Computer simulation of single burn transfers between low-Earth and halo orbits in the Sun-Earth system

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Abstract. Halo orbits of Sun-Earth system are utilized in space missions as they allow to maintain the spacecraft in an area that is stationary relative to Sun and Earth. The advantage of halo orbits is their periodicity and their form allowing the spacecraft to avoid the zones of solar interference and the Earth shadow. The transfer between a low-Earth orbit and a halo orbit around a libration point can be realized by a single-burn maneuver, which transfers the spacecraft to an orbit of stable manifold resulting in a halo orbit. An amplitude of halo orbit depends on the altitude of the parking low-Earth orbit at which the transfer maneuver is performed. This work is aimed to explore and systemize the single burn transfer options utilizing single and multiple Earth passing trajectories in the framework of the circular restricted three-body problem. The algorithms providing transfer options for the desired halo orbit and the parking orbit altitude are developed. The transfer trajectories for the Sun-Earth $L_1$ and $L_2$ halo orbits in a wide range of out-of-plane amplitudes were constructed and studied. The constructed trajectories were clustered based on the transfer time and the halo orbit amplitude.

1. Introduction
Halo orbits are the periodical solutions associated with the collinear libration points of circular restricted three body problem (CR3BP), which describes the motion of a massless point in the gravitational field produced by two massive bodies rotating around a common barycentre. The libration point orbits (LPOs) associate with the libration points $L_1$ and $L_2$ in the Sun-Earth system, provides various applications for space exploration [1]. As the dynamics of CR3BP provide a basis for the design of libration point missions, it has been studied extensively with semi-analytical [2–5] and numerical [6–10] methods. An important step of the libration point mission design is the selection of transfer options providing the way of reaching the desired LPO from a low-Earth orbit (LEO).

The natural dynamics of CR3BP provide a transfer option utilizing the stable manifold associated with LPO [11]. The advantage of this approach is that it doesn’t require any insertion maneuver to reach LPO, thus the transfer can be performed by a single burn performed at parking LEO. The drawback is that the LPO amplitude is limited since the stable manifolds of the small-amplitude orbits can not approach the Earth [12]. In particular, the halo orbit of 120000 km out-of-plane amplitude requires the transfer burn performed at about 3000 km
altitude [13]. At the same time, the small-amplitude LEOs are generally preferable because they provide smaller Sun–probe–Earth angle [14].

Recent contributions focused on transfers to small-amplitude LPOs explore the utilization of the Moons flybys patching the Earth-Moon transfers with stable manifolds of the target orbits [14–17]. A technique for an optimal transfer trajectory design for a halo orbit minimizing the halo insertion cost is ref. [13]. Srivastava et al (2018) [18] studied the transfer opportunities for the small (with the out-of-plane amplitude of 110 000 km) halo orbits around $L_1$ and $L_2$ Sun-Earth points in a complex force model involving solar radiation pressure. They showed that the desired halo orbit can be reached by single-burn transfer from the LEO with 185 km altitude by multiple Earth passes.

In the framework of this work, the single-burn transfer options from LEO to LPO were studied by computer simulations focusing on single and multiple Earth passing trajectories. The halo families for the Sun-Earth $L_1$ and $L_2$ points were calculated utilizing the bisection technique described in [10]. To achieve the transfer trajectory a small velocity perturbation in the direction of Earth was performed in multiple points of each orbit and the numerical integration in backward time was performed to construct the transfer trajectory. The achieved multiple Earth passing trajectories were divided into several classes depending on the target orbit amplitude and the transfer time. The options of reaching the small-amplitude halo orbits without lunar flybys or halo insertion maneuvers are presented.

