CEPHEID PERIOD-RADIUS AND PERIOD-LUMINOSITY RELATIONS AND THE DISTANCE TO THE LARGE MAGELLANIC CLOUD

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ABSTRACT

We have used the infrared Barnes-Evans surface brightness technique to derive the radii and distances of 34 Galactic Cepheid variables. Radius and distance results obtained from both versions of the technique are in excellent agreement. The radii of 28 variables are used to determine the period-radius (PR) relation. This relation is found to have a smaller dispersion than in previous studies, and is identical to the PR relation found by Laney & Stobie from a completely independent method, a fact which provides persuasive evidence that the Cepheid PR relation is now determined at a very high confidence level. We use the accurate infrared distances to determine period-luminosity (PL) relations in the $V$, $I$, $J$, $H$, and $K$ passbands from the Galactic sample of Cepheids. We derive improved slopes of these relations from updated LMC Cepheid samples and adopt these slopes to obtain accurate absolute calibrations of the PL relation. By comparing these relations to the ones defined by the LMC Cepheids, we derive strikingly consistent and precise values for the LMC distance modulus in each of the passbands that yield a mean value of $\mu_0$(LMC) = 18.46 ± 0.02.

By analyzing the observed dispersions of the PL relations defined by the LMC and Galactic samples of Cepheids, we disentangle the contributions due to uncertainties in the reddening, in distance measurement, and due to metallicity effects, and we estimate the intrinsic dispersion of the PL relation with the Wesenheit function. Assuming that the Galactic Cepheid distances are typically accurate to ±3% (as shown in a previous paper), and assuming an intrinsic spread in [Fe/H] of ~0.4 dex among the Cepheids of our sample as obtained by Fry & Carney, the observed dispersion of the Galactic Cepheid PL relation suggests a metallicity dependence of $\Delta\mu/\Delta\text{[Fe/H]} \approx 0.2$, about half the value suggested by Sasselov et al. from EROS data. Since this estimate of the metallicity dependence of the PL ($V$) relation is rather uncertain, however, we prefer to retain $\mu_0$(LMC) = 18.46 as our best value, but with an increased uncertainty of ±0.06, most of which is due to the uncertainty in the appropriate metallicity correction.

Our results show that the infrared Barnes-Evans technique is very insensitive to both Cepheid metallicity and adopted reddening, and therefore is a very powerful tool to derive accurate distances to nearby galaxies by a direct application of the technique to their Cepheid variables, rather than by comparing PL relations of different galaxies, which introduces much more sensitivity to metallicity and absorption corrections that are usually difficult to determine.

Subject headings: Cepheids — galaxies: distances and redshifts — infrared: stars — Magellanic Clouds — stars: distances — stars: fundamental parameters

1. INTRODUCTION

The determination of the radii and absolute magnitudes of Galactic Cepheid variables, and the establishment of the corresponding period-radius (PR) and period-luminosity (PL) relations has occupied researchers for several decades. While the correct measurement of Cepheid radii, and hence a correct knowledge of the PR relation obeyed by Cepheid variables, is important to determine the masses and other physical parameters of these variables, our ability to measure the distances to these stars critically determines our ability to scale the universe out to several megaparsecs and to lay the foundation for determining the Hubble constant. The PR relation may also turn out to be very useful for the determination of pulsational parallaxes of Cepheids in galaxies whose distances are too large to allow the observation of meaningful radial velocity curves of the variables, but still permit us to obtain good light curves and periods.

Significant progress in the determination of Cepheid distances and radii has recently been made by Fouqué & Gieren (1997, hereafter Paper I), who have calibrated an infrared version of the Barnes-Evans (BE) surface brightness technique using the $K$, $J-K$ magnitude-color combination, as well as a version using the $V$, $V-K$ combination, which was originally introduced by Welch (1994), with an accurate zero point of the surface brightness–color relations determined from a large set of interferometrically determined angular diameters of cool giants and supergiants that have become available over recent years. This new technique was applied to a sample of 16 Galactic open cluster Cepheids by Gieren, Fouqué, & Gómez (1997, hereafter Paper II), and it was demonstrated in this paper that Cepheid radii and dis-
tances can be determined with a $\pm 3\%$ accuracy from both versions of the technique, which yield identical results, within these small errors.

In this paper we extend our Cepheid sample to derive the PR relation, and period–absolute magnitude relations in optical ($V, I$) and infrared ($J, H, K$) passbands from a statistically more significant number of Galactic Cepheid variables. As we will show in later sections of this paper, our improved ability to measure the radii and distances of individual Cepheids from our infrared technique, combined with a careful selection of the stars adopted for this study, leads to relations of lower dispersion than those obtained in any previous work.

Since our Galactic Cepheid sample is not large enough to derive the slopes of the PL relations in the different passbands with the highest possible accuracy, we adopt the approach of determining the slopes from the LMC Cepheids, and use our Galactic Cepheid sample to set the zero points of the relations. These relations represent our best absolute calibrations of the Cepheid period-luminosity relations and will be derived in §5 of this paper. Comparing these Galactic PL relations to the corresponding relations defined by the LMC Cepheids, we finally proceed to derive a new distance to the LMC. We also address the open question of the metallicity dependence of the PL relation in this paper.

2. ADDITIONAL RADIUS AND DISTANCE SOLUTIONS

As in Paper II for the cluster Cepheids, we adopted the infrared $J, K$ photometry on the Carter system of Laney & Stobie (1992) for the additional Cepheids selected for this study. However, we have omitted a number of the Cepheids observed by these authors for our analysis, for one or several of the following reasons:

1. The number of $J, K$ observations is insufficient ($N < 20$).
2. The number and/or quality of available radial velocity observations is insufficient.
3. A red companion star is present (which affects the observed infrared light curves).
4. They are spectroscopic binary Cepheids with unknown orbital velocity curve (i.e., correction of the pulsational velocity curve for the orbital effect is not possible).
5. They are Cepheids with extremely small light and velocity amplitudes (for which our method produces unreliable radius and distance solutions).
6. They are double-mode Cepheids.

These selection criteria left us with the additional 18 stars listed in Table 1, and a total of 34 Cepheids, adding the 16 Cepheids studied in Paper II. While this sample is obviously smaller than the one studied by Laney & Stobie (1994, 1995), it has the advantage of being a “clean” sample, freed from stars with expected systematic or enhanced random errors in their solutions, while at the same time being still large enough to provide good statistics in the relations we are going to investigate in this paper. As in Paper II, we undertook a literature survey to determine the best available radial velocity and photometric $V$ data for the adopted Cepheids, where the database of Welch (1997) again provided valuable help. In Table 2 the sources of the adopted data are listed. For all variables, we adopted the pulsation periods given by Laney & Stobie (1992). For the color excesses of all stars we adopted the mean values given in the Fernie et al. (1997) Galactic Cepheid database, which are listed with their uncertainties in Table 3. We recall at this point that the BE technique is very insensitive to the values of the color excesses used in the analysis. For the conversion factor from radial to pulsational velocity we adopted the slightly period-dependent values resulting from the formula given by Gieren, Barnes, & Moffett (1993), as we did for the Cepheids analyzed in Paper II.

