**Dynamics Analysis of a Novel 3-PSS Parallel Robot Based on Linear Motor**

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**ABSTRACT** The traditional parallel robot based on servo rotating motor runs stable and fast, but it still has the disadvantages of low load capacity and small acceleration. Based on the characteristics of the linear motor such as high load and stable thrust, this paper optimizes the structure of traditional three degrees of freedom (DOF) parallel robot, and obtains a novel 3-Prismatic-Spherical-Spherical parallel robot. The robot has a high speed and heavy load performance with low power consumption. Then, we solved the kinematic and dynamic formulas with screw theory and Kane’s Method. Next given the Jacobian matrix. Matlab is used to verify the formulas and given the reachable workspace with the Monte Carlo Method. While verifying performance, a simple trajectory is designed and simulated by Adams. Then we have done many groups of load experiments to verify. The simulation and experimental results show that the payload is 26kg when doing gate-trajectory (25/305/25 mm) motion. It has potential application prospects in the field of high speed and heavy load.

**INDEX TERMS** 3-RSS parallel robot, linear motor, kinematics, dynamics, workspace, mechanical properties.

**I. INTRODUCTION** In recent years, parallel robots have been developed for many types [1]–[6], and applied in various industries. Compared with series robots, a parallel robot has a more compact structure, higher rigidity, stronger carrying capacity, and greater precision while possessing a small working space. In terms of statics, kinematics, dynamics, workspace, trajectory planning, and simulation experiment control, many people have presently obtained certain achievements [7]–[16].

For instance, Li and Xu [17] proposed a 3-Prismatic-Revolute-Spherical parallel manipulator with adjustable layout angle of actuators which solved the inverse, forward, and velocity kinematics problems. Fan et al. [18] derived the inverse kinematics of the 3-PRS parallel kinematic spindle platform, then analyzed the sensitivity model of the spindle platform subject to the structure parameters. Chiang and Lin [19] developed a novel 3D parallel mechanism robot manipulator with 3 vertical-axial pneumatic actuators which had a stereo vision system. After that, they used the Denavit-Hartenberg notation coordinated system investigates the inverse kinematics and the forward kinematics. On this basis, the 3D trajectory tracking control of the end effector is realized. Then they adopted Fourier series-based adaptive sliding-mode controller with H-infinity tracking performance to design the path tracking controllers of the three vertical servo pneumatic actuators.

The traditional 3-RSS parallel robot relies on the servo rotating motor to provide power [20]–[22], and the hardware alone can’t guarantee enough precision, so it needs to be compensated by the algorithm. And the power of the servo motor is limited, which cannot provide effective power for dozens of kilograms of load. But [23] proposes using linear motor [24], [25] to design parallel robot, and makes a simple verification of the feasibility. On this basis, this paper optimizes its structure, which uses linear motor as the power source, with power of 3.036KW and self-weight of 100kg. Under the condition that the power and self-weight are close to the robot proposed in this paper, the parameters of parallel robot with the maximum load of famous brands in the market are as shown in Table 1. Through the experimental verification, the load performance of the parallel robot designed in this paper is improved to 26kg, Abandoning the necessary
TABLE 1. Model of triaxial robot with maximum load of each brand.

| Brands              | Model             | Maximum Load | Cycle time(s) | Mass(Kg) | Rated power(KW) |
|---------------------|-------------------|--------------|---------------|----------|-----------------|
| Fanuc[27]           | M-3iA/12H         | 12           | -             | 160      | 12              |
| ABB[28]             | IRB 360-8/1130    | 8            | 0.6           | 120      | -               |
| Omron[29]           | Hornet 565-1720-45600 | 8          | -             | 52       | 2.2             |
| Yaskawa Electric[30] | MPP3H             | 3            | -             | 115      | 1.5             |
| Siasun[31]          | SRBD1100          | 8            | 0.58          | 78       | 5.7             |
| This paper          | LMPR              | 26           | 1.2           | 100      | 3.036           |

reducer of traditional parallel robot, which makes some contribution in the field of the heavy load of the parallel robot.

In the second section, the design of the new parallel robot is given. The kinematical derivation of the new structure is carried out in combination with the screw theory, and then obtained the Kane’s dynamic equation. The third part uses MATLAB algorithms and Simulink simulation to verify the kinematics formula. This is different from the traditional analysis method, which uses the mutual verification of forward kinematics and inverse kinematics [26]. Based on MATLAB algorithm, write program to calculate the workspace by Monte Carlo method. Then combined Adams to simulate the dynamics simulation of the robot. On the parallel robot, the test control program is written according to the simulation data and the calculation results of kinematics formula, and the simulation results are verified by many experiments. Finally, the fourth part summarizes the characteristics and future improvement direction of the new parallel robot.

