$J/\psi$ Suppression and Enhancement in Au+Au Collisions at the BNL RHIC

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Abstract

We consider the production of the $J/\psi$ mesons in heavy ion collisions at RHIC energies in the statistical coalescence model with an exact (canonical ensemble) charm conservation. The $c\bar{c}$ quark pairs are assumed to be created in the primary hard parton collisions, but the formation of the open and hidden charm particles takes place at the hadronization stage and follows the prescription of statistical mechanics. The dependence of the $J/\psi$ production on both the number of nucleon participants and the collision energy is studied. The model predicts the $J/\psi$ suppression for low energies, whereas at the highest RHIC energy the model reveals the $J/\psi$ enhancement.
The experimental program for studies of the charmonium production in nucleus–nucleus (A+A) collisions at CERN SPS over the last 15 year was mainly motivated by a suggestion of Matsui and Satz [1] to use the J/ψ as a probe of the state of matter in the early stage of the collision. The original picture [1] (see also Ref. [2] for a modern review) assumes that charmonia are produced in primary collisions of the nucleons from the colliding nuclei. The number of created charmonium states is then reduced because of inelastic interactions with the nucleons of the colliding nuclei. An additional suppression may occur due to J/ψ interaction with the secondary hadrons (‘co-movers’) [3]. The probability to destroy the charmonium state increases obviously with the number of nucleon participants \( N_p \). Similar behavior is expected when the collision energy \( \sqrt{s} \) increases because the number of produced hadrons (‘co-movers’) becomes larger. This is known as the normal J/ψ suppression. Furthermore, the J/ψ (and other charmonia) are assumed to be formed mainly from the c\( \bar{c} \) pairs with invariant mass below the D-meson threshold [4]. The fraction of these subthreshold pairs in the total number \( N_{c\bar{c}}^{\text{dir}} \) of c\( \bar{c} \) pairs (which is roughly proportional to the number of produced open charm hadrons (\( D, D^*, \Lambda_c \) etc.) also decreases with \( \sqrt{s} \). Therefore, the ratio

\[
R(N_p, \sqrt{s}) \equiv \frac{\langle J/\psi \rangle}{N_{c\bar{c}}^{\text{dir}}} \quad (1)
\]

is expected to decrease with increasing \( N_p \) and/or \( \sqrt{s} \). At large values of \( \sqrt{s} \) and \( N_p \) the formation of the quark-gluon plasma (QGP) is expected which is supposed to be signaled by the anomalous suppression [4] of the J/ψ, i.e. a sudden and strong decrease of the ratio (1) is considered as a signal of the QGP formation. Hence, a decrease of the ratio (1) is an unambiguous consequence of the standard picture [2,3].

A very different approach of the statistical J/ψ production, proposed in Ref. [5], assumes that J/ψ mesons are created at the hadronization stage similar to other (lighter) hadrons.

A picture of the J/ψ creation via c and \( \bar{c} \) coalescence (recombination) was subsequently developed within different model formulations [6–11]. Similar to the suggestion of Ref. [5], charmonium states are assumed to be created at the hadronization stage of the reaction, but they are formed due to the coalescence of c and \( \bar{c} \), which were produced by primary hard parton collisions at the initial stage.

In this paper the \( N_p \) and \( \sqrt{s} \) dependences of the ratio (1) will be studied for Au+Au collisions at RHIC energies. We use the canonical ensemble (c.e.) formulation of the statistical coalescence model (SCM) [8,11]. The number \( N_{c\bar{c}}^{\text{dir}} \) of the produced c\( \bar{c} \) pairs, which is the input for the SCM calculations, will be estimated within the perturbative QCD (pQCD). The considered pQCD+SCM approach reveals both the J/ψ suppression (at \( N_{c\bar{c}} < 1 \)) and the J/ψ enhancement (at \( N_{c\bar{c}} > 1 \)) effects.

In the framework of the ideal hadron gas (HG) model in the grand canonical ensemble (g.c.e.) formulation the hadron multiplicities are given by

\[
N_j = \frac{d_j V}{2\pi^2} \int_0^\infty k^2 dk \left[ \exp \left( \frac{\sqrt{m_j^2 + k^2} - \mu_j}{T} \right) \pm 1 \right]^{-1},
\]  

(2)
where $V$ and $T$ correspond to the volume and temperature of the HG, $m_j$ and $d_j$ denote particle masses and degeneracy factors. Eq. (2) describes the quantum HG: Bose and Fermi distributions for mesons and (anti)baryons, respectively. Quantum effects, however, are found to be noticeable for pions only, so that Eq. (2) for all other hadrons can be simplified to the Boltzmann result:

$$N_j = \frac{d_j}{2\pi^2} V \exp \left( \frac{\mu_j}{T} \right) \, T \, m_j^2 \, K_2 \left( \frac{m_j}{T} \right),$$  \hspace{1cm} (3)

where $K_2$ is the modified Bessel function.

