The enhancement of students’ mathematical representation in junior high school using cognitive apprenticeship instruction (CAI)

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Abstract. This study aims to get an in-depth understanding of the enhancement of students' mathematical representation. This study is experimental research with pretest-posttest control group design. The subject of this study is the students’ of the eighth grade from junior high schools in Bandung: high-level and middle-level. In each school, two parallel groups were chosen as a control group and an experimental group. The experimental group was given cognitive apprenticeship instruction (CAI) treatment while the control group was given conventional learning. The results show that the enhancement of students' mathematical representation who obtained CAI treatment was better than the conventional one, viewed which can be observed from the overall, mathematical prior knowledge (MPK), and school level. It can be concluded that CAI can be used as a good alternative learning model to enhance students' mathematical representation.

1. Introduction

The process of learning mathematics that optimizes all of the students’ abilities in the learning process becomes the attention of the world of mathematics education nowadays. Government Regulation of the Republic of Indonesia Number 19 the Year 2005 Clause 19 paragraph 1 states that the process of learning in educational units is held interactively, inspirational, fun, challenging, motivate learners to participate actively, and provide space with talent, interest, and the development of students' physical and psychological. Optimization of students' ability in the learning process will have an impact on the achievement of mathematical ability by the purpose of learning mathematics.

Mathematical representation ability is one of the abilities a junior high school student must have. The importance of mathematical representation ability is defined by NCTM [1] which stated that students could make connections, develop, and deepen their understanding of mathematical concepts by using various representations. Representations such as physical objects, images, diagrams, graphics, and symbols also help students communicate their thoughts. NCTM [2] stated that the use of various mathematical representations could support and deepen the students' mathematical knowledge. In fact, the mathematical representation ability of junior high school students' is still relatively low.
Some research found show that students do not have the ability of representation and mathematical abstraction as expected. Chen, Lee, and Hsu [3] argue that most of the difficulties in problem-solving occur at the representation stage. As a result, the process of translating the problem into an internal representation is the key for students to solve the problem successfully. Teachers should provide opportunities for students to explore their abilities.

The effects of teacher characteristics are caring, assertive, modeling and enthusiastic, and has high expectations. Teacher's awareness in organizing the classroom or succeeding in all teaching sections should be accomplished by earning and accommodating students' learning patterns. Assertiveness means the ability to train students' responsibilities and to apply those responsibilities to their actions. Modelling and enthusiasm in question are that teacher's belief in teaching and learning is communicated through modeling. By showing enthusiasm, teachers communicate their real-world interests in the topic [4].

Cognitive Apprenticeship Instruction (CAI) is one of learning that suits the characteristics of student-centered learning. The CAI process involves real-world problems and contextual problems. To solve problems related to the real world, students need to communicate their ideas through mathematical representation. While the components of learning CAI are: (1) modeling; (2) coaching; (3) articulation; (4) reflection; (5) scaffolding and fading; and (6) exploration [5]. The steps of CAI have various syntax according to some version of the initiator, but in this study, the most appropriate part of mathematics learning is the syntax that includes modeling, coaching, scaffolding, reflection, articulation, and exploration.

Based on the above description, the formulation of this research problem is whether the achievement of mathematical representation abilities of students who received CAI are better than students who received conventional learning (PKv) as viewed from overall and school level. So, the purpose of the research is to review the enhancement of students' mathematical representation abilities thoroughly. Currently, almost all of the energy used depends on conventional energy derived from fossil fuels which are resulting in the release of greenhouse gases such as carbon dioxide, methane, and nitrous oxide into the atmosphere increasing the ambient temperature and causing the climate change. Linearly, the higher temperatures give a severe impact on the demand for the electricity consumption regarding the peak electricity and the total electricity. To satisfy the demand, the need to build the additional power plants can increase the cost of electricity generation [1]. In fact, the global power consumption is projected to grow from 18 trillion kWh in 2006 to 32 trillion kWh by 2030. It means that there will be 4800 GW of new power plants to be developed and more than half of it is coming from the developing countries [2]. Unfortunately, the increase of electricity generation accounts for approximately 24% of the greenhouse gases emissions. Therefore, the electricity generation must be realized using the renewable energy to lower the greenhouse gases effect.

