Promoting Flexibility in the Mixed-model Assembly Line by Introducing Buffer Stocks

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Abstract: The purpose of this paper is to clarify the increase in flexibility by introducing buffer stocks that enable model-sequence changes in a mixed-model assembly line. It is shown that changing the model sequences between stations can shorten the line length for a mixed-model assembly. Numerical analysis indicates that the range of possible operation time combinations of models can be increased and the model mix can be made more flexible by introducing buffer stocks under the same line length. This phenomenon is also discussed from the viewpoint of the relationship between buffers and flexibility. A procedure for examining feasible combinations of operation times is presented.

Key Words: Mixed-model assembly line, Flexibility, Buffer stock, Re-sequencing

1. Introduction

Mixed-model assembly lines are used in many industries. A mixed-model assembly line is defined as a continuous production system, in which multiple models with nearly the same assembly method are produced together in a single line. An advantage of a mixed-model line is that a variety of products can be assembled economically in a single line.

In some cases, it is desirable to minimize the size of the facility, i.e., the length of the line, while maintaining a rate of assembly equal to the demand rate for each model type. It is also desirable to promote flexibility by both increasing the possible operation time combinations of models and changing the rate of assembly for each model.

To satisfy these requirements, manufacturers divide a line into shorter lines by installing buffer stocks to enable them to change model sequences. For example, at Toyota engine lines, a typical assembly line for one conveyor is divided into short lines. These short lines are called self-completion lines. Another example is the addition of the buffer between the paint and the assembly shops in automobile industries. Xu and Hiraki [1] proposed a design method for the self-completion line to balance work-in-process and workload. However, the effect of buffer stocks on the division of assembly lines for re-sequencing models has not yet been studied sufficiently.

In this context, the present paper aims to answer the following three questions regarding the addition of buffer stocks between stations in the mixed-model assembly line to enable manufacturers to change model sequences:

(1) Can the length of the mixed-model assembly line be shortened? If yes, what is the condition? How much can it be shortened?

(2) Can the range of possible operation time combinations of models be increased? If yes, what is the condition? How much can it be increased?

(3) Can the rate of assembly for each model be made more flexible? If yes, what is the condition?

The American Production & Inventory Control Society (APICS) [2] explains the term “flexibility” as the ability of a manufacturing system to respond quickly to external or internal changes in terms of range and time. In this paper, flexibility means the possible varieties of total operation times between models assigned to a station and the feasible range of model mixes, whereas efficiency means the line length or the size of the working area allowances of assembly lines. Buffer stock is the work-in-process planned between assembly stations in order to alter model sequences between preceding and concerned stages.

In the next section, we briefly review the existing literature. In Section 3, an analytical model is introduced. Section 4 shows that the length of the mixed-model assembly line can be shortened by installing buffer stocks, based on the relationships between the operation times for models assigned to a certain station and the required maximum working area allowance. The effect of buffer stocks on the range of possible combinations of operation times and model-mix changes is presented in Sections 5 and 6. Section 7 discusses the results from the viewpoint of the relationship between buffers and flexibility. In Section 8, the conclusions are summarized.

2. Literature Review

Many papers on mixed-model assembly lines have been reported. Important design considerations include line balancing, model sequences, and model mix. One of the main design objectives is the line length. Here we review papers that focus...
on minimizing the line length from the standpoint of model sequences.

Boysen et al. [3] reviewed and discussed three major planning approaches presented in the mixed-model assembly line literature: mixed-model assembly sequencing, car sequencing and level scheduling.

The problem of line length minimization has been treated as mixed-model assembly sequencing. Dar-El and Cother [4] described a model-sequencing algorithm and developed a lower bound for the overall line length. Dar-El and Cucuy [5] obtained an optimal sequence with minimization of the overall assembly line length for a balanced assembly line. Dar-El [6] also suggested a design strategy that can be followed by designers of mixed-model assembly lines to satisfy one or both of the following objectives: minimizing the overall line length and/or minimizing the throughput time. Bard et al. [7] presented a common mathematical framework for each possible variant of important design elements. Furthermore, Bard et al. [8] presented a genetic formulation of the problem to examine the trade-offs between line length and parts usage.

