The extended Bose–Hubbard (BH) Hamiltonian is controlled by three main physical parameters—the strength of on-site interactions, $U$, the tunnelling strength, $t$, and the interaction strength between nearest neighbour lattice sites, $V$ (ref. 1). Whereas the standard regime, in which $V$ plays a negligible role, has been thoroughly studied3–6, exploring the situation in which $V$ controls the many-body ground-state continues to seriously challenge experimental research in condensed-matter physics7. In this regime, theoretical works have predicted that long-range interactions stabilize quantum phases that spontaneously break the lattice symmetry8–18, such as density waves manifesting chequerboard (CB) or stripe solids. Such phases have been observed for fermionic systems15–17, whereas their implementation for bosons still constitutes a long-standing goal.

Theoretically, it is established that dipolar bosons in a lattice provide an ideal platform to implement the extended BH model10–12. Here, we experimentally follow this path by confining semiconductor dipolar excitons in an artificial two-dimensional square lattice. We then evidence two insulating phases, at unity and half filling of the lattice sites. The former case marks the Mott insulator (MI) regime, in which lattice sites are all occupied by a single exciton19. On the other hand, an incompressible region at half filling points towards an exciton density wave, which spontaneously breaks the lattice symmetry, favoured by nearest-neighbour (NN) repulsions. For our physical parameters, mean-field and exact diagonalization calculations predict that this phase corresponds to a CB solid. Importantly, we directly support this expectation, by measuring the thermal melting of both insulating phases, which quantitatively agree with theoretical predictions.

**Dipolar excitons in a lattice**

In recent years, different techniques for engineering tunable lattice potentials in excitonic systems have been developed, including optical20–22 or moiré lattices20–23. As illustrated in Fig. 1a, here we follow the approach devised in refs. 18,26–28 and polarize an array of gate electrodes deposited at the surface of a field-effect device embedding a GaAs double quantum well. These electrodes imprint a sinusoidally varying electric field, perpendicular to the plane of the two quantum wells, where electrons (black balls) and holes (white balls) are spatially separated to realize dipolar excitons (red arrows)29–32. With the excitonic dipoles all being aligned with the applied electric field, dipolar excitons are confined in an artificial lattice with a 250 nm period, in which they interact through strong dipolar repulsions between nearest-neighbour lattice sites then stabilize an insulating state at half filling. This characteristic feature of the extended Bose–Hubbard model exhibits the signatures theoretically expected for a chequerboard spatial order. Our work thus highlights that dipolar excitons enable controlled implementations of boson-like arrays with strong off-site interactions, in lattices with programmable geometries and more than 100 sites.
The states accessible to dipolar excitons in the lattice are directly visualized in the PL spectrum emitted in the very dilute limit, that is, when the mean density \( \bar{n} \) is around 0.2 excitons per site. Indeed, Fig. 1d shows that the spectrum then consists of two peaks separated by 150 \( \mu \)eV. This splitting matches the energy separation theoretically calculated between the two Wannier states (WS) confined in the lattice (Fig. 1b), with the lattice depth being around 250 \( \mu \)eV (Methods). Thus, we quantitatively reproduce the PL spectrum by adding two Lorentzian-like lines separated by 150 \( \mu \)eV, with each line profile being given by the spectral response of our imaging spectrometer (with 150 \( \mu \)eV full-width-at-half-maximum). We then adjust only the amplitude of the two lines that reflect the fraction of excitons occupying the corresponding WS (orange and green), as shown in Fig. 1d. On the other hand, in a denser regime, when \( \bar{n} \approx 1.3 \) excitons per site, we observe a broad peak that signals excitons occupying the continuum states accessible above the lattice potential (blue in Fig. 1b). The energy splitting between this contribution and the lattice one, around 500 \( \mu \)eV, provides an order of magnitude for the dipolar interaction strength between excitons.