2. Methods

This study is experimental research with a pretest-posttest control group design. The subject in this study is the students of eight graders at two schools; they are from high leveled school and middle level school in Bandung. In each school, two parallel group are selected to be the experimental group and the control group. The experimental group was given Cognitive Apprenticeship Instruction (CAI) treatment while the control group was done with conventional learning (PKv).

Data analysis was performed with three main stages: 1) descriptive analysis and calculate the enhancement of mathematical representation ability by using normalized gain formula; 2) test requirement of statistical analysis needed as basis in hypothesis test that is normality test and homogeneity variance test; and 3) Hypothesis test previously stated, if normality and homogeneity test is not met then the data is analyzed using non-parametric statistic.

3. Results and discussion

The results of research and discussion related to problem formulation and research objectives will be discussed below.
3.1 Mathematical Representation Ability
The analysis results of statistical descriptive of mathematical representation based on school and MPK levels are presented in Table 1.

| School Level | Class | MPK | Mathematical Representation Ability |
|--------------|-------|-----|-------------------------------------|
|              |       |     | Pre-test Mean | St. Dev | Post-test Mean | St. Dev | Enhancement Mean | St. Dev |
| High         | CAI   | High| 10.83  | 4.48   | 86.25       | 13.04   | 0.85             | 0.14    |
|              |       | Medium| 11.39  | 9.89   | 68.33       | 20.94   | 0.65             | 0.22    |
|              |       | Low    | 12.50  | 8.33   | 55.21       | 26.52   | 0.51             | 0.26    |
|              | PKv   | Total  | 11.49  | 4.94   | 70.58       | 23.06   | 0.67             | 0.24    |
| Medium       | CAI   | High  | 7.14   | 2.03   | 51.79       | 12.70   | 0.48             | 0.13    |
|              |       | Medium | 10.52  | 5.53   | 44.25       | 21.18   | 0.38             | 0.22    |
|              |       | Low    | 10.12  | 4.72   | 35.71       | 19.22   | 0.29             | 0.18    |
|              | PKv   | Total  | 9.76   | 4.94   | 44.05       | 19.60   | 0.38             | 0.20    |

3.1.1 The Pre-test of Students’ Mathematical Representation Ability Based on School Level. Based on Table 1, it can be seen that description of students' preliminary representation ability, in general, has higher mean and standard of experiment class than the preliminary representation ability of students in control class. To determine the significant difference, it is done a test between different groups. However, to ensure that there is a difference in the ability of mathematical representation at the high school level, the middle school level must meet the normality and homogeneity requirements of the pre-test data variance. The pre-test data test of students' mathematical representation ability used to test data normality is by Shapiro-Wilk Test. The normality test criteria of mathematical representation results, $H_0$ is accepted if the probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if the probability value is less than $\alpha = 0.05$. Pre-test of students' mathematical representation ability in term of high school level and medium school level have a sig. value less than $\alpha = 0.05$, this means $H_0$ is rejected.

So that data representation ability of student at the high school level and school level are not normally distributed. Because the parametric statistical test was not fulfilled, the test continued with the similarity test using a nonparametric statistics, Mann-Whitney U Test. The similarity test criteria of mathematical representation ability, $H_0$ is accepted if probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if probability value less than $\alpha = 0.05$. It can be explained that the pre-test of mathematical representation of students at high school and high school level has a bigger sig. Value more than $\alpha = 0.05$, this means $H_0$ is accepted. The conclusions obtained at each level of school that there is no difference (equal) of the mean of preliminary mathematical representation ability of students based on the school level.

The Pre-test of Students’ Mathematical Representation Ability Based on Mathematical Prior Knowledge. The recapitulation of pre-test data of students' mathematical representation based on MPK category and learning including minimum score, maximum score, mean and standard deviation. The students' preliminary ability to represent students, in general, has the higher mean and standard deviation in the experimental class than the preliminary ability of the students' representation in the control class. To determine the significant difference between different groups, the mean difference test was done. However, to ensure that there is a difference in mathematical representation ability at the high school level, the middle school level must meet the normality and homogeneity requirements.
of the pre-test data variance. The pre-test of students' mathematical representation ability to test data normality is using Shapiro-Wilk Test. The criteria of normality test, $H_0$ is accepted if the probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if the probability value is less than $\alpha = 0.05$. The students' mathematical representation ability in term of the high MPK category on CAI learning, MPK category with CAI learning and conventional learning, and the low MPK category with CAI learning has a sig. value less than $\alpha = 0.05$, this means $H_0$ is rejected. While the high MPK category with conventional learning and low MPK category with conventional learning has a higher sig value more than $\alpha = 0.05$, this means $H_0$ is accepted. So that the data of students' mathematical representation ability is viewed from high MPK category with CAI learning, MPK category with CAI learning and conventional learning, and low MPK category with CAI learning is not normally distributed. While the high MPK category with conventional learning and low MPK category with conventional learning is normally distributed.

