Basic model for traffic interweave

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Abstract. We propose a three-parameter traffic model. The system consists of a loop with two junctions. The three parameters control the inflow, the outflow (from the junctions,) and the interweave (in the loop.) The dynamics is deterministic. The boundary conditions are stochastic. We present preliminary results for a complete phase diagram and all possible phase transitions. We observe four distinct traffic phases: free flow, congestion, bottleneck, and gridlock. The proposed model is able to present economically a clear perspective to these four different phases. Free flow and congestion are caused by the traffic conditions in the junctions. Both bottleneck and gridlock are caused by the traffic interweave in the loop. Instead of directly related to conventional congestion, gridlock can be taken as an extreme limit of bottleneck. This model can be useful to clarify the characteristics of traffic phases. This model can also be extended for practical applications.

1. Introduction
Practical applications of traffic dynamics are well appreciated because of their importance to modern society. Due to the complexity of real traffic, a workable model often has numerous parameters. A lot of research has focused to explore the fundamentals of complex dynamics [1]. A basic model is indispensable in understanding the underlying dynamics. Previously, we proposed a cellular automaton model for traffic dynamics at roundabout, which has twenty four parameters [2]. In this work, we aim to reduce the number of parameters and propose a basic model for the traffic interweave.

On a simple roadway without traffic interweave, traffic dynamics is a competition between inflow and outflow which results in two phases: free flow and conventional congestion. On the complex roadway with traffic interweave, there are two more types of congestion: bottleneck and gridlock. Together, there are three types of congestion. We propose a three-parameter model to present economically a clear perspective to these distinct phases. The three parameters can each be associated to a different characteristics of traffic phases: free flow, congestion, and bottleneck. Conventional wisdom would take the gridlock as an extreme limit of congestion. We show that it can be more appropriate to consider the gridlock as an extreme limit of bottleneck. This model can be useful to clarify the characteristics of traffic phases. This model can also be extended for practical applications.

2. Model
The model system consists of a rotary with two junctions as shown in Fig. 1(a). All the roadways are single-lane and unidirectional. Vehicles move into the system through one of the straight roadway, go around the loop a few times, and leave the system from the other straight roadway.
Roadway is divided into discrete cells, on which vehicles hop with cellular automaton rules. The dynamics is deterministic and governed by the Asymmetric Simple Exclusion Processes (ASEP) [3]. If an empty cell in front is available, vehicle moves forward in the next time step. The system configuration is updated in parallel. In contrast to the deterministic dynamics, the boundary conditions are stochastic. The inflow and outflow are controlled by parameters $\alpha$ and $\beta$, respectively. When the first cell of the incoming roadway is empty, a new vehicle is added stochastically with a probability $\alpha$ in the next time step. When a vehicle reached the last cell of the outgoing roadway, that vehicle is removed stochastically with a probability $\beta$ in the next time step. Upon entering the system, each vehicle is assigned an integer $n$ which specifies the number of turns $(1 + n)$ for that vehicle to move along the loop. The parameter $n$ is taken from a Poisson process, i.e. with a probability $(\gamma^n/n!) \exp(-\gamma)$. The average number of turns in the loop is $(1 + \gamma)$. In summary, the model has three parameters: $\alpha \in (0, 1)$ controls the inflow, $\beta \in (0, 1)$ controls the outflow, and $\gamma \in (0, \infty)$ controls the interweave.

The model configuration can also be taken as a T-shaped intersection imposed with periodic boundary condition [4], as shown in Fig. 1(b). The related traffic rules can be applied directly. On all cells, except the shaded one, a vehicle has only one direction to move. The shaded cell indicates the exit junction, on which a vehicle has two choices: to exit or to go another round. The interweave implies a traffic conflict on the cell next to the shaded one, i.e. the entry junction. The model prescribes the incoming vehicles to yield to the traffic in the loop, as required by most traffic regulations. Our results show that the gridlock cannot be avoided even if all vehicles perfectly follow the well-intended traffic regulations.

![Image](image_url)

**Figure 1.** (a) System configuration, where the arrow indicates the traffic direction. (b) T-shaped intersection, where the shaded cell indicates the exit junction.

