Role of domain wall fluctuations in non-Fermi-liquid behavior of metamagnets

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Abstract
In this paper we study the resistivity temperature dependence of a three-dimensional metamagnet near the metamagnetic phase transition point. The phase transition is characterized by a phase separation of regions with high and low magnetization. We show that, in the case of weak pinning, the spin relaxation time of the domain wall, which separates the two phases, is much larger than that of the volume spin fluctuations. This opens a temperature range where resistivity temperature dependence is determined by scattering of conducting electrons by the domain wall fluctuations. We show that it leads to quasi-linear low temperature dependence of resistivity.

1. Introduction
Understanding the deviation from the Fermi liquid behavior at phase transition critical points is a current research question. The most important are deviations from quadratic temperature dependence of resistivity near quantum critical points. Theory proposes that the origin of non-Fermi-liquid behavior is scattering of conducting electrons by bosonic critical soft modes [1–3]. In the case of a ferromagnetic transition these modes are collective spin fluctuations whose relaxation time diverges near the transition point. In nearly magnetic metals, at temperatures larger than the inverse spin relaxation time, the contribution to the resistivity due to electron scattering by spin fluctuations strongly deviates from Fermi liquid quadratic dependence. It becomes linear or even saturates [4–6]. In strongly Stoner enhanced paramagnetic metals the deviation might start at very low temperature.

It was experimentally found that, in the region near the putative quantum critical point, the magnetic states of many systems experience a broadened first order phase transition with separation of different phases [7–17]. In these experiments, the temperature dependence of resistivity is characterized by non-Fermi-liquid exponents, which depend on the proximity of the system to the putative quantum critical point. Since at the first order phase transition the spin fluctuations are not critical, the theory of scattering of conducting electrons by spin fluctuations [1–3], discussed above, cannot be directly applied to describe the non-Fermi-liquid contributions to the resistivity. Alternative theories were developed to overcome some difficulties [18, 19], but complete understanding is lacking [20].

The phase separation, which is a natural phenomenon occurring at the first order phase transition, suggests the importance of considering the contributions to the resistivity due to the structural difference of separated phases and phase boundaries (domain walls) [21, 22]. We would like to point out that the nonuniform magnetic state can be considered as a distinctive phase itself [8, 21–26]. For example, at the metamagnetic transition of thin magnetic films, the magneto-dipole interaction can give rise to formation of magnetic domains of various types [21, 22, 25]. Also, a magnetic analog of a Fulde–Ferrel–Larkin–Ovchinnikov phase can develop at the first order phase transition [26]. In another example, the domains can be formed in a spatially random magnetic field. This model was proposed to explain unusual magnetotransport effects of ferromagnetic alloys [27].

In this paper, we propose a possible scenario for the appearance of bosonic critical soft modes at the magnetic first order phase transition driven by a magnetic field. The
system of our study is a three-dimensional metamagnet that splits into regions with high and low magnetization (magnetic domains) with corresponding domain walls. We show that the spin fluctuations, which are short lived inside the magnetic domains, might become critical at the domain walls. Therefore, in the spirit of theory [1–3], the scattering of conducting electrons by these domain wall fluctuations results in non-Fermi-liquid behavior of the resistivity.

The rest of the paper is organized as follows. In section 2 we give a description of the three-dimensional system of coupled conducting electrons and itinerant electrons responsible for the metamagnetic state. We propose a model where the system splits into domain walls due to spacial variation of magnetic field. In section 3 the solution for the domain wall is obtained. The dynamical magnetic susceptibility of the domain wall fluctuations is then derived. In section 4 the temperature dependence of the resistivity caused by the scattering of conducting electrons by the domain wall fluctuations is considered. We find a transition from quadratic to linear temperature dependence with increasing temperature. We give a condition of the resistivity caused by the scattering of conducting electrons by these domain wall fluctuations results in non-Fermi-liquid behavior of the resistivity.

