Quantum quench is a practical way how to bring a system out of equilibrium of thermalization among others. An experimental such as quantum computation, quantum chaos or emergence of quantum phase transitions is performed; hence, the initial state further evolves with rapid change of the control parameter to the new value of the parameter \( \lambda \). After some transition period the system approaches equilibrated regime characterized by time-independent mean values of the observables.

It is known that the equilibration process can be influenced by excited-state quantum phase transitions (ESQPTs) \([18, 15, 16, 18, 22]\). These are generalizations of ground-state quantum phase transitions (QPTs) \([28, 29]\) and primarily manifest as singularities in the level density of the excited states. They appear mostly in models describing some collective features of interacting many-body systems. In such cases, the infinite-size limit (thermodynamic limit) corresponds with the classical one \( \hbar \to 0 \) and ESQPTs are associated with the presence of stationary points in the classical version of the Hamiltonian \( H \to H_c \) \([30, 39]\).

In Ref. \([22]\) a detailed analysis of the ESQPT-induced effects on quench dynamics was performed within the class of such models derived from the Dicke model \([37–39]\). It was shown that the role of ESQPTs becomes significantly important if the dynamics is regular or weakly chaotic. In these cases the signatures of ESQPTs are clearly captured in the time evolution of survival probability of the initial state \( \langle \psi_i | \psi_i \rangle \) as well as in the time evolution of the observables. These signatures, however, depend on the details of the quench protocol as well.

Linking the specific features in the evolution of the survival probability of \( \langle \psi_i | \psi_i \rangle \) with the critical properties of the spectrum of \( H_t \) was one of the main results in Ref. \([22]\). In this paper, we show that these features can be understood from the quasiclassical perspective using the classical time evolution of the Wigner function as well as in the time evolution of the observables. The view through classical trajectories evolving in the phase space discloses the role of ESQPTs in the quench dynamics in a very intuitive way. Moreover, we show that the quantum survival probability during the equilibration as well as a typical power-law decay can be faithfully reproduced with quasiclassical techniques.

The paper is structured as follows. In Section II we introduce the model. The quasiclassical method and the quench protocols employed to probe the ESQPTs are discussed in the subsequent Section III. In Section IV we present the numerical results. Summary can be found in Section V.
II. TAVIS-CUMMINGS MODEL

A. Hamiltonian and Hilbert space structure

We consider the Tavis-Cummings Hamiltonian [38] (using the convention $\hbar = 1$)

$$H = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}} (b^\dagger J_- + b J_+),$$

(1)

which is obtained from the Dicke model [37] by applying the rotating wave approximation. This model can be intuitively understood as a simplified description of energy interaction between quantized monochromatic light with transition frequency $\omega_0$. The bosonic operators $b$, $b^\dagger$ annihilate and create photons. The response of the atoms to the radiation field is considered as collective, i.e., the individual atoms interact with the photons with the same phase factor. This assumption is valid if the spatial size of the atomic ensemble is much smaller than the wavelength of the photons. Therefore, the system of these subspaces is

$$M = 2j$$

from different $M$ subspaces. We can also express the dynamics of the system can be studied separately in any $M$ subspace by using the convention

$$\Delta \omega \geq \omega_0$$

This symmetry forbids any interaction between the states of the quasispin $j$ is linked with the total number of atoms simply as $N = 2j$ [47].

The Hamiltonian (1) is integrable. It means that apart from energy, there exists additional conserved quantity which effectively reduces the number of degrees of freedom by one. This quantity can be written as

$$M = b^\dagger b + J_z + j.$$  

(2)

It is easy to show that $[H(\lambda), M] = 0$ for any $\lambda$. It means that the Hamiltonian (1) conserves the total number of photons (term $b^\dagger b$) and atomic excitations (terms $J_z + j$). This symmetry forbids any interaction between the states from different $M$-conserving subspaces. Therefore, the dynamics of the system can be studied separately in any of these subspaces. We can also express the $M$ operator in the basis $|n\rangle |m\rangle \equiv |n, m\rangle \in \mathcal{H}_b \otimes \mathcal{H}_a$ where $n$ numbers the Fock basis in the Hilbert space $\mathcal{H}_b$ of photons and $m$ is the eigenvalue of the $J_z$ operator which determines the basis in the Hilbert space $\mathcal{H}_a$ of the atoms. Then we simply obtain $M = n + m + j$. Note that we do not distinguish explicitly between $M$ as an operator and a number as it should be always clear from the context. Considering the ranges of $n = 0, 1, 2 \ldots$ and $m = -j, -j + 1 \ldots j - 1, j$, we can number the individual subspaces by $M = 0, 1, 2 \ldots$. The dimension of these subspaces is $d = \min (M + 1, 2j + 1)$ [38].