In the case of complete chemical equilibrium the chemical potential $\mu_j$ in Eq. (2) is defined as

$$\mu_j = b_j \mu_B + q_j \mu_Q + s_j \mu_S + c_j \mu_C,$$  \hspace{1cm} (4)

where $b_j, q_j, s_j, c_j$ denote the baryonic number, electric charge, strangeness and charm of hadron $j$. The baryonic chemical potential regulates the non-zero (positive) baryonic density of the HG system created in A+A collision. The chemical potentials $\mu_S$ and $\mu_C$ should be found as the functions of $T$ and $\mu_B$ from the requirements of zero value for the total strangeness and charm in the system, and the chemical potential $\mu_Q$ from the requirement of the fixed ratio of the electric charge to the baryonic number (this ratio is defined by the numbers of protons and neutrons in the colliding nuclei).

The applications of the HG model to fitting the hadron abundances in particle and nuclear collisions revealed, however, a deviation of strange hadron multiplicities from the complete chemical equilibrium [15]. It was suggested that strange quarks and antiquarks are distributed inside hadrons according to the laws of HG equilibrium, but the total number of strange quarks and antiquarks inside the hadrons is smaller than that in the equilibrium HG and remains approximately constant during the lifetime of the HG phase. Therefore, not only the ”strange charge” $N_s - N_{\bar{s}} = 0$ but also the ”total strangeness” $N_s + N_{\bar{s}}$ is then considered as a conserved quantity. In the language of thermodynamics, it means an introduction of an additional chemical potential, $\mu_{|S|}$, which regulates this new ”conserved” number $N_s + N_{\bar{s}}$. Then the additional term, $(n_s^j + n_{\bar{s}}^j)\mu_{|S|}$, should appear in the expression (1) for $\mu_j$, where $n_s^j$ and $n_{\bar{s}}^j$ are the numbers of strange quarks and antiquarks inside hadron $j$. Introducing a notation, $\gamma_s \equiv \exp(\mu_{|S|}/T)$ [15], one can implement this additional conservation according to the following simple rule: the hadron multiplicities $N_j$ (2) are multiplied by a factor $\gamma_s^{(n_s^j + n_{\bar{s}}^j)}$, e.g. factor $\gamma_s$ appears for $K, \bar{K}, \Lambda, \bar{\Lambda}, \Sigma, \bar{\Sigma}$, factor $\gamma_s^2$ for $\Xi, \Xi$ and factor $\gamma_s^3$ for $\Omega, \bar{\Omega}$. For mesons with hidden strangeness, like $\eta, \eta', \omega, \phi$, having the wave function of the form

$$C_u|u\bar{u}\rangle + C_d|d\bar{d}\rangle + C_s|s\bar{s}\rangle$$  \hspace{1cm} (5)

1 To avoid complications we neglect the excluded volume corrections. The thermodynamical consistent way to treat the excluded volume effects was suggested in Ref. [12] (see also [13] for further details). If the excluded volume parameter is the same for all hadrons its effect is reduced only to the rescaling of the volume $V$: all particle number ratios remain the same as in the ideal hadron gas.
the factor $\gamma_s^2$ is used.

From fitting the data on the hadron yield in particle and nuclear collisions it was found that $\gamma_s \leq 1$ for all known cases. Parameter $\gamma_s$ is called therefore the strangeness suppression factor.

Recently an analogous procedure was suggested for charm hadrons \cite{7}. A new parameter $\gamma_c$ has been introduced to treat simultaneously both the open and hidden charm particles within statistical mechanics HG formulation. The multiplicities $N_j$ \cite{2} of single open charm and anticharm hadrons should be multiplied by the factor $\gamma_c$ and charmonium states by the factor $\gamma_c^2$. In contrast to the suppression of strangeness in the HG ($\gamma_s \leq 1$) one observes the enhancement of charm hadron yields in comparison to their equilibrium HG values. It means that $\gamma_c \geq 1$ and this parameter is called the charm enhancement factor \cite{7}.

