Resonances in Coupled-Channel Scattering

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Coupled-channel scattering

This talk:
Topical report of recent coupled-channel scattering results from the Hadron Spectrum Collaboration

The method:
- Build large correlation matrices with a diverse range of operators
- Extract many energy levels using the variational method
- Use these energies with extensions of Lüscher’s method to obtain infinite volume scattering amps
- Investigate the poles of the scattering amplitudes to obtain resonance information

Topics I won’t cover:
The HALQCD method, Finite Volume Hamiltonian, EFTs in a box, etc.
Coupled-channel scattering

Sounds hard... why bother?

$\alpha_0(980)$, $f_0(980)$
$\alpha_1(1260)$
$X(3872)$, and other $XYZ$ states
$N^*(1440)$, $\Lambda(1405)$, ...

All decay into multiple final states
All are resonant enhancements in multiple channels
To understand these rigorously, we need coupled-channel analyses
Extracting resonance properties

excited states seen as resonant enhancements in the scattering of lighter stable particles

\[ \rho(770) \]

\[ M(\pi^+\pi^-) \text{ GeV} \]

\[ \delta/^{\circ} \]

\[ E_{\text{cm}}/\text{GeV} \]
Extracting resonance properties

excited states seen as resonant enhancements in the scattering of lighter stable particles
Extracting resonance properties

build a large basis of operators: $\mathcal{O}^\dagger \sim \bar{\psi} \Gamma \overrightarrow{D} \ldots \overrightarrow{D} \psi$

compute large correlation matrices: $C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$

solve GEVP:

$$C_{ij}(t)v_n^j = \lambda_n(t, t_0)C_{ij}(t_0)v_n^j$$

$\bar{\psi} \Gamma \psi$

$m_\pi = 236 \text{ MeV}$
Extracting resonance properties

add in $\pi\pi$ operators using a variationally optimal pion $\pi^\dagger = \sum v_i^\pi O_i^\dagger$

combine in pairs $(\pi\pi)^\dagger = \sum C(\vec{p}_1, \vec{p}_2) \pi^\dagger(\vec{p}_1)\pi^\dagger(\vec{p}_2)$

$\vec{p}_1 + \vec{p}_2 = \vec{P}$

$[000]T_1^-$

$m_\pi = 236$ MeV
Extracting resonance properties

essential to have operators that overlap onto “meson” and “meson-meson” contributions to the physical spectrum

\[ [000] T_{1}^- \]

\[ m_\pi = 236 \text{ MeV} \]
Resonances in coupled-channel scattering

Phase shifts via the Lüscher method:

\[ \tan \delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)} \]

\[ \mathcal{Z}_{00}(1; q^2) = \sum_{n \in \mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2} \]

\[ m_\pi = 236 \text{ MeV} \]
$\rho$ resonance with moving frames

$\vec{P} = [000]$

$m_\pi = 236$ MeV
$\rho$ resonance with moving frames

$\vec{P} = [000]$
$\vec{P} = [001]$
$\vec{P} = [011]$
$\vec{P} = [111]$
$\vec{P} = [002]$

$m_\pi = 236$ MeV
$\rho$ resonance with moving frames

PRD 92 094502, arXiv:1507.02599
- for more see Antoni Woss Tuesday 26 Jul 2016 at 14:40

\[ \tilde{P} = [000] \]
\[ \tilde{P} = [001] \]
\[ \tilde{P} = [011] \]
\[ \tilde{P} = [111] \]
\[ \tilde{P} = [002] \]

one volume, 22 energy levels... lots of constraint

\[ m_\pi = 236 \text{ MeV} \]
Coupled-channel scattering

Direct extension of the elastic quantization condition derived by Lüscher

\[
\det \left[ 1 + i \rho(E) \cdot t(E) \cdot (1 + i \mathcal{M}(E, L)) \right] = 0
\]

Many derivations, **all in agreement**: 

He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-$$\frac{1}{2}$$.

