We show that a two-mode three-level cascade laser driven by external coherent fields generate intense entangled light. It turns out that external driving fields which are at resonance with the cavity modes substantially improves the intensity of the two-mode light in the cavity in a region where the squeezing and entanglement is significant making the system under consideration a viable source of bright squeezed as well as entangled light.

I. INTRODUCTION

Generation of entanglement has recently attracted great interest as it plays a key role in quantum information processing [1–5]. Particularly, much attention has been paid to generation of continuous-variable entanglement as it might be easier to manipulate than the discrete counterparts, quantum bits, in order to perform quantum information processing. In general, degree of entanglement degrades as it interacts with the environment. On the other hand, the efficiency of quantum information processing highly depends on the degree of entanglement. As a consequence, it is desirable to generate strongly entangled continuous-variable states which can survive from the environmental noise.

Schemes for generating continuous-variable states have been realized in optical parametric oscillators [6, 7]. More recently, two-mode three level cascade lasers have been proved to be a source of macroscopic bipartite entanglement as it might be easier to manipulate than the discrete counterparts, quantum bits, in order to perform quantum information processing. In general, degree of entanglement degrades as it interacts with the environment. On the other hand, the efficiency of quantum information processing highly depends on the degree of entanglement. As a consequence, it is desirable to generate strongly entangled continuous-variable states which can survive from the environmental noise.

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We consider nondegenerate three-level cascade atoms injected in to a cavity coupled to a vacuum reservoir. The atoms are initially prepared in a coherent superposition of the top and bottom levels [14, 16] or coupling these levels by external driving fields [21, 22]. On the other hand, the effect of driving the cavity mode by external coherent light has been studied recently [19]. In the experimental setting, driving the OPO cavity with seed waves in a way to improve the intensity of the signal and idler modes which resulted from the down conversion process has been realized [22]. In this work we show that the driving coherent fields considerably enhance the mean photon number, without affecting the degree of entanglement obtained from the laser system, in a region where the squeezing and entanglement is significant. This makes the system under consideration a source of intense squeezed as well as entangled light.

We derive the pertinent master equation in the linear and adiabatic approximation schemes. The resulting master equation is used to obtain equation of evolution for the first- and second- order moments of the cavity mode operators. Using the steady state solutions of these equations we study the two-mode steady state squeezing in sum and difference fields. Moreover, applying the same solutions, we analyze the entanglement properties applying the entanglement measure proposed Duan et al. [9]. Finally, we also calculate the mean photon number of the two-mode light.

II. HAMILTONIAN AND MASTER EQUATION

We consider nondegenerate three-level cascade atoms injected in to a cavity coupled to a vacuum reservoir. The atoms are initially prepared in a coherent superposition of the top and bottom levels in order to induce atomic coherence to the system. The atoms are injected into the cavity at some constant rate $r_a$ and removed after they decay spontaneously other than the middle and intermediate levels. We assume that, the transition from the upper energy level $\ket{a}$ to the intermediate level $\ket{b}$ and from level $\ket{b}$ to the lower energy level $\ket{c}$ are taken to be resonant with the cavity modes, whereas the tran-
sition \(|a\rangle \rightarrow |c\rangle\) is dipole forbidden. In turn the cavity modes are driven by two external coherent fields having the same frequency as the respective cavity modes.

\[
\mathcal{E}(\omega_c) \quad \text{Gain medium} \quad \kappa
\]

FIG. 1: (a) Scheme of a two-mode three-level cascade laser driven by two coherent fields having the same amplitude, \(\varepsilon\) but different frequencies, \(\omega_a\) and \(\omega_b\). The gain medium is ensembles of three-level cascade atoms. (b) Three-level atom in a cascade configuration.

The interaction of the cavity modes with the external driving fields and with a single three-level cascade atom is described, in the rotating wave approximation and in the interaction picture, by the Hamiltonian

\[
\hat{H} = i\varepsilon(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}) + ig(\hat{a}^\dagger |b\rangle\langle a| + \hat{b}^\dagger |c\rangle\langle b| - |a\rangle\langle b| \hat{a} - |c\rangle\langle b| \hat{b}),
\]

where \(\varepsilon\) is the amplitude of the external driving field assumed to be the same for both fields, \(g\) is the atom-cavity mode coupling constant assumed to be the same for both transitions, \(\hat{a}\) and \(\hat{b}\) are the annihilation operators for the two cavity modes. In this work, we take the initial state of a single three-level atom to be

