Warm Intermediate Inflation in $F(T)$ Gravity

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Abstract: We investigate warm intermediate scenario of the cosmological inflation in $F(T)$ gravity in the limit of high dissipation. The inflationary expansion is driven by the scalar inflaton while the gravitational dynamics follow from the $F(T)$ gravity. We calculate the relevant inflationary observables such as scalar-tensor ratio, power-spectrum indices of density perturbations and gravitational waves and the e-folding parameter. We obtain a ratio of slow-roll parameters to be constant. Our calculations support the warm-intermediate inflationary scenario in a spacetime with torsion. Moreover our results are compatible with the astrophysical observations of cosmic microwave background and Planck data.

Keywords: Cosmic microwave background; Inflation; $F(T)$ gravity; Torsion; Planck data.
I. INTRODUCTION

The hypothesis of cosmological inflation in the early Universe is useful in answering several fundamental questions of cosmology such as why the energy-matter distribution in the Universe is homogeneous; problem of fine-tuning of the initial velocities of the energy-matter (the flatness problem), the magnetic monopole problem and the horizon problem. Moreover the inflaton that drives the inflation serves as the harbinger of seed fluctuations for later large scale structure formation [1]. Observations of the cosmic microwave background (CMB) and the large-scale structure (LSS) are used to determine the spectrum of primordial seed fluctuations. This makes CMB and LSS experiments the only probes of the very early Universe. Current observations are in excellent agreement with the basic inflationary predictions: The Universe has an almost scale-invariant Gaussian power spectrum [2]. To have a better view on the essence of the late time cosmic acceleration, one geometrical approach is to use the $f(R)$ gravity. This theory was proposed by Buchdahl as a mathematical extension of the Einstein gravity in a very simple and systematic form [3]. Other possibilities include a non-zero cosmological constant or dynamical dark energy models [4]. Different aspects of dark energy models have been discussed recently (for a review see [5]).

Curvature is not a unique description of the gravity. Going beyond the Riemannian geometry, one can use torsion as an alternative for geometrical description of the gravity [6]. General relativity is constructed using a symmetric Levi Civita connection whereas teleparallel gravity is constructed from a skew-symmetric Weitzenbock connection. In recent years attentions have been focused on the generalizations of this idea as teleparallel gravity [7, 8]. Different cosmological aspects of $F(T)$ gravity such as resolution of dark energy and dark matter problems with torsion have been discussed in literature (see [9, 10] and references therein). Also some black holes solutions are also derived in $F(T)$ gravity [11].

Conventional inflation model has two distinct stages of evolution. The first stage is governed by the rapid accelerated expansion driven by the inflaton with negligible kinetic energy and a stable potential energy. In the second stage (called reheating), the inflaton decays into matter and radiation fields which is a kind of a hot Big Bang. The main problem is how we can join the universe towards the end of this era, successfully. Although both stages are driven by different physical mechanisms, the idea of ‘warm inflation’ amazingly unifies them [12]. Here inflation is described as a decay of the field into thermal component via weak and strong dissipation regimes. In this scenario, it is assumed that the radiation (produced during the inflation) keeps a constant density. This constant energy density preserves the form of the cosmological solution as a transition phase in
the form of the de-Sitter universe. To validate this proposal, we need a dissipative formalism. The dissipation coefficient $\Gamma$ is necessary to explain this heating phase completely. The full consistent model to construct this dissipation functions is fulfilled by quantum field theory tools using a two stage mechanism, applied on the interactions [32]. We need warm inflationary epoch to stop the inflation in a finite time. Dynamics of the warm inflation has some interesting features. The main aspect is, during the radiation production, some microscopic (micro-statistical) procedures happen. The average time of this cascade process goes faster than the velocity (here Hubble parameter) of the background. If we denote by $\Gamma_a$ a typical decay rate of one of these process, than it means that $\Gamma_a > H$. This implies that the quantum photon productions slows-down the dynamics of the cosmological background. It is a kind of the slow-roll approximation. This idea has motivations from the tachyonic field as well [19], the Hawking radiation [20] and via holographic principle [21].

The scale factor of the cosmological background through the inflationary era must be a power law or exponential of an intermediate form. Such forms of the scale factor possess exact solutions for fields (vector, scalar,...). One important form proposed is the following [14]

$$a(t) = \exp(A_1 t^f), \quad 0 < f < 1, \quad A_1 > 0$$

(1)

where $f$ and $A_1$ are constants. In this model, the expansion rate is between de Sitter inflationary expansion ($a(t) = \exp(\dot{H}t)$), and power-law inflationary expansion ($a(t) = t^\alpha, \alpha > 1$). The intermediate inflationary scenario has been proposed as solution for a constrained inflaton potential function in the form $V(\phi) \propto \phi^{-m}$, $m = 4(f^{-1} - 1), 0 < f < 1$ [15]. This form of the scale factor is also motivated from string/M theory as the cosmological solutions of the weak field of an effective action [16].

