Alkali metal doped $C_{60}$ ($A_x C_{60}$), $A=K$, Rb, Cs, form stable crystalline phases (fullerides) with a broad range of physical and chemical properties comprising superconductors and polymer phases. For a review, see [1]. In particular the $x = 1$ compounds [2] exhibit plastic crystalline phases with cubic rocksalt structure at high temperature ($T \geq 350$ K) and polymeric phases [3] of reduced symmetry at lower $T$. In the latter the $C_{60}$ molecules are linked through a [2+2] cycloaddition [3], a mechanism originally proposed for photo-induced polymerization [4] in pristine $C_{60}$. From X-ray powder diffraction [4] it was concluded that the crystal structure of both $K C_{60}$ and $RbC_{60}$ was orthorhombic (space group $Pmnn$). Polymerization occurs along the orthorhombic $\bar{a}$ axis (the former cubic [110] direction), where the orientation of the polymer chain is characterized by the angle $\mu$ of the planes of cycloaddition with the $\bar{c}$ axis. In the $Pmnn$ structure (Fig. 1a), these orientations are alternatively $\mu$ and $-\mu$, $|\mu| \approx 45^\circ \pm 5^\circ$. Notwithstanding this apparent structural similarity, the electronic and magnetic properties of $K C_{60}$ on one hand and $RbC_{60}$, $CsC_{60}$ on the other hand, were found to be very different [5].

Electronic and optical conductivity data [5] show that $RbC_{60}$ and $CsC_{60}$ exhibit a transition from a quasi-one dimensional metal to an insulating magnetic state near 50 K, while $K C_{60}$ stays metallic and nonmagnetic at low $T$. NMR spectra also showed marked differences between $K C_{60}$ and $Rb$-, $Cs$-polymers [6]. The contradiction between similar crystalline structures and different electromagnetic properties was resolved by single crystal X-ray diffraction and diffuse scattering studies [7]. Indeed the polymer phases of $K C_{60}$ and $RbC_{60}$ are different. While the space group $Pmnn$ is confirmed for $K C_{60}$, it is found that $RbC_{60}$ is body centered monoclinic, with space group $I2/m$. In the latter structure, the polymer chains have the same orientation $\mu$ (Fig. 1b). Electronic band structure calculations for the $I2/m$ structure have shown the importance of transverse interchain coupling for $RbC_{60}$ [1]. Recently [2] high-resolution synchrotron powder diffraction results demonstrated that $CsC_{60}$ has the same structure as $RbC_{60}$.

In this paper we study the mechanism which leads to the distinct polymer phases and we demonstrate the active role of the distinctive quadrupolar electronic polarizability of the alkali ions (cations). We start from the high temperature orientationally disordered cubic phase (space group $Fm\bar{3}m$) and describe the formation of the polymer phases as a scenario with several steps: i) the charge transfer of one electron from the alkali atom to the $C_{60}$ molecule leads to an occupation of the lowest unoccupied molecular orbital levels which are of $t_{1u}$ symmetry. Thereby the crystal field of the $C_{60}^-$ ion acquires an electronic component [8] which favors a same orientation of neighboring molecules along [110] such that the stereospecific cycloaddition occurs. ii) The cycloaddition between neighboring molecules then acts as a negative internal stress (chemical pressure) along [110]. iii) The mutual orientation of neighboring $C_{60}^-$ chains, which distinguishes between the orthorhombic and monoclinic

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The polymer phases of $AC_{60}$ form distinct crystal structures characterized by the mutual orientations of the ($C_{60}$)$_n$ chains. We show that the direct electric quadrupole interaction between chains always favors the orthorhombic structure $Pmnn$ with alternating chain orientations. However the specific quadrupolar polarizability of the alkali metal ions leads to an indirect interchain coupling which favors the monoclinic structure $I2/m$ with equal chain orientations. The competition between direct and indirect interactions explains the structural difference between $KC_{60}$ and $RbC_{60}$, $CsC_{60}$.

