Low-Energy Effective Lagrangian from Non-Minimal Supergravity with Unified Gauge Symmetry

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ABSTRACT

From general supergravity theory with unified gauge symmetry, we obtain the low-energy effective Lagrangian by taking the flat limit and integrating out the superheavy fields in model-independent manner. The scalar potential possesses some excellent features. Some light fields classified by using supersymmetric fermion mass, in general, would get intermediate masses at the tree level after the supersymmetry is broken. We show that the stability of weak scale can be guaranteed under some conditions. There exist extra non-universal contributions to soft supersymmetry breaking terms which can give an impact on phenomenological study.
1 Introduction

The minimal supersymmetric standard model (MSSM) is the most attractive candidates for the realistic theory beyond standard model. The naturalness problem is elegantly solved by the introduction of supersymmetry (SUSY)\(^\text{[1]}\). The SUSY requires new particles called ‘superpartners’. Those masses are free parameters in the MSSM\(^\text{[3]}\) but are estimated as at most order 1 TeV from the naturalness argument. The search for ‘superpartners’ is one of the main purpose in the experimental projects by the use of huge colliders, which have been planned now\(^\text{[2]}\).

It is, however, believed that the MSSM is not the ultimate theory because there are many problems not to be solved in it. Here we pick up two problems. First there are so many free parameters to be fixed only by experiments for the present. In addition to gauge couplings and Yukawa couplings, soft SUSY breaking parameters appear, i.e., gaugino masses, scalar masses and scalar trilinear couplings are all arbitrary ones. Hence the MSSM lacks predictability. Second the mechanism of SUSY breaking is unexplained. This problem is partly related to the first one since the pattern of soft SUSY breaking terms depends on the SUSY breaking mechanism.

It is expected that they are solved in more fundamental theory. Supergravity theory (SUGRA)\(^\text{[3]}\) is an attractive candidate. When we take SUGRA as an effective theory at the Planck scale \(M_{\text{Pl}}\), SUGRA has an interesting solution. There exists such a scenario\(^\text{[4]}\) that the SUSY is spontaneously broken in the so-called hidden sector and the effect is transported to the observable sector through the gravitational interaction. As a result, the soft SUSY breaking terms appear in our visible sector. The form of soft SUSY breaking terms is determined by the structure of SUGRA. A simple choice is a theory such that soft parameters take universal values at the gravitational scale \(M \equiv M_{\text{Pl}}/\sqrt{8\pi}\), e.g., the scalar potential derived from the SUGRA with a minimal Kähler function has the universal scalar mass \(m_0\) and the universal scalar trilinear coupling constant \(A\). Those values at low energy are calculated by using renormalization group equations.

The analyses based on the MSSM are energetically investigated\(^\text{[5]}\). Most of them are highly constrained by the assumption that the soft SUSY break-
ing parameters are universal at $M$ or a unification scale $M_X$. This assumption is quite interesting because the theory has high predictabilities and is testable enough, but it is difficult to say that this type of approach is completely realistic. Let us describe the reason why the universality at $M$ is not necessarily realistic. First the assumption of the universal scalar mass is motivated by the fact that the flavor-changing neutral current (FCNC) processes are suppressed experimentally[6]. However, we can relax this assumption since the suppression of FCNC processes due to SUSY particle loops requires only the degeneracy among squarks with a same flavor. Second there is no strong reason that the realistic SUGRA takes the minimal structure. In fact, the effective SUGRAs derived from superstring theories (SSTs) have, in general, non-minimal structures and they can lead to the effective theories with non-universal soft parameters[7]. Third it was pointed out that higher order corrections generally destroy the minimal form of the Kähler potential[8]. Last the effects of supersymmetric grand unified theory (SUSY-GUT) were little considered in the analysis of the running of parameters although SUSY-GUT[9] has been hopeful as a realistic theory.

We shall discuss the last point still more. The unification dogma[10] has a merit that the number of independent parameters is reduced due to a large gauge symmetry. Further SUSY $SU(5)$ GUT is supported by the LEP experiments[11] and predicts the long lifetime of nucleon consistent with the present data[12]. Thus an analysis based on the MSSM encouraged by the unification scenario seems to be hopeful. In fact, many researches have been done under the assumption that the soft SUSY breaking terms take a universal form at the GUT scale $M_X$, but this assumption is also not always realistic from the following reasons. The non-minimal SUGRA can lead to the non-universal form of soft SUSY breaking terms as described above. Even if we take a minimal SUGRA as a starting point, the radiative correction from $M$ to $M_X$ changes the universal form of SUSY breaking terms into non-universal one. In some literatures, the renormalization effects were discussed[13], but we need to consider effects on the gauge symmetry breaking further. In Ref. [14], low-energy effective theory has been derived from SUSY-GUT with non-universal soft SUSY breaking terms by integrating out superheavy fields and it is shown that new contributions to SUSY breaking terms can appear at the tree level after the breaking of unified gauge symmetry. The analyses including the effects are started recently[15].

Now we should stress the importance of studying the soft SUSY breaking
terms. The reason is that they can be a powerful probe to SUSY-GUT and/or SUGRA since the weak scale SUSY spectrum can directly reflect the physics at very high energies. For example, we can check the GUT scenario experimentally by measuring the gaugino masses[16]. Also, the scalar mass spectrum has certain “sum rules” specific to symmetry breaking patterns[17]. Therefore, the precision measurements of SUSY spectrum are very important. And it is a meaningful subject to obtain the low-energy theory in more general framework and to grasp the peculiarities concerning on the SUSY breaking terms in advance.

Various types of low-energy theories were derived as will be explained in the next section. However its low-energy theory has not been completely investigated by taking SUGRA with general structure and unified gauge symmetry as a starting point. It has been only studied in some specific cases[18][19][14]. For example, it is shown that the universality of scalar masses is preserved in the SUGRA whose Kähler potential has $U(n)$ symmetry among the $n$ chiral fields[18]. In Ref.[14], the scalar potential was derived starting from a unified theory with a certain type of non-universal soft SUSY breaking terms. Such non-universal soft terms arise if we take the flat limit of the SUGRA where the Kähler potential is a certain type of non-minimal one and the superpotential is separate from the hidden sector to the visible one. (We call this form of superpotential a hidden ansatz.)

In this paper, we derive the low-energy effective theory from non-minimal SUGRA with unified gauge symmetry. The starting SUGRA has more general structure, i.e., the Kähler potential is non-minimal and we do not impose the hidden assumption on the superpotential. Then dangerous terms, which destabilize the gauge hierarchy, generally appear at the tree level. We discuss conditions that the hierarchy is preserved, and take the flat limit and integrate out superheavy fields without identifying $M_X$ with $M$. We find various contributions to the SUSY breaking terms. Our result reduces to that obtained in Ref. [18] in the case with the minimal SUGRA. Also it is shown that it reduces to that obtained in Ref. [14] in the limit $M_X/M \to 0$ when we take a certain type of total Kähler potential.

The paper is organized as follows. In section 2, we first review the low-energy effective Lagrangians from SUGRA following the historical development. We derive the low-energy effective scalar potential starting from SUGRA with general total Kähler potential and unified gauge symmetry in section 3. In section 4, we discuss $D$-term contributions to scalar masses and
make clear the relation between our result and that in Ref.[14]. Section 5 is devoted to conclusions.

2 Historical Background

2.1 Scalar Sector in SUGRA

We begin by reviewing the scalar sector in SUGRA[3]. It is specified by two functions, the total Kähler potential $G(\Phi, \Phi^*)$ and the gauge kinetic function $f_{\alpha\beta}(\Phi)$ with $\alpha, \beta$ being indices of the adjoint representation of the gauge group. The former is a sum of the Kähler potential $K$ and (the logarithm of) the superpotential $W_{SG}$ such as

$$G(\Phi, \Phi^*) = K(\Phi, \Phi^*) + M^2 \ln |W_{SG}(\Phi)/M^3|^2.$$  \hspace{1cm} (1)

We have denoted the chiral multiplets by $\Phi^I$ and their complex conjugate by $\Phi^*_J$. The scalar potential is given by

$$V = M^2 e^{G/M^2} U + \frac{1}{2} (\text{Re} f^{-1})_{\alpha\beta} D^\alpha D^\beta,$$  \hspace{1cm} (2)

where

$$U = G^I (K^{-1})^J_I G_J - 3M^2,$$  \hspace{1cm} (3)

$$D^\alpha = G_I (T^\alpha z)^I = (z^\dagger T^\alpha) J G^J.$$  \hspace{1cm} (4)

Here $G_I = \partial G/\partial z^I$, $G^J = \partial G/\partial z^*_J$ etc, and $T^\alpha$ are gauge transformation generators. The $z^I$ is a scalar component of $\Phi^I$. Here and hereafter both $G$ and $f_{\alpha\beta}$ are regarded as functions of $z$ and $z^*$ as we take notice of the scalar potential alone. Also $(\text{Re} f^{-1})_{\alpha\beta}$ and $(K^{-1})^J_I$ are the inverse matrices of $\text{Re} f_{\alpha\beta}$ and $K^I_J$ respectively, and summation over $\alpha,...$ and $I,...$ is understood. The last equality in Eq. (4) comes from the gauge invariance of the total Kähler potential.

Let us next summarize our assumptions on the SUSY breaking. The gravitino mass $m_{3/2}$ is given by

$$m_{3/2} = \langle e^{K/2M^2} W_{SG} \rangle/M^2,$$  \hspace{1cm} (5)
where $\langle \cdots \rangle$ denotes the vacuum expectation value (VEV) of the quantity. We identify the gravitino mass with the weak scale. The $F$-auxiliary fields of the chiral multiplets $\Phi^I$ are defined as

$$F^I \equiv M e^{G/2M^2} (K^{-1})^I_J G^J.$$ (6)

We require those VEVs should satisfy

$$\langle F^I \rangle \leq O(m_{3/2} M).$$ (7)

We can show that the VEVs of the $D$-auxiliary fields become very small $\langle D^a \rangle \leq O(m_{3/2}^2)$ as will be shown in Appendix A. It follows from Eqs. (6), (8) and (7) that

$$\langle G_I \rangle, \langle G^J \rangle \leq O(M)$$ (8)

and

$$\langle U \rangle \leq O(M^2).$$ (9)

Note that we allow the non-zero vacuum energy $\langle V \rangle$ of order $m_{3/2}^2 M^2$ at this level, which could be canceled by quantum corrections. We also assume that derivatives of the Kähler potential with respect to $z$ and $z^*$ are at most of order unity (in the units where $M$ is taken to be unity), namely

$$\langle K_J^{I_1 \cdots} \rangle \leq O(1).$$ (10)

This will be justified if the Planck scale physics plays an essential role in the SUSY breaking.

We shall call the fields which induce to the SUSY breaking ‘hidden fields’, and denote those scalar components and $F$-components as $\tilde{z}^i$ and $\tilde{F}^i$, respectively. We require those VEVs should satisfy

$$\langle \tilde{z}^i \rangle = O(M)$$ (11)

and

$$\langle \tilde{F}^i \rangle = O(m_{3/2} M).$$ (12)

We shall call the rest ‘observable fields’ and denote the scalar components as $z^\kappa$. 

