Nuclear attenuation of high energy two-hadron system
in the string model

N. Akopov, L. Grigoryan, and Z. Akopov,
Yerevan Physics Institute, Br.Alikhianian 2, 375036 Yerevan, Armenia

Nuclear attenuation of the two-hadron system is considered in the string model. The two-scale model and its improved version with two different choices of constituent formation time and sets of parameters obtained earlier for the single hadron attenuation, are used to describe available experimental data for the $z$-dependence of subleading hadron, whereas satisfactory agreement with the experimental data has been observed. A model prediction for $\nu$-dependence of the nuclear attenuation of the two-hadron system is also presented.

I. INTRODUCTION

Nuclear Attenuation (NA) of high energy hadrons is a well known tool for investigation of an early stage of the hadronization process, which can not be described in the framework of the existing theory of strong interactions (perturbative QCD), because of major role of "soft" interactions. Nevertheless, there are many phenomenological models, which describe, rather qualitatively, existing experimental data for single hadron NA [10]. Also some predictions for the attenuation of multi-hadron systems leptoproduced in nuclear matter in the framework of the string model [11]-[12] were done. It was argued that measurements of NA of multi-hadron systems can remove some ambiguities in determination of the parameters describing strongly interacting systems at the early stage of particle production: formation time of hadrons and cross-section of the intermediate state interaction inside the nucleus. Recently, for the first time, data [13] on the two-hadron system NA ratio measured as a function of relative energy of the subleading hadron has been published. Therefore, in this work we attempt to describe these data based on the Two-Scale Model (TSM) [4] and Improved Two-Scale Model (ITSM) [10]. We present also predictions for the $\nu$-dependence of two-hadron system NA within the same model. Also, possible mutual screening of hadrons in string and its experimental verification is discussed.

II. THEORETICAL FRAMEWORK

In articles [11]-[12] the process of leptoproduction of two-hadron system on a nucleus with atomic mass number $A$ was theoretically considered for the first time:

$$l_i + A \rightarrow l_f + h_1 + h_2 + X,$$

(1)

where the hadrons $h_1$ and $h_2$ carry fractions $z_1$ and $z_2$ of the total available energy. The NA ratio for that process can be expressed in form:

$$R_M^{2h} = 2d\sigma_A(\nu, Q^2, z_1, z_2)/A d\sigma_D(\nu, Q^2, z_1, z_2),$$

(2)

where $d\sigma_A$ and $d\sigma_D$ are the cross-sections for the reaction (1) on nuclear and deuterium targets, respectively, $\nu$ and $Q^2$ denote the energy of the virtual photon and square of its four-momentum. One can picture the reaction (1) as shown in Fig. 1. The interaction of the lepton with nucleon occurs at the point $(b, x)$, where the intermediate state $q$ begins to propagate ($b$ and $x$ are impact parameter and longitudinal coordinate of DIS point). At some points the string breaks, and as a result the first constituents of hadrons $h_1$ and $h_2$ are produced at the points $(b, x_1)$ and $(b, x_2)$. Also, at the points $(b, x_{y1})$ and $(b, x_{y2})$ the yo-yo of the hadrons $h_1$ and $h_2$ is formed (by a "yo-yo" system we assume, that the colorless system with valence content and quantum numbers of the final hadron is formed, but without its "sea" partons).

In the string model there are simple connections between the points $x_{y1} - x_1 = x_1 - x_1 = z_1 L$ and $x_{y2} - x_2 = x_2 - x_2 = z_2 L$, where $L$ is the full hadronization length, $L = \nu/\kappa$, $\kappa$ is the string tension (string constant). The NA ratio can be presented in the following form:

$$R_M^{2h} \approx \frac{1}{2} \int d^2 b \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx_1 \int_{x_1}^{\infty} dx_2 \rho(b, x)$$

