Multiparty Quantum Key Agreement That is Secure Against Collusive Attacks

Hussein Abulkasim, Atefeh (Atty) Mashatan, Shohini Ghose

Abstract—Quantum key agreement enables remote users to fairly establish a secure shared key based on their private inputs. In the circular-type multiparty quantum key agreement protocol, two or more malicious participants can collude together to steal the private inputs of honest participants or to generate the final key alone. In this paper, we focus on a powerful collusive attack strategy in which two malicious participants in particular positions, can learn sensitive information or generate the final key alone without revealing their malicious behavior. Most of the proposed circular-type multiparty quantum key agreement protocols are not secure against this particular collusive attack strategy. As an example, we analyze the security of a recently proposed multiparty key agreement protocol to show the vulnerability of existing circular-type multiparty quantum key agreement protocols against this collusive attack. We then design a general secure multiparty key agreement model that would remove this vulnerability from such circular-type key agreement protocols and describe the necessary steps to implement this model. Our model is general and does not depend on the specific physical implementation of quantum key agreement.

Index Terms—Collusive attack, Quantum key agreement, Quantum cryptography, Secure multiparty computation.

I. INTRODUCTION

The concept of key agreement was first presented by Diffie–Hellman in 1976 [1]. It describes how two remote users are able to fairly establish a secured shared key based on their private inputs. In 1982, Ingemarsson et al. [2] extended the two-party key agreement protocol to a multiparty or group key agreement protocol. After that, several multiparty key agreement protocols have been published [3]. However, future quantum computers with sufficient power will threaten most current cryptosystems whose security mainly relies on unproven mathematical assumptions. For that reason, quantum applications in cryptography have attracted the attention of a lot of scientists and researchers in order to develop information-theoretically unconditional secure cryptosystems. One of the most common quantum cryptographic applications is quantum key distribution (QKD) [4], in which remote parties can generate a shared random key securely even in the presence of an attacker with unlimited classical or quantum computing power. Subsequently, several quantum cryptographic applications have been introduced to solve various classical security issues [19]-[31]. Recently, quantum key agreement (QKA) has attracted the attention of a lot of researchers [5]. QKA ensures fairness between the involved parties to generate a shared secure key based on their private inputs. Using the quantum teleportation protocol, Zhou et al. [5], in 2004, presented the first two-party QKA scheme.

In 2013, the two-party QKA was extended to multiparty QKA protocols [6]. Subsequently, several multiparty QKA protocols have been presented [10], [15]-[18]. In general, as noted in [6], there are three types of MQKA protocols: 1) the first type is the tree-type in which every party sends their secret data through independent quantum channels to all other parties [9]; 2) the second type is the complete-graph-type in which every participant sends a sequence of qubits to each of the others parties to encode her or his secret information. 3) while in the third type that is the circle-type (sometimes called traveling-mode) [7], [8], every party generates a random sequence of qubits and sends this sequence to another party who applies an encoding process producing a new evolved sequence of qubits and sends the new sequence to the next party; this process continues over all parties until the evolved sequence reaches the party who generates the first sequence. Compared to the other QKA types, the circle-type is more efficient and more easily achieves the property of fairness. For that reason, the QKA circle-type has been intensively investigated.

In 2016, Liu et al. [6] pointed out that all existing circle-type multiparty quantum key agreement (CT-MQKA) protocols are vulnerable to collusive attack, and asked a challenging question about the possibility of designing a secure CT-MQKA protocol. In response to this question, several CT-MQKA protocols have been proposed to avoid a collusive attack. However, in this work, we show that most of the existing CT-MQKA protocols are also not secure against a collusive attack. We study, as an example, the security of Sun et al.’s MQKA protocol (named SCWZ protocol hereafter) to show the vulnerability of the existing CT-MQKA protocols to collusive attacks. Furthermore, we design a general secure model for CT-MQKA protocols and propose the necessary steps for this model.

Hussein Abulkasim is with Ted Rogers School of Information Technology Management, Ryerson University, Toronto, Canada (e-mail: abulkasim@ryerson.ca).

Atefeh (Atty) Mashatan is with Ted Rogers School of Information Technology Management, Ryerson University, Toronto, Canada (e-mail: amashatan@ryerson.ca).

