Possible new physics signals

in $b \to s\gamma$ and $b \to sl^+l^-$

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Abstract

We consider possible new physics contributions to $b \to sl^+l^-$ assuming the new physics modifies (chromo)magnetic and electric form factors in $b \to s\gamma^*$ and $b \to sg$ with the same chirality structure as in the standard model. Parametrizing the new physics effects on $b \to s\gamma^*$ and $b \to sg$ in terms of four real parameters, one finds that there are enough region of parameter space in which the measured branching ratio for $B \to X_s\gamma$ can be accomodated, and the predicted CP violation effect could be as large as $\sim 30\%$. Moreover, the branching ratio and the forward-backward asymmetry of a lepton in $B \to X_s l^+l^-$ and the tau polarization asymmetry in $B \to X_s \tau^+\tau^-$ can be deviated from the SM predictions by a factor of $\sim 2$, which can be accessible at B factories. We also discuss these observables in a specific class of supersymmetric models with gluino-mediated flavor changing neutral current (FCNC).
I. INTRODUCTION

The missions of $B-$factories under constructions are (i) to test the CP violation in the Standard Model (SM) à la Kobayashi-Maskawa scheme [1], and (ii) to find out any new flavor violation and especially new source of CP violation beyond the KM phase in the SM with three generations. The latter is well motivated by the fact that the KM phase in the SM may not be enough to generate the baryon number asymmetry in the universe. In terms of physics viewpoint, the second mission seems more exciting one, since it could uncover a veil beyond the SM and provide an ingredient that is necessary to explain baryon number asymmetry of the universe. Then, one has to seek for a possible signal of new physics in rare decays of $B-$mesons and CP violation therein. One could choose his/her own favorite models to work out the consequences of such model to the physics issues that could be investigated at B factories. Or one could work in the effective field theory framework, in a manner as much as model-independent as possible. In the following, we choose the second avenue to study the possible signals of new physics that could be studied in detail at $B$ factories. Then we give explicit examples (that satisfy our assumptions made in the model independent analysis) in supersymmetric (SUSY) models with gluino-mediated $b \rightarrow s \gamma$ transition.

If one considers the SM as an effective field theory (EFT) of more fundamental theories below the scale $\Lambda$, the new physics effects will manifest themselves in higher dimensional operators ($\text{dim } [O] \geq 5$) that are invariant under the SM gauge group. Several groups have made a list of dimension-5 and dimension-6 operators in the last decade [2]. Assuming the lepton and baryon number conservations, there are about 80 operators that are independent with each other. It would be formidable to consider all of such operators at once, even if we are interested in their effects in $B$ physics. However, if we restrict to $b \rightarrow s \gamma$, only two operators become relevant:

$$a_L \frac{v}{\Lambda^2} \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}, \quad \text{and} \quad a_R \frac{v}{\Lambda^2} \bar{s}_R \sigma_{\mu\nu} b_L F^{\mu\nu},$$

(1)

after the electroweak (EW) symmetry breaking ($v$ is the Higgs vacuum expectation value). Here $a_{L,R}$’s are dimensionless coefficients. Thus the above operators can be recasted into the following form [3]:

$$H_{\text{eff}}(b \rightarrow s\gamma) = -\frac{G_F \lambda_t}{\sqrt{2}} [C_{7L}O_{7L} + C_{7R}O_{7R}],$$

(2)

where $\lambda_t = V_{ts}^\ast V_{tb} (= -A\lambda^2$ in the Wolfenstein parametrization [4]) and

$$O_{7L} = \frac{e}{4\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}.$$ 

(3)

The operator $O_{7R}$ is obtained from $O_{7L}$ by the exchange ($L \leftrightarrow R$). Similarly one can expect a new physics contribution to $b \rightarrow s g$:

$$H_{\text{eff}}(b \rightarrow s g) = -\frac{G_F \lambda_t}{\sqrt{2}} [C_{8L}O_{8L} + C_{8R}O_{8R}],$$

(4)

\[1\] We follow the convention of Ref. [3]
where

\[ O_{8L} = \frac{g_s}{4\pi^2} m_b s^a_L \sigma^{\mu
u} T^a_{\alpha\beta} b^\beta \gamma_\mu, \quad (5) \]

and \( O_{8R} \) is obtained from \( O_{8L} \) by the exchange \((L \leftrightarrow R)\). These two processes \( b \to s\gamma \) and \( b \to sg \) are unique in the sense that they are described in terms of only two independent operators \( O_{7(8)L} \) and \( O_{7(8)R} \) whatever new physics there are. This fact makes it easy to study these decays in a model independent manner \cite{5}.

The SM predictions for the \( C_{7,8} \) at the \( M_W \) scale are (in the limit \( m_s = 0 \))

\[
C^{\text{SM}}_{7L}(M_W) \approx -0.22, \\
C^{\text{SM}}_{7R}(M_W) = 0, \\
C^{\text{SM}}_{8L}(M_W) \approx -0.12, \\
C^{\text{SM}}_{8R}(M_W) = 0. \quad (6)
\]

Note that \( C^{\text{SM}}_{7(8)R} \) in the SM is suppressed compared to \( C^{\text{SM}}_{7(8)L} \) by \( m_s/m_b \), because \( W \) boson couples only to the left-handed fermions. Such terms proportional to \( m_s \) will be neglected in our work by setting \( m_s = 0 \) whenever they appear. On the other hand, this chirality suppression needs not be true in the presence of new physics such as Left-Right symmetric (LR) model or in a certain class of supersymmetric models with specific flavor symmetries.

Such new physics contributions can be parametrized in terms of four complex parameters,

\[
C^{\text{New}}_{7L}(m_W) = C^{\text{SM}}_{7L}(m_W) \xi_7 - 1, \\
C^{\text{New}}_{8L}(m_W) = C^{\text{SM}}_{8L}(m_W) \xi_8 - 1, \\
C^{\text{New}}_{7R}(m_W) = C^{\text{SM}}_{7L}(m_W) \xi_7^R, \\
C^{\text{New}}_{8R}(m_W) = C^{\text{SM}}_{8L}(m_W) \xi_8^R, \quad (7)
\]

where \( \xi_{7,8} \) are new complex numbers, whose phases parametrize the effects of the new sources of CP violation beyond the KM phase in the SM. The SM case corresponds to \( \xi_{7,8} = 1 \) and \( \xi_{7,8}^R = 0 \). It is convenient to define the ratio \( \chi \) as following :

\[
\chi \equiv (\xi_8 - 1)/(\xi_7 - 1) = \frac{C^{\text{New}}_{8L}(M_W)/C^{\text{SM}}_{8L}(M_W)}{C^{\text{New}}_{7L}(M_W)/C^{\text{SM}}_{7L}(M_W)}. \quad (8)
\]

In many interesting cases, this parameter \( \chi \) is real \cite{3} as assumed in this work.

Implications of new physics contributions to \( b \to sg \) have been discussed by various group in conjunction with the possible solutions for the discrepancies between theoretical expectations and the data on the semileptonic branching ratio of and the missing charms in \( B \) meson decays, and the unexpectedly large branching ratio for \( B \to \eta' + X_s \). It has been advocated that \( B(B \to X_{sg}) \approx 10\% \sim 50 \times B_{\text{SM}}(B \to X_{sg}) \) can solve these problems simultaneously \cite{5}. However, this claim is now being challenged by the new measurement \( B(B \to X_{sg}) < 6.8\% \) (90\% CL) \cite{7}. In this work, we impose this new experimental data, rather than assume that the \( B(B \to X_{sg}) \) is large enough to solve the aforementioned puzzles in \( B \) decays.

