Byz-GentleRain: An Efficient Byzantine-tolerant Causal Consistency Protocol

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Abstract. Causal consistency is a widely used weak consistency model that allows high availability despite network partitions. There are plenty of research prototypes and industrial deployments of causally consistent distributed systems. However, as far as we know, none of them consider Byzantine faults, except Byz-RCM proposed by Tseng et al. Byz-RCM achieves causal consistency in the client-server model with $3f+1$ servers where up to $f$ servers may suffer Byzantine faults, but assumes that clients are non-Byzantine. In this work, we present Byz-GentleRain, the first causal consistency protocol which tolerates up to $f$ Byzantine servers among $3f+1$ servers in each partition and any number of Byzantine clients. Byz-GentleRain is inspired by the stabilization mechanism of GentleRain for causal consistency. To prevent causal violations due to Byzantine faults, Byz-GentleRain relies on PBFT to reach agreement on a sequence of global stable times and updates among servers, and only updates with timestamps less than or equal to such common global stable times are visible to clients. We prove that Byz-GentleRain achieves Byz-CC, the causal consistency variant in the presence of Byzantine faults. We evaluate Byz-GentleRain on Aliyun. The preliminary results show that Byz-GentleRain is efficient on typical workloads.

Keywords: Causal consistency · Byzantine faults · PBFT · GentleRain · Byz-GentleRain.

1 Introduction

For high availability and low latency even under network partitions, distributed systems often partition and replicate data among multiple nodes \cite{7}. Due to the CAP theorem \cite{6} many distributed systems choose to sacrifice strong consistency and to implement weak ones.

Causal consistency \cite{1} is one of the most widely used consistency model in distributed systems. There are several variants of causal consistency in the

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literature [14, 11, 18]. They all guarantee that an update does not become visible until all its causality are visible. We informally explain it in the “Lost-Ring” example [13]. Alice first posts that she has lost her ring. After a while, she posts that she has found it. Bob sees Alice’s two posts, and comments that “Glad to hear it”. We say that there is a read-from dependency from Alice’s second post to Bob’s get operation, and a session dependency from Bob’s get operation to his own comment. By transitivity, Bob’s comment causally depend on Alice’s second post. Thus, when Carol, a friend of Alice and Bob, sees Bob’s comment, she should also see Alice’s second post. If she saw only Alice’s first post, she would mistakenly think that Bob is glad to hear that Alice has lost her ring.

There are plenty of research prototypes and industrial deployments of causally consistent distributed systems (e.g., COPS [16], Eiger [17], GentleRain [9], Cure [2], MongoDB [20], and Byz-RCM [19]). GentleRain uses a stabilization mechanism to make updates visible while respecting causal consistency. It timestamps all updates with the physical clock value of the server where they originate. Each server periodically computes a global stable time $gst$, which is a lower bound on the physical clocks of all servers. This ensures that no updates with timestamps $\leq gst$ will be generated. Thus, it is safe to make the updates with timestamps $\leq gst$ at $s$ visible to clients. A get operation with dependency time $dt$ issued to $s$ will wait until $gst \geq dt$ and then obtain the latest version before $gst$.

However, none of these causal consistency protocols/systems consider Byzantine faults, except Byz-RCM (Byzantine Resilient Causal Memory) in [19]. Byz-RCM achieves causal consistency in the client-server model with $3f + 1$ servers where up to $f$ servers may suffer Byzantine faults, and any number of clients may crash. Byz-RCM has also been shown optimal in terms of failure resilience. However, Byz-RCM did not tolerate Byzantine clients, and thus it could rely on clients’ requests to identify bogus requests from Byzantine servers [19].

In this work, we present Byz-GentleRain, the first Byzantine-tolerant causal consistency protocol which tolerates up to $f$ Byzantine servers among $3f + 1$ servers in each partition and any number of Byzantine clients. It uses PBFT [8] to reach agreement among servers on a total order of client requests. The major challenge Byz-GentleRain faces is to ensure that the agreement is consistent with the causal order. To this end, Byz-GentleRain should prevent causality violations caused by Byzantine clients or servers: Byzantine clients may violate the session order by fooling some servers that a request happened before another that was issued earlier. Byzantine servers may forge causal dependencies by attaching arbitrary metadata for causality tracking to the forward messages. To migrate the potential damages of Byzantine servers, we let clients assign totally ordered timestamps to updates in Byz-GentleRain. Utilizing the digital signatures mechanism, Byzantine servers cannot forge causal dependencies.

To preserve causality, Byz-GentleRain uses the stabilization mechanism of GentleRain. As explained above, the timestamps in Byz-GentleRain are generated by clients. However, it is unrealistic to compute a lower bound on physical clock values of an arbitrary number of clients. Therefore, each server $s$ in Byz-GentleRain maintains and periodically computes a global stable time $gst$ which
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Fig. 1: Why the servers in Byz-GentleRain need to synchronize global stable times.

is a lower bound on physical clock values of the clients it is aware of. In the following, we argue that simply refusing any updates with timestamps $\leq \text{gst}$ on each server as GentleRain does may lead to causality violations. Consider a system of four servers which are replicas all maintaining a single key $k$, as shown in Figure 1. Due to asynchrony, these four servers may have different values of $\text{gst}$. Without loss of generality, we assume that $\text{gst}_1 < \text{gst}_2 = \text{gst}_3 < \text{gst}_4$, as indicated by vertical lines. Now suppose that a new update $u : k \leftarrow 5$ with timestamp between $\text{gst}_3$ and $\text{gst}_4$ arrives, and we want to install it on $\geq 3$ servers, using quorum mechanism. In this scenario, if each server refuses any updates with timestamps smaller than or equal to its $\text{gst}$, the update $u$ can only be accepted by the first 3 servers, indicated by dashed boxes. Suppose that server 3 is a Byzantine server, which may expose or hide the update $u$ as it will. Consequently, later read operations which read from $\geq 3$ servers may or may not see this update $u$. That is, the Byzantine server 3 may cause causality violations.

