A Piecewise Objective Probabilistic Optimization Approach as Decision Making for Supplier Selection and Inventory Management With Price Discount

Sutrisno Sutrisno, Diponegoro University, Indonesia*
https://orcid.org/0000-0003-1130-0044
Sunarsih Sunarsih, Diponegoro University, Indonesia
Widowati Widowati, Diponegoro University, Indonesia

ABSTRACT

A mathematical model in the form of a piecewise objective probabilistic optimization approach was proposed in this study as the new decision-making tool to solve supplier selection and inventory management problems. The focus was on price discount and uncertain parameters such as product demand, product defect rate, and late-delivery product rate, which were approached using random variables with some known probability distribution function. Meanwhile, the decision variables contained in the model include the product volume ordered by each supplier at each time for each product type and those stored in the inventory to minimize the total operational cost in the problem. The corresponding optimization problem was solved using a probabilistic programming algorithm via the LINGO optimization tool. The computational simulation showed the proposed model provided the optimal decision, and this means it can be used as a decision-making tool by industrial practitioners.

KEYWORDS

Inventory Management, Price Discount, Probabilistic Optimization, Probabilistic Parameter, Supplier Selection

INTRODUCTION

Manufacturing industries are continuously struggling to optimize operational costs to gain more profit due to the existence of several cost components such as procurement and inventory during the industrial manufacturing process. Procurement cost is incurred during the process of purchasing raw materials or products while inventory cost is required in the warehouse to store raw materials and products. However, without loss of the generality, all kinds of materials handled in this problem were referred to as products. A manufacturer commonly has several suppliers with different requirements such as the product price, transport cost, maximum capacity to supply the product in a time, product defect rate, and late delivery product rate (Ware et al., 2014). This means it is possible to optimize the procurement cost by selecting the optimal supplier or, in more advanced technique, determining the optimal product volume to be ordered from each supplier. Meanwhile, inventory cost can be optimized by storing only the minimal product volume to satisfy demand in the warehouse. However,
in some cases, the decision-maker may prefer to set inventory at some “secure” or “target” level and this means there is the need to determine the preference level which can be termed as the inventory tracking control problem (Ignaciuk & Wieczorek, 2019; Mahmoud et al., 2010).

The effort towards obtaining the best supplier and control the inventory have been reported by several case study researches conducted in different industrial fields such as automotive manufacturing (Jamil et al., 2013), banking (Onut & Tosun, 2014), electricity and power plant (Alam et al., 2012; Tan et al., 2014; Tsui & Wen, 2014), blood distribution (M. Kumar & Kumar, 2018), medicine (Fazli-Khalaf & Nemati, 2018), steel (Oroojeni Mohammad Javad et al., 2020), humanitarian and disaster logistics (Ghorbani & Ramezanian, 2020; Olarewaju et al., 2020), petrochemical industries (Mohseni et al., 2019), and several others. Moreover, the progress of mathematical modeling in the supplier selection problem and inventory management from the simple to recently developed ones is discussed in the literature review section of this paper. However, these problems were solved separately without any integration, for example, the supplier selection was addressed in (Alhourani & Saxena, 2019) without considering inventory management. The use of one decision-making model for some simple cases i.e. without discount works has also been reported in past researches (Sutrisno, Widowati, & Solikhin, 2016; Sutrisno & Wicaksono, 2015). Therefore, this article focused on the problem with the discount.

In this paper, a new approach involving the use of decision-making tools to solve supplier selection and inventory management problems containing price discounts and probabilistic parameters is developed. This involves the application of a mathematical optimization model in a class of probabilistic programming with a piecewise objective function after which computational experiments were conducted to assess the proposed model and determine the optimal decision.

LITERATURE REVIEW

Mathematical programming is the approach mostly used as a decision-making tool to solve optimization problems. It is designed by formulating an objective function to be minimized (or maximized) subject to some constraint functions presenting the conditions to be held, and by using some optimization algorithm to achieve the optimal decision. This means it is possible to solve the best supplier and inventory management problems in the supply chain using the mathematical optimization approach. For some elementary cases of supplier selection problem i.e. those with deterministic parameters and linear objective function, a linear program model was introduced (Raupp et al., 2010; Ware et al., 2014). Moreover, some slightly advanced models were developed in linear programming by integrating with other mathematical approaches such as network process (Abbasi et al., 2013; Kara, 2011; G. K. Kumar et al., 2018; Li et al., 2013; Pan et al., 2014; Zeydan et al., 2011), fuzzy-Delphi (Wu et al., 2013), evaluation of performance (Aksoy & Öztürk, 2011; Chaharsooghi & Ashrafi, 2014), fuzzy number (Izadikhah, 2012; Sutrisno, Widowati, & Munawwaroh, 2016), deteriorating item presence (Rastogi & Singh, 2018; Yadav & Swami, 2018), fast service condition (Alegoz & Yapicioglu, 2019), and price break condition (Duan & Ventura, 2019) but the problem solved did not contain inventory management and price discount. Supplier selection problems with the presence of quantity discount have been considered in several studies however without integrating it with inventory management, and were modelled as a simple optimization such as linear programming (Farzipoor Saen, 2009).

