On the properties of the failure probability function in reliability analysis of constructions and structures with regulation of their stress-strain state

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Abstract. The properties and peculiarities of the failure probability function in dependence on the stress-strain state regulators in various (direct, inverse, optimization) problems of calculating the reliability of constructions and structures have been investigated and described. Predicting the possible changes in the probability of failure due to regulators’ varying, on the basis of the formulated signs, allows us to work out the algorithms for finding a rational and optimal regulation of the deformable system state parameters. The qualitative and quantitative illustrations of the identified properties of the failure probability function for multi-span beams with the nonlinear regulation of a bending moments are presented. The possibilities to reveal the areas of permissible values of regulators by the target reliability using a complex reliability indicator that takes into account the structure’s volume of material are shown.

1. Introduction
Regulation of the stress-strain state (SSS) of a construction, structure or their separate parts by various technical methods (force or kinematic pre-stressing, variations of the design scheme or loading etc.) is an effective mean of improving the technical and economic indicators of a construction systems, which is expressed in reducing their resource consumption (such as a materials volume or a structure cost). Therefore, there has been gained experience in applying the regulation of the state parameters of structures – forces, stresses, displacements and etc. in the practice of modern construction. Modern international [1, 2] and national [3 – 7] normative documents (standards) contain the requirements for quantifying reliability indicators, also they establish a general principles for ensuring the reliability of construction systems, up to the specified (target) reliability indexes implemented in Eurocodes considering the classes of structures. In this aspect, the necessity of developing theoretical and applied issues of reliability analysis of constructions and structures in which the regulation of their mechanical state parameters realizes is obvious. The problems of design deformable systems with regard to the influence of various regulators of their SSS in a deterministic formulation were considered in a sufficiently large number of studies. Their reviews are given particularly in [8, 9]. Theoretical foundations and applied methods for determining the reliability of engineering systems, including construction ones, are also well developed – see [10 – 15]. But studies on the special problem of the reliability of constructions and structures with a regulated mechanical state are few in number.

This work is devoted to studying of the properties of the general reliability characteristics of a system (structure) – the failure probability $P_f = 1 - P_r$ (here $P_r$ is reliability) depending on the
regulators, with taking into account the random nature of all design parameters – of the system, loads and regulators.

**The purposes** of the work are:

- research and description of the character and peculiarities of the failure probability function $P_f$ of a deformable system when varying a finite number of regulating parameters (regulators), considered as arguments of the function $P_f$, in different definition versions of the reliability analysis problems;
- qualitative and quantitative verification of the formulated properties of the $P_f$ function in a model problems.

2. **The main part**

2.1. Theoretical bases and methods of the solution of problems

2.1.1. Calculation of reliability $P_s$ or probability of failure $P_f = 1 - P_s$

The main stages of this calculation are:

1) formulation of the serviceability requirements [15] on the basis of non-failure performance general criteria (strength, stability, stiffness etc.):

$$g_j(X) < 0, \ j = 1, m,$$

where $X$ is the vector of design parameters – random variables characterizing the structure, influences (loads) and regulators; the “$<$” sign means the prevention occurrence of the corresponding limit state (if “$<$” in (1) is changing to “$=$”, then (1) sets the boundary of the allowable states area);

2) calculation of the failure probability

$$P_f = 1 - P_s = 1 - \prod_{j=1}^{m}(1 - P_{fj}) \approx \sum_{j=1}^{m} P_{fj},$$

where $P_{fj}$ is the partial probability of failure by the $j$th serviceability requirement.