B. Excited-state quantum phase transitions

We refer to the $M$-subspace with $M = N = 2j$ as the critical one. The reason is that only with this specific setting of the parameter $M$ both QPT and ESQPTs appear in the spectrum [38]. Let us define a dimensionless scaled energy $\varepsilon \equiv E/\omega_0 j$ as functions of the control parameter $\lambda$. Panel (a): Energy spectra $\varepsilon \equiv E/\omega_0 j$ as functions of the control parameter $\lambda$. Panel (b): Scaled mean value of the quasispin $z$-projection $J_z$ as a function of energy for fixed $\lambda/\lambda_c = 5$ as computed from the respective spectrum in panel (a). The red dashed line in the upper panel represents the critical energy $\varepsilon_c$. 

![Figure 1](image.png)

FIG. 1. Comparison of the critical subspace $M = 2j$ (upper row) and a non-critical one $M = j$ (lower row). Parameters used are $j = 20, \omega = 2\omega_0$. Panel (a): Energy spectra $\varepsilon \equiv E/\omega_0 j$ as functions of the control parameter $\lambda$. Panel (b): Scaled mean value of the quasispin $z$-projection $J_z$ as a function of energy for fixed $\lambda/\lambda_c = 5$ as computed from the respective spectrum in panel (a). The red dashed line in the upper panel represents the critical energy $\varepsilon_c$. 

$$\lambda_c = \frac{\Delta \omega}{2},$$

(3)

where $\Delta \omega = \omega - \omega_0$ (we consider $\omega \geq \omega_0$, there is a trivial mapping between the system with this and the reversed detuning hierarchy). On the other hand, the evolution of $\varepsilon_{q,s}(\lambda)$ is perfectly smooth in the non-critical $M = j$ subspace.
For \( \lambda > \lambda_c \) there is an ESQPT at energy \( \varepsilon_c = 1 \) in the critical subspace \([47]\) which can be also anticipated directly from Fig. 1(a). Indeed, as noted earlier, ESQPTs manifest as singularities in the level density and in our case this corresponds to the sequence of avoided crossings in the vicinity of the critical point \( \lambda_c \), along the energy \( \varepsilon = \varepsilon_c \) for \( \lambda > \lambda_c \). However, a clear evidence is provided by Fig. 1(b) where the mean value of \( J_z \) in individual eigenstates is plotted against the respective eigenenergies while parameter \( \lambda/\lambda_c = 5 \) is fixed. In the case of the critical subspace (upper panel), we see a sharp spike at energy \( \varepsilon_c \) marking the presence of an ESQPT. Note that similar figures like in panel (b) with a ‘non-analytic’ spike at \( \varepsilon_c \) can be obtained for any \( \lambda > \lambda_c \) in the critical subspace. Here we chose \( \lambda/\lambda_c = 5 \) because in the respective spectrum no apparent avoided crossings are visible for this coupling (which is due to the moderate value of \( j \)). However, the energy dependence of \( \langle J_z \rangle \) demonstrates the presence of the singularity anyway.

In order to reveal the structure of the ESQPTs, let us introduce the classical version of the Hamiltonian (1)

\[
H_{cl}(x,p) = \frac{\Delta \omega^2}{2} (x^2 + p^2) + \omega_0 (M - j) + \frac{\lambda x}{\sqrt{\omega}} \sqrt{J^2 - \left( M - j - \frac{x^2 + p^2}{2} \right)^2},
\]

where \( x \) and \( p \) form a conjugate pair of classical position and momentum. The classical limit is discussed in more detail in Appendix A. An example of the (scaled) energy profile \( h_{cl} = H_{cl}/\omega_0 j \) from Eq. (4) is plotted in Fig. 2 for the critical (upper row) and non-critical (lower row) values of \( M \). Note that the phase space is finite. This is due to the fact that Eq. (2) limits the expression \( x^2 + p^2 \leq 2M \). The respective one-dimensional cut at zero momentum \( v(x) \equiv h_{cl}(x,p = 0) \), defining a ‘quasipotential’, is also shown. The stationary point at the corresponding energy \( \varepsilon_c \) is directly visible in the critical subspace.

\section{III. QUANTUM QUENCH DYNAMICS}

\subsection{A. Survival probability}

Quantum survival probability is defined as

\[
P_{\text{qm}}(t) = |\langle \psi(t) | \psi(0) \rangle|^2 = |\langle \psi_i | e^{-iHt} | \psi_i \rangle|^2.
\]

With this quantity we can monitor the time evolution after a quench and disclose different regimes of the equilibration process. There is a direct link via Fourier transform to the distribution of the initial state in the final eigenbasis \( |E_{tk}\rangle \), the so-called strength function or local density of states, \( S(E) = \sum_k |\langle E_{tk} | \psi_i \rangle|^2 \delta(E - E_{tk}) \). Therefore, all information about the quench dynamics is fully encoded in this distribution function \([22, 49]\). The relation between the ESQPTs, strength functions and quantum survival probability can thus be established. The calculation of the quantum survival probability \([5]\) is performed exactly using diagonalization of the initial and final Hamiltonians in the finite-dimensional Hilbert space.

