Data Links Enhanced Relative Navigation for Robotic Formation Applications

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Abstract: With the rapid development and widespread application of robotic formation, relative navigation problem has attracted extensive attention. In this paper, the relative navigation problem for robotic formation applications is investigated, for which, we provide a relative navigation method based on multi-sensor fusion. First of all, a data links enhanced relative navigation scheme is proposed. Secondly, the underlying estimation problem behind the relative navigation scheme is derived. Then, a recursive relative navigation algorithm based on maximum a posterior estimation is provided for different multi-sensor combinations. Finally, simulation experiments are performed to show the effectiveness of the proposed relative navigation method.

Keywords: Robotic formation, Relative navigation, Data link, State estimation, Maximum a posterior estimation.

1. INTRODUCTION

In recent years, robotic formation application has been an active and growing research area benefiting from the evolving military and civilian demands, e.g. surveillance, reconnaissance, rescuing, docking, and in-orbit servicing, see Fosbury and Crassidis [2008], Horri and Palmer [2013], Oh et al. [2015], Jothi [2007]. As one of the fundamental research problems, formation control in multi-agent systems have received considerable attention from different disciplines and been widely investigated.

Formation control, which is one of the most actively studied topics within the realm of multi-agent systems, generally aims to steer multiple agents to achieve prescribed formation over a topology configuration network. Depending on the types of sensed variables, existing formation control approaches can be roughly classified into three categories, that is, position-based control, displacement-based control and distance-based control (see Oh et al. [2015], Han et al. [2018]). Anyhow, almost all existing strategies utilize relative position measurements as control inputs.

Nevertheless, in formation applications, there does not exist effective sensors that can directly provide a relative position measurement between two agents and GPS is still the only choice in reality. An alternative option for GPS-denied or GPS-jammed environments is to utilize research of sensor network localization problem which is addressed as a separate problem in the past.

Sensor network localization aims to determine the node position in a static or dynamic sensor network. Depending on the form of problem formulation, sensor network localization problem can be mainly classified into two categories, i.e. source localization problem (see Han et al. [2018], Jiang et al. [2017], Han et al. [2018]Han et al. [2018]Chai et al. [2013], Kexin et al. [2019]) and network localization problem(see Simonetto and Leus [2014], Lin et al. [2015], Oh and Ahn [2014]).

Source localization refers to the problem of estimating the precise location of a source or neighboring agent based on distance measurement, and for formation application, it postulates that each agent carries out specific motions during the entire process. Network localization aims to estimate the node position in a global frame with the given inter-sensor distance or angle measurement and the location of known anchors. Compared with source localization, no motion prior is needed and more suitable for formation application.

There have been significant efforts in developing algorithms and heuristics that can accurately and efficiently localize the nodes in a sensor network, see Langendoen and Reijers [2003], Mao et al. [2007]. Typical methods involve standard nonlinear filtering in Cattivelli and Sayed [2010], belief propagation techniques in Wymeersch et al. [2009] and convex relaxation optimization techniques in Simonetto and Leus [2014].

Our work differs from the literature in the sense that an sequential maximum likelihood estimation method is utilized to estimate the node position recursively and a unified framework is provided to fuse information of different sensors. In this paper, it is assumed that every robot is equipped with data links as both information exchanging media and distance measurement sensors and a few anchor nodes are assumed to be with known position.

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by setting up as static or equipping with global localization sensors.

The structure of this paper is organized as follows. In Section 2, a data links enhanced relative navigation scheme is proposed and the underlying relative navigation problem is derived. In Section 3, an estimation algorithm framework for relative navigation is provided. In Section 4, simulation experiments for the data links enhanced relative navigation method are performed to show the effectiveness of the proposed method. In Section 5, we summarize the conclusions and future work.

2. PROBLEM FORMULATION

Physical configuration and estimation methods are two key issues. In this section, we will propose a relative navigation scheme and describe the underlying estimation problem as follows.

2.1 Relative Navigation Scheme Design

There exist several types of navigation sensors for different missions or mission phases, but the main sensors for relative navigation are GPS, optical navigation sensors (ONS) and data links (DL), see Horri and Palmer [2013], Allende et al. [2015]. GPS is a type of absolute navigation sensors and ONS is a type of relative navigation sensors. As for data links, it can be multiplexed as distance sensors.

The three types of navigation sensors have different characteristics and are used in different scenarios. GPS may be the most common way for either absolute or relative navigation, but on account of its low accuracy, performs poorly in precise operation. ONS have high precision but not suitable for large scale formation. Data links are typically used to communicate, but also be used for collision detection and relative navigation. Summarize characteristics of the three types of sensors as shown in Table 1.

