Robust Transmission Design for RIS-Aided Communications with Both Transceiver Hardware Impairments and Imperfect CSI

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Abstract—Reconfigurable intelligent surface (RIS) or intelligent reflecting surface (IRS) has recently been envisioned as one of the most promising technologies in the future sixth-generation (6G) communications. In this paper, we consider the joint optimization of the transmit beamforming at the base station (BS) and the phase shifts at the RIS for an RIS-aided wireless communication system with both hardware impairments and imperfect channel state information (CSI). Specifically, we assume both the BS-user channel and the BS-RIS-user channel are imperfect due to the channel estimation error, and we consider the channel estimation error under the statistical CSI error model. Then, the transmit power of the BS is minimized, subject to the outage probability constraint and the unit-modulus constraints on the reflecting elements. By using Bernstein-type inequality and semidefinite relaxation (SDR) to reformulate the constraints, we transform the optimization problem into a semidefinite programming (SDP) problem. Numerical results show that the proposed robust design algorithm can ensure communication quality of the user in the presence of both hardware impairments and imperfect CSI.

Index Terms—Reconfigurable intelligent surface (RIS), intelligent reflecting surface (IRS), hardware impairments, imperfect channel state information (CSI).

I. INTRODUCTION

Due to the rapid development of metamaterials, reconfigurable intelligent surface (RIS), which is composed of multiple reflecting units, has recently emerged as a promising technique to improve the quality of the future wireless communications [1]–[4]. Specifically, by controlling the reflecting elements at the RIS, the reflection direction of the electromagnetic wave can be controlled accurately. Then, the reflected signals can be reconfigured to propagate towards their desired directions. As a result, RIS has been proposed to be employed in a variety of communication scenarios such as mobile edge computing [5], secrecy communication [6], and unmanned aerial vehicle-assisted communications [7].

However, most of the existing works on RIS-aided communications are based on the assumption of perfect transceiver hardware and perfect channel state information (CSI), which may not be realistic in practice. Performance degradation will be incurred by inevitable hardware impairments such as hardware aging, imperfect power amplifier, oscillators noise, low-resolution digital-to-analog converters (DACs) and imperfect analog-to-digital converters (ADCs) [8], [9]. It is noted that hardware impairments will impair the signal quality at the receiver. In [8], a robust beamforming scheme was proposed for an RIS-aided point-to-point wireless system with hardware impairments. The authors in [9] demonstrated that the hardware impairments limited the performance of RIS-aided systems and highlighted the importance of accurately modeling the transceiver hardware impairments. However, the above-mentioned works [8], [9] were based on the assumption of perfect CSI. Due to the passive property of the RIS, it is challenging to acquire accurate CSI. Recently, some existing works have studied the impact of imperfect CSI on the performance of the RIS-aided wireless systems [10], [11]. The authors in [10] proposed a robust beamforming design scheme for an RIS-aided MISO system, where the imperfect cascaded CSI was taken into account. In [11], the authors investigated the energy efficiency and power scaling laws of RIS-aided systems with imperfect CSI and transceiver hardware impairments. However, the existing literature only considered imperfect CSI or only imperfect hardware. There are no literature studying the similar optimization problems considering both the imperfect CSI and the transceiver hardware impairments.

Against the above background, we study the robust transmission design scheme that takes into account both transceiver hardware impairments and imperfect CSI. The main contributions of this work are summarized as follows: 1) To be specific, by jointly optimizing the transmit beamforming vector at the base station (BS) and the phase shifts at the RIS, the transmit power of the BS is minimized, subject to the outage probability constraint and the unit-modulus constraints on the reflecting elements. 2) By using Bernstein-Type inequality, we effectively simplify the rate outage probability constraint. Then, the beamforming vectors are obtained by utilizing the semidefinite relaxation (SDR). 3) Simulation results show that the proposed robust design can guarantee the quality of service requirements in an RIS-aided MISO wireless system with both transceiver hardware impairments and imperfect CSI.
II. SYSTEM MODEL