**Extended BH model**

Following the approach detailed in refs. 18,33, we computed the parameters of the extended Bose–Hubbard Hamiltonian. We assumed that the two WSs (WS 1 and 2 in Fig. 1b) correspond to \( s \) and \( p \) orbitals, respectively. Then, we deduced that on-site interactions have a strength greatly exceeding the lattice depth, with, for instance, \( U_{1,1} \approx 1 \) meV for excitons populating WS 1 (Methods, ‘Physical parameters of the Bose–Hubbard Hamiltonian’). As a result, lattice sites cannot be doubly occupied, because on-site interactions easily ‘expel’ excitons into the continuum. Moreover, the interaction strength \( V \) between NN lattice sites largely exceeds the tunnelling strength, \( t \), for both WS (Methods), which is necessary to stabilize symmetry-breaking phases.

**Incompressible phases in the lattice**

To detect the buildup of insulating phases in the lattice, we measured the exciton compressibility, \( \kappa \). For this, we monitored statistically the maximum of the PL spectrum \( (A_{\text{max}}) \). Precisely, we computed the average \( \bar{A}_{\text{max}} \) and standard deviation \( \sigma(A_{\text{max}}) \), which directly quantify \( \kappa \), as \( \sigma(A_{\text{max}})/\bar{A}_{\text{max}} \) is proportional to \( (k_B T)^{1/2} \) according to the fluctuation-dissipation theorem.

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**Fig. 1 Strongly interacting dipolar excitons in a lattice.** a, An array of polarized surface electrodes (gold) imprint a 250 nm period-two-dimensional lattice for dipolar excitons, made by electrons (grey) and holes (white) spatially separated in a double quantum well (DQW, white layers). b, In the lattice, dipolar excitons are confined in two WS (1 and 2). \( U_{1,1} \) and \( U_{2,2} \) denote on-site interaction strengths for excitons in WS 1 and 2, respectively, and \( U_{1,2} \) for excitons in distinct WS, whereas \( V_{1,1} \) marks the strength of dipolar repulsions between NN sites for the first WS. c, Phase diagram computed using a two-band mean-field model, highlighting that excitons can realize CB, MI and normal fluid (NF) phases. CB and MI configurations are illustrated on the left side and the vertical line highlights our lowest bath temperature. d, PL spectrum for a mean density \( \bar{n} = 0.2 \) exciton per site. The black line displays the modelled profile (\( R^2 = 98\% \)) by summing the emissions of excitons populating the WS1 and 2, orange (67% fraction) and green (33%), respectively. e, PL spectrum for \( \bar{n} = 1.3 \) excitons per site. The solid black line provides the profile reproduced (with \( R^2 = 99\% \)) by setting 54% and 46% occupations for the first and second WS, respectively (orange and green shaded regions). The blue area marks the contribution from excitons in the continuum states above the lattice. Measurements shown in d and e were performed at \( T = 330 \) mK; error bars display Poissonian noise.
Thermal melting of insulating phases

Overall, Fig. 2 provides evidence for the theoretically expected MI and CB phases at $\pi = 1$ and $\pi = 1/2$, respectively. To further support this conclusion we measured the temperature dependence of the exciton compressibility. Hence, we studied the thermal melting of the two insulating phases and compared our observations with the theoretical phase diagram, measuring cuts along the horizontal axis, as indicated in Fig. 1c (dotted lines).

For $\pi = 1$, Fig. 3a shows that $x$ increases slowly towards the level given by Poissonian fluctuations (grey points). The latter is reached for $T \gtrsim 750$ mK, in agreement with the theoretical critical temperature for the melting of the MI (vertical dashed line). In fact, the solid lines in Fig. 3a highlight that our observations accurately follow mean-field predictions, for both $x$ and the level of Poissonian fluctuations (violet and grey, respectively). Hence, we confirm the calculated magnitudes of on-site interactions $U$, as these govern the melting of MIs. This melting is possibly scrutinized by modelling the PL profile. Indeed, Fig. 3b,c indicates that the fraction of excitons with no NN (orange) increases by around 20% between 330 and 750 mK. At the same time, the population of excitons occupying the second WS or the continuum is also enhanced. These combined variations manifest that a significant fraction of empty sites are thermally activated. We attribute their emergence to the very strong on-site interaction strengths, so that thermal excitations expel excitons from the lattice (Fig. 1b). Finally, in the normal phase ($T \geq 750$ mK), we note that the various occupation fractions vary weakly.