2. Description of the model

We consider a three-dimensional metallic metamagnet where s electrons are considered to be conducting, and d electrons are responsible for the magnetic state. Coupling of conducting electrons with bosonic modes is obtained by integrating out d electrons [33]. The action describing conducting electrons in a random impurity potential V(\(r\)) is

\[
S_e = T \sum_{\Omega_n} \int dr \psi_d^\dagger(r, \Omega_n) \left( \sqrt{\frac{2}{m_e}} \frac{\partial}{\partial r} + \mu - V(r) \right) \psi_d(r, \Omega_n),
\]

where \(\psi_d\) describes electrons with mass \(m_e\), spin \(\alpha\), Matsubara frequency \(\Omega_n = \pi T (2n + 1)\) (\(T\) is temperature), and Fermi level \(\mu\), and we have set \(\hbar = 1\). We assume the impurity potential to satisfy \(\langle V(r) \rangle = 0\), and \(\langle V(r) V(r') \rangle = 1/(2\pi v_F \tau) \delta(r - r')\), where \(v_F\) is the density of electrons per spin and \(\tau\) is the electron mean free time.

The magnetic part of the free energy has two energetically unequal minima. Under application of magnetic field the system undergoes a metamagnetic phase transition when these two minima have the same energy. The value of the magnetic field at which it occurs is called the metamagnetic magnetic field; let us denote it by \(h_m\). Near the metamagnetic phase transition we approximate the free energy by two parabolas with minima at \(m(r, \tau) = \pm m_0\) corresponding to high and low magnetization states. The domain walls originate from the spatial deviation of the magnetic field from its metamagnetic value. We denote such deviation as \(h(r)\). Assuming a strong magnetic field, we consider only the longitudinal component of magnetization density \(m(r)\) (in units of \(\mu_B = 1\)) in the action. Under such assumptions the action describing the magnetization density \(m(r, \tau)\) has the form

\[
S_{mag}[m] = \int_0^{1/T} d\tau \int \frac{K}{2\gamma_0} (\nabla m(r, \tau))^2 + \frac{\alpha}{\gamma_0} (|m(r, \tau)| - m_0)^2 - h(r) m(r, \tau) + S_D,
\]

where coefficient \(K^{-1/2}\) is of the order of the electron Fermi wavelength, \(\alpha^{-1}\) is the Stoner enhancement factor, and \(\gamma_0\) is the noninteracting electron spin susceptibility, which we set to be the same for both high and low magnetization phases. This assumption greatly simplifies further calculations, and does not affect the main conclusions about the resistivity temperature dependence. In our model, the spatial distribution of the magnetic field \(h(r)\) has a Gaussian form (see the appendix for the definition and an averaging procedure of appropriate quantities).

The last term in the action (2) is the Landau damping, which is described by

\[
S_D[m] = \int d\tau' d\tau \sum_{\omega_n} m(r, \omega_n) \gamma(|r - r'|, |\omega_n|) m(r', \omega_n),
\]

where \(\omega_n\) is the bosonic Matsubara frequency. At this point it is necessary to distinguish two cases of damping depending on the regimes of electron scattering by a random impurity potential \(V(r)\) given in the expression (1). For ballistic electrons, the Fourier image of \(\gamma(r, |\omega_n|)\) is given by \(\Gamma(Q, |\omega_n|) = \frac{4\omega_n}{v_F Q}\), which is valid for large momenta \(v_F Q > |\omega_n|\) where \(v_F\) is the Fermi velocity, and \(\gamma\) is a damping constant [1, 34]. In the case of diffusive electrons, the damping is \(\Gamma(Q, |\omega_n|) = \frac{\rho_D Q^2}{DQ^2}\), which is valid for \(DQ^2 > |\omega_n|\), where \(D\) is the diffusion constant [35].

Finally, the part of the action that describes the coupling of the conducting electrons with the magnetization is

\[
S_{int} = G \int_0^{1/T} d\tau \int d\tau' s(r, \tau) m(r, \tau'),
\]

where \(s(r, \tau)\) is the operator of the spin density of the conducting electrons along the longitudinal component of the magnetization, and \(G\) is a phenomenological coupling constant.

