Chaos Analysis of the Output Game Between Multi-role Enterprises in Supply Chain

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Abstract. Based on an analysis on a variety of game models in the supply chain, this paper proposes a multi-enterprise output game model under the circumstances of information asymmetry. This paper compares two types of models from the aspects of function selection, hypothesis parameters and modeling basis, and analyzes the influence of producer's output adjustment speed parameters on the whole supply chain.

1. Introduction

With the advent of economic globalization and knowledge economy, supply chain management is widely used in manufacturing management. It is based on the demand of the customers and the market that the manufacturers develop products, purchase raw materials which are then processed into finished goods and sold to customers. With the further division of labor, the enterprises in the supply chain are more and more specialized in certain subsections along the product life cycle. There is a constant and complicated game among enterprises for the allocation of productivity and profit.

The enterprises along the supply chain are interdependent and mutually restrained. When making decisions, these enterprises are not fully rational. They need to take into consideration multiple factors such as their cost, profit, the change in market demand, decisions of their competitors and partners. Therefore, conventional decision making modeling cannot meet the challenge raised by a complex supply chain. In the next section, we construct a discrete dynamical model of three-level supply chain under the circumstances of information asymmetry and analyze its dynamic behavior in light of nonlinear dynamics.

2. Nomenclature and Model Construction

The following is a list of notations that will be used throughout the paper.

- $q_{i,t}$ is the production decision making of enterprise $i$ in period $t$.
- $P_{i,t}$ is the nonlinear inverse demand function for enterprise $i$ in period $t$.
- $C_{i,t}$ is the cost function for enterprise $i$ in period $t$.
- $\pi_{i,t}$ is the profit of enterprise $i$ in period $t$.

$$P_{i,t} = \alpha_i + \beta_i q_{i,t} - \gamma_i q_{i,t}^2$$

$$C_{i,t} = a_i + b_i q_{i,t} + c_i q_{i,t}^2$$

$$\pi_{i,t} = P_{i,t} q_{i,t} - C_{i,t}$$
Assume the retailers make adjustment on their ordering strategies in response to the changes in market demand, the distributors to the changes in the retailers’ orders, and the manufacturers to the changes in the distributors orders. Then a nonlinear dynamic model is constructed as follows:

\[
\begin{align*}
\dot{x} &= x + k_1 y [ -3 \gamma_1 x^2 + 2 (\beta_1 - c_1) x + \alpha_1 - b_1 ] \\
\dot{y} &= y + k_2 z [ -3 \gamma_2 y^2 + 2 (\beta_2 - c_2) y + \alpha_2 - b_2 ] \\
\dot{z} &= z + k_3 z [ -3 \gamma_3 z^2 + 2 (\beta_3 - c_3) z + \alpha_3 - b_3 ]
\end{align*}
\]  

(1)

3. Construction and Complex Dynamics Analysis of Compound Comparison Yield Game Model

The following is a composite comparative output game model of a three-level supply chain. In the model, the producers are not only considering the order quantity of the distributor when making the production decision, but also considering the order quantity of the retailer to the distributor, while the distributor and retailer The basis for decision making remains unchanged. The model under this condition is as follows:

\[
\begin{align*}
\dot{x} &= x + k_1 (y + z) [ -3 \gamma_1 x^2 + 2 (\beta_1 - c_1) x + \alpha_1 - b_1 ] \\
\dot{y} &= y + k_2 z [ -3 \gamma_2 y^2 + 2 (\beta_2 - c_2) y + \alpha_2 - b_2 ] \\
\dot{z} &= z + k_3 z [ -3 \gamma_3 z^2 + 2 (\beta_3 - c_3) z + \alpha_3 - b_3 ]
\end{align*}
\]  

(2)

In the yield game dynamics model, the parameters \( \alpha_i, \beta_i, \gamma_i, a_i, b_i, c_i \) are relatively deterministic, and the yield adjustment coefficients \( k_1, k_2, k_3 \) are variable. For the sake of comparison, fix the parameters as follows:

\[
\begin{align*}
\alpha_1 &= 5, \quad \beta_1 = 0.5, \quad \gamma_1 = 1, \quad b_1 = 0.5, \quad c_1 = 0.4, \\
\alpha_2 &= 5, \quad \beta_2 = 0.5, \quad \gamma_2 = 1, \quad b_2 = 0.4, \quad c_2 = 0.3, \\
\alpha_3 &= 5, \quad \beta_3 = 0.5, \quad \gamma_3 = 1, \quad b_3 = 0.3, \quad c_3 = 0.2.
\end{align*}
\]

