Towards a viable grand unified model with $M_G \sim M_{\text{string}}$ and $M_I \sim 10^{12}$ GeV

N. G. Deshpande †, B. Dutta†, and E. Keith ∗

† Institute of Theoretical Science, University of Oregon, Eugene, OR 97403
∗ Department of Physics, University of California, Riverside, CA 92521

(April, 1996)

Abstract

We present a model based on the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C$ with gauge couplings that are found to be unified at a scale $M_G$ near the string unification scale. The model breaks to the minimal supersymmetric standard model at a scale $M_I \sim 10^{12}$ GeV, which is instrumental in producing a neutrino in a mass range that can serve as hot dark matter and this scale can also solve the strong CP problem via the Peccei-Quinn (PQ) mechanism with an invisible harmless axion. We show how this model can accommodate low and high values of $\tan \beta$ and “exotic” representations that often occur in string derived models. We show that this model has lepton flavor violation which can lead to processes which are one or two orders of magnitude below the current experimental limits.
The conventional scale of supersymmetric grand unification is taken to be $M_G \sim 2 \cdot 10^{16}$ GeV, because this is where the MSSM gauge couplings are found to converge if one assumes a “dessert” between about 1 TeV and and that scale. However, in superstring theory the unification point is not a free parameter but is predicted to be a function of the gauge coupling at that scale in the $\overline{\text{MS}}$ scheme as follows [1]:

$$M_{\text{string}} \approx 7g_{\text{string}} \cdot 10^{17} \text{ GeV},$$

(1)

which predicts $M_{\text{string}}$ to be approximately 25 times greater than the conventional value of $M_G$. Stringy threshold effects have not yet proven to be at all useful in closing the gap between $M_G$ and $M_{\text{string}}$ in any realistic string model [2], and neither have weak to TeV scale MSSM thresholds. At present, it is not clear if string grand unified theories (GUTs) or models that have non-standard Kac-moody levels and hence non-conventional hypercharge normalizations may one day be able to rectify the situation [3]. However, it has been shown that extra non-MSSM matter that appears in some realistic string models can lead to a successful raising of the unification scale [2]. One obvious and attractive approach to adding extra matter, “populating the dessert,” would be to add an intermediate gauge symmetry. Such realistic string models have been built for cases where the intermediate symmetry is the flipped $\text{SU}(5) \times U(1)$ [4] or $\text{SO}(4) \times \text{SO}(6) \sim \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_C$ [5–7], and sometimes the field content of these models have been found to alleviate the discrepancy between the string and gauge unification scales [6–8]. In this letter, we shall investigate what field content and additional constraints may be required to have a model with gauge unification at the string scale, that has $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_C$ breaking to the MSSM at an intermediate scale of $\sim 10^{12}$ GeV, that allows a PQ symmetry also broken $\sim 10^{12}$ GeV with a weakly coupled axion, and where the scale of gauge symmetry breaking and the PQ symmetry breaking are determined by a single parameter in the superpotential. We will also discuss the constraint on the field content necessary to obtain, if one desires, a low value for $\tan \beta$.

An intermediate $\text{SU}(2)_R \times \text{SU}(4)_C$ breaking scale of order $10^{12}$ GeV is very attractive for two reasons: (1) if the B-L gauge symmetry is broken at around $10^{11}$-$10^{12}$ GeV, one
can easily get a neutrino mass in the interesting range of of about 3-10 eV, making it a candidate for the hot dark matter [9], and (2) if the strong CP problem is solved via the Peccei-Quinn (PQ) mechanism, this PQ symmetry is required to be broken approximately within the above window so that the axion has properties which are consistent with the lack of observation up to now and the cosmological constraints [10]. As for the scale at which the hypothetical PQ-symmetry is broken, perhaps the most elegant possibility is if it is tied in with the breaking of an intermediate gauge symmetry, so that there is only one scale between the weak and string scale to be explained. To obtain the \( \tau \)-neutrino mass in the interesting eV range without an intermediate gauge symmetry breaking scale one has to use a method that either involves a carefully chosen Yukawa coupling to an \( SU(2)_R \) triplet, which only arises for particular non-standard Kac-Moody levels, or non-renormalizable operators with \( SU(2)_R \) doublets [11]. Unlike in the case of an intermediate scale, both these methods require abandoning the attractice \( b - \tau \) unification hypothesis except in the case of the \( SU(2)_R \) doublets and high \( \tan \beta \sim m_t/m_b \) which requires greater tuning of the Higgs potential parameters and may also require \( M_G \) scale D-terms.

