AGGREGATION OF COMPOSITE SOLUTIONS: STRATEGIES, MODELS, EXAMPLES

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ABSTRACT. The paper addresses aggregation issues for composite (modular) solutions. A systemic viewpoint is suggested for various aggregation problems. Several solution structures are considered: sets, set morphologies, trees, etc. Mainly, the aggregation approach is targeted to set morphologies. The aggregation problems are based on basic structures as substructure, superstructure, median/consensus, and extended median/consensus. In the last case, preliminary structure is built (e.g., substructure, median/consensus) and addition of solution elements is considered while taking into account profit of the additional elements and total resource constraint. Four aggregation strategies are examined: (i) extension strategy (designing a substructure of initial solutions as "system kernel" and extension of the substructure by additional elements); (ii) compression strategy (designing a superstructure of initial solutions and deletion of some its elements); (iii) combined strategy; and (iv) new design strategy to build a new solution over an extended domain of solution elements. Numerical real-world examples (e.g., telemetry system, communication protocol, student plan, security system, Web-based information system, investment, educational courses) illustrate the suggested aggregation approach.

CONTENTS

1. Introduction 2
2. Auxiliary Problems and Aggregation Strategies 5
   2.1. Basic Auxiliary Problems 5
   2.2. Building of "System Kernel" 6
   2.3. Aggregation Strategies 7
3. Examined Structures, Substructure, Superstructure 9
   3.1. Sets 10
   3.2. Rankings (Layered Sets) 10
   3.3. Multisets and Morphological Sets 10
   3.4. Trees and Morphological Structures 12
4. Preliminary Illustrative Example for Notebook 13
5. Metrics and Proximities 17
   5.1. Metric/Proximity for Sets 17
   5.2. Proximity for Strings/Sequences 18
   5.3. Proximity for Rankings 18
   5.4. Proximity for Trees 21
   5.5. Proximity for Morphological Structures 23

Key words and phrases. composite solution, modular system, aggregation, consensus, agreement, median, structure, ranking, tree, morphological structure, metric, proximity, solving strategy, heuristics, combinatorial optimization, knapsack problem, multiple choice problem, clique, morphological design, decision making, systems engineering, applications.
1. Introduction

Traditional approaches to generation of new design solutions are based on modifications/improvements of existing products/systems (e.g., [37]). Often one existing product/system is used as a basic solution for modification/improvement (e.g., [101], [106], [108], [111], [112], [118], [120], [121]) (Fig. 1.1).

![Fig. 1.1. Modification/improvement of existing system](image-url)

In this article, the generation of a new modular design solution is examined as aggregation of a set of initial existing modular (composite) design solutions. The considered aggregation process is depicted in Fig. 1.2:

Find an aggregated composite solution $S^{agg}$ from a set of initial composite solutions $\{S_1, ..., S_\tau, ..., S_m\}$, i.e., $\{S_1, ..., S_\tau, ..., S_m\} \rightarrow S^{agg}$. 
Fig. 1.2. Illustration for aggregation process

Note, the considered aggregation process has to be based on an analysis of engineering and/or management application(s). An expert-based scheme (framework) for problem solving (Fig. 1.3) is used as a basic framework for aggregation of composite (modular) solutions.

Here it is necessary to point out main properties of the expert-based approach:
1. Each operation/step of the solving process has to be executed while taking into account an applied expert-based analysis of the problem situation or subsituation.
2. At each step of the solving process an expert opinion is the most important and the domain expert can correct the solving scheme and intermediate and/or resultant solutions.

Fig 1.3. Expert-based approach to problem solving

Generally, two aggregation cases have to be considered:

Case 1. Usage of elements from \( m \) initial solutions.

Case 2. Usage of elements from \( m \) initial solutions and additional design elements from an extended design domain as well.

In case 1 three solving strategies can be examined. First, the evident strategy consists in an analysis of the set of \( m \) initial solutions and building a system substructure ("system kernel" \( K \), e.g., a "basic system part" as substructure, median/consensus of the initial solutions) that can be extended or modified. Several methods can be used to build the "system kernel" \( K \): (a) substructure of the initial solutions, (b) median structure (or consensus/agreement) for the initial solutions, (c) extended substructure, (d) extended median structure (or extended consensus), (e) selection of the "best" system element for each specified system part.
Here expert judgment of expert(s) domain can be used at each stage of the solving process.

Second, a superstructure for set of $m$ initial solutions has to be designed $\Omega$ (e.g., combining the initial solutions, “covering” the initial solutions). Then, the superstructure is compressed via deletion of the less important elements.

In case 2, it is possible to consider the following:

- to build a generalized domain of design alternatives (i.e., solution elements) and
to design a new solution (including usage of new design elements).

Here the extended design domain may be obtained via the following two ways: (a) covering/approximation of $m$ initial solutions by a new design alternative domain, (b) examination of a new design alternative domain.

In the article, the following structures will be examined as basic ones: (i) sets (e.g., [93]), (ii) rankings (e.g., [29], [35], [36], [101]), (iii) set morphologies (e.g., [9], [54], [101], [105], [108], [110], [175]), (iv) trees (e.g., [57], [69], [93]).

General structure (i.e., system model) $\Lambda$ is: $\Lambda = (T, P, D, R, I)$, where the following parts are considered ([101], [105], [108]): (i) tree-like system model $T$, (ii) set of leaf nodes as basic system parts/components $P$, (iii) sets of DAs for each leaf node $D$, (iv) DAs rankings (i.e., ordinal priorities) $R$, and (v) compatibility estimates between DAs $I$.

Fig. 1.4 illustrates the generalized architecture (structure) of examined modular systems/solutions ($\Lambda$). Note the following significant structures are considered as well: (a) morphology: $\Phi = (P, D)$; (b) morphological set: $\Phi = (P, D, R, I)$; (c) morphological structure (tree): $\Phi = (T, P, D, R)$; (d) morphological structure with compatibility (i.e., with compatibility of DAs): $\Phi = (T, P, D, R, I)$.

In Table 1.1, building problems for basic substructures/superstructures are briefly pointed out.

Finally, two basis system problems are faced: (a) revelation/design of “system kernel”, (b) building an extended design alternative domain. Four solving aggregated strategies are considered:

1. **Extension strategy**: designing “system kernel” based on set of initial solutions (e.g., substructure, median/consensus) and extension of “system kernel” by additional elements;

2. **Compression strategy**: designing a superstructure of initial solutions and deletion of some its elements; and

In Fig. 1.4, the architecture of modular system ($\Lambda$) is illustrated.
(3) combined strategy (extension, deletion, and replacement operations for system elements over a preliminary aggregated solution), and
(4) design strategy: building an extended design domain of solution elements and designing a new solution.

The above-mentioned strategies are based on combinatorial models (as underlying problems): multicriteria ranking/selection, knapsack problem, multiple choice knapsack problem, combinatorial morphological synthesis.

The suggested approaches are illustrated through numerical examples including applied examples (e.g., telemetry system, communication protocol, hierarchical security system, plan of students art activity, combinatorial investment, Web-based information system, notebook, educational courses).

Table 1.1. Basic structures and bibliography references

| Initial structures | Target structure | References |
|--------------------|------------------|------------|
| Sets               | Subset, superset  | [2], [77]  |
| Strings/sequences  | Common subsequence, common supersequence, median string, consensus | [4], [5], [8], [13], [18], [21], [46], [55], [65], [74], [78], [82], [85], [126], [129], [130], [133], [135], [143], [145], [151], [153] |
| Rankings           | (i) Consensus/median | [12], [16], [35], [34], [36], [50], [72], [88], [89], [71], [91], [100], [101] |
| Trees              | (i) Agreement/consensus tree | [2], [3], [19], [52], [54], [76], [133], [146] |
| Trees              | (ii) Agreement forest | [30], [66], [138], [164], [165] |
| Trees              | (iii) Supertree    | [19], [33], [75], [142] |
| Graphs             | Common subgraph, common supergraph | [53], [81], [104], [106] |

2. Auxiliary Problems and Aggregation Strategies

2.1. Basic Auxiliary Problems. In the case of two initial solutions as element sets $A_1$ and $A_2$, Fig. 2.1 illustrates substructure $\tilde{S}_{A_1, A_2} \subseteq (A_1 \& A_2)$ and superstructure $\bar{S}_{A_1, A_2} \supseteq (A_1 \cup A_2)$ (via Venn-diagram).

Fig. 2.1. Illustration for substructure, superstructure

Superstructure $\bar{S}_{A_1, A_2} \supseteq (A_1 \cup A_2)$
Substructure $\tilde{S}_{A_1, A_2} \subseteq (A_1 \& A_2)$
Further, let $S = \{S_1, ..., S_i, ..., S_n\}$ be a set of initial solutions (structures). Let function $\rho(S_{i_1}, S_{i_2}), i_1, i_2 \in \{1, 2, ..., n\}$ be a proximity or a metric for the solutions. The following main basic auxiliary problems can be examined:

**Problem 1.** Find a maximum substructure:

$$\tilde{S} = \arg \max_{\{S'\}} (|S'|), \forall S' \in \bigcap_{i=1}^{n} S_i.$$

**Problem 2.** Find a minimum superstructure

$$\overline{S} = \arg \min_{\{S''\}} (|S''|), \forall S'' \in \bigcup_{i=1}^{n} S_i.$$

Now let us consider definitions of medians for the above-mentioned set of initial sets $S$ (e.g., [40], [82], [130], [145]):

(a) median ("generalized median") $M^g$ is:

$$M^g = \arg \min_{M \in D} \left( \sum_{i=1}^{n} \rho(M, D_i) \right),$$

where $D (D \supseteq S)$ is a set of structures of a specified kind (searching for the median is usually NP complete problem):

(b) simplified case of median (an approximation) as "set median" $M^s$ over set $S$:

$$M^s = \arg \min_{M \in S} \left( \sum_{i=1}^{n} \rho(M, S_i) \right).$$

Here a representative from $S = \{S_1, ..., S_i, ..., S_n\}$ is searched for. Computation of proximity $\rho(M, S_i)$ is usually NP-complete problem as well. Note a similar “closest string problem” is widely applied in bioinformatics (e.g., [59], [62], [63], [87], [122], [125], [163]). Finally, the following problem 3 and problem 4 can be considered:

**Problem 3.** Find "set median" $M^s$.

**Problem 4.** Find "median" ("generalized median") $M^g$.

**Problem 5.** Find an extended median/consensus structure via addition to (or correction/editing of) the basic median/structure some elements while some resource constraint(s). Here some elements are added to the median set (problem 3 or 4) while taking into account profit and required resource for the addition.

The problems are considered for the following kinds of structures: (i) sets, (ii) set morphologies, (iii) trees, and (iv) trees with set morphologies. In the case of vector-like metric/proximity $\rho(S_{i_1}, S_{i_2})$, Pareto-efficient solutions are searched for in problem 3, problem 4, and problem 5.

2.2. **Building of “System Kernel”**. The basic auxiliary problem consists in designing the “system kernel”. Several methods can be used to build the “system kernel” $K$:

(i) substructure of the initial solutions,

(ii) median structure (or consensus/agreement) for the initial solutions,
(iii) extended substructure (or extended median structure, extended consensus), and
(iv) a two-stage framework: (a) specifying a set of basic system parts (as a subset of the system parts/components), (b) selection of the “best” system elements for each specified basic system part above.

Sometimes, a subsolution (i.e., “system kernel” as substructure) is a very small subset. In this case, it is reasonable to use a special (more “soft”) method to select elements for “system kernel”. Let $S = \{S_1, ..., S_i, ..., S_n\}$ be a set of initial solutions (structures). Then, element $e$ will be included into (added to) “system kernel” if $\eta_i \geq \alpha$ (e.g., $\alpha \geq 0.5$) where $\eta_i$ is the number of initial solutions which involve element $e$. The usage of this rule will lead to an extension of the basic method for building the system substructure.

2.3. Aggregation Strategies. Four basic aggregation strategies are examined: (i) extension strategy (extension of “system kernel”), (ii) deletion strategy (compression of a superstructure for initial solutions), (iii) combined strategy (i.e., extension, deletion, and replacement operations for system elements over a preliminary aggregated solution), and (iv) design strategy (new system design).

The extension strategy is the following:

**Type I.** Extension strategy:

**Phase 1.1.** Analysis of applied problem, initial solutions, resources, solution elements.

**Phase 1.2.** Revealing from $m$ initial solutions a basis as a subsolution or “system kernel” $K$ (e.g., subset of elements, substructure, median/consensus).

**Phase 1.3.** Forming a set of additional solution elements which were not included into the basis above.

**Phase 1.4.** Selection of the most important elements from the set of additional elements while taking into account the following: (i) profit of the selected elements, (ii) total resource constraint(s), (iii) compatibility among the selected elements and elements of the basis (i.e., “system kernel”).

**Phase 1.5.** Analysis of the obtained aggregated solution(s).

Fig. 2.2 illustrates the extension strategy.
The second (compression) strategy is based on preliminary union of all elements of \( m \) initial solutions and deletion of the non-important elements to satisfy some resource constraint(s) (Fig. 2.3):

**Type II.** Compression (deletion) strategy:

*Phase 2.1.* Analysis of applied problem, initial solutions, resources, solution elements.

*Phase 2.2.* Union of all elements from \( m \) initial solutions to form a basic supersolution (e.g., superset of elements, superstructure).

*Phase 2.3.* Forming a set of element candidates to delete.

*Phase 2.4.* Selection of the most non-important elements from the set of element candidates while taking into account the following: (i) integrated profit of the compressed solution, (ii) total resource constraint(s), (iii) compatibility among the selected elements of the compressed solution.

*Phase 2.5.* Analysis of the obtained aggregated solution(s).

---

The third strategy consists in possible combination of addition operations, deletion operation, and correction (replacement) operations. The third strategy is (Fig. 2.4):

**Type III.** Combined strategy:
Phase 3.1. Analysis of applied problem, initial solutions, resources, solution elements.

Phase 3.2. Revealing from \( m \) initial solutions a basis as a subsolution or “system kernel” \( K \) (e.g., subset of elements, substructure, median/consensus).

Phase 3.3. Forming the following:
(a) a set of additional solution elements as candidates for addition \( W \) which were not included into the basis above \((|W \cap K| = 0)\),
(b) a subset \( B \subseteq K \) as candidates for deletion, and
(c) a set of element pair \( C = \{(a_u, b_v) | (a_u \in K \& b_v \in K)\} \), thus a set of correction operations is considered as replacement of \( a_u \) by \( b_v \).

Phase 3.4. Selection of the most important operations (including element addition, element deletion, and element replacement) while taking into account the following: (i) profit of the operations, (ii) total resource constraint(s), (iii) compatibility among the selected elements in the resultant solution.

Phase 3.5. Analysis of the obtained aggregated solution(s).

The fourth strategy is targeted to usage of additional elements from an extended design domain (“design space”). Here the resultant aggregated solution may involve elements which were not belonging to \( m \) initial solutions. The fourth strategy is:

Type IV. Strategy of extended design domain:

Phase 4.1. Analysis of applied problem, initial solutions, resources, solution elements.

Phase 4.2. Extension of the union of all elements from \( m \) initial solutions to form an extended design element domain (“design space”).

Phase 4.3. New design of the composite solution over the obtained design element domain while taking into account the following: (i) profit of the designed solution, (ii) total resource constraint(s), (iii) compatibility among the selected (i.e., resultant) elements of the compressed solution.

Phase 4.4. Analysis of the obtained aggregated solution(s).

Fig. 2.5 illustrates the fourth strategy. Given three initial solutions \( S_1 \), \( S_2 \), and \( S_3 \). The resultant aggregated solution \( S_{agg} \) can involve elements from solution \( S_1 \), \( S_2 \) and elements from the extended design domain.

Fig. 2.5. Illustration for aggregation design strategy

3. Examined Structures, Substructure, Superstructure

Generally, the following basic kinds of sets are examined: sets, multisets, lists, complete lists, and trees (e.g., [43, 93, 137]). In this material, our main examination is targeted to special kinds of composite structures: morphological structures.
3.1. **Sets.** Evidently, sets are basic structures. Fig. 3.1 depicts a numerical example of two initial sets $A$ and $B$.

\[
\begin{array}{c}
\text{Set } A \\
\begin{array}{cccccccccc}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
1 & 3 & 4 & 5 & 6 & 8 & 10 & 11 & 12
\end{array}
\end{array}
\quad
\begin{array}{c}
\text{Set } B \\
\begin{array}{cccccccccc}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
1 & 2 & 4 & 6 & 7 & 9 & 10 & 12
\end{array}
\end{array}
\]

Fig. 3.1. Illustration: two initial sets

Fig. 3.2 contains examples of subset $\tilde{S}_{AB}$ and superset $\overline{S}_{AB}$.

\[
\begin{array}{c}
\text{Subset } \tilde{S}_{AB} \\
\begin{array}{ccccccc}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
1 & 4 & 6 & 10 & 12
\end{array}
\end{array}
\quad
\begin{array}{c}
\text{Superset } \overline{S}_{AB} \\
\begin{array}{ccccccccccccccc}
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12
\end{array}
\end{array}
\]

Fig. 3.2. Illustration: subset and superset

Now let us consider the case of $m$ sets. Let $\{A_1, ..., A_i, ..., A_m\}$ be initial sets. Then, a subset is: $\tilde{S}_{\{A_i\}} \subseteq \bigcap_{i=1}^m A_i$. A superset is: $\overline{S}_{\{A_i\}} \supseteq \bigcup_{i=1}^m A_i$.

3.2. **Rankings (Layered Sets).** Here ranking is examined as a layered set. Let $A = \{1, ..., i, ..., n\}$ be a set of elements/items. Ranking (a partial order/partition of set $A$) is considered as linear ordered subsets of $A$ (Fig. 3.3): $A = \bigcup_{k=1}^m A(k)$, $|A(k_1) \cap A(k_2)| = 0$ if $k_1 \neq k_2$, $i_2 \preceq i_1 \forall i_1 \in A(k_1)$, $\forall i_2 \in H(k_2)$, $k_1 \leq k_2$. Set $A(k)$ is called layer $k$, and each item $i \in A$ gets priority $r_i$ that equals the number of the corresponding layer. The described partition of $A$ is be called as partial ranking, stratification, layered set (e.g., [17], [42], [98], [100], [101], [106], [115], [139], [173]).

Evidently, a linear order of elements from $A$ is ranking as well. Many years ranking problems (or sorting) have been intensively used and studied in various domains (e.g., [17], [23], [42], [29], [88], [89], [106], [115], [139], [173], [174]).

\[
\begin{array}{c}
\text{Initial elements} \\
A = \{1, ..., i, ..., n\}
\end{array}
\Rightarrow
\begin{array}{c}
A_1
\end{array}
\Rightarrow
\begin{array}{c}
A_k
\end{array}
\Rightarrow
\begin{array}{c}
A_m
\end{array}
\]

Fig. 3.3. Scheme of ranking

3.3. **Multisets and Morphological Sets.** Now let us consider two kinds of basic structures: multiset and morphological set.

Let $U$ be a universe (collection of all relevant items) and $N$ be a set of non-negative integers (e.g., $N = \{1, ..., \tau, ..., m\}$). Formally, multiset is a mapping:

$$\Gamma: U \rightarrow N.$$ 

Clearly, three basic operations can be considered for sets and multisets:
1. unions: $A \bigcup B$ (for sets $A$ and $B$, analogically for multisets),
2. intersections: $A \bigcap B$ (for sets $A$ and $B$, analogically for multisets), and
3. complements: $A \setminus B$ (for sets $A$ and $B$, analogically for multisets).

