Steady-state physics, effective temperature dynamics in holography

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Using the gauge-gravity duality, we argue that for a certain class of out-of-equilibrium steady-state systems in contact with a heat bath at a given temperature, the macroscopic physics can be captured by an effective thermodynamic description. The steady-state is obtained by applying a constant electric field that results in a stationary current flow. Within holography, we consider generic probe systems where an open string equivalence principle and an open string metric govern the effective thermodynamics. This description comes equipped with an effective temperature, which is larger than the bath temperature, and a corresponding effective entropy. For conformal or scale-invariant theories, certain scaling behaviours follow immediately. In general, in the large electric field limit, this effective temperature is also observed to obey certain generic relations with various physical parameters in the system.

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Introduction: Thermodynamics is an integral cornerstone of our understanding of the physical world, where the core principles are based on the existence of a thermal equilibrium. For systems driven out-of-equilibrium, the governing principles are much less understood; see e.g. \textsuperscript{1,2} for some general results in this direction.

However, it is extremely difficult to address such questions in a strongly coupled quantum system. The reasons are two-fold: computational and conceptual. First, given a microscopic description of a model, conventional perturbative (field theory) techniques are inadequate at strong coupling. Second, conceptual insights that may lead us to an effective description are also lacking.

In recent years, the AdS/CFT correspondence\textsuperscript{3} has emerged to be an extremely powerful tool to address aspects of strongly coupled physics. The conceptual root of this correspondence is a more general idea of holography or gauge-gravity duality\textsuperscript{4}, which, in principle, translates a well-defined quantum field theory question into a gravity one. String theory provides a large class of concrete examples where this correspondence is precise. The duality stems from two equivalent low energy descriptions of the degrees of freedom of string theory, i.e. strings and branes, one involving familiar large-$N$ (supersymmetric)-gauge theories in $d$-dimensions and the other involving classical (super)-gravity in $(d + 1)$-dimensions.

A large class of these examples correspond to a strongly coupled conformal or a scale-invariant field theory and is thus naturally equipped in describing quantum criticality. However, such examples are not necessarily restricted to scale-invariant systems; string theory gives rise to rather generic examples of large-$N$ gauge theories with a running coupling constant\textsuperscript{5}, or a confining gauge theory\textsuperscript{6}, which is qualitatively analogous to a gapped quantum many body system.

In this article, we will use the gauge-gravity duality to explore an emerging principle for a system driven out-of-equilibrium. In view of the above discussion, our results will primarily apply, but not be limited to, quantum systems at criticality. Within such theories, we will consider a non-equilibrium steady-state (NESS) situation within a probe sector which is kept in contact with a large heat bath at some given temperature $T$. The NESS in the probe sector is induced by introducing a constant external electric field that drives a constant current. We will argue that all modes in this probe sector experience an effective temperature, denoted by $T_{\text{eff}}$ and satisfying $T_{\text{eff}} > T$, with respect to which it has a purely thermal behaviour. By virtue of the probe limit, the heat flow between the probe sector and the bath is suppressed; the bath and the probe sector will have a purely thermal behaviour with respect to two different temperatures. For critical systems, this effective temperature depends only on three ingredients: (i) dimensionality $d$, (ii) the global symmetry group $G$ and (iii) how NESS is induced, namely the electric field $E$. For gapped systems, or systems with a running coupling constant, there may be additional dependences on the beta function of the gauge coupling or other dimensionful parameters in the system.

The existence of the unique effective temperature is, nonetheless, ubiquitous. The consequences of this are rather profound: it allows us to define a sensible thermodynamics with an effective entropy, obtain a fluctuation-dissipation relation and recover an otherwise thermal physics for the non-equilibrium steady-state system with an effective temperature that retains the information of how the system was driven out-of-equilibrium towards a steady-state. In this article, we will discuss the main results for a broader perspective and a technically detailed account will appear elsewhere\textsuperscript{7}.

