Impact Angle Control Guidance Law Considering the Seeker’s Field-of-View Constraint Applied to Variable Speed Missiles

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ABSTRACT Recently, multiple constraints have been considered in the design of guidance laws. Most of the existing impact angle control guidance laws considering the seeker’s field-of-view constraint are only applicable to constant speed missiles and have other limits to some extent. This paper is concerned with the field-of-view constrained impact angle control guidance law applied to variable speed missiles based on the sliding mode control theory. In the design of the sliding mode controller, we use a quadratic Lyapunov function to achieve the impact angle control. Moreover, we introduce an integral barrier Lyapunov function with an adaptive control item to address the field-of-view constraint and prevent the undesirable chattering caused by the disturbance of the nonlinear system. Then, the asymptotic stability of this close-looped system is verified. Eventually, numerical simulations are provided to demonstrate the performance of the proposed guidance law.

INDEX TERMS Adaptive control, field-of-view constraint, impact angle control, Lyapunov function, sliding mode control.

I. INTRODUCTION In recent years, with the development of the high-tech industry, precision-guided missiles have become a significant trend for sophisticated weapons. Impact angle control guidance (IACG) laws can shape the missile trajectory to enable the missile to strike the weak point of the lightly-armored target like tanks. Thus, IACG law has become an interesting issue recently.

IACG laws presented in the previous literatures can be classified into three types: optimal control based IACG laws, proportional navigation guidance (PNG) based IACG laws and sliding mode control (SMC) based IACG laws. The first type is based on the optimal control theory. A suboptimal guidance law for a reentry vehicle was proposed in [1], which is the first attempt to deal with the impact angle constraint. Ryoo et al. presented a generalized formulation of optimal IACG laws in [2] and extended this formulation to a polynomial IACG law by minimizing the energy cost weighted by time-to-go power in [3]. The second type is based on the PNG method. A two-stage PNG law was proposed to handle the impact angle constraint in [4], [5]. In [4], the PNG law consists of a constant navigation gain \( N < 2 \) in the first stage to generate an orientation trajectory to achieve all the possible impact angles, and another constant navigation gain \( N \geq 3 \) in the second stage to drive the missile to intercept the stationary target with a desired impact angle. This method was further developed for nonmaneuvering moving targets in [5]. By using the switching logic technique, Erer et al. proposed a similar two-stage guidance scheme between PNG and biased proportional navigation guidance (BPNG) to achieve the impact angle control against stationary targets by adjusting the integral value of the bias [6]. Then, the basic framework of [6] was applied to moving targets in [7]. The third type is based on the SMC theory. By defining the impact angle as the terminal line-of-sight (LOS) angle, a SMC based IACG law was proposed in [8], which guarantees that the impact angle error can converge to zero in finite time. In [9], the singular problem that may lead to the saturation of guidance command was taken into consideration on the basis of [8]. Taking the autopilot of the missile into account, Zhou et al. [10] presented a SMC based IACG law to alleviate the adverse effects of the missile dynamics on the guidance performance.

However, with the development of the strap-down system, the seeker tends to be directly fixed on the missile body instead of the inertial platform. Thus, the seeker’s look angle
may be influenced by the motion of the missile’s attitude violently. Sometimes, in order to achieve overhead attack, the missile trajectory becomes highly curved, which may drive the target to exceed the seeker’s field-of-view (FOV), leading to lock-on failure. Hence, the seeker’s FOV constraint should be taken into account when designing IACG laws. So far, there have been numerical studies on IACG laws considering the seeker’s FOV constraint. In [11], Xin et al. viewed the FOV constraint as a nonlinear optimal control problem and proposed a \(\theta - D\) control guidance law to solve this state constraint. A pure optimal IACG law considering the FOV constraint was proposed in [12]. The work of [12] was extended to a range-to-go weighted optimal IACG law with the FOV constraint in [13]. Reference [11]–[13] are based on the optimal control technique. In addition, PNG was also used to solve this problem. A bias-shaping method using the work of [6] was presented in [14]. Tekin and Erer designed a two-stage PNG law with time-varying navigation gains obtained by a numerical process to achieve the impact angle control without violating the FOV constraint [15]. Park et al. proposed a three-stage BPNG law with the impact angle constraint for intercepting nonmaneuvering targets under the FOV constraint [16]. The SMC was also applied to solve this constraint. In [17], Wang et al. proposed a SMC based guidance law considering the FOV constraint by using an integral barrier Lyapunov function to make the constrained state reach the sliding mode in finite time. Meanwhile, a nonlinear extended state observer was constructed in this paper to estimate the total disturbance caused by target maneuver. Liu et al. [18] proposed a novel back-stepping FOV constrained IACG scheme by introducing a hyperbolic tangent function and its modification to make the missile fully utilize the maximum achievable range of the seeker’s look angle. In [19], a logarithm barrier Lyapunov function based IACG law with the seeker’s FOV constraint was proposed to improve the convergence rate of the system. Although the SMC based guidance laws proposed in [17]–[19] are verified to have perfect performance in achieving the impact angle control and addressing the FOV constraint, they are only designed for constant speed missiles.

Taking the strap-down seeker missile as the research object, we derive a SMC based IACG law considering the seeker’s FOV constraint applied to variable speed missiles in this paper. Here, we view this problem as a state-constrained stabilization problem of the nonlinear system. Compared with previous studies, the main innovations and contributions of our research can be concluded as follows:

1. We take advantage of the properties of the SMC to improve the convergence rate of the nonlinear system.
2. We design two time-varying sliding surfaces and combine a traditional quadratic Lyapunov function with an integral barrier Lyapunov function in guidance law design to handle different state constraints.
3. We introduce a hyperbolic tangent function into the guidance law design to fully utilize the seeker’s allowable FOV.
4. We add an adaptive control item to the sliding mode controller to alleviate the undesirable chattering caused by the disturbance of the nonlinear system.
5. The proposed law does not use switching logic which may cause the abrupt-hopping of guidance command.
6. The proposed law is applicable for intercepting both nonmaneuvering targets and maneuvering targets. The last but the most, this method can be applied to variable speed missiles.

