Experimentally Detecting Quantized Zak Phases without Chiral Symmetry in Photonic Lattices

Zhi-Qiang Jiao,1,2 Stefano Longhi,3,4 Xiao-Wei Wang,1,2 Jun Gao,1,2 Wen-Hao Zhou,1,2 Yao Wang,1,2 Yu-Xuan Fu,1 Li Wang,1,2 Ruo-Jing Ren,1,2 Lu-Feng Qiao,1,2 and Xian-Min Jin1,2,5,∗

1Center for Integrated Quantum Information Technologies (IQIT), School of Physics and Astronomy and State Key Laboratory of Advanced Optical Communication Systems and Networks, Shanghai Jiao Tong University, Shanghai 200240, China
2CAS Center for Excellence and Synergetic Innovation Center in Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China
3Dipartimento di Fisica, Politecnico di Milano, Piazza L. da Vinci 32, I-20133 Milano, Italy
4IFISC (UIB-CSIC), Instituto de Física Interdisciplinar y Sistemas Complejos, E-07122 Palma de Mallorca, Spain
5TuringQ Co., Ltd., Shanghai 200240, China

(Dated: September 29, 2021)

Symmetries play a major role in identifying topological phases of matter and in establishing a direct connection between protected edge states and topological bulk invariants via the bulk-boundary correspondence. One-dimensional lattices are deemed to be protected by chiral symmetry, exhibiting quantized Zak phases and protected edge states, but not for all cases. Here, we experimentally realize an extended Su-Schrieffer-Heeger model with broken chiral symmetry by engineering one-dimensional zigzag photonic lattices, where the long-range hopping breaks chiral symmetry but ensures the existence of inversion symmetry. By the averaged mean displacement method, we detect topological invariants directly in the bulk through the continuous-time quantum walk of photons. Our results demonstrate that inversion symmetry protects the quantized Zak phase, but edge states can disappear in the topological nontrivial phase, thus breaking the conventional bulk-boundary correspondence. Our photonic lattice provides a useful platform to study the interplay among topological phases, symmetries, and the bulk-boundary correspondence.

Topological phases of matter are fascinating states that escape from the standard description of Ginzburg-Landau theory, exhibiting protected edge states and quantized topological properties in the bulk [1–5]. A paradigm of topological phases in condensed-matter physics is the integer quantum Hall state in a two-dimensional electron gas [1]. Topological order is of major relevance in a variety of nonelectronic systems as well, including mechanical platform, cold atoms, acoustics, trapped ions and photonics [6–15]. A central result in the theory of topological matter is that bulk topological invariants are connected to the number of protected edge states (bulk-boundary correspondence) [4, 5]. Hence topological phases can be probed either in the bulk or at the edges, as demonstrated in several recent experiments for Hermitian and even some non-Hermitian systems [16–34]. However, the common belief that a nontrivial topological phase implies the presence of protected edge states is not a general result, and even in Hermitian systems it is known that there could exist nontrivial topological phases that do not exhibit edge states at all [4, 35–37]. In such systems, edge dynamics alone cannot thus provide a safe diagnostic of a topological phase. A notable example is provided by one-dimensional (1D) and 2D lattices with broken chiral symmetry but with preserved inversion symmetry [4, 36–41]. In the Altland-Zirnbauer (AZ) classification, nontrivial topological phases are identified by three main symmetries: time-reversal, chiral and particle-hole symmetries [5, 42]. In this scheme, a topologically nontrivial 1D system should possess chiral symmetry, and the bulk topological invariant is the winding number corresponding to a quantized Zak phase [43]. A paradigmatic example is provided by the famous Su-Schrieffer-Heeger (SSH) model of polyacetylene [2, 4], where the bulk-boundary correspondence holds [4, 44].

Experimental demonstrations of quantized Zak phases in the SSH model with chiral symmetry have been reported in recent works [29, 45] using different platforms and methods to extract the Zak phase from bulk dynamics. Long-range hopping in the SSH model that breaks chiral symmetry makes the system topologically trivial. However, when inversion symmetry is preserved, the Zak phase remains quantized and one can introduce a $Z_2$ topological number which identifies two distinct topological phases, which are not caught by the AZ classification [36–38]. Interestingly, unlike the chiral SSH model, in the extended SSH model with inversion symmetry only, the quantized Zak phase can no longer predict the number of edge states since bulk-boundary correspondence cannot be established under inversion symmetry solely [4].

