Using Social Network Information in Bayesian Truth Discovery

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Abstract—We investigate the problem of truth discovery based on opinions from multiple agents who may be unreliable or biased. We consider the case where agents' reliabilities or biases are correlated if they belong to the same community, which defines a group of agents with similar opinions regarding a particular event. An agent can belong to different communities for different events, and these communities are unknown a priori. We incorporate knowledge of the agents' social network in our truth discovery framework and develop Laplace variational inference methods to estimate agents' reliabilities, communities, and the event states. We also develop a stochastic variational inference method to scale our model to large social networks. Simulations and experiments on real data suggest that when observations are sparse, our proposed methods perform better than several other inference methods, including majority voting, the popular Bayesian Classifier Combination (BCC) method, and the Community BCC method.

Index Terms—Truth discovery, social network clustering, Laplace variational inference, stochastic variational inference

1 INTRODUCTION

In crowdsourcing and social sensing [1], [2], [3], [4], [5], [6], [7], information about the same event often comes from different agents. Agents may have their own biases and produce unreliable opinions. A commonly used approach to fuse the agents' opinions together is the majority voting method, which assumes that all agents have the same reliability [8]. However, due to different backgrounds and access to prior information, agents' reliabilities or biases may vary widely. Truth discovery methods have been proposed to jointly estimate event truths and agent reliabilities.

In [7], [9], [10], [11], probabilistic models are proposed for truth discovery from binary (true or false) observations. In [9], agents are assumed to be independent of each other. An extended model is proposed in [10] and it assumes the dependency graphs of agents are disjoint trees. In [7], [11], the dependency relationship is extended to general graphs and represented by a known dependency matrix. For binary observations, the reliability of each agent is the probability an event is true given that the agent reports it to be true. For multi-ary observations, [12] uses a confusion matrix to represent the reliability of each agent, and proposes a method called Bayesian Classifier Combination (BCC) for truth discovery. However, if each agent only observes a small subset of events, it is difficult to infer its reliability. In practice, agents having similar background, culture, socioeconomic standings, and other factors, may form communities and share similar confusion matrices. The reference [13] proposed an extension of the BCC model, called the Community BCC (CBCC) model. In this model, the confusion matrix of an agent is a perturbation of the confusion matrix of its community. Both [12] and [13] estimate the confusion matrices of agents based only on the agents' observations. The papers [6] and [14] show that agents in a crowd are not independent, but are instead connected through social ties, which can provide us with important information about which community an agent belongs to.

In this paper, we consider the use of social network information to aid in truth discovery based on agents' observations. Similar to [13], we assume agents are clustered into communities for each observed event (an agent can belong to different communities for different events), and agents in the same community have similar confusion matrices. We use both the agents' observations and the social network connections among agents to jointly infer the communities and the event truths.

Truth discovery on social networks often requires analyzing massive data. However, the traditional Gibbs sampling method used in [12] and variational inference method in [13] cannot scale to a large dataset. The reason is that the entire dataset is used at each iteration of Gibbs sampling and variational inference, thus the computation of each iteration can be expensive when the dataset is large. In our paper, we develop a three-level stochastic variational inference method for our Bayesian network model (see Chapter 3 of [15]) that can scale to large networks. The truths and communities are estimated iteratively. In each iteration, instead of re-analyzing the entire dataset (which includes information about the social network and agents' observations), we use randomly sampled sub-datasets to update the target variables. Our main contributions are as follows:

- We propose a model that uses both social network information and agents' observations to jointly infer agent communities and event truths. To model the relationship between event states and the agents' observations, we use a mixed membership stochastic blockmodel [16] for the community structure and confusion matrices. Our model allows agents to switch communities when
observing different events.
- For small and medium sized networks, we develop a Laplace variational inference method to iteratively estimate the agent communities and event truths.
- We develop a three-level stochastic variational inference method for our Bayesian network model. In Section 4, we develop a three-level stochastic variational inference method that can scale to large networks. Simulation and experiment results are presented in Section 5, and we conclude in Section 6.

Notations: We use boldfaced characters to represent vectors and matrices. Suppose that $A$ is a matrix, then $A(m, \cdot)$, $A(\cdot, m)$, and $A(m, n)$ denote its $m$-th row, $m$-th column, and $(m, n)$-th element, respectively. The vector $(x_1, \ldots, x_N)$ is abbreviated as $(x_i)_{i=1}^N$ or $(x_i)_i$ if the index set that $i$ runs over is clear from the context. We use $\text{Cat}(p_1, \ldots, p_K)$, $\text{Dir}(\alpha_1, \ldots, \alpha_K)$, $\text{Unif}(a, b)$, $\text{Unif}(1, \ldots, R)$, $\text{Be}(g_0, h_0)$ and $\mathcal{N}(M, V)$ to represent the categorical distribution with category probabilities $p_1, \ldots, p_K$, the Dirichlet distribution with concentration parameters $\alpha_1, \ldots, \alpha_K$, the uniform distribution over the interval $(a, b)$, the uniform distribution over the discrete set $\{1, \ldots, R\}$, the beta distribution with shape parameters $(g_0, h_0)$, and the normal distribution with mean $M$ and covariance $V$, respectively. We use $\Gamma(\cdot)$ and $\Psi(\cdot)$ to represent the gamma function and digamma function, respectively. The notation $\sim$ means equality in distribution. The notation $y | x$ denotes a random variable $y$ conditioned on $x$, and $p(y | x)$ denotes its conditional probability density function. $E$ is the expectation operator and $E_q$ is expectation with respect to the probability distribution $q$. We use $I(a, b)$ to denote the indicator function, which equals 1 if $a = b$ and 0 otherwise. We use $|S|$ to represent the cardinality of the set $S$.

## 2 Model and notations

In this section, we present our model and assumptions. Suppose that $N$ agents observe $L$ events and each event can be in $R$ possible states. Each agent observes only a subset of events, and provides its opinions of the events’ states to a fusion center. The fusion center’s goal is to infer the true state of each event from all the agents’ opinions, and estimate the confusion matrix of each agent. We adopt the Bayesian network model shown in Fig. 1, with notations used in the model summarized in Table 1. We explain the model in detail below.

We assume that a social network connecting the $N$ agents is known. Agents in a social network tend to form communities [17] whose members have similar interests or backgrounds. Agents in the same community may be more interested in certain events, and may share the same biases or reliabilities. An agent can subscribe to the beliefs of multiple communities. Consider an agent $n$ who observes an event $l$. Suppose that it decides to adopt the belief of community $k$ when observing event $l$. Let $\hat{\omega}_k$ be the $R \times R$ confusion matrix of the community $k$ whose $r$-th row $\hat{\omega}_k(r, \cdot)$ is assumed to follow a log-normal distribution modeled as:

$$\hat{\omega}_k(r, \cdot) \sim \text{LogNormal}(M, V),$$  

where $M$ and $V$ are known hyper-parameters. Note that with the log-normal distribution assumption, $\hat{\omega}_k(r, r')$ is positive. We suppose that the $R \times R$ confusion matrix $\omega_{n,k}$

