Cognitive Radio Inspired Uplink Rate-Splitting Multiple Access

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Abstract—With the exponential increase of the number of devices in the communication ecosystem toward the upcoming sixth generation (6G) of wireless networks, more enabling technologies and potential wireless architectures are necessary to fulfill the networking requirements of high throughput, massive connectivity, ultra reliability, and heterogeneous Quality of Service (QoS). To this end, schemes based on rate-splitting multiple access (RSMA) are expected to play a pivotal role in next generation communication networks. In this work, we investigate an uplink network consisting of a primary user (PU) and a secondary user (SU) and, by introducing the concept of cognitive radio (CR) into the RSMA framework, a protocol based on RSMA is proposed. This protocol aims to serve the SU in a resource block which is originally allocated solely for the PU without negatively affecting the QoS of the PU. Moreover, a similar but simpler protocol based on successive interference cancellation is proposed. We derive closed-form expressions for the outage probability of the SU for the two proposed protocols, ensuring that there exists no negative impact for the PU. To obtain further insights, asymptotic analysis is performed and the corresponding diversity gains are presented. In the numerical results, we validate the theoretical analysis and illustrate the superiority of the proposed protocols over two benchmark schemes.

Index Terms—rate-splitting multiple access, cognitive radio, next generation multiple access, uplink network, outage probability, asymptotic analysis, diversity gain.

I. INTRODUCTION

W

ith the development of the Internet of Things (IoT) and the consequent integration of a huge amount of heterogeneous wireless devices, sharing the same orthogonal resources is necessary to achieve the goals of the next generation of wireless communication networks, including higher connectivity and data rates, as well as reduced delay and energy consumption [1], [2]. To efficiently explore enhanced networking functionalities in such complicated wireless communication scenarios and meet the heterogeneity of networking Quality of Service (QoS) requirements, enabling network slicing techniques, evolutionary multiple access mechanisms, and advanced cognitive radio (CR) technologies are required. The aforementioned approaches are capable of providing dynamic networking services, while they have the potential to reduce user interference and congestion [3]. Specifically, to address the generic networking features of the upcoming sixth generation (6G), i.e., extremely reliable and low-latency communication (ERLLC), further-enhanced mobile broadband (FeMBB), and ultra-massive machine-type communication (umMTC) [1], [3]–[6], the occupied resources could be utilized in both an orthogonal and a non-orthogonal way to provide heterogeneous 6G services. The use of this concept as the enabler of next generation multiple access schemes has attracted considerable research interest.

A. Motivation & State-of-the-Art

According to the 3GPP releases launched by the International Mobile Telecommunications for 2020 and the follow-up study based on new radio, the fundamental categories of multiple access (MA) schemes are orthogonal multiple access (OMA) and non-orthogonal multiple access (NOMA) at the physical and medium access control layer, aiming to serve multiple users in uplink or downlink wireless communication systems [7]–[9]. Specifically, conventional OMA provides communication connectivity to the participating users by allocating an orthogonal wireless communication resource in the frequency, time or spatial domain to each user. To overcome the drawback of restricted number of users resulting from the limited available orthogonal resources, NOMA can meet the requirements of high spectral efficiency and massive connectivity by breaking the orthogonality of the available resources. In terms of the interference management, the fundamental principle of OMA and NOMA rely on treating interference as noise and decoding interference, respectively, thus OMA and NOMA can be also considered as two extreme interference management strategies. To this end, a unified and general MA scheme is needed as a compromising solution to assist in coping with different communication scenarios [10].

To this end, a promising way to break orthogonality is rate-splitting multiple access (RSMA). Although RSMA enables the use of the available resources in a non-orthogonal way, it is based on a different principle compared to what is commonly referred as NOMA. Its main characteristic is its capability to provide important advantages compared to OMA in terms of connectivity, delay, throughput, energy efficiency with acceptable complexity, which do not vanish under practical conditions. In brief, the key behind realizing those
benefits is the ability of rate-splitting (RS) to partially decode interference and partially treat interference as noise by splitting messages. To this end, RSMA provides flexible decoding and, thus, a more general and robust transmission framework in comparison with the conventional NOMA mechanism.

In the existing literature, most works on RSMA investigate the downlink scenario. In downlink RSMA, the message transmitted to the users is divided into a common message and a private message. The common message is a message decoded by multiple users and the private message is a message that only a specific user intends to receive. Therefore, adjusting the split of common and private messages can control the computational complexity and the data rate achieved by RSMA. The robustness and flexibility of downlink RSMA under heterogenous traffic (unicast, multicast, broadcast) demands and user capabilities, underloaded and overloaded scenarios and different propagation conditions have been demonstrated in the existing literature. In [11], a multiple input single output broadcast downlink network consisting of two users was investigated and it was highlighted that RSMA generalizes OMA, NOMA, space division multiple access and multicasting. Moreover, in [12], the spectral and energy efficiency tradeoff of RSMA was studied in a multi-user multi-antenna downlink network. Furthermore, in [13], an optimization problem was studied aiming to maximize the sum-rate under both rate and successive interference cancellation (SIC) constraints in a downlink RSMA network. Also, in [14], the precoder design for an underloaded or critically loaded downlink was investigated in a multi-user multiple-input multiple-output (MIMO) communication system which is based on RS at the transmitter and single-stage SIC at the receivers.

