Laser-Matter Interaction: Classical Regime versus Quantum Regime

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Doppler backscattering of optical laser photons on a “flying mirror” of relativistic electrons promises to yield coherent photons with MeV-range energies. We compare the nuclear interaction of such a laser pulse with the standard atom-laser interaction. The mean-field description of atoms must be replaced by a rate equation and the classical field strength, far too faint in nuclei, by the dipole transition rate. Significant nuclear excitation occurs for photon numbers much smaller than typical for atoms. That drastically reduces the requirements on the experimental realization of a “flying mirror”.

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Introduction. This paper is triggered by recent experimental, computational and theoretical advances in the production of high-energy coherent laser pulses, based on the following mechanism [1]. A first intense laser pulse ejects electrons from a nanometer-thin Carbon foil. The electrons attain relativistic energies and form a “flying mirror”. On that mirror, a second laser pulse is Doppler backscattered [2-9]. That increases both the energy \( h\omega_0 \) and the energy spread \( \sigma \) of the photons in the second pulse by a factor \( 2\sqrt{1-\frac{v}{c}} \) where \( v \) is the velocity of the ejected electrons. In principle, photon energies in the MeV range and beyond can be reached, accompanied by energy spreads in the 50 keV range and beyond.

Doppler backscattering of photons on a “flying mirror” of electrons has produced coherent photons in the far ultraviolet regime [6] but not yet coherent MeV photons. Attaining such energies apparently requires a further step. The electrons in the flying mirror must be compressed to a mean density that is close to condensed-matter values [10]. The present work anticipates a positive solution of that problem. We accordingly consider a coherent laser pulse with a typical energy \( h\omega_0 \approx 5 \) MeV per photon and with a typical energy spread \( \sigma \) in the 50 keV range.

The prospect of a laser beam with photon energies comparable to typical nuclear excitation energies raises important questions. What is the difference between the (well-investigated) laser-atom and the (novel) laser-nucleus interaction? What conditions follow from that comparison for the experimental realization of the Doppler-backscattered laser pulse? In the main body of this Letter we answer the first of these questions. We compare the interaction with a medium-weight or heavy nucleus of a pulse as specified above with that of a standard laser pulse with an atom. The answer to the second question emerges in the form of a corollary.

Laser-induced multiphoton excitation in atoms and in nuclei are fundamentally different processes, for two reasons. (i) In atoms, the Coulomb interaction between electrons has long range and is repulsive. It can often be accommodated in terms of a mean field. In lowest order, electrons are then described as a system of independent particles [11,12]. In nuclei, the residual nucleon-nucleon interaction beyond the mean field has short range, is strong, and drives the nucleus towards equilibrium. Reactions induced on medium-weight and heavy nuclei at excitation energies in the 10 MeV range and beyond are, therefore, described by rate equations [13,14], a regime which is also exploited in atomic/molecular systems, however at comparatively larger energies [15]. (ii) For multiphoton excitation in atoms, the use of the classical approximation for the electric field strength of the laser is standard and defines the classical regime [16]. Scaling of that field strength and of the dipole operator shows that this approach does not lead to significant excitation in nuclei. In contradistinction, scaling shows that the dipole transition rate is much enhanced in nuclei. That fact, combined with the use of rate equations for equilibration, defines the quantum regime and yields a satisfactory theoretical framework for laser-nucleus reactions. The number of photons per pulse required for multiple nuclear excitation via rate processes is orders of magnitude smaller than the number needed within the classical regime for multiple atomic excitation [17]. That fact drastically reduces the requirements for a successful experimental realization of the “flying mirror”.

In what follows we address both the quantum-optics community and the nuclear-physics community. The paper is written in that spirit.

