Physical Vacuum in Superconductors

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Abstract

Although experiments carried out by Jain et al. showed that the Cooper pairs obey the strong equivalence principle, the measurement of the Cooper pairs inertial mass by Tate et al. revealed an anomalous excess of mass. In the present paper we interpret these experimental results in the framework of an electromagnetic model of dark energy for the superconductors’ vacuum. We argue that this physical vacuum is associated with a preferred frame. Ultimately from the conservation of energy for Cooper pairs we derive a model for a variable vacuum speed of light in the superconductors physical vacuum in relation with a possible breaking of the weak equivalence principle for Cooper pairs.

1 Introduction

In the present work we explore the consequences of the spontaneous breaking of gauge invariance in superconductors on the principle of equivalence for Cooper pairs. The breaking of gauge invariance in superconductors makes the frame of the superconductor a preferred frame. Thus it is natural to wonder if it is possible to observe an absolute type of motion with respect to this frame.

Assuming that the idea of a physical vacuum in superconductors defined by its energy momentum vector, with zero spatial component and non-zero energy density, is correct, one deduces that this type of vacuum is related to the superconductor’s preferred frame. Here we argue that below the superconductor’s critical temperature, the vacuum energy density in a superconductor associated with a preferred frame corresponds to an electromagnetic model of zero point energy contributing to a vacuum energy density similar to the one from cosmological origin. This model is intimately related with a discrete picture of spacetime made by Minkowski’s Diamond cells generated by the continuous process of creation and annihilation of the Cooper pairs. From this model we derive an index of refraction for the vacuum

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speed of light in the superconductor, which appears to be linked with a possible breaking of the weak equivalence principle for the Cooper pairs. Experimental results supporting these theoretical possibilities are reviewed.

In section 2, the experiment from Tate et al. measuring the Cooper pairs inertial mass and the experiment from Jain et al. testing the strong equivalence principle are analyzed with respect to a possible breaking of the weak equivalence principle. In section 3 we present the consequences of the spontaneous breaking of gauge invariance in superconductors with respect to their electromagnetic and gravitational properties. In section 4 the possibility of a variable vacuum speed of light in relation with the physical implementation of a preferred frame in superconductors is explored within a phenomenological perspective. In section 5 we review the electromagnetic zero-point dark energy model proposed by Beck, Mackey and the author, and introduce it as a possible candidate for the vacuum energy in superconductors required to physically implement locally a Lorentzian concept-type of a physical ether. In section 6 we show that a variable speed of light in superconductors reconciles the breaking of the weak equivalence principle for cooper pairs with the law of energy conservation. Finally we conclude with some avenues, which would be worth to explore.

2 Testing the Principle of Equivalence for Cooper pairs

The Principle of General Covariance, which establishes the independence of the laws of physics with respect to the physical observer’s reference frame, and which is at the foundation of Einstein’s theory of general relativity, can be formulated from two different complementary phenomenological perspectives: The Strong Equivalence Principle and the Weak Equivalence Principle.

The Strong Equivalence Principle states that locally the physical effects of a uniform gravitational field are indistinguishable from those due to an accelerating reference frame.

The Weak Equivalence Principle means the constancy of the ratio between the inertial and the gravitational mass \( m_i \) and \( m_g \) respectively.

\[
\frac{m_g}{m_i} = \iota = Cte
\]

This implies, in classical physics, that the possible motions in a gravitational field are the same for different test particles. Current experimental tests of the weak equivalence principle \[1\] \[2\], indicate that the gravitational and inertial masses of any physical body are equal to each other, \( m_g/m_i = \iota = 1 \),
within a relative accuracy of the Eötvös-factor, \( \eta(A, B) \) less than \( 5 \times 10^{-13} \).

\[
\eta(A, B) = 2 \left( \frac{m_g/m_i}_A - \frac{m_g/m_i}_B \right) < 5 \times 10^{-13} \quad (2)
\]

The Eötvös-factor is usually obtained from the measurement of the differential acceleration, \( \Delta a \), of two free falling test bodies, \( A \) and \( B \).

\[
\eta(A, B) = \frac{\Delta a}{g} \quad (3)
\]

where \( g \) is the Earth’s gravitational acceleration.

As argued by Anandan [3], the principle of equivalence cannot be demonstrated on a purely theoretical basis. Neither classical or quantum physics can derive the equivalence principle from more fundamental axioms. Thus it can only be justified by experiment. In the following sections we present two important experiments which bring relevant information about the validity of the principle of general covariance for Cooper pairs in superconductors.

2.1 Anomalous Cooper pairs inertial mass excess

In 1989 Cabrera and Tate [4, 5], through the measurement of the magnetic trapped flux originated by the London moment, reported an anomalous Cooper pair inertial mass excess in thin rotating Niobium superconductive rings:

\[
\Delta m_i = m^*_i - m_i = 94.147240(21) eV \quad (4)
\]

Here \( m^*_i = 1.000084(21) \times 2 m_e = 1.023426(21) MeV \) (\( m_e \) being the standard electron mass) is the experimentally measured Cooper pair inertial mass (with an accuracy of 21 ppm), and \( m_i = 0.999992 \times 2 m_e = 1.002331 MeV \) is the theoretically expected Cooper pair inertial mass including relativistic corrections.

