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Measurement of $D^0, \bar{D}^0$ Mixing from a Time-Dependent Amplitude Analysis of $D^0 \to K^+ \pi^- \pi^0$ Decays

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We present evidence of \( D^0 \overline{D}^0 \) mixing using a time-dependent amplitude analysis of the decay \( D^0 \rightarrow K^+ \pi^- \pi^0 \) in a data sample of 384 fb\(^{-1}\) collected with the BABAR detector at the PEP-II \( e^+ e^- \) collider at the Stanford Linear Accelerator Center. Assuming CP conservation, we measure the mixing parameters \( x'_{K\pi\pi} = [2.61^{+0.57}_{-0.60} \text{ (stat)} \pm 0.39 \text{ (syst)}] \% \), \( y'_{K\pi\pi} = [-0.06^{+0.55}_{-0.04} \text{ (stat)} \pm 0.34 \text{ (syst)}] \%. \) This result is inconsistent with the no-mixing hypothesis with a significance of 3.2 standard deviations. We find no evidence of CP violation in mixing.

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\( D^0 \overline{D}^0 \) mixing is a transition between flavor eigenstates of neutral charmed mesons \( |D^0 \rangle \) and \( |\overline{D}^0 \rangle \), and it depends upon the mass and width differences of the mass eigenstates. If mixing occurs, the physical eigenstates \( |D_{1,2} \rangle = p|D^0 \rangle \pm q|\overline{D}^0 \rangle \) (\( |p|^2 + |q|^2 = 1 \)) must have different masses \( M_{1,2} \) or widths \( \Gamma_{1,2} \). The oscillation is parametrized by \( x = 2(M_1 - M_2)/(\Gamma_1 + \Gamma_2) \) and \( y = (\Gamma_1 - \Gamma_2)/(\Gamma_1 + \Gamma_2) \), where \( 1 \) (2) refers to the nearly CP-even (odd) eigenstate. If \( CP \) is conserved, then \( |p/q| = 1 \) and \( \arg (q/pA_{f}/A_{1}) = 0 \). Here \( A_{f} \) (\( A_{1} \)) is the amplitude of the transition of the \( D^0 \) (\( \overline{D}^0 \)) to the final state \( f \). In the standard model (SM), the \( D^0 \overline{D}^0 \) mixing contribution from loop diagrams is negligible [1]. This is due to Glashow-Iliopoulos-Maiani suppression of the first two quark generations and Cabibbo-Kobayashi-Maskawa suppression of the third. Contributions from intermediate physical states that couple to both \( D^0 \) and \( \overline{D}^0 \) are difficult to predict; they are estimated to be of the order \( 10^{-3} - 10^{-2} \) [1]. Several recent studies report evidence for mixing parameters at the 1% level [2]. This is consistent with some SM expectations. As mixing is a rare process, it may be sensitive to particles and processes beyond the SM; existing measurements already pose constraints on a large number of new physics models [3]. \( CP \) violation in the charm sector is expected to be negligible in the SM; an observation would indicate contributions beyond SM.

We present the first time-dependent amplitude analysis of the \( D^0 \rightarrow K^+ \pi^- \pi^0 \) Dalitz plot to extract the mixing parameters. Previously, we studied the time dependence of \( D^0 \rightarrow K^+ \pi^- \pi^0 \) decays integrated over large regions of the Dalitz plot. We found no evidence for mixing [4]. However, since certain regions of the phase space are more sensitive to mixing than others, this analysis is more sensitive than our previous work. Two modes are reconstructed: (1) right-sign (RS) decays \( D^0 \rightarrow K^- \pi^+ \pi^0 \) from a Cabibbo-favored (CF) amplitude and (2) wrong-sign (WS) decays \( D^0 \rightarrow K^+ \pi^- \pi^0 \) from the coherent sum of a doubly Cabibbo-suppressed (DCS) amplitude and a CF amplitude produced by mixing. The interference of the two amplitudes gives rise to a linear dependence on the mixing parameters. We analyze events in which the flavor of the \( D^0 \) [5] is measured at production. We identify RS and WS decays by reconstructing the \( D^{*+} \rightarrow D^0 \pi^+_\gamma \), \( D^0 \rightarrow K \pi \pi^0 \) decay chain. The flavor of the \( D^0 \) candidate is known by the charge of the low-momentum pion (\( \pi^+_\gamma \)). The DCS and the CF amplitudes are described with isobar model [6].

