Response to defects in multi- and bipartite entanglement of isotropic quantum spin networks

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Quantum networks are an integral component in performing efficient computation and communication tasks that are not accessible using classical systems. A key aspect in designing an effective and scalable quantum network is generating entanglement between its nodes, which is robust against defects in the network. We consider an isotropic quantum network of spin-1/2 particles with a finite fraction of defects, where the corresponding wave function of the network is rotationally invariant under the action of local unitaries, and we show that any reduced density matrix also remains unaltered under the local actions. By using quantum information-theoretic concepts like strong subadditivity of von Neumann entropy and approximate quantum telecloning, we prove analytically that in the presence of defects, caused by loss of a finite fraction of spins, the network sustains genuine multisite entanglement, and at the same time may exhibit finite moderate-range bipartite entanglement, in contrast to the network with no defects.

Abstract—Isotropic spin networks are an integral component in performing efficient computation and communication tasks that are not accessible using classical systems [1, 2]. While quantum optical systems are closely associated with long distance quantum communication [3, 4], transfer of information and computation at the processor level is generally achieved using quantum networks or arrays of quantum systems [5–7]. A class of natural candidates to implement an efficient and scalable quantum network are interacting spin lattices. With the phenomenal development in optical lattices [8], solid-state qubit fabrication [9] such as nitrogen-vacancy centers [10], superconducting qubits [11], etc., implementations of controllable interacting spin lattices with desired properties have become more accessible, heralding renewed theoretical interest in the design and engineering of such quantum spin lattices [12].

A critical aspect in the study of quantum networks is the distribution of entanglement [13] between the nodes. For implementation of quantum protocols such as information transmission [5], long-range quantum teleportation in spin chains [14], and measurement-based quantum computation [6], modulation or engineered generation of entanglement between the spins on the lattice is a necessary prerequisite. In several hybrid quantum networks designed using superconducting or optomechanical cavities [15], entanglement is the key resource enabling the fidelity and speed of information transfer [16] within the network. Hence, robustness of entanglement in the presence of defects is an important requirement in the design of scalable information and computation models.

In our work, we investigate the effect of particle loss or defects in a quantum spin network on its inherent entanglement properties. Defects may destroy physical properties including entanglement in a system, and hence may adversely affect its computational and communication abilities [17]. We consider a quantum spin network consisting of spin-1/2 particles on a lattice, with an isotropic topology, of arbitrary dimensions. The interaction between the spins is such that the wave function of the spin network consists of superpositions of short-range dimer coverings [18]. We show, using information-theoretic properties such as quantum telecloning [19] and the strong subadditivity of von Neumann entropy [20] that even in the presence of a finite fraction of defects, the spin network sustains a considerable amount of genuine multisite entanglement. Moreover, we show that defects may also generate small but finite bipartite entanglement between two moderately distant sites. We also discuss potential applications of such isotropic spin systems in quantum computation.