3. Domain wall fluctuations

In this section we present a solution of the mean field equation for the domain wall and discuss fluctuations around it. Let \(\psi_d\) be...
a coordinate normal to the domain wall and \( h(x_0(p)) = 0 \), so that at \( x < x_0 \) it is a state with high magnetization, and at \( x > x_0 \) it is a low magnetization state. Here \( p \) is a two-dimensional coordinate along the domain wall. Varying the action (2), we obtain the equation for the magnetization

\[-K\frac{d^2}{dx^2}m(x) + \alpha(m(x) - m_0\text{sign}(m(x))) = \chi_0 h(x, p). \tag{5}\]

When \( h(r) \) is a slowly varying function on a scale of \( \sqrt{\chi/\alpha} \) the domain wall can be approximated as flat. With this assumption the solution of (5) describing the domain wall is

\[ m = -m_0(1 - e^{-\sqrt{\alpha/\chi}(|x - x_0(p)|)})\text{sign}(x - x_0(p)) + \frac{\chi_0 h(x, p)}{\alpha}. \tag{6}\]

Let us consider fluctuations near this solution. Taking the second derivative of the free energy we obtain the equation for the eigenfunctions, which describe fluctuations

\[ (-K\nabla^2 + \alpha)\delta m(r) = \frac{2\alpha m_0}{|\frac{d}{dx}(m(x))|}_{x=x_0} \delta(x - x_0(p))\delta m(r) = \epsilon \delta m(r). \tag{7}\]

Deriving this equation we have used the equality \( \delta(m(x)) = \delta(x - x_0(p))/|\frac{d}{dx}(m(x))|_{x=x_0} \). The delta-type potential in the equation (7) is related to the nonanalytical dependence of the free energy on the magnetization. Equation (7) has only one bounded solution, thus strongly simplifying consideration of fluctuations. With an assumption of slowly varying \( x_0(p) \) and \( \frac{d}{dx}(m(x_0(p))) \), one can search for the solution of equation (7) in the form of a plane wave in \( p \)

\[ \delta m(r) = \Psi_0(x - x_0(p))e^{iQp}, \tag{8}\]

where

\[ \Psi_0(x) = \sqrt{\beta}e^{-\frac{\beta |x - x_0|}{\chi_0}}, \tag{9}\]

and with \( \epsilon = KQ^2 + \epsilon_0 \), where \( Q \) is now a two-dimensional wavevector along the domain wall, we write

\[ \epsilon_0 = \alpha \left[ 1 - \left( 1 + \frac{\chi_0}{m_0\sqrt{\chi\alpha}} \frac{|d}{dx} h(x) \right)^2 \right], \tag{10}\]

where \( \beta = \sqrt{(\alpha - \epsilon_0)/K} \). At slowly varying \( h(r) \), the eigenvalue is small, \( \epsilon_0 \ll \alpha \), and given by

\[ \epsilon_0 = \frac{2\chi_0}{m_0\beta} \frac{|d}{dx} h(x) . \tag{11}\]

The dynamics of spin fluctuations is governed by the Landau damping (3), which for excitation described by \( \Psi_0(x) \) translates to

\[ \Gamma(Q, |\omega_n|) = \frac{\gamma |\omega_n|}{v_F} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dq \frac{e^{iQ(x - x')}\Psi_0(x)\Psi_0(x')}{\sqrt{Q^2 + q^2}}, \tag{12}\]

where as an example we used the ballistic case. At small momenta \( \beta > Q \) we have for the ballistic case

\[ \Gamma(Q, |\omega_n|) = \frac{4\gamma |\omega_n|}{\pi\beta v_F} \ln(\beta/Q). \tag{13}\]

The same procedure for the diffusive case gives

\[ \Gamma(Q, |\omega_n|) = \frac{2\gamma |\omega_n|}{DQ\beta} \tag{14}\]

