Numerical simulation of Einstein-Podolsky-Rosen experiments in a local hidden variables model.

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We simulate correlation measurements of entangled photons numerically. The model employed is strictly local. In our model correlations arise from a phase, connecting the electromagnetic fields of the two photons at their separate points of measurement. We sum up coincidences for each pair individually and model the operation of a polarizer beam splitter numerically. The results thus obtained differ substantially from the classical results. In addition, we analyze the effects of decoherence and non-ideal beam splitters. It is shown that under realistic experimental conditions the Bell inequalities are violated by more than 30 standard deviations.

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Since experimental tests of the Einstein-Podolsky-Rosen (EPR) problem became feasible \[1\,2\], the field has gained a popularity and importance, which is startling. The experiments themselves do not seem to justify this broad interest. The fact that polarization measurements of two separated photons are connected, even if the measurements are in space-like separation \[3\], is in itself problematic, but hardly accounts for the large number of publications devoted to the problem every year. The importance of the EPR problem derives from its impact on two seemingly separate fields. On the one hand, the theoretical predictions impose a limit on theoretical concepts, aimed at modifying quantum mechanics by hidden variables models \[3\,4\]. Here, the current understanding is that the experiments contradict any local model \[3\,4\]. On the other hand, entanglement of separated photons is essential for many theoretical concepts in quantum computation and quantum information \[4\,5\]. Here, the question whether quantum mechanics provides the only suitable model for these experiments, is essential, because qubits, the principal information units in these concepts, are only defined within the quantum formalism.

Theoretically, the entanglement of separated photons rests on the validity of the superposition principle. The question, whether superposition is a sufficient condition for non-locality, is open. Deutsch and Hayden showed recently that all information in an EPR experiment is stored locally, even though it cannot locally be accessed \[1\]. In a different approach it was shown that the observed connection can be interpreted as fourth-order optical correlation. The connection then arises due to the phase of the photons’ electromagnetic fields \[1\,2\,4\,5\]. All physical effects in this case are local, even though the correlations must be expressed in terms of polarizer settings at different locations.

In this paper we present a model, where the phase connection between the two photons is retained, but the superposition principle for the separate photon fields is discarded. Essentially this means that interference terms in the correlations are omitted. We shall show, however, that the model also in this case is in accordance with experimental results \[3\,4\], provided the polarizer beam splitter (PBS) is not completely ideal. The non-ideal PBS in our model can only discriminate between the horizontal and vertical polarization of photons, if the intensities of the photon’s electromagnetic field components, projected onto the orthogonal polarizer axes, differ by more than 30%. We shall show that this threshold is consistent with and necessary for the high visibility observed in state-of-the-art experiments. Furthermore, we shall establish that the Bell inequalities are violated under these conditions by more than 30 standard deviations. Single photon experiments will be treated within the same model in a separate paper.

The setup in our model consists of a source for entangled photons and two PBS with photon detectors. The source, at \(z = 0\) of the coordinate system, emits two entangled photons. Their planes of polarization are unknown. But it is known that these planes at the moment of emission are perpendicular to each other \(\Delta \varphi = \pi/2\). The polarizers are at \(z = \pm L\), where \(L\) is an integer multiple of the photons’ wavelength \(\lambda\). Since laser emissions have finite extensions in the range of a few microns \[6\], we describe each photon as an excitation of the electromagnetic field of limited extension. To simplify the description these fields shall be monochromatic. That is to say we disregard the effects of higher Fourier components, which arise from the limited extension of the fields. Then the phase along the photon’s path of propagation is equal to the phase of a monochromatic wave. For photons of circular polarization the angle of polarization at a point \(z_0\) depends on the distance from the source and the initial angle of polarization. For linear polarization only the amplitude of the field depends on the distance \(z_0\), but not the angle of polarization. This is changed, however, in EPR experiments using linear polarized light, by an optical modulator \[1\]. A modulator shifts one component of the electromagnetic field, either the horizontally or the vertically polarized one by a phase-angle \(\Delta \psi\). If the field is initially in linear polarization, it is described
by $\vec{E}(z,t) = \vec{E}_0 \cos(kz - \omega t)$, where $\vec{E}_0 = (E_{0X}, E_{0Y})$. The angle of polarization $\varphi_0 = \arctan(E_{0Y}/E_{0X})$ in this case is independent of the phase. If one component, say $E_y$, is shifted by a phase-angle $\Delta \psi$, then the field after the modulator will be:

$$\vec{E}' = (E_{0X} \cos(kz - \omega t), E_{0Y} \cos(kz - \omega t + \Delta \psi))$$  \hspace{1cm} (1)

The angle of polarization $\varphi$ is then no longer constant, but depends on the phase:

$$\varphi = \arctan \left\{ \frac{E_{0Y}}{E_{0X}} \cos \Delta \psi - \tan(kz - \omega t) \sin \Delta \psi \right\}$$  \hspace{1cm} (2)

Therefore in both cases the angle of polarization $\varphi$ depends on the location $z$ of a measurement. The measurements are performed with a PBS beam splitters and two photon detectors. This differs from using optical polarizers, because the combination PBS and detector yields not a continuous result, e.g. the field intensity after the polarizer, but a discrete one. The photon is either detected at detector 1, or at detector 2. In one case, it is said to have vertical polarization (+), in the other horizontal polarization (-). The representation of this experiment by a classical optical model is not straightforward. In Furry’s integral, for example, the correlation is described by (10).

$$P(\alpha, \beta) = \int d\varphi \cos^2(\varphi + \pi/2 - \alpha) \cos^2(\varphi - \beta)$$  \hspace{1cm} (3)

where $P(\alpha, \beta)$ is the probability of a coincidence, $\varphi$ the unknown angle of polarization, and $\alpha$ and $\beta$ are the polarizer settings. In this equation it is assumed that a single measurement can be described by the product $\cos^2(\varphi + \pi/2 - \alpha) \cos^2(\varphi - \beta)$. But this is evidently not the right representation for the dichotomic results obtained in the experiments. In our view it is partly this error, which is responsible for the deviation between the correlation curves obtained in the experiments, and the correlation curves obtained from Eq. (10). Analyzing the measurements event by event only two results are obtained in the experiments on either PBS: (+), if the angle of polarization $\varphi$ is between $\pi/4$ and $\pi/2$, and (-), if it is between 0 and $\pi/4$. In our model we formalize this feature of the experiments by a switch at $\pi/4$; numerically this is accomplished by a selection criterion based on the value of $\cos^2(\varphi)$. The model in this case encounters a difficulty, which is absent in a classical representation. It is the threshold of the PBS at the critical angle of $\pi/4$. For polarization angles close to this value the intensity in both channels of a PBS is equal. Theoretically, this would mark an event, where both polarizations, (+) and (-), are recorded for a single photon. Experimentally, such a case does not exist. Therefore we must impose a suitable threshold at this angle, to decide on the actual outcome. In our view this threshold is an experimental parameter and cannot be determined from theoretical estimates alone. We shall therefore use the experimental results, in particular the observed visibility, to determine this value. Formally, this reduces the range of the model PBS to the values $\cos^2(\varphi) \geq 0.5 + \Delta s$ and $\cos^2(\varphi) \leq 0.5 - \Delta s$, where $\Delta s$ is the polarizer threshold. This threshold is related to the intensity difference between the two channels, which a PBS can resolve in the experiments. A threshold of 0.1, for example, amounts to a resolution better than 30%.

