On the chordae structure and dynamic behaviour of the mitral valve

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We develop a fluid-structure interaction (FSI) model of the mitral valve (MV) that uses an anatomically and physiologically realistic description of the MV leaflets and chordae tendineae. Three different chordae models — complex, “pseudo-fibre”, and simplified chordae — are compared to determine how different chordae representations affect the dynamics of the MV. The leaflets and chordae are modelled as fibre-reinforced hyperelastic materials, and FSI is modelled using an immersed boundary-finite element (IB/FE) method. The MV model is first verified under static boundary conditions against the commercial FE software ABAQUS, and then used to simulate MV dynamics under physiological pressure conditions. Interesting flow patterns and vortex formulation are observed in all three cases. To quantify the highly complex system behaviour resulting from FSI, an energy budget analysis of the coupled MV FSI model is performed. Results show that the complex and pseudo-fibre chordae models yield good valve closure during systole, but that the simplified chordae model leads to poorer leaflet coaptation and an unrealistic bulge in the anterior leaflet belly. An energy budget analysis shows that the MV model with complex and pseudo-fibre chordae have similar energy distribution patterns, but the MV model with the simplified chordae consumes more energy, especially during valve closing and opening. We find that the complex chordae and pseudo-fibre chordae have similar impact on the overall MV function, but that the simplified chordae representation is less accurate. Because a pseudo-fibre chordal structure is easier to construct and less computationally intensive, it may be a good candidate for modelling MV dynamics or interaction between the MV and heart in patient-specific applications.

Keywords: mitral valve, chordae tendineae, fluid-structure interaction, immersed boundary method, finite element method
1. Introduction

Mitral valve (MV) dysfunction, including mitral valve stenosis, prolapse, and regurgitation, is one of the most common valvular heart diseases and hence has attracted significant research interest. Computational modelling of human MV function [Gao et al., 2017b] can improve our understanding of MV biomechanics [Kunzelman et al., 1997, Kunzelman et al., 1998], which is important for improving surgical procedures [Reimink et al., 1996, Cochran & Kunzelman, 1998] and medical therapies [Kheradvar et al., 2015]. However, because of the challenges of modelling the highly complex MV structure, its deformation, and its interaction with the left ventricle, only limited progress in multi-physics modelling of the MV has been made to date [Gao et al., 2017b, Sun et al., 2014, Einstein et al., 2010].

Approaches to modelling the MV often fall into two categories: structural analysis and fluid-structure interaction (FSI) analysis. The former is simpler and focuses on MV deformation in its fully closed state. The latter focuses on the whole cardiac cycle and is more computationally demanding, but provides a more complete description of valvular function. Both approaches have been used to study MV dynamics [Kunzelman & Cochran, 1992, Kunzelman et al., 1993, Kunzelman et al., 1997, Watton et al., 2008, Luo et al., 2012, Ma et al., 2013, Gao et al., 2014a, Einstein et al., 2010]. For a recent review of different MV modelling strategies, the reader is referred to [Gao et al., 2017b]. In this study, we use an immersed boundary-finite element (IB/FE) method [Griffith & Luo, 2017] to develop a dynamic MV FSI model.

The chordae tendineae in the MV apparatus, which connect the papillary muscle to MV leaflets, play an important role in MV function, especially in the systolic phase to prevent the leaflets prolapse. Studies have shown that chordae structure have a substantial impact on MV function [David et al., 1984, Reimink et al., 1996, Espino et al., 2005]. For example, David et al.1984 found that to repair chronic MV regurgitation, post-operative left ventricular function benefits from preserving the chordae structure and papillary muscles when replacing the MV. Espino et al.2005 also confirmed that anterior leaflet marginal chordae as well as commissural chordae are vital for mitral valve competence. However, few studies have investigated the difference in MV function between different chordae structures with FSI. In our previous studies, we used various simplified chordae structures, including discrete elastic “pseudo-fibre” models [Watton et al., 2008, Luo et al., 2012, Ma et al., 2013] and fully three-dimensional volumetric models [Gao et al., 2014a, Gao et al., 2017a]. However, those simplified chordae structures did not distinguish the marginal, basal, and strut chordae. As a result, our modelled anterior leaflet bulged towards the left atrium, which is a common artifact of many existing MV models.

This study aims to overcome these defects and to investigate how different chordae representations affect valvular function. To do so, we use a physiologically detailed description of the MV leaflets and chordae structure based on multi-slice CT image data [Wang & Sun, 2013]. In addition, we consider three different chordae structure representations: complex; pseudo-fibre; and simplified. The complex model uses a fully three-dimensional chordae structure based on the CT images, in which the marginal, basal, and strut chordae are all included. The pseudo-fibre model is based on the same CT data, but represented using thin elastic fibres, as in a more conventional IB approach [Ma et al., 2013]. The simplified chordae structure is based on our previous idealized chordae descriptions [Gao et al., 2014a], in which the chordae are also modelled using a fully three-dimensional volumetric solid mesh. In all these cases, anisotropic material laws are used for both the MV leaflets and the chordae.

The paper is organized as follows: Sections 2 and 3 provide details of the IB/FE method and material properties. Section 4 describes verification studies of our IB/FE model against the commercial FE package ABAQUS (https://www.3ds.com/products-services/simulia/products/abaqus/) in steady state conditions. Sections 5 and 6 provide implementation details and describe
results for FSI simulations. We compare the MV model behaviour with different chordae structures and comparisons of MV orifice area during LV systolic phase and fluid patterns during LV diastolic phase are made among those three chordae structures. We also analyse energy budget for the fully coupled MV-flow model including detailed energy distribution and its dissipation. Section 7 discusses these results and provides a concluding discussion.

