The Electroweak Phase Transition –
Standard and “Beyond”

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Abstract.
We first review why a strongly first-order phase transition needed for baryogenesis is excluded in the electroweak standard model. We also comment on some intriguing effects in the strongly interacting hot phase. In the MSSM with a light stop a strongly first-order phase transition can be achieved. It possibly proceeds in two stages.

1. Introduction

Heating up the electroweak matter of the standard model (SM) a phase transition (PT) is expected at a temperature $T_c \sim 10^{15} K (\sim 100 \text{ GeV})$ since the positive plasma mass $\sim (g_w T)^2$ of the Higgs field switches the sign of the Higgs field mass. In this simple picture the Higgs mechanism is suspended at high temperatures, the Higgs-field VEV becomes zero; naively in the “hot phase” the transversal $W$-bosons would be massless.

Such a PT should have occurred in the early universe at about $10^{-12}$ sec after the big bang. Most of the interest in it in the last years came from the observation that the electroweak interactions violate the baryon number $B$: At $T = 0$ the instanton tunneling between topologically different vacua related to $B$-violation is an immeasurably small effect $\sim e^{-8\pi^2/g_w^2}$ (unless perhaps strongly enhanced in multi-$W$ production). For $T$ below $T_c$ a

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B-violating thermodynamical transition between states with neighbouring topological quantum numbers $N_{cs}$ via an unstable 3-dimensional sphaleron configuration is Boltzmann-suppressed and has a rate/volume

$$\Gamma^\text{Higgs Phase}_B \sim (\alpha_w T)^4 e^{-S_3/T}. \quad (1)$$

$S_3$ is the sphaleron action, which can be rescaled as $v(T)/g_w \cdot 2\pi A$ where $v(T)$ is the $T$-dependent Higgs VeV and $A$ a number ($\sim 3$) only slowly varying with the electroweak parameters.

For the large $T$ of the hot phase it is argued that there is no Boltzmann suppression and that

$$\Gamma^\text{Hot Phase}_B \approx K \alpha_n^4 T^4. \quad (2)$$

(The prefactor $K$ and the power $n$ are under discussion).

Baryon minus lepton number $B - L$ is conserved in electroweak interactions. If it is zero in the very early universe, it stays zero cooling down to the PT. During the equilibrium period before the PT $B$ as well as $L$ is washed out in this case. The observed baryon asymmetry $\Delta B/\nu \gamma \sim 10^{-10}$ should then be produced during the PT. Indeed the three necessary criteria of Sakharov for producing a $B$-asymmetry may be fulfilled in the SM:

(i)baryon-number violation, (ii) $C, CP$ violation, and (iii) nonequilibrium. Concentrating on the last point in this talk we see that a first-order PT is needed.

The most attractive way to produce a $B$-asymmetry is the “charge transport mechanism”. The walls of the expanding bubbles of condensing electroweak matter contain a $C, CP$ violating phase factor and like a diaphragm produce axial charge which is transported into the hot phase in front of the bubble. There it is converted into a baryonic charge by the “hot sphaleron”—transition before the Higgs phase bubble takes over. Thus first the B-asymmetry should be produced in the PT, then it should quickly “freeze out” in the Higgs phase. Therefore the baryon number violating rate in the Higgs phase should be small compared to the inverse Hubble time $H \sim T^2/m_{\text{Planck}} \sim e^{-40} T_c$ (for $T_c \sim 100$ GeV) from where it follows that $v(T_c)/T_c \geq 1.3$ in the Boltzmann factor in (1). Thus one needs a strongly first-order PT.

