Constraining the Modified Newtonian Dynamics from spherically symmetrical hydrodynamic accretion

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ABSTRACT

The MOdified Newtonian Dynamics (MOND) is an alternative to the dark matter assumption that can explain the observed flat rotation curve of galaxies. Here hydrodynamic accretion is considered to critically check the consistency and to constrain the physical interpretation of this theory. It is found that, in case of spherically symmetrical hydrodynamic accretion, the modified Euler’s equation has real solution if the interpretation is assumed to be a modification of the law of dynamics. There is no real solution if it is assumed to be an acceleration-scale-dependent modification of the gravitational law. With the modified Euler’s equation, the steady-state mass accretion rate is found to change up to ~15 per cent. The astrophysical and cosmological implications of these results are also discussed.

Key words: accretion, accretion discs – gravitation – hydrodynamics – cosmology: theory.

1 INTRODUCTION

The rotation curve of spiral galaxies cannot be explained in terms of gravitational potential of only the visible mass with any reasonable mass-to-light ratio (e.g. Rubin & Ford 1970, 1980; Sofue 1996; Sofue & Rubin 2001; Spano et al. 2008). This indicates the presence of a significant fraction of gravitating matter with very high mass-to-light ratio. Various other observational and theoretical constraints from velocity dispersion of elliptical galaxies, baryon fraction in galaxy clusters, gravitational lensing, structure formation, cosmic microwave background (CMB) power spectrum, observation of Lyman α forest, etc. (e.g. Faber & Jackson 1976; Wu et al. 1998; Springel et al. 2005; Clowe et al. 2006; Massey et al. 2007; Viel, Bolton & Haehnelt 2009) also point out that a significant fraction of the mass of the Universe is in the form of the dark matter with very little or no electromagnetic interaction (Bertone, Hooper & Silk 2005). Though the dark matter concept is widely accepted to be an explanation of these observations, there is no general agreement on the composition and various other properties of this major constituent of the Universe (e.g. Vittorio & Silk 1984; Davis et al. 1985; Umemura & Ikeuchi 1985; Navarro, Frenk & White 1996; de Blok 2005). Though there have been indications from some of the direct and indirect detection experiments of different types of dark matters, no conclusive results showing firm detection is still reported.

However, over the time, a variety of alternative theories, with some modification of either laws of motion or law of gravitational force, have been suggested to explain some of the observations without invoking the assumption of any dark matter (e.g. Milgrom 1983a; Bekenstein & Milgrom 1984; Sanders 1986; Fahr 1990; Sanders 1997; Brownstein & Moffat 2006). A particularly successful theory in this category is the theory of the MOdified Newtonian Dynamics (MOND) proposed to explain rotation curves without the ‘hidden mass hypothesis’ (Milgrom 1983a,b). Essentially, the proposal of this theory is that the acceleration due to gravitational force is not linearly proportional to the force at very low acceleration limit. This simple modification has so far successfully explained most of the observed rotation curves for different types of galaxies (e.g. Scarpa 2003; Swaters, Sanders & McGaugh 2010), and the few cases where it fails can be explained in terms of inadequate data, large asymmetries in the velocity field and observational uncertainties (Milgrom 1991; Swaters 1999). The MOND predictions have been independently tested against observations (e.g. Sanders & McGaugh 2002; Famaey, Bruneton & Zhao 2007; Tiret & Combes 2007, and references therein) and found to be consistent. Though there have been some criticism of this theory followed by the claim of a direct proof of dark matter at cluster scale from weak lensing observations (Clowe et al. 2006), there are ways of accommodating these observations (e.g. Angus, Famaey & Zhao 2006; Angus et al. 2007), and the issue is far from being settled.

Whether or not MOND is an alternative to the dark matter scenario, there is no doubt that a complete theory of dark matter must explain this success of MOND in predicting and explaining such observations. It is, hence, increasingly important to critically verify its consistency at all scales. There are proposals to test predictions of MOND or associated theories in a variety of ways (e.g. de Lorenzi, Fuñández-Abans & Pereira 2009; Trenkel et al. 2010). In this paper, I have considered the astrophysical case of spherical accretion to check the consistency and to constrain the physical interpretation of MOND. The background is outlined in Section 2, and the outcomes in MOND regime for different modifications are
described in Section 3. Section 4 contains discussions on the results, and the conclusions are presented in Section 5.