II. PARALLEL ROBOT DESIGN

A. MECHANICAL STRUCTURE DESIGN

The Delta parallel robot’s structure includes power sources, a three-circle array of linear motor modules, drivers, controllers, and mechanical structures. The mechanical structures include an aluminum profile frame, a top beam, an aluminum alloy connector, six carbon fiber connecting rods, a top platform, and a moving platform as an end-effector. The traditional 3-RSS parallel robot uses a servo rotating electric motor as its power source, but possesses insufficient power when faced with large loads. To improve load performance, the robot abandons a traditional structural layout and use a linear motor as its power source. The mechanical diagram for such a layout is shown in Figure 1. To this end, an aluminum alloy profile was constructed in a 1.1m by 1.1m by 1.6m frame with a circle in the middle to prevent deformation, and a self-locking load wheel at each of the four bottom corners for easy movement. Three identical branches (A, B, C) formed an inverted pyramid-shaped circular array structure under the support of the connecting member, which was fixed on the top platform. Considering the excessive weight of the motor, the platform would be easily deformed, requiring the beam to be fixed at the top to solve the problem of deformation. The angle between the branches was 120°, with each branch being composed of a linear motor module and a mechanical arm. From top to bottom, there were the motor mover prismatic pair, an aluminum alloy connector, and two parallel carbon fiber rods. Connecting the connector, the rod, and the moving platform was a spherical pair. The position and speed of the moving platform in the working space were determined by the three linear motor movers relative to the original position and the moving speed. Calculate the DOF according to the Chebyshev-GrüblerKutzbach formula:

\[
F = \lambda (n - g - 1) + \sum_{i=1}^{g} f_i,
\]

where \(\lambda\) is the order of the task space, \(n\) is the number of mechanisms, \(g\) is the number of movements and \(\sum_{i=1}^{g} f_i\) is the sum of DOF for all pairs.
The space freedom of the robot was equal to 3 as per Equation (1).

B. LINEAR MOTOR AND INSTRUMENTS

A linear motor must have the good performance to ensure reliability and stability of the hardware, so a control system can achieve technical and economical requirements under safe and reliable conditions. Commonly used control methods are adaptive control, sliding mode control, robust control, proportional integral differential and intelligent control. For example: Sun et al. [32] used the Euler discretization method, a new discrete-time fractional-order sliding mode control scheme proposed ensuring ideal tracking performance for linear motor control systems. Chen et al. [33] studied the effective compensation of nonlinear electromagnetic field effects, developing a nonlinear electromagnetic field-effect adaptive robust-control algorithm that compensated for the linear motor precise motion, enabling the system to operate at higher accelerations or heavier loads. After two years, they [34] proposed a new adaptive robust control strategy based on mu-synthesis.

FIGURE 2. Linear motor module.

Referring to the above literature, we select the linear motor. Figure 2 displays the linear motor module selected for this design, including its mover (coil winding), stator (permanent magnet), grating scale, photoelectric switch, slide rail, and anti-collision block. The motor (stator, mover) model is a self-cooling LMC-E11-075-A1 [35]. The effective stroke is 260 mm, the grating scale was selected as the Renishaw’s RGH24-RGS20 Linear grating system [36], the resolution is 0.5um, and the photoelectric switch was Omron’s EE-SX674 model [37]. The performance parameters of this module are shown in Table 2. Each module of the linear motor was equipped with a set of solid high drive (GTHD-006-PV-PCI-A [38]), and the three modules were driven in parallel by the solid high motion control card (GTS-400-PV-PCI-A [39]).

C. KINEMATIC MODELING

Using screw theory [40]–[43] as a basis, this section solves the forward kinematics of the robot based on the exponential product equation. The static coordinate system O is established first, given the pose $\theta_i$ of the three motors relative to the origin $D_i$, before solving the motion spin of each joint and combining matrix transformation to solve the position of the moving platform in a static coordinate system. Using Simulink to build a parallel robot simulation block diagram, $\theta_i$ is entered to obtain the result (See “Kinematics verification based on Simulink” for the process). Compared to the calculation results of the kinematics formula and Simulink simulation results to verify the accuracy of the kinematics model.

Schematic diagram of the robot structure is shown in Figure 3; the mathematical model is established using the vector and reference system shown in Figure 4 and Figure 5. The guide rail of the linear motor intersect along the line $\overrightarrow{A_iM}$ ($i = 1, 2, 3$) at M simplifies the motor mover slider to the

