Design of SINS/LDV/OD autonomous positioning system based on carrier constraints

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Abstract. Aiming at the problem of vehicle positioning orientation technology in actual combat background, this paper studies the autonomous positioning system of SINS/LDV/OD, and designs the LDV/SINS and OD/SINS subfilters of the union kalman filter and the main filter of the autonomous positioning system. Based on the characteristics of the fault of the subfilter, the fault tolerant scheme of the autonomous positioning system was designed, and the 50Km roadster test was finally carried out. The longitude error is not more than 20m, the latitude is less than 20m, and the height error is not over 10 meters. The experimental results show that the design scheme is effective and can meet the actual operational requirements.

1. Introduction

The principle of SINS is usually adopted in the autonomous positioning system. However, the drift error of the inertial measurement component will accumulate gradually in the integral link when using the digital recursion to realize the positioning of the vector, which leads to the increase of the system positioning error [1].

There are two ways to reduce errors: one is integrated with the satellite navigation system, but the signal is greatly affected by the environment[2].The other is SINS is assisted by the measurement information of external sensors, which can effectively avoid the loss of auxiliary positioning information. Laser Doppler Velocimeter (LDV) and Odometer (OD) are commonly used external auxiliary sensors. The two methods are used together to realize high precision real-time positioning of the carrier.

2. Principle of velocity measurement

Carrier speed is one of the key parameters of the navigation system. There are four main methods for measuring carrier velocity: GPS signal receiver, LDV, OD, and accelerometer[3].

The drift error of accelerometer accumulates with time, GPS is greatly affected by environment.LDV and OD are directly mounted on the vehicle, with strong autonomy, high accuracy and good reliability.

2.1. Basic principle of vehicle LDV speed measurement

The vehicle LDV is usually equipped with dual optical differential structure. As shown in figure 1, The laser emits a laser beam, which is divided into two parallel equal-intensity lasers through a beam splitter. After the lens, the two laser beams are delivered to the surface of the object, and the reflected laser is received by the detector after the reflection mirror.
Figure 1. The principle of dual optical differential velocity measurement

The wavelength of the laser is \(\lambda\), the convergence angle of the two laser beams is \(\theta\), and the distance between the laser interference fringes displayed on the detector:

\[
\delta = \frac{\lambda}{2 \sin(\theta/2)} \quad (1)
\]

When there is a relative motion between the LDV and the ground, the scattering object's surface diffuse laser is received by the detector and converted into electrical signals. The interference between the two laser beams received by the detector varies with the intensity of the reflected laser beams, and the frequency of the interference changes is the doppler frequency. It’s assumed that the velocity of LDV relative to the scattered surface is \(\nu\), so the doppler frequency is:

\[
f_{LDV} = \frac{\nu}{\delta} = \frac{2\nu}{\lambda} \cdot \frac{\sin\theta}{2} \quad (2)
\]

The main error of LDV is the scale factor error caused by the velocity error, and the velocity error can be approximated by the correlation velocity error and calibration coefficient error caused by terrain noise.

The LDV can only measure the speed of the vehicle's forward direction. The speed of the vehicle is \(\nu^m\) when it is not considered to be skidding and jumping. Which means:

\[
\nu_r^b = \begin{bmatrix} 0 & \nu_r^b & 0 \end{bmatrix}^T \quad (3)
\]

The scale factor error is \(\delta K_L\), and the relation between the velocity measurement value of LDV and the vehicle speed is:

\[
\nu^m_L = (1 + \delta K_L)\nu_r^b \quad (4)
\]

In the above formula, \(\nu^m_L\) denotes the speed of a LDV measurement vehicle.

The error angle \(\phi = [\delta \theta \ \delta \gamma \ \delta \phi]^T\) is considered when the LDV is installed, and the error angle is treated with a small angle after calibration. The coordinate system for measuring the angular velocity of LDV is called \(m\) series, and the direction cosine matrix of \(m\) to \(b\) is:

\[
C^b_m = \begin{bmatrix}
1 & \delta \phi & -\delta \gamma \\
-\delta \phi & 1 & \delta \theta \\
\delta \gamma & -\delta \theta & 1
\end{bmatrix} \quad (5)
\]

The projection of the LDV measurement velocity in the \(b\) system is:
\[ \ddot{v}_L^b = c_m^b \dot{v}_L^m = (1 + \delta K_L) \begin{bmatrix} 1 & \delta \phi & -\delta \gamma \\ -\delta \phi & 1 & \delta \theta \\ \delta \gamma & -\delta \theta & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \delta \phi + \delta \phi \delta K_L \\ 1 + \delta K_L \\ -\delta \theta - \delta \theta \delta K_L \end{bmatrix} v_r^b \]  

Equation (6) indicates that the velocity error is related to the installation deviation angle \( \delta \phi \) and \( \delta \theta \), and has nothing to do with \( \delta \gamma \).