Applying some assumptions or conditions to a problem produces a different corresponding mathematical optimization model with the objective and constraint functions required to be adjusted to meet these conditions. In supplier selection or inventory management cases, this involved the development of models such as facility disruption assumption (Rafiei et al., 2013), piecewise holding cost condition (Sutrisno, Widowati, & Munawwaroh, 2016), deteriorating item presence (Rastogi & Singh, 2018; Yadav & Swami, 2018), fast service condition (Alegoz & Yapicioglu, 2019), and price break condition (Duan & Ventura, 2019) but these problems were solved in a deterministic environment. In an uncertain environment, a simple supplier selection problem with probabilistic parameters can be solved by implementing statistical estimation values such as maximum entropy
method (Alhourani & Saxena, 2019). Regarding the method to solve, inventory management problem was solved by using some approaches, for example, a classical method of queuing theory with the multi-supplier was employed in (Arda & Hennet, 2006), predictive control with known parameters in (Widowati et al., 2018), fuzzy optimization approach for deterministic case in (Pan et al., 2015), linear quadratic control for trajectory tracking purposes in (Luthfi et al., 2018), imperialist competitive algorithm (Alavi et al., 2016) and linear Gaussian control with uncertain demand in (Sutrisno, Widowati, & Tjahjana, 2018) but these problems were solved without any relation to supplier selection.

It is possible to solve supplier selection and inventory management problems concurrently by integrating them into one mathematical model and linking the product flow to produce an optimal decision in the one-time calculation. This has led to the development of some mathematical models under specific conditions, for example, under the fuzzy price (Sutrisno et al., 2017), fuzzy demand (Sutrisno, Widowati, Sunarsih, et al., 2018), and combination of both (Sutrisno, Widowati, & Tjahjana, 2018). These were further used to develop a more complicated model in this paper to handle more complex situations. The term “complex situations” refers to the discount on product price, transport cost, and holding cost, and the presence of several probabilistic parameters. In order to make decisions in an integrated way between supplier selection and inventory management, all involved entities including suppliers, manufacturers and warehouses need to share their data and calculating optimal decisions together. This situation belong to integrated supply chain and information system under ‘supply chain integration’ and ‘interorganisational systems’ clusters (Daneshvar Kakhki & Gargeya, 2019). Moreover, few articles on environmental effects of the industrial manufacturing process termed green supply chain management are available in several pieces of literature as reported in (Saade et al., 2019; Vörösmarty & Tátrai, 2018) and even though it was not discussed in this paper, it is an interesting subject for future research.

METHOD

Problem Definition

Assuming a manufacturer plans to purchase several types of products from different suppliers with diverse attributes for some finite future review time, probably daily, weekly, monthly, or yearly. In this situation, the supplier selection problem involves the challenges associated with selecting the best supplier for the demanded product. However, the term “best” does not necessarily mean just one supplier. Moreover, the decision variable involves the number of units of each product to be ordered from each supplier at a review time to minimize the total purchasing cost. It is also possible to store the product purchased at some review time in a warehouse with some holding costs to satisfy the demand at a future time. This means the product units to be stored in the warehouse is also a decision variable. Therefore, the following assumptions were made in discussing the problem:

1. The products did not expire during the observation or optimization periods. This assumption is common in optimization based decision-making processes, and in practice, the number of observation time periods could be reduced in order to have all products will be not expired yet after the final observation time.
2. There is a possibility of damaged units in the product purchased and these are called the defect products with the volume presented as a rate or percentage of the whole. Values for this parameter at future time are obviously uncertain as they are naturally random. Therefore, these values were approached by using probability density function under the assumption that historical data were available. The decision-maker is expected to pay a cost as a penalty for losing defect products even after they have been removed.
3. Products were sent by the supplier and received by the manufacturer at the same review time immediately after the order is placed. However, suppliers were allowed to send some of the
products purchased at the following review time and not later. These are known as late-delivered product and the rate with respect to the products purchased is also naturally uncertain. Values for the rate of late deliveries at future time were also approached by using probability density function provided that historical data were available.

4. There was a discount for the purchasing and holding costs in the piecewise price scheme and this means price per unit is cheaper for some levels of product purchasing amount. This kind of discount scheme is practicable as shown in a number of previous works (see e.g. (Basa et al., 2020; Çabuk & Erol, 2019; Hamdan & Cheaitou, 2017; Tsai & Wang, 2010)).

5. The uncertain parameters in the problem were applied as a random variable with some known probability distribution function. This means a historical data was assumed to be available for uncertain parameters which were used to formulate the corresponding probability distribution function.

6. All parties involved in the problem have exchanged all updated information that needed by the proposed decision-making model to calculate optimal decisions. Moreover, then the recent observation time period has passed, new data were added in real time and all parties can get all data immediately to calculate new decisions.

7. All decision variables were assumed to be an integer which means the product was measured to follow integer numbers. However, in practice, some decision variables basically could be non-integers while they are measured with real numbers, such as liquid and agriculture products. However, in the end, the decision-maker could make them as integers by using integer measurements such as litre and kilogram.

**METHODOLOGY**

The problem-solving procedure used in this research is shown in Fig. 1. First, the common steps of mathematical modeling including problem identification, assumption composition, decision variables, and parameter identification were employed and declared in the Problem Definition section. The next step was objective function formulation which involved piecewise function with the adaptation of a discount scheme for the unit product price, holding cost, and transport cost. This was followed by the constraint functions formulation with details described in the Mathematical Model section. Next, for numerical experiment purposes, all the parameter data were generated and the optimization problem formulated solved by using Branch and Bound algorithm combined with the generalized reduced gradient optimization algorithm in the LINGO software tool. Finally, the optimal decision was interpreted from the computation results.