In the presence of new physics contributions to \( b \to s\gamma \), there should be also generic new physics contributions to \( b \to sl^+l^- \) through electromagnetic penguin diagrams. This effect will modify the Wilson coefficient \( C_9 \) of the dim-6 local operator \( O_9 : \)

\[ O_9 \]

\[ \frac{g_s}{4\pi^2} m_b s^a_L \sigma^{\mu
u} T^a_{\alpha\beta} b^\beta \gamma_\mu. \]
\[
H_{\text{eff}}(b \to sll) \supset H_{\text{eff}}(b \to s\gamma) - \frac{G_F\lambda_t}{\sqrt{2}} \left[ C_9 O_9 + C_{10} O_{10} \right],
\]

where
\[
O_9 = \frac{e^2}{4\pi^2} (\bar{s}_L\gamma_\mu b_L) (\bar{t}\gamma^\mu t), \quad O_{10} = \frac{e^2}{4\pi^2} (\bar{s}_L\gamma_\mu b_L) (\bar{t}\gamma^\mu\gamma_5 t). \tag{10}
\]

In the SM, the Wilson coefficients \(C_{9,10}'\)'s are given by
\[
C_{9,10}^{\text{SM}}(M_W) \approx 2.01, \quad C_{9,10}^{\text{SM}}(M_W) \approx 4.55. \tag{11}
\]

Let us parametrize the new physics contribution to \(C_9\) in terms of \(\xi_9\) (or \(\chi'\)) as following:
\[
C_9^{\text{New}}(M_W) = C_9^{\text{SM}}(M_W)(\xi_9 - 1) = C_9^{\text{SM}}(M_W)\chi' (\xi_7 - 1),
\]
\[
\chi' = \frac{C_9^{\text{New}}(M_W)/C_9^{\text{SM}}(M_W)}{C_7^{\text{New}}(M_W)/C_7^{\text{SM}}(M_W)}. \tag{12}
\]

Since we assume that the new physics modifies only \(b \to s\gamma^*\) and \(b \to sg\), we have \(C_{10}(M_W) = C_{10}^{\text{SM}}(M_W)\). There is no model-independent relation between \(\xi_7\) and \(\xi_9\), although they are generated by the same Feynman diagrams for \(b \to s\gamma^*\). In Sec. IV, we will encounter examples for both \(\chi' = 0\) and \(\chi' \neq 0\) in general SUSY models with gluino-mediated flavor changing neutral current (FCNC). In principle, there are many more dim-6 local operators that might contribute to \(b \to sl^+l^-\). In the presence of so many new parameters, it is difficult to figure out which operators are induced by new physics, since we are afforded only a few physical observables, such as \(B(B \to X_s\gamma), B(B \to X_sl^+l^-), A_{FB}(B \to X_sl^+l^-)\) and the tau polarization asymmetry \(P_\tau\) in \(B \to X_s\tau^+\tau^-\). Therefore, it would be more meaningful to consider the simpler case before we take into account the most general case to figure out which operators are significantly affected by new physics.

Up to now, we considered \(O_{7,8(L,R)}, O_9\) and \(O_{10}\) relevant to \(B \to X_s\gamma, X_sg, X_sl^+l^-\), assuming new physics significantly contributes to \(b \to s\gamma^*\) and \(b \to sg\) through dim-5 operators, Eqs. (2)–(5). In doing so, five more complex numbers \((\xi_{7,8(L,R)}, \xi_9)\) have been introduced. If we further assume that the new physics does not induce new operators that are absent in the SM, we can drop \(O_{(7,8)R}\) by setting \(\xi_{7,8,R} = \xi_{8,R} = 0\), thereby reducing the number of new parameters characterizing new physics effects into three complex numbers \(\xi_{7,8,9}\)'s (or, equivalently \(\xi_7, \chi\) and \(\chi'\)). Still the number of new parameters are larger than the physical observables at our disposal. However, in many interesting cases (and especially SUSY models with gluino-mediated \(b \to s\) transition that is to be described in Sec. IV), it turns out that both \(\chi\) and \(\chi'\) are real. Therefore, we will assume that both \(\chi\) and \(\chi'\) are real hereafter, and we are end up with 4 real parameters, which we choose to be \(|\xi_7|, \Im (\xi_7), \chi\) and \(\chi'\). Then we can overconstrain these parameters from the following observables:

- the branching ratio for \(B \to X_s\gamma\) relative to the SM prediction \((R_\gamma)\)

\(^2\)The Z penguin contribution to \(b \to sl^+l^-\) is suppressed relative to the photonic penguin by a factor of \(O(M_\Delta^2/M_Z^2)\), and thus neglected in this work.
• the direct CP violation in $B \to X_s \gamma$ ($A_{CP}^{b\to s\gamma} \equiv A_{CP}$)
• the branching ratio for $B \to X_{sg}$ relative to the SM prediction ($R_g$)
• the branching ratio for $B \to X_s l^+ l^-$ relative to the SM prediction ($R_{ll}$)
• the forward-backward asymmetry in $B \to X_s l^+ l^-$ ($A_{FB}^{b\to sl\ell}$)
• the tau polarization asymmetry in $B \to X_s \tau^+ \tau^-$ ($P_{\tau}$)

At this point, it is timely to recall that there have been several works on the model-independent determination of the Wilson coefficients, $C_{7,8,9,10}$ from $R_\gamma$ and the kinematic distributions in $R_{ll}$ [9] – [11]. Our work is different from these previous works in a few aspects. First of all, we include the possibility that there is a new physics contribution to $C_7$ with a new CP violating phase ($\text{Im}(\xi_7) \neq 0$). This necessarily calls for studying the direct CP violation in $B \to X_s \gamma$ as advocated by Kagan and Neubert [5], and invalidates the most previous works on the model-independent determination of $C_{7,9,10}$’s. Secondly we include the recent experimental constraint on $R_g$, instead assuming that it can be large enough to solve the semileptonic branching ratio problem in $B$ decays. Finally, we assume that the new physics does not introduce any new operators with chiralities different from those in the SM, and simply modifies the Wilson coefficients of $O_{7,8,9}$. Thus our analysis does not consider the left-right symmetric extension of the SM.

This paper is organized as follows. In Sec. II, we give basic formulae for the relevant physical observables such as $R_\gamma$, $R_{ll}$, etc. as functions of four real parameters, $|\xi_7|$, $\text{Im}(\xi_7)$, $\chi$ and $\chi'$. In Sec. III, we present the model-independent numerical analysis for both $\chi' = 0$ and $\chi' \neq 0$ cases. We show the possible ranges of $A_{CP}$, $R_{ll}$, etc., when we impose the experimental data on $R_\gamma$, and $R_g$. In Sec. IV, we discuss explicit SUSY models with gluino-mediated FCNC that enjoy the several assumptions we make in this work. The results of this work are summarized in Sec. V.

II. RELEVANT PHYSICAL OBSERVABLES

A. $B \to X_s \gamma$ and $B \to X_{sg}$

In the SM, the branching ratios for $B \to X_s \gamma$ and $B \to X_{sg}$ are obtained including the $O(\alpha_s)$ corrections and the nonperturbative effects of $b$–quark’s Fermi motion inside the $B$ meson. Relegating the details to the recent works by Kagan and Neubert [5], we show the final expressions that will be used in the following:

$$R_\gamma \equiv \frac{B(B \to X_s \gamma)}{B_{SM}(B \to X_s \gamma)} = 1 + r_1(\chi)[\text{Re}(\xi_7) - 1] + r_2(\chi)[|\xi_7|^2 - 1], \quad (13)$$
$$R_g \equiv \frac{B(B \to X_{sg})}{B_{SM}(B \to X_{sg})} = 1 + r_3(\chi)[\text{Re}(\xi_7) - 1] + r_4(\chi)[|\xi_7|^2 - 1]. \quad (14)$$

For a real $\chi$ (and $E_{\gamma}^{\text{min}} = 1.95$ GeV for the case of $R_\gamma$), the functions $r_i$’s can be approximated by [5]
\[ r_1(\chi) \approx 0.46 + 0.020\chi - 0.0027\chi^2, \]
\[ r_2(\chi) \approx 0.11 + 0.025\chi + 0.0013\chi^2, \]
\[ r_3(\chi) \approx 0.43\chi(1-\chi) + 0.50\chi, \]
\[ r_4(\chi) \approx 0.21\chi^2. \]  

(15)

The recent CLEO data
\[ B(B \to X_s\gamma) = (3.15 \pm 0.35_{\text{stat}} \pm 0.32_{\text{syst}} \pm 0.26_{\text{model}}) \times 10^{-4}, \]  

