Electromagnetic induction and damping
– quantitative experiments using PC interface

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A bar magnet, attached to an oscillating system, passes through a coil periodically, generating a series of emf pulses. A novel method is described for the quantitative verification of Faraday’s law which eliminates all errors associated with angular measurements, thereby revealing delicate features of the underlying mechanics. When electromagnetic damping is activated by short-circuiting the coil, a distinctly linear decay of oscillation amplitude is surprisingly observed. A quantitative analysis reveals an interesting interplay of the electromagnetic and mechanical time scales.

I. INTRODUCTION

Laboratory experiments on Faraday’s law of electromagnetic induction most often involve a bar magnet moving (falling) through a coil, and studying the induced emf pulse. Several parameters can be varied, such as the velocity of the magnet, the number of turns in the coil, and the strength of the bar magnet. The observed proportionality of the peak induced emf on the number of turns in the coil and the magnet velocity provide a quantitative verification of the Faraday’s law.

Commonly it is found convenient to attach the magnet to an oscillating system, so that it passes through the coil periodically, generating a series of emf pulses. This allows the peak emf to be easily determined by charging a capacitor with the rectified coil output. A simple, yet remarkably robust setup which utilizes this concept involves a rigid semi-circular frame of aluminum, pivoted at the center (O) of the circle (see Fig. 1). The whole frame can oscillate freely in its own plane about a horizontal axis passing through O. A rectangular bar magnet is mounted at the center of the arc and the arc passes through a coil C of suitable area of cross section. The positions of the weights \(W_1\) and \(W_2\) can be adjusted to bring the mean position of the bar magnet vertically below the pivot O, and the position of coil is adjusted so that its center coincides with this mean position (\(\theta = 0\)) of the magnet. The angular amplitude can be read by means of a scale and pointer. The magnet velocity can be controlled by choosing different angular amplitudes, allowing the magnetic flux to be changed at different rates.

It is much more instructive to monitor the induced emf in the coil through a PC interface, which can be readily realized by low-cost, convenient data-acquisition modules available in the market. We have used a module based on the serial interface “COBRA-3” and its accompanying software “Measure”, marketed and manufactured by PHYWE. We specially found useful the various features of “Measure” such as “integrate”, “slope”, “extrema”, “zoom” etc. In this article we describe modified experiments designed to take advantage of the computer interface. This allows for a quantitative and pedagogical study of (i) angular (position) dependence of the magnetic flux through the coil, (ii) verification of Faraday’s law of induction and (iii) electromagnetic damping, thereby revealing delicate features of the underlying mechanics.

II. INDUCED EMF PULSE

The equation for the induced emf \(E(t)\) as a function of time \(t\) can be written as

\[
E(t) = \frac{d\Phi}{dt} = \frac{d\Phi}{d\theta} \frac{d\theta}{dt},
\]

expressing the combined dependence on the angular gradient \(d\Phi/d\theta\) and the angular velocity \(\omega(\theta) = d\theta/dt\). This is reflected in the time dependence of \(E(t)\), and a typical emf pulse is shown in Fig. 2; the pulse-shape is explained below for one quarter cycle of oscillation, starting from the extreme position of the bar magnet (\(\theta = \theta_0\)). As the magnet approaches the coil, the induced emf initially rises, then turns over and starts falling as the magnet
enters the coil and the magnetic flux begins to saturate, and finally changes sign as the magnet crosses the center of the coil ($\theta = 0$) where the flux undergoes a maximum. Thus $E$ actually vanishes at the point where the angular velocity of the magnet is maximum.

From Fig. 2 the magnitude of $E$ is seen to be significant only in a very narrow time-interval of about 200 ms, which is much smaller than the oscillation time period ($T \approx 2$ s). This implies that the magnetic flux through the coil falls off very rapidly as the magnet moves away from its center, so that $d\Phi/d\theta$ is significant only in a very narrow angular range (typically about $5^\circ$) on either side of the mean position. As $d\Phi/d\theta = 0$ at $\theta = 0$, it follows that $d\Phi/d\theta$ is strongly peaked quite close to the mean position, which accounts for the emf pulse shape seen in Fig. 2. This separation of the electromagnetic and mechanical time scales has interesting consequences on the electromagnetic damping, as further discussed in section IV.

