Polarization control of non-diffractive helical optical beams through subwavelength metallic apertures

E Lombard\textsuperscript{1}, A Drezet\textsuperscript{2,3}, C Genet\textsuperscript{1,3} and T W Ebbesen\textsuperscript{1}

\textsuperscript{1} ISIS, Laboratoire des Nanostructures, Université de Strasbourg and CNRS, 8 allée Gaspard Monge, 67000 Strasbourg, France
\textsuperscript{2} Institut Néel, CNRS-Université Joseph Fourier, 25 rue des Martyrs, 38000 Grenoble, France
E-mail: aurelien.drezet@grenoble.cnrs.fr and genet@unistra.fr

Abstract. We demonstrate experimentally a simple method for preparing non-diffractive vectorial optical beams that can display wave-front helicity. This method is based on space-variant modifications of the polarization of an optical beam transmitted through subwavelength annular rings perforating opaque metal films. We show how the description of the optical properties of such structures must account for the vectorial character of the polarization and how, in turn, these properties can be controlled by straightforward sequences of preparation and analysis of polarization states.

The phase and polarization of a light beam can be precisely tailored and manipulated using appropriate reflective or transmissive inhomogeneous (space-variant) optical masks. Recent examples have illustrated the efficiency and the richness of such methods for wave-front shaping, generating helical modes with vectorial vortices. However, these methods often require complex mask structures and holograms in order to control locally, through linear birefringence, the phase and polarization conversions of the interacting light beam \cite{1}--\cite{8}. In this paper, we demonstrate that an analogue polarization control is possible with simple designs of metallic subwavelength apertures made through opaque metal film. With high optical contrasts and high aspect ratios, subwavelength apertures can locally induce strong polarization filtering effects. We show that these effects, when considered at the level of a whole structure, turn out to be a good alternative to space-variant linear birefringence for polarization control in the visible range.

\textsuperscript{3} Authors to whom any correspondence should be addressed.
To illustrate this point, we study the transmission properties of a circular grating made of annular slits with subwavelength width milled through a thick golden (Au) film. This type of axicon-like grating has been known of for a long time, in particular as a generator of scalar non-diffractive Bessel beams [9]–[13]. More recently, due to advances in nano- and microfabrication methods, non-diffracting beams generated by annular slit arrays have been studied in the microwave regime [14]. While this work mainly focused on the non-diffracting advantages of the Bessel-like beam, it also showed the necessity of accounting for polarization aspects in the analysis. In close connection with this problem, much effort has been devoted in recent years to the study of plasmonic circular groove arrays on noble metal (the so-called plasmonic bull’s eye structure), which in the visible range allow for various effects such as extraordinary transmission and optical beaming [15, 16]. This geometry also proved to be of interest for creating evanescent Bessel beams confined on the metal surface via surface plasmon (SP) excitations [17, 18] and for the development of planar optical lens using radial polarization [19]–[21]. Recent work also considered the polarization properties of individual annular slits [18, 22], confirming therefore the potential of such geometry for polarization conversion control at the nanoscale.

Until now, however, the specific geometry of the annular subwavelength slit grating has not been studied. As shown in this paper, this geometry allows efficient wave-front shaping and polarization control. We stress that a fully vectorial description is necessary for understanding the optical properties of such a grating, since the local filtering of the incoming field polarization through each subwavelength ring implies polarization conversion channels for the transmitted field. We probe these channels by conducting a full polarization tomography of our system, in both the direct and the Fourier spaces. By selecting the proper input and output polarization states, our device generates non-diffracting optical beams that can simultaneously display a helical wave front, i.e. carry orbital angular momentum [23]. The combination of these non-diffractive and singular properties might have interesting implications for imaging techniques [24]–[26], superresolution techniques [27], quantum optics [28] and optical trapping [29, 30]. Furthermore, with annular concentric rings, SPs are resonantly excited on the grating. Our experiment, however, reveals that the symmetries of both the plasmonic and the directly transmitted field are identical and that the contribution of the former, for the chosen structure parameters, induces only small changes in the global transmission with respect to a situation where the sole contribution would be the one of the directly transmitted field. This is experimentally checked by comparing beams transmitted through similar gratings milled, respectively, in an Au film (where SPs are excited) and a tungsten (W) film (with no SP contribution).

