on qubit-qubit entanglement induced by thermal cavity field for separable and entangled initial atomic state. Note, that Fink and co-authors [42] investigated the resonant interaction between a single transmon-type multilevel artificial atom and weak thermal and coherent fields. The results was in good quantitative agreement with a generalized JaynesCummings model.

2. Model
We have a system consisting of two effective two-level atoms with transition frequency \( \omega_0 \) not-resonantly interacting with a single-mode thermal cavity field with frequency \( \omega \) via degenerate two-photon transitions in a lossless cavity. In this case one can introduce the atom-field detuning \( \Delta = \omega_0 - 2\omega \). The Hamiltonian describing such a model in the rotating wave approximation and in the interaction picture can be written as

\[
H_I = \hbar \omega_0 \sigma_1^+ + \hbar \omega_0 \sigma_2^+ + \hbar \omega a^+ a + \hbar g \sum_{i=1}^{2} (a_i^{+2} \sigma_i^- + \sigma_i^{+2}) + \sum_{i=1}^{2} \hbar a^+(a_i^+(\Gamma_2 \sigma_i^- \sigma_i^- + \Gamma_1 \sigma_i^+ \sigma_i^+)),
\]  

(1)

where \( a^+ \) and \( a \) denote the creation and annihilation operators of the field, \( \sigma_i^\pm \) are the inversion operators for \( i \)th two-level atom, \( \sigma_i^\pm = |e_i\rangle\langle g_i|, \sigma_i^\pm = |g_i\rangle\langle e_i| \) are the raising and lowering atomic operators with \( |e_i\rangle \) and \( |g_i\rangle \) being the excited and ground states of the \( i \)th atom \((i = 1, 2)\). The \( \Gamma_2 \) and \( \Gamma_1 \) are the parameters describing the intensity-dependent Stark shifts of the two levels of each atom due to the virtual transitions to the intermediate relay level, and \( g \) is the effective two-photon coupling constant between atoms and cavity.

The initial atoms state is assumed to be separable such as

\[
|\Psi(0)\rangle_A = |e, g\rangle
\]  

(2)
or entangled

\[
|\Psi(0)\rangle_A = \cos \theta |e, g\rangle + \sin \theta |g, e\rangle.
\]  

(3)

The initial cavity mode state is assumed to be the thermal one-mode state \( \rho_F(0) = \sum_n p_n |n\rangle\langle n| \), where the weight functions are \( p_n = \bar{n}^n/(1 + \bar{n})^{n+1} \). Here \( \bar{n} \) is the mean photon number in a resonator mode, \( \bar{n} = \langle \exp[\hbar \omega_0/k_B T] - 1 \rangle^{-1} \), \( k_B \) is the Boltzmann constant and \( T \) is the equilibrium resonator temperature.

For general Hamiltonian (1) the exact solution of evolution equation is too cumbersome therefore we consider in detail two special cases

3. Resonant two-atom two-photon model with equal Stark shifts for energy levels
In this section we put \( \Delta = 0 \) and \( \Gamma_2 = \Gamma_1 = \Gamma \). We intend to obtain the exact dynamics of the model under consideration. We start our investigation with the situation when two atoms interact with a resonator field prepared in a Fock state. In this case the solution to
Schrödinger equation can be obtained by expansion of the time-varying state vector in terms of the eigenfunctions of the Hamiltonian (1). Let the excitation number of the system under consideration is \( n \) \((n \geq 0)\). The of eigenfunctions and eigenvalues of Hamiltonian (1) have the form

\[
|\Psi_{in}\rangle = \zeta_{in}(X_{i1n}|g, g, n + 4\rangle + X_{i2n}|e, g, n + 2\rangle + X_{i3n}|g, e, n + 2\rangle + X_{i4n}|e, e, n\rangle)
\]

\[
(n = 0, 1, 2, \cdots ; i = 1, 2, 3, 4),
\]

where \( \zeta_{in} = 1/\sqrt{|X_{i1n}|^2 + |X_{i2n}|^2 + |X_{i3n}|^2 + |X_{i4n}|^2} \),

\[ X_{11,n} = X_{14,n} = 0, \quad X_{12,n} = -1, \quad X_{13,n} = X_{42,n} = 1, \quad X_{21,n} = \alpha_{2n}, \quad X_{22,n} = \beta_{2n}, \quad X_{32,n} = \gamma_{2n}, \]

\[ X_{31,n} = \alpha_{3n}, \quad X_{32,n} = \beta_{3n}, \quad X_{33,n} = \gamma_{3n}, \quad X_{34,n} = X_{42,n} = 1, \quad X_{41,n} = \alpha_{4n}, \quad X_{42,n} = \beta_{4n}, \quad X_{43,n} = \gamma_{4n}. \]

