Free vibration response of functionally graded carbon nanotube double curved shells and panels with piezoelectric layers in a thermal environment

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Abstract. This paper focuses on free vibration of double-curved shells and panels with piezoelectric layers in a thermal environment. Vibration characteristics of elliptical, spherical, cycloidal, and toro circular shells of revolution structures were studied in detail. These structures are made of a Carbon Nanotube (CNT) core and piezoelectric layers on the upper and lower surfaces. It was assumed here that temperature varied linearly in the thickness direction. Reissner-Mindlin and the first-order shear deformation theories were implemented to derive the governing equations of the considered structures. The distribution of nanotubes was assumed linear along the thickness direction. To solve the equation, the Generalized Differential Quadrature (GDQ) method was implemented to investigate the dynamic behavior of the structures. Finally, the effects of the boundary conditions, the thickness of piezoelectric layers, the functional distribution of CNTs, thermal environment, and two kinds of circuit (open circuit and closed circuit) were analyzed. Eigenvalue system was solved to obtain natural frequencies. Results showed that the obtained fundamental frequency of the closed circuit was smaller than that of the open circuit. Another interesting result was that the natural frequency was reduced at higher temperatures.

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1. Introduction

Shells are three-dimensional structures that are characterized by unique mechanical properties and constrained by two curved surfaces. Generally, these structures are divided into three groups such as thick, moderately thick, and thin shells [1]. Because of their excellent and unique mechanical characteristics, they are implemented in different applications including submarines, storages, tanks, automobiles, airplanes, etc. [2]. One of the investigation aspects of shells is analysis of their vibration characteristics. Therefore, considering their extensive applications to different mechanical and civil structures, it is vital to develop an accurate and reliable dynamical model for shells. Shells can not only be assumed as the 3D body, but also behave as 2D structures. In terms of their 3D applications, the governing equations are derived using the 3D elasticity theory and for 2D applications, the shell theory is implemented. In the shell theory (2D model), the mid-surface of a structure is considered to be a reference plane, and the system of equations is derived based on the assumed reference plane. The
present study used this theory to extract the governing equations. In this study, piezoelectric layers are embedded in the upper and lower surfaces. As a result, it is necessary to adjust piezoelectric applications for industries. In practice, they can be used for power generation in automobiles. When an automobile’s tires are rotating on the roads, a certain amount of energy is wasted. This kind of energy supplies the required power to automobile electronic devices. It should be mentioned that piezoelectric materials can be used to generate electrical energy when subjected to mechanical strain [3]. Another application is the tire pressure sensor, which is placed inside a wheel: by measuring the amount of force imported to the piezoelectric disc, the sensor can be an accurate indicator for regulating the tire pressure. As implied earlier, the characteristic of the piezoelectric material, which converts electrical energy into mechanical energy and vice versa, provides the potential to reap its benefit [3]. In recent years, there has been a great effort for modeling and analyzing the static and dynamic properties of the moderately thick shells. For example, Tornabene [4] probed the free vibration of anisotropic double shells using two-dimensional differential quadrature method. They implemented the first-order shear deformation theory and investigated the vibration behavior of the moderately thick structures. Viola et al. [5] conducted static analysis of double-curved shells and panels with higher order shear deformation theory. To this end, they discretized the governing equations of the structure by using the Generalized Differential Quadrature (GDQ) method. Shooshhtari and Razavi [6] used single-mode Galerkin method to study large amplitude vibration of magnetoelectro-elastic curved panels. In this research, they implemented Donnell shell theory and Gans’s laws for electrostatics and magnetostatics to derive the governing equations. Finally, they examined the first mode of vibration of this structure. Pang et al. [7] employed a semi-analytical method to analyze the free vibration of joined spherical-cylindrical-spherical shells. They utilized the Flugge thin shell theory to extract governing equations. To approximate the displacement function, unified Jacobi polynomials and Fourier series were used. Finally, the results were compared with those which were obtained from the Finite Element Method (FEM). Ront et al. [8] conducted the free vibration analysis of graphene-reinforced shells and panels in a thermal environment. In this research, the governing equations of four types of double-curved shells were solved with FEM. Moreover, materials were defined based on the Halpin-Tsai approach to showing their dependence on temperature. Finally, the results were contrasted by other studies to confirm their credits. Pang et al. [9] analyzed free vibration of the double-curved shell of revolution with a semi-analytical method. The displacements along the revolution axis were approximated with the Jacobi polynomials, and the Fourier series were used to indicate the movement along the circumferential direction. The Rayleigh-Ritz method was implemented to obtain natural frequencies. Awrejcewicz et al. [10] studied linear and nonlinear free vibrations of laminated functionally graded shallow shells. They assumed that these structures were made of Functionally Graded Material (FGM) based on power law and varied along the thickness direction. The first shear deformation theory was applied to define the displacement field. To solve equations, the Ritz and the R-functions method were utilized simultaneously. Fang et al. [11] applied Goldenveizer-Norozhikov shell theory, thin plate theory, and electro-elastic surface theory to investigate nano-sized piezoelectric double-shell structures. In this research, the surface energy effect was considered, and the governing equation was solved by using the Rayleigh-Ritz method.

Zhou et al. [12] applied an accurate approach to analyzing the free vibration of piezoelectric fiber-reinforced composite cylindrical shells in a thermal environment. They assumed that composite materials were made in two ways: first, each ply had a regular distribution and, second, reinforcements were graded functionally through the thickness direction. The governing equations were extracted by using the Hamilton method and, for solving them, an analytical approach was applied. Mallek et al. [13] analyzed the electromechanical behavior of composite shell structures with embedded piezoelectric layers. To solve the equations, they applied the 3D-shell model based on the shell elements of discrete double directors. They utilized the third shear deformation theory as the displacement field. Akbari Alishahi et al. [14] carried out the thermoelastic analysis of a functionally graded double curved shell. They assumed that the material varied in the thickness direction based on power law and, also, the piezoelectric layers were embedded in the upper and lower surfaces of the structure. They implemented the GDQ method to discretize the governing equations. They investigated the effects of temperature difference, grading index of the material, etc. Behjat et al. [15] conducted the static and dynamic analyses of a functionally graded piezoelectric plate. They hypothesized that the structure was under electrical and mechanical loads. They used the first-order shear deformation theory and the Hamilton principle to derive the governing equation. To solve the equations, they applied the FEM. The effects of the materials, boundary conditions, etc. were examined.

Although considerable researches have been devoted to vibrations of double curved structures, less attention has been paid to vibrations of smart double curved structures in a thermal condition. In the present paper, the considered structures are composed of three
main layers. They consist of a core made of the functionally graded carbon nanotube (FG-CNT) and two piezoelectric layers on the upper and lower surfaces. The rule of mixture approximates the mechanical properties of the FGM. Moreover, the electrical field was approximated as a linear function. The main focus of this study is on four types of elliptical, spherical, cycloidal, and toro circular shells and panels. These structures are assumed to be moderate thick shells, and the first-order shear deformation theory is also implemented. The developed model is then solved by using both 1D and 2D GDQ methods [14]. It should be noted that the 2D method is more complicated than the 1D mode. Based on the obtained numerical solution using GDQ methods, a parametric sensitivity analysis is conducted to show the effect of different parameters including mechanical and electrical boundary conditions, volume distribution, and thermal environment on vibrations of the considered structures.

2. Mathematical modeling

The geometric schematics and coordinate systems are shown in Figures 1–5. The \( \alpha \) and \( \beta \) values correspond to the meridional and circumferential coordinates. The position of any point in the shells is determined by \( c_0 < \alpha < \alpha_1, \beta_0 < \beta < \beta_1 \), and \( -\frac{b}{2} < z < \frac{b}{2} \). The relation of the radii of structures is defined as follows [16]. In Figures 2–4, \( \alpha_z \) is known as the axis of revolution and \( d'z' \) is the geometric axis of meridian. Further, the semi-major and semi-minor axes of the elliptical shell are denoted by \( a \) and \( b \), respectively. The distance between \( \alpha z \) and \( d'z' \) is shown with \( R_b \). The two radii of the structure are denoted by \( R_\alpha \) and \( R_\beta \).

