Index-stable compact $p$-adic analytic groups

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With an appendix by Jean-Pierre Serre.

Abstract. A profinite group is index-stable if any two isomorphic open subgroups have the same index. Let $p$ be a prime, and let $G$ be a compact $p$-adic analytic group with associated $\mathbb{Q}_p$-Lie algebra $\mathcal{L}(G)$. We prove that $G$ is index-stable whenever $\mathcal{L}(G)$ is semisimple. In particular, a just-infinite compact $p$-adic analytic group is index-stable if and only if it is not virtually abelian. Within the category of compact $p$-adic analytic groups, this gives a positive answer to a question of C. Reid. In the appendix, J-P. Serre proves that $G$ is index-stable if and only if the determinant of any automorphism of $\mathcal{L}(G)$ has $p$-adic norm 1.

Mathematics Subject Classification. Primary 20E18, 22E20; Secondary 22E60.

Keywords. Index-stable group, Just-infinite profinite group, $p$-adic analytic group, Pro-$p$ group, $p$-adic Lie lattice, Commensurator.

Introduction. Throughout, let $p$ be a prime. A pro-$p$ group $G$ is said to be powerful if $p \geq 3$ and $[G, G] \leq G^p$, or $p = 2$ and $[G, G] \leq G^4$. A finitely generated torsion-free powerful pro-$p$ group is called uniform. In his seminal paper *Groupes analytiques $p$-adiques* [12], M. Lazard obtained the following algebraic characterization of $p$-adic analytic groups: a topological group is $p$-adic analytic if and only if it contains an open uniform pro-$p$ subgroup (see [4, Theorems 8.1 and 8.18]). With every uniform pro-$p$ group $U$ one can naturally associate a $\mathbb{Z}_p$-Lie lattice $L_U$. The $\mathbb{Q}_p$-Lie algebra associated with a compact $p$-adic analytic group $G$ is defined as $\mathcal{L}(G) := L_U \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$, where $U$ is an open uniform pro-$p$ subgroup of $G$.

The following theorem is the main result of this paper.

Ilir Snopce supported by CNPq and FAPERJ.
Theorem 1. Let $G$ be a compact $p$-adic analytic group, let $\mathcal{L}(G)$ be the $\mathbb{Q}_p$-Lie algebra associated with $G$, and assume that $\mathcal{L}(G)$ is semisimple. If $H$ and $K$ are two isomorphic closed subgroups of $G$, then $|G : H| = |G : K|$. 

Remark. Soon after a preliminary version of this paper was published on the arXiv, the authors received a letter from Jean-Pierre Serre with a different proof of a more general version of Theorem 1; see Theorem 9. We thank Professor Serre for kindly agreeing to include his letter as an appendix to this paper.

A profinite group is said to be index-unstable if it contains a pair of isomorphic open subgroups of different indices; otherwise, it is said to be index-stable. This definition was introduced by C. Reid in [15], where he also raised the following question, which is still open.

Question 2. Let $G$ be a (hereditarily) just-infinite profinite group which is index-unstable. Is $G$ necessarily virtually abelian?

Recall that a profinite group $G$ is said to be just-infinite if it is infinite, and every non-trivial closed normal subgroup of $G$ is of finite index. A just-infinite profinite group $G$ is hereditarily just-infinite if every open subgroup of $G$ is just-infinite.

The following corollary gives a positive answer to Question 2 within the category of $p$-adic analytic groups.

Corollary 3. Let $G$ be a just-infinite compact $p$-adic analytic group. Then $G$ is index-stable if and only if it is not virtually abelian.
the modular function for the strong topology. Moreover, it is not difficult to see that if \( G \) is a just-infinite profinite group, then \( VZ(G) = \{1\} \) if and only if \( G \) is not virtually abelian.

The following result is a direct consequence of Corollary 3 and the above discussion.

**Corollary 4.** Let \( G \) be a just-infinite compact \( p \)-adic analytic group. If \( G \) is not virtually abelian, then \( \text{Comm}(G)_S \) is unimodular.