The structure of this paper is organized as follows: Section II constructs a planar homing guidance geometry and a variable speed missile model. The state equations of the system are established and some necessary preliminaries are given in this section. Section III derives an SMC based IACG law with the FOV constraint applied to the variable speed missile. Section IV provides numerical simulations to verify the performance of the proposed law, analyzes the simulation results and gives a discussion about the results. Eventually, Section V draws a conclusion of our research.

**FIGURE 1.** Homing guidance geometry of the missile-target engagement.

**II. PROBLEM FORMULATION**

Consider a planar homing guidance geometry of the missile-target engagement shown in Fig. 1, where \(X – O – Y\) is a Cartesian inertial reference frame. Here, the subscripts \(M\) and \(T\) represent the missile and the target respectively. \(R\) is the missile-target relative range and \(q\) is the LOS angle. \(V\) is the speed and \(A\) is the acceleration applied to the velocity vector perpendicularly. \(\theta\) and \(\varphi\) are the heading angle and the leading angle respectively. All the angles are defined as positive anticlockwise.

The kinematic equations of the missile-target engagement can be given as

\[
\dot{R} = V_T \cos \varphi_T - V_M \cos \varphi_M \tag{1}
\]

\[
R\dot{q} = V_M \sin \varphi_M - V_T \sin \varphi_T \tag{2}
\]

where \(\varphi_M = q - \theta_M, \varphi_T = q - \theta_T\). \(\dot{R}\) is the relative velocity between the missile and the target, and \(\dot{q}\) is the LOS angular rate. In this paper, both the missile and the target are regarded as mass points. However, different from above-mentioned studies, the missile’s speed \(V_M\) is not constant anymore.
To intimate the realistic missile model, we take gravity, thrust and aerodynamic drag into account. Ignoring the dynamics of the autopilot and the actuator, we construct a variable speed missile model. In a planar reference frame, the missile model is given by

\[ \dot{x} = V_M \cos \theta_M \]
\[ \dot{y} = V_M \sin \theta_M \]
\[ m \dot{V}_M = P_e - f - mg \sin \theta_M \]
\[ V_M \dot{\theta}_M = A_M - g \cos \theta_M \]

where \( x \) and \( y \) are the horizontal ordinate and the vertical ordinate of the inertial reference frame, \( P_e \) and \( f \) are the thrust and the aerodynamic drag applied to the missile, \( m \) is the mass of the missile and \( g \) is the gravitational acceleration.

Given the initial mass of the missile \( m_0 \), the total mass of fuel \( m_f \) and the consumption rate of fuel \( \dot{m} \), the mass of the missile can be given as

\[ m = \begin{cases} m_0 - \dot{m}, & 0 \leq t < \frac{m_f}{\dot{m}} \\ m_0 - m_f, & t \geq \frac{m_f}{\dot{m}} \end{cases} \]

Given the initial thrust \( P_0 \), the thrust applied to the missile can be given as

\[ P_e = \begin{cases} P_0, & 0 \leq t < \frac{m_f}{\dot{m}} \\ 0, & t \geq \frac{m_f}{\dot{m}} \end{cases} \]

Referring to [20], the aerodynamic drag is given by

\[ f = f_0 + f_i \]
\[ f_0 = C_{f0} Q S \]
\[ f_i = C_{fi} m^2 A_M^2 \]
\[ Q = \frac{1}{2} \rho V_M^2 \]

where \( f_0 \) is the zero-lift drag, \( f_i \) is the induced drag, \( C_{f0} \) and \( C_{fi} \) are the coefficients of them, \( \rho \) is the atmosphere density, \( S \) is the reference area and \( Q \) is the dynamic pressure.

Next, let \( V_R \) and \( V_q \) be the closing speed and the relative speed perpendicular to the LOS, which are given by

\[ V_R = \dot{R} \]
\[ V_q = \dot{R} \]

Differentiating (14) with respect to time \( t \), and substituting (1), (2), (6) and (13) into it, yields

\[ \dot{V}_q = -\frac{V_R V_q}{R} - (A_M - g \cos \theta_M) \cos \phi_M + d \]

\[ d = \dot{V}_M \sin \phi_M - \dot{V}_T \sin \phi_T + V_T \dot{\phi}_T \cos \phi_T \]

where \( \dot{V} \) denote the speed variation rate, \( d \) denote the disturbance caused by the maneuver of the missile and the target. We assume that there is no external force acting on the target, so the normal acceleration of the target is given by \( A_T = V_T \dot{\phi}_T \). Hence, \( d = \dot{V}_M \sin \phi_M - \dot{V}_T \sin \phi_T + A_T \cos \phi_T \).

In fact, it is exactly hard to measure the value of this disturbance directly. However, owing to the physical limit of the missile and the target, we can assume that \( \sup_{t \in [t_0, t_f]} \dot{V}_M(t) = a_M, \sup_{t \in [t_0, t_f]} \dot{V}_T(t) = a_T, \sup_{t \in [t_0, t_f]} V_T(t) = A_T \max \), where \( t_0 \) and \( t_f \) denote the initial time and the terminal time of the interception. Hence, it is reasonable to make the following assumption.