[12]
\[ B(B \to X_s\gamma) \lesssim 6.8\% \ (90\%\text{C.L.}), \]  

(16)

and the SM predictions on these decays \((3.29 \pm 0.33 \times 10^{-4})\) imply that
\[ 0.77 < R_\gamma < 1.15 \ (68\%\text{C.L.}), \]
\[ R_g \lesssim 6.8/0.2 = 34 \ (90\%\text{C.L.}). \]  

(17)

CP violation in the inclusive \(B \to X_s\gamma\) \((E^\text{min}_\gamma = 1.85\text{ Gev})\) is characterized by CP asymmetry \(A_{CP}\)
\[ A_{CP} = \frac{1}{|C_7|^2} \left\{ 1.23 \text{ Im}[C_2C_7^*] - 9.52 \text{ Im}[C_5C_7^*] + 0.10 \text{ Im}[C_2C_8^*] \right\} \% , \]
\[ = \frac{A_1(\chi) \text{ Im}(\xi_7)}{A_2(\chi) + A_3(\chi) |\xi_7|^2 + A_4(\chi) \text{ Re}(\xi_7)} \% , \]  

(18)

where
\[ A_1(\chi) \approx 0.37 - 0.18\chi, \]
\[ A_2(\chi) \approx 0.033 - 0.0034\chi + 0.000085\chi^2, \]
\[ A_3(\chi) \approx 0.018 + 0.0025\chi + 0.000085\chi^2, \]
\[ A_4(\chi) \approx 0.049 + 0.00089\chi - 0.00017\chi^2. \]  

(19)

B. \(B \to X_d l^+ l^-\)

Now let us consider the decay \(B \to X_d l^+ l^-\), which occurs through the electroweak penguin diagrams and the box diagrams in the SM. If there is a new physics beyond the SM, there would be generically dim-6 operators with chiralities different from \(O_{9,10}\) shown above through the electroweak penguin diagrams and the box diagrams. Morozumi et al. considered effects of such new operators (10 operators) on the branching ratio and the forward-backward asymmetry \((A_{FB})\) in \(b \to s l^+ l^-\). In our opinion, it would be more meaningful to consider the effects of modified \(C_{7,8}\) on the decay \(B \to X_d l^+ l^-\), since they are generically given by dim.-5 local operators. Especially the effects of \(C_7\) is enhanced by

\[ ^3\text{In this work, the Wilson coefficients without argument represent those at the scale } \mu = m_b, \text{ whereas those at the } M_W \text{ scale are written as } C_i(M_W) \text{ explicitly.} \]
1/s factor in the low s region (see the third line of Eq. (24) below). In any rate, we assume that the new physics does not introduce new operators with chiralities different from those in the SM, so that we assume that the new physics affects the $b \to sl^+ l^-$ only through modification of the $b \to s \gamma^*$. Therefore, the Wilson coefficients $C_{7,8,9}$ may change (with a new CP-violating phase), and $C_{10}$ will not be affected at all in our case.

The differential branching ratio for $b \to sl^+ l^-$ is given by [3]

$$
\frac{dB(b \to sl^+ l^-)}{d\hat{s}} = B(b \to ce\nu) \frac{\alpha^2}{4\pi^2} \left| \frac{V_{ts} V_{tb}}{V_{cb}} \right|^2 \frac{1}{f_{ph}(m_c/m_b) \kappa(m_c/m_b)} \omega(\hat{s}) \sqrt{1 - \frac{4m_f^2}{s}}
$$

where all the Wilson coefficients are evaluated at $\mu = m_b$ by the renormalization group equations, $\hat{s} = m_t^2/m_b^2$, the function $f_{ph}(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$ is the phase space factor for the semileptonic $b$ decays, and the function $\kappa(z)$ defined as

$$
\kappa(z) = 1 - 2 \frac{\alpha_s(m_b)}{3\pi} \left[ (\pi^2 - \frac{31}{4})(1 - z)^2 + \frac{3}{2} \right]
$$

is the QCD correction factor thereof. The effective Wilson coefficient $C_9^{\text{eff}}$ is defined as

$$
C_9^{\text{eff}} \equiv C_9 \tilde{\eta}(\hat{s}) + Y_{\text{pert}}(\hat{s})
$$

$$
= C_9 \tilde{\eta}(\hat{s}) + h(z, \hat{s}) \left( 3C_1^{(0)} + C_2^{(0)} + 3C_3^{(0)} + C_4^{(0)} + 3C_5^{(0)} + C_6^{(0)} \right)
$$

$$
- \frac{1}{2} h(1, \hat{s}) \left( 4C_3^{(0)} + 4C_4^{(0)} + 3C_5^{(0)} + C_6^{(0)} \right)
$$

$$
- \frac{1}{2} h(0, \hat{s}) \left( C_3^{(0)} + 3C_4^{(0)} \right) + \frac{2}{9} \left( 3C_3^{(0)} + C_4^{(0)} + 3C_5^{(0)} + C_6^{(0)} \right),
$$

where $C_i^{(0)}$s are the Wilson coefficients at $\mu = m_b$ in the leading logarithmic approximation:

$$
C_j^{(0)} = \sum_{i=1}^{8} k_{ji} \eta_i, \quad (j = 1, 2, \ldots, 6)
$$

with $\eta \equiv \alpha_s(M_W)/\alpha_s(\mu)$, and the numbers $\alpha_i$'s and $k_{ji}$'s are given in Table XXVII in Ref. [13]. The functions $\alpha$'s and $\omega$ are [14]

$$
\alpha_1(x, y, z) = \left( 1 + \frac{2z}{x} \right) \left[ -2x^2 + x(1 + y) + (1 - y)^2 \right],
$$

$$
\alpha_2(x, y, z) = \left[ -2x^2 + x(1 + y) + (1 - y)^2 \right] + \frac{2z}{x} \left[ 4x^2 - 5(1 + y)x + (1 - y)^2 \right]
$$

$$
\alpha_3(x, y, z) = \left( 1 + \frac{2z}{x} \right) \left[ -(1 + y)x^2 - (1 + 14y + y^2)x + 2(1 + y)(1 - y)^2 \right],
$$

$$
\alpha_4(x, y, z) = \left( 1 + \frac{2z}{x} \right) \left[ (1 - y)^2 - (1 + y)x \right],
$$

$$
\omega(\hat{s}) = \sqrt{[\hat{s} - (1 + \hat{m}_s)^2]} \left[ \hat{s} - (1 - \hat{m}_s)^2 \right],
$$

(24)
with $\tilde{m}_s \equiv m_s^2/m_b^2 \equiv 0$. And $\tilde{\eta}(\hat{s})$ is given by

\[
\tilde{\eta}(\hat{s}) = 1 + \frac{\alpha_s(\mu)}{\pi} \tilde{\omega}(\hat{s}),
\]

\[
\tilde{\omega}(\hat{s}) = -\frac{2}{9}\pi^2 - \frac{4}{3} \ln \hat{s} \ln(1 - \hat{s}) - \frac{5 + 4\hat{s}}{3(1 + 2\hat{s})} \ln(1 - \hat{s}) - \frac{2\hat{s}(1 + \hat{s})(1 - 2\hat{s})}{3(1 - \hat{s})^2(1 + 2\hat{s})} \ln \hat{s} + \frac{5 + 9\hat{s} - 6\hat{s}^2}{6(1 - \hat{s})(1 + 2\hat{s})}.
\]

(25)

The function $Y_{\text{pert}}(\hat{s})$ represents the $O(\alpha_s)$ corrections of the matrix elements, whose explicit form can be found at Ref. [3]. The new physics contributions can induce $b \to sg^* \to sq\bar{q}$. This will modifies the Wilson coefficients $C_{i=1-6}$’s, whose effects can be seen in the direct CP violation in the $B$ decay amplitude. However these will not affect $b \to s\gamma$ and $b \to sl^+l^-$ at the order we are working on. For the realistic prediction, one also has to include the long distance contribution through $b \to (J/\psi, \psi') + s$ followed by $(J/\psi, \psi') \to l^+l^-$. This can be taken into account by adding to the perturbative $Y_{\text{pert}}(\hat{s})$ the resonance contributions [15]:

\[
Y_{\text{res}}(\hat{s}) = \tilde{\kappa} \frac{3\pi}{\alpha^2} \sum_{i=J/\psi, \psi'} \frac{M_i\Gamma(i \to l^+l^-)/m_b^2}{\hat{s} - M_i^2/m_b^2 + iM_i\Gamma(i)/m_b^2},
\]

