The experimental data on inclusive cross sections of jet, direct photon and hadron production in $pp/\bar{p}p$ and $AA$ collisions at RHIC and Tevatron are analyzed in the framework of $z$-scaling. Results of analysis are compared with data obtained at ISR, S$pS$ and Tevatron. The properties of $z$-presentation of experimental data are verified. Physical interpretation of the scaling function $\psi(z)$ and variable $z$ is discussed.

The locality, self-similarity and fractality are argued to reflect the general structure of the colliding objects, interaction of their constituents and particle formation at small scales. The obtained results suggest that the $z$-scaling may be used as a tool for searching for new physics phenomena beyond Standard Model in hadron and nucleus collisions at high transverse momentum and high multiplicity at U70, RHIC, Tevatron and LHC.

1 Introduction

Study of particle production at large transverse momenta in high energy collisions of hadrons and nuclei is of interest to search for exotic phenomena such as quark compositeness [1], extra dimensions [2], black holes [3], fractal space-time [4] and collective phenomena such as phase transition of nuclear matter and formation of quark-gluon plasma [5].

New features of particle formation experimentally established could be crucial for precise test of Quantum chromodynamics (QCD) and Electroweak (EW) theories in perturbative and non-perturbative regimes. Many phenomenological approaches were suggested for description of regularities reflecting general properties (locality, self-similarity) of hadron and nucleus interactions at a constituent level at high energies [6]-[15].

In the present paper we use the concept of the $z$-scaling [16]-[20] for analysis of new experimental data on inclusive spectra of hadron, direct photon and jet production in $pp/\bar{p}p$ and $AA$ collisions at RHIC and Tevatron. We show that the $z$-scaling represents regularity in both soft and hard regime of particle production over a wide kinematical range for events with different centrality. The procedure for construction of the scaling function $\psi(z)$ and scaling variable $z$ for different types of produced objects is described. Both quantities are expressed via the experimentally measured inclusive cross section $E d^3\sigma/dp^3$, the multiplicity density $dN_{ch}/d\eta$ and kinematical characteristics of colliding and produced particles. Properties of $z$-scaling are used to predict particle spectra of $J/\psi, D^0, B^+$ mesons and $Z, W^+$ bosons at higher collision energies and transverse momenta. We suggest to use the $z$-scaling as a tool for searching for new physics phenomena of particle production in high transverse momentum and high multiplicity region at U70,
2 z-Scaling

Here we would like to remind basic ideas and some formulas which are important for understanding of the developed approach \[16\]-\[20\].

The idea of $z$-scaling is based on the assumption \[11\] that gross features of inclusive particle distribution of the reaction written in the symbolic form as follows

$$M_1 + M_2 \rightarrow m_1 + X$$  \tag{1}

can be described at high energies in terms of the corresponding kinematic characteristics of constituent binary collision

$$(x_1M_1) + (x_2M_2) \rightarrow m_1/y + (x_1M_1 + x_2M_2 + m_2/y)$$  \tag{2}

Here $x_1$ and $x_2$ are the fractions of the incoming 4-momenta $P_1$ and $P_2$ of the objects with the masses $M_1$ and $M_2$ carried out by constituents. The inclusive particle with the mass $m_1$ and the 4-momentum $p$ carries out the fraction $y$ of the 4-momentum of the outgoing constituent. The parameter $m_2$ is introduced to satisfy the internal conservation laws (for baryon number, isospin, strangeness, and so on). It is determined from the corresponding exclusive reaction. For example, the parameter $m_2$ for the exclusive process $p + p \rightarrow \pi^+ + (p + n)$ is equal to $m_n - m_p$ as follows from the relation $\pi^+ + (p + n) = \pi^- + p + p + (n - p)$ satisfying to the baryon and electric charge conservation laws. The parameters for inclusive $K^+, \Lambda^0$ and $\bar{p}$ production in $p+p$ collisions are equal to $m_\Lambda - m_p$, $m_K - m_p$ and $m_p$, respectively. It is assumed that the constituent interaction satisfies the energy-momentum conservation law written in the form

$$(x_1P_1 + x_2P_2 - p/y)^2 = (x_1M_1 + x_2M_2 + m_2/y)^2.$$  \tag{3}

The equation expresses locality of hadron interaction at constituent level.

