A test of general relativity from the three-dimensional orbital geometry of a binary pulsar

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Binary pulsars provide an excellent system for testing general relativity because of their intrinsic rotational stability and the precision with which radio observations can be used to determine their orbital dynamics. Measurements of the rate of orbital decay of two pulsars have been shown to be consistent with the emission of gravitational waves as predicted by general relativity1,2, providing the most convincing evidence for the self-consistency of the theory to date. However, independent verification of the orbital geometry in these systems was not possible. Such verification may be obtained by determining the orientation of a binary pulsar system using only classical geometric constraints, permitting an independent prediction of general relativistic effects. Here we report high-precision timing of the nearby binary millisecond pulsar PSR J0437−4715, which establish the three-dimensional structure of its orbit. We see the expected retardation of the pulse signal arising from the curvature of space-time in the vicinity of the companion object (the ‘Shapiro delay’), and we determine the mass of the pulsar and its white dwarf companion. Such mass determinations contribute to our understanding of the origin and evolution of neutron stars3.

Discovered in the Parkes 70-cm survey4, PSR J0437−4715 remains the closest and brightest millisecond pulsar known. It is bound to a low-mass helium white dwarf companion5,6 in a nearly circular orbit. Owing to its proximity, relative motion between the binary system and the Earth significantly alters the line-of-sight direction to the pulsar and, consequently, the orientation of the basis vectors used in the timing model (see Fig. 1). Although the physical orientation of the orbit in space remains constant, its parameters are measured with respect to this time-dependent basis and therefore also vary with time. Variations of the inclination angle, i, change the projection of the semi-major axis along the line-of-sight, \( x = a_p \sin i/c \), where \( a_p \) is the semi-major axis of the pulsar orbit.

The heliocentric motion of the Earth induces a periodic variation of \( x \) known as the annual-orbital parallax7,

\[
x_{\text{obs}}(t) = x_{\text{int}}[1 + \frac{\cot i}{d} \mathbf{r}_\odot(t) \cdot \mathbf{\Omega}'];
\]

(1)

The superscripts ‘obs’ and ‘int’ refer to the observed and intrinsic values, respectively, \( \mathbf{r}_\odot(t) \) is the position vector of the Earth with respect to the barycentre of the Solar System as a function of time, \( d \) is the distance to the pulsar, and \( \mathbf{\Omega}' = \sin \Omega_0 \mathbf{I}_0 - \cos \Omega_0 \mathbf{J}_0 \) (see Fig. 1). Similarly, the proper motion of the binary system induces secular evolution of the projected semi-major axis8,9, such that:

\[
\dot{x}_{\text{obs}} = \dot{x}_{\text{int}} - x \cot i \mu \cdot \mathbf{\Omega}';
\]

(2)

where \( \mu = \mu_\alpha \mathbf{I}_0 + \mu_\delta \mathbf{J}_0 \) is the proper motion vector with components in right ascension, \( \mu_\alpha \), and declination, \( \mu_\delta \). An apparent transverse quadratic Doppler effect (known as the Shklovskii effect) also arises from the system’s proper motion and contributes to the observed orbital period derivative10:

\[
\dot{P}_{\text{obs}} = \dot{P}_{\text{int}} + \beta P_b;
\]

(3)

where \( \beta = \mu^2 d/c \), and \( \mu = |\mu| \).

Observations of PSR J0437−4715 were conducted from 11 July 1997 to 13 December 2000, using the Parkes 64 m radio telescope. Over 50 terabytes of baseband data have been recorded with
the S2 Recorder\textsuperscript{11} and the Caltech Parkes Swinburne Recorder (CPSR)\textsuperscript{12}, followed by offline reduction at Swinburne’s supercomputing facilities. Average pulse profiles from hour-long integrations were fitted to a high signal-to-noise template\textsuperscript{13}, producing a total of 617 pulse arrival time measurements with estimated errors on the order of 100 ns.

