Topological Black Holes of (n+1)-dimensional Einstein-Yang-Mills Gravity

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Abstract

We present the topological solutions of Einstein gravity in the presence of a non-Abelian Yang-Mills field. In \((n+1)\) dimensions, we consider the \(\text{So}(n(n-1)/2-1,1)\) semisimple group as the Yang-Mills gauge group, and introduce the black hole solutions with hyperbolic horizon. We argue that the 4-dimensional solution is exactly the same as the 4-dimensional solution of Einstein-Maxwell gravity, while the higher-dimensional solutions are new. We investigate the properties of the higher-dimensional solutions and find that these solutions in 5 dimensions have the same properties as the topological 5-dimensional solution of Einstein-Maxwell (EM) theory although the metric function in 5 dimensions is different. But in 6 and higher dimensions, the topological solutions of EYM and EM gravities with non-negative mass have different properties. First, the singularity of EYM solution does not present a naked singularity and is spacelike, while the singularity of topological Reissner-Nordstrom solution is timelike. Second, there are no extreme 6 or higher-dimensional black holes in EYM gravity with non-negative mass, while these kinds of solutions exist in EM gravity. Furthermore, EYM theory has no static asymptotically de Sitter solution with non-negative mass, while EM gravity has.

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I. INTRODUCTION

The Einstein-Yang-Mills (EYM) theory, which explains the theory of a gravitating non-Abelian gauge field, may be regarded as the most natural generalization of Einstein-Maxwell gravity. The solitonic solution of this theory in 4 dimensions has been discovered by Bartnik and McKinnon for the gauge group $SU(2)$ [1]. The colored black hole solutions of this theory with $SO(3)$ gauge group have been introduced in [2], while those with $SU(2)$ gauge group have been investigated in [3–5]. These solutions have led to certain revisions of some of the basic concepts of black hole physics based on the uniqueness and no-hair theorem. It is now well-known that this theory possesses ”hairy” black hole solutions, whose metric is not a member of the Kerr-Newmann family (see [6] for a detailed review in 4 dimensions and [7] for a recent review in higher dimensions). Furthermore, unlike the Kerr-Newmann black holes, the geometry exterior to the event horizon is not determined uniquely by global charges measurable at infinity, although only a small number of parameters are required in order to describe the metric and matter field [8]. Solutions of the EYM equations in higher dimensions have been also studied in [9, 10]. These solutions were also extended in the presence of cosmological constant [11–13] and Gauss-Bonnet term [14, 17].

From stability analysis, it turns out that the solution with zero or positive cosmological constant is unstable [18], while those with negative cosmological constant are stable [14, 19]. The presence of a negative cosmological constant also invites the topological black holes into the game. Indeed, the horizon topology of an asymptotically flat black hole should be a round sphere, while in AdS space it is possible to have a black holes with zero or negative constant curvature horizon too. These black holes are referred to as topological black holes in the literature, and investigated by many authors [20–26]. All of these investigations are mainly based on Einstein-Maxwell theory, however numerical solutions have been considered in [27–29] in the presence of $SU(2)$ Yang-Mills field. It may be of interest to generalize these topological solutions for a non-Abelian matter field and investigate their properties. Indeed, the analysis of Einstein’s equation with nonlinear field sources may shed new light on the generic properties of topological solutions of Einstein’s equations. In this paper, we want to study the $(n + 1)$-dimensional topological black hole solutions of EYM gravity with a negative cosmological constant.

The outline of the paper is as follows. We give a brief review of the field equations of
EYM gravity for a semisimple gauge group in Sec. II. In Sec. III we first present the 4-dimensional solution for gauge group $SO(2,1)$ and investigate its properties, and second we introduce the 5-dimensional solution which incorporates a logarithmic term unprecedented in other dimensions. By a similar analogy we extend these solutions in $(n+1)$ dimensions with gauge group $So(n(n-1)/2-1,1)$. We finish our paper with some concluding remarks.

