Implications of tau data for CP violation in K decays

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Electroweak penguins in $K \rightarrow \pi\pi$

\[ Q_7 = \frac{3}{2} \left( \bar{s}d \right)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V+A} \]
\[ Q_8 = \frac{3}{2} \left( \bar{s}_\alpha d_\beta \right)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A} \]
\[ Q_9 = \frac{3}{2} \left( \bar{s}d \right)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V-A} \]
\[ Q_{10} = \frac{3}{2} \left( \bar{s}_\alpha d_\beta \right)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A} \]

- $Q_8$ plays a crucial role in $\epsilon'/\epsilon$
- $Q_7$ and $Q_8$ (in $K \rightarrow \pi\pi$) do not vanish in the chiral limit ($O(p^0)$)
Soft-meson theorem

\[
\lim_{q \to 0} \langle \phi^k(q) | \beta | O(0) | \alpha \rangle = \frac{i}{F} \langle \beta | [Q^k_5, O(0)] | \alpha \rangle
\]

\[
\lim_{k, p, q \to 0} \langle (\pi \pi)_{I=2} | Q_7 | K^0 \rangle = -\frac{2}{F^3} \langle O_1 \rangle_{\mu}
\]

\[
\lim_{k, p, q \to 0} \langle (\pi \pi)_{I=2} | Q_8 | K^0 \rangle = -\frac{2}{F^3} \left( \frac{1}{2} \langle O_8 \rangle_{\mu} + \frac{1}{N_c} \langle O_1 \rangle_{\mu} \right)
\]

\[
\langle O_1 \rangle_{\mu} \equiv \langle 0 | \frac{1}{2} \bar{d} \Gamma^L_{\mu} u \bar{u} \Gamma^R_{\mu} d | 0 \rangle_{\mu}, \quad \langle O_8 \rangle_{\mu} \equiv \langle 0 | \frac{1}{2} \bar{d} \Gamma^L_{\mu} \lambda_i u \bar{u} \Gamma^R_{\mu} \lambda_i d | 0 \rangle_{\mu}
\]

Donoghue, Golowich '99
\[ \Pi_{VV-AA}(q) \equiv i \int d^4 x e^{-iqx} \langle 0 | J_V(x)J_V^\dagger(0) - J_A(x)J_A^\dagger(0) | 0 \rangle, \quad J_V(A) = \bar{d} \gamma_\mu (\gamma_5) u \]

**Operator Product Expansion**

\[ \Pi(q) = \frac{\mathcal{O}_D}{Q^D} \mathrm{SVZ} \ '79 \]

- \( \mathcal{O}_{D=0} = 0 \). It vanishes at all orders in massless perturbative QCD
- \( \mathcal{O}_{D=2,4} \approx 0 \). Owing to the values of \( m_u \) and \( m_d \)
- First contribution coming from \( D = 6 \)

\[ \mathcal{O}_{D=6} \sim 4\pi \alpha_s \langle \mathcal{O}_8 \rangle_\mu + \alpha_s^2 (\tilde{C}_1 \langle \mathcal{O}_1 \rangle_\mu + \tilde{C}_8 \langle \mathcal{O}_8 \rangle_\mu), \quad \tilde{C}_i = A_i + B_i \ln(-q^2/\mu^2) \]
\( \tau \rightarrow \nu_\tau n \) with \( n \) an arbitrary non-strange hadronic state (e.g. \( \pi \) or \( \pi\pi \))

\[
M \sim G_F L^\mu \langle n | J_{W\mu} | 0 \rangle \rightarrow \frac{d\Gamma_n}{ds} \sim G_F^2 L^{\mu\nu} \langle n | J_{W\mu} | 0 \rangle \langle 0 | J^\dagger_{W\nu} | n \rangle
\]

Summing over all possible \( n \) that can be reached through \( V \) (A) current:

\[
\frac{d\Gamma_{V(A)}}{ds} \sim \text{Im} \Pi_{VV(AA)}(s)
\]

ALEPH ’14
\( \Pi(s) \) is analytic in all complex plane except for a cut in the positive real axis

**Diagram:**
- OPE: Quarks and Gluons
- \( \text{Im} \Pi \) and \( \text{Re}(s) \)
- Hadrons

**Note:**
- \( \Pi(s) \) is analytic in all complex plane except for a cut in the positive real axis.
\[
\int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \text{Im} \Pi(s) - \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi(s) = 2\pi \frac{f_{\pi}^2}{s_0} \omega(m_{\pi}^2), \quad \omega(s) \text{ analytic inside}
\]
\[ \int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \text{Im} \Pi(s) - \frac{i}{2} \int_{|s|=s_0} ds \omega(s) \Pi(s) = 2\pi \frac{f_2^2}{s_0} \omega(m_\pi^2), \quad \omega(s) \text{ analytic inside} \]

\[ \delta_{DV}[\omega(s), s_0] \equiv \frac{1}{2\pi i} \oint_{|s|=s_0} ds \omega(s) [\Pi - \Pi^{\text{OPE}}](s) = \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \frac{1}{\pi} \text{Im}(\Pi - \Pi^{\text{OPE}})(s) \]