2. Methodology

2.1. IB/FE formulation

In the IB/FE formulation of FSI used in this study [Griffith & Luo, 2017], the structure elasticity and deformation are described in Lagrangian coordinates, and the pressure, velocity, and incompressibility of the coupled fluid-structure system are described in Eulerian coordinates. Let $\Omega \subset \mathbb{R}^3$ denote the fixed physical domain occupied by the coupled fluid-structure system, and let $\Omega^s_0 \subset \Omega$ be the Lagrangian reference coordinate domain for structure. $x \in U$ are fixed Cartesian physical coordinates, $X \in \Omega^s_0$ are the fixed Lagrangian reference coordinates of the structure, and $\chi(X,t) \in U$ is the current position of material point $X$ at time $t$. Thus, the region occupied by structure at time $t$ is $\Omega_t^s = \chi(\Omega^s_0,t)$, and $\Omega^f_t = \Omega \setminus \Omega_t^s$ is the region occupied by the fluid at time $t$.

The Cauchy stress tensor $\sigma(x,t)$ is divided into two parts,

$$\sigma(x,t) = \begin{cases} \sigma^f(x,t) + \sigma^s(x,t), & x \in \Omega^s_t, \\ \sigma^f(x,t), & \text{otherwise}. \end{cases}$$

(1)

Here, $\sigma^f(x,t)$ is the Cauchy stress tensor for a viscous incompressible fluid,

$$\sigma^f(x,t) = -pI + \mu(\nabla u + \nabla u^T),$$

(2)

in which $p$ and $u$ are the material pressure and velocity, and $\mu$ is the dynamic viscosity of the fluid. The Cauchy elastic stress tensor $\sigma^s(x,t)$ can be related to the first Piola-Kirchhoff stress tensor $P^e$ using Nanson’s formula,

$$\int_{\partial V} P^e(X,t)N \, dA(X) = \int_{\partial \chi(V,t)} \sigma^s(x,t)n \, da(x),$$

(3)

in which $V \subset \Omega^s_0$ is an arbitrary Lagrangian subregion, $N$ is the outward unit vector along $\partial V$, and $n$ is the outward unit vector along $\partial \chi(V,t)$.

The equations for the IB/FE formulation of FSI used in this study are [Griffith & Luo, 2017],

$$\rho \left( \frac{\partial u(x,t)}{\partial t} + u(x,t) \cdot \nabla u(x,t) \right) = -\nabla p(x,t) + \mu \nabla^2 u(x,t) + f(x,t),$$

(4)

$$\nabla \cdot u(x,t) = 0,$$

(5)

$$f(x,t) = \int_{\Omega^s_0} F(X)\delta(x-\chi(X,t)) dX,$$

(6)

$$\int_{\Omega^s_0} F(X,t) \cdot V(X) dX = -\int_{\Omega^s_0} P^e(X,t) : \nabla X V(X) dX, \quad \forall V(X),$$

(7)

$$\frac{\partial \chi}{\partial t}(X,t) = u(\chi(X,t),t) = \int_{\Omega} u(x,t) \delta(\chi(X,t) - x) dx,$$

(8)
with $V(X)$ an arbitrary Lagrangian test function. Eq. (4) is the momentum equation for the coupled system, Eq. (5) is the incompressibility constraint, and Eqs. (6) and (8) are interaction equations that connect the Lagrangian and Eulerian coordinate systems, which use the delta function $\delta(x)$ to transfer quantities between the two frames. For problems discussed here, $\rho$ and $\mu$ are assumed constants throughout the system. Eq. (6) relates the Lagrangian elastic force density $F(X)$ and the corresponding Eulerian elastic force density $f(x,t)$. Eq. (8) implies that the no-slip condition is satisfied at the fluid-solid interface. We remark that although we used two separate set of meshes, one Eulerian and one Lagrangian, only the Eulerian momentum equation is solved for the coupled system, because the Eulerian momentum equation accounts for the momentum of the entire computational domain $\Omega = \Omega_f^t \cup \Omega_s^t$.

The applied forces ensure traction continuity at the fluid-solid interface.

### 2.2. Energy budget analysis

Now we introduce the energy budget analysis for the coupled FSI model. The energy balance of the FSI system can be written as

$$\int_{\Omega_{\text{tube}}} \rho \left( \frac{\partial u}{\partial t} \right) \cdot u \, dx + \int_{\Omega_{\text{tube}}} \rho (u \cdot \nabla) u \cdot u \, dx = \int_{\Omega_{\text{tube}}} (-\nabla p + \mu \nabla^2 u) \cdot u \, dx + \int_{\Omega_{\text{tube}}} f \cdot u \, dx,$$

in which $\Omega_{\text{tube}}$ is the interior of the stationary tube in which the MV is mounted (Fig. 3). Eq. (9) consists of the change of the kinetic energy,

$$\text{KE} = \int_{\Omega_{\text{tube}}} \rho \left( \frac{\partial u}{\partial t} \right) \cdot u \, dx = \frac{d}{dt} \left( \int_{\Omega_{\text{tube}}} \frac{1}{2} \rho u \cdot u \, dx \right),$$

the kinetic energy flux of fluid on the boundary $\partial U$,

$$\text{KF} = \int_{\Omega_{\text{tube}}} \rho (u \cdot \nabla) u \cdot u \, dx = \int_{\partial U} \frac{1}{2} \rho (u \cdot u) n \cdot da,$$

the rate of work by the applied pressure,

$$\text{WP} = \int_{\Omega_{\text{tube}}} (-\nabla p) \cdot u \, dx = \int_{\partial U} (-p) n \cdot u \, da,$$