A macroset is a (finite or infinite) set of multisets over a finite alphabet. Generally, *morphological set* is defined as follows:

**Definition 3.1.** Morphological set is a structure consisting of:
1. a finite set of integers $N = \{1, \ldots, \tau, \ldots, m\}$ (each integer $\tau$ corresponds to a system part);
2. set of elements (alternatives) for each system part $\tau$:
   $A_\tau = \{A_{\tau 1}, A_{\tau \xi}, \ldots, A_{\tau q}\}$, where $A_{\tau \xi}$ is a design alternative;
3. preference relation over elements of $A_\tau$ (or estimates upon a set of specified criteria or resultant ordinal priorities for each alternative $p(A_{\tau \xi})$); and
4. weighted (by ordinal scale) binary relation of compatibility for each pair of system parts $(\alpha, \beta) \in N$ over elements of alternative sets $A_\alpha, A_\beta$: $R_{A_\alpha, A_\beta}$.

A composite system consisting of $m$ parts is examined (Fig. 3.4) (101), (106), (110).

**Fig. 3.4. Illustration for multi-part system**

Thus, system morphology $S_A$ (or morphology $\Phi$) is defined as follows $A = \{A_1, \ldots, A_\tau, \ldots, A_m\}$ (Fig. 3.5).

**Fig. 3.5. Illustration: system morphology**

Further, numerical examples for two morphologies are presented: morphology $A^1$ (Fig. 3.6), morphology $A^2$ (Fig. 3.7), sub-morphology $\tilde{S}_{A^1, A^2}$ (Fig. 3.8), and super-morphology $S_{A^1, A^2}$ (Fig. 3.9).
In addition, tables of ordinal compatibility estimates have to be considered. The aggregation of estimates (by element) can be based on a special operation (e.g., minimal value of element). Table 3.1 presents an illustrative numerical example for compatibility estimates and their aggregation.

| Morphology $A^1$ | Morphology $A^2$ | Super-morphology $\overline{S}_{A^1,A^2}$ |
|-------------------|-------------------|------------------------------------------|
| $A_{21}$ | $A_{22}$ | $A_{21}$ | $A_{22}$ | $A_{23}$ | $A_{24}$ |
| $A_{11}$ | $2$ | $0$ | $1$ | $2$ | $1$ |
| $A_{12}$ | $3$ | $1$ | $2$ | $1$ | $2$ |
| $A_{13}$ | $2$ | $3$ | $1$ | $2$ | $2$ |

3.4. **Trees and Morphological Structures.** Now let us consider trees and morphological structures (as morphological trees). Fig. 3.10 illustrates a numerical example for two trees $T_1$ and $T_2$.

Fig. 3.10. Illustration for two initial trees

Fig. 3.11 depicts corresponding examples of supertree $\overline{T}$ and subtree $\overline{T}$. 
Now let us consider trees with morphologies. Fig. 3.12 illustrates two trees with morphologies $\Theta_1$ and $\Theta_2$.

Fig. 3.13 illustrates examples of supertree and subtree with morphologies $\overline{\Theta}$ and $\tilde{\Theta}$.

Evidently, tables of compatibility estimates and their aggregation can be considered here as well.

4. Preliminary Illustrative Example for Notebook

The preliminary illustrative applied example is targeted to representation and aggregation of three initial personal computers (notebooks). The considered general morphological structure of the notebook is presented in Fig. 4.1:

0. Notebook $S$.
1. Hardware $H$:
   1.1. Basic computation $C$:
      1.1.1. Mother board $B$: $B_1$ (P67A - C43(B3) ATX Intel), $B_2$ (MSI 870A-G54 ATX AMD), $B_3$ (ASRoot P67 EXTREME 4(B3) ATX Intel);
1.1.2. CPU $U$: $U_1$ (Intel Pentium dual-core processor T 2330), $U_2$ (Celeron dual-core processor 2330), $U_3$ (Intel core 2 T 7200);
1.1.3. RAM $R$: $R_1$ (1 GB DDR A-DATA), $R_2$ (2 GB DDR2 KINGSTON), $R_3$ (2 GB DDR3 A-DATA), $R_4$ (2 GB DDR3 HYPERX KINGSTON);
1.2. Hard drive $V$: $V_1$ (100 GB HDD), $V_2$ (120 GB HDD), $V_3$ (160 GB HDD), $V_4$ (200 GB HDD);
1.3. Video/graphic cards $J$: $J_1$ (NVIDIA GeForce CTS 300M), $J_2$ (GT 400M Series), $J_3$ (ATI Radion HD 5000 M Series);
1.4. Communication equipment (modems) $E$: $E_1$ (Internal Modem & Antenna), $E_2$ (None).

2. Software $W$:
2.1. Operation system and safety $Y$:
2.1.1. OS $O$: $O_1$ (Windows XP), $O_2$ (Windows Vista); $O_3$ (Linux).
2.1.2. Safety software $F$: $F_1$ (Norton AntiVirus), $F_2$ (AntiVirus Kaspersky).
2.2. Information processing and Internet $I$:
2.2.1. Data support and processing $D$: $D_1$ (Microsoft Office), $D_2$ (None).
2.2.2. Internet access (browser) $A$: $A_1$ (Microsoft Internet Explorer), $A_2$ (Mozilla); $A_3$ (Microsoft Internet Explorer & Mozilla).
2.3. Professional software $Z$:
2.3.1. Information processing (e.g., engineering software) $G$: $G_1$ (Matlab), $G_2$ (LabView); $G_3$ (MatCad), $G_4$ (None);
2.3.2. Special software development environment $P$: $P_1$ (C++), $P_2$ (JAVA); $P_3$ (Delphi), $P_4$ (None);
2.3.3. Special editors $L$: $L_1$ (LaTex), $L_2$ (None);
2.3.4. Games $Q$: $Q_1$ (Tetris), $Q_2$ (Solitaire), $Q_3$ (Chess).

Now a simplified illustrative example for four initial solutions $S_1$, $S_2$, $S_3$, and $S_4$ is examined (Fig. 4.2, Fig. 4.3, Fig. 4.4, Fig. 4.5). Here the tree-like structure is not changed and only leaf nodes are considered. Compatibility estimates between design alternatives are not considered. A framework of aggregation process for four notebooks is depicted in Fig. 4.6 (including two alternative methods to build the “system kernel”).
Substructure for the considered solutions $\tilde{S}$ is depicted in Fig. 4.7, superstructure $S$ is depicted in Fig. 4.8. Further, let us consider “system kernel” $K$ as an extension of substructure $\tilde{S}$ (Fig. 4.9).
Further, the aggregation strategy as modification of “system kernel” $K$ can be applied. A set of candidate modification operations are the following:

1. addition operations: 1.1. addition for $U$: $U_1$ or $U_2$ or $U_3$, 1.2. addition for $F$: $F_1$ or $F_2$, 1.3. addition for $P$: $P_2$ or $P_3$ or $P_4$;

2. correction operations: 2.1. replacement $B_1 \Rightarrow B_3$, 2.2. replacement $V_3 \Rightarrow V_4$, 2.3. replacement $A_1 \Rightarrow A_3$.

Evidently, it is reasonable to evaluate the above-mentioned modification operations (e.g., cost, profit) and to consider an optimization model. Later, corresponding optimization problems (e.g., knapsack problem, multiple choice problem) will be examined. An example of the resultant solution $S^3$ (modification of “system kernel” $K$) is shown in Fig. 4.10.

Further, the system correction process is based on the following operations:

1. addition: $A_1$, $P_1$, $L_1$;
2. deletion: $E_2$;
3. replacement: $B_1 \Rightarrow B_3$, $U_2 \Rightarrow U_1$, $O_2 \Rightarrow O_1$. 

On the other hand, building the “system kernel” can be based on multicriteria selection and/or expert judgment. Let us consider the following basic structure of “system kernel”: $B$, $U$, $R$, $V$, $O$, $F$, $D$, $G$. For each system component above, it is possible to consider a selection procedure to choose the “best” system element (while taking into account elements of the initial solution or additional elements as well). For example, we can obtain the following “system kernel”:

$K^* = B_1 \ast U_2 \ast R_2 \ast V_3 \ast E_2 \ast O_2 \ast F_2 \ast D_2 \ast G_2$.
A resultant solution $S^2$ is presented in Fig. 4.11.

Fig. 4.11. Solution $S^2$

5. Metrics and Proximities

Let us consider similarity measure between objects (in our case: sets, rankings, trees, graphs) $A_1$ and $A_2$. It is often desired that the distance measure (function) $d(A_1, A_2)$ fulfills the following properties of a metric:

1. $d(A_1, A_2) \geq 0$ (nonnegativity),
2. $d(A_1, A_1) = 0$ (identity),
3. $d(A_1, A_2) = 0 \iff A \cong B$ (uniqueness),
4. $d(A_1, A_2) = d(A_2, A_1)$ (symmetry),
5. $d(A_1, A_2) + d(A_2, A_3) \geq d(A_1, A_3)$ (triangle inequality).

If the function satisfies $d(A_1, A_2) \leq 1$ it is said to be a normalized metric.

In many applied domains, the above-mentioned conditions are too restrictive and a more weak set of properties is used. As a result, $d(A_1, A_2)$ corresponds to more weak situations, for example: (i) quasi-metrics, (ii) proximities (without property 5, e.g., proximity for rankings in [101]).

Metrics/proximities play the basic role in many important problems over structures, for example: approximation, modification, aggregation. There exist three basic approaches to similarity/proximity of objects/structures:

1. traditional metrics/distances (e.g., [22], [47], [60], [61], [128]);
2. minimum cost transformation of an object/structure into another one (edit distance) (e.g., [20], [25], [68], [97], [127], [138], [158], [171]); and
3. maximum common substructure or maximum agreement substructure (e.g., [2], [3], [25], [26], [104], [106], [136], [161]).

5.1. Metric/Proximity for Sets. Let $A = \{1, \ldots, i, \ldots, n\}$ be a set of elements. Let us consider ... for two subsets $A_1 \subseteq A$ and $A_2 \subseteq A$. The most simple case of metric by elements (i.e., distance) is the following: (while taking into account assumption $|A_1 \cup A_2| \neq 0$):

$$\rho_c(A^1, A^2) = 1 - \frac{|A_1 \cap A_2|}{|A_1 \cup A_2|}.$$  

Further, let $w_i \in (0, 1]$ be a weight of element $i \in A$. Then proximity (i.e., metric, distance) by element weights is as follows (while taking into account assumption $\sum_{i \in (A^1 \cup A^2)} w_i \neq 0$):

$$\rho_w(A^1, A^2) = 1 - \frac{\sum_{i \in (A^1 \cap A^2)} w_i}{\sum_{i \in (A^1 \cup A^2)} w_i}.$$  

Now let us consider a simple numerical example (Table 5.1) that involves initial set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and two subsets: $A^1 = \{1, 2, 4, 5\}$ and $A^2 = \{1, 2, 3, 4, 5, 6, 7\}$. Elements weights are the following: 0.5, 0.6, 0.4, 1.0, 0.7, 0.2, 0.1, and 1.0. The resultant proximities are: $\rho_c(A^1, A^2) = 1 - \frac{3}{7} = 0.571$ and $\rho_w(A^1, A^2) = 1 - \frac{3}{8/5} = 0.496$.  

...
Table 5.1. Illustrative numerical example

| i | $w_i$ | $A$ | $A^1$ | $A^2$ | $A^1 \cup A^2$ | $A^1 \cap A^2$ |
|---|---|---|---|---|---|---|
| 1 | 0.5 | * | * | * | * | * |
| 2 | 0.6 | * | * | * | * | * |
| 3 | 0.4 | * | * | * | * | * |
| 4 | 1.0 | * | * | * | * | * |
| 5 | 0.7 | * | * | * | * | * |
| 6 | 0.2 | * | * | * | * | * |
| 7 | 0.1 | * | * | * | * | * |
| 8 | 1.0 | * | * | * | * | * |

In the case of vector weights $\mathbf{w}_i = (w^1_i, \ldots, w^r_i)$, a simplified vector distance (or proximity) is:

$$\rho_\mathbf{w}(A^1, A^2) = (\rho^1_\mathbf{w}(A^1, A^2), \ldots, \rho^r_\mathbf{w}(A^1, A^2)) = (1 - \frac{\sum_{i \in (A^1 \cap A^2)} w^1_i}{\sum_{i \in (A^1 \cup A^2)} w^1_i}, \ldots, 1 - \frac{\sum_{i \in (A^1 \cap A^2)} w^r_i}{\sum_{i \in (A^1 \cup A^2)} w^r_i}).$$

5.2. Proximity for Strings/Sequences. This problem of proximity analysis is important in code design, genom studies, information processing, and mathematical linguistics. Generally, three kinds of proximity for strings/sequences are examined: (1) common part of initial strings as substring, superstring (e.g., [7], [6], [40], [65], [78], [79], [153]), (2) median string (e.g., [94], [130]), and (3) editing distance (a length/cost of a transformation/editing) (e.g., [6], [68], [97], [137], [160]).

Here basic research issues are targeted to the following: (a) complexity analysis (e.g., [153]); (b) design of polynomial algorithms (e.g., [68], [79]; and (c) design of approximation algorithms (e.g., [78]).

5.3. Proximity for Rankings. Here several types of metrics/proximities can be used, for example: 1. Kendall-Tau distance [90]; 2. distances for partial rankings (e.g., [11], [51]); 3. vector-like proximity ([98], [100], [101]). Further, Kendall-Tau distance and vector-like proximity are briefly described (the description is based on material from [101]).

Let $\|g_{ij}\|, (i, j \in A)$ be an adjacency matrix for graph $G$:

$$g_{ij} = \begin{cases} 1, & \text{if } i \succ j, \\ 0, & \text{if } i \sim j, \\ -1, & \text{if } i \prec j. \end{cases}$$

Kendall-Tau distance (metric) for graphs $G^1$ and $G^2$ is the following:

$$\rho_K(G^1, G^2) = \sum_{i < j} | g^1_{ij} - g^2_{ij} |,$$

where $g^1_{ij}, g^2_{ij}$ are elements of adjacency matrices of graphs $G^1$ and $G^2$ accordingly.

Basic definitions for vector-like proximity are the following ([98], [100], [101]). Let $\Psi(S)$ be a set of all layered structures on $A$. 
**Definition 5.1.** Let us call the first order error $\forall i \in A$, and the second order error $\forall (i, j) \in \{A \ast A | i \neq j \forall S, Q \in \Psi(S)\}$ as follows:

$$\delta^*_i (S, Q) = \pi_i (S) - \pi_i (Q),$$

$$\delta^*_{ij} (S, Q) = \pi_i (S) - \pi_j (S) - (\pi_i (Q) - \pi_j (Q)),$$

where $\pi_i (S) = l \forall i \in A(l)$ in $S$. Thus, for an estimate of a discordance between the structures $S, Q \in \Psi(S)$ with respect to $i$ and $(i, j)$, an integer-valued scale with the following ranges is obtained:

(i) $-(m - 1) \leq r \leq m - 1$ for $\delta^*_i (S, Q)$, and
(ii) $-2(m - 1) \leq r \leq 2(m - 1)$ for $\delta^*_{ij} (S, Q)$.

**Definition 5.2.** Let

$$x(S, Q) = (x^{-(m-1)}, \ldots, x^{-1}, x^{1}, \ldots, x^{m-1}),$$

$$y(S, Q) = (y^{-2(m-1)}, \ldots, y^{-1}, y^{1}, \ldots, y^{2(m-1)}),$$

be vectors of an error (proximity) $\forall S, Q \in \Psi(S)$ with respect to components $i$ (1st order), and the pairs $(i, j)$ (2nd order). The components of above-mentioned vectors are defined as follows:

$$x^r = \frac{|\{i \in A | \delta^*_i (S, Q) = r\}|}{n},$$

$$y^r = \frac{2|\{(i, j) \in \{A \ast A | i \neq j\} | \delta^*_{ij} (S, Q) = r\}|}{(n(n - 1))}.$$

Now denote a set of arguments for the components of vectors $x$ and $y$ as follows: $\Omega = \{-k, \ldots, k\}$, negative values as $\Omega^-$, and positive ones as $\Omega^+$. In addition, we will use the vectors $x$ with aggregate components of the following type (similarly, for $y$):

$$x^{k_1, k_2} = \sum_{r = k_1}^{k_2} x^r, x^{\leq -k} = \sum_{r = -(m-1)}^{-k} x^r, x^{\geq k} = \sum_{r = k}^{m+1} x^r, k > 0, x^{|r|} = x^r + x^{-r}.$$

**Definition 5.3.** Let $|\pi(S, Q)| = \sum_{r \in \Omega} x^r, |\pi(S, Q)| = \sum_{r \in \Omega} y^r$ be modules of the vectors.

Vector $x$ will be used as a basic one.

**Definition 5.4.** We will call vectors truncated ones if

(1) the part of terminal components is rejected, e.g.,

$$x(S, Q) = (x^{-(k_1)}, x^{-(k_1-1)}, \ldots, x^{-1}, x^1, \ldots, x^{k_2-1}, x^{k_2}),$$

and one or both of following conditions are satisfied: $k_1 < m - 1, k_2 < m - 1$;

(2) aggregate components are used as follows:

$$x(S, Q) = (x^{\leq k_1}, \ldots, x^{k_u-1}, x^{k_u}, \ldots, x^{k_s+1}, \ldots, x^{k_2}),$$

$$x(S, Q) = (x^{[1]}, \ldots, x^{[r]}, \ldots, x^{[k]}).$$

**Definition 5.5.** Let us call vector $x(y)$:

(a) the two-sided one, if $|\Omega^+| \neq 0$ and $|\Omega^-| = 0$;
(b) the one-sided one, if $|\Omega^+| = 0$ or $|\Omega^-| = 0$;
(c) the symmetrical one, if $-r \in \Omega^-$ exists $\forall r \in \Omega^+$, and vice versa;
(d) the modular one, if it is defined with respect of definition 5.4 (5.3).
Moreover, a pair of linear orders on the components of vectors \( x \) and \( y \) (definition 5.2) is obtained: \textit{component} \( 1(-1) \prec ... \prec \textit{component} k(-k) \).

**Definition 5.6.** \( x_1(S, Q) \succeq x_2(S, Q), \Omega(x_1) = \Omega(x_2), \forall S, Q \in \Psi(S) \), if any decreasing of weak components \( x_1 \) in the comparison with \( x_2 \) is compensated by corresponding increasing of it’s ‘strong’ components \((r, p \in \Omega^+ \text{ or } -r, -p \in \Omega^-)\):

\[
\sum_{r \geq u} x_1^r - \sum_{r \geq u} x_2^r \geq 0, \forall u \in \Omega^+(\forall -u \in \Omega^-, -r \leq -u). 
\]

Now let us consider an illustrative numerical example:

(a) initial set \( A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \),

(b) ranking \( S^1 = \{A_1^1 = \{2, 4\}, A_2^1 = \{9\}, A_3^1 = \{1, 3, 7\}, A_4^1 = \{5, 6, 8\}\} \) and

(c) ranking \( S^2 = \{A_1^2 = \{7, 9\}, A_2^2 = \{1, 3\}, A_3^2 = \{2, 5, 8\}, A_4^2 = \{4, 6\}\} \).