**MATHEMATICAL MODEL**

**Notations**

The mathematical notations used in the model are as follows:
indices:

\[ t \text{ : review time index;} \]
\[ p \text{ : product type index;} \]
\[ s \text{ : supplier alternative index;} \]
\[ i, j, k \text{ : discount level index;} \]

decision variables:

\[ X_{tp} \text{ : the amount (unit) of product type } p \text{ planned to be purchased from supplier } s \text{ at a review time } t; \]
\[ T_{tp} \text{ : the amount (unit) of product type } p \text{ planned to be stored in the warehouse at review time } t \text{ and used in a future period;} \]
\[ T_{ts} \text{ : number of deliveries from the supplier } s \text{ at a review time } t; \]
certain parameters:

\[ \begin{align*}
U_{tps}^{i} &: \text{ price at price level } i \text{ for one-unit product type } p \text{ to be purchased from supplier } s \text{ at a review time } t; \\
H_{tp}^{j} &: \text{ holding cost at discount level } j \text{ for one-unit product type } p \text{ to be stored in the inventory at a review time } t; \\
TC_{ts}^{k} &: \text{ transport cost at discount level } k \text{ for one-time delivery loading by supplier } s \text{ at a review time } t; \\
C_{tps} &: \text{ supplier } s' \text{'s maximum capacity to supply product type } p \text{ at a review time } t; \\
M_{tp} &: \text{ maximum capacity of the warehouse to store product type } p \text{ at a review time } t; \\
TRC &: \text{ maximum number for one-time delivery loading by each supplier and each time}; \\
P_{tps}^{c} &: \text{ Unit penalty cost for late delivered product } p \text{ from the supplier } s \text{ at time } t; \\
P_{tps}^{d} &: \text{ Unit penalty cost for defected product } p \text{ from supplier } s \text{ at time } t; \\
r_{tp} &: \text{ Setpoint of product } p \text{ at a time } t \text{ expected to be followed by the inventory level } I_{tp}; \\
\end{align*} \]

uncertain (probabilistic) parameters:

\[ \begin{align*}
\mathcal{L}_{tps} &: \text{ Random variable with some known probability distribution function } \mathcal{F}_{\mathcal{L}_{tps}} \text{ representing the late delivered product rate for product } p \text{ from supplier } s \text{ at a review time } t; \\
\mathcal{D}_{tps} &: \text{ Random variable with some known probability distribution function } \mathcal{F}_{\mathcal{D}_{tps}} \text{ representing the defect product rate for product } p \text{ from supplier } s \text{ at a review time } t; \\
\mathcal{D}_{tps} &: \text{ Random variable with some known probability distribution function } \mathcal{F}_{\mathcal{D}_{tps}} \text{ representing the demand amount of product } p \text{ at a review time } t. \\
\end{align*} \]
OPTIMIZATION MODEL

The discount scheme was according to the piecewise function and presented as follows for every review time $t$, supplier $s$, and product type $p$:

$$U_{tsp} = U_{tsp}^{(i)}, \quad \text{if} \quad X_{tsp}^{(i-1)} < X_{tsp}^{(i)} \leq X_{tsp}^{(i)}, \quad i = 1, 2, \ldots, k$$  \hspace{1cm} (1)

$$H_{tp} = H_{tp}^{(j)}, \quad \text{if} \quad T_{tp}^{(j-1)} < T_{tp}^{(j)} \leq T_{tp}^{(j)}, \quad j = 1, 2, \ldots, j;$$  \hspace{1cm} (2)

$$TC_{ts} = TC_{ts}^{(k)}, \quad \text{if} \quad T_{ts}^{(k-1)} < T_{ts}^{(k)} \leq T_{ts}^{(k)}, \quad k = 1, 2, \ldots, k;$$  \hspace{1cm} (3)

where $X_{tsp}^{(i)}$, $T_{tp}^{(j)}$, and $T_{ts}^{(k)}$ with $X_{tsp}^{(0)} = T_{tp}^{(0)} = T_{ts}^{(0)} = 0$ are known values representing the bound of the discount level. For example,

$$H_{tp} = \begin{cases} 
2, & \text{if } 0 < T_{tp} \leq 10, \\
1, & \text{if } 10 < T_{tp} \leq 100. 
\end{cases}$$

This means the holding cost $H_{tp}$ would be 2 per unit if the products stored in the inventory are not more than 10 units and 1 if it less or equal to 100 units. Meanwhile, the objective function was formulated as the expectation of the total cost consisting of five components explained as follows:

1. The total purchasing cost for all product types from all suppliers and at all review times was calculated as the amount of the purchased products $X_{tsp}$ times the corresponding unit price $U_{tsp}$ as shown in the following equation:

$$Z_1 = \sum_{t=1}^{T} \left( \sum_{i=1}^{S} \sum_{p=1}^{P} U_{tsp}^{(i)} \cdot X_{tsp} \right), \quad \text{if} \quad X_{tsp}^{(i-1)} < X_{tsp}^{(i)} \leq X_{tsp}^{(i)};$$

2. The total holding cost for all product types at all review times was calculated by as the amount of the stored product in the warehouse $T_{tp}$ times the corresponding holding cost $H_{tp}$ as shown in the following equation:

$$Z_2 = \sum_{t=1}^{T} \left( \sum_{p=1}^{P} H_{tp}^{(j)} \cdot T_{tp} \right), \quad \text{if} \quad T_{tp}^{(j-1)} < T_{tp} \leq T_{tp}^{(j)};$$

3. The total transportation cost from all suppliers at all review times was calculated as the number of delivery $T_{ts}$ times the transportation cost $TC_{ts}$ per one-time delivery as shown in the following relationship:
\[ Z_3 = \sum_{t=1}^{T} \left( \sum_{s=1}^{S} TC_s^{(k)} \cdot T_s \right), \text{ if } T_s^{(k-1)} \leq T_s \leq T_s^{(k)} ; \]

