Near infrared few-cycle pulses for high harmonic generation

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Abstract
We report on the development of intense tunable few-cycle pulses with central wavelengths ranging from 1.6 μm to 2 μm. These pulses were used as a proof of principle for high harmonic generation in atomic and molecular targets. In order to generate such pulses, we produced a filament in a four-bar krypton cell. Spectral broadening by a factor of two to three of a 40 fs near-infrared input pulse was achieved. The spectrally broadened output pulses were then compressed by fused silica plates down to the few-cycle regime close to the Fourier limit. The autocorrelation of these pulses revealed durations of \( \sim \) three cycles for all investigated central wavelengths. Pulses with a central wavelength of 1.7 μm and up to 430 μJ energy per pulse were employed to generate high-order harmonics in Xe, Ar, and N\(_2\). Moving to near-infrared few-cycle pulses opens the possibility of operating deeply in the non perturbative regime with a Keldysh parameter, \( \gamma \ll 1 \). Hence, this source is suitable for the study of the non perturbative tunneling regime in most generating systems used for high-order harmonic generation and attoscience.

Keywords: high harmonic generation, few-cycle pulse, non perturbative regime

(Some figures may appear in colour only in the online journal)

1. Introduction

During the interaction of intense laser fields with atoms and molecules, an electron wavepacket initially bound to the target can be ionized through different interaction processes. These ionization channels determine the evolution of the core and the electron wavepacket in the continuum. To identify which of these processes is predominant during the interaction, L V Keldysh\textsuperscript{[1]} introduced a parameter given by \( \gamma = \sqrt{I_p/2U_p} \), where \( I_p \) is the target’s ionization potential and \( U_p \) is the ponderomotive energy associated with the strong laser field employed.

In the perturbative regime, \( \gamma \gg 1 \), the target’s potential remains largely unchanged by the laser field. In this case, multi photon processes are necessary to emit a single electron wavepacket from the ground state. The electron wavepacket can absorb many photons, reaching energies just exceeding the ionization potential (multi photon ionization), or absorb many more photons to overcome the ionization potential. After ionization, the electron wavepacket in the continuum can be considered only subject to the laser field\textsuperscript{[2–4]}. When the laser field reverses half a cycle later, the electron wavepacket can be redirected to the ionic core and then rescatter (above threshold ionization\textsuperscript{[5]}).

When \( \gamma \ll 1 \), known as the non perturbative regime, the target’s potential is deformed by the strong laser field. This deformation happens at every maximum of the laser field (i.e., twice per optical cycle). Every optical half-cycle, the electron wavepacket initially bound to the target sees the tail of the Coulomb potential evolve in time, hence creating a barrier...
whose height and width change over time. Thus, the electron wavepacket reaches a greater probability of tunneling out of the system through this potential barrier. The probability of tunneling through the barrier increases with the laser field strength, to the point where the barrier drops below the binding energy level and the electron wavepacket is directly launched into the continuum (barrier suppression ionization). The latter situation is the asymptotic limit of the tunnel ionization regime and usually results in an electron wavepacket which cannot be redirected by the laser field back to the ionic core. As a result, recombination or rescattering of the electron wavepacket does not occur.

In order to understand how ionization occurs, we need to follow the transition between the different regimes of ionization. To study these transitions, most of the experiments were performed using an infrared (IR) laser field (0.8 μm) with a typical intensity of $I = 10^{14}$ W cm$^{-2}$, interacting with atoms (noble gases) [3, 4, 6, 7], molecules [8], nanoparticles [9–11], or periodic solid structures [12, 13]. For these targets, the ionization potential is kept below $\sim 25$ eV, so that ionization rates correspond to $\gamma$ values between 0.5 and 5. Under such conditions, only high $I_p$ atoms subject to a strong laser field are in the non perturbative regime. Two routes can be followed to enable the study of dynamical processes under the non perturbative regime for most atoms and molecules.