Corresponding adjacency matrices are as follows:

\[
|g_{ij}(S^1)| = \begin{pmatrix} . & -1 & 0 & -1 & 1 & 1 & 0 & 1 & -1 \\ 1 & . & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & . & -1 & 1 & 1 & 0 & 1 & -1 \\ 1 & 0 & 1 & . & 1 & 1 & 1 & 1 & 1 \end{pmatrix}.
\]

\[
|g_{ij}(S^2)| = \begin{pmatrix} . & 1 & 0 & 1 & 1 & 1 & -1 & 1 & -1 \\ -1 & . & -1 & 1 & 0 & 1 & -1 & 0 & -1 \\ 0 & 1 & . & 1 & 1 & 1 & -1 & 1 & -1 \\ -1 & -1 & -1 & . & -1 & 0 & -1 & -1 & -1 \\ -1 & 0 & -1 & 1 & . & 1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & . & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & . & 1 & 1 & 0 \\ -1 & 0 & -1 & 1 & 0 & 1 & -1 & . & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & . \end{pmatrix}.
\]

Now it is very easy to calculate Kendall-Tau distance between \( S^1 \) and \( S^2 \):

\[
\rho_K(S^1, S^2) = 31.
\]

Vector-like proximities allow more prominent description of dissimilarity between structures \( S^1 \) and \( S^2 \):

\[
\pi(S^1) = (\pi_1(S^1), ..., \pi_6(S^1)) = (3, 1, 3, 1, 4, 4, 3, 4, 2),
\]

\[
\pi(S^2) = (\pi_1(S^2), ..., \pi_6(S^2)) = (2, 3, 2, 4, 3, 4, 1, 3, 1),
\]

\[
\delta^\pi_i(S^1, S^2) = (1, -2, 1, -3, 1, 0, 2, 1, 1).
\]
dissimilarity of trees are considered (e.g., [33], [44], [2], [3], [52], [75], [141], [149], [152], [154], [155], [156]).

5.4. **Proximity for Trees.** The following approaches to proximity (similarity/dissimilarity) of trees are considered (e.g., [33], [44], [2], [3], [52], [75], [141], [149], [156], [157]):

(i) metrics (distance) (e.g., [33], [44], [149], [157]) including some special kinds of distances: alignment distance [80], isolated subtree distance [150], top-down distance [141, 167], and bottom-up distance [150];

(ii) tree edit distance (e.g., [31], [140], [148], [154], [168], [169], [170]); and

(iii) common subtree, median tree or tree agreement, consensus (e.g., [2], [3], [52], [54], [75], [76], [142], [140], [162]).

It is reasonable to note, trees are ordered structures. Thus, efficient (polynomial) computing algorithms have been suggested for building metrics/proximities of some kinds of trees (e.g., [49], [154], [153], [156]).

Further, our simplified version of two-component proximity for rooted labelled trees is considered and used. Let \( T' = (A', E') \) and \( T'' = (A'', E'') \) be two rooted labelled trees (the root is the same in both trees) where \( A' \) and \( A'' \) are the vertices, \( E' \) and \( E'' \) are the arcs. Let us consider **dominance parameter** \( \forall (a, b) \in (E' \cap E'') \). The following three **dominance cases** can be examined: (i) \( a \rightarrow b \) (a dominates b, i.e., \( a \succ b \)), (ii) \( b \rightarrow a \) (b dominates a, i.e., \( a \prec b \)), and (iii) \( a \) and \( b \) are independent. Then **dominance parameter is**:

\[
d_{(a, b) \in (E' \cap E'')} = \begin{cases} 
  d^1, & \text{if } a \rightarrow b, \\
  d^2, & \text{if } b \rightarrow a, \\
  d^3, & \text{if } a, b \text{ are independent.}
\end{cases}
\]

As a result, **change parameter** \( \forall (a, b) \in (E' \cap E'') \) can be defined as the following:

\[
p_{(a, b) \in (E' \cap E'')} = \begin{cases} 
  0, & \text{if } d_{(a, b) \in (E' \cap E'')} \text{ is not changed,} \\
  1, & \text{if } d_{(a, b) \in (E' \cap E'')} \text{ is changed.}
\end{cases}
\]

Finally, the proximity of two trees is:

\[
\mathcal{P}(T', T'') = (\rho(A', A''), \rho(E', E''))
\]

where

\[
\rho_e(A', A'') = 1 - \frac{|A' \cap A''|}{|A' \cup A''|}
\]

\[
\rho(E', E'') = \frac{\sum_{(a, b) \in (E' \cap E'')} p_{(a, b) \in (E' \cap E'')}}{|(E' \cap E'')|}
\]

Note, the coefficients \( \frac{1}{n} \) and \( \frac{2}{n(n-1)} \) were not used in \( x \) and \( y \).
Evidently, the following properties are satisfied (Freshe axioms 1 and 2):

1) \( 0 \leq \rho(E', E'') \leq 1 \), (2) \( \rho(E, E) = 0 \).

Note the described approach can be applied for digraphs as well. Now let us consider illustrative numerical examples for trees.

**Example 5.1:** Trees \( T' \) and \( T'' \) are presented in Fig. 5.2. Clearly, the general proximity is: \( \overline{\rho}(T', T'') = (0, 1) \).

![Fig. 5.2. Tree for example 1](image1)

**Example 5.2:** Trees \( T' \) and \( T'' \) are presented in Fig. 5.3. Here \( A' = A'' \) and \( \rho(A', A'') = 0 \). Table 5.2 and table 5.3 contain corresponding dominance factors for \( T' \) and \( T'' \) (changes are depicted via ’ovals’). Finally, the general proximity is: \( \overline{\rho}(T', T'') = (0, \frac{6}{28}) \).

![Fig. 5.3. Tree for example 2](image2)

**Table 5.2. Dominance factor for \( T' \)**

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | * | \( d^1 \) | \( d^1 \) | \( d^1 \) | \( d^1 \) | \( d^1 \) | \( d^1 \) | \( d^1 \) |
| 2 | − | * | \( d^3 \) | \( d^1 \) | \( d^1 \) | \( d^3 \) | \( d^3 \) | \( d^3 \) |
| 3 | − | − | − | * | \( d^3 \) | \( d^3 \) | \( d^1 \) | \( d^1 \) |
| 4 | − | − | − | − | * | \( d^3 \) | \( d^3 \) | \( d^3 \) |
| 5 | − | − | − | − | − | * | \( d^3 \) | \( d^3 \) |
| 6 | − | − | − | − | − | − | * | \( d^3 \) |
| 7 | − | − | − | − | − | − | − | * |
| 8 | − | − | − | − | − | − | − | * |
Table 5.3. Dominance factor for $T''$

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|---|----|----|----|----|----|----|----|----|
| 1 | *  | $d_1$ | $d_1$ | $d_1$ | $d_1$ | $d_1$ | $d_1$ |
| 2 | $-$ | *  | $d_3$ | $d_1$ | $d_1$ | $d_1$ | $d_3$ |
| 3 | $-$ | $-$ | $-$  | *  | $d_3$ | $d_1$ | $d_1$ | $d_3$ |
| 4 | $-$ | $-$ | $-$  | $-$ | *  | $d_3$ | $d_3$ | $d_3$ |
| 5 | $-$ | $-$ | $-$  | $-$ | $-$ | *  | $d_3$ | $d_3$ |
| 6 | $-$ | $-$ | $-$  | $-$ | $-$ | $-$ | *  | $d_3$ |
| 7 | $-$ | $-$ | $-$  | $-$ | $-$ | $-$ | $-$ | *  |
| 8 | $-$ | $-$ | $-$  | $-$ | $-$ | $-$ | $-$ | $-$ |

**Example 5.3:** Trees $T'$ and $T''$ are presented in Fig. 5.4: $A' = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A'' = \{1, 2, 3, 6, 8, 9, 10\}$, and $A' \cap A'' = \{1, 2, 3, 6, 8\}$. The proximity for vertex sets is: $\rho(A', A'') = \frac{1}{2}$.

Table 5.4 and table 5.5 contain corresponding dominance factors for $A' \cap A''$ in $T'$ and in $T''$ (changes are depicted via 'ovals'). Finally, the general proximity is: $\overline{\rho}(T', T'') = (\frac{1}{3}, \frac{1}{3})$.

![Fig. 5.4. Tree for example 3](image)

Table 5.4. Dominance factor for $T'$

|   | 1  | 2  | 3  | 6  | 8  |
|---|----|----|----|----|----|
| 1 | *  | $d_1$ | $d_1$ | $d_1$ | $d_1$ |
| 2 | $-$ | *  | $d_3$ | $d_3$ | $d_3$ |
| 3 | $-$ | $-$ | *  | $d_1$ | $d_1$ |
| 6 | $-$ | $-$ | $-$ | *  | $d_3$ |
| 8 | $-$ | $-$ | $-$ | $-$ | *  |

Table 5.5. Dominance factor for $T''$

|   | 1  | 2  | 3  | 6  | 8  |
|---|----|----|----|----|----|
| 1 | *  | $d_1$ | $d_1$ | $d_1$ | $d_1$ |
| 2 | $-$ | *  | $d_3$ | $d_3$ | $d_3$ |
| 3 | $-$ | $-$ | *  | $d_1$ | $d_1$ |
| 6 | $-$ | $-$ | $-$ | *  | $d_3$ |
| 8 | $-$ | $-$ | $-$ | $-$ | *  |

5.5. **Proximity for Morphological Structures.** Generally, morphological structures are composite ones and it is reasonable to consider the corresponding their proximity as vector-like proximity. General structure ($\Lambda$) consists of the following parts: (i) tree-like system model $T$, (ii) set of leaf nodes $P$, (iii) sets of DAs for each leaf node $D$, (iv) DAs rankings (i.e., ordinal priorities) $R$, and (v) compatibility estimates between DAs $I$. Thus, the vector-like proximity for two morphological structures $\Lambda^\alpha, \Lambda^\beta$ can be examined as follows:

$\overline{\rho}(\Lambda^\alpha, \Lambda^\beta) = (\rho'(T^\alpha, T^\beta), \rho'(P^\alpha, P^\beta), \rho'(D^\alpha, D^\beta), \rho'(R^\alpha, R^\beta), \rho'(I^\alpha, I^\beta))$. 

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**AGGREGATION OF COMPOSITE SOLUTIONS:** STRATEGIES, MODELS, EXAMPLES 23
Let us consider a simplified example: structure (Λ) is examined as a composition of two parts: (a) tree $T = (A, E)$ (i.e., set of vertices and set of arcs), (b) rankings for each leaf node $i$ ($\bigcup_i R_i$). In our case, the proximity of two morphological structures is:

$$\rho(\Lambda', \Lambda'') = (\rho(A', A''), \rho(E', E''), \rho_r(\Lambda', \Lambda'')).$$ 

Here it is assumed sets of design alternatives for leaf vertices are not changed and tables of compatibility estimates are not considered. An example illustrate the approach.

**Example 5.4:** Trees $T'$ and $T''$ are presented in Fig. 5.5: $A' = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A'' = \{1, 2, 3, 4, 5, 6, 7, 9\}$, and $A' \cup A'' = \{1, 2, 3, 4, 5, 6, 7\}$. The proximity for vertex sets is: $\rho(\Lambda', \Lambda'') = \frac{1}{2}$. Table 5.6 and Table 5.7 contain corresponding dominance factors for $A' \cup A''$ in $T'$ and in $T''$ (changes are depicted via ‘ovals’). Finally, the proximity for trees is: $\bar{\rho}(T', T'') = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$.

![Fig. 5.5. Trees with morphology for example 4](image)

Here the comparison process for rankings is based on rankings for common leaf vertices: $L(T', T'') = \{4, 5, 6, 7\}$. Thus, for each vertex above it is possible to use a metric/proximity. In our case (Fig. 14), the following metric (normalized Kendall-Tau distance) values are obtained: vertex 4: $\delta_4(\Lambda', \Lambda'') = \frac{1}{4}$, vertex 5: $\delta_5(\Lambda', \Lambda'') = \frac{1}{4}$, vertex 6: $\delta_6(\Lambda', \Lambda'') = \frac{1}{4}$, and vertex 7: $\delta_7(\Lambda', \Lambda'') = \frac{1}{4}$. The resultant general proximity for rankings can be computed as an average value:

$$\rho_r(\Lambda', \Lambda'') = \frac{\sum_{i \in L(T', T'')} \delta_i(\Lambda', \Lambda'')}{|L(T', T'')|} = 0.33.$$ 

Finally, the general proximity is:

$$\bar{\rho}(\Lambda', \Lambda'') = (\rho(A', A''), \rho(E', E''), \rho_r(\Lambda, \Lambda'')) = (0.22, 0.1, 0.33).$$

Note, proximity of compatibility estimates can be added into the proximity vector above as well.
6. UNDERLYING PROBLEMS

6.1. Multicriteria Ranking. Let \( A = \{A_1, \ldots, A_j, \ldots, A_n\} \) be a set of alternatives and \( C = \{C_1, \ldots, C_p, \ldots, C_m\} \) be a set of criteria. An estimate vector \( z_j = \{z_{j1}, \ldots, z_{jp}, \ldots, z_{jm}\} \) as a result of an assessment upon criteria above is given for each alternative \( A_j \). In addition, criteria weights \( \{\mu_p\} \) are used as well. The problem consists in comparison of the alternatives and forming for each alternative an ordinal quality estimate (priority). This problem belongs to a class of ill-structured problems by classification of H. Simon [144]. In the paper, a modification of outranking technique Electre [139] has been used (the modification was suggested in [115]).

6.2. Knapsack Problem. The basic problem is (e.g., [57], [80]):

\[
\max \sum_{i=1}^{m} c_i x_i \\
\text{s.t.} \sum_{i=1}^{m} a_i x_i \leq b, \ x_i \in \{0, 1\}, \ i = 1, m
\]

and additional resource constraints \( \sum_{i=1}^{m} a_{i,k} x_i \leq b_k; \ k = 1, t \); where \( x_i = 1 \) if item \( i \) is selected, \( c_i \) is a value ("utility") for item \( i \), and \( a_i \) is a weight (or required resource). Often nonnegative coefficients are assumed. The problem is NP-hard [57] and can be solved by the following methods: (i) enumerative methods (e.g., Branch-and-Bound, dynamic programming), (ii) approximate schemes with a limited relative error (e.g., [80]), and (iii) heuristics.
A basic version of knapsack problem with minimization of the objective function is:

$$\min \sum_{i=1}^{m} c_i x_i$$

s.t. $\sum_{i=1}^{m} a_i x_i \geq b$, $x_i \in \{0, 1\}$, $i = 1, m$.

For multiple criteria statements it is reasonable to search for Pareto-efficient solutions. Mainly, heuristics, evolutionary algorithms, dynamic programming, and local search methods are applied to multicriteria knapsack problems (e.g., [15], [48], [56], [92], [124], [172]). A recent survey on multicriteria knapsack problem is contained in [124].

6.3. Knapsack Problem and Compatibility. Now let us consider item dependence in knapsack problem [101]. Here the following is considered: binary compatibility of items as a symmetric binary relation (i.e., the selected subset has to contain only compatible items, 1 corresponds to compatibility of items and 0 correspond to incompatible case). Thus, the consideration of compatibility for knapsack problem leads to searching for a clique ("profit clique") with the following properties:

(i) maximum total profit of the selected items: $\max \sum_{i=1}^{n} c_i$,
(ii) restricted total weight of the selected items: $\sum_{i=1}^{n} a_i \leq b$.

Table 6.1 and Table 6.2 contain descriptions of numerical examples for the following problems (this example is a modification of the example from [101]):

| $i$ | $c_i$ | $a_i$ | Knapsack problem | “Profit clique” | Maximal clique |
|-----|-------|-------|------------------|----------------|---------------|
| 1   | 5     | 2     | *               | *              | *             |
| 2   | 12    | 5     | *               | *              | *             |
| 3   | 4     | 2     | *               | *              | *             |
| 4   | 2     | 1     | *               | *              | *             |
| 5   | 3     | 2     | *               | *              | *             |
| 6   | 4     | 3     | *               | *              | *             |
| 7   | 1     | 2     | *               | *              | *             |
| 8   | 1     | 3     | *               | *              | *             |
| $C$ |       |       | 26              | 23             |               |
| $A$ |       |       | 12              | 11             |               |
| $M$ |       |       | 5               | 4              | 6             |

(a) basic knapsack problem,
(b) "profit clique" (i.e., knapsack problem and compatibility of the selected items), and
(c) maximal clique (maximizing the number of selected compatible items).

The following notations are used:

(1) $M$ is the number of items in a solution (i.e., the number of the selected items);
(2) $b = 13$ is the right-side constraint (additional constraints are not applied);
(3) symbol * corresponds to a selected item; and
(4) characteristics the solution are: $C = \sum_{i=1}^{m} c_i x_i$, $A = \sum_{i=1}^{m} a_i x_i$. 

AGGREGATION OF COMPOSITE SOLUTIONS: STRATEGIES, MODELS, EXAMPLES

Clearly, the considered problems are NP-hard. Thus, heuristics are used. Multicriteria versions of “profit clique” problem can be examined and used as well.

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 1 | 1 |   |   |
| 2 | 1 | 0 | 1 | 1 | 1 | 1 |   |   |
| 3 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |   |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 |   |   |
| 5 | 0 | 0 | 1 | 0 | 1 | 1 |   | 0 |
| 6 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 7 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 8 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |

6.4. Multiple Choice Knapsack Problem. In the case of a multiple choice problem, the items (e.g., actions) are divided into groups and we select elements from each group while taking into account a total resource constraint (or constraints) (e.g., [57], [73], [86], [132]):

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

s.t. $\sum_{i=1}^{m} q_{i} \sum_{j=1}^{n} a_{ij} x_{ij} \leq b$, $\sum_{i=1}^{m} x_{ij} = 1$, $i \in [1, m]$, $x_{ij} \in \{0, 1\}$.

In the case of multicriteria description, each element (i.e., $(i, j)$) has a vector profit: $c_{ij} = (c_{i,j}^1, ..., c_{i,j}^r)$. A version of multicriteria multiple choice problem was presented in ([110], [118], [120]):

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^\xi x_{ij}, \forall \xi = 1, ..., r$$

s.t. $\sum_{i=1}^{m} q_{i} \sum_{j=1}^{n} a_{ij} x_{ij} \leq b$, $\sum_{i=1}^{m} x_{ij} = 1$, $i \in [1, m]$, $x_{ij} \in \{0, 1\}$.

Evidently, in this case it is reasonable to search for Pareto-efficient solutions (by the vector objective function above). Here the following solving schemes can be used ([118], [120]): (i) dynamic programming, (ii) heuristic based on preliminary multicriteria ranking of elements to get their priorities and step-by-step packing the knapsack (i.e., greedy approach), (iii) multicriteria ranking of elements to get their ordinal priorities and usage of approximate solving scheme (as for knapsack problem) based on discrete space of system excellence (i.e., quality lattice as in HMMD [101]). In the article, greedy heuristic above is used later.

