Estimation of the double-scattering component of the lidar return from multi-component atmosphere

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Abstract. Estimation is performed based on a theory developed by Eloranta of the double-scattering contribution to the LIDAR return from a multi-component atmosphere that may contain not only molecular (gaseous) and aerosol fractions, but other compact aerosol objects as well, such as cirrus clouds or Saharan dust layers. It is shown that the relative double-scattering component of the LIDAR return may be approximately considered as a sum of the independent relative contributions of each of the atmospheric components. Then, using appropriate models, the contribution of each component of interest is evaluated as a function of the altitude, taking into account the scattering properties of the medium under consideration, the angular divergence and the wavelength of the sensing laser beam, and the angle of view of the receiving optical system. The results obtained outline the cases when either the double scattering is negligible or corrections are necessary for the multiple scattering effects.

1. Introduction

The LIDAR sensing of the atmosphere is an effective technique for remote contactless investigation of the complex and dynamic atmospheric environment. The LIDAR return from aerosol-loaded atmosphere contains information about the properties and dynamics of the background aerosol stratification and other compact aerosol objects such as clouds, dust layers, smoke plumes, and so on. This “aerosol information” is extractable from the LIDAR return signal through the so-called LIDAR equation describing the signal as a function of the LIDAR and atmospheric characteristics. The return signal has usually been analyzed using the single-scattering LIDAR equation [1-3] that is valid when the multiple-scattering components of the LIDAR return are negligible. This equation can straightforwardly be solved [2, 3] with respect to the LIDAR line-of-sight (LOS) profiles of the atmospheric extinction and backscatter coefficients. In different cases, however, the multiple-scattering effects are not negligible and the return signal should be described by some type of multiple-scattering LIDAR equations [3-7]. These equations are not straightforwardly and unambiguously solvable, but they can provide information about the extent of the multiple-scattering effects. Such information would allow one, in principle, to remove the multiple-scatter component as
some type of background and reduce the problem to considering the single-scatter component alone [8], but at the expense of lowering the signal-to-noise ratio (SNR) of the measurement.

Because of the above-outlined importance of the knowledge about the multiple-scattering influence on the LIDAR sensing, the main purpose of the present work is to estimate, as an initial step based on an effective theory of Eloranta [7], the double-scatter contribution to the LIDAR return from a multi-component atmospheric semi-space including cirrus clouds and Saharan dust layers modeled appropriately as continuously distributed targets.

2. Basic relations and models
In general, the atmosphere is a complex medium occupying the semi-space above the ground. It may consist of many different localized finite-size objects like clouds, and continuously distributed constituents, such as the gaseous and aerosol components. For convenience, retaining their inherent compactness, the finite-size objects are appropriately modeled here as extended along all the vertical LOS. According to the approach of Eloranta [7], we consider the overall double-scatter contribution to the LIDAR return as a result of singular acts of backscattering (called here the main scattering) from each of the atmospheric components, accompanied by singular acts of forward scattering (called here the additional scattering, on the way forward or backward along the LOS) by the same component or one of the other components. Then, following the analytical procedure developed in [7], we obtain that the overall double-to-single scattering signal ratio (DSSSR) \( D(\lambda, z) = S(\lambda, z)/S(\lambda, z) \) is of the order of (but smaller than) the sum \( \sum D_i(\lambda, z) \) of the separate DSSSRs, \( D_i(\lambda, z) = S_i(\lambda, z)/S_i(\lambda, z) \), concerning the different atmospheric components. That is,

\[
D(\lambda, z) \leq \sum_i D_i(\lambda, z),
\]

where \( \lambda \) is the laser wavelength, and \( z \) is the coordinate along the LOS. For the \( i \)-th component we have (see in [7, 9])

\[
D_i(\lambda, z) = \int_0^\infty \mu_i(\lambda, x) \left( 1 - \exp\left[ -\theta_i^2 z^2 / \left( (z-x)^2 \theta_i^2 + \theta_i^2 z^2 \right) \right] \right) dx,
\]