The scaling variable $z$ is defined as follows \[20\] \[1\]

$$z = \frac{s_{1/2}}{(dN/d\eta|_0)^c \cdot m_0} \cdot \Omega^{-1}.$$  \tag{4}

Here $m_0$ is a mass constant which is fixed at the value of nucleon mass, $dN/d\eta|_0$ is a multiplicity density at (pseudo)rapidity $\eta = 0$, the parameter $c$ has physical meaning of "specific heat" of produced medium. The quantity $s_{1/2}$ is a minimal transverse kinetic energy of the constituent sub-process. It consists of two parts $s_{1/2}^\lambda$ and $s_{1/2}^\chi$ which represent the energy for creation of the inclusive particle and its recoil, respectively. The quantity $\Omega$ depends on the momentum fractions $x_1, x_2, y$ and the anomalous fractal dimensions $\delta_1$, $\delta_2$ and $\epsilon$ as follows

$$\Omega(x_1, x_2, y) = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2} (1 - y)^\epsilon.$$  \tag{5}

It is proportional to relative number of parton configurations containing constituents which carry the fractions $x_1$ and $x_2$ of the incoming momenta $P_1$ and $P_2$ and the outgoing constituent which fraction $y$ is carried out by the inclusive particle with the momentum $p$.  

\[1\]Other modifications of $z$ are described in \[16\] \[21\] \[22\] \[23\]
The fractions are determined in a way to minimize $\Omega^{-1}(x_1, x_2, y)$ taking into account the energy-momentum conservation law in the binary collision [2]. This is equivalent to the solution of the system of nonlinear equations

$$\partial \Omega(x_1, x_2, y)/\partial x_1 = 0, \quad \partial \Omega(x_1, x_2, y)/\partial x_2 = 0, \quad \partial \Omega(x_1, x_2, y)/\partial y = 0$$

with the additional condition [3].

The scaling function $\psi(z)$ is expressed in terms of the experimentally measured inclusive invariant cross section $Ed^3\sigma/dp^3$, multiplicity density $dN/d\eta$, total inelastic cross section $\sigma_{\text{in}}$ and kinematical variables (masses and momenta) characterizing the inclusive process. It can be written as follows

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{\text{in}}} J^{-1} E d^3\sigma/dp^3.$$  

(7)

Here, $s$ is the center-of-mass collision energy squared and $J$ is the corresponding Jacobian. The function $\psi(z)$ determined by (7) satisfies to the normalization condition

$$\int_0^\infty \psi(z)dz = 1.$$  

(8)

The relation allows us to interpret the $\psi(z)$ as a probability density to produce inclusive particle with the corresponding value of the variable $z$.

3 z-Scaling in $pp$ and $AA$ collisions at RHIC

In this section we analyze experimental data on particle transverse spectra obtained in $pp$ and $AA$ collisions at RHIC.

3.1 $\pi^0$ mesons

The PHENIX and STAR collaborations published the new data [24, 25] on the inclusive spectrum of $\pi^0$-mesons produced in $pp$ collisions in the central rapidity range at energy $\sqrt{s} = 200$ GeV. The transverse momenta of $\pi^0$-mesons were measured up to 20 GeV/c. The $p_T$ and $z$ presentations of data for $\pi^0$-meson spectra obtained at ISR [26]-[30] and RHIC [24, 25] are shown in Figs. 1(a) and 1(b). One can see that the PHENIX and STAR data are compatible each other over a overlapping range. The $p_T$-spectra of $\pi^0$-meson production demonstrate the strong dependence on collision energy. As seen from Fig. 1(b) the new data on $\pi^0$-meson inclusive cross sections obtained at the RHIC are in a good agreement with our earlier results [17]. The shape of the scaling functions for ISR and RHIC energies is observed to be the same. Uncertainty of relative normalization factor for cross sections is found to be 2. Based on the obtained results we conclude that the available experimental data on high-$p_T$ $\pi^0$-meson production in $pp$ collisions confirm the property of the energy independence of $\psi(z)$ in $z$ presentation.

3.2 Charged hadrons

Here we present the results of the joint analysis of experimental data on charged hadrons produced in $\bar{p}p$ [34] and $pp$ [35] collisions at different multiplicities of charged particles and different incident energies over a wide $p_T$ range.

The E735 collaboration measured the multiplicity dependence of charged hadron spectra [34] in proton–antiproton collisions at the energy $\sqrt{s} = 1800$ GeV for $dN_{ch}/d\eta =$
Figure 1: Spectra of $\pi^0$ mesons produced in $pp$ collisions in $p_T$ and $z$ presentations. Experimental data are taken from [24, 25].

2.3 – 26.2 at Tevatron. These measurements include highest multiplicity density per nucleon–(anti)nucleon collision obtained so far. The pseudorapidity range was $|\eta| < 3.25$. The data cover the transverse momentum range $p_T = 0.15 – 3$ GeV/c. The data demonstrate strong sensitivity of the spectra to the multiplicity density at high $p_T$. The independence of $\psi$ on multiplicity density $dN_{ch}/d\eta$ was observed. The result gives strong restriction on the parameter $c$ which was found to be $c = 0.25$.

The STAR collaboration obtained the new data [35] on the inclusive spectrum of charged hadrons produced in proton-proton collisions in the central rapidity range $|\eta| < 0.5$ at the energy $\sqrt{s} = 200$ GeV at RHIC. The transverse momentum spectra were measured up to 9.5 GeV/c. The data demonstrate the strong dependence of the spectra on the multiplicity density at $dN_{ch}/d\eta = 2.5, 6.0$ and 8.0. The STAR data confirm the multiplicity independence of the scaling function $\psi(z)$ established for the proton-antiproton collisions at higher energies. The corresponding value of the parameter $c$ was found to be the same 0.25.