6.5. Morphological Design. A brief description of HMMD is a typical one as follows (e.g., [101], [105], [106], [110]). The composite (modular, decomposable) system under examination consists of the components and their interconnections or compatibilities. Basic assumptions of HMMD are the following: (a) a tree-like structure of the system; (b) a composite estimate for system quality that integrates components (subsystems, parts) qualities and qualities of interconnections (hereinafter referred as ‘IC’) across subsystems; (c) monotonic criteria for the system and its components; and (d) quality of system components and IC are evaluated on the basis of coordinated ordinal scales. The designations are: (1) design alternatives
(DAs) for nodes of the model; (2) priorities of DAs \((r = 1, k; 1\) corresponds to the best level); (3) ordinal compatibility estimates for each pair of DAs \((w = 0, l; l\) corresponds to the best level). The basic phases of HMM are: 1. design of the tree-like system model (a preliminary phase); 2. generating DAs for model’s leaf nodes; 3. hierarchical selection and composing of DAs into composite DAs for the corresponding higher level of the system hierarchy (morphological clique problem); and 4. analysis and improvement of the resultant composite DAs (decisions). Let \(S\) be a system consisting of \(m\) parts (components): \(P(1), ..., P(i), ..., P(m)\). A set of design alternatives is generated for each system part above. The problem is:

\[
\text{Find a composite design alternative } S = S(1) \ast ... \ast S(i) \ast ... \ast S(m) \text{ of DAs (one representative design alternative } S(i) \text{ for each system component/part } P(i), \ i = 1, m) \text{ with non-zero IC estimates between design alternatives.}
\]

A discrete space of the system excellence on the basis of the following vector is used: \(N(S) = (w(S); n(S))\), where \(w(S)\) is the minimum of pairwise compatibility between DAs which correspond to different system components (i.e., \(\forall P_{j1}\) and \(P_{j2}\), \(1 \leq j1 \neq j2 \leq m\) in \(S\), \(n(S) = (n_{r1}, ..., n_{r1}, ..., n_{rk}\), where \(n_r\) is the number of DAs of the \(r\)th quality in \(S\) \((\sum_{r=1}^{k} n_r = m)\). As a result, we search for composite system decisions which are nondominated by \(N(S)\). Here an enumerative solving scheme (e.g., dynamic programming) is used (usually \(m \leq 6\)).

Fig. 6.1 and Fig. 6.2 illustrate the composition problem (by a numerical example for a system consisting of three parts \(S = X \ast Y \ast Z\). Priorities of DAs are shown in Fig. 6.1 in parentheses and are depicted in Fig. 6.2; compatibility estimates are pointed out in Fig. 6.2). In the example, the resultant composite decision is (Fig. 6.1, Fig. 6.2): \(S_1 = X_4 \ast Y_2 \ast Z_2\), \(N(S_1) = (2; 1, 1, 1)\).

**Fig. 6.1. Example of composition**

**Fig. 6.2. Concentric presentation**

### 7. Median/Consensus Problems and Aggregation Problems

#### 7.1. Sets

Evidently, sets are the basic structures. Let us consider the case of \(m\) sets. The extended median/consensus for \(m\) sets \(\{A_1, ..., A_i, ..., A_m\}\) is the following. Analogically, \(\forall e \in \bigcup_{i=1}^{m} A_i\) two attributes are examined: “profit”/“utility” \(c_e \geq 0\), required resource \(b_e \geq 0\). Let \(R \{A_i\} = S \{A_i\} \subseteq \bigcap_{i=1}^{m} A_i\) (or \(R \{A_i\} = M \{A_i\}\) be a basic “consensus” set and \(\sum_{e \in R_{AB}} c_e \leq b\), where \(b\) is a total resource. The problem of “extended median/consensus” is:

\[
\max_{e \in W \subseteq (\bigcup_{i=1}^{m} A_i) \setminus R \{A_i\}} \sum_{e \in W} c_e
\]
AGGREGATION OF COMPOSITE SOLUTIONS: STRATEGIES, MODELS, EXAMPLES

\[ \text{s.t.} \quad \sum_{e \in W \subseteq (\bigcup_{i=1}^{n} A_i) \setminus R_{A_i}} b_e \leq b. \]

This model corresponds to basic knapsack problem.

Further, aggregation problems for sets will be examined. First, let us consider two-set case for sets \( A, B \). Here the following proximity/metric is used: \( \rho(A, B) \geq 0, \rho(A, B) = \rho(B, A) \). For example, the following simple metric can be used: \( \rho(A, B) = \frac{|A \cap B|}{|A \cup B|} \).

A median-like subset (median-like consensus model by Kendall etc. [90]) is:

\[ M_{AB} = \arg \min_{\{M\}} (\rho(M, A) + \rho(M, B)). \]

The case of the extended median/consensus for two sets \( A, B \) is similar. Here two attributes are examined for every \( e \in (A \cup B) \): “profit”/“utility” \( c_e \geq 0 \), required resource \( b_e \geq 0 \). Let \( R_{AB} = S_{AB} \) (or \( R_{AB} = M_{AB} \)) be a basic “consensus” set and \( \sum_{e \in R_{AB}} b_e \leq b \), where \( b \) is a total resource. The problem of building the “extended median/consensus” is:

\[ \text{s.t.} \quad \sum_{e \in W \subseteq (A \cup B) \setminus R_{AB}} b_e \leq b. \]

This model corresponds to basic knapsack problem.

Now let us consider the multi-set case. The extended median/consensus for \( m \) sets \( \{A_1, \ldots, A_i, \ldots, A_m\} \) is the following. Analogically, for every \( e \in \bigcup_{i=1}^{m} A_i \) two attributes are examined: “profit”/“utility” \( c_e \geq 0 \), required resource \( b_e \geq 0 \). Let \( R_{A_i} = \tilde{S}_{A_i} \subseteq \bigcap_{i=1}^{m} A_i \) (or \( R_{A_i} = M_{A_i} \)) be a basic “consensus” set and \( \sum_{e \in R_{A_i}} b_e \leq b \), where \( b \) is a total resource. The problem of “extended median/consensus” is:

\[ \text{s.t.} \quad \sum_{e \in W \subseteq (\bigcup_{i=1}^{m} A_i) \setminus R_{A_i}} b_e \leq b. \]

This model corresponds to basic knapsack problem.

7.2. Rankings. Median/consensus of rankings is one of the basic problems in decision making (e.g., [12], [33], [34], [39], [12], [33], [34]). Here three methods are briefly described: (i) median consensus method based on assignment problem (e.g., [34], [35], [36], [95]); (ii) heuristic approach (e.g., [16], [95], [152]); and (iii) method based on multiple choice problem (e.g., [100], [101]).

**Method 1.** The median consensus method based on distance (usually: Kendall-Tau distance) and assignment problem has been studied by Cook et al. (e.g., [34], [35], [36]). Our version of the approach (for layered sets) is the following. Let \( A = \{1, \ldots, i, \ldots, n\} \) be the initial set of elements (alternatives, objects). The number of layers equals \( m \) (\( k = 1, m \)). There are \( \mu \) initial rankings of set \( A: S^1, \ldots, S^\mu \). Thus, \( S^\lambda = \bigcup_{k=1}^{m} A^\lambda_k \). Let \( r^\lambda_i \) (\( i \in A \)) be the priority of \( i \) in \( S^\lambda \), i.e., the number of corresponding layer: \( r^\lambda_i = k \) if \( i \in A^\lambda_k \).
The resultant ranking (consensus) is \( S^a = \{ A^a_1, ..., A^a_k, ..., A^a_m \} \), and corresponding consensus priorities are \( r^a_i \forall i \in A \). The following binary variable will be used:

\[
x_{ik} = \begin{cases} 
1, & \text{if } r^a_i = k \text{ or } i \in A^a_k, \\
0, & \text{otherwise.}
\end{cases}
\]

The assignment problem for finding the consensus is (case of layered set):

\[
\min \sum_{i=1}^{n} \sum_{k=1}^{m} \left( \sum_{\lambda=1}^{\mu} |r^\lambda_i - k| \right) x_{ik}
\]

\[s.t. \sum_{k=1}^{m} x_{ik} = 1, i = \overline{1, n}; x_{ik} \in \{0, 1\}.
\]

Generally, polynomial algorithms exist for basic assignment problem (e.g., \cite{57}, \cite{96}). The obtained version of assignment problem is more simple and can be solved by an evident greedy algorithm: selection of the closest layer \( \forall i \in A \). Let us consider a numerical example (Table 7.1):

| \( i \in A \) | \( r^1_i (S^1) \) | \( r^2_i (S^2) \) | \( r^3_i (S^3) \) | \( r^a_i (S^a) \) | \( \overline{r^a_i} (S^a) \) |
|---|---|---|---|---|---|
| 1 | 3 | 3 | 3 | 3 | 3 |
| 2 | 1 | 1 | 1 | 1 | 1 |
| 3 | 3 | 1 | 2 | 2 | 2 |
| 4 | 1 | 2 | 1 | 1 | 1 |
| 5 | 4 | 4 | 3 | 4 | 4 |
| 6 | 4 | 4 | 4 | 4 | 4 |
| 7 | 3 | 3 | 4 | 3 | 3 |
| 8 | 4 | 4 | 4 | 4 | 4 |
| 9 | 2 | 2 | 2 | 2 | 2 |

(a) initial set of elements \( A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \),
(b) three rankings (four layer are examined):
\( S^1 = \{ A^1_1 = \{2, 4\}, A^1_2 = \{9\}, A^1_3 = \{1, 3, 7\}, A^1_4 = \{5, 6, 8\}\} \),
\( S^2 = \{ A^2_1 = \{2, 3\}, A^2_2 = \{4, 9\}, A^2_3 = \{1, 7\}, A^2_4 = \{5, 6, 8\}\} \),
\( S^3 = \{ A^3_1 = \{2, 4\}, A^3_2 = \{3, 9\}, A^3_3 = \{1, 5\}, A^3_4 = \{6, 7, 8\}\} \).

The resultant ranking based on assignment problem above is (Table 10):
\( S^a = \{ A^a_1 = \{2, 4\}, A^a_2 = \{3, 9\}, A^a_3 = \{1, 7\}, A^a_4 = \{5, 6, 8\}\} \).

**Method 2.** A basic heuristic approach has been suggested in \cite{16}. The method is widely used (e.g., \cite{152}). Let us consider a simplified version of heuristic to find the corresponding solution \( \overline{S^a} \):

\[
\overline{r^a_i} = \begin{cases} 
[(\sum_{k=1}^{\mu} r^k_i)/\mu], & \text{if } \left( \sum_{k=1}^{\mu} r^k_i / \mu \right) - \left( \sum_{k=1}^{\mu} r^k_i / \mu \right) < 0.5, \\
\left( \sum_{k=1}^{\mu} r^k_i / \mu \right) + 1, & \text{otherwise.}
\end{cases}
\]

Thus, solution \( \overline{S^a} \) is (Table 10):
\( \overline{S^a} = \{ \overline{A^a}_1 = \{2, 4\}, \overline{A^a}_2 = \{3, 9\}, \overline{A^a}_3 = \{1, 7\}, \overline{A^a}_4 = \{5, 6, 8\}\} \).
Method 3. Now a more complicated aggregation process based on multiple choice problem is examined (initial rankings \( \{ S^\lambda | \lambda = 1, \mu \} \) are mapped into a fuzzy aggregated ranking \( S^a \)):

\[
h(S^a | \eta_o \in \{ S | \eta(S, S^\lambda) \leq \eta_o, \forall \lambda = 1, \ldots, \mu \} \rightarrow max,
\]

where \( h \) is an attribute (quality) of the resultant ('average') structure \( S^a \); \( \eta_o \) is a vector-like proximity. The problem is depicted in Fig. 7.1.

![Fig. 7.1. Aggregation](image)

The following notations are used: \( a_i \) is the number of initial structures, in which element \( i \in A_l, l = 1, \ldots, m \) (layers); vector \( \xi = (\xi_{i1}, \ldots, \xi_{il}, \ldots, \xi_{im}) \) defines frequencies of belonging of element \( i \) to layers \( \{ A_1, \ldots, A_l, \ldots, A_m \} \), where \( \xi_{il} = \frac{a_i \mu}{a_{il}} \) (it is a membership function of element \( i \) to layer \( l = 1, \ldots, m \)). Let us denote \( S^a_f \) as a set of intervals \( \{ d_i \} \). The resultant quality of \( S^a_f \) is based on the following entropy-like function:

\[
\sum_{i=1}^{n} H_i = \sum_{i=1}^{n} \frac{1}{d_i^2 - d_i^1 + 1} \rightarrow max.
\]

Next, a modular vector as the proximity is used: \( z_o = (z^1, \ldots, z^k, 0, \ldots, 0) \). Finally, the problem is:

\[
\sum_{i=1}^{n} H_i(S^a) \rightarrow max, \ z(S^\lambda, S^a) \leq z_o, \forall \lambda = 1, \ldots, \mu.
\]

With respect to zero-valued components \( z^{k+1}, \ldots, z^{k+} \), it is possible to define a set of admissible variants to intervals \( \{ d_{\theta} \} \). Thus, we reduce the model to the following modification of multiple-choice problem (57, 59):

\[
\sum_{i=1}^{n} \sum_{\theta=1}^{q} H_i(d_{\theta}) \kappa_{i\theta} \rightarrow max,
\]

\[
\sum_{r \geq p} \sum_{i=1}^{n} b_{i\theta} \kappa_{i\theta} \leq \sum_{r \geq p} z^r, p = 1, \ldots, k, \sum_{i=1}^{n} \kappa_{i\theta} = 1, i = 1, \ldots, n, \kappa_{i\theta} \in \{ 0, 1 \},
\]

where \( b_{i\theta} \) is the sum of components \( \xi_i \), which are differed from \( d_{\theta}^1 \) (\( d_{\theta}^2 \)) by \( r \). A version of the described aggregation scheme has been implemented in DSS COMBI (101, 115).

7.3. Trees. Here initial information consists in a set of trees. Usually four basic approaches are considered:

1. maximum common subtree (e.g., [2], [54]);
2. median/agreement tree (e.g., [3], [19], [52], [64], [76], [133], [146]);
3. compatible tree (e.g., [19], [64], [67]); and
4. maximum agreement forest (e.g., [30], [66], [138], [164], [165]).
Mainly, the problems above correspond to class of NP-hard problems (e.g., \[67\], \[133\]). As a result, heuristics, approximation schemes, and enumerative methods are used.

Thus, the aggregation problem for set of initial trees \( \{T\} = \{T^1, ..., T^i, ..., T^m\} \) can be considered as follows (addition strategy I):

**Stage 1.** Searching for a median-like tree (i.e., “kernel”):

\[ T^{agg} = \arg \min_{\{T\}} \left( \sum_{i=1}^{m} \rho(T, T^i) \right). \]

**Stage 2.** Generation of a set of additional elements (nodes and/or edges).

**Stage 3.** Addition of elements to \( T^{agg} \) (knapsack-like problem).

Evidently, in the case of vector-like proximity \( \rho(T', T'') \), \( T^{agg} \) has to be searched for as Pareto-efficient solution(s). On the other hand, it is reasonable to consider some heuristic algorithms for building the “kernel”, for example:

\[ K = \bigcup_{i=1}^{m-1} (T^i \cap T^{i=1}). \]

7.4. **Morphological Structures.** The basic aggregation problem for set of initial structures \( \{\Lambda\} = \{\Lambda^1, ..., \Lambda^i, ..., \Lambda^m\} \) can be considered as follows (addition strategy I):

**Stage 1.** Searching for a median-like tree (i.e., “kernel”):

\[ \Lambda^{agg} = \arg \min_{\{\Lambda\}} \left( \sum_{i=1}^{m} \rho(\Lambda, \Lambda^i) \right). \]

**Stage 2.** Generation of a set of additional elements (nodes and/or edges).

**Stage 3.** Addition of elements to \( \Lambda^{agg} \) (knapsack-like problem).

Evidently, in the case of vector-like proximity \( \rho(\Lambda', \Lambda'') \), \( \Lambda^{agg} \) has to be searched for as Pareto-efficient solution(s). Generally, morphological structures (morphological structures with compatibility) are very complicated composite structures: \( \Lambda = (T.P.D.R.I) \). Here, it may be reasonable to consider the following heuristic solving scheme:

**Stage 1.** Aggregation of sets of systems parts \( \{P\} \).

**Stage 2.** Aggregation of sets of DA (\{D\}).

**Stage 3.** Aggregation of sets of compatibility estimates \( \{I\} \).

**Stage 4.** Aggregation of tree-like models \( \{T\} \).

8. **ILLUSTRATIVE APPLIED NUMERICAL EXAMPLES**

The list of examined applied examples is presented in Table 8.1.
Table 8.1. List of examined applied examples

| Applied example                          | Aggregation strategy                                                                 | Taking into account compatibility | References for example |
|-----------------------------------------|--------------------------------------------------------------------------------------|-----------------------------------|------------------------|
| 1. On-board telemetry system            | addition (strategy I)                                                               | None                              | [116]                  |
| 2. Notebook                             | combined strategy (strategy III)                                                    | None                              |                        |
| 3. Zig-Bee protocol                     | (i) addition (strategy I)                                                           | None                              | [119]                  |
|                                          | (ii) deletion (strategy II)                                                         | None                              |                        |
| 4. Integrated security system           | addition (strategy I)                                                               | None                              |                        |
| 5. Plan of students art activity        | “kernel”-based strategy                                                             | None                              | [101]                  |
| 6. Combinatorial investment             | “kernel”-based strategy                                                             | None                              | [101]                  |
| 7. Common educational course            | (i) addition (strategy I)                                                           | None                              | [99], [101], [102], [106] |
|                                          | (ii) median-based strategy                                                          | None                              |                        |
| 8. Educational course on design         | extended strategy IV                                                                | Yes                               | [109]                  |
| 9. Configuration of car                 | median-like strategy                                                                |                                   |                        |
| 10. Applied Web-based information system| (i) kernel-based strategy                                                           | Yes                               | [114]                  |
|                                          | (ii) design strategy (IV)                                                           |                                   |                        |

8.1. **ZigBee Communication Protocol.** Here an applied example for aggregation of ZigBee protocol solutions from [119] is briefly described. A general framework for aggregation of two solutions is depicted in Fig. 8.1. A brief description of Zigbee protocol components is the following [119]:

1. Interference avoidance A: 1.1. Startup Procedure of Channel Acquisition X.
2.1. Channel Hopping J.
3. Group addressing I.
4. Centralized data collection C: 4.1. Low-overhead data collection by ZigBee Coordinator G. 4.2 Low-overhead data collection by other devices H. 4.3 Many-to-one routing Q. 4.4. 6LoWPAN multicast/broadcast support V. 4.5. Source routing P.
5. Network scalability D.
6. Message size E.
7. Standardized commissioning Z.
8. Robust mesh networking: 6LoWPAN approach U.
9. Cluster Library support L.
10. Web services support W.

The above-mentioned system components have implementation alternatives (e.g., $B_1, B_2$).

The following initial solutions are considered:
(a) an expert-based forecast $\Theta^1$ ($\tilde{S}_4$ from [119]) (Fig. 8.2),
(b) forecast based on using knapsack problem $\Theta^2$ ($\hat{\Phi}$ from [119]) (Fig. 8.3).

