Two-fold theoretical and experimental misinterpretation of luminosity in the origin of Hubble constant tension

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March 4, 2022

ABSTRACT

The comparison of the intrinsic luminosity of extragalactic objects detected at different redshifts requires the evaluation of the observed flux in a common rest-frame emission band. It is usually performed by adding a term to the observed magnitudes denominated K-correction. In this paper, the K-correction transformation has been carefully scrutinized. As a result of this inspection, it has been found that K-corrections systematically downgrades the measured energy flux by a factor of (1+z). C-correction, a more appropriate rest-frame magnitude transformation has been derived. C-correction has been applied to the re-analysis of JLA (Joint Light-curve analysis) supernovae sample. The best cosmological fit to JLA type Ia supernovae (SNIa) sample calibrated with CMB Planck h = 0.675 for the common luminosity-angular distances relation $L_2 = D_L(1+z)^2$ corresponds to a closed Universe $L_2/h_0 = 0.48, \Omega_m = 2.25$. Nevertheless, considering the new luminosity-angular distances relation $L_1 = D_A(1+z)$ that accounts for expansion lensing, the new SNIa analysis converges to $L_1(h_0 = 0.7358, \Omega_m = 0.035, \Omega_\Lambda = 0.965)$, providing a coherent explanation of the Hubble constant ($H_0$) tension between Planck and SH0ES ($h = 0.7422 \pm 0.0182$) results.

Key words. Cosmology: theory – Cosmology: observations – Galaxies: distances and redshifts – cosmological parameters – supernovae – dark energy

1. Introduction

About a century ago were established the foundations of cosmology. The field equations of general relativity (Einstein (1915)) and the FLRW metric (Friedmann (1922), Lemaître (1927, 1931), Robertson (1933) and Walker (1937)) provided the theoretical framework against data is contrasted. A proper definition of cosmological distances and an adequate treatment of data are key points to obtain true conclusions on the evolution and fate of the Universe.

Since the discovery of the expansion of the Universe by Hubble (1929), many astronomical surveys have been performed to determine the expansion rate. The goal during the eighties and well into the nineties was to measure the Hubble constant and the deceleration parameter for the Cold Dark Matter (CDM) universe model (Peebles (1982), Bond et al. (1982), Blumenthal et al. (1982), Blumenthal et al. (1984)).

Unexpectedly, in 1998 two independent groups (Riess et al. (1998), Perlmutter et al. (1999)) discovered an acceleration in the expansion of the Universe, compatible with a solution of Einstein’s field equations based on the cosmological constant $\Lambda$. A new component of the Universe – dark energy – was assumed as responsible of the accelerated expansion and the model is currently known as $\Lambda$CDM or Standard Model of cosmology. The results have been apparently confirmed with a later and more complete type Ia supernovae (SNIa) sample as JLA (Betoule et al. (2014)).

An important fact to point is that these results rest on K-correction, a process to transform the measured flux of galaxies at different bandpass to a common rest-frame band where fluxes are comparable. In the first part of this paper, it is shown that K-correction systematically downgrades the energy flux by a factor (1+z) with respect to what should be expected in an innocuous rest-frame transformation. This misleading process poisons the SNIa luminosity distance-redshift relation. Therefore, C-correction, a more appropriate rest-frame transformation is derived.

The second part of this paper focuses on determining the cosmology that best fit to luminosity distance data once the proper C-correction has been applied. To achieve this goal, the supernovae JLA sample is properly amended with C-correction. The proper C-corrected JLA sample predicts a closed universe for the standard luminosity-angular distance relation $d_L = d_A(1+z)$ (De Vicente-Albendea (2020a), De Vicente-Albendea (2020b)). A relevant derived result from this analysis is a coherent explanation of the Hubble constant tension.

The rest of the paper is organized as follows: Section 2 introduces the luminosity distance equations for the standard model and expansion lensing paradigm. Section 3 shows how K-correction introduces undesirable amounts in the measurement of flux energy. Section 4 derives C-correction, a proper innocuous rest-frame transformation. Section 5 analyses the fit of a SNIa sample to different cosmologies. In Section 6 the analysis is reproduced with the public cosmoSIS tool. Discussion is performed in Section 7. The conclusions are presented in Section 8.
2. Luminosity-Angular distances relation: Standard Model (L2) vs Expansion Lensing (L1)

Along with the main equations of cosmology there are several distances defined to link the theory with the observational data. Let us to reproduce here a brief summary of the distances and its relation with cosmological models described by relative densities \( \Omega_M, \Omega_r, \Omega_\Lambda, \Omega_k \) for matter, radiation, cosmological constant and curvature respectively (Hogg 1999).

