Reactive Power Markets for the Future Grid
Adam Potter, Rabab Haider, and Anuradha M. Annaswamy

Abstract—As pressures to decarbonize the electricity grid increase, the grid edge is witnessing a rapid adoption of distributed and renewable generation. As a result, traditional methods for reactive power management and compensation may become ineffective. Current state of art for reactive power compensation, which rely primarily on capacity payments, exclude distributed generation (DG). We propose an alternative: a reactive power market at the distribution level. The proposed market uses variable payments to compensate DGs equipped with smart inverters, at an increased spatial and temporal granularity, through a distribution-level Locational Marginal Price (d-LMP). We validate our proposed market with a case study of the New England grid on a modified IEEE-123 bus, while varying DG penetration from 5% to 160%. Results show that our market can accommodate such a large penetration, with stable reactive power revenue streams. The market can leverage the considerable flexibility afforded by inverter-based resources to meet over 40% of reactive power load when operating in a power factor range of 0.6 to 0.95. DGs participating in the market can earn up to 11% of their total revenue from reactive power payments. Finally, the corresponding daily d-LMPs determined from the proposed market were observed to exhibit limited volatility.

Keywords—reactive power, power distribution economics, power quality, distribution level market, optimal power flow

I. INTRODUCTION

With a growing demand for zero-carbon generation in the electricity sector comes an increase in grid-edge distributed energy resources (DERs), and in particular, distributed generation (DGs). The increasing penetration of these small-scale resources disrupts the existing industry practice for power dispatch, device control, and market compensation mechanisms. Traditionally, large controllable generators at the transmission level have been responsible for maintaining grid stability and power quality; however, new strategies are required for the emerging grid. While recent efforts such as the FERC Order 2222 [1] indicate a move towards accommodation of DERs, new electricity market structures are needed in order to fully embrace their high penetration and lead to a feasible framework for their integration even while preserving grid stability and power quality. This paper proposes one such market structure, with a focus on a reactive power market at the distribution level.

Reactive power is intimately connected with power quality, and its regulation is achieved in the transmission grid by altering the power factors (PFs) of large synchronous generators. The outputs of these generators are easily controllable thus permitting an efficient regulation of PF, and have therefore been sufficient for ensuring power quality and voltage stability at the transmission level. PF can be regulated at the requisite level even with a few large synchronous generators as the target PF value rarely drops below 0.95 [2]. In contrast to this practice at the transmission level, power quality in the distribution grid is typically achieved through devices such as on-load tap changers, capacitor banks, and voltage regulators, which locally inject reactive power. Such a solution however becomes quickly inadequate in a distribution grid, especially with the increased penetration of DERs. Emerging solutions that allow regulation in a wide range of PF correspond to inverter-based resources, such as DGs which are often equipped with smart inverters, and can adjust their PF using suitable power electronics control, quickly and at low costs. This in turn implies that DGs that enable such an additional flexibility must be compensated for through an appropriate market mechanism. A market structure that compensates DGs for their services, particularly for reactive power, is structurally different from the current practice [3]. In this paper, we propose such a reactive power market at the distribution grid, which builds on the authors’ earlier work in [4], [5] that proposed a retail market that enables fine granularity, in both location and in time, of power injections and corresponding payments. Unlike those works, here we focus primarily on reactive power and its role in an unbalanced distribution grid with increasing DER penetration.

Currently, reactive power markets do not exist. Any compensation for generators that provide reactive power is through capacity payments, which are used to recover the capital costs of installing the equipment necessary for providing reactive power or based on the LOC (lost opportunity cost) of not selling real power. These payments, typically cleared on a case-by-case basis, are a form of fixed payment, the alternative to which are utilization, or variable, payments. Such a practice of capacity payments often excludes small resources such as DGs, which offer significant operational flexibility at little to no cost. Further, capacity payments do not price the spatial and temporal variation in services offered by DGs, and will be insufficient in accommodating a deep penetration of DGs. Variable payments, on the other hand, enable spatial and temporal variations across the grid. In particular, approaches based on optimal power flow (OPF) allow the simultaneous determination of prices corresponding to both real and reactive power injections, enables increasing penetration of DERs, and accommodates grid physics. In this paper, we propose a reactive power market that allows variable payments based on an OPF approach.

The reactive power market proposed in this paper is a part of a distribution-level market with oversight assumed to be provided by a distribution system operator (DSO). A DSO-
centric market has several advantages, the first of which is that it enables DERs to directly participate in a local setting, and have the market location be collocated electrically, such as at a primary feeder node or a secondary feeder node. It also avoids tier bypassing, which can arise if DERs are directly dispatched into the wholesale markets in response to the bulk grid objectives without considering local constraints [6]. Our reactive power market sits at the primary feeder level (4 to 35 kV level), with any resources and uncontrollable loads residing in the secondary feeders and below aggregated upward. Each Primary Market Operator (PMO) has oversight over one primary feeder, and determines the schedule for DERs in terms of both real and reactive power injections as well as the retail electricity price of both real and reactive power, which we denote as a distribution-level Locational Marginal Price (d-LMP). Each PMO pays the participating DGs in that feeder at the d-LMP. The d-LMP also corresponds to the payment made by the loads to the PMO. As we effectively split the grid into smaller service regions, the proposed retail market has the potential to lead to less expensive and more extensive procurement and payment for reactive power resources. The study in [7] corroborates this statement as well. A DSO can be tasked with the oversight of these PMOs and perhaps be commissioned as a non-profit entity, similar to ISO/RTOs, by the corresponding Public Utility Commission.