How the one loop MSSM beta functions have to be changed at an intermediate scale to increase the scale of gauge unification has been studied in a recent paper [12]. It is found that there exist only a few acceptable solutions with a single cleanly defined intermediate scale far below the unification scale. In fact if we demand an intermediate scale as mentioned in the above paragraph, the necessary relative changes in the beta functions of the MSSM are given as follows:

\[
\Delta b_2 - \Delta b_1 = 2, \quad \Delta b_3 - \Delta b_2 = 1, \quad (2)
\]

where the hypercharge has been normalized in the standard GUT manner and 

\[ b_i = -2\pi \partial \alpha_i^{-1} / \partial \ln \mu. \]

In this paper the additional field content we choose at the scale \( M_I \) is as follows: the additional vector representation fields necessary to complete the \( SU(2)_L \times SU(2)_R \times SU(4)_C \) symmetry, 2 copies of the chiral fields \( H = (1, 2, 4) \equiv (\bar{u}_H^c, \bar{d}_H^c, \bar{E}_H^c, \bar{N}_H^c) \) and \( \bar{H} = (1, 2, 4) \equiv \)
\( (u'_H, d'_H, E'_H, N^c_H) \), and chiral singlets \( S = (1, 1, 1) \) which are necessary for the right-handed neutrinos \( N^c_i \) to acquire large Majorana masses. We also add a chiral field \( D = (1, 1, 6) \) to make all the non-MSSM Higgs modes massive along with two copies of chiral fields \( \Phi = (2, 2, 1) \), which contain the MSSM Higgs. There are of course the usual 3 MSSM matter generations that include right-handed neutrinos \( F = (2, 1, 4) \equiv (u, d, \nu, e) \) and \( \bar{F} = (1, 2, \bar{4}) \equiv (u^c, d^c, N^c, e^c) \). The SU(2)_R\times SU(4)_C gauge symmetry is broken to the U(1)\_Y\times SU(3)_c by \( \langle H \rangle = \langle \bar{H} \rangle = \langle N^c_H \rangle \sim M_I \). This causes 9 of the 21 gauge fields to become massive. The fields in \( H \) and \( \bar{H} \) which combine with the corresponding components of the gauge fields are \( u^c_H, \bar{u}^c_H, E^c_H, \bar{E}^c_H \), and a linear combination of \( N^c_H \) and \( \bar{N}^c_H \) orthogonal to that of hypercharge to make super gauge-Higgs multiplets with a common mass of the order of \( M_I \). In such a case, it is easy to see that any number of copies of \( H, \bar{H} \) that might exist and get VEVs of order \( M_I \) would not leave any massless modes except for the ones corresponding to the hypercharge generator since all of the Higgsinos corresponding to these modes acquire mass through gaugino-Higgsino-\( <\text{Higgs}> \) terms. The linear combination of \( N^c_H \) and \( \bar{N}^c_H \) corresponding to the hypercharge generator gets mass of the order of \( M_I \) through the terms \( \lambda_{H\bar{H}S}H\bar{H}S \). Note that we do not add terms like \( M_H\bar{H}H\bar{H} \) in the superpotential which cause the breaking of SUSY via F-terms. The presence of R symmetry [13] and the non-existence of a VEV for \( S \) could, for example, forbid these terms.