4. The total penalty cost expected for the late delivered products for all product types, suppliers, and at all review times was calculated by multiplying the amount of the late delivered products \( L_{ts} \cdot X_{ts} \) with the penalty cost per unit \( P_{ts}^L \) as represented in the following equation:

\[ Z_4 = E \left[ \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} P_{ts}^L \cdot L_{ts} \cdot X_{ts} \right] ; \]

5. The total penalty cost expected for defected products for all product types, suppliers, and at all review times was calculated as the amount of the defected products \( D_{ts} \cdot X_{ts} \) times the penalty cost \( P_{ts}^D \) as shown in the following equation:

\[ Z_5 = E \left[ \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{p=1}^{P} P_{ts}^D \cdot D_{ts} \cdot X_{ts} \right] ; \]

6. The gap or “error” between the normal and preferred inventory level decided by the decision-maker was defined as a quadratic form as follows:

\[ Z_6 = \sum_{t=1}^{T} \sum_{p=1}^{P} \left( I_{tp} - r_{tp} \right)^2 . \]

This objective function was used as an effort to drive the inventory level \( I_{tp} \) to the preference point \( r_{tp} \). Moreover, minimizing \( Z_6 \) means the \( I_{tp} \) decided should be as close as possible to \( r_{tp} \).

The mathematical optimization model was, therefore, formulated by combining all the cost components as shown in the following equation:

\[ \min Z = Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 \tag{4} \]

subject to the following constraint functions:

1. At review time \( t = 1 \), let the inventory level for all product types be zero and for each type, subtract the late delivered product, defect product and the amount of the product decided to be stored in the inventory from the purchased product to satisfy the demand value:

\[ \sum_{s=1}^{S} X_{ts} - \sum_{s=1}^{S} L_{ts} \cdot X_{ts} - \sum_{s=1}^{S} D_{ts} \cdot X_{ts} - I_{tp} \geq D_{tp}, \quad t = 1, \quad \forall p = 1, 2, ..., P ; \]
2. For review time \( t = 2,3,...,T \), for each product type, the inventory level at the previous review time should be added to the late delivered product at the previous review time, purchased product at the current review time while the late delivered product and defect product at the current review time as well as the amount of the product decided to be stored in the inventory was subtracted to satisfy the demand at the current review time:

\[
I_{t} = I_{t-1} + \sum_{s=1}^{S} L_{t-1} \cdot X_{t-1} + \sum_{s=1}^{S} X_{t} - \sum_{s=1}^{S} D_{s} \cdot X_{s} - I_{t}, \quad \forall t = 2,3,...,T, \quad \forall p = 1,...,P;
\]

3. The number of delivery loading at each review time from each supplier should not be more than those available:

\[
\frac{\sum_{p=1}^{P} X_{tsp}}{TRC} \leq T_{tsn}, \quad \forall t = 1,...,T, \quad \forall s = 1,...,S;
\]

where \([.]\) denotes a ceil function.

4. The inventory level should not exceed the warehouse capacity:

\[
I_{tp} \leq M_{tp}, \quad \forall t = 1,...,T, \quad \forall p = 1,...,P;
\]

5. The product ordered from a supplier should not be more than the maximum capacity:

\[
X_{tsp} \leq C_{tp}, \quad \forall t = 1,...,T, \quad \forall s = 1,...,S, \quad \forall p = 1,...,P;
\]

6. Each decision variable is an integer and non-negative:

\[
X_{tsp}, T_{ts}, I_{tp} \in \{0,1,2,...\}, \quad \forall t = 1,...,T, \quad \forall s = 1,...,S, \quad \forall p = 1,...,P.
\]

This model, however, has the following limitations that come from its assumptions and constraint functions:

1. Due to the uncertainty of parameters, the actual operational cost could be less or more than the cost provided by model. However, in the sense of the uncertain optimization, this is the best thing that the decision-maker could do in optimizing the problem.

2. The proposed model still leaves a number of decisions to be decided by the decision-maker itself. This includes the type of probability density functions for the uncertain parameters and the preferred “safe” inventory level. Therefore, decision-makers experiences would be needed in deciding the appropriate values.
3. In practical, the computational time to solve the optimization problem (4) could be longer than one review time period especially for big-scale problems, and possibly the optimal decision would be not available yet when it needs to be implemented. If so, this problem should be separated into several subproblems. However, more computational resource will be needed.

**NUMERICAL EXPERIMENT**

Numerical experiments were conducted in the laboratory with some randomly generated data to solve the problem. The specification of the computer used is important due to its effect on the computational time of the experiment. For example, a large-scale supplier selection problem involving hundreds of products and suppliers requires a complex optimization and a High-Performance Computer (HPC). However, this research made use of a personal computer with Windows 10 operating system, 4 GB memory size, and 2.8 GHz processor speed.

