Original Article

On Stability for Hybrid System under Stochastic Perturbations

Cao Tan Binh¹,*, Ta Cong Son²

¹Quy Nhon University, 170 An Duong Vuong, Quy Nhon, Vietnam
²VNU University of Science, 334 Nguyen Trai, Hanoi, Vietnam

Received 13 May 2020
Revised 22 July 2020; Accepted 30 August 2020

Abstract: The aim of this paper is to find out suitable conditions for almost surely exponential stability of communication protocols, considered for nonlinear hybrid system under stochastic perturbations. By using the Lyapunov-type function, we proved that the almost surely exponential stability remain be guaranteed as long as a bound on the maximum allowable transfer interval (MATI) is satisfied.

Keywords: Networked Control System, almost surely exponential stability, maximum allowable transfer interval, Lyapunov function.

1. Introduction

In recent years, Networked Control Systems (NCS) were addressed strongly in the control community because of its extensive applications in wireless as well as wireline. The pioneering papers were proposed by Walsh, Beldman and Bushnell [10, 11, 12]. They introduced about stability of control systems with deterministic protocol. More recently, quite many articles and literatures referred to study stability of hybrid systems by specifically showing the Lyapunov-type function and bounds on the maximum allowable transfer interval (MATI), see [1, 2, 3, 4, 8, 6, 9, 13] for more details. This paper is divided into two sections. Beside Introduction, we state Preliminary and main problem in the second section.

*Corresponding author.
Email address: caotanbinh@qnu.edu.vn
https://doi.org/ 10.25073/2588-1124/vnumap.4522
In [5], the authors solved entirely for researching the stable types of solution of hybrid systems, modelled as follows:

\[\begin{align*}
\dot{x}(t) &= f(x(t), e(t)), t \in (t_k, t_{k+1}), \\
\dot{e}(t) &= g(x(t), e(t)), t \in (t_k, t_{k+1}), \\
\dot{t}(t) &= 1, t \in (t_k, t_{k+1}), \\
\tau(t^*_k) &= 0, \\
x(t^*_k) &= x(t_k), \\
e(t^*_k) &= b_k h(k, e(t_k)) + (1 - b_k) e(t_k), k = 0, 1, 2, \ldots
\end{align*}\]

Remind here that the variable \( b_k \) belongs to the set \( \{0, 1\} \). If \( b_k = 1 \) then transmission is successful, and the protocol \( h \) determines the updated error. While if \( b_k = 0 \) then the error remains unchanged at the \( t_k \). We get a sequence \( (b_k)_{k \in \mathbb{N}} \). Let \( S := \{0, 1\} \) and the probability space \((S^1, F_k, P)\) with the sequence space

\[ S^1 := \{(b_k)_{k \in \mathbb{N}} : b_k \in S, \forall k \in \mathbb{N}\} \]

where the \( \sigma \)-algebra \( F_k := 2^S \times 2^S \times \ldots \) and the probability \( P \) satisfying

\[ P(b \in S^1 : b_k = 1) = p, \forall k \in \mathbb{N}. \]

We also assume that the random variables \( b_k \) are independently and identically distributed.

Motivated from this paper, we concern to hybrid system in which exogenously stochastic perturbation is a Wiener process. This is, up to now, one of proposed problems remain have not been solved yet. To solve the problem, we make use of tools as introduced in [5] by defining \( \tau_{MATI} \) or choosing the Lyapunov function \( W \) for protocol. We also, of course, use other tools for stochastic stability from [7] in order to support our proof.

2. Preliminary and main result

Let us now consider the perturbed hybrid system that is of form

\[\begin{align*}
\dot{x}(t) &= f_1(x(t), e(t))dt + f_2(x(t), e(t))dw(t), t \in (t_k, t_{k+1}), \\
\dot{e}(t) &= g_1(x(t), e(t))dt + g_2(x(t), e(t))dw(t), t \in (t_k, t_{k+1}), \\
\dot{t}(t) &= 1, t \in (t_k, t_{k+1}), \\
\tau(t^*_k) &= 0, \\
x(t^*_k) &= x(t_k), \\
e(t^*_k) &= b_k h(k, e(t_k)) + (1 - b_k) e(t_k), k = 0, 1, 2, \ldots
\end{align*}\]

