Radiation transport in ionizing gas flow within the quasi-steady plasma accelerator

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Abstract. Investigation of the radiation transport in the ionizing gas flow in the channel of the quasi-steady plasma accelerator is presented. The model is based on the magnetohydrodynamic (MHD) equations and equation of the radiation transport. In the MHD model the approximation of the local thermodynamic equilibrium was used in the three-component medium consisting of atoms, ions and electrons. The model of the radiation transport includes the basic mechanisms of emission and absorption for different portions of spectrum.

1. Introduction

Theoretical, numerical and experimental studies of dynamics of the ionizing gas and plasma flows constitute one of the scientific directions related with the quasi-steady plasma accelerators (QSPA) and magneto-plasma compressors (MPC) [1–6]. For these devices we explore the transonic plasma flows including the presence of an additional longitudinal magnetic field [7], the near-electrode processes caused by the Hall effect and leading to the phenomenon of the current crisis [8,9], the compressible streams, the radiation transport, the dynamics of impurities, as well as the numerical models of the ionization process corresponding to different levels of complexity [10–12]. A plenty of publications is devoted to the investigations of the ionization processes, the plasma dynamics and radiation transport (see e.g. [13–21]).

In the plasma accelerators the processes occur in the presence of the main azimuthal component of the magnetic field. Simple plasma accelerators consist of two coaxial electrodes connected to the electrical circuit. The azimuthal field is generated by electric current flowing along the inner electrode. As a result of the breakdown between electrodes the ionization front corresponding to a transition from one state of matter to another is formed. The radial plasma current flowing between electrodes and the azimuthal magnetic field provide the acceleration of plasma behind the ionization front due to the Ampere force $c^{-1}[\mathbf{j}, \mathbf{H}]$, where $\mathbf{j}$ is the current density in plasma. The ionization process and the preliminary acceleration of plasma occur particularly in the first stage of the two-stage QSPA [1–6]. Multifunctional devices are designed for fusion research, different technological applications, and are of interest for development of the perspective high-power electro-plasma thrusters.

Earlier the ionizing gas flow was considered within the framework of the quasi-one-dimensional approximation in a narrow cylindrical channel (see e.g. [10–12]). The properties of the ionizing gas streams are typically studied by means of stationary or rather quasi-stationary flows with the...
calculated steady-state solution. For steady-state flows the foundations of theory of processes at the ionization front were also developed in [22].

In this paper the numerical model of the two-dimensional axisymmetric quasi-stationary flows of the ionizing gas is based on approximation of the local thermodynamic equilibrium (LTE). New opportunities for the integrated researches and convergence of the calculation results with possibilities of experiments [2,4–6] connected with the simultaneous determination of local values of macroscopic parameters of plasma and radiation characteristics. The results of research within the framework of the three-dimensional numerical model of radiation transport in the ionizing gas flow are also presented in the work. To solve the problem of radiation transport it is necessary to take into account a number of factors associated with accuracy of the geometry description of the radiating volume and the shadow regions, with details of description of the emission spectrum and the basic mechanisms of emission and absorption, as well as to take into account possibilities of the numerical solution methods of the radiation transport equation (see e.g. [19–21]).

