A political foundation of public investment and welfare spending

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Abstract
This paper develops a political economy model to examine the implications of political selection under an authoritarian regime. We formalize the fiscal policy choice of local governments, focusing on two political selection mechanisms and their implications for public investment and welfare spending. A growth-oriented promotion system induces local officials to increase public investment, which may increase output but crowd out welfare transfers. This mimics the recent investment-driven growth in China and relatively low effort to tackle high inequality. Under a broader incentive structure, we show that it is possible for an authoritarian regime to attain the social welfare of a democracy.

1 | INTRODUCTION

One of the most intriguing problems in public economics is how government spending can be used to achieve socioeconomic objectives. Political institutions and politicians’ incentives have significant impact on the allocation of public revenue and thus have huge implications for GDP, in both democracies (e.g., Alesina et al., 1997; Glaeser & Shleifer, 2005) and authoritarian regimes (e.g., Li & Zhou, 2005; Su et al., 2012; Xu, 2011). This paper develops a model for an authoritarian regime where the perceived political promotion system offers incentives to local government officials that tilt policy toward productive public investment that stimulate GDP. We show that this public investment focus may crowd out public spending on social welfare. We also show how an authoritarian regime can revise promotion criteria to achieve the level of welfare attained under a democratic regime.
The electoral process and economic outcomes are better understood under democracy than under authoritarian regimes. The political economy literature assumes that in most democracies policies are chosen by majority voting. Politicians and political parties respond to the demand of the electorate through the median voter (or some variation) (Persson & Tabellini, 2000). In contrast, in authoritarian regimes, central government leaders are typically chosen by the leaders of the previous generation (elites or so called “elders”).\(^1\) Higher level officials appoint local government officials. As a result, local government officials are less responsible to the public, and more to upper level officials, who determine their career advancement (Maskin et al., 2000).

Clearly, different incentive structures yield different public policies. Democracies tend to focus on the electorate’s utility, allocating more expenditures that improve social welfare, as this directly affects the electorate’s well-being and thus their voting behavior. In an authoritarian regime, local government officials care relatively less about people’s current utility, and local officials are more accountable to the upper level government. This relationship between the central and local governments is a principal-agent problem. While the central government’s stated goal is to maximize social welfare, the goal of local officials is to maximize their individual interests such as career advancement. Local officials are appointed and promoted by the central government and thus behave according to the incentives provided by the central government.

For example, China achieved remarkable economic performance over the last four decades despite weak economic institutions. Many argue that China’s growth was partly due to highly active local governments (Alesina et al., 1997; Chen et al., 2005; Dong et al., 2021; Knight, 2015). Local officials devoted tremendous attention to enhancing regional economic growth (Blanchard & Shleifer, 2001; Chen et al., 2005; Wang & Wen, 2019; Zilibotti, 2017). As local governments played a crucial role in this growth, it is important to understand these officials’ incentives from a political economy perspective.\(^2\)

Public spending promotes economic development through two main channels. It can affect GDP directly as a production input and enhance GDP indirectly by lowering production costs or improving productivity. When used as transfers, public spending may also directly increase utility. We define the first type of spending as “productive public spending” and the second as nonproductive but utility-enhancing “public transfers.” Productive public spending increases the productivity of private firms, whilst public transfers directly enhance the welfare of households. Public transfers affect income inequality (Fan et al., 2020; Fleurbaey, 2009), but are subject to the law of diminishing marginal utility.

How do local governments allocate government revenue? To answer this question, the incentives of local governments must be better understood. When local public policy is made by local officials, but their career path is controlled by the central government, local officials have an incentive to make policies that satisfy the promotion criteria set up by the central government. This, in turn, provides incentives to achieve central government objectives. To address this question we build a political economy model with local government public spending on productive goods (such as infrastructure) and welfare support (such as a minimum living guarantee).

We analyze the implications of local officials’ actions for public investment, GDP, and social welfare, including how local officials react to incentives. The local government’s toolbox

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\(^1\)Lin and Wang (2017) provide a discussion of the incentives of central government leaders and the mechanisms of how their actions affect economic performance.

\(^2\)Maskin et al. (2000) note that China’s fiscal system shares features of fiscal federalism, where tax revenues and government spending are allocated by different levels of government with local governments providing public goods in local jurisdictions. In contrast, see Shinohara (2020) for a study of strategic delegation with interregional negotiations.
includes expenditure on infrastructure and transfers, and it uses these policy variables to achieve alternative objectives: maximize output or maximize welfare. We also show how an authoritarian regime can achieve the same level of welfare as a democracy under appropriate policy. The paper thus contributes to the theoretical public economics literature on the trade-offs between productive public goods and transfer payments, and the optimal allocation of government spending (e.g., Boadway & Keen, 2000; Boadway & Marchand, 1995; De la Croix & Doepke, 2003; Docquier et al., 2007; Fan et al., 2020; Glomm & Ravikumar, 1992, 1997; Possen & Slutsky, 1980; Samuelson, 1954).

The paper is organized as follows: Section 2 summarizes stylized facts that motivate assumptions in the model. Section 3 presents a simple model to illustrate how welfare support is redistributed among households. Section 4 introduces a model of political selection under an authoritarian regime with an incentive structure focused on output. Section 5 shows how an authoritarian regime with a broader incentive structure can attain the same welfare outcome as a democracy, building a link between the two political systems. The paper proceeds by presenting a series of propositions, each building a foundation and intuition for the next. Section 6 concludes.

2 | STYLIZED FACTS

This section summarizes a series of facts that justify key assumptions in the model.

**Fact 1.** China invests heavily in productive public goods.

Despite weak institutions, China has achieved remarkable economic performance with an average annual GDP growth rate of over 9% in the past 40 years (see table 1 in the Supporting Information Appendix for some indicators of the weak institutions). Figure 1 shows the magnitude of investment in infrastructure as a percent of GDP. China’s infrastructure investment is significantly higher than investment by many developed and developing countries.

**Fact 2.** Welfare spending in China is relatively low.

China has achieved the goal of eliminating extreme poverty by 2020, but a large fraction of the population continues to have very low income. China’s Dibao program provides cash support to those with income below a threshold to reduce poverty and inequality, and improve social stability. Table 2 in the Supporting Information Appendix summarizes the level of transfers, number of recipients, and national averages for rural and urban areas for 2006–2019.

**Fact 3.** China’s central government transfers about 70%–90% of its total tax revenue to subnational governments annually.

Fiscal decentralization has been a fundamental aspect of China’s transition to a market economy. China replaced its highly centralized fiscal management system (1978–1993) with a tax sharing system (1994–present) (Shen et al., 2012; Xu, 2011). Figure 2 shows that the central government’s transfers to provincial governments are large.
Regional output and social stability are key determinants of local government officials’ career advancement.

In China, subnational governments have the authority to manage the economy, but the central government largely determines the career path of subnational government officials (Knight, 2015; Maskin et al., 2000; Wang, 2019; Xu, 2011). Knight (2015) describes this as a principal-agent problem, where the central government provides incentives for local governments to achieve its objectives by focusing on two key performance criteria: GDP and social stability. Maskin et al. (2000) find that the political status of a Chinese province (measured by the number of Central Committee members) is positively correlated with the province’s economic rank. However, rapid economic growth gave rise to increased social instability. The number of officially recorded cases of civil unrest rose from under 8000 in 1993 to 180,000 in 2010 (Knight, 2015).

3 | REDISTRIBUTION RULES: TRANSFERS ONLY

The central message of this paper is that a trade-off may exist between GDP, achieved by higher infrastructure investment, and social welfare, achieved by higher household-specific lump-sum transfers. Overall, the paper considers a federal government with limited resources $G$ to spend on two objectives, productive public spending (infrastructure $G_P$) and nonproductive but utility enhancing transfers ($G_N$). Section 4 will show that a cutoff exists, which splits $G$ between...
productive and nonproductive spending. In this section we begin by introducing optimal redistribution rules for the government and households in the simplest case in which all government resources $G$ are used for transfer payments. We show that given an exogenously specified endowment distribution, an optimal rule exists for lump sum transfers that maximizes utilitarian social welfare and household utility for any distribution of household endowments, and show how the rules redistribute resources for a given amount of transfer payments.

### 3.1 Redistribution rule under utilitarian social welfare

Consider a simple economy without production. There are $N$ otherwise identical households with different levels of endowment ranging from $E_{\min}$ to $E_{\max}$. Household $i$ has an initial endowment of $E^i$, which is used to finance consumption. The endowment distribution $\Lambda_0$ has a mean $\mu_0$ and a variance $\sigma_0^2$. The government budget $G$ is redistributed to households as welfare transfers, where $g^i$ is the transfer payment for household $i$ with endowment $E^i$, and $\sum_{i=1}^N g^i = G$. A redistribution rule $\Omega = (g^1, ..., g^N)$ specifies the transfer $g^i$ for each household $i$. The fill-the-gap redistribution rule selects $g^i$ for each household $i$ such that

$$
\begin{cases}
  g^i = \gamma - E^i & \text{if } E^i \leq \gamma, \\
  g^i = 0 & \text{if } E^i > \gamma,
\end{cases}
$$

FIGURE 2 Transfer payments in central government’s total tax income. Source: National Bureau of Statistics, Reports on China’s central, local budgets 2000–2018
where $\gamma$ is the posttransfer consumption level for all households whose pre-transfer endowments are lower than $\gamma$. That is, the fill-the-gap rule ensures a minimum consumption level $\gamma$ for all households by using transfer payments that “fill the gap” between the pre-transfer endowment and $\gamma$ for all households with endowments below this minimum consumption level. Note that all households that receive positive transfer payments have the same post-transfer consumption level $\gamma$ and all households whose consumption exceeds $\gamma$ receive no transfer payments.

Consider a government that maximizes a utilitarian social welfare function $S_U = \sum_{i=1}^{N} u^i$ by solving the following problem:

$$\begin{align*}
\text{Maximize} & \quad S_U = \sum_{i=1}^{N} u^i = \sum_{i=1}^{N} u(E^i + g^i), \\
\text{subject to} & \quad \sum_{i=1}^{N} g^i \leq G.
\end{align*}$$

(1)

(2)

Households have a common utility function. Each household $i$ derives utility from their endowment and government transfer, $u^i = u(c^i) = u(E^i + g^i)$, where $u$ is assumed to be strictly concave and increasing in consumption $c$. The following proposition shows that the fill-the-gap rule is the unique optimal redistribution rule that maximizes utilitarian social welfare.

**Proposition 1.** A redistribution rule $\Omega$ maximizes utilitarian social welfare $S_U$ if and only if it is the fill-the-gap rule.

**Proof.** See Appendix A.

The intuition is straightforward. Because the utilitarian social welfare function is the sum of all utilities of every household, and the marginal utility of each dollar is higher for a household with lower consumption than for the one with higher consumption, allocating a larger transfer to a household with a lower pre-transfer endowment can generate higher social welfare than to a household with a higher pre-transfer endowment. To maximize social welfare, the marginal utility of the last dollar transferred to each household should be equal. This means that post-transfer consumption (endowment plus transfer) for all households that receive transfers should be the same.