\subsection{B. Quasiclassical method}

We will also reconstruct the quench dynamics using the quasiclassical method \([40, 41]\) defined as a classical time evolution of the Wigner distribution corresponding to the initial state

\[
W_i(x,p) = \frac{1}{\pi} \int_{-\infty}^{\infty} \psi_i^*(x + y) \psi_i(x - y) e^{2ipy} dy,
\]

where \( x, p \) are classically conjugate position and momentum. The initial state is expressed as a linear combination \( |\psi_i \rangle = \sum_n c_n | n, m = M - n \rangle \), where \( c_n \) are expansion coefficients in the \( |n, m \rangle \) basis of the whole oscillator-quasispin system. The wave function in Eq. (6) is then obtained as \( \psi_i(x) = \sum_n c_n \psi_n(x) \) where \( \psi_n(x) \) is the \( x \)-representation of the Fock state \( |n \rangle \) in the oscillator space

\[
\psi_n(x) = \langle x | n \rangle = \frac{1}{\sqrt{2^n n!\sqrt{\pi}}} e^{-x^2/2} H_n(x),
\]

with \( H_n \) denoting the Hermite polynomial. The justification of this procedure is given in Appendix A.

The propagation towards \( t > 0 \) is realized according to the classical Liouville equation with the Hamiltonian \([4]\) for a bunch of classical trajectories. The initial positions of these trajectories \( |x_k, p_k \rangle \) are located on a grid indexed by \( k \) and sampling a finite region of the phase space which

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{fig2.png}
\caption{Comparison of the critical subspace \( M = 2j \) (upper row) and a non-critical one \( M = j \) (lower row). Parameters used are \( j = 20, \omega = 2\omega_0 \). Classical Hamiltonian \( h_{cl} = H_{cl}/\omega_0 j \) in the phase space for fixed \( \lambda = 2.5 \) is shown together with the cut \( v = h_{cl}(x,p = 0) \).}
\end{figure}
is designated by non-zero values of the initial Wigner distribution. Each trajectory is associated with a specific weight corresponding to the value of the initial Wigner distribution at that phase-space point (for our choices of initial states being the ground states of $H_i$, the respective Wigner function has no negative components; therefore, its probabilistic interpretation is applicable). The trajectories are used to calculate a quasiclassical phase-space distribution at different times.

We use a numerical procedure which interpolates between individual trajectories by adding a narrow Gaussian, corresponding with the spacing of the initial grid of trajectories, to each of them. Such a width is held constant during the evolution. The trajectories are also used to directly calculate a quasiclassical analog to the quantum survival probability given as an overlap between the initial state and the evolution of the state up to the time $t$.

$$P_{cl}(t) = 2\pi \int W_i(x, p) W(x, p, t) \, dx \, dp,$$

where we integrate over the full phase space.

Common schemes for the time propagation are based on a power expansion of the position and momentum function in time where the higher derivatives are obtained from the previous time steps and the Hamilton equations of motion. Such expansions are non-converging in the case of the Tavis-Cummings model, and thus we apply a different approach. We make use of the energy conservation law which implies that each trajectory follows an equivalence line of the classical Hamiltonian in the two-dimensional phase space. First, the energy equivalence line is determined for each point $[x_k, p_k]$, which defines the initial condition for a trajectory in the phase space. The equivalence line is defined by a set of positions $[x_k^F, p_k^F]$, which are determined using standard geometrical procedures applied for a two-dimensional surface of $H_{cl}(x, p)$ defined on an equidistant grid in the $x$ and $p$ coordinates. Then the time elapsed between each two points, $t_{k+1} - t_k$, is reconstructed using the Hamiltonian equations of motion.

This quasiclassical approach to computing survival probability does not capture quantum correlations, and therefore is bound to fail whenever they become significant. The breakdown occurs whenever the semiclassical van Vleck Gutzwiller propagator starts to include more then one root. In order to obtain the long-time behavior, more sophisticated quasiclassical methods including quantum corrections must be employed, see Refs. [55][57]. The specific features of equilibration related to ESQPTs appear dominantly at earlier stages of the time evolution. Thus, the quasiclassical approach as described above represents a reliable tool for our study.

![FIG. 3. Sketch of the critical quench protocols used in the paper. Upper row: Forward quench critical protocol. Lower row: Backward quench critical protocol. The red line indicates the equal energy of the stationary point and the value $v(x)$ at the coordinate of the center of mass of the initial Wigner function.](image)

**C. Quench protocols**

In this study we always initialize the system in the ground state of $H_i$. We call the quench critical if the subsequent equilibration (i.e., time evolution according to $H_i$) is directly affected by an ESQPT. This happens if the support of the strength function $S(E)$ overlaps with the ESQPT critical energy $E_c$. In the forward quench protocols, we start with the ground state in the critical subspace for $\lambda_i = 0$ and then rapidly increase the interaction above its critical value $\lambda_i > \lambda_c$. Classically viewed, it represents a state placed initially at the bottom of the quadratic potential at $x = 0$ which subsequently starts evolving due to the fact that the potential profile has been abruptly changed. After the quench, the initial state is located at the stationary point corresponding to the ESQPT, see Fig. 3 upper row.