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**Theory of Distinct Crystal Structures of Polymerized Fullerides $AC_{60}$, $A=K$, Rb, Cs: the Specific Role of Alkalis**

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FIG. 1. Crystal structures projected onto the crystallographic ($\bar{b}, \bar{c}$) plane: (a) $Pmnn$, (b) $I2/m$. The thick bars represent the projection of the cycloaddition planes. Polymerization occurs along $\bar{a}$. The alkalis located in ($\bar{b}, \bar{c}$) planes and at $\pm a/2$ are denoted by full (+) and empty (−) circles.
structures, should depend on the intercalated alkali ions.

We first show that the orthorhombic lattice is the result of the cycloaddition. Using concepts of the theory of elasticity [3], we find that the cubic crystal is deformed into an orthorhombic one (point group \(D_{2h}\)). Taking the cubic \([110],[110]\) and \([001]\) as new axes, \(x, y\) and \(z\) axes (orthorhombic \(\vec{a}, \vec{b}\) and \(\vec{c}\)) respectively, we find the deformations

\[
\begin{align*}
\epsilon_{xx} &= \frac{K}{dc_{44}} [c_{11}(c_{11} + c_{12} + 2c_{44}) - 2c_{12}^2], \\
\epsilon_{yy} &= \frac{K}{dc_{44}} [c_{11}(c_{11} + c_{12} - 2c_{44}) - 2c_{12}^2], \\
\epsilon_{zz} &= -Kc_{12}/d, \\
\end{align*}
\]

where \(c_{ij}\) are the cubic elastic constants, \(d = c_{11}(c_{11} + c_{12}) - 2c_{12}\) and where \(K < 0\) is the uniaxial stress. Obviously \(\epsilon_{xx} < 0, \epsilon_{yy} > 0\) and \(\epsilon_{zz} > 0\) which corresponds to contraction along \(\vec{a}\) and elongations along \(\vec{b}\) and \(\vec{c}\). We next investigate the orientation of the polymer chains. Following the experimental breakthrough [14], preliminary calculations of interchain energies have been performed [15]. The interaction energy is highly sensitive to the lattice constants and a plausible scenario is that \(C_{60}\)-\(C_{60}\) interchain distances impose different relative chain orientations. However the last argument is at variance with single crystal X-ray diffraction results of pressure polymerized \(C_{60}\) [16]. There the space group is \(Pmnm\), i.e., isostructural with polymerized \(K\)C\(_{60}\). However the orthorhombic cell volume of polymerized \(C_{60}\) and the distance between a corner and the center of the cell, \(1326\ \text{Å}^3\) and \(9.956\ \text{Å}\), respectively, are closer to the corresponding values in \(\text{RbC}_{60}\), \(1314.9\ \text{Å}^3\) and \(9.852\ \text{Å}\), than to those of \(\text{KC}_{60}\), \(1298\ \text{Å}^3\) and \(9.836\ \text{Å}\). We conclude that the alkalis must play a more specific role in triggering the structural difference between \(\text{KC}_{60}\) and \(\text{RbC}_{60}\). From the theory of bilinear rotation-translation (RT) coupling between molecular rotations and lattice displacements of the counterions which plays an essential role in determining the elastic properties of ionic molecular crystals [17], one finds an effective rotation-rotation (RR) interaction that competes with the direct intermolecular RR interaction. However the lattice mediated RR interaction is found to be independent of the alkali mass and hence no distinction on basis of the different masses is possible. A further distinctive property of the alkali metal ions is the dipolar electronic polarizability, with values 0.9, 1.7 and 2.5 \(\text{Å}^3\) for \(\text{K}^+\), \(\text{Rb}^+\) and \(\text{Cs}^+\), respectively [18]. We expect that also the quadrupolar polarizability is larger for the heavy alkali metal ions than for \(\text{K}^+\). In the following we will show that the electric quadrupole interaction between \(\text{C}_{60}\) chains and the alkali ions leads to an effective orientational interaction between the \(\text{C}_{60}\) chains.