Coupling to the Electromagnetic Field. With \( S \) denoting the quantum system (atom or nucleus) and \( F \) the quantized electromagnetic field, the total Hamiltonian is

\[
H = H_S + H_F + H_{\text{int}} = H_0 + H_{\text{int}}.
\]

For atoms (nuclei), the relevant photon energy is in the eV range (the MeV range, respectively). In either case the product of wave number \( k \) and atomic (nuclear) radius \( R \) obeys \( kR \ll 1 \). That justifies the use of the dipole approximation. The interaction part of \( H \) is then \( H_{\text{int}} = -eq\vec{E}(\vec{r}) \). Here \( \vec{E} \) denotes the operator of the free electric field strength, taken at the position \( \vec{r} \) of the atomic nucleus. In the atomic case, \( \vec{q} \) denotes the sum of the position operators of the electrons relative to the atomic nucleus. In the nuclear case, \( \vec{q} \) is proportional to the difference of the centers of mass of protons and neutrons [18]. The inner (direct) product of two vectors \( \vec{a} \) and \( \vec{b} \) is written as \( \langle \vec{a} | \vec{b} \rangle \) (as \( \langle \vec{a} | \vec{b} \rangle \), respectively). With \( t \) denoting the time, we use the interaction representation where
\[ H_{\text{int}} = \exp(iH_{0t}/\hbar) H_{\text{int}} \exp(-iH_{0t}/\hbar) \] correspondingly for \( \vec{q}(t) \) and \( \vec{E}(\vec{r}, t) \). Then

\[ H_{\text{int}} = -e\vec{q}(t)\vec{E}(\vec{r}, t) . \] (2)

Needless to say, Eq. (2) applies only while the laser pulse lasts. Transients due to onset and termination of the laser pulse are neglected.

The electric field strength \( \vec{E} \) is expanded in a set of orthonormal modes, defined \( [19] \) in a large but finite cubic normalization volume of side length \( L \) with periodic boundary conditions. The modes are polarized plane waves \( L^{-3/2}\vec{e}_\lambda \exp(i\vec{k}\vec{r}) \) with discrete wave vectors \( \vec{k} \). The two polarization vectors \( \vec{e}_\lambda \) with \( \lambda = 1, 2 \) are orthogonal upon each other and upon \( \vec{k} \). For brevity we use a joint index \( k = (\vec{k}, \lambda) \) for the associated creation and annihilation operators \( a^\dagger_k \) and \( a_k \) which obey \( [a_k, a_{k'}] = 0 = [a^\dagger_k, a_{k'}], [a_k, a^\dagger_{k'}] = \delta_{kk'} \).

With \( \omega_k = c|\vec{k}| \) we then have \( H_F = \frac{1}{2} \sum_k \hbar \omega_k (a^\dagger_k a_k + a_k a^\dagger_k) \). The expansion for the electric field strength reads \( [19] \)

\[
\vec{E}(\vec{r}, t) = \sum_k \sqrt{\frac{\hbar \omega_k}{2L^3}} e_\lambda \left[ \exp(i\vec{k}\vec{r} - i\omega_k t)a_k - \text{h.c.} \right] .
\] (3)

We use a basis of coherent states. For fixed \( k \) and arbitrary complex \( \alpha_k \) the normalized coherent state \( |\alpha_k\rangle \) obeys \( a_k |\alpha_k\rangle = \alpha_k |\alpha_k\rangle \). The expectation value \( n_k \) of the number of photons in mode \( k \) and the total number \( N \) of photons in the pulse are

\[
n_k = \langle \alpha_k | a_k^\dagger a_k | \alpha_k \rangle = |\alpha_k|^2 , \quad N = \sum_k n_k . \] (4)