It is generally accepted in the calculations for (2) with the limiting conditions (1) that the concepts of resistance $R_j$, load effect (factor) $Q_j$ and the reserve of serviceability $S_j$ are used [12–15], while requirements (1) are represented as

$$S_j = R_j - Q_j > 0.$$  \hfill (3)

Load factor $Q_j$ and resistance $R_j$ in (3) generally depend on the design parameters $X$, impacts $q$ (loads and others) and regulators $V$, and moreover $X \in X$, $q \in X$, $V \in X.$ With using of the known stochastic properties of random vectors $X$, $q$ and $V$ the characteristics of the variables $Q_j$ and $R_j$ can be found – such as the bivariate distribution density function $p_{QjRj}(Q_j, R_j)$, means $\bar{Q}_j, \bar{R}_j$, coefficients of variation $\sigma_{Qj}, \sigma_{Rj}$. They determine the probabilistic characteristics of the reserve $S$ – the density $p_S(S)$, mean $\bar{S}_j$ and standard deviation $\hat{S}_j$, after which the probability of failure according to the $j$th restriction (1) is calculated as

$$P_f = P(S_j < 0) = \int_{-\infty}^{0} p_S(S_j) dS_j \hfill (4)$$

or through the reliability index [2] $\beta_j = \bar{S}_j / \hat{S}_j$ (according to [15] – safety characteristic):

$$P_{fj} = 10^{-\omega(\beta_j)}, \hfill (5)$$
the expression \( q_j (\beta_j) \) depends on the type of distribution \( p_{\beta_j} (S_j) \); for example, we use the Laplace function in the case of a normal law: \( P_{\beta_j} = 0.5 - \Phi_{0j}(\beta_j) \).

The formula for the reliability index is conveniently represented as

\[
\beta_j = (1 - \xi_j) \left[ \xi_j^2 A_{Rj}^2 + \left( \xi_j^2 A_{Qj}^2 - 2 \xi_j A_{Rj} A_{Qj} r_{RjQj} \right) \right]^{-1/2},
\]

where \( \xi_j = \overline{Q_j} / \overline{R_j} \); \( r_{RjQj} \) – correlation coefficient of \( R_j \) and \( Q_j \).

Load factors \( Q_1, \ldots, Q_m \) are expressed through effects of \( q \), regulators \( V \), and also the characteristics of the system \( X_i \) in the form

\[
Q_j = f_{Qj}(q, V, X_i), \quad j=1,m
\]

using the equations of state (algebraic or differential, linear or nonlinear, in particular, FEM equations). For linearly deformable systems with linear regulators \( V \), dependencies (7) are written as

\[
Q_j = Q_{jq} + \sum_{k=1}^{n_v} Q_{j,k-1} V_k, \quad j=1,m
\]

or in matrix form

\[
Q = \{Q_1, Q_2, \ldots, Q_m\} = Q_q + \Lambda_q V
\]

where \( Q_q = \{Q_{q1}, Q_{q2}, \ldots, Q_{qm}\} \) – vector of factors \( Q \) from the effects of \( q \);

\( V = \{V_1, V_2, \ldots, V_{n_v}\} \) – vector of regulators; \( n_v \) – number of quantities \( V \);

\( \Lambda_q = \begin{bmatrix} Q_{q1,1-1} & \ldots & Q_{q1,n_v-1} \\ \ldots & \ldots & \ldots \\ Q_{qm,1-1} & \ldots & Q_{qm,n_v-1} \end{bmatrix} \) – influence matrix of regulators \( V \) on \( Q \).

Equations (7) describe hypersurfaces, and (8) and (9) describe hyperplanes in \( n_v +1 \)-dimensional spaces \( V - Q \). If all \( Q_j \) are of the same type, then it is possible to represent (7) – (9) in the general hyperspace \( V - Q \). This is convenient for a comparative estimation of changes in \( Q \) factors in depending on the varying regulators \( V \). With heterogeneous \( Q_j \), such a combination can be achieved due to their transformation into dimensionless form [16].

The quantities \( V \) calculated by (2) – (9) can take any values, but in designs where the regulation of SSS is specifically applied, \( V \) are determined by the originally formulated requirements for the results of regulation.

2.1.2. Setting and solving the problem of regulation

The two main types of problems in calculating the regulation of SSS of structures are direct and inverse. The problem of optimal regulation may also be of practical interest and it is usually solved by successive calculations in direct or inverse formulations.

In the direct problem certain values of the regulators \( V \) used directly in the calculations from (7) – (9) are specified initially.