where \( x \in \mathbb{R}^n \) is the state of the system, \( e \in \mathbb{R}^n \) is the error at the controller, \( h \) is the update function that models the particular protocol, \( \tau \) is a timer to constrain both the transmission interval and the
transmission delay, and \( w(t) \) is a Wiener process. In this paper, suppose that \( f_1, f_2, g_1, \) and \( g_2 \) satisfy Lipschitz and linear growth conditions which guarantee the existence and uniqueness of the solution of (2). Assume furthermore that \( f_1(0,0) = f_2(0,0) = g_1(0,0) = g_2(0,0) \) and \( h(k,0) = 0 \) for all \( k \in \mathbb{N} \). So system (2) has the solution \( \xi(t) := (x(t); e(t)) = (0,0) \) corresponding to the initial value \( \xi^* := (x^*, e^*) = (0,0) \).

Now, we introduce the concept of almost surely exponential stability, which can be found in Mao [7].

**Definition 1** Consider the system (2). The solution \( \xi^* = (x^*, e^*) = (0,0) \) of (1) is called almost surely exponentially stable, if for all \( \xi_0 \)

\[
\limsup_{t \to \infty} \frac{1}{t} \log \| \xi(t,0,\xi_0,b) \| < 0, \text{ almost surely.}
\]

We need the following assumptions for the stability of network and system.

**Assumption (A1)** The probability \( p \in (0,1) \) of successful transmission of the \( k \)-th sampling time is identical for all \( k \in \mathbb{N} \) and independent of \( k \in \mathbb{N} \).

**Assumption (A2)** The stochastic perturbations \( b \) and \( w \) are mutually independent. Put \( F_b \) is the \( \sigma \)-algebra generated by \( \{b_k\}_{k \in \mathbb{N}} \), and \( F_w \) is the \( \sigma \)-algebra generated by \( \{w(t)\}_{t \geq 0} \). The system (2) defined on a probability space \( (\Omega, F, P) \) where \( F = \sigma\{F_b \cup F_w\} \). Hereafter, we use notation \( E_b(.) \) instead of \( E_b (.| F_b) \) and \( E_w(.) \) instead of \( E_w(.| F_w) \).

**Assumption (A3)** Lyapunov functions for the protocol and the perturbed system.

(i) There exist constants \( 0 < a_1, a_2, 0 < \lambda < 1 \) such that for all \( e \in \mathbb{R}^n \):

\[
a_1 \| e \|^2 \leq W(e) \leq a_2 \| e \|^2 \quad (3)
\]

\[
W(h(k,e)) \leq \lambda W(e) \quad (4)
\]

(ii) The evolution of Lyapunov function \( W \) is bounded in the sense that there exist a constant \( \alpha \geq 0, \beta \in \mathbb{R} \) and a continuous function \( H : \mathbb{R}^n \to \mathbb{R} \), such that for all \( x,e \in \mathbb{R}^n \):

\[
\frac{\partial W}{\partial e} g_1(x,e) = \left( \frac{\partial W}{\partial e}, g_1^T(x,e) \right) \leq 2\alpha W(e) + \beta H(x) \quad (5)
\]

(iii) There exist a \( C^2 \) Lyapunov function \( V \) and constants \( b_1,b_2,b_3 > 0 \) such that for all \( x,e \in \mathbb{R}^n \):

\[
b_1 \| e \|^2 \leq V(x) \leq b_2 \| e \|^2 \quad (6)
\]

\[
LV(x) := \frac{\partial V}{\partial x} f_1(x,e) + \frac{1}{2} f_2^T(x,e). \frac{\partial^2 V}{\partial x^2}. f_2(x,e) \leq -b_3 V(x), \quad (7)
\]

where
Consider the system (2). Assume that (A1), (A2) and (A3) hold. If there exist \( \eta \in (0,1) \) and \( \gamma > 0 \) as defined in (8) satisfying

\[
g_1(x,e) \frac{\partial^2 V}{\partial e^2} + g_2(x,e) \leq 2 \left[ 2\gamma \eta - b_1 W(e) - b_2 H(x) \right]
\]

for almost all \( x, e \in \mathbb{R}^n \) (10) then the solution \( \xi^* = (0,0) \) of system (2) is almost surely exponentially stable.