The chiral fields \( d_H \) and \( \bar{d}^c_H \) in these representations become massive with the help of the field \( D = (1, 1, 6) \equiv (d^c_D, \bar{d}^c_D) \). This causes \( d_H, \bar{d}^c_H, d^c_D, \) and \( \bar{d}^c_D \) to all get mass of order \( M_I \) through the superpotential terms \( \lambda_{HHD}HHD \) and \( \lambda_{H\bar{H}D}\bar{H}\bar{H}D \) when \( H \) and \( \bar{H} \) get VEVs. Note that to avoid rapid proton decay, we also need to impose a symmetry on the superpotential (for example PQ symmetry as discussed later) that forbids terms of the type \( FF\bar{D} \) and \( \bar{F}F\bar{D} \) unless the couplings are extremely small.

The existence of the field \( D \) and \( S \) are crucial to make all the Higgs modes massive. As a matter of fact, in a previous paper Ref. [6] the field content without \( D \) and one of the bidoublets have been used to raise the unification scale. This field content also satisfies Eq.(2). (Note that in SO(10): \( 10 \rightarrow (1, 1, 6) + (2, 2, 1) \).) However the choice of this minimal
field content is problematic in practice as we have already pointed out that all the non-MSSM Higgs modes do not become massive at the breaking scale $M_I$, and hence the gauge coupling renormalization group equations (RGEs) are modified beneath the scale $M_I$. This model also suggests complete third generation Yukawa coupling unification at the intermediate scale, and hence requires large $\tan\beta$, due to the existence of only one bidoublet Higgs. Instead of the second bidoublet, we could have 2 copies of fields transforming as $(2, 1, 1) + (1, 2, 1)$. As a matter of fact, in string derivations of SU(2)$_L \times$SU(2)$_R \times$SU(4)$_C$ models the exotic representations $(2, 1, 1), (1, 2, 1)$, and $(1, 1, 4) + (1, 1, \bar{4})$ tend to occur. Since the field content without D and S satisfy Eq. (2), the constraints on the additional field content is given by:

$$n_D \geq 1 , \quad n_D + n_4 = (n_\Phi - 1) + \frac{1}{2} n_2 ,$$

where $n_D$ is the number of copies of fields transforming as $(1, 1, 6)$, $n_\Phi$ is the number of fields transforming as $(2, 2, 1)$, $n_4$ is the number of copies of $(1, 1, 4) + (1, 1, \bar{4})$, and $n_2$ is the number of copies of $(2, 1, 1) + (1, 2, 1)$, which are all to be given mass of order $M_I$. Of course, Eq. (2) gives no constraint on $n_S$, the number of copies of singlet $(1, 1, 1)$ fields. It may be of interest to note that, for example, the first string derived version of the model found in Ref. [4] has $n_D = 4, n_4 = 1, n_\Phi = 4, n_2 = 10$, along with the necessary 2 copies of $H + \bar{H}$ ($N_H = 2$) at the string scale and 3 generations of $F + \bar{F}$ ($N_F = 3$), as well as several SU(2)$_L \times$SU(2)$_R \times$SU(4)$_C$ singlets. Some of these SU(2)$_L \times$SU(2)$_R \times$SU(4)$_C$ may acquire VEVs near the string scale which can break additional U(1) symmetries and may make some fields super heavy.

We now discuss how a low $\tan\beta$ scenario may be implemented in the model. To allow for the possibility of low $\tan\beta$, one needs the MSSM Higgs doublets $\phi_u$ and $\phi_d$ to not come primarily from the same bidoublet $\Phi_i$. If we assume two bidoublets $\Phi_1$ and $\Phi_2$, no difficulty would exist in the breaking of the SU(2)$_L \times$SU(2)$_R \times$SU(4)$_C$ model to the MSSM if one bidoublet was to remain light at $M_I$ and the other was to be given mass of order $M_I$. However here we want one linear combination, from the two bidoublets, of down (or up) type Higgs superfield SU(2)$_L$ doublets to remain massless and the other combination to
have mass of order $M_I$. (Otherwise, we would spoil the gauge coupling RGE analysis, which is the primary motivation for this model.) This can be accomplished through a modification of a method that has been used in conventional SO(10) GUTs. Consider adding to the model a pair of fields $H_L$ and $\bar{H}_L$ transforming as $(2, 1, 4)$ and $(2, 1, \bar{4})$, respectively, and also increasing the number of $H, H$ pairs to be $N_H = 3$ so that Eqn.(2) is still satisfied. For simplicity, we discuss the general case and will not refer to specific choices of any PQ charges for the additional field content of this paragraph. If explicit mass terms for these fields are originally forbidden, they can be generated at the intermediate scale to be of order $M_I$ through terms such as $\lambda H_L \bar{H}_L S_i$ where $S_i$ gets a VEV at the scale $M_I$ in the manner as previously discussed. In the limit of neglecting weak scale masses, the existence of the superpotential terms $W_D = \lambda_1 \bar{H} H_L \phi_1 + \lambda_2 H \bar{H} L \phi_2$ and a symmetry (for example, the superpotential can be assumed to be invariant under the transformation: $H_L = -H_L$, $\phi_1 = -\phi_1$ and $S_i = -S_i$) forbidding the terms $\lambda_1 \bar{H} H_L \phi_1 + \lambda_2 H \bar{H} L \phi_2$ would lead to the following form for the mass matrix for the SU(2)$_L$ doublets:

$$M_D = \begin{pmatrix}
\lambda_{H_L} v_{S_i} & \lambda_1 v_H & 0 \\
\lambda_2 v_{\bar{H}} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},$$

written in a basis where the rows stand for $(\bar{H}_{L\phi_u}, \phi_{1u}, \phi_{2u})$ and the columns for $(H_{L\phi_d}, \phi_{1d}, \phi_{2d})$ in obvious notation and all VEVs are of order $M_I$. This matrix naturally will have two large eigenvalues of order $M_I$, and has one massless eigenvalue which is composed of $\phi_{1d} = \phi_d$ and $\phi_{2u} = \phi_u$ and serves as the MSSM Higgs. Note that had D-parity not been broken in the model, we would not have had to have added any additional field content.

The gauge couplings in this model have the following one-loop beta functions:

$$b_i^{224} = \left( \begin{array}{cc}
-6 & 2 \\
-6 & 2 \\
-12 & 2
\end{array} \right) + n_F \left( \begin{array}{cc}
2 & 1 \\
2 & 1 \\
2 & 1
\end{array} \right) + n_\Phi \left( \begin{array}{cc}
0 & 4 \\
0 & 4 \\
0 & 4
\end{array} \right),$$

and the following two-loop beta functions:
\[
b_{ij}^{224} = \begin{pmatrix}
-24 & 0 & 0 \\
0 & -24 & 0 \\
0 & 0 & -96 \\
\end{pmatrix} + N_F \begin{pmatrix}
14 & 0 & 15 \\
0 & 14 & 15 \\
3 & 3 & 31 \\
\end{pmatrix} + n_\Phi \begin{pmatrix}
7 & 3 & 0 \\
0 & 7 & 0 \\
0 & 0 & 31 \\
\end{pmatrix} + n_D \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 18 \\
0 & 0 & 31 \\
\end{pmatrix}
\]

\[+ N_{H_L} \begin{pmatrix}
0 & 28 & 30 \\
0 & 6 & 31 \\
\end{pmatrix} + N_H \begin{pmatrix}
28 & 0 & 30 \\
6 & 0 & 31 \\
\end{pmatrix},
\]

where \(i = SU(2)_R, SU(2)_L, SU(4)_C\) respectively in the matrices, \(N_{H_L}\) is the number of field pairs (to be used for the low tan \(\beta\) scenario) transforming as \((2, 1, 4) + (2, 1, \bar{4})\), \(N_F = 3\) always, and we have left out the contributions from exotic representations since they can be easily calculated and we will not use them in our examples here. In Table 1, we show some sample gauge coupling unification results for the model in the case of \(n_\Phi = 2, n_D = 1\) and \(N_H = 2\) which requires complete third generation Yukawa coupling unification at the scale \(M_I\) and which we refer to as the high tan \(\beta\) scenario, for different values of \(\alpha_s(M_Z)\) and the effective SUSY scale \(M_S\) with \(\sin^2 \theta_W = 0.2321, \alpha(M_Z) = 1/127.9, M_I = 10^{12}\) GeV, and the top quark mass \(m_t \approx 180\) GeV. For the case that does not require \(\lambda_t = \lambda_b\) at \(M_I\), which we refer to as the low tan \(\beta\) scenario, we use \(n_\Phi = 2, n_D = 1, N_H = 3\) and \(N_{H_L} = 1\). We display \(M_G\), which we compare with the string scale prediction from Eq. (1). We do not include string threshold effects as we do not know the entire field content near the string scale, however we note from Ref. [2] that the effect of these thresholds in this model should in general tend towards having the beneficial effect of further reducing the small discrepancy between \(M_G\) and \(M_{\text{string}}\).

