Methodology for calculating power consumption of planetary mixers

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Abstract. The paper presents the methodology and equations for calculating the power consumption necessary to overcome the resistance of a dry mixture caused by the movement of cylindrical rods in the body of a planetary mixer, as well as the calculation of the power consumed by idling mixers of this type. The equations take into account the size and physico-mechanical properties of mixing material, the size and shape of the mixer’s working elements and the kinematics of its movement. The dependence of the power consumption on the angle of rotation in the plane perpendicular to the axis of rotation of the working member is presented.

1. Introduction
Planetary mixers are widely used in the production of building materials [1]. Their main purpose is to stir dry construction materials used in finishing works. This paper deals with the construction (Fig. 1) of a planetary mixer [2, 3] that differs in that it uses a cylindrical agitator moving along a complex cycloidal trajectory. Moreover, the paper considers the mixer design simplification.

2. Method and discussion
The cycloidal movement of agitators creates a pressure difference in the stirred medium, and their location along the spiral curve on a movable part of the machine provides the most complete coverage of the mixing tank, which helps to avoid the formation of dead spaces.

The use of bars with a circular cross section as kneaders allows one to reduce the energy consumption by reducing the internal and external resistance of the stirred medium.

To find the resistance value of the medium (dry mortar) by a moving cylindrical rod that has diameter $d_0$, let us regard a moving medium be as a highly viscous pseudo-liquid, with density $\gamma$, overflowing a fixed rod.
Figure 1. A planetary mixer
1 – feeder, 2 – housing cover, 3 – engine, 4 – output shaft, 5 – crank, 6 – gear, 7 – gear crown, 8 – housing, 9 – agitator, 10 – discharge unit.

According to the design scheme shown in Fig. 2, let us introduce a cylindrical coordinate system \((r, \varphi, z)\).

Figure 2. The design scheme for determining the resistance value of the medium on the outer surface of the rod

To find the resistance value on the surface of the dedicated "n" rod by the moving medium, let us select elementary area \(dS\) on the outer surface of the rod. According to the design scheme in Fig. 1, let us have:

\[
dS = \frac{d_0}{2} d\varphi dz. \tag{1}\]
where $\chi$ is a polar angle measured from the positive direction of the OX axis.

Elementary area (1) will be affected by an elementary force as follows:

$$dF_n = PdS,$$

where $P$ is the pressure of the medium at distance $z$ from the filling level of the agitator housing and it is equal to:

$$P = P_{\alpha,n} + P_c,$$

$$P_{\alpha,n} = \frac{\gamma U_n^2}{2};$$

$$P_c = \gamma g z,$$

where $g$ is the free fall acceleration;

$\gamma$ is material density.

Relation (4) is a dynamic component of the moving medium pressure, and (5) is a static component of the pressure.

Substitution (3) with regard to (4) and (5) in (2) leads to the following result:

$$dF_n = \gamma \left(\frac{U_n^2}{2} + g z\right)dS.$$

The integration of expression (6) enables one to obtain the following relation:

$$F_n = \iint \gamma \left(\frac{U_n^2}{2} + g z\right)\frac{d_0}{2} d\chi dz.$$

The evaluation of integral (7) leads to the following result:

$$F_n = \frac{d_0}{2} \int_0^{\pi/2} d\chi \int_0^H \gamma \left(\frac{U_n^2}{2} + g z\right)dz = \left(\frac{\pi}{2} \gamma U_n^2 H + \frac{\gamma g}{2} H^3\right) \frac{d_0}{2},$$

where $H$ is the loading height of the material of the cylindrical agitator housing.

The working member of this mixer moves along a complex trajectory - the hypotrochoid [4-6], whose velocity can be determined on the basis of the methods of [7], thus one obtains the equation:

$$U_n = \omega_0 (R_0 - r) \sqrt{1 + \left(\frac{r_n}{r_0}\right)^2 - 2 \frac{r_n}{r_0} \cos \left(\frac{R_0 - \varphi}{r_0}\right)}.$$

where $r_0$ is the radius of the mobile gear;

$R_0$ is the inner radius of the fixed gear;

$\varphi$ determines the change in the angle in the plane perpendicular to the rotation axis from the point with coordinates $\varphi_n$, $r_n$ to point $\varphi_n + 2\pi, r_n$.