![Fig. 1. Our proposed Bayesian network model.](image)

| Table 1 Summary of commonly-used notations. |
|-------------------------------------------|
| **Notation in Table 1** | **Description** | **Variational Parameter in Section 3** |
| $D(n, m)$ | There is a (or no) social connection between agents $n$ and $m$. | N.A. |
| $z \leftarrow (z_{n \rightarrow m})_{n,m} \in \mathbb{R}$ | $z_{n \rightarrow m}$ is the index of the community agent $n$ subscribes to under the social influence of agent $m$. | $\phi = (\phi_{n \rightarrow m,k})_{n,m}$ |
| $\beta \leftarrow (\beta_k)_{k=1}^K$ | $\beta_k$ is the social network parameter defined in (7). | $\lambda = (\lambda_k)_{k=1}^K$ |
| $\pi \leftarrow (\pi_n)_{n=1}^N \rightarrow (\pi_{n,k})_{n,k=1}^K$ | $\pi_n$ is the distribution of $s_n$, which are defined in (3) and (6). | $\psi = (\psi_{n,k})_{n,k=1}^K$ |
| $s \leftarrow (s_{n,l})_{n,l=1}^N$ | $s_{n,l}$ is the index of the community whose belief agent $n$ subscribes to when it observes event $l$. | $\psi = (\psi_{n,k})_{n,k=1}^K$ |
| $y \leftarrow (y_{n,l})_{n,l=1}^N$ | $y_{n,l}$ is the observation of agent $n$ of event $l$. | N.A. |
| $\theta \leftarrow (\theta_l)_{l=1}^L$ | $\theta_l$ is the hidden true state of event $l$. | $\nu = (\nu_l)$ |
| $\hat{\omega} \leftarrow (\hat{\omega}_k)_{k=1}^K$ | $\hat{\omega}_k$ is the confusion matrix of community $k$. | $\mu = (\mu_k)$ |
| $\omega \leftarrow (\omega_{n,k})_{n,k=1}^N$ | $\omega_{n,k}$ is the confusion matrix of agent $n$ when it subscribes to the belief of community $k$. This is a perturbed version of $\hat{\omega}_k$. | $\xi = (\xi_{n,l})_{n,l=1}^N$ |
| $\alpha$, $(g_0, h_0)$ | Known hyper-parameters defined in (1), (4), and (6), respectively. | N.A. |
of agent $n$ is then given by

$$\omega_{n,k}(r, \cdot) \sim \text{Dir}(\tilde{\omega}_k(r, \cdot))$$

(2)

for each row $r$. The model (2) allows us to correlate the individual confusion matrix $\omega_{n,k}$ for every agent $n$ who subscribes to the belief of community $k$ through the community confusion matrix $\tilde{\omega}_k$.

Let $s^l_n$ denote the index of community whose belief agent $n$ subscribes to when it observes event $l$. We model it as

$$s^l_n \mid \pi_n \sim \text{Cat}(\pi_n),$$

(3)

where $\pi_n \sim \text{GEM}(\alpha)$, and $\alpha$ is a concentration hyperparameter and GEM stands for the Griffiths, Engen and McCloskey stick breaking process [18]. Following [19], [20], we approximate the GEM process with its degree $K_s$ weak limit given by

$$\pi_n \sim \text{Dir}(\alpha/K_s, \ldots, \alpha/K_s),$$

(4)

where $K_s$ is the maximum number of communities. Let $\pi_n = (\pi_{n,k})_{k=1}^{K_s}$.

Let $\theta^l$ be the hidden true state of event $l$ with prior distribution $\theta^l \sim \text{Unif}(1, \ldots, R)$ and let $y^l_n$ denote the opinion of agent $n$ with respect to event $l$. We model the distribution of agent $n$’s opinion as:

$$y^l_n \mid \theta^l, \{\omega_{n,k}\}_{k=1}^{K_s}, s^l_n \sim \text{Cat}\left(\omega_{n,s^l_n}(\theta^l, \cdot)\right).$$

(5)

Our target is to estimate $(\theta^l)_{l=1}^L$ and $(\omega_{n,k})_{n=1}^N$ from $(y^l_n)_{n=1}^N$. To model the available social network information, we suppose that the social network graph adjacency matrix $D$ is known, where $D(n, m) = 1$ if agent $n$ and agent $m$ are connected, and $D(n, m) = 0$ otherwise. We adopt the mixed membership stochastic blockmodel (MMSB) [16] to model $D(n, m)$. In this model, we use $z_{n \rightarrow m}$ to denote the community whose belief agent $n$ subscribes to due to the social influence from agent $m$. Under the influence of different agents, agent $n$ may subscribe to the beliefs of multiple communities. If both agents $n$ and $m$ subscribe to the belief of the same community, they are more likely to be connected in the social network. We assume the following:

$$z_{n \rightarrow m} \mid \pi_n \sim \text{Cat}(\pi_n),$$

$$z_{m \rightarrow n} \mid \pi_m \sim \text{Cat}(\pi_m),$$

$$\beta_k \sim \text{Be}(g_0, h_0),$$

(6)

where $g_0, h_0 > 0$ are hyperparameters, and for $k = 1, \ldots, K_s$,

$$P(D(n, m) = 1 \mid z_{n \rightarrow m}, z_{m \rightarrow n}, \beta_k)$$

$$= \begin{cases} \beta_k, & \text{if } z_{n \rightarrow m} = z_{m \rightarrow n} = k, \\ \epsilon, & \text{if } z_{n \rightarrow m} \neq z_{m \rightarrow n}, \end{cases}$$

(7)

where $\epsilon$ is a known small constant value. We also assume that $\{z_{n \rightarrow m}\}_{n,m}$ and $\{\beta_k\}_k$ are independent. Furthermore, conditioned on $(z_{n \rightarrow m}, z_{m \rightarrow n}), D(m, n)$ is independent of $\pi_n$.

Combining (6) and (7), we have

$$p(z_{n \rightarrow m} = k \mid \pi_n, z_{m \rightarrow n}, D(n, m) = 1, \beta_k)$$

$$= \beta_k(z_{m \rightarrow n}, k) \epsilon(1-I(z_{n \rightarrow m}, k)) \pi_{n,k},$$

(8)

and

$$p(z_{n \rightarrow m} = k \mid \pi_n, z_{m \rightarrow n}, D(n, m) = 0, \beta_k)$$

$$= (1 - \beta_k)(I(z_{m \rightarrow n}, k)(1 - \epsilon)(1 - I(z_{m \rightarrow n}, k)) \pi_{n,k}. $$

(9)

### 3 Laplace Variational Inference Method

In this section, we present an approach to infer the states of events and the confusion matrices of agents, based on our model. Let $\beta = (\beta_k)_{k=1}^K$, $\pi = (\pi_{n,k})_{n,k}$, $\theta = (\theta^l)_{l=1}^L$, $\omega = (\omega_{n,k})_{n,k}$, $\tilde{\omega} = (\tilde{\omega}_k)_{k},$ and $y = (y^l_n)_{n,l}$, where $n, m \in \{1, \ldots, N\}, m \neq n,$ $k \in \{1, \ldots, K_s\},$ and $l \in \{1, \ldots, L\}$. For simplicity, let $\Omega = \{\beta, \pi, \theta, \omega, \tilde{\omega}\}$. As the closed-form of the posterior distribution $p(\Omega \mid y, D)$ is not available, we use a variational inference method [21] to approximate the posterior distribution. The variational inference method first posits a family of densities, and then iteratively updates variational parameters to select a member in the family that has minimum Kullback Leibler (KL) divergence with the posterior distribution. Compared with Markov chain Monte Carlo (MCMC) sampling, variational inference methods solve an optimization problem and thus tends to be computationally faster. However, unlike MCMC, it does not guarantee that the global optimal [22] inference is achieved. Variational inference methods are more suitable for large datasets, while MCMC is more appropriate for smaller ones. We describe our proposed variational method in detail below.