5
2.2 Effective Theories from Minimal SUGRA

The minimal SUGRA is defined as follows. The Kähler potential \( K \) has a canonical form as

\[
K = |z^\kappa|^2 + |\bar{z}^i|^2.
\]

We take the hidden ansatz for the superpotential as

\[
W_{SG} = W(z) + \bar{W}(\bar{z}).
\]

The global SUSY theory with soft SUSY breaking terms is derived by taking the flat limit, i.e., \( M \to \infty \) but \( m_{3/2} \) kept finite. The scalar potential is as follows,

\[
V_{SUSY} = |\partial \hat{W} / \partial z^\kappa|^2 + \frac{1}{2} g_\alpha^2 (z_\kappa^* (T_\alpha)_\kappa^\lambda)^2,
\]

\[
V_{Soft} = A \hat{W} + B z^\kappa \partial \hat{W} / \partial z^\kappa + H.c. + |B|^2 z_\kappa^* z^\kappa,
\]

where \( \hat{W} \) is defined as \( \hat{W} \equiv \langle \exp(\frac{K}{2M^2}) \rangle W \). \( V_{SUSY} \) stands for the supersymmetric part, while \( V_{Soft} \) contains the soft SUSY breaking terms. The \( A \) and \( B \) are the soft SUSY breaking parameters and are written as

\[
A = \langle \bar{F}^i \rangle \langle K_i \rangle / M^2 - 3m_{3/2}^*,
\]

\[
B = m_{3/2}^*.
\]

This form of SUSY breaking terms is referred to as “universal”.

The low-energy effective Lagrangian derived from the minimal SUGRA with unified gauge symmetry also has a simple structure. It was obtained by taking the flat limit and integrating out superheavy fields simultaneously on the postulation that the unification scale \( M_X \) is identified with \( M_{[18]} \). The low-energy scalar potential takes the following form,

\[
V^{\text{eff}} = V^{\text{eff}}_{SUSY} + V^{\text{eff}}_{Soft},
\]

\[
V^{\text{eff}}_{SUSY} = |\partial \hat{W}_{\text{eff}} / \partial z^\kappa|^2 + \frac{1}{2} g_\alpha^2 (z_\kappa^* (T_\alpha)_\kappa^\lambda)^2,
\]
\[ V_{\text{Soft}}^{\text{eff}} = A\hat{W}_{\text{eff}} + Bz^k \frac{\partial \hat{W}_{\text{eff}}}{\partial z^k} + H.c. + |B|^2 z^k z^k + \Delta V, \]  
\[ \Delta V \equiv -3A\hat{W}_{\text{eff}} + Az^k \frac{\partial \hat{W}_{\text{eff}}}{\partial z^k} + H.c. \]  

and it still has the same form as the original one by a suitable redefinition of the \( A \) and \( B \) parameters except the mass squared terms. Here \( z^k \) are the light scalar fields\(^4\) \( a \) is the index of generators of unbroken gauge group and \( \hat{W}_{\text{eff}} \) is the superpotential \( \hat{W} \) with the extremum values for superheavy fields plugged in. The scalar mass terms are still universal with the same mass \( B \).\(^5\)

The universal structure of the low-energy Lagrangian led to a number of strong conclusions, like the natural absence of the flavor changing neutral currents\(^6\) or the radiative breaking scenario due to the heavy top quark\(^7\). Due to these successes, the phenomenological analysis has been made in popular based on the SUSY models with the universal soft SUSY breaking terms\(^8\). However, it becomes increasingly apparent that SUGRA may not have the minimal form, and it is important to study the consequences on the low-energy effective Lagrangian.

### 2.3 Effective Theories from Non-minimal SUGRA

The scalar potential is also obtained from the non-minimal SUGRA with no superheavy fields and the result is given as\(^9\),

\[ V^{(\text{non})} = V^{(\text{non})}_{\text{SUSY}} + V^{(\text{non})}_{\text{Soft}}, \]  
\[ V^{(\text{non})}_{\text{SUSY}} = \frac{\partial \hat{W}^*}{\partial z^k} ((K^{-1})^\lambda_k) \frac{\partial \hat{W}}{\partial z^\lambda} + \frac{1}{2} g_2^2 ((K^{-1})^\lambda_k z^* (T^a)^\kappa_z z^\mu)^2, \]  
\[ V^{(\text{non})}_{\text{Soft}} = A\hat{W} + B^\kappa(z) ((K^{-1})^\lambda_k) \frac{\partial \hat{W}}{\partial z^\lambda} + H.c. + B^\kappa(z) ((K^{-1})^\lambda_k) B_\lambda(z) + C(z, z^*), \]  

\(^2\) They assumed that the supersymmetric masses of light fields from the superpotential are zero. It is straightforward to generalize their analysis into the case that the light fields have non-zero but \( O(m_{3/2}) \) masses.

\(^3\) Throughout this subsection, it is assumed that the vacuum energy \( \langle V \rangle \) vanishes. In the presence of vacuum energy, the value of scalar mass \( |B|^2 \) is replaced by \( |B|^2 + \langle V \rangle / M^2 \).
where

\[ B^\kappa(z) = m_{3/2}^3 \langle K^\kappa \rangle z^\lambda - K_j^\kappa \langle F^j \rangle, \]
\[ C(z, z^*) = -\langle \tilde{F}^i \rangle \delta^2 K_i^j \langle \tilde{F}^*_j \rangle \]

\[ + \left\{ \frac{1}{M^2} \langle \tilde{F}^i \rangle \langle K_i^j \rangle \langle \tilde{F}^*_j \rangle - 3|m_{3/2}|^2 \right\} \delta^2 K \\
+ m_{3/2}^2 \langle \tilde{F}^i \rangle \delta^2 K_i + H.c. \]

\[ - A\{m_{3/2} H(z) - \langle \tilde{F}^*_i \rangle H^i(z)\} + H.c. \] (28)

in the case that we take the hidden ansatz. Here \( \hat{W} \) is defined as

\[ \hat{W} \equiv \tilde{W} + m_{3/2} H(z) - \langle \tilde{F}^*_i \rangle H^i(z), \] (29)

where \( H \) is the holomorphic part of \( z^\kappa \) in \( K \). And \( \delta^2 K, \delta^2 K_i \) and \( \delta^2 K_i^j \) are the quantities of order \( m_{3/2}^2, m_{3/2}^2/M \) and \( m_{3/2}^2/M^2 \) in \( K, K_i \) and \( K_i^j \), respectively. Note that the SUSY breaking terms show a non-universal form.

As an excellent feature, there is a natural explanation for the origin of \( \mu \) parameter of order \( m_{3/2}^3/2 (\sim 1\text{TeV}) \)\(^2\). That is, the second and third terms in Eq. (29) correspond to \( \mu \)-term with a phenomenologically suitable order.

It is also known that the effective SUGRAs with non-minimal structure are derived from 4-dimensional string models and most of them lead to non-universal soft SUSY breaking terms\(^7\).

When the hidden ansatz is taken off, the following extra terms should be added,

\[ \frac{\partial \hat{W}^s}{\partial z_i^s} \langle (K^{-1})^i_j \rangle \frac{\partial \hat{W}}{\partial z^j} + \Delta C(z, z^*) + \langle \tilde{F}^i \rangle \frac{\partial \hat{W}}{\partial \tilde{z}^i} + H.c., \] (30)

where \( \Delta C(z, z^*) \) is a bilinear polynomial of \( z \) and \( z^* \). The magnitude of third term and its hermitian conjugate can be of order \( m_{3/2}^3 M \), and so a large mixing mass of Higgs doublets can be introduced. Hence we need to impose the condition

\[ \langle \tilde{F}^i \rangle \frac{\partial \hat{W}}{\partial \tilde{z}^i} = O(m_{3/2}^4) \] (31)

to guarantee the stability of weak scale.
The effective theories based on non-minimal SUGRA with unified gauge symmetry also have been studied in some literatures, but a complete analysis has not been carried out yet. For example, Hall et al. showed that the universality of scalar masses is preserved in the SUGRA whose Kähler potential has $U(n)$ symmetry among the $n$ chiral fields\cite{18}. Drees studied the low-energy theory based on SUGRA with a non-canonical kinetic function parametrized by one chiral field which triggers the SUSY breaking\cite{19}.

As a recent development, the effective theory has been derived from SUSY-GUT with a certain type of non-universal soft SUSY breaking terms by integrating out superheavy fields\cite{14}. This starting SUSY-GUT can be derived from a certain type of non-minimal SUGRA with unified gauge symmetry by imposing the hidden ansatz and taking the flat limit first. The low-energy effective scalar potential is obtained as follows,

\begin{align}
V_{\text{eff}}^{(\text{non})} &= V_{\text{SUSY}}^{(\text{non})} + V_{\text{Soft}}^{(\text{non})}, \\
V_{\text{SUSY}}^{(\text{non})} &= \left| \frac{\partial \hat{W}_{\text{eff}}}{\partial z^k} \right|^2 + \frac{1}{2} g_\nu^2 (z^*_k (T^\nu)_k^l z^l)^2, \\
V_{\text{Soft}}^{(\text{non})} &= A \hat{W}_{\text{eff}} + B^k(z)_{\text{eff}} \frac{\partial \hat{W}_{\text{eff}}}{\partial z^k} + H.c. \\
&\quad + B^k(z)_{\text{eff}} B_k(z)_{\text{eff}} + C(z, z^*)_{\text{eff}} + \Delta V^{(\text{non})},
\end{align}

where $\hat{W}_{\text{eff}}$, $B^k(z)_{\text{eff}}$ and $C(z, z^*)_{\text{eff}}$ are $\hat{W}$, $B^k(z)$ and $C(z, z^*)$ with the extremum values for superheavy fields plugged in, and $\Delta V^{(\text{non})}$ is a sum of extra contributions. There exist new contributions specific to SUSY-GUTs with non-universal soft SUSY breaking terms. The appearance of $D$-term contribution to the scalar masses is one example.\cite{14} In the absence of Fayet-Iliopoulos $D$-term, the conditions that sizable $D$-term contributions appear are as follows. (1) SUSY-GUT has non-universal soft SUSY breaking terms. (2) The rank of the gauge group is reduced by the gauge symmetry breaking. As the other feature, the gauge hierarchy achieved by a fine-tuning in the superpotential would be violated, in general, due to the non-universal SUSY breaking terms. It is, however, shown that it is preserved for SUSY-GUT models derived from the SUGRA with the hidden ansatz and no light observable singlets. It is also discussed some phenomenological implications

\footnote{Historically, it was demonstrated that the $D$-term contribution occurs when the gauge symmetry is broken at an intermediate scale due to the non-universal soft scalar masses in Refs.\cite{23} and its existence in a more general situation was suggested in Ref.\cite{24}.}
on the non-universal SUSY breaking terms, including the utility of sfermion masses as a probe of gauge symmetry breaking patterns and the predictions of the radiative electroweak symmetry breaking scenario and of no-scale type models.

As described in introduction, it is important to study the low-energy theory in more general framework of SUGRA because the SUSY spectrum can be a powerful probe to the physics at higher energy scales. The following subject has not been enough considered yet: to obtain the low-energy theory directly from non-minimal SUGRA with unified gauge symmetry in model-independent manner. In the following sections, we carry it out paying attention to the gauge hierarchy problem. And we discuss extra contributions to the SUSY breaking terms and the relation between our result and the previous one.