Holography and thermal physics: One hallmark of holography is the precise matching between the global symmetry group $G$ of the gauge theory with the isometry group of the gravity background. In the most familiar example of the AdS/CFT correspondence, $G \equiv SO(d, 2)$ for a gauge theory in $d$ spacetime dimensions. In re-

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cent years, interests have grown in examples where $G$ is the Lie group with Lifshitz symmetry\[8\], Schrodinger symmetry\[9\] or hyper-scaling violation\[10\] symmetry. In all these cases, the duality also comes equipped with a bulk-to-boundary correspondence which is essential to translate the computation from one description to another.

The underlying string theory (or supergravity) embedding of such geometric constructions determines the precise dictionary. This provides the gravity description which originates from the closed string sector. This is typically described by a 10-dimensional geometry of the Freund-Rubin type: e.g. $\text{AdS}_{d+1} \otimes_w M^{9-d}$, where $\text{AdS}_{d+1}$ denotes the anti-de Sitter space in $(d+1)$-dimensions and $M^{9-d}$ denotes a compact manifold. The symbol $\otimes_w$ denotes that in general the spacetime can be a warped product geometry. Typically there are also tensor matter fields of various ranks. This geometry and its reduced description for the dual gauge theory directions the probe shares, (ii) stability of the probe embedding and (iii) the physics we want to realize. This was pioneered in [13] to address the dynamics of fundamental flavours (the analogue of quarks in Quantum Chromodynamics) introduced in the background of adjoint matter (the analogue of gluons). Clearly, the full details of the 10-dimensional background play a crucial role in such embeddings and each case needs to be studied individually. For simplicity, however, we will abstract away from this detailed constructions and focus on a reduced description for the probes [13].

The probe embedding and fluctuations: Let us begin with a background $(d+1)$-dimensional metric, in the string frame, of the general form

$$ds^2 = g_{tt} dt^2 + g_{xx} dx^2 + g_{uu} du^2 + \sum_{i=2}^{d-2} g_{ii} dx^i dx^i,$$  \hspace{1cm} (1)

where we assume that the metric functions depend on only one co-ordinate $u$, which is the radial direction. In what follows we adopt the convention that $u \to 0$ is the boundary of the bulk space time. The dual field theory lives in the $\{t,x,x'\}$-plane. In this background we introduce $N_f$ number of probe Dp-branes which wrap $\{t,x,u\}$ and $m$ of the remaining space-directions (along with $(p-m-2)$ of the compact directions, which we will not elaborate on). The dynamics of these probes is determined by the Dirac-Born-Infeld (DBI) action\[14\]

$$S_{DBI} = -N_f T_p \int d^{p+1} \xi e^{-\Phi} (-\det [P|g| + F])^{1/2},$$ \hspace{1cm} (2)

where $T_p$ denotes the tension of the Dp-brane, $\xi$ denotes the collective coordinates on the brane worldvolume (including the compact directions), $\Phi$ is the dilaton field, $P|g|$ denotes the pull-back of the background metric and $F$ represents a gauge field on the worldvolume of the probe. To keep our discussions simple, we will assume that $P|g|$ is trivial, i.e. $P|g| = g$ on the probe worldvolume.

Now, the steady-state physics can be induced by having a gauge field on the worldvolume of the probe

$$A_x = -Et + a_x(u),$$ \hspace{1cm} (3)

where $E$ denotes the electric field along $x$-direction and $a_x(u)$ is a function that needs to be determined from the equation of motion. The DBI Lagrangian depends only on $u$ and $a_x$, where $'$ denotes the derivative with respect to $u$. Thus there is an integral of motion, which we call $j$. Then the equation of motion for $a_x$ gives

$$a_x'^2 = \frac{j^2}{g_{uu} g_{tt} g_{xx} + E^2} \left[ \frac{-g_{uu} (g_{tt} g_{xx} + E^2)}{j^2 g_{tt} + e^{-2\Phi} g_{tt}^2 \left( \prod_{i=2}^{m+1} g_{ii} \right)} \right].$$ \hspace{1cm} (4)