where \( \mu_i(\lambda, x) \) is the volume scattering coefficient profile of the \( i \)-th component along the LOS, \( \theta_i \) is the LIDAR angle of view, \( \theta_i(\lambda, z) \) is the angular width of the forward indicatrix peak of the \( i \)-th component, and \( \theta_i \) is the angular divergence of the sensing laser beam. Note that the quantities \( D_i(\lambda, z) \) describe in fact mathematically the additional-scattering processes. The values of the laser wavelengths considered here are \( \lambda = 337.1 \text{ nm}, 514.5 \text{ nm}, \text{ and } 1060 \text{ nm} \). The corresponding values of \( \theta_i(\lambda, z) \) are assumed independent of \( z \), that is, \( \theta_i(\lambda, z) = \theta_i(\lambda) \). For cirrus clouds (CC) they are chosen to be 1.64 mrad, 2.5 mrad and 5.15 mrad, for Saharan dust (SD) and clear (23 km visibility) or hazy (5 km visibility) atmosphere aerosol loads, 65.5 mrad, 100 mrad and 206 mrad, and for molecular atmosphere (MA), 141 rad for all the three wavelengths (see also in [7]). Four LIDAR angles of view \( \theta_i \) are considered – 2 mrad, 1 mrad, 0.4 mrad and 0.2 mrad. The laser beam divergence chosen is 0.1 mrad.

The scattering coefficient vertical profiles \( \mu_i(\lambda, z) \) employed in this work for clear, hazy and molecular atmospheres are described by the following analytical approximation [10] of the corresponding models developed by McClatchey et al. [11]:

\[
\mu_i(\lambda, z) = A_i(\lambda) / \left[ 1 + \exp\left[ (z - z_{0i}(\lambda)) / w_i(\lambda) \right] \right],
\]

where \( i = m \) for molecular atmosphere, \( i = ca \) for clear-atmosphere aerosol field, and \( i = ha \) for hazy-atmosphere aerosol field; \( A_i [\text{m}^3], z_{0i} [\text{m}], \text{ and } w_i [\text{m}] \) are the least-squares best-fit approximation
parameters. The values of these parameters are given in Table 1. The models chosen of the scattering coefficient vertical profiles of a CC or SD are described by a bell-shaped waveform [10]

\[
\mu_i(\lambda, z)/\mu_i(\lambda, z_{0,i}) = \left[1 + (z - z_{0,i})^p / w_i^p \right]^{-1},
\]

that is symmetric with respect to the position \(z_{0,i}\) of its peak value \(\mu(\lambda, z_{0,i})\) and have a characteristic width \(w_i\); \(p\) is an integer. We have considered here an SD layer (index \(i = \text{sd}\)) with parameters \(p = 4\), \(z_{0,\text{sd}} = 3500\) m, \(\mu_{4,\text{sd}}(\lambda, z_{0,\text{sd}}) = \mu_{4,\text{sd}}(z_{0,\text{sd}}) = 0.15 \times 10^3\) m\(^{-1}\) and \(w_{4,\text{sd}} = 500\) m, and two (thin and thick) CCs (index \(i = \text{cc}\)) with parameters, respectively, \(p = 4\), \(z_{0,\text{cc}} = 9500\) m, \(\mu_{4,\text{cc}}(\lambda, z_{0,\text{cc}}) = \mu_{4,\text{cc}}(z_{0,\text{cc}}) = 0.35 \times 10^3\) m\(^{-1}\) and \(w_{4,\text{cc}} = 300\) m, and \(p = 4\), \(z_{0,\text{cc}} = 15000\) m, \(\mu_{4,\text{cc}}(\lambda, z_{0,\text{cc}}) = \mu_{4,\text{cc}}(z_{0,\text{cc}}) = 1.5 \times 10^3\) m\(^{-1}\) and \(w_{4,\text{cc}} = 600\) m.

**Table 1.** Parameters of the curves approximating the vertical scattering-coefficient profiles of the molecular and aerosol (23 km and 5 km visibilities) atmospheric components.

| \(\lambda\) [nm] | \(A_m\) [m\(^{-1}\)] | \(z_{0,m}\) [m] | \(w_m\) [m] | \(A_{cc}\) [m\(^{-1}\)] | \(z_{0,cc}\) [m] | \(w_{cc}\) [m] | \(A_{ha}\) [m\(^{-1}\)] | \(z_{0,ha}\) [m] | \(w_{ha}\) [m] |
|----------------|-----------------|----------------|----------|-----------------|----------------|----------|-----------------|----------------|----------|
| 337.1          | 1.05644E-4      | 5653.13        | 5515.10  | 3.20757E-4      | 901.34287      | 876.44783 | 0.00107         | 886.55158      | 736.65616 |
| 514.5          | 3.11541E-5      | -377.72121     | 6654.69124| 2.29643E-4      | 912.98584      | 895.10918 | 0.00110         | 876.50855      | 738.06736 |
| 1060           | 1.64867E-6      | 46.13384       | 6577.02863| 9.40202E-5      | 880.20716      | 905.92918 | 4.66186E-4      | 738.91575      | 817.01461 |

### 3. Results and discussion

The results from the calculations performed using equations (2) – (4) and the corresponding parameters of interest show in general (see the figures) that, expectedly, the fractional double-scattering contribution \(D(\lambda, z)\) to the overall double-to-single scatter signal ratio is proportional to the angle of view of the LIDAR receiving optics. It is also proportional to the optical thickness of the scattering components of interest, and inversely proportional to their indicatrix peak widths; thus, \(D(\lambda, z)\) is inversely proportional to the wavelength \(\lambda\) of the sensing laser radiation.