II. FIELD EQUATIONS

The model which will be discussed here is an $(n+1)$-dimensional EYM system for an $N$-parameters gauge group $\mathcal{G}$, which is assumed to be at least semisimple with structure constants $C_{bc}^a$. The metric tensor of the gauge group is

$$\Gamma_{ab} = C_{ad}^c C_{bc}^d,$$

where the Latin indices $a, b, \ldots$ go from 1 to $N$, and the repeated indices is understood to be summed over. According to Cartan’s criteria the determinant of $\Gamma_{ab}$ is not zero, and therefore one may define

$$\gamma_{ab} \equiv -\frac{\Gamma_{ab}}{|\det \Gamma_{ab}|^{1/N}},$$

where $|\det \Gamma_{ab}|$ is the positive value of determinant of $\Gamma_{ab}$. The action of $(n+1)$-dimensional EYM gravity with negative cosmological constant $\Lambda = -n(n-1)/2l^2$ may be written as

$$I_{\text{EYM}} = \int d^{n+1}x \sqrt{-g} \left[ R + \frac{n(n-1)}{l^2} - \gamma_{ab} F_{\mu \nu}^{(a)} F^{(b)\mu \nu} \right], \quad (1)$$

where $R$ is the Ricci Scalar and $F_{\mu \nu}^{(a)}$ is the gauge field tensor defined as:

$$F_{\mu \nu}^{(a)} = \partial_\mu A_{\nu}^{(a)} - \partial_\nu A_{\mu}^{(a)} + \frac{1}{2e} C_{bc}^{a} A_{\mu}^{(b)} A_{\nu}^{(c)}. \quad (2)$$

In Eq. (2) $e$ is a coupling constant and $A_{\mu}^{(a)}$'s are the gauge potentials. Variation of the action (1) with respect to the spacetime metric $g_{\mu \nu}$ and the gauge potential $A_{\mu}^{(a)}$ yield the EYM equations as

$$F_{\mu \nu}^{(a)\lambda} \gamma_{\lambda \nu} = j^{(a)\mu}, \quad (3)$$

$$G_{\mu \nu} + \Lambda g_{\mu \nu} = 8\pi T_{\mu \nu}, \quad (4)$$

where the gauge current and the stress-energy tensor carried by the gauge fields are

$$j^{(a)\nu} = \frac{1}{e} C_{bc}^{a} A_{\mu}^{(b)} F_{\lambda \nu}^{(c)\mu}, \quad (5)$$

$$T_{\mu \nu} = \frac{1}{4\pi} \gamma_{ab} \left( F^{(a)\lambda}_{\mu} F_{\nu \lambda}^{(b)} - \frac{1}{4} F^{(a)\lambda \sigma}_{\mu} F_{\lambda \sigma}^{(b)\nu} g_{\mu \nu} \right), \quad (6)$$
respectively.

III. TOPOLOGICAL BLACK HOLES IN 4 AND 5 DIMENSIONS

The 4-dimensional static metric of a topological spacetime with a hyperbolic horizon may be written as
\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sinh^2 \theta d\varphi^2). \] (7)

Introducing the coordinates
\[ x_1 = r \sinh \theta \cos \varphi, \]
\[ x_2 = r \sinh \theta \sin \varphi, \]
\[ x_3 = r \cosh \theta, \]
and using the Wu-Yang ansatz \[30\], one obtains the explicit form of YM potentials as
\[ A^{(1)} = \frac{e}{r^2} (x_1 dx_3 - x_3 dx_1) = -e (\cos \varphi d\theta - \sinh \theta \cosh \theta \sin \varphi d\varphi), \]
\[ A^{(2)} = \frac{e}{r^2} (x_2 dx_3 - x_3 dx_2) = -e (\sin \varphi d\theta + \sinh \theta \cosh \theta \cos \varphi d\varphi), \]
\[ A^{(3)} = \frac{e}{r^2} (x_1 dx_2 - x_2 dx_1) = e \sinh^2 \theta d\varphi, \] (8)

which have the Lie algebra of \( SO(2,1) \) with nonzero structure constants and \( \gamma_{ab} \) as follows:
\[ C_{23}^{1} = C_{31}^{2} = -C_{12}^{3} = 1, \]
\[ \gamma_{ab} = \text{diag}(-1, -1, 1). \]