We use $F$ to denote a family of probability distributions over $\Omega$. Our target is to find the member in $F$ that is closest to $p(\Omega \mid y, D)$ in KL divergence. We choose $F$ to be a mean-field variational family, so that the latent variables $\beta, \pi, \theta, \omega, \tilde{\omega}$ are mutually independent and each is governed by a variational parameter. We further denote the variational parameters of the variational distribution of $\beta, \pi, \theta, \omega$ and $\tilde{\omega}$ as $\lambda = (\lambda_k)_k, \phi = ((\phi_{n,m,k})_{n,m}), \psi = ((\psi^l_{n,k})_{n,l}), \gamma = (\gamma_{n,k})_{n,k}, \nu = (\nu^l), \xi = (\xi_{n,s^l})_{n,l},$ and $\mu = (\mu_k)_k,$ respectively. Each set of parameters $\lambda, \phi, \psi, \gamma, \nu, \xi, \mu$ corresponds to a member in the mean-field variational family $F$. A distribution in $F$ is represented as

$$q(\Omega) = q(\beta; \lambda)q(\pi; \phi)q(\psi; \gamma)q(\theta; \nu)q(\omega; \xi)q(\tilde{\omega}; \mu).$$

(10)

To avoid cluttered notations, we omit the variational parameters for simplicity, e.g., we write $q(\beta)$ instead of $q(\beta; \lambda)$. In the variational inference method, we aim to find

$$q^*(\Omega) = \arg \min_{q(\Omega) \in F} D_{KL}(q(\Omega) \mid \mid p(\Omega \mid y, D)),$$

(11)

where $D_{KL}(\cdot \mid \mid \cdot)$ is the KL divergence. From [22], finding (11) is equivalent to maximizing the evidence lower bound

$$\mathcal{L}(q) \triangleq \mathbb{E}_{q(\Omega)}[\log p(\Omega, y, D)] - \mathbb{E}_{q(\Omega)}[\log q(\Omega)].$$

(12)

Since the prior distribution of $\tilde{\omega}$ and its likelihood in our model are not conjugate, we cannot use standard mean-field variational inference to update its variational parameter. Instead, we consider the Laplace variational inference approach proposed in [23], which uses Laplace approximations [24] to approximate the variational distribution of $\tilde{\omega}$ with a normal distribution. For the other variables $\{\beta, \pi, \theta, \omega\}$, the posterior distribution of each variable
is in the same exponential family as its prior distribution and we choose the variational distribution of each variable from the same exponential family as its posterior distribution. Our target is to solve (12) by updating these variational parameters iteratively. We call this the Variational Inference using Social network Information for Truth discovery (VISIT) algorithm, which is shown in Algorithm 1. In the following, we explain VISIT in detail by presenting our assumptions on the variational distribution of each variable in \( \Omega \), and deriving the procedure to iteratively update each variational parameter in our model.

Algorithm 1 VISIT (i-th iteration)

Input: Variational parameters in (i-1)-th iteration, opinions \( y \), social network data \( D \).

Output: Variational parameters in i-th iteration.

for each agent \( n \in \{1, \ldots, N\} \) do
  for each agent pair \( (n,m) \) in \( \{(n,m)\}_{m=1}^{N} \) do
    Update \( \phi_{n \rightarrow m} \) and \( \phi_{m \rightarrow n} \) using (18) and (19).
  end for
  Update \( \psi_{n} \) using (20).
  Update \( \gamma_{n} \) using (21).
  Update \( \xi_{n} \) using (26).
end for

Update \( \lambda \) using (13) and (14).
Update \( \nu \) using (24).
Update \( \mu \) using (30).
return \( \phi, \psi, \gamma, \xi, \lambda, \nu, \) and \( \mu \).

3.1 Social network parameter \( \beta \)

From our Bayesian network model in Fig. 1, we have the following: (i) For \( (n,m) \neq (n',m') \), conditioned on \( z_{n \rightarrow m}, z_{m \rightarrow n}, z_{n' \rightarrow m'}, z_{m' \rightarrow n'} \), \( \beta_{k} \), \( D(n,m) \) and \( D(n',m') \) are independent. (ii) \( \beta_{k} \) and \( z \) are independent. Thus, for \( k = 1, \ldots, K_{s} \), the posterior distribution of \( \beta_{k} \) is given by

\[
p(\beta_{k} \mid D, z) \propto \prod_{(n,m)} p(D(n,m) \mid z_{n \rightarrow m}, z_{m \rightarrow n}, \beta_{k}) p(\beta_{k}) \\
\propto \prod_{(n,m)} \beta_{k}^{D(n,m)I(z_{n \rightarrow m}, k)I(z_{m \rightarrow n}, k)} \\
\times (1 - \beta_{k})^{(1 - D(n,m))I(z_{n \rightarrow m}, k)I(z_{m \rightarrow n}, k)} \\
\times \beta_{k}^{\sum_{(n,m)} D(n,m)I(z_{n \rightarrow m}, k)I(z_{m \rightarrow n}, k) + g_{0} - 1} \\
\times (1 - \beta_{k})^{\sum_{(n,m)} (1 - D(n,m))I(z_{n \rightarrow m}, k)I(z_{m \rightarrow n}, k) + h_{0} - 1} \\
\propto Be(\eta_{k}(D, z)),
\]

where

\[
\eta_{k}(D, z) = \sum_{(n,m)} D(n,m)I(z_{n \rightarrow m}, k)I(z_{m \rightarrow n}, k) + g_{0} \\
\sum_{(n,m)} (1 - D(n,m))I(z_{n \rightarrow m}, k)I(z_{m \rightarrow n}, k) + h_{0}.
\]

We choose the variational distribution of \( \beta_{k} \) to be in the same exponential family as its posterior distribution. Let \( \lambda_{k} = (G_{k}, H_{k}) \) and the variational distribution of \( \beta_{k} \) be

\[
q(\beta_{k}) = Be(G_{k}, H_{k}).
\]

From [22], we obtain

\[
\lambda_{k} = \mathbb{E}_{q(\beta_{k})}[\eta_{k}(D, z)],
\]

with

\[
G_{k} = \mathbb{E}_{q(\beta_{k})} \left[ \sum_{(n,m)} D(n,m)I(z_{n \rightarrow m}, k)I(z_{m \rightarrow n}, k) + g_{0} \right] \\
= \sum_{(n,m)} D(n,m)\phi_{n \rightarrow m,k}\phi_{m \rightarrow n,k} + g_{0}, \quad (13)
\]

\[
H_{k} = \mathbb{E}_{q(\beta_{k})} \left[ \sum_{(n,m)} (1 - D(n,m))I(z_{n \rightarrow m}, k)I(z_{m \rightarrow n}, k) + h_{0} \right] \\
= \sum_{(n,m)} (1 - D(n,m))\phi_{n \rightarrow m,k}\phi_{m \rightarrow n,k} + h_{0}, \quad (14)
\]

where \( \phi_{n \rightarrow m,k} = q(z_{n \rightarrow m} = k) \) is defined in Section 3.2.

We also have

\[
\mathbb{E}_{q(\beta_{k})}[\log(\beta_{k})] = \Psi(G_{k} - \Psi(G_{k} + H_{k})], \quad (15)
\]

\[
\mathbb{E}_{q(\beta_{k})}[\log(1 - \beta_{k})] = \Psi(H_{k} - \Psi(G_{k} + H_{k})], \quad (16)
\]

which are used in computing the variational distributions of other parameters in our model. Recall that \( \Psi(\cdot) \) is the digamma function.