In this article, we are investigating the warm inflationary scenario in $F(T)$ theory. Previously in literature, generic inflation scenarios have been investigated in DGP gravity model [17] and Brans-Dicke gravity theory [18], to name a few. It is correct that $F(T)$ gravity generates an inflationary scenario but it is cold while we here study the warm scenario which is far to our knowledge, has not been done before.

II. THE MODEL

In literature there are a number of ways available to achieve the inflation in the very early Universe including models involving single field and multi-fields inflation, non-standard kinetic terms, vector fields and nontrivial gravitational couplings. However we study inflation via a single
scalar inflaton in $F(T)$ gravity and propose a Lagrangian as (units adopted for calculations are $16\pi G = h = c = 1$) \( S = \int d^4 x \ e(L_F + L_\gamma + L_\phi + L_{\text{int}}), \) \hspace{1cm} (2)

where \( e = \det(e^i_{\mu}) = \sqrt{-g}, \) \( e_i(x^\mu) \) are related to the metric via \( g_{\mu\nu} = \eta_{ij}e^i_{\mu}e^j_{\nu} \). where all indices run over 0,1,2,3. \( L_F, L_\gamma \) and \( L_\phi \) represent the Lagrangians for gravity model, energy-matter and the inflaton scalar field respectively. The last term \( L_{\text{int}} \) plays the role of the interaction between inflaton as the scalar player of the inflation and other fields. This scenario for inflation is called warm inflation \[12, 22\]. Specifically the total action reads

\[
S = 2\pi^2 \int dt \ a(t)^3 \left[ F(T) - \rho_\gamma + \frac{1}{2} \partial_\mu \phi^\mu + U(\phi) + L_{\text{int}} \right]. \] \hspace{1cm} (3)

Here \( a \) is the scale factor while \( H \equiv \dot{a}/a \) is the Hubble parameter. The inflaton \( \phi \) has the potential energy \( U(\phi) \) (to be determined in the later sections) and \( \rho_\gamma \) is the energy density of radiation component. The system of the field equations is

\[
S_{\mu\nu}^\rho \partial_\rho T_{TT} + \left[ e^{-1} e_i^{\alpha} \partial_\rho (e e_i^\alpha S_{\alpha}^{\rho\nu}) + T^\alpha_{\lambda\mu} S_{\alpha}^{\nu\lambda} \right] F_T + \frac{1}{4} \delta_\mu^\nu F = T^\nu_{\mu} \] \hspace{1cm} (4)

\[
\nabla_\mu \nabla^\mu \phi + U'(\phi) = 0. \] \hspace{1cm} (5)

The torsion scalar \( T \) is defined by

\[
T = S_{\rho}^{\mu\nu} T^\rho_{\mu\nu},
\]

and the components of the torsion tensor

\[
T^\rho_{\mu\nu} = e^i_{\rho} (\partial_{\mu} e^i_{\nu} - \partial_{\nu} e^i_{\mu}), \]

\[
S_{\rho}^{\mu\nu} = \frac{1}{2} (K^\mu_{\nu \rho} + \delta^\mu_{\nu} T^\theta_{\rho} - \delta^\nu_{\rho} T^\theta_{\mu}),
\]

and also for contorsion tensor

\[
K^\mu_{\nu \rho} = -\frac{1}{2} (T^\mu_{\nu \rho} + T^\nu_{\mu \rho} - T^\rho_{\mu \nu}).
\]

Here \( T_{\mu\nu} = e^a_{\mu} T_{a\nu} \) denotes the energy-momentum tensor for matter field’s Lagrangian \( L_m \), and it is defined by

\[
T_{a\nu} = \frac{1}{e} \frac{\delta L_m}{\delta e^{a\nu}}.
\]

\(^1\) Strictly speaking, \( F(T) \) is not a model of modified gravity like \( F(R), F(R, G) \) and etc. It doesn’t indicate any simple modification of Einstein-Hilbert action. In a more common form, it defines a geometry using asymmetric connections and based on the idea of teleparallelism. Commonly it stated that the later case is identical to GR. But it’s only at the level of action.
Also in the KG equation, the covariant derivatives are with respect to the metricity condition $g^{\mu \nu};_{\mu} = 0$.