First, one can increase the laser field intensity, keeping the wavelength in the IR region (0.8 μm for instance). This approach has been employed widely, owing to the development of Ti:Sa laser technologies [14]. However, it is strongly limited by the field strength that a target can withstand before its ground state is strongly depleted. One way to circumvent this limitation is to use few-cycle pulses and to control their intensity profile. In this case, one has to reconsider the Keldysh parameter in a non quasi static situation, as shown by Yudin and Ivanov [15]. They defined a general expression for non perturbative ionization rates based on a modification of the ADK theory [16], which is suitable for the case of few-cycle laser fields.

The second route is to decrease $\gamma$ by increasing wavelengths ($\lambda$) to the near-(NIR) or mid-infrared (MIR) region while maintaining low enough intensity to avoid depletion of the ground state. As a result, it is possible to study the ultrafast dynamics of charge migration involved in atoms and molecules after ionization, using high harmonic generation (HHG) [17–25]. However, analyzing HHG driven by few-cycle NIR to MIR pulses offers the possibility of extending these studies well into the nonperturbative regime and allows us to study how these charge migrations evolve while transitioning from one regime of ionization to another.

2. Employing a NIR to MIR laser field for high-order harmonic generation

In recent years, efforts have been made to push the development of multi cycle NIR to MIR femtosecond sources to drive strong field interaction, such as that found in HHG experiments. Indeed, using laser fields at longer wavelengths (i.e., in the low frequency limit to generate high order harmonics) has the potential to dramatically extend the maximum photon energy produced (cutoff position). This is because it scales with $\propto \lambda^2$ [2, 3, 26, 27] and goes beyond the carbon K-edge. Hence, the coherent XUV and x-ray radiation produced can provide a unique source to study ultrafast inner core dynamics [28], opening the possibility of producing isolated attosecond pulses in the soft x-ray regime [29, 30]. However, the efficiency of the high harmonic generation process decreases when employing laser fields at longer wavelengths as it drops with $\lambda^{-5}$ [31–33], which can limit the detection. Nevertheless, experimentally, Colosimo et al have succeeded in studying the wavelength scaling of the harmonic generation by employing multi cycle laser fields at 0.8 μm, 1.3 μm, 2 μm, and 3.6 μm [34]. In their experiments, the Keldysh parameter was varied from 1.3 to 0.3. Henceforth, they could resolve the transition from multi photon behavior to the classical limit at the longest wavelength. In addition to experimental work, the theory was adapted to the wavelength region up to 5 μm to match the experimental data for high harmonic generation [32].

To access more complex dynamics under the non perturbative regime, Ghimire et al [35] demonstrated the generation of high harmonics in periodic solid structures using multi cycle MIR laser sources. This experiment enabled access to intraband currents of these structures on the atto-second timescale. Transitions between bands of states within solids have already been described by Keldysh [1]. Thus, the transition rate of structure bands appears to be similar to that of the tunneling rate in atoms. This study has been extended further to cosine band structures for time-averaged transition rates [36].

Hawkins and Ivanov [37] have derived an analytic approximation for the transition rate in the low frequency limit to describe intraband sub-cycle electron dynamics. They showed that the transition depends on the band structure. This is due to electron acceleration after and during transition to the conduction band. Therefore, to really access time-dependent sub-cycle electron dynamics in atoms, molecules, or periodic structures, it is crucial to employ few-cycle NIR or MIR sources. In this article, we show the possibility of producing tunable few-cycle pulses via filamentation with central wavelengths tuned from 1.6 to 2 μm. As a proof of principle, we show how this source can be used to drive HHG experiments in atoms and molecules. This opens the potential to investigate sub-cycle dynamics of various targets in the non perturbative regime.

3. Perturbative versus non perturbative Keldysh parameter in the low frequency limit

When $\gamma \ll 1$, which is the case for long wavelengths (low-frequency limit) and high intensities, the ionization rate for hydrogen atoms described by Keldysh [1] coincides with the well-known tunnel ionization formula for static fields [38, 39]. In particular, the exponential dependence of the rate with respect to the field amplitude is present. When $\gamma \gg 1$,
the formula reduces to a sum describing the ionization via the simultaneous absorption of several photons. In the tunneling case (i.e., $\gamma \ll 1$), the perturbative approximation holds and the atomic structure can be reintroduced by means of both an effective $n^*$ and the usual $\ell$ and $m$ quantum numbers in the exponential prefactor [40]. This approach has been further extended to any arbitrary ionic state by Ammosov et al [16], which is today known as the ADK theory.

