Testing Contextuality in Cyclic Psychophysical Systems of High Ranks

Ru Zhang\textsuperscript{1} and Ehtibar N. Dzhafarov\textsuperscript{2}

\textsuperscript{1}Purdue University and Indiana University
zhang617@purdue.edu

\textsuperscript{2}Purdue University
ehtibar@purdue.edu

\textbf{Abstract.} Contextuality-by-Default (CbD) is a mathematical framework for understanding the role of context in systems with deterministic inputs and random outputs. A necessary and sufficient condition for contextuality was derived for cyclic systems with binary outcomes. In quantum physics, the cyclic systems of ranks $n = 5, 4, \text{ and } 3$ are known as systems of Klyachko-type, EPR-Bell-type, and Leggett-Garg-type, respectively. In earlier publications, we examined data collected in various behavioral and social scenarios, from polls of public opinion to our own experiments with psychophysical matching. No evidence of contextuality was found in these data sets. However, those studies were confined to cyclic systems of lower ranks ($n \leq 4$). In this paper, contextuality of higher ranks ($n = 6, 8$) was tested on our data with psychophysical matching, and again, no contextuality was found. This may indicate that many if not all of the seemingly contextual effects observed in behavioral sciences are merely violations of consistent connectedness (selectiveness of influences).

\textbf{Keywords:} contextuality, contextuality-by-default, cyclic systems, consistent connectedness, psychophysical matching

1 Introduction

Consider a system having two external factors $\alpha$ and $\beta$, which can be deterministically manipulated, and two random outputs $A$ and $B$ that we interpret as responses to or measurements of $\alpha$ and $\beta$, respectively. The system can belong to any empirical domain, from quantum physics to behavioral sciences. If manipulating $\beta$ does not change the marginal distribution of $A$ and manipulating $\alpha$ does not change the marginal distribution of $B$, we say that the system is consistently connected. Physicists traditionally test contextuality by assuming consistent connectedness (referred to as “no-signaling,” “no-disturbance,” etc.). However, even in quantum experiments inconsistent connectedness may occur, e.g., because of context-dependent errors in measurements. In behavioral sciences
inconsistent connectedness is ubiquitous. The Contextuality-by-Default (CbD) theory allows one to detect and measure contextuality, or to determine that a system is noncontextual, irrespective of whether it is consistently connected \[1–8\]. In quantum physics, many experiments and theoretical considerations demonstrate the existence of contextual systems \[9–15\], including in cases when consistent connectedness is violated \[8\]. By contrast, we found no evidence of contextuality in various social and behavioral data sets, from polls of public opinion to visual illusions to conjoint choices to word combinations to psychophysical matching \[16\][17\].

Most of the experimental studies of contextuality, both in quantum physics and in behavioral and social sciences, have been confined to cyclic systems. Using our introductory example, let $\alpha$ and $\beta$ vary on two levels each, denoted $\alpha_1, \alpha_2$ and $\beta_1, \beta_2$, forming four treatments that we call contexts:

\[
\begin{align*}
\text{Context 1} & \quad \text{Context 2} & \quad \text{Context 3} & \quad \text{Context 4} \\
(\alpha_1, \beta_1) & \quad (\beta_1, \alpha_2) & \quad (\alpha_2, \beta_2) & \quad (\beta_2, \alpha_1)
\end{align*}
\]

In accordance with the main CbD principle, the outputs of the system are labeled both by what they measure (respond to) and by the context in which this happens. This yields eight random variables:

\[
(A_1^1, B_1^1), (B_2^1, A_2^2), (A_3^2, B_3^3), (B_4^3, A_4^4),
\]

where the superscript of $A$ or $B$ is the index of context and the subscript of $A$ or $B$ indicates the factor level of $\alpha$ or $\beta$. It is assumed in addition that each random output is binary: $\{-1, +1\}$. The system thus formed is called a cyclic system of rank 4, by the number of distinct inputs and distinct contexts. The pairs of $A$ and $B$ with the same superscripts (i.e., recorded in the same contexts) are jointly distributed, so that, e.g., the joint probability of $A_1^1 = 1$ and $B_1^1 = 1$ is well-defined; but this is not the case for any two random outputs with different superscripts.

Note that the order in which we list the elements of the contexts, $\alpha \rightarrow \beta$ or $\beta \rightarrow \alpha$, is immaterial; we alternate the orders only to make the cyclic structure more apparent

\[
\begin{align*}
\alpha_1 & \quad \text{Context 1} & \quad \beta_1 \\
\beta_2 & \quad \text{Context 4} & \quad \alpha_2 \\
\text{Context 3} & \quad \text{Context 2}
\end{align*}
\]

In this paper we only deal with cyclic systems of even ranks, those that can be formed using the paradigm with two experimental factors $\alpha, \beta$ and two outputs in each context. A cyclic system of an even rank $2n \geq 4$ can be extracted from a design in which $\alpha$ and $\beta$ vary on $n$ levels each, denoted $\alpha_1, \alpha_2, \ldots, \alpha_n$ and $\beta_1, \beta_2, \ldots, \beta_n$. Out of $n^2$ possible treatments one extracts $n$ pairs whose elements
Cyclic Psychophysical Systems

form a cycle, e.g.,

\[
\text{Context 1 Context 2 \ldots Context (2n - 1) Context 2n} \\
(\alpha_1, \beta_1) \quad (\beta_1, \alpha_2) \quad \ldots \quad (\alpha_n, \beta_n) \quad (\beta_n, \alpha_1)
\]  

The corresponding set of \(4n\) binary \((-1, +1)\) random outputs is

\[
\{(A_1^1, B_1^1), (B_1^2, A_2^2), \ldots, (A_{2n-1}^{2n-1}, B_{2n}^{2n-1}), (B_{2n}^{2n}, A_1^1)\}.
\]

Again, the outputs sharing a context and only such outputs are jointly distributed. Clearly, for \(2n > 4\) one can form multiple cyclic systems from the same \(n \times n\) design, by permuting the \(\beta\)'s paired with the successively indexed \(\alpha\)'s.

