

\textbf{K-Nearest Neighbor Classification Using Anatomized Data}

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\textbf{Abstract—}This paper analyzes \textit{k} nearest neighbor classification with training data anonymized using \textit{anatomy}. Anatomy preserves all data values, but introduces uncertainty in the mapping between identifying and sensitive values. We first study the theoretical effect of the anonymized training data on the \textit{k}\ nearest neighbor error rate bounds, nearest neighbor convergence rate, and Bayesian error. We then validate the derived bounds empirically. We show that 1) Learning from anonymized data approaches the limits of learning through the unprotected data (although requiring larger training data), and 2) nearest neighbor using anonymized data outperforms nearest neighbor on generalization-based anonymization.

\section{I. INTRODUCTION}

Data publishing without revealing sensitive information is an important problem. Many privacy definitions have been proposed based on generalizing/suppressing data (\textit{l-diversity} \cite{27}, \textit{k}-anonymity \cite{31}, \cite{32}, \textit{t-closeness} \cite{24}, \textit{δ}-presence \cite{30}, \textit{(α,k)}-anonymity \cite{36}). Other alternatives include value swapping \cite{29}, distortion \cite{2}, randomization \cite{14}, and noise addition (e.g., differential privacy \cite{13}). Generalization consists of replacing identifying attribute values with a less specific version \cite{6}. Suppression can be viewed as the ultimate generalization, replacing the identifying value with an “any” value \cite{6}. These approaches have the advantage of preserving truth, but a less specific truth that reduces the utility of the published data.

Xiao and Tao proposed anonymization as a method to enforce \textit{l}-diversity while preserving specific data values \cite{37}. Anatomization splits instances across two tables, one containing identifying information and the other containing private information. The more general approach of fragmentation \cite{7} divides a given dataset’s attributes into two sets of attributes (2 partitions) such that an encryption mechanism avoids associations between two different small partitions. Vimercati et al. extend fragmentation to multiple partitions \cite{11}, and Tamas et al. propose an extension that deals with multiple sensitive attributes \cite{19}. The main advantage of anonymization/fragmentation is that it preserves the original values of data; the uncertainty is only in the mapping between individuals and sensitive values.

We show that this additional information has real value. First, we demonstrate that in theory, learning from anonymized data can be as good as learning from the raw data. We then demonstrate empirically that learning from anonymized data beats learning from generalization-based anonymization.

This paper looks only at instance-based learning, specifically non-parametric \textit{k} nearest neighbor classifier (\textit{k}-\textit{NN}). This focus was chosen because we have solid theoretical results on the limits of learning, allowing us to compare theoretical bounds on learning from anonymized data with learning from the underlying unprotected data. We demonstrate this for a simple approach of using the anonymized data; we simply consider all possible mappings of individuals to sensitive values as equally likely.

There is concern that anonymization is vulnerable to several attacks \cite{8}, \cite{35}. While this can be an issue, any method that provides meaningful utility fails to provide perfect privacy against a sufficiently strong adversary \cite{13}, \cite{25}. Introducing uncertainty into the anonymization process reduces the risk of many attacks, e.g, minimality \cite{18}, \cite{35}. Our theoretical analysis holds for any assignment of items to anatomy groups, including a random assignment, which provides a high degree of robustness against minimality and correlation-based attacks.

This paper has the following key contributions:

1) We define a classification task on anonymized data without violating the random worlds assumption. A violating classification task would be the prediction of sensitive attribute, a task that was found to be #P-complete by Kifer \cite{23}.
2) To our best knowledge, this is the first paper in the privacy community that studies the theoretical effect of training the \textit{k}-\textit{NN} on anonymized data. We show the anonymization effect for the error rate bounds and the convergence rate when the test data is neither anonymized nor anonymized. Inan et al. already gives a practical applications of such a learning scenario \cite{21}.
3) We show the Bayesian error estimation for any non-parametric classifier using the anonymized training data.
4) We compare the \textit{k}-\textit{NN} classifier trained on the anonymized data with the \textit{k}-\textit{NN} classifier trained on the unprotected data. In case of nearest neighbor classifier (1-\textit{NN}), we also make an additional comparison to generalization based learning scheme \cite{21}.
5) We last compare the theoretical estimation of convergence rate with the practical measurements when the
convergence rate is defined in function of $l$-diversity.

We next summarize the related work, and give a set of definitions and notations necessary for further discussion. Section IV shows error rate bounds of the non-parametric $k$-NN classifier; Section V analyzes the effect of anatomization on the Bayesian error. Section VII formulates the 1-NN convergence rate under $l$-diversity. The experimental analysis is presented in Section VII.