The results of these formulae are in excellent agreement with experimental measurements [41], accounting for cycle-average rates over one half-period of the alternating electric field, under the assumption of a sufficiently slow varying envelope between two consecutives cycles. With the possibility of producing and designing few-cycle laser pulses, which is the frame of this article, this assumption is not valid anymore. Therefore, one has to use the non perturbative ionization rate formula developed by Yudin and Ivanov [15] for any arbitrary $\gamma$:

$$\Gamma(t) = N(t) \exp \left( -\frac{E_0^2 f(t)^3}{\omega} \Phi(\gamma(t), \theta(t)) \right).$$  (1)

It explicitly takes into account the sub cycle dynamics through the envelope, $f(t)$, expressed in the time-dependent Keldysh parameter, $\gamma(t) = \gamma f(t)$, and in $N(t)$ and through the instantaneous laser phase, $\theta(t) = \omega t + \phi_0$ [15]. We applied this formula to the calculation of the ionization probability of argon by three cycle pulses generated at an intensity of $10^{14}$ W cm$^{-2}$ and for 1.7 $\mu$m, 1.8 $\mu$m, and 1.9 $\mu$m wavelengths (see figure 1). Once again, these long wavelength pulses ensure that they are in ‘better’ tunneling conditions than the usual experiments at 0.8 $\mu$m, and the three cycle of the pulses justify the use of equation (1).

Clearly, non perturbative ionization probabilities (the solid lines in figure 1) build up in a stepwise manner during the laser interaction, but do not differ much at the end of the pulse for all considered wavelengths. By comparison, ADK ionization probabilities are lower by a factor of 1.5 to 1.7, and the probabilities are separated further when the interaction is over. This shows the strong influence of the sub cycle dynamics on the ionization processes and the necessity to take them into account when a theoretical description is required in such conditions (i.e., for HHG experiments). It is worth mentioning that these two calculations asymptotically converge for $\omega \to 0$.

4. Production of tunable NIR few-cycle pulses by filamentation

Few-cycle pulses have been generated via various methods in the IR over the last decade. The two most common methods are hollow-core fiber compression [42] and filamentation [43–46]. These methods are used in a post compression scheme after the amplifier system.

These techniques have been successfully adapted to the spectral region beyond the Ti:Sa wavelength of 0.8 $\mu$m and have been investigated experimentally as well as theoretically [44, 45, 47, 48]. In the 2 $\mu$m regime, an optical parametric amplifier (OPA) scheme was used by Hauri et al generating 55 fs, 330 $\mu$J with carrier envelope stability [45]. By filamentation in a xenon cell, they managed to compress the input pulse down to 17 fs with 270 $\mu$J pulse energy. Further, supercontinua spanning up to three octaves have been demonstrated by Kartoshov et al, stretching from $\sim 0.35 \mu$m to 5 $\mu$m [49] and employing a 80 fs, 20 Hz laser with a central wavelength of 3.9 $\mu$m. Mücke et al [50] focused specifically on using 1.5 $\mu$m and managed to achieve a compression factor of three to four, with pulses as short as 19 fs and 1.5 mJ pulse energy generated in an argon cell.

In parallel to these studies, the very well-established hollow-core fiber compression technique has been used at 1.8 $\mu$m to generate $11.5$ fs pulses at 400 $\mu$J of pulse energy [51]. These studies show how post compression can be employed to produce few-cycle pulses in the NIR to MIR region. The next step is to show if conditions can be found to provide tunability of these sources.

A first study was performed using filamentation to generate tunable few-cycle pulses in the region of 1 $\mu$m–2 $\mu$m [52], but it was not clearly shown if the pulse energy was sufficient to perform strong field interaction in the non perturbative regime. We demonstrated the capability of filamentation to produce 1.6 $\mu$m–2 $\mu$m tunable few-cycle pulses [53], and we report in this article how these pulses provide sufficient energy per pulse to produce non perturbative HHG in Xe, Ar, and $N_2$ molecules.

