I. INTRODUCTION

Before the birth of the quantum chromodynamics (QCD), the possible existence of tetraquark ($qqq\bar{q}$) and pentaquark ($qqq\bar{q}\bar{q}$) had been anticipated when Gell-Mann [1] and Zweig [2] first proposed the quark model. In 1976, Jaffe studied the light tetraquark in the framework of the MIT bag model [3, 4]. Chan and Høgaasen also studied this topic in the color-magnetic spin-spin interaction from the one-gluon exchange [5]. Chao further considered the hidden-charm [6, 7] and full-charm [8] tetraquarks. Meanwhile, the pentaquark was also studied in many models, such as the color-magnetic hyperfine interaction [9, 10] and the MIT bag model [11].

In despite of the theoretical investigations, the first experimental evidence of the exotic states did not appear until 2003, when the Bell Collaboration observed the $X(3872)$ state in the exclusive $B^\pm \rightarrow K^\pm \pi^+\pi^- J/\psi$ decays [12]. Later, the CDF [13], DO [14], BaBar [15], LHCb [16], CMS [17] and BESIII [18] Collaborations confirmed this state, and the LHCb Collaboration further determined its quantum number to be $I^GJ^{PC} = 0^{+}1^{++}$ [16]. For over a decade, lots of charmonium-like $XYZ$ states have been observed, such as $Y(3940)$ [19], $Y(4140)$ [20], $Y(4260)$ [21], $Y(4360)$ [22], $Y(4660)$ [23], and so on. Many of $XYZ$ states do not fit into the conventional $q\bar{q}$ meson spectrum in the quark model. To explain their nature, theorists have interpreted some of them to be the molecular state [24, 25], hybrid meson [26, 27], tetraquark [28, 29], etc. More detailed reviews can be found in Refs. [25, 30–34] and references therein.

Compared to the tetraquark candidates, the experimental observation of the pentaquark states is more difficult. In 2015, the LHCb Collaboration measured the $\Lambda_b \rightarrow J/\psi K^- p$ decays, and observed two resonances, $P_c(4380)$ and $P_c(4450)$, in the $J/\psi p$ channel, which indicates that they have a minimal quark content of $uudc$ [35]. Very recently, the LHCb Collaboration reported the observation of three narrow peaks in the $J/\psi p$ invariant mass spectrum of the $\Lambda_b \rightarrow J/\psi Kp$ decays [36]. They found that the $P_c(4450)^+$ is actually composed of two narrow resonances, $P_c(4440)^+$ and $P_c(4457)^+$. Moreover, they also reported a new state below the $\Sigma_c D$ threshold, namely the $P_c(4312)^+$. Their masses and widths are as follows:

$$P_c(4312)^+ : M = 4311.9 \pm 0.7^{+6.8}_{-0.6} \text{ MeV},$$
$$\Gamma = 9.8 \pm 2.7^{+3.3}_{-4.5} \text{ MeV},$$

$$P_c(4440)^+ : M = 4440.3 \pm 1.3^{+4.1}_{-4.7} \text{ MeV},$$
$$\Gamma = 20.6 \pm 4.9^{+8.7}_{-10.1} \text{ MeV},$$

$$P_c(4457)^+ : M = 4457.3 \pm 0.6^{+1.1}_{-1.7} \text{ MeV},$$
$$\Gamma = 6.4 \pm 2.0^{+5.5}_{-1.5} \text{ MeV}.$$

Since their masses are slightly below the $\Sigma_c D$, $\Sigma_c^* D$ and $\Sigma_c^* D^*$ thresholds respectively, they can be interpreted as molecules composed of a charm baryon and an anti-charm meson [37–49]. For example, Chen [46] interpreted them as bound states of $\Sigma_c D$ with $J^P = 1/2^-$, $\Sigma_c^* D$ with $J^P = 3/2^-$ and $\Sigma_c^* D^*$ with $J^P = 3/2^-$, while Chen et al. [45], He [48], and Liu et al. [49] interpreted the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ as loosely bound $\Sigma_c D$ with $(I = 1/2, J^P = 1/2^-)$, $\Sigma_c^* D^*$ with $(I = 1/2, J^P = 1/2^-)$ and $\Sigma_c^* D^*$ with $(I = 1/2, J^P = 3/2^-)$.

Another interesting possibility is that some of the $P_c$ states might be tightly bound pentaquark states. The light $q\bar{q}$ pentaquark states was first studied with the color-magnetic interaction among the quarks [9, 10]. Later, Strottman used the MIT bag model to discuss this
system, where the mass spectra mostly depend on the chromomagnetic interaction between the quarks (or antiquark) [11]. The hidden-charm pentaquarks were also studied in constituent quark model [50–57].

The quark model is widely used to investigate the mass spectra of hadrons [1, 2, 58–65]. In the quark model, each quark (antiquark) carries the kinetic energy \( \sqrt{p^2 + m^2} \).

In the nonrelativistic limit, the kinetic energy reduces to \( m + p^2/2m \), and the inter-quark potential contains the lattice QCD-inspired linear confinement interaction and the short-range one-gluon-exchange (OGE) interaction. Usually the OGE interaction consists of the spin-independent color Coulomb-type terms, the spin-spin chromomagnetic interaction, the tensor interaction, and the spin-orbit interactions etc.

We can use the chromomagnetic model to study the ground state hadrons [3, 4, 61, 66–71]. In the chromomagnetic model, the mass of ground state hadrons consist of the effective quark masses and the chromomagnetic hyperfine interaction. This simple model reproduced the hyperfine splitting of hadrons quite well. Compared to the quark model, the chromoelectric interaction has been absorbed by the effective quark masses. However, the one-body effective quark masses are not enough to account for the two-body chromoelectric effects. In Ref. [72], Karlner et al. found that the color-related binding terms are needed when they considered the interactions between a heavy (anti-)quark and a strange (or heavy) quark. Similarly, Høgaasen et al. generalized the chromomagnetic model and included a chromoelectric term \( H_{CE} = - \sum_{i,j} A_{ij} \hat{\lambda}_i \cdot \hat{\lambda}_j \) to study the hidden-beauty partners of the \( X(3872) \) [73]. Note that in 1978, Fukugita et al. had already used the color and chromomagnetic interactions to investigate the pseudobaryons [9]. Chan et al. also used these interactions to study the properties of di-/tri-quarks, which are constituents of multiquarks [74].

In Ref. [75], we extended the chromomagnetic model and included the effect of color interaction. According to color algebra, we further introduced the quark pair mass parameters \( m_{qq} \) and \( m_{q\bar{q}} \) to account for both the effective quark masses \( m_q \) and the color interaction \( A_{qq} \) and \( A_{q\bar{q}} \) between the quarks. Then we used this model to calculate the masses of multi-heavy baryons. Our calculated mass of \( \Xi_{cc}^+ \), 3633.3 ± 9.3 MeV is very close to the LHCb’s experiment [3621.40 ± 0.72(stat.) ± 0.27(syst.) ± 0.14(\( \Lambda_c \)) MeV] [76].

In this paper, we systematically study the mass spectrum of the \( qq\bar{q}\bar{c} \) \((q = n, s, \text{and } n = u, d)\) pentaquarks in the extended chromomagnetic model. In Sec. II we introduce the extended chromomagnetic model and construct the wave functions of the \( qq\bar{q}\bar{c} \) pentaquarks. In Sec. IIIA we present the model parameters. Then we calculate and discuss the numerical results in Sec. IIIB. We conclude in Sec. IV.

\section{The Extended Chromomagnetic Model}

\subsection{The Hamiltonian}

In the chromomagnetic (CM) model, the mass of hadron is governed by the Hamiltonian [5, 61, 69–71]

\[ H = \sum_i m_i - \sum_{i<j} v_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \mathbf{F}_i \cdot \mathbf{F}_j, \]

where \( m_i \) is the ith constituent quark’s (or antiquark’s) effective mass, which includes the constituent quark mass, the kinetic energy, and so on, and \( \mathbf{S}_i = \sigma_i/2 \) and \( \mathbf{F}_i = \hat{\lambda}_i/2 \) are the quark spin and color operators respectively. For the antiquark, \( \mathbf{S}_q = -\mathbf{S}_q^\ast \) and \( \mathbf{F}_q = -\mathbf{F}_q^\ast \). The coefficient \( v_{ij} \) depends on the spatial wave function and the quark masses

\[ v_{ij} = \frac{8\pi}{3m_im_j} \langle \alpha_i(r)\delta^3(r) \rangle. \]

As pointed out in Ref [72, 73, 75], the effective quark masses are not enough to absorb all the two-body chromoelectric effects. To solve this problem, Høgaasen et al. generalized the chromomagnetic model by including a chromoelectric term [73]

\[ H_{CE} = - \sum_{i,j} A_{ij} \hat{\lambda}_i \cdot \hat{\lambda}_j, \]

Since

\[ \sum_{i<j} (m_i + m_j) \mathbf{F}_i \cdot \mathbf{F}_j = \left( \sum_i m_i \mathbf{F}_i \right) \cdot \left( \sum_i \mathbf{F}_i \right) - \frac{4}{3} \sum_i m_i, \]

and the total color operator \( \sum_i \mathbf{F}_i \) nullifies any colorless physical state, we introduced a new quark pair mass parameter

\[ m_{ij} = (m_i + m_j) + \frac{16}{3} A_{ij}, \]

and rewrite the model Hamiltonian as [75]

\[ H_{CM} = - \frac{3}{4} \sum_{i<j} m_{ij} \mathbf{V}_{ij}^C - \sum_{i<j} v_{ij} \mathbf{V}_{ij}^{CM}, \]

where

\[ \mathbf{V}_{ij}^C = \mathbf{F}_i \cdot \mathbf{F}_j, \]

\[ \mathbf{V}_{ij}^{CM} = \mathbf{S}_i \cdot \mathbf{S}_j \mathbf{F}_a^i \cdot \mathbf{F}_a^j, \]

are the color and CM interactions between quarks.
B. The pentaquark wave function

To investigate the mass spectra of the pentaquark states, we need to construct the wave functions. In principle, the total wave function is a direct product of the orbital, color, spin and flavor bases. Since we only consider the ground states, the orbital wave function is symmetric and irrelevant for the effective Hamiltonian [see Eq. (6)]. Moreover, the Hamiltonian does not contain a flavor operator explicitly. Thus we first construct the color-spin wave function, and then incorporate the flavor wave function to account for the Pauli principle.

