Gravity-Driven Acceleration of the Cosmic Expansion

Janna Levin

*Canadian Institute for Theoretical Astrophysics*

*Mc Lennan Labs, 60 St. George Street, Toronto, ON*

Abstract

It is shown here that a dynamical Planck mass can drive the scale factor of the universe to accelerate. The negative pressure which drives the cosmic acceleration is identified with the unusual kinetic energy density of the Planck field. No potential nor cosmological constant is required. This suggests a purely gravity driven, kinetic inflation. Although the possibility is not ruled out, the burst of acceleration is often too weak to address the initial condition problems of cosmology. To illustrate the kinetic acceleration, three different cosmologies are presented. One such example, that of a bouncing universe, demonstrates the additional feature of being nonsingular. The acceleration is also considered in the conformally related Einstein frame in which the Planck mass is constant.

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I. INTRODUCTION

In Einstein’s theory of general relativity, the gravitational constant is proposed to be a universal constant of nature, \( G = 1/M_o^2 \) where \( M_o = 10^{19} \text{ GeV} \) is the constant Planck mass. A simple extension of Einstein gravity elevates the Planck mass from a fundamental constant to a dynamical variable. The strength of gravity is then allowed to evolve as the universe evolves. Scalar-tensor theories proposed by Jordan [1] and Brans and Dicke [2] and generalized by Bergmann [3] and by Wagoner [4] incorporate such a modification of general relativity. A revival of Jordan-Brans-Dicke (JBD) gravity has occurred in the literature recently. There has been considerable interest in various JBD cosmologies and their phenomenological uses from inflationary to stringy cosmologies.

Here the early behavior of a universe empty except for the background dynamical Planck mass is studied. No potential is included. Remarkably, this investigation reveals the Planck field can accelerate the expansion of the universe. A negative pressure is always needed to accelerate the cosmic expansion. For ordinary matter, the pressure associated with kinetic energy is positive. By contrast, the nonminimal coupling of the Planck field to gravity allows for an unusual negative pressure associated with the kinetic energy of the field. It is worth stressing that there is no potential\(^1\) nor cosmological constant. This possibility was first pointed out in in references [5].

The source of the negative pressure in scalar-tensor theories of gravity can be understood heuristically. Loosely speaking, a pressure measures the negative of the change in energy with volume. If, as the universe expands, the energy contained within a unit volume decreases, the pressure is positive. Usually the energy does decrease as it takes work to power the expansion.

\(^1\)Although the nonminimal interaction of the Planck mass with gravity can in some sense be considered a potential, the energy density of the interaction is a function of derivatives of the Planck mass, and in that sense is kinetic. It is for this reason that the acceleration is dubbed kinetic driven and not potential driven.
For instance, a radiation bath redshifts and cools as it does work on the spacetime, so that the energy drops as the universe expands, \(E_{\text{rad}} \propto \rho_{\text{rad}} a^3 \propto 1/a \sim 1/V^{1/3}\) where \(a\) is the scale factor. As the energy decreases, the Hubble expansion slows. In other words, the scale factor decelerates. If instead the energy contained within a unit volume increases, then the pressure is negative. As the energy increases, the Hubble expansion quickens; that is, the universe accelerates. In standard inflation for instance \[\text{[6]},\] the energy density is constant, \(\rho \propto \Lambda\). As the scale factor grows, the energy contained within a unit volume grows, \(\rho a^3 \propto \Lambda a^3 \propto \Lambda V\), while \(p \propto -\Lambda\) is negative. The expansion gets ever faster.

Ordinarily kinetic energy has an associated positive pressure. This is not so in general scalar-tensor theories. The scalar-tensor theory can be defined entirely in terms of the kinetic coupling parameter \(\omega(m_{\text{pl}})\), where \(m_{\text{pl}}\) is the dynamical Planck mass (see \[\text{[3,1]}\]). If \(\omega\) is a positive constant, as is assumed in the original Brans-Dicke theory, then the pressure associated with the field is positive. However, if \(\omega\) grows with the Planck mass so that it satisfies a bound defined below, then the kinetic energy in the Planck field can grow as the universe expands. There is therefore a negative pressure and a corresponding acceleration of the scale factor. No potential nor cosmological constant is required. The acceleration is weak since there is a competition between the \(\omega\) effect and the redshifting in the kinetic energy.\[\text{[2]}\]

Since an acceleration is one of the key ingredients used in inflation, this feature is provocative. It is not attempted in this paper to build a model of gravity driven inflation. In fact success in such an attempt may be unlikely (see \[\text{§VI}].\) Instead a nominal condition for

\[\text{[2]}\]There is one additional possibility. If \(\omega\) is constant but negative and the Planck mass drops, then there exists a branch of the solutions which leads to an accelerated expansion, as noted in reference \[\text{[5]}.\] The kinetic energy grows as the Planck mass drops and the pressure is negative. This regime was recently considered, independently, in reference \[\text{[7]}.\] Specifically, the authors of \[\text{[7]}\] considered \(\omega = -1\) in a string-dilaton cosmology. They also found an accelerated expansion.
the theory to be pertinent for the causal physics of inflation is investigated. Although the universe may accelerate when $\omega(t) > 0$, it is demonstrated that the acceleration can be relevant for inflation only in theories for which $\omega$ is allowed to drop below zero [and then, only when the Planck mass drops as the universe evolves]. As demonstrated in §III, the energy density in a Friedman-Robertson-Walker universe is positive as long as $\omega \geq -3/2$. The range $-3/2 \leq \omega < 0$ is therefore not ruled out by the weak energy condition. The positivity of the energy density can be verified by looking in the conformally related Einstein frame as is done in §VII. These restrictions apply only for the simplest single scalar model. Chaotic alternatives have still to be explored. Further difficulties in building a sound kinetic inflationary model are discussed briefly in §VI and more fully in §V.

