Numerical studies of the saturated absorption resonances in a nonlinear spectroscopy of the degenerate atomic transitions

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Abstract. The physical processes forming the saturated absorption resonance spectra at a number of degenerate atomic transitions at their interaction with coherent optical fields and incoherent field of spontaneous radiation of excited atoms are studied numerically. The features of behavior of nonlinear resonance spectra depending on the degree of degeneration and relaxation parameters of level transitions, character of polarizations and intensities, as well as direction of propagation of optical fields are analyzed.

1. Introduction

Studies of nonlinear spectroscopic effects in the interaction of several laser fields with degenerate atomic systems have been conducted for a long time. This interest is due both to the variety of physical processes that occur in these systems and lead to narrow resonant structures in the studied spectra, and to the possibilities of their practical application in solving a number of fundamental and applied problems [1, 2].

The standard technique for theoretical consideration of the degenerate atomic systems interacting with resonant laser radiation is to divide them into two-level subsystems connected by spontaneous and induced transitions. As a result, the system of kinetic equations describing the interaction of the atomic system with radiation turns out to be a system of 2M-order (M is the maximum magnetic quantum number of levels), even without taking into account the possible coherence of the magnetic sublevels. Obtaining accurate analytical solutions of such equation systems is fraught with serious difficulties, so the solutions use approximations for a number of parameters. But this approach does not allow to follow the dynamics of the number of informative characteristics of the system to variations of parameters used in the experiments (the degree of degeneracy of the atomic transition, the intensities of the optical fields, areas of mutual orientation of the field polarizations, directions of wave propagation, etc.). This does not exclude the loss of important features of the processes under consideration, including due to the difficulty of their correct accounting, such as when taking into account the influence of light pressure [3], or when considering the effect of saturation of the medium due to its own spontaneous radiation and called the effect of self-saturation (SIR) of optical transitions [4]. Taking into account this effect even in the simple atomic systems leads to the nonlinear initial equations, which can be solved only using numerical methods [4-6]. The capabilities of modern numerical methods to a greater extent remove these difficulties and allow to analyze systems of kinetic equations with a high degree of accuracy for arbitrary
values of the intensities of both the saturating and probe fields, the orientation of their polarizations, and the total moments of the lower and upper states of atom.

This approach is implemented in this paper in relation to atomic transitions between levels with full moments $J=1$, $J=2$ in a gas medium in the presence of strong field radiation. An example of the energy level structure of transition $J_n = 1$--$J_n = 2$ is shown in figure 1. The structure of other transitions is characterized by a number of magnetic sublevels in the upper or lower states. The strong wave is assumed to be plane, monochromatic, linearly polarized (frequency $\omega$, wave vector $k$, strength of field $E$), resonant to the atomic transition. The probe wave is also monochromatic (frequency $\omega_p$, wave vector $k_p$, strength of field $E_p$) with linear polarization either orthogonal or parallel to the polarization of the strong field and is not a weak. It is assumed that the absorbing gas medium is placed in a magnetic field of strength $H$, whose magnitude can vary, and that both waves propagate collinearly to the magnetic field direction. When solving the problem, saturation of the medium with a probe wave is taken into account, assuming that it is weak compared to the strong one. The gas is assumed to be sufficiently rarefied so that collisions can be neglected. The medium is considered optically thin.

We will consider the problem in a coordinate system with the quantization axis along the direction of $\mathbf{H}$ ($\mathbf{H}$ along the Z axis). In this coordinate system, transitions with changes in the magnetic quantum number $\Delta M = \pm 1$ between levels with moments $J_m$ and $J_n$ for both fields are allowed (figure 1). It can be seen from figure 1 that the $\Lambda$- $\mu$ $V$- subsystems are formed that are related by spontaneous transitions.

When solving the problem, we will use kinetic equations for the density matrix of the atomic system. According to [1], the dynamics of diagonal element \( \rho_i = \rho_{ii} \) and nondiagonal elements \( \rho_{ik} \) of the density matrix in the relaxation constant model is described by the system of equations:

\[
\frac{d\rho_i}{dt} + \Gamma_i \rho_i = Q_i + \sum_k A_{ki} \rho_k - 2 \text{Re}(i \sum_j V_{ij} \rho_{ji}) - 2 \text{Re}(i \sum_j V_{ij}^* \rho_{ji}) \tag{1}
\]

\[
\frac{d\rho_{ik}}{dt} + (\Gamma_{ik} + i\omega_{ik}) \rho_{ik} = -i[V, \rho_i]_{ik} - i[V^*, \rho_{ik}] + R_{ik}^{(2)} \tag{2}
\]