Previously considered negligible, the annualorbital parallax has been largely ignored in experimental time-of-arrival analyses to date. However, our initial estimates of its peak-to-peak amplitude for PSR J0437−4715 (\(~400\text{ ns}\) demonstrated that it would be clearly detectable above the timing noise. As can be seen in equation 1, \(x_{\text{obs}}\) varies with a period of one year and phase determined by \(\Omega'\). Its inclusion in our timing model therefore provides a geometric constraint on \(\Omega\). We also note that the value of \(x_{\text{obs}} = (7.88 \pm 0.01) \times 10^{-14}\) observed in our preliminary studies is many orders of magnitude larger than the intrinsic \(x\) expected as a result of the emission of gravitational waves, \(x^{\text{GR}} = -1.6 \times 10^{-21}\). Neglecting \(x^{\text{int}}\), the relationship between \(i\) and \(\Omega\) defined by equation 2 is parameterized by the well determined physical parameters \(x, \dot{x}\) and \(\mu\). Also, because \(\mu\) is fortuitously nearly anti-parallel to \(\Omega'\), \(\delta i / \delta \Omega\) is close to zero, and incorporation of equation 2 in our timing model provides a highly significant constraint on the inclination angle.

The orbitalinclination parameterizes the shape of the Shapiro delay, that is, the delay due to thecurvature of space-time about the companion. In highly inclined orbits, seen more edge-on from Earth, the companion passes closer to the line-of-sight between the pulsar and the observatory, and the effect is intensified. As the relative positions of the pulsar and companion change with binary phase, the Shapiro delay also varies and, in systems with small orbital eccentricity, is given by:

\[
\Delta S = -2r \ln[1 - s \cos(\phi - \phi_0)].
\]

(4)

Here, \(s \equiv \sin i\) and \(r \equiv Gm_2/c^3\) are the shape and range, respectively, \(\phi\) is the orbital phase in radians, and \(\phi_0\) is the phase of superior conjunction, where the pulsar is on the opposite side of the companion from Earth (as shown in Fig. 1). For small inclinations, the orbit is seen more face-on from Earth, and \(\Delta S\) becomes nearly sinusoidal in form.

In the PSR J0437−4715 system, the Shapiro effect is six orders of magnitude smaller than the classical Roemer delay, the time required for light to travel across the pulsar orbit. In nearly circular orbits, the Roemer delay also varies sinusoidally with binary phase. Consequently, when modeling less inclined binary systems with small eccentricity, the Shapiro delay can be readily absorbed in the Roemer delay by variation of the classical orbital parameters, such as \(x\). For this reason, a previous attempt at measuring the Shapiro effect in the PSR J1713+0747 system\textsuperscript{14} yielded only weak, one-sided limits on its shape and range.

In contrast, we have significantly constrained the shape independently of general relativity, enabling calculation of the component of \(\Delta S\) that remains un-absorbed by the Roemer delay. The theoretical signature is plotted in Fig. 2 against post-fit residuals obtained after fitting the arrival time data to a model that omits the Shapiro effect. To our knowledge, this verification of the predicted space-time
distortion near the companion is the first such confirmation (outside our Solar System) in which the orbital inclination was determined independently of general relativity.

The complete list of physical parameters modelled in our analysis is included in Table 1. Most notably, the pulsar position, parallax distance, $d_\pi = 139 \pm 3$ pc, and proper motion, $\mu = 140.892 \pm 0.006$ mas yr$^{-1}$, are known to accuracies unsurpassed in astrometry. Although closer, $d_\pi$ lies within the 1.5 $\sigma$ error of an earlier measurement by Sandhu et al, $d_\pi = 178 \pm 26$ pc. The $d_\pi$ and $\mu$ estimates can be used to calculate $\beta$ and the intrinsic spin period derivative, $\dot{P}_{\text{int}} = \dot{P}_{\text{obs}} - \dot{\beta}P = (1.86 \pm 0.08) \times 10^{-20}$, providing an improved characteristic age of the pulsar, $\tau_c = P/(2\dot{P}_{\text{int}}) = 4.9$ Gyr. Another distance estimate may be calculated using the observed $\mu$ and $\dot{P}_b$ by solving Equation 2 for $d$, after noting the relative negligibility of any intrinsic contribution. The precision of the derived value, $d_B = 150 \pm 9$ pc, is anticipated to improve as $t^{5/2}$, providing an independent distance estimate with relative error of about 1% within the next three to four years.

