Newly observed resonance $X(4685)$: diquark-antidiquark picture

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In this work, the mass and pole residue of $X(4685)$ state with spin-parity $J^P = 1^+$ are computed by the QCD sum rule approach up to operator dimension seven based on the diquark-antidiquark configuration. For its mass we get $m_{X_{cs}} = 4607^{+36}_{-22} \text{ MeV}$ and pole residue $\lambda_{X_{cs}} = 6.19^{+0.32}_{-0.24} \times 10^{-2} \text{ GeV}^5$, which may be checked via other nonperturbative approaches as well as future experiments. As a by product, the mass of the hidden-bottom partner state of the $X(4685)$ is extracted to be both around $m_{X_{bs}} = (10604 - 10924) \text{ MeV}$ and $\lambda_{X_{bs}} = (42.2 - 53.7) \times 10^{-2} \text{ GeV}^5$, which can be searched in the $T\phi$ invariant mass distribution.

I. INTRODUCTION

As the universe cooled after the Big Bang, the building blocks of matter — the quarks and electrons of which we are all made started to form. But, we do not have complete information on dynamics of quark and gluon creating hadrons, which can be better understood by searching exotic matter beyond the traditional quark model. Many unconventional multi-quark structures have been detected a plethora charmonium-like XYZ states [12] with significant experimental improvement over the past decade. These building blocks of matter are expected to give new information on the non-perturbative aspects of QCD theory. Also in contrast to the neutral states, charged states provide evidence for tetraquark exotic states since they cannot be counted as charmonium states.

The $c\bar{c}s\bar{s}$ tetraquarks $X(4140), X(4274), X(4500)$ and $X(4700)$ were first reported by LHCb in 2016 [8] in the $J/\psi\phi$ invariant mass distribution. Very recently the LHCb collaboration has also detected two new states $Z_{cs}(4000)^+ + Z_{cs}(4220)^+$ with the quark content $c\bar{c}s\bar{s}$, and two additional states corresponding to the $c\bar{c}s\bar{s}$ called as $X(4685)$ and $X(4630)$ in three-body decay $B^+ \to J/\psi\phi K^+$ decays with the following mass and decay width values. The analysis is based on the combined proton-proton ($pp$) collision data collected using the LHCb detector in Run 1 at centre-of-mass energies $\sqrt{s}$ of 7 and 8 TeV, corresponding to a total integrated luminosity of $3fb^{-1}$, and in Run 2 at $\sqrt{s} = 13$ TeV corresponding to an integrated luminosity of $6fb^{-1}$. Among them $X(4685)$ state decaying to the $J/\psi\phi$ final state is observed with high significance. Now we’ll call this particle $X_{cs}$ for brevity:

$$m_{X_{cs}} = 4684.3 \pm 7_{-16}^{+13} \text{ MeV},$$

$$\Gamma_{X_{cs}} = 126 \pm 15_{-41}^{+37} \text{ MeV}$$

and the quantum numbers is well determined. The BESIII collaboration [9] reported the discovery of the first candidate for a charged hidden-charm tetraquark with strangeness, tentatively dubbed $Z_{cs}(3985)^-$. It is unclear whether the new $Z_{cs}(4000)^+$ tetraquark can be identified with this state. Though their masses are consistent, the width of the BESIII particle is ten times smaller. The one LHCb observed is much broader, which would make it more natural to interpret as a compact tetraquark candidate.

This paper is structured as follows. In Section II, we briefly mention about the QCD sum rules model and then determine sum rules of the heavy-light $X(4685)$ resonance in the tetraquark assumption. In Section III, the numerical analysis of sum rules are performed and the results are compared with those of the other predictions obtained in the literature. In the last section we summarize our results.

II. SUM RULES FORMALISM

Sum rules technique is a QCD based theoretical framework which incorporates nonperturbative effects universally order by order and has some advantages in exploring hadron characteristics including nonperturbative QCD, and have also been applied for the studies of the exotic hadrons. The QCD Sum rules is firstly proposed in studies [10–12] and also successfully applied to various baryonic and mesonic states [13]. According to the QCD sum rule approach, a two-point correlation function is described as

$$\Pi_{\mu\nu}(p) = i \int d^4x \ e^{ip\cdot x} \langle 0 \{ J_\mu(x), J_\nu(0) \} | 0 \rangle, \quad (1)$$

$J_\mu(x)$ is the interpolating current of the related state and $T$ is the time-ordering operator.

