Direct dark matter detection around the corner? 
Prospects in the Constrained MSSM

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Abstract. We outline the WIMP dark matter parameter space in the Constrained MSSM by performing a comprehensive statistical analysis that compares with experimental data predicted superpartner masses and other collider observables as well as a cold dark matter abundance. We find that $10^{-10}$ pb $\leq \sigma_p^{SI} \leq 10^{-8}$ pb for direct WIMP detection (with details slightly dependent on the assumptions made). We conclude that most of the 95% probability region for the cross section will be explored by future one–tonne detectors, that will therefore cover most of the currently favoured region of parameter space.

1. Introduction
Two of the most challenging questions facing particle physics today are the instability of the Higgs mass against radiative corrections (known as the “fine–tuning problem”) and the nature of dark matter. Unlike the Standard Model (SM), weak scale softly broken supersymmetry (SUSY) provides solutions to both of them. Firstly, the fine–tuning problem is addressed via the cancellation of quadratic divergences in the radiative corrections to the Higgs mass. Secondly, assuming $R$–parity, the lightest supersymmetric particle (LSP) is a leading weakly interactive massive particle (WIMP) candidate for cold dark matter (CDM). Despite these and other attractive features, without a reference to grand unified theories (GUTs), low energy SUSY models suffer from the lack of predictivity due to a large number of free parameters (e.g., over 120 in the Minimal Supersymmetric Standard Model (MSSM)), most of which arise from the SUSY breaking sector.

The MSSM with one particularly popular choice of universal boundary conditions at the grand unification scale is called the Constrained Minimal Supersymmetric Standard Model (CMSSM) [1]. The CMSSM is defined in terms of five free parameters: common scalar ($m_0$),
gaugino \((m_{1/2})\) and tri-linear \((A_0)\) mass parameters (all specified at the GUT scale) plus the ratio of Higgs vacuum expectation values \(\tan \beta\) and \(\text{sign}(\mu)\), where \(\mu\) is the Higgs/higgsino mass parameter whose square is computed from the conditions of radiative electroweak symmetry breaking (EWSB). The economy of parameters in this scheme makes it a useful tool for exploring SUSY phenomenology.

In this work we report results from a Bayesian exploration of the CMSSM parameter space, obtained through the use of Markov Chain Monte Carlo methods. In particular we focus on the prospects for direct neutralino dark matter detection with the next generation of dark matter searches. We refer the reader to \[2\] for full details. For other works applying a similar approach to the CMSSM, see \[3\] (with some relevant differences) and more recently \[4\].

2. Parameter space and data used

We restrict our analysis to the case \(\text{sign}(\mu) = +1\), as motivated by the fact that the observed anomalous magnetic moment of the muon is positive, and since the sign of the SUSY contribution to it is the same as the sign of \(\mu\). We thus consider the 8 dimensional parameter space \((\theta, \psi)\), where \(\theta\) is a vector of CMSSM parameters,

\[
\theta = (m_0, m_{1/2}, A_0, \tan \beta)
\]

while \(\psi\) is a vector of relevant SM parameters,

\[
\psi = (M_t, m_b(m_b), \alpha_{em}(M_Z), \alpha_s(M_Z)),
\]

where \(M_t\) is the pole top quark mass, \(m_b(m_b)\) is the bottom quark mass at \(m_b\), while \(\alpha_{em}(M_Z)\) and \(\alpha_s(M_Z)\) are the electromagnetic and the strong coupling constants at the \(Z\) pole mass \(M_Z\), the last three evaluated in the \(\overline{MS}\) scheme. We are not interested in constraining the value of the SM parameters, but rather in including the effect of the uncertainty in their experimental determination in our inferences. Therefore we marginalize over them in the end.

We employ a Bayesian approach coupled with a Markov Chain Monte Carlo method for exploring the CMSSM parameter space and derive posterior constraints on the neutralino mass and spin-independent scattering cross section, assuming that all of the cosmological dark matter abundance is made of neutralinos. From the CMSSM and SM parameters \(\eta\) we compute a series of derived observable quantities \(f(\eta)\): the \(W\) gauge boson mass, the effective leptonic weak mixing angle \(\sin^2 \theta_{\text{eff}}\), the anomalous magnetic moment of the muon, \(a_\mu \equiv (g-2)_\mu\), the branching ratios \(BR(B \to X_s \gamma)\) and \(BR(B_s \to \mu^+ \mu^-)\), the cosmological neutralino relic abundance \(\Omega_{\text{CDM}} h^2\), the light Higgs mass and the superpartner masses. For all of those quantities, relevant measurements (summarized in the bottom section of Table 1) or experimental limits are included via the likelihood and used to constrain high posterior probability regions of the model. The likelihood also includes estimated theoretical uncertainties in the mapping from CMSSM and SM parameters to derived quantities, another major advantage of employing a Bayesian approach (see \[2\] for details). We then compute a spin-independent dark matter WIMP elastic scattering cross section on a free proton, \(\sigma^{SI}_p\), including full supersymmetric contributions which have been derived by several groups (see \[5\] and references therein), but we do not include current constraints in the likelihood, in view of the uncertainties in the structure of the Galactic halo (\(e.g.\), existence of clumps of dark matter and therefore the value of the local halo mass density) as well as in the values of some hadronic matrix elements entering the computation of \(\sigma^{SI}_p\).

