Simplified equations for gravitational field in the vector theory of gravity and new insights into dark energy

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(Dated: April 4, 2019)

Recently, a new alternative vector theory of gravity has been proposed which assumes that universe has fixed background Euclidean geometry and gravity is a vector field that alters this geometry [Phys. Scr. 92, 125001 (2017)]. It has been shown that vector gravity passes all available gravitational tests and yields, with no free parameters, the value of cosmological constant in agreement with observations. Here we obtain substantially simplified gravitational field equations of vector gravity which are more suitable for analytical and numerical analyses. We also provide a detailed explanation why in vector gravity in the reference frame of observer that takes a snapshot of the universe at time \( t_0 \) the ratio of the cosmological constant to the critical density is equal to \( 2/3 \) at \( t = t_0 \). We also show that dark energy does not affect universe evolution in the co-evolving reference frame. Thus, in reality, universe is expanding at a continually decelerating rate, with expansion asymptotically approaching zero.

I. INTRODUCTION

Recently, a new alternative vector theory of gravity has been proposed [1]. The theory assumes that gravity is a vector field in fixed four-dimensional Euclidean space \( \delta_{ik} \) which breaks the original Euclidean symmetry of the universe and alters the space geometry. The direction of the vector gravitational field gives the time coordinate, while perpendicular directions are spatial coordinates.

Conceptually, both vector gravity and Einstein’s general relativity deal with the space-time geometry. However, in general relativity the space-time geometry itself is the (dynamic) gravitational field. The concept of vector gravity is that the fixed background geometry is altered by the vector field. Similar concept applies to any metric theory of gravity with a prior geometry [2].

Despite fundamental differences from general relativity, it has been shown that vector gravity passes all available gravitational tests [1]. In particular, vector gravity and general relativity are equivalent in the post-Newtonian limit and, thus, they both pass solar-system tests of gravity. The two theories also give the same quadrupole formula for the rate of energy loss by orbiting binary stars due to emission of gravitational waves.

In strong fields, vector gravity deviates substantially from general relativity and yields no black holes. Since the theory predicts no event horizons, the end point of a gravitational collapse is not a point singularity but rather a stable star with a reduced mass. In vector gravity, neutron stars can have substantially larger masses than in general relativity and gravitational wave detection events can be interpreted as produced by the inspiral of two neutron stars rather than black holes [1]. Vector gravity predicts that the upper mass limit for a nonrotating neutron star with a realistic equation of state is of the order of 35 \( M_\odot \) (see Sec. 13 in [1]). Stellar rotation can increase this limit to values in the range of 50 \( M_\odot \). The predicted limit is consistent with masses of compact objects discovered in X-ray binaries [3] and those obtained from gravitational wave detections [4].

Vector gravity also predicts that compact objects with masses greater than \( 10^5 M_\odot \) found in galactic centers have a non-baryonic origin and made of dark matter. It is interesting to note that their properties can be explained quantitatively in the framework of vector gravity assuming they are made of dark matter axions and the axion mass is about 0.6 meV (see Sec. 15 in [1] and Ref. [5]). Namely, supermassive objects at galactic centers are axion bubbles. The bubble mass is concentrated in a thin surface - the interface between two degenerate vacuum states of the axion field. If the bubble radius is large, surface tension tends to contract the bubble. When the radius is small, vector gravity effectively produces a large repulsive potential which forces the bubble to expand. As a result, the bubble radius oscillates between two turning points. The radius of the \( 4 \times 10^6 M_\odot \) axion bubble at the center of the Milky Way oscillates with a period of 20 mins between 1 \( R_\odot \) and 1 astronomical unit [2].

This prediction has important implication for capturing the first image of the supermassive object at the center of the Milky Way with the Event Horizon Telescope (EHT) [6]. Namely, the oscillating axion bubble produces shadow by bending light rays from the background sources only during short time intervals when bubble size is smaller or of the order of the gravitational radius \( r_g = 2GM/c^2 = 17 R_\odot \). Since typical EHT image collection time is several hours the time averaging yields a much fainter image than that expected from a static black hole in general relativity and, hence, in the time-averaged image the shadow should be almost invisible.

One should mention that first image of the Milky Way center with ALMA at 3.5 mm wavelength has been reported recently (see Fig. 5 in [7]). The resolution of the detection is only slightly greater than the size of the black hole shadow. Still, decrease in the intensity of light toward the center should be visible if the Galactic center harbors a black hole. But the image gets brighter (not dimmer) closer to the center. This agrees with the prediction of vector gravity about lack of black holes.

On the other hand, a much heavier axion bubble in
M87 galaxy ($M = 4 \times 10^9 M_\odot$) does not expand substantially during oscillations and its size remains of the order of $r_g$. As a consequence, the axion bubble in M87 produces a bubble-like shadow that is comparable to that of a static black hole. Due to bubble oscillations, the bubble shadow in M87 should vary on a timescale of a few days.

Vector gravity also provides an explanation of the dark energy as the energy of longitudinal gravitational field induced by the expansion of the universe and yields, with no free parameters, the value of $\Omega_m$ which agrees with the results of Planck collaboration [1] and recent results of the Dark Energy Survey. Moreover, recent gravitational wave polarization analysis of GW170817 [10] showed that data are compatible with vector gravity but not with general relativity [11, 12].

Similarly to general relativity, vector gravity postulates that the gravitational field is coupled to matter through a metric tensor $f_{ik}$ which is, however, not an independent variable but rather a functional of the vector gravitational field. In particular, action for a point particle with mass $m$ moving in the gravitational field reads

$$S_{\text{matter}} = -mc \int \sqrt{f_{ik}dx^idx^k}, \quad (1)$$

where $c$ is the speed of light. Action (1) has the same form as in general relativity, however, the tensor gravitational field $g_{ik}$ of general relativity is now replaced with the equivalent metric $f_{ik}$ ($f_{ik}$ is a tensor under general coordinate transformations).