We would like to note that the scaling for charged particles produced in proton–antiproton and proton–proton collisions for different multiplicities and energies is consistent with the values of the anomalous fractal dimensions $\delta_1 = 0.7$, $\delta_2 = 0.7$ and $\epsilon = 0.7$. Experimental data on multiplicity dependence of the spectra for non-identified charged hadrons obtained by the UA1 [32] and CDF [36] collaborations at the SppS and Tevatron are found to be in good agreement with the scaling function at $c = 0.25$ similarly as the E735 and STAR data.

The results of $z$-presentation for $\bar{p}p$ and $pp$ collisions are used to construct the scaling function over a wide range of $z$. There is indication that in the low $z$-range the shape of the scaling function for both collisions is the same. In the same time the asymptotic behavior of $\psi(z)$ for $\bar{p}p$ and $pp$ for high $z$ is different. The difference increases with $z$. The E735 data in $z$-presentation cover the range $z = 3 \cdot 10^{-2} – 5$. For high $z$ the STAR data demonstrate power law. In the overlapping range $z = 0.4 – 5$ the behavior of $\psi(z)$
3.3 Multiplicity dependence of particle spectra

We use the properties of $z$-scaling to predict cross sections of charged hadron production in $pp$ collisions over a wide range of transverse momenta, multiplicity densities and collision energies.

Figure 3 shows multiplicity dependence of transverse spectra of charged particles produced in $pp$ collisions in central rapidity range at $\sqrt{s} = 200$ and 11.5 GeV. The predictions are of interest for searching for phase transition of hadron matter at extremely high multiplicity (energy) density and can be verified at U70 and RHIC. We suppose that violation of $z$-scaling in particle production is a signature of such phase transition.

3.4 Dependence of $<p_T>$ on multiplicity and collision energy

The correlation between $<p_T>$ and multiplicity density of charged hadrons $dN_{ch}/d\eta$ in high-energy hadronic collisions was experimentally observed at ISR [31], SppS [32] and Tevatron [33, 34, 36]. The dependence of $<p_T>$ on collision energy $\sqrt{s}$ was experimentally established as well. It was found that the average $p_T$ grows with multiplicity, collision energy and mass of produced particle.

It is considered that the mean value of transverse momentum $<p_T>$ characterizes medium created in collisions of hadrons or nuclei. The dependence of $<p_T>$ on collision energy $\sqrt{s}$ and multiplicity density $dN_{ch}/d\eta$ is a useful tool to investigate the collective behavior of soft multi-particle production. It could give indications on phase transition of hadron matter at extremely high hadron density.

As noted in [36] the different theoretical models used for explanation of the phenomena (mechanism of multi-particle production) do not provide satisfactory predictions for existing experimental results leaving the real origin of the effect unexplained. Therefore
Figure 3: Spectra of charged hadrons produced in \(pp\) collisions for different multiplicity density \(dN_{\text{ch}}/d\eta\) in \(p_T\) presentations at RHIC (200 GeV) (a) and U70 (11.5 GeV) (b) energies. Experimental data are taken from [35].

Figure 4: Mean transverse momentum \(< p_T >\) as a function of multiplicity density \(dN_{\text{ch}}/d\eta\) (a) and collision energy \(\sqrt{s}\) (b).
Figure 5: Transverse spectra of jet production in pp collisions at $\sqrt{s} = 200$ GeV and $0.2 < \eta < 0.8$ in $p_T$ (a) and $z$ (b) presentation. Experimental data obtained by STAR collaboration are taken from [38]. MC results and $z$-scaling prediction are shown by points (+, $\sigma$, $\Delta$) and the dashed line, respectively.

The energy and multiplicity independencies of scaling function $\psi(z)$ for charged hadrons can be used to study the dependence of $<p_T>$ for non-identified charged on $dN_{ch}/d\eta$ and $\sqrt{s}$. The results obtained in previous section allow us to construct transverse momentum distributions $dN/dp_T$ over a wide kinematical range for identified hadrons and calculate mean $p_T$ as follows

$$<p_T> = \frac{\int_{p_T}^{p_{T,\text{min}}} p_T (dN/dp_T) dp_T}{\int_{p_T}^{\infty} (dN/dp_T) dp_T}. \quad (9)$$

Here $p_{T,\text{min}}$ is the minimal transverse momentum of detected particle. Figure 4 demonstrates dependence of $<p_T>$ for produced charged hadrons on multiplicity density $dN_{ch}/d\eta$ (a) and collision energy $\sqrt{s}$ (b). As seen from Fig. 4 multiplicity and energy dependencies of $<p_T>$ reveal monotonous growth with $dN_{ch}/d\eta$. Similar behavior of $<p_T>$ were found for strange particles ($K^0_S$, $\Lambda$) produced in $pp$ collisions at RHIC energies [37]. We observe no indications on jump or sharp rise of $<p_T>$ up to highest values of multiplicity density. Experimental verification of the predictions is of interest for searching for phase transition of nuclear matter.