Using this, the on-shell DBI Lagrangian is obtained to be

$$L_{\text{on}} = -e^{-2\Phi} g_{tt} \left[ \prod_{i=2}^{m+1} g_{ii} \right] \left[ \frac{-g_{uu} (g_{tt} g_{xx} + E^2)}{j^2 g_{tt} + e^{-2\Phi} g_{tt}^2 \left( \prod_{i=2}^{m+1} g_{ii} \right)} \right]^{1/2},$$ \hspace{1cm} (5)

We need to impose reality condition on the above on-shell Lagrangian. This condition determines the constant $j$:

$$E = \sqrt{-g_{tt} g_{xx}} \bigg|_{u=u_*},$$  \hspace{1cm} (6)

$$j = \sqrt{-g_{tt}} \left( \prod_{i=2}^{m+1} g_{ii} \right)^{1/2} e^{-\Phi} \bigg|_{u=u_*},$$ \hspace{1cm} (7)

where $u_*$ is obtained by solving (6) and is different from the bulk event horizon. In fact, as observed by the
bulk geometry, \( u_\ast \) is a completely unremarkable position. However, we will see that this \( u_\ast \) will play the key role for the open string degrees of freedom. Using the gauge-gravity dictionary, it can be shown that this constant \( j \) corresponds to the boundary current that is driven by the applied electric field and thus we can obtain a conductivity, \( \sigma \equiv j/E \).

So far, we have discussed the physics obtained from the classical DBI action. Let us now discuss the physics of the fluctuation modes on the probe. From the Minkowski observer’s point of view, these fluctuations are of three kinds: (i) scalar, (ii) vector and (iii) spinor. The scalar fluctuations are the transverse fluctuations of the probe brane embedding, the vector fluctuations correspond to the fluctuations of the classical gauge field on the probe and the spinors come from a supersymmetric counterpart of the DBI action in [2], which schematically consists of a standard Volkov-Akulov type term

\[
S_{VA} = -N_f T_p \int d^{D+1} \xi e^{-\Phi} \left( -\det \left[ M + i \bar{\psi} \gamma \nabla \psi \right] \right)^{1/2},
\]

where \( M = P[g] + F \) and the \( \gamma \) matrices satisfy anti-commutation relation with respect to \( P[g] \). The computation of fluctuation modes around a particular classical brane configuration will involve inverting the matrix \( M \). This will yield the following fluctuation Lagrangians at the quadratic order

\[
L_{\text{scalar}} = -\frac{\kappa}{2} e^{-\Phi} \sqrt{-\det M} S^{ab} \partial_a \varphi \partial_b \varphi + \ldots,
\]

\[
L_{\text{vector}} = -\frac{\kappa}{4} e^{-\Phi} \sqrt{-\det M} S^{ab} S_{cd} F_{ac} F_{bd} + \ldots,
\]

\[
L_{\text{spinor}} = i \kappa e^{-\Phi} \sqrt{-\det M} \psi S^{ab} \gamma_a \gamma_b \psi + \ldots
\]

Here \( \kappa \) denotes an overall constant, the details of which is not relevant for us. The fields \( \varphi, F \) and \( \psi \) represent the various fluctuation modes and the indices \( a, b \) represent the worldvolume coordinates on the probe. We have shown only the kinetic parts of the fluctuation Lagrangian; in general there can be other terms, which will not affect our conclusions.

The key observation is that all these modes perceive an effective metric, denoted by \( S \), which is different from both the background metric \( g \) and the induced metric \( P[g] \). This is the so called open string metric [16], which governs the dynamics of open string degrees of freedom propagating in a background geometry with an anti-symmetric 2-form. The open string data, in terms of the closed string data, is given by [16]

\[
S_{ab} = P[g]_{ab} - (FP[g]^{-1} F)_{ab},
\]

\[
G_s = g_s \left( -\det (P[g] + F) \right)^{1/2},
\]

where \( S \) and \( G_s \) denote the open string metric and the open string coupling; \( g \) and \( g_s \) denote the closed string metric and the closed string coupling. The structure of the fluctuations suggests an open string equivalence principle [17].