3 Derivation of the Effective Lagrangian

3.1 Basic Assumptions

We have already explained general assumptions in the hidden sector SUSY breaking scenario in subsection 2.1. We shall first add basic assumptions although parts of them would be repeated.

1. At the gravitational scale $M$, the theory is described effectively as non-minimal SUGRA with a certain unified gauge symmetry whose Kähler potential and superpotential are given as

\begin{align}
K &= K(z, z^*; \tilde{z}, \tilde{z}^*) \\
&= \tilde{K}(\tilde{z}, \tilde{z}^*) + \Lambda(z, z^*; \tilde{z}, \tilde{z}^*) \\
&\quad + H(z; \tilde{z}, \tilde{z}^*) + H.c. \tag{35}
\end{align}

and

\begin{align}
W_{SG} &= W_{SG}(z, \tilde{z}) \\
&= \tilde{W}(\tilde{z}) + W(z, \tilde{z}), \tag{36}
\end{align}

\begin{align}
W(z, \tilde{z}) &= \frac{1}{2}m_{\kappa\lambda}(\tilde{z})z^\kappa z^\lambda + \frac{1}{3!}f_{\kappa\lambda\mu}(\tilde{z})z^\kappa z^\lambda z^\mu + \cdots, \tag{37}
\end{align}
respectively. Here the dots stands for terms of higher orders in $z$. The
gauge group is not necessarily grand-unified into a simple group. The
theory has no Fayet-Iliopoulos $U(1)$ $D$-term for simplicity.\[5

2. The SUSY is spontaneously broken by the $F$-term condensation in the
hidden sector. The Planck scale physics plays an essential role in the
SUSY breaking. The hidden fields are gauge singlets and they have
the VEVs of $O(M)$. The magnitude of $W_{SG}$ and $F$-component $\tilde{F}^i$ of
$\tilde{z}^i$ are $O(m_{3/2}M^2)$ and $O(m_{3/2}M)$, respectively.

3. The unified gauge symmetry is broken at a scale $M_X$. Some observable
scalar fields have the VEVs of $O(M_X)$.

4. All the particles can be classified as heavy (with mass $O(M_X)$) or
light (with mass $O(m_{3/2})$). The light observable fields are gauge non-
singlets and have fluctuations only of $O(m_{3/2})$.

3.2 Vacuum Solutions

The scalar potential is given as

$$ V = V^{(F)} + V^{(D)}, $$

$$ V^{(F)} \equiv M^2 \exp(G/M^2)(G^I(G^{-1})_I^J G_J - 3M^2) $$

$$ = M^2 \exp(G/M^2)U, $$

$$ V^{(D)} \equiv \frac{1}{2}(\text{Ref}^{-1})_{\alpha\beta}D^\alpha D^\beta. $$

The index $I, J,...$ run all scalar species, $i, j,...$ run the hidden fields and $\kappa,\lambda,...$ run the observable fields. The $D^\alpha$’s are deformed as

$$ D^\alpha = K_\kappa (T^\alpha z)^\kappa = (z^\dagger T^\alpha)_\kappa K^\kappa $$

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5 The extension of the theory with Fayet-Iliopoulos $D$-term is straightforward. We
discuss it in Appendix B.

6 Our discussion is also applicable to the case of SUSY breaking by gaugino condensa-
tion if the freedoms are effectively replaced by some scalar multiplets whose VEVs are of
order $M$ as the models derived from SST.

7 The reason why we assume it is that there is a difficulty that radiative corrections
generally induce a large tadpole contribution to Higgs masses in several models with a
light singlet coupled to Higgs doublets renormalizably in superpotential.
from the gauge invariance of superpotential. The vacuum $\langle z^I \rangle$ and $\langle z^*_J \rangle$ are determined by solving the stationary conditions $\partial V/\partial z^I = 0$ and $\partial V/\partial z^*_J = 0$.

The conditions that the SUSY is not spontaneously broken in the observable sector are simply expressed as

$$\frac{\partial W}{\partial z^\kappa} = 0, \quad \overline{D}^\alpha = 0. \quad (42)$$

We denote the solutions of the above conditions as $z^\kappa = z^\kappa_0$. We assume that our vacuum solution $\langle z^\kappa \rangle$ is near to $z^\kappa_0$, i.e., $\langle z^\kappa \rangle = z^\kappa_0 + O(\frac{m}{2})$.

The supersymmetric fermion mass $\mu_{IJ}$ is given as

$$\mu_{IJ} = \langle Me^{G/2M^2}(G_{IJ} + \frac{G_I G_J}{M^2} - G_I (G^{-1})^I_J G^J) \rangle. \quad (44)$$

We take a basis of $z^I$ to diagonalize the SUSY fermion mass matrix $\mu_{IJ}$. Then we assume that the scalar fields are classified either as “heavy” fields $z^K, z^L, \ldots$, “light” fields $z^k, z^l, \ldots, z^i, z^j, \ldots$ such as $\mu_{KL} = O(M_X), \mu_{kl} = O(m_{3/2}), \mu_{ij} = O(m_{3/2})$ or Nambu–Goldstone fields $z^A, z^B, \ldots$ (which will be discussed just below). It is shown that the hidden fields belong to the light sector in Appendix A.

The mass matrix of the gauge bosons $(M_V^2)^\alpha\beta$ is given as

$$(M_V^2)^{\alpha\beta} = 2\langle (z^T \overline{T}^\beta)_{\kappa} K^{\kappa}_{\lambda} (T^\alpha )_{\lambda} \rangle, \quad (45)$$

up to the normalization due to the gauge coupling constants and it can be diagonalized so that the gauge generators are classified into “heavy” (those broken at $M_X$) $T^A, T^B, \ldots$ and “light” (which remain unbroken above $m_{3/2}$) $T^a, T^b, \ldots$. For the heavy generators, the fields $\langle (T^A z)^\kappa \rangle$ correspond to the directions of the Nambu–Goldstone fields in the field space, which span a vector space with the same dimension as the number of heavy generators.

We can take a basis of the Nambu–Goldstone multiplets, $z^A, z^B, \ldots$ so that

$$\sqrt{2}\langle (T^A z)^B \rangle = M_V^{AB}. \quad (46)$$

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8 Note that the superpotential is not gauge invariant under Fayet-Iliopoulos $U(1)$ transformation.

9 We can show that there exists at least such a vacuum solution in the case that the scalar potential has no flat directions in the SUSY limit.
Here the Nambu–Goldstone fields are taken to be orthogonal to the heavy and light fields such as \( \langle (T^A z)^K \rangle = 0 \) and \( \langle (T^A z)^k \rangle = 0 \). To be more precise, either real or imaginary parts of the \( z^A \)'s are the true Nambu–Goldstone bosons which are absorbed into the gauge bosons, and the other parts acquire the same mass of order \( M_X \) as that of the gauge bosons from the \( D \)-term \( V^{(D)} \) in the SUSY limit. Hence the Nambu–Goldstone multiplets belong to the heavy sector.

Let us give the procedure to obtain the low-energy effective theory.

1. We calculate the VEVs of the derivatives of the potential and we write down the potential as

\[
V = \frac{1}{2} \langle V_{IJ} \rangle \Delta z^I \Delta z^J + \cdots, \tag{47}
\]

where the scalar fields \( z^I \)'s are expanded as \( z^I = \langle z^I \rangle + \Delta z^I \) around the vacuum \( \langle z^I \rangle \).

2. When there exists a mass mixing between the heavy and light sectors, we need to diagonalize them to identify the light and heavy fields correctly.

3. We solve the stationary conditions of the potential for the heavy scalar fields while keeping the light scalar fields arbitrary and then integrate out the heavy fields by inserting the solutions of the stationary conditions into the potential. We take the flat limit simultaneously.

### 3.3 Derivatives of \( K \) and \( W \)

It is convenient to write both the Kähler potential \( K \) and the superpotential \( W_{SG} \) in terms of the variations \( \Delta z^I \) and \( \Delta z^*_J \) as follows,

\[
K = \langle K \rangle + \langle K_I \rangle \Delta z^I + \langle K^*_J \rangle \Delta z^*_J \\
+ \langle K^I_J \rangle \Delta z^I \Delta z^*_J \\
+ \frac{1}{2} \langle K_I^I \rangle \Delta z^I \Delta z^*_J + \frac{1}{2} \langle K^I_J \rangle \Delta z^*_I \Delta z^*_J \\
+ \cdots \tag{48}
\]
\[ W_{SG} = \langle W_{SG} \rangle + \langle W_{SGI} \rangle \Delta z^I + \frac{1}{2} \langle W_{SGIJ} \rangle \Delta z^I \Delta z^J + \frac{1}{3!} \langle W_{SGIJJ} \rangle \Delta z^I \Delta z^J \Delta z^J' + \cdots, \tag{49} \]

where the ellipses represent higher order terms in \( \Delta z \).

By using the expansions (48) and (49), we find the following estimations

\[
\langle G_i \rangle = O(M), \quad \langle G_K \rangle \leq O(M_X),
\]
\[
\langle G_A \rangle \leq O(m_3^{2/3}/M_X), \quad \langle G_k \rangle = 0,
\]
\[
\langle G_i^A \rangle \leq O(1), \quad \langle G_K^A \rangle \leq O(1), \quad \langle G^A_K \rangle \leq O(M_X/M),
\]
\[
\langle G^A_i \rangle \leq O(M_X/M), \quad \langle G^A_k \rangle = 0,
\]
\[
\langle G_{ij} \rangle \leq O(1), \quad \langle G_{KL} \rangle \leq O(M_{KL}/m_{3/2}), \quad \langle G_{Kj} \rangle \leq O(M_X/M),
\]
\[
\langle G_{AJ} \rangle \leq O(M_X/M), \quad \langle G_{kj} \rangle = 0,
\]
\[
\langle G_{sB} \rangle \leq O(1), \quad \langle G_{ni} \rangle \leq O(1), \tag{52} \]

where \( M_{KL} \) is the SUSY fermion mass coming from the superpotential. Here we used the assumption that our vacuum solution is near to that in the SUSY limit and a perturbative argument to derive the second relation in (50). And we used the relations (46) and \( \langle D^\alpha \rangle \leq O(m_3^{2/3}) \) to derive the third relation in (50).

