New supersymmetric source of neutrino masses and mixings

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Conventionally, neutrino masses in a supersymmetric theory arise from non-renormalizable lepton-number (L)-violating operators in the superpotential. The alternative possibility of having such operators in the Kähler potential as favoured by the SNO data provides an explanation for the smallness of \( \beta \tan \beta \) without having a large flavor mixing angle. In this letter we explore an alternative source of neutrino masses after electroweak symmetry breaking, when-
The new term $\kappa' \sin \beta \cos \beta$ goes to zero for large $\tan \beta$ so that, if only this operator were present, one could get an extra suppression in neutrino masses.

2) There is an additional $\mathcal{O}(\mu/M)$ suppression in $m_\nu$ with respect to $\nu$-masses from $W$ ($m_\nu \sim v^2/M$).

The fact that $\mu \lesssim \mathcal{O}(\text{TeV})$ (for the naturalness of electroweak breaking) implies that one can now lower significantly the scale $M$ of $L$-violation. Instead of the typical range $M \sim 10^{13} - 10^{15}$ GeV (which gives $m_\nu \sim 1 - 10^{-3}$ eV) one can easily get $M \sim 10^7 - 10^9$ GeV. These figures assume that all adimensional $\kappa$'s in are of order 1. If they are further suppressed, e.g. if they are generated at the loop level or through small couplings (maybe of non-perturbative origin) or are affected by the $\tan \beta$ suppression described above, then the scale $M$ can be much closer to the electroweak scale. This is very interesting for experimental reasons and to make contact with recent theoretical scenarios which favor such relatively light scales of new physics: these scenarios may accommodate realistic neutrino Majorana masses in a natural way, providing the relevant operators live in $K$ instead of $W$.

3. $K$ versus $W$. There can be at least two reasons for neutrino masses to originate in $K$ rather than in $W$. (As we have seen, if both sources were present and were suppressed by the same scale $M$, neutrino masses from $W$ would dominate.) First, there can be $R$-symmetries that forbid $\nu$-mass operators in $W$ (at least at the lowest level) and allow them in $K$. E.g. consider the $U(1)$-symmetry with charges $q_0 = 1$ (θ ≡ spinorial coordinate), $q_\nu + q_{H_2} \neq 0$. Then $(L \cdot H_2)^2/M$ is forbidden in $W$, but $K$ can contain operators like those of eq. (3), which generate $\nu$-masses as long as $(F^2_\nu) \sim -\mu (H_2) \neq 0$. This works fine for $\kappa'$, but no $R$-symmetry can allow $\kappa$ and forbid $(L \cdot H_2)^2$ in $W$ if $W \supset \mu H_2 H_2$. However the $\mu$-term can be induced through a Giudice-Masiero mechanism from an operator $\ol{T}H_1 H_2/M$ in the Kähler potential, where $T$ is a field driving SUSY breaking, i.e. $(F_T) \neq 0$. In this case $W$ contains a term $\sim (F_T)T$ which generates a $\mu$-term ($\mu = -(F_T)/M$) after the redefinition $T \rightarrow T - H_1 H_2/M$. Still, effective operators like $\sim (F_T)(L \cdot H_2)^2 \subset W$ or $\sim \ol{T}(L \cdot H_2)^2 \subset K$ can be allowed, but they induce $\nu$-masses of the same order. Similar conclusions are reached if the breaking of SUSY is communicated by gravitational interactions, so that $m_3/2 \sim (W_{\text{hidden}})/M_p^2$, and the effective $\mu$-term is generated by an operator $\propto H_1 H_2$ in $K$. For simplicity, we do not consider this kind of contributions to $M_\nu$. Actually, if one adds more fields to the MSSM it is easy to find some symmetry that allows a $\mu$-term and the $L$-violating operators in $K$, but forbids them in $W$. e.g. if $L$-violation is spontaneously induced by the vev of a field $\Phi$ (with lepton number $L = 2$) with a coupling $\ol{T}(L_i \cdot H_2)(L_j \cdot \ol{H}_1)$ in $K$.

Second, $L$ is an anomalous global symmetry, which is violated at the non-perturbative level. Actually, as any other global symmetry, it is expected to be violated within any fundamental theory that includes gravity (for a discussion of these points see e.g. [4]). For instance, in string theory the (global symmetry) violating effects typically appear as non-perturbative effects (e.g. worldsheet contributions). On the other hand, if these effects are non-perturbative in the string (-loop) expansion sense, they are much more likely to give sizeable contributions to the Kähler potential than to the superpotential (the latter is protected by discrete symmetries [3]). In general, it is clear that $W$ is much more protected than $K$ against non-perturbative (and of course perturbative) corrections, so it makes sense to suppose that the $L$-violating operators appear in $K$ rather than in $W$.

