Linear regression equation of mining terrain curvatures caused by hard coal excavation from a few seams and their approximated values by the use of smoothed spline

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Abstract. An article presents the own research results of curvatures shaping on the mining terrain surface. The curvatures have been caused by the hard coal exploitation which was conducted within five years in two coal seams situated at the great depth. The curvatures were observed throughout surveys carried out on the points of measuring line established parallel to the edges of mining pits. There was used a longwall excavation system with the cave-in of roof rocks. At the observational line were performed the geodetic measurements of the points heights and the distances between these points, at the time intervals amounted from three to four months. It has been selected such measuring cycle which represents the excavation influences coming from an exploitation of two coal beds. For curvatures measured after the termination of exploitation in these coal seams, it has been designated their average graph by the use of a mean-squared approximation and the smoothed spline functions.

Then, it has been determined the equation of linear regression for the measured values of curvatures and their average values. On the base of this equation, it’s possible a making of usefulness assessment of the smoothed splines to designation of average courses of measured curvatures.

1. Introduction

Underground mining exploitation of hard coal seams can cause the discontinuous deformations of terrain surface e.g. ground steps [1], sinkholes [2], [3] and/or the continuous deformations of terrain surface such as: subsidence, inclinations, curvatures, horizontal displacements and horizontal strains.

It should be emphasized that there is a some group of objects which are especially sensitive to the mining terrain curvatures. The linear and large surface objects belong to it, for instance highways, railway lines, pipelines, production and storage halls, and many others.

Builders in the mining areas, which design the objects like these and the protections on the dynamic influences of mining exploitation, take into account the values of deformations indicators (the curvatures especially [4]) forecasted by the use of selected theory of exploitation influences, e.g. the geometric-integral theories [5], [6], the cellular automata theory [7], [8].

To check the protections correctness and the credibility of made forecast, they compare the prognosis results with the average values of real deformations indicators, which are determined from the results of geodesic surveys.
The obtaining way of the average graphs of measured deformations indicators can be different. So far the orthogonal polynomials were used in an approximation of average graphs of measured subsidence, inclinations, horizontal strains [9] and curvatures [10]. The basic problems with their use are:
- an unknown, optimum and proper degree of polynomial within a deformation indicator;
- a division necessity of whole graph of deformation indicator into some parts;
- the improper results in case of curvatures.

For these reasons, in the article was shown an obtaining method of the average graphs of observed curvatures [11], [12] by the use of smoothed spline functions [13]. The measured curvatures have been determined from the results of geodesic measurements, which were conducted before and after hard coal exploitation in two coal seams, at the great depths.

To check, how much the curvatures average graph obtained from an approximation done by the use of smoothed splines corresponds their real graph, it was determined the linear regression equation between these graphs of curvatures.

2. Observed graph of curvatures
This section presents:
- an instance of hard coal exploitation which was conducted in Poland, in the Upper Silesian Coal Basin, in several coal seams, at the great depths;
- a characteristic of geodesic surveys which were carried out at the observational line established above the exploitation;
- an obtainment method of curvatures measured graph.

2.1. Hard coal exploitation
Some mining plant, located in the southern part of Poland and in the Upper Silesian Coal Basin, conducted an exploitation of hard coal deposits in two seams numbered 338/2 and 358/1 by the use of longwall system with the roof rocks cave-in.

This extraction system is regarded as the most effective in deposit obtaining and optimal in costs. Its appropriate configuration allows also to protect the building objects from the excessive mining damages [14].

Inclination of these hard coal seams was amounted to 6° in the south-eastern direction. Extraction was conducted in the years 2001 – 2006 at the large depths.

The first coal seam 338/2 was exploited in the years 2001 – 2006 by the use of five longwalls: B-1, B-2, B-3, B-4 and C-1. This hard coal seam was located at the depth of 600 m and has the thickness till 3.0 m.

Exploitation in the second coal seam 358/1 was carried out in the years 2002 – 2006 by the use of seven longwalls: B-1, B-2, B-3, B-4, B-5, B-6 and B-7. The 358/1 hard coal seam was located at the great depth of 1000 m and has the thickness till 2.9 m.

2.2. Geodesic surveys
Mining terrain curvatures, caused by an exploitation of two hard coal seams 338/2 and 358/1, were observed by the geodesic surveys conducted at the measuring line No 8. This line consisted of 53 measuring points and was located perpendicularly to the runs of the B-2 longwall in the 338/2 seam. The average length of measuring segments was amounted 35 m.

