Distributed Quantum Vote Based on Quantum Logical Operators, a New Battlefield of the Second Quantum Revolution

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Abstract. We designed two rules of binary quantum computed vote: Quantum Logical Veto (QLV) and Quantum Logical Nomination (QLN). The conjunction and disjunction from quantum computational logic are used to define QLV and QLN, respectively. Compared to classical vote, quantum computed vote is fairer, more democratic and has stronger expressive power. Since the advantage of quantum computed vote is neither the speed of computing nor the security of communication, we believe it opens a new battlefield in the second quantum revolution. Compared to other rules of quantum computed vote, QLV and QLN have better scalability. Both QLV and QLN can be implemented by the current technology and the difficulty of implementation does not grow with the increase of the number of voters.

Keywords: vote · quantum logic · quantum information

1 Introduction

Electronic vote, or e-vote, is a voting process in which ballot casting and counting is computer-aided. Since late 1990s and early 2000s, e-vote has received increasing interest and is widely applied to various situations of decision-making. Many voting protocols based on classical cryptography have been developed and successfully applied in the last two decades [17,3]. However, the security of protocols based on classical cryptography is based on the unproven complexity of some computational problems, such as the factoring of large numbers. The research in quantum computation shows that quantum computers are able to factor large numbers in a short time, which means that classical protocols based on such algorithms are insecure. To react to the risk posed by forthcoming quantum computers, a number of quantum voting protocols have been
developed in the last decade [7,24,12,9,13,10,23,26,18,22,21,14]. In these works, the ballots are still classical but they are secured by quantum methods. We call this type of vote the quantum secured vote.

While all quantum secured voting protocols have focused on the security problems of voting from the cryptographic perspective, Bao and Halpern [2] and Sun et al. [19] studied quantum vote from a social choice theoretic perspective. They designed voting rules in which ballots are in quantum states and the result of voting is calculated by using quantum operators. We call this type of vote quantum computed vote. An interesting advantage of quantum computed vote is that the quantum analogue of Arrow’s Impossibility Theorem [1] is violated in quantum computed vote. Arrow’s Impossibility Theorem is one of the most influential results in social choice theory. According to the theorem, every voting rule satisfying unanimity and independence of irrelevant alternatives must also satisfy dictatorship, which implies that a fair and democratic voting rule cannot exist. The work of Bao and Halpern [2] and Sun et al. [19] disproved Arrow’s theorem in the quantum setting. Therefore, it provides a theoretic demonstration of the advantage of quantum computed vote: quantum vote is better than classical vote in the sense that it enable the existence of fair and democratic voting rules.

In addition to fairness and democracy, another advantage of quantum computed vote is that it has better expressive power than classical vote. Classical ballots can only be in a definite state like 0 or 1, while quantum ballots can be in a superposition of |0⟩ and |1⟩ or mixed state of |0⟩⟨0| and |1⟩⟨1|. In real life, the voters usually have a mixed preference on the proposal to be voted, such as "60% agree and 40% disagree". When voters are only allowed to use 0 or 1 to express their preference, it can happen that the result of voting does not truly reveal the aggregation of the preferences of voters. For example, suppose the preference of Alice, Bob and Charlie on a proposal is 0.6 (which means that "60% agree and 40% disagree", or Alice will vote for "agree" with probability 0.6), 0.6 and 0 respectively. Then they will cast their ballots into classical state "agree", "agree" and "disagree" and the result of voting is "agree" according to classical majority vote. But intuitively, the overall probability of agree, which is the probability that the majority of the voters vote for "agree", should be $0.6 \times 0.6 = 0.36$. This is because Charlie will never vote for "agree". In order for the majority of the voters to vote for "agree", both Alice and Bob must vote for "agree", which happens with probability $0.6 \times 0.6 = 0.36$. Therefore, the result of classical vote does not truly reveal the aggregation of the preferences of voters. This inconsistency is caused by the limited expressive power of classical ballot. On the other hand, the mixed preference like "60% agree and 40% disagree" can be described by the quantum state $\sqrt{0.6}|1\rangle + \sqrt{0.4}|0\rangle$ or $0.4|0\rangle \langle 0| + 0.6|1\rangle \langle 1|$. Moreover, quantum ballot can also express entangled preference of voters. For example, Alice and Bob together can cast their ballots into state $|00\rangle + |11\rangle$, which has no analogue in classical voting. Since the main advantages of quantum computed vote is neither the speed of computing nor the security of communication. We believe quantum computed vote opens a new battlefield in the second quantum revolution.

