Induced $\mathcal{N} = 2$ composite supersymmetry in $(2+1)$ dimensions

J Alexandre, N E Mavromatos and Sarben Sarkar
Department of Physics, Theoretical Physics, King’s College London, Strand, London WC2R 2LS, UK
E-mail: sarben.sarkar@kcl.ac.uk

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Abstract. Starting from $N = 1$ scalar supermultiplets in (2+1) dimensions, we build explicitly the composite superpartners which define an $N = 2$ superalgebra induced by the initial $N = 1$ supersymmetry. The occurrence of this extension is linked to the topologically conserved current out of which the composite superpartners are constructed.

The study of $(2 + 1)$-dimensional gauge theories has been motivated by the search for an understanding of confinement [1]. The degree to which they can be of help in understanding confinement at zero temperature in three space dimensions is unclear because the nature and possible configurations of textures, instantons and other non-perturbative features such as the locality of disorder variables are very different. Their attraction is that their comparative simplicity allows a much more complete understanding of the confinement phenomenon. However, another reason for studying such theories is the interest in planar systems in high temperature superconductors [2] where the phases are believed not to be of conventional type. In particular, string-like structures, which are features of confinement in the gauge theories, occur as inhomogeneities in the charge and magnetic order [3] in certain parameter regimes of these materials. Both in the study of confinement and the condensed matter analogues, some of the reason for controversy is the room for error in the calculations that can be performed due to the intrinsic limitations of the methods themselves. It is thus important to pursue models with exact solutions. Unlike for (1+1) dimensions, where infinite-dimensional groups and the factorizability of the $S$-matrix come into play [4], dynamical supersymmetry is an important ingredient for obtaining exact information about the phases of the theory [5]. Although $N = 1$ symmetry was demonstrated for a condensed matter model [6], a lack of holomorphicity properties meant that exact non-perturbative information was not available. The situation is much improved for $N = 2$ supersymmetry. This paper demonstrates how $N = 1$ supersymmetry, which was in terms of
constituent fields, can be elevated to an $N = 2$ supersymmetry in terms of suitably constructed composite fields.

Our results may be of relevance in attempts towards an analytic understanding at a non-perturbative level of the dynamics of strongly correlated electron systems, with relevance to high-temperature superconductivity and more generally to antiferromagnetism. Also our model may be viewed as a toy model for an understanding of ideas related to the so-called scaleless limit of gauge theories [7], where the gauge fields appear dynamically from more fundamental interacting constituents in the theory.

However, in the four-dimensional setting of [7], the emergent gauge bosons (photons) appeared as Goldstone bosons of a spontaneous breakdown of Lorentz symmetry, associated with non-zero vacuum expectation values (v.e.v.) of vector fields linearizing the four-fermion Thirring interactions of the model. In three space-time dimensions, on the other hand, the photons have only a single degree of freedom and hence they are allowed, in a sense, to get a v.e.v. without breaking Lorentz symmetry. It is this fact that allows the extension of such ideas in (2+1) dimensions to incorporate supersymmetry, which is intimately related to Lorentz invariance. From a physical point of view, with relevance to strongly correlated electrons, we remark that the maintenance of Lorentz symmetry is connected with the fact that we restrict our attention to excitations near nodes of the Fermi surface of such systems [6], which exhibit relativistic behaviour, with the role of the velocity of ‘light’ played by the Fermi velocity at the node.

We will first summarize the results of [8], dealing with distinct $N = 1$ quadratic composite supermultiplets. We then expose the procedure of adding higher-order composites so as to generate the coupling between the scalar and vector supermultiplets, which implies the $N = 2$ structure of the transformations. Eventually, we give the explicit construction of the quartic composites and show that their existence is intimately linked to the topologically conserved current.