Independent of inflation, if the universe accelerates, an initial singularity may be circumvented. Loosely speaking, if the universe accelerates at its inception then the Hubble expansion is not infinite initially. Consequently, as one looks back in time to the first moment, the scale factor is not forced to zero and may approach a finite value. One example, expounded below, exhibits such nonsingular behavior.

II. THREE COSMIC ACCELERATIONS

The simplified situation of a universe empty, except for the background Planck field is studied in this paper. The metric is assumed to be a flat Friedman-Robertson-Walker (FRW) metric. The treatment is purely classical. Under these simplified conditions it is shown that an epoch of cosmic acceleration can in fact ensue. Three examples are given in which the acceleration is manifest.

The first example is of a bouncing universe. The universe begins infinitely large and contracts. It collapses down to finite size and then bounces into an accelerating phase. Alternatively, the universe could be created at the moment of the bounce in a nonsingular beginning. If this initial condition is imposed, the universe is created cold and empty except for the background Planck field. The energy density is zero initially. It thus costs no energy
to create such a universe. The Planck field begins to move and as it does so the universe begins to expand. The changing structure of gravity accelerates the expansion from zero. Eventually, the change in the parameter $\omega$ becomes more temperate and the expansion decelerates.

The second example is constructed so that $\omega$ never drops below zero, in order to assure the reader that such a model can be built with an accelerating phase. The universe begins singular in this example and decelerates as the universe evolves. As the Planck field enters a range of values, a weak burst of acceleration ensues. It turns off naturally and the universe decelerates from that moment until the end of time.

The third example is the only one in this paper which satisfies the nominal requirement relevant for the causal physics of inflation. It is not intended as a true model of inflation, only as an indication of the properties a kinetic inflationary model would have. The possible pitfalls and merits of a serious attempt at building such a model are discussed in that section.

For completeness, the kinetic acceleration is studied in the conformally related Einstein frame. In the Einstein frame, the Planck field is constant. An Einstein observer believes the universe’s expansion decelerates. However the distance between two points in space, relative to ruler lengths, agrees with the original JBD frame. Said another way, the acceleration is attributed to the relative rate of change of the length of an observer’s rulers.

Before proceeding, some of the recent cosmological settings in which scalar-tensor theories of gravity have been considered are listed. Solutions describing the universe have been found for JBD theories in vacuum [9], in a radiation-dominated era [9], [10], [11], [12], as well as in inflationary models such as (hyper)extended inflation [13] or induced gravity inflation [14]. Recently the suggestion has been made that for certain theories, the presence of nonrelativistic matter can serve to attract scalar-tensor gravity toward general relativity [15]. This idea was extended in reference [16] to low energy string theories where the dilaton plays the role of the Planck mass. It is noteworthy that the first example studied happens, quite coincidentally, to be of the form used in [15] to illustrate that general relativity is an
III. ACCELERATION FROM SCALAR-TENSOR GRAVITY

For simplicity, in this paper it is assumed that the universe is created initially devoid of all contributions to the energy-momentum tensor from ordinary matter. The universe studied is therefore empty, except for the kinetic energy density of the background Planck field. Beginning with the action for general scalar-tensor theories, a bound on the changing structure of gravity can be found such that the cosmic expansion is accelerated.

The gravitational action for a general Jordan-Brans-Dicke theory is

$$A[g_{\mu\nu}, \Phi] = \int d^4x \sqrt{-g} \left[ \Phi \frac{\omega(\Phi)}{16\pi} - \frac{\omega(\Phi)}{16\pi} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right] .$$

(3.1)

Newton’s constant $G = \Phi^{-1}$ or equivalently the dynamical Planck mass, $m_{pl}$, is related to $\Phi$ by $m_{pl} = \Phi^{1/2}$. The metric signature $(-, +, +, +)$ was used and $\mathcal{R}$ is the scalar curvature. The action (3.1) is completely general. A given theory is specified by choosing the functional form of $\omega(\Phi)$. It is explicitly enforced in (3.1) that no potential nor cosmological constant is present. Though the $\Phi\mathcal{R}$ coupling can be viewed as a potential of sorts, variation of this term with respect to the metric leads to derivatives on $\Phi$ (see eqn (3.3)). Therefore the $\Phi\mathcal{R}$ term can be identified as contributing to the kinetic energy density. A potential on the other hand would give a contribution to $T_{\mu\nu}$ proportional to $g_{\mu\nu}$.

Varying the action with respect to $g_{\mu\nu}$ gives the Einstein-like equations of motion

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = \frac{8\pi}{\Phi} T^\Phi_{\mu\nu} .$$

(3.2)

The energy-momentum tensor for $\Phi$ is given by

$$T^\Phi_{\mu\nu} = \frac{\omega(\Phi)}{8\pi \Phi} \left[ \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} (\partial_\alpha \Phi \partial^\alpha \Phi) g_{\mu\nu} \right] + \frac{1}{8\pi} (D_\mu D_\nu \Phi - g_{\mu\nu} \Box \Phi) ,$$

(3.3)

with $\Box = g^{\mu\nu} D_\mu D_\nu$, and $D_\mu$ is the covariant derivative. The first two terms in the energy momentum tensor (3.3) are analogous to those for a minimally coupled scalar field. However, unusual kinetic terms, namely the last term two terms in eqn (3.3), appear in the $\Phi$ stress.
tensor due to the variation of the nonminimal $\Phi R$ coupling in the action. These are the culprits which lead to the behavior given particular attention in this paper.

