A new analysis of the Red Giant Branch ‘Tip’ distance scale and the value of the Hubble constant

Maurizio Salaris¹ & Santi Cassisi¹,²,³

¹Max-Planck-Institut für Astrophysik, D-85740, Garching, Germany - E-Mail: maurizio@MPA-Garching.mpg.de
²Università degli studi di L’Aquila, Dipartimento di Fisica, Via Vetoio, I-67100, L’Aquila, Italy
³Osservatorio Astronomico di Collurania, Via M. Maggini, I-64100, Teramo, Italy - E-Mail: cassisi@astrte.te.astro.it

ABSTRACT
The theoretical evaluations of the Red Giant Branch Tip (TRGB) luminosity presented in Salaris & Cassisi (1997) are extended to higher metallicities, and compared with analogous independent results recently published. The present sets of stellar models agree quite well in the determination of the TRGB brightness.

Relations between TRGB bolometric and I (Cousins) magnitude, and Zero Age Horizontal Branch V magnitude with respect to the metallicity are provided by adopting empirical, semiempirical and theoretical evaluations of bolometric corrections, after a careful calibration of the zero point of the bolometric correction scales.

The comparison between our ZAHB and TRGB distance scales for galactic globular clusters presented in Paper I is now supplemented with a comparison with the HIPPARCOS distance scale set by local subdwarfs with accurate parallax determinations. The overall agreement between ZAHB and HIPPARCOS distances is quite good. The TRGB distances for globular clusters are compatible with the ZAHB distances in the limit of the small sample of red giants observed.

The ZAHB and TRGB distances to resolved galaxies are in good agreement, whereas the comparison between TRGB and Cepheid distances, computed by using the calibration suggested by Madore & Freedman (1991), reveals a systematic discrepancy of the order of 0.12 mag. The TRGB distances are systematically longer in comparison with the Cepheid ones. This result supports the case for a revision of the zero point of the Cepheid distance scale, as already suggested by other authors on the basis of HIPPARCOS parallaxes. We do not find any clear correlation of the difference between TRGB and Cepheid distances with metal content.

The application of our TRGB distance scale to NGC3379, provides a distance to the Leo I group that is about 8% higher than the one obtained by Sakai et al. (1997a) adopting the TRGB-metallicity calibration by Lee, Freedman & Madore (1993). Our distance to the Leo I group, coupled with recent independent determinations of the distance Coma cluster-Leo I, obtained differentially by means of secondary distance indicators, provides a determination of H₀ at the Coma cluster in the range: H₀ = 60±11 Km s⁻¹ Mpc⁻¹.

For choices of Ω in agreement with the observations (0.3 ≤ Ω ≤ 1) and cosmological constant equal to zero, our derived H₀ value is compatible with the most recent determinations of the galactic globular clusters ages, thus removing the long-standing conflict between the Hubble age and the age of the oldest stars in the Galaxy.

Key words: stars: evolution – globular clusters: general – galaxies: distances and redshifts – distance scale

1 INTRODUCTION
For determining the value of the Hubble constant (H₀) it is necessary to measure with high accuracy distances to galaxies sufficiently distant so that local departures from the Hubble flow are negligible. This means that one needs to extend the distance scale to distances of the order of 100 Mpc or more; to this aim different secondary distance indicators have been devised in order to evaluate the relative distances between closer galaxies and more distant ones. However, the determination of H₀ is strongly dependent on the first step of the cosmological distance ladder, that is constituted by the absolute distances of close galaxies as determined by means of the primary distance indicators.

The Cepheid period-luminosity (P-L) relation is the basis for the calibration of the extragalactic distance scale; however, Cepheids observations are restricted only to Population I systems and to late-type galaxies. An excellent alternative primary distance indicator is the TRGB; its use results particularly attractive since it is applicable to all morphological types of galaxies as long as an old stellar population is present, and it has been recently successfully applied by many authors for estimating the distances to several nearby galaxies (see e.g. Lee, Freedman & Madore 1993, hereinafter LFM93, Sakai, Madore & Freedman 1996; Soria et al. 1996; Elson 1996; Sakai et al. 1997b).

The TRGB marks the helium ignition in the degenerate He core of low-mass stars, and its luminosity depends on the He core mass, which is remarkably constant for ages larger than 2-3 Gyr (see e.g. Salaris & Cassisi 1997, hereinafter Paper I), the exact value depending on the metallicity. Moreover, the I (Cousins) magnitude of the TRGB is only weakly sensitive to the metallicity of the stellar population and it is therefore obvious to use, as suggested by
LFM93, the observed I magnitude of the TRGB stars as a distance indicator. LFM93 have provided a semiempirical calibration of this method in a large range of metallicity, and in a subsequent paper Madore & Freedman (1995) undertook a series of numerical simulations and concluded that the TRGB method can be successfully used to determine distances accurate to 0.2 mag for galaxies out to 3 Mpc using ground based telescopes, and out to 12-13 Mpc using the Hubble Space Telescope.

Very recently Sakai et al. (1997a, hereinafter SA97) have determined, by means of the TRGB method, the distance to NGC3379 (an E1 galaxy at the center of the Leo I group) and, then, taking advantage of the relative distance determination (obtained by means of secondary indicators) between the Leo I group and the Coma cluster, they determined the absolute distance to the Coma cluster and $H_0 = 68 \pm 13 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The TRGB distance to NGC3379, crucial for the derived value of $H_0$, has been obtained by adopting the LFM93 calibration of $M^\text{tip}_I$ as a function of $[M/H]$ (as usual, we consider $[M/H]=$log$(M/H)_{\text{star}} - \log(M/H)_{\odot}$, where M and H are, respectively, the global heavy element abundance and the hydrogen abundance). The LFM93 TRGB calibration is taken from Da Costa & Armandroff (1990, hereinafter DA90). DA90 provides an empirical determination of the bolometric correction for the I band, and in a subsequent paper Madore & Freedman (1995) undertook a series of numerical simulations and concluded that the TRGB distance to Leo I is determined by means of the TRGB, and by adopting a relative distance between Leo I and Coma as derived from secondary distance indicators, the value of $H_0$ is finally obtained. A summary and a discussion about the implication of our $H_0$ determination for the age of the Universe are presented in section 5.

2 THE TRGB DISTANCE SCALE

2.1 Theoretical stellar models

The models used in this paper have been already presented in Paper I. To summarize, we have determined the TRGB luminosities for stellar populations with age $t=15$ Gyr (but, as discussed before and in Paper I, the precise value of $t$ does not influence the TRGB luminosities for ages larger than a few Gyr), metallicity $-2.35 \leq [M/H] \leq -0.57$ and $Y=0.23$, by computing evolutionary tracks of low mass stars without chemical elements diffusion. As far as it concerns the physical inputs adopted in computing the stellar models, the interested reader is referred to Paper I.

To extend the TRGB luminosity calibration to higher metallicities, we have now computed also evolutionary tracks for $Z=0.01$ and $Y=0.255 ([M/H]=-0.28)$, adopting the same input physics as in Paper I.

Assuming for the Sun $M_{\text{bol, \odot}} = 4.75$ mag, in Paper I we gave a relation between $M^\text{tip}_{\text{bol}}$ and $[M/H]$ that covered all the metallicity range $-2.35 \leq [M/H] \leq -0.57$. We have now verified, by computing stellar models for the appropriate metallicity, that the same relation reproduces the computed value of $M^\text{tip}_{\text{bol}}$ at $[M/H]=-0.28$ within 0.02 mag; therefore we can safely use it for a larger metallicity range:

$$M^\text{tip}_{\text{bol}} = -3.949 - 0.178 \cdot [M/H] + 0.058 \cdot (M/H)^2$$

(1)

that covers the range $-2.35 \leq [M/H] \leq -0.28$.

It takes also automatically into account the enhancement of the $\alpha$ elements observed in galactic field halo and GC stars (see, e.g. the review by Wheeler, Sueden & Truran 1989) when considering the global metallicity $[M/H]$. In fact, as already demonstrated by Salaris, Chieffi & Straniero (1993) and verified by means of the models by Salaris & Weiss (1997, 1998), the TRGB bolometric magnitudes and ZAHB luminosities in the $[M/H]$ range spanned by the models presented in this paper as derived from $\alpha$-enhanced theoretical models, are well reproduced by scaled solar ones with the same global metallicity. For fixed values of $[\alpha/\text{Fe}] \geq 0$ and $[\text{Fe/H}]$, the global metallicity $[M/H]$ is given by (see Salaris et al. 1993):

$$[M/H] \approx [\text{Fe/H}] + \log(0.638+0.362)$$

(2)

where $\log(f)=[\alpha/\text{Fe}]$.