The underlying framework utilized to determine the market schedules and prices is based on constrained optimization, an initial version of which was proposed in [4], [5]. In the present work, we extend [4], [5] by carrying out a detailed treatment of reactive power and reactive power pricing. The case study in [4], [5] considered a specific level of penetration of DG of approximately 70%, consisting entirely of real power generation. In this paper, we significantly extend the scope, by modeling each DG as an inverter-based resource with reactive power capacities and variable PF; over a range of DG penetration from 5% to 160%. We do not consider reactive power support from flexible consumption, though our market structure can extended to accommodate them as well. We also consider an unbalanced distribution grid, not considered in [4], [5], and show how DERs can provide appropriate local reactive power support throughout the grid while optimizing network-level objectives subject to grid constraints. We place our reactive power d-LMPs in the context of industry needs and validate performance over a range of conditions, including DG penetration and inverter PF. We also discuss the implications of our market in the key areas of price volatility, revenue reliability, and supporting investment decisions.

The following are the specific contributions of the paper.

- We propose a reactive power market for DGs in the distribution grid. The market scheduling and clearing is based on a distributed optimization algorithm, Proximal Atomic Coordination, demonstrated to converge to the optimal point under suitable conditions in [4], [5], [8].
- Through a numerical case study of the New England grid modeled as a modified IEEE-123 bus, we demonstrate the following features:
  - Our market accommodates a large amount of DGs: We illustrate how reactive power support can be realized even with a high penetration of DGs. We show in particular that the revenue stream for reactive support to these DGs remain steady even as DG penetration increases from 5% to 160%, which indicates the feasibility of the proposed market at high penetrations, and even at low penetrations.
  - Our market uses all available DGs efficiently: We utilize the full flexibility of DGs to operate over a range of PF, from 0.6 to 0.95, to meet over 45% of reactive power load, even with the reactive power price per unit kVAR remaining the same. The corresponding increase in the requisite reactive power injection is appropriately compensated by an increasing percentage of the revenue from reactive power support compared to total revenue from DGs.
- Our market is stable with limited volatility: A detailed assessment of the price variations across all DGs in the IEEE 123-bus over a week shows that the price variations primarily follow load fluctuations and are otherwise minimal.

The remaining paper is organized as follows. Section II presents a review of the state of art in both theory and practice, providing a historical perspective on reactive power compensation. Section III introduces the OPF-based reactive power pricing. Section IV presents numerical evaluation of the proposed reactive power market on an IEEE-123 bus network under DG penetration of 5% to 160%, and PF operating range of 0.6 to 0.95. Section V provides concluding remarks.

II. STATE OF ART OF REACTIVE POWER COMPENSATION

The following provides a literature review of the state of art for reactive power management and pricing.

A. History of reactive power pricing

Since the US energy sector was deregulated in the 1990s, there has been much debate over the treatment of reactive power in a market setting. Early proposals for compensating reactive power (Q) aimed to address two problems: how to recover Q-enabling equipment costs and how to efficiently procure reactive power capacity. These problems were inherent to the then centralized grid predominantly consisting of synchronous generators. Early reactive power compensation methods proposed in [2] and [9] supplied no payment for generators operating at PFs in the range of 0.95 lagging, citing that synchronous generators do not incur losses within this operating range. Reactive power prices were set outside this PF range as it necessitated additional hardware and therefore recovery of equipment cost. Any further deviations imply a loss of opportunity of selling active power, and therefore the price is set based on LOC (lost opportunity cost) [10]. Much of current practice has followed this procedure. However, this approach is highly sensitive to the PF range and becomes inadequate as PF varies, which is very likely to occur with increased and varying penetration of DERs. As clearly mentioned in [3], the absence of a streamlined compensation for reactive power is anti-competitive and not recommended.
for inclusive markets, such as practices encouraged in FERC Order 2222 [1].

Much of industry’s attention was focused on the second problem of reactive power capacity procurement. As utilities rushed to ensure sufficient capacity to maintain voltage and power quality under high demand, capacity markets rose as a reliable way to incentivize reactive power capability investments. These fixed payments were preferred over variable payments for the VAR-hrs produced. In [11] and [12] OPF-based methods are used to generate reactive power prices with the optimization dual variables being used to generate reactive power prices, but noted highly volatile prices with the latter study ultimately recommending capacity payments to drive investment. Reference [13] assessed that before 1998 in the US around 80% of reactive power payout was through fixed payments while only 20% was through variable payments. It should be noted that even in this paper, which is more than 20 years old, fixed payments are recommended to be a transitional process phasing into payment purely on the utilization of reactive energy. All of these arguments clearly indicate that fixed payments are outdated and should not remain the dominant compensation method for ancillary services. These fixed payments do not reflect the true value of services provided. Further, while synchronous generators are limited in their ability to generate Q, which can result in a capacity shortage, modern DERs such as PV solar equipped with smart inverters are not. Finally, and of critical importance, capacity markets are not inclusive to DERs. Therefore, as we move towards a grid with a strong influx of DG sources with much wider PF operating range, such procurement focused capacity markets will become less important. Like real power, it is crucial that reactive power compensation transitions to market-based solutions, and accounts for value added to the grid, the spatial differences in resources, and are inclusive to DERs. In this paper, we propose a specific distributed optimization based solution that allows the determination of a reactive power pricing structure that captures a fine-grain variation of location and time.

