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On Rates of Isopycnal Dispersion at the Submesoscale

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Abstract Past studies of dispersion with float-pairs have indicated that they may remain close together for much longer when they equilibrate on the same isopycnal, presumably due to the reduced influence of vertical shear. To examine this question more closely, we use a set of 13 and 15 float pair combinations that equilibrated within 0.1 °C (≈σθ = 0.01 kg m⁻³) of each other on two density surfaces in the main thermocline in a Lagrangian dispersion study. Their average rate of separation after launch was 0.0021 ± 0.0014 ms⁻¹ (~5.5 km after 30 days). Relative dispersion is accurately expressed by <D²> = 4×10⁻⁸ exp (t/10.8) m² from start to about 30 days. Relative diffusivity (K) versus separation dropped well below the classical 4/3rds power law settling out at about 2–3 m²s⁻¹ for separations less than ~6 km, far lower than results from other float studies, but in accord with dye dispersion estimates.

Plain Language Summary The study of dispersion and relative dispersion focuses on the rates spreading and mixing in the ocean. It has numerous practical applications such as predicting the impact of an oil spill, aiding search and rescue operations, or assessing the impact of coastal run-off after severe storms. Dispersion within the ocean is also of great interest for it gives an important measure of eddy activity, lateral shear and mixing, processes that determine the observed distributions of water properties from heat and salt to CO₂ and nutrients. Sub-surface activity can be studied by tracking neutrally buoyant floats, drifting on constant density (isopycnal) surfaces, using acoustic navigation (RAFOS). But unless floats are on exactly the same surface, their relative motion may be affected by vertical shear, i.e., water at different depths moving at different speeds. In this study using float pairs that were deployed together and equilibrating on the same density surface we find that relative dispersion is very slow during the first few weeks as long as floats are closer than about 10 km. At greater separations being on exactly the same isopycnal surface matters less because their continued spread is increasingly dominated by the horizontal mesoscale eddy field.

1. Introduction

Anecdotal evidence from past Lagrangian studies has repeatedly suggested that float pairs that start on exactly the same isopycnal initially separate much more slowly than pairs with a larger vertical spread. For example, in 1993 a major Lagrangian study of the North Atlantic Current (NAC) east of Canada was undertaken to examine how this warm salty water branch from the Gulf Stream worked its way north over 10° of latitude to 50–52°N where it turns sharply to the east (Dutkiewicz et al., 2001). Isopycnal floats (Rossby et al., 1985) were ballasted and deployed in pairs to the same isopycnal to see how well one represents the other, or more generally how they separate over time. Typically, they would part ways rather quickly (with trajectories becoming independent within days to a week), but one pair stayed within ~10 km of each other in the rapid NAC as they drifted over 800 km for over 70 days (Rossby, 2016, Figure 6b). These had settled on the same isopycnal to within 0.1 °C (temperature is used as a proxy for density). In another study three carefully ballasted floats were deployed in a ~10 km triangle in the center of the Gulf Stream. They equilibrated within 0.1 °C of each other, and stayed tightly clustered, drifting from 71 to 62°W (Rossby, 2016, Figure 6a). These earlier studies illustrated the potential of isopycnal float technology to study relative dispersion in stratified fluids (LaCasce and Bower, 2000; Zhang et al., 2001), although relative dispersion was not the primary focus of those studies.

The Lagrangian Isopycnal Dispersion Experiment (LIDEX) was undertaken in 2003 to study the roles of advection and mixing along the southern edge of the low-oxygen tongue off West Africa. Based on the trajectories of 90 isopycnal floats (50 of them equipped with Aanderaa oxygen sensors) deployed at 5 sites, the two key findings were that the horizontal diffusivities are nearly twice as large in the zonal than meridional
direction, and the convergence of the meridional diffusion acting on the large-scale oxygen gradient is the primary supply of oxygen to the study region, which is balanced primarily by biological consumption (Rudnickas et al., 2019; hereafter Rd). They also reported that in both directions relative diffusivity follows the classical 4/3-law (Richardson, 1926).

