ERROR MINIMIZATION WITH GLOBAL OPTIMIZATION FOR DIFFERENCE OF CONVEX FUNCTIONS

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ABSTRACT. In this paper, a hybrid positioning method based on global optimization for difference of convex functions (D.C.) with time of arrival (TOA) and angle of arrival (AOA) measurements are proposed. Traditional maximum likelihood (ML) formulation for indoor localization is a nonconvex optimization problem. The relaxation methods can’t provide a global solution. We establish a D.C. model for TOA/AOA fusion positioning model and give a solution with a global optimization. Simulations based on TC-OFDM signal system show that the proposed method is efficient and more robust as compared to the existing ML estimation and convex relaxation.

1. Introduction. Accurate indoor positioning is an important and novel emerging technology for commercial, internet of things (IoT), public-safety and military applications. However, indoor localization is challenging because of the complex signal propagation, caused by various of obstacles such as walls, ceiling, moving person and so on. Dense multipath and non-line-of-sight (NLOS) is the main difference between indoor and outdoor localization. Global positioning system (GPS) and BeiDou system (BDS) can provide location based service (LBS) outdoor, but can’t work indoor. Recently, various techniques have been proposed for indoor positioning system such as infrared (IR) [7], ultrasound [12], radio-frequency identification (RFID) [14], WiFi [15], Bluetooth [2], sensor networks [9], ultra-wideband (UWB) [6], geomagnetic [5], vision analysis [11] and Pseudolite [13]. Each system takes advantage of a particular positioning technology or integrating some of these technologies. They make tradeoff between the performance and complexity of the IPSs.

With the increasing demand of high accuracy positioning, hybrid signal based localization is a good choice. Some nonlinear estimators including ML and nonlinear least squares (NLS) are investigated in the literature. Generally speaking, the corresponding cost functions are multi-model, so global optimization cannot be guaranteed. Convex optimization, particularly the semi-definite program (SDP) relaxation method, is a more recent positioning approach which strikes a balance between nonlinear and linear methods, namely, high accuracy and global convergence. Nonconvex optimization problem such as NLS and ML can be transformed into either a convex second-order cone program (SOCP), or a semidefinite program (SDP).

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by relaxation [1], and can be solved with exiting solutions. However, SDP and other traditional optimization methods can’t provide, in general, an approximate global solution, but only a stationary one.

The rest of this paper is organized as follows. Section 2.1 states the problem formulation and the conventional solution of the problem described in 2.2. The global optimization method for hybrid TOA/AOA measurements is discussed in Section 2.3 and its convergence and CRLB are analyzed in 3. The simulations demonstrate new method performs better and give a global solution for indoor positioning with error minimization.

2. Indoor positioning with TOA/AOA.

2.1. System model. We consider a target localization problem using hybrid TOA/AOA measurements in 2-dimensional space. There are L non-collinear sensor nodes arbitrarily deployed at surveillance region, whose positions are known as $x_i = [x_i, y_i]^T$, $(x_i \in \mathbb{R}^2), i = 1, \cdots, N$, and a target node which has unknown coordinates i.e. $u = [x, y]^T$, $(u \in \mathbb{R}^2)$. We assume that all sensor nodes’ clocks are ideally synchronized and all sensor nodes receive signal with line of sight in this paper. Each sensor node can measure the TOA and AOA of the signal transmitted from the target. Let $t_i$ and $\theta_i$ be the signal propagation delay from the target to the $i$th sensor node and angle measurement, respectively, which are modeled as:

$$
\begin{align*}
  t_i &= \frac{1}{c} \| u - x_i \| + t_0 + e_{ti} \\
  \theta_i &= \arctan \frac{y - y_i}{x - x_i} + e_{\theta i}
\end{align*}
$$

Where $c$ is the signal propagation speed, $\| \cdot \|$ denotes the Euclidean norm, $t_0$ is the local time at the sensors when the signal leaves the target, and $e_{ti}, e_{\theta i}$ is the unknown measurement error. Then the $x$ and $y$ coordinates of target node are given by

$$
\begin{align*}
  \bar{x} &= x_i + d_i \cos \theta_i = x + e_{xi} \\
  \bar{y} &= y_i + d_i \sin \theta_i = y + e_{yi}
\end{align*}
$$

