Variational problem for the Frenkel and the Bargmann-Michel-Telegdi (BMT) equations

A. A. Deriglazov

Depto. de Matemática, ICE, Universidade Federal de Juiz de Fora, MG, Brasil

We propose Lagrangian formulation for the particle with value of spin fixed within the classical theory. The Lagrangian is invariant under non-abelian group of local symmetries. On this reason, all the initial spin variables turn out to be unobservable quantities. As the gauge-invariant variables for description of spin we can take either the Frenkel tensor or the BMT vector. Fixation of spin within the classical theory implies $O(h)$-corrections to the corresponding equations of motion.

I. INTRODUCTION

Classical theories of spin are widely used (see [1-4] and references therein) in analysis of spin dynamics in various circumstances and are known to agree with the calculations based on the Dirac theory. The spin variables of the Frenkel and BMT theories obey the first-order equations of motion. On this reason, construction of the corresponding action functional represents rather nontrivial problem. Various sets of auxiliary variables have been suggested and discussed in attempts to solve the problem [5-11]. The present model is based on the recently developed construction of spin surface [12]. This represents an essentially unique $SO(n)$-invariant surface of $2n$-dimensional vector space which can be parameterized by generators of $SO(n)$-group. In [13] it has been demonstrated that $SO(3)$ spin surface leads to a reasonable model of non-relativistic spin. $SO(2,3)$ spin surface implies the model of Dirac electron [14], and represents an example of pseudoclassical mechanics [15]. Here we demonstrate that $SO(1,3)$ spin surface can be used to construct variational problem for unified description of both the Frenkel and BMT theories of relativistic spin.

In the Frenkel theory [2] we include the three-dimensional spin-vector $S^i$, $(S^i)^2 = \frac{3\hbar^2}{4}$, into the antisymmetric tensor $J^{\mu\nu} = -J^{\nu\mu}$. This required to obey the constraint

$$J^{\mu\nu}u_\nu = 0,$$

where $u_\nu$ represents four-velocity of the particle in proper-time parametrization. In the rest-frame, $u_\nu = (u_0, 0, 0, 0)$, this implies $J^{0i} = 0$, so only three components of the Frenkel tensor survive, they are $J^{ij} = 2\epsilon^{ijk}S^k$. Besides, we can impose the covariant constraint

$$J^{\mu\nu}J_{\mu\nu} = 6\hbar^2.$$

As in the rest frame $J^{\mu\nu}J_{\mu\nu} = 8(S^i)^2$, this implies the right value of three-dimensional spin, as well as the right number of spin degrees of freedom.

Frenkel tensor is equivalent to the four-vector $S^\mu = \frac{1}{4\sqrt{\omega}}\epsilon^{\mu\nu\alpha\beta}u_\nu J_{\alpha\beta}$, the latter obeys

$$S^\mu u_\mu = 0.$$

This has been taken by Bargmann, Michel and Telegdi as the basic quantity in their description of spin [16]. In terms of the BMT-vector, spin can be fixed by the constraint

$$(S^\mu)^2 = \frac{1}{8}J^2 - \frac{(Ju)^2}{4u^2} = \frac{3\hbar^2}{4}.$$

Equations for the BMT-vector can be fixed [16] from the requirements of relativistic covariance, the right non-relativistic limit and from the compatibility with above mentioned constraints. Using the proper time as the evolution parameter, they read

$$\dot{S}^\mu = \frac{\mu e}{mc}[(FS)^\mu + (SFu)u^\mu] - (\bar{u}S)u^\mu,$$

where $\mu$ stands for the anomalous magnetic moment, $m\dot{u}^\mu = f^\mu$, and $f^\mu$ is four-force.

We are interested in to formulate a variational problem for the Frenkel and BMT classical spin theories. As compare with the previous attempts [5-11], we look for the action functional which, besides of the transversality constraints [1], [10], implies also the value-of-spin constraints [3], [11]. We point out that mainly due to the absence of variational problem, canonical quantization of the Frenkel and BMT theories is not developed to date. We hope the present work may be a step towards this direction.