It is convenient to represent the vector gravitational field in terms of a unit vector $u_k$ and a scalar $\phi$ related to the field absolute value. Then in the Cartesian coordinate system of the background Euclidean space, the equivalent metric reads [3]

$$f_{ik} = -e^{-2\phi}\delta_{ik} + 2 \cosh(2\phi)u_iu_k, \quad (2)$$

while metric $\tilde{f}_{ik}$ inverse to $f_{ik}$, defined as $\tilde{f}^{ik}f_{im} = \delta^i_m$, is

$$\tilde{f}^{ik} = -e^{2\phi}\delta^{ik} + 2 \cosh(2\phi)u^iu^k, \quad (3)$$

where $\delta_{ik} = \text{diag}(1,1,1,1)$,

$$u^i = \delta^{ik}u_k, \quad u_ku^k = 1,$

and $i, k = 0, 1, 2, 3$.

The total action for the gravitational field and matter is given by

$$S = S_{\text{gravity}} + S_{\text{matter}}, \quad (4)$$

where $S_{\text{matter}}$ is the action of matter written in curvilinear coordinates with the metric $f_{ik}$. The action for the gravitational field $S_{\text{gravity}}$ is obtained from the requirement that symmetries of $S_{\text{matter}}$ and $S_{\text{gravity}}$ are the same. This requirement yields a unique answer for $S_{\text{gravity}}$ [1]

$$S_{\text{gravity}} = \frac{c^3}{8\pi G} \int dx \left[ \frac{\partial \phi}{\partial x^i} \frac{\partial \phi}{\partial x^i} \right] = \frac{c^3}{8\pi G} \int dx \left[ \delta^{ik} + (1 - 3e^{-4\phi}) u^iu^k \right] + \cosh^2(2\phi) \frac{\partial u_i}{\partial x^k} \frac{\partial u_m}{\partial x^l} \left( \delta^{im} \delta^{kl} - \delta^i \delta^{km} \right) \left( 1 + e^{-4\phi} \right) \partial \phi \frac{\partial u_m}{\partial x^k} \frac{\partial u_m}{\partial x^k} u_l, \quad (5)$$

where $G$ is the gravitational constant.

Variation of the total action (4) with respect to $\phi$ and the unit vector $u_k$ gives equations for the gravitational field (see Appendix A). Field equations (A1) and the action (4) are not generally covariant. However, they are invariant under coordinate transformations that leave the background Euclidean metric $\delta_{ik}$ intact. These transformations, in particular, include rotations of the form

$$x^0 \rightarrow x^0 + \frac{V}{\sqrt{1 + V^2/c^2}} x^1, \quad x^1 \rightarrow x^1 - \frac{V}{\sqrt{1 + V^2/c^2}} x^0, \quad (6)$$

which are analogous to the Lorentz transformations.

In vector gravity, motion of particles in gravitational field is described by the same equation as in general relativity

$$\frac{d^2 x^b}{ds^2} = \frac{1}{2} \tilde{f}^{bl} \left[ \frac{\partial f_{ik}}{\partial x^l} - \frac{\partial f_{ik}}{\partial x^l} - \frac{\partial f_{ik}}{\partial x^k} + \frac{\partial f_{ik}}{\partial x^k} \right] \frac{dx^i}{ds} \frac{dx^k}{ds}. \quad (7)$$

where $ds = \sqrt{f_{ik}dx^idx^k}$. In Eq. (7) the metric $g_{ik}$ of general relativity is replaced with the equivalent metric $f_{ik}$ and particles move along geodesics of $f_{ik}$.

One should emphasize that vector gravitational field lives in the four-dimensional Euclidean manifold and raising and lowering of indexes in the gravitational field action (5) and equations (A1) is carried out using the Euclidean metric $\delta_{ik} = \text{diag}(1,1,1,1)$. However, all non-gravitational fields and matter sense the equivalent metric $f_{ik}$, that is geometry is effectively altered by the vector gravitational field. This is why $f_{ik}$, rather than $\delta_{ik}$, appears in the equation of motion (7). The equivalent metric $f_{ik}$ constitutes a manifold describing interaction with the gravitational field. To avoid confusion between the two manifolds, we use tilde to denote quantities obtained by raising of indexes using $f_{ik}$. For example, $\tilde{f}^{ik}$ stands for the metric inverse to $f_{ik}$.

According to vector gravity, transition between Euclidean geometry of the equivalent metric and geometry of Minkowski signature occurred at the moment of Big Bang. Before the Big Bang, the vector gravitational field had no preferred direction and was undergoing quantum fluctuations. The local geometry in the pre-Big Bang era has Minkowski character and local direction of the vector field determines the time-like dimension and the equivalent metric (2). As shown in [3], the longitudinal component of the vector gravitational field is not quantized and remains classical. Hence, gravitational field does not undergo quantum fluctuations along the field direction and time is a classical object. As a result, evolution of a quantum system in time is well-defined in vector gravity.
Namely, the time derivative in the Heisenberg equation of motion has a meaning of the derivative along the direction of the vector gravitational field.