For a given household endowment distribution $\Lambda_0$, the post-transfer consumption level for beneficiary households $\gamma$ and the number of beneficiary households depend on the total amount of government budget $G$. Total transfer $G$ affects only the value of $\gamma$, not the optimal redistribution rule. That is, the optimal transfer rule $\Omega$ is independent of the size of $G$. The post-transfer consumption level $\gamma$ depends on the total available transfer budget $G$ and the household endowment distribution $\Lambda_0$. Therefore, $\gamma = \gamma(G, \Lambda_0)$. Let $f(E)$ be the probability distribution function of endowment distribution $\Lambda_0$.

$$\begin{align*}
\arg\max_{\hat{E}} \gamma = \frac{\sum_{E=E_{\min}}^{\hat{E}} Ef(E) + G}{\sum_{E=E_{\min}}^{\hat{E}} f(E)}.
\end{align*}$$

where $\gamma \geq \hat{E}$, and $\hat{E}$ is the endowment level of the transfer recipient with the highest endowment when $\gamma > \hat{E}$, or the endowment level of the nontransfer-recipient with the lowest
endowment when $\gamma = \hat{E}$. $\gamma$ and $\hat{E}$ depend on the distribution $f(E) = \Lambda_0$ and $G$, where $G = N \sum_{E=E_{\text{min}}}^{E_{\text{max}}} (\gamma - E)f(E)$. This optimal redistribution rule is illustrated in Figure 3.

This optimal redistribution rule satisfies the Fairness Axiom, which is a “no re-ranking criterion” under which a transfer payment causes no re-ranking in people’s living standards. This optimal redistribution rule guarantees that all transfer recipients will have the same posttransfer consumption, which is no higher than the consumption level of those who receive no transfers. This is important as violating this axiom would affect incentives. This axiom is widely applied in the literature on public and welfare economics.

**Fairness Axiom:** For any two households $h$ and $q$, if $E_h \geq E_q$, $E_h + g_h \geq E_q + g_q$ and $g_h \leq g_q$.

This means that for a household with a lower endowment, after government welfare transfers, total income should not exceed the income of those with a higher endowment before the transfer payment, and a richer household should not get a higher transfer than a poorer one.

### 3.2 Redistribution rule for a decisive voter

The fill-the-gap rule is the redistribution rule that maximizes the utilitarian social welfare function, which might not be the most preferred redistribution rule for each individual household. Under the fairness axiom, there will be a unique distribution rule $\Omega^*_h$, which dictates how the government redistributes goods across households. Let’s now consider the redistribution rule for a specific household $h$. The utility of household $h$ with endowment $E_h$ is $u_h(E_h + g_h)$ where $g_h$ is the transfer payment she receives. As household $h$’s utility is positively related to $g_h$, she will always prefer a higher transfer payment, $g_h$. With a government budget $G$, under the fairness axiom, her most preferred redistribution rule is equal-subsidies-for-the-poor $\Omega^*_h$. This rule selects $g^i$ for each household $i$ such that

$$
\begin{align*}
g^i & = g_h & \text{if } E_i \leq E_h \\
g^i & = E_h + g_h - E_i & \text{if } E_h < E_i \leq E_h + g_h \\
g^i & = 0 & \text{if } E_i > E_h + g_h.
\end{align*}
$$

Under this rule, all households with endowments no larger than $E_h$ receive the same amount of transfer as household $h$. Households with endowments larger than $E_h$ also receive positive transfers if their endowment is smaller than the post-transfer income of household $h$. In this case, their posttransfer income is the same as household $h$’s. Households whose endowments are larger than $E_h + g_h$ receive zero transfers. Proposition 2 shows that the equal-subsidies-for-the-poor rule $\Omega^*_h$ is the unique optimal redistribution rule for household $h$.

**Proposition 2.** For any household $h$, a redistribution rule $\Omega$ maximizes the utility of $h$ if and only if it is the equal-subsidies-for-the-poor rule $\Omega^*_h$.

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3 Whenever $\gamma$ is not an endowment level, $\sum_{E=E_{\text{min}}}^{E_{\text{max}}} (\gamma - E)f(E)$ denotes a sum over a subset of $\{E_{\text{min}}, \ldots, E_{\text{max}}\} \cup \{\gamma\}$. We use this notation throughout the paper. The following examples illustrate this redistribution rule. Suppose there are 7 households in the economy, with endowments 2, 4, 6, 7, 8, 9, and 10. If $G = 10$, under the optimal redistribution rule, only the first four households receive transfers and their post-transfer consumption is 7.25. That is, $\gamma = 7.25$ and $\hat{E} = 7$. If $G = 9$, then $\gamma = 7$ and $\hat{E} = 7$.

4 This Axiom is similar to incentive preservation by Fei (1981) and consistent with Axiom 3 in Kakwani and Lambert (1998). See Kakwani et al. (2021) for a discussion of principles of social transfer programs.
Proof. See Appendix A.

Total transfer $G$ only affects the value of $g^h$, not the redistribution rule that is optimal for household $h$. As with utilitarian social welfare functions, the optimal redistribution rule is independent of the size of total transfer $G$. In democracies where policies are made by majority voting, the redistribution rule is determined by the decisive voter $d$, who is usually the median voter. The redistribution rule will therefore be the rule that maximizes the decisive voter’s utility, $\Omega^d$. Figure 4 illustrates this redistribution rule.

For a given household endowment distribution $\Lambda_0$, the post-transfer consumption level for beneficiary households and the number of beneficiary households depend on the total amount of government budget: $G = \sum_{E=E^d} E^d f(E) + \sum_{E=E^d} E^d + g^d f(E) + g^d - E)f(E)$. A higher budget $G$ would lead to a higher level of $g^d$. As $G$ increases, more households will receive transfers. Therefore, $\frac{dg^d}{dG} > 0$ and $\frac{d^2 g^d}{dG^2} < 0$. 

FIGURE 3  Optimal redistribution rule. The horizontal axis sorts households by ascending initial endowments. The dashed curve describes initial endowments. The solid line describes households’ posttransfer consumption levels. The shape of the dashed curve depends on the initial endowment distribution

FIGURE 4  Decisive voter’s optimal redistribution rule. The horizontal axis sorts households by ascending initial endowments. The dashed curve describes initial endowments. The solid line describes households’ posttransfer consumption levels. The shape of the dashed curve depends on the initial endowment distribution
THE MODEL WITH TRANSFERS AND PUBLIC INVESTMENT

This section presents a two-period overlapping generations model in which local officials face budget and social stability constraints. We show how promotion incentives drive local officials’ public policy. There is a central government, local government officials who wish to be promoted, profit-maximizing firms, and utility-maximizing households. At the beginning of the period, a local official allocates government revenue to the two types of public spending with a given budget provided by central government. Households make a consumption-savings decision, and firms use capital and labor to produce. The central government observes aggregate economic performance in the region, and decides whether to promote the local official at the end of the period. The local official’s perception is—if GDP is higher a promotion is more likely. Consistent with fact 4, the official will not be promoted if there is social unrest regardless of economic performance.

4.1 Households

In period $t$, $L_t$ households are born. They are identical, except for different levels of endowment. The endowment distribution $\Lambda_0$ has mean $\mu_0$ and variance $\sigma_0^2$. Households live for two periods. In the first period, they work and earn the competitive wage $w_t$, and in the second period they are retired. Household utility $U(c_{1,t}, c_{2,t+1})$ is defined over nonnegative consumption, is twice continuously differentiable, strictly concave on consumption set $\mathbb{R}_+^2$, and increasing in both variables. Assume that utility takes a natural logarithmic form. For household $i$,

$$U(c_{1,t}, c_{2,t+1}) = \ln(c_{1,t}) - \ln \bar{c} + \rho \left[ \ln(c_{2,t+1}) - \ln \bar{c} \right]. \tag{3}$$

c_{1,t} is consumption in period $t$, $c_{2,t+1}$ is consumption in period $t + 1$, and $\rho$ is the discount factor. Parameter $\bar{c}$ is a socially acceptable minimum level of consumption. If consumption is below this minimum level $\bar{c}$ the household will rebel, which will induce social instability (consistent with fact 4).

During the first period, young households supply their unit of labor inelastically and earn wage $w_t$ from firms. For every household $i$, they allocate the resulting income $w_t$ and their initial endowment $E_{1,t}$ between first period consumption $c_{1,t}$ and savings $s_{1,t}$. The young may receive transfers from the local government if they meet government criteria. In the second period, all old households receive transfer $\bar{c}$ from the central government (e.g., exogenous social security at minimum consumption $\bar{c}$). The assumption that the central government provides a basic pension to old households and that the local government pays transfers to qualified young households is consistent with facts 2 and 3.

For household $i$, savings is $s_{1,t} = w_t + E_{1,t} - c_{1,t}$. Savings earn return $r_{t+1}$ in the following period and support consumption when old. Therefore, second period consumption is $c_{2,t+1} = (1 + r_{t+1} - \delta)s_{1,t}$. Assume the rate of capital depreciation $\delta$ is 1 and households know the return to capital, $r_{t+1}$. The optimal saving choice made during the first period maximizes inter-temporal utility, $s^*(w_t, r_{t+1}) = \arg \max U \left[ w_t + E_{1,t} - s_{1,t}, r_{t+1}s_{1,t} \right]$. Optimal saving $s^*(w_t, r_{t+1})$ and consumption exist and are uniquely determined. Household capital in the second period is the savings in the first period, $s_{1,t} = k_{2,t+1}$.

Thus household $i$’s budget constraints are

$^5$Functional form $\ln(c_{1,t}) - \ln \bar{c}$ allows us to measure the welfare loss due to poverty, and hence the specific subsidy needed to offset the loss. The function resembles Stone-Geary preferences $\ln(c_{1,t} - \bar{c})$, but avoids the problem that the utility of a household with consumption below $\bar{c}$ is a non-real number.
\[ c_{1,t} + k_{2,t+1}^j = I_{1,t}^j, \quad (4) \]
\[ c_{2,t+1}^j = \eta_{t+1} k_{2,t+1}^j + \bar{c}. \quad (5) \]

Let \( I_{1,t}^j = w_t + E_{1,t}^j + g_{N,t}^j \) denote total income when young, at time \( t \), where \( g_{N,t}^j \) is a lump sum transfer based on the optimal redistribution rule, which depends on income.\(^6\) The household maximizes utility subject to the two budget constraints. This results in the standard Euler equation, \( \frac{c_{1,t+1}^j}{c_{1,t}^j} = \rho r \), with \( c_{1,t}^j = \frac{\eta_{t+1} I_{1,t}^j + \bar{c}}{\eta_{t+1}(\rho + 1)} \) and \( k_{2,t+1}^j = \frac{\rho_{t+1} I_{1,t}^j - \bar{c}}{\eta_{t+1}(\rho + 1)} \).

