Trembling-Hand Perfection and Correlation in Sequential Games

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Abstract
We initiate the study of trembling-hand perfection in sequential (i.e., extensive-form) games with correlation. We introduce the extensive-form perfect correlated equilibrium (EFCE) as a refinement of the classical extensive-form correlated equilibrium (EFCE) that amends its weaknesses off the equilibrium path. This is achieved by accounting for the possibility that players may make mistakes while following recommendations independently at each information set of the game. After providing an axiomatic definition of EFCE, we show that one always exists since any perfect (Nash) equilibrium constitutes an EFCE, and that it is a refinement of EFCE, as any EFCE is also an EFCE. Then, we prove that, surprisingly, computing an EFCE is not harder than finding an EFCE, since the problem can be solved in polynomial time for general $n$-player extensive-form games (also with chance). This is achieved by formulating the problem as that of finding a limit solution (as $\epsilon \to 0$) to a suitably defined trembling LP parametrized by $\epsilon$, featuring exponentially many variables and polynomially many constraints. To this end, we show how a recently developed polynomial-time algorithm for trembling LPs can be adapted to deal with problems having an exponential number of variables. This calls for the solution of a sequence of (non-trembling) LPs with exponentially many variables and polynomially many constraints, which is possible in polynomial time by applying an ellipsoid against hope approach.

Introduction
Nash equilibrium (NE) computation in 2-player zero-sum games has been the flagship challenge in artificial intelligence for several years (see, e.g., landmark results in poker (Brown and Sandholm 2018, 2019)). Recently, increasing attention has been devoted to multi-player games, where equilibria based on correlation are now mainstream.

Correlation in games is customarily modeled through a trusted external mediator that privately recommends actions to the players. The mediator acts as a correlation device that draws action recommendations according to a publicly known distribution. The seminal notion of correlated equilibrium (CE) introduced by Aumann (1974) requires that no player has an incentive to deviate from a recommendation. This is encoded by NE conditions applied to an extended game where the correlation device plays first by randomly selecting a profile of actions according to the public distribution; then, the original game is played with each player being informed only of the action selected for her. CEs are computationally appealing since they can be implemented in a decentralized way by letting players play independently according to no-regret procedures (Hart and Mas-Colell 2000).

Computing CEs in sequential (i.e., extensive-form) games with imperfect information has received considerable attention in the last years (Celli et al. 2019, Farina, Bianchi, and Sandholm 2020). In this context, various CE definitions are possible, depending on the ways recommendations are revealed to the players. The one that has emerged as the most suitable for sequential games is the extensive-form correlated equilibrium (EFCE) of Von Stengel and Forges (2008). The key feature of EFCE is that recommendations are revealed to the players only when they reach a decision point where the action is to be played, and, if one player defects from a recommendation, then she stops receiving them in the future. Von Stengel and Forges (2008) show that EFCEs can be characterized by a polynomially-sized linear program (LP) in two-player games without chance. In the same restricted setting, Farina et al. (2019a) show how to find an EFCE by solving a bilinear saddle-point problem, which can be exploited to derive an efficient no-regret algorithm (Farina et al. 2019b). In general $n$-player games, Huang and von Stengel (2008) prove that an EFCE can be computed in polynomial time by means of an ellipsoid against hope (EAH) algorithm similar to that introduced by Papadimitriou and Roughgarden (2008) for CEs in compactly represented games (see also Gordon, Greenwald, and Marks 2008 for another algorithm). Instead, finding a payoff-maximizing EFCE is NP-hard (Von Stengel and Forges 2008). Very recently, Celli et al. (2020) provide an efficient no-regret procedure for EFCE in $n$-player games.

One of the crucial weaknesses of standard equilibrium notions, such as NE, in sequential games is that they may prescribe to play sub-optimally off the equilibrium path, i.e., at those information sets never reached when playing equilibrium strategies. One way to amend this issue is
trembling-hand perfection (Selton 1975), whose rationale is to let players reasoning about the possibility that they may make mistakes in the future, playing sub-optimal actions with small, vanishing probabilities (a.k.a. trembles). This idea leads to the NE refinement known as perfect equilibrium (PE) (Selton 1975). Other refinements have been introduced in the literature, e.g., in the quasi-perfect equilibrium of Van Damme 1984; players only account for opponents’ future trembles (see Van Damme 1991) for other examples. Trembles can also be introduced in normal-form games, leading to robust equilibria that rule out weakly dominated strategies (Hillas and Kohlberg 2002). Recently, equilibrium refinement has been addressed beyond the NE case, such as, e.g., in Stackelberg settings (Farina et al. 2018; Marchesi et al. 2019).

Trembling-hand perfection for CEs has only been studied from a theoretical viewpoint in normal-form games, by Dhillon and Mertens 1996. The authors introduce the concept of perfect CE by enforcing PE conditions in the extended game, rather than NE ones. Despite equilibrium refinements in sequential games are ubiquitous, no work addressed perfection and correlation together in such setting.\footnote{Applying the perfect CE by Dhillon and Mertens (1996) to the normal-form representation of a sequential game does not generally solve equilibrium weaknesses. This would lead to a correlated version of the normal-form PE, which is known not to guard against sub-optimality off the equilibrium path (Van Damme 1991).}

Original Contributions We give an axiomatic definition of extensive-form perfect correlated equilibrium (EFPEC), enforcing PE conditions, rather than NE ones, in the extended game introduced by Von Stengel and Forgés 2008 for their original definition of EFCE. Intuitively, this accounts for the possibility that players may make mistakes while following recommendations independently at each information set of the game. Trembles are introduced on players’ strategies, while the correlation device is defined as in classical CE notions. First, we show that an EFPEC always exists, since any PE constitutes an EFPEC, and that EFPEC is a refinement of EFCE, as any EFPEC is also an EFCE. Then, we show how an EFPEC can be computed in polynomial time in any $n$-player extensive-form game (also with chance). At first, we introduce a characterization of the equilibria of perturbed extended games (i.e., extended games with trembles) inspired by the definition of EFCE based on trigger agents, introduced by Gordon, Greenwald, and Marks (2008) and Farina et al. 2019\textsuperscript{1}. This result allows us to formulate the EFPEC problem as that of finding a limit solution (as $\epsilon \to 0$) to a suitably defined trembling LP parametrized by $\epsilon$, featuring exponentially many variables and polynomially many constraints. To this end, we show how the polynomial-time algorithm for trembling LPs developed by Farina, Gatti, and Sandholm (2018) can be adapted to deal with problems having an exponential number of variables. This calls for the solution of a sequence of (non-trembling) LPs with exponentially many variables and polynomially many constraints, which is possible in polynomial time by applying an EAH approach. The latter is inspired by the analogous algorithm of Huang and von Stengel 2008 for EFCEs, which is adapted to deal with a different set of dual constraints, requiring a modification of the polynomial-time separation oracle of Huang and von Stengel (2008).\footnote{All the omitted proofs are in Appendix.}

Preliminaries

Extensive-Form Games

We focus on $n$-player extensive-form games (EFGs) with imperfect information. We let $N := \{1, \ldots, n\}$ be the set of players, and, additionally, we let $c$ be the chance player representing exogenous stochasticity. The sequential structure is encoded by a game tree with node set $H$. Each node $h \in H$ is identified by the ordered sequence $\sigma(h)$ of actions encountered on the path from the root to $h$. We let $Z \subseteq H$ be the subset of terminal nodes, which are the leaves of the game tree. For every non-terminal node $h \in H \setminus Z$, we let $P(h) \subseteq N \cup \{c\}$ be the player who acts at $h$, while $A(h)$ is the set of actions available. The function $p_c : Z \to \{0, 1\}$ defines the probability of reaching each terminal node given the chance moves on the path from the root to that node. For every player $i \in N$, the function $u_i : Z \to \mathbb{R}$ encodes player $i$’s utilities over terminal nodes. Imperfect information is modeled through information sets (infosets). An infoset $I \subseteq H \setminus Z$ of player $i \in N$ is a group of player $i$’s nodes indistinguishable for her, i.e., for every $h \in I$, it must be the case that $P(h) = i$ and $A(h) = A(I)$, where $A(I)$ is the set of actions available at the infoset. W.l.o.g., we assume that the sets $A(I)$ are disjoint. We denote with $I_i$ the collection of infosets of player $i \in N$. For every $i \in N$, we let $A_i := \bigcup_{I \in I_i} A(I)$ be the set of all player $i$’s actions. Moreover, we let $A := \bigcup_{i \in N} A_i$. We focus on EFGs with perfect recall in which no player forgets what she did or knew in the past. Formally, for every player $i \in N$ and infoset $I \in I_i$, it must be that every node $h \in I$ is identified by the same ordered sequence $\sigma(h)$ of player $i$’s actions from the root to that node. Given two infosets $I, J \in I_i$ of player $i \in N$, we say that $J$ follows $I$, written $I \prec J$, if there exist two nodes $h \in I$ and $k \in J$ such that $h$ is on the path from the root to $k$. By perfect recall, $\prec$ is a partial order on $I_i$. We also write $I \preceq J$ whenever either $I = J$ or $I \prec J$. For every infoset $I \in I_i$, we let $C(I, a) \subseteq I_i$ be the set of all infosets that immediately follow $I$ by playing action $a \in A(I)$.