B. Implementation practices for reactive power pricing

Reactive power compensation at the transmission level is implemented using various methods. Since reactive power capabilities are deemed critical by FERC, monetary incentives are required to ensure power is delivered and the grid has sufficient capacity. Tariffs are designed as either fixed or variable payments to asset owners. However, poorly designed incentives could result in either a reactive power shortage resulting in voltage control failures, or surpluses that create monetary losses for utilities and consumers. A snapshot of these methods at various ISO/RTOs is delineated in this section.

A FERC report released in 2014 outlines two common payment methods [3]. The first is a regulatory mandate on generator reactive power support. Under the assumption that reactive power is inexpensive, all market participants are expected to help balance the grid without additional payment. This method is clearly outdated and unfair, and further, is incapable of incorporating DERs. The second method, developed by the American Electric Power Service (AEP), permits generators to report expenses explicitly tied to reactive power capabilities and settle on an appropriate fixed payment. The settlement process also gives grid operators control over capacity by recruiting and settling as necessary. This expense-based compensation is commonly used today, and is appealing because it avoids a market and the associated risks of market failures while ensuring the grid has appropriate capacity of reactive power. However, this method is generally limited to large traditional generators who can afford the cost analyses and settlement process. Additionally, unlike traditional generators, most DGs come equipped with extensive reactive power capabilities so expense-based pricing is less applicable. As mentioned earlier, the report [3] alludes to the need for retail level accommodation of DERs as their penetration increases. Without a market, recruiting many small DERs for reactive power support with the industry standard would be prohibitive and the resulting prices unsubstantiated. A well designed reactive power market could facilitate the rapid adoption of small-scale distributed grid assets.

Another point of interest is the 2020 primer from the Solar Energy Industries Association (SEIA) [14]. The primer assesses that fixed payments are critical for incentivizing reactive power generation. Figure 1 outlines reactive power revenue potential for large solar projects. All US ISO/RTOs include variable, per kVAR-yr, payment based on loss of potential revenue in the real power market, but only regions with fixed payments provide high or moderate potential for a reactive power revenue stream. Currently, if a market participant, for example DERs, did not have access to fixed payments their revenue potential would be compromised.

---

**Fig. 1:** Reactive power compensation for ISO/RTO regions in the US with corresponding estimates on the reactive power revenue potential for solar and storage in those regions [14].

The issue of loss of potential revenue in the real power market is further substantiated by data from the New England Independent System Operator (ISO-NE) settled cost for yearly reactive power capacity. In 2020, the costs varied from 1.095 $/kVAR-yr to 1.13 $/kVAR-yr; in comparison, the normal utilization price is 0.05 $/MWh (0.0057 $/kVAR-yr). As the SEIA primer suggested, current tariff structures heavily favor standby capacity over power delivery. The report further notes that while the AEP method for reactive power capacity pricing has been used for several large solar projects, it is not inclusive to residential solar. Looking at ISO-NE again, the smallest asset reported had 587 kVARs of capacity, confirm-
ing the standard is non-inclusive to smaller DGs [15]. This implies that despite fixed payments being the go-to procedure in industry practice, there is still a significant gap in the efficient integration of small-scale resources. A pricing scheme centered around fixed payments would be difficult to scale, improperly value resources, and result in tier bypassing by separating real and reactive power. In our work, we propose a reactive power market to provide variable payments for resources in the distribution grid, and argue it is effective and reliable enough to facilitate such a transition.

C. Technical Solutions for Reactive-power Pricing

A wide variety of pricing and settlement methods for reactive power have been proposed in the literature. The survey in [16] presents a review of both LOC-based and OPF-based methods for reactive power pricing schemes. These can loosely be distinguished into three categories as described in [17] based on the relationship between real and reactive power pricing. The first category includes the cost-based methods mentioned above that keep reactive power as an independent ancillary service. The dynamics of real and reactive power are assumed to be decoupled allowing voltage control to be a secondary concern to that of balancing power. Control of voltage is assumed to be possible through regulation of PFs using large generators. This solution is predicated on transmission-level resources, and the assumptions of decoupled real and reactive power are violated when PFs dip below 0.95, a plausible occurrence for DERs. This is further exacerbated by the unbalanced physics of the distribution grid.

The second category proposed in [17] includes a hierarchical approach where the first step is the clearing of a real power market followed by a reactive power market. These solutions leverage the advantages of decoupled dynamics leading to simple market-based approaches. Reactive power is priced competitively and supports existing real power markets. One such example can be found in [18] which is based on LOC of selling real power, while others such as [19] formulate a separate optimal reactive power flow (OQPF) problem to quantify its marginal value. In both cases, the use of a hierarchical structure may prevent resources that participated in the real power market from selling reactive power in the following market. This makes it difficult to leverage DG’s ability to operate at low PFs as they will be incentivized to sell real power first, thus limiting reactive power ability.

