Large Extra Dimensions, Sterile neutrinos and Solar Neutrino Data

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Solar, atmospheric and LSND neutrino oscillation results require a light sterile neutrino, $\nu_B$, which can exist in the bulk of extra dimensions. Solar $\nu_e$, confined to the brane, can oscillate in the vacuum to the zero mode of $\nu_B$ and via successive MSW transitions to Kaluza-Klein states of $\nu_B$. This new way to fit solar data is provided by both low and intermediate string scale models. From average rates seen in the three types of solar experiments, the Super-Kamiokande spectrum is predicted with 73\% probability, but dips characteristic of the 0.06 mm extra dimension should be seen in the SNO spectrum.

The four-neutrino scheme in which the solar $\nu_e$ deficit is explained by $\nu_e \rightarrow \nu_s$ (where $\nu_s$ is a sterile neutrino), the atmospheric $\nu\mu/\nu_e$ anomaly is attributed to $\nu_\mu \rightarrow \nu_\tau$, and the heavier $\nu_\mu$ and $\nu_\tau$ share the role of hot dark matter was originally proposed [1] in order to explain those three phenomena. Later the LSND experiment [2], which observed $\bar{\nu}_\mu \rightarrow \nu_e$, provided a measure of the mass difference between the nearly degenerate $\nu_\mu - \nu_s$ and $\nu_\mu - \nu_\tau$ pairs and required the three mass differences that were already present in that neutrino scheme. Exactly this same pattern of neutrino masses and mixings appears necessary to allow production of heavy elements (A > 100) by type II supernovae [3]. While qualitatively this neutrino scheme seems to explain all existing neutrino phenomena, solar neutrino observations are now sufficiently constraining that the small-angle MSW $\nu_e \rightarrow \nu_s$ explanation appears to be in some difficulty [4]. Although providing better fits to the solar data, even active-active transitions in a three-neutrino scheme do not give a quantitatively good explanation of those data. In this Letter we point out that there is a way to achieve an excellent fit and rescue the apparently needed four-neutrino model on the brand. The $\nu_B(x,y)$ is assumed to couple to the lepton doublet of the standard model $L$ and of course have a five-dimensional kinetic energy term. After electroweak symmetry breaking at scale $v_{ew}$, the $\nu_\mu - \nu_B,x$ coupling leads to $m_b = \frac{\frac{M_{pl}^2 \alpha_{em}^2}{2 y_{ew}}}{M_{pl}^4} \sim 10^{-5}$ eV, where $M_{pl}$ is the Planck mass and $h$ is the Yukawa coupling. Note that this suppression is independent of the number and radius hierarchy of the extra dimensions, provided that $\nu_B$ propagates in the whole bulk. Even if the bulk is six or higher dimensional, there is only one mm-scale dimension. The smaller dimensions will contribute only to the relationship between $M_{pl}$ and $M_*$, but their KK excitations will be very heavy and decouple from the neutrino spectrum, making the analysis as in five dimensions.

The direct Dirac or Majorana mass terms for the bulk neutrino can be forbidden by an appropriate choice of geometry and dimension of the bulk in which the $\nu_B$ resides, making an ultralight $\nu_s$ natural. For instance, in 5 dimensions the $Z_2$ orbifold symmetry under which $y \rightarrow -y$ combined with $B-L$ symmetry guarantee this for the conventional definition of charge conjugation.

In order to fit neutrino data, one needs to include new physics in the brane that will generate a Majorana mass matrix for the three standard model neutrinos of the form...
\( \delta_{ab} \) (where \( a, b = e, \mu, \tau \)). For \( \delta_{\mu\tau} \) much bigger than the other elements, the \( \nu_{e,\tau} \) in effect decouple from the \( \nu_{e,s} \) and do not affect the mixing between the bulk neutrino modes and the \( \nu_e \). Further, this leads to maximal mixing in the \( \mu-\tau \) sector, as is needed to understand the atmospheric neutrino data. If \( \delta_{\mu\tau} \sim \text{eV} \), then this provides an explanation of the LSND observations. One way to generate this pattern is to consider an \( L_e + L_\mu - L_\tau \) symmetric extension of the standard model with additional doubly-charged singlet \( (Y = 4) \) and \( Y = 2 \) triplet scalar fields. In the rest of the Letter, we focus only on the \( \nu_e - \nu_s \) sector and how we fit the solar neutrino data.