Concerning the uplink scenario, there is a subset of users that simultaneously transmit more messages than the number of users belonging in this subset. The utilized decoding order of the users’ messages is not necessarily fixed, but it can be chosen by the base station (BS), based on the instantaneous channel state information. It should be highlighted that uplink RSMA enables any point in the capacity region of the multiple access channel (MAC) to be achieved with successive decoding [15]. Specifically, in [16], the RS principle was applied in an uplink NOMA network and its performance in terms of outage probability and achievable sum rate was investigated. Moreover, in [17], an uplink NOMA network with RS consisting of two users was investigated and it was shown that the fairness among users and the outage performance are improved. Also, in [18], the performance of an uplink RSMA network with two sources was investigated in terms of outage probability and throughput, considering all possible decoding orders. In [19], an RSMA strategy for a semi-grant-free transmission was proposed to increase connectivity and reliability, while in [20], a sum-rate maximization problem was investigated in an uplink RSMA network. Furthermore, in [21], two novel cooperative-NOMA and cooperative-RSMA schemes were proposed for uplink user cooperation, the achievable rates were derived and two optimization problems were formulated to maximize the minimum rate of two users.

Moreover, considering the characteristics of spectrum sharing in the modern MA framework, the integration of MA and CR is expected to have a significant impact on coping with spectrum scarcity, as well as on meeting the satisfactory spectrum efficiency and the heterogeneity of QoS requirements in umMTC scene. Specifically, in [22], [23], CR inspired NOMA was presented where a secondary user (SU) occupies the resource block of a primary user (PU) without impacting negatively the performance of the PU. However, some restricting assumptions regarding the decoding order and the power allocation were considered. To this end, the comprehensive research on the integration of MA and CR evolves from the conventional OMA and NOMA schemes to the promising RSMA paradigm.

CR inspired protocols can be practically and efficiently utilized in sliced networks. Network slicing allows heterogeneous services to coexist in the same network architecture by allocating the resources of the network among the active services ensuring their isolation and that the targeted performance is achieved [24]. Specifically, in the context of 6G, FeMBB devices, which display a stable device activation pattern and large payloads over an extended time duration, umMTC traffic, which is characterized by a sporadic activation pattern and a fixed target rate, and ERLLC devices, which also transmit intermittently but require high reliability, are expected to coexist. Considering the intermittent nature of umMTC and ERLLC transmissions which are usually served with random access schemes, it can be useful and practical to share the resource blocks which are originally allocated to these transmissions with FeMBB ones. By treating the umMTC and ERLLC devices as PUs and the FeMBB ones as SUs, it should be ensured that the performance of the former ones is not affected.

B. Contributions

Motivated by the above, in this work, two CR inspired protocols are proposed based on RSMA and SIC for an uplink network, consisting of a BS and two users, i.e., a PU and an SU. Specifically, the contribution of this work is presented in detail below:

- We investigate an uplink network, consisting of a BS and two users, i.e., a PU and an SU. Inspired by CR, we propose two protocols based on RSMA and SIC, aiming to serve the SU in a resource block which is originally allocated solely for the PU without negatively affecting the QoS of the PU. Regarding the RSMA proposed protocol, RS is performed by the SU and the power allocation factor is derived offering advantages in a twofold way. Firstly, it is ensured that there exist no negative impact on the performance of the PU and, secondly, the achieved performance by the SU is optimized. Moreover, the SIC protocol can be considered as a similar but simpler case of the RSMA protocol. Regarding SIC, we determine the decoding order that ensures no performance degradation for the PU and the best possible performance for the SU.

- We analyze the performance in terms of outage probability for the two proposed protocols taking into account the necessity to satisfy the QoS of the PU. Considering...
the derived power allocation factors for both protocols, 
the outage probability of the SU is derived in closed-
form. It should be highlighted that the outage probability 
of the PU is the same with the single-user case, since it 
is not affected by the admission of the SU.
• To provide further insights, we perform the asymptotic 
analysis of the outage probability of the SU, extracting 
the corresponding diversity gains. The asymptotic 
analysis highlights that the proposed protocols avoid the 
floor of the outage probability which frequently occurs 
in NOMA systems Moreover, the obtained diversity gain 
over the conventional NOMA mechanism can prove the 
robustness of the proposed protocols.
• We present simulations to validate the theoretical analysis 
and also to indicate the effectiveness of the proposed proto-
colos over the conventional NOMA mechanisms which 
are illustrated as benchmarks. Besides, simulations illus-
trate that the RSMA protocol improves outage probability 
by 3 dB compared with the SIC protocol.

C. Structure
In what follows, the considered system model and the CR
inspired protocols based on RSMA and SIC are introduced in 
Section II. In Section III, the outage probability of the SU 
is investigated for the two proposed protocols and closed-
form expressions are derived. To obtain more insights, the 
asymptotic analysis is provided in Section IV, according to 
which, the corresponding diversity gains for both RSMA and 
SIC protocols are presented. Next, simulations are illustrated 
in Section V. Finally, the work is summarized in Section VI.

II. SYSTEM MODEL AND PROPOSED PROTOCOLS
We consider an uplink network consisting of a BS and two 
users, a PU and an SU. All nodes are assumed to be equipped 
with a single antenna. For this network, two CR inspired 
protocols based on RSMA and SIC are proposed where the 
SU occupies the resource block of the PU without impacting 
negatively the performance of the PU.

