b-baryon semi-tauonic decays in the Standard Model

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Abstract

Within the framework of HQET, $\Lambda_b \rightarrow \Lambda_c \tau \bar{\nu}_\tau$ and $\Omega_b \rightarrow \Omega_c^{(*)} \tau \bar{\nu}_\tau$ weak decays are studied to the order of $1/m_c$ and $1/m_b$. Helicity amplitudes are written down. Relevant Isgur-Wise functions given by QCD sum rule and large $N_c$ methods are applied in the numerical analysis. The baryonic R-ratios $R(\Lambda_c)$ and $R(\Omega_c^{(*)})$ are obtained.
I. INTRODUCTION

Recent experiments showed that there are some deviation from standard theory expectation in B-meson semi-tauonic decays [1]. The ratio \( R(D^{(*)}) \equiv \text{Br}(B \to D^{(*)}\tau\bar{\nu})/\text{Br}(B \to D^{(*)}l\bar{\nu}) \) \((l = e, \mu)\) is about \(3\sigma\) larger than the Standard Model (SM) prediction. This attracts a lot of theory attention from new physics point of view, for reviews see Refs. [2–6]. Nevertheless, a careful SM calculation is needed before any new physics conclusion can be drawn. An alternative place to further check that if there is such an anomaly is to look at b-baryon semileptonic decays. In this paper, the SM analysis of weak decays \( \Lambda_b \to \Lambda_c \tau\bar{\nu}_\tau \) and \( \Omega_b \to \Omega_c^{(*)} \tau\bar{\nu}_\tau \) is made in detail with tauon mass effects considered.

Heavy baryons are interesting both experimentally and theoretically for their own sake. They reveal important features of the heavy quark physics, and offer a good application ground of QCD. Especially experiments of LHC [7], the super B-factory [8] and BES III [9], as well as previous LEP, LEPII and Tevatron, have been collecting data on heavy baryons. Among various heavy baryons, \( \Lambda_b \) and \( \Omega_b \), as the most basic and simplest ones, have been paid much attention. Their semileptonic weak decays have already been analyzed in detail [10–23]. At the quark-gluon level, the decays are due to weak interaction which is described by the universal V-A four-fermion interaction. At the hadron level, semileptonic decay amplitudes are written in terms of hadronic matrix elements of weak currents, which are parameterized by form factors. In this paper, helicity amplitude description of hadronic transitions by Körner et al. [24–27] will be used.

The form factors of heavy hadron weak transition can be greatly simplified in the heavy quark effective theory (HQET) [28–33]. For hadrons containing a single heavy quark, HQET is the right QCD to describe them. It factorizes the perturbatively calculable part out from hadronic matrix elements. Form factors are then described by several independent universal form factors which are the so-called Isgur-Wise functions.

Nonperturbative methods are needed to calculate Isgur-Wise functions. Actually it is at this stage the uncertainty of analytical calculation is lack of control. Among various analytical nonperturbative methods, QCD sum rules [34] and the large \( N_c \) limit [35, 36] are outstanding, the former is regarded as being rooted in QCD, and the latter is just a limit of QCD. They are generally considered to be more close to real QCD. Both have reasonable and consistent ways to estimate uncertainties of calculation. Like in [12, 18, 20], we will use...
results of both the QCD sum rule and the large $N_c$ methods for these Isgur-Wise functions in the final numerical analysis. Notably $\Lambda_b$ semi-tauonic decay was also studied recently \cite{37,47}.

The outline of the paper is as follows. In section II, general description of the decays in terms of helicity amplitudes is given. In section III, form factors are expressed by Isgur-Wise functions which were obtained from the large $N_c$ and QCD sum rules. In section IV, numerical results are presented. Section V gives the summary.

II. FORM FACTORS, HELICITY AMPLITUDES AND DECAY RATES

A. Form factors

For $\Lambda_b \to \Lambda_c$ weak transition, relevant baryonic matrix elements are parameterized by form factors

\[
\langle \Lambda_c (p', s') | V^\mu | \Lambda_b (p, s) \rangle = \bar{u}_{\Lambda_c} (p', s') \left[ f_1 \gamma^\mu + i f_2 \sigma^{\mu\nu} q_\nu + f_3 q^\mu \right] u_{\Lambda_b} (p, s), \\
\langle \Lambda_c (p', s') | A^\mu | \Lambda_b (p, s) \rangle = \bar{u}_{\Lambda_c} (p', s') \left[ g_1 \gamma^\mu + i g_2 \sigma^{\mu\nu} q_\nu + g_3 q^\mu \right] \gamma_5 u_{\Lambda_b} (p, s),
\]

where $q = p - p'$ and $\sigma^{\mu\nu} = i [\gamma_\mu, \gamma_\nu] / 2$, form factors $f_i$ and $g_i$ are functions of $q^2$. It is convenient to reexpress the form factors as functions of velocities of baryons,

\[
\langle \Lambda_c (v', s') | V^\mu | \Lambda_b (v, s) \rangle = \bar{u}_{\Lambda_c} (v', s') (F_1 (\omega) \gamma^\mu + F_2 (\omega) v^\mu + F_3 (\omega) v'^\mu) u_{\Lambda_b} (v, s), \\
\langle \Lambda_c (v', s') | A^\mu | \Lambda_b (v, s) \rangle = \bar{u}_{\Lambda_c} (v', s') (G_1 (\omega) \gamma^\mu + G_2 (\omega) v^\mu + G_3 (\omega) v'^\mu) \gamma_5 u_{\Lambda_b} (v, s),
\]

where $v$ and $v'$ denote four-velocities of $\Lambda_b$ and $\Lambda_c$, respectively, $\omega = v \cdot v'$, $F_i$ and $G_i$ are functions of $\omega$.

