We compared four hole-scales that have been used to determine the hole-concentration in high-temperature cuprate superconductors. We show that the hole-scale, $P_{\rho\sigma}$-scale, based on the thermoelectric power [T. Honma et al., Phys. Rev. B70, (2004) 214517] is quantitatively consistent with spectroscopic probes for many different cuprate materials, while the other hole-scales, based on a well-known dome-shaped $T_c$-curve [M. R. Presland et al., Physica C176, 95 (1991)], the $c$-axis lattice parameter [R. Liang et al., Phys. Rev. B73, (2006) 180505(R)], and Hall coefficient [Y. Ando et al., Phys. Rev. B61, (2000) 14956(R)], are not. We show that the quantitatively different hole-scales resulted in opposite conclusion of the same experimental observations. It can also lead to different interpretations of the electronic phase diagram when comparing different physical properties in different high-$T_c$ systems. We suggest that the $P_{\rho\sigma}$-scale is the correct universal scale that works for all high-$T_c$ cuprates and it should be used for all quantitative doping dependence studies of cuprates.

**Keywords:** high-$T_c$ cuprate superconductors; hole-scale; thermoelectric power; Hall coefficient; doped-hole inhomogeneity; magnetic phase diagram.

1. Introduction

High-temperature cuprate superconductors (HTCS) have a very wide doping range that resulted in numerous doping dependent studies of various physical properties. While doping dependent studies of HTCS are becoming more and more quantitative through continuous improvements on experimental conditions, such as sample quality, experimental resolution and techniques, in the past twenty-eight years there has been, unfortunately, a misinterpretation of the corresponding data due to the use of a, although popular, quantitatively incorrect scale for determining the hole concentration. In this report, we address two issues of the doped-hole inhomogeneity and competition between magnetic order and spin density wave to demonstrate the fundamental importance of using the quantitatively correct scale to arrive at the physically correct
conclusions and to address the subtle issues when comparing the doping dependence among different HTCS materials.