Let \( E(z) \) be the function defined as:

\[
E(z) = \sqrt{\Omega_k (1+z)^2 + \Omega_r + \Omega_M (1+z)^3 + \Omega_\Lambda (1+z)^4}
\]

The Line of sight Comoving Distance \( D_C \) is defined by

\[
D_C(\Omega_k) = \frac{H_0}{H(z)} \int_0^z \frac{dz'}{E(z')}
\]

where \( \Omega_k \) remarks the dependence on relative densities and where

\[
D_H = c/H_0 = 3000h^{-1}\text{Mpc}
\]

is the Hubble distance being

\[
h = \frac{H_0}{100\text{km/s/Mpc}}
\]

the dimensionless Hubble constant used below.

The Transverse Comoving Distance \( D_M(\Omega_k) \) is defined by

\[
D_M = \begin{cases} 
\frac{D_H}{\sqrt{\Omega_k}} \sinh \left[ \sqrt{\Omega_k} D_C / D_H \right] & \text{for } \Omega_k > 0 \\
D_H \sin \left[ \sqrt{\Omega_k} D_C / D_H \right] & \text{for } \Omega_k = 0 \\
\frac{D_H}{\sqrt{\Omega_k}} \sinh \left[ \sqrt{\Omega_k} D_C / D_H \right] & \text{for } \Omega_k < 0 
\end{cases}
\]

On the other hand, the Luminosity Distance defines the relation between the bolometric flux energy \( f \) received at earth from an object to its bolometric luminosity \( L \) by means of

\[
f = \frac{L}{4\pi D_L^2}
\]

The standard luminosity-angular distances relation is given by

\[
D_{L2} = D_A (1+z)^2 \quad \text{(standard model)}
\]

Note the subscript \( L2 \) to emphasize the exponent 2 for \((1+z)\). Hereafter, we will refer to \( L2 \) there where equation 7 is assumed. Consequently, the relation between luminosity distance \( D_L \) and the transverse comoving distance \( D_M \) is given by

\[
D_{L2} = D_M (1+z) \quad \text{(standard model)}
\]

Recently, Expansion Lensing (EL) paradigm was presented. EL demonstrates that the transverse comoving distance \( D_M \) from FLRW metrics meets the inverse square law and therefore it is equal to \( D_L \).

Let \( D_{L1} = D_M \), for \((1+z)\)

The Expansion Lensing luminosity-angular distances relation is then

\[
D_{L1} = D_A (1+z) \quad \text{(expansion lensing)}
\]

Note the subscript \( L1 \) to emphasize the exponent 1 for \((1+z)\). Hereafter, we will refer to \( L1 \) there where Eq 10 is assumed. Eq. [8] and Eq. [9] provide the link between a measurable amount \( D_L \) and the densities of the components of the Universe through \( D_M(\Omega_M,\Omega_r,\Omega_\Lambda,\Omega_k) \) for \( L2 \) and \( L1 \) luminosity-angular distances relations respectively.

Fig. 1 shows the theoretical predictions for different cosmological models within \( L2 \) and \( L1 \) luminosity-angular distances relations. In principle, any of them can agree to \( A(\Lambda CDM, \Omega_M = 0.3, \Omega_\Lambda = 0.7) \), where the symbol “A” stress the angular origin of this prediction, which can be fulfilled by CMB or BAO data. Therefore, there are two alternative luminosity-angular distances relations \( L2 \) and \( L1 \) for the angular model \( A \), being already appeared tensions on the standard \( L2 \). This tension appears in the value of \( H_0 \) with differs in several between the CMB Planck \( H_0 = 0.674 \pm 0.005 \) (Aghanim et al. 2018) and supernovae SH0ES \( h = 0.7422 \pm 0.0182 \) (Riess et al. 2019).

This paper reveals the theoretical and experimental origin of \( L2 \) tension, supporting coherent arguments towards the reliability of \( L1 \). The main cosmological test related to luminosity corresponds to type Ia Supernovae probe. There are many publications providing an agreement of SNIa luminosity data with the Standard Model LCDM. Unfortunately, SNIa data reduction relies on K-correction. In this research, we have detected large systematical errors introduced by a misleading K-correction. The following sections treat the problem and its consequences.

