Designing a PI controller for Cuk converter using converter dynamics toolbox for MATLAB

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1. Introduction

Power electronics converters require negative feedback to provide a suitable output voltage or current for the load. Obtaining a stable output voltage or current in presence of disturbances like: input voltage's changes and /or output load's changes seems impossible without some form of control. Although control engineering has considerable progress over recent decades, most applications use PID controllers, because of their low price and simplicity. Generally speaking, using derivative term is not so common in power electronics converters control. Usually a P or PI controller is all that is required. Designing a classical P or PI controller for a power electronics converter is started by obtaining the model of converter. Modeling is the process of formulating a mathematical description of the system. Obtaining the mathematical model of system is the first step toward designing a controller in model base controller design techniques. Switching power converters are nonlinear variable structure systems. Various techniques can be found in literature to obtain a linear continuous Time Invariant (LTI) model of a DC-DC converter. The most well-known methods are: Current injected approach, circuit averaging and state space averaging (Middlebrook and Cuk, 1977; Kislovski et al., 1991; Mohan and Undeland, 2007; Voperian, 1990). Averaging and small signal linearization is key steps of these methods.

State space averaging described in Middlebrook and Cuk (1977) is appropriate to describe converters that work in CCM while is less suitable for converters work in DCM. The current injected method Kislovski et al. (1991) and Mohan and Undeland (2007) can do the job of modeling in either CCM or DCM. Circuit averaging gained a lot of attention recently due to its generality (Hren and Slivar, 2005).

At the time of this writing, there is no software to extract algebraic transfer function of power electronics converters. Using available commercial software only a frequency response plot can be obtained. No information about location of poles and zeros are given by software so a pencil-and-paper analysis is required to obtain the algebraic transfer function of converter. Obtaining the dynamical equation of converter in presence of circuit’s non idealities such as Equivalent Series Resistance (ESR) of capacitors and inductors, voltage drop of diodes, on resistance of MOSFETS is a tedious, time consuming and error prone task for pencil-and-paper analysis. If a parameter is changed, i.e. load, all the calculation must be done from scratch. Developed software can be used to extract dynamical equation of buck, boost, buck-boost, Cuk, SEPIC, fly back, forward and full bridge converters in presence of mentioned non-idealities. Developed software can
give both algebraic transfer function and frequency response plot of converter. Process of controller design can be started after converter’s dynamical equation extraction; MATLAB’s control system toolbox can be quite helpful for this phase. This paper shows how a control system can be designed for a Cuk converter. Cuk converter has a 4th order, i.e. has 4 poles, non-minimum phase transfer function, i.e. zeros are in RHP. It can be shown by Bode theorem that presence of RHP zeros degrades the achievable close loop performance. Presence of RHP zeros force us to use both current and voltage feedback. MATLAB’s control system toolbox is used to tune these two loops. Proposed method can be used to design controller for other non-minimum phase converters like SEPIC.

2. Related works

Foundation of State Space Averaging (SSA) was laid down in Middlebrook and Cuk (1977). The first attempt to model Discontinuous Conduction Mode (DCM) is presented in Cuk and Middlebrook (1977). Accurate small signal models for DCM operation were developed by Sun et al. (2001). A unified SSA based method to develop both CCM and DCM was developed by Suntio (2006). A comprehensive survey of the modeling issues can be found in Maksimovic et al. (2001). Application of different control methods to power electronics converters has been studied in many papers. For example, feedback linearization (Sanders et al., 1986), sliding mode control (Sira-Ramirez, 1987), PID control (Venkatanarayanan and Saravanan, 2014) and $H_\infty$ design (Rodriguez et al., 2005) has been applied to Cuk converter, Linear Matrix Inequality (LMI) control has been applied to conventional boost by Reddy et al. (2015). Discrete time controller has been designed for a boost converter in Alkrunz and Yazıcı (2016). A cascade state space controller is designed for buck mode of bidirectional dc-dc converter in Ocilka and Béreš (2010). PID control of SEPIC converter is studied in Veenalakshmi et al. (2014).

3. Working principle of Cuk converter

Topology of Cuk converter is shown in Fig. 1. Output voltage can be either smaller or larger than that of input. There is a polarity reversal on the output. Energy transfer from input source to load depends on the capacitor $C_1$. Analysis of this circuit is based on the following assumptions:

1. Both inductors (capacitors) are large enough so current in (voltage across) them are constant.
2. Circuit operating in steady state, i.e. transients has been passed.
3. Switch and diode are ideal (i.e. no resistance and voltage drop).
4. Switch is closed for time $DT$ and open for $(1-D)T$.

Corresponding circuit for switch closed and opened is shown in Figs. 2 and 3, respectively.