**Proof:** We first assume that system (2a), (2b) is almost surely exponentially stable. Consider Lyapunov-type function

\[
U(x,e) = V(x,e) + \gamma \phi(e) W(e).
\]

It follows that

\[
b_1 \|x\|^2 + V(x) \leq b_2 \|x\|^2 + a_1 \|e\|^2 \leq W(e) \leq a_2 \|e\|^2. \eta \leq \phi(e) \leq \frac{1}{\eta}.
\]

We yield

\[
b_1 \|x\|^2 + \gamma \eta a_1 \|e\|^2 \leq U(x,e) \leq b_2 \|x\|^2 + \gamma \eta^{-1} a_2 \|e\|^2
\]

and

\[
m \|x\|^2 = m \|x\|^2 \leq U(x,e) \|x\|^2 \leq M \|x\|^2 = M \|\xi\|^2
\]

where \( m = \min \{ b_1, \gamma \eta a_1 \}, M = \max \{ b_2, \gamma \eta^{-1} a_2 \} \).

By Ito’s formula and Assumption (A3), we can derive that
\[ dU(x, e, \tau) = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial e} de + \frac{\partial U}{\partial \tau} d\tau + \frac{1}{2} \left( f^T_2(x, e), \frac{\partial^2 U}{\partial x^2}, f_3(x, e) + g^T_2(x, e), \frac{\partial^2 U}{\partial e^2}, g_2(x, e) \right) dt \]
\[ = \frac{\partial V}{\partial x} [f_2(x, e) dt + f_3(x, e) dw] + \gamma \phi(\tau) \frac{\partial V}{\partial e} [g_2(x, e) dt + g_2(x, e) dw] \]
\[ + \gamma \phi(\tau) W(e) dt \tau dt + \frac{1}{2} \left( f^T_2(x, e), \frac{\partial^2 V}{\partial x^2}, f_3(x, e) + \gamma \phi(\tau) g^T_2(x, e), \frac{\partial^2 W}{\partial e^2}, g_2(x, e) \right) dt \]
\[ = LV(x) dt + \left[ \gamma \phi(\tau) \frac{\partial W}{\partial e}, g_1(x, e) + \gamma \phi(\tau) W(e) + \frac{1}{2} \gamma \phi(\tau) g^T_2(x, e), \frac{\partial^2 W}{\partial e^2}, g_2(x, e) \right] dt \]
\[ + \left[ \frac{\partial V}{\partial x}, f_2(x, e) + \gamma \phi(\tau) \frac{\partial W}{\partial e}, g_2(x, e) \right] dw \]
and
\[ \gamma \phi(\tau) \frac{\partial W}{\partial e}, g_1(x, e) + \gamma \phi(\tau) W(e) + \frac{1}{2} \gamma \phi(\tau) g^T_2(x, e), \frac{\partial^2 W}{\partial e^2}, g_2(x, e) \]
\[ = \gamma \phi(\tau) \frac{\partial W}{\partial e}, g_1(x, e) + \gamma [-2 \alpha \phi(\tau) - \gamma(\phi^2(\tau) + 1)] W(e) + \frac{1}{2} \gamma \phi(\tau) g^T_2(x, e), \frac{\partial^2 W}{\partial e^2}, g_2(x, e) \]
\[ \leq \gamma \phi(\tau) \left[ 2aW(e) + \beta H(x) \right] - 2 \alpha \phi(\tau) W(e) - \gamma \gamma^{\phi^2(\tau) + 1} W(e) \]
\[ + \gamma \phi(\tau) \left[ (2 \gamma \eta - b_1) W(e) - \beta H(x) \right] \]
\[ = 2 \gamma \alpha \phi(\tau) W(e) + \gamma \beta \phi(\tau) H(x) - 2 \gamma \alpha \phi(\tau) W(e) - \gamma \gamma^{\gamma^2(\tau) + \gamma^2(\tau)} W(e) \]
\[ + \gamma \phi(\tau) \left[ (\gamma \eta + \gamma - b_3) W(e) - \beta H(x) \right] \]
\[ \leq - \gamma \phi(\tau) b_3 W(e). \]

Therefore
\[ dU(x, e, \tau) \leq - b_3 V(x) dt - \gamma \phi(\tau) b_3 W(e) dt + \left[ \frac{\partial V}{\partial x}, f_2(x, e) + \gamma \phi(\tau) \frac{\partial W}{\partial e}, g_2(x, e) \right] dw \]
\[ \leq - b_3 U(x, e, \tau) dt + \left[ \frac{\partial V}{\partial x}, f_2(x, e) + \gamma \phi(\tau) \frac{\partial W}{\partial e}, g_2(x, e) \right] dw. \]

This implies
\[ dE_u[U(x, e, \tau)] \leq - b_3 E_u[U(x, e, \tau)] dt. \]