On the other hand, Fig. 3d reveals that for $\pi = 1/2$ $x$ increases steeply, while $T$ is enhanced. Below the critical temperature calculated for the melting of the CB phase, around 400 mK (vertical dashed line), $x$ is sub-Poissonian and then follows the (classical) variation given by Poissonian fluctuations (grey points). Again, the solid lines in Fig. 3d signal that mean-field calculations quantitatively follow our experimental observations for both $x$ and the Poissonian noise level (orange and grey, respectively). These predictions only rely on the Bose–Hubbard parameters calculated for the profile of our lattice potential, and by setting $V_{1\lambda} = 35$ μeV. This value is in good agreement with the one deduced in Fig. 2. Thereby, we confirm that $V_{1\lambda}$ is around 30 μeV, which is reasonable compared to the magnitude calculated from the theoretical profile of the lattice potential (Methods).

As for the MI phase, we studied the PL spectrum to extract the thermal variation of the exciton distribution for $\pi = 1/2$. At 330 mK, Fig. 3e shows that the PL is only due to the recombination of excitons in the first WS, with no NN interactions. Increasing $T$, we observe that the contribution at $\Delta$ above the lowest WS grows rapidly (middle panel of Fig. 3f). This higher energy PL, dominant for $T \geq 750$ mK, reflects thermal excitations of the CB, so that excitons tunnel between lattice sites and then interact with excitons in neighbouring sites. Furthermore, the bottom panel of Fig. 3f verifies that both the second WS and the continuum are weakly occupied, so that these states play a negligible role.

Conclusions

Our studies evidence that dipolar excitons can be used to controllably implement extended Bose–Hubbard Hamiltonians. On the one hand this offers a new platform to map Ising models with bosonic arrays, potentially across hundreds of lattice sites. On the other hand, for the strength of NN interactions extracted in our studies ($V_{1\lambda} = 30$ μeV), we expect a lattice supersolid phase for exciton temperatures around 10 mK, which is within experimental reach. For our current device this phase would build up in the lowest energy WS, but more exotic configurations seem accessible in the parameter space that can be explored with dipolar excitons. In particular, for shallower lattices, CB and lattice supersolids can theoretically form simultaneously in both $s$-like and $p$-like orbitals. Such multicomponent, symmetry-breaking collective states would provide a novel realm for research of quantum matter.

Figure 2a presents the variation of $\alpha(A_{\text{max}})/A_{\text{max}}$ at $T = 330$ mK, as a function of the average power of the loading laser, $P$. For two specific excitations, $P = 8$ and 17 nW (centre of orange and purple regions), Fig. 2a shows that $\alpha$ is strongly decreased compared to the level of Poissonian fluctuations (grey region). Furthermore, for $P = 17$ nW we expect that $\pi = 1$ (Methods). Noting that PL intensities differ by around twofold in Fig. 2b–d, we deduce that $\pi = 1/2$ for the former excitation. Accordingly, Fig. 2a signals two insulating phases, which extend over 100 lattice sites according to the spatially resolved PL intensity and intensity fluctuations (Extended Data Fig. 2). Importantly, we verified that the emergence of the incompressible states at $\pi = 1$ and $\pi = 1/2$ does not depend on the region of the lattice explored experimentally (Extended Data Fig. 3).