Shohini Ghose is with the Department of Physics and Computer Science, Wilfrid Laurier University, Waterloo, Canada. She also with the Institute for Quantum Computing, University of Waterloo, Waterloo, Canada (e-mail: sghose@wlu.ca)
II. THE INSECURITY OF EXISTING CT-MQKA PROTOCOLS

In this section, we show that most of the recently published works in CT-MQKA are not secure against collusive attacks [10, 15-18]. In general, there are two main collusive attack strategies, which could be applied to the CT-MQKA protocols:

1) The first collusive attack strategy
The first collusive attack strategy has been pointed out in [6, 10]. Any two dishonest participants $P_i$ and $P_j$ (where $i > j$; $i, j \in \{1, 2, ..., n\}$ and $n$ is the number of participants) in particular positions in the circle-type protocols can control the final key if their particular positions meet the following two conditions:

\[ i - j = \frac{n}{2} \text{ when } n \text{ is even}, \]  
\[ i - j = \frac{n+1}{2} \text{ or } \frac{n-1}{2} \text{ when } n \text{ is odd}. \]  

2) The second collusive attack strategy
The second collusive attack strategy, as pointed out in our previous work [8], can be described as follows. In the CT-MQKA schemes, any two dishonest participants $P_i$ and $P_j$ can steal the private inputs of an honest participant $P_k$ ($i, j, k \in \{1, 2, ..., n\}$) without being detected, if their particular positions meet one of the two following conditions:

\[ i - j = 2; \text{ then } k = i - 1; \]  
\[ j - i = 2; \text{ then } k = j - 1. \]

A. Review of SCWZ’s Protocol

In SCWZ’s protocol, there are $n$ participants and each participant $P_i$ ($i = 1, 2, ..., n$) has an $m$-bit key ($K_i$). The participants want to generate a shared secret key $K$ fairly, where $K = K_1 \oplus K_2 \oplus ... \oplus K_n$. The steps of the SCWZ’s protocol can be described as follows.

1) Preparation phase. The server generates $n$ sequences of random single-photons. Each sequence $S_i$ ($i = 1, 2, ..., n$) contains $m$ single-photons and each photon is selected randomly from the four states $\{|+, \rangle, \{|-, \rangle, \{|0\rangle, \{|1\rangle\}$, where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. The server also generates $n$ sequences of random single photons (called $C_i$), which are used as decoy photons to check the existence of eavesdroppers. Each single decoy photon is randomly selected from the states $\{|+, \rangle, \{|-, \rangle, \{|+y\rangle, \{|-y\rangle\}$, where $|+y\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm iy|1\rangle)$. The server then randomly inserts and distributes the single-photons of $C_i$ into $S_i$ getting a new sequence $S'_i$, and sends the new sequence ($S'_i$) to $P_i$.

2) Detection phase. Upon receiving $S'_i$, each participant sends an acknowledgment to the server. Then the server announces the positions of $C_i$ and their measurement bases. Each $P_i$ measures $C_i$ based on the corresponding measurement bases and stores the results. $P_i$ then randomly announces half of the measurement results of $C_i$; the server, in turn, announces the initial states of the second half of $C_i$. Then both the server and $P_i$ collaborate to compute the error rate. They end the protocol if the error rate is higher than a predefined value. Otherwise, they continue to finish the protocol.

3) After $P_i$ gets the secure sequence $S_i$, each participant performs the next sub-steps:

A. Encoding phase. $P_i$ encodes secret information ($K_i$) onto $S_i$ by applying the unitary operation $U = |0\rangle(|1\rangle - |1\rangle|0\rangle)$ when the classical bit of the secret $K_i$ is 1, and by applying the unitary operation $I = |0\rangle\langle 0| + |1\rangle\langle 1|$ when the classical bit $K_i$ is 0. $P_i$ then reorders the decoy states prepared and inserted by the server in Step (1) and reinserts them in random positions into the encoded sequence obtaining a new sequence ($S_i^{(+)}$), and sends $S_i^{(+)}$ to $P_{i+1}$.

B. Eavesdropping check phase. Upon receiving $S_i^{(+)}$, $P_{i+1}$ and $P_i$ check the security of the transmission by performing the same process indicated in step (2) between the server and $P_i$.

C. Encoding phase. After checking the security of transmission, $P_{i+1}$ encodes secret information ($K_{i+1}$) onto $S_i$ following the same rules as in step (A). $P_{i+1}$ then reorders the decoy states and reinserts them in random positions into the encoded sequence obtaining a new sequence ($S_i^{(+2)}$), and sends $S_i^{(+2)}$ to $P_{i+2}$.