Elliptical shell:

\[
R_\alpha = \frac{a^2 b^2}{\sqrt{(a^2 \sin^2 \alpha + b^2 \cos^2 \alpha)^2}}, \quad (1a)
\]

\[
R_\beta = \frac{a^2}{\sqrt{(a^2 \sin^2 \alpha + b^2 \cos^2 \alpha)}} + \frac{R_b}{\sin\alpha}, \quad (1b)
\]

\[
R_0 = R_2 \sin(\alpha). \quad (1c)
\]

Cycloidal shell:

\[
R_0 = r_c(2\alpha + \sin(2\alpha)), \quad (2a)
\]
\[ R_\alpha = 4r_c \cos(\alpha), \quad (2b) \]
\[ R_\beta = \frac{R_0}{\sin(\alpha)}. \quad (2c) \]

Circular shell:
\[ R_a = R, \quad (3a) \]
\[ R_0 = R \sin(\alpha) + R_b, \quad (3b) \]
\[ R_\beta = \frac{R_0}{\sin(\alpha)}. \quad (3c) \]

In this research, based on the first-order shear deformation theory, the displacement fields are assumed below [17]:
\[ U(\alpha, \beta, z, t) = u(\alpha, \beta, t) + z \psi_\alpha(\alpha, \beta, t), \quad (4a) \]
\[ V(\alpha, \beta, z, t) = v(\alpha, \beta, t) + z \psi_\beta(\alpha, \beta, t), \quad (4b) \]
\[ W(\alpha, \beta, z, t) = w(\alpha, \beta, t). \quad (4c) \]

In the above equations, \( u, v, \) and \( w \) are displacements of the reference plane along the meridional, circumferential, and normal directions, respectively. \( \psi_\alpha, \psi_\beta \) are the rotations around \( \alpha, \) and \( \beta \) axes, respectively. It should be noted that in the above equations \( U, V \) change linearly through the thickness and \( W \) remains constant. Therefore, this assumption can be used for the small deflection theory. The general form of the strain-displacement relationships is represented as follows [17]:
\[ \varepsilon_i = \frac{\partial}{\partial \xi_i} \left( \frac{u_i}{A_i} \right) + \frac{1}{A_i} \sum_{k=1}^{3} \frac{u_k}{A_k} \frac{\partial A_i}{\partial \xi_k} \quad (5a) \]
\[ \gamma_{ij} = \frac{1}{A_i A_j} \left[ A_i^2 \frac{\partial}{\partial \xi_j} \left( \frac{u_i}{A_i} \right) + A_j^2 \frac{\partial}{\partial \xi_i} \left( \frac{u_j}{A_j} \right) \right] \quad i \neq j \quad (5b) \]

If the above equations are extended, the relationships are developed below [17]:
\[ \varepsilon_1 = \frac{1}{A_1} \left( \frac{\partial U}{\partial \xi_1} + \frac{1}{a_1} \frac{\partial a_1}{\partial \xi_2} V + \frac{a_1}{R_1} w \right), \quad (6a) \]
\[ \varepsilon_2 = \frac{1}{A_2} \left( \frac{\partial V}{\partial \xi_2} + \frac{1}{a_2} \frac{\partial a_2}{\partial \xi_1} U + \frac{a_2}{R_2} w \right), \quad (6b) \]
\[ \gamma_{12} = \frac{A_2}{A_1} \frac{\partial}{\partial \xi_2} \left( \frac{V}{A_2} \right) + \frac{A_1}{A_2} \frac{\partial}{\partial \xi_1} \left( \frac{U}{A_1} \right), \quad (6c) \]
\[ \varepsilon_3 = \left( \frac{\partial W}{\partial z} \right), \quad (6d) \]
\[ \gamma_{21} = \frac{1}{A_2} \left( \frac{\partial W}{\partial \xi_2} \right) + A_2 \frac{\partial}{\partial z} \left( \frac{V}{A_2} \right), \quad (6e) \]
\[ \gamma_{13} = \frac{1}{A_1} \left( \frac{\partial W}{\partial \xi_1} \right) + A_1 \frac{\partial}{\partial z} \left( \frac{U}{A_1} \right). \quad (6f) \]

In the above equations, \( U, V, \) and \( W \) are the displacements substituted by Eqs. (4a)-(4c) and \( A_1, A_2, \) and \( A_3, \) are the constants defined by the following equations [18]:

---

**Figure 5.** (a) Toro circular shell, (a') cross section of toro circular shell, (b) spherical shell, (b') spherical panel, (c) elliptical shell, (c') elliptical panel, (d) cycloidal shell, and (d') cycloidal panel.
\[ A_1 = a_1 \left( 1 + \frac{z}{R_1} \right) , \quad (7a) \]
\[ A_2 = a_2 \left( 1 + \frac{z}{R_2} \right) , \quad (7b) \]
\[ A_3 = a_3 = 1. \quad (7c) \]

The ratio of thickness to radius is zero because it is assumed that the shells are moderately thick. With this assumption, the following parameters are defined [18]:

Plate:
\[ A_1 = 1. \quad (8a) \]
\[ A_2 = 1. \quad (8b) \]

Cylindrical shell:
\[ A_1 = 1, \quad (9a) \]
\[ A_2 = R. \quad (9b) \]

Double curved shell:
\[ A_1 = R_\alpha, \quad (10a) \]
\[ A_2 = R_\beta \sin(\alpha). \quad (10b) \]

By substituting Eqs. (4a)–(4c) into Eqs. (6a)–(6f), the relationship between strains and mid-surface displacements is represented below [17]:

\[ \varepsilon_\alpha = \frac{1}{A_1} \frac{\partial u}{\partial x} + \frac{v}{A_1 A_2} \frac{\partial A_1}{\partial \beta} + \frac{w}{R_\alpha}, \quad (11a) \]
\[ \varepsilon_\beta = \frac{1}{A_2} \frac{\partial v}{\partial \beta} + \frac{u}{A_1 A_2} \frac{\partial A_2}{\partial x} + \frac{w}{R_\beta}, \quad (11b) \]
\[ \gamma_{\alpha \beta} = \frac{A_2}{A_1} \frac{\partial}{\partial x} \left( \frac{v}{A_2} \right) + \frac{A_1}{A_2} \frac{\partial}{\partial \beta} \left( \frac{u}{A_1} \right). \quad (11c) \]

\[
\begin{bmatrix}
N_0 \\
N_\beta \\
N_{\alpha \beta} \\
M_\alpha \\
M_{\beta} \\
M_{\alpha \beta} \\
Q_\alpha \\
Q_{\beta}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & 0 & 0 \\
A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & 0 & 0 \\
0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 \\
B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & 0 & 0 \\
B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & 0 & 0 \\
0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \kappa A_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa A_{44}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_\alpha \\
\varepsilon_\beta \\
\gamma_{\alpha \beta} \\
\chi_\alpha \\
\chi_\beta \\
\gamma_{\alpha \beta} \\
\chi_{\alpha \beta} \\
\chi_{\beta \alpha}
\end{bmatrix}
= \begin{bmatrix}
N_0^T \\
N_\beta^T \\
N_{\alpha \beta}^T \\
M_\alpha^T \\
M_{\beta}^T \\
M_{\alpha \beta}^T \\
Q_\alpha^T \\
Q_{\beta}^T
\end{bmatrix} - \begin{bmatrix}
N_0^T \\
N_\beta^T \\
N_{\alpha \beta}^T \\
M_\alpha^T \\
M_{\beta}^T \\
M_{\alpha \beta}^T \\
Q_\alpha^T \\
Q_{\beta}^T
\end{bmatrix}, \quad (12a)
\]

\[
\begin{bmatrix}
D_\alpha \\
D_{\beta} \\
D_{\gamma z}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & \varepsilon_{15} & 0 \\
0 & 0 & \varepsilon_{24} & 0 \\
\varepsilon_{31} & \varepsilon_{32} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_\alpha \\
\varepsilon_\beta \\
\gamma_{\beta z} \\
\chi_{\alpha z} \\
\chi_{\beta z} \\
\chi_{\alpha \beta}
\end{bmatrix}
+ \begin{bmatrix}
k_{11} & 0 & 0 & k_{15} & E_\alpha & 0 \\
k_{22} & 0 & k_{22} & 0 & E_\beta & \Delta T \\
k_{33} & 0 & k_{33} & 0 & E_z & 0 \\
k_{33} & 0 & k_{33} & 0 & E_z & 0
\end{bmatrix}
\begin{bmatrix}
p_\alpha \\
p_\beta \\
p_z
\end{bmatrix}. \quad (12b)
\]

**Box I**
the bending stiffness $D_{ij}$, and the bending-extensional coupling stiffness $B_{ij}$ are defined below:

$$ (A, B, D) = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q'(1, z, z^2)dz + \int_{-\frac{h}{2} - h_p}^{\frac{h}{2} - h_p} Q^p \left(1, z, z^2\right)dz. \quad (13) $$

In the above relations, 'c' and 'p' represent core and piezoelectric parts, respectively.

$$ -\frac{h}{2} \leq z_c \leq \frac{h}{2}, \quad (14a) $$

$$ -\frac{h}{2} - h_p \leq z_p \leq -\frac{h}{2}, \quad (14b) $$

$$ \frac{h}{2} \leq z_p \leq \frac{h}{2} + h_p. \quad (14c) $$

It is worth noting that piezoelectric layers are substituted in the bottom and top layers. The components of the electrical field are defined below [20]:

$$ (E_a^c, E_b^c, E_z^c) = $$

$$ - \left( \frac{1}{A_1} \frac{2z - h}{2h} \frac{\partial \phi}{\partial \alpha}, \frac{1}{A_2} \frac{2z - h}{2h} \frac{\partial \phi}{\partial \beta}, \frac{1}{A_2} \frac{2z + h}{2h} \frac{\partial \phi}{\partial \beta}, \frac{1}{A_2} \frac{2z + h}{2h} \frac{\partial \phi}{\partial \beta}, \right). \quad (15a) $$

$$ (E_a^p, E_b^p, E_z^p) = $$

$$ - \left( \frac{1}{A_1} \frac{2z + h}{2h} \frac{\partial \phi}{\partial \alpha}, \frac{1}{A_2} \frac{2z + h}{2h} \frac{\partial \phi}{\partial \beta}, \frac{1}{A_2} \frac{2z + h}{2h} \frac{\partial \phi}{\partial \beta}, \frac{1}{A_2} \frac{2z + h}{2h} \frac{\partial \phi}{\partial \beta}, \right). \quad (15b) $$