Similarly, for the decays of $\Omega_b \to \Omega_c^{(*)}$,

\[
\langle \Omega_c (v', s') | V^\mu | \Omega_b (v) \rangle = \bar{u}_{\Omega_c} (v', s') (F'_1 \gamma^\mu + F'_2 v^\mu + F'_3 v'^\mu) u_{\Omega_b} (v, s), \\
\langle \Omega_c (v', s') | A^\mu | \Omega_b (v) \rangle = \bar{u}_{\Omega_c} (v', s') (G'_1 \gamma^\mu + G'_2 v^\mu + G'_3 v'^\mu) \gamma_5 u_{\Omega_b} (v, s), \\
\langle \Omega_c^* (v', s') | V^\mu | \Omega_b (v) \rangle = \bar{u}_{\Omega_c^{(*)}} (v', s') (N_1 v^\lambda \gamma^\mu + N_2 v^\lambda v^\mu + N_3 v^\lambda v'^\mu + N_4 v'^\mu) \gamma_5 u_{\Omega_b} (v, s), \\
\langle \Omega_c^* (v', s') | A^\mu | \Omega_b (v) \rangle = \bar{u}_{\Omega_c^{(*)}} (v', s') (K_1 v^\lambda \gamma^\mu + K_2 v^\lambda v^\mu + K_3 v^\lambda v'^\mu + K_4 v'^\mu) u_{\Omega_b} (v, s),
\]

where $u_{\Omega_c^{(*)}}$ is the Rarita-Schwinger spinor for a spin-3/2 particle.
B. Helicity amplitudes

In analyzing decays, polarization gives detailed physics information. The decay $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ can be thought of as two sub-processes $\Lambda_b \to \Lambda_c + W_{\text{off-shell}}$ and $W_{\text{off-shell}} \to \tau + \bar{\nu}_\tau$. Consider the decay $\Lambda_b \to \Lambda_c + W_{\text{off-shell}}$ in the rest system of $\Lambda_b$. $W_{\text{off-shell}}$ moves in the $+z$ direction, and $\Lambda_c$ moves in the $-z$ direction. The momentums of $\Lambda_b, \Lambda_c$ and $W_{\text{off-shell}}$ are

$$p^\mu = (m_{\Lambda_b}; 0, 0, 0), \quad p'^\mu = (E_{\Lambda_c}; 0, 0, -|\vec{q}|), \quad q^\mu = (q_0; 0, 0, |\vec{q}|),$$

respectively. The current is composed of a spin-1 and a spin-0 components. The relevant expression of polarization 4-vectors of the current is that

$$\varepsilon^\mu(t) = \frac{1}{\sqrt{q^2}} (q_0; 0, 0, |\vec{q}|), \quad \varepsilon^\mu(\pm 1) = \frac{1}{\sqrt{2}} (0; \mp 1, -i, 0), \quad \varepsilon^\mu(0) = \frac{1}{\sqrt{q^2}} (|\vec{q}|; 0, 0, q_0).$$

The $t$-label stands for the time-component of the corresponding current. Notice that in our case, the tauon mass will be taken into consideration, so the time-component of the $W_{\text{off-shell}}$ should be included. The helicity amplitudes are defined in the following,

$$H_{\lambda_2 \lambda_W}^{V,A} = M_\mu^{V,A} (\lambda_2) \varepsilon^{\ast \mu} (\lambda_W),$$

where $M_\mu^{V,A}$ stand for the matrix elements of vector and axial vector currents, $\lambda_2$ and $\lambda_W$ are the helicities of the daughter baryon $\Lambda_c$ and the $W_{\text{off-shell}}$, respectively. Helicity amplitudes are then expressed in terms of the form factors.

For $\Lambda_b \to \Lambda_c$ transition,

$$\sqrt{q^2} H_{\frac{1}{2}^+}^V = \sqrt{2m_{\Lambda_b} m_{\Lambda_c} (1 + \omega)} ((m_{\Lambda_b} - m_{\Lambda_c}) F_1 + (m_{\Lambda_b} - m_{\Lambda_c} \omega) F_2 + (-m_{\Lambda_c} + m_{\Lambda_b} \omega) F_3),$$

$$H_{\frac{1}{2}^-}^V = -2 \sqrt{m_{\Lambda_b} m_{\Lambda_c} (\omega - 1)} F_1,$$

$$\sqrt{q^2} H_{\frac{1}{2}^0}^V = \sqrt{2m_{\Lambda_b} m_{\Lambda_c} (\omega - 1)} ((m_{\Lambda_b} + m_{\Lambda_c}) F_1 + m_{\Lambda_c} (\omega + 1) F_2 + m_{\Lambda_b} (\omega + 1) F_3),$$

$$\sqrt{q^2} H_{\frac{1}{2}^+}^A = \sqrt{2m_{\Lambda_b} m_{\Lambda_c} (\omega - 1)} (- (m_{\Lambda_b} + m_{\Lambda_c}) G_1 + (m_{\Lambda_b} - m_{\Lambda_c} \omega) G_2 + (-m_{\Lambda_c} + m_{\Lambda_b} \omega) G_3),$$

$$H_{\frac{1}{2}^-}^A = -2 \sqrt{m_{\Lambda_b} m_{\Lambda_c} (\omega + 1)} G_1,$$

$$\sqrt{q^2} H_{\frac{1}{2}^0}^A = \sqrt{2m_{\Lambda_b} m_{\Lambda_c} (\omega + 1)} ((m_{\Lambda_b} - m_{\Lambda_c}) G_1 - m_{\Lambda_c} (\omega - 1) G_2 - m_{\Lambda_b} (\omega - 1) G_3).$$