The inverse problem is: for a system with known own parameters \( X_i \) and impacts \( q \) it is necessary to determine the values of the regulators \( V \), ensuring the correspondence to requirements for a selected complex of regulated parameters of the SSS \( H = \{H_1, H_2, \ldots, H_{n_H}\} \), initially formulated on the basis of certain criteria set (it is possible that \( H \cap Q \)).

Equations expressing the conditions (requirements) of regulation

\[
\Phi_i (H) = 0, \quad i=1,n_{\Phi},
\]
may be linear or non-linear, homogeneous or nonhomogeneous. The values of $H_k$ ($k = 1, n_H$) are represented by analogy with (7) as

$$H_k = F_k(q, V, X_s), \quad (11)$$

in particular, in the linear case $H = H_q + \Lambda H V$ (here $\Lambda H$ is the matrix of the influence of $V$ on $H$).

Combining (10) and (11), we obtain the regulation equations expressed through regulators $V$:

$$\psi_i(q, V, X_s) = 0, \quad i = 1, n_y. \quad (12)$$

Their solution gives the desired values of $V$. This is possible for an arbitrary realization of random variables $X_s, q$, and $V$, but it is practically to consider their expected values (means). The obtained $V$ are used further in (7) – (9), after which $P_f$ is calculated from (2) – (6).

2.2. Main properties of failure probability function $P_f$ for deformable systems with regulated stress-strain state

In the reliability analysis of structures, there are problems of two main types – direct (verification) and inverse (design). In the direct problem the probability of failure is calculated with initially specified system’s parameters and effects (loads). Inverse problems usually do not have a single solution and, as a result, reveal the areas of allowable values of the design parameters for a target (required) reliability. It is also possible to formulate the optimization problem according to the criteria of reliability (or probability of failure).

In the main reliability calculation for the systems with SSS regulators the problem of regulation may appear in both direct and inverse forms, depending on the general setting.

Further only direct reliability calculations are considered, in which the failure probability values, as a rule, are uniquely dependent on certain combinations of regulators (except for cases of strong non-linearity of equations (7), (10), (11)). Variants of problem setting are:

- **SSS regulation is carried out with unchanged loads $q$ and system parameters $X_s$;**
- **after each changing of regulators the characteristics of $X_s$ are corrected (according to limitations of strength, stability etc.) – such procedure is natural in practical calculations in the design of structures.**

In variant 1:

- a) in the case when the functions described by dependencies (7) are smooth, the failure probability function $P_f(V)$ is also smooth; its minimum corresponds to a combination of regulators that provides the most advantageous stress-strain state of the structure, taking into account the restrictions (1) or (3) (modular forms or out-forms [17] can be used to identify this combination);
- b) if the functions (7) have any singularities (discontinuities or jumps of derivatives), in particular, when the parameters of the system $X_s$ change due to the variation of the regulators $V$ (for example, reducing the rigidity during plasticity development with the bilinear dependence $\sigma \sim \varepsilon$), the $P_f(V)$ function also has features (smoothness violations).

In variant 2:

- determination of some complex of calculated parameters $X_{o0}$, in particular, the geometric characteristics of sections $G$ (possibly, $G \cap X_s$ or $G$ are other than $X_s$) is carried out according to requirements of type (3). If all $G$ are set through one common parameter $G_0$, then to find it, it is necessary to identify the object (structural element, section, dangerous point) that is the closest to the design limit state. To do this, for some combination of regulators $V$ (arbitrarily specified or corresponding to the required regulation results, according to conditions (10)), the coefficients $\xi_j$ are estimated, and by the largest of them ($\xi_0 = \max \xi_j$) from the corresponding equation of (3) type, in which $\xi_0 = \lfloor \xi \rfloor$ (here $\lfloor \xi \rfloor$ is preliminary specified), the value of $G_0$ is obtained. Note that all $\xi_j \leq \xi_0$ in this solution. With a certain combination of SSS regulators the system may be such that two or more
design objects may turn out to be “equally dangerous”, with the same \( \xi_j = \xi_0 = [\xi] \). The equality of the coefficients \( \xi \) for a certain pair of restrictions (3) when varying the regulators gives the line of intersection (edge) of two corresponding hypersurfaces, and on different sides of this line the design points of surfaces with the largest from two values are selected for finding \( G_0 \). This means that the out-form [17] for characteristics (7) or (3) (in the general case in a dimensionless form) has an edge i.e. violation of the smoothness of its hypersurface. The limit state conditions of two different objects (structural elements, sections, points) use in this case in the areas on opposite sides of an edge. As a result, as noted above, a break (edge) on the hypersurface of the failure probability \( P_f(V) \) appears too.