The third category covers simultaneous real and reactive power deployment. Typical examples are [20]–[22] that extract both real and reactive LMPs using an OPF formulation. By minimizing an objective function, the resulting Lagrange multipliers (dual variables) represent the marginal cost of real and reactive power at each load and resource, leading to a reactive power price. For example, [20] and [21] use voltage and reactive power constraints to ensure grid-feasible solutions and find the value of reactive power in meeting these constraints. By solving for both types of power simultaneously, these approaches can avoid decoupling grid dynamics and assure there is no mismatch between cleared real and reactive dispatch. Our proposed market solution belongs to this third category.

III. Reactive Power Pricing

The starting point for the reactive power market is an unbalanced distribution grid for which we are interested in determining the optimal power dispatch, both active and reactive. We utilize an OPF approach to solve this problem, whose primal and dual variables provide not only the power dispatch but also the local price of both active and reactive power.

Two distinct types of models have been used to represent the problem: modeling branch variables which leads to the Branch Flow model [23], [24], or modeling nodal variables which leads to the Bus Injection model [25], [26]. The Branch Flow model based on second order cone programming (SOCP) has proven to be advantageous in providing tight convex relaxations to the original AC-OPF problem with exactness under some conditions, and is shown to be more computationally stable than the Bus Injection model. However, both Branch Flow and Bus Injection models are typically limited to networks with radial topologies and balanced networks, and extensions to these models for unbalanced distribution grids are only valid for a small range of angle imbalances. This is a limiting assumption for many grids, especially with increasing penetration of DERs located on single-phase lines. A recent approach proposed by some of the authors of this paper, denoted as Current Injection (CI) model [27], [28], avoids this assumption and so is an ideal candidate for representing unbalanced grids with various single-phase loads and generation. The CI model uses nodal variables, similar to the Bus Injection model, but the main idea is to represent all loads and generators as nodal current injections, with all power, current, and voltage phasors represented in Cartesian coordinates. The 3-phase impedance matrix is used to describe the self and mutual inductance between phases to model the coupling of phases that are common to a distribution grid. More importantly, the key obstacle of non-convexity of the AC-OPF and the subsequent nonlinearity of SOCP and SDP convexification strategies are dealt with in the CI approach by leveraging McCormick Envelope (MCE) based convex relaxation [29] for the bilinear power relations. The MCE uses the convex hull representation of bilinear terms to render a linear OPF model. This representation requires adequate bounds on the nodal voltages and currents to ensure a tight convex relaxation. To determine these bounds, the CI approach also includes a carefully designed pre-processing algorithm which uses generation and load forecasts and grid limits to iteratively calculate tight nodal bounds [27]. The CI model has been shown to perform well on unbalanced networks with local generation, with maximum 1.2% optimality gap and 0.9% voltage error when compared to the AC-OPF for a number of use cases [27]. The CI model has also been used for solving OPF in a distributed way [28] and for voltage support from inverter-based resources in unbalanced distribution grids [30]. Due to its overall ability to model unbalanced grids and all of the aforementioned advantages including the computational simplicity of the linear model, we adopt the CI approach in this paper.
A. Problem Formulation

We denote a general distribution network as a graph \( \Gamma(N,E) \), where \( N := \{1,...,N\} \) denotes the set of nodes, \( E := \{(m,n)\} \) denotes the set of edges, and each phase is expressed as \( \phi = P, I \). The CI-OPF for the network is then written as:

\[
\begin{align*}
\min_{x} & \quad f(x) \\
\text{s.t.} & \quad AV = ZI_{\text{flow}} \quad (1a) \\
& \quad I^R = \text{Re}(A^T I_{\text{flow}}) \quad (1b) \\
& \quad I^I = \text{Im}(A^T I_{\text{flow}}) \quad (1c) \\
& \quad P_j^\phi = V_j^\phi R_j^\phi + V_j^\phi I_j^\phi \quad (1d) \\
& \quad Q_j^\phi = -V_j^\phi R_j^\phi + V_j^\phi I_j^\phi \quad (1e) \\
& \quad P_j^\phi \tan(\cos^{-1}(-pf)) \leq Q_j^\phi \leq P_j^\phi \tan(\cos^{-1}(pf)) \quad (1f) \\
& \quad P_j^\phi \leq P_j^\phi \leq P_j^\phi \quad (1g) \\
& \quad Q_j^\phi \leq Q_j^\phi \leq Q_j^\phi \quad (1h) \\
& \quad V_j^\phi, I_j^\phi \leq V_j^\phi \quad (1i) \\
& \quad V_j^\phi, I_j^\phi \leq V_j^\phi \quad (1j) \\
& \quad I_j^\phi R_j^\phi \leq I_j^\phi R_j^\phi \quad (1k) \\
& \quad I_j^\phi I_j^\phi \leq I_j^\phi I_j^\phi \quad (1l) \\
& \quad I_j^\phi I_j^\phi \leq I_j^\phi I_j^\phi \quad (1m)
\end{align*}
\]

