ABSTRACT

I present an updated discussion of the one-loop corrections to the standard top quark decay, $t \to W^+ b$, and to the charged Higgs mode, $t \to H^+ b$, within the framework of the MSSM. These higher order contributions to the top width are compared with the direct top decays into 2 and 3 real SUSY particles. It turns out that whereas the SUSY effects on $t \to W^+ b$ add up to those from standard QCD, they tend most likely to erase the conventional QCD corrections to $t \to H^+ b$. However, in the latter case the full (strong plus electroweak) SUSY quantum effects are generally so large that the QCD corrections effectively come out with the “wrong sign”. A qualitatively distinct, and quantitatively significant, “quantum signature” of that type might constitute a most sought-after imprint of “virtual” Supersymmetry, which remains sizeable enough even in the absence of any direct top decay into SUSY particles. Similar considerations are made for the hadronic partial widths of the Higgs bosons of the MSSM. While a first valuable test of these effects could possibly be performed at the upgraded Tevatron, a more precise verification would most probably be carried out in future experiments at the LHC.

\textsuperscript{1}Updated analysis based on the talks presented at SUSY 96, Univ. of Maryland, College Park, USA, May 29th-June 1st 1996; and at CRAD 96, Cracow, Poland, 1-5 August 1996.
1. Introduction

For a long time, Supersymmetry (SUSY) \(^1\) has been an extremely tantalizing candidate to extend the quantum field theoretical structure of the Standard Model (SM) of the strong and electroweak interactions while keeping all the necessary ingredients insuring internal consistency, such as gauge invariance and renormalizability. A major goal of SUSY is to produce a unified theory of all the interactions, including gravity. However, this theory calls for the existence of a host of new particles, the so-called supersymmetric particles (“sparticles”). They must be found to substantiate the theory. At present, the simplest and most popular realization of this idea, namely the Minimal Supersymmetric Standard Model (MSSM) \(^2\), is being thoroughly scrutinized by experiment and it has successfully passed all the tests up to now. As a matter of fact, the global fit analyses to a huge number of indirect precision data within the MSSM are slightly worse than in the SM, due to the large number of parameters involved, but they are still comparable to those in the SM \(^3\), a situation which is certainly not shared by all known alternative extensions of the SM, like any of the multifarious extended Technicolour models.

In the near and middle future, with the upgrade of the Tevatron, the advent of the LHC, and the possible construction of an \(e^+e^-\) supercollider (NLC), new results on top quark physics, and perhaps also on Higgs physics, will be obtained in interplay with Supersymmetry that may be extremely helpful to complement the precious information already collected at LEP from \(Z\)-boson physics \(^4\).

In this talk I propose to dwell on the phenomenology of top quark and Higgs boson decays with an eye on these future developments. A recent reanalysis of experimental studies on Higgs detection possibilities at the LHC has shown that the design of the LHC detectors is also very suitable to tag signals associated to production of supersymmetric particles \(^5\). However, even in the absence of direct detection of SUSY particles, there remain important SUSY quantum effects that have not been fully explored and that could play a fundamental role in the search for new physics. Here I wish to elaborate mainly on the kind of “high precision” measurements that may be performed on top quark and Higgs boson observables in hadron colliders in order to catalyze a potential discovery of “virtual” Supersymmetry.

But, why hadron colliders?...Well, as I have said, in part because the Tevatron has been doing a very good job, and currently the machine is being significantly improved (energy and luminosity) for the next runs (Run II, TeV33 Project,...) \(^6\), and also because of the superb LHC program that we have ahead \(^7\). Now, however important these reasons may be, there are also additional, more theoretical, arguments beyond the actual machines’ programmes and upgrading that should be taken into account. In the following I will expand a bit on these additional features.
By “high precision” measurements I mean, as usual, measurements sensitive to quantum effects. Unfortunately (but, of course, naturally) quantum effects are generally tiny since they are computed from perturbation theory. And, due to this fact, it is not easy to clutch at them unless one chooses a very clean and smart experimental environment, typically an $e^+e^-$ machine like LEP or SLC. Machines like these did offer us in the past (and still partly at present) an immaculate laboratory for electroweak physics where high precision experiments can be performed. This is both because of the relative simplicity of the $e^+e^-$ interactions and because of the possibility to collect a lot of event statistics. Not too surprisingly, therefore, LEP and SLC running at the $Z$-pole mass have revealed themselves as the greatest possible scenarios yet imagined for high precision experiments ever. Regrettably, this magnificent setup has a serious drawback: it can work at full rate only at a fixed energy $\sqrt{s} = M_Z$. And, to our distress, after all the scans searching for traces of new physics around that energy have been accomplished, they have absolutely failed to unravel anything new at all. In this respect the situation has not improved any better neither at LEP 1.61 nor at LEP 1.72 thus far, where in spite of the higher energy and high performance of the machine the event statistics became, of course, substantially diminished.

Admittedly, it would suffice collecting a few “real” non-SM events to “prove”, or at least to reasonably contend, that new physics may be around. And in this sense, “direct” new physics at LEP/SLC would be very fine. In the absence of this possibility up to the present moment, one would naturally rush to quest for “quantum signs” of new physics, in particular of Supersymmetry, by means of the indirect method of high precision measurements. Notwithstanding, this alternative approach to searching for Supersymmetry in these machines, apparently does not lead to too promising expectancies either. And this will not change during the LEP/SLC lifetime. In the end a more fundamental fact must be undermining our experimental ability to search for “virtual” Supersymmetry in an $e^+e^-$ machine. It must be that virtual SUSY effects are doomed to be small in an $e^+e^-$ environment. Of course, in the future, NLC might resolve this embarrassment very expeditiously on the basis of “brute force” production of real sparticles whatever they mass be in the range $\lesssim 1\ TeV$. But in the meanwhile the method of “quantum signatures” should be seriously considered, already at the next Tevatron run.

It is a general fact that all sorts of SUSY effects fade away when we arbitrarily increase the sparticle masses. This is nothing else but an unavoidable consequence of the decoupling theorem (DT) applied to the MSSM. This theorem has been well tested in the MSSM after a long experience of computations of supersymmetric quantum corrections to gauge boson observables, like e.g. the $W$-mass, the $Z$-width and all kinds of related observables, like $\sin^2 \theta_W$, $R_b$, $R_c$, the various asymmetries etc. 
That the DT holds in the MSSM is bound to reason; after all, the source of SUSY breaking is independent – or at least it is not directly related – to the source of spontaneous symmetry breaking (SSB) of the gauge theory. Indeed, the soft SUSY-breaking masses, which we denote collectively by $M_{SUSY}$, are $SU(2)_L \times U(1)_Y$-invariant. Therefore, since $M_{SUSY}$ can be arbitrarily large, it can quickly dominate the physical masses of all the SUSY partners. In these asymptotic conditions, the sparticle masses just increase because we are increasing the size of a dimensionful parameter; and this is precisely a sufficient condition for the decoupling theorem to hold [4].

Coming back to $e^+ e^-$ phenomenology, experience shows us that all kinds of SUSY quantum effects inherently bound to the physics of the gauge bosons come out to be very small (namely, at the level of a few percent or below); and what is more, they drop off very fast on increasing the characteristic low-energy electroweak scale, $M_{SUSY}$, which determines the soft SUSY-breaking masses of the supersymmetric partners. Therefore, aside from direct “real” sparticle production, which is always an exciting possibility (in fact, the best conceivable SUSY signature), there is little hope for being able in the future to grasp a hint of SUSY from “virtual” signatures lying behind the dynamics of gauge boson observables. We need to turn our attention to a “new” scenario, namely, one in which “quantum SUSY signatures” can be significantly larger and effectively less subdued by the decoupling theorem.

Fortunately, this scenario is at our disposal: it is the physics of real top/bottom quarks in interplay with Higgs bosons in hadron colliders (see below). Of course, I am not suggesting that the SUSY quantum effects generated in this sector of the theory may escape the consequences of the DT. I only dare to say that there are smart situations where those quantum effects are much larger (for the same values of the sparticle masses), and that the decoupling rate is slower, than in the weak gauge boson case.

### 2. $t \to W^+ b$ versus $t \to H^+ b$ in the MSSM and the renormalization of $\tan \beta$

Why do we expect potentially large virtual SUSY signatures in top quark physics? It is well known that an exception to the DT is the phenomenon of SSB, which the MSSM has in common with the SM. Here, for fixed $M_{SUSY}$, we let dimensionless parameters (such as the Yukawa couplings) go to infinity and the corresponding quantum corrections do not decouple. Now, in the MSSM, the spectrum of Higgs-like particles and of Yukawa couplings is far and away richer than in the SM. A crucial fact affecting the results of our work is that in such a framework the bottom-quark Yukawa coupling may counterbalance the smallness of the bottom mass, $m_b \simeq 4 - 5\, GeV$, at the expense of a large value of
\[ \lambda_t \equiv \frac{h_t}{g} = \frac{m_t}{\sqrt{2} M_W \sin \beta}, \quad \lambda_b \equiv \frac{h_b}{g} = \frac{m_b}{\sqrt{2} M_W \cos \beta}, \]

and can be of the same order of magnitude, perhaps even showing up in “inverse hierarchy”: \( h_t < h_b \) for \( \tan \beta > m_t/m_b \). Clearly, both at large and small values of \( \tan \beta \) the Yukawa couplings (1) can be greatly enhanced as compared to the SM. For definiteness, in our numerical analysis we will use the (approximate) perturbatively safe range

\[ \left( \frac{g m_t}{2 M_W} \right)^2 \lesssim 0.7 \lesssim \tan \beta \lesssim \left( \frac{2 M_W}{g m_b} \right)^2 \approx 60, \]

which can be fixed e.g. by tracking down the size of the squared Yukawa couplings of the CP-odd Higgs boson, \( A^0 \), to top and bottom quarks:

\[ \left( \frac{g m_b \tan \beta}{2 M_W} \right)^2 \lesssim 1 \quad (\tan \beta \lesssim 60), \]
\[ \left( \frac{g m_t \cot \beta}{2 M_W} \right)^2 \lesssim 1 \quad (\tan \beta \gtrsim 0.7). \]

Notice that the Yukawa couplings of \( A^0 \) are most suitable to fix the allowed range of \( \tan \beta \) since they do not depend on the additional mixing angle, \( \alpha \), as it would be the case for the CP-even Higgs bosons \( h^0, H^0 \).

