Non-Gaussian Corrections to Higgs Mass

in Autonomous $\lambda\phi^4_{3+1}$

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Abstract: Recent calculations in one-loop and Gaussian approximation, using the so-called autonomous renormalization scheme, indicate a comparatively massive, narrow Higgs excitation at about 2 TeV. Here I show that this result qualitatively persists in the framework of a post-Gaussian variational approximation for the pure $O(N)$-symmetric $\phi^4$-theory for $N > 1$. The method is based on nonlinear transformations of path-integral variables, and the optimization amounts to a Schwinger-Dyson-type summation of diagrams. In the case of $O(4)$, for example, I find $M_{\text{Higgs}} = 2.3$ TeV, compared to 1.9 TeV and 2.1 TeV in 1-loop and Gaussian approximation, respectively.

PACS: 11.10.Ef, 11.10.Gh, 14.80.Gt
1 Introduction

Five years ago Stevenson and Tarrach suggested certain UV-flows for the bare parameters of the $\phi^4$-model which lead to a finite Gaussian effective potential (GEP) in 3 + 1-dimensions. The method was termed autonomous renormalization scheme (ARS) because it is not directly related to the perturbative renormalization, the essential novelty being an infinite rescaling of the field. The autonomously renormalized GEP exhibits all generic features necessary for the Higgs mechanism, but it eludes triviality in that it is not equivalent to the tree-level potential.

Recently essential progress has been made in understanding the theoretical basis of the ARS and in deriving predictions for physical quantities from it. In a series of papers Consoli et al. investigated the ARS with renormalization group methods. They found asymptotic freedom in the asymmetric phase. Further, they discovered that the ARS also works in the loop approximation on the one-loop level and that definite predictions for the Higgs mass can be obtained under the natural assumption that the theory has a massless particle in the symmetric phase.

Compared with standard methods, the ARS predicts a relatively high Higgs mass at about 2 TeV, varying in the 10%-range, depending on whether the 1-loop potential or the GEP is considered, or whether couplings of the scalars to gauge bosons are taken into account or not. For instance for the $O(4)$-theory in Gaussian approximation one finds $M_{\text{Higgs}} = 2.05$ TeV.

One of the most important questions which must be asked in this context is whether the autonomous ARS has a realization beyond GEP and one-loop approximation. This is precisely the point which is tackled in this paper with a post-Gaussian variational calculation. During recent years a method has been developed, which allows to generalize the variational approach in quantum field theory to non-Gaussian trial states in the canonical formalism and non-Gaussian trial actions in the path-integral formalism, respectively. In the following I shall work in the covariant setting and restrict myself to the pure $O(N)$-theory.

2 The method

The generic (euclidean) action considered throughout the paper is given by

$$S = \frac{1}{2} \int_x \left( -\partial_\nu \phi^i \partial_\nu \phi^i + m_B^2 \phi^i \phi^i \right) + \lambda \int_x (\phi^i \phi^i)^2$$

with $\int_x = \int d^4x$. In order to approximate the effective potential, $V[\phi_c]$, I introduce the nonlinear transformation

$$\tilde{\phi}^i_1 = \tilde{\eta}_1(p) + \chi_0 \delta(p) + s \int_q c(q,r) \tilde{\eta}^a(q) \tilde{\eta}^a(r) \delta(p - q - r)$$

$$\tilde{\phi}^a = \tilde{\eta}^a(p) \quad a = 2, \ldots, N$$

(2)
which has Jacobian equal to unity, where \( \int_p = 1/(2\pi)^4 \int d^4p \), \( \delta(p) = (2\pi)^4\delta(p) \), and tildes indicate Fourier amplitudes.

Transformation (2) is appropriate for studying spontaneous symmetry breaking \( O(N) \to O(N-1) \) and has previously been used in the canonical approach [8]. Leaving the path-integral measure invariant, (2) is well suited for a variational calculation, because it can be evaluated in closed form, i.e. does not lead to a series expansion of the expectation value. The c-number \( \chi_0 \) and the correlation function \( s c(q, r) \) - the factor \( s \) is split off for normalization and as a bookkeeping device - are variational parameters.