3.5 Jets

The STAR collaboration published new data [38] on cross section of jet production in pp collisions at RHIC. The data cover the kinematical range of pseudorapidity $0.2 < \eta < 0.8$ and transverse momentum $p_T = (5 - 50)$ GeV/c.

Figure 5(a) shows inclusive differential cross section for the $p+p \rightarrow jet + X$ process at $\sqrt{s} = 200$ GeV measured by the STAR collaboration. The NLO QCD calculated results with the CTEQ6M parton distribution functions at equal factorization and renormaliza-
tion scales \( \mu_R = \mu_F = p_T \) demonstrate satisfactory agreement with the data. Comparison of the STAR data with MC results \([39]\) and predictions of \(z\)-scaling in \(z\) presentation is shown in Fig. 5(b). The sensitivity of the scaling function to the choice of the parameters \(E_{\text{seed}}, R, E_{\text{cut}}\) of the jet-finding algorithm is observed. It enhances as transverse momentum of jet decreases. The dependence of \(\eta^\text{jet} \) on \(p_T^\text{jet} \) was used for construction of the scaling function. Note that the shape of the scaling function \(\psi(z)\) for \(p_T < 10 \text{ GeV}/c\) can not be described by the power law \(\psi(z) \sim z^{-\beta}\). We see that both MC simulation results and STAR data are in a good agreement with \(z\)-scaling predictions for \(p_T^\text{jet} > 25 \text{ GeV}/c\) \((z > 180)\). The value of the slope parameter \(\beta\) is found to be \(\beta = 6.01 \pm 0.06\) for Monte Carlo results at \(\sqrt{s} = 200 \text{ GeV}\). It is compatible with \(\beta = 5.95 \pm 0.21\) obtained at lower energies \(\sqrt{s} = 38.8, 45, 63 \text{ GeV}\) \([19]\). For precise test of the asymptotic behavior of \(\psi(z)\) predicted by the \(z\)-scaling and for verification of the property in the framework of QCD measurement of jet spectra in \(pp\) collisions at RHIC energies with higher accuracy for high \(p_T\) is necessary.

### 3.6 Self-similarity of charged particle production in \(AA\)

In this section we study the multiplicity dependence of \(p_T\) and \(z\) presentations of the experimental data \([40, 41]\) on inclusive cross sections of charged hadrons produced in heavy ion collisions at RHIC. The important ingredient of \(z\)-scaling is the multiplicity density \(dN_{\text{ch}}/d\eta(s, \eta)\) as a function of the collision energy \(\sqrt{s}\) and pseudorapidity \(\eta\). The scaling variable \(z\) is proportional to \(\sqrt{s} \left[ dN_{\text{ch}}/d\eta(s, \eta) \right]^{-1}\) at \(\eta = 0\) while the function \(\psi(z)\) is expressed via multiplicity density depending on the energy \(\sqrt{s}\) and pseudorapidity \(\eta\). Using the special selection of events the transverse hadron spectra were measured by the E735 collaboration at highest \(dN_{\text{ch}}/d\eta|_0 \simeq 26\) \([34]\). The strong sensitivity of spectra to \(dN_{\text{ch}}/d\eta\) was observed to increase with \(p_T\). The difference between cross sections corresponding highest and lowest multiplicity density was found to be about order of magnitude. The value of multiplicity density of selected events for \(\bar{p}p\) collisions was larger than \(dN_{\text{ch}}/d\eta|_0/(0.5N_p)\) measured in central nucleus-nucleus collisions at AGS, \(\text{SppS}\) and RHIC. The multiplicity density in central \(PbPb\) collisions at \(\eta = 0\) at LHC energies is expected to be about 8000 particles per unit of pseudorapidity. The regime of particle production at very high multiplicity density is believed to be more preferable for searching for clear signature of QGP formation.