Similarly for $\Omega_b \to \Omega_c$ transition,
\[
\sqrt{q^2} H^W_{\frac{1}{2}t} = \sqrt{2m_\Omega m_\Omega_c (1 + \omega)} \left( (m_\Omega c - m_\Omega_b) F'_1 + (m_\Omega c - m_\Omega_b\omega) F'_2 + (-m_\Omega c + m_\Omega b\omega) F'_3 \right),
\]
\[
H^W_{\frac{1}{2}1} = -2 \sqrt{m_\Omega b m_\Omega_c (\omega - 1)} F'_1,
\]
\[
\sqrt{q^2} H^W_{\frac{1}{2}0} = \sqrt{2m_\Omega m_\Omega_c (\omega - 1)} \left( (m_\Omega b + m_\Omega c) F'_1 + m_\Omega c (\omega + 1) F'_2 + m_\Omega b (\omega + 1) F'_3 \right),
\]
\[
\sqrt{q^2} H^A_{\frac{1}{2}t} = \sqrt{2m_\Omega m_\Omega_c (\omega - 1)} \left( - (m_\Omega b + m_\Omega c) G'_1 + (m_\Omega b - m_\Omega c\omega) G'_2 + (-m_\Omega c + m_\Omega b\omega) G'_3 \right),
\]
\[
H^A_{\frac{1}{2}1} = -2 \sqrt{m_\Omega b m_\Omega_c (\omega + 1)} G'_1,
\]
\[
\sqrt{q^2} H^A_{\frac{1}{2}0} = \sqrt{2m_\Omega m_\Omega_c (\omega + 1)} \left( (m_\Omega b - m_\Omega c) G'_1 - m_\Omega c (\omega - 1) G'_2 - m_\Omega b (\omega - 1) G'_3 \right),
\]
\[
(8)
\]
and for $\Omega_b \rightarrow \Omega_c^\ast$ transition,

$$\sqrt{q_r^2} H_{\frac{1}{2}0}^{nv} = \sqrt{\frac{2}{3}} (\omega + 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)(m_{\Omega_b} - m_{\Omega_c}) N_1}$$

$$- \sqrt{\frac{2}{3}} (\omega^2 - 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)m_{\Omega_c} N_2}$$

$$- \sqrt{\frac{2}{3}} (\omega^2 - 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)m_{\Omega_b} N_3}$$

$$- \sqrt{\frac{2}{3}} \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)(m_{\Omega_b} \omega - m_{\Omega_c}) N_4}.$$

$$\sqrt{q_r^2} H_{\frac{1}{2}0}^{nA} = \sqrt{\frac{2}{3}} (\omega - 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)(m_{\Omega_b} + m_{\Omega_c}) K_1}$$

$$+ \sqrt{\frac{2}{3}} (\omega^2 - 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)m_{\Omega_c} K_2}$$

$$+ \sqrt{\frac{2}{3}} (\omega^2 - 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)m_{\Omega_b} K_3}$$

$$+ \sqrt{\frac{2}{3}} \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)(m_{\Omega_b} \omega - m_{\Omega_c}) K_4}.$$

(9)

$$\sqrt{q_r^2} H_{\frac{1}{2}1}^{nv} = \sqrt{\frac{2}{3}} (\omega + 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)(m_{\Omega_b} + m_{\Omega_c}) N_1}$$

$$- \sqrt{\frac{2}{3}} (\omega - 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)(m_{\Omega_b} - m_{\Omega_c} \omega) N_2}$$

$$- \sqrt{\frac{2}{3}} (\omega - 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)(m_{\Omega_b} \omega - m_{\Omega_c}) N_3}$$

$$- \sqrt{\frac{2}{3}} (\omega - 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)m_{\Omega_b} N_4}.$$

$$\sqrt{q_r^2} H_{\frac{1}{2}1}^{nA} = \sqrt{\frac{2}{3}} (\omega + 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)(m_{\Omega_b} - m_{\Omega_c}) K_1}$$

$$+ \sqrt{\frac{2}{3}} (\omega + 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)(m_{\Omega_b} - m_{\Omega_c} \omega) K_2}$$

$$+ \sqrt{\frac{2}{3}} (\omega + 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)(m_{\Omega_b} \omega - m_{\Omega_c}) K_3}$$

$$+ \sqrt{\frac{2}{3}} (\omega + 1) \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)m_{\Omega_b} K_4}.$$

$$H_{\frac{1}{2}1}^{nv} = - \sqrt{\frac{1}{3}} \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)N_1} + \sqrt{\frac{1}{3}} \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)N_4},$$

$$H_{\frac{1}{2}1}^{nv} = - \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega - 1)N_4},$$

$$H_{\frac{1}{2}1}^{nA} = - \sqrt{\frac{1}{3}} \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)2(\omega - 1)K_1} + \sqrt{\frac{1}{3}} \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)K_4},$$

$$H_{\frac{1}{2}1}^{nA} = \sqrt{2m_{\Omega_b} m_{\Omega_c} (\omega + 1)K_4}.$$
Other relations can be obtained by relations: $H_{\lambda_2, -\lambda_2} = H_{\lambda_2, \lambda W}$, $H_{-\lambda_2, -\lambda_2} = -H_{\lambda_2, \lambda W}$.