Because the parametric statistical test was not fulfilled, the test continued with the similarity test using a nonparametric statistics, Mann-Whitney U Test. The similarity test criteria of mathematical representation ability, $H_0$ is accepted if probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if probability value less than $\alpha = 0.05$. The students' mathematical representation ability in high MPK category, medium MPK category, and low MPK category have a bigger sig. Value more than $\alpha = 0.05$, this means $H_0$ is accepted. The conclusion obtained is that there is no difference of students' preliminary mathematical representation ability based on MPK category.

The recapitulation of students' mathematical representation ability enhancement includes minimum score, maximum score, mean and standard deviation. Students' representation enhancement, in general, has a higher mean and standard deviation in the experimental class than students' preliminary representation ability in the control class. To determine the significant difference, it was done a test between different groups. However, to ensure that there is a difference in the ability of mathematical representation at the high school level, the middle school level must meet the normality and homogeneity requirements of the pre-test data variance. The pre-test data test of students' mathematical representation ability used to test data normality is by Shapiro-Wilk Test.

The criteria of normality test of mathematical representation ability results, $H_0$ is accepted if the probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if the probability value is less than $\alpha = 0.05$. The students' overall mathematical representation ability has sig value less than $\alpha = 0.05$; this means $H_0$ is rejected. So that data representation ability of student at the high school level and school level are not normally distributed. Because the parametric statistical test was not fulfilled, the test continued with the similarity test using a nonparametric statistics, Mann-Whitney U Test. The criteria of difference test, $H_0$ is accepted if the probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if the probability value is less than $\alpha = 0.05$. Test results can be seen in Table 2.

| Table 2. The similarity test result of overall students' mathematical representation ability based on learning |
|---------------------------------------------------------------|
| Learning     | N  | Mann-Whitney Test | Sig (2-tailed) | Conclusion       |
|--------------|----|------------------|----------------|------------------|
| CAI          | 67 | 1217,500         | 0,000          | $H_0$ is rejected |
| PKV          | 68 |                  |                |                  |

Based on Table 2, it can be explained that the enhancement of students' mathematical representation as a whole has a sig value less than $\alpha = 0.05$, this means $H_0$ is rejected. The conclusion obtained is the enhancement of mathematical representation of students who obtained CAI learning is better than the enhancement of students' mathematical representation ability who obtained the conventional learning.
3.2 The enhancement of Students' Mathematical Representation Ability based on School Level

The Recapitulation of students' enhancement of representational ability in term of high and middle school levels includes minimum score, maximum score, mean and standard deviation. The enhancement of students' representational ability in the experimental class has a higher mean and standard deviation than the enhancement of students' representational ability in the control class. To determine the significant difference, a test was done between different groups. However, to ensure that there is a difference in the ability of mathematical representation at the high school level, the middle school level must meet the normality and homogeneity requirements of the pre-test data variance. The pre-test data test of students' mathematical representation ability that is used to test the data normality is using Shapiro-Wilk Test.

The normality test criteria of students' mathematical representation ability enhancement, H₀ is accepted if the probability value (sig.) is bigger than α = 0.05 and H₀ is rejected if the probability value is less than α = 0.05. The enhancement of students' mathematical representation ability from the middle school level has a sig. value less than α = 0.05, this means H₀ is rejected. While the enhancement of students' mathematical representation ability from high school level has a higher sig. Value more than α = 0.05, this means H₀ is accepted. So that the data of students' representation ability at school level is not normally distributed, while the enhancement of student representation ability at high school level is normally distributed.

Because the level of school is not normally distributed, then it is needed to test the enhancement using a nonparametric statistical test that is Mann-Whitney U test. While high school level is normally distributed, the homogeneity test of the enhancement data variance is using Levene test.