By using the equality from the gauge invariance (136), we derive the following relations,

\[
\langle G_{AK} \rangle \leq O(1/M_X), \quad \langle G_{ABI} \rangle \leq O(1/M_X),
\]
\[
\langle G_{ABC} \rangle \leq O(1/M_X), \quad \langle G_{Aki} \rangle \leq O(1/M),
\]
\[
\langle G_{ABJ} \rangle \leq O(1/M), \quad \langle G_{Aij} \rangle \leq O(1/M) \tag{53} \]

or

\[
\langle W_{SGAK} \rangle \leq O(m_{3/2}/M_X), \quad \langle W_{SGABI} \rangle \leq O(m_{3/2}/M_X),
\]
\[
\langle W_{SGABC} \rangle \leq O(m_{3/2}/M_X), \quad \langle W_{SGAKj} \rangle \leq O(m_{3/2}/M),
\]
\[
\langle W_{SGABJ} \rangle \leq O(m_{3/2}/M), \quad \langle W_{SGAij} \rangle \leq O(m_{3/2}/M). \tag{54} \]
3.4 Stability of Gauge Hierarchy

The mass squared matrices of the scalar fields are simply given by the VEVs of the second derivatives of the potential. From Eqs. (38)–(41), we get the relations,

\[ V^I_J = \frac{\partial^2 V}{\partial \phi^I \partial \phi^*_J} = \]

\[ M^2 (e^{G/M^2})^I_J U + M^2 (e^{G/M^2})^I_J U_J^J + M^2 (e^{G/M^2})^J_I U_I + M^2 e^{G/M^2} U^I_J \]

\[ + \frac{1}{2} (\text{Re} f^{-1})_{\alpha \beta, I} D^\alpha D^\beta + (\text{Re} f^{-1})_{\alpha \beta, J} D^\alpha (D^\beta)^J + (\text{Re} f^{-1})_{\alpha \beta} D^\alpha (D^\beta)^J \]

\[ + (\text{Re} f^{-1})_{\alpha \beta} D^\alpha (D^\beta)^J + (\text{Re} f^{-1})_{\alpha \beta} (D^\alpha)^J (D^\beta)^J \]  \hspace{1cm} (55)

and

\[ V_{I,J} = \frac{\partial^2 V}{\partial \phi^I \partial \phi^J} = \]

\[ M^2 (e^{G/M^2})_{I,J} U + M^2 (e^{G/M^2})_{I,J} U_J^J + M^2 (e^{G/M^2})_{J,I} U_I + M^2 e^{G/M^2} U_{I,J} \]

\[ + \frac{1}{2} (\text{Re} f^{-1})_{\alpha \beta, I} D^\alpha D^\beta + (\text{Re} f^{-1})_{\alpha \beta, J} D^\alpha (D^\beta)_I + (\text{Re} f^{-1})_{\alpha \beta} D^\alpha (D^\beta)_I \]

\[ + (\text{Re} f^{-1})_{\alpha \beta} D^\alpha (D^\beta)_{I,J} + (\text{Re} f^{-1})_{\alpha \beta} (D^\alpha)_I (D^\beta)_J. \] \hspace{1cm} (56)

By using the relations (51)–(54), the VEVs of \( V^I_J \) and \( V_{I,J} \) are estimated as\(^\ddagger\)

\[ \langle V^{(F)}_{L,K} \rangle = O(M^2_X), \langle V^{(F)}_{A} \rangle = O(m^2_{3/2}), \langle V^{(F)}_{K} \rangle = O(m^2_{3/2}), \]

\[ \langle V^{(F)}_{i,j} \rangle = O(m^2_{3/2}), \langle V^{(F)}_{B,K} \rangle = O(m^2_{3/2} M_X), \langle V^{(F)}_{I,K} \rangle = O(m^2_{3/2} M_X), \]

\[ \langle V^{(F)}_{K,J} \rangle = O(m^2_{3/2} M_X^2), \langle V^{(F)}_{A,K} \rangle = O(m^2_{3/2}), \]

\[ \langle V^{(F)}_{A,J} \rangle = O(m^2_{3/2} M_X / M), \langle V^{(F)}_{K,J} \rangle = 0 \] \hspace{1cm} (57)

and

\[ \langle V^{(F)}_{K,ijkl} \rangle = O(m^2_{3/2} M), \langle V^{(F)}_{AB} \rangle = O(m^2_{3/2}), \langle V^{(F)}_{kl} \rangle = O(m^2_{3/2} M), \]

\[ \langle V^{(F)}_{ij} \rangle = O(m^2_{3/2} M), \langle V^{(F)}_{KB} \rangle = O(m^2_{3/2} M), \langle V^{(F)}_{Kl} \rangle = O(m^2_{3/2} M), \]

\[ \langle V^{(F)}_{K,j} \rangle = O(m^2_{3/2} M), \langle V^{(F)}_{A} \rangle = O(m^2_{3/2}), \langle V^{(F)}_{A,l} \rangle = O(m^2_{3/2}), \]

\[ \langle V^{(F)}_{k,l} \rangle = 0. \] \hspace{1cm} (58)

\(^\ddagger\)For simplicity, hereafter we consider only the case that the equality holds.
respectively. The quantities of order \( m_{3/2} M \) in \( \langle V_I^{(F)} \rangle \) originates in the term \( \langle M e^{G/2M^2} G_{IJ'} \rangle \langle F^{J'} \rangle \). If \( \langle V_I \rangle \)'s are \( O(m_{3/2} M) \) for the light fields \( z^I \), the masses of light fields can get intermediate values after the diagonalization of mass matrix. The masses of those fermionic partners stay at the weak scale. The weak scale can be destabilized in the presence of the weak Higgs doublets with intermediate masses. This is so called ‘gauge hierarchy problem’. Only when \( \langle M e^{G/2M^2} G_{IJ'} \rangle \langle F^{J'} \rangle \)'s meet some requirements, the hierarchy survives. In this paper, we require the following conditions,

\[
\langle M e^{G/2M^2} G_{IJ'} \rangle \langle F^{J'} \rangle \leq O(m_{3/2}^2) \quad (59)
\]

for the light fields \( z^I \) and \( z^J \),

\[
\langle M e^{G/2M^2} G_{KIJ} \rangle \langle F^{J'} \rangle \leq O(m_{3/2} M_X), \quad (60)
\]

\[
\langle M e^{G/2M^2} G_{KJ'} \rangle \langle F^{J'} \rangle \leq O(m_{3/2} M_X^2/M), \quad (61)
\]

\[
\langle M e^{G/2M^2} G_{IJ'} \rangle \langle F^{J'} \rangle \leq O(M_X^2) \quad (62)
\]

for the heavy fields \( z^I \) and \( z^J \), and

\[
\langle M e^{G/2M^2} G_{A_{J'} \rangle \langle F^{J'} \rangle \leq O(m_{3/2}^2 M_X/M). \quad (63)
\]

The conditions (59)–(63) correspond to the statement that the magnitudes of \( \langle M e^{G/2M^2} G_{IJ'} \rangle \langle F^{J'} \rangle \) are equal to or smaller than the rest terms. The hidden ansatz trivially satisfies the above conditions. The gauge hierarchy problem has been discussed on the postulation that \( M_X \) is identified with \( M \) in Ref. [25].

The contributions from the \( D \)-term are naively estimated as follows,

\[
\langle V^{(D)}_{\kappa} \rangle = O(M_X^2), \quad \langle V^{(D)}_{\kappa \kappa} \rangle = O(M_X^2),
\]

\[
\langle V^{(D)}_{j} \rangle_{K}, \quad \langle V^{(D)}_{j K} \rangle = O(M_X^3/M), \quad \langle V^{(D)}_{j A} \rangle, \quad \langle V^{(D)}_{K j} \rangle = O(M_X^3/M),
\]

\[
\langle V^{(D)}_{j} \rangle_{i}, \quad \langle V^{(D)}_{i j} \rangle = O(M_X^4/M^2),
\]

\[
\langle V^{(D)}_{j} \rangle_{K}, \quad \langle V^{(D)}_{i j} \rangle = 0 \quad (64)
\]

where we used the relation \( \langle (D^A)_I \rangle = \langle z^T A_{\kappa} K_{I}^\kappa \rangle = O(M_X) K_{I}^A \). We require that the light fields defined by using \( \mu_{IJ} \) get no heavy masses from the \( D \)-term. For simplicity, we impose the conditions such as

\[
\langle V^{(D)}_{j} \rangle_{I} \leq \langle V^{(F)}_{j} \rangle_{I} \quad (65)
\]
\[ \langle V_{i,j}^{(D)} \rangle \leq \langle V_{i,j}^{(F)} \rangle \] (66)

for the light fields \( z^I \). They yield to the following relations

\[ \langle K^k_A \rangle, \langle K^k_A \rangle = O\left( \frac{m_3^{3/2}}{M_X^2} \right) \] (67)

and

\[ \langle K^i_A \rangle, \langle K^i_A \rangle = O\left( \frac{m_2^{3/2}}{M_X M} \right). \] (68)

The analysis could be made based on weaker requirements than (65) – (66), but we will not discuss it further to avoid a complication and a subtlety in this paper.

### 3.5 Diagonalization of Mass Matrix

The mass term is written as

\[ V^{mass} = \frac{1}{2} \langle V_{i,j} \rangle \Delta z^i \Delta z^j, \] (69)

where \( \Delta z^i = (\Delta z^K, \Delta z^L; \Delta z^A, \Delta z^B; \Delta z^i; \Delta \tilde{z}^i; \Delta \tilde{z}^j; \Delta z^k; \Delta \tilde{z}^j; \Delta z^k) \). From the discussion in the previous subsection, the orders of \( \langle V_{i,j} \rangle \) are estimated as

\[ \langle V_{i,j} \rangle = O \left( \begin{array}{ccc} M_X^2 & M_X^2 & m_{3/2} M_X \\ M_X^2 & M_X^2 & m_{3/2} M_X \\ m_{3/2} M_X & m_{3/2} M_X & m_{2/3}^2 \end{array} \right) \] (70)

for gauge non-singlet fields \( (\Delta z^K; \Delta z^A; \Delta z^k) \) and

\[ \langle V_{i,j} \rangle = O \left( \begin{array}{ccc} M_X^2 & M_X^2 & m_{3/2} M_X^2 / M \\ M_X^2 & M_X^2 & m_{3/2} M_X / M \\ m_{3/2} M_X^2 / M & m_{3/2} M_X / M & m_{3/2}^3 \end{array} \right) \] (71)
for gauge singlet fields \((\Delta z^K; \Delta z^A; \Delta \tilde{z}^i)\). As the matrix \(\langle V_{ij} \rangle\) is hermitian, it can be diagonalized by the use of a certain unitary matrix \(U^I_J\). The mass eigenstate \(\phi^I\) is related to \(\Delta z^I\) as \(\phi^I = U^I_J \Delta z^J\). We denote the heavy fields with mass \(O(M_X)\) as \(\phi^H\) and the light fields with mass \(O(m_3^2/2)\) as \(\phi^L\) where \(H = (K, A)\) and \(L = (i, k)\). Next we would like to integrate out the heavy fields \(\phi^H\). For this purpose, it is convenient to choose the variables

\[
\begin{align*}
\Delta \hat{z}^H &= (U^H_{H'})^{-1} \phi^H,
\Delta \hat{z}^L &= (U^L_{L'})^{-1} \phi^L
\end{align*}
\]

or

\[
\begin{align*}
\Delta \hat{z}^I &= \hat{U}^I_J \Delta z^J, \\
\hat{U}^I_J &\equiv \begin{pmatrix} I & (U^H_{H'})^{-1}U^H_{L'} \\ (U^L_{L'})^{-1}U^L_{H'} & I \end{pmatrix}.
\end{align*}
\]

Here we used the fact that \(\text{det}U^H_{H'} = 1 + O(m_3^2/M_X^2)\) and \(\text{det}U^L_{L'} = 1 + O(m_3^2/M_X^2)\) and neglected the higher order terms. The orders of off-diagonal elements of \(\hat{U}^I_J\) are estimated as

\[
\begin{align*}
\hat{U}^K_I &= O\left(\frac{m_3^2}{M_X}\right), \quad \hat{U}^A_I = O\left(\frac{m_3^2}{M_X}\right), \\
\hat{U}^K_J &= O\left(\frac{m_3^2}{M}\right), \quad \hat{U}^A_J = O\left(\frac{m_3^2}{M M_X}\right).
\end{align*}
\]

### 3.6 Calculation of the Effective Theory

The rest in the procedure are as follows,

1. We write down the scalar potential by using new variables \(\Delta \hat{z}^I\).
2. We take the flat limit and integrate out the heavy fields by inserting the solutions of the stationary conditions into the full potential.