To take into account the requirement of zero "charm charge" of the HG in the exact form the c.e. formulation was suggested in Ref. \cite{8}. In the c.e. formulation the charmonium multiplicities are still given by Eq.(3) as charmonium states have zero charm charge. The multiplicities \cite{9} of the open charm hadrons will, however, be multiplied by the additional ‘canonical suppression’ factor (see e.g. \cite{18}). This suppression factor is the same for all individual single charm states. It leads to the total open charm multiplicity $N_O^{ce}$ in the c.e.:

$$N_O^{ce} = N_O \frac{I_1(\gamma_c N_O)}{I_0(\gamma_c N_O)} ,$$

(6)

where $N_O$ is the total g.c.e. multiplicity of all charm and anticharm mesons and (anti)baryons calculated with Eq.(2) and $I_0, I_1$ are the modified Bessel functions. For $N_O << 1$ one has $I_1(\gamma_c N_O)/I_0(\gamma_c N_O) \simeq N_O/2$ and, therefore, the c.e. total open charm multiplicity is strongly suppressed in comparison to the g.c.e. result. For $N_O >> 1$ one finds $I_1(\gamma_c N_O)/I_0(\gamma_c N_O) \rightarrow 1$ and therefore $N_O^{ce} \rightarrow N_O$, i.e. the c.e. and the g.c.e results coincide. In high energy A+A collisions the total number of strange and antistrange hadron is much larger than unity. Hence the strangeness conservation can be considered in g.c.e. approach. The same is valid for baryonic number and electric charge. This is, however, not the case for the charm. At the SPS energies the c.e. suppression effects are always important: even in the most central Pb+Pb collisions the total number of open charm hadrons is expected to be smaller than one. It will be shown that the c.e. treatment of charm conservation remains crucially important also at the RHIC energies for the studies of the $N_p$ dependence of the open charm and charmonium production. Therefore, in what follows, the baryonic number, electric charge and strangeness of the HG system are treated according to the g.c.e. but charm is considered in the c.e. formulation where the exact "charm charge" conservation is imposed.

Hence we formulate our model as follows. The charm quark-antiquark pairs are assumed to be created at the early stage of A+A collisions. In the subsequent evolution of the system, the number of this pairs remains approximately constant and is not necessary equal to its equilibrium value. The deviation from the chemical equilibrium should be taken into account by the charm enhancement factor $\gamma_c$. The distribution of created $c\bar{c}$ pairs among open and hidden charm hadrons is regulated by statistical model according to Eq.(3) with account for the canonical suppression \cite{2}. So the statistical coalescence model in the c.e. is formulated as:

$$N^{\text{dir}}_{c\bar{c}} = \frac{1}{2} \gamma_c N_O \frac{I_1(\gamma_c N_O)}{I_0(\gamma_c N_O)} + \gamma_c^2 N_H ,$$

(7)
where $N_H$ is the total multiplicities of hadrons with hidden charm. (Note that the second term in the right-hand side of Eq.(7) gives only a tiny correction to the first term, i.e. most of the directly created $c\bar{c}$ pairs are transformed into the open charm hadrons.) To find $N_O$ and $N_H$ we use Eq.(3) for the individual hadron thermal multiplicities in the g.c.e. and take the summation over all known particles and resonances \[19\] with open and hidden charm, respectively.

Provided that $N_O$, $N_H$ and $N_{\text{dir}}^{c\bar{c}}$ are known, $\gamma_c$ can be found from Eq.(7). The $J/\psi$ multiplicity is then given by

$$\langle J/\psi \rangle = \gamma_c^2 N_{J/\psi}^{\text{tot}}.$$  

(8)

where $N_{J/\psi}^{\text{tot}}$ is given by

$$N_{J/\psi}^{\text{tot}} = N_{J/\psi} + \text{Br}(\psi') N_{\psi'} + \text{Br}(\chi_1) N_{\chi_1} + \text{Br}(\chi_2) N_{\chi_2},$$  

(9)

$N_{J/\psi}$, $N_{\psi'}$, $N_{\chi_1}$, $N_{\chi_2}$ are calculated according to Eq.(3) and $\text{Br}(\psi') \simeq 0.54$, $\text{Br}(\chi_1) \simeq 0.27$, $\text{Br}(\chi_2) \simeq 0.14$ are the decay branching ratios of the excited charmonium states into $J/\psi$.