| Performance parameter          | Unit  | numerical value |
|-------------------------------|-------|-----------------|
| Maximum thrust                | N     | 940             |
| Maximum current               | A     | 11.3            |
| Persistent thrust             | N     | 435             |
| Persistent current            | A     | 4.6             |
| Motor constant@Tmax.          | N/W1/2| 8.7             |
| Maximum Power                 | W     | 577             |
| Winding Code                  |       | A1              |
| Force constant                | N/A   | 94.4            |
| Reverse electromotive force   | V/ms-1| 77.1            |
| Interphase resistance         | ohms  | 2.8             |
| Interphase inductance         | mH    | 24.4            |
| Electrical time constant      | ms    | 8.7             |
| Maximum voltage               | V     | 330             |
| Attractive force              | KN    | 3.7             |
| Mover weight                  | Kg    | 5.0             |
| Stator weight                 | Kg/m  | 10.1            |
mobile pair $H_i(i = 1, 2, 3)$, sliding between the original $D_i$ and the positive limit point $E_i$ on the guide rail $A_iM$, $D_iE_i = 260$ mm. The angle between guide rail $A_iM(i = 1, 2, 3)$ and the top platform is $\Phi = 37^\circ$. The connection between the slider and the connecting rod is simplified as a vector $\overrightarrow{H_iC_i}(i = 1, 2, 3)$, $(HC_i = 45$ mm). It is perpendicular to the surface of the slider and $A_iM(i = 1, 2, 3)$. $C_i$’s movement trajectory is $D_iE_i$ which is parallel to $D_iE_i$, as shown in Figure 4. When the motor slider is at the home switch, three branches of $C_i$ (composed of $\Delta C_1C_2C_3$) set the static coordinate system \{O-X, Y, Z\} with center O as the coordinate origin. The link between the mobile platform and each branch is $B_1B_2B_3$, establishing a dynamic coordinate system \{P-x,y,z\} at the center point $P$ of $\Delta B_1B_2B_3$. The X and x axes of the two coordinate systems point to the $OC_1$ and $PB_1$ directions are parallel. The angle between $OC_i$, the X axis, the $PB_i$ and the x axis is equal and set to $\beta_i$ given
\[
\beta_i = \frac{2(i - 1)}{3}\pi(i = 1, 2, 3),
\]
(2)
The carbon fiber tube and ball bearing constitute a connecting rod and the length is $L = CB = 614$ mm. Two connecting rods connect in parallel and connect the moving platform to the connector. The initial pose is simplified as shown in Figure 3. $\theta_{i1}$ is the motor displacement. When the moving platform is in its initial position (the motor’s original position), $OP = Z$, then
\[
Z = \sqrt{L^2 - (a-b)^2},
\]
(3)
The motion model can be simplified as a U pair for the spherical pair at both ends of the parallel link of each branch whose rotations are around the X- and Y-axes. Figure 5 shows the model diagram. $\theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}$ is the rotation angle of each joint and $\hat{\xi}_{ij}$ is the axis motion rotation coordinated of the j-th joint of the i-th branch ($i = 1, 2, 3$, $j = 1, 2, 3, 4, 5$). The rest are same and therefore the joint unit vectors are Eq. (4):
\[
\nu_{11} = [-c\phi \ 0 \ s\phi]^T, \tag{4a}
\]
\[
\omega_{12} = [0 \ 1 \ 0]^T, \tag{4b}
\]
\[ \omega_{13} = [1 \ 0 \ 0]^T, \] 
\[ \omega_{14} = [1 \ 0 \ 0]^T, \] 
\[ \omega_{15} = [0 \ 1 \ 0]^T, \]

Taking each axis point as Eq.4:

\[ \begin{align*} q_{11} &= [a \ 0 \ 0]^T, \\ q_{12} &= [a \ 0 \ 0]^T, \\ q_{13} &= [a \ 0 \ 0]^T, \\ q_{14} &= [b \ 0 \ -Z]^T, \\ q_{15} &= [b \ 0 \ -Z]^T, \end{align*} \]

Combine Equations (4) and (5). The motion spin coordinates of each joint are available as follows:

\[ \begin{align*} \mathbf{\dot{e}}_{11} &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{bmatrix}^T, \\ \mathbf{\dot{e}}_{12} &= \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ a \end{bmatrix}^T, \\ \mathbf{\dot{e}}_{13} &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T, \\ \mathbf{\dot{e}}_{14} &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -Z \end{bmatrix}^T, \\ \mathbf{\dot{e}}_{15} &= \begin{bmatrix} 0 \\ 1 \\ 0 \\ Z \\ 0 \end{bmatrix}^T, \end{align*} \]

The kinetic equation for solving a single chain is

\[ \text{gop}(\theta) = e^{\mathbf{\dot{e}}_{11}\theta_{11}}e^{\mathbf{\dot{e}}_{12}\theta_{12}} \ldots e^{\mathbf{\dot{e}}_{15}\theta_{15}} \text{gop}(0), \]

The initial pose is transformed into

\[ \text{gop}(0) = \begin{bmatrix} I \\ p_0 \end{bmatrix}, \quad p_0 = \begin{bmatrix} 0 \\ 0 \\ -Z \end{bmatrix}, \]

\[ \text{gop}(\theta)_1 = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \delta_{14} \\ \delta_{21} & \delta_{22} & \delta_{23} & \delta_{24} \\ \delta_{31} & \delta_{32} & \delta_{33} & \delta_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]

where:

\[ \begin{align*} \delta_{11} &= c_{12}c_{15} - s_{12}s_{15}c_{13} + 14, \\ \delta_{12} &= s_{12}s_{13} + 14, \\ \delta_{13} &= c_{12}s_{13} + 14, \\ \delta_{14} &= a(1 - c_{12}) - \frac{4}{5}\theta_{11} - Zs_{12}c_{13} + b(s_{12}s_{13}c_{13} + c_{12}c_{15} + c_{12}), \\ \delta_{21} &= s_{15}s_{13} + 14, \\ \delta_{22} &= c_{13} + 14, \\ \delta_{23} &= c_{15}s_{13} + 14, \\ \delta_{24} &= bs_{13} + Zs_{13}, \\ \delta_{31} &= s_{12}c_{15} - c_{12}s_{15}c_{13} + 14, \\ \delta_{32} &= c_{12}s_{13} + 14, \\ \delta_{33} &= s_{12}c_{15} + c_{12}c_{15}c_{13} + 14, \\ \delta_{34} &= as_{12} + \frac{3}{5}\theta_{11} - Zc_{12}c_{13} + b(c_{12}s_{15}c_{13} + s_{12}c_{15} - s_{12}), \end{align*} \]

where \( S_{11} \) represents \( \sin \theta_{11} \), \( C_{11} \) represents \( \cos \theta_{11} \), and the rest is the same. So for the static platform coordinates is

\[ \begin{align*} P_{\theta_1} &= \delta_{14}, \\ P_{\theta_2} &= \delta_{24}, \\ P_{\theta_3} &= \delta_{34}. \end{align*} \]