2. The equations of the radiation plasma dynamics and formulation of the problem

In this work the modified magnetohydrodynamic (MHD) model is used within the framework of the LTE approximation including the radiation transport because the density of the radiation energy flux can influence the redistribution of energy in medium. The MHD model of the self ionizing gas flow is based on the transfer equations for the three-component medium [23] consisting of atoms, ions, and electrons, as well as on the magnetic field diffusion equation which is a consequence of Maxwell’s equations and Ohm’s law \( E = \sigma^{-1}j - c^{-1}|V, H| \) if the inertia of electrons and the displacement current are neglected. The ionization process is studied for hydrogen which is often used in experiments. The masses of atoms and ions are identical \( m_a = m_i = m \). It is known from the experimental data that the temperature at the ionization front increases up to \( 1–3 \text{ eV} \). The concentration of gas entering the channel is supposed to be sufficiently high \( n = 10^{17–10^{18}} \text{ cm}^{-3} \). It can be assumed for such parameters that a medium is quasi-neutral \( n_i = n_e \), and the velocities of the medium components are equal \( V_i = V_e = V_a = V \). Experiments and estimations also show that it is possible to consider the case of single temperature mixture. As a result of transformation of the initial equations with regard to the above assumptions we obtain the following modified set of MHD equations:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \text{div}(\rho V) = 0, \\
\rho \frac{dV}{dt} + \nabla \rho P = \frac{1}{c} [j, H], \\
\frac{d}{dt} \rho + \text{div} (\rho \rho) + P \text{ div}V = \frac{j^2}{\sigma} - \text{div}q - \text{div}W, \\
\frac{\partial H}{\partial t} = \text{rot}[V, H] - \frac{c}{\sigma} \text{rot} \frac{j}{\sigma}, \\
\frac{\partial T}{\partial t} = \gamma - \text{rot} \frac{j}{\sigma}, \\
P = P_a + P_i + P_e = (1 + \alpha) (e_p - c V) \rho T, \\
\alpha = \frac{n_a}{m_a + n_i}, \\
\xi = (1 + \alpha) c V T + \xi_I, \\
\xi_a = \frac{n_a}{m_a + n_i}, \\
q = -\kappa_e \xi_a \nabla T.
\end{align*}
\]

Here \( \rho \) is the number density of heavy particles, \( P \) is the total pressure, \( \alpha \) is the degree of ionization, \( \alpha = n_a / (n_a + n_i) \), \( \rho \) is the mass density, \( \kappa_e \) is the electron-atomic heat conductivity, and \( W \) is density of the radiation energy flux. The internal energy per unit of mass \( \varepsilon \) includes the additional term \( \varepsilon_I = \zeta \alpha / m_i \) which is responsible for the energy loss due to ionization, where \( I \) is the atom ionization energy. The Joule heating \( \frac{d}{dt} \frac{j^2}{\sigma} \) in the equation for the internal energy of the set (1) considerably exceeds the heat generated by friction with other components.

The electrical conductivity of medium in equations is equal to \( \sigma = e^2 n_e / (m_e \nu_e) \), where the average frequency of collisions of an electron with other particles is composed of the frequencies of collisions with atoms and ions: \( \nu_e = \nu_{ea} + \nu_{ei} \), \( \nu_{ea} = n_a \langle V_e \rangle S_{ea} \), \( \nu_{ei} = n_i \langle V_i \rangle S_{ei} \). Here \( S_{ea} \) and \( S_{ei} \) are the effective collision cross sections. The main heat transfer mechanisms depend on
the medium state. In the case of large degrees of ionization a significant role in the total heat transfer is played by the classical electron heat conductivity across the magnetic field. At low degrees of ionization the atomic heat conductivity makes a noticeable contribution.

In the LTE approximation the concentrations of all three components of medium and the electron temperature are connected by the Saha equation

$$\frac{n_i n_e}{n_a} = K_1(T) = \frac{2 \Sigma_i}{\Sigma_a} \left( \frac{m_e k_B T}{2 \pi \hbar^2} \right)^{3/2} \exp \left( - \frac{I}{k_B T} \right),$$

(2)

where $\Sigma_a$ and $\Sigma_i$ are the statistical sums of atom and ion. The value $K_1$ is the constant of ionization balance. The equation of the ionization balance (2) basically means that in any local volume of a dense plasma with the known values of temperature and density there is a direct process of ionization of atom from the ground state as a result of the electron impact which is counterbalanced by a inverse recombination process $A^+ + e^- \leftrightarrow A + e^- + e^-$. The Saha equation determines the structure of the three-component plasma under condition of quasi-neutrality. From the equation (2) by means of formulas $n_e = n_i = \alpha n$ and $n_a = n - n_i = (1 - \alpha) n$, where $n = n_a + n_i$ we obtain the relation for a degree of ionization

$$\alpha = -K_1(T)/(2n) + \sqrt{\left[ K_1(T)/(2n) \right]^2 + K_1(T)/n}.$$  

(3)

The main radiation characteristics are the density of the radiation energy $U$ and density of the radiation energy flux $W$, which are determined by the radiation intensity $I_{\nu}(r, \Omega)$ emitting in direction of the spatial angle $\Omega$ with the particular frequency $\nu$,

$$U(r) = \frac{1}{c} \int_0^{\infty} \int_0^{4\pi} I_{\nu}(r, \Omega) d\Omega dv,$$

$$W(r) = \int_0^{\infty} \int_0^{4\pi} I_{\nu}(r, \Omega) \Omega d\Omega dv.$$  

(4)

In MHD equations the density of the radiant energy can be neglected as it is small in comparison with a thermal or internal energy of medium ($U \ll \rho c^2$). However the radiation energy flux $W \sim cU$ can play a noticeable role in the energy redistribution.