The second option how to make the dynamics influenced by an ESQPT, is the backward quench protocol, see the lower row of Fig. 3. We start with the system in the ground state of $H_i$ where $\lambda_i > \lambda_c$, so classically the state is located at the minimum energy point with $x \neq 0$. The value of $\lambda_i < \lambda_i$ is chosen such that the point of the initial minimum is lifted up to the energy corresponding to the ESQPT energy (as indicated by the red dot-dashed line in Fig. 3). Unlike in the previous case, the initial state is not located directly at the stationary point after the quench. However, for a system with only a single degree of freedom the energy-conserving dynamics makes the trajectories explore the whole area of the phase space with the same energy (provided that it is connected). Therefore, the stationary point of the classical Hamiltonian will affect the time evolution at later
FIG. 4. Forward quench in a the non-critical subspace $M = j/2$. The parameters were chosen as $j = 200$, $\omega = 2\omega_0$, $\lambda_i = 0$, $\lambda_f = 5\lambda_c$. Panel (a): Comparison of quantum and classical survival probability. The green line indicates the power-law decay $\propto 1/\tau$ of the revivals. Panel (b): Same as in panel (a) with the linear scale. Panel (c): Snapshots of the classically evolved Wigner function for multiple values of $\tau$. The respective time points are marked by crosses in panel (a). The initial state $|\psi_i\rangle$ is the ground state of the initial Hamiltonian. The respective initial Wigner function $W_i$ is plotted in grey. Video available from Ref. [58].

IV. NUMERICAL RESULTS

A. Forward quench protocols

We start with the forward protocols with $\lambda_i = 0$ and $\lambda_f = 5\lambda_c$. From now on we will plot all results in the time domain using a scaled dimensionless quantity $\tau \equiv (\omega_0 j) t$. Figure 4 represents an example of a non-critical quench in the $M = j/2 = 100$ subspace. Quantum survival probability $P_{qm}$ in panels (a) and (b) exhibits the standard features of 'regular' quench dynamics as described in Ref. [22]. Namely, one can identify an initial Gaussian decay $\propto \exp (-\alpha \tau^2)$, $\alpha > 0$, followed by sharp revivals whose amplitudes show power-law decay $\propto 1/\tau$ leading to the final equilibrated regime.

Survival probability $P_{cl}$, reconstructed from the classical evolution of the Wigner function, is also plotted in Figs. 4(a) and (b) with the red dashed line. It faithfully reproduces the quantum version.

In Fig. 4(c) some snapshots of the classically evolved Wigner function in the phase space are presented. Initially at $\tau = 0$, the Wigner function has a Gaussian shape as it effectively corresponds to the ground state of a simple harmonic oscillator. It further evolves along closed trajectories representing equilibrium lines of the classical
Hamiltonian. At short time scales around $\tau \approx 16.7$, the Gaussian shape is still preserved and the classically evolved Wigner function is slightly displaced, which leads to a gradual decrease of $P_{cl}$. For $\tau = 166.7$, there is a zero overlap with the initial $W_i$. The shape is also modified as different trajectories propagate with different velocities. The first revival appears at circa $\tau = 333.2$ when the first period is completed. Note that the original Gaussian shape has been almost completely recovered, which corresponds with a nearly perfect revival $P_{cl} \approx 1$.

This scenario further repeats itself. Transition to the equilibrated regime is related to the fact that individual trajectories dephase as time grows. As a result, the recurrences start to overlap which is the moment when our quasiclassical method fails. This also qualitatively explains why amplitudes of the revivals start decaying. For $\tau = 1667$ it is clearly seen that the classically evolved Wigner function does not recover the initial Gaussian shape when recurring through the initial point in the phase space, which is a direct consequence of the above mentioned dephasing. In Appendix B we provide some analytic insight into the time evolution of $P_{cl}$ related to the non-critical quench. Namely, based purely on quasiclassical analysis, we reveal the origins of the initial Gaussian decay as well as of the $1/\tau$ attenuation of the revivals.

