Finite temperature effective potential
on hyperbolic spacetimes

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Abstract: The finite temperature one-loop effective potential for a scalar field defined on an ultrastatic spacetime, whose spatial part is a compact hyperbolic manifold, is studied. Different analytic expressions, especially valuable at low and high temperature are derived. Based on these results, the symmetry breaking and the topological mass generation are discussed.

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1 Introduction

One major area of research in quantum field theory in curved spacetime in the 1980s has been on interacting quantum fields and their implications ([1, 2], for a review see [3]). The central object of physical interest is the effective potential which should define the dynamics of the early universe at scales bigger than the Planck scale. In particular in new inflationary models [4, 5] the effective cosmological constant is obtained from an effective potential which includes quantum corrections to the classical potential of a scalar field [6].

Unfortunately there are a lot of difficulties in calculating the effective potential or action in a quantum field theory in a general curved spacetime [7]. Only approximations like the derivative expansion in the background field at zero [7, 8, 9] and at finite temperature [10, 11] are available.
Therefore it is very natural to deal with some specific spaces which are interesting from the cosmological viewpoint. For example the role of constant curvature [12] and of anisotropy in different Bianchi type universes [13, 14, 15, 16, 17, 18, 19, 20] has been considered in detail.

Furthermore the importance of nontrivial topology (imposed, for example, by finite temperature or compactified spatial sections) has been emphasized at several places. Apart from its well know influence on the effective mass of the field [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34] and consequently also on particle creation [35, 36], let us mention as a motivation for such considerations that possibly the universe as a whole exhibits nontrivial topology ([37, 38, 39] and references therein).

Most of the mentioned literature has been concerned with only one compact dimension (representing imaginary time or a compact spatial dimension) and with a spacetime of vanishing curvature. However, in a cosmological setting the inclusion of curvature is of course necessary. For that reason, in a previous paper [40] we started recently investigations on the influence of nontrivial topology together with nonvanishing curvature on the effective potential in a self-interacting scalar field theory. In that article we restricted ourselves to zero temperature. However, in the early universe the inclusion of nonvanishing temperature is necessary and this is the aim of the following article, which is organized as follows.

In Sec. 2 we remind the reader very briefly of the definition of the effective potential in a self-interacting scalar field theory. Afterwards different techniques are applied to find analytic results for the potential. First, in Sec. 3, the Selberg trace formula is used to derive a very compact form of the effective potential of the theory. Then, in Sec. 4, other representations of such a result, but especially useful at low and high temperature, are given. For both results the physical implications in terms of the mass of the field are discussed in Sec. 5. The main results are summarized in the conclusions.

2 One-loop effective potential of the self-interacting theory

The concept of the effective action and the effective potential is well discussed in the literature [2, 11, 6, 42] and the introduction of these quantities will be very brief.

We consider a self-interacting scalar field coupled to gravity on a four-dimensional spacetime with a compact hyperbolic spatial section \( H^3/\Gamma \), \( H^3 \) being the Lobachevsky space and \( \Gamma \) a discrete group of isometries of \( H^3 \), which is assumed to have only hyperbolic elements, so the Riemannian structure is smooth. The theory at finite temperature \( T = 1/\beta \) may be obtained by compactifying the imaginary time \( \tau \) and assuming the field \( \phi(\tau, \vec{x}) \) to fulfill periodic boundary conditions with respect to \( \tau \), this means \( \phi(\tau, \vec{x}) = \phi(\tau + \beta, \vec{x}) \). In this way, the manifold becomes an Euclidean space of the form \( M = S^1 \times H^3/\Gamma \), \( S^1 \) being a circumference.

The theory is described by the action

\[
S(\phi) = \int_0^\beta d\tau \int_{H^3/\Gamma} \left[ -\frac{1}{2} \phi \left( \frac{\partial^2}{\partial \tau^2} + \Delta_S \right) \phi + V(\phi) \right] |g|^{1/2} d^3x ,
\]

with the Laplace-Beltrami operator \( \Delta_S \) acting on the spatial section \( H^3/\Gamma \) of \( M \) and the classical potential

\[
V(\phi) = \frac{m^2 \phi^2}{2} + \frac{\xi R \phi^2}{2} + \frac{\lambda \phi^4}{24} .
\]

As usual \( R \) is the scalar curvature of \( M \), while \( \xi \) and \( \lambda \) are arbitrary dimensionless parameters (coupling constants).

