ANTI-DE SITTER QUOTIENTS,
BUBBLES OF NOTHING, AND BLACK HOLES

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Abstract

In 3+1 dimensions there are anti-de Sitter quotients which are black holes with toroidal event horizons. By analytic continuation of the Schwarzschild-anti-de Sitter solution (and appropriate identifications) one finds two one parameter families of spacetimes that contain these quotient black holes. One of these families consists of B-metrics (“bubbles of nothing”), the other of black hole spacetimes. All of them have vanishing conserved charges.

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1. Introduction

Because of the special properties of the conformal boundary of anti-de Sitter space, one can form black hole spacetimes by taking the quotient of anti-de Sitter space with discrete isometry groups. In 3+1 dimensions there are two classes of such black holes: static “topological” black holes, and a non-stationary example with a toroidal event horizon [1]. The former acquire much of their interest as members of a one parameter family of asymptotically anti-de Sitter black holes [2]. The latter can be regarded as a special case of the Ehlers-Kundt B1 metrics [3]—also known as “Kinnersley type IV.B” [4], or more catchily as “bubbles of nothing” [5, 6]—but these spacetimes do not describe black holes except in one very special case. In fact they are everywhere regular spacetimes. The main purpose of this paper is to point out that there is another one parameter family of spacetimes that does describe black holes for all values of the parameter, and which also contains the toroidal quotient black hole. Although they are free of curvature singularities, they are not everywhere regular. All three families can be locally obtained through analytic continuation from the Schwarzschild-anti-de Sitter spacetimes. The two families that we will discuss share the property that all their conserved charges, in the sense of Ashtekar and Magnon [7], are zero.

In section 2 we will recapitulate the properties of the toroidal anti-de Sitter black hole. In section 3 we present spacetimes analytically related to Schwarzschild. In section 4 we discuss the bubbles of nothing, and in section 5 the black hole family. Section 6 provides a summary.

2. A quotient black hole

Anti-de Sitter space (or adS) is a spacetime with constant non-zero curvature, conveniently regarded as the covering space of the quadric surface

\[ X^2 + Y^2 + Z^2 - U^2 - V^2 = -1 \]  

in a flat space with the metric

\[ ds^2 = dX^2 + dY^2 + dZ^2 - dU^2 - dV^2. \]
The intrinsic metric can be given as

\[ ds^2 = -\left(\frac{1+\rho^2}{1-\rho^2}\right)^2 dt^2 + \frac{4}{(1-\rho^2)^2} \left( d\rho^2 + \rho^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) , \quad (3) \]

where

\[ 0 \leq \rho < 1 , \quad 0 < \theta < \pi , \quad 0 \leq \phi < 2\pi . \quad (4) \]

Anti-de Sitter space is conformal to “one half” of the Einstein universe, and one can add a conformal boundary at \( \rho = 1 \). The conformal boundary is denoted \( \mathcal{J} \), and is itself a conformal copy of the Einstein universe in one dimension less.

In 3+1 dimensions all possible spacetimes arising by performing identifications using one-parameter subgroups of \( SO(3,2) \) have been classified [8, 10]. A black hole is obtained when the subgroup is generated by the Killing vector

\[ \xi = J_{ZU} = Z\partial_U + U\partial_Z . \quad (5) \]

At fixed \( t \) the resulting spacetime can be described as the 2+1 dimensional spinless BTZ black hole [11], rotated around a spatial axis.

The details are explained elsewhere [1, 8, 9, 10], but since it is an interesting black hole we will pause to understand it. Its global isometries are generated by anti-de Sitter Killing vectors commuting with \( J_{ZU} \). They form the Lie algebra \( SO(2) \times SO(2,1) \). A Misner singularity terminates \( \mathcal{J} \), which consists of a single component. In covering space, the event horizon is at

\[ X^2 + Y^2 - V^2 = 0 . \quad (6) \]

Points with \( X^2 + Y^2 - V^2 < 0 \) are invisible from \( \mathcal{J} \), and will therefore lie in the interior of a black hole. There is a Misner singularity also in the physical spacetime—in covering space it is a disk at \( Z = U = 0 \), which is behind the horizon. The horizon itself has some interesting properties. At fixed time \( t > 0 \) it is a torus. One of its cycles is growing, but there is another cycle of constant length. To see why this is so, recall that a Killing horizon is a light cone with a vertex on \( \mathcal{J} \)—its generators do not diverge because its vertex is infinitely far away. An example of a Killing horizon is then given by
\[ X^2 = V^2 , \quad (7) \]

and this null surface shares a one parameter family of generators with the black hole horizon. The Killing horizon is also the boundary of the causal past of a timelike curve ending up on the singular circle at \((t, r, \phi) = (\pi/2, 1, 0)\), hence it is a non-compact observer dependent event horizon. The black hole event horizon is the envelope of the set of all such observer dependent event horizons [9]. In the interior of the black hole we expect to find closed trapped surfaces, surrounded by an apparent horizon lying well inside the event horizon. This is indeed so [8].