![Diagram of aggregation for ZigBee protocol]

**Fig. 8.1. Example: aggregation for ZigBee protocol**

**Expert-based solution $\Theta^1$**

![Diagram of knapsack-based solution]

**Knapsack-based solution $\Theta^2$**

The corresponding substructure of protocol $\tilde{\Theta}$ and superstructure of protocol $\tilde{\Theta}$ are shown in Fig. 8.4 and in Fig. 8.5.

![Diagram of structure of direct expert-based forecast $\Theta^1$]

**Fig. 8.2. Structure of direct expert-based forecast $\Theta^1$ [119]**

![Diagram of forecast based on knapsack problem $\Theta^2$]

**Fig. 8.3. Forecast based on knapsack problem $\Theta^2$ [119]**
Further, let us consider aggregation processes. A list of addition operations (for strategy I) is presented in Table 8.2 (designation of operation from [119] is depicted in parentheses). Here and hereafter the following attributes (criteria) for an assessment of the addition operations (deletion operations) are used [119]: (1) cost $\Upsilon_1$; (2) required time for implementation $\Upsilon_2$; (3) performance $\Upsilon_3$; (4) decreasing a cost of maintenance $\Upsilon_4$; (5) scalability $\Upsilon_5$; (6) reliability $\Upsilon_6$; (7) mobility $\Upsilon_7$; and (8) usability value $\Upsilon_8$. An ordinal scale [1,5] is used for each criterion: 1 corresponds to “strong negative effect”, 2 corresponds to “negative effect”, 3 corresponds to “no changes”, 4 corresponds to “positive effect”, and 5 corresponds to “strong positive effect”. Priorities $\{r_i\}$ are obtained via multicriteria ranking (Electre-like method [119]).

Table 8.2. Addition operations (estimates, priorities)

| $i$ | Improvement operation | Variable | $\Upsilon_1$ | $\Upsilon_2$ | $\Upsilon_3$ | $\Upsilon_4$ | $\Upsilon_5$ | $\Upsilon_6$ | $\Upsilon_7$ | $\Upsilon_8$ | Priorities $r_i$ |
|-----|-----------------------|----------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|-----------------|
| 1   | $J_1$ ($\Phi_{16}$)   | $x_1$    | 2             | 3             | 4             | 4             | 4             | 4             | 4             | 3             | 1               |
| 2   | $B_1$ ($\Phi_{13}$)   | $x_2$    | 3             | 3             | 3             | 4             | 3             | 4             | 3             | 4             | 3               |
| 3   | $U_1$&$U_2$ ($\Phi_{17}$) | $x_3$  | 3             | 3             | 4             | 3             | 4             | 3             | 3             | 3             | 3               |
| 4   | $L_1$ ($\Phi_{3}$)    | $x_4$    | 3             | 4             | 3             | 4             | 3             | 5             | 4             | 3             | 4               |
| 5   | $W_1$ ($\Phi_{15}$)   | $x_5$    | 3             | 3             | 2             | 3             | 4             | 3             | 3             | 5             | 2               |

Thus, the addition problem (simplified knapsack problem) is:

$$\max \sum_{i=1}^{5} c_i x_i$$

s.t. $\sum_{i=1}^{5} a_i x_i \leq b, x_i \in \{0, 1\}$. 

Further, let us consider aggregation processes. A list of addition operations (for strategy I) is presented in Table 8.2 (designation of operation from [119] is depicted in parentheses). Here and hereafter the following attributes (criteria) for an assessment of the addition operations (deletion operations) are used [119]: (1) cost $\Upsilon_1$; (2) required time for implementation $\Upsilon_2$; (3) performance $\Upsilon_3$; (4) decreasing a cost of maintenance $\Upsilon_4$; (5) scalability $\Upsilon_5$; (6) reliability $\Upsilon_6$; (7) mobility $\Upsilon_7$; and (8) usability value $\Upsilon_8$. An ordinal scale [1,5] is used for each criterion: 1 corresponds to “strong negative effect”, 2 corresponds to “negative effect”, 3 corresponds to “no changes”, 4 corresponds to “positive effect”, and 5 corresponds to “strong positive effect”. Priorities $\{r_i\}$ are obtained via multicriteria ranking (Electre-like method [119]).

Table 8.2. Addition operations (estimates, priorities)

| $i$ | Improvement operation | Variable | $\Upsilon_1$ | $\Upsilon_2$ | $\Upsilon_3$ | $\Upsilon_4$ | $\Upsilon_5$ | $\Upsilon_6$ | $\Upsilon_7$ | $\Upsilon_8$ | Priorities $r_i$ |
|-----|-----------------------|----------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|-----------------|
| 1   | $J_1$ ($\Phi_{16}$)   | $x_1$    | 2             | 3             | 4             | 4             | 4             | 4             | 4             | 3             | 1               |
| 2   | $B_1$ ($\Phi_{13}$)   | $x_2$    | 3             | 3             | 3             | 4             | 3             | 4             | 3             | 4             | 3               |
| 3   | $U_1$&$U_2$ ($\Phi_{17}$) | $x_3$  | 3             | 3             | 4             | 3             | 4             | 3             | 3             | 3             | 3               |
| 4   | $L_1$ ($\Phi_{3}$)    | $x_4$    | 3             | 4             | 3             | 4             | 3             | 5             | 4             | 3             | 4               |
| 5   | $W_1$ ($\Phi_{15}$)   | $x_5$    | 3             | 3             | 2             | 3             | 4             | 3             | 3             | 5             | 2               |

Thus, the addition problem (simplified knapsack problem) is:

$$\max \sum_{i=1}^{5} c_i x_i$$

s.t. $\sum_{i=1}^{5} a_i x_i \leq b, x_i \in \{0, 1\}$. 

### Figure 8.4. Substructure of forecasts $\Theta$

### Figure 8.5. Superstructure of forecasts $\Theta$
Cost estimates are (by criterion $\Upsilon_1$) used as $\{a_i\}$, priorities $\{r_i\}$ are used as (transform to) $\{c_i\}$, and $b = 8.00$.

A resultant solution for strategy I is depicted in Fig. 8.6 ($x_1 = 1$, $x_2 = 1$, $x_3 = 0$, $x_4 = 0$, $x_5 = 1$). Here compatibility estimates between design alternatives for system components are not considered.

![Fig. 8.6. Aggregated solution $\Theta^I$ based on strategy I](image)

A list of deletion operations (for strategy II) is presented in Table 8.3 (designation of operation from [119] is depicted in parentheses).

| $i$ | Deletion operation | Variable | $\Upsilon_1$ | $\Upsilon_2$ | $\Upsilon_3$ | $\Upsilon_4$ | $\Upsilon_5$ | $\Upsilon_6$ | $\Upsilon_7$ | $\Upsilon_8$ | Priorities $r_i$ |
|-----|-------------------|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-----------------|
| 1   | $J_1 (\Phi_{16})$| $x_1$    | 2           | 3           | 4           | 4           | 4           | 4           | 4           | 3           | 1               |
| 2   | $B_1 (\Phi_{13})$| $x_2$    | 3           | 3           | 3           | 4           | 3           | 4           | 3           | 4           | 3               |
| 3   | $Q_1 (\Phi_7)$   | $x_3$    | 3           | 4           | 3           | 3           | 2           | 3           | 3           | 4           | 4               |
| 4   | $P_1 (\Phi_8)$   | $x_4$    | 3           | 4           | 4           | 3           | 4           | 3           | 3           | 4           | 3               |
| 5   | $E_3 (\Phi_{14})$| $x_5$    | 3           | 3           | 3           | 3           | 3           | 3           | 3           | 3           | 4               |
| 6   | $L_1 (\Phi_3)$   | $x_6$    | 3           | 4           | 3           | 3           | 3           | 4           | 3           | 4           | 3               |
| 7   | $U_1\&U_2 (\Phi_{17})$ | $x_7$ | 3 | 3 | 4 | 3 | 4 | 3 | 3 | 3 | 3 | 3 |

Thus, the deletion problem (knapsack problem with minimization of the objective function) is:

$$\min \sum_{i=1}^{6} c_ix_i$$

$$\text{s.t.} \sum_{i=1}^{6} a_ix_i \geq b, x_i \in \{0, 1\}.$$  

Cost estimates are (by criterion $\Upsilon_1$) used as $\{a_i\}$, priorities $\{r_i\}$ are used as (transform to) $\{c_i\}$, and $b = 8.00$.

![Fig. 8.7. Aggregated solution $\Theta^{II}$ based on strategy II](image)
8.2. **On-Board Telemetry System.** Here a numerical example for on-board telemetry system is considered from [116]. The initial morphological structure for on-board equipment is the following (Fig. 8.8):

1. On-board equipment \( S = D \times E \times P \).

   1.1. Power supply \( D = X \times Y \times Z \): 1.1.1. stabilizer \( X \): \( X_1 \) (standard), \( X_2 \) (transistorized), \( X_3 \) (high-stability); 1.1.2. main source \( Y \): \( Y_1 \) (Li-ion), \( Y_2 \) (Cd-Mn), \( Y_3 \) (Li); 1.1.3. emergency cell \( Z \): \( Z_1 \) (Li-ion), \( Z_2 \) (Cd-Mn), \( Z_3 \) (Li).

   1.2. Sensor element \( E = I \times O \times G \): 1.2.1. acceleration sensors \( I \): \( I_1 \) (ADXL), \( I_2 \) (ADIS), \( I_3 \) (MMA); 1.2.2. position sensors \( O \): \( O_1 \) (SS12), \( O_2 \) (SS16), \( O_3 \) (SS19), \( O_4 \) (SS49), \( O_5 \) (SS59), \( O_6 \) (SS94); 1.2.3. global positioning system \( G \): \( G_1 \) (EB), \( G_2 \) (GT), \( G_3 \) (LS), \( G_4 \) (ZX).

   1.3. Data processing system \( P = H \times C \times W \): 1.3.1. data storage unit \( H \): \( H_1 \) (SRAM), \( H_2 \) (DRAM), \( H_3 \) (FRAM); 1.3.2. processing unit \( C \): \( C_1 \) (AVR), \( C_2 \) (ARM), \( C_3 \) (DSP), \( C_4 \) (BM); 1.3.3. data write unit \( W \): \( W_1 \) (built-in ADC), \( W_2 \) (external ADC 12C), \( W_3 \) (external ADC SPI), \( W_4 \) (external ADC 2W), \( W_5 \) (external ADC UART(1)).

![Diagram of basic hierarchy for on-board telemetry system](image)

In [116], 24 resultant solutions have been obtained via HMMD \((A_1, ..., A_{24})\). In the example, 6 initial solutions are examined (in parentheses designation from [116] in pointed out):

- \( S_1(A_1) = D_1 \times E_1 \times P_1 = (X_2 \times Y_2 \times Z_2) \times (I_1 \times O_1 \times G_1) \times (H_2 \times C_1 \times W_2) \),
- \( S_2(A_{24}) = D_2 \times E_4 \times P_4 = (X_3 \times Y_3 \times Z_3) \times (I_3 \times O_1 \times G_1) \times (H_3 \times C_1 \times W_3) \),
- \( S_3(A_{11}) = D_1 \times E_4 \times P_3 = (X_2 \times Y_2 \times Z_2) \times (I_3 \times O_1 \times G_1) \times (H_3 \times C_1 \times W_2) \),
- \( S_4(A_{19}) = D_2 \times E_3 \times P_3 = (X_3 \times Y_3 \times Z_3) \times (I_1 \times O_3 \times G_4) \times (H_3 \times C_1 \times W_2) \),
- \( S_5(A_{23}) = D_2 \times E_3 \times P_4 = (X_3 \times Y_3 \times Z_3) \times (I_3 \times O_1 \times G_1) \times (H_3 \times C_1 \times W_2) \),
- \( S_6(A_{12}) = D_1 \times E_3 \times P_3 = (X_2 \times Y_2 \times Z_2) \times (I_3 \times O_1 \times G_1) \times (H_3 \times C_1 \times W_3) \).

In Fig. 8.9 and Fig. 8.10, supersolution and subsolution are depicted.

A resultant solution based on strategy II is depicted in Fig. 8.7 \((x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 1, x_5 = 0, x_6 = 0, x_7 = 1)\). Here compatibility estimates between design alternatives for system components are not considered.
The obtained subsolution contains only two elements. Thus, the design of “system kernel” is based on the special method with $\alpha = 0.6$ (Fig. 8.11).

Further, the addition strategy as simplified multiple choice problem is used:

$$\max \sum_{i=1}^{3} \sum_{j=1}^{2} c_{ij} x_{ij}$$

s.t. $\sum_{i=1}^{3} \sum_{j=1}^{2} a_{ij} x_{ij} \leq b$, $\sum_{j=1}^{2} x_{ij} = 1, i = \overline{1,3}; x_{ij} \in \{0, 1\}$.

A list of addition operations is presented in Table 8.4 [116]. Here cost estimates are based on expert judgment, priorities \{r$_i$\} were computed via Electre-like method in [116] and transformed to \{c$_i$\}, \(b = 9.00\).

A resultant solution is depicted in Fig. 8.12 ($x_{11} = 1$, $x_{12} = 0$, $x_{21} = 1$, $x_{22} = 0$, $x_{31} = 1$, $x_{32} = 0$). Note, compatibility estimates between design alternatives for system components $X$, $Y$, $Z$ are not considered.

### Table 8.4. Addition operations

| Improvement operation | Binary variable | Cost | Priorities $r_i$ |
|-----------------------|-----------------|------|------------------|
| $X_2$                 | $x_{11}$        | 3    | 1                |
| $X_3$                 | $x_{12}$        | 4    | 3                |
| $Y_2$                 | $x_{21}$        | 2    | 1                |
| $Y_3$                 | $x_{22}$        | 3    | 2                |
| $Z_2$                 | $x_{31}$        | 2    | 1                |
| $Z_3$                 | $x_{32}$        | 3    | 2                |
8.3. **Continuation of Example for Notebook.** Let us examine the final part of the example for notebook. Here the combined aggregation strategy (strategy III) is considered for two cases:

1. “system kernel” as an extension of substructure:
   
   \[ K' = B_1 \star R_1 \star V_3 \star J_1 \star E_1 \star O_1 \star D_1 \star A_1 \star G_1 \star L_1 \star Q_2; \]

2. “system kernel” \( K^* \) based on multicriteria selection of the “best” design alternatives for each system component.

Let us consider case 1. Here the aggregation strategy as modification of “system kernel” \( K' \) can be applied. A set of candidate modification operations are the following:

1. addition operations:
   1.1. addition for \( U \): \( U_1 \) or \( U_2 \) or \( U_3 \),
   1.2. addition for \( F \): \( F_1 \) or \( F_2 \),
   1.3. addition for \( P \): \( P_2 \) or \( P_3 \) or \( P_4 \);

2. correction operations:
   2.1. replacement \( B_1 \Rightarrow B_2 \),
   2.2. replacement \( V_3 \Rightarrow V_4 \),
   2.3. replacement \( A_1 \Rightarrow A_3 \).

Table 8.5 contains the list of modification operations above, their estimates (ordinal expert judgment) and corresponding binary variables.

| Operations | Binary variable | Cost \( a_{ij} \) | Priorities \( r_{ij} \) |
|------------|----------------|-------------------|-------------------|
| 1. Addition |                |                   |                   |
| 1.1. \( U_1 \) | \( x_{11} \) | 3 | 2 |
| 1.1. \( U_2 \) | \( x_{12} \) | 2 | 3 |
| 1.1. \( U_3 \) | \( x_{13} \) | 4 | 1 |
| 1.2. \( F_1 \) | \( x_{21} \) | 2 | 2 |
| 1.2. \( F_2 \) | \( x_{22} \) | 3 | 1 |
| 1.3. \( P_2 \) | \( x_{31} \) | 3 | 1 |
| 1.3. \( P_3 \) | \( x_{32} \) | 2 | 1 |
| 1.3. \( P_4 \) | \( x_{33} \) | 0 | 2 |
| 2. Replacement |                |                   |                   |
| 2.1. \( B_1 \Rightarrow B_3 \) | \( x_{41} \) | 4 | 1 |
| 2.1. None | \( x_{42} \) | 0 | 2 |
| 2.2. \( V_3 \Rightarrow V_4 \) | \( x_{51} \) | 3 | 1 |
| 2.2. None | \( x_{52} \) | 0 | 2 |
| 2.3. \( A_1 \Rightarrow A_3 \) | \( x_{61} \) | 2 | 1 |
| 2.3. None | \( x_{62} \) | 0 | 2 |

The following simplified multiple choice problem is used (\( c_{ij} = 3 - r_{ij} \), \( b = 11.00 \)):

\[
\max \sum_{i=1}^{6} \sum_{j=1}^{6} c_{ij} x_{ij}
\]
A resultant computer solution \( S^{1c} \) is depicted in Fig. 8.13 \((x_{11} = 1, \ x_{12} = 0, \ x_{13} = 0, \ x_{21} = 0, \ x_{22} = 1, \ x_{31} = 0, \ x_{32} = 1, \ x_{33} = 0, \ x_{41} = 0, \ x_{42} = 1, \ x_{51} = 0, \ x_{52} = 1, \ x_{61} = 1, \ x_{62} = 0)\). Here a greedy algorithm was used. Note, compatibility estimates between design alternatives for system components are not considered.

A resultant computer solution \( S^{1c} \) is depicted in Fig. 8.13 \((x_{11} = 1, \ x_{12} = 0, \ x_{13} = 0, \ x_{21} = 0, \ x_{22} = 1, \ x_{31} = 0, \ x_{32} = 1, \ x_{33} = 0, \ x_{41} = 0, \ x_{42} = 1, \ x_{51} = 0, \ x_{52} = 1, \ x_{61} = 1, \ x_{62} = 0)\). Here a greedy algorithm was used. Note, compatibility estimates between design alternatives for system components are not considered.

![Diagram](image-url)

Fig. 8.13. Solution \( S^{1c} \) based on modification of “system kernel” \( K' \)

Now let us consider case 2. Here building of “system kernel” is based on multicriteria selection and/or expert judgment. The basic structure of “system kernel” is: \( B, \ U, \ R, \ V, \ O, \ F, \ D, \ G \). For each system component above, it is possible to consider a selection procedure to choose the “best” system element (while taking into account elements of the initial solution or additional elements as well).

Table 8.6 contains design alternatives for the selected components of “system kernel” structure above including ordinal estimates (expert judgment, the smallest estimates correspond to the best situation) and the resultant priorities. The following criteria were used: cost \( (\Upsilon_1) \), usefulness \( (\Upsilon_2) \), experience \( (\Upsilon_3) \), prospective features \( (\Upsilon_4) \). As a result, the following “system kernel” \( K^* \) is obtained:

\[
K^* = B_2 \ast U_2 \ast R_3 \ast V_3 \ast E_1 \ast O_1 \ast F_2 \ast D_1 \ast G_1.
\]

Further, the system correction process is based on the following operations:

1. addition: 1.1. \( A_1 \), 1.2. \( P_1 \), 1.3. \( L_1 \);
2. deletion: 2.1. \( E_1 \);
3. replacement: 3.1. \( B_2 \Rightarrow B_3 \), 3.2. \( U_2 \Rightarrow U_1 \), 3.3. \( O_1 \Rightarrow O_3 \).

Table 8.7 contains the list of modification operations above, their estimates (ordinal expert judgment) and corresponding binary variables.