the rate of energy dissipation,

$$\text{D} = \int_{\Omega_{\text{tube}}} \mu \nabla^2 u \cdot u \, dx = -\int_{\Omega_{\text{tube}}} \mu (\nabla u + \nabla u^T) : \nabla u \, dx,$$

and the rate of change of elastic strain energy in the immersed structure,

$$\text{E} = \int_{\Omega_{\text{tube}}} f \cdot u \, dx = \int_{\Omega_{\text{tube}}} \left( \int_{\Omega_0} F(X) \delta(x - \chi(X,t)) dX \right) \cdot u \, dx$$

$$= \int_{\Omega_{\text{tube}}} \left( \int_{\Omega_0^t} u(x,t) \delta(x - \chi(X,t)) dx \right) \cdot F(X) \, dX$$

$$= \int_{\Omega_0} u(\chi(X,t),t) \cdot F(X) \, dX,$$

which can be rewritten as

$$\text{E} = -\int_{\Omega_0} \nabla u(\chi(X,t),t) \cdot F(X) \, dX,$$

with the help of Eq. (7).
2.3. **Spatial and temporal discretizations**

A Cartesian grid is used to discretize the Eulerian domain \( \Omega \). We use a staggered grid to approximate the Eulerian velocity \( \mathbf{u} = (u_x, u_y, u_z) \) and Eulerian force density \( \mathbf{f} = (f_x, f_y, f_z) \). Standard second-order accurate finite difference approximations to the divergence, gradient, and Laplace operators [Griffith, 2009] are used. The nonlinear advection terms in Eq. (4) are discretized using a version of the piecewise parabolic method (PPM) [Colella & Woodward, 1984]. We discretize the Lagrangian domain using the finite element method with trilinear (\( Q^1 \)) basis functions for the displacement and resultant force. Additional details of the discretization can be found in Griffith & Luo, 2017.

3. **Mitral valve geometry and material properties**

Our MV model includes a detailed MV anatomical geometry obtained from multi-slice CT images of a normal mitral valve at mid-diastole (when the MV is open) obtained from a 61-year-old male patient [Wang & Sun, 2013]. Three different models are built, as shown in Fig. 2. All three models have the same leaflets but different chordae representations. The first one, shown in Fig. 2(a), is referred to as the complex model. It has a CT-derived chordal structure that includes the marginal, basal, and strut chordae, with a fork-shaped geometry at chordae-leaflet connection to avoid stress concentrations. The chordae are modelled as fully three-dimensional solid bodies using a volumetric mesh discretization with a uniform cross-sectional area of 0.16 mm² [Gao et al., 2014a]. The second model, shown in Figure 2(b), is based on the same CT scan data, but here the chordae are represented as one-dimensional elastic fibres, as in more conventional IB approaches [Ma et al., 2013]. The chordae follow the same structure as in the complex model, including all three types of chordae, and a fork-shaped geometry at the chordae-leaflet connection. In other words, the pseudo-fibres are the skeleton of the complex chordae derived from the CT scan data [Wang & Sun, 2013]. We refer this as the pseudo-fibre model. The third model, shown in Fig. 2(c), uses a simplified chordae structure, as in our previous work [Gao et al., 2014a], which has a total of 16 evenly distributed chordae (6 on anterior leaflet and 10 on posterior leaflet) with the same uniform cross-sectional area of 0.16 mm². The anterolateral and posterolateral papillary muscle groups are represented by two chordae attachment movements obtained from the average of chordae attachment positions from the complex model. These chordae are also modelled as volumetric solids and run through the leaflets from the free leaflet edges to the annulus ring. Details can be found in Gao et al. 2014a. This is referred to as the simplified chordae model.

An invariant-based, anisotropic constitutive model based on the formulation proposed by [Gasser et al., 2006, Holzapfel et al., 2000] is used to describe the MV leaflets:

\[
W_{\text{leaflet}} = C_1 \{ \exp[C_2(I_1 - 3 - \log(I_3))] - 1 \} + \frac{k_1}{2k_2} \sum_{i=1}^{2} \{ \exp[k_2(I_{i4}^*-1)^2] - 1 \} + \frac{\beta}{4} \log^2(I_3),
\]

in which \( I_1 = \text{trace}(F^TF) \), \( I_3 = \text{det}(F^TF) = J^2 \), and \( F = \frac{\partial \mathbf{X}}{\partial \mathbf{X}} \) is the deformation gradient tensor. Here, \( \frac{\beta}{4} \log^2(I_3) \) is a volumetric energy that acts to penalize compressible deformations. We choose \( \beta = 500 \text{ kPa} \) in our simulations. The modified fibre pseudo-invariants are \( I_{i4}^* = \max[(\kappa I_1 + (1 - 3\kappa)I_{i4}), 1] \), and \( I_{i4} = e_i \cdot (F^TF)e_i \), using the fibre directions \( e_i \) shown in Figs. 1a and 1b. Two families of fibre are included here, whose directions as well as the parameter values are directly from biaxial experimental data for a healthy human mitral leaflets [Wang & Sun, 2013]. The modified invariant \( I_{i4}^* \) is defined to ensure that only extended fibres contribute to the strain energy function. In particular, the fibres do not support compressive loads. \( \kappa \) is a collagen fibre dispersion parameter, and \( C_1, C_2, k_1, \) and \( k_2 \) are the material parameters. The parameter values for the models are summarized in Table 1.
We define the modified elastic stress tensor $\mathbb{P}_e$ via

$$\mathbb{P}_{\text{leaflet}} = 2C_1 C_2 \exp[C_2(I_1 - 3 - \log(I_3))] \mathbb{P} - 2C_1 C_2 \exp[C_2(I_1 - 3 - \log(I_3))] \mathbb{P}^{-T}$$