**Assumption 1:** The disturbance \( d \) is above bounded. That is there exists a positive constant \( D \) satisfying

\[ |d| \leq a_M + a_T + A_T \max = D \]

where \( D \) is the upper bound of the disturbance \( d \).

Since IACG law is mainly designed for intercepting low speed targets, we should give the following assumption:

**Assumption 2:** The missile has a great speed advantage over the target (i.e., \( V_M \gg V_T \)). Here, we define a non-dimensional speed ratio between the missile and the target as

\[ \eta = \frac{V_T}{V_M} \ll 1 \]

In this paper, we only consider the vertical plane of the missile-target engagement geometry. Hence, the seeker’s FOV is defined as the range between the missile body axis and the LOS. Thus, the seeker’s look angle is the sum of the missile’s leading angle and the angle of attack (AOA). In practice, it is difficult to consider the AOA in the guidance law design. To simplify, we make the following assumption:

**Assumption 3:** The AOA of the missile is small enough to be neglected. Hence, we can use the leading angle of the missile to replace the seeker’s look angle and the FOV constraint can be given as

\[ |\psi_M(t)| < \psi_{\max} \]

where \( \psi_{\max} \) is the upper limit of the seeker’s look angle, which usually satisfies \( \psi_{\max} \in (0, \frac{\pi}{2}) \). We can further assume that the initial leading angle of the missile also satisfies \( |\psi_M(t_0)| < \psi_{\max} \). According to **Assumption 1-3**, we can make the following assumption without any loss of generality:

**Assumption 4:** Since the speed ratio satisfies \( \eta \ll 1 \), the leading angle of the missile satisfies \( |\psi_M(t)| < \psi_{\max} \) and \( \psi_{\max} \in (0, \pi/2) \), we can assume that

\[ |\sin \psi_M(t) - \eta \sin \psi_T(t)| < \sin \psi_{\max} - \eta \]

which also holds at initial time \( t_0 \).

Note that the impact angle is defined as the LOS angle at the terminal time of the interception \( t_f \) in this paper. However, some references define the impact angle as the heading angle of the missile at \( t_f \) [15], [16]. Referring to [16], we have that the impact angle defined in this paper and the heading angle of the missile at \( t_f \) satisfies the following equation:

\[ q_d = q(t_f) = \tan^{-1} \left( \frac{V_M \sin \theta_M(t_f) - V_T \sin \theta_T(t_f)}{V_M \cos \theta_M(t_f) - V_T \cos \theta_T(t_f)} \right) \]
Then, we define the state variables \( x_1 = \dot{q} - q_d \) and \( x_2 = V_q \). By using (14) and (15), we can establish the state equations of this nonlinear system, which can be given as

\[
\dot{x}_1 = \frac{1}{R} x_2 \quad (22)
\]

\[
\dot{x}_2 = -\frac{V_R}{R} x_2 - (A_M - g \cos \theta_M) \cos \varphi_M + d \quad (23)
\]

For the system expressed by (22)-(23), the guidance command \( A_M \) can be selected as the control input. Before the impact occurs, if \( A_M \) can force \( (x_1, x_2) \rightarrow 0 \), miss distance and impact angle error will converge to zero, that is \( R(t_f) \rightarrow 0 \), \( |q - q_d| \rightarrow 0 \). Here, we define a new variable as

\[
k = V_M \sin \varphi_{\text{max}} - V_T \quad (24)
\]

According to Assumption 2, it is easy to prove that \( k > 0 \). Hence, the sufficient and necessary condition for addressing the FOV constraint (i.e., \( |\varphi (t)| < \varphi_{\text{max}} \)) can be given as

\[
|\dot{x}_2| < k \quad (25)
\]

**Proof:** According to Assumption 4, we can further derive

\[
|V_M \sin \varphi_M (t) - V_T \sin \varphi_T (t)| < V_M \sin \varphi_{\text{max}} - V_T \quad (26)
\]

By using (2), (14), (24) and (26), we can easily derive \( |\dot{x}_2| < k \). Thus, (25) is a necessary condition. If \( |\dot{x}_2| < k \), we have \(-k < \dot{x}_2 < k\), which can be rewritten as

\[
V_T - V_M \sin \varphi_{\text{max}} < V_M \sin \varphi_M (t) - V_T \sin \varphi_T (t) < V_M \sin \varphi_{\text{max}} - V_T \quad (27)
\]

Then, we can further derive

\[
-V_M \sin \varphi_{\text{max}} < V_T [1 + \sin \varphi_T (t)] - V_M \sin \varphi_{\text{max}} < V_M \sin \varphi_M (t) < V_M \sin \varphi_{\text{max}} - V_T [1 - \sin \varphi_T (t)] < V_M \sin \varphi_{\text{max}} \quad (28)
\]

which can be simplified as \(-\sin \varphi_{\text{max}} < \sin \varphi_M (t) < \sin \varphi_{\text{max}}, \) that is \(|\sin \varphi_M (t)| < \sin \varphi_{\text{max}}| \).

Referring to Assumption 3, we know \( \varphi_M \in (0, \pi/2) \), then we can further derive \( |\varphi_M (t)| < \varphi_{\text{max}} \). Thus, (25) is also a sufficient condition. So far, the proof has been completed.