We can write down the Kähler potential \(K\), the superpotential \(W_{SG}\) and the \(D\)-auxiliary fields \(D^a\) in terms of the variations \(\Delta \hat{z}^I\) as follows,

\[
K = \tilde{K}(\Delta \hat{z})
\]
\[
W_{SG} = \hat{W}(\Delta \hat{z}) = \langle \hat{W} \rangle + \langle \hat{W} \rangle \Delta \hat{z} \hat{I} + \frac{1}{2} \langle \hat{W} \rangle \Delta \hat{z} \hat{I} \Delta \hat{z} \hat{J} + \ldots
\]

\[
W_{SG} = \hat{W}(\Delta \hat{z}) = \langle \hat{W} \rangle + \langle \hat{W} \rangle \Delta \hat{z} \hat{I} + \frac{1}{2} \langle \hat{W} \rangle \Delta \hat{z} \hat{I} \Delta \hat{z} \hat{J} + \ldots
\]

and

\[
D^\alpha = \hat{D}^\alpha(\Delta \hat{z}) = (\hat{K}_\lambda + \hat{K}_I \hat{U}_\lambda)(T^A)^\lambda_t((z^\alpha) + (\hat{U}^{-1})^\alpha_j \Delta \hat{z}^j),
\]

where the ellipses represent terms of higher orders and \( \hat{U}_j = \delta^j_j + \Delta \hat{U}_j \).

For a later convenience, we deform \( V^{(F)} \) as follows,

\[
V^{(F)} = \exp(\hat{K}/M^2)\left(\hat{G}_\kappa(\hat{K}^{-1})^{\kappa\lambda} \hat{G}_\lambda + \hat{G}_I(\hat{K}^{-1})^{ij} \hat{G}_j - 3\frac{\hat{W}^2}{M^2}\right) + \Delta V^{(F)},
\]

where

\[
\hat{G}_\kappa \equiv \hat{G}_\kappa + \hat{G}_I(\hat{K}^{-1})^{ij} (\hat{K})_{\mu\kappa},
\]

\[
\hat{G}_\lambda \equiv \hat{G}_\lambda + (\hat{K})_{\lambda\nu}(\hat{K}^{-1})^{\nu j} \hat{G}_j,
\]

\[
\hat{G}_I \equiv \hat{W}_I^* + \frac{\hat{K}_I}{M^2} \hat{W}^*,
\]

\[
\hat{G}_I \equiv \hat{W}_I + \frac{\hat{K}_I}{M^2} \hat{W}
\]

and

\[
(\hat{K}^{-1})^{ij} \equiv (\hat{K}^{-1})^{ij} - (\hat{K}^{-1})^{ij} (\hat{K})_{\mu\nu}(\hat{K}^{-1})^{\nu j},
\]

\[
\Delta V^{(F)} \equiv \exp(\hat{K}/M^2)\left(\hat{G}_I \Delta(\hat{U})_I^I(\hat{K}^{-1})^{IJ} \hat{G}_J + \hat{G}_I(\hat{K}^{-1})^{IJ} \hat{G}_J \Delta(\hat{U})_J^I\right) + \hat{G}_I[(\hat{K}^{-1})^{IJ} \Delta(\hat{U}^{-1})_I^J + (\hat{K}^{-1})^{IJ} \Delta(\hat{U}^{-1})_J^I]\hat{G}_J
\]

\[
+ O((\Delta U^{-1})^2).
\]
We should not confuse $(\hat{K}^{-1})^\hat{i}_j$, $\hat{G}_K$ and $\hat{G}_\lambda$ with $(\hat{K}^{-1})^\hat{i}_j$, $\hat{G}_K$ and $\hat{G}_\lambda$, respectively. (Notice that the difference of the size of hat.) Here $(\hat{K})_{\mu\nu}$ is the inverse matrix of $(\hat{K}^{-1})^\mu_\nu$. We expand $\Delta \hat{z}^I$ in powers of $m_{3/2}$ such as

$$ \Delta \hat{z}^I = \delta \hat{z}^I + \delta^2 \hat{z}^I + \cdots, \quad (88) $$

with $\delta^n \hat{z}^I = O(m_{3/2}^n/M_X^{n-1})$. We assume $\Delta \hat{z}^k = O(m_{3/2})$ for the light fields, e.g., $\delta^2 \hat{z}^k = \delta^3 \hat{z}^k = \cdots = 0$. In the same way, we expand the $\hat{G}_\lambda$, $\hat{G}_j$ and $\hat{D}^\alpha$ in powers of $m_{3/2}$ such as

$$ \hat{G}_\lambda = \delta \hat{G}_\lambda + \delta^2 \hat{G}_\lambda + \cdots, \quad (89) $$

$$ \hat{G}_j = \delta \hat{G}_j + \delta^2 \hat{G}_j + \cdots \quad (90) $$

and

$$ \hat{D}^\alpha = \delta \hat{D}^\alpha + \delta^2 \hat{D}^\alpha + \cdots. \quad (91) $$

Those orders are given as $\delta^n \hat{G}_\lambda = O(m_{3/2}^n/M_X^{n-2})$, $\delta^n \hat{G}_j = O(m_{3/2}^n/M_X^{n-2})$ and $\delta^n \hat{D}^\alpha = O(m_{3/2}^n/M_X^{n-2})$ up to the factor $O((M_X/M)^n)$.

The following relations are derived from the expansions of $\hat{G}_\lambda$ and $\hat{G}_j$

$$ \delta \hat{G}_K = \langle \hat{W}_K \rangle + \langle \hat{W}_{KL} \rangle \delta \hat{z}^L $$

$$ + \frac{1}{M^2} \langle \hat{W}_K \rangle \langle \hat{K}_K \rangle + \langle (\hat{K})_{K\nu} \rangle \langle (\hat{K}^{-1})^{\nu j} \rangle \delta \hat{G}_j, \quad (92) $$

$$ \delta^2 \hat{G}_K = \langle \hat{W}_{KL} \rangle \delta^2 \hat{z}^L + \langle \hat{W}_{KL} \rangle \delta \hat{z}^L \delta \hat{z}^C + \langle \hat{W}_{KL} \rangle \delta \hat{z}^A $$

$$ + \frac{1}{2} \langle \hat{W}_{KL} \rangle \delta \hat{z}^\lambda \delta \hat{z}^\mu $$

$$ + \frac{1}{M^2} \left( \frac{1}{2} \langle \hat{W}_{LM} \rangle \delta \hat{z}^L \delta \hat{z}^M \langle \hat{K}_K \rangle + \langle \hat{W} \rangle \langle \hat{K}_{Kj} \rangle \delta \hat{z}^j \right) $$

$$ + \langle (\hat{K})_{K\nu} \rangle \langle (\hat{K}^{-1})^{\nu j} \rangle \delta^2 \hat{G}_j + \delta (\langle (\hat{K})_{K\nu} \rangle (\hat{K}^{-1})^{\nu j} \rangle \delta \hat{G}_j, \quad (93) $$

$$ \delta \hat{G}_A = \langle (\hat{K})_{A\nu} \rangle \langle (\hat{K}^{-1})^{\nu j} \rangle \delta \hat{G}_j, \quad (94) $$

$$ \delta^2 \hat{G}_A = \langle \hat{W}_{AI} \rangle \delta \hat{z}^I + \frac{1}{2} \langle \hat{W}_{AI} \rangle \delta \hat{z}^L \delta \hat{z}^I + \frac{1}{M^2} \langle \hat{W}_{AI} \rangle \delta \hat{z}^j $$

$$ + \langle (\hat{K})_{A\nu} \rangle \langle (\hat{K}^{-1})^{\nu j} \rangle \delta^2 \hat{G}_j + \delta (\langle (\hat{K})_{A\nu} \rangle (\hat{K}^{-1})^{\nu j} \rangle \delta \hat{G}_j \quad (95) $$
and

\[ \delta \hat{G}_j = \langle \hat{W}_j \rangle + \frac{\langle W \rangle}{M^2} \langle \hat{K}_j \rangle, \]

\[ \delta^2 \hat{G}_j = \langle \hat{W}_{jI} \rangle \delta \hat{z}^I + \frac{1}{2} \langle \hat{W}_{jIJ} \rangle \delta \hat{z}^I \delta \hat{z}^J + \frac{1}{M^2} \left( \frac{1}{2} \langle \hat{W}_{LM} \rangle \delta \hat{z}^L \delta \hat{z}^M \langle \hat{K}_j \rangle + \langle \hat{W} \rangle \langle \hat{K}_{jI} \rangle \delta \hat{z}^I \right), \]

respectively. While the expansion of \( \hat{D}^A \) gives

\[ \delta \hat{D}^A = \langle \hat{K}_\lambda \rangle (T^A)_\lambda^\kappa \delta \hat{z}^\kappa 
+ (\langle \hat{K}_{IJ} \rangle \delta \hat{z}^I \langle \hat{K}_1 \rangle \delta \hat{U}^J_1 \langle T^A \rangle_\kappa^\lambda \langle \hat{z}^\kappa \rangle), \]

\[ \delta^2 \hat{D}^A = \langle \hat{K}_\lambda \rangle (T^A)_\lambda^\kappa (\delta^2 \hat{z}^\kappa + \delta (\hat{U}^{-1})_j^I \delta \hat{z}^J) 
+ (\langle \hat{K}_{IJ} \rangle \delta^2 \hat{z}^I \langle \hat{K}_{IJ} \rangle \delta \hat{z}^J \delta \hat{z}^j 
+ \langle \hat{K}_1 \rangle \delta^2 \hat{U}^I_1 \langle T^A \rangle_\kappa^\lambda \langle \hat{z}^\kappa \rangle 
+ (\langle \hat{K}_{IJ} \rangle \delta \hat{z}^I \langle \hat{K}_1 \rangle \delta \hat{U}^I_1 \langle T^A \rangle_\kappa^\lambda \langle \hat{z}^\kappa \rangle). \]

The expansions of the stationary conditions \( \partial V / \partial \hat{z}^K = 0 \) and \( \partial V / \partial \hat{z}^A = 0 \) give

\[ \langle \hat{W}_{KL} \rangle \langle (\hat{K}^{-1})^{L_\mu} \rangle \delta \hat{G}_\mu = 0, \]

\[ \langle \hat{W}_{KL} \rangle \langle (\hat{K}^{-1})^{L_\mu} \rangle \delta^2 \hat{G}_\mu = -\delta \hat{G}_\mu \langle (\hat{K}^{-1})^{\mu\lambda} \rangle \langle \hat{W}_{KL} \rangle \delta \hat{z}^\sigma + \text{const.} \]

and

\[ \langle R e f^{-1}_{\alpha\beta} \rangle \langle (\hat{z} T^\alpha)^{\bar{\mu}} \rangle \langle \hat{K}_{A\bar{\mu}} \rangle \delta \hat{D}^\beta = 0, \]

\[ \langle R e f^{-1}_{\alpha\beta} \rangle \langle (\hat{z} T^\alpha)^{\bar{\mu}} \rangle \langle \hat{K}_{A\bar{\mu}} \rangle \delta^2 D^\beta = E \delta \hat{G}_\mu \langle (\hat{K}^{-1})^{\mu\lambda} \rangle \langle \hat{W}_{\lambda\sigma A} \rangle \delta \hat{z}^\sigma + \text{const.}, \]

respectively. Here \( E \equiv \langle \exp(K/M^2) \rangle \).