The variation of the action with respect to $\Phi$ gives the equation of motion

$$-\frac{2\omega}{3} \Box \Phi + \mathcal{R} + \frac{\partial_\mu \Phi \partial^\mu \Phi}{\Phi} \left[ \frac{\omega}{\Phi} - \frac{\partial \omega}{\partial \Phi} \right] = 0$$

(3.4)

Hereafter, it is assumed that the spatial gradients in $\Phi$ are negligible so that $T_{\mu\nu}$ is homogeneous and isotropic. The Friedmann-Robertson-Walker (FRW) metric reflects this symmetry; $g_{\mu\nu} = \text{diag}(-1, a^2, a^2, a^2)$ where $a$ is the scale factor. The equation of motion for $\Phi$ becomes

$$\ddot{\Phi} + 3H \dot{\Phi} = -\frac{1}{(3 + 2\omega) \frac{d\omega}{d\Phi}} \dot{\Phi}^2$$

(3.5)

For economy of notation define

$$f(\Phi) \equiv (1 + 2\omega(\Phi)/3)^{1/2}$$

(3.6)

The $\Phi$ equation of motion has the immediate solution, of much use later,

$$\dot{\Phi} = -\frac{C}{a^3 f}$$

(3.7)

The constant of integration, $C$, can be positive, negative, or zero.

In an FRW metric, the equation of motion for the scale factor $a$ becomes

$$H^2 + \frac{\kappa}{a^2} = -\frac{\dot{\Phi}}{\Phi} H + \frac{\omega}{6} \left( \frac{\dot{\Phi}}{\Phi} \right)^2$$

(3.8)

Hereafter the metric is taken for simplicity to be flat, $\kappa = 0$. Solving eqn (3.8) for $H$ yields

$$H = -\frac{\dot{\Phi}}{2\Phi} (1 \pm f)$$

(3.9)

There are two branches for $H$ which can lead to some confusion. Notice if $f > 1$, then the Hubble expansion is positive if the upper sign is chosen for $\dot{\Phi} > 0$ and the lower sign is chosen for $\dot{\Phi} < 0$. If $f < 1$, then $H < 0$ only when $\dot{\Phi} > 0$ (for either branch) and $H > 0$ only when $\dot{\Phi} < 0$ (for either branch).
In an FRW cosmology, the energy density in the $\Phi$-field is

$$\rho_\Phi \equiv T^0_0 \Phi = \frac{\omega}{16\pi} \frac{\dot{\Phi}^2}{\Phi} - \frac{3}{8\pi} H \dot{\Phi} \ . \quad \text{(3.10)}$$

Notice if eqn (3.9) is used in the expression for $\rho_\Phi$ that

$$\rho_\Phi = \frac{3}{32\pi} \frac{\dot{\Phi}^2}{\Phi} (f \pm 1)^2 \geq 0 \ . \quad \text{(3.11)}$$

For a healthy theory, there ought to be no negative energy or ghost modes. Therefore the kinetic energy density must be positive. Classically at least, the kinetic energy density is positive in an FRW metric as long as $\omega \geq -3/2$. This can be verified by considering the conformally related Einstein frame as is done in §IV.

The pressure is $p_\Phi \delta^i_j = T^i_j \Phi$,

$$p_\Phi = \frac{\omega}{16\pi} \frac{\dot{\Phi}^2}{\Phi} - \frac{1}{8\pi} H \dot{\Phi} - \frac{\dot{\Phi}^2}{8\pi(3 + 2\omega)} \frac{\partial \omega}{\partial \Phi} \ . \quad \text{(3.12)}$$

Notice the last term can be negative if $\partial \omega / \partial \Phi > 0$. This indicates that a changing value of $\omega(\Phi)$ can lead to a negative pressure in scalar-tensor theories, in the absence of any potential. If $\omega$ is a positive constant, as was assumed in the original Brans-Dicke model, the pressure is positive. It can be verified, as argued in the introduction, that $p_\Phi$ has a contribution from $-dE_\Phi/dV$ where $E_\Phi = \rho_\Phi V$ and $V = a^3$. Thus a negative pressure signifies, in part, that the energy in a unit volume grows as the universe expands. As shown below, the changing structure of $\omega(\Phi)$ can in turn lead to an epoch of accelerated cosmic expansion, as can a constant but negative $\omega$.

**A. Condition on $\omega(\Phi)$**

The previous results can be exploited here to uncover a condition $\omega(\Phi)$ must satisfy in order to drive the scale factor to accelerate. In general the acceleration of the scale factor is given by

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi}{3\Phi} (\rho + 3p) \ . \quad \text{(3.13)}$$
While the weak energy principle requires generally that $\rho > 0$, the pressure can be negative. [The undecorated $\rho$ and $p$ will refer to unspecified energy and pressure.] In standard cosmology both $\rho$ and $p$ are always positive so that the universe always decelerates. In the inflationary cosmology by contrast, $\rho = -p$ and the universe accelerates with $\ddot{a}/a = \Lambda$. As derived above, there can be a negative contribution to the pressure from the Planck field due to the changing structure of gravity. A bound is given below on the form of $\omega(\Phi)$ which leaves the pressure negative enough to power an acceleration of the scale factor.

The results of eqns (3.10) and (3.12) are put to use in eqn (3.13). The acceleration is

$$\frac{\ddot{a}}{a} = -\frac{\omega}{3} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 + H \left( \frac{\dot{\Phi}}{\Phi} \right) + \frac{1}{2} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 \frac{\Phi}{(3 + 2\omega)} \frac{\partial \omega}{\partial \Phi}.$$  (3.14)

Using (3.6) and (3.9) in eqn (3.14) gives

$$\frac{\ddot{a}}{a} = -\frac{1}{2} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 f \left[ f \pm 1 - \frac{df}{d\ln \Phi} \frac{1}{f^2} \right].$$  (3.15)

In order for $\ddot{a} > 0$, the following condition must be satisfied:

$$f \pm 1 - \frac{df}{d\ln \Phi} \frac{1}{f^2} < 0.$$  (3.16)

If the functional form of $f = (1 + 2\omega(\Phi)/3)^{1/2}$ changes as the universe evolves such that it obeys the bound of (3.16), the scale factor of the universe will accelerate. It is easy to imagine that such a phase is entered and exited smoothly.