4. RG running. If the scale $M$ is not too close to the electroweak scale, the dominant effect of radiative corrections to neutrino masses and mixing angles is a logarithmic effect of order $\ln(M/M_W)$ that can be computed most easily by renormalization group techniques: running the operators $\kappa$ and $\kappa'$ from the scale $M$ down to $M_W$. The renormalization group equations (RGEs) for these operators can be easily extracted from the one-loop correction to the Kähler potential (computed for general non-renormalizable theories in ref. [3]). They are

$$\frac{dk}{dt} = u' \kappa' - P_E \kappa' - 2(P_E \kappa - \kappa^T P_E^T),$$

$$\frac{dk'}{dt} = u' \kappa' - P_E \kappa' - \kappa^T P_E^T,$$

where $u = \text{Tr}(3Y_L^1 Y_U + 3Y_D^1 Y_D + Y_E^1 Y_E) - 3g^2 - g'^2$, $u' = 2[\text{Tr}(3Y_L^1 Y_D + Y_E^1 Y_E) + g^2 + g'^2]$ and $P_E \equiv Y_E Y_E^T$. $Y_E$ is the matrix of leptonic Yukawa couplings, $W \supset Y_E^T H_1 \cdot L_i E_{Rj}$, $(Y_U, Y_D$ are those of $u$- and $d$-quarks). $g, g'$ are the $SU(2)_L$, $U(1)_Y$ gauge couplings, respectively.

We see that there is no mixing between $\kappa'$ and $\kappa$, as was to be expected. (A supersymmetric theory does not mix $H_2$ and $\ol{H}_1$, which make the difference between $\kappa'$ and $\kappa$.) The RGE for $\kappa$, eq. (3), is of the standard form and similar to the RGE for the neutrino mass matrix in the conventional scenario, i.e. when the masses arise from operators in $W$ [10]: there is a universal evolution given by $u'$ and a RG-change of the texture of $\kappa'$ induced by the $P_E$-terms. The stability analysis of neutrino mass splittings and mixings is similar to that of refs. [11][12]. In the following we will set $\kappa' = 0$ and focus on $\kappa$. In models with non-zero $\kappa'$ its contributions to the neutrino masses can be simply added to those of $\kappa$, with no interference between their runnings.
The structure of the RGE for $\kappa$ is quite remarkable. In order to better appreciate it, it is convenient to split $\kappa$ in its symmetric and antisymmetric parts and to remember that $\kappa_S$ contributes directly to neutrino masses, while $\kappa_A$ does not, see eq. (3). From eq. (3) we get:

$$\frac{d\kappa_S}{dt} = u\kappa_S + P_E\kappa_A - \kappa_A P_E^T,$$

$$\frac{d\kappa_A}{dt} = u\kappa_A + P_E\kappa_S - \kappa_S P_E^T + 2(P_E\kappa - \kappa^T P_E^T).$$

Both equations contain a universal piece and a potentially texture-changing piece, but for $\kappa_S$ this important last term depends only on $\kappa_A$ and not on $\kappa_S$ itself. This peculiar behaviour is the result of a cancellation between (flavour-changing) wave-function-renormalization corrections for the leptonic legs and vertex radiative corrections (coming from the non-minimal character of the Kähler potential and absent for couplings in $W$). This cancellation seems to be accidental (it does not hold at two-loops). This peculiarity of (3) leads to the first interesting result concerning radiative corrections:

3) If $\kappa_A(M) = 0$ the texture of $\kappa_S$ is quite stable. If also $\kappa'(M) = 0$, neutrino mixing angles do not run at one-loop.

This is the only instance that we know of a model for neutrino masses with such behaviour. If the full theory beyond $M$ is able to generate neutrino parameters in agreement with experiment, the often dangerous radiative corrections will not spoil this [if the conditions listed in 3) hold].

Another interesting effect that eqs. (6,7) can accommodate is the

4) radiative generation of a large mixing angle starting from a small one, in a natural way.

