Appraisal of application possibilities of smoothed splines to designation of the average values of terrain curvatures measured after the termination of hard coal exploitation conducted at medium depth

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Abstract. In paper were presented results of average values calculations of terrain curvatures measured after the termination of subsequent exploitation stages in the 338/2 coal bed located at medium depth. The curvatures were measured on the neighbouring segments of measuring line No. 1 established perpendicularly to the runways of four longwalls No. 001, 002, 005 and 007. The average courses of measured curvatures were designated based on average courses of measured inclinations. In turn, the average values of observed inclinations were calculated on the basis of measured subsidence average values. In turn, they were designated on the way of average-square approximation, which was done by the use of smoothed splines, in reference to the theoretical courses determined by the S. Knothe’s and J. Bialek’s formulas. Here were used standard parameters values of a roof rocks subsidence $a$, an exploitation rim $A_{ob}$, and an angle of the main influences range $\beta$. The values of standard deviations between the average and measured curvatures $\sigma_C$ and the variability coefficients of random scattering of curvatures $M_C$ were calculated. They were compared with values appearing in the literature and based on this, a possibility appraisal of the use of smooth splines to designation of average course of observed curvatures of mining area was conducted.

1. Introduction
The influence amount of mining exploitation on terrain surface may be expressed, among others, by means of mining area curvatures. Real values of this deformation indicator are designated based on the results of geodesic measurements conducted before, during and after termination of exploitation. These surveys are carried out on the measuring points of an observational network. Generally lengths, horizontal and vertical angles, and heights of the points are measured. They can be measured by the use of standard methods and devices or the Global Navigation Satellite Systems (GNSS), or even the Unmanned Aerial Vehicles (UAV) [1-3].

To compare the measured values of curvatures with their forecasted values, it’s necessary to eliminate a random factor from the measured curvatures, which is the highest from all deformation indicators [4]. It’s possible thanks to an average – square approximation of measured courses of curvatures. This type of approximation can be done with the use of smoothed splines, what has been presented in this article.
2. The courses of measured curvatures
This section presents a case of hard coal exploitation, which was conducted in the 338/2 coal bed at medium depth, a characteristic of geodesic measurements done on an observational line no. 1 situated above the exploitation field and an obtainment way of curvatures measured values.

2.1. Characteristic of exploitation done in the 338/2 coal bed
The data describing the geological – mining conditions in the 338/2 coal bed and an exploitation system have been shown in the Table 1.

| Table 1. The basic features of conducted exploitation |
|----------------------------------------------------|
| Exploitation feature |
| Value            |
| Coal bed         | 338/2               |
| Time /years/     | 1994 ÷ 1996         |
| Fulfillment of post–mining emptiness |
| subsidence of roof rocks [6] |
| Numbers of longwalls | 001, 002, 005, 007 |
| Interval of depth /m/ | 580 ÷ 700 |
| Length of excavations /m/ | 250 |
| Run of excavations /m/ | 750 ÷ 1080 |
| Height /m/ | 2 |
| Decline /º/ | 6 ÷ 8 |
| Direction of coal bed inclination | N – S |
| Over layers thickness /m/ | 60 |

2.2. Characteristic of geodesic measurements
Characteristic of surveys carried out on an observational line no. 1 has been presented in the Table 2.

| Table 2. The basic information about geodesic measurements |
|----------------------------------------------------------|
| Feature                                             |
| Value                                                |
| Shape of observational network                          |
| line                                                  |
| Location in relation to the longwalls                  |
| perpendicularly to the runs                            |
| Number of measuring points                             |
| 53                                                   |
| Average distance between points /m/                   |
| 37                                                   |
| Device used for height measurement                     |
| precise leveller                                       |
| Accuracy of height measurement /mm/                   |
| 0.1                                                  |
| Method of height measurement                          |
| double, geometric levelling from the middle            |
| Device used for length measurement                     |
| distance – meter                                       |
| Accuracy of length measurement /mm/                   |
| 1                                                    |
| Method of length measurement                           |
| double measurement                                     |

The points heights and segments lengths of measuring line were determined in the same time, after the termination of subsequent exploitation stages. Because of that, there were obtained the four curvatures courses measured after the ending of extraction in one, two, three and four longwalls.