In equations (1) and (2), \( \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} \) is the operator of full derivative, \( \Gamma_i \) are the relaxation constants of levels, \( \Gamma_{ik} \) are the transition half-widths, and \( Q_i \) are the excitation rates of levels, \( V \) and \( V^* \) are the operators of interaction of an atom with the strong and probe fields, that are defined as: \( V = -G \text{exp}(ikr - \omega t)) + c.c., \quad V^* = -G^* \text{exp}(ikr - \omega_p t)) + c.c. \), where \( G = dE/\hbar \), \( G^* = dE_p/\hbar \), and \( d \) is the dipole moment operator. The term \( A_{ki} \) in the equations (1) determines the spontaneous decay of the level transition probability, and the term \( R_{ik}^{(2)} \) in the equations (2) determines the spontaneous transfer of the magnetic level coherence from the upper to the lower state. In the analysis of the line shape, the quantity \( a_0 = A_{00}/\Gamma_{00} \leq 1 \) is important, which is called as a parameter of radiation branching. When the self-saturation effect is taken into account, equations (1) will include a summand, the type of which will be discussed below.

2. Theoretical model of resonant interaction of optical fields with a degenerate atomic transition

Consider the problem of the absorption spectrum of the probe field at a degenerate $m-n$ atomic transition (transition frequency $\omega_{mn}$, full moments of levels $J_m$ and $J_n$) in a gas medium in the presence of strong field radiation. The structure of the other transitions is characterized by a number of magnetic sublevels in the upper or lower states. The strong wave is assumed to be plane, monochromatic, linearly polarized (frequency $\omega$, wave vector $k$, strength of field $E$), resonant to the atomic transition. The probe wave is also monochromatic (frequency $\omega_p$, wave vector $k_p$, strength of field $E_p$) with linear polarization either orthogonal or parallel to the polarization of the strong field and is not a weak. It is assumed that the absorbing gas medium is placed in a magnetic field of strength $H$, whose magnitude can vary, and that both waves propagate collinearly to the magnetic field direction. When solving the problem, saturation of the medium with a probe wave is taken into account, assuming that it is weak compared to the strong one. The gas is assumed to be sufficiently rarefied so that collisions can be neglected. The medium is considered optically thin.

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\[
\frac{d\rho_i}{dt} + \Gamma_i \rho_i = Q_i + \sum_k A_{ki} \rho_k - 2 \text{Re}(i \sum_j V_{ij} \rho_{ji}) - 2 \text{Re}(i \sum_j V_{ij}^* \rho_{ji}) \tag{1}
\]

\[
\frac{d\rho_{ik}}{dt} + (\Gamma_{ik} + i\omega_{ik}) \rho_{ik} = -i[V, \rho_i]_{ik} - i[V^*, \rho_{ik}] + R_{ik}^{(2)} \tag{2}
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In equations (1) and (2), \( \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} \) is the operator of full derivative, \( \Gamma_i \) are the relaxation constants of levels, \( \Gamma_{ik} \) are the transition half-widths, and \( Q_i \) are the excitation rates of levels, \( V \) and \( V^* \) are the operators of interaction of an atom with the strong and probe fields, that are defined as: \( V = -G \text{exp}(ikr - \omega t)) + c.c., \quad V^* = -G^* \text{exp}(ikr - \omega_p t)) + c.c. \), where \( G = dE/\hbar \), \( G^* = dE_p/\hbar \), and \( d \) is the dipole moment operator. The term \( A_{ki} \) in the equations (1) determines the spontaneous decay of the level transition probability, and the term \( R_{ik}^{(2)} \) in the equations (2) determines the spontaneous transfer of the magnetic level coherence from the upper to the lower state. In the analysis of the line shape, the quantity \( a_0 = A_{00}/\Gamma_{00} \leq 1 \) is important, which is called as a parameter of radiation branching. When the self-saturation effect is taken into account, equations (1) will include a summand, the type of which will be discussed below.
Figure 1. Scheme of interaction of optical fields with the sublevels of the J=1–J=2 transition: strong field (solid lines), probe field (dashed lines); 1-\(a_0\) is the fraction of the spontaneous decay of sublevels of m state into other lower lying levels.

We seek solutions to the system of equations (1, 2) in the following form (approximation of the first harmonic at \(\varepsilon\)): diagonal elements as \(\rho_k = \rho_k^0 + \rho_k^\pm \exp(i(\hat{\mathbf{A}} - (\mathbf{k}_\mu - \mathbf{k})r)) + \rho_k^\pm \exp(-i(\hat{\mathbf{A}} - (\mathbf{k}_\mu - \mathbf{k})r))\); off-diagonal elements on allowed transitions as \(\rho_k = R_{ik} \exp(-i(\hat{\mathbf{A}} - (\mathbf{k}_\mu - \mathbf{k})r)) + R_{ik}^\mu \exp(-i(\hat{\mathbf{A}} - (\mathbf{k}_\mu - \mathbf{k})r)) + R_{ik}^s \exp(-i(\hat{\mathbf{A}} - (\mathbf{k}_\mu - \mathbf{k})r)) + R_{ik}^s \exp(i(\hat{\mathbf{A}} - (\mathbf{k}_\mu - \mathbf{k})r))\) and, off-diagonal elements on forbidden transitions as \(\rho_k = r_k^0 + r_k^\pm \exp(i(\hat{\mathbf{A}} - (\mathbf{k}_\mu - \mathbf{k})r)) + r_k^\pm \exp(-i(\hat{\mathbf{A}} - (\mathbf{k}_\mu - \mathbf{k})r))\), where \(\varepsilon = \omega_\mu - \omega_0, \omega_0 = 2\omega - \omega_\mu, \mathbf{k}_s = 2\mathbf{k} - \mathbf{k}_\mu\). The validity of this type of solutions in the stationary case for the transition considered in a wide range of intensities of the probe and strong waves was shown by us in [7].