In the case of one regulator \((n_V = 1)\) the \( P_f(V) \) diagram is a piecewise line with peak-like segments (figure 1, (a)) – this was shown in [18]. The qualitative view of the indicated peculiarity of the function \( P_f(V) \) graph in the case with \( n_V = 2 \) is shown in figure 1, (b) (it can be considered as a particular case when \( n_V > 2 \) and the fixed values of the other regulators are given).

(a)  
(b)  

![Figure 1. The peculiarity of the failure probability function \( P_f \) graph – break (edge) with one regulator (a), with two regulators (b).](image)

If several regulators and serviceability requirements \((n_V \geq 2, m > 2)\) are used, an equally dangerous state of three or more design objects of the system is possible as a result of improvement of the SSS by using a regulators (leveling the distributions of forces or stresses). This is accompanied by the occurrence of three or more break (edges) on the failure probability \( P_f(V) \) hypersurface converging at the top (peak), as it is shown in figure 2. If the number of edges increases, the peak sharpens and its height (\( P_f \) value) correspondingly increases.

When the regulators change within sufficiently wide limits, as well as when several independent parameters \( G \) are determined by the requirements (3), several peaks may appear on the graph of the function \( P_f(V) \). Comparable peak’s values can occur with significantly different combinations of regulators.

The nature of the \( P_f(V) \) hypersurface in the vicinity of a certain peak depends on the features of the system (more advanced design forms, which are cost-effective in terms of material consumption, can give higher levels of failure probability), as well as on the effectiveness of regulators and stochastic properties of the main design parameters of the system, regulators and loads.

The graphs of the \( P_f \) function for large \((A_{\text{min}})\) and small \((A_{\text{min}})\) coefficients of variation of the design variables are qualitatively presented in figure 3, (a). The upper graph not only has significantly larger ordinates, but also has smaller, in comparison with the lower graph, derivations \( \frac{dP_f}{dV} \) (or gradients in cases of more than one variables \( V \)) of the branches on a different sides of the vertex, which
corresponds to a wider range $D_2$ of inadmissible regulator values with a given limiting probability of failure $[P_f]$ – this is disadvantageous in terms of material intensity.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The salient feature of the failure probability function $P_f$—peak in the case of $n_V = 2$.}
\end{figure}

In cases where the coefficient $\xi_j > 1$ takes place and the reliability index $\beta_j < -2 \ldots -3$, the appearance of the failure probability values close to 1 is possible, which gives areas of hyperplanes in the form of an almost plateau with $P_f \approx 1$ (figure 3, (b)) – of course, the approach to these areas is unacceptable.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Graphs of the failure probability function for different characteristics of the variability of the design parameters (a), a possible view of $P_f(V)$ surface with the coefficient $\xi_j > 1$ (b).}
\end{figure}

The same effect would be if all conditions (3) are used without exceptions with the fixed coefficients $\xi_j = \lfloor \xi_j \rfloor$ for obtaining the parameters $G$. As a consequence a values of function $P_f(V)$ may be almost flat ($P_f(V) \approx \text{const} < 1$) with the possible deviations due to the different influence of design parameters’ coefficients of variation according to various combinations of regulators.