To cope with this problem, we need to synchronize the global stable times of servers. When a server periodically computes its $\text{gst}$, it checks whether no larger global stable time has been or is being synchronized. If so, the server will try to synchronize its $\text{gst}$ among all servers, by running PBFT independently in each partition. For each partition, the PBFT leader is also responsible for collecting updates with timestamps $\leq \text{gst}$ from $2f + 1$ servers, and synchronizing them on all servers. Once successfully synchronized, a global stable time becomes a common global stable time, denoted $\text{cgst}$, and in each partition the updates with timestamps $\leq \text{cgst}$ on all correct servers are the same. Therefore, each server can safely refuse any updates with timestamps smaller than or equal to its $\text{cgst}$.

Still, the classic PBFT is insufficient to guarantee causality, since a Byzantine leader of each partition may propose an arbitrary set of updates. To avoid this, in Byz-GentleRain the PBFT leader will also include the sets of updates it collects from $2f + 1$ servers in its PROPOSE message. A server will reject the PROPOSE message if it finds the contents of this message have been manipulated by checking hash and signatures.

Thus, we make the following contributions:
Table 1: Notations.

| Notations   | Meaning                                                                 |
|-------------|-------------------------------------------------------------------------|
| $D$         | number of data centers                                                  |
| $P$         | number of partitions                                                    |
| $r_{p}^{d}$ | the replica of partition $p$ in data center $d$                         |
| PARTITION$(k)$ | the partition that holds key $k$                                      |
| REPLICA$(p)$ | the set of replicas of partition $p$                                    |
| DATACENTER$(d)$ | the set of servers in data center $d$                                   |
| $S$         | the set of all $D \times P$ servers in the key-value store              |
| clock$_c$   | clock at client $c$                                                     |
| dt$_c$      | dependency time at client $c$                                           |
| cgst$_c$    | the maximum common global stable time known by client $c$               |
| clock$_{r}^{d}$ | clock at replica $r_{d}^{p}$                                           |
| lst$_{r}^{d}$ | local stable time at replica $r_{d}^{p}$                               |
| gst$_{r}^{d}$ | global stable time at replica $r_{d}^{p}$                              |
| cgst$_{r}^{d}$ | common global stable time at replica $r_{d}^{p}$                      |
| curr_cgst$_{r}^{d}$ | temporary common global stable time at replica $r_{d}^{p}$ during PBFT |
| VV$_{r}^{d}$ | version vector at replica $r_{d}^{p}$                                  |
| LV$_{r}^{d}$ | local stable time vector at replica $r_{d}^{p}$                        |
| Key         | the set of keys, ranged over by $k$                                     |
| Val         | the set of values, ranged over by $v$                                   |
| VVal        | the set of versioned values, ranged over by $vv$                       |
| store$_{d}^{p}$ | store maintained at replica $r_{d}^{p}$                                 |
| Store       | the union of stores at all replicas, that is, $\text{Store} \triangleq \bigcup_{1 \leq i \leq D, 1 \leq j \leq P} \text{store}_j^i$ |

- We define Byzantine Causal Consistency (Byz-CC), which is a causal consistency variant in the presence of Byzantine faults (Section 3).
- We present Byz-GentleRain, the first Byzantine-tolerant causal consistency protocol. It tolerates up to $f$ Byzantine servers among $3f + 1$ ones and any number of Byzantine clients (Section 4). All reads and updates complete in one round-trip.
- We evaluate Byz-GentleRain on Aliyun. The preliminary results show that Byz-GentleRain is efficient on typical workloads (Section 5).

Section 2 describes the system model and failure model. Section 6 discusses related work. Section 7 concludes the paper. The proofs can be found in Appendix A.

2 Model

We adopt the client/server architecture [15, 19], in which each client or server has its unique id. Table 1 summarizes the notations used in this paper.

2.1 System Model

We consider a distributed multi-version key-value store, which maintains keys in the set Key (ranged over by $k$) with values in the set Val (ranged over by $v$).
Each value is associated with a unique version, consisting of the timestamp of the update which creates this version and the id of the client which issues this update. We denote by $\text{VVal}$ (ranged over by $v$) the set of versioned values.

The distributed key-value store runs at $D$ data centers, each of which has a full copy of data. In each data center, the full data is sharded into $P$ partitions. For a key $k \in \text{Key}$, we use $\text{PARTITION}(k)$ to denote the partition that holds $k$. Each partition is replicated across $D$ data centers. For a partition $p$, we use $\text{REPLICAS}(p)$ to denote the set of replicas of $p$. For a data center $d$, we use $\text{DATA CENTER}(d)$ to denote the set of servers in $d$. We denote by $\mathcal{S}$ the set of all $D \times P$ servers in the key-value store. For each individual partition $p$, we call a set $Q$ of $2f + 1$ replicas in $\text{REPLICAS}(p)$ a quorum and denote it by $\text{quorum}(Q)$.

For convenience, we model the key-value store at replica $r_p^d$, denoted $\text{store}_p^d$, by a set of (unique) versioned values. That is, $\text{store}_p^d \subseteq \text{VVal}$. We denote by $\text{STORE}$ the union of stores at all replicas. That is, $\text{STORE} \triangleq \bigcup_{1 \leq i \leq D, 1 \leq j \leq P} \text{store}_j^i$.

The distributed key-value store offers two operations to clients:

- $\text{GET}(k)$. A get operation which returns the value of some version of key $k$.
- $\text{PUT}(k, v)$. A put operation which updates key $k$ with value $v$. This creates a new version of $k$.

We assume that each client or server is equipped with a physical clock, which is monotonically increasing. Clocks at different clients are loosely synchronized by a protocol such as NTP\footnote{NTP: The Network Time Protocol. http://www.ntp.org/}. The correctness of Byz-GentleRain does not depend on the precision of clock synchronization, but large clock drifts may negatively impact its performance.

