Analysis of the efficiency of satellite image sequences filtering

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Abstract. We considered problem of filtering multispectral images in the article. Particular attention is paid to the analysis of the efficiency of filtering image sequences. We obtained dependences of the variance of errors on the number of frames in the sequence, as well as on the coefficient of inter-frame correlation. We used doubly stochastic random fields for statistical modelling of images.

1. Introduction
One of the key features of real satellite imagery is their pronounced spatial heterogeneity [1-3]. It is associated with the variety of shapes and textures of different objects observed from space. Indeed, any satellite image of the Earth's surface contains images of different objects, for example, rivers, forests, urban buildings, agricultural lands, etc. The visual characteristics of these objects are significantly different. Attempt to describe the whole picture using the known homogeneous models (for example, autoregressive, wave, etc.) leads to errors caused by wrong representation of information.

2. Related works analysis
To overcome these errors, various pre-processing algorithms are used such as non-linear contrasting [4], noise-resistant segmentation [5], isolation of objects that have very similar properties [6-7], as well as adaptation mechanisms to local image properties [8], including those taking into account the morphological features of objects [9, 10]. However, these procedures, effective for processing individual types of images, do not allow to build adequate descriptions for satellite multispectral surveys. In works [11-12] such descriptions are based on the study of the features of the correlation function of satellite images, in particular the fact that in most cases it can be composed of the exponential correlation function of the background and local objects. Nevertheless, this approach is a good reason to limit the subsequent synthesis of processing algorithms, since it is not based on obtaining analytical or recurrence relations describing the image itself.

Doubly stochastic autoregressive RF models [13-17] were proposed as models of spatially inhomogeneous images. These models, on the one hand, make it possible to generate images with
various probabilistic properties, on the other, they are simple enough for analysis and allow us to form various processing algorithms. An example of such algorithms are the nonlinear recurrent filtering procedures described in [13]. It was shown that these procedures allow the filtering of two-dimensional spatially inhomogeneous images with significantly higher efficiency than the known linear algorithms and their modifications. In the present work, we show ways how to generalize the previously found procedures for the processing of multispectral satellite images.

3. Filtering algorithm and its investigation

Consider the task of filtering the multispectral image, which consists of a set of two-dimensional frames. Each of these frames describes the state of the earth's surface in a certain spectral band. We define the frames to be processed as a random field (RF) that is changing in discrete time and is defined on a multidimensional grid [1-3,13].

\[ \mathcal{J}_i = \{ \mathbf{j} = (j_1, j_2, j_3); j_l = 1, M_1; l = 1,2,3 \} \]

where \( j_1, j_2, j_3 \) can be regarded as spatial coordinates, and \( j_3 \) is considered to be the number of the spectral band. The RF elements are scalar values. It can be the brightness of the image at a given point. Then, the first frame of the multispectral image is described by the following autoregressive equations:

\[ x_{ij} = \sum \sum \rho_{kl} x_{kl} + \xi_{ij}^{1,1}, \quad \{(i, j); i = 1, M_1; j = 1, M_2 \}, \]

where \( \rho_{ij} \) is a neighborhood containing elements \( \{x_{kl}\} \), that define \( x_{ij} \), \( \xi_{ij}^{1,1} \) is random additive, and the coefficients \( \rho_{ij} \) are not constant, but random quantities. Let the \( \rho_{ij} \) are also defined in a similar way, for example, by following formula

\[ \rho_{ij} = \sum \sum r_{mn} \rho_{mn} + \xi_{ij}^{1,1}, \quad \{(k, l); i = 1, M_1; j = 1, M_2 \}, \]

where \( r_{ij} \) are constant coefficients characterizing the correlation characteristics of the auxiliary RF \( \{\rho_{ij}\} \).

The mathematical model defined this way is non-linear, and the process of analysis such models tends to some difficulties. Therefore, in order to reduce the material presented below and simplify its understanding, we use a simpler version of the doubly stochastic model:

\[ x_{ij} = \rho_{x_{ij}} x_{i-1,j} + \rho_{x_{ij}} x_{i,j-1} - \rho_{x_{ij}} x_{i-1,j-1} + \xi_{ij}^{1,1}, \]

where \( x_{ij} \) is simulated RF with normal distribution \( M\{x_{ij}\} = 0, M\{x_{ij}^{2}\} = \sigma_{x}^{2}; \xi_{ij}^{1,1} \) is RF of independent standard Gaussian random values (RV) \( M\{\xi_{ij}^{1,1}\} = 0. \)