$$+ \sum_{i=1}^{2} 2k_1 \kappa (I_{4i}^* - 1) \exp[k_2 (I_{4i}^* - 1)^2] \mathbb{P} + \sum_{i=1}^{2} 2k_1 (1 - 3 \kappa) (I_{4i}^* - 1) \exp[k_2 (I_{4i}^* - 1)^2] \mathbb{P} e_i \otimes e_i$$

$$+ \beta \log(I_3) \mathbb{P}^{-T}. \tag{19}$$

The term $2C_1 C_2 \exp[C_2(I_1 - 3)] \mathbb{P}^{-T}$ is included to ensure $\mathbb{P}_{\text{leaflet}}$ is zero when $\mathbb{P} = \mathbb{I}$. This modification is found to reduce the magnitude of spurious volume loss caused by the pressure discontinuities across the structure-fluid interface [Gao et al., 2014b].

In all models of the chordae tendinae, the elasticity of the chordal material is described by

$$W^{\text{chordae}} = C(I_1 - 3 - \log(I_3)) + \frac{\alpha_1}{2 \alpha_2} \left\{ \exp[\alpha_2 (I_{4i}^* - 1)^2] - 1 \right\} + \frac{\beta}{4} \log^2(I_3), \tag{20}$$

in which $C$, $\alpha_1$, and $\alpha_2$ are material parameters whose values are summarized in Table 2, and $I_1$, $I_3$ and $I_{4i}^*$ are similarly defined without dispersion, i.e. $\kappa = 0$. The fibre direction for all the chordae tendinae is simply defined along its longitudinal direction. For the complex model, all the branches have the fibre defined following its own long axis at the centre of the cross-section. As in the leaflets, we use $\beta = 500$ kPa. The resulting stress tensor is

$$\mathbb{P}_{\text{chordae}}^{\text{e}} = 2C \mathbb{P} - 2C \mathbb{P}^{-T} + 2\alpha_1 \exp[\alpha_2 (I_{4i}^* - 1)^2] \mathbb{P} e \otimes e + \beta \log(I_3) \mathbb{P}^{-T}. \tag{21}$$

Although the pseudo-fibre model describes each chord as a one-dimensional elastic fibre, the fibre model uses the same nonlinear material description as in (20), taking the cross-sectional area of the chords to be $0.04 \text{ cm} \times 0.04 \text{ cm}$. The cross-sectional area is chosen so that the total force of the pseudo-fibres on the MV equals to the total force exerted by the solid chordae.

Table 2: Material parameters of MV chordae.

|           | $C$ (kPa) | $\alpha_1$ (kPa) | $\alpha_2$ |
|-----------|-----------|------------------|------------|
| Basal chordae | 540       | 1446.2           | 22.09      |
| Marginal chordae | 540       | 200.48           | 33.83      |
| Struct chordae | 540       | 1446.2           | 22.09      |

4. IB/FE model verification

Detailed IB/FE model verification studies are performed on a disc geometry and the MV model with complex chordae under static boundary conditions. We compare the results obtained by the IB/FE model
5. Dynamic MV-fluid model implementation

In the coupled MV-fluid system, the MV is mounted on a fixed housing structure that is placed inside a rigid tube with length 16 cm and radius 3.5 cm, as shown in Figure 3. The whole structure sits inside an 8.6 cm × 8.6 cm × 16 cm computational domain filled with viscous fluid of density 1 g cm$^{-3}$. 

in static conditions to a corresponding structure-only model built in ABAQUS. The models are shown to be in good agreement; additional details are provided in Appendices A and B.
and dynamic viscosity $0.04 \text{ g-cm}^{-1} \cdot \text{s}^{-1}$. The grid sizes for both fluid and solid are chosen so that good agreement is achieved in the verification problems (see Appendix for details). For the structural discretization, we use the same grid size for the leaflets, but because of the different treatment of the chordae, the total elements/nodes are 106963/153336 for the complex model, 97093/116964 for the pseudo-fibre model, and 75292/92910 for the simplified model. For the Eulerian mesh, we use a 2-level block-structured adaptively refined grid with a refinement ratio 4 between the levels. The finest level has a grid of: $N_x = 130$, $N_y = 130$, $N_z = 200$. Hence, the fluid mesh size is in the order of $0.07 \text{ cm} \times 0.07 \text{ cm} \times 0.08 \text{ cm}$. The time step size is chosen to be $5.0 \times 10^{-6} \text{ s}$.

A physiological transvalvular pressure, shown in Figure 4, is imposed at the inlet, and the outlet pressure is held zero. The modelled cardiac cycle includes the initialization phase, MV closing, and fully closed phases, then MV opening and fully open phases. We also choose three time points (identified as times A, B, and C as shown in Figure 4) to perform a detailed analysis. To model the papillary muscle movements, the ends of chordae are constrained with displacement boundary conditions measured from experimental data similar to [Wang & Sun, 2013].

The IB/FE scheme used in this study is implemented in the open-source IBAMR software (https://ibamr.github.io), which is an adaptive and distributed-memory parallel implementation of the immersed boundary method leveraging several open-source computational frameworks, including SAMRAI (https://computation.llnl.gov/projects/samrai), PETSc (https://www.mcs.anl.gov/petsc), libMesh (http://libmesh.github.io), and hypre (http://www.llnl.gov/casc/hypre) to perform core functionality. The full simulations are run on the supercomputing services ARCHER (http://www.archer.ac.uk) and ARCHIE-WeSt (http://www.archie-west.ac.uk). Some early tests were also conducted at the Barcelona Supercomputing Centre (https://www.bsc.es). A typical simulation of one cycle using 32 processors takes about 9 days in wall-clock time on ARCHER or ARCHIE-WeSt.