With a post-fit root mean square (r.m.s.) residual of merely 130 ns over 40 months, the accuracy of our analysis has enabled the detection of annual-orbital parallax. This has yielded a three-dimensional description of a pulsar binary system and a new geometric verification of the general relativistic Shapiro delay. Only the Space Interferometry Mission (SIM) is expected to localize celestial objects with precision similar to that obtained for PSR J0437−4715 (including parallax). By the time SIM is launched in 2010, the precision of this pulsar’s astrometric and orbital parameters will be vastly improved. Observations of the companion of PSR J0437−4715 using SIM will provide an independent validation and a tie between the SIM frame and the solar-system dynamic reference frame.

We also expect that continued observation and study of this pulsar will ultimately have an important impact in cosmology. Various statistical procedures have been applied to the unmodelled residuals of PSR B1855+09 (see ref. 19 and references therein) in an effort to place a rigorous upper limit on $\Omega_g$, the fractional energy density per logarithmic frequency.
interval of the primordial gravitational wave background. As the timing baseline for PSR J0437–4715 increases, our experiment will probe more deeply into the low frequencies of the cosmic gravitational wave spectrum, where, owing to its steep power-law dependence\(^{18}\), the most stringent restriction on $\Omega_g$ can be made.

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Table 1 PSR J0437–4715 physical parameters

| Parameter                      | Value                           |
|--------------------------------|---------------------------------|
| Right ascension, \( \alpha \)  | 04\(^{h}\) 37\(^{m}\) 15\(^{s}\)7865145(7) |
| Declination, \( \delta \)      | -47\(^{\circ}\) 15\('\) 08\("\)461584(8) |
| \( \mu_\alpha \) (mas yr\(^{-1}\)) | 121.438(6)                     |
| \( \mu_\delta \) (mas yr\(^{-1}\)) | -71.438(7)                     |
| Annual parallax, \( \pi \)     | 7.19(14)                       |
| Pulse period, \( P \) (ms)     | 5.757451831072007(8)           |
| Reference epoch (MJD)           | 51194.0                        |
| Period derivative, \( \dot{P} \) (10\(^{-20}\)) | 5.72906(5)                     |
| Orbital period, \( P_b \) (days) | 5.741046(3)                    |
| \( x \) (s)                    | 3.36669157(14)                 |
| Orbital eccentricity, \( e \)  | 0.000019186(5)                 |
| Epoch of periastron, \( T_0 \) (MJD) | 51194.6239(8)                  |
| Longitude of periastron, \( \omega \) (\(^{\circ}\)) | 1.20(5)                        |
| Longitude of ascension, \( \Omega \) (\(^{\circ}\)) | 238(4)                         |
| Orbital inclination, \( i \)   | 42.75(9)                       |
| Companion mass, \( m_2 \) (M\(_{\odot}\)) | 0.236(17)                      |
| \( \dot{P}_b \) (10\(^{-12}\)) | 3.64(20)                       |
| \( \dot{\omega} \) (\(^{\circ}\)yr\(^{-1}\)) | 0.016(10)                      |

Best-fit physical parameters and their formal 1\(\sigma\) errors were derived from arrival time data by minimizing an objective function, \( \chi^2 \), as implemented in TEMPO (http://pulsar.princeton.edu/tempo). Our timing model is based on the relativistic binary model\(^{19}\) and incorporates additional geometric constraints derived by Kopeikin\(^{7,8}\). Indicative of the solution’s validity, \( \chi^2 \) was reduced by 30% with the addition of only one new parameter, \( \Omega \). To determine the 1\(\sigma\) confidence intervals of \( \Omega \) and \( i \), we mapped projections of the \( \Delta \chi^2 \equiv \chi^2(\Omega, i) - \chi^2_{\text{min}} = 1 \) contour, where \( \chi^2(\Omega, i) \) is the value of \( \chi^2 \) minimized by variation of the remaining model parameters, given constant \( \Omega \) and \( i \). Parenthesized numbers represent uncertainty in the last digits quoted, and epochs are specified using the Modified Julian Day (MJD).