Applying volt-second balance (Mohan and Undeland, 2007) to inductors lead to:

$$V_{c_1} = \frac{1}{1-D} V_s \tag{1}$$

$$V_{c_2} = -\frac{D}{1-D} V_s \tag{2}$$

Assuming an ideal capacitor, i.e. no Equivalent Series Resistance (ESR), output voltage ripple can be found as:

$$\Delta V_o = \frac{1-D}{8L_2C_2f^2} V_s \tag{3}$$

For Continuous Current Mode (CCM) operation of Cuk converter, the inductors average current must be greater than one half the changes in current. This leads to:

$$L_{1,\text{min}} = \frac{(1-D)^2R}{2Di} \tag{4}$$

$$L_{2,\text{min}} = \frac{(1-D)^2R}{2fDi} \tag{5}$$

Converter works in CCM if $L_1 > L_{1,\text{min}}$ and $L_2 > L_{2,\text{min}}$.

4. Small signal model extraction

Assume a general state space model of following form for a CCM converter:

$$\begin{align*}
\dot{x} &= A_1 x + B_1 u \\
V_o &= C_1^T x
\end{align*} \tag{6}$$
\[
\begin{align*}
\dot{x} &= A_2x + B_2u \\
v_o &= C_2^T x
\end{align*}
\]  \\
(7)

Eqs. 6 and 7 are written under switch close and switch open condition, respectively, \(x\) is state vector, i.e. capacitors' voltage and inductors' current, \(u\) is control input and \(v_o\) is output voltage of converter. Applying the averaging and perturbation to these equations leads to:

\[
\begin{align*}
\frac{d}{dt}x &= [A_1D + A_2(1-D)]\bar{x} + [(A_1 - A_2)x + (B_1 - B_2)v]d \\
\bar{v}_o &= [C_1^T + C_2^T(1-D)]\bar{x} + [(C_1^T - C_2^T)x]d
\end{align*}
\]  \\
(8)

\[
\bar{x}, \bar{d}, \bar{v}_o \]
shows small signal variables and D shows steady state duty ratio. Switch close's and switch open's equations for Cuk converter in presence of non-idealities such as capacitor's and inductor's ESR, MOSFET's on resistance and voltage drop of diode, can be written as:

\[
\begin{align*}
\frac{d}{dt}L_1 &= -\frac{1}{L_1}(r_1 + r_{sw} + r_{c1} + r_{v})L_1 + \frac{1}{L_1}v_1 \\
\frac{d}{dt}L_2 &= \frac{1}{L_2}v_2 \\
\frac{d}{dt}C_1 &= -\frac{1}{C_1}i_2 \\
\frac{d}{dt}C_2 &= -\frac{1}{C_2}i_2
\end{align*}
\]  \\
(10)

for switch closed and

\[
\begin{align*}
\frac{d}{dt}L_1 &= -\frac{1}{L_1}(r_0 + r_2 + r_{c1} + r_{v})L_1 + \frac{r_{v}}{L_1}i_2 - \frac{1}{L_1}v_c + \frac{1}{L_1}v_y \\
\frac{d}{dt}L_2 &= \frac{1}{L_2}v_2 - \frac{r_{v} + r_{v1} + r_{v2}}{L_2}i_2 - \frac{1}{L_2}v_c + \frac{1}{L_2}v_y
\end{align*}
\]  \\
(11)

\[
\frac{d}{dt}v_{C1} = \frac{1}{C_1}i_1 \\
\frac{d}{dt}v_{C2} = \frac{1}{C_2}i_2 - \frac{1}{R_{C1}C_2}v_{C2}
\]

for switch opened.

Using pencil-and-paper analysis to obtain dynamical equation of this 4th order converter is a tedious, time consuming and error prone task. A toolbox is developed for MATLAB to do the SSA procedure automatically. All what must be done by user is entering converter's parameters. Assume a Cuk converter with the following values:

\[
\text{Vin}=12V, \ g_{\text{internal}}=0.6\Omega, \ L_1=500\mu H, \ L_2=700\mu H, \ r_{L1}=10m\Omega, \ r_{L2}=21m\Omega, \ C_1=20\mu F, C_2=4\mu F, r_{c1}=10m\Omega, \ r_{c2}=5m\Omega, \ V_{\text{Diode on}}=0.7, \ g_{\text{Diode on}}=0.05 \ \Omega, \ g_{\text{MOSFET}}=40 \ \mu \Omega, \ R_{\text{Load}}=8.1\Omega.
\]

After entering these values to software (Fig. 4), following results are obtained:

\[
\begin{align*}
\frac{v_{C1}(s)}{d(s)} &= -9.09\times10^{-6}s^2 + 5.618\times10^{-5}s + 2.929\times10^{-7} \\
\frac{i_{C1}(s)}{d(s)} &= 5.013\times10^{-6}s^2 + 1.754\times10^{-5}s + 1.317\times10^{-7} \\
\frac{i_{C2}(s)}{d(s)} &= -3.44\times10^{-7}s + 9.787\times10^{-7}s + 3.27\times10^{-9}s - 3.694\times10^{-10}
\end{align*}
\]  \\
(12)

(13)

(14)

\(\frac{v_0(s)}{d(s)}\) and \(\frac{i_{L2}(s)}{d(s)}\) are not minimum phase transfer function, while \(\frac{i_{L1}(s)}{d(s)}\) is minimum phase. Suggested control structure is shown in Fig. 5.