Hence, to calculate the $J/\psi$ multiplicity in SCM we need the following information:

1) the chemical freeze-out (or hadronization) parameters $V,T,\mu_B$ for A+A collisions at high energies;
2) the number $N_{\text{dir}}^{c\bar{c}}$ of $c\bar{c}$-pairs created in hard parton collisions at the early stage of A+A reaction.

For the RHIC energies the chemical freeze-out temperature $T$ is expected to be close to that for the SPS energies: $T = 175 \pm 10$ MeV. To fix the unknown volume parameter $V$ and baryonic chemical potential $\mu_B$ we use the parametrization of the total pion multiplicity \[20\]:

$$\frac{\langle \pi \rangle}{N_p} \simeq C \frac{(\sqrt{s} - 2m_N)^{3/4}}{(\sqrt{s})^{1/4}}$$  

(10)

where $C = 1.46$ GeV$^{-1/2}$ and $m_N$ is nucleon mass. For the RHIC energies the nucleon mass in Eq.(10) can be neglected so that

$$\langle \pi \rangle \simeq C \ N_p \ (\sqrt{s})^{1/2}.$$  

(11)

Eq.(11) is an agreement with both the SPS data and the preliminary RHIC data in Au+Au collisions at $\sqrt{s} = 56$ GeV and $\sqrt{s} = 130$ GeV. The pion multiplicity (11) should be equated to the total HG pion multiplicity $N^{\text{tot}}_{\pi}$ which includes the pions coming from the resonance decays (similar to Eq.(3)). The HG parameters $V$ and $\mu_B$ are found then as the solution of the following coupled equations:

$$\langle \pi \rangle = N^{\text{tot}}_{\pi}(V,T,\mu_B) \equiv V \ n^{\text{tot}}_{\pi}(T,\mu_B),$$  

(12)

$$N_p = V \ n_B(T,\mu_B),$$  

(13)

\[\text{Note that possible (very small) contributions of particles with double open charm are neglected in Eq.(7).}\]
where \( n_B \) is the HG baryonic density. In these calculations we fix the temperature parameter \( T \). The baryonic chemical potential for Au+Au collisions at the RHIC energies is small (\( \mu_B < T \)) and decreases with collision energy. Therefore, most of the thermal HG multiplicities become close to their limiting values at \( \mu_B \to 0 \). Consequently most of hadron ratios \( N_j^{\text{tot}}/N_i^{\text{tot}} \) become independent of the collision energy. The volume of the system is approximately proportional to the number of pions:

\[
V \sim \langle \pi \rangle \sim N_p(\sqrt{s})^{1/2}.
\] (14)

Note that \( T = 170 \div 180 \text{ MeV} \) leads to the HG value of the thermal ratio:

\[
\frac{\langle \psi' \rangle}{\langle J/\psi \rangle} = \left( \frac{m_{\psi'}}{m_{J/\psi}} \right)^{3/2} \exp \left( - \frac{m_{\psi'} - m_{J/\psi}}{T} \right) = 0.04 \div 0.05,
\] (15)
in agreement with data \([16]\) in central \((N_p > 100)\) Pb+Pb collisions at the CERN SPS. This fact was first noticed in Ref. \([17]\). At small \( N_p \) as well as in p+p and p+A collisions the measured value of the \( \langle \psi' \rangle/\langle J/\psi \rangle \) ratio is several times larger than its statistical mechanics estimate (15). Our analysis of the SCM will be therefore restricted to A+B collisions with \( N_p > 100 \). We do not intend to describe the open and hidden charm production in p+p, p+A and very peripheral A+B collisions within the SCM.

The number of directly produced c\(\bar{c}\) pairs, \( N_{\text{dir}}^{\text{c}\bar{c}} \), in the left-hand side of Eq.(7) will be estimated in the Glauber approach. For \( A + B \) collision at the impact parameter \( b \), this number is given by the formula:

\[
N_{\text{dir}}^{\text{c}\bar{c}}(b) = AB T_{AB}(b) \sigma(NN \to c\bar{c} + X),
\] (16)

where \( \sigma(NN \to c\bar{c} + X) \) is the cross section of \( c\bar{c} \) pair production in \( N + N \) collisions and \( T_{AB}(b) \) is the nuclear overlap function (see Appendix for details).