Figure 5 shows the dependence of the spectra of charged hadron production in \(AuAu\) collisions on the transverse momentum \(p_T\) at the energy \(\sqrt{s} = 200 \text{ GeV}\) and over the pseudorapidity range \(|\eta| < 0.5\) for different centralities \([40]\). The data cover a wide transverse momentum range, \(p_T = 0.2 - 11 \text{ GeV}/c\). A strong sensitivity of high-\(p_T\) spectra to multiplicity density \(dN_{\text{ch}}/d\eta\) is observed. We use values of the fractal dimensions \(\delta_N = 0.7\) and \(\epsilon = 0.7\) to study the dependence of transverse spectra on multiplicity density \(dN_{\text{ch}}/d\eta\) in \(AuAu\) and \(CuCu\) collisions. The scaling behavior of the data (Fig.6(a)) are restored at \(c = 0.25\) and under the simultaneous transformation of the variable \(z\) and function \(\psi(z)\): \(z \rightarrow \alpha_p z\) and \(\psi \rightarrow \alpha_p^{-1}\psi\) \((\rho \equiv dN_{\text{ch}}/d\eta)\). The power behavior (the straight dashed line in Fig.6(a)) of the scaling function, \(\psi(z) \sim z^{-\beta}\), for high \(z\) is observed. The soft regime of particle production demonstrates self-similarity in \(z\) presentation for low \(z\) as well.

Figure 6(b) demonstrates \(z\) presentation of the transverse spectra of charged hadrons produced in \(AuAu\) and \(CuCu\) collisions in the central rapidity range at \(\sqrt{s} = 200 \text{ GeV}\).
Figure 6: Transverse momentum spectra of charged hadrons produced in \( AuAu \) and \( pp \) collisions at RHIC as a function of multiplicity density. Experimental data obtained by STAR Collaboration are taken from [40].

Figure 7: Transverse momentum spectra of charged hadrons produced in \( AuAu \) and \( CuCu \) collisions at RHIC in \( z \) presentation as a function of multiplicity density. Experimental data are taken from [40, 41].
Figure 8: Spectra of direct photons produced in $\bar{p}p$ collisions in $p_T$ and $z$ presentations. Experimental data obtained by D0 Collaboration in Run I and II are taken from [42].

as a function of multiplicity density. The data obtained by the PHOBOS collaboration [41] reach transverse momenta up to 4 GeV/c. The dashed lines are obtained by fitting of $\psi(z)$ corresponding to the STAR data [40] at $\sqrt{s} = 200$ GeV. For comparison scaling functions for different nuclei the transformation of $z \rightarrow \alpha A z$ and $\psi \rightarrow \alpha^{-1} A \psi$ ($\rho \equiv dN_{ch}/d\eta$) was used. The scaling was found to be restored at the same value of $c = 0.25$. The $z$ presentation demonstrates a good compatibility of the PHOBOS and STAR data over a kinematical range measured by the PHOBOS collaboration [41].

Thus based on the obtained results we conclude that mechanism of charged hadron production in $AuAu$ and $CuCu$ collisions at $\sqrt{s} = 200$ GeV reveals self-similarity and fractality over a wide range of transverse momentum and multiplicity density.

4 $z$-Scaling in $\bar{p}p$ collisions at Tevatron

In this section we present results of our analysis of new data obtained by the D0 and CDF Collaborations at Tevatron in Run II. We verify properties of $z$-scaling established in previous papers such as the energy and angular independence of the scaling function $\psi(z)$ for particle (hadrons, direct photons, jets) production at high $p_T$. The hypothesis of flavor independence of $\psi(z)$ for high $z$ is used for prediction of spectra of different particles ($J/\psi, \Upsilon, D^0, B^+, Z, W^+$) as well.

4.1 Direct photons

Recently the D0 collaboration published the new data [42] on inclusive cross sections of direct photons produced in $\bar{p}p$ collisions at $\sqrt{s} = 1960$ GeV. The data cover the momentum $p_T = 30 - 250$ GeV/c and pseudorapidity $|\eta| < 0.9$ range. The data together with data obtained by D0 in Run I are presented in Fig. 7(a). The strong angular dependence of cross section is observed. It increases with $p_T$ and reaches about order of magnitude at $p_T = 100$ GeV/c.

The $z$ presentation of the same data is shown in Fig. 7(b). One can see that new experimental data confirm features (the energy and angular independence of $\psi(z)$) of $z$-scaling for direct photon production in $\bar{p}p$ and $pp$ collisions established in [18, 43]. The
power law, $\psi(z) \sim z^{-\beta}$, is observed over a wide range of $z$. We consider the independence of the slope parameter $\beta$ on kinematical variables ($\sqrt{s}, p_T$ and $\eta$) as an evidence of self-similarity and fractality of photon production. It can mean that structure of a photon itself at small scales looks like structure of other particles (hadrons) characterized by fractal dimension(s) $2$.

4.2 Jets

In this section we present results of analysis of new data on inclusive cross sections of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 1960$ GeV obtained by the D0 and CDF collaborations at Tevatron [44] and compare them with our previous results [19].

Production of hadron jets at Tevatron probes the highest momentum transfer region currently accessible and thus potentially sensitive to a wide variety of new physics. The information on inclusive jet cross sections at high transverse momentum range is the basis to test QCD, to extract the parton distribution functions and to constrain uncertainties for gluon distribution in the high-$x$ range. In Run II, as mention in [45], the measurement of jet production and the sensitivity to new physics will profit from the large integrated luminosity and the higher cross section, which is associated with the increase in the center-of-mass energy from 1800 to 1960 GeV. Therefore the test of $z$-scaling for jet production in $\bar{p}p$ collisions in new kinematic range is of great interest to verify scaling features established in our previous analysis [19].