2. The electroweak phase transition in the SM

To learn about the electroweak PT one conveniently inspects an equilibrium quantity, the effective Higgs potential. The weak coupling $g_w$ is small but at high temperatures $T$ the relevant expansion parameter $g_3^2$ is the 3-dimensional gauge coupling $g_3^2 \sim g_w^2 T$ divided by some infrared (IR) scale $m_{1R}, g_3^2 = g_3^2/m_{1R}$. The most elegant way...
of separating the infrared problem is the reduction from a 4- to a time-
independent 3-dimension effective theory containing the light degrees of
freedom. Restricting the form of the effective action (derivative expansion)
the “matching” of 4- and 3-dimensional amplitudes allows to fix
its parameters. This procedure is purely perturbative and has been carried
through to 2-loop order. This loop order is necessary if one wants a few
percent accuracy ($O(g^3)$) [12]. In a first step all $n \neq 0$ Matsubara modes
with $m_n = 2\pi n T$ thus including all fermion modes ($n$ half integer) are
integrated out in the above sense. In a second step also the longitudinal
gauge bosons which have obtained a Debye mass $\sim g_w T$ in the first step
are integrated out. One ends up with the effective Langrangian

$$L_{3}^{\text{eff}} = \frac{1}{4} F_{ik}^{\mu 2} + (D_{i}^{\mu} H)^{+} (D_{i}^{\mu} H) + m_{H}^{2} H^{+} H + \lambda_{H}(H^{+} H)^{2}.$$ (3)

The U(1) part and Weinberg mixing can be neglected in this discussion
without loosing an essential point. The SU(2)-Yang-Mills Lagrangian and
the covariant derivative $D_{i}^{\mu}$ contain the gauge coupling $g_{3}^{2} = g_{w}^{2} T (1 + ...
). The Higgs parameters $m_{H}^{2}(T)$ and $\lambda_{H}(T)$ depend on the temperature.
They can be made dimensionless in the ratios

$$y = \frac{m_{H}^{2}(T)}{(g_{3}^{2})^{2}}, \quad x = \frac{\lambda_{H}(T)}{g_{3}^{4}}.$$ (4)

$y$ is related to $T - T_c$ and $x (\sim \lambda T / g_{w}^{2}$ in terms of 4-dimensional quantities) determines the nature of the phase transition. $L_{3}^{\text{eff}}$ characterizes a whole
class of theories. The specific properties of the 4-dimensional theory only
enter via the computation of $y, x$.

$L_{3}^{\text{eff}}$ as it stands as a tree level theory would give a 2nd-order PT.
But of course it has to be studied in higher perturbative order and more
than that it has to be treated as a potentially strongly interacting QFT
like QCD because of its IR behaviour. Thus the most secure way is to
discretize it on a lattice and to discuss the results of lattice calculations
[12, 13]. Still perturbation theory can provide some interesting insights.
The simple one-loop $W$-boson exchange graph in a constant Higgs field
background $\phi$ (3-dimensional, with $\phi_{4} = T^{1/2} \phi$) shown below contributes
the well-known term

$$V_{3}^{\phi^{3}} = -\frac{1}{24\pi} (g_{w}^{2} T \phi^{+} \phi)^{3/2} = -E (T \phi^{+} \phi)^{3/2}$$ (5)
to $V_{3}^{\text{eff}}$ leading already to a first-order PT. At the Higgs phase minimum $v(T)$ perturbation theory in 2-loop order (figs. 1,2) indeed compares very well with lattice results if $\tilde{g}_{3}^{2} = g_{3}^{2}/v(T) \sim cx$ is sufficiently small \cite{12,13}. Even the critical temperature $T_{c}$ which one obtains by comparing the Higgs $\phi = v(T)$ and the $\phi = 0$ IR sensitive minimum, can be determined quite reliably this way for $x \leq 0.08$. Most sensitive is the surface tension of the critical bubble where we observe \cite{12}, \cite{18} (fig. 3) strong deviations of the perturbative calculation from lattice results for $x > 0.05$.

An effective potential $V^{\text{eff}}(\phi)$ and also the Z-factor of the kinetic term is needed if one wants to calculate critical bubbles \cite{19} and sphaleron field configurations \cite{5}. Thus some perturbative expression enlarged by some nonperturbative piece is desirable.