Our structure is shown in figure 1. The grating periodicity is $\rho_0 = 700$ nm, and each of the $n = 10$ rings has a width of $w = 150$ nm and is milled through an Au film $h = 300$ nm thick. In contrast to the plasmonic bull’s eye structure made of grooves surrounding a central subwavelength hole [15], the annular rings of our mask are real apertures opened through the whole film. We consider a unit-amplitude plane wave illuminating our structure. The period is chosen to have a transmission resonance approximately at illumination wavelength $\lambda = 785$ nm, as seen on the transmission spectrum displayed in figure 1. The width $w$ of the rings is much smaller than the illumination wavelength $\lambda$ so that each point of a ring will act as a point source. We thus expect that the $n$th ring will contribute as a Dirac distribution $\delta(\rho - \rho_n)$ to the transmitted amplitude immediately after the grating ($\rho_n = n\rho_0$ is the radius of the $n$th ring, and $\rho$ is the radial distance from the optical axis in polar coordinates $\rho = \rho \hat{\rho}$ in the plane of the
Figure 1. Schematics of the experimental set-up. (a) Optical polarization tomography set-up. The source LD is a laser diode emitting at $\lambda = 785$ nm a Gaussian monomode beam. The sample is inserted inside a telescope made of two microscope objectives O1 (5×, N.A. = 0.14) and O2 (50×, N.A. = 0.42). $L_i$ are converging lenses, $P_i$ linear polarizers and $H_i$, $Q_i$, respectively, half- and quarter-wave plates. The images are recorded with a CMOS camera inserted in the C-plane for direct imaging and in the F-plane for Fourier imaging. (b) Scanning electron microscope image of the structure composed of 10 annular rings; see main text for the structure parameters. (c) Transmission spectra through such a grating, milled in Au (continuous line) and W (dashed line).

structure). However, and contrary to what occurs for $w \gg \lambda$ (e.g. see [11]), we stress that such subwavelength annular apertures in a metal mask have a strong influence on the polarization properties of the transmitted field. Indeed, given the cylindrical symmetry and the high $\lambda/w \gg 1$ radial aspect ratio of the rings, the incoming polarization is locally filtered and therefore only the radial component of the electric field, i.e. perpendicularly to the rings, will survive just after the structure. This polarization filtering is summarized in a $2 \times 2$ non-diagonal transmission matrix

$$T(\rho) \propto \sum_n \delta(\rho - \rho_n) \hat{\rho} \otimes \hat{\rho},$$

where $\otimes$ is a dyadic product. In the Cartesian coordinates of the $(x, y)$-plane, the product can be decomposed into the identity matrix summed with a mirror symmetry matrix as $2\hat{\rho} \otimes \hat{\rho} = \mathbb{I} + \mathbb{M}(\varphi)$, with

$$\mathbb{M}(\varphi) = \begin{pmatrix} \cos(2\varphi) & \sin(2\varphi) \\ \sin(2\varphi) & -\cos(2\varphi) \end{pmatrix}.$$  

Central to this paper is the fact that this $\mathbb{M}(\varphi)$ matrix is exactly of the type obtained when considering a local rotation of the optical axis of a half-wave plate, on which space-variant birefringent masks are often based [5]–[8]. An optical Fourier transform of the transmitted
field obtained from equation (1) will generate sets of plane waves propagating on a cone revolving along the optical z-axis, i.e. generating vectorial Bessel-type beams that are direct generalizations of the scalar beams obtained by Durnin et al [11, 25].

All our results stem from the tensorial shape of the transmission function, i.e. from the non-diagonal dyadic product $\hat{\rho} \otimes \hat{\rho}$ (see appendix A). The $\mathbb{T}$ matrix, with its non-diagonal elements, induces polarization conversions that must be accounted for in a full description of the optical properties of such structures. We show below that these non-diagonal elements are the ones that can produce optical vortices in the transmitted beam. So far, we have only considered the contribution of direct transmission through the annular rings. But, dealing with a subwavelength metallic grating, the excitation of delocalized SP resonances should, in principle, be considered. However, the experiment shows that the resonant coupling to SP modes does not impart to the transmitted beam any symmetries additional to that of the global structure. The slight differences between the beams transmitted through Au or W films, as seen in particular on Fourier transforms (see below), can be understood when considering the interference between the direct contribution of the rings and the SP contribution to the global transmission.