Now, it is a matter of calculation to show that the YM fields \([8]\) satisfy the YM field equation \([3]\), while the gauge currents \([5]\) don’t vanish and are
\[ j^{(1)} = \frac{e}{r^4} (\cos \varphi d\theta + \coth \theta \sin \varphi d\varphi), \]
\[ j^{(2)} = -\frac{e}{r^4} (\sin \varphi d\theta - \coth \theta \cos \varphi d\varphi), \]
\[ j^{(3)} = -\frac{e}{r^4} d\varphi, \] (9)

Here, it is worth to mention that one may perform a position-dependent gauge transformation from the gauge field \([8]\) to a set of three Abelian gauge fields which satisfy the Maxwell equation with zero current independently \([2]\). Of course the scalar
\[ F \equiv \gamma_{ab} F^{(a)\mu\nu} F_{\mu\nu}^{(b)}, \] (10)
is invariant under this transformation. In 4 dimensions, \( F = 2e^2/r^4 \) for both the solutions of YM and Maxwell equations, and therefore the solution of EYM is the same as the topological solution of Reissner-Nordstrom solution.

To find the function \( f(r) \), one may use any components of Eq. (4). The simplest equation is the \( rr \) component of these equations which can be written as

\[
[r(1 + f)]' = \frac{3}{l^2}e^2 - \frac{e^2}{r^2},
\]

where prime denotes the derivative with respect to \( r \). The solution of Eq. (11) is

\[
f(r) = -1 + \frac{r^2}{l^2} - \frac{2m}{r} + \frac{e^2}{r^2},
\]

where \( m \) is an integration constant which is related to the mass of the spacetime. Of course the above solution satisfies all the other components of the field equations. This solution has the same properties as the asymptotically AdS topological solution of EM gravity in 4 dimensions, and we do not discuss it more here. Also it is worth to mention that this solution is the counterpart of the spherical solution of EYM theory introduced in \[2\].

The 5-dimensional solution incorporates a logarithmic term unprecedented in other dimensions, and therefore we shall treat it in some details. Recently, static non-abelian black hole solutions of five-dimensional maximal (\( \mathcal{N} = 8 \)) gauged supergravity has been considered in Ref. [31]. Here, we consider the 5-dimensional, static metric with hyperbolic horizon which may be written as

\[
ds^2 = -f(r) \, dt^2 + \frac{dr^2}{f(r)} + r^2 \{ d\theta^2 + \sinh^2 \theta \left( d\varphi^2 + \sin^2 \varphi \, d\psi^2 \right) \}. \tag{13}
\]

Introducing the coordinates

\[
\begin{align*}
x_1 &= r \sinh \theta \sin \varphi \cos \psi, \\
x_2 &= r \sinh \theta \sin \varphi \sin \psi, \\
x_3 &= r \sinh \theta \cos \varphi, \\
x_4 &= r \cosh \theta,
\end{align*}
\]
and using the ansatz

\[
A^{(1)} = \frac{e}{r^2} (x_1 dx_4 - x_4 dx_1), \\
A^{(2)} = \frac{e}{r^2} (x_2 dx_4 - x_4 dx_2), \\
A^{(3)} = \frac{e}{r^2} (x_3 dx_4 - x_4 dx_3), \\
A^{(4)} = \frac{e}{r^2} (x_1 dx_2 - x_2 dx_1), \\
A^{(5)} = \frac{e}{r^2} (x_1 dx_3 - x_3 dx_1), \\
A^{(6)} = \frac{e}{r^2} (x_2 dx_3 - x_3 dx_2),
\]

