Searching for Extra Dimensions in the Early Universe

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Abstract

We investigate extra spatial dimensions \((D = 3 + \epsilon)\) in the early universe using very high resolution molecular rotational spectroscopic data derived from a large molecular cloud containing moderately cold carbon monoxide gas at \(Z \approx 6.42\). It turns out that the \(\epsilon\)-dependent quantum mechanical wavelength transitions are solvable for a linear molecule and we present the solution here. The CO microwave data allows a very precise determination of \(< \epsilon > = -0.00000657 \pm 0.0003032\). The probability that \(< \epsilon > \neq 0\) is one in 7794, only 850 million years (using the standard cosmology) after the Big Bang.

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Recently, microwave molecular CO spectroscopic transitions have been detected [1,2] from the most distant quasar currently known, SDSS J114816.64+525150.3, at a redshift $Z \approx 6.42$. The data are from rotational transitions of CO at high excitation ($J = 7 \rightarrow 6$), ($J = 6 \rightarrow 5$) [1] and ($J = 3 \rightarrow 2$) [2]. This experimental result is extremely interesting, since it has already been shown that atomic hydrogen Lyman transitions from quasars can bound the departure from the known number of space-time dimensions ($\epsilon = D_{\text{spacetime}} - 4 = D - 3$) in the early universe [3]. The interest on extra dimensions comes from the fact that they appear naturally in string theories [4]. In general, there is no known reason why the dimension of space must have the value of 3, or even be an integer. Since at present this number is three, and since in the very early universe there exists the possibility of a larger number, this indicates that it should be possible to define an effective number of space dimensions which changes continuously with time, from its very early universe value to the present one. Moreover, by looking at relics of the early universe, such as the cosmic microwave background or the light emitted by quasars with a very high redshift, it should be possible to measure deviations of the number of spatial dimensions, $D$, from its present (epoch) value of 3.

All that is known is that the present epoch value of $\epsilon$ has an exceedingly small upper bound [5]. It is an open experimental question whether $< \epsilon > = 0$ in the early universe, as is apparent in the current epoch. In reference [3], it was shown that quasar hydrogen Ly$\alpha$ and Ly$\beta$ carry information about $D = 3 + \epsilon$ dimension of space, and using available quasar database information, a possible positive experimental signal $\epsilon \neq 0$ was found for $Z_c > 4$. A major difference between this present paper and [3] is that other alternative spectroscopic signals are utilized here. The Lyman lines past Ly$\alpha$: (Ly$\beta$, Ly$\gamma$, · · · ) are subject to absorption by intervening hydrogen clouds (Lyman forest), leaving in some cases, imprecise locations of the lines. Due to absorption, the Lyman series in hydrogen becomes difficult to use for $< \epsilon >$ determination for very high $Z_c \geq 4$. However, rotational spectra of linear molecules, already starting out in the microwave region, are redshifted towards the radio, so therefore they do not suffer the intense random absorption events as optical lines. We show that they offer unique opportunities for precise determination of $D = 3 + \epsilon$ dimension of space.

The spacing of atomic and molecular energy levels varies with the dimension of space. In particular, $\epsilon$ differences between two fractal geometries give rise to corresponding $\epsilon$ differences in their quantum mechanical energies.

It is, in principle, possible to solve the Schrödinger equation in $D = 3 + \epsilon$
dimension of space \((4 + \epsilon \text{ dimension of space-time})\), using a Taylor expansion about \(D = 3\) [3].

\[
< |E| > |_{D=3+\epsilon} = < |E| > |_{D=3} + \frac{d < |E| >}{dD} |_{D=3} \epsilon + \cdots
\]  

(1)

The Hellmann-Feynman theorem is

\[
\frac{d < |E| >}{dD} |_{D=3} = < \frac{\partial H}{\partial D} |_{D=3} >
\]

(2)

where \(H\) is the D-dimensional Hamiltonian. It can be shown that a first-order Taylor expansion is always available because physical energies are continuous at \(D = 3\). Because of mathematical complexity, however, only the hydrogen atom has been presently amenable to computations of energy levels for \(\epsilon \neq 0\). We now show that another physical system, linear molecules, can have their energy levels computed for \(\epsilon \neq 0\). We are especially interested in linear molecules, such as CO, because, being the simplest molecules, they are the ones most likely to be identified in giant molecular clouds.