We note that the appearance of the intermediate breaking scale can occur through a single parameter in the singlet sector of the model. For example, consider an R symmetry invariant superpotential

\[
W = \left( \sum_{i,j=1}^{2} \lambda_{ij} H_i \bar{H}_j - r \right) S_0 + ..., \quad (7)
\]

where \(r\) is of order \(M_I\) and \(S_0\) has no VEV. It would then be the most natural case for the VEVs acquired by all four fields to be of similar order. This mechanism can easily
be extended to link the breaking of a PQ-symmetry with the breaking of the intermediate
gauge symmetry. For example, suppose a PQ-symmetry exists with $F, \bar{F}$ having a PQ charge
of 1, bidoublet(s) $\Phi_i$ which contain the MSSM Higgs doublets and has (have) PQ charge
-2, that the fields $H, \bar{H}$ have no or opposite PQ charges, singlets $S_0, S_2, S_{-2}$ exist with the
subscript denoting the PQ charge, and that R-symmetry prevents $M_{2-2}S_2S_{-2}$ mass term
from existing in the superpotential, then the following superpotential is possible:

$$W = \frac{\lambda_{ij}}{M_{Pl}} \Phi_i \Phi_j S_2 S_2 + \left( \lambda_S S_{-2} S_2 + \sum_{i,j=1}^{2} \lambda_{ij} H_i \bar{H}_j - r \right) S_0 + ...$$

(8)

where we have allowed a non-renormalizable term $[15]$ so as to not need to fine-tune the
$\mu$ parameter to a very large value or $\lambda_{ij}$ to a tiny value. We observe that we can choose the
PQ charge of $D$ to be 0 for example so as to forbid terms like $F F D$ and $\bar{F} D$
that would cause the rapid proton decay.

We note that in the model we are discussing there are no SU(2)$_R$ triplet fields, therefore
one must rely on an extended version of the seesaw mechanism $[16]$. This mechanism in this
scenario is described by the following $3 \times 3$ mass matrix for $\nu_a, N^c_a$, and singlets $S_a$ with
$a = 1, 2, 3$:

$$M_\nu = \begin{pmatrix}
0 & \lambda_{(u)} v_u & 0 \\
\lambda_{(u)}^T v_u & 0 & f v_I \\
0 & f^T v_I & M
\end{pmatrix}$$

(9)

where $\lambda_{(u)}, f,$ and $M$ are $3 \times 3$ matrices, $v_I$ is a VEV of order $M_I$, and $v_u$ is the electroweak
breaking VEV of the MSSM Higgs doublet $H_u$. Ignoring intergenerational mixing by pre-
tending $\lambda_{(u)}, f, \text{ and } M$ are diagonal, then the mass of the $a$th light Majorana neutrino is
given by $m_{\nu_a} \sim \left( \lambda_{(u)} v_u \right)^2 M_a/f_a^2 v_I^2$. To get back the usual seesaw relation and have $m_{\nu_e}$ be
in the eV range, we need $M_a \sim M_I$. This can be done by adding to the field content of
the previous paragraph three generations of singlets transforming as $S_{-1}$ and at least one $S_1$, assuming once again from R-symmetry that the explicit mass terms $M_{S_1, S_{-1}, S_1 S_{-1}}$ are
forbidden in the SU(2)$_L \times$SU(2)$_R \times$SU(4)$_C$ superpotential, and giving $H_i, \bar{H}_j$ PQ charges of
0. All this allows the terms $\lambda_{ij} \bar{F}_i H_{-1} S_{-1}$, $\lambda_{-1, -1, 2} S_{-1} S_{-1} S_2$, and $\lambda_{-1, 1, 0} S_{-1} S_1 S_0$ to be in the
superpotential, which at the intermediate scale would give $S_{-1}$ both a mass and a VEV of order $M_I$ and give the desired size to the entrees in the seesaw mixing matrix.

