Probabilistic Description of Traffic Breakdowns Caused by On-ramp Flow

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Abstract. The characteristic features of traffic breakdown near on-ramp are analyzed. To describe this phenomenon the probabilistic description regarding the jam emergence as the formation of a large car cluster on highway inside the synchronized traffic is constructed. In these terms the breakdown occurs through the formation of a certain critical nucleus in the metastable vehicle flow, which is located near the on-ramp. The strong cooperative car interaction in the synchronized traffic enables us to treat the size of critical jam nuclei as a large value and to apply to an effective one-lane model. This model assumes the following. First, the growth of a car cluster is governed by the attachment of cars to the cluster whose rate is mainly specified by the total traffic flow. Second, the cluster dissolution is determined by the car escape from the cluster whose rate depends on the cluster size directly. Third, the generation of one-car clusters (preclusters) is caused by cars entering the main road from the on-ramp. The appropriate master equation for the car cluster evolution is written and the generation rate of critical jam nuclei is found. The obtained results are in agreement with the empirical facts that the characteristic time scale of the breakdown phenomenon is about or greater than one minute and the traffic flow rate interval inside which traffic breakdowns are observed is sufficiently wide. Besides, as a new results, it is shown that the traffic breakdown probability can be analyzed, at least approximately, based solely on the data of the total vehicle flow without separating it into the vehicle streams on the main road and on-ramp when the relative on-ramp flow volume exceeds 10%–20%.

PACS. 45.70.Vn Granular models of complex systems; traffic flow – 64.60.Qb Nucleation

1 Introduction

For the last decade physics of traffic flow held attention of physical society due to two reasons. The former is its obvious importance for traffic engineering especially concerning the feasibility of attaining the limit capacities of traffic networks and quantifying it. The latter is related to the fact that vehicle ensembles on highways form a sufficiently simple example of systems with motivation being the object of new branches in modern physics. Indeed, on one hand, the individual motion of cars is affected essentially by the driver behavior in addition to the regularities of classical mechanics. So, in this sense, the vehicle ensembles are nonphysical systems. On the other hand, on the macroscopic level the vehicle ensembles exhibit a lot of properties like phase formation and phase transitions widely met in physical systems (for a review see [1]–[3]).

The traffic breakdown, i.e. the initial stage of jam formation typically near bottlenecks is an important phenomenon for traffic engineering, exactly its main characteristics determine the limit capacity of the corresponding road fragments or nodes. Its properties are sufficiently complex, traffic breakdown usually proceeds through the sequence of two phase transitions: free flow → synchronized traffic → stop-and-go pattern with a number of hysteresis and nucleation effects [4]–[5]. A detailed description of the jam formation near bottlenecks can be found in Ref. [6]. Nevertheless the basic properties of traffic breakdown are far from being understood well. In particular, it is a probabilistic phenomenon [7]–[8], i.e. traffic breakdown occurs not immediately after the vehicle flow rate attaining a certain critical value but randomly within some interval \((q_c, q_{c2})\). It is easy to explain this assuming homogeneous traffic flow to be metastable and a jam to emerge via the nucleation mechanism. A nontrivial fact is a large width of the traffic breakdown interval. According to the empirical data [5] \(q_{c1} \sim 1000–1500 \text{ v/h/l} \) (vehicle per hour per lane) and \(q_{c2} \sim 1900–2800 \text{ v/h/l} \), i.e. the ratio...
The traffic breakdown phenomenon is also characterized by a large time interval during which it develops. Namely, it is about several minutes, a typical observation time of detecting traffic breakdowns according to the traffic engineering technique. By contrast, the characteristic time scale of individual car dynamics is about or less than ten seconds.

In the previous paper [11], we have developed a simple model explaining the two last features of traffic breakdowns. Its basic point is the assumption that the synchronized phase of traffic flow develops first and traffic breakdowns occur in its own accord, comes into being via the formation of a critical jam nucleus inside this phase with strong cooperative interaction between cars. This corresponds to the recent notion about the characteristic properties of traffic flow near bottlenecks [4, 5, 6]. The size $n_0$ of such critical nuclei must be sufficiently large, $n_0 \sim 10–20$, which is justified by the analysis of single vehicle data [12] and illustrated in Fig. 1. As seen in Fig. 1, a car cluster in the synchronized traffic must span over many cars along the lane. Such a car cluster can span over all the lane on a highway, which enable us to apply to an effective one-lane approximation dealing with macrovehicles rather than real individual cars.

