Multi-Source Transmission for Wireless Relay Networks with Linear Complexity

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Abstract

This paper considers transmission schemes in multi-access relay networks (MARNs) where $J$ single-antenna sources send independent information to one $N$-antenna destination through one $M$-antenna relay. For complexity considerations, we propose a linear framework, where the relay linearly transforms its received signals to generate the forwarded signals without decoding and the destination uses its multi-antennas to fully decouple signals from different sources before decoding, by which the decoding complexity is linear in the number of sources. To achieve a high symbol rate, we first propose a scheme called Concurrent\textsubscript{S→R→D}-IC\textsubscript{D} in which all sources’ information streams are concurrently transmitted in both the source-relay link and the relay-destination link. In this scheme, distributed space-time coding (DSTC) is applied at the relay, which satisfies the linear constraint. DSTC also allows the destination to conduct the zero-forcing interference cancellation (IC) scheme originally proposed for multi-antenna systems to fully decouple signals from different sources. Our analysis shows that the symbol rate of Concurrent\textsubscript{S→R→D}-IC\textsubscript{D} is $1/2$ symbols/source/channel use and the diversity gain of the scheme is upperbounded by $M - J + 1$. To achieve a higher diversity gain, we propose another scheme called Concurrent\textsubscript{R→D}-IC\textsubscript{D} in which the sources time-share the source-relay link. The relay coherently combines the signals on its antennas to maximize the signal-to-noise ratio (SNR) of each source, then concurrently forwards all sources’ information. The destination performs zero-forcing IC. It is shown through both analysis and simulation that when $N \geq 2J - 1$, Concurrent\textsubscript{R→D}-IC\textsubscript{D} achieves the same maximum diversity gain as the full TDMA scheme in which the information stream from each source is assigned to an orthogonal channel in both links, but with a higher symbol rate.

Index Terms: Multi-access relay network, distributed space-time coding, interference cancellation, orthogonal and quasi-orthogonal designs, cooperative diversity.

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1 Introduction

Node cooperation improves the reliability and the capacity of wireless networks. Recently, many cooperative schemes have been proposed, and their multiplexing and diversity gains are analyzed [1–4]. However, most pioneer works in this area focus on cooperative relay designs without multi-user interference. It is assumed that there is a single transmission at a time or orthogonal channels are assigned to different transmissions, e.g. [1–3]. As a general network has multiple nodes each of which can be a data source, allocating an orthogonal channel to the information stream of each source is bandwidth inefficient. Therefore, concurrent transmission of information streams from multiple sources is desirable in cooperative networks to improve spectrum efficiency. Some examples on the design and performance analysis of multi-source transmission can be found in [5–8].

One model on multi-source transmission is the interference relay network [9]. Multiple parallel communication flows are supported by a common set of cooperative relays through two hops of transmission. Each source targets at one distinct destination. Two schemes using relays to resolve interference were discussed. The zero-forcing (ZF) relaying scheme designs scalar gain factors at single-antenna relays to null out interference at undesired destinations [10–12]. The minimum mean square error (MMSE) relaying scheme designs scalar gain factors to minimize interference-plus-noise power at undesired destinations [13,14]. However, both relaying schemes assume that the gain factors are first calculated at one centralized node having perfect and global channel state information (CSI), then fed back to the relays. While papers [10–14] discuss the multiplexing gain and designs of the optimal scalar gain factors, they do not provide diversity analysis. An interference relay network with multi-antenna nodes was discussed in [15], in which the authors used maximum-ratio-combining (MRC), ZF, and MMSE relaying schemes and analyzed the power-bandwidth trade-off of the network.

Another model on multi-source cooperative communication considers the scenario where several sources target at one multi-antenna destination with the help of one multi-antenna relay. The network is called multi-access relay network (MARN) [16]. We use the notation $1_J \times M_1 \times N_1$ to represent the MARN with $J$ single-antenna sources, one $M$-antenna relay, and one $N$-antenna destination. For the MARN, the source-relay link is a multi-access channel (MAC) and the relay-destination link is a point-to-point multiple input multiple output (MIMO) channel. The MARN is thus essentially a serial concatenation of the MAC and the MIMO. Both links have the potential for multi-source concurrent transmission, i.e., information streams from different sources can be simultaneously transmitted on the same channel. An intuitive scheme is to allow information streams from different sources concurrently transmitted in both links and jointly decode all sources’ information at the relay and the destination. Single source transmission schemes, e.g., distributed space time code (DSTC), can be applied straightforwardly following this idea by treating signals from different sources jointly as a higher dimension signal vector. It can be shown that this scheme achieves a symbol rate of $1/2$ symbols/source/channel.
use and the maximum diversity gain of $M$. However, with such a scheme, the decoding complexities at the relay and the destination are exponential in the number of sources, thus may become infeasible for networks with a large number of sources. For complexity considerations, we propose a linear framework for MARNs. The relay linearly transforms its received signals to generate the forwarded signals without decoding. The destination separates signals from different sources before the ML decoding of each source’s information. The decoding complexity at the destination is hence linear in the number of sources. To the best of our knowledge, MARNs with this linear framework have not been explicitly discussed in the literature. It is noteworthy that this linear framework may constrain the network optimality in some performance measures.

For single-source two-hop cooperative networks, DSTC can achieve the maximum diversity gain without any CSI at the relay [17]. For the multi-source scenario, one can use DSTC at the relay and assign the information stream of each source an orthogonal channel in both links. This scheme is denoted as TDMA$_{S \rightarrow R \rightarrow D}$, whose achievable diversity gain is $M$ for $1_J \times M_1 \times N_1$ MARNs [18]. Since interference is avoided in both links, we call this maximum diversity gain the interference-free (int-free) diversity gain. It provides a natural upperbound on the diversity gain for any multi-source transmission scheme that allows concurrent transmission of information streams from different sources. However, TDMA$_{S \rightarrow R \rightarrow D}$ has low spectrum efficiency when the number of sources is large. In [16], we proposed a multi-source transmission scheme called IC-Relay-TDMA, in which concurrent multi-source transmission is allowed in the source-relay link only. The relay, knowing the source-relay channel, performs linear interference cancellation (IC) [19–21] to decouple signals from different sources. In the relay-destination link, the relay forwards information of different sources to the destination using TDMA. To adopt the same naming system, this scheme is denoted as Concurrent$_{S \rightarrow R \rightarrow D}$-IC, instead in this paper. For a $L_J \times M_1 \times N_1$ MARN, Concurrent$_{S \rightarrow R \rightarrow D}$-IC achieves the maximum int-free diversity when $N \leq L \left(1 - \frac{J}{M} \right)$ [16]. For the MARN considered in this paper, i.e., each source has only one antenna, Concurrent$_{S \rightarrow R \rightarrow D}$-IC only achieves a diversity gain of $M - J + 1$, hence cannot achieve the maximum int-free diversity gain. Also, the TDMA method in the relay-destination link limits the symbol rate of the network.

