Novel Schemes for Directly Measuring Entanglement of General States

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An intrinsic relation between maximally entangled states and entanglement measures is revealed, which plays a role in establishing connections for different entanglement quantifiers. We exploit the basic idea and propose a framework to construct schemes for directly measuring entanglement of general states. In particular, we demonstrate that rank-1 local factorizable projective measurements, which are achievable with only one copy of entangled state involved at a time in a sequential way, are sufficient to directly determine the concurrence of arbitrary two-qubit entangled state.

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Introduction. Quantum entanglement is one of the most significant feature of quantum mechanics\cite{1}, which has attracted a lot of interest within the burgeoning field of quantum information science and its interaction with many-body physics\cite{2,3}. Entanglement measures play a central role in the theory of entanglement. It is well known that antilinearity from symmetries with time reversal operations is intrinsically non-local, which leads to a natural routine to describe and estimate entanglement\cite{4,5,6,7,8,9,10}. These entanglement measures based on nonphysical allowed transformations are usually nonlinear functions of the density matrix elements, and thus are difficult to determine directly in experiments. It is worth to point out that there exists an alternative experimental favorable class of entanglement quantifiers, which are directly defined through the measurable observables\cite{11,12}. The interesting problems are: \textit{How and why can these quantities from (anti)symmetric projections serve as entanglement quantifiers? Is there any connection between the above two different classes of entanglement measures?}

As far as determining entanglement is concerned, there are two desirable features. The first is about the parametric efficiency issue. It is inefficient and not necessary to obtain all the state parameters as quantum state tomography\cite{13}, in particular when one considers high dimensional and multipartite quantum systems. This concerns not only experimental determining entanglement itself, but is related to the general theoretical problem about extracting information efficiently from an unknown quantum state with the least measurement cost\cite{4}. Second, in many realistic scenarios, entangled particles are shared by two distant parties Alice and Bob, e.g. long distance quantum communication. It will be valuable that Alice and Bob can measure entanglement with only local operations on individual subsystems and classical communications (LOCC).

The basic ideas of measuring these entanglement measures based on nonphysical allowed transformations directly without state reconstruction\cite{14,15,16,17,18,19,20}, mainly rely on multiple copies of entangled state, i.e., a number of entangled state need to be present at the same time. This could be difficult for certain physical systems. The requisite experimental components include structure physical approximation (SPA) and interferometer circuit, the implementation of which with only LOCC is a great challenge. One may wonder \textit{Whether projective observables can also help to determine nonphysical allowed transformation based entanglement measures?}

In this paper, we address the above problems by revealing an intrinsic connection between maximally entangled states (MES) and the definitions of nonphysical allowed transformation based entanglement measures. The connection enables us to find that (anti)symmetric projections can indeed extract the properties of density matrices with nonphysical allowed transformations. This result opens the possibility to establish relations between various kinds of entanglement quantifiers\cite{4,5,7,8,9,10,11,12}. The connection also allows us to propose a framework based on local projections to design schemes for directly measuring the entanglement quantifiers from nonphysical allowed transformations. We explicitly demonstrate the benefit in determining entanglement of general two-qubit states. The most remarkable feature is that only one copy of entangled state need to be present at a time, which is distinct from other schemes using multiple copies of entangled state. As applications of our idea, we elucidate the physics underlying the first noiseless quantum circuit for the Peres-Horodecki separability criterion\cite{4,5,21}, which was obtained in\cite{21} through the mathematical analysis of polynomial invariants\cite{18}. Moreover, one can easily construct a circuit to directly measure the realignment properties of quantum states\cite{10}.

Connections between MES and entanglement measures. One useful tool in the entanglement theory is positive but not completely positive map, with antilinear conjugation as the most representative operation. Following the Peres-Horodecki separability criterion\cite{4,5}, lots of entanglement detection methods and measures based on the conjugation of density operator have been established\cite{6,7,8,9}. We pro-
pose to mathematically implement nonphysical allowed trans-
formations, in particular antilinear conjugation, with the no-
tation of MES. From an operational viewpoint, MES with
appropriate local unitary operations can be associated to
(anti)symmetric projective measurements. Thus, our result
makes a connection between antilinear conjugation and (anti)symmetric projection based entanglement quanti-
fiers [11, 12].