Equation 1 depends on the adopted initial Helium content ($Y$) since a variation of $Y$ at fixed metallicity changes the TRGB luminosity because the change of the He core mass at the He flash. Regardless of the adopted $\Delta Y/\Delta Z$ law and for reasonable choices about it, i.e. $1 < \Delta Y/\Delta Z < 5$ (see Peimbert 1993; Carigi et al. 1995), the He abundance that one has to adopt at $Z \leq 0.006$ is not substantially different from the cosmological value $Y \approx 0.23$;
the maximum variation is of about +0.03 at \( Z = 0.006 \) (for \( \Delta Y/\Delta Z = 5 \)). At \( [M/H] = -0.28 \) we have adopted \( Y = 0.255 \) assuming \( \Delta Y/\Delta Z = 2.5 \). It is possible to take into account different \( Y \) abundances around the values adopted in the present work by considering that on average \( \frac{\partial M_{tip}}{\partial Y} \) is \( \approx 1.0 \) in the metallicity range covered by Equation 1.

In Figure 1, our prescription for the bolometric magnitude of the TRGB as a function of \([M/H]\) is displayed. For the aim of comparison, similar relations as derived from the recent evolutionary models by Cassisi et al. (1997 - their 'step8', with and without He and heavy elements diffusion), Caloi et al. (1997 - no atomic diffusion) and Straniero et al. (1997 - no atomic diffusion) are also plotted. These evolutionary models have been computed using slightly different input physics with respect to our models (the reader is referred to the quoted papers for more details on this subject), and in the case of the Caloi et al. (1997) results, a completely independent evolutionary code was adopted. In the same figure the relation provided by DA90 is shown after correcting for the slightly different \( M_{\text{Bot}, \odot} \) adopted by the quoted authors. The following points are worth noticing:

i) the agreement between the different recent evolutionary results is quite good. All the theoretical relations lie within \( \approx \pm 0.05 \) mag with respect to equation 1; this provides an estimation of the internal accuracy of \( M_{\text{TRGB}}^{\odot} \) values provided by the current updated theoretical scenario;

ii) the change of the TRGB luminosity due to the inclusion of atomic diffusion - adopting the same physical inputs as in standard models - is quite negligible (see the results corresponding to the Cassisi et al. 1997 models);

iii) the slope of the relation \( M_{\text{Bot}}^{\text{TRGB}} - [M/H] \), provided by the most recent stellar models is similar to the slope suggested by DA90;

iv) there exists a difference of about 0.15 mag in the zero point between our relation and the relation provided by DA90.

2.2 The Bolometric Correction scale

In order to derive the distance modulus through the TRGB method LFM93 suggested an iterative procedure from observations in the VI Johnson-Cousins bands. Such procedure can be summarized as follows (see LFM93 for more details):

i) fixing preliminarily the distance modulus;

ii) with the fixed distance modulus determining the metallicity by measuring the dereddened \((V-I)\) color at \( M_I = -3.5 \) mag \((V-I)_0 = -3.5\) and using a relation between this color and the metallicity of the parent stellar population;

iii) obtaining the distance modulus from the observed I magnitude of the TRGB (corrected for the interstellar extinction) by adopting relations for both the TRGB bolometric magnitude as a function of metallicity and the bolometric correction to the I magnitude \((BC_I)\);

iv) iterating the previous steps until convergency is obtained between the distance modulus at step (i) and the one obtained after step (iii). Since the weak dependence of \( M_{tip}^{\odot} \) on the metallicity, convergence is generally achieved after one iteration.

![Figure 1](image)

**Figure 1.** Comparison between updated predictions of stellar evolution theory, concerning the behavior of the bolometric magnitude of the TRGB as a function of metallicity. The calibration provided by DA90 is also plotted. In all cases, a bolometric magnitude for the Sun equal to 4.75 mag, has been adopted.

When the TRGB bolometric magnitude is known from stellar models, the last ingredient necessary for the application of the TRGB method is a relation providing the bolometric correction in the I (Cousins) band. Following LFM93, an empirical \( BC_I = (V-I)_0 \) relation for RGB stars has been taken from DA90. In that paper the authors give:

\[
BC_I = 0.881 - 0.243 \cdot (V-I)_0 \tag{3}
\]

independent of the metallicity. The bolometric corrections provided by this relation are on a scale where \( BC_{V, \odot} = -0.07 \). Since \( M_{V, \odot} = 4.82 \pm 0.02 \text{ mag} \) (Hayes 1985), this implies the adoption of \( M_{\text{Bot}, \odot} = 4.75 \text{ mag} \) (as in Equation 1). We assume the quoted error on the value of \( M_{V, \odot} \) as an estimate of the error on the zero point of the bolometric correction scale.

At this point it is straightforward to derive the distance modulus of a galaxy according to the relation:

\[
(m-M)_I = I_{\text{TRGB}} + BC_I - M_{\text{tip}}^{\odot} \tag{4}
\]

The empirical \( BC_I \) provided by equation 3 were derived comparing the I magnitudes given in DA90 with the \( M_{\text{Bot}} \) values given by FPC83 for a sample of RGB stars in 8 GCs with different metallicities. By examining Figure 14 in DA90, it appears clearly that in the range of \((V-I)_0 \) values typical of the bulk of the stars considered by the authors \((V-I)_0 \) colors range from 1.0 to 1.6) and of the TRGB stars in the sample of galaxies studied in the next sections (the \((V-I)_0 \) TRGB ranges approximately from 1.3 to 2.0) there is a dispersion of the order of 0.10 mag around the least-square fit given by Equation 3. Moreover, the relation for the reddest stars is based only on a very small number of observational points.
In order to supply an independent determination of the 
$BC_I$ scale that can be safely adopted for our RGB stellar models, we also used in Paper I the theoretical $BC_I$ values derived from model atmospheres. More in detail, we used bolometric correction based on an updated version of the Kurucz’s code ATLAS9 (Castelli 1996, private communication). The $BC_I$ values were derived from the relation $BC_I = BC_V + (V-I)$, where $BC_V$ and $(V-I)$ are provided by the ATLAS9 code. Obviously the zero point of the $BC_I$ scale sets also the zero point of the $BC_V$ scale. The Kurucz $BC_V$ are normalized in such a way that the maximum value of $BC_V$ is zero, and all the other values are negative. With this choice the value of $BC_V(⊙)≈0.19$, and therefore we should have used $M_{Bol,⊙}=4.63$ mag for reproducing the observed $M_V(⊙)$ value. However, we were dealing with stars with a metallicity lower than the Sun, and therefore we decided to adopt Vega $([M/H]=-0.5$ according to Castelli & Kurucz (1994) for setting the zero point of our $BC_V$ scale. The $BC_V$ value for Vega provided by the ATLAS9 transformations presented a good agreement with the empirical value provided by Code et al. (1976), and since Code et al. (1976) empirical $BC_V$ are set on a scale where $BC_V(⊙)≈-0.07$, we adopted in Paper I the ATLAS9 transformations and $M_{Bol,⊙}=4.75$ mag.

However, a detailed comparison with the recent empirical $BC_V$ database provided by Alonso et al. (1995) - that is an extension and improvement of the work by Code et al. (1976) based on observations of many solar metallicity and metal poor Main Sequence stars, reveals that a value $M_{Bol,⊙}=4.62$ mag should be used for fitting the empirical bolometric correction scale. By assuming $M_{Bol,⊙}=4.62$ mag, we had finely reproduced the results for a large sample of metal poor stars and also the observed value for $M_V(⊙)$.

Bearing in mind this comparison (see also the discussion in De Santis 1996), in the present work we have decided to reanalyze the results obtained in Paper I. Once again, we have adopted the theoretical bolometric corrections and colors obtained with the Kurucz’s code ATLAS9, but now we have taken advantage of a new grid of model atmospheres computed with an updated version of the code (Castelli 1997, private communication; Castelli, Gratton & Kurucz 1997a, 1997b, hereinafter K97). We have verified that by adopting for $M_{Bol,⊙}$ the value 4.62 mag, with these new model atmospheres we can simultaneously match both $M_V(⊙)$ and the $M_V$ values given by the empirical $BC_V$ scale of Alonso et al. (1995). By using our stellar models and the K97 $BC_V$ and colors, the following relation has been obtained for the absolute I magnitude of the TRGB:

$$M_I^{tip} = -3.953 + 0.437 \cdot [M/H] + 0.147 \cdot [M/H]^2$$

with a correlation coefficient $r=0.99$.

For estimating the uncertainty related to the use of theoretical bolometric corrections and temperature conversion, being aware of the problems still existing with model atmospheres, we have also searched for semiempirical relations which satisfy observational constraints, independently from the DA90 $BC_I$. For this reason we have adopted here also the so called Yale transformations (Green 1988) for obtaining the theoretical $BC_I$ values. These transformations are an empirical UBVRI recalibration (independent of the DA90 $BC_I$ scale) of Vandenberg & Bell (1985) and Kurucz (1979) synthetic colors and $BC_V$, taking into account various observational constraints. The $BC_V$ values, based on a scale in which $BC_V(⊙)≈-0.07$, are in satisfactory agreement (within less than 0.04-0.05 mag, when one takes into account the difference in the assumed value of $BC_V(⊙)$) with the more recent empirical $BC_V$ by Alonso et al. (1995). By adopting these transformations (together with $M_{Bol,⊙}=4.75$ mag) and our theoretical models, we get the following relation:

$$M_I^{tip} = -4.156 + 0.157 \cdot [M/H] + 0.070 \cdot [M/H]^2$$

with $r=0.98$.