We construct the Frenkel tensor starting from angular momentum

$$J^{\mu\nu} = 2(\omega^\mu\pi^\nu - \omega^\nu\pi^\mu),$$

2 We use the Minkowski metric $\eta^{\mu\nu} = (-, +, +, +)$ and the Levi-Civita symbol with $\epsilon^{0123} = +1$.
3 We point out that $S^\mu$, being the Casimir operator of the Poincare group, has fixed value for the Poincare IRREPs as well.
4 Our $\mu = \frac{e}{c}$ of BMT, and the charge is $e < 0$.
of the spin "phase" space with the coordinates $\omega^\mu$ and the conjugate momenta $\pi^\mu$. To achieve this, we restrict dynamics of the basic variables on the spin surface determined by $SO(1,3)$-invariant equations

$$\pi^2 = a_3, \quad \omega^2 = a_4, \quad \omega \pi = 0. \quad (7)$$

As $J^{\mu \nu}J_{\mu \nu} = 8(\omega^2\pi^2 - (\omega \pi)^2) = 8a_3a_4$, an appropriate choice of the numbers $a_3$ and $a_4$ in Eq. (7) fixes the value of spin. Besides, we impose the constraints\(^5\)

$$p\omega = 0, \quad p\pi = 0, \quad (8)$$

where $p^\mu$ stands for conjugate momentum to the world-line coordinate $x^\mu$. Eqs. (8) guarantee the transversality of the Frenkel tensor. The set (7), (8) contains one first-class constraint (see below). Taking into account that each second-class constraint rules out one phase-space variable, whereas each first-class constraint rules out two variables, we have the right number of spin degrees of freedom, $8 - (4 + 2) = 2$.

Dynamics of the position variable $x^\mu(\tau)$ is restricted by the standard mass-shell condition

$$p^2 + m^2 c^2 = 0. \quad (9)$$

Our next task is to formulate the variational problem which implies these constraints. Since they are written for the phase-space variables, it is natural to start from construction of an action functional in the Hamiltonian formalism. We introduce the canonical pairs $(g_i, \pi g_i)$, $i = 1, 3, 4, 6, 7$, of auxiliary variables associated with the constraints. Then the Hamiltonian action can be taken in the form

$$S_H = \int d\tau \ p_\mu \dot{x}^\mu + \pi_\mu \dot{\omega}^\mu + \pi_{g i} \dot{g}_i - H, \quad (10)$$

$$H = \frac{1}{2}g_i(p^2 + m^2 c^2) + \frac{1}{2}g_3(\pi^2 - a_3) + \frac{1}{2}g_2(\omega^2 - a_4) + g_6(p\omega) + g_7(p\pi) + \lambda_{gi} \pi g_i. \quad (11)$$

We have denoted by $\lambda_{gi}$ the Lagrangian multipliers for the primary constraints $\pi g_i = 0$. Variation of the action with respect to $g_i$ implies\(^6\) the desired constraints (7), (8) and (9).

### II. LAGRANGIAN OF A THEORY WITH QUADRATIC CONSTRAINTS

Lagrangian of a given Hamiltonian theory with constraints can be restored within the extended Lagrangian formalism\(^17\). Our constraints (7), (8) and (9) are either linear or quadratic with respect to momenta. For this case, the general formalism can be simplified as follows. Consider mechanics with the configuration-space variables $Q^a(\tau)$, $g_{ab}(\tau) = g_{ba}$, $h^a b Q^b$ and $k_{ab} (\tau) = k_{ba}$ and with the Lagrangian action

$$S = \int d\tau \frac{1}{2} g_{ab} DQ^a DQ^b - \frac{1}{2} k_{ab} Q^a Q^b - \frac{1}{2} M(\dot{g}, h, k)(12)$$

We have denoted $DQ^a \equiv \dot{Q}^a - h^a b Q^b$, and $\dot{g}^{ab}$ is the inverse matrix of $g_{ab}$. This action can be used to produce any desired quadratic constraints of the variables $Q, P$. Indeed, denoting the conjugate momenta as $P_a$, $\pi_g$, $\pi_h$ and $\pi_k$, the equations for $P_a$ can be solved

$$P_a = \frac{\partial L}{\partial \dot{Q}^a} = g_{ab} DQ^b, \Rightarrow \dot{Q}^a = \tilde{g}^{ab} P_b + h^a b Q^b, \quad (13)$$

while equations for the remaining momenta turn out to be the primary constraints $\pi_g = \pi_h = \pi_k = 0$. The Hamiltonian reads

$$H = \frac{1}{2}g^{ab} P_a P_b + P_a h^a b Q^b + \frac{1}{2} k_{ab} Q^a Q^b + \frac{1}{2} M + \lambda_g \pi_g + \lambda_h \pi_h + \lambda_k \pi_k \quad (14)$$

Then preservation in time of the primary constraints $\pi_g$ implies the quadratic constraints $P_a, P_b, \frac{\partial H}{\partial \pi g} = 0$, and so on.