This idea has been much investigated in the literature but, as shown in [11,13], it requires a significant amount of fine-tuning in order to work. This is because, whenever radiative effects are large (as they should be), the usual RGEs for neutrino mass matrices imply that only small-mixings are infrared fixed points, while maximal mixings are not stable. The unconventional form of eq. (3) allows us to circumvent this difficulty. Consider for simplicity two-flavour oscillations $\nu_\mu \to \nu_\tau$ oscillations for atmospheric neutrinos; $\nu_e \to (\cos \theta_{atm}\nu_\mu + \sin \theta_{atm}\nu_\tau)$ oscillations for solar neutrinos. Let us start at $M$ with $[s,a = O(1)]$

$$\kappa_S(M) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}, \quad \kappa_A(M) = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix},$$

i.e. with diagonal and degenerate neutrino masses, as given by $\kappa_S$, but with a non-zero antisymmetric matrix $\kappa_A$. The mass matrix at low-energy is obtained by integrating the RGEs (3,4) and, in one-loop leading-log approximation, the result is

$$M_\nu \propto \kappa_S(M_W) = \begin{pmatrix} s' & a\epsilon \\ a\epsilon & s' \end{pmatrix},$$

with

$$s' = s \left(1 - \frac{u}{16\pi^2} \ln \frac{M}{M_W}\right), \quad \epsilon = \frac{y^2}{16\pi^2} \ln \frac{M}{M_W}.$$  

Here $y$ is some combination of leptonic Yukawa couplings, dominated by that of the $\tau$-lepton, and in the following, a prime indicates the universal RG-scaling down to $M_W$ (as for $s'$ above). $M_\nu$ in (8) has maximal mixing, which has been naturally induced by the RG-running, as promised. As long as the initial masses are not exactly degenerate, the final mixing is not exactly maximal.

On the other hand, the RG-generated neutrino mass splitting is similar to that in conventional scenarios. In particular, it should be remarked that $\epsilon$ can be of the right order of magnitude to account for the small solar mass splitting of the LMA solution, as was shown already in [3]. Actually, if $\tan \beta$ is large (and thus $y$), even the atmospheric mixing could be generated in this way.

5. Textures. Let us now explore what can be expected for the textures of the $3 \times 3$ neutrino mass matrix, $M_\nu$, and their phenomenological viability. If one starts with $\kappa'(M) = \kappa_S(M) = 0$ and any texture for $\kappa_S(M)$, then, according to the result 3), this texture will be stable under radiative corrections. However, since it is possible to generate entries of $M_\nu$ radiatively, as explained above, it is interesting to examine to what extent a realistic $M_\nu$ can be produced in this way.

The simplest case is $\kappa'(M) = \kappa_S(M) = 0$ [i.e., $K$ contains just the $(L_i \cdot L_j)(H_2 \cdot H_1)$ operator]. Then $M_\nu$ arises as a purely radiative effect [thus a smaller $M$ (by a $\sim 10^{-2}$ factor) gives $m_\nu$'s of the right order]. Denoting

$$\kappa_A(M) = \begin{pmatrix} 0 & a \\ -a & 0 \\ -b & c \end{pmatrix},$$

the one-loop leading-log integration of (3,4) gives

$$M_\nu \propto \kappa_S(M_W) = \begin{pmatrix} 0 & a\epsilon_{\mu e} & b\epsilon_{\tau e} \\ a\epsilon_{\mu e} & 0 & c\epsilon_{\tau e} \\ b\epsilon_{\tau e} & c\epsilon_{\tau e} & 0 \end{pmatrix},$$

where

$$\epsilon_{\alpha\gamma} = \frac{1}{16\pi^2} (y_\alpha^2 - y_\gamma^2) \ln \frac{M}{M_W},$$

and $y_\alpha \equiv \{h_\mu, h_\mu, h_\tau\}/\cos \beta$ are the leptonic Yukawa couplings in the matrix $Y_L$ (which can be taken to be diagonal). This is exactly a

5) (RG-generated) 'Zee'-type SUSY texture for $M_\nu$.

This texture has been confronted with experimental results and good agreement requires [3] that $c\epsilon_{\tau e} \ll b\epsilon_{\tau e} \sim a\epsilon_{\mu e}$ (plus small deviations from the pure Zee texture to avoid too maximal $\theta_{sol}$). This condition would
require the hierarchy $c \ll b$ and the tuning $b/a \sim h^2_\mu / h^2_\nu$. The origin of such hierarchies is not clear. They might result from having different mass scales in $\kappa_A$. Alternatively, it is amusing to note that if $\kappa_A$ arises as a radiative effect in the form $\kappa_A \sim \nu P_\nu A_2 - (\nu P_\nu A_2)^T$, where $A_{1,2}$ are generic antisymmetric matrices, then $a \propto h^2_\mu$, $b \propto h^2_\nu$, $c \propto h^2_\tau$, as required. In any case, the neutrino spectrum is $m_1 \simeq -m_2 \gg m_3$.