Cyclic systems of rank 2 and of odd ranks cannot be formed in this way, but they can be created in other experimental paradigms. Experimental studies of contextuality in quantum-physical cyclic systems were confined to ranks 3, 4, and 5. In behavioral and social experiments and surveys the ranks of the cyclic systems explored were 2, 3, and 4 [16, 17]. In the present study we analyze cyclic systems of ranks 4, 6, and 8.

A necessary and sufficient condition for (non)contextuality was derived for cyclic systems with binary outcomes in Refs. [4, 6, 8]. We present this condition for cyclic systems of even ranks, (4)-(5): such a system is noncontextual if and only if

\[
\Delta C = s_1 (\langle A_1^1 B_1^1 \rangle, \langle B_1^2 A_2^2 \rangle, \ldots, \langle A_{2n-1}^{2n-1} B_{2n}^{2n-1} \rangle, \langle B_{2n}^{2n} A_1^1 \rangle)
\]

\[
-\text{ICC} - (2n - 2) \leq 0
\]

with

\[
|\text{ICC} = |\langle A_1^1 \rangle - \langle A_1^{2n} \rangle| + |\langle B_1^1 \rangle - \langle B_1^{2n} \rangle| + \ldots + |\langle A_{2n-2}^{2n-2} \rangle - \langle A_{2n-1}^{2n-1} \rangle| + |\langle B_{2n-1}^{2n-1} \rangle - \langle B_{2n}^{2n} \rangle|.
\]

Here, \(s_1 (x_1, \ldots, x_k)\) is the maximum of all linear combinations \(\pm x_1 \pm \ldots \pm x_k\) with odd numbers of minuses, and \(\langle \cdot \rangle\) denotes the expected value. Recall that the values of all random variables are labeled +1 and −1. If a system is consistently connected, ICC vanishes.

### 2 Experiments

The experimental design and procedure were described in detail in Ref. [17]. Three different psychophysical matching tasks were used (Figure 1): dot position reproduction task (Experiment 1(a) and 1(b)), concentric circles reproduction task (Experiment 2(a), 2(b) and 2(c)), and floral shape reproduction task (Experiment 3(a) and 3(b)). Each of the seven experiments was conducted on three participants.

In each experimental trial, the participants were shown two stimuli on a computer screen, as shown in Fig. 1. One was a fixed stimulus, the other stimulus
was adjustable, by means of rotating a trackball. The participants were required to change this stimulus until it appeared to match the position or shape of the fixed target stimulus. Once a match was achieved, she or he clicked the button on the trackball to terminate the trial. Each stimulus was characterized by two parameters. For the target stimulus these parameters are denoted as $\alpha$ and $\beta$, and their values in each trial were generated from a pre-defined set of numbers. The values of the same parameters in the matching stimulus are denoted $A$ and $B$ (as they randomly vary for given values of $\alpha$ and $\beta$). Table 1 shows the parameters used.

![Stimuli](image)

**Fig. 1.** Stimuli used in the (a) dot position reproduction task, (b) concentric circles reproduction task, and (c) floral shape reproduction task.

| Task                        | $\alpha$          | $\beta$          | $A$            | $B$            |
|-----------------------------|-------------------|-------------------|----------------|----------------|
| Dot position reproduction   | Horizontal        | Vertical          | Horizontal     | Vertical       |
| (rectangular coordinates)   | coordinate of the target dot | coordinate of the matching dot | coordinate of the matching dot |               |
| Dot position reproduction   | Radial            | Angular           | Radial         | Angular       |
| (polar coordinates)         | coordinate of the target dot | coordinate of the matching dot | coordinate of the matching dot |               |
| Concentric circle           | Radius of         | Radius of         | Radius of      | Radius of     |
| reproduction                | the target circle 1 | the target circle 2 | the matching circle 1 | the matching circle 2 |
| Floral shape reproduction,  | Amplitude 1       | Amplitude 2       | Amplitude 1    | Amplitude 2   |
| see [8]                     | of the target shape | of the target shape | of the matching shape | of the matching shape |

**Table 1.** External factors ($\alpha, \beta$) and random outputs ($A, B$) for the three types of tasks.
The trials were separated by .5 second intervals. Each experiment took several days, each of which consisted of about 200 trials with a break in the middle. Each such section began by a practice series of 10 trials (which were not used for data analysis).

The original data sets for all the experiments are available as Excel files online (http://dx.doi.org/10.7910/DVN/OJZKKP). Each file corresponds to one participant in one experiment.

