Further results on robust $H_\infty$ control design for uncertain time-delay systems with actuator delay: application to PMSG machine

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ABSTRACT

The problem of robust $H_\infty$ state-feedback control design for a class of uncertain systems with two different time delays in the state vector and the input signal is investigated in this paper. The main feature of the paper is to develop a robust $H_\infty$ controller, which ensures the robust asymptotic stability of the system as well as the desired $H_\infty$ performance. By constructing a Lyapunov–Krasovskii functional, some sufficient conditions for the existence of the $H_\infty$ state-feedback controller is derived in terms of linear matrix inequalities. Numerical examples such as a wind energy conversion system model based on a permanent magnet synchronous generator model are provided to illustrate the effectiveness of the proposed method.

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1. Introduction

The robust control design, as a well-known context, can handle model uncertainties and information constraints for dynamical systems have received a considerable amount of researchers’ interest in the past decades. Time delay, as one of the model complexity, can be found in various engineering systems such as chemical processes, biological systems, and economic systems. The problem of delay effects on the stability of systems including delays in the state, and/or input is of interest since the delay presence may induce complex behaviours (oscillation, instability, bad performances) for the schemes (Fiagbedzi & Pearson, 1986; Liu et al., 2017). Also uncertainty parametric are often the sources of instability and poor performance in many engineering systems, that considerable attention has been devoted to the problem of stability analysis and controller synthesis for uncertain systems (Mobayen & Baleanu, 2017).

In general, the stability criteria of time-delay systems (TDS) can be divided into two categories: delay-independent type (Pal & Negi, 2017; Zhang et al., 2017) and delay-dependent type (Lee, 2017; Aleksandrov et al., 2017; Pal & Negi, 2017). The earlier type is independent of delay size; and is generally conservative, especially when a delay is small. On the other hand, studies of delay-dependent criteria have focused mainly on identical delays in neutral and discrete terms (Aleksandrov, Hu, & Zhabko, 2014; Lu, Wu, & Bai, 2014; Yang, Wang, & Wang, 2017). Sun, Liu, and Chen (2009) studied the delay-dependent stability and stabilization criteria for neutral systems with time delays. The problem of robust stability of neutral systems with mixed time-varying delays has been studied in Lakshmananr, Senthilkumar, and Balasubramaniam (2011). Karimi and Gao (2010) presented a multiple delayed state-feedback control design for exponential $H_\infty$ synchronization problem of time-delay neural networks with multiple time-varying discrete delays. Karimi (2008) studied a convex optimization method for observer-based mixed $H_2/H_\infty$ control design of linear systems with time-varying state and output delays. Li, Jing, and Karimi (2014) studied the problem of output-feedback $H_\infty$ control for a class of active quarter-car suspension systems with time delay. Theoretically the controller can be solved by using the Lyapunov function method, however, there is no universal method to construct such kinds of Lyapunov functional. Sakthivel, Sakthivel, Selvaraj, and Karimi (2017) by using Lyapunov stability method and some integral inequality techniques a new set of sufficient conditions obtained in terms of linear matrix inequality (LMI) constraints to ensure the asymptotic stability of the considered system. The authors in Xie and Han (2008) presented a new method to obtain some robust stability conditions of uncertain linear systems with interval time-varying delay. This kind of approach can reduce the conservativeness compared with the existing results. Lyapunov theory is widely employed in...
the analysis and synthesis of a control system because of its effectiveness, and LMIIs provide a powerful and
efficient numerical tool for the stability analysis and com-
putational aspects (Boyd, Ghaoui, Feron, & Balakrishnan,
1994).

Motivated by the above discussions, in this paper we
have studied the problem of delay-dependent robust
$H_\infty$ control for a class of uncertain systems with two
different time-delay parameters in the state and input
signals. The nonlinear uncertainties are supposed to be
time-varying and norm bounded. A sufficient condition
for the $H_\infty$ control problem is proposed in terms of the
LMI approach using the Lyapunov functional. Numeri-
cal examples including a wind energy conversion system
model based on a permanent magnet synchronous gen-
erator (PMSG) model are given to illustrate the feasibility
and effectiveness of the proposed results.

The main contributions of the paper is twofold: (1) sta-
tility analysis and synthesis problems of dynamical sys-
tem in the presence of mixed state and actuator delays
which can reduce the conservativeness of the model to
some extent from practical aspect; (2) dissemination of
the stability analysis and synthesis results obtained in the
paper on PMSG machine.

