A Model of Metastable EeV Dark Matter

Emilian Dudas\textsuperscript{a,}\textsuperscript{*} Lucien Heurtier\textsuperscript{b,}\textsuperscript{†} Yann Mambrini\textsuperscript{c,}\textsuperscript{‡} Keith A. Olive\textsuperscript{d,}\textsuperscript{§} Mathias Pierre\textsuperscript{e,f}\textsuperscript{¶}

\textsuperscript{a} Centre de Physique Théorique, École Polytechnique, CNRS and IP Paris, 91128 Palaiseau Cedex, France
\textsuperscript{b} Department of Physics, University of Arizona, Tucson, AZ 85721
\textsuperscript{c} Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France
\textsuperscript{d} William I. Fine Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA
\textsuperscript{e} Instituto de Física Teórica (IFT) UAM-CSIC, Campus de Cantoblanco, 28049 Madrid, Spain
\textsuperscript{f} Departamento de Física Teórica, Universidad Autónoma de Madrid (UAM), Campus de Cantoblanco, 28049 Madrid, Spain

(Dated: March 10, 2020)

We propose a model where a long-lived pseudoscalar EeV particle can be produced with sufficient abundance so as to account for the cold dark matter density, despite having a Planck mass suppressed coupling to the thermal bath. Connecting this state to a hidden sterile neutrino sector through derivative couplings, induced by higher dimensional operators, allows one to account for light neutrino masses while having a lifetime that can be much larger than the age of the Universe. Moreover, the same derivative coupling accounts for the production of dark matter in the very first instant of the reheating. Given the sensitivity of the IceCube and ANITA collaborations, we study the possible signatures of such a model in the form of Ultra-High-Energy Cosmic Rays in the neutrino sector, and show that such signals could be detected in the near future.

I. INTRODUCTION

Despite many efforts, the presence of dark matter (DM) in the Universe \cite{1} has not yet been confirmed by any direct \cite{2–4} or indirect \cite{5–9} detection signal. Recent limits severely constrain typical WIMP scenarios such as the Higgs-portal \cite{10, 11}, Z-portal \cite{12}, or even the Z'-portal \cite{13}. More complex extensions such as the minimal supersymmetric standard model \cite{14–16} also have a large part of their parameter space excluded \cite{17–19} from LHC searches \cite{20}. Direct, indirect and accelerator searches place additional constraints on these models (for a review on WIMP searches and models, see \cite{21}). As a consequence, it is important to look for alternative scenarios, including those with ultra-weak couplings such as gravitinos \cite{16, 22} or FIMP’s \cite{23} (see \cite{24} for a review), highly-decoupled dark sectors \cite{25}, or the possibility that DM production occurred in the very early stages of reheating after inflation as in SO(10) grand unification \cite{26, 27}, anomaly free U(1)' models \cite{28}, spin-2 portals \cite{29}, high scale supersymmetry \cite{30–35} or even moduli portals \cite{36}. In all of these models, it has been shown that the effects of non-instantaneous reheating \cite{31, 37, 38} and the non-instantaneous thermalization of reheating products \cite{39} on the production of DM particles are non-negligible.

On the other hand, the absolute stability of DM is usually justified by imposing a symmetry. Discrete symmetries are the most popular (R-parity in supersymmetry \cite{40}, a $Z_2$ symmetry in SO(10) unification \cite{27, 41}, a $Z_2$ symmetry in Higgs \cite{10, 11} or Z-portal \cite{12} models) and can arise from broken gauge symmetries which are exact at the Planck scale. This is not the case for continuous global symmetries which are generically violated at the Planck scale \cite{42–45}. In this case, the decay of DM is rendered possible through Planck-suppressed operators, as argued in \cite{46}.

Due to its very specific signature (monochromatic final states for a 2-body decay), a metastable candidate is regularly evoked when specific detection signals are claimed. For example, a positron excess \cite{47}, photon lines \cite{48}, high energy neutrinos in IceCube \cite{49, 50} or ultra-high energy neutrinos in ANITA \cite{51, 52}. However, in each case, the interpretation of a signal as a dark matter detection has to deal with a severe issue: justifying a long lifetime (and thus extremely tiny couplings) while at the same time finding a production mechanism able to produce the dark matter in a sufficiently large amounts to account for the PLANCK determined density of dark matter \cite{1} (implying a coupling which is not so tiny).

At first sight, it would seem that Planck-suppressed couplings of DM particles to Standard Model (SM) states could be sufficient for explaining why dark matter may be long lived on cosmological time scales. For example, one may naively expect the decay width of dark matter to be of order $\Gamma \sim \frac{m^3}{M_P^2}$, where $m$ denotes the DM mass\textsuperscript{3}. However, given the current limit from indi-

\textsuperscript{3} We will use the reduced Planck mass throughout the paper,
rect gamma [53] positron [54] or neutrino [55] detection ($\tau = \Gamma^{-1} \gtrsim 10^{25}$ seconds) one would require $m \lesssim 10$ keV which reaches the limit from Lyman-$\alpha$ or structure formation constraints [56]. It is, moreover, not an easy task to produce the requisite abundance of DM particles with such feeble couplings. Even the FIMP scenario necessitates couplings of the order of $10^{-11}$ [23], i.e., much larger than $\frac{m^2}{\mathcal{M}_P^2}$.

A potentially more natural way to couple DM to the Standard Model (SM) bath with Planck suppressed couplings is through the neutrino sector, for which there are already strong mass constraints $\sum m_\nu \lesssim 0.15$ eV [57]. Indeed, several constructions invoke a new massive scalar [58] to justify the neutrino mass through a generation of global continuous symmetries at the Planck scale, 3

However, as pointed out above, the most economical and natural way to proceed is to couple the pseudo-scalar to a sterile neutrino ($\nu_s$). After a redefinition of phases, the Lagrangian will contain terms

$$\mathcal{L} \supset \frac{1}{2} (\bar{\nu}_s \gamma^\mu Z_s \partial_\mu \nu_s - (\partial_\mu \bar{\nu}_s) &\gamma^\mu Z_s^\ast \nu_s)$$

which are of the form we consider below.