(3)

$$D(z_1, z_2, x_1 - x, x_2 - x) W_0(h_1, h_2; b, x, x_1, x_2) +$$

$$D(z_2, z_1, x_1 - x, x_2 - x) W_0(h_2, h_1; b, x, x_1, x_2) \right),$$

where $D(z_1, z_2, l_1, l_2)$ (with $l_1 < l_2$) is the distribution of the formation lengths $l_1$ and $l_2$ of the hadrons and $\rho(b, x)$ is the nuclear density function normalized to unity. $W_0$ is the probability that neither the hadrons $h_1, h_2$ nor the intermediate state leading to their production (initial and open strings) interact inelastically in the nuclear matter:

$$W_0(h_1, h_2; b, x, x_1, x_2) = (1 - Q_1 - S_1 -$$

(4)


\[(H_1 + Q_2 + S_2 + H_2 - H_1(Q_2 + S_2 + H_2)))^{(A-1)},\]

where \(Q_1\) and \(Q_2\) are the probabilities for the initial string to be absorbed in the nucleus within the intervals \((x, x_1)\) and \((x_{y1}, x_2)\), respectively. \(S_i\) \((i = 1, 2)\) is the probability for the open string containing the first constituent parton for \(h_i\) to be absorbed in the nucleus within an interval \((x_i, y_{xi})\), and \(H_i\) \((i = 1, 2)\) is the probability for the \(h_i\) to interact inelastically in the nuclear matter, starting from the point \(x_{yi}\). The probabilities \(Q_1, Q_2, S_1, S_2, H_1, H_2\) can be calculated using the general formulae:

\[P(x_{min}, x_{max}) = \int_{x_{min}}^{x_{max}} \sigma_P(b, x) dx,\]

where the subscript \(P\) denotes the particle (initial string, open string or hadron), \(\sigma_P\) its inelastic cross section on nucleon target, and \(x_{min}\) and \(x_{max}\) are the end points of its path in the \(x\) direction, as is shown in Fig. II.

III. EXPERIMENTAL SITUATION

Recently the HERMES Collaboration has obtained, for the first time, data on double hadron attenuation [13]. The following double ratio for leading and subleading hadrons has been measured:

\[R_{2M}^2(z_2) = \frac{(d^2 N(z_1, z_2)/dN(z_1))_A}{(d^2 N(z_1, z_2)/dN(z_1))_D},\]

where \(z_i = E_i/\nu, E_i\) is the energy of \(i\)-th hadron, \(A\) and \(D\) marks that interaction takes place on nuclear and deuterium targets, \(d^2 N(z_1, z_2)\) is the number of events with at least two hadrons (leading and subleading hadrons in one event). According to the experimental conditions leading hadron must have \(z_1 > 0.5\), and \(dN(z_1)\) is the number of events with at least one hadron with \(z_1 > 0.5\). In this experiment double ratio \(R_{2M}^2(z_2)\) was considered as a function of \(z_{subleading} = z_2\). Two sets of experimental data are presented:

1. the leading-subleading combinations \(++, --, +0, 0+, -0, -0, 00\) only;
2. all hadron pairs except those with invariant mass near \(\rho^0\).

In our work, we will use the first set, because, in our opinion, it contains less contribution from hadrons produced in diffractive and diffractive dissociation processes. The relevant region for the investigation of the two-hadron system NA ratio as a function of \(z_2\) is the one between \(z_2 = 0.1\) and \(z_2 = 0.4\). At \(z_2 < 0.1\) the contributions from the slow hadrons coming from the target fragmentation become large. At \(z_2 > 0.4\) it is difficult to distinguish leading and subleading hadron because in this region \(z_1 \approx z_2 \approx 0.5\).

Let us also consider the following question - to what extent are these data free from contribution of diffractive \(\rho^0\) mesons? For events \(dN(z_1)\) corresponding corrections were made by experimentalists. Here we discuss situation with \(d^2 N(z_1, z_2)\) only. At a first glance the diffractive \(\rho^0\) mesons do not give contribution in these events because of special conditions at choice of pairs of hadrons. They ensure that both hadrons in pair can not be produced from decay of a one \(\rho^0\) meson, but do not forbid that one of the hadrons in the event was produced as a result of breaking of a diffractive \(\rho^0\)-meson. Then, another hadron could be produced after the \(\rho^0\)-meson final state interaction.