Fig. 1 (a) Doping dependence of $L_c$ as a function of different hole-scales. The plotted data were extracted from Ref. 7. (b) Doping dependence of $eR_{HN}/V$ at room temperature as a function of $P_{pl}$. Inset: $eR_{HN}/V$ at room temperature as a function of $P_{Hall}$. The plotted data were extracted from Refs. 11-22. All plotted data have both $R_H$ and $S^{290}$. (c) $T_c$ of YBa$_2$Cu$_3$O$_{6+\delta}$ versus different hole-scales. (d) $T_c$ of Bi$_2$Sr$_2$La$_x$CuO$_{6+\delta}$ versus different scales. The colored line is a guide to the eyes. The plotted data were extracted from Ref. 8.
The most popular hole-scale is a measure of the hole concentration using the doping dependence of a superconducting transition temperature, $T_c$, that follows a dome-shaped $T_c$-curve with an empirical formula of $T_c = T_c^{\max} \{1 - 82.6(P_{\text{dome}} - 0.16)^2\}$, where $P_{\text{dome}}$ is the hole concentration, the value of 0.16 is the universal optimal hole concentration and $T_c^{\max}$ is a maximum in $T_c$. As seen in Fig. 1(c) where the $T_c$-curve is plotted as red circles connected by the dotted line. Instead of using $T_c$, the thermoelectric power at 290 K, $S^{290}$, was used to determine $P_{\text{dome}}$ by scaling the $S^{290}$ to the hole concentration based on the dome-shaped $T_c$-curve.\textsuperscript{2,5} We will call the hole-scale based on the dome-shaped $T_c$-curve the “dome-scale” or $P_{\text{dome}}$-scale. The dome-scale is very convenient scale for estimating the hole concentration since $T_c$ is always available and it is still very popular now. However, the dome-scale has some self-inconsistency issues. For instance, in La$_2$SrCuO$_4$, the $P_{\text{dome}}$ determined from $S^{290}$ is not corresponding to that from $T_c$.\textsuperscript{4,5} Furthermore, in YBa$_2$Cu$_3$O$_{6+\delta}$, there is a well-known plateau around $T_c = 60$ K and 90 K clearly observed in the oxygen-content dependence\textsuperscript{6} which is completely washed out in the dome-scale. To address the missing plateaus a scale for YBa$_2$Cu$_3$O$_{6+\delta}$ based on the lattice parameter along c-axis, $L_c$, was proposed by Liang et al.\textsuperscript{7} We will refer the hole concentration determined by the Liang’s approach as $P_{\text{lattice}}$ and call it “lattice-scale”. In Fig. 1(a), we plot the same $L_c$ data\textsuperscript{7} of YBa$_2$Cu$_3$O$_{6+\delta}$ as a function of doping of the different scales. In the dome-scale, $L_c$ rapidly decreases at around 1/8 with doping. In the lattice-scale the rapid decreases become smoother. Ando et al. pointed out also in Bi$_2$Sr$_2$La$_2$CuO$_{6+\delta}$ that the $P_{\text{dome}}$ determined from $S^{290}$ is not corresponding to that from $T_c$.\textsuperscript{8} They also proposed another scale based on the Hall coefficient, $R_H$.\textsuperscript{8} The scale is based on the normalized $R_H$, $eR_H/N/V$, where $V$ is a unit cell volume and $N$ is the CuO$_2$ layer number per the unit cell.\textsuperscript{8} They tried to determine the hole concentration by scaling the temperature dependence of $eR_H/N/V$ of many cuprates to that of La$_2$SrCuO$_4$. Unfortunately, the scaling actually works only in the temperature range from 200 K to 300 K. Since in this temperature range, the temperature dependence of $R_H$ becomes very weaker,\textsuperscript{4} the $eR_H/N/V$ at room temperature can be representative of their scale. We will refer the hole concentration determined by the Ando’s approach as $P_{\text{Hall}}$ and call it “Hall-scale”. In the inset of Fig. 1(b), we plot the $eR_H/N/V$ at RT as a function of $P_{\text{Hall}}$, which is equal to Sr-content in La$_2$Sr$_2$CuO$_4$. The $P_{\text{Hall}}$ is determined by scaling the value of $eR_H/N/V$ to the doping concentration of La$_2$Sr$_2$CuO$_4$ as shown in the inset of Fig. 1(b). In Fig. 1(c), we plot the $T_c$ data\textsuperscript{7} of YBa$_2$Cu$_3$O$_{6+\delta}$ as a function of doping of the different scales. According to the lattice-scale, as shown in Fig. 1(b), the dome-shaped $T_c$-curve has plateau at around 0.12. Essentially $T_c(P_{\text{lattice}})$-curve follows the dome-shaped $T_c$-curve but it is more refined such that an indication of the 60 K plateau is observable at ~0.12. Although $T_c(P_{\text{Hall}})$-curve tends to follow the $T_c(P_{\text{dome}})$-curve below 0.09, the $T_c(P_{\text{Hall}})$-curve shows a clear 60 K plateau and the optimal concentration appears at $P_{\text{Hall}} \sim 0.18$.

We proposed a hole-scale which is also based on $S^{290}$ but quantitatively different from dome-scale.\textsuperscript{5} Our proposed scale follows the experimental result that the hole concentration, $P_{\text{pl}}$, versus $S^{290}$ in Y$_{1-x}$Ca$_x$Ba$_2$Cu$_3$O$_{6+\delta}$, where the $P_{\text{pl}}$ is equal to a half of the Ca-content, is corresponding to that in La$_2$Sr$_2$CuO$_4$, where the $P_{\text{pl}}$ is equal to the Sr-content.\textsuperscript{5} It was also confirmed that the relation of $S^{290}$ versus $P_{\text{pl}}$ in the cation-doped HTCS is consistent with the hole concentration of the oxygen-doped HTCS, such as HgBa$_2$CaCu$_2$O$_{6+\delta}$ and HgBa$_2$Ca$_2$Cu$_3$O$_{6+\delta}$, and cation/oxygen co-doped HTCS, such as Bi$_2$Sr$_2$La$_2$CuO$_{6+\delta}$ and
(Hg0.5Fe0.5)Sr2(Y1-xCax)Cu2O7-δ. We will call our proposed hole-scale as the “Ppl-scale”. One of the distinct features of Ppl-scale is that the optimal hole concentration, Pplopt, depends on the HTCS materials, it distributes in the doping range from 0.16 to 0.28 and centered around 0.24 ± 0.01. This is quite distinct from the universal optimal doping concentration in the Pdome-scale. Further, for almost all HTCS, except La2-xSrxCuO4, the doping dependence of Tc does not follow a dome-shaped Tc-curve. Almost all HTCS actually follows a half-dome-shaped Tc-curve. In YBa2Cu3O6+δ, the doping dependence of Tc follows a two-plateau Tc-curve as shown in Fig. 1(c). The well-known 60 K plateau was clearly observed in Ppl-scale but absent in dome-scale. It is clear that, from Fig. 1(a), lattice-scale is quantitatively different from Ppl-scale. From Fig. 1(b), Hall-scale is also quantitatively different from Ppl-scale. In Fig. 1(b), we plot the eRHN/V of La2-xSrxCuO4 and the other materials as a function of Ppl. Although eRHN/V of the other materials roughly follows that of La2-xSrxCuO4 below Ppl ~ 0.15, eRHN/V of YBa2Cu3O6+δ, Bi2Sr2-La2CuO6+δ and Tl2Ba2CuO6+δ over Ppl ~ 0.15 deviates upward from that of La2-xSrxCuO4. In Fig. 1(d), we plot the Tc of Bi2Sr2-La2CuO6+δ against Ppl, Pdome and PHall. The optimal doping and doping dependence in the Ppl-scale are quite different from those in dome-scale and Hall-scale.

