A dynamical, confining model and hot quark stars

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Abstract

We explore the consequences of an equation of state (EOS) obtained in a confining Dyson-Schwinger equation model of QCD for the structure and stability of non-strange quark stars at finite-$T$, and compare the results with those obtained using a bag-model EOS. Both models support a temperature profile that varies over the star’s volume and the consequences of this are model independent. However, in our model the analogue of the bag pressure is $(T, \mu)$-dependent, which is not the case in the bag model. This is a significant qualitative difference and comparing the results effects a primary goal of elucidating the sensitivity of quark star properties to the form of the EOS.

Key words: Dyson-Schwinger equations; Confinement; Dynamical chiral symmetry breaking; Bag model; Quark matter equation of state; Quark stars

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In astrophysical applications of the EOS for hot stars and/or supernova explosions it is commonly assumed that dense supernova matter can be described well using a finite-temperature Hartree-Fock approximation applied to an effective nucleon-nucleon interaction, and that the star consists primarily of neutrons, protons, relativistic electrons and degenerate electron neutrinos [1]. However, for densities exceeding $2 - 3$ times nuclear saturation density, exotic phases of superdense matter are possible; e.g., the interior of compact, astrophysical objects, such as neutron stars, may be composed of strange quark
matter (SQM) [2–4]. Hitherto, however, such a scenario remains speculative as there exists little evidence to support it or the kindred hypothesis that domains of SQM might be formed under extreme conditions of temperature and density [5].

Elucidating the composition of superdense, astrophysical objects requires a knowledge of the EOS of strongly interacting matter at values of the baryochemical potential close to that expected to induce a deconfinement phase transition, and the plausibility of the results of any given study rest on the accuracy of the EOS employed, which is difficult to judge a priori. Consequently, the exploration of alternative equations of state and the identification of qualitatively consistent results is important.

Much existing research is based on the bag model EOS; e.g., [4]. Other studies include those based on a string-flip, confining quark interaction [6], which suggest that a phase of deconfined, massive quarks is possible. The absence of chiral symmetry is a defect common to all these studies. A covariant approach incorporating both confinement and chiral symmetry is necessary and a step in that direction is presented in [7], which also argues that deconfinement can occur before chiral symmetry restoration. An EOS for quark matter at finite-$T$ is provided by numerical simulations of lattice-QCD actions, however, finite chemical potential continues to present difficulties [8].

The Dyson-Schwinger equations (DSEs) have been applied successfully to the strong-interaction at $T = 0 = \mu$; i.e., to the study of confinement and dynamical chiral symmetry breaking, and hadron observables [9,10]. The generalisation to finite-($T, \mu$) is straightforward [11–14] and allows the simultaneous study of the chiral and deconfinement phase transitions. Herein we employ a simple model [15] whose bulk thermodynamic properties have recently been elucidated [13]. In many respects; e.g., the persistence of nonperturbative effects into the quark-matter domain, the thermodynamic properties of this model are qualitatively similar to those found in numerical simulations of lattice-QCD. In this note we present a first exploration of the implications of this model for the stability and structure of pure quark-matter stars.

This application requires only that we recapitulate a few important aspects of the thermodynamic properties of our simple dynamical, confining model (DC model) [13]. In the confined domain the quark pressure is zero because confinement does not allow any free quarks. The $(T, \mu)$-dependent bag pressure, which measures the difference between the pressure in the Nambu-Goldstone phase and that in the Wigner phase:

$$B(T, \mu) = P[\mathcal{S}_{NG}] - P[\mathcal{S}_W],$$

vanishes if the scalar piece of the quark self energy, $B(\tilde{p}_k)$, becomes zero. Chiral
symmetry is manifest and the quark propagator has a Lehmann representation when $B(\tilde{p}_k) = 0$, which means we have chiral symmetry restoration and deconfinement. Hence the line $B(T, \mu) = 0$ defines the phase boundary.