As for buffers in assembly lines, Scholl [9] noted three functions: to decouple stations, to reduce sequence-dependent inefficiencies through re-sequencing of work-pieces, and to extract defective parts. The present paper concentrates on the mixed-model assembly sequencing problem with buffers to reduce sequence-dependent inefficiencies, such as larger line length.

The decoupling problem in the mixed-model assembly line has been studied as the car-sequencing approach. Boysen et al. [10] surveyed papers on these functions from the standpoint of re-sequencing. As can be seen, Boysen et al. published many results on buffers for reducing sequence-dependent inefficiencies through re-sequencing of work-pieces. The re-sequencing problem is formulated as the minimal violation of a set of rules for sequencing. Boysen et al. [11] also discussed the re-sequencing problem of pull-off tables. They [12] also studied the problem of selectivity banks. The buffer between the paint and assembly shops is a well-known specific problem for reducing sequence-dependent inefficiencies. Other authors [13], [14], [15] have addressed problems in the car-sequencing approach.

The car-sequence approach does not consider variations of operation times for each model in a station, although operation times are critical for determining the line length. The car-sequencing approach is insufficient for the present study.

Miltonburg [16] and Decker [17] dealt with the planning of buffers within a mixed-model line. The positioning of buffers is examined by an exact method and three heuristics using the criterion of balanced capacity usage. For the criterion of line length, Dar-El [4] solved an object function. However, re-sequencing was not discussed. None of these papers discuss the line length with consideration of the buffer stocks between stations to enable manufacturers to change model sequences.

Furthermore, very few papers analyze the relationships among possible combinations of operation times between models, model mixes, buffer stocks and line length in mixed-model line production systems. In other words, this is a new aspect that is hardly discussed in literature.

3. Formulation

3.1 Design procedure for mixed-model

We briefly summarize the design procedure for a mixed-model assembly line in [18] and [19]. The procedure, based on the procedures, is as follows:

Step 1. Determining the cycle time. The cycle time is the output interval for models. The cycle time is calculated as the total operating time for each model during a planning period divided by the total quantities of assembly for each model.

Step 2. Determining the minimum number of work stations. This is calculated as the total operation times for each model during a planning period divided by the product of the cycle time and total quantities for the assembly of each model. The minimum number of work stations is not always realized when line balancing is incomplete.

Step 3. Preparing the precedence diagram. This presents the technical precedence relationship between work elements.

Step 4. Line balancing. Work elements are allotted to the stations in accordance with the precedence diagram. The purpose of the allotment is to minimize the number of stations.

Step 5. Determining the model sequences under some criterion.

Step 6. Determining the required working area allowance in accordance with the combinations of operation times between models and the model sequence, in order to buffer the difference between operation times between models in a station.

3.2 Analytical model

Figure 1 demonstrates the relationship between operation times, core working zone, and working area allowance. The following equations hold for the line length, the cycle time, and the allowances.

\[ L(i) = C_i + K(i) \]
\[ K(i) = W(i) + D(i) \]
\[ K = \sum_{i=1}^{N} K(i) \]

where

- \( L(i) \): Line Length of station \( i \),
- \( C_i \): Cycle time (core working zone),
- \( K(i) \): Working area allowance in station \( i \),
- \( D(i) \): Maximum work deficiency in station \( i \),
- \( K \): Total working area allowance,
- \( N \): The number of stations.

In building the system models, the following basic assumptions are made:

1. Moving operations are assumed. Models are assembled on a conveyor line. The movement speed has a constant value of 1.

2. Complete line balancing is assumed. Hence, the minimum number of work stations is achieved. In addition, sufficient working area allowances are provided. Both the idle times and the utility times defined by Thomopoulos [20] do not occur. Thus, all stations are closed.