The action (2.1) has a minimum at \( \phi = \hat{\phi} \) which satisfies the classical equation of motion

\[
- \left( \frac{\partial^2}{\partial \tau^2} + \Delta_S \right) \hat{\phi} + V'(\hat{\phi}) = 0 .
\]
Quantum fluctuations $\phi' = \phi - \hat{\phi}$ around the classical background $\hat{\phi}$ satisfy to lowest order in $\phi'$ an equation of the form

$$A\phi'(x) = \left(-\frac{\partial^2}{\partial x^2} - \Delta_S + V''(\hat{\phi})\right) \phi'(x) = \left(-\frac{\partial^2}{\partial x^2} + L\right) \phi'(x) = 0,$$  \hspace{1cm} (2.4)

where we have introduced the operator $L = -\Delta_S + V''(\hat{\phi})$.

Assuming a constant background field $\hat{\phi}$ the concept of the effective potential is well defined [20]. Formally it is given by

$$V_{\text{eff}}(\hat{\phi}) = V(\hat{\phi}) + \hbar V_{\text{T}}^{(1)}(\hat{\phi}) + \mathcal{O}(\hbar^2),$$  \hspace{1cm} (2.5)

with the purely classical part $V(\hat{\phi})$ and with the one-loop quantum corrections

$$V^{(1)}(\hat{\phi}) = \frac{1}{2\beta V_S} \ln \det(A\ell^2) = V_0^{(1)}(\hat{\phi}) + V_{\text{T}}^{(1)}(\hat{\phi}),$$  \hspace{1cm} (2.6)

where $V_S$ is the volume of the spatial hyperbolic part and $\ell$ a length renormalization scaling parameter. Moreover, we distinguished between zero ($V_0^{(1)}(\hat{\phi})$) and finite ($V_{\text{T}}^{(1)}(\hat{\phi})$) temperature contributions. Using $\zeta$-function prescription for the regularization of functional determinants [23, 14], Eq. (2.6) assumes the form

$$V^{(1)}(\hat{\phi}) = -\frac{1}{2\beta V_S} \left[\zeta'(0|A) + \zeta(0|A) \ln \ell^2\right],$$  \hspace{1cm} (2.7)

where $\zeta(s|A)$ is the $\zeta$-function associated with the operator $A$ and $\zeta'(s|A)$ its derivative with respect to $s$. More explicitly, using the Poisson summation formula, one has

$$\zeta(s|A) = \sum_{n=-\infty}^{\infty} \sum_j \left[\omega_j^2 + (2\pi n/\beta)^2\right]^{-s} = \sum_{n=-\infty}^{\infty} \zeta(s|L + [2\pi n/\beta]^2)$$  \hspace{1cm} (2.8)

$$= \frac{\beta}{\pi} \sum_{n=-\infty}^{\infty} \int_0^{\infty} \zeta(s|L + t^2) \cos n\beta t \, dt,$$  \hspace{1cm} (2.9)

$\omega_j^2$ being the eigenvalues of the operator $L$. The term $n = 0$ in Eq. (2.8) leads to the function $\zeta(s|L) = \sum_j (\omega_j^2)^{-s}$ on the spatial section. In the explicit computation, it is convenient to distinguish between $n = 0$ and $n \neq 0$. Then, using Eq. (2.9) and the Mellin representation of $\zeta$-function, we obtain

$$\zeta(s|A) - \frac{\beta\Gamma(s - \frac{1}{2})\zeta(s - \frac{1}{2}|L)}{\sqrt{\pi}\Gamma(s)} = \frac{2\beta}{\pi} \sum_{n=1}^{\infty} \int_0^{\infty} \zeta(s|L + t^2) \cos n\beta t \, dt$$  \hspace{1cm} (2.10)

$$= \frac{\beta}{\sqrt{\pi}\Gamma(s)} \sum_{n=1}^{\infty} \int_0^{\infty} t^{s-3/2} e^{-n^2\beta^2/4t} \text{Tr} e^{-tL} \, dt.$$  \hspace{1cm} (2.11)