From the previous considerations, it should be clear that a wealth of interesting new physics could potentially be unearthed from the study of Higgs-quark-quark vertices in the MSSM, basically (though not exclusively) those involving top and bottom quarks in combination with charged and neutral Higgs bosons. To start with, I shall review and update our work \[ on one-loop quantum corrections to the top quark decays \( t \to W^+ b \) and \( t \to H^+ b \) mediated by the plethora of supersymmetric partners, such as squarks, sleptons, gluinos, chargino-neutralinos and the various Higgs bosons of the MSSM, and shall compare them with the standard QCD corrections.

Whereas a simple tree-level study of \( t \to H^+ b \) is insensitive to the nature of the Higgs sector to which \( H^+ \) belongs, a careful study of the leading quantum effects on that decay could be the clue to unravel the potential supersymmetric nature of the charged Higgs. In particular, it should be useful to distinguish it from a charged Higgs belonging to a general two-Higgs-doublet model (2HDM). Now, of course, part of these quantum effects (viz. the conventional QCD corrections) cannot distinguish the structure of the underlying Higgs model. Nevertheless, their knowledge is indispensable to probe the existence of additional sources of strong virtual corrections beyond the SM.
To evaluate the relevant quantum corrections, we shall adopt the on-shell renormalization scheme \([21]\) where the fine structure constant, \(\alpha\), and the masses of the gauge bosons, fermions and scalars are the renormalized parameters \((\alpha\text{-scheme})\). Apart from the well-known \(t b W^+\) interaction, the Lagrangian describing the \(t b H^+\) vertex in the MSSM reads as follows:

\[
\mathcal{L}_{Htb} = \frac{g}{\sqrt{2} M_W} V_{tb} \left[ m_t \cot \beta P_L + m_b \tan \beta P_R \right] \bar{b} + \text{h.c.,}
\]

where \(P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\) are the chiral projector operators and \(V_{tb}\) is the corresponding CKM matrix element—henceforth we set \(V_{tb} = 1\).

The basic free parameters of our analysis concerning the electroweak sector are contained in the stop and sbottom mass matrices \((\bar{q} = \bar{t}, \bar{b})\):

\[
\mathcal{M}^2_{\bar{q}} = \begin{pmatrix}
\mathcal{M}^2_{11} & \mathcal{M}^2_{12} \\
\mathcal{M}^2_{12} & \mathcal{M}^2_{22}
\end{pmatrix},
\]

\[
\mathcal{M}^2_{11} = M^2_{\bar{q}L} + m_{\bar{q}}^2 + \cos 2\beta (T^3_q - Q_q \sin^2 \theta_W) M_Z^2,
\]

\[
\mathcal{M}^2_{22} = M^2_{\bar{q}R} + m_{\bar{q}}^2 + Q_q \cos 2\beta \sin^2 \theta_W M_Z^2,
\]

\[
\mathcal{M}^2_{12} = m_{\bar{q}} M^\eta_{LR},
\]

\[
M^{(t,b)}_{LR} = A_{(t,b)} - \mu \{ \cot \beta, \tan \beta \}.
\]

We denote by \(m_{\bar{q}_i} (\bar{q} = \bar{t}, \bar{b})\) the lightest stop/sbottom mass-eigenvalue. For the sake of simplicity, we treat the sbottom mass matrix assuming that \(\theta_b = \pi/4\), so that the two diagonal entries are equal. This is no loss of generality, since \(t \to W^+ b\) turns out to be rather insensitive to SUSY physics, and on the other hand the feature that really matters for our calculation on \(t \to H^+ b\) is that the off-diagonal element of the sbottom mass matrix is non-vanishing, so that at high \(\tan \beta\) it behaves like \(m_b M^b_{LR} \simeq -\mu m_b \tan \beta\). As for the stop mixing angle, it is determined by the remaining input parameters. Finally, in the SUSY strongly interacting sector, the only new parameter is the gluino mass, \(m_{\tilde{g}}\).

Another fundamental ingredient of our renormalization framework is the definition of \(\tan \beta = v_2/v_1\) beyond the tree-level. The quantum analysis of the standard decay of the top quark, \(t \to W^+ b\), does not require renormalization of \(\tan \beta\). Nonetheless, a full one-loop calculation of \(t \to H^+ b\) within the MSSM requires a prescription to renormalize that parameter. At one-loop, \(\tan \beta\) is not renormalized by SUSY-QCD, but of course it is affected by the electroweak corrections. Any definition will generate a counterterm, \(\tan \beta \to \tan \beta + \delta \tan \beta\), which depends on the specific renormalization condition. There are many conceivable strategies. The ambiguity is related to the fact that this parameter is just a Lagrangian parameter and as such it is not a physical observable. Its value beyond the tree-level is renormalization scheme dependent. For example, we may wish to define \(\tan \beta\) in a process-independent (“universal”) way as the ratio \(v_2/v_1\) between the
true VEV's after renormalization of the Higgs potential. But, then, we have to take care
of the induced linear terms ("tadpoles"). In other words, we have to renormalize them
away by shifting the VEV's and the mass parameters,

\[ v_i \to Z_{H_i} \left( v_i + \delta v_i \right), \]

\[ m_i^2 \to Z_{H_i} \left( m_i^2 + \delta m_i^2 \right), \]

\[ m^2_{12} \to Z_{H_1} Z_{H_2} \left( m^2_{12} + \delta m^2_{12} \right), \]

in the Higgs potential \[15\]:

\[ V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m^2_{12} \left( \epsilon_{ij} H_i^\dagger H_j^1 + \text{h.c.} \right) + \frac{1}{8} (g^2 + g'^2) \left( |H_1|^2 - |H_2|^2 \right)^2 + \frac{1}{2} g^2 |H_1^\dagger H_2|^2, \]

(8)

where \( Z_{H_i} = 1 + \delta Z_{H_i} \) on eq.(7) are the Higgs doublet field renormalization
constants:

\[ \left( \begin{array}{c} H_1^0 \\ H_1^- \end{array} \right) \to Z_{H_1}^{1/2} \left( \begin{array}{c} H_1^0 \\ H_1^- \end{array} \right), \quad \left( \begin{array}{c} H_2^+ \\ H_2^0 \end{array} \right) \to Z_{H_2}^{1/2} \left( \begin{array}{c} H_2^+ \\ H_2^0 \end{array} \right). \]

(9)

However, that shifting procedure can be manifold \[22\]. If only the VEV’s are shifted, and
the \( \overline{\text{MS}} \) scheme is adopted, then \( \tan \beta = \tan \beta(\nu) \) becomes a rapidly varying function of
the renormalization scale \( \nu \), which renders it useless \[23\]. A consistent and more convenient
choice to cancel the tadpole terms in the renormalization of the Higgs potential is to shift
both the VEV’s as well as the mass parameters in such a way that \( \delta v_1/v_1 = \delta v_2/v_2 \). From
eq(7), this obviously entails

\[ \frac{\delta \tan \beta}{\tan \beta} = \frac{1}{2} \left( \delta Z_{H_2} - \delta Z_{H_1} \right). \]

(10)

In practice, process-dependent terms are inevitable irrespective of the definition of \( \tan \beta \).
In particular, the definition of \( \tan \beta \) where \( \delta v_1/v_1 = \delta v_2/v_2 \) will also develop process-
dependent contributions, as can be seen by trying to relate the “universal” value of \( \tan \beta \)
in that scheme with a physical quantity directly read off some physical observable. For
instance, if \( M_{A^0} \) is heavy enough, one may define \( \tan \beta \) as follows:

\[ \frac{\Gamma(A^0 \to b\bar{b})}{\Gamma(A^0 \to t\bar{t})} = \tan^4 \beta \frac{m_b^2}{m_t^2} \left( 1 - \frac{4 m_t^2}{M^2_{A^0}} \right)^{-1/2} \left[ 1 + 4 \left( \frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1} \right) \right. \]

\[ \left. + 2 \left( \frac{\delta m_b}{m_b} + \frac{1}{2} \delta Z^b_L + \frac{1}{2} \delta Z^b_R - \frac{\delta m_t}{m_t} + \frac{1}{2} \delta Z^t_L - \frac{1}{2} \delta Z^t_R \right) + \delta V \right], \]

(11)

where we have neglected \( m_b^2 \ll M^2_{A^0} \), and \( \delta V \) stands for the vertex corrections to the
decay processes \( A^0 \to b\bar{b} \) and \( A^0 \to t\bar{t} \). Since the sum of the mass and wave-function
renormalization terms along with the vertex corrections is UV-finite, one can consistently
choose \( \delta v_1/v_1 = \delta v_2/v_2 \) leading to eq.(11). Hence, deriving \( \tan \beta \) from eq.(11) undoubtedly
incorporates also some process-dependent contributions. In general, process-dependent
effects can be rather important, as shown in Ref. [19]; and, therefore, they can substantially
alter the numerical comparison between the various tan β definitions presented in Ref. [24].