5. Experiment

In order to generate the few-cycle pulses needed, we employed an OPA, which generated 1.6 $\mu$m–2 $\mu$m radiation with a pulse duration of 40 fs. It was seeded by 0.8 $\mu$m, 1 kHz, 40 fs, 5 mJ pulses from a Ti:Sa amplifier. The generated NIR beam is produced with 0.7 mJ–0.9 mJ pulse energy and was loosely focused with an $f = 75$ cm concave mirror.
The 1.2 m cell was filled with 4 bar of krypton. The cell was sealed with one-mm windows with an anti reflection coating spanning 1.2 µm–2.2 µm.

In the generated filament, self-phase modulation is the main contributor to the spectral broadening, but other effects like self-steepening can create a pedestal on the blue side. Further, ionization blue shift of the fundamental wavelength and high-order Kerr effects can also be involved. We showed in a recent study how tunable spectral broadening and compression from 1.6 µm–2 µm can be produced [53]. Figure 2 shows an example of the tunable spectral broadening and compression obtained. We managed to generate three cycle pulses for several central wavelengths of the OPA. The spectrum was broadened in the filamentation process by a factor of two to three to 250–300 nm full width half maximum.

The accumulated positive group delay dispersion (GDD) was compensated by ~4 mm of fused silica. Since the fused silica has a negative GDD for these wavelengths, it was robust and straightforward to compensate for the phase. For 1.7 µm and 1.9 µm, we generated pulses of ~15 fs duration, whereas at 1.8 µm the duration was ~18 fs. The auto-correlation was very close to the Fourier limit of 12 fs for 1.7 µm and 1.9 µm and 15 fs for 1.8 µm.

We employed the 1.7 µm filament for our HHG experiment after identifying the filament size [54]. The centroid of the spectrum changed to 1.74 µm in the process. The white light in the center of the filament determines the size of the beam transverse to the propagation direction. This was estimated to be ~5 mm, whereas the longitudinal extent of the filament can only be inferred from the optical path distance needed for collimation and the focal lengths of the optics. This showed a discrepancy of 12 cm, which is thus estimated to be the longitudinal length. Selecting this spatial region ensures that we only use the few-cycle pulses for high harmonic generation in the subsequent semi-infinite cell (SIC).

The high harmonics were generated in an SIC in a vacuum chamber. The SIC was sealed by a one-mm fused silica window and a 150 µm stainless steel foil at the exit. The 15 fs pulses centered at 1.74 µm were focused by a f = 30 cm mirror (denoted FM) resulting in an interaction length of ~30 mm (cell size). The SIC output foil was drilled by the laser itself, providing an additional differential pumping stage. The pressure can subsequently be used as an additional degree of freedom to enable efficient HHG. The beam size at the entrance and exit of the cell was estimated to be 260 µm and 65 µm, respectively. Due to the fact that we focused on the output foil of the SIC, short and long trajectories for HHG can be phase matched [55, 56]. This, however, is mitigated by the long interaction region, which results in a harmonic spectrum solely due to the short trajectories. The generated HHG is subsequently redirected to a 600 lines/mm XUV grating (denoted G) at grazing incidence. The separated wavelengths are redirected with a toroidal mirror (denoted TM) onto the multi-channel plate (MCP) with phosphor screen and CCD camera. The experimental setup is shown in figure 3.