Figure 9(a) shows the new Run II data [44] on the inclusive jet cross section at $\sqrt{s} = 1960$ GeV. The pseudorapidity range covered by the D0 and CDF collaborations corresponds to $|\eta| < 0.8$ and $0.1 < |\eta| < 0.7$, respectively. The transverse momentum of jet changes from 50 up to 560 GeV. The $z$ presentation of data is shown in Fig.9(b). As seen from Fig.9(b) the D0 and CDF data are compatible each other. The energy independence of $\psi(z)$ is observed up to $z \approx 4000$. Asymptotic behavior of the scaling function is described by the power law, $\psi(z) \sim z^{-\beta}$ (the dashed line in Fig.9(b)). The slope param-

\[
\begin{align*}
\text{In paper [18] the fractal dimensions } \delta_1, \delta_2 \text{ are introduced as parameters for description of the fractal measure } z = z_0 \Omega^{-1}, \Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}.
\end{align*}
\]
Figure 10: Angular dependence of spectra of jet production in $\bar{p}p$ collisions in $p_T$ and $z$ presentations. Experimental data are taken from [44].

The angular dependence of inclusive cross section of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 1960$ GeV was investigated by the CDF collaboration [44]. The experimental data cover the rapidity range $|\eta| < 2.1$. The highest transverse energy carried by one jet was determined to be 600 GeV. As seen from Fig. 10(a) the transverse spectra demonstrate the strong dependence on pseudorapidity of produced jet. The $z$ presentation of the same data is shown in Fig. 10(b). It demonstrates the angular independence and the power behavior of $\psi(z)$. We would like to emphasize that these results are new confirmation of $z$-scaling. Jet production is usually considered as signature of hard collisions of elementary constituents (quarks and gluons). Therefore the obtained result means that interaction of constituents, their substructure and mechanism of jet formation reveal properties of self-similarity over a wide scale range (up to $10^{-4}$ Fm).

4.3 $b$-Jets

Flavor independence of $z$-scaling means that value of the slope parameter $\beta$ of the scaling function $\psi(z)$ is the same for different types of produced hadrons. The hypothesis is supported by results of analysis of hadron ($\pi^{\pm,0}, K, \bar{p}$) spectra for high $p_T$ in $pp$ and $p-A$ collisions [46]. The verification of the hypothesis is of interest for understanding of mechanism of particle production at very small scale. We assume that the transformation of a point-like quark ($u, d, s, c, b, t$) into real hadron produced at high $p_T$ is the self-similar process which is independent of the quark flavor.

The spectra of $b$-tagged jets were measured [47] by the D0 and CDF collaborations at $\sqrt{s} = 1800$ and 1960 GeV, respectively. Transverse energy of jet was measured in the range $25-400$ GeV. The data together with the CDF data obtained in Run I on inclusive cross section for non-tagged ($u_{jet}$) jets in $z$ presentation are shown in Fig. 11(a). Both data demonstrate power behavior of $\psi(z)$ for high $z$. Deviation from the power law is
observed for $z < 300^3$. The present calculation was performed\(^4\) with the values of the fractal dimensions $\delta_1 = \delta_2 = 1$. We use the universality of asymptotic behavior of $\psi(z)$ for non-tagged jets and pre-asymptotic behavior of $\psi(z)$ for $b$-tagged jets for construction of the scaling function over a wide range of $z$. Our predictions of inclusive cross sections of $b$-jet production in $\bar{p}p$ collisions at $\sqrt{s} = 200 - 14000$ GeV and the CDF data ($\circ$) are shown in Fig. 11(b).

### 4.4 $J/\psi$ mesons

Here we present results of analysis \(^{48}\) of the data \(^{49, 50}\) on inclusive cross section of $J/\psi$ production in $\bar{p}p$ collisions. The hypothesis of the flavor independence of $z$-scaling was used for the analysis. The experimental UA1 data \(^{51}\) on inclusive cross section of $\pi^0$-mesons produced in $\bar{p}p$ collisions at $\sqrt{s} = 540$ GeV were used to construct the scaling function in the asymptotic region. The scaling function of $\pi^0$ is shown by the dashed line in Fig.12(a). It is described by the power law, $\psi(z) \sim z^{-\beta}$. The value of the slope parameter $\beta$ was found to be $5.77 \pm 0.02$ over a wide $p_T$ range. To compare $z$ presentations for $J/\psi$ and $\pi^0$ the transformation of $z$ and $\psi$ in the form $z \rightarrow \alpha_F \cdot z, \quad \psi \rightarrow \alpha_F^{-1} \cdot \psi$ was applied. The coefficient $\alpha_F$ was found to be a constant (2.33). It was used to describe the overlapping region for both particles. Note that the CDF Run I and Run II data demonstrate a good matching in the region. Strong deviation of scaling function for $J/\psi$ from the power law is seen for $p_T < 10$ GeV/c. The energy independence of the scaling function was used to predict transverse spectra of $J/\psi$ production in $\bar{p}p$ collisions in the central rapidity range at the energy $\sqrt{s} = 63, 200, 630, 1800$ and 1960 GeV. The calculated results and the CDF data \(^{50}\) are shown in Fig. 12(b) by the dashed lines and points, respectively. One can see that the strong dependence of inclusive cross section on $\sqrt{s}$ enhances with $p_T$. Experimental test of the predicted results is of interest for