From a practical perspective, the quantum voting rules proposed in Bao and Halpern [2] and Sun et al. [19] to disprove Arrow’s Impossibility Theorem are too complicated to be realized with the current technology. The number of qubits that are needed to be manipulated in the voting rules is exponential in the growth of the number of voters.
Therefore, simpler voting rules are needed. In this paper, we propose two quantum voting rules that have better scalability: quantum logical veto (QLV) and quantum logical nomination (QLN). The number of qubits needed in the two rules is of constant number 3. The only quantum operation used in QLV/QLN is quantum logical conjunction/disjunction. Both of these operators are relatively simple and have been studied in-depth in the literature of quantum computational logic [6,4,3,11]. Moreover, various voting rules can be constructed by the combination of QLV and QLN without loss of scalability.

The structure of this paper is the following. We introduce elements of background knowledge in Section 2. Then in Section 3 we introduce our voting rules in detail. We conclude this paper with future work plan in Section 4.

## 2 Preliminaries

Given a Hilbert space $\mathcal{H} = \mathbb{C}^2$, we denote the set of all density operators on $\mathcal{H}$ by $D(\mathcal{H})$. Our quantum voting rules use two quantum logical operators, quantum AND and quantum OR, for ballot aggregation. The construction of the quantum AND is based on the quantum Toffoli gate [8].

**Definition 1 (Quantum Toffoli gate)** The quantum Toffoli gate is a unitary operator on $\mathbb{C}^2^3$:

$$T |x_1, x_2, x_3 \rangle = |x_1, x_2, x_1 \oplus x_2 \oplus x_3 \rangle$$

where $x_i \in \{0, 1\}$.

**Definition 2 (Quantum AND operator)** For $\rho, \sigma \in D(\mathbb{C}^2)$,

$$AND(\rho \otimes \sigma) = \text{Tr}^{1,2}(T(\rho \otimes \sigma \otimes |0\rangle \langle 0|)T^\dagger),$$

Here $\text{Tr}^{1,2}$ is the partial trace on the first and the second qubit.

The quantum AND operator is naturally generalized to multiple qubits: $\text{AND}(\rho_1 \otimes \rho_2 \ldots \otimes \rho_n) := \text{AND}(\ldots \text{AND}(\text{AND}(\rho_1 \otimes \rho_2) \otimes \rho_3) \otimes \ldots \otimes \rho_n)$.

Just like in quantum computational logic [6,4,3,11], we define the quantum NOT operator by using the Pauli X gate.

**Definition 3 (Quantum NOT operator)** For $\rho \in D(\mathbb{C}^2)$, $\text{NOT}(\rho) = X\rho X^\dagger$, where $X$ is the Pauli X operator on a single qubit: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Now we define the quantum OR operator based on quantum AND and quantum NOT.

**Definition 4 (Quantum OR operator)** For $\rho, \sigma \in D(\mathbb{C}^2)$,

$$\text{OR}(\rho \otimes \sigma) = \text{NOT}(\text{AND}(\text{NOT}(\rho) \otimes \text{NOT}(\sigma)))$$

The quantum OR operator is naturally generalized to multiple qubits: $\text{OR}(\rho_1 \otimes \rho_2 \ldots \otimes \rho_n) := \text{OR}(\ldots \text{OR}(\text{OR}(\rho_1 \otimes \rho_2) \otimes \rho_3) \otimes \ldots \otimes \rho_n)$.  