This anomalous Cooper pair mass excess has not received, so far, a satisfactory explanation in the framework of superconductor’s physics. If the gravitational mass of the Cooper pairs, \( m_g \), remains equal to the expected theoretical Cooper inertial mass, \( m_i = m_g = 0.999992 \times 2 m_e = 1.002331 MeV \), Tate’s experiment would reveal that the Cooper pairs break the WEP with an Eötvös-factor, \( \eta(E, T) = 9.19 \times 10^{-5} \gg 5 \times 10^{-13} \), obtained from eq.(2) assuming the Experimental (\( E \)) and Theoretical (\( T \)) ratios \( m_g/m^*_i = 0.999908 \) and \( m_g/m_i = 1 \) respectively. The question is thus: Is an excess of mass, similar to the one observed by Tate for the cooper pairs inertial mass, also occurring for the cooper pair’s gravitational mass?

2.2 Testing the Strong Equivalence Principle for Cooper Pairs

In 1987 Jain et al carried out an experiment to probe the SEP for Cooper pairs [6]. The experiment consisted of two Josephson junctions separated by
a height $H = 7.2cm$, connected in opposition by superconducting wires. In this experiment two effects are competing to each other: On the one side the junctions are coupled to microwave radiation from a common source, which maintains a voltage difference of $2.35 \times 10^{-21}V$ between the two junctions by means of the gravitational red shift.

$$V_u = V_l(1 - \frac{gH}{c^2})$$  \hspace{1cm} (5)

where $V_u$ and $V_l$ are the electric potentials at the upper and lower junctions respectively, and $g$ is the Earth gravitational acceleration. On the other side the strong equivalence principle predicts that the gravito-electromechanical potential $\bar{\mu}$ is constant along the connecting wires.

$$\bar{\mu} = \mu(1 + \frac{m_g gH}{m_i c^2}) = Cte$$  \hspace{1cm} (6)

where $\mu$ is the electrochemical potential, which in general will not be constant, $m_i$ and $m_g$ are the inertial and gravitational Cooper pair masses. This implies that the potential difference $V$ between the superconducting wires varies with height so that

$$V(z) = V(z = 0)(1 - \frac{m_g gH}{m_i c^2})$$  \hspace{1cm} (7)

cumulating the two effects, eq.(5) and eq.(7), we obtain the total loop emf:

$$\Delta V = V_l\left(\frac{gH}{c^2} - \frac{m_g gH}{m_i c^2}\right)$$  \hspace{1cm} (8)

which is predicted to be zero on the basis of the strong equivalence principle. Jain indeed measured the total emf to be less than $1 \times 10^{-22}$, consistent with the relativistic prediction.

Therefore Jain et al. experiment tested the strong equivalence principle for Cooper pairs, showing that using the Cooper pairs as probe masses, we also reach the conclusion that the laboratory is accelerating with respect to a local Minkowski spacetime. This plainly justifies the curved spacetime description, which has been well tested for neutral matter, to hold for Cooper pairs as well. Since Jain’s experiment tests the null result of eq.(8), this experiment also demonstrated that the inertial and gravitational mass of Cooper pairs are exactly equal to each other within an accuracy of $4\%$:

$$\frac{m_i}{m_g} = 1 \pm 0.04$$  \hspace{1cm} (9)

Unfortunately the accuracy of Jain’s experiment is not good enough to discard or confirm a difference between the inertial and the gravitational mass of Cooper pairs of 21 ppm as reported by Tate et al.
2.3 Non detection of the Gravitomagnetic London Moment in Rotating Superconductors versus breaking of the weak equivalence principle for Cooper pairs

As shown in [7], Tate’s experimental results could also be interpreted as resulting from an additional gravitomagnetic term in the Cooper pairs canonical momentum, together with the assumption that the inertial and the gravitational mass of the Cooper pairs, $m_i$ and $m_g$, remain equal to their expected theoretical values, $m_i = m_g = 0.999992 \times 2m_e = 1.002331 MeV$,

$$\pi = m_i \vec{v} + e\vec{A} + m_g \vec{A}_g$$  \hspace{1cm} (10)

Where $\vec{A}$ is the magnetic vector potential, $v$ is the Cooper pair velocity, and $\vec{A}_g$ is the gravitomagnetic vector potential, whose rotational gives the gravitomagnetic field $\vec{B}_g$:

$$\vec{B}_g = \nabla \times \vec{A}_g$$  \hspace{1cm} (11)

From Tate’s measurements one can estimate the relative value of the gravitomagnetic field, with respect to the superconductor’s angular velocity $\omega$, required to account for the anomalous Cooper pair inertial mass excess:

$$\chi = \frac{(m_i^* - m_i)}{m_g} = \frac{B_g}{2\omega} = 9.2 \times 10^{-5}$$  \hspace{1cm} (12)

This equation clearly indicates that the interpretation of Tate’s experiment in terms of a gravitomagnetic term in the Cooper pairs canonical momentum, only makes sense in the context of a breaking of the weak equivalence principle for Cooper pairs with an Eötvös-factor, $\eta(E,T) = 9.19 \times 10^{-5}$, with $\eta$ and $\chi$ related to each other in the following manner:

$$\eta(E,T) = \frac{\chi}{1 + \frac{\chi}{2}}$$  \hspace{1cm} (13)

In the case where $\chi << 1$, eq.(13) reduces simply to

$$\eta \sim \chi$$  \hspace{1cm} (14)

In the following we will refer to $\chi$ as the ”Eötvös-factor $\chi$”, and we refer to $\eta$ as the ”Eötvös-factor $\eta$”.