D. Similarly, the rest of the participants ($P_{i+2}, P_{i+3}, ..., P_{i-2}$) perform the Eavesdropping check phase and the Encoding phase indicated in steps (B) and (C).

E. Upon receiving $S_i^{(+2)}$, $P_{i-1}$ and $P_{i-2}$ check the security of transmission. If the quantum channel between $P_{i-1}$ and $P_{i-2}$ is secure, $P_{i-2}$ discards the decoy photons to get $S_i$, and informs the server of this fact.

4) When all the $P_{i-1}$ receive $S_i$, they send an acknowledgment to the server, and the server announces the measurement bases of $S_i$ to all the $P_{i-1}$. After that, each $P_{i-1}$ uses the corresponding measurement bases to measure $S_i$ obtaining $K'_i$, where $K'_i = K_i \oplus K_{i+1} \oplus ... \oplus K_{i-2}$. Finally, $P_{i-1}$ can recover the final shared secret key $K = K'_i \oplus K_{i-1}$.

B. The Collusive Attack Against CT-MQKA Protocols

In this section, we show that the SCWZ’s protocol, as an example of CR-MQKA protocols, is insecure against a collusive attack. Although the authors have presented the security analysis to prove the security of their protocol against the first model of the collusive attack mentioned in Section 2, their protocol is not secure against the second security model of collusive attack. That is to say, any two dishonest participants $P_i$ and $P_j$ in particular positions meeting the conditions in (3) and (4) can easily steal the private key of the honest participants ($P_k$).

Without loss of generality, assume we have three participants $P_1$, $P_2$, and $P_3$ and they have three private keys, e.g., $K_1 = 1000, K_2 = 0101$, and $K_3 = 1001$, respectively. And the three participants intend to share a secret key ($K$), here $K = K_1 \oplus K_2 \oplus K_3 = 0100$. We also assume that $P_1$ and $P_3$ are two
dishonest participants and they need to steal the private key of the honest one ($\mathcal{P}_1$); hence they can deduce the final key without being caught. The server generates three random sequences, e.g., $S_1 = \{\{+\}, \{0\}, \{1\}, \{\}-\}$, $S_2 = \{\{0\}, \{1\}, \{0\}, \{1\}\}$, and $S_3 = \{\{0\}, \{+\}, \{\}-\}, \{1\}\}$ each one consists of four single-photons. Also, the server generates three random sequences $\mathcal{C}_1, \mathcal{C}_2, \text{and } \mathcal{C}_3$ each one consists of four decoy single-photon states. Then the server randomly inserts the decoy state $\mathcal{C}_1 (\mathcal{C}_2/\mathcal{C}_3)$ into $S_1 = \{\{+\}, \{0\}, \{1\}, \{\}-\}$ (i.e., $S_2 = \{\{0\}, \{1\}, \{0\}, \{1\}\}$) and sends it to $\mathcal{P}_1 (\mathcal{P}_2/\mathcal{P}_3)$. After checking the security of the transmission, each participant discards the decoys and encodes their private information based on the encoding rule mentioned in Step 3.A. Subsequently, each participant sends the sequence in a circle to the other participants to encode their private inputs until the sequence is returned back to the participant.

For simplicity, we show here the circle of $S_i$ (Fig. 1.a) which will be used by the participant $\mathcal{P}_i$ to get the final key ($K_i$). First, $\mathcal{P}_1$ encodes a private input, i.e., $K_1 = 1001$ into $S_1$ getting the new sequence $S_1 = \{U\{+\}, U\{0\}, U\{1\}, U\{\}-\}$. Then $\mathcal{P}_1$ inserts some decoy photons into $S_1$ and sends it to the dishonest $\mathcal{P}_3$ instead of sending it to $\mathcal{P}_2$. After checking the security of the transmission, $\mathcal{P}_3$ discards the decoy states and gets $S_1 = \{U\{+\}, U\{0\}, U\{1\}, U\{\}-\}$. In addition, the dishonest $\mathcal{P}_1$ generates a counterfeit sequence, e.g., $S_1^f = \{\{0\}, \{0\}, \{-\}, \{-\}\}$ with decoy states and sends it to both $\mathcal{P}_2$ and $\mathcal{P}_3$. Obviously, $\mathcal{P}_2$ cannot distinguish between the genuine sequences and the counterfeit one. So, $\mathcal{P}_2$ encodes the private data, i.e., $K_2 = 0101$ into $S_1^f$ getting $S_1^f = \{\{0\}, \{0\}, \{-\}, \{-\}, \{U\{+\}\}\}$ and sends $S_1^f$ with decoy states to $\mathcal{P}_3$. After checking the security of the transmission, $\mathcal{P}_3$ discards the decoy qubits and gets $S_1^f$. $\mathcal{P}_3$ then requests the corresponding measurement bases of $S_1^f$ from $\mathcal{P}_i$ to get $K_2 = 0101$. Based on her or his private key, i.e., $K_3 = 1001$ and the private key of $\mathcal{P}_2$, $\mathcal{P}_3$ applies the corresponding unitary operations to the genuine sequence $S_1 = \{U\{+\}, U\{0\}, U\{1\}, U\{\}-\}$.  