The radiation intensity is calculated by means of equation of the radiation transport which generally has the following form

$$\frac{1}{c} \frac{\partial I_{\nu}(r, \Omega)}{\partial t} + \Omega \nabla I_{\nu}(r, \Omega) = \eta_{\nu}(r) - \kappa_{\nu}(r) I_{\nu}(r, \Omega),$$

(5)

where the radiation intensity $I_{\nu}(r, \Omega)$ corresponds to the point with the coordinate $r$. Equation (5) is written in the assumption of the isotropic scattering. Velocity of the radiation propagation is significantly higher than the specific velocities of the plasmodynamic processes. In this case the radiation field instantly adapts to distribution of the flow parameters, and it is possible to reduce the problem to solving the stationary equation of the radiation transport

$$\Omega \nabla I_{\nu}(r, \Omega) = \eta_{\nu}(r) - \kappa_{\nu}(r) I_{\nu}(r, \Omega).$$  

(6)

The absorption coefficient $\kappa_{\nu}(r)$ and emissivity $\eta_{\nu}(r)$ is dependent on the condition of medium, its density and temperature, as well as the spectral parameter $\nu$ associated with the photon energy $h \nu$. To determine their values it is necessary in general to calculate the spectrum of atom and the populations of states in atoms. The absorption coefficient and emissivity consist of three parts corresponding to

(i) absorption and emission in lines,
(ii) photo-ionization and photo-recombination, and
(iii) scattering.
Emissivity and absorption coefficient corrected on the compelled radiation are determined by the following relations [14–21]

$$\kappa_{\nu} = n \sum_{k<j,k,j=1}^{K-1} x_k \pi e^2 \left( f_{kj} \phi_{kj}(\nu) \left( 1 - \frac{n_j g_k}{n_k g_j} \right) + n \sigma_{K}^{\text{eff}}(\nu) \left[ 1 - \exp\left( -\frac{\nu}{k_B T} \right) \right] \right)$$

$$+ n \sum_{k=1}^{K-1} x_k \sigma_{\nu}^{\text{pl}}(\nu) \left[ 1 - \frac{1}{2} \left( \frac{2\pi h^2}{m_e k_B T} \right)^{3/2} \frac{n_i g_k}{n_x \Sigma_i} \exp\left( \frac{h\nu - \nu}{k_B T} \right) \right],$$

$$\eta_{\nu} = \frac{h\nu^3}{\pi c^2} \sum_{k<j,k,j=1}^{K-1} x_k g_j \pi e^2 \left( f_{jk} \phi_{jk}(\nu) + n \frac{h^3}{\pi c^2} \sigma_{K}^{\text{eff}}(\nu) \exp\left( -\frac{\nu}{k_B T} \right) \right)$$

$$+ \frac{(h\nu)^3}{4h^2\pi^2c^2} \sum_{k=1}^{K-1} n_i n_e \left( \frac{2\pi h^2}{m_e k_B T} \right)^{3/2} \sigma_{\nu}^{\text{pl}}(\nu) \frac{g_k}{\Sigma_i} \exp\left( \frac{h\nu - \nu}{k_B T} \right),$$

where $\nu$ is the photon frequency, $x_k = n_k / n$ is the relative concentration of $k$-th state of atom, $g_k$ is the statistical weight of $k$-th state of atom, $f_{kj}$ is the oscillator strength for $k \rightarrow j$ transition, $\phi_{kj}(\nu)$ is the line profile corresponding to the bound-bound transition, $\sigma_{K}^{\text{eff}}(\nu)$ is photo-ionization cross section from $k$-th state, $\sigma_{\nu}^{\text{pl}}(\nu)$ is the cross section of the inverse bremsstrahlung. The summation in formulas (7) is carried out over all states except for the state $k = K$ corresponding to ions with concentration $n_i$. The line profile taking into account the different broadening mechanisms is determined by the Voigt formula

