Computable limits of optical multiple-access communications

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Abstract: We model the optical networks as interference multiple-access channels, place computable limits on the capacity region, and evaluate the ultimate entanglement-assisted capacity of total rate. Entanglement is shown to provide strict rate-region advantages [1]. © 2022 The Author(s)

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Introduction. Entanglement can be pre-shared as assistance to boost the communication rates, as theoretically proposed [2, 3] and experimentally demonstrated [4]. Moreover, unlimited entanglement assistance (EA) in each channel use rules out the need to entangle inputs among different channel uses in the capacity evaluation of a single-sender and single-receiver channel—their entanglement-assisted (EA)1 classical capacity is additive, avoiding the superadditivity conundrum [5]. The development of an optical network further complicates the story, as multiple senders and receivers in a network can communicate simultaneously, for example over broadcast channels and multiple-access channels (MACs). In this case, the communication limit is characterized by a trade-off capacity region among multiple users. For interference bosonic Gaussian MACs (BGMACs) that model optical networks, the classical capacity formula with and without EA are known [6, 7], whereas the capacity evaluation remains a hard problem [7, 8]. In contrast to the single-sender and single-receiver case, the problem of additivity in BGMACs is complicated by the trade-off between the different senders.

In this paper, we first formulate the interference BGMAC, then provide computable limits for classical communication with and without EA and finally provide an evaluation of the ultimate EA capacity of total rate.

Communication protocol over interference BGMACs. The interference BGMAC is illustrated in Fig. 1(a). For these channels, the input modes first interfere through a beam-splitter array then the mixed mode travels through a Bosonic Gaussian channel (BGC). Formally, $\mathcal{M}$ is a concatenation $\mathcal{M} = \Psi \circ \mathcal{B}$ of an $s$-input-one-output beam-splitter $\mathcal{B}$ and a single-mode BGC $\Psi$. Upon the input modes $\hat{a}_1, \ldots, \hat{a}_s$ from the $s$ senders, the MAC $\mathcal{M}$ first combines the modes through the beam-splitter $\mathcal{B}$ to produce a mixture mode $\hat{a}_{\text{mix}} = \sum_{k=1}^s w_k |\hat{w}\rangle |\hat{a}_k\rangle$, while all other ports of the beam-splitter array are discarded, then $\hat{a}_{\text{mix}}$ goes through the single-mode BGC $\Psi$. We define the power interference ratios as $\{\eta_k = |w_k|^2/|\hat{w}|^2\}_{k=1}^s$, which sum to unity. Interference BGMACs can be classified into four fundamental classes according to the transmissivity $|\hat{w}|$ of $\Psi$, as discussed in Ref. [1].

As shown in Fig. 1(b), to communicate via the MAC $\mathcal{M}$, the $s \geq 1$ senders encode the messages on quantum systems $A_1, \ldots, A_s$ and send them to a common receiver. The common receiver measures the quantum system $B$ to decode all messages. The decoding suffers from not only the environment noise but also the interference between the multiple senders. An unassisted protocol has access to merely the contaminated output, marked in red. An EA communication protocol incorporates an EA system $A'_k$, marked in blue, pre-shared to the receiver for each sender $k$ to improve the communication rate. Specifically, the single sender ($s = 1$) case of a BGMAC reduces to a point-to-point BGC. The detailed formulae are given in Ref. [1].

For all the four classes of $\Psi$, we can respectively derive the coherent-state capacity region and the outer bounds for the unassisted case. Consider the energy constraint $\langle \hat{a}_k | \hat{a}_k \rangle \leq N_{SB}, 1 \leq k \leq s$, and dark count $N_B$ due to thermal noise. We obtain the coherent-state capacity region specified by the total communication rate of all possible groups of senders $J \subseteq \{1, \ldots, s\}$.

1We use ‘EA’ for both ‘entanglement assistance’ and ‘entanglement-assisted’.