We note that the $n = 0$ term in Eqs. (2.11) and (2.10) gives the zero temperature contribution $V_0^{(1)}(\hat{\phi})$ to the effective potential. Such a contribution, which requires renormalization, has been extensively considered in Ref. [14] to which we refer the reader for a detailed discussion. Here we shall concentrate our attention only on the effects due to non zero temperature. Using Eqs. (2.7), (2.10) and (2.11) we get

$$V_{\text{T}}^{(1)}(\hat{\phi}) = -\frac{1}{\pi V_S} \lim_{s \to 0} \sum_{n=1}^{\infty} \int_0^{\infty} \zeta'(s|L + t^2) \cos n\beta t \, dt,$$  \hspace{1cm} (2.12)

$$= -\frac{1}{\sqrt{4\pi V_S}} \sum_{n=1}^{\infty} \int_0^{\infty} t^{-3/2} e^{-n^2\beta^2/4t} \text{Tr} e^{-tL} \, dt,$$  \hspace{1cm} (2.13)

$$= \frac{1}{\beta V_S} \text{Tr} \ln \left(1 - e^{-\beta Q}\right).$$  \hspace{1cm} (2.14)
where the operator $Q = |L|^{1/2}$ has eigenvalues $\omega_j$. As it is well known, low temperature contributions to the effective potential are exponentially vanishing.

In view of a high temperature expansion, it is convenient to write Eq. (2.13) in a more useful form. To this aim we recall the Mellin transforms

$$\hat{f}(z) = \int_0^\infty x^{z-1} f(x) \, dx ,$$  \hspace{1cm} (2.15)

$$f(x) = \frac{1}{2\pi i} \int_{\text{Re} \, z = c} x^{-z} \hat{f}(z) \, dz ,$$  \hspace{1cm} (2.16)

c being a real number belonging to the strip in which $\hat{f}(z)$ is analytic, and the Mellin-Parseval identity

$$\int_0^\infty f(x)g(x) \, dx = \frac{1}{2\pi i} \int_{\text{Re} \, z = c} \hat{f}(z)\hat{g}(1-z) \, dz ,$$  \hspace{1cm} (2.17)

designed for any pair of functions $f, g$ with Mellin transforms $\hat{f}, \hat{g}$. Then choosing $f(t) = t^{-3/2} \text{Tr} \, e^{-tL}$, $g(t) = e^{-\epsilon t^2\beta^2/4t}$ and using Eqs. (2.13) and (2.17), we obtain the so called Mellin-Barnes representation for the statistical sum contribution [13]

$$V_T^{(1)}(\phi) = -\frac{1}{2\pi i} \int_{\text{Re} \, z = c} \Gamma(s-1)\zeta_R(s)\zeta(\frac{s-1}{2}|L|) \beta^{-s} \, ds .$$  \hspace{1cm} (2.18)

This is an exact expression, which is particularly useful in the derivation of the high temperature expansion of the effective potential, once the $\zeta$-function related to the operator $L$ is known. In Eq. (2.18), $\zeta_R(s)$ is the usual Riemann $\zeta$-function.

3 Selberg trace formula

The $\zeta$-function of the Laplace-Beltrami operator acting on the compact hyperbolic manifold $H^3/\Gamma$ can be related to the properties of the discrete group $\Gamma$ of isometries of $H^3$ by means of Selberg trace formula. To state such a relation, let $h(r)$ be an even and holomorphic function in a strip larger than 2 about the real axis and $h(r) = O(r^{-(3+\epsilon)})$ uniformly in this strip as $r \to \infty$. Then the Selberg trace formula reads [13, 17] (here we are considering untwisted scalar fields, namely functions such that $\phi(\gamma x) = \phi(x)$, $\forall \gamma \in \Gamma$)

$$\sum_{j=0}^\infty h(r_j) = \frac{V_F}{2\pi^2} \int_0^\infty r^2 h(r) \, dr + \sum_{k=1}^\infty \sum_{\gamma \in \Gamma} \frac{l_\gamma \hat{h}(k\gamma)}{S_\gamma(k; l_\gamma)} .$$  \hspace{1cm} (3.1)

As usual, we normalize the constant curvature to $\kappa = R/6 = -1$. In this manner all quantities will be dimensionless. Here $V_F$ is the volume of the fundamental domain $F$, which, in normal units, is just the volume of $H^3/\Gamma$, and $h$ is the Fourier transform of $h$, that is

$$\hat{h}(u) = \frac{1}{2\pi} \int_{-\infty}^\infty e^{-iru} h(r) \, dr .$$  \hspace{1cm} (3.2)