### 4.2 Firms

Firms are homogeneous and the aggregate production function is \( Y_t = F(G_{P,t}, K_t, L_t) \) in period \( t \). Following Alesina and Rodrik (1994) and Barro and Sala-i-Martin (2003), the production function has the form

\[ Y_t = A (G_{P,t})^{1-\alpha} K_t^\alpha L_t^{1-\alpha}. \quad (6) \]

\( Y_t \) is aggregate output in period \( t \), \( A \) is total factor productivity, \( G_{P,t} \) is spending on the productive public good provided by the local jurisdiction, \( K_t \) and \( L_t \) are the aggregate stocks of capital and labor in period \( t \), respectively. In per capita form, the production function is \( y_t = A (G_{P,t})^{1-\alpha} k_t^\alpha \), where \( k_t = K_t/L_t \) is capital per capita. The standard marginal conditions hold, \( w_t = A(G_{P,t})^{1-\alpha} k_t^\alpha (1 - \alpha) \) and \( \eta_t = A(G_{P,t})^{1-\alpha} a k_t^{\alpha - 1} \).

The capital stock next period per capita \( k_{t+1}^j \) follows from substituting \( w_t \) from the firm’s problem and \( E_{1,t}^j \) into \( k_{t+1}^j = \frac{\rho_{t+1} I_{1,t}^j - \bar{c}}{\eta_{t+1}(\rho + 1)} \) from the household’s problem. This gives \( k_{t+1}^j = \frac{\rho_{t+1} I_{1,t}^j - \bar{c}}{\eta_{t+1}(\rho + 1)} \), where \( E_t \) is the endowment per capita and \( g_{N,t}^j \) is the government transfer per capita. A key insight is that government transfer \( g_{N,t}^j \) contributes to the next period’s capital stock because it enables some households to be pulled out of a poverty trap and start to accumulate capital.

### 4.3 Local government

Local government officials allocate public funds to the provision of productive public goods \( (G_{P,t}) \) and welfare transfers \( (G_{N,t}) \). More productive public goods such as infrastructure increases firm productivity, resulting in higher output in the local economy. The local government official maximizes objective function \( V \), subject to a balanced budget constraint. The balanced budget constraint indicates that the local government allocates a given amount of revenue \( G \), which consistent with fact 3 is provided by the central government, between productive public goods and welfare transfers.

\[
\begin{align*}
\text{Max } V &= V(G_{P,t}, G_{N,t}), \\
\text{s. t. } G_{P,t} + G_{N,t} &= G.
\end{align*}
\]

\(^6\)Households cannot affect the amount of the transfer and take it as given when making decisions.
**Proposition 3.** Regardless of $\Lambda_0$ and the redistribution rule $\Omega$, there exists an optimal splitting rule that divides $G$ into two parts: public investment $G_{P,t}$ and transfer $G_{N,t}$.

The intuition is that $V$ assigns a real number to each combination of $G_{P,t}$ and $G_{N,t}$. Since (8) is a linear equality constraint, a split of $G$ exists that maximizes $V$. Note that $G_{P,t}$ improves households “marginal product universally, whereas transfers $G_{N,t}$ improve households” income additively and in an agent-specific manner. Therefore, the two government tools are complementary instead of perfect substitutes. The optimal redistribution rule and the optimal splitting rule are independent of each other. That is, regardless of the optimal splitting rule, the optimal redistribution rule $\Omega$ remains intact as it is invariant to the size of total transfer $G_{N,t}$.

### 4.4 Policy analysis

#### 4.4.1 Output maximizing government

The optimal splitting rule depends on the objective function of the local government official. In general, an output maximizing government tends to allocate more budget into public investment, whilst a social welfare maximizing government tends to allocate more to transfer payments. We now consider the case where local officials maximize output in their jurisdiction, subject to a balanced budget constraint.

Subject to the balanced budget constraint, $G_{P,t} + G_{N,t} = G$, the local official solves

$$\max_{G_{P,t}, G_{N,t}} y_{t+1} = A(G_{P,t})^{1-\alpha}(k_{t+1})^\alpha.$$  (9)

**Proposition 4.** For given government revenue $G$ and initial conditions $E_t$ and $k_t$, the optimal welfare support provided by a local official in period $t$ satisfies

$$G_{N,t}^* = \alpha G - L(1 + \alpha)A(1 - \alpha)^2(G - G_{N,t}^*)^{-\alpha}k_t^\alpha - L_tE_t(1 - \alpha).$$

**Proof.** See Appendix A.

The values of $G_{N,t}^*$ and $G_{P,t}^*$ depend on the values of $G, E_t$, and $k_t$. In the case that endowments are far higher than the needs for consumption, which results in a sufficiently high level of capital formation $k_{t+1}$, investing all revenue in productive public investment is preferable: $G_{P,t}^* = G$ and $G_{N,t}^* = 0$. This case is consistent with the results in this paper, but we omit this trivial case for simplicity. However, under no circumstances will $G_{N,t}^* = G$ because $G_{P,t}$ must be positive in Equation (9). It can be seen that, except for this extreme case, the optimal welfare transfer is positive from an economic output perspective. Local officials provide some level of welfare support to achieve higher GDP. Intuitively, an increase in welfare support has two opposite effects on the local economy:

- Higher transfers enable households to accumulate capital, which increases GDP.
- More welfare support decreases the provision of productive public goods, which decreases GDP.

The optimal allocation of public spending thus depends on which effect dominates. It is clear from the objective that positive welfare transfers increase output. The transfer plays two roles.
First, it can enhance the local economy’s output; low-income households can accumulate capital, inducing higher output due to more input. Secondly, it increases households’ utility and improves equality. In other words, allocating all public resources ($G$) to productive public goods will not necessarily achieve the highest possible output. Thus even when maximizing output is the objective, Proposition 3 implies that some resources will be allocated to welfare enhancing transfers. Figure 5 illustrates this.

### 4.4.2 The social stability constraint

In addition to output, another promotion criterion for local officials involves social stability. If a household’s consumption were below level $c$, it would rebel and thus threaten local social stability. Welfare transfers increase household utility and thus maintain social stability in the jurisdiction.

When the post-transfer consumption level for beneficiary households equals the socially acceptable minimum level of consumption, that is, $y = \bar{c}$, the aggregate welfare support guarantees that the consumption of every household attains at least the minimum level of consumption $\bar{c}$ and $G_{N,t} = \bar{G}_{N,t}$, where $\bar{G}_{N,t} = L_t \sum_{E=E_{\min}}^{c-w_{0}} (\bar{c} - E - w_t) f(E)$ is the minimum amount of welfare support required to maintain social stability. Under this scenario, each household whose pre-transfer income is lower than $\bar{c}$ receives a transfer that equals the difference between $\bar{c}$ and its income. Households whose pre-transfer incomes are higher than $\bar{c}$ receive zero transfers. This imposes another constraint for local officials: $G_{N,t} \geq \bar{G}_{N,t}$. This constraint ensures that social instability will not occur (i.e., households will not rebel) if welfare transfers $G_{N,t}$ are sufficiently high. In this case, the maximum provision of productive public good is thus $\bar{G}_{P,t} = G - \bar{G}_{N,t}$.

### 4.4.3 Growth with the social stability constraint

When the social stability constraint is a crucial part of the local official’s incentive structure, the local official maximizes output subject to both the balanced budget constraint and the social stability constraint. Let $\bar{G}_{N,t}$ be the welfare support that maximizes output subject only to the balanced budget constraint.

![Figure 5](image-url)  
**Figure 5** Output and productive public goods. The horizontal axis is expenditure on productive public goods $G_p$. The vertical axis is output.
Proposition 5. For given government revenue $G$ and initial conditions $E_t$ and $k_t$, the welfare support chosen by a local government depends on the initial conditions and government revenue in the local economy. Optimal welfare support is given by

$$G_{N,t}^* = \alpha G - L_t (1 + \alpha) A (1 - \alpha) + (G - \bar{G}_{N,t})^{-\alpha} k_t^\alpha$$

if $\bar{G}_{N,t} \leq \bar{G}_{N,t}$,

$$G_{N,t}^* = \bar{G}_{N,t}$$

if $\bar{G}_{N,t} > \bar{G}_{N,t}$,

where $\bar{G}_{N,t}$ satisfies $(1 + \alpha) A (1 - \alpha) + E_t (1 - \alpha) + \frac{\bar{G}_{N,t} - \alpha G}{L_t} = 0$.

Proof. See Appendix A.

When the social stability constraint binds, the local government will provide both productive public goods and welfare support. Therefore, in this case the local government provides at least a minimum level of welfare support to deter social unrest, given by $\bar{G}_{N,t} = L_t \sum_{E = E_{\text{min}}}^\alpha (\bar{c} - E - w_t) f(E)$. As a consequence, there is a lower bound on welfare transfers, $\bar{G}_{N,t}$. The optimal allocation of public spending depends on whether this lower bound binds or not.

If the minimum welfare support $\bar{G}_{N,t}$ is smaller than the output maximizing welfare support $\bar{G}_{N,t}$, then welfare support is $G_{N,t}^* = \bar{G}_{N,t}$ and spending on productive public goods is $G_{P,t}^* = G - \bar{G}_{N,t}$.

If the minimum welfare support $\bar{G}_{N,t}$ is larger than the output maximizing welfare support $\bar{G}_{N,t}$, the optimal allocations are $G_{N,t}^* = \bar{G}_{N,t}$ and $G_{P,t}^* = G - \bar{G}_{N,t}$. The local official provides the minimum level of welfare support and invests the rest in productive public goods to increase local output, and hence the probability of promotion. Figure 6 illustrates the implications of Proposition 5 for output: the optimal welfare transfer is always positive, but it may be greater than the amount that maximizes output if the stability constraint binds.

Proposition 6. When social stability is a constraint, the effect of an increase in government revenue $G$ on the welfare chosen by a local government depends on
initial conditions \( E_t \) and \( k_t \). Let \( \Phi = L_t(1 + \alpha)A(1 - \alpha)(\tilde{G}_{P,t})^{1-\alpha}k_t^\alpha + E_t \), \( L_t(1 - \alpha) + G(1 - \alpha) \). Then the following conditions hold.

**Condition 1.** If \( \Phi > G - \tilde{G}_{N,t} \), increased government revenue leads to an increase in the provision of productive public goods but a decrease in welfare support.

**Condition 2.** If \( \Phi < G - \tilde{G}_{N,t} \), increased government revenue leads to an increase in both the provision of productive public goods and welfare support.