In this work, we show that by combining the violation of global continuous symmetries at the Planck scale, while coupling DM to the neutrino sector, one can generate a large DM lifetime

$$\tau \propto \left(\frac{\mathcal{M}_P}{m_\nu}\right)^2 \frac{1}{m} \approx 10^{33} \text{ s} \left(\frac{0.05 \text{ eV}}{m_\nu}\right)^2 \left(\frac{1 \text{ GeV}}{m}\right), \quad (1)$$

in compliance with the actual experimental constraints for $\sim 1$ PeV dark matter masses. The paper is organized as follows. In section II we present our model and we compute the DM lifetime and relic abundance in section III. We propose experimental signatures in section IV. In section V, we propose a top-down model which incorporates all of the needed components for our EeV DM candidate and its coupling to the observational sector. Our conclusions are given in section VI. Appendix A contains additional details on the computation of the decay rates and Appendix B gives more detail on the UV microscopic model containing additional particles and interactions which generate, in the IR, the effective model with appropriate mass scales and couplings that we analyze in the bulk of the paper.

II. THE MODEL

Motivations

We begin with some motivation for the general and more detailed models we present below. The models we are proposing rely on a derivative coupling of a DM candidate, $a$, to matter. Indeed, axionic couplings of the type $\frac{a}{\mathcal{M}_P} \partial_\mu a$ appears in several ultraviolet constructions. For instance, in models with string or higher-dimensional inspired moduli fields $T = t + ia$ (see [36] for a more detailed study), they can couple to a sterile sector through the kinetic term as

$$\mathcal{L} \supset i \frac{1}{2} (\bar{\nu}_s \gamma^\mu Z_s \partial_\mu \nu_s - (\partial_\mu \bar{\nu}_s) &\gamma^\mu Z_s^\ast \nu_s)$$

written using a two-component notation and where $\phi = \chi e^{\frac{i\theta}{\mathcal{M}_P}}$ is the Majoron. After a redefinition of phases, $\nu_s \rightarrow e^{-\frac{i\theta}{\mathcal{M}_P}} \nu_s$, the kinetic term, $-i \nu_s \partial^\mu \partial_\mu \nu_s$, produces a coupling of the type given in Eq. (3) [50, 59].

Even in string constructions, where we can define the moduli superfield in term of the Grassmannian variables $\theta$ and $\bar{\theta}$ by $T + \bar{T} = 2t + 2 \sigma \mu \bar{\sigma} \nu \partial_\nu a$, we can show that a term $\frac{1}{(2\sigma_\nu M_P^2) \partial_\nu a \bar{\nu} s \sigma^\mu \nu_s}$ appears once expanding the Kähler metric as function of matter fields. In this case, $\alpha_s$ can be identified as $\frac{1}{(\bar{\sigma}_\nu)} \approx 10^{-2} - 10^{-3}$ in KKLT-like models [60]. As one can see, several ultraviolet constructions contains couplings of the type (3) which we use below.

The Lagrangian

Our goal in this section is to build a minimal model of metastable EeV dark matter. By minimal, we mean that we introduce the fewest number of new fields beyond those in the Standard Model with neutrino masses. We assume that DM is a pseudo-scalar field. As alluded to above, the most economical and natural way to proceed is to couple the pseudo-scalar to a sterile neutrino ($\nu_s$) and/or right-handed neutrino ($\nu_R$) sector as is the case

$\frac{1}{\sqrt{3} G_N} \approx 2.4 \times 10^{18}$ GeV.

2 Note that the lifetime of the Universe is $\sim 4 \times 10^{17}$ seconds, corresponding to $\sim 6.5 \times 10^{41}$ GeV$^{-1}$ whereas limits from indirect detection reach $\tau \gtrsim 10^{20}$ seconds ($\sim 10^{53}$ GeV$^{-1}$).

3 This coupling can be justified in models with large extra dimensions, where SM is localized on a brane, whereas gravity and SM singlets, in particular sterile neutrinos, propagates into a bulk internal space [61] and couple to an axion localized on a distant brane.
for the pseudo-scalar part of the Majoron. We consider the following Lagrangian
\[ \mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_s + \mathcal{L}_R, \]
with
\[ \mathcal{L}_\Phi = yf \Phi \bar{f} f + \left( y_\phi \phi \bar{\nu}_s \nu_s + \text{h.c.} \right), \]
\[ \mathcal{L}_s = \frac{\alpha}{M_P} \partial_\mu a \bar{\nu}_s \gamma^\mu \gamma^5 \nu_s - \left( y_s \bar{H}_L \nu_s + \frac{1}{2} m_s \bar{\nu}_s \nu_s + \text{h.c.} \right), \]
\[ \mathcal{L}_R = \frac{\alpha}{M_P} \partial_\mu a \bar{\nu}_R \gamma^\mu \gamma^5 \nu_R - \left( y_R \bar{H}_L \nu_R + \frac{1}{2} M_R \bar{\nu}_R \nu_R + \text{h.c.} \right). \]
In (7), we have included a Yukawa coupling, \( y_s \), for the sterile neutrino to a \( SU(2)_L \) Standard Model doublet, \( L \) and the Higgs doublet, \( H \), giving rise to a Dirac mass term. We also include a Majorana mass term for \( \nu_s \). We consider the pseudo-scalar part of the Majoron. We assume the following mass hierarchy (that will be justified for simplicity and suppressed flavor indices. We also assumed flavor-diagonal couplings in the SM neutrino sector, many body final state decays can dominate over two-body decays when a spin flip makes the amplitude proportional to the neutrino mass in the final state. This is reminiscent of three-body annihilation processes generated by internal bremsstrahlung which dominate over two-body annihilation processes suppressed for light fermionic final states due to spin-momentum constraints.

\[ m_s < m_D^R \ll M_R. \]

After diagonalization, we can define the 3 mass eigenstates \( \nu_1, \nu_2, \nu_3 \) by
\[ \nu_1 = \cos \theta (\nu_s + \nu_s^c) + \sin \theta (\nu_L + \nu_L^c), \]
\[ \nu_2 = \cos \theta (\nu_L + \nu_L^c) - \sin \theta (\nu_s + \nu_s^c), \]
\[ \nu_3 \sim \nu_R, \]
with
\[ \tan 2\theta = \frac{2 m_D^2 M_R}{(m_D^R)^2 + M_R m_s} \approx \frac{2 m_D^2}{m_1 + m_2}, \]
which implies that
\[ y_s \sim \sqrt{2} \frac{m_1 + m_2}{v} \approx \sqrt{2} \frac{m_2}{v} \lesssim 2.9 \times 10^{-13} \theta \]
and
\[ m_1 \approx m_s, \ m_2 \approx \frac{(m_D^R)^2}{M_R}, \ m_3 \approx M_R, \]
where the last inequality in (12) assumes a SM-like neutrino mass of \( m_2 = 0.05 \text{ eV} \).

