D4 brane probes in gauge/gravity duality

Yang Zhou

Institute of Theoretical Physics
Chinese Academy of Science
Beijing 100190, PRC

Interdisciplinary Center of Theoretical Studies
USTC, Hefei,
Anhui 230026, PRC

Abstract: We propose a DBI vertex brane + $N_c$ fundamental strings configuration for a probe baryon in the finite-temperature thermal gauge field via AdS/CFT correspondence. In particular, we investigate properties of this configuration in $\text{QCD}_4$ and warped $\text{AdS}_6 \times S^4$. We find that, in D4-D8 system, a holographic probe baryon can be described as $N_c$ fundamental strings connecting through a vertex D4 brane wrapped on $S^4$. In $\text{QCD}_4$ background, a closed vertex can exist in confined phase and can not exist in deconfined phase. In the low temperature region, screening effect still exist in confined phase like meson and the vertex D4 brane dominates the baryon mass. The lower energy state corresponds to vertex brane closer to the radial cut off position ($r = r_c$) and the higher energy state corresponds to vertex brane a little far away from the cut off position. The high energy limit of this configuration is just like the unclosed vertex brane configuration in a higher temperature deconfined phase. In warped $\text{AdS}_6 \times S^4$ background, a closed vertex can exist in deconfined phase and the vertex contains a spike, while fundamental strings are relatively short. Screening length should be defined through the distance between top position of the vertex spike and the boundary.

Keywords: D-branes, thermal field theory.
1. Introduction

In recent years, there has been much interest in studying hadrons in strongly coupled QCD in terms of AdS/CFT correspondence. One important topic is the study of holographic baryons in gravity background \[1, 2, 3, 4\]. A holographic baryon in gauge/gravity duality was first introduced by Witten \[5\], where a baryon is identified with a compact D-brane wrapped on a transverse sphere with \(N_c\) strings attached to it. Investigations of baryons in AdS/CFT have been started since ten years ago \[13, 14\], in order to find a solution with a compact vertex brane and DBI strings, but there are many challenges \[7, 8, 9, 13, 14\]. One big problem is how to get a closed brane solution for baryon vertex from the DBI+CS action in a certain gravity background which is dual to the gauge field we want to study. We construct a new configuration with a wrapped vertex brane and \(N_c\) strings to solve these problems. Furthermore, we investigate properties of this configuration in probe limit and argue that these properties may have some signals in QGP in RHIC experiment.

Recently, a simple configuration of baryon in a hot strongly coupled Super Yang Mills plasma was proposed \[25\], where the screening length of a moving baryon with finite velocity was investigated. Screening length and J-E\(^2\) behavior (J is angular momentum and E is baryon mass) of high spin baryons were analyzed \[26\]. All these investigations are in the framework of thermal SYM gauge theory/AdS black hole duality and component quarks are considered as probes. The main results of these works show that baryons in a hot strongly coupled plasma have screening length, which is similar to meson case. Boost
velocity and angular velocity dependence of screening length for baryons are both similar to those of mesons. These results are natural and reasonable, because there the vertex brane is treated as a massive point in AdS$_5$, with an action only depending on the gravity potential.

Generally, the vertex brane can be not a point in AdS$_5$, but a line(trajectory of wrapped brane on S$^5$), which has an action with a DBI term and a Chern-Simons term. Though it's difficult to find a good solution for the vertex brane in many gravity backgrounds, in the new configuration we propose, we obtain two kinds of solutions in QCD$_4$ and AdS$_6 \times S^4$ background respectively, each of which has one closed vertex brane and $N_c$ hanging strings. From new configurations constructed, we find that baryons have some special properties in the probe limit. An apparent property is that baryon mass may be dominated by the vertex brane, so the ”melting picture” of probe baryon in the quark gluon plasma is very different from the meson melting.

Another way to see the role of baryon vertex in gauge/gravity framework is through the finite quark density(or we also say baryon density) D$_{\text{color}}$-D$_{\text{flavor}}$ system. Chemical potential of finite quark density was introduced as time component of U(1) gauge field on flavor branes [18], and chemical potential of isospin density was introduced as time component of SU(2) gauge field on flavor branes. Finite quark and isospin density affect embedding of flavor brane, as well as the phase transition corresponding to meson dissociation [19, 21, 20]. In a D$_{\text{color}}$-D$_{\text{flavor}}$ system, a flavor brane can have Minkowski embedding or black hole embedding. But in the case of finite quark density, it has been argued that D$_{\text{flavor}}$ branes have to touch the black hole horizon since the strings connecting the D$_{\text{flavor}}$ branes and the horizon can be replaced by the deformation of the D$_{\text{flavor}}$ brane [18], thus there is no Minkowski embedding in the finite quark density case. In this case, a baryon vertex can end hanging strings outside of the horizon, so there is no way for a flavor brane to touch the horizon [2]. Since this argument, we can see that we can safely discuss a baryon in the probe limit in the Minkowski embedding. We should also note that a baryon in our probe limit is different from the baryons in finite density which should be considered as background baryons. In our probe limit, a baryon almost have no backreaction on flavor branes.

To find a suitable D$_{\text{color}}$-D$_{\text{flavor}}$ system for discussion, we notice the QCD$_4$ gravity background from IIA string theory. The D4 branes background of QCD$_4$ with Euclidean signature have two compactified directions and D8 branes are flavors [15]. There are two main phases, confinement and deconfinement phases in this holographic model. In the present paper, we will investigate the properties of a probe baryon in these two phases, which is an extended work on baryon probe, of the former work [24]. It’s believed that heavy-quark bound state can survive in a quark gluon plasma at a temperature which is higher than the confinement/deconfinement transition temperature [22]. We want to have a closer look at the baryon configuration in these different phases, especially in the deconfined phase of a hot quark-gluon plasma.

The present paper is organized as follows. In section 2, we review different phases of

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$^1$QCD$_4$ has the same mean as one in paper [13].
QCD$_4$ at different temperatures. We also review a point brane + strings model and a DBI brane model of a holographic baryon in phases in QCD$_4$ background and main properties of AdS$_6\times$S$^4$ background we will use. In section 3, we propose a new baryon configuration in different backgrounds. In section 4, we study the screening length, baryon mass and interacting energy in QCD$_4$ background. In the last section, we study the baryon properties and define a new screening length in warped AdS$_6\times$S$^4$ background.

2. Review of different background phases and baryon probe model

2.1 Different phases of D4 branes background

QCD$_4$ background We summarize different geometry backgrounds from the D4-D8 branes model [15]. The main success of this model is the prediction of the spectrum of low energy hadrons and their dynamics, which are non-perturbative properties of QCD. It’s a good choice for the top-down holographic QCD model since it contains chiral fermions and an apparent chiral symmetry breaking mechanism. Gluons are represented in terms of fluctuating modes of open strings on $N_c$ D4 branes. Massless quarks are represented in terms of fluctuations modes of open strings connecting $N_c$ D4 branes and $N_f$ D8(D8) branes.$^2$ The supergravity description of $N_c$ D4 branes and gauge field description of the D4 branes give the gauge/gravity duality. If $N_c >> N_f$, D8 branes which give the flavor freedom have no backreaction to the background geometry and can be considered as probes. When the excitations on D4 branes are very heavy, there will be a black hole in the bulk, which corresponds to the thermal gauge field on the boundary. At different temperatures, there appear different gravity backgrounds [27]. The confining background metric of Euclidean model at zero temperature is

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}(dt^2 + d\vec{x}^2 + f(r)dx_4^2) + \left(\frac{R}{r}\right)^{3/2}\left(\frac{dr^2}{f(r)} + r^2d\Omega_4^2\right)$$

(2.1)

with a dilaton and a 4-form RR field strength

$$e^\phi = g_s\left(\frac{r}{R}\right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{V_4}\epsilon_4.$$  