6. Results

In figure 4 from top to bottom, the HHG spectra for xenon, argon, and nitrogen are presented. The current observation
The xenon spectrum was acquired for a pressure of 41 mbar and a pulse energy of 180 μJ, which should lead to a cutoff energy of $E_{\text{cutoff}} = (3.17U_{\text{p}} + 1.3I_{\text{p}}) = 93.95$ eV ($I_p(Xe) = 12.13$ eV) at the focus, which is at the end of the cell. Estimating the maximum photon energy from the intensity at the entrance of the cell, we can expect a cutoff at 20.6 eV. The best spectrum for argon was obtained for 85 mbar and 430 μJ, potentially generating harmonics up to 200.9 eV ($I_p(Ar) = 15.76$ eV) at the focus. Considering the intensity at the entrance of the cell, we estimated a cutoff energy of 32 eV. The $N_2$ spectra, on the other hand, were generated at 105 mbar and 320 μJ pulse energy ($E_{\text{max-cutoff}} = 154.8$ eV, $I_p(N_2) = 15.58$ eV). The minimum cutoff energy should be about 28.4 eV. The pressure ratio between xenon and argon, $P_{\text{argon}}/P_{\text{xenon}} \approx 2.1$ (nitrogen molecules and argon, respectively, $P_{\text{argon}}/P_{\text{N}_2} \approx 0.81$), is close to the square root of the inverse ratio of their masses, $\sqrt{M_{\text{xenon}}/M_{\text{argon}}} = 1.73$ (\sqrt{M_{\text{N}_2}/M_{\text{argon}}} = 0.87, respectively), so that the density of emitters for HHG in the volume of interaction is comparable. This mass ratio gives an estimation of the difference of emitters in the volume of interaction from one generation gas to the other. Therefore, this stays below one order of magnitude, and hence the harmonic signal is of the same order of magnitude.

A shift is observed in the harmonic spectra generated from xenon and $N_2$ compared to the argon one. This shift can be attributed to a fundamental wavelength shift of about 50 nm. This shift of the fundamental wavelength can be due to plasma formation in the SIC [57] (xenon has a lower ionization potential than argon and nitrogen) or due to modification of the filamentation conditions.

7. Conclusion

We managed to produce tunable few-cycle pulses in the NIR region using filamentation as a post compression stage. These NIR few-cycle pulses have sufficient energy per pulse to enable the generation of high-order harmonics in the non perturbative regime. This tunable source can enable the study of transitions between ionization regimes of atomic and molecular targets in gas or condensate phases.
References

[1] Keldysh L V 1964 Zh. Eksp. Teor. Fiz. 47 1945
Keldysh L V 1965 Sov. Phys. JETP 20 1307 (Engl. transl.)

[2] Lewenstein M, Baluc P, Ivanov M Y, L’Huillier A and Corkum P B 1994 Theory of high-harmonic generation by low-frequency laser fields Phys. Rev. A 49 2117–32

[3] Corkum P B 1993 Plasma perspective on strong field multiphoton ionization Phys. Rev. Lett. 71 1994–7

[4] Schafer K J, Yang B, DiMauro L F and Kulander K C 1993 Above threshold ionization beyond the high-harmonic cutoff Phys. Rev. Lett. 70 1599–602

[5] Agostini P, Fabre F, Mainfray G, Petite G and Rahman N 1979 Free-free transitions following six-photon ionization of xenon atoms Phys. Rev. Lett. 42 1127–30

[6] L’Huillier A, Baluc P, Candel S, Schafer K J and Kulander K C 1992 Calculations of high-order-harmonic-generation processes in xenon at 1064 nm Phys. Rev. A 46 2778–90

[7] L’Huillier A and Baluc P 1993 High-order harmonic generation in rare gases with a 1-ps 1053-nm laser Phys. Rev. Lett. 70 774–7

[8] Fraser D J, Hutchinson M H R, Marangos J P, Shao Y L, Tisch J W G and Castillejo M 1995 High-harmonic generation by resonant plasmon field enhancement Nature 453 757–60

[9] Shaaran T, Cappi M F, Guichard R, Pérez-Hernández J A, Roso L, Arnold M, Siegel T, Zai A and Lewenstein M 2013 High-order-harmonic generation by enhanced plasmonic near-fields in metal nanoparticles Phys. Rev. A 87 041402

[10] Süßmann F and Kling M F 2011 Attosecond plasmonpumping streaking of localized fields near metal nanospheres Phys. Rev. B 84 121406

[11] Schiffrin A et al 2013 Optical-field-induced current in dielectrics Nature 493 70–74

[12] Apalkov V and Stockman M I 2012 Theory of dielectric nanofilms in strong ultrafast optical fields Phys. Rev. B 86 165118

[13] Moulton P F 1986 Spectroscopic and laser characteristics of Ti:Al2O3 J. Opt. Soc. Am. B 3 125