In the \( \nu_1, \nu_2 \) basis, we can rewrite the Lagrangian couplings of \( a \) and \( \Phi \) to the light neutrino sector as
\[ \mathcal{L} = a \frac{\partial_\mu a}{M_P} (\bar{\nu}_1 \gamma^\mu \gamma^5 \nu_1 - \theta (\bar{\nu}_2 \gamma^\mu \gamma^5 \nu_1 + \bar{\nu}_1 \gamma^\mu \gamma^5 \nu_2) + \mathcal{O}(\theta^2)), \]
and
\[ \mathcal{L} = y_\phi \Phi \left( \bar{\nu}_1 \nu_1 - \theta (\bar{\nu}_1 \nu_2 + \bar{\nu}_2 \nu_1) + \mathcal{O}(\theta^2) \right). \]
As one can see, our framework is similar to a double seesaw mechanism, and the coupling of the dark matter to the standard model will be highly dependent on the mixing angle \( \theta \). Even if couplings of the form in Eq. (7) may seem adhoc, they can in fact be justified by high-scale motivated models, an example of which is given in section V.

III. THE CONSTRAINTS

In this section, we consider several necessary constraints on the model. These include constraints on the lifetime from indirect detection searches, constraints on the DM abundance - that is we require a viable production mechanism, and cosmological constraints on the sterile sector from contributions to the effective number of neutrino degrees of freedom, \( N_{\text{eff}} \).

Lifetime constraints

The first constraint we apply to the model is on the lifetime of the dark matter candidate \( a \). To be a viable DM candidate, \( a \) should at least live longer than the age of the Universe. However, as was shown in [50], when dealing with long-lived decays of particles to the neutrino sector, many body final state decays can dominate over two-body decays when a spin flip makes the amplitude proportional to the neutrino mass in the final state. This is reminiscent of three-body annihilation processes generated by internal bremsstrahlung which dominate over two-body annihilation processes suppressed for light fermionic final states due to spin-momentum constraints.

[4] Using the approximation \( m_s M_R \ll (m_D^R)^2 \)
In principle, there are two lifetime limits of importance. First, the DM lifetime (to any final state) must be longer than the age of the Universe ($\tau_a > 4 \times 10^{17}$ s). Second, the lifetime must exceed $10^{29}$ s when there is an observable neutrino in the final state [63, 64]. In our case (see Appendix A for details), the dominant decay channel is indeed the three-body final state decay $\Gamma_{a\rightarrow \nu_1\nu_2 h}/Z$ and $\Gamma_{a\rightarrow \nu_1 eW}$. All three of these modes have similar amplitudes. Note that we are interested in final states where a SM particle appears, especially an active neutrino, as that gives us the most stringent constraints from experiment.\(^5\) The $\nu_1\nu_2 h$ final state is most important and we obtain (see Appendix A)

$$\Gamma_{a\rightarrow \nu_1\nu_2 h} = \frac{\alpha^2\theta^2m_a^3}{192\pi^3v^2M_p^2}(m_1 + m_2)^2,$$

implying that

$$\tau_a \gtrsim 5.5 \times 10^{28} s \left(\frac{10^{-2}}{\alpha}\right)^2 \left(\frac{10^{-5}}{\theta}\right)^2 \left(\frac{10^9 \text{ GeV}}{m_a}\right)^3,$$

for $m_1 \ll m_2 \lesssim 0.05$ eV. Note first the rather amazing result that a pseudo-scalar with mass $10^9$ GeV, has a lifetime which greatly exceeds the age of the Universe. This is due primarily to the Planck suppressed coupling and the neutrino mass (squared) in the decay rate. Note also that the lifetime of $a$ is determined by the mixing angle $\theta$ which we have normalized to $10^{-5}$ requiring a relatively small Yukawa coupling of order $10^{-18}$ from Eq. (12). The smallness of $y_b$ will be justified in section V. In this way, we avoid taking $\alpha$ excessively small.\(^6\)

Limits from [64] gives $\tau_{a \rightarrow \nu_2 \nu_2} \gtrsim 5 \times 10^{28}$ seconds whereas [63] obtained $\tau_{a \rightarrow b b} \gtrsim 10^{29}$ seconds. To be as conservative as possible, we will consider the upper limit $m_2 = 0.05$ eV for the neutrino mass and $\tau_a \gtrsim 10^{29}$ seconds throughout our work.

**Cosmological constraints**

Another important constraint comes from the relic abundance of the dark matter. Unless one heavily fine-tunes the coupling of the inflaton to $a$, the direct production of $a$ through (two-body) inflaton decay would greatly overproduce the density of $a$ whose annihilation rate would be extremely small. That is, we cannot rely on any kind of thermal freeze-out scenario. It is however possible to produce $a$ in sufficient quantities through the three-body decay of the inflaton coupled only to SM fermions and the sterile sector as in Eq. (6). This allows for a decay channel $\phi \rightarrow a\nu_1\nu_1$ as shown in Fig.1.