6. Results

6.1. Structural Deformation and Fluid Dynamics

Figure 5 shows the deformation and the maximum principal strain and stress of the three MV models at time point B. Figures 5a and 5b show that the complex and pseudo-fibre models generate similar closed valve configurations, but the simplified chordae model (Figure 5c) is quite different. The maximum displacement of the MV leaflets for complex and pseudo-fibre models are 0.85 cm and 0.80 cm, respectively. However, the anterior leaflet belly of the simplified model has an unrealistic large bulging into the left atrium, with a maximum displacement of 1.24 cm, which is shown in Figure 6c. All three models have a visible gap, which results from the regularization of the delta function [Ma et al., 2013, Gao et al., 2014a, Griffith et al., 2009]; however, this gap is much larger in the simplified model, as shown in Figure 5. The “finger-like” bumps in this case is resulted from the fact that the chordae run through the leaflets. In addition, MV orifice shape is compared between simulated results and segmented data from the same CT images mentioned in Section 3. Because of the image quality, only three phases in diastole can be segmented. Here, we choose the highest quality image and compare it to the complex chordae model. As shown in Figs. 7 and 8, the simulated MV leaflets shape matches the CT scan reasonably well despite the fact that the segmented data doesn’t have enough information near the annulus ring, but it captures more details for the rest of leaflets. For example, the bumps near the chordae-leaflet attachment are more pronounced in the segmented data than the simulated data.
In all models, higher stress regions are found in the anterior leaflet belly region, close to the annulus ring (Figures 5g, 5h, and 5i), which are in agreement with previous observations [Ma et al., 2013, Gao et al., 2014a]. However, notice that the simplified model has higher stress in the anterior leaflet than the other two models. Figure 5 suggests because the chordae stress is not included. We further compute the averaged maximum principle stress, strain, and displacement of a circular region (radius 0.5 cm) in the anterior leaflet (Figure 5a). The results are listed in Table 3. With the chordae stress included, the complex and pseudo-fibre models yield similar results, but the simplified model has a clear discrepancy.

Figures 9a, 9b, and 9c show the maximum principal stress on chordae for all three models at time point B. The complex and pseudo-fibre models have very similar stress patterns: most of the chordae in the middle experience high stress whereas the chordae attached to commissure area tend to have less stress. For the simplified model, chordae attached to anterior leaflet have high stresses because of the large stretches, and near the leaflet commissures, high stresses also are observed. The average stresses of the chordae attached to the anterolateral and posterolateral papillary muscle groups are, 1890 kPa and 2740 kPa, respectively, for the complex model, 2263 kPa and 2914 kPa for the pseudo-fibre model, 3460 kPa and 3198 kPa for the simplified model. The simplified model has higher values than the other two MV models especially for the anterolateral muscle group.

We also calculate the orifice area for each model by projecting the deformed leaflets onto the $z = 0$
FIG. 4: The inlet pressure profile with five phases: initialization; closing; fully closed; opening; and fully open phases. We further define three time points: time A is when the MV is closing; time B is when MV is fully closed and under peak pressure (110 mmHg); and time C is when the MV is fully open.

|                                | Complex chordae | Pseudo-fibre chordae | Simplified chordae |
|--------------------------------|-----------------|----------------------|--------------------|
| Maximum principal strain       | 0.177           | 0.178                | 0.172              |
| Maximum principal stress (kPa) | 148             | 154                  | 216                |
| Displacement (cm)              | 0.722           | 0.697                | 1.07               |

Table 3: Averaged maximum principle strain, stress, and displacement of the anterior leaflet in a circular region defined in Fig. 5a.

plane. The results are plotted in Figure 10. When the MV is opened at time C, the complex model has a maximum area of 5.21 cm², while the pseudo-fibre and simplified models have slightly larger areas (maximum: 5.68 cm² and 5.58 cm²) than the complex model. This is presumably because the pseudo-fibres do not constrain the MV as effectively as the solid chordae, and the simplified model does not have an effective chordae geometry, despite the fact that chordae run through the leaflets. When the MV
Fig. 5: Top row: The closed MV coloured by the displacement magnitude for MV models with (a) complex, (b) pseudo-fibre, and (c) simplified chordae models. Middle row: The closed MV coloured by maximum principal strain for MV models with (d) complex, (e) pseudo-fibre, and (f) simplified chordae. Bottom row: The closed MV coloured by maximum principal stress for MV models with (g) complex, (h) pseudo-fibre, and (i) simplified chordae.

|                          | Complex chordae | Pseudo-fibre chordae | Simplified chordae |
|--------------------------|-----------------|----------------------|--------------------|
| Orifice area at 0.165 s  | 4.93 cm²        | 5.00 cm²             | 4.86 cm²           |
| Time when orifice area = 0.52 cm² | 0.208 s   | 0.208 s             | 0.230 s            |

Table 4: MV orifice and closing speed comparison.
Fig. 6: Side view of the three models at time B: (a) the complex model, (b) the pseudo-fibre model, and (c) the simplified model. Notice the substantial bulge in the anterior leaflet in (c) shown by arrow.

Fig. 7: Comparison of leaflet shapes at MV opening: (a) the complex model at time 0.615 s and (b) the segmented MV data from CT scan.

Fig. 8: Comparison of leaflet shapes at MV opening: (a) the complex model at time 0.615 s and (b) the segmented MV data from CT scan.
is closed at time B, both the complex and pseudo-fibre chordae models have smaller orifice areas than the simplified chordae, as shown in Fig. 5. Further, by calculating the time that different models reach the same closure state (e.g. to an orifice area of 0.52 cm$^2$ at 0.165 s), we can show that the complex and pseudo-fibre models have slightly faster MV closure speeds (about 0.022 s faster) than the simplified model, as shown in Table 4.