Random variables \( \rho_{x_{ij}} \) and \( \rho_{y_{ij}} \) with a Gaussian probability distribution density can be described by the following autoregressive equations:

\[ \tilde{\rho}_{x_{ij}} = r_{1x} \tilde{\rho}_{x_{i-1,j}} + r_{2x} \tilde{\rho}_{x_{i,j-1}} - r_{1x} r_{2x} \tilde{\rho}_{x_{i-1,j-1}} + \sigma_{\rho_x} \sqrt{(1 - r_{1x}^{2})(1 - r_{2x}^{2})} \xi_{ij}^{1,1}, \]

\[ \tilde{\rho}_{y_{ij}} = r_{1y} \tilde{\rho}_{y_{i-1,j}} + r_{2y} \tilde{\rho}_{y_{i,j-1}} - r_{1y} r_{2y} \tilde{\rho}_{y_{i-1,j-1}} + \sigma_{\rho_y} \sqrt{(1 - r_{1y}^{2})(1 - r_{2y}^{2})} \xi_{ij}^{1,1}, \]
\[ \rho_{xij} = \tilde{\rho}_{xij} + m_{\rho_x}, \]
\[ \rho_{yij} = \tilde{\rho}_{yij} + m_{\rho_y}, \]

where \( r_{1x} = M\{ \tilde{\rho}_{xij} \tilde{\rho}_{x(i-1)j} \} \), \( r_{2x} = M\{ \tilde{\rho}_{xij} \tilde{\rho}_{x(i-j-1)} \} \) are correlation coefficients of a random parameter \( \tilde{\rho}_{xij} \); \( r_{1y} = M\{ \tilde{\rho}_{yij} \tilde{\rho}_{y(i-1)j} \} \), \( r_{2y} = M\{ \tilde{\rho}_{yij} \tilde{\rho}_{y(i-j-1)} \} \) are correlation coefficients of a random parameter \( \tilde{\rho}_{yij} \); \( \zeta_{\rho_{xij}} \) and \( \zeta_{\rho_{yij}} \) are normally distributed RV with \( M\{ \zeta_{\rho_{xij}} \} = 0 \), \( M\{ \zeta_{\rho_{yij}} \} = M\{ \zeta_{\rho_{xij}}^2 \} = M\{ \zeta_{\rho_{yij}}^2 \} = \sigma^2 = 1 \).

The following frames are specified by the RF \( \{ x^k \}_{ij} \), \( k = 2..N \). This RF is placed on the same grid as the first frame. Let's take into account a result of prolonged observations as the matrix \( R \) of inter-frame correlations between individual frames \( \{ x^k \} \). We will also assume that the features of registering objects on these frames ensure the closeness of intraframe correlation characteristic on all frames. Then, to simulate the second and subsequent frames, we can use the following relation:

\[ x^k_{ij} = \sum_{l=1}^k \sum_{i=1}^M R(l, k) x^l_{ij} + \sum_{i=1}^M \sum_{j=1}^N \xi_{ij} \]

where \( \xi_{ij} \) are elements of a triangular matrix \( V \), such that \( VV^T = B \), where \( B \) is covariance matrix of a separate, for example, first frame, \( \xi_{ij} \) are white noise samples with dispersion \( \sigma^2 = \prod_{p=1}^k \sqrt{1 - R(p,k)^2} \). Matrix \( V \) can be obtained by means of the well-known Cholesky transform [16].

Unfortunately, the CF of RF and the processes generated by the doubly stochastic model are generally described by very complex expressions due to the presence of double correlations between the process itself and its basic parameters. Nevertheless, the CF can be found quite simply in the important special case of a slow changes in the base RF. Then the following formula can be obtained for the CF of a one-dimensional doubly stochastic first-order model:

\[ B(k) = M\{ \sigma^2 \rho_{xij}^{[1]} \} = \sigma^2 \int_{-\infty}^{\infty} \rho_{xij}^{[1]} w(\rho) d\rho, \quad \sigma^2 \sum_{j=0}^{Q} C_k^{2j} \sigma^2 \rho_{xij}^{[2]} m_{\rho_x}^{k-2j} (2j-1)!! \]

where \( Q = k / 2 \), if \( k \) is even, and \( Q = (k-1) / 2 \), if \( k \) is odd; \( C_k^{j} = \frac{k!}{j!(k-j)!} \); \( (2j-1)!! \) is defined as the product of all natural odd numbers on the interval \([1, (2j-1)]\).