Strategies A player’s pure strategy specifies an action at every infoset of her. For every $i \in N$, the set of player $i$’s pure strategies $\pi_i$ is $\Pi_i := \times_{I \in I_i} A(I)$, with $\pi_i(I) \in A(I)$ being the action at infoset $I \in I_i$. Moreover, $\Pi := \times_{i \in N} \Pi_i$ be the set of strategy profiles specifying a strategy for each player, while, for $i \in N$, we let $\Pi_{-i} := \times_{j \neq i \in N} \Pi_j$ be the (partial) strategy profiles defining a strategy for each player other than $i$. Given $\pi_i \in \Pi_i$ and $a \in A_i$, we write $a \in \pi_i$ whenever $\pi_i$ prescribes to play $a$. Analogously, for $\pi \in \Pi$ and $a \in \Pi$, we write $a \in \pi$. Players are allowed to randomize over pure strategies by playing mixed strategies. For $i \in N$, we let $\mu_i : \Pi_i \to [0, 1]$ be a player $i$’s mixed
strategy, where \( \sum_{a \in \Pi_i} \mu_i(a) = 1 \). The perfect recall assumption allows to work with *behavior strategies*, which define probability distributions locally at each infoset. For \( i \in N \), we let \( \beta_i : A_i \to [0, 1] \) be a player \( i \)'s behavior strategy, which is such that \( \sum_{a \in A(I)} \beta_i(a) = 1 \) for all \( I \in \mathcal{I}_{-i} \).

**Additional Notation** We introduce some subsets of \( \Pi_i \) (see Figure 1 for some examples). For every action \( a \in A_i \) of player \( i \in N \), we define \( \Pi_i(a) := \{ \pi_i \in \Pi_i \mid a \in \pi_i \} \) as the set of player \( i \)'s pure strategies specifying \( a \). For every infoset \( I \in \mathcal{I}_i \), we let \( \Pi_i(I) \subseteq \Pi_i \) be the set of strategies that prescribe to play so as to reach \( I \) whenever possible (depending on players’ moves up to that point) and any action whenever reaching \( I \) is not possible anymore. Additionally, for every action \( a \in A(I) \), we let \( \Pi_i(I, a) \subseteq \Pi_i(I) \subseteq \Pi_i \) be the set of player \( i \)'s strategies that reach \( I \) and play \( a \). Given a terminal node \( z \in Z \), we denote with \( \Pi_i(z) \subseteq \Pi_i \) the set of strategies by which player \( i \) plays so as to reach \( z \), while \( \Pi(z) := \times_{i \in N} \Pi_i(z) \) and \( \Pi_{-i}(z) := \times_{j \in N \setminus \{i\}} \Pi_j(z) \). We also introduce the following subsets of \( Z \). For every \( i \in N \) and \( I \in \mathcal{I}_i \), we let \( Z(I) \subseteq Z \) be the set of terminal nodes reachable from infoset \( I \) of player \( i \). Moreover, \( Z(I, a) \subseteq Z(I) \subseteq Z \) is the set of terminal nodes reachable by playing action \( a \in A(I) \) at \( I \), whereas \( Z^{-1}(I, a) := Z(I, a) \setminus \bigcup_{J \in \mathcal{I}_i(a)} Z(J) \) is the set of those reachable by playing \( a \) at \( I \) without traversing any other player \( i \)'s infoset.

**Nash Equilibrium and Its Refinements**

Given an EFG, players’ behavior strategies \( \{\beta_i\}_{i \in N} \) constitute an NE if no player has an incentive to unilaterally deviate from the equilibrium by playing another strategy [Nash 1951]. The PE defined by Selten (1975) relies on the idea of introducing trembles in the game, representing the possibility that players may take non-equilibrium actions with small, vanishing probability. Trembles are encoded by means of Selten’s *perturbed games*, which force lower bounds on the probabilities of playing actions. Given an EFG \( \Gamma \), a pair \( (\Gamma, \eta) \) defines a perturbed game, where \( \eta : A \to (0, 1) \) is a function assigning a positive lower bound \( \eta(a) \) on the probability of playing each action \( a \in A \), with \( \sum_{a \in A(I)} \eta(a) < 1 \) for every \( i \in N \) and \( I \in \mathcal{I}_i \). Then:

\[ \text{Definition 1. Given an EFG } \Gamma, \{\beta_i\}_{i \in N} \text{ is a PE of } \Gamma \text{ if it is a limit point of NEs for at least one sequence of perturbed games } \{ (\Gamma, \eta_i) \}_{i \in N} \text{ such that, for all } a \in A, \text{ the lower bounds } \eta_i(a) \text{ converge to zero as } t \to \infty. \]

There are only a few computational works on NE refinements. For instance, Miltsens and Sørensen (2010) characterize quasi-perfect equilibria of 2-player EFGs using the sequence form (see the recent work by Gatti, Gilli, and Marchesi (2020) for its extension to \( n \)-player games) and exploit this to compute an equilibrium by solving a linear complementarity problem with trembles defined as polynomials of some parameter treated symbolically [Farina and Gatti (2017)] do the same for the PE. Recently, Farina, Gatti, and Sandholm (2018) provide a general framework for computing NE refinements in 2-player zero-sum EFGs in polynomial time. The authors show how to reduce the task to the more general problem of solving trembling LPs parametrized by some parameter \( \epsilon \), i.e., finding their limit solutions as \( \epsilon \to 0 \). Then, they provide a general polynomial-time algorithm to find limit solutions to trembling LPs. Other works study the problem of computing (approximate) NE refinements in 2-player zero-sum EFGs by employing online convex optimization techniques [Kroer, Farina, and Sandholm (2017), Farina, Kroer, and Sandholm (2017)].

**Correlation in Extensive-Form Games**

We model a correlation device as a probability distribution \( \mu \in \Delta_R \). In the classical CE by Aumann (1974), the correlation device draws a strategy profile \( \pi \in \Pi \) according to \( \mu \); then, it privately communicates \( \pi_i \) to each player \( i \in N \). This notion of CE does *not* fit well to EFGs, as it requires the players to reason over the exponentially-sized set \( \Pi_i \). Von Stengel and Forges (2008) introduced the EFCE to solve this issue. The first crucial feature of the EFCE is a different way of giving recommendations: the strategy \( \pi_i \) is revealed to player \( i \) as the game progresses, i.e., the player is recommended to play the action \( \pi_i(I) \) at infoset \( I \in \mathcal{I}_i \) only when \( I \) is actually reached during play. The second key aspect characterizing EFCEs is that, whenever a player decides to defect from a recommended action at some infoset, then she may choose any move at her subsequent infosets and she stops receiving recommendations.

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\( ^1 \) EFGs with perfect recall admit a compact strategy representation called sequence form [Von Stengel (1996)]. See Appendix A.
from the correlation device. The definition of EFCE introduced by Von Stengel and Forges (2008) (Definition 3) requires the introduction of the notion of extended game with a correlation device.

Definition 2. Given an EFG $\Gamma$ and a distribution $\mu \in \Delta_\Pi$, the extended game $\Gamma^{\text{ext}}(\mu)$ is a new EFG in which chance first selects $\pi \in \Pi$ according to $\mu$, and, then, $\Gamma$ is played with each player $i \in N$ receiving the recommendation to play $\pi_i(I)$ as a signal, whenever she reaches an infoset $I$.

The signaling in $\Gamma^{\text{ext}}(\mu)$ induces a new infoset structure. Specifically, every infoset $I \in I_i$ of the original game $\Gamma$ corresponds to many, new infosets in $\Gamma^{\text{ext}}(\mu)$, one for each combination of possible action recommendations received at the infosets preceding $I$ (this included). At each new infoset, player $i$ can only distinguish among chance moves corresponding to strategy profiles $\pi \in \Pi$ that differ in the recommendations at infosets $J \in I_i : J \preceq I$. Figure 1 shows a simple EFG with its corresponding extended game.

Definition 3. Given an EFG $\Gamma$, $\mu \in \Delta_\Pi$ defines an EFCE of $\Gamma$ if following recommendations is an NE of $\Gamma^{\text{ext}}(\mu)$. \footnote{For EFCEs, one can restrict the attention to distributions $\mu$ over reduced strategy profiles, i.e., those in which each player’s pure strategy only specifies actions at infosets reachable given that player’s moves (Vermeulen and Jansen, 1998). In the following, we stick to general, un-reduced strategy profiles since, as showed in Appendix C, these are necessary for trembling-hand perfect CEs in order to define the players’ behavior off the equilibrium path.}

Next, we introduce an equivalent characterization of EFCEs (Farina, Bianchi, and Sandholm, 2020). It is based on the following concept of trigger agent, originally due to Gordon, Greenwald, and Marks (2008).

Definition 4. Given an infoset $I \in I_i$ of player $i \in N$, an action $a \in A(I)$, and a distribution $\mu_i \in \Delta_{R_i}(I)$, an $(I, a, \mu_i)$-trigger agent for player $i$ is an agent that takes on the role of player $i$ and follows all recommendations unless she reaches $I$ and gets recommended to play $a$. If this happens, she stops committing to recommendations and plays according to a strategy sampled from $\mu_i$ until the game ends.

Then, it follows that $\mu \in \Delta_\Pi$ is an EFCE if, for every $i \in N$, player $i$’s expected utility when following recommendations is at least as large as the expected utility that any $(I, a, \mu_i)$-trigger agent for player $i$ can achieve (assuming the opponents’ do not deviate from recommendations). We provide a formal statement in Appendix B.

Computing EFCEs in $n$-player EFGs The algorithm of Huang and von Stengel (2008) relies on the following LP formulation of the problem of finding an EFCE, which has exponentially many variables and polynomially many constraints (for completeness, its derivation is in Appendix C).

\[
\begin{align*}
\max_{\mu \geq 0, v} & \sum_{\pi \in \Pi} \mu[\pi] \quad \text{s.t.} \quad A\mu + Bv & \geq 0, \\
& A^\top y & \leq -1, \\
& B^\top y & = 0, \\
& y & \geq 0,
\end{align*}
\]

where $\mu$ is a vector of variables $\mu[\pi]$ for $\pi \in \Pi$, encoding a probability distribution $\mu \in \Delta_\Pi$. Problem 1 does not enforce any simplex constraint on variables $\mu[\pi]$, and, thus, it is either unbounded or it has an optimal solution with value zero (by setting $\mu$ and $v$ to zero). In the former case, any feasible $\mu$ encodes an EFCE after normalizing it. As a result, since an EFCE always exists (Von Stengel and Forges, 2008), the following dual of Problem 1 is always infeasible:

\[
\begin{align*}
A^\top y & \leq -1, \\
B^\top y & = 0, \\
y & \geq 0,
\end{align*}
\]

where $y$ is a vector of dual variables. The EAH approach applies the ellipsoid algorithm (Grötschel, Lovász, and Schrijver, 1993) to Problem 1 in order to conclude that it is infeasible. Since there are exponentially many constraints, the algorithm runs in polynomial time only if a polynomial-time separation oracle is available. This is given by the following:

Lemma 1 (Lemma 5, Huang and von Stengel, 2008). If $y \geq 0$ is such that $B^\top y = 0$, then there exists $\mu$ encoding a product distribution $\mu \in \Delta_\Pi$ such that $\mu^\top A^\top y = 0$. Moreover, $\mu$ can be computed in polynomial time.