Phenomenological implications studied by DG '99, CDGM '01, CDGM '02

Motivation for phenomenological reanalysis

- Updated data sets
- Use of further techniques to assess systematic uncertainties
Operator Product Expansion
\[ \Pi(q) = \frac{O_D}{Q_D} \]

\[ O_{D=6} \sim 4\pi\alpha_s \langle O_8 \rangle \mu + \alpha_s^2 (\tilde{C}_1 \langle O_1 \rangle \mu + \tilde{C}_8 \langle O_8 \rangle \mu), \quad \tilde{C}_i = A_i + B_i \ln(-q^2/\mu^2) \]

Large uncertainties. We work at LO in \( \alpha_s \to O_{D=6} \sim 4\pi\alpha_s \langle O_8 \rangle \mu \)

\[
\lim_{k,p,q \to 0} \langle (\pi\pi)_{l=2} | Q_7 | K^0 \rangle = -\frac{2}{F^3} \langle O_1 \rangle \mu
\]
\[
\lim_{k,p,q \to 0} \langle (\pi\pi)_{l=2} | Q_8 | K^0 \rangle = -\frac{2}{F^3} \left( \frac{1}{2} \langle O_8 \rangle \mu + \frac{1}{N_c} \langle O_1 \rangle \mu \right)
\]

- We still need \( \langle O_1 \rangle \mu \)
- In the large-\( N_c \) limit, \( \frac{\langle Q_7 \rangle \mu}{\langle Q_8 \rangle \mu} = 0 \). Suppression confirmed by other analyses
- Taking \( \frac{\langle Q_7 \rangle \mu}{\langle Q_8 \rangle \mu} \leq \frac{1}{N_c} \to \langle Q_8 \rangle \langle O_1 \rangle \mu \leq \frac{\langle Q_8 \rangle \langle O_8 \rangle \mu}{N_c^2} \frac{1}{1-\frac{1}{N_c^2}} \)
- \( \lim_{p,q,k \to 0} \langle (\pi\pi)_{l=2} | Q_8 | K^0 \rangle \mu \approx -\frac{O_{D=6}(\mu)}{4\pi\alpha_s(\mu)F^3} \)
Revisiting $O_{D=6}$

Gonzalez-Alonso et al. ’16

- Parametrization: $\rho(s) = \kappa e^{-\gamma s} \sin(\beta(s - s_z))$, $s > \hat{s}_0$
- Random tuples $(\kappa, \gamma, \beta, s_z)$. Every tuple $\rightarrow$ a possible spectral function
- Accept those spectral functions in agreement with data and sum rules
- Every spectral function $\rightarrow$ a value of $O_6$. Used to estimate DV uncertainties
- $O_{D=6} \approx ( -3.6 \pm 1.0 ) \cdot 10^{-3} \text{GeV}^6$

Extra tests and small changes aimed to improve it Preliminary!

- Combined fit to WSRs and $O_{D=6}$ so that experimental correlation is taken into account
- Agreement with plateau observed ignoring DVs at $s_0 \sim m^2_\tau$ when pinching
- Stability by changing $\hat{s}_0$ (test of reliability) observed

$O_{D=6} = ( -3.1 \pm 1.0 ) \cdot 10^{-3} \text{ GeV}^6$ Preliminary!
\[ \mathcal{O}_{D=6} = (-3.1 \pm 1.0) \cdot 10^{-3} \text{ GeV}^6 \text{ Preliminary!} \]

\[ \lim_{p,q,k=0} \langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle_{2 \text{ GeV}} = -\frac{\mathcal{O}_{D=6}}{4\pi\alpha_s F^3} \approx (1.14 \pm 0.49) \text{ GeV}^3 \text{ Preliminary!} \]

- Good agreement with the large-\(N_c\) limit

\[ \langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle_{2 \text{ GeV}}^\text{\(N_c\)} = 2FB_0^2 = 2 \frac{M_{K^0}^4 F}{(m_d+m_s)^2} = 1.1 \text{ GeV}^3 \]

- Good agreement with the lattice, although with larger uncertainties...