Consider an RIS-aided MISO downlink system, as shown in Fig. 1, where the BS transmits signals to a single-antenna user. The BS is equipped with $N$ antennas and the RIS has $L$ reflecting elements. Then, the signal transmitted from the BS is expressed as

$$t = vs + n_t,$$  \hspace{1cm} (1)

where $s \sim CN(0, 1)$ is the data symbol and $v \in \mathbb{C}^{N \times 1}$ is the transmit beamforming vector. In (1), the hardware impairments at the BS is denoted as $n_t \sim CN(0, \beta_t \text{diag}(vv^H))$, where $\beta_t \in (0, 1)$ is the proportionality coefficient characterizing the level of the hardware impairments at the BS [8].

Let $g \in \mathbb{C}^{N \times 1}$, $H \in \mathbb{C}^{L \times N}$, and $h \in \mathbb{C}^{L \times 1}$ represent the direct channel from the BS to the user, the channel from the BS to the RIS, and the channel from the RIS to the user, respectively. At the RIS, the reflection matrix is denoted as $\Phi = \text{diag}(e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_L})$, where $\phi_i$ denotes the phase shift of the $i$-th reflecting element. Therefore, the received signal at the user can be written as

$$y = (g^H + h^H \Phi H)(vs + n_t) + n + n_r \triangleq \tilde{y} + n_r,$$  \hspace{1cm} (2)

where $n \sim CN(0, \beta_r^2)$ denotes the additive white Gaussian noise (AWGN) received at the user. The hardware impairments at the user is denoted as $n_r \sim CN(0, \beta_r, \mathcal{E}\{|\tilde{y}|^2\})$, where $\beta_r \in (0, 1)$ is the proportionality coefficient characterizing the level of the hardware impairments at the user. Let $e = [e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_L}]^T$ represent the reflecting beamforming vector containing the diagonal elements of $\Phi$. Define matrix $Q = \text{diag}(h^H)H$ as the cascaded channel matrix from the BS to the user through the RIS. Then, we can obtain that $e^HQ = h^H \Phi H$. Thus, the signal-to-noise ratio (SNR) $\gamma$ is expressed as

$$\gamma = \frac{|g^H + e^H Q|v|^2}{\mathbb{E} \{n_r n_r^*\}},$$  \hspace{1cm} (3)

where $n_e = (g^H + e^H Q)n_t + n + n_r$ is the interference noise. In (3), $\mathbb{E} \{n_r n_r^*\}$ is the interference noise power, which is given by (4) at the bottom of this page.

Due to the passive property of the RIS, it is challenging to acquire the perfect CSI. Hence, we consider the statistical CSI error, which means that the CSI estimation error is random and follows certain distribution. Then, the cascaded channel and the direct channel are respectively modeled as

$$Q = \hat{Q} + \Delta Q,$$  \hspace{1cm} (5a)

$$g = \hat{g} + \Delta g,$$  \hspace{1cm} (5b)

where $\hat{Q}$ and $\hat{g}$ respectively represent the estimated cascaded channel and the estimated direct channel, while $\Delta Q$ and $\Delta g$ are the corresponding channel estimation errors. Based on this statistical CSI error model, we assume that the channel estimation error vectors $\text{vec}(\Delta Q)$ and $\Delta g$ follow the circularly symmetric complex Gaussian (CSCG) distribution [12], and they are respectively given by

$$\text{vec}(\Delta Q) \sim CN(0, \Sigma_q),$$  \hspace{1cm} (6a)

$$\Delta g \sim CN(0, \Sigma_g),$$  \hspace{1cm} (6b)

where $\Sigma_q \in \mathbb{C}^{LN \times LN}$ and $\Sigma_g \in \mathbb{C}^{N \times N}$ are the covariance matrices of the estimation error vectors.