IV. RESULTS AND DISCUSSION

We have performed calculations for \(R_{2M}^2(z_2)\) in the framework of TSM and its improved version. The ratio \(R_{2M}^2(z_2)\) is a function of three variables \(z_1, z_2\) and \(\nu\), but for comparison with the available experimental data we present \(R_{2M}^2(z_2)\) as a function of \(z_2\) only, after integration over other variables according to the experimental conditions \((0.5 < z_1 < 1 - z_2, 0.7 < \nu < 23.5 GeV, p_t > 1.4 GeV/c)\). In this paper we also present model predictions for \(R_{2M}^2(z_2)\) as a function of \(\nu\), while performing integrations over \(z_1, z_2\) in the corresponding kinematic regions. They are chosen to be close to the conditions of the available experimental data \((0.5 < z_1 < 0.9, 0.1 < z_2 < 1 - z_1)\). The upper limit of \(z_1 < 0.9\) relates to the case of two hadrons in final state, as well as for the single hadron this limit has to be equal to unity. All theoretical curves presented here were obtained with an assumption that the final hadrons are pions. We used two expressions for Constituent Formation Time (CFT). As a first expression for CFT the equation (4.1) from [12] (marked as CFT1) was used. Second expression was taken in a form according to Ref. [13], which means, that as CFT for the first produced constituent quark of the first hadron we use \(\tau_{c1} = (1 - z_1 - z_2)L\), and for the first produced constituent quark of the second hadron \(\tau_{c2} = (1 - z_2)L\) (marked as CFT2). Three sets of parameters (including nuclear density functions) corresponding to the minimum values of \(\chi^2\) from Tables 1, 2, 3 Ref. [10] were used for calculations. The value of string tension was fixed at \(\kappa = 1\) GeV/fm. Results for the double ratio \(R_{2M}^2(z_2)\) as a function of \(z_2\) are presented on Fig. II. On panels a), c), e) three options of the theoretical curves are shown: solid curves correspond to the TSM with CFT1; dashed curves correspond to the ITSM with CFT2; dotted curves correspond to the ITSM with CFT3. According to the ideology of the string model, the transverse size of the string is much less than longitudinal one. It means that the hadrons produced from the string have close impact parameters, and could partly screen one another, which in turn must lead to the weakness of NA (partial attenuation). To study this effect and to compare with the basic supposition, that two hadrons
FIG. 1: Leptoproduction of two-hadron system on nuclear target. Details see in text.

attenuate independently (full attenuation), we consider partial attenuation in its extreme case, when two hadrons fully screen one another, and as a result the two-hadron system attenuates as a single hadron. The results of calculations within this conditions are shown in panels b), d), f). The two-hadron system will attenuate as a single hadron also when the two final hadrons appear as a result of breaking of one of the resonance. For instance, combinations $\pi^+0(\pi^-0)$ and $0+(0\pi^-)$ can be obtained as products of decay of the $\rho^0$ mesons produced via fragmentation mechanism in the nucleus, when the decay occured outside of the nucleus. Comparison with the experimental data for $z_2$-dependence shows that difference between versions is smaller than the experimental errors, consequently, different versions of model can not be distinguished by means of comparison with these data. Calculations with full and partial NA give close results. Theoretical curves quite satisfactory describe data for nitrogen. In case of krypton and xenon targets the situation is more ambiguous. While three middle points are described satisfactory, two extreme points corresponding to lower and higher values of $z_2$ are in much worse agreement. In our opinion, that possible reason could be that our model does not contain necessary ingredients for quantitative description of these points. Let us briefly discuss mechanisms which, we believe, give considerable contribution in the extreme points discussed above, but are not included in present model.

First mechanism which can lead to the increasing of the NA ratio at lower value of $z_2$ is that the part of subleading hadrons in nucleus are protons, which are produced in abundance at small $z$, and in this region they have value of NA ratio larger than unity. We will try to estimate the contribution of protons in Appendix 1. Second mechanism which can lead to the increasing of the NA ratio at lower $z_2$ is the rescattering of produced hadrons in the nucleus, in result hadrons spend part of theirs energy for production of slow hadrons. Consequently, more slow hadrons arise in nucleus, than in deuterium, and, despite absorption, the multiplicity ratio in this region can become close to and even larger than unity. Our model does not take into account the final state interactions of the produced hadrons, consequently, at present we can not calculate or estimate contribution of this mechanism. Concerning second extreme point at higher value of $z_2$, which is equal $z_2 = 0.44$, we suppose that the double hadron attenuation ratio in this point is on the order of unity because there are two more additional mechanisms, which are not included in present model. The first one had to do with the pairs of pions appearing in a result of breaking of coherently produced diffractive $\omega$-mesons, for which the coherent cross section depends proportional to the atomic mass number as $A^2$. As a result, the NA ratio for heavy nuclei raises. It is very difficult to estimate the contribution of this mechanism without implementation of additional free parameters. Second mechanism is connected with the
smallness of integration region over $z_1$, which in case of $z_2 = 0.44$ is equal to 0.06\footnote{this value of 0.06 is in fact related to the case of only two hadrons in final state, more hadrons in final state lead to the decreasing of the integration range}. The NA ratio for two-hadron system is proportional to the ratio of integrals over $z_1$ on nucleus and deuterium. Taking into account Fermi motion or nucleon-nucleon correlations can lead to the extension of the integration region in nucleus and in a result to the increasing of NA ratio (see details in Appendix 2). The model gives close results for the two-hadron system NA ratio for the krypton and xenon targets.

