Ordinary and Dilatonic Domain Walls: Solutions and Induced Space-Times

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ABSTRACT

Recent developments in unifying treatment of domain wall configurations and their global space-time structure is presented. Domain walls between vacua of non-equal cosmological constant fall in three classes depending on the value of their energy density $\sigma$: (i) extreme walls with $\sigma = \sigma_{\text{ext}}$ are planar, static walls corresponding to the supersymmetric configurations, (ii) non-extreme walls with $\sigma > \sigma_{\text{ext}}$ are expanding bubbles with two insides, (iii) ultra-extreme walls with $\sigma < \sigma_{\text{ext}}$ are bubbles of false vacuum decay. As a prototype exhibiting all three types of configurations vacuum walls between Minkowski and anti-deSitter vacua are discussed. Space-times associated with these walls exhibit non-trivial causal structure closely related to the one of the corresponding extreme and non-extreme charged black holes, however, without singularities. Recently discovered extreme dilatonic walls, pertinent to string theory, are also addressed. They are static, planar domain walls with metric in the string frame being flat everywhere. Intriguing similarities between the global space-time of dilatonic walls and that of charged dilatonic black holes are pointed out.

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1. Introduction

Domain walls\textsuperscript{1,2,3} are the most extended topological defects arising in theories with isolated, in general non-equal\textsuperscript{4,5,6} minima of the matter potential. Domain walls can form as topological defects in the early Universe in theories with isolated minima of the matter potential.\textsuperscript{7} They also form as boundaries of a true vacuum bubble created by a quantum tunnelling process\textsuperscript{8} of the false vacuum decay, as well as a boundary of the universes born from a quantum tunnelling process from nothing (quantum cosmology).\textsuperscript{9,10,11}

Recently, substantial progress\textsuperscript{12,13} has been made in understanding the space-time structure of eternal vacuum domain wall configurations. The work stemmed from an earlier discovery of supergravity walls\textsuperscript{6,14,15,16} and a subsequent study of their global space-time structure.\textsuperscript{17,18} In particular, it was recognized\textsuperscript{12,13} that the extended nature of these defects imposes strong constraints on the topology of the wall and the nature of its space-time. It turns out that the walls provide a fertile ground to study globally non-trivial space-times. These space-times were found\textsuperscript{17,12} to be closely related to the ones of certain black holes, however, without singularities.

“Ordinary” domain walls between vacua of non-equal cosmological constant fall into three classes:\textsuperscript{12} (i) extreme (supersymmetric) static, planar domain walls, (ii) non-extreme domain walls (expanding bubbles with an inertial observer inside the bubble for each side of the wall)\textsuperscript{12,2} and (iii) ultra-extreme walls (expanding bubbles\textsuperscript{4,5} of false vacuum decay\textsuperscript{19}). The energy density $\sigma_{\text{ultra}}$ of the non-[or ultra-]extreme walls is bound from below [or above] by the one $\sigma_{\text{ext}}$ of the extreme ones. Walls are thus an example\textsuperscript{12} of configurations for which supersymmetry provides a lower bound for the energy of stable wall configurations.\textsuperscript{20} A prototype which exhibits all three types of walls are walls between anti-deSitter and Minkowski vacuum. The space-times induced by such walls are non-singular with non-trivial global structure and horizons closely related to the ones of certain black holes: on the anti-deSitter [or Minkowski] side of the wall the induced non-singular space-time is closely related to the ones of the Reissner-Nordström [or Schwarzschild] black holes.\textsuperscript{12,13} In the domain wall case the role of the mass ($M$)
and the charge \((Q)\) of the black hole is played by the energy density \((\sigma)\) of the wall and the cosmological constant \((\Lambda)\) outside the wall, respectively.

Recently discovered dilatonic domain walls,\(^{21}\) on the other hand, are solutions specific to superstring theory. In such a domain wall background along with the matter fields and metric also the dilaton field\(^{22}\) changes its value. Dilatonic domain walls are of particular interest because they correspond to configurations which interpolate between isolated superstring vacua and may thus shed light on the nature and connectedness of superstring vacua.

Dilatonic domain walls are a generalization of the ordinary domain walls in an analogous way as dilatonic charged black holes\(^{23,24}\) are a generalization of “ordinary” black holes. The intriguing similarity between the space-time of the walls and the one of the corresponding black-holes reappears in the case of dilatonic walls as well.

The rest of the paper is organized in the following way. In Chapter 2 the space-time of ordinary domain walls is discussed. In Chapter 3 dilatonic walls are addressed. Brief conclusions are given in Chapter 4.