II. RELATED WORK

There have been studies in how to mine anonymized data. Nearest neighbor classification using generalized data was investigated by Martin. Nested generalization and non-nested hyperrectangles were used to generalize the data from which the nearest neighbor classifiers were trained [28]. Inan et al. proposed nearest neighbor and support vector machine classifiers using anonymized training data that satisfy $k$-anonymity. Taylor approximation was used to estimate the Euclidean distance from the anonymized training data [21]. Zhang et al. studied Naive Bayes using partially specified training data [38], proposing a conditional likelihoods computation algorithm exploring the instance space of attribute-value generalization taxonomies. Agrawal et al. proposed an iterative distribution reconstruction algorithm for the distorted training data from which a C4.5 decision tree classifier was trained. Iyengar suggested using a classification metric so as to find the optimum generalization. Then, a C4.5 decision tree classifier was trained from the optimally generalized training data [22]. Fung et al. gave a top-down specialization method (TDS) for anonymization so that the anonymized data allows accurate decision trees. A new scoring function was proposed for the calculation of decision tree splits from the compressed training data [18]. Dowd et al. studied C4.5 decision tree learning from training data perturbed by random substitutions. A matrix based distribution reconstruction algorithm was applied on the perturbed training data from which an accurate C4.5 decision tree classifier was learned [12].

None of the earlier work has provided a method directly applicable to anatomized training data. A classifier using the anonymized training data requires specific theoretical and experimental analysis, because anonymized training data provides additional detail that has the potential to improve learning; but also additional uncertainty that must be dealt with. Furthermore, previous work didn’t justify theoretically why the proposed heuristics work in empirically.

III. DEFINITIONS AND NOTATIONS

In this section, the first four definitions will recall the standard definitions of unprotected data and attribute types.

Definition 1: A dataset $D$ is called a person specific dataset for population $P$ if each instance $X \in D$ belongs to a unique individual $p \in P$.

The person specific data will be called the training data in this paper. Next, we will give the first type of attributes.

Definition 2: A set of attributes are called direct identifying attributes if they let an adversary associate an instance $X \in D$ to a unique individual $p \in P$ without any background knowledge.

Definition 3: A set of attributes are called quasi-identifying attributes if there is background knowledge available to the adversary that associates the quasi-identifying attributes with a unique individual $p \in P$.

We include both direct and quasi-identifying attributes under the name identifying attribute. First name, last name and social security number (SSN) are common examples of direct identifying attributes. Some common examples of quasi-identifying attributes are age, postal code, and occupation. Next, we will give the second type of attribute.

Definition 4: An attribute of instance $X \in D$ is called a sensitive attribute if it must be protected against adversaries from incorrectly inferring the value for an individual.

Patient disease and individual income are common examples of sensitive attributes. Unique individuals $p \in P$ typically don’t want these sensitive information to be publicly known when a dataset $D$ is released to public. Provided an instance $X \in D$, the class label is denoted by $X.C$. We don’t consider the case where $C$ is sensitive, as this would make the purpose of classification to violate privacy. Typically $C$ is neither sensitive nor identifying, although the analysis holds for $C$ being an identifying attribute.

Given the former definitions, we will next define the anonymized training data following the definition of $k$-anonymity [32].

Definition 5: A training data $D$ that satisfies the following conditions is said to be anonymized training data $D_k$: 

1) The training data $D_k$ does not contain any unique identifying attributes.
2) Every instance $X \in D_k$ is indistinguishable from at least $(k-1)$ other instances in $D_k$ with respect to its quasi-identifying attributes.

In this paper, we assume that the anonymized training data $D_k$ is created according to a generalization based data publishing method. We next define the comparison baseline classifiers.

Definition 6: A non-parametric $k$ nearest neighbor ($k$-NN) classifier that is trained on the anonymized training data $D_k$ is called the anonymized $k$-NN classifier.

Definition 7: A non-parametric $k$-NN classifier that is trained on the training data $D$ is called the original $k$-NN classifier.

The anonymized $k$-NN classifier will just be the comparison baseline in the evaluation and its theoretical discussion will not be included. We go further, requiring that there must be multiple possible sensitive values that could be linked to an individual. This requires the definition of groups [27].

Definition 8: A group $G_j$ is a subset of instances in training data $D$ such that $D = \bigcup_{j=1}^{m} G_j$, and for any pair $(G_{j1}, G_{j2})$ where $1 \leq j_1 \neq j_2 \leq m$, $G_{j1} \cap G_{j2} = \emptyset$.

Next, we define the concept of $l$-diversity or $l$-diverse given the former group definition.

