Robustness of optimal transport in disordered interacting many-body networks

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The robustness of quantum transport under various perturbations is analyzed in disordered interacting many-body systems, which are constructed from the embedded Gaussian random matrix ensembles (EGEs). The transport efficiency can be enhanced drastically, if centrosymmetry (csEGE) is imposed. When the csEGE is perturbed with an ordinary EGE, the transport efficiency in the optimal cases is reduced significantly, while in the suboptimal cases the changes are less pronounced. Qualitatively the same behavior is observed, when parity and centrosymmetry are broken by block perturbations. Analyzing the influence of the environment coupling, optimal transport is observed at a certain coupling strength, while too weak and too strong coupling reduce the transport. Taking into account the effects of decoherence, in the EGE the transport efficiency approaches its maximum at a finite nonzero decoherence strength (environment-assisted transport). In the csEGE the efficiency decays monotonically with the decoherence but is always larger than in the EGE.

I. INTRODUCTION AND MOTIVATION

Quantum efficiency assesses the transport probability of particles or excitations across a quantum system [1]. A prominent example for these systems is given by photosynthetic biomolecules, where highly efficient transport can be observed [1–5]. This system can be modeled by a network of sites and bonds [6], or more abstractly, by means of disordered random networks [7]. In general, random disorder hinders the transport due to Anderson localization. Hence, it is necessary to identify structural elements, which provide efficient quantum transport in the presence of disorder. It has been demonstrated that a specific symmetry in the Hamiltonian, called centrosymmetry, improves significantly the overall transport across the network [7–10]. Recently, these studies have been extended to interacting disordered networks, modeled by embedded Gaussian ensembles (EGEs) [11–13] and their centrosymmetric version (csEGEs) [14, 15]. The many-body interactions are reflected by the correlations in these networks. It has been shown that centrosymmetry induces additional strong correlations in these systems that enhance drastically the transport [14–16]. In particular, it has been found that in almost filled systems with a rank of interaction \( k \sim n/2 \), where \( n \) is the number of particles, high quantum efficiency is observed in almost all random realizations. While at this point it is unclear whether centrosymmetry is present in photosynthetic biomolecules, it has been implemented in the laboratory [17], and it is a necessary ingredient for efficient transfer of quantum states [18], which can be used for the state transfer between quantum processors [19, 20].

The aim of this paper is to investigate the robustness of the quantum efficiency in disordered many-body networks under various perturbations. Starting with a centrosymmetric system, we determine how noncentrosymmetric perturbations affect the quantum efficiency. Centrosymmetry is essentially parity and correlations among two sectors of different parity [16] and generates a block structure in the Hamiltonian [21]. Hence, we analyze the effect of perturbations that mix sectors with different parity as well as perturbations that mix different block sectors in the Hamiltonian. We will also investigate how the transport is affected by the coupling strength of the environment through which the excitations are injected and extracted. The transport in disordered networks can not only be enhanced by centrosymmetry but also by means of decoherence [22–25], which is certainly present to some degree in biomolecules at room temperature. Therefore, we study the effect of decoherence on the transport in disordered networks with and without centrosymmetry. This will allow us to analyze the interplay between both, centrosymmetry and decoherence.

II. MODEL AND METHODS

A. Embedded random matrix ensemble for disordered interacting systems

In this Section, we introduce the fermionic embedded Gaussian ensemble [11–13], which is used as a tool to describe the statistical properties of interacting quantum many-body systems. This ensemble has found broad applications in nuclear physics, quantum information, and quantum transport: see [26].

This ensemble is constructed in the following way [11–13, 27]. We consider a quantum system of \( n \) interacting fermions distributed over \( l \) single-particle states. As we are interested in finite quantum systems, we choose typically low values for the single-particle number \( l \). Furthermore \( 1 \leq n \leq l \) in agreement with Pauli’s exclusion principle. In the embedded random matrix ensemble,
fermionic interactions are defined by

\[ H_k = \sum_{\alpha,\gamma} v_{k;\alpha,\gamma} \Psi^\dagger_{k;\alpha} \Psi_{k;\gamma}, \]  