In the following we will rely on two coordinate description of the toroidal black hole. A coordinate system covering the region \(X^2 + Y^2 > V^2\), with \(U > 0\), in anti-de Sitter space is

\[
\begin{align*}
X &= r \cosh t \cos \phi & 0 < r < \infty \\
Y &= r \cosh t \sin \phi & 0 \leq \phi < 2\pi \\
Z &= \sqrt{r^2 + 1} \sinh \gamma & -\infty < t < \infty \\
U &= \sqrt{r^2 + 1} \cosh \gamma \\
V &= r \sinh t
\end{align*}
\]

(8)

The metric in these coordinates is

\[
ds^2 = r^2 (-dt^2 + \cosh^2 t d\phi^2) + \frac{dr^2}{r^2 + 1} + (r^2 + 1) d\gamma^2 . \quad (9)\]

The manifest Killing vectors are

\[
J_{ZU} = \partial_\gamma \quad J_{XY} = \partial_\phi . \quad (10)
\]

The black hole is obtained by making \(\gamma\) periodic, and the coordinate system then covers its exterior.

A different coordinate system, covering the region \(U^2 > Z^2\), \(U > 0\), is

\[
\begin{align*}
X &= \sqrt{r^2 - 1} \cos \chi & 1 < r < \infty \\
Y &= \sqrt{r^2 - 1} \sin \chi & 0 \leq \chi < 2\pi \\
Z &= r \sin \tau \sinh \phi & 0 < \tau < \pi \\
U &= r \sin \tau \cosh \phi \\
V &= r \cos \tau
\end{align*}
\]

(11)
These coordinates cover an entire region with spacelike $J_{ZU}$. The metric is

$$ds^2 = r^2(-d\tau^2 + \sin^2 \tau d\phi^2) + \frac{dr^2}{r^2 - 1} + (r^2 - 1)d\chi^2. \quad (12)$$

We have two manifest Killing vectors, namely

$$J_{ZU} = \partial_\phi \quad J_{XY} = \partial_\chi. \quad (13)$$

The black hole is obtained by making $\phi$ periodic. The Misner singularities are then at $\sin \tau = 0$, and the event horizon is at $r = 1/\sin \tau$.

Further local Killing vectors include

$$J_{ZV} = -\sinh \phi \partial_\tau + \cosh \phi \cot \partial_\phi \quad J_{UV} = -\cosh \phi \partial_\tau + \sinh \phi \cot \partial_\phi. \quad (14)$$

Together with $J_{ZU}$ they form a local $SO(2,1)$ algebra. Since they do not commute with $J_{ZU}$ they are not globally defined in the black hole spacetime, but we record them here since they will be of interest in section 5.

3. Analytic relatives of Schwarzschild

A useful trick, capable of producing new solutions from old, is to complexify a solution of Einstein’s equations, and then take a new real slice of the result [12]. Perhaps it is more than a trick, but if so this does not concern us now.

What does concern us is that the Schwarzschild solution has three natural relatives, obtainable by analytic continuation in this way. This gives us four solutions altogether, given in terms of one of the two functions

$$V_{\pm}(r) = \pm 1 - \frac{2m}{r} - \frac{\lambda r^2}{3}. \quad (15)$$

For the moment the cosmological constant $\lambda$ is kept arbitrary. As long as only the local geometry matters the solutions are

$$ds^2 = -V_+(r)dt^2 + \frac{dr^2}{V_+(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) = \quad (16)$$

$$= -V_+(r)dt^2 + \frac{dr^2}{V_+(r)} + r^2(\text{sphere})$$
\[ ds^2 = -V_-(r)dt^2 + \frac{dr^2}{V_-(r)} + r^2(d\theta^2 + \sinh^2 \theta d\phi^2) = \]

\[ = -V_-(r)dt^2 + \frac{dr^2}{V_-(r)} + r^2(\text{hyperbolic plane}) \quad (17) \]

\[ ds^2 = r^2(-dt^2 + \cosh^2 t d\phi^2) + \frac{dr^2}{V_+(r)} + V_+(r)d\gamma^2 = \]

\[ = r^2(\text{de Sitter}) + \frac{dr^2}{V_+(r)} + V_+(r)d\gamma^2 \quad (18) \]

\[ ds^2 = r^2(-d\tau^2 + \sin^2 \tau d\phi^2) + \frac{dr^2}{V_-(r)} + V_-(r)d\chi^2 = \]

\[ = r^2(\text{anti-de Sitter}) + \frac{dr^2}{V_-(r)} + V_-(r)d\chi^2. \quad (19) \]