Notation. The notation in this paper is quite standard.
The superscript 'T' stands for the transpose of a matrix; $R^n$
and $R^{n \times n}$ denote an $n$-dimensional Euclidean space
and the set of all $n \times n$ real matrices, respectively; $I$ is the
identity matrix of appropriate dimension; $\cdot$ is the Euclidean
vector norm, and the symmetric terms in a symmetric
matrix are denoted by $^\ast$.

2. Problem formulation

In this section, a class of dynamical systems is considered
with two different time-delay parameters in the state and
control input as follows:

$$
\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + E_1 \Delta_1(x(t), t) + E_2 \Delta_2(x(t - \tau), t) + B(f(t) + \Delta_3(f(t), t)) + E_w w(t)
$$

\hspace{30em} (1a)

$$
x(t) = \phi(t), t \in [0, \tau]
$$

\hspace{30em} (1b)

$$
z(t) = C_1 x(t)
$$

\hspace{30em} (1c)

where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$, $f(t) \in R^n$ are the
state vector and the actuator output vector, respectively, and
$\tau$ is the time delay in the state. $A_0, A_1, A_2, E_1, E_2, E_w, C_1, B$
are constant matrices with appropriate dimensions; the
uncertainties $\Delta_1(x(t), t)$, $\Delta_2(x(t - \tau), t)$ and $\Delta_3(f(t), t)$
represent the nonlinear perturbations with respect to the
current state, the delayed states of the system and the
actuator output, respectively. $\phi(t) \in R^n$ is a continuous
vector-valued initial function and $w(t)$ is an exogenous
norm-bounded disturbance.

Before proceeding further, the following definitions
and lemmas are reviewed.

Definition 2.1: The uncertain TDS of the form (1) is said
to be robustly asymptotically stable in Lyapunov sense
with an $H_\infty$ disturbance attenuation $\gamma > 0$ if system (1)
with $w(t) = 0$ is robustly stable and moreover, under zero
initial condition, there is

$$
\int_0^\infty z(t)^T z(t) dt \leq \gamma^2 \int_0^\infty w(t)^T w(t) dt
$$

(2)

Assumption 2.1: We suppose that the nonlinear uncer-
tainties of the system, i.e. $\Delta_1(x(t), t)$, $\Delta_2(x(t - \tau), t)$
$\Delta_3(u(t), t)$ are bounded, i.e.

$$
\Delta_1(x(t), t)^2 \leq c_1^2 x(t)^2
$$

(3a)

$$
\Delta_2(x(t - \tau), t)^2 \leq c_2^2 (x(t - \tau)^2
$$

(3b)

$$
\Delta_3(f(t), t)^2 \leq c_3^2 f(t)^2
$$

(3c)

where $c_i$ are positive scalars.

Remark 2.1: The nonlinear functions $\Delta_1(\cdot), \Delta_2(\cdot)\Delta_3(\cdot)$
avove can be interpreted as uncertainties in model
 dynamics representation and actuator mechanism in
practice.

Lemma 2.1: (Schur complement). Let $M, P Q$ be given
matrices such that $Q > 0$, then

$$
\begin{bmatrix}
P & M^T \\
M & -Q \\
\end{bmatrix} < 0 \iff P + M^T Q^{-1} M < 0
$$

(4)

Lemma 2.2: Let $D, E$ be real matrices of appropriate
dimensions, and $F(t)$ satisfying $F(t) F(t) \leq I$. Then, the following
inequality holds for any constant $\varepsilon > 0$:

$$
D F(t) E + E^T F(t) D^T \leq \varepsilon D D^T + \varepsilon^{-1} E^T E
$$

The problem of control synthesis with the actuator
delay we address here is as follows:

Given a prescribed level of disturbance attenuation
$\gamma > 0$ and the actuator time-delay response $d$, find a con-
trol signal $u(t)$ of the form $u(t) = K x(t)$ with the actua-
tion signal $f(t) = K x(t - d)$, where the matrix $K$ is to be
determined in the sense of Definition 2.1.
3. $H_\infty$ performance analysis

In this section, we will focus on the asymptotic stability and $H_\infty$ performance analysis for the following system:

$$
\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + A_2 x(t - d) + E_1 \Delta_1 (x(t), t) + E_2 \Delta_2 (x(t - \tau), t) + E_3 \Delta_3 (x(t - d), t) + E_w w(t)
$$

$$
x(t) = \phi(t), t \in [0, \max\{d, \tau\}]
$$

$$
z(t) = C_1 x(t)
$$

where $E_3$ is a constant matrix with appropriate dimensions.