For each Cepheid variable we obtained two radius and distance solutions, using the $K, J - K$ and the $V, V - K$ versions of the method, in exactly the same way as described in Paper II. We recall that we adopt the inverse fits in all cases for the reasons stated and investigated in Paper II. The resulting radii and distances are listed in Table 1. For most Cepheids, the agreement between the two solutions is very good, but there are a few exceptions. In order to decide on the final radius and distance to adopt for a given Cepheid, we used the plots of the linear and angular diameter variations versus phase. We found that for the pure infrared $K, J - K$ solutions, the agreement of the two curves is always excellent, while for the $V, V - K$ solutions the agreement is not so good for a fraction of the Cepheids, especially for those with the longest pulsation periods in the

| Cepheid | $K, J - K$ Radius ($R_{\odot}$) | $V, V - K$ Radius ($R_{\odot}$) | $K, J - K$ Distance (pc) | $V, V - K$ Distance (pc) |
|---------|-------------------------------|-------------------------------|--------------------------|--------------------------|
| WZ Car  | 118.8 $\pm$ 1.5               | 110.0 $\pm$ 2.0               | 4087 $\pm$ 54            | 3732 $\pm$ 66            |
| I Car   | 195.2 $\pm$ 4.6               | 162.3 $\pm$ 2.9               | 614 $\pm$ 15             | 501 $\pm$ 9             |
| VW Cen  | 96.1 $\pm$ 2.4                | 96.4 $\pm$ 5.1                | 4016 $\pm$ 99            | 3994 $\pm$ 122          |
| XX Cen  | 60.8 $\pm$ 1.8                | 59.8 $\pm$ 1.0                | 1477 $\pm$ 44            | 1430 $\pm$ 23           |
| KN Cen  | 179.8 $\pm$ 5.2               | 189.4 $\pm$ 4.0               | 3821 $\pm$ 106           | 4007 $\pm$ 80           |
| UU Mus  | 65.4 $\pm$ 1.1                | 62.5 $\pm$ 1.1                | 2923 $\pm$ 51            | 2753 $\pm$ 47           |
| U Nor   | 83.7 $\pm$ 2.1                | 80.5 $\pm$ 1.9                | 1478 $\pm$ 37            | 1388 $\pm$ 31           |
| BF Oph  | 31.9 $\pm$ 1.4                | 36.3 $\pm$ 0.5                | 718 $\pm$ 32             | 804 $\pm$ 12            |
| VZ Pup  | 122.0 $\pm$ 2.3               | 128.5 $\pm$ 3.8               | 5077 $\pm$ 92            | 5267 $\pm$ 145          |
| AQ Pup  | 167.0 $\pm$ 2.9               | 195.1 $\pm$ 8.0               | 3548 $\pm$ 62            | 4075 $\pm$ 160          |
| BN Pup  | 83.6 $\pm$ 1.2                | 80.5 $\pm$ 2.1                | 3897 $\pm$ 55            | 3689 $\pm$ 95           |
| LS Pup  | 97.6 $\pm$ 1.9                | 102.8 $\pm$ 2.3               | 5480 $\pm$ 108           | 5701 $\pm$ 121          |
| GY Sgr  | 279.1 $\pm$ 9.3               | 195.7 $\pm$ 4.1               | 3871 $\pm$ 127           | 2507 $\pm$ 55           |
| RY Sco  | 103.8 $\pm$ 3.0               | 101.1 $\pm$ 1.5               | 1287 $\pm$ 37            | 1229 $\pm$ 19           |
| T Vel   | 38.3 $\pm$ 0.8                | 39.0 $\pm$ 0.4                | 1039 $\pm$ 19            | 1046 $\pm$ 13           |
| RY Vel  | 144.8 $\pm$ 3.2               | 153.8 $\pm$ 5.5               | 2630 $\pm$ 61            | 2705 $\pm$ 94           |
| CS Vel  | 46.5 $\pm$ 3.0                | ...                          | 3488 $\pm$ 231           | ...                     |
| S Vul   | 381.9 $\pm$ 17.2              | ...                          | 5575 $\pm$ 243           | ...                     |
sample. We have seen a similar effect in the Cepheids studied in Paper II, and identified as the most likely cause a slight phase mismatch between the $V$ and $K$ light curves (which were not obtained simultaneously) used in the $V \rightarrow K$ analyses due to an increasing tendency of period variability in the long-period Cepheids, a problem that does not exist in the $K$, $J \rightarrow K$ solutions because here all photometric data were obtained contemporaneously. There is also a possibility that the $V$, $V \rightarrow K$ infrared BE technique begins to work less well for the very extended, luminous stars. If we compare the radii only for those Cepheids of the sample for which we adopted the mean of both solutions, the corresponding value is again 1.00$^\pm$0.01. This very small offset between the distances was brought the ratio even closer to unity. While one might speculate that the very small offset between the distances is a result of either method, in our case the inclusion of more Cepheids in the comparison has demonstrated that both infrared BE techniques produce identical radius results, within a very small error margin.

In Figure 1 we show the same comparison for the distances. This time the corresponding ratios are 0.983$\pm$0.013 and 0.98$\pm$0.01, respectively, and, as in the case of the radii, there is clearly no evidence that this ratio varies systematically with period. In Paper II we had found a value of 0.97, so the inclusion of more Cepheids in the comparison has brought the ratio even closer to unity. While one might speculate that the very small offset between the distances derived from the two methods might be real, it is clearly within the possible systematic errors of either method discussed in Paper II, thus justifying as an optimum way.

In Figure 2 we show the same comparison for the distances. This time the corresponding ratios are 0.983$\pm$0.013 and 0.98$\pm$0.01, respectively, and, as in the case of the radii, there is clearly no evidence that this ratio varies systematically with period. In Paper II we had found a value of 0.97, so the inclusion of more Cepheids in the comparison has brought the ratio even closer to unity. While one might speculate that the very small offset between the distances derived from the two methods might be real, it is clearly within the possible systematic errors of either method discussed in Paper II, thus justifying as an optimum way.

In Figure 1 we compare the radii determined from the two different infrared BE techniques. All stars having both solutions (32 stars) are included in this figure. To calculate the mean ratio $R_{V \rightarrow K}/R_{J \rightarrow K}$ we exclude the three shortest period stars (unreliable $K$, $J \rightarrow K$ solution) and GY Sge, for which the $V$, $V \rightarrow K$ solution is obviously unsuccessful. We then find the mean ratio $R_{V \rightarrow K}/R_{J \rightarrow K}$ to be 0.998$\pm$0.013 (27 stars). If we compare the radii only for those Cepheids of the sample for which we adopted the mean of both solutions, the corresponding value is again 1.00$\pm$0.01. This very clearly demonstrates that both infrared BE techniques produce identical radius results, within a very small error and independent of pulsation period. We note, however, that there is a tendency in Figure 1 for the scatter to increase toward longer periods, which we attribute to the problem of the nonsimultaneous $V$, $K$ photometry in the $V$, $V \rightarrow K$ solutions, as discussed above.