![Figure 1. DSSSR contributions of the modeled molecular atmosphere, clear-atmosphere aerosol component, hazy-atmosphere aerosol component, a thick CC, a thin CC, and an SD layer, versus altitude, at wavelength \(\lambda = 337.1\) nm and angle of view \(\theta = 2\) mrad.](image-url)

The concrete numerical results show that even at an angle of view of 2 mrad, the contributions to the double-to-single scattering ratio of a Saharan dust layer (\(\sim 0.15\) optical depth) and the molecular (gas) atmospheric component, as well as of the clear-atmosphere (23 km visibility) and hazy-atmosphere (5 km visibility) aerosol components are negligibly small, less than several percent, at all the wavelengths of concern here (see figure 1). Note, for instance, that the hazy-atmosphere aerosols, whose optical thickness (\(\sim 1.7\)) is near that of a thick cloud (\(\sim 2.0\)) and exceeds essentially the
thickness of a thin cloud (~ 0.23) contribute much less to the DSSSR (~ 4% at λ = 337.1 nm and θt = 2 mrad) because of their much wider indicatrix peak (figures 2 – 4).

Figure 2. DSSSR contributions of the modeled hazy-atmosphere aerosol component, versus altitude, at wavelengths λ = 337.1, 514.5 and 1060 nm and angles of view θt = 0.2 (a), 0.4 (b), 1 (c), and 2 mrad (d).

Similarly, the optical depth of the gas component is ~ 0.59 at λ = 337.1 nm, but its contribution to the DSSSR is well below one percent (figure 1). In the cases of a thin cirrus cloud (~ 0.23 optical depth) or a thick cirrus cloud (~ 2.0 optical depth), when approaching the height of the cloud, the values of Dcc(λ, z) sharply increase reaching maxima of 20 % and 200 %, respectively, a little above the cloud peak, independently of the wavelength λ and the angle of view θt (figures 3 and 4). Further, with increasing the height, a relatively slow decrease of Dcc(λ, z) is seen. Such a behavior of Dcc(λ, z) (intrinsic to some extent also to the dust layer, figure 1) is in accordance with the Ulla Wandinger’s finding [6] that the multiple-scattering-due error in the determination of the cloud extinction, using HSRLs or Raman
Figure 3. DSSSR contributions of the modeled thin CC, versus altitude, at wavelength $\lambda = 337.1$, 514.5, and 1060 nm, and angles of view $\theta_t = 0.2$ (a), 0.4 (b), 1 (c), and 2 mrad (d).

LIDAR, is maximum around the cloud base and decreases within the cloud. Thus, the additional scattering from optically dense aerosol objects with sharply forward-directed indicatrices, such as thick cirrus clouds, could essentially influence the LIDAR signal and the results of the LIDAR sensing. Certainly, this should be taken into account when interpreting the LIDAR data [8]. Nevertheless, even cleared in any way, the double-scattering effects may lead to essential lowering of the signal-to-noise ratio of determining the single-scattering return signal.

4. Conclusion
As a whole, at sufficiently small LIDAR angles of view, in the absence of optically very thick atmospheric constituents with too narrow indicatrix peaks (watery clouds, thick cirrus clouds, etc.), one may realize single-scattering regime of LIDAR sensing. In the opposite case, the results obtained should be corrected for the multiple (double and higher order) scattering effects [6, 8]. In this case, one should take into account the multiple-scattering due decrease of the signal-to-noise ratio of determining the single-scattering LIDAR signal.

The following step in our investigations on this topic will include considering the contribution to the LIDAR return of the triple, quadruple and higher-order scattering of the laser light in a multi-component atmosphere.
Figure 4. DSSSR contributions of the modeled thick CC versus altitude at wavelength $\lambda = 337.1$, 514.5, and 1060 nm, and angles of view $\theta_t = 0.2$ (a), 0.4 (b), 1 (c), and 2 mrad (d).

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