Therefore the Eötvös-factor $\chi$ and $\eta$ can be estimated not only through differential acceleration measurements during free fall experiments but also from the measurement of the gravitomagnetic Larmor theorem in rotating frames, with probe masses located in strong gravitomagnetic fields:

We are left with two alternatives:

1. We can extrapolate Jain’s experiment to the accuracy required to probe Tate’s results. This would imply that Cooper pairs do not
violate the strong and the weak equivalence principle, a null Eötvös-factor, \( \eta(E, T) = 0 \), for Cooper pairs in superconductors would then be expected. Therefore from eq.\( (13) \) we would deduce that \( \chi = 0 \). In this case the Cooper pairs would still carry out an equal excess of inertial and gravitational mass according to Tate’s result, that needs to be explained.

2. The Cooper pairs in Niobium break the weak equivalence principle with an Eötvös-factor, \( \eta(E, T) = 9.19 \times 10^{-5} \). In this case only the Cooper pairs inertial mass excess needs to be explained.

Although recent experiments from Tajmar et al.\cite{8} involving rotating superconducting rings failed at detecting the gravitomagnetic field appearing in eq.\( (12) \), we see that as indicated by eq.\( (13) \) the quotient \( \chi = B_g/2\omega \) is associated with the breaking of the weak equivalence principle for Cooper pairs, and not with a gravitomagnetic analogue of the magnetic London moment. Therefore the experiments carried out by Tajmar et al. were not designed to detect the experimental effect pertinent for the investigation of Tate’s experimental results, and which results from the correct phenomenological interpretation of eq.\( (12) \). Consequently the results from Tajmar et al. experiments cannot provide us with relevant information to decide which of the two alternatives above is the correct one.

In summary to investigate further Tate et al experimental results we should not aim at detecting a gravitomagnetic type analog of the London moment in rotating superconductors, but instead of this we should aim at testing the weak equivalence principle for Cooper pairs.

In the following we will try to demonstrate that the second alternative above is the correct one. Thus we will demonstrate that the Cooper pairs in superconductors could break the weak equivalence principle.

3 Spontaneous breaking of gauge symmetry in superconductors

Superconductivity is a macroscopic quantum phenomena associated with the formation of a condensate of electron pairs (called Cooper pairs, or superelectrons) in the crystallin lattice of certain solids below a critical temperature \( T_c \), depending upon the particular material. The Cooper pair condensate is described by one single wavefunction \( \psi(x^\mu) \) depending on the spacetime coordinates \( x^\mu = (x^0 = ct, x^1 = x, x^2 = y, x^3 = z) \).

\[
\psi(x^\mu) = \alpha(x^\mu) e^{i\beta(x^\mu)}
\]  

(15)

where \( n^* = \alpha^2 \) represents the density of Cooper pairs, and \( \beta(x^\mu) \) is the phase of the wavefunction, both, \( \alpha \) and \( \beta \) are real valued functions. The
average distance between the two electrons in a Cooper pair is known as the coherence length, $\xi_c$. Typically, the coherence length is approximately 2 orders of magnitude larger than the interatomic spacing of a solid, therefore Cooper pairs are not comparable with tightly bound electron molecules. Instead, there are many other electrons between those of a specific Cooper pair allowing for the paired electrons to change partners on a time scale, $\tau$, defined by Heisenberg’s uncertainty principle,

$$\tau \leq \hbar/\Delta(0)$$  \hspace{1cm} (16)

where $\hbar$ is Planck’s constant.

According to Bardeen, Cooper, Schriefer (BCS) theory the binding energy, $\Delta(0)$, between the electrons forming a Cooper pair and the critical temperature, $T_c$, at $T = 0$ in a given material is:

$$\Delta(0) = 1.76kT_c$$  \hspace{1cm} (17)

Where $k$ is Boltzmann constant. For low critical temperature (conventional) superconductors $T_c \sim 10K$, and $\Delta(0) \sim 1meV$.

The 4-current density of Cooper pairs in a superconductor is:

$$j^\mu = \frac{2n*e}{m_i^*} \left( -\hbar \frac{\partial \beta}{\partial x^\mu} - \frac{2e}{c} A_\mu \right)$$  \hspace{1cm} (18)

where $m_i^*$ is the mass of the Cooper pair in the interior of the superconductor, and $A_\mu(x^\mu) = (\phi, \vec{A})$ is the electromagnetic potential with time and space components being the electric scalar potential $\phi$, and the magnetic vector potential $\vec{A}$ respectively.

Superconductivity may be regarded fundamentally as being due to the spontaneous breaking of the $U(1)$ gauge symmetry. This has two fundamental consequences. first it makes the frame of the superconductor a preferred frame $\Sigma$. Second the current density relative to the superconductor vanishes. In the limit of weak gravitational fields, the preferred frame is the rest frame in which the superconductor 4-velocity is

$$t^\mu = (c_1,0,0,0)$$  \hspace{1cm} (19)

In defining the 4-velocity $t^\mu$ we have assumed a speed of light $c_1$ which can be different or equal to the classical value $c$ to allow for the possibility of corrections to the predicted relativistic effects. From eq.(18), the vanishing of the current density in the superconductor preferred frame means:

$$-\hbar \frac{\partial \beta}{\partial x^\mu} - \frac{2e}{c} A_\mu = m_i^* t^\mu$$  \hspace{1cm} (20)

The zeroth component of eq.(20) leads to the electrochemical potential $\mu$, which includes the rest-mass energy.