In the next section we will check the consistency between the empirical $BC_I$ given by DA90, the semiempirical ones from the Yale transformations and the theoretical $BC_I$ supplied by K97.

3 COMPARISON BETWEEN TRGB, RR LYRAE AND CEPHEID DISTANCE SCALES

Before using the TRGB method for determining the distance to the Leo I group, in order to assess the reliability of the theoretical TRGB luminosities, we will briefly compare in this section the TRGB distances with the distance scales set by RR Lyrae and Cepheids in GCs and nearby galaxies in which the stellar component has been resolved.

The major improvement in comparison with Paper I is that now we also adopt the Yale and the K97 transformations, after a careful calibration of the bolometric correction zero point as discussed in the previous section. Moreover we will compare our RR Lyrae distances to GC also with the distances obtained by means of the Main Sequence Fitting (MSF) technique based on nearby subdwarfs for which accurate parallaxes have been recently measured by HIPPARCOS.

In the following we will separately discuss the cases for GCs and for resolved galaxies.

3.1 Globular Clusters

In the case of galactic GCs it is possible to compare the distance scale fixed by ZAHB models, with the one derived from Equation 1. Here we have adopted the ZAHB models from Cassisi & Salaris (1997) and used in Paper I, but transformed into the observational plane by using both the Yale and K97 transformations. The relations between the ZAHB V magnitude (taken at log $T_{eff}=3.85$, that corresponds approximately to the average temperature of the RR Lyrae instability strip) and $[M/H]$ are the following:

$$M_V^{yale} = 0.921 + 0.329 \cdot [M/H] + 0.045 \cdot [M/H]^2$$

$$M_V^{k97} = 0.974 + 0.379 \cdot [M/H] + 0.062 \cdot [M/H]^2$$

for $-2.35 \leq [M/H] \leq -0.57$, with $r=1.00$ for both relations.

The difference between these two ZAHB distance scales is quite negligible, being on average equal to 0.02-0.03 mag, the ZAHB luminosities obtained by using the Yale transformations being systematically brighter. From now on we will adopt Equation 7 as our reference ZAHB RR Lyrae distance scale.
The comparison between the TRGB and the ZAHB distance scales fixed by Equations 1 and 7 has been performed, as in Paper I, by adopting the TRGB observational data by Frogel et al. (1983 - hereinafter FPC83 - and reference therein), who provided absolute bolometric magnitudes (on a scale where $BC_{V,G} = -0.07$) for many TRGB of galactic GCs. These magnitudes have been obtained empirically by directly integrating the flux from the program stars via the observed $UBVJHK$ photometry and adopting a RR Lyrae distance scale for the selected clusters.

As far as it concerns the criteria adopted for selecting the clusters in our sample, a detailed discussion can be found in Paper I. Table 1 lists, for all clusters in our sample, the values of $[Fe/H]$ and $[\alpha/Fe]$ obtained by means of spectroscopic analysis (as collected by Salaris & Cassisi 1996), the global metallicity $[M/H]$ (according to relation 2), reddening, the distance modulus (reddenning corrected) and $M_{\text{tip, Bol}}$ (obtained by modifying the values given by FPC83 for taking into account the ZAHB distance scale given by our Equation 7, and the reddenings and $[M/H]$ values we adopt).

Figure 2 shows the $M_{\text{tip, Bol}}$ values versus the global heavy elements abundance for the selected clusters, and also the theoretical relation for the TRGB luminosity (Equation 1). The vertical error bar ($\pm 0.1$ mag) for the observational points represents an average error on the distance modulus obtained from integration of relation 7 (see Cassisi & Salaris 1997) while the error on the spectroscopic determination of $[M/H]$ is typically of the order of 0.15 dex (see Paper I).

Data in Figure 2 show quite clearly that the TRGB observational points (with the exception of NGC6352) are located at lower luminosities in comparison with the theoretical relation, with an average difference of approximately 0.15-0.20 mag. However, this is exactly what is expected on the basis of simple statistical arguments (see the detailed discussion in Paper I), as soon as the evolutionary times in the upper part of the RGB and the number of stars observed in each cluster by FPC83 are taken into account. This means that the theoretical TRGB and ZAHB distance scales in GCs are in agreement within the statistical uncertainties due to the small sample of red giant stars observed.

The metal-rich GC NGC6352 is the cluster out of the whole sample which is characterized by a luminosity level higher than the theoretical value. When checking for it in the FPC83 paper one notices that the star considered to be at the TRGB in this cluster could be a field star. If this is the case the second brightest star in the FPC83 sample is $\approx 0.3$ mag fainter, and it would be located in Figure 2 below the line corresponding to the theoretical TRGB values, as expected from statistics.

As an independent check of the reliability of the distance moduli derived from Equation 7 and of the calibration of the evolutionary models, we show in Figure 3, as an example, a fit to the I-(V-I) diagrams by DA90 of the RGB in NGC6397 (lower panel) and in NGC6752 (upper panel) by using our theoretical RGB models together with the Yale transformations, the $(m - M)_0$ and $E(B-V)$ values given in Table 1, and the extinction relations by Cardelli et al. (1989). The agreement between theory and observations is quite satisfactory.

As a second check about our theoretical ZAHB distance scale for GCs we have compared our results with GC distance moduli taken from the very recent literature, derived from the MSF technique based on subdwarfs with accurate HIPPARCOS parallaxes. The sources of the GC MSF distances are Gratton et al. (1997 - we have considered the distances obtained by correcting for binary contamination to the HIPPARCOS subdwarfs sample, as displayed in column 8 of their Table 3), Reid (1997 - we have considered the dereddened distance moduli displayed in column 7 of his Table 3, derived assuming the reddening used in our paper, and then applied the corresponding extinction contribution) and Chaboyer et al. (1997).

In Figure 4 (panels a-c) we display the result of this comparison, where the error bars on the MSF distances are taken from the quoted papers. Since the various authors

| Table 1. Selected data for the sample of galactic globular clusters. |
|--------------------------|-------------|----------------|----------------|-----------------|----------------|
| Cluster                  | $[Fe/H]$   | $[\alpha/Fe]$ | $[M/H]$ | $E(B-V)$ | $(m - M)_0$ | $M_{\text{tip, Bol}}$ |
|--------------------------|-------------|----------------|----------------|-----------------|----------------|
| M71                      | -0.80       | 0.27           | -0.61          | 0.28            | 13.08          | -3.74           |
| NGC6352                  | -0.80       | 0.13           | -0.70          | 0.21            | 13.90          | -4.01           |
| 47 Tuc                   | -0.80       | 0.15           | -0.70          | 0.04            | 13.37          | -3.83           |
| NGC3362                  | -1.20       | 0.23           | -1.04          | 0.06            | 14.67          | -3.45           |
| M5                       | -1.40       | 0.30           | -1.19          | 0.03            | 14.47          | -3.37           |
| M79                      | -1.42       | 0.21           | -1.27          | 0.00            | 15.78          | -3.70           |
| NGC6752                  | -1.50       | 0.31           | -1.28          | 0.04            | 13.16          | -3.60           |
| M3                       | -1.49       | 0.26           | -1.31          | 0.00            | 15.19          | -3.53           |
| NGC6397                  | -1.88       | 0.25           | -1.70          | 0.18            | 11.99          | -3.36           |
| M68                      | -1.92       | 0.20           | -1.78          | 0.07            | 15.03          | -3.50           |
| M15                      | -2.30       | 0.30           | -2.09          | 0.10            | 15.19          | -3.53           |
adopt different procedures for the MSF, different sets of subdwarfs, different corrections for the statistical bias affecting the subdwarfs luminosities, and sometimes slightly different assumptions about the [M/H] values for both the clusters and the subdwarfs, the differences between the distance moduli obtained for the GCs in common among these three investigations give us a rough estimate of the intrinsic error of the MSF technique. To make the comparison more meaningful, we have decided to adopt for each cluster the error of the MSF technique. To make the comparison more meaningful, we have decided to adopt for each cluster the error of the MSF technique. To make the comparison more meaningful, we have decided to adopt for each cluster the error of the MSF technique. To make the comparison more meaningful, we have decided to adopt for each cluster the error of the MSF technique. To make the comparison more meaningful, we have decided to adopt for each cluster the error of the MSF technique. To make the comparison more meaningful, we have decided to adopt for each cluster the error of the MSF technique. To make the comparison more meaningful, we have decided to adopt for each cluster the error of the MSF technique.