III. MAGNETIC FLUX THROUGH THE COIL

In order to quantitatively study the magnetic flux through the coil, the “integrate” feature of the software is especially convenient. From Eq. (1) the time-integral of the induced emf directly yields the change in magnetic flux $\Delta \Phi$ corresponding to the limits of integration. If the lower limit of integration $t_1$ corresponds to the extreme position of the magnet ($\theta(t_1) = \theta_0$), where the magnetic flux through the coil is negligible (valid only for large $\theta_0$), the magnetic flux $\Phi(\theta)$ for different angular positions $\theta(t)$ of the magnet is obtained as

$$\Phi(t) \approx \int_{t_1}^{t} E(t')dt'. \quad (2)$$

Figure 3 shows a plot of $\Phi(t)$ vs. $t$ for a large angular amplitude ($\theta_0 \sim 30^\circ$). The time interval during which $\Phi(t)$ is significant ($\sim 200$ ms) is a very small fraction of the oscillation time period (about 2 sec), confirming that the magnetic flux changes very rapidly as the magnet crosses the center of the coil. As the angular velocity of the magnet is nearly constant in the central region, the time scale in Fig. 3 can be easily converted to the angular ($\theta = \omega t$) or linear ($x = R\theta$) scale. The points of inflection, where $d\Phi/dt$ (and therefore $d\Phi/d\theta$) are extremum, are at 4200 and 4300 ms, precisely where the peaks occur in the emf pulse (Fig. 2).

IV. VERIFICATION OF FARADAY’S LAW

For $\theta_0 >> 5^\circ$, the angular velocity of the bar magnet is very nearly constant in the narrow angular range near the mean position, and hence the peak emf $E_{\text{max}}$ is approximately given by

$$E_{\text{max}} \approx \left(\frac{d\Phi}{d\theta}\right)_{\text{max}} \omega_{\text{max}}. \quad (3)$$

The maximum angular velocity $\omega_{\text{max}}$ itself depends on $\theta_0$ through the simple relation (see Appendix)

$$\omega_{\text{max}} = \frac{4\pi}{T} \sin(\theta_0/2), \quad (4)$$

where $T$ is the time period of (small) oscillations. Therefore if $\theta_0/2$ (in radians) is small compared to 1, then $\omega_{\text{max}}$ is nearly proportional to $\theta_0$, and hence $E_{\text{max}}$ approximately measures the angular amplitude $\theta_0$.

Eq. (3) provides a simple way for students to quantitatively verify Faraday’s law. A plot of the peak emf $E_{\text{max}}$ (conveniently obtained using the software feature “extrema”) vs. $\omega_{\text{max}}$ (evaluated from Eq. (4)) for different angular amplitudes should show a linear dependence (for large $\theta_0$). While this behaviour could indeed be easily verified by students, the interesting deviation from linearity expected for low angular amplitudes ($\theta_0 \sim 5^\circ$), for which the $\theta$ dependence of the angular velocity $d\theta/dt$ is not negligible, turned out to be quite elusive. This
we obtain setting low. Taking the time derivative of the induced emf, and measuring the oscillation amplitude \( \theta \) pronounced. For small angles, precisely where this deviation is more delicate deviation was presumably washed out by the large percentage errors in \( \theta \) measurements, especially for small angles, precisely where this deviation is more pronounced.

An alternative approach, which eliminates the need for measuring the oscillation amplitude \( \theta_0 \), is proposed below. Taking the time derivative of the induced emf, and setting \( \theta = 0 \), where the angular velocity is maximum, we obtain

\[
\left( \frac{dE}{dt} \right)_{\theta=0} = \left( \frac{d^2\Phi}{dt^2} \right)_{\theta=0} \omega_{\text{max}}^2,
\]

which relates the slope of \( E \) at the mean position to \( \omega_{\text{max}}^2 \). As this relation holds for all amplitudes \( \theta_0 \), it may be used to obtain \( \omega_{\text{max}} \) for different angular amplitudes without the need for any angular measurement. The slope at the mean position (near zero crossing) is easily measured through linear interpolation (see Fig. 2). Thus, a plot of \( E_{\text{max}} \) vs. \( \sqrt{|(dE/dt)_{\theta=0}|} \) shows both features of interest — the linear behaviour for large angular amplitudes and the deviation for very low amplitudes. The key advantage of this plot lies in completely eliminating the errors associated with measurements of oscillation amplitudes \( \theta_0 \).