3 Quantum voting rules

We design two quantum voting rules: QLV and QLN. In both of them, we assume there is 1 proposal to be voted, \(m\) voters \(\{v_1, \ldots, v_m\}\) and \(n\) quantum voting machines \(\{M_1, \ldots, M_n\}\). Every voter’s ballot is represented by a density operator of a single qubit. The ballot in state \(|0\rangle\langle 0|\) represents “disagree” and in state \(|1\rangle\langle 1|\) represents “agree”. Every quantum voting machine is a small-scale quantum information processor. In QLV (resp. QLN), we assume that the quantum voting machine is able to execute the quantum AND (resp. OR) operator.

3.1 Quantum veto

The one-vote veto is a special type of vote, in which the proposal to be voted will be disagreed as long there is one voter who votes for “disagree”. It has been widely used by many political and economic organizations, among which the most famous is the UN Security Council’s permanent member states group.

There are some work on quantum-secured veto [27,25,15], in which the ballots are still classical, but they are encrypted by some methods of quantum cryptography in order to ensure some security properties. To the best of our knowledge, quantum veto in which ballots are in quantum state has never been studied before.

Our QLV is performed in the following steps:

1. Every voter \(v_i\) sends her/his ballot \(\rho_i \in D(\mathbb{C}^2)\) to every quantum voting machine.
2. Every quantum voting machine \(M_j\) calculate \(\text{AND}(\rho_1 \otimes \cdots \otimes \rho_m) = \rho_j\).
3. Every quantum voting machine \(M_j\) measures \(\rho_j\) by the projector \(P_1 = |1\rangle\langle 1|\). It records 1 if the result of measurement is “yes”. It records 0 if the result of measurement is “no”.
4. Every quantum voting machine sends its record to every other quantum voting machine.
5. Every quantum voting machine reads all the records it has received and outputs “Agree” if at least half of the records is 1, otherwise it outputs “Disagree”.

In order to show that QLV indeed satisfies some desirable properties of veto-like voting, we first define the winning probability of a ballot as follows.

**Definition 5 (winning probability)** For \(\rho \in D(\mathbb{C}^2)\), the winning probability of \(\rho\) is \(\text{WP}(\rho) := \text{Tr}(P_1 \rho)\).

**Lemma 1** For all \(\rho, \sigma \in D(\mathbb{C}^2)\), \(\text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_1^3(T(\rho \otimes \sigma \otimes |0\rangle\langle 0|)T^\dagger) = \text{Tr}(P_1 \rho) \cdot \text{Tr}(P_1 \sigma)\).

**Proof.** We first consider cases where \(\rho, \sigma\) ranges over the computational basis. Then we have the following cases:

1. \(\text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_1^3(T(|0\rangle\langle 0| \otimes |0\rangle\langle 0| \otimes |0\rangle\langle 0|)T^\dagger) = \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_1^3(|0\rangle\langle 0| \otimes |0\rangle\langle 0| \otimes |0\rangle\langle 0|) = \text{Tr}(|0\rangle\langle 0|) \cdot \text{Tr}(|0\rangle\langle 0|) \cdot \text{Tr} P_1(|0\rangle\langle 0|) = 1 \cdot 1 \cdot 0 = 0 = \text{Tr}(P_1 |0\rangle\langle 0|) \cdot \text{Tr}(P_1 |0\rangle\langle 0|) \).
2. \( \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_3^3(T([0] \otimes [0] \otimes [0])|0\rangle\langle 0|)T^\dagger) = \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_3^3([0] \otimes [0] \otimes [0] |0\rangle\langle 0|) \).

3. \( \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_3^3(T([1] \otimes [1] \otimes [0])|0\rangle\langle 0|)T^\dagger) = \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_3^3([1] \otimes [1] \otimes [0] |0\rangle\langle 0|) \).

4. \( \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_3^3(T([1] \otimes [1] \otimes [0])|0\rangle\langle 0|)T^\dagger) = \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_3^3([1] \otimes [1] \otimes [0] |0\rangle\langle 0|) \).