We use a representation of the Clifford algebra where all the $\gamma$-matrices are real: $\gamma^0 = -i\sigma^2$, $\gamma^1 = \sigma^1$ and $\gamma^2 = \sigma^3$ and they satisfy

\[
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad \text{with} \quad g^{\mu\nu} = (-1, 1, 1)
\]

\[
[\gamma^\mu, \gamma^\nu] = -2\epsilon^{\mu\nu\rho} \gamma_\rho.
\]  

(1)

We start from two complex $N = 1$ scalar supermultiplets $(z_a, \psi_a, f_a, a = 1, 2)$, containing the constituent fields which transform as

\[
\delta z_a = \bar{\epsilon} \psi_a
\]

\[
\delta \psi_a = \bar{\epsilon} z_a \epsilon + f_a \epsilon
\]

\[
\delta f_a = \bar{\epsilon} \bar{\psi}_a
\]  

(2)

where $\epsilon$ is a real Grassmann parameter and $\bar{\epsilon} = \epsilon^T \gamma^0$. We will use the following essential properties concerning the scalar quantities:

\[
\bar{\psi}_1 \psi_2 = (\bar{\psi}_2 \psi_1)^*
\]

\[
\bar{\psi}_1 \gamma_\mu \psi_2 = - (\bar{\psi}_2 \gamma_\mu \psi_1)^*,
\]  

(3)

and the $2 \times 2$ matrix [8]:

\[
\psi_2 \bar{\psi}_1 = - \frac{1}{2} (\bar{\psi}_1 \psi_2 + (\bar{\psi}_1 \gamma_\mu \psi_2) \gamma^\mu).
\]  

(4)

where $\bar{\psi}_a = \psi_a^\dagger \gamma^0$. 

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It was found in [8] that the quadratic composites
\[
\phi = \psi_1 \psi_2, \\
A_\mu = \psi_1 \gamma_\mu \psi_2 - z_1^* \partial_\mu z_2 + z_2 \partial_\mu z_1^*,
\]
belong respectively to an \( N = 1 \) scalar supermultiplet \((\phi, \psi, f)\) and an \( N = 1 \) vector supermultiplet \((A_\mu, \chi)\) where the composite superpartners read, in terms of the constituent fields
\[
\psi = (f_1^* - \partial z_1^*) \psi_2 + (f_2 - \partial z_2) \psi_1^*, \\
\chi = (f_1^* + \partial z_1^*) \psi_2 - (f_2 + \partial z_2) \psi_1^*, \\
f = 2(f_1^* f_2 - \partial_\mu z_1^* \partial^\mu z_2) - \psi_1^* \psi_2 - (\psi_2 \psi_1^*)^*.
\]

The transformation of \( A_\mu \) has actually the expected form up to a gauge transformation, which implies that it must be a gauge field. The fields defined in equations (5) are complex: they are not the physical degrees of freedom but have the generic composite structure that we want to study. The physical degrees of freedom form \( SU(2) \) triplets of composites which are given in [8], such that we have three \( N = 1 \) scalar supermultiplets and three \( N = 1 \) vector supermultiplets. From the three scalar composite supermultiplets, we form one real and one complex \( N = 1 \) scalar supermultiplet, that we denote by \((\phi_r, \psi_r, f_r)\) and \((\phi_c, \psi_c, f_c)\) respectively. The \( N = 1 \) composite supersymmetric transformations, found in [8], are then
\[
\delta \phi_r = \varepsilon \psi_r, \quad \delta \psi_r = (\partial \phi_r + f_r) \varepsilon, \quad \delta f_r = \varepsilon \partial \psi_r, \\
\delta \phi_c = \varepsilon \psi_c, \quad \delta \psi_c = (\partial \phi_c + f_c) \varepsilon, \quad \delta f_c = \varepsilon \partial \psi_c, \\
\delta A_\mu^a = \varepsilon \gamma_\mu \chi^a, \quad \delta \chi^a = -\frac{1}{2} F_{\mu \nu}^a \gamma^\mu \gamma^\nu \varepsilon,
\]
where \( F_{\mu \nu}^a \) is the Abelian field strength of \( A_\mu^a \) (quadratic in the constituent fields), for each gauge index \( a = 1, 2, 3 \) and the gauginos \( \chi^a \) are real.