We will study first the electric multipole interaction between polymer chains. Polymerization reduces the symmetry, the \(\text{C}_{60}\) chain is composed of units with \(D_{2h}\) symmetry. We have used a tight-binding model to study the electronic charge distribution on the \(\text{C}_{60}\) units in the chain [19]. The charge is mainly concentrated in the equatorial region of \(\text{C}_{60}\) in agreement with recent NMR results [20]. We find that only even \(l\) multipoles are allowed, in particular each \(\text{C}_{60}\) unit has an electric quadrupole. In the following we adopt a simple model of charge distribution. Using the labeling of C atoms of [20], we locate a charge of \(-0.15\) (units \(|e| = 1\)) on each bond C15-C16. These charges are fixed at a distance \(d = 3.52\ \text{Å}\) from the center of the \(\text{C}_{60}\) ball. Such a charge distribution is sufficient to obtain a quadrupole (see also Eq. (2)), the accompanying monopole is irrelevant. The chains are taken as rigid units with sole degree of freedom the rotation angle \(\mu\) about the axis \(\vec{a}\). Hence we can treat the three dimensional crystal in the polymer phase as a two dimensional problem in the \((\vec{b}\vec{c})\) plane. Here we consider one chain per unit cell with basis vectors \(\vec{r}_1 = (\vec{c}/2) + (\vec{b}/2)\) and \(\vec{r}_2 = (\vec{c}/2) - (\vec{b}/2)\). The chains are labeled by a two dimensional array \(\vec{n} = (n_1, n_2)\) and lattice vectors are given by \(\vec{X}(\vec{n}) = n_1\vec{r}_1 + n_2\vec{r}_2\), where \(n_1, n_2\) are integers. The Coulomb interaction between chains depends on their mutual orientation. We introduce symmetry adapted rotator functions (SAF’s) \(S_l(\vec{n}) = \sin(l\mu(\vec{n}))\). Symmetry of the chain for \(\mu \rightarrow \mu + \pi\) implies \(l = 2, 4, \ldots\). These functions are ungraded in \(\mu\) and \(S_2\) is maximum for \(\mu = 45^\circ\). Expanding the Coulomb interaction in terms of SAF’s we consider in lowest order of \(l\) the quadrupole-quadrupole interaction

\[
H_{SS} = -\frac{1}{2} \sum_{\vec{n}, \vec{n}'} (J_a(\vec{n}, \vec{n}') + J_b(\vec{n}, \vec{n}')) S_2(\vec{n}) S_2(\vec{n}') \quad (2)
\]

More involved molecular charge distributions would lead to higher \((l \geq 4)\) multipoles [13], but their interactions are negligible. We observe that nearest neighbor chains are shifted by a translation \(a/2\) along the \(\vec{a}\) axis with respect to each other. For a \(\text{C}_{60}\) molecule at \(\vec{n}\) as origin, \(J_a\) takes into account the interaction with the two molecules at \(\pm a/2\) on the neighboring four chains \(X(\vec{n}') = \pm \vec{r}_1\) and \(X(\vec{n}') = \pm \vec{r}_2\). \(J_b\) describes the interaction with one molecule on the chains \(X(\vec{n}') = \pm (\vec{r}_1 - \vec{r}_2)\). In two dimensional Fourier space we obtain

\[
H_{SS} = -\frac{1}{2} \sum_{\vec{q}} J(\vec{q}) S_2(\vec{q}) S_2(-\vec{q}) \quad (3a)
\]

\[
J(\vec{q}) = 8J_a \cos \left(\frac{q_a b}{2}\right) \cos \left(\frac{q_b c}{2}\right) + 2J_b \cos(q_y b) \quad (3b)
\]