In the above equation, $\phi$ is the electric potential that is equal to zero as follows:

$$ \alpha = \alpha_1, \alpha = \alpha_2, \quad \beta = \beta_1, \beta = \beta_2. $$

Elastic constants of the core and layers are represented below [21]:

$$ Q_{11} = \frac{E_{11}}{1 - \nu_{12} \nu_{21}}, \quad (16a) $$

$$ Q_{12} = \frac{\nu_{21} E_{11}}{1 - \nu_{12} \nu_{21}}, \quad (16b) $$

$$ Q_{22} = \frac{E_{22}}{1 - \nu_{12} \nu_{21}}, \quad (16c) $$

$$ Q_{66} = G_{12}, \quad (16d) $$

$$ Q^c = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}, \quad (16e) $$

$$ Q^p = \begin{bmatrix} \frac{E_p}{1 - \nu_{12}^p} \\ \frac{\nu_{21} E_p}{1 - \nu_{12}^p} \end{bmatrix}, \quad (16f) $$

$$ \begin{bmatrix} Q_{11}^p & Q_{12}^p & 0 \\ Q_{21}^p & Q_{22}^p & 0 \\ 0 & 0 & Q_{66}^p \end{bmatrix}. \quad (16g) $$

$$ Q_{11}^p = Q_{c22}^p, \quad (16h) $$

$$ Q_{66}^p = \frac{E_p}{2(1 + \nu_p)} \quad (16i) $$

$$ Q^p = \begin{bmatrix} Q_{11}^p & Q_{12}^p & 0 \\ Q_{21}^p & Q_{22}^p & 0 \\ 0 & 0 & Q_{66}^p \end{bmatrix}. \quad (16j) $$

Resultant force and momentum for the core and layers are represented as follows [21]:

$$ (N^T, M^T, F^T) = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q^c \left( \begin{array}{c} \alpha_{11} \\ \alpha_{12} \end{array} \right) (1, z, z^2) \Delta T dz $$

$$ + \int_{-\frac{h}{2} - h_p}^{\frac{h}{2} - h_p} Q^p \left( \begin{array}{c} \alpha_{11}^p \\ \alpha_{12}^p \end{array} \right) (1, z, z^2) \Delta T dz $$

$$ + \int_{\frac{h}{2}}^{\frac{h}{2} + h_p} Q^p \left( \begin{array}{c} \alpha_{11}^p \\ \alpha_{12}^p \end{array} \right) \Delta T dz. \quad (16k) $$

$$ (N^c, M^c) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z) e_{31} E_{b1}^c dz + \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z) e_{31} E_{b2}^c dz, \quad (16l) $$

$$ (N^p, M^p) = \int_{\frac{h}{2}}^{\frac{h}{2} + h_p} (1, z) e_{31} E_{b1}^p dz + \int_{\frac{h}{2}}^{\frac{h}{2} + h_p} (1, z) e_{31} E_{b2}^p dz, \quad (16m) $$

$$ (Q^c) = \int_{-\frac{h}{2}}^{\frac{h}{2}} e_{15} E_{c1}^c dz + \int_{\frac{h}{2}}^{\frac{h}{2}} e_{15} E_{c2}^c dz, \quad (16n) $$

$$ (Q^p) = \int_{\frac{h}{2}}^{\frac{h}{2} + h_p} e_{24} E_{b1}^p dz + \int_{\frac{h}{2}}^{\frac{h}{2} + h_p} e_{24} E_{b2}^p dz. \quad (16p) $$

As mentioned before, the core of the structures is made of FG-CNTs and its mechanical properties are defined below [22]:

$$ V_{cut}(z) = V^c(UD), \quad (17a) $$

$$ V_{cut}(z) = 2 \left( 1 - \frac{21d}{h} \right) V^c(FA - O), \quad (17b) $$

$$ V_{cut}(z) = \frac{4\sqrt{d}}{h} V^c(FA - x), \quad (14c) $$

$$ V_{cut}(z) = \left( 1 + \frac{2(z)}{h} \right) V^c(FA - V). \quad (17d) $$
In the following, the schematic of the CNT dispersion is shown in a matrix [23]. In the above equations, \( h \) is the thickness of structures and \( z \) denotes the direction of thickness. The total volume fraction is denoted by \( V^* \). In the UD (uniformly distributed) type model (Figure 6(a)), the distribution of CNT layer in the thickness direction remains stable. In the FGV model (Figure 6(b)), the higher and lower surfaces have the maximum and minimum distributions of CNT, respectively. The FGO model (Figure 6(c)) allocates maximum CNT to the center of the section and, thus, it becomes free of CNT on the top and bottom surfaces. The FGX model (Figure 6(d)) has the highest distribution of CNT on the top and bottom surfaces and without CNT on the middle plane. The mechanical properties of the materials are defined below [22]:

\[
E_{11} = \eta_1 V_{\text{cut}}(z)E_{11}^{\text{cut}} + V_m(z)E^m, \quad (18a)
\]

\[
\frac{E_{12}}{E_{22}} = \frac{V_{\text{cut}}(z)}{E_{22}^{\text{cut}}} + \frac{V_m(z)}{E^m}, \quad (18b)
\]

\[
\nu_{12} = V^* \nu_{12}^{\text{cut}} + V_m \nu^m, \quad (18c)
\]

\[
\nu_{21} = \nu_{12} \frac{E_{22}}{E_{11}}, \quad (18d)
\]

\[
\eta_2 = \frac{V_{\text{cut}}(z)}{G_{12}^{\text{cut}}} + \frac{V_m(z)}{G^m}, \quad (18e)
\]

\[
\rho(z) = V_{\text{cut}}(z)\rho^{\text{cut}} + V_m(z)\rho^m, \quad (18f)
\]

\[
\alpha_{11} = \frac{V_{\text{cut}}(z)E_{11}^{\text{cut}}\alpha_{11}^{\text{cut}} + V_m(z)E^m \alpha^m}{V_{\text{cut}}(z)E_{11}^{\text{cut}} + V_m(z)E^m}, \quad (18g)
\]

\[
\alpha_{22} = (1 + \nu_{12}^{\text{cut}}) V_{\text{cut}}(z)\alpha_{22}^{\text{cut}} + (1 + \nu^m) V_m(z)\alpha^m - \nu_{12}\alpha_{11}. \quad (18h)
\]

In the above equations, \( \eta_1 \), \( \eta_2 \), and \( \eta_3 \) are the first, second, and third parameters, respectively, which are all related to CNTs material. \( E_{11}^{\text{cut}}, E_{22}^{\text{cut}}, \) and \( G_{12}^{\text{cut}} \) are the young moduli and shear modulus of Single Wall Carbon Nanotubes (SWCNTs), respectively. \( \rho^{\text{cut}} \) and \( \rho^m \) are mass densities of the CNT and matrix constituents, respectively. Moreover, \( \alpha_{11} \) and \( \alpha_{22} \) are the terms for thermal expansion coefficients in \( \alpha \) and \( \beta \) directions.

In order to derive the equation of motion of the considered structure, Hamilton’s principle is implemented [24]:

\[
\int_0^t (\delta U_{\text{total}} + \delta V_{\text{total}} - \delta K_{\text{total}}) \, dt = 0. \quad (19)
\]

In the above formula, \( \delta U \), \( \delta V \), and \( \delta K \) are the virtual strain energy, the virtual potential energy, and virtual kinetic energy, respectively:

\[
\delta U_{\text{total}} = \int_A \left( \sigma_{\alpha} \delta \varepsilon_{\alpha} + \sigma_{\beta} \delta \varepsilon_{\beta} + \sigma_{\alpha \beta} \delta \gamma_{\alpha \beta} + \kappa \sigma_{\alpha \beta} \delta \gamma_{\alpha \beta} + \kappa \sigma_{\beta} \delta \gamma_{\alpha \beta} - D_{\alpha} \delta E_{\alpha} - D_{\beta} \delta E_{\alpha} - D_{\gamma} \delta E_{\alpha} \right) A_1 A_2 d\alpha d\beta, \quad (20)
\]

\[
\delta V_{\text{total}} = \int_A \left( N_a^T \left( \frac{1}{A_1^{2}} \frac{\partial U}{\partial \alpha} \frac{\partial U}{\partial \beta} \right) + N_b^T \left( \frac{1}{A_2^{2}} \frac{\partial V}{\partial \alpha} \frac{\partial V}{\partial \beta} \right) + N_c^T \left( \frac{1}{A_1 A_2^{2}} \frac{\partial W}{\partial \alpha} \frac{\partial W}{\partial \beta} \right) \right) A_1 A_2 d\alpha d\beta, \quad (21)
\]

\[
\delta K_{\text{total}} = \int_A \rho \left( \left( \frac{\partial U}{\partial \alpha} \frac{\partial U}{\partial \alpha} \frac{\partial U}{\partial \beta} \frac{\partial U}{\partial \beta} \right) + \left( \frac{\partial V}{\partial \alpha} \frac{\partial V}{\partial \alpha} \frac{\partial V}{\partial \beta} \frac{\partial V}{\partial \beta} \right) \right) A_1 A_2 d\alpha d\beta. \quad (22)
\]

