Dark Matter Candidate from Conformality

P.H. Frampton
Institute of Field Physics, Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599-3255, USA

Abstract

Abelian quiver gauge theories provide candidates for the conformality approach to physics beyond the standard model which possess novel cancellation mechanisms for quadratic divergences. A $Z_2$ symmetry (R parity) can be imposed and leads naturally to a dark matter candidate which is the Lightest Conformality Particle (LCP), a neutral spin-1/2 state with weak interaction annihilation cross section, mass in the 100 GeV region and relic density of non-baryonic dark matter $\Omega_{dm}$ which can be consistent with the observed value $\Omega_{dm} \simeq 0.24$. 
Introduction

One approach to the hierarchy, or naturalness, problem is to postulate conformality, four-dimensional conformal invariance at high energy, for the non gravitational extension of the standard model. The conformality approach suggested [1] in 1998 has made considerable progress. Models which contain the standard model fields have been constructed [4] and a model which grand unifies at about 4 TeV [5] has been examined.

Such models are inspired by the AdS/CFT correspondence [7, 8] specifically based on compactification of the IIB superstring on the abelian orbifold $AdS_5 \times S^5 / \mathbb{Z}_n$ with $N$ coalescing parallel D3 branes. A model is specified by $N$ and by the embedding $\mathbb{Z}_n \subset SU(4)$ which is characterized by integers $A_m (m = 1, 2, 3, 4)$ which specify how the $4$ of $SU(4)$ transforms under $\mathbb{Z}_n$. Only three of the $A_m$ are independent because of the $SU(4)$ requirement that $\Sigma_m A_m = 0 \pmod{n}$. The number of vanishing $A_m$ is the number $\mathcal{N}$ of surviving supersymmetries. Here we focus on the $\mathcal{N} = 0$ case.

In [9], the original speculation [1] that such models may be conformal has been refined to exclude models which contain scalar fields transforming as adjoint representations because only if all scalars are in bifundamentals are there chiral fermions and, also only if all scalars are in bifundamentals, the one-loop quadratic divergences cancel in the scalar propagator. I regard it as encouraging that these two desirable properties select the same subset of models.

Another phenomenological encouragement stems from the observation [3] that the standard model representations for the chiral fermions can all be accommodated in bifundamentals of $SU(3)^3$ and can appear naturally in the conformality approach.

In the present article I address the issue of dark matter. From studies of galactic rotation curves, large scale structure, cosmic microwave background, high redshift supernovae and other observational cosmology there is strong evidence of the need for non-baryonic dark matter. One of the most recent estimates is from WMAP3 [10] which finds a break down of the overall energy density of 4% baryons, 72% dark energy and 24% non-baryonic dark matter.

Here I define a $\mathbb{Z}_2$ symmetry for the conformality theory in the following subsection. I then show that it leads to an Lightest Conformality Particle (LCP) which is an attractive candidate for the dark matter particle.
Definition of a $\mathbb{Z}_2$ symmetry

In the quiver gauge theories, the gauge group, for abelian orbifold $AdS_5 \times S^5 / \mathbb{Z}_n$ is $U(N)^n$. In phenomenological application $N = 3$ and $n$ reduces eventually after symmetry breaking to $n = 3$ as in trinification. The chiral fermions are then in the representation of $SU(3)^3$:

\[(3, 3^*, 1) + (3^*, 1, 3) + (1, 3, 3^*)\]  \hspace{1cm} (1)

This is as in the 27 of $E_6$ where the particles break down into the following representations of the $SU(3) \times SU(2) \times U(1)$ standard model group:

\[Q, \quad u^c, \quad d^c, \quad L, \quad e^c, \quad N^c\]  \hspace{1cm} (2)

transforming as

\[(3, 2), \quad (3^*, 1), \quad (3^*, 1), \quad (1, 2), \quad (1, 1), \quad (1, 1)\]  \hspace{1cm} (3)

in a 16 of the $SO(10)$ subgroup. In addition there are the states

\[h, \quad h^*, \quad E, \quad E^*\]  \hspace{1cm} (4)

transforming as

\[(3, 1), \quad (3^*, 1), \quad (2, 1), \quad (2, 1)\]  \hspace{1cm} (5)

in a 10 of $SO(10)$ and finally

\[S\]  \hspace{1cm} (6)

transforming as the singlet

\[(1, 1)\]  \hspace{1cm} (7)