The upper bound on the effective potential derived from (2) is

\[
V(\phi_c) \leq V_A(\phi_c) = \frac{1}{(I_x)} \min \left\{ -\log N + N^{-1} \int D\eta \ e^{-S_G[\eta]} (S[\phi] - S_G[\eta]) \right\}_{\phi_c},
\]

(3)

where the available variational parameters have to be optimized under the constraint

\[
\phi_c := N^{-1} \int D\eta \ e^{-S_G[\eta]} \phi^1 = \text{fixed},
\]

(4)

i.e., with the expectation value of the field held fixed, and

\[
N = \int D\eta \ e^{-S_G[\eta]}.
\]

(5)

In the equations above the \( \phi \)'s have to be substituted with transformation (2). \( S_G \) is a quadratic test action, given by

\[
S_G[\eta] = \frac{1}{2} \int_p \left\{ \eta_1^1(p) G_L^{-1}(p) \eta_1^1(-p) + \eta_1^2(p) G_T^{-1}(p) \eta_1^2(-p) \right\},
\]

(6)

where \( G_L \) and \( G_T \) are adjustable propagators for (with respect to the direction of symmetry breaking) longitudinal and transversal modes, respectively.

### 3 Autonomous Gaussian approximation

If the nonlinear term is absent in (2), the simple identity \( \phi_c = \chi_0 \) holds and the optimization yields the (covariant) GEP. (In contrast to the canonical formalism, this procedure can be easily extended to the effective action [9], but for my present purpose the effective potential is sufficient.) In the following I am summarizing the basics of the ARS for the Gaussian approximation, and afterwards the analysis is extended to the nonlinear transformations.

The bare unoptimized GEP is given by

\[
V_G = J^L + (N-1)J^T + \frac{1}{2}m_B^2 \left[ I^L + (N-1)I^T + \phi_c^2 \right] + \lambda_B \left[ 3(I^L + \phi_c^2)^2 - 2\phi_c^4 + 2(N-1)I^L(I^T + \phi_c^2) + (N^2 - 1)(I^T)^2 \right],
\]

(7)

with

\[
J^L(T) = \frac{1}{2} \int_p \log G^{-1}_{L(T)} + \frac{1}{2} \int_p p^2 G_{L(T)}(p)
\]

(8)
and

\[ I^{LT} = \int_p G_{LT}(p) . \]  

In the case of the GEP, the exact optimal \( G \)'s in (6) have the form

\[ G_0(p; m) = \frac{1}{p^2 + m^2} , \]  

with optimization equations

\[ \Omega^2 = m_B^2 + 4\lambda_B \left( 3I_0(\Omega) + 3\phi_c^2 + (N - 1)I_0(\omega) \right) \]  

for longitudinal and

\[ \omega^2 = m_B^2 + 4\lambda_B \left( I_0(\Omega) + \phi_c^2 + (N + 1)I_0(\omega) \right) \]  

transversal mass [2]. With (10) the \( J \)'s and \( I \)'s become

\[ J(m) = -\frac{1}{2} \int_p \log G_0(p; m) - \frac{1}{2}m^2I_0(m) , \quad I_0(m) = \int_p G_0(p; m) . \]  

The ARS consists of the following UV-flows for the bare parameters:

\[ m_B^2 = \frac{m_0^2 - 12\alpha I_0(0)}{I_{-1}(\mu)}, \quad \lambda_B = \frac{\alpha}{I_{-1}(\mu)}, \quad \text{and} \quad \phi_c^2 = z_0I_{-1}(\mu)\Phi^2 , \]  

where

\[ I_{-1}(\mu) = 2 \int_p (G_0(p; \mu))^2 \]  

is a logarithmically divergent quantity which plays an important role in the following, and \( \mu \) is an arbitrary renormalization mass, a dimensional transmutation parameter for it replaces the dimensionless coupling constant. The second parameter, \( m_0 \), which turns out to be the mass at the origin, is set to zero in the following [3].

