Multi-Layer Adaptive Finite Time Super Twisting Control for Quaternion-Based Quadrotor Formation With Obstacle Avoidance

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ABSTRACT
This article investigates the trajectory tracking problem of quadrotor formation with collision avoiding. The safety distributed control strategy is introduced for quaternion-based multiple quadrotors formation to avoid collision. Then, based on super-twisting and adaptive control method, a multiple adaptive finite time super-twisting control method (MAFTSTC) is proposed, which rarely relies on the information of the formation model. The purpose of the robust controller designed in the position loop and the attitude loop is to ensure that quadrotor formation tracks the desired trajectory and maintains formation configuration in finite time. The closed-loop system stability of the novel control method is verified through Lyapunov theory. Finally, compared with traditional finite time convergence (FTC) method and non-robustness control (NRC) method, the simulation results illustrate the effectiveness of the proposed control method.

INDEX TERMS
Position control, attitude control, distributed parameter systems, nonlinear control systems, control design, angular velocity control, optimal control.

I. INTRODUCTION
The cooperative missions of quadrotors, such as rescue, tracking, positioning and observation, have attracted interests of many researchers. Compared with a single quadrotor [1], a number of quadrotors have superiority in these areas. And the formation control is considered as the key technology of the cooperative missions [2]. The objective of formation control is to guarantee multiple quadrotors to perform the dangerous and complex tasks in the hostile environment using local information only. Hence, it is of great significance to research the formation control problem of multiple quadrotors [3].

Because the quadrotor has the characteristics of multi-variable, nonlinear, strong coupling, and under-actuation, the multi-quadrotor formation control problem is challenging. Researchers are trying to deal with that by various advanced control methods [4]–[8]. However, existing literatures are mainly based upon the use of the Euler representation to describe the formation system, which improves the difficulty of control design because the Euler representation has singularities inherent to the direction cosine matrix. Adopting quaternion to represent the quadrotor formation effectively avoids this issue and some researchers proposed some control methods of the formation control modeling by quaternion [9], [10]. Whereas, the control performance of multi-quadrotor formation can be further improved, for example, the finite time convergence stability is proved only in inner-loop, but not in the outer-loop [2]. Although novel methods accomplish the convergence of multi-quadrotor formation, how to further improve the control performance is still faced with certain challenges.

External disturbances and model uncertainties are important issues influencing the formation control performance [11], [12]. Adaptive control algorithm has advantages in dealing with unknown parameters and have been employed to maintain the robustness of complex system [13]–[15]. In [16], for a class of general switched uncertain nonlinear systems with global adaptive control problem, a new adaptive control scheme is established by improving the well-known mode-dependent average dwell time to ensure the global boundedness of all signals in the closed-loop system. In [17], for the adaptive tracking control problem of a class of non-strict feedback nonlinear time-delay systems under the event triggering mechanism, an adaptive neural network controller is designed by using backstepping technology and event triggering mechanism, which eliminates the global Lipschitz condition of unknown nonlinear functions and eliminates the assumption of input state stability on measurement errors. However, the size of adaptive parameters will affect the
performance of adaptive control. A nested adaptive control method [18] provides convenience for the proof and avoids the above problems effectively by blurring parameters. In addition, the existing adaptive methods are mainly used to deal with the impact of disturbances on the system, such as [19]–[21]. Nevertheless, the above methods rely on the accurate mathematical models. For complex systems, accurate modeling is difficult to be obtained and not conducive to engineering application. Therefore, how to reduce the dependence of control theory on mathematical models under the premise of ensuring the stability and rapidness of the control system is a major problem to be solved.

The obstacle avoidance is another practical and challenging problem in the formation control because of the uncontrollability [22]–[27]. One way to solve this problem is to plan the trajectory considering the obstacle in the environment, and then design the position controller and attitude controller to track the desired trajectory [28]. However, the stability of formation control cannot be proved because of the separated designing of obstacle avoiding and trajectory tracking. The artificial potential field method is another way to avoid the obstacle [29]–[31]. Treating the obstacle as the high potential center of uncertain parameters. Moreover, the control method may impair the control performance, thereupon the existing of artificial potential field function is a part of the trajectory considering the obstacle in the environment, nevertheless, the stability of the control method is proved separated in attitude system without the obstacle [29]–[31].

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\( g \) is the acceleration of gravity, and \( m_j \) is the mass of the \( j \)th quadrotor drone. The thrust of the rotor in the \( b_j \) direction is \( T_{ij} \). \( e_3 \) represents the unit vector in the z-axis direction under the inertial system. \( R(Q_j) \) is the Rodrigue rotation matrix. \( d_{Fj} \) is external disturbances and model uncertainties for the quadrotor.

**Inner-loop:**

\[
\begin{align*}
\dot{q}_j &= \frac{1}{2} \left( \eta_j I_3 + S(q_j) \right) \omega_j \\
\dot{\omega}_j &= -\frac{1}{2} q_j^T \omega_j \\
(I_j + \delta I_j) \delta \omega_j &= \Gamma_j + d_{\Gamma j} \\
-S(\omega_j) (I_j + \delta I_j) \omega_j 
\end{align*}
\]

where \( \omega_j \) is the angular velocity of the quadrotor. \( \Gamma_j \) is the control torque of inner-loop. \( I_j = \text{diag} \left( I_{cxj}, I_{cyj}, I_{czj} \right) \) is the symmetric positive definite constant inertia matrix of the \( j \)th quadrotor and its error is \( \delta I_j = \text{diag} \left( \Delta I_{cxj}, \Delta I_{cyj}, \Delta I_{czj} \right) \). \( d_{\Gamma j} \) is sinusoidal external disturbances and model uncertainties for the quadrotor.

**Assumption 1:** The external disturbances and model uncertainties \( d_{Fj} \) and \( d_{\Gamma j} \) are bounded with the unknown upper bound.

**B. OBSTACLE AVOIDANCE MODEL**

In this part, the concept of obstacle avoidance will be explained. The area related to the \( j \)th quadrotor is shown in Fig.2. \( R_a \) is the avoidance radius, \( R_t \) is the threat radius. When the \( j \)th quadrotor enters the avoidance radius \( R_a \), that is \( \{ p_j \in \mathbb{R}^3, pk \in \mathbb{R}^3, \| p_j - p_k \| \in (R_a, R_t) \} \), where \( p_{ij} \) represents the position of obstacle. Then the quadrotor obtains the position coordinates of the obstacle and activates the obstacle avoidance module to avoid collision.