We assume here some rather generic features of the inflationary sector, and do not need to specify a particular model. We assume a coupling of the inflaton to the SM so that reheating is achieved (we assume instantaneous reheating and thermalization). If dominant, the decay rate is given by

$$\Gamma_{\phi \rightarrow th} = \frac{y_f^2N}{8\pi}m_\phi,$$

where $N$ is an effective number of final state fermionic degrees of freedom and is similar to the total number of degrees of freedom of the Standard Model. If dominant, this decay leads to a reheating temperature given by

$$T_{RH} = \left(\frac{5N}{8\pi^2}\right)^{1/4}y_f\sqrt{M_p m_\phi}.$$

In general, the relic abundance of dark matter produced in inflaton decay with a branching ratio $B_R$ can be expressed as [35]

$$\Omega_a h^2 \simeq 0.1 \left(\frac{B_R}{9 \times 10^{-16}}\right) \left(\frac{3 \times 10^{13}}{m_\phi}\right) \left(\frac{T_{RH}}{10^{10}}\right) \left(\frac{m_a}{10^9}\right),$$

where all masses are expressed in GeV. In our specific case, the partial width for producing the DM candidate $a$ is the three body decay width

$$\Gamma_{\phi \rightarrow a\nu_1\nu_1} = \frac{\alpha^2y_f^2}{24\pi^3} \left(\frac{m_\phi}{M_p}\right)^2 m_\phi,$$

and the branching ratio for $\Phi \rightarrow \bar{\nu}_1\nu_1 a$ is given by

$$B_R = \frac{\Gamma_{\phi \rightarrow \bar{\nu}_1\nu_1 a}}{\Gamma_{\phi \rightarrow th}} \simeq \frac{5 \times 10^{-16}}{N} \left(\frac{\alpha}{10^{-2}}\right)^2 \left(\frac{y_\phi}{y_f}\right)^2 \left(\frac{m_\phi}{3 \times 10^{13}}\right)^2,$$

---

\(^5\) Note that the dominant 2-body decay has $\nu_1\nu_1$ in the final state. But for $m_\phi \theta > 10^{-5}m_1$, the three body partial width is always larger (see Eq. (47) in Appendix A).

\(^6\) Indeed, the way we wrote the Planck mass coupling $\frac{\alpha}{M_p}$ imposes $\alpha \lesssim 1$ to avoid large transplanckian BSM scales.
where we have assumed that the total rate is dominated by the two-body decay to SM fermions, or equivalently that $N y_f^2 \gg y_\phi^2$.

\[
\Omega_\chi h^2 \simeq 0.1 \times \frac{125}{N} \left(\frac{\alpha}{5 \times 10^{-2}}\right)^2 \frac{1}{y_\phi} \frac{y_f^2}{y_\phi^2} \left(\frac{m_\Phi}{3 \times 10^{13} \text{ GeV}}\right) \left(\frac{T_{\text{RH}}}{10^{11} \text{ GeV}}\right) \left(\frac{m_a}{10^9 \text{ GeV}}\right), \tag{23}
\]

We note that the expression (23) does not depend on the parameter $\theta$, in contrast to the lifetime of $a$ (16). Indeed, the dominant decay channel of the inflaton to neutrinos involves only the lighter state, whereas mixing proportional to $\theta$ is compulsory for decays with $\nu_2$ in the final state. We also note that to produce the Planck determined abundance of EeV DM, we need $T_{\text{RH}} \sim 10^{11}$ GeV when the Yukawa couplings, $y_\Phi$ and $y_f$ are similar.

\section*{Constraints on $N_{\text{eff}}$}

It is also important to consider the contribution of neutrino sector present in our model to the overall expansion rate of the universe. In principle, adding a new light degree of freedom, would increase the effective number of light neutrinos which is strongly constrained by the CMB and BBN. The current upper limit is [65]

\[
\Delta N_{\text{eff}} < 0.17 \quad (95\% \text{ CL}), \tag{24}
\]

where $\Delta N_{\text{eff}} = N_{\text{eff}} - 3$. However, a completely sterile neutrino which would never equilibrate with the SM bath would only contribute a small fraction of a neutrino to $\Delta N_{\text{eff}}$ since its energy density gets greatly diluted compared to the energy density of SM neutrinos [66]. In cases where a light sterile neutrino (or right-handed $\nu_R$) mixes with the active left-handed neutrinos $\nu_L$, a non-negligible contribution to $N_{\text{eff}}$ may result.

Using Eq. (24), we can derive an upper limit on the mixing angle, $\theta$, by noting that interaction rates for $\nu_1$ are the same as those of active neutrinos, $\nu_2$, suppressed by $\theta^2$. Therefore, $\nu_1$ will decouple at a higher temperature, $T_{d_1}$, than that of $\nu_2$, $T_{d_2}$. Indeed we can approximate $T_{d_1} \theta^{2/3} = T_{d_2} = 2 \text{ MeV}$. As a result, the ratio of the temperatures of $\nu_1$ and $\nu_2$ at $T_{d_2}$ will be given by

\[
\left(\frac{T_1}{T_2}\right)^3 = \frac{43}{4N(T_{d_1})}, \tag{25}
\]

Implementing the expression for the branching ratio into Eq. (20) we obtain

\[
\Delta N_{\text{eff}} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{43}{4N(T_{d_1})}\right)^{4/3}. \tag{26}
\]

For example, the upper limit in Eq. (24), yields $T_1/T_2 < 0.64$ and $N(T_{d_1}) > 162/4$ implying that the decoupling temperature should be greater than $\Lambda_{\text{QCD}}$. That is decoupling should take place before the QCD transition in the early universe. Thus we arrive at an upper limit,

\[
\theta < \left(\frac{T_{d_2}}{T_{d_1}}\right)^{3/2} = \left(\frac{2\text{MeV}}{\Lambda_{\text{QCD}}}ight)^{3/2} \lesssim 1.5 \times 10^{-3}, \tag{27}
\]

for $\Lambda_{\text{QCD}} = 150 \text{ MeV}$.

As one can see, in the cosmologically viable region of interest, the value of $y_\phi$ (and $\theta$) are too weak to be constrained by $N_{\text{eff}}$ (at the 2$\sigma$ level). In contrast, the 1$\sigma$ upper limit to $N_{\text{eff}}$ is 0.05 [65], and in that case, $T_1/T_2 < 0.47$ and $N(T_{d_1}) > 407/4$ implying that the decoupling temperature should be as large as $m_a$ (that is greater than all SM masses). In this case, the limit on $\theta$ is significantly stronger, $\theta < (2\text{MeV}/m_\chi)^{3/2} \approx 4 \times 10^{-8}$. Because the number of degrees of freedom varies slowly with temperature above $\Lambda_{\text{QCD}}$ the limit on $\theta$ varies quickly with $\Delta N_{\text{eff}}$. At the value of $\theta \approx 1.5 \times 10^{-6}$ corresponding to $\alpha = 0.05$, we would predict $T_{d_1} \approx 15 \text{ GeV} > m_b$, implying that $\Delta N_{\text{eff}} \approx 0.062$, which may be probed in future CMB missions. In other words, demanding that our model satisfies cosmological constraints therefore favours the region with $\alpha \sim 1$ which is more natural from the model-building point of view.