The polarization conversion channels can be probed through a polarization tomography of the structure, following a methodology that we have already presented [31, 33]. We only recall that it consists in measuring the intensities of the transmitted beam after the incident beam has been prepared in, and the transmitted beam has been projected on, one of the six polarization states $\hat{a}$: the four linear polarization states $\hat{x}, \hat{y}, \hat{p} = (\hat{x} + \hat{y})/\sqrt{2}, \hat{m} = (\hat{x} - \hat{y})/\sqrt{2}$ and the two circular polarization states $\hat{L} = (\hat{x} + i\hat{y})/\sqrt{2}, \hat{R} = (\hat{x} - i\hat{y})/\sqrt{2}$. The experimental set-up of figure 1 thus consists of a $\lambda = 785$ nm laser collimated beam at the input $I$-plane $z_I$, which is weakly focused by the objective $O_1$ on the grating, after passing through polarization elements preparing it in a defined polarization state. The beam transmitted through the structure is collected by a second objective $O_2$ and subsequently polarization analyzed before being imaged on a CMOS camera in the back focal $C$-plane $z_C$ of a conjugating lens. An additional lens performs a Fourier transform of the output beam, which we image by placing the camera in its back focal $F$-plane $z_F$. Formally, the transfer matrix $\mathbb{G}$ associated with the full optical set-up links the incoming electric field vector $\mathbf{E}_{\text{in}}$ at a point $\mathbf{r}' = (\rho', z_I)$ in the input plane to the transmitted electric field vector $\mathbf{E}_{\text{out}}$ recorded at the point $\mathbf{r} = (\rho, z_k)$ in the chosen output ($k = C, F$) plane. We write

$$\mathbf{E}_{\text{out}}(\mathbf{r}) = \int \mathbb{G}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}_{\text{in}}(\mathbf{r}')d^2\rho'.$$  \hspace{1cm} (3)

The $\mathbb{G}$ matrix is the combination between the transmission matrix $\mathbb{T}$ of the grating and the transfer matrices associated with the optical elements of the set-up. Given equation (1), $\mathbb{G}$ is a symmetric matrix with $\mathbb{G}_{ij}(\mathbf{r}, \mathbf{r}') = \mathbb{G}_{ji}(\mathbf{r}, \mathbf{r}')$.

The polarization analysis sequence projects the transmitted field on a chosen polarization state $\hat{a}$ and thus selects a component $E_{\alpha}(\mathbf{r}) = \hat{a} \cdot \mathbf{E}(\mathbf{r})$. In this experiment, we acquire images of the transmitted beam with a camera in transverse ($x, y$) planes at $z = z_C$ and $z = z_F$. Such images display space-variant intensities $I_{\alpha}(\mathbf{r}) = \langle E_{\alpha}(\mathbf{r})E_{\alpha}^*(\mathbf{r}) \rangle$ over the beam cross-sections, revealing the transverse modal structure of the transmitted beam. This performs an imaging polarization tomography. We regroup all the measured intensity patterns into a $6 \times 6$ matrix, corresponding to all the possible choices of polarization preparation and analysis. By convention, each image of the matrix will be labeled as $\mathcal{M}_{ab}$, where $a$ and $b$ belong to the set.
Figure 2. Intensity distributions recorded in the C-plane (first line) and the F-plane (second line) with three different input polarizations: two linear states $\hat{x}$, $\hat{p}$ and one right-circular state $\hat{R}$. No polarization analysis is performed here.

$[\hat{x}, \hat{y}, \hat{p}, \hat{m}, \hat{L}, \hat{R}]$. In fact, due to the symmetric structure of $G$, this matrix will be symmetric too.