In order to achieve effective obstacle avoidance, this article proposes an obstacle avoidance function \( \Delta_{ij}(l) \)

\[
\Delta_{ij}(l_{ij}) = \begin{cases} 
\frac{4(R_{a}^2 - R_{a}^2)(R_{a}^2 - l_{ij}^2)}{(l_{ij}^2 - R_{a}^2)} & \text{if } l_{ij} \in (R_a, R_d) \\
0 & \text{if } l_{ij} \notin (R_a, R_d)
\end{cases}
\]

where \( l_{ij} \) is the distance between the obstacle and the quadrotor \( l_{ij} = \| p_i - p_j \| \).

**C. GRAPH THEORY**

This article mainly studies formation control of a group of quadrotors, which is a leader-follower structure consisting of a leader and \( n \) followers.

In this article, the communication topology among \( n \) quadrotors is described by a fixed and directed graph \( G = (V, \varepsilon, A) \), where \( V = \{ v_i, i = 1, 2, \ldots, n \} \) is the node set and represents quadrotors, \( \varepsilon \subseteq V \times V \) is the set of edges. \( A = \{ a_{jk} \} \subseteq \mathbb{R}^{n \times n} \) is the weighted adjacency matrix of \( G \). For \( A \), if there is an edge between agent \( j \) and \( k \), then \( a_{jk} = a_{kj} = 1 \), otherwise, \( a_{jk} = a_{kj} = 0 \). The set of neighbor of node \( v_i \) is denoted as \( N_i = \{ j: (v_i, v_j) \in \varepsilon \} \). Let \( D = \text{diag} \{ d_1, \ldots, d_n \} \) denotes the degree matrix of directed graph \( G \) and the Laplacian matrix of directed graph \( G \) is \( L = D - A \).

The reference attitude is represented by a virtual leader, and the connection weight between the \( i \)th quadrotor and the leader is expressed by \( b_i, i \in \Gamma \). If the information of the leader can be provided to the leader quadrotor, then \( b_i > 0 \), otherwise \( b_i = 0 \). To be convenient, set \( B = \text{diag} \{ b_1, \ldots, b_n \} \).

**Assumption 2:** The communication topology for \( n \) quadrotors is connected at the initial time.

**III. FORMATION CONTROL DESIGN**

For the control objective mentioned above, a robust distributed control method is proposed. The control design procedure is divided into three sections in this section. First, a finite time formation control with obstacle avoidance is designed for outer-loop system (4) to form and maintain the formation structure. Then, a finite time attitude tracking control is proposed for quaternion-based inner-loop system (5). At last, based on the stability analysis of the two loops, the finite time convergence characteristic of whole closed system is verified.

**Lemma 1** [34]: Consider the following nonlinear system \( \dot{x}(t) = f(x(t)) + g(x(t))u + \omega \). Suppose there exists an open neighborhood \( \mathbb{Z} \) of the origin, a \( C^1 \) positive-definite function \( V : \mathbb{Z} \to \mathbb{R} \) and real number \( \alpha > 0 \), such that \( \dot{V} + \alpha V^2 \) is negative semidefinite on \( \mathbb{Z} \). Then the origin is a finite-time stable equilibrium of nonlinear system.

**A. FORMATION CONFIGURATION CONTROL DESIGN WITH OBSTACLE AVOIDANCE**

In this section, the leader quadrotor will track the desired trajectory, which have been given in advance. Define error function \( e_{ij} = \sum_{k=0}^{n} a_{jk} (p_j - p_k - D_{jk}) \), in which \( D_{jk} \) is the formation distance. In order to reduce the dependence of control method on the precise model, Eq.(4) is rewritten as

\[
\dot{p}_j = v_j \\
m\dot{v}_j = b_{or}(t) + \left( 1 - \sum_{k=0}^{n} a_{jk} \right) d_{Fj}
\]
\[-T_i R(\tilde{Q}_i)e_3 + \left(1 - \sum_{k=0}^{n} a_{jk}\right) mge_3\]
\[-\mathbf{v}_j^T \sum_{i=1}^{n} f_i \int_0^t \left\| p_j - p_{bi} \right\| \Delta g_i(l_{ij}) l_{ij} d l_{ij}\]
\[-\mathbf{v}_j^T \sum_{i=1}^{n} f_i \int_0^t \left\| p_j - p_{bi} \right\| \Delta g_i(l_{ij}) l_{ij} d l_{ij}\]
\[\frac{v_j}{v_j^T v_j}\]

where \(b_{or}(t)\) is a continuous and twice differentiable term and defined as
\[b_{or}(t) = \sum_{k=0}^{n} a_{jk} d_{Fj} + \sum_{k=0}^{n} a_{jk} mge_3\]
\[+ \mathbf{v}_j^T \sum_{i=1}^{n} f_i \int_0^t \left\| p_j - p_{bi} \right\| \Delta g_i(l_{ij}) l_{ij} d l_{ij}\]
\[+ \mathbf{v}_j^T \sum_{i=1}^{n} f_i \int_0^t \left\| p_j - p_{bi} \right\| \Delta g_i(l_{ij}) l_{ij} d l_{ij}\]

where \(b_{or}(t)\) satisfies \(\left\| b_{or}(t) \right\| < b_{or_0}, \left\| \dot{b}_{or}(t) \right\| < b_{or_1}, \left\| \ddot{b}_{or}(t) \right\| < b_{or_2}\). Define the sliding mode as
\[s_j(t) = \lambda_1 e_{vj} + \dot{e}_{vj}\]
\[-\dot{\mathbf{S}}(v_j)^{-1} \sum_{i=1}^{n} \int_0^t \left\| p_j - p_{bi} \right\| \Delta g_i(l_{ij}) l_{ij} d l_{ij} E_{a}\]

where \(\lambda_1\) is a constant and \(\lambda_1 > 0, E_{a} = [1 1 1]^T\). \(\dot{\mathbf{S}}(v_j) : \mathbb{R}^3 \rightarrow \mathbb{R}^{3 \times 3}\) is the diagonal matrix operator and defined as
\[\dot{\mathbf{S}}(v_j) = \begin{bmatrix} v_{ix} & 0 & 0 \\ 0 & v_{iy} & 0 \\ 0 & 0 & v_{iz} \end{bmatrix}\]