Figure 2b,d compares the PL spectra radiated at $\pi = 1$ and $\pi = 1/2$. We first note that, for the latter case (Fig. 2b), the maximum of the PL lies at the energy of the first WS (see left vertical line), whereas for $\pi = 1/2$ it does not coincide with any WS energy (Fig. 2d). Instead, the PL maximum lies at $\Delta = 100$ μeV above the energy of the first WS. As we theoretically expect that, for $\pi = 1/2$ and 1, excitons occupy the first WS and realize CB and MI phases, respectively, we deduce that $\Delta = 4V_{1\lambda}$. $V_{1\lambda}$ denoting the strength of NN interactions for the first WS. Figure 2 thus yields $V_{1\lambda} = 25 \pm 5$ μeV.

We have scrutinized the buildup of the two insulating phases by modelling the PL spectra emitted for every average filling $\pi$. Figure 2c then shows that for $\pi = 1/2$ the fraction of excitons with no NN interaction is maximized, reaching around 90% for $P = 8$ nW (Fig. 2d and Extended Data Fig. 4), as expected for a CB solid. Increasing $P$ breaks this distribution, and remarkably for $\pi = 1$ we observe a characteristic feature of MIs, namely that the fraction of excitons interacting with four NN is maximized suddenly. For $P = 17$ nW, it reaches around 90% of the exciton population (Fig. 2b and Extended Data Fig. 4), underlining that excitons are spontaneously ordered with one exciton per site.
**Fig. 3 | Melting of insulating phases.** a, Temperature variation of the compressibility $\kappa$ measured for $\pi = 1$ at the maximum of the PL spectrum (black points) and its theoretical variation deduced from mean-field calculations (violet line). Grey points and the grey line display the measured and theoretical level of Poissonian fluctuations, respectively. b, PL spectra at $T = 330$ and 750 mK for $\pi = 1$. Black lines provide modelled profiles ($R^2 = 98$ and 97%) with orange and violet areas marking the contributions from excitons in the first WS, without and with NN interactions, respectively. Excitons occupying the continuum (blue) and the second WS (green) are also shown. c, Temperature dependence of the exciton fractions occupying the first WS without (orange) or with (violet) NN interactions, together with the fraction of excitons occupying the continuum (blue) and the second WS (green). d–f, Results for the same experiments as in a–c for temperature variation of compressibility (d), PL spectra (e) and temperature dependence of exciton fraction (f) for $\pi = 1/2$. Error bars display our statistical precision in a–d, the Poissonian noise level in e–f and the standard deviation in c–f.

**Online content**

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Device and experimental procedure

The 250 nm period electrostatic lattice was realized using a heterostructure described in ref. 18. It was based on two 8 nm wide GaAs quantum wells, separated by a 4 nm AlGaAs barrier. The quantum wells were positioned 200 nm below the surface of the field-effect device in which they were embedded, and 150 nm above a conductive layer that served as the electrical ground. The lattice potential was engineered using the procedure detailed in the Supplementary Information of ref. 18. Then, we designed and realized gate electrodes deposited at the surface of the heterostructure. These were polarized at around 1 V in our experiments, yielding a periodically varying electric field perpendicular to the quantum well plane. This field imprintes the lattice potential of the excitons due to their interaction with the permanent electric dipole of the excitons. The latter has an amplitude around 12 e nm, where e denotes the electron charge, so that the lattice depth is about 250 μeV for 1 V applied across our 350 nm thick field-effect device.

In our studies, electronic carriers were injected into the lattice potential using a laser excitation at resonance with the direct exciton absorptions of each quantum well. The laser excitation was set with a rectangular profile at the surface of our device, with an area equal to approximately 10 × 5 μm². We then studied spectrally the PL re-emitted by dipolar excitons in the 3 × 2.5 μm² central region (Figs. 2 and 3), the horizontal extension being set by the width of our spectrometer's slit given our optical magnification. This area corresponds to over 100 lattice sites. Moreover, PL spectra were acquired with a grating of 1,800 lines per mm. The PL spectra were then sampled with 15 μeV precision so that the narrowest profiles that could possibly be measured, for example, a laser line, were Lorentzian-like with a 150 μeV full-width-at-half-maximum. In Figs. 1–3, we assign this profile to the emission of the first and second WS, as well as for the profile of the first WS shifted by Δ. The energy position of each contribution was fixed throughout our analysis, so that we only adjusted the amplitudes. Finally, note that the PL was acquired during a 100 ns long time interval, starting 300 ns after extinction of the loading laser pulse, whereas the exciton radiative lifetime exceeds 700 ns (ref. 20). Hence, we ensured that excitons were thermalized at the bath temperature and that the concentration of photo-injected excess carriers was minimized 19,20.