3.2 Community membership indicators \( z \)

Consider two agents \( n \) and \( m \) with \( D(n,m) = 1 \). From our Bayesian network model in Fig. 1, for each \( k = 1, \ldots, K_{s} \), we have

\[
p(z_{n \rightarrow m} = k \mid \pi_{n}, z_{m \rightarrow n}, D(n,m) = 1, \beta_{k}) \\
\propto p(D(n,m) = 1 \mid z_{n \rightarrow m} = k, \pi_{n}, z_{m \rightarrow n}, \beta_{k}) \\
\times p(z_{n \rightarrow m} = k \mid \pi_{n}, z_{m \rightarrow n}, \beta_{k}) \\
= p(D(n,m) = 1 \mid z_{n \rightarrow m} = k, z_{m \rightarrow n}, \beta_{k})p(z_{n \rightarrow m} = k \mid \pi_{n}) \\
= \beta_{k}^{I(z_{n \rightarrow m}, k)}(1 - I(z_{n \rightarrow m}, k))^{\pi_{n}, k}.
\]

Therefore, \( p(z_{n \rightarrow m} \mid \pi_{n}, z_{m \rightarrow n}, D(n,m) = 1, \beta_{k}) \) is a categorical distribution, which is an exponential family with natural parameter

\[
\left( \log \left( \beta_{k}^{I(z_{n \rightarrow m}, k)}(1 - I(z_{n \rightarrow m}, k))^{\pi_{n}, k} \right) \right) \\
- \log \left( \sum_{k=1}^{K_{s}} (\beta_{k}^{I(z_{n \rightarrow m}, k)}(1 - I(z_{n \rightarrow m}, k))^{\pi_{n}, k}) \right) \right) K_{s} \\
\]

We let the variational distribution of \( z_{n \rightarrow m} \) to be in the same exponential family as its posterior distribution, namely a categorical distribution. Assume its categorical probabilities are \( \phi_{n \rightarrow m,k} \). From [22], we obtain

\[
\log \phi_{n \rightarrow m,k} \\
= \mathbb{E}_{q(\beta_{k})}[\log(\beta_{k}^{I(z_{n \rightarrow m}, k)}(1 - I(z_{n \rightarrow m}, k))^{\pi_{n}})] \\
- \mathbb{E}_{q(\pi_{n})}[\log(\sum_{k=1}^{K_{s}} (\beta_{k}^{I(z_{n \rightarrow m}, k)}(1 - I(z_{n \rightarrow m}, k))^{\pi_{n}, k}))].
\]

(17)
which implies

\[ \phi_{n \rightarrow m, k} \]

\[ \propto \exp\{ E[q(\beta_n, \pi_n, z_{m-n}) \mid \log(\beta_{n,k}) \{1 - l(z_{m-n,k})\} \pi_{n,k}] \} \]

\[ = \exp\{ \phi_{m-n,k} E[q(\beta_k) \mid \log(\beta_k)] + (1 - \phi_{m-n,k}) \log(\epsilon) \}

\[ + E[q(\pi_n) \mid \log(\pi_{n,k})] \} \}

\[ \propto \exp\{ \phi_{m-n,k} (E[q(\beta_k) \mid \log(\beta_k)] - \log(\epsilon)) \}

\[ + E[q(\pi_n) \mid \log(\pi_{n,k})] \}, \]

(18)

where \( E[q(\beta_k) \mid \log(\beta_k)] \) and \( E[q(\pi_n) \mid \log(\pi_{n,k})] \) are computed using (15) and (22) in the sequel, respectively.

Similarly, if \( D(n, m) = 0 \), we have

\[ p(z_{n-m} = k \mid \pi_n, z_{m-n}, D(n, m) = 0, \beta_k) \]

\[ \propto (1 - \beta_k)^0 l(z_{m-n,k}) \{1 - l(z_{m-n,k})\} \pi_{n,k}, \]

and

\[ \phi_{n \rightarrow m, k} \propto \exp\{ E[q(\beta_n, \pi_n, z_{m-n}) \mid \log((1 - \beta_k)^0 l(z_{m-n,k}) \{1 - l(z_{m-n,k})\} \pi_{n,k}] \}

\[ = \exp\{ \phi_{m-n,k} E[q(\beta_k) \mid \log(1 - \beta_k)] \}

\[ + (1 - \phi_{m-n,k}) \log(1 - \epsilon) + E[q(\pi_n) \mid \log(\pi_{n,k})] \}

\[ \propto \exp\{ \phi_{m-n,k} (E[q(\beta_k) \mid \log(1 - \beta_k)] - \log(1 - \epsilon)) \}

\[ + E[q(\pi_n) \mid \log(\pi_{n,k})] \}, \]

(19)

where \( E[q(\beta_k) \mid \log(1 - \beta_k)] \) is computed in (16) in the sequel.

3.3 Event community indices \( s \)

From our Bayesian network model in Fig. 1, for \( k = 1, \ldots, K_s \), the posterior distribution of \( s_{n}^{l} \) is

\[ p(s_{n}^{l} = k \mid \pi_n, \theta_l, \omega_{n,k}) \]

\[ \propto \gamma_{n,k} \]

\[ \propto \gamma_{n,k} \]

\[ \propto \gamma_{n,k} \]

\[ = \omega_{n,k}(\theta_l, \omega_{n,k}). \]

Similar to Section 3.2, we let the variational distribution of \( s_{n}^{l} \) to be a categorical distribution with categorical probabilities \( (\psi_{n,k}^{l})_{k=1}^{K_s} \). We then have

\[ \psi_{n,k}^{l} \propto \exp\{ E[\pi_n \mid \log(\omega_{n,k}(\theta_l, \omega_{n,k}))] \}

\[ = \exp\{ E[q(\theta_l)] \}

\[ + E[q(\pi_n) \mid \log(\pi_{n,k})] \}

\[ = \exp\{ \Psi(\xi_{n,k}(\theta_l, \omega_{n,k}) - \Psi(\sum_{r'=1}^{R} \xi_{n,k}(\theta_l, r')) \}

\[ + E[q(\pi_n) \mid \log(\pi_{n,k})] \}

\[ = \exp\{ \sum_{r=1}^{R} \nu_{r}(r) \}

\[ \Psi(\xi_{n,k}(r, \omega_{n,k})) - \Psi(\sum_{r'=1}^{R} \xi_{n,k}(r', r')) \}

\[ + E[q(\pi_n) \mid \log(\pi_{n,k})] \}

\[ = \exp\{ \sum_{r=1}^{R} \nu_{r}(r) \}

\[ \Psi(\xi_{n,k}(r, \omega_{n,k})) - \Psi(\sum_{r'=1}^{R} \xi_{n,k}(r', r')) \}

\[ + E[q(\pi_n) \mid \log(\pi_{n,k})] \}, \]

(20)

where \( E[q(\omega_{n,k}(\theta_l, \omega_{n,k})) \mid \log(\omega_{n,k}(\theta_l, \omega_{n,k}))] \) and \( E[q(\pi_n) \mid \log(\pi_{n,k})] \) are computed in (27) and (22), respectively, and \( \nu_{r}(r) \) is defined in (23).