We show in figure 2 the C- and F-intensity profiles obtained by inserting the camera, respectively, in the back focal plane of the conjugating lens and the back focal plane of the Fourier lens. In the C-plane, we form an image of the grating while in the F-plane an optical Fourier transform is done. These profiles were generated by preparing a given $\hat{x}$, $\hat{p}$ or $\hat{R}$ input polarization state and were measured without any polarization analysis of the transmitted beam. Therefore, the additional profiles associated with the three other input polarization states can be immediately deduced from the present images by mirror symmetry. It is interesting to note that the Fourier intensity profiles have smoother distributions. This is directly related to the conical wave vector spectrum that an annular aperture generates from a scalar point of view. Accounting for polarization does not quantitatively change that argument. The images acquired in the conjugated C-plane are more complex, essentially due to the fact that an image is formed through the convolution of the object with the two-dimensional (2D) Fourier transform of the pupil function of the imaging optical system (here the $O_2$ objective and the conjugating lens $L_2$). When an optical Fourier transform of the formed image is then realized, the convolution is resolved into a simple product of two Fourier transforms: one associated with the optical system, the other with the object. When the optical elements have sufficiently large transverse dimensions$^5$, the former can be readily integrated as a 2D Fourier transform of Gaussian type, becoming a simple prefactor of the Fourier transform of the object (i.e. with no influence on the Fourier imaging) [32].

To obtain better insight into the physics involved, we consider separately the two complementary spaces associated with the C- and F-planes. Starting from the C-plane, it

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$^4$ Since we measure intensities, circular polarization states are degenerated when combined with linear polarization states in a preparation or analysis sequence. Taken together, these relations reduce the number of independent elements of the tomography matrix to 16. However, for the sake of completeness, we will always acquire in our experiments all 36 possible distributions.

$^5$ This is in fact a discussion in terms of large Fresnel numbers with transverse dimensions of the optics $D \gg 2\sqrt{\lambda f}$. $f$ corresponding to associated focal lengths. The Fresnel number associated with our experiment lies in this limit.
Figure 3. Intensity distributions corresponding to the full optical polarization tomography performed in the $C$-plane. Each distribution is labelled in the main text as $M_{ab}$, with $a$ and $b$ belonging to the set $[\hat{x}, \hat{y}, \hat{p}, \hat{m}, \hat{L}, \hat{R}]$.

is clear from the dyadic product $\hat{\rho} \otimes \hat{\rho}$ that the components of the transmitted beam should obey simple symmetry relations, which in turn govern the symmetries of the recorded intensity distributions. Indeed, with a linearly polarized input beam, the transmitted intensity distribution must have a dipolar symmetry $I(\rho) = I(\rho) \cos^2 \phi$, where $\phi$ is the polar angle between the direction of the incoming polarization and the observation point $\rho$ in the $C$-plane. Similarly, with a circularly polarized beam, the intensity distribution must take an azimuthal symmetry, in agreement with experiment (see figure 2). To complete our study in the $C$-plane, we then perform a full polarization imaging tomography of the transmitted beam. The resulting images are shown in figure 3. Three $M_{ab}$ image subspaces can be distinguished, with $a$ and $b$ belonging to the subsets $[\hat{x}, \hat{y}]$, $[\hat{p}, \hat{m}]$ and $[\hat{L}, \hat{R}]$, respectively. For the linearly polarized subspaces (i.e. $[\hat{x}, \hat{y}]$ and $[\hat{p}, \hat{m}]$) we note, as expected from the dyadic product, that the effect of the polarization analysis on the diagonal elements is to reduce the angular spread of the intensity distributions from dipolar distributions to $\cos^4 \phi$ ones. The cross-polarized elements display quadrupolar patterns $\cos^2 \phi \sin^2 \phi$. Crossed terms between these two subspaces reveal more complex $\cos^2 \phi(1 + \sin 2\phi)$ patterns, differently oriented and showing no full extinction in the perpendicular direction. Looking at the crossed terms between these linearly polarized states and the circularly polarized states shows that one recovers the dipolar angular patterns one had for linear polarized states without any analysis. Images of the $[\hat{L}, \hat{R}]$ subspace have the expected
azimuthal symmetry. We note that the diagonal elements of this subspace display intensity distributions with maxima of intensity in the center, while the non-diagonal ones display minima, showing doughnut-type modes. This is associated with the generation of zeroth-order Bessel modes for $M_{aa}$ with $a = \hat{L}, \hat{R}$ and second-order ones for $M_{ab}, M_{ba}$ with $a = \hat{L}, b = \hat{R}$ with an $\exp(\pm 2i\psi)$ phase singularity, as will be discussed below in the $F$-plane.