We further note that for all \( a, b \in \{0, 1\} \) with \( a \neq b \), it holds that \( \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_3^3(T([a] \otimes [b] |\sigma \otimes [0] \langle 0|)T^\dagger) = 0 \). This plus the fact that \( T \) is a linear operator implies that \( \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3 P_3^3(T(\rho \otimes \sigma \otimes [0] \langle 0|)T^\dagger) = \text{Tr}(P_1 \rho) \cdot \text{Tr}(P_1 \sigma) \) for all \( \rho, \sigma \in D(C^2) \).

Lemma 2 For \( \rho, \sigma \in D(C^2) \), \( \text{WP}(AND(\rho \otimes \sigma)) = \text{WP}(\rho) \cdot \text{WP}(\sigma) \).

Proof. \( \text{WP}(AND(\rho \otimes \sigma)) = \text{Tr}(P_1 AND(\rho \otimes \sigma)) = \text{Tr}(P_1 \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3(T(\rho \otimes \sigma \otimes [0] \langle 0|)T^\dagger) = \text{Tr}(P_1 \text{Tr}^1 \otimes \text{Tr}^2 \otimes \text{Tr}^3(T(\rho \otimes \sigma \otimes [0] \langle 0|)T^\dagger) = \text{Tr}(P_1 \rho) \cdot \text{Tr}(P_1 \sigma) = \text{WP}(\rho) \cdot \text{WP}(\sigma) \).

The following theorem states that QLV is indeed a veto-like voting rule.

Theorem 1 For every quantum voting machine, it records 0 with probability 1 iff at least one voter’s ballot is in state \( |0\rangle\langle 0| \).

Proof. A quantum voting machine records 0 with probability 1 iff it records 1 with probability 0 iff \( \text{WP}(AND(p_1 \otimes \cdots \otimes p_m)) = 0 \) iff \( \text{WP}(\rho_1) \cdot \cdots \cdot \text{WP}(\rho_m) = 0 \) iff \( \text{WP}(\rho_i) = 0 \) for some voter \( v_i \).

Remark 1 After Step 4 in QLV, every quantum voting machine gets the records of all quantum voting machines. Those records is a collection of 0s and 1s. The collection is a probability distribution on \( \{0, 1\} \) according to the winning probability of \( \text{AND}(p_1 \otimes \cdots \otimes p_m) \). Every quantum voting machine has the same collection.

3.2 Quantum nomination

The quantum nomination is dual to quantum veto, in which the proposal to be voted will be agreed as long as there is one voter who votes for “agree”. Intuitively, this type of voting can be understood as that a candidate is nominated as long as there is one voter who nominates her/him. Classical nomination has been widely used in many political and economic elections. Nomination-like vote has also been used in some TV programs. For example, in The Voice of China, which is a Chinese reality television singing competition, a contestant get elected in the blind audition phase as long as there is at least one coach who votes for her/him.

To the best of our knowledge, quantum nomination in which ballots are in quantum state has never been studied before. Our QLN operates in the following steps:

1. Every voter \( v_i \) sends her/his ballot \( \rho_i \) to every quantum voting machine.
2. Every quantum voting machine $M_f$ calculates $OR(\rho_1 \otimes \cdots \otimes \rho_m) = \rho^f$.
3. Every quantum voting machine measures $\rho^i$ using the projector $P_1 = |1\rangle\langle 1|$. It records 1 if the result of measurement is “yes”. It records 0 if the result of measurement is “no”.
4. Every quantum voting machine sends its record to every other voting machine.
5. Every quantum voting machine reads all the records it has received and outputs “Agree” if at least half of the records is 1. Otherwise it outputs “Disagree”.

The following lemmas and theorem demonstrate that quantum nomination is indeed a voting rule for nomination-like vote.