To go from the metric (16) to (19), say, we use

\[ t \to \chi, \quad r \to ir, \quad \theta \to \tau, \quad \phi \to i\phi, \quad m \to -im. \quad (20) \]

The others are similarly related. The local isometry group of these metrics is \( R \times SO(3) \) for the first case, and \( R \times SO(1,2) \) for the other three. Since the coordinate ranges are as yet undecided, the global isometry groups and the topologies of the underlying spacetimes remain to be found.

The metric (16) is the Schwarzschild, or Schwarzschild-(anti)-de Sitter, solution. The metric (17) is not interesting as it stands, but if the isometries are used to turn the hyperbolic plane into a closed Riemann surface it describes the topological black holes [1, 2]. The metric (18) describes a completely regular spacetime, once the coordinate singularity at \( V_+(r) = 0 \) has been dealt with appropriately [3]. It is referred to as a “bubble of nothing”, and will be discussed in the next section. The metric (19) has been called an “anti-bubble” [13], and is the topic of our section 5.
4. Bubbles of nothing

We will now discuss the “bubble of nothing” metric (18) in some detail, although we will keep the discussion fairly brief because many of the things we will say can be found in the literature already, also for the case \( \lambda < 0 \) [14, 15]. When \( m = 0 \) it reduces to the anti-de Sitter metric (9), which describes the exterior of a toroidal black hole if \( \gamma \) is periodic [16]. For all other values of \( m \) there is a coordinate singularity at \( V(r) = 0 \), and a carefully adjusted periodicity in \( \gamma \) is necessary if one is to obtain an everywhere regular spacetime [3]. It has topology \( \mathbb{R}^3 \times S^1 \).

To be precise, introduce a new radial coordinate \( \sigma \) through

\[
\sigma^2 \equiv V_+(r) = 1 - \frac{2m}{r} - \frac{\lambda r^2}{3} = (r - r_+) \frac{r^2 + r_+ r + r_+^2 + 1}{r}.
\]  

(21)

In the last step we actually assumed that \( \lambda < 0 \), and defined the largest positive root \( r_+ \). Then

\[
dl^2 = \frac{dr^2}{V_+(r)} + V_+(r) d\gamma^2 = \frac{4d\sigma^2}{V_+^2} + \sigma^2 d\gamma^2.
\]  

(22)

This metric will be regular at the origin if and only if \( \gamma \) is periodic with

\[
0 \leq \gamma < \frac{4\pi r_+}{3r_+^2 + 1}.
\]  

(23)

The periodicity in \( \phi \) is arbitrary, although a period which is a multiple of \( 2\pi \) is preferred because it allows a globally defined isometry group \( SO(2) \times SO(2, 1) \).

Consider the case \( \lambda = 0 \). The first detailed discussion of this spacetime was given by Witten [5], who was interested in its five dimensional version as an example of an asymptotically flat solution with vanishing mass. It is not asymptotically Minkowski however; as noted by Aharony et al. [6] it does not admit a sensible \( \mathcal{G} \). One can understand this either as a consequence of a general theorem—an asymptotically simple spacetime cannot have a null infinity with topology \( T^2 \times \mathbb{R} \) [17]—or by inspection of the special case \( m = 0 \). When \( m = 0 \) the spacetime is obtained by identifying points in Minkowski space, using a translation. For Minkowski space \( \mathcal{G}^+ \) has topology \( S^2 \times \mathbb{R} \). Spatial translations act along the null generators on \( \mathcal{G}^+ \), except
Figure 1: Penrose diagram of the adS black hole (left), and of the corresponding B metric. Each point is a torus, except for the points on the dashed lines which are circles. The observer dependent horizons cannot be seen here.

for an equator’s worth of generators that are left untouched. When the identifications are carried through this will result in closed null curves on $\mathcal{J}$.

When $\lambda < 0$ the situation is very different. In this case the intrinsic geometry of $\mathcal{J}$ is independent of $m$. In the coordinates used to express the metric, we choose the conformal factor

$$\Omega = \frac{1}{r}. \tag{24}$$

This ensures that $\mathcal{J}$ is a totally geodesic surface at $\Omega = 0$, with respect to the unphysical metric $\hat{g}_{ab} = \Omega^2 g_{ab}$. Its intrinsic metric is

$$\hat{s}_{\Omega=0}^2 = -dt^2 + \cosh^2 t d\phi^2 + d\gamma^2. \tag{25}$$

Although this is de Sitter space times a circle, the conformal properties are quite different from that of de Sitter space itself [10].

