Neutrinos and Cosmology: an update

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Abstract. We review the current cosmological status of neutrinos, with particular emphasis on their effects on Big Bang Nucleosynthesis, Large Scale Structure of the universe and Cosmic Microwave Background Radiation measurements.

Keywords: neutrinos, big bang nucleosynthesis, large scale structure of the universe, cosmology

INTEGRATION

The Standard Cosmological Model predicts the existence of a neutrino background (CνB) filling the universe with densities of the order $n_\nu \approx 110$ cm$^{-3}$ per flavor. Neutrino properties are rather difficult to be probed experimentally, due to the weakness of neutrino interactions which, especially at low energies, makes hopeless at present any perspective of direct detection of the CνB. Nevertheless, neutrinos are one of the most abundant relics of the primordial universe and played a key role in different stages of its evolution. Several cosmological observables are then sensitive to neutrinos, and can be used to put bounds on their properties.

Given their extremely low interaction rate, the natural out-of-equilibrium driving force of the expansion of the Universe pushed neutrinos to decouple from the thermal bath very early, when the temperature was $\theta(1 \text{ MeV})$. This temperature is close to the electron mass $m_e$, setting the scale of the electron/positron annihilation, and both are close to the $\theta(0.1 \text{ MeV})$ scale of the synthesis of the light nuclei via thermonuclear fusion. So, Big Bang Nucleosynthesis (BBN) is a privileged laboratory for the CνB studies. In particular, it is sensitive to the $\nu$ (weak) interactions as well as to the shape of the $\nu_e - \bar{\nu}_e$ phase space distributions entering the $n \leftrightarrow p$ inter-conversion rates. Apart from the energy density due to the extra (i.e. non electromagnetic) relativistic degrees of freedom, the BBN tests the dynamical properties of the neutrinos in a thermalized (almost) CP-symmetric medium.

Other cosmological probes are Cosmic Microwave Background (CMB) anisotropies or the Large Scale Structure (LSS) of the universe, which are, however, sensitive only to the CνB gravitational interaction. The role of neutrinos as the dark matter (DM) particles has been widely discussed since the early 1970s. For values of neutrino masses much larger than the present cosmic temperature one finds a contribution in terms of the critical density $\Omega_\nu \approx 0.0108 h^{-2} \sum m_\nu / eV$, $h$ being the Hubble parameter in units of 100 Km s$^{-1}$ Mpc$^{-1}$. Nowadays, we know that neutrinos cannot constitute all the DM ($\Omega_\text{DM} \approx \Omega_m \sim 0.3 \left[ \begin{array}{c} \text{[1]} \end{array} \right]$), and the main question is how large the contribution of neutrinos can be, deducing $\Omega_\nu$ from their contribution to cosmological perturbations. In fact, neutrino background erases the density contrasts on wavelengths smaller than a mass-dependent free-streaming scale. Neutrinos of sub-eV mass behave almost like a relativistic species for CMB considerations and therefore the power spectrum suppression can be seen only in LSS data. Even if neutrino mass influences only slightly the spectrum of CMB anisotropies, it is crucial to combine CMB and LSS observations, because the former give independent constraints on the cosmological parameters, and partially remove the parameter degeneracy that would arise in an analysis of the LSS only.

NEUTRINOS AND COSMOLOGY

At temperatures above $\theta(1 \text{ MeV})$, neutrinos are in thermal equilibrium with the thermal bath and their distribution is a perfect Fermi-Dirac one,

$$f_{\nu a}(y) = \frac{1}{e^{y-\xi a} + 1},$$ (1)

where $y \equiv p/T_\nu$ and $\xi a \equiv \mu_a/T_\nu$ (here $\mu_a$ is the chemical potential of the flavor $\alpha$, which is neglected in the standard scenario). As the temperature goes down, the universe expansion prevents weak interactions from maintaining neutrinos in equilibrium and they decouple. As a first approximation, the neutrino decoupling can be described as an instantaneous process taking place around 2-4 MeV, without any overlap in time with $e^\pm - e^\pm$ annihilation. All flavors would then keep Fermi-
Dirac distributions (both neutrino momenta and temperature redshift identically with the universe expansion), but the neutrino temperature $T_{\nu}$ will not benefit of the entropy release from $e^+ - e^-$ annihilations. The asymptotic ratio $T / T_{\nu}$ for $T \ll m_\nu$ can be evaluated in an analytic way, and turns out to be $(11/4)^{1/3} \approx 1.401$.