(26)

with $\tilde{\kappa} = -1$. To avoid the large contributions from the $J/\psi$ and $\psi'$ resonances, we consider the following two regions: the low $\hat{s}$ region, $1 \text{ GeV}^2 < \hat{s} < 8 \text{ GeV}^2$ for $b \to se^+e^-$ case, and the high $\hat{s}$ region, $0.6 < \hat{s} < 1$ for $b \to s\tau^+\tau^-$. Using these informations, it is straightforward to evaluate $R_{ll}$:

\[
R_{ll} \equiv \frac{B(B \to X_s l^+l^-)}{B_{\text{SM}}(B \to X_s l^+l^-)}.
\]

(27)

For the decay $B \to X_se^+e^-$,

\[
B(B \to X_se^+e^-) = B_1 + B_2|C_9|^2 + B_3\text{Re}(C_9) + B_4\text{Im}(C_9)
+ B_5|C_7|^2 + B_6\text{Re}(C_7^*C_9) + B_7\text{Re}(C_7) + B_8\text{Im}(C_7),
\]

(28)

with

\[
B_1 \approx 1.89, \quad B_2 \approx 0.07, \quad B_3 \approx 0.19, \quad B_4 \approx 0.007,
B_5 \approx 4.07, \quad B_6 \approx 0.68, \quad B_7 \approx 0.87, \quad B_8 \approx 0.034 \quad (\times 10^{-6}).
\]

(29)

For the decay $B \to X_s\tau^+\tau^-$,

\[
B(B \to X_s\tau^+\tau^-) = D_1 + D_2|C_9|^2 + D_3\text{Re}(C_9) + D_4\text{Im}(C_9)
+ D_5|C_7|^2 + D_6\text{Re}(C_7^*C_9) + D_7\text{Re}(C_7) + D_8\text{Im}(C_7),
\]

(30)

with

\[
D_1 \approx 12.6, \quad D_2 \approx 0.87, \quad D_3 \approx -1.51, \quad D_4 \approx 1.66,
D_5 \approx 6.24, \quad D_6 \approx 4.58, \quad D_7 \approx -4.21, \quad D_8 \approx 4.32 \quad (\times 10^{-8}).
\]

(31)
Another interesting observable at B factories is the forward-backward asymmetry of the lepton in the center of mass frame of the lepton pair:

\[
A_{FB} \equiv \frac{\int_0^1 d(\cos \theta) - \int_0^1 d(\cos \theta)}{\int_0^1 d(\cos \theta) + \int_0^1 d(\cos \theta)} \frac{d^2 B/d\hat{s}d\cos \theta}{d^2 B/d\hat{s}d\cos \theta} = -3 \omega(\hat{s}) \sqrt{1 - 4\hat{m}_t^2/\hat{s}} C_{10} \text{Re} \{\hat{s}[C_9 + Y(\hat{s})] + 2C_7\} / \{|C_9 + Y(\hat{s})|^2 \alpha_1 + C_{10}^2 \alpha_2 + (4/\hat{s})C_7^2 \alpha_3 + 12\alpha_4 \text{Re}C_7[C_9 + Y(\hat{s})]\},
\]

where \(\theta\) is the angle between the positively charged lepton and the \(B\) flight direction in the rest frame of the dilepton system. For the decay \(B \to X_s e^+e^-\), the integrated forward-backward asymmetry is given by

\[
A_{FB}(ee) = \frac{E_1 + E_2 \text{Re}(C_9) + E_3 \text{Re}(C_7)}{B(B \to X_s e^+e^-)},
\]

where

\[
E_1 \approx 0.21, \ E_2 \approx 0.14, \ E_3 \approx 1.69 \ (\times 10^{-6}).
\]

For the decay \(B \to X_s \tau^+\tau^-\),

\[
A_{FB}(\tau\tau) = \frac{F_1 + F_2 \text{Re}(C_9) + F_3 \text{Re}(C_7)}{B(B \to X_s \tau^+\tau^-)},
\]

where

\[
F_1 \approx -0.86, \ F_2 \approx 1.26, \ F_3 \approx 3.66 \ (\times 10^{-8}).
\]

The last observable we discuss is the tau polarization asymmetry \(P_\tau(\hat{s})\) in \(B \to X_s \tau^+\tau^-\) defined as \[16\]

\[
P_\tau(\hat{s}) \equiv \frac{\frac{dB}{d\hat{s}} \lambda = -1 - \frac{dB}{d\hat{s}} \lambda = +1}{\frac{dB}{d\hat{s}} \lambda = -1 + \frac{dB}{d\hat{s}} \lambda = +1} = \frac{-2\omega(\hat{s}) \sqrt{1 - 4\hat{m}_t^2/\hat{s}} C_{10} \text{Re}[(1 + 2\hat{s})\{C_9 + Y(\hat{s})\} + 6C_7]}{\{|C_9 + Y(\hat{s})|^2 \alpha_1 + C_{10}^2 \alpha_2 + (4/\hat{s})C_7^2 \alpha_3 + 12\alpha_4 \text{Re}C_7[C_9 + Y(\hat{s})]\}}.
\]

The integrated tau polarization asymmetry \(P_\tau\) can be expressed as

\[
P_\tau = \frac{T_1 + T_2 \text{Re}(C_9) + T_3 \text{Re}(C_7)}{B(B \to X_s \tau^+\tau^-)},
\]

where

\[
T_1 \approx -1.99, \ T_2 \approx 2.83, \ T_3 \approx 7.31 \ (\times 10^{-8}).
\]

Since \(B\) decays into the tau pair probes high \(m_{\tau\tau}(> 3.554 \text{ GeV})\) region, the observable \(P_\tau\) is sensitive to the deviation of \(C_9\) from their SM values which dominates the \(B \to X_s \tau^+\tau^-\) at high \(s\) region.
III. MODEL-INDEPENDENT ANALYSIS

Now we are ready to do a model-independent analysis using the formulae obtained in the previous section. There are two different cases depending on \( \chi' = 0 \) or not. In principle, any new physics contributing to magnetic form factor in \( b \to s\gamma \) may affect the electric form factor as well. Therefore one would expect generically \( \chi' \neq 0 \). However this needs not be necessarily true as discussed in the next section (the case (i)). So we discuss \( \chi' = 0 \) and \( \chi' \neq 0 \) separately in this section.

Our strategy is the following: impose the experimental data on \( R_\gamma \) and \( R_\tau \):

- **E1**: \( 0.77 < R_\gamma < 1.15 \) as in Ref. [2]
- **E2**: \( R_\gamma < (6.8\%/0.2\%) = 34 \)

For given \( \chi \) and \( \chi' \), these constraints (E1) and (E2) determine the allowed region in the complex \( \xi_7 \) plane. Then, in the allowed \( \xi_7 \) plane, one can calculate other physical observables, \( A_{CP}, R_{ll}, A_{FB}(b \to sll) \) and \( P_\gamma \). Because the number of observables are greater than the number of unknown parameters (one complex number \( \xi_7 \) and two real numbers \( \chi \) and \( \chi' \)), one can overconstrain these 4 real parameters. If there is no consistent solution, there would be a few possibilities: \( \chi \) and/or \( \chi' \) may be complex, \( C_{10} \) is modified by new physics effects, or one has to enlarge the operator basis by including operators with different chiralities from those in the SM, as in Ref. [5].

