STRANGE-QUARK VECTOR CURRENT PSEUDOSCALAR-MESON TRANSITION FORM FACTORS

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Abstract

Similarly to the electromagnetic pseudoscalar-meson transition form factors one can define also strange-quark vector current pseudoscalar-meson transition form factors, contributing only to a behaviour of the isoscalar parts of the previous ones. Their explicit form is found by constructing unitary and analytic models of the strange pseudoscalar-meson transition form factors dependent only on \( \omega \) and \( \phi \) coupling constant ratios as a free parameters. Numerical values of these ratios are then determined from the corresponding pseudoscalar-meson transition form factors by employing the \( \omega-\phi \) mixing and a special assumption on the coupling of the quark components of vector-meson wave functions to flavour component of currents under consideration.

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1 Introduction

During the last years there was an experimental effort [1]-[3] to confirm non-zero contributions of sea strange quark-antiquark pairs to the structure of nucleons, which are built by nonstrange up and down quarks. The results of those experiments were

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values of the nucleon strange electric and magnetic form factors (FF’s), or of their combinations at nonzero values of the four-momentum transfer squared \( t = -Q^2 \).

On the other hand there are various theoretical approaches [4]-[8] in the framework of which one can predict strange electric and magnetic or strange Dirac and Pauli FF’s of nucleons. One of these approaches [8] utilizing the unitary and analytic models of electromagnetic (EM) structure of hadrons [9], appeared in a description of the scarce experimental information on nucleons to be successful and it can be directly extended also to the pseudoscalar-meson transition FF’s \( F_{\gamma P}(t) \).

The idea consists in the following. If the unitary and analytic models, with all known properties of the EM pseudoscalar-meson transition FF’s are constructed \( F_{\gamma P}^{EM}(t) = f[t; a_\rho, a_\omega, a_{phi}] \), where the free parameters \( a_\rho = (f_{\rho \gamma P}/f_{\rho}^{EM}) \), \( a_\omega = (f_{\omega \gamma P}/f_{\omega}^{EM}) \), \( a_{phi} = (f_{\phi \gamma P}/f_{\phi}^{EM}) \) are determined by a comparison of the model with all existing data on \( |F_{\gamma P}^{EM}(t)| \) in space-like and time-like region simultaneously, and unitary and analytic models of the same inner structure (besides the asymptotic behaviour and normalization) with all known properties of the strange-quark vector current pseudoscalar-meson transition FF’s are established \( F_{\gamma P}^{s}(t) = g[t; b_\omega, b_{phi}] \) with unknown parameters \( b_\omega = (f_{\omega \gamma P}/f_{\omega}^{s}) \), \( b_{phi} = (f_{\phi \gamma P}/f_{\phi}^{s}) \), then the latter parameters are determined from the known \( a_\omega, a_{phi} \) by the relations [4]

\[
\begin{align*}
b_\omega &= -\sqrt{6} \frac{\sin \epsilon}{\sin (\epsilon + \theta_0)} a_\omega \\
b_{phi} &= -\sqrt{6} \frac{\cos \epsilon}{\cos (\epsilon + \theta_0)} a_{phi},
\end{align*}
\]

where \( \epsilon = 3.7^\circ \) is deviation from the ideally \( \omega-\phi \) mixing angle \( \theta_0 = 35.3^\circ \).

In the next section we review briefly the unitary and analytic model of EM pseudoscalar-meson transition FF’s. The section 3 is devoted to a prediction of behaviours of strange-quark vector current pseudoscalar-meson transition FF’s. In the last section we present conclusions and discussion.
2 EM Pseudoscalar-meson transition form factors

The EM pseudoscalar-meson transition FF’s are understood to be functions $F_{\gamma P}^{EM}(t)$ describing any $\gamma^* \to \gamma P$ transition, where $P$ can be $\pi^0$, $\eta$ and $\eta'$. Only recently a progress in the EM pseudoscalar-meson transition FF’s was achieved [10] thanks to the sophisticated unitary and analytic model of EM structure of hadrons [9] and an appearance of a new experimental information, especially in the time-like region [11].