The Concurrent$_{S \rightarrow R}$-IC$_{R}$ scheme uses the relay to remove interference from different sources. For the considered MARN, the multi-antenna destination also has the capability of IC. In this paper, we propose two schemes in which IC is conducted at the destination rather than the relay. This is desirable for networks with powerful destinations such as the uplink of cellular systems. The first protocol allows information streams from different sources simultaneously transmitted in both links. The relay conducts DSTC to linearly transform its received signals without decoding. The destination performs IC to separate signals from different sources. Hence, this protocol is called Concurrent$_{S \rightarrow R \rightarrow D}$-IC$_{D}$. For the second protocol, the sources time-share the source-relay link. The relay obtains soft-estimates of the symbols from each source by MRC, encodes soft-estimates of each source by one DSTC, then concurrently forwards all sources’ DSTCs. The destination performs IC to decouple
signals from different sources. Since information streams of different sources are simultaneously transmitted in the relay-destination link only, we call this protocol $\text{Concurrent}_{R \rightarrow D} \cdot \text{IC}_{D}$. A brief comparison of the proposed protocols with TDMAT\textsc{s→r→d} and $\text{Concurrent}_{S \rightarrow R} \cdot \text{IC}_{R}$ in symbol rate, diversity gain, and CSI requirements is illustrated in Table 1. Contributions of the proposed protocols are summarized as follows.

1. The proposed protocols fit the linear framework: linear processing without decoding at the relay and linear decoding complexity in the number of sources at the destination. Furthermore, they are applicable to the interference relay network.

2. CSI feedback, which is necessary for ZF and MMSE relaying schemes \cite{10,11,13,14}, is not required for the protocols proposed in this paper.

3. We perform rigorous analysis on the diversity gain of the proposed protocols, which to the best of our knowledge, is not provided for related work on multi-source cooperative networks.

4. Concurrent $S \rightarrow R \rightarrow D \cdot \text{IC}_{D}$ achieves a symbol rate of $1/2$ symbols/source/channel use, the highest among the linear schemes in Table 1. Since the symbol rate of each source is independent of the number of sources, the throughput of the network grows linearly with the number of sources without increasing the bandwidth. With rigorous analysis, the diversity gain is shown to be upperbounded by $M - J + 1$.

5. Concurrent $R \rightarrow D \cdot \text{IC}_{D}$ achieves a symbol rate of $\frac{1}{M + T}$ symbols/source/channel use in conjunction with a diversity gain of $\min\{M, \lfloor \frac{M}{J} \rfloor (N - J + 1)\}$ ($\lfloor x \rfloor$ denotes the maximum integer not greater than $x$). When $N \geq 2J - 1$, Concurrent $R \rightarrow D \cdot \text{IC}_{D}$ achieves the maximum int-free diversity gain. Compared with TDMAT\textsc{s→r→d}, it has a higher symbol rate with no penalty on the diversity gain for networks satisfying $N \geq 2J - 1$. Compared with Concurrent $S \rightarrow R \cdot \text{IC}_{R}$ \cite{16}, it has the same symbol rate but has advantage in the diversity gain for the $(1J \times M)$ MARN.

The rest of the paper is organized as follows. Section 2 introduces the network model. Section 3 presents Concurrent $S \rightarrow R \rightarrow D \cdot \text{IC}_{D}$ and analyzes its diversity gain. In Section 4 Concurrent $R \rightarrow D \cdot \text{IC}_{D}$ is proposed and its performance is studied. Section 5 provides the numerical results. Conclusions are given in Section 6. Involved proofs are presented in appendices.

Notation: For a matrix $A$, denote its $(i, j)$th entry as $a_{ij}$. $A^t$, $A^*$, and $\overline{A}$ are the transpose, Hermitian, and conjugate of $A$, respectively. $\|A\|$ is the Frobenius norm of $A$. For two matrices $A$ and $B$ of the same dimension, $A \succ B$ means that $A - B$ is positive definite. $I_n$ is the $n \times n$ identity matrix. $0_{m \times n}$ is the $m \times n$ matrix of all zeros. When $m = n$, $0_{n \times n}$ is simplified as $0_n$. $f(x) = o(x)$ means $\lim_{x \rightarrow 0^+} \frac{f(x)}{x} = 0$. $E[x]$ denotes the expected value of the random variable $x$. 
2 Network Model

Consider a MARN with \( J \) single-antenna sources, one \( M \)-antenna relay, and one \( N \)-antenna destination, where there is no direct connection from the sources to the destination. This MARN is denoted as a \( 1_J \times M_1 \times N_1 \) MARN. We further assume that both the numbers of relay antennas and destination antennas are no less than the number of sources, i.e., \( J \leq \min\{M, N\} \). This condition is to guarantee full IC at the destination, the details of which will be shown later. The condition can be realized by user admission control in the upper-layers. We assume that both the relay and the destination know the value of \( J \).

Denote the channel coefficient from Source \( j \) (\( j = 1, \ldots, J \)) to the \( i \)-th (\( i = 1, \ldots, M \)) relay antenna as \( f^{(j)}_i \), and the channel coefficient from the \( i \)-th relay antenna to the \( n \)-th (\( n = 1, \ldots, N \)) destination antenna as \( g_{in} \). Assume that all channel coefficients are i.i.d. circularly symmetric \( \mathcal{CN}(0, 1) \) distributed. In addition, we assume a block-fading model with coherent interval \( T \). The noises at each relay antenna and destination antenna are modeled as additive white Gaussian noise (AWGN) with zero mean and unit power. Throughout the paper, we assume global and perfect CSI at the destination. The CSI requirement at the relay depends on the scheme. In Section 3, the proposed protocol does not need any CSI at the relay; in Section 4, the relay needs only backward CSI, i.e., the channel information from all sources to itself. The required backward CSI can be acquired by training [18, 22]. No CSI feedback is required for either protocol. To focus on the diversity gain performance, we assume that all sources and the relay have the same average power constraint. Further, all nodes are assumed to be perfectly synchronized at the symbol level.

3 The Protocol of Concurrent\( S \rightarrow R \rightarrow D \)-IC\( D \)

In this section, we propose a protocol that allows concurrent transmission of information streams from different sources in both the source-relay link and the relay-destination link to achieve the symbol rate of \( 1/2 \) symbols/source/channel use. The protocol is thus called Concurrent\( S \rightarrow R \rightarrow D \)-IC\( D \). Based on the linear framework introduced in Section 1, we need to design the linear signal processing at the relay and the destination. Since DSTC requires only a linear transformation at the relay and achieves the maximum diversity gain in single-source relay networks [2, 17], we propose to use DSTC for the MARN to gain protection against channel fading. At the destination, the IC method [19, 21], originally proposed for multi-user MAC to decouple interfering signals [23], is used to separate information of different sources. In Subsection 3.1, we describe the details of the protocol. Subsection 3.2 provides the diversity gain analysis. Subsection 3.3 contains the discussion on the condition for full IC at the destination and the symbol rate.
3.1 Protocol Description

We first describe Concurrents→R→D-IC\(_D\) in the 1\(_2\) × 2\(_1\) × N\(_1\) MARN with two single-antenna sources, one double-antenna relay, and one N-antenna destination; then consider the 1\(_2\) × 4\(_1\) × N\(_1\) MARN followed by its generalization to 1\(_J\) × M\(_1\) × N\(_1\) MARNs.