\[
|\Psi_A(0)\rangle = C_a|a\rangle + C_c|c\rangle
\]

and the corresponding density operator is

\[
\hat{\rho}_A(0) = \rho_{aa}^{(0)}|a\rangle\langle a| + \rho_{ac}^{(0)}|a\rangle\langle c| + \rho_{ac}^{(0)*}|c\rangle\langle a| + \rho_{cc}^{(0)}|c\rangle\langle c|,
\]

where \(\rho_{aa}^{(0)} = |C_a|^2\) and \(\rho_{cc}^{(0)} = |C_c|^2\) are respectively the probabilities for the atom to be initially in the upper and lower levels and \(\rho_{ac}^{(0)} = C_aC_c = \rho_{ac}^{(0)*}\) represents the initial atomic coherence of the atom.

We are interested in the evolution of the cavity modes only. This can be achieved by tracing the atom-cavity modes density operator over atomic variables. The time evolution of the density operator for the cavity modes \(\hat{\rho}(t)\) can, in general, be written as

\[
\frac{d}{dt} \hat{\rho}(t) = -i \text{Tr}_A[\hat{H}, \hat{\rho}_A(t)].
\]

Now employing Eqs. (1)-(3) and taking into the damping of the cavity modes by the vacuum reservoir, we obtain the master equation for the cavity modes, in the linear and adiabatic approximation schemes, to be of the form

\[
\frac{d}{dt} \hat{\rho} = \varepsilon(\hat{a}^\dagger \hat{a} - \hat{a}^\dagger \hat{a}) + \varepsilon(\hat{b}^\dagger \hat{b} - \hat{b}^\dagger \hat{b}) + \frac{A\rho_{aa}^{(0)}}{2}(2\hat{a}^\dagger \hat{a} - \hat{a} \hat{a}^\dagger - \hat{a}^\dagger \hat{a}) + \frac{\kappa}{2}(2\hat{a} \hat{a}^\dagger - \hat{a}^\dagger \hat{a} \hat{a} - \hat{a} \hat{a}^\dagger) + \frac{A\rho_{ac}^{(0)}}{2}(\hat{a} \hat{b}^\dagger + \hat{a}^\dagger \hat{b} - 2\hat{a} \hat{b}^\dagger) + \frac{A\rho_{cc}^{(0)}}{2}(\hat{b} \hat{b}^\dagger + \hat{b}^\dagger \hat{b} - 2\hat{b} \hat{b}^\dagger),
\]

where

\[
A = 2g^2 r_a / \gamma^2
\]

is the linear gain coefficient, \(\gamma\) is the spontaneous decay rate assumed to be the same for all the three levels, \(\kappa\) is the cavity mode damping constant which we assumed it to be the same for each cavity mode for convenience.

### III. SQUEEZING IN THE SUM AND DIFFERENCE FIELDS

In this section, the squeezing properties of the two-mode light in the cavity produced by the two-mode three-level cascade laser is analyzed. The squeezing properties a two-mode light can be investigated by introducing sum and difference fields of the two cavity modes and analyzing their respective quadrature variances.

We define the sum operator of the two modes as

\[
\hat{c} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{b}).
\]

The corresponding quadrature operators are

\[
\hat{c}_+ = \hat{c}^\dagger + \hat{c}, \quad \hat{c}_- = i(\hat{c}^\dagger - \hat{c})
\]

Based on these definitions of the quadrature operators the two-mode light represented by the sum operator is said to be in a two-mode squeezed state provided that either of the variances of these operators, \(\Delta c_+^2\) or \(\Delta c_-^2\), should be less than that of the vacuum level, which is unity. The variances of the quadrature operators can be expressed as

\[
\Delta c_\pm^2 = \langle c_\pm^2 \rangle - \langle c_\pm \rangle^2.
\]

We wish to calculate the quadrature variances at steady state. Thus, using the steady state solutions obtained at the Appendix, the steady state quadrature variances are found to be

\[
\Delta c_\pm^2 = \frac{A^2(1 - \eta^2) + (2\kappa + A + A\eta)(2\kappa + A\eta \pm A\sqrt{1 - \eta^2})}{2(2\kappa + A\eta)(\kappa + A\eta)}.
\]
Similarly we can define the difference operator of the two modes as

\[ \hat{d} = \frac{1}{\sqrt{2}}(\hat{a} - \hat{b}) \]