3. Revising K-correction

Extracting uniform data among the complete supernovae sample is a complex process due the eventuality of the phenomenon
and the large range of cosmological redshifts at which they may appear. A key process of data reduction is the k-correction. The goal of k-correction is to transform a magnitude measured on the observed-frame bandpass \( R \), to a common rest-frame bandpass \( Q \) where the magnitudes of objects with different redshifts are comparable. Along the years, different authors have provided formulae to compute k-correction for galaxies (Oke & Sandage (1968)) and supernovae (Kim et al. (1996), Nugent et al. (2002)).

Leaving apart the zero point terms, let us to reproduce Eq. 2 from Kim et al. (1996) as

\[
K_{m}^{Q} = 2.5 \log(1+z) + 2.5 \log \left( \frac{\int F(\lambda) S_{e}(\lambda) d\lambda}{\int F(\lambda/1+z) S_{e}(\lambda) d\lambda} \right)
\]  

where the superscript \( m \) has been added to indicate magnitude correction. Let us to arrange the terms for the analysis in the form

\[
K_{m}^{Q} = 2.5 \log \int \frac{F(\lambda) S_{e}(\lambda) d\lambda}{F(\lambda) S_{e}(\lambda) d\lambda} = 2.5 \log \int \frac{F(\lambda) S_{e}(\lambda) d\lambda}{F(\lambda) S_{e}(\lambda) d\lambda'}
\]

where

\[
\lambda' = \lambda/(1+z) \quad \text{and} \quad d\lambda' = d\lambda/(1+z)
\]

and \( F_{e} \) is the spectral energy distribution (SED) in the emission or rest-frame. In the simplest case where there is no absorption, \( S_{e}(\lambda) = S_{\lambda}(\lambda) = 1, K_{m}^{Q} = 0 \), one obtains

\[
\int F(\lambda) d\lambda = \int F(\lambda') d\lambda'
\]

expression that enters in deep conflict with a simple change of variable based on Eq. [13] that should give

\[
\int F(\lambda) d\lambda = \int F(\lambda') (1+z) d\lambda' = \int (1+z) F(\lambda') d\lambda'
\]

Therefore, k-correction constructs – through Eq. [14] – the spectral energy distribution (SED) in the rest-frame \( F_{e}(\lambda') \) by simply blue-shifting the SED from the observed-frame \( F(\lambda) \) to the rest-frame \( F_{e}(\lambda') \), and assuming the misleading transformation

\[
F_{e}(\lambda') = (1+z) F(\lambda')
\]  

rather than

\[
F_{e}(\lambda') = F(\lambda')
\]

Fig. 2 shows how any integration of \( F_{e}(\lambda) \) in the observed-frame (i.e. the measured energy) is degraded by k-correction into a lower value in the corresponding rest-frame \( F_{e}(\lambda') \) integration.

In the next section, C-correction is derived. C-correction includes a more appropriate SED rest-frame representation accounting for Eq. [17], which preserves the measured energy in rest-frame transformations.

## 4. C-correction: a more appropriate rest-frame transformation

In this section, C-correction is derived. C-correction performs a proper SED rest-frame representation.

Let \( f_{Q} \) be the flux measured in the observed-frame bandpass \( R \). We are interested in obtaining \( f_{Q} \), the flux represented in the rest-frame bandpass \( Q \). C-correction is computed as the ratio between \( I_{Q} \) and \( I_{R} \), being \( I_{Q} \) the rest-frame SED \( F_{e}(\lambda_{o}) \) integrated in the rest-frame bandpass \( Q \), and being \( I_{R} \) the observed-frame SED \( F_{e}(\lambda_{o}) \) integrated in the observed-frame bandpass \( R \). That is

\[
f_{Q} = \frac{I_{Q}}{I_{R}} = C_{RQ} f_{R}
\]

where

\[
C_{RQ} = \frac{I_{Q}}{I_{R}} = \frac{\int Q(\lambda_{o}) F_{e}(\lambda_{o}) d\lambda_{o}}{\int R(\lambda_{o}) F_{e}(\lambda_{o}) d\lambda_{o}}
\]
A key question is how the observed-frame \( F_o(\lambda_o) \) is transformed into the rest-frame \( F_r(\lambda_r) \). To establish this relation, we focus on the simplest hypothetical case where no bandpass absorption exists, i.e. \( R = Q = 1 \). In that case, \( C_{RQ} \) should be equal to one to preserve the measured energy in an innocuous transformation between frames. Thus

\[
\int F_o(\lambda_o) d\lambda_o = \int F_r(\lambda_r) d\lambda_o
\]