3.1.1 Concurrents→R→D-IC\(_D\) for the 1\(_2\) × 2\(_1\) × N\(_1\) MARN

The protocol of Concurrents→R→D-IC\(_D\) consists of two steps as shown in Fig. 1. During the first step, each source collects two symbols \(s_1^{(j)}\) and \(s_2^{(j)}\) independently and uniformly from the constellation \(S\). Source \(j\) transmits a vector of two symbols \(\mathbf{x}^{(j)} = [s_1^{(j)} \ s_2^{(j)}]^t\) and both sources transmit concurrently. The received signal vector at the \(i\)-th relay antenna can be expressed as

\[
\mathbf{r}_i = \sqrt{P} \mathbf{x}^{(1)} f_i^{(1)} + \sqrt{P} \mathbf{x}^{(2)} f_i^{(2)} + \mathbf{v}_i,
\]

where \(\mathbf{v}_i\) denotes the 2 × 1 AWGN vector at the \(i\)-th relay antenna. The relay uses Alamouti DSTC \([17]\) to generate its output signal vector at the \(i\)-th antenna as,

\[
\mathbf{t}_i = \sqrt{\frac{P}{4P + 2}} (\mathbf{A}_i \mathbf{r}_i + \mathbf{B}_i \mathbf{r}_j), \quad i = 1, 2,
\]

where \(\sqrt{\frac{P}{4P + 2}}\) normalizes the average power at the relay to \(P\) and \(\mathbf{A}_i\) and \(\mathbf{B}_i\) are the 2 × 2 encoding matrices based on Alamouti design \([24]\):

\[
\mathbf{A}_1 = \mathbf{I}_2, \quad \mathbf{B}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{A}_2 = \mathbf{B}_1 = 0_2.
\]

From (2), the output vector \(\mathbf{t}_i\) is a linear transformation of the input vector \(\mathbf{r}_i\), i.e., the relay signal processing is a linear transformation. Since this linear transformation is independent of the channels, the relay does not need any CSI.

During the second step, the relay transmits \(\mathbf{t}_i\) from its \(i\)-th antenna, and \(\mathbf{t}_1\) and \(\mathbf{t}_2\) are concurrently transmitted. Denote the sampled signal at the \(n\)-th antenna of the destination and time slot \(\tau\) as \(x_{\tau n}\). Using the special structure of the Alamouti design, an equivalent system can be obtained as

\[
\begin{bmatrix} x_{1n} \\ x_{2n} \end{bmatrix} = \sqrt{\frac{P^2}{4P + 2}} \sum_{j=1,2} \begin{bmatrix} f_1^{(j)} g_{1n} - f_2^{(j)} g_{2n} \\ f_2^{(j)} g_{2n} f_1^{(j)} g_{1n} \end{bmatrix} \begin{bmatrix} s_1^{(j)} \\ s_2^{(j)} \end{bmatrix} + \sqrt{\frac{P}{4P + 2}} \begin{bmatrix} g_{1n} v_{11} \\ g_{1n} v_{21} \end{bmatrix} + \begin{bmatrix} -g_{2n} v_{12} \\ g_{2n} v_{12} \end{bmatrix} + \begin{bmatrix} w_{1n} \\ w_{2n} \end{bmatrix},
\]

where \(w_{\tau n}\) denotes the AWGN at the \(n\)-th destination antenna and time slot \(\tau\) and \(\mathbf{u}_n\) denotes the equivalent noise vector at the \(n\)-th antenna of the destination. The 2 × 2 equivalent channel matrix \(\mathbf{H}_{\tau n}^{(j)}\) for Source \(j\) has Alamouti structure, i.e.,

\[
\mathbf{H}_{\tau n}^{(j)} = \begin{bmatrix} |f_1^{(j)} g_{1n}|^2 & |f_2^{(j)} g_{2n}|^2 \end{bmatrix} \mathbf{I}_2.
\]
Note that the equivalent system equation in (4) is similar to that of a MAC with two double-antenna users except that the noise vector is correlated. Using the IC techniques proposed for MAC in [20], the destination can fully decouple the information streams from different sources and separately decode the information of each source. Without loss of generality, we discuss how the destination decodes the information of Source 1.

To cancel the symbols of Source 2, the destination calculates \( \hat{x}_n = \frac{2H^{(2)*}}{\|H_n^{(2)}\|^2} \hat{x}_n - \frac{2H^{(2)*}}{\|H_n^{(N)}\|^2} \hat{x}_N \) for \( n = 1, \ldots, N-1 \). Define \( \hat{x} = [\hat{x}^*_1, \ldots, \hat{x}^*_N]^t \), which is a \( 2N \times 1 \) vector, and \( X = [\hat{x}^*_1, \ldots, \hat{x}^*_{N-1}]^t \), which is a \( (2N-2) \times 1 \) vector. The IC process can be represented in a matrix form as

\[
\hat{X} = B\hat{x} = \sqrt{\frac{P^2}{4P + 2}} BH_1 \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} + Bu_n,
\]

(5)

where the \( (2N-2) \times 2N \) matrix \( B \) is the IC matrix, the \( 2N \times 2 \) matrix \( H_1 \) denotes the equivalent channel matrix for Source 1, and the \( (2N-2) \times 1 \) vector \( n \) denotes the remaining equivalent noise vector. \( B, H_1, \) and \( u \) are given as

\[
B = \begin{bmatrix} \frac{2H^{(2)*}}{\|H_n^{(2)}\|^2} & 0_2 & \cdots & 0_2 & -\frac{2H^{(2)*}}{\|H_n^{(N)}\|^2} \\
0_2 & \frac{2H^{(2)*}}{\|H_n^{(2)}\|^2} & \cdots & 0_2 & -\frac{2H^{(2)*}}{\|H_n^{(N)}\|^2} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0_2 & \cdots & \cdots & \frac{2H^{(2)*}}{\|H_n^{(2)}\|^2} & -\frac{2H^{(2)*}}{\|H_n^{(N)}\|^2} \end{bmatrix},
H_1 = \begin{bmatrix} H_1^{(1)} \\ H_2^{(1)} \\ \vdots \\ H_{N-1}^{(1)} \end{bmatrix},
u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}.
\]

(6)

It can be shown that the equivalent noise vector \( n \) is Gaussian but not white. With straightforward calculation, the \( (2N-2) \times (2N-2) \) covariance matrix of \( n \) can be obtained as

\[
R_n = \frac{P}{4P + 2} B\hat{G}^*B^* + BB^*,
\]

(7)

where

\[
\hat{G} = \begin{bmatrix} \hat{G}_1^t & \cdots & \hat{G}_N^t \end{bmatrix}^t, \quad \hat{G}_n = \begin{bmatrix} g_{1n} & 0 & g_{2n} & 0 \\ 0 & \tilde{y}_{1n} & 0 & \tilde{y}_{2n} \end{bmatrix}.
\]

(8)