More accurate calculations by solving the kinetic equations have been performed, and they show a partial entropy transfer to the neutrino plasma. As a consequence, the neutrino distributions get distorted, since this transfer is more efficient for larger neutrino momenta. In [2, 3] it was shown that with a very good approximation the distortion in the $\alpha$-th flavor can be described as

$$f_{\nu\alpha}(x,y) \simeq \frac{1}{e^y + 1} \left( 1 + \sum_{i=0}^{3} c_i^{\alpha}(x)y^i \right),$$

with $x \equiv m_\nu / T_{\nu}$. The electron neutrinos get a larger entropy transfer than the $\mu$ and $\tau$, since they also interact via charged currents with the $e^\pm$ plasma. The effective ratio $T / T_{\nu} \approx 1.3984$ is slightly lower than the instantaneous decoupling estimate.

The incomplete decoupling of neutrinos also induces a modification in the contribution of neutrinos to the energy density. By fully consistently including order QED corrections to the photon and $e^\pm$ equation of state, in [2] the energy density in the neutrino fluid is found to be enhanced by 0.935% (for $\nu_e$) and 0.390% (for $\nu_\mu$ and $\nu_\tau$). A refined treatment, also including the effects of three-flavor neutrino oscillations, has been recently provided in [2].

The contribution of neutrinos to the total relativistic energy density of the universe is usually parameterized via the “effective number” of neutrinos, $N_{\text{eff}}$.

$$\rho_{\nu} = N_{\text{eff}} \frac{7}{8} \left( \frac{T_{\nu}}{T} \right)^4 \rho_T.$$  (3)

$N_{\text{eff}}$ measures neutrino energy density in “units” of the energy density of massless neutrinos with zero chemical potential, but it can in principle receive a contribution from other (relativistic) relics. For three massless neutrinos with zero chemical potential and in the limit of instantaneous decoupling, $N_{\text{eff}} = 3$. The inclusion of entropy transfer between neutrinos and the thermal bath modifies this number to about 3.04 at the CMB epoch.

Shortly after neutrino decoupling the temperature reaches the value of the neutron-proton mass difference, and weak interactions are no longer fast enough to maintain equilibrium among nucleons: a substantial final neutron fraction survives, however, down to the phase of nucleosynthesis where all neutrons become practically bound in $^4\text{He}$ nuclei. The predicted value of the $^4\text{He}$ mass abundance, $Y_\alpha$, is poorly sensitive to the nuclear network details and has only a weak, logarithmic dependence on

![FIGURE 1. The relative correction to the $n \rightarrow p$ (solid line) and $p \rightarrow n$ (dashed line) total rates, due to the neutrino distortion (see Ref. [1] for details).](image-url)

the baryon fraction of the universe, $\omega_b = \Omega_b h^2$, being fixed essentially by the ratio of neutron to proton number density at the onset of nucleosynthesis. This in turn crucially depends on the weak rates and on the (standard or exotic) neutrino properties.