The final protocol is described as follows:

1. The server generates $n$ sequences $S_i (i = 1, 2, ..., n)$, with each sequence containing $m+n\ell$ single qubits. The server records the position of each single qubit. Every qubit is selected randomly from the four quantum states $\{|+, -\rangle\} = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle), \{-\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle), |0\rangle, |1\rangle\}$. The server also generates $n$ sequences of random single qubits (called $\mathcal{C}_i$), which are used as decoy states to check the existence of eavesdroppers. Every single decoy qubit is randomly selected from the four quantum states $\{|+, -\rangle, |0\rangle, |1\rangle\}$. Then the server inserts $\mathcal{C}_i$ into $S_i$ producing a new sequence $S_i'$, and sends the new sequence ($S_i'$) to $\mathcal{P}_i$.

2. Upon receiving $S_i'$, every participant sends an acknowledgment to the server. Then the server announces the positions of $\mathcal{C}_i$ and their measurement bases. Every $\mathcal{P}_i$ measures $\mathcal{C}_i$ based on the corresponding measurement bases and stores the results. $\mathcal{P}_i$ then randomly announces half of the measurement results of $\mathcal{C}_i$; the server, in turn, announces the initial states of the second half of $\mathcal{C}_i$. Then both the server and $\mathcal{P}_i$ collaborate to compute the error rate. They end the protocol if the error rate is higher than a predefined value. Otherwise, $\mathcal{P}_i$ discards $\mathcal{C}_i$ from $S_i'$ getting $S_i$ and continues to Step 3.

3. After every $\mathcal{P}_i$ gets the secure sequence $S_i$, each $\mathcal{P}_i$ performs the next substeps:

a) Encoding phase. $\mathcal{P}_i$ encodes the secret information ($K_i'$) onto $S_i$ by applying the unitary operation $I = |0\rangle\langle 0| + |1\rangle\langle 1|$ when the classical bit $K_i$ is 0, and by applying the unitary operation $U = |0\rangle\langle 1| - |1\rangle\langle 0|$ if the classical bit $K_i$ is 1 (see also Table 1).

b) Detecting the external attack phase. For detecting external eavesdroppers, $\mathcal{P}_i$ generates a sequence of random single qubits ($\mathcal{C}_i$), which are used as decoy qubits to check the existence of eavesdroppers in the quantum channel between $\mathcal{P}_i$ and $\mathcal{P}_{i+1}$ (note, the symbol $+$ in “$i + 1$” represents the additional mod $n$). Every single decoy qubit is randomly selected from the four quantum states $\{|+, -\rangle, |0\rangle, |1\rangle\}$. Then $\mathcal{P}_i$ discards $\mathcal{C}_{i+1}$ into $S_i$ producing a new sequence $S_i$, and sends the new sequence ($S_i'$) to $\mathcal{P}_{i+1}$. As in Step (2), $\mathcal{P}_i$ and $\mathcal{P}_{i+1}$ share the information of $\mathcal{C}_{i+1}$ to measure it; then, they collaborate to compute the error rate. $\mathcal{P}_i$ and $\mathcal{P}_{i+1}$ end the protocol if the error rate is higher than a predefined value. Otherwise, $\mathcal{P}_{i+1}$ discards $\mathcal{C}_{i+1}$ from $S_i$ obtaining $S_i$ and continues to the next process.