It is natural to define a $\mathbb{Z}_2$ symmetry $R$ which commutes with the $SO(10)$ subgroup of $E_6 \to O(10) \times U(1)$ such that $R = +1$ for the first 16 of states. Then it is mandated that $R = -1$ for the 10 and 1 of SO(10) because the following Yukawa couplings must be present to generate mass for the fermions:

\[16_f 16_f 10_s, \quad 16_f 10_f 16_s, \quad 10_f 10_f 1_s, \quad 10_f 1_f 1_s, \quad 1_f 1_f 1_s\]  \hspace{1cm} (8)

which require the $R$ assignments for the scalars $R = +1$ for $10_s, 1_s$ and $R = -1$ for $16_s$. In $E(6)$ various possibilities for R parity including this one were analysed in [11].

The LCP is the lightest linear combination of the three neutral components of $E$, $E^*$ and $S$. It is expected to have mass $\sim 1$ TeV and is a WIMP candidate for dark matter.
Annihilation Cross-Section

The LCP act as cold dark matter WIMPs, and the calculation of the resultant energy density follows a well-known path. Here I follow the procedure in [12].

The LCP decouple at temperature $T_*$, considerably less than their mass $M_{LCP}$; I define $x_* = M_{LCP}/T_*$. Let the annihilation cross-section of the LCP at decoupling be $\sigma_*$. Then the dark matter density $\Omega_{dm}$, relative to the critical density, is estimated as

$$\Omega_{dm} h^2_{75} = \frac{\tilde{g}_*^{1/2}}{g_*} x_*^{3/2} \left( \frac{3 \times 10^{-38} \text{cm}^2}{\sigma_*} \right)$$  \hspace{1cm} (9)

where $h_{75}$ is the Hubble constant in units of $75\text{km/s/Mpc}$. $g_* = (g_b + \frac{7}{8} g_f)$ is the effective number of degrees of freedom (d.o.f.) at freeze-out for all particles which later convert their energy into photons; and $\tilde{g}_*$ is the number of d.o.f. which are relativistic at $T_*$. 

Thus, to estimate the non-baryonic dark matter density arising from LCPs, I need estimates of five quantities occurring in Eq.(9): $h_{75}, \tilde{g}_*, g_*, x_*$ and $\sigma_*$, and to this I now turn.

I start with $h_{75}$ where the central value from WMAP3 [10] is $H_0 = 72\text{km/s/Mpc}$ and so a good estimate of $h_{75}$ is $h_{75} = 72/75 = 0.96$.

For the energy ranges I consider, $\tilde{g}_* = g_*$ and depends on the freeze-out temperature $T_*$. We consider masses in the range $30\text{GeV} \leq M_{LCP} \leq 2\text{TeV}$. Since $x_* = M_{LCP}/T_*$ is relatively insensitive to $M_{LCP}$, as we shall see shortly, always within the values $20 \leq x_* \leq 30$, the freeze-out temperatures of relevance will be in the range $1\text{GeV} \leq T_* \leq 100\text{GeV}$.

For these $T_*$ we compute:

For $100\text{GeV} \geq T_* \geq 10\text{GeV}$:

$$g_* = 86.25$$  \hspace{1cm} (10)

For $10\text{GeV} \geq T_* \geq 3\text{GeV}$:

$$g_* = 75.75$$  \hspace{1cm} (11)

For $3\text{GeV} \geq T_* \geq 1\text{GeV}$:

$$g_* = 61.75$$  \hspace{1cm} (12)
The value of $x_*$ may be estimated using the formula [13]

$$X = 0.038 \sqrt{g_*} M_{\text{Planck}}^2 M_{\text{LCP}} \sigma_*$$ (13)

$$x_* = \ln X - \frac{1}{2} \ln \ln X$$ (14)

I have already estimated $g_*$. I use $M_{\text{Planck}} = 10^{19} GeV$.