The critical quench, as depicted in Fig. 5, shows different time evolution. The most striking difference is the long initial survival and the absence of the large region after initial decay where the survival probability would vanish as in the non-critical quench. This is a consequence of the ESQPT, see Ref. [22]. Comparing $P_{qm}$ and $P_{cl}$ in panel (a), one can see that the quasiclassical approach overestimates the quantum survival probability significantly at early stages. Such a discrepancy is not observed in the case of non-critical dynamics (Fig. 4(a)) which leads us to a conclusion that this artifact is not explained by the semiclassical limit approximation ($j \to \infty$) nor as a numerical error. Rather, we anticipate a genuine quantum phenomenon taking place near the stationary point. It remains an open question whether such a phenomenon could be addressed using the advanced quasiclassical method which takes into account interference between different roots of the van Vleck Gutzwiller propagator [57]. Nevertheless the main features of the quench dynamics are still captured within our approach.
Figure 5(c) provides an explanation of the ESQPT-induced effect on the survival probability from a quasi-classical viewpoint. This time the initial Wigner function $W_i$ at $\tau = 0$ is placed directly at the stationary point of the classical Hamiltonian (this point gives rise to the ESQPT at the quantum level). If a classical trajectory lies at exactly the same energy as the stationary point, the passage through it is infinitely slow. Similarly, the trajectories in the adjacent region are significantly slowed down here which effectively stabilizes the initial state at the beginning. This also leads to a rapid dephasing of the individual trajectories already during the first period, cf. Fig. 4(c) and Fig. 5(c) at the times $\tau = 166.7$ and $\tau = 333.2$. Due to the fact that a part of the classically evolved Wigner function is stuck at the initial point, there is a small overlap for any $\tau > 0$.

### B. Backward quench protocols

In the backward protocols we set $\lambda_i = 5\lambda_c$ and the initial ground state $|\psi_i\rangle$ is obtained from the diagonalization of the Hamiltonian $H_i$. The respective Wigner function in $(x,p)$ variables corresponding to the classical Hamiltonian is then computed using QuTiP. See Eqs. (6) and (7), and Appendix A for more details. We will fix $M = 2j$ so the dynamics will take place in the subspace with the ESQPT. By precise tuning of $\lambda_f < \lambda_i$, the quench dynamics can be influenced by the ESQPT (critical quench) or cannot (non-critical quench).

An example of a non-critical quench is depicted in Fig. 6. Because the initial Wigner function $W_i$ can be well approximated by a Gaussian, the situation is very similar to Fig. 4. One can observe an initial Gaussian decay and subsequently a series of revivals decaying as $1/\tau$ in panels (a) and (b), again, see Ref. [22] for details. Panel (c) shows a few snapshots of the classical time evolution of the Wigner function.

If the final value of the control parameter is finely tuned to $\lambda_f = 1.544\lambda_c$, the initial Wigner function occupies a part of the phase space with the same energy as the energy of the stationary point. This critical backward quench is depicted in Fig. 7. The Gaussian profile $P \propto \exp(-\alpha t^2)$ of the initial decay is still present in panels (a) and (b). Indeed, the stationary point does not affect the initial evolution as it is located elsewhere in the
phase space. However, when the trajectories reach that location, some get trapped there similarly as in the forward critical quench. This leads to a rapid dephasing and so no revivals appear in the survival probability. Faster dephasing then essentially leads to faster transition to the equilibrated regime. Indeed, Fig. 7(c) for $\tau = 10^4$ shows that most of the available phase space is already covered with trajectories.

V. SUMMARY

Quench dynamics was studied in the class of integrable Dicke-type models. We showed that quasiclassical technique based on the Wigner phase-space distribution function can faithfully reproduce quantum survival probability during different stages of equilibration. The method breaks down at the Ehrenfest time scale when the Wigner distribution spreads over a considerable fraction of the accessible phase space before the onset of the equilibrated regime. Up to that time, all our results are in agreement with those in Ref. [22] obtained by a pure quantum analysis. The evolution of quasiclassical survival probability in the specific stages was directly linked with both qualitative and quantitative features of the dynamics of the classically evolved Wigner function in phase space. In particular, the power law $\propto \tau^{-1}$ decay of revivals in non-critical quenches was explained by an analytic quasiclassical calculation (Appendix B).

The effects of ESQPTs on the dynamics of the system were demonstrated at the quasiclassical level. ESQPTs are induced by stationary points of the classical Hamiltonian. From the viewpoint of the classical evolution, the trajectories with the same or approximately the same energy as these stationary points get trapped in their vicinity. Depending on the type of the quench protocol, this leads either to stabilization of the initial state (forward quench) or, on the contrary, to faster transition to the equilibrated regime (backward quench).

It was already proposed in Ref. [22] that dynamical ‘fingerprints’ such as those reported here could be used to detect ESQPTs experimentally as their direct spectroscopic measurement is in many cases impossible. Therefore, the intuitive view of the quench dynamics through classical trajectories as presented in this paper brings additional benefit for interpretation of eventual experi-
mentical results. On the other hand, considering purely classical dynamics has its limitations. Especially close to the stationary points, quantum correlations affect the dynamics already at early stages. So, despite the good overall match, the details of the quantum survival probability cannot be captured by our method. This is in compliance with the standard textbook knowledge that quasiclassical approach fails in the vicinity of stationary points. Applications of more sophisticated quasiclassical techniques in relation to the ESQPTs remain open for future studies.