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\(^3\)We assume that for $s$- and $c$-tagged jets the deviation will be smaller and for $t$-tagged jets larger than for $b$-tagged jets.

\(^4\)The calculation procedure is described in \(^{19}\).
understanding of mechanism of $J/\psi$ production and properties of $z$-scaling.

4.5 $D^0$ mesons

Data on open charm production can provide understanding of mechanism of particle formation depending on the flavor quantum number and test of QCD predictions. They could give additional constraints on parton distribution and fragmentation functions of charmed quark. Such data allow us to study flavor dependent features of $z$-scaling.

The inclusive charm meson cross sections in $\bar{p}p$ collisions were measured by the CDF collaboration in the central rapidity $|y| < 1$ and transverse momentum $p_T = 5−20$ GeV/c range at $\sqrt{s} = 1960$ GeV [52]. Here we present results of analysis in the framework of $z$-scaling of the data corresponding to reconstructed decay mode $D^0 \rightarrow K^-\pi^+$. It was established that the prompt fraction of $D^0$ meson production for each $p_T$ bin is $(86.6 \pm 0.4)\%$. The hypothesis of the flavor and energy independence of $z$-scaling for $D^0$ meson production was used in the analysis. The data [51] on inclusive cross section of $\pi^0$-mesons produced in $\bar{p}p$ collisions at $\sqrt{s} = 540$ GeV and the CDF data [52] were used for construction of the scaling function of $D^0$ mesons over a wide range of $z$. The results of analysis are shown in Fig. 13(a). The dashed lines are obtained by fitting the data [51 52] in $z$ presentation. Figure 13(b) demonstrates transverse spectra of prompt $D^0$ meson production in $\bar{p}p$ collisions over the kinematical range $\sqrt{s} = 63−14000$ GeV, $p_T = 5−40$ GeV/c and $\theta_{cms} = 90^0$ predicted by $z$-scaling. The obtained results is of interest for comparison with QCD predictions and experimental data over a wider range of $p_T$.

4.6 $B^+$-mesons

Results of the first direct measurements of the $B$ meson differential cross sections in $\bar{p}p$ collisions at $\sqrt{s} = 1800$ GeV by measuring the mass and momentum of the $B$ meson decaying into exclusive final states were presented in [53]. The cross section was measured in the central rapidity region $|y| < 1$ for $p_T(B) > 6.0$ GeV/c. Here we analyze in the framework of $z$-scaling the data on transverse spectrum of $B^+$ mesons reconstructed via
Figure 13: Spectra of $D^0$ mesons produced in $\bar{p}p$ collisions in $z$ and $p_T$ presentations. Experimental data are taken from [51, 52].

Figure 14: Spectra of $B^+$ mesons produced in $\bar{p}p$ collisions in $z$ and $p_T$ presentations. Experimental data are taken from [51, 53].
4.7 $\Upsilon(1S)$-mesons

Features of heavy flavor production in high energy hadron collisions at high $p_T$ can be related to new physics phenomena at small scales. Understanding of flavor origin and search for similarity of particle properties depending on the additive quantum numbers (strangeness, charm, beauty, top) is fundamental problem of particle physics.

The differential cross sections of $\Upsilon$ production in $\bar{p}p$ collisions in the rapidity range $|y| < 0.4$ in $1S, 2S$ and $3S$ states at $\sqrt{s} = 1800$ GeV are presented in [54]. The three resonances were reconstructed through the decay $\Upsilon \to \mu^+\mu^-$. Transverse momentum of $\Upsilon$ was measured over the range $p_T = 0.5 - 20$ GeV/c. The shape of $p_T$ spectrum was found to be the same for all states. The data were noted to be important for the investigation of the bound state production mechanisms in $\bar{p}p$ collisions.