From Eqs. (102), (104), (106) and (100), we find \( \delta \hat{z}^K = 0 \) by using \( \langle \delta \hat{z}^K \rangle = 0 \). Eq. (101) gives the solution for \( \delta^2 \hat{G}_K \) as

\[ \delta^2 \hat{G}_K = \langle \hat{G}_K \rangle - \langle (\hat{K}_{KL}) \rangle \langle \hat{W}^{-1} \rangle^{KL} \delta \hat{G}_\mu \langle (\hat{K}^{-1})^{\mu\lambda} \rangle \langle \hat{W}_{MKL} \rangle \delta \hat{z}^l \]

where a constant factor of \( \delta^2 \hat{G}_K \) is denoted as \( \langle \hat{G}_K \rangle \). From Eqs. (77), (98) and (102), we find \( \delta \hat{D}^A = 0 \) and \( \delta \hat{z}^A = 0 \). By using the relations \( \langle W_{ABk} = \)
$O(m_{3/2}/M_X)$ and $\langle \hat{W}_{ABi} \rangle = O(m_{3/2}/M)$, we can show that $\delta^2 \hat{D}^A$ is a constant independent of the light fields. Therefore we will denote it by $\langle \hat{D}^A \rangle$.

Now it is straightforward to calculate the scalar potential $V^{\text{eff}}$ in the low-energy effective theory by substituting the solutions of the stationary conditions for the heavy fields. The result can be compactly expressed if we define the effective superpotential $\hat{W}_\text{eff}$ as

$$
\hat{W}_\text{eff}(z) = \frac{1}{2!} \mu_{kl} \delta \hat{z}^k \delta \hat{z}^l + \frac{1}{3!} \hat{h}_{klm} \delta \hat{z}^k \delta \hat{z}^l \delta \hat{z}^m, \tag{105}
$$

where

$$
\hat{\mu}_{kl} \equiv E^{1/2} \left( \langle \hat{W}_{kl} \rangle + \frac{\langle \hat{W} \rangle}{M^2} \langle \hat{K}_{kl} \rangle - \langle \hat{K}_{kli} \rangle \langle (\hat{K}^{-1})^{ij} \rangle \delta \hat{G}_j \right) + (m''_{3/2})_{kl}, \tag{106}
$$

$$
\hat{h}_{klm} \equiv E^{1/2} \langle \hat{W}_{klm} \rangle. \tag{107}
$$

Then we can write down the scalar potential of effective theory as

$$
V^{\text{eff}} = V^{\text{eff}}_{\text{SUSY}} + V^{\text{eff}}_{\text{Soft}}, \tag{108}
$$

$$
V^{\text{eff}}_{\text{SUSY}} = \frac{\partial \hat{W}^*_{\text{eff}}}{\partial \hat{z}^k} \langle (\hat{K}^{-1})^{kl} \rangle \frac{\partial \hat{W}_{\text{eff}}}{\partial \hat{z}^l} + \frac{1}{2} g_a^2 \langle (\hat{K}_{kl}) \hat{z}^i (T^a)^{kl} \rangle^2, \tag{109}
$$

$$
V^{\text{eff}}_{\text{Soft}} = A \hat{W}_{\text{eff}} + B_k (\hat{z})_{e_{\text{eff}}} \langle (\hat{K}^{-1})^{kl} \rangle \frac{\partial \hat{W}_{\text{eff}}}{\partial \hat{z}^l} + H.\text{c.}
$$

$$
+ B_k (\hat{z})_{e_{\text{eff}}} \langle (\hat{K}^{-1})^{kl} \rangle B_l (\hat{z})_{e_{\text{eff}}} + C(\hat{z})_{e_{\text{eff}}}
$$

$$
+ \Delta \hat{V} + \Delta V^{(\text{F})}, \tag{110}
$$

where $\Delta \hat{V} + \Delta V^{(\text{F})}$ is a sum of contributions such as

$$
\Delta \hat{V} = \Delta \hat{V}^{(\text{F})} + \Delta \hat{V}^{(\text{D})} + \Delta \hat{V}_1 + \Delta \hat{V}_1^{(\text{D})}, \tag{111}
$$

\[
\Delta \hat{V}_0^{(\text{F})} \equiv E \left( - \delta^2 \hat{G}_K \langle (\hat{K}^{-1})^{KL} \rangle \delta^2 \hat{G}_L + \delta^2 \hat{G}_A \langle (\hat{K}^{-1})^{AB} \rangle \delta^2 \hat{G}_B 
\right.
\]

$$
+ \delta \hat{G}_A \langle (\hat{K}^{-1})^{AB} \rangle \hat{F} \hat{G} \hat{B} + H.\text{c.}
$$

$$
+ \delta^2 \hat{G}_k \langle (\hat{K}^{-1})^{kL} \rangle \left( \langle \hat{W}_{Lk} \rangle \delta \hat{z}^k + \frac{1}{2} \langle \hat{W}_{Lkl} \rangle \delta \hat{z}^k \delta \hat{z}^l + \cdots \right) \tag{112}
$$

\[\text{Here we omitted the terms irrelevant to the gauge non-singlet fields } \delta \hat{z}^k \text{ and the terms whose magnitudes are less than } O(m_{3/2}^4).\]
\[
\Delta \hat{V}_0^{(D)} \equiv \langle \text{Re} f_{AB}^{-1} \rangle \langle \hat{D}^A \rangle \langle \hat{K}_{1,\lambda} \rangle \delta \hat{z}^I (T^B)^{\lambda}_{\kappa} \delta \hat{z}^\kappa, \tag{112}
\]

\[
\Delta \hat{V}_1^{(F)} \equiv \left( m_{3/2}^{'''} \right)_{kk} \delta \hat{z}^k \langle (\bar{K}^{-1})_{kl} \rangle \left( \frac{\partial \hat{V}_{\text{eff}}}{\partial \hat{z}^l} - (m_{3/2}^{''''})_{lm} \delta \hat{z}^m \right) + \left( m_{3/2} + m_{3/2}^{''''} \right)_{lm} \delta \hat{z}^m + H.c. \tag{113}
\]

\[
+ E \{ \delta \hat{G}_K \langle \langle \hat{K}^{-1} \rangle \rangle_{k} \delta^{\lambda} \hat{G}_B + H.c. + \delta \hat{G}_K \delta^{\lambda} \langle \langle \hat{K}^{-1} \rangle \rangle_{\kappa} \delta \hat{G}_\lambda + \delta \hat{G}_\kappa \delta \langle \langle \hat{K}^{-1} \rangle \rangle^{k} \delta^{\lambda} \hat{G}_K + H.c. \}, \tag{114}
\]

\[
\Delta \hat{V}_1^{(D)} \equiv \langle \text{Re} f_{AB}^{-1} \rangle \langle \hat{D}^A \rangle \langle \hat{K}_{1,\lambda} \rangle \delta \hat{z}^I \delta \hat{z}^J (T^B)^{\lambda}_{\kappa} \langle \hat{z}^\kappa \rangle, \tag{115}
\]

\[
\Delta \hat{\mathcal{V}}^{(F)} = E[\text{Const.}]^L \frac{1}{2} \langle \hat{W}_{klt} \rangle \delta \hat{z}^k \delta \hat{z}^l + H.c., \tag{116}
\]

\[
E[\text{Const.}]^L = E[\delta \hat{G}_I \delta \langle \hat{U} \rangle_{j}^I \langle \langle \hat{K}^{-1} \rangle \rangle^{jL} + \delta \hat{G}_I \langle \langle \hat{K}^{-1} \rangle \rangle^{jI} \delta \langle \hat{U} \rangle_{j}^L + \delta \hat{G}_I \langle \langle \hat{K}^{-1} \rangle \rangle^{jL} \langle \langle \hat{K}^{-1} \rangle \rangle^{jI} \langle \langle \hat{K}^{-1} \rangle \rangle^{jL} \rangle]. \tag{117}
\]

The quantities with a prime such as \( \delta^{\lambda} \hat{G}_B \) mean that the terms proportional to \( \delta^2 \hat{z}^I \) are omitted. The ellipses in Eq. (112) represent other terms in \( \delta^2 \hat{G}_K - \langle \hat{W}_{kLt} \rangle \delta^2 \hat{z}^L \). (Refer Eq. (93).) The soft SUSY breaking parameters \( A, \ B_k(z)_{\text{eff}} \) and \( C(\hat{z})_{\text{eff}} \) are given as

\[
A = m_{3/2}^{'''} - 3m_{3/2}^{''}, \tag{118}
\]

\[
B_k(\hat{z})_{\text{eff}} = (m_{3/2}^{'''} + m_{3/2})_{kl} \delta \hat{z}^l ; \tag{119}
\]

\[
C(\hat{z})_{\text{eff}} = E \delta \hat{G}_I \langle \langle \hat{K}^{-1} \rangle \rangle^{jI} \left( \frac{1}{3!} \langle \hat{W}_{jlll} \rangle \delta \hat{z}^l \delta \hat{z}^l \delta \hat{z}^l + \frac{\langle \hat{W} \rangle}{M^2} \delta^{\lambda} \hat{K}_j \right) + H.c.
\]

\[
+ E \left( \delta \hat{G}_I \delta^{\lambda} \langle \langle \hat{K}^{-1} \rangle \rangle^{jI} \delta \hat{G}_j + \frac{\langle \hat{W} \rangle}{M^2} \delta^{\lambda} \hat{K} \right) 
\]

\[
- \left( m_{3/2}^{'''} \right)_{kI} \langle \langle \hat{K}^{-1} \rangle \rangle^{kI} \langle \langle \hat{K}^{-1} \rangle \rangle^{kI} \delta \hat{z}^l \delta \hat{z}^l 
\]

\[
- \left( m_{3/2}^{'''} \right)_{kl} \langle \langle \hat{K}^{-1} \rangle \rangle^{kI} \langle \langle \hat{K}^{-1} \rangle \rangle^{kI} \delta \hat{z}^l \delta \hat{z}^l 
\]

\[
- \left( m_{3/2}^{''''} \right)_{kl} \langle \langle \hat{K}^{-1} \rangle \rangle^{kI} \langle \langle \hat{K}^{-1} \rangle \rangle^{kI} \delta \hat{z}^l \delta \hat{z}^l + H.c. \right \}
\]

\[
+ A \left[ \frac{\langle \hat{W} \rangle}{M^2} \langle \hat{K}_{kl} \rangle - \langle \hat{K}_{kl} \rangle \langle \langle \hat{K}^{-1} \rangle \rangle^{jI} \delta \hat{G}_j + \left( m_{3/2}^{''''} \right)_{kl} \delta \hat{z}^k \delta \hat{z}^l \right. \tag{120}
\]

23
where

\[(m_{3/2})_{kl} = \frac{E^{1/2}\langle \hat{W} \rangle}{M^2} \langle \hat{K}_{kl} \rangle, \quad (121)\]

\[m'_{3/2} = \frac{E^{1/2}\langle \hat{K}_{k\bar{l}} \rangle ((\hat{K}^{-1})^{ij}) \delta \hat{G}_j}{M^2}, \quad (122)\]

\[(m''_{3/2})_{kl} = -\frac{E^{1/2}\langle \hat{K}_{k\bar{l}} \rangle ((\hat{K}^{-1})^{\lambda}) \delta \hat{G}_\lambda}{M^2}, \quad (123)\]

\[(m'''_{3/2})_{kl} = -\frac{E^{1/2}\langle \hat{K}_{\kappa\bar{l}} \rangle ((\hat{K}^{-1})^{A}) \delta \hat{G}_A}{M^2}, \quad (124)\]

\[(m''''_{3/2})_{kl} = -\frac{E^{1/2}\langle \hat{K}_{\kappa\bar{l}} \rangle ((\hat{K}^{-1})^{\lambda}) \delta \hat{G}_\lambda}{M^2}. \quad (125)\]

The $\Delta \hat{V}_0^{(D)}$ and $\Delta \hat{V}_1^{(D)}$ come from the $D$-term of the heavy gauge sector and are referred to as the $D$-term contributions, while the others are called the $F$-term contributions.