Where $e_{xi}$ and $e_{yi}$ represent the error of $x$ and $y$ of the $i$th sensor, respectively, which are nonlinear function of $e_{ti}$ and $e_{\theta i}$ in 1

2.1.1. Traditional solutions. Then the positioning error of $u = [x, y]^T$ can be written as :

$$
\begin{align*}
  e_{x_i}^2 + e_{y_i}^2 &= \left( (x_i + d_i \cos \theta_i - x)^2 + (y_i + d_i \sin \theta_i - y)^2 \right)
\end{align*}
$$

Where the auxiliary variable $e_i^2 = e_{x_i}^2 + e_{y_i}^2$ is introduced. The target localization can be written as

$$
\begin{align*}
  \min_{\zeta} \sum_{i=1}^{N} q_i \\
  \text{subject to } y = A\zeta \\
  q_i = e_i^2, i = 1, 2, \cdots, N \\
  \bar{R} = u^T u
\end{align*}
$$
Where
\[
y = \begin{pmatrix}
d_1^2 + x_1^T x_1 + 2d_1 x_1 \\
d_2^2 + x_2^T x_2 + 2d_1 x_2 \\
\vdots \\
d_N^2 + x_N^T x_1 + 2d_1 x_N
\end{pmatrix}
\]  
\tag{5}

\[
A = \begin{pmatrix}
2 (x_1 + d_1 \vartheta_1)^T & -1 & 1 & 0 & \cdots & 0 \\
2 (x_2 + d_2 \vartheta_2)^T & -1 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
2 (x_N + d_N \vartheta_N)^T & -1 & 0 & 0 & \cdots & 1
\end{pmatrix}
\]  
\tag{6}

\[
\zeta = [u, \bar{R}, \bar{q}]^T
\]  
\tag{7}

With \(\bar{q} = [e_1^2, e_2^2, \cdots, e_N^2]\) and \(\zeta_i = [\cos \theta_i, \sin \theta_i]^T\). Note that the optimization problem described in 4 is still nonconvex as the constraints is nonconvex. Hence, it is still difficult to obtain the solution. ML and convex relaxation methods can’t provide, in general, an approximate global solution, but only a stationary one.

2.2. Global optimization of D.C. for error minimization. The constraint function \(y = A\zeta\) in 4 is a typical nonconvex optimality problem, where the goal function with and the inequality constraints represented by the difference of convex functions discussed in [10]. And the goal function in 4 can be rewriter as
\[
\min_u E(u) := u^T u - \Xi(t, \theta) u + \gamma(t, \theta)
\]  
\tag{8}

where \(\Xi(t, \theta) = 2 (x_i + d_i \vartheta_i)^T \gamma(t, \theta) = d_i^2 + x_i^T x_i + 2d_i x_i, \) The function \(E(u)\) in 8 is called a D.C. function, and \(u \in \{(x, y) | x, y \in \mathbb{R}^n\}\). We employ the auxiliary problem [3], and the equality \(F(u, \bar{R}) = F(u, \bar{R}) + \Lambda(u) - \Lambda(u)\) where
\[
F(u, \bar{R}) := \max_u \{u^T u - \bar{R}\} \text{ and } \Lambda(u) = \Xi(t, \theta) u
\]  
\tag{9}

then the global optimization of 4 can be given by
\[
\min_u F(u, \bar{R}), \bar{R} \in \mathbb{R}
\]  
\tag{10}

Suppose that the coordinate \(\Theta = [x_0, y_0]^T\) is a solution to 10 and \(\bar{R} := u^T u|_{\Theta}\), then for every pair \((\xi, \Phi) \in IR^n \times IR\), such that
\[
\Lambda(\xi) = \Xi(t, \theta) = \Phi
\]  
\tag{11}

The following inequality holds
\[
F(u, \bar{R}) + \Lambda(u) - \Phi \geq \Lambda'(\xi)
\]  
\tag{12}

With the subgradient \(\Phi'(\xi) = \partial(\Phi(\xi))\) at the point \(\xi\). So the global optimality condition of 10 hold
\[
0 \in \partial(F(\xi, \bar{R}) + \Lambda(\xi)) - \Phi'(\xi) + N(\xi; \mathcal{S})
\]  
\tag{13}