For the gauge field in [3], the open string metric in \( \{ \tau, u \} \)-plane is obtained to be

\[
ds_{\text{om}}^2 = S_{tt} d\tau^2 + \left( \frac{S_{uu} S_{tt} - S_{ut}^2}{S_{tt}} \right) du^2,
\]

where \( \tau = t + f(u) \), with \( f'(u) = S_{ut}/S_{tt} \). The presence of the electric field results in an event horizon of the open string metric at the location \( u = u_\ast \), which is obtained by solving [4], and thus \( S_{tt}(u_\ast) = 0 \). The fact that all open string modes perceive this event horizon implies the existence of a universal effective temperature that is obtained by first Euclideanizing [14], periodically compactifying the Euclidean time-direction and identifying the inverse period as the effective temperature. Note that, since \( u_\ast \) is different form the bulk event horizon, we cannot make Euclidean \( g \) and Euclidean \( S \) regular simultaneously. The period of the \( U(1) \)-symmetric time-direction chosen in order to avoid a conical singularity crucially depends on where the geometry pinches off. The fact that we cannot make both \( g \) and \( S \) simultaneously regular is a signature of the underlying physics that the bath and the probe sector are not in a thermal equilibrium.

Assuming that the original background metric in [1] is isotropic and also \( g_{ii} = g_{xx} = 1/u^2 \) [13], this effective temperature, given in terms of the background data \( \{ T, E, \beta \} \), can be recast in the following general form

\[
T_{\text{eff}} = \frac{1}{4\pi} \left[ \frac{(2E^2 u + g_{tt})(2E^2 u (m + \beta) + g_{tt})}{-g_{tt} g_{uu}} \right]^{1/2}
\]

where \( \beta(u) := (d\Phi)/d(\log u) \) represents the beta function corresponding to the running dilaton. Assuming \( \beta = 0 \), from [15] it is clear that the sufficient condition to violate \( T_{\text{eff}} > T \) is to have \( E^2 < 0 \), which will violate weak energy condition for the matter field on the classical probe profile. This is expected on physical grounds from the dual field theory perspective, since maintaining the steady-state continually pumps energy into the system and if there is a notion of an effective temperature it should be higher than the bath temperature. In the presence of a non-trivial \( \beta \), this inequality \( T_{\text{eff}} > T \) involves another data from the background geometry, i.e. how the dilaton behaves. It can be explicitly checked that for all known geometries this inequality is satisfied, however, at present this can be shown as a case-by-case analysis [17].

The propagation of fluctuations in a DBI-background can be analyzed using the method of characteristics, as introduced by Boillat (see e.g. [19] and [20] for a more recent discussion). Subsequently, it can be shown that the open string causal structure is determined by the so called Boillat metric, which belongs to the same conformal class as the open string metric. Typically, the Boillat light-cone lies inside the Einstein light-cone as long as an weak energy condition is satisfied by the matter field on the probe [20]. This light cone structure is another manifestation of the \( T_{\text{eff}} > T \) inequality.
This effective temperature is accompanied by an effective entropy as well. In [21] the Helmholtz free energy for such an NESS probe system was conjectured for a particular system. A natural generalization of this proposal can be given by

$$F_H = N_f T_p \int_0^{u_*} d\eta \xi (L_{\omega} - j a'_z), \quad (16)$$

where we have evidently written down the expression in the full 10-dimensional geometry and $F_H$ denotes the Helmholtz free energy. It can be noted that the conjectured free energy is essentially a Legendre transformation of the on-shell Euclidean probe action. Using thermodynamic identities, one can now recover the effective entropy by computing $s_{\text{eff}} = - (\partial F_H / \partial T_{\text{eff}})$. The effective entropy depends more crucially on the details of the specific 10-dimensional construction.

