Modified Gravity

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Batz sur Mer  October  2010
Outline

1- Modified Gravity and/or Dark Energy?

2- Large Scale Structures

3- Laboratory Tests
Modified Gravity?
Underlying hypothesis: the validity of General Relativity.

What does this mean?
The theory of gravity is General Relativity, described in a Lagrangian way:

\[ S_{\text{Einstein–Hilbert}} = \frac{1}{16\pi G_N} \int d^4 x \sqrt{-g} R \]

This action depends on the Ricci curvature $R$ and Newton’s constant. Matter couples in a minimal way:

\[ S_{\text{matter}} = \int d^4 x \sqrt{-g} \mathcal{L}_m(\psi^i, g_{\mu\nu}) \]

In this talk, we will see various ways of modifying this action, all motivated by the discovery (in 1998) that the universe undergoes a recent phase of cosmic acceleration.
Evidence: The Hubble Diagram

The explosion of high red-shift SN Ia (standard candles):

\[ q_0 \equiv -\frac{a_0 \ddot{a}_0}{(\dot{a}_0)^2} \simeq -0.67 \pm 0.25 \]

Within General Relativity, link to matter and dark energy

\[ q_0 = -\Omega_\Lambda + \frac{1}{2} \Omega_m \sim -0.67 \]

Dark Energy must exist!
The Cosmic Microwave Background

Fluctuations of the CMB temperature across the sky lead to acoustic peaks and troughs, snapshot of the plasma oscillations at the last scattering

The position of the first peak:

\[ l_1 \approx \frac{220}{\sqrt{\Omega_\Lambda + \Omega_m}} \]

The universe is spatially flat

\[ \Omega_\Lambda + \Omega_m = 1 \]

\[ \Omega_\Lambda = \frac{2}{3} \left( \frac{1}{2} - q_0 \right) \sim 0.78 \]
The acceleration of the expansion of the universe may be interpreted in *four different ways*:

1) The acceleration is entirely due to the presence of a constant vacuum energy (*cosmological constant*). Anthropic principle and/or string theory?

2) The acceleration results from the existence of a new type of matter: *dark energy*.

3) What is seen as acceleration is in fact a misinterpretation of data and really we must face a *modification of gravity* at large enough scales.

4) There is no real acceleration. We just live in a *void* surrounded by more matter. No copernican principle stands.
Gravity Tests

- Deviations from Newton’s law are parametrised by:

\[ \phi_N = -\frac{G_N}{r} (1 + 2\alpha^2 e^{-r/\lambda}) \]

The tightest constraint on \( \alpha \) comes from the Cassini probe measuring the Shapiro effect (time delay):

\[ \alpha^2 \leq 1.210^{-5}, \quad \lambda \geq 9 - 10 \text{ AU} \]

- Strong constraints follow from the tests of the weak equivalence principle (universality of free fall):

\[ \eta_{AB} = \frac{|a_A - a_B|}{|a_A + a_B|} \leq (0.3 \pm 1.8)10^{-13} \]

Schlamminger et al. Phys. Rev. Lett. 100,041101 (2008)
An infinite class of modified gravity models can be considered:

\[ S_{MG} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}) \]

These Lagrangian field theories fall within the category of higher derivative theories.

Ostrogradski’s theorem states that these theories are generically plagued with ghosts. Quantum mechanically, this implies an explosive behaviour with particles popping out of the vacuum continuously.
A large class is ghost-free though, the $f(R)$ models:

$$S_{MG} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R)$$
f(R) vs Scalar-Tensor Theories

f(R) totally equivalent to an effective field theory with gravity and scalars

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G_N} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{Pl}} g_{\mu\nu}) \right) \]

The potential \( V(\phi) \) is directly related to f(R).

\[ V(\phi) = m_{Pl}^2 \frac{R f' - f}{2 f'^2}, \quad f' = e^{-2\phi/\sqrt{6}m_{Pl}} \]

...... and a large coupling!

\[ \alpha = \frac{1}{\sqrt{6}} \]
Chameleons

Chameleon field: field with a matter dependent mass

A way to reconcile gravity tests and cosmology:

Nearly massless field on cosmological scales

Massive field in the laboratory scales

J. Khoury and A. Weltman Phys. Rev. Lett. 69 (2004) 044026

Ph. Brax et al. PRD 70 (2004) 123518
Chameleon Effective Theory

Effective field theories with gravity and scalars

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G_N} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, A^2(\phi)g_{\mu\nu}) \right) \]

\[ \alpha = \frac{d \ln A}{d(\kappa_4 \phi)} \]

An example: \[ A(\phi) = e^{\kappa_4 \beta \phi}, \quad \kappa_4^2 = 8\pi G_N = m_{Pl}^{-2} \]
The Chameleon Mechanism

When coupled to matter, scalar fields have a matter dependent effective potential

\[ V_{eff}(\phi) = V(\phi) + \rho_m A(\phi) \]
No shell

\[ F_\phi = -\beta \frac{m}{m_{\text{Pl}}} \nabla \phi, \quad \beta = \frac{m_{\text{Pl}}}{M} \]

Thin shell
F(R)-Chamelelon Cosmology

Late time acceleration

Lurking cosmological constant

Electron kick during BBN

Solution including kicks
Solution neglecting kicks

Ph. Brax et al.
PRD 70 (2004)
123518
Linear Growth

- For larger scales, gravity may be modified. A test of modified gravity can be obtained by studying the growth of structures at low redshift (in the linear regime).

- This is most sensitive to the behaviour of the growth factor on sub-horizon scales and the ratio of the two Newton potentials:

  \[ f = \frac{d \ln \delta}{d \ln a}, \quad \gamma = \frac{d \ln f}{d \ln \Omega_m} \]

- In general relativity, the slip function and the growth index are known to be:

  \[ \gamma \approx 0.55, \quad \eta = 1 \]
At the background level, chameleon models and their siblings the f(R) models behave like a pure cosmological constant.

Fortunately, this is not the case at the perturbation level where the growth factor evolves like:

$$\delta'' + \mathcal{H}\delta' - \frac{3}{2} H^2 (1 + \frac{2\beta^2}{1 + \frac{m^2 a^2}{k^2}}) \delta = 0$$

The new factor in the brackets is due to a modification of gravity depending on the comoving scale k.

This is equivalent to a scale dependent Newton constant.
Everything depends on the comoving Compton length:

\[ \lambda_c = \frac{1}{ma} \]

Gravity acts in an usual way for scales larger than the Compton length

\[ \delta \sim a \]

Gravity is modified inside the Compton length with a growth:

\[ \delta \sim a^{\nu}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta^2)}}{2} \]
Everything depends on the time dependence of $m(a)$. If $m$ is a constant then the Compton length diminishes with time. So a scale inside the Compton length will eventually leave the Compton length.

On the other hand, for chameleons the Compton length increases implying that scales enter the Compton length.

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**Modified gravity** $\rightarrow$ **General Relativity**

$z = z^*$

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**General Relativity** $\rightarrow$ **Modified gravity**

$z = z^*$
The growth index of CDM has a pronounced deviation from General Relativity.

Its behaviour depends on both $z^*$ and the strength of the coupling $\alpha_B B$. 
Bispectrum for equilateral configurations (i.e. normalised three point function) at second order in perturbation theory (non-Gaussianity).

**General Relativity**

**Modified gravity.**

With F. Bernardeau, to appear
With B. Li (to appear)
Laboratory Experiments?

- Scalar fields and chameleons could be experimentally detected. Different types of experiments have been proposed and carried out:
  
  - **Casimir force** experiments could be sensitive to a new scalar field force.
  
  - **Helioscopes** (CAST(CERN)......) could detect scalars emitted from the inner sun.
  