The dynamical susceptibility of one domain wall fluctuations is represented in the form

\[ \chi(Q, \omega) = \int \frac{d^2Q}{(2\pi)^2} e^{-iQ(p - p')}\Psi_0(x)\Psi_0(x') \chi(Q, \omega), \tag{15}\]

where

\[ \chi(Q, \omega) = \frac{\chi_0}{\epsilon_0 + KQ^2 + \Gamma(Q, i\omega)}. \tag{16}\]

We now assume that there is a finite number of domain walls in the system. The positions of the domain walls are given by zeros of spatially varying magnetic field \( h(r) \), which we assume to be obeying the Gaussian distribution. The procedure of averaging over the magnetic field is given in the appendix to the paper. In the following, we outline its steps. First, we consider a locally flat domain boundary. This can be justified if the scale along the wall \( L_1 \sim \sqrt{K / \epsilon_0} \sim \frac{1}{\rho \sqrt{\epsilon_0}} \) is much smaller than the domain wall curvature. Second, we consider domain walls to be independent from each other, meaning that overlap of eigenfunctions (9) of neighboring domain walls is exponentially small. This allows us to separately average over the domain wall direction and position, and to introduce the concentration of the walls \( n_W \) in the definition of the domain wall susceptibility (16) as \( \chi \sim \frac{n_W}{\rho} \chi \). The averaging of considered quantities over random values of \( \epsilon_0 \) does not need a cutoff at small \( \epsilon_0 \) and might be approximated by substituting the average value of \( \epsilon_0 \) into the susceptibility.

According to (16), the relaxation time of domain wall fluctuations is proportional to \( \epsilon_0^{-1} \) and is much larger than the relaxation time of the volume fluctuations, which is proportional to \( \alpha^{-1} \). At small values of \( \epsilon_0 \) (see expression (11)), the contribution of the domain wall fluctuations to the total susceptibility of the system can be approximated as

\[ \delta \chi \sim n_W \int dx \int dx' \Psi_0(x)\Psi_0(x') \chi(0, 0) \sim \frac{\chi_0 n_W}{\beta\epsilon_0}. \tag{17}\]

Moreover, depending on the parameters, it can be of the same order as the volume susceptibility \( \frac{\pi a}{\rho} \) when the ratio of domain wall concentration to domain wall thickness \( \frac{n_W}{\rho} \) is of the order of \( \frac{\epsilon_0}{\alpha} \).

### 4. Resistivity temperature dependence

Let us consider the contribution to the resistivity due to domain wall scattering of conducting electrons. We will consider three temperature dependent contributions to the resistivity. They are due to scattering of conducting electrons by fluctuations of domain walls, due to the domain walls’ shape change with the temperature variation, and due to variation of concentration of the domain walls \( n_W \). The first two contributions are considered in this section and have a common nature. The third contribution depends
on the position of the system on the phase diagram. We discuss its contribution in the conclusions of the paper. A contribution to the resistivity due to a mechanism of electron scattering by spin fluctuations is obtained in second order perturbation theory in interaction (4) and is expressed through the imaginary part of the averaged susceptibility as [1, 5]

\[
\rho(T) = R_0 \frac{1}{T} \int_0^{2\pi} \frac{dq}{p_F} q^2 \int_{-\infty}^{\infty} d\omega \frac{\omega}{\sinh^2(\omega/2T)} \text{Im} \chi(q, \omega),
\]

(18)

with \(R_0 = \frac{m_e G^2 v}{2\pi^2}\), where \(v\) is the density of states of conducting electrons per spin, and \(p_F\) and \(n\) are the Fermi momentum and density of conduction electrons respectively.