In this experiment the oscillations were started with a large initial angular amplitude, and during the gradual decay of oscillations, the peak voltages \( E_{\text{max}} \) (on both sides of the mean position) and the slope \( (dE/dt)_{\theta=0} \) were measured for a large number of pulses, so as to cover the full range from large to very small angular amplitudes. Fig. 4 shows this plot of the averaged \( E_{\text{max}} \) vs. \( \sqrt{|(dE/dt)_{\theta=0}|} \), clearly showing the deviation from linearity for low angular amplitudes. To provide an idea of the angular amplitude scale, the peak voltage of \( \sim 6 \) V corresponds to an angular amplitude \( \theta_0 \approx 35^\circ \), so that the deviations become pronounced when \( \theta_0 \approx 5^\circ \).

FIG. 4. Plot of \( E_{\text{max}} \) vs. \( \sqrt{|(dE/dt)_{\theta=0}|} \), showing the deviation from a straight line at low angular amplitudes.

Although an exponential decay of amplitude is more commonly encountered in damped systems, the plot of the normalized peak voltage \( V_{\text{max}}(t)/V_{\text{max}}(0) \) vs. \( t \) shows a distinctly linear decay (Fig. 5). To distinguish the electromagnetic damping from other sources (friction, air resistance etc.) the same experiment was repeated in the open-circuit configuration, in which case electromagnetic damping is absent. In this case the amplitude decay is, as expected, much weaker, but significantly it is still approximately linear.

A quantitative analysis of the energy loss provides an explanation for this nearly linear decay in both cases. We first consider the electromagnetic energy loss. Neglecting radiation losses, the main source of energy loss is Joule heating in the coil due to the induced current. Integrating over one cycle we have

V. ELECTROMAGNETIC DAMPING

To study the nature of the electromagnetic damping in the oscillating system, the coil was short-circuited through a low resistance (220 \( \Omega \)). The oscillations were started with a large initial amplitude (still \( \theta_0/2 \ll 1 \)), and the voltage \( V(t) \) across the resistor was studied as a function of time. As the oscillations decayed, the peak voltages \( V_{\text{max}} \) for sample pulses were recorded at roughly equal intervals on the time axis. This voltage \( V(t) \) is proportional to the current through the circuit, and hence to the induced emf \( E(t) \). As the peak emf \( E_{\text{max}} \) is approximately proportional to the oscillation amplitude (except when the amplitude becomes too small), a plot of \( V_{\text{max}} \) vs. \( t \) actually exhibits the decay of the oscillation amplitude with time.

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FIG. 5. The normalized peak voltage \( V_{\text{max}}(t)/V_{\text{max}}(0) \) vs. time \( t \) for the short-circuit (\(+\)) and open-circuit (\(\Diamond\)) cases, showing nearly linear fall off.
\[ \Delta E_{\text{one cycle}} = \int i^2 R \, dt = \frac{1}{R} \int \mathcal{E}^2 \, dt \]
\[ = \frac{1}{R} \int \left( \frac{d\Phi}{d\theta} \right) \left( \frac{d\theta}{dt} \right)^2 \, dt , \quad (6) \]

where \( R \) is the coil resistance. This may be further simplified since \( d\Phi/d\theta \) is significant only in a narrow angular range near \( \theta = 0 \) and rapidly vanishes outside. Now, for amplitudes not too small, the angular velocity \( d\theta/dt \) is nearly constant (\( \approx \omega_{\text{max}} \)) in this narrow angular range, and therefore taking it outside the integral, we obtain

\[ \Delta E_{\text{one cycle}} \approx \frac{\omega_{\text{max}}}{R} \int \left( \frac{d\Phi}{d\theta} \right)^2 \, d\theta . \quad (7) \]