$$\mu = m_i^* c_1 c + 2eA_0$$  \hspace{1cm} (21)
In field theory the spontaneous breaking of gauge invariance leads to massive photons via the Higgs mechanism. Massive photons can also be understood as a consequence of the possibility of a preferred frame in superconductors. In this case, in the superconductor, the Maxwell equations transform to the so called Maxwell-Proca equations, which are given by

\[
\nabla \vec{E} = \frac{\rho^*}{\epsilon_0} - \frac{1}{\lambda_\gamma^2} \phi \tag{22}
\]

\[
\nabla \vec{B} = 0 \tag{23}
\]

\[
\nabla \times \vec{E} = -\dot{\vec{B}} \tag{24}
\]

\[
\nabla \times \vec{B} = \mu_0 \rho^* v_s + \frac{1}{c^2} \ddot{\vec{E}} - \frac{1}{\lambda_\gamma^2} \vec{A} \tag{25}
\]

Where \( \vec{E} \) is the electric field, \( \vec{B} = \nabla \times \vec{A} \) is the magnetic field, \( \epsilon_0 \) is the vacuum electric permittivity, \( \mu_0 = 1/\epsilon_0 c^2 \) is the vacuum magnetic permeability, \( \phi \) is the scalar electric potential, \( \vec{A} \) is the magnetic vector potential, \( \rho^* = 2en^* \) is the Cooper pairs fluid electric density, \( v_s \) is the cooper pairs velocity, and \( \lambda_\gamma = h/m_\gamma c \) is the photon’s Compton wavelength, which is equal to the London penetration depth \( \lambda_L = \sqrt{\frac{m}{\mu_0 n^* e}} \). Superconductor’s properties like the Meissner effect, and the London moment can be derived from this set of equations \[10\].

The possibility of a preferred reference frame in superconductors also allows to linearize Einstein Field Equations with a cosmological constant \[11\]. This leads to the set of de Sitter gravitoelectromagnetic equations, which include a massive graviton.

\[
\nabla \vec{g} = +3\pi G \rho - \frac{2}{3} \Lambda \varphi \tag{26}
\]

\[
\nabla \vec{B}_g = 0 \tag{27}
\]

\[
\nabla \times \vec{g} = -\frac{\partial \vec{B}_g}{\partial t} \tag{28}
\]

\[
\nabla \times \vec{B}_g = -\frac{4\pi G}{c^2} \rho v_s + \frac{1}{c^2} \frac{\partial \vec{g}}{\partial t} - \frac{2}{3} \Lambda \vec{A}_g \tag{29}
\]

Where \( \vec{g} \) is the gravitational field, \( \vec{B}_g = \nabla \times \vec{A}_g \) is the gravitomagnetic field, \( \epsilon_{0g} = 1/3\pi G \) is the vacuum gravitational permittivity, \( \mu_{0g} = 4\pi G/c^2 \) is the vacuum gravitomagnetic permeability, \( \varphi \) is the scalar gravitational potential, \( \vec{A}_g \) is the gravitomagnetic vector potential, \( \rho \) is the superconductor’s physical vacuum mass density, \( v_s \) is the cooper pairs velocity, \( \Lambda \) is the cosmological constant, and \( \lambda_g \) is the Compton wavelength of the massive graviton.

\[
\frac{1}{\lambda_g^2} = \left( \frac{m_pc}{\hbar} \right)^2 = \frac{2\Lambda}{3} \tag{30}
\]
The de Sitter gravitoelectromagnetic set of equations is only valid locally, at the origin of the preferred reference frame attached to the superconductor, and since repulsive gravitational fields are predicted by eq. (26) they only apply to the energy density of the superconductor’s physical vacuum. Taking the rotational of eq. (29), and solving the resulting differential equation for the 1-dimensional case of a superconducting ring, rotating with angular velocity $\omega$, we find the gravitomagnetic Larmor theorem [12] for Cooper pairs:

$$B_g = 2\omega \mu_0 g \rho \lambda_g^2 \tag{31}$$

The interpretation of a non-zero cosmological constant in Einstein field equations as the physical possibility of privileged coordinate frames without breaking the strong equivalence principle, was already debated by Rayski in [15]. Although the strong equivalence principle is the backbone of the principle of general covariance, the gravitational analogue of electromagnetic gauge invariance is the weak equivalence principle [13]. Therefore the spontaneous breaking of gauge invariance in superconductors would appear together with the breaking of the weak equivalence principle for Cooper pairs. Since the breaking of gauge invariance is affecting the photon rest mass, is the existence of a preferred frame attached to a superconductor affecting also the constant value of the speed of light in vacuum?

4 Vacuum speed of light in superconductors

Starting from Consoli and Costanzo idea that the physical vacuum might be defined by its energy momentum vector, with zero spatial component and non-zero energy density for the time component [20][21], one fixes the 4-velocity of the vacuum medium in a similar manner as we defined above the preferred frame attached to the superconductor, eq. (19):

$$t^\mu(S') \equiv (c',0,0,0) \tag{32}$$

where the vacuum speed of light $c'$ is not necessarily equal to its classical value $c$ or to the superconductor’s preferred frame vacuum speed of light $c_1$.