Local Minkowski geometry allows for the field fluctuations which occur on a Planck scale. Averaging over a small four-dimensional volume with size much larger than Planck length and assuming that fluctuations are isotropic yields

\[ \langle u_k \rangle = 0, \quad \langle u_i u_k \rangle = \frac{1}{4} \delta_{ik}, \]

\[ \langle f_{ik} \rangle = \frac{e^{-2\phi}}{4} \left( e^{4\phi} - 3 \right) \delta_{ik}. \quad (8) \]

That is before the Big Bang the equivalent metric has Euclidean character on “macroscopic” scales much larger than the Planck length.

Big Bang is the point of quantum phase transition at which the gravitational field vector acquires nonzero expectation value on the “macroscopic” scales \( \langle u_k \rangle \neq 0 \). This expectation value serves as a transition order parameter. We choose coordinate axis \( \bar{x}_0 \) along the direction of \( \langle u_k \rangle \). In the disordered phase of universe the spatial average of \( u_0^2 \) is

\[ \langle u_0^2 \rangle = \frac{1}{4}. \]

Deviation of \( \langle u_0^2 \rangle \) from 1/4 by fluctuations can result in the signature flip of \( \langle f_{ik} \rangle \). Amplitude of the fluctuation which produces the signature flip depends on the local value of \( \phi \). Namely, spatial averaging of Eq. (2) yields that if \( e^{4\phi} < 3 \) the signature flip (phase transition) occurs if

\[ \langle u_0^2 \rangle = \frac{1}{1 + e^{4\phi}} > \frac{1}{4}. \]

At this point \( \langle f_{00} \rangle \) changes sign from negative to positive. For \( e^{4\phi} > 3 \) the signature flip occurs at

\[ \langle u_0^2 \rangle = 1 - \frac{3}{1 + e^{4\phi}} > \frac{1}{4}. \]

At this point \( \langle f_{aa} \rangle \) (\( a = 1, 2, 3 \)) change sign from positive to negative.

According to Eq. (8), for

\[ e^{4\phi} = 3 \]

the average equivalent metric vanishes before the Big Bang \( \langle f_{ik} \rangle = 0 \) and the signature flip occurs when \( \langle u_0^2 \rangle \) only slightly deviates from the value in the disordered phase of universe

\[ \langle u_0^2 \rangle = \frac{1}{4} + \Delta, \]

where \( \Delta \) is a small positive number. This deviation creates a nonzero average equivalent metric with Minkowski character

\[ \langle f_{ik} \rangle = \frac{2\Delta}{\sqrt{3}} \text{diag} \left( 1, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3} \right). \quad (9) \]

In the pre-Big Bang era \( \phi \) is inhomogeneous in the four-dimensional space. Big Bangs occur at points where \( e^{4\phi} = 3 \). At such points a small ordering of \( u_k \) caused by fluctuations produces average equivalent metric with Minkowski character yielding instability toward generation of matter and gravitational waves and onset of the inflation stage [1]. Our universe began from one of such points. Subsequent universe expansion resulted in exponentially large deviation of \( \phi \) from the initial value such that shortly after Big Bang \( e^{-\phi} \gg 1 \). If we disregard exponentially small terms \( e^{\phi} \) compared to the exponentially large terms of the order of \( e^{-\phi} \) then gravitational field is no longer “absolute”. Namely, shift of \( \phi \) by a constant is equivalent to rescaling of coordinates.

In vector gravity the gravitational field is not coupled to itself through the equivalent metric and “feels” the background geometry. This is a typical feature of metric theories of gravity with a prior geometry [2]. As a consequence, equations for the vector gravitational field contain the background geometry \( \delta_{ik} \); while equations of matter motion contain only the equivalent metric \( f_{ik} \). Despite this, in vector gravity gravitational waves travel with the speed of light [1]. This is usually not the case in other alternative theories of gravity. After the multimessenger detection of the GW170817 coalescence of neutron stars [10], where light and gravitational waves were measured to travel at the same speed with an error of \( 10^{-15} \), many alternative theories of gravity were excluded [13].

In 1965 Weinberg found within a perturbative dynamical framework that Maxwell’s theory of electromagnetism and Einstein’s theory of gravity are essentially the unique Lorentz invariant theories of massless particles with spin 1 and 2 respectively [14]. In vector gravity, the graviton is a spin–1 massless particle. It was shown in [1] that quantization of linearized equations of vector gravity yields a theory equivalent to QED, which agrees with the Weinberg’s findings.

One should also mention that the Weinberg-Witten theorem [15] stating that massless particles with spin \( j > 1/2 \) can not carry a Lorentz-covariant current, while massless particles with spin \( j > 1 \) cannot carry a Lorentz-covariant stress-energy does not apply to vector gravity. The reason is the same why Weinberg-Witten theorem does not forbid photons, namely, both spin–1 photons in QED and spin–1 gravitons in vector gravity carry no conserved charge.

It is remarkable that field equations of vector gravity [11] can be solved analytically for arbitrary static mass distribution [1]. Namely, if point masses are located at \( r_1, r_2, \ldots, r_N \) then exact solution of the field equations for the equivalent metric is

\[ f_{ik} = \begin{pmatrix} e^{2\phi} & 0 & 0 & 0 \\ 0 & -e^{-2\phi} & 0 & 0 \\ 0 & 0 & -e^{-2\phi} & 0 \\ 0 & 0 & 0 & -e^{-2\phi} \end{pmatrix}, \quad (10) \]
where
\[ \phi(r) = -\frac{m_1}{|r - r_1|} - \cdots - \frac{m_N}{|r - r_N|} \] (11)
and \( m_l \) \((l = 1, \ldots, N)\) are constants determined by the value of masses. Solution (10) shows lack of black holes in vector gravity.

However, in a general case, field equations (11) are complicated. The main purpose of the present paper is to simplify gravitational field equations (11) and make them more suitable for analytical and numerical analyses. In addition, section IV of this paper provides new insights into dark energy.