The complex channel coefficient of the \(i\)-th user with \(i \in \{P, S\}\) is denoted by \(h_i\). We assume Rayleigh fading, thus, 
\(|h_i|^2\) follows the exponential distribution with rate parameter 
equal to one. Furthermore, the received signal-to-noise ratio 
(SNR) at the BS from the \(i\)-th user can be denoted as
\[
\gamma_i = \gamma_0 l_i |h_i|^2, 
\]
where \(\gamma_0\) and \(l_i\) denote the average received SNR of the \(i\)-th 
user at the reference distance \(d_0\) and the path loss coefficient 
between the \(i\)-th user and the BS, respectively. Specifically, \(l_i\) 
is given by
\[
l_i = \left(\frac{d_i}{d_0}\right)^{-\alpha} 
\]
with \(d_i\) and \(u\) being the distance between the \(i\)-th user and the 
BS and the path loss exponent, respectively. Therefore, the 
average received SNR \(\gamma_i\) follows the exponential distribution 
with rate \(\lambda_i = \frac{1}{\gamma_0 l_i}\).

A. Cognitive radio inspired RSMA
As it is also shown in Fig. 1(a), for the proposed RSMA-
based protocol, the message of one user needs to be split in 
order to achieve the capacity region [15]. It is assumed that 
the PU transmits a single message, while the SU transmits two 
messages in a way that does not have a negative impact to the 
performance of the PU. Thus, the received message at the BS 
can be written as
\[
y = \sqrt{\alpha} l_p h_p (x_1^P + x_2^P) + \sqrt{(1-\alpha)} l_s h_s (x_1^S + x_2^S) + \sqrt{\lambda_p P_T x_p} + n, \tag{3}
\]
where \(p_i\) denotes the transmitted power of the \(i\)-th user. 
Moreover, \(\alpha \in [0, 1]\) and \(n\) denote the power allocation factor 
and the additive white Gaussian noise at the BS with zero 
mean and variance \(\sigma^2\), respectively. Furthermore, \(x_1^P, x_2^P\) and 
\(x_1^S, x_2^S\) denote the first message of the SU, the second message 
of the SU and the message of the PU, respectively. Without loss 
of generality, it is assumed that \(x_1^P\) is decoded first. Thus, when 
decoding the message of the PU, \(x_2^P\) is handled as interference. 
This decoding order provides flexibility and ensures that the 
performance of the PU is not negatively affected by properly 
selecting the power allocation factor \(\alpha\).

B. Cognitive radio inspired SIC
By using the main principles of cognitive radio inspired 
multiple access, a similar protocol based on SIC is proposed 
which can be considered as a simpler special case of the 
proposed RSMA-based protocol. The main principle of cognitive
radio inspired SIC is also illustrated in Fig. 1(b). In this SIC-based protocol, the SU transmits only one message utilizing a power allocation factor $c$ in order to avoid a negative impact to the performance of the PU. Thus, the received message at the BS can be written as

$$y = \sqrt{1_{P_P}} h_P x_P^P + \sqrt{c S_{PS}} h_S x_S + n. \tag{4}$$

The decoding order for the SIC protocol to also ensure that the PU is not negatively affected is given by

$$\mathcal{F} = \{ \text{SU, PU} \}, \quad \frac{\gamma_P}{\gamma_P + 1} \geq \theta_S$$

$$\mathcal{F} = \{ \text{PU, SU} \}, \quad \text{otherwise.} \tag{5}$$

### III. PERFORMANCE ANALYSIS

In this section, the performance of the protocols based on RSMA and SIC is investigated and closed-form expressions for the outage probability are derived.

#### A. Analysis of cognitive radio inspired RSMA

Regarding the protocol based on RSMA, the achievable rate of the PU can be written as

$$R_P = B \log_2 \left( 1 + \frac{\gamma_P}{1 - \alpha} \gamma_S + 1 \right), \tag{6}$$

with $B$ being the bandwidth of this system. Since the performance of the PU must not be negatively affected, the constraint $\gamma_P \geq \theta_P$, where $\theta_1 = 2^{R_{1,th}} - 1$ with $R_{1,th}$ being the target rate, must be satisfied if possible. Thus, $\alpha$ is derived as follows

$$\alpha = \begin{cases} 
0, & \frac{\gamma_P}{\gamma_P + 1} \geq \theta_P \\
1, & \frac{\gamma_P}{\gamma_P + 1} < \theta_P 
\end{cases} \tag{7}$$

As stated in (7), outage occurs in the message of the PU only when $\alpha = 1$, thus the outage probability of the PU is given by

$$P_P = 1 - e^{-\lambda_P \theta_P}. \tag{8}$$

The achievable rate of the SU is given by

$$R_S = B \log_2 \left( 1 + \frac{\alpha \gamma_S}{\gamma_P + (1 - \alpha) \gamma_S + 1} \right) + B \log_2 \left( 1 + \frac{\alpha \gamma_S}{\gamma_P + (1 - \alpha) \gamma_S} \right). \tag{9}$$

The main contribution of this work is to calculate the outage probability of the SU, which is defined as the union of three mutually exclusive events. The first event occurs when the message of the PU cannot be decoded and the message of the SU also cannot be decoded be treating as interference the undecoded PU’s message. The second event occurs when the message of the PU is decoded successfully by treating the message of the SU as interference, but consequently the message of the SU cannot be decoded. These two events describe the cases that no splitting is performed. Finally, the third event occurs when the message of the PU can be solely decoded, but it cannot be decoded with the whole SU’s message as interference, and consequently the achievable rate of the SU is below the target rate of the SU by using the optimal values of the power allocation factor $\alpha$ given in (7).