The corresponding eigenvalues are

\[ \varepsilon_{1n} = 2hg(2\beta + n\beta), \]

\[ \varepsilon_{2n} = hg \left( 2(2 + n)\beta + \frac{2(27(5 + 2n)\beta + \Omega_n)^{1/3}}{3^{2/3}} + \frac{2(7 + n(5 + n) + 4\beta^2)}{(27(5 + 2n)\beta + \Omega_n)^{1/3}} \right), \]

\[ \varepsilon_{3n} = h\text{Re} \left( 2(2 + n)\beta + \frac{i(i + \sqrt{3})(27(5 + 2n)\beta + \Omega_n)^{1/3}}{3^{2/3}} - \frac{i(-i + \sqrt{3})(7 + n(5 + n) + 4\beta^2)}{(27(5 + 2n)\beta + \Omega_n)^{1/3}} \right), \]

\[ \varepsilon_{4n} = h\text{Re} \left( 2(2\beta + n\beta) + \frac{(1 - i\sqrt{3})(-7 + 4n - n^2 - 4\beta^2)}{4 \cdot 2^{2/3}(135\beta + 54n\beta + \Omega_n)^{1/3}} - \frac{8(1 + i\sqrt{3})(135\beta + 54n\beta + \Omega_n)^{1/3}}{3 \cdot 2^{1/3}} \right). \]

Here we use the following notations

\[ \Omega_n = \sqrt{(135\beta + 54n\beta)^2 + (21 - 15n - 3n^2 - 12\beta^2)^3}/4, \]

\[ \alpha_{in} = -\left( -24n\beta - 14n^2\beta - 2n^3\beta + 12E_{in} + 7nE_{in} + n^2E_{in} \right)/\left( \sqrt{2} + 3n + n^2\sqrt{2} + 7n + n^2(8\beta + 2n\beta - E_{in}) \right), \]

\[ \beta_{in} = \frac{\left( \zeta_n(2n\beta - E_{in}) - \mu_n(\nu_n\beta - E_{in}) \right)(-\lambda_n + (\zeta_n\beta - E_{in})(\nu_n\beta - E_{in}))}{\left( \zeta_n(\sqrt{\nu_n}\zeta_n - \sqrt{\nu_n}(-\lambda_n + (\zeta_n\beta - E_{in})(\nu_n\beta - E_{in})) \right)} - \frac{(\zeta_n(2n\beta - E_{in}) - \mu_n(\nu_n\beta - E_{in}))}{\sqrt{2} + 3n + n^2(8\beta + 2n\beta - E_{in})}/\zeta_n, \]

\[ \gamma_{in} = -\left( \zeta_n(2n\beta - E_{in}) - \mu_n(\nu_n\beta - E_{in}) \right)/\left( \sqrt{\nu_n}\zeta_n - \sqrt{\nu_n}(-\lambda_n + (\zeta_n\beta - E_{in})(\nu_n\beta - E_{in})) \right) \]

\( (i = 2, 3, 4), \)

\[ \zeta_n = -(12 + 7n + n^2), \quad \lambda_n = ((n + 4)(n + 3), \quad \mu_n = (n + 2)(n + 1), \quad \nu_n = 2(n + 4), \]

\[ \xi_n = 2(n + 2), \quad \beta = \Gamma/g. \]

Using the formulae (4), (5) we derived the exact expression for the time-dependent density matrix \( \rho(t) \) in the ”dressed states” representation. The evident form of this expression is too cumbersome to present here. Taking a trace over the field variables we obtained the reduced atomic density operator \( \rho_A(t) = Tr_F \rho(t) \). For two-qubit system described by the density
operator $\rho_A(t)$, a measure of entanglement or negativity can be defined in terms of the negative eigenvalues $\mu_i$ of partial transpose of a reduced atomic density matrix $(\rho_A^T)$

$$\varepsilon = -2 \sum \mu_i.$$  

(6)