We now make use of the optical factorization argument discussed above and perform the imaging polarization tomography not in the $C$- but in the $F$-plane. In this transverse Fourier $k$ plane, the mode generation is distilled from the optical imaging system and after a polarization filtering process, the transverse structure of the accessible beams can be expressed only in terms of exact zeroth- and second-order Bessel modes, in other words in terms of non-diffracting beams [11].

In the subwavelength approximation leading to equation (1), this is easily seen formally when evaluating the 2D Fourier transform of the transmission matrix of the structure $T[k] = \int T(\rho) \exp(-ik \cdot \rho) d^2\rho / (2\pi)^2$. In polar coordinates $(k_x = k \cos(\psi), k_y = k \sin(\psi))$, this transform reads as

$$T[k] \propto \sum_n \rho_n (J_0[k\rho_n] I - J_2[k\rho_n] M(\psi)),$$

where $I$ is the unity matrix, $M(\psi)$ is the mirror symmetry matrix already mentioned evaluated at the polar angle $\psi$ (see appendix A), and $J_0$ and $J_2$ are Bessel functions of zeroth- and second-order, respectively. The experimental images are presented in figure 4 and display intensity distributions $I[k]$ that can be easily evaluated as $M_{ab} = |\hat{b}^\dagger \cdot \mathbb{T} \cdot \hat{a}|^2$, with $\hat{a}$ and $\hat{b}$, the chosen prepared and analyzed polarization states, respectively. We present in figure 4 simulated intensity distributions of the polarization matrix recorded in the $F$-plane, which compare very well with the experimental data of figure 4. The simulations are performed by evaluating, in the Fresnel approximation, the optical Fourier transform of a $50 \times$ magnified image of the structure (i.e. as formed in the $C$-plane). The complex amplitude of the transmitted field at a point $\rho$

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**Figure 4.** (a) Experimental $M_{ab}$ intensity distributions corresponding to the full optical polarization tomography performed in the $F$-plane, with the same labelling convention as given in figure 3. (b) Simulated $M_{ab}$ distributions corresponding to the experimental $F$-plane tomography; see panel (a).
of the $F$-plane is proportional to the Fourier transform given by equation (4) evaluated at the spatial frequency $k = 2\pi \rho / \lambda f$. The focal length of the Fourier lens is $f = 150$ mm. The same method applied to the results of figure 2 leads also to the same agreement.

Before briefly commenting on the distributions for linearly polarized states in both preparation and analysis, we shall compare closely the experimental images of figure 4(a) with the simulated ones of figure 4(b). This comparison is made by looking, in figure 5, at cuts along the $k_x$ direction on experimental and simulated $M_{\hat{x}\hat{x}}$ images. This close look reveals some differences in the patterns. Interestingly, the same cut done on an experimental $M_{\hat{x}\hat{x}}$ image recorded through a W film, on which absolutely no SPs can propagate, exactly follows the simulated one. This allows us to conclude that the optical differences observed for the Au grating must be associated with the SP contribution interfering with the contribution associated with the field directly transmitted through the rings. This interference is, however, such that the optical structure of the transmitted beam can still be well understood by only considering the direct contribution to the image formation. With this in mind, we can identify two types of mode for the linearly polarized states. The elongated ones show along their long axis a central spot associated with $J_0$ terms and two satellite lobes associated with $J_2$ terms. The exact structure (as e.g. relative strength of the Bessel contributions) depends on which polarization states are chosen. The non-diagonal patterns in the $[\hat{x}, \hat{y}]$ and $[\hat{p}, \hat{m}]$ subspaces are determined on $\sum_n \rho_n \sin(2\psi) J_2[k\rho_n]$ for the former and $\sum_n \rho_n \cos(2\psi) J_2[k\rho_n]$ for the latter, i.e. on pure second-order Bessel contributions modulated by a four-fold angular symmetry. More interesting is the $[\hat{L}, \hat{R}]$ subspace. Interfering $\rho_n J_0[k\rho_n]$ amplitudes define diagonal elements with a central maximum in intensity. The non-diagonal elements, corresponding to polarization conversions, are determined from interfering $\rho_n \exp(\pm 2i\psi) J_2[k\rho_n]$ wave fronts. The phase term can be factorized out of the summation and eventually corresponds to a helical structure of the resulting wave front with an orbital helicity of $|\ell| = 2$. This shows that simple preparation or analysis sequences can give a helical wave front to the optical beam transmitted through the grating, and demonstrates the efficiency of a polarization tomography set-up for controlling the
Figure 6. Compared intensity cross-sections between (a) the Gaussian illumination beam and (c) the zeroth-order Bessel beam associated with the $M_{\hat{L}\hat{L}}$ distribution for a single ring of radius $10\rho_0$. These cross-sections evolve as the camera is moved away from the $F$-plane along the optical axis. For clarity, each cross section in panels (a) and (c) is normalized to its maximum intensity. The corresponding evolutions of these maxima along the optical axis are plotted in panels (b) and (d).