2. Space-time of Ordinary Domain Walls

A convenient way to describe the gravitational field is in the rest frame of the wall, \textit{i.e.}, by using comoving coordinates of observers sitting on the wall. Hence, the wall is placed at a fixed \(z\)-coordinate, and the metric is static in the \((t, z)\) directions transverse to the wall. Assuming maximal symmetry associated with the space-time internal to the wall, the metric is taken to be homogeneous and isotropic and geodesically complete in the \((\rho, \phi)\) surfaces parallel to the wall. Since the extrinsic curvature is independent of the wall’s proper time, one can show\(^ {13}\) that the metric is:

\[
\begin{align*}
\text{ds}^2 & = A(z)[dt^2 - dz^2 - \beta^{-2} \cosh^2(\beta t) d\Omega^2_2] ,
\end{align*}
\]

with \(A(z) > 0\) and \(d\Omega^2_2 \equiv [1 - (\beta \varrho)^2]^{-1} d(\beta \varrho)^2 + (\beta \varrho)^2 d\phi^2\). In the extreme limit, \(\beta \to 0\), the \((\rho, \phi)\) surface becomes a plane with \(\varrho\) and \(\phi\) planar polar coordinates.
With $\beta \neq 0$, the $(\rho, \phi)$ hyperspace is the surface of a three-dimensional sphere, that is, its topology is $S^2$. In this case the coordinate $\rho = \beta^{-1} \sin \theta$ is compact. The scalar curvature of the spatial $S^2$ is $2\beta^2 A(z)^{-1} [\cosh(\beta t)]^{-2}$. The constant $z$ section with $\beta \neq 0$ is $(2+1)$-dimensional de Sitter space-time (dS$_3$), which is geodesically complete.$^{13}$ Note, that the extended nature of the wall imposes strong constraints on the topology of the wall; the wall can be either a planar, static configuration (Eq.(2.1) with $\beta = 0$), or a time dependent spherical bubble (Eq.(2.1) with $\beta \neq 0$).

(i) Extreme Walls ($\beta = 0$) induce a static, conformally flat space-time. Such walls turn out to correspond to supersymmetric configurations between isolated supersymmetric minima of the matter potential in $4d$, $N = 1$ supergravity theory.$^{6,14}$ Supersymmetric minima have either zero (Minkowski space-times ($M_4$)) or negative (anti-deSitter space-times ($AdS_4$)) cosmological constant $\Lambda$, which is related to the value of the superpotential ($W$) and Kähler potential ($K$) at the minimum in the following way: $\Lambda = -3\kappa^2 e^{\kappa K} |W|^2$. Here, $\kappa \equiv 8\pi G$.

The existing static domain walls between such minima have been classified:$^{14}$ there are two types of $AdS_4$--$AdS_4$ walls (Type II and Type III) and an $AdS_4$--$M_4$ wall (Type I). There are no static (supersymmetric) domain walls between two $M_4$--$M_4$ vacua. In the following I will discuss only Type I walls and their generalizations. This is a prototype of the walls where on one side of the wall the space-time is asymptotically $M_4$; such walls may thus have implications for the observable world. Generalizations to other examples, including the walls between deSitter vacua, are discussed in Refs.12,13.

Extreme, thin, $M_4$--$AdS_4$ walls, centered at $z = 0$, have the energy density, $\sigma_{\text{ext}}$, and the conformal factor $A(z)$ in (2.1) of the following form:$^{6,14}$

$$\sigma_{\text{ext}} = 2\kappa^{-1} \alpha,$$

$$A(z) = (\alpha z - 1)^{-2}, \quad z < 0; \quad A(z) = 1, \quad z > 0.$$  

(2.2)
Here, $\alpha \equiv \kappa e^{\frac{1}{2}K}|W| = (-\Lambda/3)^{1/2}$. The *horo-spherical* coordinates on the AdS$_4$ side are discussed in Refs. 14,18. The field equations for the matter and metric are coupled first order rather than second order differential equations, thus allowing for a straightforward solution for any thickness of the wall.$^6,14$

