Role of Four-Quark Operators in the Inclusive $\Lambda_b$ Decays

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Abstract
We compute by QCD sum rules the matrix elements of the relevant four-quark operators appearing in the expression of the $\Lambda_b$ inclusive decay rates at the order $1/m_b^3$. The results suggest that $1/m_b^3$ corrections are not responsible of the observed difference between the lifetime of $\Lambda_b$ and $B_d$. 

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1 Introduction

An interesting problem of the present-day heavy quark physics is represented by the measured difference between the $\Lambda_b$ baryon and $B_d$ meson lifetimes: $\tau(\Lambda_0^b) = 1.18 \pm 0.07 \text{ ps}$ and $\tau(\bar{B}^0) = 1.56 \pm 0.05 \text{ ps}$ [1]. As a matter of fact, the deviation from unity, at the level of 20%, of the ratio $\tau(\Lambda_b)/\tau(B_d)$: $\tau(\Lambda_0^0)/\tau(\bar{B}^0) = 0.75 \pm 0.05$, is in contradiction with the naive expectation that, at the scale of the $b$ quark mass, the spectator model should describe rather accurately the decays of the hadrons containing one heavy quark.

The new measurement by the Delphi Collaboration at LEP of the average $b$–baryon lifetime: $\tau(b - \text{baryon}) = 1.25 \pm 0.11 \pm 0.05 \text{ ps}$ [2], together with the CDF result $\tau(\Lambda_b) = 1.33 \pm 0.16 \pm 0.07 \text{ ps}$ [3], although pointing towards a larger value of $\tau(\Lambda_b)/\tau(B_d)$, still confirms that this ratio is considerably different from unity.

In principle, the ratio $\tau(\Lambda_b)/\tau(B_d)$ can be computed in QCD. As a matter of fact, a field theoretical approach has been developed for the analysis of the inclusive decay rates of the hadrons $H_Q$ containing one heavy quark $Q$ [4, 5]. The method is based on an expansion in the inverse heavy quark mass $m_Q$, in the framework of the operator product expansion (OPE), and it has provided us with several hints on the hierarchy of the lifetimes of these hadronic systems. The calculation, however, involves the matrix elements of a number of high dimensional quark and gluon operators. For the $D = 5$ operators defined below such matrix elements have been theoretically calculated, or can be inferred from the experimental measurements. As for the $D = 6$ operators, in the case of heavy mesons their matrix elements can be obtained invoking the factorization ansatz, and therefore it is possible to express them in terms of quantities such as the leptonic constant $f_B$, whose estimates can be found in the literature. On the other hand, the heavy baryon matrix elements of the $D = 6$ operators cannot be obtained by factorization, and indeed they have been estimated only in the framework of constituent quark models, with uncertainties whose size is difficult to assess. Since the $D = 6$ $\Lambda_b$ matrix elements may be responsible of the large difference between $\tau(\Lambda_b)$ and $\tau(B_d)$, it is interesting to compute them by field theoretical approaches; this paper is devoted to a calculation based on the method of QCD sum rules.

Before reporting on this calculation, let us briefly summarize the main aspects of the QCD analysis of the inclusive decay widths of the heavy hadrons. The starting point is the transition operator $\hat{T}(Q \rightarrow X_f \rightarrow Q)$ [4]:

$$\hat{T} = i \int d^4 x \ T[\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)]$$

(1)
describing an amplitude with the heavy quark $Q$ having the same momentum in the initial and final state. $\mathcal{L}_W$ is the effective weak Lagrangian governing the decay $Q \to X_f$. The inclusive width the hadron $H_Q$ can be obtained by averaging $\hat{T}$ over $H_Q$ and taking the imaginary part of the forward matrix element:

$$\Gamma(H_Q \to X_f) = \frac{2 \text{Im} < H_Q | \hat{T} | H_Q >}{2 M_{H_Q}}. \quad (2)$$