Based on (5), Source 1’s information can be recovered using the maximum-likelihood (ML) decoding rule

\[
\arg \min_{s_1^{(1)}, s_2^{(1)}} \left( X - \sqrt{\frac{P^2}{4P + 2}} BH_1 \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} \right)^* R_n^{-1} \left( X - \sqrt{\frac{P^2}{4P + 2}} BH_1 \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} \right).
\]

(9)

Next, we show that the ML decoding in (9) can be decoupled into two symbol-wise ML decodings. It suffices to show that \( H_1^*B^*R_n^{-1}BH_1 \) is a diagonal matrix. Notice that Alamouti structure [24] is closed under matrix addition, matrix multiplication, and scalar multiplication. Since the \( 2 \times 2 \) submatrices in \( B, H_1, \) and \( \hat{G} \) have Alamouti structure from (6) and (8), the matrix \( H_1^*B^*R_n^{-1}BH_1 \) also has Alamouti structure in addition to being Hermitian. Generally, it can be shown that any Hermitian Alamouti matrix is diagonal with equal
diagonal entries. Therefore, $H_1^*B^*R_n^{-1}Bh$ is diagonal. The ML decoding in (9) can be decomposed to two procedures of symbol-wise decoding as

$$
\text{arg max}_{s_i^{(1)}} 2\text{Re} \left( h_1^*B^*R_n^{-1}x_s^{(1)} \right) - \sqrt{\frac{P^2}{4P + 2}} h_1^*B^*R_n^{-1}Bh |s_i^{(1)}|^2,
$$

$$
\text{arg max}_{s_i^{(2)}} 2\text{Re} \left( h_2^*B^*R_n^{-1}x_s^{(2)} \right) - \sqrt{\frac{P^2}{4P + 2}} h_2^*B^*R_n^{-1}Bh |s_i^{(2)}|^2,
$$

where $h_i$ denotes the $i$-th column of $H_1$. Similarly, the destination can cancel the symbols of Source 1 and decode the information of Source 2. Four procedures of symbol-wise ML decoding are needed in total to decode both sources’ information.

### 3.1.2 Concurrent $S \rightarrow R \rightarrow D$-ICD for the $1_J \times 4_1 \times N_1$ MARN

This subsection describes Concurrent $S \rightarrow R \rightarrow D$-ICD in the MARN with four relay antennas and $J$ sources. During the first step, Source $j$ transmits a $4 \times 1$ vector consisting of four symbols, i.e., $[s_1^{(j)} s_2^{(j)} s_3^{(j)} s_4^{(j)}]^T$, and all sources transmit concurrently. The $i$-th relay antenna receives a $4 \times 1$ vector $r_i$. The relay performs DSTC with quasi-orthogonal design [17]. The $4 \times 1$ forwarded vector $t_i$ is generated as $t_i = c(A_1r_i + B_1r_i)$, where $c = \sqrt{\frac{P}{4JP+1}}$ is to constrain the power of the relay to $P$; $A_i$ and $B_i$ are DSTC encoding matrices with quasi-orthogonal design [17][24]:

$$
A_1 = I_4, \ A_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \ B_4 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ A_2 = A_3 = B_1 = B_4 = 0_4.
$$

During the second step, the relay concurrently forwards $t_i$. The received signal at the $n$-th destination antenna can be written as

$$
\begin{bmatrix} x_{1n} \\ x_{2n} \\ x_{3n} \\ x_{4n} \end{bmatrix} = \sqrt{\bar{P}c} \sum_{j=1:J} \begin{bmatrix} s_1^{(j)} \\ -s_2^{(j)} \\ -s_3^{(j)} \\ s_4^{(j)} \end{bmatrix} \begin{bmatrix} f_1^{(j)} \\ f_2^{(j)} \\ f_3^{(j)} \\ f_4^{(j)} \end{bmatrix} g_1n + c \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} v_{21} \\ v_{22} \\ v_{23} \\ v_{24} \end{bmatrix} \begin{bmatrix} g_{2n} \\ g_{3n} \\ g_{4n} \\ w_{4n} \end{bmatrix} + \begin{bmatrix} v_{31} \\ v_{32} \\ v_{33} \\ v_{34} \end{bmatrix} \begin{bmatrix} g_{2n} \\ g_{3n} \\ g_{4n} \\ w_{4n} \end{bmatrix},
$$

where $x_{\tau n}$ denotes the sampled signal at the $n$-th destination antenna and time slot $\tau$. It can be observed that $S^{(j)}$ has quasi-orthogonal structure due to the DSTC at the relay. Using the IC techniques in [21], we can break
the system into two equivalent Alamouti systems as
\[
\begin{bmatrix}
    x_{1n} + x_{4n} \\
    x_{2n} + x_{3n}
\end{bmatrix} = \sqrt{P_c} \sum_{j=1}^{J} \begin{bmatrix}
    f_{1j} g_{1n} + f_{4j} g_{4n} \\
    f_{2j} g_{2n} - f_{3j} g_{3n}
\end{bmatrix} \begin{bmatrix}
    s_{1j} + s_{4j} \\
    s_{3j} - s_{2j}
\end{bmatrix} + u_n^+ \quad (11)
\]
\[
\begin{bmatrix}
    x_{1n} - x_{4n} \\
    x_{2n} + x_{3n}
\end{bmatrix} = \sqrt{P_c} \sum_{j=1}^{J} \begin{bmatrix}
    f_{1j} g_{1n} - f_{4j} g_{4n} \\
    f_{2j} g_{2n} + f_{3j} g_{3n}
\end{bmatrix} \begin{bmatrix}
    s_{1j} - s_{4j} \\
    -s_{3j} - s_{2j}
\end{bmatrix} + u_n^- \quad (12)
\]
where \( u_n^+ \) and \( u_n^- \) denote the equivalent noise vector for each system. They have the following expressions:
\[
\begin{align*}
    u_n^+ &= c \begin{bmatrix}
        \frac{(v_{11} + v_{41})g_{11}}{v_{21} - v_{31}} \\
        \frac{(v_{21} + v_{31})g_{11}}{v_{21} - v_{31}}
    \end{bmatrix} + \begin{bmatrix}
        \frac{(-v_{32} + v_{42})g_{32}}{v_{21} - v_{31}} \\
        \frac{(-v_{12} + v_{42})g_{42}}{v_{21} - v_{31}}
    \end{bmatrix} + \begin{bmatrix}
        \frac{(-v_{43} + v_{23})g_{33}}{v_{21} - v_{31}} \\
        \frac{(-v_{44} - v_{14})g_{43}}{v_{21} - v_{31}}
    \end{bmatrix} + \begin{bmatrix}
        \frac{(v_{44} + v_{14})g_{13}}{v_{21} - v_{31}} \\
        \frac{(-v_{34} + v_{24})g_{43}}{v_{21} - v_{31}}
    \end{bmatrix} + \begin{bmatrix}
        \frac{w_{1n} + w_{4n}}{w_{2n} - w_{3n}} \\
        \frac{w_{1n} - w_{4n}}{w_{2n} - w_{3n}}
    \end{bmatrix}
\end{align*}
\]
\[
\begin{align*}
    u_n^- &= c \begin{bmatrix}
        \frac{(v_{11} - v_{41})g_{11}}{v_{21} + v_{31}} \\
        \frac{(v_{21} + v_{31})g_{11}}{v_{21} + v_{31}}
    \end{bmatrix} + \begin{bmatrix}
        \frac{(-v_{22} - v_{32})g_{22}}{v_{21} + v_{31}} \\
        \frac{(-v_{12} - v_{42})g_{42}}{v_{21} + v_{31}}
    \end{bmatrix} + \begin{bmatrix}
        \frac{(-v_{43} - v_{23})g_{33}}{v_{21} + v_{31}} \\
        \frac{(-v_{44} - v_{14})g_{43}}{v_{21} + v_{31}}
    \end{bmatrix} + \begin{bmatrix}
        \frac{(v_{44} + v_{14})g_{13}}{v_{21} + v_{31}} \\
        \frac{(-v_{34} + v_{24})g_{43}}{v_{21} + v_{31}}
    \end{bmatrix} + \begin{bmatrix}
        \frac{w_{1n} - w_{4n}}{w_{2n} + w_{3n}} \\
        \frac{w_{1n} + w_{4n}}{w_{2n} + w_{3n}}
    \end{bmatrix}
\end{align*}
\]