Differentiating \(s_j(t)\) along the trajectory of system (7), it yields
\[\dot{s}_j(t) = \lambda_1 \dot{e}_{vj} + \ddot{e}_{vj}\]
\[-\sum_{i=1}^{n} f_i \Delta g_i(l_{ij}) l_{ij} d l_{ij}\]
\[+ \mathbf{v}_j^T \sum_{i=1}^{n} f_i \int_0^t \left\| p_j - p_{bi} \right\| \Delta g_i(l_{ij}) l_{ij} d l_{ij}\]
\[\frac{v_j}{v_j^T v_j}\]

Substituting (7) into (11), it follows that
\[\dot{s}_j(t) = b_{or}(t) - \sum_{k=0}^{n} a_{jk} F_j + \frac{n}{m_{j}} f_i \Delta g_i(l_{ij}) l_{ij} d l_{ij}\]
\[+ \lambda_1 \sum_{k=0}^{n} a_{jk} (v_j - v_k) - \sum_{k=0}^{n} a_{jk} \dot{v}_k\]

Theorem 1: Consider the outer-loop subsystem (7), assumption 1-2 and the obstacle model (6) in the environment, the displacement controller \(F_j\) is designed as:
\[F_j = \left(-m_j \sum_{i=1}^{n} f_i \Delta g_i(l_{ij}) l_{ij} d l_{ij}\right) + \frac{n}{m_{j}} f_i \Delta g_i(l_{ij}) l_{ij} d l_{ij}\]
\[+ \lambda_1 \sum_{k=0}^{n} a_{jk} (v_j - v_k) - \sum_{k=0}^{n} a_{jk} \dot{v}_k\]

where \(n, f_i\) are both constants and satisfy \(n > 0, f_i > 0\). \(\chi(x) = \left\{\begin{array}{ll} \frac{1}{\varepsilon_0} & x 
otin \mathbb{R} \\ 0 & x = 0 \end{array}\right.\). \(\varepsilon_0\) is a constant greater than 0, and it is designed to avoid singularity.

During the sliding motion \(s_j(t) \equiv 0\), the so-called equivalent control \(F_{eq}(t)\) must take on average to maintain sliding. So, there is \(\|F_{eq}(t)\| < \|\dot{b}_{or}(t)\|\).

To obtain the estimate value of \(F_{eq}(t)\), the following low-pass filter is designed below
\[\hat{F}_{eq}(t) = \frac{1}{\kappa_1}(F_{eq}(t) - \hat{\hat{F}}_{eq}(t))\]

where \(\kappa_1 > 0\) is a small constant, which aims to make \(\hat{\hat{F}}_{eq}(t) \rightarrow F_{eq}(t)\). Assuming that \(1 > \phi_{or_1} > \phi_{or_0} > 0\), it follows that \(\|\hat{\hat{F}}_{eq}(t)\| < \phi_{or_1} \|F_{eq}(t)\| + \phi_{or_0}\).

The equivalent control is used to construct the adaptive algorithm for \(h(t)\) in this article. \(\dot{h}(t)\) is a varying scalar term shown as follows.
\[\dot{h}(t) = -\rho_{or}(t) \| \delta(t) \|\]

where \(\rho_{or}(t)\) is a varying scalar and defined as \(\rho_{or}(t) = r_0 + \rho_{or}(t)\). \(r_0\) is a fixed positive scalar. In order to ensure the security of the objective, there is
\[h(t) > \frac{1}{\beta_{or}} \| F_{eq}(t) \| + \mu_{or}\]

where \(0 < \beta_{or} < 1, \mu_{or} > 0\) are constants and chosen to make
\[\frac{1}{\beta_{or}} \| \hat{F}_{eq}(t) \| + \frac{\mu_{or}}{2} > \| F_{eq}(t) \|\]
ensure \( r(t) + r_0 > \hat{h}(t) \).

\[
\dot{r}(t) = \begin{cases} 
\gamma \|\delta(t)\| + r_0 \sqrt{\gamma} \|\xi(t)\| & \text{if } \|\delta(t)\| > \delta_0 \\
r_0 \sqrt{\gamma} \|\xi(t)\| & \text{otherwise}
\end{cases}
\tag{18}
\]

where \( \delta_0 \) is a design scalar, \( \delta(t) \) is an error function, which denotes the error of the first layer adaptive participation estimates value and is defined as

\[
\delta(t) = h(t) - \frac{1}{\beta_{or}} \|\tilde{F}_{eq}(t)\| - \mu_{or}
\tag{19}
\]

where \( \|\tilde{F}_{eq}(t)\| < q_{or} b_{or} \), \( \xi(t) \) denotes the error between the maximum value of \( r(t) \) and \( r(t) \). It is defined as

\[
\xi(t) = q_{or} b_{or} - r(t)
\tag{20}
\]

in which \( q_{or} \) is a constant and \( q_{or} > 1 \).

**Proof:** Choose the following Lyapunov function candidate

\[
V_1 = V_{11} + V_{12}
\tag{21}
\]

\[
V_{11} = \frac{1}{2} \delta(t)^2 + \frac{1}{2\gamma} \xi(t)^2
\tag{22}
\]

\[
V_{12} = \frac{1}{2} \xi'(t)
\tag{23}
\]

Considering the piecewise adaptive law of \( \delta(t) \), the following calculation of \( V_1 \) will be divided into two steps.

First, if \( \|\delta(t)\| > \delta_0 \), according to (20), the first-order derivative of \( \xi(t) \) will be got as

\[
\dot{\xi}(t) = -\dot{r}(t) = -\gamma \|\delta(t)\| - r_0 \sqrt{\gamma} \frac{\|\xi(t)\|}{\xi(t)}
\tag{24}
\]

Then, with adaptive law of \( \delta(t) \)

\[
\dot{V}_{11} = \delta \dot{h}(t) - \frac{1}{\beta_{or}} \|\tilde{F}_{eq}(t)\|
+ \frac{1}{\gamma} \xi(-\gamma \|\delta\| - r_0 \sqrt{\gamma} \frac{\|\xi\|}{\xi})
- (r_0 + r(t)) \|\delta\| - \frac{1}{\beta_{or}} \|\tilde{F}_{eq}(t)\| - \xi \|\delta\|
- r_0 \frac{\|\xi\|}{\sqrt{\gamma}}
\leq -(r_0 + q_{or} b_{or} - \xi)(\|\delta\| + \frac{\|\delta\|}{\beta_{or}} \|\tilde{F}_{eq}(t)\|
- \xi \|\delta\| + \frac{r_0}{\sqrt{\gamma}} \|\xi\|
\leq -r_0 \|\delta\| - \frac{q_{or} b_{or} 2}{\beta_{or}} \|\delta\| + \xi \|\delta\| + \frac{q_{or} b_{or} 2}{\beta_{or}} \|\delta\|
- \xi \|\delta\| + \frac{r_0}{\sqrt{\gamma}} \|\xi\|
\tag{25}
\]

