Scotogenic $U(1)_\chi$ Dirac Neutrinos

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Abstract

The standard model of quarks and leptons is extended to include the gauge symmetry $U(1)_\chi$ which comes from $SO(10) \rightarrow SU(5) \times U(1)_\chi$. The radiative generation of Dirac neutrino masses through dark matter is discussed in two examples. One allows for light Dirac fermion dark matter. The other allows for self-interacting scalar dark matter with a light scalar mediator which decays only to two neutrinos.
**Introduction**: Whereas neutrinos are usually assumed to be Majorana, there is yet no experimental evidence, i.e. no definitive measurement of a nonzero neutrinoless double beta decay. To make a case for neutrinos to be Dirac, the first is to justify the existence of a right-handed neutrino $\nu_R$, which is not necessary in the standard model (SM) of quarks and leptons. An obvious choice is to extend the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ to the left-right symmetry $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)/2}$. In that case, the $SU(2)_R$ doublet $(\nu, e)_R$ is required, and the charged $W^+_R$ gauge boson is predicted along with a neutral $Z'$ gauge boson.

A more recent choice is to consider $U(1)_\chi$ which comes from $SO(10) \to SU(5) \times U(1)_\chi$, with $SU(5)$ breaking to the SM. Assuming that $U(1)_\chi$ survives to an intermediate scale, the current experimental bound on the mass of $Z_\chi$ being about 4.1 TeV \cite{1,2}, then $\nu_R$ must exist for the cancellation of gauge anomalies. Now $\nu_R$ is a singlet and $W^+_R$ is not predicted. In this context, new insights into dark matter \cite{3,4} and Dirac neutrino masses \cite{5} have emerged. In particular, it helps with the following second issue regarding a Dirac neutrino mass. Since neutrino masses are known to be very small, the corresponding Yukawa couplings linking $\nu_L$ to $\nu_R$ through the SM Higgs boson must be very small. To avoid using such a small coupling, a Dirac seesaw mechanism \cite{6,7} is advocated in Ref. \cite{5}. The alternative is to consider radiative mechanisms, especially through dark matter, called scotogenic from the Greek ‘scotos’ meaning darkness. Whereas the original idea \cite{8} was applied to Majorana neutrinos, one-loop \cite{9,10} and two-loop \cite{11} examples for Dirac neutrinos already exist in the context of the SM. For a generic discussion of Dirac neutrinos, see Ref. \cite{12}, which is patterned after that for Majorana neutrinos \cite{13}. Here two new $U(1)_\chi$ examples are shown. One allows for light Dirac fermion dark matter. The other allows for self-interacting scalar dark matter with a light scalar mediator which decays only to two neutrinos.

**First Scotogenic $U(1)_\chi$ Model**: The particle content follows that of Ref. \cite{5} except for the
addition of $\zeta \sim (1, 15)$ from the 672 of $SO(10)$. This is used to break $U(1)_\chi$ without breaking global lepton number. The fermions are shown in Table 1 and scalars in Table 2.

| Fermion | $SO(10)$ | $SU(5)$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_\chi$ | $Z^A_2$ | $Z^B_2$ | $Z^C_2$ | $Z^D_2$ |
|---------|----------|---------|-----------|-----------|-----------|-------------|---------|---------|---------|---------|
| $d^c$   | 16       | 5*      | 3*        | 1         | 1/3       | 3           | +       | +       | +       | +       |
| $(\nu, e)$ | 16       | 5*      | 1         | 2         | -1/2      | 3           | +       | +       | +       | +       |
| $(u, d)$ | 16       | 10      | 3         | 2         | 1/6       | -1          | +       | +       | +       | +       |
| $u^c$   | 16       | 10      | 3*        | 1         | -2/3      | -1          | +       | +       | +       | +       |
| $e^c$   | 16       | 10      | 1         | 1         | 1         | -1          | +       | +       | +       | +       |
| $\nu^c$ | 16       | 1       | 1         | 1         | 0         | -5          | -       | -       | -       | -       |
| $N$     | 126*     | 1       | 1         | 1         | 0         | 10          | -       | +       | -       | +       |
| $N^c$   | 126      | 1       | 1         | 1         | 0         | -10         | +       | +       | -       | -       |

Table 1: Fermion content of model.