In this Letter, we report on the experimental demonstration of quantized Zak phases in the extended SSH model with broken chiral symmetry and demonstrate that the bulk topological invariant cannot predict the number of edge states. The two-band insulating system is built on a photonic chip under a continuous-time quantum walk framework. Waveguide lattices are a natural platform to study topological properties, both in the bulk and at the edges, in virtue of photon dynamics [17, 18, 46–49]. The quantized Zak phase can be directly detected from bulk dynamics by means of the beam displacement method [29, 39], while edge state dynamics can be visualized by boundary excitation of the lattice [18, 39]. We constructed an extended SSH lattice using a zigzag design to introduce weak next nearest neighbor couplings (NNNCs) in order to break the chiral symmetry [37], as shown in Fig.1. By tuning the intercell and intracell hopping amplitudes, we observe quantization of the Zak phase. The nontrivial topological edge states are initially protected by inversion sym-
and B breaks the chiral, but not the inversion symmetry pro-
vide arrangement. The long-range hopping in sublattices A
= A sublattices, odd sites
ponents site lattice of even number with staggered hopping coeffi-
tions (NNNCs) 1 = 1 sublattices, odd sites

become mapped into the longitudinal photon propagation in the
lattices. The temporal evolution of the insulating system can
localize and flow inside the bands as the NNNCs is increased

where: NNNCs 1, 1 = 1 sublattices, odd sites

and even sites

As shown Fig.1, we realize an extended
SSH model with NNNCs 1, 1 using a binary zigzag wave-
guide arrangement. The long-range hopping in sublattices A
and B breaks the chiral, but not the inversion symmetry pro-
vided that 1 = 1 [36, 37]. The tight-binding Hamiltonian
of the photonic lattice reads [11],

where 1 (n) and 1 (n) are the creation (annihilation) op-

 Sergeant 3

To measure the Zak phase in our photonic platform, we
use the mean displacement method [29, 39], looking at the
asymptotic mean spatial displacement of a time-evolved wave
packet corresponding to a single-site excitation in the bulk of
the lattice. We define the mean wave packet displacement at
propagation distance z (corresponding to evolution in time
t = z/c) as

\[
\mathcal{P}(z) = \sum_{n} n \left| A_n(z) \right|^2 + \left| B_n(z) \right|^2,
\]

where

\[
A_n(z) = A_n(z) e^{i \phi(z)}
\]

and

\[
B_n(z) = B_n(z) e^{i \phi(z)}
\]

are the field amplitudes in sublattice A (B) in the
n-th cell. The averaged mean wave packet displacement
in the interval (0, Z) is defined by

\[
\mathcal{P}(Z) = \frac{1}{Z} \int_0^Z \mathcal{P}(z) dz.
\]

For large Z, \( \mathcal{P}(Z) \) approaches a constant asymptotic value
[29, 39]. Remarkably, when the two-band system displays

FIG. 1. Extended SSH lattice. Schematic of a bipartite lattice in a
zigzag geometry, comprising two sublattices A and B with staggered
coupling constants 1 and 2 and with next nearest neighbor cou-
lings (NNNCs) 1 and 1 = 1. The NNNCs break the chiral
symmetry of the lattice while preserving the inversion symmetry.

The system displays chiral (1) and inversion (R) symmetry
whenever 1 = 1 1 and R = 1 1 [5]. For a two-band system, the two symmetries are defined
by the unitary operators 1 = 1 and R = 1 , thus a lattice
with chiral symmetry requires 1 = 1 = 1 = 1, i.e. 1 = 1 = 0, while inversion symmetry only requires
1 = 1. As a result, an equal NNNCs strength inside sublat-
tices makes the chiral symmetry broken, while inversion sym-
metry is preserved. The Zak phase, that is, the Berry phase ac-
cumulated by an eigenstate adiabatically transporting through
the whole Brillouin zone, is a bulk property of the insulating
system that is quantized in the presence of chiral symmetry.
When the chiral symmetry is broken but the system still pos-
sesses inversion symmetry, i.e. for 1 = 1, the NNNCs
terms modify the energy spectrum of the bulk Hamiltonian
(2) but not the corresponding eigenfunctions, so that quanti-
zation of the Zak phase is preserved by inversion symmetry.
In this case the Zak phase in the two bands takes the same
value, given by

\[
\gamma = \frac{1}{2} \int_{-\pi}^{\pi} \frac{\partial \phi}{\partial k} = \pi \mathcal{W}
\]

where \( \mathcal{W} = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{\partial \text{Im} \phi}{\partial k} \) is the winding number asso-
ciated to the motion of the vector \( \text{Im} \phi(\mathbf{n}) = (\cos \phi, -\sin \phi, 0) \).