Therefore, the outage probability of the SU for the proposed RSMA protocol is given by

$$P_S^{RSMA} = \Pr \left( \gamma_P < \theta_P \cap \frac{\gamma_S}{\gamma_P + 1} < \theta_S \right) + \Pr \left( \frac{\gamma_P}{\gamma_P + 1} \geq \theta_P \cap \gamma_S < \theta_S \right) + \Pr \left( \theta_P \leq \gamma_P < \theta_P (\gamma_S + 1) \cap O_S \right), \tag{10}$$

where the term $O_S$ denotes the event that

$$\log_2 \left( 1 + \frac{\gamma_S - \frac{\theta_P}{\gamma_P} + 1}{\gamma_P + \frac{\theta_P}{\gamma_P}} \right) + \log_2 \left( \frac{\gamma_P}{\theta_P} \right) < \frac{R_{S,th}}{B}. \tag{11}$$

In the following theorem, the closed-form expression for outage probability of the SU is derived.

**Theorem 1.** The outage probability of the SU for the proposed RSMA protocol is given by

$$P_S^{RSMA} = 1 - e^{-\lambda_P \theta_P} - \lambda_P e^{-\lambda_S \theta_S} \left( 1 - e^{-\mathcal{F}_P \theta_P} \right) + \frac{\lambda_S e^{-\lambda_P \theta_P}}{\mathcal{F}_P} \left( 1 - e^{-\mathcal{F}_P \theta_S} \right) + \frac{-\lambda_P \theta_P e^{-\lambda_S \theta_S}}{\mathcal{F}_P} \left( e^{-\mathcal{F}_P (\theta_S + 1)} - e^{-\mathcal{F}_P} \right) - A,$$

where

$$A = \begin{cases} 
\lambda_P \theta_S e^{-\lambda_S M_{SP}}, & \lambda_S = \lambda_P \\
\lambda_P e^{-\lambda_S M_{SP}} \left( 1 - e^{-\lambda_P (\theta_S + 1) \theta_P} \right), & \lambda_S \neq \lambda_P
\end{cases} \tag{13}$$

where $\mathcal{F}_S = \lambda_S \theta_S + \lambda_P$, $\mathcal{F}_P = \lambda_P \theta_P + \lambda_S$, and $M_{SP} = \theta_S \theta_P + \theta_S + \theta_P$.

**Proof.** The proof is provided in Appendix A. \hfill $\square$

#### B. Analysis of cognitive radio inspired SIC

Similarly to the RSMA protocol, the power allocation factor $c$ for the SIC protocol in order to avoid a negative impact to the performance of the PU is given by

$$c = \begin{cases} 
\frac{\gamma_P}{\gamma_P + 1}, & \max \left\{ \theta_P, \frac{\gamma_S}{\theta_S} - 1 \right\} \leq \gamma_P < \theta_P (\gamma_S + 1) \\
1, & \text{otherwise}
\end{cases} \tag{14}$$

According to (5), the outage probability of the SU can occur only when the PU is decoded first and can be defined as the union of three mutually exclusive events. The first two events are the same with the ones for the RSMA protocol. Finally, the third event occurs when the message of the PU can be solely decoded, but it cannot be decoded by treating the SU’s message with $c = 1$ as interference, and consequently, using the optimal values of the power allocation factor $\alpha$ given in (14), the received SNR of the SU is below the corresponding
The outage probability of the SU for the proposed SIC protocol is given by

\[ P_{S}^{\text{SIC}} = \Pr \left( \gamma_P < \theta_P \cap \gamma_S(\gamma_P + 1) < \theta_S \right) + \Pr (\gamma_P \geq \theta_P (\gamma_S + 1) \cap \gamma_S < \theta_S) + \Pr \left( \frac{\gamma_S}{\gamma_P + 1} < \theta_S \cap \gamma_P \leq \gamma_P (\gamma_S + 1) \cap \gamma_P < \theta_P (\theta_S + 1) \right). \]  

(15)

In the following theorem, the closed-form expression for the outage probability of the SU for the SIC protocol is derived.

**Theorem 2.** The outage probability of the SU for the proposed uplink SIC protocol can be expressed as

\[ P_{S}^{\text{SIC}} = 1 - \frac{\lambda_S \theta_S e^{\lambda_P}}{F_S} \left(e^{-F_S} \theta_S - e^{-F_S (\theta_P + 1)}\right) - \frac{\lambda_P e^{-\lambda_S \theta_S}}{F_S} (1 - e^{-F_S \theta_P}) + e^{-\lambda_P} (e^{-\lambda_S \theta_S} - e^{-\lambda_S \theta_S}) - e^{-\lambda_S \theta_S} e^{-\lambda_P \theta_P}, \]

(16)

where \( G_S = \theta_P \theta_S + \theta_P + 1, K_P = \lambda_P \theta_P (\theta_S + 1), \) and \( K_S = \lambda_S \theta_S (\theta_P + 1). \)

**Proof.** The proof is provided in Appendix B. \( \Box \)

Both (12) and (16) are utilized in the next Section to extract useful insights for the performance of the considered network.