There is one other possibility which can lead to $\ddot{a} > 0$. The specific combination of a constant but negative $\omega$ ($f < 1$) with the minus branch in eqn (3.16) can give $\ddot{a} > 0$. As mentioned below eqn (3.9), this corresponds to an accelerated expansion only if $\dot{\Phi} < 0$. The particular combination $\omega = -1$, $\dot{\Phi} < 0$ with the minus branch, was studied in reference [7] in the context of string theory. In that reference, the authors considered the role of the accelerated expansion in the graceful exit problem. Perhaps if $\omega(\Phi)$ is allowed to vary, as opposed to being constrained to the value $-1$, a graceful exit will be easier to execute.

Incidentally, if there is a contribution to the energy-momentum tensor from ordinary matter of the perfect fluid form $T^\mu_\nu_{\text{matter}} = \text{diag}(-\rho, p, p, p)$ and a potential is included for the $\Phi$ field, then the full acceleration is given by the lengthy expression
\[ \frac{\ddot{a}}{a} = -\frac{4\pi}{3\Phi} \left[ \rho + 3p + \frac{(\rho - 3p)}{(1 + 2\omega/3)} \right] - \frac{\omega}{3} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 + H \left( \frac{\dot{\Phi}}{\Phi} \right) + \frac{1}{2} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 \frac{\Phi}{(3 + 2\omega)} \frac{\partial \omega}{\partial \Phi} \]  
(3.17)

\[ + \frac{8\pi}{3\Phi} \left[ V - \frac{1}{(1 + 2\omega/3)} \left( 2V - \Phi \frac{\partial V}{\partial \Phi} \right) \right] . \]  
(3.18)

This puts a more complicated bound on \( f \) which is undoubtedly harder to meet. This possibility was discussed in references [5].

Three separate examples of gravity driven accelerations are considered following a comment on the causal physics relevant for inflation.

**B. Inflation and the Horizon Problem**

The horizon problem questions how our entire observable universe could appear so homogeneous and isotropic. In standard cosmology such a large volume encompasses many regions which were causally disconnected at early times. The smoothness of the universe across these regions appears to defy causal microphysics. Inflation resolves this quandry by blowing up a region, causally connected at early times, large enough to envelop everything as far as the eye can see.

Before facing the demands of sufficient inflation, a much weaker condition can be used to severely restrict the range of \( \omega(\Phi) \) pertinent to inflation. During an epoch of acceleration, constant comoving scales cross outside of the Hubble radius \( H^{-1} \) only to cross back inside during a decelerating phase. The key to resolving the horizon problem is for the scales which cross inside today to have been causally connected before they crossed out. Thus a nominal condition for the acceleration to be relevant for inflation is simply that

\[ d_\gamma > H^{-1} . \]  
(3.19)

The distance a photon travels, \( d_\gamma \), defines the extent of a causally connected region. If this condition is not met, then none of the scales which cross outside \( H^{-1} \) during the acceleration are causally connected. This is much weaker than a sufficient inflation condition. It is shown here that in the simplest case of the single scalar model studied, the nominal condition (3.19) can be met only if \( \omega(\Phi) < 0 \) and if \( \Phi \) drops.
First consider the equation of motion (3.8) rewritten as
\[ \left( H + \frac{\dot{\Phi}}{2\Phi} \right)^2 = \frac{1}{4} f^2 \left( \frac{\dot{\Phi}}{\Phi} \right)^2. \] (3.20)

Taking the square-root and reexpressing this equation gives
\[ \frac{d\ln(\Phi a^2)}{dt} = \pm f \frac{\dot{\Phi}}{\Phi}. \] (3.21)

Assume for now that the upper sign holds for \( \Phi \) growing and the lower sign for \( \Phi \) decreasing. Using (3.7) and integrating over \( dt \) gives
\[ \Phi a^2 = |C| \int_{t'} \frac{dt'}{a'} \] (3.22)

up to a constant of integration. Notice that from this we can deduce the particle horizon distance; that is, the distance a photon has traveled since the beginning of time,
\[ d_{\gamma} = \frac{\Phi a^3}{|C|}, \] (3.23)

up to a constant of integration. For ease of comparison, \( H \) can be written as
\[ H = \frac{|C|}{(2a^3\Phi)} \frac{(f \pm 1)}{f}. \] (3.24)

Using the above two expressions eqn (3.19) becomes \( f < \pm 1 \). Since \( f = (1 + 2\omega/3)^{1/2} \) is always positive, the condition is impossible to meet if \( \Phi \) grows. If \( \Phi \) drops, then (3.19) demands
\[ \omega < 0. \] (3.25)

According to (3.11) the classical energy density is positive in an FRW metric as long as \( \omega \geq -3/2 \). Thus \( \omega < 0 \) is not forbidden, at least not by the weak energy condition.

Physically, constraint (3.23) says the acceleration is terribly weak. From eqn (3.15) and (3.7) it can be seen that \( \ddot{a} \) is always suppressed by \( (a^3\Phi)^2 \). Roughly, if \( \Phi \) grows then the acceleration is weakened relative to the scenario if \( \Phi \) drops. This gives a feel for why \( \Phi \) growing is prohibited entirely but \( \Phi \) dropping allows a narrow range of interest. Only the third example delineated below will fall within this range.
Not all possible branches were considered above. More generally, all possibilities can be summarized by the condition

\[ f < \pm \left( \frac{1 - \delta}{1 + \delta} \right) \tag{3.26} \]

where the constant of integration dropped from (3.22) is included in \( \delta = a_i^2 \Phi_i / a^2 \Phi \). The subscript \( i \) denotes initial values and \( \delta \) is always positive. When \( f > 1 \), the condition \( H > 0 \) enforces the branches chosen below eqn (3.21). When \( f < 1 \), the condition \( H > 0 \) can only be met when \( \Phi \) drops. Therefore the case of \( \Phi \) growing is totally excluded by the proceeding arguments. When \( \Phi \) drops and \( f < 1 \), both branches are allowed in eqn (3.21). For the plus branch, \( \delta < 1 \) and for the minus branch, \( \delta > 1 \). The restriction (3.26) on \( f \) is more severe.