The spins of the pentaquark states can be 1/2, 3/2 and 5/2. In the \((qq\otimes q\bar{q})\otimes q\bar{q}\) configuration, the possible color-spin wave functions are listed as follows,

1. \(J^p = 1/2^+\):

\[
\begin{align*}
\beta_1^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_2^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_3^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_4^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_5^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_6^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_7^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_8^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_9^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_{10}^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_{11}^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_{12}^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_{13}^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_{14}^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}, \\
\beta_{15}^{1/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{1/2}^{1/2}.
\end{align*}
\]

(9)

2. \(J^p = 3/2^+\):

\[
\begin{align*}
\beta_1^{3/2} &= \left|(q_1 q_2)^6 q_3 q_4 q_5\right|_{3/2}^{1/2}, \\
\beta_2^{3/2} &= \left|(q_1 q_2)^6 q_3 q_4 q_5\right|_{3/2}^{1/2}, \\
\beta_3^{3/2} &= \left|(q_1 q_2)^6 q_3 q_4 q_5\right|_{3/2}^{1/2}, \\
\beta_4^{3/2} &= \left|(q_1 q_2)^6 q_3 q_4 q_5\right|_{3/2}^{1/2}, \\
\beta_5^{3/2} &= \left|(q_1 q_2)^6 q_3 q_4 q_5\right|_{3/2}^{1/2}, \\
\beta_6^{3/2} &= \left|(q_1 q_2)^6 q_3 q_4 q_5\right|_{3/2}^{1/2}.
\end{align*}
\]

(10)

3. \(J^p = 5/2^-\):

\[
\begin{align*}
\beta_1^{5/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{5/2}^{1/2}, \\
\beta_2^{5/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{5/2}^{1/2}, \\
\beta_3^{5/2} &= \left|(q_1 q_2)^0 q_3 q_4 q_5\right|_{5/2}^{1/2},
\end{align*}
\]

(11)

where the superscript 1, 3, 6 or 8 denotes the color, and the subscript denotes the spin 0, 1, 1/2, 3/2 or 5/2. These wave functions have definite symmetry under the exchange of the first two quarks. \((q_1 q_2)^0\) and \((q_1 q_2)^3\) are symmetric, while \((q_1 q_2)^6\) and \((q_1 q_2)^8\) are antisymmetric.

Next we consider the flavor wave function. Taking the Pauli principle into account, we can obtain four types of total wave functions.

1. Type A \(\{\text{Flavor} = \{(nnsQQ)^1=1, ssnQQ\}\}\):

\[
\begin{align*}
\text{(a) } J^p &= 1/2^-: \\
\Psi_{A1}^{1/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_1^{1/2}, \\
\Psi_{A2}^{1/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_2^{1/2}, \\
\Psi_{A3}^{1/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_3^{1/2}, \\
\Psi_{A4}^{1/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_4^{1/2}, \\
\Psi_{A5}^{1/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_5^{1/2}, \\
\Psi_{A6}^{1/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_6^{1/2}, \\
\Psi_{A7}^{1/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_7^{1/2}, \\
\Psi_{A8}^{1/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_8^{1/2},
\end{align*}
\]

(12)

\[
\begin{align*}
\text{(b) } J^p &= 3/2^-: \\
\Psi_{A1}^{3/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_1^{3/2}, \\
\Psi_{A2}^{3/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_2^{3/2}, \\
\Psi_{A3}^{3/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_3^{3/2}, \\
\Psi_{A4}^{3/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_4^{3/2}, \\
\Psi_{A5}^{3/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_5^{3/2}, \\
\Psi_{A6}^{3/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_6^{3/2}, \\
\Psi_{A7}^{3/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_7^{3/2}, \\
\Psi_{A8}^{3/2} &= q_1 q_2 q_3 q_4 q_5 \otimes \beta_8^{3/2}.
\end{align*}
\]

(13)
c) $J^P = 5/2^-$:

$$\Psi_{A_1}^{5/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{5/2}$$,
$$\Psi_{A_2}^{5/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{5/2},$$

(14)

2. Type B [Flavor = $(nnsQQ)^I=0$]:

(a) $J^P = 1/2^-$:

$$\Psi_{B_1}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{1/2}^1$$,
$$\Psi_{B_2}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{1/2}^2$$,
$$\Psi_{B_3}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{1/2}^3$$,
$$\Psi_{B_4}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{1/2}^4$$,
$$\Psi_{B_5}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{1/2}^5$$,
$$\Psi_{B_6}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{1/2}^6$$,
$$\Psi_{B_7}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{1/2}^7,$$

(15)

(b) $J^P = 3/2^-$:

$$\Psi_{B_1}^{3/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{3/2}^1$$,
$$\Psi_{B_2}^{3/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{3/2}^2$$,
$$\Psi_{B_3}^{3/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{3/2}^3$$,
$$\Psi_{B_4}^{3/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{3/2}^4$$,
$$\Psi_{B_5}^{3/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{3/2}^5$$,

(16)

(c) $J^P = 5/2^-$:

$$\Psi_{B_1}^{5/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{5/2}^1$$,

(17)

3. Type C [Flavor = $(nnsQQ)^I=3/2, sssQQ)$]:

(a) $J^P = 1/2^-$:

$$\Psi_{C_1}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \frac{1}{\sqrt{2}} \left( \beta_{3/2}^1 - \beta_{1/2}^1 \right)$$,
$$\Psi_{C_2}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \frac{1}{\sqrt{2}} \left( \beta_{3/2}^2 - \beta_{1/2}^2 \right)$$,
$$\Psi_{C_3}^{1/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{1/2}^3$$,

(b) $J^P = 3/2^-$:

$$\Psi_{C_1}^{3/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \frac{1}{\sqrt{2}} \left( \beta_{3/2}^3 - \beta_{1/2}^3 \right)$$,
$$\Psi_{C_2}^{3/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{3/2}^4$$,
$$\Psi_{C_3}^{3/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{3/2}^5$$,

(19)

(c) $J^P = 5/2^-$:

$$\Psi_{C_1}^{5/2} = q_1 q_2 q_3 Q_4 Q_5 \otimes \beta_{3/2}^6$$,

(20)

4. Type D [Flavor = $(nnsQQ)^I=1/2$]:

(a) $J^P = 1/2^-$:

$$\Psi_{D_1}^{1/2} = \frac{1}{\sqrt{2}} \left[ \{|n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{6}^1 \right.$$
$$- |n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1}^2 \left], \right.$$ \ 
$$\Psi_{D_3}^{1/2} = \frac{1}{2} \left[ |n_1n_2\} n_3 Q_4 Q_5 \otimes \left( \beta_{3/2}^1 + \beta_{1/2}^2 \right) \right.$$
$$+ |n_1n_2\} n_3 Q_4 Q_5 \otimes \left( \beta_{3/2}^2 - \beta_{1/2}^1 \right) \left] \right.$$ \ 
$$\Psi_{D_4}^{1/2} = \frac{1}{\sqrt{2}} \left[ \{|n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1/2}^3 \right.$$
$$+ |n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1/2}^4 \left], \right.$$ \ 
$$\Psi_{D_5}^{1/2} = \frac{1}{\sqrt{2}} \left[ \{|n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1/2}^5 \right.$$
$$+ |n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1/2}^6 \left], \right.$$ \ 

(21)

(b) $J^P = 3/2^-$:

$$\Psi_{D_1}^{3/2} = \frac{1}{\sqrt{2}} \left[ \{|n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{6}^3 \right.$$ \ 
$$- |n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1}^3 \left], \right.$$ \ 
$$\Psi_{D_2}^{3/2} = \frac{1}{\sqrt{2}} \left[ \{|n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{6}^2 \right.$$ \ 
$$- |n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{2}^3 \left], \right.$$ \ 
$$\Psi_{D_3}^{3/2} = \frac{1}{2} \left[ |n_1n_2\} n_3 Q_4 Q_5 \otimes \left( \beta_{3/2}^1 + \beta_{3/2}^2 \right) \right.$$ \ 
$$+ |n_1n_2\} n_3 Q_4 Q_5 \otimes \left( \beta_{3/2}^2 - \beta_{3/2}^1 \right) \left] \right.$$ \ 
$$\Psi_{D_4}^{3/2} = \frac{1}{\sqrt{2}} \left[ \{|n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1/2}^3 \right.$$ \ 
$$+ |n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1/2}^4 \left], \right.$$ \ 
$$\Psi_{D_5}^{3/2} = \frac{1}{\sqrt{2}} \left[ \{|n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1/2}^5 \right.$$ \ 
$$+ |n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{1/2}^6 \left], \right.$$ \ 

(22)

(c) $J^P = 5/2^-$:

$$\Psi_{D_1}^{5/2} = \frac{1}{\sqrt{2}} \left[ \{|n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{5/2}^1 \right.$$ \ 
$$- |n_1n_2\} n_3 Q_4 Q_5 \otimes \beta_{5/2}^2 \left], \right.$$ \ 

(23)

where $\{n_1n_2\} \equiv (n_1n_2)^I=1$ and $|n_1n_2\} \equiv (n_1n_2)^I=0$. If we use $\Psi_{X,a}^I$ to denote the total wave function of X-type, the preceding wave functions can be rewritten as

$$|\Psi_{X,a}^I\rangle = \sum_{\mu} \sum_k |F_{\mu} \otimes \beta_{k}^I\rangle S_{X,\mu,ak},$$

(24)
where $F_\mu$ is the flavor wave function and

$$S_{X,\mu,ak} = \langle F_\mu \otimes \beta_k^2 \mid \Psi^I_{X,a} \rangle \tag{25}$$

is the coefficient in Eqs. (12)–(23).

To obtain the mass spectrum of the pentaquark, we insert the Hamiltonian (6) into the total wave functions,

$$\langle \Psi^I_{X,b} \mid H_{CM} \mid \Psi^I_{X,a} \rangle = -\frac{3}{4} \sum_{i<j} m_{ij} \langle \Psi^I_{X,b} \mid V^C_{ij} \mid \Psi^I_{X,a} \rangle$$

$$- \sum_i v_i \langle \Psi^I_{X,b} \mid V^CM_{ij} \mid \Psi^I_{X,a} \rangle. \tag{26}$$

Since $V^C_{ij}$ and $V^CM_{ij}$ are independent of the flavor, we have (I = C, CM)

$$\langle \Psi^I_{X,b} \mid V^C_{ij} \mid \Psi^I_{X,a} \rangle = \sum_{\mu \nu} \sum_{kl} \langle F_\nu \mid F_\mu \rangle \langle \beta_l^2 \mid V_{ij}^C \mid \beta_k^2 \rangle S_{X,\mu,ak} S_{X,\nu,bl}. \tag{27}$$

Diagonalizing the Hamiltonian, we can obtain the mass spectrum and the eigenvectors.