modal structure of the beam transmitted through subwavelength apertures. Note that the sign of this helicity can be chosen with $M_{\hat{L}\hat{R}}$ associated to $\ell = 2$ and $M_{\hat{R}\hat{L}}$ to $\ell = -2$. It is easy to check that in the case of circularly polarized input beams, the total angular momentum of the light is conserved. Also, as shown below, the topological charge $\ell$ of the generated optical vortex is conserved during propagation due to the non-diffracting nature of the beam.

The non-diffracting properties of these beams generated through polarization control is another fundamental consequence of the present analysis. It was indeed the non-diffractive nature of the Bessel beams that motivated the work of Durnin et al [11]. The trends are similar here, but since we are considering vectorial beams with polarization conversion control, we expect new features, potentially interesting for some applications (such as imaging or optical trapping). To study the non-diffractive properties of vectorial Bessel beams, we start with a simple subwavelength annular ring. We have prepared a mask consisting of a single ring of radius $\rho_{10} = 10\rho_0$, with a width $w = 150$ nm and perforated through the same Au film as that used for the grating structure. In figure 6, we compare the evolution of the transverse profiles of the beam transmitted through the single ring with that of those of a Gaussian beam corresponding to the illumination beam imaged when no sample is inserted inside the telescope. We prepare and analyze in the same optical state $\hat{a} = \hat{b} = \hat{L}$ so that the transmitted beam through the ring is a zeroth-order Bessel beam. Starting from the back focal plane of the Fourier lens, we move the camera over more than $50$ cm along the optical axis. The scans clearly show the robustness of the Bessel beam with respect to the Gaussian one, as already seen in [25]. Keeping the same prepared $\hat{a}$ and analyzed $\hat{b}$ polarization state, we repeat the scanning experiment using the grating structure of figure 1. As seen in figure 7, the transverse evolution is now
Figure 7. Simulated evolutions of intensity cross-sections for the grating structure of figure 1 as the camera is moved away from the $F$-plane along the optical axis. Panel (a) corresponds to the zeroth-order Bessel-like $\mathcal{M}_{RR}$ distribution and panel (d) to the second-order Bessel-like $\mathcal{M}_{LR}$ distribution. Corresponding normalized experimental results are shown in panels (b) and (e) with the actual evolution of the maximum intensities along the optical axis in panels (c) and (f).