Solvable just-infinite \( p \)-adic analytic pro-\( p \) groups are irreducible \( p \)-adic space groups; in particular, they are virtually abelian (cf. [11]). Hence, they contain many pairs of open subgroups of different indices that are isomorphic to each other. In contrast, by [11, Lemma III.11], an unsolvable just-infinite \( p \)-adic analytic pro-\( p \) group \( G \) has the remarkable property of not being isomorphic to any of its proper closed subgroups. C. Reid generalized this result to all profinite groups by proving that a just-infinite profinite group \( G \) that contains an open proper subgroup isomorphic to \( G \) is virtually abelian (see [15, Theorem E]). The following corollary shows that within the category of compact \( p \)-adic analytic groups a much stronger result holds.

**Corollary 5.** Let \( G \) be an unsolvable just-infinite \( p \)-adic analytic pro-\( p \) group. Then \( G \) is index-stable.

Open pro-\( p \) subgroups of \( p \)-adic Chevalley groups form a rich source of unsolvable (hereditarily) just-infinite \( p \)-adic analytic pro-\( p \) groups; for \( n \geq 2 \), the first congruence subgroup \( \text{SL}_n^1(\mathbb{Z}_p) \) of \( \text{SL}_n(\mathbb{Z}_p) \) and its open subgroups are typical examples of such groups. More generally, given a simple finite dimensional \( \mathbb{Q}_p \)-Lie algebra \( \mathcal{L} \), any open pro-\( p \) subgroup of \( \text{Aut}(\mathcal{L}) \) is an unsolvable (hereditarily) just-infinite \( p \)-adic analytic pro-\( p \) group (see [11, Proposition III.9]). Moreover, given an unsolvable just-infinite \( p \)-adic analytic pro-\( p \) group \( G \), there is a semisimple \( \mathbb{Q}_p \)-Lie algebra \( \mathcal{L} \) such that \( G \) is an open subgroup of \( \text{Aut}(\mathcal{L}) \) (cf. [11, Section III.9]).

By [6, Proposition 6.1], every solvable just-infinite pro-\( p \) group other than \( \mathbb{Z}_p \) has torsion. Thus, a non-procyclic torsion-free just-infinite \( p \)-adic analytic pro-\( p \) group must be unsolvable. Hence, we can deduce the following result, which is the correct formulation of [14, Conjecture 2.10].

**Corollary 6.** Let \( G \) be a non-procyclic torsion-free just-infinite \( p \)-adic analytic pro-\( p \) group. Then \( G \) is index-stable.

Next, we observe that Theorem 1 relies on its Lie-algebra counterpart, which is an interesting result on its own. By definition, a \( \mathbb{Z}_p \)-Lie lattice is a \( \mathbb{Z}_p \)-Lie algebra the underlying module of which is finitely generated and free. A \( \mathbb{Z}_p \)-Lie lattice \( L \) is said to be index-stable if for any pair \( M \) and \( N \) of isomorphic finite-index subalgebras of \( L \), we have \( |L : M| = |L : N| \).

**Theorem 7.** Let \( L \) be a \( \mathbb{Z}_p \)-Lie lattice, and assume that \( L \otimes_{\mathbb{Z}_p} \mathbb{Q}_p \) is semisimple as \( \mathbb{Q}_p \)-Lie algebra. Then \( L \) is index-stable.

An \( n \)-dimensional \( \mathbb{Z}_p \)-Lie lattice \( L \) is said to be just-infinite if every non-zero ideal of \( L \) has dimension \( n \). The following corollary proves (the correct formulation of) [14, Conjecture 2.9].
Corollary 8. Let $L$ be a just-infinite $\mathbb{Z}_p$-Lie lattice, and assume that $\dim L > 1$. Then $L$ is index-stable.

We remark that Corollary 5 and Corollary 8 have applications to the study of self-similar actions of hereditarily just-infinite $p$-adic analytic pro-$p$ groups using the language of virtual endomorphisms (cf. [14]).

We close the introduction by stating the theorem proved by Serre in the appendix.

**Theorem 9** (Serre). Let $G$ be a compact $p$-adic analytic group, and let $\mathcal{L}(G)$ be the $\mathbb{Q}_p$-Lie algebra associated with $G$. Then $G$ is index-stable if and only if all automorphisms $s$ of $\mathcal{L}(G)$ satisfy $|\det(s)| = 1$, where $|\cdot|$ is the $p$-adic norm of $\mathbb{Q}_p$.

The following corollary was suggested by the referee, to whom we are thankful.