By using Hamilton’s principle, the motions of equations are obtained below:

\[
\frac{1}{A_1 A_2} \frac{\partial}{\partial \alpha} (A_2 N_\alpha) - \frac{1}{A_1} \frac{\partial A_2}{\partial \alpha} N_\beta + \frac{1}{A_2} \frac{\partial A_1}{\partial \beta} N_\beta + \frac{Q_a}{R_a} = \frac{1}{A_1 A_2} N_a^T \frac{\partial}{\partial \alpha} \left( \frac{A_2}{A_1} \frac{\partial A_2}{\partial \alpha} \right) + \frac{1}{A_1 A_2} N_b^T \frac{\partial}{\partial \beta} \left( \frac{A_2}{A_1} \frac{\partial A_2}{\partial \beta} \right) + \frac{1}{A_1 A_2} M_c^T \frac{\partial}{\partial \beta} \left( \frac{A_2}{A_1} \frac{\partial A_2}{\partial \beta} \right) \right), \quad (23a)
\]

\[
\frac{1}{A_1} \frac{\partial N_\beta}{\partial \beta} + \frac{1}{A_1} \frac{\partial A_2}{\partial \alpha} N_{\alpha \beta} + \frac{1}{A_1 A_2} \frac{\partial}{\partial \beta} \left( (A_2 N_{\alpha \beta}) + \frac{Q_\beta}{R_\beta} \right) = \frac{1}{A_1 A_2} N_a^T \frac{\partial}{\partial \alpha} \left( \frac{A_2}{A_1} \frac{\partial A_2}{\partial \alpha} \right) + \frac{1}{A_1 A_2} N_b^T \frac{\partial}{\partial \beta} \left( \frac{A_2}{A_1} \frac{\partial A_2}{\partial \beta} \right) \right). \quad (23b)
\]
\begin{align}
  &+ \frac{1}{A_1A_2} M^T_{\alpha} \frac{\partial}{\partial \alpha} \left( A_2 \frac{\partial \psi_3}{\partial \alpha} \right) \\
  &+ \frac{1}{A_1A_2} M^T_{\beta} \frac{\partial}{\partial \beta} \left( A_2 \frac{\partial \psi_3}{\partial \beta} \right) + I_0 \ddot{\psi}_3 + I_1 \ddot{\psi}_1, \quad (23b) \\
  - \frac{N_\alpha}{R_\alpha} \frac{N_\beta}{R_\beta} + \frac{1}{A_1A_2} \frac{\partial}{\partial \alpha} Q_\alpha A_2 + 1 \frac{\partial}{\partial \beta} Q_\beta \\
  &= \frac{1}{A_1A_2} N^T_{\alpha} \frac{\partial}{\partial \alpha} \left( A_\beta \frac{\partial A_\alpha}{\partial \alpha} \right) + \frac{1}{A_1A_2} \\
  &= \frac{1}{A_1A_2} N^T_{\alpha} \frac{\partial}{\partial \alpha} \left( A_\beta \frac{\partial v}{\partial \alpha} \right) + I_0 \ddot{\omega}, \quad (23c) \\
  \frac{1}{A_1A_2} \frac{\partial}{\partial \alpha} (A_2 M_\alpha) - \frac{1}{A_1A_2} \frac{\partial A_2}{\partial \alpha} M_\beta + \frac{1}{A_1A_2} \frac{\partial}{\partial \beta} M_{\alpha\beta} \\
  + Q_\alpha = \frac{1}{A_1A_2} M^T_{\alpha} \frac{\partial}{\partial \alpha} \left( A_2 \frac{\partial \psi_\alpha}{\partial \alpha} \right) \\
  + \frac{1}{A_1A_2} M^T_{\beta} \frac{\partial}{\partial \beta} \left( A_2 \frac{\partial \psi_\beta}{\partial \beta} \right) + I_1 \ddot{\psi}_1, \quad (23d) \\
 \frac{1}{A_1A_2} \frac{\partial}{\partial \beta} \left( A_2 M_\beta \right) + \frac{1}{A_1A_2} \frac{\partial}{\partial \alpha} (A_2 M_{\alpha\beta}) \\
  + Q_\beta = \frac{1}{A_1A_2} M^T_{\beta} \frac{\partial}{\partial \beta} \left( A_2 \frac{\partial \psi_\beta}{\partial \beta} \right) \\
  + \frac{1}{A_1A_2} M^T_{\alpha} \frac{\partial}{\partial \alpha} \left( A_2 \frac{\partial \psi_\alpha}{\partial \alpha} \right) + I_1 \ddot{\psi}_1, \quad (23e) \\
  \frac{1}{R_\alpha R_\beta} \frac{\partial}{\partial \alpha} \left( \frac{1}{R_\alpha} \int_{\beta}^{b+h_p} D_{\alpha} \left( \frac{2z-h}{2h_p} \right) dz \right) \\
  + \frac{1}{R_\alpha R_\beta} \frac{\partial}{\partial \beta} \left( \frac{1}{R_\beta} \int_{\beta}^{b+h_p} D_{\beta} \left( \frac{2z-h}{2h_p} \right) dz \right) \\
  \frac{1}{R_\alpha R_\beta} - \frac{1}{R_\alpha R_\beta} \int_{\beta}^{b+h_p} \frac{D_{\beta}}{h_p} dz = 0. \quad (23f)
\end{align}

\section{Solution procedure}

\subsection{GDQ method in one dimension}

In this work, the GDQ method is applied to solve the equations of motion of the considered structures. In order to solve the equations, they are assumed constant in the circumferential direction. As a result, Eqs. (24a)–(24g) are used to reduce 2D equations into 1D [25].

\begin{align}
  u(\alpha, \beta, t) &= u(\alpha, t) \cos(n\beta), \quad (24a) \\
  v(\alpha, \beta, t) &= v(\alpha, t) \sin(n\beta), \quad (24b) \\
  w(\alpha, \beta, t) &= w(\alpha, t) \cos(n\beta), \quad (24c) \\
  \psi_\alpha(\alpha, \beta, t) &= \psi_\alpha(\alpha, t) \cos(n\beta), \quad (24d) \\
  \psi_\beta(\alpha, \beta, t) &= \psi_\beta(\alpha, t) \sin(n\beta), \quad (24e) \\
  \phi_\alpha(\alpha, \beta, t) &= \phi_\alpha(\alpha, t) \cos(n\beta), \quad (24f) \\
  \phi_\beta(\alpha, \beta, t) &= \phi_\beta(\alpha, t) \sin(n\beta). \quad (24g)
\end{align}

Based on this method, grid points along the reference surface are given as follows [26]:

\begin{equation}
  \alpha_i = \left( 1 - \cos \left( \frac{i-1}{N-1} \pi \right) \right) \frac{(\alpha_1 - \alpha_0)}{2} + \alpha_0. \quad (25)
\end{equation}

The final equation of motion is approximated by using the 1D GDQ method. Details of this method are given below [26].

\begin{align}
  \frac{\partial^n f}{\partial \alpha^n} &= \sum_{k=1}^{N} c_{ik} f_k, \quad (26a) \\
  M^{(1)}(\alpha_k) &= \prod_{j=1, j \neq k}^{N} (\alpha_k - \alpha_j), \quad (26b) \\
  c_{ij} &= \frac{M^{(1)}(\alpha_i)}{(\alpha - \alpha_j)M^{(1)}(\alpha_j)}, \quad (26c)
\end{align}
\[ \begin{align*}
  c_{ij}^{n} &= n(c_{ij}^{1} \ast (c_{ii}^{(n-1)} - c_{ij}^{(n-1)})/(\alpha_i - \alpha_j)) \\
  i, j &= 1, 2, ..., n = 2, 3, ..., N - 1, \quad (26d) \\
  c_{ii}^{n} &= - \sum_{j=1, j \neq i}^{N} c_{ij}^{n} \\
  i, j &= 1, 2, ..., N, \quad n = 1, 2, 3, ..., N - 1. \quad (26e)
\end{align*} \]

For analyzing the shell structures, three boundary conditions are presumed being displayed in Table 1.