The annihilation cross section $\sigma_*$ for LCPs at freeze-out may be estimated using analogs of the Feynman graphs used in [14]. A naive estimate of $\sigma_*$ follows from the dimensional formula [15] $\sigma_* \sim G_F^2 T_*^2$, but we shall use a more detailed calculation, see Eq. (15) below. From Eq. (9) and the estimates for $h_{75}, g_*, \tilde{g}_*, x_*$ already given, the cross-section must satisfy $\sigma_* \geq 3 \times 10^{-35} \text{cm}^2$ with the lower bound saturating $\Omega_{dm}$ and a smaller cross-section being unacceptable because it leads to too much dark matter. Empirical bounds [17] require $M_{\text{LCP}} \geq 43.1 \text{GeV}$. The allowed range is generically [17–21] $43.1 \text{GeV} \leq M_{\text{LCP}} \leq 1 \text{TeV}$ (see below).

One important contributing Feynman graph is $Z$ exchange in the direct channel which gives, by itself, the cross-section

$$\sigma_*(X \bar{X} \to f \bar{f}) = \frac{1}{128 \pi M_X^2} [\alpha^2 + \beta^2]^2 \left[ \frac{g_2(M_Z)^4}{16 \cos^4 \theta_W(M_Z)} \frac{M_f^2 M_{\chi}^2}{M_Z^4} \right]$$ (15)

in which $\alpha, \beta$ are defined by

$$\Phi_{\text{LCP}} = \alpha E^0 + \beta \bar{E}^0 + \gamma S^0$$ (16)

so that $\alpha, \beta$ are coefficients of doublets and $\gamma$ is coefficient of a singlet.

Let the mass of the fermion $f$ be $M_f = f_{10} \times 10 \text{GeV}$. Then using $\alpha_2(M_Z) = g_2(M_Z)^2/4\pi = 0.0339$, $\sin^2 \theta_W(M_Z) = 0.231$, $M_Z = 91.19 \text{GeV}$, $1(\text{GeV})^{-2} \equiv 3.894 \times 10^{-28} \text{cm}^2$, leads, independently of $M_X = M_{\text{LCP}}$, to $\sigma_*(X \bar{X} \to f \bar{f}) = 2.68 \times 10^{-38} [\alpha^2 + \beta^2]^2 \times (f_{10})^2 \text{ cm}^2$.

For the top quark $f \equiv t$ we find for $f_{10} = 17.2$ that $\sigma_* = 6.61 \times 10^{-36} [\alpha^2 + \beta^2]^2 \text{cm}^2$.

To generate all the dark matter we require

$$f_{10}[\alpha^2 + \beta^2] \leq 36.1$$ (17)

which suggests for reasonable values (not very close to pure singlet) $M_{\text{LCP}} < 1 \text{TeV}$.
Fermion mass hierarchy

In the conformality approach, the Yukawa couplings at the conformal scale, usually \( \sim 4 \text{ TeV} \), are of order one. Thus, when the electroweak \( SU(2) \times U(1) \) is broken it is natural that all quarks and charged leptons would acquire mass comparable to the weak scale. Although this is valid for the top quark, all the other fermion masses in the standard model are smaller; this is the fermion hierarchy problem which also affects estimation of the LCP mass.

Conformality does not predict this fermion hierarchy but can accommodate it by adding soft mass terms after breaking \( U(3)^n \to U(3)^3 \to SU(3) \times SU(2) \times U(1) \to SU(3) \times U(1) \) (For details of the gauge symmetry breaking see the trinification analysis in [22–24]). The soft terms must be fine-tuned significantly to cancel the mass acquired in gauge symmetry breaking. For the up and down quarks such tuning is 1 part in \( 10^4 \) while for the electron it is 1 part in 250,000. Regretfully alternative approaches have no more predictivity about masses than here.

Because of this conceptual question, it is merely assumed that the LCP is at \( \sim 100 \text{ GeV} \); there is every reason to believe this is a possible outcome. An improved understanding of spontaneous breaking of conformal symmetry using the ideas of [25] may shed light on the mass spectrum.

Discussion

The LCP is a viable candidate for a cold dark matter particle which can be produced at the LHC. To produce all of the nonbaryonic cold dark matter \( \Omega_{LCP} = \Omega_{dm} \approx 0.24 \) requires that the mass of the LCP be in the range 43.1 GeV (ALEPH) \( \leq M_{LCP} \leq 1 \text{ TeV} \).

The distinction from other dark matter candidates will require establishment of the \( U(3)^3 \) gauge bosons, extending the 3-2-1 standard model and the discovery that the LCP is in a bifundamental representation thereof.