Most excitingly, lattice calculations \cite{18} show a “crossover” behaviour (qualitatively predicted in ref. \cite{21}, \cite{22}) in this phase diagram for $x \geq 0.11$ (corresponding to $m_{H} \geq m_{W}$): The first order PT fades away at such values of $x$. Of course a strongly first-order PT with $v(T_{c})/T_{c} > 1$ avoiding the sphaleron erasure of $B$-asymmetry requires $x \leq 0.03 - 0.04$ much below the crossover. Indeed, a careful perturbative analysis of $x$ in terms of the physical Higgs mass $m_{H}$ and the top mass gives \cite{10}

$$x \sim \frac{1}{8} \frac{m_{H}^{2}}{m_{W}^{2}} + c \frac{m_{\text{top}}^{4}}{m_{W}^{4}} \quad (6)$$

and shows that the above limit excludes SM baryogenesis for any $m_{H}$!

Still the analysis of the SM at high $T$ based on the Lagrangian (3) and its lattice regularization leads to results very interesting by themselves: In the hot phase there is a confining linear potential – about the same as in pure YM theory \cite{13}. On the lattice one can measure a rich spectrum (fig. 4) of $W$-balls and of “Higgs hadrons” which are QCD-type bound states of Higgses rather than quarks. One should keep in mind that the masses of these bound states are 3-dimensional correlation masses. These masses can also be calculated \cite{23} in a relativistic bound-state model with a confining potential and compare very well with lattice results. One can call spin 0 and spin 1 states Higgses and massive $W$-bosons respectively, but one should emphasize that the hot phase is not another Higgs phase although there are no massless vector bosons as in the naive picture mentioned in the introduction. It would be interesting to have a complete model of these bound states in the whole phase diagram ($y$ versus $x$) and to compare it with lattice results. It is also desirable to have a concrete model \cite{23} of how the perturbative effective potential is modified by a nontrivial vacuum structure including gauge-field condensates at small values of $\phi$. This is particularly important if the perturbative potential and its Higgs minimum are small as they are in the case of $x$ values in the crossover region and beyond.
Figure 1. The 1- (lower) and 2-loop (upper curves) effective potential at $T_c$ for $x = 0.12$ in units of $v(T)$ in different $\xi$-covariant background gauges (from ref. [14]). The phase transition becomes stronger and the gauge dependence diminishes in the 2-loop results.

Figure 2. $g_s^2(T_c)/(g_w v(T_c)) \sim g_w T_c / v(T_c)$ as a function of $x$ (1-loop: upper, 2-loop: lower curves) (ref. [14]); for $x > 0.04$ one has $v(T_c)/T_c < 1$. 
Figure 3. from ref. [15]) The perturbatively calculated interface tension $\sigma$ (including $Z$-factor effect and gauge variations) vs. $x$ compared to lattice data from ref. [12] (squares), ref. [16] (triangles) and ref. [17] (circles).

Figure 4. (ref. [20]) Lattice results for the correlation masses for $0^{++}$ \((J^{PC})\), $1^{--}$ and $2^{++}$ operators (at $x = 0.0239$). Dark points indicate purely "gluonic" operators. Whereas in the Higgs phase only $H$, $W$ and multiple ($2W, \ldots$) are seen, in the hot phase one observes confinement and a completely different massive spectrum.
We conclude that the high T electroweak standard theory does not provide a first-order phase transition which is strong enough for baryogenesis and that it even vanishes for $m_H \gtrsim m_W$. It is also very questionable if standard CP violation is large enough. Still a lot of know-how also concerning $B$-asymmetry production has accumulated. Thus if one does not want to go back to $(B-L)$ violating asymmetry production in GU theories, it might be attractive to consider variants of the SM.

3. Variants of the SM, the MSSM with a light stop

It is widely accepted that the SM is an effective theory to be embedded in a deeper theory including the Planck scale. It is not clear, however, if at all and how it should be varied at the electroweak scale considering the great experimental success of the SM. Taking a pragmatic view baryogenesis at the electroweak scale requires a strongly first-order PT with $v(T_c)/T_c > 1$. In 1-loop order one has $v(T)/T \sim E/\lambda_T$, where $E$ is defined in eq. (5). This can be increased by

(i) increasing the $E$-prefactor of the “$\phi^3$-term” having more light bosons in the loop. (below (5))

(ii) decreasing the coupling $\lambda_T = \lambda_3/T$.