\section*{Results}

We note at this point that the combination of Eqs. (16) and (23) seem to point toward a natural region of the parameter space with $\alpha \simeq 10^{-2}$, $\theta \simeq 10^{-6}$, and $T_{\text{RH}} \simeq 10^{11}$ GeV which corresponds to $y_\phi \simeq y_f \simeq 10^{-5}$ from Eq. (23).
In order to explore this region of the parameter space, we performed a scan on the set of parameters \( \{y_s, \theta, m_s\} \), while fixing \( M_R = 10^{12} \text{ GeV} \) and requiring that \( m_2 = 0.05 \text{ eV} \).

We show in Fig. 2 a scan of the plane \((\theta, m_1 \approx m_s)\) after diagonalization of the mass matrix of Eq. (9). For all points considered, we have fixed the DM mass, \( m_\nu = 1 \text{ EeV} \), the DM lifetime, \( \tau_a = 10^{29} \text{ s} \), the active neutrino mass, \( m_2 = 0.05 \text{ eV} \), and the inflaton mass, \( m_\phi = 3 \times 10^{13} \text{ GeV} \). We consider three values of \( \alpha \) as labelled. In the simple case where the inflaton couples equally to the sterile neutrino \( \nu_1 \) and SM fermions \((y_0 = y_f)\), from Eq. (23) we can satisfy \( \Omega_\alpha h^2 \simeq 0.12 \) for different values of \( \alpha \) by compensating with a different value the reheating temperature. For the values of \( \alpha \) chosen, we require \( T_{RH} = 10^{13}, 10^{11} \) and \( 10^{9} \text{ GeV} \) as indicated on the figure. The position of the lines of constant \( \alpha \) is determined by setting the lifetime to the experimental limit of \( \tau_a = 10^{29} \text{ s} \) which can be read from Eq. 16 (with \( \tau_a \approx \Gamma_{a \rightarrow R_h} \)). Note that for \( m_1 \lesssim m_2 \), the lifetime can be approximated by Eq. (17) which is independent of \( m_1 \) which explains why the lines are mostly vertical in the depicted plane. Moreover, since the lifetime is proportional to \((\alpha \theta)^2\), the choice of \( \alpha \) determines \( \theta \) for constant \( \tau_a \). For each point of the scan, the value of the corresponding Yukawa coupling \( y_s \) is indicated by the colored bar.

![FIG. 2: Points in the \((\theta, m_1)\) parameter space for the case of an EeV dark matter candidate with a cosmological lifetime of \(10^{29} \text{ seconds}\). The red lines correspond to three different values of \(\alpha \) \((5 \times 10^{-1}, 5 \times 10^{-2} \text{ and } 5 \times 10^{-3})\) and their position in the plane is explained in the text.](image)

As was anticipated in the previous subsection, in the region of interest the correction to \( \Delta N_{\text{eff}} \) is quite small as compared to the 95% CL upper limit \( \Delta N_{\text{eff}} < 0.17 \), which corresponds to the dashed blue contour in Fig. 2.

We also see in the figure, that the coupling \( y_s \) should be quite small (\( \mathcal{O}(10^{-18}) \) in the region of interest). We will justify this small coupling in section V. Note that the reason the contribution of \( \nu_s \) to \( N_{\text{eff}} \) is small, is precisely because the coupling, \( y_s \) (and mixing angle) is small.

**IV. SIGNATURES**

**Neutrino Scattering in the IceCube Detector**

One clear signature of the model discussed above would be a monochromatic neutrino signal that could be observed by the IceCube, or the ANITA collaborations. In Ref. [51], the case of a scalar DM particle decaying into light right-handed neutrinos was studied and it was shown that the decay of an EeV DM particle followed by the scattering of the RH neutrino within the Earth’s crust could lead to visible signals both for ANITA and IceCube for a mixing angle and DM lifetime of order \( \tau_a / \theta^2 \lesssim 10^{27} \text{ s} \). As we have seen, the region of the parameter space that is favored in our model lies towards smaller values of the mixing angle \( \theta \lesssim 10^{-5} \) and \( \tau_a \gtrsim 10^{29} \text{ s} \), leading to a ratio \( \tau_a / \theta^2 \gtrsim 10^{39} \text{ s} \). This would indicate that our model cannot be detected in searches for anomalous upward-propagating cosmic rays.

In contrast, searches for downward-propagating ultra-high-energy (UHE) cosmic rays are better suited for signatures of the model discussed here. The IceCube collaboration has reported limits on the decay of dark-matter particles with masses reaching up to a few hundred PeV to active neutrinos. Furthermore, it was shown in Refs. [64, 68] that the creation of electroweak showers from the decay of a heavy DM state into neutrinos at very high energy might be constrained at lower energies since the secondary products of such a shower might be visible in the form of a diffuse flux of neutrinos or photons at low energy. These studies led us in the previous sections to impose that the DM lifetime is larger than \( \tau_a \gtrsim 10^{29} \text{ s} \). In this section we determine the region of parameter space that might be probed experimentally by IceCube, either in the form of direct scattering of UHE neutrinos within the detector, or from secondary electroweak showers which would arrive on Earth at lower energies.
a Navarro-Frenk-White (NFW) profile [69]:

$$\rho_{\text{DM}}(r) \propto \frac{1}{\left(\frac{r}{r_s}\right)^{2}} \left[1 + \frac{\left(\frac{r}{r_s}\right)^{2}}{1 + \left(\frac{r}{r_s}\right)^{2}}\right]^{\alpha}, \quad (28)$$

where \( r_s = 24 \) kpc and the dark-matter density distribution is normalized to equal \( \rho_0 = 0.3 \) GeV cm\(^{-3}\) in the vicinity of the solar system [70]. Following Ref. [51], the dark-matter flux, averaged over solid angle, is

$$\langle \Phi \rangle \simeq 1.6 \times 10^{-16} \text{ cm}^{-2} \text{ s}^{-1} \left(\frac{10^{29} \text{ s}}{\tau_{\text{DM}}(\nu_N)}\right) \left(\frac{1 \text{ GeV}}{m_{\text{DM}}(\nu_N)}\right). \quad (29)$$