III. PROBLEM FORMULATION AND ROBUST BEAMFORMING DESIGNS

A. Problem Formulation

In this work, the transmit power is minimized by jointly optimizing the transmit beamforming and the reflecting phase shifts, subject to the unit-modulus constraints on the reflecting elements and the outage probability constraint $\tau$, $\tau \in (0, 1]$. Thus, we can express the optimization problem as

$$\min_{\substack{v, e}} \|v\|^2_2$$  \hspace{1cm} (7a)

s.t. $|e^H \Phi| = 1, \forall l = 1, \ldots, L,$  \hspace{1cm} (7b)

$$\text{Pr} \left\{ \log_2(1 + \gamma) \geq R \right\} \geq 1 - \tau,$$  \hspace{1cm} (7c)

where $\|\cdot\|$ denotes the Euclidean norm. The rate outage probability constraint (7c) guarantees that the receiver can decode desired information at a data rate of $R$ no less than the probability $1 - \tau$.

Due to the fact that the constraints (7b) are non-convex and the constraint (7c) has no simple closed-form expression [13], it is challenging to solve Problem (7). In order to address it, we firstly perform mathematical transformations on constraint (7c).

Firstly, we can rewrite the rate outage probability as

$$\text{Pr} \left\{ \log_2(1 + \gamma) \geq R \right\} = \text{Pr} \left\{ 1 + \gamma \geq 2^R \right\}$$

$$= \text{Pr} \left\{ (g^H + e^H Q)vv^H(Q^H e + g) \geq (2^R - 1)\Lambda \right\}$$

$$= \text{Pr} \left\{ (g^H + e^H Q)\Lambda(Q^H e + g) - (1 + \beta_r)\delta^2 \leq 0 \right\},$$  \hspace{1cm} (8)

$$\mathbb{E} \{n_r n_r^*\} = \mathbb{E} \{(g^H + e^H Q)n_t + n + n_r)(n_t^H(Q^H e + g) + n^* + n_r^*)

= (g^H + e^H Q)\mathbb{E} \{n_t n_t^H\} (Q^H e + g)

+ \beta \mathbb{E} \{(g^H + e^H Q) vs + (g^H + e^H Q)n_t + n)(s^*v^H(Q^H e + g) + n_t^H(Q^H e + g) + n^*)

= (g^H + e^H Q) (s^*v^H + (1 + \beta_r)\Delta \text{diag}(vv^H)) (Q^H e + g) + (1 + \beta_r)\delta^2 \leq \Lambda.$$

(4)
where \( A = \left[ \frac{1}{2} - \beta_r \right] v v^H - (1 + \beta_r) \beta_i \text{diag}(v v^H) \). By substituting (5) into (8), the rate outage probability in (8) is further written as

\[
\Pr \left\{ (g^H + e^H Q) A (Q^H e + g) > (1 + \beta_r) \delta^2, \right\} = \Pr \left\{ (\Delta g^H + e^H \Delta Q) A (\Delta Q^H e + \Delta g) \right\} \\
+ 2 \text{Re} \left\{ (g^H + e^H Q) A (\Delta Q^H e + \Delta g) \right\} \\
+ (g^H + e^H Q) A (Q^H e + g) > (1 + \beta_r) \delta^2 \}
\]

(9)

By defining \( E = e^H \), the first term in (9) is rewritten as

\[
(\Delta g^H + e^H \Delta Q) A (\Delta Q^H e + \Delta g) \\
= \Delta g^H A \Delta g + 2 \text{Re} \left\{ (\Delta g^H (A \otimes e^T) \text{vec}(\Delta Q^*) \right\} \\
+ \text{vec}^T (\Delta Q^*) (A \otimes e^T) \text{vec}(\Delta Q^*)
\]