Correlation function can be computed in two different ways; it can be expressed with regard to hadronic degrees of freedom such as the masses, decay constants, form factors, the coupling constants and so forth sandwiching complete sets of hadronic states with the same quantum numbers into the correlation function. This side is called as “phenomenological” (or physical/hadronic) side. It can also be computed according to quark-gluon degrees of freedom in the deep Euclidean region i.e., when $q^2 \to \infty$. That side is named as “theoretical” (or OPE/QCD) side. As usual we calculate the hadronic parameters by equating the physical representation of the two point function with the Operator Product

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Expansion (OPE) using Borel transformation in terms of momentum to suppress the contributions coming from the higher states and continuum.

**Phenomenological Side:** The first stage of our computation is to obtain the phenomenological side for the considered state. In order to apply the QCD sum rules to investigate \(X(4685)\) meson, first we insert a complete set of states between the two currents in the \(T\)-product definition in Eq. (1). By placing the complete set of intermediate states and performing integral over \(x\), the phenomenological side of the correlation function can be written in the following form:

\[
\Pi_{\mu\nu}^{\text{phen}}(p) = \frac{\langle 0 | J_\mu | X_{cs}(p) \rangle \langle X_{cs}(p) | J_\nu^\dagger | 0 \rangle}{(p^2 - m_{X_{cs}}^2)} + \ldots ,
\]

where "dots" stands for the contribution of higher states and the continuum. To continue we define the pole residue as following matrix element:

\[
\langle 0 | J_\mu | X_{cs}(p) \rangle = \lambda_{X_{cs}} \varepsilon_\mu,
\]

where \(\varepsilon_\mu\) denotes the polarization vector satisfying following relation:

\[
\varepsilon_\mu \varepsilon_\nu = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_{X_{cs}}^2}.
\]

Below we present the correlation function belonging to the physical side in terms of the ground state mass and pole residue:

\[
\Pi_{\mu\nu}^{\text{phen}}(p) = \frac{\lambda_{X_{cs}}^2}{m_{X_{cs}}^2 - p^2} \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_{X_{cs}}^2} \right) + \ldots .
\]

Here, we select the first structure, i.e. including \(g_{\mu\nu}\) terms.

**Theoretical Side:** To carry out the calculations, the second stage of the evaluation is to define the theoretical side in the framework of QCD sum rules writing the two-point correlation function firstly as in Eq. (1). The possible tetraquark interpolating current with \(J^P = 1^+\) for the \(X_{cs}\) can be constructed as:

\[
J^e_{\mu} = e^{abc} e_{dec} \left( s^T C \gamma_5 c_b \right) \left( \bar{s} d \gamma_\mu C c^T \right),
\]

where \(a\) and \(b\) are color indices and \(C\) denotes the charge conjugation.

Inserting the interpolating current in the correlation function in Eq. (1) and employing Wick theorem after contractions, we obtain the theoretical (OPE) part of the correlation function expressed in terms of the light and heavy quark propagators. So after some algebra, the expression is obtained as follows:

\[
\Pi_{\mu\nu}^{\text{OPE}}(p^2) = i C C' \int d^4x \ e^{ipx} \left[ Tr \left( \gamma_\mu \tilde{S}^e_c(x) \gamma_\nu \right) \times S^{d'd}(x) \right] \left[ Tr \left( \gamma_5 \tilde{S}^{aa'}_s(x) \gamma_5 S^{b'b'}_c(x) \right) \right],
\]

here

\[
C = \varepsilon_{abc} \varepsilon_{dec}, \quad C' = \varepsilon_{a'b'c'} \varepsilon_{d'd'e'c'},
\]

and we employed short-hand notation \(\tilde{S}^e(x) = CS^{ij}\) in Eq. (7). In calculations, it is appropriate to employ the \(x\)-space definition of the light quark propagators, while using momentum-based expressions for the heavy quarks. The heavy quark propagator’s explicit form is [13]:

\[
S^{ab}_Q(x) = i \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \left\{ \frac{\delta_{ab} (k + m_Q)}{k^2 - m_Q^2} - \frac{g G^{\alpha\beta}_{ab} (k + m_Q)}{4} \frac{\sigma_{\alpha\beta}(k + m_Q)}{(k^2 - m_Q^2)^2} + \ldots \right\}.
\]