Significant steps towards a general 3-body quantization condition have been made
- see Stephen Sharpe on Tuesday 26 Jul 2016 at 15:40 for the latest
Determinant

\[ t = (\pi\pi \rightarrow \pi\pi) \]

\[ \det \left[ 1 + i \rho \cdot t \cdot (1 + iM(L)) \right] \]

\[ \delta/\circ \]

\[ \delta_{\pi\pi} \]
\[ t = (\pi \pi \rightarrow \pi \pi) \]

\[ \text{Determinant} \]

\[ \text{det} [1 + i \cdot \rho \cdot t \cdot (1 + i \cdot M(L))] \]
Determinant

\[
t = \begin{pmatrix}
\pi\pi \rightarrow \pi\pi & 0 \\
0 & K\bar{K} \rightarrow K\bar{K}
\end{pmatrix}
\]

\[
\text{det}\left[1 + i\rho \cdot t \cdot (1 + iM(L))\right]
\]

\[
\delta /^\circ
\]

\[
\pi\pi
\]

\[
K\bar{K}
\]

\[
\eta 0.5
\]

David Wilson

Resonances in coupled-channel scattering
\[ t = \begin{pmatrix}
\pi\pi \rightarrow \pi\pi & \pi\pi \rightarrow K\bar{K} \\
K\bar{K} \rightarrow \pi\pi & K\bar{K} \rightarrow K\bar{K}
\end{pmatrix} \]

\[ \det \left[ 1 + i \rho \cdot t \cdot (1 + i M(L)) \right] \]
Determinant

$$t = \begin{pmatrix} \pi \pi \to \pi \pi & \pi \pi \to K \bar{K} \\ K \bar{K} \to \pi \pi & K \bar{K} \to K \bar{K} \end{pmatrix}$$

$$\text{det}[1 + i \rho \cdot t \cdot (1 + i M(L))]$$

$$\delta/\circ \quad 180$$

$$\eta \quad 0.5$$

$$a_t E_{cm}$$

Resonances in coupled-channel scattering
Determinant

\[ t = \begin{pmatrix} \pi \pi \rightarrow \pi \pi & \pi \pi \rightarrow K\bar{K} \\ K\bar{K} \rightarrow \pi \pi & K\bar{K} \rightarrow K\bar{K} \end{pmatrix} \]

\[ \text{det} \left[ 1 + i \rho \cdot t \cdot (1 + iM(L)) \right] \]

\[ \delta^{\pi \pi} \]

\[ \delta^{K\bar{K}} \]

\[ \eta \]

\[ a_t E_{cm} \]
Amplitude parameterization

\[
\mathbf{t} = \begin{pmatrix}
\pi\pi \rightarrow \pi\pi & \pi\pi \rightarrow K\bar{K} \\
K\bar{K} \rightarrow \pi\pi & K\bar{K} \rightarrow K\bar{K}
\end{pmatrix}
\]

\[
\det [\mathbf{1} + i\mathbf{\rho}(E) \cdot \mathbf{t}(E) \cdot (\mathbf{1} + i\mathbf{M}(E, L))] = 0
\]
determinant condition:
- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations
- Constrained problem when \(\#(\text{energy levels}) > \#(\text{parameters})\)
- Essential amplitudes respect unitarity of the \(S\)-matrix

\[
S^\dagger S = \mathbf{1} \quad \rightarrow \quad \text{Im} \mathbf{t}^{-1} = -\mathbf{\rho}
\]
\[
\rho_{ij} = \delta_{ij} \frac{2k_i}{E_{cm}}
\]

\(K\)-matrix approach:
\[
\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\mathbf{\rho}
\]
\text{e.g.: } \quad K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}
ρ resonance into the coupled-channel region

\[ K\bar{K} \]

\[ \rho \]

\[ m_\pi = 236 \text{ MeV} \]
\( \rho \) resonance into the coupled-channel region

\[ \rho \rightarrow \pi \pi, K \bar{K} \]

\[ m_\pi = 236 \text{ MeV} \]

**Diagram:**
- \( \delta_1^{\pi\pi} \) and \( \delta_1^{K\bar{K}} \) angles as functions of \( a_t \bar{E}_{\text{cm}} \)
- \( \eta \) parameter

