Periodic-phase-diagram similarity method for weak signal detection

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Abstract
The periodic-phase-diagram similarity method is proposed to identify the frequency of weak harmonic signals. The key technology is to find a set of optimal coefficients for Duffing system, which leads to the periodic motion under the influence of weak signal and strong noise. Introducing the phase diagram similarity, the influences of strong noise on the similarity of periodic phase diagram are discussed. The principle of highest similarity of periodic phase diagram with the same frequency is detected by discussing the persistence of similarity of periodic motion phase diagram under the strong noise and the periodic-phase-diagram similarity method is constructed. The weak signals of early fault and strong noise are input into Duffing system to obtain the identified system. The stochastic subharmonic Melnikov method is extended to obtain the conditions of the optimal coefficients for the identified system. Based on the results, the constructed frequency conversion harmonic weak signals are considered to form a datum periodic system. With the change of frequency in the datum periodic system, the phase diagram similarity of the two constructed systems can be calculated. Based on the periodic-phase-diagram similarity method, the frequency of weak harmonic signals can be identified by the principle of highest similarity of periodic phase diagram with the same frequency. The results of numerical simulation and the early fault diagnosis results of actual bearings verify the feasibility of the periodic-phase-diagram similarity method. The accuracy of the detection effect is 97%, and the minimum signal-to-noise ratio is -80.71 dB.

KEYWORDS
periodic system, signal-to-noise ratio, stochastic subharmonic Melnikov method, strong noise, weak signal
1 | INTRODUCTION

Most of the rolling bearing failures appear as early minor faults which can cause equipment downtime or major accidents and are not easy to find in the early stages. Most of the early minor fault signals are submerged in strong noise, which is called weak signal. Therefore, detecting weak fault signal is very important and complex. The traditional rolling bearing fault signal detection method first filters the collected fault signals, suppresses the noise signal and extracts useful fault signal features. However, in the early stage of the fault, compared with the noise, the fault signal is relatively weak. While eliminating the signal noise, it is possible to filter out the useful signal which can characterize the fault and then draw the wrong detection conclusion. Therefore, the traditional fault detection methods have high requirements on the signal-to-noise ratio of the detected signal, and the application of early weak fault detection methods is limited.

With the development of nonlinear theory, researchers already proposed nonlinear detection methods in which nonlinear systems are used to detect weak signals. This type of signal detection method extracts the characteristics of weak signals, not by eliminating or suppressing noise, but by using the characteristics of some nonlinear systems to detect weak signals in strong noise. In the early weak fault detection methods of rolling bearings, the stochastic resonance and the chaotic oscillator are widely used. The parameter sensitivity of the chaotic oscillator and the immunity to noise make it have certain advantages in detecting weak fault signals with lower signal-to-noise ratio. At present, the types of chaotic oscillators used in weak fault diagnosis include Duffing oscillator, Lorenz oscillator, and some coupled chaotic oscillators, which depend on the characteristic quantities of the chaotic system during the transition between "chaotic state" and "period state". In the judgment of chaotic state, for qualitative analysis methods, the critical point is difficult to be determined, and the threshold selection is not accurate enough. Although most quantitative analysis methods can be used to obtain good results, there are some problems such as complex algorithms and large calculations, which are difficult to be used in engineering practice for fault diagnosis.

In this paper, it is found that in the periodic state, the influence of noise on the phase diagram of the system is minimal. On this basis, the stochastic subharmonic Melnikov function is used to give the condition of maintaining periodic motion and determine the system parameters of the periodic recognition system. The phase diagram similarity is introduced to determine the highest similarity of the system phase diagram at the same frequency. The periodic-phase-diagram similarity method is constructed to identify the frequency. The experimental results of numerical simulation and the early fault diagnosis results of actual bearings prove that the phase diagram similarity method based on periodic motion can be used in frequency identification under strong noise background, which can achieve better results (lower signal-to-noise ratio (SNR) = −80.71 dB) than the conventional detection (SNR = −45.8497 dB obtained in Ref. 8).