II. SIMPLIFIED EQUATIONS FOR GRAVITATIONAL FIELD

Due to expansion of the universe the spatial scale \( a = e^{-\phi} \) has been magnified in an exponentially large factor. Thus, shortly after Big Bang \( e^{-\phi} \) became an exponentially large number \((e^{-\phi} \gg 1)\). Therefore, one can disregard exponentially small terms \( e^\phi \) compared to the exponentially large terms of the order of \( e^{-\phi} \).

As a result of cosmological expansion the gravitational field became approximately uniform in the entire universe. We denote the coordinate along the average direction of the gravitational field as \( x^0 = ct \). It determines the cosmological reference frame. With the exponential accuracy one can take \( u_0 \approx 1 \), while \( u^\alpha \) is of the order of \( e^{2\phi} \).

Taking into account that \( e^{-\phi} \gg 1 \) the gravitational field action (5) in the cosmological reference frame reduces to

\[ S_{\text{gravity}} = \frac{c^3}{16\pi G} \int d^4x \left[ -\frac{1}{2} R + \frac{1}{4} (\nabla \phi)^2 \right] \]

\[ -3e^{-4\phi} (D_t \phi)^2 + 2e^{-4\phi} \nabla \phi \cdot D_t u - e^{-8\phi} (D_t u)^2 \] (16)

where \( D_t = \frac{\partial}{\partial t} + (u \cdot \nabla) \) (17)
is the local time derivative. Recall that in vector gravity the time coordinate is given by the direction of the four-vector \( u^k \) which in the present approximation reduces to \( u^k = (1, u) \). The local time derivative (17) is the derivative along the four-vector \( u^k \), namely, \( D_t = u^k \partial / \partial x^k \). On the other hand, in the present approximation, derivatives in the directions perpendicular to \( u^k \) reduce to combinations of \( \partial / \partial x^\alpha \) (\( \alpha = 1, 2, 3 \)).

Action (10) is invariant under transformations
\[ \phi \rightarrow \phi + \phi_0, \quad r \rightarrow e^{\phi_0}r, \quad t \rightarrow e^{-\phi_0}t, \quad u \rightarrow e^{2\phi_0}u, \] (18)
where \( \phi_0 \) is an arbitrary constant.

One can show that action (16), up to irrelevant surface term, can be written as

\[ S_{\text{gravity}} = -\frac{c^3}{16\pi G} \int d^4x \left[ \sqrt{-f} R + \frac{1}{2} e^{-8\phi} (D_t u)^2 \right] \] (19)

where \( R \) is the Ricci scalar calculated from the equivalent metric \( f_{ik} \), \( f = \det(f_{ik}) \) and \( \sqrt{-f} = e^{-2\phi} \). The first term in Eq. (19) is the Einstein-Hilbert action of general relativity in which GR metric \( g_{ik} \) is replaced with the equivalent metric \( f_{ik} \).

Variation of the total action (11), where \( S_{\text{gravity}} \) is given by Eq. (16), with respect to \( \phi \) and \( u \) yields equations for the gravitational field. Variation of \( S_{\text{matter}} \) can be calculated using formula (16)

\[ \delta S_{\text{matter}} = -\frac{1}{2c} \int d^4x \sqrt{-f} T^{ik} \delta f_{ik}, \]
where $T^{ik}$ is the energy-momentum tensor of matter. Variation of the action \( \mathcal{A}(\phi, u) \) yields the following equations for the gravitational field \( (\phi, u) \) in the cosmological reference frame

\[ \Delta \phi + 3 e^{-4\phi} [D_t + \text{div} \ u - 2D_t \phi] D_t \phi - e^{-4\phi} \text{div}(D_t u) \]

\[ -\frac{1}{2} e^{-4\phi} \text{curl}^2 u + e^{-8\phi} (D_t u)^2 = \frac{8\pi G}{c^4} \left(T^{00} - \frac{e^{-2\phi}}{2} T \right), \tag{20} \]

\[ \nabla \text{div} u - \Delta u + 4(\nabla \phi \cdot \nabla)u + 4 [D_t \phi - \text{div} \ u - D_t] \nabla \phi \]

\[ -e^{-4\phi} \left[(8D_t \phi - \text{div} \ u - D_t) D_t u + \nabla u^\beta \cdot D_t u_{\beta} \right] = \frac{16\pi G}{c^4} j, \tag{21} \]

where $T = T^{mk} f_{mk}$ is the trace of the energy-momentum tensor and

\[ j^\alpha = T^{00} - u^\alpha T^{00}. \]

Equations (20) and (21) are invariant under transformations (18) which can be used to eliminate the cosmological background $\phi_{\text{com}}$. One can also obtain Eqs. (20) and (21) directly from the gravitational field equations (11) in the limit $e^{-\phi} \gg 1$. Namely, equation with $i = 0$ yields Eq. (20). To obtain Eq. (21) one should take equation for $i = \alpha$ and subtract equation for $i = 0$ multiplied by $u^\alpha$.