**Description of the backscattered laser pulse.** We use three assumptions. First we assume \( [19] \) that the density matrix \( \rho(\{\alpha_k\}) \) of the laser pulse is stationary in time. Stationarity is plausible for pulse lengths considered here that are about two orders of magnitude larger than the wave length. Stationarity is equivalent to phase-averaging over the amplitudes \( \alpha_k \) and implies \( \text{Tr}[\rho a_k] = 0 \) and \( \text{Tr}[\rho a_k^\dagger a_k] = n_k \delta_{kk'} [19] \). For stationary fields the expectation value of the field strength, therefore, vanishes, and the field strength must be defined via the square root of the intensity. We write \( \vec{E} = \vec{E}^+ + \vec{E}^- \) where \( \vec{E}^+ (\vec{E}^-) \) contains the annihilation (the creation) operators, respectively. Then

\[
\text{Tr}[\rho \vec{E}^- (\vec{r}', t') \langle \vec{E}^+(\vec{r}, t) = \sum_k \frac{\hbar \omega_k}{2L^3} a_k^\dagger \langle \vec{e}_\lambda \exp[i(\vec{k}\vec{r}' - \vec{k}\vec{r} - i\omega_k(\vec{r}' - \vec{r})] . \] (5)

Second we assume that the laser is sufficiently monochromatic. We may then replace in Eq. (5) \( \omega_k \) by the mean frequency \( \omega_0 \) by \( \vec{k} \) by \( \vec{k}_0 \) and \( \vec{e}_\lambda \) by \( \vec{e}_{\lambda_0} \). Respectively. We use Eq. (3). Then the right-hand side of Eq. (5) factorizes and can be written as the product \( E_{\text{cl}}^- (\vec{r}', t') \langle \vec{E}^+_{\text{cl}} (\vec{r}, t) \) of the components

\[
\vec{E}^\pm_{\text{cl}} (\vec{r}, t) = \sqrt{N \frac{\hbar \omega_0}{2L^3}} \vec{e}_{\lambda_0} \exp\{\pm i\vec{k}_0 \vec{r} - i\omega_0 t\} \] (6)
of the classical electric field strength. Third, we assume that the “flying mirror” is not exactly planar. Then the backscattered pulse is not completely collinear but has a finite aperture.

**Classical Regime.** The expression

\[
H_{\text{int}} (\vec{r}, t) = -e\vec{q}(t)\vec{E}_{\text{cl}} (\vec{r}, t) \] (7)

for the interaction Hamiltonian defines the classical regime. We have to bear in mind, however, several restrictions. First, Eqs. (6) and (7) apply only in a restricted domain of space and time as defined by the coherence properties of the laser pulse. For a laser-induced nuclear reaction, quenching of the original laser pulse in the direction of propagation by Doppler backscattering results in a pulse that has approximately the shape of a circular disk with lateral radius \( r \approx \) several \( \mu \text{m} \) (the value for the pulse prior to backscattering) and length \( l \approx \hbar c/\sigma = 100 \) wave lengths so that \( l \ll r \). Accordingly, the lateral spread in momentum space is much smaller than the value \( k_0 \). The second restriction emerges when we consider terms of higher order than the first in \( H_{\text{int}} \). The transition probability contains a sum of traces of \( \rho(\vec{E}^+)^m (\vec{E}^-)^n \), with integer \( m, n \). Stationarity of \( \rho(\{\alpha_k\}) \) implies that only terms with \( m = n \) differ from zero. Use of the classical Hamiltonian (7) is possible only if for all \( n, \) \( \text{Tr}[\rho(\vec{E}^+)^m (\vec{E}^-)^n] \) factorizes in the same manner as does the right-hand side of Eq. (5). Then the field is fully coherent (the \( n^{\text{th}}\)-order correlation functions factorize for all \( n \)). That is the case for coherent laser beams where the field is generated by an “essentially classical source” \( [19] \). The third restriction is the most severe one. It arises because (as we have just noted) in the quantum approach the operators \( \vec{E}^+ + \vec{E}^- \) always contribute with equal powers to the transition probability. The corresponding property does not hold for the classical Hamiltonian where the transition probability does contain nonvanishing terms proportional to \( (\vec{E}^+_{\text{cl}})^m (\vec{E}^-_{\text{cl}})^n \) with \( m \neq n \). Dipole transitions connect only states of opposite parity. That constraint somewhat reduces the number of combinations \( (\vec{E}^+_{\text{cl}})^m (\vec{E}^-_{\text{cl}})^n \) with \( m \neq n \). Nevertheless the semiclassical approximation (7) is at best semiquantitatively correct.