The Penrose diagrams of these spacetimes are given in fig. 1. They do not make full justice to the causal structure, because an observer may end up anywhere on the singular circle that terminates $\mathcal{J}$. This gives rise to observer dependent non-compact Killing event horizons [9, 6], which can be seen only if we rotate the Penrose diagram around a vertical axis. This latter operation also explains why the $m \neq 0$ solutions are referred to as “bubbles of nothing” in the literature: the vertical axis of rotation should be placed so that the dashed curve turns into a de Sitter hyperboloid, and there is simply nothing in the center of the picture [6].
What is the mass? The proper definition of conserved charges for asymptotically anti-de Sitter spacetimes has been the subject of a large body of recent research, as one can see by consulting, say, Gibbons et al. [18] or Papadimitriou and Skenderis [19]. Our case is a relatively simple one since its conformal boundary is even dimensional and conformally flat. Therefore we will rely on the definitions by Ashtekar and Magnon [7], which are based on $\mathcal{F}$. When its cross sections are closed 2-manifolds, $\mathcal{F}$ will always admit conserved charges. The explicit expressions were given by Ashtekar and Magnon—who assumed that the topology is $S^2 \times \mathbb{R}$, but their charges remain conserved in our case. They are expressed in terms of an asymptotic Killing vector together with the rescaled Weyl tensor

$$K_{abcd} = \frac{1}{\Omega}C_{abcd} = \Omega C_{abcd}.$$  \hspace{1cm} (26)

More precisely we need its electric and magnetic parts

$$E_{ab} = K_{acbd}n^c n^d \quad B_{ab} = \star K_{acbd}n^c n^d, \quad n_a = \nabla_a \Omega.$$ \hspace{1cm} (27)

For the metric at hand we find the non-vanishing components

$$E_{tt} = m \quad E_{\phi\phi} = -m \cosh^2 t \quad E_{\gamma\gamma} = 2m.$$ \hspace{1cm} (28)

The magnetic part vanishes identically, which means that $\mathcal{F}$ is conformally flat.

The conserved charges are defined by

$$Q = \oint E_{ab} \xi^a dS^b,$$ \hspace{1cm} (29)

where the integral is taken over a toroidal cross section of $\mathcal{F}$, and $\xi$ is an asymptotic Killing vector, hence a conformal Killing vector of the metric (25). But the most general such vector field available is a linear combination of the four vector fields

$$\xi_1 = \cos \phi \partial_t - \tanh t \sin \phi \partial_\phi \quad \xi_3 = \partial_\phi$$
$$\xi_2 = \sin \phi \partial_t + \tanh t \cos \phi \partial_\phi \quad \xi_4 = \partial_\gamma.$$ \hspace{1cm} (30)
Provided that \( \phi \) is periodic with period \( 2\pi n \) all of these exist globally, but it is easily checked that all the conserved charges vanish. Hence the mass of these bubbles of nothing is zero, even though superenergies and quasi-local masses are non-zero. The mass of the toroidal black hole is also zero. This reminds us of the C-metric, which has zero ADM mass even though its Bondi news tensor is non-vanishing [20].

5. Black hole spacetimes

Now consider the metric (19), with \( \lambda = -3 \):

\[
d s^2 = r^2 ( - d\tau^2 + \sin^2 \tau d\phi^2 ) + \frac{d r^2}{V_- (r)} + V_- (r) d\chi^2 , \quad V_- (r) = r^2 - 1 - \frac{2m}{r} . \tag{31}
\]

The coordinates \( \tau \) and \( \phi \) cover a maximal causal diamond in a 1+1 dimensional anti-de Sitter space. When \( m = 0 \) and the coordinate \( \phi \) is made periodic the metric is that of the quotient black hole, in the coordinate system where its metric takes the form (12). The coordinate \( \chi \) must be periodic in order to avoid a coordinate singularity at \( V_- (r) = 0 \). The appropriate period is

\[
0 \leq \chi < \frac{4 \pi r_+}{3 r_+^2 - 1} , \tag{32}
\]

where \( r_+ \) is the largest real root of \( V_- (r) \). The range of the radial coordinate is then \( r > r_+ \). It is not hard to see that

\[
2m = r_+^3 - r_+ . \tag{33}
\]

Hence \( m \) can have either sign, but we make the restriction that \( r_+ \) be positive because if \( r \) is allowed to take the value zero a curvature singularity arises. Since \( r_+ \) is also the largest root we find that

\[
m \geq - \frac{1}{3 \sqrt{3}} \quad \quad r_+ \geq \frac{1}{\sqrt{3}} . \tag{34}
\]