There are further points to be mentioned:

(iii) The 2-loop contributions to $V_3$ are very important and, in the SM, lead to a considerable strengthening of the PT (fig. 1).

(iv) A delay in the PT towards lower $T_c$ strengthens the PT.

(v) The rescaled sphaleron factor $A$ mentioned after Eq. (1) may be increased in some variants.

(vi) Models with a tree level $\phi^3$-type term (like in NMSSMs) may be interesting.

Here we concentrate on the first point and argue [24]-[30] that in the Minimal Supersymmetric SM (MSSM) the $\phi^3$-type term in the effective potential is increased if $\tilde{t}_R$, the superpartner of the right handed top, is rather light. This enhancement is due to a diagram as shown below eq. (3) but now with a stop in the loop. Its coupling to the $\phi^2$ background is given by the Yukawa coupling $h^2_t$. To strengthen this term $h_t$ should be large and the thermal mass

$$m^2_{u_3} \sim m^2_u + cT^2$$

of $\tilde{t}_R$ should be small in the $\phi=0$-phase. The physical $\tilde{t}_R$ mass is $m^2_{\tilde{t}_R} = m^2_u + m^2_{\tilde{t}_{top}}$, where $m^2_u$ is the SUSY-breaking scalar mass$^2$ and $m_{\tilde{t}_{top}} \sim h_t^2 \phi^2$. In renormalisation group equations starting with some $m^2_u$ at the Planck scale one indeed observes a decrease in $m^2_u$ much stronger than for other particles. Large $h_t$ means rather small $\tan \beta = v_1/v_2$. It is also convenient
to make one Higgs doublet heavy postulating a large axial Higgs mass $m_a$. The Higgs mass $m_H$ is then fixed by $\tan \beta$ and $m_a$.

If now the SUSY-partner particles and one Higgs doublet combination are heavy enough, they can be “integrated out”, and one ends up at the same type of 3-dimensional theory (3) but with different relations between 4- and 3-dimensional parameters [10, 28, 29, 30]. It turns out that one can get $v(T_c)/T_c > 1$ for $m_H \lesssim 70/75$ GeV (fig. 5). One can even go further and discuss a very small or even negative $m_{2u}$. However, to do this properly [31], the stop field $U = \tilde{t}_R$ cannot be integrated out. It has to be included in the 3-dimensional Lagrangian adding to (3) a term

$$L_{3\text{stop}} = \frac{1}{4} G_{ik}^2 + (D_i^a U)^+(D_i^a U) + m_{u3}^2 U^+ U + \lambda_{u3} (U^+ U)^2 + \gamma_3 H^+ H U^+ U$$

where $G$ is the SU(3) Yang-Mills field strength and $D_i^a$ the color covariant derivative, $m_{u3}^2$ and $\lambda_{u3}$ are the mass and the coupling like in (3), and the last term is a mixing with, at tree level, $\gamma_3 \sim T h_t^2 \sin^2 \beta$. We have calculated the corresponding perturbative potential. 2-loop effects are important since

Figure 5. (from ref. [28]). The parameter $x = \lambda_3/g_3^2$ dependent on the axial mass $m_A$, the Higgs mass $m_k$ (resp. $\tan \beta$) and the SUSY breaking $m_u$ in the MSSM.
only in this order gluon and Higgs exchange in a $\tilde{t}_R$-loop come into play.

This leads to a much stronger PT. Going with $-m^2_{u_1} = \tilde{m}^2_u$ to 70 GeV, the upper limit for $m_H$ to obtain $v(T_c)/T_c > 1$ becomes $m_H \leq 100$ GeV (fig. 6), still avoiding an unstable Higgs vacuum at $T = 0$. Here we have put $\tilde{A}_t = 0$ (see [24] for its definition) for simplicity and in order to obtain maximal effect.