In Figure 2 we show the same comparison for the distances. This time the corresponding ratios are 0.983$\pm$0.013 and 0.98$\pm$0.01, respectively, and, as in the case of the radii, there is clearly no evidence that this ratio varies systematically with period. In Paper II we had found a value of 0.97, so the inclusion of more Cepheids in the comparison has brought the ratio even closer to unity. While one might speculate that the very small offset between the distances derived from the two methods might be real, it is clearly within the possible systematic errors of either method discussed in Paper II, thus justifying as an optimum way.
TABLE 3
ADOPTED CEPHEID MEAN MAGNITUDES AND COLOR EXCESSES

| Cepheid     | $\langle V \rangle$ | $\langle B \rangle - \langle V \rangle$ | $\langle I \rangle$ | $\langle J \rangle$ | $\langle H \rangle$ | $\langle K \rangle$ | $E_{B-V}$ | $\sigma(E_{B-V})$ |
|-------------|---------------------|----------------------------------------|---------------------|---------------------|---------------------|---------------------|----------|------------------|
| EV Sct      | 10.131              | 1.182                                  | 8.694               | 7.666               | 7.170               | 7.028               | 0.663    | 0.016            |
| SZ Tau      | 6.530               | 0.852                                  | 5.564               | 4.831               | 4.408               | 4.311               | 0.326    | 0.013            |
| QZ Oph      | 8.866               | 0.908                                  | 7.893               | 7.137               | 6.734               | 6.622               | 0.307    | 0.021            |
| BF Oph      | 7.332               | 0.856                                  | 6.411               | 5.699               | 5.284               | 5.176               | 0.278    | 0.017            |
| T Vel       | 8.032               | 0.934                                  | 7.010               | 6.225               | 5.768               | 5.642               | 0.300    | 0.019            |
| CV Mon      | 10.306              | 1.337                                  | 8.684               | 7.402               | 6.791               | 6.576               | 0.750    | 0.019            |
| V Cen       | 6.823               | 0.872                                  | 5.810               | 5.074               | 4.628               | 4.508               | 0.282    | 0.017            |
| CS Vel      | 11.688              | 1.345                                  | ...                 | ...                 | 8.838               | 8.232               | 8.018    | 0.762            |
| BB Sgr      | 6.932               | 0.985                                  | 5.840               | 5.100               | 4.639               | 4.510               | 0.303    | 0.012            |
| U Sgr       | 6.685               | 1.091                                  | 5.455               | 4.858               | 4.091               | 3.952               | 0.434    | 0.007            |
| S Nor       | 6.426               | 0.945                                  | 5.414               | 4.729               | 4.274               | 4.161               | 0.194    | 0.008            |
| XX Cen      | 7.818               | 0.982                                  | 6.750               | 5.992               | 5.530               | 5.407               | 0.261    | 0.013            |
| V340 Nor    | 8.375               | 1.151                                  | 7.151               | 6.271               | 5.731               | 5.586               | 0.323    | 0.010            |
| UU Mus      | 9.783               | 1.147                                  | 8.489               | 7.530               | 6.990               | 6.828               | 0.458    | 0.044            |
| U Nor       | 9.229               | 1.622                                  | 7.358               | 5.930               | 5.237               | 4.990               | 0.923    | 0.040            |
| BN Pup      | 9.889               | 1.194                                  | 8.549               | 7.624               | 7.076               | 6.922               | 0.449    | 0.018            |
| LS Pup      | 10.447              | 1.230                                  | 9.064               | 8.093               | 7.517               | 7.354               | 0.481    | 0.010            |
| VW Cen      | 10.242              | 1.347                                  | 8.766               | 7.655               | 7.014               | 6.819               | 0.451    | 0.023            |
| VY Car      | 7.460               | 1.164                                  | 6.275               | 5.463               | 4.944               | 4.804               | 0.287    | 0.020            |
| RY Sco      | 8.016               | 1.480                                  | 6.300               | 4.998               | 4.365               | 4.143               | 0.696    | 0.047            |
| RZ Vel      | 7.089               | 1.129                                  | 5.852               | 4.979               | 4.460               | 4.308               | 0.320    | 0.012            |
| WZ Sgr      | 8.023               | 1.404                                  | 6.530               | 5.402               | 4.763               | 4.565               | 0.486    | 0.027            |
| WZ Car      | 9.255               | 1.149                                  | 7.946               | 7.008               | 6.456               | 6.290               | 0.379    | 0.007            |
| VZ Pup      | 9.631               | 1.158                                  | 8.280               | 7.370               | 6.828               | 6.668               | 0.461    | 0.019            |
| SW Vel      | 8.121               | 1.151                                  | 6.834               | 5.934               | 5.393               | 5.233               | 0.360    | 0.010            |
| T Mon       | 6.123               | 1.168                                  | 4.978               | 4.185               | 3.653               | 3.525               | 0.221    | 0.016            |
| RY Vel      | 8.372               | 1.367                                  | 6.841               | 5.702               | 5.122               | 4.928               | 0.573    | 0.013            |
| AQ Pup      | 8.669               | 1.337                                  | 7.119               | 6.099               | 5.481               | 5.297               | 0.565    | 0.018            |
| KN Cen      | 9.855               | 1.622                                  | 7.992               | 6.515               | 5.755               | 5.489               | 0.775    | 0.043            |
| L Car       | 3.735               | 1.260                                  | 2.593               | 1.766               | 1.211               | 1.092               | 0.163    | 0.017            |
| U Car       | 6.281               | 1.178                                  | 5.045               | 4.193               | 3.669               | 3.521               | 0.294    | 0.014            |
| SV Vul      | 7.243               | 1.465                                  | 5.746               | 4.668               | 4.077               | 3.920               | 0.504    | 0.026            |
| GY Sge      | 10.208              | 2.215                                  | ...                 | ...                 | 5.722               | 4.889               | 4.597    | 1.258            |
| S Vul       | 8.968               | 1.898                                  | ...                 | ...                 | 5.534               | 4.830               | 4.599    | 0.782            |

Fig. 1.—Ratio of the radii obtained from the $V, V-K$ version of our technique to those obtained from the pure infrared $K, J-K$ version, plotted against the pulsation period. The mean ratio is 1.00 with a very low uncertainty, and there is no dependence on period.

Fig. 2.—Same as Fig. 1, but for the distances. The mean ratio is $0.98 \pm 0.02$, and there is no dependence on period. The slightly increased scatter toward the longer periods is probably due to increased problems with the correct phase alignment between the $V$ and $K$ light curves for the longest period stars in the sample, which show an enhanced tendency for period variability.
choice the averaging of the distances, as well as the radii, from both infrared BE techniques, as long as there are no clear reasons in particular cases to exclude one of the solutions.