### III. NUMERICAL RESULTS

#### A. Parameters

In Ref. [75], we have carefully extracted the parameters of the extended chromomagnetic model from the ground state mesons and baryons. Specifically, the parameters $m_{q\bar{q}}$ and $v_{q\bar{q}}$ are extracted from the mesons. The $m_{q\bar{q}}$ and $v_{q\bar{q}}$ with at most one heavy quark are extracted from the light and singly heavy baryons, and those with two heavy quarks are estimated from a quark model consideration. With these parameters, we calculated the mass of $\Xi_{cc}$ to be 3633.3 ± 9.3 MeV, which is very close to the LHCb's result, $M_{\Xi_{cc}} = 3621.40 \pm 0.72$ MeV [76]. All parameters are listed in Table I. In this work, we use the same parameters to study the mass spectrum of the $S$-wave $qqqc\bar{c}$ pentaquark states.

#### B. The hidden-charm pentaquarks

##### 1. The $nnnc\bar{c}$ system

The calculated eigenvalues and eigenvectors of the $nnnc\bar{c}$ state are listed in Table II. First we consider the $nnnc\bar{c}$ state with isospin $I = 1/2$. The lowest state has mass of 3097.0 MeV with $J^P = 1/2^-$. This state, $\sum_i b_i \Psi^I_{D5}$ with

$$\{b_i\} = \{0.111, -0.112, 0.013, 0.001, 0.987\}, \tag{28}$$

has a dominant component of $\Psi^I_{D5}$. Notice that in the $nnnc\bar{c}$ configuration, $\Psi^I_{D5}$ can be written as a direct product of a baryon and a meson,

$$\Psi^I_{D5} = N \otimes \eta_c. \tag{29}$$

In other word, this state couple almost completely to the $N\eta_c$ scattering state. Therefore it has probably a very broad width and is just a part of the continuum. It is worth stressing that this kind of state also exists in the calculation of the $qqqc\bar{c}$ tetraquark, where the lowest state couple strongly to a heavy charmonium and a light meson [28, 77]. Moreover, the states of 4024.2 MeV (with $J^P = 1/2^-$) and 4028.2 MeV (with $J^P = 3/2^-$) couple strongly to $N$ and $J/\psi$ channel. The above states are also scattering states. We label these scattering states in the fifth column of Table II. The situation of the $nnnc\bar{c}$ states with $I = 3/2$ is similar. There are four low mass states. The lowest one, 4217.5 MeV with $J^P = 3/2^+$, is a scattering state of $\Delta$ and $\eta_c$, and the other three states, 4320.8 MeV with $J^P = 1/2^-$, 4336.0 MeV with $J^P = 3/2^-$ and 4336.8 MeV with $J^P = 3/2^-$, couple very strongly to $\Delta$ and $J/\psi$.

After identifying the scattering states, the other states are genuine pentaquarks. We plot their relative position in Fig. 1(a). For simplicity, we use $P_{c}(m, I, J^P)$ to denote the $nnnc\bar{c}$ pentaquark states. From Table II, we see that the lightest state is $P_{c}(4327,0,1/2,1/2^-)$. This state is very close to the recently observed $P_{c}(4312)$. If the future experiment does confirm the quantum number of $P_{c}(4312)$ to be $1/2^-$, it is likely a tightly bound pentaquark state. The next lowest state $P_{c}(4367,4,1/2,3/2^-)$ corresponds to the $P_{c}(4380)$, which has the same quantum number [35] and may be a tightly bound pentaquark as well. Moreover, there is a partner state $P_{c}(4372,4,1/2,1/2^-)$ in this region. If $P_{c}(4380)$ truly corresponds to $P_{c}(4367,4,1/2,3/2^-)$, this partner state should also exit, which can be searched for in future experiment. In higher energy region, we find the $P_{c}(4476,3,1/2,3/2^-)$ and $P_{c}(4480,9,1/2,1/2^-)$, which can be identified with $P_{c}(4440)$ and $P_{c}(4457)$ [36]. Above 4.5 GeV, there are $P_{c}(4524,5,1/2,3/2^-)$ and $P_{c}(4540,0,1/2,5/2^-)$. The $nnnc\bar{c}$ pentaquark with isospin $I = 3/2$ are all above 4.6 GeV.

Besides the mass spectrum, the eigenvectors also provide important information about the decay properties [3, 11, 53, 78]. We can calculate the overlap between the pentaquark and a particular baryon $\times$ meson state. Then we can determine the decay amplitude of the pentaquark into that particular baryon $\times$ meson channel. To calculate the overlap, we transform the eigenvectors of the pentaquark states into the $nnn\otimes n\bar{c}$ configuration. Normally, the $nnn$ and $n\bar{c}$ components inside the pentaquark can be either of color-singlet or of color-octet (see Sec. II B). The former one can easily dissociate into a $S$-wave meson and a $S$-wave baryon (the so-called “OZI-superallowed” decays [3]). The latter one cannot fall apart without the gluon exchange. For sim-
TABLE I. Parameters of the $q\bar{q}$ and $qq$ pairs (in unit of MeV).

| $m_{n\bar{n}}$ | $m_{n\bar{s}}$ | $m_{s\bar{s}}$ | $m_{n\bar{c}}$ | $m_{c\bar{c}}$ | $m_{n\bar{b}}$ | $m_{c\bar{b}}$ | $m_{n\bar{b}}$ |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 615.95        | 794.22        | 936.40        | 1973.22       | 2076.14       | 3068.53       | 5313.35       | 5403.25       |
| 477.92        | 298.57        | 249.18        | 106.01        | 107.87        | 85.12         | 33.89         | 36.43         |
| 724.85        | 906.65        | 1049.36       | 2079.96       | 2183.68       | 3171.51       | 5412.25       | 5494.80       |
| 305.34        | 212.75        | 195.30        | 62.81         | 70.63         | 56.75         | 19.92         | 8.47          |

(a) $I = \frac{1}{2}$ (solid) and $I = \frac{3}{2}$ (dashed) $nncc\bar{c}$ states

(b) $I = 0$ (solid) and $I = 1$ (dashed) $nssc\bar{c}$ states

(c) $ssnc\bar{c}$ states

(d) $sssc\bar{c}$ states

FIG. 1. Mass spectra of the $qqq\bar{c}$ pentaquark states. The dotted lines indicate various meson-baryon thresholds and the long solid lines in (a) indicate the observed $P_c$ states. The masses are all in unit of MeV.

plicity, we follow Ref. [3, 11] and focus on the “OZI-superallowed” decays in this work. The corresponding eigenvectors in the $nncc\bar{c}$ configuration are listed in Table III. For the color-singlet part, we can rewrite the base states as a direct product of a baryon and a meson. For each decay mode, the branching fraction is proportional to the square of the coefficient of the corresponding component in the eigenvectors, and also depends on the phase space. For the two body $L$-wave decay, its partial
width reads [79]

$$\Gamma_i = \gamma_i \alpha \frac{k^{2L+1}}{m^{2L}} |c_i|^2,$$

(30)

where \(\alpha\) is an effective coupling constant, \(\gamma_i\) is a quantity determined by the decay dynamics, \(m\) is the mass of the parent particle, \(k\) is the momentum of the daughter particles in the rest frame of the parent particle, and \(c_i\) is the coefficient of the corresponding component. For the decay processes which we are interested in, \((k/m)^2\) is of \(\mathcal{O}(10^{-2})\) or even smaller. Thus we only consider the \(S\)-wave decays since the higher wave decays are all suppressed. Employing the eigenvectors in Table III, we can calculate the value of \(k \cdot |c_i|^2\) for each decay process (see Table IV). Next we have to estimate the \(\gamma_i\). Generally, \(\gamma_i\) depends on the spatial wave functions of the initial pentaquark and final meson and baryon, which are different for each decay process. In the quark model,
The values of $k \cdot |c_i|^2$ for the $nnnc\bar{c}$ pentaquark states (in unit of MeV).

| $I$ | $J^P$ | Mass | $\Delta J/\psi$ | $\Delta \eta_c$ | $N J/\psi$ | $N \eta_c$ | $\Sigma_c^0 \bar{D}^*$ | $\Sigma_c^+ \bar{D}$ | $\Sigma_c \bar{D}^*$ | $\Sigma_c^0 \bar{D}$ | $\Lambda_c \bar{D}^*$ | $\Lambda_c \bar{D}$ |
|-----|-------|------|-----------------|-----------------|------------|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $1/2$ | $1/2^-$ | 4601.9 | 27.8 | 8.8 | 177.8 | 72.8 |
|       |       | 4717.1 | 11.1 | 11.1 | 254.5 | 36.3 | 6.2 |
| $3/2^-$ |       | 4633.0 | 2.5 | 13.7 | 179.1 | 105.7 | 33.9 |
| $1/2$ | $1/2^-$ | 4327.0 | 4.8 | 14.6 | 0.0 | 36.9 | 27.8 | 0.5 |
|       |       | 4372.4 | 6.3 | 5.1 | 0.0 | 1.5 | 2.1 | 123.5 |
|       |       | 4480.9 | 4.0 | 1.2 | 0.0 | 26.0 | 7.7 | 84.3 | 6.3 |
| $3/2^-$ |       | 4367.4 | 3.7 | 0.0 | 0.0 | 0.0 | 52.4 |
|       |       | 4476.3 | 0.1 | 0.0 | 6.3 | 64.3 | 33.2 |
|       |       | 4524.5 | 2.8 | 0.0 | 18.0 | 19.8 | 31.3 |
| $5/2^-$ |       | 4546.0 | 92.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

TABLE V. The partial width ratios for the hidden-charm decays of the $nnnc\bar{c}$ pentaquark states. For each state, we chose one mode as the reference channel, and the partial width ratios of the other channels are calculated relative to this channel. The masses are all in unit of MeV.

| $I$ | $J^P$ | Mass | $\Delta J/\psi$ | $\Delta \eta_c$ | $N J/\psi$ | $N \eta_c$ | $\Sigma_c^0 \bar{D}^*$ | $\Sigma_c^+ \bar{D}$ | $\Sigma_c \bar{D}^*$ | $\Sigma_c^0 \bar{D}$ | $\Lambda_c \bar{D}^*$ | $\Lambda_c \bar{D}$ |
|-----|-------|------|-----------------|-----------------|------------|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $1/2$ | $1/2^-$ | 4601.9 | 27.8 | 0.05 | 1.0 | 0.4 |
|       |       | 4717.1 | 7.0 | 1.0 | 0.2 |
| $3/2^-$ |       | 4633.0 | 5.3 | 3.1 | 1 |
| $1/2$ | $1/2^-$ | 4327.0 | 0.0 | 1.3 | 1 | 0.02 |
|       |       | 4372.4 | 0.0 | 0.7 | 1 | 57.6 |
|       |       | 4480.9 | 0.0 | 0.3 | 0.09 | 1 | 0.07 |
| $3/2^-$ |       | 4367.4 | 0.0 | 0.0 | 1 |
|       |       | 4476.3 | 0.0 | 0.2 | 1.9 | 1 |
|       |       | 4524.5 | 0.58 | 0.63 | 1 |
| $5/2^-$ |       | 4546.0 | 1 | 0 | 0 |

TABLE VI. The partial width ratios for the open charm decays of the $nnnc\bar{c}$ pentaquark states. For each state, we chose one mode as the reference channel, and the partial width ratios of other channels are calculated relative to this channel. The masses are all in unit of MeV.