In the approximation of rotating optical fields, the system of equations (1, 2) for the density matrix is reduced to a system of equations with respect to the coefficients \(\rho_i^0, \rho_i^\pm, R_{ik}, R_{ik}^0, R_{ik}^\mu, R_{ik}^s\). Given the Hermitian nature of these coefficients, we write only independent equations. From equation (1) follows a system of equations for the populations of the i-th sublevel of the lower n and the k-th sublevel of the upper m state:

\[
\frac{d\rho_i^0}{dt} + \Gamma_n \rho_i^0 = Q_i + \sum_k A_{ki} \rho_k^0 + 2 \text{Re}(\sum_k G_{ik} R_{ik}^\mu) + 2 \text{Re}(\sum_k G_{ik}^\mu R_{ik}^0) \tag{3}
\]

\[
\frac{d\rho_i^+}{dt} + (\Gamma_n + i(\varepsilon - (k_\mu - k)\nu))\rho_i^+ = \sum_k A_{ki} \rho_i^+ + i \sum_k (G_{ik}^\mu R_{ik}^0 - G_{ik} R_{ik}^\mu + G_{ik}^\mu R_{ik}^\mu) \tag{4}
\]

\[
\frac{d\rho_k^0}{dt} + \Gamma_m \rho_k^0 = Q_k + 2 \text{Re}(\sum_i G_{ki} R_{ik}^\mu) + 2 \text{Re}(\sum_i G_{ki}^\mu R_{ik}^0) \tag{4}
\]

\[
\frac{d\rho_k^+}{dt} + (\Gamma_m + i(\varepsilon - (k_\mu - k)\nu))\rho_k^+ = i \sum_i (G_{ki}^\mu R_{ik}^0 - G_{ki} R_{ik}^\mu - G_{ki} R_{ik}^\mu) \tag{4}
\]

From equations (2) we obtain the following systems of equations for the coefficients \(R_{ik}, R_{ik}^\mu, R_{ik}^s\) for polarizations at allowed transitions:
The equations for the coefficients $r_{ik}^0$, $r_{ik}^+$ of polarizations at forbidden transitions between magnetic sublevels of the lower or upper states are the next:

$$\begin{align*}
\frac{dR_{ik}^0}{dt} + (\Gamma_{mn} - i\Omega_{ik})R_{ik}^0 &= -iG_{ik}^0(\rho_l^0 - \rho_k^0) - iG_{ik}^s(\rho_l^s - \rho_k^s) + iG_{il}^0\rho_l^0 + iG_{il}^s\rho_l^s + iG_{ik}^0r_{ik}^+ \\
\frac{dR_{ik}^+}{dt} + (\Gamma_{mn} - i\Omega_{ik})R_{ik}^+ &= -iG_{ik}^0(\rho_l^s - \rho_k^0) - iG_{ik}^s(\rho_l^0 - \rho_k^s) + iG_{il}^0\rho_l^0 + iG_{il}^s\rho_l^s + iG_{ik}^0r_{ik}^+ \\
\end{align*}$$

Here indices $i$ and $k$ denote sublevels of different states and indices $l$ and $k$ denote sublevels of the same state. In equations (3) - (5) $\Gamma_n$ and $\Gamma_m$ are the relaxation constants of the lower and upper levels, $\Gamma_{mn}$ is the homogeneous half-width of the transition line, $\Omega_{ik} = (\omega - \omega_{ik} - k\nu)$ and $\Omega_{ik}' = (\omega_{ik} - \omega_{ik} - k\nu)$ are the detunings of frequencies of the strong and probe fields from the transition frequencies $\omega_{ik}$ between the magnetic sublevels of the upper and lower states with regard to the Doppler shift, and $\nu$ is the velocity vector of atom.