The actual measurement speed of LDV is expressed as:

\[ \hat{v}_L^b = v_r^b + \delta v_r^b \]  

In the formula, \( \hat{v}_L^b \) is the carrier velocity of the actual measurement of LDV, \( v_r^b \) is the carrier velocity of LDV measurement in the ideal state, and \( \delta v_r^b \) is the measurement error.

Ignore the high order in \( \delta v_r^b \):

\[ \delta v_r^b = [\delta \phi \ \delta K_L \ -\delta \theta]^T v_r^b \]  

According to the above analysis, the LDV velocity error equation is:

\[ \dot{X}_L = F_L X_L + G_L W_L \]  

Where, the state vector \( F_L = [\delta \phi \ \delta K_L \ -\delta \theta] \), \( X_L = [v_x, v_y, v_z] \).

**2.2. Principle of OD measurement**

OD is used for position navigation. By measuring the winding number of the wheel, the ratio relation between the number of turns of the wheel and the distance between the driving distance is measured, and the instantaneous coefficient is determined to determine the driving distance.

The OD velocity formula is:

\[ S_O = K_O \times N_O \]  

\[ \Delta S_O = V_O = K_O \times \Delta N_O \]  

In the formula, \( K_O \) is the calibration coefficient, \( N_O \) is the number of wheel turns in the odometer, \( S_O \) is the driving distance of the car, \( V_O \) is the speed of the car, and \( \Delta N_O \) is the number of wheel turns in the unit time.

In the period \( k \) to \( k+1 \), the odometer measures the projection of the rotation winding number in \( b \) system:

\[ \Delta S_{O,k+1}^n = C_{b,k}^n \begin{bmatrix} 0 & \Delta S_{O,k} & 0 \end{bmatrix}^T \]  

The dead-reckoning model is:

\[ \begin{align*}
L_{O,k} &= L_{O,k-1} + \frac{\Delta S_{O,k}^n(N)}{R_n + h_{O,k-1}} \\
\lambda_{O,k} &= \lambda_{O,k-1} + \frac{\Delta S_{O,k}^n(E) \sec L_{O,k-1}}{R_n + h_{O,k-1}} \\
h_{O,k} &= h_{O,k-1} + \Delta S_{O,k}^n(U)
\end{align*} \]  

The error sources of the model are two: installation angle error and scale factor error. The installation angle error \( \phi_O \) can be regarded as a constant after the calibration. The scale factor error \( \delta K_O \) can be expressed by the first-order markov process.
\[
\delta K_o = e^{-\tau} \delta K_{O,k-1} + W_{O,k-1} \tag{14}
\]

In the formula, \( \tau \) is the relevant time of markov process, \( T \) is the odometer output cycle, and \( \phi_0 \times \) is the angle error of OD installation.

The projection model of OD actually in \( b \) system \([4]\) is:

\[
\Delta \hat{S}_{O,k}^n = C_{h,k-1} \left[ I - (\phi_0 \times) \right] (1 + \delta K_o) \Delta S_{O,k}^n \tag{15}
\]

The estimated error of OD position is \( \delta \Delta S_{O,k}^n \):

\[
\delta \Delta S_{O,k}^n = \Delta \hat{S}_{O,k}^n - \Delta S_{O,k}^n
\]

\[
= \Delta S_{O,k}^n \times \phi_o + \Delta S_{O,k}^n \delta K_o \tag{16}
\]

The actual dead-reckoning equation is:

\[
\begin{align*}
\hat{L}_{O,k} &= \hat{L}_{O,k-1} + \frac{\Delta \hat{S}_{O,k}^n (N)}{R_M + h_{O,k-1}} \\
\hat{\lambda}_{O,k} &= \hat{\lambda}_{O,k-1} + \frac{\Delta \hat{S}_{O,k}^n (E) \sec \hat{L}_{O,k-1}}{R_N + h_{O,k-1}} \\
\hat{h}_{O,k} &= \hat{h}_{O,k-1} + \Delta \hat{S}_{O,k}^n (U)
\end{align*}
\tag{17}
\]