In order to elevate the \( N = 1 \) supersymmetry to an \( N = 2 \) (Abelian) supersymmetry, we should couple the complex composite scalar supermultiplet to one of the composite vector supermultiplets, which we denote as \((A_\mu, \chi)\). For this, we will construct explicitly the covariant derivative of \( \phi_c \) by adding to the complex quadratic scalar field a higher-order composite which will generate the minimal coupling. We take into account the quartic contribution only and neglect the higher orders: we are looking for a quartic composite scalar \( M \) whose supersymmetric transformations generate the quartic fermion \( \Lambda \) and the quartic auxiliary field \( F \) such that, under the transformations (2), one has
\[
\delta M = \varepsilon \Lambda, \\
\delta \Lambda = (-i \lambda \phi_c + \partial M + F) \varepsilon, \\
\delta F = \varepsilon (-i \lambda \psi_c + \partial \ Lambda)
\]
We shall find that the transformation (8) of the fermion is satisfied in the gauge defined by the complex equation
\[
\partial^\nu (A_\nu \square \phi_c) = 0,
\]
which implies actually two gauge conditions. This is possible in (2+1) dimensions, since a gauge field has one physical degree of freedom [9]. The transformation (8) of the auxiliary field will be satisfied up to irrelevant operators: the expected transformation is obtained if we set higher-order derivative operators to zero. This approximation consists of neglecting derivative interactions
between the composites that are ignored in the framework of the low-energy effective theories that we wish to describe. Such terms will not affect the infrared universality class of the model we are interested in.

Making the substitutions:

\[
\begin{align*}
\phi_c & \rightarrow \Phi = \phi_c + gM \\
\psi_c & \rightarrow \Psi = \psi_c + g\Lambda \\
f_c & \rightarrow F = f_c + g\mathcal{F},
\end{align*}
\]

where \( g \) is a dimensionful constant, we shall then obtain the expected covariant derivatives

\[
\begin{align*}
D_\mu \Phi &= (\partial_\mu - igA_\mu)\Phi = \partial_\mu \phi_c - igA_\mu \phi_c + g\partial_\mu M + \text{sixth order composite} \\
D_\mu \Psi &= (\partial_\mu - igA_\mu)\Psi = \partial_\mu \psi_c - igA_\mu \psi_c + g\partial_\mu \Lambda + \text{sixth order composite}
\end{align*}
\]

in the (supersymmetry) transformation laws of the complex fermion \( \Psi \) and auxiliary field \( F \) respectively. Next, we shall construct the complex gaugino \( \lambda = \psi_r + i\chi \) and obtain from equations (7) the following transformations:

\[
\begin{align*}
\delta \Phi &= \epsilon \Psi, \quad \delta \Psi = (D\Phi + F)\epsilon, \\
\delta \phi_r &= \bar{\epsilon} \psi_r, \quad \delta A_\mu = \bar{\epsilon} \gamma_\mu \chi, \\
\delta \lambda &= \left( \partial \phi_r + f_r - \frac{i}{2} F_{\mu\nu} \gamma^\mu \gamma^\nu \right) \epsilon \\
\delta f_r &= \bar{\epsilon} \bar{\psi}_r, \quad \delta F \simeq \epsilon D\Psi,
\end{align*}
\]

which, when \( \epsilon \) is a complex parameter, constitute an off-shell set of \( N = 2 \) supersymmetric transformations for an Abelian Higgs model [10].

At this point we note that in [10], where the authors consider ‘elementary’ fields and not composites, the superpartners transform under an on-shell set of \( N = 2 \) transformations where the equations of motion of the auxiliary fields are used, in the supersymmetric Abelian Higgs model. In this paper we shall concentrate only on the algebraic aspects of the composite supersymmetric transformations and will not consider the detailed dynamics of this model, and the associated physical consequences in connection with strongly correlated electron systems. These topics have been briefly mentioned in [8], and a more detailed analysis will be postponed to a forthcoming publication.

We also remark that the extension of the composite \( N = 1 \) transformations (7) to an \( N = 2 \) non-Abelian superalgebra would involve too many gauge conditions. Indeed, besides the addition of a quartic scalar, in order to generate the covariant derivatives, this extension would also consist of adding quartic gauge fields \( A^{a}_{\mu} \), which would transform into quartic gauginos \( \Sigma^{a} \) and are such that

\[
\begin{align*}
\delta A^{a}_{\mu} &= \bar{\epsilon} \gamma_\mu \Sigma^{a} \\
\delta \Sigma^{a} &= f^{abc} A^{b} A^{c} \epsilon,
\end{align*}
\]

so as to generate the non-Abelian field strength in the transformation of the gauginos (\( f^{abc} \) are the \( SU(2) \) structure constants). We found that the transformations (13) of the quartic gauginos are satisfied provided we impose two new constraints on each colour of the gauge field. This shows that a non-Abelian model, such as the one considered in [11], cannot be obtained with the composite procedure developed here.