The equations for the coefficients $r_{ik}^0$, $r_{ik}^+$ of polarizations at forbidden transitions between magnetic sublevels of the lower or upper states are the next:

$$\begin{align*}
\frac{d\delta r_{ik}^0}{dt} + (\Gamma_{ik} + i\omega_{ik})\delta r_{ik}^0 &= i(G_{il}R_{ik}^0 - G_{ik}^sR_{li}^+ + G_{il}^0R_{ik}^0 - G_{ik}^sR_{li}^0 + \delta \gamma_{ik}^0) \\
\frac{d\delta r_{ik}^+}{dt} + (\Gamma_{ik} + i(\omega_{ik} + (\epsilon - (k\mu - k)\nu)))\delta r_{ik}^+ &= i(G_{il}^0R_{ik}^0 - G_{ik}^sR_{li}^0 + G_{il}^sR_{ik}^0 + \delta \gamma_{ik}^+) \\
\end{align*}$$

Here indices $i$, $k$ denote sublevels of the one state, and indices $l$ denote sublevels of the another state; $\omega_{ik}$ is the frequency of transitions between magnetic sublevels of the lower $n$ or upper $m$ states; $\Gamma_{ik}$ is the half-width of the lines of these transitions: $\Gamma_{ik} = \Gamma_n$ (for the lower state) and $\Gamma_{ik} = \Gamma_m$ (for the upper state). The terms $\delta \gamma_{ik}^0$, $\delta \gamma_{ik}^+$ in (6) determine the spontaneous transfer of magnetic coherence from the upper state to the lower state, they are present in equations for the lower state and are absent in equations for the upper state. The type of these terms depends on the moments of transition levels. Below, we will consider solutions of the system of equations (3) - (6) under steady-state conditions in cases of the counter ($k\mu - k = 0$) or unidirectional ($k\mu - k = 0$) waves with close frequencies ($\omega = \omega_{ik}$).

In general, taking into account the finite transverse size of the light beam, the coordinate derivatives are significant if the average flight time of atom through the light beam is comparable to or less than the lifetime of the longest-lived state (in our case, the lower state), i.e. $1/\Gamma_n \geq d/\nu_T$. At $\nu_T = 10^5$ cm/s and the transverse size of the light beam $d \sim 0.1$ - 1 cm (typical for experimental conditions), we have an estimate value of $\Gamma_n \sim 10^7 - 10^8$ s$^{-1}$. For the smaller widths, in particular, for transitions from the ground state, it is necessary to use spatially inhomogeneous solutions of the equations for the density matrix. Our calculations have shown that the solutions of the spatially inhomogeneous problem involving the ground state of an atom are qualitatively consistent with the results of a spatially homogeneous one, in which the relaxation constant of the lower level $\Gamma_n$ is replaced by an average flight width determined by the transverse size $d$ of the light beam and the most likely particle velocity $\nu_T$. Therefore, results of spatially homogeneous solutions of equations (3) - (6) are given below.

The question of manifestation of the self-saturation effect in the shape of the saturated absorption resonance at degenerate transitions is considered on the basis of the model [4]. According to this model, in the equations (1, 3, 4) for diagonal elements of the density matrix, the following terms $
abla_{nm}^{MM}(\rho_n^M - \rho_n^M)$ are added, describing transitions induced by spontaneous radiation of excited atoms. The equations for non-diagonal terms (5, 6) remain the same. The value $\nabla_{nm}^{MM}$ determines the frequency of induced transitions between the magnetic sublevels of the lower and upper states. The total
sublevel frequency of self-saturation is defined as: $\nu_{nm} = \sum_{M'M} \nu_{nm}^{M'M} = g_{nm} <\rho_{nm}>$, where $g_{nm}$ is determined by parameters of atomic transition, and the transverse size of interaction region as: $g_{nm} = (\lambda_{nm} A_{nm}/2)^2 d_\perp / kv_T$ and $<\rho_{nm}>$ is the full number of particles in the excited state of atom. The dependence of the self-saturation frequency $\nu_{nm}$ on the total population of the upper level leads to the nonlinear nature of the stationary system of equations. When solving the equations, the parameter $a_{nm} = g_{nm} N_{nm}/\Gamma_m$ occurs, where $N_{nm}$ is the difference in the population of levels in the absence of optical fields. The parameter $a_{nm}$ characterizes the relative frequency of transition self-saturation, its maximum value in model [4] can reach the value $\sim 2$.

3. Results of numerical solution of initial equations for a number of transitions

The steady state systems of equations (3) - (6) were solved numerically for transitions with a change full of moments $J=0$-$J=1$, $J=1$-$J=2$, $J=2$-$J=3$ by varying the widths of the atomic levels, the branching parameter $a_0$, and the intensities of optical fields using the following level relaxation constants $\Gamma_m = 5.5 \times 10^3$ s$^{-1}$, $\Gamma_n = 10^{-3} + 10^3$ s$^{-1}$ and $\Gamma_{mn} = (\Gamma_m + \Gamma_n)/2$. The widths of the levels varied from the indicated above to the values $\Gamma_m = \Gamma_n = \Gamma_{mn}$, while maintaining a constant value $\Gamma_{na}$. The branching parameter $a_0$ varied within $0.1-1$. Line widths of forbidden transitions between magnetic sublevels of the lower and upper levels were assumed equal to the widths of the lower and upper levels. The Doppler line width was taken equal to $kv_T = 5.2 \times 10^8$ s$^{-1}$, and the integration interval of the particle velocities was $\pm 3kv_T$ with a step of $\Delta kv_T/kv_T = 10^{-3} \times 10^4$. The saturation parameters of the strong $\kappa_s$ and probe $\kappa_p$ fields were taken as $\kappa_s = 2(2E/2\sqrt{5}h)^2 (\Gamma_{mn}\Gamma_n)$, $\kappa_p = 2(E_{\mu_s}/\sqrt{3}h)^2 (\Gamma_{mn}\Gamma_n)$, where $E$ and $E_{\mu}$ are the strengths of the circular components of the strong and probe fields, $d$ is the reduced matrix element of the dipole moment of transition, $\gamma_{mn} = \Gamma_m + \Gamma_n - A_{nm}$. The values of the saturation parameters were varied within $\kappa_s = 0.1 \div 50$ and $\kappa_p \leq \kappa_s$.

