String Stability Control Strategy Analysis of Mixed Traffic Flow with the CIVs and NCVs

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Abstract. With the development of vehicle-to-infrastructure cooperation system, a mixed traffic phenomenon with non-connected vehicles (NCVs) and connected and intelligent vehicles (CIVs) will exist over a long period of time. Therefore, the mixed traffic flow stability control has become a hot topic in the future. In order to improve the string stability in the complex and changeable internet of vehicles environment, it is necessary to propose the optimal control method of string stability in the mixed traffic flow. In this paper, NCV and CIV car-following modes are employed to propose a local platoon control method of the connected vehicle, which can achieve the purpose of optimizing the mixed traffic flow stability. Two types of local mixed platoon are considered when the effective communication distance with two vehicles in the vehicle-to-vehicle (V2V) communication. Numerical simulations results show that our proposed string stability control strategy has the effectiveness in the improvement of the mixed traffic flow stability.

Keywords. Mixed traffic flow; local platoon; vehicle-to-vehicle communication; connected and intelligent.

1. Introduction
In recent years, the characters of mixing traffic flow have been studied by many scholars. In the mixed traffic flow, the mixed traffic flow can improve traffic flow stability and reduce rear-end collision risk when the vehicle platoon exists the corresponding proportions of connected and intelligent vehicles (CIVs) [1-6]. Therefore, traffic flows with non-connected vehicles (NCVs) and CIV will become the normal state with the development of vehicle-to-vehicle (V2V) communication in the near future. Therefore, improving the mixed traffic flow stability becomes a important problem in the field of intelligent transportation systems.

To improve the mixed traffic flow stability, many scholars have done a lot of research in this field. Such as, Chen et al. [7] analyzed the string stability in the mixed traffic with automated and manual driving vehicles. Zheng et al. [8] studied the effect of automated vehicles on platoon stability in the mixed traffic using a stochastic car-following model. Huang et al. [9] studied the cooperative control strategy for heterogeneous in the internet of vehicles. Wang et al. [10] investigated the effect of multiple front vehicles’ optimal velocity on the mixed traffic flow stability based on a new car-following model. Although many research papers have analyzed the the evolution of mixed traffic...
flow based on some car-following models, few research papers have proposed a string stability control strategy for the NCV and the CIV traffic flows. Therefore, we propose a local platoon control method to improve the mixed traffic flow stability.

In Section 2, NCV and CIV car-following models are introduced. In Section 3, the Local string stability constraints are derived. In Section 4, numerical simulations verify the validity of our proposed local platoon stability control method for improving the global platoon stability. Conclusions are given in Section 5.

2. NCV and CIV Car-following Models

NCV and CIV car-following models are proposed by Zhang et al. [11] as:

\[ \ddot{a}_n^{\text{non-connected}}(t+\tau) = \alpha(1+\xi)\left(S_n(t) - S_D\right), \xi \sim N\left(\mu, \sigma^2\right), \]
\[ \ddot{a}_n^{\text{connected}}(t+\tau) = \alpha(S_n(t) - S_D) + \sum_{i=1, j \neq n-1}^m \left(\gamma_i \dot{a}_{n-i}^{\text{non-connected}}(t) + \gamma_i \dot{a}_{n-i}^{\text{connected}}(t)\right) \]
\[ S_n(t) = 1 - \frac{0.15v_n(t)}{D_n(t)} + \frac{(v_n(t) - v_{n-1}(t))(v_n(t) + v_{n-1}(t))}{1.5g \cdot D_n(t)}, \]

where \( S_D \) is desired safety margin, \( D_n(t) \) is the spacing between the \( n \)-th vehicle and the \( n \)-th vehicle, \( v_n(t) \) is the \( n \)-th vehicle’s velocity; \( v_{n-1}(t) \) is the \( n \)-th vehicle’s velocity; \( m_1 \) and \( m_2 \) are the number of NCVs and CIVs at the front and rear of the \( n \)-th vehicle, and \( \beta_i \) and \( \gamma_i \) indicate the feedback gain coefficients, respectively; \( g = 9.8 \text{ m/s}^2 \).

We consider the communication range with radius of two vehicles in the car-following process. There are two types of local mixed platoons, as shown in figure 1.