Hence, the objective of the guidance law we propose can be concluded as: designing a proper \( A_M \) to achieve the stabilization of the nonlinear system given by (22) and (23) under the state constraint given by (25) with (24) in finite time. For the purpose of designing a guidance law to achieve the above-mentioned objective, it is necessary to give the following preliminaries:

**Definition 1 (see [21]):** A Barrier Lyapunov Function (BLF) is a scalar function \( V (x) \), defined with respect to the system \( \dot{x} = f (x) \) on an opening region \( F \) containing the origin that is continuous, positive definite and has continuous first-order partial derivatives at every point of \( F \). It has the following two properties:

1. \( V (x) \rightarrow \infty \) as \( x \) approaches the boundary of \( F \).
2. There exists a positive constant \( b \) satisfying \( V (x (t)) \leq b \) for \( \forall t \geq 0 \) along the solution of \( \dot{x} = f (x) \) and for \( x (0) \in F \).

A BLF can be classified into the symmetric form and the asymmetric form, as illustrated in Fig. 2. Moreover, according to previous references, BLFs consist of three types: the tangent BLF (tBLF) [22], the integral BLF (iBLF) [23] and the logarithm BLF (lBLF) [24].

**Lemma 1 (see [21]):** For any positive constant \( k_{b_1} \), let

\[
Z_{i_1} := \{ z_1 \in \mathbb{R} : -k_{b_1} < z_1 < k_{b_1} \} \subset \mathbb{R} \text{ and } N := \mathbb{R}^l \times Z_1 \subset \mathbb{R}^{l+1} \text{ be open sets. Consider the system
}
\]

\[
\dot{z}_1 = h (t, \beta) \quad (29)
\]

where \( \beta := [\omega, z_1]^T \in N, h := \mathbb{R}^+ \times N \rightarrow \mathbb{R}^{l+1} \) is piecewise continuous in \( t \) and locally Lipschitz in \( z \), uniformly in \( t \), on \( \mathbb{R}^+ \times N \). Suppose that there exist functions \( U : \mathbb{R}^l \rightarrow \mathbb{R}^+ \) and \( V_1 : Z_1 \rightarrow \mathbb{R}_+ \), continuously differentiable and positive definite in their respective domains, such that

\[
V_1 (z_1) \rightarrow \infty \text{ as } z_1 \rightarrow -k_{b_1} \text{ or } z_1 \rightarrow k_{b_1} \quad (30)
\]

\[
\gamma_1 (\|\omega\|) \leq U (\omega) \leq \gamma_2 (\|\omega\|) \quad (31)
\]

where \( \gamma_1 \) and \( \gamma_2 \) are class \( K_\infty \) functions.

Let \( V (\beta) := V_1 (z_1) + U (\omega) \), and \( z_1 (0) \) belong to the set \( z_1 \in (-k_{b_1}, k_{b_1}) \). If the following inequality holds:

\[
\dot{V} = \frac{\partial V}{\partial \beta} h (t, \beta) \leq 0 \quad (32)
\]

then \( z_1 (t) \) remains in the open set \( z_1 \in (-k_{b_1}, k_{b_1}) \) for \( \forall t \in [0, \infty) \). The proof can be found in [21].

**Remark 1:** As mentioned above, the state variables can divided into the free state and the constrained state. The leading advantage of the BLF is that it can prevent the constrained state from reaching its boundaries. Hence, the constrained state \( x_2 \) requires a BLF while the free state \( x_1 \) may only involve a quadratic Lyapunov function (QLF).

**Definition 2 (see [17], [23]):** Consider the following function:

\[
V_i (s_i, \alpha_{i-1}) = \int_0^{s_i} \frac{\sigma k_i^2}{k_i^2 - (\sigma + \alpha_{i-1})^2} d\sigma, \quad i = 1, 2, \ldots, n
\]

(33)

\[FIGURE 2. The schematic diagram: (a) symmetric BLF; (b) asymmetric BLF.\]
where, $s_i = x_i - \alpha_{i-1}$, $\alpha_{i-1}$ is a continuously differentiable function satisfying $|\alpha_{i-1}| \leq A_i < \kappa$, for some positive $A_i$. Then, $V_i(s_i, \alpha_{i-1})$ can be defined as an iBLF. The partial derivative of $V_i(s_i, \alpha_{i-1})$ with respect to $\alpha_{i-1}$ can be given as
\[
\frac{\partial V_i}{\partial \alpha_{i-1}} = s_i \left[ \frac{k_i^2}{k_i^2 - x_i^2} - \psi(s_i, \alpha_{i-1}) \right]
\] (34)
where $\psi(s_i, \alpha_{i-1})$ is a function defined in a neighborhood of zero, which is given by
\[
\psi(s_i, \alpha_{i-1}) = \frac{k_i}{2s_i} \ln \left( \frac{k_i + x_i}{k_i - x_i} \frac{k_i - \alpha_{i-1}}{k_i + \alpha_{i-1}} \right)
\] (35)
The proof can be found in [23], which is omitted here for saving space.

III. DESIGN OF iBLFqG LAW

In this section, we design a SMC based ICAG law considering the FOV constraint by using a QLF and an iBLF with an adaptive control item to stabilize the nonlinear system given by (22)-(23) under the constraint given by (25) with (24).

As mentioned above, the FOV constraint we need to solve is transformed into a time-varying symmetric state constraint $|x_2| < k$. For the sake of guidance law design, it is necessary to introduce a symmetric bounded function. Here, we introduce a hyperbolic tangent function given in [18], which is defined as
\[
\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}
\] (36)
The properties of the hyperbolic tangent function $\tanh(x)$ are as follows:
1) $-1 < \tanh(x) < 1$ holds for all $x \in \mathbb{R}$
2) $x \tanh(x)$ is a positive definite function.