[14] Yudin G L and Ivanov M Y 2001 Nonadiabatic tunnel ionization: Looking inside a laser cycle Phys. Rev. A 64 013409

[15] Ammosov M V, Delone N B and Krainov V P 1986 Tunnel ionization of complex atoms and atomic ions by an alternating electromagnetic field Sov. Phys. JETP 64 1191–4

[16] Baker S, Robinson J S, Haworth C A, Teng H, Smith R A, Chirilă C C, Lein M, Tisch J W G and Marangos J P 2006 Probing proton dynamics in molecules on an attosecond time scale Science 312 424–7

[17] Marangos J P, Baker S, Kajumba N, Robinson J S, Tisch J W G and Torres R 2007 Dynamic imaging of molecules using high order harmonic generation Phys. Chem. Chem. Phys. 10 35–48

[18] Lein M 2007 Molecular imaging using recolliding electrons J. Phys. B: At. Mol. Opt. Phys. 40 R135R173

[19] Iatani J, Levesque J, Zeudler D, Nikiura H, Pépin H, Kieffer J C, Corkum P B and Villeneuve D M 2004 Tomographic imaging of molecular orbitals Nature 432 867–71

[20] Hässler S et al 2010 Attosecond imaging of molecular electronic wavepackets Nat. Phys. 6 200–6

[21] Torres R et al 2010 Extension of high harmonic spectroscopy in molecules by a 1.300 nm laser field Opt. Express 18 3174–80

[22] Lock R M, Zhou X, Li W, Murnane M M and Krapejek H C 2009 Measuring the intensity and phase of high-order harmonic emission from aligned molecules Chem. Phys. 366 2232

[23] Smirnova O, Mairesse Y, Patchkovskii S, Dudovich N, Villeneuve D, Corkum P B and Ivanov M Y 2009 High harmonic interferometry of multi-electron dynamics in molecules Nature 460 972–7

[24] Zait A et al 2013 Molecular internal dynamics studied by quantum path interferences in high order harmonic generation Chem. Phys. 414 184–91

[25] Krause J, Schafer K and Kulander K 1992 High-order-harmonic-generation from atoms and ions in the high-intensity regime Phys. Rev. Lett. 68 5353–8

[26] Lewenstein M, Salières P and L’Huillier A 1995 Phase of the atomic polarization in high-order harmonic generation Phys. Rev. A 52 4747–54

[27] Drescher M, Hentschel M, Kienberger R, Uiberacker M, Yakovlev V, Scrini A, Westerwalbesloh T, Kleineberg U, Heinzmann U and Krausz F 2002 Time-resolved atomic inner-shell spectroscopy Nature 419 803–7

[28] Chen M-C et al 2013 Generation of bright isolated attosecond soft x-ray pulses driven by multi-cycle mid-infrared lasers arXiv:1401.0240 [physics]

[29] Popmintchev T et al 2012 Bright coherent ultrahigh harmonics in the keV x-ray regime from mid-infrared femtosecond lasers Science 336 1287–91

[30] Tate J, Auguste T, Muller H G, Salières P, Agostini P and DiMauro L F 2007 Scaling of wave-packet dynamics in an intense mid infrared field Phys. Rev. Lett. 98 013901

[31] Pérez-Hernández J A, Roso L and Plaja L 2009 Harmonic generation beyond the strong-field approximation: the physics behind the short-wave-ultraviolet scaling laws Opt. Express 17 9891–903

[32] Yavuz I, Altun Z and Topcu T 2012 Wavelength scaling of high-order-harmonic-generation efficiency by few-cycle laser pulses: Influence of carrier-envelope phase Phys. Rev. A 86 043836

[33] Colosimo P et al 2008 Scaling strong-field interactions towards the classical limit Nat. Phys. 4 386–9

[34] Ghimire S, DiChiara A D, Sistrunk E, Agostini P, DiMauro L F and Reis D A 2010 Observation of high-order harmonic generation in a bulk crystal Nat. Phys. 7 138–41

[35] Gruzdev V E 2007 Photoionization rate in wide band-gap dielectrics Phys. Rev. A 75 043836

[36] Oppenheimer J R 1928 Three notes on the quantum theory of atomic polarization in high-order harmonic generation Phys. Rev. 66 063842