![Fig. 3.—Period-radius relation defined by 28 Galactic Cepheid variables. The plotted line is a least-squares fit to the data. For the sake of completeness, excluded stars at short and long periods (see text) are added with a different symbol (cross).](image)

### TABLE 4

**Adopted Infrared Radii and Distances of Galactic Cepheids**

| Cepheid   | log $P$ (days) | $R_{ad}$ ($R_\odot$) | log $R_{ad}$ | $d_{ad}$ (pc) |
|-----------|----------------|-----------------------|--------------|---------------|
| EV Sct    | 0.490          | 32.5 ± 0.5            | 1.512        | 1634 ± 25     |
| SZ Tau    | 0.4981         | 27.7 ± 0.5            | 1.442        | 415 ± 8       |
| QZ Nor    | 0.5783         | 38.5 ± 0.5            | 1.585        | 1656 ± 24     |
| BF Oph    | 0.6094         | 35.8 ± 2.0            | 1.554        | 793 ± 40      |
| T Vel     | 0.6665         | 38.9 ± 0.4            | 1.653        | 725 ± 8       |
| CV Mon    | 0.7307         | 40.2 ± 0.7            | 1.604        | 1514 ± 32     |
| V Cen     | 0.7399         | 45.0 ± 1.5            | 1.653        | 725 ± 8       |
| CS Vel    | 0.7712         | 46.5 ± 3.0            | 1.667        | 3488 ± 231    |
| BB Sgr    | 0.8220         | 44.4 ± 0.4            | 1.647        | 704 ± 7       |
| U Car     | 0.8290         | 48.8 ± 0.3            | 1.688        | 594 ± 4       |
| S Nor     | 0.9892         | 70.9 ± 0.9            | 1.851        | 963 ± 11      |
| Xx Cen    | 1.0395         | 60.8 ± 1.8            | 1.784        | 1477 ± 44     |
| V340 Nor  | 1.0526         | 79.7 ± 4.7            | 1.901        | 1993 ± 119    |
| UU Mus    | 1.0658         | 64.0 ± 1.4            | 1.806        | 2831 ± 120    |
| U Nor     | 1.1019         | 81.9 ± 1.6            | 1.913        | 1425 ± 44     |
| BN Pup    | 1.1358         | 82.8 ± 1.3            | 1.918        | 2845 ± 90     |
| LS Pup    | 1.1506         | 99.7 ± 2.6            | 1.999        | 5578 ± 110    |
| VW Cen    | 1.1771         | 96.2 ± 1.9            | 1.983        | 4007 ± 77     |
| VY Car    | 1.2766         | 109.4 ± 1.5           | 2.039        | 1922 ± 38     |
| RY Vel    | 1.3079         | 101.6 ± 1.3           | 2.007        | 1241 ± 24     |
| RZ Vel    | 1.3097         | 121.8 ± 2.3           | 2.086        | 1713 ± 20     |
| WZ Sgr    | 1.3394         | 122.2 ± 1.2           | 2.087        | 1788 ± 17     |
| WZ Car    | 1.3620         | 115.6 ± 6.0           | 2.063        | 3945 ± 246    |
| VZ Car    | 1.3650         | 123.7 ± 2.9           | 2.092        | 5132 ± 86     |
| SW Vel    | 1.3698         | 117.7 ± 2.1           | 2.071        | 2499 ± 65     |
| T Mon     | 1.4318         | 133.4 ± 4.1           | 2.125        | 1304 ± 40     |
| RR Vel    | 1.4489         | 144.8 ± 3.2           | 2.161        | 2630 ± 61     |
| AQ Pup    | 1.4787         | 167.0 ± 2.9           | 2.223        | 3548 ± 62     |
| KN Cen    | 1.5319         | 179.8 ± 5.2           | 2.255        | 3821 ± 106    |
| I Car     | 1.5507         | 195.2 ± 4.6           | 2.290        | 614 ± 15      |
| U Car     | 1.5889         | 167.5 ± 3.1           | 2.224        | 1636 ± 29     |
| SV Vul    | 1.6536         | 250.7 ± 8.2           | 2.399        | 2918 ± 97     |
| GY Sge    | 1.7134         | 279.1 ± 9.3           | 2.446        | 3871 ± 127    |
| S Vul     | 1.8378         | 381.9 ± 17.2          | 2.582        | 5575 ± 243    |

### 3. THE PERIOD-RADIUS RELATION

When we plot in Figure 3 the PR relation from the radius data in Table 4, we note that the three longest period Cepheids of our sample (SV Vul, GY Sge, and S Vul) lie clearly above the very tight relation defined by all the other stars. While we cannot exclude the possibility that the radii of these stars are correct, we prefer to conclude that our radius determinations overestimate the true radii in these cases and eliminate these stars in establishing the PR relation. There are several justifications for suspecting a problem with these very long period stars: first, the pronounced disagreement between the $K$, $J-K$ and the $V$, $V-K$ solutions for GY Sge, which might imply that part of the problem is also with the (adopted) $K$, $J-K$ solution, and not only with the $V$, $V-K$, solution for very luminous supergiants with very extended atmospheres; second, there is clearly a problem of a variable period for SV Vul (Bersier et al. 1994; Berdinkov 1997a) and GY Sge and S Vul (Berdinkov 1997b) which may have caused a systematic error in our solutions. Particularly in the case of GY Sge and S Vul, we were not able to find a single period that represents satisfactorily the different sets of photometric data available in the literature. Finally, the range of periods for which our infrared BE techniques are calibrated (see Paper I) is up to 40 days, and, while there is no particular reason to suspect that the calibration is different for Cepheids of longer periods, it is just these three stars that lie outside the calibration range. There is clearly less confidence in the radius and distance solutions of these Cepheids, and we therefore feel that it is wise to exclude them from the calibration of the PR relation. We therefore do not think that the curvature in the PR relation (Fig. 3) that may be suggested by these stars is real.

At the other extreme of the period spectrum, there is a possible ambiguity as to the pulsation modes of the three shortest period stars in our sample, which are EV Sct, SZ Tau, and QZ Nor. Evidence for all of these Cepheids to be first-overtone pulsators has been brought forward repeatedly in the literature, and is supported by the near-sinusoidal shapes of their light curves, but an uncertainty with regard to the mode identification remains, and we therefore choose to omit these stars as well from the discussion of the PR relation.

From the remaining 28 Cepheids of our sample, we find

$$\log R = 0.750(±0.024) \log P + 1.075(±0.007)$$

as the resulting period-radius relation, with a dispersion of $\sigma = 0.036$ and a correlation coefficient of $r = 0.987$. This relation is plotted in Figure 3. It is identical to the one found by Laney & Stobie (1995, hereafter LS95) from an application of the maximum likelihood method to a somewhat larger Cepheid sample which includes the present one, which is

$$\log R = 0.751(±0.026) \log P + 1.070(±0.008),$$

with a dispersion of $\sigma = 0.051$ (we note that LS95 have used a constant $p$-factor of 1.36, which corresponds to our value for intermediate-period stars. The same choice would have made the slope of our PR relation steeper, but by a very small amount well within the errors). While many of the data of the individual Cepheids, in particular the infrared photometry, are shared by both studies, the methods of deriving the radii are completely independent, and we thus...
feel that the perfect agreement of our result with LS95 provides extremely persuasive evidence that we have now established the true PR relation obeyed by classical Cepheids in our Galaxy, with a very high degree of confidence and within the small errors of the coefficients stated above. We also note that in a direct star-to-star comparison of our radii to those derived by LS95 (both radii normalized to the same $p$-factor), the radii agree to better than 10% in all cases except one (CV Mon), and in most cases to better than 5%, without any dependence on period, which reassures us that we are now able to measure very accurate radii of individual Cepheid variables using infrared photometry and both the Barnes-Evans and the maximum likelihood techniques.