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Appendix A: Classical limit of the Tavis-Cummings Hamiltonian

The classical limit of the Hamiltonian (11) can be obtained through the mapping

\[
(J_x, J_y, J_z) = \left( \sqrt{j^2 - j_z^2} \cos \phi, \sqrt{j^2 - j_z^2} \sin \phi, j_z \right),
\]

(A1)

\[
(b, b^\dagger) = \frac{1}{\sqrt{2}} (\hat{x} + i \hat{p}, \hat{x} - i \hat{p}),
\]

(A2)

where \((\hat{x}, \hat{p})\) and \((\phi, j_z)\) are two pairs of mutually conjugate coordinates and momenta [15]. Considering that in the \(j \to \infty\) limit these quantities can be replaced by ordinary numbers, we obtain the classical Hamiltonian

\[
H_{cl} = \omega \frac{\hat{x}^2 + \hat{p}^2}{2} + \omega_0 j_z + \lambda \sqrt{\frac{3}{2}} \sqrt{j^2 - j_z^2} (\hat{x} \cos \phi - \hat{p} \sin \phi).
\]

(A3)

The conserved quantity [2], which is rewritten as

\[
M = \frac{\hat{x}^2 + \hat{p}^2}{2} + j_z + j,
\]

(A4)

connects the degrees of freedom of the atom and field subsystems. Separation of a single effective degree of freedom is achieved by the canonical transformation [17]

\[
\begin{align*}
(x, p) &= \left( \cos \phi - \sin \phi \right) \hat{x} + \left( \sin \phi \cos \phi \right) \hat{p}, \\
(\varphi, M) &= \left( \phi + M - j \right) \hat{x} + \left( M - j \right) \hat{p},
\end{align*}
\]

(A5)

which leads to the Hamiltonian (11). Since angle \(\varphi\) does not appear in the Hamiltonian, the quantity \(M\) (and of course also \(M\)) is an integral of motion.

As we see in Eq. (A5), the angle \(\phi\) from the quasispin representation (A1) determines the relation of the new coordinate-momentum pair \((x, p)\) describing the coupled atom-field system to the old one \((\hat{x}, \hat{p})\) characterizing the field subsystem alone. However, it turns out that this angle is completely arbitrary. To show this, we first note that quantum expectation values \(\langle J_x \rangle\) and \(\langle J_y \rangle\) cannot determine \(\phi\) as they vanish in any eigenstate of the Hamiltonian with a fixed eigenvalue of parity \(P = \epsilon \Delta M\).

The discrete symmetry under this parity transformation applies to a wide class of Dicke-type Hamiltonians including the present one. Moreover, the Tavis-Cummings Hamiltonian (11) possesses also a continuous symmetry under the transformation \(U(\alpha) = e^{i\alpha(M-j)}\) with arbitrary angle \(\alpha \in (0, 2\pi)\). This results in a gauge rotation \((b, b^\dagger) \to (e^{-i\alpha} b, e^{i\alpha} b^\dagger)\) of the boson operators and a simultaneous counter-rotation \(J^{\mu} \to e^{i\alpha J^{\mu} \sin \varphi} e^{-i\alpha J^{\mu} \cos \varphi}\) of the quasispin operators with \(k = x, y, z\) [15]. In fact, the conservation of quantity \(M\) follows from this symmetry. In the coordinate-momentum form the transformation reads as follows,

\[
\begin{align*}
\left( \frac{\hat{x}'}{\hat{p}'} \right) &= \left( \cos \alpha \sin \alpha \right) \left( \hat{x} \right) \\
\left( J^{x'}_{x}, J^{y'}_{y} \right) &= \left( \cos \alpha - \sin \alpha \cos \alpha \right) \left( J^{x}_{x}, J^{y}_{y} \right),
\end{align*}
\]

(A7)

where formula (A7) is of the same form as the canonical transformation (A5). Therefore, any preselected value of angle \(\phi\) can be altered to any other value \(\phi'\), particularly to \(\phi' = 0\), by applying the gauge transformations in Eqs. (A7) and (A8) with \(\alpha = \phi - \phi'\).

With this background we are ready to accept that the classical Hamiltonian (11) can be written with \((x, p)\) replaced by the original coordinate-momentum pair \((\hat{x}, \hat{p})\). A direct derivation avoiding canonical transformation in Eqs. (A5) and (A6) is possible via inserting \(j_z\) calculated from the constraint (A4) into the classical Hamiltonian (A3) with \(\phi = 0\). In view of the above considerations, this can be interpreted merely as a choice of a particular ‘gauge’ in which \((x, p)\) of the coupled system coincides with \((\hat{x}, \hat{p})\) of the field subsystem.