In general, the design of a sliding mode controller contains two steps. For our method, the first step is selecting proper sliding surfaces to make the close-looped nonlinear system that we establish has a well-behaved dynamic performance on them, and the second step is designing a guidance law to force the sliding surfaces converge to zero in finite time and maintain at zero till the end. For this paper, in the first step, in order to achieve the interception under a desired impact angle without violating the FOV constraint, we design two sliding surfaces $s_1$ and $s_2$, which is defined as the two tracking errors of this system.
\[
s_1 = q - q_d = x_1
\] (37)
\[
s_2 = V_q - V_{q_d} = x_2 - V_{q_d}
\] (38)
where $V_{q_d}$ is the desired value of $V_q$. Here, in order to address the FOV constraint $|x_2| < k$, we introduce a hyperbolic tangent function into the design of $V_{q_d}$, which can be given as
\[
V_{q_d} = -k_a \tanh(k_1 s_1)
\] (39)
where $k_1 > 0$ and $k_a = k - \varepsilon$, where $\varepsilon$ is a small positive constant such that the FOV can be utilized as much as possible under the guidance law we design.

In the second step, firstly we construct a quadratic Lyapunov functions $V_1$ to drive $s_1$ to converge to zero in finite time, which is given by
\[
V_1 = \frac{1}{2} s_1^2
\] (40)
Differentiating (40) with respect to time, yields
\[
\dot{V}_1 = s_1 \dot{s}_1
\] (41)
Differentiating (37) and substituting (22) and (38), the time derivative of $s_1$ is given by
\[
\dot{s}_1 = \ddot{s}_1 = \frac{V_{q_d}}{R} + \frac{s_2}{R}
\] (42)
Substituting (42) into (41), we have
\[
\dot{V}_1 = s_1 \dot{s}_1 = \frac{s_1 V_{q_d}}{R} + \frac{s_1 s_2}{R}
\] (43)

Note that there exists a cross term $\frac{V_{q_d}}{R}$ in (43), but it can be cancelled in the following design. Substituting (39) into (43), we have
\[
\dot{V}_1 = s_1 \dot{s}_1 = -k_a s_1 \tanh(k_1 s_1) + s_1 s_2
\] (44)

As can be seen, once we can successfully drive $s_2$ to converge to zero in finite time, $V_q$ can track its desirable value $V_{q_d}$ accurately such that the symmetric state constraint $|x_2| < k$ can be guaranteed. That is the seeker’s FOV constraint is not violated. According to Lemma 1, we need to construct a BLF to solve this constraint. Here, we design an iBLF, which can be given as
\[
V_2 = \int_{0}^{s_2} \frac{k^2 \alpha}{k^2 - (\alpha + \alpha_1)^2} d\alpha + \frac{1}{2\lambda} \hat{D}^2
\] (45)
where $\alpha_1 = x_2 - s_2$. It is noted that we add an adaptive control term to alleviate the chattering caused by the disturbance $d$. Here, we define the adaptive tracking error as
\[
\hat{D} = D - \hat{D}
\] (46)
where $\hat{D}$ is the estimated value of $D$.

As mentioned above, $\alpha_1 = V_{q_d}$, which is continuously differentiable and satisfies $|\alpha_1| \leq k_a < k$. In the following design, we use $V_{q_d}$ to replace $\alpha_1$. Differentiating (39) with respect to time, the derivative of $V_{q_d}$ can be given as
\[
\dot{V}_{q_d} = -k_a k_1 \left[ 1 - \tanh^2(k_1 s_1) \right] \dot{s}_1
\] (47)
Substituting (39) and (42) into (47), yields
\[
\dot{V}_{q_d} = -k_a k_1 \frac{1}{R} \left[ 1 - \tanh^2(k_1 s_1) \right] \left[ -k_a \tanh(k_1 s_1) + s_2 \right]
\] (48)
Here, referring to (34)-(35), we derive
\[
\frac{\partial V_2}{\partial \alpha_1} = s_2 \left[ \frac{k^2}{k^2 - x_2^2} - \psi(s_2, \alpha_1) \right]
\] (49)
\[ \psi(s_2, \alpha_1) = \frac{k}{2s_2} \ln\left(\frac{k + x_2}{k - x_2}\right) \]  

Differentiating (45) with respect to time and substituting (49), yield

\[ \dot{V}_2 = \frac{k^2s_2^2}{k^2 - x_2^2} + s_2 \left( \frac{k^2}{k^2 - x_2^2} - \psi \right) \cdot \dot{V}_{q_d} + \frac{1}{\lambda} \ddot{D} \]  

By (23) and (38), the time derivative of \( s_2 \) is given by

\[ \dot{s}_2 = -\frac{V_R V_q}{R} - (A_M - g \cos \theta_M) \cos \varphi_M + d - \dot{V}_{q_d} \]  

Let \( K = \frac{k^2}{k^2 - x_2^2} > 0 \) and substituting (52) into (51), yield

\[ \dot{V}_2 = Ks_2 \left[ -\frac{V_R V_q}{R} - (A_M - g \cos \theta_M) \cos \varphi_M + d - \dot{V}_{q_d} \right] + s_2 (K - \psi) \cdot \dot{V}_{q_d} + \frac{1}{\lambda} \ddot{D} \]  

Next, we define \( V = V_1 + V_2 \) as the ultimate form of the Lyapunov function in the second step of the guidance law design. By (44) and (51), the derivative of \( V \) is given by

\[ \dot{V} = Ks_2 \left[ -\frac{V_R V_q}{R} - (A_M - g \cos \theta_M) \cos \varphi_M + d - \dot{V}_{q_d} \right] + s_2 (K - \psi) \cdot \dot{V}_{q_d} + \frac{1}{\lambda} \ddot{D} - \frac{k_{a}s_1 \tanh (k_1 s_1)}{R} + \frac{s_1 s_2}{R} \]  

Hence, the guidance command \( A_M \) is designed as

\[ A_M = g \cos \theta_M - \frac{1}{\cos \varphi_M} \times \left( \frac{V_R V_q}{R} + \frac{\psi V_{q_d}}{K} - k_2 s_2 - \dot{D} \text{sign} (s_2) + \frac{s_1}{K R} \right) \]  

where \( k_2 \) is a positive constant.