Making a change of variable \( \lambda_o \) by \( \lambda_r \) on the second part of Eq. 20 according to

\[
\lambda_o = (1+z)\lambda_r \quad \text{and} \quad d\lambda_o = (1+z)d\lambda_r
\]

we have

\[
\int F_o(\lambda_r) d\lambda_r = \int F_r(\lambda_r)(1+z)d\lambda_r
\]

and therefore

\[
F_r(\lambda_r) = (1+z)F_o(\lambda_r)
\]

Fig. 2 shows how the C-correction transformation (Eq. 23) or Eq. [17] in [Kim et al. (1996)] notation) from the observed-frame \( F_o(\lambda_o) \) to the rest-frame \( F_r(\lambda_r) \) preserves the measured energy, i.e. the integrated area below the observed-frame SED curve. Therefore, C-correction \( (C_{RQ}) \) is the transformation to be applied to convert flux energy between frames.

Multiplying by \(-2.5\) and taking logarithms in Eq. 19, we can obtain the expressions in magnitudes

\[
m_{Q} = C_{RQ}^m + m_R
\]

where

\[
C_{RQ}^m = 2.5 \log \frac{\int R(\lambda_o)F_o(\lambda_o)d\lambda_o}{\int Q(\lambda_r)F_r(\lambda_r)d\lambda_r}
\]

Finally, the relation between k-corrections and C-corrections for magnitudes becomes

\[
C_{RQ}^m = K_{RQ}^m - 2.5 \log(1+z)
\]

5. **SNIa COSMOLOGICAL FIT WITH C-CORRECTION**

SNIa appears as the explosion of a carbon-oxygen white dwarf in a binary system as it reaches the Chandrasekhar limit due to accretion. SNIa provide a very uniform population of exploding stars with similar absolute magnitude at peak luminosity. Nowadays, some studies show a correlation between the peak luminosity and how quickly the type Ia light curve decays in the 15 days after maximum light in the B band. Based on this property, they are now considered standardizable candles by means of some nuisance parameters not fully understood yet (Betoule et al. [2014]). In the last three decades, numerous SNIa data analyses have been carried out. In this section, a basic analysis of the luminosity distance-redshift relation on JLA sample

\[
\gamma = h \sqrt{L}
\]

Once the optimal value \( \gamma_{opt} \) is obtained for the best fit, it can be transformed to Hubble constant \( h_{opt} \) considering \( \gamma_{cmb} \) as the best fit to \( A(\Omega_M = 0.315, \Omega_\Lambda = 0.685, h_{\text{Planck}} = 0.674) \) by means of

\[
h_{opt} = \frac{\gamma_{opt}}{\gamma_{(\Omega_M=0.315, \Omega_\Lambda=0.685)}} h_{\text{Planck}}
\]
The value of $m_b$ in Eq. (27) is the rest frame B band magnitude at the peak. Two sets of $d_L$ values from JLA data are generated depending on whether K-correction or C-correction is applied. In the first case, $m_b$ takes the value of the rest-frame magnitude $m_b^0$ provided by the catalog that incorporates, according to the common practice, the K-correction prescribed by Oke & Sandage (1968) and Kim et al. (1996). In the second case, the C-correction described in the previous section is applied (Eq. 26). The luminosity distance is evenly fitted along 20 redshift bins within the range $z = (0.0, 1.0)$. The algorithm searches for the values of $\gamma$, $\Omega_M$ and $\Omega_L$ from $d_L$ (Eq. 4, Eq. 8 and Eq. 9) that minimize

$$
\chi^2 = \sum_n \frac{(d_L(z_i) - D_L(z_i))^2}{\sigma_i^2}
$$

where $d_L(z_i)$ is the luminosity distance from JLA sample averaged over the redshift bin $i$. Note also that it is assumed $\Omega_k = 0$ as in the standard model.