The cross section of \( c\bar{c} \) pair production in \( N+N \) collisions can be calculated in the pQCD. Such calculations (in the leading order of the pQCD) were first done in Ref. \([22]\). We use the next-to-leading order result presented in Ref. \([23]\). This result was obtained with GRV HO \([24]\) structure functions, the \( c\)-quark mass and renormalization scale were fixed at \( m_c = \mu = 1.3 \text{ GeV} \) to fit the available experimental data. We parametrize the \( \sqrt{s} \)-dependence of the cross section for \( \sqrt{s} = 20 \div 200 \text{ GeV} \) as:

\[
\sigma(pp \to c\bar{c}) = \sigma_0 \left( 1 - \frac{M_0}{\sqrt{s}} \right)^\alpha \left( \frac{\sqrt{s}}{M_0} \right)^\beta,
\] (17)

with \( \sigma_0 \approx 3.392 \mu b \), \( M_0 \approx 2.984 \text{ GeV} \), \( \alpha \approx 8.185 \) and \( \beta \approx 1.132 \). Note that free parameters of the pQCD calculations in Ref. \([23]\) were fitted to the existing data, therefore, our parametrization (17) is also in agreement with data on the total charm production in p+p collisions.

The average number of participants (‘wounded nucleons’) in A+B collisions at impact parameter \( b \) is given by \([23]\)

\[
N_p(b) = A \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \ T_A(\sqrt{x^2 + y^2}) \left[ 1 - \left( 1 - \sigma_{NN}^{\text{inel}} T_B(\sqrt{x^2 + (y-b)^2}) \right)^B \right] + B \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \ T_B(\sqrt{x^2 + (y-b)^2}) \left[ 1 - \left( 1 - \sigma_{NN}^{\text{inel}} T_A(\sqrt{x^2 + y^2}) \right)^A \right],
\] (18)
where \( T_A(\vec{s}) \) (\( T_B(\vec{s}) \)) is the nuclear thickness function for the nucleus \( A \) (\( B \)) and \( \sigma_{NN}^{inel} \) is the total inelastic cross section of \( N+N \) interaction. To parametrize the \( \sqrt{s} \)-dependence of \( \sigma_{NN}^{inel} \) we made the assumption that in the energy range \( \sqrt{s} = 10 \div 200 \) GeV, it is proportional to the total \( NN \) cross section \( \sigma_{NN} \) and use the standard fit for \( \sigma_{NN} \) \[^3\] : 

\[
\sigma_{NN}^{inel} \approx 0.7 \sigma_{NN} \approx 0.7 \left( X_{pp}s^\epsilon + Y_{pp}s^{-\eta_1} - Y_{2pp}s^{-\eta_2} \right),
\]

where \( \epsilon = 0.093, \eta_1 = 0.358, \eta_2 = 0.560, X_{pp} = 18.751, Y_{pp} = 63.58 \) and \( Y_{2pp} = 35.46 \).

Eqs. (\[^3\] and (\[^8\]) provide parametric dependence of the number of produced \( \bar{c}c \) pairs on the number of participating nucleons, \( N_{\bar{c}c}^{dir} = N_{\bar{c}c}^{dir}(N_p) \), which is shown in Fig 1 for Au+Au collisions. It is seen that the dependence is represented by a straight line in the double-logarithmic scale, so that

\[
N_{\bar{c}c}^{dir} \sim (N_p)^k
\]

for \( N_p > 50 \). We find that \( k = 1.31 \div 1.34 \)^3. Using Eq.\(^[7]\) one finds then the following behavior of \( N_{\bar{c}c}^{dir} \) at high energies:

\[
N_{\bar{c}c}^{dir} \sim (N_p)^k(\sqrt{s})^3.
\]

Now we are able to calculate the ratio \( R \) \[^4\] in the SCM and study its dependence on \( N_p \) and \( \sqrt{s} \). The dependence of \( R \) on the number of participants is shown in Fig 2. It is seen that the ratio has qualitatively different behavior at different energies. At the lowest RHIC energy \( \sqrt{s} = 56 \) GeV, the SCM predicts decreasing of the ratio with the number of participants (\( J/\psi \) suppression). In contrast, at the highest RHIC energy \( \sqrt{s} = 200 \) GeV the ratio increases with the number of participant (\( J/\psi \) enhancement) for \( N_p > 100 \). Both suppression (at \( N_p < 150 \)) and enhancement (at \( N_p > 200 \)) are seen at the intermediate RHIC energy \( \sqrt{s} = 130 \) GeV.

