Standardizing reverberation-measured Mg II time-lag quasars, by using the radius-luminosity relation, and constraining cosmological model parameters

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ABSTRACT

We use 78 reverberation-measured Mg II time-lag quasars (QSOs) in the redshift range $0.0033 \leq z \leq 1.89$ to constrain cosmological parameters in six different cosmological models. The basis of our method is the use of the radius-luminosity or $R-L$ relation to standardize these 78 Mg II QSOs. In each cosmological model we simultaneously determine $R-L$ relation and cosmological model parameters, thus avoiding the circularity problem. We find that the $R-L$ relation parameter values are independent of the cosmological model used in the analysis thus establishing that current Mg II QSOs are standardizable candles. Cosmological constraints obtained using these QSOs are significantly weaker than, but consistent with, those obtained from a joint analysis of baryon acoustic oscillation (BAO) observations and Hubble parameter $[H(z)]$ measurements. So, we also analyse these QSOs in conjunction with the BAO + $H(z)$ data and find cosmological constraints consistent with the standard spatially-flat $\Lambda$CDM model as well as with mild dark energy dynamics and a little spatial curvature. A larger sample of higher-quality reverberation-measured QSOs should have a smaller intrinsic dispersion and so should provide tighter constraints on cosmological parameters.

Key words: (cosmology:) cosmological parameters – (cosmology:) observations – (cosmology:) dark energy – (galaxies:) quasars: emission lines

1 INTRODUCTION

It is a well-established fact that our Universe is currently undergoing accelerated cosmological expansion (Farooq et al. 2017; Scolnic et al. 2018; Planck Collaboration 2020; eBOSS Collaboration 2021). This observational fact can be explained by general relativistic cosmological models if we include dark energy in them. The simplest cosmological model that is consistent with this observation is the standard spatially-flat $\Lambda$CDM model (Peebles 1984). In this model, dark energy in the form of the cosmological constant $\Lambda$ contributes $\sim 70\%$ of the current cosmological energy budget, non-relativistic cold dark matter (CDM) contributes $\sim 25\%$, and almost all of the remaining $\sim 5\%$ is contributed by non-relativistic baryons. This model is consistent with most observational data but a little spatial curvature and mild dark energy dynamics are not ruled out. So, in this paper, in addition to the $\Lambda$CDM model, we consider two dynamical dark energy models, one being the widely-used but physically-incomplete XCDM parametrization which parametrizes dynamical dark energy as an $X$-fluid and the other is the physically-complete $\phi$CDM model which models dynamical dark energy as a scalar field. In each case we consider flat and non-flat spatial hypersurfaces to also allow for possibly non-zero spatial curvature of the Universe.$^1$

These models are mostly tested using well-established cosmological probes such as cosmic microwave background (CMB) anisotropy data, baryon acoustic oscillation (BAO) observations, Hubble parameter $[H(z)]$ measurements, and Type Ia supernova (SNIa) apparent magnitude data. CMB anisotropy data probe the $z \sim 1100$ part of redshift space and are the only high-redshift data.

$^1$ Recent observational constraints on spatial curvature are discussed in Farooq et al. (2015), Chen et al. (2016), Rana et al. (2017), Ooba et al. (2018a,c), Yu et al. (2018), Park & Ratra (2019a,c), Wei (2018), DES Collaboration (2019), Li et al. (2020), Handley (2019), Efostathiou & Gratton (2020), Di Valentino et al. (2021), Velasquez-Toribio & Fabris (2020), Vagnozzi et al. (2020, 2021), KiDS Collaboration (2021), Arjona & Nesseris (2021), Dhawan et al. (2021), and references therein.
BAO data probe redshift space up to $z \sim 2.3$, the highest $z$ reached by the better-established lower-redshift probes. These are limited sets of cosmological data and a number of observationally-viable cosmological models make very similar predictions for these probes, so to establish a more accurate standard cosmological model and to obtain tighter cosmological parameter constraints we need to use other astronomical data.

A significant amount of work has been done to develop new cosmological probes. This work includes use of HII starburst galaxy observations which extend to $z \sim 2.7$ (Mania & Ratra 2012; Chávez et al. 2014; González-Morán et al. 2019, 2021; Cao et al. 2020, 2021a; Johnson et al. 2021), quasar (QSO) angular size measurements which extend to $z \sim 7.5$ (Risaliti & Lusso 2015, 2019; Khadka & Ratra 2020a,b, 2021a; Yang et al. 2020; Lusso et al. 2020b; Li et al. 2021; Liu et al. 2021), and gamma-ray burst (GRB) data that extend to $z \sim 8.2$ (Amati et al. 2008, 2019; Samushia & Ratra 2010; Wang et al. 2016; Demianski et al. 2019; Fana Dirirsa et al. 2019; Khadka & Ratra 2020c; Khadka et al. 2021).

An additional new method that can be used in cosmology is based on QSOs with a measured time delay between the quasar ionizing continuum and the Mg II line luminosity. This technique is referred to as reverberation mapping and it makes use of the tight correlation between the variable ionizing radiation powered by the accretion disc and the line-emission that originates in the broad-line region (BLR) optically-thick material located farther away that efficiently reprocesses the disc continuum radiation (Blandford & McKee 1982). We refer to these reverberation-mapped sources as Mg II QSOs. We use Mg II QSOs to constrain cosmological dark energy models for the following reasons: (i) The current reasonably large number, 78, of studied Mg II QSOs at intermediate $z$ (Czerny et al. 2019; Homayouni et al. 2020; Martínez-Aldama et al. 2020a, for overviews). Some attempts have previously been made to use reverberation-measured Mg II QSOs to determine constraints on cosmological parameters since the time delay measurement allows one to obtain the source absolute luminosity (see Panda et al. 2019; Martinez Aldama et al. 2020a, for overviews). Some attempts have previously been made to use reverberation-measured QSOs in cosmology (Martínez-Aldama et al. 2019; Czerny et al. 2021; Zajaček et al. 2021), and so far an overall agreement has been found with the standard $\Lambda$CDM cosmological model for Mg II QSOs (Martínez-Aldama et al. 2019), combined MgII and Mg II sources (Czerny et al. 2021), and Mg II QSOs alone (Zajaček et al. 2021).

In this paper, we use 78 Mg II QSOs — the largest set of such measurements to date — to simultaneously constrain cosmological parameters and $R - L$ relation parameters — done here for Mg II QSOs for the first time — allows us to avoid the circularity problem. This is the problem of having to either assume $\beta$ and $\gamma$ to use the $R - L$ relation and data to constrain cosmological model parameters, or having to assume a cosmological model (and parameter values) to use the measurements to determine $\beta$ and $\gamma$. Since we determine $\beta$ and $\gamma$ values in six different cosmological models, we are able to test whether Mg II QSOs are standardizable candles. 3 We find that the $R - L$ relation parameters are independent of the cosmological model in which they were

\[ R \propto L^\beta \]

Using the definition of the ionization parameter for a BLR cloud, $U = Q(H) / [4 \pi c n(H)]$, where $Q(H)$ is the hydrogen-ionizing photon flux in $cm^{-2}s^{-1}$. $R$ is the cloud distance from the continuum source, and $n(H)$ is the total hydrogen density. Assuming that $U n(H) = const$ for BLR clouds in different sources, we obtain $R \propto L^{1/2}$. 3 This is one reason why we study a number of different cosmological models.

\[ \frac{\sigma_{\text{int}}}{\sigma_{\text{tot}}} \approx 0.13 \]
Constraints from Mg II QSO data

2 MODELS

We constrain cosmological model parameters by comparing model predictions to cosmological measurements at known redshift \( z \). We consider six different dark energy cosmological models, three with flat spatial geometry and three with non-flat spatial geometry. For the observations we consider, model predictions depend on the Hubble parameter — the cosmological expansion rate — a function that depends on \( z \) and on the cosmological parameters of the model.