Jiang and Leyton-Brown (2015) show how, given a product distribution $\mu$ computed as in Lemma 1 it is possible to recover, in polynomial time, a violated constraint for Problem 1 corresponding to some strategy profile $\pi \in \Pi$. This, together with some additional technical tricks ensuring that $B^\top y = 0$ holds (see Huang and von Stengel, 2008 for more details), allows to apply the ellipsoid algorithm to Problem 1 in polynomial time. Since the problem is infeasible, the algorithm must terminate after polynomially many iterations with a collection of violated constraints, which correspond to polynomially many strategy profiles. Then, solving (in polynomial time) Problem 1 with the variables $\mu$ restricted to these strategy profiles gives an EFCE of the game. Let us also remark that the EFCE obtained in this way has support size polynomial in the size of the game.

Trembling-Hand Perfection and Correlation

We are now ready to show how trembling-hand perfection can be injected into the definition of EFCE so as to amend its weaknesses off the equilibrium path (see the following for an example). We generalize the approach of Dhillon and Mertens (1996) (restricted to CEs in normal-form games) to the general setting of EFCEs in EFGs. The core idea is to use the PE rather than the NE in the definition of CE. Thus:

Definition 5. Given an EFG $\Gamma$, a distribution $\mu \in \Delta_\Pi$ is an extensive-form perfect correlated equilibrium (EFCE) if following recommendations is a PE of $\Gamma^{\text{ext}}(\mu)$.

The definition of EFCE crucially relies on the introduction of trembles in extended games, i.e., it takes into account the possibility that each player may not follow action recommendations with a small, vanishing probability. In the following, given a perturbed EFG $\Gamma(\eta)$ and $\mu \in \Delta_\Pi$, we denote with $\Gamma^{\text{ext}}(\mu, \eta)$ a perturbed extended game in which
the probability of playing each action is subject to a lower bound equal to the lower bound \( \eta(a) \) of the corresponding action \( a \in A \) in \( \Gamma \). By recalling the definition of PE (Definition 1) and the structure of perturbed extended games, it is easy to infer the following characterization of EFPCEs:

**Lemma 2.** Given an EFG \( \Gamma \), a distribution \( \mu \in \Delta_{II} \) is an EFPCE of \( \Gamma \) if following recommendations constitutes NEs for at least one sequence of perturbed extended games \( \{\Gamma_{\text{ext}}(\mu, \eta_t)\}_{t \in \mathbb{N}} \) such that, for all \( a \in A \), the lower bounds \( \eta_t(a) \) converge to zero as \( t \to \infty \).

We remark that, with an abuse of terminology, we say that players follow recommendations in a perturbed extended game \( \{\Gamma_{\text{ext}}(\mu, \eta_t)\}_{t \in \mathbb{N}} \) whenever they play strategies which place all the residual probability (given lower bounds) on recommended actions. In the following sections, we crucially rely on the characterization of EFPCEs given in Lemma 2 in order to derive our computational results. First, we show an example of EFPCE and prove some of its properties.

**Example of EFPCE** Consider the EFG in Figure 1 (Left) and lower bounds \( \eta_t : A \to (0, 1) \) for \( t \in \mathbb{N} \), with \( \eta_t(a) \to 0 \) as \( t \to \infty \) for all \( a \in A \). First, notice that player 1 is always better off playing action \( a \) at the root infoset \( 1 \), since she can guarantee herself a utility of 1 by selecting \( c \) at the following \( 1 \), while she can achieve at most \( \frac{1}{2} \) by playing \( b \). Thus, any EFPCE of the game (as well as any EFC) must recommend \( a \) at \( 1 \) with probability 1. Then, in the sub-game reached when playing \( a \) at \( 1 \), it is easy to check that recommending the pairs of actions \( (c, m) \), \( (c, n) \), and \( (d, m) \) each with probability \( \frac{1}{3} \) is an equilibrium, as no player has an incentive to deviate from a recommendation, even with trembles (see Appendix E for more details). The correlation device described for \( f_0 \) is sufficient to define an EFC, as recommendations at infosets \( Y \), \( K \), and \( L \) are not relevant given that they do not influence players’ utilities at the equilibrium \( (b \) is never recommended). However, they become relevant for EFPCEs, since, in perturbed extended games, these infosets could be reached due to a tremble with probability \( \eta_t(b) \). Then, player 2 must be told to play \( p \) at \( Y \), because her utility is always 1 if she plays \( p \), while it is always 0 for \( o \). Moreover, with an analogous reasoning, player 1 must be recommended to play \( e \) and \( h \) at \( K \) and \( L \), respectively. In conclusion, we can state that \( \mu \in \Delta_{II} : \mu(ac, mp) = \mu(aceh, mp) = \mu(adeh, mp) = \frac{1}{3} \) is an EFPCE.

**Properties of EFPCEs** We characterize the relation between EFPCEs and other equilibria, also showing that EFPCEs always exist and represent a refinement of EFCEs.

**Theorem 1.** This relation holds: PE \( \subseteq \) EFPCE \( \subseteq \) EFCE.

**Theorem 2.** The following relations hold:
- \( \text{EFPCE} \not\subseteq \text{NE} \) and \( \text{NE} \not\subseteq \text{EFPCE} \);
- \( \text{EFPCE} \cap \text{NE} = \text{PE} \).

**NEs of Perturbed Extended Games** We provide a characterization of NEs of perturbed extended games \( \{\Gamma_{\text{ext}}(\mu, \eta_t)\}_{t \in \mathbb{N}} \), useful for our main algorithmic result on EFPCEs given in the following section. Specifically, we give a set of easily interpretable conditions which ensure that following recommendations is an NE of \( \{\Gamma_{\text{ext}}(\mu, \eta_t)\}_{t \in \mathbb{N}} \). These are crucial for the derivation of the LP exploited by our algorithm. Our characterization is inspired by that of EFCEs based on trigger agents (see Lemma 4 in Appendix B). However, the presence of trembles in extended games requires some key changes, which we highlight in the following.

First, we introduce some additional notation. Given a perturbed extended game \( \{\Gamma_{\text{ext}}(\mu, \eta_t)\}_{t \in \mathbb{N}} \), we let \( \xi^\eta(z, \pi) \) be the probability of reaching a node \( z \in Z \) when a strategy profile \( \pi \in \Pi \) is recommended and players’ obey to recommendations, in presence of trembles defined by \( \eta \). Each \( \xi^\eta(z, \pi) \) is obtained by multiplying probabilities of actions in \( \sigma(z) \), which are those on the path from the root to \( z \). For each \( a \in \sigma(z) \), two cases are possible: either \( a \) is prescribed by the recommended \( \pi \) and played with its maximum probability given \( \eta \); or it is not, which means that a tremble occurred with probability \( \eta(a) \). Formally, letting \( f \{a \in \pi\} \) be an indicator for the event \( a \in \pi \), for every \( z \in Z \) and \( \pi \in \Pi \):

\[
\xi^\eta(z, \pi) := \prod_{a \in A : a \in \sigma(z)} \eta(a)^{f\{a \in \pi\}} \eta(a)^{1-f\{a \in \pi\}} p_c(z),
\]

In the following, we denote the sets of equilibria with their corresponding acronyms (e.g., NE is the set of all NEs of a game).
where, for $a \in A(I)$, we let $\tilde{\eta}(a) := 1 - \sum_{a' \neq a \in A(I)} \eta(a')$ be the maximum probability assignable to $a$ given $\eta$. Moreover, for every player $i \in N$, infoset $I \in I_i$, terminal node $z \in Z(I)$ reachable from $I$, and strategy profile $\pi \in \Pi$, we let $\xi^I(\pi(I), \pi) \xi^I(\pi(I), \pi)$ be defined as $\xi^I(\pi(I), \pi)$ excluding player $i$’s actions leading from $I$ to $z$, i.e., with the product restricted to actions $a \in \sigma(I) \cup (\sigma(z) \setminus A_i)$. Analogously, for every player $i$’s strategy $\pi_i \in \Pi(I, I)$, we let $\xi^I(\pi(I), \pi_i)$ be defined for player $i$’s actions $a \in A_i \cap (\sigma(z) \setminus \sigma(I))$ from $I$ to $z$.

Following recommendations is an NE of the perturbed extended game $(\Gamma^{ext}(\mu), \eta)$ if, for every player $i \in N$, infoset $I \in I_i$, and action $a \in (I)$, player $i$’s utility when obeying to the recommendation $a$ at $I$ is at least as large as the utility achieved by any $(I, a, \mu_i)$-trigger agent. The fundamental differences with respect to EFCE are: (i) an infoset $I$ could be reached even when actions recommended at preceding infosets do not allow it (due to trembles); and (ii) trigger agents are subjects to trembles, which means that they may make mistakes while playing the strategy sampled from $\mu_i$.

For any terminal node $z \in Z$, the probability of reaching it when following recommendations is:

$$q^I_\mu(z) := \sum_{\pi \in \Pi} \xi^I(z, \pi) \mu(\pi),$$

where the summation accounts for the probability of reaching $z$ for every possible $\pi$. The sum is over $\Pi$ rather than $\Pi(z)$ as for EFCE (see Equation (9) in Appendix B), since, due to trembles, $z$ could be reached even when $\pi \notin \Pi(z)$.