From Figure 4 one can easily notice that on average there is a good agreement between our ZAHB distance scale and the HIPPARCOS MSF distances. For the most metal poor (and more distant) clusters displayed in the figure, the Reid (1997) data seem to disagree systematically with our ZAHB distances, but it is worth noting that the same M68 distance derived by Gratton et al. (1997) nicely agrees with the ZAHB distance. In particular, in the case of M5 and NGC6752 the distance moduli derived from the MSF by the three different groups are almost identical, and the agreement with the ZAHB distance scale is almost perfect.

We can therefore finally conclude that for GCs the HIPPARCOS, TRGB and ZAHB distance scales are in agreement one with each other within the present errors.

With the distance scale set by Equation 7, it is now possible to recalibrate Equation 7 of Paper I, which provides [M/H] as a function of (V −I)$_0$−3.5, based on a subsample of the GCs listed in Table 1 for which DA90 provide V−(V−I) diagrams. The new relation is

$$[M/H] = -39.270 + 64.687 \cdot [(V − I)_0, −3.5]$$

$$-36.351 \cdot [(V − I)_0, −3.5]^2 + 6.838 \cdot [(V − I)_0, −3.5]^3$$

(9)

with r=0.99. As in Paper I, this relation has been obtained by imposing that the (V−I)$_0$,−3.5 values have to be monotonously increasing for increasing metallicity.

3.2 Resolved galaxies

When considering resolved galaxies we can compare (as in Paper I, LFM93) our TRGB distance moduli with RR Lyrae and Cepheid distances. The observational database used in this comparison is the same one as in Paper I, with the additional data for Sextans B taken from Sakai et al. (1997b) and new BVRI data for the Cepheids in NGC3109 from Musella et al. (1997). The Cepheid distance scale is the one set by the P-L relations provided by Madore & Freedman (1991), with the zero point set by a LMC distance modulus of 18.50 mag and E(B − V) = 0.10. The extinction law (Cardelli et al. 1989), adopted for correcting the apparent RR Lyrae and TRGB distance moduli for the extinction, is the same adopted by Madore & Freedman (1991) and employed in all the Cepheid distance determinations used in this comparison.

In Table 2 we report the distance modulus determinations as obtained with the three different methods. The various columns provide the following data: (1) the name of the object; (2) the reddening; (3) the observed I magnitude; (4) the mean RGB metallicity, as obtained by adopting Equation 9 and the distance moduli in column 5; (5) the true distance modulus estimated by using the TRGB method and the DA90 $BC_I$; (6) the intrinsic Cepheid distance; (7) the true distance obtained by using the mean RR Lyrae luminosity; (8) as in column (7) but for an average metallicity of the RR Lyrae population [M/H]=−1.5 (see below); (9) the distance modulus obtained by applying the TRGB method and Equation 6 (the metallicities derived in this way are on average ≈0.05-0.10 dex lower than the values in column 4); (10) the distance modulus obtained applying the TRGB method and Equation 5 (also in this case the metallicities are on average ≈0.05-0.10 dex lower than the values in column 4).

Estimates of the individual errors associated to the Cepheid, TRGB and RR Lyrae distances are given in Table 3. The errors on (m − M)$_0$,$_{Cepheid}$ are taken from the corresponding papers (in the case of NGC3109 we have used the new Cepheid observations by Musella et al. (1997) and applied the method outlined in Madore & Freedman (1991) for deriving the distance and the related error) without taking into account the contribution due to the uncertainty in the adopted zero point of the P-L relation (the distance modulus of the LMC, set at (m − M)$_0$=18.50) since we want to check if there exists a discrepancy between the Cepheid distances set by this zero point and the TRGB and RR Lyrae theoretical distance scales.

As for the errors on the TRGB distances, we have considered the error associated to the detection of $IT_{TRGB}$ as given in the original papers, to this it has been added statistically the contribution due to an error on the metallicity.
Table 2. Selected parameters for a sample of resolved galaxies (see text).

| Galaxy   | E(B−V) | I_{TRGB} | [M/H] | (m−M)_{DA90}^{0,TRGB} | (m−M)_{0,Crph} | (m−M)_{0,RR} | (m−M)^{-1.5}_{0,RR} | (m−M)_{Yale}^{0,TRGB} | (m−M)_{K97}^{0,TRGB} |
|----------|--------|----------|-------|------------------------|----------------|--------------|---------------------|------------------------|---------------------|
| LMC      | 0.10   | 14.60    | -1.0  | 18.60                  | 18.50          | 18.54        | 18.64               | 18.64                  |
| NGC6822  | 0.28   | 20.05    | -1.7  | 23.61                  | 23.62          |              |                     |                        |
| NGC185   | 0.19   | 20.30    | -1.0  | 24.12                  | 24.06          | 24.15        | 24.15               | 24.15                  |
| NGC147   | 0.17   | 20.40    | -0.9  | 24.27                  | 24.17          | 24.28        | 24.29               | 24.29                  |
| IC1613   | 0.02   | 20.25    | -1.2  | 24.43                  | 24.42          | 24.41        | 24.45               | 24.47                  |
| M31      | 0.08   | 20.55    | -0.9  | 24.56                  | 24.44          | 24.56        | 24.67               | 24.62                  |
| M33      | 0.10   | 20.95    | -2.0  | 24.82                  | 24.63          | 24.85        | 24.78               | 24.92                  |
| WLM      | 0.02   | 20.85    | -1.5  | 24.97                  | 24.92          |              |                     |                        |
| NGC205   | 0.035  | 20.45    | -0.9  | 24.54                  | 24.90          | 25.01        | 24.62               | 24.61                  |
| Sex A    | 0.03   | 21.79    | -1.9  | 25.88                  | 25.85          |              |                     |                        |
| Sex B    | 0.015  | 21.60    | -1.6  | 25.72                  | 25.69          |              |                     |                        |
| NGC3109  | 0.04   | 21.55    | -1.5  | 25.61                  | 25.60          |              |                     |                        |

Figure 4. Comparison between different GC distance moduli, obtained by using the MSF with HIPPARCOS subdwarfs and our ZAHB distance scale (Equation 7). In each panel, the ZAHB distance modulus has been computed by using the same [Fe/H] values adopted by the corresponding MSF author and [α/Fe]=0.30 (see text for more details).

Figure 5. Comparison between different TRGB distances obtained by adopting the DA90 BC with the results from LFM93 given in section 2.2. Moreover, it has been also accounted for the contribution due to the uncertainty on the extinction: in the case it is discussed in the original papers, we have adopted the value given by the authors; if the value for the reddening is taken from Burstein & Heiles (1984), we have considered the correspondent error as given by the quoted authors, and then translated it into an error on the extinction by adopting the reddening law previously quoted.

In the case of RR Lyrae distances, the errors were derived from the observational errors on the mean brightness of the RR Lyrae sample observed in each individual galaxy, and by adding statistically the contribution due to the error related to the procedure followed for converting this mean brightness of the RR Lyrae sample into the corresponding ZAHB magnitude (see Paper I for details about the proce-
Figure 6. Comparison between different distances for the selected sample of resolved galaxies, obtained using the TRGB together with the DA90, Yale and K97 $BC_I$ values

Table 3. Estimates of the individual errors associated to the TRGB, Cepheid and RR Lyrae distances for the sample of galaxies in Table 2 (see text).

| Galaxy   | $\Delta (m - M)_{0,\text{TRGB}}$ | $\Delta (m - M)_{0,\text{Ceph}}$ | $\Delta (m - M)_{0,\text{RR}}$ |
|----------|---------------------------------|---------------------------------|---------------------------------|
| LMC      | 0.14                             | 0.12                             |                                 |
| NGC6822  | 0.14                             | 0.17                             | 0.20                            |
| NGC185   | 0.32                             | 0.20                             |                                 |
| NGC147   | 0.14                             | 0.20                             |                                 |
| IC1613   | 0.22                             | 0.13                             | 0.20                            |
| M31      | 0.18                             | 0.10                             | 0.20                            |
| M33      | 0.18                             | 0.09                             | 0.20                            |
| WLM      | 0.14                             | 0.11                             | 0.20                            |
| NGC205   | 0.22                             | 0.15                             |                                 |
| Sex A    | 0.14                             | 0.15                             |                                 |
| Sex B    | 0.13                             | 0.25                             |                                 |
| NGC3109  | 0.14                             | 0.15                             |                                 |

dure). We just notice that in this paper, when RR Lyrae observations have been performed in the $g$ Thuan-Gunn band (as in the case of NGC185, NGC147, IC1613, NGC205), the relation by Kent (1985) for transforming the $g$ magnitudes into $V$ magnitudes has been adopted. It is also important to remember that the $[M/H]$ values used for obtaining the distance moduli given in column 7 of Table 2 are derived from RGB stars, and correspond to an average metallicity of this stellar population. In principle this metallicity could not correspond to the RR Lyrae average metal content, especially for highest and lowest values of $[M/H]$ displayed in Table 2, due to the low probability that metal-poor and metal-rich RGB stars evolve during their He central burning phase through the RR Lyrae instability strip (as for example in the case of the metal poor GC M92 and the metal rich one 47 Tuc). For roughly estimating the uncertainty introduced by the unknown metallicity of the RR Lyrae population, the distance moduli obtained assuming for the RR Lyrae stars an average metallicity equal to $[M/H]=-1.5$ - adopted as a reasonable estimate of the average metallicity for the galactic GC RR Lyrae population - have been also reported (with the unique exception of the LMC; in this case the correct metallicity for the considered RR Lyrae stars is taken into account, see Paper I) in Table 2 (column 8).