**Corollary 10.** Let $G$ be a $p$-adic analytic group and let $\mathcal{L}(G)$ be the $\mathbb{Q}_p$-Lie algebra associated with $G$. If all automorphisms $s$ of $\mathcal{L}(G)$ satisfy $|\det(s)| = 1$, then $G$ is unimodular. On the other hand, if some automorphism $s$ of $\mathcal{L}(G)$ has $|\det(s)| \neq 1$, then there exists a $p$-adic analytic group $H$ that contains $G$ as an open subgroup and such that $H$ is not unimodular.

**Remark 11.** The proof of Theorem 9 (see the appendix) relies on $p$-adic integration, for which the reader may consult, for instance, [9, Section 7.4]. Now, denote by (*) the condition on $\mathcal{L}(G)$ in the statement of Theorem 9. We note that there exist $\mathbb{Q}_p$-Lie algebras that satisfy condition (*) but are not semisimple. J.L. Dyer constructed a nilpotent Lie algebra of dimension 9 and nilpotency class 6 with the property that its group of automorphisms is unipotent [5]. In general, by a result of G. Leger and E. Luks [13, Theorem (*)], if the automorphism group $\text{Aut}(\mathcal{L})$ of a nilpotent Lie algebra $\mathcal{L}$ of dimension $> 1$ is nilpotent, then it is unipotent. J. Dixmier and W.G. Lister constructed an example of a nilpotent Lie algebra $\mathcal{M}$ of dimension 8 and nilpotency class 3 such that $\text{Aut}(\mathcal{M})$ is not nilpotent but the derivation algebra $\text{Der}(\mathcal{M})$ is nilpotent [3]. The latter condition implies that any automorphism of $\mathcal{M}$ has eigenvalues that are roots of unity. Clearly, all of these Lie algebras satisfy condition (*).

**Corollary 12.** Let $G$ be a compact $p$-adic analytic group of dimension $> 1$, let $\mathcal{L}$ be the $\mathbb{Q}_p$-Lie algebra associated with $G$, and assume that $\mathcal{L}$ is nilpotent. If $\text{Aut}(\mathcal{L})$ is nilpotent or $\text{Der}(\mathcal{L})$ is nilpotent, then $G$ is index-stable.

1. **Proofs of the main results.** In this section, we prove Theorem 1, Theorem 7, Corollary 3, Corollary 8, and Corollary 10.

**Proof of Theorem 7.** Let $M$ and $N$ be finite-index subalgebras of $L$, and let $\varphi : M \to N$ be an isomorphism of $\mathbb{Z}_p$-Lie lattices. Let $\mathcal{L} := L \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ and fix a $\mathbb{Z}_p$-basis $\{a_1, \ldots, a_d\}$ of $L$. Then, clearly, $\{a_1, \ldots, a_d\}$ is a $\mathbb{Q}_p$-basis of $\mathcal{L}$. Let $\kappa : \mathcal{L} \times \mathcal{L} \to \mathbb{Q}_p$, defined by $\kappa(x, y) = \text{tr}(\text{ad}(x)\text{ad}(y))$, be the Killing form of $\mathcal{L}$. Denote by $A$ the matrix representing $\kappa$ with respect to the given basis; in other words, $A = (A_{ij})$, where $A_{ij} = \kappa(a_i, a_j)$ for $1 \leq i, j \leq d$. Since $\mathcal{L}$ is a semisimple Lie
algebra over a field of characteristic 0, by Cartan’s criterion [10, Page 69], we have that \( \kappa \) is a non-degenerate bilinear form; in particular, \( \det(A) \neq 0 \). Let \( \{m_1, \ldots, m_d\} \) be a \( \mathbb{Z}_p \)-basis of \( M \); then \( \{\varphi(m_1), \ldots, \varphi(m_d)\} \) is a \( \mathbb{Z}_p \)-basis of \( N \).