where \( x = [I^R, I^I, V^R, V^I, P, Q, I_{\text{flow}}] \) is the decision vector for the CI-OPF problem; \( I, V, P, Q \) denote the vector of nodal current injections, voltages, and real/reactive power injections respectively; \( I_{\text{flow}} \) denotes the vector of line currents; \( A \in \mathbb{R}^{3N \times 3N} \) is the 3-phase graph incidence matrix; \( Z \) is the system impedance matrix. We use upper and lower limits of a variable \( x \); \( \text{Re}(\cdot) \) and \( \text{Im}(\cdot) \) denote the real and imaginary components of a complex number \( x \); \( \bar{\pi} \) and \( \underbar{\pi} \) denote the upper and lower limits of a variable \( x \); \( \text{Re}(\cdot) \) and \( \text{Im}(\cdot) \) denote the real and imaginary components of a complex number. Constraint \( 1b \) describes Ohm’s law, \( 1c-1i \) describe Kirchhoff’s Current Law, and \( 1e-1m \) are the definitions of real and reactive power. Constraint \( 1g-1m \) models the multiphase inverter of a DG, where the ratio of \( P \) and \( Q \) determine the PF setting. Constraints \( 1e-1m \) are for all nodes \( j \in N \) and per each phase \( \phi \in \mathcal{P} \).

The objective function \( f(x) \) in \( (1a) \) is chosen in this paper to be a combination of Social Welfare and line losses and is given by:

\[
f(x) = \sum_{j \in G} \left( a_j^P P_j^2 + b_j^P P_j + a_j^Q Q_j^2 + b_j^Q Q_j \right) - \sum_{j \in L} \left( \alpha_j^P (P_j - P_j^*)^2 + \alpha_j^Q (Q_j - Q_j^*)^2 + \zeta \sum_{j \in N} (R_{ij} I_{\text{flow}}) \right)
\]

where the first two terms represent generator cost and customer load disutility, and the third term represents line losses. In \( 2 \), \( N_G \) and \( N_L \) are the set of generator and load nodes respectively, \( \zeta \) describes the tradeoff between economic and energy efficiency objectives, \( a_j^P, b_j^P, a_j^Q, b_j^Q \) are generating cost coefficients, and \( \alpha_j^P, \alpha_j^Q \) are load disutility coefficients. All nodes \( j \in N_G \) and \( j \in N_L \) represent active DG and DR participants in the distribution grid, respectively. Node 1 is treated as the point of common coupling with the transmission grid, and reflects the wholesale price of electricity, which is the LMP \( \lambda^f \) from the WEM. Thus for \( j = 1 \), \( a_j^P = 0 \) and \( b_j^P = \lambda^f \). The cost coefficients related to reactive power are chosen as \( a_j^Q = 0 \), \( b_j^Q = 0.1 b_j^P \). The motivation for these choices comes from \( 3 \) which cites that reactive price powers are often one-tenth of that of real power. It should be noted that the weighted combination of the social welfare and line losses is not standard practice, but is included here as line losses are more significant for an optimal functioning of a distribution grid compared to a transmission grid. The resulting solutions of the CI-OPF problem forms the backbone of our proposed reactive power market.

B. Reactive Power Pricing

The prices for real and reactive power of a DER at node \( j \) are chosen as the marginal costs of generators/consumers and therefore determined using the dual variables of the CI-OPF model. In particular, the Lagrange multipliers corresponding to \( (1e) \) and \( (1l) \) are used to determine the price for real and reactive power respectively. While the discussions below are focused almost entirely on the reactive power component, the solutions above encompass both the real and reactive power dispatch simultaneously.

Each market clearing sets the reactive power dispatch, \( Q_{j,t} \) for node \( j \) and time \( t \), and produces the corresponding dual variable \( \mu_j^{Q,t} \). This dual variable sets the basis for the time-varying reactive power price \( \bar{\mu}_j^{Q,t} \), which we denote as the d-LMP, determined for node \( j \) as:

\[
\bar{\mu}_j^{Q,t} = \frac{\sum_{t \in T} \mu_j^{Q,t} Q_{j,t}}{\sum_{t \in T} Q_{j,t}}
\]

This d-LMP varies daily, calculated as a weighted average of the dual variable from the OPF problem in \( (1) \). Thus for a 5-minute retail market clearing period, the set \( T \) includes 288 points. Such a time-varying price allows DGs to adjust their generation and/or PF settings and DRs to shift their consumption behavior, all in a coordinated manner so that the DSO can accurately recover costs. Rather than exposing end-use customers and DER owners to the complete dynamics of the electricity system by using \( \mu_j^{Q,t} \) as a real-time d-LMP, as wholesale market participation models (like FERC 2222) would do with a corresponding \( L^Q \), the averaging procedure in \( (3) \) allows the price volatility to be contained. In the next section we present a numerical case study of the reactive power market, and further discuss the pricing model.