We should consider the renormalization effects for the soft SUSY breaking parameters and diagonalize the scalar mass matrix $V_{kl}$ to derive the weak scale SUSY spectrum, which is expected to be measured in the near future.

4 Features of the Effective Lagrangian

The effective theory obtained in the previous section has some excellent features. We discuss two topics.

4.1 Chirality Conserving Mass

We discuss a chirality-conserving mass term $(m^2)_{kl}$, namely the coefficient of $\delta \hat{z}^k \delta \hat{z}^l$. They are easily extracted from $V_{\text{Soft}}^{\text{eff}}$ and given by

\[(m^2)_{kl} = (m^2)_{0kl} + (\Delta \hat{V}_0)_{kl} + (\Delta \hat{V}_1)_{kl}, \quad (126)\]

\[(m^2)_{0kl} \equiv \frac{\partial}{\partial \hat{z}^k} B_m(\hat{z})_{eff} \langle (\hat{K}^{-1})^{m\bar{n}} \rangle \frac{\partial}{\partial \hat{z}^l} B_m(\hat{z})_{eff} + \frac{\partial^2}{\partial \hat{z}^k \partial \hat{z}^l} C(\hat{z})_{eff}, \quad (127)\]

\[(\Delta \hat{V}_0)_{kl} \equiv \frac{\partial^2}{\partial \hat{z}^k \partial \hat{z}^l} \Delta \hat{V}_0^{(F)} + \langle \text{Ref}_{AB}^{-1} \hat{D}^A \rangle \langle \hat{K}_{\kappa\lambda} \rangle (T_B)^\lambda_\kappa, \quad (128)\]

\[(\Delta \hat{V}_1)_{kl} \equiv \frac{\partial^2}{\partial \hat{z}^k \partial \hat{z}^l} \Delta \hat{V}_1^{(F)} + 2\langle \text{Ref}_{AB}^{-1} \hat{D}^A \hat{K}_{kl} \rangle (T_B)^\lambda_\kappa \langle \hat{z}^\kappa \rangle. \quad (129)\]
The term \((m_0^2)_{ki}\) is present before the heavy sector is integrated out and so it respects the original unified gauge symmetry. On the other hand, other terms coming from \(\Delta \hat{V}\) can pick up effects of the symmetry breaking.

The last terms in Eqs. (128) and (129) are the \(D\)-term contributions. We discuss the conditions of those existence. The non-zero VEV of the \(D\)-term is allowed for a \(U(1)\) factor, \textit{i.e.} a diagonal generator from the gauge invariance. And the \(D\)-term for an unbroken generator cannot have its VEV. Thus it can arise when the rank of the gauge group is reduced by the gauge symmetry breaking. The \(D\)-term contribution is proportional to the charge of the broken \(U(1)\) factor and gives mass splittings within the same multiplet in the full theory. We can rewrite \(\delta^2 \hat{D}^A = \langle \hat{D}^A \rangle\) as

\[
\langle \hat{D}^A \rangle = 2(M^{-2}_V)^{AB} E \delta \hat{G}_\kappa \delta \hat{G}_\lambda \{ G^\mu_{\kappa \lambda} (\hat{z} T^B)_{\bar{\mu}} + G^{\bar{\mu} \kappa} (T^B)_{\lambda \bar{\mu}} \}
\]

by using the gauge invariance. We can see that the VEVs vanish up to \(O(m_0^4/2 (M_X/M)^n)\) when the Kähler potential has the minimal structure. Hence the sizable \(D\)-term contribution can appear only when the Kähler potential has a non-minimal structure.

The other terms in Eqs. (128) and (129) are related to the \(F\)-terms. They can be neglected in the case that the superpotential couplings are weak and the \(R\)-parity conservation is assumed. Therefore phenomenologically the \(D\)-term contribution to the scalar masses is important to probe SUSY-GUT models because it can give an additional contribution to squarks, sleptons and Higgs bosons\[17\].

### 4.2 Specific Case

Finally we discuss the relation between our result and that in Ref.\[14\]. For later convenience, we list up features in the approach of Ref.\[14\].

1. The starting theory is a unified theory obtained by taking the flat limit of SUGRA with a certain type of total Kähler potential, so the terms of order \(m_{3/2}^4 (M_X/M)^n\) are neglected. Since the unification scale \(M_X\) is now believed to be lower than the gravitational scale \(M\) from LEP data\[14\], this procedure can be justified in such a model. However it will be important when higher order corrections are to be considered. Then we must incorporate threshold effects and loop effects.
2. The scalar fields have canonical kinetic terms. It was assumed that the SUSY fermion mass matrix and the kinetic function can be diagonalized simultaneously, i.e., the relation $\langle \hat{K}_{\kappa \lambda} \rangle = \delta_{\kappa \lambda}$ is imposed.

3. The *Hidden* assumption on superpotential was taken because it was purposed to discuss consequences independent of the details of each models. The stability of gauge hierarchy is automatically guaranteed under this assumption.

4. The heavy-light mixing, in general, can occur after soft SUSY breaking terms are incorporated. Then we must re-define the scalar fields by diagonalizing the mass matrix. It was assumed that there is no heavy-light mixing after SUSY breaking.

   We shall derive the previous one $V_{\text{eff}}^{(\text{non})}$ from our scalar potential $V_{\text{eff}}$ by refering to the list.

1. When we take the limit $M_X/M \rightarrow 0$, we find that some terms vanish. For example, $(m_{3/2}^\prime)^{-\prime}_{3/2}$ and $(m_{3/2}^\prime)^{-\prime}_{3/2}$ vanish and

   $$\Delta \hat{V}_1 \rightarrow E \{ \delta \hat{G}_R \langle (\hat{K}^{-1})^{KB} \rangle \delta \hat{G}_B + H.c. \}. \quad (131)$$

2. Next we impose the condition $\langle \hat{K}_{\kappa \lambda} \rangle = \delta_{\kappa \lambda}$. Then $\Delta \hat{V}_1$ and some other terms vanish.

3. Further we take the hidden ansatz $\langle \hat{W}_{j\ldots k\ldots} \rangle = 0$. Then the trilinear coupling constant is reduced to

   $$AE^{1/2} \langle \hat{W}_{klm} \rangle + \frac{1}{2} (m_{3/2}^* + m_{3/2}^* \delta_{kk}) \langle (\hat{K}^{-1})^{km} \rangle \langle \hat{W}_{ntm} \rangle. \quad (132)$$

4. When we take a model with no heavy-light mixing, $\Delta V^{(F)}$ does not exist. We can find an ansatz for the Kähler potential that the heavy-light mixing does not occur in the gauge non-singlet sector after taking the flat limit. For example, the ansatz

   $$K = K^{(H)}(z^H, z^H; \bar{z}, \bar{z}^*) + K^{(k)}(z^k, z^k; \bar{z}, \bar{z}^*) \quad (133)$$

fulfills our requirement.

We find that $V_{\text{eff}}$ reduces to $V_{\text{eff}}^{(\text{non})}$ after the above procedures.
5 Conclusions

We have derived the low-energy effective Lagrangian from SUGRA with non-minimal structure and unified gauge symmetry in model-independent manner. The starting SUGRA is more general one than those considered before. The total Kähler potential has a non-minimal structure based on the hidden sector SUSY breaking scenario. We have distinguished between the scales $M_X$ and $M$.

It is important to investigate its consequences at low-energy because the non-minimal SUGRA appears naturally in many circumstances. For example, SSTs lead to the non-minimal SUGRA effectively. Even if SUGRA have the minimal structure at the tree level, it can get renormalized and as a result, in general, become non-minimal.

We have calculated the scalar potential by taking the flat limit and integrating out the heavy sector. The result is summarized in Eqs. (103) – (123). We found new contributions to the soft terms reflected to the non-minimality and the breaking of unified gauge symmetry. In particular, the sizable $D$-term contributions generally exist in the scalar masses when the rank of the gauge group is reduced by the gauge symmetry breaking and the Kähler potential has a non-minimal structure. Its phenomenological implications were discussed in Ref.[17]. Another important point is the gauge hierarchy problem. Many SUSY-GUT models achieve the small Higgs doublet masses by a fine-tuning of the parameters in the superpotential. If the SUSY breaking due to the hidden field condensations is turned on, a SUSY breaking Higgs mass term can become heavy and the weak scale can be destabilized. We have shown that the masses of light fields remain at the weak scale if the couplings of hidden-sector fields to visible-sector fields in the superpotential satisfy certain requirements.

We have derived the results in Ref.[14] by taking some limit and conditions. We also have studied the SUGRA with Fayet-Iliopoulos $D$-term and derived the low-energy effective theory.