Second, if \( \|\delta(t)\| \leq \delta_0 \), according to \( \xi(t) \) in (20), it can be given by the first-order derivative.

\[
\dot{\xi}(t) = -\dot{r}(t) = -r_0 \sqrt{\gamma} \frac{\|\xi(t)\|}{\xi(t)}
\tag{26}
\]

Based on \( \delta(t) \), there is

\[
\dot{V}_{11} = \delta \dot{h}(t) - \frac{1}{\beta_{or}} \|\tilde{F}_{eq}(t)\|
+ \frac{1}{\gamma} \xi(-r_0 \sqrt{\gamma} \frac{\|\xi\|}{\xi})
- (r_0 + r(t)) \|\delta\| - \frac{1}{\beta_{or}} \|\tilde{F}_{eq}(t)\| - r_0 \frac{\|\xi\|}{\sqrt{\gamma}}
\leq -(r_0 + q_{or} b_{or} - \xi)(\|\delta\| + \frac{\|\delta\|}{\beta_{or}} \|\tilde{F}_{eq}(t)\|
- \xi \|\delta\| + \frac{r_0}{\sqrt{\gamma}} \|\xi\|
\leq -r_0 \|\delta\| - \frac{q_{or} b_{or} 2}{\beta_{or}} \|\delta\| + \xi \|\delta\| + \frac{q_{or} b_{or} 2}{\beta_{or}} \|\delta\|
- \xi \|\delta\| + \frac{r_0}{\sqrt{\gamma}} \|\xi\|
\tag{27}
\]

From (25) and (27), if \( \xi(t) \leq 0 \), it follows that \( \dot{V}_{11} \leq -r_0 \|\delta\| - \frac{r_0}{\sqrt{\gamma}} \|\xi\| \).

According to the definition of \( \dot{r}(t) \) in (17), \( r(t) > 0 \) all the time, then

\[
\xi(t) < \frac{q_{or} b_{or} 2}{\beta_{or}}
\tag{28}
\]

Take \( \delta(t) \) as the X-axis, \( \xi(t) \) as the Y-axis to establish the plane rectangular coordinate system, \( \frac{b_{or}}{2} \) as the width, greater than \( \frac{q_{or} b_{or} 2}{\beta_{or}} \) as the height to construct an ellipse and the expressions are as follows

\[
\dot{V}_{11} = \{(\delta, \xi) : V_{11}(\delta, \xi) < \psi\}
\tag{29}
\]

\[
\psi = \frac{1}{2} (\frac{q_{or} b_{or} 2}{\beta_{or}})^2
\tag{30}
\]

Inside the ellipse, a rectangular of that \( \delta_0 \) in width and \( \frac{q_{or} b_{or} 2}{\beta_{or}} \) in height, as before, \( \dot{V}_{11} \leq -r_0 \|\delta\| - \frac{r_0}{\sqrt{\gamma}} \|\xi\| \), outside of the rectangle

\[
\Theta = \left\{(\delta, \xi) : |\delta| < \delta_0, 0 \leq \xi < \frac{q_{or} b_{or} 2}{\beta_{or}}\right\}
\tag{31}
\]

which encloses the rectangular region given by (30), if the parameter \( \mu_{or} \) can be chosen to satisfy

\[
\frac{1}{4} \mu_{or}^2 > \delta_0^2 + \frac{1}{\gamma} \left(\frac{q_{or} b_{or} 2}{\beta_{or}}\right)^2
\tag{32}
\]

there is \( \Theta \subset \dot{V}_{11} \).
According to the certification process in [18], we can draw a conclusion that \( t > t_0, | \delta | < \frac{| \mu_0 |}{2} \) in finite time. Then, based on the definition of \( \delta(t) \) given by (18), it follows that
\[
\| \delta(t) \| = \left\| h(t) - \frac{1}{\beta_{or}} \| \tilde{F}_{eq}(t) \| - \mu_{or} \right\| < \frac{1}{2} \mu_{or} \quad (33)
\]
Therefore, the following inequality can be concluded:
\[
h(t) = \frac{1}{\beta_{or}} \| \tilde{F}_{eq}(t) \| + \mu_{or} > \| F_{eq}(t) \|
= \| b_{or}(t) \| \quad (34)
\]
Therefore, there is \( \dot{V}_{11} \leq -r_0 \| \delta \| - \frac{r_0}{\sqrt{\beta}} \| \xi \| \) in the whole region.

Based on (20), there is
\[
\dot{V}_1 = \dot{V}_{11} + \dot{V}_{12}
\leq \dot{s}^T \tilde{s} - r_0 \| \delta \| - \frac{r_0}{\sqrt{\beta}} \| \xi \|
\leq (b_{or}(t) - (h(t) + r_0) \chi (t)) - r_0 \| \delta \|
\leq -r_0 \sqrt{\beta} \| \xi \|
\leq -r_0 \| \delta \| - r_0 \| s \| - r_0 \| \delta \|
\leq -e_1 \sqrt{2} V_1^T \quad (35)
\]
where \( e_1 = \min \left\{ \varphi_1, r_0, \frac{r_0}{\sqrt{\beta}} \right\} \).

As lemma 1, the finite time stability of outer-loop is proved.

The outer-loop proof is thus completed.

Remark 1: In order to ensure the good performance of the system, \( f_1 \) is chosen as \( f_1 > 0 \). The scalar \( \delta_0 \) needs to be larger than noise or computational errors which is selected as a small positive constant. The defined parameters \( \mu_{or} \) and \( \beta_{or} \) are the safety factors, \( q_{or} \) reflects the accuracy associated with the estimated equivalent control. By selecting the value of the adaptive gain \( \gamma \) can always satisfy inequality (32). In order to ensure good convergence of the system, \( \varphi_1 \) and \( \lambda_1 \) are chosen as \( \varphi_1 > 0, \lambda_1 > 0, r_0, \phi_{or1}, \phi_{or2} \) are constants to meet to proof.