Equations (20) and (21) for the scalar $\phi$ and the three dimensional vector $u$ are the main equations of the vector theory of gravity. They determine the equivalent metric which in the cosmological Cartesian coordinates reads

\[ f_{ik} = \begin{pmatrix}
  e^{2\phi} - e^{-2\phi} u^2 & e^{-2\phi} u x & e^{-2\phi} u y & e^{-2\phi} u z \\
  -e^{-2\phi} u x & -e^{2\phi} & 0 & 0 \\
  e^{-2\phi} u y & 0 & -e^{-2\phi} & 0 \\
  e^{-2\phi} u z & 0 & 0 & -e^{-2\phi} 
\end{pmatrix}. \]

The inverse metric is given by

\[ f^{00} = e^{-2\phi}, \quad f^{0\alpha} = e^{-2\phi} u^\alpha, \quad f^{\alpha\beta} = e^{-2\phi} u^\alpha u^\beta - e^{-2\phi} \delta^{\alpha\beta}. \]

Equation of motion of massive particles in gravitational field can be obtained from Eq. (7) or directly from Lagrange’s equation $\frac{d}{dt} \frac{\partial L}{\partial \dot{\phi}} = \frac{\partial L}{\partial \phi}$ which yields

\[ \frac{dp}{dt} = -mc^2 \left( e^{2\phi} + e^{-2\phi} \left( \frac{\nabla \phi}{c} - u \right)^2 \right) \nabla \phi - cp^\beta u_{\beta}, \tag{22} \]

where $p$ is the particle generalized momentum and $d/dt = \partial/\partial t + \nabla \cdot \nabla$ is the total time derivative.

One should remember, however, that Eqs. (20) and (21) do not describe phenomena related to gravitational waves. Equations for the radiative part of the gravitational field (which is quantized) depend on the vacuum state of the field. Recall that vector gravity assumes that quantum of gravitational field (graviton) is a composite particle assembled from fermion-antifermion pairs. If fermion states are empty (moment of the Big Bang) then radiative part of the field is described by Eqs. (20) and (21). However, this vacuum is unstable toward generation of matter and filling the fermion states. Such instability is the mechanism of matter generation at the Big Bang. Shortly after the Big Bang the fermion states become filled and matter generation comes to an end. For the filled vacuum emission of a gravitational wave corresponds to creation of fermion-antifermion hole pairs out of the filled fermion states. In this case the radiative part of the field is described by Eq. (21) with the opposite sign of $j$. Namely, linearized equation for the transverse radiative field (div $u = 0$) far from the sources reads

\[ \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \right) u = -\frac{16\pi G}{c^4} j_{\text{tr}}, \tag{23} \]

where $j_{\text{tr}}$ is the transverse part of $j$. For the filled vacuum the energy of gravitational waves is positive.

### III. ENERGY DENSITY FOR GRAVITATIONAL FIELD

If action of the system has the form

\[ S = \frac{1}{c} \int d^4 x L \left( A_\alpha, \frac{\partial A_\alpha}{\partial x^\alpha} \right), \]

where the Lagrangian density $L$ is some function of the quantities $A_\alpha$, describing the state of the system, and of their first derivatives, then energy density of the system $w$ can be calculated using formula

\[ w = \sum_l \frac{\partial A_l}{\partial x^0} \frac{\partial L}{\partial \frac{\partial A_l}{\partial x^0}} - L. \tag{24} \]

Equation (16) yields

\[ L_{\text{gravity}} = \frac{c^4}{8\pi G} \left[ -\left( \nabla \phi \right)^2 + \frac{e^{-4\phi}}{4} \text{curl}^2 u \right. \]

\[ \left. -3e^{-4\phi} (D_t \phi)^2 + 2e^{-4\phi} \nabla \phi \cdot D_t u - \frac{e^{-8\phi}}{4} (D_t u)^2 \right]. \]

Using Eq. (21), we obtain the following expression for the energy density of gravitational field

\[ w_{\text{field}} = \frac{c^4}{8\pi G} \left[ (\nabla \phi)^2 - \frac{3e^{-4\phi}}{c^2} (\partial_t \phi)^2 \right. \]

\[ \left. -e^{-4\phi} \frac{1}{4c^2} \text{curl}^2 u - \frac{e^{-8\phi}}{4c^2} (\partial_t u)^2 - 2e^{-4\phi} \nabla \phi \cdot (u \cdot \nabla) u \right]. \]
\[ +3e^{-4\phi}(u \cdot \nabla \phi)^2 + \frac{e^{-8\phi}}{4}((u \cdot \nabla)u)^2 \]  \hspace{1cm} (25)

This expression is valid for the vacuum of empty fermion states. For such vacuum the energy of gravitational waves is negative which yields instability toward generation of matter with positive energy and gravitational waves with negative energy. As soon as fermion states are filled the gravitational wave energy becomes positive. In particular, for filled vacuum the energy density for a weak transverse gravitational wave is \[^{[1]}\]

\[
w_{\text{tr}} = \frac{e^4}{32\pi G} \left[ \frac{1}{c^2} \left( \frac{\partial u}{\partial t} \right)^2 + \text{curl}^2 u \right].
\]

For matter of density \( \rho \) moving with velocity \( V \) the Lagrangian density

\[ L_{\text{matter}} = -\rho c^2 \sqrt{e^{2\phi} - e^{-2\phi} \left( \frac{V}{c} - u \right)^2} \]
gives the following expression for the matter energy density

\[ w_{\text{matter}} = \rho c^2 e^{2\phi} + \frac{1}{2} \rho c^2 e^{-3\phi} \left( \frac{V^2}{c^2} - u^2 \right). \hspace{1cm} (26)\]

Equation (26) yields that up to terms quadratic in \( u \) and \( V \)

\[ w_{\text{matter}} = \rho c^2 e^{4\phi} + \frac{1}{2} \rho c^2 e^{-3\phi} \left( \frac{V^2}{c^2} - u^2 \right). \]

IV. ON THE NATURE OF DARK ENERGY

According to vector gravity, dark energy is the energy of longitudinal gravitational field induced by the expansion of the universe \[^{[1]}\]. Universe expansion generates matter current which causes small deviations of the four-vector gravitational field \( A^k \) from the average cosmological direction. These deviations yield nonzero cosmological constant \( \Lambda \) in the universe evolution equation if the time axis is fixed by the direction of \( A^k \) at a certain moment \( t_0 \).