Comparing the Hamiltonian of our interest (11) with the expression (14), let us take $Q^a = (x^\mu, \omega^\nu)$, $P_a = (p^\mu, \pi^\nu)$,

$$\tilde{g}^{ab} = \begin{pmatrix} g_1 & g_7 & g_3 \end{pmatrix}, \quad h^a b = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad k_{ab} = \begin{pmatrix} g_4 & 0 & 0 \\ 0 & g_4 & 0 \\ 0 & 0 & g_4 \end{pmatrix},$$

where $g_i = g_i \eta^{\mu \nu}$ and so on. Besides, we take the "mass" term in the form $M = g_1 m^2 c^2 - g_3 a_3 - g_4 a_4$. With this choice, the equation (14) turns into the desired Hamiltonian (11). So the corresponding Lagrangian action reads from (12) as follows

$$S = \int d\tau \frac{1}{2A} \left[ g_1(Dx^2 - 2g_7(Dx \omega) + g_1 \omega^2) \right] - \frac{1}{2} g_1 m^2 c^2 + \frac{1}{2} g_3 a_3 - \frac{1}{2} g_4(\omega^2 - a_4). \quad (15)$$

We have denoted $A = \det \tilde{g} = g_1g_3 - g_7^2$, $Dx^\mu = \dot{x}^\mu - g_6 \omega^\mu$.

### III. FREE THEORY

Equations for the canonical momenta $p^\mu$ and $\pi^\mu$ of the theory (15)

$$p^\mu = \frac{g_3}{A} Dx^\mu - \frac{g_1}{A} \omega^\mu, \quad \pi^\mu = -\frac{g_7}{A} Dx^\mu + \frac{g_1}{A} \omega^\mu, \quad (16)$$

can be resolved as follows

$$\dot{x}^\mu = g_1 P^\mu + g_7 \pi^\mu + g_6 \omega^\mu, \quad \dot{\omega}^\mu = g_7 P^\mu + g_3 \pi^\mu \quad (17)$$

---

5. While we work with conjugate momenta $p^\mu$ instead of four velocity $u^\mu$, our spin-vector obeys the BMT-condition\(^3\), see Eq. (8) below.

6. The equation $\omega \pi = 0$ appears as the third-stage constraint, see Eq. (9) below.
while equations for the remaining momenta imply the primary constraints, \( \pi_{g1} = 0 \). Using these equations in the expression \( \dot{p}x + \pi \dot{\omega} - L \), we immediately obtain the Hamiltonian (11). Preservation in time of the primary constraints implies the following chains of higher-stage constraints:

\[
\pi_{g1} = 0 \Rightarrow p^2 + m^2c^2 = 0. \tag{18}
\]

\[
\pi_{g3} = 0, \Rightarrow \pi^2 - a_3 = 0 \]
\[
\pi_{g4} = 0, \Rightarrow \omega^2 - a_4 = 0 \Rightarrow \pi \omega = 0, \Rightarrow \]
\[
g_4 = \frac{a_3}{a_4} g_3, \Rightarrow \lambda g_4 = \frac{a_3}{a_4} \lambda g_3. \tag{19}
\]

\[
\pi_{g7} = 0 \Rightarrow p\tau = 0, \Rightarrow g_6 = 0, \Rightarrow \lambda g_6 = 0. \tag{20}
\]

The constraints \( p^2 + m^2c^2 = 0 \) and \( \pi^2 - a_3 + \frac{a_3}{a_4} (\omega^2 - a_4) = 0 \) form the first-class subset. This indicates that the action (10) is invariant under the two-parametric group of local transformations. It is composed by the standard reparametrizations as well as by the following transformations with the parameter \( \gamma(\tau) \):

\[
\delta \omega^\mu = \gamma g_3 \pi^\mu, \quad \delta \pi^\mu = -\gamma g_4 \omega^\mu, \quad \delta g_3 = (\gamma g_3), \quad \delta g_4 = (\gamma g_4), \quad \delta g_6 = \gamma g_6, \quad \delta g_7 = -\gamma g_7, \quad \delta \lambda g_4 = (\gamma g_4). \tag{21}
\]

Note that \( \pi^\mu, J^{\mu\nu} \) and \( S^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} p_\nu J^\rho_\sigma \) are \( \gamma \)-invariant quantities.

