Modelling and Analysis of a Virtual Prototype of a Rotary Machine in MSC.ADAMS

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Abstract. The article presents an analysis of the virtual model of Laval (Jeffcott) rotor in the software environment MSC.ADAMS. The parameters describing the stability of the rotor operation were monitored and evaluated, i.e. critical angular velocity and trajectory of the rotor center of gravity (orbit). The results were compared with the values measured on the experimental equipment, as well as with the values obtained by analytical calculation. The paper further presents a simulation in which the second critical velocity was reached.

1. Introduction

Analysis and diagnostics of rotating body phenomena is a complex process, not quite easy to solve. For high power transmission, rotary machines are often designed as high-velocity, with the aim of reducing their energy consumption, weight and increasing reliability. The observed phenomenon is mainly the oscillation of the system, which is significantly affected by bearing in bearings, the effects of external forces, the distribution of discs on the shafts, the weight of the device, velocity, acceleration and many other parameters [1-4]. It should be noted that the beginning of the study of rotor dynamics dates back to the second half of the 19th century, when due to the ever-increasing velocity it was necessary to include the angular velocity and its effects in the overall dynamic analysis. Föppl tried to clarify these phenomena first, later Belluzzo, Stodola and finally Jeffcott [1,5]. At present, the results of work in the field of rotor system dynamics and interactions of flexible members are published in a number of journal articles and in professional publications [3-10]. Commercial programs are often used to investigate the dynamic properties of rotor systems. Therefore, it is necessary to have a suitable mathematical model that will allow the observed physical phenomena to be understood, analysed and evaluated in the current context [11].

2. Laval rotor

Based on the reality that real physical events cannot be modeled exactly due to their complexity, we chose a simplified model of a rotary machine, which was designed by the Swedish engineer Gustaf de Laval [12-14]. The model consists of a rigid disk mounted perpendicular to the axis of rotation in the middle of the length of a flexible shaft held in rigid rotary bonds. We neglect the weight of the shaft.
Despite the fact that it is a simplified model of a real rotor, it is possible to qualitatively clarify many important issues related to real rotors [12]. In Figure 1, point C presents the geometric center of the elastic shaft [1]. The point P represents the center of gravity of the disc.

The position and velocity of the point P can be expressed in the form:

$$\mathbf{r}_P(t) = \begin{bmatrix} x_C(t) + \varepsilon \cos(\Omega t) \\ y_C(t) + \varepsilon \sin(\Omega t) \end{bmatrix}$$  \hspace{1cm} (1)

$$\mathbf{r}_P'(t) = \begin{bmatrix} \dot{x}_C(t) - \varepsilon \Omega \sin(\Omega t) \\ \dot{y}_C(t) + \varepsilon \Omega \cos(\Omega t) \end{bmatrix}$$  \hspace{1cm} (2)

Kinetic energy $E_K$ and potential energy $E_P$ then they take form:

$$E_K = \frac{1}{2} m (\dot{x}_P^2 + \dot{y}_P^2) = \frac{1}{2} m (\dot{x}_C^2 + \dot{y}_C^2 + \varepsilon^2 \Omega^2 + 2\varepsilon \Omega \{ -\ddot{x}_C \sin(\Omega t) + \ddot{y}_C \cos(\Omega t) \})$$  \hspace{1cm} (3)

$$E_P = \frac{1}{2} k (x_C^2 + y_C^2)$$  \hspace{1cm} (4)

Lagrange equations can be written in the form:

$$\frac{d}{dt} \left( \frac{\partial (E_K - E_P)}{\partial q_i'} \right) - \frac{\partial (E_K - E_P)}{\partial q_i} = Q_i,$$  \hspace{1cm} (5)

whereas $q_i$ are Lagrange's generalized coordinates, here $x_C$ and $y_C$.

In case the external forces act on the point $P$ in the $xy$ plane (e.g. the weight of the rotor during horizontal rotation), the forces $Q_i$ can be obtained by considering the virtual displacement of point $C$ [\(\delta x_C, \delta y_C\)]$^T$. Since the angular velocity is given by the drive, i.e. angle $\theta = \Omega t$, is not a function of generalized coordinates, the virtual displacement of the point $P$ is [\(\delta x_P, \delta y_P\)]$^T$ and virtual work $\delta \mathcal{L}$ force $F$ with components $F_x$ and $F_y$ acting on point $P$ is equal to:

$$\delta \mathcal{L} = F_x \delta x_C + F_y \delta y_C.$$

(6)