3. Results for neutralino direct detection

Our Bayesian approach allows to easily compute the posterior pdf for the cross section or any other derived variable. In Fig. 1 we present the 2-dimensional posterior pdf for \(\sigma^{SI}_p\) and \(m_\chi\),
Table 1. Experimental mean $\mu$ and standard deviation for the nuisance parameters (top section) and derived observable quantities (bottom section, including a theoretical uncertainty describing the imprecise mapping of CMSSM and SM parameters onto observable quantities) used in the analysis.

| Nuisance parameter | Mean value | Uncertainty |
|--------------------|------------|-------------|
| $M_t$              | 172.7 GeV  | 2.9 GeV     |
| $m_b(m_b)$ MS      | 4.24 GeV   | 0.11 GeV    |
| $\alpha_s(M_Z)$ MS | 0.1186     | 0.002       |
| $1/\alpha_{em}(M_Z)$ MS | 127.958   | 0.048       |

| Derived observable | Mean value | Uncertainty |
|--------------------|------------|-------------|
| $M_W$              | 80.425 GeV | 34 MeV     |
| $\sin^2 \theta_{\text{eff}}$ | 0.23150   | $16 \times 10^{-5}$ |
| $\delta a_{\mu}^{\text{SUSY}} \times 10^{10}$ | 25.2      | 9.2         |
| BR($B \to X_s \gamma$) $\times 10^4$ | 3.39      | 0.30        |
| $\Omega \chi h^2$ | 0.119      | 0.009       |

Figure 1. The 2–dimensional probability density in the neutralino mass and spin–independent cross section plane in the CMSSM (all other parameters marginalized) with the contours containing 68% and 95% probability also marked. Current 90% experimental upper limits are also shown. A large fraction of the high–probability region lies just below current constraints and it will be probed by the next generation of dark matter searches, starting from the focus point region (horizontal region at $\sigma^{SI}_p \sim 10^{-8}$).

The biggest, banana–shaped region of high probability (68% regions delimited by the internal solid, blue curve) shows a well–defined anticorrelation between $\sigma^{SI}_p$ and $m_\chi$. It results from two allowed regions in the CMSSM parameter space: the bulk and stau coannihilation region and from the $A$–resonance at large $\tan \beta$. This region covers roughly the range $10^{-10} \lesssim \sigma^{SI}_p \lesssim 10^{-8}$ pb

with all other parameters marginalized over. For comparison, we also show current CDMS–II [6], Edelweiss–I [7] and UKDMC ZEPLIN–I [8] 90% CL upper limits, but we stress that this constraint has not been used in our analysis.
and $200 \lesssim m_\chi \lesssim 700$ GeV. In both cases the dominant contribution to $\sigma_p^{SI}$ comes from a heavy Higgs exchange. At small $m_\chi \lesssim 100$ GeV we notice a small vertical band of fairly low probability density ($\lesssim 0.2$) at small $\sigma_p^{SI}$, a region where the light Higgs resonance contribute to reducing $\Omega h^2$ at small $m_{1/2}$ to meet the WMAP measurement. Finally, we can see a well pronounced region of high probability at $\sim 0.6 \times 10^{-8}$ pb for $m_\chi \lesssim 420$ GeV which at low $m_\chi$ partly overlaps with the previous region. At 95% this region extends up to $m_\chi \lesssim 720$ GeV for fairly constant $\sigma_p^{SI}$. This “high” $\sigma_p^{SI}$ band results from the focus point (FP) region, basically independently of $\tan \beta$. This result has to be interpreted carefully, since there are large uncertainties associated with FP region, in particular with its location in the $(m_{1/2}, m_0)$ plane. Despite those outstanding questions, we believe that it is safe to expect that the FP will be the first to be probed by dark matter search experiments.

After marginalizing over all other parameters, we obtain the following 1–dimensional region encompassing 95% of the total probability:

$$0.5 \times 10^{-10} \text{ pb} < \sigma_p^{SI} < 3.2 \times 10^{-8} \text{ pb} \quad (95\% \text{ region}).$$

Currently running experiments (most notably CDMS–II but also Edelweiss–II and ZEPLIN–II) should be able to reach down to a few $\times 10^{-8}$ pb, on the edge of exploring this FP region. A future generation of “one–tonne” detectors is going to reach down to $\sigma_p^{SI} \gtrsim 10^{-10}$ pb, thus exploring much of the 95% interval.

We have checked that the above result is rather robust with respect to a change in the prior range used in the analysis or to the exclusion of the constrain coming from $g−2$.

4. Conclusions

We have presented a detailed investigation of the prospects for dark matter detection in the framework of the CMSSM parameter using state–of–the–art Bayesian methods. We have shown that the WIMP dark matter direct detection elastic scattering cross section $\sigma_p^{SI}$ presents a wide spread of values (below today’s limits) at around $10^{-9}$ pb and a strong anticorrelation with $m_\chi$. In addition, a region at relatively large $\sigma_p^{SI} \simeq 1.6 \times 10^{-8}$ pb, and fairly independent of $m_\chi$, appears to be a feature of the focus point region (despite large theoretical uncertainties) and will be the first to be tested in direct detection experiments.

We conclude that a large fraction of the high–probability parameter space of neutralino dark matter in the CMSSM will be within reach of currently running upgraded dark matter detector, and most of it will be explored by future one–tonne detectors. Our result is thus highly suggestive of a possible detection by the next generation of direct dark matter searches.

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