Fig. 1: Schematic of (a) an interference BGMAC and (b) entanglement-assisted communication through quantum multiple-access channel. See also Fig. 2 in Ref. [7].
where \( g(x) \) is the von Neumann entropy of a thermal state of mean photon number \( x \). Also the outer bounds for the rates \( \{ R_k \}_{k=1}^s \)

\[
R_k \leq g(\sum_{k=1}^s |w|^2 N_{S,k} + N_B) - g(N_B), \quad 1 \leq k \leq s, \tag{2}
\]

\[
\sum_{k=1}^s R_k \leq g(\sum_{k=1}^s \eta_k |w|^2 N_{S,k} + N_B) - g(N_B). \tag{3}
\]

The outer-bound region is a more computation-friendly benchmark than the exact formula in Ref. [6]. Any EA protocol surpassing the outer-bound region possesses a provable advantage over all unassisted protocols. Incidentally, for the total rate \( J = U \equiv \{ 1, 2, \ldots, s \} \), we find that the coherent-state capacity achieves the outer bound.

Similar to the unassisted case, we have also obtained the outer bound for the EA case [1]. Here we show the evaluation of the ultimate capacity (of total rate). The evaluation is allowed for by Theorem 2 in Ref. [1], which states that the EA classical capacity of total rate \( C_{E,U}(N) \) over an \( s \)-sender phase-insensitive BGMAC (which includes interference BGMAC) \( \mathcal{N} \) is additive and achieved by an \( s \)-partite two-mode squeezed vacuum (TMSV) state. Note that this capacity differs from the capacity of a point-to-point BGC, because here the encoding is more constrained as the \( s \) senders are forbidden to collaborate. The above theorem immediately circumvents most computation complexity due to super-additivity and non-Gaussianity and enables numerical evaluation. As shown in Fig. 2(a), we see a logarithmic EA advantage in the total rate, similar to the point-to-point communication over thermal-loss channel [3]. Surprisingly, the conjugate amplifier BGMAC yields a much larger advantage than the others, which see a logarithmic EA advantage in the total rate, similar to the point-to-point communication over thermal-loss the two-mode squeezed vacuum (TMSV) state. Note that this

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N = \text{BGMAC (conjugate amplifier)}
\]

\[
N = \text{BGMAC (sender phase-insensitive BGMAC (which includes interfer-}
\]

\[
C_{coh,J} = g \left[ \sum_{k=1}^s |w|^2 \eta_k N_{S,k} + N_B \right] - g(N_B), \tag{1}
\]

\[
\text{benchmark than the exact formula in Ref. [6]. Any EA protocol surpassing the outer-bound region possesses a provable advantage over all unassisted protocols. Incidentally, for the total rate}
\]

\[
J = \text{U = \{1, 2, 3\}. Colored from blue to red for \( \Psi \) being the thermal-loss channel, the AWGN channel, the amplifier channel, and the conjugate amplifier channel.}
\]

\[
N_{S1} = N_S/2, N_{S2} = N_S/3, N_{S3} = N_S/6, \eta_1 = 1/2, \eta_2 = 1/3, \eta_3 = 1/6. \text{The gain/loss of} \Psi \text{ is} |w|^2 = 0.1, 1, 0.1, 0.1, 0.1 \text{ with dark count } N_B = 0.1 + \max\{|\langle w |^2 - 1\rangle (1 - \delta)\} = 0.1, 0.1, 0.2, 0.1 \text{ respectively. (b) Optimization of the Gaussian-state rate regions of} s = 2 \text{-sender interference BGMACs with} \Psi \text{ being a thermal-loss channel. Signal brightness} N_{S1} = 10^{-3}, N_{S2} = 2 \times 10^{-3}, \text{interference ratio} \eta_1 = 1/3, \eta_2 = 2/3, |w|^2 = 0.1, N_B = 0.1. \text{Colored from blue to red at the 1, 5, 20, 50th progress steps of the numerical optimization.}
\]

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\text{Conclusion. We prove the additivity and provide the formula of the ultimate EA capacity of total rate in an interference BGMAC, which is the theoretical model of optical networks. The corresponding optimum EA input state is an} s \text{-partite TMSV state. Meanwhile, we propose outer bounds of general unassisted interference BGMACs. Results for general phase-insensitive BGMAC can be found in Ref. [1]. Equipped with our capacity formula, we show that an EA protocol using the TMSV outperforms the upper bound of the unassisted protocols to an appreciable extent, and Gaussian entanglement has been sufficient to surpass the classical outer bounds. The additivity of the whole capacity region of phase-insensitive BGMACs is still an open question.}
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