The destination uses \( J - 1 \) antennas to cancel the symbols of \( J - 1 \) interfering sources by the multi-user IC technique in [20] for each Alamouti system in (11) and (12), thus decouple information streams from different sources. The destination then recovers information of each source separately using the ML decoding. Since the detailed formulas can be found in [21], we do not repeat the IC procedure here.

### 3.1.3 Concurrent_{S→R→D}-IC_{D} for \( 1_j \times M_1 \times N_1 \) MARNs

To use the protocol in MARNs with a general \( M \), each source transmits a vector of \( 2^n \) symbols, with \( 2^n \) the minimum number that is no less than \( M \). The relay designs the DSTC using the first \( M \) columns of a \( 2^n \times 2^n \) quasi-orthogonal space-time block code (STBC) with ABBA structure [25] [26]. The destination separates the system into \( 2^{n-1} \) Alamouti systems and decouples the signals from different sources using the IC procedure in [16][21]. Each source’s information can be decoded separately at the destination. The decoding complexity is thus linear in the number of sources.

### 3.2 Diversity Gain Analysis

In this subsection, we analyze the diversity gain of Concurrent_{S→R→D}-IC_{D}. Due to the concatenation of the channels, a direct diversity analysis from the system equation [5] is challenging. Instead, we work on an equivalent representation for the tractability of the analysis. The equivalent representation captures the effect of the IC at the destination to the first step of transmission. For Concurrent_{S→R→D}-IC_{D}, although the ZF IC procedure is conducted at the destination, there is a virtual ZF at the relay and a dimension reduction filtering at the destination. We first derive this system representation in Subsection 3.2.1 then prove an upperbound of the diversity gain on Concurrent_{S→R→D}-IC_{D} in Subsection 3.2.2.
3.2.1 An equivalent representation for Concurrent\textsubscript{S→R→D-IC\textsubscript{D}} in $1_J \times M_1 \times N_1$ MARNs

Since the network parameters and the processing at the relay and the destination are statistically equivalent for all sources, the diversity gains of all sources are identical. Thus, we only focus on Source 1.

For the simplicity of the presentation, we first look at the system equation of the $1_2 \times 2_1 \times N_1$ MARN in (5). Notice that each entry in the channel matrix $\mathbf{B}\mathbf{H}_1$ is a rational function of the channel coefficients of both links. Then, the entries are neither independent nor Gaussian. This complicates the diversity gain analysis. In the following, we derive an equivalent system representation to decouple the channel concatenation, which will help the diversity gain analysis. The system equation in (5) can be rewritten as

$$\mathbf{X} = \sqrt{\frac{P^2}{4P + 2}} \mathbf{B}\hat{\mathbf{G}}\mathbf{F}^{(1)} \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} + \mathbf{n},$$

(13)

where $\hat{\mathbf{G}}$ is defined in (8) and $\mathbf{F}^{(j)} \triangleq \begin{bmatrix} f_1^{(j)} & 0 & 0 & f_2^{(j)} \\ 0 & f_1^{(j)} & f_2^{(j)} & 0 \end{bmatrix}^t$. Note that the IC matrix $\mathbf{B}$ zero-forces the channels of Source 2, i.e., $\mathbf{B}\mathbf{G}\mathbf{F}^{(2)} = 0$. In other words, $\hat{\mathbf{G}}$ nulls out $\mathbf{F}^{(2)}$. Then, the rows of $\hat{\mathbf{G}}$ are in the null space of the column space of $\mathbf{F}^{(2)}$. Therefore, the channel matrix in (13) is invariant if $\mathbf{F}^{(1)}$ is first projected onto the null space of $\mathbf{F}^{(2)}$, i.e., $\hat{\mathbf{G}}\mathbf{F}^{(1)} = \hat{\mathbf{G}}\Phi\mathbf{F}^{(1)}$, where $\Phi$ is the projection matrix to the null space of $\mathbf{F}^{(2)}$, i.e.,

$$\Phi = \mathbf{I}_4 - \frac{2\mathbf{F}^{(2)}\mathbf{F}^{(2)*}}{\text{tr} (\mathbf{F}^{(2)}\mathbf{F}^{(2)*})}.$$  

Thus, (13) can be rewritten as

$$\mathbf{X} = \sqrt{\frac{P^2}{4P + 2}} \hat{\mathbf{G}}\Phi\mathbf{F}^{(1)} \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} + \mathbf{n},$$

(14)

This new system equation can be interpreted as follows. Symbols $s_1^{(1)}$ and $s_2^{(1)}$ are first transmitted through channel $\mathbf{F}^{(1)}$ to the relay. Then, ZF operation $\Phi$ is conducted to null out the information of Source 2. After that, signals are forwarded through channel $\hat{\mathbf{G}}$ and the destination applies a filter $\mathbf{B}$ to reduce the dimension of the received signal vector from $2N \times 1$ to $2(N - 1) \times 1$. The virtual ZF at the relay and the dimension reduction at the destination are due to the ZF IC at the destination. A diagram illustrating this process is shown in Fig. 2.