For any $(I, a, \mu_i)$-trigger agent, the probability of reaching $z \in Z(I)$ when the agent ‘gets triggered’ is defined as:

$$p^I_{\mu, \mu_i}(z) := \left( \sum_{\pi_i \in \Pi_i(a)} \xi^I(z, I, \pi) \mu(\pi) \right) \left( \sum_{\tilde{\pi}_i \in \Pi_i(I)} \xi^I(\tilde{\pi}_i) \mu_i(\tilde{\pi}_i) \right),$$

where the first summation is over $\Pi_i(a)$ instead of $\Pi_i(I, a)$ (as in the EFCE, see Equation (B) in Appendix B) since it might be the case that the agent is activated also when the recommended strategy $\pi_i$ does not allow to reach infoset $I$.

Finally, the overall probability of reaching $z \in Z(I)$ is:

$$\eta^{I, a}_{\mu, \mu_i}(z) := p^I_{\mu, \mu_i}(z) + \sum_{\pi_i \in \Pi_i \setminus \Pi_i(a)} \xi^I(z, \pi) \mu(\pi),$$

where the first term is for when the agent ‘gets triggered’, while the second term accounts for the case in which the agent is not activated (the two events are independent).

**Theorem 3.** Given a perturbed extended game $(\Gamma^{ext}(\mu), \eta)$, following recommendations is an NE of the game if for every $i \in N$ and $(I, a, \mu_i)$-trigger agent for player $i$, it holds that:

$$\sum_{I \in I_i} \left( \sum_{\pi_i \in \Pi_i(a)} \xi^I(z, \pi) \mu(\pi) \right) u_i(z) \geq \sum_{I \in I_i} p^I_{\mu, \mu_i}(z) u_i(z).$$

Computing an EFPC E in $n$-player EFGs

We provide a polynomial-time algorithm to compute an EFPC E in $n$-player EFGs (also with chance). The algorithm is built on three fundamental components: (i) a trembling LP (with exponentially many variables and polynomially many constraints) whose limit solutions define EFPCs; (ii) an adaption of the algorithm by Farina, Gatti, and Sandholm (2018) that finds such limit solutions by solving a sequence of (non-trembling) LPs; and (iii) a polynomial-time EAH procedure that solves these LPs.

**Trembling LP for EFPCs** It resembles the EFCE LP in Problem 1. In this case, the constraints appearing in the LP ensure that following recommendations is an NE in a given sequence of perturbed extended games, by exploiting the characterization given in Theorem 3. Then, Lemma 2 allows to conclude that the limit solutions of the trembling LP define EFPCEs. In the following, we assume that a sequence of perturbed extended games $\{\Gamma^{ext}(\mu), \eta_t\}_{t \in \mathbb{N}}$ is given. For every player $i \in N$, infoset $I \in I_i$, and action $a \in A(I)$, we introduce a variable $u[i, I, a]$ to encode player $i$’s expected utility when following the recommendation to play $a$ at $I$ in the perturbed extended game $(\Gamma^{ext}(\mu), \eta_t)$. These variables are defined by the following constraints:

$$u[i, I, a] = \sum_{z \in Z(I)} \left( \sum_{\pi_i \in \Pi_i(a)} \xi^I(z, \pi) \mu(\pi) \right) u_i(z)$$

$$\forall i \in N, \forall I \in I_i, \forall a \in A(I).$$

Then, we introduce constraints that recursively define variables $v[i, I, a, J]$ for every infoset $J \in I_i : I \not\subseteq J$. These encode the maximum expected utility obtained at infoset $J$ by trigger agents associated with $I$ and $a$. To this end, we also need some auxiliary non-negative variables $w[i, I, a, J, a']$, which are defined for every player $i \in N$, infoset $I \in I_i$, action $a \in A(I)$, infoset $J \in I_i : I \not\subseteq J$ following $I$ (this included), and action $a' \in A(J)$ available at $J$.

$$v[i, I, a, J] - w[i, I, a, J, a'] \geq$$

$$\sum_{z \in Z(J \cup a')} \left( \sum_{\pi_i \in \Pi_i(a)} \xi^I(z, \pi) \mu(\pi) \right) u_i(z) +$$

$$\sum_{K \in \mathbb{C}(J \cup a')} v[i, I, a, K] - \sum_{a'' \in A(K)} \eta(t(a'')) w[i, I, a, K, a'']$$

$$\forall i \in N, \forall I \in I_i, \forall a \in A(I), \forall J \in I_i : I \not\subseteq J, \forall a' \in A(J).$$

Intuitively, each auxiliary variable $w[i, I, a, J, a']$ represents a penalty on $v[i, I, a, J]$ due to the possibility of trembling by playing a (possibly) sub-optimal action $a' \in A(J)$ at $J$. Indeed, whenever $a'$ is an optimal action at infoset $J$, then $w[i, I, a, J, a']$ is set to 0 in any solution; otherwise, $w[i, I, a, J, a']$ represents how much utility is lost by playing $a'$ instead of an optimal action (see Figure 3 Right) for an
By substituting the expression of \( u \) step, for a fixed value of the parameter \( (2018) \) can be used, with the only difference that, at each step, the incentive constraints are:

\[
\begin{align*}
\text{(1, a): } & \quad v[1, i, a, b] - v[1, i, a, a] \geq v[1, i, a, b] + v[1, i, a, K] - \eta(c)u[1, i, a, c] - \eta(f)a[1, i, a, b] \\
\text{(1, b): } & \quad v[1, i, a, b] - v[1, i, a, b] \geq U_i(z_3) \\
\text{(4a): } & \quad v[1, i, a, K] - v[1, i, a, K] \geq U_i(z_3) \\
\text{(4b): } & \quad v[1, i, a, K] - v[1, i, a, K, c] \geq U_i(z_3) \\
\text{(5a): } & \quad v[1, i, a, K] - v[1, i, a, K, f] \geq U_i(z_3) \\
\text{(5b): } & \quad v[1, i, a, K] - v[1, i, a, K] \geq U_i(z_3) \\
\text{(5c): } & \quad v[1, i, a, K] - v[1, i, a, K, c] \geq U_i(z_3) \\
\text{(5d): } & \quad v[1, i, a, K] - v[1, i, a, K, f] \geq U_i(z_3)
\end{align*}
\]

Figure 3: (Left) Simple EFG. (Right) Example of Constraints (4)–(5); we let \( U_1(z) := \sum_{\pi \in \Pi} \xi(z, I, \pi) \mu[\pi] u_1(z) \) for every \( z \in Z \). Variables \( v[1, i, a, \cdot] \) encode the optimal utility of trigger agents associated to \( i, a \) at infosets following \( \top \) (without trembles). Variables \( w[1, a, \cdot, \cdot, \cdot] \) account for penalties due to trembles. To see this, fix \( \mu[\pi]\). Assume that \( U_1(z_4) > U_1(z_3) \) and consider the constraints for \( (K, c) \) and \( (K, f) \). Then, it must be \( v[1, i, a, K] = U_1(z_4) \) and \( w[1, i, a, K, f] = 0 \), which implies \( w[1, i, a, K, c] = U_1(z_3) \) (constraint for \( (1, a) \)). Similarly, assuming \( U_1(z_2) > U_1(z_1) \), it must be \( v[1, i, a, I] = U_1(z_2) \), \( w[1, i, a, j, c] = 0 \), and \( w[1, i, a, j, c] = U_1(z_2) - U_1(z_1) \). An analogous reasoning holds at infosets upwards in the game tree.

Theorem 4. Given a perturbed extended game \( (\Gamma_{\text{ext}}(\mu), \eta_{\pi}) \), if Constraints (3), (4), and (5) can be satisfied for the vector \( \mu[\pi] \) of variables \( \mu[\pi] \) encoding the distribution \( \mu \), then following recommendations is an NE of \( (\Gamma_{\text{ext}}(\mu), \eta_{\pi}) \).

By substituting the expression of \( u[i, I, a] \) (given by Constraints (3) and (5)) into Constraints (4), we can formulate the following trembling LP parameterized by \( t \in \mathbb{N} \):

\[
\begin{align*}
\max_{\mu \geq 0, \nu, u, v \geq 0} & \quad \sum_{\pi \in \Pi} \mu[\pi] \text{ s.t.} \\
A_i \mu + Bv + C_i u & \geq 0,
\end{align*}
\]

where \( A_i \) is the analogous of matrix \( A \) in Problem [1], \( \mu \) is a vector whose components are the variables \( w[i, I, a, j, a'] \), and \( C_i \) is a matrix defining the constraints coefficients for these variables. Notice that the coefficients of variables in \( \nu \) (as defined by \( B \)) are the same as in Problem [1].

Limit Solutions of Trembling LP Problem [6] can be cast into the framework of Farina, Gatti, and Sandholm (2018) by defining sequences of lower bounds \( \eta_{\pi} \) by means of vanishing polynomials in a parameter \( \epsilon \rightarrow 0 \). As a result, the polynomial-time algorithm by Farina, Gatti, and Sandholm (2018) can be used, with the only difference that, at each step, for a fixed value of the parameter \( \epsilon \) (i.e., particular lower bounds \( \eta_{\pi} \)), it needs to solve an instance of Problem [6] featuring exponentially many variables. Provided that the latter can be done in polynomial time, the polynomiality of the overall procedure is preserved, since the bounds on the running time provided by Farina, Gatti, and Sandholm (2018) do not depend on the number of variables in the LP.

EAH Procedure In order to solve Problem [6] for a particular lower bound function \( \eta_{\pi} \) in polynomial time, we can apply a procedure similar to the EAH algorithm by Huang and von Stengel (2008). Notice that Problem [6] is always unbounded, since there always exists a distribution \( \mu \in \Delta_{\Pi} \) such that following recommendations is an NE of the perturbed extended game \( (\Gamma_{\text{ext}}(\mu), \eta_{\pi}) \) (such \( \mu \) is an EFC of the corresponding perturbed, non-extended game).