In the deconfined domain, each massless quark species $i = u, d$ contributes an amount $(\tilde{p}_k = (\tilde{p}, \omega_k + i\mu), \omega_k = (2k + 1)\pi T, p = |\tilde{p}|)$

$$P_i(T, \mu_i) = \frac{2N_c T}{\pi^2} \int_0^\infty dp \ p^2 \left\{ \ln \left| \beta^2 \tilde{p}_k^2 \hat{C}(\tilde{p}_k) \right| - 1 + \Re \left( \frac{1}{\hat{C}(\tilde{p}_k)} \right) \right\},$$  \hspace{1cm} (2)

$$\hat{C}(\tilde{p}_k) = \frac{1}{2} \left( 1 + \sqrt{1 + \frac{2\eta^2}{p_k^2}} \right),$$  \hspace{1cm} (3)

to the pressure, which is normalised to zero at the phase boundary. Here $\eta \approx 1 \text{ GeV}$ is a mass-scale set [15] by requiring a good description of $\pi$ and $\rho$ masses. In calculating this pressure a modified free particle dispersion relation arises: $e(p; \mu_i) = \kappa(p; \mu_i) + p_i$. It is characterised by a new, dynamically-determined energy-scale $\kappa(p; \mu_i)$, with $\kappa(0, 0) \approx 0.6 \text{ GeV}$, which appears because even in the deconfined phase the vector piece of the dressed-quark self energy is not trivial; i.e., $\hat{C}(\tilde{p}_k) \neq 1$ in Eq. (2). $\kappa(0, 0)$ is a single, characteristic, nonperturbative scale that measures the deviation of $\hat{C}(\tilde{p}_k)$ from 1 and its slow approach to the asymptotic limit: $\hat{C}(\tilde{p}_k) = 1$. The quark pressure in the Wigner phase is therefore $(e_\pm(p, \mu_i) := e(p, \mu_i) \pm \mu_i)$

$$P_i(T, \mu_i) = \frac{N_c}{\pi^2} T \int_0^\infty dp \ p^2 \left\{ \ln \left[ 1 + e^{-\beta e_-} \right] + \ln \left[ 1 + e^{-\beta e_+} \right] \right\}. \hspace{1cm} (4)$$

The persistence of nonperturbative effects in the deconfined domain at finite-$T$ agrees qualitatively with the findings of lattice-QCD simulations [18]. It entails that the ultrarelativistic limit is reached slowly. Therefore, it is important to keep all three dressing functions in the propagator and model their dependence on their arguments in a qualitatively accurate manner.

For the following numerical calculations we use the EOS in Eq. (4) with the simplification of neglecting the $\mu$-dependence of $\kappa$, which reduces the computational effort significantly. As illustrated in [13], this is a good approximation for $\mu < 3\mu_c$. (In our model, $\mu_c(T = 0) \approx 0.28 \eta$ and $\mu_c(T) \approx \text{constant}$ for $T < 0.6 \eta$.) We will refer to this equation of state as EOS\text{DC}.

The total pressure of nonstrange quark matter in this model was calculated in [13]. It is obtained as the sum of the u- and d-quark contributions

$$P_{\text{DC}}(T, \mu_u, \mu_d) = P_u(T, \mu_u) + P_d(T, \mu_d) \hspace{1cm} (5)$$
and is depicted in Fig. 4 therein. The inclusion of strange quarks awaits an answer to the question of whether the simple Ansatz for the gluon propagator used in \[13\] can model 3-flavour QCD accurately. It is not necessary for our present, exploratory purposes and its qualitative impact can be anticipated.

Our model is characterised by deconfinement and chiral symmetry restoration as a dynamical result following the introduction of the external mass-scales: \((T, \mu)\), and, as noted above, the quark pressure as a thermodynamic potential, is zero in the confined domain. Consequently the contribution of quarks to all other thermodynamic quantities: entropy density, number densities, etc., as derivatives of this potential with respect to the set of thermodynamic variables \((T, \mu)\), also vanishes. In the confinement domain all the thermodynamic quantities are determined by hadronic degrees of freedom.