In the initial position, each branch has the same pose and a circular array centered on the Z-axis. Therefore, the transformation matrix of the i-th branch relative to the first branch about the Z-axis rotation \( \alpha \) angle is:

\[ \mathbf{R}_i = \text{Rot}(Z, \alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, \]

And the unit vector of each joint axis corresponding to the first branch, as well as the first branch and the points taken on each axis, can be expressed as Eq. (13):

\[ \begin{align*} v_{ij} &= R_{ij}v_{i1}, \\ \omega_{ij} &= R_{ij}\omega_{i1}, \end{align*} \]

The rotational momentum of each branch joint is expressed as Eq. (14):

\[ \begin{align*} \mathbf{\dot{e}}_{ij} &= \begin{bmatrix} \omega_{ij} \\ v_{ij} \end{bmatrix} \times \omega_{ij}. \end{align*} \]

The ends of each branch meet at the moving platform, so the ends of each branch have the same posture as follows:

\[ \text{gop}(\theta)_1 = \text{gop}(\theta)_2 = \text{gop}(\theta)_3. \]

The relationship between each joint variable and the motor input variable \( \theta_{11}, \theta_{21}, \theta_{31} \) can be obtained from Equations (15). In this way, the relationship between the pose of the moving platform and the input variables of the linear motors can be derived.

**D. JACOBIAN MATRIX AND VELOCITY KINEMATICS**

In the view of screw theory’s Velocity formulas [35] and the unit motion spin coordinate definition, the motion subrotation coordinates corresponding to the rotating joint and the moving joint are Eq. (16):

\[ \begin{align*} \mathbf{\dot{x}} &= \begin{bmatrix} \omega' \\ v' \end{bmatrix}, \\ \mathbf{\dot{x}}' &= \begin{bmatrix} \omega' \\ v' \end{bmatrix}. \end{align*} \]

where \( \omega' \) is the unit vector corresponding to the direction of the axis of the rotating joint in the current pose and \( q_i' \) is the point position vector on the axis of the current pose, which can be written as Eq.(17):

\[ \begin{align*} \omega_i' &= e^{\omega_i \theta_1} e^{\omega_i \theta_2} \ldots e^{\omega_i \theta_{i-1}} \omega_i, \\ \mathbf{q}_i' &= e^{\mathbf{\dot{\mathbf{x}}}_i \theta_1} e^{\mathbf{\dot{\mathbf{x}}}_i \theta_2} \ldots e^{\mathbf{\dot{\mathbf{x}}}_i \theta_{i-1}} \mathbf{q}_i. \end{align*} \]
where $q_i(0)$ is the position vector of a point on the axis of the initial pose.

Substituting Equations (4) and (5) into Equations (16) and (17) yields

\[
\begin{align*}
\xi'_{11} &= \begin{bmatrix} 0 & 0 & -4/5 & 0 & 3/5 \end{bmatrix}^T, \quad (18a) \\
\xi'_{12} &= \begin{bmatrix} 0 & 1 & -3/5 \theta_{11} & 0 & a - 4/5 \theta_{11} \end{bmatrix}^T, \quad (18b) \\
\xi'_{13} &= \begin{bmatrix} c_{12} & -s_{12} & 0 & \dot{\theta}_{11} & 0 \end{bmatrix}^T, \quad (18c) \\
\xi'_{14} &= \begin{bmatrix} c_{12} & -s_{12} & \dot{\theta}_{12} & \dot{\theta}_{13} & \dot{\theta}_{14} \end{bmatrix}^T, \quad (18d) \\
\xi'_{15} &= \begin{bmatrix} \dot{\theta}_{15} & c_{13+14} & \dot{\theta}_{16} & \dot{\theta}_{17} & \dot{\theta}_{18} & \dot{\theta}_{19} \end{bmatrix}^T, \quad (18e)
\end{align*}
\]

where:

\[
\begin{align*}
\dot{\theta}_{11} &= \frac{3}{5} \theta_{11} c_{12} - s_{12} \left( \frac{4}{5} \theta_{11} - a \right) \\
\dot{\theta}_{12} &= -Zs_{12} s_{13} \\
\dot{\theta}_{13} &= as_{12} + \frac{3}{5} \theta_{11} c_{12} - \frac{4}{5} \theta_{11} s_{12} - Zc_{13} \\
\dot{\theta}_{14} &= -Zc_{12} s_{13} \\
\dot{\theta}_{15} &= s_{12} s_{13+14} \\
\dot{\theta}_{16} &= c_{12} s_{13+14} \\
\dot{\theta}_{17} &= -c_{13+14} \left[ \frac{3}{5} \theta_{11} + s_{12} (a-b) - Zc_{12} c_{13} \right] + Zc_{12} s_{13+14} \\
\dot{\theta}_{18} &= s_{13+14} \left[ \frac{3}{5} \theta_{11} (s_{12} - c_{12}) + a - b - ac_{12} \right] \\
\dot{\theta}_{19} &= c_{13+14} \left[ \frac{3}{5} \theta_{11} + bc_{12} + a (1-c_{12}) \right] - Zs_{12} c_{14}.
\end{align*}
\]