In practice, one may eventually like to fix the on-shell renormalization condition on
tan β in a more physical way, viz. by relating it to some concrete physical observable,
so that it is the measured value of this observable that is taken as an input rather than
the VEV’s of the Higgs potential. Following this practical attitude, we choose as a phys-
ical observable the decay width of the charged Higgs boson into τ-lepton and associated
neutrino:
\[ \Gamma(H^+ \rightarrow \tau^+ \nu_\tau) = \frac{\alpha m_\tau^2 M_H}{8M_W s_W} \tan^2 \beta. \] (12)
This should be a very good choice, since this decay is the leading decay mode of \( H^+ \) at
high tan β, which is the regime where \( t \rightarrow H^+ b \) is competitive with \( t \rightarrow W^+ b \). This
definition produces the following counterterm [19]:
\[ \frac{\delta \tan \beta}{\tan \beta} = \frac{\delta v}{v} - \frac{1}{2} \delta Z_{H^+} + \cot \beta \delta Z_{HW} + \Delta_\tau, \] (13)
where \( v^2 = v_1^2 + v_2^2 \), and
\[ \delta Z_{H^\pm} = +\Sigma'_{H^\pm}(M_{H^\pm}^2), \]
\[ \delta Z_{HW} = \frac{\Sigma_{HW}(M_{H^\pm}^2)}{M_W^2}, \] (14)
are the field renormalization constants associated to \( H^+ \) and the \( H^+-W^+ \) mixing. The
term \( \Delta_\tau \) stands for a cumbersome expression containing the full set of MSSM corrections
to the decay \( H^+ \rightarrow \tau^+ \nu_\tau \).

Any definition of tan β is in principle as good as any other; and, in spite of the fact
that the corrections themselves may drag along some dependence on the choice of the
particular definition, the physical observables should not depend at all on that choice.
Notwithstanding, it can be a practical matter what definition to use in a given context.
For example, our definition of tan β given in eq. (12) should be most adequate to analyze
the decay \( t \rightarrow H^+ b \) (and so for charged Higgs masses \( M_{H^\pm} < m_t - m_b \)) and large tan β
since, then, \( H^+ \rightarrow \tau^+ \nu_\tau \) is the dominant decay of \( H^+ \), whereas the definition based on
eq(11) requires also a large value of tan β (to avoid an impractical suppression of the \( b\bar{b} \)
mode); moreover, in order to be operative, it also requires a much heavier charged Higgs
boson, for it turns out that \( M_{H^\pm} \approx M_{A^0} > 2m_t \) when the decay \( A \rightarrow t\bar{t} \) is kinematically
open in the MSSM. If however \( m_t + m_b < M_{H^\pm} < 2m_t \), the definition based on
eq(11) cannot be used whereas the alternative definition (12) can again be useful, e.g. to analyze \( H^+ \rightarrow t\bar{b} \) [25].

Within our context, we use eq. (13) for \( \delta \tan \beta/\tan \beta \) in order to compute the one-loop
corrections to the decay \( t \rightarrow H^+ b \). The relative importance of the process-dependent
contributions associated to our definition of tan \( \beta \) is studied in Ref. [19]. Here I shall not split any more the oblique and non-oblique contributions, and shall present only the full result. After explicit calculation of all the counterterms in our renormalization scheme, as well as of the full plethora of MSSM corrections to the three-processes: \( t \rightarrow W^+ b \), \( t \rightarrow H^+ b \) and \( H^+ \rightarrow \tau^+ \nu_\tau \), we are ready to present the outcome of our analysis. We do it in terms of the relative shift with respect to the corresponding tree-level width \( \Gamma^{(0)} \):

\[
\delta = \frac{\Gamma_{W,H} - \Gamma_{W,H}^{(0)}}{\Gamma_{W,H}^{(0)}} \equiv \frac{\Gamma(t \rightarrow \{W^+,H^\}\ b) - \Gamma^{(0)}(t \rightarrow \{W^+,H^\}\ b)}{\Gamma^{(0)}(t \rightarrow \{W^+,H^\}\ b)},
\]  

(15)

In what follows we understand that \( \delta \) defined by eq. (15) are the corrections relative to the tree-level width \( \Gamma^0 \equiv \Gamma^{(0)}_\alpha \) in the \( \alpha \)-scheme. The corresponding correction with respect to the tree-level width in the \( G_F \)-scheme is simply given by [16]

\[
\delta(G_F) = \delta^{MSSM} - \Delta r^{MSSM},
\]  

(16)

where \( \Delta r^{MSSM} \) was object of a particular study [11, 12] and therefore it can be easily incorporated, if necessary. Note, however, that \( \Delta r^{MSSM} \) is already tightly bound by the present experimental data on \( M_Z = 91.1863 \pm 0.0020 \text{ GeV} \) at LEP [4] and the ratio \( M_W/M_Z \) in \( p\bar{p} \), which lead to \( M_W = 80.356 \pm 0.125 \text{ GeV} \). Therefore, even without doing the exact theoretical calculation of \( \Delta r \) within the MSSM, we already know from

\[
\Delta r^{exp} = 1 - \frac{\pi \alpha}{\sqrt{2} G_F} \frac{1}{M_W^2 (1 - M_W^2 / M_Z^2)} = 0.040 \pm 0.018,
\]

(17)

that \( \Delta r^{MSSM} \) must lie in this experimental interval. Since, as we have argued before, quantum effects in the MSSM do preserve the DT, we expect (and we know [11, 12]) that for the present values of the sparticle masses \( \Delta r^{MSSM} \leq \Delta r^{SM} \leq \Delta r^{exp} \).

We plot in Figs.1a-1d the results of the SUSY corrections to \( t \rightarrow W^+ b \) within the framework of the MSSM [3]. For a better understanding of the various types of corrections, we introduce the following definitions:

- (i) The supersymmetric electroweak (SUSY-EW) contribution from genuine (\( R \)-odd) sparticles, i.e. from sfermions (squarks and sleptons), charginos and neutralinos: \( \delta_{SUSY-EW} \).
- (ii) The electroweak contribution from non-supersymmetric (\( R \)-even) particles: \( \delta_{EW} \).

It is composed of two distinct types of effects, namely, those from the MSSM Higgs and Goldstone bosons (collectively called “Higgs” contribution, and denoted \( \delta_H \)) plus the oblique SM effects [21] from conventional fermions (\( \delta_{SM} \)):

\[
\delta_{EW} = \delta_H + \delta_{SM}.
\]

(18)

\(^3\)These results update the analyses presented in Refs. [16, 17].
The remaining non-supersymmetric electroweak effects are subleading and are neglected.

- (iii) The strong supersymmetric QCD (SUSY-QCD) contribution from squarks and gluinos: $\delta_{\text{SUSY-QCD}}$.

- (iv) The total supersymmetric contribution from sparticles:

\[
\delta_{\text{SUSY}} = \delta_{\text{SUSY-QCD}} + \delta_{\text{SUSY-EW}}.
\]  
(19)

- (v) The standard QCD correction from quarks and gluons: $\delta_{\text{QCD}}$.

- (vi) The total MSSM contribution, $\delta_{\text{MSSM}}$, namely, the net sum of all the previous contributions:

\[
\delta_{\text{MSSM}} = \delta_{\text{SUSY}} + \delta_{\text{EW}} + \delta_{\text{QCD}}.
\]  
(20)

We see from Fig.1a that even for very large (or very small) values of $\tan \beta$ – within the perturbative range (2) – the SUSY quantum corrections to $t \to W^+ b$ are rather tiny, already in the $\alpha$-scheme. In the $G_F$-scheme, the corrections are still smaller, for $\delta$ in the $\alpha$-scheme is comparable to $\Delta r$ and there is a cancellation between the two terms in eq.(13). We point out that the SUSY-QCD effects on $t \to W^+ b$ are basically insensitive to $\tan \beta$ (the only dependence being in the masses of the squarks). Thus the evolution of $\delta_{\text{SUSY}}$ with $\tan \beta$ in Fig.1a is essentially due to the electroweak component. Unfortunately, $\delta_{\text{SUSY}}$ stays all the time within a few percent for $\tan \beta$ values in the range (2) and typical sparticle masses of $\mathcal{O}(100) \text{ GeV}$. The precise dependence on the squark and gluino masses is detailed in Figs.1b-1d. For the sake of better clarity, in Fig.1d we have split up $\delta_{\text{SUSY}}$ into the $\delta_{\text{SUSY-QCD}}$ and $\delta_{\text{SUSY-EW}}$ components. In this figure, we have also included the standard QCD correction (dotted horizontal line); $\delta_{\text{QCD}}$ reaches $-10\%$ and stays basically constant in the present experimental $m_t$ range (i.e. $m_t = 175 \pm 6 \text{ GeV}$). Moreover, as it is plain from Fig.1d, $\delta_{\text{QCD}}$ is generally dominant over the SUSY-QCD and the SUSY-EW correction. The transient dominance of the SUSY-QCD corrections near $m_{\tilde{g}} = 75 \text{ GeV}$ (see the downward spike in Fig.1d) is related to a threshold effect on the two and three point functions at $m_{\tilde{g}} = m_t - m_{\tilde{t}_1}$. Similar threshold behaviours are recorded in Figs.1b and 1c involving other sparticle masses.

The smallness of the SUSY corrections to $t \to W^+ b$ may be somewhat surprising, and it was not obvious a priori, due to the appearance of enhanced top and bottom quark Yukawa couplings (1) scattered over the plethora of Higgs-quark-quark and chargino-quark-squark diagrams involved in the calculation. Already in the pure SM context [20], where one could also expect large electroweak corrections of order $\alpha_W m_t^2 / M_W^2$, the actual corrections turn out to be rather small. Specifically, they are positive and stay within
+(4−5)% in the α-scheme, and in a narrow interval around +1.7% in the $G_F$-scheme. These results hold for the present values of the top quark mass, the variation with the SM Higgs mass being very mild. We point out that, due to the SUSY constraints, the extra Higgs effects from the two-doublet Higgs sector of the MSSM (relatively to the SM) are very small, typically one order of magnitude smaller than the SM Higgs effects [27].