The time-dependent decay rate depends on both the DCS amplitude \( A_{f}(s_{12}, s_{13}) = \langle f | \mathcal{H}^{\text{DCS}} | D^0 \rangle \) and the CF amplitude \( \overline{A}_{f}(s_{12}, s_{13}) = \langle f | \mathcal{H}^{\text{CF}} | \overline{D}^0 \rangle \), where \( s_{12} = m_{K^+}^2 \), \( s_{13} = m_{D^0}^2 \), and \( f = K^+ \pi^- \pi^0 \). In the limit \( |x|, |y| \ll 1 \) and defining \( \delta_f(s_{12}, s_{13}) = \arg [A_{f}(s_{12}, s_{13}) \overline{A}_{f}(s_{12}, s_{13})] \),

\[
\frac{dN_f(s_{12}, s_{13}, t)}{ds_{12}ds_{13}dt} = e^{-\Gamma_f t}\left\{ |A_{f}|^2 + |A_{f}| |\overline{A}_{f}| y \cos \delta_f - x \sin \delta_f \right\} (\Gamma_f) + \frac{x^2 + y^2}{4} |\overline{A}_{f}|^2 (\Gamma_f)^2 \right].
\]

The first term in Eq. (1) is the DCS contribution to the WS rate; the second term arises from the interference between DCS and mixing CF amplitudes; the third term is a pure mixing contribution. We determine the CF amplitude \( \overline{A}_{f} \) in a time-independent Dalitz plot analysis of the RS decay sample and use it in the analysis of the WS sample. The DCS amplitude \( A_{f} \) is extracted along with the mixing parameters using a fit to the WS data that separates the time dependence across the Dalitz plot from its overall rate. The time dependence is manifest in

\[
\frac{dN_f(s_{12}, s_{13}, t)}{ds_{12}ds_{13}dt} \propto e^{-\Gamma_f t}
\left\{ |A_{f}|^{\text{DCS}}|^2 + |A_{f}|^{\text{DCS}} |A_{f}|^{\text{CF}} |y \cos \delta_f - x \sin \delta_f |(\Gamma_f) + \frac{x^2 + y^2}{4} |A_{f}|^{\text{CF}}^2 (\Gamma_f)^2 \right].
\]
The interference terms in Eqs. (1) and (2) produce a variation in average decay time as a function of position in the WS Dalitz plot that is sensitive to the complex amplitudes of the resonant isobars as well as the mixing parameters. The change in the average decay time and the interference between the $D^0 \rightarrow K^{*+} \pi^-$ and $D^0 \rightarrow \rho^- K^+$ amplitudes are the origin of our sensitivity to mixing. For both $A_f(A_f^{CF})$ and $A_f(A_f^{DCS})$, one complex amplitude must be fixed arbitrarily; the strong interaction phase difference $\delta_{K^{*} \pi^\pm \nu}$ between the DCS $D^0 \rightarrow \rho^- K^+$ and the CF $D^0 \rightarrow K^+ \rho^-$ cannot be determined in this analysis. As a result, we are sensitive to $x$ and $y$ in the form

$$
\begin{align*}
\chi_{K^{*} \pi^\pm \nu} &= x \cos \delta_{K^{*} \pi^\pm \nu} + y \sin \delta_{K^{*} \pi^\pm \nu}, \\
\chi_{K^{*} \pi^\pm \nu} &= y \cos \delta_{K^{*} \pi^\pm \nu} - x \sin \delta_{K^{*} \pi^\pm \nu}.
\end{align*}
$$

A nonzero value of $x_{K^{*} \pi^\pm \nu}$ or $y_{K^{*} \pi^\pm \nu}$ would signify mixing. In general, $\delta$ differs among decay modes.