In this report, we analyzed the doping dependent data of HTCS by using of Ppl-scale, and compared our conclusions with that derived from other hole-scales. Ppl is determined by comparing the value of S290 data with Ppl-scale, which is more reliable. As the second method, for YBa2Cu3O6+δ, Ppl is determined by comparing the value of Tc with the two-plateau Tc(Ppl)-curve as shown in Fig. 1(c). For HgBa2CuO4+δ, the value of Ppl is determined by comparing the corresponding value of Tc in the paper with Tc-curve determined from S290 data by using Ppl-scale. As the third method, we can determine Ppl from either the in-plane conductivity, σab, or out-of-plane conductivity, σc, since there is a universal doping dependence of σab and σc on Ppl. We always selected the paper that reports the value of S290 first and used the data with the value of Tc second. We also report Rb of Y1-xCa0.5Ba0.5CuO6 with no chain. The details of sample preparation were reported in Refs. 5,9,26. The Hall effect was measured by dc method under a magnetic field up to 7 T or by physical property measurement system (PPMS, Quantum Design). Both results were consistent. In analyzing the reported data by Ppl-scales, we selected the data with either S290 or Tc reported in the literature.

2. Quantitative comparison of Ppl-scale with the other working hole-scales

In Fig. 2, we plot the hole concentration determined by many spectroscopic probes as a function of Ppl. For comparison we also plot the corresponding hole concentration of typical HTCS determined by the dome-scale and the lattice-scale as color-coded lines and dotted line, respectively. The hole concentration measured by near edge x-ray absorption fine structure, NEXAFS, and nuclear quadruple resonance, NQR, are consistent with Ppl within a error band of ± 0.01, consistent with the uncertainty of the Ppl-scale. Further,
we also plot $P_{\text{Hall}}$ of various HTCS materials. $P_{\text{pl}}$ corresponds well to the hole concentration determined by NEXAFS and NQR. In contrast, the $P_{\text{dome}}$, $P_{\text{lattice}}$ and $P_{\text{Hall}}$ scales are clearly deviate substantially from $P_{\text{pl}}$. Here, we demonstrate the first example that completely opposite conclusions resulted when using the $P_{\text{pl}}$-scale and $P_{\text{dome}}$-scale to analyze data. While many HTCS are shown to be having an inhomogeneous doped-hole concentration the absence of doped-hole inhomogeneity in YBa$_2$Cu$_3$O$_{6+\delta}$ seems to be the only exception. Bobroff et al. reported that the NQR spectrum of YBa$_2$Cu$_3$O$_{6+\delta}$ showed that the inhomogeneity in the doping distribution is quite small. However we need to point out that all the conclusions in Ref. 34 were based on the $P_{\text{dome}}$-scale.