This paper is organized as follows. In the next part, the influence of noise on the similarity of phase diagrams under periodic and chaotic states is discussed and the persistence of the similarity of phase diagrams of periodic motion under strong noise background is given. In Section 3, the stochastic subharmonic Melnikov function is used to give the condition of maintaining periodic motion and determine the system parameters of the periodic identification system. The principle of the phase diagram similarity method based on periodic motion is initially investigated. And then the periodic-phase-diagram similarity method is constructed to identify the frequency. In Section 4, a numerical simulation experiment is conducted to explore the accuracy of the method under the influence of different noise intensities. Finally, the effectiveness of the periodic-phase-diagram similarity method is verified by the early fault diagnosis results of real bearings in Section 5, which provides a new idea for parameter identification of weak harmonic signals in actual engineering.

2 | PHASE DIAGRAM SIMILARITY

Consider Duffing system, which leads to

$$\begin{align*}
\dot{x} &= y, \\
\dot{y} &= x - x^3 + f \cos \omega t - \mu y,
\end{align*}$$

where $x = x(t)$ is the displacement at time $t$, $y$ is the first derivative of $x$ with respect to time $t$, and $f, \omega, \mu$ represent the parameters of Duffing system.

White Gaussian noise is added into system (1), which leads to

$$\begin{align*}
\dot{x} &= y, \\
\dot{y} &= x - x^3 + f \cos \omega t - \mu y + \sigma \dot{\nu},
\end{align*}$$

where $\nu$ is the standard Gaussian white noise, $\sigma$ is the noise intensity and the power spectral density of white Gaussian noise is $K = \sigma^2/2\pi$.

To measure the similarity between the phase diagrams, the mean square error

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} \left( (x_i - \tilde{x}_i)^2 + (y_i - \tilde{y}_i)^2 \right),$$

can be introduced to represent the similarity of the phase diagrams, where $x_i, y_i, \tilde{x}_i, \tilde{y}_i$ are the values of $x, y$ in system (2) under different working conditions; $n$ is the number of points on the phase diagram.

Assuming $f = 1, \omega = 1$, and $\mu = 0.5$, system (2) is in periodic motion (as shown in Figure 1A). When $f = 0.7, \omega = 1$, and $\mu = 0.5$, system (2) is in a state of chaotic motion (as shown in Figure 1B).

It can be seen from Figure 1A that in the periodic state, the phase diagram similarity of the noise-free system and the noise-free system is MSE = 0.031. From Figure 1B, it is not difficult to calculate it in the chaotic state the phase diagram similarity of the
noise system and the noise-free system is MSE = 1.110. Therefore, in the periodic state, the noise is considered in system (1) and there are

\[ x \approx \bar{x}, y \approx \bar{y}, \dot{y} \approx \dot{\bar{y}}. \]  

(4)

But, for the chaotic state, Equation (4) does not hold. The main reason is the sensitivity of the chaos to the initial value. Hence, the persistence of the similarity of phase diagrams of periodic motion under strong noise background is shown.

Moreover, let \( f = 1, \mu = 0.5, \) and \( \sigma = 0.1, \) system (2) is in periodic motion at both \( \omega = 0.8 \) and \( \omega = 1 \) (as shown in Figure 2). It can be seen that in the periodic state, the phase diagram similarity of systems with the same frequency is MSE = 0.003 (see Figure 2A), and the similarity of the phase diagrams of the different frequency systems is MSE = 1.965 (see Figure 2B).

Therefore, in the periodic state, the different frequencies are considered in system (2) and there are

\[ x \neq \bar{x}, y \neq \bar{y}, \dot{y} \neq \dot{\bar{y}}. \]  

(5)

In summary, the phase diagram similarity can be introduced to determine the highest similarity of the system phase diagram at the same frequency. And the stochastic subharmonic Melnikov function will be used to give the condition of maintaining periodic motion.

### 3 | PERIODIC-PHASE-DIAGRAM SIMILARITY METHOD

Considering that the weak harmonic signal affected by Gaussian noise is \( r \cos \omega t + \sigma \Gamma, \) the identified system is
\[
\begin{align*}
\dot{x} &= y, \\
\dot{y} &= x - x^3 + f \cos \omega t - \mu y + r \cos \omega_1 t + \sigma \Gamma, \\
\end{align*}
\]  

(6)

Obviously, the system

\[
\begin{align*}
\dot{x} &= y, \\
\dot{y} &= x - x^3,
\end{align*}
\]  

(7)

has two centers (±1, 0) and a saddle point (0, 0).