**IV. EXAMPLE 1: BOUNCING UNIVERSE**

As a first example, take \( \Phi \) to grow, so that the strength of gravity weakens as the universe evolves. An \( \omega(\Phi) \), or equivalently \( f(\Phi) \), which leads to a universe with a bounce is chosen below. The acceleration is not sufficient to be of interest for inflation. The interesting quality to this universe is the nonsingularity. Before proceeding with the details of the solution, it is worth sketching some properties of the resultant cosmology. The evolution can be traced back to an infinitely large universe. The space collapses down to finite size and then bounces into an accelerated expansion. Eventually it settles down into a decelerating phase.

If instead the universe begins at the moment of the bounce, the evolution is the same but the physical picture is different. The universe begins nonsingular with zero energy density. The last property has appeal since it costs nothing to create a universe with zero energy, cold and empty. The Hubble expansion is initially zero. As the Planck mass moves, the expansion accelerates. Eventually the acceleration ends and the expansion slows forever.

The general sketch outlined above can be substantiated by solving the equations of motion of the previous section with

\[ f(\Phi) = \frac{1}{\left[ 2 \ln(\Phi/\Phi_0) \right]^{1/2}}, \tag{4.1} \]
where $\Phi$ is an arbitrary constant and again $f = (1+2\omega/3)^{1/2}$. As already discussed positivity of the energy density demands $\omega \geq -3/2$ or $f \geq 0$. This condition is automatically met here. Example (4.1) gives an acceleration of

$$\frac{\ddot{a}}{a} = \frac{1}{2} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 f \geq 0 .$$

(4.2)

It is interesting to note that the form for $f$ in eqn (4.1) is the same form recently considered in [15] to demonstrate that the presence of matter fields can attract scalar-tensor theories toward general relativity.

In this example, $\Phi$ grows and the strength of gravity weakens with time. For $\dot{\Phi} > 0$ consider the minus branch in eqn (3.9) for $H$

$$H = \frac{\dot{\Phi}}{2\Phi} (f - 1) .$$

(4.3)

Eqn (4.3) can be integrated over $dt$ to find

$$a = \bar{a} \left( \frac{\Phi}{\bar{\Phi}} \right)^{1/2} \exp \left[ - \left( \frac{\ln \Phi/\bar{\Phi}}{2} \right)^{1/2} \right] ,$$

(4.4)

where $\bar{a}$ is the arbitrary constant of integration.

The Planck field $\Phi$ and the scale factor $a$ can be related to cosmic time parametrically. Define a new variable $y$ from

$$\Phi = \bar{\Phi} \exp(-y^2) .$$

(4.5)

In terms of $y$ the scale factor is

$$a = \bar{a} \exp \left[ \frac{y^2}{2} - \frac{y}{\sqrt{2}} \right] .$$

(4.6)

Substituting this expression into the first integral of motion (3.7) and using $f = 1/(\sqrt{2}y)$ gives the following integral

$$I \equiv - \int_1^{\sqrt{2}} \exp \left[ \frac{1}{2} \left( y' - \frac{3}{\sqrt{2}} \right)^2 \right] dy' = e^{9/4}K \Delta t ,$$

(4.7)

where $\Delta t \equiv t - t_i$ and $K = |C|/(\bar{a}^3\bar{\Phi})$. The limit of integration is chosen so that $\Delta t = 0$ at $\omega = 0$ ($f = 1$). Integrating $I$ gives the elapse of cosmic time in terms of the parameter $y$.
\[ \Delta t = t e^{-9/4} \sqrt{\frac{\pi}{K}} \left[ \text{erf}(i(y/\sqrt{2} - 3/2)) - \text{erf}(i) \right]. \quad (4.8) \]

If \( y \) is allowed to run from \(+\infty\) to \(-\infty\), then time runs from \(-\infty\) to \(+\infty\).

In figure 1, the scale factor and the Planck field \( \Phi \) are shown as functions of time. In figure 2, the Hubble constant is drawn with time. At time equals minus infinity, \( \Phi = 0 \) and \( f = 0 \). The scale factor was infinite and \( H = -\infty \). The universe, huge at minus infinity, contracts down to finite size and then rebounds into an expanding phase. The bounce occurs at \( \Delta t = 0 \) with \( \Phi = e^{-1/2}\bar{\Phi} \) and \( f = 1 \).

Alternatively one could start the universe with \( f = 1 \); that is, impose the initial condition that the universe pop into existence at \( \Delta t = 0 \) with \( f = 1 \) (\( \omega = 0 \)). Initially then \( \Phi_i = e^{-1/2}\bar{\Phi} \) and \( a_i = \bar{a}e^{-1/4} \). Notice also that \( \rho_{\Phi} = 0 \) and therefore \( H = 0 \). As time marches forward, \( \Phi \) grows, \( f \) grows, and the universe begins to expand. The universe had to accelerate to go from \( H = 0 \) to an expanding cosmology.

At the value \( \bar{\Phi} \), \( \Phi \) hits a maximum and then begins to drop. As it does so the acceleration ends and the Hubble expansion slows. The onset of deceleration is demonstrated in figure 2. Eventually the universe expands to infinite size, the expansion slows asymptotically to zero. As \( \Phi \) drops toward zero, \( f \) also approaches zero and \( \omega \) approaches \(-3/2\). The kinetic coupling \( \omega \) never falls below \(-3/2\) and therefore the energy density is always positive.