3. The cyclic model sequence is assumed.
(4) Operation times are static. They do not vary stochastically.

(5) The capacity of buffer stocks is not limited. The required space for buffer stocks is sufficiently small compared to the total line length.

(6) After changes in operation time allotment, the cycle time is not varied. Rebalancing is conducted and completed.

(7) After changes to the model mix, the cycle time is not varied. The cycle time does not have to be changed, since the assembly time for each module is assumed to be the same. The total quantity of assembly for each model is not altered. The quantity for assembling each model is adjusted according to the altered product mix. Rebalancing is conducted and completed.

From assumption (1), the difference in total operation times between models assigned to a station is absorbed through the working area allowance given to the station. Assumptions (2), (3), and (4) assure that the worker completes the final operation of a cycle for a model sequence at the end of the core working zone, or the cycle time (Figure 1). Assumption (5) is not necessarily strong. The required size of buffer stocks depends on the number of models. If the number of models is small, only a small capacity is required. For example, in the case of two models, the smallest pull-off table is sufficient. Figure 2 shows a pull-off table. By pulling off a model to the table and then pushing it on the line, a new model sequence is achieved.

From assumptions (5), (6), and (7), the working area allowance is evaluated as the main criterion in this study.

Thus, the problem is reduced to that of the mixed-model assembly line composed of two stations. Figure 3 shows the two systems considered in this paper. The system shown in Figure 3(a) is a simplified illustration of the assembly line divided into two shorter lines by buffer stocks. The model sequences can be altered at the station of concern ("concerned station") from the preceding station. All cyclic permutations of models can be considered as possible model sequences from assumption (5).

![Fig. 3 Two system models](image)

4. Shortening the line length

The working area allowance depends on the model sequences. The number of model sequences equals the number of circular permutations, since cyclic model sequences are assumed (Assumption (3)). In the case of two models and model-mix ratio 2:1, the number of sequences becomes 2. The two model sequences are AAABB and ABABA. In the circular permutation, AAABB is identical to ABABA, BAAAB, and BBAAB, and ABABA is identical to AABBA, BAAAB, BABAA, and BAAAB. The allowances are simulated as shown in Figure 1. The results are summarized in Table 1.

For example, if the model sequence at the preceding station is AAABB for some reasons, such as setup times, and if the model sequence at the concerned station is altered to ABABA with buffer stocks, then the line length is shortened from 12 to 8 by 4 (33%), for example, when a=14 and b=4.

From the results shown in Table 1, the following knowledge is obtained.

(1) As the difference in assigned operation times between models becomes larger, the difference in the required working area allowance between two model sequences is
Table 1  Possible combinations of operation times and working area allowance for model-mix ratio

| Model and Sequence | Possible Combinations of Operation Times (a and b) | Required Allowance |
|--------------------|---------------------------------------------------|--------------------|
| Model A            | 16 14 12 10 8 6 4 2                              |                    |
| Model B            | 1 4 7 10 13 16 19 22                             |                    |
| Sequence AAABB     | 18 12 6 0 6 12 18 24                             |                    |
| Sequence ABABA     | 12 8 4 0 4 8 12 16                             |                    |

extended. If the assigned operation times for models are equal, then no working area allowance is required.

(2) In other words, when non-optimal model sequences in terms of working area allowance must be considered at a preceding station for some reason, such as setup times, the total working area allowance can be reduced by installing buffer stocks at the concerned station. The reduction is the larger, the operation time difference between models is the larger. However, the rate of line shortening is a constant 33%, except when the operation times are equal to the cycle time.

(3) In this small example case, the results demonstrate that the optimal model sequences in terms of the working area allowance do not depend on the operation times assigned to a station.