The absorption line shape of the probe field is determined by the relation:

$$\alpha_{\mu} / \alpha_0 = -\Gamma_{mn} \left( \frac{\text{Re} \left( \sum_{1, k} R_{ik} G_{ik}^* / |G_{ik}|^2 \right)}{\text{Re} \left( \sum_{1, k} R_{ik} G_{ik}^* / |G_{ik}|^2 \right)} \right)$$

(7). Here $\langle...\rangle$ denotes averaging with respect to the Maxwell velocity distribution of particles, and $\alpha_0 = 4\pi\omega_0 d^2/c_s \Gamma_{nm}$ is the resonant absorption cross section.

We calculated also the dependences of the probe wave absorption in the line center (at $\Omega = \Omega_0 = 0$) on the level splitting (so-called magnetic scanning spectra). Such dependences were recorded in experiments [9, 10], and narrow structures in the spectra were also named as electromagnetically induced transparency (EIT) and electromagnetically induced absorption (EIA) resonances.

To identify the physical causes of dependence of the saturated absorption and magnetic scanning resonances on the mutual orientation of the polarizations of light waves, we studied the contributions to the form of these resonances the next processes: magnetic coherence induced at the lower levels of the $\Lambda$-transition scheme; magnetic coherence induced at the upper levels in the $V$-scheme with its subsequent transfer to the lower state levela; nonlinear polarization at the Raman frequency $\omega_\nu = 2\omega - \omega_\mu$. Note that it is the transfer of magnetic coherence from the upper to lower levels that causes, according to the authors [11], the peak form of resonance (the EIA effect) in the spectra of magnetic scanning in the spectroscopy of unidirectional waves. To determine the contributions of the above processes the corresponding terms $R_{ik}^{(2)}$, or $R_{ik}^{(3)}$ were excluded from the system of stationary equations (3)-(6).

As result of modeling, it turned out that in the case of counter waves, the features of the absorption spectrum of the probe wave are due to the incoherent effect of saturation of the level populations of an atomic transition by the strong and probe wave fields, and coherent processes are strongly suppressed, as in the analytical calculations of simple transitions [1, 2]. The contribution of coherent processes is significant in the case of unidirectional waves, and the value of the contribution depends on the nature of transition (open or closed), the level relaxation constants and can qualitatively affect on the shape of the absorption spectrum.
3.1. Case of the counter propagating waves

The characteristic behavior of the saturated absorption resonance shapes of the probe wave (NR) in counter propagating waves are shown in figure 2 and figure 3 for the transition J=2-J=2. Qualitatively, the same forms of nonlinear resonance are realized at the transition with the level moments J=1.

At low intensities of the strong field (at saturation parameters $k_s \leq 1$), a traditional dip in the center of Doppler absorption line at the probe field intensities (at saturation parameters $k_p < k_s$) is observed. When intensity of the strong field corresponds $k_s > 1$, the increase of the probe field intensity leads to drastic changes in the shape of the NR (figure 2). At small intensities of the probe field, the NR shape has the form of a dip in the center of line (curve 1). An increase in the probe field intensity leads to a non-uniform frequency decrease in the absorption line (curves 2-4). Changes in the absorption shape are relatively small near the line center and more significant in the line wings, which lead to deformation of the line contour and its effective narrowing. At the same time, as the intensity of the probe field increases, the saturation resonance (dip) near the line center narrows with a decrease in its contrast, and then inverts to the absorption peak due to the illumination of the line's wings (curves 3, 4). At certain ratios of intensities of the strong and probe fields, the doublet structure appears at the peak of absorption. The spectral width of the peak decreases with increasing intensity of the probe field, while the peak amplitude significantly exceeds the amplitude of the lining.

Action of a strong field on the shape of an inverted NR (peak) is shown in figure 3. Here, an increase in the intensity of a strong field leads to a decrease in the amplitude of NR, an increase in its width, and the manifestation of a doublet structure.

Figure 2. Shape of resonance at $k_s=50$ and $k_p=5\times10^{-3}$ (1), 1.1 (2), 1.5 (3), 2.1 (4).

Figure 3. Shape of resonance at $k_p=2$ and $k_s=10^2$ (1), 25 (2), 50 (3), 100 (4).