Besides the constraints, the action implies the Hamiltonian equations

\[
\dot{x}^\mu = g_1 p^\mu, \quad \dot{p}^\mu = 0, \quad \dot{\omega} = g_3 \pi^\mu, \quad \dot{\pi}^\mu = -g_4 \frac{a_3}{a_4} \omega^\mu. \tag{22}
\]

Obtaining these equations, we have used the constraints (19) and (20). The functions \( g_1(\tau) \) and \( g_3(\tau) \) can not be determined neither with the constraints nor with the dynamical equations. It implies the functional ambiguity in solutions to the equations of motion (22), besides the integration constants, solution depends on these arbitrary functions. The ambiguity of \( x^\mu \) due to \( g_1 \) reflects the reparametrization invariance, while the ambiguity of \( \omega^\mu \) and \( \pi^\mu \) due to \( g_3 \) is related with the \( \gamma \)-symmetry. According to the general theory of singular systems (18–20), the variables with ambiguous dynamics do not represent the observable quantities. So, our next task is to find candidates for observables, which are variables with unambiguous dynamics. Equivalently, we can look for the gauge-invariant variables. As the physical variables of the spin-sector, we can take either the Frenkel tensor or the BMT-vector, both turn out to be \( \gamma \)-invariant quantities. The ambiguity related with reparametrizations can be removed in the standard way: we assume that the functions \( x^{x_\tau}(\tau) \) represent the physical variables \( x^\mu(\tau) \) in the parametric form. As it should be, dynamics of the physical variables is unambiguous

\[
\frac{dx^i}{dt} = e^i_j p^j, \quad \frac{dp^i}{dt} = 0, \quad \frac{dJ^{\mu
u}}{dt} = dS^\mu = 0. \tag{23}
\]

According to the equations (18–20), the variables obey also the desired constraints \( p^2 + m^2c^2 = 0, \quad J^{\mu\nu} p^\mu = 0, \quad J^2 = 8a_3 a_4, \quad S^\mu p_\mu = 0, \quad S^2 = 4m^2c^2 a_3 a_4. \)

IV. INTERACTION WITH UNIFORM ELECTROMAGNETIC FIELD

Let us consider the spinning particle with electric charge \( e \) and the anomalous magnetic moment \( \mu \). We take the Hamiltonian of interacting theory in the form

\[
H = \frac{1}{2} g_1 (p^2 - \frac{e\mu}{2c} F^{\mu\nu} J^{\mu\nu} + m^2 c^2) + \frac{1}{2} g_3 (\pi^2 - a_3) + \frac{1}{2} g_4 (\omega^2 - a_4) + g_6 (P\omega) + g_7 (P\pi) + \lambda g_\xi \pi_{g1}. \tag{24}
\]

We consider uniform electromagnetic field, \( F^{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu = \text{const} \). We have denoted \( P^\mu = p^\mu - \frac{e\mu}{2c} A^\mu \). In contrast to \( p^\mu \), the \( (1)-\text{invariant} \) quantities \( P^\mu \) have non-vanishing Poisson brackets, \( \{ P^\mu, P^\nu \} = \xi F^{\mu\nu} \). We point out that the \( \gamma \)-symmetry survives in the interacting theory even for nonuniform field.

The Hamiltonian (24) implies the constraints (19) and the mass-shell condition

\[
\mathcal{P}^2 = \frac{e\mu}{2c} F^{\mu\nu} J^{\mu\nu} + m^2 c^2 = 0, \tag{25}
\]

while instead of (20) we obtain

\[
\mathcal{P}\omega = 0, \Rightarrow g_6 = g_1 \frac{e(\mu - 1)}{c^3 M^2} (\mathcal{P} F) \Rightarrow \lambda g_6 \sim \lambda g_1.
\]

\[
\mathcal{P}\pi = 0, \Rightarrow \mathcal{P} \pi = -g_1 \frac{e(\mu - 1)}{c^3 M^2} (\mathcal{P} F) \Rightarrow \lambda g_7 \sim \lambda g_1. \tag{26}
\]

We have denoted \( M^2 = m^2 - \frac{e(2\mu + 1)}{4c^4} F^{\mu\nu} J^{\mu\nu} \). The constraints imply the useful consequence

\[
g_6 (\omega F\mathcal{P}) + g_7 (\pi F\mathcal{P}) = 0. \tag{27}
\]

This equation can be used to verify that the quantities \( F^{\mu\nu} J^{\mu\nu} \), \( M^2 \) and \( \mathcal{P}^2 \) represent the integrals of motion.