For a chiral system, \( \gamma = 0 \) corresponds to the topological
trivial phase (1 > 1) without edge states in a chain with
open boundaries, while \( |\gamma| = \pi \) corresponds to the nontriv-
ial topological phase (1 < 1) with two zero-energy edge
states (which do not hybridize in the thermodynamic limit).
To change \( |\gamma| \) from \( \pi \) to 0, i.e. to remove edge states, one
needs to close and reopen the gap at 1 = 1, where the two
bands touch at k = ±\pi. For a system with broken chiral sym-
metry but with inversion symmetry, the eigenfunctions (and
thus the Zak phase \( \gamma \)) are independent of the NNNCs, how-
ever the energy bands depend on the NNNCs and gap closing
can occur for a strong NNNC without bound touching (owing
to the indirect nature of the gap [37,39]). As a consequence,
edge states in the topological nontrivial phase \( |\gamma| = \pi \) can de-
localize and flow inside the bands as the NNNCs is increased
[36,37,39], which is impossible in an insulating system with
chiral symmetry owing to the bulk-boundary correspondence.

To measure the Zak phase in our photonic platform, we
use the mean displacement method [29,39], looking at the
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\]

For large Z, \( \mathcal{P}(Z) \) approaches a constant asymptotic value
[29,39]. Remarkably, when the two-band system displays
distribution versus propagation distance $z$. (a) and (c) show the numerically computed light intensity for details). The results are depicted in Fig. 2. Figures various output planes are recorded on a CMOS camera (see laser at 780 nm wavelength, and the intensity distributions at are probed by exciting the central waveguide with a coherent from 0 mm to 9.5 mm with an interval of 0.5 mm. The arrays lattices with a series of evolution lengths increasing gradually the averaged mean displacement, we fabricate the photonic system is an insulator with a direct open gap. To calculate in inversion symmetry, one has $W = 2\overline{P}(Z)$ for large $Z$ [39]. Hence a measurement of the mean beam displacement provides a measure of the winding number $W$, and thus of the Zak phase $\gamma$.

The binary waveguide lattice in the zigzag geometry shown in Fig. 1 is manufactured on a photonic chip by the femtosecond laser direct-writing technique [50–52]. We prepare four photonic chips to realize four kinds of lattices in different topological phases and tuned NNNCs strengths. Each array comprises 20 unit cells with open boundaries. The hopping coefficients are determined by the evanescent mode coupling between adjacent waveguides, which can be tailored by controlling waveguide spacing [46]. Once the hopping parameters are fixed, the geometry structure of the lattice can be easily generated. The first chip realizes a topological trivial insulator with the hopping coefficients $J_1 = 0.30$, $J_2 = 0.13$, and $J_A = J_B = 0.05$ (in units of 1/mm), corresponding to intercell distance 13.2 $\mu$m, intracell distance 11.2 $\mu$m, and NNNCs distance 16.5 $\mu$m. The second chip realizes a topological nontrivial lattice, with intercell distance 10.7 $\mu$m, intracell distance 14.4 $\mu$m, and NNNCs distance 16.5 $\mu$m, corresponding to $J_1 = 0.09$, $J_2 = 0.36$ and $J_A = J_B = 0.05$ (in units of 1/mm). In both cases, the weak NNNCs ensures that the system is an insulator with a direct open gap. To calculate the averaged mean displacement, we fabricate the photonic lattices with a series of evolution lengths increasing gradually from 0 mm to 9.5 mm with an interval of 0.5 mm. The arrays are probed by exciting the central waveguide with a coherent laser at 780 nm wavelength, and the intensity distributions at various output planes are recorded on a CMOS camera (see [46] for details). The results are depicted in Fig. 2. Figures 2 (a) and (c) show the numerically computed light intensity distribution versus propagation distance $z$ in the topologically trivial (a) and nontrivial (c) phases for a evolution length of 15 mm. The mean displacement $P(z)$ versus $z$ is obtained from the recorded intensity distributions at various propagation distances [insets in Figs. 2(b) and (d)]. The experimental results are shown in Figs. 2 (b) and (d). In these plots, the red dots refer to the experimental measurements of $P(z)$, while the blue solid curves correspond to the simulated results obtained from the intensity maps of Figs. 2 (a) and (c). The averaged mean displacements $\overline{P}(Z)$ at $Z = 9.5$ mm, obtained from the fitting theoretical curves of $P(z)$ and using Eq.(5), are depicted by the horizontal gray dashed lines in Figs. 2(b) and (d). In the topological trivial lattice [panel (b)], one obtains $\overline{P}(Z) = 0.001 \pm 0.05$, in good agreement with the theoretical winding number $W = 0$. In the topological nontrivial lattice [panel (d)], a winding number $W \approx -1$ can be estimated from the measured mean displacement value $\overline{P}(Z) = -0.455 \pm 0.08$. Uncertainties in $\overline{P}(Z)$, indicated by the $\pm$ error bars, are estimated by assuming possible error in the coupling constants of the lattices, arising from fabrication imperfections of the geometric structure, that can slightly deviate from the target values and thus change the fitting curves $P(z)$ and the corresponding mean value $\overline{P}(Z)$.