(10)

with corresponding quadrature operators

\[ \hat{d}_+ = \hat{d}^\dagger + \hat{d}, \quad \hat{d}_- = i(\hat{d}^\dagger - \hat{d}). \]

(11)

The variances of these quadrature operators are:

\[ \Delta d^2_\pm = \langle d^2_\pm \rangle - \langle d_\pm \rangle^2. \]

(12)

In the same way, the difference mode is said to be in a two-mode squeezed state if either \( \Delta d^2_+ < 1 \) or \( \Delta d^2_- > 1 \). On account of Eqs. (10) and (11) along with the steady state solutions at the Appendix, the quadrature variances of take the form

\[ \Delta d^2_\pm = \frac{A^2(1 - \eta^2) + (2\kappa + A + A\eta)(2\kappa + A\eta \mp A\sqrt{1 - \eta^2})}{2(2\kappa + A\eta)(\kappa + A\eta)} \]

(13)

where \( \eta = \rho^{(a)}_{cc} - \rho^{(a)}_{aa} \) which relates the initial probabilities for an atom to be in the upper and lower levels.

According to Eq. (9) and (13), all quadrature variances are independent of the parameter \( \varepsilon \) which represents the driving coherent fields. This shows that the driving coherent fields do not have any effect on the degree of squeezing of the two-mode light. This is due to the fact that the external coherent fields do not introduce additional coherence to the system which is believed to be the source of squeezing in three-level cascade lasers \cite{14, 16, 18, 19}. Comparing Eqs. (9) and (13), we easily to see that \( \Delta c^2_+ = \Delta d^2_- \) and \( \Delta c^2_- = \Delta d^2_+ \). It is found that exactly the same amount of squeezing exhibited in the minus quadrature of the sum mode and in the plus quadrature for the difference mode. It is however worth mentioning that this statement is valid only at steady state. In the following we explore how the two mode squeezing can be optimized by varying the three parameters in Eqs. (9) and (13). In Fig. 2, we plot \( \Delta c^2_\pm \) for different values of the linear gain coefficient.

In Fig. 3, we plot \( \Delta c^2_\pm \) for different values of the linear gain coefficient. It is possible to see from this plots that the two-mode squeezing increases with the linear gain coefficient as previously established \cite{12, 13, 19}. Moreover, the value of \( \eta \) at which the maximum squeezing occurs decreases to zero as the linear gain coefficient increases. Therefore, two-mode squeezing can be optimized by choosing small values of \( A/\kappa \).

IV. ENTANGLEMENT BETWEEN THE TWO CAVITY MODES

In this section we demonstrate a continuous-variable bipartite entanglement between the two cavity modes \( a \) and \( b \). Entanglement is solely a property of quantum mechanics which is related to the inseparability of the combined density matrix of a system into the density matrices of the individual system. Inseparability criteria for continuous variable bipartite states have been pro-
posed by various authors \[9, 24\]. In this work, we apply entanglement criterion introduced by Duan et al. \[9\] which is sufficient and necessary condition for Guassian states and sufficient, in general to detect bipartite continuous variable entanglement. According to this criterion, a quantum state of a system is said to be entangled if the sum of the variances of the EPR-like quadrature operators satisfy the inequality

\[ \Delta u^2 + \Delta v^2 < 2(z^2 + \frac{1}{z^2}), \]  

(14)

where

\[ \hat{u} = |z|\hat{x}_a + \frac{1}{z}\hat{x}_b \]  

(15)

and

\[ \hat{v} = |z|\hat{p}_a - \frac{1}{z}\hat{p}_b \]  

(16)

with \( \hat{x}_a = (\hat{a}^\dagger + \hat{a})/\sqrt{2}, \hat{x}_b = (\hat{b}^\dagger + \hat{b})/\sqrt{2}, \hat{p}_a = i(\hat{a}^\dagger - \hat{a})/\sqrt{2}, \) and \( \hat{p}_b = i(\hat{b}^\dagger - \hat{b})/\sqrt{2} \) in which \( z \) is a non-zero real number. In the following, we choose \( z = -1 \) so that the upper bound of the inequality Eq. (14) to be 2. This is not an optimal choice in general but it is sufficient to compare with other inequality. With \( z = -1 \), the quadrature operators between which we want to demonstrate bipartite entanglement are simply the squeezed quadrature operators for the sum and difference modes, \( \hat{u} = \hat{c}_- \) and \( \hat{v} = \hat{d}_+ \). We already calculated the variances of these quadrature operators in the previous section. Thus on account of Eqs. (9) and (13), the sum of the variances of \( \hat{u} \) and \( \hat{v} \) becomes

\[ \Delta u^2 + \Delta v^2 = 2\Delta c_+^2 = 2\Delta d_+^2 \]  