### B. INNER-LOOP SYSTEM CONTROL DESIGN

In this section, the aim is to design the attitude controller for inner-loop system such that the error \( \tilde{\eta}_1, \tilde{\eta}_2 \) and \( \tilde{\xi} \) converge to 0 and \( \tilde{\eta}_1 \) converge to 1 in finite time. Define the angular velocity error as
\[
\omega_{dj} = \omega_j - R (\tilde{Q}) \omega_{dj} \quad (37)
\]
where \( \omega_{dj} \) is the desired angular velocity defined as
\[
\omega_{dj} = 2 \left[ \eta_{dj} \eta_3 + S (q_{dj}) \right] Q_{dj} \quad (38)
\]
In the inner loop system, considering the practical application of the control method in engineering, in order to reduce the dependence on the precise model, Eq.(5) is rewritten as
\[
\left\{
\begin{align*}
\dot{\eta}_j &= \frac{1}{2} (\eta_j I_3 + S (q_j)) \omega_j \\
\dot{\omega}_j &= \frac{1}{2} d^T_j \omega_j \\
(I_{ijj} + \Delta I_{ijj}) \omega_j &= (I_{ijj} + \Delta I_{ijj}) \omega_j + \Gamma_j
\end{align*}
\right. \quad (39)
\]
where \( b_{ir} (t) \) is a continuous and twice differentiable term and defined as
\[
b_{ir}(t) = (I_{ijj} + \Delta I_{ijj})^{-1} \dot{d}_{ir}
\quad (40)
\]
It yields that
\[
\dot{\sigma}_j(t) = \frac{\lambda_2}{2} (\tilde{\eta}_j I_3 + S (\tilde{q}_j)) \tilde{\omega}_j + \omega_j
\quad (41)
\]
with Eq.(40), there is
\[
\dot{\sigma}_j(t) = b_{ir}(t) + \frac{\lambda_2}{2} (\tilde{\eta}_j I_3 + S (\tilde{q}_j)) \tilde{\omega}_j
\quad (42)
\]
where \( \sigma_j(t) \) is the attitude tracking controller \( \Gamma_j \) is defined as
\[
\Gamma_j = (I_{ijj} + \Delta I_{ijj})(-g(t) + \varphi_2) \chi (\sigma(t))
\quad (43)
\]
where \( \varphi_2 > 0, \lambda_2 > 0 \).

During the sliding motion \( \sigma_j(t) \equiv 0 \), the so-called equivalent control \( \Gamma_{eq}(t) \) must take on average to maintain sliding. So to make sure that \( \dot{\sigma}_j(t) = 0, \sigma_j(t) = 0, \Gamma_{eq}(t) \) must satisfy
\[
\Gamma_{eq}(t) = -b_{ir}(t). \quad \text{So, there is} \quad \| \Gamma_{eq}(t) \| \leq \| b_{ir}(t) \|
\]
To obtain the estimate value of \( \Gamma_{eq}(t) \), the following low-pass filter is designed below.
\[
\hat{\Gamma}_{eq}(t) = \frac{1}{k_2} (\Gamma_{eq}(t) - \hat{\Gamma}_{eq}(t)) \quad (44)
\]
where \( \kappa_2 > 0 \) is a small constant, which aims to make \( \hat{\Gamma}_{eq}(t) \rightarrow \Gamma_{eq}(t) \). Assuming that \( 1 > \phi_{ir} > 0, \phi_{i0} > 0 \), so it follows that \( \| \hat{\Gamma}_{eq}(t) \| - \| \Gamma_{eq}(t) \| < \phi_{ir} \| \Gamma_{eq}(t) \| + \phi_{i0} \).

The equivalent control is used to construct the adaptive algorithm for \( g(t) \) in this article. \( g(t) \) satisfies the adaptive algorithm proposed in this article and \( \dot{g}(t) \) is a varying scalar term. The specific size is

\[
\dot{g}(t) = -\rho_{ir}(t) \| \theta(t) \| \tag{46}
\]

where \( \rho_{ir}(t) \) is a varying scalar defined as \( \rho_{ir}(t) = z_0 + \dot{z}(t) \), \( z_0 \) is a fixed positive scalar. In order to ensure the security of the objective, there is

\[
g(t) > \frac{1}{\beta_{ir}} \| \hat{\Gamma}_{eq}(t) \| + \mu_{ir} \tag{47}
\]

where \( 0 < \beta_{ir} < 1, \mu_{ir} > 0 \) are constants and chosen to make

\[
\frac{1}{\beta_{ir}} \| \hat{\Gamma}_{eq}(t) \| + \mu_{ir} > \| \Gamma_{eq}(t) \| \tag{48}
\]

The evolution of \( \dot{z}(t) \) will also satisfy an adaptive law, which aims to make the two adaptive \( g(t) \) and \( z(t) \) converge to a fixed value in the finite time. The rate of change of \( g(t) \) depends on the time-varying parameter \( z(t) \), which can adapt itself to ensure \( z(t) + z_0 > \dot{g}(t) \).

\[
\dot{z}(t) = \begin{cases} 
\sigma \| \theta(t) \| + z_0 \sqrt{\sigma} \frac{\| \xi(t) \|}{\xi(t)} & \text{if } \| \theta(t) \| > \theta_0 \\
\frac{z_0 \sqrt{\sigma}}{\| \xi(t) \|} & \text{otherwise}
\end{cases} \tag{49}
\]

\( \theta_0 \) is a design scalar, \( \theta(t) \) is an error function, which denotes the error between the first layer adaptive participation evaluates value and is defined as

\[
\theta(t) = g(t) - \frac{1}{\beta_{ir}} \| \hat{\Gamma}_{eq}(t) \| - \mu_{ir} \tag{50}
\]

in which \( \| \hat{\Gamma}_{eq}(t) \| < q_{ir} \beta_{ir} \| \xi(t) \| \). \( \xi(t) \) denotes the error between the maximum value of \( z(t) \) and \( z(t) \). It is defined as

\[
\zeta(t) = \frac{q_{ir} \beta_{ir}}{\beta_{ir}} - z(t) \tag{51}
\]

in which \( q_{ir} \) is a constant and \( q_{ir} > 1 \).