There is a single FF for each $\gamma^* \to \gamma P$ transition to be defined by a parametrization of the matrix element of the EM current $J_{\mu}^{EM} = \frac{2}{3}\bar u \gamma_{\mu} u - \frac{1}{3}\bar d \gamma_{\mu} d - \frac{1}{3}\bar s \gamma_{\mu} s$

$$\langle P(p)\gamma(k)|J_{\mu}^{EM}|0\rangle = \epsilon_{\mu\nu\alpha\beta}^{\gamma P} e^\alpha k^\beta F_{\gamma P}^{EM}(t),$$

where $\epsilon^\alpha$ is the polarization vector of the photon $\gamma$, $\epsilon_{\mu\nu\alpha\beta}^{\gamma P}$ appears as only the pseudoscalar-meson belongs to the abnormal spin-parity series. Every $F_{\gamma P}^{EM}(t)$ for $P = \pi^0, \eta, \eta'$ in the framework of the unitary and analytic model of the EM structure of hadrons takes the form

$$F_{\gamma P}^{EM}(t) = F_{\gamma P}^{I=0}[V(t)] + F_{\gamma P}^{I=1}[W(t)]$$

with

$$F_{\gamma P}^{I=0}[V(t)] = \left(\frac{1-V^2}{1-V_N^2}\right)^2 \left\{ \frac{1}{2} F_{\gamma P}^{EM}(0) H(\omega') + [L(\omega) - H(\omega')] a_{\omega} + [L(\phi) - H(\omega')] a_{\phi} \right\}$$

$$F_{\gamma P}^{I=1}[W(t)] = \left(\frac{1-W^2}{1-W_N^2}\right)^2 \left\{ \frac{1}{2} F_{\gamma P}^{EM}(0) H(\rho) + [L(\rho) - H(\rho')] a_{\rho} \right\}$$

where $V(W)$ is the conformal mapping

$$V(t) = i \sqrt{\frac{q^{I=0} + q}{q^{I=0} - q}} \sqrt{\frac{q^{I=0} + q}{q^{I=0} - q}}$$

$$q = [(t-t_0)/t_0]^{1/2}; \quad q^{I=0} = [(t_{in}^{I=0} - t_0)/t_0]^{1/2}$$

of the four-sheeted Riemann surface in $t$-variable into one $V$-plane ($W$-plane),

$$F_{\gamma P}^{EM}(0) = \frac{2}{\alpha m_P} \sqrt{\frac{\Gamma(P \to \gamma\gamma)}{\pi m_P}},$$
$t_0 = m_{\pi^0}^2, t_{in}^{I=0}$ and $t_{in}^{I=1}$ are the effective square-root branch points including in average contributions of all higher important thresholds in both, isoscalar and isovector case, respectively, and

$$L(s) = \frac{(V_N - V_s)(V_N - V_s^*)(V_N - 1/V_s)(V_N - 1/V_s^*)}{(V - V_s)(V - V_s^*)(V - 1/V_s)(V - 1/V_s^*)},$$

$$s = \omega, \phi, \quad V_N = V(t)_{|t=0}$$

$$H(\omega') = \frac{(V_N - V_{\omega'})(V_N - V_{\omega'}^*)(V_N + V_{\omega'})(V_N + V_{\omega'}^*)}{(V - V_{\omega'})(V - V_{\omega'}^*)(V + V_{\omega'})(V + V_{\omega'}^*)},$$

$$L_\rho = \frac{(W_N - W_\rho)(W_N - W_\rho^*)(W_N - 1/W_\rho)(W_N - 1/W_\rho^*)}{(W - W_\rho)(W - W_\rho^*)(W - 1/W_\rho)(W - 1/W_\rho^*)}; \quad W_N = W(t)_{|t=0}$$

$$H_\rho' = \frac{(W_N - W_\rho')(W_N - W_\rho')(W_N + W_\rho')(W_N + W_\rho')}{(W - W_\rho')(W - W_\rho')(W + W_\rho')(W + W_\rho')}.$$