Let us first consider the case with \( \chi' = 0 \). In Fig. 4, we show the scattered plots of various observables as functions of \( R_\gamma \) for \( \chi = 0 \). The SM case is denoted by a square, possible values in our model are represented by dots, whereas the filled circles represent the case where there is no new CP violating phase, namely \( \text{Im} \xi_7 = 0 \), but \( \text{Re} \xi_7 \neq 0 \). Implications of these figures are clear. For example, the CP asymmetry in \( b \to s\gamma \) cannot be larger than \( \sim \pm 8\% \) if \( \chi = 0 \), and \( R_{ee} \) can be anywhere between 0.98 to 2.2. For comparison, let us discuss the minimal SUGRA model with universal soft mass terms at GUT scale, in which typical values of \( \chi \) and \( \chi' \) are \( \chi \sim 1 \) and \( \chi' \lesssim 0.05, \chi \approx 0.05 \) respectively [14]. Therefore, the predictions in the minimal SUGRA model are very close to the dots in Fig. 4. Namely, in the SUGRA case, there are two bands for the possible \( R_{ee} \) for a given \( R_\gamma \), whereas in our case, \( R_{ee} \) can be anywhere in between because of the presence of a new CP-violating phase given by Arctan(Im\( \xi_7/\text{Re} \xi_7 \)).

In Figs. 3 and 5, we show similar plots for \( \chi = 5 \) and \( \chi = -5 \), respectively. This choice of \( \chi \) covers a large class of new physics as discussed in Ref. [5]. Implications of these figures are almost the same as Fig. 1, except that there is now rather strong constraint from \( B \to X_{sg} \) (E2). In this case we can have larger direct CP violation in \( B \to X_{s\gamma} \) up to \( 10 - 30\% \). Also the (E2) constraint removes substantial parts of available \( R_{ee}, A_{FB}(ee) \) and \( R_{\tau\tau} \) as shown in Fig. 3 (\( \chi = 5 \)), compared to Fig. 4 where the constraint (E2) was not imposed. This effect is much more prominent for negative \( \chi \) as shown in Fig. 3 (\( \chi = -5 \)). For example, the \( A_{FB}(ee) - R_\gamma \) correlation is almost identical to the case with vanishing new phase \( \text{Im} \xi_7 = 0 \). From Figs. 4, 5, it is clear that the existence of a new CP violating phase not only can generate a large CP asymmetry in \( b \to s\gamma \), but can it also induce quite a lot deviations of various observables in \( b \to sl^+l^- \) for \( l = e, \mu \) and \( \tau \). For \( \chi = 0 \) and \( \chi = 5 \), deviations of the observables \( A_{CP}, R_{ee}, A_{FB}(ee), R_{\tau\tau} \) from their SM values can be large enough that they can be clearly observed at future B factories, whereas deviations of other observables \( A_{FB}(\tau\tau) \)
and $P_\tau$ from their SM values are rather small that it would be very difficult to measure them. For $\chi = -5$, only $A_{\text{CP}}$ and $A_{\text{FB}}(ee)$ shows substantial deviations from the SM values because of the (E2) constraint again. If the experimental data on $A_{\text{FB}}(\tau\tau)$ and $P_\tau$ show large deviations from their SM values, it would indicate that $\chi$ and/or $\chi'$ are complex, or some new physics contributes to $C_{10}$ (with a possibly new CP violating phase), and/or even generates $O_{7(8), R}$ and possibly other dimension-6 $bsll$ operators with different chiralities from $O_9$ in the SM.

The nonvanishing $\chi'$ does not affect $b \to s\gamma$ and $b \to sg$ so that the allowed region in the complex $\xi_7$ plane remains the same as before, for a given $\chi$. However, it does change the observables related with $B \to X_s l^+ l^-$, and we show them in Fig. 4 for $(\chi, \chi') = (5, 0.3)$, where we chose $\chi' = 0.3$ that is typical in the gluino-mediated SUSY models considered in the next section. The $R_{ee}, R_{\tau\tau}, P_\tau$ dependence on $R_\gamma$ differ from those in Fig. 4, and the possible deviations of these observables from their SM values are smaller if $\chi' = 0.3$. If there is no new CP violating phase, the differences are so tiny that one may not be able to distinguish two cases in practice.

The message of this model-independent study is that the previous methods [8]-[11] has to be enlarged to include a new observable $A_{\text{CP}}$ that could be sensitive to a new CP violating phase. In the presence of such a new phase, simple correlations among various observables in $B \to X_s l^+ l^-$, and we show them in Fig. 4 for $(\chi, \chi') = (5, 0.3)$, where we chose $\chi' = 0.3$ that is typical in the gluino-mediated SUSY models considered in the next section. The $R_{ee}, R_{\tau\tau}, P_\tau$ dependence on $R_\gamma$ differ from those in Fig. 4, and the possible deviations of these observables from their SM values are smaller if $\chi' = 0.3$. If there is no new CP violating phase, the differences are so tiny that one may not be able to distinguish two cases in practice.

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There are some models in the literature which fall into these two categories. As discussed below, the case (i) does not contribute to $C_9$ so that $\xi_9 = 1$ (or, $\chi' = 0$). On the other hand, the case (ii) contributes to $C_9$ as well as to $\xi_7$ and $\xi_8$. Also, there would be generically other contributions from $H^- - t, \chi^- - \tilde{t}$ and $\chi^0 - \tilde{d}_k$ loops. If these loop effects are competent with the gluino-mediated loop effects we consider in the following, then our assumption that both $\chi$ and $\chi'$ are real would not be true any longer. In the following, we assume that these (SUSY) electroweak loops are indeed negligible compared to the gluino mediated FCNC loop amplitudes. The latter is enhanced by $\alpha_s/(G_F m_{3\tilde{t}}^2)$, as usually assumed. However there is a suppression factor in the latter case, the mixing angle in the squark sector given by $\Gamma_{\tilde{d}GL}^d$ (or, $(\delta_{23}^d)_{LL}$ in the mass insertion approximation). Also the heavy squark-gluino loops will be suppressed compared to the charged Higgs - top, chargino - stop and neutralino - down squarks, unless all the SUSY particles have similar masses so that squark and gluinos are not too heavy. So one has to keep in mind that our assumption may break down for too small mixing angle in the squark sector or too heavy squark/gluino. With this caveat in mind, new physics contributions considered here depend on only one new phase so that $\chi$ and $\chi'$ are real, as assumed in the previous section.

In order to estimate the $\xi_{7,8,9}$ in the generalized SUSY models with gluino-mediated FCNC, we consider both the vertex mixing (VM) method and the mass insertion approximation (MIA). The latter approximation is good, when squarks are almost degenerate. The corresponding expressions can be obtained from the former expressions by taking a suitable expansion in $\Delta \tilde{m}^2 \equiv ((M^d)^2 - \tilde{m}^2)$, where $\tilde{m}$ is a suitable average mass of almost degenerate squarks. On the other hand, in the scenario in which the SUSY FCNC and SUSY CP problem are solved by decoupling of the (nearly degenerate) first two generation squarks such as in the effective SUSY models, there is a large hierarchy between the first two and the third squarks so that the MIA is no longer a good approximation. In such case, we have to resort to the VM method.