Using (14) in (7) one obtains

\[ V_G = \alpha I_{-1}(\mu) \left( A\Omega^4 + B\omega^4 + C\Omega^2\omega^2 + D\Omega^2 z_0\Phi^2 + E\omega^2 z_0\Phi^2 + z_0^2\Phi^4 \right) + \text{finite} + \mathcal{O}(1/I_{-1}) \]  

with

\[ A = \frac{1}{8\alpha} + \frac{3}{4}, \quad B = \frac{N - 1}{8\alpha} + \frac{N^2 - 1}{4}, \quad C = 2(N - 1), \quad D = -3, \quad E = -(N - 1) , \]  

i.e. all quartic and quadratic divergencies are cancelled, leaving behind a logarithmically divergent expression. In order to end up with a finite effective potential eventually, also the logarithmically divergent term, proportional to \( I_{-1} \), has to vanish in the minimum with respect to the variational masses. This amounts to an equation for \( \alpha \) which can be solved analytically. The result is given by:

\[ \alpha = \frac{1}{4(1 + \sqrt{N + 3})} . \]
The remaining finite terms in (7) are

\[ V_G(\Phi) = \frac{1}{8\pi^2} z_0^2 \Phi^4 \left( X \log(\Phi^2/\mu^2) - Y \right) \]  

(19)

with

\[ X = \frac{\sqrt{N+3}}{(2 + \sqrt{N+3})(1 + \sqrt{N+3})^2}. \]  

(20)

For \( X > 0 \) the potential is stable, has massless excitations at \( \Phi = 0 \) and an asymmetric vacuum with massive particles. (The constant \( Y \) can be calculated but is not important for the following considerations.) The longitudinal mass - which is to be with the Higgs mass in the asymmetric minimum - is given by

\[ \Omega^2 = z_0 \sigma_L \Phi^2 \]  

(21)

It is worthwhile to mention that the stability condition \( X > 0 \) is automatically satisfied, once the solution for \( \alpha \) exists. In (16) all quantum corrections have to add up in order to produce a negative contribution that cancels the positive \( z_0^2 \Phi^4 \). The term proportional to \( X \) is solely generated by those subleading terms, which are produced when the longitudinal and transversal masses are eliminated in favor of \( \mu \). The latter is done with the help of the relation

\[ I_{-1}(m) = I_{11}(\mu) - \frac{1}{8\pi^2} \log \frac{m^2}{\mu^2}, \]  

(22)

and the log-term yields the \( \log(\Phi^2/\mu^2) \) in (19). All other subleading terms contribute only to \( Y \). Thus, \( X > 0 \) is a direct consequence of the minus sign in (22).

The finite factor \( z_0 \) that enters the wavefunction renormalization in (14) can be calculated from the renormalization condition

\[ \left. \frac{d^2V}{d\Phi^2} \right|_{\Phi=\Phi_v} = \left. \Omega^2 \right|_{\Phi=\Phi_v} = z_0 \sigma_L \Phi_v^2, \]  

(23)

where \( \Phi_v \) is the value of the field in the asymmetric minimum. This leads to

\[ z_0 = \pi^2 \frac{\sigma_L}{X} = \pi^2 \frac{10 + 6\sqrt{N+3} + 2N}{\sqrt{N+3}}. \]  

(24)

If now the well-known value \( \Phi_v = 0.246 \) TeV is inserted in (21), \( M_{Higgs} \) can be computed; the table below shows the results.