As the angular integral is nearly independent of the initial amplitude \( \theta_0 \), and therefore of \( \omega_{\text{max}} \), the energy loss per cycle is proportional to \( \omega_{\text{max}} \), and therefore to \( \sqrt{E} \). On a long time scale \( (t >> T) \), we therefore have

\[ \frac{dE}{dt} = -k\sqrt{E} . \quad (8) \]

Integrating this, with initial condition \( E(0) = E_0 \), we obtain

\[ \sqrt{E_0} - \sqrt{E} \propto t \]
\[ \Rightarrow \omega_{\text{max}}^0 - \omega_{\text{max}} \propto t \]
\[ \Rightarrow E_{\text{max}} - E_{\text{max}} \propto t , \quad (9) \]

indicating linear decay of the peak emf, and therefore of the amplitude, with time.

We now consider the energy loss in the open-circuit case, where the damping is due to frictional losses. A frictional force proportional to velocity, as due to air resistance at low velocities, will result in an exponential decay of the oscillation amplitude. However, a function of the type \( e^{-at} \) did not provide a good fit. On the other hand, assuming a constant frictional torque \( \tau \) at the pivot, which is not unreasonable considering the contact forces at the pivot, we obtain for the energy loss in one cycle,

\[ \Delta E_{\text{one cycle}} = \int \tau \, d\theta = \tau \, 4\theta_0 \]
\[ \propto \omega_{\text{max}} \]
\[ \propto \sqrt{E} . \quad (10) \]

This is similar to the earlier result of Eq. (8) for electromagnetic damping, yielding a linear decay of the oscillation amplitude with time, which provides a much better fit with the observed data, as seen in Fig. 5. The deviation from linearity at large times is presumably due to a small air resistance term. In fact, if a damping term \(-k'E\) due to air resistance is included in Eq. (8), the differential equation is easily solved, and the solution provides an excellent fit to the data. Finally, another term \(-k''E^{3/2} \) should be included in Eq. (8), arising from

the reduction in the average centripetal force \( Ml\omega^2\theta_0^2/2 \) with the oscillation amplitude \( \theta_0 \), which decreases the frictional force at the pivot due to the reduction in the normal reaction.

\[ \begin{align*} \text{VI. SUMMARY} \end{align*} \]

A pedagogically instructive study of electromagnetic induction and damping is made possible by attaching a PC interface to a conventional setup for studying Faraday’s law. By eliminating all errors associated with angular measurements, the novel method applied for the verification of Faraday’s law reveals delicate features associated with the underlying mechanics. A quantitative analysis of the distinctly linear decay of oscillation amplitude due to electromagnetic damping reveals an interesting interplay of the electromagnetic and mechanical time scales.

\[ \begin{align*} \text{VII. APPENDIX} \end{align*} \]

If the system is released from rest with an angular displacement \( \theta_0 \), then from energy conservation

\[ \frac{1}{2} I\omega_{\text{max}}^2 = Mgl(1 - \cos\theta_0) , \quad (11) \]

where \( M \) is the mass of the system, \( I \) its moment of inertia about the pivot O, and \( l \) the distance from O to the centre of gravity. For small oscillations, the equation of motion is \( I\ddot{\theta} = -(Mgl)\theta \), so that the time period is given by

\[ T = 2\pi \sqrt{\frac{I}{Mgl}} . \quad (12) \]

Eliminating \( I/Mgl \) from these two equations, we obtain

\[ \omega_{\text{max}} = \frac{4\pi}{T} \sin(\theta_0/2) . \quad (13) \]

\[ \begin{align*} \text{ACKNOWLEDGEMENT} \end{align*} \]

Helpful assistance from Shri B. D. Gupta, B. D. Sharma, and Manoj Kumar is gratefully acknowledged.

\[ ^1 \text{P. Rochon and N. Gauthier, “Induction transducer for recording the velocity of a glider on an air track,” Am. J. Phys. 50, 84-85 (1982).} \]
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5 PHYWE Systeme GmbH, Göttingen, Germany.
6 Identifying the time $t_i$ for a given induced emf pulse $\mathcal{E}(t)$ is a good exercise for students.