Isotropic spin network.—Let us consider a quantum network consisting of N spin-1/2 particles, such that each particle is surrounded by R nearest neighbors (NNs). The network is isotropic, which implies that all bonds between equidistant sites are equal in magnitude, and the lattice appears the same from the perspective of any lattice site. Moreover, the quantum state of the network is invariant, up to a global phase, under identical local unitaries, i.e., the spin state is rotationally invariant. Such a spin network is in principle equivalent to the spin liquid phases of certain antiferromagnetic strongly-correlated systems [21–23], such that the state consists of equal-weight superposition of all possible dimer coverings, where a dimer between any two NN spin-1/2 particles is given by $|\psi_{ij}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$. The state of the N-spin quantum network can then be defined as $|\Psi_N\rangle = \sum_{k} \langle \psi_{ij}, k \rangle_k$, where N is even, and $\{|\psi\rangle\}$ is the kth (defect-free) covering with $N/2$ dimers of the entire lattice. Since $|\psi_{ij}\rangle$ is rotationally invariant under operations of the form $U \otimes U$, where U is a unitary acting on a single spin-1/2 particle, the entire state, $|\Psi_N\rangle$, is rotationally invariant under the local operation, $U \otimes U^\otimes N$ [22]. In presence of defects, description of the state can be mapped to a three-level basis or the qudit Hilbert space [24]. To accommodate the defect, each node of the network is represented by $\{|\nu_0\rangle, |\nu_1\rangle\}$, where $|\nu_0\rangle$ denotes a node with no spin particle (a defect). The occupied node is $|\nu_1\rangle$, representing the spin-1/2 particle with the two-level basis $\{|\uparrow\rangle, |\downarrow\rangle\}$. Hence, the overall state of the network can be expressed by the joint three-level basis, $\{|\nu_0\rangle, |\nu_1\rangle, \uparrow\rangle, |\downarrow\rangle\} \to \{|\zeta_0\rangle, |\zeta_1\rangle, |\zeta_2\rangle\}$. In this new basis, a dimer between two NN spins is defined as $|\psi_{ij}\rangle = \frac{1}{\sqrt{2}}(|\zeta_1\rangle - |\zeta_2\rangle)$ and a defect at node j is written as $|\zeta_0\rangle_j$. We note that the (un)primed elements of the basis represent the spin (un)occupied sites, corresponding to $(|\nu_0\rangle)$ and $|\nu_1\rangle$. For an N-spin network containing P nodes with defects, the overall state can be written as $\Psi_N^P = \sum_k \langle \psi_{ij} \rangle_k \otimes \langle \phi_i \rangle_k$, where $\{|\phi_i\rangle\}$ is a basis, such that $I_{ij}$ is the nodes
containing the $\mathcal{P}$ defects. The rest of the sites are covered by dimers. $D = \mathcal{P} / N$ gives us the defect density of the network.

The state of the quantum network with finite defects, $\Psi^P$, is invariant under the three-level local unitary operation $\bar{U} \otimes \bar{N} = (1 \otimes \bar{U}) \otimes \bar{N}$, where $\bar{U}$ is a $2 \times 2$ unitary operator acting on the spin space $\{|\zeta_1\rangle, |\zeta_2\rangle\}$, and each node of any term of $\Psi^P$ contains either a defect or is part of a dimer. The invariance of $\Psi^P$ is significant in analysing the entanglement properties of the quantum network, and provides the following lemma.

**Lemma:** The reduced state, $\rho^{(x)}$, for any nodes, obtained by tracing over all but $x$ nodes from the state $\Psi^P$ is invariant under the action of $\bar{U} \otimes \bar{x}$.

**Proof:** The reduced state of $x$ nodes is obtained by tracing out the remaining $\bar{x} = N - x$ nodes, which can be written as $\rho^{(x)} = \text{Tr}_x[|\Psi\rangle\langle\Psi|] = \sum_x |\langle \phi_x |\Psi \rangle|^2$, where $\{|\phi_x \rangle\}$ forms a complete basis over the system of $\bar{x}$ sites. Now, $\rho^{(x)} = \sum_x |\langle \phi_x |\Psi \rangle|^2$, due to the invariance property of $\Psi$. Therefore, $\rho^{(x)} = \sum_x |\langle \phi_x |\Psi \rangle|^2 = \bar{U} \otimes \bar{x} = \bar{U} \otimes \bar{x} |\phi_x \rangle \langle \phi_x | = \sum_x |\langle \phi_x |\Psi \rangle|^2$, where $|\phi_x \rangle = \bar{U} \otimes \bar{x} |\phi_x \rangle$ forms another basis of the system of $\bar{x}$ nodes.

The above lemma allows us to find the invariant reduced density matrices in the quantum spin network, and we especially focus our attention on the reduced states of single and two nodes. Let $\rho^{(1)}$ be a single node reduced state, so that it satisfies $\bar{U} \rho^{(1)} \bar{U} = \rho^{(1)}$, as shown by the above lemma. Let us denote the elements of $\rho^{(1)}$ and $\bar{U}$ by $\rho^{(1)}_{ij}$ and $u_{ij}$, respectively, where $\{|i, j\rangle\} = \{1, 2, 3\}$. Now for $\bar{U}$ we know that $u_{ij} = \delta_{ij}$, for the two $2 \times 2$ unitary operators $U$, we get the relation $u_{ij} u_{ij}^* = \delta_{kk}$, where $\{|k, k\rangle\} = \{2, 3\}$.