Definition 9: A set of groups is said to be $l$-diverse if and
where $y$ yields the

In the theoretical analysis, we assume that all the training data

A non-parametric $k$-NN classifier is known to fit well on discrete

If $d$ is even will be ignored, because such cases include the tie

Then an instance of the form:

For the sake of simplicity, $A_{id}$ will denote the identifying attributes $A_1 \cdots A_d \in IT$. $T$ stands for a test data which is not processed by any anonymization and generalization method. $X$ will be an instance of the test data $T$. $d(U, V)$ is the quadratic distance metric for a pair of instances $U$ and $V$ in metric space $M$. $X'_{Nl}(k)$ denotes the set of $k$ number of nearest neighbors of $X$ in $D$ that the original $k$-NN classifier uses while $X'_{Nl}(k)$ denotes the set of $k$ number of nearest neighbors of $X$ in $D_A$ that the anatomized $k$-NN classifier uses. $X_i$ will interchangeably be an instance of $D$ or $D_A$ and $X_j$ will interchangeably be an instance of $X'_{Nl}(k)$ or $X'_{Nl}(k)$. In case of $k = 1$, we will use $X'_{Nl}$ and $X'_l$ for the nearest neighbors in $D$ and $D_A$. $X$ is the random variable with probability distribution $P(X)$ from which $X_i$ and $X_j$ are drawn. Training and test instances will be column vectors in format of $(A_1, ..., A_d, A_j)$. $C$ is the class attribute in $D$ and $D_A$ with binary labels 1 and 2. Given the training data $D$ and the class label $i$, $q_i(X)$, $P_i(X)$ and $P_i$ stand for the posterior probability, the likelihood probability and the prior probability respectively. If the anatomized training data $D_A$ is used, $q_{A_i}(X)$, $P_{A_i}(X)$ and $P_{A_i}$ are the symmetric definitions for the class label $i$. $R(X'_{Nl}(k), X)$ is the error rate when $X \in T$ is classified using $X'_{Nl}(k)$. If $X'_{Nl}(k)$ is used to classify $X$, $R_A(X'_{Nl}(k), X)$ will be the error rate. When $X_j \cong X$ hold for all $X_j \in X'_{Nl}(k)$, we denote the error rate by $R^k(X)$ in Equation 1 [15].

$$R^k(X) = \sum_{i=1}^{k+1/2} \frac{1}{i} \left(\frac{2i-2}{i-1}\right)q_i(X)q_2(X)^i \quad \text{Eqn. 1}$$

$R^k_A(X)$ is the error rate when $X_j \cong X$ hold for all $X_j \in X'_{Nl}(k)$. $R^k_A(X)$ can trivially be derived from Eqn. 1 by substituting $q_i(X)$ with $q_{A_i}(X)$. The Bayesian errors given $X$ are denoted by $R^*(X)$ and $R^*_A(X)$ when $X_j \cong X$ holds for all $X_j \in X'_{Nl}(k)$ and $X_j \in X'_{Nl}(k)$. Eqn. 2 computes $R^*_A(X)$ [15].

$$R^*(X) = \min\{q_1(X), q_2(X)\} \approx \sum_{i=1}^{\infty} \frac{1}{i} \left(\frac{2i-2}{i-1}\right)q_1(X)q_2(X)^i \quad \text{Eqn. 2}$$
training data and anatomized training data respectively. We will denote \( R^3(X) \) and \( R^3_A(X) \) by \( R(X) \) and \( R_A(X) \) for convenience. Similarly, \( R \) and \( R_A \) will denote \( R^1 \) and \( R^1_A \). Further notations and definitions will be given in the paper if necessary.

IV. ERROR BOUNDS OF ANATOMIZED k-NN

In this section, we will first show the error bounds for the anatomized 1-NN classifier. We will then discuss the extension to the anatomized k-NN classifier for all odd \( k > 1 \). We give only proof sketches due to space limitations.

We first give Corollary 1 which is critical for the error bounds of the anatomized 1-NN classifier.

**Corollary 1: Convergence of the nearest neighbor in the anatomized training data** Let \( X \in T \) and \( X_1, \ldots, X_N \in D_A \) be i.i.d. instances taking values separable in any metric space \( M \subset \mathbb{R}^{d+1} \). Let \( X_N \) be the nearest neighbor of \( X \) in \( D_A \). Then, \( \lim_{N \to \infty} X_N = X \) with probability one.

We can intuitively say that Corollary 1 should hold for the anatomized training data \( D_A \) if it already holds for the training data \( D \). For the nearest neighbor \( X_N \) in \( D \) of \( X \), there are \( l \) instances in the anatomized training data \( D_A \) including \( X_N \) itself. Assuming very large training data size \( (N \to \infty) \), \( X_N \) must still be the closest instance to \( X \) in the anatomized training data \( D_A \). The \( l-1 \) incorrect instances are expected to remain far and \( X_N = X \) should eventually hold.

We now give a sketch of the proof or Corollary 1. Let \( S_X(r) = \{X \in M : d(X, \bar{X}) \leq r\} \) be the sphere with radius \( r > 0 \) centered at \( X \). Let’s consider that \( X \) has a sphere \( S_X(r) \) with non-zero probability. Therefore, for any radius \( \delta > 0 \) and any fixed \( l \geq 0 \);

\[
P\{\underset{i=1,\ldots,N}{\min} d(X_i, X) \geq \delta\} = [1 - P(S_X(\delta))]^N \leq \lim_{N \to \infty} [(1 - \rho(S_X(\delta))]^N = 0
\]

Since \( d(X_i, X) \) is monotonically decreasing in terms of \( i \) for all \( X_i \in D_A \), we can conclude that \( \lim_{N \to \infty} X_N = X \) holds with probability 1. The rest of proof follows the denseness of the set \( \mathbb{Q} \) in the set \( \mathbb{R} \) according to Cover et al. [9].