This model, however, has left the question about the jam origin or, what is the same, about the source of one-car clusters (preclusters) beyond the consideration. Typically, traffic breakdowns are caused by the influence of different bottlenecks, in particular, on- or off-ramps. Exactly vehicles entering or leaving traffic flow on the main road are source of jam nuclei. Investigation of the effect of the on-ramp flow on the traffic breakdown is the subject of the present work. As a particular result we would like to find an justification for the analysis of traffic breakdown phenomena near on-ramps using only the data of the total traffic flow leaving the on-ramp region. This is a typical way of analyzing real empirical data because of the lack of statistics.

\[ \frac{\langle v_n v_0 \rangle - \langle v_0 \rangle^2}{\langle v_n^2 \rangle - \langle v_0 \rangle^2} \]

\( n_0 \)

\( q_{-1} = q_{-2} \sim 50\%–100\% \). The traffic breakdown phenomenon is also characterized by a large time interval during which it develops. Namely, it is about several minutes, a typical observation time of detecting traffic breakdowns according to the traffic engineering technique. By contrast, the characteristic time scale of individual car dynamics is about or less than ten seconds.

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Finally, the detachment rate from the car cluster of size $n$ runs from 1 to $\infty$. Time scales $\tau_0 < \tau_\infty$ characterize the detachment rate from small clusters and sufficiently large ones, respectively. Their values can be estimated from the relations $q_{c1} \approx 1/\tau_\infty$ and $q_{c2} \approx 1/\tau_0$, where $q_{c1}$ and $q_{c2}$ are the traffic flow rates (per lane) such that a jam cannot form at all for $q < q_{c1}$ and a jam emerges immediately when the flow rate $q$ exceeds $q_{c2}$ (see Section 1). In these estimates we do not distinguish between the flow rate on the main road $q_{in}$ and the total flow rate $q_{in} + q_{on}$, because typically $q_{on} \ll q_{in}$. Applying to the available empirical data we set, for example, $q_{c1} \sim 1500$ v/h/l and $q_{c2} \sim 2500$ v/h/l whence get $\tau_0 \sim 1.5$ sec and $\tau_\infty \sim 2.5$ sec. Besides, as also discussed in Section 1, a scale $n_0$ dividing the car clusters into small and large ones is much greater than unity, $n_0 \sim 10-20$, due to the cooperative interaction of cars in the synchronized traffic. The form of the $w_-(n)$-dependence is schematically shown in Fig. 3.

The model under consideration describes only the initial stage of jam emergence, i.e. the formation of the jam critical nucleus. When the size $n$ of a car cluster exceeds a certain critical value $n_c$ it undergoes the irreversible growth giving rise to the jam formation. Within the frameworks of the adopted description this effect is taken into account by the following “boundary” condition imposed on the distribution function $P(n,t)$ taken at a sufficiently distant point $N_b \gg n_0$:

\[
P(N_b,t) = 0. \tag{7}
\]

Naturally, in this case equation (6) holds at points $2 \leq n \leq N_b - 1$. The system of equations (4–7) makes up the proposed model.

In what follows the assumption

\[
\frac{1}{\tau_\infty} \approx q_{c1} < q_{in} < q_{c2} \approx \frac{1}{\tau_0}, \tag{8}
\]

will be adopted. In other words, we will confine our consideration to the case when the traffic breakdown can occur but the homogeneous state of the vehicle flow is locally stable. Exactly in this case the traffic breakdown exhibits the probabilistic behavior. Then assuming the traffic flow rate on the on-ramp and the main road to be fixed the
Here the critical car cluster size $n_c$ is specified by equation (21), $\Omega(n_c)$ is the “growth” potential (20) taken at the point $n = n_c$, and the constant $\beta_c \approx 1$ is determined by expansion (22). The car clusters have to overcome exactly the potential barrier $\Omega(n_c)$ for their growth to become irreversible.