**Proof:** Choose the following Lyapunov function candidate

\[
V_2 = V_{21} + V_{22} \tag{52}
\]

\[
V_{21} = \frac{1}{2\gamma} \| \xi(t) \|^2 + \frac{1}{2} \| \theta(t) \|^2 \tag{53}
\]

\[
V_{22} = \frac{1}{2} \sigma \| \sigma \| \tag{54}
\]

If \( \| \theta(t) \| > \theta_0 \), according to \( \zeta(t) \) in (51), we can get the first-order derivative of \( \zeta(t) \) as

\[
\dot{\zeta}(t) = -\dot{z}(t) = -\sigma \| \theta(t) \| - z_0 \sqrt{\sigma} \frac{\| \xi(t) \|}{\xi(t)} \tag{55}
\]

Based on the Lyapunov function (53) and (55), the first-order derivative of \( V_{21} \) is got as

\[
\dot{V}_{21} = \beta + \frac{1}{\sigma} \zeta \dot{\zeta} = \beta (\hat{g}(t) - \frac{1}{\beta_{ir}} \| \hat{\Gamma}_{eq}(t) \|) + \frac{1}{\sigma} \zeta(-z_0 \sqrt{\sigma} \frac{\| \xi(t) \|}{\xi(t)}) \tag{56}
\]

\[
= -z_0 \sqrt{\sigma} \frac{\| \theta(t) \|}{\beta_{ir}} \| \Gamma_{eq}(t) \| - z_0 \sqrt{\sigma} \frac{\| \xi(t) \|}{\xi(t)} \tag{57}
\]

\[
\leq -z_0 \| \theta(t) \| - \frac{q_{ir} \beta_{ir}}{\beta_{ir}} \| \theta(t) \| + \| \theta(t) \| \| \Gamma_{eq}(t) \| \tag{58}
\]

\[
= -z_0 \| \theta(t) \| - \frac{z_0 \sqrt{\sigma}}{\xi(t)} \| \xi(t) \| \tag{59}
\]

Form (58), if \( \zeta(t) \leq 0 \), then \( \dot{V}_{21} \leq -z_0 \| \theta(t) \| - \frac{z_0 \sqrt{\sigma}}{\xi(t)} \| \xi(t) \| \).

According to the definition of \( \dot{z}(t) \), there is \( z(t) > 0 \), then it follows that

\[
\zeta(t) < \frac{q_{ir} \beta_{ir}}{\beta_{ir}} \tag{60}
\]

Take \( \theta(t) \) as the X-axis, \( \zeta(t) \) as the Y-axis to establish the plane rectangular coordinate system, \( \frac{h}{2} \) as the width, greater
than \( \frac{q_{ir}b_{ir2}}{\beta_{ir}} \) as the height to construct an ellipse the expressions are as follows
\[
\tilde{V}_{21} = \{ (\theta, \xi) : V_{21}(\theta, \xi) < \nu \} \quad (60)
\]
\[
\nu = \frac{1}{2} \theta_0^2 + \frac{1}{\sigma} \left( \frac{q_{ir}b_{ir2}}{\beta_{ir}} \right)^2 \quad (61)
\]

Inside the ellipse, a rectangular of that \( \theta_0 \) in width and \( \frac{q_{ir}b_{ir2}}{\beta_{ir}} \) in height, as before, \( \tilde{V}_{21} \leq -z_0 \| \theta \| - \frac{z_0}{\sqrt{\sigma}} \| \xi \| \), outside of the rectangle
\[
\Phi = \left\{ (\theta, \xi) : |\theta| < \theta_0, 0 \leq \xi < \frac{q_{ir}b_{ir2}}{\beta_{ir}} \right\} \quad (62)
\]
which encloses the rectangular region given by (62), if the parameter \( \mu_{ir} \) can be chosen to satisfy
\[
\frac{1}{4} \mu_{ir}^2 > \theta_0^2 + \frac{1}{\sigma} \left( \frac{q_{ir}b_{ir2}}{\beta_{ir}} \right)^2 \quad (63)
\]
there is \( \Phi \subseteq \tilde{V}_{21} \).

Similar with outer-loop subsystem, a conclusion can be drawn that \( t > t_0, |\theta| < \frac{b_{ir2}}{2 \beta_{ir}} \) in finite time.

Then, based on the definition of \( \tilde{V}(t) \) given by (40), there is
\[
\| \tilde{V}(t) \| = \| g(t) - \frac{1}{\beta_{ir}} \tilde{\Gamma}_{eq}(t) - \mu_{ir} \| < \frac{1}{2} \mu_{ir} \quad (64)
\]
Therefore, according to (47), the following inequality can be concluded:
\[
g(t) \geq \frac{1}{\beta_{ir}} \tilde{\Gamma}_{eq}(t) + \frac{\mu_{ir}}{2} > \| \tilde{\Gamma}_{eq}(t) \|
\]
\[
= \| \tilde{b}_{ir}(t) \| \quad (65)
\]
Then,
\[
\dot{\tilde{V}}_{11} \leq -z_0 \| \theta \| - \frac{z_0}{\sqrt{\sigma}} \| \xi \| \text{ in the whole region.}
\]
\[
\dot{\tilde{V}}_2 = \dot{\tilde{V}}_{21} + \dot{\tilde{V}}_{22}
\]
\[
\leq \sigma^T \ddot{\theta} - z_0 \| \theta \| - \frac{z_0}{\sqrt{\sigma}} \| \xi \|
\]
\[
= (b_{ir}(t) - g(t)) \| \sigma \| - \varphi_2 \| \sigma \|
\]
\[
- z_0 \| \theta \| - \frac{z_0}{\sqrt{\sigma}} \| \xi \|
\]

if \( g(t) > \| \tilde{b}_{ir}(t) \| \), there is \( \dot{\tilde{V}}_2 < -\varphi_2 \| \sigma \| - z_0 \| \theta \| - \frac{z_0}{\sqrt{\sigma}} \| \xi \|
\]
Therefore, the second derivative of \( V_2 \) satisfies \( \dot{V}_2 < -\varphi_2 \| \sigma \| - z_0 \| \theta \| - \frac{z_0}{\sqrt{\sigma}} \| \xi \| \), it can be rewritten as
\[
\dot{V}_2 \leq -\varepsilon_2 \sqrt{2} V_{0.5} \quad (66)
\]
where \( \varepsilon_2 = \min \left\{ \varphi_2, z_0, \frac{z_0}{\sqrt{\sigma}} \right\} \)

As lemma 1, the finite time stability of inner-loop is proved.

