An application of multicriteria optimization in selection of the two-speed two-carrier planetary gear trains

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Abstract. This paper outlines the optimum selection procedure for a two-speed planetary gear train with four external shafts based on two design and operational criteria: radial dimensions and efficiency. The paper outlines how to quickly determine the structure and important basic parameters of two-speed planetary gear trains that meet predefined transmission requirements. The procedure is followed by a numerical example in which the optimal two-speed planetary gear train is selected, defined by the teeth number of the central gears, modules and transmission ratios. The relevance of the presented results is reflected in the fact that these transmissions offer significant advantages in systems which require speed change under load.

1. Introduction

Compared to conventional gear trains, planetary gear trains (PGTs) offer numerous benefits, and over the past several decades their use has proliferated significantly in a variety of branches in mechanical engineering. Multi-stage planetary gear trains are built by linking one or two shafts between two different PGT stages from single-stage gear trains. A special multi-stage PGT type is a two-speed two-carrier PGT consisting of two coupling shafts and four external shafts. There are many important characteristics of this type of compound gear train, the most notable being the possibility of speed changes under load from the operator or control unit (machine tools, cranes...). The basic part of the paper is the review of possible choice of the transmission for any defined application by using a specially developed computer program. The choice between the computer obtained variants is then made by comparative analysis of the solutions.

Until recently, two-speed planetary change-gears with four external shafts and two coupled shafts were not investigated systematically. In general, some sporadic reviews of selected schemes were carried out in [1-4].

The review of 15 schemes of reversible transmissions of this type with orientation values for transmission ratio and achievable efficiencies is presented in [5]. The characteristics of some schemes are presented in [6] and [7], however the methodology of optimal scheme choice was not included. Symbolic review of structural schemes without detailed analysis is given in [7] and the authors describe the torque method, which can be employed for their analysis. The torque method was developed by Arnaudov as the principal tool of systematic analysis of multi-speed PGTs and the reader is referred to [8], where the method is explained and all source references are cited. The detailed systematic research carried out between 2006 and 2011 laid the foundation of the methodology for the selection of optimal PGT structures which was implemented in a computer program – all possible
schemes of these transmissions and their working regimes are laid out by Troha in [9] and [10]. The choice of the optimal variant of a two-speed PGT intended for use in the driveline of fishing boat propulsion is given in [11].

The main objective of this paper is to demonstrate the capabilities of the computer program DVOBRZ that has been developed for the purpose of analysis and optimization of two-speed PGTs. This is also followed with an example in which the optimal solution determined by design parameters is obtained.

After the introductory section this paper is structured as follows. Section 1 introduces the synthesis methodology for two-speed planetary gear trains. Section 2 presents an analysis of numerical results obtained from the developed computer program, which is also complemented with a brief discussion. Section 3 presents the paper's conclusion.

2. Methodology of synthesis of two-speed planetary gear trains

2.1. Two-carrier planetary gear train structures and labelling method

In cases where two-speed transmissions are required, a mechanism obtained by connecting two simple planetary gear trains of type 2k-h, variant A [5] is one of the best suited design solutions. A symbolic representation of a simple planetary gear train with a Wolf-Amaudov symbol, with the relations of torques acting on the shaft is outlined in [7]. Joining two shafts of one planetary gear train with two shafts of another planetary gear train forms a mechanism with four external shafts in total, Fig. 1.

![Symbolic representation of a compound planetary gear train](image)

Figure 1. Symbolic representation of a compound planetary gear train with four external shafts).

We will refer to such an assembly hereafter as a compound gear train, and we will also refer to the two connected PGTs as gear train I (stage I) and gear train II (stage II).

In research published in [7] it was shown that there are a total of 120 different compound trains of this type, namely: 24 variants with brakes on single external shafts, 24 variants with brakes on the coupling shafts and 72 variants with brakes on both coupling and single shafts. Each of the 120 variants of two-speed planetary gearboxes has specific shifting capabilities.

By placing brakes on two shafts, a braking system is established in which activation of one brake (i.e. prevention of any shaft rotation) shifts the power flow through the planetary gear train, which also causes a change in the transmission ratio.

Each variant has its own features that determine the conversion possibilities. It could be assumed that some variants work either as reducers or as multipliers in both speeds, while other variants work in one speed as reducers and in the other as multipliers. Additionally, when the speed is changed, some variants change the direction of rotation, while others keep the direction of rotation during speed changing. Only the basic transmission ratios (i.e. ideal torque ratios) depend on the transmission ratio of the compound planetary gear train. It is important that some of these transmissions are capable of changing the transmission ratio under load. An example of transmission choice will be presented in the next section for a particular application.