Lastly, we consider a signal of the model. Recently, GUT scale physics have been shown to be significant sources of lepton flavor violation in SUSY grand unification with a universal SUSY soft breaking boundary condition appearing near the reduced Planck scale \[17\] \[19\]. However the parameter space where these signals could be observed is somewhat constrained by the experimental constraints of $b \rightarrow s\gamma$ \[20\]. More recently, it has also been shown that even if the universal boundary condition is taken at the GUT scale, that an intermediate gauge symmetry breaking can also be a significant source of lepton violation \[22\]. We now show that the rate of $\mu \rightarrow e\gamma$ can be within two orders of magnitude of experiment in the model discussed in this letter. For some parameter space, it can be above the experimental limit. In $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_C$ gauge symmetry, the quarks and leptons are unified. Hence, the $\tau$-neutrino Yukawa coupling is the same as the top Yukawa coupling. Through the RGE’s, the effect of the large $\tau$-neutrino Yukawa coupling is to make the third generation sleptons lighter than the first two generations, thus mitigating the GIM cancellation in one-loop leptonic flavor changing processes which involve virtual sleptons.

The superpotential terms which will be responsible for giving the SM fermion masses have the following form :

$$W_Y = \lambda_{F_u} F \Phi_2 \bar{F} + \lambda_{F_d} F \Phi_1 \bar{F}, \quad (10)$$

where all group and generation indices have been suppressed. $\Phi_1$ and $\Phi_2$ are the two bidoublets. We have assumed that $\Phi_2$ contains the MSSM Higgs doublet which gives masses to the up quarks and Dirac masses for the neutrinos and $\Phi_1$ contains the doublet which gives masses to the down quarks and the charged leptons. Now we give the RGEs for the soft SUSY breaking parameters which we need for the intermediate gauge symmetry. First of all, there are gaugino masses $M_i$ corresponding to each $g_i$. Secondly, corresponding to each tri-linear superpotential coupling $\lambda_i$ there is a tri-linear scalar term with the coupling $A_i \lambda_i$ at $M_G$. Finally there are soft scalar mass terms $m_i^2$ for each of the the fields $F_{L,R}$, and $\Phi_{1,2}$. 


\[ \mathcal{D} \lambda^2_{Fug} = - \sum_i c_i^{(\lambda F)} g_i^2 + (4 + 4 \delta_{g3}) \lambda^2_{F1}, \]  
(11)

\[ \mathcal{D} \lambda^2_{Fdg} = - \sum_i c_i^{(\lambda F)} g_i^2 + 4 \delta_{g3} \lambda^2_{F1}, \]  
(12)

\[ \mathcal{D} M_i = b_i g_i^2 M_i, \]  
(13)

\[ \mathcal{D} A_{Fug} = \sum_i c_i^{(\lambda F)} g_i^2 M_i + (4 + 4 \delta_{g3}) \lambda^2_{F1} A_{F1}, \]  
(14)

\[ \mathcal{D} A_{Fdg} = \sum_i c_i^{(\lambda F)} g_i^2 M_i + 4 \delta_{g3} \lambda^2_{F1} A_{F1}, \]  
(15)

\[ \mathcal{D} m^2_{F,\bar{F}} = - \sum_i c_i^{(F,\bar{F})} g_i^2 M_i^2 + 2 \lambda^2_{F1} X \delta_{g3}, \]  
(16)

\[ \mathcal{D} m^2_{\Phi_1} = - \sum_i c_i^{(\Phi)} g_i^2 M_i^2, \]  
(17)

\[ \mathcal{D} m^2_{\Phi_2} = - \sum_i c_i^{(\Phi)} g_i^2 M_i^2 + 4 \lambda^2_{F1} X, \]  
(18)