**Theorem 3.1:** Under Definition 2.1 and Assumption 2.1, uncertain TDS (1) is asymptotically stable and fulfills $H_\infty$ performance condition (2) if there exist positive-definite matrices $P, Q_1, Q_2$ and $\epsilon_i, i = 1, 2, 3$, satisfying the following LMI:

$$
\begin{bmatrix}
\dot{M} & PA_1 & PA_2 & PE_1 & PE_2 & PE_3 & C_1^T
\end{bmatrix} = 
\begin{bmatrix}
-Q_1 + \epsilon_1 C_1^T & 0 & 0 & 0 & 0 & 0
\end{bmatrix} < 0
$$

where

$$
\dot{M} = A_0^T P + PA_0 + Q_1 + Q_2 \epsilon_1 C_1^T + \epsilon_1^{-1} P E_1 E_1^T P
$$

**Proof:** Choose the following Lyapunov function:

$$
v(t) = v_1(t) + v_2(t) + v_3(t)
$$

with

$$
v_1(t) = x^T(t) P x(t)
$$

$$
v_2(t) = \int_{t-\tau}^{t} x^T(r) Q_1 x(r) dr
$$

$$
v_3(t) = \int_{t-d}^{t} x^T(r) Q_2 x(r) dr
$$

where $P = P^T \geq 0, Q_i = Q_i^T \geq 0$ ($i = 1, 2$).

Calculating the time-derivative of $v(t)$ we have the following time-derivatives of $v_i(t), i = 1, 2, 3$, that

$$
\dot{v}_1(t) = [A_0 x(t) + A_1 x(t - \tau) + A_2 x(t - d) + E_1 \Delta_1 (x(t), t) + E_2 \Delta_2 (x(t - \tau), t) + E_3 \Delta_3 (x(t - d), t) + E_w w(t)]^T P x(t) + x^T(t) P [A_0 x(t) + A_1 x(t - \tau) + A_2 x(t - d) + E_1 \Delta_1 (x(t), t) + E_2 \Delta_2 (x(t - \tau), t) + E_3 \Delta_3 (x(t - d), t) + E_w w(t)]
$$

$$
= x^T(t) (A_0^T P + PA_0) x(t) + x^T(t - \tau) A_1^T P x(t) + x^T(t - d) A_2^T P x(t) + \Delta_1^T (x(t), t) E_1^T P x(t) + \Delta_2^T (x(t - \tau), t) E_2^T P x(t) + \Delta_3^T (x(t - d), t) E_3^T P x(t) + w^T(t) E w^T(t) P x(t) + x^T(t) P A_1 x(t - \tau) + x^T(t) P A_2 x(t - d) + x^T(t) P E_1 \Delta_1 (x(t), t) + x^T(t) P E_2 \Delta_2 (x(t - \tau), t) + x^T(t) P E_3 \Delta_3 (x(t - d))
$$

$$
\dot{v}_2(t) = x^T(t) Q_1 x(t) - x^T(t - \tau) Q_1 x(t - \tau)
$$

$$
\dot{v}_3(t) = x^T(t) Q_2 x(t) - x^T(t - d) Q_2 x(t - d)
$$

Then, from (7)–(9) and Assumption 2.1, one obtains

$$
\dot{v}(t) \leq x^T(t) A_0^T P + PA_0 + Q_1 + Q_2 \epsilon_1 C_1^T + \epsilon_1^{-1} P E_1 E_1^T P + x^T(t - \tau) (-Q_1 + \epsilon_2 C_2^T) x(t - \tau) + x^T(t - d) (-Q_2 + \epsilon_3 C_3^T) x(t - d)
$$

$$
+ x^T(t) A_1^T P x(t) + x^T(t) A_2^T P x(t) + x^T(t) P E_1 \Delta_1 (x(t), t) + x^T(t) P E_2 \Delta_2 (x(t - \tau), t) + x^T(t) P E_3 \Delta_3 (x(t - d))
$$

$$
\int_{0}^{\infty} \left[ \dot{v}(t) x^T(t) - \gamma^2 \omega^T(t) \omega(t) \right] dt
$$

In the following, the $H_\infty$ disturbance attenuation in (2) can be written as

$$
J(t) = \int_{0}^{\infty} \left[ v(t) z(t) - \gamma^2 \omega^T(t) \omega(t) \right] dt
$$