2 BACKGROUND

In MOND, the Newtonian equation of dynamics $F = ma$ is modified to $F = m_{\mu}\alpha$, where $\mu = \mu(a)$ is a dimensionless parameter. This modification is significant at very low acceleration regime (below an acceleration of $a_0 \approx 10^{-10}$ m s$^{-1}$). It is proposed that $\mu \approx a/a_0$ for $a < a_0$ and $\mu = 1$ for $a > a_0$, and the exact form of $\mu(a)$ may not have any serious consequences. Thus the motion due to gravitational force will be governed by

$$ F_\mu = \frac{G M m}{r^3} = m_{\mu}\alpha. \quad (1) $$

For the low acceleration at a large distance from the centre of a galaxy, this will imply $a = v^2/r = \sqrt{G M a_0/r}$ giving rise to a flat rotation curve. See Sanders & McGaugh (2002) for a comprehensive critical review of the theory, a more generalized formulation and its implications, its observational supports and other details. Also see Bekenstein (2004) and references therein for the details of the ‘MOND-inspired’ relativistic, generalized theory of gravitation called tensor–vector–scalar gravity (TeVeS). For the purpose of this work, I will only use this theory to be phenomenological as summarized in equation (1) to investigate its possible implications in the case of spherically symmetrical hydrodynamic accretion.

Note that in equation (1), $\mu$ can be written as a modification of either the inertial term ($F_\mu = m_{\mu}\alpha$) or modification of the gravitational force term ($F_\mu = \mu m a$). Though they lead to the same result for the rotation curve, the physical implication is significantly different. There may be situations where these two interpretations lead to drastically different results. Note that this phenomenological description of the dynamics is consistent with the non-relativistic limit of TeVeS with spherical symmetry (Bekenstein 2004).

3 ACCRETION IN MOND REGIME

Here I have considered spherically accretion to check if, in the MOND regime, there is any change of physical conditions from that of the Newtonian scenario. In the Newtonian case, the governing equations for spherically symmetrical hydrodynamic steady-state accretion are the continuity equation and the Euler’s equation

$$ \frac{1}{r^2} \frac{d}{dr} (\rho u r^2) = 0, \quad (2) $$

$$ \frac{d u}{d r} = -\frac{1}{\rho} \frac{d P}{d r} - \frac{G M}{r^2}. \quad (3) $$

where $u(r)$ is the radial inward velocity, $P(r)$ and $\rho(r)$ are pressure and density related by an equation of state $P = K \rho^\gamma$ and polytropic index $\gamma$, and $M$ is the mass of accretor. The sound speed in the medium $c_s(r)$ is related to $P$ and $\rho$ as $c_s^2 = dP/d\rho$.

Starting with a boundary condition $\rho_\infty$ and $c_s\infty$ at a very large distance from the central accretor, equations (2) and (3) can be solved for a given $M$ and $\gamma$ to derive the steady-state density and velocity profile. The solution of astrophysical interest is a unique transonic solution with a mass accretion rate of $\dot{M} = 4\pi \lambda \gamma (\rho_\infty G^2 M^2/c_s^\infty)^2$, where $\lambda$ is a dimensionless constant. Note that the acceleration at the sonic point is of the order of $c_s\infty^2/GM$. For a solar mass accretor and a typical ambient sound speed of $\sim 10$ km s$^{-1}$, this is $\sim 10^{-7}$, very much comparable to the MOND acceleration constant $a_0$. Keeping this in mind, it is useful here to introduce a dimensionless parameter $\tilde{a}_0 = a_0/(c_s\infty^2/GM)$ and to study the behaviour of the system for different values of this parameter.

In case of the hydrodynamic accretion, the dynamics is governed by the interplay of three terms – pressure, gravitational force and inertia. In the aforementioned two different interpretations of the MOND modification, namely modification of gravitational force and modification of the Newtonian dynamics, these three terms change in different ways. Thus, considering this case of hydrodynamic accretion gives us a chance to study any possible difference that may arise in these two interpretations.