For initial atomic states (2), (3) the partial transpose of reduced atomic density operator $\rho_A(t)$ has the form

$$\rho_A^T(t) = \begin{pmatrix} U(t) & 0 & 0 & H(t)^* \\ 0 & W(t) & 0 & 0 \\ 0 & 0 & V(t) & 0 \\ H(t) & 0 & 0 & R(t) \end{pmatrix}.$$  

(7)

The elements of matrix (7) for initial atomic state (2) are

$$U = \sum_n p_n |Z_{11,n}(t)|^2, \quad W = \sum_n p_n |Z_{21,n}(t)|^2, \quad V = \sum_n p_n |Z_{31,n}(t)|^2,$$

$$R = \sum_n p_n |Z_{11,n}(t)|^2, \quad H = \sum_n p_n Z_{21,n}(t)Z_{31,n}(t)^*.$$  

(8)

Here

$$Z_{ij,n} = e^{-iE_{1n}t/\hbar} \zeta_{1n} Y_{jin} X_{1in} + e^{-iE_{2n}t/\hbar} \zeta_{2n} Y_{jin} X_{2in} +$$

$$+ e^{-iE_{3n}t/\hbar} \zeta_{3n} Y_{jin} X_{3in} + e^{-iE_{4n}t/\hbar} \zeta_{4n} Y_{jin} X_{4in} \quad (i = 1, 2, 3, 4).$$  

(9)

where $Y_{jin} = w_{jn}X_{jin}^*$. These for initial atomic states (3) have too cumbersome form to present in this paper.

Matrix (7) has only one eigenvalue, which may take a negative value. As a result we have

$$\varepsilon(t) = \sqrt{(U(t) - R(t))^2 + 4|H(t)|^2} - U(t) - R(t).$$  

(10)

We obtained the exact expressions for negativity (10). These contain sums by photon number $n$ from zero to infinity with thermal weight functions $p_n$. The convergence of such series is well studied. Therefore, we can easily achieve any accuracy of computations required for subsequent comparison with experimental data when they are obtained.

The results of numerical calculations of negativity (10) are shown in Fig. 1 and Fig. 2.

The negativity for a separable initial atomic state (2) is plotted in Fig. 1 as a function of a scaled time $gt$ with fixed mean photon number and different values of Stark shift. We take the mean photon number $\bar{n} = 0.1$. One can easily find that as a scaled Stark shift $\beta$ increases, higher entanglement is obtainable. In Fig. 2, we plot the negativity as a functions of a scaled time $gt$ for initial entangled atomic state (4). As in the previous cases we put $\bar{n} = 0.1$. For entangled initial atomic state the inclusion of the Stark shift leads to stabilization of entanglement oscillations.

4. Not-resonant two-atom two-photon model with zero Stark shifts for energy levels

In this section we put $\Gamma_2 = \Gamma_1 = 0$. Let the excitation number of the system under consideration is $n$ ($n \geq 0$). Then, the eigenfunctions of the Hamiltonian (1) can be written as (4) with

$$X_{11} = 0, \quad X_{12} = -1, \quad X_{13} = 1, \quad X_{14} = 0, \quad X_{i1,n} = -\frac{2\sqrt{2} + 3n + n^2\sqrt{12} + 7n + n^2}{24 + 14n + 2n^2 - \delta_{in} - \varepsilon_{in}^2},$$

(11)
Figure 1. The negativity as a function of $gt$ for initial atomic state (3) with $\beta = 0$ (solid) and $\beta = 1$ (dashed). The mean photon number $\bar{n} = 0.1$.