For a conformal theory, where $\Phi = 0$ and where the 10-dimensional background is a simple product manifold, this entropy can be written as a Bekenstein-Hawking type formula involving the open string data $\{G_s, S\}$. Recall that a probe action scales as $g_s^{-1}$, where $g_s$ is the closed string coupling. Thus, the most natural generalization of an area-law can be put forth in the following form

$$s_{\text{eff}} = \frac{1}{G_s} \text{Area}(\mathcal{S}) \bigg|_{u = u_*}, \quad (17)$$

where $\alpha$ is a constant (independent of $u_*$) that depends on the details of the brane construction and $\text{Area}(\mathcal{S})$ denotes the area of the open string metric event horizon. Also note that both $G_s$ and $\text{Area}(\mathcal{S})$ are functions of the radial coordinate $u$. It can be checked that the formula in (17), applied to a probe system in equilibrium with the bath, reduces to the same entropy as one obtains by computing the Helmholtz free energy of the probe sector.

Some special results: Let us discuss some specific results.

(i) For a system with $\Phi = 0$, and assuming that $g_{xx} = 1/u^2 = g_{ii}$, the effective entropy is observed to obey a simple relation with the conductivity:

$$s_{\text{eff}} \propto \sigma^{(m+1)/(m-1)}.$$ (ii) If we assume that the dual field theory is Lorentz invariant, i.e., $g_{tt} \rightarrow 1/u^2$ as $u \rightarrow 0$, and also $g_{xx} = 1/u^2 = g_{ii}$, then in the limit of strong electric field (i.e., $E \gg T_R$), the effective temperature takes a very generic form

$$T_{\text{eff}} = \frac{E}{\pi} \left[ 1 + \frac{\partial \log \sigma}{\partial \log E} \right]^{1/2}, \quad (18)$$

In obtaining the above relation in (18), we have used the conductivity obtained from (6) and (7). A given physical system obeying such relations perhaps bears the possibility of a holographic dual description.

Let us now briefly discuss some results that can be obtained for particular class of backgrounds. An AdS$_{d+1}$-Schwarzschild background is given by

$$g_{tt} = -\frac{1}{u^2} \left( 1 - \frac{u^d}{u_H^d} \right) = -\frac{R^2}{u^4 g_{uu}}, g_{xx} = g_{ii} = \frac{1}{u^2}, \quad (19)$$

where $R$ is the radius of the AdS-space. The dilaton vanishes for this background. The background temperature is obtained to be: $T = d/(4\pi u_H R)$. Correspondingly the effective temperature is obtained from the general formula in (15) and setting $d = 4$, $m = 1$ and $\Phi = 0$, we recover the results discussed in [22]. In the large and small electric field limits, obtained by setting $T_R \ll E$ and $T_R \gg E$, we get $T_{\text{eff}} \approx E^{1/2} \sqrt{2(m+1)/(2\pi R)}$ and $T_{\text{eff}} \approx T$, irrespective of the value of $d$. These scaling behaviours are simple consequences of the underlying conformal symmetry of the system.

Let us now discuss another special geometry, namely the Lifshitz geometry. This geometry corresponds to a dual field theory with non-relativistic scale invariance under $t \rightarrow \lambda^z t, x \rightarrow \lambda x$, where $z$ is known as the dynamical exponent of the system. Such geometries can be viewed as infrared fixed points that describe low energy quantum criticality, possibly relevant for condensed matter systems. The metric components take the form

$$g_{tt} = -\frac{1}{u^{2z}}, g_{uu} = \frac{1}{u^2}, g_{xx} = g_{ii} = \frac{1}{u^2}, \quad (20)$$

where, for convenience, the radius of the background is set to unity. For such a geometry, the analogue of the relation in (18) takes the following form

$$T_{\text{eff}} = \frac{E(1+z)}{2\pi} \left[ 1 + \frac{\partial \log \sigma}{\partial \log E} \right]^{1/2}, \quad (21)$$

A generalization of the Lifshitz metrics is the so called hyper-scaling violation geometries, which are covariant under scale transformation and is characterized by both a dynamical exponent and a hyperscaling violation exponent. For this class of geometries as well, one can obtain relations similar to the one in (21).