Figure 11 shows the flow rate through the MV. As suggested by the orifice area changes detailed in Fig. 10, faster MV closure leads to smaller regurgitant flow volumes: 8.66 mL for the complex model and 8.69 mL for the pseudo-fibre model. By contrast, the simplified model has a significantly higher regurgitation volume of 12.81 mL.

Figure 12 shows the streamlines of the three models at time C when the MV is fully open. Two flow jets are formed as shown by the arrows (Figure 12a, 12c, and 12e), one close to the anterior leaflet and the other close to the posterior leaflet. The complex and pseudo-fibre models have similar maximum filling velocities (227.1 cm/s and 228.9 cm/s, respectively), and both are lower than that of the simplified model (260 cm/s). Figures 12b, 12d, and 12f show the corresponding flow vectors around the leaflets. It can be seen that all three models are associated with vortices behind the leaflet tips. Because the pseudo-fibre chordae are modelled as one-dimensional elastic fibres, in the continuum equations, they would not interact with the fluid. In practice, however, interaction between the flow and the chordae is mediated by regularized delta functions, and the finite extent of the delta function kernel imbues the chordae with an effective thickness. Further, at the present mesh resolutions, the Eulerian grid spacing is comparable to the cross-sectional dimensions of the pseudo-fibres. Consequently, from the standpoint of the fluid dynamics, the effective numerical thickness of the pseudo-fibre chordae is similar to that of the volumetric models. Under further grid refinement, it would be necessary to fix the support of the regularized kernel function associated with the pseudo-fibre model, as done in prior work on modelling the interaction of thin rods with fluids using the generalized IB method [Lim et al., 2008, Griffith & Lim, 2012], to provide the pseudo-fibres with a fixed physical thickness.

Figure 13 shows the pressure fields for the complex model at different time points. The other two models yield similar results.
Fig. 10: Orifice area change.

6.2. Energy Analysis

To further quantify the dynamics of these different models, we perform an energy analysis. Figures 14 and 15 show the energy budget for the complex model. It is clear that in the fully open phase, kinetic energy dominates the energy distribution and consumes most of the input work (WP-KF). In the closing and opening phases, the MV strain energy (E) starts to have a bigger impact on the energy distribution. The large oscillations in E are associated with the strong motion of the MV during these phases. In the fully closed phase, the coupled system is settling down, and all energy terms gradually decay to zero. Similar energy budget patterns are found in the other two cases. Table 5 summarizes all the energy budget terms at time A (closing) and C (fully open). It shows that 80% of input work is used to increase the kinetic energy at time C, and that the MV consumes 94% of the total energy from both the input work and the kinetic energy at time A.

We further define the energy dissipation function,

\[ \Phi = -\mu (\nabla u + (\nabla u)^T) : \nabla u, \]  

(22)
ON THE CHORDAE STRUCTURE AND DYNAMIC MODELLING OF THE MV

Table 5: Energy budget (in J/s) for the complex model at times A and C.

|          | WP-KF  | KE     | D                  | E                   |
|----------|--------|--------|--------------------|---------------------|
| A (MV Closing) | 1.82   | -0.38  | -0.153 (6% of WP-KF-KE) | -2.06 (94% of WP-KF-KE) |
| C (MV Fully Open) | 0.759  | 0.608 (80% of WP-KF) | -0.0975 (13% of WP-KF) | -0.0532 (7.0% of WP-KF) |

Fig. 11: Flow rate through the MV.

and plot the dissipation field for the MV models at time C (fully open) in Fig. 16. We can see that high dissipation exists close to the MV leaflets and chordae structure. Energy is also dissipated at a higher rate close to the atrium side of leaflets due to the high shear stress. Figures 16g, 16h, and 16i show a sliced view of the dissipation field associated with the chordae. Overall, the complex and pseudo-fibre models have similar energy dissipation patterns near the chordae structure. However, the simplified model seems to cause a greater dissipation when the MV is opening.

Figure 17 shows the total energy dissipation rate $D$ against time. When the MV is closing or opening,
D increases, as to be expected. Again, the complex and pseudo-fibre models have similar patterns, but the simplified model introduces a higher energy dissipation rate. On the other hand, during the MV closure phase, when the inlet pressure increases quickly, the pseudo-fibre model has the highest dissipation rate (-0.31 J/s), compared to the complex and simplified ones, which are -0.23 J/s and -0.21 J/s, respectively.
Figure 13: Pressure field for the complex model at (a) time A (closing), (b) time B (peak adverse pressure), and (c) time C (fully open).

Figure 18 plots the rate of MV strain energy (E) for the three MV models in MV closing and fully closed phases; results in other phases are similar in all the models. It can be seen that the complex and pseudo-fibre models have similar patterns overall, with negative peaks of -3.43 J/s and -3.17 J/s, respectively, when the MV is closing. However, the simplified model leads to much greater oscillations and a negative peak of -5.44 J/s. This is consistent to the fact that simplified chordae model undergoes a larger structural deformation, as is reflected in the displacement contours in Fig. 5c.

Figure 19 shows the input energy rate for the three models in the closing and fully closed phases; results in other phases are similar in all the models. As in Fig. 18, the complex and pseudo-fibre models have similar trends with maximum magnitudes: 2.07 J/s and 2.01 J/s, whereas the input energy rate of the simplified model is much greater (2.81 J/s). In other words, the simplified model requires a greater input energy compared to the other two MV models during the cardiac cycle.