The coordinates of the metric (2.1) with $\beta = 0$ and $A(z)$ in (2.2) are not geodesically complete; geodesic extensions have been given with emphasis on the Type I walls in Ref. 17 and Type II walls in Ref. 18. Namely, on the AdS$_4$ side the time-like geodesics reach $(t = \infty, z = -\infty)$ in a finite proper time $\tau = \alpha^{-1} \arcsin(1/\epsilon)$. Here, $\epsilon$ is the energy per unit mass of the test particle. Therefore, $(t, z)$ coordinates are not geodesically complete on the AdS$_4$ side; there is a Cauchy horizon at $(t = \infty, z = -\infty,)$. The most symmetric geodesic extension (see Figure 1) comprises of a system of an infinite lattice of semi-infinite $M_4$ space-times separated by an AdS$_4$ core. It turns out that the $(t, r)$ line elements near the Cauchy horizon of the Reissner-Nordström (RN) black hole and $(t, z)$ line element on the AdS$_4$ side of the wall are identical.$^{17}$ Note also a similarity between the global space-time structure (see Figure 1) of the wall and that of the extreme RN black hole.$^{25}$ However, the time-like singularities of the RN space-time are replaced by the domain walls.

(ii) **Non-Extreme and Ultra-Extreme Walls** induce space-time (2.1) with $\beta \neq 0$. In the case of thin walls one employs Israel’s formalism of singular hypersurfaces$^{26}$ which determines the matching conditions across the wall region. Einstein’s field equations and Israel’s matching conditions as applied to this case$^{13}$ yield two types of the solutions (depending on the sign of parameter $\beta$) with energy density...
and conformal factor of the following type:

\[
\sigma_{\text{ultra}}^{\text{non}} = 2\kappa^{-1}[(\alpha^2 + \beta^2)^{1/2} + \beta],
\]

\[
A(z) = \beta^2 \alpha^{-2} [\sinh(\beta z - \beta z')]^{-2} \quad z < 0; \quad A(z) = e^{-2\beta z} \quad z > 0.
\]

(2.3)

where \(e^{2\beta z'} \equiv [\alpha^2 + 2\beta^2 + 2\beta(\beta^2 + \alpha^2)^{1/2}]/\alpha^2 \equiv \delta\) is determined by \(A(0) \equiv 1\).

The solution with \(\beta > 0\) of Eq.(2.3) represents a non-extreme wall. It can be shown that in the non-extreme wall region the potential barrier associated with the scalar field is larger than that for the corresponding extreme domain wall, which implies that \(\sigma_{\text{non}} > \sigma_{\text{ext}}\). E.g., within \(N = 1\) supergravity theory, such a wall can be realized as a wall interpolating between a supersymmetric \(M_4\) vacuum and an \(\text{AdS}_4\) vacuum with supersymmetry spontaneously broken.

In the rest frame of the wall the non-extreme walls exhibit cosmological horizons\(^{12,13}\) on both the \(\text{AdS}_4\) and \(M_4\) sides. Namely, a particle with energy per unit mass \(\epsilon \geq 1\), freely falling at constant \(\theta\) and \(\phi\) in the \(z \rightarrow \mp \infty\)-direction has a finite proper time\(^{12}\) \(\tau = \alpha^{-1} \{\arcsin\{[1 + (\epsilon \alpha/\beta)^2]^{-1/2}(\delta + 1)/(\delta - 1)\} - \arcsin[1 + (\epsilon \alpha/\beta)^2]^{-1/2}\}\) and \(\tau = \beta^{-1}[\epsilon - (\epsilon^2 - 1)^{1/2}]\), respectively. As \(\beta \rightarrow 0\), the cosmological horizon on the \(\text{AdS}_4\) side becomes a Cauchy horizon (as in the extreme wall space-time) with \(\tau = \alpha^{-1} \arcsin(1/\epsilon)\), while the \(M_4\) side becomes geodesically complete. It turns out\(^{12}\) that \((t, z)\) line element on the \(\text{AdS}_4\) side of the wall is identical to \((t, r)\) line element near the horizon of the non-extreme RN black hole.