The full expressions for the Wilson coefficients $C_{7,8,9}$ due to the FCNC gluino exchange diagrams are [14]

\[ C_{7SUSY}^{SUSY}(M_W) = -\frac{8\pi \alpha_s}{9\sqrt{2} G_F m_{3\tilde{t}}^2 \lambda_t} \sum_{I=1}^{6} x_I (\Gamma_{GL}^d f_2(x_I)), \]
\[ C_{8SUSY}^{SUSY}(M_W) = -\frac{\pi \alpha_s}{\sqrt{2} G_F m_{3\tilde{t}}^2 \lambda_t} \sum_{I=1}^{6} x_I (\Gamma_{GL}^d f_2(x_I)) \times \left[ (\Gamma_{GL}^d)_{I3} f_2(x_I) + (\Gamma_{GR}^d)_{I3} \frac{m_3}{m_b} f_4(x_I) \right], \]
\[ C_{9SUSY}^{SUSY}(M_W) = \frac{16\pi \alpha_s}{9\sqrt{2} G_F m_{3\tilde{t}}^2 \lambda_t} \sum_{I=1}^{6} x_I (\Gamma_{GL}^d f_6(x_I)), \]  

(40)

---

After we submitted this paper, there appeared works which considered these effects in the most general MSSM [18], in the minimal supergravity scenario [19] and its modified versions [20].
where $x_i \equiv m_{\tilde g_i}^2 / m_{\tilde d_i}^2$. $\Gamma_{GL}^d$ and $\Gamma_{GR}^d$ determine the $\tilde g - \tilde d_i - d_j$ vertices as follows:

$$\mathcal{L} = -g_s \sqrt{2} (T^a)_{\alpha \beta} \overline{G^a} \left[ (\Gamma_{GL}^d)_{ij} P_L + (\Gamma_{GR}^d)_{ij} P_R \right] d_{ji} d^*_{i3}, \tag{41}$$

with $I, J = 1, 2, \ldots 6$ and $i, j = 1, 2, 3$. They are related with the mixing matrix elements diagonalizing the down-squark mass matrix via $d_I = (U_D)_{ij} (d_L, d_R)^T$, $U_D M_{\tilde d}^2 U_D^\dagger$ = diagonal, with the following identification:

$$(\Gamma_{GL}^d)_{ij} = (\tilde U_D)_{ij}, \quad (\Gamma_{GR}^d)_{ij} = -(\tilde U_D)_{i,j+3}. \tag{42}$$

The functions $f_i$’s are given by \cite{14}

$$f_1(x) = \frac{x^3 - 6x^2 + 3x + 2 + 6x \ln x}{12(x - 1)^4},$$

$$f_2(x) = \frac{2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x}{12(x - 1)^4},$$

$$f_3(x) = \frac{x^2 - 4x + 3 + 2 \ln x}{2(x - 1)^3},$$

$$f_4(x) = \frac{x^2 - 1 - 2x \ln x}{2(x - 1)^3},$$

$$f_6(x) = \frac{-11x^3 + 18x^2 - 9x + 2 + 6x^3 \ln x}{36(x - 1)^4}. \tag{43}$$

The corresponding expressions in the MIA is obtained from the above expressions by making a Taylor expansion around $\tilde m$ as follows: $x_i = (\tilde m^2 + \Delta m_{\tilde d_i}^2) / \tilde m^2$ and using the unitarity condition for $\tilde U_D$. This way one can recover the results in Ref. \cite{17}. For completeness, we present the resulting expressions below:

$$C_{7}^{SUSY}(M_W) = \frac{8\pi\alpha_s}{9\sqrt{2} G_F \tilde m^2 \lambda_t} \left[ (\delta_{23}^d)_{LL} M_3(x) + (\delta_{23}^d)_{LR} \frac{m_{\tilde g}}{m_b} M_1(x) \right],$$

$$C_8^{SUSY}(M_W) = \frac{\pi\alpha_s}{\sqrt{2} G_F \tilde m^2 \lambda_t} \left[ (\delta_{23}^d)_{LL} \left( \frac{1}{3} M_3(x) + 3M_4(x) \right) + (\delta_{23}^d)_{LR} \frac{m_\tilde g}{m_b} \left( \frac{1}{3} M_1(x) + 3M_2(x) \right) \right],$$

$$C_9^{SUSY}(M_W) = \frac{16\pi\alpha_s}{9\sqrt{2} G_F \tilde m^2 \lambda_t} (\delta_{23}^d)_{LL} P_1(x). \tag{44}$$

The functions $M_{1,3}(x)$ and $P_1(x)$ are defined as

$$M_1(x) = \frac{1 + 4x - 5x^2 + (4x + 2x^2) \ln x}{2(1 - x)^4},$$

$$M_2(x) = \frac{-x^2 5 - 4x - x^2 + (2 + 4x) \ln x}{2(1 - x)^4},$$

$$M_3(x) = \frac{-1 + 9x + 9x^2 - 17x^3 + (18x^2 + 6x^3) \ln x}{12(x - 1)^5},$$

$$M_4(x) = \frac{-1 - 9x + 9x^2 + x^3 - 6(x + x^2) \ln x}{6(x - 1)^5},$$

$$P_1(x) = \frac{1 - 6x + 18x^2 - 10x^3 - 3x^4 + 12x^3 \ln x}{18(x - 1)^5}. \tag{45}$$
In order to estimate the \( \xi_7, \chi \) and \( \chi' \), we assume that the (23) mixing is the same order of the corresponding CKM matrix element with an unknown new phase \( \phi \sim O(1) \). For example, \( \delta_{23} \sim |\lambda_t| \times e^{i\phi} = A X^2 e^{i\phi} \) with \( \lambda = \sin \theta_c = 0.22 \) for both cases (i) and (ii), and similarly for \( \Gamma_{GL,GR} \). Then it is clear that \( \chi \) and \( \chi' \) are real in the MIA both in the cases (i) and (ii). In case of the VM approximation, the relevant model is the effective SUSY model where only the third family squark can be lighter than \( \sim 1 \) TeV so that \( x_1, x_2 \approx 0 \) and we may keep only terms proportional to \( x_3 \) in the summation over \( I = 1 - 6 \) in Eqs. (40). Then, the \( \chi \) and \( \chi' \) are real again, as assumed in the previous section. Finally, in the following subsection, we will consider only two observables \( A_{CP} \) and \( R_{ee} \) for simplicity among several observables considered in the previous section. These two observables will be sufficient for us to find out the generic features considered in the previous section in the specific SUSY models with gluino mediated FCNC.

**B. Case (i) : (LR) insertion**

Let us first discuss the case (i) : (LR) insertion. Since the flavor changing (LR) mixing terms are not generated by SUSY breaking in the limit of vanishing Yukawa couplings, they are proportional to the corresponding Yukawa couplings. Therefore, the mass insertion approximation is always appropriate, and we consider the (LR) insertion only in the MIA. From Eqs. (44), one gets

\[
\begin{align*}
\xi_7 &= 1 + \frac{1}{C_{7L}^{SM}(m_W)} \frac{8\pi\alpha_s}{9\sqrt{2}G_F m_t^2 \lambda_t} (\delta_{23})_{LR} m_t^2 M_1(x), \\
\chi &= \frac{9}{8} C_{8L}^{SM}(M_W) \left[ \frac{1}{2} M_1(x) + 3 M_2(x) \right], \\
\chi' &= 0.
\end{align*}
\]

Note that \( \chi \) and \( \chi' \) are functions of \( x \) only, whereas \( \xi_7 \) depends on \( \tilde{m}, x \) and also on (\( \delta_{23} \))\(_{LR} \). Therefore, for a fixed \( x \) and assuming \( |\delta_{LR}| = |\lambda_t| \), one can calculate the \( A_{CP} \) as a function of \( \tilde{m} \) and \( \phi \) with the constraints (E1) and (E2). The result is that only \( x \leq 1 \) is consistent with the constraints (E1) and (E2). As \( x \) increases, the contribution to \( R_t \) and/or \( R_s \) get(s) too large.