Here we take the LIDEX study one step further by limiting the dispersion analysis to float pairs that settle on the same isopycnal to within 0.1°C (≈0.01 kg m\(^{-3}\); see Supplemental Figure S1). The objective is to reduce the effect of vertical shear as much as possible. Unfortunately, we cannot preclude its presence entirely; from a hydrographic section we estimate typical vertical shear in the main thermocline to be \(\sim 1.5 \times 10^{-4} \text{s}^{-1}\) or about 1.5 mm s\(^{-1}\) per 0.01 kg m\(^{-3}\). Nonetheless, this is so far as we know the first attempt at a systematic study of dispersion of floats on the same isopycnal surface, what we call iso-isopycnal dispersion. The key quantities we seek to determine are the average rate of separation, \(D\), as a function of time \(<D(t)>\); the relative dispersion, \(<D(t)^2>\); and diffusivity as a function of separation, \(K(D)\). The next section gives a brief summary of the field program and how the float data were prepared for this study. The results are given in section 3. This is followed by a discussion in section 4.

2. The Field Program

The LIDEX took place in the eastern tropical Atlantic about 800 km south-southwest of the Cape Verde Islands along the southern edge of the low oxygen tongue that extends west from Africa. The objective was to employ isopycnal floats to study the roles of advection and mixing in maintaining the observed oxygen pattern. They were released in clusters at five sites arranged 1.5° apart in a cross pattern centered at 8.5°N 28 W. At each site 18 isopycnal floats were deployed, 10 on the 27.1 and 8 on the 27.3 kg m\(^{-3}\) density surfaces, for a total of 90 floats. The floats ballasted for the 27.1 surface were also equipped with Aanderaa oxygen sensors. The float missions were 600 days long.

Due to slight variations in ballasting the spread in temperature between the warmest and coldest floats in a group were up to 0.9°C for the 27.1 and 0.5°C for the 27.3 kgm\(^{-3}\) layer or roughly 70 and 110 m spread in depth (Figure S1). It is this spread in density space we circumvent by selecting those subsets that equilibrate within 0.1°C of each other on either density surface (roughly within 10 and 20 m on the upper and lower surfaces, resp.). From a total of 50 and 40 floats deployed on the two surfaces, 13 and 15 iso-isopycnal pairs could be constructed. Figure 1 shows the first 80 days trajectories of all floats deployed at site Rossby et al. 10.1029/2021GL093526 2 of 8

All 18 floats that were deployed at each of the 5 sites were released within minutes of each other to ensure that their drifts at depth started with little separation. Four RAFOS sound sources, pinging 4 times per day, provided the acoustic navigation. The absolute tracking accuracy was <3 km while the relative accuracy and tracking resolution were roughly 1 km, somewhat better for the deeper floats due to less multipath variability. The reader is referred to Rd for a comprehensive report and discussion of the LIDEX as a whole.

3. Results

The impact of vertical shear on relative dispersion is expected to be most keenly felt at short times before the floats have spread so far apart that mesoscale eddy dispersion is likely to dominate (and which we will see). Thus, the focus here is on the first few months. The trajectories in Figure 1 show a striking presence of inertial, and near-inertial oscillations, superimposed on the more gradual spreading. We analyze these first with a concern or suspicion that these could potentially play a role in dispersion.