The main idea to calculate the r.h.s of eq.(2) is to set up an operator product expansion for the transition operator $\hat{T}$ in terms of local operators $O_i$:

$$\hat{T} = \sum_i C_i O_i \quad (3)$$

with $O_i$ ordered according to their dimension, and the coefficients $C_i$ containing appropriate inverse powers of the heavy quark mass $m_Q$. The lowest dimension operator appearing in (3) is $O_3 = \bar{Q}Q$. The next gauge and Lorentz invariant operator is the $D = 5$ chromo-magnetic operator $O_G$: $O_G = \bar{Q} \frac{g_2}{2} \sigma_{\mu\nu} G^{\mu\nu} Q$, whose hadronic matrix element

$$\mu_G^2(H_Q) = \frac{< H_Q | \bar{Q} \frac{g_2}{2} \sigma_{\mu\nu} G^{\mu\nu} Q | H_Q >}{2 M_{H_Q}} \quad (4)$$

measures the coupling of the heavy quark spin to the spin of the light degrees of freedom in the hadron $H_Q$, and therefore is responsible of the mass splitting between hadrons belonging to the same $s_\ell$ multiplet ($s_\ell$ is the total angular momentum of the light degrees of freedom in $H_Q$). In the case of $b$-flavoured hadrons this mass difference has been measured, both for mesons ($M_{B^*} - M_B = 42.0 \pm 0.6 \text{ MeV}$ [3]) and $\Sigma_b$ baryons ($M_{\Sigma_b^*} - M_{\Sigma_b} = 56 \pm 16 \text{ MeV}$ [7]).

The matrix element of $\bar{Q}Q$ over $H_Q$ can be obtained using the heavy quark equation of motion, expanded in the heavy quark mass:

$$\bar{Q}Q = \bar{Q} \gamma^0 Q + \frac{O_G}{2 m_Q^2} - \frac{O_\pi}{2 m_Q^2} + O(m_Q^{-3}) \quad (5)$$

$O_\pi$ is the kinetic energy operator $O_\pi = \bar{Q} (i \tilde{D})^2 Q$ whose matrix element

$$\mu_\pi^2(H_Q) = \frac{< H_Q | \bar{Q} (i \tilde{D})^2 Q | H_Q >}{2 M_{H_Q}} \quad (6)$$

measures the average squared momentum of the heavy quark inside $H_Q$. On the other hand, the $H_Q$ matrix element of $\bar{Q} \gamma^0 Q$ is unity (modulo the covariant normalization of the states).
The number of independent operators appearing in (3) increases if the $1/m_Q^3$ term is considered. Such operators can be identified in the four-quark operators of the type

$$O_0^f = \bar{Q} \Gamma q \bar{q} \Gamma Q$$

(7)

where $\Gamma$ is an appropriate combination of Dirac and color matrices.

In this way, a complete classification of the various contributions to the inclusive decay rates can be obtained for the different hadrons $H_Q$. In the expression for $\Gamma(H_Q \rightarrow X_f)$:

$$\Gamma(H_Q \rightarrow X_f) = \Gamma_{0}^{f} \left[ A_{0}^{f} + \frac{A_{2}^{f}}{m_{Q}^{2}} + \frac{A_{3}^{f}}{m_{Q}^{3}} + \ldots \right]$$

(8)

the $A_{i}^{f}$ factors, that together with $\Gamma_{0}^{f}$ depend on the final state $X_f$, include perturbative short-distance coefficients and nonperturbative hadronic matrix elements incorporating the long range dynamics. The partonic prediction for the width in (8) corresponds to the leading term $\Gamma_{part}(H_Q \rightarrow X_f) = \Gamma_{0}^{f} A_{0}^{f}$, with $A_{0}^{f} = 1 + c^{f} \alpha_{s}/\pi + O(\alpha_{s}^{2})$ and $\Gamma_{0}^{f} \simeq m_{Q}^{5}$; differences among the widths of the hadrons $H_Q$ emerge at the next to leading order in $1/m_{Q}$ and are related to the different value of the matrix elements of the operators $O_i$ of dimension larger than three.

It is important to notice the absence of the first order term $m_{Q}^{-1}$ in (8), a result obtained by Chay, Georgi and Grinstein [8], and Bigi, Uraltsev and Vainshtein [9].

The occurrence of operators of the type in eq.(7) is an appealing feature of the expansion (8), as far as the determination of the inclusive widths is concerned. As a matter of fact, contrarily to the $D = 5$ operators $O_G$ and $O_{\pi}$ which are spectator blind, the $D = 6$ operators give different contributions when averaged over hadrons belonging to the same $SU(3)$ light flavour multiplet, and therefore they are responsible of the different lifetime of, e.g., $B^{-}$ and $B_{s}$, $\Lambda_{b}$ and $\Xi_{b}$. The spectator flavour dependence is related to the mechanisms of weak scattering and Pauli interference [4], both suppressed by the factor $m_{Q}^{-3}$ with respect to the parton decay rate.