which takes into account interactions between \( k \)-fermions (\( 1 \leq k \leq n \)). The \( \Psi^\dagger_{k;\alpha} \) is a collective creation operator of \( k \)-particles. When applied to the state \( |0\rangle \) it generates a quantum state \( \Psi^\dagger_{k;\alpha} |0\rangle \) of \( k \)-particles distributed in \( l \) levels in the specific configuration denoted by \( \alpha \). For instance, if \( l = 6 \) and \( k = 2 \), one possible \( \alpha \) configuration is \( \Psi^\dagger_{2;6} |0\rangle = a^\dagger_1 a^\dagger_2 |0\rangle = |1,0,1,0,0,0\rangle \), where \( a^\dagger_2 \) is a fermionic creation operator. By convention, the indices of the \( a^\dagger_j \) are arranged in increasing order. The corresponding annihilation operator \( \Psi_{k;\gamma} \) is constructed analogously. The coefficients \( v_{k;\alpha,\gamma} \) are independent identically distributed Gaussian variables with zero mean and unit variance. Finally, the sum in Eq. (1) runs over all distinct configurations \( \alpha \) and \( \gamma \) of \( k \)-particles distributed in \( l \) single particle states.

A natural basis to represent the interaction Hamiltonian \( H_k \) is the occupation number basis, which corresponds to the set \( \{|\mu\rangle = \Psi^\dagger_{k;\mu} |0\rangle |\mu \in S\rangle \} \), where \( S \) is the set of all the possible ways in which we can distribute \( n \)-particles in \( l \)-single particle levels. This representation of the Hamiltonian \( H_k \) can be interpreted as a disordered network, where each site represents an \( n \)-body many-particle state |\( \mu \rangle \). The total number of sites in the network is determined by the dimension of the Hilbert space \( N = \binom{l}{n} \). A pair of sites is coupled if the matrix element \( \langle \nu | H_k | \mu \rangle \neq 0 \). An example for such a network is shown in Figure 1. For the construction of the csEGEs, which is based on preserving the centrosymmetry at the one-particle level, we refer to Refs. [14] [15].

![Figure 1. Network representation of Hamiltonians from the EGE (a) and csEGE (b). Excitations are injected (\( \Sigma_{in} \)) and extracted (\( \Sigma_{out} \)) through two specific states of the system. Dashed lines indicate negative values, solid lines positive values.](image)

In [14] [15] we show that optimal transport properties are obtained for both the EGE and the csEGE if the total number of particles is \( n = l - 1 \) and \( k \sim n/2 \). In this case, centrosymmetry implies [10] [21]

\[ [H^{(cs)}_k, J_N] = 0. \]  

The exchange matrix \( J_N \) is defined by \( J_{ij} = \delta_{i,N-j+1} \), where \( \delta_{ik} \) is the Kronecker delta. The centrosymmetric Hamiltonian \( H^{(cs)}_k \) (in the occupation number basis) attains the block structure

\[ H^{(cs)}_k = \begin{pmatrix} A & C^T \\ C & J_{N/2}AJ_{N/2} \end{pmatrix}, \]

where \( A, J_{N/2} \) and \( C \) are matrices of dimension \( N/2 \times N/2 \) and \( A = A^T \), \( C^T = J_{N/2}CJ_{N/2} \). Using the orthogonal transformation \( \Omega \)

\[ \Omega = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -J_{N/2} \\ 1 & J_{N/2} \end{pmatrix}, \]

\( H^{(cs)}_k \) can be cast in a block diagonal form

\[ \Omega H^{(cs)}_k \Omega^T = \begin{pmatrix} A - J_{N/2}C & 0 \\ 0 & A + J_{N/2}C \end{pmatrix}. \]

Furthermore, the eigenvectors of \( H^{(cs)}_k \) fulfill

\[ J_N |v\rangle = |v\rangle, \quad J_N |w\rangle = -|w\rangle, \]

where half of the eigenvectors are symmetric (|\( v\rangle \)), and the other half are skew-symmetric (|\( w\rangle \)). In this context, a vector in the occupation number basis obeys parity if it fulfills either of the two equations in Eq. (6). Therefore, using Eq. (5), we see that \( H^{(cs)}_k \) has parity, revealed in its block structure, and correlations between different sectors of parity.