The generalized forces can then be calculated as:

$$Q_i = \frac{\partial \delta \mathcal{L}}{\partial \delta q_i}$$

(7)

By performing the given derivatives, still considering rotation $\Omega$ as constant, we obtain the following equations of motion:
\[
\begin{align*}
\left\{ \begin{array}{l}
mx_c(t) + kx_c(t) = me\Omega^2 \cos(\Omega t) + F_x(t) \\
my_c(t) + ky_c(t) = me\Omega^2 \sin(\Omega t) + F_y(t)
\end{array} \right.
\end{align*}
\]
whereas the forces \( F_x(t) \) and \( F_y(t) \) are considered to be generalized functions of time, the forces causing instability having the same value of amplitude.

We get a general solution of the previous equation by adding a solution of a homogeneous equation (as a supplement to the functions):

\[
\begin{align*}
\left\{ \begin{array}{l}
m\ddot{x}_c(t) + kx_c(t) = 0 \\
m\ddot{y}_c(t) + ky_c(t) = 0
\end{array} \right.
\end{align*}
\]

to the particular integral of the complete equation. With this equation it is possible to describe the free oscillation of a perfectly balanced Laval rotor, while equation (15) describes the response to static imbalance and the response to the action of external forces in the \( xy \) plane. Due to the linearity, it is possible to analyze separately the response to imbalance and the response to static forces.

As mentioned at the beginning, it is possible to use coordinates \( x_p \) and \( y_p \) point \( P \) as generalized coordinates. In this case, the position of point \( C \) can be described as:

\[
\overrightarrow{C-O} = \mathbf{r}_C(t) = \begin{bmatrix} x_c(t) \\ y_c(t) \end{bmatrix} = \begin{bmatrix} x_p(t) - ec\cos(\Omega t) \\ y_p(t) - es\sin(\Omega t) \end{bmatrix}.
\]

The following then applies to kinetic and potential energy:

\[
E_K = \frac{1}{2}m(\ddot{x}_p^2 + \ddot{y}_p^2);
\]

\[
E_p = \frac{1}{2}k(x_p^2 + y_p^2 + \varepsilon^2 - 2\varepsilon[x_pc\cos(\Omega t) + y_pc\sin(\Omega t)]).
\]

Subsequent use of Lagrange functions and generalized forces \( F_s \) and \( F_i \) in Lagrange’s equations we get the equations of motion of a point \( P \):

\[
\begin{align*}
\left\{ \begin{array}{l}
m\ddot{x}_p(t) + kx_p(t) = ke\cos(\Omega t) + F_x(t) \\
m\ddot{y}_p(t) + ky_p(t) = ke\sin(\Omega t) + F_y(t)
\end{array} \right.
\end{align*}
\]

The critical velocity is the characteristic velocity at which the system resonates [15].

The bending stiffness of the shaft is characterized by the parameter stiffness \( k \). It applies to the area of elastic deformations:

\[
k = \frac{3Edh^4}{4l^3}
\]

whereas \( E \) is Young’s modulus, \( l \) distance of rotational bonds and \( dh \) diameter of the rotor disc system [14]. The following applies to the critical angular velocity:

\[
\omega = \frac{k}{\sqrt{m}}
\]

into which by substituting equation (14) we get the expression for the calculation of the critical angular velocity:

\[
\omega = \sqrt{\frac{3Edh^4}{4l^3m}}
\]

3. Modelling of a rotary machine in MSC.ADAMS

A virtual prototype (VP) of a Laval rotor was modeled in the environment of the program for solving the dynamics of bound mechanical systems MSC.ADAMS (Figure 2) [16-18].

The model consists of a rigid disk mounted perpendicular to the axis of rotation in the middle of the length of a flexible shaft with negligible weight modeled in the environment of the Adams Flex module, mounted in rigid rotary bonds. To obtain real results, the input physical and design parameters
were taken from the experimental equipment Rotor Kit [13,14]. They are: material density - steel S235, disc and shaft weight, disc and shaft diameter, disc width, bearing point distance from disc and modulus of elasticity. The input kinematic parameter was the angular velocity with constant angular acceleration (rotor start-up).

![Figure 2. Laval rotor model in MSC.ADAMS.](image)

The simulation was performed in time 7 s with the number of steps 10000. The results were presented in a postprocessor environment. We observed the deflection of the shaft center of gravity in the direction of the y-axis of the global coordinate system. At the same time, the angular velocity of the shaft was plotted (Figure 3).