Let \( B = (B_{ij}) \) and \( C = (C_{ij}) \) be the change-of-basis matrices defined by \( m_j = \sum_i B_{ij} a_i \) and \( \varphi(m_j) = \sum_i C_{ij} a_i \). Note that \( |L : M| = p^{v_p(\det(B))} \) and \( |L : N| = p^{v_p(\det(C))} \), where \( v_p \) is the \( p \)-adic valuation (one way to prove this claim is to recall that there exist a basis \( \{b_i\} \) of \( L \) and non-negative integers \( \{k_i\} \) such that \( \{p^{k_i} b_i\} \) is a basis of \( M \); similarly for \( N \); cf. [8, Lemma 10.7.2]). The matrices of \( \kappa \) with respect to the bases \( \{m_i\} \) and \( \{\varphi(m_i)\} \) are \( B^T A B \) and \( C^T A C \), respectively. Since the automorphism of \( \mathcal{L} \) induced by \( \varphi \) preserves the Killing form, \( B^T A B = C^T A C \). Taking the determinant on both sides, and recalling that \( \det(A) \neq 0 \), we see that \( v_p(\det(B)) = v_p(\det(C)) \), and the theorem follows. \( \square \)

**Proof of Corollary 8.** Since \( L \) is a just-infinite \( \mathbb{Z}_p \)-Lie lattice of dimension greater than 1, it is not difficult to see that \( L \otimes_{\mathbb{Z}_p} \mathbb{Q}_p \) is a simple \( \mathbb{Q}_p \)-Lie algebra. Hence, the corollary follows from Theorem 7. \( \square \)

A \( \mathbb{Z}_p \)-Lie lattice \( L \) is called **powerful** if \( |L, L| \subseteq pL \) for \( p \) odd, or \( |L, L| \subseteq 4L \) for \( p = 2 \).

With a uniform pro-\( p \) group \( G \) one may associate a powerful \( \mathbb{Z}_p \)-Lie lattice \( L_G \) in the following way: \( G \) and \( L_G \) are identified as sets, and the Lie operations are defined by

\[
g + h = \lim_{n \to \infty} (g^{p^n} h^{p^n})^{p^{-n}},
\]

\[
[g, h]_{\text{Lie}} = \lim_{n \to \infty} [g^{p^n}, h^{p^n}]^{p^{-2n}} = \lim_{n \to \infty} (g^{-p^n} h^{p^n} g^{p^n} h^{p^n})^{p^{-2n}}.
\]

On the other hand, if \( L \) is a powerful \( \mathbb{Z}_p \)-Lie lattice, then the Campbell-Hausdorff formula induces a group structure on \( L \); the resulting group is a uniform pro-\( p \) group. If this construction is applied to the \( \mathbb{Z}_p \)-Lie Lattice \( L_G \) associated with a uniform group \( G \), one recovers the original group. Indeed, the assignment \( G \mapsto L_G \) gives an isomorphism between the category of uniform pro-\( p \) groups and the category of powerful \( \mathbb{Z}_p \)-Lie lattices (see [4, Theorems 4.30 and 9.10]). Recall that every compact \( p \)-adic analytic group contains an open subgroup that is a uniform pro-\( p \) group (see [4, Corollary 8.34]). As we already mentioned in the introduction, the \( \mathbb{Q}_p \)-Lie algebra associated with a compact \( p \)-adic analytic group \( G \) is defined as \( \mathcal{L}(G) := L_U \otimes_{\mathbb{Z}_p} \mathbb{Q}_p \), where \( U \) is an open uniform pro-\( p \) subgroup of \( G \) (see [4, Section 9.5]). A key invariant of a \( p \)-adic analytic group \( G \) is its dimension as a \( p \)-adic manifold, denoted by \( \dim(G) \). Algebraically, \( \dim(G) \) can be described as \( d(U) \), where \( U \) is any uniform open pro-\( p \) subgroup of \( G \), and \( d(U) \) denotes the minimal cardinality of a topological generating set for \( U \).