...We still can turn the table→ use the lattice to improve other tau-based results
Input from the lattice

\[ \langle Q_7 \rangle_{3 \text{GeV}} = 0.36 \pm 0.03 \]

\[ \langle Q_8 \rangle_{3 \text{GeV}} = 1.6 \pm 0.1 \]

Excellent SD knowledge of \( \Pi(s) \)

\[ \Pi(q^2) = \frac{O_D}{Q_D} \]

- \( O_{D=0} = 0 \). It vanishes at all orders in massless perturbative QCD
- \( O_{D=2,4} \approx 0 \). Owing to the values of \( m_u \) and \( m_d \)
- \( O_{D=6} \approx 4\pi\alpha_s \langle O_8 \rangle_\mu + \alpha_s^2 (\tilde{C}_1 \langle O_1 \rangle_\mu + \tilde{C}_8 \langle O_8 \rangle_\mu) \), \( \tilde{C}_i = A_i + B_i \ln(-q^2/\mu^2) \)

\( \langle O_1 \rangle_\mu \) and \( \langle O_8 \rangle_\mu \) known from lattice input + DG relations

Beyond the percent level!

Other applications

- \( O_{D=8} = -(0.7 \pm 0.6) \cdot 10^{-2} \text{ GeV}^8 \text{ Prel.} \)
- Bound to New Physics (see G-A’s talk)
Conclusions Preliminary results

\[ \langle (\pi\pi)_{I=2}|Q_8|K^0\rangle_{2\text{GeV}} = (1.14 \pm 0.49) \text{GeV}^3 \]

Dispersive relations \(\rightarrow\) very precise predictions where p-QCD and \(\chi PT\) do not work!
BACK UP
Dispersion relations with polynomial kernels at NLO in $\alpha_s$

$$\Pi^{(1+0)}(Q^2 = -q^2) = \sum_{p=D/2} a_p(\mu) + b_p(\mu) \ln \frac{Q^2}{\mu^2}$$

$$\int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \text{Im} \Pi(s) = - \int_{s_0}^{\infty} \frac{ds}{s_0} \omega(s) \text{Im} \Delta_{DV}(s) - \frac{\pi}{(-s_0)^p} \sum_{n=0}^{\infty} \sum_{p=1}^{\infty} c_n d_p^{(n)}$$

$$+ 2\pi \frac{f_\pi^2}{s_0} \omega(m_\pi^2),$$

with $\omega(s) = \sum_n c_n x^n$,

$$d_p^{(n)} = \begin{cases} a_p^M & \text{if } p = n + 1 \\ \frac{b_p}{n-p+1} & \text{if } p \neq n + 1 \end{cases},$$

$$a_p^M(\mu, s_0) = a_p(\mu) + b_p(\mu) \ln \left( \frac{s_0}{\mu^2} \right).$$
Sum rules in the chiral limit

\[ \omega^{(2,0)}(s) \equiv \left(1 - \frac{s}{s_0}\right)^2 \]

\[
\int_{s_0}^{s_0} \frac{ds}{s_0} \left(1 - \frac{s}{s_0}\right)^2 \text{Im } \Pi(s) = 2\pi \frac{f_\pi^2}{s_0} - 4\pi \frac{f_\pi^2 m_\pi^2}{s_0^2} + 2\pi \frac{f_\pi^2 m_\pi^4}{s_0} + \pi \frac{a_3}{s_0^3} + \delta \omega^{(2,0)}(s_0),
\]

\[
\int_{s_0}^{s_0} \frac{ds}{s_0} \left(1 - \frac{s}{s_0}\right)^2 \text{Im } \Pi_{m_q=0}(s) = 2\pi \frac{F^2}{s_0} + \pi \frac{a_3^0}{s_0^3} + \delta \omega^{(2,0),0}(s_0),
\]

\[
\int_{s_0}^{s_0} \frac{ds}{s_0} \left(1 - \frac{s}{s_0}\right)^2 (\text{Im } \Pi(s) - \text{Im } \Pi_{m_q=0}(s)) \approx 2\pi \frac{f_\pi^2 - F^2}{s_0}.
\]

\[ 2\pi \frac{f_\pi^2 - F^2}{s_0} \sim 0.0028, \quad \pi \frac{|a_3|}{s_0^3} \sim 0.0015 \]
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| $\hat{s}_0 (\text{GeV}^2)$ | 1.25 | 1.4 | 1.55 | 1.7 | 1.9 |
|---------------------------|------|-----|------|-----|-----|
| $a_3 (10^{-3} \text{GeV}^6)$ | $-5.3^{+0.7}_{-0.5}$ | $-5.1^{+0.7}_{-0.5}$ | $-5.3^{+0.5}_{-0.3}$ | $-3.7^{+1.3}_{-0.9}$ | $-3.8^{+1.8}_{-1.0}$ |