The Hamiltonian of Equation (7) can be written as

\[
H(x, y) = \frac{y^2}{2} - \frac{x^2}{2} + \frac{x^2}{4} = h,
\]  

(8)

where \( h \) is a constant.

For each set of the given initial conditions, the value of \( h \) can be calculated, which uniquely corresponds to a closed curve. There is a periodic orbit with \( h \) as the parameter in the entire phase plane:

\[
\begin{align*}
q^h(t) &= (x^h(t), y^h(t)) = \sqrt{\frac{2}{2 - k^2}} \text{cd} \left( \frac{t}{\sqrt{2 - k^2}}, k \right), \\
&= \sqrt{\frac{2}{2 - k^2}} \text{sn} \left( \frac{t}{\sqrt{2 - k^2}}, k \right), \\
q^h(t) &= -q^h(t),
\end{align*}
\]  

(9)

where \( \text{sn}, \text{cn}, \text{and dn} \) are the Jacobi elliptic functions, \( k \) is the modulus of the Jacobi elliptic functions and \( h = \frac{k^2 - 1}{2 - k^2} \).

The stochastic subharmonic Melnikov function corresponding to the subharmonic orbit of system (6) with period \( T(k) = \frac{n\pi}{m} \) is

\[
M^{h/2}(t_0) = -\bar{M} + \bar{M}_1(t_0) + \bar{M}_2(t_0) + \bar{M}(t_0),
\]  

(10)

where \( \bar{M}, \bar{M}_1(t_0) \) and \( \bar{M}_2(t_0) \) are the means of stochastic subharmonic Melnikov process of \( \mu y, f \cos \omega t, \) and \( r \cos \omega_1 t. \bar{M}(t_0) \) satisfies

\[
E[M(t_0)] = E \left[ \int_{t_0}^{T(k_0)} \sigma_\omega(t) \Gamma(t + t_0) \, dt \right] = 0.
\]  

(11)

According to Ref. 8, in the mean-square sense, the stochastic subharmonic Melnikov function can be detected as

\[
-\bar{M} + \bar{M}_1^2(t_0) + \bar{M}_2^2(t_0) + E[\bar{M}(t_0)^2] = 0,
\]  

(12)

where

\[
M = -\frac{4\left(2 - k^2\right)E(k) - 8k^2F(k)}{3(2 - k^2)^{3/2}},
\]

\[
\bar{M}_1 = f \sqrt{2 \pi \text{sech}(\sqrt{2 - k^2}F(k) \omega) \sin \omega_0 t},
\]

\[
\bar{M}_2 = r \sqrt{2 \pi \text{sech}(\sqrt{2 - k^2}F(k) \omega) \sin \omega_1 t},
\]

(13)

and

\[
C(\omega) = \frac{\sigma_\omega}{\pi} \sqrt{2} \pi \text{sech}(\sqrt{2 - k^2}F(k) \omega) \left[ T \left(\log(1 + e^{-TZ}) + \log(1 + e^{TZ}) + T \text{tanh} \left(\frac{TZ}{2}\right)\right)\right] - \Theta,
\]  

(14)

\[
I_0 = \frac{70}{10} \left(\text{sech}^2\left(\sqrt{2 - k^2}F(k) \omega\right) \right) \frac{E(t)}{\sigma_\omega(2 - k^2)^{3/2}} \]  

(15)

where \( C(\omega) \) is the transfer function between \( \Gamma \) and \( \bar{M}(t_0) \), the period is

\[
T = T(k) = 2\sqrt{2 - k^2} F(k), \quad Z = \sqrt{2 - k^2} F(k), \quad k^2 = 1 - k^2, \quad F'(k) = F(k'), \quad F(k) \) is the first type of complete elliptic integral, \( \text{Li}_1(z) \) is the multi-logarithmic function, and \( K \) is the power spectral density of Gaussian white noise.

According to the subharmonic bifurcation theorem, if \( M^{h/2}(t_0) \) has a simple zero point, system (6) does have a subharmonic solution with a period of \( T(k) = \frac{n\pi}{m} \).