To calibrate the average exciton density, as in ref. 18, we monitored the temporal dynamics of the PL energy following the laser excitation. For that we used a region without lattice potential so that dipolar excitons realize a homogeneous fluid. Comparing the PL energy detected 300 ns after the laser pulse with the one at much longer delays, we deduced the magnitude of repulsive dipolar interactions that translate into the average exciton density. Thus, for P = 33 mW we observed that the energy shift was bound to 150 μeV, so that the exciton density was about 6 × 10⁷ cm⁻². In a 250 nm period lattice potential, this translates into n = 2 excitons per lattice site. Accordingly, we deduce that n = 1 for P = 17 mW, as we verified that the average exciton density varies linearly with the power of the loading laser pulse.

Physical parameters of the extended Bose–Hubbard Hamiltonian

At low filling (n ≤ 1), we observed experimentally (see, for example, Fig. 3e) that continuum states are barely occupied, such that our system is theoretically well described by a two-band extended BH model. In its most generic form, the corresponding Hamiltonian is given by $H = \sum_{i} h_i + \sum_{i,j} h_{ij}$ (see ref. 23), with the on-site terms $h_i = \sum_{\alpha,\beta} v_{\alpha\beta} d_i^{\alpha\dagger} b_i^{\beta} + h.c.$ and the term between NN sites $h_{ij}$ given by $h_{ij} = -t_{ij} (d_i^{\alpha\dagger} b_j^{\beta} + b_i^{\beta\dagger} d_j^{\alpha} + h.c.) + \sum_{\alpha,\beta,\gamma,\delta} v_{\alpha\beta\gamma\delta} a_i^{\alpha\dagger} b_j^{\beta\dagger} b_j^{\delta} a_i^{\gamma}$. Here, Greek (Latin) indices represent band (site) degrees of freedom, $b_i^{\alpha}$ ($d_i^{\alpha}$) are bosonic annihilation (creation) operators and $a_i^{\alpha\dagger}$ refers to hermitian conjugate. The dominant interaction terms are density–density interactions, $U_{11} = U_{1111}, U_{12} = U_{1222}$ and $U_{22} = U_{2211} + U_{1122} + U_{2222}$ for the on-site interactions, as well as $V_{11} = V_{1111}, V_{12,21} = V_{2211}$ and $V_{22} = V_{2222}, V_{1122}$ for NN interactions. Other interaction channels that mix bands have also been included within our calculations based on exact diagonalization (see "Exact diagonalization calculations", but are disregarded within the mean-field description (see "Mean-field calculations").

