Analysis of follower motion law and optimization of fitting function in reverse design of cam

Zhaoping Ji
School of Mechanical Engineering, Nantong Vocational University, Nantong 226007, P. R. China
E-mail: zpji@foxmail.com

Abstract. Because of its compact structure and reliable performance, the cam mechanism has been widely used in automatic machinery, such as automatic tool changing system, packaging machinery, automatic production line, automatic switching system and so on. At present, a variety of common drive and control devices have been developed with arc cam as the core component. The cam indexing mechanism is a new type of transmission device to realize high speed and high precision intermittent indexing motion. It is widely used and has been studied by more and more scholars at home and abroad, and has achieved remarkable results. At present, its theoretical research has matured. However, because the profile of indexing cam is a non-spiral surface in space, it brings inconvenience to the design and manufacture of cam. When the arc cam is damaged and the design parameters of the original arc cam cannot be known, the original arc cam profile can only be obtained by measuring limited data. In the original parameterized design method, the profile of the arc cam is determined by the motion law of the follower. If the motion law curve of the follower can be obtained, the contour curve of the arc cam can be obtained in reverse. So the problem of solving the profile surface of the cam can be transformed into a follower which meets the requirements of the curve of motion law. Taking the CAM Z175 as an example, this paper describes in detail the whole process of motion rule analysis, optimization of the motion law curve and simulation verification in CAM reverse design.

1. Introduction
With the rapid development of society and the development of science and technology, all kinds of automatic machinery is needed much higher requirement for performance. Cam mechanism is widely used in all kinds of machines because of its high precision and high efficiency [1]. However, the cam components in the mechanism are extremely easy to wear during long-term use, so they are replaced frequently. For the reasons of protecting, these machines generally only give schematic diagram and structure diagram, and do not give the parameters of components. It takes a long time to contact foreign manufacturers to get accessories, and sometimes foreign manufacturers may not sell them alone. In order to replace the cam components when needed and reduce the production cost of enterprises, manufacturers are eager to reverse design fragile cam parts and try to produce qualified parts. Using reverse design and optimizing on this basis is the best way to solve such problems. Following is an example of the worn grooved cam Z175 (Figure 1) to discuss how to analyze the motion law of cam follower in reverse design, optimize the fitting function and verify the simulation.
2. Cam follower motion law and wear analysis problem description

2.1. Cam follower motion law
In the reverse design of cam, the motion law of follower is obtained according to the shape of the fitted surface. The motion law of follower can only be close to the original motion law. Therefore, in the reverse design, the fitting function should be adjusted according to the actual situation. Under the premise of not violating the law of lift movement of the follower, the acceleration of the follower is minimized and the impact on the cam is reduced, so as to reduce the wear and improve the service life of the cam.

The overall motion law of the follower of groove cam Z175 is as follows:

1) The curved surface of the long and near stops is projected as arc curve. The follower is in a static state in these two stages. The lift \( h \) of the follower is 0, the velocity \( V \) is 0, and the acceleration \( a \) is 0.

2) The projection curves of the surface of the push and return sections are non-circular curves. The motion law of the follower needs to be analyzed according to the fitting function and optimized according to the wear condition.

2.2. Cam wear condition
The cam controls the loading and unloading motion of the workpiece as shown in Figure 2. In the push section, since the cam follower holds the workpiece, the load increases from the minimum to the maximum, and the impact is strong, so the push section is seriously worn. For the return section, the wear of the cam is much smaller than that of the push section because of loose clamping, relatively small load, insufficient change of acceleration and weak impact. Others, such as the far stop section and the near stop section, have very slight wear due to the constant acceleration of the follower. According to the wear situation, it is necessary to optimize the curve fitting function of the surface projection when analyzing the movement law of the follower, so as to reduce the impact, reduce the cam wear and improve the service life.

Figure 1. Basic structure of grooved cam Z175.
3. Analysis and optimization of motion law of cam follower

3.1. Analysis of the Motion Law of the Third-Order Fitting Function of Least Squares Method

First, based on the point cloud data obtained by the 3D laser scanner, the characteristics and size data of the grooved cam are reversed. The numerical analysis method such as least squares method and Lagrangian difference method is used to focus on the curve projection curve function of the grooved cam push segment [2-3]. Based on the above method, the best fitting function of the curve projection curve of the cam derivation segment can be obtained as the least squares third-order polynomial fitting function, as follows:

\[ y = 0.0008x^3 - 0.066x^2 + 1.958x + 62.2903 \]