For \( x = 0.3 \) and \( x = 0.8 \), the allowed range of \( A_{CP} \) and \( R_{ee} \) as functions of \( \phi \) are shown in Fig. 3 and Fig. 4, respectively, along with the constant \( \tilde{m} \) contours. In the present case where the MIA is appropriate, one also has to take into account the constraints on squark masses from CDF (\( \tilde{m} > 230 \) GeV for \( x = 1 \)) \cite{21} and D0 (\( \tilde{m} > 260 \) GeV for \( x = 1 \)) \cite{22}. For \( x \neq 1 \), one can read off the allowed mass range for the squark mass from the \((m_{\tilde{g}}, \tilde{m})\) exclusion plot \cite{23}. Roughly speaking, \( \tilde{m} > 300 \) GeV for \( x = 0.3 \) and \( \tilde{m} > 200 \) GeV for \( x = 3.0 \). Fig. 3 (a) \((x = 0.3)\) for which \( \chi = 1.73, \chi' = 0 \) indicates that the direct CP asymmetry \( A_{CP}^{b \to s \gamma} \) is in the range \( \lesssim 1.3\% \) for the squark mass \( \tilde{m} = 400 - 1000 \) GeV and the new CP violating phase \( \phi = 0 - 0.4\pi \). This asymmetry is probably too small to be observed. But for the same range of \( \tilde{m} \) and \( \phi \), the \( R_{ee} \) can be as large as 2.1 (see Fig. 3 (b)). So \( b \to s e^+ e^- \) is more sensitive to the (LR) mixing than the direct CP asymmetry in \( b \to s \gamma \) if \( x = 0.3 \). From Fig. 3 (a) \((x = 0.8)\) for which \( \chi = 5.47 \) and \( \chi' = 0 \), the \( A_{CP} \) is in the range \( 8 - 11\% \) for \(|\phi| = 0.2 - 0.35\pi \). It seems that there is a definite lower bound to the
\(A_{CP}\), but this is an artifact due to our choice of \(\tilde{m} < 1\) TeV. For heavier \(\tilde{m}\) it vanishes very slowly (see Fig. 6 (a) and the following paragraph). However if all the squarks (including the third family squarks) are heavier than \(O(1)\) TeV, the motivation for the low energy SUSY is lost, since the fine tuning problem is reintroduced. Therefore we think that the condition \(\tilde{m} \lesssim 1\) TeV is a reasonable requirement in the scenarios for the soft SUSY breakings where the MIA is valid. With this caveat, the predicted values for \(A_{CP}\) are within reach of the B factories. The impact on \(R_{ee}\) is less striking than the \(x = 0.3\) case, but there is still a modest enhancement up to 1.44 of \(R_{ee}\) over its SM value which may be also detectable at B factories.

One interesting feature of the \((LR)\) mixing case is that the observables we show in Figs. 5 and 6 can probe the effects of very heavy squark masses \(\tilde{m} = 400\) (710) – 1000 GeV for \(x = 0.3\) (0.8). Moreover, the heavier squarks can generate larger \(A_{CP}\), which may be in conflict with the naive expectation based on the decoupling of heavy particles in SUSY models. However, this is just an artifact of our requirement \(\tilde{m} < 1\) TeV, as described at the end of the previous paragraph. This is because we have fixed \(x\), since the heavier squark mass \(\tilde{m}\) for a fixed \(x\) implies the heavier gluino mass \(m_{\tilde{g}}\). Therefore the \(\xi_7\) decreases rather slowly as \(\tilde{m}\) increases with a fixed \(x\) because of the \(m_{\tilde{g}}\) factor in the numerator of the second term. In Fig. 7 (a) and (b), we plot the direct CP asymmetry \(A_{CP}\) and \(R_{ee}\) as functions of \(\tilde{m}\) for \(x = 0.8\). We fixed \(\phi = 0.3 \pi\) and 0.5\(\pi\). If \(\phi\) changes its sign, the direct CP asymmetry \(A_{CP}\) also changes its sign. We observe that \(A_{CP}\) is maximized around \(\tilde{m} = 1\) TeV or so. The effects of heavy squarks decouple very slowly for \(A_{CP}\) in the \((LR)\) mixing case. On the contrary, the effect on \(R_{ee}\) is larger for the lighter squark mass as usual.

C. Case (ii) : \((LL)\) insertion

Next let us consider the case (ii) : \((LL)\) insertion. In this case, the SUSY breaking terms are the main source of the flavor changing \((LL)\) mixing, which are not related with the Yukawa couplings in principle. Therefore, the MIA may not be always valid, depending on the superparticle spectra. For example, a class of models [24], [25] falls into this case where the \((LL)\) mixing dominates. These models [24] [25] predict that (23) mixing is order of \(\lambda^2\). The mass spectra of the down-squarks in the model [24] are nearly degenerate, whereas in the model [25] only the \(\tilde{t}_{L,R}, \tilde{b}_L\), gauginos and the lightest neutral Higgs are relatively light compared to \(\sim 1\) TeV. Therefore, one can use the MIA for the first models [24], whereas one has to use the vertex mixing for the second model [25].

Below, we will consider the MIA case first. In the mass insertion approximation,

\[
\begin{align*}
\xi_7 &= 1 + \frac{1}{C_{7L}^{SM}(m_W)} \frac{8\pi\alpha_s}{9\sqrt{2} G_F \tilde{m}^2 \lambda_t} (\delta_{23})_{LL} M_3(x), \\
\chi &= \frac{9}{8} \frac{C_{7L}^{SM}(M_W)}{C_{8L}^{SM}(M_W)} \left( \frac{1}{2} M_3(x) + 3 M_4(x) \right) M_3(x), \\
\chi' &= 2 P_1(x) \frac{C_{7L}^{SM}(M_W)}{M_3(x)} C_{9}^{SM}(M_W).
\end{align*}
\]

In this case we consider two different choices for \(|(\delta_{23})_{LL}|\) in order to compare our results with other existing literatures : \(|(\delta_{23})_{LL}| = |\lambda_t| [20] and \(|(\delta_{23})_{LL}| = O(1) [27]. As before,
one imposes the experimental informations on $R_\gamma$ and $R_\gamma$, and gets the allowed regions for $A_{CP}$ and $R_{ee}$ for a given phase $\phi$, as well as the direct search limit on the squark mass from CDF and D0. For $x = 0.3$, 1.0 and 3.0, the $(\chi, \chi') = (7.28, 0.19), (5.25, 0.27)$ and $(3.83, 0.40)$, respectively. Therefore, the overall features of various observables will be close to Fig. 4, except that $\xi_7$ should be fixed to some definite value.

In case $|\langle \delta_{23} \rangle_{LL}| = |\lambda_3|$, there are no visible new physics effects on $A_{CP}$ and $R_{ee}$, and the CP violating dilepton signals can be complimentary to our study. One always have

$$|A_{CP}| \lesssim 1\%, \quad |R_{ee} - 1| \lesssim 0.01 \quad (\text{for } |\langle \delta_{23} \rangle_{LL}| = |\lambda_3|)$$

(48)
after all the experimental constraints are imposed. However there may be some visible deviation in $A_{FB}(ee)$ as inferred from Fig. 4.

If $|\langle \delta_{23} \rangle_{LL}| \sim O(1)$ as assumed in Ref. [28], then one expects that $A_{CP}$ can be as large as ±10% to ±15% for $|\phi| = \pm 0.3\pi$ for $x \sim 1 - 0.3$, although the $R_{ee}$ does not change very much from its SM value (Figs. 8 and 9). If $x$ gets larger, the $A_{CP}$ gets smaller and eventually becomes undetectable (e.g., $A_{CP} \lesssim 2\%$ for $x = 3$, if we impose $m > 200$ GeV). In Ref. [27], it was noted that this new CP violating phase could result in the CP violation in the decay amplitudes for $B \to (\phi, \pi^0) + K_S$ at the level of $0.1 - 0.7$ of the SM amplitude depending on the squark mass. There would be some intrinsic theoretical uncertainties in such estimates of nonleptonic exclusive $B$ decays. On the contrary, the direct CP violation in $b \to s\gamma$ can provide independent informations on $|\langle \delta_{23} \rangle_{LL}|$ with less theoretical uncertainties, since we are dealing with the inclusive decay rate. In any rate the observable $A_{CP}$ can play an important role in probing a new CP violating phase in $B$ decays if the condition $|\langle \delta_{23} \rangle_{LL}| \sim O(1)$ is met.