Next, Theorem 1 shows the error bounds of the anatomized 1-NN classifier using Corollary 1.

**Theorem 1: Error Rate Bounds of the anatomized 1-NN classifier** Let \( M \subset \mathbb{R}^{d+1} \) be a metric space. Let \( P_{A_1}(X) \) and \( P_{A_2}(X) \) be the likelihood probabilities of \( X \) such that \( P_A(X) = P_{A_1}P_{A_1}(X) + P_{A_2}P_{A_2}(X) \) with class priors \( P_{A_1} \) and \( P_{A_2} \). Last, let’s assume that \( X \) is either a point of non-zero probability measure or a continuity point of \( P_{A_1}(X) \) or \( P_{A_2}(X) \). Then the nearest neighbor has the probability of error \( R_A \) with the bounds

\[
R_A^s \leq R_A \leq 2R_A^s
\]

(4)

where \( R_A^s \) denotes the Bayesian error when the anatomized training data \( D_A \) is used.

We now give a sketch of proof for Theorem 1. Let \( R_A(X_{N'1}, X) \) denote the probability of error for a pair of instances \( X \in T \) and \( X_{N'1} \in D_A \). Since Corollary 1 shows that \( \lim_{N \to \infty} X_{N'1} = X \) always hold, 5 is derived from 1 by substituting \( k \) with 1 and \( q_i(X) \) with \( q_{A_i}(X) \).

\[
\lim_{N \to \infty} R_A(X_{N'1}, X) = R_A(X) = 2q_{A_1}(X)q_{A_2}(X)
\]

(5)

The rest of the derivation follows Cover et al. using 1 2 9.

Extending 4 from the anatomized 1-NN classifier to the anatomized k-NN classifier for all odd \( k > 1 \) follows the steps in Corollary 1 and Theorem 1. The key is to show that \( \lim_{N \to \infty} X_{N'1} = X \) holds for all \( X_{N'1} \in X_{N'1}(k) \). The rest is to derive an expression of \( R_k^s(X) \) as in 4 for all odd \( k > 1 \) and show that \( R_k^s(X) \) is always less than \( 2R_A^s \) and \( R_k^{s-2}(X) \). We exclude this derivation due to space limitations, but the derivation follows from the original k-NN classifier analysis in [15]. The anatomized k-NN classifier has the bound 6

\[
R_k^s \leq \cdots \leq R_A^k \leq R_A \leq 2R_A^k
\]

(6)

for all odd \( k > 1 \).

Note that the Bayesian errors \( R_A^s \) and \( R^s \) are not always same due to the \( l \)-diverse groups of the anatomization. The \( l \)-diverse groups cause new likelihood \( P_{A_1}(X) \) and eventually posterior probabilities \( q_{A_i}(X) \). \( R_A^s \) thus differ from 2 because 4 uses \( q_i(X) \) instead of \( q_{A_i}(X) \). The next section formulates this change.

V. BAYESIAN ERROR ON ANATOMIZED TRAINING DATA

Since it is impossible to know the exact Bayesian error, many Bayesian error estimation techniques were suggested [4], [10], [15]. In this section, the Bayesian error will be estimated for binary classification using Parzen density estimation. Although such estimation would be very interesting for multi-label classification, the theoretical analysis on unprotected data only covers binary classification [11]. The Parzen density estimation approach, which is easier to derive than the \( k \) nearest neighbor density estimation approach, will follow Fukunaga [15] and Fukunaga et al. [16]. Both approaches show the same behavior in terms of the Bayesian estimation that makes the discussion general enough for any non-parametric density based binary classification method [15]. We first give three axioms and a lemma.

**Axiom 1:** Given the anatomized training data \( D_A \) and the training data \( D \); let \( P_i \) and \( P_{A_i} \) be the class priors for class labels \( i = \{1, 2\} \). Then, \( P_i = P_{A_i} \) is always true.

**Axiom 2:** Let \( P_i P_i(X.A_i) = P_i P_2(X.A_i) \) and \( P_{A_i} P_{A_i}(X.A_i) = P_{A_i} P_{A_2}(X.A_i) \) be \( P(X.A_i) \) and \( P_{A_i}(X.A_i) \) respectively. Given the anatomized training data \( D_A \) and the training data \( D \); let \( P(X.A_i) \) and \( P_{A_i}(X.A_i) \) be the smooth joint densities of identifying attributes \( A_i \). Then, \( P(X.A_i) = P_{A_i}(X.A_i) \) is always true.

