Automated design tools for piezoelectric transformer-based power supplies

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Abstract: This paper proposed a design approach for piezoelectric-transformer-based power supplies. Initially, an analytical approach is used to derive the Mason equivalent circuit parameters of a PT. This is coupled with a semi-automated procedure to decide the appropriate physical dimensions of the PT. The procedure is applied to a typical low-power power supply specification, whereupon a specific transformer is design, verified using COMSOL and used as a simulated converter in SPICE. Results show that the design can be successfully implemented while maintaining efficient zero-voltage switching operation.

1 Introduction

Piezoelectric transformers (PTs) can be used in resonant power converters to replace many of the passive components. With appropriate topology decisions and careful design, it is possible to eliminate the magnetic components; the only power capacitors required are for DC links and smoothing. PTs transform the electrical resonant properties with mechanical resonance and use the piezoelectric effect to move energy efficiently between the electrical and mechanical domains. The resulting transformers have a number of useful features which are valuable in a number of applications: they are highly efficient, small, and exhibit exceptionally low EMI and high EMC. Ongoing materials research also offers new avenues to exploit PT-based power converters, where the PT can be designed for a particular application [1, 2].

However, despite their many advantages, the complexity and multidisciplinary nature of the design process means that PT power converters have only limited market penetration and only a small number of researchers are currently active in the area. To rectify the situation, a framework for the design of PT power converters is required to reduce barriers to adoption, particularly among non-experts, by abstracting the materials and mechanical design from electrical circuit. Here, we apply a first-order PT design process to an electrical specification and compare the results to finite element simulation of the mechanical aspects in COMSOL and the electrical aspects in SPICE.

2 Radial mode piezoelectric transformers

This paper considers radial mode PTs, although with suitable adjustments to the underlying physics, the technique can be applied to a range of PT topologies. Radial mode PTs are formed by attaching piezoelectric discs (typically made from PZT) in the form shown in Fig. 1.

A PT (radial or otherwise) can be modelled electrically as a specific resonant circuit called a Mason equivalent circuit. It is shown, together with a typical half-bridge driver and resistive load, in Fig. 2. The initial objective is to take a set of physical parameters from a specific design of a PT and calculate the various parameters of the equivalent circuit analytically.

2.1 Input and output capacitance

Ultimately, these capacitors are simple parallel plate capacitances between the electrodes and are, therefore, calculated from

\[ C = \frac{\varepsilon_0 A}{t} \]  

where \( A = 2\pi r \) is the cross-sectional area of the disc, radius \( r \); \( \varepsilon_33 \) is the permittivity in the vertical direction and \( t \) is the thickness of the layer. If there are multiple layers, \( n \), then these layers are arranged electrically in parallel and, therefore, the total capacitance is the sum of all layers. As all input layers are the same thickness, \( t_{in} \) and radius, and all output layers are the same thickness, \( t_{out} \), and radius, the input and output capacitances can be written

\[ C_{in} = \frac{\pi r^2 n_{in} \varepsilon_{33}}{t_{in}} \]  
\[ C_{out} = \frac{\pi r^2 n_{out} \varepsilon_{33}}{t_{out}} \]

Adjustments are sometimes required to account for electrodes that are not identical in size to the discs. These are neglected in this analysis.

2.2 Turns ratio

The turns ratio is a property of the direct piezoelectric and converse piezoelectric effect. It can be shown [3] that the turns ratio for a PT may be approximated by the ratio of input to output layers, i.e.

\[ N = \frac{n_{in}}{n_{out}} \]

2.3 Resonant tank

The resonant tank is ultimately a model of the mechanical resonance of the piezoelectric material. The resonant frequency can be calculated from the eigen frequency of the whole PT. Radial mode resonances can be found at the first solution to

\[ \frac{J_1(R)}{J_1(R)} = 1 - \frac{\rho}{\rho_{th}} \]

where

\[ \rho_{th} = \frac{R}{T} \sqrt{\frac{1}{\sigma}} \]
where \( \sigma \) is the Poisson’s ratio for the material and \( J_0 \) and \( J_1 \) are the zero- and first-order Bessel functions of the first kind, respectively. \( Y \) is the Young’s modulus of the material and \( \rho \) is its density. The particular value of inductance relates to the mass of the material as

\[
L_i = \frac{m R^2 + \sigma^2 - 1}{2 \sigma^2 d_0 n_0} \times \frac{s_1 \sigma}{2\pi r d \sigma n_0}
\]