Thus, we only need to provide a polynomial-time separation oracle for the always-infeasible dual of Problem [6] which reads:

\[
\begin{align*}
A_i^\top y & \leq -1 \\
B^\top y & = 0 \\
C_i^\top y & \geq 0 \\
y & \geq 0,
\end{align*}
\]

where the vector of dual variables \( y \) has the same role as in Problem [2] since the constraints of the primal problems are indexed on the same sets. Notice that constraints \( C_i^\top y \geq 0 \) are polynomially many. As a result, one can always check whether one of these constraints is violated in polynomial time and, if this is the case, output one such constraint as a violated inequality. This allows to focus on separation oracles for the other constraints. Then, the required one is given by the following lemma, an analogous of Lemma [4].

Lemma 3. If \( y \geq 0 \) is such that \( B^\top y = 0 \), then there exists \( \mu \) encoding a product distribution \( \mu \in \Delta_{\Pi} \) such that \( \mu^\top A_i^\top y = 0 \). Moreover, \( \mu \) can be computed in poly-time.

The proof of Lemma [5] follows the same line as that of Lemma 5 by Huang and von Stengel (2008) (see Huang 2011 for its complete version) and it is based on the CE existence proof by Hart and Schmeidler (1989).

Discussion and Future Works

We started the study of trembling-hand perfection in sequential games with correlation, introducing the EFFCE as a refinement of the EFCE that amends its weaknesses off the equilibrium path. This paves the way to a new research line, raising novel game-theoretic and computational challenges.
As for EFPCEs, an open question is whether compact correlated strategy representations, like the EFCE-based correlation plan by [Von Stengel and Forges (2008)], are possible in some restricted settings, such as 2-player games without chance. This would enable the optimization over the set of EFPCEs in polynomial time. The main challenge raised by EFPCEs with respect to EFCEs is that the former require to reason about general, un-reduced strategy profiles.

Another possible future work is to extend our analysis to other CE-based solution concepts, such as the normal-form CE and the agent-form CE (see [Von Stengel and Forges 2008] for their definitions). This raises the interesting question of how different trembling-hand-based CEs are able to amend weaknesses off the equilibrium path.

Finally, an interesting direction is to consider different ways of refining CE-based equilibria in sequential games, such as, e.g., using quasi-perfection [Van Damme 1984].

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References
Aumann, R. J. 1974. Subjectivity and correlation in randomized strategies. Journal of mathematical Economics 1(1): 67–96.

Brown, N.; and Sandholm, T. 2018. Superhuman AI for heads-up no-limit poker: Libratus beats top professionals. Science 359(6374): 418–424.

Brown, N.; and Sandholm, T. 2019. Superhuman AI for multiplayer poker. Science 365(6456): 885–890.

Celli, A.; Marchesi, A.; Bianchi, T.; and Gatti, N. 2019. Learning to correlate in multi-player general-sum sequential games. In Advances in Neural Information Processing Systems, 13076–13086.

Celli, A.; Marchesi, A.; Farina, G.; and Gatti, N. 2020. No-regret learning dynamics for extensive-form correlated equilibrium. Advances in Neural Information Processing Systems 33.

Dhillon, A.; and Mertens, J. F. 1996. Perfect correlated equilibria. Journal of economic theory 68(2): 279–302.

Farina, G.; Bianchi, T.; and Sandholm, T. 2020. Coarse Correlation in Extensive-Form Games. In AAAI Conference on Artificial Intelligence, 1934–1941.

Farina, G.; and Gatti, N. 2017. Extensive-Form Perfect Equilibrium Computation in Two-Player Games. In AAAI Conference on Artificial Intelligence, 502–508.

Farina, G.; Gatti, N.; and Sandholm, T. 2018. Practical exact algorithm for trembling-hand equilibrium refinements in games. In Advances in Neural Information Processing Systems, 5039–5049.

Farina, G.; Kroer, C.; and Sandholm, T. 2017. Regret minimization in behaviorally-constrained zero-sum games. In International Conference on Machine Learning, 1107–1116.

Farina, G.; Ling, C. K.; Fang, F.; and Sandholm, T. 2019a. Correlation in Extensive-Form Games: Saddle-Point Formulation and Benchmarks. In Advances in Neural Information Processing Systems, 9229–9239.

Farina, G.; Ling, C. K.; Fang, F.; and Sandholm, T. 2019b. Efficient Regret Minimization Algorithm for Extensive-Form Correlated Equilibrium. In Advances in Neural Information Processing Systems, 5187–5197.

Farina, G.; Marchesi, A.; Kroer, C.; Gatti, N.; and Sandholm, T. 2018. Trembling-hand perfection in extensive-form games with commitment. In International Joint Conferences on Artificial Intelligence Organization, 233–239.

Farina, G.; and Sandholm, T. 2020. Polynomial-time computation of optimal correlated equilibria in two-player extensive-form games with public chance moves and beyond. Advances in Neural Information Processing Systems 33.

Gatti, N.; Gilli, M.; and Marchesi, A. 2020. A characterization of quasi-perfect equilibria. Games and Economic Behavior 122: 240–255.

Gordon, G. J.; Greenwald, A.; and Marks, C. 2008. No-regret learning in convex games. In International Conference on Machine Learning, 360–367.

Grötschel, M.; Lovász, L.; and Schrijver, A. 1993. Geometric algorithms and combinatorial optimization. Springer.

Hart, S.; and Mas-Colell, A. 2000. A simple adaptive procedure leading to correlated equilibrium. Econometrica 68(5): 1127–1150.

Hart, S.; and Schmeidler, D. 1989. Existence of correlated equilibria. Mathematics of Operations Research 14(1): 18–25.

Hillas, J.; and Kohlberg, E. 2002. Foundations of strategic equilibrium. Handbook of Game Theory with Economic Applications 3: 1597–1663.

Huang, W. 2011. Equilibrium computation for extensive games. Ph.D. thesis, Citeseer.

Huang, W.; and von Stengel, B. 2008. Computing an extensive-form correlated equilibrium in polynomial time. In International Workshop on Internet and Network Economics, 506–513. Springer.

Jiang, A. X.; and Leyton-Brown, K. 2015. Polynomial-time computation of exact correlated equilibrium in compact games. Games and Economic Behavior 91: 347–359.

Kroer, C.; Farina, G.; and Sandholm, T. 2017. Smoothing Method for Approximate Extensive-Form Perfect Equilibrium. In International Joint Conference on Artificial Intelligence, 295–301.

Marchesi, A.; Farina, G.; Kroer, C.; Gatti, N.; and Sandholm, T. 2019. Quasi-perfect stackelberg equilibrium. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 33, 2117–2124.

Miltersen, P. B.; and Sørensen, T. B. 2010. Computing a quasi-perfect equilibrium of a two-player game. Economic Theory 42(1): 175–192.
Nash, J. 1951. Non-cooperative games. *Annals of Mathematics* 286–295.

Papadimitriou, C. H.; and Roughgarden, T. 2008. Computing correlated equilibria in multi-player games. *Journal of the ACM (JACM)* 55(3): 1–29.

Selten, R. 1975. Reexamination of the perfectness concept for equilibrium points in extensive games. *International Journal of Game Theory* 4(1): 25–55.

Van Damme, E. 1984. A relation between perfect equilibria in extensive form games and proper equilibria in normal form games. *International Journal of Game Theory* 13(1): 1–13.

Van Damme, E. 1991. *Stability and Perfection of Nash Equilibria*, volume 339. Springer.

Vermeulen, D.; and Jansen, M. 1998. The reduced form of a game. *European Journal of Operational Research* 106(1): 204–211.

Von Stengel, B. 1996. Efficient computation of behavior strategies. *Games and Economic Behavior* 14(2): 220–246.

Von Stengel, B.; and Forges, F. 2008. Extensive-form correlated equilibrium: Definition and computational complexity. *Mathematics of Operations Research* 33(4): 1002–1022.
Appendix

A Sequence-Form Representation for Perfect-Recall EFGs

The number of pure strategies $|\Pi_i|$ of each player $i \in N$ may be exponentially large in the size of an EFG, preventing the development of scalable computational tools using them. Moreover, the same holds for reduced pure strategies, which only specify actions at infosets that are reachable given the player’s past moves. This problem is circumvented by the sequence form introduced by [Von Stengel (1996)], where each player selects a sequence of actions rather than a pure strategy. For any node $h \in H$, we let $\sigma_i(h)$ be the ordered sequence of actions of player $i \in N$ on the path from the root of the game tree to $h$. We recall that, given the perfect recall assumption, all nodes in an infoset $I \in I_i$ of player $i \in N$ define the same sequence $\sigma_i(I)$ of player $i$’s actions, i.e., it holds $\sigma_i(h) = \sigma_i(I)$ for all $h \in I$. Moreover, $\sigma_i(I)$ can be extended by any action $a \in A(I)$, defining a new player $i$’s sequence $\sigma_i(I)a$. Thus, by introducing the empty sequence to represent the paths in the game tree in which a player does not play, the set of sequences available to player $i$’s sequence $\sigma_i$’s expected utility when following recommendations is at least as large as the expected utility that any $(I, a, \mu_i)$-trigger agent for player $i$ can achieve (assuming the opponents’ do not deviate from recommendations).