**Single-Photon Process.** The quantum regime is here defined by the use of rate equations. We consider dipole-induced one-step photon absorption in the quantum system \( S \) from an initial state \( |i\rangle \) to a final state \( |f\rangle \). The process involves only \( \vec{E}^+(\vec{r}, t) \). The trace of the square of the transition matrix element then carries the factor \( \text{Tr}[\rho \vec{E}^-] \langle \vec{E}^+ \rangle \). As shown above, the transition is induced by the monochromatic electric field of Eq. (6). The calculation of the rate is, except for the factor \( \sqrt{N} \), identical to that for a single photon of frequency \( \omega_0 \), and is standard \( [20] \). First-order time-dependent perturbation theory yields the transition amplitude \( h_{i\rightarrow f}(t) \). For the square of that expression, we use the long-wavelength limit. The approximation (6) holds only within a finite domain of space and time, and it is necessary to sum over the finite width of the photon \( \vec{k} \), including the finite width of the aperture. That is done using \( [19] \) \( \sum_k \rightarrow (L^3/(2\pi)^3) \int d^3 k \). The last step involves the limit \( L \rightarrow \infty \) and removes the normalization volume \( L^3 \). For an on-shell transition \( (E_f - E_i = \hbar \omega_0) \) the dipole
absorption rate \( R_{\text{dip}} = (1/t) \sum_k |b_{k\to f}(t)|^2 \) is then given by
\[
R_{\text{dip}} = \frac{e^2}{\hbar c} N \frac{2}{\pi \omega} k_0^3 \sum f |\langle \vec{q} \vec{e} \lambda_0 |\vec{f}|\rangle|^2.
\]
(8)

We have used spherical polar coordinates. The sum indicates that the integral over the solid angle, i.e., the aperture of the pulse, the average over initial spin directions, and the sums over polarization directions and final spin directions have yet to be carried out. The aperture of the pulse is determined by the backscattering process. We assume that the experiment can be driven in such a way that the aperture becomes quite small and only weakly dependent on \( k_0 \). Then the solid-angle integral, though strongly suppressing the rate in comparison with its full solid-angle value, does not remove the \( k_0^3 \)-dependence of expression (8). That dependence differs from the one of a completely collinear optical laser (21) where the rate is linear in \( k_0 \) because one considers only the integral over the spread of the photon frequency \( \omega \).

The dipole width is \( \Gamma_{\text{dip}} = \hbar R_{\text{dip}} \). Without the factor \( N \), the dipole width in nuclei is of the order 100 eV \cdot 1 \text{ keV}. For \( N = 10^3 - 10^4 \) photons in the pulse, the nuclear dipole width is boosted into the MeV domain. Such values were used in the calculations of Refs. [22, 23].

**Multiphoton Process.** An exact treatment of a laser-induced multiphoton process would use time-dependent perturbation theory for \( H_{\text{int}}(t) \) and the eigenstates of \( H_S \) as intermediate states. Such an exact treatment may, however, be impractical or even beyond reach in interacting many-body systems. Two approximation schemes may be used: a mean field–approach combined with the classical approximation for the electric field (i), and rate equations (iii).