Here

\[
G^{\alpha\beta}_{ab} = G^{A\beta}_{ab} \delta_{ab}, \quad G^2 = G^{A\alpha}_{ab} G^{A\alpha}_{ab},
\]

where \(a, b = 1, 2, 3\) are the color index and \(A, B, C = 1, 2 \ldots 8\) are flavor indices, \(t^A = \lambda^A/2, \lambda^A\) are the Gell-Mann matrices and the gluon field strength tensor \(G^{\alpha\beta}_{ab} \equiv G^{A\alpha}_{ab}(0)\) is fixed at \(x = 0\). The light quark propagator is defined as:

\[
S^{ab}_q(x) = i \delta_{ab} \frac{1}{2\pi^2 x^4} - \delta_{ab} \frac{m_q}{4\pi^2 x^2} - \delta_{ab} \frac{\langle \bar{q} q \rangle}{12} - i \frac{\delta_{ab} x^2 \langle \bar{q} g q \rangle}{1152} + i \frac{\delta_{ab} x^2 \langle \bar{q} g q \rangle}{7776} \langle \bar{q} S^{aa'}_s(x) \rangle \gamma_5 S^{bb'}_c(x) \rangle,
\]

Then taking into account tensor structure of \(\Pi_{\mu\nu}^{\text{OPE}}(p)\) we can write:

\[
\Pi_{\mu\nu}^{\text{OPE}}(p^2) = \lambda_{X_{cs}}^2 \frac{p_\mu p_\nu}{m_{X_{cs}}^2} + \Pi_0^{\text{OPE}}(p^2)(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}),
\]

here \(\Pi_0^{\text{OPE}}(p^2)\) and \(\Pi_1^{\text{OPE}}(p^2)\) are invariant functions. The sum rules for the mass and pole residue of \(X_{cs}\) and its \(b\)-partner can be extracted after equating the same structures in both \(\Pi_{\mu\nu}^{\text{phen}}(p)\) and \(\Pi_{\mu\nu}^{\text{OPE}}(p)\). To continue our evaluations, we select the same structures for each one at the later stage. The invariant function \(\Pi_{\mu\nu}^{\text{OPE}}(p^2)\) corresponding to this structure can be represented as the dispersion integral

\[
\Pi_{\mu\nu}^{\text{OPE}}(p^2) = \int_{M^2}^\infty ds \frac{\rho_{\text{OPE}}(s)}{s - p^2}.
\]
where $M^2 = 4(m_c + m_s)^2$ and $\rho^{\text{OPE}}(s)$ is the two-point spectral density. It includes terms with two different contents and can be classified as

$$\rho^{\text{OPE}}(s) = \rho^{\text{Pert.}}(s) + \rho^{qq}\langle g(s) \rangle + \rho^{GG}\langle g^2 G^2 \rangle(s) + \rho^{<q\bar{q}g>}(s) + \rho^{<q\bar{q}g>^2}(s) + \rho^{<qq>^2}(s).$$

(14)

here the first term denotes perturbative contribution and the other terms denote non-perturbative contributions. The imaginary parts of the $\Pi_n$ functions lead us to the spectral densities $\rho_n(s)$, i.e., $\rho_n(s) = \frac{1}{2}Im[\Pi_n]$. Due to the lengthy expressions, only the perturbative part $\rho^{\text{Pert.}}(s)$ of the spectral function is given as:

$$\rho^{\text{Pert.}}(s) = \int_0^1 dx_1 dx_2 \frac{(\alpha s - m_c^2 \beta_1 \beta_2)^2}{3072 \alpha \pi^6 \beta^8} \times \left[35\alpha^2 \eta^3 s^2 - 2\alpha m_c \beta \left(6\eta m_c \beta (5x_1 + 4x_2) + 13\eta^2 m_c \beta \right) + 3m_c^2 \beta \left(4m_c m_s \beta^2 (2x_1 + 2x_2) \right) + \eta m_c^2 \beta^2 + 2m_s^2 (x_1^2 + 2x_1 + 1 ) \right],$$

(15)

here

$$x_1 + x_2 - 1 = \alpha,$$

$$x_2 - 1 = \beta,$$

$$x_1 - 1 = \sigma,$$

$$x_1 + x_2 = \zeta,$$

$$x_1 x_2 = \eta.$$

After applying the Borel transformation on the variable $p^2$ to both the phenomenological and OPE sides of the equality, subtracting the contribution of higher resonances and continuum states and assuming the quark-hadron duality, we find the required sum rules.

