ν_µ ↔ ν_τ vs ν_µ ↔ ν_s solutions for the atmospheric neutrino problem

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The ν_µ ↔ ν_τ and ν_µ ↔ ν_s solutions to the atmospheric neutrino problem are compared with Superkamiokande data. Both the solutions with a large mixing angle seem to be consistent with the data.

1. Introduction

Recent atmospheric neutrino data by Superkamiokande [1–3] provide strong evidence for neutrino oscillations. It has been shown [1–3] that atmospheric neutrino data favor ν_µ ↔ ν_τ oscillations with maximal mixing, rather than ν_µ ↔ ν_e. However, ν_µ disappearance alone does not imply uniquely a ν_µ ↔ ν_τ solution and there is another solution ν_µ ↔ ν_s, where ν_s denotes a sterile neutrino. In this talk some aspects of the ν_µ ↔ ν_τ and ν_µ ↔ ν_s solutions are discussed.

In the past there has been a prejudice against the ν_µ ↔ ν_s solution to the atmospheric neutrino problem. The argument [4] was based on big bang nucleosynthesis which gives a condition ∆m^2 sin^4\(\theta^2\) < 10^{-4} eV^2 in order for sterile neutrinos not to be in thermal equilibrium. However, there was a loophole in this argument. Foot and Volkas [5] have shown that large lepton asymmetries will suppress ν_s ↔ ν_x neutrino oscillations. Interestingly, given certain conditions, the required lepton asymmetries can actually be created by the oscillations themselves [3]. So there is no longer any obstruction to ν_µ ↔ ν_s as a solution to the atmospheric neutrino anomaly.

Many models [6,7] have been proposed which predict large or maximal active-sterile mixing. Among others, Foot and Volkas have been obsessed by exact parity symmetric models [7] and this was the main motivation of [3] in which ν_µ ↔ ν_s was examined in detail by fitting to the contained events of the Superkamiokande atmospheric neutrino data for 414 days.

2. Analysis of the Superkamiokande contained events for 414 days

The survival probability \(P(\nu_\alpha \leftrightarrow \nu_\alpha)\) is obtained by solving the Schrödinger equation for neutrino evolution including matter effects. It is given by

\[
i \frac{d}{dx} \begin{pmatrix} \nu_\mu(x) \\ \nu_\tau, s(x) \end{pmatrix} = M \begin{pmatrix} \nu_\mu(x) \\ \nu_\tau, s(x) \end{pmatrix},
\]

where

\[
M \equiv U \text{diag}(0, \Delta m^2/2E)U^{-1} + \text{diag}(0, A_{\tau, s}(x)),
\]

is the MNS mixing matrix [9], \(x\) is the distance traveled, \(\Delta m^2\) the difference in squared masses, \(\theta\) the vacuum mixing angle and \(\nu_{\mu, \tau, s}(x)\) the wavefunctions of the neutrinos. The quantities \(A_{\tau, s}(x)\) are the effective potential differences generated through the matter effect [10]:

\[
A_\tau(x) = 0
\]

and, for electrically neutral terrestrial matter [11]

\[
A_s(x) = \frac{1}{\sqrt{2}} G_F N_n(x),
\]

where \(G_F\) is the Fermi constant, \(N_n(x)\) is the number density of neutrons along the path of the neutrino. It is this matter effect \(A_{\tau, s}\) that make a difference between the ν_µ ↔ ν_τ and ν_µ ↔ ν_s oscillations. For antineutrinos the sign of \(A_s\) is reversed.

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The way to obtain the numbers of events and evaluate $\chi^2$ is described in \cite{8}, where two quantities have been introduced to perform a $\chi^2$ analysis. One is the double ratio \cite{12}

$$ R \equiv \frac{(N_{\mu}/N_e)_{\text{osc}}}{(N_{\mu}/N_e)_{\text{no-osc}}} $$

where the quantities $N_{e,\mu}$ are the numbers of $e$-like and $\mu$-like events. The numerator denotes numbers with oscillation probability obtained by \cite{3}, while the denominator the numbers expected with oscillations switched off. The other one is the quantity on up-down flux asymmetries for $\alpha$-like ($\alpha=e,\mu$) events and is defined by

$$ Y_\alpha \equiv \frac{(N_\alpha(\cos \Theta < -0.2)/N_\alpha(\cos \Theta > 0.2))_{\text{osc}}}{(N_\alpha(\cos \Theta < -0.2)/N_\alpha(\cos \Theta > 0.2))_{\text{no-osc}}} $$

where $\Theta$ is the zenith angle, $N_\alpha(\cos \Theta < -0.2)$ and $N_\alpha(\cos \Theta > 0.2)$ are the number of upward and downward going events, respectively. $\chi^2$ with the double ratio $R$ is defined by

$$ \chi^2_{\text{atm}}(R) = \sum_E \left( \frac{R^{SK} - R^{th}}{\delta R^{SK}} \right)^2, $$

and $\chi^2$ with the up-down asymmetry $Y_\alpha$ is defined by

$$ \chi^2_{\text{atm}}(Y) = \sum_E \left[ \left( \frac{Y^{SK}_\mu - Y^{th}_\mu}{\delta Y^{SK}_\mu} \right)^2 + \left( \frac{Y^{SK}_e - Y^{th}_e}{\delta Y^{SK}_e} \right)^2 \right], $$

where the sum is over the sub-GeV and multi-GeV cases, the measured Superkamiokande values and errors are denoted by the superscript “SK” and the theoretical predictions for the quantities are labeled by “th”.