The points heights measurements were carried out in reference to the ‘Kronstadt’ altitude system, by the use of precise leveller with an accuracy of 0.1 mm. It was used the double geometric levelling from the middle of measuring segments.

The distances between the subsequent observational points were measured by the use of distance-meter device with an accuracy of 1 mm.
2.3. Obtainment method of curvatures observed graph

On the base of differences of points heights, which were measured before an exploitation has been stared and after an exploitation ending of two coal seams, there were calculated the subsidence of measuring points from the following formula:

\[ S_{i}^{\text{meas}} = H_{i}^{A} - H_{i}^{B}, \]  

where:

- \( H_{i}^{A} \) – the absolute height of the \( i \) point after termination of exploitation \([mm]\);
- \( H_{i}^{B} \) – the absolute height of the \( i \) point before beginning of exploitation \([mm]\);
- \( S_{i}^{\text{meas}} \) – measured subsidence of the \( i \) point \([mm]\).

The subsidence graph of measuring points, registered along the observational line \( N^8 \) after the termination of exploitation in the 338/2 and 358/1 coal seams, has been shown at the Figure 1.

![Figure 1. The subsidence graph noticed along the measuring line \( N^8 \) and caused by exploitation of two coal seams (338/2, 358/1)](image)

The curvatures values, which were observed on two neighbouring segments of measuring line \( N^8 \), have been calculated from the subsidence of three neighbouring points divided by an average length of these segments:

\[ C_{i-1,i,i+1}^{\text{meas}} = \frac{2S_{i-1}^{\text{meas}} - 2S_{i}^{\text{meas}} + S_{i+1}^{\text{meas}}}{l_{i-1,i}^{\text{meas}} + l_{i,i+1}^{\text{meas}}}, \]  

where:

- \( C_{i-1,i,i+1}^{\text{meas}} \) – curvature measured on the \( i-1,i,i+1 \) segments of measuring line \([10^{-6} \cdot 1/m]\);
- \( l_{i-1,i}^{\text{meas}} \) – measured length of the \( i-1,i \) segment \([m]\);
- \( S_{i}^{\text{meas}} \) – measured subsidence of the \( i \) point \([mm]\);
- \( i \) – measuring point.
The curvatures graph of measuring sections, registered along the observational line \( N^8 \) after the termination of exploitation in the 338/2 and 358/1 coal seams, has been shown at the Figure 2.

![Figure 2. The curvatures graph noticed along the measuring line \( N^8 \) and caused by exploitation of two coal seams (338/2, 358/1) \( C_{\text{max}}^\text{meas} = 134.72 \cdot 10^{-5} \text{1/m} \) \( C_{\text{min}}^\text{meas} = -85.37 \cdot 10^{-5} \text{1/m} \)](image)

3. Average graph of curvatures
This section contains:

- an approximation of average graph of measured subsidence, which was done by the use of smoothed spline functions and minimization criterion of differences between the average and model values of subsidence;
- a manner of obtainment of curvatures average graph.

3.1. Approximation of average graph of the measured subsidence
To determine the average graph of curvatures, which were observed after the exploitation end of the 338/2 and 358/1 coal seams, first it’s necessary to do an approximation of average graph of measured subsidence [15], [16].

The average values of measured subsidence were calculated by the use of smoothed splines. There were used the \( R \) computer programme and the \textit{smooth.spline} function. Basic arguments of this function are:

- \( df \) – number of the freedom degree assuming the values from 0 to \( n \), where \( n \) is number of measuring points;
- \( spar \) – smoothing parameter of approximating function assuming the values from 0 to 1, where 0 denotes a total lack of smoothing and approximating function assumes the form of approximated function;
- \( \lambda \) – penalized criterion;
- \( cv \) – cross validation, when \( cv = \text{truth} \) then we deal with the leave-one-out validation, if \( cv = \text{false} \) then we deal with the general cross validation (GCV).