The textures become more involved once $\kappa_S$ and $\kappa'$ are non-zero. Still, it is nice to note that one could generate all the mixings radiatively. E.g., starting with $\kappa' = 0$ (for simplicity) and $\kappa_S = \text{diag}(s_1, s_2, s_3)$ it is clear that the low-energy mass matrix

$$M_\nu \propto \kappa_S(M_W) = \begin{pmatrix} s'_1 & a\epsilon_{\mu e} & b\epsilon_{\tau e} \\ a\epsilon_{\mu e} & s'_2 & c\epsilon_{\mu \tau} \\ b\epsilon_{\tau e} & c\epsilon_{\mu \tau} & s'_3 \end{pmatrix}, \quad (14)$$

contains the exact number of parameters necessary to fit the three $\nu$-masses and mixing angles. One could go further and demand the absence of unnatural fine-tunings between the entries of $\kappa_S$. An interesting possibility is $s'_1 \simeq s'_2 \sim s'_3 \sim c$ (which can be taken $\sim 1$); $b \simeq 0$. Then the masses are quasi-degenerate, $m_1 \simeq m_2 \simeq m_3 \equiv m_0 \lesssim 0.3$ eV to avoid too fast $0\nu\beta\beta$-decay, and the atmospheric angle and mass splitting are radiatively generated if $\tan \beta$ is large, $\sim 34$ (0.3 eV/$m_0$). However, to avoid a solar splitting as large as the atmospheric one requires a certain fine-tuning, $|s'_1 - s'_2| \sim |c\epsilon_{\mu \tau}|$. The solar angle remains undetermined at this stage, but is generated once $\kappa_S$ is perturbed with (e.g. two-loop) corrections in all entries. Then, it is in principle easy to accommodate the solar mixing and mass splitting with no further tunings. So, we conclude that

6) To agree with experiment, textures $\kappa_S(14)$ require some tuning of parameters, which is moderate in the quasi-degenerate case.

One can be less ambitious and try to generate just part of the neutrino mixings and mass splittings radiatively, for example, those of the solar sector, assuming that the physics beyond the scale $M$ fixes the atmospheric sector. Let us then start with $\kappa' = 0$, $\kappa_A \neq 0$ at the scale $M$ and

$$\kappa_S(M) \propto M_\nu(M) = m_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 + \Delta/2 & \Delta/2 \\ 0 & \Delta/2 & 1 + \Delta/2 \end{pmatrix},$$

(with $m_0^2 \gg \Delta m^2_{\text{atm}}$) which gives maximal atmospheric mixing and the right atmospheric mass splitting for $\Delta = \Delta m^2_{\text{atm}} / m_0^2$ while the solar mixing angle and splitting are both zero. At low-energy, besides the universal re-scaling just mentioned, $\kappa_S$ gets a RG-correction of the form (12) which drives the solar parameters to some non-zero values. If $a, b, c$ are all of the same order of magnitude we can neglect the $\epsilon_{\mu e}$ term ($\epsilon_{\mu e} \ll \epsilon_{\tau e} \simeq \epsilon_{\tau \mu}$) and the only free parameters left are $b$ and $c$ (for a given $M$). The generated solar mass splitting is $\Delta m^2_{\text{sol}} \simeq 2 m_0^2 \sqrt{2 b^2 + c^2} \epsilon_{\tau e}$, which can be inside the experimental range quite naturally. The solar angle is a function of $r \equiv c/|b|:

$$\tan^2 \theta_{\text{sol}} \simeq 1 + r^2 + r^2 / 2 + r^2.$$

For $|c| \ll |b|$, $r \to 0$ and the solar angle gets exactly maximal. This is disfavoured now, but one gets values of $\tan^2 \theta_{\text{sol}}$ inside the experimental interval for $-1.0 \leq r \leq -0.19$. (The atmospheric angle is not disturbed much by the radiative effects and stays nearly maximal while a non-zero CHOOZ angle of order $\epsilon_{\tau e}$ is generated which stays comfortably below the experimental upper bound $\sin \theta_{\tau e} < 0.24$ (3)). We conclude that

7) $\Delta m^2_{\text{sol}}$ and $\theta_{\text{sol}}$ of LAMS$\text{W}$ solar oscillations could have a purely low-energy (RG) origin.

In such scenario, the more fundamental theory beyond $M$ should account for the parameters that control the oscillations of atmospheric neutrinos only. This would relax some of the demands on that theory (e.g. on flavour symmetries).

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