The values of the relative widths of different decay modes are listed in Tables V and VI.

First we consider the $I = 1/2$ case. The lowest state, $P_c(4327.0, 1/2, 1/2^-)$, has two hidden charm decay modes, namely $N J/\psi$ and $N \eta_c$. Their partial decay width ratio is

$$\frac{\Gamma[P_c(4327.0, 1/2, 1/2^-) \to N \eta_c]}{\Gamma[P_c(4327.0, 1/2, 1/2^-) \to N J/\psi]} = 3.0,$$

which indicates that the partial decay width of the $N \eta_c$ channel is larger than that of the $N J/\psi$. On the other hand, $P_c(4327.0, 1/2, 1/2^-)$ also has open charm decay modes. From Table IV, we see that $\Sigma_c \bar{D}$ and $\Lambda_c \bar{D}^*$ are its dominant decay modes. It is worth stressing that the calculated mass of this state is just several MeV higher than the threshold of $\Sigma_c \bar{D}$ (4321 MeV); considering the error of the model (Taking $\Xi_{cc}$ for example, our calculation differs from the experiment by 12 MeV [75]),

spatial wave functions of the ground state scalar and vector meson are the same. And in the heavy quark limit, $\Sigma_c$ and $\Sigma_c^*$ have the same spatial wave function. Furthermore, the spatial wave function of $\Lambda_c$ does not differ much from that of $\Sigma_c$. Then for each pentaquark,

$$\gamma_{\Delta J/\psi} = \gamma_{\Delta \eta_c},$$

$$\gamma_{N J/\psi} = \gamma_{N \eta_c},$$

and

$$\gamma_{\Sigma_c^0 \bar{D}^*} = \gamma_{\Sigma_c^+ \bar{D}} = \gamma_{\Sigma_c \bar{D}^*} = \gamma_{\Sigma_c^0 \bar{D}} = \gamma_{\Lambda_c \bar{D}^*}. \quad (33)$$

The values of the relative widths of different decay modes are listed in Tables V and VI.
this state may probably lie below the ΣcD threshold and thus cannot decay into this channel. If we assume that the \( P_c(4327,0,1/2,1/2^-) \) corresponds to the observed \( P_c(4312) \) state, we have

\[
\frac{\Gamma[P_c(4312)\rightarrow N\eta_c]}{\Gamma[P_c(4312)\rightarrow N/\psi]} = 3.1. \tag{35}
\]

If the \( P_c(4312) \) is observed in the \( N\eta_c \) channel, and its partial decay width is larger than that of the \( N/\psi \) channel, then the \( P_c(4312) \) is very likely a tightly bound pentaquark which corresponds to the \( P_c(4327,0,1/2,1/2^-) \). If the \( P_c(4312) \) does not appear in the \( N\eta_c \) channel, or its partial decay width is much smaller than that of the \( N/\psi \) channel, the \( P_c(4312) \) may not be a tightly bound pentaquark. Moreover,

\[
\frac{\Gamma[P_c(4312)\rightarrow \Lambda_c\bar{D}]}{\Gamma[P_c(4312)\rightarrow \Lambda_c\bar{D}^*]} = 0.02. \tag{36}
\]

We hope the future experiments can search for the \( P_c(4312) \) in the \( N\eta_c \) and \( \Lambda_c\bar{D}^* \) channels.

Next we consider the two states in the vicinity of \( P_c(4308) \). The \( P_c(4367.4,1/2,3/2^-) \) only has one hidden charm decay mode \( N/\psi \), while the \( P_c(4372.4,1/2,1/2^-) \) can decay to both \( N/\psi \) and \( N\eta_c \). Moreover,

\[
\frac{\Gamma[P_c(4372.4,1/2,1/2^-)\rightarrow N\eta_c]}{\Gamma[P_c(4372.4,1/2,1/2^-)\rightarrow N/\psi]} = 0.8. \tag{37}
\]

Thus this state can also be found in the \( N\eta_c \) channel. On the other hand, \( P_c(4367.4,1/2,3/2^-) \) can only decay to \( \Lambda_c\bar{D}^* \), and \( P_c(4372.4,1/2,1/2^-) \) decays dominantly to \( \Lambda_c\bar{D} \).

Then we consider the \( P_c(4476.3,1/2,3/2^-) \) and \( P_c(4480.9,1/2,1/2^-) \). Both of them couple weakly to the hidden charm channel(s). Note that the former state can only decay to \( N/\psi \) while the latter state can also decay to \( N\eta_c \) which can be used to distinguish the two states. In the open charm channels, both of the two states decay dominantly to the \( \Lambda_c\bar{D} \) channel. The \( \Sigma_c\bar{D}^* \) mode is also important for \( P_c(4476.3,1/2,3/2^-) \). The mass difference between the \( \Sigma_c\bar{D}^* \) threshold (4462 MeV) and the two states is only \( \sim 10 \) MeV, which is within the error of the CM model. The two states probably lie below the \( \Sigma_c\bar{D}^* \) threshold and cannot decay through this mode. \( P_c(4476.3,1/2,3/2^-) \) can also decay to \( \Sigma_c\bar{D} \) with a not-so-small fraction. If \( P_c(4476.3,1/2,3/2^-) \) and \( P_c(4480.9,1/2,1/2^-) \) truly correspond to the \( P_c(4440) \) and \( P_c(4457) \) respectively, we have

\[
\frac{\Gamma[P_c(4440)\rightarrow \Sigma_c\bar{D}]}{\Gamma[P_c(4440)\rightarrow \Lambda_c\bar{D}^*]} = 0.16, \tag{38}
\]

\[
\frac{\Gamma[P_c(4457)\rightarrow N\eta_c]}{\Gamma[P_c(4457)\rightarrow N/\psi]} = 0.29, \tag{39}
\]

and

\[
\Gamma[P_c(4457)\rightarrow \Sigma_c\bar{D}] : \Gamma[P_c(4457)\rightarrow \Lambda_c\bar{D}^*] \tag{40}
\]

Finally, we consider the two states over 4.5 GeV. We see that \( P_c(4524.5,1/2,3/2^-) \) may also be observed in the \( N/\psi \) channel, while \( P_c(4546.0,1/2,5/2^-) \) can only decay to this mode through higher partial waves, which is suppressed. The dominant decay modes of \( P_c(4524.5,1/2,3/2^-) \) are \( \Sigma_c^*\bar{D}, \Sigma_c\bar{D}^* \) and \( \Lambda_c\bar{D}^* \). Note that the \( \Sigma_c^*\bar{D}^* \) mode has the largest coefficient in the eigenvector, but this mode is suppressed by phase space. Then the \( P_c(4546.0,1/2,5/2^-) \) can only decay to \( \Sigma_c^*\bar{D}^* \).

There are three \( nnncc \) pentaquark states with \( I = 3/2 \). Their masses are all above 4.6 GeV. As shown in Table III, since their couplings to \( \Delta J/\psi \) are not very small, they can be observed in the \( \Delta J/\psi \) channel in the future experiments. We also calculate the partial decay width ratio of each mode. For \( P_c(4601.9,3/2,1/2^-) \) and \( P_c(4717.1,3/2,1/2^-) \) states, we have

\[
\Gamma[\Sigma_c\bar{D}^* : \Gamma[\Sigma_c\bar{D} : \Gamma[\Delta J/\psi = 0.05 : 1 : 0.4 \tag{41}
\]

and

\[
\Gamma[\Sigma_c\bar{D} : \Gamma[\Sigma_c\bar{D} : \Gamma[\Delta J/\psi = 7.0 : 1 : 2 \tag{42}
\]

respectively. And for \( P_c(4633.0,3/2,3/2^-) \), we have

\[
\Gamma[\Delta J/\psi : \Gamma[\Delta J/\psi = 1 : 5.5 \tag{43}
\]

and

\[
\Gamma[\Sigma_c\bar{D} : \Gamma[\Sigma_c\bar{D} : \Gamma[\Delta J/\psi = 5.3 : 3.1 : 1. \tag{44}
\]

In both cases, the \( P_c \) states have a large decay fraction to the open charm channels. Since all \( P_c \) states are observed in the \( N/\psi \) channel, it is very helpful if the future experiments can search for the open charm channels.

2. The \( nnncc \) system

Now we turn to the \( nnncc \) systems. The mass spectrum of the \( nnncc \) system is listed in Table VII. Similar to the \( nnncc \) case, we first identify the scattering states composed of a \( nns \) baryon and a charmonium. For the \( I = 0 \) case, the \( \Lambda\otimes\eta_c \) scattering state corresponds to the spin-1/2 state. The \( \Lambda\otimes J/\psi \) scattering states can be of spin-1/2 and -3/2. The latter one has a mass of 4209.5 MeV, while the former one is more complex. Actually, there are two states correspond to the spin-1/2 \( \Lambda\otimes J/\psi \) scattering state. Their masses are 4197.4 MeV and 4208.6 MeV respectively. Since they all have large fractions of color-octet components (57% and 46%), we still consider them as pentaquarks. We also reproduce most of the scattering states with \( I = 1 \). The scattering state of \( \Sigma_c^* \) and \( \eta_c \) has \( J^P = 1/2^- \) (3/2^-) and mass 4145.5 MeV (4366.8 MeV). And the \( \Sigma\otimes J/\psi \) scattering states can be of \( J^P = 1/2^- \) (4264.9 MeV) and \( J^P = 3/2^- \) (4269.7 MeV). We only reproduce two \( \Sigma^*\otimes J/\psi \) scattering states, namely the \( J^P = 1/2^- \) one.
to label these scattering states. In the following, we will
tessively. They also have large fractions of color-octet com-

pentaquark states. We also plot all the meson-baryon

In Fig. 1(b), we show the relative position of the

case, there

cay constrains of the isospin conservation and kinetics.