In order to investigate geodesically complete space-times for the non-extreme walls, the metric (2.3) with is transformed\(^{12}\) to the inertial spherical \(M_4\) and \(\text{AdS}_4\) coordinates on the respective sides. Introducing \(t = \beta^{-1} e^{-\beta z} \sinh \beta t\) and \(r = \beta^{-1} e^{-\beta z} \cosh \beta t\) brings the line element on the \(M_4\) side to the spherically symmetric form \(ds^2 = dt^2 - dr^2 - r^2 d\Omega_2^2\). The bubble at \(z = 0^+\) lives on the hyperbolic trajectory \(r^2 - t^2 = \beta^{-2}\) (see Figure 2). On the \(\text{AdS}_4\) side, one maps to the spherically symmetric Einstein cylinder coordinates\(^{25}\). This transformation is done in three steps: (i) \(\ln \Xi = \beta (z - z')\), with
0 ≤ Ξ ≤ 1. (ii) \( T = \Xi \sinh(\beta t) \) and \( R = \Xi \cosh(\beta t) \). (iii) \( T ± R = \tan[(t_c ± \psi)/2] \). The line element on the AdS\(_4\) side \((z < 0)\) becomes \( ds^2 = (\alpha \cos \psi)^{-2}(dt_c^2 - d\psi^2 - \sin^2 \psi d\Omega_2^2) \), where \(-\pi ≤ t_c ± \psi ≤ \pi\) and \(0 ≤ \psi ≤ \pi/2\). The bubble at \(z = 0^-\) again lives on a hyperbolic trajectory \(R^2 - T^2 = -\delta^{-1}\) (see Figure 2).

The \((t, z)\) chart is an interpolating map which covers the space-time on both sides of the non-extreme wall region. To complete the space-time (see Figure 2), one extends on one side onto pure M\(_4\). On the AdS\(_4\) side, the most symmetric periodic extension yields a lattice structure of walls. Notice that now on the AdS\(_4\) [M\(_4\)] side, the Penrose diagram bears similarities to the one of a non-extreme RN [Schwarzschild] black hole, however, without singularities. The wall region of the AdS\(_4\) diagram is linked by a wall region of the corresponding M\(_4\) diagram. At \(t = 0\) the bubble has a radius \(\beta^{-1}\) which then increases as as \(\cosh(\beta t)/t\). Since the radius of the bubble \(\beta^{-1}A(z)^{1/2} \cosh(\beta t)\) decreases as one moves spatially away from the bubble in both \(z\) directions, observers on both sides are inside a bubble.

Figure 2 Penrose-Carter diagram for the most symmetric extension of the non-extreme M\(_4\)-AdS\(_4\) wall (the bubble with two insides). The diagrams on the right hand side are those on the M\(_4\) side of the bubble with compactified coordinates \(u', v' = 2\tan^{-1}[\beta(t ± R)]\). The solid curved lines are the world lines of the wall bubble (its anti-podal points). The dotted lines represent cosmological horizons in the wall’s rest frame coordinates \((z, t)\). The left diagram is the one of the AdS\(_4\) side. It is a part of pure AdS\(_4\) cylinder\(^{27}\) as seen in the Einstein cylinder coordinates \((t_c, \psi)\). The vertical boundaries correspond to \(\psi = \pi/2\) and the time direction \(t_c\) is upward. The solid curved lines are again the world lines of the wall (its anti-podal points), sweeping one half of the fundamental domain of the pure AdS\(_4\). The dotted lines represent cosmological horizons in the wall’s rest frame coordinates \((t, z)\). The two types of the diagrams are identified across the wall region.
The solution of Eq. ((2.3)) with $\beta < 0$ describes an ultra-extreme wall. For these walls the potential barrier associated with the scalar field is smaller than that of the extreme walls which means $\sigma_{ultra} < \sigma_{ext}$ and the metric blows up on the $M_4$ side. Ultra-extreme walls exhibit the same causal structure on the $AdS_4$ side as the non-extreme wall. However, the $M_4$ side is geodesically complete in the $(t, z)$-coordinates. The $M_4$ side is the complement of the $M_4$ side.

The Minkowski side is on the outside of the ultra-extreme bubble because the radius $|\beta|^{-1} A(z)^{1/2} \cosh(\beta t)$ increases with $z$ on the $z > 0$ side. On the $AdS_4$ side; however, the radius decreases away from the wall, and thus $AdS_4$ is on the inside just as for the non-extreme solution. Since $\sigma_{ultra} < 2\kappa^{-1}\alpha$, i.e. below the Coleman-De Luccia bound, the ultra-extreme solution for $t \geq 0$ describes the classical evolution of a bubble of true $AdS_4$ vacuum created by the quantum tunnelling process of false vacuum decay. At $t = 0$ the bubble is formed with radius $|\beta|^{-1}$, expands as $\cosh(\beta t)$, and inevitably hits all time-like observers on the $M_4$ side.
3. Dilatonic Domain Walls

Dilatonic walls\(^{21}\) are pertinent to the study of 4\(d\) superstring vacua with the dilaton always arising as an inherent part of the supergravity fields. Thus, in the domain wall background (between isolated minima of the matter potential) not only the metric, but also the dilaton field may change the value.