To study edge dynamics, we excited the two lattices at the left/right edge waveguides and recorded the intensity distributions at the output facets of the chips, as shown in Fig. 3 (c) and (d). For a topological trivial lattice, there are no edge states and the energy spectrum, shown in Fig. 3(a), is formed by two bands without gapped states; this results in a clear delocalization of light far from the initially excited boundary waveguides, as shown in Fig. 3(c). Note that the energy spectrum is not symmetric around $E = 0$ because of the broken chiral symmetry introduced by the NNNCs. In the topological nontrivial lattice, two gapped states, corresponding to two localized edge states at the left and right boundaries of the lattice, clearly emerge in the energy spectrum [Fig. 3(b)], resulting in

![Fig. 2. Averaged mean displacement and bulk topological number. (a) (c) is the simulated light intensity dynamics in the bulk excitation of trivial (nontrivial) topological lattices. The light intensities of each waveguide are displayed versus the propagation distance $z$ on a pseudo color map. The topological trivial phase ($W = 0$) corresponds to $J_1 > J_2$, while $J_1 < J_2$ corresponds to the topological nontrivial phase ($W = -1$). (b) and (d) are the averaged mean displacements in trivial and nontrivial cases. The solid blue curves show the theoretical fitting curves of the spatial beam displacement $P(z)$ versus $z$. The experimental results are labeled by red dots. Every dot corresponds to the mean beam displacement obtained from the 40-site recorded images (examples at $z = 6.5$ mm are shown in insets of figures). The averaged mean displacement of trivial (nontrivial) lattice is 0.005 (0.455), highlighted by the gray horizontal dashed line.](image-url)
FIG. 3. The energy spectrum and edge dynamics. (a) and (b) show the energy spectrum of the trivial and nontrivial lattices, respectively. In (a) there is an energy gap between the two bands without any gapped state, while in (b) there are two gapped (edge) states, partly protected by inversion symmetry. (c) and (d) are the normalized intensity distributions (NID) of the 40-site lattices in the trivial and nontrivial phases, respectively, as measured at the output plane of the chips. The photonic lattices are excited in the two edges labeled by the dashed gray circles.

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FIG. 4. Delocalization of edge states in the topological phase. (a,b) Intensity light patterns at the output facets of nontrivial topological lattices, initially excited at the two edges, imaged by a CMOS camera. In (a) \( J_A > J_2/2 \) (open gap), while in (b) \( J_A < J_2/2 \) (closed gap). (c) Normalized light intensity distribution (NID) versus lattice site \( n \) (a) (blue) and (b) (red) after \( Z = 18 \) mm propagation distance. For the strong NNNCs case the energy gap closes, causing the light to delocalize in the bulk of the lattice (shown in red columns).

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mission of Shanghai Municipality (STCSM) (20JC1416300, 2019SHZDZX01), Shanghai Municipal Education Commission (SMEC)(2017-01-07-00-02-E00049). X.-M.J. acknowledges additional support from a Shanghai talent program. X.-M.J. acknowledges support from a Shanghai talent program and support from Zhiyuan Innovative Research Center of Shanghai Jiao Tong University.

* xianmin.jin@sjtu.edu.cn

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