$$\phi_{kj}(\nu) = \frac{\gamma_e}{\pi \gamma_e^2} \int_{-\infty}^{\infty} \frac{\exp(-s^2) ds}{(\nu - \nu_{kj} - sD)^2 + (\gamma_e)^2},$$

where $D = \nu_{kj} V_a / c$ is the Doppler width, $V_a$ is the characteristic velocity of atom, $\gamma_e$ is the total width caused by interaction of atom with radiation and the surrounding ions and electrons. The Voigt line profile has form of a bell-shaped curve for which the characteristic width corresponds to the state lifetime. The effective technique of calculation of the Voigt profile according to (8) is described in [20]. In the presented calculations the profiles of the Lyman alpha and beta lines are described in more detail. The width of these lines is given by several spectral groups to determine form of a core and wings of lines. The lines in other part of spectrum are described within one group. All spectrum is divided in more than 300 spectral groups.

The interaction with electrons is an essential factor for the typical concentration of plasma $n \sim 10^{15} - 10^{17}$ cm$^{-3}$ and temperature $T \sim 1$ eV. In order to calculate the value of $\gamma_e$ in this case it is possible to consider only the broadening due to the electron impact and to use the formulas presented in [15, 18]. The cross sections of bound-free and free–free processes, the oscillator strength $f_{kj}$ for the bound-bound transitions are also calculated by means of the known relations.

In case of the local thermodynamic equilibrium the population of states are connected by the Saha equation (2) and the Boltzmann’s formula

$$x_k = \frac{n_a g_k}{\sum_{\alpha} n_\alpha} \exp\left( -\frac{E_k - E_1}{k_B T} \right),$$

where $E_k = Z^2 e^2 / (2a_0 k^2)$ is the energy levels of hydrogen atom and $a_0$ is the Bohr radius.

The numerical solution of problem with equations (1) and (3) is carried out in dimensionless variables. As units of measurement we will choose a length of channel $L$, the characteristic concentration or gas density at the inlet of accelerator channel $n_0$ ($\rho_0 = m n_0$) and temperature $T_0$. The characteristic value of the azimuthal magnetic field $H_0$ at the inlet to channel is defined by discharge current in device $J_p$ so that $H_0 = 2 J_p / (c R_0)$ where $R_0$ is the characteristic radius.
of channel. These values allow to form the units of pressure $P_0 = H_0^2/(4\pi)$, velocity $V_0 = H_0/\sqrt{4\pi \rho_0}$, time $t_0 = L/V_0$, electric field $E_0 = H_0V_0/c$, and plasma current $j_0 = cH_0/(4\pi L)$.

The set of the MHD equations in the dimensionless variables contains such dimensionless parameters as the ratio of the characteristic gas pressure to magnetic one $\beta = 8\pi P_0/H_0^2 (P_0 = k_3 n_0 T_0)$, parameter $T^* = 1/(k_3 T)$, and magnetic viscosity $\nu_m = 1/\text{Re}_m = c^2/(4\pi LV_0 \sigma)$ which is inversely proportional to the magnetic Reynolds number. In this case the magnetic viscosity contains values $\sigma_{10}$ and $\sigma_{20}$ which are expressed in terms of the initial dimensional parameters and physical constants

$$\nu_m = \frac{1}{\text{Re}_m} = \frac{1}{\sigma_1} + \frac{1}{\sigma_2}, \quad \sigma_1 = \frac{\alpha \sigma_{10}}{(1-\alpha) \sqrt{T}}, \quad \sigma_2 = \sigma_{20} T^{3/2}.$$ 

Influence of the thermal conductivity on the ionizing gas flow is insignificant as a whole because the dimensionless value of thermal conductivity is very small. The transition to the dimensionless values of the radiation energy flux is carried out by means of the following unit of measurement $W_0 = V_0 H_0^2/(4\pi)$.