Table 2. The values of $\alpha$ and $\beta$ used in the analysis of the experiments. {...} are lists of values; [...] and (...) are intervals from which the values were chosen randomly.

| Experiment                        | $\alpha \times \beta$ |
|-----------------------------------|------------------------|
| Rectangular sub-design of Experiment 1(a) | {32 px, 64 px} × {32 px, 64 px} |
| Polar sub-design of Experiment 1(a)     | {53.67 px, 71.55 px} × {63.43 deg, 26.57 deg} |
| Rectangular design of Experiment 1(b)   | [20 px, 80 px] × [20 px, 80 px] |
| Polar sub-design of Experiment 1(b)     | [40 px, 90 px] × [30 deg, 60 deg] |
| Experiment 2(a)                      | {16 px, 56 px, 64 px} × {48 px, 72 px, 80 px} |
| Experiment 2(b)                      | {12 px, 24 px} × {18 px, 30 px} |
| Experiment 2(c)                      | [18 px, 48 px] × [56 px, 86 px] |
| Experiment 3(a)                      | [-18 px, 10 px, 14 px] × [-16 px, -12 px, 20 px] |
| Experiment 3(b)                      | [-30 px, 30 px] × [-30 px, 30 px] |

2.1 Participants

All the participants were students at Purdue University. The first author of this paper, labeled as P3, participated in all the experiments. Participants P1 and P2 participated in Experiments 1(a) and 2(a), and Participants P4 and P5 in Experiments 1(b), 2(b), 2(c), 3(a), and 3(b). All participants were about 25 years old and had normal or corrected to normal vision.

2.2 Stimuli and Procedure

Visual stimuli consisting of curves and dots were presented on a flat-panel monitor. The diameter of the dots and the width of the curves was 5 pixels (px). The stimuli were grayish-white on a comfortably low intensity background. The participants viewed the stimuli in darkness using a chin rest with a forehead support at the distance of 90 cm from the monitor, making 1 screen pixel approximately 62 sec arc.

Experiment 1 In Experiment 1(a), each trial began with presenting two circles with a dot in the first quadrant of each circle (as shown in Figure 1(a)). The dot
in the upper left circle was fixed at one of randomly chosen six positions. These
six positions contained a $2 \times 2$ “rectangular” sub-design: \{32 px, 64 px\} $\times$ \{32 px, 64 px\} and a $2 \times 2$ “polar” sub-design: \{53.67 px, 71.55 px\} $\times$ \{63.43 deg, 26.57 deg\}, as shown in Table 2. The coordinates were recorded using the center of the circle as the origin. The adjustable-position dot was in the bottom right circle. The task was to move the bottom right dot by rotating the trackball to a position that matched that of the fixed one. There were 1200 trials overall.

Experiment 1(b) was identical to Experiment 1(a) except the horizontal coordi-

cate and vertical coordinate of the target dot were random integers drawn

down from the interval [20 px, 80 px), as shown in Table 2. This “rectangular” design

also contained a “polar” sub-design [40 px, 90 px) $\times$ [30 deg, 60 deg). The overall

number of trials for the “rectangular” design was 1800, for the polar sub-design about 900.

Experiment 2 In each trial of Experiment 2(a), the target stimulus on the left

consisted of two concentric circles and a dot in their center. The radii of circle 1

and circle 2 were randomly chosen from the sets \{16 px, 56 px, 64 px\} and \{48 px, 72 px, 80 px\}, respectively (Table 2). At the beginning of each trial the right

stimulus was a dot. The participants had to reproduce the target stimulus by

rotating the trackball to “blow up” two circles from that dot one by one. They

had the freedom to produce the inner or the outer circle first. Once the first

matching circle was produced, the participants clicked a button on the trackball

to confirm this circle and then the program enabled them to “blow up” the other

circle. After the second circle was created, the trial was terminated by clicking

the same button. There were 1800 trials overall.

Experiment 2(b) was identical to Experiment 2(a) except that in each trial the

radii of the target circles were randomly chosen from four possibilities \{12 px, 24 px\} $\times$ \{18 px, 30 px\}. There were 1600 trials overall.

Experiment 2(c) was identical to Experiment 2(a) except that in each trial

the radii of the target circles were integers randomly chosen from \[18 px, 48 px\]

$\times$ \{56 px, 86 px\}. There were 1800 trials overall.

Experiment 3 Two floral shapes (Figure 1(c)) were presented simultaneou-

sly in each trial in Experiment 3(a). The target one was on the left. The right one

was modifiable. Each floral shape was generated by a function

\[
\begin{align*}
    x &= \cos(0.02 \pi \Delta)[70 + \alpha \cos(0.06 \pi \Delta) + \beta \cos(0.1 \pi \Delta)], \\
    y &= \sin(0.02 \pi \Delta)[70 + \alpha \cos(0.06 \pi \Delta) + \beta \cos(0.1 \pi \Delta)],
\end{align*}
\]

where $\Delta$ is polar angle and $x$ and $y$ are the horizontal and vertical coordinates

(in pixels). For a matching floral shape, $\alpha, \beta$ are replaced with $A, B$, respectively.

The amplitudes $\alpha, \beta$ of the target shape were randomly chosen from the sets \{-18 px, 10 px, 14 px\} and \{-16 px, -12 px, 20 px\}, respectively. The two amplitudes

of the right shape were randomly initialized from the interval [-35 px, 35 px). The
participants were asked to match the left shape by modifying the right shape by rotating the trackball. There were 1800 trials overall.

Experiment 3(b) was identical to Experiment 3(a) except that the two amplitudes of the target shape were randomly chosen integers from the interval $[-30\,\text{px}, 30\,\text{px})$.

3 Results

In each experiment the matching points that were too far from the target values were considered outliers and they were removed from data analysis. The outliers made less than 1% of all data. Ref. [17] briefly reported how contextuality for cyclic systems of rank 4 was tested using the data collected from our seven experiments. In this paper, we present the contextuality test for rank 4 in greater details, and add the analyses for cyclic systems of ranks 6 and 8, using the same data.