Figure 5 shows the difference between the TRGB distance moduli obtained by adopting the DA90 $BC_I$ and the corresponding results from LFM93 (where the same $BC_I$ are used). It is clear from the figure that there is a systematic offset by on average 0.15 mag, our distances being larger.

We have then displayed in Figure 6 the difference between the TRGB distance moduli obtained adopting respectively the DA90, Yale or K97 $BC_I$. The average difference $(m - M)_{\text{TRGB,DA90}} - (m - M)_{\text{TRGB,Yale}}$ is equal to -0.06 mag, while the $(m - M)_{\text{TRGB,DA90}} - (m - M)_{\text{TRGB,K97}}$ is equal to -0.08mag.

The difference between the TRGB distances - obtained by adopting alternatively the DA90, the Yale, or the K97 $BC_I$ - and the RR Lyrae distances is shown in Figure 7. In all cases there is no statistically significant correlation between $(m - M)_{\text{TRGB}} - (m - M)_{\text{RR Lyrae}}$ and
clusters, we adopt in present work (see Paper I for a detailed explanation).

Figure 8. Comparison between different distances for the selected sample of resolved galaxies, obtained by using the TRGB (by means of the DA90, Yale and K97 BC) and the Cepheid distance scales. The error bars associated to each individual point are also displayed. The long dashed line in each panel corresponds to the weighted average difference between TRGB and Cepheid distance moduli. Its associated error is also shown.

\((m - M)_{RRLyrae}\). By using the DA90 BC for the TRGB theoretical luminosity and by neglecting the very discrepant point corresponding to NGC205 - see also the discussion in LFM93 about this galaxy -, if one considers for the RR Lyrae stars the same metallicity as derived from the RGB stars, a weighted average difference \((m - M)_{TRGB} - (m - M)_{RRLyrae}\) of 0.03 ± 0.10 mag has been obtained. When considering (with the exception of the LMC) an average metallicity \([M/H] = -1.5\), a weighted average difference \((m - M)_{TRGB} - (m - M)_{RRLyrae}\) of -0.01 ± 0.10 mag is derived.

In the case of the Yale or the K97 bolometric corrections, the same quantity \((m - M)_{TRGB} - (m - M)_{RRLyrae}\) ranges between 0.05 ± 0.10 and 0.09 ± 0.10 mag depending on the assumed RR Lyrae metallicity. One can therefore conclude that the RR Lyrae and TRGB distance scales agree quite well. Their average difference is statistically consistent with a value equal to zero when considering our sample of resolved galaxies, spanning a metallicity range (for the RGB stellar populations) by ≈1 dex.

It is worth noting the large difference which exists between our evaluation (18.54 mag) and the LFM93 result (18.28 mag) for the LMC distance modulus based on the RR Lyrae distance scale. The origin of such discrepancy has to be related both to the difference (≈0.1 mag) between our RR Lyrae distance scale and the one adopted by LFM93, and to the different observational data for the LMC clusters, we adopt in present work (see Paper I for a detailed discussion on this point).

Figure 9. Average difference \((m - M)_{0,TRGB} - (m - M)_{0,Cepheid}\) for the seven irregular galaxies quoted in the text, as a function of the \([O/H]\) abundance in their HII regions (see text for more details).

In Figure 8 (panels a-c) we display the difference between the distance moduli obtained by adopting the TRGB and the Cepheid distance scale. Also in this case there is no statistically significant correlation between \((m - M)_{TRGB} - (m - M)_{Cepheid}\) and \((m - M)_{Cepheid}\). The points corresponding to the individual galaxies are always located above the line corresponding to a difference equal to zero. The weighted average differences \((m - M)_{TRGB} - (m - M)_{Cepheid}\) obtained by adopting for the TRGB distance scale the Equation 1 and the DA90 BC, or Equation 6 or Equation 5, are equal to 0.08 ± 0.07 mag (very similar to the result obtained in Paper I with a smaller galaxy sample), 0.13 ± 0.07 mag and 0.16 ± 0.07 mag, respectively (corresponding to the long dashed line in each panel of the Figure 8). It is interesting to notice that in all of the three cases the systematic difference from zero is statistically significant.

The case of Sex B deserves a brief comment. According to Sakai et al. (1997b) the Cepheid distance modulus is 25.69 ± 0.25 mag (as displayed in table 2) but, as discussed in their paper, if one short period Cepheid in their sample (possibly an overtone pulsator) is excluded from the analysis, it is possible to find a solution with a reddening equal to zero and a distance modulus equal to 25.82 mag. In this case, the difference \((m - M)_{TRGB} - (m - M)_{Cepheid}\) for Sex B would be quite different. We have therefore recomputed the average differences \((m - M)_{TRGB} - (m - M)_{Cepheid}\) excluding Sex B or adopting the higher value for its distance modulus. However in both cases the average differences \((m - M)_{TRGB} - (m - M)_{Cepheid}\) for all the sample
of galaxies are changed by not more than 0.01 mag.

If one considers the full range of values obtained by adopting the three different sets of $BC_I$ as an estimate of the uncertainty in the $BC_I$ scale, we derive an mean difference $(m - M)_{TRGB} - (m - M)_{Cephid} = 0.12$ mag. It is interesting to note that this difference between TRGB and Cepheid distances is in agreement with very recent results obtained by adopting HIPPARCOS parallaxes, which show that the LMC distance modulus (and therefore the zero point of the Cepheid calibration) could be larger than 18.50. LMC distance moduli equal to 18.60±0.07 mag, 18.65±0.07 mag, 18.60±0.20 mag, 18.70±0.10 mag, 18.56±0.08 mag are derived respectively by Gratton et al. (1997), Reid (1997), Whitelock et al. (1997), Feast & Catchpole (1997), Oudmaijer et al. (1997), by adopting different calibrators and different techniques.

When considering these results, it is worth to bear in mind that the TRGB luminosities determined for the sample of galaxies displayed in Table 2 are based on observations of a very large number of RGB stars, much larger than in the case of the FPC8333 observations of GC RGB. Therefore, according to the discussion presented in Paper I and following the results of the statistical analysis performed by Madore & Freedman (1995), in the case of the RGB star sample observed for each individual galaxy we can compare directly the observed and predicted TRGB I luminosities, without any statistical uncertainty due to the small number of observed stars, provided that the considered star sample contains ‘real’ RGB stars belonging to the target galaxy. However, one has always to be aware of the fact that in observations of these galaxies, crowding and potential contaminants (as background galaxies, foreground stars, an AGB stellar population) have a systematic effect on the determination of $I_{TRGB}$, and that they have to be carefully treated for obtaining a reliable TRGB determination. According to Madore & Freedman (1995) the influence of background/foreground contamination can be strongly reduced when working as far out in the halo of the galaxies as possible. They give also a set of criteria and a method for deriving consistently TRGB luminosities for resolved galaxies. However, their method is not adopted in all of the TRGB observations collected in Table 2: these results come from different authors, and have been obtained by adopting different statistical procedures for determining the TRGB position.

Keeping in mind these warnings about the problems and the heterogeneity of the $I_{TRGB}$ observations, it is a safe conclusion to say that in the limit of the accuracy of the present determinations of $I_{TRGB}$ in resolved galaxies, the comparison between theoretical TRGB distance scale and Cepheid distances with the zero point set by a LMC distance modulus equal to 18.50 mag shows a statistically significant systematic difference, TRGB distances being higher.

Since the existence of this difference, we also performed a test for assessing whether it could be correlated with the metallicity of the Cepheids. Since the Madore & Freedman (1991) calibration of the P-L relation is independent of [M/H], the presence of such a correlation could be an indication of the need to correct the Cepheid distances also for metallicity effects (but see also the discussion in De Santis 1997 and Madore & Freedman 1998). For example, in a recent analysis of the P-L relation of a sample of 481 Cepheids in the LMC and SMC, Sasselov et al. (1997) derived a linear relation between the distance modulus correction ($\delta \mu$) to apply to the distances computed with a metallicity-independent calibration, and the metallicity of the Cepheid population, of the form $\delta \mu = 0.4(\text{[Fe/H]} + 0.3)$. The problem in performing this comparison is the lack of direct determinations of Cepheids metallicities for the sample of galaxies in Table 2, with the only exception of the LMC. Of course, it is not correct to use the metallicities derived from the RGB stars (reported in column 4 of Table 2), since they are typical of old stellar populations, while the Cepheids are much younger stars.