2.3. Determination of measured curvatures courses
Based on the measurements of points heights before and after the ending of next exploitation stages in coal bed 338/2, can be obtained the subsidence of particular points from the following formula (1) [7]:

$$W_{i}^{k+1} = H_{i}^{k+1} - H_{i}^{k},$$  \hspace{1cm} (1)

where: $H$ – height of measuring point; $W$ – measured subsidence; \( i \) – number of measuring point; \( k \) – number of measuring cycle.
The measured courses of subsidence were presented at the Figure 1.

![Figure 1](image1.png)

**Figure 1.** The subsidence courses, which were observed after the end of next excavation stages in coal bed 338/2

On the basis of subsidence calculated in three neighboring points, it can be obtained a curvature of two segments situated between these points from the formula (2) [7]:

\[
C_{i,i+1,i+2}^{k+1} = 2 \cdot \frac{W_{i+2}^{k+1} - 2W_{i+1}^{k+1} + W_i^{k+1}}{I_{i,i+1} + I_{i+1,i+2}},
\]  

(2)

where: \(C\) – measured curvature; \(W\) – measured subsidence; \(i\) – number of measuring point; \(k\) – number of measuring cycle; \(I\) – measured length of segment.

The measured courses of curvatures were presented at the Figure 2.

![Figure 2](image2.png)

**Figure 2.** The curvatures courses, which were observed after the end of next excavation stages in coal bed 338/2
3. The average courses of measured curvatures
This paragraph presents the obtainment way of average values of measured curvatures using of smoothed spline functions to that and the basic assumptions of smooth.spline function from the R programme.

3.1. Calculation of average values of measured curvatures
The average values of curvatures observed after the ending of exploitation in next longwalls were calculated based on the formula (3) [8], [9]:

$$ C_{i,i+1,i+2}^{\text{aver}} = 2 \cdot \frac{W_{i+2}^{\text{approx}} - 2W_{i+1}^{\text{approx}} + W_i^{\text{approx}}}{l_{i,i+1} + l_{i,i+2}} , $$

where: $C_{i,i+1,i+2}^{\text{aver}}$ – average value of curvature; $W_i^{\text{approx}}$ – approximated value of subsidence; $i$ – number of point; $l$ – measured length of segment.

The average courses of measured curvatures calculated in reference to the Knothe’s (dots) and Bialek’s (squares) formulas were presented at the Figure 3.

![Figure 3. Observed curvatures average courses calculated regarding to their model values obtained on the basis of the Knothe’s and Bialek’s formulas](image)

To compare the measured curvatures with their average values, there were calculated:
- $r_C$ – correlation coefficient;
- $\sigma_C$ – standard deviation of curvatures;
- $P_C$ – percentages of the extreme measured values of curvatures which correspond the extreme average values of measured curvatures.

The values of variability coefficient of random dispersion of curvatures $M_C$ were calculated as a standard deviation of curvatures $\sigma_C$ divided by the module of average value of measured extreme curvature.

These values have been juxtaposed in the Table 3.
Table 3. The correlation between the measured and average values of curvatures

| Parameter   | Reference | After longwall 001 | After longwalls 001 and 002 | After longwalls 001, 002 and 005 | After longwalls 001, 002, 005 and 007 |
|-------------|-----------|--------------------|-----------------------------|---------------------------------|----------------------------------------|
|             |           | $r_c$              | $r_c$                       | $r_c$                           | $r_c$                                  |
| $\sigma_c$ [10^{-6} 1/m] |           | 9.33               | 20.45                       | 28.37                           | 23.88                                  |
| $M_c$ [%]   |           | 17.35              | 58.99                       | 138.40                          | 114.95                                 |
| $P_{C_{max}}$ [%] |           | 65.09              | 37.90                       | 22.51                           | 23.96                                  |
| $P_{C_{min}}$ [%] |           | 57.73              | 36.71                       | 27.03                           | 28.25                                  |

3.2. Approximation of average values of measured subsidence
The average courses of observed subsidence were calculated on the way of least – square approximation, which was done by the use of smoothed splines. The R computer programme was used for that with the smooth.spline function.

The basic parameters of this function are:

- $df$ – the number of freedom degree, values from the interval $<0, n>$, where: $n$ – the number of observational points;
- $spar$ – an approximating function smoothing parameter, values from the interval $<0, 1>$, where: $0$ – a total lack of smoothing (approximating function takes the form of approximated function);
- $\lambda$ – a penalized criterion ($PC$);
- $cv$ – the cross validation, when: $cv = truth$ it occurs the leave-one-out validation, if $cv = false$, then occurs the general cross validation ($GCV$);
- $nknots$ – the number of knots, if $all.knots = false$, then the value is smaller than the number of observational points.