![Figure 1. Optimization target diagram of local mixed platoon.](image)

3. Local String Stability Constraints

According to figure 1, we define all vehicles with the same velocity \( v^* \) and the given desired gap \( y^* \) in the steady-state. Then we have:

\[ Y_n(t) = \ddot{Y}_n(t) + y^*, V_n(t) = \ddot{V}_n(t) + v^*, V_{n-1}(t) = \ddot{V}_{n-1}(t) + v^* \]
where $\tilde{Y}_n(t)$ and $\tilde{V}_n(t)$ is the disturbance of gap and velocity, respectively.

According to equation (1), the gap of the steady-state can be obtained as:

$$y^* = \frac{v^* \tau_y}{1 - SM_\rho}$$

First, NCV car following model is linearized under the steady-state flow solution, we have:

$$r \tilde{V}_n(t) + \tilde{V}_n(t) = f_{v_n} \tilde{V}_{n-1}(t) + f_{y_n} \tilde{Y}_n(t) + f_{v_n} \tilde{V}_n(t)$$

where $f_{v_n}$, $f_{y_n}$, and $f_{v_n}$ denote the partial derivatives at $(v'_{n-1}, y', v_n)$ for the speed of vehicle $(n-1)$, the gap of two vehicles, and the speed of vehicle $n$, respectively.

Furthermore, equation (4) can be written as follows:

$$r \frac{d^2 \tilde{V}_n(t)}{dt^2} + \frac{d \tilde{V}_n(t)}{dt} = f_{v_n} \frac{d \tilde{V}_n(t)}{dt} + f_{y_n} \tilde{V}_n(t) + f_{v_n} \tilde{V}_n(t)$$

Then, the transfer function $T_i(s)$ is shown using the Laplace transform as

$$T_i(s) = \frac{f_{v_n} \cdot s + f_{y_n}}{\tau \cdot s^3 + s^2 - f_{v_n} \cdot s + f_{y_n}}$$

where

$$f_{v_n} = \frac{\partial f}{\partial v_n} \bigg|_{v'_n, y'_n} = -\alpha(1 + \mu) \frac{\tau \cdot d + v^*}{y \cdot d}$$

$$f_{y_n} = \frac{\partial f}{\partial y_n} \bigg|_{v'_n, y'_n} = \alpha(1 + \mu) \frac{v^* \tau_y}{(y^*)^2}$$

$$f_{v_{n-1}} = \frac{\partial f}{\partial v_{n-1}} \bigg|_{v'_n, y'_n} = \alpha(1 + \mu) \frac{v^*}{y \cdot d}$$

Furthermore, CIV car-following model is simplified, and CIV only accept the motion state of the preceding vehicle as feedback control term. Then the gain coefficients of feedback control of equation (1) $\beta_i$ and $\gamma_{m_i-j}$ are also 0.

The simplified CIV car following model is linearized under the steady-state flow solution, we have:

$$r \frac{d^2 \tilde{V}_n(t)}{dt^2} + \frac{d \tilde{V}_n(t)}{dt} = f_{v_n} \frac{d \tilde{V}_n(t)}{dt} + f_{y_n} \tilde{V}_n(t) + f_{v_n} \tilde{V}_n(t) + \gamma_{i} \frac{d \tilde{V}_{n-1}(t)}{dt}$$

Then, the transfer function $T_i(s)$ is shown using the Laplace transform as

$$T_i(s) = \frac{\gamma_i \cdot s^2 + f_{v_n} \cdot s + f_{y_n}}{\tau \cdot s^3 + s^2 - f_{v_n} \cdot s + f_{y_n}}$$
The input of the system is the velocity disturbance of the leading vehicle of local platoon, and the output of the system is the velocity disturbance of the tail vehicle of local platoon. Then the transfer function $T(s)$ can be written as:

$$T(s) = \frac{\gamma_1 \cdot s f_{y_{m-1}} + s f_{y_n} + \tilde{f}_y}{\tau \cdot s f_{y_{m-1}} + s f_{y_n} + \tilde{f}_y}$$

Then, the corresponding transfer function $\tilde{T}(s)$ should be satisfied as:

$$\left\| \tilde{T}(s) \right\|_\infty = \sup_{z \in [0, +\infty)} \tilde{T}(iz) \leq 1$$

We have

$$\left\| T(iz) \right\| = \left| \frac{-i\tau \cdot z^3 - z^2 - if_{y_{m-1}} \cdot z + f_{y_n} \cdot z^2 + f_{y_{m-1}} \cdot iz + \tilde{f}_y}{-i\tau \cdot z^3 - z^2 - if_{y_{m-1}} \cdot z + f_{y_n} \cdot z + \tilde{f}_y} \right| \leq 1, z \in [0, +\infty)$$

According to equations (3), (7), (10), and (13), the stability condition of the local platoon is determined by the feedback gain coefficient and the expected speed of the platoon.