As shown in \[^{[1]}\], in vector gravity the equivalent metric

\[ f_{ik} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{pmatrix} \hspace{1cm} (27)\]

obeys evolution equation of the standard FLRW cosmology with the cosmological term \( \Lambda \). For example, for cold universe equation for the scaling factor \( a(t) \) in the metric \[^{[2]}\] reads

\[-\frac{d^2}{dt^2}a^2(t) = \frac{16\pi G}{3} \left( \frac{\rho}{2a^3(t)} - \Lambda \right), \hspace{1cm} (28)\]

where \( \rho \) and \( \Lambda \) are independent of time (\( \Lambda \) is usually called \( \rho_{\Lambda} \), the equivalent density). The constant \( \rho \) has a meaning of matter density for \( a = 1 \). Eq. (25) describes evolution of the spatially averaged metric which is uniform and isotropic (depends only on time).

In vector gravity, as well as in general relativity, the cosmological term is introduced into the action for the averaged metric using symmetry arguments. Namely, symmetry arguments yield that spatial averaging can give an effective gravitational field action with an additional cosmological term of the form

\[ S_{\text{cosm}} = -c \Lambda \int d^4x \sqrt{-f}, \hspace{1cm} (29)\]

where \( \Lambda \) is a constant independent of the gravitational field and \( f = \det(f_{ik}) \). The cosmological term \[^{[2]}\] appears in vector gravity due to replacement of the exact inhomogeneous equations with the equations for the averaged metric which is spatially uniform and isotropic. The value of \( \Lambda \) can be obtained by matching the effective nonlinear evolution equation \[^{[28]}\] with the exact linearized inhomogeneous equations which do not contain \( \Lambda \). Since gravitational field equations in vector gravity are not generally covariant, the value of the cosmological constant \( \Lambda \) depends on a coordinate system.

In \[^{[1]}\] we obtained \( \Lambda \) in the reference frame of an observer on Earth that takes a snapshot of the universe from a fixed point \( O \) of the four-dimensional space at time \( t_0 \). In such frame, \( t \) is a coordinate of the Cartesian coordinate system in which the background Euclidean metric is equal to \( \delta_{ik} \) and the direction of the time axis is determined by the location of the observer at time \( t_0 \). Namely, the observer interprets the time axis as the instantaneous direction of the gravitational field four-vector \( A^k \) at the point \( O \). Thus, location of the observer at time \( t_0 \) fixes the time coordinate in the entire four-dimensional space and from the perspective of the observer the universe evolves along this time coordinate according to Eq. (28) with nonzero \( \Lambda \).

It is shown in \[^{[1]}\] (by averaging the exact linearized inhomogeneous equations without cosmological term) that in such coordinate system in the vicinity of the observational point \( O \) the scaling factor for the cold universe satisfies condition

\[ \frac{d^2}{dt^2}a^2(t_0) = \frac{8\pi G \rho}{a^3(t_0)} \hspace{1cm} (30)\]

which yields the following value of the cosmological constant

\[ \Lambda = \frac{2\rho}{a^3(t_0)}, \hspace{1cm} (31)\]

where \( a(t_0) \) is the value of the scaling factor at the space-time position of the observer \( O \).

Thus, vector gravity predicts that

\[ \frac{\Lambda}{\rho_{\text{critical}}(t_0)} = \frac{2}{3} \hspace{1cm} (32)\]
where \( \rho_{\text{critical}}(t_0) = \rho/a^3(t_0) + \Lambda \) is the critical density of the cold universe at the moment \( t_0 \) when the observer takes a snapshot of the universe. Here we shed more light on this result.

In vector gravity, gravitational field is a four-vector \( A^k \) in a fixed background four-dimensional Euclidean space \( x^i \) \((i, k = 0, 1, 2, 3)\). Let us assume that we have solved the exact inhomogeneous field equations for the whole universe and find \( A^k(x^i) \). The vector \( A^k \) predominantly points in the same direction everywhere, however there are deviations from this direction that depend on \( x^i \). These deviations, in particular, are caused by the universe expansion itself.

In the cosmological model we approximate universe as homogeneous and isotropic. In such a model the vector field \( A^k(x^i) \) is replaced by its average over spatial coordinates. But what is the time coordinate and what are the spatial coordinates? Recall that in the background Euclidean space all coordinates are equivalent. At this stage we must specify what are the spatial coordinates, that is specify the reference frame in which we perform averaging.

Let us consider an observer located at a point \( y^j \) in the four-dimensional Euclidean space. Direction of \( A^k \) at this point is the time coordinate from the perspective of this observer. By making coordinate transformation along the lines of Eq. (6) one can make the \( x^0 - \)axis parallel to \( A^k \) and denote \( x^0 = ct \), and \( x^1, x^2, x^3 \) as spatial coordinates \( r \). Thus, position of the observer fixes the time and spatial coordinates in the whole Euclidean space. This is a global Cartesian coordinate system associated with the point \( y^j \). Observer on Earth takes a snapshot of the whole universe in this global coordinate system. Namely, the observer averages \( A^k \) over \( r \) and obtains \( \bar{A}^k(t) = \langle A^k(x^i) \rangle_r \) in this coordinate system.

If the observer is located at a different point \( z^i \) the direction of \( A^k \) at this point is different and, thus, division into time \( t \) and spatial coordinates \( r \) will not be the same. As a consequence, the function \( \bar{A}^k(t) \) will be different because it is obtained by averaging of \( A^k \) over different coordinates. Thus, equation for the universe evolution in the cosmological model depends on the reference frame.