Potentially phenomenologically viable superstring vacua are described by an effective 4\(d\) \(N = 1\) supergravity theory. The scalar part of the effective Lagrangian involves the metric \(g_{E\mu\nu}\), the dilaton \(S \equiv e^{-2\phi} + ia\) (written in this form as a scalar part of the chiral superfield), matter matter fields and gauge fields. I do not include gauge fields; however, since the dilaton does couple to gauge fields the study of charged dilatonic walls is also interesting. Matter fields, scalar components of a chiral superfield interpolate between isolated minima of the matter potential.

In the Einstein frame the effective Lagrangian of the theory can be written in terms of the superpotential and Kähler potential. Superpotential \(W_0(T)\) is a holomorphic function of the matter fields \(T\), only, \(i.e.,\) to all orders in string loops it does not depend on the dilaton.\(^{29}\) In the Kähler potential \(K\) the dilaton couples in a specific way:

\[
K = -\kappa^{-1} \log(S + S^*) + K_0(T, T^*).
\]

The imaginary part (axion) of the dilaton field can be put to zero \((a = 0)\); this turns out to be the solution of field equations for the dilatonic domain walls anyway.

A natural frame to which strings couple is the string frame, \(i.e.,\) the frame of the sigma model expansion of the string effective action. In this case the metric in the string frame, \((g_s)_{\mu\nu}\), and the one in the Einstein frame, \((g_E)_{\mu\nu}\), are related as:

\[
(g_s)_{\mu\nu} = e^{2\phi}(g_E)_{\mu\nu}.
\]

The scalar part of the Lagrangian is of then of the form:

\[
L_s = \sqrt{-g_s}e^{-2\phi}\left[\frac{1}{2\kappa}R_s - 2\kappa^{-1}g_s^{\mu\nu}\partial_\mu \phi \partial_\nu \phi + T_0 - \frac{V_0}{2}\right]
\]  

(3.1)

where \(T_0V_0\) correspond, respectively, to the kinetic energy and the potential of the matter fields \(T\), only. Static planar domain walls have been found between
isolated supersymmetric minima of the matter potential $V_0$, whose value at the supersymmetric minimum is related to $W_0$ and $K_0$ as: $V_0 \equiv -2\kappa e^{\kappa K_0}|W_0|^2 \leq 0$. In the Einstein frame the metric is conformally flat (see the metric Ansatz (2.1) with $\beta = 0$) with the conformal factor $A_E(z)$. The scalar field $T(z)$ and the dilaton $\phi(z)$ also depend on $z$, only.

The minimal energy (supersymmetric) solution satisfies three first order coupled differential equations. Thus, straightforward solutions can be found for any thickness of the wall (for explicit examples see Ref.21). Three types of the solutions are classified according to the value of the potential $V_0 \leq 0$ on either side of the wall. Again, there are no static walls where on both side of the wall $V_0 = 0$. In particular, I will describe the walls where on one side of the wall $V_0 = 0$ and on the other side of the wall $V_0 < 0$, i.e., the $AdS' - M'$ walls. $M'$ refers to the Minkowski space with $V_0 = 0$ and $AdS'$ refers to a type of anti-deSitter space-time with $V_0 < 0$.

In this case the thin wall solution, located at $z = 0$ has the explicit form:

$$
\sigma_{\text{dil}} = \sqrt{2}\kappa^{-1}\alpha,
A_E(z) = e^{-\sqrt{2}\alpha z}, \quad z < 0; \quad A_E(z) = 1, \quad z > 0.
$$

(3.2)

where $\alpha \equiv \kappa e^{\kappa K_0/2}|W_0| = (-\kappa \tilde{V}_0/2)^{1/2}$. The solution was obtained by normalizing the conformal factor $A_E(0) = 1$ and choosing the boundary condition $e^{2\phi(0)} = 1$. A more general boundary condition $e^{2\phi(0)} = e^{2\phi_0}$ allows for a family of one parameter solutions.21

The energy density of ordinary supersymmetric domain walls is of a similar form (see Eq.(2.2)): $\sigma_{\text{ext}} = 2\kappa^{-1}\alpha$ where $\alpha \equiv \kappa e^{\kappa K_0/3}|W_0| = (-\kappa V_0/3)^{1/2}$ is defined in terms of $W_0$ and $K_0$ in the same way as above. An additional factor $1/\sqrt{2}$ in the case of dilatonic walls is associated with the dilaton contribution to the quantity $e^{\kappa K_0/3}|W_0| = 1/\sqrt{2} \times e^{\kappa K_0/3}|W_0|$. Namely, the boundary condition $e^{2\phi(z_0)} = 1$ ensures that the effective cosmological constant on each side of the wall is by a factor of $1/\sqrt{2}$ less negative, thus decreasing the energy density of the wall by a factor of $1/\sqrt{2}$. There is a parallel relation24 between the mass $M$ and the charge $Q$ for extreme
RN black holes \((M = Q)\) and extreme charged dilatonic black holes \((M = Q/\sqrt{2})\). In the domain wall case the role of the charge is is played by the parameters \(\alpha\) associated with the value of the matter potential at each minimum.