### 2.2 Failure Model

Clients and servers are either correct or faulty. Correct clients and servers obey their protocols, while faulty ones may exhibit Byzantine behaviors\footnote{[8]}, by deviating arbitrarily from their protocols.

We assume asynchronous point-to-point communication channels among clients and servers. Messages may be delayed, duplicated, corrupted, or delivered out of order. We do not assume known bounds on message delays. The communication network is fully connected. We require that if the two ends of a channel are both correct and the sender keeps retransmitting a message, then the message can eventually be delivered.

We also assume the channels are authenticated. Clients and servers can sign messages using digital signatures when needed. A message $m$ signed by a client $c$ or a replica $r_p^d$ is denoted by $\langle m \rangle_c$ or $\langle m \rangle_p^d$, respectively. We denote by $\text{valid}(m)$ that $m$ is valid in signatures. We also use a cryptographic hash function $\text{hash}()$, which is assumed to be collision-resistant: the probability of an adversary producing inputs $m$ and $m'$ such that $\text{hash}(m) = \text{hash}(m')$ is negligible $\footnote{[5,8]}$.\footnote{NTP: The Network Time Protocol. http://www.ntp.org/}
3 Byzantine Causal Consistency

Causal consistency variants in the literature are defined based on the happens-before relation among events. However, they are not applicable to systems that allows Byzantine nodes, particularly Byzantine clients. We now adapt the happens-before relation in Byzantine-tolerant systems, and define Byzantine Causal Consistency (Byz-CC) as follows. For two events \( e \) and \( f \), we say that \( e \) happens before \( f \), denoted \( e \happensBefore f \), if and only if one of the following three rules holds:

- **Session-order.** Events \( e \) and \( f \) are two operation requests issued by the same correct client, and \( e \) is issued before \( f \). We denote it by \( e \happensBefore f \). We do not require session order among operations issued by Byzantine clients.

- **Read-from relation.** Event \( e \) is a put request issued by some client and \( f \) is a get request issued by a correct client, and \( f \) reads the value updated by \( e \). We denote it by \( e \happensBefore f \). Since a get of Byzantine clients may return an arbitrary value, we do not require read-from relation induced by it.

- **Transitivity.** There is another operation request \( g \) such that \( e \happensBefore g \) and \( g \happensBefore f \).

If \( e \happensBefore f \), we also say that \( f \) causally depends on \( e \) and \( e \) is a causal dependency of \( f \). A version \( vv \) of a key \( k \) causally depends on version \( vv' \) of key \( k' \), if the update of \( vv \) causally depends on that of \( vv' \). A key-value store satisfies Byz-CC if, when a certain version of a key is visible to a client, then so are all of its causal dependencies.

4 The Byz-GentleRain Protocol

As discussed in Section 1, it is the clients in Byz-GentleRain that are responsible for generating totally ordered timestamps for updates. Specifically, when a client issues an update, it assigns to the update a timestamp consisting of its current clock and identifier.

As in GentleRain, we also distinguish between the updates that have been received by a server and those that have been made visible to clients. Byz-GentleRain guarantees that an update can be made visible to clients only if so are all its causal dependencies. The pseudocode in Algorithms 1–3 dealing with Byzantine faults is underlined.

4.1 Key Designs

In Byz-GentleRain, both clients and servers maintain a common global stable time \( cgst \). We denote the \( cgst \) at client \( c \) by \( cgst_c \) and that at replica \( r_d \) by \( cgst_d \). We maintain the following invariants that are key to the correctness of Byz-GentleRain:

INV (I): Consider \( cgst_c \) at any time \( \sigma \). All updates issued by correct client \( c \) after time \( \sigma \) have a timestamp \( \geq cgst_c \).
Algorithm 1 Operations at client $c$

1: procedure get$(k)$
2: var $ts ← \max\{dt, cgst\}$
3: var $p ← \text{PARTITION}(k)$
4: send get$_{-\text{REQ}}(k, ts)$ to replicas$(p)$
5: wait receive \{get$_{-\text{ACK}}(v_i, cgst_i) \mid r_p^i ∈ Q\} = M$ for a quorum $Q$
6: $cgst ← \max\{cgst, \min r_p^i \in Q cgst_i\}$
7: $v ←$ the majority $v_i$ in $M$
8: return $v$
9: procedure put$(k, v)$
10: var $p ← \text{PARTITION}(k)$
11: wait clock $> cgst$
12: send (put$_{-\text{REQ}}((k, v, clock, c)_d))_p$ to replicas$(p)$
13: wait receive \{put$_{-\text{ACK}}(cgst_i) \mid r_p^i ∈ Q\} = M$ for a quorum $Q$
14: $cgst ← \max\{cgst, \min r_p^i \in Q cgst_i\}$
15: $dt ←$ clock
16: return ok

Inv (II): Consider $cgst_p^d$ at any time $\sigma$. No updates with timestamps $\leq cgst_p^d$ will be successfully executed at $>f$ correct replicas in replicas($p$) after time $\sigma$.

Inv (III): Consider a $cgst$ value. For any two correct replicas $r_p^d$ and $r_p^i$ (where $i ≠ d$) of partition $p$, if $cgst_p^d ≥ cgst$ and $cgst_p^i ≥ cgst$, then the updates with timestamps $\leq cgst$ in store$_p^d$ and store$_p^i$ are the same.

Byz-GentleRain further enforces the following rules for reads and updates:

Rule (I): For a correct replica $r_p^d$, any updates with timestamps $> cgst_p^d$ in store$_p^d$ are invisible to any clients.

Rule (II): Any correct replica $r_p^d$ will reject any updates with timestamps $\leq cgst_p^d$.

Rule (III): For a read operation with timestamp $ts$ issued by client $c$, any correct replica $r_p^d$ that receives this operation must wait until $cgst_p^d ≥ ts$ before it returns a value to client $c$.