(10)

where \( (a) \) holds due to \( \text{Tr} \{ A_1 B_1 C_1 D_1 \} = (\text{vec}(D_1^T))^T (C_1^T \otimes A_1) \text{vec}(B_1) \) [14], and \( (b) \) holds by assuming the channel estimation error vectors \( \Delta g = \zeta_g \tilde{g} \sim \mathcal{CN}(0, \tilde{g}_2 I_N) \) and \( \text{vec}(\Delta Q) = \zeta_q i_q \sim \mathcal{CN}(0, i_q I_{L,N}) \), where \( i_g \) and \( i_q \) are the CSCG random vectors, \( \zeta_g \) and \( \zeta_q \) are the constants which measure the relative amount of CSI uncertainties, and \( I_N \) and \( I_{L,N} \) are the \( N \times N \) identity matrix and \( L \times N \) identity matrix, respectively. In (10), \( i \) and \( M \)

are respectively defined as

\[
i = [(\hat{g}_g)^H I_g]^H, M = \left[ \begin{array}{ccc}
\zeta_g \hat{g}_g A (A \otimes e^*) \\
\zeta_q \text{vec}^T (e (g^H + e^H Q) A)
\end{array} \right].
\]

Similarly, the second term of (9) is rewritten as

\[
2 \text{Re} \left\{ (g^H + e^H Q) A (\Delta Q^H e + g) \right\} \\
= 2 \text{Re} \left\{ \zeta_g (g^H + e^H Q) A \hat{g}_g + \text{vec}^T (e (g^H + e^H Q) A) i_q^* \right\} \\
\Rightarrow 2 \text{Re} \left\{ \bar{m}^H i \right\},
\]

(12)

where \( \bar{m} = \left[ \begin{array}{c}
\zeta_g (g^H + e^H Q) A \\
\zeta_q \text{vec}^T (e (g^H + e^H Q) A)
\end{array} \right]^H, \) and \( (c) \) is obtained by using \( \text{Tr} \{ A_1 B_1 \} = \text{vec}^T (A_1) \text{vec}(B_1) \) [14]. Then, the constraint (7c) can be simplified as

\[
\Pr \left\{ i^H \bar{M} i + 2 \text{Re} \left\{ \bar{m}^H i \right\} + \bar{m} \geq 0 \right\} \geq 1 - \tau,
\]

(13)

where \( \bar{m} = (g^H + e^H Q) A (Q^H e + g) - (1 + \beta_r) \delta^2 \). Then, we will further transform the constraint (7c) by utilizing Lemma 1 in [13], which is given by the following lemma.

**Lemma 1:** (Bernstein-Type Inequality: Lemma 1 in [13])

Assume \( a^H M a + 2 \text{Re} \left\{ m^H a \right\} + m \), where \( M \in \mathbb{H}^{n \times n}, m \in \mathbb{C}^{n \times 1}, m \in \mathbb{R} \) and \( a \in \mathbb{C}^{n \times 1} \sim \mathcal{CN}(0, I) \). For any \( \tau \in (0, 1] \), \( x \) and \( y \) are slack variables, then, we have the following relationship:

\[
\Pr \left\{ a^H M a + 2 \text{Re} \left\{ m^H a \right\} + m \geq 0 \right\} \geq 1 - \tau
\]

(14)

\[
\Rightarrow \begin{cases}
\text{vec}(M) \leq x, \\
\sqrt{2m} \leq y, \quad x, y \geq 0
\end{cases}
\]

By introducing the auxiliary variables \( a \) and \( b \), we apply the Bernstein-Type Inequality in Lemma 1 to transform the original constraint (7c) as

\[
\begin{cases}
\text{Tr} \{ M \} - 2 \text{ln}(1/\tau) a - \text{ln}(1/\tau) b + \bar{m} \geq 0, \\
\sqrt{2\bar{m}} \leq x, \quad x, b \geq 0
\end{cases}
\]

(15)

The variables \( \text{Tr} \{ M \} \), \( \| M \|_F \) and \( \| \bar{m} \|_2^2 \) in constraints (15) can be further simplified as

\[
\begin{align}
&\text{Tr} \{ M \} = \zeta_g^2 + \zeta_q^2 L \text{Tr} \{ A \}, \\
&\| \text{vec}(M) \|_2 = \| \text{vec}(g^H + e^H Q) A \|_2 = \| \text{vec}(e (g^H + e^H Q) A) \|_2^2 = \| a \|_2^2. \\
&\| \bar{m} \|_2^2 = \| (g^H + e^H Q) A \|_2^2 + \| e (g^H + e^H Q) A \|_2^2 = \| a \|_2^2. \\
&\| \text{vec}(M) \|_2 = \text{Tr} \{ M \} - 2 \text{ln}(1/\tau) a - \text{ln}(1/\tau) b + \bar{m} \geq 0,
\end{align}
\]