### 4 Non-Gaussian contributions

I am switching on the nonlinear term now, leaving the correlation function \( c(p,q) \) subject to optimization. The expectation value can be calculated straightforwardly. The unoptimized effective potential becomes

\[ V_A = V_G + \Delta V = V_G + s^2 \chi_7 + \frac{1}{2} m_B s^2 \chi_2 + \lambda_B \left\{ 4\phi_c s \chi_3 + 2s^2 \chi_5 \right\} + s^2 \left[ 6 I_L + 2(N-1) I_T + 6 \phi_c^2 \right] \chi_2 + 4\phi_c s^3 \chi_4 + 3s^4 (\chi_6 + \chi_2) \]  

(25)

(26)
where $V_G$ is the Gaussian result, with yet undetermined longitudinal and transversal propagators, and

\[
\chi_2 = 2(N-1) \int_{pq} c^2(p,q) G_T(p) G_T(q), \quad (27)
\]

\[
\chi_3 = 2(N-1) \int_{pq} c(p,q) G_T(p) G_T(q), \quad (28)
\]

\[
\chi_4 = 8(N-1) \int_{pqr} c(p,q) c(p,r) c(q,-r) G_T(p) G_T(q) G_T(r), \quad (29)
\]

\[
\chi_5 = 8(N-1) \int_{pqr} c(p,q) c(q,r) G_T(p) G_T(q) G_T(r), \quad (30)
\]

\[
\chi_6 = 16(N-1) \int_{pqrs} c(p,q) c(p,r) c(q,s) c(r,s) G_T(p) G_T(q) G_T(r) G_T(s), \quad (31)
\]

\[
\chi_7 = (N-1) \int_{pq} (p+q)^2 c^2(p,q) G_T(p) G_T(q). \quad (32)
\]

It is the term linear in $s$, proportional to $\chi_3$, that assures an improved energy compared to the GEP, at least on the level of the bare theory. Neglecting for the present the higher-order terms proportional to $\chi_4$ and $\chi_6$ - the justification for that will be given below -, the optimization can be carried out, reducing the problem to a Schwinger-Dyson-type integral equation for the optimal correlation function $c(p,q):$

\[
c(p,q) = G_0(p+q; \Omega) \left( 1 - 8\lambda_B \int_r [c(p,r) + c(q,r)] G_T(r) \right), \quad (33)
\]

with

\[
\Omega^2 = m^2_B + 4\lambda_B \left( (N-1)I^T + 3(I^L + s^2\chi^2 + \phi^2_c) \right) \quad (34)
\]

and where

\[
s = -4\lambda_B\phi_c \quad (35)
\]

has been chosen for convenient normalization.

The solution of (33) leads to a net energy-decreasing term

\[
\Delta V = -8\lambda_B^2\phi_c^2 \bar{\chi}_3 + \mathcal{O}(s^4) \quad (36)
\]

where $\bar{\chi}_3$ is (28) with optimal correlator. The $\chi$-integrals satisfy

\[
2\bar{\chi}_7 + \Omega^2 \bar{\chi}_2 + 4\lambda_B \bar{\chi}_5 = \bar{\chi}_3. \quad (37)
\]

The iteration of (33) reveals that (36) contains an infinite series of contributions to the two-point vertex function - the first four graphs are depicted in Fig. 1 - , thus demonstrating that the approximation goes far beyond the Gaussian approximation which sums up the “cactus” graphs.
5 Renormalization

Dimensional analysis shows that the bare correction $\Delta V$ is quadratically divergent. As a consequence, it necessitates a correction to the Gaussian mass counterterm in (14). As discussed in [13], however, any such correction to $m_B^2$ multiplies the whole mass term,

$$\langle \phi^i \phi^i \rangle = \left( I^L + s^2 \chi_2 + \phi_c^2 + (N - 1)I^T \right), \quad (38)$$

thus generating new (quartically and quadratically) divergent terms which have no direct counterpart in the expectation value. This dilemma originally led to the notion of the instability of variational calculations [11] in general and the ARS in particular [12]. With the help of the diagrammatic representation, however, a deeper understanding of the problem and, eventually, a straightforward solution can be obtained.