As for the differences in the lifetime of mesons and baryons, they could already arise at the order $m_{Q}^{-2}$, due both to the chromomagnetic contribution and to the kinetic energy term in (2). In particular, the kinetic energy term is responsible of the difference for systems where the chromomagnetic contribution vanishes, namely in the case of $\Lambda_{b}$ and $\Xi_{b}$ having the light degrees of freedom in $S-$ wave. However, the results of a calculation of $\mu_{\pi}^{2}$ for mesons [10] and baryons [11] support the conjecture, put forward in [12], that the
kinetic energy operator has the same matrix element when computed on such hadronic systems. The approximate equality of the kinetic energy operator on $B_d$ and $\Lambda_b$ can also be inferred by considering that, to the leading order in $1/m_Q$, $\mu^2_n(\Lambda_b)$ can be related to $\mu^2_n(B_d)$ and to the heavy quark masses by the expression (which assumes the charm mass $m_c$ heavy enough for a meaningful expansion in $1/m_c$) [13]:

$$\mu^2_n(\Lambda_b) - \mu^2_n(B_d) \simeq \frac{m_b m_c}{2(m_b - m_c)} [(M_B + 3M_{B^*} - 4M_{\Lambda_b}) - (M_D + 3M_{D^*} - 4M_{\Lambda_c})]. \quad (9)$$

Using present data and the CDF measurement $M_{\Lambda_b} = 5623 \pm 5 \pm 4 \text{ MeV}$ [14] (in [1] the value $5639 \pm 15 \text{ MeV}$ is reported) eq. (9) gives

$$\mu^2_n(\Lambda_b) - \mu^2_n(B_d) \simeq 0.002 \pm 0.024 \text{ GeV}^2,$$

where the error mainly comes from the error on $M_{\Lambda_b}$. The QCD sum rule outcome for $\mu^2_n(\Lambda_b)$ is $\mu^2_n(B_d) \simeq \mu^2_n(\Lambda_b) \simeq 0.6 \text{ GeV}^2$, with an estimated uncertainty of about 30%. This result implies that the differences between meson and baryon lifetimes should occur at the $m_Q^{-3}$ level, thus involving the four-quark operators in eq.(7). They can be classified as follows [15]:

$$O^{q}_{V-A} = \bar{Q}_L\gamma_\mu q_L \bar{q}_L\gamma_\mu Q_L$$
$$O^{q}_{S-P} = \bar{Q}_R q_L \bar{q}_L Q_R$$
$$T^{q}_{V-A} = \bar{Q}_L\gamma_\mu \lambda^a \bar{q}_L \gamma_\mu \lambda^a Q_L$$
$$T^{q}_{S-P} = \bar{Q}_R \lambda^a \frac{1}{2} q_L \bar{q}_L \lambda^a Q_R \quad (10)$$

with $q_{R,L} = \frac{1\pm\gamma_5}{2} q$ and $\lambda_a$ the Gell-Mann matrices.

For mesons, the vacuum saturation approximation can be used to compute the matrix elements of the operators in (10):

$$<B_q|O^{q}_{V-A}|B_q>_{VAS} = \left(\frac{m_b + m_q}{M_{B_q}}\right)^2 <B_q|O^{q}_{S-P}|B_q>_{VAS} = \frac{f_B^2 M_{B_q}^2}{4} \quad (11)$$
$$<B_q|T^{q}_{V-A}|B_q>_{VAS} = <B_q|T^{q}_{S-P}|B_q>_{VAS} = 0. \quad (12)$$

Therefore, the matrix elements are expressed in terms of quantities such as $f_B$ and the quark masses, and the resulting numerical values can be used in the calculation of the lifetimes, with the only caveat concerning the accuracy of the factorization approximation [15].