The model behaviours can also be seen in the integrated energy budgets. Table 6 shows the energy budgets integration during the period when the MV is closing (from 0.165 s to 0.25 s). The complex and pseudo-fibre models produce similar results. Part of the fluid kinetic energy is dissipated due to viscosity (40% and 46%), and the rest is transferred into the MV strain energy (43% and 38%), and to overcome the pressure loading (17% and 16%). However, the simplified model shows a totally different energy distribution: much more strain energy (60% of KE-P-F) is gained from not only the kinetic energy but also the external input energy. Compared to other two models, it also stores about twice as much strain energy during MV closing phase, and requires more input energy.
Fig. 14: Energy budget for the complex model during the initialization and closing.

Table 6: Integrated energy distribution (in J) over the MV closing phase.
Fig. 15: Energy budget for the complex model during fully closed, opening and fully open phases.
FIG. 16: Isovolume plots for energy dissipation function ($\Phi$) for (a) the complex model, (b) pseudo-fibre model, and (c) simplified model. The corresponding two-dimensional sliced views and contour plots around the chordal cross-sections of $\Phi$ are shown in (d,e,f) and (g, h, i), respectively.
FIG. 17: Dissipation Energy Rate (D)
FIG. 18: Rate of Strain Energy Change (E) in MV closing and fully closed phases.
FIG. 19: Total Input Energy Rate (WP-KF) in MV closing and fully closed phases.
7. Discussion

This paper describes an FSI model of the mitral valve that uses an anatomically and physiologically realistic description of the MV leaflets and chordae tendineae that is based on an immersed boundary method with finite element elasticity (IB/FE) [Griffith & Luo, 2017]. Our model is verified by comparing the structural results between the IB/FE model and a structure-only model implemented using ABAQUS. In this study, several advances have been made compared with our previous studies [Ma et al., 2013, Gao et al., 2014a]: (1) we use physiologically-detailed MV leaflets and chordae structures that are based on CT image data; (2) we compared three different chordae structure models and their effects on the MV function; and (3) the FSI simulation of the MV dynamics also allows us to conduct an energy budget analysis of the dynamic MV and the flow for the first time.

A realistic MV geometry is necessary for modelling MV dynamics, but it is almost impossible to reconstruct the chordae structure from low-resolution imaging data. Consequently, simplified MV models are widely used [Kunzelman et al., 1997, Prot & Skallerud, 2009, Stevanella et al., 2009, Ma et al., 2013, Gao et al., 2014a]. However, researchers started to notice the limitation of such simplifications. For example, Wang et al. [Wang & Sun, 2013], using a purely structural model, showed that simplified chordae geometry leads to billowing leaflets and incomplete MV closure compared with complex chordae structures. With the help of advanced imaging techniques, more realistic chordae structures are now being reported [Wang & Sun, 2013, Toma et al., 2016, Lee et al., 2013, Lee et al., 2015]. Our work goes further to compare the effects of three different chordae structure modelling approaches on overall MV function. In particular, because the chordae structure has often been represented by connections of elastic springs [Griffith et al., 2009, Ma et al., 2013, Luo et al., 2012] when simulating MV dynamics using immersed boundary method, it is interesting to compare the difference between the volumetric chordae and the pseudo-fibre type representation.

The most notable result of this study is that the complex and pseudo-fibre chordae produce very similar dynamics and kinematics. However, the simplified chordae models tend to have a better performance in preventing the regurgitation, which can be seen from the orifice area profile (Fig. 10) and the volumetric flow rate (Fig. 11). It is also interesting to note that all of our MV models do not shown prolapse, because of the realistic hyperelastic constitutive law used for the leaflets and volumetric structure model which automatically includes the bending stiffness. This is in agreement with our previous study [Luo et al., 2012], which showed that bending in the MV leaflets plays an important role. The complex and pseudo-fibre models also show consistent results with other published results of MV studies. Wang et al. Wang & Sun, 2013 reported the average maximum principal stress is approximately 160 kPa on a midsection of anterior leaflet at the systolic peak. Kunzelman et al.2007 used a FE model to find that the maximum principal stress on the anterior leaflet was 254 kPa with a transvalvular pressure of 98 mmHg. Gao et al. 2014a also reported the peak average stress is 115 kPa and 169 kPa, along the radial and circumferential directions, in the anterior belly region. These results are comparable to our model predictions, as shown in Table 3.

The majority of MV modelling research focuses on the mechanics of the MV or the hemodynamics inside the left ventricle [Wang & Sun, 2013, Gao et al., 2014a, Gao et al., 2017b], and comparatively few reported studies have evaluated MV function from an energetic perspective [Choi et al., 2014]. Akins et al. 2008 pointed out that evaluating energy loss during the cardiac cycle can help assess the impact of valve performance on cardiac function. Choi et al. 2014 also attempted to calculate the energy dissipation in a simplified LV-MV model to study the MV performance. Using the energy budget
analyses reported in Eqs. (10-17), our work is the first effort to consider the whole cycle of the realistic MV model and FSI from an energetic perspective. This energy analysis reveals that when the MV is fully open, most of the input energy from left atrial contraction is used to increase the kinetic energy, which is then used to pump blood into the left ventricle from the left atrium. However, when the MV is closing or opening, the MV leaflet elastic energy becomes dominant.

The energy distributions also show that the chordae structure plays an important role. In our simulations, even with different mesh elements, the complex chordae and pseudo-fibre chordae lead to very similar energy results, as seen in Figs. 16–19. Moreover, Table 6 shows that during early LV systolic phase until the time when the aortic valve opens, most of the kinetic energy is reduced via viscous dissipation and increases in the MV strain energy. However, the simplified chordae introduces more energy exchanges overall, especially when the MV is closing or opening, which can be seen from the larger magnitude peaks in Figs. 18 and 19. Further, Table 6 shows that much more strain energy is stored in the simplified chordae, which may be partially responsible for the unrealistic bulging. As a result, the input energy from left ventricular contraction is also substantially increased in this model. The slower MV closing process in the simplified model also leads to a sign change in the input energy contribution in Table 6, indicating a different energy distribution from the other two models.