Predicting the nature and features of the probability function of failure based on the peculiarities and properties formulated above allows in the process of structure design to look for areas of regulators’ values in which the probability of failure does not exceed the allowable level $[P_f]$. It is
essential, especially for optimal regulation of the system’s SSS, to take into account the material consumption of the structure, which is achieved by using the integrated dimensionless indicator [18], which takes into account also the volume of material $v$ and the load parameter $q$, along with the probability of failure $P_f$:

$$\rho = \rho^* \cdot v^*, \quad (13)$$

where $\rho^* = P_f / P_f^0$; $v^* = v_0^* / q^*$; $q^* = \bar{q} / \bar{q}_0^*$; $\bar{P}_f^0, \bar{v}_0, \bar{q}_0^*$ – baseline values.

2.3. Example

The problem of estimating the reliability of structures with regulated parameters was considered in application to statically determinate beams with arbitrary number of spans (figure 4).

Setting of the regulation problem:

- the bending moments in characteristic sections of the beam are taken as the regulated parameters: $H \equiv M = \{M_1, M_2, \ldots, M_{nM}\}$;
- the regulators are the structural and geometric characteristics of the beam’s design scheme – the relative coordinates of the hinges and the ratio of the lengths of the spans $V = \{v_1, v_2, \ldots, v_n\}$ (here $\bar{q}_i = \tilde{q}_i / l_i$; $\bar{v}_i = \tilde{l}_i / l_i$), $n_V = n_{\alpha} + n_v$ (figure 4);
- the influence of the selected regulators on the regulated parameters (moments) is non-linear [9];
- the regulation requirements are set in the form of a ratio between the means of bending moments:

$$\bar{M}_1 = -k_1\bar{M}_2; \quad \bar{M}_2 = -k_2\bar{M}_3; \ldots; \quad \bar{M}_j = -k_j\bar{M}_{j+1}; \ldots; \quad \bar{M}_{nM} = -k_{nM}\bar{M}_{nM+1} \quad (14)$$

Or

$$\begin{cases} |\bar{M}_1| = k_1 |\bar{M}_2| \\
|\bar{M}_2| = k_2 |\bar{M}_3| \\
\vdots \\
|\bar{M}_j| = k_j |\bar{M}_{j+1}| \\
|\bar{M}_{nM}| = k_{nM} |\bar{M}_{nM+1}| \end{cases} \quad (15)$$

where $k_1, k_2, \ldots, k_j$ – coefficients of regulation that are associated with regulators $V$, but they are more convenient for use in this example ($V$ values can be expressed in $k$).

Note that in the case at issue ((15) are homogeneous) the condition $n_M = n_V + 1$ should be satisfied.

![Figure 4](image)

**Figure 4.** The design schemes of beams with type …s-h-s-h… (a), type …s-s-h-s-s-h-s-h… (b) (s – support, h – hinge).

When the coefficients of regulation $k$ are assigned then the means of regulators obtain from the restrictions (1) or (2) with use of a state equations, nonlinear in the considered problem, while the
expected values of the moments contained in a conditions (1), (2) are expressed in dependence of the loads, regulators and beam parameters also from the state equations (nonlinear).

Fundamentally important is that the main design variables were presented in a dimensionless form, which formed the basis of a rational approach to solving the problem.

With known (specified) coefficients of variability of the design parameters it is possible to calculate the system’s failure probabilities corresponding to the varying coefficients of regulation.

The two-span beam was considered (figure 5, (a)). Regulators are $\alpha_1 = \bar{a}_1/\bar{l}_2$, $\alpha_2 = \bar{a}_2/(v\bar{l}_2)$, $v = \bar{l}_1/\bar{l}_2$ ($n_v = 3$), they correspond to the regulation coefficients $k_1$, $k_2$, $k_3$; regulated parameters $H = \{M_1, M_2, ... M_4\}$ ($n_H = 4$); serviceability requirements – by stresses in sections with moments $M_1, M_2, ... M_4$ ($m = 4$) when the beam is in an elastic stage of work.

Problem statement is in accordance with the above variant 2 with one geometrical parameter $G_0$ (the cross section is constant along the beam).

![Figure 5](image_url)

**Figure 5.** The design scheme of the beam (a), bending moments means diagram (b), the graph of the failure probability function in dependence on the coefficients of regulation (c).