So, the error model for \( k \) moment's reckoning error is:

\[
\begin{bmatrix}
\delta L_{O,k} \\
\delta \lambda_{O,k} \\
\delta h_{O,k}
\end{bmatrix} =
\begin{bmatrix}
\delta \hat{L}_{O,k} \\
\delta \hat{\lambda}_{O,k} \\
\delta \hat{h}_{O,k}
\end{bmatrix} = M_{O,1} \delta \hat{p}_{O,k-1} + M_{O,2} \delta \Delta S_{O,k}^n
\]

\[
M_{O,1} =
\begin{bmatrix}
1 & 0 & \frac{\Delta S_{O,k}^n (N)}{(R_M + h_{O,k-1})^2} \\
\frac{\Delta S_{O,k}^n (E) \sec L_{O,k-1} \tan L_{O,k-1}}{R_N + h_{O,k-1}} & 1 & -\frac{\Delta S_{O,k}^n (E) \sec L_{k-1}}{(R_N + h_{O,k-1})^2} \\
0 & 0 & 1
\end{bmatrix}
\tag{18}
\]

\[
M_{O,2} =
\begin{bmatrix}
0 & \frac{1}{R_M + h_{O,k-1}} & 0 \\
\frac{\sec L_{k-1}}{R_N + h_{O,k-1}} & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\tag{19}
\]

According to the above analysis, the DR error equation is:

\[
\hat{X}_O = F_O X_O + G_O W_O \tag{21}
\]
Where, the state vector is:
\[ X_O = \begin{bmatrix} L \ 
\lambda \\ h \\
\phi_x \ 
\phi_y \ 
\phi_z \ 
\delta K_o \end{bmatrix} \] (22)

The coefficient matrix is:
\[
F_O = \begin{bmatrix}
M_{O,1} & M_{O,2} (\Delta S_{O,k}^n \times) & M_{O,3} \Delta S_{O,k}^n \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{T}{\tau} e^{-\frac{T}{\tau}}
\end{bmatrix}
\] (23)

\( G_O \) and \( W_O \) are noise coefficient matrix and noise, which can be selected according to the actual situation.

3. **Basic principles of federal kalman filter**

Kalman filtering is divided into concentrated filtering and distributed filtering\(^5\). Centralized filtering has a large amount of calculation and poor fault tolerance. The output reliability of the system is poor. Therefore, this paper adopts the decentralized filtering, namely the federal kalman filter, to construct the filter of the integrated navigation system.

3.1. **Algorithm of federal filter**

The federal kalman filter contains both subfilter and main filter two-stage filtering structure. In order to ensure the consistency of information transmission, the information of each sub-filter is distributed according to the following principles.

Assuming that the total noise \( Q^{-1} \) in the filtering process is allocated to each sub-filter and the main filter, so:
\[
Q^{-1} = \sum_{i=1}^{N} Q^{-1} + Q^{-1}_m
\] (24)

While,
\[
Q_i = \beta_i Q
\] (25)

So,
\[
Q^{-1} = \sum_{i=1}^{N} \beta_i Q^{-1} + \beta_m Q^{-1}
\] (26)

\[
\sum_{i=1}^{N} \beta_i + \beta_m = 1
\] (27)

The initial information of the state is allocated according to the principle of information distribution:
\[
P^{-1} = P^{-1}_1 + P^{-1}_2 + \cdots + P^{-1}_N + P^{-1}_m = \sum_{i=1}^{N} \beta_i P^{-1} + \beta_m P^{-1}
\] (28)

According to the above principle of information distribution, the global optimal estimation can be expressed as follows, on the premise that the main filter and the sub-filter are independent of each other.
\[
\begin{cases}
\dot{X}_s = P_s \sum_{i=1}^{N} P^{-1}_i \dot{X}_i \\
X_s = (\sum_{i=1}^{N} P^{-1}_i)^{-1}
\end{cases}
\] (29)
In the formula, \( \hat{X}_g \) represents the global estimation state, and \( P_g \) represents the minimum covariance matrix of \( \hat{X}_g \) estimation error.

3.2. Structure and performance analysis of federal filters

The federal filter is generally divided into subfilter and main filter, as shown in figure 2.