For any $\mu \in \Delta_H$ and $(I, a, \mu_i)$-trigger agent, we define the probability of reaching a terminal node $z \in Z(I)$ as:

$$p_{\mu, \hat{\mu}_i}(z) := \sum_{\pi_i \in \Pi_i(I, a)} \sum_{\pi_{-i} \in \Pi_{-i}(z)} \mu(\pi_i, \pi_{-i}) \sum_{\hat{\pi}_i \in \Pi_i(z)} \hat{\mu}_i(\hat{\pi}_i) p_c(z),$$

which accounts for the fact that the agent follows recommendations until she receives the recommendation of playing $a$ at $I$, and, thus, she ‘gets triggered’ and plays according to $\hat{\pi}_i$ sampled from $\hat{\mu}_i$ from $I$ onwards. Moreover, the probability of reaching a terminal node $z \in Z$ when following the recommendations is defined as follows:

$$q_{\mu}(z) := \sum_{\pi \in \Pi(z)} \mu(\pi) p_c(z).$$

Then, the following lemma provides the trigger-agent-based characterization of EFCEs:

**Lemma 4** (Farina, Bianchi, and Sandholm (2020)). Given an EFG $\Gamma$, $\mu \in \Delta_H$ is an EFCE of $\Gamma$ if for every $i \in N$ and $(I, a, \mu_i)$-trigger agent for player $i$, it holds that:

$$\sum_{z \in Z(I, a)} q_{\mu}(z) u_i(z) \geq \sum_{z \in Z(I)} p_{\mu, \mu_i}(z) u_i(z).$$

B Characterization of EFCEs Using Trigger Agents

We provide a formal statement of the characterization of EFCEs based on trigger agents (see Definition 4, originally introduced by Gordon, Greenwald, and Marks (2008) and Farina et al. (2019a) (see also Farina, Bianchi, and Sandholm (2020) for a more general treatment). We recall that such characterization is based on the fact that $\mu \in \Delta_H$ is an EFCE if, for every $i \in N$, player $i$’s expected utility when following recommendations is at least as large as the expected utility that any $(I, a, \mu_i)$-trigger agent for player $i$ can achieve (assuming the opponents’ do not deviate from recommendations).

For any $\mu \in \Delta_H$ and $(I, a, \mu_i)$-trigger agent, we define the probability of reaching a terminal node $z \in Z(I)$ as:

$$p_{\mu, \mu_i}^{I, a}(z) := \sum_{\Pi_i(I, a)} \mu(\pi_i, \pi_{-i}) \sum_{\hat{\pi}_i \in \Pi_i(z)} \hat{\mu}_i(\hat{\pi}_i) p_c(z),$$

which accounts for the fact that the agent follows recommendations until she receives the recommendation of playing $a$ at $I$, and, thus, she ‘gets triggered’ and plays according to $\hat{\pi}_i$ sampled from $\hat{\mu}_i$ from $I$ onwards. Moreover, the probability of reaching a terminal node $z \in Z$ when following the recommendations is defined as follows:

$$q_{\mu}(z) := \sum_{\pi \in \Pi(z)} \mu(\pi) p_c(z).$$

C LP Formulation for the Set of EFCEs in $n$-Player EFGs

We show how to derive the LP formulation (Problem 1) for the set of EFCEs in $n$-player EFGs originally introduced by Huang and von Stengel (2008), using the characterization of EFCEs based on trigger agents (see Definition 4 and Lemma 4).

In the following, we assume that a probability distribution $\mu \in \Delta_H$ is encoded by means of variables $\mu(\pi)$, defined for $\pi \in \Pi$. For every player $i \in N$, infoset $I \in I_i$, and action $a \in A(I)$, we introduce a variable $u[i, I, a]$ representing player $i$’s expected utility when following the recommendation to play $a$ at infoset $I$. These variables are defined by the following constraints:

$$u[i, I, a] = \sum_{z \in Z(I, a)} \sum_{\pi \in \Pi(z)} \mu(\pi) p_c(z) u_i(z) \quad \forall i \in N, \forall I \in I_i, \forall a \in A(I).$$
Then, we need to introduce constraints which ensure that following recommendations guarantees a utility at least as large as that achieved by any \((I, a, \mu, J)\)-trigger agent. For every infoset \(J \in \mathcal{I}_t\) such that \(I \preceq J\), we introduce a variable \(v[i, I, a, J]\) that encodes the maximum expected utility obtained at infoset \(J\) by trigger agents associated with \(I\) and \(a\). We can recursively define variables \(v[i, I, a, J]\) as follows:

\[
v[i, I, a, J] \geq \sum_{z \in \mathbb{Z}^{+}(J, a')} \left( \sum_{\pi_i \in \Pi_i(I, a)} \mu[\pi_i, \pi_{-i}] \right) p_c(z)u_i(z) + \sum_{K \in C(J, a')} v[i, I, a, K]\]

\[
\forall i \in N, \forall I \in \mathcal{I}_t, \forall a \in A(I), \forall J \in \mathcal{I}_t : I \preceq J, \forall a' \in A(J),
\]

where we notice that the first summation is over the set of terminal nodes which are reachable from \(J\) by playing \(a'\) without traversing any other player’s infosets. The following incentive constraints complete the formulation:

\[
u[i, I, a, J] = \sum_{I'} v[i, I', a, I] \\
\forall i \in N, \forall I \in \mathcal{I}_t, \forall a \in A(I).
\]

A direct application of Lemma 4 and LP duality is enough to prove that Constraints (10), (11), and (12) correctly characterize the set of EFCEs (formally, it is enough to follow steps similar to those in the proof of Theorem 4, with the only difference that Constraints (13) in the inner maximization problems and the corresponding dual variables \(w[i, I, a, J, a']\) are missing).

By substituting the equalities in Constraints (10) and (12) into Constraints (11), we obtain the following set of linear constraints, which are equivalent to those introduced by Huang and von Stengel (2008):

\[
A\mu + B\nu \geq 0,
\]

where \(\mu\) is the vector whose components are the variables \(\mu[\pi]\) for \(\pi \in \Pi\), while \(\nu\) is the vector of variables \(v[i, I, a, J]\) indexed by \(i \in N, I \in \mathcal{I}_t, a \in A(I), J \in \mathcal{I}_t : I \preceq J\). Moreover, the matrices \(A\) and \(B\) encode the coefficients appearing in Constraints (10) and (11). Specifically, non-zero entries of \(A\) are products \(p_c(z)u_i(z)\), while those of \(B\) are either 1 or \(-1\).

### D Discussion on EFPCES and Un-Reduced Strategies

Next, we discuss the reasons why EFPCES need un-reduced strategy profiles in order to be defined consistently.

First, we remark that, as discussed by Von Stengel and Forges (2008), restricting the definition of probability distributions \(\mu\) to reduced strategy profiles (i.e., those in which each player’s pure strategy only specifies actions at infosets reachable given that player’s moves, see Vermeulen and Jansen [1998] for a formal definition) is sufficient for the characterization of the classical notions of correlated equilibria. Intuitively, the reason is that, at the equilibrium, each player follows recommendations issued by the correlation device, and, thus, the latter does not need to specify action recommendations for the player at those infosets that are never reached when following recommendations at the preceding infosets of the same player.

This is no longer the case if we introduce trembles in the game, which make all the infosets reachable with positive probability even when committing to following recommendations. As a result, the correlation device has to be ready to issue action recommendations everywhere in the game. Then, when defining EFPCES, we cannot restrict the attention to probability distributions over reduced strategy profiles, and un-reduced ones are necessary. The EFG in Figure [1] (Left) provides an example where un-reduced strategies are necessary to express EFPCES. As shown in the main text, any EFPCES of the game must recommend to play \(a\) at \(t\), while, at the same time, it is crucial to define recommendations also at infosets \(K\) and \(L\), in order to achieve optimality off the equilibrium path. Clearly, this is incompatible with reduced strategies, as any player 1’s reduced strategy prescribing \(a\) at \(t\) does not specify anything at infosets \(K\) and \(L\), which are unreachable when playing \(a\) at \(t\).

### E Detailed Examples of EFPCES

Consider the EFG in Figure [1] (Left) and lower bound functions \(\eta_t : A \to (0, 1)\) for \(t \in \mathbb{N}\), with \(\eta_t(a)\) converging to zero as \(t \to \infty\) for each \(a \in A\). First, let us notice that, without trembles, player 1 is always better off playing action \(a\) at the root infoset 1, since she can guarantee herself a utility of 1 by selecting \(c\) at the following infoset 1, while she can achieve at most a utility of \(\frac{1}{2}\) by playing \(b\). Thus, any EFPCES of the game (as well as any EFCE) must recommend \(a\) at \(t\) with probability 1, since there is no way player 1 can be incentivized to play \(b\). Then, in the sub-game reached when playing \(a\) at \(t\), it is easy to check that recommending the pairs of actions \((c, m)\), \((c, n)\), and \((d, m)\) each with probability \(\frac{1}{3}\) is an equilibrium, as each player has no incentive to deviate from each possible recommendation, even in presence of trembles. As an example, consider the case in which player 1 is told to play action \(c\) at \(J\). Then, by following the recommendations, she gets a utility equal to:

\[
\begin{align*}
2 \cdot \frac{1}{3} (1 - \eta_t(n)) (1 - \eta_t(d)) + 3 \cdot \frac{1}{3} (1 - \eta_t(n)) \eta_t(d) &+ 1 \cdot \frac{1}{3} \eta_t(n) (1 - \eta_t(d)) + 0 \cdot \frac{1}{3} \eta_t(n) \eta_t(d) + \\
1 \cdot \frac{1}{3} (1 - \eta_t(m)) (1 - \eta_t(d)) &+ 0 \cdot \frac{1}{3} (1 - \eta_t(m)) \eta_t(d) + 2 \cdot \frac{1}{3} \eta_t(m) (1 - \eta_t(d)) + 3 \cdot \frac{1}{3} \eta_t(m) \eta_t(d),
\end{align*}
\]
where the first sum is for the case in which \((c, m)\) is recommended, while the second one is for \((c, n)\). Each term appearing in a sum is for one of the four possible outcomes that may result when following recommendations subject to trembles. Instead, player 1’s utility if deviating to \(d\) at \(i\) is:

\[
\begin{align*}
3 \left( \frac{1}{3} (1 - \eta_t(n)) (1 - \eta_t(c)) + 2 \cdot \frac{1}{3} (1 - \eta_t(n)) \eta_t(c) + 0 \cdot \frac{1}{3} \eta_t(n) (1 - \eta_t(c)) + 1 \cdot \frac{1}{3} \eta_t(n) \eta_t(c) \right) \\
0 \left( \frac{1}{3} (1 - \eta_t(m)) (1 - \eta_t(c)) + 1 \cdot \frac{1}{3} (1 - \eta_t(m)) \eta_t(c) + 3 \cdot \frac{1}{3} \eta_t(m) (1 - \eta_t(c)) + 2 \cdot \frac{1}{3} \eta_t(m) \eta_t(c) \right).
\end{align*}
\]