![Fig. 1. Sensing and communication topology of robots formation](image)

Table 1. Characteristics of navigation sensors

| Sensor | Noise (1σ) | Application Scene         |
|--------|------------|----------------------------|
| GPS    | 50cm ~ 3m  | Autonomous formation flying|
| ONS    | 0.1cm ~ 10cm | Precise 3D operations     |
| DL     | 5cm ~ 30cm | Collision avoidance       |

In Table 1, each type includes many specific sensors with similar principles, and so, the noise term only provides a rough range. Specific tasks require particular sensors, and no single sensor can meet the needs of all tasks. It is necessary to explore innovative combinations of different sensors for complex application scenarios.

As described in Ranger [1996], data link network has the measurement ability of formation topology, but cannot determine the transformation relative to global or local frame. It needs to rely on GPS or ONS to align with the reference frame. On account of that, the relative navigation scheme based on data links can be categorized into three types:

1. Set at least three robots static as base stations;
2. Equip GPS for at least three robots;
3. Equip ONS for at least three robots;

The three types of relative navigation schemes reduce dependence on expensive GPS or ONS and are especially important for large scale formation applications. Scheme (1) ~ (3) can all be modeled as network localization problem and we will give detailed description in the following.

2.2 Measurement Model Description

The sensing and communication topology of robots formation system is as shown in Fig. 1, where the blue boxes are common nodes and the red boxes are anchor nodes. Amongst, \(F^G_b\) and \(F^G_j\) (\(i, j \in 1, 2 \cdots N\)) are local frames of robot \(i\) and robot \(j\) respectively, and \(F_w\) is the global frame.

It is known that both absolute and relative navigation sensors provide position measurements, just in different reference frames. If there are no absolute navigation sensors, a local frame \(F^G_b\) can be chosen as the global reference frame and measurements of other relative navigation sensors can be represented in \(F^G_b\). In some sense, absolute and relative navigation sensors are equivalent.

On account of that, establish a unified measurement model for absolute and relative navigation sensors as below:

\[
\eta_i = p_i - p_r + \delta_{\eta_i} \tag{1}
\]

where \(\eta_i\) is the measurement of relative position, \(p_i\) is the position of the reference frame origin, \(p_r\) is the location of robot \(i\), and \(\delta_{\eta_i}\) is the random noise. The measurement model of data links can be expressed as:

\[
\rho_{ij} = ||p_i - p_j|| + \delta_{\rho_{ij}} \tag{2}
\]

where \(p_i\) and \(p_j\) are positions of robot \(i\) and \(j\), \(\rho_{ij}\) is the measurement of distance between robot \(i\) and \(j\), and \(\delta_{\rho_{ij}}\) is the random noise.

2.3 Relative Navigation Problem Description

The motion process of robotic formation is expressed as the state flow diagram Fig. 2(a) shows. The states of
robotic formation at moment $k$ are defined as a set of independent states $P_k := \{p_i\}_k$, the observations of robotic formation are defined as a set of independent measurements $(Y_k, \Gamma_k) := (\{y_i\}_k, \{p_{ij}\}_k)$ and the control inputs of robotic formation are defined as a set of independent inputs $V_k := \{v_i\}_k$. Each term in $\{s_i\}_k$ represent the state of robot $i$ at time $k$, and $i \in 1, 2, \cdots, N$ is the robot ID in robotic formation.

The motion process of robotic formation can be modeled as a first-order Markov process as shown in Fig. 2(b). To simplify the derivation process, we use $x_k, u_k, z_k$ to represent the state, measurements and control inputs respectively, as follows:

$$x_k := P_k$$
$$u_k := V_k$$
$$z_k := (Y_k, \Gamma_k).$$

The states, control inputs and observations of robotic formation from moment 1 to $T$ are defined as $X := \{x_k\}_{k=1:T}$, $U := \{u_k\}_{k=1:T}$ and $Z := \{z_k\}_{k=1:T}$ respectively. Since that the motion process obeys Markov property, the full probability formula can be expanded to:

$$p(X | U, Z) = \prod_{k=1}^{T} p(x_k | x_{k-1}, u_k, z_k).$$

(3)

An important insight from equation (3) is that the global estimation problem can be handled recursively because the constraints exist only between adjacent states. This means that an online estimator for relative navigation problem can be used.

3. STATE ESTIMATION FOR RELATIVE NAVIGATION SYSTEM

In this section, we derive a recursive state estimation framework based on optimization methods for relative navigation problems described in Fig. 2(b).