In these models the Hubble parameter can be expressed as

\[
H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{k0}(1 + z)^2 + \Omega_{DE}(z)},
\]

where \( H_0 \) is the Hubble constant, \( \Omega_{DE}(z) \) is the dark energy density parameter, and \( \Omega_{m0} \) and \( \Omega_{k0} \) are the non-relativistic matter and curvature energy density parameters. In the spatially-flat models \( \Omega_{k0} = 0 \). For analyses of the BAO + \( H(z) \) and QSO + BAO + \( H(z) \) data, we express \( \Omega_{m0} \) in terms of the current values of the cold dark matter density parameter \( \Omega_{\chi} \) and the baryon density parameter \( \Omega_b \); \( \Omega_{m0} = \Omega_{\chi} + \Omega_b \), and use \( \Omega_b h^2 \) and \( \Omega_{\chi} h^2 \) as free parameters [here \( h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) \)] instead of \( \Omega_{m0} \). As discussed in Sec. 4, QSO data alone cannot constrain \( H_0 \), which in this case is set to \( 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \); for the BAO + \( H(z) \) and QSO + BAO + \( H(z) \) data analyses \( H_0 \) is a free parameter to be determined from the data. The dark energy density evolves as a power of \( (1 + z) \) in four of the six models we study. In these models \( \Omega_{DE}(z) = \Omega_{DE0}(1 + z)^{\alpha} \) where \( \Omega_{DE} \) is the dark energy equation of state parameter (defined below) and \( \Omega_{DE0} \) is the current value of the dark energy density parameter.

In the ΛCDM model \( \omega_X = -1 \) so \( \Omega_{DE} = \Omega_{DE0} = \Omega_{\Lambda} \), and dark energy is the standard cosmological constant. The current values of the three ΛCDM model energy density parameters obey the energy budget equation \( \Omega_{m0} + \Omega_{k0} + \Omega_{\Lambda} = 1 \). For the QSO-only data analyses we fix \( H_0 \) and in the spatially-flat ΛCDM model we take \( \Omega_{m0} \) to be the free parameter while in the non-flat ΛCDM model \( \Omega_{m0} \) and \( \Omega_{k0} \) are the free parameters.

In the ΛCDM parametrization dark energy is parametrized as an ideal X-fluid with equation of state parameter \( \omega_X \) being the ratio of the X-fluid pressure and energy density. Here \( \Omega_{DE0} = \Omega_{\Lambda0} \) is the current value of the X-fluid dynamical dark energy density parameter. The current values of the three ΛCDM parametrization energy density parameters obey the energy budget equation \( \Omega_{m0} + \Omega_{k0} + \Omega_{\lambda0} = 1 \). The X-fluid energy density decreases with time when \( \omega_X > -1 \). For the QSO-only data analyses we fix \( H_0 \) and in the spatially-flat ΛCDM parametrization we take \( \Omega_{m0} \) and \( \omega_X \) to be the free parameters while in the non-flat ΛCDM parametrization, \( \Omega_{m0}, \Omega_{k0}, \) and \( \omega_X \) are the free parameters. In the limit \( \omega_X \to -1 \) the ΛCDM parametrization reduces to the ΛCDM model.

In the \( \phi \)CDM model (Peebles & Ratra 1988; Ratra & Peebles 1988; Pavlov et al. 2013) dynamical dark energy is a scalar field \( \phi \). Here the dynamical dark energy scalar field density parameter \( \Omega_{DE} \) is determined by the potential energy density of the scalar field. In this paper we assume an inverse power law scalar field potential energy density

\[
V(\phi) = \frac{1}{2} k m_p^2 \phi^{-\alpha}.
\]

In this equation \( m_p \) is the Planck mass, \( \alpha \) is a positive parameter \( [\Omega_{DE} = \Omega_\phi(z, \alpha)] \) is the scalar field dynamical dark energy density parameter, and the constant \( k \) is determined by using the shooting method to ensure that the current energy budget constraint \( \Omega_{m0} + \Omega_{k0} + \Omega_\phi(z = 0, \alpha) = 1 \) is satisfied.

For this potential energy density, the equations of motion for a spatially homogeneous scalar field and FLRW metric tensor are

\[
\ddot{\phi} + \frac{3}{a} \dot{a} \dot{\phi} - \frac{1}{2} k a^2 \phi^{-\alpha-1} = 0,
\]

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3m_p^2} (\rho_m + \rho_\phi) - \frac{k}{a^2}.
\]

Here \( a \) is the scale factor, an overdot denotes a derivative with respect to time, \( k \) is negative, zero, and positive for open, flat, and closed spatial geometries (corresponding to \( \Omega_{k0} = 0, = 0, < 0 \), \( \rho_m \) is the non-relativistic matter energy density, and the scalar field energy density

\[
\rho_\phi = \frac{m_p^2}{32\pi} [\dot{\phi}^2 + k m_p^2 \phi^{-\alpha}] .
\]

We numerically integrate eqs. (3) and (4), compute \( \rho_\phi \), and then compute \( \Omega_\phi(z, \alpha) \) from

\[
\Omega_\phi(z, \alpha) = \frac{8\pi \rho_\phi}{3m_p^2 H_0^2}.
\]

For the QSO-only data analyses we fix \( H_0 \) and in the spatially-flat \( \phi \)CDM model we take \( \Omega_{m0} \) and \( \alpha \) to be the free parameters and in the non-flat \( \phi \)CDM model, \( \Omega_{m0}, \Omega_{k0}, \) and \( \alpha \) are the free parameters. In the limit \( \alpha \to 0 \) the \( \phi \)CDM model reduces to the ΛCDM model.

3 DATA

We use three different Mg II QSO compilations, as well as BAO and \( H(z) \) data. The Mg II QSO data sets are summarized in Table 1, which lists the number of QSOs in each sample, and the covered redshift range. These data are listed in Table A1 where for each source the name, \( z \), measured QSO flux for the Mg II line \( (F_{3000}) \), and rest-frame time-delay \( (\tau) \) are listed.

4 Recent observational constraints on the \( \phi \)CDM model are discussed in Ayyanajeshvili et al. (2015), Solá Peracaula et al. (2018, 2019), Zhai et al. (2017), Ooba et al. (2018b, 2019), Park & Ratra (2018, 2019b, 2020), Sangwan et al. (2018), Singh et al. (2019), Ureña-López & Roy (2020), Sinha & Banerjee (2021), and references therein.
Table 1. Summary of the Mg II QSO data sets.

| Data set     | Number | Redshift range |
|--------------|--------|---------------|
| Mg II QSO-69 | 69     | [0.0033, 1.89] |
| Mg II QSO-9  | 9      | [0.00703, 1.7496] |
| Mg II QSO-78 | 78     | [0.0033, 1.89] |

**Mg II QSO-69 sample.** This sample includes the first 69 QSOs listed in Table A1. These data were originally analyzed and described in several publications. The Mg II QSO-69 sample contains 69 QSOs including those from the most recent Mg II Sloan Digital Sky Survey Reverberation Mapping data set (SDSS-RM, 57 sources; Homayouni et al. 2020), from previous SDSS-RM results (6 sources; Shen et al. 2016, where one source is included in the more recent SDSS-RM sample), several luminous quasars, in particular CTS252 (Lira et al. 2018), CTS C30.10 (Czerny et al. 2019), HE 0413-4031 (Zajaček et al. 2020), and HE 0435-4312 (Zajaček et al. 2021), and two older International Ultraviolet Explorer (IUE) measurements of the low-luminosity QSO NGC 4151 based on two separate campaigns in 1988 and 1991 (Metzroth et al. 2006). The redshift range of this sample is 0.0033 ≤ z ≤ 1.89, while the 3000 Å luminosity of QSOs in the Mg II QSO-69 sample covers four orders of magnitude, 42.83 ≤ log L3000,[ erg s⁻¹] ≤ 46.79. Both the low- and high-luminosity sources are beneficial for better determining the R – L correlation relation. The Pearson correlation coefficient for the whole sample is r = 0.63 with p = 5.60 × 10⁻⁵, while the Spearman correlation coefficient is r = 0.47 with p = 4.52 × 10⁻⁵, where p expresses a two-sided p-value. The RMS intrinsic scatter reaches σint = 0.30 dex for the standard R – L relation, but it drops for the highly-accrediting subsample, especially for extended versions of the R – L relation (Zajaček et al. 2021). The sample is relatively homogeneous, with ≈ 93% of the sources coming from the most recent SDSS-RM sample (Homayouni et al. 2020) and ≈ 9% of the sources from the previous SDSS-RM sample (Shen et al. 2016). This means that for most of the sources a consistent approach was used to infer the significant time-delay, mostly using the JAVELIN method that makes use of the dumped random walk approach in fitting the continuum light curve (Kelly et al. 2009; MacLeod et al. 2010; Kozlowski et al. 2010; Zu et al. 2011, 2013, 2016) as well as the CREAM that uses a random walk spectral density prior of P(f) ∝ f⁻² for the driving ionizing continuum (Starkey et al. 2016). The remaining sources were analyzed typically by a combination of other methods, including a standard interpolation and discrete cross-correlation functions (ICCF and DCF, including the z-transformed DCF), the χ² method, and measures of data randomness/regularity (see Czerny et al. 2013; Chelouche et al. 2017; Zajaček et al. 2019, 2021, for overviews and applications to data). The scatter along the RL correlation may be systematically increased due to the uncertainties of the time-delay analysis. For the largest SDSS-RM sample, Homayouni et al. (2020) analyzed the sample of 193 quasars in the redshift range of 0.35 < z < 1.7, where they identified 57 significant time lags with the average false-positive rate of 11%, 24 sources out of them are further identified as a “golden” sample with the false-positive rate of 8%. In the older SDSS-RM sample of 6 quasars, the false-positive rate is comparable, at the level of ~ 10% – 15% for the reported significant lags (Shen et al. 2016). For the individual sources, a combination of more methods was typically employed to identify the consistent Mg II time delay, which was backed up by alias mitigation using bootstrap, pair-weighting, or Timmer-Koenig light-curve modelling, see e.g. Zajaček et al. (2021).