(2.2)

where $f(r) = 1 - \left(\frac{r}{r_c}\right)^3$. The duality relation is $R^3 = \pi g_s N_c \epsilon_4^3$. $g_s, N_c, l_s$ are string coupling constant, color number and string length scale, respectively. $\vec{x} = x_{1,2,3}$, $r$ is the radial coordinate and $\Omega_4$ is four angle coordinates, in the $x_{5,6,7,8,9}$ space. $V_4 = 8\pi^2/3$ is the volume of the unit S$^4$ and $\epsilon_4$ is the volume 4-form. To cancel the conical singularity at $r = r_c$, the period of $\delta x_4$ in the compactified direction must be

$$\delta x_4 = \frac{4\pi}{3}\left(\frac{R}{r_c}\right)^{1/2}.$$  

(2.3)

The Kaluza-Klein mass is defined as

$$M_{KK} = \frac{2\pi}{\delta x_4} = \frac{3}{2}\left(\frac{r_c}{R}\right)^{1/2}.$$  

(2.4)

$^2$We do not consider these light quarks in the same way of the work [24] and just consider a probe baryon composed with heavy quarks in our present work.
In this phase, we can obtain the glueball and meson spectra by computing the fluctuation of background supergravity fields and fields on the flavor branes respectively. Their spectra are discrete, which shows that the system is confined. We regard the Hawking temperature of the background as the temperature of thermal field. We have \( T = 1/\beta \), where \( \beta \) is the period of Euclidean time. At high temperature region, the phase is deconfined. The background geometry contains a black hole:

\[
\text{ds}^2 = \left( \frac{R}{r} \right)^{3/2} \left( f(r) dt_E^2 + d\vec{x}^2 + dx_4^2 \right) + \left( \frac{R}{r} \right)^{3/2} \left( \frac{dr^2}{f(r)} + r^2 d\Omega_4^2 \right)
\]

(2.5)

with a dilaton and a 4-form RR field strength

\[
e^\phi = g_s \left( \frac{R}{r} \right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{V_4} \epsilon_4.
\]

(2.6)

where \( R^3 = \pi g_s N_c l_s^3 \), \( f(r) = 1 - \left( \frac{r_0}{r} \right)^2 \). The \( t_E \) must have period

\[
\delta t_E = \frac{4\pi}{3} \left( \frac{R^3}{r_0} \right)^{1/2} = \frac{1}{T}.
\]

(2.7)

This background metric has the same form as the confining one (2.1), only with the exchanging of \( t_E \) and \( x_4 \).

**Warped AdS\(_6\) × S\(^4\) background** Massive type IIA supergravity admits a warped AdS\(_6\) × S\(^4\) vacuum solution, which is expected to be dual to an \( \mathcal{N}=2 \), D=5 super-conformal Yang-Mills theory. This background is supported by Ramond-Ramond field strengths, in addition with a non-constant dilaton. The AdS\(_6\) × S\(^4\) metric is warped, with a warped factor which becomes singular on the equator of S\(^4\). Thus the geometry really corresponds to a hemisphere instead of the full S\(^4\). This background arises as the near-horizon geometry of a semi-localized D4-D8 system \([30, 31]\). In the string frame, this solution is given by \([2, 3]\)

\[
\text{ds}^2_{10} = (\cos \theta)^{-1/3} \left[ \text{ds}^2_{\text{AdS}_6} + 2g^{-2}(d\theta + \sin^2 \theta d\Omega_3^2) \right].
\]

(2.8)

Four form field strength and the dilaton take the forms

\[
F_{(4)} = \frac{5\sqrt{2}}{6} g^{-3}(\cos \theta)^{1/3} \sin^3 \theta d\theta \wedge \Omega_{(3)} \quad \text{e}^\phi = (\cos \theta)^{-5/6},
\]

(2.9)

where \( \Omega_{(3)} \) is three sphere volume form. The AdS black hole metric is

\[
\text{ds}^2_{\text{AdS}_6} = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2(\Sigma_{i=1}^4 dx^i dx^i)
\]

(2.10)

where \( f(r) = g^2 r^2 - \mu \), \( g \) and \( \mu \) are two parameters independent on \( r \).

**2.2 Point brane + hanging strings configuration for baryon**

Holographic baryons have configurations of hanging strings attached to a compact vertex brane. While baryons in the boundary field are composed of external heavy quarks. In probe limit, each component quark attached with a fundamental string is considered as a probe string in the bulk. Since these fundamental strings connect with a vertex brane, we
can see this vertex brane as a probe. In recent works \cite{25, 26}, the configuration of open strings+compact D5 brane wrapped on the $S^5$ were analyzed. The background is given by

$$ds^2 = -f(r)dt^2 + \frac{r^2}{R^2}d\Omega_4^2 + \left( \frac{R}{f(r)} \right)^{3/2} \left( dt_E^2 + f(r)dx_4^2 \right) + \frac{R}{f(r)} \left( \frac{dr^2}{f(r)} + r^2 d\Omega_4^2 \right).$$  \hspace{1cm} (2.11)

where $f(r) = \frac{r^2}{R^2} \left( 1 - \frac{r_0^4}{r^4} \right)$. The action of baryon is summation of $N_c$ fundamental strings and D5 brane

$$S_{\text{total}} = \sum_{i=1}^{N_c} S_{\text{string}}^{(i)} + S_{D5}. \hspace{1cm} (2.12)$$

where the action of D5 is given by a massive point action in gravity field

$$S_{D5} = \frac{\mathcal{V}(r_c) TV_5}{(2\pi)^5 \alpha'^3}. \hspace{1cm} (2.13)$$

The screening length of baryon with finite moving speed in a plasma was computed \cite{25}. High spin baryons in the $AdS_5 \times S^5$ were investigated and the angular velocity dependence of screening length and $J-E^2$ behavior were computed numerically \cite{26}. However, all these computations were done in the conformal and supersymmetric background, and the D5 brane is treated as a massive point in $AdS_5$. The QCD$_4$ background is a good choice for the non-supersymmetric and non-conformal background. We can construct a new configuration with a DBI D4 brane and $N_c$ strings in this background.

### 2.3 A brane model for baryon vertex

Ten years ago, people believed that main information of a holographic baryon was hidden in the vertex brane. We can indeed get interesting information of a holographic baryon from the DBI+CS action of a compact brane. We take a confining background for example as follow:

$$ds^2 = \left( \frac{r}{R} \right)^{3/2} \left( dt_E^2 + d\vec{x}^2 + f(r)dx_4^2 \right) + \left( \frac{R}{r} \right)^{3/2} \left( \frac{dr^2}{f(r)} + r^2 d\Omega_4^2 \right) \hspace{1cm} (2.14)$$

where $r$ is the radial coordinate and the embedding function of the compact D4 brane wrapped on the $S^4$ can only be determined by $r(\theta)$. And the gauge field on the single D4 brane can also be written as $A(\theta,t)$ for symmetry. The action of the D4 brane obtained from the induced metric is given by \cite{14, 13}

$$S_{D4} = -T_4 \int d^5\xi \sqrt{-\det(g + F)} + T_4 \int A_{(1)} \wedge G_{(4)} \hspace{1cm} (2.15)$$

From this action, we can solve the equation of motion for gauge field and find the embedding function $r(\theta)$.

It is argued that a closed curve $r(\theta)$ corresponds to a real baryon vertex. The vertex brane dynamics can be also solved in a deconfined phase, but there is no closed solution \cite{14}. However, there is abundant evidence to support that heavy-quark bound states can survive in a quark-gluon plasma \cite{22}, which is in the deconfined phase from the lattice results. So there is an apparent paradox. To solve this problem, we try the deconfined warped AdS black hole background in subsection 3.3 in the present paper, where the warped AdS example reflects more faithfully the physics expected in actual QCD.
3. DBI brane + $N_c$ strings configuration in different phases

In this section, we propose a DBI brane + $N_c$ strings configuration for a holographic baryon probe and investigate its properties in different phases of D4 branes background.