There are 18 channels that the

with mass 4466.7 MeV and the \( J^P = 5/2^- \) one with mass
4487.8 MeV. For the spin-3/2 \( \Delta \otimes J/\psi \) case, there
are two \( J^P = 3/2^- \) states couple strongly to \( \Sigma^* \otimes J/\psi \).

Their masses are 4485.9 MeV and 4488.4 MeV respectively.
They also have large fractions of color-octet components (37\% and 62\%). Thus we consider them as pentaquarks. For clarity, we add a fifth column in Table VII to label these scattering states. In the following, we will use \( P_{c,s}(m, I, J^P) \) to denote the \( nnc\bar{c} \) pentaquark states.

In Fig. 1(b), we show the relative position of the \( nnc\bar{c} \) pentaquark states. We also plot all the meson-baryon

thresholds which they can decay to through quark re-

arrangement. Compared to the \( nnc\bar{c} \) case, the \( nnc\bar{c} \) system has larger numbers of states and decay patterns. There are 18 channels that the \( nnc\bar{c} \) pentaquarks may decay to. From the figure, we can easily identify the de-

ay constrains of the isospin conservation and kinetics.

To have a more quantitative description of the decays, we transform the eigenvectors to the \( nnc\bar{c}\otimes c \) and \( nnc\bar{c}\otimes \bar{c} \) configurations. The color-singlet components are shown in Tables VIII–IX. Using the eigenvectors, we calculate

| System | \( J^P \) | Mass | Eigenvectors | Scattering state |
|--------|--------|------|--------------|-----------------|
| \( (nnc\bar{c})^{I=1} \) | \( \frac{1}{2}^- \) | 4145.5 | \{0.095, –0.017, –0.170, 0.108, –0.020, –0.0002, 0.003, –0.975\} | \( \Sigma_c(4177) \) |
| | | 4264.9 | \{0.037, 0.115, 0.079, 0.038, 0.135, 0.005, –0.979, –0.014\} | \( \Sigma J/\psi(4290) \) |
| | | 4442.8 | \{–0.122, 0.224, 0.837, –0.190, 0.351, 0.144, 0.134, –0.190\} | \( \Sigma^* J/\psi(4481) \) |
| | | 4466.7 | \{–0.086, 0.199, 0.173, 0.043, –0.180, –0.942, 0.006, –0.033\} | \( \Sigma^* J/\psi(4481) \) |
| | | 4522.2 | \{–0.565, –0.169, –0.141, –0.760, –0.178, –0.008, –0.105, –0.108\} | |
| | | 4612.6 | \{0.019, –0.437, 0.454, 0.188, –0.732, 0.138, –0.106, 0.034\} | |
| | | 4696.3 | \{–0.621, 0.566, –0.084, 0.408, –0.260, 0.229, 0.017, –0.005\} | |
| | | 4808.1 | \{–0.512, –0.598, 0.019, 0.412, 0.437, –0.140, –0.012, –0.007\} | |
| \( (nnc\bar{c})^{I=0} \) | \( \frac{1}{2}^- \) | 4086.1 | \{–0.126, –0.059, 0.022, 0.146, 0.001, 0.002, 0.979\} | \( \Lambda c(4100) \) |
| | | 4197.4 | \{0.045, 0.350, 0.130, 0.547, 0.361, 0.652, –0.059\} | |
| | | 4208.6 | \{–0.038, 0.381, 0.250, 0.479, 0.136, –0.735, –0.057\} | |
| | | 4386.6 | \{–0.208, 0.102, –0.327, 0.435, –0.797, 0.095, –0.078\} | |
| | | 4465.0 | \{0.735, 0.323, –0.572, –0.036, 0.040, –0.091, 0.132\} | |
| | | 4489.6 | \{0.152, –0.763, –0.255, 0.502, 0.238, –0.112, –0.096\} | |
| | | 4607.0 | \{0.612, –0.179, 0.649, 0.085, –0.397, 0.076, 0.041\} | |
| | \( \frac{3}{2}^- \) | 4209.5 | \{0.041, 0.088, 0.024, –0.033, 0.994\} | \( \Lambda J/\psi(4212) \) |
| | | 4387.3 | \{0.101, 0.074, –0.402, –0.906, –0.031\} | |
| | | 4501.5 | \{0.521, 0.335, 0.743, –0.242, –0.077\} | |
| | | 4603.6 | \{–0.845, 0.258, 0.396, –0.249, –0.006\} | |
| | | 4656.0 | \{0.037, 0.899, –0.360, 0.239, –0.065\} | |
| | \( \frac{5}{2}^- \) | 4680.6 | \{1\} | |

TABLE VII. Pentaquark masses and eigenvectors of the \( nnc\bar{c} \) systems. The masses are all in unit of MeV.
In the quark model, the spatial wave functions of the ground state scalar and vector meson are the same. And the same spatial wave function of $\Sigma^*$ does not differ much from that of $\Sigma$. In the heavy quark limit, $\Sigma_s$ and $\Sigma^*_s$ have the same spatial wave function. Similarly, the $\Xi_c^*$ and $\Xi'_c^*$ have the same spatial wave function, and their spatial wave functions do not differ much from that of $\Xi_c$. Thus for each $nns\tilde{c}\bar{c}$ pentaquark

$$\gamma_{\Lambda_c\bar{D}_s^*} = \gamma_{\Lambda_c\bar{D}_s},$$

(45)

and

$$\gamma_{\Xi_c^*\bar{D}^*} = \gamma_{\Xi_c\bar{D}^*} = \gamma_{\Xi'_c\bar{D}^*} = \gamma_{\Xi_c\bar{D}^*} \approx \gamma_{\Xi'_c\bar{D}^*} = \gamma_{\Xi_c\bar{D}^*}. \tag{49}$$

The relative partial widths of different decay modes are listed in Tables XI–XIII.

From the eigenvectors, we find a new type of scattering state, which consists of a charm baryon plus an anti-charm meson. The $P_{c,s}(4584.9, 1, 3/2^-)$ has 82% of the $\Sigma_c\bar{D}_s^*$ component, while both the $P_{c,s}(4636.2, 1, 3/2^-)$ and $P_{c,s}(4644.3, 1, 5/2^-)$ have more than 85% of the $\Sigma'_c\bar{D}_s^*$ component. Some other states, namely the $P_{c,s}(4386.6, 0, 1/2^-)$, $P_{c,s}(4387.3, 0, 3/2^-)$,
TABLE X. The values of $k \cdot |c_i|^2$ for the $nnsc\bar{c}$ pentaquark states (in unit of MeV).

| $I\ J^P$ | $\Sigma^*J/\psi$ | $\Sigma^*\eta_c$ | $\Sigma J/\psi$ | $\Sigma\eta_c$ | $\Lambda J/\psi$ | $\Lambda\eta_c$ | $\Xi_c\bar{D}^*$ | $\Xi_c\bar{D}$ | $\Xi_c^0\bar{D}^*$ | $\Xi_c^0\bar{D}$ | $\Xi_c\bar{D}^*\Xi_c\bar{D}$ |
|----------|-------------------|------------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\frac{1}{2}^+$ | 4424.8 | 0 | 9.4 | 24.9 | 0 | 0 | 157.1 | 0 | 0 | 0 | 34.5 |
| 4522.0 | 0.02 | 7.1 | 9.3 | 0 | 0 | 0.008 | 0 | 0 | 81.8 | 0.17 | 31.2 |
| 4612.6 | 9.7 | 8.7 | 1.1 | 0 | 97.0 | 0.013 | 0 | 31.5 | 3.5 | 46.7 | 14.5 |
| 4696.3 | 34.4 | 0.27 | 0.03 | 0.001 | 85.8 | 44.6 | 8.3 | 16.4 | 12.7 | 274.6 | 59.9 |
| 4808.1 | 16.0 | 0.15 | 0.05 | 178.0 | 26.7 | 4.7 | 265.1 | 12.9 | 8.7 | 22.3 | 1.8 |
| $\frac{3}{2}^-$ | 4485.9 | 57.4 | 1.3 | 3.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.001 |
| 4488.4 | 42.9 | 0.26 | 5.0 | 0 | 17.6 | 0 | 0 | 0 | 0 | 18.0 |
| 4584.9 | 0.23 | 0.30 | 2.2 | 0 | 1.0 | 172.1 | 0 | 0.2 | 0 | 25.9 |
| 4636.2 | 0.14 | 1.7 | 3.1 | 99.4 | 5.5 | 1.8 | 0 | 16.4 | 20.1 | 43.0 |
| 4728.8 | 2.7 | 14.9 | 0.02 | 43.5 | 67.2 | 14.4 | 149.9 | 81.8 | 34.4 | 10.3 |
| $\frac{5}{2}^-$ | 4644.3 | 0.02 | 158.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 4.0 | 0 | 1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4208.6 | 0 | 1.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4386.6 | 4.9 | 4.3 | 0 | 0.02 | 0 | 0 | 0 | 0 | 0 | 11.6 |
| 4465.0 | 5.5 | 14.0 | 4.1 | 0.14 | 0 | 0 | 20.0 | 0 | 158.4 |
| 4489.6 | 8.7 | 7.6 | 1.2 | 160.5 | 0 | 0 | 31.8 | 18.1 | 36.2 |
| 4607.0 | 4.8 | 1.6 | 91.3 | 7.0 | 0 | 62.0 | 13.1 | 19.6 | 8.3 |
| $\frac{3}{2}^-$ | 4387.3 | 0.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4501.5 | 4.2 | 30.7 | 0 | 0 | 0 | 0 | 10.3 |
| 4603.6 | 0.03 | 37.9 | 0 | 16.0 | 5.6 | 289.2 |
| 4656.0 | 3.7 | 31.7 | 26.8 | 31.0 | 40.4 | 20.1 |
| $\frac{5}{2}^-$ | 4680.6 | 163.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$P_{c,s}(4608.6,0,5/2^-)\text{ and } P_{c,s}(4442.8,1,1/2^-)\text{ states, also have quite large fractions of the color-singlet open charm components. They are expected to be broad. But we still cannot rule out the possibility that they are pentaquark states. To obtain a more definite conclusion, one needs to consider the dynamics inside the pentaquark, which is beyond the present work.}