The proof in the appendix shows that if \( \Phi > G - \tilde{G}_{N,t} \), government revenue is positively related to the optimal provision of productive public goods, but negatively related to welfare support. Under Condition 1, \( \tilde{G}_{N,t} > \tilde{G}_{P,t} \). Therefore, the welfare support provided by the local government is \( \tilde{G}_{N,t} \). Note that the government budget constraint is balanced. Therefore, for Condition 1, spending on productive public goods is lower than the provision of public goods when there is no stability constraint (\( \tilde{G}_{P,t} < \tilde{G}_{P,t} \)), and additional government revenue will be allocated to \( G_{P,t} \). That is, the local government provides the lowest level of welfare support and allocates the rest to productive public goods. The increase in the provision of productive public goods, \( G_{P,t} \), raises the wage, resulting in a smaller minimum welfare transfer required to maintain social stability. For Condition 2, when \( \tilde{G}_{N,t} < \tilde{G}_{N,t} \), both productive public goods and welfare transfers (\( G_{P,t} \) and \( G_{N,t} \)) increase when government revenue \( G \) increases.

Proposition 6 shows that for local governments, more revenue from the central government \( (G) \) does not necessarily lead to more welfare support. In a local region in which Condition 1 holds, higher government revenue \( G \) will provide more productive public goods, which will increase the wage for all households. Therefore, some households whose incomes were marginally lower than the socially acceptable minimum level \( \bar{c} \), would now have incomes higher than \( \bar{c} \) and thus no longer need transfers. However, the total income of households whose private income is still lower than \( \bar{c} \) remains unchanged. The key insight is that when Condition 1 holds, an increase in \( G \) does not automatically lead to higher welfare transfers.\(^7\) In Section 5 we consider a revision to the promotion criteria that provides an incentive to increase welfare transfers.

In summary, consistent with fact 1, our results show that an “output focused” political promotion criterion provides local government officials with an incentive to tilt resources provided by the central government toward productive public goods to stimulate the local economy. Productive public goods such as infrastructure enter the production function as a factor of production, attenuating diminishing returns to private capital and raising local output. High investment in productive public goods comes at the cost of welfare spending that could directly improve the well-being of households, especially for those in the lower end of the income distribution.

## 5 | PUBLIC INVESTMENT AND SOCIAL WELFARE

The previous section analyzed the public policy choice of local officials when the perceived criterion for promotion by the central government is local output. In this section we study the policy choice of the median voter and then show that by broadening the promotion criterion

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\(^7\)This mechanism is different from Jahandideh (2020), where the representative agent prefers more distributive policies as exogenous resource revenue increases.
from a narrow focus on local output, an authoritarian political system can induce policy that mimics the welfare outcomes achieved by democratic regimes.

5.1 The median voter

In democracies, spending on productive public goods and welfare is determined by majority voting. Each household votes for their most preferred allocation \( G_{P,t} \) and \( G_{N,t} \). As shown in Remark A1 in Appendix A, the preferences for productive government spending of all households are single-peaked. The median voter theorem then implies a Condorcet winner exists and it is given by the \( G_{P,t} \) preferred by the household with the median level of endowment. Total welfare support \( G_{N,t} \) will then be redistributed using the equal-subsidies-for-the-poor redistribution rule where the decisive voter is the household with the median level of endowment.\(^8\)

Let \( E_{1,t}^{mv} \) be the median endowment level in the distribution. The median voter's problem is to

\[
\max_{c_{1,t}^{mv}, c_{2,t+1}^{mv}} U\left(c_{1,t}^{mv}, c_{2,t+1}^{mv}\right) = \ln(c_{1,t}^{mv}) + \rho \ln(c_{2,t+1}^{mv})
\]

subject to

\[
c_{1,t}^{mv} + k_{2,t+1}^{mv} = w_t + E_{1,t}^{mv} + g_{N,t}^{mv}
\]

\[
c_{2,t+1}^{mv} = n_{t+1} k_{2,t+1}^{mv}.
\]

At an optimum, it follows that

\[
c_{1,t}^{mv} = \frac{w_t + E_{1,t}^{mv} + g_{N,t}^{mv}}{(\rho + 1)},
\]

\[
\frac{c_{2,t+1}^{mv}}{c_{1,t}^{mv}} = \rho n_{t+1},
\]

\[
k_{2,t+1}^{mv} = \frac{\rho \left(w_t + E_{1,t}^{mv} + g_{N,t}^{mv}\right)}{(1 + \rho)}.
\]

Substitute \( w_t \) from the firm's problem and Equations (11)–(13) into objective (10) to get the indirect utility for the household with \( E_{1,t}^{mv} \), where \( g_{N,t}^{mv} \) is the transfer received by the household with \( E_{1,t}^{mv} \).\(^9\)

5.2 Social welfare

To analyze the policy choices of local officials when more attention is given to social welfare rather than simply to local output, this section provides a framework that explicitly accounts for social welfare rather than a socially acceptable minimum level of consumption \( \bar{c} \). The performance of local officials is now measured by local economic performance and a measure of life satisfaction, using the Gini social welfare function proposed by Sen (1974) and

\(^8\)In a country where median income is very close to mean income, it is theoretically possible that the median post-transfer income is higher than the mean post-transfer income. However, we only consider the realistic case where median post-transfer income is no higher than the mean post-transfer income.

\(^9\)This yields \( V^{mv} = (1 + \rho) \ln \left[ A(G_{P,t})^{\rho} + n_{t+1}^{mv} (1 - \alpha) + k_{1,t}^{mv} + g_{N,t}^{mv} \right] + \rho (1 - \alpha) \ln \frac{G_{N,t}}{A(G_{P,t})^{\rho} + n_{t+1}^{mv} (1 - \alpha) + E_{1,t}^{mv} + g_{N,t}^{mv}} + \rho \ln \left( \frac{\rho}{\rho + 1} \right) - \ln (\rho + 1) + \rho \ln (4 \alpha) \).
developed by Kakwani and Son (2016) and Kakwani et al. (2019). This social welfare function reflects both absolute inequality and relative inequality.\textsuperscript{10}

The Gini social welfare function, $S_G$, is

$$S_G = \mu (1 - \text{Gini}),$$

(14)

where $\mu$ is mean income and \text{Gini} is the Gini index $\text{Gini} = \frac{1}{2N^2} \sum_{i=1}^N \sum_{j=1}^N |x_i - x_j|$, with $x_i$ and $x_j$ denoting the income of households $i$ and $j$ respectively.\textsuperscript{11} In this social welfare function, equality is the percentage of welfare loss. If there is no inequality, social welfare is $\mu$, mean income. Larger inequality leads to lower social welfare.

**Proposition 7.** A redistribution rule maximizes Gini social welfare $S_G$ if and only if it is the fill-the-gap rule.

**Proof.** See Appendix A.

Let $V$ be the local official’s broader objective function, where performance is now assessed by next period’s output, $y_{t+1} = A(G_{P,t})^{1-\alpha}(k_{t+1})^\alpha$, and social welfare function $S_G$, with weight $\eta$ on each component:

$$\max_{G_{P,t}, G_{N,t}} V = \eta[A(G_{P,t})^{1-\alpha}(k_{t+1})^\alpha] + (1 - \eta)S_G.$$  

(15)

There is also a balanced budget constraint and a redistribution rule. For $\eta \in (0, 1)$, optimal spending on productive public goods and transfers, $G_{P,t}^*$ and $G_{N,t}^*$, thus depends on government revenue $G$, initial conditions $E_t$ and $k_t$, and social welfare weight $\eta$.\textsuperscript{12}

Under the equal-subsidies-for-the-poor rule, a higher $g^d$ reduces inequality measured by the Gini. A lower Gini decreases the welfare loss from inequality and increases the likelihood a local official will be promoted. To finance a higher $g^d$, more government revenue $G$ is allocated to $G_{N,t}$. The value placed on local output is measured by $\eta$, with a lower value indicating the local government cares more about social welfare.

Let $G_{P,t}^{V1}$ ($G_{N,t}^{V1}$) be the optimal spending on productive goods (transfers) when $\eta = 1$, $G_{P,t}^{V0}$ ($G_{N,t}^{V0}$) be optimal spending on productive goods (transfers) when $\eta = 0$, and $G_{P,t}^{V}$ ($G_{N,t}^{V}$) be the optimal spending on productive goods (transfers) chosen by local officials when social welfare is included in the objective, $(0 < \eta < 1)$. Note that $\eta = 1$ corresponds to the model with the narrow output criterion and $\eta = 0$ corresponds to the case where only social welfare is maximized. When $0 < \eta < 1$, both local economic performance and life satisfaction are valued. Optimal productive public good spending is

- $G_{P,t}^{V1}$: when output is maximized.
- $G_{P,t}^{V0}$: when social welfare is maximized.
- $G_{P,t}^{mv}$: when chosen by the median voter.

\textsuperscript{10}See Fleurbaey (2008, 2009) for a review of measures of social welfare.

\textsuperscript{11}See Sen and Foster (1997). In this social welfare function, inequality is measured by the Gini coefficient. A broader class of such social welfare functions use similar measures of inequality.

\textsuperscript{12}Differentiating $V = \eta[A(G_{P,t})^{1-\alpha}(k_{t+1})^\alpha] + (1 - \eta)\left(1 - \alpha\right)A(G_{P,t})^{1-\alpha}k_t^\alpha + E_t + g_{N,t}\right)(1 - \text{Gini})$ gives:

$$\frac{\partial [A(G_{P,t})^{1-\alpha}(k_{t+1})^\alpha]}{\partial G_{P,t}} + (1 - \eta)(1 - \alpha)A'(G_{P,t})^{-1+\alpha}k_t^\alpha + (1 - \eta)(1 - \alpha)A'(G_{P,t})^{-1+\alpha}k_t^\alpha + E_t + g_{N,t}\left(\frac{d \text{Gini}}{d G} \right)_{G_{P,t}} + E_t = 0.$$
Proposition 8 summarizes the optimal public goods allocations under each objective.

**Proposition 8.** For local official objective function, \( V = \eta [A(G_{P,t})^{1-\alpha}(k_{t+1})^\alpha] + (1 - \eta)S_G, \)

(i) If \( \eta \in [0, 1], G_{N,t}^{V_1} \leq G_{N,t}^V \leq G_{N,t}^{V_0} \) and \( G_{P,t}^{V_0} \leq G_{P,t}^V \leq G_{P,t}^{V_1} \).

(ii) When \( G_{P,t}^{V_0} \leq G_{P,t}^{mv}, \) there exists an \( \eta \in [0, 1] \) such that \( G_{N,t}^V = G_{N,t}^{mv} \) and \( G_{P,t}^V = G_{P,t}^{mv} \).

**Proof.** See Appendix A. \( \square \)

Proposition 8 indicates that when both economic performance and social welfare are valued, the optimal level of welfare support provided by a local government increases compared with the scenario of maximizing output only; and local officials could choose an \( \eta \) such that the optimal level of welfare support matches the welfare support chosen by majority voting. Implications for output follow directly, where \( y(G_{P,t}^{V_1}) \geq y(G_{P,t}^{mv}) > y(G_{P,t}^{V_0}) \) when \( G_{P,t}^{V_0} < G_{P,t}^{mv} \leq G_{P,t}^{V_1} \).

The first panel of Figure 7 illustrates

the outputs from the three levels of productive public spending under each objective function: (i) output maximization \( (V^1) \); (ii) social welfare maximization \( (V^0) \); and (iii) median voter utility maximization \( (V^{mv}) \). The dotted lines show the provision of the public good, \( G_{P,t} \).