The vacuum saturation approach cannot be employed for baryons; in this case a direct calculation of the matrix elements is required, for example using constituent quark models.
A simplification can be obtained for $\Lambda_b$, as noticed in [15], using color and Fierz identities and introducing the operators

$$\tilde{O}_{V-A}^i = Q_L^i \gamma_\mu Q_L^j \bar{q}_L^i \gamma^\mu \bar{q}_L^j$$

(13)

and

$$\tilde{O}_{S-P}^i = Q_L^i \bar{q}_L^j \bar{q}_L^i Q_R^i$$

(14)

($i$ and $j$ are color indices). As a matter of fact, the $\Lambda_b$ matrix elements of the operators in (10) can be expressed in terms of $<\Lambda_b|\tilde{O}_{V-A}^i|\Lambda_b>$ and $<\Lambda_b|O_{V-A}^i|\Lambda_b>$, modulo $1/m_Q$ corrections contributing to subleading terms in the expression for the inclusive widths.

The matrix element of $\tilde{O}_{V-A}^i$ and $O_{V-A}^i$ can be parametrized as

$$<\tilde{O}_{V-A}^i>_{\Lambda_b} = \frac{<\Lambda_b|\tilde{O}_{V-A}^i|\Lambda_b>}{2M_{\Lambda_b}^2} = \frac{f_B^2 M_B}{48} r$$

(15)

and

$$<\Lambda_b|O_{V-A}^i|\Lambda_b> = -\tilde{B} <\Lambda_b|\tilde{O}_{V-A}^i|\Lambda_b>$$

(16)

with $\tilde{B} = 1$ in the valence quark approximation.

For $f_B = 200$ MeV and $r = 1$ eq (13) corresponds to the value: $<\tilde{O}_{V-A}^i>_{\Lambda_b} = 4.4 \times 10^{-3}$ GeV$^3$. In ref.[16] the $\Lambda_c$ matrix element of $\tilde{O}_{V-A}^i$ has been computed using a bag model and a nonrelativistic quark model; the results $<\tilde{O}_{V-A}^i>_{\Lambda_c} \simeq 0.75 \times 10^{-3}$ GeV$^3$ and $<\tilde{O}_{V-A}^i>_{\Lambda_c} \simeq 2.5 \times 10^{-3}$ GeV$^3$, correspond to $r \simeq 0.2$ and $r \simeq 0.6$, respectively. An analysis using the model in [17] can also be found in [5].

Larger values of the matrix elements have been advocated by Rosner [18] using the values of the mass splitting $\Sigma_b^* - \Sigma_b$ and $\Sigma_c^* - \Sigma_c$, and assuming that the $\Lambda_b$ and $\Sigma_b$ wave functions are similar: $r \simeq 0.9 \pm 0.1$ taking $M_{\Sigma_b}^2 - M_{\Sigma_b} = M_{\Sigma_c}^2 - M_{\Sigma_c}$, or $r \simeq 1.8 \pm 0.5$ using the Delphi measurement in [4].

It is worth supplementing the information from constituent quark models by estimates based on field theoretical approaches, for example QCD sum rules. As a matter of fact, a large value of $r$, namely $r \simeq 4 - 5$, would explain the difference between $\tau(\Lambda_b)$ and $\tau(B_d)$ [13]. As we shall see, the application of the QCD sum rule method to the calculation of the matrix element of an operator of high dimension presents a number of disadvantages; nevertheless, interesting and quite reliable information can be obtained for $<\Lambda_b|\tilde{O}_{V-A}^i|\Lambda_b>$. 
2 QCD sum rule calculation of $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$

A quantitative estimate of the matrix element $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$ can be obtained by the method of QCD sum rules [19] applied to a suitable correlator in the heavy quark effective theory (HQET). Let us consider the three-point correlation function:

$$\Pi_{CD}(\omega, \omega') = (1 + \gamma)_{CD} \Pi(\omega, \omega') = i^2 \int dxdy < 0|T[J_C(x)\hat{O}_{V-A}^q(0)\bar{J}_D(y)]|0 > e^{i\omega(v \cdot x) - i\omega'(v \cdot y)}$$

(17)

of the spin $\frac{1}{2}$ local fields $J(x)$ and $\bar{J}(y)$ ($C$ and $D$ are Dirac indices) and of the operator $\hat{O}_{V-A}^q$ in eq.(13). The variable $\omega$ ($\omega'$) is related to the residual momentum of the incoming (outgoing) baryonic current:

$$p^\mu = m_b v^\mu + k^\mu$$

(18)

with $k^\mu = \omega v^\mu$.