Such equation for the scaling factor \( a(t) = e^{-\varphi} \) was obtained in [1] using the averaging procedure outlined above. This procedure yields uniform and isotropic equivalent metric \([27]\) and for non relativistic matter the evolution equation for \( a(t) \) is given by Eq. (28), or after time integration

\[
\dot{a}^2(t) = \frac{8\pi G}{3} \left( \frac{\rho}{a^3(t)} + \Lambda \right),
\]

where \( \rho \) and \( \Lambda \) are independent of time. However, the value of \( \Lambda \) depends on the observer’s reference frame. Local direction of \( A^k \) at the observer’s position determines the time coordinate \( t \) and the observer sees evolution of the universe \( a(t) \) as a function of this time coordinate.

Equation (31) for \( \Lambda \) has been obtained in [1] for non-relativistic matter (present universe). Using similar procedure one can show that for the radiation-dominated universe the answer will be

\[
\Lambda = \frac{2\rho}{a^3(t_0)},
\]

that is at time \( t_0 \) we also obtain

\[
\Omega_\Lambda(t_0) = \frac{\Lambda}{\rho_{\text{critical}}(t_0)} = \frac{2}{3} \approx 0.67,
\]

where for the radiation-dominated universe \( \rho_{\text{critical}}(t) = \rho/a^3(t) + \Lambda \).

The ratio (31) is independent of the universe equation of state and on the time \( t_0 \) the observer measures \( \Lambda \). However, the critical density of the universe \( \rho_{\text{critical}}(t) \) depends on time. Eq. (31) holds only at the time \( t_0 \) for which direction of the gravitational field four-vector \( A^k \) coincides with the \( t \)-axis. When the observer looks back in time by detecting light coming from distant parts of the universe the direction of \( A^k \) deviates from the \( t \)-axis at the position of the light sources. As a result, at \( t \neq t_0 \) Eq. (31) is not satisfied.

Eq. (34) is a constraint obtained in vector gravity on the evolution equation of the standard FLRW cosmology (28) in the reference frame of an observer that takes a snapshot of the universe at time \( t_0 \). \( \Omega_\Lambda(t_0) = 2/3 \) is a prediction of vector gravity which is consistent with the experimental results of Planck collaboration [8] and the Dark Energy Survey 0.686 ± 0.02.

Next we show that obtained results are also consistent with the Big Bang nucleosynthesis (and other standard processes in the early universe such as recombination, etc.). To this purpose we must consider evolution of the scaling factor in a locally co-evolving reference frame, rather than in the global inertial frame associated with the fixed Euclidean background. Dark energy (universe expansion) changes direction of \( A^k \) with time. If we choose a local non-inertial coordinate system such that the time coordinate always points along the direction of \( A^k \) then this is a co-evolving frame which is the proper frame to study Big Bang nucleosynthesis.

Next we show that in the co-evolving frame the cosmological constant in Eq. (28) is equal to zero. Therefore, in vector gravity the Big Bang nucleosynthesis proceeds the same way as in general relativity which yields that contribution from the cosmological term is negligible for the early universe. To be specific, we will assume non-relativistic matter, however, the answer is valid for a general equation of state.

It is convenient to work with the equivalent metric linearized near the Minkowski space-time. The linearized version of Eq. (28) reads [1]

\[
3 \langle h_{00} \rangle + 16\pi G\Lambda = 8\pi G \rho(t_0),
\]

where \( \langle h_{00} \rangle \) is a component of the spatially averaged met-
and \( \rho(t_0) = \rho_0/T^3(t_0) \).

In Ref. [1] it has been shown that components of the linearized spatially inhomogeneous metric before averaging

\[
\langle f_{ik} \rangle = \begin{pmatrix}
1 + h_{00} & h_{01} & h_{02} & h_{03} \\
h_{01} & -1 + h_{00} & 0 & 0 \\
h_{02} & 0 & -1 + h_{00} & 0 \\
h_{03} & 0 & 0 & -1 + h_{00}
\end{pmatrix}
\]

(36)

obey equations

\[
\Delta h_{00} + 3 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - 2 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^3} = \frac{8 \pi G}{c^4} T_{00},
\]

(37)

\[
\left( \frac{\partial^2}{\partial x^0 \partial x^0} - \Delta \right) h_{0\alpha} + \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^3} - 2 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} = \frac{16 \pi G}{c^4} T_{0\alpha}.
\]

(38)

Using the continuity equation for matter

\[
\frac{\partial T_{00}}{\partial x^0} + \frac{\partial T_{0\alpha}}{\partial x^\alpha} = 0
\]

one can reduce Eqs. (37) and (38) to [1]

\[
\frac{1}{c^2} \ddot{h}_{00} = \Delta h_{00} - \frac{8 \pi G}{c^2} \rho(t, r),
\]

(39)

\[
\frac{\partial^2 h_{0\alpha}}{\partial x^0 \partial x^\alpha} = 2 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^3}.
\]

(40)

Eq. (40) has the following local solution valid in the linear approximation

\[
h_{0\alpha}(t, r) = \frac{2}{3c} \ddot{h}_{00}(t_0)(t - t_0)x^\alpha.
\]

(41)

The coordinates \( x^0 = ct \) and \( x^\alpha (\alpha = 1, 2, 3) \) determine the global inertial reference frame in the Euclidean space. At the observer’s position \( x^\alpha = 0 \) in the linear approximation \( h_{0\alpha} = 0 \). That is the gravitational field 4-vector points along the \( x^0 \)-axis.