3.1 Recursive State Estimation

As shown in equation (3), the global state estimation problem degenerates into a series of local state estimation problem. Then, the conditional probability density function $p(x_k | x_{k-1}, u_k, z_k)$ at time $k$ is the only concern. Based on the maximum a posterior estimation criteria, the optimal estimation for $x_k$ is acquired by

$$\hat{x}_k^* = \arg \max_{x_k} p(x_k | x_{k-1}, u_k, z_k)$$

(4)

where $\hat{x}_k^*$ is the optimal estimation. Maximum a posterior estimation can be rewritten using Bayesian inference:

$$\hat{x}_k^* = \arg \max_{x_k} \frac{p(z_k | x_k, x_{k-1}, u_k)p(x_k | x_{k-1}, u_k)}{p(z_k | x_{k-1}, u_k)}$$

$$\simeq \arg \max_{x_k} p(z_k | x_k, x_{k-1}, u_k)p(x_k | x_{k-1}, u_k)$$

$$\simeq \arg \max_{x_k} p(z_k | x_k)p(x_k | x_{k-1}, u_k)$$

(5)

where $p(z_k | x_k)$ is a likelihood term, $p(x_k | x_{k-1}, u_k)$ is a prior term and $p(z_k | x_{k-1}, u_k)$ is assumed to be subject to uniform distribution.

Observation $z_k$ is typically composed of a set of independent observations measured by different sensors, such as $z_k := \{z_{si}^k | s = 1, 2, \cdots, M\}$. Maximum a posterior estimation of the multi-sensor fusion can then be derived as follows:

$$\hat{x}_k^* = \arg \max_{x_k} p(x_k | x_{k-1}, u_k) \prod_s p(z_{si}^k | x_k)$$

(6)

Assuming that noises of the likelihood term and the prior term obey Gaussian distribution, the optimization problem in equation (6) is equivalent to:

$$\hat{x}_k^* \simeq \arg \min_{x_k} -\log p(x_k | x_{k-1}, u_k) \prod_s p(z_{si}^k | x_k)$$

$$\simeq \arg \min_{x_k} ||r_u||^2_{\Sigma_u} + \sum_s ||r_{z_s}^k||^2_{\Sigma_{z_s}}$$

(7)

where $\Sigma_u$ and $\Sigma_{z_s}$ are the covariances of observations and control inputs, $r_{z_s}^k$ and $r_u$ are the residuals of the likelihood term and the prior term respectively. This method is especially useful for multi-sensor fusion system, which can add any independent observations into $\sum_s ||r_{z_s}^k||^2_{\Sigma_{z_s}}$ as in equation (7).

3.2 Covariance Update Mechanism

To make the recursive estimation possible, proper covariance estimation methods need to be designed. The intuitive idea is to find a measure of the quadratic problem inspired by marginalization tricks, see Barfoot [2017]. The cost function of relative navigation estimation problem is defined as

$$L = ||r_u||^2_{\Sigma_u} + \sum_s ||r_{z_s}^k||^2_{\Sigma_{z_s}}$$
and the second-order Taylor expansion of $L$ at the optimal value $\hat{x}_k$ is
\[
L \approx L(\hat{x}_k) + J \cdot (x_k - \hat{x}_k) + \frac{1}{2} (x_k - \hat{x}_k)^T H (x_k - \hat{x}_k)
\]
where $J$ and $H$ are the Jacobian and Hessian matrices of cost function $L$ respectively. If the state variable $x_k$ obeys a Gaussian distribution, the covariance of $x_k$ can be approximated by
\[
E[(x_k - x_k^* )^2] \approx H^{-1}
\]
when there is an inverse matrix $H^{-1}$, see Segal and Weinstein [1989] and Zhu et al. [2018]. Intuitively, the smaller the curvature, the greater the uncertainty.

In practical applications, cost function $L$ is only known implicitly, or the explicit second derivative is difficult to calculate, so Hessian matrix must be numerically computed, e.g. by finite difference method. The numerical calculation of $H$ are given as follows:
\[
H^{(i)}(\theta) = \left[ \frac{\partial}{\partial \theta_i} \frac{\partial L(\theta)}{\partial \theta_j} \right]_{\theta = \theta^*} \\
\approx \frac{1}{\Delta \theta_j} \left[ \frac{\partial L(\theta)}{\partial \theta_i} \theta = \theta^* + \Delta \theta_j/2 - \frac{\partial L(\theta)}{\partial \theta_i} \theta = \theta^* - \Delta \theta_j/2 \right]
\]
\[
\approx \frac{1}{\Delta \theta_j} \left[ \frac{L(\theta^* + \Delta \theta_j/2) - L(\theta^* + \Delta \theta_j/2)}{\Delta \theta_i} \right]
\]
where $\theta$ is an alternative expression of $x_k$; $\theta_i$ and $\theta_j$ represent the $i$-th and $j$-th element of $x_k$ respectively.