When we use a two-dimensional RF, an approximate with a slow change in the base RF can be found as follows:

\[ B(k1, k2) = M\{ \sigma^2 \rho_{xij}^{[1]} \rho_{yij}^{[1]} \}. \]

The parameters \( \rho_{xij} \) and \( \rho_{yij} \) are independent. So we can write:

\[ B(k1, k2) = \sigma^2 \int_{-\infty}^{\infty} \rho_{xij}^{[1]} w(\rho_{xij}) d\rho_{xij} \int_{-\infty}^{\infty} \rho_{yij}^{[2]} w(\rho_{yij}) d\rho_{yij} = B(k1)B(k2) / \sigma^2, \]

where \( \rho_{xij}^{(k1)} \) and \( \rho_{yij}^{(k2)} \), \( k1, k2 = 1, 2,... \) - correlation coefficients between adjacent samples along the axes \( x \) and \( y \). Thus, for an approximate calculation of the CF of a two-dimensional doubly stochastic RF one can use the multiplication of two one-dimensional CFs \( B(k1), B(k2) \).
Let us consider the vector of estimations \( \bar{x}_{ij} = (\bar{x}_{ij}^1, \bar{p}_{ij}^1, \bar{p}_{ij}^2, \ldots, x_{ij+1}^N)^T \) consisting of \( 3(M_1 + 1) + N - 1 \) elements, where
\[
\bar{x}_{ij} = (x_{ij}, x_{ij-1}, \ldots, x_{N_1}, x_{i-1, j}, \ldots, x_{i, j-1})^T,
\]
\[
\bar{p}_{ij} = (\rho_{ij}, \rho_{ij-1}, \ldots, \rho_{i1}, \rho_{iM_1}, \ldots, \rho_{1j}, \rho_{1j-1}, \ldots, \rho_{11})^T.
\]
Using this vector, we can rewrite the model of the multispectral image in the form
\[
\bar{x}_{ij} = R \bar{x}_{ij-1} + z_{ij},
\]
where \( \bar{z}_{ij} = (\varepsilon_{ij}^1, 0, \ldots, 0, \sigma_{ij} 0, \ldots, 0, \sum_{i=1}^{M_1} \sum_{j=1}^{M_2} b_{ij} \varepsilon_{ii}^1, \ldots, \sum_{i=1}^{M_1} \sum_{j=1}^{M_2} b_{ij} \varepsilon_{N-1}^{i+1})^T;
\]
\[
R_{ij} = \begin{pmatrix}
\varphi_{ij} & 0 & 0 & 0 \\
0 & \varphi_{ij} & 0 & 0 \\
0 & 0 & \varphi_{ij} & 0 \\
R_N & 0 & 0 & 0
\end{pmatrix}, \quad R_{ij} = \begin{pmatrix}
\rho_{ij} & \rho_{i-1,j} & -\rho_{i-1,j} & \rho_{ij} \\
1 & 0 & 0 & 0 \\
0 & \ldots & \ldots & \ldots \\
0 & 0 & 1 & 0
\end{pmatrix}, \quad R_{ij, \rho} = \begin{pmatrix}
r_{i1} & \ldots & r_{i2} & -r_{i1}r_{i2} \\
1 & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots \\
0 & \ldots & 1 & 0
\end{pmatrix};
\]
\[
R_N = \begin{pmatrix}
R(1,1) & 0 & \ldots & 0 \\
R(1,2) & R(2,2) & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
R(1,N) & R(1,N) & \ldots & R(N,N)
\end{pmatrix}.
\]

Figure 1 shows process of formation of two two-dimensional correlated frames of the conditional multispectral image. A feature of this process is the use of only one first frame as a reference for estimating intra-frame spatial correlation characteristics. Accordingly, most part of the vector \( \bar{x}_{ij} \) are elements of the first frame and parameters that determine the intra-frame correlation. And only the last element of the vector \( \bar{x}_{ij} \) matches the element \( x_{ij+1}^2 \), located on the second frame at the point with coordinates \( (i, j-1) \).