Now, the operation under local unitaries of the form $\bar{U}$ gives us $\rho^{(1)}_{ij} = (\bar{U} \rho^{(1)} \bar{U})_{ij} = \sum_{j'=k} u_{ij'} \rho^{(1)}_{ij'} u_{kj}^*$. To satisfy the invariance under locally operations the final state, $\rho^{(1)}_{ij}$, must be equal to the initial state, $\alpha^{(1)}_{ij}$. Therefore, upon expanding, we obtain the condition, $\rho^{(1)}_{ij} = \rho^{(1)}_{ij} u_{ij} \rho^{(1)}_{ij} u_{ij}^* = \alpha^{(1)}_{ij}$. Now, for the unitary operator $\bar{U}$ it is known that $u_{ij} u_{ij}^* = \delta_{kk}$, where $\{|k, k\rangle\} = \{2, 3\}$. Hence, the invariance $\rho^{(1)}_{ij}$, is satisfied for all states, if and only if the single-site density matrix is diagonal, i.e., $\rho^{(1)}_{ij} = \rho_{ii}$, such that $\sum_i \rho_{ii} = 1$, and $\rho_{ij} = 0$. Therefore, the single site reduced density matrix at site $a$ is given by

$$\rho^{(1)}_{a} = \text{diag}\{p_1, p_2/2, p_2/2\} = p_1 |\zeta_0\rangle \langle \zeta_0 | + p_2 I_2 / 2,$$

where $I_2 = (0 \otimes I_2)$, with $I_2$ being the identity matrix in the spin space $\{|\zeta_1\rangle, |\zeta_2\rangle\}$. Note that the single-site density matrix in the absence of defects is just $I_2 / 2$.

Similarly, one can obtain the analytical expression for the reduced two-node states invariant under the local unitary operation, $\bar{U} \otimes \bar{U}$. For an arbitrary pair of nodes $a$ and $b$, the reduced density matrix is given by

$$\rho^{(2)}_{ab} = p_1 |\zeta_0 \rangle \langle \zeta_0 | + p_2 I_4 / 4 + p_3 W(q),$$

where $p_i' \geq 0 \forall i$, and $\sum_i p_i' = 1$. The diagonal matrix $I_4 = \sum_i \langle \zeta_0 \rangle \langle \zeta_0 | + \langle \zeta_0 \rangle \langle \zeta_0 | + \langle \zeta_0 \rangle \langle \zeta_0 | + \langle \zeta_0 \rangle \langle \zeta_0 |$ is the Werner state [25] in $\mathbb{C}^2 \otimes \mathbb{C}^2$, where $\mathbb{C}$ is spanned by $\{|\zeta_1\rangle, |\zeta_2\rangle\}$, so that $W(q) = q |\psi\rangle \langle \psi | + (1 - q) I_4 / 4$, with $|\psi\rangle \langle \psi | = 1 / 4 (|\zeta_2\rangle \langle \zeta_2 | - |\zeta_2\rangle \langle \zeta_2 |)$ being the spin dimer.

$1 - q, q \leq 1$, and $I_4 = \sum_{i,j=1}^{2} \langle \zeta_i \rangle \langle \zeta_i | \langle \zeta_j \rangle \langle \zeta_j |$ being the identity matrix in the $\mathbb{C}^2 \otimes \mathbb{C}^2$ space. Hence, Eqs. (1) and (2) give us the analytical expressions for the single- and two-node reduced density matrices of the spin network with finite defects represented by the state $|\Psi\rangle_{N-\bar{x}}$. We note that the single- and two-node reduced states are dependent on the defect density, $D = \mathcal{P} / N$. For instance, for $D = 0, p_2 = p_3', 1$, which corresponds to a spin network with no defects. Ideally, for large networks with a small number of defects, $D \ll 1$, $p_1 \ll p_2, p_3' \gg (p_1', p_2')$.