For general $J$ and $M$, with the same argument, the ZF at the destination induces a virtual ZF at the relay followed by a dimension reduction at the destination. It can be shown that the dimensions of $\mathbf{F}^{(1)}, \hat{\mathbf{G}}, \mathbf{B}$ are $2^{n+1} \times 2^n$, $2^n N \times 2^n+1$, and $2^n (N - J + 1) \times 2^n N$, respectively, where $2^n$ is the minimum number no less than $M$. The virtual ZF operation at the relay nulls out the information from Sources 2 to $J$. The dimension reduction filter $\mathbf{B}$ decreases the dimension of the received signal vector from $2^n N \times 1$ to $2^n (N - J + 1) \times 1$. Although the new system representation looks more complicated than the original one, it simplifies the diversity gain analysis.
3.2.2 Diversity gain upperbound

Diversity gain is defined as the negative of the asymptotic slope of the bit error rate (BER) with respect to the average transmit SNR in the high SNR regime. In [27], it is shown that for a communication system represented by the equation \( y = hs + n \) where \( h \), \( s \), and \( n \) are the channel vector, scalar symbol, and noise vector, respectively, diversity gain can be calculated using the outage probability of the instantaneous normalized receive SNR \( \gamma \) as

\[
\frac{d}{\epsilon} = \lim_{\epsilon \to 0^+} \frac{\log P(\gamma < \epsilon)}{\log \epsilon},
\] (15)

where the instantaneous normalized receive SNR is defined as \( \gamma = h^*R_n^{-1}h \) and \( R_n \) is the covariance matrix of \( n \). This technique is usually easier than the direct calculation based on the error rate, thus is used in this paper to analyze the diversity gain of Concurrent\(_{S-R-D}\)-IC\(_D\). Based on the equivalent system equation in [14], the following theorem is proved.

**Theorem 1.** In \( 1, J \times M_1 \times N_1 \) MARNs, the diversity gain of Concurrent\(_{S-R-D}\)-IC\(_D\) is upperbounded by \( M - J + 1 \).

**Proof.** See Appendix A. 

Theorem 1 can be intuitively explained as follows. Since the first step transmission is a MAC with an \( M \)-antenna receiver and the virtual ZF operation at the relay requires the use of \( J - 1 \) antennas to null out the information of \( J - 1 \) sources, the diversity gain achievable after the virtual ZF is no higher than \( M - J + 1 \). The second step transmission and the dimension reduction at the destination cannot improve the diversity gain of the first step. Therefore, the protocol has at most a diversity gain of \( M - J + 1 \). When \( J = 2 \) and \( M = 2 \), the following diversity result can be obtained as a special case of Theorem 1.

**Corollary 1.** In the \( 1, 2 \times 2_1 \times N_1 \) MARN, the diversity gain of Concurrent\(_{S-R-D}\)-IC\(_D\) is upperbounded by 1.

3.3 Discussion

In this subsection, we discuss the properties of Concurrent\(_{S-R-D}\)-IC\(_D\), including the condition on the network parameters for full IC, the symbol rate, and its comparison with existing schemes. Finally, we present its possible applications in the interference relay network.

First we consider the condition on the network parameters to achieve full IC. From the IC procedure in [21], at least \( J \) receive antennas are required to decouple signals from \( J \) source. Thus, Concurrent\(_{S-R-D}\)-IC\(_D\) requires the number of destination antennas to be no less than the number of sources. In addition, a condition on the number of relay antennas is required. To show this, we start with the example in the \( 1, 2 \times 2_1 \times N_1 \) MARN.
From \([14]\), the equivalent channel vector experienced by \(s_1^{(1)}\) in the source-relay link is the first column of \(F^{(1)}\), i.e., \([f_1^{(1)} 0 0 f_2^{(1)}]^t\), which is a \(4 \times 1\) vector in a 2-dimension subspace. It is discussed in Subsection 3.2.1 that the IC operation at the destination creates a virtual ZF operation at the relay. Then, the equivalent channel vector at the relay can be projected onto the null space of the equivalent channel vector of at most one interfering source. In other words, the virtual ZF operation at the relay can null out interference from at most one source and the network can allow at most two sources to transmit simultaneously. In general \(1_J \times M_1 \times N_1\) MARNs, the equivalent channel vector at the relay is a \(2^n + 1 \times 1\) vector in a \(M\)-dimension subspace. The virtual ZF operation at the relay can null out at most \(M - 1\) information streams from interfering sources. Thus, the number of relay antennas also needs to be no less than the number of sources. With Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\), the \(1_J \times M_1 \times N_1\) MARN admits at most \(\min\{M, N\}\) sources to concurrently transmit information.

Now we discuss the symbol rate of the scheme. The multi-source transmission in Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\) improves the spectrum efficiency of the network. In the first step, each source sends a vector of \(T = 2^n\) symbols in \(T\) channel uses where \(2^n\) is the minimum number no less than \(M\). Using DSTC with quasi-orthogonal design at the relay \([17]\), another \(T\) channel uses are required for the second step. Overall, \(2T\) channel uses are required to send \(T\) symbols from end to end. Thus, the symbol rate is \(1/2\) symbols/source/channel use, which is independent of the number of sources.

Next, we compare the diversity gain, symbol rate, and CSI requirements at the relay of Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\) with two existing schemes: TDMA\(S \rightarrow R \rightarrow D\) and Concurrent\(S \rightarrow R \rightarrow IC\(R\) [16], that also fit the linear framework. The results are shown in Table 1. Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\) achieves a higher symbol rate compared to Concurrent\(S \rightarrow R \rightarrow IC\(R\) and TDMA\(S \rightarrow R \rightarrow D\). For Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\), the throughput of the network grows linearly with the number of sources; while for the other two, the throughput of the network is \(\frac{J}{1 + J}\) for Concurrent\(S \rightarrow R \rightarrow IC\(R\) and \(1/2\) for TDMA\(S \rightarrow R \rightarrow D\), which has an upper bound when \(J\) grows large. However, from Theorem 1, the diversity gain of Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\) is upper bounded by \(M - J + 1\), thus is inferior to TDMA\(S \rightarrow R \rightarrow D\) and no better than Concurrent\(S \rightarrow R \rightarrow IC\(R\). For Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\), diversity gain degradation is necessary to trade for a higher symbol rate. Regarding CSI, the relay does not need any channel information for Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\) or TDMA\(S \rightarrow R \rightarrow D\), while for Concurrent\(S \rightarrow R \rightarrow IC\(R\), the relay needs to know the source-relay channels.

Finally, Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\) can be applied in more general network models. Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\) can be used in MARNs with distributed relay antennas. From \([2]\), the forwarded signal from a relay antenna \(t_i\) only depends on its own received signal \(r_i\). No cross-talk between relay antennas is needed. Thus, the relay antennas do not have to be collocated to conduct the scheme. It is the total number of relay antennas that matters. Furthermore, Concurrent\(S \rightarrow R \rightarrow D\)-IC\(D\) can also be straightforwardly used in the interference relay network with multi-antenna destinations. The protocol description shows that each destination can use its multi-antennas to decouple the information streams of multi-sources and decode the information of its interest as long as the
numbers of destination antennas and distributed relay antennas are no less than the number of sources.

4 The Protocol of Concurrent $R \rightarrow D$-IC$_D$

Although Concurrent$_{S \rightarrow R \rightarrow D}$-IC$_D$ improves the spectrum efficiency of MARNs, it cannot achieve the maximum int-free diversity. In this section, we propose another protocol that has the potential of achieving the same int-free diversity gain but with a higher symbol rate compared to TDMA$_{S \rightarrow R \rightarrow D}$. In $1_J \times M_1 \times N_1$ MARNs, the source-relay link has $M$ independent channel paths for each source and the relay-destination link has $MN$ independent channel paths. The diversity gain is thus bottlenecked by the source-relay link. We propose to use TDMA in the source-relay link to achieve the maximum diversity gain, and in the relay-destination link, concurrent transmission of information streams from different sources is designed to improve the symbol rate. We denote this protocol as Concurrent$_{R \rightarrow D}$-IC$_D$. In Subsection 4.1, we present details of the protocol. We analyze the diversity gain of the protocol in Subsection 4.2 and compare it with other schemes in Subsection 4.3.