To solve this problem we have decided to adopt, as representative of the Cepheids original chemical composition, the $\text{[O/H]}$ determinations for the HII regions of the parent galaxies. We have restricted our analysis to the 7 dwarf irregular galaxies (LMC, NGC3109, NGC6822, IC1613, WLM, Sex A and Sex B) within the sample displayed in Table 2; this because the presence of metallicity gradients in the disk of spiral galaxies, while (as discussed in Skillman et al.1989) in dwarf irregular galaxies it seems that the dispersion in the $\text{[O/H]}$ abundances of HII regions is quite small. Moreover, as discussed by Luck & Lambert (1992) in the case of the LMC and SMC, the $\text{[O/H]}$ values determined from the HII regions agree well with the $\text{[O/H]}$ values determined for the Cepheids and supergiants. In the case of the LMC they provide for the Cepheids and supergiants $\text{[O/H]} = -0.70$, in good agreement with the value $\text{[O/H]} = -0.58$ as derived from the LMC HII regions by Skillman et al. (1989).

Figure 9 displays the difference $(m - M)_{TRGB} - (m - M)_{Cephid}$ (obtained from Table 2 after averaging the values corresponding to the three different sets of $BC_I$) as a function of $\text{[O/H]}$ for the Cepheids in the 7 irregular galaxies previously quoted, spanning a range of almost 1 dex in $\text{[O/H]}$. The empty circle indicates the point corresponding to Sex B if a Cepheid distance modulus of 25.82 mag is assumed (see previous discussion); the typical error on $\text{[O/H]}$ is of ±0.20 dex.

The $\text{[O/H]}$ ratios for the 7 irregular galaxies are from Skillman et al. (1989), with the exception of NGC3109 for which we used the results by Hunter & Gallagher (1985). The $\text{[O/H]}$ values are derived assuming (as in Skillman et al. 1989) $12 + \log(O/H)_{\odot} = 8.92$ (Lambert 1978). Of course, when using the quantity $\text{[O/H]}$ for obtaining the correct (qualitatively and quantitatively) ranking in metal content for the Cepheids in the galaxies in our sample, we are assuming that the $\text{[O/Fe]}$ ratio is the same in all the galaxies considered, irrespective of the $\text{[O/H]}$ absolute value.

We have performed a simple statistical analysis with the data displayed in Figure 9, computing the linear correlation coefficient $r_a$ between $(m - M)_{TRGB} - (m - M)_{Cephid}$ and $\text{[O/H]}$. To be conservative, we have chosen to accept the existence of a linear relation between these two quantities only when the probability $P$ to derive a value $\geq r_a$ from a random sample of $(m - M)_{TRGB} - (m - M)_{Cephid}$ values is less than 5%. Adopting for Sex B the Cepheid distance modulus displayed in Table 2 we find $P \approx 15\%$, while $P \approx 5\%$ if a Cepheid distance modulus of 25.82 mag is adopted. We have also performed the same test excluding Sex B from the sample; in this case we have considered only 6 galaxies, obtaining again $P \approx 15\%$. 


Due to the dependence of the result of this simple statistical analysis on the distance to Sex A, and to the quite large error bars associated to the observational points, we conclude that, in the limit of the small sample considered and of our assumptions on the Cepheids metal content, there is no clear evidence for a linear correlation between $(m - M)_{TRGB} - (m - M)_{Cepheid}$ and Cepheid metallicity.

4 THE LEO I GROUP TRGB DISTANCE AND THE VALUE OF $H_0$

Using the TRGB distance scale set by our evolutionary models, we now determine the value of $H_0$ in the same way as recently done by Tanvir et al (1995), SA97, Thomsen et al (1997), Hjorth & Tanvir (1997), Gregg (1997). This means that at first we determine the TRGB distance to the Leo I group of galaxies. This group is much more compact than, for instance, the Virgo cluster and from this point of view it is better suited for a determination of this important cosmological parameter. However, also if the Leo I group is not affected by the uncertainty in distance introduced by the depth of the Virgo cluster, it is not sufficiently far away for the local peculiar velocity field being only a small fraction of its recession velocity. Therefore we will use the Leo I distance as the zero point for the distance to the Coma cluster, that is sufficiently far away for determining $H_0$ with only a small indetermination due to the local velocity field. The distance to the Coma cluster will be determined adopting purely differential estimates (by means of secondary distance indicators) of the relative distance Leo I-Coma.

4.1 The TRGB distance to Leo I

The Leo I group consists of five dominant galaxies: NGC3351 [M95,SB(r)ab], NGC3368 [M96, SAB(rs)ab], NGC3377[E5/6], NGC3379 [E1], NGC3384 [SB(s)0]; it is relatively nearby, compact, with a full line-of-sight depth estimated to be $\approx 8\%$ compared to its distance, assuming spherical symmetry (Tanvir et al.1995). Its mean recession velocity is of the order of 700 km/s (SA97). Schneider (1989) shows a strong argument in favour of the five dominant galaxies being in close physical proximity, probably all within 0.5 Mpc or less from each other. In particular the group contains a ring of intergalactic neutral hydrogen which is orbiting around the close pair NGC3379 and NGC3384, and seems to be interacting with NGC3368.

Very recently, SA97 detected the TRGB in NGC3379, by means of HST WFPC2 observations. They placed the observed TRGB at $I=26.32\pm0.05$ mag, assumed $A_I=0.02$ mag (Burstein & Heiles 1982), a negligible internal reddening in NGC3379 (due to the location 6' west from the NGC3379 nucleus of the target field), and a metallicity $[M/H]=-0.68\pm0.40$ as derived from the $(B - I)$ color at the target field according to Sodeman & Thomsen (1994), using the relation between $[M/H]$ and $(B - I)$ by Couture et al. (1990).

By adopting the quoted values for extinction, metallicity and TRGB location, we derive a distance modulus $(m - M)_{0,3379}=30.44\pm0.13$ mag when using the DA90 $BC_1$, $(m - M)_{0,3379}=30.53\pm0.09$ mag when using Equation 6 (Yale transformations) and $(m - M)_{0,3379}=30.48\pm0.13$ mag when using Equation 5 (K97 transformations). It is worth noticing that the distance modulus obtained by adopting the DA90 $BC_1$ is 0.14 mag higher than the distance modulus derived by SA97. This difference is due exclusively to the use of the updated TRGB brightness-[M/H] relation given by Equation 1.

The error budget used in deriving the uncertainty on the distance to NGC3379 is reported in Table 4. The contributions corresponding to the different error sources are added in quadrature to obtain the total uncertainty.

At this point, by considering the difference among the three NGC3379 distance moduli as an indication of the error due to the uncertainty associated to the bolometric correction scale, we adopt a final value $(m - M)_{0,3379}=30.46\pm0.16$ mag, that spans all the range of distance moduli allowed by the three sets of $BC_1$. This distance modulus corresponds to a linear distance $d_{3379}=12.4\pm0.9$ Mpc, that is $\approx 8\%$ higher than the value derived by SA97.

It is now interesting to compare our derived distance to the Leo I group with the results from Cepheid observations. By adopting the HST observations of 7 Cepheid stars in NGC3368 by Tanvir et al. (1995), using the Madore & Freedman (1991) calibration (the same used in the previous section), and by correcting for the photometric zero point as discussed in SA97, the Cepheid distance modulus to Leo I is $(m - M)_{0,3368}=30.36\pm0.13$ mag (the error budget is derived from Tanvir et al. (1995) by excluding the contribution due to the possible systematic error on the LMC distance, since this is exactly what we are trying to determine in the present study). The difference between the best value of the distance to Leo I as derived from the Cepheids and our analogous quantity as derived from the TRGB, is in agreement with the result of the same comparison performed with the sample of 12 galaxies listed in Table 2.

More recently Graham et al. (1997) discovered, through HST observations, 49 probable Cepheids in NGC3351; their derived distance modulus (adopting the Madore & Freedman (1991) calibration) is 30.01$\pm$0.16 mag (once again, the error does not include the contribution due to the uncertainty on the distance of the LMC). This value is much lower than distance derived from the TRGB, and also lower than the NGC3368 Cepheid distance. This difference in the Cepheid distance moduli of NGC3351 and NGC3368 would imply a reciprocal distance of $\approx 1.8$ Mpc, difficult to reconcile, as already discussed by Gregg (1997), with the results by Schneider (1989). Waiting for other results about Cepheids in the Leo I group, we consider this discrepancy as a measure of the uncertainty on the present distance estimates by means of the Cepheid P-L relation.