3.1 DBI brane + $N_c$ strings in confined QCD$_4$ background

We start from the confined background:

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}(dt^2 + dx_4^2) + \left(\frac{R}{r}\right)^{3/2}\left(\frac{dr^2}{f(r)} + r^2dΩ_3^2\right). \quad (3.1)$$

This metric with periodic Euclidean time and a compactified space direction $x_4$ corresponds to four dimensional boundary thermal field. We consider a static baryon in the thermal field as hanging open strings attached to a D4 brane wrapped on the $S^4$. Open strings connect flavor branes and the single compact D4 brane. In this configuration, we denote the world volume coordinates of D4 brane as $(t, θ, α, β, γ)$. And the embedding function is $r = r(t, θ, α, β, γ)$ and the U(1) gauge field on the D4 brane is $A_μ = A_μ(t, θ, α, β, γ)$. The induced metric on D4 brane is

$$ds_{D4}^2 = -\left(\frac{r}{R}\right)^{3/2}dt^2 + \frac{R}{r}\left[\frac{r'^2}{f(r)} + r^2(sin^2 θ)D(θ)^2\right]dθ^2 + r^2sin^2 θdΩ_3^2, \quad (3.2)$$

The action of the compact D4 brane can be given as

$$S_{D4} = -T_4\int d^5ξ e^{-φ}\sqrt{-det(g + F)} + T_4\int A(1)∧G(4), \quad (3.3)$$

where, $g_{μν}$ is the induced metric on the D4 brane. $F = dA$, $T_4 = 1/(gs(2π)^4l_s^4)$ is D4 brane tension, and the last term is Wess-Zumino term. We suppose that the D4 brane wrapped on the $S^4$ has a SO(4) symmetry, and the embedding function and gauge field depend only on $(t, θ)$. For a static configuration, we have $r = r(θ)$ and $A_t = A_t(θ)$. We can rewrite the action by performing Legendre transformation \[^3\] to

$$\mathcal{H} = T_4Ω_3R^3\int dθ\sqrt{r'^2 + f(r)^{-1}r''(θ)^2}\sqrt{D(θ)^2 + sin^6 θ}, \quad (3.4)$$

where the displacement $D$ satisfies

$$\partial_θ D = -3sin^3 θ, \quad (3.5)$$

which comes from the equation of motion of the gauge field. Solving equation(3.5), we get

$$D(θ) = 3cos θ - cos^3 θ - 2 + 4ν. \quad (3.6)$$

$ν$ in the constant of integration is a parameter $0 ≤ ν ≤ 1$, which controls the number of Born-Infeld strings emerging from the D4-brane at each pole of the $S^4$ ($θ = π$ and $θ = 0$) \[^3\] \[^4\]. We find that the spikes at $θ = 0$ and $θ = π$ have the same asymptotic

[^3]: Since this is a static solution, we ignore the time component.
‘tension’ as \( \nu N_c \) and \( (1 - \nu)N_c \) fundamental strings, respectively. We hope that the solution which contains a vertex brane with excited string-like spikes can be obtained from the total action of vertex brane. But there remain lots of challenges \([13, 12, 9, 11]\). So we always use FBC to cancel the singularity of vertex brane.

Now we give the numerical result in Figure 1. We set \( r'(0) = 0 \) and consider \( \nu = 0 \), which means that \( N_c \) fundamental strings all connect with the north pole of \( S^4 \). The \( N_c \) fundamental strings hang from flavor brane (we just consider single flavor). By the following string embedding

\[
\tau = t, \quad \sigma = r, \quad \rho = \sqrt{x_1^2 + x_2^2} = \rho(r), \quad x_3 = \text{constant},
\]

where \( x_i(i = 1, 2, 3) \) is the boundary spatial direction, the action can be written as

\[
S_{\text{string}} = \frac{1}{2\pi\alpha'} \int dt \int_{r_e}^{r_N} dr \sqrt{-\det[h_{ab}]},
\]

thus the string world sheet Lagrangian density is

\[
\mathcal{L} = \sqrt{-\det[h_{ab}]},
\]

where \( h_{ab} = g_{\mu\nu} \frac{\partial x^a}{\partial x^\mu} \frac{\partial x^b}{\partial x^\nu} \). The total action is

\[
S_{\text{total}} = \sum_{i=1}^{N_c} S^{(i)}_{\text{string}} + S_{D4}.
\]

To obtain the force balance condition (FBC), we rewrite the action of the D4 brane:

\[
S'_{D4} = \int dt \mathcal{H}
\]

Extremizing \( S_{\text{total}} \) with respect to \( r_e \), we get FBC

\[
\sum_{i=1}^{N_c} H^{(i)} \bigg|_{r_e} = \Sigma,
\]
where
\[ H^{(i)} = L^{(i)} - \rho^{(i)} \frac{\partial L^{(i)}}{\partial \rho^{(i)}}, \quad (3.13) \]
\[ \Sigma \equiv \frac{2\pi\alpha' \cdot \partial S'_{D4}}{T} \partial r_e = 2\pi\alpha' \frac{\partial H}{\partial r_e}. \quad (3.14) \]
For a given \( r_0 \) we can get the value of \( r_e = r(\pi) \) from the EOM of \( r(\theta) \). Then we can get the embedding function of \( N_c \) strings with FBC.

We consider a D8 brane as a flavor brane. The interaction between the D8 brane and the background \( N_c \) D4 branes makes the D8 brane embed nontrivially in the confined geometry. The DBI action of the D8 brane is
\[ S_{D8} = T_8 \int d^9 \xi \sqrt{-\det(g + F)}, \quad (3.15) \]
where the D8 brane extends \( (t, \vec{x}, x_4, \Omega_4) \). The gauge field fluctuations on the flavor brane always correspond to vector mesons in the boundary field. In the present paper, we ignore these fluctuations. After the ansatz of the D8 brane, the embedding function is only determined by \( x_4(r) \) and we can obtain the solution by solving the equation of motion with an initial condition. In the D4-D8 system, the heavy quarks have no backreaction to the system, so we ignore the pull force of hanging strings to the flavor brane. As discussed in our former work [26], we can obtain solutions and define screening length.

Let us turn to the solution of the compact D4 brane we obtained in Figure 1. We will get a very large \( r_e \) if we change the initial \( r(0) \) to a suitable one. The cusp of the D4 brane will replace the fundamental strings and touch the flavor brane if \( r_e \geq r_A \). But, since we assume that \( N_c \) probe quarks (not dynamic) have no backreaction to D8, here \( N_c \) hanging strings do not need to be replaced by the deformed D8 brane. It’s very different from the case of finite quarks density where the deformed flavor brane always replaces many open strings connected with the horizon. In experiments, component quarks in probe baryons which can survive in the QGP are much heavier than the background quarks or gluons in quark gluon plasma. There will be some apparent properties of the probe in the medium, especially when we boost or rotate it. These properties are important signals for hot strongly coupled plasma.