4.1 Protocol Description

Since Concurrent$_{R \rightarrow D}$-IC$_D$ uses TDMA in the source-relay link, the main challenge in the design is to allow concurrent transmission of multi-sources in the relay-destination link, and decouple the multiple information streams at the destination. Our proposed protocol requires the relay to know its channels with all sources, which can be obtained by training, and does not require CSI feedback. It fits the linear framework introduced in Section 1. In what follows, we first explain the protocol for the $1_J \times (2J)_1 \times N_1$ MARN, followed by the $1_J \times (4J)_1 \times (N)_1$ MARN, then extend the design to the general case of $1_J \times M_1 \times N_1$ MARNs.

4.1.1 Concurrent$_{R \rightarrow D}$-IC$_D$ for the $1_J \times (2J)_1 \times N_1$ MARN

In this subsection, we describe Concurrent$_{R \rightarrow D}$-IC$_D$ for the MARN in which the number of relay antennas is twice that of the number of sources. The protocol of Concurrent$_{R \rightarrow D}$-IC$_D$ consists of two steps as shown in Fig. 3. During the first step, two symbols randomly selected from one constellation $S$ are collected by Source $j$ to form a vector as $s^{(j)} = [s_1^{(j)} \ s_2^{(j)}]^T$. Source $j$ uses time slots $2j - 1$ and $2j$ to send $s^{(j)}$. In other words, sources transmit to the relay in TDMA. In time slots $(2j - 1)$ and $2j$, the $i$-th relay antenna overhears

$$
\begin{bmatrix}
    r^{(2j-1)i} \\
    r^{(2j)i} \\
    r^{(j)}_i
\end{bmatrix} = \sqrt{P} \begin{bmatrix}
    f_i^{(j)} \\
    s^{(j)} \\
    v^{(j)}_i
\end{bmatrix} + \begin{bmatrix}
    u^{(2j-1)i} \\
    u^{(2j)i} \\
    v^{(j)}_i
\end{bmatrix}, i = 1, \ldots, 2J, j = 1, \ldots, J,
$$
where \( r_{ri} \) and \( v_{ri} \) denote the received signal and the AWGN at the \( i \)-th relay antenna and time slot \( \tau \), respectively. The relay coherently combines signals at each antenna to maximize the SNR of Source \( j \)'s transmission and obtains a soft estimate of \( s^{(j)} \) as,

\[
\hat{s}^{(j)} = \frac{\sum_{i=1:2J} f_i^{(j)} r_i^{(j)}}{\sum_{i=1:2J} |f_i^{(j)}|^2} + \sqrt{P} \hat{s}^{(j)} + \frac{\sum_{i=1:2J} f_i^{(j)} v_i^{(j)}}{\hat{\nu}^{(j)}},
\]

(16)

where the \( 2 \times 1 \) noise vector \( \hat{\nu}^{(j)} \) has i.i.d. \( \mathcal{CN}(0, \sum_{i=1:2J} |f_i^{(j)}|^2)^{-1} \) entries. The relay uses Alamouti DSSTCs [17] to encode the soft estimate of \( s^{(j)} \) into

\[
[t_{(2j-1)}(2j)] = \sqrt{\frac{P}{MP + M}} \left[ A_1 \hat{r}^{(j)} B_2 \hat{r}^{(j)} \right],
\]

(17)

where \( \sqrt{\frac{P}{MP + M}} \) is to constrain the average relay power to \( P \); the encoding matrices \( A_1 \) and \( B_2 \) are given in [5]. From (16) and (17), the relay generates the signal \( t_i \) by a linear transformation from its received signal \( r_i^{(j)} \).

During the second step, the relay forwards the \( 2 \times 1 \) vector \( t_i \) using its \( i \)-th antenna. All relay antennas transmit simultaneously to realize concurrent transmissions of all sources’ information streams. From (17), we can see that each source is assigned two antennas and \( J \) Alamouti DSSTCs are concurrently transmitted to the destination. Denote \( y_{\tau n} \) as the received signal at time slot \( \tau \) and the \( n \)-th antenna at the destination. An equivalent system can be obtained as

\[
\begin{bmatrix}
y_{1n} \\
y_{2n} \\
y_n
\end{bmatrix} = \sqrt{\frac{P^2}{MP + M}} \sum_{j=1:J} \begin{bmatrix}
g_{(2j-1)n} & -g_{(2j)n} \\
g_{(2j)n} & g_{(2j-1)n}
\end{bmatrix} \begin{bmatrix}
s_1^{(j)} \\
s_2^{(j)}
\end{bmatrix} + \sqrt{\frac{P}{MP + M}} \sum_{j=1:J} G_{jn} \begin{bmatrix}
v_1^{(j)} \\
v_2^{(j)}
\end{bmatrix} + \begin{bmatrix}
w_{1n} \\
w_{2n} \\
w_n
\end{bmatrix},
\]

(18)

where \( \hat{v}_i^{(j)} \) is the \( i \)-th entry of \( \hat{v}^{(j)} \) in (16). Note that the \( 2 \times 2 \) equivalent channel matrix \( G_{jn} \) has Alamouti structure. The equivalent system equation in (18) is similar to that of a multi-user multi-antenna MAC system except that the equivalent noises are not white. By applying the multi-user IC schemes in [20], the destination can iteratively cancel the symbols of \( J - 1 \) interfering sources using signals at any \( J - 1 \) antennas. For full IC, \( N \geq J \) is required. Here, we provide a compact matrix representation of this algorithm, which is not provided in [20],[21], because the resulting equations are needed for the diversity analysis. Without loss of generality, we show how the destination cancels the information of Sources 2 to \( J \) and obtains int-free observations of Source 1 in \( J - 1 \) iterations.

Stack \( \hat{y}_n \) to obtain \( \hat{y} = [\hat{y}_1^*, \ldots, \hat{y}_N^*]^* \) and let \( G_j = [G_{j1}^*, \ldots, G_{jN}^*]^* \) for \( j = 1, \ldots, J \). The iterative process is described as follows:
- **Initialization**: \(G(0) = \begin{bmatrix} G_1 & \ldots & G_J \end{bmatrix}, \hat{y}(0) = \hat{y}.\)

- **For the \(i\)-th iteration**: \(i = 1, \ldots, J - 1\)
  
  1. Form the \(2(N - i) \times 2(N - i + 1)\) IC matrix \(B(i)\) as

  \[
  B(i) = \begin{bmatrix}
  -2G_i^*J_i+1,i(i-1) \left[\frac{2G_i^*J_i+1,i(i-1)}{\|G_iJ_i+1,i(i-1)\|^2}\right] & 0_2 & \ldots & 0_2 \\
  -2G_i^*J_i+1,i(i-1) \left[\frac{2G_i^*J_i+1,i(i-1)}{\|G_iJ_i+1,i(i-1)\|^2}\right] & 0_2 & \ldots & 0_2 \\
  \vdots & \vdots & \ddots & \vdots \\
  -2G_i^*J_i+1,i(i-1) \left[\frac{2G_i^*J_i+1,i(i-1)}{\|G_iJ_i+1,i(i-1)\|^2}\right] & 0_2 & \ldots & 0_2
  \end{bmatrix}, \quad (19)
  \]

  where the \(2 \times 2\) matrix \(G_{p,q}(i)\) denotes the \((p, q)\)-th \(2 \times 2\) submatrix of \(G(i)\).