Therefore, since in the end the full Higgs effects are negligible we may set, in good approximation,

$$\delta_{EW} \simeq \delta_{SM} \simeq \tilde{\Sigma}'_W(M_W^2) \simeq -\frac{\tilde{\Sigma}'_W(0)}{M_W^2} = \Delta r^{SM},$$

where we have used the fact that $\Delta r$ constitutes the bulk of the renormalized wavefunction factor $\tilde{\Sigma}'_W(M_W^2)$ associated to the $W$-field. Indeed, the unrenormalized factor $\Sigma'_W(M_W^2)$ does not contain the leading SM fermionic contributions. These are brought along by the corresponding counterterm $\delta Z_W^2$:

$$\tilde{\Sigma}'_W(M_W^2) = \Sigma'_W(M_W^2) - \delta Z_W^2,$$

with

$$\delta Z_W^2 = \left. \frac{\Sigma_\gamma(k^2)}{k^2} \right|_{k^2=0} + \frac{2c_W}{s_W} \Sigma_\gamma(0) + \frac{c_W^2}{s_W^2} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right),$$

$$\simeq -\Delta \alpha + \frac{c_W^2}{s_W^2} \delta \rho + \ldots \simeq -\Delta r + \ldots,$$

where we have formally neglected UV-divergent terms which cancel in the full expression (22). Similarly, we have used the formal approximation

$$\delta M_{W,Z}^2 = -\Sigma_{W,Z}(M_{W,Z}^2) \simeq -\Sigma_{W,Z}(0) + \ldots$$

(we dismiss terms other than leading fermionic contributions) and the standard formulae

$$\Delta r = \Delta \alpha - \frac{c_W^2}{s_W^2} \delta \rho + (\Delta r)_{rem.},$$

$$\Delta \alpha = \frac{\Sigma_\gamma(k^2)}{k^2} \bigg|_{k^2=M_Z^2} - \frac{\Sigma_\gamma(k^2)}{k^2} \bigg|_{k^2=0},$$

$$\delta \rho = \frac{\Sigma_W(0)}{M_W^2} - \frac{\Sigma_Z(0)}{M_Z^2},$$

which we consider within the same leading fermionic approximation. The oblique contribution (21) cancels out in the $G_F$-scheme — Cf. eq.(16) — but, as mentioned before, in the SM one is still left with a non-oblique remainder of about +1.7% which is basically unaltered by the extra Higgs effects of the MSSM.

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4The notation is as in eqs. (27), (31) and (32) of Ref. [1].
From the foregoing we have $\delta_{SM} \simeq \Delta r^{SM} \simeq +(3 - 4)\%$. However, by inspection of Fig.1a we see that typically the SUSY correction reads $\delta_{SUSY} = -(2 - 4)\%$, i.e. it is negative and may cancel to a large extent against the previous SM contribution. Thus the upshot is that

$$\delta_{MSSM} \simeq \delta_{SUSY} + \delta_{SM} + \delta_{QCD} \simeq \delta_{QCD}.$$  \hfill (26)

In other words, the total MSSM correction to $\Gamma(t \rightarrow W^+ b)$ is mostly the standard QCD correction. For this reason we have not plotted the total $\delta_{MSSM}$ in Fig.1 but only the relevant SUSY part. At the end of the day, the net SUSY quantum signature potentially hidden behind the conventional top quark decay is fairly disappointing. This scanty result is reminiscent of the analogue one with $Z$-boson physics mentioned above, the “problem” being that in both cases the original vertex is a gauge boson-fermion-fermion interaction where the SUSY quantum effects are doomed to be very small\textsuperscript{5}.

The case with the decay $t \rightarrow H^+ b$ is quite another story, the corrections being much larger (Cf. Figs.2 and 3a-3d) than in the decay $t \rightarrow W^+ b$. Most important, the genuine SUSY part of these corrections can also be very large and consequently the total MSSM correction is highly modulated by the SUSY components. The only feature that $t \rightarrow H^+ b$ has in common with $t \rightarrow W^+ b$ is that the total Higgs contribution ($\delta_H$) is also very small (Fig.3a).

In Fig.2 we show the partial width $\Gamma(t \rightarrow H^+ b)$ including all the MSSM effects. This quantity is a physical observable and so is independent of our particular renormalization scheme. In the same figure we have simultaneously plotted the tree-level width and the conventional QCD-corrected width (without any sort of SUSY effect), just to show that in the presence of virtual SUSY effects the physical value of that observable could be significantly different. As it is plain from Fig.2 (see also Fig.3a), already without SUSY the standard QCD effects are rather large and have opposite sign to the SUSY effects. Finally, in Fig.2 we have also included the MSSM-corrected partial width of the conventional top quark decay, $\Gamma(t \rightarrow W^+ b)$. As noted above, for a typical choice of sparticle masses, the SUSY part of the correction to that decay is very small, and cancels in part against the standard electroweak correction, $\delta_{EW} \simeq \Delta r^{SM}$. The evolution with tan\(\beta\) of $\Gamma(t \rightarrow W^+ b)$ is very mild as compared with that of $\Gamma(t \rightarrow H^+ b)$.

To appraise the relative importance of quantum SUSY physics on $\Gamma(t \rightarrow H^+ b)$, in Fig.3a we chart the various types of individual MSSM corrections using the very same notation defined above for the standard decay $t \rightarrow W^+ b$. The dependence on the squark

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\textsuperscript{5}For instance, it is well-known that the “$R_b$ anomaly”, in its worse episode, could not be completely accounted for in the MSSM, not even pushing the bottom quark Yukawa coupling up to the maximum allowed value by perturbation theory\textsuperscript{28}. And to cure that anomaly it only required an additional SUSY correction to the $Z \rightarrow b\bar{b}$ partial width of less than +3%, which the MSSM could hardly afford! As expected, the size (in absolute value) of the maximum SUSY correction demanded to $\Gamma(Z \rightarrow b\bar{b})$ is of the same order, though obviously a bit smaller, than the maximum SUSY effect found on $\Gamma(t \rightarrow W^+ b)$. 

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and gluino masses is exhibited in Figs. 3b-3d. A distinctive feature of these corrections stands out immediately: viz. they grow very fast with $\tan \beta$. For example, from Fig.3a we read $\delta_{\text{MSSM}} \simeq +27\%$ for $\tan \beta = 35 \simeq m_t/m_b$; and at $\tan \beta \simeq 50$, which is the preferred value claimed by $SO(10)$ Yukawa coupling unification models \cite{24}, the correction is already $\delta_{\text{MSSM}} \simeq +55\%$. These are truly large corrections which occur for values of the sparticle masses of $O(100)$ GeV. The evolution with $m_{\tilde{b}_1}$ and $m_{\tilde{t}_1}$ shows a slow decoupling (Figs.3b,c) while the dependence on $m_{\tilde{g}}$ is such that, locally, the SUSY-QCD corrections slightly increase with $m_{\tilde{g}}$ (Cf. Fig.3d) and eventually decouple (not shown).

Quite remarkably, the decoupling rate turns out to be so slow that one may reach $m_{\tilde{g}} \sim 1$ TeV without yet undergoing dramatic suppression.

It is patent from Figs.2-3 that, after including the MSSM radiative corrections, the branching ratio of the charged Higgs mode can be of order 50\% for $\tan \beta$ near a typical $SO(10)$ point $\tan \beta = 45 - 50$. To be sure, such large values of the branching ratio of the Higgs mode are not in contradiction with the known data on top decays at the Tevatron, as explained in Ref.\cite{19}. In contrast, with only standard QCD effects, the branching ratio of $t \rightarrow H^+ b$ could barely reach 20\% for the same values of the parameters. It is thus crystal clear that if those SUSY quantum signatures are there, they could hardly be missed in the next generation of experiments!

I shall not dwell on the complicated structure of the full one-loop MSSM effects \cite{19}. Nevertheless, a few remarks may rapidly convince us that important corrections on the decay width of $t \rightarrow H^+ b$ were, indeed, to be expected. This can be understood from the counterterm structure of the charged Higgs-fermion-fermion vertex, eq.(4), which is very different from that of the gauge boson-fermion-fermion vertex. In fact, the corresponding counterterm Lagrangian is the following:

$$\delta \mathcal{L}_{Hbt} = \frac{g}{\sqrt{2} M_W} H^- \bar{b} [\delta C_R m_t \cot \beta P_R + \delta C_L m_b \tan \beta P_L] t + \text{h.c.}, \quad (27)$$

with

$$\delta C_R = \frac{\delta m_t}{m_t} - \frac{\delta v}{v} + \frac{1}{2} \frac{\delta Z_{H^+}}{Z_{H^+}} + \frac{1}{2} \frac{\delta Z_L}{Z_L} + \frac{1}{2} \frac{\delta Z_R}{Z_R} - \frac{\delta \tan \beta}{\tan \beta} + \delta Z_{HW} \tan \beta,$$

$$\delta C_L = \frac{\delta m_b}{m_b} - \frac{\delta v}{v} + \frac{1}{2} \frac{\delta Z_{H^+}}{Z_{H^+}} + \frac{1}{2} \frac{\delta Z_L}{Z_L} + \frac{1}{2} \frac{\delta Z_R}{Z_R} + \frac{\delta \tan \beta}{\tan \beta} - \delta Z_{HW} \cot \beta, \quad (28)$$

where in our scheme $\delta \tan \beta$ is given by eq.(13).

Clearly, from eq.(28) large SUSY effects on $t \rightarrow H^+ b$ are attainable at high $\tan \beta$, mainly because of the presence of the counterterm $\delta m_b/m_b$. This counterterm receives large contributions from both SUSY-QCD (strong) and SUSY-EW (electroweak) loops. Formally, we have here the same one-loop threshold effect from massive particles that one has to introduce in order to correct the ordinary massless contributions (i.e. to correct the standard QCD running bottom quark mass, see below) in SUSY GUT models \cite{29}. 


In these models, a non-vanishing sbottom mixing may lead to important SUSY-QCD quantum effects on the bottom mass: \( m_b = m_{b}^{\text{GUT}} + \Delta m_b \), where \( \Delta m_b \) is proportional to \( M_{LR}^b \rightarrow -\mu \tan \beta \) at sufficiently high \( \tan \beta \). In our case, however, the bottom mass is an input parameter for the on-shell scheme and the effect obviously has a different physical meaning.