The first diagram in the series Fig.1, the so-called barred circle, has the close relative shown in Fig. 2, a vacuum diagram that may be generated from the barred circle by attaching two external lines and connecting them. In second-order $\delta$-expansion [10], for example, both diagrams are present and the renormalization works as Fig. 2 contains the barred circle as a divergent subgraph. In the variational calculation, however, the situation is quite different. Fig. 2 is not generated automatically. This can be verified by considering the effective potential at the origin where the optimization can be carried out exactly [8]. Consequently, the counterterm for the renormalization of the barred circle causes (quartically) divergent subtractions which are actually supposed to renormalize vacuum graphs.

There are two possible solutions to this problem. Firstly, one can think of an ansatz which generates both diagrams. This can in principle be achieved by a transformation like (2) with cubic nonlinear term of the form $\eta^1 \eta^a \eta^b$. However, this transformation has nontrivial Jacobian and thus makes the evaluation of the ansatz much more complicated and the applicability of the variational method questionable. Secondly, as suggested in [13], one can choose a sub-optimal form of the variational parameters, just generating the divergence, which otherwise would be caused by Fig. 2 and which then can be subtracted by the mass counterterm.

The ansatz is given by

$$G_L(p) = G_0(p; \Omega) - s^2 \xi(p) \quad (39)$$

with

$$\xi(p) = 2(N - 1) G_T(p) \int_q c^2(p, q) G_T(q) \quad (40)$$

for the longitudinal propagator and

$$G_T(p) = G_0(p; \omega) \quad (41)$$

for the transversal propagator.

With (39) one obtains

$$I^L = I_0(\Omega) - s^2 \chi_2 \quad (42)$$
and
\[ J^L = J(\Omega) + \frac{1}{2}\Omega^2 s^2 \chi_2 + \mathcal{O}(s^4) \] (43)

The next step is to set
\[ s = -4\lambda_B \left( \phi_c - \sqrt{-\Delta(\Omega) - (N-1)\Delta(\omega)} \right) \] (44)

where
\[ \Delta(m) = I_0(m) - I_0(0) = -\frac{1}{2}m^2 I_{-1}(\mu) + \frac{1}{16\pi^2} \left( \log \frac{m^2}{\mu^2} - 1 \right). \] (45)

As opposed to (33) the ansatz (44) generates a term proportional to
\[ \left( \Delta(\Omega) + (N-1)\Delta(\omega) \right) \chi_3, \] (46)

whereas on account of the identity (37) terms containing the square root of the \( \Delta \)'s cancel among each other.

Finally, by demanding that the excitation at the origin remains massless, the bare mass
\[ m_B^2 = -12\lambda_B I_0(0) + 16\lambda_B^2 \bar{\chi}_3|_{\phi_c=0} \] (47)

is obtained. As a result of (33), (44), and (47) the effective potential is given by
\[ V_A = V_G - 8\lambda_B^2 (\bar{\chi}_3 - \bar{\chi}_3|_0) \left( \Delta(\Omega) + (N-1)\Delta(\omega) + \phi_c^2 \right) + \mathcal{O}(\phi_c^4). \] (48)

This expression is not yet finite, but all power divergencies have been removed. As I shall demonstrate below, the only remaining divergencies are logarithmic and, like in the GEP-case, they can be cancelled by adjusting the parameter \( \alpha \) in \( \lambda_B \).

Before I proceed, the terms \( \mathcal{O}(\phi_c^4) \) have to be discussed. From (44) and (44) it follows that \( s^2 \propto 1/I_{-1} \). Analyzing the integrals which occur in \( \mathcal{O}(\phi_c^4) \) (like \( \chi_4, \chi_6 \), and other contributions produced by (39)), one finds that all these terms are at most \( \mathcal{O}(1/I_{-1}) \) and, thus, they are not affecting the finite parts of the effective potential in the ARS.

6 Evaluation of the effective potential

In this section the Schwinger-Dyson equation (33) will be discussed. The equation is in a sense the higher-order analogue to the self-consistency equation of the Gaussian approximation. The integral equation corresponding to the latter is extremely simple and can be solved algebraically. With (33) the situation is more complicated. The technical problems are closely related to the ones occurring in the 1/N expansion [4]. Like there, subsets of of diagrams, containing chains of bubbles of different length, can in principle be summed up. In the following I am not proceeding along those lines, however, because there are good reasons to believe that the series converges
rapidely when the factor $\alpha$ (which is the expansion parameter in the integral equation) assumes its physical value, where logarithmic divergencies are cancelled.