![Figure 2. General structure of the federal filter](image)

In the process of designing the Federal Kalman Filter, the global total noise \( Q^{-1} \) is allocated to the main filter and the subfilter, the distribution coefficient of \( \beta_i \) can take different values for different types of filter structure\(^6\). The typical structure is shown in table 1. FDI refers to system fault detection and isolation ability, and FR refers to the system failure recovery capability.

| Serial number | \( \beta_m \) | \( \beta_i \) | Reset mode | the subfilter FDI | The main filter FDI | main System FR |
|---------------|----------------|----------------|-------------|-------------------|-------------------|----------------|
| 1             | 1              | 0              | Zero type   | poor              | top               | medium         |
| 2             | 1/(N+1)        | 1/(N+1)        | reset       | medium            | medium            | medium         |
| 3             | 0              | 1/N            | reset       | medium            | poor              | medium         |
| 4             | 0              | 1/N            | no reset    | top               | top               | Top            |
| 5             | 1              | 0              | reset       | poor              | top               | Poor           |
| 6             | 0.5            | 0.5            | reset       | medium            | medium            | medium         |

3.3. The principle and method of system-level fault detection

Once the integrated autonomous positioning system fails, the navigation accuracy will be seriously affected. Therefore, in this paper, a system-level fault detection and isolation method based on \( \chi^2 \) test\(^7\) is adopted to find isolated fault sources.

\( \hat{X}_k \) is obtained by the measured value \( Z_k \) through Kalman filter, which is affected by the system failure; \( \hat{X}_k^s \) is obtained by the state recursion with prior information, which is independent of measurement and is not affected by system failure. The difference between \( \hat{X}_k \) and \( \hat{X}_k^s \) can be used for fault detection and isolation. \( \hat{X}_k \) and \( \hat{X}_k^s \) can be calculated using the following formula:
\[
\begin{align*}
\hat{X}_k &= \left[I - K_k H_k \right] \Phi_{k,k-1} \hat{X}_{k-1} + K_k Z_k \\
\hat{X}_0 &= X^0 \\
P_{k|k-1} &= \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^T \\
P_k &= \left[I - K_k H_k \right] P_{k|k-1} \\
P_0 &= P^0 \\
K_k &= P_{k|k-1} H_k^T \left[H_k P_{k|k-1} H_k^T + R_k \right]^{-1} \\
\hat{X}_k^S &= \Phi_{k,k-1} \hat{X}_{k-1}^S \\
\hat{X}_k^S &= X^0 \\
P_{k|k-1}^S &= \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^S + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^T \\
P_0^S &= P^0
\end{align*}
\] (30)

In equation (30), \(X^0\) is a Gaussian random vector, so \(X_k\), \(\hat{X}_k\), \(\hat{X}_k^S\) are Gaussian vectors, and the estimation error is defined as follows:

\[
\begin{align*}
e_{ik} &= (X_k - \hat{X}_k) - (X_k^S - \hat{X}_k^S)
\end{align*}
\] (32)

When the system fails, \(E\{e_{ik}\} \neq 0\). Making binary hypothesis for \(e_{ik}\):

\[
\begin{align*}
H_0: \text{No fault} & \quad E\{e_{ik}\} = 0, E\{e_{ik} e_{ik}^T\} = P_{ik} \\
H_1: \text{fault} & \quad E\{e_{ik}\} = \mu, E\{(e_{ik} - \mu)(e_{ik} - \mu)^T\} = P_{ik}
\end{align*}
\]

Suppose the fault detection function is:

\[
\lambda_k = e_{ik}^T P_{ik}^{-1} e_{ik}
\] (33)

The criteria for failure determination:

\[
\begin{align*}
\lambda_k > T_D & \quad \text{accept } H_1, \text{ fault} \\
\lambda_k \leq T_D & \quad \text{accept } H_0, \text{ no fault}
\end{align*}
\] (34)

The value of \(T_D\) in the fault decision criteria can be checked, see the references\[8\].

4. Filter design of autonomous positioning system

The initial navigation accuracy of strapdown inertial navigation system is very high, but it decreases gradually with the increase of time. Therefore, SINS/LDV/OD autonomous positioning system based on vehicle constraint model is constructed.