In the following sections, we explain how these invariants and rules are implemented and why they are important to the correctness.

4.2 Client Operations

To capture the session order, each client maintains a dependency time $dt$, which is the clock value of its last put operation. When a client $c$ issues a get operation on key $k$, it first takes as $ts$ the minimum of its dependency time $dt_c$ and common global stable time $cgst_c$ (line 12). Then it sends a get$_{-\text{REQ}}$ request with $ts$
Algorithm 2 Operation execution at $r_d^p$

1: when received GET_REQ($k$, $ts$) from client $c$
2:     wait until cgst $\geq ts$
3:     $v \leftarrow$ the value of key $k$ with the largest timestamp $\leq ts$ in store
4:     send (GET_ACK($v$, cgst))$^p_d$ to client $c$

5: when received (PUT_REQ(($k$, $v$, $cl$, $c$)))$^c$ from client $c$
6:     pre: $cl \geq lst$
7:     wait clock $\geq cl$
8:     var vv $\leftarrow$ ($k$, $v$, $cl$, $c$)$_c$
9:     store $\leftarrow$ store $\cup$ {$vv$}
10: send (PUT_ACK(cgst))$^p_d$ to client $c$
11: send (REPLICATE(vv))$^p_d$ to replicas($p$) \{ $r_d^p$ \}

4.3 Operation Executions at Replicas

When a replica $r_d^p$ receives a GET_REQ($k$, $ts$) request from some client $c$, it first waits until cgst $\geq ts$ (line 2). This implements Rule III and is used to ensure the session guarantee on client $c$ and eventual visibility of updates to $c$. Then the replica obtains the value $v$ of key $k$ in store which has the largest timestamp before $ts$, breaking ties with client ids (line 3). Finally, it sends a signed GET_ACK response, along with the value $v$ and its current cgst, to client $c$ (line 4).
When a replica $r_d^p$ receives a put_req$(k, v, cl, c)$ request from client $c$, it first checks the precondition $cl \geq curr\_cgst_d^p$ (line 2.9). This enforces Rule 11 and prevents fabricated updates with timestamps $\leq cgst_d$ from now on. If the precondition holds, the replica adds the new versioned version $vv \triangleq (k, v, cl, c)_c$ signed by $c$ (line 2.8) to $store_d^p$ (line 2.9). Then, the replica sends a signed put_ack response, with its $cgst_d^p$, to client $c$ (line 2.10). Finally, it broadcasts a signed replicate message with $vv$ to other replicas in partition $p$ (line 2.11).

4.4 Metadata

Replica States Each replica $r_d^p$ maintains a version vector $VV_d^p$ of size $D$, with each entry per data center. For data center $d$, $VV_d^p[j]$ is the timestamp of the last update that happens at $r_d^p$. For data center $i \neq d$, $VV_d^p[i]$ is the largest timestamp of the updates that happened at replica $r_d^p$ and have been propagated to $r_d^p$. For fault-tolerance, we compute the local stable time $lst_d^p$ at replica $r_d^p$ as the $(f+1)$-st minimum element of its $VV_d^p$.

Each replica $r_d^p$ also maintains a list vector $LV_d^p$ of size $P$, with each entry per partition. For partition $1 \leq j \leq P$, $LV_d^p[j]$ is the largest $lst$ of replica $r_d^p$ of which $r_d^p$ is aware. We compute the global stable time $gst_d^p$ at replica $r_d^p$ as the minimum element of its $LV_d^p$. That is, $gst_d^p \triangleq \min_{1 \leq j \leq P} LV_d^p[j]$.

Each replica $r_d^p$ periodically synchronizes their $gst_d^p$ with others via PBFT, and maintains a common global stable time $cgst_d^p$. We discuss it in Section 4.5.

Propagation As in GentleRain, Byz-GentleRain propagates and updates metadata in the background. Once a new version $vv$ is created at replica $r_d^p$, the replica sends a signed replicate message to other replicas of partition $p$ (line 2.11).

When replica $r_d^p$ receives a replicate message from another replica $r_i^p$ in data center $i \neq d$, it stores $vv$ in its $store_d^p$ (line 3.2), and updates $VV_d^p[i]$ to $vv.cl$ if the latter is larger (line 3.3).

Each replica $r_d^p$ periodically computes its $lst_d^p$ (line 3.5) and sends a signed broadcast message to the servers of the data center $d$ (line 3.6).

When replica $r_d^p$ receives a broadcast message from another replica $r_d^p$ in data center $d$, it updates $LV_d^p$ and $gst_d^p$ accordingly (lines 3.8 and 3.9).

If the new $gst_d^p$ is larger than $cgst_d^p$ (line 3.10), the replica $r_d^p$ sends a signed newcgst$(gst_d^p)$ message to all servers $S$ of the key-value store (line 3.11).

To ensure liveness, a replica $r_d^p$ periodically (e.g., at time interval $\Delta t$; line 3.13) sends a signed hb$(\Delta t)$ heartbeat to $replicas(p)$ (line 3.14). When replica $r_d^p$ receives a heartbeat message from replica $r_i^p$, it updates its $VV_d^p[i]$ to $clock$ if the latter is larger (line 3.16).