  - **Optical cavity experiments** are looking for birefringence and afterglow effects (BMV, ALP (DESY), GammeV (Fermilab), ADMX (Seattle)......)
Casimir Force Experiments

- Measure force between
  - Two parallel plates
  - A plate and a sphere
The Casimir Force

- We focus on the plate-plate interaction in the range:

\[ m_c^{-1} \leq d \leq m_b^{-1} \]

- The force is algebraic:

\[ \frac{F_{\phi}}{A} \sim \Lambda^4 (\Lambda d)^{-\frac{2n}{n+2}} \]

- Dark energy sets a typical scale:

\[ \Lambda^{-1} \sim 82 \mu m \]
Behaviour of Chameleonic Pressure for $V = \Lambda_0^4(1+\Lambda^n/\phi^n)$; $n = 1$

Chameleonic Pressure: $(V(\phi_c) - \Lambda_0^4)^{-1} F/\Lambda$

Separation of plates: $m_c d$

- Constant force behaviour
- Power-law behaviour
- Exponential behaviour

$d = m_c^{-1}$ for $d = m_b^{-1}$
Detectability

- The Casimir forces is also an algebraic law implying:

\[
\frac{F_\phi}{F_{\text{cas}}} \sim \frac{240}{\pi^2} (\Lambda d)^{\frac{2(n+4)}{n+2}}
\]

- This can be a few percent when d=10\,\mu m and would be 100% for d=30 \,\mu m
500 kE granted to carry out experiments in Amsterdam over the next 4 years.
Chameleons Coupled to Photons

- Chameleons may couple to electromagnetism:

  \[ \mathcal{L}_{\text{optics}} = \frac{e^2}{M \gamma} F_{\mu\nu} F^{\mu\nu} \]

- Cavity experiments in the presence of a constant magnetic field may reveal the existence of chameleons. The chameleon mixes with the polarisation orthogonal to the magnetic field and oscillations occur (like neutrino oscillations)

- The coherence length \( z_{\text{coh}} = \frac{2\omega}{m^2} \) depends on the mass in the optical cavity and therefore becomes pressure and magnetic field dependent:

  \[ \rho = \rho_m + \frac{B^2}{2} \]
Induced Coupling

\[ L_{\text{eff}} = \frac{e^2}{3(4\pi)^2 M_{\text{matter}}} \phi F_{ab} F^{ab} \]

\[ \beta_m = \frac{m_p}{M_{\text{matter}}}, \quad \beta_\gamma = \frac{m_p}{M_\gamma} \]

When the coupling to matter is universal, and heavy fermions are integrated out, a photon coupling is induced. Other contribution from conformal anomaly too.

\[ M_\gamma = \frac{3(4\pi)^2}{e^2} M_{\text{matter}} \]
1. Vacuum Magnetic Dichroism and Birefringence

- Send linearly polarized laser beam through transverse magnetic field ⇒ measure changes in polarization state:
  - rotation (dichroism)
  - ellipticity (birefringence)

[Brandi et al. '01]
Ellipticity predictions: \( n = 1 \) & \( \Lambda = 2.3 \times 10^{-3} \) eV

- **PVLAS @ 5.5T**
- **PVLAS @ 2.3T**
- **BMV @ 11.5T**

**PVLAS 07 @ 2.3T upper bound**

**Expected BMV sensitivity**

Ph.B., C. van de bruck, A. C. Davis, D. Shaw.
Afterglow

Axion-like particles, once generated can go through the wall and then regenerate photons on the other side.

Chameleons cannot go through but can stay in a jar once the laser has been turned off and then regenerate photons.

Even in the absence of magnetic fields, chameleons can induce atomic transitions and lead to an afterglow phenomenon (to be tested with ALPS (DESY)).
GammeV (Fermilab) and ADMX (Seattle) cover a large part of the parameter space. Constraints from BMV (Toulouse) too. Best constraints from Chase (Fermilab).

Theoretical constraints from atomic physics (1s-2s, Lamb shift) and particle physics (Z width)
Conclusions

- Gravity may be altered. Cosmic acceleration?

- Locally, Newtonian gravity can be restored: chameleon effect (and extensions Damour-Polyakov, Vainshtein ...).

- Effects on large scale structures: exciting prospects.

- Possible detectability with Casimir and optical experiments.

- Extensions to other models (Galileons, dilatons....)
$z = 0.32$

Reyes, R. et al. 2010, Nature, 464, 256-258.