At last, the following relation is needed, $H_{\lambda_2 \lambda W} = H_{\lambda_2 \lambda W} - H_{\lambda_2 \lambda W}$. Decay rates can be given in terms of these helicity amplitudes.

C. Decay rates

The differential decay rate $d\Gamma/d\omega$ is obtained as following [25, 26],

$$d\Gamma(\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau) = \frac{G_F^2}{(2\pi)^3} |V_{cb}|^2 \frac{(q^2 - m_\tau^2)^2 m_{\Lambda_b}^2 \sqrt{\omega^2 - 1}}{12m_{\Lambda_b}q^2} \times \left[ \left(1 + \frac{m_\tau^2}{2q^2}\right)|H_{\frac{1}{2}1}|^2 + \left(1 + \frac{m_\tau^2}{2q^2}\right)|H_{-\frac{1}{2}1}|^2 + \left(1 + \frac{m_\tau^2}{2q^2}\right)|H_{\frac{1}{2}0}|^2 \right]$$

$$+ \left(1 + \frac{m_\tau^2}{2q^2}\right)|H_{-\frac{1}{2}0}|^2 + \frac{3m_\tau^2}{2q^2}|H_{\frac{1}{2}t}|^2 + \frac{3m_\tau^2}{2q^2}|H_{-\frac{1}{2}t}|^2 \right] \frac{d\Gamma_{T_+}}{d\omega} + \frac{d\Gamma_{T_-}}{d\omega} + \frac{d\Gamma_{L_+}}{d\omega} + \frac{d\Gamma_{L_-}}{d\omega} + \frac{d\Gamma_{t_+}}{d\omega} + \frac{d\Gamma_{t_-}}{d\omega},$$

where $G_F$ is the Fermi coupling constant, $V_{cb}$ is the CKM matrix element, and $\frac{d\Gamma_{T_\pm}}{d\omega}$, $\frac{d\Gamma_{L_\pm}}{d\omega}$, and $\frac{d\Gamma_{t_\pm}}{d\omega}$ are defined as the transverse, longitudinal and time-component contribution to the decay rate with $\pm$ denoting the final baryon helicity.

Following the same method, we get that for $\Omega_b \to \Omega_c \tau \bar{\nu}_\tau$,

$$d\Gamma(\Omega_b \to \Omega_c \tau \bar{\nu}_\tau) = \frac{G_F^2}{(2\pi)^3} |V_{cb}|^2 \frac{(q^2 - m_\tau^2)^2 m_{\Omega_b}^2 \sqrt{\omega^2 - 1}}{12m_{\Omega_b}q^2} \times \left[ \left(1 + \frac{m_\tau^2}{2q^2}\right)|H'_{\frac{1}{2}1}|^2 + \left(1 + \frac{m_\tau^2}{2q^2}\right)|H'_{-\frac{1}{2}1}|^2 + \left(1 + \frac{m_\tau^2}{2q^2}\right)|H'_{\frac{1}{2}0}|^2 \right]$$

$$+ \left(1 + \frac{m_\tau^2}{2q^2}\right)|H'_{-\frac{1}{2}0}|^2 + \frac{m_\tau^2}{2q^2}\left( |H'_{\frac{1}{2}t}|^2 + |H'_{-\frac{1}{2}t}|^2 \right) \right] \frac{d\Gamma_{T_+}}{d\omega} + \frac{d\Gamma_{T_-}}{d\omega} + \frac{d\Gamma_{L'_+}}{d\omega} + \frac{d\Gamma_{L'_-}}{d\omega} + \frac{d\Gamma_{t'_+}}{d\omega} + \frac{d\Gamma_{t'_-}}{d\omega},$$

(11)
and for $\Omega_b \to \Omega_c^* \tau \bar{\nu}_\tau$,

$$
\frac{d\Gamma(\Omega_b \to \Omega_c^* \tau \bar{\nu}_\tau)}{d\omega} = \frac{G_F^2}{(2\pi)^3} |V_{cb}|^2 \frac{(q^2 - m_{\tau}^2)^2}{12m_{\Omega_b}q^2} \sqrt{\omega^2 - 1} \times \left[ \left( 1 + \frac{m^2_\tau}{2q^2} \right) |H''_{+\frac{1}{2}}|^2 + \left( 1 + \frac{m^2_\tau}{2q^2} \right) |H''_{-\frac{1}{2}}|^2 + \left( 1 + \frac{m^2_\tau}{2q^2} \right) |H''_{\frac{1}{2}}|^2 \right] \\
+ \left( 1 + \frac{m^2_\tau}{2q^2} \right) |H''_{-\frac{1}{2}}|^2 + \left( 1 + \frac{m^2_\tau}{2q^2} \right) |H''_{\frac{1}{2}}|^2 + \left( 1 + \frac{m^2_\tau}{2q^2} \right) |H''_{0}|^2 \\
+ \frac{m^2_\tau}{2q^2} \left( |H''_{+\frac{1}{2}}|^2 + |H''_{-\frac{1}{2}}|^2 \right) \right]
$$

(12)

where $\frac{d\Gamma''_{T_{2\pm}}}{d\omega}$ and $\frac{d\Gamma''_{T_{1\pm}}}{d\omega}$ correspond to $H''_{\pm\frac{1}{2}\pm1}$ and $H''_{\pm\frac{1}{2}\pm1}$, respectively.