The amplitudes entering the WS analysis are described as a sum of isobar components $A_j$ that are parametrized with Breit-Wigner functions, $A_j^{CF/DCS} = \sum_{j=1}^{n_{CF/DCS}} a_j e^{i\delta_j} A_j(m_{K^{*} \pi^\pm \nu}, m_{K^{*} \pi^\pm \nu})$, where $a_j$ and $\delta_j$ are the strong interaction amplitudes and phases of the $j$th resonant amplitude [6]. For the $K \pi S$-wave component, we use a parametrization derived from $K - \pi$ scattering data [7], which has a $K^0(1430)$ resonance plus an effective nonresonant component. The mass and width of the resonances are taken from the world average [8].

We analyze a data sample of 384 fb$^{-1}$ collected with the BABAR detector [9] at the PEP-II $e^+ e^-$ collider at the Stanford Linear Accelerator Center near a center-of-mass energy of 10.58 GeV. Charged tracks are reconstructed with a silicon-strip detector (SVT) and a drift chamber (DCH), both in a 1.5 T magnetic field. Particle identification is based on measurements of ionization energy loss ($dE/dx$) in the SVT and DCH together with measurements from a Cherenkov ring-imaging device. Photon energies are measured with a CsI(Tl) calorimeter. All selection criteria, the fit procedure, and the systematic error analysis are finalized before we search for evidence of mixing in the data.

Selection criteria, identical for the RS and WS samples, are based partly on Ref. [4]. The $\pi^+$ candidates must have a transverse momentum $p_T^{\pi^+} > 0.12 \text{ GeV}/c$, where LAB indicates the laboratory frame, and reject electrons using $dE/dx$ measurements. We use kinematic selection criteria to eliminate electrons from pair conversions. The energies of photon candidates used to form $\pi^0$ are required to be greater than 0.1 GeV; the invariant mass of photon pairs.

FIG. 1 (color online). Dalitz plots for the (a) RS and (b) WS $D^0$ samples. The reconstructed (c) $D^0$ mass and (d) $\Delta m$ distributions for the WS sample requiring, respectively, (c) $0.1449 < \Delta m < 0.1459 \text{ GeV}/c^2$ and (d) $1.8495 < m_{K^{*} \pi^\pm \nu} < 1.8795 \text{ GeV}/c^2$. The fit results are shown by the superimposed curves. The light histogram represents the mistag background, while the dark histogram shows the combinatorial background.
must be in the range $0.09 < m_{\phi} < 0.16 \text{ GeV}/c^2$. We require the $\pi^0$ momentum $p_{\pi^0}^{\text{LAB}}$ to be greater than 0.35 GeV/c. The reconstructed invariant mass for the $D^0$ candidates must have $1.74 < m_{K\pi\pi^0} < 1.98 \text{ GeV}/c^2$. The $\pi^0$ and $D^0$ masses are then set equal to their nominal values [8], and the $D^*$ is refitted [10] with the constraint that its production point lies within the beam spot region. The $D^{*+}$ invariant mass and $D^0$ measured decay time $t_{K\pi\pi^0}$ are derived from this fit. We require $0.139 < \Delta m < 0.155 \text{ GeV}/c^2$, where $\Delta m = m_{K\pi\pi^0} - m_{\phi}$. To reject $D^*$ candidates from $B$ decays, we require the $D^0$ center-of-mass momentum to be greater than 2.4 GeV/c. For events containing multiple $D^*$ candidates with shared tracks, the candidate that yields the most probable fit for the decay chain is used. The three-dimensional flight path determines $t_{K\pi\pi^0}$ and its uncertainty $\sigma_t$. For signal events, the typical value of $\sigma_t$ is 0.23 ps; we accept $D^*$ candidates with $\sigma_t < 0.50$ ps. The $K^+$ and $\pi^-$ tracks dominate the decay-vertex resolution.