2. Models and Methods

2.1. CR TBP

This work utilized circular restricted three-body problem (CRTBP) as a framework for spacecraft motion simulation. A spacecraft is considered to be moving in a gravitational field of two larger bodies (Sun and Earth), which move in circular orbits around a common barycentre. After rotating coordinate system connected with major bodies is introduced (Fig. 1), the motion of a spacecraft can be described by the system of equations (1),

$$\begin{align*}
\ddot{x} - 2\dot{y} &= \frac{\partial U}{\partial x}, \\
2\dot{x} + \dot{y} &= \frac{\partial U}{\partial y}, \\
\ddot{z} &= \frac{\partial U}{\partial z},
\end{align*}$$

where

$$U(x, y, z) = \frac{1}{2}(x^2 + y^2) + \frac{\mu_S}{r_S} + \frac{\mu_E}{r_E},$$

$$\mu_S = \frac{m_S}{m_S + m_E}, \quad \mu_E = \frac{m_E}{m_S + m_E},$$

$$r_S = \sqrt{(x + \mu E)^2 + y^2 + z^2}, \quad r_E = \sqrt{(x - \mu S)^2 + y^2 + z^2}$$

Figure 1: Coordinate system

where $m_S$ and $m_E$ are the Sun and the Earth masses, $r_S$ and $r_E$ are the distances from the probe to the Sun and the Earth respectively.

The libration points are well-known stationary solutions of CR3BP, which are placed on the x-axis (collinear: $L_1$, $L_2$, and $L_3$), and on the lighter body orbit (triangular: $L_4$, $L_5$). No other analytical solutions except the stationary ones are known, thus numerical are utilized for the computation of LPOs. As the linear dynamics of the collinear libration points is of type center×center×saddle [4], the solutions of the linearized system involve the harmonic and exponential parts. The nonlinear system inherits this character, thus every periodical
or quasiperiodical orbit around the libration point has the associated stable and unstable asymptotic manifolds. The stable manifold consists of the orbits, which asymptotically approach the orbit around a libration point, thus the orbits of the stable manifold can be used as the transfer trajectories. Due to their instability, the computation of the orbits around the libration point requires special numerical algorithms preventing the slipping of the numerical solutions to the unstable manifold orbit.

In this work, the halo orbits around the Sun-Earth $L_1$ and $L_2$ points were studied (Fig. 2). The range of out-of-plane amplitudes was $[0 \text{ km}, 1.844 \times 10^6 \text{ km}]$ for the $L_1$ orbit family and $[0.602 \text{ km}, 1.846 \times 10^6 \text{ km}]$ for the $L_2$ family. The halo orbits were constructed using the bounding planes method described in [10].

![Figure 2: $L_1$ and $L_2$ halo families](image)

2.2. Construction of LEO-LPO transfers with fixed LPO amplitude

This problem can be formulated as follows. Having a given halo orbit we want to describe the transfer possibilities between LEOs and this orbit, specifically - what altitude values of LEOs are possible.

To solve this problem the following algorithm was implemented. A number of equally spaced orbit’s points are taken. For every point a small perturbation is applied in such way, that the spacecraft is no longer stays on the orbit but moves towards the Earth. Then the numerical integration is performed until either the specified integration time limit is exceeded or the spacecraft exits the proximity of a libration point. While the integration is performed the orbits’ pericenters are tracked which are considered to be the potential transition points with the LEO. When the pericenters are found, new points of LPOs for generating extra pericenters are selected by the following rule. For each pair of neighboring points from initially taken orbit’s points, points of the orbit between these two are taken if for any of pair’s points LEO transitions were found. Then, the sequence, described above, is performed for these points too.

2.3. Construction of LEO-LPO transfers with fixed LEO altitude

The second problem is the opposite in some sense. For a given LEO altitude value we want to find halo orbits which are reachable from such LEOs.

To find these orbits we use data received using the previous algorithm, which is orbits points and corresponding LEOs altitudes. Then for every halo orbit we find pairs of neighbor points such that, for one of the points the corresponding LEO altitude value is slightly smaller than needed and for another one is slightly bigger. Then, using bisection method, we can find the point on the orbit between these two which corresponds to the exact (with the desired precision) needed LEO’s altitude.
3. Results and discussion

To illustrate the transfer possibilities for both halo orbit families, several maps (Fig. 4 - 9), which represent the relation between halo orbit amplitude \( A_z \), LEO altitude and transition time, were produced. Each point on any of the maps correspond to one pericenter found by one of the mentioned algorithms.