**Proof of Theorem 1.** Let \( H \) and \( K \) be two isomorphic closed subgroups of \( G \). If one of these subgroups, say \( H \), has infinite index in \( G \), then \( \dim(H) < \dim(G) \). Hence \( \dim(K) < \dim(G) \), and therefore \( |G : K| \) is infinite as well. Since \( G \) is virtually pro-\( p \), it is not difficult to see that \( |G : K| \) and \( |G : H| \) coincide as supernatural numbers.
Now suppose that $H$ and $K$ are open subgroups in $G$. Let $\varphi : H \rightarrow K$ be an isomorphism, and let $U$ be an open uniform subgroup of $G$; note that, since $G$ is compact, $U$ is of finite index in $G$. Let $U_H$ be a uniform open subgroup of $H$. Then $U_K := \varphi(U_H)$ is a uniform open subgroup of $K$. Moreover, $|H : U_H| = |K : U_K|$. Choose a positive integer $p^m$ such that $V_H := (U_H)^{p^m}$ and $V_K := (U_K)^{p^m}$ are contained in $U$; by [4, Theorem 3.6], $V_H$ and $V_K$ are uniform. Let $L_U$ be the $\mathbb{Z}_p$-Lie lattice associated with $U$. Then the $\mathbb{Q}_p$-Lie algebra associated with $G$ is given by $\mathcal{L}(G) = L_U \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$, which is semisimple by assumption. Since $U_H$ and $U_K$ are isomorphic, we have that $V_H$ and $V_K$ are isomorphic open uniform subgroups of $U$. This implies that $L_{V_H}$ and $L_{V_K}$ are isomorphic $\mathbb{Z}_p$-Lie sublattices of $L_U$ of finite index. By Theorem 7, $|L_U : L_{V_H}| = |L_U : L_{V_K}|$. Hence, by [4, Proposition 4.31], $|U : V_H| = |U : V_K|$. Clearly, this implies that $|G : V_H| = |G : V_K|$. Now, since $|H : U_H| = |K : U_K|$ and $|U_H : V_H| = |U_K : V_K|$, it follows that $|G : H| = |G : K|$, as desired.

Proof of Corollary 3. Clearly, if $G$ is virtually abelian, then $G$ is index-unstable. Suppose that $G$ is not virtually abelian. By [7, Theorem 3.6], a just-infinite profinite group is either a branch profinite group or it contains an open normal subgroup which is isomorphic to the direct product of a finite number of copies of some hereditarily just-infinite profinite group. Note that $G$ cannot be a branch profinite group since, by [4, Theorem 8.33], $G$ is of finite rank (that is, there exists a positive integer $d$ such that every closed subgroup of $G$ can be generated topologically by at most $d$ elements). Hence, $G$ contains an open normal subgroup $H$ which is isomorphic to the direct product of a finite number, say $k$, of copies of some hereditarily just-infinite uniform pro-$p$ group $U$; in particular, $H$ is uniform. Note that if $U$ is solvable, then $U$ is isomorphic to $\mathbb{Z}_p$, and consequently $G$ is virtually abelian, which is a contradiction. Hence, $U$ is an unsolvable hereditarily just-infinite uniform pro-$p$ group. By [6, Proposition F], the associated $\mathbb{Q}_p$-Lie algebra $\mathcal{L}(U) = L_U \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ is simple. Thus, the $\mathbb{Q}_p$-Lie algebra $\mathcal{L}(G) = L_H \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ associated with $G$ is semisimple since it is isomorphic to the direct sum of $k$ copies of $\mathcal{L}(U)$. Now Theorem 1 yields that $G$ is index-stable.

Proof of Corollary 10. We sketch the proof of the second statement. Let $s$ be an automorphism of $\mathcal{L}(G)$ such that $|\det(s)| \neq 1$. There exist open compact subgroups $U$ and $V$ of $G$, and an isomorphism $\varphi : U \rightarrow V$ such that $(d\varphi)_e = s$, where $e$ is the identity of $G$. Let $\mu$ be a left-invariant Haar measure on $G$. From $|\det(s)| \neq 1$, it follows that $\mu(U) \neq \mu(V)$. Let $H$ be the HNN-extension of $G$ associated with $\varphi$, that is, $H = \langle G, t \mid tut^{-1} = \varphi(u) \forall u \in U \rangle$, where $t \in H$ is the stable letter (see, for instance, [2, page 188]); then $tUt^{-1} = V$ in $H$. From [2, Proposition 8.B.10], we see that there exists a unique topology on $H$ that makes it a topological group such that $G$ is (identified with) an open subgroup of $H$. It follows that $H$ is $p$-adic analytic. If $H$ was unimodular, it would follow that $\mu(U) = \mu(V)$, giving a contradiction.

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Appendix: A letter from Jean-Pierre Serre.