Note that the precise form of the interacting Lagrangian $\mathcal{L}_{\text{int}}$ is not known, however a dissipation term $\Gamma \dot{\phi}$ is introduced in the dynamical field equation of the inflaton \[^{22}\]. Earlier $\Gamma$ was considered as a phenomenological function, but very recently starting from the first principles, the general dissipation coefficient in low-temperature warm inflation has been derived \[^{23}\]. The implications of the above model were recently explored in \[^{24}\] via Noether symmetry approach and in \[^{25}\] via energy conditions. In \[^{24}\] we showed that such a model admits $F(T) \sim T^{3/4}$, $V(\phi) \sim \phi^2$ and can drive cosmic acceleration in the late time evolution of the Universe while crossing the phantom divide line at the present time. In the present article, we discuss its implications in the very early Universe, particularly cosmological inflation. The Friedmann-Robertson-Walker (FRW) metric representing a spatially flat, homogeneous and isotropic spacetime is

\[ ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2). \] (6)

The modified Friedmann equations are \[^{7}\] (for a review see \[^{27}\])

\[ 12H^2 F_T + F = \rho_\phi + \rho_\gamma, \] (7)

\[ 48H^2 \dot{H} F_{TT} - 4F_T(3H^2 + \dot{H}) - F = p_\phi + p_\gamma, \] (8)

The torsion scalar reads as $T = -6H^2$, and associated inflaton’s density and pressure are

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + U(\phi), \] (9)

\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - U(\phi), \] (10)

where $\rho_\phi$, $p_\phi$, show the energy density and effective pressure of inflaton, while $\rho_\gamma$ and $p_\gamma$ are correspondingly energy density and pressure of the radiation. The KG equation defines a unique causal dynamics for the scalar field with a frictional term \[^{22}\]

\[ \ddot{\phi} + 3H \dot{\phi} + U'(\phi) = -\Gamma \dot{\phi}, \] (11)

where we absorbed the interaction of inflaton with all other existing fields during inflation using the dissipation factor $\Gamma$. We mention here that from the dynamical point of the view, Warm inflation is a phase transition with a dissipation auxiliary sub-dominant mechanism. There is a possibility to compute this factor for different fields (scalar, fermion) using quantum field theory \[^{26}\]. Note that equation \(^{11}\) is inhomogeneous due to presence of a decay or dissipation term on the right hand side. The term $-\Gamma \dot{\phi}$ in \(^{11}\) shows the decaying nature of the field and its conversion to the
thermal component. Eq. (11) is a special case of the generalized Langevin equation [28]. In general Γ is a dynamical parameter and can not be taken constant, but later we will investigate a regime in which the form of H(t) allows to take the rate of dissipation as a very slowly varying function of time. It turns out that cosmic inflation was de Sitter-like with ρφ + pφ ≈ 0 (or wφ ≈ −1) [29]. To model a cosmic accelerated expansion in very early Universe, it is convenient to use scalar fields and we employ the same strategy. Such cosmic inflaton is quasi-stable and can decay to other forms of energy like radiation and matter in the process of reheating as discussed in conventional inflationary models. Moreover the quantum fluctuations in a quantum inflaton will serve as seeds for later structure formation. The dynamical equations are described by:

\[ \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\Gamma \dot{\phi}^2, \]

(12)

\[ \dot{\rho}_\gamma + 4H \rho_\gamma = \Gamma \dot{\phi}^2. \]

(13)

Here Γ = f(ϕ, TA) may be taken general as a function of the inflaton ϕ, or the average temperature TA, or both [30]. Furthermore we suppose that Γ > 0, as the Second Law of Thermodynamics must be valid even in the inflationary era. In inflationary epoch, the Friedmann equation (7) reduces to

\[ 12H^2 F_T + F \approx \rho_\phi. \]

(14)

The perturbations of the scalar field at the quantum level were the initial seeds in the Universe and which culminated in the large scale structure formation [31]. It is believed that quantum field theoretic version of warm inflation resolves the horizon and flatness problem [32]. We consider R as the decay (or dissipation) rate defined by

\[ R = \frac{\Gamma}{3H}. \]

(15)

Here the decay rate R is a dynamical quantity. To estimate R we need the form of Γ. In accordance to the results of the QFT the interacting supersymmetric theory [33], the decay rate can be written as

\[ \Gamma \simeq C_\phi T^3_{\phi^2}. \]

(16)

where C_ϕ = 0.64h^4N in which N = N_χN_{decay}^2. Here N_χ is related to the superfield’s decay (further details can be seen in [34] and references therein). It is one possible form of the dissipation parameter. Other forms are different and they need separate calculations (beyond the scope of our work). We study two models:

\[ \Gamma \simeq \frac{T^3_\phi}{\phi^2}, \quad \text{Model-I.} \]

(17)

\[ \Gamma \simeq \Gamma_1 = \text{const}, \quad \text{Model-II.} \]

(18)
Here $T_*$ is bath’s temperature. Just as a historical fact we mention here that in the original proposal of warm inflation, the authors at that time, could not specify the precise form of $\Gamma$ due to the lack of a detailed model [22]. Later on taking motivation from super-string theory and quantum field theoretic approach, a suitable forms of $\Gamma$ were derived [23, 26, 33].