4. Numerical Simulation

Based on the stability condition of the local platoon (equation (13)), the reasonable selection of the feedback control gain coefficient can be obtained by analyzing the stability region between the feedback control gain coefficient and the expected velocity. Then all types of local platoon can reach the steady-state under the given expected velocity, thereby achieving the purpose of optimizing the stability control of mixed traffic flow.

4.1. Local Platoon of I Type

According to equation (13), $\left\| T(iz) \right\| = \left| \frac{-i\gamma_1 \cdot z^3 - z^2 - if_{y_{m-1}} \cdot z + f_{y_n} \cdot z^2 + f_{y_{m-1}} \cdot iz + \tilde{f}_y}{-i\tau \cdot z^3 - z^2 - if_{y_{m-1}} \cdot z + f_{y_n} \cdot z + \tilde{f}_y} \right| \leq 1, z \in [0, +\infty)$ when $m = 1$.

Substituting equations (3) and (10) to $\left\| T(iz) \right\|$, the string stability constraint can be calculated, and then the range of stability region of feedback gain coefficient $\gamma_1$ can be obtained under the given expected velocity (20 m/s), as shown in figure 2. Local platoon can keep a stable car-following state under the given expected velocity (20 m/s) when $0.045 < \gamma_1 < 0.53$. Therefore, optimal control results of feedback gain coefficients for type I local platoon: $0.045 < \gamma_1 < 0.53$.

4.2. Local Platoon of II Type

Equation (13) can be written when $m=2$ as:
\[
[T(iz)] = \left( \begin{array}{c}
 f_{y_{n+1}} \cdot iz + f_{y_n} \\
 -i\tau \cdot z^3 - z^2 - if_{y_n} \cdot z + f_{y_n}
\end{array} \right) \cdot \frac{-\gamma_{1}(s) - z^2 + \tilde{f}_{y_{n+1}} \cdot iz + \tilde{f}_{y_n}}{-i\tau \cdot z^3 - z^2 - if_{y_n} \cdot z + f_{y_n}} \leq 1, z \in [0, +\infty)
\] (14)

Figure 2. Relationship between feedback gain coefficient and string stability constraints for local platoon of type I.

Likewise, we can derive the string stability constraint, and then the range of stability region of feedback gain coefficient \( \gamma_1 \) can be obtained under the given expected velocity (20 m/s), as shown in figure 3. Local platoon can keep a stable car-following state under the given expected velocity (20 m/s) when \( 0.1 < \gamma_1 < 1.38 \). Therefore, Optimal control results of feedback gain coefficients for type II local platoon: \( 0.1 < \gamma_1 < 1.38 \).

Figure 3. Relationship between feedback gain coefficient and string stability constraints for local platoon of type II.
In conclusion, we choose $\gamma_i=0.2$ under the local platoon stability optimization control strategy. There is a vehicle platoon with 50 vehicles, which is composed of the ICV and NCV randomly according to 50% proportion. Next, the effectiveness of the proposed local platoon stability control method is verified by numerical simulation.

As can be seen from figure 4a, the velocities have a large fluctuation in the platoon with the increase of the number of vehicles in the mixed traffic flow. Under the local platoon stability control strategy, the fluctuations of velocities in the platoon are kept within $\pm 2$ m/s, as shown in figure 4b. Results show that feedback control can improve the string stability in mixed traffic environment. Therefore, the local platoon stability control method can optimize the global platoon stability to a certain extent.

![Vehicle number vs Velocity (m/s)](a) Without local platoon control strategy  
![Vehicle number vs Velocity (m/s)](b) With local platoon control strategy

**Figure 4.** Velocity evolution in the mixed traffic flow.

5. Conclusions
The CIV and NCV car-following model are employed to analyze the string stability in the complex and changeable internet of vehicles environment. We presented a local platoon control method in the mixed traffic flow. And local string stability constraints are derived for different two types local platoon. The relationship between feedback gain coefficient and string stability constraints are obtained. Numerical simulations results show that our proposed local platoon stability control method can optimize the global platoon stability to a certain extent.

**Acknowledgments**
This work was supported by the National Key R&D Program of China No. 2018YFB1601100 and No. 2018YFC0807500, Chinese Postdoctoral Science Foundation Science (No. 2019M660407).

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