The substitution of (9) in (8) after minor changes results in:

$$F_n = H d_0 \frac{\pi}{4} \omega_0 (R_0 - r) \left[\omega_0 (R_0 - r)^2 \left(1 + \left(\frac{r_n}{r_0}\right)^2 - 2 \frac{r_n}{r_0} \cos \left(\frac{R_0 - \varphi}{r_0}\right)\right) + g \cdot H\right].$$
Let us express value $H$ in terms of the height of mixer housing $H_0$ according to the relation:

$$ H = H_0 \cdot \psi, \quad (11) $$

where $\psi$ is the load factor of the planetary mixer casing.

The magnitude of the elementary work to overcome resistance force (10) when moving cylindrical rods along the trajectory by amount $dl_n$ will be equal to:

$$ dA_n = F_n \cdot dl_n, \quad (12) $$

where $dl_n$ is determined based on the following equation (13) by the following value:

$$ L_n = (R_0 - r_0) \int_{\varphi_n}^{\varphi_n + 2\pi} \sqrt{1 + \left(\frac{r_n}{r_0}\right)^2 - 2 \frac{r_n}{r_0} \cos \left(\frac{R_0}{r_0} \varphi\right)} \, d\varphi, \quad (13) $$

where $L_n$ is the path length passed by the "$n$" rod per one complete revolution of the mobile gear of the planetary mixer:

$$ dl_n = (R_0 - r_0) \cdot \sqrt{1 + \left(\frac{r_n}{r_0}\right)^2 - 2 \frac{r_n}{r_0} \cos \left(\frac{R_0}{r_0} \varphi\right)} \, d\varphi. \quad (14) $$

Substitution (10) and (14) with regard to (12) and (13) leads to the following result:

$$ dA_n = \frac{\pi H_0 \psi d_0}{4} \cdot \left[\omega_0^2 (R_0 - r_0)^3 \cdot \left(1 + \left(\frac{r_n}{r_0}\right)^2 - 2 \frac{r_n}{r_0} \cos \left(\frac{R_0}{r_0} \varphi\right)\right)^{\frac{3}{2}} + gH_0 \psi (R_0 - r_0) \cdot \left(1 + \left(\frac{r_n}{r_0}\right)^2 - 2 \frac{r_n}{r_0} \cos \left(\frac{R_0}{r_0} \varphi\right)\right)^{\frac{1}{2}}\right] \, d\varphi. \quad (15) $$

The work required to overcome resistance force (10) by cylindrical rods during their movement in the loose medium per one full revolution performed by the mobile gear along the inner surface of the cylindrical mixer housing can be determined if one integrates (15) over the range of $\varphi = \varphi_n$ to $\varphi = \varphi_n + 2\pi$:

$$ A_n = \frac{\pi H_0 \psi d_0}{4} \cdot \left[\omega_0^2 (R_0 - r_0)^3 \cdot D_n (r_n, \varphi_n) + gH_0 \psi (R_0 - r_0)S_n (r_n, \varphi_n)\right], \quad (16) $$

with the following notations:

$$ D_n (r_n, \varphi_n) = \int_{\varphi_n}^{\varphi_n + 2\pi} \left(1 + \left(\frac{r_n}{r_0}\right)^2 - 2 \frac{r_n}{r_0} \cos \left(\frac{R_0}{r_0} \varphi\right)\right)^{\frac{3}{2}} \, d\varphi, \quad (17) $$

$$ S_n (r_n, \varphi_n) = \int_{\varphi_n}^{\varphi_n + 2\pi} \sqrt{1 + \left(\frac{r_n}{r_0}\right)^2 - 2 \frac{r_n}{r_0} \cos \left(\frac{R_0}{r_0} \varphi\right)} \, d\varphi. \quad (18) $$
Integrals (17) and (18) in the general case (for arbitrary \( r_n \) and \( r_0 \) values) are not expressed in terms of elementary \( \varphi \) functions. Therefore, their numerical values can be found by specifying concrete numerical values of constructive parameters \( r_n, r_0, \varphi_n \).

Power \( N_c \), required to complete work (16), can be found on the basis of the following equality:

\[
N_c = \omega_0 \sum_{n=0}^{n_0} A_n .
\]  

(19)

Substitution (16) in (19) results in the following:

\[
N_c = \frac{\pi \nu H_0 \psi \rho \omega_0}{4} \left[ \omega_0^2 (R_0 - r_0)^3 \sum_{n=0}^{n_0} D_n (r_n, \varphi_n) + g H_0 \psi (R_0 - r_0)^3 \sum_{n=0}^{n_0} S_n (r_n, \varphi_n) \right].
\]  

(20)

Fig. 3 shows the graphical dependence of the power determined by formula (20), subject to the load factor of the mixer housing. Analysis of the graphical dependence shown in Fig. 2 shows that as load factor \( \psi \) of the mixer's capacity increases, the power consumption increases at one and the same rotation speed.