In order to solve the optimization problem (4), the codes were written in LINGO 18.0 programming language while a generalized reduced gradient was combined with the branch and bound algorithm inbuilt in the LINGO 18.0 software. The generalized reduced gradient was used for the corresponding quadratic programming while the branch and bound was to determine the integer solution of the decision variables. With due consideration for the problem defined in Section 3, there are four suppliers \( \{s_1, s_2, s_3, s_4\} \), three product types \( \{p_1, p_2, p_3\} \), and five review times \( t \in \{1, 2, 3, 4, 5\} \). Let \( \mathcal{N}(a, b) \) denotes the normal probability distribution with mean \( a \) and variance \( b \), the probability distribution function for the probabilistic parameters were designed as follows:

\[
L_{tp} \sim \mathcal{N}(0.02, 0.01), \quad D_{tp} \sim \mathcal{N}(0.03, 0.02), \quad \mathbb{D}_{tp} \sim \mathcal{N}(200, 20) \quad F_{tp},
\]

for every \( t = 1, \ldots, 5 \), \( s = 1, \ldots, 4 \), \( p = 1, \ldots, 3 \). there were three discount levels, DL1, DL2, and DL3 and the bound for each is stated as follows:

\[
U_{tp} = \begin{cases} 
U_{tp}^{(1)}, & \text{if } \mathcal{X}_{tp} \leq 10, \\
U_{tp}^{(2)}, & \text{if } 10 < \mathcal{X}_{tp} \leq 15, \\
U_{tp}^{(3)}, & \text{if } \mathcal{X}_{tp} > 15, 
\end{cases}
\]  

(5)

\[
H_{tp} = \begin{cases} 
H_{tp}^{(1)}, & \text{if } \mathcal{T}_{tp} \leq 5, \\
H_{tp}^{(2)}, & \text{if } 5 < \mathcal{T}_{tp} \leq 10, \\
H_{tp}^{(3)}, & \text{if } \mathcal{T}_{tp} > 10, 
\end{cases}
\]  

(6)

\[
TC_{ts} = \begin{cases} 
TC_{ts}^{(1)}, & \text{if } \mathcal{T}_s \leq 5, \\
TC_{ts}^{(2)}, & \text{if } 5 < \mathcal{T}_s \leq 10, \\
TC_{ts}^{(3)}, & \text{if } \mathcal{T}_s > 10, 
\end{cases}
\]  

(7)
where $U_{isp}^{(i)}$, $H_{iy}^{(i)}$, and $TC_{is}^{(i)}$ for all review times, supplier alternatives, and product types are provided in Tables 1, 2, and 3 respectively. The reference point for inventory level was set to zero i.e. $r_{tp} = 0, \forall t, \forall p$ and this means the decision-maker wanted the number of the product stored in the warehouse to be as less as possible. The maximum number of products for one-time delivery loading was 100 units and the value for all the remaining parameters is provided in Tables 4 and 5.

### Table 1. Product unit price with discount level

| Supplier | Product | P1     | P2     | P3     |
|----------|---------|--------|--------|--------|
|          |         | DL1    | DL2    | DL3    | DL1    | DL2    | DL3    | DL1    | DL2    |
| S1       | 20      | 19     | 18     | 30     | 28     | 28     | 50     | 48     | 49     |
| S2       | 22      | 21     | 20     | 30     | 29     | 28     | 49     | 48     | 48     |
| S3       | 20      | 20     | 18     | 30     | 29     | 28     | 50     | 48     | 48     |
| S4       | 20      | 18     | 18     | 30     | 29     | 28     | 50     | 49     | 49     |

### Table 2. Holding cost per unit with a discount level

| Product | Discount Level |
|---------|----------------|
|         | DL1  | DL2  | DL3  |
| P1      | 2.00 | 1.00 | 0.5  |
| P2      | 1.00 | 0.50 | 0.5  |
| P3      | 2.00 | 1.50 | 0.5  |

### Table 3. Transportation cost with discount level

| Supplier | Discount Level |
|----------|----------------|
|          | DL1  | DL2  | DL3  |
| S1       | 80   | 70   | 65   |
| S2       | 100  | 90   | 80   |
| S3       | 90   | 90   | 80   |
| S4       | 80   | 60   | 50   |

### Table 4. Values of some parameters

| Parameter         | Product |
|-------------------|---------|
|                   | P1      | P2      | P3      |
| Max Inventory (unit) | 20      | 20      | 20      |
| Defect Cost       | 2       | 3       | 2       |
| Delay Cost        | 0.5     | 1       | 0.5     |
The problem considered was solved for six review times which were split into two periods due to the computer’s capacity and computational time limit and this is the limitation of the experiment. The periods 1 to 2 were solved first and the inventory level obtained was used as the initial for 3 and 4 while the value recorded after this stage was also used as the initial for 5 and 6. Meanwhile, the demand value was generated randomly in the LINGO solver. Furthermore, four samples for each scenario and the optimal decision for scenario-1 and scenario-2 are shown in Figures 2-5. The minimal expected total cost for scenario-1 was 5395.786 at review times 1 to 2 plus 5354.476 for 3 to 4 and 5340.506 for 5 to 6 while the minimal expected objective function value for scenario-2 was 6860.167 for 1 to 2 and this was added to 6805.658 and 6788.843 for review times 3 to 4 and 5 to 6 respectively.

Figure 2 shows that at review time 1, the decision-maker ordered 15 units of P2 and 5 units of P3 from supplier S1, 4 units of P2 and 16 units of P3 from S2, 2 units of P2 and 18 units of P3 from S3, and 14 units of P1, 25 units of P2 and 17 units of P3 from supplier S4. Meanwhile, the number of products ordered from the supplier represents the discount level, for example, at review time 1, the 15 units of P2 from S1 means it was at a discount level 3 and the product price was, therefore, 18 per unit.

Figure 4 shows the proposed model for scenario-1 case decided to store 10 units for P2 and 19 units of P3 both at a discount level 3 with holding cost 0.5 per unit in the warehouse at review time 1. Meanwhile, the optimal decision for review time 2 was derived analogously from Figures 3 and 4 and this was the period the model decided not to store any product in the warehouse due to the experiment’s restriction. Therefore, the same results were recorded for review times 3 to 4 and 5 to 6.