If the baryonic currents $J$ and $\bar{J}$ have non-vanishing projection on the $\Lambda_b$ state

$$< 0|J_C|\Lambda_b(v) >= f_{\Lambda_b}(\psi_v)_C$$

(19)

($\psi_v$ is a spinor for a $\Lambda_b$ of four-velocity $v$) with the parameter $f_{\Lambda_b}$ representing the coupling of the current $J$ to the $\Lambda_b$ state, the matrix element $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$ can be obtained by saturating the correlator (17) with baryonic states, and considering the double pole contribution in the variables $\omega$ and $\omega'$:

$$\Pi_{had}^{\omega, \omega'} = \langle \hat{O}_{V-A}^q \rangle_{\Lambda_b} f_{\Lambda_b}^2 \frac{1}{2} \frac{1}{(\Delta_{\Lambda_b} - \omega)(\Delta_{\Lambda_b} - \omega')} + \ldots$$

(20)

at the value $\omega = \omega' = \Delta_{\Lambda_b}$. The mass parameter $\Delta_{\Lambda_b}$ represents the binding energy of the light degrees of freedom in $\Lambda_b$ in the static color field generated by the $b$–quark:

$$M_{\Lambda_b} = m_b + \Delta_{\Lambda_b}$$

(21)

and must be derived within the same QCD sum rule theoretical framework.

A suitable interpolating field for $\Lambda_b$, in the infinite heavy quark mass limit, has been proposed by Shuryak [20]. It is set up by a function of the quark fields:

$$J_C(x) = \epsilon^{ijk}(q^{Ti}(x)\Gamma q^j(x))(h_v^k)_C(x)$$

(22)

where $T$ means transpose, $i, j$ and $k$ are color indices, and $C$ is the Dirac index of the effective heavy quark field $h_v(x)$ related to the Dirac field $Q$ by:

$$h_v(x) = e^{im_v x} \frac{1 + \gamma}{2} Q(x)$$

(23)
The matrix $\tau$ is the $\Lambda_b$ light flavour matrix:

$$
\tau = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
$$

(24)
corresponding to zero isospin. As implied by the spectroscopy of baryons containing one heavy quark in the limit $m_b \to \infty$, in the $\Lambda_b$ the light diquark is in a relative $0^+$ spin-parity state; this feature can be described by the current (22) by two possible choices of the Dirac matrix $\Gamma$:

$$
\Gamma^{(1)} = C\gamma_5, \quad \Gamma^{(2)} = C\gamma_5\gamma^0
$$

(25)
(for a $\Lambda_b$ at rest) where $C$ is the charge conjugation matrix. In principle, the currents obtained using the $\Gamma^{(\ell)}$ matrices in (25), and a linear combination

$$
\Gamma = C\gamma_5(1 + b\gamma^\ell)
$$

(26)
can be used in (17). As discussed in [11], there are arguments in favour of the choice $b = 1$ in (26); we shall come to this point below.

Also the coupling $f_{\Lambda_b}$ can be derived by a sum rule, considering the two-point correlator:

$$
H_{CD}(\omega) = (1 + \gamma^\ell)_{CD}H(\omega) = i \int dx < 0|T[J_C(x)\bar{J}_D(0)]|0 e^{i\omega(v \cdot x)}
$$

(27)
saturated by a set of baryonic states, in correspondence to the pole at $\omega = \Delta_{\Lambda_b}$:

$$
H^{\text{had}}(\omega) = \frac{f_{\Lambda_b}^2}{2} \frac{1}{(\Delta_{\Lambda_b} - \omega)} + \ldots
$$

(28)
Therefore, the calculation of $f_{\Lambda_b}$ can be carried out following the same QCD sum rule approach in HQET; this analysis can be found in the literature [11, 21].

Let us consider the correlators (17) and (27). In the Euclidean region, for negative values of $\omega, \omega'$ the correlation functions (17) and (27) can be computed in QCD, in terms of a perturbative contribution and of vacuum condensates. The results can be written in a dispersive form:

$$
\Pi^{OPE}(\omega, \omega') = \int d\sigma d\sigma' \frac{\rho_{\Pi}(\sigma, \sigma')}{(\sigma - \omega)(\sigma' - \omega')}
$$

(29)
$$
H^{\text{OPE}}(\omega) = \int d\sigma \frac{\rho_H(\sigma)}{(\sigma - \omega)}
$$

(30)
where possible subtraction terms have been omitted. The spectral function of (17) read in HQET as

\[ \rho_{\Pi}(\sigma, \sigma') = \rho_{\Pi}^{(pert)}(\sigma, \sigma') + \rho_{\Pi}^{(D=3)}(\sigma, \sigma') < \bar{q}q > + \rho_{\Pi}^{(D=4)}(\sigma, \sigma') \frac{\alpha_s}{\pi} G^2 > + \rho_{\Pi}^{(D=5)}(\sigma, \sigma') < \bar{q}g\sigma Gq > + \rho_{\Pi}^{(D=6)}(\sigma, \sigma')(< \bar{q}q >)^2 + \ldots \] ; (31)

a similar expression can be given for \( \rho_{H}(\sigma) \).