Fluctuation-dissipation relations: One of the profound consequences of the open string equivalence principle is the existence of a fluctuation-dissipation relation involving the effective temperature. In a system at thermal equilibrium, fluctuation-dissipation relations connect the linear relaxation response of the system to a small perturbation around the equilibrium configuration. In such a relation was obtained for the current noise in a $(2+1)$-dimensional theory in a steady-state. Because of the open string equivalence principle, here we argue that all open string degrees of freedom in a holographic theory obey such a relation with respect to $T_{\text{eff}}$. The above claim can be explicitly verified by calculating the boundary Schwingere-Keldysh two-time correlator using the holographic description, as outlined in [22].
Kruskal patch of the corresponding bulk spacetime. For a typical black hole geometry, fluctuation-dissipation relations physically arise from the underlying black hole thermodynamics. For the modes in (9)-(11), this underlying thermodynamics is governed by the open string metric event horizon and thus a corresponding fluctuation-dissipation result is imminent.

**Conclusions:** In this article, we have argued that for the holographic probe systems at NESS the physics remains thermal with an effective temperature. The corresponding particle distribution functions $n_{\epsilon}(T_{\text{eff}})$ at an energy $\epsilon$ are expected to be given by a Bose-Einstein or a Fermi-Dirac statistics involving this effective temperature. The governing principles are the existence of an open string metric event horizon and an open string equivalence principle. The origin of this effective temperature is rooted in the combined effect of thermal fluctuation from the bath and a Schwinger pair production which sets off the NESS. We further note that the thermal nature of a similar system was noticed earlier in [24,25] in describing phase transitions, and in [26], where it was argued that the charge transport properties on the probe can be obtained from a membrane-paradigm-type description on the open string metric event horizon, further adding to the claim of the theme of this article.

In [27], using probe branes rotating along the compact directions, it was argued that there is an effective thermal description on the probe with a Hawking temperature different from the background one. The corresponding effective temperature was realized as an event horizon on the worldvolume of the probe. It is known that for such probe brane systems, T-duality symmetry of string theory transforms a spatial boost of the brane to a gauge field on the worldvolume. Thus, our NESS situation is qualitatively a T-dual description of [27]. It is interesting to note that an underlying thermodynamic principle remains intact in these two seemingly unrelated systems and the intricacies of string theory, such as the T-duality, translates the induced metric event horizon to an open string metric event horizon. This perhaps also insinuates the deep connection between geometry and thermodynamics [28] that may go beyond Einstein gravity. For a recent article involving both an electric field and a rotation on the worldvolume, see [29].

On the other hand, our analysis suggests that for systems out-of-equilibrium and at least in a steady-state, an effective thermal description emerges rather universally. This is evidently true for a probe sector in a large-$N$ gauge theory with a holographic dual, irrespective of any further details of the system. Interestingly, a thermal nature for an inherently nonequilibrium process has also been observed elsewhere: in quantum critical systems (a large-$N$ Bose-Fermi Kondo model), e.g. [30], or in aging glass system [31]. These observations perhaps suggest a more ubiquitous presence of an effective thermodynamical description for systems out-of-equilibrium, even outside the lore of gauge-gravity duality.

By construction, our holographic analysis is applicable for a fundamental probe sector in a large-$N$ SU($N$) gauge theory. More recent progress on holography has made it possible to understand gravitational dual descriptions for $O(N)$-type gauge theories, which was originally proposed in [32]. It is an intriguing question whether such an effective thermodynamics persists for an analogous NESS situation there.

Our discussions have also been limited to the probe limit. Beyond this limit, the probe (fundamental) sector and the bath (adjoint) undergoes heat exchange since $T_{\text{eff}} > T$. Eventually, after the electric field is turned off, they will reach a thermal equilibrium. This should be realized as the open string equivalence principle merging with the closed string equivalence principle and having the same event horizon in the end. It will be very interesting to explicitly construct such a time-dependent geometric example where this physics is manifest, perhaps along the lines of [33].

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It is straightforward to check that any known geometry that has a well-defined holographic correspondence, satisfy this assumption. Relaxing the assumption of isotropy will result in a slightly more involved algebraic expression, which we do not pursue here.

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