Substituting Equation (18) into the Jacques matrix,

\[
\begin{align*}
J_{ST}^s (\theta) &= \begin{bmatrix} \xi'_{11} & \xi'_{12} & \xi'_{13} & \xi'_{14} & \xi'_{15} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & c_{12} & c_{12} & \dot{\theta}_{15} \\
0 & 1 & 0 & 0 & c_{13+14} \\
0 & 0 & -s_{12} & -s_{12} & \dot{\theta}_{16} \\
-\frac{4}{5} & -\frac{3}{5} \theta_{11} & 0 & \dot{\theta}_{12} & \dot{\theta}_{17} \\
0 & 0 & \dot{\theta}_{11} & \dot{\theta}_{13} & \dot{\theta}_{18} \\
\frac{3}{5} & a - \frac{4}{5} \theta_{11} & 0 & \dot{\theta}_{14} & \dot{\theta}_{19} \end{bmatrix}, \quad (20)
\end{align*}
\]

The speed of the end of the first branch can be rewritten as follows:

\[
\begin{align*}
\dot{V}_{ST}^s &= J_{ST}^s (\theta) \dot{\theta} \\
&= \begin{bmatrix} 1 & 0 & c_{12} & c_{12} & \dot{\theta}_{15} \\
0 & 1 & 0 & 0 & c_{13+14} \\
0 & 0 & -s_{12} & -s_{12} & \dot{\theta}_{16} \\
-\frac{4}{5} & -\frac{3}{5} \theta_{11} & 0 & \dot{\theta}_{12} & \dot{\theta}_{17} \\
0 & 0 & \dot{\theta}_{11} & \dot{\theta}_{13} & \dot{\theta}_{18} \\
\frac{3}{5} & a - \frac{4}{5} \theta_{11} & 0 & \dot{\theta}_{14} & \dot{\theta}_{19} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\dot{\theta}_{ij} &= \text{the joint velocity of the } n\text{-th joint and the } i\text{-th column of the Jacobian matrix is the rotational momentum of the } i\text{-th joint in the current static attitude of the robot.}
\end{align*}
\]

For the second and third branches, the rotation matrix $R_{ij}$ is introduced. In view of Equations (16) and (20), the Jacobian matrix of the second and third branches are respectively as follows:

\[
\begin{align*}
2J_{ST}^s (\theta) &= \begin{bmatrix} \xi'_{21} & \xi'_{22} & \xi'_{23} & \xi'_{24} & \xi'_{25} \end{bmatrix}, \quad (22a) \\
3J_{ST}^s (\theta) &= \begin{bmatrix} \xi'_{31} & \xi'_{32} & \xi'_{33} & \xi'_{34} & \xi'_{35} \end{bmatrix}. \quad (22b)
\end{align*}
\]

The speed of the second branch and the end of the third branch are Eq. (23):

\[
\begin{align*}
2\dot{V}_{ST} &= 2J_{ST}^s (\theta) \dot{\theta}, \\
3\dot{V}_{ST} &= 3J_{ST}^s (\theta) \dot{\theta}.
\end{align*}
\]

The ends of all three branches are all moving platforms, making the speed is equal as follows,

\[
\dot{V}_{ST} = 1\dot{V}_{ST}^s = 2\dot{V}_{ST}^s = 3\dot{V}_{ST}^s.
\]

**E. KANE’S DYNAMICS MODELING**

In this section, based on kinematic analysis, Kane equation [44], [45] is used to derive its dynamic equation. Each branch of the parallel robot is mainly composed of servo linear motor, connecting rod and joint. For each branch, the generalized active force and generalized inertial force of each branch are determined.

Establish a joint link coordinate system on the center of mass of each member, the i-th centroid of the k-th branch is: $^kC_i$, as shown in Figure 6. The unit vector on the rotation axis of each joint is Equation (4). With the Kane’s method, For each joint: Kinematic pair is $^kN_{ij}$, the object Jacobian matrix $^kJ_{ij}$, active power spinor $^kF_{\xi i}$, angular velocity $\omega_{ci}$, joint acceleration $^1a_{ji}$, angular acceleration $\alpha_{ci}$, mass-center acceleration $a_{ci}$. Take the first branch as an example:

Where object Jacobian matrix $J_{ki}$ is Eq.(25):

\[
\begin{align*}
J_{ki} &= \begin{bmatrix} k_{n11} & k_{n12} & k_{n13} & k_{n14} & k_{n15} \end{bmatrix}, \quad (25a) \\
J_{11} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (25b) \\
J_{12} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (25c) \\
J_{13} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (25d) \\
J_{14} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (25e) \\
J_{15} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (25f)
\end{align*}
\]