(17)

\[ = \frac{A^2(1 - \eta^2) + (2\kappa + A + A\eta)(2\kappa + A\eta - A\sqrt{1 - \eta^2})}{(2\kappa + A\eta)(\kappa + A\eta)} \]

We immediately notice that, this particular entanglement measure is directly related the two-mode squeezing, as previously reported elsewhere \[12, 13\]. This direct relationship shows that whenever there is a two-mode squeezing in the system there will be entanglement in the system as well. It also follows that the degree of entanglement does not depend on the external driving coherent fields. This is attributed to the fact that the coherent fields do not introduce additional atomic coherence to the system, as the same is true for the case of squeezing. In Fig. 4, we show results for the sum of the squeezed quadrature variances indicating a clear violation of the inequality and hence the entanglement between the modes. It can easily be seen that the degree of entanglement increases with the rate at which the atoms are injected into the cavity, \( A \). The entanglement however disappears at two extreme value of \( \eta \), one at \( \eta = 0 \) which corresponds to maximum injected atomic coherence, \( \rho^{(0)}_{ac} = 1/2 \) and the other at \( \eta = 1 \) corresponds to no atomic coherence, \( \rho^{(0)}_{ac} = 0 \).

FIG. 4: Plots of \( \Delta u^2 + \Delta v^2 \) of the two-mode light in the cavity as steady state versus \( \eta \) and for different values of the linear gain coefficient.

V. MEAN PHOTON NUMBER OF THE CAVITY MODES

In order to know how intense the produced light is, we calculate the total mean photon number of the cavity modes. In terms of the sum and difference operators of the cavity modes, the mean photon number can be written as

\[ \langle \hat{N} \rangle = \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{b}^\dagger \hat{b} \rangle. \]

(18)

Using the definitions of the sum and difference operators (Eqs. (6) and (10)), the mean photon number becomes

\[ \langle \hat{N} \rangle = \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{b}^\dagger \hat{b} \rangle. \]

(19)

Now, with the aid of Eqs. (A19) and (A20) the mean photon number takes the form

\[ \langle \hat{N} \rangle = \frac{A(1 - \eta)(2\kappa + A + A\eta)}{4(\kappa^2 + \kappa A\eta)} - \frac{A^3\eta(1 - \eta^2)}{4(\kappa^2 + \kappa A\eta)(2\kappa + A\eta)} \]

\[ + \frac{4\varepsilon^2[A^2(1 - \sqrt{1 - \eta^2}) + 2(\kappa^2 + \kappa A\eta)]}{(\kappa^2 + \kappa A\eta)^2}. \]

(20)

In Eq. (20) the term that contain \( \varepsilon \) represents the contribution from the external driving coherent fields to the total mean photon number. Graphically, the effect of the coherent fields on the mean photon number of the cavity modes is shown in Figs. 5 and 6. Fig. 5, indeed, clearly indicates that the mean photon number of the cavity modes increases with the amplitude of the coherent driving fields. In order to clearly see by what extent the coherent fields enhance the mean photon number over the laser system, we plotted in Fig. 6 the mean photon number versus the parameter \( \eta \) in the absence and presence of the coherent fields. It is quite interesting to note
driven by two external coherent fields at resonance with the cavity modes, where the atomic coherence is induced by initial superposition of top and bottom levels. We have demonstrated two-mode squeezing in the sum and difference fields with equal amount of squeezing in each field. In addition, we have shown that the two cavity modes are strongly entangled and the degree of entanglement is directly related to the two-mode squeezing. We also found that the effect of the external coherent fields is to increase the mean photon numbers considerably in a region where the squeezing and entanglement are strong making the system under scrutiny a viable source of a bright macroscopic squeezed as well as entangled light. The degree of squeezing and entanglement can be optimized by choosing small values of the cavity damping constant, for large values of the linear gain coefficient, and by initially preparing slightly more atoms in the lower level than in the upper level.