**Bipartite entanglement properties.** We begin by investigating the bipartite entanglement properties of the state, given by Eq. (2), between any two arbitrary sites $(a$ and $b)$ of the quantum spin network. The condition of positive partial transposition (PPT) [26] is given by $p'_2(1 - 3q) / 4 \geq 0$. Therefore, the density matrix of two arbitrary sites in the spin network have a negative-partial-transpose (NPT) and hence entangled if $q > 1 / 3$, which is the same as that for a two-qubit Werner state. To obtain a more specific criteria for the bipartite entanglement properties of the spin network with defects, one needs to estimate the bounds on the parameter $q$ in terms of the defect in the network. To this effect, we use a QIT concept called quantum teleportation [19], which combines the concept of quantum teleportation [27] and quantum cloning [28].

While teleportation provides the fidelity with which a quantum state can be transferred to $M$ parties using shared bipartite entanglement and classical communication, quantum cloning provides the optimal fidelity with which $M$ copies of a quantum state can be prepared. Consider a site $A$ with $M$ sites surrounding it, given by $\{B_i\}$, and the $M$ states, $\rho_{AB_i}$. We suppose that an ancillary system in an arbitrary quantum state, $|\alpha\rangle$, is brought near the site $A$. This can be teleported to the site $B_i$, using the channel $\rho_{AB_i}$, with some optimal fidelity, $F_{tele}$. If all $\rho_{AB_i}$’s are the same (equidistant neighbors in an isotropic state), then the state $|\alpha\rangle$ can be teleported to $M$ sites with fidelity $F_{tele}$. However, using an optimal cloning machine, $M$ copies of a $d$-dimensional quantum state, $|\alpha\rangle$, can only be produced with a fidelity, $F_{clo} = d F_{tele}$. Therefore, $F_{clo} \leq F_{tele}$, and we obtain an upper bound on the fidelity with which $M$ copies of a quantum state can be remotely prepared.

Let us now consider the quantum state of the system under consideration as the required resource for the remote protocol where an unknown qutrit, $|\alpha\rangle$, is brought near site $a$, and $M$ copies of it are to be prepared at $M$ sites, $\{b_i\}$, on the isotropic network. The $M$ equivalent two-site density matrices, $\rho_{AB_i}$, are of the form given in Eq. (2). The fidelity of teleporting $|\alpha\rangle$ from near site $a$ to site $b_i$, is obtained from the maximal singlet fraction, $F = \max \langle \psi^* | \Lambda(\rho_{AB_i}) | \psi^* \rangle$, where the maximization is over all local operations and classical communication protocols, $\Lambda$, and where $|\psi^*\rangle$ is a maximally entangled state in
\( \mathbb{C}^d \otimes \mathbb{C}^d \). The teleportation fidelity is then given by \( F_{\text{tele}} = (Fd + 1)/(d + 1) \) [28], where \( d = 3 \) for the qutrit. For \( |\psi^x\rangle = \frac{1}{\sqrt{M}} (|\zeta_1^z\rangle - |\zeta_2^z\rangle + |\zeta_3^z\rangle) \) and for \( \Lambda \) as the identity operation, \( F' = \langle \psi^x | \rho_{ab}^{(2)} | \psi^x \rangle \leq F \). Using Eq. (2), for \( \rho_{ab}^{(2)} \), we obtain \( F' = p'_1/3 + (p'_3/3)(3q + 1)/2 \). Therefore, for \( d = 3 \), we obtain

\[
F_{\text{tele}} \geq \frac{(F' + 1)}{(d + 1)} = \frac{p'_1 + p'_3(3q + 1)/2 + 1}{4}, \quad \text{and (3)}
\]

\[
F_{\text{clo}} = \frac{2M + (d - 1)}{M(d + 1)} = \frac{1}{2} + \frac{1}{2M}. \quad \text{(4)}
\]