Considering $(t) = C_1 x(t)$ we can obtain

$$
x^T(t) C_1^T C_1 x(t) \leq \gamma^2 \omega^T(t) \omega(t)
$$

From (10)–(12), it can be shown that

$$
\dot{v}(t) + x^T(t) C_1^T C_1 x(t) - \gamma^2 \omega^T(t) \omega(t) < \xi^T(t) \Theta \xi(t)
$$

or

$$
x^T(t) [A_0^T P + PA_0 + Q_1 + Q_2 \epsilon_1 C_1^T + \epsilon_1^{-1} P E_1 E_1^T P + \gamma^2 \omega^T(t) \omega(t)]
$$

$$
+ x^T(t - \tau) (-Q_1 + \epsilon_2 C_2^T) x(t - \tau) + x^T(t - d) (-Q_2 + \epsilon_3 C_3^T) x(t - d)
$$

$$
+ x^T(t) A_1^T P x(t) + x^T(t) A_2^T P x(t) + x^T(t) P E_1 \Delta_1 (x(t), t) + x^T(t) P E_2 \Delta_2 (x(t - \tau), t) + x^T(t) P E_3 \Delta_3 (x(t - d))
$$

$$
+ x^T(t) P A_1 x(t - \tau) + x^T(t - d) A_2 P x(t)
$$
\[ + x^T(t) P A_2 x(t - d) + x^T(t) P E_w w(t) + w^T E_w^T P x(t) - \gamma^2 \omega^T(t) \omega(t) < \varsigma^T(t) \Theta \varsigma(t) \]

where \( \varsigma(t) = [x^T(t), x^T(t - \tau), x^T(t - d), w^T(t)]^T \), and

\[
\Theta = \begin{bmatrix}
M & PA_1 & PA_2 & PE_w \\
* & -Q_1 + \varepsilon_2 c_2^2 \beta & 0 & 0 \\
* & * & -Q_2 + \varepsilon_3 c_3^2 \beta & 0 \\
* & * & * & -\gamma^2 I
\end{bmatrix} < 0
\] (15)

With \( M = A_0^T P + PA_0 + Q_1 + Q_2 + \varepsilon_1 c_1^2 \beta + \varepsilon_1^{-1} P E_1 T P + \varepsilon_2^{-1} P E_2 T P + \varepsilon_3^{-1} P E_3 T P + C_1^T C_1 \).

By using Schur complement in Lemma 1, we can rewrite (15) in the form of (5), which completes the proof. \( \blacksquare \)

4. Control design

This section is devoted to design the robust \( H_\infty \) control for TDS in (1) in the presence of actuator delay.

**Theorem 4.1:** Consider uncertain TDS (1) with state delay \( \tau \) and the actuator delay \( d \). The system (1) with a control input (16) is asymptotically stable and satisfies disturbance attenuation level \( \gamma > 0 \), if there exist positive-definite matrices \( \tilde{P} \tilde{Q}_1 \tilde{Q}_2 \) and matrix \( \tilde{K} \) and positive scalars \( \varepsilon_1 \varepsilon_2 \varepsilon_3 \) such that the following LMI holds:

\[
\begin{bmatrix}
PA_0^T + A_0 \tilde{P} + \tilde{Q}_1 + \tilde{Q}_2 & A_1 \tilde{P} & B \tilde{K} & E_w & E_1 \\
-\tilde{Q}_1 & 0 & 0 & 0 & 0 \\
0 & -\tilde{Q}_2 & 0 & 0 & 0 \\
0 & 0 & -\gamma^2 I & 0 & 0 \\
0 & 0 & 0 & -\varepsilon_1 I & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 \\
0 & * & 0 & 0 & 0
\end{bmatrix} < 0
\] (16)

Then, the control feedback gain \( K \) is obtained by

\[
K = \tilde{K} \tilde{P}^{-1}
\] (17)

**Proof:** By substituting the control signal (16) in (1), we have

\[
\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + B K x(t - d) + E_1 \Delta_1(x(t), t) + E_2 \Delta_2(x(t - \tau), t) + B \Delta_3(K x(t - d), t) + E_w w(t)
\] (18)

According to Theorem 3.1 and by replacing \( A_2 \) with \( B K \) and \( E_3 \) with we can obtain

\[
\begin{bmatrix}
\tilde{M} & PA_1 & PBK & PE_w & PE_1 & PE_2 & PB & C_1^T \\
* & -Q_1 + \varepsilon_2 c_2^2 \beta & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -Q_2 + \varepsilon_3 c_3^2 \beta & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & -\varepsilon_1 I & 0 & 0 & 0 \\
* & 0 & 0 & 0 & 0 & -\varepsilon_1 I & 0 & 0 \\
* & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I & 0 \\
* & 0 & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0
\] (19)