### 3.2 DBI brane + \( N_c \) strings in deconfined QCD\(_4\) background

As the temperature rises up, the D4-D8 system will undergo a first order phase transition, where the gluonic degree of freedom get deconfined. The corresponding background geometry becomes:
\[ ds^2 = \left( \frac{r}{R} \right)^{3/2} (f(r) dt^2 + d\vec{x}^2 + dx_4^2) + \left( \frac{R}{r} \right)^{3/2} \left( \frac{dr^2}{f(r)} + r^2 d\Omega_4^2 \right) \quad (3.16) \]
The investigation of baryon probes in this phase is similar to the investigation in confined phase. The induced metric on the vertex D4 brane is
\[ ds^2_{D4} = -\left( \frac{r}{R} \right)^{3/2} f(r) dt^2 + \left( \frac{R}{r} \right)^{3/2} \left[ \left( \frac{r^2}{f(r)} + r^2 \right) d\theta^2 + r^2 \sin^2 \theta d\Omega_3^2 \right] \quad (3.17) \]
The DBI action of the compact D4 brane is
\[ S_{D4} = -T_4 \int d^5 \xi e^{-\phi} \sqrt{-\det(g + F)} + T_4 \int A_{(1)} \wedge G_{(4)}, \] (3.18)

Thus using the same method in the confined case, we obtain the energy function
\[ H = T_4 \Omega_3 R^3 \int d\theta \sqrt{f(r) r^2 + r'^2} \sqrt{D(\theta)^2 + 6 \sin^6 \theta}, \] (3.19)

Among solutions of \( r(\theta) \) from the above Lagrangian, we cannot find a closed one. \( r(\pi) \) of all solutions run to infinity. But we can still try to get the baryon configuration as in [13] where the background has no \( x_4 \) direction and becomes Euclidean effectively. And an effective baryon there there has a transformed Lagrangian which is similar to (3.4).

### 3.3 DBI brane + \( N_c \) strings in warped \( AdS_6 \times S^4 \)

Massive type IIA supergravity admits a warped \( AdS_6 \times S^4 \) vacuum solution, which is expected to be dual to an \( \mathcal{N}=2, D=5 \) super-conformal Yang-Mills theory. We study a DBI brane + \( N_c \) strings configuration for baryon in this background. In Eq.(2.8), we note that the metric is singular at \( \theta = \frac{\pi}{2} \), thus \( \theta \) covers \([0, \frac{\pi}{2})\) in a hemisphere instead of the full \( S^4 \). We shall study DBI vertex brane + hanging strings configuration in this geometry, which corresponds to a baryon state in \( \mathcal{N}=2, D=5 \) super-conformal Yang-Mills theory. We denote coordinates of three sphere as \((\alpha, \beta, \gamma)\) and world volume coordinates of a vertex D4 brane as \((\tau, \xi^1, \xi^2, \xi^3, \xi^4)\). By the following consistent ansatz that describes the embedding D4 brane
\[ \tau = t, \quad \xi^1 = \theta, \quad \xi^2 = \alpha, \quad \xi^3 = \beta, \quad \xi^4 = \gamma, \quad r = r(\xi^1) \] (3.20)
we write the induced metric of D4 brane
\[ ds_{D4} = (\cos \xi^1)^{-1/3} [-f(r) d\tau^2 + \left( \frac{r'^2(\xi^1)}{f(r)} + 2g^{-2} \right) d\xi^1 + 2g^{-2} \sin^2 \xi^1 d\Omega^2_3]. \] (3.21)

The world volume action of D4 brane contains a DBI term and a Chern-Simons term
\[ S_{D4} = S_{DBI} + S_{CS}, \] (3.22)
where
\[ S_{DBI} = -T_4 \int d\tau d^4 \xi \sqrt{-\det(g + F)} \]
\[ = -T_4 g_s \Omega_3 \left( \frac{\sqrt{2}}{g} \right)^3 \int d\tau d\xi^1 \sin^3(\xi^1) \]
\[ \times \sqrt{r''(\xi^1) + 2g^{-2} f(r) - \cos(\xi^1)^2} \frac{F_{\xi^1}}{4}. \] (3.23)

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\(^4\)We do not consider this kind of unclosed solutions here due to the following reasons. One is that for not very heavy probe quarks these solutions are usually not regarded as baryons since quarks and gluons are deconfined in this phase. Another reason is that we don’t know how to analyze properties of baryon probes in detail by these solutions, though we can also think that for very heavy probe quarks, this D4-brane “spike” by itself represents a bundle of \( N \) strings and these solutions can be regarded as baryons within which the quarks have coalesced and are no longer individually discernible [13].
The Chern-Simons coupling term is
\[ S_{CS} = T_4 \int A \wedge \mathcal{P}(F_4) \]
\[ = -\frac{5\sqrt{2}}{6} g^{-3} \Omega_3 \int d\tau d\xi^1 A_t \cos(\xi^1)^{1/3} \sin^3(\xi^1), \]
where \( \Omega_3 \) is the volume of unit \( S^3 \). We obtain the vertex D4 brane Lagrangian density along \( \xi^1 \)
\[ \mathcal{L}_{D4} = \sin^3(\xi^1) \cos(\xi^1)^{1/3} \sqrt{[r'(\xi^1) + 2g^{-2} f(r)] \cos(\xi^1)^{-2/3} - \frac{F_{\xi^1}^2}{2}} + \frac{5}{24} A_t. \] (3.25)

Thus the action of vertex D4 brane can be written as
\[ S = -T_4 \Omega_3 g_s \frac{\sqrt{2}}{g} \int d\tau \mathcal{L}_{D4}. \] (3.26)

The equation of motion of \( A_t \) is given by
\[ \frac{\partial \mathcal{L}_{D4}}{\partial F_{\xi^1}} = -D(\xi^1), \] (3.27)
where
\[ \partial_{\xi^1} D(\xi^1) = \frac{5}{12 g_s} \sin^3(\xi^1) \cos(\xi^1)^{1/3}. \] (3.28)

To solve \( D(\xi^1) \), we denote \( y = \sin(\xi^1) \), then \( D(\xi^1) \) is given by
\[ D(y) = \frac{5}{12 g_s} \frac{1}{1729} 3y[-(1 - y^2)^{2/3}(135 + 105y^2 + 91y^4) + 135F[1/2, 1/3, 3/2, y^2]] + C \] (3.29)

The Legendre transformation of the Lagrangian can help to eliminate the gauge field. We obtain an energy function of the embedding coordinate \( r(\xi^1) \) only:
\[ \mathcal{H} = -T_4 \Omega_3 g_s \frac{\sqrt{2}}{g} \int d\xi^1 \sqrt{[r'(\xi^1) + 2g^{-2} f(r)] \cos(\xi^1)^{-2/3} [\sin^6(\xi^1) \cos(\xi^1)^{1/3} + D^2]} \] (3.30)

From this Lagrangian, we get the EOM of \( r(\xi^1) \)
\[ \frac{\partial \mathcal{H}}{\partial r} - \partial_{\xi^1} \frac{\partial \mathcal{H}}{\partial r^\xi^1} = 0 \] (3.31)

To see the result quickly, we solve the EOM numerically in Figure 2. From Figure 4, we find that for \( \xi^1 \in [0, \frac{\pi}{2}] \), \( r \) has finite values. Since the hemisphere geometry corresponds to the background with \( \theta \in [0, \frac{\pi}{2}] \), \( \xi^1 \) can only take values between 0 and \( \frac{\pi}{2} \). The warped AdS black hole corresponds to a deconfined gauge field theory, so we obtain closed vertex D4 brane solutions from the DBI+CS action naturally.
4. Baryon properties in QCD$_4$ background

Baryon probes in quark gluon plasma were first investigated in works \cite{6, 25, 26}, where the background is AdS$_5 \times$S$^5$ dual to the $\mathcal{N} = 4$ supersymmetric Yang-Mills gauge field, and the vertex D5 brane sits a point in the AdS space in \cite{25, 26}. In D4-D8 system, we consider that the trajectory of the vertex D4 brane is not a trivial point and the embedding function depends on $r(\theta)$. We are interested in the properties of baryon probe of this configuration. We analyze the screening length and baryon mass behavior in this section.

4.1 Screening length and baryon melting

In the former work \cite{25, 26}, screening length of baryon was defined as the largest value of the boundary quark separation when we change the position of the bottom of hanging strings $r_e$. From our solutions, we find the trajectory of vertex brane in the bulk radial direction is a finite line. We defined the position of cusp of vertex brane as the furthest position which can be probed by the baryon. Thus, we obtain the $r_e$ dependence of the quark separation $l_q$ and get the value of screening length $l_s$.