3.1 Testing Contextuality for Rank 4

In the “rectangular” sub-design of Experiment 1(a), the experimental conditions, represented in the rectangular coordinates, are $(\alpha_1, \beta_1)$, $(\alpha_1, \beta_2)$, $(\alpha_2, \beta_1)$, and $(\alpha_2, \beta_2)$, where $\alpha_1 = 32\,\text{px}$, $\alpha_2 = 64\,\text{px}$, $\beta_1 = 32\,\text{px}$, and $\beta_2 = 64\,\text{px}$.

The “polar” sub-designs of Experiment 1(a) and Experiment 2(b) also have a $2 \times 2$ structure, and they are presented as cyclic systems analogously.

Experiment 2(a) or Experiment 3(a) have $3 \times 3$ factorial designs, and we extracted 9 cyclic systems of rank 4 from each of them.

The “rectangular” design of Experiment 1(b) and the “polar” sub-designs of Experiment 1(b), Experiment 2(c), and Experiment 3(b) have external factors spanning certain intervals. In order to have a cyclic system of rank 4, each interval was dichotomized into two subintervals. For instance, four experimental conditions $(\alpha_{i_1}, \beta_{i_2})$, $i_1, i_2 \in \{1, 2\}$, are formed if one chooses $\alpha_1 = [20\,\text{px}, 50\,\text{px})$, $\alpha_2 = [50\,\text{px}, 80\,\text{px})$, $\beta_1 = [20\,\text{px}, 50\,\text{px})$, and $\beta_2 = [50\,\text{px}, 80\,\text{px})$. Of course other cut-off points can be chosen to dichotomize the intervals. In this paper, we only report the results from the midpoint-dichotomized data sets.

Irrespective of the experiment, the random outputs $A_{i_1 i_2}, B_{i_1 i_2}$ should each be dichotomized. The two values for each random variable were defined by choosing a value $a_{i_1}$ and a value $b_{i_2}$ and computing

$$A_{i_1 i_2}^* = \begin{cases} 
+1 & \text{if } A_{i_1 i_2} > a_{i_1} \\
-1 & \text{if } A_{i_1 i_2} \leq a_{i_1} 
\end{cases}, \quad A_{2 i_2}^* = \begin{cases} 
+1 & \text{if } A_{2 i_2} > a_{2} \\
-1 & \text{if } A_{2 i_2} \leq a_{2} 
\end{cases},$$

$$B_{i_1 1}^* = \begin{cases} 
+1 & \text{if } B_{i_1 1} > b_1 \\
-1 & \text{if } B_{i_1 1} \leq b_1 
\end{cases}, \quad B_{i_2 2}^* = \begin{cases} 
+1 & \text{if } B_{i_2 2} > b_2 \\
-1 & \text{if } B_{i_2 2} \leq b_2 
\end{cases}.$$
and analogously for \(a_2\) and \(b_2\). For each choice of the quadruple, we applied the test [6]-[7] to the distributions of the obtained \(A^*\) and \(B^*\) variables.

No positive \(\Delta C\) was observed, indicating the absence of contextuality in the rank 4 cyclic system for each participant in each experiment.

We present an example to illustrate how the test of (non)contextuality was conducted. For participant P3 in the “polar” sub-design of Experiment 1(a), one choice of the quadruple was \((a_1, a_2, b_1, b_2) = (72 \text{ px}, 67 \text{ px}, 60 \text{ deg}, 23 \text{ deg})\). The distributions of the random outputs for the four contexts indexed as in (1) are presented in Table 3, where the numbers in the grids are joint probabilities and the numbers outside are marginal probabilities.

**Table 3.** Distributions of the random outputs for the cyclic system of rank 4, P3 in the “polar” sub-design of Experiment 1(a).

| Context 1 | \(B_{11} > b_1\) | \(B_{11} \leq b_1\) | Context 2 | \(B_{21} > b_1\) | \(B_{21} \leq b_1\) | Context 3 | \(B_{22} > b_2\) | \(B_{22} \leq b_2\) | Context 4 | \(B_{12} > b_2\) | \(B_{12} \leq b_2\) |
|-----------|------------------|------------------|-----------|------------------|------------------|-----------|------------------|------------------|-----------|------------------|------------------|
| \(A_{11} > a_1\) | \(.0056\) | \(.0056\) | \(A_{21} > a_2\) | \(.6403\) | \(.3399\) | \(.9802\) |
| \(A_{11} \leq a_1\) | \(.3944\) | \(.9944\) | \(A_{21} \leq a_2\) | \(.0099\) | \(.0099\) | \(.0198\) |

Using the formulas

\[
\langle XY \rangle = (+1)(+1)\Pr(X = 1, Y = 1) + (+1)(-1)\Pr(X = 1, Y = -1) + (-1)(+1)\Pr(X = -1, Y = 1) + (-1)(-1)\Pr(X = -1, Y = -1)
\]

and

\[
\langle X \rangle = (+1)\Pr(X = 1) + (-1)\Pr(X = -1),
\]
to have, in reference to [6]-[7],

\[
s_1(\langle A_{11}^* B_{11}^* \rangle, \langle B_{21}^* A_{21}^* \rangle, \langle A_{22}^* B_{22}^* \rangle, \langle B_{12}^* A_{12}^* \rangle) = s_1(.2112, .3004, .1578, .0164) = .653,
\]

\[
\text{ICC} = |\langle A_{11}^* \rangle - \langle A_{12}^* \rangle| + |\langle B_{11}^* \rangle - \langle B_{21}^* \rangle| + |\langle A_{21}^* \rangle - \langle A_{22}^* \rangle| + |\langle B_{22}^* \rangle - \langle B_{12}^* \rangle|
\]

\[
= |(-.9016) - (-.9888)| + |(.2) - .3004| + |.9604 - .9912| + |.1666 - (-.0056)|
\]

\[
= .0872 + .5004 + .0308 + .1722 = .7906.
\]

With \(2n - 2 = 4 - 2 = 2\) we obtain

\[
\Delta C = -2.1376 < 0,
\]
no evidence of contextuality.