where \(q_y\) and \(q_z\) are components along the original orthorhombic axes \(\vec{b}\) and \(\vec{c}\). With the simple model of charge distribution for the \(\text{C}_{60}\) units and the orthorhombic lattice constants of Table I, we have calculated the quadrupolar interaction energies \(J_a\) and \(J_b\) quoted in Table I. From Eq. (3a) we find that \(J(\vec{q})\) is negative and maximum in absolute value at the boundary of the
Brillouin zone (BZ). For all three compounds we have $| J(\vec{q}_Z) | > | J(\vec{q}_r) |$, where $\vec{q}_Z = (0, 2\pi/c)$ is a BZ boundary vector and $\vec{q}_r = (0, 0)$ is the BZ center. This result is independent of the strength of the molecular quadrupoles but is a consequence of the orthorhombic lattice. The dominance of $J(\vec{q}_Z)$ leads to a condensation of $S_2(\vec{q})$ for $\vec{q} = \vec{q}_Z$: $\langle S_2(\vec{q}_Z) \rangle = \eta/\sqrt{N}$. Here $\eta$ is the order parameter amplitude and $N$ is the number of chains (i.e., lattice points in the $(b\bar{c})$ plane). Condensation at $\vec{q}_Z$ implies that the chains in a same basal plane ($\bar{a}b\bar{c}$) of the orthorhombic lattice all have the same orientation, but the orientation alternates in neighboring planes at distance $c/2$. This is the “antiferrorotational” structure $Pmmn$, Fig. 1a. We find that the quadrupole interaction between C$_{60}$ chains leads to $Pmmn$ for KC$_{60}$, RbC$_{60}$, CsC$_{60}$, irrespective of the different orthorhombic lattice constants. For RbC$_{60}$ and CsC$_{60}$ the experimental result is $I2/m$ [11,12]. We now include the role of the alkali ions with their distinctive polarizability. It is known from work on the ammonium halides [2] that the indirect interaction of two NH$_4^+$ tetrahedra via the polarizable halide ions plays an essential role in determining the various crystalline phases of the ammonium halides NH$_4$X, X=Cl, Br, I [23]. However in the present problem, by symmetry the dipolar polarizability of the alkali metal ions is irrelevant and we have to resort to the quadrupolar polarizability. Since the C$_{60}$ units in a polymer chain are rigidly linked in a same orientation, the C$_{60}$ chains produce coherent electric field gradients which induce an anisotropic (quadrupolar) deformation of the electron shell of the alkali. We model the corresponding charge distribution of each alkali by a symmetric linear dumbbell centered on lines along $\bar{a}$ directions. On a same line, these induced dumbbells are parallel with their axis perpendicular to $\bar{a}$ and a same orientation angle $\nu$ with the $\bar{c}$ axis. We consider chains of alkali dumbbells, where the rigid chain aspect is not imposed by intrachain interactions but by the surrounding C$_{60}$ chains. The orientational motion of an alkali dumbbell is characterized by the SAF’s $s_2(\vec{n}) = \sin(2\nu(\vec{n}))$, where $\vec{n}$ is again a two dimensional array labeling the chains. The quadrupole-quadrupole interaction between the C$_{60}$ chains and the surrounding alkali is then given by

\[
H_{ss} = \sum_{\vec{n}} \left( \lambda_b(\vec{n}, \vec{n}') + \lambda_c(\vec{n}, \vec{n}') \right) S_2(\vec{n}) s_2(\vec{n}'),
\]  

(4)

where $\lambda_b(\vec{n}, \vec{n}')$ accounts for the two alkalis at $\pm a/2$ on the chains $X(\vec{n}') = \pm (\vec{r}_1 - \vec{r}_2)/2$ and where $\lambda_c(\vec{n}, \vec{n}')$ describe the interaction with one alkali on the chains $X(\vec{n}') = \pm (\vec{r}_1 + \vec{r}_2)/2$. In Fourier space we have

\[
H_{ss} = \sum_{\vec{q}} \lambda(\vec{q}) S_2(\vec{q}) s_2(-\vec{q}),
\]  

(5a)

and

\[
\lambda(\vec{q}) = 4\lambda_b \cos \left( \frac{q_yb}{2} \right) + 2\lambda_c \cos \left( \frac{q_cb}{2} \right).\]

(5b)

We take dumbbells with charges $q_A$ at distances $\pm d_A$ from the center. Here $A$ refers to K, Rb, or Cs. The numerical values of $d_A$ (Table I) are the average radii of valence electron $d$ shells calculated with atomic wave functions $3d_{3/2}$, $4d_{3/2}$, $5d_{3/2}$ for K+, Rb+ and Cs+ respectively. We consider $d$-shells because they can support an electric quadrupole moment. We observe that the values for Cs+ and Rb+ are close to each other but differ from K+. The alkali ions are isoelectronic with the rare gas atoms Ar, Kr, Xe. There the role of excided d states has been found to be important in the explanation of the face centered cubic structure [22]. With the same value $q_A = 0.15|e|$ for the three cases, we have calculated the interaction energies $\lambda_b$ and $\lambda_c$ quoted in Table I. The quantity $|\lambda(q)|$ is maximum for $\vec{q} = \vec{q}_r$ in contradistinction with $|J(\vec{q})|$. The intra ionic restoring forces which oppose the deformation of the electron shells of the alkalis are described by a sum of single particle energy terms

\[
H_{ss} = g_A \sum_{\vec{n}} s_2(\vec{n}) = g_A \sum_{\vec{q}} s_2(\vec{q}) s_2(-\vec{q}),
\]  

(6)

with $g_A > 0$. The self energy $g_A$ is inversely proportional to the quadrupolar electronic polarizability and hence $g_{Cs} < g_{Rb} < g_{K}$ (see Table I). These concepts are inspired from the shell model of lattice dynamics [24].