In addition, the fluctuations give a temperature dependent contribution to the average magnetization of the domain wall. The fluctuation part of and magnetization is determined by a derivative \(\delta m(r) = \frac{\delta \Delta E}{\delta r}\) of and fluctuation part of the free energy \(\Delta E = \frac{1}{2} \sum_{\alpha \beta} Q \ln (\epsilon + \Gamma(Q, \omega_\alpha))\). This leads to a change of the domain wall profile and gives an additional temperature dependence of and resistivity, which is proportional to \(\nabla^2 m(Q) \delta m(Q, T)\). The sum of the two contributions to the resistivity is then given by

\[
R = \frac{4\pi^2 R_0}{p_F^4} \times \int_0^{2\pi} d\omega \left( \frac{\omega}{T \sinh^2(\omega/2T)} - 2 \coth \left( \frac{\omega}{2T} \right) + 2 \right) \times \int d^2Q \text{Im} \chi(Q, \omega).
\]

(19)

The term proportional to \((- \coth(\frac{\omega}{2T}) + 1)\) is due to \(\delta m(Q, T)\). The domain wall susceptibility here is given by the expression (16) with a substitution \(x \rightarrow \frac{2\pi}{T} x\) already made.

Our calculations show that, despite the different expressions for damping in ballistic (13) and diffusive (14) regimes, the temperature dependence of resistivity has the same analytical form for both of them. It is quadratic at low temperatures and linear at higher. The transition temperature is proportional to the inverse relaxation time of the domain wall fluctuations. For the ballistic scattering regime at temperatures \(T < T_0\) the resistivity has a quadratic temperature dependence

\[
R = \frac{2\pi \beta R_0 \rho_m \chi_0}{KT_0} \frac{\gamma T^2}{p_F^4},
\]

(20)

and at higher temperatures \(T > T_0\) the dependence becomes linear

\[
R = \frac{2\pi \beta R_0 \rho_m \chi_0}{KT_0} \frac{T}{K},
\]

(21)

where \(T_0 = \gamma \beta \sqrt{\epsilon_0}/(5 \ln(\beta^2 K/\epsilon_0))\). In the case of diffusive scattering we get that the resistivity temperature dependence has the same form with \(T_0\) modified to \(T_0 = \gamma D \beta \sqrt{K}/\epsilon_0\). The transition from quadratic \(T^2\) to linear \(T\) dependence corresponds to the transition from quantum to thermal fluctuations of the domain walls [36].

We would like to note that the same temperature dependence holds for scattering by volume spin fluctuations, except for the difference of effective \(T_0\) in ballistic and diffusive cases [34, 35]. Let us compare the obtained contribution to the resistivity (20) with one due to volume spin fluctuations \(\varrho_{vol}(T)\). In the considered temperature range, the contribution of volume spin fluctuations is quadratic and is given by \(\varrho_{vol}(T) \sim R_0 \rho \beta^2 T^2/(2\pi \sqrt{\alpha})\) [1, 5]. Therefore, the ratio of contributions of domain wall fluctuations to volume spin fluctuations, \(R(T)/\varrho_{vol}(T) \sim \frac{2\pi}{\sqrt{\alpha}}\), can be of the order of unity.

5. Interaction correction

At low temperatures, an important resistivity temperature dependence is related to weak localization and electron interaction corrections [28]. Here we are going to discuss the contribution to the conductivity originating from the interplay between electron inelastic scattering by domain wall fluctuations and elastic scattering by impurities. The triplet channel contribution to the conductivity after disorder averaging is given by [31, 37]

\[
\delta \sigma = 2\pi e^2 v_F^2 v G^2 \int_0^{2\pi} d\omega \frac{\partial}{\partial \omega} \left( \frac{\sigma \coth(\omega/2T)}{2\pi T} \right)
\]

\[
\times \int d^2q |B(q, \omega)|^2 \chi(q, \omega),
\]

(22)

where we also averaged over the positions of the domain walls, after which the equation above became isotropic. In the following, we will only be interested in the temperature dependent terms of the contribution (22).

In the ballistic regime \(T \tau > 1\) one can use the following approximation: \(B(q, \omega) \approx 2/(v \sqrt{2q})\), which is valid for \(v q > |\omega|\). The expression for the susceptibility of the domain wall (16) at small momenta \(q < \beta\) is \(\chi(q, \omega) = 4\pi\rho (\epsilon_0 + Kq^2 + i/4\sqrt{2q}/(\tau v \beta)) \ln (\omega/Kq^2)^{-1}\).