**Axiom 3:** Let \( P_i P_i(X.A_s) = P_i P_2(X.A_s) \) and \( P_{A_i} P_{A_i}(X.A_s) = P_{A_i} P_{A_2}(X.A_s) \) be \( P(X.A_s) \) and \( P_{A_i}(X.A_s) \) respectively. Given the anatomized training data \( D_A \) and the training data \( D \); let \( P(X.A_s) \) and
Let $P_A(X,A_i)$ be the smooth densities of sensitive attribute $A_i$. Then, $P(X,A_i) = P_A(X,A_i)$ is always true.

Axioms 1, 2 and 3 are obvious due to the following: provided a sample of size $N$ drawn from a probability distribution $P$, repeating every instance for fixed $l > 0$ times and obtaining a sample of size $Nl$ does not change the probability distribution $P$. The estimated parameters $\hat{\mu}$ and $\hat{\sigma}^2$ of distribution $P$ remain same.

Lemma 1: Given the anatomized training data $D_A$ and the training data $D$, let identifying attributes $A_{id}$ and the sensitive attribute $A_i$ be independent. Then, $P(A_i|X) = P(A_i)$ is always true under the axioms 2 and 3. Using axioms 2 and 3, the proof of lemma 1 is straightforward.

Using axioms 1-3 yield the Theorem 2: Using lemma 2, we will assume that $R_A^* = R^*$ holds asymptotically for Bayesian errors.

Theorem 2: Let $M \subset \mathbb{R}^{d+1}$ be a metric space. Let $P_{A_1}(X)$ and $P_{A_2}(X)$ be the smooth probability density functions of $X$. Let $P_1$ and $P_2$ be the class priors such that $P_A(X) = P_{A_1}P_A(X) + P_{A_2}P_A(X)$. Similarly, let $P_1(X)$ and $P_2(X)$ be the smooth probability density functions of $X$ such that $P(X) = P_1P_1(X) + P_2P_2(X)$ with class priors $P_1$ and $P_2$. Let $h_A(X) = -\ln\frac{P_{A_1}(X)}{P_{A_2}(X)}$ and $h(X) = -\ln\frac{P_1(X)}{P_2(X)}$ be the classifiers with biases $\Delta h_A(X)$ and $\Delta h(X)$ respectively. Let $t = \ln\frac{P_{A_2}}{P_{A_1}} = \ln\frac{P_2}{P_1}$ be the decision threshold with threshold bias $\Delta t$. Let $\epsilon_A > 0$ be the small change in the likelihood probabilities $P_1(X)$ which results in $P_{A_1}(X)$, $t$ be $\ln(P_1/P_2)$ and $t = t_A$ be true due to axiom 1. Therefore, we have 9 and 10 as the likelihood densities in the anatomized training data $D_A$ using lemma 1.

\[
P_{A_1}(X) = P_1(X) + \epsilon_A \\
P_{A_2}(X) = P_2(X) - \epsilon_A
\]

Using 9 and 10 in the Taylor approximations of $E\{\hat{P}_A(X)\}$ and $\text{Var}\{\hat{P}_A(X)\}$ results in the approximations of $E\{\Delta h_A(X)\}$ in 11.

\[
E\{\Delta h_A(X)\} \approx E\{h(X)\} \\
+ \epsilon_A \Delta t r^2 \frac{\alpha_1(X)}{P_1(X)} + \epsilon_t \frac{\alpha_2(X)}{P_2(X)} - \epsilon_A \Delta t r^2 \frac{\alpha_2(X)}{P_1(X)} + \epsilon_t \frac{\alpha_1(X)}{P_2(X)}
\]

and $E\{\Delta h^2_A(X)\}$ in 12.

\[
E\{\Delta h^2_A(X)\} \approx E\{h^2(X)\} \\
- \epsilon_A \Delta t r^2 \frac{\alpha_1(X)}{P_1(X)} + \epsilon_t \frac{\alpha_2(X)}{P_2(X)} \\
+ \epsilon_A \Delta t r^2 \frac{\alpha_2(X)}{P_1(X)} - \epsilon_t \frac{\alpha_1(X)}{P_2(X)}
\]

where $\epsilon_A > 0$ is the small change in the likelihood probabilities $P_1(X)$ which results in $P_{A_1}(X)$, $t$ be $\ln(P_1/P_2)$ and $t = t_A$ be true due to axiom 1. Therefore, we have 9 and 10 as the likelihood densities in the anatomized training data $D_A$ using lemma 1.

VI. ANATOMIZED 1-NN CONVERGENCE

We discuss the error rate of the anatomized 1-NN classifier when the anatomized training data $D_A$ has finite size $Nl$. We will then derive the convergence rate from the former error rate. The discussion here won’t be generalized to the anatomized $k$-NN classifier since the finite size training data performance of $k$-NN classifiers are not generalized to $k > 2$ in the pattern recognition literature 10. 11. Also, only binary classification will be considered due to space limitations.
From Theorem 3, we intuitively expect a faster convergence rate than the original 1-NN classifier’s one. For \( N \) number of instances in training data \( D \), using the anatomized training data \( D_A \) reduces the variance of any classifier’s Bayesian error estimation. Therefore, there are fewer possible models to consider for a given sample size which eventually means a faster convergence to the asymptotic result. Theorem 4 extends the analysis of Fukunaga et al. [15, 17].