An observer attached to the superconductor preferred frame, $\Sigma$, would witness locally two possible vacuum speeds of light $c$ and $c'$, this will affect locally the laws of special relativity for this observer. The diagonal form of the interval of universe with respect to $\Sigma$ before the material becomes superconductor is the standard one:

$$ds^2 = c^2 dt^2 - dl^2 \tag{33}$$

When the material becomes superconductor the same observer in sigma will observe a different diagonal form for the interval of universe $ds$ that
will change to $ds'$ according to the superconductor vacuum refractive index $N_{\text{vacuum}}$:

$$\left(\frac{ds}{ds'}\right)^2 = N_{\text{vacuum}} = \frac{c}{c'}$$  \hspace{1cm} (34)$$

Therefore the Minkowski interval of universe in the superconducting state will be:

$$ds^2 = \frac{c}{c'}\left[(2cc' - c^2)dt - dl^2\right]$$  \hspace{1cm} (35)$$

From this interval we deduce that the effective Lorentzian speed of light that will set the usual relativistic effects for the superconductor will be:

$$c_{\text{eff}}^2 = 2cc' - c^2 = 2c_g^2 - c^2$$  \hspace{1cm} (36)$$

where $c_g = cc'$ is the geometric mean between $c$ and $c'$.

The time dilatation relative to the superconductor will be expressed in function of this effective speed of light.

$$dt = \frac{d\tau}{\sqrt{1 - \frac{v^2}{c_{\text{eff}}^2}}}$$  \hspace{1cm} (37)$$

Accordingly the law of length contraction will be.

$$dl = d l_0 \sqrt{1 - \frac{v^2}{c_{\text{eff}}^2}}$$  \hspace{1cm} (38)$$

The increase of mass-energy with velocity will be:

$$E = \frac{m_0 c_{\text{eff}}^2}{\sqrt{1 - \frac{v^2}{c_{\text{eff}}^2}}}$$  \hspace{1cm} (39)$$

The spacetime metric is only diagonal with respect to $\Sigma$. The relative velocity $v$ between an observer $S'$, located outside the volume of the superconductor, and $\Sigma$ introduces off diagonal elements $g_{0i}$ in the metric relative to $S'$. By starting with the diagonal and isotropic form eq.(35), one obtains

$$g_{0i} \sim 2(N_{\text{vacuum}} - 1)\frac{v_i}{2c' - c}$$  \hspace{1cm} (40)$$

Since the components of the gravitomagnetic vector potential $\vec{A}_g = (A_{g1}, A_{g2}, A_{g3})$ results from the spacetime metric components $g_{0i}$

$$A_{gi} = -cg_{0i}$$  \hspace{1cm} (41)$$

And since the gravitomagnetic field $\vec{B}_g$ is obtained from the rotational of the gravitomagnetic vector potential

$$\vec{B}_g = \nabla \times \vec{A}_g$$  \hspace{1cm} (42)$$
Multiplying eq. (40) by $2c' - c$ and taking the rotational of both sides of the equation we obtain

$$-2 \frac{B'_g}{2\omega} + \frac{B_g}{2\omega} \sim 2(N_{\text{vacuum}} - 1)$$

(43)

where $B'_g = \nabla \times (-c'g_0i)$ is the gravitomagnetic field existing in the superconductor due to the non-classical vacuum. At this stage of our rational the non-classical Eötvös-factor $\chi' = B'_g/2\omega$ is an unknown quantity.

We reach the conclusion that the refractive index of the vacuum depends on the Eötvös-factor, $\chi = B_g/2\omega$, and vis versa.

$$\chi' + \frac{\chi}{2} \sim N_{\text{vacuum}} - 1$$

(44)

Therefore a preferred frame resulting from a breaking of gauge invariance in a superconductor would lead simultaneously to a breaking of the weak equivalence principle for Cooper pairs and to a variable speed of light in the superconductor’s vacuum. The experimental detection of one of this effects should imply the existence of the other. The next question that naturally arise is about the physical nature of the vacuum in the superconductor.

5 Electromagnetic zero-point dark energy and discrete spacetime in superconductors

A non-vanishing cosmological constant can be interpreted in terms of a non-vanishing vacuum energy density, $\rho_0$.

$$\rho_0 = \frac{c^4}{8\pi G} \Lambda \sim 10^{-29} \text{ g cm}^{-3} \simeq 3.88 \text{ eV/mm}^3$$

(45)

where $\Lambda = 1.29 \times 10^{-52} [\text{m}^{-2}]$ is the cosmological constant [14]. The cosmological constant is a good candidate to account for the dark-energy resulting from the latest cosmological observations reporting an accelerated expansion of the universe according to the following equation of state: $\rho_0 = -p$, where $p$ is the pressure. In [16] Beck and Mackey have developed an electromagnetic model of vacuum energy in superconductors. This model is based on bosonic vacuum fluctuations creating a small amount of vacuum energy density. One assumes that in the superconducting phase the photons, with zero point energy $\varepsilon = \frac{1}{2}h\nu$, contribute to a vacuum energy density, $\rho^*$, similar to the vacuum energy density resulting from the cosmological constant, eq. (45), and to which we will often refer as "electromagnetic zero-point dark energy". This vacuum energy density depends on a certain frequency cutoff $\nu_c$.

$$\rho^* = \frac{\pi h}{2 c^3 \nu_c^4}$$

(46)
In [16] the formal attribution of a temperature $T$ to the graviphotons is done by comparing their zero-point energy with the energy of ordinary photons in a bath at temperature $T$:

$$\frac{1}{2} \hbar \nu = \frac{\hbar \nu}{e^{\frac{\hbar \nu}{kT}} - 1}$$  \hspace{1cm} (47)$$