B. Nonequilibrium Green’s function method for quantum transport

The transport of fermionic excitations in disordered networks generated from the EGE or csEGE is studied by means of the nonequilibrium Green’s function method. We summarize briefly the essential equations. A detailed description can be found in Refs. [29] [32].

The Green’s function of the system is defined as

\[ G(E) = (E - H - \Sigma_{in} - \Sigma_{out})^{-1}, \]

where \( E \) is the excitation energy and \( H \) the system Hamiltonian (for example, a member of the EGE or the csEGE). The self-energy matrix elements

\[ \Sigma_{in,r,s} = -i\eta \delta_{r,s} \delta_{r,s}, \]

\[ \Sigma_{out,r,s} = -i\eta \delta_{r,s} \delta_{r,s}, \]

describe the effect of coupling the system to an external environment (or reservoir) through which the excitations are injected (\( in \)) and extracted (\( out \)) with rate \( \propto \eta \). Note that the self-energies have only one nonvanishing matrix element for \( r = s = in/out \). Transport is studied between the state \( |in\rangle = |1,1,\ldots,1,0,0,\ldots,0\rangle \), where all the fermions are shifted to the left, and the state \( |out\rangle = |0,0,\ldots,0,1,1,\ldots,1\rangle \), where all the fermions are shifted to the right. Note that |\( out \rangle \) is related to |\( in \rangle \) by
centrosymmetry, \( |\text{out} \rangle = J_N \, |\text{in} \rangle \). We consider such centrosymmetric related states because transport is optimal among them \[16\]. Considering other pairs of centrosymmetric states yields the same results.

A Fourier transform from the energy to the time domain shows that the matrix elements of the Green’s function \( G_{i,j}(t) \) describe the response of the state \( j \) at time \( t \) after a \( \delta(t) \) excitation of the state \( i \) at time \( t = 0 \) \[29, 30\]. Hence, the Green’s function describes the propagation of excitation through the many-body states of the quantum system. A similar situation is found in photosynthetic complexes, where an excitation is injected at a specific site, called the antenna, and extracted after a certain time at another specific site, called the sink \[33\].

The transmission probability between the states \( |\text{in} \rangle \) and \( |\text{out} \rangle \) is given by \[51\]

\[
T(E) = 4 \text{Tr}[\text{Im}(\Sigma_{\text{in}})G \text{Im}(\Sigma_{\text{out}})G^\dagger].
\] (10)

The ensemble-averaged total current, which can be driven through the system, is given by \[15\]

\[
\langle I \rangle = \int_{-\infty}^{\infty} dE T(E) \; .
\] (11)

This quantity will be used below to benchmark the efficiency of quantum transport in the system.

### III. RESULTS AND DISCUSSION

In this Section, we present the results of perturbing the transport in disordered interacting systems. In our previous work \[14, 15\], we have shown that optimal transport is obtained in a system of \( l \) states if these states are occupied with \( n = l - 1 \) fermions interacting via \( k \sim n/2 \)-body interactions. Hence, we will focus our investigations on the optimal case \( (l, n, k) = (6, 5, 3) \). Unless otherwise stated, all ensembles comprise \( 10^4 \) realizations.

#### A. Mixture of csEGE and EGE

We add to a centrosymmetric Hamiltonian \( H_k^{(\text{cs})} \) a non-centrosymmetric perturbation \( H_{k'} \) by means of the model

\[
H_T = \sqrt{1-\epsilon} H_k^{(\text{cs})} + \sqrt{\epsilon} H_{k'},
\] (12)

where \( \epsilon \in [0, 1] \) controls the strength of the perturbation. Both Hamiltonians have the same values for \( l \) and \( n \). Such a mixture of ensembles is a paradigmatic case, because in many situations the system is composed of one- and two-body interactions \[35, 36\]. However, here we investigate a much broader parameter space, because \( k \) can vary between 1 and \( n \). Note that the perturbation strength is scaled by a square root in order to keep the spectral span and the current constant in the case that both Hamiltonians are from the same ensemble with the same \( k = k' \), see for example the horizontal curves in Figure 3. In the subsequent figures \( \epsilon \) is varies from 0 to 1 with steps of size \( \Delta \epsilon = 10^{-2} \). For each value of \( \epsilon \) we calculate the corresponding ensemble average \( \langle I \rangle \).