If we average \( f_{ik} \) over the global spatial coordinates \( x^\alpha \) we find that \( \langle f_{ik} \rangle \) is diagonal. Averaging Eq. (39) yields

\[
\langle \ddot{h}_{00} \rangle = -8 \pi G \rho(t_0).
\]

(42)

Matching this with Eq. (35), we obtain \( \Lambda = 2 \rho(t_0) \neq 0 \).

Next, we make transformation to the co-evolving coordinates in which spatially nonuniform metric (36) is diagonal in the local region. Recall that for an infinitesimal transformation of coordinates

\[
x'^k = x^k + \xi^k
\]

the metric transforms as

\[
g'_{ik} = g_{ik} - \xi_{i;k} - \xi_{;i}.
\]

For the present case the covariant derivatives can be replaced with partial derivatives.

Transformation of coordinates

\[
x'^\alpha = x^\alpha - \frac{1}{3} h_{00}(t_0)(t - t_0)^2 x^\alpha,
\]

(43)

\[
x'^0 = x^0 + \frac{c}{9} \ddot{h}_{00}(t_0)(t - t_0)^3,
\]

(44)

yields a diagonal metric

\[
f'_{ik} = \begin{pmatrix}
1 + \dot{h}_{00} & 0 & 0 & 0 \\
0 & -1 + \ddot{h}_{00} & 0 & 0 \\
0 & 0 & -1 + \ddot{h}_{00} & 0 \\
0 & 0 & 0 & -1 + \ddot{h}_{00}
\end{pmatrix},
\]

where

\[
\ddot{h}_{00} = h_{00} - \frac{2}{3} \ddot{h}_{00}(t_0)(t - t_0)^2.
\]

Taking the second order time derivative we find that \( \ddot{h}_{00} \) obeys differential equation

\[
\frac{d^2 \ddot{h}_{00}}{dt^2} = -\frac{1}{3} \ddot{h}_{00}(t_0).
\]

Plug in here Eq. (42) gives

\[
3 \frac{d^2 \ddot{h}_{00}}{dt^2} = 8 \pi G \rho(t_0),
\]

that is \( \ddot{h}_{00} \) obeys Eq. (35) with \( \Lambda = 0 \). So, in the co-evolving frame \( \Lambda = 0 \) and the universe expansion is decelerating.

V. SUMMARY

In vector gravity universe evolution is described by the equation of the standard FLRW cosmology with a cosmological term \( \Lambda \). However, contrary to general relativity, the value of the cosmological constant \( \Lambda \) in vector gravity depends on a coordinate system.

According to vector gravity, the Euclidean geometry of the universe is altered by the vector gravitational field \( A^k \). The direction of \( A^k \) gives the time coordinate, while perpendicular directions are spatial coordinates. Universe expansion yields change of the direction of \( A^k \) with time. This change appears as the cosmological constant (dark energy) in the evolution equation if we look at the
universe globally in the fixed inertial reference frame of the background Euclidean space. This is what an observer on Earth does when he takes a snapshot of the universe tacitly assuming that the time coordinate was always pointing in the same direction given by the direction of $A^k$ at the moment of observation $t_0$.

In this special coordinate system associated with the observer on Earth the $t$–axis points in the direction of $A^k$ only at time $t_0$. When the observer looks back in time by detecting light coming from distant parts of the universe the direction of $A^k$ deviates from the $t$–axis at the position of the light sources. As a result, in this coordinate system the universe evolves as if there is a nonzero cosmological constant $\Lambda = 2\rho_{\text{critical}}(t_0)/3$, where $\rho_{\text{critical}}(t_0)$ is the critical density of the universe at the moment $t_0$.

However, in the local co-evolving reference frame, in which the time coordinate is evolving together with the universe always aiming in the direction of $A^k$, the dark energy produces no effect on the universe evolution, and, as a result, it does not alter the Big Bang nucleosynthesis and galaxy formation.

Relation between Hubble parameters $H$ in the fix and co-evolving frames can be obtained by applying the coordinate transformation \ref{ordinate} and \ref{ordinate2}, which yields that in the vicinity of $t_0$

$$H' = H - \frac{2}{3} \dot{h}_{00}(t_0)(t - t_0). \quad (45)$$

Eq. \ref{expparam} shows that rate of change of Hubble parameter is different in the two frames. Thus, universe expansion can be accelerating in one frame, and decelerating in the other. However, in both frames the spatially averaged metric has the same form given by Eq. \ref{metric}.

Present analysis also answers the question about the fate of the universe which is determined by the evolution of the scaling factor in the co-evolving frame. Namely, the universe will expand forever at a continually decelerating rate, with expansion asymptotically approaching zero. This is what is expected for spatially flat universe in absence of exotic forms of energy.

Exponential expansion of the universe at the moment of Big Bang allows us substantially simplify equations for the gravitational field and reduce them to a simple form \ref{field1}, \ref{field2}. Apart from being much simpler, these equations have more symmetries than original field equations \ref{A}. In particular, Eqs. \ref{field1} and \ref{field2} are invariant under transformations \ref{transf}.

Simplified gravitational field equations \ref{field1} and \ref{field2} open a perspective for a rapid development of the vector theory of gravity and expand the class of problems for which analytical solutions can be obtained. E.g., one can use them to find, in the framework of vector gravity, stationary metric produced by a spinning mass in cylindrical or spherical geometries.

Acknowledgments

This work was supported by the Air Force Office of Scientific Research (Award No. FA9550-18-1-0141), the Office of Naval Research (Award Nos. N00014-16-1-3054 and N00014-16-1-2578) and the Robert A. Welch Foundation (Award A-1261).
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