**Mg II QSO-9 sample.** This sample includes the last 9 QSOs listed in Table A1. These data are from Yu et al. (2021). They measured 9 significant Mg II lags using the first five years of data from the Dark Energy Survey (DES, e.g. Flaugher et al. 2015) - Australian DES (OzDES, e.g. Lidman et al. 2020) reverberation mapping project. The measurement sample spans the redshift range ~ 1.1 – 1.7. The lags are consistent with both the Hβ R – L relation determined by Bentz et al. (2013) and the Mg II R – L relation of Homayouni et al. (2020). For 9 Mg II time delays, the median false-positive rate is 4%.

**Mg II QSO-78 sample.** This sample is the union of the Mg II QSO-69 and the Mg II QSO-9 samples. For the united sample, the Pearson correlation coefficient between τ and L3000 is r = 0.63 with p = 6.68 × 10⁻¹⁰ and the Spearman correlation coefficient is s = 0.50 with p = 4.06 × 10⁻⁶, hence the correlation along the R – L is slightly enhanced by adding MgII QSO-9 to the MgII QSO-69 sample. After the sample enlargement, the RMS scatter decreases only by ~ 1.68% from ~ 0.30 dex to ~ 0.29 dex.

In this paper, we also use 31 H(z) and 11 BAO measurements. The H(z) data redshift range is 0.07 ≤ z ≤ 1.965 and the BAO data redshift range is 0.0106 ≤ z ≤ 2.33. The H(z) data are given in Table 2 of Ryan et al. (2018) and the BAO data are listed in Table 1 of Khadka & Ratna (2021b). Cosmological constraints obtained from the Mg II QSO samples are consistent with those obtained from the BAO + H(z) data so we also jointly analyse the Mg II QSO-78 and BAO + H(z) data sets.

4 METHODS

The R – L correlation relates the rest-frame time-delay of the Mg II broad line and the monochromatic luminosity of the QSO. For the sources used in this paper, this correlation can be seen in Fig. 1. The R – L relation is usually expressed in the form

\[ \log \left( \frac{\tau}{\text{day}} \right) = \beta + \gamma \log \left( \frac{L_{3000}}{10^{44} \text{erg s}^{-1}} \right), \]

where \( \log L_{3000} \) and \( \tau \) are the monochromatic luminosity of the quasar at 3000 Å in the rest frame in units of erg s⁻¹ and the rest-frame time-delay of the Mg II line in units of day. Here \( \beta \) and \( \gamma \) are the correlation model free parameters and need to be determined from the data.

The measured quantities are the time delay and the quasar flux. Expressing the luminosity in terms of the flux we obtain

\[ \log \left( \frac{\tau}{\text{day}} \right) = \beta + \gamma \log \left( \frac{F_{3000}}{10^{44} \text{erg cm}^{-2} \text{s}^{-1}} \right) + 2\gamma \log \left( \frac{D_L}{\text{cm}} \right), \]

where \( F_{3000} \) is the measured quasar flux at 3000 Å in units of erg cm⁻² s⁻¹ and \( D_L(z, p) \) is the luminosity distance in units of
where we set time-delays by using the likelihood function (D’Agostini 2005)

and the cosmological parameters $p$ of the cosmological model under study (see Sec. 2). The luminosity distance is

$$d_L(z, p) = \frac{H_0 \sqrt|Ω_0|}{(1 + z)} \frac{d \tau}{dz'},$$

where

$$g(z) = H_0 \sqrt|Ω_0| \int_0^z dz' \frac{d \tau'}{H(z')}$$

and $H(z)$ is the Hubble parameter which is given in Sec. 2 for each cosmological model.

In a given cosmological model, eqs. (8) and (9) can be used to predict the rest-frame time-delay of the Mg II line for a quasar at known redshift. We can then compare the predicted and observed time-delays by using the likelihood function (D’Agostini 2005)

$$\ln(LF) = -\frac{1}{2} \sum \left[ \frac{\ln(r_{obs}^{X_i}) - \ln(r_{obs}^{X_i})}{\sigma_i^2} \right]^2 + \ln(2\pi\sigma_i^2)$$

Here $\ln = \ln_{10}, r_{X,i}^{obs}(p)$ and $r_{X,i}^{obs}(p)$ are the predicted and observed time-delays at redshift $z_i$, and $\sigma_i^2 = \sigma_{\log r_{obs,i}}^2 + y^2 \sigma_{\log F_{3000,i}}^2 + \sigma_{\text{ext}}^2$, where $\sigma_{\log r_{obs,i}}$ and $\sigma_{\log F_{3000,i}}$ are the measurement error on the observed time-delay ($r_{X,i}^{obs}(p)$) and the measured flux ($F_{3000}$) respectively, and $\sigma_{\text{ext}}$ is the intrinsic dispersion of the $R-L$ relation.

QSO data alone cannot constrain $H_0$ because of the degeneracy between the correlation intercept parameter $\beta$ and $H_0$, so in this case we set $H_0$ to 70 km s$^{-1}$Mpc$^{-1}$.

To determine cosmological model and $R-L$ parameter constraints from QSO-only data, we maximize the likelihood function given in eq. (11) and determine the best-fit values of all the free parameters and the corresponding uncertainties. The likelihood analysis for each data set and cosmological model is done using the Markov chain Monte Carlo (MCMC) method implemented in the MontePython code (Brinckmann & Lesgourgues 2019). Convergence of the MCMC chains for each parameter is determined by using the Gelman-Rubin criterion ($R - 1 < 0.05$). For each free parameter we assume a top hat prior which is non-zero over the ranges given in Table 2.

Table 2. Summary of the non-zero flat prior parameter ranges.

| Parameter | Prior range |
|-----------|-------------|
| $Ω_0 h^2$ | [0, 1]      |
| $Ω_0 \Omega$ | [0, 1] |
| $Ω_0$    | [0, 1]      |
| $Ω_{\Lambda}$ | [−2, 1] |
| $\omega_X$ | [−5, 0.33] |
| $\alpha$ | [0, 10]     |
| $\sigma_{\text{ext}}$ | [0, 5] |
| $\beta$  | [0, 10]     |
| $\gamma$ | [0, 5]      |

To determine cosmological model parameter constraints from BAO + $H(z)$ data we use the method described in Khadka & Ratra (2021b). To determine cosmological model and $R-L$ relation parameter constraints from QSO + BAO + $H(z)$ data we maximize the sum of the ln likelihood function given in eq. (11) and the BAO + $H(z)$ ln likelihood function given in eqs. (12) and (13) of Khadka & Ratra (2021b).