Then the production game dynamics model becomes

\[
\begin{align*}
\dot{x} &= x + k_1 (y + z) [ -3 x^2 + 0.2 x + 4.5 ] \\
\dot{y} &= y + k_2 z [ -3 y^2 + 0.4 y + 4.6 ] \\
\dot{z} &= z + k_3 z [ -3 z^2 + 0.6 z + 4.7 ]
\end{align*}
\]  

(3)

The following is a numerical simulation of the system, and one of them is still considered. The coefficient \( k_i \) changes and the coefficients \( k_2, k_3 \) are fixed. When \( k_2 = 0.03, k_3 = 0.02 \), simulate the influence of the change of the production adjustment speed of the manufacturer on the evolution route of the complexity of the system, analyze the evolution route of the production decision of the three roles in the supply chain through the bifurcation diagram, and explain the simulation results from the perspective of the supply chain.

The step size is 0.0003. For different values of \( k_1 \), the system 1-5 is iterated 1000 times to obtain the bifurcation diagram. The manufacturer’s output and output adjustment coefficient is in the interval of \( k_1 \in (0, 0.1065) \), and the production decision is stable. However, as \( k_1 \) increases, the first branching occurs at \( k_1 = 0.1065 \), the second branching occurs at \( k_1 = 0.1245 \), and the third branching occurs at \( k_1 = 0.1305 \), and finally enters the chaotic state.
Figure 1 production bifurcation diagram of the manufacturer at $k_2 = 0.03$, $k_3 = 0.02$

Figure 2 decision-making diagram of distributors and retailers at $k_2 = 0.03$, $k_3 = 0.02$

The stable area of Figure 1 is greatly reduced, and the value of $k^i$ where the branching point is moved from 0.2082 to 0.1065, indicating that when the manufacturer decides to consider a role retailer, the production decision will become more unstable, so narrowing the links in the commercial sector is also a powerful measure to stabilize production. But as you can see from Figure 2, the decisions of distributors and retailers have been stable. In addition, if the speed of production adjustment is reduced by the producer in the production decision, the chaotic state will be delayed. Because the simple rationality of distributors and retailers makes them react only to subordinate needs, the decision is in a stable state.

Control the system 3-2, add control signals $\mu$ to the system, and get the following control model:

$$
\begin{align*}
x' &= \mu x + k_1 (y + z)(-3x^2 + 0.2x + 4.5) + (1 - \mu)x \\
y' &= \mu y + k_2 z(-3y^2 + 0.4y + 4.6) + (1 - \mu)y \\
z' &= \mu z + k_3 z(-3z^2 + 0.6z + 4.7) + (1 - \mu)z
\end{align*}
$$

Figure 3 is a plot of producer X yield when the control parameter $\mu = 0.9$. When $k^i$, the first branch occurs in the system at 0.1185, the second branch appears at 0.1388, and the third occurs at 0.144. Compared with Figure 1, without adding the control parameter $\mu$, the stable region is enlarged. When the control parameter $\mu=0.8$, $k^i$ shows the first branching occurs in the system at 0.1335 and the
second branching occurs in 0.1568. When the control parameter $\mu=0.7$, only one branching occurs at 0.1538, which proves that this parameter the system has stabilized.

![Figure 3 bifurcation diagram of production of manufacturer X at $\mu = 0.9$](image)

### 4. Summary

In this paper, a game models of compound comparative production are constructed, and the complex dynamic analysis is done. Compared with the data of the previous game model, we can draw a conclusion: When a producer makes a production decision, if he considers the demand across levels, he may lose the stability of the decision more quickly. The output response of each enterprise in the three role supply chain to the anti-controller is studied, and the corresponding anti-control model is constructed, which puts forward another solution to alleviate the bullwhip effect.

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