Fig. 3 shows how the misleading K-corrected data follow the standard $L_2(\Lambda CDM, 0.3, 0.7)$ prediction. In this case the nuisance parameters have been included to reproduce faithfully the fiducial SNIa results. Note that in spite of the good agreement, it is a misleading result since both data treatment and theoretical prediction have some unnoticed flaws.

Fig. 3(a) shows how the outlook changes completely when the proper C-correction is applied. In this case, the agreement of SNIa data with $L_2(\Lambda CDM, 0.3, 0.7)$ becomes too poor and the best fit within the standard L2 becomes $L_2(h_{L2} = 0.48, \Omega_M = 2.25)$, i.e. a deeply closed universe. Therefore, a huge discordance appears between L2 luminosity probes and angular ones as CMB, enlarging the Hubble tension. This discordance points directly to a fault of the standard L2 luminosity-angular distances relation.

Fig. 3(b) shows the best fit for JLA with C-correction within the luminosity-angular distances relation $L_1$. It corresponds to $L_1(\Omega_L = 0.965, \Omega_M = 0.035)$. Regarding the value of $h$ one can calibrate $\gamma_{opt} = 4443$ by Eq. 30 with the value of $\gamma = 3962$ obtained in the fit to $L_1(\Omega_L = 0.685, \Omega_M = 0.315)$ whose Hubble constant is known by Planck satellite, $h_{Planck} = 0.674$. Thus, the Hubble constant for SNIa probe with C-corrections in $L_1$ is

$$
h_{L1} = \frac{4443}{3962} (0.674 \pm 0.0050) = 0.7558 \pm 0.0056
$$

which is compatible with the value $h_{SH0ES} = h = 0.7422 \pm 0.0182$ measured by Riess et al. (2019) in the local universe. On the other hand, the corresponding baryonic matter for $h_{L1}$, calibrated with the values $(h_{Planck} = 0.674, \Omega_b = 0.049)$ of Planck satellite can be computed as

$$
\Omega_b = \left(\frac{h_{Planck}}{h_{L1}}\right)^2 0.049 = 0.039
$$

that is close to the value $\Omega_b = 0.035$ obtained in the fit of SNIa data to $L_1$, leaving small margin for non-baryonic matter.

### 6. Redundant analysis with CosmoSIS

Complementary to these results, the CosmoSIS (Zuntz et al. (2015)) framework has been applied to SNIa JLA sample. Example 5 of Cosmosis performs the JLA analysis for a flat universe obtaining the $\Lambda CDM$ prediction within the Standard Model (Howlett et al. (2012), Lewis et al. (????), Betoule et al. (2014)). Note that these data include the misleading K-correction treatment. Example 5 has been slightly modified to account for C-correction. In addition, the file cambinterface.F90 from cosmosis-standardlibrary has been modified including the luminosity distance relation $D_L = D_M$ derived from the expansion lensing paradigm. The cosmological parameters obtained deactivating the nuisance parameters are $\Omega_M = 0.033, \Omega_L = 0.967$, $h = 0.7372$, close to those provided above. Fig. 5 shows the 1σ
angular distance drives to a closed universe to JLA supernovae sample within the standard luminosity-frame transformation is derived. The application of C-correction supernovae data analysis results. calibrated luminosity distance-redshift relation, compromising previous (1 + z).

Therefore, a coherent explanation on the Hubble constant tension is demonstrated within the expansion lensing paradigm. Large systematics or misconceived modeling details in one or both methods still remains to be solve to further convergence between both methods.

Acknowledgements. Funding support for this work was provided by the Autonomous Community of Madrid through the project TEC2SPACE-CM (S2018/NMT-4291).