The similarity test criteria, H₀ is accepted if the probability value (sig.) is bigger than α = 0.05 and H₀ is rejected if the probability value is less than α = 0.05. The enhancement of students' mathematical representation in the middle school level has a sig. value less than α = 0.05, this means H₀ is rejected. The conclusion is that the enhancement of mathematical representation ability of students who obtained CAI learning is better than the enhancement of mathematical representation ability of students who obtained conventional learning. Because the high school level is normally distributed, the homogeneity test of increased data variance using Levene test is used.

The criteria of homogeneity test, H₀ is accepted if the probability value (sig.) is bigger than α = 0.05 and H₀ is rejected if the probability value is less than α = 0.05. The enhancement of students' mathematical representation with CAI learning and conventional learning at high school level has a higher sig. Value than α = 0.05, this means H₀ is accepted. It can be concluded that the enhancement of mathematical representation ability at high school level varies homogeneously. Furthermore, to know the difference between enhancement mean of mathematical representation ability of high school students.

The similarity test criteria, H₀ is accepted if the probability value (sig.) is bigger than α = 0.05 and H₀ is rejected if the probability value is less than α = 0.05. The enhancement of students' mathematical representation at high school level has a sig. value less than α = 0.05, this means H₀ is rejected. The conclusion is that the enhancement of mathematical representation ability of students who obtained CAI learning is better than the enhancement of mathematical representation ability of students who obtained conventional learning.

3.3 The enhancement of Students' Mathematical Representation Ability Based on Preliminary Mathematical Ability

The recapitulation of pre-test data of students' mathematical representation based on MPK category and learning includes minimum score, maximum score, mean and standard deviation. The ability of students' mathematical representation in term of high MPK category with CAI learning has a sig. value less than α = 0.05, this means H₀ is rejected. While MPK category is high in conventional learning, MPK category is on CAI learning, and conventional learning, low MPK category on CAI learning and conventional learning have a higher sig. value than α = 0.05. So the data of students' representation ability of high MPK category with CAI learning is not normally distributed, whereas MPK category is
high with conventional learning, MPK category with CAI learning and conventional learning, low MPK category with CAI learning and conventional learning are normally distributed.

Because MPK category is not normally distributed, the mean difference test is done using nonparametric statistic, Mann-Whitney U test. While medium and low MPK categories are normally distributed, a homogeneity test of data variance is done using Levene test. The mean difference test criteria, $H_0$ is accepted if the probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if the probability value is less than $\alpha = 0.05$. The enhancement of students' mathematical representation in high MPK category has a sig. value less than $\alpha = 0.05$, this means $H_0$ is rejected. The conclusion is that the enhancement of students' mathematical representation who obtained CAI learning is better than the enhancement of mathematical representation ability of the students who obtained the conventional learning. Because the medium and low MPK categories are normally distributed, the homogeneity test of enhancement data variance is done using the Levene test.

Test Criteria of $H_0$ is accepted if the probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if the probability value is less than $\alpha = 0.05$. The data of students' mathematical representation enhancement in MPK category has a higher sig. Value more than $\alpha = 0.05$, this means $H_0$ is accepted. While the data enhancement of students' mathematical representation ability in the low category has a sig. value less than $\alpha = 0.05$, this means $H_0$ is rejected. It can be concluded that the data enhancement of students' representation ability in MPK category varies homogeneously. While the data enhancement of students' representation ability in low MPK category does not vary homogeneously.

Furthermore, to know the difference of the mean of data enhancement of students' mathematical representation ability in MPK category is done using T-test, whereas, in low MPK category, it is done using T-test'. The mean difference test criteria, $H_0$ is accepted if the probability value (sig.) is bigger than $\alpha = 0.05$ and $H_0$ is rejected if the probability value is less than $\alpha = 0.05$. The data enhancement of students' mathematical representation ability with CAI learning and conventional learning in medium MPK category and low MPK category have a sig. value less than $\alpha = 0.05$, this means $H_0$ is rejected. The conclusion is that the enhancement of mathematical representation ability of students who obtained CAI learning better is better than the enhancement of mathematical representation ability of the students who obtained conventional learning.