Two other recent efforts to calibrate the Cepheid PR relation are the work of Ripepi et al. (1997) and that of Krockenberger, Sasselov, & Noyes (1996). While Ripepi et al. use a modified version of the CORS method (Caccin et al. 1981), which makes use of two optical color indices ($B-V$ and $V-R$), Krockenberger et al. have devised a method of the Baade-Wesselink type, which makes use of the Fourier coefficients of the observables, and have applied it again using optical photometry (on the Geneva system) in their analysis. In both studies, the resulting slopes of the PR relation are much shallower than our result, close to 0.60. We suspect that in both studies the problem is not with the techniques but with the use of optical photometry in the application to Cepheid variables, which is not able to provide correct estimates of the surface brightness—apparently even in the case of the CORS method, which tries to remedy the problems by the introduction of a second color index. On the other hand, it is interesting to note that the PR relation derived by Gieren, Barnes, & Moffett (1989), which is based on optical $(V-R)$ Barnes-Evans radii of 100 Galactic variables is very close to the relation found from infrared photometry. LS95 have tried to explain this as a coincidence in the selection of the variables, but this seems unlikely in view of the large sample used by Gieren et al. The results in Paper II in which we have also derived the radii from our newly calibrated optical version of the BE method shed some light on this question. They indicate that one can have large systematic errors in individual optical BE radii (up to $\sim 30\%$), but since these systematic errors can apparently have either sign, it is possible that they cancel out, to a large degree, in the determination of a mean PR relation based on a large number of stars. This possibly explains why the Gieren et al. (1989) PR relation is close to the true relation found from infrared photometry, but exhibits a much larger dispersion than the relation found in this paper, because of a strong contribution of observational scatter to the total dispersion.

The dispersion of our infrared-based Cepheid PR relation is smaller than in any previous study and is close to the dispersion expected from the finite width of the Cepheid instability strip, with the mass being the third parameter in the full period-radius-mass relationship. Given that the mass depends sensitively on the radius, approximately as $M \sim R^{2.5}$, and that individual radii of Cepheids could not be determined with an accuracy better than $\sim 10\%$, it has hitherto not been possible to derive good individual masses for Cepheid variables via the PRM relation (e.g., Gieren 1989). However, the Cepheid radii based on the new infrared BE technique are on average accurate to 3% and should therefore make it possible, for the first time, to derive Cepheid masses from the pulsational PRM relation that are accurate to better than 10%. This will result in important progress in the determination of this most fundamental stellar parameter for supergiant stars.

4. OPTICAL AND NEAR-INFRARED PERIOD-LUMINOSITY RELATIONS

We now turn to the discussion of the Cepheid period-luminosity relation in the optical $V, I$ and in the near-infrared $J, H, K$ passbands, which we will derive from our new infrared distances to our Galactic Cepheid calibrating sample. In order to convert the distances into absolute magnitudes, we have to adopt mean magnitudes and absorption corrections for the variables. For all variables in our sample, we have adopted the intensity mean magnitudes in $V, J, H, K$ as given by Laney & Stobie (1993). The intensity mean magnitudes in the $I$ band were derived from Caldwell & Coulson (1987), adding a constant $-0.03$ mag correction to their magnitude means to convert them into intensity means, a correction that was found appropriate from tests on several Cepheids of different periods and light amplitudes. All these data, together with the adopted color excesses of the stars from the Fernie et al. database and the intensity mean $B-V$ colors (again from Laney & Stobie 1993) are given in Table 3.

The absorption corrections were calculated from $A_i = R_i (E(B-V))$ using the following expressions:

$$R_V = 3.07 + 0.28(B-V) + 0.04E(B-V),$$
$$R_I = 1.82 + 0.205(B-V) + 0.022E(B-V),$$
$$R_J = 0.764,$$
$$R_H = 0.450,$$
$$R_K = 0.279.$$

The (constant) $R$-values for the infrared passbands were adopted from the work of Laney & Stobie (1993), while the expressions for the ratios of total to selective absorption in the optical $V$ and $I$ bands were adopted from Caldwell & Coulson (1987) and Laney & Stobie (1993). We then calculated the absolute magnitudes in the different passbands from the absorption-corrected intensity mean magnitudes and the true distance moduli of the stars as determined from their distances given in Table 4. The resulting absolute magnitudes in the optical $V$ and $I$ bands, together with their uncertainties due to the combined effect of distance and reddening uncertainty, are given in Table 5, while in Table 6 we give the infrared absolute magnitudes of the stars and their uncertainties. From these data, we derived period–absolute magnitude relations by least-squares fits to the same subset of 28 Cepheids discussed in the previous section, with the results for slope, zero point, and dispersion of the relations as given in Table 7. We display these relations in Figures 4, 5, 6, 7, and 8. The observed dispersion in the PL relations defined by our Galactic Cepheid sample decreases from 0.21 mag in the $V$ band to 0.17 mag in $K$, and the corresponding observed total widths of the relations decrease from about 0.7 mag in $V$ to 0.5 mag in $K$.

In order to judge the improvement in the accuracy of our measurement of Cepheid distances, it is instructive to compare the present Galactic $V$-band PL relation to the one derived from the visual surface brightness technique by Gieren, Barnes, & Moffett (1993). The total width, observed...
to be ~1.1 mag in the Gieren et al. (1993) relation, has now decreased to ~0.7 mag, and the dispersion of the relation has decreased from 0.3 to 0.2 mag. This huge improvement is almost exclusively due to the improvement in distance measurement, since there has been little change in the adopted absorption corrections.

One very important point which could not yet be properly addressed in refers to the metallicity sensitivity of our infrared method to measure Cepheid distances. From model atmosphere studies, like that of & GustafssonBell (1989), we can conclude that the effect of a changing metal-

![Fig. 4](image1.png)

**Fig. 4.** The $V$-band period-luminosity relation defined by the infrared Barnes-Evans distances of 28 Galactic Cepheids. The error bars correspond to the combined effect of distance and absorption uncertainties on the absolute magnitudes. The plotted line has the slope obtained from a sample of LMC Cepheids (see text and Table 10). Excluded stars at short and long periods (see text) are added with a different symbol (cross).

![Fig. 5](image2.png)

**Fig. 5.** Same as Fig. 4, but for the $I$ (Cousins system) passband.
There has been evidence before (e.g., that Cepheids have a genuine metallicity spread which is not homogeneous with respect to metallicity. The amount of this spread may be surprisingly large, there has been a supposed crude indicator of metallicity) correlate with the dispersion that is metallicity-related (see discussion in Giridhar 1986; the plot of metallicity versus Galactocentric distance turns out to be basically a scatter plot. Thus we do not expect that the galactocentric distances of our Cepheids (as a supposed crude indicator of metallicity) correlate with the residuals of the absolute magnitudes from the mean K-band PL relation (which are the best suited for the problem because they are almost unbiased by possible errors in the color excesses), and effectively this is not the case. It is therefore not possible to detect a metallicity dependence of our distance measuring method in this way. However, the small dispersion of the PL relation itself already tells us that, given the relatively large ~0.4 dex spread among the metallicities of the Cepheids of our sample, the distances cannot depend strongly on metallicity, because this would have to show up as an additional, significant observational scatter in the PL relation. While there probably is a contribution to the dispersion that is metallicity-related (see discussion in Giridhar 1986).