The studies of the self-saturation effect in case of counter propagating waves at transitions with level moments J=2 and J=1 showed that for small values of saturation parameters the probe and strong field ($k_p / k_s << 1$), when the resonance was traditional (with dip in the line center), the self-saturation effect with growth of parameter $a_{mn}$ lead to a decrease in the amplitude, width and contrast of the dip relative to the Doppler lining. Action of the self-saturation on the shape of the inverted NR (figure 2, curve 4) is more diverse. At certain ratios of the level relaxation constants and field intensities, an increase in the contrast of the inverted resonance can be observed, then its leveling, followed by a uniform subsidence of the absorption line. The resonance contours at $k_s = 50$, $k_p = 2$ and different values of $a_{mn}$ are presented in figure 4. It can be seen that the self-saturation effect is selective in the resonance spectrum and qualitatively different for values $a_{mn}<<1$ and $a_{mn} \sim 1$. At values of $a_{mn} \leq 0.4$ (curves 1-3), there is a weak effect near the line center and an increase of absorption in the line wings (curves 2, 3). At the same time,
the NR expands, the contrast of its peak decreases (~by 5 times), and the doublet structure appears. At $a_{mn} \approx 1$ (curves 4-6), the effect of self-saturation becomes decisive near the line center. Here, an increase in the $a_{mn}$ leads to a decrease in absorption contour, to disappearance of the peak's doublet structure, narrowing of its spectrum (~2 times), and to a change in its contrast.

![Figure 4](image-url)

**Figure 4.** Shape of resonance at values $k_s=50$, $k_p=2.0$ and $a_{mn}=10^{-3}$ (1); 0.16 (2); 0.4 (3); 0.8 (4); 1.2 (5); 2.0 (6).

Note that the agreement of the results of calculations and experimental data [8] indicates in favor of the saturating effect of the probe field as the reason for the unusual behavior of the saturated absorption resonance under experimental conditions.

3.2. Case of the unidirectional waves

The main features of the saturated absorption resonances and their forming processes in case of unidirectional waves are shown in the example of transitions $J=1-J=1$ and $J=1-J=2$. Here the shapes of saturated absorption resonances will be determined both by the effect of saturation of the atomic level populations, and by coherent processes in the system of degenerate transition levels in the field of strong and probe waves. The size and shape of the coherent processes depend on the values of the total moments and level relaxation constants, on the branching parameter $a_0$, values of the level splittings, on the intensity and direction of polarizations of the strong and probe fields.

Characteristic spectra of nonlinear resonances at these transitions are shown in figure 5 and figure 6 for two types of transitions (closed and open) in the absence of level splitting for the open ($a_0=0.5$) and closed ($a_0=1$) under action of optical fields with linear parallel or orthogonal polarizations.

In case of the $J=1-J=1$ transition (figure 5) and optical fields with **orthogonal polarizations** the resonance appears on the Doppler absorption line of the probe wave near the frequency $\Omega_0 = \Omega$ as a narrow dip against the background of a wide absorption peak of small amplitude (curves 1, 2). At fixed field intensities, the width of the narrow dip is determined by the relaxation constant of the lower level $\Gamma_n$, and the width of the peak structure and the position of the maxima are determined by the polarization relaxation constant $\Gamma_{mn}$. In this case, the amplitudes of peaks and dips depend on the values of the branching parameter $a_0$. Changing the value of $a_0$ in the range $a_0=1.0$ leads to a decrease both in the amplitude of the narrow dip and the amplitude of the peak structure until it disappears (curves 1, 2).

In case of fields with **parallel polarizations**, the nonlinear resonance manifests itself as a wide dip (with half-width $\Gamma_{mn}$), within which a narrow peak (with half-width $\Gamma_n$) of a small amplitude is formed (curves 3, 4). This resonance structure is observed at the open (curve 3) and closed transition (curve 4).
**Figure 5.** Shapes of saturated absorption resonance for $\Omega = 0$, $k_s = 1$, $k_p = 0.001$, $\Gamma_n = 0.1 \Gamma_m$, $a_0 = 1$ (1, 4), $a_0 = 0.5$ (2, 3). Solid lines (1, 2) represent orthogonal polarizations, and dotted lines (3, 4) represent parallel polarizations of the fields.

In case of the $J=1$-$J=2$ transition, the nonlinear resonance has a different structure. Here, a wide dip is formed on the Doppler absorption contour of the probe wave near the frequency $\Omega_\mu = \Omega$ and a narrow structure is formed in the center (figure 6). The narrow structure manifests itself as a peak at the closed transition (curves 1, 2), and in the form of dip at the open transition (curves 3, 4). Parameters of the main dip and narrow structures depend differently on the orientation of the polarizations of optical fields.