![Figure 3. The course of the angular velocity of the shaft (blue) and the trajectory of the center of gravity in the y-axis (red).](image)

Since the shaft start-up is not set from zero angular velocity due to the convergence of the solver, we observe strongly unstable waveforms. After stabilization, the course of angular velocity is linear - it increases in proportion to time.

Figure 4 presents the area of stable operation. The value of the displacement of the monitored center of gravity is approximately in the interval <73.5 ; 75.3> μm. So it is about to amplitude approximately 2 μm. They start at velocity 140 rad/s up to a velocity 170 rad/s (Figure 3), the rotor works in stable operation with negligible deviations, which corresponds to the time from the first to about the fourth second.
Figure 4. Stable running area.

After the 4th second, the amplitude increases - the deviations start to increase. At a velocity of 187.4 rad/s, they reach a maximum and range from <65.2 ; 83.5> μm. The amplitude in this case is 18.3 μm, which corresponds to a nine-fold increase in the displacement value compared to stable operation.

With a further increase in the angular velocity of the shaft, the displacement decreases over time from approximately 5.75 s to 6.2 s. Here, the transition through resonance occurs, and subsequently the values of the deflection reach lower values (Figure 5).

Figure 5. Critical turns range.

The cause of this sudden instability is the achievement of critical velocity 187.42 rad/s.

These phenomena can also be observed when plotting the orbits of the shaft center of gravity at times of 0.4 s, where the state of instability of the solver in a non-circular trajectory and a large amplitude manifests itself. In the 3rd s, a steady state is evident, in the 5th s the amplitude also increases due to the increase of the velocity, and in 5.7 s resonance occurs (Figure 6).
Substituting the parameters into equation (16) we get the value of the critical angular velocity:

$$\omega = \sqrt{\frac{3E\pi d_h^4}{4l^3m}} = \sqrt{\frac{3.2.1.10^{11} \pi \cdot 10^{-12}}{4.0,5^3 \cdot (0.85 + 0.3)}} = 184,72 \text{ rad/s}$$

(17)

Table 1 presents a comparison of the results of the critical angular velocity in the individual analyzes and also the percentage deviation from the results obtained in MSC.ADAMS. The higher value obtained by the model from MSC.ADAMS is due to the non-consideration of the real damping that occurs in the experimental model.

Table. 1 Comparison of critical angular velocity results.

| Analysis                     | Critical angular velocity [rad/s] | Deviation from result in MSC.ADAMS [%] |
|------------------------------|----------------------------------|---------------------------------------|
| Experiment (Rotor Kit)       | 179                              | -4.5                                  |
| Analytical solution          | 184,72                           | -1.4                                  |
| Simulation in MSC.ADAMS      | 187,42                           | 0                                     |

The lower frequency in the experiment is due to the presence of damping in the real test state (testing stand).

Figure 6. Shaft center of gravity orbits.
Figure 7 plots the simulation output in which the 1st and 2nd critical velocities are recognizable. The first critical velocity is 187 rad/s and the second 377 rad/s. The red graph shows the magnitude of the center of gravity shift. The blue graph represents the magnitude of the velocity of the center of gravity and the green line represents the linear increase of the angular velocity of the rotor shaft (run up). Extreme values of displacements and velocities present the presence of critical velocities. The modal shape of the oscillation (shaft deflection) at the first and second critical velocities is shown in Figure 8.

Figure 7. The second critical velocity of the system

Figure 8. Modal shapes (shaft deflection) a) 1. critical velocity b) 2. critical velocity.

Conclusion
A model of the Laval rotor was created in the MSC.ADAMS environment and simulations were performed, while the flexible shaft with a rigid disk was placed in fixed rotary bonds. As the angular velocity of the shaft increased, the deviations of the reference point, which was the center of gravity in the center of the axis of rotation, changed. This process corresponded to the start-up of the rotor. Increased deviations of the center of gravity position or increased values of its velocity correspond to the critical rotor velocity. The system enters a state of resonance here. This state was confirmed by theoretical calculation. Since the geometry and dynamic parameters of the modeled rotor were identical with the parameters of the real rotor presenting the Rotor Kit, the performance is also compared with the experiment, where the value of critical velocity was found to be about 4.5% lower. We explain this condition by the presence of damping in the bearings on the test rig, the effect of which was not included in the calculation. Finally, a simulation was presented in which the second critical velocity was reached.

We want to consider this model as a starting point for further analysis of the dynamics of rotary machines. E.g. obliquely mounted disk on the shaft, resp. moving the disc closer to the bearing.
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