Figure 5 depicts the dependence of the critical cluster size $n_c(\Delta)$ as well as the growth barrier $\Omega(n_c) := \Omega_c(\Delta)$ on the total traffic flow rate $q_{total} := q_{in} + q_ramp$ in units of the overcriticality

$$\Delta := \frac{q_{total} - q_{c1}}{q_{c2} - q_{c1}}$$

that were obtained using Ansatz (4). The corresponding form of the critical cluster generation rate $G_c$ is (see Appendix A expression (28))

$$G_c = \frac{q_{ramp}}{2\pi n^2} \sqrt{\frac{q_{c2}}{q_{c1}}} \left( \frac{q_{c2}}{q_{c1}} \right)^{\frac{\Delta^2}{2}} n_0n_{c2} - q_{c1}.$$  

Figure 6 exhibits the characteristic features of the obtained critical cluster generation rate depending on the volumes of the on-ramp flow and the total vehicle flow.

\section{3 Conclusion}

The present paper developed a probabilistic description of the traffic breakdown phenomena near an on-ramp. Previously [11] we have proposed a simple probabilistic model for the traffic breakdown that explains two facts observed empirically (see Refs [3,4] as well as for a review Ref. [5]). The former is the large width of the traffic flow rate interval $(q_{c1},q_{c2})$ wherein the traffic breakdown phenomena are observed, namely, $(q_{c2} - q_{c1})/q_{c1} \sim 50\%-100\%$. The latter is the fact that the time interval during which traffic breakdown develops is about several minutes whereas the characteristic time scale of the individual car dynamics is about or less than ten seconds. However the origin of traffic breakdown has been left beyond the analysis. Typically different kinds of bottlenecks, for example, on- and off-ramps, are responsible for the jam emergence. The main purpose of the given paper was to take into account directly the effect of on-ramp within this probabilistic description. Besides we would like to explain why the empirical analysis tackling the traffic breakdown near on-ramps can be performed dealing solely with the data of the total traffic flow without separating it into main road and on-ramp streams. It is a typical situation because of the lack of statistics.

According to the modern notion of jam emergence it proceeds mainly through the sequence of two phase transitions: free flow $\rightarrow$ synchronized mode $\rightarrow$ stop-and-go pattern [5]. Both of these transitions are of the first order, i.e. they exhibit breakdown, hysteresis, and nucleation effects [4]. However, in the jam formation the second transition typically plays the leading role (for a detailed analysis see Ref. [5]).

Therefore the proposed model assumes, as the basic point, that a jam develops inside a certain mode of traffic flow with strong cooperative car interaction. In other words, cars entering the main road from the on-ramp give rise to a cooperative phase of car motion. Exactly inside this phase jam nuclei occur and lead to the irreversible jam formation when their size exceeds randomly a certain
critical value. Due to the cooperative car interaction this critical size must be much larger than unity, as it follows from the available single vehicle data \[10]. Besides, in this case a critical jam nucleus has to span over all the lane, which enabled us to use an effective one-lane approximation actually dealing with macrovehicles rather than real cars.

The effect of complex interaction between the cars moving on the main road and entering the neighborhood of the on-ramp with a jam nucleus located near the on-ramp is taken into accounts as follows. A jam nucleus is treated as a cluster of cars moving sufficiently slow near the on-ramp. Each car entering the on-ramp neighborhood with a car cluster attaches itself to it. In this approach the rate of the cluster growth \(w_++\) is considered to be determined completely by the vehicle flow on the main road entering the on-ramp region as well as by the traffic flow on the on-ramp. The car detachment process is described by the escaping rate \(w_-(n)\) depending on the cluster size \(n\) and decreasing with \(n\). Therefore the facts that cars can avoid a small jam nucleus by changing the lanes and overtaking it as well as can escape it also changing the lanes are allowed for by the \(w_-(n)\)-dependence.

What is new in the model under consideration in comparison with the previous one \[11\] is the assumption that one-car clusters, i.e. car preclusters being the initial state of jam nuclei are due to cars entering the main road from the on-ramp. Expression \[14\] or its particular form \[11\] specifies the desired generation rate \(G_\alpha\) of the critical jam nuclei depending, in particular, on the on-ramp flow rate. The obtained result is illustrated in Fig. \(6\).