**Remark 2:** The scalar \( \theta_0 \) needs to be larger than noise or computational errors which is selected as a small positive constant. The defined parameters \( \mu_{ir} \) and \( \beta_{ir} \) are the safety factors. \( q_{ir} \) reflects the accuracy associated with the estimated equivalent control. By selecting the value of the adaptive gain \( \sigma \) can always satisfy inequality (62). In order to ensure good convergence of the system, \( \varphi_2, \lambda_2 \) are chosen as \( \varphi_2 > 0, \lambda_2 > 0, z_0, \varphi_{ir1}, \varphi_{ir2} \) are constants to meet to proof.

### C. CLOSED LOOP SYSTEM STABILITY ANALYSIS

The position and attitude of the quadrotor are strongly coupled in the dynamics, it is necessary to analyze the quadrotor closed-loop stability.

**Theorem 3:** Considering the closed loop system controlled by the outer loop (7), inner loop (39) and the adaptive control law (15), (46) with \( \varphi_1 > 0, r_0 > 0, \varphi_2 > 0, z_0 > 0 \), the closed-loop system is input-state stable in finite convergence time.

**Proof:** Consider the Lyapunov function candidate of closed-loop system as follows:
\[
V_0 = V_1 + V_2 \quad (68)
\]
where \( V_1 \) and \( V_2 \) are the Lyapunov function of outer-loop system and inner-loop system. Combined with the above analysis and the analysis of Theorem 1 and Theorem 2, there is
\[
\dot{V}_0 = \dot{V}_1 + \dot{V}_2
\]
\[
\leq - \min \left\{ \varphi_1, r_0, \frac{r_0}{\sqrt{\nu}} \right\} \sqrt{2} V_{0.5}^i
\]
\[
- \min \left\{ \varphi_2, z_0, \frac{z_0}{\sqrt{\sigma}} \right\} \sqrt{2} V_{0.5}^i \quad (69)
\]
It follows that
\[
\dot{V}_0 \leq - \min \left\{ \min\{\varphi_1, r_0, \frac{r_0}{\sqrt{\nu}} \}, \min\{\varphi_2, z_0, \frac{z_0}{\sqrt{\sigma}} \} \right\} \sqrt{2} \left( V_{1.5}^i + V_{2.5}^i \right)
\]
\[
\leq - \min \left\{ \min\{\varphi_1, r_0, \frac{r_0}{\sqrt{\nu}} \}, \min\{\varphi_2, z_0, \frac{z_0}{\sqrt{\sigma}} \} \right\} \sqrt{2} V_{0.5}^i \quad (70)
\]
From the lemma 1, the whole closed-loop system will stay convergence in finite time.

Now, the closed loop proof is thus completed.

### IV. SIMULATION

In this section, two simulations were performed. First, the method in this article was compared with the traditional FTC method and NRC to verify the effectiveness, accuracy and robustness of the proposed method. Then the proposed method is applied for the system in practical considering the measurement noises. In the simulation, we assume that the structure and model parameters of the quadrotor are same with each other.

#### A. SIMULATION I

1) **PARAMETERS SETTING**

The communication topology diagram is shown in Fig.3, where the node indexed by 1 is the leader quadrotor. And some stationary obstacles are existed in the environment. The simulation verifies that the quadrotors can maintain formation.
flying when there are multiple obstacles. The control laws of FTC are shown as
\[
U_j = \lambda m \left( v_j - v_k \right) + m \left( g e_3 - \ddot{v}_k \right) + m(K_1 \ddot{\omega}^j + K_2 \text{sgn}(s)) \sum_{k=0}^{n} a_{jk}
\]
(71)
\[
Z_j = -\frac{1}{2} I_{lj}(\ddot{q}_j) + \text{sgn}(s) \dot{q}_j + S(\omega_j) I_{lj} \omega_j - I_{lj} R(\ddot{q}_j) \omega_j
\]
(72)

And the other compared control law of NRC is \( U_j \) and \( Z_j \) without Symbolic function.

The physical parameters of the quadrotor formation are designed as follows. The inertia matrices of the quadrotor is \( \text{diag} \{ 0.039 \ 0.039 \ 0.12 \} \) kg \cdot m^2, its error is \( \Delta I_j = \text{diag} \{ 0.001 \ 0.0015 \ 0.0018 \} \). The mass of the quadrotor is \( m = 1 \) kg, and the gravitational acceleration \( g \) is 9.8 m/s^2. The avoidance radius is defined as \( R_d = 4 \) m and the threat radius is \( R_t = 1 \) m. The threat radius and avoidance radius are equal with each obstacle.

Other quadrotors follow the leader quadrotor with communication graph, and the reference trajectory of the leading quadrotor is set as
\[
p_d = \left[ \begin{array}{c} -5 \sin \left( \frac{t}{2\pi} \right) \\ 5 \sin \left( \frac{t}{2\pi} \right) \\ -0.5t \end{array} \right]^T.
\]

The formation distance \( D_{jk} \) corresponds to the communication topology Fig.3, for \( j = 1, 2, 3, 4, 5 \), \( k = 1, 2, 3, 4, 5 \), \( D_{10} = [ 0 \ 0 \ 0 ]^T \), \( D_{12} = [ 0 \ 9 \ 9 ]^T \), \( D_{23} = [ 0 \ 9 \ -9 ]^T \), \( D_{34} = [ 0 \ -9 \ -9 ]^T \), \( D_{24} = [ 0 \ 0 \ -18 ]^T \), \( D_{45} = [ 0 \ 9 \ 0 ]^T \).

The control parameters are same to each quadrotor. And the outer-loop controller parameters of each quadrotor are chosen as \( \kappa_1 = 10, \beta_{or} = 0.99, \mu_{or} = 0.15, \varphi_1 = 18, \epsilon_0 = 0.01, f_1 = [ 0 \ 0.015 \ 0.001 ]^T \). The inner-loop controller parameters are chosen as \( \kappa_2 = 1, \beta_{ir} = 0.99, \mu_{ir} = 0.15, \varphi_2 = 18 \). The parameters of potential function are set to be \( \lambda_1 = 5, \lambda_2 = 5, r_0 = 0.5, \gamma = 800, \delta_0 = 0.01, z_0 = 0.5, \sigma = 800, \phi_0 = 0.01 \).