4.5 Synchronization of Global Stable Time

Each individual partition $p$ independently runs PBFT [8] to reach agreement on a common global stable time $cgst$ and the same set of updates before $cgst$ across
Algorithm 3 Updating metadata at replica $r_d^p$

1: when received $\langle$REPLICATE$((vv))\rangle_i^p$  
2: $\text{store} \leftarrow \text{store} \cup \{vv\}$  
3: $\text{VV}[i] \leftarrow \max\{\text{VV}[i], vv\cdot\text{cl}\}$

4: procedure BROADCAST() \Comment*[r]{Run periodically}  
5: $\text{lst} \leftarrow \max\{\text{lst}, \text{the } (f+1)\text{-st minimum element of } \text{VV}[i]\}$  
6: send $\langle$bc($\text{lst}$)$\rangle_d^p$ to DATACENTER($d$)

7: when received $\langle$bc($\text{lst}$)$\rangle_d^j$  
8: $\text{LV}[j] \leftarrow \max\{\text{LV}[j], \text{lst}\}$  
9: $\text{gst} \leftarrow \max\{\text{gst}, \min_{1 \leq j \leq P} \text{LV}[j]\}$  
10: if $\text{gst} > \text{cgst}$ then  
11: send $\langle$newcgst($\text{gst}$)$\rangle_d^p$ to $\mathcal{S}$

12: procedure HEARTBEAT() \Comment*[r]{Run periodically}  
13: pre: clock $\geq \text{VV}[d] + \Delta$  
14: send $\langle$hb($\text{clock}$)$\rangle_d^p$ to REPLICAS($p$)

15: when received $\langle$hb($\text{clock}$)$\rangle_d^i$  
16: $\text{VV}[i] \leftarrow \max\{\text{VV}[i], \text{clock}\}$

We follow the pseudocode of single-shot PBFT described in [5], and refer its detailed description and correctness proof to [5]. In the following, we elaborate the parts specific to synchronization of global stable time; see the pseudocode underlined in Algorithm 4.

When replica $r_d^p$ receives a new cgst($\text{gst}$) message, it first checks whether $\text{gst} \leq \text{lst}_d^p$ as expected and $\text{gst} > \text{curr cgst}_d^p$ which means that no smaller global stable time has been or is being synchronized (line 4:2). If so, it sets curr cgst to $\text{gst}$ (line 4:3). Now the replica stops accepting updates with timestamps $<$ curr cgst $p$ (line 2:6). Then it triggers a newview action with a view larger than curr view $p$ (line 4:4).

As in classic PBFT [5][8], the newview(view) action can also be triggered spontaneously, due to timeout, or by failure detectors. When it is triggered at a replica $r_d^p$, the replica will send a signed newleader message to the leader leader($\text{view}$) of view in REPLICAS($p$) (line 4:10). The newleader message carries both curr cgst$^p_d$ and prepared store$^p_d$ which is the set of updates collected in prepared view$^p_d$.

When replica $r_d^p$ receives a set $M$ of newleader messages from a quorum $Q$ of REPLICAS($p$), it selects as its proposal from $M$ the set store$_j$ of collected updates that is prepared in the highest view, say view$_i$ (line 4:15), or, if there are no such store$_j$, its own proposal. In the latter case, the replica sets its curr cgst$^p_d$ to the maximum of cgst$_i$ in $Q$ that are $\leq$ lst$^p_d$ (line 4:17). Then, it sends a signed
Algorithm 4 Updating cgst at replica $r^p_d$ (see Table A1 in Appendix A for the definitions of ValidNewLeader and safe_propose that are adapted from [9].)

$$\text{safe\_collect}(\text{store}) \triangleq \forall \text{COLLECT\_ACK}((\omega, st_i) \in \text{store}, \forall (\omega, \ldots, \omega)_c = u \in st_i, \text{valid}(u))$$

1: when received \{(NEW\_CGST(gst))\}_u^r = m
2: pre: $gst \leq \text{lst} \land gst > \text{curr\_cgst}$
3: $\text{curr\_cgst} \leftarrow gst$
4: NEWVIEW(view) with $view > \text{curr\_view}$

5: upon NEWVIEW(view)
6: pre: $view > \text{curr\_view}$
7: $\text{curr\_view} \leftarrow view$
8: voted $\leftarrow$ false
9: send (NEWLEADER(curr\_view, prepared\_view, curr\_cgst, prepared\_store, cert))\_q^r
10: to leader(curr\_view)

11: when received \{(NEWLEADER(view, view\_i, cgst\_i, store\_i, cert))\}_p^r | r^p_i \in Q\} = M
12: from a quorum $Q$
13: pre: curr\_view = view $\land$ leader(view) $= r^p_d \land (\forall m \in M. \text{ValidNewLeader}(m))$
14: if $\exists j, \text{view}_j = \max\{\text{view}_i | r^p_i \in Q\} \neq 0$ then
15: send (PROPOSE(view, store\_i, M))\_q^r to REPLICAS(p)
16: else
17: $\text{curr\_cgst} \leftarrow \max\{\text{cgst\_i} | r^p_i \in Q \land \text{cgst\_i} \leq \text{lst}_c\}$
18: send (COLLECT(curr\_cgst))\_q^r to REPLICAS(p)
19: wait receive \{(COLLECT\_ACK(curr\_cgst, st\_i))\}_p^r | r^p_i \in Q\} = \text{store}
20: from a quorum $Q'$ satisfying safe\_collect(store)
21: send (PROPOSE(view, store\_i, M))\_p^r to REPLICAS(p)

22: when received \{(PROPOSE(view, store\_i, M))\}_p^r = m
23: pre: curr\_view = view $\land$ voted $\leftarrow$ false $\land$ safe\_propose(m) $\land$ safe\_collect(store)
24: $\text{curr\_store} \leftarrow \text{store}$
25: voted $\leftarrow$ true
26: send (PREPARE(view, hash(curr\_store)))\_q^r to REPLICAS(p)

27: when received \{(PREPARE(view, h))\}_p^r | r^p_i \in Q\} = C from a quorum $Q$
28: pre: curr\_view = view $\land$ voted $\leftarrow$ true $\land$ hash(curr\_store) = h
29: prepared\_view $\leftarrow$ curr\_view
30: prepared\_store $\leftarrow$ curr\_store
31: cert $\leftarrow$ C
32: send (COMMIT(view, h))\_p^r to REPLICAS(p)