### IV. Asymptotic Analysis

To obtain more insights for the outage performance of the SU in the proposed protocols, asymptotic analysis is performed and the corresponding diversity gains are presented in this section. First, the asymptotic expression for the outage probability of the SU in the high SNR regime can be obtained by assuming extremely high received SNRs, i.e., \( \gamma_i \to \infty, i \in \{P,S\}. \) For the proposed RSMA and SIC protocols, the corresponding diversity gain in terms of the outage probability can be defined as \( [25] \)

\[ \Delta_i = - \lim_{\gamma \to \infty} \frac{\log_{10} P_{i}}{\log_{10} \gamma}. \]  

(17)

This, the exponential functions can be approximated by applying Taylor expanded when satisfies \( x \to 0, i.e., e^{-x} \approx 1 - x \) and \( e^{x} \approx 1 + x. \)

Specifically, the asymptotic expression for the outage probability of SU in the considered RSMA and SIC protocols are presented in the following propositions.

**Proposition 1.** For the CR inspired RSMA protocol, the asymptotic expression for the outage probability of the SU is given by

\[ P_{S}^{\text{RSMA, \infty}} = \lambda_S \theta_S + \lambda_P \theta_P \lambda_S \theta_S (1 + \theta_S). \]  

(18)

Moreover, the proposed RSMA protocol achieves diversity order equal to one which implies that no error floor exists.

**Proof.** The first two events of the outage probability of the SU for the RSMA protocol shown in (26) and (27) can be respectively approximated in the high SNR regime as follows

\[ P_{S}^{\text{RSMA, \infty}} \approx \lambda_P \theta_P \lambda_S \theta_S, \]  

(19)

and

\[ P_{S,J2}^{\text{RSMA, \infty}} \approx (1 - \lambda_P \theta_P) \lambda_S \theta_S. \]  

(20)

Moreover, the two cases considered for the third event \( P_{S,J3}^{\text{RSMA, \infty}} \) in (28), i.e., \( \lambda_P = \lambda_S \) and \( \lambda_P \neq \lambda_S, \) result in the same asymptotic expression which can be written as

\[ P_{S,J3}^{\text{RSMA, \infty}} \approx \lambda_P \theta_P \lambda_S \theta_S (1 + \theta_S \theta_P + \theta_S + \theta_P). \]  

(21)

Then, by jointly considering the asymptotic analysis for the three outage events, the diversity order is given by

\[ \Delta_{S}^{\text{RSMA}} = \min\{d_{S,J1}, d_{S,J2}, d_{S,J3}\} = 1, \]  

(22)

where \( \Delta_{S,J1} = 2, \Delta_{S,J2} = \min\{1,2\} = 1, \) and \( \Delta_{S,J3} = 2. \) Therefore, (18) has been derived and the corresponding discussion has been validated, which completes the proof. \( \Box \)

**Proposition 2.** For the SIC protocol, the asymptotic expression for the outage probability of the SU is given by

\[ P_{S}^{\text{SIC, \infty}} = \lambda_S \theta_S + \lambda_P \theta_P \lambda_S \theta_S^2 (1 + \theta_S \theta_P + 2 \theta_P). \]  

(23)

The diversity gain achieved by the proposed SIC protocol is equal to one, which implies that no error floor exists.

**Proof.** Since the first two events of the outage probability in the SIC protocol are the same with the ones in RSMA protocol, the same diversity order can be obtained, i.e., \( P_{S,J1}^{\text{SIC, \infty}} = P_{S,J1}^{\text{RSMA, \infty}} \) and \( P_{S,J2}^{\text{SIC, \infty}} = P_{S,J2}^{\text{RSMA, \infty}}. \) Regarding the third event in the SIC protocol, the asymptotic expression can be written as

\[ P_{S,J3}^{\text{SIC, \infty}} \approx \lambda_P \theta_P \lambda_S \theta_S^2 (1 + \theta_S \theta_P + 2 \theta_P). \]  

(24)

Thus, the diversity gain is given by \( \Delta_{S,J3} = 2. \) Following similar procedure as that for the RSMA protocol, the proof is completed. \( \Box \)

### V. Simulation Results

In this section, Monte Carlo simulations are performed to validate the derived expressions. Regarding the selected parameters, unless stated otherwise, we set the distances of the two users \( d_{ij}/d_0 = 2.5 \) and \( d_i/d_0 = 1. \) Also, we set the path loss exponent \( u = 2, \) and the normalized target rates \( R_{S,0}/B = R_{P,0}/B = 2.5 \) bps/Hz.