3.4 Community weights \( \pi \)

Since conditioned on \( \pi_n = (\pi_{n,k})_{k=1}^{K_s}, \{s_{n}^{l}\}_{l=1}^{N} \) are independent, therefore we have

\[ p(\pi_n \mid \{s_{n}^{l}\}_{l=1}^{N}, \{z_{m-n}\}_{m=1, n \neq m}^{N}) \]

\[ \propto \prod_{l=1}^{L} p(s_{n}^{l} \mid \pi_n) \prod_{m=1, n \neq m}^{N} p(z_{m-n} \mid \pi_n)p(\pi_n) \]

\[ \propto \text{Dir} \left( \begin{pmatrix} \frac{\alpha}{K_s} + \sum_{m=1, n \neq m}^{N} I(z_{m-n}, k) + \sum_{l=1}^{L} I(s_{n}^{l}, k) \end{pmatrix}_{k=1}^{K_s} \right). \]

We let the variation distribution of \( \pi_n \) to be in the same exponential family as its posterior distribution by letting \( q(\pi_n) \triangleq \text{Dir}(\gamma_n), \) where \( \gamma_n \) is a vector of \( K_s \) elements with \( k \)-th element being

\[ \gamma_{n,k} = \frac{E[q(\{s_{n}^{l}\}_{l=1}^{N}, \{z_{m-n}\}_{m=1, n \neq m}^{N}) \mid \alpha \left[ \sum_{k=1}^{K_s} \right] \pi_n]}{\sum_{m=1, n \neq m}^{N} I(z_{m-n}, k) + \sum_{l=1}^{L} I(s_{n}^{l}, k)} \]

(21)

We also have

\[ E[q(\pi_n) \mid \log(\pi_{n,k})] = \Psi(\gamma_{n,k}) - \Psi(\sum_{k=1}^{K_s} \gamma_{n,k}), \]

(22)

which is used in Sections 3.2 and 3.3.

3.5 Event states \( \theta \)

For \( r = 1, \ldots, R \), the posterior distribution of \( \theta_l \) is

\[ p(\theta_l = r \mid \{y_{n}^{l}\}_{n=1}^{N}, \omega, \{s_{n}^{l}\}_{n=1}^{N}) \]

\[ = \prod_{n=1}^{N} p(\theta_l = r \mid \omega_{n,k}^{l} = \omega, s_{n}^{l} = \omega, p(\theta_l). \]

where \( p(y_{n}^{l} \mid \theta_l = r = \omega_{n,k}^{l} = \omega, s_{n}^{l} = \omega, \theta_l \sim \text{Unif}(1, \ldots, R). \) Thus,

\[ p(\theta_l = r \mid \{y_{n}^{l}\}_{n=1}^{N}, \omega, \{s_{n}^{l}\}_{n=1}^{N}) \]

\[ \propto \sum_{n=1}^{N} \omega_{n,s_{n}^{l}}(r, y_{n}^{l}). \]

We let the variational distribution of \( \theta_l \) be a categorical distribution, and denote

\[ q(\theta_l = r) \triangleq \nu_{r}(r), \]

(23)
where
\[
\nu^l(r) = \exp \left\{ \sum_{n=1}^{N} \mathbb{E}_{q_1(n, s_n^l(r), s_n^l)} \left[ \log \left( \omega_{n, s_n^l(r), y_n^l} \right) \right] \right\}
\]
\[
= \exp \left\{ \sum_{n=1}^{N} \mathbb{E}_{q_1(n, s_n^l(r), s_n^l)} \left[ \mathbb{E}_{q_2(n, s_n^l(r), s_n^l)} \left[ \log \left( \omega_{n, k(r), y_n^l} \right) \right] \right] \right\}
\]
\[
= \exp \left\{ \sum_{n=1}^{N} \mathbb{E}_{q_1(n, s_n^l(r), s_n^l)} \left[ \Psi(\xi_{n, s_n^l(r), y_n^l}) - \Psi(\sum_{r'=1}^{R} \xi_{n, s_n^l(r), r'}) \right] \right\}
\]
\[
= \exp \left\{ \sum_{n=1}^{N} \sum_{k=1}^{K} \psi_{n, k}^l \left[ \Psi(\xi_{n, k(r), y_n^l}) - \Psi(\sum_{r'=1}^{R} \xi_{n, k(r), r'}) \right] \right\}. 
\]

The penultimate equation holds because
\[
\mathbb{E}_{q_1(n, s_n^l(r), s_n^l)} \log \left( \omega_{n, k(r), y_n^l} \right) = \mathbb{E}_{q_1(n, s_n^l(r), s_n^l)} \mathbb{E}_{q_2(n, s_n^l(r), s_n^l)} \log \left( \omega_{n, k(r), y_n^l} \right)
\]
\[
= \Psi(\xi_{n, k(r), y_n^l}) - \Psi(\sum_{r'=1}^{R} \xi_{n, k(r), r'}),
\]
which is the expectation of logarithm of \(\omega_{n, k(r), y_n^l}\), the \((r, y_n^l)\)-th element of the confusion matrix of agent \(n\) when it is in community \(k\) (see (27)).

3.6 Agent confusion matrices \(\omega\)

The posterior distribution of \(\omega_{n, k}(r, \cdot)\) is
\[
p(\omega_{n, k}(r, \cdot) | \{y_n\}_{n=1}^{L}, \{s_n\}_{n=1}^{L}, \theta, \tilde{\omega}_k(r, \cdot)) \propto \prod_{l \in \{1, \ldots, L\}} p(y_n^l | \omega_{n, k}(r, \cdot), s_n^l, \theta) \cdot p(\omega_{n, k}(r, \cdot) | \tilde{\omega}_k(r, \cdot))
\]
\[
= \prod_{l=1}^{L} \omega_{n, k}(r, y_n^l) \mathcal{I}(s_n^l, k \mid \theta^l, r) \text{Dir}(\tilde{\omega}_k(r, \cdot))
\]
\[
= \text{Dir} \left( \zeta(r, r') \beta_{r'}^{-1} \right),
\]
where \(\zeta(r, r') = \tilde{\omega}_k(r, r') + \sum_{l=1}^{L} I(y_n^l, r') I(s_n^l, k) I(\theta^l, r)\).

Letting the variational distribution of \(\omega\) be the same as its posterior distribution, we have
\[
q(\omega_{n, k}(r, \cdot)) \approx \mathcal{N}(\mu_k(r, \cdot), -\frac{1}{\nabla^2 \log(q(\mu_k(r, \cdot))}}),
\]
where \(\mu_k(r, \cdot)\) is given by
\[
\mu_k(r, \cdot) = \arg \max_{\omega_{n, k}(r, \cdot)} \log(q(\omega_{n, k}(r, \cdot))),
\]
which can be solved using the gradient descent algorithm.

4 Three-level Stochastic Variational Inference Method

The traditional variational inference method needs to analyze the whole dataset in each iteration and thus does not scale well to large datasets. To mitigate this, we propose the use of stochastic optimization and a three-level Stochastic VISIT (S-VISIT) algorithm to update parameters using randomly sampled subsets. The pseudo code of our top-level algorithm is shown in Algorithm 2.

The variational parameters corresponding to the variables in our model in Fig. 1 can be divided into three levels as follows:

(i) Agent pair level: The variational parameters \((\theta_{n \rightarrow m}, \gamma_{n \rightarrow m}) \neq m\) corresponding to the community membership indicators \((z_{n \rightarrow m})_{n \neq m}\) provide information about the social relationships between agents.