III. HQET WITH QCD SUM RULE AND LARGE $N_c$

A. HQET

The form factors in HQET can be simplified in terms of Isgur-Wise functions. For $\Lambda_b \to \Lambda_c$ at the leading order of heavy quark expansion, there is only one Isgur-Wise function $\xi(\omega)$ \cite{30,32},

$$
\left\langle \Lambda_c \left( v', s' \right) \left| \bar{h}_v^{(c)}(x) \Gamma h_v^{(b)}(x) \right| \Lambda_b(v, s) \right\rangle = \xi(\omega) \bar{u}_{\Lambda_c}(v', s') \Gamma u_{\Lambda_b}(v, s),
$$

(13)

where $h_v^{(Q)}$ denotes the heavy quark field defined in the HQET with velocity $v$, and $\Gamma$ stands for any gamma matrices. $\xi(\omega)$ is normalized at the zero recoil, $\xi(1) = 1$.

When $1/m_Q$ correction is taken into consideration, another Isgur-Wise function $\chi(\omega)$ and a mass parameter $\bar{\Lambda}$ appear. The subleading Isgur-Wise function $\chi(\omega)$ is defined by

$$
\left\langle \Lambda_c \left( v' \right) \left| \bar{h}_v^{(c)}(x) \Gamma h_v^{(b)}(x) \right| \Lambda_b(v) \right\rangle = \frac{\bar{\Lambda}}{m_Q} \chi(\omega) \bar{u}_{\Lambda_c}(v') \Gamma u_{\Lambda_b}(v),
$$

(14)

where $\bar{\Lambda}$ is the heavy baryon mass in HQET, $\bar{\Lambda} = m_{\Lambda_Q} - m_Q$. 

8
Including $\alpha_s$ and $\Lambda_{\text{QCD}}/m_{c,b}$ corrections, the form factors are given as following \[30–32\],

\[
\begin{align*}
F_1 &= C(\mu)\xi(\omega) + C(\mu) \left( \frac{\Lambda}{2m_c} + \frac{\Lambda}{2m_b} \right) \left[ 2\chi(\omega) + \xi(\omega) \right], \\
G_1 &= C(\mu)\xi(\omega) + C(\mu) \left( \frac{\Lambda}{2m_c} + \frac{\Lambda}{2m_b} \right) \left[ 2\chi(\omega) + \frac{\omega - 1}{\omega + 1} \xi(\omega) \right], \\
F_2 &= G_2 = -C(\mu) \frac{\Lambda}{m_c(\omega + 1)} \xi(\omega), \\
F_3 &= -G_3 = -C(\mu) \frac{\Lambda}{m_b(\omega + 1)} \xi(\omega),
\end{align*}
\]  

\[15\]

where the perturbative QCD coefficient in the leading logarithmic approximation is

\[
C(\mu) = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} \left[ \frac{\alpha_s(m_c)}{\alpha_s(\mu)} \right]^{a_L(\omega)},
\]  

\[16\]

and $a_L(\omega) = \frac{8}{27} [\omega r(\omega) - 1]$, $r(\omega) = \frac{1}{\sqrt{\omega^2 - 1}} \ln \left( \frac{\omega + \sqrt{\omega^2 - 1}}{\sqrt{\omega^2 - 1}} \right)$.

For $\Omega_{b(c)}^{(s)}$ cases, similarly, based on the standard tensor method \[30, 33\], we denote $\Omega_Q$ and $\Omega_\bar{Q}$ as $B_\mu^1$ and $B_\mu^2$ respectively,

\[
B_\mu^1(v, s) = \frac{1}{\sqrt{3}} (\gamma_\mu + v_\mu) \gamma^5 u(v, s), \quad B_\mu^2(v, s) = u_\mu(v, s).
\]

\[17\]

In the leading order of heavy quark expansion, the fourteen form factors are reduced to two Isgur-Wise functions which are defined as,

\[
\left\langle \Omega_c^M \left| \overline{h}_v^{(c)} \Gamma h_B^{(b)} \right| \Omega_b^N \right\rangle = C \bar{B}_\mu^M \Gamma B^N \left[ -g^{\mu\nu} \xi_1(\omega) + v^\mu v^\nu \xi_2(\omega) \right].
\]

\[18\]