Although the results presented in Figs. 2-3 include the full correction from all possible one-loop diagrams in the MSSM (see Ref.\cite{19} for a full list), it is illustrative to pick up the leading contributions stemming from the \( \delta m_b/m_b \) term\footnote{Previous partial results can be found in the literature \cite{30}, but in no one of them the leading effects were clearly recognized nor the relative importance of the various contributions was fully assessed.}. This is most easily done from the diagrams in the electroweak-eigenstate basis. From mixed LR-sbottoms and gluino loops in Fig.4a we find the finite term

\[
\left( \frac{\delta m_b}{m_b} \right)_{\text{SUSY-QCD}} = C_F \frac{\alpha_s(m_t)}{2\pi} m_{\tilde{g}} M_{LR}^b I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) \\
\quad \rightarrow \quad -\frac{2\alpha_s(m_t)}{3\pi} m_{\tilde{g}} \mu \tan \beta I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}),
\]  

(29)

where \( C_F = (N_c^2 - 1)/2 N_c = 4/3 \) is a colour factor. The last result holds for sufficiently large \( \tan \beta \). We have introduced the positive-definite function

\[
I(m_1, m_2, m_3) = \frac{m_1^2 m_2^2 \ln m_3^2 - m_2^2 m_3^2 \ln m_1^2 + m_2^2 m_3^2 \ln m_1^2}{(m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_1^2 - m_3^2)}.
\]

(30)

As an aside, we observe that the so-called light gluino scenario would not favour the charged Higgs decay of the top quark since eq.(29) vanishes for \( m_{\tilde{g}} = 0 \). Still, sizeable electroweak supersymmetric effects could be around.

The bulk of the SUSY electroweak corrections is also buried in \( \delta m_b/m_b \). They are induced by \( \tan \beta \)-enhanced Yukawa couplings of the type \( (1) \). Specifically, from loops involving mixed LR-stops and mixed charged higgsinos (Cf. Fig.4b), one finds:

\[
\left( \frac{\delta m_b}{m_b} \right)_{\text{SUSY-Yukawa}} = -\frac{h_t h_b}{16\pi^2} \frac{\mu}{m_t} M_{\text{LR}}^t I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu) \\
\quad \rightarrow \quad -\frac{h_t^2}{16\pi^2} \mu \tan \beta A_t I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu),
\]  

(31)

where again the last expression holds for large enough \( \tan \beta \).

Similarly, the main source of process-dependent effects lies in the corrections generated by the \( \tau \)-mass counterterm, \( \delta m_\tau/m_\tau \), and can be easily picked out in the electroweak-eigenstate basis (see Fig.4c-4d) much in the same way as we did for the \( b \)-mass counterterm. There are, however, some differences, as can be appraised by comparing the diagrams in Figs.4a,b and c,d, where we see that in the latter case the effect derives from diagrams
involving $\tau$-sleptons with gauginos or mixed gaugino-higgsinos. An explicit computation of the leading pieces yields

$$
\frac{\delta m_\tau}{m_\tau} = \frac{g'^2}{16\pi^2} \mu M' \tan \beta I(m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}, M')
+ \frac{g^2}{16\pi^2} \mu M \tan \beta I(\mu, m_{\tilde{\nu}_\tau}, M),
$$

where $g' = g s_W/c_W$ and $M'$, $M$ are the soft SUSY-breaking Majorana masses associated to the bino $\tilde{B}$ and winos $\tilde{W}^\pm$, respectively, and the function $I(m_1, m_2, m_3)$ is again given by eq. (30).

We may now ascertain the reason why $t \to W^+ b$ and LEP physics cannot generate comparably large quantum SUSY signatures. The counterterm configuration associated to vertices involving gauge bosons and conventional fermions [21] does not involve the term $\delta m_b/m_b$ nor any similar structure, so that one cannot expect enhancements of the type mentioned above. Therefore, SUSY-QCD corrections to $t \to W^+ b$ are not foreseen to be particularly significant in this case, but just of order $\alpha_s(m_t)/4\pi$ without any enhancement factor. This is borne out by the numerical analysis in Fig.1. The only hope for gauge boson interactions with top and bottom quarks to develop sizeable radiative corrections is to appeal to large non-oblique corrections triggered by the Yukawa terms (1). However, even in this circumstance the results are rather disappointing. For, at large $\tan \beta \geq m_t/m_b$, the bottom quark Yukawa coupling (the only relevant one in these conditions) gives a contribution of order

$$
\frac{\alpha_W}{4\pi} \frac{m_b^2}{M_W^2} \tan^2 \beta \geq \frac{\alpha_W}{4\pi} \frac{m_t^2}{M_W^2},
$$

which is numerically very close to the strong contribution $\alpha_s(m_t)/4\pi$. In contrast, the SUSY-QCD effect associated to (29) is of order

$$
C_F \frac{\alpha_s}{4\pi} \tan \beta,
$$

where we have approximated $I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{\tau}}) \simeq 1/(2\tilde{m}^2)$ in the limit where there is no large hierarchy between the sparticle masses, i.e. all of them being of order $\tilde{m}$. The ratio between (34) and (33) is

$$
C_F \left( \frac{\alpha_s}{\alpha_W} \right) \left( \frac{M_W}{m_t} \right)^2 \tan \beta = O(1) \tan \beta.
$$

Consequently, in the entire high $\tan \beta$ regime below the perturbative limit (2), the SUSY-QCD effects on the $t b H^\pm$-vertex are expected to be a factor of order $\tan \beta$ larger than the SUSY-QCD and Yukawa coupling effects on $t b W^\pm$ and $b\bar{b} Z$.

Similar considerations apply for the SUSY electroweak corrections from (31). Even though they are subleading as compared to the strong supersymmetric effects, they are generally larger than the electroweak supersymmetric effects on gauge boson observables.
As a matter of fact, they embody the (non-negligible) 20% residual MSSM contribution left over after the SUSY-QCD and standard QCD corrections cancel out at \( \tan \beta \gtrsim 30 \) (see Fig.2a).

The dominant MSSM effects on \( \Gamma_H \) are, by far, the QCD and SUSY-QCD ones. As noted above, even the ordinary QCD effects [31] can be very large here. At large \( \tan \beta \) they are approximately given by

\[
\delta_{QCD} = -\frac{2 \alpha_s(m_t)}{\pi} \left( \frac{8\pi^2 - 15}{36} + \ln \frac{m_t^2}{m_b^2} \right) \approx -62\% \quad (\tan \beta >>> \frac{\sqrt{m_t/m_b}}{6}). \tag{36}
\]

The exact \( O(\alpha_s) \) formula gives slightly below \(-60\%\), and for better accuracy one should also include renormalization group improvement. The big log factor \( \ln m_t^2/m_b^2 \) is the necessary building block to construct the running b-quark mass at the top quark scale, which is the energy scale of the process. The QCD correction is of course negative since \( \alpha_s(q^2) \) is asymptotically free and so it significantly decreases from \( q^2 = m_b^2 \) to \( q^2 = m_t^2 \).

We note that there can be a strong competition between QCD and SUSY-QCD effects. For \( \mu < 0 \) (which in practice is the only tenable possibility [19]) \( \delta_{QCD} \) and \( \delta_{SUSY-QCD} \) have opposite signs (compare with the leading SUSY-QCD term, eq.(29)). Therefore, there is a crossover point of the two strongly interacting dynamics where the conventional QCD loops are fully cancelled by the SUSY-QCD loops. This leads to the funny situation noted above, namely, that the total MSSM correction is given by just the subleading, albeit non-negligible, electroweak supersymmetric contribution: \( \delta_{MSSM} \approx \delta_{SUSY-EW} \). The crossover point occurs at \( \tan \beta \gtrsim m_t/m_b \), where \( \delta_{SUSY-EW} \gtrsim 20 \). For larger and larger \( \tan \beta > m_t/m_b \), the total (positive) MSSM correction grows very fast, since the SUSY-QCD loops largely overcompensate the standard QCD corrections. As a result, the net effect on the partial width appears to be opposite in sign to what might naively be “expected” (i.e. the QCD sign). This should leave an indelible imprint on the quantum dynamics of the decay mode \( t \to H^+ b \) which could be crucial to identify the SUSY nature of \( H^\pm \). While a first significant test of these effects could possibly be performed at the upgraded Tevatron, a more precise verification would most likely be carried out in future experiments at the LHC.

A typical common set of inputs has been chosen in Figs.2-3 such that the supersymmetric electroweak corrections do reinforce the strong supersymmetric effects (SUSY-QCD). For this set of inputs, the total MSSM correction to the partial width of \( t \to H^+ b \) is positive for \( \tan \beta > 20 \) (approx.). This is because the leading SUSY-QCD correction (at high \( \tan \beta \)) is proportional to \(-\mu \) (Cf. eq.(29)) and we are concentrating our numerical analysis on just the \( \mu < 0 \) case.

Together with \( A_t > 0 \), this yields \( A_t \mu < 0 \), which is a sufficient condition [32] for the MSSM prediction of \( BR(b \to s \gamma) \) to be compatible with experiment in the presence
of a relatively light charged Higgs boson (as the one participating in the top decay under study). We recall that charged Higgs bosons of $\mathcal{O}(100)\,\text{GeV}$ interfere constructively with the SM amplitude and would render a final value of $BR(b \to s\,\gamma)$ exceedingly high. Fortunately, this conclusion can be circumvented in the MSSM since the alternative contribution from charginos and stops tends to cancel the Higgs contribution provided that $A_t \mu < 0$. Furthermore, one must also require relatively light values for the masses of the lightest representatives of these sparticles, as well as high values of $\tan \beta$; hence one is led to a set of conditions which fit in with nicely to build up a favourable scenario for the decay $t \to H^+ b$.