**Theorem 3:** Let \( M \in \mathbb{R}^{d+1} \) be a metric space. Let \( P_{A_1}(X) = P_{A_1}P_{A_2}(X) + P_{A_2}P_{A_2}(X) \). Let \( q_{A_1}(X) \) and \( q_{A_2}(X) \) be the smooth probability density functions of \( X \). Let \( P_{A_1} \) and \( P_{A_2} \) be the class priors such that \( P_{A_1}(X) = P_{A_1}P_{A_1}(X) + P_{A_2}P_{A_2}(X) \). Let \( q_{A_1}(X) \) and \( q_{A_2}(X) \) be the smooth posterior probability densities such that \( q_{A_1}(X) + q_{A_2}(X) = 1 \) and \( Nl \to \infty \). Let \( q_{A_1}(X_{Ni}) \) and \( q_{A_2}(X_{Ni}) \) be the smooth posterior probability densities such that \( q_{A_1}(X_{Ni}) + q_{A_2}(X_{Ni}) = 1 \) and \( Nl \to \infty \). Let \( \delta > 0 \) be the difference between \( q_{A_1}(X) \) and \( q_{A_2}(X) \) for class labels \( i = \{1, 2\} \). Let \( d(X_{Ni}, X) \) be the quadratic distance with matrix \( A \) and \( \rho \) be the calculated value of \( d(X_{Ni}, X) \). Let \( R_A \) be the error rate of the anatomized 1-NN classifier when \( Nl \to \infty \). Last, let \( R_{AN} \) be the error rate of the anatomized 1-NN classifier when \( Nl \to \infty \). Then,

\[
R_{AN} \approx R_A + \beta \frac{1}{(Nl)^{\frac{d+1}{2}}} E_X \{|A| \frac{\pi(d+1)}{\pi} \text{tr} \{AB(X)\} \} \tag{13}
\]

where \( \beta \) is

\[
\beta = \frac{\Gamma(d+\frac{1}{2})\Gamma(\frac{d+1}{2}) + 1}{\pi(d+1)} \tag{14}
\]

and \( B(X) \) is

\[
B(X) = P_{A}^{\frac{d+1}{2}}(X)[q_{A_2}(X) - q_{A_1}(X)]
\times \left[ \frac{1}{2} \nabla^2 q_{A_1}(X) + P_{A}^{-1}(X) \nabla P_{A}(X) \nabla^T q_{A_1}(X) \right] \tag{15}
\]

We will give here a summary of proof. We first define \( q_{A_1}(X_{Ni}) \) in function of \( q_{A_1}(X) \pm \delta \) such that \( q_{A_1}(X_{Ni}) + q_{A_2}(X_{Ni}) = 1 \) holds. Then, \( R_{AN} \) is written in function of \( R_A \) and \( \delta \). The result is

\[
R_{AN} = R_A + E[[q_{A_2}(X) - q_{A_1}(X)]\delta] \tag{16}
\]

where \( E[[q_{A_2}(X) - q_{A_1}(X)]\delta] \) is

\[
E\{[q_{A_2}(X) - q_{A_1}(X)]\delta \} = E_X \{E_\rho [E_{X_{Ni}} \{q_{A_2}(X) - q_{A_1}(X) | \delta, \rho, X \} | X] \} \tag{17}
\]

a 3-step expectation in [17]. The rest of the proof follows Fukunaga [15]. The key deviation of the anatomized training data \( D_A \) from the training data \( D \) results from the step 2. In step 2, the nearest neighbor density estimation is done on \( Nl \) training instances instead of \( N \) training instances. Thus, the expectation with respect to \( \rho \) gives [18]

\[
E\{\rho^2\} \approx \frac{1}{P_{A}^{\frac{d+1}{2}}(X)\pi|A|^{\frac{d+1}{2}}} \frac{1}{(Nl)^{\frac{d+1}{2}}} \tag{18}
\]

Using [18] expectation with respect to \( X \) in [17] (step 3) according to Fukunaga [15] results in [13]. Table I gives a summary of theoretical analysis, including a comparison between the anatomized training data \( D_A \) and the training data \( D \).