Figure 6. (from ref. [31]) The critical temperatures $T_c$ of the three transitions $0 \rightarrow \phi$, $0 \rightarrow \chi$, $\chi \rightarrow \phi$ for $\tan \beta = 3, 12$ (thin lines) and $\tan \beta = 5$ (thick lines). The two-stage transition would occur to the left of the crossing point of the three critical curves. Also the boundary $x \leq 0.04$ for strongly first-order PT $0 \rightarrow \phi$ is indicated.
Figure 7. (from ref. [31]) The 1- and 2-loop effective potentials in $<U_3> = \chi$ and $<H> = \phi$ directions ($\tan \beta = 5$). Here $m_{\tilde{t}_R} = 158.3$ GeV is chosen such that the critical temperatures $T^{2\text{-loop}}_c$ of the two-phase transitions are equal.
Most interestingly we also can obtain\textsuperscript{[31]} a two-stage (figs. 6, 7) PT in a certain range of $m_{t_{\tilde{t}}} \approx 155 - 160$ GeV. There is a first-order PT to the “colored” minimum (strictly speaking, in a gauge theory $<U^+U> \neq 0$ and not $<U> \neq 0$!) and then at lower $T$ a transition to the Higgs vacuum. This delays the second $B$-asymmetry generating PT towards lower $T$ and thus increases\textsuperscript{[31]} $v(T)/T$. But now one has to make sure that the transition rate does not become too small. Fortunately, in the intermediate phase the sphaleron is Boltzmann-unsuppressed contrary to the 2-Higgs-two-stage PT considered previously\textsuperscript{[32]} and thus allows strong $B$-violation.

All this is based on perturbation theory and should be checked by lattice calculations. Experience tells us that perturbative calculations are o.k. for a strongly first-order PT. Observing gauge and $\mu$-dependence in our calculation\textsuperscript{[32]} (fig. 7) as well as noting the steep tree level potential between the minima, doubts are, however, allowed.

In conclusion of the last part it can be said that the MSSM variant of the SM allows a strongly first-order PT for Higgs masses as large as 95 GeV. Even a two-stage PT is possible. Given such a strong PT it is of interest to develop further the machinery of producing $B$-asymmetry in front of expanding bubbles which also requires the discussion of $CP$ violation in the model.

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$V(\chi)/T^4$

- 2-loop, $\mu=T$, $\zeta=0$
- 2-loop, RG-improved
- 2-loop, $\zeta=1$
- 1-loop, $\mu=T$, $\zeta=0$

$T_c^{2\text{-}l}=92.43$ GeV
$T_c^{\zeta=1}=94.89$ GeV
$T_c^{\text{RG}}=93.30$ GeV
$T_c^{1\text{-}l}=97.93$ GeV
\[ V(\phi)/T^4 \]

- 2-loop, \( \mu=T, \xi=0 \)
- 2-loop, RG-improved
- 2-loop, \( \xi=1 \)
- 1-loop, \( \mu=T, \xi=0 \)

- \( T_{c^{2-l}} = 92.43 \text{ GeV} \)
- \( T_{c^{1-l}} = 96.50 \text{ GeV} \)
- \( T_{c^{RG}} = 94.13 \text{ GeV} \)
- \( T_{c^{\xi=1}} = 92.64 \text{ GeV} \)
confinement $m_3^2/g_3^4=0.089$

Higgs $m_3^2/g_3^4=-0.02$
$\tan \beta = 2$, $m_U = 150$
$m_h = 70$, $m_U = 150$
$m_h = 75$, $m_U = 50$
$m_h = 70$, $m_U = 50$

strong enough for baryogenesis
$\tan \beta = 12$ ($m_H \sim 97$ GeV)

$\tan \beta = 5$ ($m_H \sim 92$ GeV)

$\tan \beta = 3$ ($m_H \sim 82$ GeV)

strong enough for baryogenesis

$\tan \beta = \tan \beta$