Calculation of the Underground Muon Intensity Crouch Curve from a Parameterization of the Flux at Surface

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Abstract: Utilizing only the vertical muon intensity of the Gaisser parameterization of the muon flux at the surface and propagating this energy spectrum underground according to statistical ionization and radiative energy losses, it is possible to calculate the underground muon intensity Crouch curve. In addition, the primary spectral index of the Gaisser parameterization can be adjusted from $E^{-2.7}$ to $E^{-2.643}$ simply by minimizing the deviation from the Crouch curve. For chemical compositions other than standard rock, the propagation of the spectrum underground can be repeated with a different muon energy loss in the material. The resulting underground muon intensity curve represents a consistent conversion of the Crouch curve to the local rock, fully accounting for the energy dependence of the muon $dE/dx$.

Gaisser Parameterization of Muon Flux at Surface

The Gaisser parameterization of the cosmic induced muon flux at surface is an approximate extrapolation formula valid when muon decay is negligible ($E_\mu > 100/\cos \theta$ GeV) and the underground detector zenith angle $\theta$ can be assumed identical to the production angle in the upper atmosphere ($\theta < 70^\circ$). Not including small contributions from charm and heavier flavors, which are negligible for energies below 10 TeV, the Gaisser parameterization is [1][2],

$$\frac{dN_\mu}{dE_\mu} \approx 0.14 \cdot \frac{E_\mu^{-2.7}}{cm^2 \cdot sr \cdot GeV} \times \left( \frac{1}{1 + \frac{1.1 E_\mu \cos \theta}{115 GeV}} + \frac{0.054}{1 + \frac{1.1 E_\mu \cos \theta}{850 GeV}} \right),$$

(1)

where the two terms give the contribution of pions and charged kaons. The energy spectrum steepens gradually to reflect the primary spectrum in the $10-100$ GeV range, and steepens further at higher energies because pions with $E_\pi >$ critical energy $\epsilon_\pi \approx 115$ GeV tend to interact in the atmosphere before they decay. Asymptotically ($E_\mu \gg 1$ TeV), the energy spectrum of atmospheric muons is one power steeper ($E^{-3.7}$) than the primary spectrum ($E^{-2.7}$). For the following calculations we only consider the vertical muon intensity for which $\cos \theta = 1$.

Muon Range Underground

The statistical energy loss of muons, traversing an amount $X$ of matter, with energies far above the Bethe-Bloch minimum is given as

$$-\frac{dE_\mu}{dX} = a(E_\mu) + \sum_{n=1}^{3} b_n(E_\mu) \cdot E_\mu,$$

(2)

where $a$ is the collisional term (i.e. ionization mostly due to delta-ray production) and the second term accounts for the three radiative muon energy loss processes: 1. Bremsstrahlung, 2. pair production and 3. photonuclear interactions. In Table 1 these energy loss parameters are listed for standard rock. The critical energy where ionization losses equal radiative losses in standard rock is approximately 0.6 TeV. As the energy dependencies of the $a$ and $b$ parameters are relative mild, they are often assumed to be constant and the then resulting simplified differential equation can be easily solved by using an exponential function [2]. However, for our purpose of precisely determining the average muon range underground for each value of surface
energy, the $a$ and $\Sigma b$ values are parameterized with the following functions

$$a(E_{\mu}) = A_0 + (A_1 \cdot \log_{10} E_{\mu}[GeV]) \quad (3)$$

$$\Sigma b(E_{\mu}) = B_0 + (B_1 \cdot \log_{10} E_{\mu}[GeV]) + (B_2 \cdot (\log_{10} E_{\mu}[GeV])^2) \quad (4)$$

Table 2 lists the fitted coefficients for standard rock. In addition, a Geant4 [4] based simulation is performed, tracking muons through a block of standard rock, defined as a mixture of CaCO$_3$ and MgCO$_3$ (with a density of 2.65 g/cm$^3$ and mass fractions of 52% O, 27% Ca, 12% C and 9% Mg). The comparison for the simulated $\Sigma b$ parameter as a function of energy and the parameterization in Eq. 4 validates that the fractional difference is less than 3% over the entire energy range.

We used Eq. 2 to numerically compute the propagation of the muon by stepping ($\Delta X = 1 g/cm^2$) through standard rock. Thus, for each initial value of muon energy, we determined, at what value of depth in meter-water-equivalent the muon ranges out. Fig. 1 shows the computed muon range in mwe of standard rock as a function of muon surface energy from 10 GeV to 10 TeV. Finally, the muon range $X$ in standard rock can be simply parameterized as

$$X[mwe] = p_0 \cdot \log_e \{ (p_1 \cdot E_{\mu}[GeV]) + p_2 \}, \quad (5)$$

with the fitted parameters listed in Table 3.