To help the reader understanding the formula, we briefly explain the double series on the right hand side. The symbol $\mathcal{P}$ denotes the set of all primitive closed geodesics in the compact manifold, namely, those geodesics passing only once through any given point. If $\gamma \in \mathcal{P}$ then its length is denoted by $l_\gamma$. The integer $k$ is the winding number of any non primitive geodesic. Hence, the sum over all primitive geodesics followed by the sum over $k$, which appears in the trace formula, is nothing but the sum over all geodesics of the compact manifold. This is expected because every geodesic must contribute to the trace. Now to every closed primitive geodesic $\gamma$, with length $l_\gamma$, we can associate an isometry, also denoted $\gamma$, namely, the unique isometry such
that the hyperbolic distance \( d(x, \gamma x) = l_\gamma \). Here \( x \in H^3 \) is any point whose projection in the quotient manifold belongs to \( \gamma \). Thus \( \gamma \in PSL(2, \mathbb{C}) \) (the isometry group of \( H^3 \)) and so it can be conjugated to a diagonal matrix, whose upper entry we call \( a_\gamma \) (the lower entry is then \( a_\gamma^{-1} \)). A simple calculation will show that \( l_\gamma = \ln(|a_\gamma|) \) and finally \( S_3(k; l_\gamma) = |a_\gamma^k - a_\gamma^{-k}|^2 \). This is seen to be essentially the Jacobian determinant of the parallel transport around \( \gamma \) (see Ref. [18] for details).

In Eq. (3.1), the sum on the left hand side is over the spectrum of the Laplace operator with eigenvalues \( -\lambda_j \), and \( r_j = \sqrt{\lambda_j - 1} \), if \( \lambda_j > 1 \), while \( r_j = i\sqrt{1 - \lambda_j} \) if \( \lambda_j < 1 \). Finally, the integral is the contribution of those geodesics which are contractible to a point, i.e. it is the direct path contribution to the trace.

In order to compute \( \text{Tr} \exp(-tL) \) by Selberg trace formula, we may choose \( h(r) = \exp(-t[r^2 + M^2]) \), whose Fourier-transform reads

\[
\hat{h}(u) = \frac{Me^{-tM^2-u^2/4t}}{\sqrt{4\pi t}} \quad (3.3)
\]

Here we have set

\[
M^2 = V''(\phi) - \kappa = m^2 + \frac{\lambda\phi^2}{2} + \left( \xi - \frac{1}{6} \right) R. \quad (3.4)
\]

By a simple calculation we obtain

\[
\text{Tr} \exp(-tL) = V_F e^{-tM^2} + \frac{e^{-tM^2}}{(4\pi t)^{3/2}} \sum_{p} \sum_{k=1}^{\infty} \frac{l_\gamma e^{-k^2\lambda_j^2/4t}}{S_3(k; l_\gamma)}, \quad (3.5)
\]

\[
\zeta(s|L) = \frac{V_F}{(4\pi)^{3/2}} \frac{\Gamma(s - 3/2)M^{3-2s}}{\Gamma(s)} + \sum_{p} \sum_{k=1}^{\infty} \frac{l_\gamma}{S_3(k; l_\gamma)} \left[ \frac{kl_\gamma}{2M} \right]^{s-1/2} \frac{K_{s-1/2}(Mkl_\gamma)}{\pi^{1/2} \Gamma(s)}, \quad (3.6)
\]

while the effective potential reads

\[
V_T^{(1)}(\phi) = -\frac{M^4}{2\pi^2} \sum_{n=1}^{\infty} \frac{K_2(n\beta M)}{(n\beta M)^2}
- \frac{M^2}{\pi V_S} \sum_{n=1}^{\infty} \sum_{p} \sum_{k=1}^{\infty} \frac{l_\gamma K_1 \left( M \sqrt{n^2\beta^2 + k^2\lambda_j^2} \right)}{S_3(k; l_\gamma) M \sqrt{n^2\beta^2 + k^2\lambda_j^2}}, \quad (3.7)
\]

where \( K_\nu(s) \) represents the modified Bessel function. From Eq. (3.6) we immediately get the residue of \( \zeta(s|L) \) in the poles \( s = 3/2 - k \) \((k = 0, 1, 2 \ldots)\). That is

\[
\text{Res}(\zeta(s|L); s = 3/2 - k) = \frac{(-1)^k M^{2k}}{(4\pi)^{3/2}k!} \frac{V_F}{\Gamma(3/2 - k)}. \quad (3.8)
\]

To complete this section we introduce the Selberg \( \Xi \) and \( Z \) functions, which contain all topological informations. They are defined by means of formula

\[
\Xi(s) = \frac{d}{ds} \ln Z(s) = \sum_{p} \sum_{k=1}^{\infty} \frac{l_\gamma e^{-(s-1)kl_\gamma}}{S_3(k; l_\gamma)}, \quad (3.9)
\]

and can be found for example in Ref. [16] to which we ones more refer the reader for more details. Since the number of closed primitive geodesics with a given length \( l_\gamma \) is asymptotically
\( l_\gamma^{-1} \exp(2l_\gamma) \) and \( S_\delta(k;l_\gamma) \simeq \exp(kl_\gamma) \), the series defining the \( \Xi(s) \)-function is convergent for \( \text{Re} s > 2 \). The Selberg function \( Z(s) \), on the other hand, is entire of order 3 and has zeroes at the points \( s_j = 1 \pm ir_j \), whose order is the eigenvalues multiplicity.