### 3.2. GDQ method in two dimensions

For 2D solution, the approximations are considered in the two following directions [4]:

\[
\begin{align*}
  \frac{\partial \varphi}{\partial x} &= \sum_{k=1}^{N} c_{ik}^{n}u_{k,j} \\
  \frac{\partial \varphi}{\partial y} &= \sum_{m=1}^{N} c_{jm}^{n}u_{m,j}.
\end{align*} \tag{27a}
\]

\[
\begin{align*}
  \frac{\partial \varphi}{\partial x} &= \sum_{k=1}^{N} c_{ik}^{n}u_{k,j} \\
  \frac{\partial \varphi}{\partial y} &= \sum_{m=1}^{N} c_{jm}^{n}u_{m,j}.
\end{align*} \tag{27b}
\]

Based on this method, grid points along the reference surface are considered as follows:

\[
\begin{align*}
  \alpha_i &= \left(1 - \cos\left(\frac{i - 1}{N-1}\pi\right)\right)\left(\alpha_1 - \alpha_0\right)/2 + \alpha_0, \quad (28a) \\
  \beta_j &= \left(1 - \cos\left(\frac{j - 1}{M-1}\pi\right)\right)\left(\beta_1 - \beta_0\right)/2 + \beta_0. \quad (28b)
\end{align*}
\]

\[
\begin{align*}
  M^{(1)}(\alpha_k) &= \prod_{j=1, j \neq k}^{N} (\alpha_k - \alpha_j), \quad (29a) \\
  M^{(1)}(\beta_k) &= \prod_{j=1, j \neq k}^{N} (\beta_k - \beta_j). \quad (29b)
\end{align*}
\]

### Table 1. Boundary conditions of the shell structures.

| Type | Condition | Equations |
|------|-----------|-----------|
| C-C  | \( i = 1, i = N \) | \( u_i = v_i = w_i = M_{oi} = \psi_{bi} = 0 \) |
| C-S  | \( i = 1, i = N \) | \( u_i = v_i = w_i = M_{oi} = \psi_{bi} = 0 \) |
| S-S  | \( i = 1, i = N \) | \( u_i = v_i = w_i = M_{oi} = \psi_{bi} = 0 \) |

### Table 2. Boundary conditions of the panel structures.

| Type | Condition | Equations |
|------|-----------|-----------|
| C-C-C-C | \( i = 1, 2, ..., M, j = 1, 2, ..., N \) | \( u_{ij} = v_{ij} = w_{ij} = \psi_{okij} = \psi_{bjkj} = 0 \) |
| C-S-C-S | \( i = 1, j = 1, 2, ..., M \) | \( u_{ij} = v_{ij} = w_{ij} = M_{oi} = \psi_{bi} = 0 \) |
|      | \( i = 1, 2, ..., M, j = 1 \) | \( u_{ij} = v_{ij} = w_{ij} = \psi_{okij} = M_{bjij} = 0 \) |

In the above equation, the following can be proposed:

\[
\begin{align*}
  \{m\} &= \{ u \quad v \quad w \quad \psi_{\alpha} \quad \psi_{\beta} \}^T, \quad (31a) \\
  \{E\} &= \{ \phi_{b} \quad \phi_{t} \}^T. \quad (31b)
\end{align*}
\]

Indices of \( b \) and \( t \) indicate the bottom and top layers, respectively. By omitting the electrical variables,
the final form of the equation will be introduced as follows [27]:

$$[M]\{\ddot{m}\} + ([K_{mm}]-[K_{EE}][K_{mE}]^{-1}[K_{Em}])\{m\} = 0.$$  

(32)

To explore the effect of electrical properties, two boundary conditions including open and closed circuits will be considered. The electrical condition is closed if the upper and lower surfaces are ground; otherwise, it is open.

$$[M]\{\ddot{m}\} + ([K_{mm}])\{m\} = 0.$$  

(33)

Eq. (32) or (33) is separated into two equations:

$$[K_{dd}]\{m_d\} + [K_{bd}]\{m_d\} = [M]\{\ddot{m}_d\}.$$  

(34a)

$$[K_{bb}]\{m_b\} + [K_{bd}]\{m_d\} = 0.$$  

(34b)

After omitting $\{m_d\}$, the final form is as follows:

$$[K_{dd}] - [K_{dd}][K_{bb}]^{-1}[K_{bd}]\{m_d\} = M\{\ddot{m}_d\}.$$  

(35)

In the above equation, $b$ and $d$ indices present the boundary and inside domains, respectively. In Eqs. (32) and (33), $m_b$ and $m_d$ are defined as below:

For 1D solution:

$$\{m_b\} = \{u_1, u_N, v_1, v_N, w_1, w_N, \}
\psi_{\alpha 1}, \psi_{\alpha N}, \psi_{\beta 1}, \psi_{\beta N}\}.$$  

(36a)

$$\{m_d\} = \{u_2, u_3, \ldots, u_{N-1}, v_2, v_3, \ldots, v_{N-1}, \}

w_2, w_3, \ldots, w_{N-1}, \psi_{a 2}, \psi_{a 3}, \ldots, \psi_{a N-1}; \psi_{\beta 2}, \psi_{\beta 3}, \ldots, \psi_{\beta N-1}\}.$$  

(36b)

For 2D solution:

$$\{m_{dd}\} = \{a_{22}, a_{23}, \ldots, a_{2N-1}, a_{32}, a_{33}, \ldots, a_{3N-1}, \ldots, \}

a_{(N-1)1}, a_{(N-1)2}, \ldots, a_{(N-1)(N-1)}\}^T.$$  

(37a)

$$a = \{u, v, w, \psi_{\alpha}, \psi_{\beta}\}.$$  

(37b)

$$\{m_d\} = \{m_{du}, m_{dv}, m_{dw}, m_{d\psi_{\alpha}}, m_{d\psi_{\beta}}\}^T.$$  

(37c)

$$\{m_{bb}\} = \{a_{11}, a_{12}, \ldots, a_{1N}, a_{21}, a_{22}, \ldots, a_{2N}, a_{N-1}1, a_{N-1}2, \ldots, a_{N-1}N\}.$$  

(37d)

$$\{m_{bb}\} = \{m_{bw}, m_{bw}, m_{bw}, \psi_{\beta 1}, m_{bw}, \psi_{\beta 2}, \ldots, m_{bw}, \psi_{\beta N-1}\}^T.$$  

(37e)

For free vibration, $\{m_d\} = me^{i\omega t}$ is assumed. Therefore, the final form which provides vibration frequency is given below:

$$\left(\left[\begin{array}{c}
K_{dd} - [K_{dd}][K_{bb}]^{-1}[K_{bd}]
\end{array}\right] + [M]\omega^2\right)\{m_d\} = 0.$$  

(38)

4. Numerical analysis and discussion

In the current study, the free vibration responses of the FG-CNT double-curved shells and panels embedded in piezoelectric layers are studied. At the first stage, results are compared to those of other studies. Polymethyl methacrylate is chosen as a matrix with the following material properties [28].

$$E_m = (3.52 - 0.003 \Delta T) \text{ GPa},$$

$$\nu_m = 0.34 \quad \rho = 1150 \text{ kg/m}^3.$$

Here, $\Delta T = T_0$. The mechanical properties of SWCNT at different temperatures are given in Tables 3 and 4. Also, the material characteristics of PZT-5A are displayed in Table 5.

For validation, data results are initially obtained from shells and panels via the GDQ method and, then, they are compared with those of other works. To investigate validity, the materials including zirconia (ceramic) and aluminum (metal) are added to the FGM model. Young’s modulus, Poisson’s ratio, and mass density are: $E_c = 168 \text{ GPa}$, $\nu = 0.3$, and $\rho_c = 5700 \text{ kg/m}^3$, respectively, for zirconia; they are

| T (K) | E_{11}^{CNT} (TPa) | \alpha_{22}^{CNT} \times 10^{-5} (\) | E_{22}^{CNT} (TPa) | \alpha_{11}^{CNT} \times 10^{-5} (\) | G_{12}^{CNT} (TPa) | \nu_{12}^{CNT} | \rho (kg/m^3) |
|-------|---------------------|----------------------------------|---------------------|----------------------------------|------------------|------------------|------------------|
| 300   | 5.64                | 5.16                             | 7.08                | 3.45                             | 1.91             | 0.17             | 1400             |
| 500   | 5.53                | 5.01                             | 6.93                | 4.53                             | 1.95             | 0.17             | 1400             |
| 700   | 5.47                | 5.01                             | 6.86                | 4.53                             | 1.96             | 0.17             | 1400             |
Table 4. The parameters of (10,10) armchair Single Wall Carbon Nano Tube (SWCNT) obtained from molecular dynamics [32].

| $V_{CNT}'$ | $\eta_1$ | $\eta_2$ | $\eta_3$ |
|-------------|----------|----------|----------|
| 0.12        | 0.137    | 1.022    | 0.792    |
| 0.17        | 0.142    | 1.626    | 0.792    |
| 0.18        | 0.141    | 1.585    | 0.792    |

$E_m = 70$ GPa, $\nu_m = 0.3$, and $\rho_m = 2707$ kg/m$^3$, respectively, for aluminum. The volume fraction of FGM is considered as:

$$FGM_1(a/b/c/p): V_1 = \left[1 - a\left(\frac{1}{2} + \frac{z}{h}\right) + b\left(\frac{1}{2} - \frac{z}{h}\right)\right]^p,$$

$$FGM_11(a/b/c/p): V_2 = \left[1 - a\left(\frac{1}{2} - \frac{z}{h}\right) + b\left(\frac{1}{2} + \frac{z}{h}\right)\right]^p.$$%

It is worth noting that for validation, the spherical shell is considered and the results are compared to those found in [29]. The geometrical properties of the spherical shell are assumed as follows:

$$\varphi_0 = \frac{\pi}{8}, \quad \varphi_1 = \frac{\pi}{2}, \quad h = 0.05, \quad R = 1.$$%

The results for this purpose are provided in Table 6.