The initial position of the quadrotor from 1 to 5 is \( P_1 = [1 \ 3 \ 2]^T m \), \( P_2 = [2 \ 10 \ 8]^T m \), \( P_3 = [3 \ 19 \ -1]^T m \), \( P_4 = [2 \ -10 \ -10]^T m \), \( P_5 = [1 \ 19 \ -1]^T m \). The initial speed of each quadrotor drone is zero. The initial quaternion of each quadrotor is \( Q = [0 \ 0 \ 0 \ 1]^T \), and expected value is \( Q_d = [0 \ 0 \ 0 \ 1]^T \).
The initial angular velocity of each quadrotor are set to be zero. Sinusoidal wave disturbances for each quadrotor are randomly considered. The initial angular velocity of each quadrotor are set to be zero. Disturbances for each quadrotor are randomly considered

\[
\begin{align*}
&\bar{d}_j = \begin{bmatrix} \sin(t) & \sin(t) & \sin(t) \end{bmatrix}^T \\
&\bar{d} = \begin{bmatrix} \sin(\pi t) & \sin(\pi t) & \sin(\pi t) \end{bmatrix}^T
\end{align*}
\]

2) RESULTS ANALYSIS
The simulations are demonstrated with MAFTSTC, FTC and NRC control methods. The simulation time in this article was conducted for 100s. Fig. 4 shows the trajectories of five quadrotors and three obstacles in a three-dimensional coordinate system. With the distance between the quadrotor and the obstacle shown in Figs.8-10, the obstacle avoidance process of each quadrotor can be observed. And the quadrotor formation can maintain the formation configuration with obstacles existing in environment by the proposed control scheme. The curve of X-axis, Y-axis, and Z-axis coordinates changed with time are shown in Figs.5-7. Figs. 11-15 show the comparison of the position error between the MAFTSTC, FTC and NRC control methods. The formation meets obstacles at the
20s, 40s, and 80s, respectively. It can be seen that when fluctuations occur due to obstacle avoidance, the position error of MAFTSTC is always smaller than FTC, which is about \( \frac{1}{3} \) of FTC shown in Figs. 11-15. It can be seen clearly in Figs. 11-15 that the convergence accuracy of MAFTSTC is significantly better than that of NRC. Figs.8-10 illustrates the distances between five quadrotors and three obstacles are larger than the collision radius all the time which effectively guarantee the safety of quadrotor formation. Figs.16-20 shows the quaternion error of five quadrotors under the three control methods. The convergence rate of MAFTSTC is obviously faster than that of FTC. FTC takes about 6s to converge, while MAFTSTC only takes about 2s. Compared with NRC, MAFTSTC also has higher convergence accuracy. In the enlarged view of Figs.16-20, the fluctuation of MAFTSTC is smaller than NRC, which means the robustness of MAFTSTC proposed in this article can be guaranteed.

**B. SIMULATION II**

In this part, in order to further illustrate the effectiveness of this method in practical applications, the Parameter errors are considered to simulate the actual situation to illustrate the practical application value of this method. Therefore, we have added measurement errors \( \Delta \omega_j \) and \( \Delta v_j \) to the model. Then, the rigid-body dynamics of the \( j \)th quadrotor is described as

\[
\begin{align*}
\dot{p}_j &= v_j + \Delta v_j \\
\dot{m}v_j &= mge_3 - T_jr(\tilde{Q}_j)e_3 + d_{f_j} \\
\dot{q}_j &= \frac{1}{2}(\eta_j I_3 + S(q_j))(\omega_j + \Delta \omega_j) \\
\dot{\eta}_j &= -\frac{1}{2}q^T_j(\omega_j + \Delta \omega_j) \\
(\hat{l}_j + \Delta \hat{l}_j)\omega_j &= \Gamma_j + d_{\Gamma_j} \\
-S((\omega_j + \Delta \omega_j)(\hat{l}_j + \Delta \hat{l}_j))(\omega_j + \Delta \omega_j)
\end{align*}
\]

(73) (74)
1) PARAMETERS SETTING
For convenience, we choose the value of $\Delta \omega_j$ and $\Delta v_j$ as $\Delta \omega_j = [0.01 \sin(t) \ 0.01 \sin(t) \ 0.01 \sin(t)]^T$, $\Delta v_j = [0.01 \sin(t) \ 0.01 \sin(t) \ 0.01 \sin(t)]^T$ and The initial position of the quadrotor from 1 to 5 is $P_{1} = [0 \ 0 \ 0]^T$, $P_{2} = [3 \ 4 \ 4]^T$, $P_{3} = [7 \ 7 \ 8]^T$, $P_{4} = [9 \ 10 \ -4]^T$, $P_{5} = [4 \ -2 \ -8]^T$. The quadrotor formation distance $D_i$ corresponding to the communication topology is in Fig.38, $D_{1} = [3 \ 4 \ 4]^T$, $D_{2} = [4 \ 4 \ 4]^T$, $D_{3} = [2 \ 3 \ -12]^T$, $D_{4} = [9 \ 10 \ -4]^T$, $D_{5} = [-5 \ -12 \ -4]^T$. The expected trajectory of the quadrotor is $p_d = [0.3t \ -0.4t \ -10 \sin(\frac{t}{2})]^T$.

2) RESULT ANALYSIS
From Fig.21 with the control method in this article quadrotor formation can avoid obstacles and track the desired route.
trajectory considering measurement errors. Figs. 22-24 show the detailed trajectory of the quadrotors with parameter errors. Figs.28-32 and Figs. 33-37 show the position error and quaternion error of the quadrotors, respectively. With the existing of measurement errors, the position and quaternion errors are convergence to zero with high speed and high
convergence accuracy. Figs. 25-27 show the distance between each quadrotor and the obstacle. It can be seen the safe distance is always maintained between the quadrotors and the obstacle, which shows that the method in this article can still avoid obstacles under the condition of parameter error exists.

V. CONCLUSION
In this article, the multi-quadrotor formation control was designed subject to satisfy robustness and avoid collisions. The safety of formation is accomplished by adopting the potential energy function which guarantees the distance between quadrotors and obstacles to be larger than collision radius. Then a novel adaptive super twisting controller is designed for quaternion-based quadrotor formation to form and maintain the formation construction with reduction of the dependence on the model. According to Lyapunov stability theory, it is proved that the proposed formation control
method can guarantee the multi-quadrotor system to converge to the desired construction in finite time, meanwhile to maintain the predefined patterns with obstacles avoidance. Finally, simulations are carried out to verify effectiveness of the proposed approach in controlling the multi-quadrotor formation flying in the obstacle environment. In the future works, the fixed time convergence formation control of multi-quadrotor with obstacle avoidance and inter-quadrotor avoidance will be considered.

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