**Table 6**

| Cepheid     | log $P$ (days) | $M_J$ (mag) | $\sigma(M_J)$ (mag) | $M_H$ (mag) | $\sigma(M_H)$ (mag) | $M_K$ (mag) | $\sigma(M_K)$ (mag) |
|-------------|----------------|-------------|---------------------|-------------|---------------------|-------------|---------------------|
| EV Sct      | 0.4901         | -3.907      | 0.035               | -4.194      | 0.034               | -4.223      | 0.033               |
| SZ Tau      | 0.4981         | -3.508      | 0.043               | -3.829      | 0.042               | -3.870      | 0.042               |
| QZ Nor      | 0.5783         | -4.193      | 0.035               | -4.499      | 0.032               | -4.559      | 0.032               |
| BF Oph      | 0.6094         | -4.009      | 0.111               | -4.337      | 0.110               | -4.398      | 0.110               |
| T Vel       | 0.6665         | -4.098      | 0.027               | -4.461      | 0.025               | -4.536      | 0.024               |
| CV Mon      | 0.7307         | -4.072      | 0.048               | -4.448      | 0.047               | -4.534      | 0.046               |
| V Cen       | 0.7399         | -4.443      | 0.027               | -4.801      | 0.025               | -4.873      | 0.025               |
| CS Vel      | 0.7712         | -4.457      | 0.146               | -4.824      | 0.145               | -4.908      | 0.144               |
| BB Sgr      | 0.8220         | -4.369      | 0.024               | -4.735      | 0.023               | -4.813      | 0.022               |
| U Sgr       | 0.8290         | -4.616      | 0.016               | -4.973      | 0.015               | -5.038      | 0.015               |
| S Nor       | 0.9892         | -5.337      | 0.026               | -5.731      | 0.025               | -5.811      | 0.025               |
| XX Cen      | 1.0395         | -5.054      | 0.066               | -5.434      | 0.065               | -5.513      | 0.065               |
| V340 Nor    | 1.0526         | -5.481      | 0.130               | -5.916      | 0.130               | -6.005      | 0.130               |
| UU Mus      | 1.0658         | -5.080      | 0.098               | -5.476      | 0.094               | -5.560      | 0.093               |
| U Nor       | 1.1019         | -5.544      | 0.074               | -5.947      | 0.069               | -6.037      | 0.068               |
| BN Pup      | 1.1358         | -5.643      | 0.053               | -6.050      | 0.052               | -6.127      | 0.051               |
| LS Pup      | 1.1506         | -6.006      | 0.044               | -6.431      | 0.043               | -6.512      | 0.043               |
| VW Cen      | 1.1771         | -5.704      | 0.046               | -6.203      | 0.043               | -6.321      | 0.042               |
| VY Car      | 1.2766         | -6.175      | 0.046               | -6.604      | 0.044               | -6.695      | 0.043               |
| RY Sco      | 1.3079         | -6.003      | 0.055               | -6.417      | 0.047               | -6.520      | 0.044               |
| RZ Vel      | 1.3097         | -6.434      | 0.027               | -6.853      | 0.025               | -6.950      | 0.025               |
| WZ Sgr      | 1.3394         | -6.231      | 0.030               | -6.718      | 0.024               | -6.833      | 0.022               |
| WZ Car      | 1.3620         | -6.262      | 0.135               | -6.695      | 0.135               | -6.796      | 0.135               |
| WZ Pup      | 1.3650         | -6.533      | 0.039               | -6.930      | 0.037               | -7.012      | 0.036               |
| SW Vel      | 1.3698         | -6.330      | 0.057               | -6.758      | 0.056               | -6.856      | 0.056               |
| T Mon       | 1.4318         | -6.560      | 0.068               | -7.022      | 0.067               | -7.113      | 0.067               |
| RV Vel      | 1.4489         | -6.836      | 0.051               | -7.236      | 0.050               | -7.332      | 0.050               |
| AQ Pup      | 1.4787         | -7.083      | 0.040               | -7.523      | 0.039               | -7.611      | 0.038               |
| KN Cen      | 1.5319         | -6.988      | 0.068               | -7.505      | 0.063               | -7.638      | 0.061               |
| I Car       | 1.5507         | -7.300      | 0.055               | -7.803      | 0.054               | -7.894      | 0.053               |
| U Car       | 1.5889         | -7.101      | 0.040               | -7.532      | 0.038               | -7.630      | 0.038               |
| SV Vul      | 1.6536         | -8.042      | 0.075               | -8.475      | 0.073               | -8.546      | 0.072               |
| GY Sge      | 1.7154         | -8.178      | 0.115               | -8.616      | 0.089               | -8.693      | 0.078               |
| S Vul       | 1.8378         | -8.794      | 0.103               | -9.253      | 0.098               | -9.350      | 0.096               |

**Table 7**

| Band       | Slope          | ZP (log $P = 1$) | rms (mag) | Correlation Coefficient | N  |
|------------|----------------|-----------------|-----------|-------------------------|----|
| V          | $-3.037 \pm 0.138$ | $-4.058 \pm 0.040$ | 0.209     | 0.974                   | 28 |
| I (Cousins)| $-3.329 \pm 0.132$ | $-4.764 \pm 0.037$ | 0.194     | 0.981                   | 27 |
| J (Carter) | $-3.436 \pm 0.114$ | $-5.185 \pm 0.033$ | 0.173     | 0.986                   | 28 |
| H (Carter) | $-3.562 \pm 0.115$ | $-5.580 \pm 0.033$ | 0.175     | 0.987                   | 28 |
| K (Carter) | $-3.598 \pm 0.114$ | $-5.664 \pm 0.033$ | 0.173     | 0.987                   | 28 |
§ 5), this small effect is almost certainly due to a (slight) metallicity dependence of the Cepheid absolute magnitudes, and not an artifact of the technique we use to measure the distances. The small size of any metallicity-related effect is demonstrated by Figure 9, where we have plotted the $M_K$ residuals against the $[\text{Fe/H}]$ values for the eight stars in common with the Fry & Carney open cluster Cepheid sample; clearly there is no detectable correlation of these residuals with metallicity.

5. IMPROVED ABSOLUTE CALIBRATION OF CEPHEID PL RELATIONS AND THE DISTANCE TO THE LMC

Since our Galactic Cepheid sample is relatively small, we can obtain a more accurate determination of the slopes of the PL relations in the different passbands by using Magellanic Cloud Cepheids. This assumes, of course, that the slope of the PL relation (in all passbands) is universal, a question which still awaits an exhaustive empirical check but which has some supportive evidence to the moment (e.g., Musella, Piotto, & Capaccioli 1997; Sasselov et al. 1997) and seems to be firmly supported by theoretical expectations (e.g., Stothers 1988). While the LMC has a relatively small intrinsic depth in the line of sight and there is the possibility of correcting LMC Cepheid magnitudes for the tilt of the LMC bar (Caldwell & Laney 1991), the situation is more complicated for the SMC Cepheids, and for this reason we will derive PL relations only from the Cepheids in the Large Magellanic Cloud. We will then fit the slopes found from the LMC Cepheid samples to the Galactic relations and this way obtain our best absolute...
calibrations of the Cepheid PL relations in the \( V, I, J, H, \) and \( K \) passbands. Comparison of these relations to the corresponding relations in the LMC will yield a LMC distance value in each of these passbands, from which a best mean LMC distance will be derived. While this distance still bears some dependence on metallicity and the adopted LMC Cepheid reddening corrections, we will present evidence that this has only a small effect (of the order of a few hundredths of a magnitude) on our adopted LMC distance.