For model comparisons, we compute the Akaike and Bayes Information Criterion (AIC and BIC) values,

$$AIC = x^2_{\text{min}} + 2d,$$

$$BIC = x^2_{\text{min}} + d \ln N,$$

where $x^2_{\text{min}} = -2 \ln(LF_{\text{max}})$. Here $N$ is the number of data points, $d$ is the number of free parameters, and the degree of freedom $df = N - d$. AIC and BIC penalize free parameters, while $x^2_{\text{min}}$ does not, with BIC more severely penalizing larger $d$ (than AIC does) when $N \geq 7.4$, as is the case for all data sets we consider here. We also compute the differences, $\Delta AIC$ and $\Delta BIC$, with respect to the spatially-flat LCDM model $AIC$ and $BIC$ values. Positive $\Delta AIC$ or $\Delta BIC$ values indicate that the flat LCDM model is favored over the model under study. They provide weak, positive, and strong evidence for the flat LCDM model when they are in $[0, 2]$, $[2, 6]$, or $> 6$. Negative $\Delta AIC$ or $\Delta BIC$ values indicate that the model under study is favored over the flat LCDM model.

5 RESULTS

5.1 Mg II QSO-69, Mg II QSO-9, and Mg II QSO-78 data constraints

Results for the Mg II QSO-69, QSO-9, and QSO-78 data sets are given in Tables 3 and 4. The un marginalized best-fit parameter values are listed in Table 3 and the marginalized one-dimensional best-fit parameter values and limits are given in Table 4. The one-dimensional likelihood distributions and the two-dimensional likelihood contours for the Mg II QSO-69 and Mg II QSO-78 data sets are shown in blue and olive, respectively, in Figs. 2–4 and corresponding plots for the Mg II QSO-9 data set are shown in blue in Figs. 5–7.

The Mg II QSO-9 data set is small and so constraints derived using these data have larger error bars than those determined from the QSO-69 data. From Table 4 and Figs. 2–7, we see that the QSO-9 and QSO-69 constraints are consistent and so it is reasonable to use the combined QSO-78 data to constrain parameters.

From Table 4 we see that the $R-L$ relation parameters $\beta$ and...
Figure 2. One-dimensional likelihood distributions and two-dimensional likelihood contours at 1σ, 2σ, and 3σ confidence levels using Mg II QSO-69 (blue), Mg II QSO-78 (olive), and BAO + H(z) (red) data for all free parameters. Left panel shows the flat ΛCDM model. The black dotted vertical lines are the zero acceleration lines with currently accelerated cosmological expansion occurring to the lower left of the lines. Right panel shows the non-flat ΛCDM model. The black dotted sloping line in the Ω_m0 − Ωₖ0 subpanel is the zero acceleration line with currently accelerated cosmological expansion occurring to the lower left of the line. The black dashed horizontal or vertical line in the Ωₖ0 subpanels correspond to Ωₖ0 = 0.

Figure 3. One-dimensional likelihood distributions and two-dimensional likelihood contours at 1σ, 2σ, and 3σ confidence levels using Mg II QSO-69 (blue), Mg II QSO-78 (olive), and BAO + H(z) (red) data for all free parameters. Left panel shows the flat XCDM parametrization. The black dotted curved line in the ωₓ − Ωₖ0 subpanel is the zero acceleration line with currently accelerated cosmological expansion occurring below the line and the black dashed straight lines correspond to the ωₓ = −1 ΛCDM model. Right panel shows the non-flat XCDM parametrization. The black dotted lines in the Ωₖ0 − Ωₚ0, ωₓ − Ωₖ0, and ωₓ − Ωₚ0 subpanels are the zero acceleration lines with currently accelerated cosmological expansion occurring below the lines. Each of the three lines is computed with the third parameter set to the BAO + H(z) data best-fit value given in Table 3. The black dashed straight lines correspond to the ωₓ = −1 ΛCDM model. The black dotted-dashed straight lines correspond to Ωₖ0 = 0.
Figure 4. One-dimensional likelihood distributions and two-dimensional likelihood contours at 1σ, 2σ, and 3σ confidence levels using Mg II QSO-69 (blue), Mg II QSO-78 (olive), and BAO + $H(z)$ (red) data for all free parameters. The $\alpha = 0$ axes correspond to the $\Lambda$CDM model. Left panel shows the flat $\Lambda$CDM model. The black dotted curved line in the $\alpha - \Omega_{\text{m,eff}}$ subpanel is the zero acceleration line with currently accelerated cosmological expansion occurring to the left of the line. Right panel shows the non-flat $\Lambda$CDM model. The black dotted lines in the $\Omega_{\Lambda} - \Omega_{\text{m,eff}}$, $\Omega_{\text{m,eff}}$, and $\Omega_{\text{m,eff}} - \Omega_m$ subpanels are the zero acceleration lines with currently accelerated cosmological expansion occurring below the lines. Each of the three lines is computed with the third parameter set to the BAO + $H(z)$ data best-fit value given in Table 3. The black dashed straight lines correspond to $\Omega_m = 0$.

Figure 5. One-dimensional likelihood distributions and two-dimensional likelihood contours at 1σ, 2σ, and 3σ confidence levels using Mg II QSO-9 (blue), and BAO + $H(z)$ (red) data for all free parameters. Left panel shows the flat $\Lambda$CDM model. The black dotted vertical lines are the zero acceleration lines with currently accelerated cosmological expansion occurring to the left of the lines. Right panel shows the non-flat $\Lambda$CDM model. The black dotted sloping lines in the $\Omega_{\text{m,eff}} - \Omega_m$ subpanel is the zero acceleration line with currently accelerated cosmological expansion occurring to the lower left of the line. The black dashed vertical or horizontal line in the $\Omega_m$ subpanels correspond to $\Omega_m = 0$. 
Figure 6. One-dimensional likelihood distributions and two-dimensional likelihood contours at 1σ, 2σ, and 3σ confidence levels using Mg II QSO-9 (blue), and BAO + $H(z)$ (red) data for all free parameters. Left panel shows the flat $\Lambda$CDM parametrization. The black dotted curved line in the $\omega_X - \Omega_{00}$ subpanel is the zero acceleration line with currently accelerated cosmological expansion occurring below the line and the black dashed straight lines correspond to the $\omega_X = -1$ $\Lambda$CDM model. Right panel shows the non-flat $\Lambda$CDM parametrization. The black dotted lines in the $\Omega_{00} - \Omega_{00}$, $\omega_X - \Omega_{00}$, and $\omega_X - \Omega_{00}$ subpanels are the zero acceleration lines with currently accelerated cosmological expansion occurring below the lines. Each of the three lines is computed with the third parameter set to the BAO + $H(z)$ data best-fit value given in Table 3. The black dashed straight lines correspond to the $\omega_X = -1$ $\Lambda$CDM model. The black dotted-dashed straight lines correspond to $\Omega_{00} = 0$.

Figure 7. One-dimensional likelihood distributions and two-dimensional likelihood contours at 1σ, 2σ, and 3σ confidence levels using Mg II QSO-9 (blue), and BAO + $H(z)$ (red) data for all free parameters. The $\alpha = 0$ axes correspond to the $\Lambda$CDM model. Left panel shows the flat $\phi$CDM model. The black dotted curved line in the $\alpha - \Omega_{00}$ subpanel is the zero acceleration line with currently accelerated cosmological expansion occurring to the left of the line. Right panel shows the non-flat $\phi$CDM model. The black dotted lines in the $\Omega_{00} - \Omega_{00}$, $\alpha - \Omega_{00}$, and $\alpha - \Omega_{00}$ subpanels are the zero acceleration lines with currently accelerated cosmological expansion occurring below the lines. Each of the three lines is computed with the third parameter set to the BAO + $H(z)$ data best-fit value given in Table 3. The black dashed straight lines correspond to $\Omega_{00} = 0$. 
y for each data set, QSO-9, QSO-69, and QSO-78, have values that are independent of the cosmological model assumed in the analysis. This validates the basic assumption of the R – L relation and means that these sources can be used as standardizable candles to constrain cosmological model parameters. For these three data sets, the best-fit values of β are ~ 1.7 and the best-fit values of γ are ~ 0.3. The Mg II R – L relation is thus shallower than the value predicted by the simple photoionization model (γ = 0.5). This is not a problem from a photoionization point of view because it appears that the broad Mg II line is emitted towards the outer part of the BLR and it exhibits a weaker response to the continuum variation than do the Balmer emission lines (Guo et al. 2020); see however Zajaček et al. (2020) for a significant correlation coefficient of ~ 0.8 and the presence of the intrinsic Baldwin effect for the luminous quasar HE 0413-4031. In addition, the Mg II line is a resonance line that is mostly collisionally excited, while Balmer lines are recombination lines. This can qualitatively affect the slope of the R – L relation for the Mg II line in comparison with the Balmer lines. However, Martínez-Aldama et al. (2020b) and Zajaček et al. (2021) found that by separating the sample into low and high accretors, it is possible to recover the expected value in both cases, i.e. the slope increases from ~ 0.3. This result supports the existence of the R – L correlation for Mg II QSOs, which is also consistent with the theoretical findings of Guo et al. (2020), who predict the existence of the global Mg II R – L correlation, while the weaker response of Mg II to the continuum variations can affect the R – L correlation slope for some individual sources, but apparently not all, or the epochs of correlated line light curve may be interrupted by a decorrelated light curve (BLR “holidays”; see also the study of NGC 5548: Dehghanian et al. 2019, for an example). Given that there is a significant Mg II QSO R – L correlation, as long as there are no significant unaccounted-for errors, an R – L relation slope ~ 0.3 (instead of ~ 0.5) does not invalidate the cosmological usage of Mg II QSOs. Another free parameter of the R – L relation is the intrinsic dispersion (σ_{ext}). The minimum value of σ_{ext}, ~ 0.25 dex, is obtained using the Mg II QSO-9 data set and the maximum value of σ_{ext}, ~ 0.3 dex, is obtained using the Mg II QSO-69 data set.