(14)

one obtains:

\[
A^{(1)} = -e \sin \varphi \cos \psi d\theta - e \sinh \theta \cosh \theta (\cos \varphi \cos \psi d\varphi - \sin \varphi \sin \psi d\psi), \\
A^{(2)} = -e \sin \varphi \sin \psi d\theta - e \sinh \theta \cosh \theta (\cos \varphi \sin \psi d\varphi + \sin \varphi \cos \psi d\psi), \\
A^{(3)} = -e \cos \varphi d\theta + e \sinh \theta \cosh \theta \sin \varphi d\varphi, \\
A^{(4)} = e \sinh^2 \theta \sin^2 \varphi d\psi \\
A^{(5)} = -e \sinh^2 \theta (\cos \psi d\varphi - \sin \varphi \cos \psi d\psi), \\
A^{(6)} = -e \sinh^2 \theta (\sin \psi d\varphi + \sin \varphi \cos \varphi \cos \psi d\psi).
\]

(15)

The non-zero structure constants of the group are:

\[
C_{24}^1 = C_{35}^1 = C_{41}^2 = C_{36}^2 = C_{51}^3 = C_{62}^3 = 1, \\
C_{56}^4 = C_{21}^4 = C_{64}^5 = C_{31}^5 = C_{45}^6 = C_{32}^6 = 1,
\]

(16)

which show that the gauge group is isomorphic to \( So(3,1) \). Again, it ia a matter of calculations to show that the gauge currents are

\[
j^{(1)} = \frac{2e}{r^4} \left( \sin \varphi \cos \psi d\theta + \coth \theta \cos \varphi \cos \psi d\varphi - \frac{\coth \theta}{\sin \varphi} \sin \psi d\psi \right), \\
j^{(2)} = \frac{2e}{r^4} \left( \sin \varphi \sin \psi d\theta + \coth \theta \cos \varphi \sin \psi d\varphi - \frac{\coth \theta}{\sin \varphi} \cos \psi d\psi \right), \\
j^{(3)} = -\frac{2e}{r^4} (\cos \varphi d\theta + \coth \theta \sin \varphi d\varphi), \\
j^{(4)} = -\frac{2e}{r^4} d\psi, \\
j^{(5)} = \frac{2e}{r^4} (\cos \psi d\varphi - \cot \varphi \sin \psi d\psi), \\
j^{(6)} = \frac{2e}{r^4} (\sin \psi d\varphi + \cot \varphi \cos \psi d\psi).
\]

(17)
Calculating the left hand-side of YM equation (3), one obtains exactly the same expressions as (17). Thus the gauge fields (15) satisfy the YM field equation (3). Using the definition of the metric tensor of the gauge group, one obtains:

$$\gamma_{ab} = \text{diag}(-1, -1, -1, 1, 1, 1),$$

The value of the invariant $F$ for the YM fields (15) is

$$F_{YM} = \frac{6e^2}{r^4},$$

while for spherically symmetric solutions of Maxwell theory is

$$F_{Max} = \frac{2e^2}{r^6}.$$  

Comparison of the invariant $F$ for YM and Maxwell fields given in Eqs. (18) and (19) shows that one cannot introduce a position-dependent transformation from the non-Abelian gauge fields to a set of Abelian ones which satisfy the Maxwell equation. This guarantees that the Yasskin theorem which states that the solutions of EYM and EM theories are the same [2] does not hold in 5 dimensions. Also, it is worth to mention that the $r$-dependence of the components of energy momentum tensor for EYM and EM theories are not the same. This point has been discussed in details in the next section.

The $rr$ component of the field equation (4) reduces to

$$[r^2(1 + f)]' = \frac{4}{l^2}r^3 - \frac{2e^2}{r},$$

with the solution

$$f(r) = -1 - \frac{3m}{r^2} - \frac{2e^2 \ln(r)}{r^2} + \frac{r^2}{l^2},$$

where $m$ is an integration constant which is related to the mass of the spacetime. Of course, the metric function (20) satisfies all the other components of the EYM field equations.

In order to study the general structure of the solution given in Eq. (20), we first look for curvature singularities. It is easy to show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r = 0$ and so the above metric given by Eqs. (13) and (20) has an essential singularity at $r = 0$. Since the function $f(r)$ goes to $\infty$ as $r$ goes to zero and becomes $+\infty$ as $r$ goes to infinity, the singularity is timelike. Seeking possible black hole solutions, we turn to look for the existence of horizons. The horizon(s) is (are) located at the roots of $g^{rr} = f(r) = 0$. The function $f(r)$ may have zero, one or two real roots. Denoting the largest real root of
FIG. 1: $f(r)$ versus $r$ for $n = 4, l = 1, e = .2, m < m_{\text{ext}} < 0, m = m_{\text{ext}} < 0, m_{\text{ext}} < m < 0$ and $m > 0$ from up to down, respectively.