c) Detecting the internal attack phase. Upon confirming that the communication between $\mathcal{P}_i$ and $\mathcal{P}_{i+1}$ is secure against the external attackers, the server randomly selects $\ell$ single-qubits as decoy qubits from $S_i$, by announcing their positions, and asks $\mathcal{P}_i$ to publicly
announce the unitary operations that were applied to the \( \ell \) qubits. Then, the server announces the measurement bases of the \( \ell \) qubits to \( P_{i+1} \). \( P_{i+1} \) measures the \( \ell \) qubits using the corresponding measurement bases. Based on the measurement results, the measurement bases and the applied unitary operations, \( P_{i+1} \) can judge whether the \( \ell \) qubits are genuine or not. If not, \( P_{i+1} \) ends the protocol. Otherwise, the participants do the following: i) \( P_{i+1} \) discards the \( \ell \) qubits selected by the server, from \( S_{i-1} \); ii) the server also discards the corresponding \( \ell \) qubits from \( S_i \); iii) every \( P_i \) discards the corresponding classical bits from their private keys \( K'_i \).

d) After discarding the \( \ell \) qubits and the corresponding classical bits, \( P_{i+1} \) encodes the secret information \( (K'_i) \) onto \( S_i \) as in Step (3.a), then inserts some random decoy states \( (C_{ph+1}) \) into \( S_{i-1} \) producing \( S_{i-1} \). Then, \( P_{i+1} \) sends \( S_{i-1} \) to \( P_{i+2} \).

e) Upon receiving \( S_{i-1} \), \( P_{i+1} \) and \( P_{i+2} \) collaborate to check the security of communication by performing the Step (3.a – 3.d); then \( P_{i+2} \) encodes her or his information and sends the new sequences to the next participants. This process continues until \( P_i \) receives the secure quantum message \( (S_{i-1}) \) from \( P_{i-1} \); here, the symbol “–” in “\( i-1 \)” represents the subtraction mod \( n \).

Fig. 1. An example of a three-party QKA protocol. Any two dishonest participants in particular positions can steal the private input of an honest participant.

IV. ILLUSTRATION OF THE PROPOSED PROTOCOL

For simplicity, suppose we have three participants \( P_1 \), \( P_2 \), and \( P_3 \) and they want to generate a shared secret key \( K = K_1 \oplus K_2 \oplus K_3 \) with length \( m \) (e.g., \( m = 3 \)). The three participants \( P_1 \), \( P_2 \), and \( P_3 \) have three private keys \( K'_1 \), \( K'_2 \), and \( K'_3 \), respectively, with length \( m+\ell \), e.g., \( m+n \ell = 3 + (3 \times 3) = 12 \); here \( n \ell \) is the number of decoy states for checking the security of all quantum channels in one complete circle, and for the \( n \) circle it will be \( n \times n \ell \). Here, there are three complete circles for three participants, and the number of decoy qubits for checking the security of all quantum channels is \( n \times n \ell = 9 \ell \). Also, we assume that, \( K'_1 = 00001101101 \), \( K'_2 = 111011101000 \), and \( K'_3 = 110011010110 \).

The server generates a sequence of quantum states contains 12 random states (e.g., \( S_1 = |0,0,0,1,0,1,0,1,0,1,0,0\rangle \) for the first circle and sends it to \( P_1 \). \( P_1 \) checks the security of the transmission with the server as in Step (2). Based on her/his private data \( (K'_i) \), \( P_i \) applies the unitary operations \( \{I, I, I, I, I, I, I, I, U, U, U, U\} \) to \( S_1 \) getting \( S_{i-2} = |0,0,0,1,0,1,0,1,0,0,0,0\rangle \). \( P_1 \) then inserts some decoy qubits into \( S_{i-2} \) and sends it to \( P_2 \). \( P_2 \) then performs Step (3.b) to detect the external attack. As in Step (3.c), the server chooses random \( \ell \) states (e.g., \( \ell = 1 \)) from \( S_i \) and announce the position of \( \ell \) (e.g., the position of last state in \( S_i \)) to \( P_1 \) and \( P_2 \). The server then asks \( P_1 \) to announce the unitary operation that was applied to \( \ell \), and asks \( P_2 \) to announce the measurement result of the corresponding states in \( S_{i-2} \) (i.e., \( -|\rangle \)), respectively. Based on the announced information \( (|\rangle, U, -|\rangle) \), the server can judge whether \( P_2 \) has received genuine information or not. Then, the server and \( P_2 \) discard the last sequence from \( S_1 \) and \( S_{i-2} \) getting new updated sequences \( S_1 = (|0,0,0,1,0,1,0,1,0,0,0,0\rangle \) and \( S_{i-2} = (|0,0,0,1,0,1,0,1,0,0,0,0\rangle \), respectively. Also, all participants update their private keys by discarding the corresponding classical bits. The updated private keys of \( P_1 \), \( P_2 \), and \( P_3 \) become \( K'_1 = 00001101101 \), \( K'_2 = 111011101000 \), and \( K'_3 = 1100110110111 \), respectively. They also consume two quantum states (e.g., the last two states) for checking the quantum channel between \( (P_2 \) and \( P_3 \) and \( (P_3 \) and \( P_i \) ). The updated private keys after completing one circle are as follows: \( K'_1 = 00001101101 \), \( K'_2 = 111011101000 \), and \( K'_3 = 1100110110111 \). And the updated private keys after completing the three circles are as follows: \( K'_1 = 000 \), \( K'_2 = 111 \), and \( K'_3 = 110 \). Now, \( |K| = |K'_1| = |K'_2| = |K'_3| = |K'_1| = |K'_2| = |K'_3| \). Finally, each participant can get the final key \( K = K_1 \oplus K_2 \oplus K_3 = 000 \oplus 111 \oplus 110 = 001 \). Note, for simplicity, we assumed that each time, the server chooses the last state for checking the security of communication; but the
selected positions should be completely random.