The Hamiltonian is

\[
H_r = B(L)L^2
\]

(3)

where \(L\) is the body (molecule)-fixed rotational angular momentum and \(B\) is the principal rotational constant (equal to \(\frac{1}{2I}\), where \(I\) is the principal moment of inertia). Because of centrifugal stretching, and higher order effects, \(B = B(L)\). We need to generalize this energy to \(D = 3 + \epsilon\) fractal space. In quantum mechanics, \(L^2\) is a second order Casimir invariant operator, \(C_2\). The generalized rotation operator [6] that correctly incorporates higher order effects is

\[
\mathcal{H}_r = B(C_2)C_2
\]

(4)

\[
B(C_2) = B_0 + B_1C_2
\]

(5)

where \(B_0\) and \(B_1\) are constants for zero vibrational quanta states (as is the astrophysical situation). In general, the second order Casimir invariant is [7]:

\[
C_2 = f_{jk}^i f_{il}^j X^k X^l = H_i G_{ij} H_j + \sum_{\text{all roots}} E^\alpha E_{-\alpha}
\]

(6)
where \( f_{jk} \) are the structure constants, \( X^k \) are generators, and \( C_2 \) commutes with all generators. The Racah formula for the eigenvalue of \( C_2 \) for any irreducible representation is easily derived by letting \( C_2 \) act on the state with highest weight \( \Lambda \). The result is:

\[
C_2 = (\Lambda, \Lambda + 2\delta)
\]

where \( \delta = (1, 1, ..., 1) \) in the Dynkin basis. The scalar product of any two weights can be written as:

\[
(\Lambda, \Lambda') = \sum_{ij} a'_j G_{ij} a_j
\]

where \( G_{ij} \) is a symmetric tensor whose elements can be computed for each simple group, and which are given in Table 7 of reference [7], and the \( a_i \) are the Dynkin components of \( \Lambda \).

In our case we want to find the Casimir invariant for the \((L, 0, ..., 0)\) totally symmetric representation. The choice of representation depends, of course, on the way in which we want to generalize angular momentum, and the correct choice, we argue, would preserve the symmetry properties (in this case this would mean to keep the completely symmetric coupling) when generalizing to larger dimensions.

For odd space dimensions \( D = 2n + 1, n = 1, 2, ..., \) the algebra is \( B_n \), and for even space dimensions \( D = 2n, n = 1, 2, ..., \) it is \( D_n \). We can calculate \( C_2 \) using equations (7) and (8), and the \( a_j \) values given by \((L, 0, ..., 0)\):

\[
C_2 = (L, 0, ..., 0)G(B_n \text{ or } D_n)(2 + L, 2, 2, ..., 2)
\]

which gives, by simple matrix multiplication, both for \( B_n \) and \( D_n \):

\[
C_2 = L(L + D - 2)
\]

The above formulae can be analytically continued to any non-integer number of spatial dimensions.

The pure rotational energies of linear molecules in \( D \)-dimension space are then:

\[
H_{rot} = [B_0 + B_1(L(L + 1))]L(L + 1) + \{2B_1L^2(L + 1) + B_0L\}c
\]

It is a simple matter to determine \( B_0 = 57.635968 \) GHz and \( B_1 = -1.835 \times 10^{-4} \) GHz for CO, giving Table 1.
Rotational rest-frame $\epsilon$ values

| Transition | in GHz          | in mm            |
|------------|-----------------|------------------|
| $7 \rightarrow 6$ | 806.651719 +57.584588$\epsilon$ | .371650429-0.026531074 $\epsilon$ |
| $6 \rightarrow 5$ | 691.47309 +57.598534$\epsilon$ | .433556218-0.036114496 $\epsilon$ |
| $3 \rightarrow 2$ | 345.795991 +57.62716$\epsilon$ | .86696337-0.14480092 $\epsilon$ |

Table 1: Pure $D$-dimensional rotational levels for CO

If $\epsilon \neq 0$, the result is a change in each transition by a unique amount. The effect is unmistakable: even a very tiny $\epsilon$ will be easily detectable.