Hamiltonian equations for the basic variables read

\[
\dot{x}^\mu = g_1 u^\mu, \quad \dot{p}^\mu = g_1 \frac{e}{c} (F u)^\mu, \tag{28}
\]

\[
\dot{\omega} = g_1 \frac{e}{c} (F \omega) + g_3 \pi^\mu + g_7 \mathcal{P}^\mu, \tag{29}
\]

\[
\dot{\pi}^\mu = g_1 \frac{e}{c} (F \pi)^\mu - \frac{a_3}{a_4} g_3 \omega^\mu - g_6 \mathcal{P}^\mu, \tag{30}
\]

where the four-velocity \( u^\mu \) is (see Eq. (26))

\[
u^\mu = \mathcal{P}^\mu + \frac{g_7}{g_1} \pi^\mu + \frac{g_6}{g_1} \omega^\mu = \mathcal{P}^\mu + \frac{e(\mu - 1)}{2c^3 M^2} (J F \mathcal{P})^\mu. \tag{31}
\]

Hence the interaction leads to modification of the Lorentz-force equation. Only for the "classical" value of anomalous momentum, \( \mu = 1 \), the constraints (26) would be the same as in the free theory, \( g_6 = g_7 = 0 \). Then the
four-velocity coincides with \( \mathcal{P} \). When \( \mu \neq 1 \), the difference between \( u \) and \( \mathcal{P} \) is proportional to \( x_{\mu} \sim \frac{u_{\mu}}{u^\nu u_{\nu}} \). All the basic variables have ambiguous evolution. \( x^{\mu} \) and \( \mathcal{P}^{\mu} \) have one-parametric ambiguity due to \( g_{1} \) while \( \omega \) and \( \pi \) have two-parametric ambiguity due to \( g_{1} \) and \( g_{2} \).

The quantities \( x^{\mu} \), \( \mathcal{P}^{\mu} \) and the Frenkel tensor \( J^{\mu\nu} \) are \( \gamma \)-invariants. Their equations of motion form a closed system

\[
\dot{x}^{\mu} = g_{1} \left[ \mathcal{P}^{\mu} + \frac{e(\mu-1)}{2c^{3}M^{2}} (JFP)^{\mu} \right], \quad \dot{\mathcal{P}}^{\mu} = \frac{e}{c} e(F^{\mu})(\mathcal{P}), \quad (31)
\]

\[
\dot{J}^{\mu\nu} = g_{1} \frac{e}{c} \left[ \mu F^{[\nu} J^{\alpha]\mu J^{\beta}] + \frac{\mu-1}{c^{2}M^{2}} \mathcal{P}^{[\nu} J^{\beta]} (FP) \right], \quad (32)
\]

The remaining ambiguity due to \( g_{1} \) presented in these equations reflects the reparametrization symmetry of the theory. Assuming that the functions \( x^{\mu}(t) \), \( p^{\nu}(t) \) and \( J^{\mu\nu}(t) \) represent the physical variables \( x^{\mu}(t) \), \( p^{\nu}(t) \) and \( J^{\mu\nu}(t) \) in the parametric form, their equations read

\[
\frac{dx^{\mu}}{dt} = e^{\mu}_{\nu}, \quad \frac{dp^{\nu}}{dt} = \frac{e(F^{\mu})_{\nu}}{e}, \quad \frac{dJ^{\mu\nu}}{dt} = \frac{J^{\mu\nu}}{g_{1}^{\mu\nu}}. \quad \text{As it should be, they have unambiguous dynamics.}
\]

Since \( J^{\mu\nu} \mathcal{P}_{\nu} = 0 \), the Frenkel tensor is equivalent to the BMT-vector constructed as follows:

\[
S^{\mu} = \frac{1}{4\sqrt{-\mathcal{P}^{2}}} \epsilon^{\mu\nu\alpha\beta} \mathcal{P}_{\nu} J_{\alpha\beta} \equiv \frac{1}{4\sqrt{-\mathcal{P}^{2}}} \epsilon^{\mu\nu\alpha\beta} u_{\nu} J_{\alpha\beta}. \quad (33)
\]