We consider boosted and rotating quarks and use the following background:\footnote{Corresponding hanging strings rotate rigidly in the bulk, which is like the meson case in work \cite{24} and different from the strings with rotating ends in work \cite{39}.

$$\begin{align*}
ds^2 &= (\frac{r}{R})^{3/2} (-dt^2 + dx_3^2) + (\frac{r}{R})^{3/2} (d\rho^2 + \rho^2 d\theta^2) + (\frac{R}{r})^{3/2} \frac{dr^2}{f(r)}
\end{align*}$$

The metric is invariant when it is boosted in $x_3$ direction. So properties of a baryon are independent on wind in $x_3$ direction. We now focus on baryon configurations rotating in the $x_1-x_2$ plane at angular velocity $\omega$. For symmetry, each string can be described by $\rho(r)$. By the following consistent ansatz

$$\begin{align*}
\tau &= t , \quad \sigma = r , \quad \theta = \omega t , \quad \rho = \rho(r) ,
\end{align*}$$

Figure 2: Embedding function of D4 brane in $r - \xi^1$ plane
We can write the action of a single string is:

\[ S_{\text{string}} = \frac{1}{2\pi\alpha'} \int dt \int_{r_e}^{r_A} dr \sqrt{-\det[h_{ab}]} \]  

(4.3)

\( h_{ab} \) can be read from the induced metric

\[ ds^2 = \left( \frac{r}{R} \right)^{3/2} (\rho^2 \omega^2 - 1) dt^2 + \left[ \left( \frac{R}{r} \right)^{3/2} \rho^2 + \left( \frac{R}{r} \right)^{3/2} \frac{1}{f} \right] dr^2 . \]  

(4.4)

The string world sheet Lagrangian density

\[ \mathcal{L}_{\text{string}} = \left( \frac{r}{R} \right)^{3/2} \sqrt{(1 - \rho^2 \omega^2)(\rho^2 + \frac{R^3}{r^3 - r_e^3})} . \]  

(4.5)

To solve the equation of motion

\[ \left[ \frac{\partial}{\partial \rho(r)} - \frac{\partial}{\partial r} \frac{\partial}{\partial \rho(r)} \right] \mathcal{L} = 0 , \]  

(4.6)

We need two boundary conditions \( \rho(r_e) \) and \( \rho'(r_e) \). Where the constraint at \( r_e \) can be obtained from the FBC in eqa (3.11):

\[ \mathcal{L} - \rho \frac{\partial \mathcal{L}}{\partial \rho'} \bigg|_{r_e} = \frac{2\pi\alpha'}{N_c} \frac{\partial H}{\partial r_e} , \]  

(4.7)

The FBC turns out to be

\[ \left. \left( \frac{r}{R} \right)^{3/2} \frac{\partial}{\partial \rho(r)} \frac{\partial}{\partial \rho'(r)} \right|_{r_e} = \frac{8\pi\alpha' T_3 \Omega_3 R^3}{N_c} \sqrt{\frac{f^{-1} r'}{r^2 + f^{-1} r'^2}} \bigg|_{r=r(\theta=\pi)} . \]  

(4.8)

Finally, by analyzing the solutions we define the screening length as the critical value of the boundary quark separation and find the \( \omega \) dependence of screening length \( l_s \). The screening length \( l_s \) shows that if heavy quarks have enough kinetic energy, they may break away from each other. It means that baryons dissociate.

To obtain the screening length of the DBI brane + strings configuration, we calculate the \( r_e \) dependence of \( l_q \) numerically which has been discussed more carefully in [26]. The results with different \( \omega \) are shown in Figure 3. We choose the maximal value of \( l_q \) (also the critical value) as the screening length for baryons. And we think that only the right part of each curve beside the highest point contains the points which correspond to real baryon configurations. The \( \omega \) dependence of \( l_s \) can be given in Figure 4.

### 4.2 Baryon mass and interaction potential

Now we want to give the definition of baryon mass and interaction potential. In a very general way, baryon mass is given by summation of the energy of \( N_c \) strings and the vertex brane.

\[ E_{\text{total}} = N_c E_{\text{string}} + E_{D4} , \]  

(4.9)
Figure 3: $r_e$ dependence of $l_q$ at different values of $\omega$. Curves I, II, III correspond to $\omega = 0, 0.5, 1$ respectively. Values of $l_q$ and $r_e$ are determined assuming $r_0$ as unit.

Figure 4: $\omega$ dependence of $l_s$.

where

$$E_{\text{string}} = \omega \frac{\partial L}{\partial \omega} - L, \quad E_{D4} = \mathcal{H},$$

(4.10)

The string Lagrangian is

$$L = \frac{1}{2\pi \alpha'} \int_{r_e}^{r_\Lambda} dr \left( \frac{r}{R} \right)^{3/2} \sqrt{\left(1 - \rho^2 \omega^2\right)\left(\rho'^2 + \frac{R^3}{r^3 - r_e^3}\right)}.$$  

(4.11)

In order to obtain the interaction potential, we analyze the free quarks case, in which $N_c$ strings hang from the boundary to $r_c$ and compact D4 brane almost wrapped on the $r = r_0$. The so called interaction potential is given by subtracting the energy of the free strings and the corresponding vertex brane. Assuming the radial position of D8 is $r_\Lambda$, the radial
Figure 5: $l_q$ dependence of interaction potential of $N_c$ quarks. Curves I, II, III correspond to $\omega = 0.3, 0.5, 0.8$ respectively. For simplicity, the value of $E_I$ in the figure is actually $E_I \ast (2\pi\alpha' / N_c)$ in our paper.

Distance of $r_c$ and D8 is $r_\Lambda - r_c$, and the energy of single quark is given by

$$E_q = \frac{1}{2\pi\alpha'} \int_{r_c}^{r_\Lambda} dr,$$

and the energy of initial brane which is very close to the “cut off position” ($r = r_c$) is given by

$$\mathcal{H}_0 = T_4 \Omega_3 R^3 r_0 \int d\theta \sqrt{D(\theta)^2 + \sin^6 \theta}.$$

Then the interaction potential of baryon is

$$E_I = E_{\text{total}} - N_c E_q - \mathcal{H}_0.$$

We give the numerical results about $l_q$ dependence of $E_I$ at different values of $\omega$ in Figure 5. The Figure 5 shows that we should choose the low energy branch for a given $l_q$. Comparing with Figure 3, we find that the low energy branch in Figure 5 corresponds to the left branch in Figure 3. It implies that we should choose the smaller $r_c$ for a given $l_q$. The main reason is that larger $r_c$ corresponds to larger D4 brane energy. Actually, for a given $r_0$ larger than a critical value, the D4 brane will be enlarged to the boundary D8 brane and the length of hanging strings becomes zero. Thus we can not obtain a closed brane configuration, which is similar to the deconfined case. When the background becomes closer to a deconfined one, quarks will try to run away from each other. In our confined background, it corresponds to the vertex D4 brane enlarged to attach to the boundary brane.

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6Actually, there is no free quark in this confined phase, to calculate the interaction energy in the string picture, we should subtract mass of $N_c$ quarks, which are not real physical objects in this confined phase but a reference.

7We should note that interaction potential here is different from binding energy, the rigid rotating effect of strings and the vertex brane contribute more energy to make $E_I$ positive.
4.3 Discussion

After a simple discussion of point brane + $N_c$ strings in AdS$_5 \times S^5$ in a former work [26], we extend the investigation to a new configuration of a DBI brane + $N_c$ strings in QCD$_4$ background. Our investigation implies that some properties such as screening length still exists in the new configuration. But the baryon mass and interaction potential are very different from the simple model with a point vertex brane in the AdS bulk. In the QCD$_4$ background, there is no horizon and we find that the minimal energy state corresponds to the point-like D4 brane in the AdS gravity background, connected to $N_c$ hanging strings with the largest length. It appears very different from the $N_c$ strings + point D5 brane as before [26, 25], where the energy of the D5 brane depends only on $r$. The energy of D4 brane dominates the interaction potential in this case.