References
Aghanim, N. et al. 2018, VI. Cosmological parameters
Betoule, M. e. a., Kessler, R., Guy, J., et al. 2014, Astronomy & Astrophysics, 568, A22
Blumenthal, G. R., Faber, S., Primack, J. R., & Rees, M. J. 1984, Nature, 311, 517
Blumenthal, G. R., Pagels, H., & Primack, J. R. 1982, Nature, 299, 37
Bond, J. R., Szalay, A. S., & Turner, M. S. 1982, Physical Review Letters, 48, 1636
De Vicente-Albendea, J. 2020a, arXiv preprint astro-ph/2003.05307
De Vicente-Albendea, J. 2020b, arXiv preprint astro-ph/2003.06139
Einstein, A. 1915, Sitzung der physikalisch-mathematischen Klasse, 25, 844
Friedmann, A. 1922, Zeitschrift fur Physik, 10, 377
Hogg, D. W. 1999, arXiv preprint astro-ph/9905116
Howlett, C., Lewis, A., Hall, A., & Challinor, A. 2012, Astropart. Phys, 4, 1201
Hubble, E. 1929, Proceedings of the National Academy of Sciences, 15, 168
Kim, A., Goobar, A., & Perlmutter, S. 1996, Publications of the Astronomical Society of the Pacific, 108, 190
Lemaître, G. 1927, Ann. Soc. Sci. Bruxelles, Ser. 1, 47, 49
Lemaître, G. 1931, Monthly Notices of the Royal Astronomical Society, 91, 483
Lewis, A., Challinor, A., & Lasenby, A. 2000, Astrophysics, 1, 353, 473
Nugent, P., Kim, A., & Perlmutter, S. 2002, Publications of the Astronomical Society of the Pacific, 114, 803
Oke, J. B. & Sandage, A. 1968, The Astrophysical Journal, 154, 21
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, The Astrophysical Journal, 517, 565
Riess, A. G., Casertano, S., Yuan, W., Macri, L. M., & Scolnic, D. 2019, The Astrophysical Journal, 876, 85
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, The Astronomical Journal, 116, 1009
Robertson, H. P. 1933, Reviews of modern Physics, 5, 62
Walker, A. G. 1937, Proceedings of the London Mathematical Society, 2, 90
Zuntz, J., Paterno, M., Jennings, E., et al. 2015, Astronomy and Computing, 12, 45

Fig. 5: Best fit for SNIa JLA sample with Cosmosis once C-correction and expansion lensing L1 have been applied.

and 2σ contours for $H_0$ and $\Omega_M$. The test with nuisance parameters drives to similar conclusion given below.

7. Discussion
Note that in the present analysis, $h$ value is extracted by fitting JLA SNIa data up to redshift $z \sim 1$ and calibrating with Planck $H_0$ measurement. The value obtained is close to the one measured by SH0ES. Therefore, a coherent link between the values of the Hubble constant in L1 is found providing a new insight into the Hubble constant tension. The two sets of data CMB and SNIa are fitting to different cosmologies within L1, giving therefore different but coherent values of $H_0$. In this situation, one or both datasets should have remaining unnoticed systematic effects that makes them diverge. In the case of SNIa, one can consider the common nuisance parameters coming from the correlation between the peak luminosity and light curve decays. Nevertheless, other relation is required since the current one drive to a universe with $\Omega_L \sim 1$ that goes against the existence of at least some percentage of (baryonic) matter. On the side of CMB and other angular cosmological probes, many nuisance parameters are introduced depending on the assumptions of the underlying cosmology. Testing dark energy plus baryonic matter model ($\Lambda$CDM) on these probes should be of great interest given the results presented in this paper.

8. Conclusions
The $\Lambda$CDM model emerged at the end of the last century as the response to the apparent acceleration in the universe expansion found by two independent groups in supernovae type Ia data analysis. A new component of the Universe, the dark energy, was included in the standard model to explain such behaviour.

This paper revises k-correction, a key process of supernovae data reduction. It is demonstrated that k-correction systematically downgrades the measured supernovae flux by a factor $(1 + z)$. This misleading transformation poisons the cosmological luminosity distance-redshift relation, compromising previous supernovae data analysis results.

C-correction, a more appropriate supernovae magnitude rest-frame transformation is derived. The application of C-correction to JLA supernovae sample within the standard luminosity-angular distance drives to a closed universe $L2(h = 0.48, \Omega_M = 0.25)$ enlarging the discrepancy with CMB (angular) probe. On the contrary, the analysis within the expansion lensing paradigm produces $L1(h = 0.7558, \Omega_M = 0.035, \Omega_L = 0.965)$ when $h$ is calibrated with Planck cosmology $L1(h = 0.674, \Omega_M = 0.315, \Omega_L = 0.685)$, i.e., the SH0ES $H_0$ value is closely recovered when calibrating with Planck $H_0$.

Therefore, a coherent explanation on the Hubble constant tension is demonstrated within the expansion lensing paradigm. Large systematics or misconceived modeling details in one or both methods still remains to be solve to further convergence between both methods.