Fig. 4: Entering Cuk converter's parameters to developed software
As seen in Fig. 5, there are two control loops: inner loop which control inductor L1’s current and outer loop which control output voltage. These two loops must be designed properly in order to achieve desired performance. Inner loop must be designed first. Pole-zero diagram of $\frac{i_{L1}}{d}$ is shown in Fig. 6.

![Pole-zero diagram of $\frac{i_{L1}}{d}$](image)

**Fig. 6:** Pole-zero diagram of $\frac{i_{L1}}{d}$

As shown in Fig. 6, $\frac{i_{L1}}{d}$ is minimum phase. Using MATLAB’s control system toolbox, following values are found for controller parameters:

$$K_p, \text{ inner loop} = 1.748,$$
$$K_i, \text{ inner loop} = 0.$$

Where $K_p$ and $K_i$ show proportional gain and integral term gain of PI controller. Close loop step response of inner loop is shown in Fig. 7.

![Close loop step response of inner loop](image)

**Fig. 7:** Close loop step response of inner loop ($K_p, \text{inner loop} = 1.748, K_i, \text{inner loop} = 0$)

Close loop transfer function of inner loop is:

$$H_{cL, \text{inner loop}} = \frac{-8.92 \times 10^4 s^4 + 2.95 \times 10^3 s^2 + 23.2 \times 10^2 s + 2.302 \times 10^1}{s^4 + 4.53 \times 10^3 s^2 + 5.309 \times 10^2 s + 2.71 \times 10^1}$$  \hspace{1cm} (15)

Using converter dynamics toolbox for MATLAB, $\frac{v_o(s)}{i_{L1}(s)}$ is found as:

$$\frac{v_o(s)}{i_{L1}(s)} = -1.783 \times 10^3 s^2 + 7.091 \times 10^2 s - 5.863 \times 10^2$$ \hspace{1cm} (16)

Using control system toolbox to tune the outer loop lead to:

$$K_p, \text{ outer loop} = 0,$$
$$K_i, \text{ outer loop} = -275.$$

Close loop step response of outer loop is shown in Fig. 8.

![Close loop step response of outer loop](image)

**Fig. 8:** Close loop step response of $\frac{v_o(s)}{v_{ref}(s)}$

As shown in Fig. 8 close loop step response has zero steady state error.

5. Simulation results

Simulink diagram of Cuk converter with designed controllers is shown in Fig. 9.

Performance of designed controllers is tested with the aid of following scenario: Load resistance goes from $R_{load} = 8 \Omega$ to $R_{load} = 2.7 \Omega$ at $t = 20 \text{ms}$, input source voltage changes from $V_S = 12V$ to $V_S = 6V$ at $t = 40 \text{ms}$ and reference voltage has been changed from $V_{ref} = 14V$ to $V_{ref} = 10V$ at $t = 80 \text{ms}$. This scenario is summarized in Table 1. Simulation results are shown in Fig. 10.

As seen in Fig. 10, output has zero steady state error. Controller keeps output voltage constant despite of changes in load resistance and input voltage.
6. Conclusion

Control theory plays an important role in power electronics. Providing a stable output voltage despite changes in load and input voltage is not achievable without the use of control theory. Control design for converters like Cuk, SEPIC and Zeta is more challenging due to high order of converter and presence of RHP zeros. Designing a controller for Cuk converter is the aim of this paper. Converter’s dynamical equations are extracted using converter dynamics toolbox for MATLAB. A controller is designed for obtained equations using MATLAB’s control system toolbox. Designed controller is tested in Simulink environment. Simulation results showed the performance of designed controller.

Table 1: Test scenario

| Parameter's name | Time | Initial value | Final value | Final – Initial |
|------------------|------|---------------|-------------|----------------|
| Rload            | 20 ms| 8.1 Ω         | 2.7 Ω       | +67%           |
| Vs               | 40 ms| 12 V          | 6 V         | +50%           |
| Vref             | 80 ms| 14 V          | 10 V        | -40%           |

Fig. 10: Simulation results

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