3.1. Inertial Oscillations

To quantify the conspicuous inertial oscillations seen in Figure 1, we use complex demodulation, essentially a least square fit of an oscillation to the expected period (set by starting latitude), to estimate their amplitude and frequency. Jitter in the acoustic tracking prevents accurate amplitude and phase estimation when the radius of orbital motion is significantly less than 1 km. To provide a sense of the quality of the tracking data, Figure 2 shows in panel a) latitude as a function of time for two deep floats deployed at site...
3 (8.5°N 28°W). The mesoscale eddy field dominates the longer time scales with rapid inertial oscillations superimposed much of the time. Panel b) shows amplitude (in km) and panel c) the phase of the oscillations based on a 3.38 days inertial period at 8.5°N latitude. The amplitude and phase are estimated in 2 steps, first high-pass filter the trajectories (5 days cut-off) to remove the mesoscale; second a least-square fit (LSF) of a sinusoid to the residuals based on the inertial period, 12 h/sine (latitude), at launch (4.1, 3.38, 2.88 days at 7, 8.5 and 10°N, resp.). Each fit spans 42 points (10.5 days), repeated every 21 points (5.25 days). The upward slope of phase with time indicates that the oscillations have a slightly shorter period than expected for the latitude they are at. The actual period is shown in panel d): it hovers close to 3.1–3.2 days (dots plotted only if radius >0.8 km). Figure 3 shows histograms of amplitude and periods for all floats on the two surfaces. These show the probability of a float looping at a certain period in relation to the expected inertial period for the latitude it is at. As expected, inertial oscillations have periods near or slightly less than the inertial period with the floats on the deeper surface showing less spread (which may reflect less jitter in acoustic navigation at these depths). In both cases there are examples of periods longer than the local inertial period. Some may be misfits (poor LSF), but some appear to be real (Figure 2d). We assume this is possible where the mesoscale eddy field has a negative relative vorticity $\zeta$ so as to create an effective inertial frequency less than $f$ (Kunze, 1985). Given a nominal latitude of 8.5°N, a 1 km radius oscillation implies a 0.022 ms$^{-1}$ orbital velocity. The overall average speed of the circular motions is 0.025 and 0.018 ms$^{-1}$ for the two layers. The point is that these velocities are substantial compared to their initial rates of separation (next).

4. **Pair Separation Rates**

The mesoscale activity in the eastern tropical Atlantic is modest with 0.05 and 0.04 ms$^{-1}$ typical RMS velocities (excluding inertial oscillations) at 500 and 800 m depths. The fact that these are only a factor 2 greater than the inertial oscillation velocities explains why the orbital motion is so conspicuous along their
trajectories. However, the rate at which pairs separate when they are closer than approximately 6 km of one another is more than an order magnitude less than either the average mesoscale velocity field or the inertial motions. The average distance between all paired floats as a function of time is shown in Figure 4. Panel a) shows iso-isopycnal separation \(<D>\) for the two density surfaces (13 and 15 pairs, resp). The right panel b)
shows the corresponding dispersion $<D^2>$. What stands out is that over the first 100 days, $<D>$ as a function of time is virtually identical for the two density surfaces, lending significance to the pattern of separation: mean separation distance on both density surfaces grows to $\sim 5.5$ km after 30 days ($0.0021 \pm 0.0014$ ms$^{-1}$), thereafter linearly about 3x faster to $\sim 48$ km at 100 days since start. The growing error bars with increasing separation reflect the increasing influence of the mesoscale velocity field. Given the evident similarity of $<D>$ and $<D^2>$ for both isopycnals, we combine both datasets into one with 28 pairs, represented by the black lines, to reduce uncertainty. The left panel of Figure 5 shows the black dispersion curve again, now on a linear-log plot. The peaks and valleys during these 30 days appear to be generated by the residual inertial motion in separation distance (roughly a 3.5 day period) perhaps due to the fact three of the 5 sites are at the same latitude. This is substantiated by the red curve where we have filtered out the inertial oscillations

![Image 1](image1.png)

**Figure 4.** Panel a shows mean iso-isopycnal separation as a function of time. It includes standard error bars for all pairs for the two density surfaces (red and green) while black combines the two into one population. Panel b shows relative dispersion $<D^2>$ as a function of time for the two density surfaces (red and green) and combined into one population (black). In both panels the blue line incorporates all possible float pairs from both surfaces.