(i) Multiphoton Process in Atoms. The interaction Hamiltonian \( H_{\text{int}} \) is a (sum of) single-particle operator(s). That fact and the long range of the Coulomb interaction suggests using a mean-field approximation. Theoretical work on multiphoton processes in laser–atom interactions is, therefore, often based upon a combination of the independent-particle picture and the classical approximation (7) for \( H_{\text{int}} \). Each electron moves independently under the influence of the classical electric field strength \( \vec{E}_{\text{cl}} \). The field strength (5) is usually determined phenomenologically. Empirically, the product \( e R |\vec{E}_{\text{cl}}| \) ranges from meV values to maximum values in the eV range for very strong lasers. Such strong fields deform the nuclear Coulomb potential, and electrons may be set free by tunnel ionization. For a laser pulse of frequency \( \omega = 1 \text{ eV} / \hbar \), of length \( \hbar c / (10^{-2} \text{ eV}) \) and with lateral width of the order of \( 10^{-6} \text{ m} \), Poynting’s theorem shows that for \( e |\vec{E}| \), values in the range of \( (0.001 - 1) \text{ eV} / (10^{-8} \text{ cm}) \) are attained for photon numbers \( N \) in the range \( N = 10^6 - 10^{12} \). The classical approach has important advantages: It offers qualitative and intuitive insights into the way the laser interacts with atoms. It allows for further important and useful approximations such as the Keldysh formalism (see, for instance, Ref. [24, 26]). The time-dependent Schrödinger equation can be solved stepwise numerically while the solution of the full quantum problem would be much less transparent.

(ii) Nuclei: Scaling. The classical approximation (7) has little bearing for laser-nucleus interactions because the attainable classical field strengths are too faint. We show that using a scaling argument and a numerical simulation.

For the scaling we take an optimistic view, grossly overestimating the field strength. We use Poynting’s theorem \( \vec{E}_{\text{cl}} = N \hbar \omega / (2V_L) \). We assume that all \( N \) photons of the initial laser pulse are coherently Doppler backscattered on the “flying mirror” described above. That increases both the mean photon energy \( \hbar \omega \) and the energy spread \( \sigma \) of the photons to values of about 5 MeV and 50 keV, respectively, i.e., by a factor \( 10^7 \). We assume that the volume \( V_L \) of the laser pulse is quenched in the direction of propagation by that same factor while the lateral width remains unchanged. Then \( |\vec{E}_{\text{cl}}| \) scales with the factor \( 10^7 \). For a medium-weight or heavy nucleus, the nuclear radius is about four orders of magnitude smaller than the radius of the atom. The interaction Hamiltonian (7) thus scales with a factor \( 10^3 \). For a strong atomic laser, \( e R |\vec{E}_{\text{cl}}| \) is of the order eV. For the nuclear case scaling gives \( e R |\vec{E}_{\text{cl}}| \approx 1 \text{ keV} \). Typical excitation energies for low-lying states in medium-weight or heavy nuclei are two orders of magnitude larger than that figure, and three orders of magnitude larger in light nuclei. Within the classical approximation, efficient nuclear excitation seems therefore unlikely. Indeed, with \( e R |\vec{E}_{\text{cl}}| \approx 1 \text{ keV} \) and the duration time of the pulse given by \( \hbar / \sigma \), the nuclear transition amplitude is of order \( 1 \text{ keV} / \sigma \approx 1/50 \) and the transition probability is of order \( 4 \cdot 10^{-4} \).

Numerical calculations of photon absorption by \(^{16}\text{O}\) using the interaction Hamiltonian (7) confirm that scenario. We use a three-dimensional time-dependent Hartree-Fock code [27] based on the Skyrme energy density functional and a cutoff factor to account for the finite duration of the laser pulse, writing \( H_{\text{int}} = H_{\text{int}} \cos^2[\pi(t - T_{\text{pulse}})/(2T_{\text{pulse}})] \), where \( T_{\text{pulse}} \) is the half width of the pulse. The arguments given above suggest that we use \( e |\vec{E}_{\text{cl}}| = 10^{-4} \text{ MeV/fm} \) and \( T_{\text{pulse}} = 10^{-19} \text{ s} \). That faint field strength would require extreme precision in the numerical calculations. We use the fact that we are deeply in the linear regime and rescale \( |\vec{E}_{\text{cl}}| \) and \( T_{\text{pulse}} \), keeping the fluence, i.e., the product \( |\vec{E}_{\text{cl}}| T_{\text{pulse}}^2 \) constant. Our calculations were performed for \( |\vec{E}_{\text{cl}}| = 0.033 \text{ MeV/fm} \) and \( T_{\text{pulse}} = 2000 \text{ fm/c} \). Figure 1 shows the energy absorbed from the photon field versus time for a photon energy \( \hbar \omega = 10 \text{ MeV} \). Dividing that number by 10 MeV gives the average number of absorbed photons as \( 5 \cdot 10^{-6} \). That is actually an overestimate because we have enhanced \( |\vec{E}_{\text{cl}}| \) by a factor 30. On the other hand, larger absorption rates are expected for photon energies that are close to the energy of the dipole resonance and for heavier nuclei. In any case, these results confirm our estimates and show that the classical approximation (7) does not yield significant photon absorption in nuclei.