As a means of elucidating the consequences of this model we compare our results with the oft employed bag-model (BM), in which the EOS is

\[
P_{\text{BM}}^{\text{BM}}(T, \mu_u, \mu_d) = P_{u}^{\text{UR}}(T, \mu_u) + P_{d}^{\text{UR}}(T, \mu_d) - B_P. \tag{6}
\]

\[
P_{i}^{\text{UR}}(T, \mu_i) = \frac{g_i}{24\pi^2} \left( \mu_i^4 + 2\pi^2 \mu_i^2 T^2 + \frac{7}{15} \pi^4 T^4 \right) \tag{7}
\]

is the pressure of a massless, ultrarelativistic gas for a fermion of species \(i\), \(i = u, d, e\). For the bag-model we have \(N_c = 3\) and \(g_i = 2N_c\), \(i = u, d\), and the bag constant, \(B_P\), is \((T, \mu)\)-independent. It is introduced by hand to play the role of an external pressure necessary to “confine” the (ideal) Fermi gas of quarks, and its \((T, \mu)\)-independence entails a strong first-order deconfinement transition at all values of \((T, \mu)\).\(^1\) Herein we use \(B_P = 57\) MeV/fm\(^3\) \[4\] and refer to this equation of state as EOS\(^{\text{BM}}\).

Using the two different equations of state, Eqs. (5) and (6), it is straightforward to obtain all the thermodynamic quantities: the partial densities \(n_i = \partial P/\partial \mu_i\), the entropy density \(s = \partial P/\partial T\), the energy density \(\varepsilon = -P + Ts + \sum \mu_i n_i\), etc. The only modification of the pressure necessary for the case of neutron star matter in \(\beta\)-equilibrium is, in both models, to add the contribution of the electron component, which for massless-\(e^-\) is given by

\[
P_e(T, \mu_e) = P_{e}^{\text{UR}}(T, \mu_e), \quad g_e = 2. \tag{8}
\]

Figure 1 depicts the quark pressure as a function of the energy density in our model and in the bag model. The essential differences are clear. In bag-model-like approaches the energy density has a large finite value when the total

\(^1\)This is at odds with the expectation that the deconfinement transition is second order at \(\mu = 0\) \[11,13\]. Also, since the bag surface breaks chiral symmetry explicitly, the chiral symmetry restoring transition is outside the scope of the model.
Fig. 1. Total pressure of quark matter as a function of the energy density for the DC-model at $T = 0$ (solid line) and at $T = 50$ MeV (dashed line). The dotted line for the massless bag-model is valid at all temperatures.

pressure is zero as a direct consequence of the assumed $(T, \mu)$-independence of $B_P$. In our case, the energy density at the phase boundary in the deconfined domain, defined by $\mu_c(T)$, is negligibly small over the temperature interval $0 \leq T \leq 50$ MeV, which is the domain relevant to the problem of quark stars. This feature is a direct manifestation of the $(T, \mu)$-dependence of $B$, which provides a more realistic representation of the quark-dynamics of the phase transition. One also observes the markedly slower approach to the ultraviolet limit in our model. This means that EOS$^{BM}$ is much stiffer than EOS$^{DC}$, whose softness is consistent with expectations arising from numerical simulations of lattice-QCD.

Most relativistic studies of pure quark-matter stars have been performed for matter at $T \ll T_F$, where $T_F$ is the Fermi temperature. However, in the early stages of the life of a neutron/quark star; e.g., just after a supernova explosion, an initial temperature $T \sim T_F$ is possible. The cooling of the star via neutrino emission requires minutes [19], which is a time-scale much greater than that required to establish $\beta$-equilibrium. It is therefore important to determine whether for temperatures $T \sim T_F$ the stability criterion (Chandrasekhar limit) for such proto-stars requires modification.