The locally anti-de Sitter case is at \( r_+ = 1 \). With this restriction the curvature remains bounded everywhere, but since we make \( \phi \) periodic there are Misner singularities at \( \tau = 0 \) and \( \tau = \pi \).
The Misner singularities will be present also on the conformal boundary $\mathcal{J}$. Indeed, as a three dimensional manifold $\mathcal{J}$ is independent of $m$, and the discussion of the anti-de Sitter case $m = 0$ can be taken over verbatim [10]. In particular there will be an event horizon bounding the region of spacetime that can be seen from $\mathcal{J}$. It can be located by solving for radial null geodesics ending up at the Misner singularity at $\tau = \pi$. This means that a point with coordinates $(r, \tau)$ belongs to the horizon only if

$$\int_0^\pi d\tau = \int_r^\infty \frac{dr}{r\sqrt{V(r)}} = \int_r^\infty \frac{dr}{\sqrt{r\sqrt{r^3 - r - 2m}}}.$$  (35)

This is an elliptic integral of the first kind, degenerating to an elementary integral at $r_+ = 1/\sqrt{3}$, $r_+ = 1$, and $r_+ = 2/\sqrt{3}$.

![Figure 2: This is $\tau_+$ as a function of $r_+$; these quantities are defined in the text.](image)

We are especially interested in $\tau_+$, which is the value of $\tau$ at which the event horizon is born. Clearly

$$\tau_+ = \pi - \int_{r_+}^\infty \frac{dr}{\sqrt{r\sqrt{r^3 - r - 2m}}}.$$  (36)

The numerical solution is displayed in Fig. 2. The fact that

$$\tau_+ \sim \pi - \frac{\text{constant}}{r_+}$$  (37)

in the limit of large $r_+$ is easily verified analytically. Note also that $\tau_+$ diverges to negative infinity at $r_+ = 1/\sqrt{3}$. Negative values of $\tau_+$ are not
realized; when they occur they mean that no complete spatial slice, going all the way down to $r_+$, is visible from $\mathcal{I}$.

The causal structure of these spacetimes can be displayed by Penrose diagrams where each point is a torus. See Fig. 3. As in the previous cases these diagrams do less than full justice to the causal structure because the observer dependent horizons cannot be seen. On the other hand the dependence of the causal structure on $r_+$, and implicitly on $m$, is seen clearly. When $m$ is large the diagram is very tall, and when $m$ is close to its lower bound it becomes a rectangle whose width is much larger than its height.

![Penrose diagrams for the black hole family.](image)

**Figure 3:** Penrose diagrams for the black hole family. They depend on $r_+$, and hence on $m$, in a characteristic way.

Locally the isometry group of the solution is $\mathbb{R} \times SO(2,1)$, just as for the bubbles of nothing. However, on examination one finds that the local Killing vectors are given in terms of the intrinsic coordinates by eqs. (13-14). But the global structure of the solution breaks the symmetries described by eqs. (14), so the global isometry group is only $SO(2) \times SO(2)$. The case $m = 0$ is exceptional in that it has more symmetries, not given by these Killing vectors.

The Ashtekar-Magnon conserved charges are therefore very easy to compute. Just as for the bubble of nothing, the electric part of the rescaled Weyl tensor is diagonal (and its magnetic part vanishes), but this time there are no global Killing vector having a "time" component, and so the Ashtekar-
Magnon conserved charges are zero. None of them seem to deserve the name “mass”.

6. Summary

We have observed that the toroidal anti-de Sitter black hole [1] can be regarded as a member of two different one parameter families of spacetimes, both of them obtainable by analytic continuation from the Schwarzschild-anti-de Sitter solution, if suitable topological identifications are made. One of these families contains regular spacetimes called “bubbles of nothing”, and they have been discussed in some detail elsewhere [3, 5, 6]. The other family consists of black hole spacetimes, with Misner singularities to terminate \( \mathcal{M} \), but without curvature singularities. The global isometry group of the latter is just \( SO(2) \times SO(2) \), and the causal structure is similar to that of the anti-de Sitter black hole itself. Both families have vanishing conserved charges in the sense of Ashtekar and Magnon [7], hence the mass (if any) is zero. We argued that this is unsurprising.

We think that these results are of some interest as examples of model spacetimes; perhaps they can find some use in the context of the adS/CFT correspondence. However, although the locally anti-de Sitter black hole exists in all dimensions, it is not obvious to us how to generalize the entire black hole family to dimensions higher than four.
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