4.1. Structure design of autonomous positioning system

In the vehicle autonomous positioning system, the SINS is used as a common reference system, and LDV and OD are added, and the Federated Kalman filter is constructed by combining the vehicle auxiliary information, as shown in figure 3.
4.2. Sub-filter design

According to the SINS error model, the position, velocity and attitude error are selected, as well as the gyro and accelerometer drift to form the 15-dimensional state vector, and construct the error equation of SINS. The SINS error equation is written as follows:

$$\dot{\hat{X}}_S = F_S \hat{X}_S + G_S W_S \tag{35}$$

In the equation,

Coefficient matrix of state vector:

$$F_S = \begin{bmatrix} M_2 & M_1 & 0_{b3} & 0_{b3} & 0_{b3} \\ G_1 & G_2 & G_3 & 0_{b3} & C^s \\ 0_{3b} & 0_{3b} & 0_{3b} & 0_{153} \\ 0_{15b} & 0_{15b} & 0_{15b} & C^n \end{bmatrix} \tag{36}$$

Assuming that:

$$R_v = 6378137.0 \quad f = 1/298.275 \quad \omega \beta = 15^\circ / h \quad R_i = R_e (1 - 2f + 3f^2 \sin^2 L) + h ,$$

$$R_\delta = R_e (1 + f^2) L + h \quad R_h = R_e + h .$$

System noise:

$$W_S = \begin{bmatrix} w_{gx} & w_{gy} & w_{gz} & w_{rx} & w_{ry} & w_{rz} & w_{ax} & w_{ay} & w_{az} \end{bmatrix} \tag{37}$$

Coefficient matrix of noise matrix:

$$G_S = \begin{bmatrix} 0_{3x3} & 0_{3x3} & C^n \\ 0_{3x3} & 0_{3x3} & 0_{3x3} \\ C^n & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & I_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & I_{3x3} \end{bmatrix} \tag{38}$$
In the formula, $C^n_b$ represents the direction cosine matrix from the carrier coordinate system to the navigation coordinate system.

1) SINS/LDV subfilters

This sub-filter adopts the method of centralized filtering, and selects the error equation of SINS (35) and the error equation of LDV (9) as the state equation of the subfilter. The result of the velocity reduction between SINS and LDV is used as the observation quantity of the subfilter, and the measurement equation of the subsystem is established.

**Equation of state:**

\[
\begin{bmatrix}
\dot{X}_S^g \\
\dot{X}_L^g
\end{bmatrix} =
\begin{bmatrix}
F_S & 0 \\
0 & F_L
\end{bmatrix}
\begin{bmatrix}
X_S^g \\
X_L^g
\end{bmatrix} +
\begin{bmatrix}
G_S & 0 \\
0 & G_L
\end{bmatrix}
\begin{bmatrix}
W_S^g \\
W_L^g
\end{bmatrix}
\]

**Measurement equation:**

\[
Z_1 = HX + V = \begin{bmatrix} H_1 \\ H_1^T \end{bmatrix}
\begin{bmatrix}
X_s \\
X_l
\end{bmatrix} + V_1
\]

In the formula, $Z = [S - V]$, $H_1 = [0_{3x3} I_{3x3} 0_{12x3}]$, $H_1^T = -I_{3x3}$, $V_1$ is the noise measurement array of the subsystem, which is independent of the system noise, and its variance matrix $R_1$ is:

\[
R_1 = \text{diag}\left[(0.04m/s)^2 \quad (0.04m/s)^2 \quad (0.04m/s)^2\right]
\]

2) SINS/OD subfilters

This sub-filter adopts the method of centralized filtering, the error equation of SINS (35) and the error equation of DR (21) are selected as the state equation of the subfilter when the subfilter is established by means of concentrated filtering. Based on the difference of the position between the two, the measurement equation of the subsystem is established.

**Equation of state:**

\[
\begin{bmatrix}
\dot{X}_S^g \\
\dot{X}_O^g
\end{bmatrix} =
\begin{bmatrix}
F_S & 0 \\
0 & F_O
\end{bmatrix}
\begin{bmatrix}
X_S^g \\
X_O^g
\end{bmatrix} +
\begin{bmatrix}
G_S & 0 \\
0 & G_O
\end{bmatrix}
\begin{bmatrix}
W_S^g \\
W_O^g
\end{bmatrix}
\]

**Measurement equation:**

\[
Z_2 = HX + V = \begin{bmatrix} H_2 \\ H_2^T \end{bmatrix}
\begin{bmatrix}
X_s \\
X_0
\end{bmatrix} + V_2
\]