where (26a)–(26o), as shown at the next page.
\(1_{n1} = [-c\phi\ 0\ s\phi\ 0\ 0\ 0]^T, \)  
\(1_{n2} = [-c\phi\ 0\ s\phi\ 0\ 0\ 0]^T, \)  
\(1_{n22} = [0\ 1\ 0\ 0\ 0\ 0]^T, \)  
\(1_{n31} = \begin{bmatrix} s_{12}\phi - c_{12}\phi \\ -s_{13}\ (c_{12}\phi + c\phi{s}_{12}) \\ c_{13}\ (c_{12}\phi + c\phi{s}_{12}) \\ Zs_{13}\ (c_{12}\phi + c\phi{s}_{12}) \\ \frac{Z(\ (c_{12}\phi - s_{12}\phi) \ - (c_{12}\c_{13}\phi + c\phi{s}_{12}) \ (a + b) \ )}{2} \\ \frac{s_{13}\ (a + b) \ (c_{12}\phi + c\phi{s}_{12})}{2} \end{bmatrix}, \)  
\(1_{n32} = [0\ c_{13}\ s_{13} - \frac{Z}{2}c_{13}\ - \frac{a + b}{2}s_{13}\ - \frac{a + b}{2}c_{13}]^T. \)  
\(1_{n33} = [1\ 0\ 0\ 0\ Z \ 0]^T. \)  
\(1_{n41} = \begin{bmatrix} s_{12}\phi - c_{12}\phi \\ -c_{13}s_{14} + c_{14}s_{13} \ (c_{12}\phi + c\phi{s}_{12}) \\ (c_{13}\phi - s_{13}s_{14})(c_{12}\phi + c\phi{s}_{12}) \\ Z(c_{13}s_{14} + c_{14}s_{13})(c_{12}\phi + c\phi{s}_{12}) \\ -(c_{12}\phi - s_{12}s_{14}) + c\phi{s}_{12}(c_{13}\phi - s_{13}s_{14})(a - b) - Z(c_{12}\phi - s_{12}\phi) \ - (a - b)(c_{13}s_{14} + c_{14}s_{13})(c_{12}\phi + c\phi{s}_{12}) \end{bmatrix}. \)  
\(1_{n42} = \begin{bmatrix} 0 \\ c_{13}s_{14} - s_{13}s_{14} \\ c_{13}s_{14} + c_{14}s_{13} \\ -Z(c_{13}s_{14} - s_{13}s_{14}) \\ -(a - b)(c_{13}s_{14} + c_{14}s_{13}) \end{bmatrix}. \)  
\(1_{n43} = [1\ 0\ 0\ 0\ Z\ 0]^T. \)  
\(1_{n44} = [1\ 0\ 0\ 0\ 0\ 0]^T. \)  
\(1_{n51} = \begin{bmatrix} s\phi (c_{15}s_{12} + c_{12}(c_{13}c_{14}s_{15} - s_{13}s_{14}s_{15})) - c\phi (c_{12}c_{15} - s_{12}(c_{13}c_{14}s_{15} - s_{13}s_{14}s_{15})) \\ -c_{13}s_{14} + c_{14}s_{13} \ (c_{12}\phi + c\phi{s}_{12}) \\ c\phi (c_{12}s_{15} + s_{12}(c_{13}c_{14}s_{15} - c_{15}s_{13}s_{14})) - s\phi (s_{12}s_{15} - s_{12}(c_{13}c_{14}s_{15} - c_{15}s_{13}s_{14})) \\ Z(c_{13}s_{14} + c_{14}s_{13}) (c_{12}\phi + c\phi{s}_{12}) \\ -(a - b) \ (c\phi (c_{12}s_{15} + s_{12}(c_{13}c_{14} - s_{13}s_{14})) - s\phi (s_{12}s_{15} - c_{12}c_{15}(c_{13}c_{14} - s_{13}s_{14}))) \ - (a - b)(c_{13}s_{14} + c_{14}s_{13})(c_{12}\phi + c\phi{s}_{12}) \ (2\a - b) \end{bmatrix}. \)  
\(1_{n52} = \begin{bmatrix} s_{15}\ (c_{13}s_{14} + c_{14}s_{13}) \\ c_{13}s_{14} - s_{14}s_{13} \\ c_{15}(c_{13}s_{14} + c_{14}s_{13}) \\ -Z(c_{13}s_{14} - s_{14}s_{13}) \\ (c_{13}s_{14} + c_{14}s_{13}) (b_{15} - 2ac_{15} + 2zs_{15}) \ (a - b) \ (c_{13}s_{14} - s_{14}s_{13}) \end{bmatrix}. \)  
\(1_{n53} = [c_{15}\ 0\ -s_{15}\ 0\ Zc_{15} + c_{15} \ (a - b)\ 2 | 0]^T. \)  
\(1_{n54} = \begin{bmatrix} c_{15}\ 0\ -s_{15}\ 0\ b \ (2s_{15}) \ 0 \end{bmatrix}^T. \)  
\(1_{n55} = [0\ 1\ 0\ 0\ 0\ b\ (2)]^T. \)
The active force rotation for each joint of the first branch chain is obtained as follows:

\[
\begin{align*}
1F_{11} & = [0 \quad -m_1g_s_{11} \quad m_1g_c_{11} \quad -\tau_1 \quad 0 \quad 0]^T, \\
1F_{22} & = [0 \quad 0 \quad m_2g \quad \tau_1 \quad 0 \quad 0]^T, \\
1F_{33} & = [m_3g_s_{13} \quad m_3g_c_{13} \quad \tau_1 \quad 0 \quad 0]^T, \\
1F_{44} & = [0 \quad 0 \quad m_4g \quad \tau_1 \quad 0 \quad 0]^T, \\
1F_{55} & = [0 \quad 0 \quad m_5g \quad -\tau_1 \quad 0 \quad 0]^T, \\
\end{align*}
\]

(27)