Initial data: $l = 5$ m; $q = 5$ kN/m; $A_t = 0,01$; $A_q = 0,02$; $A_{\sigma_0} = 0,07$; $A_W = 0,01$; $A_{\sigma_1} = 0,001$; $A_{\sigma_2} = 0,001$; $A_{\sigma_2} = 0,001$; $A_v = 0,01$; $[\xi_0] = 0,7$.

The stress variation coefficients at dangerous points of the design cross sections which are needed for calculating the failure probability were determined by the statistical linearization method [15]. The results of the calculations of $P_f$ are presented graphically in figure 5, (c) for a fixed value $k_3 = 1$.

The graphs clearly show the predicted above specific (peak-like) nature of the failure probability dependence on the regulation coefficients. For systems with one and two regulators this feature was discovered earlier, here it is confirmed with three regulators. At point $A$ ($k_1 = 1$, $k_2 = 1$, $k_3 = 1$), which corresponds to the most favorable regulation from the position of minimizing the consumption of materials, the maximum value of the failure probability is obtained.

For the same beam, the complex characteristic (13) of the failure probability was calculated; its dependence on regulators is presented in figure 6, (a). Comparison of this graph with the graph of the probability of failure according to figure 5, (b) shows that taking into account the material’s consumption leads to the changes in the mutual arrangement and the ordinates ratio for the surfaces characterizing the probability of failure.
The influence was studied of design parameters’ variability characteristics on the type and features of the function $P_f$ graphs. It is known that the coefficients of variation of loads can vary in fairly wide intervals – much larger than the coefficients of variation of the strength and geometric characteristics. For comparison, the following values were taken:

$0.05 < A_q < 0.15$; $0.0005 < A_t < 0.015$; $0.01 < A_w < 0.02$; $0.001 < A_{at/2} < 0.02$; $0.001 < A_v < 0.02$; $0.03 < A_{su} < 0.1$. The dependence of $P_f$ on the coefficients of variation is presented in figure 7.

For comparison, the following values were taken: $0.05 < A_q < 0.15$; $0.0005 < A_t < 0.015$; $0.01 < A_w < 0.02$; $0.001 < A_{at/2} < 0.02$; $0.001 < A_v < 0.02$; $0.03 < A_{su} < 0.1$. The dependence of $P_f$ on the coefficients of variation is presented in figure 7.

The direct problem of calculating the reliability of the beam was also solved by varying the regulators without changing the geometric characteristics of the sections (for a fixed value of the material strength characteristic) – variant 1 (see above). The graph of the failure probability function $P_f$ in a relatively narrow zone of $k_1$ and $k_2$ changes (figure 8) is a smooth surface with a distinct concavity and the minimum point at $k_1 = k_2 = 1$. In the peripheral areas degeneracy is observed in the
almost plane form with \( P_f \approx \text{const.} \) For larger values of \( k_1 \) and \( k_2 \) the increasing of the \( P_f \) is observed (not shown in figure 8).

![Figure 8. Graph of the failure probability function in direct formulation of problem.](image)

The results of solving the direct problem of reliability analysis of the model system (beam) are in concordance with the predictions on the basis of detected specific features of the failure probability function.

3. Conclusion
1. The analysis has been performed and a description are presented on the main properties and peculiarities of the failure probability function of deformable systems (constructions and structures), in which the stress-strain state is regulated by various methods. The direct and inverse statements of the problems of calculating the reliability and regulation are considered.

2. A method, a design apparatus and algorithms for determining the reliability of beam systems with a different number of non-linear regulators, in various problem statements, including the procedure of using the dimensionless random variables to simplify calculation techniques, are developed. The obtained results of beams’ reliability calculations correspond to the predictions of the properties of the failure probability function based on the formulated properties and features.

3. The quantitative estimates of an influence of the design parameters’ variability characteristics on the probability of failure are presented. The reliability indicators have been calculated using a complex criterion that takes into account the material consumption of the structure with SSS regulators.

4. The results obtained in this work make it possible to set and solve the problems of optimal regulation of the SSS of constructions and structures with regard to the requirements of reliability.

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