The number of events predicted for IceCube, assuming a fiducial volume of \( V_{\text{IC}} \approx (1\text{km})^3 \) and an exposure time of \( T_{\exp} = 3142.5 \) days, is given by the relation

$$N_{\text{events}} = V_{\text{IC}} \cdot \rho_{\text{Ice}} \cdot \mathcal{N}_A \cdot T_{\exp} \cdot \sigma_{\nu_N}(E_{\nu}), \quad (30)$$

where the density of the ice is taken to be \( \rho_{\text{Ice}} = 0.92 \) g cm\(^{-3}\), \( \mathcal{N}_A \) is Avogadro’s constant and we estimate the deep-inelastic scattering cross-section \( \sigma_{\nu_N} \) of neutrinos scattering off nuclei using the results of Ref. [71]

$$\log_{10} \left( \sigma_{\nu_N}(E)/\text{cm}^2 \right) = \sum_{i=0}^{3} p_i \log_{10} (E/eV)^i, \quad (31)$$

with \( p_0 = -53.5(-54.1), \ p_1 = 2.66(2.65), \ p_2 = -0.129(-0.112) \) and \( p_3 = 0.00182(0.00175) \) for charged current (and neutral current) interactions, respectively. This yields the final result

$$N_{\text{events}} \approx 0.2 \times \left(\frac{10^{29} \text{ s}}{\tau_a}\right) \left(\frac{1 \text{ GeV}}{m_a}\right) \left(\frac{\sigma_{\nu_N}(m_a/2)}{2.6 \times 10^{-33} \text{cm}^2}\right). \quad (32)$$

Therefore, in the region of the parameter space which we have considered, \( \tau_a \gtrsim 10^{29} \) s, the number of events that IceCube might see within the detector is expected to be of order one. Therefore, it is reasonable to suppose that increasing the exposure time by a factor of a few could lead to the detection of such signal in the relatively near future.

**Secondary Electroweak Shower Detection**

In Ref. [64], limits on the lifetime of a dark-matter particle decaying into active neutrinos have been derived from IceCube data by studying the secondary showers that would be produced by electroweak states at lower energies. We used the limit of Ref. [64] on the lifetime \( \tau_a \) as a function of the dark-matter mass \( m_a \) in order to translate it into a limit on the mixing angle \( \theta \) for a fixed set of parameters.

**Results**

Our results are summarized in Fig. 3 where we have fixed the value of \( \alpha \) to one of our previous benchmark points, \( \alpha = 5 \times 10^{-2} \), and fixed the active neutrino mass to be \( m_2 = 0.05 \text{ eV} \gg m_1 \). The green-shaded area indicates parameter values in the \( m_a - \theta \) plane for which the lifetime of dark matter would be shorter than the age of the universe. This occurs only at large values of both \( m_a \) and \( \theta \) and would be excluded by the lack of events at IceCube. The purple-shaded area excludes the region of the parameter space in which the lifetime of dark matter is shorter than the limit derived in Ref. [64]. Finally the blue lines indicate the value of the mixing angle \( \theta \) as a function of the DM mass, \( m_a \) which would lead to 1 event (solid line) or 10 events (dashed line) in the IceCube detector given the exposure time \( T_{\exp} = 3142.5 \) days. As one can see, our benchmark point (yellow star) corresponding to \( \alpha = 5 \times 10^{-2}, m_2 = 0.05 \text{ eV} \gg m_1 \) with \( m_a = 1 \text{ EeV} \) and \( \tau_a = 10^{29} \) s (corresponding to \( \theta \approx 1.5 \times 10^{-6} \)) is flirting with the experimental limits we presented above, suggesting that the prospect for discovery or exclusion of this benchmark is quite high for IceCube, especially for dark-matter masses ranging from the PeV scales to EeV scales.

![FIG. 3: Regions of the $m_a - \theta$ parameter plane that are excluded by astrophysical constraints obtained by neutrino detectors (shaded purple). In the green shaded region, the lifetime of the DM candidate is shorter than the age of the Universe. Along the solid (dashed) blue line, the number of events expected by Icecube in its exposure time is $1$ (10). The yellow star indicates the benchmark point $m_a = 1 \text{ EeV}$ and $\theta \approx 1.5 \times 10^{-6}$.](image-url)

As we have seen, current IceCube data is already on the edge of discovery of EeV dark matter. In its next phase,
starting next year, the IceCube collaboration will be able
to probe the EeV scale with much better sensitivity for
an observable signal.

V. TOWARDS A MICROSCOPIC APPROACH

In this section we develop a toy microscopic model that
could justify our assumed hierarchy given in Eq. (10).
In fact, such a hierarchy can be generated naturally by
the spontaneous breaking of a global $U(1)$ symmetry and
the generation of non-renormalizable operators at low-
energy. We introduce a set of heavy Weyl fermion pairs
$\tilde{\psi}_i, \psi_i$ with $i = 1, 4$ and a complex scalar field $S$
whose charges are given in Table I. One can integrate out
these heavy fermions and obtain effective interactions between
the scalar $S$ and the different neutrino species, as can be
seen from Fig. 4.

\begin{align}
\langle S \rangle & \langle S \rangle \langle S \rangle \langle S \rangle \langle S \rangle \\
\nu_s & \psi_1 \tilde{\psi}_1 \psi_2 \tilde{\psi}_2 \psi_3 \tilde{\psi}_3 \psi_4 \tilde{\psi}_4 \nu_s \\
\langle S \rangle & \langle S \rangle \langle S \rangle \\
\nu_s & \tilde{\psi}_1 \psi_1 \tilde{\psi}_2 \psi_2 \tilde{\psi}_3 \psi_3 \tilde{\psi}_4 \psi_4 \nu_s \\
\langle S \rangle & \langle S \rangle \\
\nu_s & \nu_s \end{align}

FIG. 4: Feynman diagrams involving the heavy fermions
{$\{\psi_i, \tilde{\psi}_i\}_{i=1,4}$, the scalar $S$ and the different neutrino species.