One of the most important aspect in cardiac energy analysis is to see how energy loss (from viscosity, turbulence, and flow separation) affect cardiac function, especially under pathological conditions. Although the energy dissipation term tends to have much smaller magnitude compared with other energy terms, this is the irreversible loss of the system, and is thus important over the longer term. Figure 16 gives quantified description of energy loss for the fluid fields of different models. A higher dissipation rate is always found near the structure because of the high shear, particularly during opening. The simplified model shows a different energy dissipation pattern and leads to greater energy dissipation compared to other two models, which suggests decreased efficiency.

This study has several limitations. One of these is that homogeneous material properties are assumed for MV leaflets. Some studies [Kunzelman & Cochran, 1992, May-Newman & Yin, 1998] have suggested that this is not realistic, and regionally dependent material parameters should be considered in future works. Although we don’t have sufficient experimental data at hand to directly compare with our CT-scan based MV models at additional time points in the cardiac cycle, our results are consistent with published results of MV studies. Finally, we have shown that the energy budget analysis for the complex flow field has a great potential to provide useful pathological indicators; however, additional critical analyses will be required before it can be used for physical insights into differences between healthy and pathological cases. This requires extensive evaluation on how the energy budget is affected by sufficient number of pathological cases. Finally, we have not attempted to study pathological changes either in the chordae structure or the MV leaflets. However, with the framework developed, it is our future aim to model various scenarios with clinical significance.

8. Conclusions

We have studied the effects of different chordae structures in a dynamic mitral valve model that includes a detailed description of the valve leaflets and chordae. Our computational approach includes fluid-structure interaction, anisotropic hyperelastic constitutive modelling, and energy budget analysis. The simulation has been carefully verified against the commercial software ABAQUS under static loading conditions. Our results show that detailed chordae structure is superior than our previously used simplified chordae structure, which, even when modelled as three-dimensional structures and embedded through the leaflets, leads to a poor leaflets coaptation, large bulging on the anterior leaflet belly, and
different energy transferring behaviour. On the other hand, the pseudo-fibre representation is a good approximation of the more realistic three-dimensional complex solid chordae construction. Indeed, similar mitral valve closure shape, strain and stress patterns, and energy distribution patterns are observed using either the complex or the pseudo-fibre models. This suggests that pseudo-fibres may be a reasonable choice to model mitral chordae in future.

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A Verification Test 1: deformation of a circular disc under constant pressure

Model verification is first done on a simple axis symmetric geometry, as shown in Figs. 20a and 20b. It is a disc-shaped structure with radius of 0.4 cm and thickness of 0.03 cm. In the IB/FE model, the disc is immersed in a viscous fluid with density of 1 g·cm⁻³ and dynamic viscosity of 1 g·cm⁻¹·s⁻¹.

A constant pressure of -20 mmHg is applied on the top surface of the structure in both cases, and in the IB/FE model, periodic conditions are applied on the boundaries of the computational domain Ω. Eq.(18) is used for describing the disc material property. Two families of fibres are defined simply along the circumferential direction and radial direction, respectively.

Figures 21a and 21b show the distribution of displacement magnitudes obtained from IB/FE and ABAQUS. The maximum displacement norm is 11.11 mm in the IB/FE model, which is slightly less
than the value in the ABAQUS model, 11.13 mm. It is clear that two plots have almost the same patterns. The relative difference in the displacements is computed via

\[
\text{relative difference} = \frac{\text{norm of displacement difference}}{\text{average displacement norm from ABAQUS}} \times 100\%.
\]  

Figure 21c shows regions that have relative difference less than 2%, except for the inner edge, the relative difference is very small in most of the disc.

Figures 22a and 22b show the logarithmic maximum principal strain. The maximum values are 0.2143 in the IB/FE model and 0.2485 in the ABAQUS model. Using the same relative difference definition given by Eq. (23), Figure 22c shows all the nodes with relative difference below 5%.

B Verification Test 2: deformation of MV under constant pressure

We use the MV with complex chordae structure to compare the IB/FE and the ABAQUS models under static loading conditions. In the IB/FE model, the MV is immersed in a 5 cm × 4 cm × 6 cm box filled with the same viscous fluid as in the disc case. A constant pressure of -30 mmHg is applied on the atrial
side of the leaflets, the annulus ring is fixed by using stiff tethering forces, and displacement boundary conditions are applied to the chordae ends. Periodic boundary conditions are applied to boundaries of the computational domain $\Omega$. The same structural constraints and loading conditions are also used in the ABAQUS model. Because of the large deformation in MV model, dynamic explicit analysis in ABAQUS is used. Eq. (18) is used for the leaflets, but a Neo-Hookean material is used for the chordae structure

$$W = C(I_1 - 3),$$

with $C = 740 \text{ kPa}$.

![Displacement plot from (a) IB/FE and (b) ABAQUS.](image)

Fig. 23: Displacement plot from (a) IB/FE and (b) ABAQUS.

Figures 23a and 23b show the distribution of displacement magnitudes from the IB/FE and the ABAQUS models when the MV reaches a equilibrium state by maintaining constant loading for a long time. The maximum values are 1.1372 cm in the IB/FE model, and 1.0847 cm in the ABAQUS model. Both models give good agreement in the MV closure shapes, with a mean difference of 0.0689 cm, except for slightly bigger gaps in the IB/FE model as shown in Fig. 23a due to the Eulerian mesh density and the use of delta function in FSI model.

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