where \( g \) refers to generation and \( i \) refers to the gauge,

\[ c^{(\lambda F)} = \left(3, 0, \frac{15}{2}\right), \quad c^{(F)} = \left(3, 0, \frac{15}{2}\right), \quad c^{(\bar{F})} = \left(0, 3, \frac{15}{2}\right), \quad c^{(\Phi)} = (3, 3, 0), \]

\[ X \equiv m^2_F + m^2_{\bar{F}} + M^2_{\Phi_1} + A^2_{\Phi_1}, \]

and we have used

\[ \mathcal{D} \equiv \frac{16 \pi^2}{2} \frac{d}{dt}, \]

where \( t = \ln (\mu/\text{GeV}) \) with \( \mu \) being the scale. At the scale \( M_G \), we assume a universal form to the soft SUSY breaking parameters i.e. all gaugino masses \( M_i(M_G) = m_{1/2} \), all tri-linear scalar couplings \( A_i(M_G) = A_0 \), and all soft scalar masses \( m^2_i(M_G) = m_{0}^2 \). At the scale \( M_I \), we match the intermediate gauge symmetry breaking effective theory parameters with the MSSM parameters in the usual fashion. We run all the RGEs according to the MSSM [21] down to the top scale. Details of \( \mu \rightarrow e\gamma \) with an intermediate symmetry are presented in Ref. [22]. As an example, we show the specific case of the universal gaugino masses \( m_{1/2} = 145 \text{ GeV} \) and universal tri-linear soft breaking parameter \( A_0 = 0 \) at the unification scale, \( \alpha_s(M_Z) = 0.119 \), \( M_G = 10^{17.87} \text{ GeV} \), \( \alpha_G = 1/12.4 \), \( m_t = 176 \text{ GeV} \) and \( m_b = 4.35 \text{ GeV} \), and with minimal field content for low \( \tan \beta = 2 \) in Fig. 1. We have plotted the function

\[ l_r \equiv \log_{10} \left( \frac{B}{B_{\text{exp}}} \right), \]  
(19)
where $B$ is the predicted $\mu \to e\gamma$ branching ratio and $B_{\text{exp}} = 4.9 \cdot 10^{-11}$ being the experimental 90% confidence limit upper bound on the branching ratio. With $A_0 = 0$, $A_i$ is always negative at the weak scale. Consequently we find that $\mu < 0$ gives a greater branching ratio.

In summary, we have discussed a model that allows gauge coupling unification in the vicinity if the string scale and can have an intermediate $\SU(2)_L \times \SU(2)_R \times \SU(4)_C$ gauge symmetry breaking scale $M_I$ of order $10^{12}$ GeV, which is useful for producing a $\tau$-neutrino with a mass of a few eV and solving the strong CP problem via a PQ symmetry whose breaking produces a harmless axion. We have shown that a range of field content is allowed by the model as given by Eq. (3) to satisfy Eqn. (2) which predicts $M_G/M_I \approx 10^{6 \pm 2}$ GeV, and we have shown that Eqn. (1) which gives the string prediction between unification scale mass and gauge couplings are approximately satisfied for some choices of field content. We have also discussed what $\SU(2)_L \times \SU(2)_R \times \SU(4)_C$ singlets and global U(1) charges may be useful to take advantage of $M_I \sim 10^{12}$ GeV and make a single parameter in the singlet sector responsible for the scale $M_I$.

Recently, an intermediate scale at $\sim 10^{12}$ GeV has been advocated to produce monopoles that would explain the high energy cosmic ray spectrum. Since our model factors into U(1) group at $M_I$, it would be natural for such monopoles to arise in this model. It may be of interest to see if such monopoles satisfy the requirements of relic abundance required for the suggested mechanism.

We are grateful to K. S. Babu and E. Ma for invaluable comments and discussions. This work was supported by Department of Energy grants DE-FG06-85ER 40224 and DE-FG02-94ER 40837.
REFERENCES

[1] V. A. Kaplunovsky, Nucl. Phys B307 145, (1988); ERRATUM-ibid.B382 436, (1992).