Assuming that the fermions $\{\psi_i, \tilde{\psi}_i\}_{i=1,4}$ have masses
of the same order of magnitude $M_i \sim M$ where $M$ is some
mass scale close to the Planck scale, after integrating out
the heavy fermions, one obtains the effective low energy
the Lagrangian

\begin{align}
\mathcal{L}_{U(1)}^{\text{eff}} & \supset \frac{\alpha}{M_P^2} \partial_\mu \bar{\psi}_s \gamma^\mu \gamma_5 \psi_s - \left( \frac{1}{2} \bar{\psi}_s \gamma^\mu \gamma_5 \psi_s + \frac{1}{2} M R \bar{\psi}_R \psi_R \\
& + \frac{1}{2} m_{SR} \bar{\psi}_R \psi_R + y_S H \bar{\psi}_L \nu_s + y_R H H \bar{\psi}_L \psi_R + \text{h.c.} \right) \tag{35}
\end{align}

where we defined

\begin{align}
m_s & = 2 h_s \frac{\langle S \rangle^5}{M_T^4}, \quad M_R \equiv 2 h_R \langle S \rangle, \\
y_s & = h_{LS} \frac{\langle S \rangle^2}{M_T^4}, \quad y_R \equiv \lambda_R^L.
\end{align}

If we assume all of the couplings $\lambda_i \sim 0.1$, with the exception of $\lambda_L^L$ which we take to be $\sim 10^{-4}$, and a heavy mass
scale $M \approx M_P$ and a symmetry breaking scale of

\begin{align}
\langle S \rangle & \approx 5 \times 10^{12} \text{ GeV},
\end{align}

we naturally get the desired hierarchy of scales

\begin{align}
m_s & \approx 2 \times 10^{-6} \text{ eV}, \quad M_R \approx 10^{12} \text{ GeV} \\
y_s & \approx 4 \times 10^{-18}, \quad y_R \approx 0.1,
\end{align}

which approximates the favored parameter space of our
model. Note that in addition to the model we have previously studied, there is an additional mixing term $h_{SR} \frac{\langle S \rangle^3}{M_T^2} \bar{\psi}_R \psi_R$ in the seesaw mass matrix, but we checked
that for $h_{SR} \sim 10^{-3}$ this term does not perturb the see-
saw mechanism or our mass hierarchy.

Due to the charge assignment, a coupling of the inflaton
of the type $\Phi \bar{\psi}_s \psi_s$ in Eq. (6) is not allowed be-
cause of the neutrality of $\Phi$ under this $U(1)$. It is
then impossible to generate a sufficiently large quanti-
ty of dark matter through the decay process shown
in Fig. 1. However, the production of dark matter via
the 3-body decay of the inflaton could be made possible
by considering a term like $\mu_\Phi \Phi |S|^2$, with $\mu_\Phi$
being a dimensionful parameter in addition to the term
$h_{SR} \frac{\langle S \rangle^3}{M_T^2} \bar{\psi}_R \psi_R$ with $h_{SR} \sim 0.1$, as depicted in Fig. (5). In this case, we expect the
effective inflaton decay coupling to be $y_\Phi \sim \mu_\Phi \frac{h_{SR}}{M_S/M_R} (M_S/M_R) \sim 10^{-5}$ for $h_{SR} \sim 0.1,$
$\mu_\Phi/M_R \sim 0.1$ and $(M_S/M_R) \sim 1$ if $M_S \sim M_R$ as we expect, $M_S$ being the mass of the heavy scalar.

VI. CONCLUSION

The most commonly considered mass ranges for dark
matter have been either WIMPs with masses between 100
GeV to 1 TeV, or axions with masses much less than an
In this section, we present a microscopic scenario in which the mass hierarchy that we introduced in Sec. (3), after using the four-component notations
\[ \nu_s \rightarrow (\nu_s^0) \], \[ \nu_R \rightarrow (\nu_R^0) \], \[ \nu_L \rightarrow (0 \bar{\nu}_L) \], (4)
eV. Despite a vigorous search program neither have yet been discovered. Therefore it is natural to open up the possible mass range for new searches for dark matter. Indeed there is a lot of effort going into sub-GeV candidates and the prospects for new direct detection experiments. Here we have explored another regime of dark matter masses of an EeV.

In this paper, we have shown that we can reconcile the dark matter lifetime, which requires extremely reduced couplings, with a natural production mechanism. The long lifetime is possible when Planck-mass suppressed operators (generated, for example, by the breaking of a global symmetry) are combined with tiny neutrino masses. The induced lifetime respects the strongest indirect detection limits once the dark matter is coupled to the neutrino sector. Moreover, despite the feebleness of the coupling, we showed that inflaton decay into dark matter can be sufficient to produce a relic abundance compatible with PLANCK results. Furthermore, we showed that the next generation of neutrino telescopes will be able to probe such heavy dark matter in the near future.

**Acknowledgments:** The authors want to thank especially Marcos Garcia for very insightful discussions. This work was supported in part by the France-US PICS MicroDark and the ANR grant Black-ds-String ANR-16-CE31-0004-01. The work of MP was supported by the Spanish Agencia Estatal de Investigación through the grants FPA2015-65929-P (MINECO/FEDER, UE), PGC2018-095161-B-I00, IFT Centro de Excelencia Severo Ochoa SEV-2016-0597, and Red Consolider MultiDark FPA2017-90566-REDC. The research activities of L.H. are supported in part by the Department of Energy under Grant DE-FG02-13ER41976(de-sc0009913). This work was partially performed at the Aspen Center for Physics, which is supported by NationalScience Foundation grant PHY-1607611. The work of LH has been partially performed during the workshop "Dark Matter as a Portal to New Physics" supported by Asia Pacific Center for Theoretical Physics. KO and MP would like to thank the Lawrence Berkeley National Laboratory for its hospitality during part of the realization of this work. MP and LH would also like to thank the Paris-Saclay Particle Symposium 2019 with the support of the P2I and SPU research departments and the P2IO Laboratory of Excellence (program "Investissements d’avenir" ANR-11-IDEX-0003-01 Paris-Saclay and ANR-10-LABX-0038), as well as the IPht. This project has received funding/support from the European Unions Horizon 2020 research and innovation programme under the Marie Skodowska-Curie grant agreements Elusives ITN No. 674896 and InvisiblesPlus RISE No. 690575. The work of KO was supported in part by the DOE grant de-sc0011842 at the University of Minnesota.