These explanations justify the procedure used in this paper to calculate the Wigner function (6). The coordinate and momentum operators in any fixed-M subspace of \(\mathcal{H}_b \otimes \mathcal{H}_a\) are written in analogy to those in the Fock space \(\mathcal{H}_b\) of the field subsystem, cf. Eq. (A2), but in the form that strictly conserves the value of \(M\):

\[
x = \frac{1}{\sqrt{2}} (b^\dagger L_- + b L_+), \quad p = \frac{1}{\sqrt{2}} (b^\dagger L_- - b L_+).
\]

(A9)

Here \(L_{\pm} = [J_{+} J_{-}(J_z \pm 1)]^{-1/2} J_{\pm}\) are normalized ladder operators in \(\mathcal{H}_b\) which compensate changes of the boson number \(n\) induced by \(b^\dagger b\) and \(b\). It can be shown that for very large \(j\) and \(|m| \ll j\) the operators in Eq. (A9) approximately satisfy the expected commutation relation \([x, p] = i\). Under these conditions the relation \(b^\dagger J_{-} + b J_{+} \approx x \sqrt{j^2 - j_z^2}\) immediately transforms the interaction term of the Hamiltonian (11) into
The time change $\tilde{\tau}$ as well as the expression of the initial state wave function according to initial Wigner function has a form of a Gaussian centered (the same symbol as for the operator) is given by of the position operator in Eq. (A9) with an eigenvalue states $\psi_{n}$ relation for one-dimensional harmonic oscillator eigenfunctions.

its classical limit in Eq. (4). Using the known recursive relation for one-dimensional harmonic oscillator eigenstates $\psi_{n}(x)$ we can easily prove that the eigenvector of the position operator in Eq. (A9) with an eigenvalue $x$ (the same symbol as for the operator) is given by $|x\rangle = \sum_{n} \psi_{n}(x)^{\ast}|n, m = M - n\rangle$. This results in the expression of the initial state wave function according to Eq. (7) and the text formula above it.

Appendix B: Quasiclassical analytic evaluation of a non-critical quench

In this appendix we derive the Gaussian initial decay as well as the $\propto 1/t$ power-law attenuation of the revivals in the case of a non-critical quench. We assume that the initial Wigner function has a form of a Gaussian centered at point $(x_0, p_0)$ in the phase space, i.e.,

$$W_{i} = N \exp \left[ -\frac{(x - x_0)^{2}}{2\sigma_{x}^{2}} \right] \exp \left[ -\frac{(p - p_0)^{2}}{2\sigma_{p}^{2}} \right], \quad (B1)$$

where $\sigma_{x}$ and $\sigma_{p}$ are the respective dispersions and $N$ is a proper normalization. As the evolution always starts from rest, from now on we set $p_0 = 0$. To make further calculations feasible, we employ the following approximations.

i. Around the initial point $(x_0, p = 0)$ there is a domain of size $\sigma_{x}\sigma_{p} = \hbar$ corresponding to the quantum uncertainty. In the classical limit $\hbar \to 0$ the size of this region is small so we assume the energy equivalence lines to be approximately parallel with the $p$ axis within the phase space initially covered by the probability density, see Fig. 8.

ii. We also assume that the classical Hamiltonian can be approximated linearly within this small region. The classical potential $V(x) = H_{cl}(x, p = 0)$ is approximately a linear function in the vicinity of $x_0$, i.e., $V(x)|_{x = x_0} = E_0 + V'(x_0)(x - x_0)$, where $E_0 = H_{cl}(x_0, 0)$ and $V'(x_0) = dV/dx|_{x = x_0}$. The kinetic energy, on the other hand, is represented by a constant as $dH_{cl}/dp = 0$. This means that in this region $H_{cl}(x, p)$ is approximately constant in $p$.

These assumptions can be justified by a detailed investigation of $H_{cl}(x, p)$, from Eq. (4). As a result, we write the classical Hamiltonian around $(x_0, 0)$ as

$$H_{cl}(x, p) = E_0 + V'(x_0)(x - x_0), \quad (B2)$$

where $E_0 = H_{cl}(x_0, 0)$ and $V'(x_0) = dV/dx|_{x = x_0}$.

Now we define classical survival probability for a given energy shell $E$

$$P_{cl}(t, E) = 2\pi \int W_{i}(x, p)W(x, p, t) \delta[H_{cl}(x, p) - E] \, dx \, dp. \quad (B3)$$

Apparently, by integration of $P_{cl}(t, E)$ over the energy domain, one recovers $P_{cl}(t)$ from Eq. (8). Let $\tau(E)$ be a period of the trajectory. $P_{cl}(t, E)$ acquires non-zero values only when the trajectory is recurring through the initial region. This means that further on we can restrict our evaluation to the times which are approximately integer multiples of the period $t \approx n\tau(E)$. According to the assumption i., the trajectory evolves only in $p$ direction, so one can write

$$W(x, p, t) = N \exp \left[ -\frac{(x - x_0)^{2}}{2\sigma_{x}^{2}} \right] \exp \left[ -\frac{(p - \dot{p}(t - n\tau))^{2}}{2\sigma_{p}^{2}} \right], \quad t \approx n\tau(E). \quad (B4)$$

The time change $\dot{p}$ is linked with the properties of the potential as $\dot{p} = -V'(x_0)$. We also directly inserted the time argument in the form $t - n\tau(E)$ which reflects the periodicity.