The vertex mixing case can be obtained from Eq. (40) by following identifications:

$$\langle \Gamma^d_{GL} \rangle_{I,j} = \begin{cases} (V_L)_{i,j} & (\text{for } I = 1, 2, 3) \\ 0 & (\text{otherwise}), \end{cases}$$

$$\langle \Gamma^d_{GR} \rangle_{I,j} = \begin{cases} 0 & (\text{for } I = 1, 2, 3) \\ -(V_R)_{i,j} & (\text{otherwise}). \end{cases}$$

(49)

In the $(LL)$ mixing case we consider here, one has $(V_R)_{ij} = \delta_{ij}$. Also, we assume that $|(V_L)_{23}| = O(0.1)$ with a new phases of $O(1)$. This assumption is motivated by a recent model by Kaplan et al. [28], which is a SUSY model of flavor based on the single $U(1)$ generating the fermion spectra as well as communicating SUSY breaking to the visible sector. In this model, only the third generation squarks are lighter than $\sim 1$ TeV, and the 1st and the 2nd generation squarks simply decouple. Therefore, we can keep only the third family squarks ($I = 3$) in the sum over the squark mass eigenstates, since others are all heavier than $O(1)$ TeV and/or the relations (40),(49) hold. After one imposes the experimental informations on $R_\gamma$ and $R_\gamma$, one gets the allowed regions for $A_{CP}$ and $R_{ee}$ for a given phase $\phi$, as shown in Figs. 10 (a)-(c) for $x = 0.3, 1.0$ and 3.0. The corresponding values of $(\chi, \chi')$ are $(9.27, 0.16), (7.50, 0.20)$ and $(6.20, 0.26)$, respectively. In Fig. 10, we superposed the contours for three different values of $\tilde{m}_3 = 100, 200, 1000$ GeV. In the case only the third generation squarks are light, the strongest bound on the lighter stop comes from LEP experiments [29], and $\tilde{m}_3 = 100$ GeV is not excluded yet by LEP experiment. Therefore, one expects that $A_{CP}$ can be as large as 6 % to 12 % for $\phi \sim 0.7\pi$ radian for $x = 0.3, 1.0$
and 3.0 respectively, although the $R_{ee}$ does change little: $1 \lesssim R_{ee} \lesssim 1.1$. However, this large $A_{CP}$ quickly diminishes as $\tilde{m}_3$ gets heavier, and $|A_{CP}| \lesssim 2\%$ for $\tilde{m}_3 \sim 200$ GeV.

Therefore, it is very difficult to see the effects of the $(LL)$ insertion in the effective SUSY models (for which the VM method is valid) as well as the case of almost degenerate squarks (for which MIA is valid) from $R_{ee}$. In other words, the $(LL)$ insertion can generate a large direct CP violation in $b \to s\gamma$ if there is a new CP violating phase associated with the squark mass matrix, $(M^d_{LL})^2$, whereas there can be no significant change in $R_{ee}$ compared with the SM case. Also the deviation from the SM diminish very quickly as stop gets heavier. Practically speaking, it would be impossible to notice the new physics effects if $\tilde{m}_3$ with the SM case. Also the deviation from the SM diminish very quickly as stop gets heavier.

For larger $|\delta_{LL}| \sim O(1)$, direct CP violations in nonleptonic $B$ decays through $\Delta B = 1$ penguin operators can provide additional informations [24]. Again, different channels are sensitive to different types of new physics, and it will be helpful to study as many modes as possible in order to find out new physics signals at B factories.

V. CONCLUSIONS

In conclusion, we considered the possible new physics effects on the $b \to sl^+l^-$ through the modified $b \to s\gamma$ vertex. The CP violation in $b \to s\gamma$ can be very different from the SM expectation ($A_{CP}(\text{SM}) \simeq 0$), and the branching ratio and $A_{FB}$ in $b \to sl^+l^-$ can be affected by the new physics contributing to $b \to s\gamma$. In particular, the usual model-independent extraction of the Wilson coefficients $C_{7,9,10}$ may be useless in the presence of new physics that modifies the $C_{7,8}$ with a new CP-violating phase (namely, Im $(\xi_t) \neq 0$). Therefore, not only is the CP asymmetry in $b \to s\gamma$ a sensitive probe of new physics that might be discovered at $B-$factories, but also it is indispensable for the model-independent analysis of $b \to sl^+l^-$. Search for $A_{CP}$ is clearly warranted at $B-$factories.

We also considered specific models which satisfy our assumptions made in the model-independent analysis: namely, generalized SUSY models with gluino-mediated FCNC. In the case of $(LR)$ mixing, $R_{ee}$ can be enhanced compared to the SM value. Also the direct asymmetry $A_{CP}$ can be as large as $8-11\%$ for $x = 0.8$ and $\tilde{m}_1 = 1$ TeV. In this case, the direct asymmetry $A_{CP}$ is sensitive to the heavy squark masses, since the decoupling occurs very slowly, beyond $\tilde{m}_1 = 1.1$ TeV (see Fig. 7 (a)). Also there is a lower bound on $A_{CP}$ since all the squarks cannot be simultaneously heavier than $O(1)$ TeV. This is quite an interesting feature of the $(LR)$ mixing scenario. In the $(LL)$ mixing case, there is no observable effects both for $A_{CP}$ and $R_{ee}$ if $|\delta_{LL}| = |\lambda_t|$. But there can be an appreciable amount of $A_{CP}$ upto $\pm 15\%$, if $|\delta_{LL}| = O(1)$ in the MIA. In the $(LL)$ mixing with the VM approximation, one may be observable $A_{CP}$ upto $6-12\%$ depending on $x$ and the new CP violating phase $\phi$ for $|(V_L)_{23}| \sim 0.1$ which is the typical values in the model by Kaplan et al. [28].

Our study is also complimentary to other previous works, e.g., the dilepton asymmetry considered by Randall and Su [26], and the CP asymmetry in the decay amplitudes for...
nonleptonic $B$ decays considered by Ciuchini et al. [27]. It is very important to measure various kinds of CP asymmetries at B factories, especially those CP asymmetries which (almost) vanish in the SM like the direct asymmetry in $b \to s\gamma$ and dilepton asymmetry, in order to probe new CP violating phase(s) that may be necessary for us to understand the baryon number asymmetry of the universe. Different channels may be sensitive to different parameter values in new physics, and thus can provide independent informations on new physics.

While we were preparing this manuscript, we received a preprint by Chua et al. [30] considering the CP-violation in $b \to s\gamma$ in supersymmetric models. It somewhat overlaps with Sec. IV of our present work. But they did not consider the $b \to sg$ constraint, and get somewhat larger $A_{CP}$ asymmetry than our work.

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FIG. 1. Scattered plots for $A_{CP}$, $R_{ee}$, $A_{FB}(ee)$, $R_{\tau\tau}$, $A_{FB}(\tau\tau)$ and $P_{\tau}$ as functions of $R_{\gamma}$ for $\chi = \chi' = 0$. The SM values are marked as a filled square.
FIG. 2. The same as Fig. 1 for $\chi = 5$ and $\chi' = 0$. The SM values are marked as a filled square.
FIG. 3. The same as Fig. 1 for $\chi = -5$ and $\chi' = 0$. The SM values are marked as a filled square.
FIG. 4. The same as Fig. 1 for $\chi = 5$ and $\chi' = 0.3$. The SM values are marked as a filled square.
FIG. 5. Possible values of $A_{\text{CP}}$ and $R_{\text{ee}}$ as functions of the new phase $\phi$ in the $(LR)$ mixing case with $x = 0.3$ (i.e., $\chi = 1.73$ and $\chi' = 0$). We assume that $|\langle \delta_{23} \rangle_{LR}| = \lambda_t$. 
FIG. 6. The same as Fig. 5 with $x = 0.8$ (i.e., $\chi = 5.47$ and $\chi' = 0$).
FIG. 7. (a) $A_{CP}$ and (b) $R_{ee}$ as functions of the squark mass $\tilde{m}$ in the (LR) mixing case with $x = 0.8$. 
FIG. 8. Possible values of $A_{CP}$ and $R_{ee}$ as functions of the new phase $\phi$ in the $(LL)$ mixing case with $x = 0.3$ (i.e., $\chi = 7.28$ and $\chi' = 0.19$). We assume that $|\langle \delta_{23}^{d} \rangle_{LL}| = 1$. 
FIG. 9. Possible values of $A_{CP}$ and $R_{ee}$ as functions of the new phase $\phi$ in the (LL) mixing case with $x = 1$ (i.e., $\chi = 5.25$ and $\chi' = 0.27$). We assume that $|(\delta_{23}^d)_{LL}| = 1.$
FIG. 10. The $A_{CP}$ contours in the $(\tilde{m}, \phi)$ plane for (a) $x = 0.3$, (b) $x = 1$ and (c) $x = 3$ in the $(LL)$ insertion case using the vertex mixing method.