V. THE SECURITY ANALYSIS

This section presents detailed security analyses for both the external eavesdropping and internal attacks.

A. External eavesdropping

In the proposed protocol, the decoy state technique is used to prevent external eavesdroppers from attacking the protocol. To achieve that, a sequence of single decoy qubits is randomly selected from the states \(|\pm\rangle, |\mp\rangle, |0\rangle, |1\rangle\) and then inserted in random positions into the secret message. The eavesdropper (Eve) cannot distinguish between the decoy-states and secret message states. Eve may try to entangle a secret message state with an auxiliary quantum state \(|\alpha\rangle\) by applying a unitary operation \(U_0\) as follows:

\[
U_0|0\rangle|\epsilon\rangle = a_0|0\rangle|\epsilon_{00}\rangle + a_2|1\rangle|\epsilon_{01}\rangle,
\]

\[
U_0|0\rangle|\epsilon\rangle = a_1|0\rangle|\epsilon_{00}\rangle + a_2|0\rangle|\epsilon_{01}\rangle,
\]

\[
U_1|0\rangle|\epsilon\rangle = \frac{1}{2}|\pm\rangle(\alpha_1|e_{00}\rangle + \alpha_2|e_{01}\rangle + \alpha_3|e_{10}\rangle + \alpha_4|e_{11}\rangle) + |\mp\rangle(\alpha_1|e_{00}\rangle - \alpha_2|e_{01}\rangle + \alpha_3|e_{10}\rangle - \alpha_4|e_{11}\rangle),
\]

\[
U_1|0\rangle|\epsilon\rangle = \frac{1}{2}|\pm\rangle(\alpha_1|e_{00}\rangle + \alpha_2|e_{01}\rangle - \alpha_3|e_{10}\rangle - \alpha_4|e_{11}\rangle) + |\mp\rangle(\alpha_1|e_{00}\rangle - \alpha_2|e_{01}\rangle - \alpha_3|e_{10}\rangle + \alpha_4|e_{11}\rangle).
\]

In (5) and (6), \(|\alpha_1|^2 + |\alpha_2|^2 = 1\) and \(|\alpha_3|^2 + |\alpha_4|^2 = 1\). Also, \(|\epsilon_{00}\rangle, |\epsilon_{01}\rangle, |\epsilon_{10}\rangle\), and \(|\epsilon_{11}\rangle\) are four ancilla states decided by Eve. To pass the external eavesdropping detection phase, Eve sets \(a_2 = a_3 = 0\), if the targeted quantum state is \(|0\rangle\) or \(|1\rangle\), and \((\alpha_1|e_{00}\rangle - \alpha_2|e_{01}\rangle - \alpha_3|e_{10}\rangle + \alpha_4|e_{11}\rangle) = 0\). If the targeted quantum state is \(|\pm\rangle\) or \(|\mp\rangle\), the proposed protocol is not open to the Trojan horse attack since all information is sent in a one-way manner [32].