The mass matrix for the \( \nu_e \), \( \nu_s \) sector can be written

\[
\begin{pmatrix}
\nu_e & \nu_{0B} \\
\nu_{0B}^* & \nu_{B,0}
\end{pmatrix}
\begin{pmatrix}
\delta_{ee} & m & \sqrt{2} m \\
m & m & 0 \\
\sqrt{2} m & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
\nu_{0B} \\
\nu_{B,0}
\end{pmatrix},
\]  

where \( \nu_{B,0}^* \) represents the KK excitations, and the off-diagonal term \( \sqrt{2} m \) is actually an infinite row vector of the form \( \frac{1}{\sqrt{2} m} (1, 1, \ldots) \). The operator \( \partial_5 \) stands for the diagonal and infinite KK mass matrix whose \( n \)-th entry is given by \( n/R \). Using this short-hand notation makes it easier to calculate the exact eigenvalues \( m_n \) and the eigenstates of this mass matrix,

\[
m_n = \delta_{ee} + \pi m^2 R \cot((\pi m_n/R)).
\]  

The equation for eigenstates is

\[
N_n \nu_n = \nu_e + \frac{m}{m_n} v_{0B} + \frac{\sqrt{2} m (m_n v_{B,0} - \partial_5 v_{B,0}^*)}{m_n^2 - \partial_5^2},
\]  

where the sum over the KK modes in the last term is implicit. \( N_n \) is the normalization factor given by

\[
N_n^2 = 1 + m^2 \pi^2 R^2 + \frac{(m_n - \delta_{ee})^2}{m^2}.
\]  

Note that in the limit of \( \delta_{ee} = 0 \), the \( \nu_e \) and \( \nu_{B,0} \) are two, two-component spinors that form a Dirac fermion with mass \( m \). The KK modes come in pairs of mass \( m_n = \pm kn_0 R \), with \( kn_0 \) a positive integer, and they couple to the \( \nu_e \) approximately as \( m_n R \). Once we include the effect of \( \delta_{ee} \neq 0 \), they become Majorana fermions with masses given by \( m_1 \approx +\delta_{ee}/2 + m \) and \( m_2 \approx +\delta_{ee}/2 - m \), and they are maximally mixed; i.e., the two mass eigenstates are \( \nu_{1,2} \approx \nu_e \pm \nu_{B,0}/\sqrt{2} \). Thus the now produced in a weak interaction process evolves, it oscillates to the \( \nu_{B,0} \) state with an oscillation length \( D \approx E/(2m_\delta) \), which for natural values of \( m, \delta_{ee} \) gives \( D \) of order of the Sun-Earth distance so that our model leads to vacuum oscillation ("VO") of the solar neutrinos. Furthermore, since the \( \nu_e \) also mixes with the KK modes of the bulk neutrinos with a \( \delta m^2 \sim 10^{-5} \text{ eV}^2 \), this brings in the MSW resonance transition of \( \nu_e \) to \( \nu_{B,0,KK} \) modes at higher energies.

The second way to achieve the same phenomenon is to use a much higher string scale associated with local \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) symmetry of the left-right symmetric model in the brane, coupled to a single \( \nu_B \) in the large fifth dimensional bulk. Using the usual fermion content of the left-right models and breaking the right-handed symmetry by Higgs doublets, one obtains the seesaw type matrix in the presence of the infinite tower of KK modes. The heavy right-handed neutrino now decouples, leading to the following neutrino mass matrix in the one generation case (the basis is \( \nu_e v_{0B} v_{B,0}^* - v_{B,0}^* \))

\[
\frac{1}{M} \begin{pmatrix} m^2 + \Delta & m\alpha \sqrt{2}\alpha^2 & 0 \\ m\alpha \sqrt{2}\alpha^2 & \sqrt{2}\alpha^2 & 0 \end{pmatrix},
\]