Moreover, for the sake of comparison, other SIC-based mechanisms are plotted as benchmark, i.e., a two-user uplink NOMA system with CSI-based SIC and QoS-based SIC, termed “CSI” and “QoS”, respectively [22]. Specifically, “CSI” scheme denotes the case that the SU is permitted to access the channel, via which the PU transmits as if it solely occupied, thus the outage probability of the PU and the SU in this scheme can be expressed as \( P_{P}^{\text{CSI}} = \Pr (\gamma_P < \theta_P) \) and \( P_{S}^{\text{CSI}} = \Pr (\gamma_S < \theta_S (\gamma_P + 1)), \) respectively [22]. It should be highlighted that the performance of the PU for this benchmark scheme is the same with the proposed protocols, since it is not affected by the SU, and, in all figures, it is termed as “PU”. In the “QoS” scheme, the PU is decoded first and the SU is treated as interference, thus the outage probability of the PU and the SU are given by \( P_{P}^{\text{QoS}} = \Pr (\gamma_P < \theta_P (\gamma_S + 1)) \) and \( P_{S}^{\text{QoS}} = \Pr (\gamma_P < \theta_P (\gamma_S + 1)) + \Pr (\gamma_P > \theta_P (\gamma_S + 1), \gamma_S < \text{threshold. Therefore, the outage probability of the SU for the proposed SIC protocol is given by \( P_{S}^{\text{SIC}} = \Pr \left( \gamma_P < \theta_P \cap \gamma_S(\gamma_P + 1) < \theta_S \right) + \Pr (\gamma_P \geq \theta_P (\gamma_S + 1) \cap \gamma_S < \theta_S) + \Pr \left( \frac{\gamma_S}{\gamma_P + 1} < \theta_S \cap \gamma_P \leq \gamma_P (\gamma_S + 1) \cap \gamma_P < \theta_P (\theta_S + 1) \right). \) **Theorem 2.** The outage probability of the SU for the proposed uplink SIC protocol can be expressed as

\[ P_{S}^{\text{SIC}} = 1 - \frac{\lambda_S \theta_S e^{\lambda_P}}{F_S} \left(e^{-F_S} \theta_S - e^{-F_S (\theta_P + 1)}\right) - \frac{\lambda_P e^{-\lambda_S \theta_S}}{F_S} (1 - e^{-F_S \theta_P}) + e^{-\lambda_P} (e^{-\lambda_S \theta_S} - e^{-\lambda_S \theta_S}) - e^{-\lambda_S \theta_S} e^{-\lambda_P \theta_P}, \]

(16)

where \( G_S = \theta_P \theta_S + \theta_P + 1, K_P = \lambda_P \theta_P (\theta_S + 1), \) and \( K_S = \lambda_S \theta_S (\theta_P + 1). \)

**Proof.** The proof is provided in Appendix B. \( \Box \)

Both (12) and (16) are utilized in the next Section to extract useful insights for the performance of the considered network.
The SU is plotted versus different SNR values $\gamma$ when the PU is decoded. Since in this case the SU is treated as interference the other three schemes in terms of the outage performance of the PU, since in this case the SU is treated as interference when the PU is decoded.

As illustrated Fig. 2, the outage probability of the PU and the SU is plotted versus different SNR values $\gamma_{0S} = \gamma_{0P} = \gamma_{0}$ for the proposed protocols and the two baselines. Intuitively, the simulations and analytical results demonstrate that both of the protocols based on RSMA and SIC outperform the baselines. Specifically, the proposed protocols can efficiently avoid the outage probability error floors which occur in each benchmark schemes. Moreover, the asymptotic results for the SU for both protocols reveal that the same diversity gains are obtained by the PU and the SU in the high SNR region. It can also be observed that the RSMA protocol outperforms the SIC protocol, since in RSMA the transmitted message is split, thus any point of the capacity region can be achieved instead of solely the corner points achieved with SIC. Under these parameter settings, it is apparent that when the value of $\gamma_0$ is around 20 dB, 3 dB SNR gain can be obtained by the RSMA protocol compared to the proposed SIC one, i.e., double power should be transmitted in the SIC protocol to achieve the same outage performance with the RSMA one.

To obtain more insights on the proposed protocols in terms of outage probability versus the transmitted SNRs, the outage performance versus $\gamma_{0S}$ with $\gamma_{0P} = 22$ dB is depicted in Fig. 3(a), while the outage performance versus $\gamma_{0P}$ with $\gamma_0 = 15$ dB is plotted in Fig. 3(b). Moreover, the other parameters in Fig. 3(b) are set as $d_{b}/d_0 = 2$, and $R_{V,sh}/B = R_{V,sh}/B = 1.5$ bps/Hz. Regarding the outage performance of the SU, the RSMA protocol outperforms the other schemes. It should also be highlighted that error floor occurs in the benchmark schemes, while in the proposed protocols it is avoided as proven in the previous section. Also, the performance of the PU is the same for all schemes highlighting the fact that the PU is not negatively affected.

To investigate the impact of target rates on users’ performance, the outage probability versus the target rates of the SU and the PU are illustrated in Figs. 4(a) and 4(b), respectively. Specifically, Fig. 4(a) is plotted by setting $\gamma_{0S} = \gamma_{0P} = 25$ dB, $d_{v}/d_0 = 2$, $d_{s}/d_0 = 1$, and $R_{v,sh}/B = 2$ bps/Hz. In Fig. 4(b), we set $\gamma_{0S} = \gamma_{0P} = 30$ dB, $d_{v}/d_0 = 2.5$, $d_{s}/d_0 = 1$, and $R_{v,sh}/B = 2.5$ bps/Hz. Moreover, in both Fig. 4(a) and Fig. 4(b), the outage probability of the SU in the CR inspired protocols increases with the increase of the normalized target rate. With the increase of $R_{v,sh}/B$ as shown in Fig. 4(a), more power should be transmitted by the SU to achieve the increasing target rate. Accordingly, in Fig. 4(b), with the increase of $R_{v,sh}/B$, more resources should be allocated to the PU to avoid performance degradation, thus the available resource block for the SU is limited. Furthermore, the constant performance of the PU in Fig. 4(a) reveals there exists no negative impact to the PU when serving the SU in the resource block of the PU.