For each \( k = 1, 2, \ldots \), integrating both sides of (16) from \( t_{k-1}^+ \) to \( t_k \), we get
\[ E_u[U(x(t_k, b), e(t_k, b), \tau(t_k))] \leq E_u[U(x(t_{k-1}^+, b), e(t_{k-1}^+, b), \tau(t_{k-1}^+))] + \int_{t_{k-1}^+}^{t_k} E_u[-b_3 U(x, e, \tau)] dt \]
\[ \leq E_u[U(x(t_{k-1}^+, b), e(t_{k-1}^+, b), \tau(t_{k-1}^+))]. \]

If at time \( t_k \) transmission is successful, i.e. if \( b_3 = 1 \), then
\[ U(x(t_k^+, b), e(t_k^+, b), \tau(t_k^+)) \leq V(x(t_k^+, b)) + \eta^{-2} \gamma \phi(\tau(t_k^+)) W(e(t_k^+, b)). \]
On the other hand if transmission fails, i.e. if \( b_k = 0 \) then
\[
U(x(t^*_k, b), e(t^*_k, b), \tau(t^*_k)) \leq V(x(t_k, b)) + \eta^2 \gamma \phi(\tau(t_k)) W(e(t_k, b)).
\]
These give
\[
E_0 \left\{ E_0 \left[ U(x(t^*_k, b), e(t^*_k, b), \tau(t^*_k)) \right|x(t^*_k, b), e(t^*_k, b) \right] \right\} \leq p \left\{ E_0 \left[ V(x(t_k, b)) + \eta^2 \lambda \phi(\tau(t_k)) E_0 \left[ W(e(t_k, b)) \right] \right) \right\} + (1-p) \left\{ E_0 \left[ V(x(t_k, b)) + \eta^2 \gamma \phi(\tau(t_k)) E_0 \left[ W(e(t_k, b)) \right] \right) \right\} < E_0 \left\{ U(x(t_k, b), e(t_k, b), \tau(t_k)) \right\} - \kappa \eta E_0 \left[ W(e(t_k, b)) \right]
\]
where \( \kappa := 1-(1-p+\rho \lambda)\eta^2 \).