| TABLE I: Summary of Theoretical Analysis |
|------------------------------------------|
| Training Data \( D \) | Anatomized Training Data \( D_A \) | Notations |
|------------------------|-------------------------------------|-------------|
| \( k \)-NN Error Rate Bounds | \( R^*_t \leq \cdots \leq R^*_2 \leq R \leq 2R^*_t \) | \( R^*_t \leq \cdots \leq R^*_2 \leq R_A \leq 2R^*_t \) |
| 1-NN Convergence Rate | \( O(1/N^{2/d+1}) \) | \( O(1/(Nl)^{2/d+1}) \) |
| Bayesian Error Estimation | \( R^* + a_1t^2 + a_2t^4 + a_3 t^{-(d+1)} \) | \( R^* + \epsilon_A a_2t^2 + \epsilon_A a_5t^4 - \epsilon_A a_2 t^{-(d+1)} \) |

\( R^* \): 1-NN error rate (\( D \))
\( R^*_t \): \( k \)-NN error rate (\( D \))
\( R^*_A \): Bayesian error (\( D_A \))
\( R^*_t \): \( k \)-NN error rate (\( D_A \))
\( R^*_A \): Bayesian error (\( D_A \))

**VII. EXPERIMENTS AND RESULTS**

**A. Preprocessing, Setup and Implementation**

We evaluate the anatomized \( k \)-NN classifier using cross validation on the Adult, Bank Marketing, IPUMS datasets from UCI collection [5] and on the Fatality (fars) dataset from Keel repository [3].

In the adult dataset, we predicted the income attribute. The instances with missing values were removed and features selected using the Pearson correlation filter (CfsSubsetEval) of Weka [34]. After preprocessing, we had 45222 instances with 5 attributes education, marital status, capital gain, capital loss and hours per week and the class attribute income. The other datasets were used without feature selection. In IPUMS, we predicted whether a person is veteran or not. After removing the N/A and missing values for veteran information, there were 148585 instances with 59 attributes. In the Fatality dataset, we predicted whether a person is injured or not in a car accident based on 29 attributes. Since the class attribute was non-binary in the original data, the instances with class labels “Injured_Severity_Unknown”, “Died_Prior_to_Accident” and “Unknown” were removed and the binary class values
“Injured” vs “Not_Injured” were created. The former removal resulted in 91085 instances. In the Bank Marketing dataset, we predicted whether a person replied positively or negatively to the bank’s phone marketing campaign. The dataset is used with 41188 instances and 20 attributes.

In the Adult, Bank Marketing and IPUMS datasets, education (educrec in IPUMS) was deemed sensitive whereas the remaining attributes were quasi-identifying attributes. Education had many discrete values which lets all samples satisfy $l$-diversity when $l = 2, 3$. In the Fatality dataset, “POLICE_REPORTED_ALCOHOL_INVOLVEMENT” was the sensitive attribute whereas rest of the attributes were quasi-identifying attributes. This was the only discrete attribute in the dataset other than class attribute that is not a typical quasi-identifying attribute such as state, age, zipcode.

Weka (same version of Inan et al. [21]) was used to implement the $k$-NN classifier [34]. The anonymization algorithm was implemented by us following Xiao et al. [37]. All the anonymized training data were created from identifier and sensitive tables using the merge function of R. The error rates were measured on each test fold according to the definition in Weka implementation. When we compared the anonymized 1-NN with anonymized 1-NN, we also used the same generalization hierarchies that Inan et al. used. The statistical tests following Kumar et al. are provided [33].

In Figure 1 the general trend is that anonymized 1-NN has the smallest error rates and anonymized 1-NN has the largest error rates. The average error rates for anonymized 1-NN and anonymized 1-NN classifiers are 0.3132 and 0.204 for $k = l = 2$ and 0.3132 and 0.2324 for $k = l = 3$. Meanwhile, the original 1-NN has average error rate of 0.2456. When $k = l = 2$, the anonymized 1-NN has significantly lower error rates than the original 1-NN at the confidence intervals 0.99, 0.98, 0.95, 0.9 and 0.8. When $k = l = 3$, the anonymized 1-NN has significantly lower error rates than the original 1-NN at the confidence interval 0.99. This is a surprising and an interesting result showing the practical interpretation of Theorem 2 in Section V. Theorem 2 shows that the Bayesian error of the anonymized training data $D_A$ has smaller variance term than the Bayesian error of the training data $D$. Hence, a model which is overfitted on the training data $D$ is likely to be left out in the search space if the model is trained from the anonymized training data $D_A$.

The anonymized 1-NN has significantly lower error rate than the anonymized 1-NN at the confidence intervals 0.99 and 0.98 when $k = l = 2$, and at the confidence interval 0.99 when $k = l = 3$. The results aren’t statistically significant for confidence intervals smaller than 0.95 or 0.99, as the anonymized 1-NN consistently doesn’t fit one fold’s training data. Its high error rate results in a significant increase in sample variance, reducing the statistical confidence. When we analyzed this training data, we noticed that the instance values were generalized to the root values of the generalization hierarchies which could eliminate the decision boundary in the original data. This observation emphasizes the anatomy’s advantage for keeping the original attribute values despite diversifying the sensitive attribute values within a group.