3.1 Modification of gravitational force

In this case, I consider the modification of the gravitational force of the form $F' = F/\mu = m a$. As mentioned earlier, this can be derived from TeVeS at non-relativistic limit assuming spherical symmetry (Bekenstein 2004). With this modification, the continuity equation will remain unchanged from equation (2), but the Euler’s equation will be modified to

$$ \frac{d u}{d r} = -\frac{1}{\rho} \frac{d P}{d r} - \frac{G M}{\mu r^2}. \quad (4) $$

where $\mu = \mu(a/a_0)$ and $a = \sqrt{G M a_0/r}$. Note that, in MOND regime, with $\mu(a/a_0) = a/a_0 < 1$, the gravitational term will be $\sqrt{G M a_0/r}$, whereas in the low-acceleration regime with $ma = 1$, it will be same as the regular Newtonian term $GM/r^2$. Using equation (2) and the equation of state, $P$ and $\rho$ can be eliminated from equation (4) in terms of $u$, and can be rewritten as

$$ \frac{(u^2 - c_s^2)}{u} \frac{d u}{d r} = \frac{2c_s^2}{r} - \frac{G M}{\mu r^2}. \quad (5) $$

With an initial condition $\rho_i$ and $a_i$ at a small radius $r_i$, equation (5), along with the equation of state and the continuity equation, can be solved numerically for the density and velocity profile. Note that for typical astrophysical condition, $a$ at small radius is significantly larger than $a_0$. So, for the inner region, the MONDian solution will not differ from the Newtonian solution. Here also, like the Newtonian case, velocity at $r_i$ should have a unique value for the solution to pass through the sonic point and to give the right accretion rate. For a given $\gamma$ and $\tilde{a}_0$, equation (5) can be solved with same initial conditions for both the Newtonian and the MONDian case. For the Newtonian case, at large radius, $\rho$ and $c_s$ tend asymptotically to a constant value $\rho_\infty$ and $c_s\infty$, respectively, and the velocity $u \sim r^{-2}$ tends to zero. Interestingly, as shown in Fig. 1, for MONDian case, equation (5) does not have this asymptotic solution at large radius. Fig. 1 shows the Newtonian and MONDian solutions for $\gamma = 7/5$ and $\tilde{a}_0 \approx 0.3, 1.0$ and $3.0$. The top and bottom panels show the density and the velocity field, respectively. Both the density and the velocity profile deviate from the Newtonian solution and diverge away from the asymptotic solution at large radius. In these plots, the density and the velocity values are scaled by $\rho_\infty$ and $c_s\infty$ (of the Newtonian solution), and the radius is scaled by $r_i = GM/c_s^\infty$.

One can understand this result analytically with the following arguments. In the MOND regime, with $\mu = a/a_0 < 1$, equation (5) can be written in terms of $u' = \frac{d u}{d r}$ as

$$ u' = u \left( \frac{\sqrt{\frac{2c_s^2}{r}} - \frac{\sqrt{\mu a_0}}{r}}{(u^2 - c_s^2)} \right). \quad (6) $$

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Figure 1. Density and velocity profile in the MOND regime for the modified gravitational force case. Here $\gamma = 7/5$ and $\tilde{a}_0 \approx 0.3, 1.0$ and $3.0$ (for $c_{\infty}$ of the Newtonian solution). Solid line is for the Newtonian solution.

Since both the term in numerator scales as $1/r$, with decreasing $c_s$, $u'$ changes sign at large radius. Thus, $u/c_s$ is no more a monotonically decreasing function with increasing radius, and the MONDian solution diverges from the Newtonian solution. It implies that there is no solution of the flow with $\rho$, $u$ and $c_s$ having the right physical boundary condition.

3.2 Modification of dynamics

In this case also, the continuity equation will remain as in equation (2). But since $F = m\mu a$, the modified Euler’s equation will be

$$u\mu = -\frac{1}{\rho} \frac{dP}{dr} - \frac{GM}{r^2}. \tag{7}$$

Following a similar numerical analysis as in the earlier case, one can solve for $u(r)$ and $\rho(r)$ from equations (2) and (7). As shown in Fig. 2, one of the solutions in this case starts to deviate from that of the Newtonian one at a large radius and have asymptotic value of $\rho_{\infty}$ and $c_{s\infty}$ lower than that of the Newtonian value with same mass accretion rate. The other solution becomes imaginary after a certain distance. Fig. 2 shows the Newtonian and these two MONDian solution for $\gamma = 7/5$ and $\tilde{a}_0 \approx 1$. The deviation of $\rho_{\infty}$ from that of the Newtonian one depends on the value of both $\gamma$ and $\tilde{a}_0$.