For the combined Mg II QSO-78 data, σ_{ext} ~ 0.29 dex. This is smaller than the σ_{ext} ~ 0.39 dex for the best available gamma-ray burst data set of 118 standardizable-candle GRBs spanning 0.3399 ≤ z ≤ 8.2 (Khadka et al. 2021) and a little larger than the σ_{ext} ~ 0.24 dex for the best available QSO X-ray and UV flux data set of 1019 standardizable-candle QSOs spanning 0.009 ≤ z ≤ 1.479 (Khadka & Ratra 2021b).

### Table 3. Unmarginalized one-dimensional best-fit parameters for Mg II QSO and BAO + H(z) data sets. For each data set, ΔAIC and ΔBIC values are computed with respect to the AIC and BIC values of the flat ΛCDM model.

| Model     | Data set          | Ω_bh^2 | Ω_c h^2 | Ω_m h^2 | Ω_de | α     | H_0^a | σ_{ext} | β     | γ  | d.o.f. | 2 ln(L_{max}) | AIC | BIC | ΔAIC  | ΔBIC |
|-----------|-------------------|--------|---------|---------|-------|-------|-------|---------|-------|----|-------|---------------|-----|-----|-------|-------|
| Mg II QSO-69 | -                 | 0.155  | -       | -       | -     | -     | 0.288 | 1.667   | 0.290 | 65 | 29.56 | 37.56         | 46.50 | -   | -     | -     |
| Flat Mg II QSO-78 | -                 | 0.138  | -       | -       | -     | -     | 0.281 | 1.666   | 0.283 | 74 | 30.16 | 38.16         | 47.58 | -   | -     | -     |
| ΛCDM Mg II QSO-9 | -                 | 0.804  | -       | -       | -     | -     | 0.207 | 1.254   | 0.002 | 5  | -0.874| 7.126          | 7.91  | -   | -     | -     |
| B+H^b       | 0.024  0.119 298  | -      | -       | 0.119   | 0.298 | -     | 0.691 | -       | -     | 39 | 23.66 | 29.66         | 34.87 | -   | -     | -     |
| Q+B+H^c     | 0.024  0.119 300  | -      | -       | 0.119   | 0.300 | -     | 0.689 | 0.285   | 1.685 | 0.293| 115   | 53.96          | 63.96 | 77.90 | -     | -     |
| Mg II QSO-69 | -                 | 0.357  | -1.075  | -       | -     | -     | 0.274 | 1.612   | 0.364 | 64 | 23.50 | 33.50         | 46.47 | -1.83 | -     | -     |
| Non-flat Mg II QSO-78 | -                 | 0.391  | -1.119  | -       | -     | -     | 0.270 | 1.623   | 0.354 | 73 | 25.40 | 35.40         | 47.18 | -2.76 | -0.40 | -     |
| ΛCDM Mg II QSO-9 | -                 | 0.664  | 0.759   | -       | -     | -     | 0.211 | 2.157   | 0.001 | 4  | -0.88 | 9.12           | 10.11 | 2.00  | 2.20  | -     |
| B+H^b       | 0.025  0.114 294  | 0.021  | -       | -       | 0.294 | -     | 0.670 | -       | -     | 38 | 23.60 | 31.60         | 38.55 | 1.94  | 3.68  | -     |
| Q+B+H^c     | 0.024  0.117 298  | 0.012  | -       | 0.117   | 0.298 | -     | 0.667 | 0.278   | 1.679 | 0.291| 114   | 53.98          | 65.98 | 82.70 | 2.02  | 4.80  |

**Notes:**

1. km s^{-1} Mpc^{-1}. H_0 is set to 70 km s^{-1} Mpc^{-1} for the QSO-only data analyses.
2. BAO + H(z).
3. Mg II QSO-78 + BAO + H(z).

### Constraints from Mg II QSO data

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Table 4: Marginalized one-dimensional best-fit parameters with 1σ confidence intervals, or 1σ or 2σ limits, for the Mg II QSO and BAO + $H(z)$ data sets.

| Model                  | Data                        | $\Omega_b h^2$ | $\Omega_c h^2$ | $\Omega_m$ | $\Omega_{\Lambda}$ | $\omega_X$ | $\sigma$ | $H_0$ | $\sigma_{\text{ref}}$ | $\beta$ | $\gamma$ |
|------------------------|-----------------------------|----------------|----------------|------------|-------------------|------------|--------|------|------------------------|--------|---------|
| Flat $\Lambda$CDM      | Mg II QSO-69                | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
|                        | Mg II QSO-78                | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
|                        | Mg II QSO-9                 | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Non-flat $\Lambda$CDM  | Mg II QSO-69                | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
|                        | Mg II QSO-78                | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
|                        | Mg II QSO-9                 | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |

In our analyses $\Omega_\Lambda$ is a derived parameter and in each case $\Omega_\Lambda$ chains are derived using the current energy budget equation $\Omega_\Lambda = 1 - \Omega_m - \Omega_b$ (where $\Omega_{\Lambda 0} = 0$ in the flat $\Lambda$CDM model). From these chains, using the python package getdist (Lewis 2019), we determine best-fit values and uncertainties for $\Omega_\Lambda$. We also use this python package to plot the likelihoods and compute the best-fit values and uncertainties of the free parameters.

b $\text{km s}^{-1}\text{Mpc}^{-1}$. $H_0$ is set to 70 $\text{km s}^{-1}\text{Mpc}^{-1}$ for the QSO-only data analyses.

c $\text{BAO} + H(z)$.

d Mg II QSO-78 + BAO + $H(z)$. 

Non-flat $\Lambda$CDM

| Model                  | Data                        | $\Omega_b h^2$ | $\Omega_c h^2$ | $\Omega_m$ | $\Omega_{\Lambda}$ | $\omega_X$ | $\sigma$ | $H_0$ | $\sigma_{\text{ref}}$ | $\beta$ | $\gamma$ |
|------------------------|-----------------------------|----------------|----------------|------------|-------------------|------------|--------|------|------------------------|--------|---------|
| Mg II QSO-69           | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Mg II QSO-78           | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Mg II QSO-9            | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| BAO+H                  | 0.024 $^{+0.003}_{-0.003}$  | 0.119 $^{+0.008}_{-0.008}$ | 0.290 $^{+0.05}_{-0.05}$ | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Q+B+H                  | 0.024 $^{+0.003}_{-0.003}$  | 0.119 $^{+0.008}_{-0.008}$ | 0.290 $^{+0.05}_{-0.05}$ | -          | -                 | -          | -      | -    | -                      | -      | -       |