**Equation:**
- \( m_\pi = 236 \text{ MeV} \)

**Reference:**
- PRD 92 094502, arXiv:1507.02599
Resonances in coupled-channel scattering

\[ \rho \text{ resonance pole} \]

\[ t_{ij} \sim \frac{c_i c_j}{s_0 - s} \]

\[ m_\pi = 236 \text{ MeV} \]
An $a_0$ resonance

$\pi\eta$-$K\bar{K}$-$\pi\eta'$

$I = 1 \quad J = 0$

- for more see Jozef Dudek, Tuesday 26 Jul 2016 at 16:50

PRD 93 094506, arXiv:1602.05122

Figure 58. Partial wave analysis of the $K^-\pi^+\pi^-\rightarrow K^-\pi^+\pi^-$ amplitudes deduced from the LASS results of Fig. 57, showing the magnitude and phase for the S, P and D-waves. [103]

Figure 59. Mass distribution for $\pi^0\eta$ from the GAMS experiment [11] on $\pi^-p\rightarrow(\pi^0\eta)n$, where both $\pi^0$ and $\eta$ are detected in the $\gamma\gamma$ decay mode. Partial wave analysis reveals that the $J = 0$ wave has two possible resonances $a_0(980)$ and $a_0(1430)$ in this mass region.

GAMS, Alde et al PLB 203 397, 1988.

$m_\pi = 391$ MeV
An $a_0$ resonance

$\pi \eta - K\bar{K} - \pi \eta'$

$m_\pi = 391$ MeV
**a₀ resonance - two channel region**

**πη-K̅K**

using 47 energy levels

\[
K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}
\]

\[
m = (0.2214 \pm 0.0029 \pm 0.0004) \cdot a_t^{-1}
\]

\[
g_{\pi \eta} = (0.091 \pm 0.016 \pm 0.009) \cdot a_t^{-1}
\]

\[
g_{K \bar{K}} = (-0.129 \pm 0.015 \pm 0.002) \cdot a_t^{-1}
\]

\[
\gamma_{\pi \eta, \pi \eta} = -0.16 \pm 0.24 \pm 0.03
\]

\[
\gamma_{\pi \eta, K \bar{K}} = -0.56 \pm 0.29 \pm 0.04
\]

\[
\gamma_{K \bar{K}, K \bar{K}} = 0.12 \pm 0.38 \pm 0.08
\]

\[
\chi^2/N_{\text{dof}} = \frac{58.0}{47-6} = 1.41
\]

\[
m_\pi = 391 \text{ MeV}
\]
a_0 resonance - two channel region

S-wave \( \pi \eta - K \bar{K} \)

from 47 energy levels

\( m_\pi = 391 \text{ MeV} \)
Resonances in coupled-channel scattering

\[ m_\pi = 391 \text{ MeV} \]

\[ t_{ij} \sim \frac{c_i c_j}{s_0 - s} \]
Other calculations

Coupled $\pi K - \eta K$

Combined S & P-wave analysis
80 energy levels from 3 volumes
arXiv:1406.4158, PRL 113 (2014) no.18, 182001

Coupled $D\pi - D\eta - D_s\bar{K}$

Combined S & P-wave analysis
37 coupled channels in S-wave
47 energy levels from 3 volumes
arXiv:1607.????

$m_\pi = 391$ MeV

- Graham Moir, Thursday 28 July 2016 at 15:00
The $f_0(500)/\sigma$ resonance

elastic scattering with vacuum quantum numbers $\pi\pi$ in $I = 0, J = 0$
The $f_0(500)/\sigma$ resonance

elastic scattering with vacuum quantum numbers $\pi\pi$ in $I = 0$, $J = 0$

$m_\pi = 236$ MeV

$m_\pi = 391$ MeV
The $f_0(500)/\sigma$ resonance

- see Raul Briceño, Tuesday 26 July 2016 at 15:20
arXiv:1607.05900
Future directions

two-body coupled-channel

\( f_0(980) \)
\( D\bar{D} \)
\( D\bar{D}^* \)
\( N\pi \)
\( \gamma a \rightarrow bc \)