(ii) Agent level: The variational parameters \(\psi_n, \gamma_n, \text{ and } (\xi_{n, s_n^l(r), r'})\) correspond to agent \(n\)'s event community indices \(\{s_n^l\}_l\), community weights \(\pi_n\), and confusion matrices \((\omega_{n, k} \neq k)\).
(iii) Social network and event level: The variational parameters \( \{\lambda_k\}_k \) and \( \{\nu^l\}_l \) corresponding to the social network parameters \( \{\beta_k\}_k \) and event states \( \theta \).

**Algorithm 2 S-VISIT (i-th iteration)**

**Input:** Variational parameters in \((i-1)\)-th iteration, opinions \( y \), social network data \( D \).

**Output:** Variational parameters in \(i\)-th iteration.

Sample a subset \( S_n \) from agent set \( \{1, \ldots, N\} \).

**Social network and event level start:**

for each agent \( n \) in \( S_n \) do

**Agent level start:**

Sample a subset \( S_p \) from pair set \( \{(n,m)_{m=1,m\neq n}\} \).

for each agent pair \( (n,m) \) in \( S_p \) do

**Agent pair level start:**

Update \( \phi_{n\rightarrow m} \) and \( \phi_{m\rightarrow n} \) using (18) and (19).

**Agent pair level end.**

end for

Update \( \psi_n \) using (20).

Update \( \gamma_n \) using (32).

Update \( \xi_n = (\xi_n, k) \) using (26).

**Agent level end.**

end for

Update \( \lambda \) using (35).

Update \( \nu \) using (36).

Update \( \mu \) using (38).

**Social network and event level end.**

return \( \phi, \psi, \gamma, \xi, \lambda, \nu, \) and \( \mu \).

Consider \( \gamma_{n,k} \), the variational parameter of \( \pi_{n,k} \), as an example. We fix the rest of variational parameters and calculate the natural gradient [26] of (12) with respect to \( \gamma_{n,k} \):

\[
\nabla_{\gamma_{n,k}} \mathcal{L}(q) = \frac{\alpha}{K_s} + \sum_{m=1,m\neq n}^{N} \phi_{n\rightarrow m,k} + \sum_{l=1}^{L} \psi_{n,k} - \gamma_{n,k}.
\]

Let \( \gamma_{n,k}^{(i)} \) be the update of \( \gamma_{n,k} \) at the \(i\)-th iteration. If we let \( \nabla_{\gamma_{n,k}} \mathcal{L}(q) = 0 \) and update \( \gamma_{n,k} \), we need to use the whole set of \( \{\phi_{n\rightarrow m,k}\}_{m=1,m\neq n}^{N} \). Instead of using (31) to update \( \gamma_{n,k} \), we can use stochastic approximation to randomly sample a subset \( S_p \) from the set \( \{(n,m)_{m=1,m\neq n}\} \) and update \( \gamma_{n,k} \) by using

\[
\gamma_{n,k}^{(i)} = \gamma_{n,k}^{(i-1)} + \rho^{(i)}(\hat{\gamma}_{n,k}^{(i)} - \gamma_{n,k}^{(i-1)}),
\]

where

\[
\hat{\gamma}_{n,k}^{(i)} = \frac{\alpha}{K_s} + \frac{1}{|S_p|} \sum_{(n,m)\in S_p} \phi_{n\rightarrow m,k} + \sum_{l=1}^{L} \psi_{n,k}^{(i)}.
\]

and \( \rho^{(i)} \) is the step size at \(i\)-th iteration.

In (33) we only use the randomly sampled subset \( \{\phi_{n\rightarrow m,k}\}_{(n,m)\in S_p}^{N} \) instead of the whole set \( \{\phi_{n\rightarrow m,k}\}_{m=1,m\neq n}^{N} \) to perform the update. If the sequence of step sizes \( \{\rho^{(i)}\}_i \) of all the iterations satisfies

\[
\sum_{i=1}^{\infty} \rho^{(i)} = \infty \text{ and } \sum_{i=1}^{\infty} (\rho^{(i)})^2 < \infty, \tag{34}
\]

then from [27], \( \gamma_{n,k}^{(i)} \) converges to a local optimum.

Similarly, we can update the other two variational parameters \( \lambda_k \) and \( \nu^l(r) \), which correspond to the global parameters \( \beta_k \) and \( \theta^l \) respectively. At each iteration, \( |S_n| \) agents are randomly selected from the agent set \( \{1, \ldots, N\} \).

For \( r = 1, \ldots, R \), we have the following equations:

\[
G_k^{(i)} = G_k^{(i-1)} + \rho^{(i)}(\hat{G}_k^{(i)} - G_k^{(i-1)}),
\]

\[
H_k^{(i)} = H_k^{(i-1)} + \rho^{(i)}(\hat{H}_k^{(i)} - H_k^{(i-1)}),
\]

\[
\lambda_k^{(i)} = [G_k^{(i)}, H_k^{(i)}]^T,
\]

\[
\nu^l(r) = \nu^l(r) + \rho^{(i)}(\hat{\nu}^l(r) - \nu^l(r)), \tag{35}
\]

where

\[
\hat{G}_k^{(i)} = \frac{N}{|S_n|} \sum_{n \in S_n} \sum_{(n,m)\in S_p} D(n,m) \phi_{n\rightarrow m,k} \phi_{m\rightarrow n,k} + g_0,
\]

\[
\hat{H}_k^{(i)} = \frac{N}{|S_n|} \sum_{n \in S_n} \sum_{(n,m)\in S_p} (1-D(n,m)) \phi_{n\rightarrow m,k} \phi_{m\rightarrow n,k} + h_0,
\]

\[
\hat{\nu}^l(r) \propto \exp \left\{ \frac{N}{|S_n|} \sum_{n \in S_n} \sum_{k=1}^{K_s} \sum_{l=1}^{L} \psi_{n,k}^{(i)} \left( \Psi(\xi_n^{(i)}(r, y_n)) - \Psi(\xi_{n,k}^{(i)}(r, y_n)) \right) \right\}.
\]

When updating \( \mu_k^{(i)}(r, \cdot) \), the variational parameter of \( \tilde{\omega}_k(r, \cdot) \), instead of using (28), we use its noisy but unbiased estimator, which is given by

\[
\tilde{q}^{(i)}(\tilde{\omega}_k(r, \cdot)) = \exp \left\{ \frac{N}{|S_n|} \sum_{n \in S_n} \log \left( \Gamma(\tilde{\omega}_k(r, r')) - \Gamma(\tilde{\omega}_k(r, r')) \right) \right\}.
\]

Then we update

\[
\mu_k^{(i)}(r, \cdot) = \arg \max_{\tilde{\omega}_k(r, \cdot)} \log[\tilde{q}^{(i)}(\tilde{\omega}_k(r, \cdot))], \tag{37}
\]

We use the gradient descent algorithm to find \( \mu_k^{(i)}(r, \cdot) \) in (38). At each iteration, multiple gradient descent steps are conducted using the sub-dataset to update \( \mu_k^{(i)}(r, \cdot) \), which is then set as the initial value in the next iteration.
5 Simulation and Experiment Results

In this section, we present simulations and real data experiments to evaluate VISIT and S-VISIT methods. For comparison, we adopt three state-of-the-art methods as the baseline methods, namely majority voting, BCC, and CBCC. We demonstrate that our methods outperform the baseline methods for inferring event states as well as confusion matrices when agents either remain in the same community or switch communities when observing different events.