To evaluate the strength of the different on-site and inter-site interactions, $U$ and $V$, respectively, we proceeded as detailed in refs. 18,22. Relying on the spatial profiles of the Wannier wavefunctions expected for our lattice potential with around 250 μeV depth, we estimated the magnitudes of $U$ and $V$. For the former, dominant terms were the density–density on-site interactions, namely $U_{11} = 1$ meV and $U_{12} = 500$ μeV for the WS 1 and 2, which both exceed the exciton confinement depth in the lattice and can then not be measured. Also, we find that $U_{12} = 200$ μeV. Density–density interactions are also the dominant inter-site interactions, and for the first WS our calculations yield $V_{11} = 15$ μeV. This magnitude is two times smaller than the one deduced from our experiments. Nevertheless, in Figs. 2 and 3, within the framework of the extended Bose–Hubbard model, we attribute the energy difference between MI and CB phases to dipolar interactions between excitons confined in NN sites only. Thereby we neglect longer range contributions. These need to be included to possibly compare the value calculated from the lattice profile with the one extracted from our measurements. The value calculated for $V_{11}$ then has to be multiplied by a Madelung constant, of around 6/4, and is thus effectively increased to 22 μeV, in reasonable agreement with the measurements shown in Figs. 2 and 3. Moreover, it should be noted that the lattice potential possibly confines excitons more weakly than expected. In this case, the overlap between Wannier wavefunctions localized in neighbouring lattice sites would increase rapidly, resulting in larger values of $V_{11}$. Finally, for the tunnelling strength between nearest-neighbour lattice sites, our model calculations yield $t_{1} = 1$ μeV and $t_{2} = 7$ μeV for the first and second WS, respectively.

**Exact diagonalization calculations**

For the theoretical description we considered the extended two-band Bose–Hubbard Hamiltonian, relying on the theoretically expected parameters for $t_{ij}, U_{\alpha\beta\gamma\delta}$ and $V_{\alpha\beta\gamma\delta}$ but with increased amplitude for density–density inter-site interactions, to match the experimentally measured value of $V_{11}$ in the first WS. Specifically, we use $V_{11} = 35$ μeV, $V_{12} = 250$ μeV and $V_{22} = 40$ μeV.

We obtain the full eigenspectrum of the Hamiltonian by applying ED on a supercell, spanned by the two Lenstra–Lenstra–Lovász (LLL)-reduced supercell vectors $L_{1} = (2,2)$ and $L_{2} = (2,−2)$, with $a$ denoting the lattice period, which contains eight sites of the square lattice. In the ED calculation, we fix the particle number to four excitons (half filling). From the eigenspectrum, we calculate thermal expectation values of observables of interest. A hallmark of CB order is a peak of the structure factor at $k_{CB} \equiv \frac{(p, p)}{L}$. Indeed, the structure factor in the first WS, defined as $S_{n}(k) = \sum_{\alpha\beta\gamma\delta} \langle n_{\alpha}(k) n_{\beta}(k) \rangle e^{i k \cdot R_{\alpha}}$, with $R_{\alpha}$ the lattice vector connecting sites $\alpha$ and $\beta$, is found to exhibit a pronounced peak at $k_{CB}$ (Extended Data Fig. 1a), with a value $S_{n}(k_{CB})$ that remains more than twice as large as any other value of $S_{n}$ (up to temperatures as large as $T = 400$ mK (Extended Data Fig. 1b).

**Mean-field calculations**

Larger system sizes were studied in the mean-field approximation, which reduces the Hamiltonian to a sum of single-site terms, $H_{MF} = \sum_{i} h_{i} + \sum_{i} U_{ii} n_{i}^{2} + \sum_{i,j} V_{ij} n_{i} n_{j}$, with $h_{i} = \sum_{\alpha,\beta} v_{\alpha\beta} d_{i}^{\alpha\dagger} b_{i}^{\beta} + h.c.$, and $U_{ii} = U_{1111}, U_{ij} = U_{1122} + U_{1212} + U_{2212} + U_{2222}$ for the on-site interactions, as well as $V_{11} = V_{1111}, V_{12,21} = V_{2211}$ and $V_{22} = V_{2222}, V_{1122}$ for NN interactions. Other interaction channels that mix bands have also been included within our calculations based on exact diagonalization (see "Exact diagonalization calculations", but are disregarded within the mean-field description (see "Mean-field calculations").
of the self-consistent mean-field Hamiltonian, we calculated the phase diagram, shown in Fig. 1c, and the compressibility $\kappa$, shown in Fig. 3. Fixing the chemical potential such that the filling corresponds to $n = 1$ or $n = 1/2$, the temperature dependence of $\kappa$ shown in Fig. 3a-d matches quantitatively with the experimental data. Moreover, Extended Data Fig. 1c plots the population imbalance between sublattices whose non-zero values theoretically characterize the CB phase.