4.2 From Leo I to Coma and the value of $H_0$

The second step toward the determination of $H_0$ is the relative distance between Leo I and Coma. According to the recent analysis by Colless & Dunn (1996), the Coma cluster is likely to consist of two components: the main cluster centered around NGC4874 and NGC4889, with a mean recession velocity $cz=6853$ km s$^{-1}$ and a subgroup around NGC4839 characterized by a mean value of $cz=7339$ km s$^{-1}$. The virial mass of the main cluster results to be around one order of magnitude higher than the subgroup mass.
Table 4. Error budget used in deriving the uncertainty on the distance to NGC3379. The three different values for the error due to the uncertainty on the metallicity correspond to the three different sets of bolometric corrections used. The errors on the galactic extinction, TRGB measurement and WFPC2 photometric zero point come from SA97.

| Source                        | Error (mag) |
|-------------------------------|-------------|
| Galactic extinction           | ±0.02       |
| TRGB measurement              | ±0.05       |
| Photometric zero point        | ±0.04       |
| [M/H] (±0.40 dex)             | ±0.09 (DA90) ±0.03 (Yale) ±0.09 (K97) |
| Y (±0.03)                     | ±0.03       |
| BC1 zero point                | ±0.02       |
| Theoretical calibration       | ±0.05       |
| Total uncertainty             | ±0.13 (DA90) ±0.09 (Yale) ±0.13 (K97) |

By adopting a relative distance between the Coma cluster main component and Leo I \((m - M)_{0,\text{Coma}} - (m - M)_{0,\text{Leo}} = 4.73 \pm 0.13\) mag as derived from the diameter-velocity dispersion data by Faber et al. (1989) and \(cz = 6853 \pm 100\) km s\(^{-1}\), as done by SA97, we obtain:

\[(m - M)_{0,\text{Coma}} = 35.19 \pm 0.21\text{mag}\]

and a linear distance \(d_{\text{Coma}} = 109 \pm 10\) Mpc (in the error budget it has been taken into account the error on the Leo I distance modulus, the uncertainty on the relative distance Coma-Leo I and an error of \(\pm 0.04\) mag that takes into account the r.m.s. depth of the Leo I group as adopted by Tanvir et al. (1995)), and finally \(H_0 = 63 \pm 6\) km s\(^{-1}\) Mpc\(^{-1}\). This value is lower by 5 Km s\(^{-1}\) Mpc\(^{-1}\) than the value derived by SA97 with the same method, the same observational data for Leo I, the same relative distance Coma-Leo I, the same recession velocity for the Coma cluster, but an old calibration of the TRGB theoretical luminosities.

However, the recession velocity for the Coma cluster, adopted by SA97, is the value given by Colless & Dunn (1996), which corresponds to the heliocentric recession velocity, not to the cosmicologic recession velocity. We have therefore transformed the heliocentric recession velocity to the centroid of the Local Group and corrected for the motion of the Local Group relative to the cosmic background radiation in the direction of Coma \((272\) km s\(^{-1}\) according to Staveley-Smith & Davies (1989), to which we attribute an error by \(\pm 100\) Km s\(^{-1}\)). Moreover, we have corrected for the peculiar motion \((V_p)\) of the cluster as estimated by Han & Mould (1992): \(V_p = +66 \pm 428\) Km s\(^{-1}\) (the median value of their three solutions for \(V_p\) has been adopted). So a cosmic recession velocity for the Coma cluster equal to \(cz = 7068 \pm 440\) km s\(^{-1}\) is finally obtained.

Moreover, we have searched in the literature for recent independent determinations of the relative distance Coma-Leo I; the most recent results are by Gregg (1997), Thomsen et al. (1997) and Hjorth & Tanvir (1997).

Gregg (1997) determined diameter-velocity dispersion relations in B,V,K bands for NGC3377, NGC3379 and NGC3384 in the Leo I group from published photometry and kinematic data. These relations, whose slopes in the three colors are in good agreement with those for 24 galaxies in the main component of the Coma cluster, yield an estimate of the Coma-Leo I distance ratio \(d_{\text{Coma}}/d_{\text{Leo}} = 8.84 \pm 0.23\). This value, coupled with our TRGB distance to the Leo I group gives a linear distance \(d_{\text{Coma}} = 110 \pm 11\) Mpc. Thomsen et al. (1997) applied the surface brightness fluctuations technique for deriving the relative distance between NGC3379 and NGC4881 in the main component of the Coma cluster. They derived \((m - M)_{0,\text{Coma}} - (m - M)_{0,\text{Leo}} = 4.89 \pm 0.30\) mag, which gives \((m - M)_{0,\text{Coma}} = 35.35 \pm 0.34\) mag and a linear distance \(d_{\text{Coma}} = 117 \pm 20\) Mpc. Hjorth & Tanvir (1997) determined a distance ratio \(d_{\text{Coma}}/d_{\text{Leo}} = 9.5 \pm 0.7\) through the construction of the fundamental plane of the Leo I group. This distance ratio provides \(d_{\text{Coma}} = 118 \pm 18\) Mpc.

Considering the cosmic recession velocity previously given: \(cz = 7068 \pm 440\) km s\(^{-1}\), these three distances give values of the Hubble constant equal to \(H_0 = 64 \pm 7, 60 \pm 11\) Km s\(^{-1}\) Mpc\(^{-1}\) and \(60 \pm 10\) Km s\(^{-1}\) Mpc\(^{-1}\), respectively; in good agreement within each other.

5 SUMMARY AND CONCLUSIONS

The main results presented in this paper can be summarized as follows:

1) we have presented theoretical relations between \(M_{\text{tip}}\) or \(M_{\text{btip}}\) and metallicity, covering the range \(-2.35 \leq [\text{M/H}] \leq -0.28\). These relations, obtained from evolutionary stellar models computed with updated input physics, expand the metallicity range covered by the relations presented in Paper I and by the old calibration by LFM93. We use the empirical \(BC1\) values from DA90, the semiempirical \(BC1\) from the Yale transformations and the purely theoretical ones from K97 when deriving galaxies distance moduli from their TRGB I (Cousins) magnitude. A particular attention has been devoted to the correct calibration of the zero point of the bolometric correction scales.

2) the ZAHB models presented in Cassisi & Salaris (1997) have been transformed to the observational V-(B-V) plane by adopting the Yale and K97 transformations. The relations obtained adopting these two sets of transformations agree quite well;

3) the GC distance scale set by the HIPPARCOS subdwarfs agrees well, within the errors, with the distance scale set by our ZAHB models for RR Lyrae stars. At the same time TRGB and ZAHB distance scales for GC agree within the statistical uncertainties associated to the location of the TRGB in GC;

4) with the distance scale set by our ZAHB models, we have recalibrated the relation \([\text{M/H]}- (V-I)_{0.385} \text{mag}\) given in Paper I;

5) when considering stellar populations in resolved galaxies, the RR Lyrae distance scale agrees well with the TRGB; the mean difference between the two distance scales is statistically compatible with a value equal to zero;

6) when using the available observational sample of TRGB determinations in resolved galaxies, the comparison between TRGB and Cepheid distance scales suggests the revision of the Madore & Freedman (1991) zero point for the Cepheid distances, that is set by a LMC distance modulus equal to 18.50 mag. In particular, we find that TRGB distances are on average 0.12 mag larger than the ones determined from
that this difference is linearly correlated with the metallicity of the Cepheid population;

7) we have determined the distance to NGC3379 in the Leo I group by using the observational TRGB data by SA97 and our theoretical TRGB models. We obtain a distance of 12.4±0.9 Mpc, that is 8% higher than the value obtained by SA97 using the old LFM93 calibration of the theoretical TRGB luminosities;

8) we have used the most recent determinations of the relative distance between Leo I and Coma cluster from Gregg (1997), Thomsen et al. (1997) and Hjorth & Tanvir (1997), obtaining $d_{\text{Coma}}=110\pm11$, $117\pm20$ and $118\pm18$ Mpc, respectively;

9) from these Coma cluster distances by assuming a cosmologic recession velocity for the main component of the cluster $cz=7068\pm440$ km s$^{-1}$, we obtain: $H_0=64\pm7$, $60\pm11$ and $60\pm10$ km s$^{-1}$ Mpc$^{-1}$, respectively. The final error on $H_0$ is computed by taking into account many different sources of errors in the evaluation of the Leo I and Coma distance moduli, and the uncertainty on the Coma recession velocity.

These results indicate clearly that the use of the TRGB as a primary distance indicator, calibrated by means of updated stellar models, provides a distance scale systematically lower in comparison with the determinations obtained by adopting the older TRGB calibration by LFM93 or the Cepheid distance scale by Madore & Freedman (1991). Moreover, the agreement between TRGB, ZAHB and HIPPARCOS subdwarfs distances comfortably assesses the reliability of our theoretical models.

What are the implications of these $H_0$ values for the age of the universe?