Non-flat $\phi$CDM

| Model                  | Data                        | $\Omega_b h^2$ | $\Omega_c h^2$ | $\Omega_m$ | $\Omega_{\Lambda}$ | $\omega_X$ | $\sigma$ | $H_0$ | $\sigma_{\text{ref}}$ | $\beta$ | $\gamma$ |
|------------------------|-----------------------------|----------------|----------------|------------|-------------------|------------|--------|------|------------------------|--------|---------|
| Mg II QSO-69           | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Mg II QSO-78           | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Mg II QSO-9            | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| BAO+H                  | 0.024 $^{+0.006}_{-0.006}$  | 0.081 $^{+0.007}_{-0.007}$ | 0.266 $^{+0.03}_{-0.03}$ | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Q+B+H                  | 0.024 $^{+0.006}_{-0.006}$  | 0.081 $^{+0.007}_{-0.007}$ | 0.266 $^{+0.03}_{-0.03}$ | -          | -                 | -          | -      | -    | -                      | -      | -       |

Non-flat $\phi$CDM

| Model                  | Data                        | $\Omega_b h^2$ | $\Omega_c h^2$ | $\Omega_m$ | $\Omega_{\Lambda}$ | $\omega_X$ | $\sigma$ | $H_0$ | $\sigma_{\text{ref}}$ | $\beta$ | $\gamma$ |
|------------------------|-----------------------------|----------------|----------------|------------|-------------------|------------|--------|------|------------------------|--------|---------|
| Mg II QSO-69           | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Mg II QSO-78           | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Mg II QSO-9            | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| BAO+H                  | 0.024 $^{+0.006}_{-0.006}$  | 0.081 $^{+0.007}_{-0.007}$ | 0.266 $^{+0.03}_{-0.03}$ | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Q+B+H                  | 0.024 $^{+0.006}_{-0.006}$  | 0.081 $^{+0.007}_{-0.007}$ | 0.266 $^{+0.03}_{-0.03}$ | -          | -                 | -          | -      | -    | -                      | -      | -       |

Non-flat $\phi$CDM

| Model                  | Data                        | $\Omega_b h^2$ | $\Omega_c h^2$ | $\Omega_m$ | $\Omega_{\Lambda}$ | $\omega_X$ | $\sigma$ | $H_0$ | $\sigma_{\text{ref}}$ | $\beta$ | $\gamma$ |
|------------------------|-----------------------------|----------------|----------------|------------|-------------------|------------|--------|------|------------------------|--------|---------|
| Mg II QSO-69           | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Mg II QSO-78           | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Mg II QSO-9            | -                           | -              | -              | -          | -                 | -          | -      | -    | -                      | -      | -       |
| BAO+H                  | 0.024 $^{+0.006}_{-0.006}$  | 0.081 $^{+0.007}_{-0.007}$ | 0.266 $^{+0.03}_{-0.03}$ | -          | -                 | -          | -      | -    | -                      | -      | -       |
| Q+B+H                  | 0.024 $^{+0.006}_{-0.006}$  | 0.081 $^{+0.007}_{-0.007}$ | 0.266 $^{+0.03}_{-0.03}$ | -          | -                 | -          | -      | -    | -                      | -      | -       |
The scatter $\sigma_{\text{cut}}$ appears to be driven by the accretion-rate as shown by Zajaček et al. (2020) and Zajaček et al. (2021). In principle, the scatter could partially be mitigated by adding an independent observational quantity to the RL relation correlated with the accretion rate, see Martínez-Aldama et al. (2020b) for the analysis using the relative Fe II strength or fractional AGN variability parameters. This would, however, add one more nuisance parameter besides $\beta$ and $\gamma$ in the fitting scheme, and the overall effect on constraining cosmological parameters needs to be studied in detail in a future study. Furthermore, a homogeneous time-delay analysis applied to all the sources may also help to mitigate a fraction of the scatter, especially for a larger sample, since some sources exhibit more comparable peaks in correlation space, see e.g. Czemy et al. (2019), which creates a systematic uncertainty in the time-delay determination.

From Figs. 2–4 we see that for the Mg II QSO-78 data set the likelihoods favor the part of cosmological model parameter space that is consistent with currently-accelerating cosmological expansion, with the non-flat $\Lambda$CDM model being somewhat of an outlier.

From Table 4, for the Mg II QSO-69 data set, the minimum value of $\Omega_{m0}$, 0.240$^{+0.450}_{-0.170}$ is obtained in the spatially-flat $\Lambda$CDM model with the maximum value of $\Omega_{m0}$, 0.681$^{+0.219}_{-0.301}$, is in the spatially non-flat $\Lambda$CDM model. These data cannot constrain $\Omega_{m0}$ in the flat XCDM parametrization or the non-flat $\phi$CDM model. For the Mg II QSO-9 data, the value of $\Omega_{m0}$ is determined to be $> 0.088$ and $> 0.126$, at 2$\sigma$, in the flat and non-flat $\Lambda$CDM model respectively. These data cannot constrain $\Omega_{m0}$ in the four other models. For the Mg II QSO-78 data, the minimum value of $\Omega_{m0}$, 0.270$^{+0.210}_{-0.297}$ is in the non-flat $\Lambda$CDM model. These data cannot constrain $\Omega_{m0}$ in the flat XCDM parametrization or the non-flat $\phi$CDM model. All $\Omega_{m0}$ values obtained using these QSO data sets are consistent with those from BAO + $H(z)$ data or other well-established cosmological probes such as CMB anisotropy or Type Ia supernova measurements. In Fig. 8 we plot the Hubble diagram of the 78 Mg II QSOs and this figure shows that this QSO Hubble diagram is consistent with that of a flat $\Lambda$CDM model with $\Omega_{m0} = 0.3$.

From Table 4 and Figs. 2–4, we see that currently-available Mg II QSO data set at most only weak constraints on $\Omega_{\Lambda}$, $\Omega_{k0}$, $\omega_X$, and $\alpha$.

Table 3 lists, for all three QSO data sets, the values of AIC, BIC, and their differences, $\Delta$AIC and $\Delta$BIC, with respect to the AIC and BIC values for the spatially-flat $\Lambda$CDM model. From the AIC and BIC values, for the Mg II QSO-69 and Mg II QSO-78 data sets, the most favored case is the non-flat XCDM parametrization while non-flat $\phi$CDM is least favored. From the AIC and BIC values, for the Mg II QSO-9 data set, the most favored case is the flat $\Lambda$CDM model while the non-flat XCDM parametrization and the $\phi$CDM model are least favored. From the AIC values, only in the non-flat XCDM parametrization do the Mg II QSO-69 and Mg II QSO-78 data sets provide strong evidence against the spatially-flat $\Lambda$CDM model. From the BIC values, the Mg II QSO-69 and Mg II QSO-78 data sets provide strong evidence against only the non-flat $\phi$CDM model.

5.2 BAO + $H(z)$ and Mg II QSO-78 + BAO + $H(z)$ data constraints

The BAO + $H(z)$ data results listed in Tables 3 and 4 are from Khadka & Ratra (2021b) and are discussed in Sec. 5.3 of that paper. These BAO + $H(z)$ results are shown in red in Figs. 2–7 and 9–11. In this paper, we use these BAO + $H(z)$ results to compare with cosmological constraints obtained from the Mg II QSO data sets to see whether the Mg II QSO results are consistent or not with the BAO + $H(z)$ ones. This provides us with a qualitative idea of the consistency (inconsistency) between the Mg II QSO results and those obtained using better-established cosmological probes which favor $\Omega_{m0} > 0.3$.

In Figs. 2–4 we see that the cosmological constraints from QSO-78 data and those from BAO + $H(z)$ data are mutually consistent. It is therefore not unreasonable to jointly analyze these data. Since the Mg II QSO-78 data cosmological constraints are significantly less restrictive than those that follow from BAO + $H(z)$ data, adding the QSO-78 data to the mix will not significantly tighten the BAO + $H(z)$ cosmological constraints. Results from the Mg II QSO-78 + BAO + $H(z)$ data set are given in Tables 3 and 4. The un-marginalized best-fit parameter values are listed in Table 3 and the one-dimensional marginalized best-fit parameter values and limits are given in Table 4. Corresponding one-dimensional likelihood distributions and two-dimensional likelihood contours are plotted in blue in Figs. 9–11.