We find that, at temperatures \(T > \pi v \rho \beta \epsilon_0/(2\gamma) \approx T_0\) and \(T < v \sqrt{\epsilon_0}/K\), the contribution to the conductivity is logarithmic in temperature

\[
\delta \sigma = -\left( \frac{2\pi^2 \gamma}{2\sqrt{2\gamma}} e^2 v G^2 n_W \right) \sqrt{\epsilon_0/K v T} \ln \left( \frac{\sqrt{\epsilon_0/K}}{v T} \right).
\]

(23)

At higher temperatures \(T > v \sqrt{\epsilon_0}/K\) and \(T < 2\pi v \rho \beta\), the contribution to the conductivity becomes linear

\[
\delta \sigma = \left( \frac{5\chi_0}{12\pi^2 \gamma} e^2 v G^2 n_W \right) \epsilon T.
\]

(24)

As the temperature increases further, the contribution decays as \(1/T\).

Next, let us discuss the diffusive regime at \(T \tau < 1\). In this case \(B(q, \omega) = 4 D \beta \rho \sqrt{\alpha}/(D \beta q)\) and \(\chi(q, \omega) = 4\pi\rho (\epsilon_0 + Kq^2 + i2\sqrt{2q}/(D \beta q))^{-1}\) are approximated at small momenta \(q < \beta\). The greatest singular contribution arises from
\[ \sqrt[\omega(t)]{TD} < q < \min(\beta, 1/\ell), \quad \text{and} \quad q > \sqrt{e_0/K}, \] where \( \ell \) is the electron’s mean free path. Calculations show that at temperatures \( p = \min(\Omega_1, \Omega_2, 1/\tau) > T, \) where \( \Omega_1 = \frac{\beta^2 D}{2K^2 \beta^2}, \Omega_2 = \sqrt{2} \beta^2 D K / \gamma, \) the contribution to the conductivity is given by
\[
\delta \sigma = - \left( \frac{2^{7/2} \gamma_0}{9 \pi^3 \gamma^2} \right) \ln \left( \frac{n_T}{\Omega_1} \right),
\]

Expressions (23)–(25) have a two-dimensional-like [28, 31] temperature behavior while obtained for a three-dimensional interacting electron system. Incidentally, all of the expressions have a non-Fermi-liquid contribution to the conductivity. One can relate the obtained results for the resistivity as
\[
\frac{\delta \sigma}{\sigma} = \left( \frac{2^{7/2} \gamma_0}{9 \pi^3 \gamma^2} \right) \ln \left( \frac{n_T}{\Omega_1} \right).
\]

6. Conclusions

To conclude, we have proposed a possible scenario of a non-Fermi-liquid temperature behavior of resistivity of a metallic metamagnet undergoing a first order phase transition. The present paper is motivated by experiments (see section 1 of the paper), where it was observed that a system close to a putative quantum critical point shows a non-Fermi-liquid temperature dependence of resistivity in a wide range of parameters, such as temperature, pressure or magnetic field. In this case the theory [1–3] of electron scattering on critical spin fluctuations fails to explain the experimental observations.

In the present paper we point out the importance of the phase separation in the properties of a system close to a putative quantum critical point. In the studied model, the metamagnet undergoes a phase separation to magnetic domains with corresponding domain walls. It is shown that in the case of weak pinning, when \( \alpha \gg \epsilon_0 \) (see expression (11)), the relaxation time of domain wall fluctuations is much larger than the relaxation of volume spin density. It is important that in this regime the domain wall fluctuations are more critical than the volume fluctuations. Therefore, the scattering of conducting electrons on these domain wall fluctuations results in non-Fermi-liquid dependence of resistivity, which is linear in temperature (see expression (21)). In this temperature range, the contribution of electron scattering to volume fluctuations is quadratic in temperature (of a Fermi liquid type) [1, 5]. Moreover, as we have shown, the non-Fermi-liquid contribution (21) can be a dominant one when the density of domain walls increases. We also considered the interaction correction to the conductivity due to an interplay of impurity and spin fluctuation scattering. Overall, expressions (21), (23)–(25) are the main results of the presented paper.