By means of the above functions, it is possible to rewrite the topological contribution to \( \zeta(s|L) \). In fact we obtain

\[
\zeta(s|L) = \frac{V_F}{(4\pi)^{3/2}} \frac{\Gamma(s-3/2)M^{3-2s}}{\Gamma(s)} 
+ \frac{\sin \pi s M^{1-2s}}{\pi} \int_1^\infty \Xi(1 + Mu)(u^2 - 1)^{-s} \, du,
\]

from which easily follows

\[
\zeta'(0|L) = \frac{V_F M^3}{6\pi} - \ln Z(M + 1) .
\]

Now, replacing \( M^2 \) with \( M^2 + t^2 \) in the latter formula and using Eq. (2.12), we obtain the nice representation

\[
V_T^{(1)}(\phi) = -\frac{M^4}{2\pi^2} \sum_{n=1}^{\infty} \frac{K_2(n\beta M)}{(n\beta M)^2} 
+ \frac{1}{\pi V_S} \sum_{n=1}^{\infty} \int_0^\infty \ln Z \left( 1 + \sqrt{|t^2 + M^2|/|\kappa|} \right) \cos n\beta t \, dt ,
\]

where normal units have been restored.

4 Low and high temperature expansions

The low temperature behaviour of the theory can be directly obtained from Eq. (3.14) since the asymptotic expansion of modified Bessel function is well known. In fact we have

\[
V_T^{(1)}(\phi) \sim -\frac{1}{2V_S} \frac{M^2}{(2\pi)^{1/2}} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{l_\gamma \exp(-n\beta M \sqrt{1 + (kl_\gamma/n\beta)^2})}{S_\delta(k;l_\gamma) \left( n\beta M \sqrt{1 + (kl_\gamma/n\beta)^2} \right)^{3/2}} 
\times \left[ 1 + \frac{3}{4 \left( n\beta M \sqrt{1 + (kl_\gamma/n\beta)^2} \right)} \right] - \frac{M^4}{2(2\pi)^{3/2}} \sum_{n=1}^{\infty} \frac{e^{-n\beta M}}{(n\beta M)^{5/2}} .
\]

We see that the leading term comes from the topological part.

In order to get the high temperature expansion of the effective potential, we shall perform the integration in Eq. (2.18). To this aim we observe that the integrand function in Eq. (2.18) has simple poles at the points \( s = 4,2,1,-2k \) \((k = 1,2,\ldots)\) and a double pole at \( s = 0 \). Hence integrating this function along a path enclosing all the poles, we obtain a series expansion in powers of \( \beta \), whose coefficients are related to the residues of \( \zeta \)-function, which have been computed in Sec. 3 (see also Ref. [49]). After a straightforward computation, using Eq. (3.10) we obtain the high temperature expansion (here \( \gamma \) is the Euler-Mascheroni constant)

\[
V_T^{(1)}(\phi) \sim \frac{\pi^2 T^4}{90} + \frac{M^2 T^2}{24} - \left[ \frac{M^3}{12\pi} - \frac{M}{2V_S} \ln Z(1 + M) \right] T
- \frac{M^4}{32\pi^2} \left[ \ln \frac{M}{4\pi T} + \gamma - \frac{3}{4} \right]
- \frac{M^2}{2\pi V_S|\kappa|^{1/2}} \int_1^\infty \Xi(1 + tM|\kappa|^{-1/2}) \sqrt{t^2 - 1} \, dt + O(T^{-2}) ,
\]
which in the case of flat, topological trivial space, is in agreement with results of Ref. [50].

It is interesting to note that all coefficients of the negative powers of $T$ are proportional to the residues of $\zeta(s|L)$ and so they do not depend on the topology. Moreover, the topological term independent of $T$ in the latter formula is the same, but the sign, as the topological contribution to the zero temperature effective potential, also after renormalization (see Ref. [40]). This means that topology enters the high temperature expansion of the effective potential only with a term proportional to $T$.