Then, substituting (55) into (54), we have

\[ \dot{V} = Ks_2 \left[ \frac{\psi V_{q_d}}{K} - k_2 s_2 - \dot{D} \text{sign} (s_2) - \frac{s_1}{K R} + d - \dot{V}_{q_d} \right] + s_2 (K - \psi) \cdot \dot{V}_{q_d} + \frac{1}{\lambda} \ddot{D} - \frac{k_{a}s_1 \tanh (k_1 s_1)}{R} + \frac{s_1 s_2}{R} \]  

The adaptive law is designed as

\[ \dot{D} = \lambda K \left| s_2 \right| \]  

Differentiating (46) with respect to time and substituting (57), yield

\[ \dot{D} = -\dot{D} = -\lambda K \left| s_2 \right| \]  

Substituting (46) and (58) into (56) and further simplifying it, yield

\[ \dot{V} = -Kk_2 s_2^2 - K \left( D - \dot{D} \right) \left| s_2 \right| + Kds_2 + \frac{1}{\lambda} \ddot{D} - \frac{k_{a}s_1 \tanh (k_1 s_1)}{R} \]  

Referring to the second property of \( \tanh (x) \), we have \( k_{1}s_1 \tanh (k_1 s_1) \geq 0 \). Since \( k_1 > 0, k_{a} > 0 \), we have \( -k_{a}s_1 \tanh (k_1 s_1) \leq 0 \). Moreover, since \( K > 0 \) and \( k_2 > 0 \), we can derive \( \dot{V} < 0 \) by (59), which indicates that this nonlinear system is asymptotically stable under the guidance command given by (55). According to Lemma 1, the FOV constraint can be guaranteed.

It is noted that there exists a sign function in the guidance command given by (55), which may also cause undesirable chattering. Hence, we need to design a novel continuous function to replace the sign function in order to obtain a continuous guidance command, which can be given as

\[ \partial (s_2) = \begin{cases} \sin \tau s_2, & \left| s_2 \right| \leq \chi \\ \text{sign} (s_2), & \left| s_2 \right| > \chi \end{cases} \]  

where the product of \( \tau \) and \( \chi \) is \( \pi \).

### IV. NUMERICAL SIMULATIONS

This section presents numerical simulations of different engagement scenarios to demonstrate the performance of the proposed law. For the variable speed missile model, we need to give the following parameters which are shown in Table 1.

| Symbol       | Quantity               | Value       |
|--------------|------------------------|-------------|
| \( V_{u_s} \) | initial velocity       | 30m/s       |
| \( \theta_{u_s} \) | initial head angle   | 30          |
| \( A_{\text{max}} \) | limit of guidance command | 10g (1g = 9.8 m/s²) |
| \( \rho_s \) | initial position       | (-10000m,0) |
| \( \varphi_{\text{max}} \) | limit of lead angle   | 45          |
| \( m_s \)   | initial mass           | 150kg       |
| \( m_f \)   | fuel mass              | 50kg        |
| \( \dot{m} \) | assumption rate of fuel | 10kg/s     |
| \( \rho_0 \) | initial thrust         | 30000N      |
| \( S \)     | reference area         | 1           |

Besides, referring to [20], the zero-lift drag coefficient \( C_{f_0} \) and the induced drag coefficient \( C_{f_i} \) are given by

For \( C_{f_0} \)

\[ C_{f_0} = \begin{cases} 0.02, & \text{M < 0.93} \\ 0.02 + 0.2(M - 0.93), & \text{M < 1.03} \\ 0.04 + 0.06(M - 1.03), & \text{M < 1.10} \\ 0.0442 - 0.007(M - 1.10), & \text{M ≥ 1.10} \end{cases} \]  

For \( C_{f_i} \)

\[ C_{f_i} = \begin{cases} 0.2, & \text{M < 1.15} \\ 0.2 + 0.246(M - 1.15), & \text{M ≥ 1.15} \end{cases} \]
where $M$ is the Mach number. It is defined as $M = \frac{V_M}{V_S}$, where $V_S = 340\, \text{m/s}$ is the sound speed. In addition, when the flight altitude of the missile satisfies $y \in [0, 20000\, \text{m}]$, the atmosphere density is given by

$$\rho(y) = 1.15579 - 1.058 \times 10^{-4} y + 3.725 \times 10^{-9} y^2 - 6.0 \times 10^{-14} y^3$$  \hspace{1cm} (63)

Next, we give the following parameters employed for the proposed law. They are given as follows: $k_1 = 3$, $k_2 = 20$, $\varepsilon = 1$, $\lambda = 0.001$, $\chi = 0.01$, $\tau = \frac{\pi}{2}/\chi$.

We select the fourth-order Runge-Kutta scheme as the numerical solver with a fixed-step size of 10ms and we set the following three conditions to terminate the simulation:

1) The missile touches the ground.
2) The missile-target distance is less than 3m.
3) The closing speed of the missile becomes positive.

