Modeling of Non-Newtonian resin flows in Composite Microstructures

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Abstract. The multi-scale modeling of the flow of the non-Newtonian viscosity resin in the composite structure is based on the asymptotic homogenization method, and a so-called local problem is obtained. The finite element method was used to calculate the local problem, and the distribution of single hole velocity, pressure and non-Newtonian viscosity was obtained.

1. Introduction

Plastic is a very widely used polymer composite applications, the main component is a resin. In the plastic manufacturing process, a resin-based polymer solution is usually injected into a specific mold. Especially for composites with resin as a reinforcing material, the flow of non-Newtonian resin in the composite microstructure is very important [1]. Therefore, in order to optimize these processes, numerical modeling of the flow of non-Newtonian resins in composite microstructures has become very important [2].

2. Main assumptions of the model

2.1. Geometric model

Consider a three-dimensional orthogonal composite structure [3]. (Fig. 1). It is assumed that the composite structure is characterized by periodicity. Wherein red indicates a solid material, and transparent color indicates a polymer reinforcing material (resin polymer solution).

2.2. The model assumptions

Take the following assumptions regarding the properties of the phases: (1) fluids are isotropic non-Newtonian viscous incompressible media; (2) the porous skeleton is assumed to be non-deformable, i.e. his movement is not considered; (3) fluid movement is isothermal; (4) mass forces are absent.

We introduce the following notation: $l_0$ - characteristic size of periodicity cell (PC) $V_0$; $x_0$ - characteristic size of the global entire porous medium $V_0$; $\kappa = l_0/x_0 << 1$ - small parameter; $\xi = \chi/\kappa$ - dimensionless local coordinates varying within a periodicity cell (PC) $V_0$.

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3. Principal equations

The motion of non-Newtonian resins in the composite occupying the region \( V \), within the framework of the assumptions made, is described by the system of the incompressible Navier-Stokes equations, which, together with the adhesion conditions on a solid surface, have the following form [4]:

\[
\nabla \cdot \vec{v} = 0 \quad (1)
\]

\[
\rho \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \nabla \cdot \mu \left( \nabla \otimes \vec{v} + \nabla \otimes \vec{v}^T \right) \quad (2)
\]

\[
\vec{v} \big|_{\text{BC}} = 0 \quad (3)
\]

\[
p \big|_{t=t_0} = p_0 \quad (4)
\]

where \( \rho \) - density; \( \vec{v} \) - velocity of fluid; \( p \) - pressure; \( \mu \) - non-newtonian viscosity and \( \otimes \) - tensor product. The strain rate tensor is defined as follow:

\[
D = \frac{1}{2} \left( \nabla \otimes \vec{v} + \nabla \otimes \vec{v}^T \right)
\]

The AI and AV models of Voigt isotropic viscous media [4] are considered, and non-Newtonian viscosity satisfies the following defining relation [1]:

\[
\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \left(1 + \lambda^2 I_2^2 \right)^{\frac{\eta - 1}{2}} \quad (5)
\]

which depends on the second invariant of the strain rate tensor [5]:

\[
I_2(D) = \sqrt{2D \cdot \bar{D}}
\]

Then we used a dimensionless form, where:

\[
\bar{\rho} = \frac{\rho}{\rho_0} \quad . \quad \bar{\vec{v}} = \frac{\vec{v}}{v_0} \quad . \quad \bar{p} = \frac{p}{p_0} \quad . \quad \bar{t} = \frac{t}{t_0} \quad . \quad \bar{\vec{x}} = \frac{x}{x_0} \quad . \quad \bar{t_0} = \frac{x_0}{v_0}
\]

Here \( \bar{\rho}, \bar{\vec{v}}, \bar{p} \) are the dimensionless pressure, density and velocity of fluid, respectively; \( p_0, \rho_0, v_0 \) are their typical magnitudes. \( t_0 \) is the dimensionless time.

The equations for the flow of an incompressible fluid can be written in a dimensionless form as follows (the symbol is omitted below):

\[
\nabla \cdot \bar{\vec{v}} = 0 \quad (6)
\]
\[
\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -Eu \nabla p + \frac{1}{\text{Re}} \nabla \left( \nabla \vec{v} + \nabla \vec{v}^T \right) \tag{7}
\]

\[
\mu = s + (1-s)\left[1 + Cu^2T^2_1\right]^{\frac{\eta-1}{\eta}} \tag{8}
\]

\[
\left| \vec{v} \right|_2 = 0 \ , \ p\big|_{t=0} = p_0 \tag{9}
\]

where \( Eu = \frac{p_0}{\rho_0 v_0^2} \) - Euler number, \( \text{Re} = \frac{\rho_0 v_0 x_0}{\mu_0} \) - Reynolds number, \( Cu = \frac{\lambda v_0}{x_0} \) - Carreau number.

4. Asymptotic averaging

In the framework of the method of asymptotic averaging[6], the differentiation of quasi-periodic functions is performed in accordance with the rule of differentiation of a complex function:

\[
\nabla f \rightarrow \nabla_x f + \frac{1}{\kappa} \nabla_{\xi} f \tag{10}
\]

where \( \nabla_x, \nabla_{\xi} \) - Hamilton operators with respect to \( \vec{x} \) and \( \vec{\xi} \) coordinates, respectively.