Figure 2. Typical spherical accretion density and velocity profile in the MOND regime for the modified dynamics case. Here $\gamma = 7/5$ and $\tilde{a}_0 \approx 1$ (for $c_{\infty}$ of the Newtonian solution). Solid line is for the Newtonian solution. Other two broken lines correspond to the two different roots of equation (9). One of the roots is imaginary at large radius, but the other one satisfies physical boundary condition requirement.

Eliminating $P$ and $\rho$ using equation (2) and the equation of state, equation (7) can be rewritten as

$$\frac{\mu u^2 - c_s^2 u'}{u} \frac{du}{dr} = \frac{2c_s^2}{r} - \frac{GM}{r^2}. \tag{8}$$

In the MOND regime, where $\mu = a/a_0 = -a\frac{du}{dr}$, this also becomes a quadratic equation in $u'$,

$$\frac{u^2}{a_0} a' + c_s^2 u' + \left(\frac{2c_s^2}{r} - \frac{GM}{r^2}\right) = 0, \tag{9}$$

where the condition of a real solution for $u'$ is

$$\frac{u^2}{c_s^4} \leq \frac{a_0}{4} \left(\frac{2c_s^2}{r} - \frac{GM}{r^2}\right)^{-1}. \tag{10}$$

This is an upper limit condition to $u$ and does not contradict the required physical condition of velocity tending to zero at large radius. Thus, in this case, there exist a physically meaningful solution where $u$ tends to zero at large radius, whereas density and sound speed asymptotically tend to $\rho_{\infty}$ and $c_{s\infty}$, respectively.

Effectively, for this interpretation, the general nature of the solution in MOND regime does not change from that of the Newtonian solution. In details, however, the mass accretion rate changes to $\dot{M} = 4\pi\tilde{\lambda}(\gamma, a_0)\rho_{\infty} G^2 M^2/c_{s\infty}$, where $\tilde{\lambda}$ is a dimensionless factor.
This is further investigated by evaluating the mass accretion rate parameter $\dot{\Lambda}$ as a function of acceleration parameter $\tilde{a}_0$ for different equation of state with polytropic index $\gamma$.

As shown in Fig. 3, the steady-state mass accretion rate may change by up to \( \sim 15 \) per cent. However, in case of astrophysical accretion, the accretor mass, ambient density and sound speed are often not so well determined to observationally distinguish this change between Newtonian and MONDian accretion.

**4 DISCUSSIONS**

The results derived in Section 3 are for the spherically symmetrical and non-magnetized hydrodynamic accretion. While considering these results, it should be kept in mind that, in reality, astrophysical accretion is a complex phenomenon. These assumptions of steady state, spherical symmetry and no importance of angular momentum, and ignoring the possible role of self-gravity and magnetic field are for the simplicity of this semi-analytical investigation. However, in the low-acceleration regime far away from the accretor, the effect of magnetic field, self-gravity and angular momentum may be negligible, and the large-scale accretion may be approximated as hydrodynamic, non-magnetized, spherical accretion on to a central compact accretor. Thus, the exact solution may get modified due to these complications, but the general nature of the solution is expected to remain unchanged.

The other point to note is that the pressure term in the Euler's equation is assumed to be not affected by the MONDian modification. Fundamentally, this term, which arises form the random motion and change of momentum of the particles, is expected to get modified in a similar way as the inertia term. In that case, equation (7) will be reduced to equation (4), and, there will be no real solution in the MOND regime. The way out is to consider equation (1) as the governing equation not for random motion but only for bulk motion of system with symmetry. This scenario is consistent with the observation that the random motion and acceleration of the gas does not alter the galaxy rotation curve either.

**5 CONCLUSIONS**

For the case of spherically symmetrical hydrodynamic accretion in MOND regime, it is shown here that physically meaningful solution exists only for the interpretation of the modification of dynamics but not of the gravitational law. At a phenomenological level, this modification should not be for random motion but for bulk motion only. Given the uncertainty on various parameters, the change of accretion rate is not significant to distinguish between the Newtonian and the MONDian scenario.

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