This condition is equivalent to

$$\hbar \nu = \ln 3kT$$  \hspace{1cm} (48)$$

Substituting the critical transition temperature $T_c$ specific to a given superconductive material in Eq.(48), we can calculate the critical frequency characteristic for this material:

$$\nu_c = \ln 3\frac{kT_c}{\hbar}$$  \hspace{1cm} (49)$$

For example, for Niobium with $T_c = 9.25K$ we get $\nu_c = 0.212$ THz, which when used as a cutoff frequency in eq.(49) leads to a vacuum energy $\rho^* = 0.49meV/mm^3$ inside the superconductor. Substitution of eq.(49) in eq.(46) leads to the law defining the density of electromagnetic zero-point dark energy in function of the superconductor’s critical temperature, $T_c$.

$$\rho^* = \frac{\pi \ln^4 3}{2} \frac{k^4}{(ch)^3} T_c^4$$  \hspace{1cm} (50)$$

If the zero-point electromagnetic dark energy has the same equation of state has the cosmological dark energy, $\rho_0$, then $\rho^*$ should exert a tiny negative pressure on the superconductor. By substitution of eq.(30), eq.(45) and eq.(50) in eq.(31) we obtain the Eötvös-factor, $\chi$, quantifying the level of breaking of the WEP for the Cooper pairs in a given superconductor in function of the superconductor’s critical temperature, $T_c$.

$$\chi = \frac{3}{2} \frac{\rho^*}{\rho_0} = \frac{3\ln^4 3}{8\pi} \frac{k^4 G}{c^3 \hbar^3 \Lambda} T_c^4.$$  \hspace{1cm} (51)$$

Remarkably, this equation connects the five fundamental constants of nature $k, G, c, \hbar, \Lambda$ with measurable quantities in a superconductor, $\chi$ and $T_c$.

We may define a Planck-Einstein temperature scale $T_{PE}$ as

$$T_{PE} = \frac{1}{k} \left( \frac{c^7 \hbar^3 \Lambda}{G} \right)^{1/4} = 60.71K.$$  \hspace{1cm} (52)$$

and the corresponding Planck-Einstein length

$$l_{PE} = \frac{\hbar}{M_{PEc}} = \left( \frac{\hbar G}{c^3 \Lambda} \right)^{1/4} = 0.037[mm]$$  \hspace{1cm} (53)$$
which is of the same order of magnitude as the Cooper pairs coherence length \( \xi_c \). Eq.\((51)\) can then be written as \[9\]

\[
\chi = \frac{3\ln^4 3}{8\pi} \left( \frac{T_c}{T_{PE}} \right)^4.
\] (54)

Substituting the critical transition temperature of Niobium, \( T_c = 9.25K \), in Eq.\((54)\) we find the following Eötvös-factor \( \chi \) for superconductive Niobium:

\[
\chi = 9.35 \times 10^{-5}
\] (55)

The above theoretically predicted value is close to the measured value in Cabrera and Tate’s experiment, Eq.\((12)\):

\[
\chi = \left( \frac{m^* - m_i}{m_g} \right) = \frac{B_g}{2\omega} = 9.2 \times 10^{-5}
\] (56)

In summary we found that by extending the initial Beck and Mackey model of electromagnetic dark energy to superconductor’s critical temperatures, different from the critical temperature associated with the cosmological constant cutoff frequency, we predict an Eötvös-factor \( \chi \) for Cooper pairs in superconducting Niobium very close to the one estimated from Tate et al experiment. This is a very encouraging result with respect to our interpretation, in section 2.3, of the Cooper pairs inertial mass excess being related with a breaking of the weak equivalence principle for these particles.

The attempt to resolve the inverse cosmological constant problem in [17], where formally the cosmological constant comes out 120 orders of magnitude too small, leads to assume that the spacetime volume filled by Cooper pairs in a superconductor is made of Planck-Einstein cells having a 4-volume, \( l_{PE}^4 \), which will statistically fluctuate according to:

\[
\Delta V \sim \sqrt{V} l_{PE}^2
\] (57)

Since the density of vacuum energy associated with the cosmological constant \( \rho_0 \), is canonically conjugated with the universe four-volume \( V \), we can formulate the following 4-dimensional Heisenberg uncertainty principle for the cosmos.

\[
\Delta \rho_0 \Delta V \sim \hbar c
\] (58)

By substitution of \( \rho_0 \) and \( \Delta V \) in eq.\((58)\), by the electromagnetic zero-point dark energy density, eq.\((50)\), and the SC’s four volume fluctuations, eq.\((57)\) respectively, we deduce that the superconductor’s Eötvös-factor \( \chi \) statistically fluctuates according to the quantum fluctuations of the Cooper pairs’s discrete spacetime volume:

\[
\chi \sqrt{V} \sim \frac{\pi^2}{3} l_{PE}^2
\] (59)
A discrete structure of the spacetime volume spanned by the Cooper pairs \[\text{[18]},\] in terms of Planck-Einstein cells, with volume fluctuating around the Planck-Einstein volume, \(l_{PE}^4\), can find a fundamental basis in relation with the Unruh effect for finite lifetime inertial observers \[\text{[19]}.\] For an observer with constant acceleration and infinite lifetime, the vacuum state of a suitable quantum field theory in Minkowski spacetime appears as a thermal equilibrium state with temperature

\[T_U = \frac{\hbar a}{2\pi kc}\]  \hspace{1cm} (60)