From Eqs. (12) and (14), we obtain

$$\dot{\phi}^2 = -\frac{4\dot{H}(F_T + 2TF_{TT})}{R + 1}. \quad (19)$$

Here the choice of $F(T)$ must be astrophysically viable which means that it must be in conformity with the observational data. We assume that in inflationary epoch, radiation is prone to decay on average on certain cosmic time scales i.e. $\dot{\rho}_\gamma \ll 4H\rho_\gamma$, and $\dot{\rho}_\gamma \ll \Gamma \dot{\phi}^2$. From (13) we get the modified radiation density as

$$\rho_\gamma = \frac{\Gamma \dot{\phi}^2}{4H}. \quad (20)$$

In other words, if the decay is much rapid than cosmic expansion $\Gamma \gg H$, (keeping $\dot{\phi}^2 > 0$ to avoid ghosts) the radiation density will keep on increasing. In warm inflationary scenario, the period of inflation continuously (but very rapidly) dilutes to the radiation density. But due to presence of dissipation, the depletion of radiation is compensated. Inserting (19) in (20) we obtain

$$\rho_\gamma = -\frac{\Gamma \dot{H}(F_T + 2TF_{TT})}{H(R + 1)}, \quad (21)$$

which can be written as $\rho_\gamma = C_\gamma T_*^4$, where $C_\gamma = \pi^2 g_* / 30$. Here $g_*$ denotes the countable numbers of relativistic degeneracy of states and $T_*$ is the average temperature for the background of thermal bath given by

$$T_* = \left( -\frac{\Gamma \dot{H}(F_T + 2TF_{TT})}{HC_\gamma(R + 1)} \right)^{1/4}. \quad (22)$$

Now from (7) and (33) we obtain the inflaton scalar potential

$$U(\phi) = -2TF_T + F + \frac{\dot{H}}{R + 1}(F_T + 2TF_{TT})\left(2 + \frac{\Gamma^2}{H}\right). \quad (23)$$

We consider the following model of modified gravity [35]

$$F(T) = T + \alpha\sqrt{-T} + \beta. \quad (24)$$

The first linear term indicates the Lagrangian for $F(T)$ gravity and is dynamically equivalent to the general relativity at the level of action. The second term denotes a ghost dark energy and performs a role of cosmological constant and the last term $\beta$ is a constant. We mention here that
\( F(T) \) represents the geometrical part of our model. There is no dark energy in our model. Strictly speaking, in the warm inflation scenario and in inflationary era when the inflation is driven by inflaton field, we can safely neglect the dark energy density. So, the only thing which we need is a background with a definite geometry, here is the spacetime with torsion and radiation and inflaton. So, this \( F(T) \) can be considered just as geometry. It is not necessary for \( F(T) \) model to mimic the equation of state of any kind of dark energy. But it can be reconstructed mathematically as a toy model of a type of dark energy ghost dark energy if and only if we neglect all matter fields. In our context, we used this form of \( F(T) \) because it is a viable model. However we redefine the field equation in terms of effective FRW field equations as the following:

\[
3H^2 = \rho_{F(T)} + \Sigma \rho_i, \quad \rho_{F(T)} = -F(T) - T + 2TF_T(T).
\] (25)

We observe that in the absence of any matter field (beyond our inflationary scenario with matter fields like radiation, inflaton) \((24)\) gives us \( \rho_{F(T)} = \beta \). It has the meaning of a mathematical reconstruction of a type of dark energy which is not interesting for us in this paper. If we investigate dark energy this form of energy density makes us worry. But here in inflationary era, we do not need any dark energy component. Further more \( \rho_{F(T)} \) does not indicate any dark energy. So absence of such variable \( \rho_{F(T)} \) does not sense any problem for our model. We used \( F(T) \) just for gravity of the model. For dynamical evolution we have inflaton and radiation. Note that in inflationary models, radiation and inflaton are the most important fields. The action ansatz \((24)\) is different form a pure cosmological constant. It can be reconstructed from different points of view \([36–40]\). So this form of \( F(T) \) is one of the most physically viable models of \( F(T) \) proposed by us as the first time \([35]\). It reduces to the TEGR with a cosmological constant. Even if we treat with \( F(T) \) as a model of dark energy this simple model is able to explain warm inflation in a generalized teleparallel gravity.

In order to recover the intermediate scenario, we make the scale factor

\[
a(t) = A \exp \left[ X(1 + \omega(t - t_0))Y \right],
\]

\[
X = \frac{H_0(2n + 1)}{2\omega(1 + n)}, \quad Y = \frac{2n + 2}{2n + 1},
\]

which is very similar to the intermediate inflationary scenario \([42]\). Since we do not need any further assumption on acceleration in this inflationary era, we choose \( t_0 = 0 \). Thus our ansatz is viable as it yields an intermediate inflationary scenario in \( F(T) \) gravity. Using this scale factor we have:

\[
\dot{H} = \frac{c}{(-T)^n}
\] (27)
where $c < 0$ and $n$ are constants. Indeed this equation is in agreement with $T = -6H^2$ if and only if $a(t) = A \exp \left[ X(1 + \omega(t - t_0))Y \right]$, since:

$$
\dot{H} = \frac{c}{(-T)^n} \to \dot{H}H^{2n+1} = \frac{c}{6^n} \to H(t) = \left( \frac{c(2n + 1)}{6^n} (t - t_0) \right)^{1/(2n+1)} \quad (28)
$$

$$
\frac{d\log a(t)}{dt} = \left( \frac{c(2n + 1)}{6^n} (t - t_0) \right)^{1/(2n+1)} \to a(t) = A \exp \left[ X(1 + \omega(t - t_0))Y \right].
$$

$$
X = \frac{H_0(2n + 1)}{2\omega(1 + n)}, \quad Y = \frac{2n + 2}{2n + 1}.
$$

This is the same assumption which we used in our paper. To have a correct dimension, we see $\frac{c}{6^n} = O(H_0^{2n+2})$, so it is adequate to define a new parameter $k_1 = \frac{c}{6^n H_0^{2n+2}}$, such that the Hubble parameter reads as

$$
\dot{H} = k_1 \frac{H_0^{2n+2}}{H^{2n}}, \quad k_1 < 0. \quad (29)
$$

The scale factor and Hubble parameter is suitably chosen so that it is consistent with the $F(T)$ gravity and the intermediate expansion. The scale factor is necessary to perform the analysis and therefore working with a hypothetical scale factor may not be consistent with the inflationary scenario. Hence we picked the intermediate scale factor which is also consistent with astrophysical observations.

The slow-roll parameters which provide a necessary (but not sufficient) condition for inflation are defined as

$$
\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = -\frac{\ddot{H}}{H\dot{H}}, \quad (30)
$$

In the present context, using (29), the slow-roll parameters yield

$$
\epsilon = -k_1 \left( \frac{H_0}{H} \right)^{2n+2}, \quad \eta = 2nk_1 \left( \frac{H_0}{H} \right)^{2n+2}. \quad (31)
$$

The first very interesting observation from (31) is that the ratio between two slow-roll parameters remains constant, free from the model of $F(T)$ or time evolution of the Hubble parameter i.e.

$$
\frac{\eta}{\epsilon} = -2n. \quad (32)
$$

Also since always $H < H_0$ \footnote{The Hubble parameter after the end of inflation is assumed to be larger compared to its value at any later time.}, so to have a small set of the parameters we must have $k_1 << 1$. Since $\eta = \frac{m_\phi^2}{3H^2}$, our model implies two possible situations: either $H \to \infty$ (it corresponds to the $\epsilon \cong \eta \cong 0$) and finite $m_\phi \neq 0$ or $m_\phi = 0$ and $H$ is finite. Inflationary expansion stops when $\rho_\phi \cong \rho_\gamma$ and the field violates the slow-roll approximations $\dot{H} + H^2 \approx 0$, ($\ddot{a} \approx 0$) or $\epsilon(\phi_e) = 1$. 
In high dissipative regime for our $F(T)$ model, we have:

$$\rho_\gamma = -3\dot{H}(F_T + 2TF_{TT}), \quad (33)$$

The condition of the inflation implies that $\ddot{a} < 1$. It corresponds to that $\epsilon < 1$. Now we compute the number of e-folding using the standard definition:

$$N(t) = \int_{t_1}^{t_2} H(t)dt. \quad (34)$$

where $t_1$ and $t_2$ correspond to time of start and end of inflationary period respectively. We need also a perturbation theory for our model. In the flat FRW background, this perturbation can be computed just using the perturbation of the inflaton $\phi$. In the warm inflation the expression of the fluctuations of the inflaton reads

$$< \delta \phi >_{\text{thermal}} = \left(\Gamma HT^2\right)^{1/4}. \quad (35)$$

Now for power spectrum of the primordial cosmic radiation, a more general expression reads as the following [46]

$$P_R = R^{1/2} \left(\frac{H^2}{2\pi \phi}\right)^2 \frac{T_*}{H}. \quad (36)$$

Here, the starred quantities correspond to the evaluated parameters at horizon crossing [47]. As mentioned in [47], that in the regime when particle production is not so high, it is possible to ignore $n_s$ from the power spectrum calculations. This power spectrum is a function of the wavelength. From observational data of WMAP7 [48] we know that for $k = 0.002\text{Mpc}^{-1}$ the following value is accepted:

$$P_R = (2.445 \pm 0.096) \times 10^{-9}. \quad (37)$$

The scalar spectral (or power spectrum) index $n_s$ is given by\(^3\)

$$n_s - 1 = \frac{d \ln P_R}{d \ln k}. \quad (38)$$

The conventional inflationary models predict that the initial density perturbations have a Gaussian distribution, and their power spectrum index is, $n_s \approx 1$. The early COBE experiment results turned out to be in agreement with the above prediction [50]. As is well-known in the literature,