(iii) Multiphoton Process in Nuclei: Rate Equations. When we describe a single photon by setting \( N = 1 \) in Eq. (6), the resulting classical electric field strength is entirely negligible for exciting nuclei. Nevertheless, the absorption of
single photons with energies in the MeV range is an important process in nuclei. The apparent discrepancy is trivially resolved: Single-photon absorption is described not within the classical approximation (7) but in terms of the transition rate (8). The rate is proportional to \((\hbar \omega_0)^3\) and to \(R^2\) and, therefore, scales as \((10^7)^3(10^{-3})^2\) \(\approx 10^{13}\) while we have shown above that the square of the interaction Hamiltonian scales as \((10^7)^2(10^{-3})^2\) \(= 10^6\). The additional factor \(10^7\) appearing in the rate not only explains the strength of photon absorption in nuclei; The estimate below Eq. (8) suggests that for \(N \gg 1\) the dipole transition rate also plays a dominant role in laser-induced multiphoton absorption provided the process is described in terms of rate equations.

When would that be the case? In nuclei the strong residual short-range interaction (i.e., the nucleon-nucleon interaction beyond the mean field) drives the system towards statistical equilibrium (the “compound nucleus”). In a series of two-body collisions, the dipole mode excited by photon absorption mixes with other modes having (nearly) the same excitation energy and carrying the same quantum numbers until all such modes are occupied with equal weight. The characteristic time scale is that of a nucleon-nucleon collision induced by the residual interaction \([28, 29]\). The collision time \(\tau_{\text{coll}} \approx \lambda / v_F\) is the ratio of the nucleon mean free path \(\lambda \approx 4 – 6\) fm and of the Fermi velocity \(v_F = \sqrt{2E_F/m}\) where \(m\) is the nucleon mass and \(E_F \approx 30\) MeV is the Fermi energy. Thus, \(\tau_{\text{coll}} \approx 5 \cdot 10^{-23}\) s or \(\hbar/\tau_{\text{coll}} \approx 10\) MeV. We have shown below Eq. (8) that for \(N \approx 10^3 – 10^4\) the dipole width attains values in the MeV range, comparable with \(\hbar/\tau_{\text{coll}}\). In other words, the nucleus readjusts to absorption of a single photon within a time that is comparable to that for the absorption of the next photon. Therefore, laser-induced multiphoton absorption in nuclei must take account of relaxation processes.

In medium-weight and heavy nuclei, the only practicable way of doing that consists in using rate equations. At excitation energies above approx. 10 MeV, these nuclei are so complex that dynamical details cannot be followed in practice, and a statistical description must be used. The approach is standard in the theory of preequilibrium processes induced by projectiles of several 10 MeV kinetic energy impinging on nuclei \([14]\). Equilibration is described in terms of the change in time of the average occupation probabilities of classes of states. Such classes can, for instance, be defined by the number of particle-hole excitations out of the ground state. The associated rate equations involve \(\hbar/\tau_{\text{coll}}\) and density-of-states factors. Rates for laser-induced photon absorption and emission derived from \(R_{\text{dip}}\) couple the rate equations at different excitation energies. That is the approach used in Ref. [30] for the perturbative regime \((\Gamma_{\text{dip}} \ll \hbar/\tau_{\text{coll}})\) and in Refs. [22, 23] in the quasidiabatic regime \((\Gamma_{\text{dip}} \approx \hbar/\tau_{\text{coll}})\). We stress that a rate equation may be used even for excitation out of the ground state \([30]\). For \(N \approx 10^3 – 10^4\) that yields an excitation probability of order unity.