As a technical point, notice that as \( \Phi \) reaches its maximum \( \bar{\Phi} \), \( f = \infty \). Although this looks singular, the equations of motion remain regular. To put this another way, it is often said that the Planck mass is frozen and thus scalar-tensor theories are driven to general relativity in the limit of \( \omega \to \infty \). More correctly, scalar-tensor theories are driven to general relativity in the limit of \( \omega \to \infty \) and \( \omega^{-2}d\omega/d\Phi \to 0 \). In terms of \( f \) this means \( f \to \infty \) and \( f^{-3}df/d\Phi \to 0 \), as can be seen by studying the right hand side of equation of motion (3.5):

\[ \ddot{\Phi} + 3H \dot{\Phi} = -\frac{1}{f} \frac{df}{d\ln \Phi} \frac{\dot{\Phi}^2}{\Phi} \]  \( \propto \frac{1}{f^3} \frac{df}{d\Phi} \). \quad (4.9)
Here, although \( f \to \infty \), \( f^{-3} df/d\Phi \) does not approach zero. Therefore the motion of \( \Phi \) is not locked in when \( f \to \infty \). As \( \Phi \) reaches \( \bar{\Phi} \), \( \dot{\Phi} \) passes through zero while at this moment

\[
\dot{\Phi}|_{\Phi=\bar{\Phi}} = -K^2 \bar{\Phi} < 0 ,
\]

where \( K = |C|/(\bar{a}^3 \bar{\Phi}) \). Since \( \ddot{\Phi} < 0 \), \( \Phi \) has hit a maximum at \( \bar{\Phi} \) and then begins to decrease. This is effected by \( C \to -C \) in the equations of motion.

**V. EXAMPLE 2: WEAK BURST OF ACCELERATION**

More generally, a whole family of scalar-tensor gravity models can be investigated with forms for \( f(\Phi) \) similar to the last example. Consider the following

\[
f = 1 + 2 \frac{b}{(\ln(\bar{\Phi}/\Phi))^{1/n}} .
\]

Again, \( \bar{\Phi} \) is an arbitrary integration constant and \( f = (1 + 2\omega/3)^{1/2} \). Take \( \Phi \) growing again and integrate the Hubble equation as before

\[
a = \bar{a} \exp \left[ -\frac{n}{n-1} b (\ln(\bar{\Phi}/\Phi))^{\frac{n-1}{n}} \right] .
\]

The constant of integration is \( \bar{a} \). As an example which avoids the terrain of \( \omega < 0 \), let \( n = 1/2 \). Then \( f(\Phi) \) of (5.1) is greater than 1 for all values of \( \Phi \). The scale factor becomes \( a = \bar{a} \exp(b/\ln(\bar{\Phi}/\Phi)) \). If \( \Phi = 0 \) initially, the universe begins with infinite Hubble expansion, infinite energy density, and decelerates as it evolves.

Depending on \( n \) and \( b \) the theory specified by (5.1) may lead to an acceleration, for some range of \( \Phi \). For \( b = 50 \) for example and \( n = 1/2 \) an accelerating phase is entered briefly, for a spirt, and then deceleration begins again. The acceleration occurs for values of \( \Phi \) roughly between \( 0.24\bar{\Phi} \) and \( 0.81\bar{\Phi} \).

The Planck field \( \Phi \) grows forever, asymptotically slowing to zero as \( \Phi \to \bar{\Phi} \) and \( f \to \infty \). The scale factor grows infinitely large and the Hubble expansion slows asymptotically to zero.
VI. EXAMPLE 3: KINETIC INFLATION

The purpose of this third example is to offer an explicit scalar-tensor theory which meets the nominal requirement of §III B that $d_\gamma > H^{-1}$. Here it is demonstrated that if $\omega$ is allowed to drop below 0, but still greater than $-3/2$, then the scales which cross outside of $H^{-1}$ during the acceleration are causally connected. This is not intended as a model of inflation. In fact, inflation in this context may not succeed. The question of inflation is given more complete attention in a subsequent paper [8]. A list of the conspicuous weaknesses in a realistic attempt at successful inflation can be compiled: (1) Spatial gradients in the Planck field have explicitly been neglected throughout. This is equivalent to the assumption that the universe begin homogeneous and isotropic. Such an assumption is always made for inflation. When inflation is powered by a potential this supposition may be justified. However, it is not clear that the assumption can be justified in the case of a kinetic inflation. The acceleration is so weak it may be unable to dilute inhomogeneities (or flatten the universe). (2) An obvious reheating mechanism, or rather heating mechanism, does not reveal itself. Perhaps fluctuations in the Planck field could be coaxed into creating the hot soup of the early universe. (3) In the conformally related Einstein frame, the universe appears to decelerate as shown in the next section. However the condition of sufficient inflation demands the Einstein frame scale factor accelerate [8]. Unless some unforeseen subtleties resolve this, successful completion of inflation looks unlikely. More optimistically, as the structure of gravity evolves it can be arranged quite simply that the acceleration ends, providing a long sought for graceful exit. Hopefully this possibility would not be so fine tuned as to make the exit graceless.

Regardless of the application to the initial condition problems of cosmology, it is still worthwhile to demonstrate the scale crossings as such effects could be pertinent to the issue of density perturbations. Consider the theory given by

$$f(\Phi) = \ln(\Phi/\bar{\Phi}) .$$

(6.1)

As in the proceeding examples, $\bar{\Phi}$ is an arbitrary constant and $f = (1+2\omega/3)^{1/2}$. Integrating
the Einstein-like eqn (3.8) gives the scale factor as a function of Φ,

\[ a = \bar{a} \left( \frac{\Phi}{\bar{\Phi}} \right)^{1/2} \exp \left[ -\frac{1}{4} \left( \ln(\Phi/\bar{\Phi}) \right)^2 \right] \]  

(6.2)

For concreteness take the Planck mass to begin infinite. Initially the scale factor is zero and the universe is created singular and decelerating.

Condition (3.16) states that the universe accelerates if

\[ 1 + \ln(\Phi/\bar{\Phi}) - \frac{1}{\left( \ln(\Phi/\bar{\Phi}) \right)^2} < 0 \]  

(6.3)

Eqn (6.3) is only satisfied for values of Φ below roughly \( e^{1/\bar{\Phi}} \), or in terms of \( \omega \), roughly \( \omega < 0 \). Therefore the universe decelerates from \( \Phi = \infty \) down to \( \Phi = e^{1/\bar{\Phi}} \). Below that the universe accelerates for a burst. In order to prevent a true ghost from developing the behavior has to be shut off artificially so that \( \omega \) does not drop below the forbidden \(-3/2\).