We extract the signal and background yields from a binned extended maximum likelihood fit to the $m_{K\pi\pi^0}$ and $\Delta m$ distributions [Figs. 1(c) and 1(d)]. For subsequent analysis, we retain $D^*$ candidates in the signal region, $0.1449 < \Delta m < 0.1459 \text{ GeV}/c^2$ and $1.8495 < m_{K\pi\pi^0} < 1.8795 \text{ GeV}/c^2$. Our final RS (WS) sample is composed of 658 986 (3009) events with a purity of 99% (50%). The efficiency of the signal region selection is 54.6%.

The RS sample is used to determine the CF isobar model parameters $\sigma_{j}^{\text{CF}}$ and $\delta_{j}^{\text{CF}}$, as well as the decay time resolution function, which is parametrized as a sum of three Gaussian functions with a common mean, with widths given by the per event $\sigma_t$ times a different scale factor for each Gaussian. We account for the reconstruction efficiency in the determination of the $\sigma_{j}^{\text{CF}}$ and $\delta_{j}^{\text{CF}}$. The reconstructed RS signal decay time distribution [Fig. 2(a)] is described by a probability density function (PDF) consisting of an exponential function convolved with the resolution. The resolution function parameters and $D^0$ lifetime are determined in an unbinned maximum likelihood fit. The mean value of the resolution function is found to be $4.2 \pm 0.7 \text{ fs}$, and it is consistent with the magnitude expected from instrumental effects. The associated systematic uncertainty is determined by setting the value to zero. We determine the $D^0$ mean lifetime to be $[409.9 \pm 0.8\text{(stat only)}] \text{ fs}$, in agreement with the world average $[410.1 \pm 1.5\text{(stat + syst)}] \text{ fs}$ [8].

The $D^0$ candidates in the WS signal region can be divided into three categories: signal events, combinatorial

![FIG. 2](color online). (a) Proper time distribution for RS events with the fit result superimposed. The distribution of background events is shown by the shaded histogram. (b) Proper time distribution for WS events. (c), (d) $m_{K\pi}^2$ and $m_{K\pi\pi^0}$ projections with superimposed fit results (line). The light histogram represents the mistag background, while the dark histogram shows the combinatorial background.
background, and incorrectly tagged RS events (mistag), each one described by its own PDF whose parameters are
determined in an unbinned maximum likelihood fit. During
the fit procedure, the number of events in each category is
fixed to the value obtained from the fit to the \( m_{K\pi\pi} \) and
\( \Delta m \) distributions.

The PDF describing the WS decay rate as a function of
the Dalitz plot variables is convolved with the \( j_{K\pi\pi} \) resolution
function. The DCS amplitudes and phases for each resonance,
along with the mixing parameters, are deter-
mined in the fit. The CF Dalitz plot amplitudes arising from
mixing are taken from the fit to the RS sample previously
described. The mistag events contain correctly recon-
structed RS \( D^0 \) decays; as the \( \pi_j^\pm \) has no influence in the
decay chain fit, the \( D^0 \) lifetime of those events is also
correct. Therefore, the mistag events are parametrized
using an empirical PDF obtained from the RS data for
both the lifetime and the Dalitz plot variables. The PDF
describing the combinatorial background is constructed by
averaging the \((s_{12}, s_{13}, f_{K\pi\pi})\) distributions obtained from
the WS \( m_{K\pi\pi} \) sidebands: this accounts for correlations
between those three variables that might be present in the
data. We describe the \( \sigma \), distribution for signal and back-
ground using an empirical PDF from the RS data.