As the maxim “we get what we measure” suggests, the narrow output maximization performance criterion leads to high public good provision and high average output associated with China’s experience. In the middle panel under the broader social welfare criterion, maximum

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13 See the Supporting Information Appendix for an application to the Chinese economy.

14 For simplicity, the figure illustration does not include the trivial case when \( G_{P,t}^{V_1} = G_{P,t}^{mv} = G \).
social welfare occurs at $G_{P,V}^0$, with less public good provision relative to the two other objectives (and more transfers). Finally, the utility of the median voter is maximized at $G_{P,V}^{mv}$.  

6 | CONCLUSION

This paper develops a model to analyze trade-offs between public investment and welfare spending. The model can explain China’s remarkable investment-led growth despite weak economic institutions. We show that the optimal allocation of public spending depends on initial economic conditions, and the incentive structure and resources provided by the central government. The model provides a political foundation for why local governments in China used to focus narrowly on GDP and why their recent emphasis on broader objectives could achieve economic outcomes comparable to democratic regimes.

Two allocations of public spending are possible. In the first case, the local government provides welfare support at the lowest possible level that deters social unrest (i.e., only to poor households with income below a socially acceptable minimum). In the second case, more welfare support is provided to increase social welfare. When the central government broadens local officials’ incentives to include both GDP and social welfare, local governments optimally increase welfare support. We show how officials can achieve the same level chosen by majority voting in a democracy.

We use stylized facts from China to motivate key assumptions, but our political economy model applies to other countries. For example, the relatively autocratic East Asian Tigers, South Korea, Taiwan, and Singapore, jump started development by focusing on productive public investment (both human and infrastructure). In democracies voters may prioritize welfare transfers, whilst it is easier for autocratic governments to focus on productive investment and GDP, at the cost of welfare transfers.

Several extensions would be interesting. We consider only direct welfare transfers, which increase savings and thus contribute to GDP. Public expenditure on education would enhance human capital accumulation and have a positive effect on the quality of labor supply. Health services would improve worker productivity. Welfare expenditures on education and health increase the effective labor supply and optimal welfare spending. Infrastructure improves access to education and health and thus also indirectly increases GDP and optimal welfare spending.

With regard to the government’s objective and redistribution, we find that the fill-the-gap rule is the unique redistribution rule that maximizes the utilitarian social welfare function. Many other types of social preferences are possible. Pivato (2020) considers rank-additive social welfare functions and “population ethics,” the tradeoff between the number of future people and their quality of life. Mongin and Pivato (2021) consider a Rawlsian maximin rule, with and without uncertainty. Gersbach et al. (2021) consider redistribution that maximizes aggregate utility under a different political mechanism—a heterogeneous legislature that bargains over public expenditure. Dai et al. (2019) consider two regions in which each government maximizes local welfare with a different discount rate, and show that migration may cause redistribution to occur from impatient to patient regions. Further research that characterizes how such different redistribution rules are linked to economic primitives, such as the endowment

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15In Figure 7c, the position of $V^{mv}(g_{P,V}^1)$ and $V^{mv}(g_{P,V}^2)$ is for illustrative purposes only.
distribution and the size of the government’s budget in our model, could provide insight into tackling inequality.

Finally, the effect of government spending on labor market participation could also be considered. Some welfare transfers disincentivize labor supply (poor households might choose not to work and rely on transfers; maintaining social stability would require more welfare spending). On the other hand, welfare expenditures on family services such as child care could increase working hours (Rogerson, 2007). Such a virtuous circle is often observed in Nordic countries where welfare expenditures are productive. In addition, our production function assumes that public and private capital are complementary and an increase in productive public expenditures increases the marginal product of private capital. The elasticity of substitution is unity. Instead, public capital might crowd out private investment. As our model suggests, the empirical specification of such parameters is important for understanding differences in development and standards of living.

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APPENDIX A: PROOF OF PROPOSITIONS

Proof of Proposition 1. Let \( x^i = E^i + g^i \) for each household \( i \). For any profile of household endowments \( (E^1, ..., E^n) \) and any \( G, S_U \) assigns some real number to each redistribution rule \( \Omega = (g_1, ..., g_n) \). Since constraint (2) is a linear equality constraint, this means that there exists at least one solution to the government’s social welfare maximization problem for \( S_U \).

Let \( \Omega \) be the redistribution rule that maximizes social welfare. To reach a contradiction, assume that there exist households \( j \) and \( k \) such that \( g_j > 0 \) and \( x^j > x^k \). Since \( u \) is an increasing function, \( u(x^j) > u(x^k) \). Let \( \varepsilon \) be an arbitrarily small number such that \( x^j - \varepsilon \geq x^k + \varepsilon \) and \( \varepsilon \leq g^j \). From the assumption that \( u \) is increasing and strictly concave, \( u(x^j - \varepsilon) + u(x^k + \varepsilon) > u(x^j) + u(x^k) \). This implies that a feasible transfer exists from household \( j \) to household \( k \) that increases \( S_U \). This is a contradiction, since \( \Omega \) maximizes \( S_U \), there cannot exist households \( j \) and \( k \) such that \( g_j > 0 \) and \( x^j > x^k \). This implies that no household \( j \) that receives a positive transfer \( g_j > 0 \) can attain a higher consumption level \( x^j \) than any other household. Thus, every household that receives a positive transfer attains the same consumption level, which is weakly lower than the consumption level of households that do not receive any transfers. Since \( \Omega \) satisfies these properties, it is the fill-the-gap rule. Since \( \Omega \) exists and since \( \Omega \) must be the fill-the-gap rule, the fill-the-gap rule is the unique redistribution rule that maximizes utilitarian social welfare.

\( \square \)

Proof of Proposition 2. Let \( \Omega \) be a redistribution rule that maximizes household \( h \)'s utility. To reach a contradiction, assume that \( \Omega \neq \Omega^h \) and let \( g^i \) and \( \bar{g}^i \) be the transfer each household \( i \) receives under \( \Omega \) and \( \Omega^h \), respectively. Since \( \Omega \) maximizes the utility of \( h \), it follows that \( g^h \geq \bar{g}^h \). If \( \Omega \neq \Omega^h \), then at least one household \( j \) must exist such that

(a) \( E^j \leq E^h \) and \( g^j \neq \bar{g}^h \), or
(b) \( E^h \leq E^j \leq E^h + \bar{g}^h \) and \( g^j \neq E^h + \bar{g}^h - E^j \), or
(c) \( E^j > E^h + \bar{g}^h \) and \( g^j \neq 0 \).
In case (a), if $g^j < g^h$ the fairness axiom is violated since $g^h \geq g^j > g^j$. Hence, $g^j > g^h$. The fairness axiom requires that $g^j \geq g^j$ for all households $i$ such that $E^i \leq E^j$. Since $G$ is fixed, some household $i$ must exist for whom $E^i > E^j$ and $g^i < g^j$. As $g^i = 0$ for all $i$ such that $E^i > E^h + g^h$, we either have $E^h < E^i \leq E^h + g^h$ or $E^i \leq E^h$. If:

- $E^h < E^i \leq E^h + g^h$, we have that $E^i + g^i = E^h + g^h$: $g^i < g^j$ implies that $E^i + g^i < E^h + g^h = E^h + g^h$ and $g^h \geq g^h$ implies that $E^i + g^i < E^h + g^h \leq E^h + g^h$. This violates the no-reranking criterion which requires that $E^i + g^i \geq E^h + g^h$.
- $E^i \leq E^h$, first note that $g^i = g^h$. Since $g^i < g^j$, it follows that $g^i < g^j = g^h \leq g^h$. This violates the fairness axiom which requires that $g^i \geq g^h$.

Thus, the type of household described in case (a) can not exist.

In case (b), if $g^j < E^h + g^h - E^j$, $E^j + g^j < E^h + g^h \leq E^h + g^h$ which violates the no-reranking criterion. Thus, $g^j > E^h + g^h - E^j = g^j$. First note that $E^j + g^j = E^j + g^j$ for all $i$ such that $E^j \leq E^j \leq E^h + g^h$. Since $g^j > g^j$, $E^j + g^j > E^j + g^j$. The no-reranking condition requires that $E^j + g^j \leq E^j + g^j$. Thus, $g^j > g^j$ for all $i$ such that $E^j \leq E^j \leq E^h + g^h$. Since $G$ is fixed, there must exist some household $i$ for whom $g^j < g^j$ and either $E^j < E^j$ or $E^j > E^h + g^h$. The latter case can be dismissed immediately since $g^j = 0$ whenever $E^j > E^h + g^h$ and transfer payments can not be negative. When $E^j < E^j$, it must either be the case that $E^j < E^h$ or $E^j \geq E^h$. If:

- $E^j < E^j$, then $g^j = g^j$. Thus, we have that $g^j < g^j = g^h \leq g^h$. This violates the fairness axiom, which requires that $g^j \geq g^h$.
- $E^j \geq E^j$, $E^j + g^j = E^j + g^h$. Since $g^j < g^j$ and $g^j \geq g^j$, $E^j + g^j < E^j + g^j = E^j + g^j \leq E^j + g^j$. This violates the no-reranking criterion which requires that $E^j + g^j \geq E^j + g^j$.

Thus, the type of household described in case (b) can not exist.

In case (c), as transfer payments can not be negative, $g^j > 0$. Since $G$ is fixed, some household $i$ must exist for whom $g^j < g^j$. It can not be the case that $E^j > E^h + g^h$ since $g^j = 0$ whenever $E^j > E^h + g^h$ and transfer payments can not be negative. If:

- $E^h < E^j \leq E^h + g^h$, $E^j + g^j = E^h + g^j$. Since $g^j < g^j$ and $g^j \geq g^j$, $E^j + g^j < E^j + g^j = E^h + g^h$. This violates the no-reranking criterion, which requires that $E^j + g^j \geq E^h + g^h$.
- $E^j < E^h$, $g^j = g^j$. Since $g^j < g^j$ and $g^j \geq g^j$, we have that $g^i < g^j = g^h \leq g^h$. This violates the fairness axiom, which requires that $g^j \geq g^h$.

Thus, the type of household described in case (c) can not exist. We have shown that $\Omega \neq \Omega^h$ always results in a contradiction. Therefore, if $\Omega$ is a redistribution rule that maximizes household $h$’s utility, then it must be the equal-subsidies-for-the-poor rule $\Omega^h$.

It now remains to show that $\Omega^h$ maximizes the utility of household $h$. Note that given a distribution of endowments, the utility of household $h$ only depends on $g^h$. Given $E^h$, the utility function of household $h$ assigns a real number to every possible value of $g^h$. Since $\Omega^h$ satisfies the fairness axiom, the set of redistribution rules that satisfy the fairness axiom is nonempty. Give a distribution of endowments, each redistribution rule assigns some value to $g^h$ and consequently corresponds to some level of utility for household $h$. 
Thus, at least one redistribution rule exists that maximizes the utility of household \( h \) over all rules in the set of rules that satisfy the fairness axiom. This establishes the existence of a redistribution rule that maximizes the utility of household \( h \). As demonstrated above, if a redistribution rule maximizes household \( h \)'s utility, it must be \( \Omega^h \). These two statements jointly imply that \( \Omega^h \) maximizes household \( h \)'s utility.