Other studies have assumed $T = \text{constant}$ across a hot quark star [4], although an internal heating mechanism necessary to effect an isothermal distribution is not elucidated. This simplification leads to the conclusion that the maximum mass of a quark star is increased by a few percent when the temperature is increased from $T = 0$ to 50 MeV. However, we argue that only local thermal and hydrodynamical equilibrium can reliably be assumed in the early stages of the evolution of a quark star, and hence that the temperature profile must
vary over the star.

For clarity we enumerate the assumptions employed in our calculations:

(i) The quark star is a spherically symmetric, compact object, in which the matter is in local hydrodynamical and thermodynamical equilibrium with the self-consistently determined gravitational field. The pressure profile is therefore determined by a solution of the Tolman-Oppenheimer-Volkoff equation

\[
\frac{dP(r)}{dr} = -G(\varepsilon(r) + P(r))\frac{m(r) + 4\pi P(r)r^3}{r(r - 2Gm(r))},
\]

where \(G\) is the gravitational constant and \(m(r)\) is the mass accumulated inside a distance \(r\) from the center of the star, defined by

\[
\frac{dm(r)}{dr} = 4\pi r^2\varepsilon(r).
\]

(ii) The central baryon number densities are higher than the densities assumed to correspond to the confinement-deconfinement phase transition: \(n > 2 - 3 n_0\), where \(n_0 = 0.17 \text{ fm}^{-3}\) is the nuclear saturation density, and the central temperatures \(T < 20 - 50 \text{ MeV}\) are typical of neutron stars newly formed in a supernova explosion [19,20]. The radius of the star \(R\) is obtained from the condition \(P(R) = 0\) and the total mass is then \(M = m(R)\).

(iii) All the components of the hot matter (quark and lepton species: \(i = u, d, e^-\)) are in chemical equilibrium with respect to the \(\beta\)-decay process: \(d + \nu_e \leftrightarrow u + e^-\), such that

\[
\mu_e - \mu_{\nu_e} = \mu_d - \mu_u > 0.
\]

Hereafter we set \(\mu_{\nu_e} = 0\), which assumes the neutrinos escape quickly from the star.

(iv) The conditions of charge neutrality: \(n_e = (2n_u - n_d)/3\) and the conservation of baryon number

\[
n = \frac{1}{3}(n_u + n_d)
\]

are fulfilled. Thermodynamical consistency requires that the thermodynamic functions obey Gibbs’ law, which for conserved baryon number is

\[
d\left(\frac{\varepsilon}{n}\right) - Td\left(\frac{s}{n}\right) + Pd\left(\frac{1}{n}\right) = 0.
\]
Fig. 2. Composition of quark matter star in β-equilibrium with electrons: DC-model (left panels); bag-model (right panels). Temperatures (in MeV): $T = 0$ (thin solid lines), $T = 1$ (long dashed lines), $T = 5$ (dashed lines), $T = 20$ (dot-dashed lines) and $T = 50$ (dotted lines). $e^-$ (lower panels), and $u$- and $d$-quark (upper panels) concentrations are shown as a function of the energy density. The critical energy densities, $\varepsilon_c$, for the bag-model are temperature independent. In the DC-model they are negligibly small and only $\varepsilon_c(T = 50 \text{ MeV})$ can be shown in the figure.

The entropy per particle remains constant with respect to volume-deformations under the forces of gravity such that the second term in Eq. (13) vanishes and the star profile is determined by

$$\frac{s}{n} \left( T(r), \mu(r) \right) = \frac{s}{n} \left( T(0), \mu(0) \right).$$

(14)

With these assumptions we have specified that complete set of equations: (9)-(14), necessary to determine the structure of a star with given central temperature and chemical potentials once the EOS of the hot and dense matter is specified. We have employed them to obtain nonstrange quark star configurations using EOS$^{\text{DC}}$ and also EOS$^{\text{BM}}$ for comparison.