The following multiple choice problem is used \((c_{ij} = 3 - r_{ij}, \ b = 9.00)\):

\[
\text{max} \sum_{i=1}^{7} \sum_{j=1}^{2} c_{ij}x_{ij}
\]

s.t. \( \sum_{i=1}^{7} \sum_{j=1}^{2} a_{ij}x_{ij} \leq b, \sum_{j=1}^{2} x_{ij} = 1 \forall i = 1,7; x_{ij} \in \{0, 1\} \).

A resultant computed solution \( S^{2c} \) is depicted in Fig. 8.14 \((x_{11} = 1, \ x_{12} = 0, \ x_{21} = 0, \ x_{22} = 1, \ x_{31} = 1, \ x_{32} = 0, \ x_{41} = 1, \ x_{42} = 0, \ x_{51} = 1, \ x_{52} = 0, \ x_{61} = 0, \ x_{62} = 1, \ x_{71} = 1, \ x_{72} = 0)\). Here a greedy algorithm was used. Note, compatibility estimates between design alternatives for system components are not considered.
Table 8.6. Design alternatives, estimates and priorities

| Design alternatives | Criteria | Priority |
|---------------------|----------|----------|
|                     | $\Upsilon_1$ | $\Upsilon_2$ | $\Upsilon_3$ | $\Upsilon_4$ | $r_i$ |
| $B_1$               | 3        | 2        | 1        | 2        | 2    |
| $B_2$               | 2        | 2        | 1        | 2        | 1    |
| $B_3$               | 4        | 1        | 1        | 1        | 2    |
| $U_1$               | 3        | 2        | 1        | 2        | 2    |
| $U_2$               | 2        | 2        | 1        | 3        | 1    |
| $U_3$               | 4        | 1        | 1        | 1        | 2    |
| $R_1$               | 1        | 3        | 1        | 3        | 3    |
| $R_2$               | 2        | 2        | 2        | 3        | 3    |
| $R_3$               | 2        | 2        | 2        | 2        | 1    |
| $R_4$               | 3        | 1        | 3        | 1        | 2    |
| $V_1$               | 1        | 2        | 1        | 3        | 3    |
| $V_2$               | 2        | 1        | 1        | 2        | 2    |
| $V_3$               | 3        | 1        | 2        | 1        | 1    |
| $V_4$               | 4        | 1        | 2        | 1        | 2    |
| $E_1$               | 2        | 1        | 2        | 1        | 1    |
| $E_2$               | 0        | 2        | 1        | 2        | 2    |
| $O_1$               | 2        | 2        | 1        | 2        | 1    |
| $O_2$               | 3        | 1        | 2        | 2        | 2    |
| $O_3$               | 1        | 1        | 3        | 1        | 2    |
| $F_1$               | 2        | 2        | 1        | 3        | 2    |
| $F_2$               | 3        | 1        | 2        | 1        | 1    |
| $D_1$               | 1        | 1        | 1        | 1        | 1    |
| $D_2$               | 0        | 2        | 2        | 2        | 2    |
| $G_1$               | 4        | 1        | 1        | 1        | 1    |
| $G_2$               | 4        | 2        | 3        | 2        | 3    |
| $G_3$               | 3        | 2        | 3        | 2        | 3    |
| $G_4$               | 2        | 3        | 2        | 3        | 2    |

Fig. 8.14. Solution $S^{2c}$
Table 8.7. Modification operations

| Operations | Binary variable | Cost | Priorities |
|------------|----------------|------|------------|
| 1. Addition |                |      |            |
| 1.1. $A_1$ | $x_{11}$       | 1    | 1          |
| 1.1. None  | $x_{12}$       | 0    | 3          |
| 1.2. $P_1$ | $x_{21}$       | 3    | 2          |
| 1.2. None  | $x_{22}$       | 0    | 3          |
| 1.3. $L_1$ | $x_{31}$       | 1    | 1          |
| 1.3. None  | $x_{32}$       | 0    | 3          |
| 2. Deletion |                |      |            |
| 2.1. $E_1$ | $x_{41}$       | 1    | 1          |
| 2.2. None  | $x_{42}$       | 0    | 2          |
| 3. Replacement |            |      |            |
| 3.1. $B_2 \Rightarrow B_3$ | $x_{51}$ | 4 | 1          |
| 3.1. None  | $x_{52}$       | 0    | 3          |
| 3.2. $U_2 \Rightarrow U_1$ | $x_{61}$ | 3 | 2          |
| 3.2. None  | $x_{62}$       | 0    | 3          |
| 3.3. $O_1 \Rightarrow O_3$ | $x_{71}$ | 1 | 1          |
| 3.3. None  | $x_{72}$       | 0    | 3          |

8.4. Integrated Security System. Here a numerical example for integrated security system is considered from [117]. The initial morphological structure is the following (Fig. 8.15):

0. Integrated security system $S = A \ast B \ast O$.
1. Closed-circuit television (CCTV) $A = J \ast D$:
   1.1. Cameras $J$: conventional $J_1(2)$, “Tirret” $J_2(2)$, “varifocal” $J_3(3)$, and “auto-house” $J_4(1)$.
   1.2. Lighting $D$: natural $D_1(3)$, natural and guard $D_2(2)$, and natural, guard, and alarm $D_3(1)$.
2. Access control $B = G \ast U \ast V$:
   2.1. Access to territory $G$: card $G_1(1)$, radio-pendant $G_2(3)$, and biometry $G_3(2)$.
   2.2. Access to building $U$: card $U_1(1)$, radio-pendant $U_2(3)$, and biometry $U_3(2)$.
   2.3. Access to premises $V$: card $V_1(1)$, code $V_2(2)$, biometry $V_3(2)$, $V_4 = V_1 \& V_2(2)$, $V_5 = V_1 \& V_3(3)$, and $V_6 = V_1 \& V_2 \& V_3(3)$.
3. Burglar alarm $O = X \ast Y \ast Z$:
   3.1. Border-line 1 based on some principles $X$: single physical principle $X_1(1)$, two physical principles $X_2(2)$, and three physical principles $X_3(3)$.
   3.2. Border-line 2 based on some principles $Y$: single physical principle $Y_1(1)$, two physical principles $Y_2(3)$, and three physical principles $Y_3(2)$.
   3.3. Border-line 3 based on some principles $Z$: single physical principle $Z_1(1)$, two physical principles $Z_2(3)$, and three physical principles $Z_3(2)$.

In [117], two resultant solutions have been obtained (via HMMD):

$S_1 = A_1 \ast B_1 \ast O_1 = (J_2 \ast D_1) \ast (G_1 \ast U_1 \ast V_1) \ast (X_3 \ast Y_3 \ast Z_3)$,
$S_2 = A_1 \ast B_1 \ast O_1 = (J_2 \ast D_1) \ast (G_1 \ast U_1 \ast V_1) \ast (X_1 \ast Y_1 \ast Z_1)$. 
AGGREGATION OF COMPOSITE SOLUTIONS: STRATEGIES, MODELS, EXAMPLES 43

In Fig. 8.16 and Fig. 8.17, supersolution and subsolution are depicted.

Here, the obtained subsolution can be used as “system kernel”, i.e., \( K = \tilde{\Theta} \). Further, the addition strategy as simplified multiple choice problem is used:

\[
\max \sum_{i=1}^{3} \sum_{j=1}^{2} c_{ij} x_{ij}
\]

\[
s.t. \sum_{i=1}^{3} \sum_{j=1}^{2} a_{ij} x_{ij} \leq b, \quad \sum_{j=1}^{2} x_{ij} = 1 \forall i = 1, 3; x_{ij} \in \{0, 1\}.
\]

A list of addition operations is presented in Table 8.8 [117]. Here cost estimates are based on expert judgment, priorities \( \{r_i\} \) were computed via Electre-like method in [117] and transformed to \( \{c_i\}, b = 7.00 \).

A resultant solution is depicted in Fig. 8.18 (\( x_{11} = 1, x_{12} = 0, x_{21} = 1, x_{22} = 0, x_{31} = 1, x_{32} = 0 \)). Note, compatibility estimates between design alternatives for system components \( X, Y, Z \) are not considered.
Table 8.8. Addition operations

| Improvement operation | Binary variable | Cost | Priorities $r_i$ |
|-----------------------|-----------------|------|------------------|
| $X_1$                 | $x_{11}$        | 3    | 1                |
| $X_3$                 | $x_{12}$        | 4    | 3                |
| $Y_1$                 | $x_{21}$        | 2    | 1                |
| $Y_3$                 | $x_{22}$        | 3    | 3                |
| $Z_1$                 | $x_{31}$        | 2    | 1                |
| $Z_3$                 | $x_{32}$        | 3    | 3                |

Fig. 8.18. Resultant aggregated solution

8.5. **Common Educational Course.** Here the following initial sets are considered (Table 8.9):

(i) initial set of educational modules for a basic course on combinatorial optimization $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$,

(ii) initial set of educational modules for a course on combinatorial optimization for students in “communication systems” $A^1 = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14\} \subseteq A$, and

(iii) initial set of educational modules for a course on combinatorial optimization for students in “information systems” $A^2 = \{1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 15\} \subseteq A$.

The aggregation problem is:

Design a common course for students in “communication systems” and “information systems” $A^0 \subseteq A$ while taking into account weights of educational modules ($\{w_i\}, \{w^1_i\}, \{w^2_i\}$).

Here two solving strategies are considered:

1. Addition strategy (strategy I).
2. Median-based strategy.

Let us examine the first case. The following “system kernel” is considered: $K = A^1 \cap A^2 = \{1, 2, 3, 4, 5, 7, 9, 10, 12\}$. The set of elements for addition is:

$$B = \{i \in A \setminus (A^1 \cap A^2)| (w^1_i \geq 0.73) \cup (w^2_i \geq 0.73)\} = \{6, 8, 11, 14, 15\}.$$ 

Here, three design versions (alternatives) are considered $i \in B$: $V^0_i$ (None), $V^1_i$ (compressed version), and $V^2_i$ (normal version) ($j = 1, 3$). Table 8.10 contains the description of the addition elements and corresponding versions.
Table 8.9. Illustrative numerical example

| i  | Topic                     | $A$ | $w_i(A)$ | $A^1$ | $w_i(A^1)$ | $A^2$ | $w_i(A^2)$ | $A^1 \cup A^2$ | $A^1 \cap A^2$ |
|----|---------------------------|-----|-----------|-------|-------------|-------|-------------|-----------------|-----------------|
| 1  | Optimization             | *   | 1.0       | *     | 0.6         | *     | 0.6         | *               | *               |
| 2  | Complexity issues        | *   | 1.0       | *     | 0.7         | *     | 0.7         | *               | *               |
| 3  | Selection/sorting        | *   | 1.0       | *     | 0.6         | *     | 1.0         | *               | *               |
| 4  | Knapsack problem         | *   | 1.0       | *     | 0.4         | *     | 1.0         | *               | *               |
| 5  | Clustering               | *   | 1.0       | *     | 1.0         | *     | 1.0         | *               | *               |
| 6  | Multiple-choice problem  | *   | 1.0       | *     | 0.0         | *     | 0.76        | *               |                 |
| 7  | Spanning tree            | *   | 1.0       | *     | 1.0         | *     | 0.3         | *               | *               |
| 8  | Routing                  | *   | 1.0       | *     | 1.0         | *     | 0.0         | *               |                 |
| 9  | Assignment               | *   | 1.0       | *     | 0.8         | *     | 0.4         | *               | *               |
| 10 | Scheduling               | *   | 1.0       | *     | 0.4         | *     | 0.8         | *               | *               |
| 11 | TSP                      | *   | 1.0       | *     | 0.75        | *     | 0.0         | *               |                 |
| 12 | Covering                 | *   | 1.0       | *     | 0.8         | *     | 0.7         | *               | *               |
| 13 | Steiner tree             | *   | 1.0       | *     | 0.7         | *     | 0.0         | *               |                 |
| 14 | Graph coloring           | *   | 1.0       | *     | 0.73        | *     | 0.0         | *               |                 |
| 15 | SAT problem              | *   | 1.0       | *     | 0.0         | *     | 0.8         | *               |                 |
| 16 | Alignment                | *   | 1.0       | *     | 0.0         | *     | 0.0         |                 |                 |

Thus, the addition problem (multiple choice problem) is:

$$\max \sum_{\kappa=1}^{5} \sum_{j=1}^{3} c_{\kappa j} x_{\kappa j}$$

s.t. $\sum_{\kappa=1}^{5} \sum_{j=1}^{3} \alpha_{\kappa j} x_{\kappa} \leq b$, $\sum_{j=1}^{3} x_{\kappa j} \leq 1 \forall \kappa, x_{\kappa j} \in \{0, 1\}$.
Here \( c_{ij} = \max(w^1_i, w^2_j) \), estimate of \( a_{ij} \) is based on expert judgment (Table 8.10), and \( b = 3.50 \).

A resultant solution based on strategy I is depicted in Fig. 8.19 \((x_{11} = 0, x_{12} = 0, x_{13} = 1, x_{21} = 0, x_{22} = 0, x_{23} = 1, x_{31} = 1, x_{32} = 0, x_{33} = 0, x_{41} = 0, x_{42} = 1, x_{43} = 0, x_{51} = 0, x_{52} = 0, x_{53} = 1)\). Evidently, normal version \( V^3_i \) is used for elements of “system kernel” \( K \). The solving process was based on a greedy algorithm. Here compatibility estimates between design alternatives for system components were not considered.

![Fig. 8.19. Aggregated common course (addition strategy)](image)

In the second case, Pareto-efficient median-solutions \( \{A^0\} \) are searched for through the following two-component vector criterion \( \overline{\alpha} = (\rho^1(A^0, A^1), \rho^2(A^0, A^2)) \) (by weights \( \{w^1_i\} \) and \( \{w^2_i\} \)):

\[
\min \rho^1(A^0, A^1) = 1 - \frac{\sum_{i \in (A^0 \cap A^1) w^1_i}{\sum_{i \in (A^0 \cup A^1) w^1_i}}, \min \rho^2(A^0, A^2) = 1 - \frac{\sum_{i \in (A^0 \cap A^2) w^2_i}{\sum_{i \in (A^0 \cup A^2) w^2_i}}.
\]

Generally, the problem of searching for the median belongs to class of NP-hard problems. Let us consider an approximation heuristic. Initial elements for the median are the following (i.e., deletion of element 16):

\[\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}\]

Thus, cardinality of search space is: \(|\{A^0\}| = 2^{15}\).

First, let us assume that elements \( \{1, 2, 5\} \) will be included into each median-solution (to decrease problem dimension) and, as a result, \(|\{A^0\}| = 2^{12}\).

Second, elements from set \( \{3, 4, 9, 10, 11, 12\} \) will be included into each median solution as well and, as a result, \(|\{A^0\}| = 2^6 = 64\).

Finally, only six submedian-subsolutions are selected for examination:

\[A^{01} = \{7, 8, 13, 14, 15\}, A^{02} = \{6, 8, 13, 14, 15\}, A^{03} = \{6, 7, 13, 14, 15\}, A^{04} = \{6, 7, 8, 14, 15\}, A^{05} = \{6, 7, 8, 13, 15\}, A^{06} = \{6, 7, 8, 13, 14\}\]

Table 92 contains the subsolutions \((\kappa = 1, 6)\), corresponding values of vector \( \overline{\alpha}(A^{0\kappa}) = (\rho^1(A^{0\kappa}, A^1), \rho^2(A^{0\kappa}, A^2)) \), and information on inclusion into the layer of Pareto-efficient solutions. Here parts of \( A^1 \) and \( A^2 \) are considered, which correspond to \( \{6, 7, 8, 13, 14\} \) (for \( A^1 \)) and \( \{6, 7, 13, 15\} \) (for \( A^2 \)).

Thus, the following preference relation is obtained over the six subsolutions above: \( A^{01} \succeq A^{06} \) and \( A^{04} \succeq A^{05} \succeq A^{03} \succeq A^{02} \).

Finally, subsolutions \( A^{01} \) and \( A^{04} \) are Pareto-efficient ones (Table 8.11, Fig. 8.20).
Table 8.11. Candidate-subolutions

| \( \kappa \) | Candidate subsolution \( A^{0\kappa} \) | \( \rho^1(A^{0\kappa}, A^1) \) | \( \rho^2(A^{0\kappa}, A^2) \) | Inclusion into Pareto-layer |
|---------|-------------------------------|----------------|----------------|-----------------------------|
| 1       | \( A^{01} = \{7, 8, 13, 14, 15\} \) | 0.00           | 0.40           | *                           |
| 2       | \( A^{02} = \{6, 8, 13, 14, 15\} \) | 0.30           | 0.05           |                             |
| 3       | \( A^{03} = \{6, 7, 13, 14, 15\} \) | 0.30           | 0.00           |                             |
| 4       | \( A^{04} = \{6, 7, 8, 14, 15\} \) | 0.21           | 0.00           | *                           |
| 5       | \( A^{05} = \{6, 7, 8, 13, 15\} \) | 0.22           | 0.00           |                             |
| 6       | \( A^{06} = \{6, 7, 8, 13, 14\} \) | 0.00           | 0.43           |                             |

The resultant (extended) Pareto-efficient solutions \( \hat{A}^{01} \) and \( \hat{A}^{04} \) are presented in Fig. 8.21 and in Fig. 8.22 (normal versions of course components are used).

**8.6. Plan of Students Art Activity.** The considered numerical example is a small part of the example of students plan from [101]. The initial morphological structure is the following (Fig. 8.23):

1. Plan of students art activity \( S = I \star J \star U \):
   1.1 dance \( I \): \( I_1 \) (none), \( I_2 \) (ball dance), \( I_3 \) (ensemble);
   1.2 music \( J \): \( J_1 \) (none), \( J_2 \) (classic), \( J_3 \) (jazz), \( J_4 \) (singing);
1.3 theatre $U$: $U_1$ (none), $U_2$ (actor), $U_3$ (producer), $U_4$ (technical worker), $U_5$ (author).

The following criteria were used for assessment of the design alternatives: cost $C_1$, opportunity to meet new friends $C_2$, opportunity to meet boy friend or girl friend $C_3$, accordance to personal inclinations $C_4$, usefulness for future career $C_5$, usefulness for health $C_6$, and usefulness for future life $C_7$. The resultant priorities of design alternatives (via Electre-like technique) are shown in parentheses (Fig. 60). Thus, three resultant solutions are the following (via HMMD):

$S_1 = I_2 \star J_3 \star U_2$, $S_2 = I_3 \star J_3 \star U_2$, and $S_3 = I_3 \star J_2 \star U_5$.

In Fig. 8.24 and Fig. 8.25, supersolution and subsolution are depicted. Note, the subsolution contains only one element. Thus, the design of “system kernel” is based on the special method with $\alpha = 0.6$ (Fig. 8.26). Finally, the obtained “system kernel” $K$ can be considered as the resultant solution.

8.7. Combinatorial Investment. The considered numerical example was presented in [101]. The initial morphological structure is the following (Fig. 8.27):
1. composite portfolio $S = A \ast B \ast L$:

1.1 short-time investment $A$: $A_1$ (state bonds), $A_2$ (bank deposit), $A_3$ (speculation on the stock exchange), $A_4$ (oil shares);

1.2 middle-time investment $B$: $B_1$ (state bonds), $B_2$ (bank deposit), $B_3$ (immovables), $B_4$ (jewelry), $B_5$ (shares in biotechnology);

1.3 long-time investment $L$: $L_1$ (state bonds), $L_2$ (bank deposit), $L_3$ (antique), $L_4$ (shares of airspace companies).