### 3.2 Testing Contextuality for Rank 6

Both Experiment 2(a) and Experiment 3(a) have 3×3 designs: \{α_1, α_2, α_3\} × \{β_1, β_2, β_3\}. From each of them we extracted one cyclic system of rank 6,

\[
\begin{align*}
\text{Context 1} & \quad \text{Context 2} & \quad \text{Context 3} & \quad \text{Context 4} & \quad \text{Context 5} & \quad \text{Context 6} \\
(α_1, β_1) & \quad (β_1, α_2) & \quad (α_2, β_2) & \quad (β_2, α_3) & \quad (α_3, β_3) & \quad (β_3, α_1)
\end{align*}
\]

and labeled the random outputs \(A, B\) accordingly.

The “rectangular” design of Experiment 1(b) and the “polar” sub-designs of Experiment 1(b), Experiment 2(c), and Experiment 3(b) are the systems with quasi-continuous factors. These factors were discretized into three levels by using the one-third quantile and the two-third quantile of each interval as cut-off points. For instance, a 3×3 design is formed in the “rectangular” design of Experiment 1(b) according to this rule: \(α_1 = [20 \text{ px}, 40 \text{ px}), \ α_2 = [40 \text{ px}, 60 \text{ px}), \ α_3 = [60 \text{ px}, 80 \text{ px}), \ β_1 = [20 \text{ px}, 40 \text{ px}), \ β_2 = [40 \text{ px}, 60 \text{ px}), \ \text{and} \ β_3 = [60 \text{ px}, 80 \text{ px})\).

Again, the random outputs should be dichotomized in each experiment. We chose \(α_1\), and a value \(b_{i_2}, i_1, i_2 \in \{1, 2, 3\}\), and defined

\[
\begin{align*}
A^*_1 & = \begin{cases} +1 & \text{if } A_{1i_2} > a_1 \\ -1 & \text{if } A_{1i_2} \leq a_1 \end{cases}, & A^*_2 & = \begin{cases} +1 & \text{if } A_{2i_2} > a_2 \\ -1 & \text{if } A_{2i_2} \leq a_2 \end{cases}, \\
A^*_3 & = \begin{cases} +1 & \text{if } A_{3i_2} > a_3 \\ -1 & \text{if } A_{3i_2} \leq a_3 \end{cases}, & B^*_{i_1} & = \begin{cases} +1 & \text{if } B_{i_1} > b_1 \\ -1 & \text{if } B_{i_1} \leq b_1 \end{cases}, \\
B^*_{i_2} & = \begin{cases} +1 & \text{if } B_{i_2} > b_2 \\ -1 & \text{if } B_{i_2} \leq b_2 \end{cases}, & B^*_{i_3} & = \begin{cases} +1 & \text{if } B_{i_3} > b_3 \\ -1 & \text{if } B_{i_3} \leq b_3 \end{cases}.
\end{align*}
\]

We chose \(a_1\) as any integer between \(\max(\min A_{11}, \min A_{13})\) and \(\min(\max A_{11}, \max A_{13})\), \(b_1\) as any integer between \(\max(\min B_{11}, \min B_{21})\) and \(\min(\max B_{11}, \max B_{21})\), and analogously for \(a_2, a_3, b_2,\) and \(b_3\) for the experiments with discrete factor points (Experiment 2(a) and Experiment 3(a)). For the experiments with quasi-continuous factors, we chose \(a_1, a_2, a_3, b_1, b_2, b_3\) as every third integer within the corresponding range. The total number of the rank-6 systems thus formed varied from 18,000 to 31,905,600 per experiment per participant.

For each such choice of the sextuple \((a_1, a_2, a_3, b_1, b_2, b_3)\) we conducted the test \((6)-(7)\) numerous times, on the order of \(10^7\) in some data sets. No positive \(\triangle C\) was observed for the systems of rank 6 extracted from the “rectangular” design of Experiment 1(b), “polar” sub-design of Experiment 1(b), Experiment 2(a), Experiment 2(c), Experiment 3(a), and Experiment 3(b). We conclude that there was no evidence of contextuality in the investigated cyclic systems of rank 6.

We present an example of how the test \((6)-(7)\) was conducted. For participant P1 in Experiment 2(a), in which \(\{α_1, α_2, α_3\} × \{β_1, β_2, β_3\} = \{16 \text{ px}, 56 \text{ px}, 64 \text{ px}\} × \{48 \text{ px}, 72 \text{ px}, 80 \text{ px}\}\), one choice of the sextuple was \((a_1, a_2, a_3, b_1, b_2, b_3) = \{(16 \text{ px}, 56 \text{ px}, 64 \text{ px}, 48 \text{ px}, 72 \text{ px}, 80 \text{ px})\}. The distributions of the random outputs for the six contexts indexed as in \((6)\) are presented in Table \((4)\).
Table 4. Distributions of the random outputs for the cyclic system of rank 6, P1 in Experiment 2(a).