This result can be derived from the integration along the worldline of the observer of the interaction term between a detector and the vacuum. Martinetti and Rovelli \[\text{[19]}\] argued that for an inertial observer (i.e. with zero acceleration) with finite-proper lifetime \(\tau\), the finite Minkowski spacetime region with 4-volume \((ct)^4\), called a Diamond, appears as a thermal equilibrium state with temperature

\[T_D = \frac{2\hbar}{\pi k\tau}\]  \hspace{1cm} (61)

\(T_D\) is designated as the Diamond temperature associated with the spacetime Diamond shaped region spanned by the inertial observer in Minkowski spacetime. Let us assimilate a Cooper pair, in a given superconductor, to an inertial observer with (periodic) proper lifetime \(\tau\) given by the substitution of eq.(17) in eq.(16)

\[\tau \leq \frac{\hbar}{1.76kT_c} \sim t_{PE} = \frac{l_{PE}}{c}\]  \hspace{1cm} (62)

Substitution of eq.(62) in eq.(61) gives a lower limit for the Diamond temperature of Cooper pairs.

\[T_D \sim \frac{3.52}{\pi}T_c = 1.12T_c\]  \hspace{1cm} (63)

Substituting eq.(63) in the expression of the electromagnetic zero-point dark energy quanta, eq.(48) we get

\[\hbar\nu_c = \frac{\ln 3}{1.12}kT_D = 0.98kT_D \sim kT_D\]  \hspace{1cm} (64)

This equation means that we can assimilate a quanta of electromagnetic zero-point dark energy in a superconductor with a Planck-Einstein sized, \(l_{PE}^4\), Cooper pair Diamond cell.

Since the gravitational dark energy quantum condensate is related with zero-point fluctuations, the physical model of the superconductor’s vacuum, presented above violates the principle of energy conservation. This fact is also expected from a breaking of the weak equivalence principle for Cooper
pairs, which also leads to non-conservation of energy-momentum. Would a simultaneous breaking of the weak equivalence principle for Cooper pairs, together with a variable speed of light in the superconductor reconcile the electromagnetic zero-point dark energy model in superconductors with the law of energy conservation?

6 Variable speed of light and breaking of the weak equivalence principle in superconductors

In figure (1) we plotted the value of the Eötvös-factor $\chi$, eq.(54), in function of the superconductor’s critical temperature. From this curve we see that the difference between the inertial and the gravitational mass of the Cooper pairs should continuously increase with the increase of the superconductor’s critical temperature. If the speed of light remains constant the difference between the electrons rest mass energy before forming the Cooper pair and the rest mass energy of a Cooper pair should progressively increase with the superconductors critical temperature. Therefore a breaking of the weak equivalence principle for Cooper pairs with a constant speed of light in the superconductors would violate the law of energy conservation. Would a variable speed of light predicted by eq.(44) compensate for the Cooper pairs inertial mass increase in a way that would preserve energy conservation?

In the following we will impose that the rest mass energy of the Cooper pairs should be conserved independently of the breaking of the weak equivalence principle. In other words the Cooper pairs theoretical and experimental (as measured by Tate) rest mass energy should be equal to each other. Setting the magnetic term in the electrochemical potential of Cooper pairs in eq.(21) equal to zero, and defining the superconductor 4-velocity $t_\mu(c_1,0,0,0)$, if the Cooper pairs rest mass energy is conserved we should have

$$m_i c^2 = cc_1 m^*_i$$  \hspace{1cm} (65)

where $m_i$ is the Cooper pairs theoretical inertial mass and $m^*_i$ is the experimental Cooper pairs mass measured by Tate et al. Setting $v = 0$ in eq.(39) we obtain the proper rest mass energy of Cooper pairs in the Cooper pairs rest frame, i.e., in the superconductor preferred frame $\Sigma$.

$$m^*_i cc_1 = m^*_i c(2c' - c)$$  \hspace{1cm} (66)

Note that from this equation we deduce that the speed of light associated with the superconductors preferred frame is:

$$c_1 = 2c' - c.$$  \hspace{1cm} (67)

Setting eq.(65) equal to eq.(66) we get

$$m_i c^2 = m^*_i (2cc' - c^2)$$  \hspace{1cm} (68)
From eq. (68) we deduce the superconductor’s vacuum refractive index in function of the Eötvös-factor $\chi$.

$$N_{\text{vacuum}} - 1 = \frac{\chi}{\chi + 2}$$  \hspace{1cm} (69)

which we can also express in function of the Eötvös-factor $\eta$ using eq. (13)

$$N_{\text{vacuum}} - 1 = \frac{\eta}{2}$$  \hspace{1cm} (70)

Substituting eq. (54) in eq. (69) we obtain the variation of the superconductor’s vacuum refractive index with respect to the superconductor’s critical temperature $T_c$.

$$N_{\text{vacuum}} - 1 = \frac{1}{1 + \frac{16\pi}{3\ln^3 3} \left( \frac{T_{PE}}{T_c} \right)^4}$$  \hspace{1cm} (71)

From eq. (69) we deduce the speed of light $c'$ associated with the superconductor’s vacuum, in function of the Eötvös-factor $\chi$.

$$c' = c \frac{1 + \frac{\chi}{2}}{1 + \chi}$$  \hspace{1cm} (72)

which we can also express in function of the Eötvös-factor $\eta$

$$c' = \frac{2c}{\eta + 2}$$  \hspace{1cm} (73)

Solving eq. (73) with respect to $c'$ we obtain the variation of the superconductor’s vacuum velocity $c'$ with respect to the superconductor’s critical temperature $T_c$.

$$c' = c \left[ \left( 1 + \frac{16\pi}{3\ln^3 3} \left( \frac{T_{PE}}{T_c} \right)^4 \right)^{-1} + 1 \right]^{-1}$$  \hspace{1cm} (74)

As $T_c$ tends to infinity $c'$ tends to $c/2$

Setting eq. (69) equal to eq. (44) we deduce the non-classical Eötvös-factor $\chi' = B_0'/2\omega$

$$\chi' = \frac{\chi^2}{2(\chi + 2)}$$  \hspace{1cm} (75)

We see that $\chi'$ is a second order term with respect to $\chi$.