### 5.1. LMC Cepheid PL Relations in \( V, I, J, H, \) and \( K \)

A first and very important step is to define the LMC Cepheid samples to adopt for our purpose. This task was greatly facilitated by the LMC Cepheid database put at our disposal by J. A. R. Caldwell. After inspecting the available data and their quality, we decided to use, in the \( V \) and \( I \) bands, the sample of Tanvir (1997) of 53 LMC Cepheids with photometry in both bands and \( \log P < 1.8 \). We have improved the periods and intensity means in \( V \) and \( I \) for nine Cepheids of this sample from new, high-quality light curves obtained for these variables by Moffett et al. (1997). We have also used individual redenngs for 25 Cepheids from the Caldwell database that were obtained as described by Laney & Stobie (1994). For the remaining Cepheids, we have adopted Caldwell's average reddening value of \( E(B-V) = 0.07 \), except for the Cepheids in the field of the cluster NGC 1850, for which we adopted 0.15, as recommended by Sebo & Wood (1995). Correction for absorption has been made according to the following equations, where the mean \( R_I \) value appropriate to Cepheid colors was taken from Gieren & Fouqué (1993) and the value \( A_I/A_V = 0.592 \) from Tanvir (1997):

\[
\langle V_0 \rangle = \langle V \rangle - 3.26E(B-V),
\]

\[
\langle I_0 \rangle = \langle I \rangle - 1.93E(B-V).
\]

The adoption of individual redenngs slightly improves the dispersion about the mean PL relations, from 0.233 to 0.204 in \( V \) and from 0.164 to 0.150 in \( I \). Determination of accurate individual redenngs to each Cepheid in the sample should make it possible to recover the smaller dispersion of the reddening-dependent Wesenheit function (see below). Small corrections for the tilt of the LMC against the plane of the sky have not been applied because they did not significantly improve the dispersion. For the sake of clarity, we list in Table 8 the final sample for the LMC Cepheid PL solutions in the \( V \) and \( I \) bands, with the adopted values of the periods, absorption-corrected intensity mean magnitudes, and extinctions. The PL relations resulting from least-squares fits to these data are given in Table 10. The slopes we find are indistinguishable from the values of Tanvir (1997).

Unfortunately, the \( J, H, \) and \( K \)-band coverage of Tanvir’s sample of Cepheids is far from complete, and we therefore prefer to use the Laney & Stobie (1994) sample as a starting point for the infrared bands. This sample contains 19 LMC Cepheids with good infrared light curves and 33 with few-phase IR data, adopted from Welch et al. (1987) and transformed to the Carter system. In fact, seven other Cepheids have IR data from Welch et al. but have not been retained by Laney & Stobie. As they do not seem to increase the dispersions of the infrared PL relations, we prefer to adopt the complete sample of 59 Cepheids, and transform the Welch et al. data following the Laney & Stobie precepts for system conversion and dereddening. Extinction values for these seven additional stars have also been provided by Caldwell. Again, no tilt corrections were applied, as they did not improve the dispersions about the PL relations. We list in Table 9 the final sample of LMC Cepheids adopted for the infrared PL solutions with the adopted periods, absorption-corrected intensity mean magnitudes, and extinctions. The PL relations in \( J, H, \) and \( K \) were derived from least-squares fits to these data and are given in Table 10. They differ significantly from the PL relations given in Laney & Stobie (1994), because these authors mixed LMC, SMC, and Galactic Cepheid samples.
In order to find out the intrinsic dispersion of the PL relation, and the contribution of uncertainties in reddening and distance to the observed, total dispersions in the different passbands, we constructed the $V-I$ Wesenheit function for the LMC Cepheid sample. In this sample, the contribution of distance errors to the observed dispersion is negligible (since all stars are basically at the same distance), but there will be a contribution due to errors in the adopted absorption corrections. These errors are removed to a large extent if one uses the reddening-independent Wesenheit function defined as

$$W = V_0 - R(V_0 - I_0) = V - R(V - I), \quad (10)$$

where $R$ is defined as $A_V/(A_V - A_I)$ and is obtained as the slope of the fit of the $V$-band PL relation residuals to the residuals from a mean $V_0 - I_0$ period-color relation. Using the data of Table 8, we determined $R = 2.34 \pm 0.22$, which is close to the expected value of 2.45 which corresponds to the reddening correction. Using this value in the Wesenheit function and plotting $W$ against log $P$, we find a relation whose dispersion has decreased to 0.113 mag, which might then interpret as the intrinsic dispersion of the $V$-band PL relation. A very similar Wesenheit relation is displayed as Figure 3 in Tanvir (1997). A corresponding plot of the $K$-band Wesenheit function against log $P$ yields a relation whose rms dispersion is 0.114 mag, and there is no gain as compared to the $K_0$ PL relation, which is expected, since $K$ the corrections for absorption for our LMC Cepheid sample are negligible. This finding is consistent with our interpretation of the intrinsic dispersion of the Cepheid PL relation being $\sim 0.11$ mag, and this value seems to remain much the same as going from $V$ to $K$.

Unlike the LMC Cepheid sample, the dispersions observed in the Galactic Cepheid sample PL relations do contain a significant contribution due to errors in the distance measurements of the individual Galactic Cepheids. Building a $V-I$ Wesenheit function for the Galactic Cepheid sample in the same way as done above for the LMC sample, the $W$ versus log $P$ relation is found to have a dispersion of 0.17, as compared to the 0.21 mag dispersion shown by the Galactic $V$-band PL relation, and very similar to the dispersion found in the $K$-band PL relation, where contributions from reddening errors are also negligible. Since we know the intrinsic dispersion of the $V$-band PL relation from the LMC sample (see above), the remaining dispersion we observe should be due to errors in the dis-
tances, and perhaps to a metallicity-related effect. This remaining dispersion is 0.12 mag, and corresponds to an error of $\pm 5\%$ in the distances if the error is completely due to distance uncertainties. From the results of Paper II we know, on the other hand, that the expected uncertainty of a distance is $\sim \pm 3\%$, so there might be a small, metallicity-related contribution of the same order that is probably due not to a dependence of our technique on metallicity but rather to a slight systematic dependence of Cepheid absolute magnitudes on metallicity as found by the EROS results (Sasselov et al. 1997), which seems to amount to $\sim 0.06$ mag in the metallicity range of $\Delta[Fe/H] \approx 0.4$ dex covered by the Cepheids of our Galactic sample, and which is small enough to be hidden in Figure 9 (and smaller than the metallicity dependence of the $V$-band PL relation suggested by Sasselov et al. 1997). As a note of caution, this (rough) estimate of the metallicity dependence of the PL relation assumes that the intrinsic width of the instability strip is the same in LMC and the Galaxy.