**Sudden Regime.** The results of the present paper cast new and unexpected light on the (so far unexplored) sudden regime \(\Gamma_{\text{dip}} \gg \hbar/\tau_{\text{coll}}\). In that regime photoabsorption is so fast that nuclear relaxation may become negligible. But then the use of rate equations is not justified. And we have shown above that the classical approximation (7) does not yield significant excitation because the electric field strength is too faint. Significant nuclear excitation by laser-induced photon absorption is, therefore, possible only when between any two absorption processes, the nucleus does have time to relax. In other words, in the sudden regime the characteristic time scale for photon absorption is set by the collision time \(\tau_{\text{coll}}\), not by \(\hbar/\Gamma_{\text{dip}}\). That suggests that in the sudden regime, nuclear excitation processes are only quantitatively but not qualitatively different from the ones in the quasidiabatic regime \([23]\).

**Conclusions.** Within the dipole approximation, we have compared laser-induced photon absorption processes in atoms and in nuclei, assuming laser pulses of equal photon numbers but different mean energies per photon \((0.5\) eV versus \(5\) MeV) and energy spreads \((0.005\) eV versus \(50\) keV). If the lateral width of the laser is unchanged, scaling shows that the electric field strength \(|\vec{E}_{\text{cl}}|\) scales with the factor \(10^7\) while the system radius scales with a factor \(10^{-4}\). Therefore, the square of a typical dipole matrix element of the interaction Hamiltonian \(\vec{E}_{\text{cl}}\) scales with \(|\vec{E}_{\text{cl}}|^2 R^2 \propto 10^6\) while the dipole transition rate \([8]\) scales with \((\hbar \omega_0)^3 R^2 \propto 10^{11}\). That fact renders the transition rate a much more important concept for the laser-nucleus interaction than for the laser-atom interaction.

We have addressed two approximation schemes for quantum many-body systems that allow for a practicable treatment of multi-photon absorption processes. These are: (i) The mean-field approach, i.e., use of the time-dependent Schrödinger equation for single-particle motion, combined with the classical approximation (7) for the electric field strength. This approach is characteristic for atoms, for which the classical electric field strength of the laser may be so strong as to significantly deform the nuclear Coulomb potential, giving rise to ionization. We have shown that in nuclei, the scaled field strength is too faint to cause significant excitation. (ii) Rate equations that describe strongly interacting systems which relax between any two subsequent photon absorption processes. Rates are fundamentally a quantum concept. We have shown that rate processes yield significant nu-
clear excitation carrying the system far above yrast \[23\]. Conversely, the transition rate is relatively much less important in atoms. That fact coincides with the smaller role of relaxation processes in nuclei. The need to use rate equations in nuclei casts new light on the case where the rate for photon absorption is much bigger than the relaxation rate (the “sudden” regime). Here the nuclear relaxation rate (not the photon absorption rate) should define the characteristic time scale for the entire process.

The number \( N \approx 10^4 \) of photons required \[23\] to generate significant multiple photon absorption in nuclei via rate processes is much smaller than the number \( 10^{10} \) of photons per pulse carried by a medium-intensity optical laser. Thus, coherent Doppler backscattering of a tiny fraction of all photons in the original atomic laser pulse suffices to generate significant nuclear excitation. That fact reduces the requirements imposed on the experimental realization of the “flying mirror” and strongly enhances the likelihood of a successful start of experimental laser-nucleus reaction processes.

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