The point of this example is to consider the causal structure of the universe. Using the results of §III B the particle horizon, \( d_\gamma = \int_{t_i}^t dt/a \), is

\[ d_\gamma = \frac{\Phi a^3}{|C|} \]  

(6.4)

and the Hubble radius is

\[ H^{-1} = \frac{2\Phi a^3}{|C|} \frac{f}{f + 1} \]  

(6.5)

Comparing the two shows \( d_\gamma > H^{-1} \) when \( f \) drops below 1, or in other words when \( \omega \) drops below 0. In figure 3, the Hubble radius \( H^{-1} \) is drawn versus the scale factor in a logarithmic plot. Also shown are an array of physical scales, constant in the comoving frame; i.e. \( \lambda = a\lambda_{\text{constant}} \). During the early deceleration, \( H^{-1} \) grows and scales cross inside the Hubble radius. When the era of acceleration is entered, \( H^{-1} \) drops. As in the inflationary picture, constant comoving scales cross outside the Hubble radius while the expansion accelerates. Since \( d_\gamma \) exceeds \( H^{-1} \), the scales crossing outside \( H^{-1} \) are indeed causally connected.

To reiterate, this example demonstrates explicitly a kinetic acceleration which might be relevant for inflation. As it was argued in §III B would have to be the case, \( \omega \) drops below zero as the Planck mass drops.
VII. EINSTEIN FRAME

The kinetic driven acceleration can be studied in the conformally related Einstein frame where the theory of gravity is the usual Einstein theory with a fundamental Planck scale \( M_o = 10^{19} \) GeV. [A study of inflation in the conformally related frame will not be presented here but will be presented in [8].] However different the world appears, there is no experiment observers in two conformally related frames could perform which would distinguish one universe from a conformally related counterpart. All experiments involve the comparison of scales, whether it be the length scale of a ruler or the unit of time read on a clock. In the Jordan-Brans-Dicke (JBD) frame, it was implicitly assumed that ruler lengths were constant and thus there was meaning to the physical length units. In other words, the JBD scale factor truly contains complete information about how distances, constant in comoving coordinates, evolve in physical units. If the rulers were constant in the JBD frame then they would not be constant under the conformal transformation. Since in the Einstein frame rulers therefore change with time, the physical length units change with time. The scale factor does not contain complete information. Another scale factor must be introduced which accounts for the changing meaning of a physical length unit. Once this is done, the outcome of all experiments can be compared and must be the same.

Perform a conformal transformation on the metric

\[
g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu},
\]

where \( \Omega = M_o/\Phi^{1/2} \). Under the conformal transformation the action becomes

\[
A = \int d^4 x \sqrt{-\tilde{g}} \left[ \frac{M_o^2}{16 \pi} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_{\mu} \Psi \partial_{\nu} \Psi \right]
\]

The field \( \Psi \) is tantamount to a rewriting of the \( \Phi \) field:

\[
\Psi \equiv \frac{M_o}{\sqrt{8 \pi}} \int \frac{(\omega + 3/2)^{1/2}}{\Phi} d\Phi.
\]

Notice that \( \Psi \) is real and the energy density in the \( \Psi \)-field, \( \rho_\psi = \dot{\Psi}^2/2 \) is positive as long as \( \omega \geq -3/2 \). The momentum associated with the field, \( p_\psi = \rho_\psi \), is always positive.
In addition to the conformal transformation, perform the coordinate transformation

\[ d\tilde{t} = \Omega^{-1} dt \quad (7.4) \]

\[ \tilde{a} = \Omega^{-1} a \quad (7.5) \]

so that the spacetime interval can be written in the usual FRW form,

\[ d\tilde{s}^2 = \Omega^{-2} ds^2 \quad (7.6) \]

\[ = \left[ - (\Omega^{-1} dt)^2 + (\Omega^{-1} a)^2 d\vec{x}^2 \right] \quad (7.7) \]

\[ = \left[ - d\tilde{t}^2 + \tilde{a}^2 d\vec{x}^2 \right]. \quad (7.8) \]

The metric in \((\tilde{t}, \tilde{a}\vec{x}^i)\) coordinates is thus \(\tilde{g}_{\mu\nu} = (-1, \tilde{a}^2, \tilde{a}^2, \tilde{a}^2)\). The evolution of the scale factor and of \(\Psi\) can be found directly in the Einstein frame;

\[ \tilde{a} \propto \Delta \tilde{t}^{1/3} \quad (7.9) \]

\[ \tilde{\Psi} \propto \pm \ln(\Delta \tilde{t}) \quad (7.10) \]

where the upper sign refers to \(\Psi\) growing and the lower sign to \(\Psi\) dropping. Clearly the scale factor always decelerates. The solution seems to know nothing about \(\omega(\Phi)\) and thus does not distinguish between different scalar-tensor theories until an observer is included.

At first glance it seems all information about the acceleration of the spacetime is lost in the Einstein picture where the FRW universe is filled with an ordinary, minimally coupled scalar field. There is no acceleration. However, in order to properly pose and answer questions, an observer, a test particle, must be included. The test particle carries rulers and clocks with which to make observations.

Include in the action a term for a test particle:

\[ A_{\text{tester}} = \int m \left[ - g_{\mu\nu} dx^\mu dx^\nu \right]^{1/2}. \quad (7.11) \]

In the JBD frame, \(m\) is the constant mass of the observer. Under the conformal transformation the action becomes
\[ A_{\text{tester}} = \int \tilde{m} \left[ -\tilde{g}_{\mu\nu} dx^\mu dx^\nu \right]^{1/2}. \quad (7.12) \]

where \( \tilde{m} = \Omega m \). This means that the length scale of our test particle is

\[ \tilde{\lambda} = \Omega^{-1} \lambda, \quad (7.13) \]

and \( \lambda = 1/m \) is the constant wavelength of the observer in the JBD frame. Better said, the observer carries clocks and rulers. The rulers, being constants in the original JBD frame, have variable lengths in the Einstein frame. The observer’s rulers scale as indicated by \( (7.13) \), \( \tilde{L}_{\text{ruler}} = \Phi^{1/2} M_o L_{\text{ruler}} \). As \( \Phi \) evolves, so does the length of the ruler.