The result of this perturbation is shown in Figure 2. The ensemble-averaged total current \( \langle I \rangle \) is plotted as a function of the parameter \( \epsilon \). We observe that in the case of optimal transport \( (k \sim 3) \) any perturbation reduces drastically and rapidly the transport. In particular, a weak perturbation \( (\epsilon = 0.25) \) with \( k' = 2, 3, 4 \) reduces the total current by approximately 30%. On the other hand, in the case of suboptimal transport \( (k = 1, 5) \) the effect of the perturbation is much weaker. Note that the case \( k = 1 \) can also be interpreted as lifting the degeneracy of the single-particle states. We observe that perturbing with \( k' = 2, 3, 4 \) initially degrades the current (because centrosymmetry is broken), while stronger perturbations again enhance the transport (because the transport is generally better for \( k' = 2, 3, 4 \)). Note that Figure 5 also confirms our previous findings \[15\] that the

![Figure 2. Perturbation of centrosymmetric Hamiltonian with a noncentrosymmetric Hamiltonian. The mean current \( \langle I \rangle \) is plotted as a function of the perturbation strength \( \epsilon \). In each figure the value interaction rank \( k \) for the csEGE is kept constant, while the interaction rank \( k' \) for the EGE perturbation is varied; see the inset. In the case of optimal transport \( (k = 3) \) any perturbation reduces drastically the transport efficiency.](image)
transport efficiency in centrosymmetric systems ($\epsilon = 0$) is higher than in noncentrosymmetric systems ($\epsilon = 1$). The total current as a function of the two parameters $k$ and $k'$ shows several symmetries \( \langle I(k,k') \rangle = \langle I(n-k,k') \rangle = \langle I(n-k,n-k') \rangle \). This is a consequence of the way in which the ensemble is defined [see Eq. (1)] and has nothing to do with particle-hole symmetry [15, 27]. These symmetries will also appear in the perturbations discussed below.

When two Hamiltonians from the EGE or from the csEGE are mixed, as shown in Figure 3, we observe a transition between the cases of optimal transport ($k \sim 3$) and suboptimal transport ($k = 1, 5$). Moreover, it is confirmed clearly that centrosymmetry (top curves) enhances significantly the transport efficiency compared to noncentrosymmetric systems (bottom curves).

**B. Breaking parity and centrosymmetry by block perturbations**

Taking into account the block diagonal form of the Hamiltonian Eq. (5), the matrix that breaks parity can be written as

\[
O H_B O^T = \begin{pmatrix} 0 & B \\ B & 0 \end{pmatrix},
\]  

where for simplicity we consider $B$ as a member of the Gaussian orthogonal ensemble. It is evident that the off-diagonal blocks mix different parity sectors in Eq. (5). In the occupation number basis, this perturbation takes the form

\[
H_B = \begin{pmatrix} B & 0 \\ 0 & -J_{N/2} BJ_{N/2} \end{pmatrix},
\]

and thus, parity breaking in the basis, where $H$ has block structure, is equivalent to a diagonal perturbation by blocks in the occupation number basis of $n$ particles. We model parity breaking by

\[
H_T = \sqrt{1 - \epsilon} H_k^{(cs)} + \sqrt{\epsilon} H_B.
\]

Figure 4 shows the effect of parity breaking on the current $\langle I \rangle$ as a function of $\epsilon$. For all values of $k$, parity breaking reduces significantly the current. In the case of optimal transport ($k \sim 3$), the current decays approximately linearly (after a very short seemingly quadratic decay) and approaches for $\epsilon \approx 0.8$ the corresponding values of the EGE. For $k = 1, 5$ this values is obtained already for $\epsilon \approx 0.2$. For $\epsilon \to 1$ a strong reduction of the current is observed, because the Hamiltonian of the system $H_T = H_B$ consists of two independent blocks. As the injecting and extracting reservoirs ($|\text{in}\rangle$ and $|\text{out}\rangle$) are located in different blocks, the two reservoirs are effectively decoupled and transport gets completely suppressed ($\langle I \rangle = 0$).