From Table 4, the minimum value of $\Omega_b h^2$ is found to be $0.024^{+0.003}_{-0.001}$ in the spatially-flat $\Lambda$CDM model while the maximum value of $\Omega_b h^2$ is $0.032^{+0.007}_{-0.001}$ in the spatially non-flat $\phi$CDM model. The minimum value of $\Omega_b h^2$ is $0.081^{+0.018}_{-0.014}$ and is obtained in the spatially flat $\phi$CDM model while the maximum value of $\Omega_b h^2$ is found to be $0.119^{+0.007}_{-0.008}$ in the spatially-flat $\Lambda$CDM model. The minimum value of $\Omega_{k0}$ is $0.266^{+0.024}_{-0.022}$ in the spatially-flat $\phi$CDM model and the maximum value of $\Omega_{k0}$ is $0.290^{+0.015}_{-0.015}$ in the spatially-flat $\Lambda$CDM model. As expected, these results are almost identical to those obtained using BAO + $H(z)$ data.

7 In the spatially non-flat $\phi$CDM model, $\Omega_{k0}(z, \alpha)$ is obtained from the numerical solutions of the equations of motion and its current value always lies in the range $0 \leq \Omega_{k0}(0, \alpha) \leq 1$. This restriction on $\Omega_{k0}(0, \alpha)$ cannot be seen in the non-flat $\phi$CDM model plots in Figs. 4 and 7 in the form of straight-line contour boundaries in the $\Omega_{m0} - \Omega_{k0}$ subpanels.
Figure 9. One-dimensional likelihood distributions and two-dimensional likelihood contours at 1σ, 2σ, and 3σ confidence levels using Mg II QSO-78 (gray), BAO + $H(z)$ (red), and Mg II QSO-78 + BAO + $H(z)$ (blue) data for all free parameters. Left panel shows the flat ΛCDM model and right panel shows the non-flat ΛCDM model. The black dashed straight lines in the right panel correspond to $\Omega_{k0} = 0$.

Figure 10. One-dimensional likelihood distributions and two-dimensional likelihood contours at 1σ, 2σ, and 3σ confidence levels using Mg II QSO-78 (gray), BAO + $H(z)$ (red), and Mg II QSO-78 + BAO + $H(z)$ (blue) data for all free parameters. Left panel shows the flat XCDM parametrization. Right panel shows the non-flat XCDM parametrization. The black dotted straight lines in both panels correspond to the $\omega_X = -1$ ΛCDM models. The black dashed straight lines in the $\Omega_{k0}$ subpanels in the right panel correspond to $\Omega_{k0} = 0$. 
From Table 4, in the flat \( \Lambda \)CDM model, the value of the spatial curvature energy density \( \Omega_k \) is \( 0.700^{+0.017}_{-0.015} \). In the non-flat \( \Lambda \)CDM model, the value of the spatial curvature energy density \( \Omega_k \) is \( 0.675^{+0.092}_{-0.079} \).

For analyses that involve the BAO + \( H(z) \) data, the Hubble constant \( H_0 \) is a free parameter. From the Mg II QSO-78 + BAO + \( H(z) \) data, the minimum value of \( H_0 \) is \( 65.2 \pm 2.1 \) km s\(^{-1}\)Mpc\(^{-1}\) in the spatially-flat \( \varphi \)CDM model while the maximum value of \( H_0 \) is \( 69.3 \pm 1.8 \) km s\(^{-1}\)Mpc\(^{-1}\) in the spatially-flat \( \Lambda \)CDM model.

From Table 4, the values of the spatial curvature energy density parameter \( \Omega_k \) are \( 0.031^{+0.094}_{-0.120} \), \( -0.110^{+0.130}_{-0.130} \), and \( -0.090^{+0.100}_{-0.120} \) in the non-flat \( \Lambda \)CDM, XCDM, and \( \varphi \)CDM model respectively. These are consistent with flat spatial hypersurfaces and also with mildly open or closed ones.

From Table 4, in the flat XCDM parametrization, the value of the dynamical dark energy equation of state parameter \( (\omega_X) \) is \( -0.750^{+0.150}_{-0.100} \) while in the non-flat XCDM parametrization \( \omega_X \) is \( -0.700^{+0.140}_{-0.079} \). In the flat \( \varphi \)CDM model, the scalar field potential energy density parameter \( (\alpha) \) is \( 1.510^{+0.620}_{-0.820} \) while in the non-flat \( \varphi \)CDM model \( \alpha \) is \( 1.660^{+0.670}_{-0.850} \). In these four dynamical dark energy models, dynamical dark energy is favored at \( 1.7\sigma - 3.8\sigma \) statistical significance over the cosmological constant.

From Table 3, from the \( AIC \) and \( BIC \) values, the most favored model is flat \( \varphi \)CDM while non-flat \( \Lambda \)CDM is least favored. From the \( \Delta AIC \) values, all models are almost indistinguishable from the spatially-flat \( \Lambda \)CDM model. From the \( \Delta BIC \) values, the non-flat \( \Lambda \)CDM, XCDM, and \( \varphi \)CDM models provide positive evidence for the spatially-flat \( \Lambda \)CDM model.

### 6 CONCLUSION

In this paper, we use the \( R - L \) relation to standardize Mg II QSOs. Analyses of different Mg II QSO data sets using six different cosmological dark energy models show that the \( R - L \) relation parameters are model-independent and that the intrinsic dispersion of the \( R - L \) relation for the whole Mg II QSO data set is \( \sim 0.29 \) dex which is not very large for only 78 QSOs. So, for the first time, we have shown that one can use the \( R - L \) relation to standardize available Mg II QSOs and thus use them as a cosmological probe.

We determined constraints on cosmological model parameters using these Mg II QSO data and found that these constraints are significantly weaker than, and consistent with, those obtained using BAO + \( H(z) \) data. In Fig. 8 we show that the 78 Mg II QSOs have a Hubble diagram consistent with what is expected in the standard spatially-flat \( \Lambda \)CDM model with \( \Omega_{k0} = 0.3 \). This differs from the results of the QSO X-ray and UV flux data compiled by Risaliti & Lusso (2019) and Lusso et al. (2020).

The constraints obtained from the joint analyses of Mg II QSO data and the BAO + \( H(z) \) measurements are consistent with the current standard spatially-flat \( \Lambda \)CDM model but also do not rule out slight spatial curvature. These data weakly favor dynamical dark energy over the cosmological constant.

The current Mg II QSO data set contains only 78 sources and covers the redshift range \( 0.0033 \leq z \leq 1.89 \). Future detections of significant time-delays of the BLR emission of Mg II QSOs will increase the number of sources over a larger redshift extent, which will further constrain the Mg II QSO \( R - L \) relation, in

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8 Khadka & Ratra (2019) found that only about half of the Lusso et al. (2020) QSO flux sources, about a 1000 QSOs at \( z \leq 1.5 \), were standardizable and that cosmological constraints from these QSOs were consistent with what is expected in the standard \( \Lambda \)CDM model.
particular its slope. A large increase of suitable sources is expected from the Rubin Observatory Legacy Survey of Space and Time that will monitor about 10 million quasars in six photometric bands during its 10-year lifetime. We hope that such an improved data set will soon provide tighter cosmological constraints, as well as allow for a comparison with constraints from QSO X-ray and UV flux measurements which currently are exhibiting some tension with standard flat ΛCDM model expectations.

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DATA AVAILABILITY

The data analysed in this article are listed in Table A1 of this paper.