The form factors are expressed as \[49\],

\[19\]

\[
\begin{align*}
F'_1 &= -\frac{\omega}{3} C(\mu)\xi_1 + \frac{\omega^2 - 1}{3} C(\mu)\xi_2, & G'_1 &= -\frac{\omega}{3} C(\mu)\xi_1 + \frac{\omega^2 - 1}{3} C(\mu)\xi_2, \\
F'_2 &= \frac{2}{3} C(\mu)\xi_1 + \frac{2(1 - \omega)}{3} C(\mu)\xi_2, & G'_2 &= \frac{2}{3} C(\mu)\xi_1 + \frac{-2(1 + \omega)}{3} C(\mu)\xi_2, \\
F'_3 &= \frac{3}{3} C(\mu)\xi_1 + \frac{3(1 - \omega)}{3} C(\mu)\xi_2, & G'_3 &= \frac{3}{3} C(\mu)\xi_1 + \frac{2(1 + \omega)}{3} C(\mu)\xi_2, \\
N_1 &= \frac{1}{\sqrt{3}} C(\mu)\xi_1 + \frac{\omega - 1}{\sqrt{3}} C(\mu)\xi_2, & K_1 &= \frac{1}{\sqrt{3}} C(\mu)\xi_1 + \frac{\omega + 3}{\sqrt{3}} C(\mu)\xi_2, \\
N_2 &= 0, & K_2 &= 0, \\
N_3 &= 0 + \frac{2}{\sqrt{3}} C(\mu)\xi_2, & K_3 &= 0 + \frac{-2}{\sqrt{3}} C(\mu)\xi_2, \\
N_4 &= \frac{-2}{\sqrt{3}} C(\mu)\xi_1 + 0, & K_4 &= \frac{2}{\sqrt{3}} C(\mu)\xi_1 + 0.
\end{align*}
\]
B. QCD sum rule and Large $N_c$

Isgur-Wise functions and the mass parameters should be calculated by nonperturbative methods. In this work, we make use of results from QCD sum rule \cite{11, 13} and large $N_c$ methods \cite{12, 18, 20, 50}.

1. QCD sum rule

Within HQET, the QCD sum rule method gives the following results \cite{11, 13},

$$
\xi(\omega) = 1 - \rho^2(\omega - 1), \quad \rho^2 = 1.35 \pm 0.12,
$$
$$
\chi(\omega) \approx O(10^{-2}),
$$
$$
\tilde{\Lambda} \approx 0.79 \pm 0.05 \text{GeV}.
$$

2. Large $N_c$

In the large $N_c$ limit, the leading Isgur-Wise function $\xi(\omega)$ and the mass parameter $\tilde{\Lambda}$ are given as \cite{50}

$$
\xi(\omega) = 0.99 \exp[-1.3(\omega - 1)], \quad \tilde{\Lambda} \approx 0.87 \text{ GeV}.
$$

This $\xi$ is actually a realization of $\delta$ function \cite{12}. Ref. \cite{12} further showed that $\chi(\omega) = 0$ in the large $N_c$ limit.

In the large $N_c$ limit, Isgur-Wise functions $\xi_1$ and $\xi_2$ can be expressed by $\xi(\omega)$ \cite{51, 52},

$$
\xi_1(\omega) = \xi(\omega), \quad \xi_2(\omega) = \frac{\xi(\omega)}{1 + \omega}.
$$

To the order of $1/m_Q$, sub-leading Isgur-Wise functions can also be written as $\xi(\omega)$ \cite{18, 20}.
We finally obtain the form factors as

\[ F_1' = -C(\mu) \frac{1}{3} \xi(\omega) - \frac{1}{3} C(\mu) \xi(\omega) \left[ \frac{\Omega}{2m_c} + \frac{\Omega}{2m_b} \right], \]

\[ F_2' = C(\mu) \frac{4 \xi(\omega)}{3(1 + \omega)} + C(\mu) \frac{\xi(\omega)}{3(1 + \omega)} \left[ -\frac{\Omega}{m_c} + \frac{2\Omega}{m_b} \right], \]

\[ F_3' = C(\mu) \frac{4 \xi(\omega)}{3(1 + \omega)} + C(\mu) \frac{\xi(\omega)}{3(1 + \omega)} \left[ \frac{2\Omega}{m_c} - \frac{\Omega}{m_b} \right], \]

\[ G_1' = -C(\mu) \frac{1}{3} \xi(\omega) + C(\mu) \frac{1}{3} \xi(\omega) \left[ \frac{\Omega}{2m_c} + \frac{\Omega}{2m_b} \right] \left( 1 - \frac{\omega}{1 + \omega} \right), \]

\[ G_2' = C(\mu) \frac{\Omega}{3m_c} \left( \frac{1}{1 + \omega} \right) \xi(\omega), \]

\[ G_3' = -C(\mu) \frac{\Omega}{3m_b} \left( \frac{1}{1 + \omega} \right) \xi(\omega), \]

\[ N_1 = C(\mu) \frac{-2 \xi(\omega)}{\sqrt{3}(1 + \omega)} + C(\mu) \frac{-\xi(\omega)}{\sqrt{3}(1 + \omega)} \left[ \frac{\Omega}{m_c} + \frac{\Omega}{m_b} \right], \]

\[ K_1 = 0, \quad N_2 = 0, \quad K_2 = C(\mu) \frac{2}{\sqrt{3}} \xi(\omega) \frac{\Omega}{m_c} \left( \frac{1}{1 + \omega} \right)^2, \]

\[ N_3 = C(\mu) \frac{2 \xi(\omega)}{\sqrt{3}(1 + \omega)} + C(\mu) \frac{\xi(\omega)}{\sqrt{3}(1 + \omega)} \left[ \frac{\Omega}{m_c} + \frac{\Omega}{m_b} \right], \]

where \( \bar{\Omega} = m_{\Omega_Q} - m_Q. \)

\[ IV. \text{ NUMERICAL RESULTS} \]