Two of the lowest $nnsc\bar{c}$ pentaquark states are the $P_{c,s}(4197.4,0,1/2^-)\text{ and } P_{c,s}(4208.6,0,1/2^-)$. From Fig. 1(b), we see that they can only decay to $\Lambda\eta_c$, thus they should have narrow widths. However, their wave functions have large overlaps with the $\Lambda J/\psi$, and their predicted masses are just below the $\Lambda J/\psi$ threshold. Considering the error of the present model, their masses can probably be larger than the $\Lambda J/\psi$ threshold. In that case, they will decay easily to $\Lambda J/\psi$ and be broader. Both $P_{c,s}(4386.6,0,1/2^-)\text{ and } P_{c,s}(4387.3,0,3/2^-)$ decay dominantly to $\Lambda J/\psi$. But $P_{c,s}(4386.6,0,1/2^-)$ can also decay to $\Lambda\eta_c$, with

$$\Gamma[P_{c,s}(4386.6,0,1/2^-)\to \Lambda\eta_c] = 0.87.$$  \hspace{1cm} (50)

$\Gamma_{\Lambda J/\psi} : \Gamma_{\Lambda\eta_c} = 1 : 2.6,$ \hspace{1cm} (51)

$\Gamma_{\Lambda J/\psi} : \Gamma_{\Lambda\eta_c} = 1 : 0.03,$ \hspace{1cm} (52)

$\Gamma_{\Xi_c^0\bar{D}^*} : \Gamma_{\Xi_c^0\bar{D}} = 1 : 0 : 7.9.$ \hspace{1cm} (53)

And the $P_{c,s}(4489.6,0,1/2^-)$ has

$\Gamma_{\Lambda J/\psi} : \Gamma_{\Lambda\eta_c} = 1 : 0.87,$ \hspace{1cm} (54)

$\Gamma_{\Lambda J/\psi} : \Gamma_{\Lambda\eta_c} = 1 : 134.1,$ \hspace{1cm} (55)

$\Gamma_{\Xi_c^0\bar{D}^*} : \Gamma_{\Xi_c^0\bar{D}} = 1 : 0.57 : 1.1.$ \hspace{1cm} (56)

The $P_{c,s}(4465.0,0,1/2^-)\text{ and } P_{c,s}(4489.6,0,1/2^-)$ have the same quantum numbers and decay channels, but we can still use their relative size of partial decay widths to distinguish them. For $P_{c,s}(4465.0,0,1/2^-)$, we have

$\Gamma_{\Lambda J/\psi} : \Gamma_{\Lambda\eta_c} = 1 : 2.6,$ \hspace{1cm} (51)

The $P_{c,s}(4489.6,0,1/2^-)$ has

$\Gamma_{\Lambda J/\psi} : \Gamma_{\Lambda\eta_c} = 1 : 0.03,$ \hspace{1cm} (52)

$\Gamma_{\Xi_c^0\bar{D}^*} : \Gamma_{\Xi_c^0\bar{D}} = 1 : 0 : 7.9.$ \hspace{1cm} (53)

And the $P_{c,s}(4489.6,0,1/2^-)$ has

$\Gamma_{\Lambda J/\psi} : \Gamma_{\Lambda\eta_c} = 1 : 0.87,$ \hspace{1cm} (54)

$\Gamma_{\Lambda J/\psi} : \Gamma_{\Lambda\eta_c} = 1 : 134.1,$ \hspace{1cm} (55)

$\Gamma_{\Xi_c^0\bar{D}^*} : \Gamma_{\Xi_c^0\bar{D}} = 1 : 0.57 : 1.1.$ \hspace{1cm} (56)
TABLE XI. The partial width ratios for the hidden-charm decays of the $nnsc$ pentaquark states. The masses are all in unit of MeV.

| $IJ^P$ | Mass | $\Sigma^*J/\psi$ | $\Sigma^*\eta_c$ | $\Sigma J/\psi$ | $\Sigma\eta_c$ | $\Lambda J/\psi$ | $\Lambda\eta_c$ |
|--------|------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| $1 \frac{1}{2}^-$ | 4422.8 | 0 | 1 | 2.7 |
|       | 4522.2 | 0.002 | 1 | 1.3 |
|       | 4612.6 | 1.1 | 1 | 0.12 |
|       | 4696.3 | 1 | 0.008 | 0.0008 |
|       | 4808.1 | 1 | 0.009 | 0.003 |
| $\frac{3}{2}^-$ | 4855.9 | 18.5 | 0.4 | 1 |
|       | 4488.4 | 8.6 | 0.05 | 1 |
|       | 4584.9 | 0.10 | 0.14 | 1 |
|       | 4636.2 | 0.05 | 0.54 | 1 |
|       | 4728.8 | 1 | 5.5 | 0.007 |
| $\frac{5}{2}^-$ | 4644.3 | 1 |

| $0 \frac{1}{2}^-$ | 4197.4 | 0 | 1 |
|       | 4208.6 | 0 | 1 |
|       | 4386.6 | 1 | 0.87 |
|       | 4465.0 | 1 | 2.6 |
|       | 4489.6 | 1 | 0.87 |
|       | 4607.0 | 1 | 0.33 |
| $\frac{3}{2}^-$ | 4387.3 | 1 |
|       | 4501.5 | 1 |
|       | 4603.6 | 1 |
|       | 4656.0 | 1 |
| $\frac{5}{2}^-$ | 4680.6 |

TABLE XII. The partial width ratios for the $nnc\otimes s\bar{c}$ open charm decays of the $nnsc$ pentaquark states. The masses are all in unit of MeV.

| $IJ^P$ | Mass | $\Sigma^*\bar{D}_s$ | $\Sigma\bar{D}_s$ | $\Sigma_s\bar{D}_s$ | $\Sigma_{l}\bar{D}_s$ | $\Lambda\bar{D}_s$ | $\Lambda_s\bar{D}_s$ |
|--------|------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| $1 \frac{1}{2}^-$ | 4422.8 | 0 | 0 | 1 |
|       | 4522.2 | 0 | 0 | 1 |
|       | 4612.6 | 0 | 1 | 0.0001 |
|       | 4696.3 | 0.00002 | 1 | 1 |
|       | 4808.1 | 37.7 | 5.6 | 1 |
| $\frac{3}{2}^-$ | 4859.9 | 0 | 0 | 0 |
|       | 4488.4 | 0 | 1 | 0 |
|       | 4584.9 | 0 | 1 | 176.1 |
|       | 4636.2 | 17.9 | 1 | 0.33 |
|       | 4728.8 | 0.65 | 1 | 0.21 |
| $\frac{5}{2}^-$ | 4644.3 | 1 |

| $0 \frac{1}{2}^-$ | 4197.4 | 0 | 0 |
|       | 4208.6 | 0 | 0 |
|       | 4386.6 | 0 | 1 |
|       | 4465.0 | 1 | 0.03 |
|       | 4489.6 | 1 | 134.1 |
|       | 4607.0 | 13.1 | 1 |
| $\frac{3}{2}^-$ | 4387.3 | 0 |
|       | 4501.5 | 1 |
|       | 4603.6 | 1 |
|       | 4656.0 | 1 |
| $\frac{5}{2}^-$ | 4680.6 |

3. The $sssc$ and $sssc$ systems

The $sssc$ system is similar to the $I = 1$ $nnsc$ system. We present their mass spectra in Table XIV. As indicated in the last column, we reproduce the scattering states of $\Xi_{c6}$ (4288.0 MeV with $J^P = 1/2^-$), $\Xi J/\psi$ (4406.0 MeV with $J^P = 1/2^-$ and 4413.7 MeV with $J^P = 3/2^-$), $\Xi^*\eta_c$ (4509.4 MeV with $J^P = 3/2^-$) and $\Xi^*J/\psi$ (4604.7 MeV with $J^P = 1/2^-$, 4630.6 MeV with $J^P = 3/2^-$ and 4631.7 MeV with $J^P = 5/2^-$). In the following, we will use $P_{c,ss}(m, J^P)$ to denote the $sssc$ pentaquark.

We plot the relative position of the $sssc$ pentaquark states and all the relevant meson-baryon thresholds in Fig. 1(c). We also transform the eigenvectors to the $ssc\otimes n\bar{c}$ and $nsc\otimes n\bar{c}$ configurations. The color-singlet components are listed in Table XV. The only state with $J^P = 5/2^-$, $P_{c,ss}(4790.0, 5/2^-)$, lies over all thresholds and

$$P_{c,ss}(4790.0, 5/2^-) = 0.94487\Omega_{c}^*\otimes \bar{D}^* + \cdots .$$ (57)
TABLE XIII. The partial width ratios for the $nsc\otimes n\bar{c}$ open charm decays of the $nsc\bar{c}$ pentaquark states. The masses are all in unit of MeV.

| $I$ | $J^P$ | $\Xi_c^*\bar{D}^*$ | $\Xi_c\bar{D}$ | $\Xi_c^*\bar{D}^*$ | $\Xi_c\bar{D}$ | $\Xi_c\bar{D}$ | $\Xi_c\bar{D}$ |
|-----|-------|---------------------|----------------|---------------------|----------------|----------------|----------------|
| 1   | $\frac{1}{2}^-$ | 4424.8 | 0 | 0 | 0 | 0 | 1 |
|     | 4522.2 | 0 | 0 | 2.6 | 0.006 | 1 |
|     | 4612.6 | 0 | 2.2 | 0.24 | 3.4 | 1 |
|     | 4696.3 | 0.14 | 0.27 | 0.21 | 4.6 | 1 |
|     | 4808.1 | 151.4 | 7.4 | 5.0 | 12.8 | 1 |
| $\frac{3}{2}^-$ | 4485.9 | 0 | 0 | 0 | 1 |
|     | 4488.4 | 0 | 0 | 0 | 1 |
|     | 4584.9 | 0 | 0.007 | 0 | 1 |
|     | 4636.2 | 0.38 | 0.49 | 1 |
|     | 4728.8 | 14.6 | 8.0 | 3.4 | 1 |
| $\frac{5}{2}^-$ | 4644.3 | 0 |
| 0   | $\frac{1}{2}^-$ | 4197.4 | 0 | 0 | 0 | 0 | 0 |
|     | 4208.6 | 0 | 0 | 0 | 0 | 0 |
|     | 4386.6 | 0 | 0 | 0 | 0 | 1 |
|     | 4450.0 | 0 | 0 | 1 | 0 | 7.9 |
|     | 4496.6 | 0 | 0 | 0.88 | 0.50 | 1 |
|     | 4607.0 | 0 | 7.4 | 1.6 | 2.4 | 1 |
| $\frac{3}{2}^-$ | 4387.3 | 0 | 0 | 0 | 0 |
|     | 4501.5 | 0 | 0 | 0 | 0 | 1 |
|     | 4603.6 | 0 | 1 | 0.35 | 18.0 |
|     | 4656.0 | 0.87 | 1 | 1.3 | 0.65 |
| $\frac{5}{2}^-$ | 4680.6 | 1 |

It is a scattering state of $\Omega_c^*\bar{D}^*$. Its dominant decay mode is $\Omega_c^*\bar{D}^*$ and it should be broad. In Table XVI, we calculate the values of $k \cdot |c_i|^2$ of the $sssc\bar{c}$ pentaquark states. Similar to the $nnsc\bar{c}$ and $nsc\bar{c}$, for each $sssc\bar{c}$ pentaquark state,

$$\gamma_{\Xi_c^*J/\psi} = \gamma_{\Xi_c^*\eta_c} = \gamma_{\Xi_cJ/\psi} = \gamma_{\Xi_c\eta_c},$$  \hspace{1cm} (58)

$$\gamma_{\Omega_c^*\bar{D}^*} = \gamma_{\Omega_c^*\bar{D}} = \gamma_{\Omega_c\bar{D}^*} = \gamma_{\Omega_c\bar{D}},$$  \hspace{1cm} (59)

$$\gamma_{\Xi_c^*\bar{D}^*} = \gamma_{\Xi_c^*\bar{D}} = \gamma_{\Xi_c\bar{D}^*} = \gamma_{\Xi_c\bar{D}},$$  \hspace{1cm} (60)

The calculated partial decay width ratios are listed in Tables XVII–XIX.