Appendix A: Equation of evolution of cavity mode operators

In this Appendix, we derive, applying the master equation obtained in Sec. II, the equation of evolution for the cavity mode operators. We also calculate the steady state solutions of the resulting equations. The equation of evolution of an operator $\hat{O}$ in the Schrodinger picture is expressible as

$$\frac{d}{dt} \langle \hat{O} \rangle = Tr \left( \frac{d\hat{\rho}}{dt} \hat{O} \right). \quad (A1)$$

Using the master equation, Eq. (5a) and Eq. (5b), we obtained the following equations:

$$\frac{d}{dt} \langle \hat{a} \rangle = -\frac{\mu_a}{2} \langle \hat{a} \rangle - \frac{A\rho_{ac}^{(0)}}{2} \langle \hat{b} \rangle + \varepsilon, \quad (A2)$$

$$\frac{d}{dt} \langle \hat{b} \rangle = -\frac{\mu_b}{2} \langle \hat{b} \rangle + \frac{A\rho_{ac}^{(0)}}{2} \langle \hat{a} \rangle + \varepsilon, \quad (A3)$$

$$\frac{d}{dt} \langle \hat{a}^2 \rangle = -\mu_a \langle \hat{a}^2 \rangle - A\rho_{ac}^{(0)} \langle \hat{a}\hat{b} \rangle + 2\varepsilon \langle \hat{a} \rangle, \quad (A4)$$

$$\frac{d}{dt} \langle \hat{b}^2 \rangle = -\mu_b \langle \hat{b}^2 \rangle + A\rho_{ac}^{(0)} \langle \hat{a}\hat{b} \rangle + 2\varepsilon \langle \hat{b} \rangle, \quad (A5)$$

$$\frac{d}{dt} \langle \hat{a}\hat{a} \rangle = -\mu_a \langle \hat{a}\hat{a} \rangle - A\rho_{ac}^{(0)} \langle \hat{a}\hat{b} \rangle + 2\varepsilon \langle \hat{a} \rangle + \varepsilon (\langle \hat{a} \rangle \langle \hat{a} \rangle + (\langle \hat{a} \rangle)^2)$$

$$+ \varepsilon (\langle \hat{a} \rangle + \langle \hat{a} \rangle), \quad (A6)$$

$$\frac{d}{dt} \langle \hat{b}\hat{b} \rangle = -\mu_b \langle \hat{b}\hat{b} \rangle + A\rho_{ac}^{(0)} \langle \hat{a}\hat{b} \rangle + 2\varepsilon \langle \hat{b} \rangle + \varepsilon (\langle \hat{b} \rangle + \langle \hat{b} \rangle), \quad (A7)$$
\[
\frac{d}{dt}(\hat{b}^\dagger) = -\mu(\hat{b}^\dagger) + \frac{A\rho_{ac}^{(0)}}{2}((\hat{a}^2) - (\hat{b}^1)^2) \\
+ \varepsilon((\hat{a}^\dagger) + (\hat{b}^1)), \tag{A8}
\]

\[
\frac{d}{dt}(\hat{a}b) = -\mu(\hat{a}b) + A\rho_{ac}^{(0)}((\hat{a}^\dagger \hat{a}) - (\hat{b}^1 \hat{b})) \\
+ \frac{1}{2}A\rho_{ac}^{(0)} + \varepsilon((\hat{a}) + (\hat{b})), \tag{A9}
\]
in which
\[
\mu_a = \kappa - A\rho_{aa}^{(0)}, \quad \mu_b = \kappa + A\rho_{bc}^{(0)}, \\
\mu = \frac{1}{2}[2\kappa + A(\rho_{cc}^{(0)} - \rho_{aa}^{(0)})]. \tag{A10}
\]

It proves to be convenient to introduce a parameter that relates the probability for an atom to be in the upper and lower levels, such that
\[
\rho_{aa}^{(0)} = \frac{1 - \eta}{2}, \quad \rho_{cc}^{(0)} = \frac{1 + \eta}{2} \tag{A11}
\]
and
\[
\rho_{ac}^{(0)} = \frac{1}{2}\sqrt{1 - \eta^2}. \tag{A12}
\]