3. Direct $t \to$ SUSY decays

Due to the large mass of the top quark, there is plenty of phase space available for two-body and multibody decays. As for the direct decays of the top quark into two (R-odd) SUSY particles the leading modes are the following [33, 34]. If there is a light gluino window, $m_{\tilde{g}} = \mathcal{O}(1)\,\text{GeV}$, a pure SUSY-QCD decay is conceivable:

$$t \to \tilde{t}_1 \tilde{g},$$

but, then, that top decay would be overwhelming and should have been observed at the Tevatron. Therefore, since a light stop is recommended [3], we shall consider that decay unlikely. On the SUSY electroweak side we have some decays which cannot be ruled out yet:

$$t \to \tilde{b}_a \chi^+_i,$$

and

$$t \to \tilde{t}_a \chi^0_\alpha.$$

Notice that several final states with chargino-neutralinos could be kinematically permitted among the possible ones ($a = 1, 2; i = 1, 2; \alpha = 1, \ldots, 4$). In Figs. 5a-5b we illustrate a case where some of these two-body decays could be very important as compared to the SM decay. In particular, the most favoured modes involve, not the lightest, but the next-to-lightest chargino-neutralino, the reason being that the latter are more higgsino-like and so have larger Yukawa couplings of the form (1) than the former.

Be as it may, it could well happen that our most cherished SUSY two-body decays are not kinematically allowed or are significantly suppressed in some regions of parameter space. In this circumstance, some supersymmetric three-body decays could still be significant. Only very recently a systematic study of the direct top decays into three (R-odd) SUSY particles has been made available in the literature, see Ref.[34]. These 3-body decays could be important not only because they may lead to exotic, highly non-standard,
signatures (see Tables I and II of Ref. [34]) but also from the point of view of a future determination of the total top quark width, $\Gamma_t$.

Indeed, for a consistent treatment of the “quantum signatures” embodied in the observable $\Gamma_t$ at second order of perturbation theory (in the strong and electroweak gauge couplings) one should include the tree-level contributions from all possible three-body decays of the top quark in the MSSM. As it happens, the contribution of some of these three-body decays turns out to be comparable to the largest SUSY quantum effects on the standard top quark decay, $t \rightarrow W^+ b$.

From the point of view of an inclusive model-independent measurement of the total top-quark width, $\Gamma_t$, the future $e^+ e^-$ supercollider (NLC) should be a better suited machine. In an inclusive measurement, all thinkable non-SM effects would appear on top of the corresponding SM effects already computed in the literature [26]. As shown in Ref. [35], one expects to be able to measure the top-quark width in $e^+ e^-$ supercolliders at a level of $\sim 4\%$ on the basis of a detailed analysis of both the top momentum distribution and the resonance contributions to the forward-backward asymmetry in the $t\bar{t}$ threshold region.

Although there are a roster of supersymmetric three-body decays of the top quark that could be kinematically open in the light of the present sparticle bounds, only a few can be of interest. In Ref. [34], we arrived at the conclusion that the (potentially) most relevant modes are the following:

IV. $t \rightarrow b \chi^0 \chi^+_i$

VIII. $t \rightarrow b \tilde{g} \chi^+_i$

IX. $t \rightarrow b \bar{t}_b \bar{b}_a$

X. $t \rightarrow b \bar{\tau}^+ \bar{\nu}_\tau$,

which we assign the same numbering as in Ref. [34]. Let us denote by $\delta_i$ ($i = IV, VIII, IX, X$) the partial width of these supersymmetric three-body decays of the top quark relative to the SM partial width, $\Gamma_{SM} \equiv \Gamma(t \rightarrow W^+ b)$: i.e.

$$\delta_i = \frac{\Gamma(t \rightarrow 3 \text{ body})}{\Gamma_{SM}}.$$  \hspace{1cm} (41)

In Fig.5c, $\delta_{IV}$ is studied as a function of $\tan \beta$ for parameter values close to those used in Section 2. In this frame of inputs, Decay IV becomes more suppressed as compared to the pay-off delivered in optimizing conditions [34], but still it can reach a few percent, and thus be non-negligible as compared to the ordinary QCD corrections to the standard top width $\Gamma(t \rightarrow W^+ b)$. Decay VIII on the other hand, can yield $\delta_{VIII} = 2\%$ (not shown) in similar conditions, but it requires light gluinos.
As for the Decay IX, it is a very interesting three-body decay. Nevertheless it can be relevant only if the 2-body channel $H^+ \to \tilde{t} \tilde{b}$ is kinematically forbidden, otherwise the Higgs decay width becomes too large and it has a dramatic suppression on the relevant 3-body mode. In Fig.5d we study the evolution of $\delta_{IX}$ as a function of $\tan \beta$. Among the four amplitudes contributing to this decay, the most relevant one is the Higgs mediated amplitude, since it contains the Higgs-stop-sbottom coupling which can be very large. For parameter values comparable to those in Section 2, $\delta_{IX}$ can reach a few percent and thus also compete with the quantum effects on $t \to W^+ b$. Decay X is similar to IX but it is numerically less significant.

Finally, we mention that one can envision situations where all these 3-body decays can be further enhanced [34]. In particular, Decay IX is extremely sensitive to large values of $\mu$, $A_b$ and $\tan \beta$, which in appropriate conditions could make it competitive with any of the aforementioned two-body modes.

4. Hadronic Higgs decays in the MSSM

We can easily convince ourselves of the relevance of addressing the issue of the width of a Higgs boson. Just notice that if a heavy neutral Higgs is discovered and is found to have a narrow width, it would certainly not be the SM Higgs, whilst it could be a SUSY Higgs. For, a heavy enough SM Higgs boson is expected to rapidly develop a broad width through decays into gauge boson pairs whereas the SUSY Higgs bosons cannot in general be that broad since their couplings to gauge bosons are well-known to be suppressed [15]. In compensation, their couplings to fermions (especially to heavy quarks) can be considerably augmented. Thus the width of a SUSY Higgs should to a great extent be given by its hadronic width; even so a heavy $H^0$ and $A^0$ is in general narrower than a SM Higgs of the same mass. Alternatively, if the discovered neutral Higgs is sufficiently light that it cannot decay into gauge boson pairs, its decay width into relatively heavy fermion pairs such as $\tau^+ \tau^-$, and especially into $b \bar{b}$, could be much larger than that of the SM Higgs, because of $\tan \beta$-enhancement of the fermion couplings [15]. Hence, it becomes clear that the hadronic width may play a very important role in the study of the MSSM higgses, already at the tree-level.

Let us now consider the Higgs decays most sensitive to our SUSY quantum signatures. If the charged Higgs mass turns out to be larger than $m_t - m_b$, and the top quark decay $t \to H^+ b$ is thus kinematically forbidden, still interesting information could be extracted from virtual Higgs contribution to $t \to b \tau \bar{\nu}_\tau$ [14] plus SUSY corrections. However, for very large Higgs masses, we could still move to LHC physics and try to extract information from the real charged Higgs decay $H^+ \to t \bar{b}$. Similarly, there are two regimes of Higgs
masses \((M_{A^0} < 2 m_t \text{ and } M_{A^0} > 2 m_t)\) where the neutral Higgs can decay into hadrons, basically \(\Phi^i \rightarrow q \bar{q}\) with \(\Phi^i = A^0, h^0, H^0\) and \(q = t, b\). In the MSSM, \(h^0\) cannot decay into \(t \bar{t}\), but \(H^0\) and \(A^0\) can decay both into \(b \bar{b}\) and perhaps also into \(t \bar{t}\), depending on their masses. These \(q \bar{q}\) modes are the dominant decays of the charged and neutral Higgs particles in the MSSM. Although a fully-fledged one-loop analysis of the MSSM effects on these hadronic modes is not available within one and the same renormalization framework, the main features from SUSY-QCD are known.

We define for the Higgs decays a relative correction, \(\delta\), similar to eq. (15). In Figs. 6a-6b we deal with the strong (gluino mediated) SUSY effects \((\delta_{\tilde{g}})\) on \(H^+ \rightarrow t \bar{b}\) [38]. We show the dependence of \(\delta_{\tilde{g}}\) on the higgsino-mixing parameter \(\mu\) and on the lightest sbottom mass. In Figs. 7a-7b we illustrate the behaviour of the strong SUSY effects on the hadronic neutral modes, as a function of \(M_{A^0}\) and various \(\tan \beta\). The other Higgs masses are determined by the usual MSSM Higgs mass relations. Since we are not including the electroweak supersymmetric effects, we have used the tree-level mass relations.

In Fig. 7 we have simultaneously plotted the standard (gluon mediated) QCD corrections \((\delta_g)\). The QCD effects for \(H^+ \rightarrow t \bar{b}\) are also important (see Fig. 5 of Ref.[38]). In all Higgs decays (charged and neutral) large supersymmetric effects are obtained at high \(\tan \beta\) which could significantly modulate the conventional QCD corrections. Also noticeable is the high sensitivity of these decays to the higgsino-mixing parameter \(\mu\). We observe that for the \(b \bar{b}\) final states, the dominant part of these corrections again originates from the bottom mass counterterm, eq.(29). For the \(t \bar{t}\) final states, instead, the leading corrections come from the vertex form factors; however, in this case the partial width becomes smaller the higher is \(\tan \beta\). Finally, we emphasize that in general all these effects remain substantial (> 10 – 20%) even for all sparticle masses well above the LEP 200 discovery range.

While we have just shown the impact of the standard QCD and SUSY-QCD corrections, it is legitimate to worry about the larger and far more complex body of electroweak quantum effects, especially those coming from enhanced Yukawa couplings. For the neutral Higgs bosons, this issue has already been addressed in other renormalization frameworks [22], but there is no corresponding study of \(H^+ \rightarrow t \bar{b}\). A full analysis within our renormalization scheme is under way and will be presented elsewhere [25]. Our preliminary results indicate that the electroweak supersymmetric corrections are not negligible but are definitely subdominant, except in the (unlikely) light gluino scenario.