5. Baryon properties in warped AdS$_6 \times S^4$ background

After analyzing the properties of holographic baryon probe in QCD$_4$ background, we want to see properties of our solution in warped AdS$_6 \times S^4$. We consider boosted and rotating quarks and use the following background:

$$ds^2 = (\cos \theta)^{-1/3}[-(g^2 r^2 - \mu/r^3)dt^2 + r^2 dx_3^2 + \frac{dr^2}{(g^2 r^2 - \mu/r^3)} + r^2(d\rho^2 + \rho^2 d\theta^2)]$$  \hspace{1cm} (5.1)

If we stand in the rest frame of quarks, we will feel a moving plasma wind. We now focus on baryon configurations rotating in the $x_1 - x_2$ plane with angular velocity $\omega$ in the plasma moving with a wind velocity $v = -\tanh \eta$ in the $x_3$ direction. For symmetry, each of strings can be described by $\rho(r)$. By the following consistent ansatz

$$\tau = t, \quad \sigma = r, \quad \theta = \omega t, \quad \rho = \rho(r), \quad x_3 = x_3(\sigma),$$  \hspace{1cm} (5.2)

We can write the action of a single string is:

$$S_{\text{string}} = \frac{1}{2\pi \alpha'} \int dt \int_{r_e}^{r_{\text{top}}} dr \sqrt{-\det[h_{ab}]} = \frac{1}{2\pi \alpha'} \int dt \int_{r_e}^{r_{\text{top}}} dr L_{\text{string}},$$  \hspace{1cm} (5.3)

$h_{ab}$ can be read from the induced metric on the string world sheet (more details can be read from appendix). All hanging strings attach to the highest point of vertex solution in $r$ direction, then the FBC turns to be

$$L - \rho \frac{\partial L}{\partial \rho} \bigg|_{r_e} = \frac{2\pi \alpha'(\cos \theta)^{1/3}}{N_c} \frac{\partial H}{\partial r_e},$$  \hspace{1cm} (5.4)

where $r_e = r_{\text{top}}(\text{with } r'(\xi^1) = 0)$ in the brane solution in Figure 4. To calculate the $\frac{\partial H}{\partial r_e}$, we rewrite the energy function of D4 brane as

$$H = -T_4 \Omega_3 g_s \left(\frac{\sqrt{2}}{g}\right)^3 \int dr \sqrt{[1 + 2g^{-2}(\xi^1)^2(r^2)] \cos(\xi^1)^{-2/3}[\sin^6(\xi^1) \cos(\xi^1)^{2/3} + D^2]^3} \hspace{1cm} (5.5)$$
Figure 6: $r_e$ dependence of $l_q$ in AdS$_6 \times$S$^4$ background.

We denote
\[
\mathcal{L} = \sqrt{[1 + 2g^{-2}f(r)\xi'(r)]\cos(\xi^1)^{-2/3}[\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2]}.
\] (5.6)

Then the force in $r$ direction at the highest point is given by
\[
\mathcal{L} - \xi'' \frac{\partial \mathcal{L}}{\partial \xi'} = \sqrt{\cos(\xi^1)^{-2/3}\sin^6(\xi^1)\cos(\xi^1)^{2/3} + D^2}.
\] (5.7)

The FBC supplies the constraint between $r_e$ and $\rho'(r_e)$. For a given $r(\xi^1 = 0)$, we can obtain a maximum value of $r$ in a vertex brane solution, then we can get the string solutions. By analyzing these string solutions in this AdS$_6 \times$S$^4$ background, we plot the curve of $r_e$ dependence of $l_q$ by numerical calculation in Figure 6. We note that there is no maximal value of boundary quark separation. There are two reasons for this phenomenon: one is that hanging strings in this AdS$_6 \times$S$^4$ background do not have similar behaviors as in AdS$_5 \times$S$^5$; the other is that spike of vertex brane eliminates the critical behavior of hanging strings. We note that the spike solution of the vertex D4 brane tries to replace the hanging strings. This is a signal to show that, in this configuration, the vertex brane dominates the properties of baryons.

In this case, we should choose another way to define a “critical length” (which can be used to judge whether the baryon is physical, but very different from the usual screening length). We note in the QCD$_4$ deconfined phase, baryon vertex solutions always touch the boundary brane directly and can not form closed ones, thus we argue that this touching process corresponds to baryon dissociation in deconfined medium.\footnote{Usually, we know that there is no baryon states in deconfined phase, since quarks and gluons are deconfined. From the holographic solutions, we see that solutions in deconfined phase are always unclosed. Thus we think that even if quarks can form a state with the unclosed solution, it is not a baryon.} From this point of view, we can consider the distance between the top of the vertex brane and the bottom of the boundary brane as another parameter to judge whether baryons dissociate.

In the AdS$_6 \times$S$^4$ background, through analyzing the solutions of D4 vertex brane, we indeed find a maximal value of $r_{top}$ ( $r_{top} = r_e$ is the radial top position of each solution

In the QCD$_4$ deconfined phase, baryon vertex solutions always touch the boundary brane directly and can not form closed ones, thus we argue that this touching process corresponds to baryon dissociation in deconfined medium. From the holographic solutions, we see that solutions in deconfined phase are always unclosed. Thus we think that even if quarks can form a state with the unclosed solution, it is not a baryon.
\[ r(\theta) \] when we change initial conditions \( r(\xi^1 = 0) \).\(^{10}\) Thus, we can define the maximal top position \( l_m = \max\{r_{\text{top}}\} \) as a critical position. And the critical distance between \( l_m \) and boundary is \( l_c = r_\Lambda - l_m \). We can use this critical distance to judge whether a baryon state can exist due to the following reason. Among these vertex solutions with a spike in AdS\(_6\times\text{S}^4\) background, we can see from Figure 3 that sizes of baryons are always very small compared with Figure 3. Baryons in this background almost stand in a spatial point because the spikes of vertex brane replace hanging strings to some extent. In this case, the vertex brane dominates main properties of baryons. We can use baryon energy to judge whether it is physical or not physical. From numerical results, we find that larger \( r_{\text{top}} \) corresponds to larger baryon energy, and a solution with \( l_m \) has the largest energy. Beyond this \( l_m \), there is no solution with a spike, so we discard those solutions here. Thus \( l_m \) gives a upper limit of baryon energy, which is like the usual screening analysis of holographic baryons. We can understand this point from the rough relation \( E_m \sim \frac{1}{r_\Lambda - l_m} \), where \( E_m \) is the maximal energy of baryons. To avoid possible confusion, we wish to emphasize that \( l_m \) or \( l_c \) are not lengths in the gauge theory, but radial parameters in the gravity dual.

Our investigation shows that vertex branes dominate properties of baryons in these configurations. So the velocity dependence of baryon screening should be obtained by studying on the vertex brane, and high spin baryon should be described by the vertex brane with inner \( J \) charge.

6. Conclusion and discussion

How to understand the confinement and calculate hadron spectrum are considered as two biggest problems in QCD (or non-perturbative QCD exactly). So far we know little about the non-perturbative world and have almost no general powerful tool to study the strongly coupling problem. AdS/CFT correspondence, which is usually called gauge/gravity duality in general, is believed as a useful framework to study these problems. In the experiment side, many people believe there exists a QGP (quark gluon plasma) state in RHIC, which is a strongly coupled quark and gluon thermal state like a fluid, investigated in many works \[35 \ 34 \ 36 \ 38 \]. How to describe this QGP and understand the strongly coupled behavior is still a problem, though it is very useful for solving confinement and hadron spectrum problem. It is believed that heavy quark bound state can be alive in QGP, including \( J/\psi \) meson and some multi-quark bound states. We call these multi-quark bound states baryons, though they may be different from baryons we see when they survive within QGP. Using meson or baryon as a probe is the simplest method to study the properties of the strongly coupled quark gluon state.