As discussed earlier, \( F_{\text{tele}} \leq F_{\text{clo}} \), and using Eqs. (3) and (4), we obtain an upper bound on the parameter \( q \) in \( \rho_{ab}^{(2)} \), given by \( q \leq \frac{1}{3} \left( \frac{2}{p'_1 - 1} \right) - \frac{2}{p'_1} \left( p'_1 - \frac{1}{N} \right) \). As we know the two-site density matrix is entangled if \( q > 1/3 \), the above relation provides us important indicators about the bipartite entanglement between two sites of the lattice. For relatively small number of defects in the lattice, \( p'_1 / p'_3 \approx 0 \). Hence, \( q \leq (1/3)(1 + 2/M + \delta) \), where \( \delta = 2/p'_3 - 2 \), with \( \delta \to 0 \) as \( p'_3 \to 1 \). Hence, the upper bound on \( q \) decreases as the number of copies, \( M \), increases. For example, let us consider an isotropic 2D square lattice with a low number of defects. We consider the teleporting of a qutrit from a site to its four NNs (\( M = 4 \)), so that the bound on \( q \) for NN two-party reduced density matrices is given by \( q \leq 1/2 + \delta/3 \). Hence, the NN state with highest bipartite entanglement for the isotropic 2D square lattice is given by Eq. (2), with \( q = 1/2 + \delta/3 \). Similarly, let us consider the teleporting of a qutrit to \( N \) nodes \( \{b_1, b_2, \ldots, b_N\} \), \( x \) edges away from node \( a \), such that \( M = N \). Alternatively, one may consider \( R \) to be the number of nodes contained in the area formed by concentric circles of radius \( r > x - x' \) and \( r \leq x + x' \), where \( x' \ll x \). For large \( x \), all the \( R \) nodes can be considered as equidistant from node \( a \). \( R \) increases with \( x \), and we have \( q \leq 1/3 + \delta/3 \). We observe that as \( \delta \to 0 \), \( q \leq 1/3 \) and the states \( \rho_{ab}^{(2)} (i = 1, 2, \ldots, R) \) are separable, since \( \rho_{ab}^{(2)} \) is then a mixture of three unentangled states. Figure 1 shows that the permissible upper limit on bipartite entanglement, as quantified by logarithmic negativity [26, 29], is finite for any two sites in the isotropic lattice with defects but decreases as the number of equidistant isotropic pairs \( (M) \) increases.

**Multipartite entanglement properties.**—We now show that an \( N \)-spin network with \( P \) defective nodes, given by \( |\Psi\rangle_N \), such that \( P < N \), is always genuine multipartite entangled. For a pure multiparty quantum state to be genuinely multipartite entangled, it must be entangled across all possible bipartitions of the system. This requires that the reduced density matrices across all possible bipartitions must necessarily be mixed. From Eqs. (1) and (2), for \( D < 1 \), we observe that the reduced single- and two-node reduced states are always mixed. Hence, the state \( |\Psi\rangle_N \) is entangled across all single:rest and two:rest bipartitions. Now we need to show that \( |\Psi\rangle_N \) is entangled across the other possible bipartitions. Consider the reduced state, \( \rho^{(x)} = \text{Tr}_x(|\Psi\rangle\langle\Psi|)^{(N)}_{\text{rest}} \), for any arbitrary but fixed set of \( x \) nodes. Let us assume that \( \rho^{(x)} \) is pure and thus \( |\Psi\rangle_N \) is separable along the \( x : (N - x) \) bipartition. Let \( x = (x \setminus 1) \cup y \), where \( y \) is one specific node. Since, the spin network is isotropic, there shall always exist an equivalent but spatially different set of \( x' \) nodes, such that both \( x' \) and \( x \) contain an equal number of nodes and the node \( y \) is common to both reduced sets. For an example, see Fig. 2. Hence, \( x' = (x' \setminus 1) \cup y \). Now, \( \rho^{(x')} \) is also pure, by the symmetry of the isotropic lattice. Applying the strong subadditivity of von Neumann entropy (\( S(\cdot) \)) [20], we obtain \( S(\rho^{(x'\setminus 1)} + S(\rho^{(x'\setminus 1)}) \leq S(\rho^{(x'\setminus 1)} + y) + S(\rho^{(x'(x')\setminus 1)} + y) \). As \( \rho^{(x')} \) and \( \rho^{(x')} \) are pure, \( S(\rho^{(x')} + y) = S(\rho^{(x')} + y) = 0 \). Since \( S(\rho) \) is non-negative, \( S(\rho^{(x'\setminus 1)}) = S(\rho^{(x'\setminus 1)}) = 0 \), and this implies that \( S(\rho^{(x'\setminus 1)}) = S(\rho^{(x'\setminus 1)}) = 0 \), and hence that \( |\Psi\rangle_N \) is genuine multipartite entangled for spin networks with finite defects. The above method can also be extended to prove that infinite spin networks are entangled across infinite line of bipartition. For example, the isotropic spin state, \( |\Psi\rangle \), defined on an infinite 2D lattice is always entangled across infinite lines on the lattice. This proves that the entire multiparty state of the isotropic spin network is genuinely multipartite entangled, except in the extreme case where all spins are lost (\( D = 1 \)), so that the state becomes a product of vacuum nodes, given by \( |\zeta_1^{(1)}\rangle \otimes |\zeta_2^{(2)}\rangle \cdots \otimes |\zeta_0^{(N)}\rangle \).