This label of a compound two-speed two-carrier train transmits information unambiguously about the structural scheme (the way two PGTs are connected) and the layout variant (the role of each shaft in the transmission) and is determined from the program DVOBRZ. The label may also contain brake engagement information; for example, S16V2Br2 indicates the S16V2 compound train's operating regime when brake 2 is engaged [9]. An alphanumerical label (S11…S56) is assigned to structural
schemes, which indicates the way in which the shafts of the first and the second PGT are connected, while the term layout variant refers to the particular placement of the input and output shaft and of the brakes. Consequently, any designation assigned to a particular PGT conveys information about both the structural scheme and the layout variant, and represents in essence a combination of the two.

2.2. The principle of synthesis of a two-speed planetary gear train

To ensure that the required transmission ratios \( i_{k1} \) and \( i_{k2} \) are realized, all possible variants that meet the imposed constraints are searched and this creates a set of feasible solutions. Fig. 2 shows a graphical explanation of this principle.

The intervals of ideal torque ratios need to be defined at the beginning as: \( t_{I_{\text{min}}} \ldots t_{I_{\text{max}}} \) and \( t_{II_{\text{min}}} \ldots t_{II_{\text{max}}} \). The intervals of the required transmission ratios should also be known: \( I_1 \) and \( I_2 \).

The graphic representation of the transmission ratio dependence on the ideal torque ratios (thus forming surface areas) is shown in Fig. 2a. Corresponding transmission ratio intervals \( I_1 \) and \( I_2 \) (\( i_{k1} \in I_1 \) and \( i_{k2} \in I_2 \) must be fulfilled) are shown on the vertical axis in figure 2b. Also, an arbitrary two-speed PGT as an example is presented in figure 2.

Figure 2. The principle of synthesis of a two-speed planetary gear train.

For every possible combination of ideal torque ratios, the computer program DVOBRZ, specifically developed for this purpose, calculates the values of the transmission ratio functions for each design structure of a two-speed PGT. The program then checks if the calculated transmission ratios fall within the user-defined \( I_1 \) and \( I_2 \) intervals, and if such torque ratio combinations exist, the program extracts them as possible solutions. The transmission ratio effected by the activation of brake Br1 is marked as \( i_{Br1} \) and the transmission ratio effected by the activation of brake Br2 is marked as \( i_{Br2} \).

After all the obtained solutions are stored, the program is able to compare them according to the defined relevant criteria (e.g. minimal radial dimensions, maximum equivalent efficiency etc.) [9, 11-13].

3. Numerical example and discussion

The procedure is shown on an example problem to better illustrate how this program operates. We shall assume a two-speed PGT needs to be synthesized, subject to the following kinematic, design and operational requirements which present input data:

- the transmission ratio intervals: \( 5 \leq i_{k1} \leq 5.2 \) and \( 3.5 \leq i_{k2} \leq 3.6 \), or \( -5.2 \leq i_{k1} \leq -5 \) and \( -3.6 \leq i_{k2} \leq -3.5 \),
- no reversal of rotation between the two output speeds: sign \( i_{Br1} \) = sign \( i_{Br2} \),
- the number of teeth of the sun gears: \( z_{1I} = z_{1II} = 18 \),
- the number of planet gears in a single gear train: \( k = 3 \),
- torque on the input shaft: \( T_A = 50 \) Nm,
- input number of revolution \( n_A = 500 \) min\(^{-1}\)
- frequency of operation at nominal power: \( \alpha_1 = 0.5 \) and \( \alpha_2 = 0.5 \), respectively for the required transmission ratios,
- material of all gears 16MnCr5,
- the main optimality criterion is minimisation of radial dimensions.
Based on the requirements and assumptions listed above, the DVOBRZ program lists six possible two-speed PGT design structures as acceptable solutions, with the main parameters and kinematic capabilities summarized in Table 1 ($n_A$ – rotational speed of the input shaft, $n_2$ – relative rotational speed of the faster satellites, comparing satellites in two stages). In the next step (also by using the DVOBRZ software) it is possible to determine whether the required transmission ratio is achieved with the activation of brake Br1 or brake Br2. Transmission ratios $i_{Br1}$ can be either upper or lower surface, as well as $i_{Br2}$.

With the obtained data of ideal torque ratios $t_I$ and $t_{II}$, transmission ratios $i_{Br1}$ and $i_{Br2}$, the teeth number of the ring gears $z_{3I}$ and $z_{3II}$, as well as of the pitch diameters of the ring gears $d_{3I}$ and $d_{3II}$, the program then determines the corresponding designation of the PGT and the required order of brake engagements that produces the calculated transmission ratios.