(16)

Then, Problem (7) can be approximated as

\[
\min_{v, e, a, b} \| v \|_2^2 \]

s.t.

\[
\begin{align}
&|e| = 1, v_{l} = 1, ..., L, \\
&\zeta_g^2 + \zeta_q^2 L \text{Tr} \{ A \} - 2 \text{ln}(1/\tau) a - \text{ln}(1/\tau) b + \bar{m} \geq 0, \\
&\| \text{vec}(M) \|_2 \leq x, \quad x, b \geq 0.
\end{align}
\]

(17a)

(17b)

(17c)

(17d)

(17e)

B. Optimizing the Transmit Beamforming Vector

Due to the fact that the vector \( v \) and the vector \( e \) are coupled together, Problem (17) is non-convex. To solve it, we use the alternating optimization (AO) technique to optimize the vector \( v \) and the vector \( e \). Firstly, we optimize \( v \) when \( e \) is fixed. By defining \( V = v v^H \), Problem (17) is reformulated as

\[\text{Problem (17)}\]
By adopting the similar method in [10], we introduce a slack variable \( \hat{\mu} \) to modify (8) as
\[
\min_{v, a, b} \quad \text{Tr} \{ V \} \\
\text{s.t.} \quad \mathbf{1} \mathbf{c} \text{, } \mathbf{1} \mathbf{d} \text{, } \mathbf{1} \mathbf{e} \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{Tr} \{ \mathbf{M} \} - 2 \ln(1/\rho)a - \ln(1/\rho)b + \hat{\mu} \geq 0, (18a)
\]
\[
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{Tr} \{ \mathbf{E} \} \geq 0, \text{rank}(V) = 1. (18b)
\]
\[
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{rank}(\hat{\mathbf{M}}) \geq n, (18c)
\]

By utilizing the semidefinite relaxation (SDR) method [10], the constraint \( \text{rank}(V) = 1 \) is removed. Therefore, the resulting semidefinite programming (SDP) problem can be effectively solved by utilizing the CVX tools [15]. Finally, the optimal \( v \) can be obtained from \( V \) by using Gaussian randomization method [10].

### C. Optimizing the Phase Shifts Vector

In this subsection, we will optimize the phase shifts vector \( e \) when the transmit beamforming vector \( v \) is fixed. Due to the fact that the objective function \( ||v||^2_3 \) of Problem (17) does not contain the phase shifts vector \( e \), Problem (17) reduces to a feasibility check problem when the variables \( \{v, a, b\} \) are fixed. By adopting the similar method in [10], we introduce a slack variable \( \hat{\mu} \geq 0 \), and it can achieve a strictly larger rate than the target rate \( R \) for the user and accelerate the convergence. Hence, the rate outage probability in constraint (5) is modified as
\[
\Pr \{ (g^H + e^H Q) A(Q^H e + g) - (1 + \beta_v) \delta^2 - \mu \geq 0 \}. (19)
\]

Then, by performing the same transformations for (19) again, we can obtain the approximation of (19) as
\[
\left\{ \begin{array}{l}
\text{Tr} \{ \tilde{\mathbf{M}} \} - 2 \ln(1/\rho)a - \ln(1/\rho)b + \hat{\mu} \geq 0, \\
\end{array} \right. (19a)
\]
where \( \hat{\mu} = (g^H + e^H Q) A(Q^H e + g) - (1 + \beta_v) \delta^2 - \mu. \)