| Context 1 | $B_{11} > b_1$, $B_{11} \leq b_1$ | Context 2 | $B_{21} > b_1$, $B_{21} \leq b_1$ |
|-----------|-----------------------------------|-----------|-----------------------------------|
| $A_{11} > a_1$ | \text{.2124, \ .2487, \ .4611} | $A_{21} > a_2$ | \text{.2353, \ .1041, \ .3394} |
| $A_{11} \leq a_1$ | \text{.1917, \ .3472, \ .5389} | $A_{21} \leq a_2$ | \text{.1538, \ .5068, \ .6606} |
| \text{.4041, \ .5959} | \text{.3891, \ .6109} | \text{.3891, \ .6109} | \text{.3891, \ .6109} |

| Context 3 | $B_{22} > b_2$, $B_{22} \leq b_2$ | Context 4 | $B_{32} > b_2$, $B_{32} \leq b_2$ |
|-----------|-----------------------------------|-----------|-----------------------------------|
| $A_{22} > a_2$ | \text{.1221, \ .0814, \ .3394} | $A_{32} > a_3$ | \text{.2703, \ .5586, \ .3288} |
| $A_{22} \leq a_2$ | \text{.1628, \ .6337, \ .7965} | $A_{32} \leq a_3$ | \text{.1982, \ .7470, \ .6712} |
| \text{.2849, \ .7151} | \text{.4685, \ .5316} | \text{.4685, \ .5316} | \text{.4685, \ .5316} |

| Context 5 | $B_{33} > b_3$, $B_{33} \leq b_3$ | Context 6 | $B_{13} > b_3$, $B_{13} \leq b_3$ |
|-----------|-----------------------------------|-----------|-----------------------------------|
| $A_{33} > a_3$ | \text{.0702, \ .0468, \ .1170} | $A_{13} > a_1$ | \text{.1321, \ .981, \ .3302} |
| $A_{33} \leq a_3$ | \text{.0409, \ .8421, \ .8830} | $A_{13} \leq a_1$ | \text{.1651, \ .5047, \ .6698} |
| \text{.1111, \ .8889} | \text{.2972, \ .7028} | \text{.2972, \ .7028} | \text{.2972, \ .7028} |

We have

$$s_1 (\langle A_{11}^* B_{11}^* \rangle, \langle B_{21}^* A_{21}^* \rangle, \langle A_{22}^* B_{22}^* \rangle, \langle B_{32}^* A_{32}^* \rangle, \langle A_{33}^* B_{33}^* \rangle, \langle B_{13}^* A_{13}^* \rangle) = s_1 (.1192, .4843, .5116, .4865, .8246, .2736) = 2.4613,$$

$$\text{ICC} = |\langle A_{13} \rangle - \langle A_{11} \rangle| + |\langle B_{11} \rangle - \langle B_{21} \rangle| + |\langle A_{22} \rangle - \langle A_{21} \rangle| + |\langle B_{22} \rangle - \langle B_{32} \rangle| + |\langle A_{32} \rangle - \langle A_{33} \rangle| + |\langle B_{33} \rangle - \langle B_{13} \rangle|$$

$$= |-.0778 - (-.3396)| + |-.1918 - (-.2218)| + |-.3212 - (-.593)| + |-.4302 - (-.0632)| + |-.3424 - (-.766)| + |-.7778 - (-.4056)|$$

$$= .2618 + .03 + .2718 + .367 + .4236 + .3722 = 1.7264,$$

$$2n - 2 = 6 - 2 = 4,$$

whence

$$\Delta C = -3.2651 < 0,$$

no evidence of contextuality.

3.3 Testing Contextuality for Rank 8

The “rectangular” design of Experiment 1(b) and the “polar” sub-designs of Experiment 1(b), Experiment 2(c), and Experiment 3(b) have quasi-continuous factors. Each factor in each experiment was discretized into four levels in order to form a rank 8 cyclic system. Three points should be chosen for each factor to make this discretization. We chose the first quartile point, the second quartile (median) point, and the third quartile point of each interval. For instance, a 4×4 design is formed in Experiment 3(b): $\alpha_1 = [-30 \text{ px}, -15 \text{ px})$, $\alpha_2 = [-15 \text{ px}, 0 \text{ px})$, $\alpha_3 = [0 \text{ px}, 15 \text{ px})$, $\alpha_4 = [15 \text{ px}, 30 \text{ px})$, $\beta_1 = [-30 \text{ px}, -15 \text{ px})$, $\beta_2 = [-15 \text{ px}, 0 \text{ px})$, $\beta_3 = [0 \text{ px}, 15 \text{ px})$, $\beta_4 = [15 \text{ px}, 30 \text{ px}).$
Cyclic Psychophysical Systems

\[ \beta_2 = [-15\text{ px}, 0\text{ px}), \beta_3 = [0\text{ px}, 15\text{ px}), \text{ and } \beta_4 = [15\text{ px}, 30\text{ px}). \]

A cyclic system of rank 8 was extracted from each experiment:

\[
\begin{align*}
\text{Context 1} & \quad (a_1, b_1) \\
\text{Context 2} & \quad (b_1, a_2) \\
\text{Context 3} & \quad (a_2, b_2) \\
\text{Context 4} & \quad (b_2, a_3) \\
\text{Context 5} & \quad (a_3, b_3) \\
\text{Context 6} & \quad (b_3, a_4) \\
\text{Context 7} & \quad (a_4, b_4) \\
\text{Context 8} & \quad (b_4, a_1)
\end{align*}
\]

with the random outputs labeled accordingly.