Finally, we note that no new constraints arise if we consider higher-order composites: the quartic composite can be seen as the truncation of a series of composite operators which lead
to the exact covariant derivatives if they are resummed. The gauge condition (9) gives rise to a gauge fixing term in the Lagrangian which is also the truncation of a series of gauge fixing terms.

We now proceed to discuss in detail the derivation of the above results, namely to construct explicitly the quartic-order superpartners which lead to the transformations (12). First we note that, in order for the expression (11) of the covariant derivative to be consistent, the mass dimension of $g$ should be $-1$, such that the mass dimension of $M$ should be 3. Let us consider the following possibilities:

$$
\begin{align*}
\Box M(1) &= \phi_c \partial^\mu A_\mu \\
\Box M(2) &= A_\mu \partial^\mu \phi_c \\
\Box M(3) &= \bar{\psi}_c \chi.
\end{align*}
$$

(14)

The supersymmetric transformation of $M(1)$ under (2) leads to $\delta M(1) = \bar{e} \Lambda(1)$ with

$$
\Box \Lambda(1) = \partial^\mu A_\mu \psi_c + \phi_c \partial \chi.
$$

(15)

Taking into account the properties (3), we find that the transformation of $\Box \Lambda(1)$ reads

$$
\delta \Box \Lambda(1) = (\partial (\phi_c \partial^\mu A_\mu) - \phi_c \partial \partial A + f_c \partial^\mu A_\mu) \varepsilon + (\partial \chi \bar{\psi}_c - \psi_c \partial \bar{\psi}_c \chi) \varepsilon.
$$

(16)

Finally, the property (4) gives

$$
\delta \Box \Lambda(1) = \partial (\partial M(1) + \partial M(2) + \partial M(3)) \varepsilon - (\partial \partial \Lambda - \partial \Lambda) \varepsilon.
$$

(17)

If we define $\delta M(2) = \bar{e} \Lambda(2)$ and $\delta M(3) = \bar{e} \Lambda(3)$ and proceed in a similar way, we find

$$
\begin{align*}
\Box \Lambda(2) &= \partial \phi_c \chi + A_\mu \partial^\mu \psi_c \\
\Box \Lambda(3) &= (f_c - \partial \phi_c) \chi + (\partial \Lambda - \partial A^\mu) \psi_c,
\end{align*}
$$

(18)

and the transformations of $\Box \Lambda(2)$ and $\Box \Lambda(3)$ are

$$
\begin{align*}
\delta \Box \Lambda(2) &= \partial \Box M(2) \varepsilon - (\partial \partial \psi_c \chi) \varepsilon + (A_\mu \partial^\mu f_c - \partial^\mu \phi_c \partial A_\mu + \phi_c \partial A_\mu f_c - \partial \phi_c \partial A_\mu) \varepsilon \\
\delta \Box \Lambda(3) &= \partial \Box M(3) \varepsilon + \partial (\partial \psi_c \chi) \varepsilon + (2 \partial \phi_c \partial A_\mu + 2 \partial A_\mu \partial^\mu \phi_c - 2 \partial \mu \phi_c \partial A_\mu - 2 \partial \phi_c \partial A_\mu) \varepsilon.
\end{align*}
$$

(19)

If we define the scalar $\tilde{M}$ to be the linear combination

$$
\tilde{M} = M^{(1)} + 2 M^{(2)} + M^{(3)},
$$

(20)

and note that $\delta \tilde{M} = \bar{e} \tilde{\Lambda} = \bar{e} (\Lambda_1 + 2 \Lambda_2 + \Lambda_3)$, then we find

$$
\delta \Box \tilde{\Lambda} = (- \partial \partial \Lambda - 2 \partial \Lambda \partial A^\mu \chi) \varepsilon,
$$

(21)

where the auxiliary field $\tilde{\mathcal{F}}$ satisfies

$$
\Box \tilde{\mathcal{F}} = 2 A_\mu \partial^\mu f_c + f_c \partial^\mu A_\mu - \partial^\mu \bar{\psi}_c \chi.
$$

(22)

In order to generate the expected minimal coupling $\Box (\phi_c \Lambda)$, we still need to add to (21) the operator $- \Box \phi_c \Lambda$, which will be obtained by the introduction of the scalar $M^{(4)}$ defined by the following equation

$$
\partial^\rho \Box M^{(4)} = \epsilon^{\mu \nu \rho} f_c \partial_\mu A_\nu.
$$