At a closed transition (curves 1, 2), amplitude and width of the main dip are less (about twice) for orthogonal polarizations of fields, and amplitudes of the narrow peaks are significantly greater than for parallel polarizations of the fields. At the same time, wide maxima are formed in the wings of the main dip (curve 2). At the open transition, the change of polarization directions does not change parameters of the main dip, but leads to increase in the amplitude of the narrow structure with parallel polarizations of the fields (curves 3, 4). An increase of the transition openness (reducing the value of $a_0$) with fixed other parameters reduces amplitude and increases width of the main dip and decreases amplitudes of maxima in the dip wings. In this case, a narrow peak in the line center (curves 1, 2) is converted into a narrow dip (curves 3, 4). Moreover, the peak inversion occurs at $a_0 \sim 0.7$ for orthogonal polarizations and already at values $a_0 \sim 0.95$ for parallel polarizations of fields.

The calculations showed that the amplitude and width of the main dip was determined by the relaxation constant $\Gamma_{mn}$ and values of the saturation parameters, but parameters of the narrow structure was determined by the relaxation constant $\Gamma_n$. A decrease of value $\Gamma_n$ lead to a decrease in the width and to an increase in the amplitude of the resonance narrow structure.
Figure 6. Shapes of saturated absorption resonance on closed ($a_0 = 1$, curves 1, 2) and open ($a_0 = 0.5$, curves 3, 4) transitions for $\Omega_s = 0$, $\kappa_s = 1$, $\kappa_p = 0.001$, $\Gamma_n = 0.02\Gamma_m$. Curves (1, 3) represent parallel polarizations, and curves (2, 4) represent orthogonal polarizations of fields.

An increase in the strong wave intensity for any field polarizations leads to an increase in both amplitude and width of the main dip, and amplitudes of the narrow structures. Moreover, the broadening effect is greater for parallel polarizations. At the same time, the width of the narrow dip also increases, but the width of the peak does not change practically.

Action of the probe field intensity on the shape of the nonlinear resonance is shown in a different way (see figure 7). For all types of transitions, the increase in the probe field intensity leads both for parallel and orthogonal polarizations of fields at saturation parameters $\kappa_p \leq \kappa_s$ to a decrease of the Doppler absorption contour, as well as to a decrease in the amplitudes and to an increase in the widths of main dip and narrow resonance structures (curves 1-3). Radical changes in the resonance shape are detected when the intensity of the probe field exceeds the intensity of the saturating field. In this case, at saturation parameters $\kappa_p > \kappa_s \sim 1$ a resonance of the opposite sign occurs in the center of the expanded narrow structure (EIT or EIA resonances, curves 4, 5). Note that a similar transformation of the EIA resonances into EIT resonances and vice versa was observed experimentally in [12].

Studies of the formation processes of the nonlinear resonance spectrum at degenerate transitions showed that the main contribution is made by two- and three-level V-transition schemes formed by sublevels with maximum magnetic number. They determine the characteristic features of the resonance spectrum, including the shape of its narrow structures. The contributions of transitions between other magnetic sublevels are much smaller. An example of the spectra contributions of individual components transition $J=1$-$J=2$ in the absence of level splitting is shown in figure 8 for parallel field polarizations (the total contour is shown in figure 6, curve 1). The difference in the amplitudes of the Doppler linings of the spectra is caused by the difference of the oscillator strengths, and the difference in the resonance
shapes is related to the different degrees of openness of these transitions, because the transitions between the sublevels $M=\pm 1 \rightarrow M'=\pm 2$ are closed, whereas the transitions between the other sublevels are open.

Figure 7. Dependence of the resonance shape on the saturation parameter of the probe field at parallel polarizations of fields; $\kappa_s = 1$, $\kappa_p = 0.001(1)$; $0.5(2)$; $0.95(3)$; $5(4)$; $10(5)$.

Calculations of the contribution of the magnetic coherence transfer from the upper level state to the lower state (contribution term $R^{(2)}_{ik}$ in equation (2)) in the shape of saturated absorption resonance showed this contribution depends on the field saturation parameters, branching ratio $a_0$, level splittings and directions of field polarizations. The transfer contribution is maximum for a closed transition in the absence of level splitting. While for parallel field polarizations the transfer leads to an increase of the magnetic coherence of the lower levels and a decrease in the probe wave absorption near the line center, whereas at orthogonal polarizations it decreases the level magnetic coherence levels and increases the absorption. For figures 5, 6, the maximum contribution of the magnetic coherence transfer to the resonance peak amplitude at orthogonal field polarizations is $\sim 10 \%$, whereas at parallel polarizations it is smaller by a factor of more two. The calculations showed that the transfer of magnetic coherence from the upper to the lower levels cannot cause the formation of narrow (with a lower level width) structures in the nonlinear resonance spectrum, as is generally believed in the literature for a long time [11].