Substituting eq. (72) in the equation of the effective Lorentzian speed of light eq. (36), we deduce this speed in function of the Eötvös-factor $\chi$.

$$c_{\text{eff}} = \frac{c}{\sqrt{1 + \chi}}$$  \hspace{1cm} (76)

In a similar manner as above we can express $c_{\text{eff}}$ in function of the Eötvös-factor $\eta$

$$c_{\text{eff}} = c \left( \frac{2 - \eta}{2 + \eta} \right)^{1/2}$$  \hspace{1cm} (77)

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Substituting eq.\((54)\) in eq.\((76)\) we obtain the variation of \(c_{\text{eff}}\) with respect to the superconductor’s critical temperature \(T_c\).

\[
\frac{c_{\text{eff}}}{c} = \sqrt{1 + \frac{3 \ln^2 3}{8\pi} \left(\frac{T_c}{T_{PE}}\right)^4} \quad (78)
\]

As \(T_c\) tends to infinity \(c_{\text{eff}}\) tends to zero.

Substituting eq.\((74)\) in eq.\((67)\) we obtain the speed of light \(c_1\) with respect to the superconductor’s preferred frame \(\Sigma\) in function of the superconductor’s critical temperature \(T_c\).

\[
c_1 = c\left(2\left(1 + \frac{16\pi}{3\ln^4 3}\left(\frac{T_{PE}}{T_c}\right)^4\right)^{-1} + 1\right)^{-1} - 1 \quad (79)
\]

Figure \(2\) displays the plots of \(c'\) eq.\((74)\), \(c_{\text{eff}}\) eq.\((78)\) and \(c_1\) eq.\((79)\).

7 Conclusions

In conclusion the Cooper pairs bosonic condensate in superconductors seems to generate a physical medium similar to a superconducting "ether" constituted by a discrete set of Minkowski’s Diamond cells, each diamond cell representing a quanta of electromagnetic zero-point dark energy in the superconductor. A variable vacuum speed of light in superconductors would appear in relation with a breaking of the weak equivalence principle, ultimately resulting from a spontaneous breaking of gauge invariance leading to a preferred frame in superconductors. We have seen that a variable vacuum speed of light in superconductors would allow to reconcile a breaking of the weak equivalence principle for Cooper pairs with the principle of energy conservation.

Since the breaking of the weak equivalence principle is only affecting the Cooper pairs it is very difficult to detect this effect with macroscopic superconducting samples, because the Cooper pairs are only contributing marginally to the total inertial and gravitational mass of the sample. Although this is a major difficulty to test the weak equivalence principle for Cooper pairs, the results presented in the present work lead to recommend to carry out new versions of Jain and Tate experiments with improved accuracy in order to measure the inertial and the gravitational mass of Cooper pairs and therefore to measure the Eötvös factor for Cooper pairs with improved accuracy. Streamlined with this recommendation it would be important to carry out the non-null version of Jain’s experiment suggested by Anandan in \[3\].

The possibility of two different vacuum speeds of light \(c\) and \(c'\), with \(c/2 \leq c' \leq c\), for an observer attached to the superconductor’s preferred frame affects locally the classical diagonal form of the Minkowski’s spacetime metric relative to the superconductor’s preferred frame. We have shown
that the effective speed of light $c_{eff}$ setting the relativistic effects for the superconductor with respect to an observer external to the superconductor, is a function of the geometric mean between the two possible speeds of light eq. $c_{eff}^2 = 2cc' - c^2 = 2c_0^2 - c^2 < c$. Therefore we cannot accelerate a superconductor until it approaches asymptotically the classical vacuum speed of light $c$. Contrary to ordinary matter, the speed of a superconductor can only approach the effective speed of light $c_{eff} < c$ depending on the fourth power of the superconductor’s critical temperature. to illustrate this result let us think about an hypothetical superconductor with an infinite critical temperature, according to the theory presented here it would be impossible to communicate a relative speed different from zero to the superconductor. so to speak, this superconductor would be in an absolute state of rest.

It is interesting to note that the main ether drift experiments referenced by Consoli in [21] contain optical superconducting cavities. A possible question raised by the present work is: Are the observed anisotropy in the speed of light in Hermann [22] and Muller [23] experiments due to the presence of superconducting optical cavities in their experiments?

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Figure 1: Eötvös-factor $\chi$, quantifying the level of breaking of the weak equivalence principle for Cooper pairs, in function of the superconductor’s critical temperature $T_c$, eq. (54).
Figure 2: Decrease of the speed of light associated with the superconductor electromagnetic dark energy vacuum medium, $c'$ eq. (74); decrease of the speed of light in the superconductor’s preferred frame $c_1$ eq. (79); and decrease of the effective Lorentzian speed of light $c_{eff}$ eq. (78) defining the usual relativistic effects for the superconductor; in function of the superconductor’s critical temperature.
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