To confirm that the LCP is the dark matter particle would, however, require direct detection of dark matter. In bolometric experiments with a small number events the LCP will appear similar to other WIMPs but with high statistics the unique couplings of the LCP will be distinguishable.

It has been established that conformality can provide (i) naturalness without one-loop quadratic divergence for the scalar mass [9] and anomaly cancellation [26]; (ii) precise unification of the coupling constants [5, 6]; and (iii) a viable dark matter candidate. It

\#1 Note that the trinification here has the major difference from that proposed in [22] and studied in [23, 24] that unification is at a Teravolt, not a Yottavolt, scale.
remains for experiment to determine whether quiver gauge theories with gauge group $U(3)^3$ or $U(3)^n$ with $n \geq 4$ are employed by Nature.

Acknowledgements

I thank my colleagues E. Di Napoli and R. Rohm for discussions. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-97ER-41036.

References

[1] P.H. Frampton, Phys. Rev. D60, 041901(R) (1999). [hep-th/9812117] See also [2,3].
[2] P.H. Frampton and W.F. Shively, Phys. Lett. B454, 49 (1999). [hep-th/9902168]
[3] P.H. Frampton and C. Vafa. [hep-th/9903226]
[4] P.H. Frampton, Phys. Rev. D60, 085004 (1999). [hep-th/9905042] *ibid* D60, 121901 (1999). [hep-th/9907051]
[5] P.H. Frampton, Mod. Phys. Lett. A18, 1377 (2003). [hep-ph/0208044] See also [6].
[6] P.H. Frampton, R.M. Rohm and T. Takahashi, Phys. Lett. B567, 265 (2003). [hep-ph/0302074]
[7] J. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998). [hep-th/9711200]
[8] E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998). [hep-th/9802150]
  S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B428, 105 (1998). [hep-th/9802109]
[9] X. Calmet, P.H. Frampton and R.M. Rohm, Phys. Rev. D72, 055003 (2005). [hep-th/0412176]
[10] D.N. Spergel, *et al*, Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology. (March 17, 2006). [astro-ph/0603449]
  L. Page, *et al*, Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Polarization results. (March 17, 2006). [astro-ph/0603450]
  G. Hinshaw, *et al*, Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Temperature results. (March 17, 2006). [astro-ph/0603451]
  [http://map.gsfc.nasa.gov/mm/pub_papers/threeyear.html](http://map.gsfc.nasa.gov/mm/pub_papers/threeyear.html)
[11] E. Ma, Phys. Rev. Lett. 60, 1363 (1988).

[12] G. Jungman, M. Kamionkowski and K. Griest, Phys. Rep. 267, 195 (1996). [hep-ph/9506380]

[13] E.W. Kolb and M.S. Turner, The Early Universe, Addison-Wesley (1990).

[14] K. Griest, Phys. Rev. D38, 2357 (1988).

[15] J.E. Gunn, B.W. Lee, I. Lerche, D.N. Schramm and G. Steigman, Ap. J. 223, 1015 (1978). Reprinted in [16].

[16] Particle Physics and Cosmology: Dark Matter. Editor: M. Srednicki. North-Holland. (1990).

[17] ALEPH Collaboration (A. Heister, et al.) Phys. Lett. B583, 247 (2004).

[18] M. Drees and M.M. Nojiri, Phys. Rev. D47, 376 (1993). [hep-ph/9207234]

[19] F. Franke and S. Hesselbach, Phys. Lett. B387, 535 (1996). [hep-ph/9606291]

[20] P.N. Pandita, Phys. Rev. D53, 566 (1996). [hep-ph/9412247]

[21] K. Huitu, J. Laamanen and P.N. Pandita, Phys. Rev. D67, 115009 (2003). [hep-ph/0303262]

[22] S.L. Glashow, in Proceedings of the Fifth Workshop on Grand Unification, Editors: K.Kang, H. Fried and P.H. Frampton, World Scientific (1984). pages 88-94.

[23] K.S. Babu, X.G. He and S. Pakvasa, Phys. Rev. D33, 763 (1986).

[24] J. Sayre, S. Wiesenfeldt and S. Willenbrock, Phys. Rev. D73, 035013 (2006). [hep-ph/0601040]

[25] P.H. Frampton, Mod. Phys. Lett. A21, 893 (2006). [hep-th/0511265]

[26] E. Di Napoli and P.H. Frampton, Phys. Lett. B638, 374 (2006). [hep-th/0603065]