\(^7\)See Ref.[37] for a recent comprehensive study of the two-body Higgs decays at the tree-level in the MSSM.

\(^8\)If the Higgs particles of the MSSM can decay into real squarks, radiative corrections to the partial widths can also be significant [12]. However, the SUSY corrections to the more conventional hadronic modes \(H^+ \rightarrow t \bar{b}\) and \(\Phi^i \rightarrow q \bar{q}\) remain large even if sparticles are heavy enough that Higgs bosons cannot decay directly into them [38].
In spite of the fact that we are focusing on decay processes, we should not forget that the $t\,b\,H^\pm$-vertex, and similar $\Phi^i\,q\,\bar{q}$ neutral Higgs vertices, can also be involved in the production mechanisms (see Fig.8) and undergo significant renormalization by SUSY effects. In hadron machines an actual measurement of the hadronic partial widths and in general of the effective hadronic vertices $t\,b\,H^\pm$ and $\Phi^i\,q\,\bar{q}$ ($q = t, b$) should be feasible. Let us briefly remind of the five basic mechanisms for neutral Higgs production in a hadron collider\cite{13}. They have been primarily described for the the SM Higgs, $H^0_{SM}$, but can be straightforwardly extended to the three neutral higgses, $\Phi^i$, of any 2HDM:

- (i) Gluon-gluon fusion: $g\,g \to \Phi^i$;
- (ii) $WW(ZZ)$ fusion: $q\,\bar{q} \to q\,\bar{q}\,\Phi^i$;
- (iii) Associated $W(Z)$ production: $q\,\bar{q} \to W(Z)\,\Phi^i$;
- (iv) $t\,\bar{t}$ fusion: $g\,g \to t\,\bar{t}\,\Phi^i$, and
- (v) $b\,\bar{b}$ fusion: $g\,g \to b\,\bar{b}\,\Phi^i$.

It is well-known\cite{14} that, in the SM, $g\,g \to H^0_{SM}$ fusion provides the dominant contribution over most of the accessible range. Nevertheless, for very large (obese) SM Higgs mass ($M_{H^0_{SM}} > 500\,GeV$), the $WW(ZZ)$-fusion mechanisms eventually takes over; the rest of the mechanisms are subleading, and in particular $b\,\bar{b}$ fusion is negligible in the SM. Remarkably enough, this situation could drastically change in the MSSM. For instance, whereas one-loop $g\,g$-fusion in the SM is dominated by a top quark in the loop, this is not always so in the MSSM where the new couplings turn out to enhance, at high $\tan\beta$, the $b$-quark loops and make them fully competitive with the top quark loops. Another example: $b\,\bar{b}$ fusion, which is negligible in the SM, can be very important in the MSSM at large $\tan\beta$. As a matter of fact, for large enough $\tan\beta$, the $b\,\bar{b}$-fusion cross-section can be larger than that for any mechanism for producing a SM Higgs boson of similar mass.

What about the quantum effects on these production vertices? The conventional QCD corrections to $g\,g \to H^0_{SM}$ are known to be large\cite{15}. A similar conclusion holds for an obese SM Higgs boson produced at very high energies by means of the $WW(ZZ)$-fusion mechanisms; here, again, non-negligible radiative effects do appear\cite{16}. Therefore, the production cross-section for $H^0_{SM}$ is expected to acquire valuable quantum corrections both for light and for heavy Higgs masses. This is not so for the corresponding width. In fact, only for a heavy SM Higgs, namely, with a mass above the vector boson thresholds, the corrections to its decay width can be of interest; for a light SM Higgs, instead, light enough that it cannot decay into gauge boson pairs, the decay width is very small and thus the corresponding quantum effects are of no practical interest.
In contradistinction to the SM case, the hadronic vertices $H^\pm t b$ and $\Phi^i q \bar{q}$ could be the most significant interactions for MSSM higgses irrespective of the value of the Higgs masses. In fact, these vertices can be greatly enhanced and large radiative corrections could further shape the effective structure of these interactions, as for example in the charged Higgs mechanisms sketched in Fig.8. In some of these mechanisms a Higgs boson is produced in association, but in some others (fusion processes) the Higgs boson enters as a virtual particle. Now, however different these production processes might be, all of them are sensitive to the effective structure of the $H^\pm t b$ and $\Phi^i q \bar{q}$ vertices. While it goes beyond the scope of this report to compute the SUSY corrections to the production processes themselves, we have at least faced the detailed analysis of a partial decay width which involves one of the relevant production vertices. In this way, a definite prediction is made on the properties of a physical observable and, moreover, this should suffice both to exhibit the relevance of the SUSY quantum effects and to demonstrate the necessity to incorporate these corrections in a future, truly comprehensive, analysis of the cross-sections, namely, an analysis where one would include the quantum effects on all the relevant production mechanisms within the framework of the MSSM. Some steps in this direction have already been given \cite{47}. For this reason I think that in the future a precise measurement of the various (single and double) top quark production cross-sections \cite{8} will be able to detect or to exclude the $t b H^\pm$-vertex as well as the vertices $\Phi^i q \bar{q}$ involving the neutral Higgs particles of the MSSM and the third generation quarks $q = t, b$.

I conclude the discussion on Higgs physics with a remark on $e^+ e^-$ colliders and super-colliders. Thus far we have mostly elaborated on the Higgs strategies at hadron colliders, such as the Tevatron and especially the LHC. Now of course, in a future NLC, searches for quantum SUSY signatures should also be feasible provided that the following processes can be handled\cite{42}:

\begin{align*}
e^+ e^- &\rightarrow Z h^0 (H^0), \\
e^+ e^- &\rightarrow A h^0 (H^0), \\
e^+ e^- &\rightarrow H^+ H^-.
\end{align*}

The observed cross-sections for these processes are equal to the production cross-sections times the Higgs branching ratios. Hence, in an $e^+ e^-$ environment, one aims more at a measurement of the various branching ratios (or, more precisely: ratios of branching ratios) of the fermionic Higgs decay modes rather than of the partial widths themselves. For instance, in a $e^+ e^-$ machine one would naturally address the measurement of

$$BR(\Phi^i \rightarrow b \bar{b})/BR(\Phi^i \rightarrow \tau^+ \tau^-).$$

\footnote{Other $e^+ e^-$ Higgs boson production processes providing complementary information involve e.g. radiation of Higgs bosons off of top quarks, $ZZ$ fusion or $\gamma \gamma$ collisions, which could also be sensitive to SUSY quantum contributions.}
This observable should receive large SUSY-QCD corrections if $\Phi_i \to b\bar{b}$ proves to be, as we have seen, very sensitive to the strong supersymmetric effects. Unfortunately, among the processes (12) only $e^+ e^- \to Z h^0$ should obviously be accessible to a typical NLC of $\sqrt{s} = 500 GeV$, and maybe even to LEP 200. This is because $m_{h^0} < 150 GeV$ in the MSSM whereas the other Higgs particles can be much heavier. If the latter is indeed the case, and so $M_{A^0}$ is considerably large ($> 400 GeV$), then also the SUSY corrections to $h^0 \to b\bar{b}$ – which we could measure e.g. by means of the the $e^+ e^-$ observable (13) – turn out to be very small (see Fig.7a) and in that event nothing firm could be decided on whether a hypothetically observed $h^0$ with mass $m_{h^0} < 150 GeV$ would be a SUSY Higgs or just the SM Higgs. In contrast, as also seen in Fig.7a, the SUSY corrections to $H^0 \to b\bar{b}$ and $A^0 \to b\bar{b}$ remain much larger, especially at high $\tan \beta$, so that the form factors associated to the vertices ($H^0$, $A^0$) $q\bar{q}$ are sensitive to SUSY corrections in a much wider kinematical range, probably well within the reach of the LHC\(^{[10]}\). Thus, unless all of the MSSM Higgs particles happen to be relatively light, or the nominal NLC energy turns out to be amply superior to $M_{A^0} + M_{H^0} \simeq 2 M_{A^0}$ – a requirement entailing $\sqrt{s} > 2 TeV$ in order to cover the worst possible scenario among the admissible ones – the ability of $e^+ e^-$ super-colliders to fully explore the MSSM Higgs sector would be seriously hampered. Still, if $\sqrt{s} \lesssim 1 TeV$, at least three out of the five possible Higgs final states in eq.(12) could perhaps be accessible\(^{[11]}\).

In general, hadron colliders should have in spite of their own limitations a wider covering and also prove greater sensitivity to SUSY quantum signatures. This is because, as repeatedly emphasized, the main hadronic decay modes involve the same basic dynamics as in the production modes, namely on shell and off-shell Higgs-quark-quark vertices, respectively, so that all Higgs bosons with mass below 1 TeV can be singly produced in a hadron machine and some or all of the production mechanisms are greatly sensitive to SUSY virtual effects.

5. Conclusions

In this talk I have discussed the main top and Higgs decays in the MSSM by emphasizing the role played by the quantum effects as a means to discriminate the potential SUSY dynamics underlying those decays. The impact of direct top decays into sparticles has also been assessed and compared its relative importance with the SUSY quantum effects. While on the one hand the generally small SUSY effects found out on the standard top

\(^{10}\)Higgs bosons unaccessible to LHC, i.e. much heavier than 1 TeV, could hardly be considered as natural objects in the MSSM.

\(^{11}\)For example, the European NLC is nominally planned to cover a range $\sqrt{s} = 500 - 800 GeV$\(^{[15]}\), maybe extendable up to 1 TeV.
quark decay, \( t \rightarrow W^+ b \), unfortunately suggest that only a forlorn attempt could be made to patch up some SUSY physics out of top quark decay dynamics, the potentially large SUSY effects unveiled on the alternative decay \( t \rightarrow H^+ b \) should on the other hand raise a message of hope for SUSY physics in hadron colliders. This conclusion is borne out also by our discussion on charged and neutral Higgs decay processes and Higgs production mechanisms which could be highly sensitive to supersymmetric quantum effects.