Because of nonlinear terms in (20) it should be changed to a linear form by pre-multiplying and post-multiplying \( \text{diag} \{ p^{-1} p^{-1} p^{-1} 1 1 1 1 \} \) to the matrix inequality (20), and we can obtain

\[
\begin{bmatrix}
[1, 1] & A_1 \tilde{P}^{-1} & B \tilde{K} \tilde{P}^{-1} \\
& -\tilde{P}^{-1} Q_1 \tilde{P}^{-1} & 0 \\
& +\varepsilon_2 c_2^2 \tilde{P}^{-1} p^{-1} & 0 \\
& \varepsilon_3 c_3^2 \tilde{P}^{-1} p^{-1} & 0
\end{bmatrix} < 0
\] (20)

where \( [1, 1] = \tilde{P}^{-1} A_0^T \tilde{P} + A_0 \tilde{P}^{-1} + \tilde{P}^{-1} Q_1 \tilde{P}^{-1} + \tilde{P}^{-1} Q_2 \tilde{P}^{-1} + \varepsilon_1 c_1^2 \tilde{P}^{-1} p^{-1} + \varepsilon_2 c_2^2 \tilde{P}^{-1} p^{-1} + \varepsilon_3 c_3^2 \tilde{P}^{-1} p^{-1} - \gamma^2 I \).

Let \( \tilde{P}^{-1} = \tilde{P} \tilde{Q}_1 \tilde{P}^{-1} \tilde{Q}_2 = \tilde{P}^{-1} Q_2 \tilde{P}^{-1} \tilde{K} = K \tilde{P}^{-1} \) and using Schur complement in Lemma 2.1, we can obtain the LMI in (17). This completes the proof. \( \blacksquare \)

**Remark 4.1 (The case of without input delay):** Consider the following TDS with a delay in state vector only:

\[
\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + E_1 \Delta_1(x(t), t)
\] (16)
Then, in the case of delay-free state-feedback control $f(t) = u(t) = Kx(t)$, the system (21a)–(21c) can be represented as

$$\dot{x}(t) = (A_0 + BK)x(t) + A_1x(t - \tau) + E_1\Delta_1(x(t), t) + E_2\Delta_2(x(t - \tau), t) + B\Delta_3(Kx(t), t) + E_ww(t).$$

(22a)

$$x(t) = \phi(t), t \in [0, \tau]$$

(21b)

$$z(t) = C_1x(t)$$

(21c)

Then, it can be similarly shown that the stability analysis and synthesis for system (22) will be met under a disturbance attenuation level $\gamma > 0$ if there exist positive-definite matrices $\bar{P}$, $\bar{Q}_1$, and matrix $R$ and positive scalars $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ such that the following LMI holds:

$$\begin{bmatrix}
[1, 1] & A_1\bar{P} & E_w & E_1 & E_2 & B & \bar{P}C_1^T & \varepsilon_1C_1^T & \bar{P} & 0 & 0 \\
0 & -\bar{Q}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\gamma^2I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\varepsilon_1I & -\varepsilon_2I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\varepsilon_2I & -\varepsilon_3I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\
\end{bmatrix} < 0$$

(23)

where $[1, 1] = \bar{P}A_0^T + A_0\bar{P} + BK + \bar{K}^TB^T + \bar{Q}_1$. And, the control feedback gain $K$ is obtained by $K = \bar{K}\bar{P}^{-1}$.

### 5. Numerical example

In this section, the applicability of the proposed method is validated by three examples in the sequel.

**Example 5.1:** In this example, we consider the simulation of a wind energy conversion system model based on a PMSG model. It can be shown that by utilization of Park’s transformation to the $(abc)$ coordinate frame PMSG model and linearizing the nonlinear model about an operating point, the following linear model in the $d$–$q$ coordinate frame model can be obtained (Mittal, Sandhu, & Jain, 2012):

$$\begin{bmatrix}
\dot{i}_d(t) \\
\dot{i}_q(t) \\
\dot{\omega}_r(t)
\end{bmatrix} =
\begin{bmatrix}
g_1 & g_2 & 0 \\
-g_2 & g_1 & 0 \\
0 & g_3 & 0
\end{bmatrix}
\begin{bmatrix}
i_d(t) \\
i_q(t) \\
\omega_r(t)
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & P_{g1}^\tau \\
0 & 0 & g_4 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\begin{bmatrix}
u_d(t) \\
u_q(t) \\
w(t)
\end{bmatrix}