To dichotomize the outputs, we chose a value \( a_{i1} \) and a value \( b_{i2} \), \( i_1, i_2 \in \{1, 2, 3, 4\} \) to define

\[
\begin{align*}
A_{i12}^* & = \begin{cases} +1 & \text{if } A_{i12} > a_{i1} \\ -1 & \text{if } A_{i12} \leq a_{i1} \end{cases}, & A_{2i2}^* & = \begin{cases} +1 & \text{if } A_{2i2} > a_2 \\ -1 & \text{if } A_{2i2} \leq a_2 \end{cases}, \\
A_{3i2}^* & = \begin{cases} +1 & \text{if } A_{3i2} > a_3 \\ -1 & \text{if } A_{3i2} \leq a_3 \end{cases}, & A_{4i2}^* & = \begin{cases} +1 & \text{if } A_{4i2} > a_4 \\ -1 & \text{if } A_{4i2} \leq a_4 \end{cases}, \\
B_{i11}^* & = \begin{cases} +1 & \text{if } B_{i11} > b_{i1} \\ -1 & \text{if } B_{i11} \leq b_{i1} \end{cases}, & B_{i12}^* & = \begin{cases} +1 & \text{if } B_{i12} > b_{i2} \\ -1 & \text{if } B_{i12} \leq b_{i2} \end{cases}, \\
B_{i13}^* & = \begin{cases} +1 & \text{if } B_{i13} > b_3 \\ -1 & \text{if } B_{i13} \leq b_3 \end{cases}, & B_{i14}^* & = \begin{cases} +1 & \text{if } B_{i14} > b_4 \\ -1 & \text{if } B_{i14} \leq b_4 \end{cases}.
\end{align*}
\]

For each rank 8 cyclic system, we chose \( a_1 \) as every sixth integer between \( \max(\min A_{11}, \min A_{14}) \) and \( \min(\max A_{11}, \max A_{14}) \), \( b_1 \) as every sixth integer between \( \max(\min B_{11}, \min B_{21}) \) and \( \min(\max B_{11}, \max B_{21}) \), and analogously for \( a_2, a_3, a_4, b_2, b_3, \) and \( b_4 \). The total number of the rank-8 systems thus formed varied from 432 to 6,453,888 per experiment per participant.

For each thus obtained octuple we conducted the test (6)-(7). No positive \( \Delta C \) was observed. When testing the “polar” sub-design of Experiment 1(b), each context contained only about 50 data points. Even with such small sample sizes, no positive \( \Delta C \) was observed. We conclude that there is no contextuality in all the investigated cyclic systems of rank 8.

To give an example, for participant P4 in Experiment 3(b), one choice of the octuple was \((a_1, a_2, a_3, b_1, b_2, b_3, b_4) = (-21 \text{ px}, -6 \text{ px}, 6 \text{ px}, 21 \text{ px}, -21 \text{ px}, -9 \text{ px}, 9 \text{ px}, 21 \text{ px})\). The distributions of the random outputs for the eight contexts are presented in Table 5.
whence noncontextuality boundaries are generally breached in quantum physics [8]. In systems as contextual or noncontextual. Experimental data suggest that the Contextuality-by-default is a mathematical framework that allows to classify

| Table 5. Distributions of the random outputs for the cyclic system of rank 8, P4 in Experiment 3(b). |
|-----------------------------------------------|
| **Context 1 | \( B_{11} > b_1 \), \( B_{11} \leq b_1 \) | **Context 2 | \( B_{21} > b_1 \), \( B_{21} \leq b_1 \) |
| \( A_{11} > a_1 \) | .1532 | .2823 | .4355 | \( A_{21} > a_2 \) | .1619 | .2667 | .4286 |
| \( A_{11} \leq a_1 \) | .1855 | .3790 | .5645 | \( A_{21} \leq a_2 \) | .1905 | .3810 | .5715 |
| .3387 | .6613 | .3524 | .6477 |
| **Context 3 | \( B_{22} > b_2 \), \( B_{22} \leq b_2 \) | **Context 4 | \( B_{32} > b_2 \), \( B_{32} \leq b_2 \) |
| \( A_{22} > a_2 \) | .2759 | .2155 | .4914 | \( A_{32} > a_3 \) | .4130 | .1739 | .5869 |
| \( A_{22} \leq a_2 \) | .2586 | .2500 | .5086 | \( A_{32} \leq a_3 \) | .1957 | .2174 | .4131 |
| .5345 | .4655 | .6087 | .3913 |
| **Context 5 | \( B_{33} > b_3 \), \( B_{33} \leq b_3 \) | **Context 6 | \( B_{43} > b_3 \), \( B_{43} \leq b_3 \) |
| \( A_{33} > a_3 \) | .2736 | .3208 | .5944 | \( A_{43} > a_4 \) | .2460 | .3095 | .5555 |
| \( A_{33} \leq a_3 \) | .1604 | .2453 | .4057 | \( A_{43} \leq a_4 \) | .1667 | .2778 | .4445 |
| .4340 | .5661 | .4127 | .5873 |
| **Context 7 | \( B_{44} > b_4 \), \( B_{44} \leq b_4 \) | **Context 8 | \( B_{14} > b_4 \), \( B_{14} \leq b_4 \) |
| \( A_{44} > a_4 \) | .3209 | .3134 | .6343 | \( A_{14} > a_1 \) | .1619 | .2571 | .4190 |
| \( A_{44} \leq a_4 \) | .1493 | .2164 | .3657 | \( A_{14} \leq a_1 \) | .2381 | .3429 | .5810 |
| .4702 | .3298 | .64 | .6 |