Figure 3 presents the pitch diameter of the larger ring gear with the corresponding equivalent efficiencies and, in a separate diagram, the pitch diameter ratios of each design structure as shown in Table 1. We shall define here the pitch diameter ratio of a two-speed PGT as

$$D_{3,rel} = \max(d_{3I}, d_{3II}) \min(d_{3I}, d_{3II})$$

The value of this parameter by definition always exceeds 1.

The value of the pitch diameter ratio is important as it indicates the housing shape. The housing can be of a cylindrical shape if it is close to one. In other cases, the form of housing must be used gradually.

The equivalent efficiency $\eta_{eq}$ is calculated by using equation (2) as

$$\eta_{eq} = \alpha_{Br1} \eta_{Br1} + \alpha_{Br2} \eta_{Br2}$$

where $\eta_{Br1}$ is the efficiency when brake Br1 is activated, $\eta_{Br2}$ is the efficiency when brake Br2 is activated, and $\alpha_{Br1}$ and $\alpha_{Br2}$ are the frequencies of operation at nominal power for their respective transmission ratios.

The equivalent efficiency can be precisely defined only when the transmission operates at nominal power with both speeds.

By closer examination of figure 3 and table 1, it is clear that the PGT S16V2 is the optimal solution according to the criterion of minimal radial dimensions of planetary gear sets.

On the other hand, when maximization of the equivalent efficiency is taken as the guiding criterion, the optimal solution is the S36V4 PGT.
Since minimal radial dimensions are the main criterion in this example, the optimal solution is S16V2.

By activating brake Br1 the input of the system is the sun gear of the first stage, the output is the sun gear of the second stage. The first stage carrier and the second stage ring gear are motionless, figure 4a.

![Figure 4. Power flow through the transmission](image)

The second stage operates in three-shaft mode, but without any resistance, which means it is idling. Because the carrier is reactive in the first stage, the power is transmitted to the first stage ring gear and then to the second stage sun gear, i.e. the power output B. There is no power branching, so the losses occur in just one PGT stage and only one power sink is present.

The power flow when Br2 is activated and brake Br1 disengaged is presented in figure 4b. The second stage carrier is motionless and the system's input element is the sun gear of the first stage. The power is divided into two components: one component is transmitted to the ring gear and the other to the carrier. The output element is the sun gear of the second stage, where both branches of the power flow finally meet and sum up to the total of the power output. Both planetary stages are activated in this case.

This example clearly outlines the procedure needed in the selection of a kinematic structure designed to operate optimally within a defined mechanical system. In the next step it is necessary to apply multicriteria optimization methods in order to define the PGT with design parameters [11].
Table 1. Possible two-speed PGT design structures as acceptable solutions.

| Designation | Kinematic diagram | $t_I$ | $n_I$ | $z_{III}$ | $z_{III}$ | $i_{Br1}$ | $i_{Br2}$ | $n_{II}/n_A$ | $m_I$ | $m_{II}$ |
|-------------|------------------|------|------|---------|---------|----------|----------|-------------|------|-------|
| S16V2       | ![Diagram](image1) | 3.5  | 3    | 63      | 54      | -3.5     | -5       | 0.8         | 1.75 | 2     |
| S33V6       | ![Diagram](image2) | 2.5  | 4.17 | 45      | 75      | 5.167    | 3.5      | 1.075       | 2    | 1.75  |
| S13V2       | ![Diagram](image3) | 5    | 3.17 | 90      | 57      | -5       | -3.56    | 0.534       | 1.75 | 2     |
| S36V4       | ![Diagram](image4) | 7.83 | 4.17 | 141     | 75      | 5.167    | 3.511    | 0.509       | 1.125 | 1.75  |
| S55V12      | ![Diagram](image5) | 3.5  | 5.17 | 63      | 93      | -3.5     | -5.167   | 0.8         | 1.75 | 1.75  |
| S55V5       | ![Diagram](image6) | 7.67 | 2.5  | 138     | 45      | 3.5      | 5.194    | 1.077       | 1.25 | 2.25  |

4. Conclusions
This paper presents the possibilities of a computer program designed to optimize the choice of two-speed planetary two-carrier gear trains. Optimization is possible only if the desired transmission ratios are defined in both operating regimes, along with the torque and number of input shaft revolutions, as well as some other input data required by the computer program. The final selection of the transmission's kinematic structure and brake layout depends primarily on the main governing criterion...
chosen by the designer among a potentially large number of feasible solutions. A numerical example demonstrates this principle, showing how conflicting criteria yield different solutions as optimality priorities change.

The procedure presented is a vital part of the optimal design of planetary gear transmissions, but other components such as shafts and clutches also need to be studied to fully investigate and optimize the functionality of the entire assembly.

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