It can be seen from Fig. 4 and Fig. 5, that for both \( L_1 \) and \( L_2 \) points 3 main transition "windows" exist: 20-25 days, 140-180 days and 240-360, with different halo orbit amplitudes corresponding to these windows.

The ranges of amplitudes of reachable orbits for different transition lengths and their dependency on LEO altitude can be seen more clearly if the order of axes is changed (Fig. 6, Fig. 7). It becomes visible, that the dependencies are almost identical for \( L_1 \) and \( L_2 \): small orbits \((A_z < 125 000 \text{ km})\) can not be reached in less then \( \approx 150 \text{ days} \), large orbits \((A_z > 1 400 000 \text{ km})\) are not reachable in less then \( \approx 180 \text{ days} \), and the ranges of orbits, reachable in these time intervals are not dependent on LEO altitude. The range of amplitudes of halo orbits reachable in about 25 days increases approximately 1.25 times from \((350 000 \text{ km}, 950 000 \text{ km})\) to \((250 000 \text{ km}, 1 000 000 \text{ km})\) as the altitude of LEO changes from 0 to 2500 km. The orbits, with the amplitudes between ranges reachable in \( \approx 150 \text{ days} \) and ranges reachable in \( \approx 25 \text{ days} \), could
be approached in $\approx 300$ days. Finally, halo orbits with $A_z$ amplitudes equal to $\approx 1\, 200\, 000$ km could be reached in $\approx 300$ days also.

When transition options for a fixed LEO altitude value are represented (Fig. 8 and Fig. 9), different types of possible transitions become even more visible, as points, representing these transitions divide into several clusters, with different transition trajectory shapes for each cluster ($xy$ projections of trajectories are shown). The clusters are enumerated in the descending order in respect to number of orbits in each cluster. Clusters 2-3 and 6-8 are of the greatest interest since they increase the range of approachable halo orbits compared to generally studied transition trajectories (cluster 1) by utilizing several passes around the Earth.

4. Summary
This paper systemizes the single burn, single and multiple Earth passing transfer options in the framework of the circular restricted three-body problem for halo orbits around $L_1$ and $L_2$ points of Sun-Earth system. Two algorithms for finding transfer options for the desired halo orbit and
the parking orbit altitude were developed in order to achieve this goal. Using the data collected with these algorithms, following conclusions can be made about the approachability of the halo orbits in the Sun-Earth system. The shortest transfer option, taking about 25 days is available for the halo orbits of $A_z$ amplitude in the range of $(350,000 \text{ km}, 1,000,000 \text{ km})$. The transfer to larger or smaller halo orbits would need multiple Earth passing and take longer time. Orbits with $A_z < 125,000 \text{ km}$ could be reached in $\approx 150$ days; orbits with $A_z \in (125,000 \text{ km}, 250,000 \text{ km})$ could be reached in $\approx 300$ days; orbits with $A_z \in (250,000 \text{ km}, 350,000 \text{ km})$ could be reached in $\approx 25$ or $\approx 300$ days, depending on the altitude of LEO. The transfers for the orbits corresponding to the $A_z$ range of $1,000,000 \text{ km}$ to $1,400,000 \text{ km}$ were not found. The exclusion is the orbits of $1,200,000 \text{ km} A_z$, which can be reached in about 300 days. Larger amplitude halo orbits are reachable by the direct single burn transfer: the transfer time for the orbits having the $A_z$ in the ranges of $(1,400,000 \text{ km}, 1,600,000 \text{ km})$ and $(1,600,000 \text{ km}, 1,800,000 \text{ km})$ are 180 and 300 days correspondingly. The obtained $A_z$ ranges and corresponding transfer durations are similar for the both investigated libration options.

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