Using Eq. (B2) we define $x(E) = (E - E_0)/V'(x_0) + x_0$ and evaluate the integral

$$P_{cl}(t, E) = 2\pi \int W_{i}(x, p)W(x(E), p, t) \, dp \quad (B5)$$

$$= 2\pi N^{2} \sqrt{\pi\sigma_{p}} \exp \left[ -\left( \frac{E - E_0}{V'(x_0)\sigma_{x}} \right)^{2} \right] \exp \left[ -\left( \frac{V'(x_0)}{2\sigma_{p}} \right)^{2} (t - n\tau(E))^{2} \right].$$
As already noted, to obtain $P_{cl}(t)$ equivalent to Eq. (8), we have to integrate over the energy domain. Before we do so, we need to insert one more assumption on the function $\tau(E)$. We will consider the simplest non-trivial dependence

$$\tau(E) = \tau(E_0) + \beta(E - E_0),$$  \hspace{1cm} (B6)

where $\tau(E_0)$ is the period of the trajectory passing through $x_0$ (which has energy $E_0$) and $\beta$ is real positive number. Now we can write

$$P_{cl}(t) = \frac{2\pi N^2 \sigma_p \pi}{\sqrt{A^2 + B^2}} \exp \left[ -A^2 B^2 \frac{t}{A^2 + B^2} C^2(t) \right],$$  \hspace{1cm} (B7)

with

$$A = \frac{1}{V'(x_0) \sigma_x}, \quad B = \frac{V'(x_0)n^2}{2\sigma_p},$$  \hspace{1cm} (B8)

$$C(t) = \frac{t - \tau(E_0)n}{n\beta}.$$  \hspace{1cm} (B9)

The integration (B7) yields

$$P_{cl}(t) = \frac{2\pi N^2 \sigma_p \pi}{\sqrt{A^2 + B^2}} \exp \left[ \frac{-A^2 B^2}{A^2 + B^2} C^2(t) \right].$$  \hspace{1cm} (B10)

If we focus on the initial decay, we set $n = 0$. In that case $BC(t) = V'(x_0)t/2\sigma_p$ and $B = 0$. So the initial time evolution of survival probability is

$$P_{cl}^{n=0}(t) = \frac{2\pi N^2 \sigma_p \sigma_x}{4\sigma_p^2} V'(x_0) \pi \exp \left[ -\frac{V'(x_0)^2}{4\sigma_p^2} t^2 \right],$$  \hspace{1cm} (B11)

which corresponds with the observed initial Gaussian decay.

Further on, let us reveal the origin of $1/t$ attenuated revivals. The maxima of (B10) are reached when the exponential factor becomes equal to one. This is obtained for $t = \tau(E_0)n$ which makes coefficient $C(t)$ vanish. For such values of $t$, the center of $W(x,p,t)$ is exactly recurring through the initial point $x_0$. With these assumptions

we write

$$\log P_{cl}^{n=t/\tau}(n) = \log 2\pi N^2 \sigma_p \pi - \frac{1}{2} \log (A^2 + B(n)^2)$$

$$= \log 2\pi N^2 \sigma_p \pi - \log B(n)$$

$$- \frac{1}{2} \log \left( 1 + \frac{A^2}{B(n)^2} \right).$$  \hspace{1cm} (B12)

Note that now we investigate these revivals in survival probability $P_{cl}^{n=t/\tau}(n)$ as a function of the number of recurrences. Now we suppose that $A \ll B(n)$ which according to Eq. (B8) leads to

$$\frac{\sigma_p}{\sigma_x} \frac{2}{V'(x_0)} \ll n\beta.$$  \hspace{1cm} (B13)

This inequality is generally fulfilled if the potential is ‘steep enough’ around $x_0$. Taking into account the condition (B13), the last term in Eq. (B12) can be neglected; thus, one obtains

$$P_{cl}^{n=t/\tau}(n) = \frac{2N^2 \sigma_p^2 \pi}{V'(x_0)\beta n}.$$  \hspace{1cm} (B14)

Recalling $n = t/\tau(E_0)$, Eq. (B14) shows the power-law decay 1/t of the revivals.

In the case of the critical forward quench, this derivation fails (mainly) because the assumption ii. is not fulfilled. Indeed, $x_0$ is a stationary point of the classical potential $V'(x_0) = 0$. In the critical backward quench, however, both the assumptions i. and ii. are reasonably fulfilled; therefore, the initial Gaussian decay (B11) is observed. The 1/t power-law decay of the revivals is not observed, because the assumption on the linear scaling of the period with energy (B6) does not hold. This is a direct result of the dephasing of the trajectories at the stationary point of $H_{cl}$.

For analytical insight into computation of the survival probability from the quantum perspective we refer the reader to Ref. [61]. The results are in agreement with those derived in this appendix.

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