$f(r)$ by $r_+$, we consider first the case that $f(r)$ has one real root. In this case $f'(r)$ vanishes at

$$r_{\text{ext}} = \frac{1}{2} \left( 1 + \sqrt{1 + \frac{8e^2}{l^2}} \right).$$

The value of mass for which the metric function has one real root may be obtained as

$$m_{\text{ext}} = -\left\{ \frac{e^2}{3} \left[ \ln \left( \frac{l^2 + l\sqrt{l^2 + 8e^2}}{4} \right) - \frac{1}{2} \right] + \frac{l^2 + l\sqrt{l^2 + 8e^2}}{24} \right\}.$$  \hspace{1cm} (22)

Then, the metric of Eqs. (13) and (20) presents a naked singularity provided $m < m_{\text{ext}}$, an extreme black hole for $m = m_{\text{ext}}$ and a black hole with two horizons if $m > m_{\text{ext}}$. Figure 1 shows the diagram of $f(r)$ for various values of the mass parameter.

One may note that although the metric function (11) has a logarithmic term which is absent in the 5-dimensional Reissner-Nordstrom solution, the properties of these two solutions are the same. The Hawking temperature of the black holes can be easily obtained by requiring the absence of conical singularity at the horizon in the Euclidean sector of the black hole solutions. One obtains

$$T_+ = \frac{2r_+^4 - l^2 r_+^2 - e^2 l^2}{2\pi l^2 r_+^3},$$ \hspace{1cm} (23)

which vanishes for the extreme black hole with $m = m_{\text{ext}}$ given in Eq. (22).
IV. HIGHER DIMENSIONAL SOLUTIONS

We assume that the metric has the following form:

\[ ds^2 = -f(r) \, dt^2 + \frac{dr^2}{f(r)} + r^2 \left( d\theta^2 + \sinh^2 \theta \, d\Omega_{n-2}^2 \right), \]  

(24)

where \( d\Omega_{n-2}^2 \) is the line element of \((n-2)\)-sphere. In order to obtain the gauge fields, we use the coordinates

\[
x_1 = r \sinh \theta \prod_{j=1}^{n-2} \sin \varphi_j,
\]

\[
x_i = r \sinh \theta \cos \varphi_{n-i} \prod_{j=1}^{n-i-1} \sin \varphi_j; \quad i = 2...n-1,
\]

\[
x_n = r \cosh \theta,
\]

and the ansatz

\[
A^{(a)} = \frac{e}{r^2} (x_i dx_n - x_n dx_i); \quad a = i = 1...n-1,
\]

\[
A^{(b)} = \frac{e}{r^2} (x_i dx_j - x_j dx_i); \quad i < j,
\]

(25)

where \( b \) runs from \( n \) to \( n(n-1)/2 \). It is a matter of calculation to show that the Lie algebra of the gauge group is \( So(n(n-1)/2 - 1, 1) \) with the following \( \gamma_{ab} \):

\[
\gamma_{ab} = \epsilon_a \delta_{ab}; \quad \text{no sum on} \ a,
\]

where \( \epsilon_a \) is

\[
\epsilon_a = \begin{cases} 
-1 & 1 \leq a \leq n-1 \\
1 & n \leq a \leq \frac{n(n-1)}{2} 
\end{cases}
\]

(26)

One can also shows that the above gauge fields (25) satisfy the YM field equation (3), while the gauge currents do not vanish.