By defining \( \tilde{\mathbf{E}} = \left[ \begin{array}{cc} \mathbf{e} \mathbf{e}^H & \\mathbf{e} \end{array} \right], \)
\[
\hat{\mu} = \text{Tr} \{ \tilde{\mathbf{E}} \} + g^H \mathbf{A} \hat{g} - (1 + \beta_v) \delta^2 - \mu, \quad \text{where} \quad \mathbf{B} = \left[ \begin{array}{cc} \mathbf{Q} \mathbf{A} \mathbf{Q}^H & \mathbf{Q} \mathbf{A} \hat{g} \\
\hat{g} \mathbf{A} \mathbf{Q}^H & \hat{g} \mathbf{A} \hat{g} \end{array} \right].
\]

In order to obtain the variable \( \tilde{\mathbf{E}} \), the constraint (17d) is reformulated as
\[
\left( c^2 + \nu^2 \right) \| \mathbf{A} \|_F^2 + 2(\nu^2 + \xi^2) \text{Tr} \{ \mathbf{C} \tilde{E} \} + \mathbf{g}^H \mathbf{A} \mathbf{A}^H \mathbf{g} \leq a^2, (21)
\]
where \( \mathbf{C} = \left[ \begin{array}{cc} \mathbf{Q} \mathbf{A} \mathbf{A}^H \mathbf{Q}^H & \mathbf{Q} \mathbf{A} \mathbf{A}^H \mathbf{g} \\
\mathbf{g}^H \mathbf{A} \mathbf{A}^H \mathbf{Q}^H & 0 \end{array} \right]. \)

Since constraint (21) is non-convex, we need to linearize it by using the first-order linear approximation. Then, the constraint (21) is reformulated as
\[
\left( c^2 + \nu^2 \right) \| \mathbf{A} \|_F^2 + 2(\nu^2 + \xi^2) \text{Tr} \{ \mathbf{C} \tilde{E} \} + \mathbf{g}^H \mathbf{A} \mathbf{A}^H \mathbf{g} \leq 2 a^{(n)} + |a^{(n)}|^2, (22)
\]
where \( a^{(n)} \) on the right hand side of the above inequality is the optimal solution in the \( n \)-th iteration. By optimizing \( \tilde{\mathbf{E}} \) to make the achievable rate larger than the target rate \( R \), Problem (18) has more space to reduce the transmit power. Therefore, the phase shifts optimization problem is reformulated as
\[
\max_{\mathbf{E}, \rho, a, b} \quad \mu \\
\text{s.t.} \quad \left( \frac{c^2}{\nu^2} + \xi^2 \right) \| \mathbf{A} \|_F^2 + 2(\nu^2 + \xi^2) \text{Tr} \{ \mathbf{C} \tilde{E} \} + \mathbf{g}^H \mathbf{A} \mathbf{A}^H \mathbf{g} \leq a^2, \quad (23a)
\]
\[
\| \mathbf{E} \|_{F} \geq 0, \| \mathbf{E} \|_{F} \geq 0, \quad \text{rank}(\mathbf{E}) = 1, (23b)
\]
\[
\mu \geq 0, a \geq 0, \quad (23c)
\]
\[
\tilde{\mathbf{E}} \geq 0, \text{rank}(\tilde{\mathbf{E}}) = 1, l = 1, \ldots, L + 1. \quad (23d)
\]

where \( \tilde{\mathbf{E}} \) denotes the \((i,j)\)th element of \( \tilde{\mathbf{E}} \). To solve Problem (23), the SDR is employed. Then, Problem (23) becomes a convex SDP problem. By making use of the CVX tools, we can obtain the optimal solution \( \tilde{\mathbf{E}} \) to Problem (23). The optimal phase shifts vector \( e \) can be also obtained from the optimal \( \tilde{\mathbf{E}} \) by using Gaussian randomization techniques. Algorithm 1 presents the overall algorithm of AO method for solving Problem (17).

**Algorithm 1 AO algorithm for Problem (17)**

**Require:** Initial iteration number \( n = 0 \), \( e^{(0)} \) and \( v^{(0)} \).

1: repeat
2: Given \( e^{(n)} \), calculate \( v^{(n+1)} \) from Problem (18);
3: Given \( v^{(n+1)} \), calculate \( e^{(n+1)} \) from Problem (23);
4: Set \( n \leftarrow n + 1 \);
5: until The value \( ||v||^2_3 \) converges.