Let us discuss how the contribution related to the dynamics of domain walls might be the dominant one. In the temperature range considered in this paper, the contribution of volume fluctuations to the resistivity is always \( \sim T^2 \). It is reasonable to assume that the additional resistivity due to elastic scattering of the conduction electrons by the domain walls is proportional to \( n_w \). Depending on the position of the average magnetic field relative to a metamagnetic value, \( h_m(T) \) (see section 2 for definition), the concentration \( n_w \) can increase or decrease with the temperature. For example, when \( h_m(T) \) is an increasing function of temperature, \( n_w \) decreases with temperature if the average magnetic field is smaller than \( h_m(T = 0) \). In the considered temperature range, when the dependence \( h_m(T) \) on temperature is determined by volume fluctuations, the concentration \( n_w \) quadratically changes with the temperature. In the case when \( n_w \) decreases with temperature there can be a cancelation of the quadratic temperature dependence of the volume contribution to the resistivity by the contribution of the domain walls \( n_w(T) \). In the case of cancelation, the obtained results (21), (23) and (24) will be dominant, and the total resistivity will have quasi-linear non-Fermi-liquid temperature dependence.

One of the assumptions we made in this paper is the flatness of the domain walls. Another important point is related to the type of magnetic domain structure. We considered magnetic domain walls, which are independent of each other with random positions described by a Gaussian distribution. Possible arrangement of domain walls into a periodic structure, such as striped or hexagonal, will certainly change the dynamical susceptibility of the spin fluctuations, and therefore contribute differently to the temperature behavior of the resistivity. We address the elaboration of these assumptions to future research.

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Appendix. Averaging over random surfaces

Here we discuss the procedure of averaging over the random positions of domain walls and random values of \( \epsilon_0 \). We assume that the magnetic field is described by a Gaussian distribution, such that
\[
\langle \Delta h(r)\Delta h(r') \rangle = A \exp(-\zeta (r - r')^2)
\]
and \( \langle \Delta h(r) \rangle = 0 \), where \( \Delta h(r) = h(r) - h_m \). When the parameter \( \zeta \) is small, the magnetic field becomes a slowly varying random function \( h(r) \). A typical boundary condition in such a random field is smooth. The positions of the domain walls are defined by the following equation:
\[
h(r) = h_m + \Delta h(r) = 0.
\]
Note that under this definition we neglect cases when in small closed regions there is no solution of the mean field equation (5) for the favorite phase, or the energy associated with it is too high.

The Fourier transform of any quantity \( V(r) \) which is nonzero near the domain wall surface and slowly varying along it is calculated as
\[ V(q) = \int \mathrm{d}r \exp(\mathrm{i}qr) V(r) \]
\[ = \int \mathrm{d}S \exp(\mathrm{i}qr) V(qn(r_S), r_S) \]
\[ = \int \mathrm{d}r V(qn(r), r) \exp(\mathrm{i}qr) \left( \frac{\mathrm{d}}{\mathrm{d}r} h(r) \right) \delta(h(r)), \quad (A.3) \]

where \( r_S \) is a point on the surface, and \( n(r_S) \) is a vector normal to the surface at the point \( r_S \) defined as \( n(r) = \frac{\partial h(r)}{\partial} \). In \((A.3)\) \( V(qn(r), r) \) is a one-dimensional Fourier transform in the direction normal to the surface.

We can now average the dynamical susceptibility of the number of domain walls. For one domain wall the susceptibility is given by the expression (15). The scale along the surface of the domain wall that we are interested in is \( \frac{L_{\parallel}}{\sqrt{r_0}} \sim \frac{1}{\sqrt{\epsilon_0}} \), here \( \delta h \sim \sqrt{\lambda} \sim \frac{\eta m}{\lambda} \) and \( \epsilon_0 \) is estimated as
\[ \epsilon_0 \sim \frac{\chi(0)}{m_0 \alpha R} \frac{\mathrm{d}h}{\mathrm{d}x} \sim \frac{\sqrt{\alpha K}}{\sqrt{\xi}}. \quad (A.4) \]