In all experiments of this kind decoherence of the two separate laser beams should play an important role. Decoherence in the experiments has essentially two origins: (i) The random deviation of the phase of laser beams from the phase of monochromatic waves; and (ii) the random deviation of the location of source and analyzers due to thermal vibration. In the experiments the correlations were independent of distance variations. Therefore the laser beams are assumed to be ideal. But, as shown above, the angle of polarization depends on the location of the measurement. This means that we have to include thermal vibrations in the model. This is done by a random variation of the optical path of a photon from its source to the analyzer. The length of the random segment is determined by the level of decoherence. 100% decoherence, for example, means that half a wavelength, or a phase angle of $\pi$, is random. Correspondingly, for 10% decoherence a phase angle of $\pi/10$ is random.

Numerically, the procedure is as follows. 10000 pairs of photons are emitted from their common source. Their initial angle of polarization is created by a random number generator, the random number [0,1] is mapped onto the interval [0,2$\pi$]. We add an angle of $\pi/2$ for one photon of each pair. The polarization angle is set equal to the initial phase angle $\psi_0$. Both photons cover an optical path of $L \pm \Delta L$, where $\Delta L$ is the random segment due to decoherence, created by a separate random input. After covering the path to the analyzers, both photons are measured. The variables in this measurement are the phase angle $\psi$ (= the hidden variable), and a polarizer angle $\alpha$. We formalize the actual measurement by a switch $\cos^2(\psi - \alpha) \geq 0.5 + \Delta s$ (+), and $\cos^2(\psi - \alpha) \leq 0.5 - \Delta s$ (-). The setting of polarizer 2 remains unchanged during the whole simulation. After 10000 pairs have been recorded, we change the angle $\alpha$ by $\pi/100$. Compared to the experimental practice, where the PBS remains unchanged, and only the speed of the optical modulator is altered, the only difference should be a phase shift of the correlation function. In all figures we report only the coincidences $N_{++}$.

Initially we simulated an ideal measurement. The fields of the two laser beams are fully coherent throughout the distance between the two polarizers, and the experimental devices are supposed to have ideal characteristics. The result of this simulation is shown in Fig. (1). The simulation differs from the theoretical curve obtained by Eq. (3). This difference is due to the digital measurements, on which our model is based. While Furry's integral (10) predicts a minimum of $N_{max}/8$ for the correlation curve, where $N$ is the number of photon pairs, we obtain zero. The visibility $(N_{max} - N_{min})/(N_{max} + N_{min})$
for an ideal measurement is therefore 1, contrary to 0.5, obtained by the integration. The simulation also differs from the prediction in quantum mechanics, which is described by $N/2\sin^2(\alpha - \beta)$. The correlation curve in the simulation has a different functional form: while it is sinusoidal in quantum mechanics, it is a saw tooth in our model. This difference is due to the omission of interference terms, or the disregard for superposition. The maximum $(N/2)$ and the minimum (zero), however, are equal in both cases. We also simulated the effect of the polarizer threshold $\Delta s$ on the result of ideal measurements. The threshold $\Delta s$ was varied from 0.0 to 0.2. Here, we find that the threshold reduces the absolute yield of coincidences in a simulation and increases the width of the minimum at the angles 0 and $\pi$. This effect is equal to a retarded onset of the correlation function at its minimum position.

![Graph showing correlation function](image)

FIG. 1. Coincidence $(N_{++})$ counts of 10000 emitted photon pairs. The ideal measurement (visibility 1.0) has the shape of a saw-tooth.

The polarizers in current measurements are more than 400 m apart [5]. Furthermore, there is no vibration damping or cooling to very low temperatures involved in such a measurement. This feature of the measurements is bound to cause random motion of system components. From surface science the range of motion without damping can be estimated, it should be for an isolated surface no less than a few nanometers or more than one percent of the photon’s wavelength. Considering that we deal with three coupled components and optical paths in between it seems safe to increase this estimate by one order of magnitude. In this case we have to include random motion of our system in the range of about 5-10% of the wavelength. This translates, in our model, into a decoherence rate of 10-20% (100% means that half a wavelength of the photon’s optical path is random). Simulations with a decoherence rate of 10%, 50% and 100% are shown in Fig. 2.