The last class of the hidden-charm pentaquark is the $sssc\bar{c}$ system. They are similar to the $nnsc\bar{c}$ states with isospin $I = 3/2$. We present their mass spectra in Table XX. We find three scattering states (4736.0 MeV with $J^P = 1/2^-$, 4767.5 MeV with $J^P = 3/2^-$ and 4768.6 MeV with $J^P = 5/2^-$) which couple very strongly to the $\Omega J/\psi$ and a scattering state (4645.1 MeV with $J^P = 3/2^-$) which couples strongly to the $\Omega \eta_c$. We will focus on the other $sssc\bar{c}$ pentaquark states. To study their decay properties, we transform their wave functions to the $sssc\bar{c}$ configuration (see Table XXI). And we also plot their relative position in Fig. 1(d), along with all possible decay channels. We find that they are all above the open charm thresholds and have large overlap with the $\Omega_c^{(*)} \otimes \bar{D}^{(*)}$ component. Thus they should all be very broad. The partial decay width ratios can be found in Tables XXII–XXIV.

IV. CONCLUSIONS

In this work, we have systematically studied the mass spectrum of the hidden charm pentaquark in the framework of an extended chromomagnetic model. In addition to the chromomagnetic interaction, the effect of color interaction is also considered in this model. With the eigenvectors obtained, we have further investigated the decay properties of the pentaquarks.

For the $nnsc\bar{c}$ pentaquark with $I = 1/2$, we find that the masses of the experimentally observed $P_c$ states are compatible with such pentaquark states. The lowest state $P_c(4327.0, 1/2, 1/2^-)$ corresponds to the $P_c(4312)$. This state has two hidden charm channels, namely the $NJ/\psi$ and $N\eta_c$ channels. And its partial decay width of the $N\eta_c$ mode is larger than that of the $NJ/\psi$ mode. In the open charm decay channel, $P_c(4327.0, 1/2, 1/2^-)$ decays dominantly to the $\Lambda_c\bar{D}^*$ mode. We hope the future experiments can search for the $P_c(4312)$ in the $N\eta_c$ and $\Lambda_c\bar{D}^*$ channels.

There are two states, $P_c(4367.4, 1/2, 3/2^-)$ and $P_c(4372.4, 1/2, 1/2^-)$, in the vicinity of the $P_c(4380)$. $P_c(4367.4, 1/2, 3/2^-)$ decays into the $NJ/\psi$ and $\Lambda_c\bar{D}^*$ modes, while the other hidden charm (like $N\eta_c$) or open charm decay modes are all suppressed. Its partner state, $P_c(4372.4, 1/2, 1/2^-)$ can decay into both $NJ/\psi$ and $N\eta_c$ modes. And their partial decay widths are comparable. In the open charm channel, $P_c(4372.4, 1/2, 1/2^-)$ decays dominantly to the $\Lambda_c\bar{D}^*$ mode. If $P_c(4380)$ truly corresponds to the $P_c(4367.4, 1/2, 3/2^-)$, this partner state should also exist, which can be searched for in future experiments.

In higher mass region, we find $P_c(4476.3, 1/2, 3/2^-)$ and $P_c(4480.9, 1/2, 1/2^-)$. They may correspond to the $P_c(4440)$ and $P_c(4457)$ respectively. Both of them can only decay to $NJ/\psi$ while the latter state can also decay to $N\eta_c$, which can be used to distinguish the two states. In the open charm channels, both of them decay dominantly to the $\Lambda_c\bar{D}^*$. And the $P_c(4476.3, 1/2, 3/2^-)$ can also decay to $\Sigma_c\bar{D}$ with a not-so-small fraction.

Moreover, we predict two states above 4.5 GeV, namely $P_c(4524.5, 1/2, 3/2^-)$ and $P_c(4546.0, 1/2, 5/2^-)$. Like the observed $P_c$ states, $P_c(4524.5, 1/2, 3/2^-)$ can also be ob-
TABLE XIV. Pentaquark masses and eigenvectors of the $ssnc\bar{c}$ systems. The masses are all in unit of MeV.

| System | $J^P$ | Mass |  \begin{array}{c|c|c|c} 
    \hline
    \text{Mass} \text{ Eigenvector} & \text{Mass} \text{ Eigenvector} & \text{Mass} \text{ Eigenvector} & \text{Mass} \text{ Eigenvector} \\
    \hline
    ssnc\bar{c} & \frac{1}{2}^- & 4288.0 & \begin{pmatrix} 0.123, -0.021, -0.169, 0.115, -0.019, -0.0001, 0.003, -0.971 \end{pmatrix} & \Xi_\eta(4302) \\
    & & 4406.0 & \begin{pmatrix} -0.043, -0.160, -0.095, -0.043, -0.146, 0.004, 0.069, 0.015 \end{pmatrix} & \Xi J/\psi(4415) \\
    & & 4573.4 & \begin{pmatrix} 0.222, -0.379, -0.781, 0.179, -0.303, 0.068, -0.171, 0.199 \end{pmatrix} \\
    & & 4604.7 & \begin{pmatrix} -0.050, 0.239, -0.087, 0.130, -0.231, -0.928, 0.003, 0.024 \end{pmatrix} & \Xi^* J/\psi(4630) \\
    & & 4621.7 & \begin{pmatrix} -0.700, -0.198, -0.167, -0.605, -0.205, -0.035, -0.136, -0.123 \end{pmatrix} \\
    & & 4728.5 & \begin{pmatrix} 0.157, -0.625, 0.561, 0.075, -0.492, -0.090, -0.111, -0.046 \end{pmatrix} \\
    & & 4787.6 & \begin{pmatrix} 0.479, -0.330, -0.051, -0.578, 0.480, -0.306, 0.010, -0.001 \end{pmatrix} \\
    & & 4902.2 & \begin{pmatrix} -0.434, -0.484, -0.013, 0.479, 0.564, -0.174, 0.007, 0.004 \end{pmatrix} \\
    \hline
    \text{pn} \frac{3}{2}^- & 4413.7 & \begin{pmatrix} -0.042, 0.058, 0.119, -0.038, -0.001, -0.003, -0.990 \end{pmatrix} & \Xi J/\psi(4415) \\
    & & 4509.4 & \begin{pmatrix} -0.141, 0.018, 0.008, 0.100, -0.020, -0.985, 0.007 \end{pmatrix} & \Xi^* \eta(4517) \\
    & & 4614.5 & \begin{pmatrix} 0.548, -0.582, -0.350, 0.469, 0.041, -0.047, -0.118 \end{pmatrix} \\
    & & 4630.6 & \begin{pmatrix} -0.027, -0.034, -0.007, 0.073, -0.996, 0.031, -0.004 \end{pmatrix} & \Xi^* J/\psi(4630) \\
    & & 4715.2 & \begin{pmatrix} 0.374, 0.804, -0.363, 0.284, -0.015, -0.013, -0.023 \end{pmatrix} \\
    & & 4769.1 & \begin{pmatrix} -0.460, -0.091, -0.849, -0.226, 0.006, 0.034, -0.079 \end{pmatrix} \\
    & & 4819.0 & \begin{pmatrix} -0.570, 0.033, 0.9998, 0.795, 0.077, 0.162, 0.007 \end{pmatrix} \\
    \hline
    \text{np} \frac{3}{2}^- & 4631.7 & \begin{pmatrix} -0.006, 0.99998 \end{pmatrix} & \Xi^* J/\psi(4630) \\
    & & 4790.0 & \begin{pmatrix} 0.99998, 0.006 \end{pmatrix} \\
    \hline
  \end{array} |

TABLE XV. The eigenvectors for the $ssnc\bar{c}$ pentaquark states. The masses are all in unit of MeV.

| $J^P$ | Mass | \begin{array}{c|c|c|c|c|c|c|c|c|c} 
    \hline
    \text{Mass} & \text{Mass} & \text{Mass} & \text{Mass} & \text{Mass} & \text{Mass} & \text{Mass} & \text{Mass} & \text{Mass} & \text{Mass} \\
    \hline
    1/2^- & 4573.4 & 0.068 & 0.019 & -0.112 & -0.180 & -0.725 & -0.038 & 0.065 & 0.426 & -0.397 & 0.017 \\
    & 4621.7 & -0.035 & -0.123 & -0.426 & -0.526 & -0.078 & 0.246 & 0.312 & 0.003 & 0.135 & -0.492 \\
    & 4728.5 & -0.091 & -0.046 & -0.542 & 0.401 & 0.158 & 0.335 & -0.298 & -0.052 & -0.447 & -0.112 \\
    & 4787.6 & -0.306 & 0.010 & -0.001 & 0.164 & -0.653 & 0.260 & 0.085 & -0.534 & 0.247 & 0.075 & -0.007 \\
    & 4902.2 & -0.174 & 0.007 & 0.004 & 0.673 & 0.191 & 0.078 & 0.602 & 0.187 & 0.074 & 0.019 & 0.030 \\
    3/2^- & 4614.5 & 0.041 & -0.047 & -0.118 & -0.012 & -0.751 & -0.065 & -0.047 & 0.410 & 0.042 & 0.449 \\
    & 4715.2 & -0.015 & -0.013 & -0.023 & 0.096 & 0.162 & 0.848 & -0.101 & -0.053 & -0.447 & 0.276 \\
    & 4769.1 & 0.006 & 0.034 & -0.079 & -0.702 & -0.310 & 0.313 & 0.486 & 0.070 & -0.109 & -0.257 \\
    & 4819.0 & 0.077 & 0.162 & 0.007 & 0.665 & -0.323 & 0.208 & 0.445 & -0.330 & 0.200 & 0.090 \\
    \end{array} |

served in the $NJ/\psi$ channel. In the open charm channel, it decays dominantly into $\Lambda_c D^*$, while the $\Sigma_c D$, $\Sigma_c D^*$ modes are also important. On the other hand, $P_c(4546.0, 1/2, 5/2^-)$ can only decay to $\Sigma_c D^*$, all other decay modes are suppressed.