A simple calculation shows that the first quantity is greater than or equal to the second one as the lower bounds approach zero. Analogous conditions hold for other recommendations at infosets \(x\) and \(j\). Notice that, when lower bounds are zero, the conditions above collapse to the classical incentive constraints for EFCE. The correlation device described up to this point is sufficient to define an EFCE, as recommendations at infosets \(y, k,\) and \(l\) are not relevant given that they do not influence players’ utilities at the equilibrium \((b\) is never recommended). However, in perturbed extended games, these infosets could be reached due to a tremble which happens with probability \(\eta_t(b)\), and, thus, recommendations at such infosets become relevant. Then, it is easy to check that player 2 must be told to play \(p\) at \(y\), because her utility is always 1 if she plays \(p\), while it is always 0 when playing \(o\). Moreover, with an analogous reasoning, player 1 must be recommended to play \(e\) at \(k\) and \(l\), respectively. As an example, consider the case in which player 1 is recommended to play \(e\) at \(k\). Then, her utility would be \(\frac{1}{2} \cdot (1 - \eta_t(f)) + 0 \cdot \eta_t(f)\), while she would get \(0 \cdot (1 - \eta_t(e)) + \frac{1}{2} \cdot \eta_t(e)\) by deviating to \(e\). Similar conditions hold for infosets \(y\) and \(l\). In conclusion, we can state that the following distribution \(\mu \in \Delta_{I_t}\) defines an EFCE:

\[
\mu(aceh, mp) = \mu(aceh, np) = \mu(adeh, mp) = \frac{1}{3}.
\]

Let us remark that this is not the only EFCE of the game, as there are other ways of correlating players’ behavior at infosets \(x\) and \(j\) while satisfying the required incentive constraints. For example, setting

\[
\mu(aceh, mp) = \mu(aceh, np) = \mu(adeh, mp) = \mu(adeh, np) = \frac{1}{4}
\]

defines a valid EFCE that results from a PE of the game (where players play uniform strategies at infosets \(x\) and \(j\)).

### F Proofs of Theorems and Lemmas

In this section, we provide the complete proofs of Theorems 1, 2, 3, 4, and Lemma 5.

#### Theorem 1. This relation holds: \(PE \subseteq EFCE \subseteq EFCE\).

**Proof.** Clearly, \(EFCE \subseteq EFCE\) holds since any PE of \(\Gamma^{ext}(\mu)\) is also an NE. As for the other direction, let \(\{\beta_t\}_{t \in N}\) be a PE of \(\Gamma^{ext}\) obtained for a sequence of perturbed games \(\{(\Gamma, \eta_t)\}_{t \in N}\) and a corresponding sequence of NEs in these games, namely \(\{\beta_t\}_{t \in N}\) for \(t \in N\), where each \(\beta_t\) is a well-defined player \(i\)’s behavior strategy in \((\Gamma, \eta_t)\), i.e., it holds \(\beta_i(a) \geq \eta_t(a)\) for all \(t \in N, i \in N,\) and \(a \in A_i\). Let \(\mu \in \Delta_{I_t}\) be such that, for every \(\pi \in \Pi\), it holds \(\mu(\pi) = \prod_{t \in N} \prod_{i \in I_t} \beta_i(\pi_i(I_t))\). Consider the extended game \(\Gamma^{ext}(\mu)\), where we denote with \(I^{ext}_t\) the set of all infosets of player \(i \in N\), one for each infoset \(I \subseteq I_t\) of \(\Gamma\) and possible combination of recommendations received by \(i\) at the infosets \(J \subseteq I_t\). Overloading the notation, for each infoset \(I \in I^{ext}_t\) of the extended game, we use \(I\) as well to denote the corresponding infoset in the original game. We also use \(A(I)\) as the set of actions available at \(I \in I^{ext}_t\). Let \(\{(\Gamma^{ext}(\mu), \eta_t)\}_{t \in N}\) be the sequence of perturbed extended games resulting from \(\{(\Gamma, \eta_t)\}_{t \in N}\). Furthermore, for each \(t \in N\) and player \(i \in N\), we define a player \(i\)’s behavior strategy for \((\Gamma^{ext}(\mu), \eta_t)\) such that, at each infoset \(I \in I^{ext}_t:\)

- all the residual probability given the lower bounds \(1 - \sum_{a \in A(I) : a \neq \pi_i(I)} \eta_t(a)\) is placed on the action \(\pi_i(I)\) which is recommended at \(I\); and
- all the other, non-recommended actions \(a \in A(I) : a \neq \pi_i(I)\) are played with probabilities equal to their corresponding lower bounds \(\eta_t(a)\).

Intuitively, these strategies encode the fact that players follow recommendations in the perturbed extended games \((\Gamma^{ext}(\mu), \eta_t)\), where trembles prevent them to perfectly obey to recommendations. Given the definition of \(\mu\) and the fact that each \(\{\beta_t\}_{t \in N}\) constitutes an NE for the perturbed game \((\Gamma, \eta_t)\), we can conclude that the behavior strategies defined above constitute NEs for the perturbed extended games \((\Gamma^{ext}(\mu), \eta_t)\). Thus, any limit point of the sequence defined by such behavior strategies for \(t \in N\) is a PE of \(\Gamma^{ext}(\mu)\). Moreover, by definition, any limit point prescribes to play recommended actions, which shows that \(\mu\) defines an EFCE of \(\Gamma\), proving that \(PE \subseteq EFCE\).

#### Theorem 2. The following relations hold:

- \(EFCE \notin NE \text{ and } NE \notin EFCE\);
- \(EFCE \cap NE = PE\).
Proof. Let us start with the first bullet point. We consider the EFG in Figure 1 (Left) in order to provide examples that prove the two relations. Notice that, in such game, player 1 is always better off playing action \( a \) at the first infoset \( I \), since she can guarantee herself to get at least 1 by playing \( c \) at \( I \), while she can achieve at most 1 by playing action \( b \). Then, it is easy to check that, in any NE of the game, the players play behavior strategies \( \beta_1 \) and \( \beta_2 \) such that:

- \( \beta_1(a) = 1 \) and \( \beta_1(b) = 0 \), while \( \beta_1(c) = \beta_1(d) = \frac{1}{2} \); and
- \( \beta_2(m) = \beta_2(n) = \frac{1}{2} \).

The players’ behavior at other infosets can be any, as it does not affect players’ utilities at the equilibrium (given that infosets \( Y, K, \) and \( L \) are never reached due to \( \beta_1(b) = 0 \)). As we have shown in the main text, one EFPCE of the game is the distribution \( \mu \in \Delta_\Pi \) such that

\[
\mu(aceh, mp) = \mu(adeh, mp) = \mu(aceh, np) = \frac{1}{3},
\]

which enforces each player to follow recommendations, even in presence of trembles. Clearly, this distribution \( \mu \) cannot come up from players’ behavior strategies, and, thus, it cannot result from an NE. This shows that EFPCE \( \not\subset \) NE. Moreover, notice that any NE such that \( \beta_1(f) > 0 \) cannot determine a distribution \( \mu \in \Delta_\Pi \) which is an EFPCE, since it would be the case that action \( f \) is recommended with positive probability when reaching infoset \( K \) (due to trembles). However, player 1 cannot have any incentive to follow such recommendation, as she can gain a utility of 1 instead of 0 by deviating to \( e \). This proves that NE \( \not\subset \) EFPCE.

As for the second bullet point, notice that all the EFPCEs \( \mu \in \Delta_\Pi \) which are also NEs must be such that \( \mu \) is obtained from some players’ behavior strategies defining an NE. As a result, by definition of EFPCE, we can conclude that such behavior strategies are indeed PEs.

Theorem 3. Given a perturbed extended game \((\Gamma^\text{ext}(\mu), \eta)\), following recommendations is an NE of the game if for every \( i \in N \) and \((I, a, \mu_i)\)-trigger agent for player \( i \), it holds that:

\[
\sum_{z \in Z(I)} \left[ \left( \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, \pi) \mu(\pi) \right) u_i(z) \right] \geq \sum_{z \in Z(I)} p^{n,I,a}_{\mu,\hat{\mu}_i}(z) u_i(z).
\]

Proof. Given the definitions of \( q^n_{\mu}(z) \), \( p^{n,I,a}_{\mu,\hat{\mu}_i}(z) \), and \( y^{n,I,a}_{\mu,\hat{\mu}_i}(z) \), following recommendations is an NE of \((\Gamma^\text{ext}(\mu), \eta)\) if for every \( i \in N \) and \((I, a, \hat{\mu}_i)\)-trigger agent for player \( i \), it holds that:

\[
\sum_{z \in Z} q^n_{\mu}(z) u_i(z) \geq \sum_{z \in Z \setminus Z(I)} q^n_{\mu}(z) u_i(z) + \sum_{z \in Z(I)} y^{n,I,a}_{\mu,\hat{\mu}_i}(z) u_i(z).
\]

Equivalently, we can write:

\[
\sum_{z \in Z(I)} q^n_{\mu}(z) u_i(z) \geq \sum_{z \in Z(I)} y^{n,I,a}_{\mu,\hat{\mu}_i}(z) u_i(z)
\]

\[
\sum_{z \in Z(I)} \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, \pi) \mu(\pi) + \sum_{\pi_i \in \Pi_i \setminus \Pi_i(a)} \xi^\eta(z, \pi) \mu(\pi) \]  

\[
\]  

\[
\sum_{z \in Z(I)} \left[ \left( \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, \pi) \mu(\pi) \right) u_i(z) \right] \geq \sum_{z \in Z(I)} p^{n,I,a}_{\mu,\hat{\mu}_i}(z) u_i(z),
\]

which proves the result.