Therefore, \( L_{\parallel} \sim \frac{1}{\sqrt{\lambda}} \sqrt{\beta/\alpha} \sqrt{\xi} \) is smaller than the radius of the surface curvature \( \frac{L_{\parallel}}{\sqrt{r_0}} \sim \sqrt{\xi/\beta} \ll 1 \), and under an averaging procedure we can use expression (15). Then the average susceptibility is
\[ \chi(q, \omega) = \int \mathrm{d}^3R \int \mathrm{d}b \Pi(R, b) \times \exp(\mathrm{i}qR) b^2 \Psi_0(qn) \Psi_0(-qn) \times \int \mathrm{d}^2Q \frac{\exp(iQR)}{2\pi} \chi(Q, \omega), \quad (A.5) \]

where \( \Psi_0(q) = 2\beta^{3/2}(q^2 + \beta^2) \) is a Fourier transform of \( \Psi_0(x) \) defined by \((9)\), and
\[ \Pi(R, b) = \left\{ \delta(h(r_1)) \delta(h(r_2)) \left[ b - \frac{1}{2} \left( \frac{\mathrm{d}}{\mathrm{d}r_1} h(r_1) \right. \right. \right. \]
\[ \left. \left. \left. + \frac{\mathrm{d}}{\mathrm{d}r_2} h(r_2) \right) \right] \right\}. \quad (A.6) \]

At \( R \ll \sqrt{\xi} \), \( b \) is a normal to the surface and therefore is also normal to \( R \). Therefore, in \((A.5)\) we can separately average over the direction and the value of \( b \), considering \( \Pi(R, b) \) only in the limit of \( R \sqrt{\xi} \ll 1 \). In the case of a Gaussian distribution, \( \Pi(R, b) \) is calculated analytically as
\[ \Pi(R, b) = \frac{n_w}{16\pi R \sqrt{\xi}} P(b_{\perp})P(b_{\parallel}), \quad (A.7) \]

with
\[ P(b_{\perp}) = \frac{1}{2\pi A_{\xi}} \exp \left( -\frac{b_{\perp}^2}{2A_{\xi}} \right) \quad (A.8) \]

and
\[ P(b_{\parallel}) = \frac{3}{\sqrt{\pi} A_{\xi}^2 R^4} \exp \left( -\frac{3b_{\parallel}^2}{A_{\xi}^2 R^4} \right), \quad (A.9) \]

where \( b_{\perp} \) and \( b_{\parallel} \) are perpendicular and parallel components to \( R \) respectively. The quantity \( n_w \) has the meaning of domain wall concentration and is defined as
\[ n_w = \frac{1}{V} \left( \frac{\int \mathrm{d}r \delta(h(r)) \left( \frac{\partial}{\partial} h(r) \right)}{\delta(h(r))} \right) \sim \frac{\sqrt{\xi}}{\pi} \exp \left( -\frac{h_m^2}{A} \right), \quad (A.10) \]

where \( V \) is the volume of the system. The factor \( b^2 \) under the integral of expression \((A.5)\) is estimated as \( b^2 \sim \epsilon_0^2 \). So averaging of quantities such as susceptibility does not diverge at small \( \epsilon_0 \) and can be approximated by inserting the average \( \epsilon_0 \). Finally, the averaged susceptibility over the domain wall positions is given by the next expression:
\[ \chi(q, \omega) = \frac{1}{4\pi} \int \mathrm{d}^2n \Psi_0(qn) \Psi_0(-qn) \times \frac{n_w \chi_0}{\epsilon_0 + \frac{1}{2}KQ^2 + \frac{1}{2}Q^2 + \frac{1}{2}Q^2 + \frac{1}{2}(Q, \omega)}, \quad (A.11) \]

where \( \epsilon_0 \) is approximated by its average value. At small momenta \( q < \beta \) the Fourier transform \( \Psi_0(qn) \) is approximated as \( \Psi_0 \sim 2/\sqrt{\beta} \).

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