In Fig. 5, the outage probability is plotted versus the normalized user distance. Specifically, the impact of the normalized distance between the SU and the BS on the outage probability by setting $d_{v}/d_0 = 3$ is illustrated in Fig. 5(a), while the impact of the normalized distance between the PU and the BS with $d_{s}/d_0 = 3$ on the outage probability is depicted in Fig. 5(b). For both subfigures, the corresponding parameters are set as $\gamma_{0S} = \gamma_{0P} = 35$ dB and $R_{v,sh}/B = R_{v,sh}/B = 3$ bps/Hz. Generally, the proposed RSMA protocol outperforms the SIC protocol and the two benchmarks. In Fig. 5(a), the outage probability of SU in “QoS” illustrates a non-monotonic behavior, since the two events that are summed to calculate $P_{S}^{QoS}$ have different monotonicity with respect to the distance. Moreover, an interesting observation is that the proposed RSMA and SIC protocols, and the “QoS” scheme achieve the same performance as the normalized distance between the SU and the BS increases, since the outage probability of the SU for these three schemes is mainly affected by the term $Pr(\gamma_{P} > \theta_{P}(\gamma_{S} + 1), \gamma_{S} < \theta_{S})$. Regarding Fig. 5(b), it is apparent that the outage probability of the SU for the “QoS” benchmark and the PU for all schemes increases with the increase of $d_{v}/d_{0}$, because the increase of $d_{v}/d_{0}$ leads to the decrease of $\gamma_{P}$ resulting in the increase of the outage probability. The non-monotonicity illustrated by the outage probability of the SU for the RSMA and SIC protocols can be also justified by the different monotonicity of the three events in (10) and (15), respectively. Moreover, the outage probability of the SU for the proposed RSMA and SIC protocols approaches the one of the “CSI” scheme with the increase of $d_{v}/d_{0}$, since the term $Pr(\gamma_{S} < \theta_{S}(\gamma_{P} + 1))$ dominates the outage events of these schemes.

VI. CONCLUSIONS

In this paper, an uplink network consisting of one BS and two users, i.e., a PU and an SU, is investigated and two CR inspired protocols based on RSMA and SIC are proposed. Without impacting negatively the performance of the PU, the concept of CR allows the SU to share the communication
resource block which is originally solely occupied by PU, by utilizing and deriving the appropriate decoding order and power allocation factor. To this point, closed-form expressions for the outage probability of the SU for both protocols are derived. Moreover, to obtain further insights for the RSMA and SIC protocols in the high SNR region, the asymptotic analysis of the SU's outage probability is presented. Utilizing the asymptotic analysis, the corresponding diversity gains are obtained and the non-zero results reveal that both protocols avoid the error floor of the SU’s outage performance. Finally, numerical results are presented to demonstrate the accuracy of the theoretical analysis and illustrate that the proposed protocols outperform two benchmark NOMA schemes also inspired by CR.

**APPENDIX A**

**Proof of Theorem 1**

As shown in (10), three terms are summed to derive the outage probability of the SU for the proposed RSMA protocol, thus

$$P_{S} = P_{S, J_1} + P_{S, J_2} + P_{S, J_3}. \quad (25)$$
The first term can be expressed as

\[
\mathcal{P}_{S,J1}^{\text{RSMA}} = \int_0^{\theta_p} \lambda_p e^{-\lambda_p y} \int_0^{\theta_p(y+1)} \lambda_S e^{-\lambda_S x} dx dy \\
= 1 - e^{-\lambda_p \theta_p} - \frac{\lambda_p e^{-\lambda_p \theta_p}}{\lambda_S \theta_S + \lambda_p} \left( 1 - e^{-\theta_p(\lambda_p \theta_p + \lambda_S)} \right).
\]

Moreover, the second term is calculated as

\[
\mathcal{P}_{S,J2}^{\text{RSMA}} = \int_0^{\theta_p} \lambda_S e^{-\lambda_p y} \int_{\theta_p(y+1)}^{\infty} \lambda_p e^{-\lambda_p x} dx dy \\
= \frac{\lambda_S e^{-\lambda_p \theta_p}}{\lambda_p \theta_p + \lambda_S} \left( 1 - e^{-\theta_p(\lambda_p \theta_p + \lambda_S)} \right).
\]

Finally, the third term can be expressed as

\[
\mathcal{P}_{S,J3}^{\text{RSMA}} = \Pr \left( \theta_p \leq \gamma_p < \theta_p(\gamma_S + 1) \right) \\
\cap \left( \frac{\gamma_p}{\theta_p} + \frac{\gamma_S - \gamma_p}{\theta_p} + 1 < \theta_S + 1 \right) \\
= \Pr \left( \theta_p \leq \gamma_p < (\theta_S + 1)\theta_p \right) \\
\cap \left( \frac{\gamma_p}{\theta_p} - 1 < \gamma_S \leq (\theta_S + 1)(\theta_p + 1) - \gamma_p - 1 \right) \\
= \int_{\theta_p}^{\theta_p(\theta_p + 1)} e^{-\lambda_S y} dy \\
\times \int_{\gamma_p - 1}^{\gamma_p(\theta_p + 1) - y - 1} \lambda_S e^{-\lambda_S x} dx dy.
\]

If \( \lambda_p \neq \lambda_S \), (28) is given by

\[
\mathcal{P}_{S,J3}^{\text{RSMA}} = \frac{\lambda_p \theta_p e^{\lambda_S}}{\lambda_p \theta_p + \lambda_S} \left( e^{-(\lambda_p \theta_p + \lambda_S)} - e^{-(\lambda_p \theta_p + \lambda_S)(\theta_S + 1)} + \frac{\lambda_p e^{-\lambda_p \theta_p}}{\lambda_p \theta_p + \lambda_S} \left( e^{-(\lambda_p \theta_p + \lambda_S)} - e^{-(\lambda_p \theta_p + \lambda_S)(\theta_S + 1)} \right) \right)
\]

To this end, by plugging (26), (27), and (29) or (30) into (25), then the proof is completed.