Another perturbation is the breaking of centrosymmetry, modeled by

\[
H_T = \sqrt{1 - \epsilon} H_k^{(cs)} + \sqrt{\epsilon} H_D,
\]

where $H_D$ (in the occupation number basis) is defined as

\[
H_D = \begin{pmatrix} 0 & D \\ D^T & 0 \end{pmatrix}.
\]

$D$ is a real square matrix with Gaussian normal variables in each entry and hence, generally not symmetric.

Figure 5 shows the effect of centrosymmetry breaking by applying the off-diagonal block perturbation with the matrix $D$. For all $k$ the current decreases until its final value. In the case of optimal transport ($k \sim 3$) the current decreases $\sim 50\%$, while for $k = 1, 5$ it decreases only $\sim 33\%$. 

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**Figure 3.** Mixing two Hamiltonians from the csEGE (top curves) and from the EGE (bottom curves). The systems pass between the cases of optimal transport ($k \sim 3$) and suboptimal transport ($k = 1, 5$). In general, the transport in the csEGE is much more efficient than in the EGE.
C. Contact influence in coherent transport

A parameter that is often ignored in studies of quantum efficiency is the influence of the coupling of the environment (or reservoirs) to the central system. Hence, we analyze in Figure 6 how the current is affected by the parameter $\eta$ [see Eq. (8)], which parametrizes the coupling strength of the two real reservoirs. The left column indicates the case of the EGE and the right column for the csEGE. In the top row the scale of $\eta$ is linear, while in the bottom row it is logarithmic.

The main result is that the current is maximal for a specific finite value of $\eta$, which depends on $k$. The current decreases if the coupling is weakened or if the coupling gets too strong. Comparing the two columns, it can be observed that for a fixed value of $\eta$ the csEGE provides a higher current than the EGE. Note that the described properties are similar to the superradiance transition \[37\] and the transport can be understood also in terms of the transfer time through the system. Hence, we can interpret the optimal coupling strength as the one where the transfer time matches the rate at which the excitations are injected and extracted.

D. Transport in the EGE and csEGE in presence of decoherence

We study the effects of decoherence on the transport efficiency, comparing in particular its interplay with centrosymmetry. In order to take into account the effects of decoherence, we use Büttiker’s approach of fictitious reservoirs, where excitations are absorbed and re-injected after randomization of their phase \[38\]. This idea has been generalized by Pastawski to a continuous distribution of fictitious probes \[39\]. Following this work we attach to each state $|\mu\rangle$ a fictitious reservoir that is modeled by the self-energy

$$
\Sigma_{\mu_{r,s}} = -i\nu \delta_{r,0} \delta_{r,s},
$$

which also have to be taken into account in the Green’s function Eq. (7). The coupling strength $\nu$ of the virtual reservoirs determines the decoherence strength. The system now comprises of two real reservoirs (in, out), through which the excitations are injected and extracted, as well as $N$ virtual reservoirs, which model the effects
Figure 7. Transmission $T(E)$ for a realization from the EGE (a) and csEGE (b) under the effect of decoherence. The decoherence strength is controlled by the parameter $\nu$. The total current, given by the area below the curves, is indicated in the inset. In general, decoherence smooths out the transmission resonances. In the EGE, the current attains its maximum for a finite nonzero value of $\nu$ (decoherence assisted transport). In the csEGE, the decoherence always reduces the current.

Figure 8. Total current $\langle I \rangle$ as a function of the decoherence strength $\nu$. In all cases $l = 6$, while the values of $(n, k)$ are indicated in the inset of each figure. (a) corresponds to parameters of optimal transport, (b) corresponds to a completely uncorrelated disordered network and, (c) corresponds to a many-particle system with one-body interactions. In the EGE (blue curves), the current approaches its maximum at a finite nonzero value of $\nu$, which is known as environment-assisted transport. In the csEGE (red curves), the current decays monotonically with increasing decoherence strength but is always larger than in the EGE. Hence, the correlations induced by centrosymmetry enhance the transport much more than decoherence, which suppresses Anderson localization but also breaks correlations.