The interesting feature of decoherence is that it renders the resulting distribution more sinusoidal than the correlation function of an ideal measurement. The fully decoherent simulation proves that the correlation, in our model, depends only on a phase connection between the two photons.

If decoherence due to thermal vibrations of the system components is assumed to be about 10%, we may estimate the actual threshold $\Delta s$ in the experiments from the experimentally obtained visibility. In the 1998 experiment the visibility was reported to be 97% [5]. This is not in keeping with an ideal combination PBS/photon detectors, since a threshold $\Delta s = 0.0$, under the condition of 10% decoherence, leads to a visibility of only 86%. This value is also lower than the visibility of $\approx 88\%$, reported in the 1981 experiment [7]. From the experimental facts we have to conclude that the threshold $\Delta s$ cannot be zero. This conclusion is also consistent with previous considerations, which established that a finite threshold is necessary to decide on the outcome of a polarization measurement using a PBS. Then the only remaining question is the actual value of $\Delta s$. We only obtain a visibility of close to 97%, as in the 1998 experiments, if $\Delta s \approx 0.1$.

![Graph comparing ideal and non-ideal measurements](image)

FIG. 2. Coincidence counts for non-ideal measurements. Decoherence renders the resulting curves (10%, 50%) sinusoidal, the fully decoherent curve (100%) shows that correlations in our model are only due to the connecting phase.

In Fig. 3 (a) we show the result of our simulation under these conditions. It can be seen that the correlation curve in this case is very similar to the ideal curves from quantum mechanical models, it is also virtually identical to the curves obtained in the 1998 experiment [5]. It has been claimed that these experiments rule out "objective local theories" or "realistic local theories" [8-19]. The basis of this claim was the assertion that within both classes of theories the Bell inequalities are not violated in such an experiment. To show that our model violates the Bell inequalities, we simulate the counts at four selected angular positions of the polarizers $\alpha$ and $\beta$ (0, 45, 22.5, 67.5). These positions yield the maximum violation of Bell’s inequalities in the standard framework. We performed the simulations for varying threshold values from 0.0 to 0.2, and for a fixed decoherence rate of 10%. For every setting we performed 10 separate runs, each with 10000 pairs of photons. Fig. 3 (b) gives the result of our simulation. The violation was computed according to the version of Clauser et al. (CHSH) [20]. As shown, it increases with increasing threshold. Furthermore, the limit of violation is close to 2.0 (CHSH value of 3.90) in the final setting. For a threshold of 0.1 and a decoherence rate of 10%, which we found close to the experimental conditions, the CHSH value is $2.69 \pm 0.02$, as compared to $2.73 \pm 0.02$ reported in the 1998 experiments [5]. The results are therefore in excellent agreement. Finally, we
Simulated Bell violations over the whole range of polarizer angles, using the formulation employed in Aspect’s 1982 experiments. Also in this simulation decoherence was 10%, the number of photon pairs was 10000. The result of this simulation is shown in Fig. 3 (c). It can be seen that it is virtually identical, for realistic conditions, to the result obtained in the experiment. Given the fact that our model is a local and realistic model of the experiments, the claim that these EPR experiments rule out any “realistic local theory” is only valid, if the experiments are in fact ideal. Which we think they are not. But instead of using current experimental results as arguments how much or how little the various no-go theorems apply, we shall suggest controlled experiments in future publications. There, the non-ideality can be related to model parameters, and these in turn allow extrapolating the experimental results to ideal situations. We think that this procedure may ultimately provide a better understanding of the experiments than exists at present.

A referee has asked whether the author actually “believes” that this model “describes nature”. I think this is not a scientific question. Every model, which correctly describes an experiment, describes nature. The only relevant question is whether one model has advantages compared to another one. Given that this model is local, it is realistic, and it explains the observed correlation in a simple manner, it has at least three advantages compared to the standard model in quantum mechanics. Whether it is correct on a more general level, depends on the results of subsequent experimental research.

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