An important consequence of the difference between these equations of state highlighted in Fig. 1, is a strong modification at low densities of the composition of quark matter in β-equilibrium with electrons, depicted in Fig. 2. Due to the low critical energy density in our model; i.e, the energy density at deconfinement (e.g., $\varepsilon_c(T = 50 \text{ MeV}) = 1.4 \text{ MeV/fm}^3$), we observe a transition to isospin-symmetric matter as the energy density is decreased, whereas in the bag model, with $\varepsilon_c = 4B_P = 228 \text{ MeV/fm}^3$, the partial fractions of the quark and lepton species are little changed over the relevant energy domain.
Fig. 3. Upper Figure – Density profiles for adiabatic star configurations: DC-model (left panel); bag-model (right panel), for central temperatures $T(0) = 0$ (solid lines) and $T(0) = 50$ MeV (dotted lines). For each model the total baryon number is kept fixed: $N_{\text{DC}} = 0.866 \, N_{\odot}$ and $N_{\text{BM}} = 5.766 \, N_{\odot}$. Lower Figure – Temperature profile for adiabatic quark star configurations with $T(0) = 50$ MeV and central density $n(0) = 7.2 \, n_0$.

In Fig. 3 we plot the density and temperature profiles for a quark star with fixed baryon number:

$$N = 4\pi \int_0^R dr \, r^2 \, \frac{n(r)}{[1 - 2 \, m(r) \, G/r]^2},$$  \hspace{1cm} (15)$$

where $m(r)$ is the mass distribution, Eq. (10), and $n(r)$ is the baryon-number density, Eq. (12). Our reference measure for $N$ is $N_{\odot} := M_{\odot}/m_N$, with $m_N$ the nucleon mass. These figures provide a further comparison between the two models. EOS$^{\text{DC}}$ leads to profiles that approach zero smoothly at the surface.
of the star, whereas using EOS\textsuperscript{BM} the density and temperature are nonzero at the surface. This is a direct consequence of the difference between constant $B_P$ and $(T, \mu)$-dependent $B$, and how they characterise the phase transition. For a fixed central density, since the density profile vanishes at the surface, the total number of particles in our model is less. The calculated difference in surface temperature means that the luminosity of a star described by EOS\textsuperscript{DC} is less than that of a quark-matter star described by EOS\textsuperscript{BM}.

Our analysis of the stability of quark stars is illustrated in Fig. 4. At $T = 0$ in the bag-model the maximum attainable mass is approximately three-times more than that in our model. This difference is primarily the result of the fact that, when the pressure is zero, the energy density in the bag-model is large whereas in our model it is small, as depicted in Fig. 1.

In a real star the quark core is surrounded by a hadron shell, which will contribute a finite amount to the total pressure. In the bag-model the $(T, \mu)$-independent bag constant mimics this effect to some indeterminable extent.

In contrast, EOS\textsuperscript{DC} is obtained systematically and excludes by construction the contribution of hadrons; i.e., it only describes the contribution of quarks to the pressure. Hadrons provide an additional, additive contribution that is calculable in our framework; e.g., as proposed in [21]. The absence of this contribution in our present calculation is the origin of the low maximum-possible quark-star mass in our model: EOS\textsuperscript{DC} yields the maximum mass of a pure quark-matter star.

At finite-$T$ the maximum radius of a quark star is approximately the same in both models: $R \approx 8$ km in our model and $R \approx 10$ km in the bag-model. The lower panel in Fig. 4 shows that increasing $T$ leads to a reduction in both the maximum radius, $R$, and mass of a quark star. This is because increasing $T$ increases the compressibility; i.e., at finite-$T$ the pressure increases less rapidly with density, and hence a given gravitational mass occupies less volume. This same effect, which involves an increase in the central density, entails that the maximum mass of a stable quark star is reduced, as depicted in Fig. 4.