Hence, from Equation (12), we can obtain the stochastic subharmonic Melnikov function threshold

\[
f > \frac{\pi(2 - k^2)E(k) - 8k^2F(k)}{3(2 - k^2)^{3/2}} - \text{sech}(\sqrt{2 - k^2}F(k) \omega) \]  

(16)

According to the characteristics of the measured simple harmonic weak signal, the variable frequency \( \omega_0 \) test signal \( r \cos \omega_0 t \) is constructed and a datum system is obtained as follows:

\[
\begin{align*}
\dot{x}_i &= y_i, \\
\dot{y}_i &= x_i^3 + f \cos \omega t - \mu y_i + r \cos \omega_1 t,
\end{align*}
\]  

(17)

From Equation (10), the subharmonic Melnikov function of system (17) yields

\[
-\bar{M} + \bar{M}_1(t_0) + \bar{M}_2(t_0) = 0,
\]  

(18)

which leads to the subharmonic Melnikov function threshold
If $f, \omega, \mu, \text{and } r_1$ are chosen to satisfy Equations (16) and (19), the identified system and the datum system can keep in a periodic motion state under the influences of weak signal, variable frequency $\omega_s$ test signal and strong noise. Through the numerical solution method with the same initial value and step length, $x_i, y_i, \bar{x}_i, \bar{y}_i, \bar{y}_i (i = 1, 2, \ldots, n)$ of the identified system (6) and the datum periodic system (17) can be obtained, where $n$ is the total number of iteration steps.

Since the system is in a periodic state, when the frequency $\omega$ of the test signal is equal to the frequency $\omega_1$ of the signal ($\omega_s = \omega_1$), the phase diagram similarity yields

$$f > \frac{\mu(4(2 - k^2)E(k) - 8k^2F(k))}{3\sqrt{2} \pi \alpha(2 - k^2)^{3/2} \text{sech} \sqrt{2 - k^2} F(k) \omega} - \frac{\omega \text{sech} \sqrt{2 - k^2} F(k) \omega}{\omega_1 \text{sech} \sqrt{2 - k^2} F(k) \omega_1}. \quad (19)$$

When the frequency $\omega$ of the test signal is equal to the frequency $\omega_1$ of the signal ($\omega_s = \omega_1$), the phase diagram similarity satisfies

$$\text{MSE} = \frac{\sum_{i=1}^{n} \sqrt{(x_i - \bar{x}_i)^2 + (y_i - \bar{y}_i)^2}}{n} > 0. \quad (20)$$

From Equations (20) and (21), we know that when the frequency $\omega_s$ of the test signal is equal to the frequency $\omega_1$ of the signal ($\omega_s = \omega_1$), MSE is the smallest. Therefore, the frequency of weak signals can be identified by the value of MSE. Hence, the periodic-phase-diagram similarity method can be constructed to identify the frequency.

The steps of the periodic-phase-diagram similarity method are described as follows:
1. Based on the predicted digital characteristics, the subharmonic Melnikov method is used to explore and determine the parameters in the system (6).
2. Introduce the constructed weak harmonic signal into the system (2), the datum periodic system is obtained.
3. The phase diagram similarity is used to determine the frequency of weak harmonic signals.

4 | NUMERICAL SIMULATION

The periodic-phase-diagram similarity method provides a general procedure for identifying the frequency of weak harmonic signals, which will be demonstrated by identifying the weak harmonic signals of different noise intensity.

According to Equations (16) and (19), let \( f = 1, \omega = 1, \mu = 0.5, r = 0.02 \) and the identified system is

\[
\begin{align*}
\dot{x} &= y, \\
y &= x - x^3 + \cos t - 0.5y + r \cos \omega_1 t + \sigma t,
\end{align*}
\]

the datum periodic system is

\[
\begin{align*}
\dot{x}_s &= y_s, \\
y_s &= x_s - x_s^3 + \cos t - 0.5y_s + 0.02 \cos \omega_1 t.
\end{align*}
\]

When \( r = 0.02 \) and \( \omega_2 = 0.8 \), the identified effect under different noise intensities is shown in Figure 3.