![Image 2](image2.png)

**Figure 5.** Panel (a) Linear-log plot of relative dispersion $<D^2>$ as a function of time for the first 60 days. $T_e$, the e-folding time for the first 30 days is $\sim 10.8$ days. Red line represents same data with inertial oscillations filtered from first 30 days. Blue line represents relative dispersion based on all possible pairs. Panel (b) Diffusivity $K = (1/4)d<D^2>/dt$ as a function of separation. The dotted and solid lines correspond to ±1 and 2 days first differences of $<D^2>$. The red lines represent iso-isopycnal pairs, the blue lines to all possible pairs. (Inertial oscillations removed for solid red line.) The thin black line $0.000375 x D^{4/3}$ summarizes the Rudnickas et al. (2019) findings.
for the first 30 days. The thin black line with an e-folding time of 10.8 days fits the result well. The right panel shows diffusivity, $K = \frac{\partial}{\partial t} \left\langle D^2 \right\rangle$, plotted as a function of separation from 2 to 100 km. From about 20 to 100 km the plot follows the classical 4/3-law (as indicated by the thin black line), reflecting the results reported by Rd. For separations less than 6 km, $K \sim 2–3 \text{ m}^2\text{s}^{-1}$, nearly an order of magnitude smaller than expected from the 4/3-law. The blue line is based on all possible float pairs within each group (i.e., no restriction to being on exactly the same density surface), which conceals the apparent transition between slower and faster diffusion regimes.

5. Discussion

Two scales dominate the float trajectories: inertial oscillations and the mesoscale velocity field. Almost certainly the “gracefulness” of the trajectories in Figure 1 can be attributed to the very large radius of deformation at these low latitudes, 100–140 km (Rd). But the inertial motions are so conspicuous this begs this question whether they are simply superimposed or play an active role in dispersion. Given the large amplitude of the inertial motions, it is tempting to think that these play a significant role in the observed dispersion, slow though this is, i.e., that the rate of separation is a residual effect from the inertial motions. In the next paragraph we explore this question further but conclude that we find no compelling evidence for this.

By working with float pairs within 0.1 C (roughly = 10 m in the vertical at 500 m) we are reducing the effect of vertical shear substantially, but we cannot claim that it has been eliminated for two reasons. First, with a $\sim 1.5 \times 10^{-4} \text{ s}^{-1}$ typical geostrophic shear there could be a $\sim 1–2 \text{ mms}^{-1}$ relative velocity; this is close to the average rate of pair separation we observe during the first 30 days. Second, the accuracy of the temperature measurement is ±0.1 C. The pressure measurement is a bit better at ±5 dbar. In any case, further reducing the density difference between pairs would be expected to lead to an even slower rate of separation than observed, which was already below what would be expected from the 4/3-law scaling. Close inspection of individual pairs shows that the transition from a slower to a faster rate of separation ranges between as early as 10 and 50 days but applies to roughly half of the pairs from both layers, panel a in “The remaining pairs stay within 5–15 km of each other for at least 80 days (panel b). The near-absence of an inertial signal in most of the separation time series implies highly coherent motion at small separations. Even at larger D, relative inertial motion is often or largely absent leaving the question unanswered: What drives the transition from a slow to faster rate of separation? We note that the inertial oscillations are not present in either temperature (not surprising given that the floats are isopycnal) or pressure suggesting little coupling of the oscillations with the density field. Further, comparison of separation with difference in either temperature or pressure shows no evidence of a connection, implying that it is not driven or caused by slippage due to stratification. Altogether it appears that the inertial oscillations simply radiate through mesoscale velocity field. A similar conclusion was obtained for surface drifters in the Gulf of Mexico (Veron-Bera and LaCasce, 2016). Left unanswered is the question of what happens when oscillations with slightly different periods interact. We are not aware of any literature on this topic. It should be noted that the velocity field appears to be in geostrophic balance. Given $U \sim 0.05 \text{ ms}^{-1}$ typical velocities, the Coriolis parameter ($f$) $\sim 1.2e^{-5}$, and a characteristic length scale (not separation) $L \sim 50 \text{ km}$, a characteristic Rossby number ($U/f\cdot L$) would be $\sim 0.05$.