where \( \alpha \simeq \frac{h_B M_{ rupture}}{M_{KK}^2}, m = h_\text{t} v_w, M = v_\text{t}^2/\lambda_s, \) and \( v_R \) is the scale of \( SU(2)_R \) breaking. \( \Delta \) denotes the radiative correction induced due to the extrapolation from the \( v_R \) scale to the weak scale. The lowest eigenvalue of this mass matrix is \( \sim \frac{\alpha^2}{M(\alpha^2 + \alpha^2)} \), and the next lowest one is \( \frac{m^2 + \alpha^2}{M} \). For \( m, \alpha \sim 1 - 5 \text{ MeV} \) (similar to the first generation fermion mass) and \( M \approx 10^9 \text{ GeV} \), we get this eigenvalue to be of order \( 10^{-6} - 2.5 \times 10^{-5} \text{ eV} \). Its square is therefore in the range where the VO solution to the solar neutrino puzzle can be applied. Also for \( m \approx \alpha, \) the mixing angle between the zero eigenvalue mode and this mode is maximal. Thus this model has properties similar to the first model for neutrinos, and below we carry out our fit to solar neutrino data using the latter.

For propagation in solar matter, the eigenvectors and eigenvalues can be found by replacing the squared mass matrix, \( M^2 \), for the neutrinos, with \( M^2 + H \), where \( H = 2E\rho_e \) when acting on \( \nu_e \) (where \( \rho_e = \sqrt{3} G_{F}(n_e - n_n)/2) \), and zero on sterile neutrinos. Defining

\[
w = \frac{E\rho_e}{m_n \delta_{ee}} + \sqrt{1 + \left( \frac{E\rho_e}{m_n \delta_{ee}} \right)^2},
\]

\((w = 1 \text{ in vacuum}) \) Eq. 3 becomes

\[
m_n = w\delta_{ee} + \pi m^2 R \cot((\pi m_n/R)).
\]
first node of the survival probability, $P_{ee}$, to suppress the $^7$Be. Going up in energy toward $^8$B neutrinos, $P_{ee}$, which in the VO case would have risen to very near one, is reduced by the small-angle MSW transitions to the different KK excitations of the bulk sterile neutrinos, as is clear from Fig. 1. This is a new way to fit the solar neutrino data in models with large extra dimensions.

To do the fit, we studied the time evolution of the $\nu_e$ state with a program supplied by W. Haxton [10], but updated to use a recent solar model [11] and modified to do all neutrino transport within the Sun numerically, using no adiabatic approximation. Changes were also necessary for oscillations into sterile neutrinos and to generalize beyond the two-neutrino model, for up to 14 neutrinos contribute for the solutions we considered.

For comparison with experimental results, tables of detector sensitivity for the Chlorine and Gallium experiments were taken from Bahcall’s web site [11]. The Neutrino Energy (MeV) is 0.451 ± 0.016. The best fits were with $R \approx 58 \mu m$, $mR$ around 0.0094, and $\delta_{ee} \sim 0.84 \times 10^{-7}$ eV$^2$, corresponding to $4m^2 \sim 0.53 \times 10^{-11}$ eV$^2$, giving average $P_{ee}$ for Chlorine, Gallium, and water of 0.383, 0.533, and 0.450, respectively, and the $P_{ee}$ energy dependence shown in Fig. 1. For two-neutrino oscillations, the mixing angle is $\sin^2 2\theta$, whereas here the coupling between $\nu_e$ and the first KK excitation replaces $\sin^2 2\theta$ by $4m^2 R^2 = 0.00035$.

Vacuum oscillations between the lowest two mass eigenstates nearly eliminate electron neutrinos with energies of 0.63 MeV/(2n+1) for $n = 0, 1, 2, \ldots$. Thus Fig. 1 shows nearly zero $P_{ee}$ near 0.63 MeV, partly eliminating the $^7$Be contribution at 0.862 MeV, and giving a dip at the lowest neutrino energy. MSW resonances with mass pairs of higher KK states start causing the third and fourth eigenstates to be significantly occupied above $\sim 0.8$ MeV, the fifth and sixth eigenstates above $\sim 3.7$ MeV, the 7'th and 8'th above $\sim 8.6$ MeV, and the 9'th and 10'th above $\sim 15.2$ MeV. Fig. 1 shows dips in $P_{ee}$ just above these energy thresholds.