Fig. 3 shows the prediction of model for $\nu$-dependence of double ratio $R_{2h}^{2h}$ for nitrogen, krypton and xenon targets. On the panels a), c), e) three varieties of the theoretical curves are shown: solid curves correspond to the TSM with CFT1; dashed curves correspond to the ITSM with CFT2; dotted curves correspond to the ITSM with CFT1. On panels b), d), f) are shown the same curves calculated with additional condition that only first produced hadron attenuates (partial attenuation). Easy to see, that curves corresponding to the full and partial attenuation have different behavior at low values of $\nu$. In Fig. 3 it is shown, as example, case of krypton only. Curves marked as in Fig. 3. Lower curves correspond to the case of full attenuation and upper curves correspond to the case that only first produced hadron attenuates (partial attenuation). The measurement of NA ratio in the region of $\nu$ from 3$GeV$ to 10$GeV$ is allowed to verify a supposition about possible mutual screening of hadrons in string. We think that such experiment can be useful for comparison with the results obtained at RHIC by STAR Collaboration\cite{15}, which state that two hadrons from one jet absorbed more weakly than two hadrons from away-side jets.

#### V. CONCLUSIONS

- String model\cite{11, 12} gives natural and simple mechanism for description of the two-hadron sys-
tem NA, which allows to describe the available experimental data for \( z \)-dependence of subleading hadron on a satisfactory level, using the sets of parameters obtained in Ref. [10] for single hadron NA.

- Comparison with the experimental data for \( z_2 \)-dependence show that difference between versions of model is smaller than experimental errors, consequently, they can not be distinguished by means of comparison with these data.

- Double ratio considered as a function of partial energy of subleading hadron \( z_2 \) has weak sensitivity to the mutual screening of hadrons

- It is of certain interest to also study other aspects of the two-hadron system production in nuclear medium. In particular we propose to measure the \( \nu \)-dependence of NA, because, as we have shown, it is more sensitive to the mutual screening of hadrons than the \( z_2 \)-dependence. Investigation of the \( \nu \)-dependence in the region of \( \nu \) from 3GeV to 10GeV will allow better understanding of questions connected with possible mutual screening of hadrons in the string. Corresponding measurements can be performed at HERMES and JLab.

- Estimations show that agreement with the experimental data can be improved by means of inclusion of additional mechanisms which were not included in the model we have presented.

As a last remark concerning the description of data for two-hadron system attenuation in other models.

There are at least two theoretical works which attempt to describe data for two-hadron system NA: first of them based on BUU transport model [16], and second based on the so called energy loss model [17].
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APPENDIX 1

In this appendix we will discuss considerable difference between our model and the experimental data at the point \( z_2 = 0.09 \) for krypton nucleus, and will try to understand the cause of this discrepancy. Our model, as mentioned above, takes into account pions only. But in heavy nuclei many slow protons are produced, in addition to pions (in this discussion we do not distinguish kaons from pions). We want to show that by including in consideration these protons one can improve agreement with data. Let us present the two-hadron system NA ratio in the form:

\[
R_{2h}^M = (1 - \alpha)R_{2\pi}^M + \alpha R_{\pi P}^M ,
\]

where \( \alpha \) is the part of the events which contain pairs consisting from a fast pion and a slow proton, \( R_{2\pi}^M \) and \( R_{\pi P}^M \) are two-hadron system NA ratios in case when a pair consist of two pions and pion-proton, respectively. It is convenient to introduce a parameter \( \beta = R_{\pi P}^M / R_{2\pi}^M \). Then \( R_{2h}^M \) can be rewritten in the form:

\[
R_{2h}^M = (1 + \alpha(\beta - 1))R_{2\pi}^M .
\]