The first derivative is as follows:

\[ y' = 0.0024x^2 - 0.132x + 1.958 \]

The second derivative is as follows:

\[ y'' = 0.048x - 0.132 \]  \hspace{1cm} (2-1)

Let \( x = \rho \cos \phi \), \( y = \rho \sin \phi \). The fitting function is transformed from rectangular coordinate system to polar coordinate system, so as to facilitate the analysis of follower motion law. Among them, \( \rho \) is the cam direction and \( \phi \) is the cam angle. Since the grooved cam is a counter-center cam, \( \rho \) can be regarded as the lift of the follower, and \( \phi \) can be regarded as the corner and the central angle of the cam projection curve with respect to the axis. According to the definition of the polar coordinate system, the meaning of time derivation is as follows: \( \rho' = v, \) \( \phi' = a \) is the speed of the follower, \( a \) is the acceleration of the follower, \( \phi'' = \omega \), \( \omega \) is the angular velocity of the cam, and there are \( \rho \omega = \phi \). The derivatives of the curved surface projection function are transformed into polar coordinate functions. The first and second derivatives of the time in the polar coordinate form of \( Y \) are as follows:

\[ y' = (\rho \cos \phi)' = v \cos \phi - \phi \sin \phi \]
\[ y'' = (v \cos \phi - \phi \sin \phi)' = a \cos \phi - \omega \sin \phi - \phi \omega \cos \phi \]  \hspace{1cm} (2-2)

The acceleration of the follower can be obtained by formula (2-2) substitution formula (2-1):

\[ a = (0.048 \omega + \omega \tan \phi + \phi \omega) \frac{0.132}{\cos \phi} \]

Among them, since the cam is rotating at a constant speed, \( \omega \) is a constant.

According to the analysis of cam, the angle \( \omega \) of cam push section is 60°. As shown in Figure 3, let's set \( \omega = 1 \) and W at intervals of 1°. Use the EXCEL chart function to observe and analyze the acceleration of the follower. It can be seen from the analysis that the least-squares third-order fitting function is closest to the cam surface projection curve, and the situation in Figure 3 can reflect the motion and impact of the cam more realistically.
Figure 3 shows that,

(1) Because the acceleration change at the junction of the push section and the near stop section is small, the impact is small and the wear is little. As shown in Figure 2, this is consistent with the wear of existing cams.

(2) During the process of pushing the follower, the acceleration gradually increases, and the sudden increase of the acceleration periodically occurs, which causes a large impact. In addition, the cam is responsible for the partial contour of the workpiece loading and unloading movement, and the working load is large, which will further increase the impact. Both of the above cases produce severe wear on the curved surface of the cam groove. As shown in Figure 2, this is consistent with the wear of existing cams.

(3) The acceleration $a$ at the junction of the push segment and the far stop segment is changed from a certain value to 0, and there is also a certain flexible impact, but it is much smaller than the impact during the pushing process, and also causes some wear. As shown in Figure 2, this is consistent with the wear of existing cams.

(4) The projection curve of the return section surface is completely symmetrical with the push section, and the law of acceleration change is also completely symmetrical with the push section. But because the cam is in the unloading movement of the workpiece, the working load is much smaller than that in the push, so the impact caused by the change of the follower acceleration is much smaller, and the wear of the groove surface in this section will be very small [4-5]. As shown in Figure 2, this is consistent with the wear of existing cams.

In summary, although the shape of the curved surface is the closest to the shape of the existing cam surface if the third-order fitting function of the least squares method is used to reconstruct the projection curve of the projection segment surface, the impact of the follower on the cam cannot be reduced, and the wear condition cannot be improved.

3.2. Optimization of projection curve fitting function of groove cam surface pushing section

The optimization of the projection curve fitting function not only reduces the impact of the follower on the cam surface during the pushing section to effectively improve the wear of the segment surface, but also takes into account that the fit curve is similar to the existing cam. Because the curvature change of the second-order fitting function of the least square method is small, the impact caused by it must be reduced. The least squares second-order fitting function can be used to fit the curve projection curve of the cam pushing section.