If in a comparison of (3) with existing data masses and width of all vector-mesons under consideration are fixed at the table values, then other free parameters of the model acquire the following values:

$$\pi^0: \quad \chi^2/ndf = 0.79; \quad t_{in}^{I=0} = 0.9714 GeV^2; \quad t_{in}^{I=1} = 1.0198 GeV^2; \quad (6)$$

![Figure 1: $\pi^0$ transition form factor](image)

$$\left(f_{\omega\gamma}\pi^0/f_{\omega EM}\right) = 0.0120 \pm 0.0002; \left(f_{\phi\gamma}\pi^0/f_{\phi EM}\right) = -0.0002 \pm 0.0001;$$

$$\left(f_{\rho\gamma}\pi^0/f_{\rho EM}\right) = 0.0208 \pm 0.0006;$$

$$\eta: \quad \chi^2/ndf = 1.08; \quad t_{in}^{I=0} = 0.6081 GeV^2; \quad t_{in}^{I=1} = 0.6299 GeV^2; \quad (7)$$
Figure 2: $\eta$ transition form factor.

\[(f_{\omega \gamma \eta}/f_{\omega}^{EM}) = 0.0201 \pm 0.0020; (f_{\phi \gamma \eta}/f_{\phi}^{EM}) = -0.0013 \pm 0.0001; \]
\[(f_{\rho \gamma \eta}/f_{\rho}^{EM}) = 0.0119 \pm 0.0012\]

Figure 3: $\eta'$ transition form factor

\[\eta': \ \chi^2/ndf = 1.29; \ \ t^{I=0}_{in} = 1.0106 GeV^2; \ \ t^{I=1}_{in} = 0.9578 GeV^2; \]
\[(f_{\omega \gamma \eta'}/f_{\omega}^{EM}) = -0.1049 \pm 0.0011; (f_{\phi \gamma \eta'}/f_{\phi}^{EM}) = 0.0757 \pm 0.0017; \]
\[(f_{\rho \gamma \eta'}/f_{\rho}^{EM}) = 0.0859 \pm 0.0009\]

and a prediction of behaviours of the corresponding FF’s and their comparison with exiting data are graphically presented in Figs. 1-3.
3 Strange pseudoscalar-meson transition form factors

The strange-quark vector current pseudoscalar-meson transition FF’s \( F_{\gamma P}^s(t) \) can be defined analogically to (2) by the parametrization

\[
\langle P(p)\gamma(k)|J^s_\mu \rangle = \epsilon_{\mu\alpha\beta\gamma} p^\alpha k^\beta F_{\gamma P}^s(t)
\]

where \( J^s_\mu = \bar{s}\gamma_\mu s \) is the strange-quark vector current.

Since the isospin of the strange quark \( s \) is zero, then the strange-quark vector current pseudoscalar-meson transition FF’s \( F_{\gamma P}^s(t) \) can contribute only to the isoscalar parts of \( F_{\gamma P}^{EM}(t) \), from where it directly follows that \( F_{\gamma P}^s(t) \) are saturated (unlike \( F_{\gamma P}^{EM}(t) \)) only by isoscalar vector-mesons. However, since the total strangeness of \( P \) and \( \gamma \) is zero, then their normalizations take the form

\[
F_{\gamma P}^s(0) = 0.
\]

The asymptotic behaviours of the strange pseudoscalar-meson transition FF’s are

\[
F_{\gamma P}^s(t)_{|t|\to\infty} \sim t^{-3}
\]

as there are another two \( \bar{s}s \) quarks contributing to the structure of \( P \).

Analytic properties of \( F_{\gamma P}^s(t) \) are identical with analytic properties of \( F_{\gamma P}^{I=0}(t) \).