**A. INTERCEPTING A STATIONARY TARGET**

This case compares the performance of the proposed law with the BPNG law presented in [16] against a stationary target. The target is located at (0,0). In [16], numerical simulations have been done to verify the BPNG law is an effective IACG law against constant speed targets considering the seeker’s FOV constraint for a variable speed missile model, so it is also applicable to a stationary target. However, it is not applicable to the maneuvering target. The BPNG law is selected as the comparison group in this case because it is a typical method for solving this problem. We choose $\varphi_d = 60^\circ$ to do this simulation and Fig. 3 shows the simulation results for these two laws. The simulation results show that both laws successfully steer the variable speed missile to intercept the stationary target without violating the seeker’s FOV constraint. However, as we can see from Fig. 3 (a), the BPNG law drives the missile to fly a longer trajectory, which leads to a longer time to finish the interception. Impact times read 29.57s and 29.05s for the BPNG law and the proposed law respectively. Table 2 gives the values of miss distance and impact angle error for each guidance law. Obviously, the guidance precision of the proposed law is higher than that of the BPNG law.

![FIGURE 3. Intersecting a stationary target: (a) trajectory; (b) relative distance; (c) LOS angle; (d) heading angle; (e) leading angle; (f) guidance command.](image-url)
Since the target is assumed to be stationary, \( V_T = 0 \). Referring to (21), we have \( q_d = \theta_M(t_f) \). As can be seen from Fig. 3 (c) and (d), the result is consistent with the theoretical analysis. As we can see from Fig. 3 (e), none of these three cases violate the FOV constraint. Fig. 3 (f) shows that guidance commands under both laws are continuous and converges to nearly zero at final time. However, there exists an abrupt-hopping at about 0.19s and there exists some chattering afterwards in the guidance command under the BPNG law due to the use of the switching logic.

### B. INTERCEPTING A CONSTANT SPEED TARGET

The target is assumed to be moving at a constant speed of 10m/s along the \( Y \) axis in this case and simulations for a range of impact angles \( q_d = -30^\circ, -60^\circ, -90^\circ \) are provided. As can be seen from Fig. 4, the proposed law can steer the

| Law     | Miss Distance | Impact Angle Error |
|---------|---------------|--------------------|
| BPNG    | 0.268m        | 1.27e-005          |
| Proposed| 0.120m        | 5.62e-006          |
missile to intercept the constant speed moving target with negligible impact angle error and miss distance within the FOV constraint. The Impact times are 25.13s, 28.67s and 31.42s for \( q_d = -30^\circ, -60^\circ, -90^\circ \) respectively. We observe that the larger \( q_d \) is, the longer impact time is, which leads to a longer trajectory shown in Fig. 4 (a).

The values of miss distance and impact angle error for each \( q_d \) are shown in Table 3. As can be seen from Table 3, all the miss distances are less than 0.3m and all the impact angle errors are less than 1e − 003°. Thus, the guidance precision of the proposed law is sufficient for the engineering demand.

### TABLE 3. Values of miss distance and impact angle error for each \( q_d \).

| \( q_d \) | Miss Distance | Impact Angle Error |
|---------|---------------|--------------------|
| -30°    | 0.145m        | 5.66e−004°        |
| -60°    | 0.125m        | 5.20e−005°        |
| -90°    | 0.266m        | 1.81e−005°        |

As can be seen from Fig. 4 (g), the QLF successfully drives \( s_1 \) to zero which leads \( q \) to track \( q_d \) accurately. Fig. 4 (h) shows that \( s_2 \) converges to zero in finite time under the iBLF, which forces \( V_q \) to converge to its desired value \( V_{qd} \). As we introduce a hyperbolic tangent function into the design of \( V_{qd} \), the saturation characteristic of the function makes \( V_{qd} \) not exceed its boundary values \( \pm k_d \) such that the seeker’s FOV constraint is not violated for each \( q_d \) as shown in Fig. 4 (e). As can be seen, especially for \( q_d = -60^\circ, -90^\circ \), the seeker’s available FOV is fully utilized, which is caused by choosing a small \( \varepsilon \) for the hyperbolic tangent function such that \( V_q \) can approach its boundary values \( \pm k \). As we can see from Fig. 4 (f), the guidance command for each \( q_d \) is continuous without any obvious chattering during the whole reaching phase due to the use of adaptive control method. It is because the adaptive control item we add to the design of the guidance command can estimate the total disturbance of this system and serve as a compensation item for the chattering caused by the disturbance.

### C. INTERCEPTING A MANEUVERING TARGET

The first two cases have demonstrated the performance of the proposed law against nonmaneuvering targets. In this case, we will apply it to a maneuvering target. The target is assume to be moving with an initial speed \( V_{T_0} = 10 \)m/s and an initial heading angle \( \theta_{T_0} = 100^\circ \). Moreover, the target has an acceleration which can be decomposed into two components: \( a_{xT} = -2 \)m/s² along the X axis and \( a_{yT} = 1 \)m/s² along the Y axis. The simulation results for \( q_d = -30^\circ, -60^\circ, -90^\circ \) are shown in Fig. 5.

The simulation results illustrate that the proposed law is also effective for the maneuvering target. As can be seen from Fig. 5 (a), (b), (c) and (e), the variable missile successfully intercepts the target with negligible impact angle error and miss distance within the FOV constraint for each \( q_d \). Note that the truth still holds true that the larger \( q_d \) is, the longer impact time is. Impact times read 22.86s, 25.51s and 26.91s for \( q_d = -30^\circ, -60^\circ, -90^\circ \) respectively. We observe that \( q_d \neq \theta_{M}(t_f) \) from Fig. 5 (c) and (d), which is caused by \( V_T \neq 0 \) according to (21).