**Proof of Proposition 4.** Recall from section 4.2 that \( y_{t+1} = A(G_{P,t})^{1-\alpha}(k_{t+1})^\alpha \) and \( k_{t+1} = \frac{\rho r_{t+1}A(G_{P,t})^{1-\alpha}k_t^\alpha + E_t + \varphi \eta_t}{(1 + \rho)\alpha r_{t+1}}. \) Two countervailing effects are at work with respect to \( y_{t+1} \). First, given government budget \( G \), productive and nonproductive government spending, \( G_{P,t} \) and \( G_{N,t} \), are rivals. A higher level of \( G_{N,t} \) leads to a lower level of \( G_{P,t} \), and higher \( G_{N,t} \) leads to worse performance measured by \( y_{t+1} \). Second, a higher level of government transfers \( \varphi \) contributes to \( k_{t+1} \) and thus better \( y_{t+1} \) performance.

From output objective (5) and the balanced budget constraint, the sign of \( \frac{\partial y_{t+1}}{\partial G_{P,t}} \) is given by the sign of \( (1 + \alpha)A(1 - \alpha)^2G_{P,t}^{1-\alpha}k_t^\alpha + E_t(1 - \alpha) + \frac{G(1 - \alpha)}{L_t} - \frac{G_{N,t}^2}{L_t} \). Define this expression as \( f(G_{P,t}) \).

Let \( \overline{G}_{P,t} \) be the level of productive public spending such that \( f(\overline{G}_{P,t}) = 0 \). Therefore, \( (1 + \alpha)A(1 - \alpha)^2(\overline{G}_{P,t})^{1-\alpha}k_t^\alpha - \frac{\overline{G}_{P,t}}{L_t} < 0 \) and thus \( L_t(1 + \alpha)A(1 - \alpha)^3k_t^\alpha < (\overline{G}_{P,t})^\alpha < (G_{P,t})^\alpha \), if \( G_{P,t} \in (\overline{G}_{P,t}, G) \). For \( G_{P,t} \in (\overline{G}_{P,t}, G) \), \( f'(G_{P,t}) = (1 + \alpha)A(1 - \alpha)^3G_{P,t}^{1-\alpha}k_t^\alpha - \frac{1}{L_t} < 0 \). Therefore, \( f(G_{P,t}) < 0 \) and \( \frac{dy_{t+1}}{dG_{P,t}} < 0 \), for \( G_{P,t} \in (\overline{G}_{P,t}, G) \).

Define \( \hat{G}_{P,t} = \left[L_t(1 + \alpha)A(1 - \alpha)^3k_t^\alpha\right]^{1/\alpha} \). For \( G_{P,t} \in (\hat{G}_{P,t}, \overline{G}_{P,t}) \), we have that \( (G_{P,t})^\alpha > L_t(1 + \alpha)A(1 - \alpha)^3k_t^\alpha \). Therefore, \( f'(G_{P,t}) = (1 + \alpha)A(1 - \alpha)^3G_{P,t}^{1-\alpha}k_t^\alpha - \frac{1}{L_t} < 0 \) and \( f(G_{P,t}) > f(\overline{G}_{P,t}) = 0 \). That is, for \( G_{P,t} \in (\hat{G}_{P,t}, \overline{G}_{P,t}), \frac{dy_{t+1}}{dG_{P,t}} > 0 \).

If \( G_{P,t} < \hat{G}_{P,t}, G_{P,t}^\alpha < (\hat{G}_{P,t})^\alpha \), \( L_t(1 + \alpha)A(1 - \alpha)^3k_t^\alpha < L_t(1 + \alpha)A(1 - \alpha)^3k_t^\alpha \). That is, \( (1 + \alpha)A(1 - \alpha)^2G_{P,t}^{1-\alpha}k_t^\alpha - \frac{G_{P,t}}{L_t} > 0 \). For \( G_{P,t} \in (0, \hat{G}_{P,t}), \frac{dy_{t+1}}{dG_{P,t}} > 0 \).

Since \( \frac{dy_{t+1}}{dG_{P,t}} > 0 \) for \( G_{P,t} \in (0, \overline{G}_{P,t}) \) and \( \frac{dy_{t+1}}{dG_{P,t}} < 0 \) for \( G_{P,t} \in (\overline{G}_{P,t}, G) \), \( y_{t+1} \) is strictly quasiconcave in \( G_{P,t} \). Therefore, one unique maximum exists and \( y_{t+1} \) is maximized at \( \overline{G}_{P,t} \). Welfare support \( \overline{G}_{N,t} \) is \( G - \overline{G}_{P,t} = \alpha G - L_t(1 + \alpha)A(1 - \alpha)^2(G - \overline{G}_{N,t})^{1-\alpha}k_t^\alpha - L_tE_t(1 - \alpha) \).

**Proof of Proposition 5.** This follows immediately from Proposition 4. The optimal welfare support depends on whether \( \overline{G}_{N,t} \) is \( < \) or \( > \) than the output maximizing welfare support \( \overline{G}_{N,t} \).

**Proof of Proposition 6.** From Proposition 4, if \( \Phi < G - \overline{G}_{N,t} \), optimal provision of productive public goods and welfare support satisfy
Clearly, government revenue is positively related to given public spending $G$:

$$dG_{P,t}^* = \frac{1 - \alpha}{1 + \alpha}A(1 - \alpha)^2(G_{P,t}^*)^{-1}k_t^a + E_tL_t(1 - \alpha) + G(1 - \alpha),$$

$$dG_{N,t}^* = \frac{\alpha}{1 + \alpha}A(1 - \alpha)^2(G - G_{N,t}^*)^{-1}k_t^a - L_tE_t(1 - \alpha).$$

If $\Phi > G - \overline{G}_{N,t}$, the optimal provision of productive public goods and welfare support satisfy $G_{P,t}^* = G - \overline{G}_{N,t} = G - L_t\sum_{E=E_{min}}^{E_{max}}(\bar{c} - E - w_i)f(E)$ and $G_{N,t}^* = \overline{G}_{N,t} = L_t\sum_{E=E_{min}}^{E_{max}}(\bar{c} - E - w_i)f(E)$. Therefore,

$$\frac{dG_{P,t}^*}{dG} = 1 - L_tA(1 - \alpha)^2(G_{P,t}^*)^{-1}k_t^a \sum_{E=E_{min}}^{E_{max}}f(E) > 0,$$

$$\frac{dG_{N,t}^*}{dG} = -A(1 - \alpha)^2(G - G_{N,t}^*)^{-1}k_t^a \sum_{E=E_{min}}^{E_{max}}f(E) < 0.$$
\[
\frac{d^2V_{mv}}{d(G_{p,t})^2} = \frac{(1 + \rho)\left[(1 - \alpha)^2(-\alpha)A(G_{p,t})^{-\alpha-1}k_t^\alpha + \frac{d^2g_{nt}}{d(G_{p,t})^2}\right]}{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t}^m + g_{n,t}^m}
\]

\[
\rho(1 - \alpha)\left\{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t} + g_{n,t}\right\}\left\{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t} + g_{n,t}\right\}^2
\]

\[
\frac{\rho(1 - \alpha)}{G_{p,t}^2} \left\{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t} + g_{n,t}\right\}^2.
\]

Note that \(A(1 - \alpha)^2(G_{p,t})^{-\alpha}k_t^\alpha + G_{p,t} \frac{dg_{nt}}{dG_{p,t}}\) could be positive or negative. We will show that \(\frac{d^2V_{mv}}{d(G_{p,t})^2}\) < 0 in both cases.

**Case (a).** \(A(1 - \alpha)^2(G_{p,t})^{-\alpha}k_t^\alpha + G_{p,t} \frac{dg_{nt}}{dG_{p,t}} > 0\). As \(\frac{d^2g_{nt}}{d(G_{p,t})^2} < 0\), then \(A(1 - \alpha)^2(-\alpha)(G_{p,t})^{-\alpha-1}k_t^\alpha + \frac{d^2g_{nt}}{d(G_{p,t})^2} < 0\). As \(g_{n,t} = \frac{G_{p,t}}{L_{t}}\), then

\[
\frac{dg_{nt}}{dG_{p,t}} = \frac{1}{L_{t}} \quad \text{and} \quad G_{p,t} \frac{dg_{nt}}{dG_{p,t}} = -\frac{G_{p,t}}{L_{t}}. \quad \text{Therefore,} \quad g_{n,t} = \frac{G_{p,t}}{L_{t}} > -\frac{G_{p,t}}{L_{t}} = G_{p,t} \frac{dg_{nt}}{dG_{p,t}},
\]

and thus

\[
\left\{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t} + g_{n,t}\right\} > \left\{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha)^2 + E_{1,t} + g_{n,t}\right\} \left\{(A(1 - \alpha)^2(G_{p,t})^{-\alpha}k_t^\alpha + G_{p,t} \frac{dg_{nt}}{dG_{p,t}}\right\}
\]

Since \(A(1 - \alpha)^2(G_{p,t})^{-\alpha}k_t^\alpha + G_{p,t} \frac{dg_{nt}}{dG_{p,t}} > 0\), it follows that

\[
\left\{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t} + g_{n,t}\right\} > \left\{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha)(1 - \alpha + \alpha^2) + E_{1,t} + g_{n,t}\right\} > \left\{(A(1 - \alpha)^2(G_{p,t})^{-\alpha}k_t^\alpha + G_{p,t} \frac{dg_{nt}}{dG_{p,t}}\right\}^2 \quad \text{and} \quad \frac{d^2V_{mv}}{d(G_{p,t})^2} < 0.
\]

**Case (b).** \(A(1 - \alpha)^2(G_{p,t})^{-\alpha}k_t^\alpha + G_{p,t} \frac{dg_{nt}}{dG_{p,t}} < 0\).

\(\frac{d^2V_{mv}}{d(G_{p,t})^2}\) can be decomposed into five terms:

\[
\frac{d^2V_{mv}}{d(G_{p,t})^2} = \frac{(1 + \rho)\left[(1 - \alpha)^2(-\alpha)A(G_{p,t})^{-\alpha-1}k_t^\alpha + \frac{d^2g_{nt}}{d(G_{p,t})^2}\right]}{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t}^m + g_{n,t}^m}
\]

\[
+ \frac{\rho(1 - \alpha)\left\{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t} + g_{n,t}\right\}^2}{A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t} + g_{n,t}} - \frac{\rho(1 - \alpha)}{G_{p,t}^2}
\]

\[
- \frac{(1 + \rho)\left\{A(1 - \alpha)^2(G_{p,t})^{-\alpha}k_t^\alpha + \frac{d^2g_{nt}}{d(G_{p,t})^2}\right\}^2}{\left\{(A(G_{p,t})^{-\alpha}k_t^\alpha(1 - \alpha) + E_{1,t} + g_{n,t}\right\}^2}
\]