![Figure 1. Processing of multispectral image.](image)

We can obtain the following two-dimensional nonlinear filter by using these relations and the method of recurrent nonlinear filtering
\[
\hat{x}_{ij} = \hat{x}_{ij} + B_{ij} (z_{ij} - \hat{x}_{ij}),
\]
where $\hat{X}_{xij}$ is subset of elements of the extrapolation vector $\hat{X}_{xij}$, corresponding to the observation vector $z_{ij}$, $(i, j)$; $B_{ij} = P_{ij}C^TD_{ij}^{-1}$; $C = (1,0,0,...,0,1,...,1)$; $D_{ij} = CP_{ij}C^T + \sigma_n^2$;

$$P_{ij} = M\left(\hat{X}_{xij} - \bar{X}_{xij}\right)\left(\hat{X}_{xij} - \bar{X}_{xij}\right)^T = \phi'\left(\hat{X}_{y_{i+1}}\right)P_{y_{i+1}}\phi'\left(\hat{X}_{y_{i-1}}\right)^T + V_{y_{i+1}}$$

Variance of the filtering error at each step $P_{ij} = (E - B_{ij}C)P_{ij}$. It should be noted that the peculiarity of this filter is the fact that at each of its iterations there is a repeated recount of the elements making up the vector $\hat{X}_{xij}$. It is also necessary to take into account that at each point with coordinates $(i,j)$ there is more than one scalar observation $z_{ij}$, it usually contents the vector of such observations, whose values correspond to the values of the multispectral image at the point $(i,j)$ on the corresponding frame.

Figure 2 shows the dependence of the filtering efficiency on the number of frames of the multispectral image and the coefficient of inter-frame correlation. In this case, Figure 2(a) shows the filtering errors that occur when processing a single frame ("dashed line") and when using multiple frames (up to 10). Figure 2(b) shows the dependence of the filtering error on the coefficient of inter-frame correlation $R$. In the latter case, it was assumed that $R$ is the same for any pair of different two-dimensional frames entering the multispectral image. In this case, the curve (1) corresponds to the case of simultaneous processing of two frames, the curve (2) corresponds to processing of four frames, the curve (3) corresponds to processing of 10 frames.

Figure 3 shows fragments of individual frames of three test multispectral satellite images obtained from the Landsat 8 spacecraft. We used eight spectral bands from Landsat in each processed multispectral image. The level of inter-frame correlation was from 0.61 (bands of "SWIR-Blue") to 0.99 ("Blue-Green" bands).

In order to analyze the efficiency of the algorithms found, these images were mixed with white noise of varying intensity so that the signal-to-noise ratio $q = \frac{\sigma_n^2}{\sigma_s^2}$ varied from 2 to 10. Then its filtering was successively performed using a different number of spectral bands. Table 1 shows the ratio of the variance of filtering errors to the variance of the image ($\text{MSE}/\sigma_s^2$). Left image on figure 3 corresponds to Image 1, middle image corresponds to Image 2 and right image corresponds to Image 3.
Table 1. Filtering efficiency (as a variance of filtering error).

| Number of bands used for filtering | Image 1 | Image 2 | Image 3 |
|-----------------------------------|---------|---------|---------|
| q=2                               | 0.083   | 0.067   | 0.053   |
| q=5                               | 0.042   | 0.036   | 0.031   |
| q=10                              | 0.033   | 0.029   | 0.026   |
| q=2                               | 0.068   | 0.056   | 0.048   |
| q=5                               | 0.036   | 0.029   | 0.026   |
| q=10                              | 0.032   | 0.027   | 0.028   |
| q=2                               | 0.028   | 0.024   | 0.021   |
| q=5                               | 0.020   | 0.024   | 0.020   |
| q=10                              | 0.019   | 0.019   | 0.019   |
| q=2                               | 0.008   | 0.007   | 0.006   |
| q=5                               | 0.006   | 0.005   | 0.004   |
| q=10                              | 0.005   | 0.004   | 0.003   |

Analysis of the data allows to conclude that with the increase in the number of spectral bands used for processing, the effectiveness of this treatment is substantially increased. So the use of all available eight spectral bands allows to reduce the variance of the filtering error by an average of 80% compared to the results of processing a single frame.

4. Conclusion
In the present work we have synthesized the nonlinear three-dimensional recurrent filter that allows processing real multispectral images. The corresponding algorithms were performed. Numerical characteristics of the efficiency of this filter are obtained, which make it possible to recommend it for use on real satellite material.

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**Acknowledgments**

The work was supported by RFBR and Ulyanovsk region, project №16-41-732027.