To answer to this question we have to assume a value for the mass density parameter $\Omega = \Omega_m + \Omega_\Lambda$, where the two terms take into account respectively the contribution of the mass density and of the vacuum energy density. Independently of the constraints from the simple standard flat inflationary models with $\Omega = \Omega_m = 1$, the values of $\Omega_m$ as observationally determined adopting different methods suggest $0.3 \leq \Omega_m \leq 1.0$ (see, e.g., Bartelmann et al. 1998, Dekel 1997, Steigman et al. 1997, Perlmutter et al. 1997). Assuming conservatively $\Omega_\Lambda = 0$, and considering $H_0=60\pm11$ that reproduces all the range of values for the Hubble constant previously given, we can consider the two extreme cases:

i) flat universe with $\Omega_m = 1$; $H_0=60\pm11$ km s$^{-1}$ Mpc$^{-1}$ provides an age of the universe $t \approx 11\pm2.5$ Gyr;

ii) open universe with $\Omega_m = 0.3$; $H_0=60\pm11$ km s$^{-1}$ Mpc$^{-1}$ gives $t \approx 13\pm2.5$ Gyr. The use of a cosmological constant different from zero increases in both cases the age of the universe with respect to the quoted values.

These results nicely agree with the most recent determinations of the GC ages, also in the case of the most restrictive constraint $\Omega_m = 1$, $\Omega_\Lambda = 0$. For instance, Salaris et al. (1997) and Salaris & Weiss (1997) find for the older GC $t_{GC} \approx 12\pm1$ Gyr, while Gratton et al. (1997) and Chaboyer et al. (1997) obtain respectively $t_{GC} \approx 11.8\pm2$ Gyr and $t_{GC} \approx 11.5\pm1.3$ Gyr.

This concordance between age of the GC and age of the universe as derived by adopting a spectrum of reasonable choices of $\Omega$, suggests that the long-standing conflict between the Hubble age and GC ages is resolved when adopting updated stellar models for deriving the GC ages and for calibrating the TRGB distance scale.

ACKNOWLEDGMENTS

It is a pleasure to thank S.Schindler for illuminating discussions about the properties of clusters of galaxies, M. Groenewegem for supplying his results about the LMC distance modulus in advance of publication, and for useful suggestions about the distance scale problem, Matthias Bartelmann and Doris Neumann for discussions and useful references about recent determinations of $\Omega$. A. Weiss is warmly thanked for encouragement and fruitful discussions during all the development of this work. We warmly thank G. Bono, for reading a preliminary version of this paper and for his helpful suggestions. We are also grateful to F. Castelli for many interesting discussions and for providing updated bolometric corrections and color transformations. The referee, N. Tanvir, is warmly thanked for many useful and pertinent remarks that improved the quality of the paper.

The work of one of us (M.S.) was carried out as part of the TMR Programme (Marie Curie Research Training Grants) financed by the EC.

REFERENCES

Alonso A., Arribas S. & Martinez-Roger C 1995, A&A, 297, 197
Bartelmann M., Huss A., Colberg J.M., Jenkins A. & Pearce F.R. 1998, A&A, 330, 1
Burstein D. & Heiles C. 1982, AJ, 87, 1165
Burstein D. & Heiles C. 1984, ApJS, 54, 33
Caloi V., D’Antona F. & Mazzitelli I. 1997, A&A, 320, 823
Cassisi S., Castellani V., Degl’Innocenti S. & Weiss A. 1997, A&A, in press
Castelli F & Kurucz R.L. 1994, A&A, 281, 817
Castelli F., Gratton R.G. & Kurucz R.L. 1997a, A&A, 318, 841
Castelli F., Gratton R.G. & Kurucz R.L. 1997b, A&A, 324, 432
Chaboyer B., Demarque P., Kernan P.J. & Krauss L.M. 1997, ApJ, submitted to
Code A.D., Davis J., Bless R.C. & Hanbury Brown R. 1976, ApJ, 203, 417
Colless M. & Dunn A.M. 1996, ApJ, 458, 435
Coute J., Harris W.E. & Alright J.W.B. 1990, ApJ, 73, 671
De Costa G.S. & Armandroff T.E. 1990, AJ, 100, 162
De Santis R. 1997, in 'Galaxy Scaling Relations: Origins, Evolution and Applications', ed. L. Da Costa (Springer) in press
Dekel A. 1997, in Press
Dekel A. 1997, in 'Galaxy Scaling Relations: Origins, Evolution and Applications', ed. L. Da Costa (Springer) in press
De Santis R. 1996, A&A, 306, 755
De Santis R. 1997, A&A, submitted to
Elson R. A. W. 1996, MNARS, 286, 732
Faber S.M. et al.1989, ApJS, 69, 763
Feast M.W. & Catchpole R.M. 1997, MNARS, 286, L1
Frogel J.A., Persson S.E. & Cohen J.G. 1983, ApJS, 53, 713
Gratton R.G., Fusi Pecci F., Carretta E., Clementini G., Corsi C.E. & Lattanzio M. 1997, ApJ, 491, 749
Green E.M. 1988, in 'Calibration of Stellar Ages', A.G. Davis (ed. (L. Davis Press) p. 81
Gregg M.D. 1997, New Astronomy, 1, 363

Hubble constant from the TRGB distances

13

...
Han M. & Mould J. 1992, ApJ 396, 453
Hayes D.S., IAU Symp. 111, p.225
Hunter D.A. & Gallagher J.S. 1985, ApJS, 58, 533
Hjorth J. & Tanvir N.R. 1997, ApJ, 482, 68
Kent S.M. 1985, PASP, 97, 165
Kurucz R.L. 1979, ApJS, 40, 1
Lambert D.L. 1978, MNRAS, 182, 249
Lee M.G., Freedman W. & Madore B.F. 1993, ApJ, 417, 553
Lee Y.-W., Demarque P. & Zinn R. 1990, ApJ, 350, 155
Luck R.E. & Lambert D.L. 1992, ApJS, 79, 303
Madore B. F. & Freedman W.L. 1991, PASP, 103, 933
Madore B. F. & Freedman W.L. 1995, AJ, 109, 1645
Madore B. F. & Freedman W.L. 1998, ApJ, 492, 110
Musella I., Piotto G. & Capaccioli M. 1997, AJ, 114, 976
Oudumaier R.D., Groenewegen M.A.T. & Schrijver H. 1997, MNRAS, in press
Peimbert M. 1993, Rev. Mex. Astr. Astrofis., 27, 9
Perlmutter S. et al.1997, ApJ, 483, 565
Reid I.N. 1997, AJ, 114, 161
Sakai S., Madore B.F. & Freedman W.L. 1996, ApJ, 461, 713
Sakai S., Madore B.F., Freedman W.L., Lauer T.R., Ajhar E.A.,
Baum W.A. 1997a, ApJ, 478, 49
Sakai S., Madore B.F. & Freedman W.L. 1997b, ApJ, 480, 589
Salaris M. & Cassisi S. 1996, A&A, 305, 858
Salaris M. & Cassisi S. 1997, MNRAS, 289, 406
Salaris M. & Weiss A. 1997, A&A 327, 107
Salaris M. & Weiss A. 1998, in preparation
Salaris M., Chieffi A. & Straniero O. 1993, ApJ, 414, 580
Sasselov D.D. et al. 1997, A&A, 324, 471
Schneider S.E. 1989, ApJ 343, 94
Skillman E.D., Kennicutt R.C. & Hodge P.W. 1989, ApJ, 347, 875
Sodeman M. & Thomsen B. 1994, A&A, 292, 425
Soria R. et al. 1996, ApJ, 465, 79
Staveley-Smith L. & Davies R.D. 1989, MNRAS, 241, 787
Steigman G., Hata N. & Felten J.E. 1997, preprint astro-ph/9708014
Straniero O., Chieffi A. & Limongi M. 1997, ApJ, 490, 425
Sweigart A. V. & Gross P. G. 1978, ApJS, 36, 405
Tanvir N.R., Shanks T., Ferguson H.C. & Robinson D.R.T.
1995, Nature, 377, 27
Thomsen B., Baum W.A., Hammergren M. & Worthey G. 1997, ApJ, 483, L37
Vandenbergh D.A. & Bell R.A. 1985, ApJS, 58, 561
Wheeler J.C., Sneden C. & Truran J. W. 1989, ARAA, 252, 179
Whiteoek P.A., van Leeuwen F. & Feast M.W. 1997, preprint astro-ph/9706096

This paper has been produced using the Blackwell Scientific Publications \TeX macros.
heavy solid – present work

$M_{\text{bol}}(\text{TRGB})$

Da Costa & Armandroff 90

△ Cassisi et al. 97 – diffusion

▲ Cassisi et al. 97 – standard

□ Caloi et al. 97

■ Straniero et al. 97

$[\text{M/H}]$
NGC6752

$[M/H]=-1.28$

$(m-M)_I=13.22$

$E(V-I)=0.05$

NGC6397

$[M/H]=-1.70$

$(m-M)_I=12.27$

$E(V-I)=0.23$
\( \Delta (m-M)_{0,\text{TRGB}} \)

- \( (m-M)_0(\text{DA90}) - (m-M)_0(\text{Yale transf.}) \)
- \( (m-M)_0(\text{DA90}) - (m-M)_0(\text{K97 transf.}) \)