[2] K. R. Dienes and A. E. Faraggi, Phys. Rev. Lett. 75 2646, (1995); K. R. Dienes and A. E. Faraggi, Nucl. Phys. B457 409, (1995); K. R. Dienes, hep-th/9602043;

[3] I. Antoniadis, J. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B194 231, (1987); ibid. B205 459, (1988), ibid. B208 209, (1988).

[4] I. Antoniadis and G.K. Leontaris, Phys. Lett. B216 333, (1989).

[5] I. Antoniadis, G.K. Leontaris and J. Rizos, Phys. Lett. B245 161, (1990).

[6] A. Murayama and A. Toon, Phys. Lett. B318 298, (1993).

[7] G.K. Leontaris and N. D. Tracas, hep-ph/9511280.

[8] S.F. King, Phys. Lett. B325 129, (1994).

[9] R. Shafer and Q. Shafi, Nature (London) 359, 199 (1992).

[10] M. Dine, W. Fischler and M. Srednicki, Physics lett. B104 199 (1981).

[11] F. Vissani and A. Y. Smirnov, Physics Lett. B341 173 (1994); A. Brignole, H. Murayama, and R. Rattazzi, Physics Lett. B335 345 (1994); D-G. Lee and R. N. Mohapatra, Phys. Rev. D52 4125, (1995).

[12] S. P. Martin and P. Ramond, Phys. Rev. D51 6515, (1995).

[13] J. Wess and J. Bagger, Supersymmetry and Supergravity, (Princeton university Press, 1983).

[14] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 74 2418, (1995).

[15] E. J. Chun, Phys. Lett. B348 111, (1995).

[16] R. N. Mohapatra, Phys. Rev. Lett. 56 561, (1986), R. N. Mohapatra and J. W. F. Valle,
Phys. Rev D34 1642, (1986), D-G Lee, R. N. Mohapatra, Phys. Rev. D52 4125, (1995).

[17] R. Barbieri and L. J. Hall, Phys. Lett. B338 212, (1994).

[18] R. Barbieri, L. J. Hall, and A. Strumia, Nucl. Phys B445 219, (1995).

[19] P. Ciafaloni, A. Romanino, and A. Strumia, IFUP-TH-42-95.

[20] T. V. Duong, B. Dutta and E. Keith, OITS-591, UCRHEP-T154, hep-ph 9510441 (to appear in Phys. Lett. B).

[21] For example, see: V. Barger, M. Berger, and P. Ohmann, Phys. Rev. D47 1093, (1993); V. Barger, M. Berger, and P. Ohmann, Phys. Rev. D49 4908, (1994).

[22] N. G. Deshpande, B. Dutta and E. Keith, hep-ph/9512398.

[23] T. W. Kephart and T. J. Weiler, astro-ph/9505134.
Table captions

Table 1: The gauge unification scale $M_G$ and string scale $M_{\text{string}}$ (as predicted in Eqn. (1)) are shown for different values of $\alpha_s(M_z)$ and effective SUSY scale $M_S$ in both the low and the high tan $\beta$ models described in the text.

Figure captions

Fig. 1: $l_r \equiv \log_{10}(B/B_{\exp})$ is plotted as a function of of the universal at $M_G$ scale soft mass $m_0$.

The solid line corresponds to $\mu > 0$, while the dashed line corresponds to $\mu < 0$.

$\lambda_{F_{4G}} = 1.25$ and $m_{1/2} = 140$ GeV for both lines.
| $\tan \beta$ | $\alpha_s(M_Z)$ | $M_s(GeV)$ | $1/\alpha_G$ | $M_G(GeV)$ | $M_{string}(GeV)$ |
|------------|----------------|------------|--------------|------------|------------------|
| high       | 0.1258         | 175        | 20.34        | $10^{18.26}$ | $10^{17.74}$     |
| high       | 0.1187         | $10^3$     | 22.10        | $10^{17.92}$ | $10^{17.72}$     |
| low        | 0.1192         | 175        | 10.65        | $10^{17.83}$ | $10^{17.84}$     |
| low        | 0.1144         | $10^3$     | 12.85        | $10^{18.05}$ | $10^{17.88}$     |
Fig. 1