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### APPENDIX A: MG II QSO DATA

Table A1: Reverberation-mapped Mg II QSO samples. For each source, columns list: QSO name, redshift, continuum flux density at 3000 Å, and measured rest-frame time delay. The first 68 sources are from Martínez-Aldama et al. (2020b), the source in boldface is from Zajaček et al. (2021), and the last 9 sources are from Yu et al. (2021).

| Object | $z$     | $\log \left(F_{3000}/\text{erg s}^{-1}\text{cm}^{-2}\right)$ | $\tau$ (day) |
|--------|---------|-------------------------------------------------------------|--------------|
| 018    | 0.848   | $-13.1412 \pm 0.0009$                                       | 125.9         |
| 028    | 1.392   | $-12.4734 \pm 0.0004$                                       | 65.7          |
| 038    | 1.383   | $-12.3664 \pm 0.0003$                                       | 120.7         |
| 044    | 1.233   | $-13.04308 \pm 0.0013$                                      | 65.8          |
| 102    | 0.861   | $-12.5575 \pm 0.0005$                                       | 86.9          |
| 114    | 1.226   | $-11.8369 \pm 0.0003$                                       | 188.6         |
| 118    | 0.715   | $-12.2592 \pm 0.0006$                                       | 102.2         |
| 123    | 0.891   | $-12.8942 \pm 0.0009$                                       | 81.6          |
| 135    | 1.315   | $-12.8122 \pm 0.0005$                                       | 93.0          |
| 158    | 1.478   | $-13.2376 \pm 0.0012$                                       | 119.1         |
| 159    | 1.587   | $-12.7139 \pm 0.0006$                                       | 324.2         |
| 160    | 0.36    | $-12.8441 \pm 0.0013$                                       | 106.5         |
| 170    | 1.163   | $-12.6802 \pm 0.0005$                                       | 98.5          |
| 185    | 0.987   | $-12.8039 \pm 0.0094$                                       | 387.9         |
| 191    | 0.442   | $-13.0544 \pm 0.0012$                                       | 93.9          |
| 228    | 1.264   | $-13.2697 \pm 0.0011$                                       | 37.9          |
| 232    | 0.808   | $-13.1895 \pm 0.0014$                                       | 273.8         |
| 240    | 0.762   | $-13.3270 \pm 0.0021$                                       | 17.2          |
| 260    | 0.995   | $-12.4126 \pm 0.0004$                                       | 94.9          |
| 280    | 1.366   | $-12.5531 \pm 0.0003$                                       | 99.1          |
| 285    | 1.034   | $-13.2539 \pm 0.0020$                                       | 138.5         |
| 291    | 0.532   | $-13.2471 \pm 0.0016$                                       | 39.7          |
| 294    | 1.215   | $-12.4272 \pm 0.0004$                                       | 71.8          |
| 301    | 0.548   | $-12.8782 \pm 0.0011$                                       | 136.3         |
| 303    | 0.821   | $-13.3066 \pm 0.0013$                                       | 57.7          |
| 329    | 0.721   | $-11.9680 \pm 0.0007$                                       | 87.5          |
| 338    | 0.418   | $-12.9969 \pm 0.0013$                                       | 22.1          |
| 419    | 1.272   | $-12.9765 \pm 0.0011$                                       | 95.5          |
| 422    | 1.074   | $-13.0946 \pm 0.0011$                                       | 109.3         |
| 440    | 0.754   | $-12.5157 \pm 0.0004$                                       | 114.6         |
| 441    | 1.397   | $-12.5772 \pm 0.0004$                                       | 127.7         |
| 449    | 1.218   | $-12.9299 \pm 0.0013$                                       | 119.8         |
| 457    | 0.604   | $-13.4805 \pm 0.0029$                                       | 20.5          |
| 459    | 1.156   | $-12.8737 \pm 0.0011$                                       | 122.8         |
| 469    | 1.004   | $-12.1222 \pm 0.0002$                                       | 224.1         |
| 492    | 0.964   | $-12.3786 \pm 0.0004$                                       | 92.0          |
| 493    | 1.592   | $-12.2173 \pm 0.0004$                                       | 315.6         |
| 501    | 1.155   | $-12.9728 \pm 0.0009$                                       | 44.9          |
| 505    | 1.144   | $-13.0625 \pm 0.0011$                                       | 94.7          |
| 522    | 1.384   | $-12.9671 \pm 0.0006$                                       | 115.8         |
| 556    | 1.494   | $-12.6492 \pm 0.0005$                                       | 98.7          |
| 588    | 0.998   | $-12.1158 \pm 0.0002$                                       | 74.3          |
| 593    | 0.992   | $-12.7093 \pm 0.0006$                                       | 80.1          |
| 622    | 0.572   | $-12.6232 \pm 0.0005$                                       | 61.7          |
| 645    | 0.474   | $-12.7268 \pm 0.0009$                                       | 30.2          |
| 649    | 0.85    | $-13.0437 \pm 0.0013$                                       | 165.5         |
| 651    | 1.486   | $-12.9434 \pm 0.0011$                                       | 76.5          |
| 675    | 0.919   | $-12.5273 \pm 0.0005$                                       | 139.8         |
| 678    | 1.463   | $-12.8267 \pm 0.0007$                                       | 82.9          |
Table A1: continued.

| Object                     | $z$  | $\log \left( F_{3000} / \text{erg s}^{-1} \text{ cm}^{-2} \right)$ | $\tau$ (day) |
|----------------------------|------|-------------------------------------------------|--------------|
| 709                        | 1.251| $-12.9586 \pm 0.0010$                          | 85.4$^{+17.1}_{-19.3}$ |
| 714                        | 0.921| $-12.8296 \pm 0.0012$                          | 320.1$^{+11.3}_{-15.2}$ |
| 756                        | 0.852| $-13.1462 \pm 0.0023$                          | 315.3$^{+16.4}_{-16.4}$ |
| 761                        | 0.771| $-12.6395 \pm 0.0024$                          | 102.1$^{+3.4}_{-3.4}$  |
| 771                        | 1.492| $-12.4477 \pm 0.0004$                          | 31.3$^{+3.1}_{-4.0}$  |
| 774                        | 1.686| $-12.5786 \pm 0.0004$                          | 58.9$^{+13.7}_{-10.1}$ |
| 792                        | 0.526| $-13.5353 \pm 0.0030$                          | 111.4$^{+20.0}_{-20.0}$ |
| 848                        | 0.757| $-13.3199 \pm 0.0015$                          | 65.1$^{+29.4}_{-29.4}$ |
| J141214                    | 0.4581| $-12.2526 \pm 0.00043$                         | 36.7$^{+10.3}_{-12.8}$ |
| J141018                    | 0.4696| $-13.1883 \pm 0.00506$                         | 32.3$^{+12.9}_{-9.3}$  |
| J141417                    | 0.6037| $-13.4926 \pm 0.0029$                          | 29.1$^{+3.6}_{-3.6}$  |
| J142049                    | 0.751| $-12.7205 \pm 0.0009$                          | 34.0$^{+6.7}_{-12.0}$  |
| J141650                    | 0.5266| $-13.2586 \pm 0.00198$                         | 25.1$^{+2.0}_{-2.0}$  |
| J141644                    | 0.4253| $-12.8667 \pm 0.0015$                          | 17.2$^{+2.7}_{-2.7}$  |
| CTS252                     | 1.89  | $-11.6068 \pm 0.09142$                         | 190.0$^{+59.0}_{-10.0}$ |
| NGC4151                    | 0.0033| $-9.5484 \pm 0.18206$                          | 8.6$^{+1.2}_{-1.0}$   |
| NGC4151                    | 0.0033| $-9.5484 \pm 0.18206$                          | 5.3$^{+1.0}_{-1.0}$   |
| CTS300                     | 0.9005| $-11.5825 \pm 0.026$                           | 564.0$^{+109.0}_{-21.0}$ |
| HE0413-4031                | 1.3765| $-11.3203 \pm 0.0434$                          | 302.9$^{+23.7}_{-19.0}$ |
| HE0435-4312                | 1.2231| $-11.5754 \pm 0.036$                           | 296.0$^{+14.0}_{-10.0}$ |
| J025225.52+003405.90       | 1.6245| $-12.6489 \pm 0.05203$                         | 198.8$^{+18.96}_{-19.03}$ |
| J021612.83-044634.10       | 1.5604| $-12.7064 \pm 0.04075$                         | 51.4$^{+13.37}_{-8.85}$ |
| J033553.51-275044.70       | 1.57774| $-12.3688 \pm 0.04223$                         | 48.1$^{+23.05}_{-8.82}$ |
| J003710.86-444048.11       | 1.06703| $-12.1225 \pm 0.04395$                         | 191.7$^{+28.97}_{-18.47}$ |
| J003207.44-433049.00       | 1.53278| $-12.5873 \pm 0.02829$                         | 146.9$^{+7.43}_{-0.92}$ |
| J003015.00-430333.52       | 1.64984| $-12.7328 \pm 0.04608$                         | 185.5$^{+13.35}_{-12.72}$ |
| J003052.76-430301.08       | 1.42754| $-12.5959 \pm 0.03380$                         | 166.7$^{+12.00}_{-12.28}$ |
| J003234.33-431937.81       | 1.64058| $-12.5643 \pm 0.03011$                         | 248.8$^{+11.00}_{-11.00}$ |
| J003206.50-425325.22       | 1.7496| $-12.7498 \pm 0.09277$                         | 157.8$^{+12.79}_{-4.95}$ |

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