(7)

where \( m \) is the mass of the PT, \( d \) is the direct piezoelectric constant matrix, and \( s \) is the mechanical compliance matrix. \( C_1 \) and \( R_1 \) can be calculated with reference to the quality factor and standard resonant circuit equations

\[
C_1 = \frac{1}{a_0 L_1}
\]

(8)

\[
R_1 = \frac{L_1}{Q C}
\]

(9)

The quality factor, \( Q \), is a material and construction property related to the losses in the vibration. In practice, it is determined empirically.

3 Design process

Using (2)–(9), we are able to parameterise a PT based on materials and construction. Using this process, we have developed a procedure to design a PT. Fig. 3 shows the design process.

Starting with an intended specification, a designer selects the turns ratio and maximum number of layers (which relates to budget and manufacturing capability). The thickness ratio and total thickness must then be selected. In our previous research [4], we have discovered a critical design criterion which limits the range of acceptable capacitances. Since, from (1), capacitance and thickness are related, the thickness decisions must be checked against this criterion. Finally, a radius and material is selected and a fully equivalent circuit generated. Each decision in this process can have impact on the final design, but allows the designer to parameterise the equivalent circuit.

The design process is iterative – having developed an initial design, the electrical properties must be checked against the needs of the electrical system and design modifications made as appropriate.

3.1 Design of a 100 V DC to 12 V AC converter

To demonstrate the design process, we followed through the design of a 1 W converter, taking a 100 V DC link supply to a 12 V pk AC supply. This implies that a 72 \( \Omega \) resistor will be used as the load. The PT is to be built using PZT-5H, with physical constants given in [5].

Resonant circuits need dead time between their high and low driving periods. In PTs, the large input capacitor means that a significant dead time is required to charge the input to the DC link voltage. In previous work, we have shown that with a dead time of 25% of the period, the largest chance of achieving ZVS is attained [4]. This means the input voltage to the PT is approximately trapezoidal, and the first harmonic has an amplitude of 0.6 of the DC-link voltage (i.e. 60 V in this case). This suggests an ideal turn ratio of \( N_{\text{ideal}} = 12/60 = 0.2 \). To allow overheads to account for losses and off-resonant operation, a turns ratio of \( N = 0.25 \), meaning layers \( n = 1 \) and \( n_{\text{out}} = 4 \).

To ensure compliance with the critical design criterion in [4], the thicknesses need to be adjusted such that

\[
\frac{C_{\text{in}}}{N C_{\text{out}}} \leq \frac{2}{N}
\]

(10)

\[
\frac{t_{\text{out}}}{t_{\text{in}}} \leq \frac{2N}{\pi}
\]

(11)

This gives a maximum ratio of 0.16. Choosing a total thickness of 4 mm for ease of manufacture, therefore, sets \( t_{\text{in}} = 2.5 \) mm and \( t_{\text{out}} = 0.38 \) mm per layer.

Finally, a radius and material is required. We use PZT-5H as it is a hard material (less displacement for a given stress, reducing losses). To achieve a resonant frequency around the 100 kHz mark (which leads to reasonably sized PTs and electrically appropriate circuits), we select \( r = 7 \) mm. That gives a resonant frequency of 142.2 kHz.

The full-circuit parameters predicted from (1)–(9) are \( R_1 = 6.2 \) \( \Omega \), \( C_1 = 360 \) pF, \( L_1 = 3.5 \) mH, \( C_{\text{in}} = 1.8 \) nF, \( C_{\text{out}} = 49 \) nF and \( N = 0.25 \).