- see Gavin Cheung, Monday 25 July 2016 at 14:55
- see Luka Leskovec on 29 July 2016 at 17:50

further operator structures - glueball, tetraquark, ...
- see Briceño et al, Phys.Rev.Lett. 115 (2015)
- see Stephen Sharpe, Tuesday 26 July 2016 at 15:40

formalism for three-body and beyond
- needed for higher energies
- needed to get closer to the physical mass
Promising results so far ...  
... but a lot still to do
$m_\pi = 391 \text{ MeV}$
The $f_0(500)/\sigma$ resonance

$m_\pi = 236$ MeV

$m_\pi = 391$ MeV

$|p|$
The $f_0(500)/\sigma$ resonance

- see Raul Briceño, Tuesday 26 Jul 2016 at 15:20
The $f_0(500)/\sigma$ resonance

from J. R. Pelaez, arXiv:1510.00653
The $f_0(500)/\sigma$ resonance

from J. R. Pelaez, arXiv:1510.00653

Figure 7: Left panel: Compilation of $f_0(500)$ or $\sigma$ resonance poles in the 2012 RPP edition. The light gray area stands for the uncertainty assigned to the poles from 1996 until 2010. The darker gray rectangle is the new uncertainty estimated in the summary tables, Eq.(1). Right panel: Detail of the new uncertainty band. We only show the four "most advanced dispersive analyses" according to the 2012 RPP and as a darker area the "more radical" and "restricted range of $f_0(500)$ parameters", Eq.(2), if one averages those four analyses. The dashed rectangle corresponds to the "conservative dispersive estimate" $p_s = 449 + 22 + 16 i (275 \pm 12) \text{MeV}$ which, as explained in the text, takes into account that the differences between these four dispersive approaches are of systematic nature.

Of course, this was well known to the RPP authors and that is why in the 2012 RPP "Note on scalar mesons below 2 GeV", they suggested that "One might also take the more radical point of view and just average the most advanced dispersive analyses, [97, 111, 116, 117]... " which we show in the right panel of Fig.7, "which provide a determination with minimal bias". By averaging the values obtained in those four references a more restricted range of parameters is estimated at the 2012 RPP:

\[ p_s = (446 \pm 6) + (276 \pm 5) \text{MeV} \text{(RPP2012 restricted range)} \]  

In the left panel of Fig.7, the area covered by this "restricted" uncertainty would be almost imperceptible within the darker rectangle, and hence we show in the right panel an expanded view of the darker rectangle and just the "most advanced dispersive analyses" according to the 2012 RPP. Thus the "restricted range of parameters" corresponds to the smallest and even darker rectangle in the middle of the plot.

At the risk of being annoying, these uncertainties may now be too small, since the differences between those four determinations are more of a systematic than statistical nature. Thus, weighting them as if the uncertainties and differences were statistical to obtain an even smaller uncertainty is somewhat optimistic. Moreover, an uncertainty of less than 3% is hard to achieve due to isospin breaking effects, which are not incorporated into these formalisms (except maybe in the experimental uncertainties), and to the absence of $4\pi$ channels, although we have seen above that this effect is very small. A suggestion would be to take as a conservative dispersive estimate the band that covers [111] and [116], since the pole in [97] was not really calculated with the analytic extension of Roy equations but from the phenomenological representation and can be considered superseded by the results of [111]. In addition the result of [117] lies within this estimate (it uses results from the [97, 111] group as input). That is:

\[ p_s = 449 + 22 + 16 i (275 \pm 12) \text{MeV} \text{(Conservative dispersive estimate)} \]
An $a_0$ resonance

$\pi\eta-K\bar{K}-\pi\eta'$

$E_{cm}=0.17, 0.19, 0.21, 0.23, 0.25, 0.27, 0.29$

$\pi\eta$, $q\bar{q}$, $K\bar{K}$, $\pi\eta'$

$A_1^+$

$L/\alpha_s$

$m_\pi = 391$ MeV

- for more see Jozef Dudek, Tuesday 26 Jul 2016 at 16:50
Resonances in coupled-channel scattering
An $a_0$ resonance - three channel region

$m_\pi = 391$ MeV