It should be noted that these results are obtained without piezoelectric materials and at room temperature. For validation of the panel analysis, the toro circular panel is analyzed. Then, the results were compared with those found in [30]. The obtained results are explained in Table 7. For this purpose, the materials are assumed as FGM and the volume fraction is given below:

$$FGM_1 = \left(\frac{1}{2} + \frac{z}{h}\right)^p, \quad FGM_11 = \left(\frac{1}{2} - \frac{z}{h}\right)^p.$$%

Moreover, the geometrical properties of this structure are given below:

$$R = 3, \quad h = 0.2, \quad \varphi_0 = \frac{\pi}{3}, \quad \varphi_1 = \frac{2\pi}{3}, \quad \varphi_2 = \frac{2\pi}{3}, \quad \varphi_3 = \frac{2\pi}{3},$$

$$R_6 = 1.5, \quad D_m = \frac{E_m h^3}{12(1 - \nu_m^2)}.$$

Table 5. Mechanical and electrical properties of PZT-5A [33].

| $E$ (GPa) | $\nu$ | $\rho$ (kg/m$^3$) | $\varepsilon_{0}$ (m) | $\varepsilon_{0}$ (m) | $K_{11}$ | $K_{22}$ | $K_{33}$ |
|-----------|-------|-------------------|------------------------|------------------------|--------|--------|--------|
| 63        | 0.35  | 7600              | $e_{31} = -7.29$       | $e_{32} = -7.29$       | 1.53   | 1.53   | 1.5    |
|           |       |                   | $e_{15} = 12.322$     | $e_{14} = 12.322$     |        |        |        |

Table 6. Comparison of frequencies (HZ) for the Functionally Graded Material (FGM) spherical shell with different boundary conditions. Geometrical properties of the spherical shell are: $\varphi_0 = \frac{\pi}{8}, \quad \varphi_1 = \frac{\pi}{2}, \quad h = 0.05, \quad R = 1$.

| $P$ | $n$ | $S-S$ Present | $S-S$ Ref. [29] | $C-C$ Present | $C-C$ Ref. [29] |
|-----|-----|---------------|-----------------|---------------|-----------------|
|     |     |               |                 |               |                 |
|     |     | $FGM_1$ $(a = 1/b = 0/c/p = 0.6)$ | $FGM_1$ $(a = 1/b = 0/c/p = 0.6)$ | $FGM_11$ $(a = 1/b = 0/c = 1/p = 0.6)$ | $FGM_11$ $(a = 1/b = 0/c = 1/p = 0.6)$ |
| 1   | 1   | 827.82        | 827.89          | 910.55        | 910.69          |
| 2   | 2   | 886.88        | 886.97          | 849.65        | 894.68          |
| 3   | 3   | 881.63        | 881.64          | 889.43        | 889.45          |
|     |     | 797.35        | 797.41          | 887           | 887.13          |
| 2   | 2   | 851.91        | 851.92          | 859.37        | 859.37          |
| 3   | 3   | 848.52        | 848.53          | 856.54        | 856.57          |
|     |     | 815.15        | 815.21          | 908.66        | 908.88          |
| 2   | 2   | 887.92        | 887.95          | 893.78        | 893.81          |
| 3   | 3   | 885.54        | 885.56          | 888.30        | 888.32          |
|     |     | 790.75        | 790.81          | 885.80        | 885.93          |
| 2   | 2   | 852.66        | 852.68          | 858.73        | 858.76          |
| 3   | 3   | 850.57        | 850.59          | 855.74        | 855.76          |
Table 7. Comparison of frequency parameters $\Omega = \omega R^2 \sqrt{\frac{E h}{\rho m}}$ for Functionally Graded Material (FGM) toro circular panel. Geometrical properties of this structure are: $R = 3$, $h = 0.2$. $\varphi_0 = \frac{\pi}{2}$, $\varphi_1 = \frac{2\pi}{3}$, $\varphi = \frac{2\pi}{3}$, $R_0 = 1.5$.

| Type   | $\varphi$ | $P$ | CCC | CCC | Present | Ref. [29] | Present | Ref. [30] |
|--------|------------|-----|-----|-----|---------|-----------|---------|-----------|
| $FGM_1$ | $\frac{\pi}{2}$ | 0.5 | 63.05 | 63.06 | 59.47 | 60.38 |
|        | 10          | 60.48 | 61.41 | 58.08 | 59.06 |
| $FGM_2$ | $\frac{\pi}{2}$ | 0.5 | 56.90 | 57.77 | 56.66 | 57.53 |
|        | 10          | 55.45 | 56.30 | 55.16 | 56.01 |
| $FGM_3$ | $\frac{\pi}{2}$ | 0.5 | 56.07 | 56.79 | 56.06 | 56.74 |
|        | 10          | 54.53 | 55.26 | 54.52 | 55.23 |

To check the accuracy for the structures with piezoelectric materials, the results are obtained for a square plate. The plate is made of FG-CNT and embedded in the piezoelectric layer on the top and bottom surfaces. The results are reported in Table 8. The piezoelectric material considered in this study is PZT-5A. Of note, the parameters of the plate structure are:

$A_1 = 1$, $A_2 = 1$, $R_0 = \infty$, $R_3 = \infty$.

Subsequently, by substituting these parameters into the governing equations, the results are obtained for the plate. The geometrical properties of the plate are given below:

$x_1 = 0.4$, $y_1 = 0.4$, $h = 0.05$, $h_p = 0.1$.

4.1. Case study result

In this section, the effects of electrical and mechanical boundary conditions, the distribution of CNTs, thermal environment, the volume fraction of CNTs, and the geometrical characteristics of the shells and panels are studied. Tables 9-12 present the fundamental frequency of the spherical, toro circular, cycloidal, and elliptical shells that are reinforced by CNTs and integrated with piezoelectric layers. The natural frequencies of the four structures with three boundary conditions including C-C, C-S, and S-S are shown in Tables 9-12. Based on these results, the switching of the electrical boundary conditions (shifting from closed circuit to open circuit) leads to increase in the amount of the fundamental frequencies. Following the comparison of the results of the CNTs pattern distributions, it becomes clear that the FG-X type of CNTs distribution has the highest frequency, while the FG-O pattern has the lowest frequency value. It is clear that among the three cases of the boundary conditions, C-C shells have higher natural frequencies than those with C-S and S-S boundary conditions. Furthermore, Tables 13-16 show that by changing the

Table 8. Comparison of frequency parameters

$\Omega = \omega \sqrt{\frac{E h}{\rho m}}$ for CCC plate integrated with PZT-5A layers. The geometrical properties of the plate are:

$x_1 = 0.4$, $y_1 = 0.4$, $h = 0.05$, $h_p = 0.1$.

| $V_{CNT}$ | $\lambda$ | Condition | CCC | CCC | Present | Ref. [20] |
|-----------|----------|-----------|-----|-----|---------|-----------|
| 0.12 FGO  | 0.5      | Open      | 5.32 | 5.46 |
| 0.12 FGO  | 0.5      | Close     | 5.19 | 5.23 |
| 0.17 FGO  | 0.5      | Close     | 5.60 | 5.72 |
| 0.17 FGO  | 0.5      | Close     | 5.47 | 5.52 |
| 0.28 FGO  | 0.5      | Close     | 5.84 | 5.93 |
| 0.28 FGO  | 0.5      | Close     | 5.54 | 5.67 |
| 0.28 FGO  | 0.5      | Close     | 5.88 | 5.98 |
| 0.12 FGV  | 0.5      | Close     | 5.73 | 5.80 |
| 0.28 FGV  | 0.5      | Close     | 6.34 | 6.63 |
| 0.28 FGV  | 0.5      | Close     | 6.16 | 6.28 |
| 0.12 FGV  | 0.5      | Close     | 6.17 | 6.27 |
| 0.17 FGV  | 0.5      | Close     | 5.97 | 6.09 |
| 0.28 FGV  | 0.5      | Close     | 6.68 | 6.78 |
| 0.17 FGV  | 0.5      | Close     | 5.46 | 6.62 |
| 0.28 FGV  | 0.5      | Close     | 7.33 | 7.42 |
| 0.12 UD   | 0.5      | Close     | 5.79 | 5.91 |
| 0.17 UD   | 0.5      | Close     | 5.62 | 5.70 |
| 0.28 UD   | 0.5      | Close     | 6.31 | 6.32 |
| 0.12 UD   | 0.5      | Close     | 6.03 | 6.14 |
| 0.28 UD   | 0.5      | Close     | 6.77 | 6.86 |
| 0.17 UD   | 0.5      | Close     | 6.54 | 6.71 |
### Table 9. Fundamental frequencies of the spherical shell.
Geometrical parameters are: $\varphi_0 = \frac{\pi}{3}$, $\varphi_1 = \frac{2\pi}{3}$, $h = 0.05$ m, $R = 1$ m, $h_p = 0.1$ h, $n = 1$.