33: when received \{(COMMIT(view, h))\}_p^r | r^p_i \in Q\} from a quorum $Q$
34: pre: curr\_view = prepared\_view = view $\land$ hash(curr\_store) = h
35: store $\leftarrow$ $\bigcup$ \{\text{st\_i} | (COLLECT\_ACK(cgst\_i, st\_i))\}_p^r $\in$ curr\_store\}
36: if cgst $< cgst$ then
37: cgst $\leftarrow$ cgst
38: store $\leftarrow$ store $\cup$ \{(\omega, cl, c) \in store | cl $>$ curr\_cgst\}
39: when received \{(COLLECT(cgst))\}_p^r = m
40: pre: cgst $\leq$ lst $\land$ cgst $\geq$ curr\_cgst
41: curr\_cgst $\leftarrow$ cgst
42: st $\leftarrow$ \{(\omega, cl, c) \in store | cl $\leq$ curr\_cgst\}
43: send (COLLECT\_ACK(cgst, st))\_d to $r^p_i$
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collect($\text{curr cgst}_p$) message to $\text{REPLICAS}(p)$ (line 3.18), and waits to receive enough $\text{COLLECT}_\text{ACK}$ messages.

When replica $r_d^p$ receives a $\text{COLLECT}(\text{cgst})$ message from replica $r_i^p$ and $\text{cgst}$ passes the precondition (line 4.10), it first sets its $\text{curr cgst}_d^p$ to $\text{cgst}$ (line 4.11). Now the replica stops accepting updates with timestamps $< \text{curr cgst}_d^p$ (line 2.6).

Then it sends a signed $\text{COLLECT}_\text{ACK}(\text{cgst}, st)$ message back to $r_i^p$ (line 4.13), where $st$ is the set of updates with timestamps $\leq \text{curr cgst}_d^p$ in its store$^p_d$ (line 4.12).

The replica $r_d^p$ waits to receive a set, denoted store, of $\text{COLLECT}_\text{ACK}$ messages from a quorum $Q'$ of $\text{REPLICAS}(p)$. We require the messages in store carry the same $\text{curr cgst}$ as in the corresponding $\text{COLLECT}$ message and the signatures of all the collected updates be valid (i.e., $\text{safe collect}(\text{store})$ holds). Then, it sends a signed $\text{PROPOSE}$ message with store as its proposal to $\text{REPLICAS}(p)$ (line 4.21).

When replica $r_d^p$ receives a $\text{PROPOSE}$ message from replica $r_i^p$, it also checks the predicate $\text{safe collect}(\text{store})$ (line 4.23). After setting $\text{curr cgst}_d^p$ to store, it sends a signed prepared message to $\text{REPLICAS}(p)$. When replica $r_d^p$ receives a set $C$ of prepared messages from a quorum $Q$ of $\text{REPLICAS}(p)$, both its $\text{curr view}_d^p$ and $\text{curr store}_d^p$ are prepared (lines 4.29 and 4.30). The certification $C$ is also remembered in $\text{cert}_d^p$ (line 4.31). They will be sent to new leaders in view changes to ensure agreement across views (line 4.10). Then the replica sends a signed commit message to $\text{REPLICAS}(p)$ (line 4.32).

When replica $r_d^p$ receives a set of commit message from a quorum $Q$ of $\text{REPLICAS}(p)$, it computes store as the union of the sets of updates $st$ collected from each $r_i^p$ in $\text{curr store}_d^p$ (line 4.35). If $\text{cgst}_d^p$ is smaller than the $\text{cgst}$ in $\text{curr store}_d^p$, the replica sets $\text{cgst}_d^p$ to this $\text{cgst}$ (line 4.36), and replaces the set of updates with timestamps $\leq \text{curr cgst}_d^p$ in store$^p_d$ with the new store (line 4.38).

5 Evaluation

We evaluate Byz-GentleRain in terms of performance, throughput, and latency of remote update visibility. We also compare Byz-GentleRain to Byz-RCM.

5.1 Implementation and Setup

We implement both Byz-GentleRain and Byz-RCM in Java and use Google’s Protocol Buffers\(^6\) for message serialization. We implement the key-value stores as $\text{HashMap}$, where each key is associated with a linked list of versioned values. The key-value stores hold 300 keys in main memory, with each key of size 8 bytes and each value of size 64 bytes.

We run all experiments on 4 Aliyun\(^7\) instances running Ubuntu 16.04. Each instance is configured as a data center, with 1 virtual CPU core, 300 MB memory, and 1G SSD storage. All keys are shared into 3 partitions within each data center, according to their hash values.

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\(^6\) Protocol Buffers: https://developers.google.com/protocol-buffers

\(^7\) Alibaba Cloud: https://www.alibabacloud.com/
5.2 Evaluation Results

Figure 2 shows the system throughput and the latency of get and put operations of both Byz-GentleRain and Byz-RCM in failure-free scenarios. We vary the get : put ratios of workloads. First, Byz-RCM performs better than Byz-GentleRain, especially with low get : put ratios. This is because Byz-RCM assumes Byzantine fault-free clients and is signature-free. In contrast, Byz-GentleRain requires clients sign each put request. Second, it demonstrates that Byz-GentleRain is quite efficient on typical workloads, especially for read-heavy workloads. Third, the performance of Byz-GentleRain is closely comparable to that of Byz-RCM, if digital signatures are omitted deliberately from Byz-GentleRain; see Figures 2b and 2c. Finally, Figure 2d shows the latency of put visibility, which gets higher and higher with more and more put operations.

We also evaluate Byz-GentleRain in several typical Byzantine scenarios. Generally, both Byzantine clients and replicas may fail by crash or send arbitrary messages. Particularly, we consider (1) Byzantine clients that may send get req and/or put req requests with incorrect timestamps (line 1:4 and line 1:12), and (2) Byzantine replicas that may broadcast different global stable time cgst to replicas in different partitions (line 3:11).