So the physical dynamics can be described using \( S^{\mu} \) instead of \( J^{\mu\nu} \). Using the identities

\[
J^{\mu\nu} = -\frac{2}{\sqrt{-\mathcal{P}^{2}}} \epsilon^{\mu\nu\alpha\beta} \mathcal{P}_{\alpha} S_{\beta}, \quad \epsilon^{\mu\nu\alpha\beta} J_{\alpha\beta} = \frac{4}{\sqrt{-\mathcal{P}^{2}}} \mathcal{P}^{[\mu} S^{\nu]} \quad (34)
\]

to represent \( J^{\mu\nu} \) through \( S^{\mu} \) in Eq. \( (31) \), we obtain the closed system of equations for \( \gamma \)-invariant quantities

\[
\dot{x}^{\mu} = g_{1} \left[ \mathcal{P}^{\mu} + \frac{e(\mu-1)}{2c^{3}M^{2}} \epsilon^{\mu\nu\alpha\beta} (FP)^{\nu} \mathcal{P}_{\alpha} S_{\beta} \right], \quad (35)
\]

\[
\dot{S}^{\mu} = g_{1} \frac{e}{c} \left[ (FS)^{\mu} + \frac{1}{\mathcal{P}^{2}} (SFP) \mathcal{P}^{\mu} \right] - \frac{1}{\mathcal{P}^{2}} (\dot{\mathcal{P}} S) \mathcal{P}^{\mu}. \quad (36)
\]

These equations are written in an arbitrary parametrization of the world-line.

V. CONCLUSION

In this work we have specified the construction of spin surface [12] for the case of \( SO(1, 3) \)-group. On this base, we have constructed the Lagrangian action [14] which describe the particle with fixed value of spin interacting with uniform electromagnetic field. Due to the constraints [7], [8], the number of physical degrees of freedom in the spin-sector is equal to 2, as it should be. The basic spin-space coordinates \( \omega^{\mu} \), \( \pi^{\nu} \) are gauge non-invariant variables, hence they do not correspond to the observable quantities. We can take the antisymmetric tensor [9] as an observable quantity. For an appropriate choice of the parameters \( a_{3}, a_{4} \), this obeys both the transversality constraint [11] and the value-of-spin constraint [2]. Its dynamics is governed by the Frenkel-type equation [32]. Equivalently, we can take the vector [33] as an observable quantity. This is subject to the constraints [4], [1] and obeys the Bargmann-Michel-Telegdi equations of motion [46].

VI. ACKNOWLEDGMENTS

This work has been supported by the Brazilian foundation FAPEMIG.

[1] A. J. Silenko and O. V. Teryaev, Phys. Rev. D76 (2007) 061101.
[2] L. M. Slad, Physics Letters A 374 (2010) 1209.
[3] E. V. Arbuzova, A. E. Lobanov, and E. M. Murachikova, Phys. Rev. D 81 (2010) 045001.
[4] A. Kar and S. G. Rajasek, Ann. Phys. 326 (2011) 958.
[5] J. Frenkel, Z. fur Physik 37, (1926) 243.
[6] A. O. Barut Electrodynamic and Classical Theory of Fields and Particles, (MacMillan, New York 1964).
[7] W. G. Dixon, Nuovo. Cim. 34 (1964) 317.
[8] A. J. Hanson and T. Regge, Ann. Phys. 87 (1974) 498.
[9] P. Grassberger, J. Phys. A: Math. Gen. 11 (1978) 1221.
[10] A. P. Balachandran, G. Marmo and A. Stern, Phys. Lett. B89 (1980) 199.
[11] G. Cognola, R. Soldati, L. Vanzo and S. Zerbini, Phys. Lett. B 104 (1981) 67.
[12] A. A. Deriglazov, Ann. Phys. 327 (2012) 398.
[13] A. A. Deriglazov, Mod. Phys. Lett. A 25 (2010) 2769.
[14] A. A. Deriglazov, Phys. Lett. A 376 (2012) 309.
[15] A. A. Deriglazov, Phys. Lett. A 377 (2012) 13.
[16] V. Bargmann, L. Michel and V.L. Telegdi, Phys. Rev. Lett. 2 (1959) 435.
[17] A. A. Deriglazov, J. Phys. A 40 (2007) 11083; J. Math. Phys. 50 (2009) 012907–1–15.
[18] P. A. M. Dirac, Can. J. Math. 2 (1950) 129; P. A. M. Dirac, Lectures on Quantum Mechanics (Yeshiva University, New York, 1964).
[19] D. M. Gitman and I. V. Tyutin, Quantization of Fields with Constraints (Springer-Verlag, Berlin, 1990) p. 36.
[20] A. A. Deriglazov, Classical Mechanics, Hamiltonian and Lagrangian Formalism (Springer-Verlag, Berlin Heidelberg, 2010).