## APPENDIX

### A. DECAY RATES

In this appendix, we provide some relevant details concerning the computation of the dark matter decay rate. From Eq. (14), we see that up to \( O(\theta^2) \) there are two two-body final state decay channels to consider

\[
\Gamma_{a \rightarrow \nu_1 \nu_1} = \frac{\alpha^2 m_a m_1^2}{\pi M_P^2}
\]

\[
\Gamma_{a \rightarrow \nu_1 \nu_2} \simeq \frac{\alpha^2 \theta^2 m_a (m_1 + m_2)^2}{2 \pi M_P^2}
\]

where we neglected some threshold factors which are negligible in the limit \( m_a \gg m_2, m_1 \). The direct decay of \( a \) to two SM-like neutrinos is suppressed by \( \theta^4 \).

There are also three-body final state decays which involve a Higgs, \( W^\pm \), or \( Z \) in the final state. These couple...
to neutrinos in the $\nu_1, \nu_2$ basis through

$$\mathcal{L} = - \frac{g}{4c_W} Z^\mu (\bar{\nu}_2 \gamma^\mu \gamma_5 \nu_2 + \theta (\bar{\nu}_2 \gamma^\mu \gamma_5 \nu_1 + \bar{\nu}_1 \gamma^\mu \gamma_5 \nu_2) + \mathcal{O}(\theta^2))$$

$$- \frac{g}{\sqrt{2}} \left( \bar{N}_2 \gamma^\mu e_L W^\mu_+ + \bar{e}_L \gamma^\mu N_2 W^- \right) \quad$$

$$+ \theta (\bar{\nu}_1 \gamma^\mu e_L W^\mu_+ + \bar{e}_L \gamma^\mu \nu_1 W^-) + \mathcal{O}(\theta^2)) \quad (41)$$

where we used $H = (v_h + h)/\sqrt{2}$ in unitary gauge, where $h$ is the Higgs real scalar field. This leads to the three-body decay width with a Higgs in the final state, which is of the same order than the partial width

$$\Gamma \sim \frac{G_F m^2}{16\pi^3} \left( m_1 + m_2 \right)^2 \quad (42)$$

$$\Gamma \propto \frac{G_F m^2}{16\pi^3} \left( m_1 + m_2 \right)^2 \quad (43)$$

which naturally leads to smaller widths than the 3-body decay modes.

### B. THE MICROSCOPIC MODEL

We provide here the microscopic Lagrangian which allows us to derive the $U(1)$ invariant effective theory in Eq. (33) of Section V. Using a two-component notation, the most general renormalizable and $U(1)$ invariant Lagrangian that one can write involving the fields of the model in Section V is

$$\mathcal{L}_{UV} \supset - \left( \lambda_1^{S} S \nu_{i} \psi_{1} + \lambda_2^{S} \bar{S} \bar{\nu}_{i} \psi_{2} + \lambda_3^{S} \bar{S} \bar{\psi}_{2} \psi_{3} + \lambda_4^{S} \bar{\psi}_{3} \psi_{4} \right.$$  

$$+ \lambda_2^{S} \bar{S} \bar{\psi}_{4} \nu_{s} + \lambda_3^{S} \bar{S} \bar{\psi}_{2} \nu_{R} + \lambda_4^{S} \bar{S} \bar{\psi}_{3} \nu_{R} + \lambda_5^{S} \bar{S} \bar{H} \nu_{L} \psi_{3} \right.$$  

$$+ \lambda_6^{S} \bar{H} \nu_{L} \nu_{R} + h.c.) - \sum_{i=1}^{4} M_{i} \bar{\psi}_{i} \psi_{i} - V(S) \right.$$  

$$+ \frac{\alpha}{\Gamma} \frac{\partial_{\mu} \mu \left( \bar{\nu}_{s} \bar{\gamma}_{\mu} \nu_{s} + \bar{\nu}_{R} \bar{\gamma}_{\mu} \nu_{R} \right)}{\mu} \right). \quad (50)$$

Taking a common mass $M \equiv M$ for all the supermassive fermions $\left\{ \psi_{i}, \bar{\psi}_{i} \right\}_{i=1,4}$, one can integrate them out to obtain the effective operators

$$\mathcal{L}_{\text{eff}} \supset - \frac{\lambda_1^{S} \lambda_4^{S} \lambda_3^{S} \lambda_2^{S} \lambda_1^{S} M^4}{M^4} S^{5} \nu_{s} \nu_{s} \nu_{R} \nu_{s},$$

$$- \frac{\lambda_2^{S} \lambda_3^{S} \lambda_4^{S} \lambda_5^{S} M^2}{M^2} S^{3} \nu_{R} \nu_{R} \nu_{R} - \frac{\lambda_1^{S} \lambda_4^{S} \lambda_3^{S} \lambda_2^{S} \lambda_1^{S} M^2}{M^2} S^{2} \bar{H} \nu_{L} \nu_{s} + h.c. \quad (51)$$

One thereafter obtains at low energy the Lagrangian of Eq. (33), written using four-component notation

$$\nu_{s} \rightarrow \left( \begin{array}{c} \nu_{s} \\ 0 \end{array} \right), \quad \nu_{R} \rightarrow \left( \begin{array}{c} \nu_{R} \\ 0 \end{array} \right), \quad \nu_{L} \rightarrow \left( 0 \bar{\nu}_{L} \right), \quad (52)$$

and introducing the effective couplings of Eq. (34).

In full generality, certain interaction or mass terms could be added to the Lagrangian of Eq. (50) while preserving the symmetries of the model. However, we checked that the presence of such terms do not modify the structure of the effective theory we introduce in Sec. V but simply generate additional contributions to the relations of Eq. (34).

* emilian.dudas@polytechnique.edu
1 heurtier@email.arizona.edu
‡ yann.mambrini@th.u-psud.fr
§ olive@umn.edu
* mathias.pierre@uam.es