The star configurations shown in the upper panels of Fig. 3 can be considered as the initial and final states in the thermal evolution of a star. For a stable quark-star characterised by the quoted fixed total baryon numbers we find the following masses

\begin{align*}
T \text{ (MeV)} & \quad 50 \quad 0 \\
\text{DC} & \quad 0.55 M_\odot \quad 0.54 M_\odot \\
\text{BM} & \quad 1.69 M_\odot \quad 1.62 M_\odot.
\end{align*}

(16)
Fig. 4. Upper Figure – Gravitational mass of quark stars as a function of the central energy density: DC-model (upper panel); bag-model (lower panel). At a central temperature of $T(0) = 50\text{ MeV}$ (dashed lines) the maximum mass is lowered by $\sim 20\%$ when compared to the $T(0) = 0$ case (solid lines). Lower Figure – Gravitational mass of quark stars as a function of the radius. At $T(0) = 50\text{ MeV}$ the objects are more compact than when $T(0) = 0$.

Hence, in cooling, 2-4\% of the gravitational mass of a quark star is radiated.

Our study demonstrates that the properties of a quark-matter star are sensitive to the EOS and the temperature profile across the star, and to emphasise this we recapitulate our main results. Some are common to both models and should therefore be reliable predictions: (i) When the temperature $T(r)$ varies across the star the maximum allowable mass of a quark star is reduced as the central temperature, $T(0)$, is increased. Comparing $T(0) = 0$ and $50\text{ MeV}$, the reduction is $\sim 20\%$. In contrast, if the temperature is assumed to be constant across the star then the maximum allowable mass increases slightly with increasing $T(0)$. (ii) At $T(0) = 50\text{ MeV}$ the maximum attainable radius of a
pure quark star is $R \sim 8 - 10$ km, with $EOS^{DC}$ giving the least upper bound. (iii) In cooling a quark star radiates 2-4% of its gravitational mass.

There are, however, significant disagreements and the predictions of both models should be viewed cautiously in these cases: (i) Using $EOS^{DC}$ the maximum mass of a pure quark-matter star is $M \sim 0.6 - 0.7 M_\odot$, which is approximately the same as the maximum mass of a star composed of an ideal neutron gas. It is only one-third of the maximum mass attainable using $EOS^{BM}$. (ii) The temperature vanishes at the surface of an $EOS^{DC}$ star whereas in an $EOS^{BM}$ star the surface temperature is approximately 60% of the central temperature. (iii) The energy density at the surface of a quark star described by $EOS^{DC}$ is less than that of a star described by $EOS^{BM}$. Hence, in an $EOS^{DC}$ star the electron density fraction is higher and the quark matter is approximately isospin-symmetric at the surface, whereas in an $EOS^{BM}$ star the composition remains unchanged for all relevant energy densities.

Our study is based on a particularly simple gluon propagator. Those that use a more sophisticated model [11,14] lead to qualitatively the same behaviour of the quark propagator and hence to a similar EOS for pure quark-matter. We anticipate only small quantitative modifications, such as a slight stiffening of the EOS and concomitant improvement of its correspondence with lattice-QCD simulations. Therefore our analysis provides a qualitatively instructive first exploration of the properties of quark matter when the EOS is calculated in a dynamical model that describes chiral symmetry restoration and deconfinement simultaneously.

An important next step is the inclusion of hadronic bound states and their contribution to the pressure, which is a systematic but complex extension. Insofar as we have explicitly neglected hadrons, our equation of state is only directly relevant to the idealised case of a pure quark-matter star, and what might be considered as the physically unreasonable aspects of our results are primarily the consequence of their omission. This extension is necessary if we are to make statements about the evolution of real stars. Hence, at present, the mimicking of these effects in $EOS^{BM}$, although uncontrolled, makes it a firmer foundation for a phenomenology of real stars. Nevertheless, incipient in $EOS^{DC}$ is a constructive alternative that does not contain hidden degrees of freedom and can be improved systematically: $EOS^{DC}$ holds the promise of an unaffected, phenomenologically acceptable description of hot and dense nuclear matter.

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