In several papers (see [5, 6] and references therein) the value of $Y_\alpha$ has been computed by improving the evaluation of the weak rates including electromagnetic radiative corrections, finite mass corrections and weak magnetism effects, as well as the plasma and thermal radiative effects. In particular in [7] it has been also considered the effect of the neutrino spectra distortions and of the process $\gamma + p \rightarrow p + e^+ + n$, which is kinematically forbidden in vacuum, but allowed in the thermal bath. The latter is shown to give a negligible contribution, while neutrino distortions have a significant influence on the rates for different reasons: a) the larger mean energy of $\nu_e$ induces a $\delta T_{\nu_e}$ and (indirectly, through a decrease in $\rho_{\text{c.m.}}$) a $\delta T$; b) the ratio $T / T_{\nu_e}$, which enters the weak rates, is changed. Moreover, the time-temperature relationship is changed and so the time at which BBN starts.

The total effect on the rates are shown in Figure 1. Even though one would expect effects up to $\mathcal{O}(1\%)$, the spectral distortion and the changes in the energy density and $T_{\nu}(T)$ conspire to almost cancel each other, so that $Y_p$ is changed by a sub-leading $\mathcal{O}(0.1\%)$. This effect is of the same order of the predicted uncertainty coming from the error on the measured neutron lifetime, $\tau_n = 885.7 \pm 0.8$ s [8], and has to be included in quoting the theoretical prediction, $Y_p = 0.2481 \pm 0.0004$ (1\,$\sigma$, for $\omega_b = 0.023 \pm 0.001$).

Apart from the uncertainty on $\omega_b$, the $^2\text{H}$, $^3\text{He}$ and $^7\text{Li}$ abundance predictions are mainly affected by the nuclear
reaction uncertainties. An updated and critical review of the nuclear network and a new protocol to perform the nuclear data regression has been widely discussed in \[7\], to which we address for details.

A different scenario, the Degenerate BBN (DBBN) one, has received a new attention in the last few years, especially when the first data on CMB seemed to indicate a tension between the determination of \( \omega_b \) from CMB and standard BBN \[8\]. Such a tension could, in fact, be relaxed assuming that the number of \( \nu_\alpha \) be different, i.e. \( \mu_\alpha \neq 0 \) in Eq.(1). In this degenerate case, changes are expected: a) in the weak rates (a reduction in \( Y_p \)), since a positive \( \xi_e \) enhances \( n \to p \) processes with respect to the inverse ones, and modifies the initial condition on the \( n/p \) ratio at \( T \gg 1 \) MeV; b) in the expansion rate (an increase in \( Y_p \)), since non zero \( \xi_\alpha \)'s contribute to total \( N_{\text{eff}} \) as

\[
N^\text{DBBN}_{\text{eff}} \approx N_{\text{eff}} + \sum_\alpha \left[ \frac{30}{7} \left( \frac{\xi_\alpha}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi_\alpha}{\pi} \right)^4 \right]. (4)
\]

When several neutrino species are degenerate, both the effects might combine and particular values of \( \xi_\alpha \)'s exist for which the predictions of DBBN are still in good agreement with the observational data on the abundances of primordial elements. Notice that BBN is more sensitive to neutrino degeneracy than CMB or LSS, due to the further effect played by the \( v_e - \bar{v}_e \) distributions in the weak rates, while the latter are only sensitive to the extra energy density present in the \( \xi \neq 0 \) case.

Earlier claims of discrepancies between BBN and CMB have been largely overcome by new data, and it was recently realized \[10\] that the flavor oscillations in the primordial plasma induced by (presently determined) mass differences and mixing angles from atmospheric and solar neutrinos almost equalize the three asymmetry parameters \( \xi_i \). Still, exotic models, where both a common relatively large \( \xi \) and a \( N_{\text{eff}} \neq 3.04 \) exist, have been considered (“hidden relativistic degrees of freedom” \[11\] \[12\]), and were shown to be compatible with the data. However, if one sticks to the scenario with \( N_{\text{eff}} \) fixed by standard Physics, introduces no sterile species, and assumes the CMB prior on \( \omega_b \), BBN turns into a powerful "leptometer", constraining the common \( \xi \) to an unprecedented accuracy even under conservative assumptions for \( Y_p \) \[13\]. This provides an indirect consistency check for the sphaleron mechanism at electroweak phase transition, predicting baryon and lepton asymmetries of the same order.