Figure 2. Optimal number of each product type ordered from each supplier at each review time (scenario-1)
Figure 3. Optimal number of each product type ordered from each supplier at each review time (scenario-2)

Figure 4. Inventory level (scenario-1)

Figure 5. Inventory level (scenario-2)
The formulated model and simulation experiment results provided the following managerial insights:

1. It is possible to optimize any kind of supplier selection and inventory management problems in manufacturing, farming, and energy-storing industries and several others by implementing the formulated model with due consideration for the assumptions.
2. A decision-maker can modify the parameters used in the model such as the number of product types, supplier alternatives, and discount levels based on requirements.
3. In the simulation experiment, the reference point for inventory level was zero and this can be changed to another value to move the inventory to the desired point which is used as the reference level to prevent stock out.
4. The availability of a more powerful or high-performance computer is necessary to solve the model for all the review time without splitting for optimization.

CONCLUSIONS AND FURTHER WORKS

A decision-making tool was proposed in the form of a piecewise-objective probabilistic optimization model to solve supplier selection and inventory management problems with a discount scheme for the purchasing price, storing cost and transporting cost as well as the use of some uncertain parameters. A simulation-based experiment was conducted to show the working process of the proposed model and an optimal decision on the amount of the products to be purchased from suppliers and stored in the warehouse was achieved. Moreover, the minimal expected cost to be incurred was also determined. The proposed decision-making tool was tested with numerical experiments and found applicable in manufacturing industries.

Future studies should focus on more complicated problems involving fuzzy parameters, nonlinear discount schemes, etc. Moreover, other parties connected to supplier selection and inventory management problems in the supply chain such as carrier services, production plans, etc are also recommended to be integrated. Moreover, relating to Information Systems, when historical data are available online, this proposed model could be implemented as a real-time decision-making tool by gathering all information from all parties including suppliers, carriers, warehouses, and production units. This, however, still needs some works in order to make it working in an online environment.

ACKNOWLEDGMENT

This research was supported by Faculty of Science and Mathematics, Diponegoro University [grant number 1960/UN7.5.8/PP/2020].
REFERENCES

Abbasi, M., Hosnavi, R., & Tabrizi, B. (2013). An Integrated Structure for Supplier Selection and Configuration of Knowledge-Based Networks Using QFD, ANP, and Mixed-Integer Programming Model. *Journal of Industrial Engineering*, 2013, 1–8. doi:10.1155/2013/407573

Aksoy, A., & Öztürk, N. (2011). Supplier selection and performance evaluation in just-in-time production environments. *Expert Systems with Applications*, 38(5), 6351–6359. doi:10.1016/j.eswa.2010.11.104

Alam, M. B., Pulikki, R., Shahi, C., & Upadhyay, T. P. (2012). Economic Analysis of Biomass Supply Chains: A Case Study of Four Competing Bioenergy Power Plants in Northwestern Ontario. *ISRN Renewable Energy*, 2012, 1–12. 10.5402/2012/107397

Alavi, S., Azad, N., Heydar, M., & Davoudpour, H. (2016). Integrated production, inventory, and location-Allocation decisions in designing supply chain networks. *International Journal of Information Systems and Supply Chain Management*, 9(4), 22–42. doi:10.4018/IJISSCM.2016100102

Alegoz, M., & Yapicioglu, H. (2019). Supplier selection and order allocation decisions under quantity discount and fast service options. *Sustainable Production and Consumption*, 18, 179–189. doi:10.1016/j.spc.2019.02.006

Alhourani, F., & Saxena, U. (2019a). Supplier Selection Under Conditions of Uncertainty. *International Journal of Information Systems and Supply Chain Management*, 12(4), 42–54. doi:10.4018/IJISSCM.2019100103

Arda, Y., & Hennet, J. C. (2006). Inventory control in a multi-supplier system. *International Journal of Production Economics*, 104(2), 249–259. doi:10.1016/j.ijpe.2004.09.008

Bahroun, M., Harbi, S., & Bouchriha, H. (2019). A new approach for ordering decision under uncertainties: A case study in the retail supply chain. *International Journal of Logistics Systems and Management*, 32(3–4), 392–413. doi:10.1504/IJLSM.2019.098325

Basa, G., Becker, T., & Kedir, A. (2020). Single Item Supplier Selection and Order Allocation Problem with a Quantity Discount and Transportation Costs. *Momona Ethiopian Journal of Science*, 12(1), 20–38. doi:10.4314/mejs.v12i1.2

Çabuk, S., & Erol, R. (2019). Modeling and Analysis of Multiple-Supplier Selection Problem with Price Discounts and Routing Decisions. *Applied Sciences (Basel, Switzerland)*, 9(3480), 1–12. doi:10.3390/app9173480

Chaharsooghi, S. K., & Ashrafi, M. (2014). Sustainable Supplier Performance Evaluation and Selection with Neofuzzy TOPSIS Method. *International Scholarly Research Notices*, 2014(434168), 1–10. doi:10.1155/2014/434168 PMID:27379267

Daneshvar Kakhki, M., & Gargeya, V. B. (2019). Information systems for supply chain management: A systematic literature analysis. *International Journal of Production Research*, 57(15-16), 5318–5339. doi:10.1080/00207543.2019.1570376