Theorem 4. Given a perturbed extended game \((\Gamma^\text{ext}(\mu), \eta_1)\), if Constraints (3), (4), and (5) can be satisfied for the vector \( \mu \) of variables \( \mu[i] \) encoding the distribution \( \mu \), then following recommendations is an NE of \((\Gamma^\text{ext}(\mu), \eta_1)\).
Proof. By Theorem [3] following recommendations is an NE of $(\Gamma^{ext}(\mu), \eta_i)$ if the vector $\mu$ of variables $\mu[\pi]$ encoding the distribution $\mu$ satisfies the following constraints (form here on, we omit the subscript $t$ for the ease of notation):

$$\sum_{z \in Z(I)} \left[ \left( \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, \pi) \mu(\pi) \right) u_i(z) \right] = \sum_{z \in Z(I)} p^{n,I,a}_{\mu,\hat{\mu}_i}(z) u_i(z) \quad \forall i \in N, \forall I \in \mathcal{I}_i, \forall a \in A(I)$$

$$\hat{\mu}_i \in \arg \max_{\hat{\mu}_i \in \Delta_{\Pi_i(I)}} \left\{ \sum_{z \in Z(I)} p^{n,I,a}_{\mu,\hat{\mu}_i}(z) u_i(z) \right\} \quad \forall i \in N, \forall I \in \mathcal{I}_i, \forall a \in A(I),$$

where we replaced quantifications over all player $i$’s pure strategies $\hat{\mu}_i \in \Delta_{\Pi_i(I)}$ with inner maximizations, by introducing auxiliary variables $\hat{\mu}_i$ for each player $i \in N$, infoset $I \in \mathcal{I}_i$, and action $a \in A(I)$. Next, let us notice that, as long as the objective to be maximized in each inner problem is the sum $\sum_{z \in Z(I)} p^{n,I,a}_{\mu,\hat{\mu}_i}(z) u_i(z)$ (which only contains terms referred to terminal nodes reachable from $I$), strategies $\hat{\mu}_i \in \Delta_{\Pi_i(I)}$ can be replaced with realization plans $x_i : \Sigma \to [0, 1]$ such that $x_i(\sigma_i(I)) \equiv 1$ (i.e., where the probability of reaching infoset $I$ given player $i$’s moves is 1). This holds thanks to the equivalence between mixed strategies and realization plans (Von Stengel [1996]). As a result, for every player $i \in N$, infoset $I \in \mathcal{I}_i$, and action $a \in A(I)$, we can write each inner maximization problem as follows:

$$\max \sum_{z \in Z(I)} \left( \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, I, \pi) \mu(\pi) \right) u_i(z) x_i[\sigma_i(z)] \quad \text{s.t. } (13a)$$

$$x_i[\sigma_i(I)] = 1 \quad (13b)$$

$$x_i[\sigma_i(J)] = \sum_{a' \in A(J)} x_i[\sigma_i(J)a'] \quad \forall J \in \mathcal{I}_i : I \preceq J, \forall a' \in A(J) \quad (13c)$$

$$x_i[\sigma_i(J)a'] \geq \eta(a') x_i[\sigma_i(J)] \quad \forall J \in \mathcal{I}_i : I \preceq J, \forall a' \in A(J) \quad (13d)$$

$$x_i[\sigma_i(J)] \geq 0 \quad \forall J \in \mathcal{I}_i : I \preceq J, \forall a' \in A(J),$$

$$x_i[\sigma_i(J)a'] \geq 0 \quad \forall J \in \mathcal{I}_i : I \preceq J, \forall a' \in A(J),$$

where $x_i[\sigma_i(I)]$ and $x_i[\sigma_i(J)a']$ are variables encoding a player $i$’s realization plan restricted to sequences extending $\sigma_i(I)$ (these are the only variables needed, since Objective $[13a]$ does not depend on the realization plan probabilities of other sequences). We also notice that the trembles associated with player $i$’s actions at infosets $J \in \mathcal{I}_i : I \preceq J$ (managed by the terms $\xi^\eta(z, I, \pi_i)$ in the definition of $p^{n,I,a}_{\mu,\hat{\mu}_i}(z)$) are encoded by Constraints $[13d]$, which ensure that each action $a' \in A(J)$ is played with probability $\frac{x_i[\sigma_i(J)a']}{x_i[\sigma_i(J)]} \geq \eta(a')$ (given that the denominator is non-null). The dual of Problem $[13]$ reads as follows:

$$\min v[i, I, a, \emptyset] \quad \text{s.t. } (14a)$$

$$v[i, I, a, \emptyset] \geq v[i, I, a, J] + \sum_{a' \in A(J)} \eta(a') w[i, I, a, J, a'] \quad (14b)$$

$$v[i, I, a, J] - w[i, I, a, J, a'] \geq \sum_{z \in Z(J, a')} \left( \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, I, \pi) \mu(\pi) \right) u_i(z) +$$

$$+ \sum_{K \in C(J, a')} \left( v[i, I, a, K] + \sum_{a'' \in A(K)} \eta(a'') w[i, I, a, K, a''] \right) \quad \forall J \in \mathcal{I}_i : I \preceq J, \forall a' \in A(J) \quad (14c)$$

$$w[i, I, a, J, a'] \geq 0 \quad \forall J \in \mathcal{I}_i : I \preceq J, \forall a' \in A(J),$$

where $v[i, I, a, \emptyset]$ is the dual variable associated to Constraint $[13b]$, $v[i, I, a, J]$ for $J \in \mathcal{I}_i : I \preceq J$ are the dual variables associated to Constraints $[13c]$, and $w[i, I, a, J, a']$ for $J \in \mathcal{I}_i : I \preceq J$ and $a' \in A(J)$ are the dual variables associated to Constraints $[13d]$. By using the fact that variable $v[i, I, a, \emptyset]$ appears only in Constraint $[13b]$ and by changing sign to variables
$w[i, I, a, J, a']$, we can re-write Problem (14) as follows:

$$\min v[i, I, a, I] - \sum_{a' \in A(I)} \eta(a')w[i, I, a, I, a'] \quad \text{s.t.} \quad \text{(15a)}$$

$$\begin{align*}
v[i, I, a, J] - w[i, I, a, J, a'] & \geq \sum_{z \in Z^-(J,a')} \left( \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, I, \pi)\mu(\pi) \right) u_i(z) + \\
& + \sum_{K \in C(J,a')} \left( v[i, I, a, K] - \sum_{a'' \in A(K)} \eta(a'')w[i, I, a, K, a''] \right) \quad \forall J \in \mathcal{I}_i : I \leq J, \forall a' \in A(J) \\
w[i, I, a, J, a'] & \geq 0 \quad \forall J \in \mathcal{I}_i : I \leq J, \forall a' \in A(J).
\end{align*} \quad \text{(15b)}$$

Then, we can remove the inner maximization problems by enforcing strong duality, i.e., we add constraints equating Objective (13a) and Objective (15a). Noticing that Objective (13a) is equal to $\sum_{z \in Z(I)} \eta^\mu, I, a, \pi(z)u_i(z)$, we obtain the following set of linear constraints:

$$\sum_{z \in Z(I)} \left[ \left( \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, \pi)\mu(\pi) \right) u_i(z) \right] = v[i, I, a, I] - \sum_{a' \in A(I)} \eta(a')w[i, I, a, I, a'] \quad \forall i \in N, \forall I \in \mathcal{I}_i, \forall a \in A(I)$$

$$v[i, I, a, J] - w[i, I, a, J, a'] \geq \sum_{z \in Z^-(J,a')} \left( \sum_{\pi_i \in \Pi_i(a)} \xi^\eta(z, I, \pi)\mu(\pi) \right) u_i(z) + \\
+ \sum_{K \in C(J,a')} \left( v[i, I, a, K] - \sum_{a'' \in A(K)} \eta(a'')w[i, I, a, K, a''] \right) \quad \forall i \in N, \forall I \in \mathcal{I}_i, \forall a \in A(I), \forall J \in \mathcal{I}_i : I \leq J, \forall a' \in A(J)$$

$$w[i, I, a, J, a'] \geq 0 \quad \forall i \in N, \forall I \in \mathcal{I}_i, \forall a \in A(I), \forall J \in \mathcal{I}_i : I \leq J, \forall a' \in A(J).$$

By introducing variables $u[i, I, a]$ we get to the result. \qed

**Lemma 3.** If $y \geq 0$ is such that $B^\top y = 0$, then there exists $\mu$ encoding a product distribution $\mu \in \Delta_{I\Pi}$ such that $\mu^\top A_i^\top y = 0$. Moreover, $\mu$ can be computed in poly-time.

**Proof.** The proof follows the same line as the proof of Lemma 5 of Huang and von Stengel (2008) (its complete version can be found in Huang (2011)). This is an extension of the CE existence proof by Hart and Schmeidler (1989) to the case of EFCE. It is based on the construction of an auxiliary 2-player zero-sum EFG, where player 1 plays first by selecting a strategy profile $\pi \in \Pi$, and player 2 plays second by choosing an infoset $I \in I_i$ of some player $i \in N$, an action $a \in A(I)$, and a combinations of actions at following infosets $J \in I_i : I \leq J$ (intuitively, player 2 chooses a trigger agent corresponding to $I$ and $a$, together with a possible trigger agent’s behavior). It is easy to see that, for our Problem(7) variables in $y$ have the same meaning as in Lemma 5 of Huang and von Stengel (2008), i.e., they represent valid player 2’s strategies in the auxiliary game. This is because they satisfy the same linear restrictions $B^\top y = 0$. As a result, the only difference is in the coefficients of the exponentially-many constraints, which, in our case, are defined by the (perturbed) matrix $A_i$, rather than $A$. These define the payoffs in the auxiliary game. In particular, following steps analogous to those by Huang (2011) we can conclude that, in the auxiliary game, player 2’s expected payment to player 1 when the latter plays $\pi \in \Pi$ is given by the entry of $A_i^\top y$ corresponding to $\pi$. Then, the proof follows the same reasoning as that of Huang (2011) to prove the result. \qed