Here, we first compare the doped-hole concentration determined by nuclear magnetic resonance, NMR, using $P_{\text{pl}}$-scale and $P_{\text{dome}}$-scale. In Fig. 3, we plot $^{89}$Y Knight shift, $^{89}K_r$, at room temperature, RT, of Y-based HTCS, as a function of $P_{\text{dome}}$ in the top panel and same $^{89}K_r$ as a function of $P_{\text{pl}}$ in the lower panel. In Fig. 3(a), we also draw the two lines with the linear slopes of 580 ppm/hole and 823 ppm/hole reported in Refs. 30, 31. While each linear slope may represent the limited data selected in the corresponding paper, it...
clearly showed that the doping dependence of $^{89}\text{K}_s$ cannot be represented by a straight line using $P_{dome}$-scale when we use all available $^{89}\text{K}_s$ at RT in Fig. 3(a). This suggests a large ambiguity of using $^{89}\text{K}_s$ data to determine hole concentration based on the $P_{dome}$-scale. However, in the $P_{pl}$-scale as shown in Fig. 3(b), all $^{89}\text{K}_s$ at RT lie on one straight line represented by $555P_{pl} - 68$. The deviation at $P_{pl} < 0.07$ may come from the AF phase. The $^{89}\text{K}_s$ increases linearly with $P_{pl}$ within the error bar of the $P_{pl}$-scale. However $^{89}\text{K}_s$ is barely linearly propositional to $P_{dome}$ with a large scattering.

In Ref. 34, from $^{89}\text{Y}$ NMR Fourier transform spectra at 300 K, the doping distribution, $\Delta P_{dome}$, of the slightly overdoped YBa$_2$Cu$_3$O$_7$ and the underdoped YBa$_2$Cu$_3$O$_{6.6}$ were deduced by using the relation of the slope of 580 ppm/hole based on the $P_{dome}$-scale. The
full width at half maximum (FWHM) of $^{89}$Y NMR spectra for YBa$_2$Cu$_3$O$_{6+\delta}$ led to a conclusion that the doping distribution $\Delta P_{\text{dome}}$ < 0.025 for YBa$_2$Cu$_3$O$_7$ and $\Delta P_{\text{dome}}$ < 0.01 for YBa$_2$Cu$_3$O$_{6.6}$ as shown in Fig. 4(a). These values are much smaller than $\Delta P_{\text{dome}}$ = 0.1 in La$_{2-x}$Sr$_x$CuO$_4$ and Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. However if we calculate the doping distribution using $P_{\text{pl}}$-scale, $\Delta P_{\text{pl}}$, from their FWHM and the relation of $555P_{\text{pl}} - 68$, the $\Delta P_{\text{pl}}$, as seen in Fig. 4(b), of YBa$_2$Cu$_3$O$_{6.6}$ is 0.07 and that of YBa$_2$Cu$_3$O$_7$ is ~0.017. We re-plot Fig. 1(a) by converting $P_{\text{dome}}$ into $P_{\text{pl}}$. The red line shows the double plateau $T_c$-curve, with $T_c^{\text{max}}$ = 93 K, reported in Ref. 9.

As a second example, we demonstrate the universal nature of our quantitative scale by focusing on the translational symmetry preserving magnetic order ($T_{\text{mag}}$) recently observed in YBa$_2$Cu$_3$O$_{6+\delta}$ by polarized neutron in the pseudogap state. In the heavily underdoped regime where incommensurate spin-density-wave, SDW, order ($T_{\text{SDW}}$) exists in the YBa$_2$Cu$_3$O$_{6+\delta}$, the $T_{\text{mag}}$ seems to be much reduced, suggesting that they are competing with
each other.47,48 Same magnetic order is also observed in the HgBa2CuO4+δ,49 although the SDW has not been reported in the same system yet. Depending on how the hole concentration is estimated, the competition between magnetic order and SDW cannot be pinned down in the HgBa2CuO4+δ.50 In Fig. 5(a), we reproduce the magnetic phase diagram of Fig. 6 in Ref. 50, YBa2Cu3O6+δ,46-48,51 Y0.85Ca0.15Ba2Cu3O6+δ,46 and HgBa2CuO4+δ.49,50 In Ref. 46, the data of YBa2Cu3O6+δ and Y0.85Ca0.15Ba2Cu3O6+δ are plotted as a function of P_{lattice} and that of HgBa2CuO4+δ is plotted as a function of P_{dome}, determined from the S290 data9 according to dome-scale. The different scales used by different groups present difficulties to compare experimental observations from different group and, furthermore, add substantial confusions in case a conclusion was drawn without properly address the issue of using a quantitatively consistent hole-scale. To evaluate the relation between T_{mag} and T_{SDW}, we also plot T_{SDW} reported in Ref. 48 into Fig. 5(a). According to Ref. 50 if the Fig. 5(a) is correct, it would necessarily imply that a competition between the SDW and the q = 0 magnetic order is either absent in HgBa2CuO4+δ or will only commence at much lower doping than that in YBa2Cu3O6+δ. On the other hand, if the inset of Fig. 5(a) is correct, the q = 0 order in HgBa2CuO4+δ has not yet been investigated to a hole concentration as low as P_{lattice} ~ 0.073 of YBa2Cu3O6+δ, and no conclusion can be drawn at this point regarding its competition with the SDW order. In Fig. 5(b), we plot the same data set as a