Where \(\partial(F(\xi, \bar{R}) + \Lambda(\xi))\) is the subdifferential of \(F(u, \bar{R}) + \Lambda(u)\) at \(\xi\), and \(N(\xi; \mathcal{S})\) is the normal cone at \(\xi\) to \(\mathcal{S}\) [8]. The global optimization of error minimization can be summarized in Table 1.
Table 1. Global optimization of error minimization

| INPUT: $t_1, \theta_1 \in IR$ |
|-----------------------------|
| For $k = 1, \ldots, N$ do |
| Find $u_k$ approximately by solving the problem |
| $\partial (F(u_k, 8) + \Lambda(u_k)) - \Lambda'(u_k) + N(u_k; S) = 0$ |
| Find $\Phi_{k+1} \in \Phi(u_k)$ by solving the problem |
| minimize $F(u_k, 8) + \Lambda(u_k) - \Phi_{k+1}$ |
| End for |
| OUTPUT: $u_{N+1}$ |

3. Convergence and CRLB of Global optimization method.

3.1. Convergence discussion. Let us discuss the convergence of the global optimization method. Recall that the function $\Lambda(u)$ is convex. So an element $u' \in IR$ is a critical point of the function $E(u)$ defined in 8, and $\partial (F(u', 8) + \Lambda(u')) \cap \partial \Lambda(u') \neq \emptyset$. We have $\nabla E(u') = 0$, then

$$
\partial (F(u', 8) + \Lambda(u')) := \cup_{u \in IR} \partial (F(u, 8) + \Lambda(u)) = \{ u' \in IR : \partial (F(u', 8) + \Lambda(u')) \neq \emptyset \}
$$

(14)

So $F(u, 8) + \Lambda(u) \in IR$ is a coercive of superior order if

$$
\lim_{\|u\| \to +\infty} \frac{F(u, 8) + \Lambda(u)}{\|u\|} = +\infty
$$

(15)

According to [15], the Fenchel conjugate $(F(u, 8) + \Lambda(u))^*$ is a finite convex function. Therefore, for any $u' \in IR, \partial (F(u, 8) + \Lambda(u))^*$ is nonempty.

3.2. Cramer-Rao lower bound. CRLB as the optimal performance indicator for unbiased estimator is widely applied in the localization and positioning system. In this section, the error bounds of the localization will be analyzed by using the joint TOA/AOA measurements based on Fisher Information Matrix (FIM). For simplicity, we assume that $e_{ti}$ and $e_{\theta i}$ in the 1 follow the Gaussian distribution with the mean zero and variance $\sigma_1^2$ and $\sigma_2^2$, respectively, and the joint TOA/AOA measurements follow the bivariate normal distribution. Thus the Probability Distribution Function (PDF) of the TOA/AOA measurements with respect to $u$ is

$$
f_u(\text{TOA/AOA}) = \prod_{i=1}^{N} \frac{1}{2\pi\sigma_1\sigma_2} \sqrt{1 - \rho_{12}^2} \exp \left( -\frac{\bar{\omega}_{12}}{2(1 - \rho_{12}^2)} \right)
$$

(16)

Where $\rho_{12}$ is the correlation coefficient of $t_i$ and $\theta_i$, $\bar{\omega}_{12} = \frac{\xi_1^2}{\sigma_1^2} + \frac{\xi_2^2}{\sigma_2^2} - 2\rho_{12}\frac{\xi_1}{\sigma_1}\frac{\xi_2}{\sigma_2}$, and based on 1, we have

$$
\begin{align*}
\xi_1 &= t_i - \frac{1}{c}\|u - x_i\| + t_0 \\
\xi_2 &= \theta_i - \arctan \frac{y - y_i}{x - x_i}
\end{align*}
$$

(17)

If $\bar{u}_i$ is calculated from the unbiased estimated of the joint TOA/AOA measurements at the $i$th location $u$. Then, the CRLB tells us that [4]

$$
\text{Var}(\bar{u}) \geq [F^{-1}(u)]
$$

(18)
Where \( I(u) \) is the Fisher Information Matrix:

\[
I(u) = -\frac{\partial^2 \ln f_u(\text{TOA}/\text{AOA})}{\partial u^2}
\]  

(19)

Where \( f_u(\text{TOA}/\text{AOA}) \) is the PDF of \( u \) with respect to \( t_i \) and \( \theta_i \). Let

\[
I(u) = \begin{pmatrix}
I_{xx}(u) & I_{xy}(u) \\
I_{yx}(u) & I_{yy}(u)
\end{pmatrix}
\]