According to the point cloud data obtained by the previous 3D laser scanner, the numerical analysis method such as least square method and Lagrangian difference method is used to focus on fitting the projection curve function of the cam groove pushing section surface. The second-order fitting function of the least squares method of the surface projection curve of the cam pushing section is as follows:
Its first derivative is:

\[ y' = 0.0256x - 0.6152 \]

Its second derivative is:

\[ y'' = 0.0256 \quad (2-3) \]

Same as before, convert second derivative to polar form and substitute \((2-2)\) into \((2-3)\). The change rule of the acceleration of the follower in the pushing section with the rotation angle under the second-order fitting curve is as follows:

\[ a = \frac{0.0256}{\cos \phi} + \tan \phi + \phi \omega \]

Similarly, as shown in Figure 4, it is still set \(\omega = 1\) and the value of \(\phi\) is separated by 1°. Observe and analyze the change of the acceleration \(a\) of the follower with \(b\) by using the EXCEL chart function, and at the same time, the acceleration \(a\) of the follower is compared with the change of the third-order fitting function of the least squares method.

![Figure 4](image)

**Figure 4.** Comparison of the second-order fitting curve and the third-order fitting curve on follower acceleration.

Figure 4 shows that,

1. The second-order fitting curve greatly reduces the acceleration of the follower in the entire pushing section, and the impact caused by it is inevitably greatly reduced, and the wear of the cam in pushing section is greatly reduced.
2. The sudden change in acceleration during the push process is reduced by nearly 2.5 times, which will definitely improve the impact. Under the same working load, the wear caused by the sudden change is also greatly reduced.
3. The second-order fitting curve is further reduced in the flexible impact with the near-stop section and the far-stop section, which is also beneficial to the improvement of the cam wear condition. According to the above analysis, under the requirement of reducing the cam wear and approaching the original cam groove surface shape as close as possible, the fitting function of the surface projection curve of the cam pushing section is determined as:

\[ y = 0.0128x^2 - 0.6152x + 88.8931 \]

The surface of the cam return section is symmetric with the push section, so the fitting function of the return section is the mirror function of the push section. It is as follows:

\[ y = 0.0128x^2 + 0.6152x + 88.8931 \]
4. Motion simulation of groove cam mechanism

After completing the reverse of the feature data of each part of the groove cam, 3D CAD software can be used to reconstruct the 3D CAD model and perform the motion simulation of the mechanism [6-7]. This article uses PTC Creo Parametric 4.0 software for modeling and motion simulation and verify the correctness of the cam reverse design. Through the motion simulation, the law of the lift motion of the groove cam is obtained. It is shown in Figure 5.

![Figure 5. Lifting motion law of groove cam.](image_url)

The lift motion law of Figure 5 shows that the lift of the groove cam a plane in the push section, the far stop section, the return section and the near stop section is in the loop of “upward→stop→down→stop”. The lift of each section is consistent with the original cam lift far-moving law and the travel distance, so the accuracy of the reverse design cam is higher [8].

5. Conclusion

Some SMEs using cam equipment have certain difficulties in the reverse design process due to the limitations of design optimization capabilities [9]. In this paper, the groove cam Z175 is taken as an example to elaborate a set of methods for the analysis of the motion law of the follower, the fitting function optimization and the motion simulation in the cam reverse design. This method can provide some technical assistance for enterprises in reverse design.

References

[1] Zhang Xia, Li Zhen, Tao Shuai, Yang Huadong. CAM reverse measurement experiment device and experiment method [J]. Experiment Technology and Management Journal, 2020, 10:25-29.
[2] Wang Bintuan, Zhang Zuoquan, Zhao Pingfu. A brief tutorial on numerical analysis[M]. Beijing: Tsinghua University press, 2012, 8:81-85.
[3] Zheng Jiming, Zhu Wei, Liu Yong, Fang Changjie. Numerical Analysis[M]. Beijing: Tsinghua University press, 2016, 12:17-21.
[4] Wang Hui. Mechanical Design Course Design (Third Edition)[M]. Beijing: Peking University press, 2011, 5:121-125.
[5] Cheng Daxian. Mechanical Design Manual (5th Edition, Volume 3, Part 1)[M]. Beijing: Chemical Industry Press, 2001, 2:65-70
[6] Beijing zalldy Technology Co., Ltd.[Z]. Creo 4.0 Mold Design Tutorial, 2018, 3:21-25.
[7] Li Tengxun, Lu Jie. Computer Aided Design (AutoCAD 2009 Tutorial)[M]. Beijing: Tsinghua University press, 2009, 5:48-51.
[8] Cao Jintang. Cam mechanism design[M]. Beijing: China machine press, 1985, 4:161-165.
[9] Chen Zhongshi, Zhou Zechan, Liu Weilin. The connecting rod of innovation design based on reverse engineering technology [J]. Mechanical Engineer Journal, 2019, 9:93-98.