The findings of the study indicate that the enhancement of mathematical representation ability of students who received cognitive apprenticeship instruction is higher than students learning with conventional model reviewed from overall, MPK, and school level. The ability of mathematical representation in this research is the ability of students to create mathematical models; students' ability to draw graphs or tables to solve linear equations system of two variables; students' ability to compose stories or written texts based on presented representations; Students' ability to create equations from a given chart; And the student's ability to solve and summarize the problem of a linear equation system of two variables with written text.

The enhancement of students' mathematical representation ability who received CAI is better than students who received conventional learning for all indicators. This is possible because the CAI stages provide more opportunities for students to develop their mathematical abilities in their group. Student-centred instruction is characterized more by interaction and communication in learning, which is intended to develop the mathematical identity of the student [6]. The student's worksheet based on cognitive apprenticeship instruction helps students in building students' understanding of the material they are studying. The cognitive apprenticeship instruction stage is depicted in the student worksheet.

In a student-centered learning process, teachers act as facilitators and mediators to make students understand the material being studied. CAI with its stages provides students with greater opportunities to build on their knowledge. At the modeling stage, students get an overview and examples to understand the material. The material is presented about the real world of student's daily life. Problems are presented with things students have ever experienced. In line with Brown's opinion, Solomon & Williams [7], our evolving understandings, of who we are and of what we do, shape our use of mathematics and thus our understandings of what mathematics is. Moreover, public images of mathematics pull in some directions that produce alternative conceptions of mathematics. These
disparities of vision result in much variety in how mathematics is materialized in everyday activity. This, of course, helps the student in solving the given problem and makes the students eager in learning. It can be seen from the student's enthusiasm in doing the assigned task. Also, a very prominent part of this stage is that students can build representational ability on indicators to create mathematical models. The modeling contained in the worksheet indirectly builds and develops students' mathematical representation abilities. For example in building the understanding about gradient, then on student's worksheets are presented images of people who are climbing the hill. Climbing the hill was once experienced by students on scout activities so that the atmosphere up the hill can be felt by the students. Then at the coaching stage, the teacher helps provide an understanding that the slope in mathematics is called the gradient. Next, the students write the gradient in a mathematical symbol. Teacher continues to pay attention and facilitate the development of students, while students perform the task.

When students have difficulty in understanding the given problem, even though it has passed the modeling stage, the teacher helps gradually. This happens at the scaffolding stage of the cognitive apprenticeship instruction. The teacher gradually directs according to the level of difficulty that students experience in their group. The main aspiration of teaching in the Zone of Proximal Development is to see mathematics teachers being actively engaged in their teaching-learning practices with the prospect of helping them becoming self-directed mathematics teachers. This suggests that the interpretation of the meaning of the Zone of Proximal Development points to the fact that effective teaching focuses on the transformation of socially constructed knowledge into that which is individually owned. This type of teaching assumes a specific paradigm of adult-teacher/teacher-learner interaction where the role of the adult-teacher is that of a collaborator and co-constructor of the knowledge being constructed (Verenikina & Chinnappan in Nature and Jbrin) [8]. Casem [9]. Teachers should use scaffolding strategy in combination with some traditional methods in teaching Mathematics. But first, they need to reconceptualize their role as facilitators in the development of the students' mathematical constructions rather than the sole source of mathematical knowledge while employing scaffolding in the classrooms. In line with Casem, Yusepa [10] recommends that the provision of assistance in the learning process should be made in stages (scaffolding).

The next part, students, write back about the material that has been studied with their language following the student worksheet guide. This is done at the articulation stage. For example, students rewrite gradient definition and rewrite how to solve linear equations of two variables. Not only write, but students also convey directly at the presentation in front of the class. Student's activity in comparing the results of work with other groups is also done. So at this stage students can express their difficult experiences and also how to overcome the obstacles. Students exchange information that is facilitated by the teacher. In CAI this is done at reflection stage. Alternately in the groups, students present the results of their work along with the findings obtained.

To strengthen students' understanding of the material that has been studied, at the exploration stage students make a complete problem with the answer. Of course, this also supports the student's representation ability on composing stories or written text.

4. Conclusion
Based on the results and discussion of this study, it can be concluded that the enhancement of mathematical representation ability of students who obtained CAI learning is better than the students who obtained PKV regarding 1) overall, 2) school level and 3) mathematical prior knowledge. The enhancement of the mathematical representation of students who obtained CAI in high MPK category is in high qualification, while the enhancement of students' mathematical representation ability in the medium and low category is in medium qualification.

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