We have then

\[
s_1(\langle A_{11}^1 B_{11}^1 \rangle, \langle B_{21}^1 A_{21}^1 \rangle, \langle A_{22}^1 B_{22}^1 \rangle, \langle B_{32}^1 A_{32}^1 \rangle, \langle A_{33}^1 B_{33}^1 \rangle, \langle B_{43}^1 A_{43}^1 \rangle, \langle A_{44}^1 B_{44}^1 \rangle, \langle B_{14}^1 A_{14}^1 \rangle)
\]

\[= s_1(.0644,.0857,.0518,.2608,.0377,.0476,.0746,.0096) = .613,\]

\[
\text{ICC} = |\langle A_{11} \rangle - \langle A_{14} \rangle| + |\langle B_{11} \rangle - \langle B_{21} \rangle| + |\langle A_{22} \rangle - \langle A_{21} \rangle| + |\langle B_{22} \rangle - \langle B_{32} \rangle|
\]
\[+ |\langle A_{33} \rangle - \langle A_{32} \rangle| + |\langle B_{33} \rangle - \langle B_{32} \rangle| + |\langle A_{44} \rangle - \langle A_{43} \rangle| + |\langle B_{44} \rangle - \langle B_{43} \rangle|
\]
\[= |.129 - (.162)| + |-.3226 - (.2952)| + |-.0172 - (.1428)| + |.069 - .2174|
\]
\[+ |.1888 - .1738| + |-.1746 - (.132)| + |.2686 - (.1110)| + |-.2 - (.0596)|
\]
\[=.033 + .0274 + .1256 + .1484 + .015 + .0426 + .1576 + .1404 = .6902,\]

whence

\[\Delta C = -6.0772 < 0,\]

no evidence of contextuality.

4 Conclusions

Contextuality-by-default is a mathematical framework that allows to classify systems as contextual or noncontextual. Experimental data suggest that the noncontextuality boundaries are generally breached in quantum physics [8]. In
Ref. [16][17] we reviewed several behavioral and social scenarios to conclude that none of them provided evidence for contextuality. By examining the psychophysical data collected in our laboratory, we found no contextuality for cyclic systems of different ranks, including high ranks (6 and 8) that have never been analyzed before. We suspect that it may be generally true that human and social behaviors are not contextual in the same sense in which quantum systems are.

Acknowledgments

This research has been supported by NSF grant SES-1155956, AFOSR grant FA9550-14-1-0318.

References

1. Dzhafarov, E. N., & Kujala, J. V. (2014). A qualified Kolmogorovian account of probabilistic contextuality. Lecture Notes in Computer Science 8369, 201-212.
2. Dzhafarov, E. N., & Kujala, J. V. (2014). Embedding quantum into classical: Contextualization vs conditionalization. PLoS One 9(3): e92818. doi: 10.1371/journal.pone.0092818.
3. Dzhafarov, E. N., & Kujala, J. V. (2014). Contextuality is about identity of random variables. Physica Scripta T163, 014009.
4. Dzhafarov, E. N., & Kujala, J. V., & Larsson, J.-Å. (2015). Contextuality in three types of quantum-mechanical systems. Foundations of Physics 7, 762-782.
5. Dzhafarov, E. N., Kujala, J. V., & Cervantes, V. H. (2016). Contextuality by Default: A brief overview of ideas, concepts, and terminology. Lecture Notes in Computer Science 9535, 12-23.
6. Kujala, J. V., & Dzhafarov, E. N. (2015). Proof of a conjecture on contextuality in cyclic systems with binary variables. Foundations of Physics 46, 282-299.
7. Kujala, J. V., & Dzhafarov, E. N. (2015). Probabilistic Contextuality in EPR/Bohm-type systems with signaling allowed. In Dzhafarov, E. N., Jordan, J. S., Zhang, R., & Cervantes, V. H. (Eds). Contextuality from Quantum Physics to Psychology pp. 287-308. New Jersey: World Scientific.
8. Kujala J. V., Dzhafarov, E. N, & Larsson J.-Å. (2015). Necessary and sufficient conditions for extended noncontextuality in a broad class of quantum mechanical systems. Physical Review Letters 115, 150401. (arXiv:1407.2886)
9. Bell, J. (1964). On the Einstein-Podolsky-Rosen paradox. Physics 1, 195-200.
10. Kochen, S., & Specker, E. P. (1967). The problem of hidden variables in quantum mechanics. Journal of Mathematics and Mechanics 17, 59–87.
11. Clauser, J. F., Horne, M. A., Shimony, A., & Holt, R. A. (1969). Proposed experiment to test local hidden-variable theories. Physical Review Letters 23, 880-884.
12. Fine, A. (1982). Joint distributions, quantum correlations, and commuting observables. Journal of Mathematical Physics 23, 1306-1310.
13. Fine, A. (1982). Hidden variables, joint probability, and the Bell inequalities. Physical Review Letters 48, 291-295.
14. Leggett, A. J., & Garg, A. (1985). Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks? Physical Review Letters 54, 857–860.
15. Klyachko, A. A., Can, M. A., Binicioğlu, S., and Shumovsky, A. S. (2008). Simple test for hidden variables in spin-1 systems. Physical Review Letters 101(2): 020403.
16. Dzhafarov, E. N., Kujala, J. V., Cervantes, V. H., Zhang, R., & Jones, M. (2016). On contextuality in behavioural data. Philosophical Transactions of the Royal Society A 374: 20150234. (arXiv:1508.04751)
17. Dzhafarov, E. N., Zhang, R., & Kujala, J. V. (2015). Is there contextuality in behavioral and social systems? Philosophical Transactions of the Royal Society A 374, 20150099. (arXiv:1504.07422)