As shown in Figure 3, the noise intensity ranges from 0.2 to 0.8, and the frequencies are all successfully identified for \( r = 0.02 \) and \( \omega_1 = 0.8 \). The results of numerical simulation prove that the periodic-phase-diagram similarity method can be used to identify the frequency of weak harmonic signals under the background of strong noise. Moreover, the lowest threshold of the signal-to-noise ratio\(^8\) is given by

\[
\text{SNR} = 10 \log \frac{r^2}{2\sigma^2} = 10 \log \frac{(0.02)^2}{2 \times (0.8)^2} = -80.71 \text{ dB.} \tag{24}
\]

Hence, the lowest threshold of the SNR of the weak signal is obtained, which is lower than \( \text{SNR} = -45.8497 \text{ dB} \) obtained in Ref. 8.

5 | APPLICATION

To apply the theoretical results in engineering practice, a group of experimental fault data for fast rotating bearings will be detected by the periodic-phase-diagram similarity method.

In this experiment, the Dongfang Institute vibration signal acquisition instrument (INV3018A) and DASP software were used to collect the data of the comprehensive performance test bench of railway bearings, and the sampling frequency was set to 51200 Hz. The driving motor speed is 1200 r min\(^{-1}\). The bearing type is FAG F-80781109 TAROL 130/240-B-TVP. Artificial processing defects are used to simulate the fault of the outer ring of the bearing. The fault size, length and depth are 5, 1, and 0.7 mm. During the experiment, the acceleration sensor

\[
\begin{array}{c}
\text{Acceleration sensor} \\
\end{array}
\]

\[
\text{FIGURE 4} \quad \text{Experimental device for fast rotating bearings.} \\
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\]

\[
\begin{array}{c}
\text{FIGURE 5} \quad \text{Frequency identification of weak fault signal wave}
\end{array}
\]

\[
\text{TABLE 1} \quad \text{Identification results of fault frequency of bearing outer ring}
\]

| Method                                | Identification results | Absolute error | Accuracy | Identification time |
|---------------------------------------|------------------------|----------------|----------|---------------------|
| Periodic-phase-diagram similarity method | 889.0 rad s\(^{-1}\)   | 28 rad s\(^{-1}\) | 97.0%    | 402 s               |
| Variable scale-convex-peak method     | 890.64 rad s\(^{-1}\)  | 26.36 rad s\(^{-1}\) | 97.5%    | 1300 s              |
is installed at the top of the end cover of the bearing as shown in Figure 4.

According to Ref. 8, the failure frequency of the outer ring of the bearing can be estimated to be about 917 rad s⁻¹.

The result of the identification is shown in Figure 5. There is a minimum value at a frequency of 889.0 rad s⁻¹, which proves that there is a weak fault signal with a frequency of 889.0 rad s⁻¹ in the collected data. It can be seen from Table 1 that the absolute error is 28 rad s⁻¹ and the relative error is 28/917 = 3.0%.

Although it is slightly higher than the 2.5% obtained in Ref. 8, the identification efficiency is greatly improved. Furthermore, in practical engineering applications, we need to identify weak signals quickly and accurately. But, within a certain error range, the identification time should be shortened as much as possible.

Hence, in the comprehensive evaluation of accuracy and efficiency, this method is no less than the variable scale convex peak method, and it is easier to realize the construction of software platform.

6 | CONCLUSIONS

In this paper, the periodic-phase-diagram similarity method was proposed to identify the frequency of the faulty weak harmonic signals under the background of strong noise. Based on the stochastic subharmonic Melnikov method, the appropriate parameters were selected to make the identification system always remain in a periodic state under the influence of weak signals and noises. Furthermore, the frequency conversion harmonic weak signals were constructed and the datum periodic systems were given. Based on the principle of highest similarity of periodic phase diagram with the same frequency, the periodic-phase-diagram similarity method was constructed. Moreover, the feasibility of the periodic-phase-diagram similarity method was verified by the results of numerical simulation. And the early fault diagnosis results of actual bearings prove that the periodic-phase-diagram similarity method can achieve better results (lower SNR = −80.71 dB) than the conventional detection (SNR = −45.8497 dB obtained in Ref. 8).

Meanwhile, the identification time is shortened from 1300 to 402 s, which provides a new idea for parameter identification of weak harmonic signals in actual engineering.

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CONFLICT OF INTEREST

The authors declare that there are no conflict of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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