**Data availability**

Source data are provided with this paper.

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**Author contributions**

K.B. and L.P. realized the GaAs bilayer and C.L. and F.D. designed and fabricated the gate electrodes to realize the 250 nm period electrostatic lattice. C.L. and F.D. performed all experiments and data analysis. C.L., U.B., T.G, R.W.C., T.S., M.L., M.H. and F.D. contributed to the theoretical developments. All authors contributed to writing the manuscript. F.D. directed the project.

**Competing interests**

The authors declare no competing interests.

**Additional information**

Correspondence and requests for materials should be addressed to F. Dubin.

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Extended Data Fig. 1 | Theoretical hallmarks of CB order. a, Lowest band structure factor $S_1(k)$ at $T=100$ mK obtained by exact diagonalisation of a 8 site square lattice (Betts cluster) with periodic boundary conditions. It exhibits a dominant peak at quasi-momentum $k = (\pi/a, \pi/a)$, which is a characteristic signature of CB order. A second strongly suppressed quasi-peak lies at $k = (0, 0)$ (due to finite size effects), corresponding to a homogeneous liquid without any density order. b, $|S_1(\pi/a, \pi/a)|$ (black) and $|S_1(0, 0)|$ (blue) are plotted versus temperature $T$. Up to $T \lesssim T_c = 420$ mK, the structure factor signalling CB order remains at least twice as large as the structure factor for a homogeneous liquid. c, CB order parameter deduced from mean-field calculations as a function of the chemical potential $\mu$ and temperature ($T = 4, 125, 247, 389, 450$ mK in blue, violet, black, red and green respectively). The order parameter is given by the population difference $|n_A - n_B|$ between two sub-lattices $A$ and $B$ of the square lattice. Below around 410 mK $|n_A - n_B|$ is significant manifesting CB order.
Extended Data Fig. 2 | Spatially resolved PL intensity and intensity fluctuations. **a**, Spatial variations of the PL intensity $A_{\text{max}}$ (black line) and $\sigma(A_{\text{max}})/A_{\text{max}}$ (violet bars) measured at $T = 330$ mK and $P = 17$ nW, that is for the MI phase. Both $A_{\text{max}}$ and $\sigma(A_{\text{max}})/A_{\text{max}}$ vary weakly in the $3 \mu$m central region of the laser excited region, evidencing that the MI phase is homogeneous across more than 100 lattice sites. Outside this region we note that $\sigma(A_{\text{max}})/A_{\text{max}}$ increases steeply while $A_{\text{max}}$ drops, which signals that excitons realise a normal fluid. **b**, Same measurements obtained for $P = 8.2$ nW, that is for the CB phase. Results are extracted from the experiments reported in Fig. 2.
Extended Data Fig. 3 | Exciton compressibility versus average lattice filling. Fluctuations of the maximum of the PL intensity ($\sigma(A_{\text{max}})/A_{\text{max}}$) as a function of the power $P$ of the loading laser, in a different region of our two-dimensional square lattice. As for Fig. 2, experimental results are obtained by statistically analysing a series of 10 measurements for every value of $P$. Remarkably we recover that two insulating phases emerge for $P = 7$ and 14.4 nW, in good agreement with the findings discussed in the main text. Experiments were realised at $T = 330$ mK; error bars display statistical confidence while the level of Poissonian fluctuations is given by the grey shaded region.
Extended Data Fig. 4 | Residuals at π=1/2 and 1. a, PL spectrum measured at π=1/2 (top) together with the modelled profile (black line). The bottom panel displays the residuals between modelled and measured profiles (black line), compared to the amplitude of poissonian fluctuations (grey area). b, Same measurements for π= 1. Experimental results are taken from the data reported in Fig. 2b–d.