It turns out that the solution for the conformal factor \(A_E(z)\) and the dilaton field imply:

\[
A_s(z) \equiv A_E(z)e^{2\phi(z)} = 1 \quad (3.3)
\]

_everywhere_ in the domain wall background.

Therefore, the metric factor \(A_s(z)\) in the string frame is flat everywhere. Although there is a nontrivial matter potential, the dilaton field adjusts itself in the domain wall background in such a way as to leave the string metric flat; strings do not “feel” the wall.

The Penrose diagram for such walls in the \((t, z)\) plane is given on Figure 4. The \(M'\) side corresponds to Minkowski space-time. On the \(AdS'\) side (see Eq.(3.2) ) both the dilaton field and the metric curvature blow up as \(z \to -\infty\). However, this singularity is an infinite geodesic distance away. Note, a formal similarity with the Penrose diagram \(^{23,24}\) for the \((r, t)\) plane of the extreme charged dilatonic black hole.

Generalizations of the solutions to the case with a nonpertubatively induced dilaton potential \(^{32}\) should also be addressed. The case with supersymmetry broken spontaneously in the matter part of the potential \((V_0)\) should also be addressed. This bears similarities to the case of non-extreme of charged dilatonic black holes.
with \( M \neq Q/\sqrt{2} \). In these cases the wall need not be static anymore \(^{21}\) and the
global space-time structure of such walls is of special interest.

4. Conclusions

Progress in unifying treatment of eternal vacuum domain wall solutions and
their global space-time structure was presented. As a prototype, the space-time
of vacuum walls between Minkowski and anti-deSitter space-times was addressed.
While extreme (supersymmetric) walls are planar and static configurations, the
non-extreme walls correspond to a bubble with two insides and have energy density
bounded from below by the one of the extreme wall. Since the energy density of the
extreme domain wall is equal to the Coleman-De Luccia bound \(^{19}\), supersymmetry
provides a lower bound \(^{20}\) for a (non-extreme) domain wall separating vacua which
are stable against quantum tunnelling. On the other hand, the ultra-extreme wall,
which has energy density lower than the one of the extreme wall, corresponds to
the classical evolution of a bubble of true AdS\(_4\) created by the decay of the false
M\(_4\) vacuum.

In addition, dilatonic walls, specific to isolated 4d superstring vacua, were
discussed. Extreme (supersymmetric) dilatonic domain walls correspond to static
configurations between isolated supersymmetric minima of the matter potential.
 Everywhere in the domain wall background the dilaton field adjusts itself in a way
as to leave metric in the string frame flat. There are intriguing similarities between
extreme dilatonic walls and extreme charged dilatonic black holes.

Presented progress in the study of eternal domain wall configurations provides
a step towards a theoretical foundation for addressing cosmological implications of
domain walls, in particular, those arising in supergravity and superstring theories
as supersymmetric configurations or as configurations between isolated minima
with supersymmetry spontaneously broken.

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26. W. Israel, Nuovo Cimento \textbf{44B}, 1 (1966).

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31. A superpotential for the dilaton can, however, be induced non-perturbatively. Also, the Kähler potential receives one-loop corrections due to mixed Yang-Mills-σ model anomalies. See, G. Cardoso and B. Ovrut, Nucl. Phys. B369, 351 (1992); J.P. Derendinger, S Ferrara, C Kounnas and F. Zwirner, Nucl. Phys. B372, 145 (1992). The above effects should eventually be included in the full treatment of the theory.

32. Analogous solutions for dilatonic black holes with a massive dilaton were studied by R. Gregory and J. Harvey, Enrico Fermi Preprint EFI-92-49 and and J. Horn and G. Horowitz, Santa Barbara Preprint UCSBTH-92-17, while 2d black holes with a massive dilaton were studied by M. McGuigan, C.Nappi, and S. Yost, Nucl. Phys. B375, 421 (1992).
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