**Figure 3.** The graphical dependence of the power on the rotational speed of the mobile gear

1- consumption power with load factor \( \psi=70\% \);

2- consumption power with load factor \( \psi=50\% \);

3- consumption power with load factor \( \psi=30\% \).

Given that the rotation speed is \( \omega=4 \text{ c}^{-1} \) and the load factor is \( \psi=30\% \), the consumption power is \( N=6.2 \text{ Watt} \), and if the load factor is \( \psi=70\% \) it grows up to \( N=30.5 \text{ Watt} \), i.e. it is 5 times larger. This can be due to the fact that when the volume of the loaded material increases, the resistance of the stirred medium also increases. Besides, as \( \omega \) frequency increases to a value of \( \omega = 8 \text{ c}^{-1} \) and at the same load values of \( \psi = 30\% \) and \( \psi = 70\% \), power consumption increases in the first case (position 3 in Fig. 2) to a value of \( N = 18.8 \text{ Watt} \), i.e. it is 3 times greater than that at a frequency of \( \omega = 4 \text{ c}^{-1} \). In the second case (position 1 in Fig. 2), power consumption value is \( N = 76.1 \text{ Watt} \), which is 2.5 times more than that at a frequency of \( \omega=4 \text{ c}^{-1} \). This is due to the fact that when moving the rods in the same loading volume, the resistance to movement at high speeds substantially (3 and 2.5 fold) increases. That is obvious and confirms the adequacy of the equations obtained (20).

Energy required to drive the mechanical part of the planetary mixer when there is no material is determined by the expression that looks like:
\[ E_{sp} = \frac{\sum_{n=0}^{n_0} M_n^2}{2I_0}, \]  
\[ I_0 \] is the inertia of rotating parts of the planetary mixer; 
\[ M_n \] is the impulse of the mobile gear with cylindrical rods. 
Values determining expression (21) are equal to:

\[ I_0 = I_d + I_u, \]  
\[ M_n = M_c U_n \rho_n, \] 

Here \( I_d \) is inertia of the mobile gear. Assuming that the mobile gear is a hollow thin-walled cylinder, let us find that:

\[ I_d = \frac{m_d}{2} r_0^2, \]  
where \( m_d \) is the mass of the moving part,
\[ I_c \] is the inertia of a set of cylindrical rods:

\[ I_c = \sum_{n=0}^{n_0} I_{nc}, \]  
where \( I_{nc} \) is the inertia of \( n \) cylindrical rod that is equal to:

\[ I_c = \frac{m_c d_0^2}{4} + m_c \rho_n^2, \]  
where \( m_c \) is the mass of one cylindrical rod; 
\( d_0 \) is the diameter of a cylindrical rod; 
\( \rho_n \) is the distance from the axis of rotation to \( n \) - a cylindrical rod. The square value of the given distance is:

\[ \rho_n^2 = x_n^2 + y_n^2. \] 

The substitution of the parametric hypotrochoid equations along which the \( n \) cylindrical rod (27) and moves in leads to the following result:

\[ \rho_n^2 = \Delta^2 \cos^2 \varphi + r_n^2 \cos^2 \frac{\Delta}{r_0} \varphi + 2r_n \cos \varphi \cos \left( \frac{\Delta}{r_0} \varphi \right) + \Delta^2 \sin^2 \varphi + r_n^2 \sin^2 \frac{\Delta}{r_0} \varphi - \\
- 2\Delta r_n \sin \varphi \sin \left( \frac{\Delta}{r_0} \varphi \right) = \Delta^2 + r_n^2 + 2\Delta r_n \cos \left( \frac{R_0}{r_0} \varphi \right), \]  
where:

\[ \Delta = R_0 - r_0; \]
\[ M_0 = m_u + m_c \cdot n_0, \]
The change in the speed of \( n \) cylindrical rod when the mobile gear is rotated is determined by the relation:

\[
U_n = \omega_n \rho_n. \tag{31}
\]

Substitution (24) and (25) with regard to (26) leads to the following result:

\[
I_0 = \frac{m_d}{2} r_0^2 + \frac{m_d d_0^2}{8} n_0 + m \sum_{n=0}^{n_0} \rho_n^2. \tag{32}
\]