B. Internal attack

In the QKA protocols, a collusive attack is the most powerful internal attack in which two or more dishonest participants collude together to extract sensitive information or generate the final key alone without revealing their malicious behavior. In this subsection, we show that the proposed model is immune to collusive attacks, such that any group of dishonest participants trying to perform a collusive attack (including the two attack strategies mentioned in section 2) will be detected immediately. Indeed, dishonest participants rely mainly on two important processes to successfully achieve the collusive attack: 1) sharing information about the carrier quantum states that will be used to encode the private data and generate the final key, 2) deceiving the honest participants to deduct their private data by sending forged data. Therefore, to prevent the collusive attack, dishonest participants should be prevented from conducting these two processes. In our protocol, a semi-honest server is used, as indicated in Step (1), to generate the initial quantum states \((S_1)\) that will be used to encode the private inputs of the participants. The server shares \((S_1)\) with all participants after they receive the encoded data. In that case, all participants use the shared information to deduce the final key fairly. Also, the server checks the security of the quantum channel between every two participants and makes sure that the receiver has received genuine quantum states. Using these two processes, the protocol guarantees that the honest participant has received genuine data, and the dishonest participants cannot obtain useful information to generate the final key alone or steal the private inputs of honest participants.

VI. CONCLUSION

In this work, we showed that most of the existing circular-type multiparty quantum key agreement protocols are insecure against a specific type of collusive attack. We analyzed the security of a recently proposed circular-type multiparty quantum key agreement protocol to demonstrate the vulnerability of such protocols. Then, we proposed a general secure quantum key agreement model to avoid the different types of collusive attacks. We showed that the proposed protocol could generate the final key correctly, and that the proposed protocol is secure against all known collusive attack strategies.

REFERENCES

[1] W. Diffie, and M. Hellman, “New directions in cryptography,” IEEE transactions on Information Theory, vol. 22, no. 6, pp. 644-654, 1976.
[2] I. Ingemarsson, D. Tang, and C. Wong, “A conference key distribution system,” IEEE Transactions on Information Theory, vol. 28, no. 5, pp. 714-720, 1982.
[3] J. Pieprzyk, and C.-H. Li, “Multiparty key agreement protocols,” IEEE Proceedings-Computers and Digital Techniques, vol. 147, no. 4, pp. 229-236, 2000.
[4] C. H. Bennett and G. Brassard, “Quantum cryptography: Public key distribution and coin tossing,” in Proc. IEEE Int. Conf. Computer Systems and Signal Processing., Bangalore, India, 1984, pp. 175–179.
[5] N. Zhou, G. Zeng, and J. Xiong, “Quantum key agreement protocol,” Electronics Letters, vol. 40, no. 18, pp. 1149-1150, 2004.
[6] B. Liu, D. Xiao, H.-Y. Jia, and R.-Z. Liu, “Collusive attacks to “circle-type” multi-party quantum key agreement protocols,” Quantum Information Processing, vol. 15, no. 5, pp. 2113-2124, 2016.
[7] L. Wang, and W. Ma, “Quantum key agreement protocols with single photon in both polarization and spatial-mode degrees of freedom,” Quantum Information Processing, vol. 16, no. 5, pp. 130, 2017.
[8] H. Abulkasim, A. Farouk, H. Alsuaigh, W. Hamdan, S. Hamad, and S. Ghose, “Improving the security of quantum key agreement protocols with single photon in both polarization and spatial-mode degrees of freedom,” Quantum Information Processing, vol. 17, no. 11, pp. 316, 2018.

### TABLE I

| Unitary operations / quantum states | | | |
|-----------------|-----------------|-----------------|-----------------|
| \(0 \implies I\) | \(|0\rangle\) | \(|1\rangle\) | \(|\pm\rangle\) | \(|\mp\rangle\) |
| \(1 \implies U\) | \(−|1\rangle\) | \(|0\rangle\) | \(|\pm\rangle\) | \(|\mp\rangle\) |

The unitary operation \(I\) represents 0 and the unitary operation \(U\) represents 1.
[9] R.-H. Shi, and H. Zhong, “Multi-party quantum key agreement with bell states and bell measurements,” Quantum information processing, vol. 12, no. 2, pp. 921-932, 2013.

[10] Z. Sun, R. Cheng, C. Wu, and C. Zhang, “New Fair Multiparty Quantum Key Agreement Secure against Collusive Attacks,” Scientific reports, vol. 9, no. 1, pp. 1-8, 2019.

[11] Y.-H. Zhou, J. Zhang, W.-M. Shi, Y.-G. Yang, and M.-F. Wang, “Continuous-variable multiparty quantum key agreement based on third party,” Modern Physics Letters B, vol. 34, no. 06, pp. 2050083, 2020.