Formulation of the problem includes the boundary conditions at the electrodes and at the inlet and outlet of the accelerator channel. We assume that at the inlet of channel ($z = 0$) the plasma is supplied with the known values of the density $\rho(r) = f_1(r)$ and the temperature $T(r) = f_2(r)$. Not solving an additional electric circuit equation it is possible to assume that the current is kept constant and comes into the system only through electrodes. Then at $z = 0$ we have $j_z = 0$ or $r H_r = r_0 = \text{const}$ where $r_0 = R_0/L$. The subsonic inflow comes along a certain direction for example along the coordinate lines. If one takes into account a finite electrical conductivity in the parabolic part of set of the MHD equations then the boundary conditions for the magnetic field are necessary at all borders.

The boundary conditions at electrodes $r = r_\alpha(z)$ and $r = r_\kappa(z)$ forming the wall of the channel are based on the assumptions that the electrode surfaces are equipotential ($E_r = 0$) and impermeable for plasma ($V_n = 0$). These boundary conditions are traditional for numerical models of dynamics of the plasma streams and the ionizing gas flows in channels of plasma accelerators.

The free flowing out of plasma is supposed at the outlet for the studied transonic flows. In the middle part of the channel there occurs a transition of the flow velocity through velocity of the fast magnetosonic waves or velocity of a signal $C_S[2]$ which in the absence of a longitudinal magnetic field or in the presence of only azimuthal component of field is equal to

$$C_S = \sqrt{C_{G_0}^2 + C_A^2}, \quad C_{G_0}^2 = \gamma P/\rho, \quad C_A^2 = H^2/\rho.$$ 

Various boundary conditions for the radiation transport equation were used in the calculations. The simplest boundary condition assumes the absence of radiation falling from the outside and $I_\nu = 0$ on the electrode surface and at the inlet of channel. In some cases the accelerator walls can play a role of the additional radiation source. Radiation of the accelerator walls can be included in calculations using the data on the degree of blackness of metals depending on temperature [17].

The numerical solution of equations (1) is performed in the domain in variables $(z, r)$, which correspond to the form of channel represented in figures 1 and 2. Considered two-dimensional numerical model of axisymmetric flows of the ionizing gas and plasma involves the solution splitting by the coordinate directions and by the physical factors taking into account the mixed type of the MHD equations. The finite-difference scheme with flux correction (see e.g. [24]) is used to calculate the hyperbolic part of the MHD equations.

The parabolic part of the MHD equations contains the coefficients of the electrical and heat conductivity which vary greatly for the considered ionizing gas flows on the ionization front together with the thermodynamic parameters of medium defined by the magnetic gas dynamics.
A reliable way to solve the problem with strongly changing coefficients is based on the flux variant of the back-substitution method [25].

In accordance with equations (4) and (6)–(9) the problem of the radiation transport in flow of the ionizing gas and plasma should be solved in the three-dimensional formulation for an axisymmetric flow in the accelerator channel. It is easy to obtain a grid for the 3D problem of the radiation transport rotating the initial grid in plane of the variables \((z, r)\) by 360 degrees around the axis of channel with a certain step. The radiation intensity has to be determined in different directions for the further calculation of the integral values in relations (4) in any node or cell of a three-dimensional grid. For this purpose an additional angular grid is built in the azimuth and polar angles. The splitting of the complete spatial angle \(\Omega = 4\pi\) into elements of an angular grid is made by the method providing the uniform distribution of rays in all directions. For each node in calculation we use as a rule 440 rays for the complete spatial angle.

The ray tracing is carried out in accordance with the method of the long characteristics in order to determine the points of the crossing of a ray with the faces of cells in the three-dimensional grid and the position of the crossing of a ray with one of the boundaries of the three-dimensional computational domain. It is possible to assume the absorption coefficient and emissivity being constant within grid cell. In this case the coefficients \(\kappa_p\) and \(\eta_p\) are calculated by the average value of density and temperature in the center of the cell. The invisible shadow regions are excluded from calculation of the radiation energy flux for the given node of grid in the tracing process of the computational domain by rays emerging from any node of the grid. The method of characteristics [19] allows to take into account the details of the accelerator channel geometry. If the characteristic sequentially passes through regions with different optical properties then it is necessary to provide continuity of the solution across the boundary between the regions. As a result we will receive a solution on characteristic passing through an arbitrary number of homogeneous regions with known coefficients of absorption and emissivity. The more detailed formulation of the problem is presented in [26].