| Scalar  | $SO(10)$ | $SU(5)$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_\chi$ | $Z^A_2$ | $Z^B_2$ | $Z^C_2$ | $Z^D_2$ |
|---------|----------|---------|-----------|-----------|-----------|-------------|---------|---------|---------|---------|
| $(\phi^0_1, \phi^-_1)$ | 10       | 5*      | 1         | 2         | -1/2      | -2          | +       | +       | +       | +       |
| $(\phi^+_2, \phi^0_2)$ | 10       | 5       | 1         | 2         | 1/2       | 2           | +       | +       | +       | +       |
| $(\eta^+, \eta^0)$     | 144      | 5       | 1         | 2         | 1/2       | 7           | +       | +       | -       | -       |
| $\sigma$             | 16       | 1       | 1         | 1         | 0         | -5          | +       | -       | +       | -       |
| $\zeta$              | 672      | 1       | 1         | 1         | 0         | 15          | +       | +       | +       | +       |

Table 2: Scalar content of model.

New fermions $N, N^c$ belonging to $126^*, 126$ respectively are added per family, as well as a Higgs doublet from 144 and a singlet from 16. Note that their $Q_\chi$ charges are fixed by the $SO(10)$ representations from which they come. It should also be clear that incomplete $SO(10)$ and $SU(5)$ multiplets are considered here (which is the case for all realistic grand unified models). Since $\Phi^i_1$ transforms exactly like $\Phi_2$, the linear combination $\Phi = (v_1 \Phi^i_1 + v_2 \Phi_2) / \sqrt{v_1^2 + v_2^2}$ is the analog of the standard-model Higgs doublet, where $\langle \phi_{1,2}^0 \rangle = v_{1,2}$. An important $Z_2$ discrete symmetry is imposed so that $\nu^c$ is odd and all other SM fields are even, preventing thus the tree-level Yukawa coupling $(\nu \phi^0 - e \phi^+) \nu^c$. This $Z_2$ symmetry is respected by all dimension-four terms of the Lagrangian. It will be broken softly by the
dimension-three trilinear term $\mu \sigma \Phi^\dagger \eta$ (in cases B and D) or the $m_N N N^c$ mass term (in cases A and C). This allows the one-loop diagram of Fig. 1 to generate a radiative Dirac neutrino mass. Cases C and D allow the Yukawa coupling $\zeta \nu^c N^c$ which would violate lepton number, hence only cases A and B will be considered. In case A, the quartic scalar term $\zeta \sigma^3$ is allowed. Hence the would-be dark U(1) symmetry is reduced to $Z_3$, i.e. $\omega$ for $\sigma, N^c$ and $\omega^2$ for $\eta, N$, where $\omega = \exp(2\pi i/3)$. In case B, it is forbidden, so the model possesses a dark U(1) symmetry, i.e. $D = 1$ for $\sigma, N^c$ and $-1$ for $\eta, N$. In either case, there is still a conserved lepton symmetry, i.e. $L = 1$ for $\nu, N$ and $L = -1$ for $\nu^c, N^c$. The idea of using a scalar which breaks a gauge U(1) symmetry by 3 units, so that a global U(1) symmetry remains was first discussed in Ref. [14] and then used for $B - L$ in Ref. [15]. There have been also studies [16, 17, 18], using dimension-five operators, i.e. $(\nu \phi^0 - e \phi^+)^c S/\Lambda$ where $\nu^c$ carries a new charge which forbids the dimension-four term but the singlet scalar $S$ carries a compensating charge which allows the dimension-five term.

To compute the neutrino mass of Fig. 1, note first that it is equivalent to the difference of the exchanges of two scalar mass eigenstates

$$\chi_1 = \cos \theta \sigma - \sin \theta \bar{\eta}^0, \quad \chi_2 = \sin \theta \sigma + \cos \theta \bar{\eta}^0,$$

where $\theta$ is the mixing angle due to the $\bar{\phi}^0 \eta^0 \sigma$ term. Let the $\nu_i N^c \eta^0$ Yukawa coupling be $h_{ik}^\nu$.