\(^3\) Alternative definitions of $n_s$ are $n_s = 1 + 2\eta - 6\epsilon$, and $n_s = 1 + 4\frac{\dot{H}}{H^2} - \frac{\ddot{H}}{H^2}$. [49].
the necessary number of e-fold should be between 60 – 80, to produce our observable Universe \[1\]. For perfect Gaussianity \((n_s = 1)\), however CMB is not completely Gaussian and the non-Gaussian (i.e. \(n_s\) deviates from unity by small amounts) features point to some deeper physical mechanisms that still need to be understood \[51\]. Our model predicts a non-Gaussianity of CMB which varies for differently picked e-folding numbers. Here \(d \ln k(\phi) = dN(\phi) = (H/\dot{\phi})d\phi\) i.e. wave number interval is related with number of e-fold parameter. Following \[52\], if the tensor perturbations are generated during inflation, than it would produce gravitational wave. The detection of these primordial gravitational waves is already underway by LISA, BBO and DECIGO \[53\] and also the value of Planck data \(n_s \approx 0.96\) \[54\]. The corresponding gravitational wave power spectrum becomes

\[
P_g(\phi) = 4 \left(\frac{H}{2\pi}\right)^2.
\]  

(39)

Single inflaton slow roll scenario is falsifiable on the basis of the following future observations: (1) CMB has large non-Gaussian features, (2) Non-zero iso-curvature perturbations and (3) Large running of the scalar spectrum. The scalar-tensor ratio \(r(\phi)\) is given by

\[
r(\phi) = \frac{P_g}{P_R}.
\]  

(40)

Also the spectral index for tensor perturbations is:

\[
n_t = -2\epsilon.
\]  

(41)

It is adequate here to compare \(n_s\), and \(r\) with the observational data of pre-Planck \[54\]. As it is analyzed through Planck, we know that \[54, 58\]: \(n_s \approx 0.9603 \pm 0.0073\) (at 68% confidence limit), \(n_s \approx 0.961 \pm 0.007\) from pre-Planck data and finally \(r < 0.11\) (at 95% confidence limit) and \(r < 0.22\) from WMAP7.

### III. INTERMEDIATE INFLATION

As we mentioned before our \(F(T)\) based model of inflation naturally leads to the scale factor as the following Hubble parameter:

\[
H = H_0(1 + \omega t)^{(2n+1)^{-1}}, \quad \omega = k_1 H_0 (2n + 1).
\]  

(42)

This is similar to the intermediate form of the Hubble parameter if we shift the time and identify the parameters \(H_0, \omega\) with the parameters \(A, f\) of the intermediate inflation model. For our model,
the e-folding number reads as
\[ N = \int_{t_1}^{t} H dt = \frac{1}{2k_1(1+n)} \left( (1 + \omega t)^{(2n+2)/(2n+1)} - (1 + \omega t_1)^{(2n+2)/(2n+1)} \right). \] (43)
Here \( t_1 \) denotes the initializing time of the inflation and \( t \) the ending time.

A. Model-I: \( \Gamma = \Gamma_0 t_0^{\gamma/2} \)

In slow roll approximation and in the high dissipation regime \( R >> 1 \), when (14) is valid, using (16) and (22) and by using (42) we have the following solutions for \( H \), inflaton \( \phi \):
\[ \phi(t) = \phi_0 e^{\theta(1+\omega t)(4n+3)/(2(2n+1))}. \] (44)
\[ H = H_0 \theta^{-2/(4n+3)} \left( \log \frac{\phi}{\phi_0} \right)^2/(4n+3). \] (45)
Here \( \tau_0^{-2} = \frac{4C_r H_0^3/4}{1} \), \( \theta = \pm \frac{2^{(2n+1)}}{\eta_0(3+4n)k_1 H_0} \) and \( \Gamma_0 = \text{const} \). Using (31),(30) the slow roll parameters reads are written as
\[ \epsilon = -k_1 \theta^{2(2n+2)/(4n+3)} \left( \log \frac{\phi}{\phi_0} \right)^{-2(2n+2)/(4n+3)}. \] (46)
\[ \eta = 2n k_1 \theta^{2(2n+2)/(4n+3)} \left( \log \frac{\phi}{\phi_0} \right)^{-2(2n+2)/(4n+3)}. \] (47)

The energy density corresponds to the radiation is given by
\[ \rho_\gamma = \hat{\rho} \left( \log \frac{\phi}{\phi_0} \right)^{-3/(4n+3)}. \] (48)
Where \( \hat{\rho} = \frac{\Gamma_0^{3/4} H_0^{1/2} \left(-k_1/C_r\right)^{3/4}}{4\pi^2} \theta^{-3/(4n+3)} \).