IV. RESULTS

Our reactive power market proposed for a distribution grid is evaluated in this section using an IEEE 123-bus network. The network is modified to include clusters of rooftop photovoltaic (PV) units at up to 27 of the 123 nodes. To emulate varying levels of DG penetration, PV clusters are incrementally
added at various nodes. Each PV cluster is chosen to have a capacity of 25-80kW, a range that represents an addition of 3 to 10 small residential installations or larger projects such as carports or community solar. To appropriately assess the performance of our reactive power market, we introduce a few metrics. We define DG penetration as

$$\text{DG penetration} = \frac{\sum_{N_G} \text{Nameplate Capacity}}{\sum_{N_L} \text{Average Load}}$$  \hspace{1cm} (4)

We also introduce a second penetration metric which accounts for the capacity factor of renewable resources, the DG energy penetration, as

$$\text{DG energy penetration} = \frac{\sum_{t \in T} \sum_{j \in N_G} S_{j,t}}{\sum_{t \in T} \sum_{j \in N_L} S_{j,t}}$$  \hspace{1cm} (5)

where $S$ is apparent power.

The ratio of the reactive power revenue to the total revenue for DGs is defined as

$$\text{Q-Revenue Ratio} = \frac{1}{|N_G|} \sum_{j \in N_G} \left( \frac{\mu_j^Q \sum_{t \in T} Q_{j,t}}{\mu_j^Q \sum_{t \in T} Q_{j,t} + \sum_{t \in T} \mu_j^P P_{j,t}} \right)$$  \hspace{1cm} (6)

where $| \cdot |$ denotes the cardinality of set $N_G$, i.e. the number of generator nodes. That is, the Q-revenue ratio is the average fraction of the reactive power payout compared to the total payout from real and reactive power, over the network.

The DG-Q utilization is defined as

$$\text{DG-Q utilization} = \frac{\sum_{t \in T} \sum_{j \in N_G} Q_{j,t}}{\sum_{t \in T} \sum_{j \in N_L} Q_{j,t}}$$  \hspace{1cm} (7)

That is, the DG-Q utilization is the fraction of reactive power loads that are served by DGs. Similarly, the DG-Q utilization is defined as

$$\text{DG-P utilization} = \frac{\sum_{t \in T} \sum_{j \in N_G} P_{j,t}}{\sum_{t \in T} \sum_{j \in N_L} P_{j,t}}$$  \hspace{1cm} (8)

The remaining load (for both P and Q) is assumed to be served by the transmission system.

In our simulations, the cluster capacity range was chosen so as to emulate a range of 5% to 160% DG penetration, which can reflect the trend that is anticipated over the coming decade especially in the context of more aggressive DG adoption scenarios. For New England, the current and forecasted DG penetration by 2030 is 4.3% and 14%, respectively [31]. Each DG is assumed to be equipped with a smart inverter which operates at a flexible PF, with appropriate power electronic control [32]. Rather than fixing the PF, we enforce a lower limit for each unit, permitting the market to determine the inverter setting. We model all inverters as having the same PF limits. The baseline PV generation data $\alpha_{PV}(t)$ is taken from the NREL SAM dataset [33], and is assumed to be the same for each PV cluster. The load profile for each node is modelled with real-time data from ISO-NE [15], and perturbed for each load at node $j$ as $\alpha_{P}(t) = \alpha_{D}(t)\delta_{j,t}$ with $\delta_{j,t} \sim N(0, 0.1)$. Wholesale electricity prices are obtained from ISO-NE’s real-time market to determine $\lambda$. With these modifications introduced into the IEEE-123 bus network, the reactive power price $\bar{\mu}_j$ and the reactive power injection $Q_j$ are determined using the OPF problem in [1]-[2].

We assess the performance of our proposed retail market by considering the following scenarios:

a) with increasing PV penetration; and

b) with varying lower limit on PF operating range

We also quantify the performance of our retail market by evaluating the variation in reactive power pricing over a 24-hour period.

A. Increasing DG Penetration

In this section we investigate the behaviour of the reactive power market for a range of DG penetration of 5% to 160% obtained from the modified IEEE-123 bus. This corresponds to a DG energy penetration range of 0.5% to 35%. All inverters were assumed to have a PF limit of 0.9.

Figure 2 shows the variation in $\bar{\mu}_j^Q$ and Q revenue, as a function of the DG penetration. It is clear that $\bar{\mu}_j^Q$ remains relatively constant even as the DG penetration increases from 5% to 160%. We also observe that the Q-revenue ratio remains relatively steady around 5%. As DG-penetration changes in the range of 5% to 160%, the Q-revenue ratio declines hovers around 4-5%, before increasing to 6% as DG penetration increases to 160%. Figure 3 shows the DG-Q and DG-P utilization over the course of a day, for a DG penetration of 160% (equivalent to a DG energy penetration of 35%). When solar generation peaks around mid-day, almost 100% of real power load can be met with local generation. This saturation in real power means that any additional DG capacity added to the grid will be used for meeting reactive power load. This shift, wherein DGs move from supplying primarily real power to reactive power, results in an increase in Q-revenue ratio. This is seen in Fig. 2 by the Q-revenue ratio inflection point at high DG penetration. Notably, even at this high penetration, the reactive power price $\bar{\mu}_j^Q$ remains steady. Together, these results show that as more DGs are added, they can support the grid’s real and reactive power needs without hurting the revenue potential of other DGs. In this way, our market maintains consistent payouts for reactive power.