(24)

with $g_1 = (-R_s + \Delta R_s)/(L_d + \Delta L_d)g_2 = P\omega^* / 2$ and $g_3 = -\Psi_m P/4Jg_4 = 4P/(2L_d + \Delta L_d) - P\omega^* / 2$ where $u_d$ and $u_q$ are the $d$- and $q$-axis stator voltage components; $i_d$ and $i_q$ are the $d$- and $q$-axis stator current components, respectively; and $L_d$, $R_s$ are the stator inductance and resistance, respectively. $\Psi_m$ is the flux, $\omega_r$ is the rotor electrical angular speed and $P$ the number of the poles.

Assuming small deviation of resistance and inductance values from nominal corresponding values, we can represent the system (24) in the following form:

$$\dot{x}(t) = x(t) + \Delta_1(x(t)) + \Delta_2(x(t - \tau)) + w(t)$$

(25)

where $x(t) = [i_d(t) i_q(t) \omega_r(t)]^T$ and $\Delta_1(x(t)) = \Delta_1(x(t))$. Consider the system (25) with the parameters of the WECS model in Table 1 and $C_1 = [1 1 1]$. For $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$, $\tau = 0.5$, $\gamma = 11.794$, by solving LMI (23) and in accordance to Remark 4.1, we can show the behaviour of input control signal in Figure 1 and state responses of both open-loop and closed-loop systems in Figures 2–4.

**Example 5.2:** Consider the system (1) with the following state-space matrices:

$$A_0 = \begin{bmatrix}
-2 & 0 \\
0 & -3
\end{bmatrix}, A_1 = \begin{bmatrix}
0.5 & -0.1 \\
-0.2 & -0.3
\end{bmatrix}, E_1 = \begin{bmatrix}
0 \\
0
\end{bmatrix}.$$
Figure 1. Input control signals.

Figure 2. State trajectory of $d$-axis current (with control–without control).

Figure 3. State trajectory of $q$-axis current (with control–without control).
According to Theorem 4.1, the corresponding LMI is solved using Matlab LMI Toolbox, then the following solutions can be computed in the case of $\gamma = 7.7449$:

$P = \begin{bmatrix} 21.2175 & -16.5189 \\ -16.5189 & 20.0661 \end{bmatrix}$,

$Q_1 = \begin{bmatrix} 71.0401 & -0.8853 \\ -0.8853 & 76.7651 \end{bmatrix}$,

$Q_2 = \begin{bmatrix} 74.5346 & 4.9586 \\ 4.9586 & 69.1344 \end{bmatrix}$

With the state-feedback control gain $K = [-14.7550 - 14.2538]$. In Figures 5–7, time behaviour of the disturbance, state system and control input are depicted. Both system stability and disturbance attention effect can be observed from the figures.

**Example 5.3:** Consider the model of system (22) with the following parameters:

$\begin{bmatrix} A_0 & 0 \\ 0 & A_1 \end{bmatrix} = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -0.1 & 1 \\ -1 & -3 \end{bmatrix}$,

$E_1 = E_2 = E_3 = E_w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, D = [1]$

By choosing $\tau = 0.5, \gamma = 0.7, d = 0.1$ and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$ we can obtain a feasible solution to LMI (24),
and compute the gain of state-feedback controller $K = [0.4692 \quad -1.9617]$.

Then, for two different values of $d = 0.1$ and $d = 0$ we can show the behaviour of the states system and control input in Figures 8 and 9.

6. Conclusion

In this paper, the problem of robust $H_\infty$ state-feedback control design for a class of uncertain systems with two different state delays has been considered. The Lyapunov stability theory and LMI have been used to guarantee the robust stabilization of the system under consideration. Also, a $H_\infty$ state-feedback controller has been explicitly computed. Moreover, an extension of the proposed problem to the case of delay in both the system state and input signals as the system actuator is presented and the corresponding control signal is developed. Finally, numerical examples such as a wind energy conversion system model based on a PMSG model are given to show the validity of the proposed methods. As further work, the method proposed in this paper will be examined within the event-triggered mechanism, see for instance Li, Shen, Liu, and Huang (2017), and Wang, Wang, Shen, Li, and Alsaadi (2018).

Disclosure statement

No potential conflict of interest was reported by the author.

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