| $V_{CNT}^*$ Type Condition | C-C | C-S | S-S |
|-----------------------------|-----|-----|-----|
| 0.12 FGX Open               | 709.45 | 689.76 | 682.39 |
| Close                       | 709.01 | 689.31 | 682.22 |
| 0.17 FGX Open               | 791.89 | 766.53 | 756.08 |
| Close                       | 791.60 | 766.23 | 755.84 |
| 0.28 FGX Open               | 880.71 | 846.03 | 830.13 |
| Close                       | 880.58 | 845.89 | 830.03 |
| 0.12 FGV Open               | 695.31 | 685.59 | 650.00 |
| Close                       | 694.92 | 685.26 | 650.37 |
| 0.17 FGV Open               | 771.04 | 757.38 | 731.12 |
| Close                       | 770.79 | 757.19 | 719.96 |
| 0.28 FGV Open               | 846.99 | 824.41 | 771.87 |
| Close                       | 846.88 | 824.34 | 771.79 |
| 0.12 FGO Open               | 688.79 | 688.19 | 657.11 |
| Close                       | 688.39 | 672.78 | 656.81 |
| 0.17 FGO Open               | 761.90 | 741.61 | 720.18 |
| Close                       | 761.65 | 741.34 | 719.99 |
| 0.28 FGO Open               | 833.33 | 804.70 | 776.16 |
| Close                       | 833.11 | 804.58 | 776.07 |

### Table 10. Fundamental frequencies of toro circular shell.
Geometrical parameters are: $\varphi_0 = \frac{\pi}{3}$, $\varphi_1 = \frac{2\pi}{3}$, $h = 0.2$ m, $h_p = 0.1$ h, $R_b = 1.5$ m, $R = 3$ m.

| $V_{CNT}^*$ Type Condition | C-C | C-S | S-S |
|-----------------------------|-----|-----|-----|
| 0.12 FGX Open               | 254.63 | 254.21 | 251.21 |
| Close                       | 254.57 | 252.33 | 251.12 |
| 0.17 FGX Open               | 282.18 | 280.13 | 278.91 |
| Close                       | 282.15 | 280.08 | 278.85 |
| 0.28 FGX Open               | 318.90 | 317.17 | 316.07 |
| Close                       | 318.88 | 317.15 | 316.04 |
| 0.12 FGV Open               | 251.64 | 251.60 | 251.59 |
| Close                       | 251.55 | 251.51 | 251.50 |
| 0.17 FGV Open               | 278.77 | 278.47 | 278.42 |
| Close                       | 278.71 | 278.41 | 278.35 |
| 0.28 FGV Open               | 314.97 | 313.76 | 313.53 |
| Close                       | 314.93 | 313.72 | 313.48 |
| 0.12 FGO Close              | 250.13 | 249.05 | 248.80 |
| 0.17 FGO Close              | 277.05 | 276.20 | 275.96 |
| 0.28 FGO Close              | 312.97 | 312.36 | 312.21 |

### Table 11. Fundamental frequencies of cycloidal shell.
Geometrical parameters are: $\varphi_0 = \frac{\pi}{3}$, $\varphi_1 = \frac{2\pi}{3}$, $h = 0.1$ m, $h_p = 0.1$ h, $r_c = 1$ m, $R_b = 2$ m.

| $V_{CNT}^*$ Type Condition | C-C | C-S | S-S |
|-----------------------------|-----|-----|-----|
| 0.12 FGX Open               | 261.36 | 242.99 | 214.83 |
| Close                       | 261.14 | 242.88 | 214.70 |
| 0.17 FGX Open               | 297.95 | 278.23 | 248.40 |
| Close                       | 297.79 | 278.15 | 248.30 |
| 0.28 FGX Open               | 340.85 | 317.08 | 286.92 |
| Close                       | 340.79 | 317.02 | 286.86 |
| 0.12 FGV Open               | 236.75 | 213.09 | 190.21 |
| Close                       | 236.67 | 213.04 | 190.19 |
| 0.17 FGV Open               | 268.04 | 242.84 | 218.54 |
| Close                       | 267.98 | 242.80 | 218.53 |
| 0.28 FGV Open               | 302.53 | 276.82 | 249.91 |
| Close                       | 302.50 | 276.79 | 249.90 |
| 0.12 FGO Open               | 229.57 | 209.49 | 186.78 |
| Close                       | 229.43 | 209.35 | 186.63 |
| 0.17 FGO Open               | 239.30 | 238.26 | 214.27 |
| Close                       | 239.20 | 238.15 | 214.15 |
| 0.28 FGO Open               | 291.7 | 270.07 | 243.96 |
| Close                       | 291.64 | 269.99 | 243.85 |

### Table 12. Fundamental frequencies of elliptical shell.
Geometrical parameters are: $\varphi_0 = \frac{\pi}{3}$, $\varphi_1 = \frac{5\pi}{3}$, $h = 0.2$ m, $a = 3$ m, $b = 2$ m, $R_b = 0$ m, $h_p = 0.1$ h, $n = 1$.

| $V_{CNT}^*$ Type Condition | C-C | C-S | S-S |
|-----------------------------|-----|-----|-----|
| 0.12 FGX Open               | 231.68 | 216.74 | 202.51 |
| Close                       | 231.64 | 216.69 | 202.46 |
| 0.17 FGX Open               | 253.77 | 239.11 | 224.93 |
| Close                       | 253.75 | 239.08 | 224.89 |
| 0.28 FGX Open               | 271.42 | 258.49 | 245.69 |
| Close                       | 271.41 | 258.47 | 245.68 |
| 0.12 FGV Open               | 219.33 | 200.40 | 183.11 |
| Close                       | 219.22 | 200.38 | 183.09 |
| 0.17 FGV Open               | 239.86 | 220.52 | 202.49 |
| Close                       | 239.85 | 220.51 | 202.48 |
| 0.28 FGV Open               | 257.19 | 238.64 | 220.77 |
| Close                       | 257.18 | 238.63 | 220.76 |
| 0.12 FGO Close              | 216.10 | 201.20 | 187.53 |
| 0.17 FGO Close              | 236.17 | 220.95 | 206.81 |
| 0.28 FGO Close              | 253.11 | 238.16 | 223.98 |
| 0.28 FGO Close              | 253.09 | 238.14 | 223.96 |
Table 13. Fundamental frequencies of spherical panel
\( \varphi_0 = \frac{\pi}{3}, \ \varphi_1 = \frac{\pi}{3}, \ \varphi = \frac{2\pi}{3}, \ h = 0.05 \text{ m}, \ R = 1 \text{ m}, \ h_p = 0.1 \text{ h} \).

| \( V_{\text{C}NT} \) | Type | Condition | C-C-C-C | C-S-C-S |
|-------------------|------|-----------|--------|--------|
| 0.12              | FGX  | Open      | 811.12 | 784.00 |
|                   |      | Close     | 810.55 | 783.41 |
| 0.17              | FGX  | Open      | 901.86 | 871.27 |
|                   |      | Close     | 901.51 | 870.94 |
| 0.28              | FGX  | Open      | 996.07 | 988.97 |
|                   |      | Close     | 995.94 | 988.85 |
| 0.12              | FGV  | Open      | 777.66 | 758.08 |
|                   |      | Close     | 777.20 | 757.60 |
| 0.17              | FGV  | Open      | 852.34 | 831.81 |
|                   |      | Close     | 852.04 | 831.54 |
| 0.28              | FGV  | Open      | 923.00 | 897.71 |
|                   |      | Close     | 922.82 | 897.55 |
| 0.12              | FGO  | Open      | 755.56 | 737.08 |
|                   |      | Close     | 755.14 | 736.68 |
| 0.17              | FGO  | Open      | 821.97 | 800.44 |
|                   |      | Close     | 821.76 | 800.25 |
| 0.28              | FGO  | Open      | 885.10 | 857.49 |
|                   |      | Close     | 885.03 | 857.42 |

Table 14. Fundamental frequencies of the toro circular panel. Geometrical parameters are: \( \varphi_0 = \frac{\pi}{3}, \ \varphi_1 = \frac{3\pi}{4}, \ \varphi = \frac{4\pi}{5}, \ h = 0.2 \text{ m}, \ R = 3 \text{ m}, \ R_b = 1.5 \text{ m}, \ h_p = 0.1 \text{ h} \).