The results of the time-dependent fit of the RS data, the
\( \alpha_j^{\text{DCS}}, \beta_j^{\text{DCS}} \) and fit fractions \( f_j \) [6], are given in Table I. The fit fraction of the nonresonant contribution to the \( K^- S \)
wave is absorbed into the \( K^+ \) (1430) and \( K^0 \) (1430) fit
fractions. Projections of the fit results are shown in
Figs. 2(b)–2(d). The change in log-likelihood \((\Delta \ln L)\)
between the fit with mixing and with no mixing
\((x_j^0/r_0 = y_j^0/r_0 = 0)\) is 13.5 units, including
systematic uncertainties. For 2 degrees of freedom, the
confidence level for the no-mixing hypothesis is 0.1%.
Equivalently, this constitutes evidence for \( D^0 \)–\( \bar{D}^0 \) at
the 3.2 standard deviation level.

To derive the values of \( x_j^0 \) and \( y_j^0 \), we first determine
\( r_0^2 = [5.25 \pm 0.25 \text{(stat)} \pm 0.12 \text{(syst)}] \times 10^{-3} \) using

\[
\begin{equation}
 r_0^2 = N_{WS} \left[ N_{RS} \left( 1 + \frac{\bar{y}A^2 - \bar{y}B^2 + \frac{\bar{x}^2 + \bar{y}^2}{2}}{2} \right) \right]
\end{equation}
\]

with \( A^2(B^2) = \int \text{Re(Im)}[A_j^{(DCS)} + A_j^{(CF)}] d\xi_3 d\xi_1 \). \( N_{WS} \) (\( N_{RS} \))
is the number of WS (RS) signal events in the sample.
We then generate \( 10^5 \) \((x_j^0/r_0, y_j^0/r_0)\) points in ac-
cordance with the fit covariance matrix, assuming
Gaussian errors (width given by the total uncertainty in-
cluding systematics). For each point, we compute \( r_0 \)
using Eq. (4) and determine values for \( x_j^0 \) and \( y_j^0 \).

Using a Bayesian approach, by integrating the likelihood
function with respect to \( x_j^0 \) and \( y_j^0 \), assuming a flat
prior distribution, we obtain \( x_j^0 = [2.61 \pm 0.57 \text{(stat)} \pm 0.39 \text{(syst)}] \) \% and
\( y_j^0 = [-0.06 \pm 0.55 \text{(stat)} \pm 0.34 \text{(syst)}] \) \% with a correlation of \(-0.75\).

Extensive validation of this fitting procedure is per-
formed using Monte Carlo (MC) experiments based on
the PDF shapes and DCS amplitudes extracted from data.
The validation studies are performed over the range
\([-0.6, 0.6]\) for both \( x_j^0/r_0 \) and \( y_j^0/r_0 \). These studies
demonstrate that the fit correctly determines the mixing
parameters to within a small offset of 0.2–0.3\( \sigma \), where \( \sigma \) is
the statistical uncertainty. These small biases are a conse-
quence of the relatively small size of our data sample
and become negligible if MC samples with higher statistics
are used. We correct the final result for this offset.

Sources of systematic uncertainty for \( x_j^0/r_0 \)
and \( y_j^0/r_0 \), related to the choice of the isobar model
and the experimental assumptions, are considered. For
each effect we refit the data with an alternative assumption
and extract the overall correlated uncertainty for the fitted
parameters. We estimate the Dalitz model uncertainties
\([0.38\sigma (0.35\sigma)]\), where \( \sigma \) is the statistical uncertainty,
by varying the mass and the width of each resonance
within the experimental assumptions, are considered. For each
effect we refit the data with an alternative assumption and
extract the overall correlated uncertainty for the fitted
parameters. We estimate the Dalitz model uncertainties
\([0.38\sigma (0.35\sigma)]\), where \( \sigma \) is the statistical uncertainty,
by varying the mass and the width of each resonance
within their error and by using alternative parametrizations for
the isobar components \( A_j \) in the fit: the largest error arises from
uncertainties in the \( K^\ast \) and \( \rho \) parameters and from
uncertainties in the parametrization of the \( K^- S \) wave.