At the lowest order in \( \alpha_s \) the diagrams contributing to \( \rho_{\Pi}(\sigma, \sigma') \) are depicted in fig.1. The perturbative contribution to the spectral function, obtained computing by the Cutkosky rule the imaginary part of the diagram in fig.1a, has the following expression:

\[ \rho_{\Pi}^{pert}(\sigma, \sigma') = \frac{3}{32 \pi^6} (1 + b^2) \left\{ \theta(\sigma - \sigma') \sigma'^2 \left( \frac{\sigma'^2}{105} - \frac{\sigma\sigma'}{30} \right) + (\sigma \leftrightarrow \sigma') \right\} . \] (32)

A comment is in order. The operator \( \tilde{O}_{V-A}^q \) in the correlator (17) could give rise to non-spectator contributions through diagrams where two of the quark fields appearing in it are contracted in a tadpole. Such contributions, as noticed in [15], do not affect the differences in lifetime for the various heavy hadrons, therefore they can be omitted assuming a normal ordering in the four-quark operators. Such contributions vanish when the non-perturbative spectral functions are computed.

The contributions proportional to the quark condensate and to the mixed quark-gluon condensate can be derived by computing a class of diagrams of the type in fig.1b-d, with the result:

\[ \rho_{\Pi}^{(D=3)}(\sigma, \sigma') = -\frac{b}{16 \pi^4} \left\{ \theta(\sigma - \sigma') \sigma'^2 (\sigma - \sigma')^2 + \theta(\sigma' - \sigma) \sigma^2 (\sigma - \sigma')^2 + \sigma^2 \sigma'^2 \right\} \] (33)

\[ \rho_{\Pi}^{(D=5)}(\sigma, \sigma') = \frac{b}{256 \pi^4} \left\{ \theta(\sigma - \sigma') (\sigma^2 + 9 \sigma'^2 - 10 \sigma\sigma') + \theta(\sigma' - \sigma) (\sigma'^2 + 9 \sigma^2 - 10 \sigma\sigma') + 3 \sigma^2 + 3 \sigma'^2 + 8 \sigma\sigma' \right\} . \] (34)

Notice that in the adopted Fock-Schwinger gauge no gluon can be emitted from heavy quark leg in the infinite heavy quark mass limit.

Terms proportional to the four-quark condensate come from diagrams of the type in fig.1e, assuming factorization of the matrix element \( < \bar{q}qqq > \):

\[ \rho_{\Pi}^{(D=6)}(\sigma, \sigma') = \frac{1 + b^2}{96 \pi^2} \left\{ \sigma'^2 \delta(\sigma) + \sigma^2 \delta(\sigma') + \sigma\sigma' \delta(\sigma - \sigma') \right\} . \] (35)
We do not include the gluon condensate contribution $\rho_\Pi^{(D=4)}(\sigma, \sigma')$ since it represents the low momentum component of gluon exchange diagrams omitted in the calculation of the perturbative term.

The inclusion of additional contributions proceeds in a similar way: we shall discuss in the following the consequences of the neglect of such terms.

A sum rule for $\langle \tilde{O}_{V-A}^q \rangle_{\Lambda_b}$ can be derived by equating, according to the QCD sum rule strategy, the hadronic and the OPE representations of the correlator (17). Moreover, invoking a global duality ansatz, the contribution of the higher resonances and of the continuum in $\Pi^{\text{had}}$ in (20) can be modeled as the QCD contribution outside the region (duality region) $0 \leq \omega \leq \omega_c$, $0 \leq \omega' \leq \omega_c$, with $\omega_c$ an effective threshold. Finally, the application of a double Borel transform to both the $\Pi^{\text{OPE}}$ and $\Pi^{\text{had}}$ representation of the correlator (17) in the momenta $\omega, \omega'$:

$$
B(E_1) \frac{1}{\sigma - \omega} = \frac{1}{E_1} e^{-\sigma/E_1}, \quad B(E_1) \frac{1}{\Delta - \omega} = \frac{1}{E_1} e^{-\Delta/E_1},
$$