5.3. The Distance of the LMC and the Intrinsic Calibration of the PL Relation in $V$, $I$, $J$, $H$, and $K$

The determination of an accurate and reliable distance to the LMC is a fundamental step in the extragalactic distance scale. Recent results from various distance indicators show that the range of values for $\mu_0$ (LMC) is from about 18.3 from Hipparcos proper-motion–based RR Lyrae distances (Fernley et al. 1997) to 18.70 based on Hipparcos trigonometric parallax measurements of a sample of nearby Galactic Cepheids (Feast & Catchpole 1997), with a SN 1987A ring upper limit on the LMC distance modulus of 18.44 lying between these extremes (Gould & Uza 1997). As a consequence, the true LMC distance is still uncertain at the 20% level, which is a very unsatisfactory situation. Furthermore, Cepheid-based LMC distance moduli tied to the ZAMS-fitting method and a traditional Pleiades distance modulus of 5.57 have now to be revised in accordance with the new Pleiades distance modulus value of 5.33 obtained from Hipparcos data (Mermilliod et al. 1997; van Leeuwen & Hansen Ruiz 1997), which brings these LMC distance estimates (e.g., Laney & Stobie 1994) close to 18.3, a value similar to the one derived from the RR Lyrae distances. To make things even worse, the Hipparcos results on several nearby open cluster distances have cast serious doubts on the small intrinsic dispersion among the locations of open cluster main sequences on which this method rests (Mermilliod et al. 1997). In view of this situation, a determination of the distance to the LMC from yet another independent method such as the one used by us is clearly very important.

Although the slopes of the LMC PL relations in Table 10 differ from the best-fit slopes from our 28 Galactic calibrators, we attribute this difference to small-number statistics. Indeed, looking at Figures 4–8, where the Galactic data are displayed with the LMC relations superimposed, we see that the difference in the slopes may not be significant. Because the LMC samples are larger and the dispersion of the LMC relations are smaller than those of their Galactic counterparts, we force the LMC slopes to the Galactic sample to establish an absolute zero point of the PL relations, and an absolute distance to the LMC in each band.

In order to take account of the variable accuracies of the distances of our Galactic calibrating Cepheids, we have taken a weighted mean of the LMC distance moduli calculated from each Galactic Cepheid, in each band. The uncertainty of this weighted mean is the quadratic sum of the weighted dispersion divided by the square root of the number of Cepheids (28 in all bands except $I$, which has 27), and of the mean error of the intercept of the corresponding LMC PL relation. Results are given in the last column of Table 10 for each band, and the agreement among the distance moduli derived from the different bands is striking. From a weighted mean of these values, we obtain as the final distance modulus of the LMC

$$\langle \mu_0(\text{LMC}) \rangle = 18.46 \pm 0.20$$.

Subtracting this value from the intercept of the LMC PL relation in each band yields our adopted absolute calibrations, which now do not depend on any assumed LMC distance and mean extinction, and may be used to calibrate, for instance, results from HST in external galaxies if the metallicity is not too far from solar or LMC values. The uncertainty of the absolute intercept of our adopted PL relations is the quadratic sum of the corresponding uncertainty in the LMC PL relation intercept, and of the mean error of the LMC distance modulus. With this, our final absolute calibrations of the Cepheid PL relations in the various bands are then

$$M_V = -2.769(\pm 0.073)(\log P - 1.0) - 4.063(\pm 0.034),$$

$$M_I = -3.041(\pm 0.054)(\log P - 1.0) - 4.767(\pm 0.029),$$

$$M_J = -3.129(\pm 0.052)(\log P - 1.0) - 5.240(\pm 0.028),$$

$$M_H = -3.249(\pm 0.044)(\log P - 1.0) - 5.628(\pm 0.026),$$

$$M_K = -3.267(\pm 0.042)(\log P - 1.0) - 5.701(\pm 0.025).$$

Comparison of these relations to the multiwavelength PL solutions of Madore & Freedman (1991) shows that the slopes of the $V$- and $I$-band relations are almost identical, but that the present zero points are 0.10 mag fainter in both bands. The dispersions of the present relations are significantly smaller, and so are the uncertainties on the coefficients of equations (11) and (12). The $J$-, $H$-, and $K$-band relations of Madore & Freedman yield almost identical absolute magnitudes at $\log P = 1.0$, but their slopes are significantly larger than ours, as are the dispersions of their relations. We attribute this difference to the larger number of stars and improved photometry we have been able to use in our solutions.

As noted before, our way of deriving the LMC distance modulus assumes that there is no metallicity effect on the PL relation. To correct for the mean metallicity difference between the Galactic and the LMC Cepheid samples of $\sim 0.3$ dex, we might adopt the small 0.02 mag shift found by Laney & Stobie (1994), or the larger 0.14 mag shift following from the results of Sasselov et al. (1997). Our own
results favor a correction of 0.06 mag, with an uncertainty of about the same size. We therefore prefer to retain \( \mu_0(\text{LMC}) = 18.46 \) as our best value, but with an increased uncertainty of \( \pm 0.06 \), most of which is due to the uncertainty in the metallicity correction. Obviously, one would like to reduce the uncertainty of the metallicity correction to the LMC distance modulus, and a very promising way of doing this is to apply the infrared Barnes-Evans method directly to LMC Cepheids, taking advantage of the fact that distances measured with this method are almost completely independent of both absorption and metallicities of the target Cepheids. Such a program is currently underway and should yield the true distance modulus of the LMC with an accuracy of \( \sim 0.02 \) mag.

### 6. CONCLUSIONS

We have determined the radii and distances of 34 Galactic Cepheid variables from the infrared Barnes-Evans surface brightness technique of Fouqué & Gieren (1997). We find that the two versions of the technique produce radii that agree to better than 1% and distances that agree at the 2% level, which is within the total uncertainty of both versions of the method. The radius data are used to construct a period-radius relation that shows a dispersion of only \(+0.036\) in \( \log R \) about the mean relation, smaller than in any previous determination. We use the infrared distances of the variables to determine the period-luminosity relations in the optical \( V \) and \( I \), and in the near-infrared \( J \), \( H \), and \( K \) passbands, and again find smaller dispersions in any of these relations than in previous studies. In order to obtain, in each of the passbands, absolute calibrations of the PL relation that are as accurate as possible, we determine the slopes from larger LMC Cepheid samples that show PL relations of smaller dispersions, due to a negligible contribution of distance uncertainties to the observed dispersions. Adopting the slopes defined by the LMC samples, we then use the Galactic Cepheid sample to determine the absolute zero point of the PL relation in each passband. Comparing the Galactic PL relations to the ones defined by the LMC samples, we find values of the true, absorption-corrected metallicity independent, in agreement with model atmosphere predictions; this feature, together with the insensitivity of the method to adopted absorption corrections, makes it an almost ideal instrument to determine the true distances to several nearby galaxies with high accuracy, and thus make a very important contribution toward an improved calibration of the local extragalactic distance scale.

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