5 Phase transitions of the system

The relevant quantity for analyzing the phase transitions of the system is the mass of the field. The quantum corrections to the mass are defined by means of equation

$$V^{(1)}(\dot{\phi}) = \Lambda_{eff} + \frac{1}{2}(m_0^2 + m_T^2)\dot{\phi}^2 + O(\dot{\phi}^4),$$ (5.1)

where $\Lambda_{eff}$ (the cosmological constant) in general represents a complicated expression not depending on the background field $\dot{\phi}$ while $m_0$ and $m_T$ represent the zero and finite temperature quantum corrections to the mass $m$. As has been shown in Ref. [40], $m_0$ has curvature and topological contributions, which help to break the symmetry (for $\xi < 1/6$). On the contrary, here we shall see that $m_T$ always helps to restore the symmetry.

By evaluating the second derivative with respect to $\dot{\phi}$ at the point $\dot{\phi} = 0$ of Eqs. (2.14) and (3.12), we obtain for $m_T$ the two equations

$$m_T^2 = \frac{\lambda}{2V_S} \text{Tr} \frac{e^{-\beta Q_0}}{Q_0(1 - e^{-\beta Q_0})},$$

$$= \frac{\lambda M_0^2}{4\pi^2} \sum_{n=1}^{\infty} \frac{K_1(n\beta M_0)}{n\beta M_0}$$

$$+ \frac{\lambda}{2\pi V_S|\kappa|} \sum_{n=1}^{\infty} \int_0^{\infty} \frac{\Xi(1 + \sqrt{[t^2 + M_0^2]/|\kappa|})}{\sqrt{[t^2 + M_0^2]/|\kappa|}} \cos n\beta t dt,$$ (5.3)

where $Q_0 = Q_{\phi=0} = | - \Delta_S + m^2 + \xi R |^{1/2}$ and $M_0 = M_{\phi=0} = | m^2 + (\xi - 1/6) R |^{1/2}$. From the exact formula, Eq. (5.2), we see that the finite temperature quantum corrections to the mass are always positive, their strength mainly depending on the smallest eigenvalues of the operator $Q_0$. This means that such a contribution always helps to restore the symmetry. The second very interesting expression, Eq. (5.3), gives the mass in terms of geometry and topology of the manifold. In fact, the $\Xi$-function is strictly related to the isometry group $\Gamma$, which realizes the non trivial topology of $M$.

To go further, we compute the corrections to the mass in the low and high temperature limits. Using Eqs. (4.1) and (4.2) we obtain

$$m_T^2 \sim \frac{\lambda}{4(2\pi)^{3/2}} \frac{M_0^2}{(n\beta M_0)^{3/2}} \sum_{n=1}^{\infty} \frac{e^{-n\beta M_0}}{(n\beta M_0)^{3/2}}$$

$$+ \frac{\lambda}{4V_S} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{1}{S_3(k; l_n)} \left( \frac{n\beta \sqrt{1 + (kl_n/n\beta)^2}}{4(n\beta M_0 \sqrt{1 + (kl_n/n\beta)^2})} \right)^{1/2}$$

$$\times \left[ 1 + \frac{1}{4(n\beta M_0 \sqrt{1 + (kl_n/n\beta)^2})} \right],$$ (5.4)
\[ m_T^2 \sim \frac{\chi T^2}{24} - \frac{3M_0\chi T}{8\pi} + \frac{|\kappa|^{-1/2}}{4M_0^2 V_S} \left[ \ln Z(1 + M_0|\kappa|^{-1/2}) + M_0|\kappa|^{-1/2}\ln(1 + M_0|\kappa|^{-1/2}) \right] M_0\chi T , \] 

(5.5)

which are valid for low and high temperature respectively.

6 Conclusions

In this paper we considered a self-interacting scalar field living on an ultrastatic spacetime whose spatial part is a compact hyperbolic manifold. We were especially interested in the finite temperature effective potential of the theory, which serves as the effective cosmological constant in new inflationary models. As we have seen, the main technical tool to obtain the effective potential was the Selberg trace formula, Eq. (3.1). Analytical expressions for the effective potential suitable in the low, Eq. (4.1), and high temperature limit, Eq. (4.2), have been obtained. Whereas at \( T = 0 \) for \( \xi < \frac{1}{6} \) quantum corrections to the classical potential can help to break symmetry [10], the results show that, if the temperature is high enough, quantum corrections always help to restore the symmetry (see Eq. (5.5)).

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