Figure 3 demonstrates the impacts of various Byzantine failures on the system throughput of Byz-GentleRain. On the one hand, the Byzantine failures of types
(1) and (2) above has little impact on throughput. On the other hand, frequently sending arbitrary messages, such as NEW_CGST or PROPOSE messages, does hurt throughput. This is probably due to the signatures carried by these messages.

6 Related Work

As far as we know, Byz-RCM [19] is the only causal consistency protocol that considers Byzantine faults. It achieves causal consistency in the client-server model with $3f + 1$ servers where up to $f$ servers may suffer Byzantine faults, and any number of clients may crash. Byz-RCM has also been shown optimal in terms of failure resilience. However, Byz-RCM did not tolerate Byzantine clients, and thus it could rely on clients’ requests to identify bogus requests from Byzantine servers [19].

Linde et al. [14] consider peer-to-peer architecture. A centralized server maintains the application data, while clients replicate a subset of data and can directly communicate with each other. They analyze the possible attacks of clients to causal consistency (the centralized server is assumed correct), derive a secure form of causal consistency, and propose practical protocols for implementing it.

Liskov and Rodrigues extend the notion of linearizability [10] and define BFT-linearizability in the presence of Byzantine servers and clients [15]. They also design protocols that achieve BFT-linearizability despite Byzantine clients. The protocols require $3f + 1$ replicas of which up to $f$ replicas may be Byzantine. They are quite efficient for linearizable systems: Writes complete in two or three phases, while reads complete in one or two phases.

Auvolat et al. [3] defines a Byzantine-tolerant Causal Order broadcast (BCO-broadcast) abstraction and proposes an implementation for it. However, as a communication primitive for replicas, BCO-broadcast does not capture the get/put semantics from the perspective of clients. Thus, it does not prevent Byzantine clients from violating causality.
7 Conclusion

We present Byz-GentleRain, the first causal consistency protocol which tolerates up to $f$ Byzantine servers among $3f+1$ servers in each partition and any number of Byzantine clients. The preliminary experiments show that Byz-GentleRain is quite efficient on typical workloads. Yet, more extensive large-scale experiments on more benchmarks are needed. We will also explore optimizations of our synchronization protocol in Algorithm 4 in future work.

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Table A1: Predicates in Algorithm 4 (adapted from [5]).

\[
\text{prepared}(\text{view}, h, M) \triangleq \exists Q. \text{quorum}(Q) \land M = \{ \text{prepared}(\text{view}, h, M)p \mid r_p \in Q \}
\]

\[
\text{ValidNewLeader}((\text{newleader}(\text{view}, \text{view}, \text{cgst}, \text{cert}))p) \triangleq \text{view}_i < \text{view} \land (\text{view}_i \neq 0 \implies \text{prepared}(\text{view}_i, \text{hash}(\text{store}), M))
\]

\[
\text{safe_propose}((\text{propose}(\text{view}, \text{store}, M))p) \triangleq r_p = \text{leader}(\text{view}) \land \exists Q, \text{view}, \text{store}, M. \text{quorum}(Q) \land M = \{ (\text{newleader}(\text{view}, \text{view}_i, \text{cgst}_i, \text{store}_i, \text{cert}_i))p \mid r_p \in Q \} \land (\forall m \in M. \text{ValidNewLeader}(m)) \land (\exists j. \text{view}_j \neq 0) \implies (\exists j. \text{view}_j = \max \{ \text{view}_i \mid r_p \in Q \} \land \text{store} = \text{store}_j))
\]

A Correctness of Byz-GentleRain

We show that Byz-GentleRain satisfies Byz-CC. We assume that single-shot PBFT is correct and refer its detailed correctness proof to [5]. Table A1 gives the definitions of the predicates \text{ValidNewLeader} and \text{safe_propose} used in Algorithm 4, which are also adapted from [5].

Remark 1. In the following, we use \( R \) and \( W \) to denote the set of \text{get} and \text{put} operations, respectively. We also define \( O \triangleq R \cup W \) to denote the set of all operations.

For a variable, e.g., \( \text{clock}_c \) at client \( c \), we refer to its value at time \( \sigma \) by, e.g., \((\text{clock}_c)_{\sigma}\).

According to the description of Algorithms 1 and 2,

**Lemma 1.** Rule II–Rule III are maintained by Byz-GentleRain.

**Lemma 2.** Inv II–Inv III are maintained by Byz-GentleRain.

**Proof.** Inv II holds due to line 1:11. Inv II holds due to the read rule at line 2:6. By the correctness of single-shot PBFT [5], Inv III holds.

**Definition 1 (Timestamps).** We use \( ts(o) \) to denote the timestamp of operation \( o \), which is defined as follows:

- For a \text{get} operation \( o \), \( ts(o) \) refers to the value of “ts” at line 1:2.
- For a \text{put} operation \( o \), \( ts(o) \) refers to the value of “cl” at line 1:12.

**Lemma 3.**

\((w \xrightarrow{m} w' \land w \in W \land w' \in W) \implies ts(w') > ts(w).\)

**Proof.** Suppose that \( w' \) is issued by client \( c \) at time \( \sigma' \).

\( ts(w) \leq (\text{dt}_c)_{\sigma'} < (\text{clock}_c)_{\sigma'} = ts(w'). \)
Lemma 4.

\[(r \stackrel{so}{\rightarrow} w \land r \in R \land w \in W) \implies ts(w) > ts(r).\]

*Proof.* Suppose that \(r\) and \(w\) are issued by correct client \(c\) at time \(\sigma_r\) and \(\sigma_w\), respectively. By line 12

\[ts(r) = \max\{(cgst_c)_\sigma, (dt_c)_\sigma\} .\]

By INV 1

\[ts(w) > (cgst_c)_{\sigma_w} > (cgst_c)_{\sigma_r}.\]

Moreover,

\[ts(w) > (dt_c)_{\sigma_r} \geq (dt_c)_{\sigma_r}.\]

Putting it together yields

\[ts(w) > ts(r).\]

Lemma 5.