5.1 Agents remain in the same community when observing different events

As both BCC and CBCC require each agent to have a unique confusion matrix when observing different events, we first consider the scenario where agents remain in the same community for all the events.

5.1.1 Synthetic Data Generation

The event states are selected from a set of $R = 6$ states. We use both opinions from agents and the social network to infer the event states. We set the number of agents, events, and communities to $N = 80$, $L = 200$, and $K = 4$, respectively. We set $\beta_k = 0.9$ for $k = 1, \ldots, K$. Note that $K$ is used here to generate the synthetic dataset and is smaller than the maximum number of communities $K_s$ we set in our inference method. For agents $1 : 20$, $21 : 40$, $41 : 60$ and $61 : 80$, we set $\pi_n$ to be $(\frac{9}{30}, \frac{1}{30}, \frac{1}{30}, \frac{1}{30})$, $(\frac{9}{30}, \frac{4}{30}, \frac{1}{30}, \frac{1}{30})$, respectively. We sample $z_{nm} = s_n, D_i(n, m)$, and $y_{in}^j$ from (6), (3), (7), and (5), respectively.

It is worth emphasizing that we keep agents in the same community for all the events as required by both BCC and CBCC. Thus, we set $s_1 = \ldots = s_N$ and sample their common value from a categorical distribution with parameter $\pi_n$, which means the confusion matrix of each agent remains the same across different events. Let the confusion matrix $\omega_n$ have the same value $d_k$ on its diagonal and every non-diagonal element be $1 - d_k/R - 1$, where $d_k$ equals 0.05, 0.1, 0.5, 0.2 for $k = 1, \ldots, 4$. Let $A$ be an $N \times L$ observation matrix, whose $(i,j)$-th element $A(i,j)$ is equal to 1 if agent $i$ observes event $j$, and $A(i,j)$ is equal to 0 otherwise. Let $n$ be the proportion of zero elements in $A$. We call $n$ the sparsity of $A$. We generate synthetic datasets for 5 sparsity values, which are 0.7, 0.75, 0.8, 0.85, and 0.9, respectively.

5.1.2 Truth discovery accuracy

In the simulation, we set the maximum number of communities $K_s$ to be 10. We set hyper-parameters $g_0, h_0, M, V$ and $\alpha$ to be 1, 1, 2, 2, 2, 2, 2, 2, 0.7Z and 0.1 respectively, where $Z$ is a $6 \times 6$ identity matrix. For different sparsity values, we generate different data sets and conducted 50 Monte Carlo experiments for each method and each sparsity value of the observation matrices. We define the accuracy score as the number of correct estimated events, divided by the total number of events and then averaged over all Monte Carlo experiments.

The result is shown in Fig. 2. We observe that BCC, CBCC, VISIT and S-VISIT have better performance than the majority voting method because these methods model the confusion matrices of agents and can better explain the quality of observations. VISIT, S-VISIT and CBCC have better performance than BCC because they take into account the community of the agents and can thus better estimate the confusion matrix when the observation matrix is sparse. VISIT outperforms CBCC by almost 20% because VISIT uses information about the social network, which improves the clustering performance. The accuracy score of S-VISIT is lower than VISIT, as expected. With increasing sparsity $n$, the accuracy scores of all methods decrease. It is noteworthy that when the observation matrix is very sparse, the performance of all the methods can be worse than majority voting.

5.1.3 Estimation of the confusion matrices

Let $\omega_n^*$ be the true confusion matrix of agent $n$. The mean square error (MSE) in estimating $\omega_n'$ is defined as:

$$\text{MSE} = \frac{\sum_{n=1}^{N} \sum_{l=1}^{L} \sum_{r=1}^{R} |\omega_{n,s_n'}(r,r') - \omega_n^*(r,r')|^2}{N \cdot L \cdot R \cdot R}.$$  

(39)

For CBCC and BCC, as the community indices remain the same when observing different events, we denote the estimated confusion matrix as $\omega_n'$ and calculate the MSE as

$$\text{MSE} = \frac{\sum_{n=1}^{N} \sum_{r=1}^{R} \sum_{r'=1}^{R} |\omega_{n,s_n'}(r,r') - \omega_n^*(r,r')|^2}{N \cdot R \cdot R}.$$  

(40)

We then take the average of the MSEs over 50 Monte Carlo experiments. The result is shown in Fig. 3. Our methods have better performance than BCC and CBCC. When the observation matrix is sparse, our methods and CBCC collect the observations in the same community to estimate the community confusion matrix, which is then used to estimate confusion matrices for the agents belonging to that community. This yields better results than BCC, which does not assume any community structure. Compared with CBCC, our methods are better able to estimate the communities because we use social network information to aid in the procedure.
5.2 Agents switch communities when observing different events

In some applications, agents may subscribe to the beliefs of different communities when observing different events. In this subsection, we evaluate the performance of our proposed methods when agents switch communities when observing different events.

5.2.1 Synthetic Data Generation

We use the same settings as Section 5.1.1 except when sampling the community indices. In Section 5.1.1, we let \( s_{1:n} = \ldots = s_{L:n} \) and sample their common value from a categorical distribution with parameter \( \pi_n \). In this subsection, we sample each \( s_{l:n} \) independently from \( \pi_n \). Therefore, the confusion matrices of an agent may vary across different events.

5.2.2 Truth discovery accuracy

From Fig. 4, we observe that the performance of S-VISIT and VISIT are better than the three baseline methods. This is because our methods account for the agents switching community beliefs over different events, while BCC and CBCC both assume that agents do not switch communities.

5.2.3 Estimation of the confusion matrices

From Fig. 5, we observe that VISIT method performs the best, and the performance of our S-VISIT method is also better than the three baseline methods.

5.3 Real data

We next compare the performance of our proposed methods on two real datasets against the three baseline methods.

5.3.1 Movie ranking dataset

The real dataset consists of movie evaluations from IMDB, which provides a platform where individuals can evaluate movies on a scale of 1 to 10. If a user rates a movie and clicks the share button, a Twitter message is generated. We then extract the rating from the Twitter message. In our first experiment, we divide the movie evaluations into 4 levels: bad (0-4), moderate (5,6), good (7,8), and excellent (9,10). In our second experiment, we divide the movie evaluations into 2 levels: bad (0-5), good (6-10). We treat the ratings on the IMDB website as the event truths, which are based on the aggregated evaluations from all users, whereas our observations come from only a subset of users who share their ratings on Twitter. Using the Twitter API, we collect information about the follower and following relationships between individuals that generate movie evaluation Twitter messages. To better show the influence of social network information on event truth discovery, we delete small sub-networks that consist of less than 5 agents. The final dataset we use consists of 2266 evaluations from 209 individuals on 245 movies (events) and also the social network between these 209 individuals. Similar to [29], [30], we regard the social network to be undirected as both follower or following relationships indicate that the two users have similar taste. The social network is shown in Fig. 6.

The performance of different methods are shown in Table 2. We observe that our proposed methods perform better than the other benchmark methods in both experiments.

1. http://www.imdb.com/
5.3.2 Russian president rumor dataset

The dataset from [11] contains rumors regarding the Russian president Vladimir V. Putin, who had not appeared in public for more than one week up to 14 March 2015. Various speculations including false reports were posted on Twitter. The Twitter following relationships are provided in the dataset. We select the Twitter users with more than two followers to form the social network. As a Twitter user only reports an event to be true if he believes that it has occurred, to enhance the complexity of the dataset, for each event we randomly choose 20% of users in the social network who did not post any report about that event, and set their observations about the event to be opposite of the ground truth. The performance of the different methods are shown in Table 3. We observe that VISIT has the best performance, followed by S-VISIT and CBCC.