![Fig. 5 Magnetic phase diagram of YBa2Cu3O6+δ, Y1-xCaxBa2Cu3O6+δ, Bi2Sr2CaCu2O8+δ and HgBa2CuO4+δ. The plotted data were extracted from Refs. 46-51. (a) Main figure and the inset are corresponding to those of Fig. 6 in Ref. 47, respectively. The carrier concentration for YBa2Cu3O6+δ is based on lattice-scale.7 The carrier concentration for HgBa2CuO4+δ is determined from S290 data23 according to dome-scale in the main figure and from the Tc(P_{lattice})-curve of YBa2Cu3O6+δ in the inset. (b) All carrier concentration is based on the P_{pl-scale}. P_{pl} is 0.25 for YBa2Cu3O6+δ and 0.235 for HgBa2CuO4+δ.9,10 Tc of YBa2Cu3O6+δ are coming from Refs. 47,53,54.](image-url)
function of the reduced doped-hole concentration $P_{pl}/P_{plopt}$, and compare them with universal electronic phase diagram (UEPD) reported in Ref. 9 based on the $P_{pl}$-scale. According to the $P_{pl}$-scale, while $T_{mag}$ of both YBa$_2$Cu$_3$O$_{6+\delta}$ and HgBa$_2$CuO$_{4+\delta}$ occurs at the upper pseudogap temperature for $P_{pl}/P_{plopt}$ > 0.4. But, the $T_{mag}$ of YBa$_2$Cu$_3$O$_{6+\delta}$ rapidly decreases with undoping over $P_{pl}/P_{plopt}$ ~ 0.4 where SDW appears, although there is no data of HgBa$_2$CuO$_{4+\delta}$ with $P_{pl}/P_{plopt}$ < 0.4. Regarding the question in Ref. 50, our answer is that $P_{pl}$ of HgBa$_2$CuO$_{4+\delta}$ investigated is still too high to be compare with YBa$_2$Cu$_3$O$_{6+\delta}$ for addressing the competition between $T_{mag}$ and $T_{SDW}$. More importantly, it is clear that the magnetic phase diagram of HgBa$_2$CuO$_{4+\delta}$ is essentially the same as that of YBa$_2$Cu$_3$O$_{6+\delta}$ for $P_{pl}/P_{plopt}$ > 0.4 and translational symmetry preserving magnetic order, $T_{mag}$, is intimately related to the upper pseudogap reported in our UEPD.

Finally, we want to point out some recent theoretical approaches to the hole-scale.\textsuperscript{55-57} The $P_{pl}$-scale is based on a universal doping dependence of $S^{290}$.\textsuperscript{5,9} Although the dome-scale is based on the universal optimal hole-concentration of 0.16,\textsuperscript{1} it demonstrates another universal doping dependence of $S^{290}$, except of La$_{2-x}$Sr$_x$CuO$_4$.\textsuperscript{2} In order to compare the theoretical works with the hole-scales, it is convenient to use a characteristic doped-hole concentration, $P_{pl}$, where a sign change of $S^{290}$ occurred. Experimentally, the value of $P_{pl}$ is universally ~0.25 in the $P_{pl}$-scale,\textsuperscript{5,9} but ~0.16 in the dome-scale.\textsuperscript{2,4} Theoretically, the doping dependence of $S^{290}$ is reproduced within the Hubbard model with the moderate on-site repulsion\textsuperscript{55} and using a simplified Hubbard model of a bad metal.\textsuperscript{56} In both cases, $P_{pl}$ ~ 0.2. Furthermore, the doping dependence of $S^{290}$ at 0 K, including the sign change, is attributed to a possible underlying quantum critical point.\textsuperscript{57} Unfortunately neither could distinguish the dome-scale from $P_{pl}$-scale.

3. Conclusions

In summary, we have demonstrated the quantitative consistency and the use of the universal $P_{pl}$-scale. Independent of any theoretical considerations we shall emphasize that $P_{pl}$-scale is consistent with the carrier concentration estimated by many different experimental probes in many cuprates. We suggest that the $P_{pl}$-scale should be used for all quantitative doping dependent studies of HTCS.

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