The same computation can be done for overburdens made up of other rock compositions like e.g. for more dense rock ($\rho \approx 2.85$ g/cm$^3$) at the Soudan iron mine in northern Minnesota, US (c.f. again Fig. 1 and Table 3). In order to easily transform the muon energy loss parameters $a_s$ and $b_s$ in standard rock (average nuclear properties $Z_s = 11$, $\bar{A}_s = 22$ [2]) to $a'$ and $b'$ in other rock (e.g. Soudan $Z' = 12$, $\bar{A}' = 24$ [5]), the $a$ and total $b$ parameter at a given muon energy can be scaled as

$$a'(E_{\mu}) = \frac{Z'/\bar{A}'}{Z_s/\bar{A}_s} \cdot a_s(E_{\mu}) \quad (6)$$

$$\Sigma b'(E_{\mu}) = \left( \frac{Z'^2/\bar{A}'}{Z_s/\bar{A}_s} \cdot 0.9 + 0.1 \right) \cdot \Sigma b_s(E_{\mu}) \quad (7)$$

The muon energy loss from Bremsstrahlung and pair production can be scaled with $Z^2/\bar{A}$ to the first order [5], whereas the simulated contribution from photonuclear interactions to $dE/dX$ has only a very weak dependence on the nuclear properties in rock and is herein assumed constant per mwe. Furthermore, the MC simulation shows that in the region of interest from 250 GeV to 10 TeV the photonuclear interactions account for a constant fraction (10%) of all radiative muon energy losses in standard rock. The scaled $a$ and $\Sigma b$ values can be parameterized again with the functions in Eq. 3, 4 and the average muon range underground for each value of surface energy can be computed again in the same procedure as described above (c.f. again Table 2 and 3).

Table 1: Fitted coefficients for the parameterizations of the ionization (Eq. 3) and the total radiative (Eq. 4) average muon energy loss in standard and Soudan rock.

| $E_{\mu}$ [GeV] | $a_{ion}$ [MeV cm$^2$/g] | $b_{DIS}$ [10$^{-18}$ cm$^2$/g] | $b_{pair}$ [10$^{-18}$ cm$^2$/g] | $b_{brems}$ [10$^{-18}$ cm$^2$/g] | $\Sigma b$ |
|-----------------|------------------------|---------------------|-------------------|-------------------|------------------|
| 10$^2$          | 2.44                   | 1.10                | 0.90              | 0.90              | 1.90             |
| 10$^3$          | 2.68                   | 1.44                | 2.07              | 0.41              | 3.92             |
| 10$^4$          | 2.93                   | 1.62                | 2.27              | 0.46              | 4.35             |

Table 2: Average muon energy loss parameters calculated for standard rock [3][2]

|                      | $A_0$       | $A_1$      | $B_0$   | $B_1$   | $B_2$   |
|----------------------|-------------|------------|---------|---------|---------|
| standard rock        | 1.925       | 0.252      | 0.358   | 1.711   | -0.178  |
| Soudan rock          | 1.925       | 0.252      | 0.393   | 1.878   | -0.195  |

Table 3: Fitted coefficients for the parameterization of the average muon range in standard and Soudan rock.

|                      | $p_0$ [mwe] | $p_1$ [GeV$^{-1}$] | $p_2$   |
|----------------------|-------------|--------------------|---------|
| standard rock        | 2298.2      | 0.001920           | 0.99809 |
| Soudan rock          | 2098.9      | 0.002119           | 0.99789 |

Calculating the Muon Intensity Underground

Together with Eq. 1, which is the Gaisser parameterization of the differential intensity of vertical muons at the surface ($\cos \theta = 1$), and Eq.
Refining the Gaisser Parameterization by Comparison with Crouch Curve

we tried to further improve the agreement between the calculation and the Crouch curve by refining the primary spectral index ($\gamma$) and the absolute normalization ($c$) of the Gaisser parameterization of the muon flux at the surface (substitute $E^{-2.7}$ with $c \cdot E^\gamma$ in Eq. 1). In a Newtonian iteration procedure, the two variables $\gamma$ and $c$ were varied before each new calculation, such that the new values at $\sim 1500 \text{ mwe}$ and $\sim 9500 \text{ mwe}$ are in optimum agreement with Crouch and the amplitude of the fractional differences for slant depth values in between is minimal (c.f. Fig. 3). The best fit values obtained by this procedure are $\gamma_{fit} = -2.643$ for the primary spectral index and $c_{norm} = 80.5\%$ for the normalization factor (c.f. Fig. 4, 2). We found the maximum fractional difference between the optimized calculation and the Crouch curve to be 6%, which is of the order of the uncertainty on the measured Crouch curve.

Discussion

The fact that the parameterized muon flux at the surface in combination with the propagated muon energy loss in the rock reproduces the 'world average' Crouch curve implies that the underlying physics, as described in this paper, is well un-
Calculating the Crouch Curve

Figure 3: Fractional difference between the integral vertical muon intensity underground according to the Crouch curve and the optimized calculation for slant depth values in standard rock.

understood. Our optimized Gaisser parameterization, with an absolute value of the primary spectral index of slightly less than 2.7 and a ~ 20% lower muon intensity at surface, is in reasonable agreement with experimental data [2]. Crouch [6] converted the underground muon intensity data to depth values in standard rock utilizing an approximate mathematical solution of the differential Eq. 2 assuming the $a$ and $b$ parameters to be constant [2], which becomes less valid at increasing depths. It is possible now to perform the herein described calculation in other rock than standard rock, yielding new underground muon intensity curves for the local rock composition.

Summary

Taking the vertical muon intensity of the refined Gaisser parameterization of the muon flux at the surface (80.5% · $E^{-2.643}$ instead of $E^{-2.7}$) and propagating this energy spectrum underground according to statistical ionization and radiative energy losses yields a good fit to the underground muon intensity Crouch curve. The obtained agreement is for most part better than the uncertainty of ~ 5% associated with the Crouch function. For chemical compositions other than standard rock, a consistent computation of the underground muon intensity curve can be repeated, fully accounting for the energy dependence of the muon $dE/dx$ in the local rock. This can yield a better determination of a map of the overburden of an underground muon detector, by normalizing the measured intensity to the computed distribution as a function of slant depth in the local rock. Furthermore, the methods described herein also allow for a more precise extrapolation of the underground muon energy back to the surface.

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