(20)

Thus the maximum accuracy of the two dimensional localization can be given by the CLRB

\[
\text{Var}(\bar{u}) \geq \frac{I_{xx}(u) + I_{yy}(u)}{\det[I(u)]}
\]

(22)

Where \( \det[I(u)] = I_{xx}(u)I_{yy}(u) - I_{xy}(u)^2 \), and

\[
I_{xx}(u) = -\frac{1}{2(1 - \rho_{12}^2)} \left[ A_1 \frac{\partial \xi_1}{\partial x} + A_2 \frac{\partial^2 \xi_1}{\partial x^2} + A_3 \frac{\partial \xi_2}{\partial x} + A_4 \frac{\partial^2 \xi_2}{\partial x^2} \right]
\]

\[
I_{xy}(u) = -\frac{1}{2(1 - \rho_{12}^2)} \left[ B_1 \frac{\partial \xi_1}{\partial x} + B_2 \frac{\partial^2 \xi_1}{\partial x \partial y} + B_3 \frac{\partial \xi_2}{\partial x} + B_4 \frac{\partial^2 \xi_2}{\partial x \partial y} \right]
\]

\[
I_{yy}(u) = -\frac{1}{2(1 - \rho_{12}^2)} \left[ C_1 \frac{\partial \xi_1}{\partial y} + C_2 \frac{\partial^2 \xi_1}{\partial y^2} + C_3 \frac{\partial \xi_2}{\partial y} + C_4 \frac{\partial^2 \xi_2}{\partial y^2} \right]
\]

(23)

\[
I_{yx}(u) = I_{xy}(u)
\]

And

\[
A_1 = \frac{2 \xi_1}{\sigma_1^2} - \frac{2 \rho_{12} \xi_2}{\sigma_1 \sigma_2} \quad A_2 = \frac{2 \xi_1}{\sigma_1^2} - \frac{2 \rho_{12} \xi_2}{\sigma_1 \sigma_2} \\
A_3 = \frac{2 \xi_2}{\sigma_2^2} - \frac{2 \rho_{12} \xi_1}{\sigma_1 \sigma_2} \quad A_4 = \frac{2 \xi_2}{\sigma_2^2} - \frac{2 \rho_{12} \xi_1}{\sigma_1 \sigma_2} \\
B_1 = \frac{2 \xi_1}{\sigma_1^2} - \frac{2 \rho_{12} \xi_2}{\sigma_1 \sigma_2} \quad B_2 = \frac{2 \xi_1}{\sigma_1^2} - \frac{2 \rho_{12} \xi_2}{\sigma_1 \sigma_2} \\
B_3 = \frac{2 \xi_2}{\sigma_2^2} - \frac{2 \rho_{12} \xi_1}{\sigma_1 \sigma_2} \quad B_4 = \frac{2 \xi_2}{\sigma_2^2} - \frac{2 \rho_{12} \xi_1}{\sigma_1 \sigma_2}
\]

(24)

\[
C_i = B_i, \ i = 1, 2, 3, 4
\]

4. Simulation and analysis of positioning accuracy.

4.1. Simulation environment. In this section, simulations are conducted to evaluate the performance of the proposed methods. We exploit the wireless insite software based on ray tracing to generate the TOA and AOA measurements including 25 paths in the scenario shown in fig.1 where 7 sensor nodes in red circle are configured.
4.2. Positioning accuracy and CRLB analysis. We evaluate the performance of proposed method by Monte Carlo simulations and fig.2 shows the cumulative probability distribution positioning error, and we know most of the positioning error of the proposed method is below 2 meter. When positioning error is about 1 meter, the probability is 95%.

We consider the root mean square error (RMSE) versus the variances of TOA and AOA measurements represented by $\sigma_\theta$ and $\sigma_t$ respectively. From Fig.3, we can know that the proposed method outperforms the traditional ML and TOA based location and match with the optimal performance predicted by CRLB.
5. Conclusions. In this paper, Global optimization theory for D.C. to minimize the positioning is proposed and conformed efficient and more robust than convex relaxation based location method. The global condition, convergence and CRLB are induced. New method improves the indoor positioning performance by matching with CRLB. Simulation results show that the proposed methods can provide superior performance and matching with the performance predicted by CRLB. The extension of this work to sensor node selection deserves further investigation for higher accuracy and more robustness.

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