(36)

(and similar for $\omega'$ with the Borel parameter $E_2$) allows us to remove the subtraction terms appearing in (29), that are polynomials in the variables $\omega$ or $\omega'$ (the Borel transform of a polynomial vanishes). Moreover, the convergence of the OPE is factorially improved by the transform, and the contribution of the low-lying resonances in $\Pi^{\text{had}}$ is enhanced for low values of the Borel variables. The symmetry of the spectral functions in $\sigma, \sigma'$ suggests the choice $E_1 = E_2 = 2E$ where $E$ is the Borel parameter appearing in the QCD sum rule analysis of the two-point function (27). The final expression for the matrix element reads:

$$
\frac{f_{\Lambda_b}^2}{2} (1 + b)^2 \exp\left(-\frac{\Delta_{\Lambda_b}}{E}\right) \langle \tilde{O}_{V-A}^q \rangle_{\Lambda_b} = \int_0^{\omega_c} \int_0^{\omega_c} d\sigma d\sigma' \exp\left(-\frac{\sigma + \sigma'}{2E}\right) \rho_\Pi(\sigma, \sigma').
$$

(37)

We use the standard values of the condensates appearing in (31): $< \bar{q}q > = (-240 \, \text{MeV})^3$ and $< \bar{q}g \sigma Gq > = m_0^2 < \bar{q}q >$, with $m_0^2 = 0.8 \, \text{GeV}^2$ [19]. The threshold parameter $\omega_c$ has been fixed in the QCD sum rule determination of $f_{\Lambda_b}$ and $\Delta_{\Lambda_b}$ [11]: $\omega_c = 1.1 - 1.3 \, \text{GeV}$. The parameter $b$ appearing in the baryonic current $J$ in (22) has also been fixed in [11] studying the $\Lambda_b$ matrix element of the kinetic energy operator. The choice $b = 1$ allowed to obtain $f_{\Lambda_b} = (2.9 \pm 0.5) \times 10^{-2} \, \text{GeV}^3$ and $\Delta_{\Lambda_b} = 0.9 \pm 0.1 \, \text{GeV}$.

Using this set of parameters, we derive from eq.(37) the result depicted in fig.2. A stability window is observed, starting at a value of the Borel variable $E \approx 0.2 \, \text{GeV}$ and

\footnote{In [11] the value $< \bar{q}q > = (-230 \, \text{MeV})^3$ has been used. The numerical results for $f_{\Lambda_b}$ and for $\Delta_{\Lambda_b}$, within the quoted uncertainty, are not affected by this choice.}
continuing towards large values of $E$; in this range the result for $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$ is independent of the external parameter $E$. It is known, however, that large values of the Borel variable do not provide us with interesting information, since in this region the sum rule is sensitive to the continuum model. Our sum rule (33) is particularly affected by this problem, due to the high dimension of the spectral density. Therefore, we are forced to consider a narrow region of $E$, close to the values $E \simeq 0.2 - 0.3$ GeV; this is the Borel region already considered in the QCD sum rule analysis of $f_{\Lambda_b}$ and $\mu_{\pi}^2(\Lambda_b)$ [11]. The variation of the sum rule result with $E$ and with the continuum threshold $\omega_c$ provides us with an estimate of the accuracy of the numerical outcome. We find:

$$\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b} \simeq (0.4 - 1.20) \times 10^{-3} \text{ GeV}^3,$$  

(38)
a result corresponding to the parameter $r$ in the range: $r \simeq 0.1 - 0.3$.