Since the median post-transfer income is no larger than the mean post-transfer income among all households, \(E_{1,t} + g_{n,t}^m \geq E_{1,t} + g_{n,t}\).
As \( E_{1,t} + g_{N,t} \geq E_{1,t}^{mv} + g_{N,t}^{mv} \) and \( \frac{d^2 g_{N,t}}{dG_{P,t}} < 0 \),

\[
(1 + \rho) \left[ (1-\alpha)^2 \rho_{t}^{1-\alpha} k_{t}^{1-\alpha} \frac{d^2 g_{N,t}}{dG_{P,t}} \right] + \frac{\rho(\alpha - 1) [(1-\alpha)^2 (-\alpha) A(G_{P,t})^{-\alpha - 1} k_{t}^{\alpha - 1}]}{A(G_{P,t})^{-\alpha + 1} k_{t}^{\alpha} (1 - \alpha) + E_{i,t} + g_{N,t}} \leq \frac{(1 + \rho) [(1-\alpha)^2 (-\alpha) A(G_{P,t})^{-\alpha - 1} k_{t}^{\alpha - 1}]}{A(G_{P,t})^{-\alpha + 1} k_{t}^{\alpha} (1 - \alpha) + E_{i,t} + g_{N,t}}.
\]

As \( [(1-\alpha)^2 (-\alpha) A(G_{P,t})^{-\alpha - 1} k_{t}^{\alpha - 1}] < 0 \) and \( \frac{d^2 g_{N,t}}{dG_{P,t}} < 0 \), it follows that

\[
(1 + \rho) \left[ (1-\alpha)^2 \rho_{t}^{1-\alpha} k_{t}^{1-\alpha} \frac{d^2 g_{N,t}}{dG_{P,t}} \right] + \frac{\rho(\alpha - 1) [(1-\alpha)^2 (-\alpha) A(G_{P,t})^{-\alpha - 1} k_{t}^{\alpha - 1}]}{A(G_{P,t})^{-\alpha + 1} k_{t}^{\alpha} (1 - \alpha) + E_{i,t} + g_{N,t}} \leq \frac{(1 + \rho) [(1-\alpha)^2 (-\alpha) A(G_{P,t})^{-\alpha - 1} k_{t}^{\alpha - 1}]}{A(G_{P,t})^{-\alpha + 1} k_{t}^{\alpha} (1 - \alpha) + E_{i,t} + g_{N,t}} < 0.
\]

Therefore, in the decomposition of \( \frac{d^2 g_{N,t}}{dG_{P,t}} \) the sum of the first two terms is negative.

The third term \( -\frac{\rho(\alpha - 1)}{(G_{P,t})} \) is negative. It remains to find the sign of the last two terms,

\[
\begin{align*}
&\frac{(1 + \rho) \left[ A(1-\alpha)^2 G_{P,t}^{-\alpha - 1} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} \right]^2}{[A(G_{P,t})^{-\alpha + 1} k_{t}^{\alpha} (1 - \alpha) + E_{i,t} + g_{N,t}]^2} + \frac{\rho(\alpha - 1) \left[ A(1-\alpha)^2 G_{P,t}^{-\alpha - 1} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} \right]^2}{[A(G_{P,t})^{-\alpha + 1} k_{t}^{\alpha} (1 - \alpha) + E_{i,t} + g_{N,t}]^2} \\
&\geq \frac{(1 + \rho) \left[ A(1-\alpha)^2 G_{P,t}^{-\alpha - 1} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} \right]^2}{[A(G_{P,t})^{-\alpha + 1} k_{t}^{\alpha} (1 - \alpha) + E_{i,t} + g_{N,t}]^2}.
\end{align*}
\]

Clearly, the denominators are all positive. If \( A(1-\alpha)^2 (G_{P,t})^{-\alpha} k_{t}^{\alpha} + G_{P,t} \frac{d^2 g_{N,t}}{dG_{P,t}} < 0 \), then \( A(1-\alpha)^2 (G_{P,t})^{-\alpha} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} < 0 \). Consider the two terms in the numerator. As \( \frac{d^2 g_{N,t}}{dG_{P,t}} < \frac{d^2 g_{N,t}}{dG_{P,t}} < 0 \),

\[
A(1-\alpha)^2 (G_{P,t})^{-\alpha} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} < A(1-\alpha)^2 (G_{P,t})^{-\alpha} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} < 0.
\]

Therefore,

\[
\begin{align*}
A(1-\alpha)^2 (G_{P,t})^{-\alpha} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} > A(1-\alpha)^2 (G_{P,t})^{-\alpha} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} \right] \geq (1 + \rho) \left[ A(1-\alpha)^2 (G_{P,t})^{-\alpha} k_{t}^{\alpha} + \frac{d^2 g_{N,t}}{dG_{P,t}} \right]^2,
\end{align*}
\]

it follows that
\[(1 + \rho)\left[ A(1 - \alpha)^2(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}} \right]^2 + \rho(\alpha - 1)\left[ A(1 - \alpha)^2(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}} \right]^2 \]

\[> (1 + \rho)\left[ A(1 - \alpha)^2(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}} \right]^2 + \rho(\alpha - 1)\left[ A(1 - \alpha)^2(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}} \right]^2 \]

\[= \left[ A(1 - \alpha)^2(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}} \right]^2 (1 + \rho\alpha) > 0.\]

Therefore, the sum of the last two terms is negative. We have determined the sign of all five terms and clearly, \( \frac{dV_{mv}}{d(G_{P,t})^2} < 0. \)

We have concluded that \( \frac{d^2V_{mv}}{d(G_{P,t})^2} < 0 \) in both cases. This means that there exists \( G_{P,t}^\text{mv} \) such that \( G_{P,t}^\text{mv} \in \arg\max(V_{mv}) \) and \( \frac{dV_{mv}}{dG_{P,t}} \bigg|_{G_{P,t}=G_{P,t}^\text{mv}} = 0. \) The first derivative \( \frac{dV_{mv}}{dG_{P,t}} \) is

\[
\frac{dV_{mv}}{dG_{P,t}} = \frac{(1 + \rho)G_{P,t}(1 - \alpha)^2A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}}{A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}} \frac{\rho(\alpha - 1)A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}}{A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}} \frac{\rho(1 - \alpha)A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}}{A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}} \frac{1}{A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}}.
\]

The optimum, \( G_{P,t}^\text{mv} \) satisfies:

\[
(1 + \rho)G_{P,t}^\text{mv}(1 - \alpha)^2A(G_{P,t}^\text{mv})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}} = \rho(1 - \alpha)A(G_{P,t}^\text{mv})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}.
\]

Recall that in Proposition 2, only some households receive transfers while others will not. For each household \( j \) who does not receive transfers, \( g_{N,t}^j = 0. \) For household \( j, \) the first derivative is

\[
\frac{dV_{mv}}{dG_{P,t}} = \frac{(1 + \rho)(1 - \alpha)^2A(G_{P,t})^{-\alpha}k^\alpha_t}{A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}} \frac{\rho(1 - \alpha)^2A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}}{A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}} \frac{\rho(1 - \alpha)A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}}{A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}} \frac{1}{A(G_{P,t})^{-\alpha}k^\alpha_t + \frac{dg_{N,t}}{dG_{P,t}}}.
\]

As \( g_{N,t} > G_{P,t}^\text{mv}, dG_{P,t} \), \( \frac{dV_{mv}}{dG_{P,t}} \) > 0. Therefore, the optimal \( G_{P,t}^j \) for household \( j, \) is \( G_P^j = G. \)

For each household \( i \) who receives transfers, \( g_{N,t}^j > 0. \) If \( E_{1,t}^i \leq g_{N,t}^i, g_{N,t}^i = g_{N,t}^i \) and \( \frac{dg_{N,t}^i}{dG_{P,t}} = \frac{dg_{N,t}^i}{dG_{P,t}} \). If \( E_{1,t}^m < E_{1,t}^i \leq E_{1,t}^m + g_{N,t}^i, g_{N,t}^i = E_{1,t}^m + g_{N,t}^m - E_{1,t}^i \) and \( \frac{dg_{N,t}^i}{dG_{P,t}} = \frac{dg_{N,t}^m}{dG_{P,t}}. \) Therefore, for household \( i \) who receives transfers, the first derivative is
This is significantly large such that it is worthwhile for all public revenue to be allocated to productive public investment. As \( \frac{dG_{m,v}}{dG_{j,t}} < \frac{dg_{N,t}}{dG_{j,t}} < 0 \),

\[
(1 - \alpha)^2 A(G_{j,t})^{-\alpha} k_i^\alpha + \frac{dg_{N,t}}{dG_{j,t}} > 0. \text{ From Proposition 4, the sign of \( \frac{dy_{i,t}}{dG_{j,t}} \) is given by the sign of }
\]

\[
(1 + \alpha)A(1 - \alpha)^2 G_{j,t}^{1-\alpha} k_i^\alpha + E_i(1 - \alpha) + \frac{G(1-\alpha)}{L_i} - \frac{G_{j,t}}{L_i}. \text{ In this case, } (1 + \alpha)A(1 - \alpha)^2 G_{j,t}^{1-\alpha} k_i^\alpha + E_i(1 - \alpha) + \frac{G(1-\alpha)}{L_i} - \frac{G_{j,t}}{L_i} > (1 - \alpha)^2 A(G_{j,t})^{-\alpha} k_i^\alpha + \frac{dg_{N,t}}{dG_{j,t}} > 0.
\]

Therefore, \( \frac{dy_{i,t}}{dG_{j,t}} > 0. G_{j,t} = G_{m,v} = G. \)

\* If \( (1 - \alpha)^2 A(G_{j,t})^{-\alpha} k_i^\alpha + \frac{dg_{N,t}}{dG_{j,t}} < 0 \), we find \( G_{j,t} \) where \( \frac{dy_{i,t}}{dG_{j,t}} = 0. \) We therefore have

\[
\frac{dV_i}{dG_{j,t}} = \frac{(1 + \rho)(1 - \alpha)^2 A(G_{j,t})^{-\alpha} k_i^\alpha + \frac{dg_{N,t}}{dG_{j,t}}}{A(G_{j,t})^{-\alpha} k_i^\alpha(1 - \alpha) + E_i + g_{N,t}} + \frac{\rho(1 - \alpha)^2 A(G_{j,t})^{-\alpha} k_i^\alpha + \rho(1 - \alpha)E_i + g_{N,t} - G_{j,t}\frac{dg_{N,t}}{dG_{j,t}}(g_{N,t})}{G_{j,t} A(G_{j,t})^{-\alpha} k_i^\alpha(1 - \alpha) + E_i + g_{N,t}} = 0.
\]

**Remark A1.** Observe that \( G_{j,t} \) is positively related to \( E_i. \) That is, among those who receive transfers, households with higher endowments prefer a larger \( G_{j,t}. \) Households who do not receive transfers prefer \( G_{j,t} = G. \) Following nearly identical steps as for \( V_{m,v}, \) \( \frac{dV_i}{dG_{j,t}} < 0 \) for any household \( i \) who receives transfers. Furthermore, as shown above, \( \frac{dV_i}{dG_{j,t}} > 0 \) for any household \( j \) who receives transfers. This implies that households have single-peaked preferences over \( G_{j,t}. \) The median voter theorem then implies that a Condorcet winner exists and it is given by the \( G_{j,t} \) preferred by the median voter. As established above, households with higher endowment prefer a weakly higher \( G_{j,t}. \) This means that the household with the median endowment is the median voter.