Duan, L., & Ventura, J. A. (2019). A Dynamic Supplier Selection and Inventory Management Model for a Serial Supply Chain with a Novel Supplier Price Break Scheme and Flexible Time Periods. *European Journal of Operational Research*, 272(3), 979–998. doi:10.1016/j.ejor.2018.07.031

Esmaeili, M., & Ghobadi, S. N. (2018). A game theory model for pricing and supplier selection in a closed-loop supply chain. *International Journal of Procurement Management*, 11(4), 472–494. doi:10.1504/IJPM.2018.092771

Farzipoor Saen, R. (2009). Suppliers selection in volume discount environments in the presence of both cardinal and ordinal data. *International Journal of Information Systems and Supply Chain Management*, 2(1), 69–80. doi:10.4018/jisscm.2009010105

Fazli-Khalaf, M., & Nemati, N. G. (2018). A socially responsible supplier selection model under uncertainty: Case study of pharmaceutical department of an Iranian hospital. *International Journal of Logistics Systems and Management*, 32(1), 69–90. doi:10.1504/IJLSM.2019.097074

Ghorbani, M., & Ramezanian, R. (2020). Integration of carrier selection and supplier selection problem in humanitarian logistics. *Computers & Industrial Engineering*, 144(106473), 1–16. doi:10.1016/j.cie.2020.106473
Hamdan, S., & Cheaitou, A. (2017). Green supplier selection and order allocation with incremental quantity discounts. *International Conference on Modeling, Simulation, and Applied Optimization, ICMSAO 2017*. doi:10.1109/ICMSAO.2017.7934913

Ignaciuk, P., & Wieczorek, Ł. (2019). Networked Base-Stock Inventory Control in Complex Distribution Systems. *Mathematical Problems in Engineering*, 2019(3754367), 1–14. doi:10.1155/2019/3754367

Izadikhah, M. (2012). Group Decision Making Process for Supplier Selection with TOPSIS Method under Interval-Valued Intuitionistic Fuzzy Numbers. *Advances in Fuzzy Systems*, 2012, 1–14. doi:10.1155/2012/407942

Jamil, N., Besar, R., & Sim, H. K. (2013). A Study of Multicriteria Decision Making for Supplier Selection in Automotive Industry. *Journal of Industrial Engineering*, 2013(841584), 1–22. doi:10.1155/2013/841584

Kara, S. S. (2011). Expert Systems with Applications Supplier selection with an integrated methodology in unknown environment. *Expert Systems with Applications*, 38(3), 2133–2139. doi:10.1016/j.eswa.2010.07.154

Kumar, G. K., Rao, M. S., & Rao, V. V. S. K. (2018). Supplier Selection and Order Allocation in Supply Chain. *Materials Today: Proceedings*, 5(5), 12161–12173. doi:10.1016/j.matpr.2018.02.194

Kumar, M., & Kumar, D. (2018). Green logistics decision support system for blood distribution in time window. *International Journal of Logistics Systems and Management*, 31(3), 420–447. doi:10.1504/IJLSM.2018.095824

Li, Z., Wong, W. K., & Kwong, C. K. (2013). An Integrated Model of Material Supplier Selection and Order Allocation Using Fuzzy Extended AHP and Multiobjective Programming. *Mathematical Problems in Engineering*, 2013, 1–14. doi:10.1155/2013/363718

Lu, Z., Sun, X., Wang, Y., & Xu, C. (2019). Green supplier selection in straw biomass industry based on cloud model and possibility degree. *Journal of Cleaner Production*, 209, 995–1005. doi:10.1016/j.jclepro.2018.10.130

Luthfi, M. F., Sutrisno, & Widowati. (2018). Stock Control of Single Product Inventory System with Imperfect Delivery by Using Robust Linear Quadratic Regulator. *Proceedings - 2018 4th International Conference on Science and Technology, ICST 2018*. doi:10.1109/ICSTC.2018.8526862

Mahmoud, M. S., Al-Turki, U. M., & Selim, S. Z. (2010). Tracking policies for a class of dynamic production-inventory systems. *Journal of the Franklin Institute*, 347(9), 1689–1703. doi:10.1016/j.jfranklin.2010.08.003

Mohseni, M., Abdollahi, A., & Siadat, S. H. (2019). Sustainable Supply Chain Management Practices in Petrochemical Industry Using Interpretive Structural Modeling. *International Journal of Information Systems and Supply Chain Management*, 12(1), 22–50. doi:10.4018/IJISSCM.2019010102

Olanrewaju, O. G., Dong, Z. S., & Hu, S. (2020). Supplier selection decision making in disaster response. *Computers & Industrial Engineering*, 143(106412), 1–10. doi:10.1016/j.cie.2020.106412

Onut, S., & Tosun, S. (2014). An Integrated Methodology for Supplier Selection under the Presence of Vagueness: A Case in Banking Sector, Turkey. *Journal of Applied Mathematics*, 2014, 1–14. doi:10.1155/2014/283760

Oroojeni Mohammad Javad, M., Darvishi, M., & Oroojeni Mohammad Javad, A. (2020). Green supplier selection for the steel industry using BWM and fuzzy TOPSIS: A case study of Khouzestan steel company. *Sustainable Futures*, 2(100012), 1–11. doi:10.1016/j.sftr.2020.100012

Pan, W., Wang, F., Guo, Y., & Liu, S. (2015). A Fuzzy Multiobjective Model for Supplier Selection under Considering Stochastic Demand in a Supply Chain. *Mathematical Problems in Engineering*, 2015, 1–8. doi:10.1155/2015/174585