The dominant pattern of a slow separation rate transitioning to a faster one occurs at different times after deployment and applies to only half of all pairs within the first 80 days considered here. Yet when all float pairs are assembled the “bi-linear” pattern of growth prevails, Figure 4a. The picture that emerges is a weak and variable strain transitioning to a more rapid rate as one of the floats is advected by the local velocity field perhaps toward an internal “front” or line of convergence. How soon this transition happens would depend upon proximity and time to such an encounter. Floats not on exactly the same isopycnal will separate more rapidly as indicated by the blue line in Figure 4a. We take this to reflect the action of vertical shear.

The relative dispersion $<D^2(t)>$ exhibits a very different character from separation. Here, Figure 5a, as in Figure 4b, the distinction between isopycnal dispersion and iso-isopycnal dispersion is striking. It is only the latter group that evinces exponential dispersion. This must reflect the 2-dimensional character of iso-isopycnal motion. The 10.8 days e-folding time for the first 30 days is nearly an order of magnitude longer than for relative dispersion at the surface: 2–3 days in the Gulf of Mexico (LaCasce and Ohlmann, 2003) and $\sim 0.5$ days in the Nordic Seas (Koszalka et al., 2009). But these studies took place in far more energetic
wind-driven surface waters compared to the stratified essentially 2-dimensional iso-isopycnal field examined here. Morel and Larchevêque (1974) and Er-El and Peskin (1981) found exponential dispersion with e-folding times of 1.35 and ∼1.5 days at 200 and 150 mbar levels respectively in the southern hemisphere atmosphere. The factor 10 longer e-folding time observed here surely must be related to the restriction to iso-isopycnal pairs dispersing slowly in a very nearly 2-dimensional flow field imposed by stratification on the one hand, and the large scales on the other.

The relative diffusivity K estimated from the iso-isopycnal pairs (red line) is substantially less than that estimated from all float pairs (blue line) for the smallest resolvable separations, <6km, Figure 5b. The dashed line for D < 10^2 m has a D^2 slope; it appears to fit the data well at these small separations and is consistent with the exponential dispersion D^2 ~ exp(t/T) in Figure 5a since K = d/dt (D^2) ~ 1/T•D^2. At the smallest scale resolved here (∼2 km) K ~ 2–3 m^2 s^-1 growing to ∼15 m^2 s^-1 at 10 km and by 20 km falling in line with the 4/3-law obtained by Rd. These results accord well with Ledwell et al. (1998), who estimate K ~ 2 m^2 s^-1 for 1–10 km separations. At 30–300 km scales they estimate K = 10^3 m^2 s^-1, similar to what Rd obtained for separations greater than ∼100 km. Significantly, the distinction of iso-isopycnal diffusivity appears to be largely lost at separations greater than ∼10 km where it merges with the blue line. For separations >20 km both fall in line with the 4/3-law even though this scale is still nearly an order of magnitude less than the radius of deformation (100–140 km). The Ledwell et al. (1998) study also took place in the quiet eastern North Atlantic (∼26°N, 29°W). This is relevant for LaCasce and Bower (2000), who, in their reanalysis of several float studies in the western North Atlantic, also find diffusivities with D^4/3 dependency but in all cases at far higher levels than reported here, perhaps due to higher eddy kinetic energy (EKE) levels. This EKE-dependence has been pointed out before (Price, in a figure shown in Rossby et al., 1983). When the separations become comparable to the radius of deformation, horizontal shear dominates vertical shear to such a degree that the details of ballasting become unimportant (LaCasce and Bower, 2000).

To summarize, it takes on average about 30 days for truly isopycnal (iso-iso) pairs to separate to ∼6 km. During this phase, the flow field appears to be controlled less by the mesoscale velocity and more by local 2-dimensional straining the nature of which will need study. The exponential character relative dispersion suggests that strain is proportional to separation (LaCasce, 2008). At 10–20 km separations float pairs become increasingly subject to the cascading of the mesoscale eddy velocity field defined by the radius of deformation to smaller and smaller scales. At these larger scales, the contribution of vertical shear to further separation plays an insignificant role.

Data Availability Statement

All RAFOS float data can be obtained at the AOML RAFOS data repository (https://www.aoml.noaa.gov/phod/float_traj/).

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