Number of e-folding reads from the (49) as the following:
\[ N = \frac{\theta^{-2(2n+2)/(4n+3)}}{2k_1(1+n)} \left( \left( \log \frac{\phi}{\phi_0} \right)^{2(2n+2)/(4n+3)} - \left( \log \frac{\phi_1}{\phi_0} \right)^{2(2n+2)/(4n+3)} \right). \] (49)

At the beginning of the inflation using \( \epsilon(\phi_1) = 1 \) we have:
\[ \phi_1 = \phi_0 e^{\theta(-k_1)(4n+3)/(2(2n+2))}. \] (50)

Now we will compute the e-folding in terms of the inflaton (or vise vera):
\[ \phi = \phi_0 e^{\theta \frac{N}{N_e} - u}(4n+3)/(2(2n+2)) \] (51)
Here \( N_e = \frac{\theta^{-2(2n+2)/(4n+3)}}{2k_1(1+n)} \) and \( u = k_1 \theta (\frac{3+4n}{3+4n}) \).
Now using this last equation, the expressions of the power-spectrum, spectral index (scalar+tensor) and finally the tensor-scalar ratio can be calculated using (36,38,40,41) as the following:

\[ P_R = P_0 \left( \frac{N}{N_c} - u \right) \frac{1}{4(2n+1)(4n+3)} \frac{10n^2 - 15n - 6}{2(2n+1)(4n+3)} e^{-3(\frac{N}{N_c} - u)^{4(n+3)}}. \quad (52) \]

\[ P_g = \frac{H_0^2 \theta^{-4/(4n+3)}}{\pi^2} \left( \frac{N}{N_c} - u \right)^{1/(n+1)}. \quad (53) \]

\[ n_s = 1 - \frac{H_0^2 \theta^{-4/(4n+3)}}{\tau_0 \phi_0} \left( \frac{N}{N_c} - u \right)^{-1/(2n+2)} e^{-\left(\frac{N}{N_c} - u\right)^{4(n+3)}} \left( \frac{10n^2 - 15n - 6}{2(2n+1)(4n+3)} \left( \frac{N}{N_c} - u \right)^{-\frac{4n+3}{4n+7} + 3} \right)^{4(n+3)} \quad (54) \]

\[ n_t = \frac{2k_1 \theta^{2(2n+2)/(4n+3)}}{\frac{N}{N_c} - u}. \quad (55) \]

Here

\[ P_0 = \frac{\tau_0^2}{4\pi^2 \sqrt{3\phi_0}} \left( 3k_1 \right)^{5/8} H_0^{-\frac{3\phi_0+11}{2n+1}} \theta \left( \frac{1}{\frac{N}{N_c} - u} \right)^{\frac{10n^2 - 15n - 6}{2(2n+1)(4n+3)}}. \quad (56) \]

And also the scalar-tensor ratio reads:

\[ r(N) = \frac{H_0^2 \theta^{-4/(4n+3)}}{\pi^2 P_0} \left( \frac{N}{N_c} - u \right) \left( \frac{N}{N_c} - u \right)^{\frac{n+2+10n^2}{8(2n+1)(2n+3)}} \left( \frac{N}{N_c} - u \right)^{(4n+3)/(2(2n+2))}. \quad (57) \]

Graphics of (54), (57) are presented in figures 1 and 2. We set the parameters as \( n = -\frac{3}{2} \) to compare with the parameters in the usual intermediate inflationary models \( \omega = 1, f = \frac{2n+2}{2n+1}, A = \frac{(2n+1)H_0}{2n+2} \).

**B. Model-II: \( \Gamma \simeq \Gamma_1 \)**

In this case and in the high dissipation regime, the solutions for inflaton and Hubble are written as the following:

\[ \phi = \phi_0 + \theta_1 (1 + \omega t) \theta_1^{1-2n}. \quad (58) \]

\[ H = H_0 \left( \frac{\phi - \phi_0}{\theta_1} \right)^{\frac{3}{2-n}}. \quad (59) \]

Here \( \theta_1 = \frac{2(2n+1)}{\tau_1 \omega(2n+3)}, \tau_1 = \sqrt{\frac{\Gamma_1 (2n+1)}{12H_0^2 \omega}} \). So, the slow role parameters read:

\[ \epsilon = -k_1 \left( \frac{\phi - \phi_0}{\theta_1} \right)^{\frac{4(n+1)}{2n+1}}. \quad (60) \]

\[ \eta = 2nk_1 \left( \frac{\phi - \phi_0}{\theta_1} \right)^{\frac{4(n+1)}{2n-1}}. \quad (61) \]
FIG. 1: 

(Left) Spectral scalar index $n_s$ versus $N$ given by (54). The data is confirmed by $\Gamma_0 = C^{1/6} = 70, n = -\frac{1}{2}, k_1 \sim 7 \times 10^7$. Our model gives us $n_s(N = 60) \sim 0.95$ in a reasonable agreement with the WMAP7 and for pre Planck $n_s < 0.961$. (Right) Spectral index versus ratio of power spectra. We predict that $n_s|_{0.69} \approx 0.12$ in agreement with observational data.