Again, one may calculate the value of the invariant \( F \) for the \( n(n-1)/2 \) non-Abelian gauge fields (25) and \( n(n-1)/2 \) Abelian gauge fields of Maxwell equation with spherical symmetry as

\[
F_{YM} = \frac{(n-1)(n-2)e^2}{r^4},
\]

(27)

\[
F_{Max} = \frac{2e^2}{r^2(n-1)}.
\]

(28)
Thus, one cannot introduce a transformation from the non-Abelian gauge fields to a set of Abelian ones which satisfy the Maxwell equation. This shows that the solutions of EYM theory are not the same as EM theory in $(n + 1)$ dimensions with $n > 3$. Also, one may show that the $r$-dependence of the components of energy momentum tensor for EYM and EM theories are not the same. Using the definition of the energy-momentum tensor (6), one obtains:

\[
T^t_t = T^r_r = -(n - 2)(n - 1)e^2 \frac{2}{2r^4},
\]

\[
T^\theta_\theta = T^{\varphi_i}_{\varphi_i} = -(n - 2)(n - 5)e^2 \frac{2}{2r^4},
\]

(29)

while the energy-momentum of Maxwell field may be written as:

\[
(T^t_t)_{\text{Max}} = (T^r_r)_{\text{Max}} \sim -\frac{e^2}{r^{2(n-1)}},
\]

\[
(T^\theta_\theta)_{\text{Max}} = (T^{\varphi_i}_{\varphi_i})_{\text{Max}} \sim \frac{e^2}{r^{2(n-1)}},
\]

(30)

As one may note by comparing Eqs. (29) and (30), the $r$-dependence of energy-momentum of these two fields are different for $n > 3$. Also the tangential components of these two energy momentum tensor differ by a minus sign for $n > 5$ and is zero for $n = 5$. These differences show that the Yasskin theorem which states that the solution of EYM and EM theories are the same [2] is only true in 4 dimensions. Also, as we see below these differences drastically change the properties of the solutions.

The $rr$-component of the field equation (4) reduces to

\[
[r^{n-2}(1 + f)]' = \frac{n}{l^2}r^{n-1} - (n - 2)e^2r^{n-5} = 0.
\]

(31)

Integrating Eq. (31), one obtains

\[
f(r) = -1 + \frac{r^2}{l^2} - \frac{(n - 1)m}{r^{n-2}} - \frac{(n - 2)e^2}{(n - 4)r^2}; \quad n \neq 4.
\]

(32)

Unlike the topological Reissner-Nordstrom solutions in 6 and higher dimensions, the singularity at $r = 0$ for the solutions with non-negative mass is spacelike, and therefore it is unavoidable. This is due to the fact that $f(r)$ approaches to $-\infty$ as $r$ goes to zero and goes to $+\infty$ at large $r$. These solutions present black holes with one horizon. The spacetime of Eqs. (24) and (32) with negative mass presents a naked singularity if $m < m_{\text{ext}} < 0$, but
FIG. 2: $f(r)$ versus $r$ for $n = 6, l = 1, e = .2, m < m_{\text{ext}}, m = m_{\text{ext}}, m_{\text{ext}} < m < 0$ and $m > 0$ from up to down, respectively.

an extreme black hole for $m = m_{\text{ext}} < 0$ and a black hole with inner and outer horizons provided $m > m_{+\text{ext}}$, where $m_{\text{ext}}$ is

$$m_{\text{ext}} = -2 \frac{r_{\text{ext}}^{n-4}}{n-1} \left( \frac{r_{\text{ext}}^4}{(n-2)l^2} + \frac{e^2}{n-4} \right),$$

$$r_{\text{ext}} = \sqrt{\frac{n-2}{2n}} l \left\{ 1 + \sqrt{1 + \frac{4ne^2}{(n-2)l^2}} \right\}^{1/2}. \tag{33}$$

Figure 2 shows $f(r)$ in term of $r$ for various values of $m$.