The leading (logarithmic) divergence of the diagrams generated by (33) is proportional to $I_{-1}^k$, where $k$ is the number of vertices. With the help of (45) one finds that $\Delta V \propto I_{-1}$, like in the Gaussian approximation. I have calculated explicitly the contributions from the first and second diagram in Fig. 1. The leading divergence of these graphs is given by

$$-\frac{1}{8} (\Omega^2 + 2\omega^2) I_{-1}^2 \quad \text{and} \quad -\frac{2}{3} (\Omega^2 + 2\omega^2) I_{-1}^3,$$

respectively.

With the ansatz

$$\Omega^2 = \sigma_L z_0 \Phi^2, \quad \omega^2 = \sigma_T z_0 \Phi^2$$

the divergent part of the effective potential again has the form (14). The coefficients read

$$A' = A - \Delta, \quad B' = B - 2(N-1)\Delta,$$
$$C' = C - (N+1)\Delta, \quad D' = D + 4\Delta \quad E' = E + 2\Delta$$

with

$$\Delta = (N-1)\alpha \left(1 - \frac{16}{3} \alpha + \mathcal{O}(\alpha^2)\right)$$

and where the unprimed letters stand for the Gaussian coefficients (17). One derives three equations from the conditions that the term proportional $I_{-1}$ vanishes in the minimum with respect to the variational masses. These equations can be solved numerically giving $\alpha, \sigma_L,$ and $\sigma_T$ as functions of $N$.

Eventually also the finite parts of the effective potential can be obtained. They again have the form (19), and the factor $X$ can be calculated on account of (22) directly from the leading divergence. The results for the Higgs mass are given in the table.

7 Discussion of results

I have demonstrated that within the $O(N)$-theory the autonomous renormalization scheme exists beyond the one-loop and Gaussian variational approximation. This result gives some confidence in the ARS and makes it unlikely that one is dealing with an artifact of the GEP. The optimized nonlinear transformations lead to a Schwinger-Dyson-type integral equation, formally summing up a series of diagrams that goes far beyond the one that is taken into account by the Gaussian method. The integral equation has been approximated by the second iteration, which contains diagrams with up to three loops. An estimate of the changes due to four-loops indicates that the error of the approximation is less than 3%.

The results for the Higgs mass are shown in the table. For low values of $N$, the mass is higher than in one-loop and Gaussian approximation. For instance, for $N = 4$ it lies 10% above
the Gaussian and 20% above the one-loop prediction. For higher $N$ it drops substantially below the Gaussian results.

| $N$ | Gaussian | Non-Gaussian |
|-----|----------|--------------|
| 1   | 2.19     | –            |
| 2   | 2.13     | 2.21         |
| 4   | 2.05     | 2.27         |
| 7   | 1.97     | 2.22         |
| 10  | 1.95     | 2.08         |
| 20  | 1.84     | 1.70         |
| 100 | 1.69     | 1.34         |

Table 1 Values of Higgs mass (in TeV) for different values of $N$ from Gaussian and non-Gaussian approximation.

The present calculation shows that the error in the Higgs mass due to the approximative treatment of the self interaction lies in the same order of magnitude as corrections due to interactions with gauge bosons and fermions. Hence, if the phenomenological consequences of the ARS are to be taken seriously, one clearly has to confirm them by alternative improved approximations, like the conventional loop expansion, the $\delta$-expansion [10], or the optimized expansion [15].
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Figure captions

**Fig. 1** Four contributions in the series of diagrams generated by the Schwinger-Dyson equation (33). Thin (thick) lines represent transversal (longitudinal) propagators.

**Fig. 2** Vacuum diagram that contains the “barred circle” as divergent subgraph.