much more complex, and is actually reminiscent of a Talbot diffraction behavior, with self-imaging effects as one moves along the optical axis at basically every multiple of the focal length of the Fourier lens. Similar interference effects are found for cross-circular polarization preparation and analysis. These experiments again compare well with our simulation. This time, a propagation phase $\exp(-i\pi(n\rho_0)^2(z-f)/\lambda f^2)$ is included, where $z$ is the longitudinal distance between the detector plane and the back focal plane of the Fourier lens of focal length $f$. This phase factor enters, for each ring, the interfering summation of equation (4) and the optical Fourier transform is evaluated in the Fresnel approximation. The interference between multiple rings does not affect the non-diffracting properties. In particular for the cross-polarized terms, the $|\ell| = 2$ helical wave front is preserved. The convolution between these Talbot-like effects and the Bessel symmetry of the generated modes is also responsible for interesting longitudinal sub-structures. We note in particular in the $\hat{a} = \hat{b} = \hat{R}$ set-up that a small 3D optical cavity can be created at about $z \sim f$, as seen in figure 7. Such a cavity could be useful in 3D optical nanoscopy, for example [34].
In conclusion, we have shown that it is essential to account for the vectorial character of an optical beam when it is transmitted through subwavelength apertures. At these scales, one expects space-variant modifications of the polarization. Our results therefore go beyond the scalar description of the generation of non-diffracting Bessel beams. In this context, we have demonstrated that imaging polarization tomography is an interesting tool to control transmitted mode structures, and to induce, for example, helical wave fronts. Our experiments reveal the slight contribution of SP modes to the transmission, which, in our cases, do not induce any symmetry breaking. The situation is actually richer if the plasmonic coupling channel imprints on the transmitted beam specific additional symmetries. This issue will be considered in a future study. Fascinating experiments have explored the potential of such complex plasmonic structures when observing how a plasmon-based spin–orbit interaction can be responsible for a polarization symmetry breaking at the level of the transmitted beam [18]. Such work is opening new routes for plasmonic subwavelength devices, with a promising mixture of fundamental and applied issues.

Appendix A

The dyadic product $\hat{\rho} \otimes \hat{\rho}$ can be decomposed into the identity matrix summed with a mirror symmetry matrix as $2\hat{\rho} \otimes \hat{\rho} = I + \mathbb{M}(\phi)$, with

$$\mathbb{M}(\phi) = \begin{pmatrix} \cos(2\phi) & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) \end{pmatrix},$$

i.e.

$$\mathbb{M}(\phi) = \frac{1}{x^2 + y^2} \begin{pmatrix} x^2 - y^2 & 2x \cdot y \\ 2x \cdot y & y^2 - x^2 \end{pmatrix}. \quad (A.2)$$

The mirror matrix is here associated with a symmetry axis making an angle $\phi$ with the $x$-axis.

The 2D Fourier transform of the transmission matrix $\mathbb{T}(\rho)$ is by definition

$$\mathbb{T}[k] = \int \frac{d^2 \rho}{(2\pi)^2} \mathbb{T}(\rho) e^{-ik \cdot \rho}$$

$$= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{+\infty} \rho \frac{d\rho}{2\pi} \mathbb{T}(\rho) e^{-ik \rho \cos(\psi - \psi)}, \quad (A.3)$$

where $\psi$ is the angle between the vector $k$ and the $x$ axis. With $\mathbb{T}(\rho) = \sum_n \delta(\rho - \rho_n) \hat{\rho} \otimes \hat{\rho}$, we obtain

$$\mathbb{T}[k] = \sum_n \rho_n \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-ik \rho_n \cos(\psi - \psi)} \left( I + \mathbb{M}(\phi) \right). \quad (A.4)$$

By definition, Bessel functions of order $m$ are expanded as

$$J_m(x) = \frac{1}{i^m} \int_0^{2\pi} \frac{d\phi}{2\pi} e^{ix \cos(\phi)} e^{im\phi}, \quad (A.5)$$

from which we deduce

$$\int_0^{2\pi} \frac{d\phi}{2\pi} e^{ix \cos(\phi - \psi)} \begin{pmatrix} \cos(m\phi) \\ \sin(m\phi) \end{pmatrix} = i^m J_m(x) \begin{pmatrix} \cos(m\psi) \\ \sin(m\psi) \end{pmatrix}, \quad (A.6)$$
Inserting equation (A.4) into equation (A.6) directly leads to
\[ T[k] \propto \sum_n \rho_n (J_0[k\rho_n]I - J_2[k\rho_n]M(\psi)) . \] (A.7)

In Fourier space a useful representation of matrix \( M(\psi) \) is
\[ M(\psi) = \frac{1}{k_x^2 + k_y^2} \left( k_x^2 \cdot k_y - k_y^2 \cdot k_x \right) . \] (A.8)

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