The transmission through the system is now given by the D’Amato-Pastaswki model

$$T(E) = T_{\text{in},\text{out}} + \sum_{ij} T_{\text{in},i} R_{ij} T_{j,\text{out}}, \quad \text{(19)}$$

where

$$R_{ij}^{-1} = \begin{cases} -T_{ij}, & i \neq j, \\ \sum_{k \neq i} T_{ik}, & i = j. \end{cases}$$

The transmission $T_{ij}$ from reservoir $i$ to reservoir $j$ can be calculated by means of Eq. (10) using the corresponding self-energies.

Figure 7 shows the transmission $T(E)$ for a typical realization from the EGE (a) and csEGE (b). In the case of coherent transport ($\nu = 0$), centrosymmetry generates several resonances of perfect transmission ($T(E) = 1$); see our previous work [14] for details. When the decoherence increases, the transmission resonances are smoothed out. The total current $\langle I \rangle$ as a function of the decoherence strength $\nu$ is shown for various system parameters $(n, k)$ in Figure 8. In the EGE (blue curves), the current obtains its maximum for a finite nonzero value of $\nu$. This is the decoherence assisted transport [22, 23], where the loss of the height of the resonance peaks is overcompensated by the broadening of the resonances (c.f. Figure 7) and hence, the environment fosters transport. In the csEGE (red curves), the total current decreases monotonically under the effect of decoherence. In the same way as decoherence suppresses Anderson localization and fosters transport, in the present case it also destroys the correlations induced by centrosymmetry. In spite of this, it can be observed clearly that centrosymmetry enhances transport much more than decoherence, as manifested by the total current, which in the csEGE is always larger than in the EGE. Finally, for strong decoherence ($\nu = 50$) the transport is completely blocked in both ensembles. We emphasize that these results apply...
to the ensemble-averaged current. For specific values of the energy, as can be read in Figure 7, the transmission can be enhanced or decreased by decoherence depending on the actual value of the energy considered. In particular, close to a resonance we observe that decoherence may increase the transmission, while far from it the transmission is suppressed. This behavior was noticed already by D’Amato and Pastawski [9] cf. Fig 3] for a certain noncentrosymmetric system.

IV. CONCLUSIONS

We have studied the robustness of the transport efficiency in disordered interacting many-body quantum systems, addressing in particular the role of centrosymmetry. The efficiency has been quantified by the average total current $\langle I \rangle$ that can be driven through the system.

We have analyzed how the transport efficiency is affected when a centrosymmetric $k$-body EGE is perturbed by a $k'$-body noncentrosymmetric one, see Figure 2. It was found that in the optimal cases ($k \sim 3$) the efficiency is reduced significantly, while in the suboptimal cases the efficiency is less affected. When two Hamiltonians from the csEGE or from the EGE are mixed [see Figure 3] it is clearly observed that the transport in centrosymmetric systems is always better than in the corresponding noncentrosymmetric systems. We have studied the effect of block perturbations that break parity and centrosymmetry, see Figure 4 and Figure 5. It was found that, similarly to the case of mixing csEGE with EGE, the transport efficiency decays to a minimal value. Investigating the effect of the coupling strength $\eta$ to the environment, we have shown in Figure 6 that the transport efficiency approaches a maximum at a specific value of $\eta$, whereas too weak and too strong coupling hinders the transport. Finally, analyzing the interplay of decoherence and centrosymmetry in Figure 7 and Figure 8 we have found that in the EGE the transport efficiency can be enhanced by decoherence, which is known as environment-assisted transport. In the csEGE the efficiency is reduced monotonically by decoherence, and therefore there is no signature of environment-assisted transport. We interpret such suppression of transport as a consequence of decoherence affecting the correlations induced by centrosymmetry and parity. Yet, the resulting net current is always higher than for the noncentrosymmetric ensemble.

The results about decoherence are interesting in various aspects. While it is not clear if centrosymmetry is present in efficient photosynthetic biomolecules, it certainly defines an alternative for the design of efficient transport devices. The resulting transport properties in presence of centrosymmetry are an improvement over those by environment-assisted transport, and may likely exceed also the superradiance controlled by disorder [41]. These results could be experimentally tested using finite discrete optical lattices or in spin chains with NMR techniques; see [17]. Thus, centrosymmetry represents a valuable option worth considering for optimal transport.

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