The least – square approximation was done regarding to the subsidence model values. These values were obtained by the use of the EDN – OPN computer programme and with an application of the Knothe’s formula [10] with the typical values of its parameters ($a = 0.8$; $tg\beta = 2.0$; $A_{obr} = 0$) and the Bialek’s formula with the typical values of its parameters ($a = 0.8$; $tg\beta = 2.3$; $A_{obr} = 0.15$). The detailed information about the reasons of such procedure were presented in the works [11], [12].
The parameters values of subsidence approximation measured after termination of the subsequent excavation stages have been juxtaposed in the Table 4.

| Parameter | Reference of approximation | After longwall 001 | After longwalls 001 and 002 | After longwalls 001, 002 and 005 | After longwalls 001, 002, 005 and 007 |
|-----------|----------------------------|-------------------|-----------------------------|---------------------------------|--------------------------------------|
|           | the Knothe’s formula       | the Bialek’s formula | | | |
| $df$      | 11.88                      | 23.07             | 7.24                        | 10.54                           | 8.23                                 |
| $spar$    | 0.47                       | 0.32              | 0.63                        | 0.55                            | 0.66                                 |
| $\lambda$| $3.67 \times 10^{-3}$      | $2.02 \times 10^{-6}$ | $3.90 \times 10^{-4}$      | $7.73 \times 10^{-5}$          | $4.39 \times 10^{-4}$               |
| $PC$ [mm²] | 5841                      | 225               | 135434                      | 35184                           | 254332                               |

The average courses of observed subsidence, determined regarding to their model values calculated by the use of the Knothe’s (dots) and Bialek’s (squares) formulas with the parameters standard values, were presented at the Figure 4.

Figure 4. The average courses of subsidence observed after the end of next excavation stages in coal bed 338/2
Dependence between the observed and average values of subsidence has been presented by: the correlation coefficient \( r_w \), the subsidence standard deviation \( \sigma_w \) and the percentage of the extreme observed value of subsidence which corresponds the extreme average value of observed subsidence \( P_w \). That was also calculated the variability coefficient of subsidence random dispersion \( M_w \). It was obtained as the subsidence standard deviation divided by the module of the extreme average value of observed subsidence.

| Parameter | After longwall 001 | After longwalls 001 and 002 | After longwalls 001, 002 and 005 | After longwalls 001, 002, 005 and 007 |
|-----------|-------------------|-----------------------------|---------------------------------|-----------------------------------|
| \( r_w \) | 0.9978            | 0.9935                      | 0.9923                          | 0.9944                           |
| \( \sigma_w \) [mm] | 13.11              | 58.93                       | 73.56                           | 64.83                            |
| \( M_w \) [%] | 2.42               | 4.64                        | 4.92                            | 4.12                             |
| \( P_w \) [%] | 94.25              | 96.48                       | 98.47                           | 100.28                           |
|             |                   |                             |                                 |                                   |

| Parameter | After longwall 001 | After longwalls 001 and 002 | After longwalls 001, 002 and 005 | After longwalls 001, 002, 005 and 007 |
|-----------|-------------------|-----------------------------|---------------------------------|-----------------------------------|
| \( r_w \) | 0.9978            | 0.9935                      | 0.9923                          | 0.9944                           |
| \( \sigma_w \) [mm] | 13.11              | 58.93                       | 73.56                           | 64.83                            |
| \( M_w \) [%] | 2.42               | 4.64                        | 4.92                            | 4.12                             |
| \( P_w \) [%] | 94.25              | 96.48                       | 98.47                           | 100.28                           |
|             |                   |                             |                                 |                                   |

### 4. Discussion of the results

The data, which were presented in the Table 4, show that the smoothing parameter values \( spar \) of the function averaging the subsidence observed courses increase together with the growth of excavation range: from 0.47 after the termination of extraction in the one longwall to 0.68 after the ending of exploitation in the three longwalls (in case of approximation done regarding to the Knothe’s formula), and from 0.32 after the first period of excavation to 0.61 after the third extraction period (in case of approximation done regarding to the Bialek’s formula). It can indicate that the approximating function reproduces the empirical data in a lesser degree when the excavation escalates. This thesis is confirmed by the penalized criterion values (a residual sum of squares). They also increase with the increasing of extraction period number.