(23)
Note that the occurrence of this scalar is specific to $(2+1)$ dimensions, as it is proportional to
the topologically conserved current $J^\rho = \epsilon^{\mu\nu\rho} \partial_\mu A_\nu$, which plays a central role in the
elevation of an $N = 1$ supersymmetry to an extended $N = 2$ supersymmetry [12]. If we consider the
identity (1), we find that
$$\delta \Box M^{(4)} = \Box^2 M^{(4)} = \Box (f_c (\partial^\nu A_\nu - \Box A)),$$
which will be used in the supersymmetric transformations of $M^{(4)}$. The gaugino $\Lambda^{(4)}$, defined
by $\delta M^{(4)} = \overline{\epsilon} \Lambda^{(4)}$, satisfies
$$\partial^\rho \Box \Lambda^{(4)} = \epsilon^{\mu\nu\rho} \left( \partial_\mu A_\nu \psi_c + f_c \gamma_\nu \partial_\mu \chi \right),$$
such that
$$\Box \Lambda^{(4)} = (\partial^\nu A_\nu - \Box A) \psi_c - 2 f_c \partial_\nu \chi.$$  

The supersymmetric transformation of $\Lambda^{(4)}$ then gives, when using (4),
$$\delta (\Box \Lambda^{(4)}) = \left( (\partial^\nu A_\nu - \Box A)(\partial f_c + \Box f_c) - 2 f_c \partial (\partial^\nu A_\nu - \Box A) \right) \epsilon + \epsilon^{\mu\nu\rho} \left( \partial_\nu \overline{\psi}_c \gamma^\rho \chi - \partial^\rho \overline{\psi}_c \partial_\rho \chi \right) \gamma_\rho \epsilon$$
$$= \Box \left( -\Box \phi_c A + \Box M^{(4)} + \Box \mathcal{F}^{(4)} \right) \epsilon + \partial^\nu (A_\nu \Box \phi_c) \epsilon,$$
where the quartic auxiliary field $\mathcal{F}^{(4)}$ satisfies
$$\partial^\rho \Box \mathcal{F}^{(4)} = \epsilon^{\mu\nu\rho} A_\mu \partial_\nu \psi_c + 2 \partial_\nu (f_c \partial^\nu A_\nu) + f_c (\Box A^\rho - \partial^\rho \partial^\nu A_\nu)$$
$$+ \partial_\nu \overline{\psi}_c \gamma^\rho \partial^\sigma \chi - \partial^\rho \overline{\psi}_c \partial_\rho \chi + \epsilon^{\mu\nu\rho} \partial_\nu \overline{\psi}_c \partial_\mu \chi.$$

We now assume the gauge choice
$$\partial^\nu (A_\nu \Box \phi_c) = 0,$$
which allows us to write the transformation of $\Box \Lambda^{(4)}$ in the expected form, i.e.
$$\delta (\Box \Lambda^{(4)}) = \left( -\Box \phi_c A + \Box M^{(4)} + \Box \mathcal{F}^{(4)} \right) \epsilon,$$
such that the final quartic spinor that has the expected transformation (8) is
$$\Lambda = i (\tilde{\Lambda} + \Lambda^{(4)}),$$
and the quartic scalar $M$ and auxiliary field $\mathcal{F}$ are given by
$$M = i (\tilde{M} + M^{(4)})$$
$$\mathcal{F} = i (\tilde{\mathcal{F}} + \mathcal{F}^{(4)}).$$

Let us now consider the auxiliary field $\mathcal{F}$. It can be easily checked that
$$-i \Box^2 \mathcal{F} = \Box \left( 2 A_\mu \partial^\mu f_c + f_c \partial^\mu A_\mu \right) + \epsilon^{\mu\nu\rho} \partial_\mu A_\mu \partial_\nu \Box \phi_c - \partial^{\mu} \left( \Box \overline{\psi}_c \gamma_\mu \chi + \partial^\nu \overline{\psi}_c \gamma_\nu \partial_\mu \chi \right).$$