Table 4 gives the values of miss distance and impact angle error for each \( q_d \). As can be seen from Table 4, all the miss distances are less than 0.5m and all the impact angle errors are less than 1e − 002°. Hence, the proposed law still has a high guidance precision against the maneuvering target. In addition, with the help of the additional adaptive term in the design of \( A_M \), there is no obvious chattering appearing in the guidance command for each \( q_d \) as well. From Fig. 5 (f), we further observe that the guidance commands fail to converge to zero for \( q_d = 60^\circ, 90^\circ \). This is because the compensation item for gravity \( g \cos \theta_M \) in \( A_M \) is relatively large. The analyses and explanations about the simulations which are the same as Case B are omitted here.

### D. DISCUSSION

In this section, we provide three cases of numerical simulations to verify the performance of the proposed law.

In the first case, we assume the target is stationary and compare the proposed law with the BPNG law proposed in [16]. The results illustrate that although both laws could steer the variable speed missile to intercept the stationary target successfully without violating the FOV constraint, the proposed law has the following advantages:

1. The proposed law can make the missile fly a shorter distance to intercept the target and its guidance precision is higher.
2. The proposed law can effectively avoid the undesirable chattering and the abrupt-hopping of guidance command caused by the disturbance.
3. The proposed law is applicable against not only stationary targets and constant speed targets but also maneuvering moving targets.

In the second and the third case, the proposed law is applied to interception of a constant speed target and a maneuvering target respectively under a range of impact angles \( q_d = -30^\circ, -60^\circ, -90^\circ \). The results show that the proposed law still have a good performance. The variable speed missile can intercept the targets with a high precision without violating the seeker’s FOV constraint. Through the analysis of the simulation results for these two cases, we find that the impact time is affected by \( \max(\theta) \) from Fig. 5 (c) and (d), which is caused by \( V_T \neq 0 \). Moreover, with the help of the additional adaptive control term, the guidance commands are continuous without any obvious chattering or abrupt-hopping during the whole reaching phase. Finally, we note that the proposed Lyapunov functions can successfully drive the two sliding surfaces to converge to zero in finite time.

Referring to [17], the achievable range of \( q_d \) is determined by \( \psi_{max} \). The truth is the larger \( \psi_{max} \) is, the wider the achievable range of \( q_d \) is and if \( \psi_{max} \) arrives 80°, the range can be \(( -180^\circ, 180^\circ )\). However, we select \( \psi_{max} = 45^\circ \) for
practical demand. Hence, we only do the simulations for a limited range of impact angles in the second case and the third case. Here, if $q_d$ is chosen quite larger, it may exceed its achievable range such that the proposed law will not have such perfect guidance performance. We have not mastered the allowable maximum impact angle for the proposed law but we know the estimation of this value is another difficult course which is beyond our research scope.

As mentioned above, most existing IACG laws with the FOV constraint suffer the abrupt-hopping or obvious chattering of guidance command due to the application of the switching logic. Our method effectively solve this drawback.
by using an adaptive control technique. In fact, without its help, the switching of the sliding mode may also cause undesirable chattering. Ding et al. proposed a quasi-continuous polynomial high-order sliding mode (HOSM) controller of a simple form [25] to reduce the chattering phenomenon and further proposed a second-order sliding mode (SOSM) controller subject to mismatched term [26]. In addition, the disturbance observer is also an effective method to solve this problem [27].

Note that our method is based on the assumption that the dynamics of the autopilot and the actuator can be ignored. However, in some particular cases, the system lag is large enough to affect the performance of the proposed law and we should take it into consideration. Once the autopilot and the actuator are considered in the missile model, we have to face a realistic problem of fault diagnosis and fault tolerant control which is well handled in [28] and [29]. In addition, we assume that the AOA of the missile is small enough to be neglected. But for practical missile models, sometimes the AOA of the missile cannot be neglected and we should also take it into consideration. Zhao et al. [30] has proposed an integrated strap-down missile guidance and control law and taken the AOA into account when formulating the problem.

In this paper, we only focus on the impact angle constraint and the seeker’s FOV constraint which are both state constraints. However, we ignore the input constraint that is the normal acceleration saturation in the design of the proposed law. We directly establish a threshold of the guidance command and assume that the normal acceleration is always limited by this threshold. For realistic missile models, the normal acceleration is affected by many factors and if the input saturation occurs, the stability of the close-looped system will be influenced severely. To handle this problem, we can introduce a novel SOSM controller with the input constraint presented in [31].

V. CONCLUSION

In this paper, we propose an SMC based IACG law considering the seeker’s FOV constraint applied to variable speed missiles. To handle this guidance problem, we select \( x_1 = q - q_d \) and \( x_2 = V_q \) as the state variables and transformed this problem into a state-constrained stabilization problem of a close-looped nonlinear system. Based on the framework of the SMC theory, we design two sliding surfaces \( s_1 \) and \( s_2 \) for solving different state constraints and propose a QLF and an iBLF to drive \( s_1 \) and \( s_2 \) to converge respectively. Note that we utilize an adaptive control technique to compensate the total disturbance of the system caused by the maneuver of the missile and the target such that no obvious chattering or abrupt-hopping are observed in the guidance command of the missile. Eventually, we provide numerical simulations for three cases to demonstrate the performance of the proposed law. Compared with previous works, our method has the following advantages: maintaining the seeker’s lock-on condition without using any switching logic, preventing the adverse chattering from appearing in the guidance command, being applicable against maneuvering targets and can be applied to variable speed missiles. However, our research still has some limits as mentioned above, which need to be coped with in our future work.

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TABLE 4. Values of miss distance and impact angle error for each \( q_d \).

| \( q_d \) | Miss Distance | Impact Angle Error |
|--------|---------------|--------------------|
| -30°   | 0.326m        | 1.77e–003°         |
| -60°   | 0.353m        | 7.36e–003°         |
| -90°   | 0.425m        | 4.88e–003°         |
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