To sum up, the generalized active forces of the five joints of the first branch chain are as follows:

\[
\begin{align*}
1F_1 & = 1F_{11}^T n_{11} + 1F_{12}^T n_{21} + 1F_{13}^T n_{31} + 1F_{14}^T n_{41} + 1F_{15}^T n_{51}, \\
1F_2 & = 1F_{21}^T n_{12} + 1F_{22}^T n_{22} + 1F_{23}^T n_{32} + 1F_{24}^T n_{42} + 1F_{25}^T n_{52}, \\
1F_3 & = 1F_{31}^T n_{13} + 1F_{32}^T n_{23} + 1F_{33}^T n_{33} + 1F_{34}^T n_{43} + 1F_{35}^T n_{53}, \\
1F_4 & = 1F_{41}^T n_{14} + 1F_{42}^T n_{24} + 1F_{43}^T n_{34} + 1F_{44}^T n_{44} + 1F_{45}^T n_{54}, \\
1F_5 & = 1F_{51}^T n_{15} + 1F_{52}^T n_{25} + 1F_{53}^T n_{35} + 1F_{54}^T n_{45} + 1F_{55}^T n_{55}, \\
\end{align*}
\]

(28)

For the i joint of the k branch, principal vector of inertia force \(kR_{ci}^e\) and principal moment of inertia \(kT_{ci}^e\) of inertial force can be obtained as Eq.(29a)–(29e), as shown at the bottom of the page and Eq.(30a)–(30e), as shown at the bottom of the next page. Moreover, In conclusion, the generalized inertial forces of the five joints are Eq.(31):

\[
\begin{align*}
1F_1^* & = 1F_{11}^T n_{11} + 1F_{12}^T n_{21} + 1F_{13}^T n_{31} + 1F_{14}^T n_{41} + 1F_{15}^T n_{51}, \\
1F_2^* & = 1F_{21}^T n_{12} + 1F_{22}^T n_{22} + 1F_{23}^T n_{32} + 1F_{24}^T n_{42} + 1F_{25}^T n_{52}, \\
1F_3^* & = 1F_{31}^T n_{13} + 1F_{32}^T n_{23} + 1F_{33}^T n_{33} + 1F_{34}^T n_{43} + 1F_{35}^T n_{53}, \\
1F_4^* & = 1F_{41}^T n_{14} + 1F_{42}^T n_{24} + 1F_{43}^T n_{34} + 1F_{44}^T n_{44} + 1F_{45}^T n_{54}, \\
1F_5^* & = 1F_{51}^T n_{15} + 1F_{52}^T n_{25} + 1F_{53}^T n_{35} + 1F_{54}^T n_{45} + 1F_{55}^T n_{55}, \\
\end{align*}
\]

(31)

Combining the generalized active force and the generalized inertial force, the Kane’s equation of the first branch chain...
is Eq.(32):

\[
\begin{align*}
1F_1 + 1F^*_1 &= 0 \\
1F_2 + 1F^*_2 &= 0 \\
1F_3 + 1F^*_3 &= 0 \\
1F_4 + 1F^*_4 &= 0 \\
1F_5 + 1F^*_5 &= 0,
\end{align*}
\]

Introduce rotation matrix Equation (11) for the second and third branches, in the same way, we can get Kane’s equation of two and three branches.

For the manipulator of parallel robot, each branch chain is connected to the same moving platform, so the generalized acceleration of the end of each branch chain is equal. By combining the three acceleration formulas, the constraint
TABLE 3. The linear motor movement parameters.

| GROUP | 1ST (mm) | 2ND (mm) | 3RD (mm) |
|-------|----------|----------|----------|
| 1     | 40       | 28       | 60       |
| 2     | 53       | 106      | 80       |
| 3     | 70       | 80       | 90       |
| 4     | 90       | 85       | 130      |
| 5     | 150      | 132      | 174      |
| 6     | 116      | 202      | 170      |
| 7     | 180      | 162      | 224      |
| 8     | 167      | 231      | 61       |
| 9     | 68       | 21       | 173      |
| 10    | 14       | 185      | 254      |

TABLE 4. Moving platform position.

| POINTS | X (mm)       | Y (mm)       | Z (mm)       |
|--------|--------------|--------------|--------------|
| 1      | 4.5306       | 29.1623      | -510.0282    |
| 2      | 44.0873      | -26.1142     | -552.6623    |
| 3      | 16.6978      | 9.8269       | -531.6066    |
| 4      | 21.0680      | 46.1127      | -577.5607    |
| 5      | 4.8943       | 46.8267      | -631.6041    |
| 6      | 88.1081      | -39.5432     | -637.7775    |
| 7      | 20.7369      | 75.8975      | -665.6868    |
| 8      | -9.9624      | -182.2812    | -611.6518    |
| 9      | 41.5577      | 151.7238     | -545.7972    |
| 10     | 228.3529     | 93.9586      | -590.5708    |

III. SIMULATION AND EXPERIMENTAL RESULTS

In this section, Simulink is used to verify the calculation results of the previous kinematic formula. The simple motion planning is performed. The theoretical simulation is performed by Adams and verified by the actual machine test. The experimental data are compared and analysed.

conditions of the closed-loop system can be established. Combining with the above formula, \( \dot{\theta} \) and \( \ddot{\theta} \) can be extracted, sorted and combined, and we can be obtained the expression of the standard vector matrix:

\[
M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau, \tag{33}
\]

where: \( M(\theta) \in \mathbb{R}^{n \times n} \) represents inertia matrix, \( C(\theta, \dot{\theta}) \in \mathbb{R}^{n \times n} \) represents centrifugal force and Coriolis force matrix, \( G(\theta) \) represents gravity vector, \( \tau \in \mathbb{R}^n \) represents motor input vector.