The supersymmetric transformation of $\Box^2 \mathcal{F}$ is then
$$\delta (\Box^2 \mathcal{F}) = \overline{\epsilon} \Box^2 (i \Box \psi_c + \Box \Lambda) + \overline{\epsilon} S,$$
where the spinor $S$ reads
$$S = \partial_\nu f_c \partial_\mu \chi - \Box f_c \Box \chi - 2 \partial^\mu \phi_c \partial_\mu \chi$$
$$+ \Box \left( 2 \Box \psi_c + 4 \partial^\mu \partial_\mu \psi_c + \Box \Box \psi_c - \Box A^\mu \partial_\mu \psi_c \right) + \partial^\alpha \left( \partial^\nu A_\nu - \Box A \right) (\gamma_\alpha \Box \psi_c - \partial_\alpha \Box \psi_c) - \epsilon^{\mu\nu\rho} \partial_\mu A_\nu \partial_\rho \psi_c.$$
As we can now see, ignoring the contribution of $S$ amounts to neglecting higher-order derivative operators that can be omitted in the context of low energy effective theories. Hence, up to such terms, one obtains the standard supersymmetry transformation of the auxiliary field:

$$\delta F \simeq \epsilon (-i \mathcal{A} \psi_c + \mathcal{B} \Lambda).$$

(36)

Finally, the quartic scalar $M$ that we were looking for satisfies:

$$-i \partial^\rho \Box M = \epsilon^{\mu\nu\rho} f_c \partial_\mu A_\nu + \partial^\rho \left( 2 \partial^\mu \phi_c A_\mu + \phi_c \partial^\mu A_\mu + \overline{\psi}_c \chi \right).$$

(37)

This completes the off-shell closure of the $N = 2$ supersymmetric algebra for the composite operators at a quartic order in the constituent fields.

As we have seen, with the specific choice of composite operators discussed here, of quartic order in the constituent spinon and holon fields\(^1\), we have arrived at an $N = 2$ supersymmetric extension which coupled the two $N = 1$ supersymmetries of the quadratic composites of [8]. The coupling was done in a way consistent with a supersymmetric Abelian Higgs model [10]. The emergence of an $N = 2$ supersymmetry algebra at a composite level is consistent with the elevation of $N = 1$ supersymmetry to $N = 2$ in the constituent theory [6], due to the existence of topological currents in (2+1) dimensions.

It should be remarked at this stage that, from a physical point of view, one starts from a microscopic lattice system, an appropriately extended $t - j$ model [6] with nodes in its Fermi surface. Upon assuming spin-charge separation one arrives at a continuum low-energy theory of nodal spinons and holon excitations, which has the form of a relativistic $CP^1$-$\sigma$-model (magnon-spinons) coupled to Dirac-like fermions (holon degrees of freedom). At supersymmetric points in the microscopic model phase space [6] one recovers a supersymmetric theory between the spinon and holon constituents without any dynamical gauge fields. This is the constituent theory. The non-dynamical gauge fields simply express contact interactions between spinons and holons.

Although at first sight the model appears to have only an $N = 1$ supersymmetry, it actually has a hidden $N = 2$ supersymmetry due to its low dimensionality (2+1 dimensions), for reasons stated above. At the composite operator level, obtained after integrating out the non-dynamical gauge fields, one generates dynamical gauge fields, made out of appropriate combinations of spinon and holons. It is interesting to notice that the choice of composite operators made in this paper seem to necessitate an Abelian nature of the gauge field involved in the $N = 2$ supersymmetric multiplet. We were unable to find a choice of composite operators that generate the full non-Abelian $SU(2)$ supersymmetric model of [11].

In view of this Abelian nature of the gauge group an interesting question arises as to whether the composite Abelian gauge field, which emerges from this construction, is compact or not. The two cases lead to very different non-perturbative dynamics [14], for instance as far as confinement properties of the three-dimensional gauge theory are concerned. Such matters will be investigated in more detail in future works.

We believe that this work, together with our earlier works on the subject [6, 8], opens up a way for a formal discussion of exact non-perturbative results on the phase-space dynamics of strongly correlated electron systems, even if the latter lie away from such supersymmetric points in realistic situations. If one understands analytically some aspects of the non-perturbative dynamics at supersymmetric points, then one might hope to use such knowledge to draw conclusions for the theory away from these points, where supersymmetry is explicitly broken. We intend to

\(^1\) The form of the quadratic parts (in the constituent fields) of these operators was motivated by microscopic lattice system considerations [13, 6].
continue working along these lines with a view to applying the results to realistic condensed-matter situations of relevance to high-temperature superconductivity, and more generally to antiferromagnetic systems. The rich phenomenology of such systems will always be our guiding principle in discussing various models.

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