B. Varying Power Factors

In this scenario, we vary the PF operating range of all inverters, varying minimum PF from 0.6 to 0.95. We calculate the PF per DG $k$ as

$$PF_k = \cos \left( \arctan \left( \frac{\sum_{\phi \in P} Q_{k,\phi}}{\sum_{\phi \in P} P_{k,\phi}} \right) \right)$$

This is a realizable scenario as DGs like PV are being increasingly equipped with smart inverters with advanced power electronics making them capable of operating in such a range. With the same OPF procedure outlined in Section III, we once again determined the Q-revenue ratio by changing the PF in the inverter P-Q model [15], and determining the overall solution from [1]-[2].
Fig. 2: As the penetration of DGs increases, both the average price and Q-revenue ratio remain relatively stable. The price of reactive power is not strongly dependent on the PV penetration, making it more reliable for the transitioning grid. The revenue ratio shows that even as DGs are added, reactive power remains a reliable source of revenue.

Fig. 3: With a high DG penetration of 160%, DGs nearly saturate the grid in the middle of the day. At the same time, DGs cover more than half of the reactive power load; much of the remaining load is covered by capacitor banks.

Fig. 4: Our market enables DGs to sell reactive power while operating at lower PFs. When DGs are leveraged at lower PFs, they can provide substantial support, in this case providing almost half of the grid’s reactive power load.

Q-revenue ratio increases from 3% to 10.5%. This implies that DGs operating at lower PFs can anticipate up to 10% of their total revenue to come from reactive power payments, which is a significant amount. Clearly, a retail market for reactive power, such as the one proposed in this paper, is essential to appropriately compensate DGs for their services.

Fig. 5: Enabling DGs to operate at low PFs increases the fraction of their revenue from reactive power. DGs that are permitted to have PFs as low as 0.6 sourced over 10% of their revenue from our market’s reactive power payments.

These results show that our market will utilize the full flexibility in PF operating range offered by DGs through the variable payments, something not possible under current industry practice of fixed pricing.

C. Market Performance over a Week

To better understand the market performance over a range of conditions, we simulate the market over the week of June 27, 2021 to July 3, 2021, using demand and LMP data from ISO-NE. This week is particularly interesting because it coincided with the end of a heat wave in New England and thus represents a larger than average variation in grid demand. For this simulation, a DG penetration of 120% was used, corresponding to the addition of PV clusters at 20 nodes...
in the network, each with an inverter with a minimum PF of 0.9.

Figure 6 shows the resulting $\bar{\mu}_j^Q$ values over the week, for all 20 DGs. The significant price differences between nodes speaks to the importance of d-LMPs due to their variability in both time and location. The temporal variation in price exhibited over the week further motivates the need for time-varying prices in order to reflect the varying grid conditions, especially with increased DER-penetration.

![Figure 6: The daily prices of reactive power vary spatially across nodes and temporally over a week, but maintain the same order of magnitude. Prices remain relatively consistent, creating a stable and reliable revenue stream for DGs.](image)

Figure 7: The hourly dual variable $\mu_{j,t}^Q$ shows significant volatility over the course of a single day. The proposed daily d-LMP is approximately 3.3 less more volatile than the hourly price, while both result in the same payout. Price volatility is a key concern for reactive power markets [11], [12], and motivates the weighted average pricing model introduced in (3). The prices in Figure 6 are calculated using the dual variables $\mu_{j,t}^Q$, which are shown in Figure 7 for a single DG over a 24-hr simulation. Compared to the stable daily prices, the underlying hour-to-hour price exhibits large price variations, something that is undesirable for a market. We can further quantify this price volatility using the coefficient of variation $c_v$ as defined below:

$$c_v = \frac{std(\mu_{j,t}^Q)}{\mu_j^Q}$$ (9)

The average $c_v$ for $\bar{\mu}_j^Q$ is 0.42, approximately 3.3 times less volatile than the hourly $c_v$ of 1.39. While the total payout to DERs is the same for the hourly duals $\mu_{j,t}^Q$ or using the daily d-LMPs $\bar{\mu}_j^Q$, the daily d-LMPs are significantly less volatile. Averaging over each day, therefore, is chosen for our reactive power d-LMP, $\bar{\mu}_j^Q$, in order to contain the volatility and provide a reliable revenue stream for DERs. A choice of a larger period such a month or a year would suppress useful granularity in prices, and render an inefficient pricing mechanism.

D. Key Observations

Through these simulations and analyses, the following key observations were made:

- The reactive power market introduced and evaluated in this paper can incentivize DERs to participate in reactive power support with Q-revenue streams of up to 10% of total revenue (Figure 5).
- The limited price volatility of the daily Q d-LMPs (Figure 6) and the stable revenue streams under varying DG penetration (Figure 2) can instill confidence in the market, through the energy transition. This could encourage additional DG enrollment into the retail market, and also help drive investment decisions for additional DER adoption.
- Our market allows DGs to contribute a significant amount of grid support by leveraging a wide range of PFs. In simulation, DGs accounted for 42% of reactive power when allowed to operate at PFs as low as 0.6 (Figure 4).