**Lemma 3** For $\rho \in D(\mathbb{C}^2)$, $WP(\text{NOT}(\rho)) = 1 - WP(\rho)$.

**Proof.** $WP(\text{NOT}(\rho)) = WP(X(\rho)X^\dagger) = WP(X(\rho)X) = \text{Tr}(P_1 X(\rho)X) = \text{Tr}(XP_1 X(\rho))$.

By simple calculation we have $P_0 := |0\rangle\langle 0| = XP_1 X$ and $P_0 + P_1 = I$. Therefore,

$\text{Tr}(XP_1 X(\rho)) = \text{Tr}(P_0(\rho)) = \text{Tr}((I - P_1)\rho) = \text{Tr}(\rho - P_1 \rho) = \text{Tr}(\rho) - \text{Tr}(P_1 \rho) = 1 - WP(\rho)$.

**Lemma 4** For $\rho, \sigma \in D(\mathbb{C}^2)$, $WP(\text{OR}(\rho \otimes \sigma)) = WP(\rho) + WP(\sigma) - WP(\rho) \cdot WP(\sigma)$.

**Proof.** $WP(\text{OR}(\rho \otimes \sigma)) = WP(\text{AND}(\text{NOT}(\rho) \otimes \text{NOT}(\sigma))) = 1 - WP(\text{AND}(\text{NOT}(\rho) \otimes \text{NOT}(\sigma))) = 1 - WP(\text{NOT}(\rho)) \cdot WP(\text{NOT}(\sigma)) = 1 - (1 - WP(\rho)) \cdot (1 - WP(\sigma)) = 1 - (1 - WP(\rho)) - WP(\sigma) + WP(\rho) \cdot WP(\sigma) = WP(\rho) + WP(\sigma) - WP(\rho) \cdot WP(\sigma) = WP(\rho) + WP(\sigma) - WP(\rho) \cdot WP(\sigma)$.

**Theorem 2** For every quantum voting machine, it records 1 with probability 1 iff at least one voter’s ballot is in state $|1\rangle\langle 1|$.

**Proof.** We only prove cases in which there are only two voters. The case with multiple voters can be generalized straightforwardly.

($\Leftarrow$) $WP(\text{OR}(|1\rangle\langle 1| \otimes \sigma)) = WP(|1\rangle\langle 1|) + WP(\sigma) - WP(|1\rangle\langle 1|) \cdot WP(\sigma) = 1 + WP(\sigma) - WP(\sigma) = 1$. The case in which $\sigma = |1\rangle\langle 1|$ is similar.

($\Rightarrow$) Assume $\sigma$ is not state $|1\rangle\langle 1|$ and $\rho$ is not state $|1\rangle\langle 1|$. Then $WP(\sigma) < 1$ and $WP(\rho) < 1$. Then $WP(\text{OR}(\rho \otimes \sigma)) = WP(\rho) + WP(\sigma) - WP(\rho) \cdot WP(\sigma) = WP(\rho)(1 - WP(\sigma)) + WP(\sigma) < 1 - WP(\sigma) + WP(\sigma) = 1$.

### 3.3 Extension and Application

In this subsection we use AND and OR to build other voting rules and study some mathematical properties of the quantum computed vote.

**Logical formulas and voting rules** The essential feature of QLV and QLN is determined by the logical operators they use. It turns out that different quantum voting rules can be defined by different combinations of logical operators. We illustrate some of them in the following examples.

**Example 1 (role-weighted vote)** Suppose $v_1$ is a professor of quantum information, $v_2$ and $v_3$ are two associate professors of quantum information. Then the following formula determines a voting rule based on the voters’ roles:
According to the above formula, as long as the professor votes for “agree”, the proposal will be agreed. Otherwise both of the two associate professors needs to vote for “agree” in order for the proposal to be agreed.