3.3 Relative Navigation Algorithm

In Section 3.1, a unified estimation framework is proposed to deal with the relative navigation problem of data link enhanced relative navigation systems. Although there may be multiple types of absolute or relative navigation sensors, all sensors may be modeled by equation (1) regardless of physical characteristics. In this section, we design the relative navigation algorithm based on equation (1) and (2), and the algorithm flow is as shown in Algorithm 1.

In Algorithm 1, we use the Gaussian-Newton method to solve the weighted least square problem efficiently, see Madsen et al. [2004]. For the nonlinear optimization problem, an initialization step is applied based on the measurements of absolute or relative navigation sensors, and then the optimal solution can be found.

4. SIMULATION EXPERIMENTS

4.1 Simulation Experiment Setup

As described in Section 2.1, here we mainly study two types of data link enhanced relative navigation systems in this paper, as follows:

(1) Scheme 1: Relative navigation system based on fusion of ONS and data links;
(2) Scheme 2: Relative navigation system based on fusion of GPS and data links;

In the simulation experiments, the number of robots in robotic formation is set to 5, and the accuracy of different sensors is set according to the previous analysis, as shown in Table 2. The robotic formation is located in the $x - y$ plane and moves along a line.

Then we conduct simulation experiments based on Algorithm 1 for Scheme 1 and Scheme 2 respectively, and the results are illustrated below.
Robots Spatial Distribution

Fig. 3. The spatial distribution estimation for robotic formation at moment \( k \) (Fig (a) and (b) show estimation results of the robotic formation space distribution based on accurate ONS and inaccurate GPS respectively.

4.2 Relative Navigation Estimation Experiment

The estimation results for the two schemes are as shown in Fig. 3 ~ Fig. 5, where Fig. 3 represent the spatial distribution of robotic formation in \( x-y \) plane at a certain moment, Fig. 4 represent the offset error of estimation in the motion process, and Fig. 5 represent the statistic of random error for the robotic formation.

The topology of the robotic formation can be well estimated (black nodes) whether based on accurate ONS or inaccurate GPS, and it is far better than pure measurements of absolute navigation sensors (blue nodes in Fig. 3(b)) and almost the same as pure measurements of relative navigation sensors (blue nodes in Fig. 3(a)). Fig. 5 indicates that the topology of robotic formation is well estimated for the total process regardless of the motion of robotic formation.

Relative navigation estimates based on imprecise GPS have larger offset errors than relative navigation estimates based on precise ONS. Compensate the offset error manually, and then the transformed formation configuration (green nodes) is almost identical to the true value (red nodes), as shown in Fig. 3(b). The offset error can also be seen in Fig. 4. The offset error \( (\Delta x, \Delta y, \Delta \theta) \) in Scheme 1 is much smaller than that in Scheme 2 regardless of the motion of robotic formation, meaning that the estimates based on data links and GPS are not well aligned with the reference frame.

Analyzing the experiment results, the distance measurements of data link can keep relatively accurate estimation of the robotic formation topology, and ONS or GPS are mainly used to align the rigid topology with the reference frame. The accuracy of ONS or GPS determines the offset error, which is the main determinant of whether it can be applied in practice.

Besides, from the perspective of practicability, the combination of data links and ONS makes large scale formation applications possible because only a few relative observations are required. It has great advantages compared to solutions with only ONS, thanks to its low cost, high reliability, and high precision.

| Sensors | Noise (1σ) |
|---------|-----------|
| Data links | 10 cm |
| ONS | 5 cm |
| GPS | 50 cm |

Table 2. Experiment parameters setup
5. CONCLUSIONS

In this paper, we propose a data links enhanced relative navigation scheme that reduces the dependence on relative or absolute navigation sensors by reusing the navigation functions of data links. Besides, an estimation algorithm for relative navigation problems is derived, which is especially useful for multi-sensor fusion systems. According to experiment results, the offset error is the main source of error and mainly determined by the noise level of absolute or relative navigation sensors. In the future, we will focus on the physical characteristics of sensors and further study the topology measurement problem of data links.

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