### Table 3

| Levels | Majority Voting | BCC | CBCC | S-VISIT | VISIT |
|--------|-----------------|-----|------|---------|-------|
| 2      | 0.69            | 0.66| 0.70 | 0.73    | 0.75  |
| 4      | 0.50            | 0.51| 0.53 | 0.55    | 0.56  |

6 Conclusion

In this paper, we have proposed a truth discovery method based on social network communities. Similar to other community based methods, our method performs better than methods without considering communities when the observation matrix is sparse. We incorporate information about the social network into the truth discovery framework to improve the truth discovery performance and accuracy of estimating the agents’ confusion matrices. We have developed a Laplace variational method and a three-level stochastic variational inference method to infer our non-conjugate model, with simulation and experimental results suggesting that the performance of our proposed approaches are better than several other inference methods, including majority voting, the popular BCC and CBCC methods. Unlike BCC and CBCC, in our model, each agent can subscribe to the beliefs of different communities when observing different events.

### References

[1] D. R. Karger, S. Oh, and D. Shah, “Efficient crowdsourcing for multi-class labeling,” ACM SIGMETRICS Performance Evaluation Review, vol. 41, no. 1, pp. 81–92, 2013.
[2] Q. Kang and W. P. Tay, “Sequential multi-class labeling in crowdsourcing: A Ulam-Renyi game approach,” in IEEE/WIC/ACM Int. Conf. on Web Intelligence, Leipzig, Germany, Aug. 2017.
[3] D. Acemoglu, M. A. Dahleh, I. Lobel, and A. Ozdaglar, “Bayesian learning in social networks,” Review of Economic Studies, vol. 78, no. 4, pp. 1201–1236, Mar. 2011.
[4] J. Ho, W. P. Tay, T. Q. S. Quek, and E. K. P. Chong, “Robust decentralized detection and social learning in tandem networks,” IEEE Trans. Signal Process., vol. 63, no. 19, pp. 5019 – 5032, Oct. 2015.
[5] W. P. Tay, “Whose opinion to follow in multiphoythesis social learning? A large deviations perspective,” IEEE J. Sel. Topics Signal Process., vol. 9, no. 2, pp. 344 – 359, Mar. 2015.
[6] M. L. Gray, S. Suri, S. S. Ali, and D. Kulkarni, “The crowd is a collaborative network,” in ACM Conf. Computer-Supported Cooperative Work & Social Computing, 2016, pp. 134–147.
[7] C. Huang and D. Wang, “Topic-aware social sensing with arbitrary source dependency graphs,” in ACM/IEEE Int. Conf. Inform. Process. Sensor Networks, 2016, pp. 1–12.
[8] Y. Li, J. Gao, C. Meng, Q. Li, L. Su, B. Zhao, W. Fan, and J. Han, “A survey on truth discovery,” arXiv:1505.02463, 2015.
[9] D. Wang, L. Kaplan, H. Le, and T. Abdelzaher, “On truth discovery in social sensing: A maximum likelihood estimation approach,” in ACM/IEEE Int. Conf. Inform. Process. Sensor Networks, 2012, pp. 233–244.
[10] D. Wang, M. T. Amin, S. Li, T. Abdelzaher, L. Kaplan, S. Gu, C. Pan, H. Liu, C. C. Aggarwal, and R. Ganti, “Using humans as sensors: an estimation-theoretic perspective,” in ACM/IEEE Int. Conf. Inform. Process. Sensor Networks, 2014, pp. 35–46.
[11] S. Yao, S. Hu, S. Li, Y. Zhao, L. Su, L. Kaplan, A. Yener, and T. Abdelzaher, “On source dependency models for reliable social sensing: Algorithms and fundamental error bounds,” in IEEE Int. Conf. Distributed Computing Syst., 2016, pp. 467–476.
[12] H. C. Kim and Z. Ghahramani, “Bayesian classifier combination,” in Artificial Intell. and Stat., 2012, pp. 619–627.
[13] M. Venanzi, J. Guiver, G. Kazai, P. Kohli, and M. Shokouhi, “Community-based Bayesian aggregation models for crowdsourcing,” in Int. Conf. World Wide Web, 2014, pp. 155–164.
[14] M. Yin, M. L. Gray, S. Suri, and J. W. Vaughan, “The communication network within the crowd,” in Int. Conf. World Wide Web, 2016, pp. 1293–1303.
[15] D. Koller and N. Friedman, Probabilistic Graphical Models: Principles and Techniques (Adaptive Computation and Machine Learning series). The MIT Press, 2009.
[16] R. M. Airoldi, D. M. Blei, S. E. Fienberg, and E. P. Xing, “Mixed membership stochastic blockmodels,” J. Mach. Learning Research, vol. 9, no. Sep, pp. 1981–2014, 2008.
[17] S. Fortunato and D. Hric, “Community detection in networks: A user guide,” Physics Rep., vol. 659, pp. 1–44, 2016.
[18] Y. W. Teh, M. I. Jordan, M. J. Beal, and D. M. Blei, “Hierarchical Dirichlet processes,” J. Am. Stat. Assoc., 2012.
[19] H. Ishwaran and M. Zarepour, “Exact and approximate sum representations for the Dirichlet process,” Canadian J. Stat., vol. 30, no. 2, pp. 269–283, 2002.
[20] B. E. Fox, E. B. Sudderth, M. I. Jordan, and A. S. Willsky, “A sticky HDP-HMM with application to speaker diarization,” Ann. Appl. Stat., pp. 1020–1056, 2011.
[21] D. D. Hoffman, D. M. Blei, C. Wang, and J. Paisley, “Stochastic variational inference,” J. Mach. Learning Research, vol. 14, no. 1, pp. 1303–1347, 2013.
[22] D. M. Blei, A. Kucukelbir, and J. D. McAuliffe, “Variational inference: A review for statisticians,” J. Amer. Stat. Assoc., 2017.
[23] C. Wang and D. M. Blei, “Variational inference in nonconjugate models,” J. Mach. Learning Research, vol. 14, no. Apr, pp. 1005–1031, 2013.
[24] L. Tierney, R. E. Kass, and J. B. Kadane, “Fully exponential Laplace approximations to expectations and variances of nonpositive functions,” J. Amer. Stat. Assoc., vol. 84, no. 407, pp. 710–716, 1989.
[25] C. M. Bishop, Pattern Recognition and Machine Learning. Springer, 2006.
[26] S. I. Amari, “Natural gradient works efficiently in learning,” Neural Computation, vol. 10, no. 2, pp. 251–276, 1998.
[27] H. Robbins and S. Monro, “A stochastic approximation method,” Ann. Math. Stat., pp. 400–407, 1951.
[28] J. Yang and W. P. Tay. (2018) Using social network information to discover truth of movie ranking. [Online]. Available: https://doi.org/10.21979/N9/L5TRW
[29] S. Fortunato, “Community detection in graphs,” Physics Rep., vol. 486, no. 3, pp. 75–174, 2010.
[30] P. K. Gopalan and D. M. Blei, “Efficient discovery of overlapping communities in massive networks,” Proc. Nat. Academy Sci., vol. 110, no. 36, pp. 14 534–14 539, 2013.

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