We now evaluate the experimental data [1,2]. The theoretical procedure to determine $\epsilon$ and the error $\delta \epsilon$ is given in [3]. Once two rotational lines are identified, then $\epsilon$ is derivable by

$$\epsilon = \frac{\tau_M \lambda_1 - \lambda_0}{a - b \tau_M}$$  \hspace{1cm} (12)

where

$$\tau_M = \frac{\lambda_{0M}}{\lambda_{1M}}$$

with $\lambda_{0M}$, $\lambda_{1M}$ the two measured redshifted rotational lines, which have molecular cloud rest-frame wavelengths $\lambda_0 + a \epsilon$, $\lambda_1 + b \epsilon$, where $\lambda_0$, $\lambda_1$ are the present epoch (i.e. laboratory) transition wavelengths. In this equation, $a$ and $b$ are read off from Table 1.

Note that here, the error in the determination of the coefficients that multiply $\epsilon$ (labeled $\delta a$, $\delta b$ in [3]) is zero, because the generalized pure rotation operator has already included the higher order terms.

The instrumental precision centered on the observed frequencies is 50 MHz for $(J = 3 \rightarrow 2)$ [2] and 5 MHz for both $(J = 7 \rightarrow 6)$, $(J = 6 \rightarrow 5)$ [1]. The observed widths of the lines in km/s is 279 for both $(J = 7 \rightarrow 6)$, $(J = 6 \rightarrow 5)$ and 320 (50 MHz) for $(J = 3 \rightarrow 2)$. We reduce the data to Table 2, using the speed of light $c = 2.99792458 \times 10^{11}$ mm/s.

The procedure in [3] uses pairs of lines from the same source, so the three lines here yields the unique case of three separate determinations of $\epsilon$ and its error $\delta \epsilon$. In Table 3, we give the results for the three possible line combinations.

The average values are
### Processed data

| Transition | $\lambda_{\text{obs}}$ (mm) | $\Delta\lambda_{\text{obs}}$ (mm) | $\Delta\lambda_{\text{resol}}$ (mm) |
|------------|-----------------------------|-----------------------------------|-----------------------------------|
| $7 \to 6$  | 108.729(9)/c                 | 2.56601(-3)                       | 1.26794(-4)                       |
| $6 \to 5$  | 93.204(9)/c                  | 2.99343(-3)                       | 1.725526(-4)                      |
| $3 \to 2$  | 46.610(9)/c                  | 6.865475(-3)                      | 6.865475(-3)                      |

Table 2: CO lines with experimental precision ($\Delta$). The number in parenthesis is the exponent, i.e. (-3) $\equiv 10^{-3}$

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| microwave pair | $\epsilon$ | $\delta\epsilon$ |
|----------------|-------------|-------------------|
| $(J = 7 \to 6)$ with $(J = 6 \to 5)$ | -0.000012043 | 0.118937801 |
| $(J = 7 \to 6)$ with $(J = 3 \to 2)$ | -0.000004379 | 0.084521824 |
| $(J = 6 \to 5)$ with $(J = 3 \to 2)$ | -0.000003283 | 0.096631339 |

Table 3: $\epsilon$ and error $\delta\epsilon$

\[
\begin{align*}
<\epsilon> & = -0.00000657 \\
<\delta\epsilon> & = 0.10003032
\end{align*}
\]

Tuning $<\delta\epsilon>$ as the standard deviation, one can perform the statistical Z-test [8]. This predicts that the probability of $\epsilon \neq 0$ to be one in 7794.

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