### 3. Results

If all vehicles go around the loop just one time, i.e. $\gamma = 0$, there is no traffic interweave. The model reduces to ASEP on a straight roadway. The simulation results for the density $\rho$ on the loop are shown in Fig. 2(a). When $\alpha < \beta$, traffic is free-flowing and $\rho$ depends on $\alpha$ only. When $\alpha > \beta$, traffic is congested and $\rho$ depends on $\beta$ only. An abrupt transition along $\alpha = \beta$ separates the free flow phase and the congestion phase. When vehicles go around the loop more than one times, traffic in the loop will interweave with the incoming traffic. Two new traffic phases can be observed. At $\gamma = 0.5$, vehicles go around the loop one and a half times on average, where 61% of vehicles go around the loop one time, 30% two times, 8% three times, and 1% four times. The results are shown in Fig. 2(b). Four different phases can be observed. Besides the free flow and congestion, the plateau indicates a bottleneck phase and the narrow stripe with a maximum density $\rho = 1$ indicates a gridlock phase. When both $\alpha$ and $\beta$ are larger than the critical value, the plateau emerges and $\rho$ is independent of both $\alpha$ and $\beta$. When $\beta$ becomes
very small, the severe congestion leads to the gridlock. The density reaches the maximum $\rho = 1$ on the loop and also on the incoming roadway; while the outgoing roadway becomes empty. When the traffic interweave further increases, the gridlock phase becomes dominant. At $\gamma = 1$, vehicles go around the loop two times on average. The ratio of vehicles travelling around the loop one time decreases to 37% and the ratios of multi-time traveller increase accordingly. The results are shown in Fig. 3(c), where we observe only free flow and gridlock. The free flow phase is limited to $\alpha < \beta$ and small $\alpha$.

Figure 2. Density on the loop as a function of inflow and outflow $\rho(\alpha, \beta)$: (a) $\gamma = 0$; (b) $\gamma = 0.5$; (c) $\gamma = 1$.

Figure 3. Density evolution in the consecutive transitions: (a) $\rho(\beta)$ with fixed $\alpha = 0.3$ and $\gamma = 0.5$; (b) $\rho(\beta)$ with fixed $\alpha = 0.6$ and $\gamma = 0.5$; (c) $\rho(\gamma)$ with fixed $\alpha = 0.4$ and $\beta = 0.6$; (d) $\rho(\gamma)$ with fixed $\alpha = 0.6$ and $\beta = 0.4$;

This model presents four steady phases: Free flow ($F$), Congestion ($C$), Bottleneck ($B$), and Gridlock ($G$). Simulation results in Fig. 2 indicate various kind of phase transitions as the three control parameters ($\alpha$, $\beta$, $\gamma$) vary. Basically the traffic conditions become worse and the free
flow can no longer be maintained when either $\alpha$ increases, $\beta$ decreases, or $\gamma$ increases. First we consider the simple transition, which involves only two phases. As $\alpha$ increases, we observe three abrupt transitions ($F \rightarrow C$, $F \rightarrow B$, $F \rightarrow G$). As $\beta$ decreases, we observe two abrupt transitions ($F \rightarrow C$, $F \rightarrow G$) and one smooth transition ($B \rightarrow G$). As $\gamma$ increases, we observe one smooth transition ($C \rightarrow G$). Then we consider the consecutive transition, which involves three phases. As $\alpha$ increases, there is no consecutive transition. As $\beta$ decreases, we observe two consecutive transitions ($F \rightarrow C \rightarrow G$, $B \rightarrow C \rightarrow G$). As $\gamma$ increases, we observe two consecutive transitions ($F \rightarrow B \rightarrow G$, $C \rightarrow B \rightarrow G$). Typical results are shown in Fig. 3. As $\alpha$ increases, all three types of congestion can be the final phase. As $\beta$ decreases or $\gamma$ increases, gridlock becomes the dominate final phase. Bottleneck cannot be the final phase. Congestion can be the final phase only if there is no interweave ($\gamma = 0$).

4. Discussions

This model is able to differentiate the four traffic phases. The free flow is characterized by the inflow ($\alpha$). The conventional congestion is caused by a direct comparison between inflow ($\alpha$) and outflow ($\beta$), i.e. $\alpha > \beta$, and characterized by the outflow ($\beta$). The bottleneck is caused mainly by the interweave ($\gamma$), i.e. independent of both $\alpha$ and $\beta$. The model also presents the gridlock, which often occurs in real traffic but is totally absent in ASEP. We observe two kinds of gridlock: one as a narrow stripe and the other as a plateau in Fig. 2. The stripe phase is a gridlock transited from the conventional congestion as shown in Figs. 3(a) and 3(b); while the plateau phase is a gridlock transited from the bottleneck as shown in Figs. 3(c) and 3(d). The stripe phase appears only at the boundary of the parameter space. In the limit $\beta \rightarrow 0$, the conventional congestion is the final phase provided there is no interweave $\gamma = 0$. With interweave $\gamma \neq 0$, the conventional congestion becomes unstable. The gridlock can be triggered when the density on the loop is high. As a result, the high density in the outgoing roadway takes a drastic turn to reduce to zero in the limit $\beta \rightarrow 0$, as shown in Figs. 3(a) and 3(b). In contrast, the plateau phase occupies a finite region of the parameter space. All densities evolve gradually from bottleneck to gridlock, as shown in Figs. 3(c) and 3(d). Unlike the other three traffic phases, the gridlock implies the loss of mobility which cannot be restored. Thus the observation of gridlock will depend on the time scale. Typical results are shown in Figs. 4(a) and 4(b).

Figure 4. Density on the loop at various time scales $T$, where the simulation runs a period of $T$ before taking the average: (a) the same as Fig. 3(b); (b) the same as Fig. 3(d).

5. References

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