\[(w \stackrel{so}{\rightarrow} r \land w \in W \land r \in R) \implies ts(r) \geq ts(w).\]

*Proof.* Suppose that \(r\) are issued by correct client \(c\) at time \(\sigma\). By line 12

\[ts(r) = \max\{(cgst_c)_\sigma, (dt_c)_\sigma\} \geq (dt_c)_\sigma.\]

By line 115

\[(dt_c)_\sigma \geq ts(w).\]

Thus,

\[ts(r) \geq ts(w).\]

Lemma 6.

\[(r \stackrel{so}{\rightarrow} r' \land r \in R \land r' \in R) \implies ts(r') \geq ts(r).\]

*Proof.* Suppose that \(r\) and \(r'\) are issued by correct client \(c\) at time \(\sigma\) and \(\sigma'\), respectively. By line 12

\[ts(r) = \max\{(cgst_c)_\sigma, (dt_c)_\sigma\},\]

and

\[ts(r') = \max\{(cgst_c)_{\sigma'}, (dt_c)_{\sigma'}\}.\]

Moreover,

\[(cgst_c)_{\sigma'} \geq (cgst_c)_\sigma \land (dt_c)_{\sigma'} \geq (dt_c)_\sigma.\]

Thus,

\[ts(r') \geq ts(r).\]

Lemma 7.

\[o \stackrel{so}{\rightarrow} o' \land o \in O \land o' \in O \implies ts(o') \geq ts(o).\]
Proof. By Lemmas 3–6.

Lemma 8.

\[ w \xrightarrow{r} r \land w \in W \land r \in R \implies ts(r) \geq ts(w). \]

Proof. By the read rule at line 27.

Lemma 9.

\[ o \xrightarrow{o'} o \in O \land o' \in O \implies ts(o') \geq ts(o). \]

Proof. By Lemmas 7 and 8.

Lemma 10. Consider \( r \in R \) and \( w \in W \). Suppose \( r \) reads from some value at a correct replica \( r_p^d \) at time \( \sigma \) (line 23). If \( w \) would be added to \( \text{store}^p_d \) at a later time than \( \sigma \) (line 24), then \( ts(w) > ts(r) \).

Proof. By Rule 1:

\[ (\text{cgst}^p_d)_\sigma \geq ts(r). \]

By INV 11:

\[ ts(w) > (\text{cgst}^p_d)_\sigma. \]

Thus,

\[ ts(w) > ts(r). \]

Lemma 11. Consider \( r \in R \) and \( w \in W \). Suppose the successful \( r \) returns at time \( \sigma \) (line 23) and the successful \( w \) starts at a later time than \( \sigma \) in partition \( p \). Then \( \neg(w \xrightarrow{r}) \).

Proof. By lines 16 and 13, there is a correct replica at which \( r \) obtains its value (line 23) before \( w \) is added to the store (line 29). By Lemma 10:

\[ ts(w) > ts(r). \]

By Lemma 9:

\[ \neg(w \xrightarrow{r}). \]

Theorem 1. Byz-GentleRain satisfies Byz-CC. That is, when a certain put operation is visible to a client, then so are all of its causal dependencies.

Proof. By Lemmas 11 and 13.

Lemma 12. Suppose a put operation \( w \) successfully returns in partition \( p \) at time \( \sigma \) (line 13). Then, it will eventually be in \( \text{store}^p_i \) for each correct data center \( i \).

Proof. By line 13 and line 20, there is a correct replica in partition \( p \) at which \( w \) is added to the store (line 29) before it is sent to the PBFT leader in the collect_ack message (line 43). By the correctness of single-shot PBFT [5], it will eventually be in \( \text{store}^p_i \) for each correct data center \( i \).
Lemma 13. Let $w$ be a successful put operation. Then, eventually for each correct replica $r^i_j$ ($1 \leq i \leq D, 1 \leq j \leq P$), $\text{cgst}^i_j \geq \text{ts}(w)$.

Proof. Suppose $w$ successfully returns in partition $p$. Then it is added to the stores of at least $f+1$ correct replicas in $\text{replicas}(p)$. By Algorithm 3, eventually for each correct replica $r^i_j$, $\text{lst}^i_j \geq \text{ts}(w)$ and $\text{gst}^i_j \geq \text{ts}(w)$. By Algorithm 4, $\text{cgst}^i_j \geq \text{ts}(w)$.

Lemma 14. Suppose a correct replica sends a new \text{CGST}(gst) message (line 3:11). Then, there is a $\text{cgst} \geq \text{gst}$ such that eventually for each correct replica $r^i_j$ ($1 \leq i \leq D, 1 \leq j \leq P$), $\text{cgst}^i_j \geq \text{cgst}$.

Proof. A byzantine replica can propose $\text{gst}^p_d$ to replica $r^p_d$, but it will take effect only when $\text{gst}^p_d < \text{lst}^p_d$ maintained by $r^p_d$. If $\text{gst}^p_d$ is smaller than a $\text{gst}'$ broadcast by a correct replica, then it will be overwritten at $r^p_d$. And eventually, there must be a $\text{gst}$ version $\geq r^p_d$ broadcast by a correct replica. Thus it will eventually be overwritten by a correct $\text{gst}$ version.

Lemma 15. Consider $r \in R$ and $w \in W$. Suppose the successful $r$ returns at time $\sigma$ (line 4:9) and the successful $w$ starts at a later time than $\sigma$ in any partition $j \neq p$. Then $\neg(w \leadsto r)$.

Proof. By Algorithm 4, $\text{ts}(w) > \text{lst}$ at each replica who accept $w$. By Algorithm 4, $\text{tsr} \leq \text{cgst}$ at each replica who reply it. Since $\text{lst}$ at any correct correct server is an upbound of all the $\text{cgst}$ at all correct replicas, $\text{ts}(w) > \text{lst} > \text{cgst} > \text{ts}(r)$. So $\neg(w \leadsto r)$. 