Next, we calculate the steady solutions by setting the time derivatives of the cavity mode variable to zero. By simultaneously solving the steady state solutions, we find the following:
\[
\langle \hat{a} \rangle = \frac{\varepsilon[2\kappa + A(1 + \eta) - A\sqrt{1 - \eta^2}]}{\kappa^2 + \kappa A\eta}, \tag{A13}
\]
\[
\langle \hat{b} \rangle = \frac{\varepsilon[2\kappa - A(1 - \eta) - A\sqrt{1 - \eta^2}]}{\kappa^2 + \kappa A\eta}, \tag{A14}
\]
\[
\langle \hat{a}^2 \rangle = \frac{4\varepsilon^2(2\kappa + A(1 + \eta) - A\sqrt{1 - \eta^2})}{(\kappa^2 + \kappa A\eta)(2\kappa - A(1 - \eta))} \\
- \frac{\varepsilon^2 A\sqrt{1 - \eta^2}(2\kappa + A(1 + \eta))}{(\kappa^2 + \kappa A\eta)^2} \\
- \frac{\varepsilon^2 A^3(1 - \eta^2)(2 - \sqrt{1 - \eta^2})}{(2\kappa - A(1 - \eta))(\kappa^2 + \kappa A\eta)^2}, \tag{A15}
\]
\[
\langle \hat{b}^2 \rangle = \frac{4\varepsilon^2(2\kappa - A(1 - \eta) + A\sqrt{1 - \eta^2})}{(\kappa^2 + \kappa A\eta)(2\kappa + A(1 + \eta))} \\
+ \frac{\varepsilon^2 A\sqrt{1 - \eta^2}(2\kappa - A(1 - \eta))}{(\kappa^2 + \kappa A\eta)^2} \\
+ \frac{\varepsilon^2 A^3(1 - \eta^2)(2 - \sqrt{1 - \eta^2})}{(2\kappa + A(1 + \eta))(\kappa^2 + \kappa A\eta)^2}. \tag{A16}
\]
\[
\langle \hat{a}^\dagger \rangle = \frac{\varepsilon^2 (A(1 - \eta) - A^2)}{(\kappa^2 + \kappa A\eta)^2} \\
- \frac{\varepsilon^2 A\sqrt{1 - \eta^2}(2\kappa + A(1 + \eta))}{(\kappa^2 + \kappa A\eta)^2} \\
+ \frac{\varepsilon^2 A^3(1 - \eta^2)(2 - \sqrt{1 - \eta^2})}{(2\kappa + A(1 + \eta))(\kappa^2 + \kappa A\eta)^2} \tag{A17}
\]
\[
\langle \hat{b} \rangle = \frac{\kappa A(1 + \eta)(2\kappa + A(1 + \eta))}{4(\kappa^2 + \kappa A\eta)(2\kappa + A(1 + \eta))} \\
+ \frac{\varepsilon^2 A\sqrt{1 - \eta^2}(2\kappa + A(1 + \eta))}{(\kappa^2 + \kappa A\eta)^2} \\
- \frac{\varepsilon^2 A^3(1 - \eta^2)(2 - \sqrt{1 - \eta^2})}{(2\kappa + A(1 - \eta))(\kappa^2 + \kappa A\eta)^2} \tag{A18}
\]
\[
\langle \hat{b}^\dagger \rangle = \frac{\kappa A^2(1 - \eta^2)}{4(2\kappa + A\eta)(\kappa^2 + \kappa A\eta)} \\
- \frac{\varepsilon^2 A\sqrt{1 - \eta^2}(A(1 + \eta) - A\sqrt{1 - \eta^2})}{(2\kappa - A(1 - \eta))(\kappa^2 + \kappa A\eta)^2} \\
+ \frac{\varepsilon^2 A^3(1 - \eta^2)(2 - \sqrt{1 - \eta^2})}{(2\kappa + A(1 + \eta))(\kappa^2 + \kappa A\eta)^2}. \tag{A19}
\]
\[
\langle \hat{b}^2 \rangle = \frac{\kappa A^2(1 - \eta^2)}{4(2\kappa + A\eta)(\kappa^2 + \kappa A\eta)} \\
- \frac{\varepsilon^2 A\sqrt{1 - \eta^2}(A(1 + \eta) - A\sqrt{1 - \eta^2})}{(2\kappa - A(1 - \eta))(\kappa^2 + \kappa A\eta)^2} \\
+ \frac{\varepsilon^2 A^3(1 - \eta^2)(2 - \sqrt{1 - \eta^2})}{(2\kappa + A(1 + \eta))(\kappa^2 + \kappa A\eta)^2}. \tag{A20}
\]

It is worth mentioning that these solutions are obtained provided that \( \eta > 0 \).
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