Parameter \( \beta \) can be defined from the data on single hadron attenuation, if one can assume that the fast and the slow hadrons in the event are produced independently (correlation between them can be neglected). Then \( \beta \approx R_{\pi P}^M / R_{2h}^M \), where \( R_{\pi P}^M \) and \( R_{2h}^M \) are single hadron NA ratios for proton and pion respectively. In addition to the \( z_2 = 0.09 \) point we make estimates for the next point \( z_2 = 0.15 \) also, because contribution of protons in this point can still be considerable. Using available data for krypton [18], we have extrapolated it in the point \( z = 0.09(0.15) \) and obtained \( R_{\pi P}^M \approx 1.5(1.425) \) and \( R_{2h}^M \approx 0.85(0.85) \), which gave \( \beta \approx 1.765(1.676) \). Then we determined the parameter \( \alpha \) from eq. (8). For \( z_2 = 0.09(0.15) \) the model...
gives $R_{2h}^{2h} \approx 0.89(0.87)$, while the experimental value is $R_{2h}^{2h} \approx 1.2(0.92)$ which gives $\alpha \approx 0.47(0.087)$. The results of calculations with the model improved by means of inclusion of the protons, as was discussed above, are shown in Fig. 5a for the krypton case. Three theoretical curves are shown: the solid curve corresponds to the TSM with CFT1; dashed curve corresponds to the ITSM with CFT2; dotted curve corresponds to the ITSM with CFT1. Such a value of $\alpha$ for the first point seems to be too big. But one can note that still, this correction works in the right direction and improves agreement of theory with the experimental data even if real value of parameter $\alpha$ is smaller than that following from our estimate. In Fig. 5b the dependence on value of parameter $\alpha$ at $z_2 = 0.09$ is shown. As an example we take the case of TSM with CFT1 calculated for three values of $\alpha$ equal to 0.47 (upper curve), 0.30 (middle curve), 0.10 (lower curve). Simultaneously we proportionally changed values of $\alpha$ in the next experimental point also. The estimates with other versions of the model give close results.

APPENDIX 2

As mentioned in text, the Fermi motion of nucleons in nucleus and the presence of nucleon-nucleon correlations (fluctons) can improve agreement with the experimental data at $z_2 = 0.44$ by means of extending the range of integration.

Let us first consider the influence of the Fermi motion on increase of $z_{\text{max}}$. We would like to remind the reader, that in case of scattering on rest nucleon $z_{\text{max}} = 1$. The total centre of mass energy of the secondary hadrons $W$, and in case of taking into account Fermi momentum we mark it as $W_F$. Then, after averaging over the angle between virtual photon and nucleon momenta, it can be presented as:

$$W_F^2 = M^2 - Q^2 + 2E_N\nu = M^2 - Q^2 + 2M\nu_{\text{eff}},$$  \hspace{1cm} (9)

where $M$ and $E_N$ are mass and energy of nucleon respectively, $\nu_{\text{eff}}$ is the value of $\nu$ which give on the rest nucleon value of $W$ equal $W_F$. Then for $z_{\text{max}}$ we obtain:

$$z_{\text{max}} = \frac{\nu_{\text{eff}}}{\nu} = \frac{E_N}{M}.$$  \hspace{1cm} (10)

Now we can estimate the influence of the Fermi momentum. If we take the average Fermi momentum for middle and heavy nucleus to be equal to
0.25 GeV/c \[19\], then: \( z_{\text{max}} = 1.035 \) and NA ratio with taking into account that also the integration region for single hadrons will be increased (about 10%), \( R_{3f}^{2h} \) at \( z_2 = 0.44 \) must be multiplied on factor 1.42.

As an alternative mechanism the nucleon-nucleon correlations could be considered in the framework of the flucton mechanism. That means, that a virtual photon scatters on fluctons with the masses \( M, 2M, 3M, \) etc. If one takes into account only one- and two-nucleon correlations, one obtains:

\[
z_{\text{max}} = \frac{\nu_{\text{eff}}}{\nu} \approx 1 + \alpha_{fl}, \quad (11)
\]

where \( \alpha_{fl} \) is the probability of scattering on the two-nucleon flucton. If, for the sake of an estimate, one takes \( \alpha_{fl} = 0.01 \) then \( z_{\text{max}} = 1.01 \) and the NA ratio \( R_{3f}^{2h} \) at \( z_2 = 0.44 \) must be multiplied by a factor of 1.17. We see that both mechanisms give considerable improvement for the last \( z_2 \) point.

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