For the knowledge (3), generally, the working area allowance depends upon the operation times. Figure 4 shows an example where the optimal model sequence is different for each station. In Figure 4, three models A, B and C are assembled under the cycle time 10, where a, b and c denote operation times for A, B and C allotted to the stations. The model mix is assumed to be 3:2:2. Hence, the following holds: 3a+2b+2c=70. In Figure 4(a), model sequences are optimized with the buffer stock in each station, whereas in Figure 4(b) and 4(c), assembly is conducted in the same model sequence without buffer stocks between the two stations. In Figure 4(a), the total working area allowance of 48 is achieved by installing buffer stocks, whereas it is 54 and 56 with no buffer stocks.

Based on the obtained results, a new procedure for designing a mixed-model assembly line system is presented in Figure 5. That is, two processes and one judgment are added to the traditional design procedure for the mixed-model assembly line, as follows. 1) Once a model sequence is determined in the traditional procedure, the line length, or required working area allowance, is calculated. 2) If the allowance exceeds the given upper limit, it is determined that buffer stocks should be installed between stations in consideration of the model sequence alteration. 3) After positioning the buffer stock, the working area allowance with the altered model sequence is revised and the procedure returns to 1), as shown in Figure 5.

Table 2 shows a numerical example of the design process proposed in this paper. It is assumed that the model sequence AAABB is necessary in stages 5 and 6 for reasons such as setup times. Space limits mean the total allowance must be less than or equal to 70. Without buffer stocks, the model sequence AAABB is necessary throughout all eight stages, and the sequence requires a total allowance of 96. This case does not meet the allowance condition. If the buffer stock is installed between stations 4 and 5, the model sequence AABAB can be used from stages 1 to 4. This leads to an allowance of 84, based on the results shown in Table 2. However, the condition of length 70 is still not met. If further buffer stock is added between stations 6 and 7, the allowance of 70 is achieved, since model sequence AABAB is applied in both stations 7 and 8.
6. Increasing the range of possible operation time combinations of models

In this section, we examine the effect of installing buffer stocks for increasing the range of possible operation time combinations for the models. Increasing the range of possible operation time combinations are required in some cases, such as changes in the operation time allotment, for example, resulting from continuous improvements. Table 3 presents feasible combinations of operation times increased by buffer stocks, when the working area allowance is 12. In Table 3, shaded boxes indicate feasible combinations of operation times when the working area allowance is 12.

From Table 3 it can be seen that the area of possible operation times becomes wider under the same working area allowances by installing buffer stocks and changing the model sequence from AAABB to ABABA. For example, if the model sequence in the preceding station is AAABB, and if the model sequence of the concerned station is altered to ABABA, then the possible combinations of operation times can be increased from 6 < a < 4 (width: 8) to 4 < a < 16 (width: 12), which is a 50% increasing in width.

Figure 6 presents a procedure for examining the feasibility of combinations of operation times when the working area allowance is 12. The results indicate the following. In the case of altered model mix 3:2, if the model sequence in the preceding station is AAABBB or ABABB, then the range of feasible combinations of operation times is twice the maximum by installing the buffer stocks and changing the model sequence. In the case of altered model mix 3:3, if the model sequence in the preceding station is AAAABB or AAABAB, then the range of feasible combinations of operation times is increased by approximately 50% of the maximum in the working area allowance by installing the buffer stocks and changing the model sequence.

In Table 4, shaded boxes indicate feasible combinations of operation times when the working area allowance is 12. The results indicate the following. In the case of altered model mix 3:2, if the model mix is altered to 4:2 (increase of one unit in A) and 3:3 (a decrease of one unit in B). The working area allowance for the concerned station is set as 12 in the standard product mix 3:2. From Table 3, when the working area allowance is 12, operation time a can vary from 6 (b=16) to 14 (b=4) in the model sequence AAABB, and from 4 (b=19) to 16 (b=1) in the model sequence ABABAB.

Table 4 presents the possible combinations of operation times between models A and B by model sequences under the altered model-mix ratios. The relationships 3a + 3b = 60 (model mix 3:3) and 4a + 2b = 60 (model mix 4:2) hold between a and b from the abovementioned conditions. The number of model sequences equals the number of circular permutations, and becomes 3 in both cases of model-mix ratios 3:3 and 4:2.