Similarly, there are qualitatively different dependencies of \( R \) on the collision energy for small \( (N_p = 100) \) and for large \( (N_p = 350) \) number of the participants. This can be seen in Fig 3. Non-monotonic dependence of the ratio \( R \) on \( \sqrt{s} \) is expected at \( N_p = 100 \). At \( N_p = 350 \), the ratio \( R \) increases monotonically with \( \sqrt{s} \) at all RHIC energies \( \sqrt{s} = 56 \div 200 \) GeV. The minimum of \( R \) in this case corresponds to the energy region between the SPS and RHIC: \( \sqrt{s} \approx 30 \) GeV.

To understand the behavior of \( R \) it is instructive to study the limiting cases: \( N_{\bar{c}c}^{dir} << 1 \) and \( N_{\bar{c}c}^{dir} >> 1 \). Neglecting the hidden-charm term in Eq.\(^[7]\) one finds for \( N_{\bar{c}c}^{dir} << 1 \):

\[
N_{\bar{c}c}^{dir} \sim \frac{1}{4} \gamma_c^2 N_O^2,
\]

hence,

\[
R \equiv \frac{\langle J/\psi \rangle}{N_{\bar{c}c}^{dir}} \sim \frac{N_{J/\psi}^{tot}}{N_O^2/4} \sim \frac{1}{V} \sim N_p^{-1} \left( \sqrt{s} \right)^{-1/2}.
\]

\[^3\] It is interesting to note that for the most central \( A + A \) collisions \( (N_p \approx 2A) \), \( N_{\bar{c}c}^{dir} \) has approximately the same dependence on the atomic weight of the colliding nuclei: \( N_{\bar{c}c}^{dir} \sim A^{2/3} \sim (N_p)^{1/3} \).
Eq. (23) shows $1/V$ universal suppression of the ratio $R$. This ratio decreases as $N_p^{-1}$ and $(\sqrt{s})^{-0.5}$ with the increasing number of participants and collision energy, and the shape of this $J/\psi$ suppression is essentially independent of the functional dependence of $N_{cc}^{\text{dir}}$ on $N_p$ and $\sqrt{s}$.

If $N_{cc}^{\text{dir}} >> 1$ one finds from Eq. (23):

$$N_{cc}^{\text{dir}} \simeq \frac{1}{2} \gamma_c N_O,$$

so that $\gamma_c \simeq 2N_{cc}^{\text{dir}}/N_O \sim N_{cc}^{\text{dir}}/V$ and, hence,

$$R \equiv \frac{\langle J/\psi \rangle}{N_{cc}^{\text{dir}}} \simeq \frac{\gamma_c N_{cc}^{\text{dir}}/V}{\gamma_c N_O/2} \sim N_{cc}^{\text{dir}}/N_p \sim N_{cc}^{\text{dir}}/V \sim N_p^{k-1} (\sqrt{s})^{\beta-1/2}.$$

According to Eq. (23) the ratio $R$ increases with both $N_p$ and $\sqrt{s}$. The $J/\psi$ enhancement takes place due to the fact that the number of primary nucleon-nucleon collisions grows faster than the number of participants ($N_{cc}^{\text{dir}} \sim (N_p)^k$, $k > 1$) and because the pion multiplicity (and therefore the volume of the system) is less sensitive to the collision energy ($\langle \pi \rangle \sim (\sqrt{s})^{1/2}$) than the number of $c\bar{c}$ pairs ($N_{cc}^{\text{dir}} \sim (\sqrt{s})^\beta$, $\beta > 1/2$).

It is seen from Fig. 1 that $N_{cc}^{\text{dir}} < < 1$ at the lowest RHIC energy for small numbers of participants, hence the SCM predicts the $J/\psi$ suppression. In contrast, for the highest RHIC energy and large $N_p$ the opposite limit $N_{cc}^{\text{dir}} >> 1$ is reached. This leads to the $J/\psi$ enhancement.

In conclusion, the production of the $J/\psi$ mesons is studied in Au+Au collisions at the RHIC energies in the statistical coalescence model with the exact charm conservation. The $c\bar{c}$ quark pairs are assumed to be created in the primary hard parton collisions and their number is estimated within the pQCD. At the hadronization stage the $c\bar{c}$ quarks are distributed among the open charm and charmonium particles according to the hadron gas statistical mechanics in the canonical ensemble formulation.