Systematic uncertainties related to the number of signal
and background events \([0.15\sigma (0.22\sigma)]\) are evaluated by
varying them according to their statistical uncertainties.
Similarly, the definition of the signal region, the \( \sigma \), require-
ment, and the selection of the best \( D^- \) candidate are varied.
The effect on the mixing parameters is \( 0.50\sigma (0.37\sigma) \).

Variations in efficiency across the Dalitz plot contribute
systematic uncertainties of \( 0.09\sigma (0.10\sigma) \). The \( j_{K\pi\pi} \) reso-
lution function parameters are varied within their errors.
The offset is also set to zero. The systematic effect is \( 0.11\sigma (0.09\sigma) \). The total systematic error on \( x_j^0/r_0 \) \((y_j^0/r_0)\) is
\( 0.66\sigma (0.57\sigma) \).

The same procedure is applied separately to the WS
\( D^0 \)-tagged (+) and \( \bar{D}^0 \)-tagged (−) events to search for

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**TABLE I.** Fit results for the WS \( D^0 \) data sample. The total fit fraction is 102\% and the \( \chi^2/\text{ndof} \) is 188/215. The results for
\( x_j^0/r_0 \) and \( y_j^0/r_0 \) include statistical and systematic errors; their total linear correlation is \(-0.34\).

| Resonance | \( \alpha_j^{\text{DCS}} \) (\% fixed) | \( \beta_j^{\text{DCS}} \) (\% fixed) | \( f_j \) (\%) |
|-----------|----------------------------------|----------------------------------|---------------|
| \( \rho(770) \) | 1 (fixed)                          | 0 (fixed)                          | 39.8 ± 6.5    |
| \( K^0(1430) \) | 0.088 ± 0.017                      | -17.2 ± 12.9                       | 2.0 ± 0.7     |
| \( K^0(1430) \) | 6.78 ± 1.00                        | 69.1 ± 10.9                        | 13.1 ± 3.3    |
| \( K^+(892) \) | 0.899 ± 0.005                      | -171.0 ± 5.9                       | 35.6 ± 5.5    |
| \( K^0(1430) \) | 1.65 ± 0.59                        | -44.4 ± 18.5                       | 2.8 ± 1.5     |
| \( K^0(892) \) | 0.398 ± 0.038                      | 24.1 ± 9.8                         | 6.5 ± 1.4     |
| \( \rho(1700) \) | 5.4 ± 1.6                          | 157.4 ± 20.3                       | 2.0 ± 1.1     |
| \( x_j^0/r_0 \) | 0.353 ± 0.091 ± 0.066              |                                  |               |
| \( y_j^0/r_0 \) | -0.002 ± 0.090 ± 0.057             |                                  |               |
CP violation in mixing or interference. We find 
\[ \chi_K^{\pi\pi} = (2.53^{+0.54}_{-0.63} \pm 0.39)\%, \quad \chi_K^{\pi\pi} = (0.05^{+1.63}_{-0.67} \pm 0.50)\%, \]
\[ \chi_K^{\pi\pi} = (3.55^{+0.73}_{-0.63} \pm 0.65)\%, \quad \chi_K^{\pi\pi} = (0.54^{+0.40}_{-0.36} \pm 0.41)\%, \]
respectively, and thus observe no evidence for CP violation. The correlation between \( \chi_K^{\pi\pi} \) and \( \chi_K^{\pi\pi} \) is \(-0.69 (-0.66)\).

Our data are inconsistent with the no-mixing hypothesis with a significance of 3.2 standard deviations including systematic uncertainties and thus present evidence of mixing. For the rotated mixing parameters, we find \( \chi_K^{\pi\pi} = (2.61^{+0.57}_{-0.68} \pm 0.39)\% \) and \( \chi_K^{\pi\pi} = (0.06^{+0.55}_{-0.64} \pm 0.34)\% \) with a correlation of \(-0.75\). These values are consistent with our previous result [4] and with some SM estimates. No evidence for CP violation is found.

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