Therefore the scale factor \( \tilde{a} \) does not tell all about physical scales in the Einstein frame. The true scale factor is not \( \tilde{a} \) but in some sense, \( \tilde{R} \), the distance in ruler units of two points at one unit of comoving separation, defined by

\[ \tilde{R} = \frac{\tilde{a}}{\tilde{L}_{\text{ruler}}}. \quad (7.14) \]

In terms of JBD quantities \( \tilde{R} = R = \frac{a}{L_{\text{ruler}}} \). The distance in ruler units is a conformal invariant. In fact, all ratios of lengths should be conformally invariant. Both JBD and Einstein observers agree on the numerical value of \( \tilde{R} \).

The Einstein observer can define the expansion parameter, \( \tilde{H}_R = \tilde{R}'/\tilde{R} \) where \( ' \) represents derivatives with respect to \( \tilde{t} \). The parameter \( \tilde{H}_R \) accounts not only for the expansion of the spacetime, but also for the change in the units of distance. In the JBD frame, \( \dot{a} = H a \) grows during the gravity driven acceleration. In terms of Einstein variables, \( H a = \tilde{H}_R \tilde{a} \).

The observer in the Einstein frame agrees this quantity grows. The JBD observer sees the separation between two points at fixed comoving distance grow at an accelerated rate due to the cosmic expansion. Einstein observers interpret this acceleration as due to the relative rate at which rulers change and not just the expansion of spacetime.

As a last comment, the Einstein frame picture allows for a singularity in the scale factor. For the bouncing universe, where there is no singularity in the JBD-frame, this signals the break down of the conformal factor; that is, the singularity is in the conformal factor. The
Einstein time can be found as a function of the parameter $y$ by integrating the coordinate transformation,

$$
\Delta \tilde{t} = \left( \frac{\Phi_1}{2M_o} \right) \frac{2}{3K} e^{-3y/\sqrt{2}}.
$$

(7.15)

At $y = \infty$, $\Delta \tilde{t} = 0$ and the scale factor vanishes in the Einstein frame but is infinite in the JBD frame. The (inverse) conformal transformation relating $\tilde{a}$ to $a$ through $\tilde{a} = \Omega^{-1} a$, diverges at $y = \infty$; $\Omega^{-1} = \Phi^{1/2}/M_o \rightarrow \infty$.

**VIII. SUMMARY**

In a theory with a dynamical Planck mass the kinetic energy density can have an associated negative pressure. The pressure is negative when the kinetic coupling factor, $\omega$, grows with Planck mass according to the bound defined in the paper. The pressure is also negative for a branch of the solutions when the Planck mass drops and $\omega$ is a negative constant. By comparison, the pressure of a free minimally coupled field is always positive.

As a result of the unusual negative pressure, the cosmic expansion can accelerate. The acceleration is unique as it is driven solely by the kinetic energy in the Planck field. No potential nor cosmological constant is present.

It is suggestive that scalar-tensor gravity alone can accelerate the scale factor. An accelerated scale factor is one of the key ingredients used in inflation. This hints of a gravity driven or kinetic inflation. Still, the acceleration by itself does not lead to a successful inflationary model. For a successful inflationary model, the universe must inflate enough to solve the cosmological horizon, flatness, and monopole problems. There are indications this may be impossible. For instance, it is worrisome that no acceleration is apparent in the conformally related Einstein frame. This suggests the acceleration is not meaningful for inflation.

In this paper, it was shown that the weakest minimal requirement that the acceleration be relevant for inflation can only be satisfied in a theory for which $\omega$ drops below zero as the Planck mass drops. Although this strongly restricts the range of theories, the possibility is
not ruled out by the weak energy condition. Positivity of the energy density in an FRW universe demands only that $\omega \geq -3/2$. Before an attempt is made at building a model of kinetic inflation a few questions must be addressed. For one, it must be shown that the universe remain smooth even after spatial gradients are included. Further, a viable heating mechanism is needed. Lastly, the Einstein picture must be shown to be consistent.

For general illustration, examples were pursued which demonstrated an era of accelerated expansion. As an interesting consequence, a nonsingular cosmology with a bounce was found. The universe could be created nonsingular at the moment of the bounce. In this picture, the cosmos begins cold and empty as the energy density is zero. The formation of the universe from nothing would be energetically free, which suggests a small barrier to spontaneous creation. Although the creation of a universe from nothing is an inherently quantum process, it is worth noting that quantum gravity energy scales are not entered since the energy is zero while the Planck scale is finite. As the Planck mass begins to roll, the universe begins to expand. This scenario offers a departure from an initial big bang. A transition from this cold and quiet beginning to the fiery soup of the early universe remains to be pursued.

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Figure Captions

Figure 1: The scale factor and the Planck field $\Phi$ are drawn as functions of time. Notice the scale factor begins infinite and contracts. At $\Delta t = 0$, the universe bounces into an expanding phase.

Figure 2: The Hubble expansion $H$ grows and the universe accelerates as $\Phi$ grows to $\bar{\Phi}$. However, while $\Phi$ decreases $H$ decreases and the universe enters a decelerating phase.
Figure 3: The Hubble radius, $H^{-1}$ is drawn versus the scale factor in a logarithmic plot. Also shown are an array of physical scales, constant in the comoving frame; i.e. $\lambda = a\lambda_{\text{constant}}$. For a time, the Hubble radius increases indicating the universe decelerates. It reaches a maximum and the decreases while the universe accelerates. During deceleration, scales cross inside $H^{-1}$. During acceleration, scales cross outside $H^{-1}$. 
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