**Example 2 (majority vote)** Majority vote is probably the most popular voting rule in our society. The quantum majority vote for three voters can be determined by the following formula:

\[
OR((\text{AND}(\rho_1 \otimes \rho_2)) \otimes (\text{AND}(\rho_2 \otimes \rho_3)) \otimes (\text{AND}(\rho_1 \otimes \rho_3))).
\]

Indeed, if at least two voters vote for “agree”, then the proposal will be agreed according to the above formula. On the other hand, if the preference of the voters is 0.6, 0.6 and 0 respectively, then they can set \( \rho_1 = \rho_2 = 0.4|0\rangle\langle 0| + 0.6|1\rangle\langle 1| \) and \( \rho_3 = |0\rangle\langle 0| \). Then \( WP(OR((\text{AND}(\rho_1 \otimes \rho_2)) \otimes (\text{AND}(\rho_2 \otimes \rho_3)) \otimes (\text{AND}(\rho_1 \otimes \rho_3)))) = 0.36 \).

**Embedding probabilistic ballot into quantum ballot** Let \( r \in [0,1] \), \( |\theta_r\rangle = \sqrt{1-r}|0\rangle + \sqrt{r}|1\rangle \) and \( \Theta_r := |\theta_r\rangle\langle \theta_r| \). Then \( WP(\Theta_r) = r \) and we call the quantum ballot \( \Theta_r \) a canonical representation of the probabilistic ballot \( r \). Therefore, if a voter’s preference is \( r \), then he can set his ballot into the pure state \( \Theta_r \) to represent his preference. In this way all probabilistic ballot \( r \) can be represented by a quantum ballot \( \Theta_r \).

Nevertheless, not all quantum ballot can be represented by probabilistic ballot. For example, ballots in the entangled state \( \frac{|00\rangle + |11\rangle}{\sqrt{2}} \) cannot be represented by probabilistic ballots. We demonstrate this fact by the following observations.

**Observation 1** If two quantum ballots in state \( \frac{|00\rangle + |11\rangle}{\sqrt{2}} \) are submitted to QLV, then the proposal will be agreed with probability \( \frac{1}{2} \). The same probability will appear when they are submitted to QLN.

**Observation 2** There are no probabilistic ballot \( x, y \in [0,1] \) such that when they are submitted to veto/nomination, the proposal will be agreed with probability \( \frac{1}{2} \).

*Proof.* Suppose \( x, y \in [0,1] \) are two probabilistic ballots and they produce probability \( \frac{1}{2} \) in both veto and nomination. Then \( xy = \frac{1}{4} \) and \( x+y-xy = \frac{1}{2} \). Hence \( x+y = 1 \). Then we know \( x(1-x) = \frac{1}{2}, x^2 - x + \frac{1}{2} = 0 \). But there is no real \( x \) satisfies \( x^2 - x + \frac{1}{2} = 0 \).

In fact, two quantum ballots \( \rho_1, \rho_2 \) in states \( \rho_1 = \frac{1+i}{2}|0\rangle + \frac{1-i}{2}|1\rangle \) and \( \rho_2 = \frac{1-i}{2}|0\rangle + \frac{1+i}{2}|1\rangle \) also produce the same probability in QLV and QLN as the quantum ballots in state \( \frac{|00\rangle + |11\rangle}{\sqrt{2}} \). Therefore, there even exist non-entangled quantum ballots which cannot be represented by probabilistic ballots.
4 Conclusion and future work

We have designed two rules of binary quantum computed vote: QLV and QLN. In both of them ballots are cast into quantum states. The conjunction and disjunction from quantum computational logic are used to define quantum veto and quantum nomination, respectively. Compared to other rules of quantum computed vote, QLV and QLN have advantages in scalability. Both of them can be physically realized by the current technology and the difficulty of physical realization does not grow with the increase of the number of voters. They can also be combined to define other interesting and useful quantum voting rules without loss of scalability.

In the future, we will be interested in the physical realization of quantum veto and nomination. For example, an ion trap quantum computer is a good candidate because the realization of the quantum Toffoli gate with trapped ions has been successful since 2009 [16]. We also plan to study quantum veto and nomination in the situation where some quantum voting machines suffer from faulty behaviour such as crash failure or Byzantine failure. In these situations we will use quantum blockchain [20,21] as a platform to execute quantum veto and nomination.

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