The main conclusion of the present paper is the following. When the vehicle flow rate, \(q_{\text{ramp}}\), exceeds 10–20\% of the total traffic flow rate, \(q_{\text{total}}\), the characteristics of the traffic breakdown depend weakly on \(q_{\text{ramp}}\) individually. So in this case the traffic breakdown phenomenon can be analyzed, at least semiquantitatively, using solely the total flow rate data. Otherwise, \(q_{\text{ramp}} \ll q_{\text{total}}\), the details of partitioning the traffic flow rate into the main and on-ramp streams are the factor. In addition, the given model, as previous one \[11\], explains the large width of the vehicle flow rate interval \((q_{c1}, q_{c2})\) wherein traffic breakdowns are observed. It relates the critical values \(q_{c1}, q_{c2}\) to the dependence of the car detachment rate \(w_-(n)\) on the cluster size \(n\), namely, \(q_{c1} = w_-(\infty)\) and \(q_{c2} = w_-(1)\). Since a car cluster forms inside the synchronized traffic where the car cooperative interaction is strong the characteristic value \(n_0\) separating car clusters into “small” and “large” is much greater than unity. Therefore the ratio \((w_1-w_\infty)/w_\infty\) has to be about unity. Besides, as follows from expression \[13\], the characteristic time scale of the traffic breakdown development is about \(\tau_{\text{bd}} = \sqrt{2\pi n_0(q_{c1}/q_{\text{ramp}})\tau_\infty} \gtrsim 1\) min for \(q_{\text{ramp}} \sim 0.2q_{c1}\). This estimate explains ones more why the traffic breakdown phenomena are typical detected within 5–15 minute intervals.

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Fig. 8. Schematic illustration of the growth potential and the corresponding characteristic values of the cluster size.

Continuum approximation

When the inequalities
\[ w_-(\infty) < w_+ < w_-(1), \]
\[ (w_+ - w_-(\infty)) n_0, (w_-(1) - w_+) n_0 \gg 1 \]
hold expression (17) is simplified. Since the characteristic scale \( n_0 \) on that the quantity \( w_-(n) \) exhibits substantial decrease is large, \( n_0 \gg 1 \), the sums in expression (17) can be replaced by the corresponding integrals. Indeed, let us introduce the “growth potential”

\[ \Omega(n) := \sum_{q=1}^{n} \ln \left( \frac{w_-(q)}{w_+} \right). \]

Its form is illustrated in Fig. 8. In particular, the growth potential attains the maximum at the point \( n_c \) being the root of the equation

\[ w_-(n_c) = w_+ \]

and playing the role of the critical size of jam nuclei.

In the vicinity of the point \( n_c \) the value \( w_-(n) \) can be treated as a function of the continuous argument \( n \) and approximated by the expression

\[ w_-(n) \approx w_+ \left[ 1 - \beta_c \frac{(n - n_c)}{n_c} \right], \]

where \( \beta_c \approx 1 \) is a constant about unity. Since the main contribution to the first sum in expression (17) is due to a certain neighborhood of the point \( n_c \) we approximate the growth potential as follows

\[ \Omega(n) = \Omega(n_c) - \beta_c \frac{(n - n_c)^2}{2n_c}. \]

Substituting formula (23) into expression (17) and replacing the sum running over \( p \) by the corresponding integral over the continuous variable \( p \) we get

\[ G_c \approx cw_+ \sqrt{\frac{\beta_c}{2\pi n_c}} \exp \{-\Omega(n_c)\}. \]

Formula (24) is the desired expression for the generation rate of jam critical nuclei.

In particular, in the given limit for Ansatz (1) the critical size \( n_c \) of jam nuclei is specified by the expression

\[ \phi(n_c) = \Delta, \]

where the traffic flow overcriticality measure \( \Delta \) was introduced by formula (10) and we have set \( q_{c1} = 1/\tau_{\infty} \) and \( q_{c2} = 1/\tau_0 \). Using in addition Ansatz (1) we get from formulae (20) and (22) the expressions for the parameter

\[ \frac{\beta_c}{n_c} = \frac{(q_{c2} - q_{c1})}{q_{total}} \frac{\Delta^2}{n_0} \]

and for the critical potential barrier

\[ \Omega(n_c) = -n_0 \left[ \frac{(q_{c2} - q_{c1})}{q_{c1}} \ln \Delta + \frac{q_{c2}}{q_{c1}} \ln \left( \frac{q_{c2}}{q_{total}} \right) \right] \]

Substitution of expressions (26) and (27) into (24) yields

\[ G_c \approx cw_+ \sqrt{\frac{\beta_c}{2\pi n_c}} \exp \left\{ -\frac{2n_0(q_{c2} - q_{c1})}{q_{c1}} \Delta^2 \right\}, \]

which is exactly formula (11) due to relation (2).

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