3.2 COMSOL simulation

A 2D axisymmetric COMSOL simulation was performed using the physical parameters derived earlier. The vibration mode of the transformer can be seen in Fig. 4. The lighter shading indicates increased vibration velocity towards the edge of the transformer. The dark centre and radial symmetry indicate that this transformer is operating in radial mode. Equivalent circuit parameters were derived by taking the impedance at each side of the transformer with the other side short-circuited. For example, the input impedance was measured as shown in Fig. 5.
From this data, the equivalent circuit values (excluding $C_{\text{out}}$ and $N$) were calculated through curve fitting the equivalent circuit and noting that the minimum occurs at ($\omega_0$, $R_1$). A similar exercise was carried out for the output impedance, which gives $C_{\text{out}}$ and the resonant tank admittance scaled by $N^2$. Since the tank admittance must be the same at both sides, $N$ can be calculated.

Performing this operation on the PT model yields the following circuit parameters. In Table 1, they are compared with the values expected from (1) to (9).

The resonant tank values are not significantly different and result in a resonant frequency of 141 and 132 kHz for the equations and COMSOL model, respectively. The differences in the particular values of $C_1$ and $L_1$ relate to the energy stored in the PT under stress and are likely due to imperfections in radial oscillation. The value of $R_1$ relates to losses and is likely to be an underestimation in both cases as mounting damping is not modelled.

The marked differences in $C_{\text{in}}$ and $C_{\text{out}}$ are less easily explained. The referred capacitor ratio, $C_{\text{in}}$ ($N^2C_{\text{out}}$), is similar at 0.62 and 0.57 for the equations and model, respectively, both fulfilling the critical criterion. The reason for the difference in absolute value warrants further investigation outside the scope of this paper.

### 3.3 SPICE simulation

A SPICE simulation was performed using the arrangement indicated in Fig. 1 and the COMSOL-derived PT characteristics shown in Table 1. The analysed schematic is shown in Fig. 6. The frequency was adjusted to give the correct output voltage, resulting in switching frequency of 145 kHz and a dead time of 25% of the period. Fig. 7 shows the input and output voltages. The results show that ZVS has been achieved: during the dead time, the resonant current charges or discharges the input capacitor to the high or low DC voltage, as appropriate.

| $C_{\text{in}}$, nF | 1.8 | 0.63 |
| $R_1$, Ω    | 6.2 | 2.2  |
| $L_1$, mH    | 3.5 | 6.5  |
| $C_1$, pF    | 360 | 222  |
| $N$          | 0.25 | 0.35 |
| $C_{\text{out}}$, nF | 49  | 9.54 |

**Table 1** Comparison of equivalent circuit parameters

**Design choices**

**Proposed design procedure**

**COMSOL simulation of designed PT showing layered structure. Lighter shading indicates higher vibration velocity.**

**Impedance plot from COMSOL.**
4 Discussion and conclusion

This paper has introduced a first-order approach to a semi-automated design process for piezoelectric-transformer power supplies. The process and its future expansion will reduce barriers to PT adoption by abstracting complex materials science from the electrical design, thereby allowing non-expert power electronics engineers to use PTs with confidence. A comparison of the analytical design approach with a COMSOL model showed good agreement, and a SPICE implementation using the Mason equivalent circuit demonstrated that a standard low power specification can be met.

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6 References

[1] Woodward, D.I., Reaney, I.M., Eitel, R.E., et al. ‘Crystal and domain structure of the BiFeO$_3$-PbTiO$_3$ solid solution’, J. Appl. Phys., 2003, 94, (5), pp. 3313–3318
[2] Woodward, D.I., Knudsen, J., Reaney, I.M.: ‘Review of crystal and domain structures in the PbZr$_x$Ti$_{1-x}$O$_3$ solid solution’, Phys. Rev. B, 2005, 72, (10), 104110
[3] Horsley, E.L.: ‘Modelling and analysis of radial mode piezoelectric transformers and inductor-less resonant power converters’. PhD Thesis, The University of Sheffield, Sheffield, 2011
[4] Foster, M.P., Davidson, J.N., Horsley, E.L., et al. ‘Critical design criterion for achieving zero voltage switching in inductorless half-bridge-driven piezoelectric-transformer-based power supplies’, IEEE Trans. Power Electron., 2016, 31, (7), pp. 5057–5066
[5] Erturk, A., Imran, D.J.: ‘Piezoelectric energy harvesting’ (Wiley, Chichester, 2011), Appendix E