The metric of the extreme black hole near horizon may be written as

$$ds^2 = -C_0(r - r_{\text{ext}})^2 dt^2 + \frac{dr^2}{C_0(r - r_{\text{ext}})^2} + r^2 \left( d\theta^2 + \sinh^2 \theta d\Omega_{n-2}^2 \right),$$

where

$$C_0 = \frac{f''(r_{\text{ext}})}{2} = \frac{4n}{l^2} \frac{1 + \frac{e^2}{(n-2)^2l^2} + \sqrt{1 + \frac{4ne^2}{(n-2)^2l^2}}}{\left( 1 + \sqrt{1 + \frac{4ne^2}{(n-2)^2l^2}} \right)^2}.$$
A. Thermodynamical properties

The Hawking temperature is given by

\[ T_+ = \frac{nr_+^4 - (n - 2)l^2r_+^2 - (n - 2)e^2l^2}{4\pi l^2r_+^3}, \]

which vanishes for \( m = m_{\text{ext}} \). The entropy is given by

\[ S = \frac{V_{n-1}r_+^{n-1}}, \]

where \( V_{n-1} \) is the volume of the constant \( t \) and \( r \) hypersurface with radius 1. Now we want to compute the mass of the system. One may note that the ADM mass diverges as in the case of solutions introduced in Ref. [32]. Using the first law of thermodynamics, one may obtain the thermodynamical mass (see Ref. [9] for more details) through the use of the relation \( T_+ = (\partial M_T/\partial S)_e \) as

\[ M_T = \frac{(n - 1)^2V_{n-1}}{16\pi}m. \]

The above equation shows that the parameter \( m \) may be denoted as the mass parameter as mentioned before. It is worth to mention that these results are valid in all dimensions.

V. CLOSING REMARKS

In this paper, we introduced the topological black holes of Einstein-Yang-Mills theory with hyperbolic horizon and investigate their properties. The topological solution of EYM gravity in 4 dimensions is not a new solution, and is exactly the same as the topological solution of EM theory. This is due to the fact that one can perform a position-dependent gauge transformation, from the \( \text{So}(2, 1) \) gauge fields of YM theory and a set of three Abelian gauge fields of Maxwell theory. But, we showed that one cannot perform a position-dependent gauge transformation from the \( \text{So}(n(n - 1)/2 - 1, 1) \) gauge fields of YM theory and a set of Abelian gauge fields of Maxwell equation in 5 and higher dimensions. Also, we noted that the components of energy-momentum tensor of YM fields are proportional to \( r^{-4} \) which are drastically different from the components of energy-momentum of a Maxwell field which are proportional to \( r^{-2(n-1)} \). Indeed, not only the \( r \)-dependence of the components of energy-momentum of the \( \text{So}(n(n - 1)/2 - 1, 1) \) YM fields \( (n > 3) \) is different from the energy-momentum components of a \( U(1) \) gauge field, but also they are different in a minus sign.
which has a drastic effect on the properties of the solutions [see the minus sign in front of the last term in the metric function (32)]. That is, the EYM solutions in 5 and higher dimensions are new topological solutions.

These solutions in higher dimensions with negative mass have the same properties as the solutions in EM gravity. Here, it is worth to compare the distinguishing features of non-negative mass solutions of EYM and EM gravities in 6 and higher dimensions. First, these solutions of EYM theory do not present a naked singularity, while those of EM gravity do. That is, the solutions of EYM theory respect the cosmic censorship hypothesis. Second, the singularity in the case of EYM black hole is spacelike and therefore unavoidable, while the singularity of EM black holes are timelike. Third, the solutions in EYM gravity cannot present an extreme black hole, while those of EM gravity have extreme black hole solutions. Furthermore, unlike the Reissner Nordstrom black holes, the geometry exterior to the event horizon is not determined by a global charge measurable at infinity.

Also, it is worth to mention that one does not have a static asymptotically de Sitter solution in 6 and higher-dimensional EYM gravity with non-negative mass, while in EM theory this kind of solution exists. This is due to the fact that the metric function of higher-dimensional Einstein-Yang-Mills gravity in the presence of a positive cosmological constant, $\Lambda = n(n-1)/l^2$, may be written as

$$f(r) = -1 - \frac{r^2}{l^2} - \frac{(n-1)m}{r^{n-2}} - \frac{(n-2)e^2}{(n-4)r^2}; \quad n \geq 5$$

which is negative everywhere for $m \geq 0$, and therefore the solution is not static.

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