Also, the radiation density reads:

$$\rho_\gamma = \frac{-3H_0\omega}{2n+1} \left( \frac{\phi - \phi_0}{\theta_1} \right)^{\frac{4n}{2n-1}}.$$  

(62)

So, the e-folding from (19) reads:

$$N = \frac{1}{2k_1(n + 1)} \left[ \left( \frac{\phi - \phi_0}{\theta_1} \right)^{\frac{4(n+1)}{1-2n}} - \left( \frac{\phi_1 - \phi_0}{\theta_1} \right)^{\frac{4(n+1)}{1-2n}} \right].$$  

(63)

The starting inflaton’s magnitude is:

$$\phi_1 = \phi_0 + \theta_1(2nk_1)^{\frac{-2(n-1)}{4(n+1)}}.$$  

(64)

So, it is possible to rewrite the inflaton in terms of the $N$:

$$\phi = \phi_0 + \theta_1(2k_1(1 + n))N + \frac{1}{2nk_1} \left[ \frac{1-2n}{4(1+n)} \right]^{\frac{1}{4(1+n)}}.$$  

(65)

So, the temperature is obtained:

$$T_\ast = \left( \frac{-3H_0\omega}{C_\gamma} \right)^{\frac{14}{14}} \left( \frac{\phi - \phi_0}{\theta_1} \right)^{\frac{n}{n-1}}.$$  

(66)

Now we can write the spectrum and indexes functions as the following list:

$$P_R = P_1 \left[ (2k_1(1 + n))N + \frac{1}{2nk_1} \right]^{\frac{2}{3}}.$$  

(67)
Here

\[ P_1 = \frac{H_0^{5/2}}{4\pi^2 \sqrt{3}} \Gamma_1^{1/2} \left( \frac{-3H_0^2\omega}{C_\gamma} \right)^{1/4}. \]  

(68)

\[ P_g = \frac{H_0^2}{\pi^2} \left[ (2k_1(1+n))N + \frac{1}{2nk_1} \right]^{1+n}. \]  

(69)

\[ n_t = -4nk_1 \left[ (2k_1(1+n))N + \frac{1}{2nk_1} \right]^{-1}. \]  

(70)

The scalar to tensor ratio

\[ r = \frac{H_0^2}{\pi^2 P_1} \left[ (2k_1(1+n))N + \frac{1}{2nk_1} \right]^{-\frac{1+3n}{\pi(1+n)}}. \]  

(71)

So, the index \( n_s \) reads:

\[ n_s = 1 - \frac{3(n+1)}{\theta_1 \tau_1 H_0(2n-1)} \left[ (2k_1(1+n))N + \frac{1}{2nk_1} \right]^{\frac{1}{\pi(1+n)}}. \]  

(72)

We check the data for (71,72). We set \( \Gamma_1 = C_\gamma^{1/6}, \ n = \frac{5}{4} \).

\[
\begin{array}{c}
\text{FIG. 2: (Left) Spectral scalar index } n_s \text{ versus } N \text{ given by (68). The data is confirmed by } \Gamma_0 = C_\gamma^{1/6}, C_\gamma^{1/6} = 70, n = -\frac{5}{4}, k_1 \sim 7 \times 10^7. \text{ Our model gives us } n_s(N = 50) \sim 0.95 \text{ in a reasonable agreement with the WMAP7 and for pre Planck } n_s < 0.97. \text{ (Right) Spectral index versus ratio of power spectra. We predict that } n_s|_{0.69} \approx 0.95 \text{ in agreement with observational data.}
\end{array}
\]
IV. CONCLUSION

In summary, for the first time in literature, we investigated cosmological warm inflation model in the framework of teleparallel gravity. We introduced a canonical inflaton to act as an inflaton field. We calculated the relevant inflationary parameters such as scalar-tensor ratio, power-spectrum indices for density perturbations and gravitational waves and e-folding parameter. We get a constant ratio of two slow-roll parameters whose value depends on our ansatz. Like generic inflationary models, ours also predicts a gravitational wave background with a power spectrum. Our calculations support the warm-intermediate inflationary scenario. Moreover our results are compatible with compatible with astrophysical observations of cosmic microwave background and pre Planck. Our principal motivation is firstly we proposed warm inflation in torsion based spacetime, for the first time using a scalar field. But the main result which it differs our work from any previous work is that our numerical results and out proposed mechanism gives us \( n_s(N = 60) \sim 0.95 \) in a reasonable agreement with the WMAP7 and for pre Planck \( n_s < 0.961 \). So, in agreement with data we success to warm inflation in F(T). This is the main credit of the present work for publication. We mention here that a better and viable model of warm inflation can be investigate if we use a dynamical form of \( F(T) = T + \alpha \sqrt{-T} + \sqrt{-T} \log(-T) + \beta \). It means we want \( \rho_{F(T)} = \rho_{DE} \) to be proportional to \( H(t) \). Although as we explained it in details, in our warm inflation we can safely neglect the effect of dark energy in favor of other dominant matter fields, but this suggestion will be carried out as a separate paper.

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