The studies of manifestation of the self-saturation effect in the form of nonlinear resonances at degenerate transitions in case of unidirectional waves showed that this effect acts both in the amplitude and width of the main dip, as in counter waves, and in a specific way on the characteristics of narrow resonance structures formed by coherent processes. The characteristic manifestation of the self-saturation effect is shown in figure 9 for a closed ($a_0 = 1$) transition between levels with moments $J=1$.
Figure 8. Spectra of the contributions of transitions between the magnetic sublevels $M = \pm 1 \rightarrow M' = \pm 2$ (1), $M = 0 \rightarrow M' = \pm 1$ (2) and $M = \pm 1 \rightarrow M' = 0$ (3) to the resonance at parallel field polarizations; $\Omega_s = 0$, $\Omega_H = 0$, $k_s = 1$, $k_p = 0.001$, $\Gamma_n = 0.02 \Gamma_m$.

for parallel and orthogonal polarizations of fields. It follows that the effect leads with increasing values of $a_{mn}$ at both types of field polarizations, firstly, to reduce the amplitude of the wings of the Doppler absorption line, and, secondly, to decrease amplitudes of the resonance structures (as peak and dip) and, thirdly, to the broadening of the resonance structures. Moreover, the expansion of the SIR field on narrow resonance structures is more pronounced in orthogonal polarizations (curve 4). A similar action of self-saturation on the resonance shape is observed at open transitions ($a_0 < 1$).

However, at closed transition ($a_0 = 1$) for any polarizations of waves are found features in the manifestation of the SIR effect: if the absorption in the line wings decreases sequentially with the increase of $a_{mn}$, then near the center of the line, first there is an increase in absorption (curves 1-3), and then its sequential decrease (curves 4-5). In the case of open transition (with $a_0 = 0.5$) the SIR effect with an increase of $a_{mn}$ leads to a sequential decrease in the absorption both in the wing and near the center of resonance line.

Studies have shown [13] that differences in the effect of SIR on spectra of nonlinear resonance near the line center at closed and open transitions due to the difference of the contributions of the magnetic coherence induced by linearly polarized optical fields at the lower levels of these transitions, in term of destruction of her by the incoherent SIR field. The contributions of the level magnetic coherence to the form of nonlinear resonance are shown in a narrow range near the line center (figure 6, curves 1, 4). The effect of the SIR field destroys the magnetic coherence of the lower levels and leads to an increase in the absorption of the probe field near the line center at both types of transitions. Moreover, at a closed transition at low self-saturation frequencies, the increase in absorption due to the destruction of the level magnetic coherence prevails over the decrease in absorption due to the formation of a uniform saturation band [4] (figure 9, curves 1-3). In this case, the changes in the absorption for parallel polarizations are greater than for orthogonal polarizations. With a further increase in the frequency of self-saturation, the
contribution of the uniform SIR band begins to prevail over the contribution of magnetic coherence, and
the absorption behavior near the center of the line turns out to be the same as in the line wings (curves
4, 5). At the open transition, even at low self-saturation frequencies, the contribution of the uniform
saturation band to the absorption prevails over the contribution of the effect of breaking of the level
magnetic coherences by the SIR field. Therefore, here the nature of the self-saturation effect is the same
both in the center and in the wings of the absorption line.

**Figure 9.** Shape of resonance due to self-saturation effect: $a_0=1; k_s=1. k_p=0.001; \Gamma_n=0.1\Gamma_m; a_{mn}=0.01$
\(\div 0.1(1); 0.22 \quad (2); 0.35(3); 0.5(4); 0.75 (5).\) Solid lines are orthogonal polarizations, dotted lines are
parallel polarizations of fields.

4. Conclusion

Thus, the application of numerical methods for solving problem of interaction of complex atomic
transitions with several resonant optical fields allows us to study the spectra of nonlinear resonances of
saturated absorption, as well as the processes that form them in a wide range of changes of the atomic
transition parameters (level relaxation constants, level splittings), directions of polarizations and
intensities of optical fields. This approach allowed us to describe the anomalous behavior of the
nonlinear resonance shapes in experiments [8, 12], when the optical field intensities do not meet the
conditions of the probe field theory [1, 2]. The qualitative dependence of the type of nonlinear resonance
spectrum on the direction of propagation of light waves and the degree of openness (branching
parameter) of the degenerate transition is revealed.

It is shown that the features of the resonance spectra at counter propagating waves are formed mainly
in two-level transition schemes and are determined by the effects of saturation and splitting of levels,
the contribution of coherent processes is small. It is the specifics of population beats of the level
relaxation that determine the type of narrow resonance structures: a peak at the closed transitions and a
dip at the open transitions. The contributions of the level magnetic coherences depend on the orientation
of the strong and probe field polarizations and are shown as additives. The main contribution is made by the lower state levels, the contributions of the magnetic coherence transfer effect from the upper to lower state are small and appear only near the line center. This result requires a revision of the long-accepted interpretation of the mechanism [11] for the formation of EIA resonances found in experiments [9].

In conclusion, we note that only the numerical modeling method allows us to study the self-saturation effect in the shape of nonlinear resonances at degenerate atomic transitions, as well as to identify the specifics of its manifestation in cases of counter and unidirectional waves.

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