Rafiei, M., Mohammadi, M., & Torabi, S. A. (2013). Reliable multi period multi product supply chain design with facility disruption. *Decision Science Letters*, 2(2), 81–94. doi:10.5267/j.dsl.2013.02.002

Rastogi, M., & Singh, S. R. (2018). A production inventory model for deteriorating products with selling price dependent consumption rate and shortages under inflationary environment. *International Journal of Procurement Management*, 11(1), 36–52. doi:10.1504/IJPM.2018.088614

Raupp, F. M. P., Monteiro, M. M., & Leal, J. E. (2010). A four-type decision-variable MINLP model for a supply chain network design. *Mathematical Problems in Engineering*, 2010(450612), 1–16. doi:10.1155/2010/450612
Saade, R., Thoumy, M., & Sakr, O. (2019). Green supply chain management adoption in Lebanese manufacturing industries: An exploratory study. *International Journal of Logistics Systems and Management, 32*(3–4), 520–547. doi:10.1504/IJLSM.2019.098334

Sutrisno, W., & Wicaksono, P. A. (2015). Optimal Strategy for Multi-product Inventory System with Supplier Selection by Using Model Predictive Control. *Procedia Manufacturing, 4*, 208–215. doi:10.1016/j.promfg.2015.11.033

Sutrisno, W., & Munawwaroh, D. A. (2016). Hybrid mathematical model of inventory system with piecewise holding cost and its optimal strategy. *ICAMIMIA 2015 - International Conference on Advanced Mechatronics, Intelligent Manufacture, and Industrial Automation, Proceeding - In Conjunction with Industrial Mechatronics and Automation Exhibition, IMAE, 29–33*. 10.1109/ICAMIMIA.2015.7507996

Sutrisno, W., & Tjahjana, R. H. (2018). Fuzzy Expected Value Based Model to Solve Integrated Supplier Selection and Inventory Control Problem in Fuzzy Environment. *International Journal of Supply Chain Management, 7*(3), 24–30.

Sutrisno, W., Tjahjana, R. H. (2018). Single Product Inventory Control Considering Unknown Demand Using Linear Quadratic Gaussian. *2018 IEEE International Conference on Robotics, Biomimetics, and Intelligent Computational Systems (Robionetics)*, 17–20. doi:10.1109/ROBIONETICS.2018.8674677

Sutrisno, W., & Tjahjana, H. (2017). Expected value analysis for integrated supplier selection and inventory control of multi-product inventory system with fuzzy cost. *AIP Conference Proceedings*, (020038), 1–7. doi:10.1063/1.5016672

Sutrisno, W., Widowati, , & Solikhin, . (2016). Optimal Strategy for Integrated Dynamic Inventory Control and Supplier Selection in Unknown Environment via Stochastic Dynamic Programming. *Journal of Physics: Conference Series, 725*(1), 1–11. doi:10.1088/1742-6596/725/1/012008

Sutrisno, W., Widowati, , Sunarsih, , & Kartono, . (2018). Expected value based fuzzy programming approach to solve integrated supplier selection and inventory control problem with fuzzy demand. *IOP Conference Series. Materials Science and Engineering, 300*(012009), 1–8. doi:10.1088/1757-899X/300/1/012009

Tan, Z., Ju, L., Yu, X., Zhang, H., & Yu, C. (2014). Selection Ideal Coal Suppliers of Thermal Power Plants Using the Matter-Element Extension Model with Integrated Empowerment Method for Sustainability. *Mathematical Problems in Engineering, 2014*, 1–11. doi:10.1155/2014/302748

Tsai, W. C., & Wang, C. H. (2010). Decision making of sourcing and order allocation with price discounts. *Journal of Manufacturing Systems, 29*(1), 47–54. doi:10.1016/j.jmsy.2010.08.002

Tsui, C.-W., & Wen, U.-P. (2014). A Hybrid Multiple Criteria Group Decision-Making Approach for Green Supplier Selection in the TFT-LCD Industry. *Mathematical Problems in Engineering, 2014*, 1–13. doi:10.1155/2014/709872

Vörösmarty, G., & Tátrai, T. (2018). Green supply management in the public and private sector in Hungary. *International Journal of Procurement Management, 12*(1), 41–55. doi:10.1504/IJPM.2019.096996

Ware, N. R., Singh, S. P., & Banwet, D. K. (2014b). A mixed-integer non-linear program to model dynamic supplier selection problem. *Expert Systems with Applications, 41*(2), 671–678. doi:10.1016/j.eswa.2013.07.092

Widowati, T., Tjahjana, R. H., Sutrisno, , & Saputra, A. (2018). Robust model predictive control for inventory system with uncertain demand using linear matrix inequalities. *Journal of Physics: Conference Series, 1025*(1), 1–10. doi:10.1088/1742-6596/1025/1/012089

Wu, C.-M., Hsieh, C.-L., & Chang, K.-L. (2013). A Hybrid Multiple Criteria Decision Making Model for Supplier Selection. *Mathematical Problems in Engineering, 2013*(324283), 1–8. doi:10.1155/2013/324283

Yadav, A. S., & Swami, A. (2018). Integrated supply chain model for deteriorating items with linear stock dependent demand under imprecise and inflationary environment. *International Journal of Procurement Management, 11*(6), 684–704. doi:10.1504/IJPM.2018.095650

Zeydan, M., Çolpan, C., & Çobanoğlu, C. (2011). A combined methodology for supplier selection and performance evaluation. *Expert Systems with Applications, 38*(3), 2741–2751. doi:10.1016/j.eswa.2010.08.064