[12] W.-J. Liu, Z.-Y. Chen, S. Ji, H.-B. Wang, and J. Zhang, “Multi-party semi-quantum key agreement with delegating quantum computation,” International Journal of Theoretical Physics, vol. 56, no. 10, pp. 3164-3174, 2017.

[13] H. Cao, and W. Ma, “Multi-party traveling-mode quantum key agreement protocols immune to collusive attack,” Quantum Information Processing, vol. 17, no. 9, pp. 219, 2018.

[14] W.-c. Huang, Y.-k. Yang, D. Jiang, C.-h. Gao, and L.-j. Chen, “Designing Secure Quantum Key Agreement Protocols Against Dishonest Participants,” International Journal of Theoretical Physics, vol. 58, no. 12, pp. 4093-4104, 2019.

[15] Z. Sun, C. Wu, S. Zheng, and C. Zhang, “Efficient Multiparty Quantum Key Agreement With a Single S d S-Level Quantum System Secure Against Collusive Attack,” IEEE Access, vol. 7, pp. 102377-102385, 2019.

[16] H.-N. Liu, X.-Q. Liang, D.-H. Jiang, G.-B. Xu, and W.-M. Zheng, “Multi-party quantum key agreement with four-qubit cluster states,” Quantum Information Processing, vol. 18, no. 8, pp. 242, 2019.

[17] H.-N. Liu, X.-Q. Liang, D.-H. Jiang, Y.-H. Zhang, and G.-B. Xu, “Multi-party quantum key agreement protocol with bell states and single particles,” International Journal of Theoretical Physics, vol. 58, no. 5, pp. 1659-1666, 2019.

[18] C.-M. Tang, M. Zhang, L. Huang, Z.-q. Hu, J.-N. Zhu, Z. Xiao, Z. Zhang, Q.-x. Lin, X.-L. Zheng, and S.-L. Wu, “CircRNA 000203 enhances the expression of fibrosis-associated genes by derepressing targets of miR-26b-5p, Col1a2 and CTGF, in cardiac fibroblasts,” Scientific reports, vol. 7, no. 1, pp. 1-9, 2017.

[19] H. Abulkasim, A. Farouk, S. Hamad, A. Mashatan, and S. Ghose, “Secure dynamic multiparty quantum private comparison,” Scientific reports, vol. 9, no. 1, pp. 1-16, 2019.

[20] H. Abulkasim, H. N. Alsouqaih, W. F. Hamdan, S. Hamad, A. Farouk, A. Mashatan, and S. Ghose, “Improved dynamic multi-party quantum private comparison for next-generation mobile network,” IEEE Access, vol. 7, pp. 17917-17926, 2019.

[21] W. Wu, Q. Cai, S. Wu, and H. Zhang, “Cryptanalysis of He’s quantum private comparison protocol and a new protocol,” International Journal of Quantum Information, vol. 17, no. 03, pp. 1950026, 2019.

[22] R. Qi, Z. Sun, Z. Lin, P. Niu, W. Hao, L. Song, Q. Huang, J. Gao, L. Yin, and G.-L. Long, “Implementation and security analysis of practical quantum secure direct communication,” Light: Science & Applications, vol. 8, no. 1, pp. 1-8, 2019.

[23] L. Li, and Z. Li, “A verifiable multiparty quantum key agreement based on bivariate polynomial,” Information Sciences, vol. 521, pp. 343-349, 2020.

[24] H. Abulkasim, and A. Alotaibi, “Improvement on ‘Multiparty Quantum Key Agreement with Four-Qubit Symmetric W State’,” International Journal of Theoretical Physics, vol. 58, no. 12, pp. 4235-4240, 2019.

[25] R.-h. Shi, and M. Zhang, “privacy-preserving Quantum sealed-bid Auction Based on Grover’s search Algorithm,” Scientific reports, vol. 9, no. 1, pp. 1-10, 2019.

[26] N. Ban, and N. Y. Halpern, “Quantum voting and violation of Arrow’s impossibility theorem,” Physical Review A, vol. 95, no. 6, pp. 062306, 2017.

[27] A. Chowdhury, P. Vezio, M. Bonaldi, A. Borrielli, F. Marino, B. Morana, G. Prodi, P. Sarro, E. Serra, and F. Marin, “Quantum signature of a squeezed mechanical oscillator,” Physical Review Letters, vol. 124, no. 2, pp. 023601, 2020.