3. Radiation filed in the ionizing gas flow

In experiments the continuous inflow of gas at the inlet of accelerator is provided at a certain value of pressure \(P_0\) close in order of magnitude to \(P_0 \approx 10\) Torr in small installations and accelerators of the first stage of the QSPA to ensure the ionization and preliminary acceleration of plasma [2,4–6]. The results of the numerical study of the radiation transport presented further showed that the radiation in a particular frequency range coming from the emerging ionization front can penetrate deeply into the volume of the entering gas. Therefore its temperature can differ from the initial temperature resulting in some arbitrariness in the setting of parameter \(T_0\).

The parameters of stream change in the discharge process depending on values of the discharge current in the electric circuit. The characteristic time \(t_0 = L/V_0\) corresponds to the time of flight of particles in the channel of accelerator and the time to reach the quasi-stationary state in the calculations. This time is much less than the duration of the capacitor bank discharges in plasma accelerators. Therefore the research of the main regularities of flows of the ionizing gas and plasma is based usually on the formulation of problem with the specified and fixed value of the discharge current without taking into account the processes in the electrical circuit. Respectively we assume \(J_0 = \text{const}\) in the numerical model.

The calculation of the quasi-steady flow of the ionizing gas for the following initial parameters of problem \(n_0 = 4 \times 10^{17} \text{ cm}^{-3}, T_0 = 750\) K, \(J_0 = 50\) kA, \(L = 10\) cm which correspond to the dimensionless parameters \(\beta = 0.12, r_0 = 0.33, \sigma_{10} = 553.9,\) and \(\sigma_{20} = 0.03\) is presented in figures 1, 2 and 3. At the channel inlet the plasma is supplied with the prescribed distribution of density and temperature: \(\rho(r) = 1, T(r) = 1\) at \(z = 0\) in the considered case. The two-dimensional temperature distribution \(T(z, r)\) in the axisymmetric plasma stream is given in figure 1 corresponding to the plane of the variables \((z, r)\). The dashed curves or the level lines of
The density distribution and the vector velocity field are presented in figure 2. The scale of velocity vectors is determined by the length of vector $V^* = 10 \, V_0$ which is indicated in the figure. In this case the unit of velocity is equal to $V_0 = 1.03 \times 10^6$ cm/s for the initial parameters mentioned above.

The solution of a radiation transport problem allows to get a more complete understanding of the flow dynamics of the radiating medium, character of the radiation transport corresponding
Table 1. Directional diagram of radiation for different parts of spectrum and points in channel.

| $h\nu_1 = 8.0\text{(eV)}$ | $h\nu_2 = 10.2\text{(eV)}$ | $h\nu_3 = 13.6\text{(eV)}$ |
|--------------------------|--------------------------|--------------------------|
| $I_{\nu} = 10^6$        | $I_{\nu} = 5 \times 10^3$ | $I_{\nu} = 5 \times 10^4$ |
| $A_1$                    | $A_1$                    | $A_1$                    |
| $I_{\nu} = 4 \times 10^6$| $I_{\nu} = 10^4$         | $I_{\nu} = 10^5$         |
| $A_2$                    | $A_2$                    | $A_2$                    |
| $I_{\nu} = 4 \times 10^6$| $I_{\nu} = 2 \times 10^3$| $I_{\nu} = 10^5$         |
| $A_3$                    | $A_3$                    | $A_3$                    |

The densities of the radiation energy $U$ and energy flux $W$ are determined by the integrals (4) of the radiation intensity $I_{\nu}(r, \Omega)$ which are taken over the spatial angles and over the radiation spectrum. Respectively the characteristic features of the radiation intensity distribution in different directions for certain parts of spectrum at the various points of channel were studied firstly.

Table 1 shows the typical directional diagram of radiation in the channel section plane $(z, r)$ for the three points marked in figure 3. Point $A_1$ is located far from the ionization front in the incoming flow of the weakly ionized gas. The radiation emerging from the front can penetrate into this region in a certain range of photon frequencies with the mean free path which is greater than and comparable with the characteristic length of the channel. Points $A_2$ and $A_3$ are located in the vicinity of the front respectively on the left and right of it.