In the formula, $Z_2 = [P_s - P_o]$, $H_2 = [I_{3x3} 0_{3x4}]$, $H_2^T = [-I_{3x3} 0_{3x4}]$, $V_2$ is the noise measurement array of the subsystem, which is independent of the system noise, and its variance matrix $R_2$ is:

\[
R_2 = \text{diag}\left[(0.001')^2 \quad (0.001')^2 \quad (10m)^2\right]
\]

### 4.3. Design of main wave device

1) Discretization of the system:

The equation of state and the measurement equation established by Kalman filter are continuous and cannot be operated on the computer. Therefore, the established state equation and the measurement equation are discretized first. Discrete formula of continuous equation:

\[
X_k = \Phi_{k,k-1}X_{k-1} + \Gamma_{k,k-1}W_{k-1}
\]

\[
Z_k = H_kX_k + V_k
\]

The formula for $F(t)$ to $\Phi_{k,k-1}$ in the continuous equation is:
\[ \Phi_{k,k-1} = \sum_{n=0}^{\infty} \left[ F(t_k)T \right]^n / n! \quad (44) \]

(2) distribution of information:
\[ P_{ij}^{-1} = \beta_i P_{ii}^{-1}, \quad Q_{ij}^{-1} = \beta_i Q_i^{-1}, \quad \sum_{i=1}^{3} \beta_i = 1 \quad (45) \]

(3) filtering of main system:
The subsystem adopts the centralized Kalman filter, the filtering period is 1 second. The main filter takes 10 seconds to perform an information fusion calculation, and the fusion algorithm:
\[
\begin{align*}
\hat{X}_m &= P_m \sum_{i=1}^{3} P_{ii}^{-1} \hat{X}_i \\
\hat{P}_m &= \left( \sum_{i=1}^{3} P_{ii}^{-1} \right)^{-1}
\end{align*}
\quad (46)\]

4.4. Fault tolerance design of the system

The concrete structure of fault tolerant system is shown in figure 4. Fault detection of sub-filters can detect and isolate faulty sub-filters. The error estimate of the remaining subfilters is normally entered into the main filter, and the error estimation of the system can be carried out, and the result of the main filter can be used to repair and detect the sub-filter of the fault\cite{10}. And then provide precise information about navigation.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The structure of SINS/LDV/OD fault tolerant system based on carrier constraints}
\end{figure}

Suppose the error estimates for the three sub-filters are: \( \hat{X}_1, P_1, \hat{X}_2, P_2, \hat{X}_3, P_3 \), when the system works normally, the overall error estimate is:
\[
\hat{X}_g = P_g \left( P_1^{-1} \hat{X}_1 + P_2^{-1} \hat{X}_2 + P_3^{-1} \hat{X}_3 \right) \quad (47)
\]
\[
P_g = (P_1^{-1} + P_2^{-1} + P_3^{-1})^{-1} \quad (48)
\]

When the subfilter fails, the result \( \hat{X}_i, P_i \) of the failure of the subfilter is to be removed from the overall system estimation error expression to achieve the purpose of the isolation system failure.

Taking the failure of subfilter 1 as an example, the estimation error of the main filter is:
\[
\hat{X}_g = P_g (P_{21} \hat{X}_1 + P_{31} \hat{X}_3)
\]

(49)

\[
P_g = (P_{31}^{-1} + P_{51}^{-1})^{-1}
\]

(50)

The results of failure subsystem 1 are estimated by the following formula:

\[
\hat{Z}_1 = H_1 \hat{X}_g
\]

(51)

In the same way, the results of failure subsystem 2 and 3 are estimated by the following formula:

\[
\hat{Z}_2 = H_2 \hat{X}_g, \hat{Z}_3 = H_3 \hat{X}_g
\]

(52)

5. Experimental verification

In order to verify the validity and positioning accuracy of SINS/LDV/OD autonomous positioning system based on carrier constraints, vehicle validation was carried out in Xi'an area. The test car and SINS are shown in figure 5.

![Figure 5. The test car and SINS.](image)

Install OD on vehicle. The error of OD calibration coefficient is 0.01309, the error of installation angle is 0.70031', error of yaw angle is 0.72031' and the error of pitch angle is -0.08559'. After the vehicle was started, the SINS were initially aligned for 5min, and the calibration coefficient error was calibrated within two minutes.