Incidentally, we point out that our MSSM analysis of \( t \rightarrow H^+ b \), followed by \( H^+ \rightarrow \tau^+ \nu_\tau \), could have consequences for \( \tau \)-lepton physics at the Tevatron. In particular, the bounds on the parameters \((\tan \beta, M_{H^\pm})\) involved in the charged Higgs decay of the top quark could be significantly affected by quantum SUSY signatures. In fact, the identification of the decay \( H^+ \rightarrow \tau^+ \nu_\tau \) could be a matter of measuring a departure from the universality prediction for all lepton channels, in the sense that the experimental signature for \( t\bar{t} \rightarrow H^+ H^- b\bar{b} \) would differ from that of \( t\bar{t} \rightarrow W^+ W^- b\bar{b} \) by an excess of final states with two \( \tau \)-leptons and two b-quarks and large transverse missing energy.

In practice, \( \tau \)-identification is possible at the Tevatron and there are recent analyses in the literature \cite{49, 50} finding excluded regions in the \((M_{H^\pm}, \tan \beta)\)-plane on the basis of unsuccessful Tevatron searches for \( \tau \)-lepton violations of universality. However, these exclusion plots are not fully watertight and must be revised \cite{51} in the light of our MSSM results on \( t \rightarrow H^+ b \). Moreover, they should also be complemented with the highly competing information extracted from the recent calculation of SUSY corrections to low-energy semileptonic \( B \)-meson decays \cite{52} where, again, a similar pattern of large SUSY quantum corrections is found \cite{12}.

Our general conclusion is extremely encouraging for Tevatron and LHC physics: In view of the potentially large size and large variety of manifestations, quantum effects on physical processes involving \( H^\pm t b \), \( \Phi^i b\bar{b} \) and/or \( \Phi^i t\bar{t} \) interactions could be the clue to the discovery of “virtual” Supersymmetry.

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\(^{12}\)By the same token, one should not conclude – as in Ref.\cite{24} – that the high tan\( \beta \) solution \cite{28} to the (remnant!) \( R_b \) “anomaly” is strongly disfavoured, without re-computing “à la SUSY” the excluded portion of the \((M_{H^\pm}, \tan \beta)\)-plane. In particular, since the high tan\( \beta \) approach to \( R_b \) is extremely sensitive to the value of the CP-odd Higgs mass around 60 GeV \cite{28}, even a slight modification of the allowed region in the \((M_{H^\pm}, \tan \beta)\)-plane induced by the SUSY quantum effects could be crucial \cite{53}.\
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**Figure Captions**

- **Fig.1** Total SUSY correction δ_{SUSY} (strong and electroweak) to the standard top quark decay, t → W^+ b, for given values of the other parameters. In the figure, A_t = A_b ≡ A. Evolution of δ_{SUSY} with: (a) tan β; (b) the lightest sbottom mass, m_{\tilde{b}_1}; (c) the lightest stop mass, m_{\tilde{t}_1}; (d) the gluino mass, m_{\tilde{g}}. In (d) we have split up the SUSY correction into SUSY-EW and SUSY-QCD effects and have also included the standard QCD correction (horizontal dotted line). As a rule, and unless stated otherwise, the parameters not explicitly defined in a figure have common values with the other figures. The masses of the top and bottom quarks are m_t = 175 GeV and m_b = 5 GeV, respectively.

- **Fig.2** The partial widths for the two decays t → W^+ b and t → H^+ b including all MSSM effects, versus tan β, for a common set of input parameters. We have simultaneously plotted the tree-level width and the QCD-corrected width (without SUSY effects) of the decay t → H^+ b. For the standard decay, t → W^+ b, the SUSY correction is very small and exhibits a very mild evolution with tan β for the typical set of parameters given.

- **Fig.3** As in Fig.1, but for the MSSM corrections to the charged Higgs decay of the top quark, t → H^+ b. Particularly, in (a) we have detailed the SUSY-EW, standard EW, SUSY-QCD, standard QCD, and total MSSM contributions.
• **Fig.4** Leading SUSY-QCD (a) and SUSY-EW (b) contributions to $\delta m_b/m_b$ in the electroweak-eigenstate basis. Similarly, (c) and (d) show two independent sources of leading SUSY-EW contributions to $\delta m_\tau/m_\tau$.

• **Fig.5** (a) Partial width of the 2-body SUSY decay $t \to \tilde{b}_a \chi_i^+$, as a function of $\tan \beta$, relative to that of the SM decay $t \to W^+ b$; (b) As in (a), but for the decay $t \to \tilde{t}_a \chi_0^0$; (c) Partial width of the 3-body SUSY decay $t \to b \chi_0^0 \chi_1^+$, as a function of $\tan \beta$, relative to that of the SM decay $t \to W^+ b$; (d) As in (c), but for the decay $t \to b \tilde{t}_b \tilde{b}_a$. In all cases, the results are given for the kinematically allowed mass eigenvalues ($a, b = 1, 2; i = 1, 2; \alpha = 1, ..., 4$) compatible with the given set of inputs.

• **Fig.6** Gluino-mediated SUSY-QCD corrections ($\delta_{\tilde{g}}$) to the partial width of the top quark decay of a charged Higgs boson, $H^+ \to t \tilde{b}$, for fixed values of the other parameters. Evolution of $\delta_{\tilde{g}}$ with: (a) the higgsino-mixing parameter, $\mu$; (b) the lightest sbottom mass, $m_{\tilde{b}_1}$.

• **Fig.7** Standard QCD ($\delta_g$) and SUSY-QCD ($\delta_{\tilde{g}}$) corrections to the hadronic partial widths $\Gamma(\Phi^i \to q \bar{q})$ ($q = b, t$) of the neutral Higgs bosons: (a) Dependence of $\Gamma(\Phi^i \to b \bar{b})$ on the CP-odd Higgs mass, $M_{A^0}$, for fixed values of $\tan \beta$. The other Higgs masses are determined by the usual mass relations in the MSSM Higgs sector. (b) As in (a), but for the $\Gamma(A^0 H^0 \to t \bar{t})$. Here $A_t = 400 \text{GeV}$.

• **Fig.8** Typical diagrams for top quark and charged Higgs production in hadron colliders involving the relevant $t b H^\pm$-vertex. Similar diagrams can be devised for the neutral Higgs boson $\Phi^i q q$-vertices ($q = t, b$) – see Ref. [39].
\[
\delta_{\text{SUSY}} \quad (\mu, M) = (-150, 150) \text{ GeV} \\
\delta_{\text{SUSY}} \quad (-100, 150) \text{ GeV}
\]

\[
m_{\tilde{t}} = 150 \text{ GeV} \\
m_{\tilde{t}} = 100 \text{ GeV} \\
m_{\tilde{t}} = m_{\tilde{\nu}} = 200 \text{ GeV} \\
A = 300 \text{ GeV} \\
m_{\tilde{g}} = 300 \text{ GeV}
\]

\[
t \to W^+ b
\]

\[
\begin{align*}
\text{(a)} & \\
\text{(b)} & \\
\text{(c)} & \\
\text{(d)} & 
\end{align*}
\]

\textbf{Fig. 1}
$M_{H^+} = 120$ GeV, $m_{b_1} = 150$ GeV

$M = 150$ GeV, $A = 300$ GeV

$m_g = 300$ GeV, $m_u = m_v = 200$ GeV

$m_{t_1} = 100$ GeV

$\mu = -150$ GeV

---

$\Gamma_0 (t \rightarrow H^+ b)$

$\Gamma_{QCD} (t \rightarrow H^+ b)$

$\Gamma_{MSSM} (t \rightarrow H^+ b)$

$\Gamma (t \rightarrow W^+ b)$

---

$\tan \beta$

---

Fig. 2
Fig. 3
Fig. 4
\( t \rightarrow b_1 \chi_1^+ \)
\( \mu = -150 \text{ GeV} \)
\( M = 100 \text{ GeV} \)
\( m_{b_1} = 70 \text{ GeV} \)

\( \Gamma / \Gamma_{SM} \)
\( \tan \beta \)

\( t \rightarrow b_1 \chi_1^+ \)
\( \mu = -150 \text{ GeV} \)
\( M = 100 \text{ GeV} \)
\( m_{b_1} = 70 \text{ GeV} \)

\( \Gamma / \Gamma_{SM} \)
\( \tan \beta \)

\( t \rightarrow b \chi \) - (c)
\( m_b = m_t = 120 \text{ GeV} \)
\( A_t = A_b = 300 \text{ GeV} \)
\( M = 100 \text{ GeV} \)

\( \Gamma / \Gamma_{SM} \)
\( \tan \beta \)

\( t \rightarrow b \tilde{t}_1 \tilde{b}_1 \)
\( M_{H^+} = 120 \text{ GeV} \)
\( m_{\tilde{b}_1} = 200 \text{ GeV} \)
\( A_t = -A_b = 300 \text{ GeV} \)
\( M = 150 \text{ GeV} \)

\( \Gamma / \Gamma_{SM} \)
\( \tan \beta \)

Fig. 5
Fig. 6
Fig. 7

(a) $\delta_{g, g}$ vs. $M_{A^0}(\text{GeV})$ for different values of $\tan\beta$. The lines represent different decay modes: $A^0 \rightarrow b\bar{b}$ (dashed), $H^0 \rightarrow b\bar{b}$ (solid), and $h^0 \rightarrow b\bar{b}$ (dotted). The legend shows the decay modes and their corresponding markers.

(b) $\delta_{g, g}$ vs. $M_{A^0}(\text{GeV})$ for different values of $\tan\beta$: $\delta_g \tan\beta = 4$ (filled circle), $\delta_g \tan\beta = 10$ (filled diamond), $\delta_g \tan\beta = 20$ (filled square), and $\delta_g$ (filled triangle).
Fig. 8