Diagrams show the radiation intensity in different directions at three points $A_1$, $A_2$, $A_3$ and for three portions of spectrum: $h\nu_1 = 8$ eV, $h\nu_2 = 10.2$ eV, and $h\nu_3 = 13.6$ eV. The scale of vectors for each diagram in table 1 is determined by the characteristic length of vectors depicted above diagrams. The corresponding values of the radiation intensity $I_{\nu}$ referred to the unit interval of photon energy and expressed in eV are indicated above all the characteristic vectors. The dimensionality of the intensity for all values indicated in table is equal to $[I_{\nu}] = \text{erg}/(\text{cm}^2\text{s}\text{eV})$.

Photons with energy of $h\nu_3 = 13.6$ eV correspond to the portion of the spectrum of the recombination radiation. The spectral absorption coefficient $\kappa_{\nu}$ is inversely proportional to the mean free path of a photon $l_{\nu} \sim 1/\kappa_{\nu}$. The photons corresponding to a recombination part of the spectrum have the mean free path exceeding the size of the channel. Therefore the influence of radiation from the ionization front and other regions for the photon energy $h\nu \geq 13.6$ eV is observed in all channel points.
The brightest radiance is observed in vicinity of the ionization front in accordance with experiments. The radiation going from the front generally determines a radiation orientation in various points of channel in the wide range of photon energy. The mean free path of photons for the energy \( h\nu = 10.2 \) eV corresponding to the Lyman \( \alpha \)-line is relatively small. It leads to the particular corrections in the orientation of the spectral line radiation which is almost isotropic at points located far from the ionization front in flow of the weakly ionized gas. However the radiation in lines coming from the front makes a significant contribution to the radiation field at points located respectively on the left and right of the front in regions with a relatively high degree of ionization. The values of intensity at these points for the photon energy corresponding to the Lyman \( \alpha \)-line are substantially higher than the intensity values for other parts of spectrum.

The comparisons with the experimental data on spectroscopy of streams in channels of plasma accelerators are possible subsequently. The developed model allows to determine the spectral intensity of radiation in any frequency range or the photon energy for any rays emitted from the plasma volume in the direction of the detector.

In figure 3 the two-dimensional distribution of the radiation energy density \( U \) and the vector field of the radiation energy flux density \( W \) in the axially symmetric flow of the ionizing gas are presented in the plane of variables \((z, r)\). This radiation field corresponds to the distributions of temperature and density shown in figures 1 and 2. The figure 3 demonstrates the distribution of the dimensionless value \( \tilde{U} \) which is related to the radiation energy density by means of the following relationship \( \tilde{U} = 10^4cU/W_0 \) where \( c \) is the light velocity, and the unit of the radiation energy flux density is equal to \( W_0 = V_0H_0^2/(4\pi) = 5.43 \times 10^{11} \text{ erg}/(\text{cm}^2 \text{s}) \) for the initial parameters \( n_0, T_0, J_p \), and \( L \) specified above. The scale of vectors \( W \) is determined by the length of the vector \( W_\ast = 2 \times 10^{-4}W_0 \) which is indicated in figure.

It is clearly seen that region with relatively high values of radiation energy density is located in vicinity of the ionization front. The radiation energy flux is directed to all sides from the given region including the direction to the incoming flow of the weakly ionized gas. It naturally leads to its preliminary ionization directly ahead of front, and also in depth of volume of the entering gas.

4. Conclusion
The study of the flows of the ionizing gas and plasma in the channel of the coaxial plasma accelerator was carried out on the basis of the set of the MHD and radiation transport equations.
The results for two-dimensional axisymmetric flows of the ionizing gas and the radiation transport within the framework of three-dimensional model were obtained in the approximation of local thermodynamic equilibrium. The radiation field and integral radiation characteristics in the ionizing gas flows in the channel of the plasma accelerator were determined. It was found that the radiation in lines brings an essential contribution to the total radiation field. The radiation energy density has a maximum values in the vicinity of the ionization front. At the same time the radiation energy flux changes its direction at the transition through the ionization front.

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