With regard to (28), relation (32) looks like:

\[
I_0 = \frac{m_d}{2} r_0^2 + \frac{m_d d_0^2}{8} n_0 + \Delta m \sum_{n=0}^{n_0} \left( 1 + \frac{r_n^2}{\Delta^2} + 2 \frac{r_n}{\Delta} \cos \frac{R_0}{r_0} \varphi \right). \tag{33}
\]

Substitution (30), (31), and (28) into expression (23) gives the following result:

\[
M_n = (m_d + m_c \cdot n_0) \omega_n \rho_n^2 = (m_d + m_c \cdot n_0) \omega_n \Delta \left( 1 + \frac{r_n^2}{\Delta^2} + 2 \frac{r_n}{\Delta} \cos \frac{R_0}{r_0} \varphi \right). \tag{34}
\]

Let us introduce the following notation:

\[
\Omega_n(\varphi) = 1 + \frac{r_n^2}{\Delta^2} + 2 \frac{r_n}{\Delta} \cos \frac{R_0}{r_0} \varphi. \tag{35}
\]

With regard to (35), the formula (34) looks like:

\[
M_n = (m_d + m_c \cdot n_0) \omega_n \Delta \Omega_n(\varphi). \tag{36}
\]

Substitution (36) and (33) in (21) with regard to (35) leads to the following result:

\[
E_{\text{ef}} = \frac{(m_d + m_c \cdot n_0)^2 \Delta \omega_0^2 \sum_{n=0}^{n_0} \Omega_n^2(\varphi)}{2 \left( \frac{m_d}{2} r_0^2 + \frac{m_d d_0^2}{8} n_0 + \Delta m \sum_{n=0}^{n_0} \Omega_n(\varphi) \right)}. \tag{37}
\]

Based on (37), desired power \( N_m \), [8] imposed to drive the mechanical part of the planetary mixer will be determined by the following relation:

\[
N_m = E_{\text{ef}} \omega_0. \tag{38}
\]

Formula (38) with regard to (37) eventually looks like:

\[
N_m = \frac{(m_d + m_c \cdot n_0)^2 \Delta \omega_0^2 \sum_{n=0}^{n_0} \Omega_n^2(\varphi)}{m_d r_0^2 + \frac{m_d d_0^2}{4} n_0 + 2 \Delta m \sum_{n=0}^{n_0} \Omega_n(\varphi)}. \tag{39}
\]

Thus, for structural parameters of a laboratory mixer, namely:
\[ n_0 = 11; \quad m_c = 0.07 \text{ kg}; \quad m_d = 11.05 \text{ kg}; \quad d_0 = 0.008 \text{ m}; \quad R_0 = 0.175 \text{ m}; \quad r_0 = \frac{2}{3}R_0; \]

\[ \varphi_k = \frac{3}{2}\pi; \quad \omega = 10 \text{ c}^{-1}; \quad r_1 = 0.023 \text{ m}; \quad r_2 = 0.092 \text{ m}, \]

expression (39) looks like:

\[
N_m = \frac{0.5\left(0.0882 + 0.1658 \cos \frac{3}{2}\phi + 0.0782 \cos \frac{3}{2}\phi\right)\omega_0^3}{0.7253 + 0.0052 \cos \frac{3}{2}\phi}. \tag{40}
\]

Resulting relation (40) determines the change in the power consumption, depending on the angle of rotation of the mobile gear of the planetary mixer. On the basis of (40), let us find the average value of the power consumption by the mechanical part of the planetary mixer for one complete revolution:

\[
\overline{N_m} = \frac{1}{2\pi} \int_0^{2\pi} N_m d\phi. \tag{41}
\]

The calculation of integral (41) with regard to (40) enables the following:

\[
\overline{N_m} = 0.0874\omega_0^3. \tag{42}
\]

Obtained relation (42) for the parameters of the laboratory planetary mixer determines the average power consumption when there is no loaded material, depending on frequency \( \omega_0 \) of the mobile gear running-in.

Figure 4. Graphical dependence of the power consumption change on the rotation angle of the mobile gear: 1 – power consumption of the 4th rod, 2 – power consumption of the 8th rod, 3 – power consumption of the 1st rod.

Fig. 4 shows the graphical dependence of power consumption \( N \) on rotation angle \( \varphi \).

3. Conclusion

Thus, one can conclude that the further the rod from the center of the gear, the more power it consumes, which is due to the more intense resistance of the medium for the rods located at a distance, which is further from the center. For rods 1, 4 and 8, the power consumption increases by an average
of 0.1 Watt. The more operating elements are installed and the further they are from the central part of the gear wheel, the more electricity will be consumed for stirring.

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