In this section, we provide numerical results to evaluate the performance of our proposed algorithm. We assume that the BS, the RIS, and the user are respectively located at \((0 \text{ m}, 0 \text{ m})\), \((90 \text{ m}, 0 \text{ m})\), and \((90 \text{ m}, 5 \text{ m})\) in a two-dimensional plane. The large-scale fading of the channels are modeled as \( PL = -30 - 10 \log_{10} d \), where \( \alpha \) is the path loss exponent and \( d \) is the link distance in meter. In this work, we set \( \alpha = 3 \) and \( \alpha = 4 \) for the cascaded channel and the direct channel, respectively. The small-scale fading is assumed to be Rician distributed. For simplicity, the Rician factor is assumed to be 5. We respectively define \( c^2 = \delta^2 \| \text{vec}(Q) \|_2^2 \) and

![Fig. 2: Outage probability versus the channel uncertainty level \( \delta \) when \( N = \text{2} \) and \( M = \text{24} \).](image-url)
\[ C_{\text{f}}^2 = \delta_c^2 \left\| g \right\|^2_f, \]
where \( \delta_c \in [0, 1] \) is the channel uncertainty level. We assume that the BS and the user have the same level of the hardware impairments, i.e., \( \beta_s = \beta_r = \beta \). The other parameters are set as: noise power of \( \delta^2 = -80 \) dBm, target rate of \( R = 1.5 \) bit/s/Hz, convergence tolerance of \( \epsilon = 10^{-6} \), outage probability of \( \tau = 0.01 \).

We illustrate the advantage of our proposed robust beamforming design by comparing it with the following schemes:

- \text{HWI/Nonrobust CSI of } [3]: It only considered the impact of the transceiver hardware impairments in the transmission beamforming design, while the channel estimation error was ignored, which corresponds to the beamforming design scheme in [3].
- \text{CSI/Nonrobust HWI of } [10]: The channel estimation error was only considered in the robust transmission design, while the hardware impairments were ignored, which corresponds to the beamforming design scheme in [10].
- \text{Nonrobust CSI HWI of } [3]: Both channel estimation error and hardware impairments were ignored during the transmission design, which corresponds to the beamforming design scheme in [3].

Fig. 2 illustrates the outage probability versus the CSI uncertainty level. As seen from Fig. 2, the outage probabilities of both \text{“CSI/Nonrobust HWI of } [10] \text{” and “HWI/Nonrobust CSI of } [3] \text{” gradually increase with the increase of channel estimation error. In particular, the outage probabilities of \text{“Nonrobust CSI HWI of } [3] \text{” are always 1, which is due to the fact that the transceiver hardware impairments always deteriorate the received signal quality. By contrast, our proposed robust design method can always ensure that the outage probability is 0 with both transceiver hardware impairments and imperfect CSI. Fig. 3 depicts the outage probability versus the transceiver hardware impairments level. We can also find that our proposed robust design can keep the outage probability at 0. The system performances for the other three schemes become worse with the increase of transceiver hardware impairments level.

Fig. 4 depicts the transmit power versus the number of RIS reflecting elements. Compared our proposed robust design with other three schemes and \text{“Perfect CSI HWI”}, which means perfect CSI acquisition and ideal hardware, it is observed that the transmit power of our proposed design is higher than others. Since the robust transmission design takes into account both the hardware impairments and the imperfect CSI, the BS needs higher power to transmit data in order to compensate for the signal loss.

V. Conclusions

This work studied an RIS-aided wireless system, where the impacts of both transceiver hardware impairments and imperfect CSI were considered. Specifically, the transmit power of the BS is minimized, subject to the outage probability constraint and the unit-modulus constraints on the reflecting elements. By adopting the Bernstein-Type Inequality, we reformulated the constraints into the tractable forms. Then, the reformulated problem was solved via the AO framework. Simulation results demonstrated that the robustness of the proposed transmission design with both hardware impairments and imperfect CSI.

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