From (16) it follows that
\[
E_0 \left[ U(\xi(t, b), \tau(t)) \right] = E_0 \left[ U(x(t, b), e(t, b), \tau(t)) \right] < e^{-b(t-t_0)} E_0 \left[ U(\xi(t^*_k, b), \tau(t^*_k)) \right].
\]
Taking expectation in \( b \), we obtain
\[
e^{b\lambda} E_0 \left\{ E_0 \left[ U(\xi(t, b), \tau(t)) \right] \right\} < e^{b\lambda} E_0 \left\{ E_0 \left[ U(\xi(t^*_k, b), \tau(t^*_k)) \right] \right\}
\]
and
\[
0 \leq e^{b\lambda} E_0 \left\{ E_0 \left[ U(\xi(t^*_k, b), \tau(t^*_k)) \right] \right\} = e^{b\lambda} E_0 \left\{ E_0 \left[ U(\xi(t^*_k, b), \tau(t^*_k)) \right] \xi(t_k, b) \right\}
\]
\[
\leq ME_0 \left\| \xi_{t_0} \right\|^2 - \kappa \eta \sum_{i=0}^{k} e^{b\lambda} E_0 \left\{ E_0 \left[ W(e(t_i, b)) \right] \right\}.
\]
From (12), (19) and (20), it follows that
\[
me^{b\lambda} E_0 \left\{ E_0 \left[ \xi(t, b) \right] \right\} \leq e^{b\lambda} E_0 \left\{ E_0 \left[ U(\xi(t, b), \tau(t)) \right] \right\} < e^{b\lambda} E_0 \left\{ E_0 \left[ U(\xi(t^*_k, b), \tau(t^*_k)) \right] \right\} \leq ME_0 \left\| \xi_{t_0} \right\|^2.
\]
Hence
\[
E_0 \left\{ E_0 \left[ \xi(t, b) \right] \right\} \leq \frac{M}{m} E_0 \left\| \xi_{t_0} \right\|^2 e^{-b\lambda}, \forall t \geq 0.
\]
From the system (2), we have
\[
x(t) = x(t_0) + \int_{t_0}^{t} f_1(s) ds + \int_{t_0}^{t} f_2(s) dw(s)
\]
and
\[
e(t) = e(t_0) + \int_{t_0}^{t} g_1(s) ds + \int_{t_0}^{t} g_2(s) dw(s).
\]
In addition, the conditions \( f_1(0,0) = f_2(0,0) = g_1(0,0) = g_2(0,0) \) lead to exist a positive constant \( K \) such that
\[
\left\| f_1(x,e) \right\|^2 \left\| f_2(x,e) \right\|^2 \leq K \left\| (x,e) \right\|^2
\]
\[
\left\| g_1(x,e) \right\|^2 \left\| g_2(x,e) \right\|^2 \leq K \left\| (x,e) \right\|^2
\]
Therefore, we obtain
\[
E_n \|x(t)\|^2 \leq 3 \left[ E_n x^2(t_k) + E_n \left( \int_{t_k}^t f(s,ds)^2 \right) + E_n \left( \int_{t_k}^t f^2(ds) \right) \right] \\
\leq 3 \left[ E_n x^2(t_k) + (t-t_k) \int_{t_k}^t E_n f^2(ds) + \int_{t_k}^t E_n f^2(ds) \right] \\
= 3 \left[ E_n x^2(t_k) + (\bar{\tau} + 1) K \int_{t_k}^t E_n \|\xi(s)\|^2 ds \right] \\
\leq 3 \left[ E_n x^2(t_k) + (\bar{\tau} + 1) K \int_{t_k}^t E_n \|\xi(s)\|^2 ds \right]
\]
and
\[
E_n \|e(t)\|^2 \leq 3 \left[ E_n e^2(t_k) + (\bar{\tau} + 1) K \int_{t_k}^t E_n \|\xi(s)\|^2 ds \right].
\]
As a result
\[
E_n \|x\|^2 = E_n \|(x,e)\|^2 = E_n \|x(t)\|^2 + E_n \|e(t)\|^2 \leq 3 \left[ E_n \|\xi(t_k)\|^2 + 2(\bar{\tau} + 1) K \int_{t_k}^t E_n \|\xi(s)\|^2 ds \right].
\]
Hence
\[
E_n \left( \sup_{t \in [t_k, t_{k+1}]} \left[ E_n \|\xi(t,b)\|^2 \right] \right) \leq 3 \left[ E_n E_n \|\xi(t_k,b)\|^2 + 2(\bar{\tau} + 1) K \int_{t_k}^t E_n \|\xi(s,b)\|^2 ds \right] \\
\leq 3 \left[ \frac{M}{m} E_n \|\xi\|^2 + 2(\bar{\tau} + 1) K \int_{t_k}^t E_n \|\xi\|^2 e^{-b_3} ds \right] \\
\leq 3 \left[ \frac{1}{b_3} K(\bar{\tau} + 1)(1 - e^{-b_3}) \right] \frac{M}{m} E_n \|\xi\|^2 e^{-b_3} \\
\leq C e^{-b_3},
\]
where
\[
C = 3 \left[ \frac{1}{b_3} K(\bar{\tau} + 1)(1 - e^{-b_3}) \right] \frac{M}{m} E_n \|\xi\|^2.
\]
Applying Chebyshev’s inequality, we get
\[
P \left( b : \sup_{t \in [t_k, t_{k+1}]} E_n \|\xi(t,b)\|^2 > e^{\frac{b_3}{2}(t_{k+1})} \right) \leq \frac{E_n \left( \sup_{t \in [t_k, t_{k+1}]} E_n \|\xi(t,b)\|^2 \right)}{e^{\frac{b_3}{2}(t_{k+1})}} \\
\leq C e^{-\frac{b_3}{2}(t_{k+1})}.
\]
Since \( t_0 = 0 \) and \( 0 < \delta < t_{k+1} - t_k \leq \bar{\tau} \), it is clear that
\[
\sum_{k=0}^{\infty} e^{\frac{b_3}{2}(t_{k+1})} = e^{\frac{b_3}{2}t_0} + e^{\frac{b_3}{2}t_1} + \ldots + e^{\frac{b_3}{2}(t_{k+1} - t_k)} + \ldots < \infty.
\]
Using Borel-Cantelli’s lemma argument (see Mao [7]) to conclude that there exist a set $\Omega_1$ with $P(\Omega_1) = 1$ and an integer-value random variable $k_0$ such that for every $b \in \Omega_1$ we have

$$\sup_{t \in t_k \cup_t} E \xi(t, b) \leq e^{\frac{b_k}{2}}, \forall k \geq k_0(b).$$