Lemma 1 Given an $n$-partite operator $A$ on the Hilbert space
$\mathcal{H} = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n$, with the dimension $\dim(\mathcal{H}_i) = d_i$, the
maximally entangled state of $d_i \otimes d_i$ bipartite system is
denoted as $|S_i\rangle = \sum_{s=0}^{d_i-1} |s\rangle s / \sqrt{d_i}$, then

$$(A \otimes I_{1\ldots n})|S\rangle = (I_{1\ldots n} \otimes A^T)|S\rangle \quad \text{with} \quad |S\rangle = \bigotimes_{i=1}^n |S_i\rangle_{ii}$$

Moreover, we have

$$\text{tr}A = (\prod_{i=1}^n d_i)(|S\rangle (I_{1\ldots n} \otimes A)|S\rangle) \quad \text{(1)}$$

Eq.(1) is the generalization of the fact that qubit operator
can travel through singlets, which has been used to investi-
gate the localizable entanglement properties of valence bond
states [22]. Here, from a different perspective, we view
$A$ itself as a density matrix $\rho$ rather than an operator on quan-
tum states, lemma 1 thus indicates that with the notation of
MES, we can mathematically implement the (partial) trans-
pose (conjugation) of arbitrary quantum states. Eq.(2) is
another key point, which enables us to extract the properties of
transformed density operators through the projective measure-
ments associated to MES.

Remark 1: Our idea is quite different from the SPA, in
which the transpose of quantum state is approximated by a
completely positive map [12, 15]. It is worth to point out that,
in lemma 1, $A$ can be a density operator of arbitrary dimen-
sional multipartite quantum systems.

Theorem 1 For a general quantum state $\rho$ on the Hilbert
space $\mathcal{H} = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n$, we denote the antilinear trans-
formation of $\rho$ as $\rho_u = (U_1 \otimes \cdots \otimes U_n) \rho^* (U_1^\dagger \otimes \cdots \otimes U_n^\dagger)$ and
$|S_u\rangle = (I \otimes U_1)|S\rangle$, it can be seen that

$$(\rho \otimes \rho)|S_u\rangle = (I_{1\ldots n} \otimes \rho_u)|S_u\rangle \quad \text{with} \quad |S_u\rangle = \bigotimes_{i=1}^n |S_{ui}\rangle_{ii}$$

This will lead to

$$\text{tr}(\rho \rho_u) = \left(\prod_{i=1}^n d_i\right) \cdot \text{tr}\left( \bigotimes_{i=1}^n P^{(i)}_u (\rho \otimes \rho) \right) \quad \text{(3)}$$

where $U_i$ are local unitary operations, and $P^{(i)}_u =
|S_{ui}\rangle_{ii}\langle S_{ui}|$ are projections on two copies of the $i$-th subsystem.

Remark 2: Theorem 1 can help us to establish connections
between antilinearity and (anti)symmetric projections. The
result is quite general, e.g. $U_i$ can be arbitrary local unitary
operators, and it is applicable for high dimensional situations
by using appropriate $U_i$ or reducing the projections of high
dimensional bipartite systems into a sum of two-qubit projec-
tions [11]. It also provides an intuitive meaning of the Woot-
ters’ concurrence, which can be linked with the success prob-
ability of establishing MES via entanglement swapping fol-
lowing the above theorem.

Novel schemes for measuring entanglement. Besides the
theoretical interest, with the above connection we find that
projective observables can help to determine these nonphysi-
tical transformation based entanglement quantifiers with much
less experimental efforts. We first demonstrate how to di-
rectly measure the concurrence of general states [6, 8, 9],
and then explicitly illustrate the physics underlying the first
noiseless circuit for the Peres-Horodecki separability criterion
[4, 5, 7, 21] following the present idea. Finally, we construct
a simple circuit for the realignment separability criterion [10].

1. Scheme for directly measuring the concurrence of general
states. The concurrence family of entanglement measures are
defined through the eigenvalues $\lambda_j$ of $\rho \rho_u$ as in theorem 1. In
order to determine these eigenvalues, quantum state tomog-
raphy need to obtain $(d_1 \cdots d_n)^2 - 1$ parameters, while direct
strategy without state reconstruction only need to measure the
moments $m_k = \sum_j \lambda_j^k$, the number of which is $d_1 \cdots d_n$, and
thus it is quadratically efficient.

Lemma 2 The moments of $\rho \rho_u$ can be obtained as follows

$$m_k = (d_a d_b)^k \text{tr}\left( (P^{(a)} \otimes P^{(b)}) V_{a_2 \cdots a_{2k}} V_{b_2 \cdots b_{2k}} \bigotimes_{i=1}^{2k} (\rho)_{a_i b_i} \right) \quad \text{(5)}$$

$P^{(s)} = P^{(s_1 s_2)} \otimes \cdots \otimes P^{(s_{2k-1} \cdots s_{2k})}$, and $V_{a_2 \cdots a_{2k}}$ are $k$-circle
permutations $(s = a, b)$.