In Table 4, shaded boxes indicate feasible combinations of operation times when the working area allowance is 12. The results indicate the following: the rate of assembly for each model can be made more flexible under the same line length by installing buffer stocks that alter the model sequence in the preceding station. As the model sequence gets farther from the optimum of the concerned station in line length, or working area allowance, the more the rate of assembly can be made more flexible.

For example, when the concerned station operates with working area allowance 12 before alteration of the model from mix 3:2, if the preceding station operates under model sequence AAABB after alteration of the model mix to 3:3, the combination of operation times a=5 and b=15 (as a result of rebalancing) is infeasible without buffer stock. However, by alternating the model sequence at the concerned station to AABABB or

| Model mix | Model and sequence | Possible combinations of operation times (a&b) |
|-----------|--------------------|-----------------------------------------------|
| 3:2       | Model A            | 2 4 6 8 10 12 14 16                            |
|           | Model B            | 22 19 16 13 10 7 4 1                           |
|           | AAABB              | 24 18 12 6 0 6 12 18                           |
|           | ABABA              | 16 12 8 4 0 4 8 12                           |

Table 2 Example of the proposed design process (▼ indicates inserted buffer stock)

| Station | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------|---|---|---|---|---|---|---|---|
| Model A | 16| 14| 12| 10| 8 | 6 | 4 | 2 |
| Model B | 1 | 4 | 7 | 10| 13| 16| 19| 22|

Table 3 Possible combinations of operation times increased by buffer stocks for model mix 3:2
ABABAB, the combination becomes feasible.

In the same way, when the concerned station operates with working area allowance 12 before alteration of the model from mix 3:2, if the preceding station operates under model sequence AAAABB after alteration of the model mix to 4:2, the combination of operation times a=4 and b=22 (as a result of rebalancing) is infeasible without buffer stock. However, by altering the model sequence at the concerned station to AABAAB, the combination becomes feasible, while the combination still remains infeasible by alternating the model sequence at the concerned station to AAABAB.

7. Discussion: buffer and flexibility

Newman et al.[21] discussed general relationships between buffers and flexibility. They suggested a dynamic equilibrium model that illustrates the trade-offs and interrelationships between manufacturing flexibility and buffers such as inventory, lead time, and capacity. This paper suggests that the flexibility is promoted by setting buffer stocks for changing model sequences in the mixed-model assembly line. Figure 7 demonstrates the schematic relationship between flexibility and buffer stocks in this paper. The figure is based on the seesaw model presented in [21]. The results of this paper provide an example of the trade-offs between buffers and flexibility for a mixed-model assembly line.

8. Conclusions

In this paper, the relationships between operation times for products assigned to a certain stage and the required maximum working area are analysed. Based on the results, this paper answers the following three questions for installing buffer stocks: 1) Can the length of the mixed-model assembly line be shortened? 2) Can the range of possible operation time combinations of models be increased? 3) Can the rate of assembly for each model be made more flexible? The answers obtained for each question are as follows.

For question 1), the line length can be shortened by installing buffer stock for altering the model sequence in the preceding station. As the model sequence is not the optimum of the concerned station in line length, the length can be shortened. A 33% reduction of the required working area allowance is ob-
served in this paper. For question 2), the range of operation time combinations of models can be increased under the same line length by installing buffer stock for altering the model sequence in the preceding station. As the model sequence is not the optimum of the concerned station in line length, the range can be increased. A increased range of possible processing times of 50% in the working allowance is confirmed in this paper. For question 3), the rate of assembly for each model can be made more flexible under the same line length by installing buffer stock for altering the model sequence in the preceding station. As the model sequence becomes farther from the optimum of the concerned station in line length, the more the rate of assembly for each model can be made more flexible.

The above effects are discussed through the buffer and flexibility framework in order to provide insight to the results of this paper.

Further analysis on three or more product models is one of the future issues. In addition, more precise discussion is needed for concrete buffer systems such as pull-off tables.

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