There are three $nnnc\bar{c}$ pentaquark states with $I = 3/2$, their masses are all over 4.6 GeV. They can decay into the $\Delta J/\psi$ channel, while $P_c(4633.0, 3/2, 3/2^-)$ can also decay to $\Delta \eta_c$. In the open charm channel, $P_c(4601.9, 3/2, 1/2^-)$ decays dominantly to the $\Sigma_c D^*$ and $\Sigma_c D$ modes, $P_c(4717.1, 3/2, 1/2^-)$ decays dominantly to the $\Sigma_c D^*$ and $\Sigma_c D^*$ modes, and $P_c(4633.0, 3/2, 3/2^-)$ can decay to the $\Sigma_c D^*$, $\Sigma_c D$ and $\Sigma_c D^*$ modes.

We have also used this model to explore the $nnnc\bar{c}$, $ssnc\bar{c}$ and $ssnc\bar{c}$ pentaquark states. With the obtained eigenvectors, we further explore the hidden and open charm decays of these pentaquark states. We hope that
TABLE XVI. The values of $k \cdot |c_i|^2$ for the ssnc¯c pentaquark states (in unit of MeV).

| $J^P$ | Mass | $\Xi^* J/\psi$ | $\Xi^* \eta_c$ | $\Xi J/\psi$ | $\Xi \eta_c$ | $\Omega_c^+ D^+$ | $\Omega_c^- D^-$ | $\Omega_c D$ | $\Omega_c^0 D_s$ | $\Xi^0 D_s^*$ | $\Xi^+ D_s^*$ | $\Xi^- D_s^*$ | $\Xi D_s$ |
|-------|------|----------------|----------------|-------------|-------------|----------------|----------------|-------------|----------------|---------------|---------------|---------------|-------------|
| $^{1/2}$ | 4573.4 | 0 | 16.0 | 28.5 | 0 | 81.7 | 0 | 0 | 44.6 | 0 | 0.15 |
| | 4621.7 | 0 | 11.7 | 11.9 | 0 | 0 | 2.2 | 0 | 0 | 0.003 | 5.5 | 155.2 |
| | 4728.5 | 3.6 | 9.7 | 2.0 | 0 | 38.4 | 15.2 | 0 | 26.4 | 1.7 | 116.4 | 10.1 |
| | 4787.6 | 54.0 | 0.08 | 0.001 | 4.7 | 188.4 | 48.3 | 1.9 | 136.2 | 45.5 | 3.9 | 0.04 |
| | 4902.2 | 23.0 | 0.04 | 0.01 | 248.4 | 25.0 | 512.4 | 24.9 | 5.0 | 0.32 | 0.95 |
| $^{3/2}$ | 4614.5 | 0 | 0.97 | 8.6 | 0 | 0 | 3.8 | 0 | 0 | 55.4 |
| | 4715.2 | 0.10 | 0.11 | 0.41 | 0 | 11.3 | 116.9 | 0 | 1.3 | 48.1 | 42.2 |
| | 4769.1 | 0.02 | 0.82 | 5.3 | 0 | 53.4 | 38.2 | 37.9 | 2.9 | 5.1 | 43.7 |
| | 4819.0 | 3.7 | 21.0 | 0.05 | 142.9 | 68.0 | 22.3 | 75.3 | 75.1 | 22.0 | 164.1 |
| | 4790.0 | 0.02 | 170.0 | 60.4 |

TABLE XVII. The partial width ratios for the hidden charm decays of the ssnc¯c pentaquark states. The masses are all in unit of MeV.

| $J^P$ | Mass | $\Xi^* J/\psi$ | $\Xi^* \eta_c$ | $\Xi J/\psi$ | $\Xi \eta_c$ |
|-------|------|----------------|----------------|-------------|-------------|
| $^{1/2}$ | 4573.4 | 0 | 1 | 1.8 |
| | 4621.7 | 0 | 1 | 1.02 |
| | 4728.5 | 0.38 | 1 | 0.20 |
| | 4787.6 | 1 | 0.001 | 0.0003 |
| | 4902.2 | 1 | 0.002 | 0.0006 |
| $^{3/2}$ | 4614.5 | 0 | 0.11 | 1 |
| | 4715.2 | 0.23 | 0.26 | 1 |
| | 4769.1 | 0.004 | 0.16 | 1 |
| | 4819.0 | 1 | 5.7 | 0.01 |
| | 4790.0 | 1 |

future experiments in LHCb and other collaborations can search for these states.

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TABLE XVIII. The partial width ratios for the $\Lambda s_{c}\bar{n}c$ open charm decays of the $ssnc$ pentaquark states. The masses are all in unit of MeV.

| $J^P$ | Mass (MeV) | $\Omega_c^+ D_s^-$ | $\Omega_c^0 D_s^-$ | $\Omega_c^0 D_s^+$ | $\Omega_c^-' D_s^-$ |
|-------|------------|-----------------|-----------------|-----------------|-----------------|
| $\frac{1}{2}^-$ | 4573.4 | 0 | 0 | 1 | 0.003 |
| | 4621.7 | 0 | 0 | 1 | 28.1 |
| | 4728.5 | 0.10 | 1 | 0.39 | 1 |
| | 4902.2 | 46.0 | 4.6 | 1 |
| $\frac{3}{2}^-$ | 4614.5 | 0 | 0 | 0.0005 | 1 |
| | 4715.2 | 0 | 1 | 10.3 |
| | 4769.1 | 0 | 1 | 0.72 |
| | 4819.0 | 2.1 | 1 | 0.33 |
| $\frac{5}{2}^-$ | 4790.0 | 1 |

TABLE XIX. The partial width ratios for the $nsc\bar{s}c$ open charm decays of the $ssnc$ pentaquark states. The masses are all in unit of MeV.

| $J^P$ | Mass (MeV) | $\Xi_c^\pm D_s^\mp$ | $\Xi_c^{0} D_s^\pm$ | $\Xi_c^{0} D_s^0$ | $\Xi_c^{++} D_s^-$ |
|-------|------------|-----------------|-----------------|-----------------|-----------------|
| $\frac{1}{2}^-$ | 4573.4 | 0 | 0 | 1 |
| | 4621.7 | 0 | 0.0005 | 1 |
| | 4728.5 | 15.3 | 1 | 67.4 |
| | 4787.6 | 0.04 | 3.0 | 0.09 |
| | 4902.2 | 42.4 | 5.0 | 0.06 |
| $\frac{3}{2}^-$ | 4614.5 | 0 | 1 | 0.19 |
| | 4715.2 | 0 | 1 | 35.9 |
| | 4769.1 | 12.9 | 1 | 14.9 |
| | 4819.0 | 1.0 | 1 | 0.29 |
| $\frac{5}{2}^-$ | 4790.0 | 1 |

TABLE XX. Pentaquark masses and eigenvectors of the $sscc$ systems. The masses are all in unit of MeV.

| System | $J^P$ | Mass (MeV) | Eigenvector | Scattering state |
|--------|-------|------------|-------------|------------------|
| $sscc$ | $\frac{1}{2}^-$ | 4736.0 (0.164, −0.386, 0.908) | $\Omega_c^+ \Lambda$ (4769) |
| | $\frac{1}{2}^-$ | 4984.4 (0.756, −0.542, −0.367) |
| | $\frac{1}{2}^-$ | 5009.4 (0.633, 0.747, 0.203) |
| | $\frac{3}{2}^-$ | 4645.1 (−0.190, −0.021, −0.982) | $\Omega_c^0 \Lambda$ (4565) |
| | $\frac{3}{2}^-$ | 4767.5 (−0.082, −0.996, 0.037) | $\Omega_c^+ \Lambda$ (4769) |
| | $\frac{3}{2}^-$ | 4924.1 (0.978, −0.087, −0.187) |
| $\frac{5}{2}^-$ | 4768.6 (1) | $\Omega_c^+ \Lambda$ (4769) |

TABLE XXI. The eigenvectors for the $sscc$ pentaquark states. The masses are all in unit of MeV.

| $J^P$ | Mass (MeV) | Eigenvector |
|-------|------------|-------------|
| $\frac{1}{2}^-$ | 4894.4 | 0.367 |
| | 5009.4 | 0.203 |
| $\frac{3}{2}^-$ | 4924.1 | −0.187 |

TABLE XXII. The values of $k \cdot |c_i|^2$ for the $sscc$ pentaquark states (in unit of MeV).

| $J^P$ | Mass (MeV) | Value (MeV) |
|-------|------------|-------------|
| $\frac{1}{2}^-$ | 4894.4 | 70.7 |
| | 5009.4 | 30.2 |
| $\frac{3}{2}^-$ | 4924.1 | 4.5 |

TABLE XXIII. The partial width ratios for the hidden charm decays of the $sscc$ pentaquark states. The masses are all in unit of MeV.

| $J^P$ | Mass (MeV) | Value |
|-------|------------|-------|
| $\frac{1}{2}^-$ | 4894.4 | 1 |
| | 5009.4 | 1 |
| $\frac{3}{2}^-$ | 4924.1 | 6.1 |
TABLE XXIV. The partial width ratios for the open charm decays of the $sscc\bar{c}$ pentaquark states. The masses are all in unit of MeV.

| $J^P$ | Mass | $\Omega_s^+D_s^-$ | $\Omega_s^+D_s^-$ | $\Omega_c^+D_s^-$ | $\Omega_c^+D_s^-$ |
|-------|------|------------------|------------------|------------------|------------------|
| $\frac{3}{2}^-$ | 4894.4 | 0.01 | 1 | 0.2 |
| | 5009.4 | 10.2 | 1 | 0.2 |
| $\frac{3}{2}^-$ | 4924.1 | 4.3 | 3.3 | 1 |

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