V. SUMMARY AND CONCLUDING REMARKS

Recently, the Biden administration has revealed as part of its climate strategy it aims to reach 40% solar generation by 2035 [34]. Reaching such an aggressive goal will require new strategies that do not rely on a shrinking number of centralized generators for power quality support. DERs represent a growing resource capable of significant grid support contributions through their adjustable PFs. However, current market practices are inadequate to integrate and compensate small grid-edge devices, particularly for reactive power. Our proposed distribution level retail market achieves this by enabling DERs to provide grid services and be remunerated at a fair retail rate.

In this paper, we have proposed a reactive power market for DGs in the distribution grid. The market scheduling and clearing is based on PAC, a distributed optimization algorithm, which has been shown to converge in other works. Through a numerical case study of a modified IEEE-123 bus, we demonstrated that our market accommodates a large amount of DGs with a DG penetration anywhere between 5% and 160%, with the revenue stream remaining steady. We also showed that this market utilizes the full flexibility of DGs with the ability to operate over a range of PF, from 0.6 to 0.95 thereby meeting over 45% of reactive power load, with the reactive power price per unit kVAR remaining the same. We also showed through a detailed assessment of the price variations across all DGs in the IEEE 123-bus over a week shows that the price variations...
are non-volatile; they primarily follow load fluctuations and otherwise exhibit little fluctuations.

The proposed market is structured so as to simultaneously clear real and reactive power with the resulting solution serving as the DER dispatch, and the duals providing a basis for real and reactive power pricing. The reactive power market incentivizes DERs to enroll and participate in reactive power support by offering a reliable additional source of income through daily Q-dLMPs. The results from our numerical study show that this revenue stream remains dependable even as DG penetration rapidly grows. By building confidence with stable prices, as shown in Section IV, additional DG enrollment may perhaps be encouraged into the retail market and help drive investment decisions for additional DER adoption. Our results appear promising in this regard and permit better DER integration and utilization to meet distribution grid objectives, and can enable deeper DER penetration.

REFERENCES

[1] Federal Energy Regulatory Commission, “Order 2222: Participation of distributed energy resources aggregations in markets operated by RTO and ISO,” April 2021.
[2] J. Zhong and K. Bhattacharya, “Reactive power management in deregulated electricity markets - A review,” in 2002 IEEE Power Engineering Society Winter Meeting, Conference Proceedings, vol. 2, pp. 1287–1292, January 2002.
[3] Federal Energy Regulatory Commission, “Payment for reactive power - Deregulation staff report,” tech. rep., April 2014.
[4] R. Haider, S. Baros, Y. Wasa, J. Romovay, K. Uchida, and A. Annaswamy, “Towards a retail market for distribution grids,” IEEE Transactions on Smart Grids, 2020. doi:10.1109/TSG.2020.2996565
[5] R. Haider, D. A’chiardi, V. Venkataramanan, S. Srivastava, A. Bose, and A. M. Annaswamy, “Reinventing the utility for distributed energy resources: A proposal for retail electricity markets,” Advances in Applied Energy, vol. 2, p. 100026, 2021.
[6] L. Kristov, P. De Martini, and J. D. Taft, “A tale of two visions: Designing a decentralized transactive electric system,” IEEE Power Energy Magazine, vol. 14, pp. 63–69, May 2016.
[7] J. Zhong, E. Nobile, A. Bose, and K. Bhattacharya, “Localized reactive power markets using the concept of voltage control areas,” IEEE Transactions on Power Systems, vol. 19, pp. 1555–1561, August 2004.
[8] J. Romovay, G. Ferro, R. Haider, and A. M. Annaswamy, “A distributed proximal atomic coordination algorithm,” IEEE Transactions on Automatic Control, 2021. doi:10.1109/TAC.2021.3053907
[9] K. Bhattacharya and J. Zhong, “Reactive power as an ancillary service,” IEEE Transactions on Power Systems, vol. 16, pp. 294–300, May 2001.
[10] A. C. Rueda-Medina and A. Padilha-Feltrin, “Distributed generators as providers of reactive power support—a market approach,” IEEE Transactions on Power Systems, vol. 28, pp. 490–502, February 2013.
[11] J. Weber, T. Overbye, P. Sauer, and C. DeMarco, “A simulation based approach to pricing reactive power,” in Proceedings of the Thirty-First Hawaii International Conference on System Sciences, vol. 3, pp. 96–103 vol.3, January 1998.
[12] J. Barquin Gil, T. San Roman, J. Alba Rios, and P. Sanchez Martin, “Reactive power pricing: a conceptual framework for remuneration and charging procedures,” IEEE Transactions on Power Systems, vol. 15, pp. 483–489, May 2000.
[13] S. Ahmed and G. Strbac, “A method for simulation and analysis of reactive power market,” IEEE Transactions on Power Systems, vol. 15, pp. 1047–1052, August 2000.
[14] Michael Borgatti, Adrian Kimbrough, Steven Sharper, “Reactive power compensation: Unlocking new revenue opportunities for solar and storage projects,” tech. rep., Solar Energy Industries Association, July 2020.
[15] ISO-NE, “Real-time and historical data for informed market decisions,” [Online].Available:https://www.iso-ne.com/markets-operations/iso-express/2020
[16] A. Safari, P. Salyani, and M. Hajiloo, “Reactive power pricing in power markets: a comprehensive review,” International Journal of Ambient Energy, vol. 41, pp. 1548–1558, November 2020.