The data shown in the Table 5 indicate that the correlation coefficient between the observed and average values of subsidence \( r_w \) fall down with the growth of excavation range. This coefficient has reached the maximal values after the first exploitation period (0.9978 in case of the Knothe’s formula and 0.9999 for the Bialek’s formula). The smallest values have been reached after the third excavation stage (0.9923 for the Knothe’s formula and 0.9967 in case of the Bialek’s formula). As can be seen its values are high and don’t fall down below 0.99.

The excavation progress in the time causes that the values of standard deviation of subsidence \( \sigma_w \) grow together with the amount of exploited longwalls. Its values show that it is almost two times smaller in case of the Bialek’s formula than for the Knothe’s formula.
Regarding to the values of subsidence correlation coefficient $r_W$ and subsidence standard deviation $\sigma_W$ it can be seen, that the precision of mapping of observed courses of subsidence by their average courses decreases with the extraction range growth. Influence for it has got a random factor. Its participation in the subsidence observed values increases with the exploitation progress. This statement is confirmed by the growing values of variability coefficient of random dispersion of subsidence $M_W$. The values of $M_W$, as in case of $\sigma_W$, are two times smaller for the Bialek’s formula than for the Knothe’s formula.

As can be seen, the percentages of maximal values of observed subsidence which correspond the maximal values of average subsidence $P_W$ are greater for more developed excavation.

The correlation coefficients between the measured courses of curvatures and their average courses $r_C$, juxtaposed in the Table 3, decrease after each exploitation stage (as in case of subsidence) and the decrease of these values is considerable. Their values amount from 0.9498 to 0.5125 (for the Knothe’s formula) and from 0.9809 to 0.5962 (for the Bialek’s formula). As can be seen, these values are definitely lower than the values of correlation coefficient of subsidence $r_W$.

The conventional deviation of curvatures $\sigma_C$ increases with the exploitation development, from $9.33 \times 10^{-6}$ 1/m after the first longwall to $28.37 \times 10^{-6}$ 1/m after the three longwalls (the Knothe’s formula) and from $4.85 \times 10^{-6}$ 1/m after the first longwall to $26.62 \times 10^{-6}$ 1/m after the three longwalls (the Bialek’s formula). Notwithstanding, the differences between the values of standard deviations calculated for the curvatures in case of the Knothe’s and Bialek’s formulas aren’t so visible like for the values of standard deviations calculated for the subsidence.

The values of variability coefficient of curvatures random dispersion $M_C$ are higher for the Knothe’s formula than for the Bialek’s formula. They are much bigger than the values occurring in the literature [13]. The maximal value 138.4 % after the third excavation stage is four times bigger than 33.5 % identified by A. Kowalski.

In case of curvatures the $P_C$ values are very small and go down when an exploitation is developing. It means that the extreme average values of measured curvatures are much smaller than extreme values of observed curvatures.

5. Conclusions
It’s possible to obtain the average courses of mining area curvatures by the use of smoothed splines functions in case, when hard coal excavation was conducted at the medium depth, but taking into account below restrictions.

An approximating function (here a smoothed spline) should be parameterized by a shape parameter (here $spar$). This parameter has got an influence on fitting accuracy of approximating function to the empirical data.

The obtaining method of average courses of measured curvatures, which was presented above, requires to do an approximation of average courses of measured subsidence first. This method allows also to obtain the average courses of measured inclinations [11], [14].

In proposed manner of obtaining of average courses of measured curvatures should be use an average – square approximation. This approximation kind eliminates a random factor from the measured values of curvatures in a good degree.

The minimizing of squares residual sum should be referred to the model courses. Thanks to that, it’s possible to obtain the average measured courses similar to the model courses. It’s especially important in case of curvatures.

Projection accuracy of the measured courses by the average courses is dependent on an adopted model of deformation forecasting [7], [10], [15], [16].

Approximation reference to the Bialek’s formula gives the better results than the Knothe’s formula. It follows from that the Bialek’s formula is a three – parametric model. It takes into account an excavation rim, which narrows an exploitation field [17]. For this purpose has been defined the $A_{obj}$ parameter. It should be emphasized that the Knothe’s formula is a two – parametric model [18], [19].
The values of standard deviations of subsidence and curvatures increase with the development of exploitation in the time.

The values of variability coefficient of curvatures random dispersion \( M_c \) significantly exceed its values occurring in the literature, even four times more. It’s related to a high random factor which occurs in the measured values of curvatures. An average – square approximation isn’t able to eliminate this factor. It’s also caused by the typical values of models parameters adopted to the calculation of subsidence model courses.

The results improvement can be obtained by the use of parameters values, which have been designated directly from the geodesic measurements results [20].

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