The method of subfilter failure simulation is: during the experiment, the specific sensor stops collecting data during a fixed period of time, and other sensors work normally.

The test of the vehicle started from (108.764, 34.03) and ended at (108.606, 34.02), with a total distance of 56.6km, taking 112min, of which 424s was still. The driving route is shown in figure 6.

![Figure 6. Test track of the car.](image)

![Figure 7. Test position error of the car](image)

Figure 7 shows the position error of the actual vehicle running. As can be seen from figure 7, the longitude error in the experiment is no more than 20m. The latitude error is no more than 10m and the height error is no more than 5m.
During the experiment, the LDV fault in the autonomous positioning system was simulated and the other equipment was working normally. LDV data collection was not recorded when the car was tested in 1500s to the 1800s. The output position error of subfilter 1 is shown in figure 8, and the speed error is shown in figure 9.

When the subfilter 1 fails, the LDV error is estimated. As can be seen from the figure 8 longitude error no more than 20 m, latitude error is not more than 25 m, height error is not more than 10 m, can be seen from the figure 9 east and north to velocity error is not more than 0.1 m, days to the velocity error is not more than 0.05 m.

The OD failure in the autonomous positioning system is simulated, and the other equipment works normally. When the sports car test 2500s to 2800s, OD data collection was not recorded. The output position error of subfilter 3 is shown in figure 10, and the speed error is shown in figure 11.

When the subfilter 2 fails, the estimation error of the navigation position is estimated. Can be seen from the figure 10 error is not more than 10 m, longitude latitude error no more than 20 m, height error is not more than 10 m, can be seen from the figure 11 east and north to velocity error is not more than 0.1 m, days to the velocity error is not more than 0.05 m.

When the subfilter 3 fails, the estimation error of the navigation position is estimated. Can be seen from the figure 12 error is not more than 10 m, longitude latitude error no more than 20 m, height error is not more than 10 m, can be seen from the figure 13 east and north to velocity error is not more than 0.1 m, days to the velocity error is not more than 0.05 m.
During the experiment, the Angle sensor fault in the autonomous positioning system is simulated and the other equipment works normally. When the sports car test 3500s to 3800s, the Angle sensor was not recorded to collect data. The position error of the output of subfilter 3 is shown in figure 12, and the velocity error is shown in figure 13. When the subfilter 3 fails, the subfilter 3 error is estimated. As can be seen from the figure 12 longitude error no more than 20 m, latitude error no more than 20 m, height error no more than 5 m, can be seen from the figure 13 east and north to velocity error is not more than 0.1 m, days to the velocity error is not more than 0.05 m.

![Figure 14](image) location error of main filter

![Figure 15](image) speed error of main filter

All the equipment that simulates the autonomous positioning system works normally. In the car test, the position error of the main filter is shown in figure 14, and the speed error is shown in figure 15. When the federal filter works normally, it can be seen from figure 12 that the longitude error is not more than 5m, the latitude error is not more than 5m, and the height error is not more than 4m. From figure 13, it can be seen that the error of east direction, north direction and sky speed is not more than 0.05m.

As can be seen from fig. 10 to fig.17, the position and velocity error of the subfilter are estimated and compensated by the information fusion technology when the subfilter fails. Can be seen from the sports car test result: the information fusion technology to fault sensor error estimation and compensation, to keep autonomous positioning system error in a reasonable range, to ensure the system to work normally.

The results show that the error caused by the LDV fault is smaller than the error caused by the OD fault, and the effect on the combined navigation is small. At the same time, with the extension of time, the positioning accuracy of the combination system tends to be stable, but the error always exists. Combination of design positioning system were analyzed, and found that the fusion algorithm of error equation of defects, lead to actual compensation and the theoretical calculation of bias, fusion algorithm need to be further improved. Through the experiment, it is proved that the integrated navigation system designed in this paper is reasonable and effective.

6. Conclusion

The principle of LDV/OD velocity measurement is described in this paper, and its error source is analyzed. According to the error sources of LDV and OD, the error equations of SINS, LDV and OD were established. This paper introduces the basic principle of federal Kalman filter is designed based on constraint model of SINS/LDV/OD carrier independent federal Kalman filter positioning system, and its state equation and measurement equation is given, as well as the system fault tolerance design. Finally, the car test is carried out, and the experimental results show that the designed federal filter is reasonable and effective.

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