The following criteria were used for assessment of the design alternatives: possible profit $\Upsilon_1$, risk $\Upsilon_2$, prestige $\Upsilon_3$, possibility for continuation $\Upsilon_4$, possibility to establish a new company $\Upsilon_5$, obtaining a new experience $\Upsilon_6$, possibility to organize a new market $\Upsilon_7$, possibility to obtain “name” $\Upsilon_8$, and connection with previous activity $\Upsilon_9$.

The resultant priorities of design alternatives (via Electre-like technique) are shown in parentheses (Fig. 8.27). Thus, four resultant solutions are the following (via HMMD):

$S_1 = A_4 \ast B_3 \ast L_1$, $S_2 = A_2 \ast B_5 \ast L_1$, $S_3 = A_2 \ast B_3 \ast L_4$, and $S_4 = A_2 \ast B_5 \ast L_4$.

In Fig. 8.28 and Fig. 8.29, supersolution and subsolution are depicted. Note, the subsolution does not contain elements (empty). Thus, the design of “system kernel” is based on the special method with $\alpha = 0.6$ and selection of the best elements for system parts: $B_5$ for components $B$ and $L_4$ for components $L$ (Fig. 8.30). Finally, the obtained “system kernel” $K$ can be considered as the resultant solution.

8.8. **Configuration of Applied Web-based Information System.** Here aggregation of solution for applied Web-based information system is considered [114]. The tree-like model of the system is depicted in Fig. 8.31.

DAAs for system components are the following [114]:

(1) server for DBs $J$: PC ($J_1$), Supermicro ($J_2$), and Sun ($J_3$);

(2) server for applications $E$: on server of DBs ($E_1$), Sun ($E_2$), Supermicro ($E_3$), and PC ($E_4$);
(3) Web-server W: Apache HTTP-server \((W_1)\), Microsoft IIS \((W_2)\), Beaweblogic \((W_3)\), Web Sphere \((W_4)\), and Weblogic cluster \((W_5)\);

(4) DBMS D: Oracle \((D_1)\), Microsoft SQL \((D_2)\), and designed SQL \((D_3)\); and

(5) operation system O: Windows 2000 server \((O_1)\), Windows 2003 \((O_2)\), Solaris \((O_3)\), FreeBSD \((O_4)\), and RHEL AS \((O_5)\).

In [114], the design problem was solved for three basic applied situations: (a) communication provider (Fig. 8.32), (b) corporate application (Fig. 8.33), and (c) academic application (Fig. 8.34).

\[
S = A \times B
\]

\[
S_1^1 = A_1^1 \times B_1^1 = (J_2 \times E_2) \times (W_1 \times D_3 \times O_3)
\]

\[
S_2^2 = A_2^2 \times B_2^2 = (J_2 \times E_2) \times (W_2 \times D_2 \times O_2)
\]

\[
S_3^3 = A_3^3 \times B_3^3 = (J_2 \times E_2) \times (W_1 \times D_2 \times O_5)
\]

Hardware

\[
A = J \times E
\]

Software

\[
B = W \times D \times O
\]

\[
A_1^1 = J_2 \times E_2
\]

\[
B_1^1 = W_1 \times D_3 \times O_3
\]

\[
B_2^2 = W_2 \times D_2 \times O_2
\]

\[
B_3^3 = W_1 \times D_2 \times O_5
\]

Fig. 8.32. Communication provider [114]

\[
S = A \times B
\]

\[
S_1^1 = A_1^1 \times B_1^1 = (J_1 \times E_1) \times (W_1 \times D_3 \times O_5)
\]

\[
S_2^2 = A_2^2 \times B_2^2 = (J_1 \times E_1) \times (W_2 \times D_3 \times O_2)
\]

\[
S_3^3 = A_3^3 \times B_3^3 = (J_1 \times E_1) \times (W_1 \times D_3 \times O_5)
\]

\[
S_4^4 = A_4^4 \times B_4^4 = (J_2 \times E_2) \times (W_2 \times D_3 \times O_2)
\]

Hardware

\[
A = J \times E
\]

Software

\[
B = W \times D \times O
\]

\[
A_1^1 = J_1 \times E_1
\]

\[
B_1^1 = W_1 \times D_3 \times O_5
\]

\[
B_2^2 = W_2 \times D_3 \times O_2
\]

\[
B_3^3 = W_2 \times D_3 \times O_2
\]

Fig. 8.33. Corporate application [114]
Now let us consider an aggregation problem:

To obtain an aggregated solution while taking into account all three application situations.

Here two simplified heuristic solving strategies are examined:

(i) aggregation of composite solutions for three considered cases;
(ii) aggregation of information at the first stage of the solving process (aggregation of rankings for DAs of system components J, E, W, D, O) and usage of HMMD to solve the design problem.

Let us describe the first strategy. Here the solving process is based on median/kernel-based strategy (Fig. 8.35). The resultant solution is (an approximate median/kernel):

\[ S = (J_2 \ast E_2) \ast (W_1 \ast D_3 \ast O_2). \]

The second solving strategy is based on preliminary aggregation of rankings (Fig. 8.36, Fig. 8.37, Fig. 8.38, Fig. 8.39, Fig. 8.40) and a new design.
Fig. 8.36. Aggregation for $J$

Fig. 8.37. Aggregation for $E$

Fig. 8.38. Aggregation for $W$

Fig. 8.39. Aggregation for $D$

Fig. 8.40. Aggregation for $O$
After the usage of HMMD for the aggregated rankings of DAs (for system components \( J, E, W, D, O \)), the following four resultant composite DAs are obtained (Fig. 8.41, compatibility estimates are contained in [114]):

(a) \( S_1^{agg} = A_1 \times B_1 = (J_2 \times E_2) \times (W_1 \times D_2 \times O_5) \),
(b) \( S_2^{agg} = A_1 \times B_2 = (J_2 \times E_2) \times (W_1 \times D_3 \times O_5) \),
(c) \( S_3^{agg} = A_2 \times B_1 = (J_3 \times E_2) \times (W_1 \times D_2 \times O_5) \), and
(d) \( S_4^{agg} = A_2 \times B_2 = (J_3 \times E_2) \times (W_1 \times D_3 \times O_5) \).

\[
S = A \times B \\
S_1^{agg} = A_1 \times B_1 = (J_2 \times E_2) \times (W_1 \times D_2 \times O_5) \\
S_2^{agg} = A_1 \times B_2 = (J_2 \times E_2) \times (W_1 \times D_3 \times O_5) \\
S_3^{agg} = A_2 \times B_1 = (J_3 \times E_2) \times (W_1 \times D_2 \times O_5) \\
S_4^{agg} = A_2 \times B_2 = (J_3 \times E_2) \times (W_1 \times D_3 \times O_5)
\]

For structure \( \Lambda^1 \) (Fig. 8.42), (1) course on systems engineering (structure \( \Lambda^1 \)) ([101], [102]) (Fig. 8.42); (2) course on information engineering (structure \( \Lambda^2 \)) ([99], [101]) (Fig. 8.43); and (3) course on hierarchical design (structure \( \Lambda^3 \)) ([101], [102], [113]) (Fig. 8.44). The following general types DAs (i.e., of the corresponding educational module) are examined for each leaf node of the presented hierarchical models (Fig. 8.42, Fig. 8.43, and Fig. 8.44): “absence” of the educational module \( X_0 \), compressed information on the educational module \( X_1 \), teaching at a medium level \( X_2 \), serious teaching \( X_3 \), and serious teaching with a special student research work/project \( X_4 \). Compatibility estimates for three examined morphological structures above are presented in Tables (expert judgment): (i) structure \( \Lambda^1 \): Table 8.12, Table 8.13; (ii) structure \( \Lambda^2 \): Table 8.14, Table 8.15, Table 8.16; and (iii) structure \( \Lambda^3 \): Table 8.17, Table 8.18, Table 8.19.

After usage of HMMD, the following composite DAs are obtained:

For structure \( \Lambda^1 \) (Fig. 8.42):

\[
D_1 = L_3 \times G_2 \times C_2 \times M_2, \ N(D_1) = (2; 3, 0, 1), \\
D_2 = L_4 \times G_2 \times C_1 \times M_2, \ N(D_2) = (1; 2, 2, 0), \\
Q_1 = A_3 \times B_2, \ N(Q_1) = (3; 2, 0, 0), \ Q_2 = A_4 \times B_3, \ N(Q_2) = (3; 2, 0, 0); \\
S_1^1 = D_1 \times Q_1 = (L_3 \times G_2 \times C_2 \times M_2) \times (A_3 \times B_3), \\
S_1^2 = D_1 \times Q_2 = (L_3 \times G_2 \times C_2 \times M_2) \times (A_4 \times B_4), \\
S_2^1 = D_2 \times Q_1 = (L_4 \times G_2 \times C_1 \times M_2) \times (A_3 \times B_3), \\
S_2^1 = D_2 \times Q_2 = (L_4 \times G_2 \times C_1 \times M_2) \times (A_4 \times B_4).
\]

8.9. Modular Educational Course on Design. Here aggregation of three educational courses (morphological structures) is examined: (1) course on systems engineering (structure \( \Lambda^1 \)) ([101], [102]) (Fig. 8.42); (2) course on information engineering (structure \( \Lambda^2 \)) ([99], [101]) (Fig. 8.43); and (3) course on hierarchical design (structure \( \Lambda^3 \)) ([101], [102], [113]) (Fig. 8.44). After usage of HMMD, the following composite DAs are obtained:

For structure \( \Lambda^1 \) (Fig. 8.42):

\[
D_1 = L_3 \times G_2 \times C_2 \times M_2, \ N(D_1) = (2; 3, 0, 1), \\
D_2 = L_4 \times G_2 \times C_1 \times M_2, \ N(D_2) = (1; 2, 2, 0), \\
Q_1 = A_3 \times B_2, \ N(Q_1) = (3; 2, 0, 0), \ Q_2 = A_4 \times B_3, \ N(Q_2) = (3; 2, 0, 0); \\
S_1^1 = D_1 \times Q_1 = (L_3 \times G_2 \times C_2 \times M_2) \times (A_3 \times B_3), \\
S_1^2 = D_1 \times Q_2 = (L_3 \times G_2 \times C_2 \times M_2) \times (A_4 \times B_4), \\
S_2^1 = D_2 \times Q_1 = (L_4 \times G_2 \times C_1 \times M_2) \times (A_3 \times B_3), \\
S_2^1 = D_2 \times Q_2 = (L_4 \times G_2 \times C_1 \times M_2) \times (A_4 \times B_4).
\]
\[ S_1^1 = D_2 \ast Q_2 = (L_4 \ast G_2 \ast C_1 \ast M_2) \ast (A_4 \ast B_4). \]

II. For structure \( \Lambda^2 \) (Fig. 8.43):
\[ I_1 = G_2 \ast C_2, \quad N(I_1) = (3; 2, 1, 0); \]
\[ O_1 = H_2 \ast W_2 \ast M_2, \quad N(O_1) = (2; 3, 0, 0), \]
\[ O_2 = H_2 \ast W_2 \ast M_2, \quad N(O_2) = (3; 2, 1, 0); \]
\[ Q_1 = V_3 \ast J_3 \ast Y_2 \ast R_3, \quad N(Q_1) = (3; 4, 0, 0); \]
\[ S_2^1 = I_1 \ast Q_1 = (G_2 \ast C_2 \ast E_2) \ast (H_2 \ast W_2 \ast M_2) \ast (V_3 \ast J_3 \ast Y_2 \ast R_3), \]
\[ S_2^2 = I_1 \ast O_2 \ast Q_1 = (G_2 \ast C_2 \ast E_2) \ast (H_2 \ast W_2 \ast M_2) \ast (V_3 \ast J_3 \ast Y_2 \ast R_3). \]

III. For structure \( \Lambda^4 \) (Fig. 8.44):
\[ I_1 = G_2 \ast C_2, \quad N(I_1) = (3; 2, 0, 0); \]
\[ O_1 = H_2 \ast W_2 \ast M_2 \ast U_1, \quad N(O_1) = (3; 3, 1, 0), \]
\[ O_2 = H_3 \ast W_2 \ast M_2 \ast U_1, \quad N(O_2) = (2; 4, 0, 0); \]
\[ Q_1 = D_2 \ast J_1 \ast Y_2 \ast Z_3, \quad N(Q_1) = (3; 3, 1, 0); \]
\[ S_2^3 = I_1 \ast Q_1 = (G_2 \ast C_2) \ast (H_2 \ast W_2 \ast M_2 \ast U_1) \ast (F_4 \ast J_1 \ast Y_2 \ast Z_3), \]
\[ S_2^3 = I_1 \ast O_2 \ast Q_1 = (G_2 \ast C_2) \ast (H_3 \ast W_2 \ast M_2 \ast U_1) \ast (F_4 \ast J_1 \ast Y_2 \ast Z_3). \]

Evidently, it is possible to aggregate the obtained composite solutions (Fig. 8.45).

On the other hand, let us consider the following extended aggregation strategy IV.

General structure \( (\Lambda) \) consists of the following parts: (i) tree-like system model \( T \),
(ii) set of leaf nodes \( P \), (iii) sets of DAs for each leaf node \( D \), (iv) DAs rankings
(i.e., ordinal priorities) \( R \), and (v) compatibility estimates between DAs \( I \).
Thus, a vector proximity for two structures \( \Lambda^\alpha, \Lambda^\beta \) can be examined as follows:
\[ \Xi(\Lambda^\alpha, \Lambda^\beta) = (\rho^\alpha(T^\alpha, T^\beta), \rho^\alpha(P^\alpha, P^\beta), \rho^\alpha(D^\alpha, D^\beta), \rho^\alpha(R^\alpha, R^\beta), \rho^\alpha(I^\alpha, I^\beta)). \]

As a result, \( \Lambda^{agg} \) has to be searched for as Pareto-efficient solution(s) by the
vectors \( \Xi(\Lambda^{agg}, \Lambda^i) \forall i \in \{1\} \) where index \( i \) corresponds to an initial solution. This
problem is very complicated. Let us consider a simplified solving framework:

Phase 1. Aggregation of basic initial data for initial structures:
1.1. aggregation of morphological structures including the following:
   (1.1.1.) aggregation of sets of leaf nodes,
   (1.1.2.) aggregation of sets of DAs for each leaf node,
   (1.1.3.) aggregation of DAs rankings, and
   (1.1.4.) aggregation of compatibility estimates for DAs sets;
1.2. aggregation tree-like structures.

Phase 2. New hierarchical design.
Thus, the following stages are considered for examined three structures \( \Lambda^1, \Lambda^2, \Lambda^3 \):

Stage 1. Aggregation of leaf node sets for the initial structures (Fig. 8.46);
Stage 2. Aggregation of morphological structure:
   2.1. aggregation of sets of DAs for each leaf node (by each leaf node
        while taking into account addition of DAs which correspond to “absence”, i.e., aggregation of
        rankings with extension of DAs sets) (Fig. 8.47, Fig. 8.48, Fig. 8.49, Fig. 8.50,
        Fig. 8.51, Fig. 8.52, Fig. 8.53, Fig. 8.54, Fig. 8.55, Fig. 8.56, Fig. 8.57, Fig. 8.58,
        Fig. 8.59, Fig. 8.60, Fig. 8.61, Fig. 8.62);
   2.2. aggregation of interconnection (compatibility estimates) for DAs sets (Tables 8.20, 8.21, 8.22, 8.23;
        selection of a maximal value or expert judgment);
Stage 3. Building of an aggregation tree-like structure (Fig. 8.63); and
Stage 4. Hierarchical design of an integrated course (as an aggregated solution)
(Fig. 8.63).

Here the aggregated superstructure \( \bar{X} \) (Fig. 8.63) has been obtained via expert judgment.
$S^1 = D \ast Q$
$S^1_1 = D_1 \ast Q_1 = (L_3 \ast G_2 \ast C_2 \ast M_2) \ast (A_3 \ast B_3)$
$S^1_2 = D_1 \ast Q_2 = (L_3 \ast G_2 \ast C_2 \ast M_2) \ast (A_4 \ast B_4)$
$S^1_3 = D_2 \ast Q_1 = (L_4 \ast G_2 \ast C_1 \ast M_2) \ast (A_3 \ast B_3)$
$S^1_4 = D_2 \ast Q_2 = (L_4 \ast G_2 \ast C_1 \ast M_2) \ast (A_4 \ast B_4)$

Methodology Applied examples/student projects

$D = L \ast G \ast C \ast M$

$Q = A \ast B$

$D_1 = L_3 \ast G_2 \ast C_2 \ast M_2$
$D_2 = L_4 \ast G_2 \ast C_1 \ast M_2$

$Q_1 = A_3 \ast B_3$
$Q_2 = A_4 \ast B_4$

Life cycle System analysis, design, selection

$M_1 = (2)$
$M_2 = (2)$
$M_3 = (2)$

System trajectory cycle

$A_1 = (1)$
$A_2 = (2)$
$A_3 = (3)$
$A_4 = (3)$

$B_1 = (1)$
$B_2 = (2)$
$B_3 = (1)$
$B_4 = (1)$

Fig. 8.42. Course on systems engineering (structure $\Lambda^1$)

$S^2 = I \ast O \ast Q$
$S^2_1 = I_1 \ast O_1 \ast Q_1 = (G_2 \ast C_2 \ast E_2) \ast (H_3 \ast W_2 \ast M_2) \ast (V_3 \ast J_3 \ast Y_2 \ast R_3)$
$S^2_2 = I_1 \ast O_2 \ast Q_1 = (G_2 \ast C_2 \ast E_2) \ast (H_2 \ast W_2 \ast M_2) \ast (V_3 \ast J_3 \ast Y_2 \ast R_3)$

Methodology

$I = G \ast C \ast E$
$I_1 = G_2 \ast C_2 \ast E_2$

System analysis, design, selection

$G_1 = (3)$
$C_2 = (1)$
$E_3 = (2)$

$G_2 = (1)$
$C_5 = (1)$
$E_2 = (2)$

$G_3 = (2)$
$C_4 = (2)$
$E_3 = (3)$

Combinatorial optimization Applied examples/student projects

$O = H \ast W \ast M$

$Q = V \ast J \ast Y \ast R$

$O_1 = H_3 \ast W_2 \ast M_2$
$O_2 = H_2 \ast W_2 \ast M_2$

$Q_1 = V_3 \ast J_3 \ast Y_2 \ast R_3$

$H = H_2 \ast W_1 \ast M_1 \ast V_2 \ast J_2 \ast Y_2 \ast R_2 \ast H_3 \ast W_2 \ast M_2 \ast V_3 \ast J_3 \ast Y_3 \ast R_3 \ast R_4 \ast R_5 \ast R_6 \ast R_7 \ast R_8 \ast R_9 \ast R_{10}$

Fig. 8.43. Course on information system (structure $\Lambda^2$)
$S^3 = I \ast O \ast Q$

$S^3_1 = I_1 \ast O_1 \ast Q_1 = (G_2 \ast C_2) \ast (H_2 \ast W_2 \ast M_2 \ast U_1) \ast (F_4 \ast J_1 \ast Y_2 \ast Z_3)$

$S^3_2 = I_1 \ast O_1 \ast Q_2 = (G_2 \ast C_2) \ast (H_3 \ast W_2 \ast M_2 \ast U_1) \ast (F_3 \ast J_1 \ast Y_2 \ast Z_3)$