Let us come to neutrino role in structure formation. In general, neutrinos tend to stream freely across gravitational potential wells, and to erase density perturbations. Free-streaming is efficient on a characteristic scale \( \lambda_J \) called the Jeans length, corresponding roughly to the distance on which neutrinos can travel in a Hubble time. For ultra-relativistic neutrinos, \( \lambda_J \) is by definition equal to the Hubble radius \( c/H \), but for non-relativistic ones it is lower than \( c/H \). Neutrinos with masses smaller than approximately 0.3 eV are still relativistic at the time of last scattering, and their direct effect on the CMB perturbations is identical to that of massless neutrinos. In the intermediate mass range from \( 10^{-3} \) eV to 0.3 eV, the transition to the non-relativistic regime takes place during structure formation, and the matter power spectrum will be directly affected in a mass-dependent way. Wavelengths \( \lambda \) smaller than the current value of the neutrino \( \lambda_\nu \) are suppressed by free-streaming. The largest wavelengths, which remain always larger than the neutrino \( \lambda_\nu \), are not affected. Finally, there is a range of intermediate \( \lambda \) which become smaller than the neutrino \( \lambda_\nu \) for some time, and then encompass it again: these scales smoothly interpolate between the two regimes. The net signature in the matter power spectrum \( P(\lambda, t) \) is a damping of all wavelengths smaller than the Hubble scale at the time \( \tau_0 \) of the transition of neutrinos to a non-relativistic regime.

Then, for \( \lambda \ll \lambda_J \), if \( \Omega_\nu \ll \Omega_m \) the suppression is given roughly by the factor

\[
\frac{\Delta P}{P} \approx -8 \frac{\Omega_\nu}{\Omega_m}. \quad (5)
\]

that is by the ratio between neutrino and matter energy densities.

Notice that one can somehow play with both \( N_{\text{eff}} \) and \( \sum m_\nu \) and find models which give excellent fits of the data. In fact, models with massive neutrinos have suppressed power at small scale. Adding relativistic energy further suppresses power at scales smaller than the horizon at matter-radiation equality. For the same matter density such a model would therefore be even more incompatible with data. However, if the matter density is increased together with \( m_\nu \) and \( N_{\text{eff}} \), data can be described very nicely (see, for example Figure 3 in \[14\]).

**COMPARISON WITH DATA**

Still few years ago, the BBN theory together with the observations of the abundances of primordial nuclides were used to determine the baryon fraction of the universe, \( \omega_b \). Nowadays \( \omega_b \approx 0.023 \) is fixed to better than 5% accuracy by detailed CMB anisotropies analysis \[11\], thus leaving the BBN as an over-constrained and very predictive theory. Once \( \omega_b = 0.023 \pm 0.001 \) is plugged into the BBN theory, the prediction for the deuterium, which is the nuclide most sensitive to \( \omega_b \), nicely fits the range of the observed values in high redshift, damped Ly-\( \alpha \) QSO systems \[15\], thus offering a remarkable example of internal consistency of the current cosmological scenario. Moreover, the predictions of other light nuclei
which at least qualitatively agree with the observed values are likely to put constraints on the Galactic chemical evolution ($^3$He) or on the temperature scale calibration or depletion mechanisms in PopII halo stars ($^7$Li).

The determination of $Y_p$ is usually performed by extrapolating to zero metallicity the measurements done in dwarf irregular and blue compact galaxies. The typical statistical errors are of the order of 0.002 (i.e., at the 1% level), but the systematics are such that in the recent re-analysis [16] the authors argue for the conservative range $0.232 \leq Y_p \leq 0.258$, i.e., a 1 $\sigma$ error of $\delta Y_p/5\%$.