Similarly, for local officials the augmented objective function is

\[
V = \eta A \left( \frac{\rho}{1 + \rho} \right)^\alpha G_{j,t}^{1-\alpha} A(1 - \alpha) G_{j,t}^{1-\alpha} k_i^\alpha + E_i + g_{N,t} + (1 - \eta)A(1 - \alpha) G_{j,t}^{1-\alpha} k_i^\alpha + E_i + g_{N,t}(1 - Gini).
\]
The first order condition is

$$\frac{dV}{dG_{P,t}} = \eta A\left(\frac{\rho}{1+\rho}\right)^{\alpha}(G_{P,t})^{1-\alpha}[A(1-\alpha)G_{P,t}^{1-\alpha}k_t^\alpha + E_t + g_{N,t}]^{\alpha-1}\left(A(1-\alpha)^2G_{P,t}^{-\alpha}k_t^\alpha + \frac{d\theta_{N,t}}{dG_{P,t}}\right)$$

$$+ \eta A\left(\frac{\rho}{1+\rho}\right)^{\alpha}(G_{P,t})^{-\alpha}[A(1-\alpha)G_{P,t}^{-\alpha}k_t^\alpha + E_t + g_{N,t}]^{\alpha-1}(1-\alpha)[A(1-\alpha)G_{P,t}^{-\alpha}k_t^\alpha + E_t + g_{N,t}]$$

$$- (1-\eta)\left\{[A(1-\alpha)G_{P,t}^{-\alpha}k_t^\alpha + E_t + g_{N,t}](\frac{dG_{\text{Gini}}}{dG_{P,t}})\right\} = 0.$$

From this first order condition, $G_{P,t}^V$, $G_{P,t}^V$ and $G_{P,t}^V$ satisfy three equations:

$$\eta A\left(\frac{\rho}{1+\rho}\right)^{\alpha}(G_{P,t})^{1-\alpha}\left[A(1-\alpha)(G_{P,t})^{1-\alpha}k_t^\alpha + E_t + g_{N,t}\right]^{\alpha-1}\left(A(1-\alpha)^2(G_{P,t})^{-\alpha}k_t^\alpha + \frac{d\theta_{N,t}}{dG_{P,t}}\right)$$

$$+ \eta A\left(\frac{\rho}{1+\rho}\right)^{\alpha}(G_{P,t})^{-\alpha}[A(1-\alpha)(G_{P,t})^{-\alpha}k_t^\alpha + E_t + g_{N,t}]^{\alpha-1}(1-\alpha)[A(1-\alpha)(G_{P,t})^{-\alpha}k_t^\alpha + E_t + g_{N,t}]$$

$$= (1-\eta)\left\{[A(1-\alpha)(G_{P,t})^{1-\alpha}k_t^\alpha + E_t + g_{N,t}](\frac{dG_{\text{Gini}}}{dG_{P,t}})\right\} = 0,$$

$$A(1-\alpha)(G_{P,t})^{1-\alpha}k_t^\alpha + g_{N,t} = 0,$$

$$A(1-\alpha)(G_{P,t})^{1-\alpha}k_t^\alpha + g_{N,t} = 0.$$
From Equation (18),
\[-\alpha G^V_{P,t} \left[ A(1 - \alpha)^2 (G^V_{P,t})^{-\alpha} k^\alpha_t + \frac{d g_{N_I}^{mv}}{d G_{P,t}} \right] = (1 - \alpha) [A(G^V_{P,t})^{1-\alpha} k^\alpha_t (1 - \alpha) + E_{1,t} + g_{N_I,t}].\]

Therefore,
\[\frac{d V^{mv}}{d G_{P,t}} \bigg|_{G_{P,t} = G^V_{P,t}} = \frac{(1 - \alpha)^2 A(G^V_{P,t})^{-\alpha} k^\alpha_t + \frac{d g_{N_I}^{mv}}{d G_{P,t}}}{A(G^V_{P,t})^{1-\alpha} k^\alpha_t (1 - \alpha) + E_{1,t} + g_{N_I,t}}\]

\[= \frac{\rho(A(G^V_{P,t})^{-\alpha} k^\alpha_t (1 - \alpha)) \left( \frac{d g_{N_I}^{mv}}{d G_{P,t}} \right)_{G_{P,t} = G^V_{P,t}} - \frac{d g_{N_I}^{mv}}{d G_{P,t}}}{A(G^V_{P,t})^{-\alpha} k^\alpha_t (1 - \alpha) + E_{1,t} + g_{N_I,t}}\]

\[+ \frac{\rho(E_{1,t} + g_{N_I,t}) \left( (1 - \alpha)^2 A(G^V_{P,t})^{-\alpha} k^\alpha_t + \frac{d g_{N_I}^{mv}}{d G_{P,t}} \right)_{G_{P,t} = G^V_{P,t}}}{A(G^V_{P,t})^{-\alpha} k^\alpha_t (1 - \alpha) + E_{1,t} + g_{N_I,t}}\]

\[- \frac{\rho(E_{1,t} + g_{N_I,t}) \left( (1 - \alpha)^2 A(G^V_{P,t})^{-\alpha} k^\alpha_t + \frac{d g_{N_I}^{mv}}{d G_{P,t}} \right)_{G_{P,t} = G^V_{P,t}}}{A(G^V_{P,t})^{-\alpha} k^\alpha_t (1 - \alpha) + E_{1,t} + g_{N_I,t}}\]

Note here that \( A(1 - \alpha)^2 (G^V_{P,t})^{-\alpha} k^\alpha_t + \frac{d g_{N_I}^{mv}}{d G_{P,t}} \bigg|_{G_{P,t} = G^V_{P,t}} > 0 \). Under the equal-subsidies-for-the-poor rule, as only some households received transfers, \( \frac{d g_{N_I}^{mv}}{d G_{P,t}} \bigg|_{G_{P,t} = G^V_{P,t}} > \frac{d g_{N_I}^{mv}}{d G_{P,t}} \bigg|_{G_{N_I,t} = G^V_{N_I,t}} \) and thus
\[\frac{d g_{N_I}^{mv}}{d G_{P,t}} \bigg|_{G_{P,t} = G^V_{P,t}} > 0.\]

This implies that
\[\left( 1 - \alpha \right)^2 A(G^V_{P,t})^{-\alpha} k^\alpha_t + \frac{d g_{N_I}^{mv}}{d G_{P,t}} \bigg|_{G_{P,t} = G^V_{P,t}} < 0.\]

We then get that
\[\left[ (A(G^V_{P,t})^{1-\alpha} k^\alpha_t (1 - \alpha)) \left( \frac{d g_{N_I}^{mv}}{d G_{P,t}} \right)_{G_{P,t} = G^V_{P,t}} - \frac{d g_{N_I}^{mv}}{d G_{P,t}} \right]_{G_{P,t} = G^V_{P,t}} < 0 \]

and
\[(E_{1,t} + g_{N_I,t}) \left( (1 - \alpha)^2 A(G^V_{P,t})^{-\alpha} k^\alpha_t + \frac{d g_{N_I}^{mv}}{d G_{P,t}} \right)_{G_{P,t} = G^V_{P,t}} - (E_{1,t} + g_{N_I,t}) \left( (1 - \alpha)^2 A(G^V_{P,t})^{-\alpha} k^\alpha_t + \frac{d g_{N_I}^{mv}}{d G_{P,t}} \right)_{G_{P,t} = G^V_{P,t}} < 0.\]

Therefore, at \( G^V_{P,t} \), we have that \( \frac{d V^{mv}}{d G_{P,t}} < 0. \) To attain the level of \( G_{P,t} \) where \( \frac{d V^{mv}}{d G_{P,t}} = 0 \), \( G_{P,t} \) must decrease and thus \( G^V_{P,t} > G^{mv}_{P,t} \). If \( \eta = 1 \) and the central government cares only about output in the local area, \( G^V_{N_I,t} < G^{mv}_{N_I,t} \), and the welfare support provided is lower than that provided in a democracy.

Note that \( G^V_{P,t} = G^V_{P,t} \). As shown in the proof of Proposition 4, \( Y_{t+1} \) is strictly quasiconcave in \( G_{P,t} \) and has a unique maximum point \( G^V_{P,t} \). As \( G^{V}_{P,t} < G^V_{P,t} \) and \( G^{mv}_{P,t} < G^V_{P,t} \). This implies that \( G^{V}_{P,t} \neq G^V_{P,t} \) and \( G^{mv}_{P,t} \neq G^V_{P,t} \). Since \( G^V_{P,t} \) is the unique maximum point, \( G^{V}_{P,t} \neq G^V_{P,t} \) and \( G^{mv}_{P,t} \neq G^V_{P,t} \), it follows that \( y(G^V_{P,t}) > y(G^{V}_{P,t}) \) and \( y(G^{V}_{P,t}) > y(G^{mv}_{P,t}) \).

From Equation (19), \( G^{V}_{P,t} \) depends on the economy’s Gini. When \( G^{V}_{P,t} \leq G^{mv}_{P,t}, G^{V}_{N,I,t} \geq G^{mv}_{N,I,t} \), \( G^{mv}_{N,I,t} \in \{G^{V}_{N,I,t}, G^{V}_{P,t}\} \). As argued above, \( \frac{\partial G^{V}_{P,t}}{\partial \eta} > 0 \) and Equation (17) is a continuous function of
$G_{p,t}^V$. There exists $\eta \in [0, 1)$ such that the welfare support is equivalent to that provided in a democracy. Therefore, setting the promotion criteria to a combination of output and Gini social welfare gives the local official an incentive to allocate the two types of public spending in the same way as in a democracy. Under this scenario, $y(G_{p,t}^V) > y(G_{p,t}^{mv}) \geq y(G_{p,t}^{V0})$. Note that in the case where the capital formation $k_t$ is sufficiently large, median voter’s choice of public investment is the same as that of the output-maximizing public investment. That is $G_{p,t}^{V1} = G_{p,t}^{mv} = G$. This implies that $y(G_{p,t}^{V1}) = y(G_{p,t}^{mv})$ and $\eta = 1$.

\footnote{There is another possible scenario where $G_{p,t}^{V1} > G_{p,t}^{mv}$. We have $G_{p,t}^{V1} < G_{p,t}^{mv}$, where the welfare support chosen the median voter is no lower than that provided with only social welfare consideration. We do not focus on this case.}