In the Borel scheme, the action of the Borelization operator is given by $B^2(p^2 \to \tau) (1/(s-p^2)) = e^{-s\tau}$ and the OPE side of the correlation function can be composed as perturbative and non-perturbative parts:

$$\Pi^{\text{OPE}}(p^2) = \Pi^{\text{Pert.}}(p^2) + \Pi^{(q\bar{q})}(p^2) + \Pi^{(qG)}(p^2) + \Pi^{<q\bar{q}G>^2}(p^2)$$

(16)

where $\text{Pert.}$ denotes the perturbative part and the upper indices $(q\bar{q})$, $(G G)$, $(q G q)$, $(\bar{q} q \bar{q})^2$ and $(\bar{q} q \bar{q} g^2 G^2)$ represent the contributions of quark, gluon and mixed condensates, respectively.

Now using these definitions, transferring the continuum contribution to the QCD part, applying Borel transformation to both parts of the sum rules and equating them, pole residue sum rule for the axial-vector meson $X_{cs}$ up to the dimension-seven condensates is written as follows:

$$\lambda_{X_{cs}}^2 e^{-m_{X_{cs}}^2/M^2} = \int_{M^2}^{s_0} ds \rho^{\text{OPE}}(s)e^{-s/M^2}$$

(17)

and then taking the derivative of Eq. $[17]$ in terms of $(-1/M^2)$ we reach the mass sum rule of $X_{cs}$:

$$m_{X_{cs}}^2 = \int_{M^2}^{s_0} dss \rho^{\text{OPE}}(s)e^{-s/M^2}.$$  

(18)

where $s_0$ is the effective threshold for the onset of higher states and $M$ is the auxiliary Borel parameter. The next step is to carry out the numerical analysis to determine the values of hadronic parameters of resonance $X_{cs}$ and also replace $c$ quark with $b$ quark to obtain the $b$-partner $X_{bs}$ of $X_{cs}$ in tetraquark picture.

Let's perform numerical analysis to complete the computation considering $X_{cs}$ and its $b$-partner in diquark-antidiquark picture.

### III. MASS AND POLE RESIDUE ANALYSIS

After getting mass and pole residue sum rules as in Eq. $[17]$ and Eq. $[18]$, we will perform the numerical analyses to calculate observable characteristics of the hadronic ground state. We first give all the input values that are relevant for our calculation in this section. For the quark masses, we employ $m_s = (93^{+11}_{-5})$ MeV, $m_b = (1.27 \pm 0.02)$ GeV and $m_b = (4.18^{+0.06}_{-0.02})$ GeV $[14]$. $m_{b}^2 = 0.8 \pm 0.2$ GeV $[15]$. $\langle q\bar{u} \rangle$ or $\langle q\bar{d} \rangle = -(0.24 \pm 0.01)^3$ GeV $^3$, $\langle 0|s|0 \rangle = m_0^2$ $[10]$ $[13]$ and $\langle q\bar{q} G^2 \rangle = 0.012$ GeV $^4$ $[17]$.

According to the idea of the sum rule the Borel mass $M$ is an unphysical parameter, which is not related with the ground state mass. Therefore we have to show that resonance mass curve should be stable with changing $M$ values. This is a criterion for the reliability of obtained sum rules. However it is not enough for us, we must provide another two norm that always need to be tested to guarantee the validity of the sum rules:

- **The first one** is the lower bound of the Borel window which is fixed by the convergence of the OPE. But, it is very hard to calculate up to high orders due to our lack of knowledge on the high-dimensional condensates, so it is not possible to define a severe convergence standard. Instead, we use the contribution of the highest dimensional term is less than 20% of all the OPE terms (see Fig. [1]):

$$\frac{\Pi^{\text{Dim7}}}{\Pi^{\text{All terms}}} < 0.2$$

- **The latter one** is the upper bound of the Borel window which is determined from the relative contribution of the pole terms to the continuum as presented in Fig. [2]. The most frequently used condition is:

$$\frac{\Pi(s_0, M^2)}{\Pi(\infty, M^2)} < 0.5.$$
Also, we obtain the best result for the continuum threshold using \((m + 0.5 \text{ GeV}) \leq \sqrt{s_0} \leq (m + 0.55 \text{ GeV})\). After analysis we get the following working regions for the \(X_{cs}\):

\[
M^2 \in [4.4 - 4.8] \text{ GeV}^2, \\
s_0 \in [26.8 - 27.5] \text{ GeV}^2.
\]

Finally we present our results in Table I and II:

| \(m_{X_{cs}} \text{ (MeV)}\) | \(m_{X_{bs}} \text{ (MeV)}\) |
|------------------------|------------------------|
| **Our results** | 4607^{+36}_{-22} | 10604 - 10924 |
| **Experiment** | 4684.3 ± 7^{+13}_{-16} | – |

### Table II: Predictions for the pole residues of \(X_{cs}\) and \(X_{bs}\) state.

| \(\lambda_{X_{cs}} \text{ (GeV}^5)\) | \(\lambda_{X_{bs}} \text{ (GeV}^5)\) |
|------------------------|------------------------|
| **Our results** | 6.19^{+0.32}_{-0.24} \times 10^{-2} | (42.2 - 53.7) \times 10^{-2} |
| **Experiment** | – | – |

Further to assure the stability region we plot the resonance mass and pole residue versus \(M^2\) in Figs. 3 and 4. In Ref. [18] \(X_{cs}\) is analysed as the first excited state of \(X(4140)\) in tetraquark scenario and obtained the mass as \(M_X = 4.70 \pm 0.12 \text{ GeV}\) and the pole residue \(\lambda_X = (1.08 \pm 0.17) \times 10^{-1} \text{ GeV}^3\) assigning the quantum numbers \(J^{PC} = 1^{++}\).
IV. SUMMARY

Hadron spectroscopy continues to be a rich area of fundamental exploration today, with results from collider experiments over the past two decades revealing the existence of multi-quark states more exotic than the familiar mesons and baryons. The emergence of exotic hadrons in experiments has provided new challenges for QCD. Testing QCD at high precision is a key to refine fundamental exploration today, with results from collider experiments revealing the existence of multi-quark states more exotic than the familiar mesons and baryons. The emergence of exotic hadrons in experiments has provided new challenges for QCD. Among them tightly bound colored diquark plays a fundamental role in hadron spectroscopy. Thanks to the remarkable achievements of LHC, a great amount of data on hadrons is accumulated. The first LHCb upgrade is currently in progress and data taking will commence at the beginning of LHC Run 3 in 2022, with a second upgrade phase planned to gather a much larger data set by 2030. New tetraquark discoveries will have triggered the debate on the multi-quark states and allowed us to complete the hadron spectrum.

Testing QCD at high precision is a key to refine our understanding of strongly interacting matter, especially to explain the nature of tetraquark binding mechanisms. In this study, we analyse the very recently discovered $X(4685)$ state and its $b$-partner using the QCD Sum rules method including operators up to dimension seven. Within the error of uncertainties, our result $m = 4607^{+36}_{-22}$ MeV falls in the experimental measurement, that is $4684.3 \pm 7^{+13}_{-16}$ MeV. Additionally we search for the $b$-partner of in the range of $10604\text{ MeV} \leq m_{X_{bs}} \leq 10924 \text{ MeV}$. For the pole residues we obtain $\lambda_{X_{bs}} = 6.19^{+0.32}_{-0.24} \times 10^{-2}$ (GeV$^5$), $\lambda_{X_{bs}} = (42.2 - 53.7) \times 10^{-2}$ (GeV$^5$) which can be used to study the electromagnetic, weak or strong decays of handled states as an important input parameter. From this result, we conclude that the assignments $J^P = 1^+$ and $c\bar{c}s\bar{s}$ for the quantum numbers and quark structure of this state works well. Moreover hypothetical resonance $X_{bs}$ can be detected in future experiments. We hope to see whether other facilities confirm the LHCb observations and provides a new horizon for our understanding of the exotic structures in quantum chromodynamics (QCD).

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