Figure 2. The negativity as a function of $gt$ for entangled initial atomic state (4) with $\theta = \pi/4$ and $\beta = 0$ (solid) and $\beta = 1$ (dashed). The mean photon number $\bar{n} = 0.1$.

$$X_{i2,n} = X_{i3,n} = -\frac{\sqrt{2} + 3n + n^2(\delta + \varepsilon_{in})}{24 + 14n + 2n^2 - \delta \varepsilon_{in} - \varepsilon_{in}^2}, \quad X_{i4,n} = 1 \quad (i = 2, 3, 4).$$

The corresponding eigenvalues are

$$\varepsilon_{1n} = 0, \quad \varepsilon_{2n} = -\frac{2^{1/3}}{3 A_n} \left( B_n - A_n^2/2^{2/3} \right),$$

$$\varepsilon_{3n} = \frac{1}{3 \times 2^{2/3} A_n} \text{Re} \left( \left( 1 + i\sqrt{3} \right) Y_n - \left( 1 - i\sqrt{3} \right) A_n^2/2^{2/3} \right),$$

$$\varepsilon_{4n} = \frac{1}{3 \times 2^{2/3} A_n} \text{Re} \left( \left( 1 - i\sqrt{3} \right) B_n - \left( 1 + i\sqrt{3} \right) A_n^2/2^{2/3} \right),$$

where

$$A_n = \left( Z_n + \sqrt{Z_n^2 + 4 \left( -84 - 60n - 12n^2 - 3\delta^2 \right)^3} \right)^{1/3},$$

$$B_n = -84 - 60n - 12n^2 - 3\delta^2, \quad Z_n = -216 \delta \left( 5/2 + n \right).$$

For separable initial atomic states (2) or entangled state (3) the partial transpose of reduced atomic density matrix has the form (7). The elements of matrix (7) for initial atomic state $|e, g\rangle$ have the form (8), where $C_{ij,n}$ and $X_{ij,n}$ ($i, j = 1, 2, 3, 4$) described by the formulae (9) and (11) correspondingly. These for entangled initial state (3) are too cumbersome to present in this paper. The negativity for considered states has the form (10).

The problem of computational accuracy was discussed in the previous section. The results of numerical calculations of negativity (10) for considered model are shown in Figs.3-6.

The negativity for separable initial atomic state (2) is shown in Fig. 3 as a function of a scaled time $gt$ for small detunings and fixed value of mean photon number $\bar{n} = 0.1$. One can see from Fig. 3 that for a slight detunings, as the parameter $\delta$ increases, higher entanglement is obtainable. In Fig. 4 and Fig. 5 we compare the dependence of entanglement on detuning for one- and two-photon two two-atom Jaynes-Cummings models. Fig. 4 shows that for resonant atom-field interaction the entanglement induced by nonlinear two-photon interaction is larger than that induced by linear one-photon interaction. This result has been earlier derived by Zhou and co-authors [21]. But, as one can see from Fig. 5, the situation is opposite for non-resonant
atom-field interaction. In the presence of the atom-field detuning the entanglement induced by two-photon interaction is smaller than that induced by one-photon interaction.

The time dependence of negativity for entangled initial atomic state (3) is presented in Fig. 6 as a function of a scaled time $gt$ for fixed value of mean photon number $\bar{n} = 1$. One can see from Fig. 6 that the inclusion of the atom-field detuning stabilizes the negativity oscillations. For resonant atom-field interaction situation the effect of the sudden death of the entanglement takes place. But the inclusion of a detuning eliminates this phenomenon.

5. Conclusion
In this paper, we investigated the influence of Stark shift and atom-field detuning on entanglement between two two-level atoms not resonantly interacting with a thermal one-mode field of a lossless resonator via degenerate two-photon transitions. We showed that for a separable
initial atomic state $|e, g\rangle$ a slight detuning or Stark shift may greatly enhance the amount of atom-atom entanglement induced by a thermal field. We compared the influence of the detuning on atomic entanglement for two-atom one- and two-photon Jaynes-Cummings model. The results showed that for not-resonant atom-field interaction the entanglement induced by nonlinear two-photon interaction is smaller than that induced by one-photon interaction in contrast to the resonant interaction situation. In the last case, as have been earlier shown by Zhou et al. [6], the entanglement induced by nonlinear interaction is larger than that induced by linear interaction. The results showed that for Bell-type entangled initial atomic state the Stark shift and detuning have great impact on the atom-atom entanglement evolution. As these parameters increase, there are an appreciable decrease in the amplitudes of the negativity oscillations, i.e. the stabilization of entanglement takes place. In the case of relatively intensive thermal fields the inclusion of detuning eliminates the effect of the sudden death of entanglement.

6. References
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