In the gauge/gravity duality framework, we calculate properties of the probe in the classic gravity background. From the strong/weak duality, we know these results are always suitable for the probe in the strongly coupled background in the field side. A lot of works have been done on the meson spectrum and meson melting process in different

\[^{10}\text{We ignore solutions with } r_{\text{top}} > \max\{r_{\text{top}}\}, \text{ because there is no spike in these solutions. This seems just the result from the numerical calculation, we have not found some physical reasons.}\]
gauge/gravity systems. When we consider baryon in the present work, we find following interesting results:

1) In D4-D8 system, a holographic probe baryon can be described as $N_c$ fundamental strings connect through a vertex D4 brane wrapped on $S^4$.

2) In QCD$_4$ background, a closed vertex can exist in confined phase and can not exist in deconfined phase. In the low temperature region, screening effect still exist in confined phase like meson and the vertex D4 brane dominates the baryon mass. The lower energy state corresponds to vertex brane closer to cut off position ($r = r_c$) and the higher energy state corresponds to vertex brane closer to boundary (or flavor brane exactly). We think it is reasonable, because the high energy limit of this configuration is just like the unclosed vertex brane configuration in a higher temperature deconfined phase.

3) In warped AdS$_6 \times S^4$ background, a closed vertex can exist in deconfined phase and the vertex contains a spike, and fundamental strings are relatively short. Screening length should be defined through the distance between top position of the vertex spike and the boundary.

Related extended works can be done in the future. One is finding more evidence from the experiment data to support live multi quark bound state in quark gluon plasma. Another is calculating some special parameters of QGP through baryon probe and comparing them with experiment data. A clear picture of baryon melting is needed and energy loss of baryon probe is also a very interesting unsolved problem [40, 41, 42, 43, 44].

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Appendix

We denote coordinates in rest frame of quarks as $(t', x'_{3})$, then we have

$$\begin{align*}
    dt &= dt' \cosh \eta - dx'_{3} \sinh \eta , \\
    dx_{3} &= -dt' \sinh \eta + dx'_{3} \cosh \eta .
\end{align*}$$

(6.1)

The boosted metric is given by

$$ds^2 = (\cos \theta)^{-1/3} [Adt^2 + 2Bt\,dx_{3} + C x_{3}^2 + \frac{dr^2}{(g^2 r^2 - \frac{\mu}{r^3})} + r^2 (d\rho^2 + \rho^2 d\theta^2)]$$

(6.2)

where

$$\begin{align*}
    A &= -(g^2 r^2 - r^2 - \frac{\mu}{r^3}) \cosh^2 \eta - r^2 ; \\
    B &= (g^2 r^2 - r^2 - \frac{\mu}{r^3}) ; \\
    C &= -(g^2 r^2 - r^2 - \frac{\mu}{r^3}) \sinh^2 \eta + r^2 ;
\end{align*}$$

(6.3)
In the warped AdS5×S4 background, \( h_{ab} \) can be read from the induced metric

\[
ds^2_{r,a} = (\cos \theta)^{-1/3} [(A + \sigma^2 \rho^2 \omega^2) d\tau^2 + 2B x'_3 d\tau d\sigma + (C x'^2_3 + \frac{1}{g^2 \sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2 \rho^2) d\sigma^2] \, , \tag{6.4}
\]

The string Lagrangian density

\[
\mathcal{L}_{\text{string}} = (\cos \theta)^{-1/3} \sqrt{(A + \sigma^2 \rho^2 \omega^2)(C x'^2_3 + \frac{1}{g^2 \sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2 \rho^2) - B^2 x'^2_3} \, . \tag{6.5}
\]

When the wind velocity \( v = 0(\eta = 0) \), then \( x'_3(\sigma) = 0 \). The Lagrangian becomes

\[
\mathcal{L}_{\eta=0} = (\cos \theta)^{-1/3} \sqrt{(-g^2 \sigma^2 + \frac{\mu}{\sigma^3} + \sigma^2 \rho^2)(\frac{1}{g^2 \sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2 \rho^2)} \, . \tag{6.6}
\]

When angular velocity \( \omega = 0 \), the Lagrangian becomes

\[
\mathcal{L}_{\omega=0} = (\cos \theta)^{-1/3} \sqrt{\left(\frac{1}{g^2 \sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2 \rho^2\right) + (AC - B^2) x'^2_3} \, . \tag{6.7}
\]

The EOM of \( x_3 \):

\[
(\cos \theta)^{1/3} \frac{\partial \mathcal{L}_{\text{string}}}{\partial x'_3} = \frac{[(A + \sigma^2 \rho^2 \omega^2) C - B^2] x'_3}{\sqrt{(A + \sigma^2 \rho^2 \omega^2)(C x'^2_3 + \frac{1}{g^2 \sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2 \rho^2) - B^2 x'^2_3}} = F \, , \tag{6.8}
\]

where \( F \) is a constant. The new Lagrangian containing no \( x'_3(\sigma) \) is given by

\[
L = \sqrt{\frac{F^2(\frac{1}{g^2 \sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2 \rho^2)}{(A + \sigma^2 \rho^2 \omega^2) C - B^2 - F^2} + (A + \sigma^2 \rho^2 \omega^2)\left(\frac{1}{g^2 \sigma^2 - \frac{\mu}{\sigma^3}} + \sigma^2 \rho^2\right)} \, . \tag{6.9}
\]

All hanging strings attach to the highest point, then the FBC turns to be

\[
L - \rho^2 \left. \frac{\partial L}{\partial \rho} \right|_{r_e} = \frac{2\pi \rho'(\cos \theta)^{1/3} \partial \mathcal{H}}{N_c}, \tag{6.10}
\]

where \( r_e = r_{\text{max}} \) (with \( r'(\xi^1) = 0 \)) in the brane solution. To calculate the \( \frac{\partial \mathcal{H}}{\partial r_e} \), we rewrite the new Lagrangian of D4 brane as

\[
\mathcal{H} = -T_4 \Omega_3 g_s \left(\frac{\sqrt{2}}{g}\right)^3 \int d\xi^1 \sqrt{[r'(\xi^1) + 2g^{-2} f(r)] \cos(\xi^1)^{-2/3}[\sin^6(\xi^1) \cos(\xi^1)^{2/3} + D^2]} \tag{6.11}
\]

\[
= -T_4 \Omega_3 g_s \left(\frac{\sqrt{2}}{g}\right)^3 \int dr \sqrt{[1 + 2g^{-2} f(r)\xi'^1(r)] \cos(\xi^1)^{-2/3}[\sin^6(\xi^1) \cos(\xi^1)^{2/3} + D^2]} \, .
\]

We denote

\[
\mathcal{L} = \sqrt{[1 + 2g^{-2} f(r)\xi'^1(r)] \cos(\xi^1)^{-2/3}[\sin^6(\xi^1) \cos(\xi^1)^{2/3} + D^2]} \, . \tag{6.12}
\]

Then the force in \( r \) direction at the highest point is given by

\[
\mathcal{L} - \xi'^1(\partial \mathcal{L})_{\xi'^1} = \frac{\sqrt{\cos(\xi^1)^{-2/3}[\sin^6(\xi^1) \cos(\xi^1)^{2/3} + D^2]} \, |_{\xi'^1=0}}{\sqrt{1 + 2g^{-2} f(\xi'^1)}} = \sqrt{\cos(\xi^1)^{-2/3}[\sin^6(\xi^1) \cos(\xi^1)^{2/3} + D^2]} \, . \tag{6.13}
\]

\[
\xi'^1(r) = \frac{\partial \xi'^1}{\partial r} \, .
\]
References

[1] K. Ghoroku and M. Ishihara, “Baryons with D5 Brane Vertex and k-Quarks,” Phys. Rev. D 77, 086003 (2008) [arXiv:0801.4216 [hep-th]].