[37] Opperheiner J R 1928 Three notes on the quantum theory of aperiodic effects Phys. Rev. 31 66–81

[38] Landau L D and Lifshitz E M 1965 Quantovaya mehanika (Quantum Mechanics) (Reading, MA: Addison-Wesley) (Eng. transl)

[39] Peremolov A M, Popov V S and Terent’ev M V 1966 Ionization of atoms in an alternating electric field Sov. Phys. JETP 23 924
[41] Larochelle S F J, Talebpour A and Chin S L 1998 Coulomb effect in multiphoton ionization of rare-gas atoms J. Phys. B: At. Mol. Opt. Phys. 31 1215–24
[42] Nisoli M, de Silvestri S, Svelto O, Szipőcs R, Ferencz K, Spielmann C, Sartania S and Krausz F 1997 Compression of high-energy laser pulses below 5 fs Opt. Lett. 22 522–4
[43] Guandalini A, Eckle P, Anscombe M, Schlup P, Biegert J and Keller U 2006 5.1 fs pulses generated by filamentation and carrier envelope phase stability analysis J. Phys. B: At. Mol. Opt. Phys. 39 S257
[44] Couairon A and Mysyrowicz A 2007 Femtosecond filamentation in transparent media Phys. Rep. 441 47–189
[45] Hauri C P et al 2007 Intense self-compressed, self-phase-stabilized few-cycle pulses at 2um from an optical filament Opt. Lett. 32 868–70
[46] Berge L, Skupin S, Nuter R, Kasparian J and Wolf J-P 2007 Ultrashort filaments of light in weakly ionized, optically transparent media Rep. Prog. Phys. 70 16331713
[47] Voronin A A, Ališauskas S, Mücke O D, Pugžlys A, Baltuška A and Zheltikov A M 2011 High-energy-throughput pulse compression by of f-axis group-delay compensation in a laser-induced filament Phys. Rev. A 84 023832
[48] Bergé L 2008 Self-compression of 2um laser filaments Opt. Express 16 21529–43
[49] Kartashov D, Ališauskas S, Pugžlys A, Voronin A, Zheltikov A, Petarca M, Béjot P, Kasparian J, Wolf J-P and Baltuška A 2012 White light generation over three octaves by femtosecond filament at 3.9 μm in argon Opt. Lett. 37 3456–8
[50] Mücke O D, Ališauskas S, Verhoeff A J, Pugžlys A, Baltuška A, Smilgevičius V, Pocius J, Gimin纳斯 L, Danielius R and Forget N 2009 Self-compression of millijoule 1.5 μm pulses Opt. Lett. 34 2498–500
[51] Schmidt B E, Béjot P, Giguerre M, Shiner A D, Trallero-Herrero C, Bisson É, Kasparian J, Wolf J-P, Villeneuve D M, Kieffer J-C, Corkum P B and Légaré F 2010 Compression of 1.8 μm laser pulses to sub two optical cycles with bulk material Appl. Phys. Lett. 96 121109
[52] Trisorio A, Grabielle S, Divall M, Forget N and Hauri C P 2012 Self-referenced spectral interferometry for ultrashort infrared pulse characterization Opt. Lett. 37 2892–4
[53] Driever S et al 2013 Tunable 1.6–2 μm near infrared few-cycle pulse generation by filamentation Appl. Phys. Lett. 102 191119
[54] Zaïr A, Guandalini A, Schapper F, Holler M, Biegert J, Gallmann L, Couairon A, Franco M, Mysyrowicz A and Keller U 2007 Spatio-temporal characterization of few-cycle pulses obtained by filamentation Opt. Express 15 5394–404
[55] Salières P et al 2001 Feynman path-integral approach for intense-laser-atom interactions Science 292 902–5
[56] Zaïr A et al 2008 Quantum path interferences in high-order harmonic generation Phys. Rev. Lett. 100 143902
[57] Giammanco F, Pirri A, Brandi F, Barkauskas M and Ubachs W 2005 Measurements of chirp-induced frequency shift in high-order harmonic generation in xenon Laser Phys. 15 328–33