We introduce the operator of the mean value \( \langle \cdot \rangle \) over the PC \( V_\xi \) for quasiperiodic in local \( \vec{\xi} \in V_\xi \) coordinates of the functions \( f \):

\[
\langle f \rangle = \frac{1}{\phi_p V_p} \int_{V_p} f \, dV \tag{11}
\]

where \( \phi_p = \int_{V_p} dV \) is the volume fraction of the liquid in the PC (porosity), and \( |V_\xi| \) is the volume of the PC \( V_\xi \).

However, due to periodicity of the structure, the solution may be represented in the form of asymptotic expansion in terms of parameter \( \kappa \) [6,7]:

\[
\vec{v} = \vec{v}^{(0)}(\vec{x}, \vec{\xi}) + \kappa \vec{v}^{(1)}(\vec{x}, \vec{\xi}) + \kappa^2 \vec{v}^{(2)}(\vec{x}, \vec{\xi}) + \cdots \tag{12}
\]

\[
p = p^{(0)}(\vec{x}) + \kappa p^{(1)}(\vec{x}, \vec{\xi}) + \kappa^2 p^{(2)}(\vec{x}, \vec{\xi}) + \cdots \tag{13}
\]

\[
\mu = \mu^{(0)}(\vec{x}, \vec{\xi}) + \kappa \mu^{(1)}(\vec{x}, \vec{\xi}) + \kappa^2 \mu^{(2)}(\vec{x}, \vec{\xi}) + \cdots \tag{14}
\]

Having substituted expansions (12)-(14) into Equations (1)-(5), then having collected in them terms at the same powers of \( \kappa \) and putting terms at the lowest powers of \( \kappa \) equal to zero, we obtain the local problems of the zero level ‘over the periodicity cell’:
\[ \nabla_\xi \cdot \tilde{v}^{(0)} = 0 \]  
(15)

\[-\nabla_\xi p^{(1)} + \eta_0 \nabla_\xi \cdot (2\mu^{(0)} D^{(0)}) = \nabla_\xi p^{(0)} \]  
(16)

\[ D^{(0)} = \frac{1}{2} \left( \nabla_\xi \otimes \tilde{v}^{(0)} + \nabla_\xi \otimes \tilde{v}^{(0)T} \right) \]  
(17)

\[ \mu^{(0)} = s + (1-s) \left( 1 + Cu^2 Y^{(0)} \right)^{\frac{n-1}{2}}, \ Y^{(0)} = 2D^{(0)} \cdot D^{(0)} \]  
(18)

\[ \tilde{v}^{(0)} \big|_{\text{ext}} = 0, \ \langle p^{(1)} \rangle = 0, \ \| \tilde{v}^{(0)} \| = 0, \ \| p^{(1)} \| = 0 \]  
(19)

where unknown \( \tilde{v}^{(0)} \) and \( p^{(1)} \), the symbol \( \| \cdot \| \) denotes the periodicity conditions. \( \nabla_\xi p^{(0)} \) is considered as “input data”. And \( \text{Re} = \kappa^0 \text{Re}_0, \ \text{Eu} = \kappa^2 \text{Eu}_0 \) and \( Cu = \kappa^0 Cu_0 \). Then \( \eta_0 = \frac{1}{\text{Re}^0 \text{ Eu}^0} \).

5. Results of Computational modeling

Next, the finite element method will be used to solve the local problems (16)-(19). We only consider finite elements that meet LBB conditions [8]. Here, the chemical industry raw material benzene is taken as an example for numerical study, so \( \eta_0 = 0.0652 \).

| Parameter          | Value          | Parameter          | Value          |
|--------------------|----------------|--------------------|----------------|
| Minimum pressure   | -0.30925494    | Maximum pressure   | 0.30260100     |
| Velocity \( V_1 \) | 0.12414077     | Velocity \( V_2 \) | 9.06195241 \times 10^{-5} |
| Velocity \( V_3 \) | 4.74660504 \times 10^{-6} | Viscosity \( \mu^{(0)} \) | 0.69590102     |
| Porosity           | 0.62303864     | Permeability       | 0.12414077     |

Tab. 1. Results for non-Newtonian fluids \( n = 0.25 \) at \( \nabla_\xi p^{(0)} = (1,0,0)^T \).
Fig. 1. $V$-periodic porous structure.

Fig. 2. Distribution of the velocity $V_1$.

Fig. 3. Distribution of the velocity $V_2$.

Fig. 4. Distribution of the velocity $V_3$.

Fig. 5. Distribution of the pressure $P$.

Fig. 6. Distribution of the viscosity $\mu$. 
6. Conclusions

Multi-scale modeling of non-Newtonian viscosity resin flow in a composite structure using an asymptotic homogenization method was developed. And the finite element method is used to calculate the local problem for the non-Newtonian viscosity resin flow in a composite microstructure.

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