For the purpose of designation of proper value of the \( spar \) smoothing parameter [17], it was applied a minimization criterion of differences between the approximated and model values of subsidence...
(the least squares method). Model values of subsidence were obtained from the Bialek’s formula with the values of its parameters determined from the results of surveys: \( a = 0.890; \) \( \tan \beta = 2.616; \) \( A_1 = 0.227. \)

In the Table 1 have been juxtaposed the parameters values of the smooth.spline function and made approximation.

| Parameter | Value |
|-----------|-------|
| \( Df \)  | 20.95 |
| \( Spar \) | 0.41  |
| \( A \)   | \( 4.53 \cdot 10^6 \) |
| \( GCV \) | 403.36 |
| \( \sum (S_{\text{mod}} - S_{\text{approx}})^2 [\text{mm}^2] \) | 858891.36 |
| \( \sum (S_{\text{meas}} - S_{\text{approx}})^2 [\text{mm}^2] \) | 6148.87 |

At the Figure 3 has been shown an average graph of subsidence caused by exploitation of two hard coal seams, determined on the way of approximation by the use of smoothed splines.

Figure 3. Approximated graph of subsidence measured at the observational line No 8 after the ending of exploitation in two coal seams (338/2, 358/1)

3.2. Average graph of the observed curvatures

The average graph of the observed curvatures has been determined on the base of an average graph of measured subsidence. The average values of measured curvatures have been obtained from the same formula as the measured values of curvatures, but instead the measured values of subsidence have been used the approximated values of measured subsidence:

\[
C_{\text{aver}} = 2 \cdot \frac{S_{\text{approx}} - 2S_{\text{meas}} + S_{\text{approx}}}{S_{\text{meas}} + S_{\text{meas}} + S_{\text{meas}}},
\]

where:
average value of curvature measured on the $i-1,i; i,i+1$ segments of measuring line \(10^{-6} \cdot 1/m\);

- measured length of the $i-1,i$ segment \([m]\);

- approximated value of subsidence measured in the $i-1$ point \([mm]\);

- measuring point.

Figure 4 shows an average graph of curvatures caused by exploitation of two hard coal seams, which was obtained from an average graph of measured subsidence.

![Figure 4. Average graph of curvatures observed at the measuring line No 8 after the ending of exploitation in two coal seams (338/2, 358/1)](image)

4. **Linear regression equation between the observed and average graphs of curvatures**

The correlation between the measured and average graphs of curvatures has been shown in the Table 2 by the use of:

- the $P_C$ percentages of the extreme measured values of curvatures which correspond the extreme average values of observed curvatures;
- the $\sigma_C$ standard deviation between the observed and average values of curvatures;
- the $R_C$ correlation coefficient.

| Parameter        | Value  |
|------------------|--------|
| $P_{C_{max}}$ [%] | 51.76  |
| $P_{C_{min}}$ [%] | 68.57  |
| $\sigma_C$ \(10^{-6} \cdot 1/m\) | 25.29  |
| $R_C$            | 0.8122 |
For the purpose of relationship showing of the measured values of curvatures with their average values, it has been determined a linear regression equation. This correlation has been presented at the Figure 5.

![Figure 5. Linear regression between the average and observed values of curvatures at the measuring line No. 8 after the ending of exploitation in the 338/2 and the 358/1 coal seams](image)

5. Conclusions

Figures 1 and 3 indicate that the maximum value of average subsidence (-3867.70 mm) is almost equal the maximum measured subsidence (-3868.00 mm) and has been occurred almost in the same point (point number 819b, about 816 meters from the beginning of measuring line) like the maximum value of measured subsidence (point number 820, about 853 meters from the beginning of measuring line). Maximum subsidence has been observed above middle of the B-2 longwall in the 338/2 coal seam.

Figures 2 and 4 show that the minimum values of observed and average curvatures have been occurred in the same places (point number 831, about 1418 meters from the line beginning, the run end of the B-2 longwall in the 338/2 coal seam).

However, the maximum value of average curvatures has been occurred in the different place than the maximum value of observed curvatures (point number 827, about 1258 meters from the first point of measuring line instead point number 828, about 1287 meters from the first measuring point). The maximum curvatures have been noted near the southern run of the B-4 longwall in the 358/1 seam.

From the data juxtaposed in the Table 2 results that the smoothed splines functions better reflect a minimum observed curvature (almost in 69 %) than a maximum observed curvature (only in 52 %).

The value of standard deviation between the observed and average curvatures equals to almost 40 % of the extreme values of average curvatures.

The linear regression equation between the measured and average curvatures has a form: \( C_{\text{aver}} = 0.5571 C_{\text{meas}} \). It means that the average values of observed curvatures are about 44.29 % smaller than the measured values of curvatures. Nevertheless, the value of correlation coefficient should be considered as high (more than 0.81). That means, that relationship of the average curvatures with the measured curvatures is rather strong and definitely better than in case of exploitation of three hard coal seams [18].

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