Methodology

$I = G \ast C$

$I_1 = G_2 \ast C_2$

System design, analysis, making,

Decision selection,

$G_1(3)$ $C_2(1)$
$G_2(1)$ $C_3(1)$
$G_3(2)$ $C_4(2)$

Applied examples/student projects

$Q = F \ast J \ast Y \ast Z$

$Q_1 = F_4 \ast J_1 \ast Y_2 \ast Z_3$

Software Information center

Student career

Investment

$F_0(3)$ $J_0(1)$ $Y_0(3)$ $Z_0(2)$
$F_1(3)$ $J_1(2)$ $Y_2(1)$ $Z_1(2)$
$F_4(1)$ $J_4(3)$ $Y_4(2)$ $Z_3(1)$

Combinatorial optimization

$O = H \ast W \ast M \ast U$

$O_1 = H_2 \ast W_2 \ast M_2 \ast U_1$

$O_2 = H_3 \ast W_2 \ast M_2 \ast U_1$

Knapsack Assignment Composition Routing

$H_2(2)$ $W_1(2)$ (clique) $U_0(2)$
$H_3(1)$ $W_2(1)$ $M_1(2)$ $U_1(1)$
$H_4(2)$ $W_3(3)$ $M_2(1)$ $U_4(3)$

Fig. 8.44. Course on hierarchical design (structure $\Lambda^3$)

Fig. 8.45. Aggregation of solutions
Table 8.12. Compatibility

| $L_2$ | $G_2$ | $C_1$ | $C_2$ | $C_3$ | $M_1$ | $M_2$ | $M_3$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 2     | 0     | 3     | 2     | 0     | 3     | 2     | 0     |
| 2     | 3     | 0     | 2     | 1     | 3     | 2     |
| 1     | 3     | 0     | 3     | 3     | 0     | 2     | 3     |
| 1     | 3     | 2     | 1     | 3     | 2     |
| 0     | 2     | 3     | 0     | 2     | 3     |

Table 8.13. Compatibility

| $B_2$ | $B_3$ | $B_4$ |
|-------|-------|-------|
| $A_2$ | 3     | 2     | 1     |
| $A_3$ | 2     | 3     | 2     |
| $A_4$ | 1     | 2     | 3     |

Table 8.14. Compatibility

| $C_2$ | $C_3$ | $C_4$ | $E_1$ | $E_2$ | $E_3$ |
|-------|-------|-------|-------|-------|-------|
| $G_1$ | 2     | 1     | 0     | 3     | 1     | 0     |
| $G_2$ | 3     | 0     | 1     | 1     | 3     | 2     |
| $G_3$ | 0     | 3     | 3     | 0     | 2     | 3     |
| $C_2$ |       |       |       | 3     | 3     | 0     |
| $C_3$ |       |       |       | 3     | 3     | 3     |
| $C_4$ |       |       |       | 0     | 2     | 3     |

Table 8.15. Compatibility

| $W_1$ | $W_2$ | $W_3$ | $M_1$ | $M_2$ |
|-------|-------|-------|-------|-------|
| $H_2$ |       |       | 3     | 3     |
| $H_3$ |       |       | 2     | 3     |
| $H_4$ |       |       | 0     | 2     |
| $W_1$ |       |       | 3     |
| $W_2$ |       |       | 3     |
| $W_3$ |       |       | 3     |

Table 8.16. Compatibility

| $J_2$ | $J_3$ | $J_4$ | $Y_0$ | $Y_2$ | $R_2$ | $R_3$ |
|-------|-------|-------|-------|-------|-------|-------|
| $V_2$ | 3     | 2     | 2     | 3     | 3     | 3     |
| $V_3$ | 2     | 3     | 3     | 3     | 3     | 3     |
| $J_2$ |       |       |       | 3     | 2     | 3     |
| $J_3$ |       |       |       | 3     | 3     | 2     |
| $J_4$ |       |       |       | 3     | 3     | 2     |
| $Y_0$ |       |       |       | 2     |
| $Y_2$ |       |       |       | 2     | 3     |

Table 8.17. Compatibility

| $C_2$ | $C_3$ | $C_4$ |
|-------|-------|-------|
| $G_1$ | 2     | 1     |
| $G_2$ | 3     | 0     |
| $G_3$ | 0     | 3     |

Table 8.18. Compatibility

| $J_0$ | $J_1$ | $J_4$ | $Y_0$ | $Y_2$ | $Y_4$ | $Z_0$ | $Z_2$ | $Z_4$ | $Z_6$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $F_0$ | 3     | 1     | 1     | 3     | 1     | 0     | 3     | 2     | 2     | 0     |
| $F_1$ | 0     | 3     | 3     | 0     | 3     | 3     | 1     | 3     | 3     | 3     |
| $F_2$ | 0     | 3     | 3     | 0     | 3     | 3     | 0     | 2     | 3     | 3     |
| $J_0$ |       |       |       |       |       | 3     | 1     | 0     | 3     | 2     |
| $J_1$ |       |       |       |       |       | 0     | 3     | 3     | 1     | 3     |
| $J_4$ |       |       |       |       |       | 0     | 3     | 3     | 0     | 2     |
| $Y_0$ |       |       |       |       |       | 3     | 2     |
| $Y_2$ |       |       |       |       |       | 1     | 3     |
| $Y_4$ |       |       |       |       |       | 0     | 2     |

Table 8.19. Compatibility

| $W_1$ | $W_2$ | $W_3$ | $M_1$ | $M_2$ | $U_0$ | $U_1$ | $U_4$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $H_2$ |       |       | 3     | 3     | 2     | 3     | 3     |
| $H_3$ |       |       | 1     | 2     | 3     | 1     | 3     |
| $H_4$ |       |       | 0     | 2     | 3     | 0     | 3     |
| $W_1$ |       |       | 3     | 2     | 3     |
| $W_2$ |       |       | 3     | 3     | 2     |
| $W_3$ |       |       | 3     | 3     | 2     |
| $M_1$ |       |       | 1     | 3     | 3     |
| $M_2$ |       |       | 1     | 3     | 3     |
Fig. 8.46. Aggregation of leaf node sets

Fig. 8.47. Aggregation for $L$

Fig. 8.48. Aggregation for $G$

Fig. 8.49. Aggregation for $C$

Fig. 8.50. Aggregation for $E$
Fig. 8.57. Aggregation for $J$

Fig. 8.58. Aggregation for $R$

Fig. 8.59. Aggregation for $Y$

Fig. 8.60. Aggregation for $Z$

Fig. 8.61. Aggregation for $A$

Fig. 8.62. Aggregation for $B$
Fig. 8.63. Superstructure for aggregated course (structure $\bar{\Lambda}$)

\[ S = I \ast O \ast X \]
\[ S_1 = I_1 \ast O_1 \ast X_1 \]
\[ S_2 = I_2 \ast O_1 \ast X_1 \]
\[ S_3 = I_3 \ast O_1 \ast X_1 \]

Methodology
\[ I = L \ast G \ast C \ast E \]
\[ I_1 = L_3 \ast G_2 \ast C_2 \ast E_2 \]
\[ I_2 = L_3 \ast G_2 \ast C_3 \ast E_2 \]
\[ I_3 = L_4 \ast G_3 \ast C_3 \ast E_2 \]

| Life cycle | System analysis, design | Decision making, selection | Systems reengineering |
|------------|-------------------------|----------------------------|-----------------------|
| $L_2(2)$   | $G_1(3)$                | $C_1(2)$                   | $E_1(2)$              |
| $L_3(1)$   | $G_2(1)$                | $C_2(1)$                   | $E_2(2)$              |
| $L_4(2)$   | $G_3(2)$                | $C_3(1)$                   | $E_3(3)$              |

Applied examples/student projects
\[ X = V \ast F \ast J \ast R \ast Y \ast Z \]
\[ X_1 = V_3 \ast F_4 \ast J_3 \ast R_3 \ast Y_2 \ast Z_3 \ast A_3 \ast B_3 \]

| Analysis of components | Software Inf. center | Improvement of inf. center career | Student investment | System trajectory life cycle |
|------------------------|----------------------|-----------------------------------|-------------------|-----------------------------|
| $V_0(2)$               | $F_0(3)$             | $J_0(1)$                          | $R_0(3)$          | $Z_0(2)$                    |
| $V_3(1)$               | $F_4(1)$             | $J_3(1)$                          | $R_3(1)$          | $Z_3(3)$                    |

Combinatorial optimization
\[ O = H \ast W \ast M \ast U \]
\[ O_1 = H_3 \ast W_2 \ast M_2 \ast U_1 \]

| Knapsack Assignment | Composition (clique) | Routing |
|---------------------|----------------------|---------|
| $H_2(2)$            | $W_1(2)$             | $M_1(2)$|
| $H_3(1)$            | $W_2(1)$             | $M_2(1)$|
| $H_4(2)$            | $W_3(2)$             | $M_3(3)$|

Table 8.20. Compatibility

|       | $W_1$ | $W_2$ | $W_3$ | $M_1$ | $M_2$ | $M_3$ | $U_0$ | $U_1$ | $U_4$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $H_2$ | 3     | 3     | 3     | 3     | 3     | 3     | 3     | 3     | 3     |
| $H_3$ | 1     | 2     | 3     | 1     | 3     | 3     | 3     | 3     | 3     |
| $H_4$ | 0     | 2     | 3     | 0     | 3     | 3     | 3     | 3     | 3     |
| $W_1$ |       |       |       | 3     | 2     | 2     | 3     | 2     | 2     |
| $W_2$ |       |       |       | 3     | 3     | 2     | 2     | 2     | 2     |
| $W_3$ |       |       |       | 3     | 3     | 2     | 2     | 3     | 3     |
| $M_1$ |       |       |       |       | 1     | 0     | 2     |       |       |
| $M_2$ |       |       |       |       | 0     | 3     | 3     |       |       |
| $M_3$ |       |       |       |       | 0     | 2     | 3     |       |       |
After usage of HMMD, the following composite DAs are obtained for the resultant aggregated structure ($\mathcal{X}$) (Fig. 8.63):

\[
\begin{align*}
X_1 &= V_3 \star F_4 \star J_3 \star R_3 \star Y_2 \star Z_3 \star A_3 \star B_3, \\
N(X_1) &= (3; 8, 0, 0), \\
O_1 &= H_3 \star W_2 \star M_2 \star U_1, \\
N(O_1) &= (2; 4, 0, 0); \\
I_1 &= L_3 \star G_2 \star C_3 \star E_2, \\
N(O_1) &= (2; 4, 0, 0), \\
I_2 &= L_3 \star G_2 \star C_3 \star E_2, \\
N(O_1) &= (2; 4, 0, 0), \\
I_3 &= L_4 \star G_3 \star C_3 \star E_2, \\
N(O_1) &= (3; 2, 2, 0);
\end{align*}
\]
\[
S_1 = I_1 \star O_1 \star X_1 = (L_3 \star G_2 \star C_2 \star E_2) \star (H_3 \star W_2 \star M_2 \star U_1) \star (V_3 \star F_4 \star J_3 \star R_3 \star Y_2 \star Z_3 \star A_3 \star B_3),
\]
\[
S_2 = I_2 \star O_1 \star X_1 = (L_3 \star G_2 \star C_3 \star E_2) \star (H_3 \star W_2 \star M_2 \star U_1) \star (V_3 \star F_4 \star J_3 \star R_3 \star Y_2 \star Z_3 \star A_3 \star B_3),
\]
\[
S_3 = I_3 \star O_1 \star X_1 = (L_3 \star G_3 \star C_3 \star E_2) \star (H_3 \star W_2 \star M_2 \star U_1) \star (V_3 \star F_4 \star J_3 \star R_3 \star Y_2 \star Z_3 \star A_3 \star B_3).
\]

8.10. **Configuration of Car in Electronic Shopping.** Recently, many products have a complex configuration and a buyer can often generate a product configuration that is more useful for him/her. An approach to electronic shopping based on product configuration (morphological approach) has been suggested in [109]. Here let us consider a multi-choice scheme for selection of structured product/system in electronic shopping with a resultant aggregation (Fig. 8.64).

![Diagram](image)

Fig. 8.64. Multi-choice scheme of electronic shopping

Now the following example is described (at a conceptual level). An initial morphological structure \( \Lambda \) of a car is the following (Fig. 8.65) (in real application, this structure can be considered as a result of processing the selected products/solutions):

0. Car \( S = A \star B \star C \).
1. Main part \( A = E \star D \):
   1.1. Engine \( E \): diesel \( E_1 \), gasoline \( E_2 \), electric \( E_3 \), hydrogenous \( E_4 \), and hybrid synergy drive HSD \( E_5 \);
   1.2. Body \( D \): sedan \( D_1 \), universal \( D_2 \), jeep \( D_3 \), pickup \( D_4 \), and sport \( D_5 \).
2. Mechanical part \( B = X \star Y \star P \star Z \):
   2.1. gear box \( X \): automate \( X_1 \), manual \( X_2 \);
   2.2. suspension \( Y \): pneumatic \( Y_1 \), hydraulic \( Y_2 \), and pneumohydraulic \( Y_3 \);
   2.3. drive \( P \): front-wheel drive \( P_1 \), rear-drive \( P_2 \), all-wheel-drive \( P_3 \).
3. Safety part \( C = O \star G \):
   3.1. \( O \): “absence” \( O_0 \), electronic \( O_1 \);
3.2. Safety subsystem $G$: “absence” $G_0$, passive $G_1$, active $G_2$.

$\Lambda = A \ast B \ast C$

- **Main part**
  - $A = E \ast D$

- **Engine**
  - $E_1$
  - $E_2$
  - $E_3$
  - $E_4$
  - $E_5$

- **Body**
  - $D_1$
  - $D_2$
  - $D_3$
  - $D_4$
  - $D_5$

- **Safety part**
  - $C = O \ast G$

- **Security system**
  - $O_0$
  - $O_1$

- **Mechanical part**
  - $B = X \ast Y \ast P$

- **Gearbox**
  - $X_1$
  - $X_2$

- **Suspension**
  - $Y_1$
  - $Y_2$

- **Drive**
  - $P_1$
  - $P_2$
  - $P_3$

Fig. 8.65. General structure of car $\Lambda$

The following initial solutions (obtained by users) are considered:

- $S_1^1 = E_1 \ast D_1 \ast X_1 \ast Y_1 \ast P_1 \ast O_1 \ast G_1$, $S_1^2 = E_5 \ast D_1 \ast X_1 \ast Y_1 \ast P_1 \ast O_1 \ast G_2$, $S_2^1 = E_2 \ast D_1 \ast X_2 \ast Y_1 \ast P_1 \ast O_0 \ast G_1$, $S_2^2 = E_2 \ast D_3 \ast X_1 \ast Y_2 \ast P_3 \ast O_1 \ast G_0$, and $S_3^2 = E_2 \ast D_5 \ast X_1 \ast Y_3 \ast P_1 \ast O_1 \ast G_1$.

An aggregated solution can be considered as simplified aggregation of five sets above:

$S_{agg} = E_2 \ast D_1 \ast X_1 \ast Y_1 \ast P_1 \ast O_1 \ast G_1$.

9. Conclusion

In the article, a systemic viewpoint to aggregation of modular solutions is firstly presented. The aggregation strategies have been suggested and considered: (i) extension strategy (addition of elements to a “system kernel”), (ii) compression strategy (deletion of elements of a superstructure), (iii) combined extension/compression strategy (addition, deletion, and replacement of elements in “system kernel”), and (iv) strategy of a new design (while taking into account new elements). The examined strategies have not been previously discussed in the literature. Our material corresponds to the first step in the investigated domain (aggregation of modular
solutions). The considered aggregation approaches are useful for engineering domains, management, computer science and information technology.

It is necessary to point out, close aggregation problems have been intensive studied and used in several domains: (a) decision making (aggregation of rankings to obtain a consensus, aggregation of preferences) (e.g., [35], [36], [166]); (b) integration of organizational structures in organization science (e.g., [10], [38], [39], [41], [58]); and (c) integration of information (database schema integration, integration of knowledge base structures, integration of catalogs, merging and integration of ontologies) (e.g., [1], [14], [32], [123], [131], [134], [159]).

In the article, many illustrative applied examples are presented. It is necessary to point out fundamentals of the described approaches correspond to practice (e.g., engineering, management). Thus, the suggested composite solving schemes and their local parts have to be based on a combination of mathematical combinatorial models and expert judgment of domain experts. Our approach for modeling the complex modular solutions/systems is based on the following four layers: (i) system hierarchy as tree-like structure, (ii) system components (leaf nodes of the tree-like mode above), (iii) sets of design alternatives for each system components, (iv) compatibility between design alternatives of different system components. Thus, models, problems, and algorithmic schemes are examined for the above-mentioned structures: proximities/metrics, substructures (including median/consensus, etc.), aggregation strategies/frameworks. The considered solving strategies are based on combinatorial problems (multicriteria ranking/selection, knapsack problem, multiple choice problem, morphological combinatorial synthesis, building median/consensus for structures).

The suggested aggregation problems may be useful in the following situations: (a) design processes based on uncertainty (e.g., fuzzy sets, stochastic models) can lead to generating a set (as a grid) of design solutions (here the problem under uncertainty can be approximated by a set of deterministic problems and, further, their solutions can be aggregated); (b) scenario-based methods can lead to a set of design solutions which can be aggregated. Note, complicated methods and models are required for dynamical modeling of complex systems, for example: (a) various kinds of Petri nets, (b) dynamical graphs. Our material does not involve this type of studies.

In the future, the following research directions can be considered:
1. usage of the suggested aggregation approaches for various kinds of system configuration solutions (e.g., set of selected elements, assignment/allocation solutions, network-like solutions [110]);
2. usage of other types of metrics/proximities for structures;
3. additional theoretical and applied studies of formal approaches to aggregation problems based on aggregation of system tree-like models and/or general system structures (i.e., multicriteria searching for median-like tree(s) and/or median-like structure(s) as Pareto-efficient structured solution(s));
4. examination of more complicated design problems for several resultant aggregated solutions (e.g., as “product line”);
5. examination of various models (for example, building of median/consensus), including multicriteria problem statements and usage of special quality domains/spaces, e.g., lattice-like quality domain/space ([101], [103], [106]);
6. examination and usage of other structures for system modeling, e.g., pyramids \( [24, 70] \);
7. taking into account uncertainty;
8. usage of artificial intelligence methods in problem solving processes;
9. investigation of new applications;
10. usage of the suggested aggregation problems as auxiliary underlaying sub-problems in information integration/fusion (for example, for approximation of alignment-like problems in bioinformatics, image processing; multi-source fusion of structured information);
11. special examination of aggregation problems for aggregation of solutions in combinatorial optimization problems (e.g., routing, scheduling, traveling salesman problem, assignment/allocation, graph coloring, covering);
12. usage of the suggested aggregation approaches in electronic shopping (and recommender systems) as the following two-stage framework: (i) an user is selecting a preliminary set of modular products (by his/her choice) from product catalogs, (ii) a computer system (i.e., special design support service) is constructing an aggregated product (selection of the best product from the preliminary product set is a simplified case);
13. it may be very interesting to apply the suggested aggregation approaches to drug design (generally, to combinatorial chemistry); and
14. usage of the suggested aggregation approaches in education (engineering, management, computer science and information technology).

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