Decreasing of the $\langle J/\psi \rangle$ to $N_{cc}^{\text{dir}}$ ratio with increasing the number of nucleon participants $N_p$ is found at the lowest $\sqrt{s} = 56$ GeV RHIC energy. At fixed small number of participants ($N_p \approx 100$) the ratio decreases with $\sqrt{s}$ between the lowest ($\sqrt{s} = 56$ GeV) and the intermediate ($\sqrt{s} = 130$ GeV) RHIC energies. This is in a qualitative agreement with the standard picture [1,2] of the $J/\psi$ suppression. In contrast, a rise of the $\langle J/\psi \rangle$ to $N_{cc}^{\text{dir}}$ ratio with the collision energy is predicted for central $Au+Au$ collisions. Moreover, at the highest RHIC energy, the ratio is expected to grow with the number of participants, which is in a drastic contradiction with the standard picture. The reason for this that in the standard picture the hidden charm mesons are supposed to be created exclusively in the primary (hard) nucleon-nucleon collisions. It is assumed that all other interaction can only destroy them. Especially strong suppression of the charmonia is expected in the quark-gluon plasma (‘anomalous $J/\psi$ suppression’). In distinction to this standard approach, the statistical coalescence model considers a possibility for the charmonium states to be formed from $c$ and $\bar{c}$ at the stage of the quark-gluon plasma hadronization. This possibility definitely cannot be ignored, when the number of produced $c\bar{c}$ pairs per $A+A$ collision becomes large: $N_{cc} >> 1$ (this happens for the central Au + Au collisions at the highest RHIC energy). In this case
the $c$ and $\bar{c}$ pairs initially produced in different hard collision processes can recombine into a hidden charm meson. Therefore, an increase of the $\langle J/\psi \rangle$ to $N_{c\bar{c}}^{dir}$ ratio should be expected. The hot quark gluon plasma is most probably formed at high RHIC energies and this destroy all primarily produced charmonium states \cite{27}. However, the hadronization of the quark gluon plasma within the SCM reveals itself in the $J/\psi$ enhancement rather than suppression. Another interesting phenomena may also take place: when the number of $c\bar{c}$ pairs becomes large, two $c$ quarks (or two $\bar{c}$) can combine with a light (anti)quark and form a double charmed (anti)baryon. These baryons are predicted by the quark model but have not been observed yet. We expect that double (and probably triple) charmed baryons may be discovered in the $Au + Au$ collisions at RHIC \cite{27}.

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**APPENDIX: NUCLEAR GEOMETRY**

The spherically symmetrical distribution of the nucleons in the $Au - 197$ nucleus can be parametrized by the two-parameter Fermi function \cite{28} (this parametrization is also known as Woods-Saxon distribution):

$$
\rho(r) = \rho_0 \left[ 1 + e^{x} \left( \frac{r - c}{a} \right) \right]^{-1} \quad (A1)
$$

with $c \approx 6.38$ fm, $a \approx 0.535$ fm and $\rho_0$ is given by the normalization condition:

$$
4\pi \int_0^\infty dr r^2 \rho(r) = 1. \quad (A2)
$$

The nuclear thickness distribution $T_A(b)$ is defined by the formula

$$
T_A(b) = \int_{-\infty}^\infty dz \rho \left( \sqrt{b^2 + z^2} \right), \quad (A3)
$$

and the nuclear overlap function is defined as

$$
T_{AB}(b) = \int_{-\infty}^\infty dx \int_{-\infty}^\infty dy T_A \left( \sqrt{x^2 + y^2} \right) T_B \left( \sqrt{x^2 + (y - b)^2} \right). \quad (A4)
$$

From Eq.$(A2)$, one can deduce that the above functions satisfy the following normalization conditions:

$$
2\pi \int_0^\infty db b T_A(b) = 1, \quad 2\pi \int_0^\infty db b T_{AB}(b) = 1. \quad (A5)
$$
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FIG. 1. $N_{\text{c} \bar{c}}^{\text{dir}}$ versus $N_p$ for $\sqrt{s} = 56, 130, 200$ GeV.
FIG. 2. $\langle J/\psi \rangle / N_{\text{dir}}^{c\bar{c}}$ versus $N_p$ for $\sqrt{s} = 56, 130, 200$ GeV. The vertical line shows the lower bound of the applicability domain of the SCM.
FIG. 3. $\langle J/\psi \rangle/N_{\text{dir}}$ versus $\sqrt{s}$ for $N_p = 100$ and 350.