That means

$$E \xi(t, b) \leq e^{\frac{b_k}{2}}, \forall t \in (t_k, t_{k+1}), \forall k \geq k_0(b).$$

Similarly to argument as above, using Borel-Cantelli’s lemma again, there exist a set $\Omega_2$ with $P(\Omega_2) = 1$ and an integer-value random variable $k_1$ such that for every $w \in \Omega_2$ we have

$$\|\xi(t, b)\| \leq e^{\frac{b_k}{2}}, \forall t \in (t_k, t_{k+1}), \forall k \geq k_1(w).$$

Let $k_2 = \max\{k_0, k_1\}$, $\Omega_0 = \Omega_1 \cap \Omega_2$, we have $P(\Omega_0) = 1$ and

$$\|\xi(t, b)\| \leq e^{\frac{b_k}{2}}, \forall t \in (t_k, t_{k+1}), \forall k \geq k_2(w), (b, w) \in \Omega_0.$$}

Consequently

$$\limsup_{t \to \infty} \frac{1}{t} \log \|\xi(t, b)\| \leq \frac{b_1}{8} < 0.$$}

The proof is completed.

**Remark 3** The inequalities (5) and (10) are existent. In fact, we choose $g_1(x, e) = g_2(x, e) = e$, $W(e) = \|e\| = (e_1^2 + e_2^2)^{1/2}$ and $\beta = 0$. Then we have

$$\frac{\partial W}{\partial e} g_1(x, e) = \left(\frac{\partial W}{\partial e}, g^T_1(x, e)\right) = \left(e_1^2 + e_2^2\right)^{1/2} \leq 2 \alpha W(e), \forall \alpha \geq 0,$$

Moreover,

$$g_2^T \frac{\partial^2 W(e)}{\partial e^2} g_2 = 0 \leq 2(2\gamma - \beta_3) W(e),$$

as long as $2\gamma - \beta_3 \geq 0$.

**References**

[1] D. Carnevale, A. R. Teel, D. Nesic, Further results on stability of networked control systems: a lyapunov approach, In 2007 American Control Conference, IEEE. (2007) 1741-1746.

[2] D. Carnevale, A. R. Teel, D. Nesic, A Lyapunov proof of an improved maximum allowable transfer interval for networked control systems, IEEE Transactions on Automatic Control. 52 (2007) 892-897.

[3] D. Christmann, On the behavior of black bursts in tick-synchronized networks, Techn. Ber. 337 (2010).
[4] M. B. Cloosterman, N. Van de Wouw, W. P. M. H. Heemels, H. Nijmeijer, Stability of networked control systems with uncertain time-varying delays, IEEE Transactions on Automatic Control. 54 (2009) 1575-1580.

[5] L. H. Duc, D. Christmann, R. Gotzhein, S. Siegmund, F. Wirth, The stability of try-oncediscard for stochastic communication channels: Theory and validation, In 2015 54th IEEE Conference on Decision and Control (CDC). (2015) 4170-4175.

[6] W. P. M. H. Heemels, D. Nesic, A. R. Teel, N. Van de Wouw, Networked and quantized control systems with communication delays, In Proceedings of the 48th IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference. (2009) 7929-7935.

[7] X. Mao, Stochastic differential equations and applications, Elsevier, 2007.

[8] D. Nesic, D. Liberzon, A unified framework for design and analysis of networked and quantized control systems, IEEE Trans. Automatic Control. 54 (2009) 732-747.

[9] P. Naghshtabrizi, J. P. Hespanha, A. R. Teel, Stability of delay impulsive systems with application to networked control systems, Transactions of the Institute of Measurement and Control. 32 (2010), 511-528.

[10] G. C. Walsh, O. Bdeliman, L. G. Bushnell, Asymptotic behavior of nonlinear networked control systems, IEEE transactions on automatic control. 46 (2001) 1093-1097.

[11] G. C. Walsh, O. Bdeliman, L. G. Bushnell, Error encoding algorithms for networked control systems, Automatica. 38 (2002) 261-267.

[12] G. C. Walsh, H. Ye, L. G. Bushnell, Stability analysis of networked control systems, IEEE transactions on control systems technology. 10 (2002) 438-446.

[13] L. Zhang, Y. Shi, T. Chen, B. Huang, A new method for stabilization of networked control systems with random delays, IEEE Transactions on automatic control. 50 (2005) 1177-1181.