Numerical results for \( \Lambda_b \to \Lambda_c l\bar{\nu}_l \) and \( \Omega_b \to \Omega_c^{(*)} l\bar{\nu}_l \) \((l = e, \mu, \tau)\) can be obtained now. In the calculation it takes \( m_{\Lambda_b} = 5.62 \text{ GeV}, m_{\Lambda_c} = 2.23 \text{ GeV}, m_{\Omega_b} = 6.07 \text{ GeV}, m_{\Omega_c} = 2.70 \text{ GeV}, m_{\Omega_{c2}} = 2.77 \text{ GeV}, |V_{cb}| = 0.04 \) and \( G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2} \) \([53]\). And \( m_c = 1.44 \text{ GeV}, m_b = 4.83 \text{ GeV}, \mu = 0.47 \text{ GeV} \) \([31, 32]\). \( \omega \) is in the range \( 1 \leq \omega \leq \frac{m_{\Lambda_c}^2 + m_{\Omega_c}^2 - m_{\tau}^2}{2m_{\Lambda_c}m_{\Omega_c}} \) for the \( \Lambda_b \to \Lambda_c \) decay and \( 1 \leq \omega \leq \frac{m_{\Omega_{c2}}^2 + m_{\Omega_{c2}}^2 - m_{\tau}^2}{2m_{\Omega_{c2}}m_{\Omega_{c2}}} \) for \( \Omega_b \to \Omega_c^{(*)} \).

Tauonic decay distributions are plotted in Figs.1-7. Fig.1 presents the \( \Lambda_b \to \Lambda_c \tau\bar{\nu}_\tau \) differential decay rate, both QCD sum rule and large \( N_c \) results are given for comparison, with the uncertainty of the QCD sum rule considered. The two results are close to each other, especially in the low recoil region. In Figs.2 and 3, we display the \( \omega \) dependence of \( \Lambda_b \to \Lambda_c \tau\bar{\nu}_\tau \) partial differential rates \( T, L, t \) and the total differential rate. The transverse rate \( T \) dominates in the low recoil region while the longitudinal rate \( L \) dominates in the
large recoil region. Fig. 2 is for the QCD sum rule method. And Fig. 3 is that from the large \( N_c \) method. Figs. 4-7 show the corresponding plots for \( \Omega_b \to \Omega_c^{(*)} \tau \bar{\nu}_\tau \) decays for the large \( N_c \) limit. For the partial decay distribution of \( \Omega_b \to \Omega_c^{(*)} \tau \bar{\nu}_\tau \) (Fig. 7), what should be discussed is that the \( t_+ \) channel is almost 0, and the \( L_\pm \) channels are almost the same. As for the tauonic decay, time-components should be considered specifically, because they are absent in the massless charged lepton case. In the \( \Lambda_b \) case, time-component is still small. However, in the \( \Omega_b \to \Omega_c \) case, time-component gets comparatively larger. In the \( \Omega_b \to \Omega_c^{(*)} \) case, time-component gets to be even much larger and begins to dominate in the large recoil region.

The decay rates are obtained by \( \omega \) integration. For the \( \Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau \) decay, we obtain the total decay rate, the branching ratio, and the R-ratio in the following from the QCD sum rule,

\[
\Gamma (\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau) = 1.16 \pm 0.05 \ (\rho^2) \pm 0.004 \ (\bar{\Lambda}) \times 10^{-14} \text{ GeV} , \\
\text{Br} (\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau) = (2.59 \pm 0.09) \% \times \left( \frac{\tau (\Lambda_b)}{1.47 \times 10^{-12} \text{sec}} \right) , \\
R (\Lambda_c) = \frac{\Gamma (\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau)}{\Gamma (\Lambda_b \to \Lambda_c \mu \bar{\nu}_\mu)} = (33.1 \pm 1.4) \% ,
\]

where uncertainties are due to the error of QCD sum rules in Eq. (20). For the large \( N_c \) case,

\[
\Gamma (\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau) = 1.22 \times 10^{-14} \text{ GeV} , \\
\text{Br} (\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau) = 2.73 \% \times \left( \frac{\tau (\Lambda_b)}{1.47 \times 10^{-12} \text{sec}} \right) , \\
R (\Lambda_c) = \frac{\Gamma (\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau)}{\Gamma (\Lambda_b \to \Lambda_c \mu \bar{\nu}_\mu)} = 29.2 \% .
\]

The error of the large \( N_c \) result is estimated to be \( 1/N_c \sim 30\% \) in general. However, the uncertainty of \( R \) which is what we are really interested in, is supposed to be smaller because of the cancellation in the ratios [12]. Thus, the uncertainty in \( R (\Lambda_c) \) is estimated as small as \( \sim 10\% \).

Table I lists results for the \( \Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell \) semileptonic decay. Experimental data [53] and results from the quark model [37], HQET [44], and lattice QCD [21] are also listed for comparison. From the table one can see that the large \( N_c \) result is somewhat larger than other theoretical results, as far as central values are concerned. The QCD sum rule result is very close to that of the quark model. Nevertheless within the 2\( \sigma \) uncertainty, all the results
are still consistent with each other.