**APPENDIX B**

**PROOF OF THEOREM 2**

Similarly to the RSMA case, the outage probability of the SU for the proposed SIC protocol is derived by summing three terms, i.e.,

\[
\mathcal{P}_{S,J3}^{\text{SIC}} = \mathcal{P}_{S,J1}^{\text{SIC}} + \mathcal{P}_{S,J2}^{\text{SIC}} + \mathcal{P}_{S,J3}^{\text{SIC}}.
\]

It should be highlighted that \( \mathcal{P}_{S,J1}^{\text{SIC}} = \mathcal{P}_{S,J1}^{\text{RSMA}} \) and \( \mathcal{P}_{S,J2}^{\text{SIC}} = \mathcal{P}_{S,J2}^{\text{RSMA}} \). Subsequently, the third term can be rewritten as

\[
\mathcal{P}_{S,J3}^{\text{SIC}} = \Pr \left( \frac{\gamma_S \theta_p}{\theta_S} - 1 \leq \gamma_p < (\theta_p(\theta_S + 1)) \right) \\
\times \Pr \left( (\theta_p(\theta_p + 1)) \right) < \gamma_S < (\theta_S \theta_S) + \Pr \left( \theta_p \leq \gamma_p < (\theta_p(\theta_S + 1)) \right) \\
\times \Pr \left( (\theta_S(\theta_p + 1)) \right) + \Pr \left( \theta_p \leq \gamma_p < (\theta_p(\gamma_S + 1)) \right) \Pr \left( \gamma_S < (\theta_S) \right).
\]
After some algebraic manipulations, (32) is given by

$$P_{S, i | S} = \frac{1}{\theta_i} \left( 1 + e^{-\lambda_i (\theta_i + \theta)} \right) \left( 1 + e^{-\lambda_i (\theta_i + \theta)} \right)$$

To this end, the outage probability of the SU for the SIC protocol can be derived by substituting (26), (27), and (33) into (31), thus the proof is completed.

REFERENCES

[1] Z. Zhang, Y. Xiao, Z. Ma, M. Xiao, Z. Ding, X. Lei, G. K. Karagiannidis, and P. Fan, “6G wireless networks: Vision, requirements, architecture, and key technologies,” IEEE Veh. Technol. Mag., vol. 14, no. 3, pp. 28–41, 2019.

[2] Y. Liu, S. Zhang, X. Mu, Z. Ding, R. Schober, N. Al-Dhahir, H. V. Poor, and X. Shen, “Evolution of NOMA Toward Next Generation Multiple Access (NGMA) for 6G,” IEEE J. Sel. Areas Commun., pp. 1–1, 2022.

[3] A. Shahraki, M. Abbasi, M. J. Piran, and A. Taherkordi, “A Comprehensive Survey on 6G Networks: Applications, Core Services, Enabling Technologies, and Future Challenges,” 2021. [Online]. Available: https://arxiv.org/abs/2101.12475

[4] E. J. D. Santos, R. D. Souza, and J. L. Rebelatto, “Rate-Splitting Multiple Access for URLLC Uplink in Physical Layer Network Slicing with eMBB,” IEEE Trans. Veh. Technol., vol. 69, no. 6, pp. 163178–163187, 2021.

[5] O. Dizdar, Y. Mao, Y. Xu, P. Zhu, and B. Clerckx, “Rate-splitting multiple access for enhanced URLLC and eMBB in 6G,” in 2021 17th International Symposium on Wireless Communication Systems (ISWCS). IEEE, 2021, pp. 1–6.

[6] S. A. Tegos, P. D. Diamantoulakis, A. S. Lioumpas, P. G. Sarigiannidis, and G. K. Karagiannidis, “A Novel Rate-Splitting Multiple Access Protocol for Unveiling the Importance of SIC in NOMA Systems—Part II: State of the Art and Future Research Challenges,” IEEE Access, vol. 9, pp. 55 765–55 784, 2021.

[7] L. Zheng and D. Tse, “Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels,” IEEE Trans. Inf. Theory, vol. 49, no. 5, pp. 1073–1096, 2003.

[8] B. Clerckx, Y. Mao, R. Schober, and H. V. Poor, “Rate-Splitting Unifying SDMA, OMA, NOMA, and Multicasting in MISO Broadcast Channel: A Simple Two-User Rate Analysis,” IEEE Wireless Commun. Lett., vol. 9, no. 3, pp. 349–353, Mar. 2020.

[9] A. Krishnamoorthy and R. Schober, “Downlink MIMO-RMSA with Successive Null-Space Precoding,” 2021. [Online]. Available: https://arxiv.org/abs/2107.08294

[10] B. Rimoldi and R. Urbanke, “A Rate-Splitting Approach to the Gaussian Multiple-Access Channel,” IEEE Trans. Inf. Theory, vol. 42, no. 2, pp. 364–373, Mar. 1996.