[2] Y. Seo and S. J. Sin, “Baryon Mass in medium with Holographic QCD,” JHEP 0804, 010 (2008) [arXiv:0802.0568 [hep-th]].

[3] K. Ghoroku, M. Ishihara, A. Nakamura and F. Toyoda, “Multi-quark baryons and color screening at finite temperature,” arXiv:0806.0195 [hep-th].

[4] K. Sfetsos and K. Siampos, arXiv:0807.0236 [hep-th].

[5] E. Witten, “Baryons and branes in anti de Sitter space,” JHEP 9807, 006 (1998) [arXiv:hep-th/9805112].

[6] M. Chernicoff and A. Guijosa, “Energy loss of gluons, baryons and k-quarks in an N = 4 SYM plasma,” JHEP 0702, 084 (2007) [arXiv:hep-th/0611155].

[7] B. Janssen, Y. Lozano and D. Rodriguez-Gomez, “The baryon vertex with magnetic flux,” JHEP 0611, 082 (2006) [arXiv:hep-th/0606264].

[8] S. A. Hartnoll and R. Portugues, “Deforming baryons into confining strings,” Phys. Rev. D 70, 066007 (2004) [arXiv:hep-th/0405214].

[9] Y. Imamura, “On string junctions in supersymmetric gauge theories,” Prog. Theor. Phys. 112, 1061 (2004) [arXiv:hep-th/0410138].

[10] H. Hata, T. Sakai, S. Sugimoto and S. Yamato, “Baryons from instantons in holographic QCD,” arXiv:hep-th/0701280.

[11] N. Kim, “Intersecting noncommutative D-branes and baryons in magnetic fields,” Phys. Rev. D 62, 066002 (2000) [arXiv:hep-th/0002086].

[12] J. Gomis, A. V. Ramallo, J. Simon and P. K. Townsend, “Supersymmetric baryonic branes,” JHEP 9911, 019 (1999) [arXiv:hep-th/9907022].

[13] C. G. Callan, A. Guijosa, K. G. Savvidy and O. Tafjord, “Baryons and flux tubes in confining gauge theories from brane actions,” Nucl. Phys. B 555, 183 (1999) [arXiv:hep-th/9902197].

[14] C. G. Callan, A. Guijosa and K. G. Savvidy, “Baryons and string creation from the fivebrane worldvolume action,” Nucl. Phys. B 547, 127 (1999) [arXiv:hep-th/9810092].

[15] T. Sakai and S. Sugimoto, “Low energy hadron physics in holographic QCD,” Prog. Theor. Phys. 113, 843 (2005) [arXiv:hep-th/0412141]. T. Sakai and S. Sugimoto, “More on a holographic dual of QCD,” Prog. Theor. Phys. 114, 1083 (2006) [arXiv:hep-th/0507073].

[16] D. K. Hong, M. Rho, H. U. Yee and P. Yi, “Chiral dynamics of baryons from string theory,” Phys. Rev. D 76, 061901 (2007) [arXiv:hep-th/0701276].

[17] D. K. Hong, M. Rho, H. U. Yee and P. Yi, “Dynamics of Baryons from String Theory and Vector Dominance,” JHEP 0709, 063 (2007) [arXiv:0705.2632 [hep-th]].

[18] S. Kobayashi, D. Mateos, S. Matsuura, R. C. Myers and R. M. Thomson, “Holographic phase transitions at finite baryon density,” JHEP 0702 (2007) 016 [arXiv:hep-th/0611099].

[19] J. Erdmenger, M. Kaminski and F. Rust, “Holographic vector mesons from spectral functions at finite baryon or isospin density,” Phys. Rev. D 77 (2008) 046005 [arXiv:0710.0334 [hep-th]].
[20] J. Erdmenger, M. Kaminski, P. Kerner and F. Rust, arXiv:0807.2663 [hep-th].
[21] J. Mas, J. P. Shock, J. Tarrio and D. Zoakos, arXiv:0805.2601 [hep-th].
[22] P. de Forcrand et al. [QCD-TARO Collaboration], “Meson correlators in finite temperature lattice QCD,” Phys. Rev. D 63, 054501 (2001) [arXiv:hep-lat/0008005].
[23] H. Liu, K. Rajagopal and U. A. Wiedemann, “An AdS/CFT calculation of screening in a hot wind,” Phys. Rev. Lett. 98, 182301 (2007) [arXiv:hep-ph/0607062].
[24] K. Peeters, J. Sonnenschein and M. Zamaklar, “Holographic melting and related properties of mesons in a quark gluon plasma,” Phys. Rev. D 74, 106008 (2006) [arXiv:hep-th/0606195];
[25] C. Athanasiou, H. Liu, K. Rajagopal “Velocity dependence of baryon screening in a hot strongly coupled plasma” arXiv:0801.1117 [hep-th].
[26] M. Li, Y. Zhou and P. Pu, “High spin baryon in hot strongly coupled plasma,” arXiv:0805.1611 [hep-th].
[27] O. Aharony, J. Sonnenschein and S. Yankielowicz, “A holographic model of deconfinement and chiral symmetry restoration,” Annals Phys. 322, 1420 (2007) [arXiv:hep-th/0604161].
[28] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, “Baryons from supergravity,” JHEP 9807, 020 (1998) [arXiv:hep-th/9806158].
[29] H. Liu, K. Rajagopal and Y. Shi, “Robustness and Infrared Sensitivity of Various Observables in the Application of AdS/CFT to Heavy Ion Collisions”, arXiv:0803.3214[hep-ph]
[30] A. Brandhuber and Y. Oz, “The D4-D8 brane system and five dimensional fixed points,” Phys. Lett. B 460, 307 (1999) [arXiv:hep-th/9905148].
[31] D. Youm, “Localized intersecting BPS branes,” arXiv:hep-th/9902208.
[32] M. Cvetic, H. Lu and C. N. Pope, “Gauged six-dimensional supergravity from massive type IIA,” Phys. Rev. Lett. 83, 5226 (1999) [arXiv:hep-th/9906221].
[33] Z. W. Chong, H. Lu and C. N. Pope, “Rotating strings in massive type IIA supergravity,” arXiv:hep-th/0402202.
[34] J. Erdmenger, M. Haack, M. Kaminski and A. Yarom, arXiv:0809.2488 [hep-th].
[35] K. Dusling, J. Erdmenger, M. Kaminski, F. Rust, D. Teaney and C. Young, JHEP 0810, 098 (2008) [arXiv:0808.0957 [hep-th]].
[36] R. G. Cai and Y. W. Sun, JHEP 0809, 115 (2008) [arXiv:0807.2377 [hep-th]].
[37] S. Pu and Q. Wang, arXiv:0810.5271 [hep-ph].
[38] R. G. Cai, Z. Y. Nie and Y. W. Sun, arXiv:0811.1665 [hep-th].
[39] K. B. Fadafan, H. Liu, K. Rajagopal and U. A. Wiedemann, arXiv:0809.2869 [hep-ph].
[40] S. Peigne and A. V. Smilga, arXiv:0810.5702 [hep-ph].
[41] C. Marquet, arXiv:0810.2572 [hep-ph].
[42] P. Arnold and W. Xiao, arXiv:0810.1026 [hep-ph].
[43] P. M. Chesler, K. Jensen, A. Karch and L. G. Yaffe, arXiv:0810.1985 [hep-th].
[44] S. Peigne and A. Peshier, Phys. Rev. D 77, 114017 (2008) [arXiv:0802.4364 [hep-ph]].