**Discussion.**—The single- and two-node reduced states in Eqs. (1) and (2), respectively, are defined for the quantum spin network with finite defects. Without defects, the network can be mapped to the two-qubit space, spanned by \( \{|\zeta_1^{(1)}\rangle, |\zeta_2^{(2)}\rangle\} \). The single-node reduced density matrix is then \( I_2/2 (p_1 = 0, p_2 = 1) \) in Eq. (1) and the two-node reduced state is given by the Werner state, \( \mathcal{W}(q) (p'_1 = p'_2 = 0, p'_3 = 1) \). Since, the single-node state is always mixed, using the approach discussed earlier, but applied to the case of zero defects, it can be shown...
that the quantum spin network is always genuine multisite entangled. Finite defects in the network do not destroy the multipartite entanglement, but in the extreme case, where all spins are lost ($D = 1$).

In the absence of defects, the condition of bipartite entanglement between any two arbitrary nodes reduces to $q \leq (1/3)(1 + 2/M)$, since $\delta = 0$. Again, considering the example of an isotropic 2D square lattice, in the limit $M \to \infty$, we obtain $q \leq 1/3$, and the upper bound ensures that the system has no long-range bipartite entanglement [22]. However, in the presence of finite defects, the upper bound may allow a small but finite entanglement, as $q \leq (1/3)(1 + \delta)$. Hence, presence of defects in quantum spin networks may not qualitatively affect the presence of genuine multipartite entanglement in the system but may permit the presence of finite bipartite entanglement between moderately-distant sites, in contrast to the spin network with no defects.

In summary, our work primarily aims at highlighting the response to defects of the distribution of entanglement, both bipartite as well as multipartite, in a particular prototype of quantum spin networks. Our results show that presence of finite defects do not affect the presence of multisite entanglement properties of the network, while, interestingly, finite moderate-range bipartite entanglement may emerge. This shows that isotropic spin networks that can be obtained as dimerized ground states of spin Hamiltonians provide a robust model for implementation of quantum protocols in presence of defects. Importantly, recent experimental developments allow scalable simulation of these spin networks for practical applications [8–11].

The spin network considered in our work is a useful tool in realization of solid-state quantum computation protocols, in particular as realistic dimer models for fault tolerant topological computation [30, 31]. The state of the isotropic quantum spin network with nearest-neighbor dimer bonds, such as the ground states of the dimer model on the triangular lattice [33], serves as an Abelian topological phase that can host anyons in low excitations [32] and is closely related to Kitaev’s toric code model [31]. Moreover, the state under study is closely related to interpolations of the projected entangled pair states [34–36], which are valuable resources for measurement-based quantum computation [6]. Such spin network states, which are possible gapped ground states of antiferromagnetic Hamiltonians in a spin lattice, under specific parameter regimes, provide the necessary resource for measurement-based quantum computation [37, 38] and may potentially be more accessible as compared to 2D cluster states in many-body architectures where the latter do not appear as natural ground states. The accessibility of the state discussed in our work, for both topological and measurement-based computation, is primarily due to symmetry protected topological order [39] in these states, which has received considerable attention in the literature [40].

In recent years, experimental realization of spin network states with nearest neighbor dimer bonds have been achieved using photonic lattices [41–43]. In particular, generation of spin-1 Affleck, Kennedy, Lieb and Tasaki (AKLT) system and corresponding implementation of single-qubit and two-qubit CNOT gates have been achieved using linear optics operations and photon detection [41, 42]. Moreover, dimer states such as the resonating valence states have also been simulated using polarization states of photons [43] and ultracold atoms in optical lattices [44].

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