Proof. The $k$-cycle permutation $V^{(k)}|\phi_1\rangle|\phi_2\rangle \cdots |\phi_k\rangle =
|\phi_k\rangle|\phi_1\rangle \cdots |\phi_{k-1}\rangle$ is the key element for spectrum measure-
ment based on the property $\text{tr}(V^{(k)} \otimes A_i) = \text{tr}(A_k \cdots A_1)$
[23]. As in lemma 1, using $2k$ copies of entangled state and
with the notation of MES, we can mathematically have $k$
copies of $\rho \rho_u$. Thus, one can get Eq.(5) with the above two facts.

Remark 3: For simplicity, we only give the formulations
for bipartite systems, lemma 2 however is valid for general
multipartite states. Our results provide a simple and general
framework to design schemes for directly measuring the con-
currence family of entanglement measures.

If we use the similar interferometer circuit for spectrum measure-
ment as usual, $m_k$ can be obtained by controlled $k$-
circle permutation and experimental feasible antisymmetric
projections, which means that half of controlled-swap oper-
ations are saved compared with controlled 2$k$-circle permuta-
tion. Since $m_k$ are real, we do not have to measure the whole
interference pattern in order to obtain the visibility [14, 15].

Nevertheless, the implementation of interferometer circuit by
LOCC is still complicated. The experimental efforts can be reduced if no interferometer circuit is required. We demonstrate the benefit of our framework in the case of general two-qubit states by showing that only rank-1 local factorizable projective measurements are required, which then leads to another interesting feature that we do not have to manipulate a number of entangled state at a time, even the starting point of our scheme is also based on multiple copies of entangled state.

Consider a general two-qubit state $\rho$, its entanglement can be quantified by the Wootters’ concurrence as $C = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where $\lambda_2$s are the square roots of the eigenvalues of $\rho \bar{\rho}$ in the decreasing order, with $\bar{\rho} = (\sigma_y \otimes \sigma_y) \rho^*(\sigma_y \otimes \sigma_y)$. Before proceeding, we first introduce some notations as $|\varphi_i\rangle = \otimes_{j=1}^2 |S_j\rangle^{2j-1,2j}$. We conclude that $\rho \bar{\rho}$ can be quantified by the Wootters’ concurrence as $C = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where $\lambda_2$s are the square roots of the eigenvalues of $\rho \bar{\rho}$ in the decreasing order, with $\bar{\rho} = (\sigma_y \otimes \sigma_y) \rho^*(\sigma_y \otimes \sigma_y)$. Before proceeding, we first introduce some notations as $|\varphi_i\rangle = \otimes_{j=1}^2 |S_j\rangle^{2j-1,2j}$. 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formalism. Every pure state $|\psi\rangle$ has an MPS representation, and thus can be generated in a sequential way, i.e.

\[ V_{[2k]} \cdots V_{[1]} |\phi_L\rangle_C |0 \cdots 0\rangle_{1 \cdots 2k} = |\phi_R\rangle_C |\psi\rangle_{1 \cdots 2k}, \]

where $|\phi_L\rangle_C$ and $|\phi_R\rangle_C$ are the initial and final state of an auxiliary system, e.g. cavity mode or atoms. $V_{[i]}$ is a unitary interaction between qubit $i$ and $C$. By reversing the above procedure, we can obtain the rank-1 local projective observable $\langle \psi|\langle \phi_{2k}|\phi\rangle |\psi\rangle$ of Eq.(6) in a similar sequential way as in Fig.1. First, the auxiliary system is prepared in $|\phi_R\rangle$, entangled pairs are generated one by one, pass through and interact with $C$, then measured along the $\hat{z}$ basis. Only if the results of all steps are 0, we need to measure the auxiliary system

with $M_C = |\phi_L\rangle_C \langle \phi_L|$. Otherwise, if the result of any step is not 0, we restart the iteration and do not need to generate all the $2k$ copies. Our rough estimation shows that in comparison with quantum state tomography, for each observable, the involved qubits are a little more (5/4 and 4/3 vs. 1); however, the total number of entangled states need to be generated is even less (95/12 vs. 9).

**Remark 5:** All current schemes for directly measuring entanglement also raise an interesting problem: *How does entanglement play its role in reducing the measurement cost in extracting information from an unknown quantum state?*

**Conclusions.** Maximally entangled state retains its fundamental role in the entanglement theory, which provides an approach to investigate the connections between different entanglement quantifiers. With the notation of maximally entangled states, one can mathematically implemented nonphysical allowed transformations of quantum states. This enables us to design novel schemes for directly measuring various kinds of entanglement quantifiers. The benefit is explicitly demonstrated for general two-qubit states, in which only rank-1 local projective measurements are required. It is parametrically efficient without increasing the requirement for state generation over quantum state tomography.

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