![Figure 1: First one-loop diagram for scotogenic $U(1)_\chi$ Dirac neutrino mass.](image)
and the $\nu_j^c N_k \sigma$ Yukawa coupling be $h_{jk}^R$, then the Dirac neutrino mass matrix is given by

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik}^L h_{jk}^R \sin 2\theta M_k}{16\pi^2} \left[ \frac{m_2^2}{m_2^2 - M_k^2} \ln \frac{m_2^2}{M_k^2} - \frac{m_1^2}{m_1^2 - M_k^2} \ln \frac{m_1^2}{M_k^2} \right],$$  \hspace{1cm} (2)

where $m_{1,2}$ are the masses of $\chi_{1,2}$ and $M_k$ is the mass of $N_k$. If $|m_2^2 - m_1^2| \ll m_2^2 + m_1^2 = 2m_0^2 \ll M_k^2$, then

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{\sin 2\theta (m_2^2 - m_1^2)}{16\pi^2 M_k} h_{ik}^L h_{jk}^R \left[ \ln \frac{M_k^2}{m_0^2} - 1 \right].$$  \hspace{1cm} (3)

This expression is of the radiative seesaw form. On the other hand, if $M_k \ll m_{1,2}$, then [19]

$$(\mathcal{M}_\nu)_{ij} = \frac{\sin 2\theta \ln(m_2^2/m_1^2)}{16\pi^2} \sum_k h_{ik}^L h_{jk}^R M_k.$$  \hspace{1cm} (4)

This is no longer a seesaw formula. It shows that the three Dirac neutrinos $\nu$ have masses which are linear functions of the three light dark Dirac fermions $N$. This interesting possibility opens up the parameter space in the search for fermion dark matter with masses less than a few GeV.

Consider the annihilation of $N \bar{N} \rightarrow \nu \bar{\nu}$ through $\chi_1$ exchange, assuming that $\theta$ is very small in Eq. (1). The cross section $\times$ relative velocity is

$$\sigma \times v_{\text{rel}} = \frac{(h^R)^4}{32\pi^2} \frac{m_N^2}{(m_1^2 + m_2^2)^2}.$$  \hspace{1cm} (5)

As an example, let $m_N = 6$ GeV, $m_1 = 100$ GeV, $h^R = 0.92$, then this is about 1 pb, which is the correct value for $N$ to have the observed dark-matter relic abundance of the Universe, i.e. $\Omega h^2 = 0.12$. In Eq. (4), let $\sin 2\theta = 10^{-4}$, $h^L = 10^{-4}$, and $m_2 = 115$ GeV, then $m_\nu = 0.1$ eV, as desired.

At the mass of 6 GeV, the constraint on the elastic scattering cross section of $N$ off nuclei is about $2.5 \times 10^{-44}$ cm$^2$ from the latest XENON result [37]. This puts a lower limit on the mass of $Z_\chi$, i.e.

$$\sigma_0 = \frac{4m_N^2}{\pi} \left[ Z f_P + (A - Z) f_N \right]^2 A \leq 2.5 \times 10^{-8} \text{ pb},$$  \hspace{1cm} (6)
where
\[ f_P = g_{Z_\chi}^2 N_V (2u_V + d_V) / M_{Z_\chi}^2, \quad f_N = g_{Z_\chi}^2 N_V (u_V + 2d_V) / M_{Z_\chi}^2, \]
(7)
and \( Z = 54, A = 131 \) for xenon. In \( U(1)_\chi \), the vector couplings are
\[ N_V = \sqrt{5} / 2, \quad u_V = 0, \quad d_V = -1 / \sqrt{10}. \]
(8)
Using \( \alpha_\chi = g_{Z_\chi}^2 / 4\pi = 0.0154 \) from Ref. [3], the bound \( M_{Z_\chi} > 4.5 \) TeV is obtained.

**Second Scotogenic \( U(1)_\chi \) Model**: Using two new fermion singlets and one fermion doublet, with a different \( Z_2 \), another one-loop diagram is obtained in Fig. 2. The relevant particles are shown in Table 3. Again, the \( Z_2 \) symmetry forbids the would-be tree-level Yukawa coupling