| \( V_{\text{C}NT} \) | Type | Condition | C-C-C-C | C-S-C-S |
|-------------------|------|-----------|--------|--------|
| 0.12              | FGX  | Open      | 286.34 | 282.31 |
|                   |      | Close     | 286.13 | 282.11 |
| 0.17              | FGX  | Open      | 325.13 | 320.35 |
|                   |      | Close     | 324.97 | 320.20 |
| 0.28              | FGX  | Open      | 378.00 | 374.78 |
|                   |      | Close     | 377.98 | 374.69 |
| 0.12              | FGV  | Open      | 279.06 | 273.77 |
|                   |      | Close     | 278.83 | 273.57 |
| 0.17              | FGV  | Open      | 316.16 | 309.02 |
|                   |      | Close     | 315.99 | 308.87 |
| 0.28              | FGV  | Open      | 351.02 | 349.00 |
|                   |      | Close     | 350.98 | 348.96 |
| 0.12              | FGO  | Open      | 276.36 | 273.44 |
|                   |      | Close     | 276.33 | 273.22 |
| 0.17              | FGO  | Open      | 313.27 | 309.50 |
|                   |      | Close     | 313.10 | 309.34 |
| 0.28              | FGO  | Open      | 338.65 | 336.17 |
|                   |      | Close     | 338.64 | 336.15 |

Table 15. Fundamental frequency of the cycloidal panel. Geometrical parameters are: \( \varphi_0 = \frac{\pi}{3}, \ \varphi_1 = \frac{3\pi}{4}, \ \varphi = \frac{4\pi}{5}, \ h = 0.1 \text{ m}, \ R_b = 2 \text{ m}, \ r_c = 1 \text{ m}, \ h_p = 0.1 \text{ h} \).

| \( V_{\text{C}NT} \) | Type | Condition | C-C-C-C | C-S-C-S |
|-------------------|------|-----------|--------|--------|
| 0.12              | FGX  | Open      | 260.68 | 260.07 |
|                   |      | Close     | 260.51 | 259.90 |
| 0.17              | FGX  | Open      | 294.51 | 294.15 |
|                   |      | Close     | 294.42 | 294.05 |
| 0.28              | FGX  | Open      | 330.13 | 328.82 |
|                   |      | Close     | 330.10 | 328.80 |
| 0.12              | FGV  | Open      | 235.61 | 235.45 |
|                   |      | Close     | 235.52 | 235.37 |
| 0.17              | FGV  | Open      | 262.57 | 262.38 |
|                   |      | Close     | 262.52 | 262.32 |
| 0.28              | FGV  | Open      | 290.30 | 289.08 |
|                   |      | Close     | 290.46 | 289.04 |
| 0.12              | FGO  | Open      | 225.62 | 225.49 |
|                   |      | Close     | 225.54 | 225.41 |
| 0.17              | FGO  | Open      | 210.32 | 219.09 |
|                   |      | Close     | 210.27 | 219.06 |
| 0.28              | FGO  | Open      | 274.39 | 272.87 |
|                   |      | Close     | 274.38 | 272.86 |

Table 16. Fundamental frequency of the elliptical panel. Geometrical parameters are: \( \varphi_0 = \frac{\pi}{5}, \ \varphi_1 = \frac{3\pi}{5}, \ \varphi = \frac{\pi}{3}, \ h = 0.2 \text{ m}, \ R_b = 0, \ h_p = 0.1 \text{ h}, \ a = 3, \ b = 2 \).

| \( V_{\text{C}NT} \) | Type | Condition | C-C-C-C | C-S-C-S |
|-------------------|------|-----------|--------|--------|
| 0.12              | FGX  | Open      | 257.83 | 257.38 |
|                   |      | Close     | 257.72 | 257.26 |
| 0.17              | FGX  | Open      | 289.32 | 278.35 |
|                   |      | Close     | 289.23 | 278.33 |
| 0.28              | FGX  | Open      | 307.81 | 295.62 |
|                   |      | Close     | 307.80 | 295.61 |
| 0.12              | FGV  | Open      | 240.54 | 250.32 |
|                   |      | Close     | 240.48 | 250.26 |
| 0.17              | FGV  | Open      | 272.96 | 265.89 |
|                   |      | Close     | 272.94 | 265.87 |
| 0.28              | FGV  | Open      | 288.01 | 279.80 |
|                   |      | Close     | 288.02 | 279.78 |
| 0.12              | FGO  | Open      | 241.96 | 239.20 |
|                   |      | Close     | 241.83 | 239.17 |
| 0.17              | FGO  | Open      | 263.09 | 255.56 |
|                   |      | Close     | 263.67 | 255.54 |
| 0.28              | FGO  | Open      | 278.23 | 268.94 |
|                   |      | Close     | 278.22 | 268.93 |
electrical state to open circuit, fundamental frequencies of the panels rise. All of the case studies show that as the CNT volume fraction increases, the fundamental frequencies increase. Furthermore, among the four possible graded patterns of the CNTs, the FG-X type of CNT distribution produces higher frequencies and the lowest frequencies are relevant to the FG-O pattern. As expected, among the two boundary conditions, CCCC panels have higher natural frequencies than the CSCS panels. Figures 7-14 show the fundamental frequencies of structures in a thermal environment. To investigate the thermal effect, the distribution function of CNT is assumed based on FG-O pattern and the volume fraction is 0.12. According to these figures, the fundamental frequencies are reduced at higher temperatures. Thus, by increasing the ratio of the piezoelectric layer thickness to the core thickness, the effect of temperature is negligible and can be ignored. Thus, the difference between fundamental frequencies at different temperatures is reduced.

5. Conclusion

In this study, the General Differential Quadrature (GDQ) method was implemented to examine the free vibration of double curved structures. The core of the structures is made of Carbon Nanotubes (CNTs). The mechanical properties were obtained based on the modified rule of mixture. The upper and lower surfaces were covered with piezoelectric layers. The
Figures 12, 13, and 14 show the fundamental frequency of the toroidal, cycloidal, and elliptical panels, respectively, with varying thickness ratios of the piezoelectric layer to the core layer. The numerical results are employed to examine the effects of boundary conditions, volume fractions, electrical condition, thickness of piezoelectric layers, and kinds of structures. It was observed that the frequency of the whole structures rose when the FGX pattern was used. In addition, the frequency rose when the volume fraction increased. The open circuit condition has a higher frequency than the closed circuit. In the high mode number, the difference between open circuit and closed circuit frequencies widens. The effect of the mechanical boundary conditions on the fundamental frequencies is studied; accordingly, the clamped-clamped boundary condition has a higher frequency than others. The effect of a thermal environment is investigated. Due to the increase in the environment temperature, the frequency parameter tends to decrease. By increasing the thickness of piezoelectric layers, the effect of the thermal environment is reduced.

**Nomenclature**

- $\alpha$: Value of the meridional direction
- $\beta$: Value of the circumferential direction
- $z$: Value of the thickness direction
- $z_c$: Value of the piezoelectric thickness direction
- $h$: Thickness of the structures
- $h_p$: Thickness of piezoelectric
- $R_b$: Distance between geometric axis and revolution axis
- $a$: Semi major axis
- $b$: Semi minor axis
- $R_a$: Radius in the meridional direction
- $R_\beta$: Radius in the circumferential direction
- $R_0$: Horizontal radius
- $V_{tot}$: Potential energy
- $r_c$: Radius of the produced circle of cycloid curve
- $U$: Displacement in the meridional direction
- $V$: Displacement in the circumferential direction
- $W$: Displacement in the thickness direction
- $u$: Displacement of reference plane in the meridional direction
- $v$: Displacement of reference plane in the circumferential direction
- $w$: Displacement of reference plane in the thickness direction
- $\Psi_a$: Rotation around $\alpha$ axis
- $\Psi_\beta$: Rotation around $\beta$ axis
- $A_i$: Lamé parameters
- $N$: Resultant force
- $A_{ij}$: Extensional stiffness
- $B_{ij}$: Bending-extensional coupling stiffnesses
- $D_i$: Electrical displacements
- $P_{ij}$: Pyroelectric property
\[
\begin{align*}
\epsilon_{ij} & \quad \text{Piezoelectric stress constant} \\
p & \quad \text{Piezoelectric} \\
c & \quad \text{Core} \\
\alpha_{ij} & \quad \text{Thermal expansion} \\
\Delta T & \quad \text{Temperature gradient} \\
\gamma^{+} & \quad \text{Total volume fraction} \\
\eta_{i} & \quad \text{Efficiency parameter} \\
\nu & \quad \text{Poisson's coefficient} \\
\rho & \quad \text{Density} \\
E_{ij} & \quad \text{Young modulus} \\
U_{\text{tot}} & \quad \text{Strain energy} \\
K_{\text{tot}} & \quad \text{Kinetic energy} \\
I_{i} & \quad \text{Moment of inertia} \\
C_{ij}, C_{p ij} & \quad \text{Weighting coefficients} \\
M(\alpha) & \quad \text{Legendre polynomial} \\
[M] & \quad \text{Mass matrix} \\
[K] & \quad \text{Stiffness matrix} \\
\chi & \quad \text{Curvature} \\
M & \quad \text{Moment resultant} \\
D_{ij} & \quad \text{Bending stiffness} \\
T & \quad \text{Thermal} \\
k_{ij} & \quad \text{Dielectric permittivity} \\
\phi & \quad \text{The electric potential}
\end{align*}
\]
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