It is worth observing that the various contributions to the spectral function in (38) have different signs and similar sizes, and that cancellations occur among the various terms. This is a common feature of the sum rule analyses of heavy baryon systems [11, 21, 22], since the loop terms are suppressed by phase-space factors and are comparable with the nonperturbative corrections. A different procedure can be followed to soften this problem, using a partial resummation of the nonperturbative corrections through nonlocal condensates [23]. Another possibility to test of the numerical result (38) consists in assuming a local quark-hadron duality prescription [24], that amounts to calculate the matrix elements of $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$ and $f_{\Lambda_b}$ by free quark states produced and annihilated by the baryonic currents in (17) and (27), and then averaging on a duality interval in $\omega, \omega'$. The resulting equation for $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$ is simply given by:

$$\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b} = \frac{\int_0^{\omega_c} \int_0^{\omega_c} d\sigma d\sigma' \rho_{\text{pert}}^{11}(\sigma, \sigma') \exp\left(-\frac{\sigma + \sigma'}{2E}\right)}{\frac{1}{20\pi^4} \int_0^{\omega_c} d\sigma \sigma^5 \exp\left(-\frac{\sigma}{E}\right)},$$  

(39)

where in the denominator the perturbative spectral function of the two-point correlator in eq.(27) appears. It is possible to check the numerical outcome for the binding energy $\Delta_{\Lambda_b}$ by this method: the result is reported in fig.3a-b where it is shown that the same value for such a parameter is obtained from 2 and 3-point sum rules. As for $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$, the result is depicted in fig.3c; it corresponds to the value: $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b} \simeq (0.6 - 1.2) \times 10^{-3} \text{ GeV}^3$ in agreement with the result (38).

Let us consider, now, the parameter $\tilde{B}$ in eq.(16). As it emerges considering the diagrams in fig.1, in our computational scheme only valence quark processes are taken
into account, and therefore a sum rule for the matrix element in (16) would produce the result $\tilde{B} = 1$. The calculation of the contribution corresponding to the diagrams of the type in fig.1 immediately confirms this conclusion.

3 Conclusions

Within the uncertainties of the method, we have obtained by QCD sum rules small values for the matrix elements of $\langle \tilde{O}_{V-A}^q \rangle_{\Lambda_b}$ and $\langle \tilde{O}_{V-A}^q \rangle_{\Lambda_b}$, comparable with the outcome of constituent quark models. Results smaller then the value obtained using the mass splitting $\Sigma^{*}_b - \Sigma_b$ suggest that an experimental confirmation of this mass difference is required.

From our results we conclude that the inclusion of $1/m_Q^3$ terms in the expression of the inclusive widths does not solve the puzzle represented by the difference between $\tau(\Lambda_b)$ and $\tau(B_d)$. As a matter of fact, using the formulae in (15) for the lifetime ratio, the value in eq.(38) together with $\tilde{B} = 1$ gives:

$$\tau(\Lambda_b)/\tau(B_d) \geq 0.94 . \quad (40)$$

It seems to us unlikely that the next order contribution $m_Q^{-4}$ can solve the problem. If the measurement of $\tau(\Lambda_b)$ and $\tau(B_d)$ will be confirmed in future, we feel that a reanalysis of the problem will be required, as suggested for example in [25]. Meanwhile, it is interesting that new data are now available for other $b$–flavoured hadrons, e.g. $\Xi_b$ [1], although with errors too large to perform a meaningful comparison with $\Lambda_b$. Such new information will be of paramount importance for the complete study of the problem of the beauty hadron lifetimes, an argument that definitely deserves further investigation.

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FIGURE CAPTIONS

Fig. 1
Set of diagrams contributing to the spectral function (31). Diagram contributing to the perturbative term (a), to the $D = 3$ term (b), to the $D = 5$ term (c,d), to the $D = 6$ term (e). The thick lines correspond to the $b$-quark propagator in HQET.

Fig. 2
Sum rule (37) for the matrix element $\langle \tilde{O}_{V-A} \rangle_{\Lambda_b}$ as a function of the Borel variable $E$. The curves refer to the threshold parameter $\omega_c = 1.1 \, GeV$ (continuous line), $\omega_c = 1.2 \, GeV$ (dashed line), $\omega_c = 1.3 \, GeV$ (dotted line).

Fig. 3
Sum rule (39) for the matrix element $\langle \tilde{O}_{V-A} \rangle_{\Lambda_b}$ obtained using the local duality ansatz. In (a) and (b) the mass parameter $\Delta$, obtained from the two-point sum rule and from the sum rule for the four quark operators is depicted. The curves refer to $\omega_c = 1.1 \, GeV$ (continuous line), $\omega_c = 1.2 \, GeV$ (dashed line), $\omega_c = 1.3 \, GeV$ (dotted line).
Fig. 1
Fig. 1
Fig. 1
Fig. 2
Fig. 3