C. Anatomized $k$-NN vs. Original $k$-NN

In this section, we compare the anatomized $k$-NN classifier with the original $k$-NN classifier. The comparison doesn’t include the anonymized $k$-NN classifier because Inan et al.’s work considers only the anonymized 1-NN classifier [21]. Its extension to $k > 1$ cases is beyond the scope of this work. Although we ran the experiments for anatomized 3-NN, 5-NN, 7-NN and 9-NN classifiers on the Adult, Bank Marketing, Fatality and IPUMS datasets, we give the results on the larger Fatality and IPUMS datasets due to space limitations. We again include the cases of $l = 2$ and $l = 3$. Figure 2 plots the error rate distributions of 3-NN and 5-NN classifiers on Fatality dataset, and 7-NN and 9-NN classifiers on IPUMS data.

In the Fatality data, the anatomized 3-NN and 5-NN classifiers outperform the original 3-NN and 5-NN classifiers at the confidence intervals 0.99 and 0.98 when $l = 2$. The anatomized 5-NN classifier also outperforms the original 5-NN classifier at the confidence interval 0.95 when $l = 2$. In contrast, the original 3-NN and 5-NN classifiers outperform the anatomized 3-NN and 5-NN classifiers when $l = 3$, although not to a statistically significant level. For 3-NN classifiers, the average error rates are 0.0128, 0.0135 and
0.0132 for $D_A$ with $l = 2$, $D_A$ with $l = 3$ and original data respectively. On the other hand, the average error rates of 5-NN classifier on $D_A$ with $l = 2$, $D_A$ with $l = 3$ and original data are 0.0119, 0.0122 and 0.0122 respectively.

In the IPUMS data, the original 7-NN classifier outperforms the anatomized 7-NN classifier at the confidence intervals 0.99, 0.98, 0.95, 0.9 when $l = 2$ and $l = 3$. On the other hand, the original 9-NN classifier outperforms the anatomized 9-NN classifiers at the confidence interval 0.99 when $l = 2$ and $l = 3$. For 7-NN classifiers, the average error rates are 0.1567, 0.1586 and 0.1549 for $D_A$ with $l = 2$, $D_A$ with $l = 3$ and original data respectively. The average error rates of 9-NN classifier on $D_A$ with $l = 2$, $D_A$ with $l = 3$ and original data are 0.1552, 0.1568 and 0.1542 respectively.

In conclusion, the anatomized and original $k$-NN classifiers have similar statistically significant error rates for multiple values of $l$. These results confirm the theoretical analysis that we made in the earlier sections.

D. Convergence Behavior

We now compare the anatomized 1-NN classifier versus the original 1-NN classifier on convergence behavior. We create 5 partitions from the Adult (after preprocessing), Bank Marketing, Fatality and IPUMS datasets. Each partition is used as test data, and the remaining 4 partitions are used incrementally for training. Our objective is to show how the parameter $l$ in anatomized training data change the error rates when the training data size is increased incrementally. Figure 3 plots the average error rates for the original training data, the anatomized training data with $l = 2$, the anatomized training data with $l = 3$; and the theoretical error rate in function of the training data size.

We can’t know the asymptotical $R_A$ practically for theoretical error rates. We thus make the following estimation for the theoretical result. For each dataset, we set the $R_A$ to the minimum of the error rates in the specific dataset’s results. We then calculate the rate $\frac{1}{(Nl)^{d+1}}$ from the $N$, $d$ and $l$ values.
that we set in the experiments. Using the $R_A$ and $\frac{1}{(N/l)^{1+l}}$, we computed the respective bias and eventually the theoretical error rate according to the respective training data size and $l$.

The measured error rates in Figure 3 show a convergence that is similar to the one that theoretical error rates show. Given the largest training data size $\frac{4N}{5}$, 0.015, 0.004, 0.008 and 0.0085 are approximately the maximum deviations of measured error rates from the theoretical error rates for the Adult, Bank Marketing, Fatality and the IPUMS datasets respectively. We can also see that the convergence of error rate does not make much difference between the original data, anatomized data with $l = 2$ and the anatomized data with $l = 3$. In all types of training data, the convergence rate of 1-NN classifier is slow.

VIII. Conclusion

This work demonstrates the feasibility of $k$-NN classification using training data protected by $l$-anonymity. We show that the asymptotic error bounds are the same for anonymized data as for the original data. Perhaps surprisingly, the proposed 1-NN classifier has a faster convergence to the asymptotic error rate than the convergence of 1-NN classifier using the training data without anonymization. In addition, the analysis suggests that the Bayesian error estimation for any non-parametric classifier using the anonymized training data reduces the variance term of the Bayesian error estimation, although it is hard to define the characteristic of the bias term.

Experiments on multiple datasets confirm the theoretical convergence rates. These experiments also demonstrate that proposed $k$-NN on anonymized data approaches or even outperforms $k$-NN on original data. In particular, the experiments on well known Adult data show that 1-NN on anonymized data outperforms learning on data anonymized to the same anonymity levels using generalization.

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