Via the same procedure, the result of the $\Omega_b \to \Omega_c^{(*)} \tau \bar{\nu}_\tau$ decay is obtained by using the large $N_c$ method in the following,

$$\Gamma (\Omega_b \to \Omega_c \tau \bar{\nu}_\tau) = 4.83 \times 10^{-15} \text{ GeV},$$

$$\text{Br} (\Omega_b \to \Omega_c \tau \bar{\nu}_\tau) = 1.21\% \times \left( \frac{\tau (\Omega_b)}{1.65 \times 10^{-12}\text{sec}} \right),$$

$$R (\Omega_c) = \frac{\Gamma (\Omega_b \to \Omega_c \tau \bar{\nu}_\tau)}{\Gamma (\Omega_b \to \Omega_c \mu \bar{\nu}_\mu)} = 30.4\%.$$ 

And

$$\Gamma (\Omega_b \to \Omega_c^{*} \tau \bar{\nu}_\tau) = 1.27 \times 10^{-14} \text{ GeV},$$

$$\text{Br} (\Omega_b \to \Omega_c^{*} \tau \bar{\nu}_\tau) = 3.18\% \times \left( \frac{\tau (\Omega_b)}{1.65 \times 10^{-12}\text{sec}} \right),$$

$$R (\Omega_c^{*}) = \frac{\Gamma (\Omega_b \to \Omega_c^{*} \tau \bar{\nu}_\tau)}{\Gamma (\Omega_b \to \Omega_c^{*} \mu \bar{\nu}_\mu)} = 33.2\%.$$ 

Like in the $\Lambda_b$ decay, the $1/N_c$ uncertainty for $R (\Omega_c^{(*)})$ is expected to be $\sim 10\%$.

TABLE I. Results for the $\Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell$ decay.

| Decay                  | Sum rule | Large $N_c$ | Ref.[37]  | Ref.[21]  | Ref.[44]  | Experiment[53] |
|------------------------|----------|-------------|-----------|-----------|-----------|----------------|
| $\Lambda_b \to \Lambda_c (e, \mu) \bar{\nu}(e, \mu)$ |          |             |           |           |           |                |
| $\Gamma \times 10^{14}$ | $3.50 \pm 0.30$ | $4.2 \pm 0.4$ | $3.13$    | $2.26 \pm 0.14$ |           |                |
| $\text{Br} (%)$        | $7.83 \pm 0.60$ | $9.3 \pm 0.9$ | $7.0$     | $5.06 \pm 0.32$ |           | $6.2_{-1}^{+1.4}$ |
| $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ |          |             |           |           |           |                |
| $\Gamma \times 10^{14}$ | $1.16 \pm 0.05$ | $1.2 \pm 0.1$ | $0.91$    | $0.753 \pm 0.033$ |           |                |
| $\text{Br} (%)$        | $2.59 \pm 0.09$ | $2.7 \pm 0.3$ | $2.0$     | $1.68 \pm 0.07$ |           |                |
| $R (\Lambda_c)$        | $0.33 \pm 0.01$ | $0.29 \pm 0.03$ | $0.29$    | $0.3328 \pm 0.0102$ | $0.324 \pm 0.004$ |                |

1 Covariant confined quark model.
2 Lattice QCD.
3 HQET to $O(\Lambda_{QCD}^2/m_c^2)$, form factors are determined by fitting to LHCb and lattice QCD data.

V. SUMMARY

In this paper, $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ and $\Omega_b \to \Omega_c^{(*)} \tau \bar{\nu}_\tau$ semileptonic decays have been calculated within the Standard model systematically. In the analysis with lepton-mass effects considered, helicity amplitudes have been given, form factors are expanded in HQET to the
order of $\Lambda_{QCD}/m_b$ and $\Lambda_{QCD}/m_c$, the Isgur-Wise functions obtained by QCD sum rule and large $N_c$ methods have been applied in the calculation. We have obtained decay rates, decay distributions, and R-ratios for $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ and $\Omega_b \to \Omega_c^{(*)} \tau \bar{\nu}_\tau$ decays. The R-ratio $R(\Lambda_c) \simeq (33 \pm 1)\%$ (QCD sum rule), $R(\Lambda_c) \simeq (29 \pm 3)\%$ (large $N_c$ QCD), $R(\Omega_c) \simeq 30.4\%$ (large $N_c$ QCD), and $R(\Omega_c^{(*)}) = 33.2\%$ (large $N_c$ QCD) with an estimated 10% uncertainty for the large $N_c$. These results will be checked by experiments in the near future, such as LHCb, to see if there is any new physics in these decays.
FIG. 3. Partial decay distributions in various helicities of $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ in the Large $N_c$ QCD. $1 \leq \omega \leq 1.31$

FIG. 4. The differential decay rate of $\Omega_b \to \Omega_c \tau \bar{\nu}_\tau$, $1 \leq \omega \leq 1.25$

FIG. 5. Partial decay distributions in various helicities of $\Omega_b \to \Omega_c \tau \bar{\nu}_\tau$. T, L, t stand for $\frac{d\Gamma^T}{d\omega}$, $\frac{d\Gamma^L}{d\omega}$, and $\frac{d\Gamma^t}{d\omega}$ which are defined in Eq. (11), $1 \leq \omega \leq 1.25$
FIG. 6. The differential decay rate of $\Omega_b \rightarrow \Omega_c^* \tau \bar{\nu}_\tau$, $1 \leq \omega \leq 1.23$

FIG. 7. Partial decay distributions in various helicities of $\Omega_b \rightarrow \Omega_c^* \tau \bar{\nu}_\tau$, $T$, $L$, $t$ stand for $\frac{d\Gamma_{T \pm}}{d\omega}$, $\frac{d\Gamma_{L \pm}}{d\omega}$, $\frac{d\Gamma_{t \pm}}{d\omega}$ which are defined in Eq. (12), $1 \leq \omega \leq 1.23$.

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