In Table 1 the present bounds on the effective number of neutrinos from various analyses are presented, together with the type of data used. The most stringent bounds come from CMB alone (Deuterium+Helium), while CMB and BBN-Deuterium are less effective in constraining $N_{\text{eff}}$. Some differences are due to (slightly) different databases or assumptions.

Table 2 shows the bounds on $\xi$. The interval from Ref. [12] is broader than what in Ref. [11], since in the first case only a prior from CMB instead of all data is used in the analysis. The third line shows the bound obtained in [13] assuming only standard physics, while the previous two bounds assume no prior on $N_{\text{eff}}$.

Finally, Table 3 shows, with the same notations, the upper bound on the neutrino mass. As can be gauged from this table, a fairly robust bound on the sum of neutrino masses is at present somewhere around 1 eV, depending on the specific priors and data sets used.

In conclusion, the complementarity among different fields of cosmology (CMB, BBN, LSS) can be used to test the role of neutrinos from very early epochs (redshift $z \approx 10^{10}$) down to relatively recent history ($z \approx$ a few).

### Table 1. Bounds on $N_{\text{eff}}$ (2 $\sigma$) from different analyses

| Ref. | Bound on $N_{\text{eff}}$ | Data used |
|------|--------------------------|-----------|
| [11] | $1.8 \leq N_{\text{eff}} \leq 3.7$ | CMB, BBN |
| [12] | $1.3 \leq N_{\text{eff}} \leq 6.1$ | CMB, BBN(D) |
| [13] | $1.6 \leq N_{\text{eff}} \leq 3.6$ | BBN(D+$Y_p$) |
| [14] | $1.4 \leq N_{\text{eff}} \leq 6.8$ | CMB, LSS, HST |
| [15] | $1.9(2.3) \leq N_{\text{eff}} \leq 7.0(3.0)$ | CMB, LSS, (+BBN) |
| [16] | $1.7 \leq N_{\text{eff}} \leq 3.0$ | CMB, BBN |
| [17] | $N_{\text{eff}} \leq 4.6$ | CMB, BBN |
| [18] | $1.90 \leq N_{\text{eff}} \leq 6.62$ | CMB, LSS, HST |

### Table 2. Bounds on $\xi$ (2 $\sigma$) from different analyses

| Ref. | Bound on $\xi$ | Data used |
|------|----------------|-----------|
| [11] | $-0.10 \leq \xi \leq 0.25$ | CMB, BBN |
| [12] | $-0.13 \leq \xi \leq 0.31$ | BBN($D+Y_p$)+prior on $\omega_b$ |
| [13] | $-0.05 \leq \xi \leq 0.07$ | BBN($Y_p$)+prior on $\omega_b$, $N_{\text{eff}}$ |

### Table 3. Bounds on $\sum m_\nu$ (2 $\sigma$) from different analyses

| Ref. | Bound on $\sum m_\nu$ | Data used |
|------|------------------------|-----------|
| [11] | $\leq 0.69$ | CMB, LSS, Ly$\alpha$ |
| [12] | $\leq 1.01$ | CMB, LSS, HST, SNla |
| [13] | $\leq 0.65$ | CMB, LSS, HST, Ly$\alpha$ |
| [14] | $0.56^{+0.30}_{-0.26}$ | CMB, LSS, $f_{\text{gas}}$, XLF |
| [15] | $\leq 1.7$ | CMB, LSS |
| [16] | $\leq 0.75$ | CMB, LSS, HST |
| [17] | $\leq 1.0(0.6)$ | CMB, LSS (+HST, SNla) |
| [18] | $\leq 0.47$ | CMB, LSS, HST, SNla, Ly$\alpha$ |

New nuclear rate measurements and a better understanding of possible systematics affecting primordial abundance determination (BBN) and more data together with an increased precision (CMB and LSS) will give us the opportunity to constrain standard neutrino properties as well as to test new physics in the neutrino sector, gaining at the same time a deeper insight on the physics of the early universe.

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