Figure 15 demonstrates results of our analysis of experimental data of $\Upsilon(1S)$ production in $\bar{p}p$ collisions. The scaling function of $\Upsilon(1S)$ (see Fig.15(a)) was constructed using both the $\Upsilon(1S)$ [54] and $\pi^0$ [51] transverse spectra. For low $z$ the scaling function $\psi(z)$ deviates very strongly from the power law shown by the dashed line. The slope parameter of $\psi(z)$ changes the sign with positive on negative at $z \approx 3$. Our predictions of inclusive cross sections over a range $\sqrt{s} = 63 - 14000$ GeV, $\theta_{\text{cms}} = 90^\circ$ and $p_T = 1 - 40$ GeV/c are shown in Fig. 15(b). Comparison of the obtained spectra with experimental data and QCD results are of interest for verification of the flavor independence of $z$-scaling and test
of mechanisms of vector meson production described in the QCD theory.

4.8  \(Z^0, W^+\) bosons

Vector \(Z\) and \(W\) bosons are carriers of electroweak and strong interactions. Therefore high precise data on transverse spectra for both \(Z\) and \(W\) bosons produced in high energy \(\bar{p}p\) collisions can provide an important tests and a direct confirmation of the unified model of the weak, electromagnetic and strong interactions (the Standard Model). Quantum chromodynamics ascribes the transverse momentum of the vector bosons produced in \(\bar{p}p\) collisions to associated production of one or more gluons or quarks with the boson. Therefore data on the differential cross sections for boson production can provides an important test of our understanding of mechanism of boson production described in the framework of QCD. The large mass of the vector bosons assures a large energy scale for probing perturbative QCD with good reliability. It also provides bounds on parametrizations of the parton distribution functions used to describe the nonperturbative regime of QCD processes. Deviations from the prediction for high \(p_T\) could indicate new physics phenomena beyond the Standard Model.

Measurements of the differential cross sections for \(Z\) and \(W\) boson production as a function of transverse momentum in \(\bar{p}p\) collisions at \(\sqrt{s} = 1800\) GeV over a range \(p_T = 1 - 200\) GeV/c are presented in [55, 56]. The \(W\) and \(Z\) bosons were detected through their leptonic decay modes (\(W \rightarrow e\nu, Z \rightarrow e^+e^-\)).

We assume that asymptotic behavior of the scaling function of vector bosons and direct photons produced in \(\bar{p}p\) collisions is similar. It describes by the power law for high \(z\) with the same value of the slope parameter \(\beta\) of \(\psi(z)\) for both particles. Direct confirmation of the similarity of direct and virtual photon (Drell-Yan pair) and vector boson production for high \(p_T\) is considered as an important feature of constituent interactions.

The asymptotic behavior of \(\psi(z)\) of direct photons was used to construct \(\psi(z)\) of vector bosons over a wide range of \(z\). Figures 16(a) and 17(a) demonstrate \(z\) presentation of data on inclusive transverse spectra for vector bosons [55, 56] and direct photons [18]. Note that the overlapping range of \(\psi(z)\) for the \(W^+\) boson and \(\gamma\) is large enough. Nevertheless the direct measurements of transverse spectra of the \(W^+\) bosons for \(p_T > 200\) GeV/c are necessary for verification of the assumption. Figures 16(b) and 17(b) show the dependence of inclusive cross sections of \(Z\) and \(W^+\) bosons produced in \(\bar{p}p\) collisions over a range \(\sqrt{s} = 200 - 14000\) GeV, \(p_T = 1 - 500\) GeV/c and \(\theta_{\text{cms}} = 90^\circ\).

5  Conclusions

The results of recent analysis in the framework of \(z\)-scaling of experimental data on inclusive cross sections of particle production in \(pp, AA\) and \(\bar{p}p\) collisions at high \(p_T\) at RHIC and Tevatron were reviewed. Physical concept of \(z\)-scaling and interpretation of the function \(\psi(z)\) and variable \(z\) were discussed. It was shown that the generalized concept of \(z\)-scaling allows us to study the multiplicity dependence of particle spectra and restore the multiplicity independence of the scaling function. The properties of \(z\) presentation of experimental data were verified. We consider that the properties established in the present analysis of data reflect general features of the structure of the colliding objects, interaction of their constituents and mechanism of particle formation.

The hypothesis on universality of asymptotic behavior of the scaling function (for \(\pi^0\)-mesons, direct photons) was used for construction of \(\psi(z)\) for \(J/\psi, \Upsilon(1S), D^0, B^+, Z, W^+\)
Figure 16: Spectra of $Z^0$ bosons produced in $\bar{p}p$ collisions in $z$ and $p_T$ presentations. Experimental data are taken from [43, 55].

Figure 17: Spectra of $W^+$ bosons produced in $\bar{p}p$ collisions in $z$ and $p_T$ presentations. Experimental data are taken from [43, 56].
particles and predictions of transverse spectra over a wide range of $\sqrt{s}$ and $p_T$. The obtained results support main idea of $z$-scaling based on property of self-similarity of particle interactions for high $p_T$.

We consider that the $z$-scaling may be used as a tool for searching for new physics phenomena beyond the Standard Model in hadron and nucleus collisions at high transverse momentum and high multiplicity at RHIC, Tevatron and LHC.

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