The expected energy dependence of $P_{ee}$ is compared with Super-K data [1] in Fig. 2. The uncertainties are statistical only. The parameters used in making Fig. 2 were chosen to provide a good fit to the total rates only; they were not adjusted to fit this spectrum. Combining spectrum data with rates using the method described in Ref. [1] gives $\chi^2 = 14.0$ for the spectrum predicted from the fit to total rates. With 18 degrees of freedom, the probability of $\chi^2 > 14.0$ is 73%. A fit with $\delta_{ee}$ constrained to be very small to eliminate vacuum oscillations increased the best fit $\chi^2$ from 3.4 to 4.4. The same parameters then used with the Super-K spectrum gave $\chi^2 = 18.7$ (probability 41%). This is comparable to $\chi^2 = 19.0$ for an energy independent spectrum.

![Energy dependence of the $\nu_e$ survival probability](image1)

FIG. 1. Energy dependence of the $\nu_e$ survival probability when $R = 58 \mu m$, $mR = 0.0094$, and $\delta_{ee} = 0.84 \times 10^{-7}$ eV$^2$. The dot-dashed part of the curve assumes the radial dependence in the Sun for neutrinos from the pp reaction, the solid part assumes $^{15}$O radial dependence, and the dashed part assumes $^8$B radial dependence.

![Electron Neutrino Survival Probability](image2)

FIG. 2. Super-Kamiokande 1258-day measured [1] energy spectrum (error bars) and predicted (curve) using the parameters of Fig. 1 without fitting these data.
The seasonal effect was computed for a few points on the Earth’s orbit. If $r$ is the distance between the Earth and the Sun, \[ \theta = 1 + \epsilon \cos (\theta - \theta_0), \] where $r_0$ is one astronomical unit, $\epsilon = 0.0167$ is the orbital eccentricity, and $\theta - \theta_0 \approx 2\pi (t - t_0)$, with $t$ in years and $t_0 = \text{January 2}, 4h 52m$. Table I shows very small seasonal variation.

Not only is the seasonal effect very difficult to observe, but also the smallness of the mixing angle makes day-night effects hard to measure. In addition, the mass of the electron neutrino, which consists mainly of eigenstates of mass $3 \times 10^{-5}$ eV, is undetectable directly or by neutrinoless double beta decay. The latter process measures an effective neutrino mass, but even contributions to that from the $\nu_\mu$ and $\nu_\tau$ must be so small as to make detection very unlikely, although other conjectured processes unrelated to neutrino mass could cause this decay.

On the other hand, the dimension size of 0.06 mm, suggested by the average rates of the three types of solar experiments should be detectable by gravity experiments. The present best limit on such effects is less than a factor of four from that value.

The large size of the extra dimension raises issues about cosmological and supernova limits from the effects of high KK states of both sterile neutrinos and gravitons, despite uncertainties in the understanding of the complex regimes of the early universe and the supernova core. For sterile neutrino limits, this phenomenology is aided because there is a single KK tower based on an exceedingly small mass, the VO $\Delta m^2$ is an order of magnitude smaller than usual, and for MSW the equivalent $\sin^2 \theta$ is more than an order of magnitude smaller than for standard fits. For the global B-L model, the universe re-heating temperature could be very low ($> 0.7$ MeV works cosmologically), reducing production of high KK states. The high string scale of the local B-L model would appear to avoid all these constraints, however. More complete investigation of these constraints may enable choosing between these quite different models, both of which provide this new way to explain solar, atmospheric, and LSND oscillation data and may give the first evidence for an extra large dimension.

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TABLE I. Predicted seasonal variations in $\nu_e$ fluxes, excluding the $1/r^2$ variation. The model assumed $\mu_0 = 0.32 \times 10^{-2}$ eV, $m_0 = 0.34 \times 10^{-4}$ eV, and $\delta_{ee} = 0.78 \times 10^{-7}$ eV.

| $\theta - \theta_0$ | Chlorine | Gallium | Water |
|---------------------|----------|---------|-------|
| 0 (January 2)       | 0.3787   | 0.5144  | 0.4635|
| $\pm \pi/2$         | 0.3762   | 0.5121  | 0.4633|
| $\pi$ (July 4)      | 0.3747   | 0.5082  | 0.4631|

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