![Figure 2: Second one-loop diagram for scotogenic \( U(1)_\chi \) Dirac neutrino mass.](image)

| particle | \( SO(10) \) | \( SU(5) \) | \( SU(3)_C \) | \( SU(2)_L \) | \( U(1)_Y \) | \( U(1)_\chi \) | \( Z_2 \) | \( Z_4^L \) | \( Z_2^{(D)} \) |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( (\nu, e) \) | 16 | 5* | 1 | 2 | -1/2 | 3 | + | i | + |
| \( \nu^c \) | 16 | 1 | 1 | 1 | 0 | -5 | - | -i | + |
| \( (E^+, E^0) \) | 10 | 5 | 1 | 2 | 1/2 | 2 | - | 1 | - |
| \( S_1 \) | 45 | 24 | 1 | 1 | 0 | 0 | - | 1 | - |
| \( S_2 \) | 45 | 24 | 1 | 1 | 0 | 0 | + | 1 | - |
| \( \sigma \) | 16 | 1 | 1 | 1 | 0 | -5 | - | -i | - |
| \( \zeta' \) | 126 | 1 | 1 | 1 | 0 | -10 | + | -1 | + |
| \( \zeta'' \) | 2772 | 1 | 1 | 1 | 0 | -20 | + | 1 | + |

Table 3: Fermion and scalar content of model.

\( \phi^0 \nu \nu^c \), but is softly broken by the \( S_1 S_2 \) mass term, whereas \( S_1^2 \) and \( S_2^2 \) are allowed Majorana
mass terms. The $U(1)_\chi$ gauge symmetry is broken by $\zeta''$ and since it couples to $(\zeta')^2$ and $\zeta'$ couples to $\sigma^2$, the residual symmetry of this model is $Z_4$ \cite{20, 21, 22, 23, 24}, which enforces the existence of Dirac neutrinos, and the dark symmetry is $Z_2$, i.e. $(-1)^{Q_\chi+2j}$ as pointed out in Refs. \cite{3, 25}, as shown in Table 3.

In this second model, the scalar $\sigma$ is a pure singlet, whereas in the first model, it must mix with $\eta^0$ which is part of a doublet. Because of the $\zeta'\sigma\sigma$ interaction, it is a self-interacting dark-matter candidate \cite{26} which can explain the flatness of the core density profile of dwarf galaxies \cite{27} and other related astrophysical phenomena. The light scalar mediator $\zeta'$ decays dominantly to $\nu^c\nu^c$ so it does not disturb \cite{28} the cosmic microwave background (CMB) \cite{29}, thus avoiding the severe constraint \cite{30} due to the enhanced Sommerfeld production of $\zeta'$ at late times if it decays to electrons and photons, as in most proposed models. This problem is solved if the light mediator is stable \cite{31, 32, 33} or if it decays into $\nu\nu$ through a pseudo-Majoron in the singlet-triplet model of neutrino mass \cite{34}. A much more natural solution is for it to decay into $\nu^c\nu^c$ as first pointed out in the prototype model of Ref. \cite{35} and elaborated in Refs. \cite{3, 5}. Here it is shown how it may arise in the scotogenic Dirac neutrino context using $U(1)_\chi$. The connection of lepton parity to simple models of dark matter was first pointed out in Ref. \cite{36}. To obtain three massive Dirac neutrinos, there are presumably also three $\sigma$’s. Only the lightest is stable, the others would decay into the lightest plus $\zeta'$ which then decays into two neutrinos. Typical mass ranges for $\sigma$ and $\zeta'$ are

$$100 < m_\sigma < 200 \text{ GeV}, \quad 10 < m_{\zeta'} < 100 \text{ MeV},$$

as shown in Ref. \cite{35}. Lastly, the conjugate fermions to $(E^+, E^0)$ are also assumed, to allow them to have invariant Dirac masses and to cancel the gauge $U(1)_\chi$ anomalies.

**Concluding Remarks :** The $U(1)_\chi$ gauge symmetry and a suitably chosen particle content with a softly broken $Z_2$ symmetry are the ingredients for the radiative generation of Dirac neutrino masses through dark matter. Both the symmetries for maintaining the Dirac nature
of neutrinos and the stability of dark matter are consequences. In the first example, because the breaking of $U(1)_\chi$ is by 3 units of lepton number through the relationship

$$15(B - L) = 12Y - 3Q_\chi,$$

(10)

global $U(1)$ lepton number remains, whereas the dark symmetry is either $Z_3$ or $U(1)$. The dark-matter candidate is a Dirac fermion which may be light. In the second example, the lepton symmetry is $Z_4$ and the dark parity is $(-1)^{Q_\chi+2j}$. The dark-matter candidate is a complex scalar which has self-interactions through a light scalar mediator which decays only into two neutrinos. Both cases are interesting variations of basic dark matter, and will face further scrutiny in future experiments.

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