It is believed that the measurements of SUSY spectrum at the weak scale can be useful in probing physics at SUSY-GUT and/or SUGRA, if the SUSY breaking scenario through the gauge-singlet sector in SUGRA is realized in nature. Hence the precision measurements should be carried out by the colliders in the near future.
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A Consequences of $\langle \partial V/\partial z^I \rangle = 0$

In this appendix, we give some consequences of the stationary conditions $\langle V \rangle \equiv \langle \partial V/\partial z^I \rangle = 0$. From Eq. (2), we find

$$V_I = M^2 (e^{G/M^2})_I U + M^2 e^{G/M^2} U_I + \frac{1}{2} (Ref^{-1})_{\alpha\beta,I} D^\alpha D^\beta$$

$$+ (Ref^{-1})_{\alpha\beta,D^\alpha (D^\beta)_I}$$

$$= G_I e^{G/M^2} U + M^2 e^{G/M^2} \left\{ G_{IJ} (K^{-1})_J^I G'' - G'' (K^{-1})_J^I K''_{IJ} + G_I \right\}$$

$$+ \frac{1}{2} (Ref^{-1})_{\alpha\beta,I} D^\alpha D^\beta + (Ref^{-1})_{\alpha\beta,I} D^\alpha (z^I T^3)_{J} K''_{IJ}.$$  (134)

Let us now multiply $(T^\alpha z)^I$ to the above, or project on a heavy-real direction. Using the identities derived from the gauge invariance of the total Kähler potential

$$G_{IJ}(T^\alpha z)^J + G_J(T^\alpha)^J_I - K''_{IJ} (z^I T^3)_{J} = 0,$$  (135)

$$G_{IJ,J'}(T^\alpha z)^J + G_{IJ,(T^\alpha)^J_I} + G_{J,J'}(T^\alpha)^{J'}_I$$

$$- (z^I T^3)_{J} K''_{IJ} = 0,$$  (136)

$$K''_{IJ}(T^\alpha z)^J + K''_{IJ}(T^\alpha)^J_I - [G'' (z^I T^3)_{J}]^I_1 = 0,$$  (137)

we obtain

$$V_I(T^\alpha z)^I = M^2 e^{G/M^2} (2 + U/M^2) D^\alpha - F^I F^*_J (G'' (z^I T^3)_{J})^I_1$$

$$+ \frac{1}{2} (Ref^{-1})_{\beta\gamma,I} (T^\alpha z)^I D^\beta D^\gamma$$

$$+ (Ref^{-1})_{\beta\gamma}(T^\alpha z)^I K''_{IJ} (z^I T^3)_{J} D^\beta.$$  (138)
Taking its VEV, we find
\begin{align*}
0 &= m_{3/2}^2 (2 + \langle U \rangle / M^2) \langle D^\alpha \rangle - \langle F^I \rangle \langle F^*_I \rangle \langle (G'^{\alpha} (z^\dagger T^\alpha)_{\mu})^T_I \rangle \\
&+ \frac{1}{2} \langle (R e f^{-1})_{\beta, \gamma, I} (T^\alpha z)^I \rangle \langle D^\beta \rangle \langle D^\gamma \rangle \\
&+ \frac{1}{2} \langle (R e f^{-1})_{\beta, \gamma} (M^2_V)^{\alpha \gamma} \langle D^\beta \rangle \rangle,
\end{align*}
(139)
where $(M^2_V)^{\alpha \beta} = 2 \langle (z^\dagger T^\beta) J (T^\alpha z)^J \rangle$ is, up to the normalization due to the gauge coupling constants, the mass matrix of the gauge bosons. Recalling that $(M^2_V)^{\alpha \beta}$ are assumed to be $O(M_X^2)$ for broken generators of the GUT symmetry, we conclude
\[ \langle D^\alpha \rangle \leq O(m_{3/2}^2), \]
(140)
as the first three terms of Eq. (139) are already of order $m_{3/2}^2 M_X^2$ or less. It is noteworthy that quite a similar equation to (140) is obtained for the case of a non-linear realization of the gauge symmetry.

From Eqs. (4) and (140), we find
\[ \langle G^A \rangle \leq O(m_{3/2}^2 / M_X). \]
(141)
By using the relations (50), we find
\[ \langle F^*_i \rangle = O(m_{3/2} M), \]
(142)
\[ \langle F^*_\kappa \rangle \leq O(m_{3/2} M_X). \]
(143)

We now return to $\langle V_I \rangle = 0$ itself. Taking the VEV of Eq. (134) and using the relations (50) and (140), we find
\[ \langle M e^{G/2M^2} G_{IJ} \rangle \langle F^I \rangle = O(m_{3/2}^2 M), \]
(144)
\[ \langle M e^{G/2M^2} G_{I\lambda} \rangle \langle F^I \rangle \leq O(m_{3/2}^2 M_X). \]
(145)
Since $\langle M e^{G/2M^2} G_{IJ} \rangle = \mu_{IJ} + O(m_{3/2})$, the above reads
\[ \mu_{IJ} \langle F^I \rangle = O(m_{3/2}^2 M), \]
(146)
\[ \mu_{I\lambda} \langle F^I \rangle \leq O(m_{3/2}^2 M_X). \]
(147)
Since we assume that $\mu_{KL}$ is $O(M_X)$ for heavy complex fields, we find
\[
\langle F^K \rangle \leq O(m_{3/2}^2).
\] (148)

The relation $\mu_{ij} = O(m_{3/2}^3)$ is derived from the above relations (142) and (146). Therefore, in our convention, the hidden-sector fields are contained in the light sector.

We can derive the following formula for the $D$-term condensation
\[
\langle D^{\alpha} \rangle = 2(M_v^{-2})^{\alpha\beta} \langle F^I \rangle \langle F^*_J \rangle \langle (G^{I'} (z^{1/3})^{1/2})_I \rangle 
\]
\[
+ \langle G^{I'}_I \rangle (T^{1/3})^J_1.
\] (149)

from Eq.(139). We shall discuss the condition that sizable $D$-term condensations of $O(m_{3/2}^3)$ exist. In the case with minimal Kähler potential, the formula (149) turns into simpler form as
\[
\langle D^{\alpha} \rangle = 2(M_v^{-2})^{\alpha\beta} \langle F^I \rangle \langle F^*_J \rangle \langle T^{1/3} \rangle^J_1.
\] (150)

It is shown that the $\langle D^{\alpha} \rangle$ is estimated as less than $O(m_{3/2}^4/M_X^2)$ because $\langle F^K \rangle \leq O(m_{3/2}^2)$ and $\langle F^A \rangle = m_{3/2} \langle G^A \rangle \leq O(m_{3/2}^3/M_X)$. This result also holds in the presence of Fayet-Iliopoulos $D$-term. Hence we find that the existence of non-minimal Kähler potential is essential to the appearance of sizable $D$-term condensations.

\section{SUGRA with Fayet-Iliopoulos $D$-term}

In this appendix, we investigate the low-energy theory derived from SUGRA with Fayet-Iliopoulos $D$-term. This subject has not been completely examined in the literatures \cite{26,27}. At first, we give comments about the SUGRA with Fayet-Iliopoulos $D$-term. (1) It necessarily has local $U(1)_R$ symmetry\cite{28}. It is expected that this symmetry become a key to solve the problem of baryon and lepton number violation\cite{29}. On the other hand, the anomaly cancellation condition can give a strong constraint on a model

\footnote{Note that a careful analysis tells us that $\langle F^K \rangle \leq O(m_{3/2}^2 (z^K)/M_K)$, where $M_K$ is the mass of $z^K$ from the superpotential. Thus as far as $\langle z^K \rangle \sim M_K$, the VEV of its $F$-term is always small $\sim m_{3/2}^2$.}
building \[27\]. (2) Fayet-Iliopoulos D-term \[30\] is generated by one-loop effects in SSTs with anomalous \(U(1)\) symmetry \[31\]. The anomalies are cancelled by Green-Schwarz mechanism \[32\]. Hereafter we denote Fayet-Iliopoulos \(U(1)\) symmetry as \(U(1)_R\).

Let us explain our starting point. The gauge group is \(G = G_{SM} \times U(1)_R\) where \(G_{SM}\) is a standard model gauge group \(SU(3)_C \times SU(2)_L \times U(1)_Y\). Two types of chiral multiples exist. One is a set of \(G_{SM}\) singlet fields denoted as \(\tilde{z}_i\). Some of them have non-zero \(U(1)_R\) charge and induce to the \(U(1)_R\) breaking. We assume that the SUSY is broken by the \(F\)-term condensations of chargeless \(\tilde{z}'s\). The second one is a set of \(G_{SM}\) non-singlet fields \(z_\kappa\). For simplicity, we treat all \(z_\kappa's\) as light fields. Of course, we can generalize the case that the gauge group is \(G = G_U \times U(1)_R\) where \(G_U\) is a unified group.

The scalar potential is given as

\[
V = V^{(F)} + V^{(D)},
\]

\[
V^{(F)} \equiv M^2 \exp(G/M^2)(G/(G^{-1})^IGJ - 3M^2),
\]

\[
V^{(D)} \equiv \frac{1}{2} (RRef^{-1})_{\alpha\beta}D^\alpha D^\beta + \frac{1}{2}(D^R)^2,
\]

where the index \(I, J,...\) run all scalar species and \(D^R \equiv g_R G_1(Q^R z)^I\). Here we denote the gauge coupling constant and \(U(1)\) charge of \(U(1)_R\) as \(g_R\) and \(Q_R\), respectively. We find that \(D^R\) contains a constant term \(2g_RM^2\) since

\[
D^R = g_R(K_1(Q^R z)^I + 2M^2),
\]

where we used the fact that the superpotential carries \(U(1)_R\) charge 2, i.e.,

\[
\frac{\partial W_{SG}}{\partial z}(Q^R z)^I = 2W_{SG}.
\]

It is easy to find that Fayet-Iliopoulos D-term \[30\] exists in the second term of \(V^{(D)}\). Note that the coefficient of Fayet-Iliopoulos D-term is fixed from the \(U(1)_R\) symmetry and this fact is essential to the conclusion that no sizable D-term contribution to scalar masses appears in the SUGRA with the minimal Kähler potential.

The \(U(1)_R\) is broken by the condensations of \(\tilde{z}\) because \(V^{(D)}\) is a dominant part in \(V\). The orders of those VEVs are estimated as \(\langle \tilde{z} \rangle = O(M)\). Hence the breaking scale of \(U(1)_R\) is of order \(M\).
Now we compute the scalar potential of the low-energy effective theory by taking the flat limit and integrating out the heavy fields in $\tilde{z}$'s simultaneously. The $D$-term contribution is added to the scalar masses in comparison with the result in Ref.\[21\]. We write it down in the form that the scalar masses are read off,

$$V^{(FI)} = V_{SUSY}^{(FI)} + V_{Soft}^{(FI)} + \Delta V^{(FI)},$$\hspace{1cm} (156)

$$V_{SUSY}^{(FI)} = \frac{\partial \hat{W}}{\partial z^*_\kappa} (\langle (K^{-1})^\lambda_\kappa \rangle_{\hat{W}}) \frac{\partial \hat{W}}{\partial z^*_\lambda} + \frac{1}{2} g_a^2 (\langle (K^\lambda_\kappa z^*_\lambda (T^\alpha)^\mu_\mu z^\mu) \rangle_{\hat{W}})^2,$$\hspace{1cm} (157)

$$V_{Soft}^{(FI)} = \hat{W} + B^a(z) (\langle (K^{-1})^\lambda_\kappa \rangle_{\hat{W}}) \frac{\partial \hat{W}}{\partial z^*_\lambda} + H.c.$$\hspace{1cm} (158)

$$\Delta V^{(FI)} = \frac{\partial \hat{W}}{\partial \tilde{z}^*_i} (\langle (K^{-1})^j_i \rangle_{\hat{W}}) \frac{\partial \hat{W}}{\partial \tilde{z}^*_j} + \langle \tilde{F}^i \rangle \frac{\partial \hat{W}}{\partial \tilde{z}^*_i} + H.c.$$\hspace{1cm} (159)

up to constant terms and higher order terms of $O(m^5_3/2/M)$. Here $B^a(z)$, $C(z, z^*)$, $\hat{W}$ and $\tilde{W}$ have been already defined in subsection 2.3.

We find that the $U(1)_R$ $D$-term contribution to scalar masses which can destroy universality among scalar masses at $M$. (Its existence was suggested in ref.\[26\], but we have proved it by deriving the full low-energy scalar potential from SUGRA directly.) As its contribution is proportional to the $U(1)_R$ charge, the $U(1)_R$ charge of matters can be known from the measurements of weak scale SUSY spectrum.

Our procedure and result are applicable to the effective SUGRAs derived from SSTs. It is known that there are many string models with $G \times U(1)_n$\[33\] and some models generate Fayet-Iliopoulos $D$-term\[34\]. Therefore it is important to search for a realistic string model by taking care of extra $U(1)$ symmetries.

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