The results of the $\chi^2$ fits are displayed in Figs.1–8. In Figs. 1 and 2, $\chi^2$ is plotted against $\Delta m^2$. For $\nu_\mu \leftrightarrow \nu_\tau$, $\chi^2$ does not experience a deep minimum at the best fit point with respect to $\Delta m^2$ particularly when the $R$’s are excluded from the fit. In general, for geometrical reasons, atmospheric neutrino analysis does not constrain $\Delta m^2$ very precisely. Note that the situation is slightly different in case of $\nu_\mu \leftrightarrow \nu_s$. Figure 3 shows the allowed region of $(\sin^2 2\theta, \Delta m^2)$ at

Figure 1. $\chi^2$ as a function of $\Delta m^2$.
(a): $\nu_\mu \leftrightarrow \nu_\tau$, $\sin^2 2\theta = 1$;
(b): $\nu_\mu \leftrightarrow \nu_\tau$, $\sin^2 2\theta = 0.8$;
(c): $\nu_\mu \leftrightarrow \nu_s$, $\sin^2 2\theta = 1$;
(d): $\nu_\mu \leftrightarrow \nu_s$, $\sin^2 2\theta = 0.8$, $\Delta m^2 > 0$;
(e): $\nu_\mu \leftrightarrow \nu_s$, $\sin^2 2\theta = 0.8$, $\Delta m^2 < 0$.

Figure 2. The same as Figure 1 but with $R$ data excluded from the fit.
Figure 3. The allowed region in the $(\sin^2 2\theta, \Delta m^2)$ plane for the $\nu_\mu \leftrightarrow \nu_\tau$ scenario.

Figure 4. The same as Figure 3 but with $R$ data excluded from the fit.

Figure 5. The allowed region in the $(\sin^2 2\theta, \Delta m^2)$ plane for the $\nu_\mu \leftrightarrow \nu_s$ scenario with $\Delta m^2 > 0$.

Figure 6. The same as Figure 5 but with $R$ data excluded from the fit.
Figure 7. The allowed region in the $(\sin^2 2\theta, \Delta m^2)$ plane for the $\nu_\mu \leftrightarrow \nu_s$ scenario with $\Delta m^2 < 0$.

Figure 8. The same as Figure 7 but with $R$ data excluded from the fit.

various confidence levels for the $\nu_\mu \leftrightarrow \nu_\tau$ scenario. Maximal mixing provides the best fit, and $\Delta m^2$ values in the $10^{-3}$ to $10^{-2} \text{eV}^2$ range are favored. Note that the confidence levels are defined in the usual way by

$$\chi^2 = \chi^2_{\text{min}} + \Delta \chi^2$$

where $\Delta \chi^2 = 2.3, 4.6, 6.2, 11.8$ for the 1$\sigma$, 90% C.L., 2$\sigma$ and 3$\sigma$ allowed region respectively. Our $\chi^2_{\text{min}}$ for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations is $\chi^2_{\text{min}} = 4.5$ for 4 degrees of freedom. This is quite a good fit to the data (allowed at the 35% level). In Figure 4 we show the allowed region considering just the asymmetries instead of using both the asymmetries and the $R$ ratios. Note that in this case there are 4 data points and 2 free parameters which gives 2 degrees of freedom.

Figures 5–8 show the corresponding results for the $\nu_\mu \leftrightarrow \nu_s$ scenario. If $\Delta m^2 > 0$, smaller values of $\Delta m^2$ are disfavored because the matter effect moves both $R$ and $Y$ away from the measured values, but if $\Delta m^2 < 0$, then smaller values of $\Delta m^2$ and $\sin^2 2\theta$ are permitted at the 90% confidence level. The value of $\chi^2_{\text{min}}$ for the $\nu_\mu \leftrightarrow \nu_s$ scenario is $\chi^2_{\text{min}} = 5.1$ for 4 degrees of freedom. This is similar to $\nu_\mu - \nu_\tau$ case and also represents quite a good fit (which is allowed at 28%).

To summarize, both the solutions $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ provide a good fit to the contained events of the Superkamiokande atmospheric neutrino data.

3. Other analyses

There have been several proposals to distinguish the $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ oscillations. Matter effects in $\nu_\mu \leftrightarrow \nu_s$ oscillations in upward going muon data were first analyzed by Akhmedov, Lipari and Lusignoli [13] and more recently by Lipari and Lusignoli [14]. It has been pointed out by Liu, Smirnov [15] and Liu, Mikheyev, Smirnov [16] that signatures due to parametric enhancement in $\nu_\mu \leftrightarrow \nu_s$ oscillations may be seen in upward going muon data. Vissani and Smirnov [17] proposed to look at the ratio $(\pi^0$-events)/(two ring events). Learned, Pakvasa and Stone [18] suggested that the up-down asymmetry (upward going $\pi^0$-events)/(downward going $\pi^0$-events) can...
tell a difference. Hall and Murayama proposed a similar technique to use the up-down asymmetry in the multi-ring events. Kajita mentioned the ratio \( \frac{\pi^0 \text{-events}}{\text{e-like events}} \) which should in principle enable us to distinguish. All these analyses seem to be still inconclusive and we need more statistics and accurate knowledge on nuclear cross sections to draw a conclusion.

4. Conclusions

We have demonstrated that matter effects in the Earth have a significant role to play in comparing and contrasting the \( \nu_\mu \leftrightarrow \nu_\tau \) and \( \nu_\mu \leftrightarrow \nu_s \) solutions to the atmospheric neutrino anomaly with Superkamiokande data. So far both solutions provide a good fit to the data and we need more statistics to be conclusive. We hope that non-accelerator experiments such as Superkamiokande will distinguish them before future long baseline experiments with emulsion techniques give direct evidence.

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