Dear MM. Noseda and Snopce,

I have just seen your arXiv paper on “Index-stable compact $p$-adic analytic groups”. Your proof uses Lazard’s very nice results, but in fact these results are not necessary: $p$-adic integration is enough. Let me explain:

Let $L$ be a finite-dimensional Lie algebra over $\mathbb{Q}_p$. Denote by $|x|$ the $p$-adic norm of $\mathbb{Q}_p$. Consider the following property of $L$:

For every automorphism $s$ of $L$, we have $|\det(s)| = 1$ (*).

This is true, for instance, if $L$ is semisimple since $\det(s) = \pm 1$, which is the case you consider.

Assume property (*). Let $n = \dim L$. Let $u$ be a non-zero element of $\wedge^n L^*$, where $L^*$ is the $\mathbb{Q}_p$-dual of $L$. Let $G$ be a compact $p$-adic analytic group with Lie algebra $L$. Then $u$ defines a right-invariant differential form $\omega_u$ on $G$ of degree $n$. The corresponding measure $\mu_u = |\omega_u|$ is a non-zero right-invariant positive measure on $G$, hence is a Haar measure since $G$ is compact. Property (*) implies that $\mu_u$ is invariant by every automorphism of $L$. Hence every isomorphism of $G$ onto another group $G'$ carries $\mu_u$ (for $G$) into $\mu_u$ (for $G'$). This implies that, if $G, G'$ are open subgroups of some compact $p$-adic group $G''$, then they have the same index - as wanted.

Conversely, if (*) is not true for some $s$, and if $G''$ is compact with Lie algebra $L$, $s$ defines a local automorphism of $G''$, and if $G$ is a small enough open subgroup of $G''$, it is transformed by $s$ into another open subgroup $G'$, and the ratio $(G'' : G)/(G'' : G')$ is equal to $|\det(s)|$, which is $\neq 1$.

Best wishes,

J-P. Serre

References

[1] Barnea, Y., Ershov, M., Weigel, T.: Abstract commensurators of profinite groups. Trans. Amer. Math. Soc. 363, 5381–5417 (2011)
[2] Cornulier, Y., de la Harpe, P.: Metric Geometry of Locally Compact Groups. EMS Tracts in Mathematics, vol. 25. European Mathematical Society, Zürich (2016)
[3] Dixmier, J., Lister, W.G.: Derivations of nilpotent Lie algebras. Proc. Amer. Math. Soc. 8, 155–158 (1957)
[4] Dixon, J.D., Du Sautoy, M.P.F., Mann, A., Segal, D.: Analytic Pro-$p$ Groups, 2nd edn. Cambridge Studies in Advanced Mathematics, vol. 61. Cambridge University Press, Cambridge (1999)
[5] Dyer, J.L.: A nilpotent Lie algebra with nilpotent automorphism group. Bull. Amer. Math. Soc. 76(1), 52–56 (1970)
[6] González-Sánchez, J., Klopsch, B.: Analytic pro-$p$ groups of small dimensions. J. Group Theory 12, 711–734 (2009)
[7] Grigorchuk, R.I. Just infinite branch groups. In: New Horizons in Pro-$p$ Groups, Progr. Math., vol. 184, pp. 121–179. Birkhäuser Boston, Boston, MA (2000)

[8] Grillet, P.: Algebra. Wiley, Hoboken (1999)

[9] Igusa, J.: An Introduction to the Theory of Local Zeta Functions. AMS/IP Studies in Advanced Mathematics, vol. 14. American Mathematical Society, Providence, RI; International Press, Cambridge, MA (2000)

[10] Jacobson, N.: Lie algebras. Dover Publications, Mineola (1962)

[11] Klaas, G., Leedham-Green, C.R., Plesken, W.: Linear pro-$p$ Groups of Finite Width. Lecture Notes in Mathematics, vol. 1674. Springer, Berlin (1997)

[12] Lazard, M.: Groupes analytiques $p$-adiques. Publ. Math. Inst. Hautes Études Sci. 26, 5–219 (1965)

[13] Leger, G., Luks, E.: On nilpotent groups of algebra automorphisms. Nagoya Math. J. 46, 87–95 (1972)

[14] Noseda, F., Snopce, I.: On self-similarity of $p$-adic analytic pro-$p$ groups of small dimension. J. Algebra 540, 317–345 (2019)

[15] Reid, C.D.: On the structure of just infinite profinite groups. J. Algebra 324, 2249–2261 (2010)

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Received: 18 July 2020

Revised: 22 September 2020

Accepted: 7 October 2020.