Taking into account all the abovementioned properties in a construction of the unitary and analytic models of \( F_{\gamma P}^s(t) \) we start with the corresponding VMD parametrization

\[
\bar{F}_{\gamma P}^s(t) = \sum_{i=\omega,\phi,\omega'} \frac{m_i^2}{m_i^2 - t} \left( f_{\gamma P}/f_i^s \right)
\]

where \( f_i^s \) is a coupling of the strangeness current to vector meson \( i=\omega, \phi, \omega' \) and we use the FF denotation \( \bar{F}_{\gamma P}^s(t) \) as it has still the VMD asymptotic behaviour.

Requirement of the normalization (10) leads to the expression

\[
\bar{F}_{\gamma P}^s(t) = \left[ \frac{m_\omega^2}{m_\omega^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t} \right] b_\omega + \left[ \frac{m_\phi^2}{m_\phi^2 - t} - \frac{m_{\omega'}^2}{m_{\omega'}^2 - t} \right] b_\phi.
\]
Then analogically to (3) the unitary and analytic model of $\tilde{F}_{\gamma P}^s(t)$ takes the form

$$\tilde{F}_{\gamma P}^s(t) = \left( \frac{1 - V^2}{1 - V_N^2} \right)^2,$$

but still with the VMD asymptotics. However, taking into account a change of the exponent in the asymptotic term

$$\left( \frac{1 - V^2}{1 - V_N^2} \right)^2 \rightarrow \left( \frac{1 - V^2}{1 - V_N^2} \right)^{2n}, \quad n = 1, 2, 3, ..$$

leading to the change of the asymptotic behavior

$$|t| \rightarrow \infty \sim t^{-1} \rightarrow |t| \rightarrow \infty \sim t^{-n}$$

of any unitary and analytic FF, one can multiply both sides of (14) by the factor

$$\left( \frac{1 - V^2}{1 - V_N^2} \right)^n$$

and redefine the FF

$$F_{\gamma P}^s(t) = \tilde{F}_{\gamma P}^s(t) \left( \frac{1 - V^2}{1 - V_N^2} \right)^4,$$

Figure 4: Strange $\pi^0$ transition form factor
in order to achieve the unitary and analytic model of $F^s_{\gamma P}(t)$ with the required asymptotic behaviour (11) and dependent only on unknown $b_\omega$ and $b_\phi$ to be determined by the relations (1) from the values of $a_\omega$, $a_\phi$ given by (6)-(8).

Now taking into account the numerical values (6)-(8) and utilizing relations (1) one gets for

$$
\pi_0 : \ (f_{\omega\gamma\pi_0}/f^s_\omega) = +0.0062; \quad (f_{\phi\gamma\pi_0}/f^s_\phi) = +0.0006; \quad (18)
$$

$$
\eta : \ (f_{\omega\gamma\eta}/f^s_\omega) = -0.0050; \quad (f_{\phi\gamma\eta}/f^s_\phi) = +0.0041;
$$

$$
\eta' : \ (f_{\omega\gamma\eta'}/f^s_\omega) = +0.0263; \quad (f_{\phi\gamma\eta'}/f^s_\phi) = -0.2386
$$

and a prediction of behaviours of the corresponding strange pseudoscalar-meson transition FF’s are graphically presented in Figs. 4-6.
4 Conclusions and discussion

The method of a behaviour of strange-quark vector current nucleon FF behaviours, which is interesting in relation to an experimental effort to confirm non-zero contributions of sea strange quark-antiquark pairs to the nucleon structure, is extended to pseudoscalar-meson transition FF’s. An explicit form of strange-quark vector current of pseudoscalar-meson transition FF’s is found by constructing unitary and analytic models dependent only on the $\omega$ and $\phi$ coupling constant ratios as only unknown parameters. Their numerical values are determined from the corresponding coupling constant ratios of the EM pseudoscalar-meson transition FF’s by employing the $\omega$-$\phi$ mixing and a special assumption on the coupling of the quark components of vector-meson wave functions to flavour components of quark-current under consideration.

However, we don’t know how to measure the strange pseudoscalar-meson transition FF’s.

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