The direct interchain coupling of alkali quadrupoles is numerically found to be small and will be neglected. We consider the total interaction $H = H_{SS} + H_{ss}$. The induced alkali quadrupoles follow adiabatically the motion of the C$_{60}$ chains. For a given configuration $\{S_2(\vec{q})\}$ of the latter, we minimize $H$ with respect to $s_2(\vec{q})$ and find

\[
s_2(\vec{q}) = -\frac{1}{2} \frac{\lambda(q)}{g_A} S_2(\vec{q}),
\]  

(7)

Substituting this result for $s_2$ in $H$ we get

\[
H = \frac{1}{2} \sum_{\vec{q}} \vec{J}(\vec{q}) S_2(\vec{q}) S_2(-\vec{q}),
\]  

(8a)

where

\[
\vec{J}(\vec{q}) = J(\vec{q}) + C(\vec{q}),\]

(8b)

and

\[
C(\vec{q}) = -\frac{1}{2} \frac{\lambda^2(\vec{q})}{g_A}.
\]  

(8c)
The coupling to the alkalis leads to an effective orientational interaction $C(q)$ between $C_{60}$ chains. This interaction is attractive and since $|C(q)|$ with $\lambda(q)$ is maximum at $q = q_0$, it favors a “ferrorotational” structure (space group $I2/m$). On the other hand the direct quadrupolar interaction between $C_{60}$ chains with $|J(q_0)| > |J(q_1)|$ favors the antiferrorotational structure. The system of interacting polymer chains described by Eq. (8) chooses the lowest energy structure which is $Pmnn$ or $I2/m$ depending on whether $|J(q_0)| > |J(q_1)|$, i.e. $J_a > \lambda_b \lambda_c / g_A$ or the opposite holds respectively. Using the numerical values from Table I, we have plotted $J(q)$ for $q = (q_y = 0, q_z)$ in Fig. 2. For $K\text{C}_{60}$ with the small polarizability of the K$^+$ ion, the direct interchain interaction $J(q_0)$ dominates and leads to $Pmnn$ while for $\text{RbC}_{60}$ and even more for $\text{CsC}_{60}$ the alkali mediated interaction $C(q)$ dominates and leads to $I2/m$. A condensation of $S_{2}(q)$ at the BZ center leads, via coupling to the center of mass displacements of the alkali ions (bilinear RT-coupling) to $\epsilon_{qz}$ shear modes and hence to a deviation of the $\langle \hat{b}, \hat{c} \rangle$ angle $\alpha$ from $90^\circ$ in the $I2/m$ structure. However this RT-coupling is not the driving process of the monoclinic structure. Indeed, the deviations of $\alpha$ are very small.

In conclusion we have shown that the cycloaddition between $C_{60}$ units leads to an orthorhombic lattice structure. The concomitant symmetry reduction produces a quadrupolar electric charge distribution on the $C_{60}$ units. The interaction between $C_{60}$ chains has two competing components: a direct quadrupole-quadrupole interaction $J(q)$ and an indirect one $C(q)$. The latter is mediated by the induced quadrupoles on the polarized alkali metal ions. The direct quadrupolar interaction drives the antiferrorotational structure $Pmnn$ while the indirect one yields the ferrorotational structure $I2/m$. In $\text{RbC}_{60}$ and $\text{CsC}_{60}$ with larger electronic polarizability and quadrupolar radius $d_A$ of the alkalis, $\lambda_b \lambda_c / g_A > J_a$, the ferrorotational structure is realized while in $\text{KCl}_{60}$, $J_a > \lambda_b \lambda_c / g_A$, the antiferrorotational structure is realized. Within the present theory, the alkalis play a specific role beyond the function of lattice spacers. The study of alkali specific effects is a problem of broad interest and likely to be relevant for the understanding of superconducting fullerenes.

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