A. KINEMATICS VERIFICATION BASED ON MATLAB

To verify the derived kinematics formula, this paper uses the Simulink module of Matlab for its simulation. Using the Multibody toolbox under Simscape and modeling the parallel robot simplification, according to the mathematical model of Figure 5, gives the simulation block diagram shown in Figure 7. Since the three branches of the robot are in a circular array structure, the first branch is modeled first. The motor’s simplified moving pair (A, B, C) stroke is 260 mm, the moving platform and kinematic pair are connected by the spherical pair, and the connecting rod is Li = 614 mm.

Randomly select 10 groups of motor motion position parameters, such as in Table 3. Set the motor slider motion position through Simulink simulation, and feedback the rotation angle of each joint in the model. Substitute the above parameters into Equation (11), and the calculation results are consistent with Table 4. The equations in Section II. C are verified to be correct.

Using the Monte Carlo method. The travel range of the three linear motors is 0-260 mm. For each motor, a pose is collected every 10 mm. According to the arrangement, \( 27 \times 27 \times 27 = 19683 \) sets of data can be obtained. Each set of data corresponds to a pose of the moving platform in the motion space. According to the kinematics program, the coordinates of these poses relative to the static coordinate system is calculated. Write the program, draw these points in the static coordinate system by using the interpolation method.
FIGURE 9. Physical figure. (a) Path planning. (b) Load figure.

FIGURE 10. Simulation parameters for the motor. (a) First motor’s motion parameters. (b) Second motor’s motion parameters. (c) Third motor’s motion parameters.

in MATLAB as shown in Figure 8(a), draw the envelope surface according to the outermost point, the space enclosed by this envelope surface is the robot’s workspace, which is shown in Figure 8(b).
B. KINEMATICS VERIFICATION BASED ON ADAMS AND EXPERIMENTAL VERIFICATION

In order to make a performance comparison under the initial conditions, the traditional PID controller of [46] is introduced in this paper without any optimization. Firstly, the $K_p$ is adjusted slowly to avoid high frequency oscillation. Under the condition of satisfying the response speed, by adjusting, the speed feed-forward coefficient and eliminating the zero drift method; the reverse gap compensation can further improve the control accuracy.

$$U(t) = K_p \times \left( e(t) + K_i \times \int_0^t e(t) \, dt + K_d \frac{de(t)}{dt} \right).$$

(34)

Before using the load, Adams was used to simulate the feasibility of the experiment. A 260N load was added to the moving platform in the positive direction of the Z-axis. The planned path trajectory is shown in Figure 9(a), and the standard gate type (25 mm/305 mm/25 mm) was used for the grab motion. Starting from the initial position $A_0$, the vertical movement was first performed at 25 mm up to point $B_0$, then horizontally moved 305 mm to $C_0$ before being moved down to $D_0$. After the grab, it returned to the initial position $A_0$ along the original path. To ensure accuracy, 4 points ($A_0$, $B_0$, $C_0$, $D_0$) were taken on the path. The simulation step size was 3000/s, and the coordinates of each point in the static coordinate system are shown in Table 5. According to the simulation results, the cycle time was 1 s. The speed, acceleration, and required power of the three linear motors, $D_i$, movers are as showed in Figure 10 (a, b, c). In Figure 10, the moving platform moved from the origin to $A_0$ within 0–1 seconds and the gate shaped motion started from $A_0$ to $D_0$ within 1–2 seconds before returning to $A_0$. It can be seen from the Figure 10 that the maximum speed required for the motor during operation was $v_{max} = 1.2m/s$, the maximum acceleration was $a_{max} = 16m/s^2$, the maximum required power was $F_{max} = 320N$. Compared with the linear motor parameters of Table 2, it was found that the rated parameters of the motor were not reached, rendering the experiment feasible.
TABLE 5. Coordinates of path acquisition points.

| POINTS | X (mm) | Y (mm) | Z (mm) |
|--------|--------|--------|--------|
| A₀     | 152.5  | 0      | -575   |
| B₀     | 152.5  | 0      | -550   |
| C₀     | -125..5| 0      | -550   |
| D₀     | -125.5 | 0      | -575   |

Fix the load on the mobile platform with 10, 15, 20, 26 kg dumbbells as shown in Figure 9(b), and start to move according to the planned gate-path. A black-and-white checkerboard (6 mm * 6 mm in size) is installed on the moving platform as the calibration, and using a high-speed camera to capture the motion track at 160-200 frames per second. Fourier transform the sampling data; filter the noise with the moving average filter, the filtered v-t image of moving platform shows in Figure 11.

IV. CONCLUSION
In this paper, we proposed a 3-PSS three-DOF space parallel robot. Three identical branches were arranged in a circular array around a central axis, and a servo-linear motor was used as the power source. The kinematics of the robot was analyzed using the theory of spin amount, and derived the Jacobian matrix, kinematic velocity and Kane’s Dynamic equation. Using MATLAB to write algorithms and Simulink simulation verification to verify the kinematic formula and give the motion space by Monte Carlo method. Under the action of load F = 260N, the planned path is a standard gate-trajectory motion (25 mm/305 mm/25 mm). The dynamic simulation is carried out in ADAMS, and the simulation results are verified by experiments. The experimental parameters and simulation data are fully fitted. The results show that the parallel robot has good power and speed transmission performance. Compared with the maximum load robots of all brands in Table 1, the load is increased by about 2 times, and the power loss also has certain advantages. In line with expectations. It has potential application in the field of high speed and heavy load.

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