  2. Cancel the symbols of Source \(J - i + 1\) by calculating \(\hat{y}(i) = B(i)\hat{y}(i - 1)\).

  3. Form the \(2(N - i) \times 2J\) remaining equivalent channel matrix \(G(i)\) as \(G(i) = B(i)G(i - 1)\).

  Note that \(\hat{y}(i)\) is the \(2(N - i) \times 1\) signal vector after cancelling the information Source \(J - i + 1\). After \(J - 1\) iterations, \(\hat{y}(J - 1)\) only contains the information of Source 1 and has dimension \(2(N - J + 1) \times 1\). Let

  \[B = \prod_{i=1:J-1} B(i)\].

  This iterative IC process can be expressed as a linear operation on \(\hat{y}\) as

  \[
  \hat{y}(J - 1) = B\hat{y} = \sqrt{\frac{P}{MP + M}}BG_1 \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} + \sqrt{\frac{P}{MP + M}}BG_1n^{(1)} + B\hat{w}. \quad (20)
  \]

  where \(\hat{w} = [\hat{w}_1^* \ldots \hat{w}_M^*]^T\) and \(n\) denotes the equivalent noise vector after IC. Note that \(n\) is Gaussian but not white. After straightforward calculation, its covariance matrix can be calculated as

  \[
  R_n = \sum_{i=1:M} \frac{c^2_i}{f_i^{(1)}}BG_1G_1^*B + BB^*, \quad (21)
  \]

  where \(c_1 = \sqrt{\frac{P}{MP + M}}.\) The ML decoding of Source 1’s information can be performed as

  \[
  \arg\min_{s_1^{(1)},s_2^{(1)} \in S} \left( B\hat{y} - \sqrt{P}c_1BG_1 \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} \right)^* R_n^{-1} \left( B\hat{y} - \sqrt{P}c_1BG_1 \begin{bmatrix} s_1^{(1)} \\ s_2^{(1)} \end{bmatrix} \right). \quad (22)
  \]

  Since the \(2 \times 2\) submatrices of \(B, G_1,\) and \(R_n\) have Alamouti structure, (22) can be further decoupled into two procedures of symbol-wise decoding following the similar argument in Subsection 3.1.1. Likewise, the information of the other \(J - 1\) sources can be decoupled and decoded. In total, \(2J\) symbol-wise ML decoding procedures are required to decode all sources’ information. Therefore, the decoding complexity is linear in the number of sources.
4.1.2 Concurrent$_{R→D}$-IC$_D$ for the $1_J \times (4J)_1 \times N_1$ MARN

In this subsection, we describe Concurrent$_{R→D}$-IC$_D$ for the MARN where the number of relay antennas is four times the number of source antennas. During the first step, Source $j$ collects four symbols $s_i^{(j)}$ ($i = 1, \ldots, 4$), in which $s_1^{(j)}, s_2^{(j)} \in S$ and $s_3^{(j)}, s_4^{(j)} \in S'$. The constellation $S'$ is obtained by rotating $S$ [28,29]. Source $j$ transmits a vector of these four symbols to the relay in four time slots, and sources timeshare the source-relay link. The relay obtains a soft estimate of each symbol from Source $j$ by coherently combining signals at different antennas as in (16), then linearly transforms this soft estimate $\hat{r}^{(j)}$ into a quasi-orthogonal DSTC by $[t_{4j-3} t_{4j-2} t_{4j-1} t_{4j}] = c_2 \begin{bmatrix} A_1 \hat{r}^{(j)} B_2 \tilde{r}^{(j)} B_3 \tilde{r}^{(j)} A_4 \hat{r}^{(j)} \end{bmatrix}$, where the scalar $c_2 = \sqrt{\frac{P}{M_P+M}}$ normalizes the average power at the relay to $P$; and $A_j$ and $B_i$ are the DSTC encoding matrices [17], as given in (10).

During the second step, information streams from all sources are concurrently forwarded to the destination by sending $t_i$ from the $i$-th antenna. With this design, the relay uses four of its antennas to forward the quasi-orthogonal DSTC of each source and the information of all sources is forwarded concurrently. Denote $y_{\tau n}$ and $w_{\tau n}$ as the sampled signal and noise at the $n$-th destination antenna and time slot $\tau$, respectively. Following the analysis in [21], two equivalent Alamouti systems can be obtained as

\[
\begin{align*}
y_n^{1+} &= \sqrt{P} c_2 \sum_{j=1}^{J} \begin{bmatrix} g_{1n}^{(j)} + g_{2n}^{(j)} g_{3n}^{(j)} - g_{4n}^{(j)} \end{bmatrix} \begin{bmatrix} s_1^{(j)} + s_4^{(j)} \\ -s_3^{(j)} - s_2^{(j)} \end{bmatrix} + c_2 \sum_{j=1}^{J} G_{jn}^+ \begin{bmatrix} \hat{v}^{(j)+} \end{bmatrix} + \begin{bmatrix} w_{1n}^{1+} + w_{4n}^{1+} \\ w_{2n}^{1+} - w_{3n}^{1+} \end{bmatrix}, \\
y_n^{1-} &= \sqrt{P} c_2 \sum_{j=1}^{J} \begin{bmatrix} g_{1n}^{(j)} - g_{4n}^{(j)} g_{2n}^{(j)} + g_{3n}^{(j)} \end{bmatrix} \begin{bmatrix} s_1^{(j)} - s_4^{(j)} \\ -s_2^{(j)} - s_3^{(j)} \end{bmatrix} + c_2 \sum_{j=1}^{J} G_{jn}^- \begin{bmatrix} \hat{v}^{(j)-} \end{bmatrix} + \begin{bmatrix} w_{1n}^{1-} - w_{4n}^{1-} \\ w_{3n}^{1-} + w_{2n}^{1-} \end{bmatrix},
\end{align*}
\]

where $g_{kn}^{(j)}$ ($k = 1, 2, 3, 4$) denotes the four channel paths from the four relay antennas that forward Source $j$'s DSTC to the $n$-th destination antenna, i.e., $g_{kn}^{(j)} = g_{(4j-4+k)n}$; $\hat{v}_k^{(j)}$ denotes the equivalent noises at the relay, which can be shown to be i.i.d. $\mathcal{CN}(0, \left(\sum_{i=1:M} \left| f_i^{(j)} \right|^2\right)^{-1})$. By applying the multi-user IC proposed for quasi-orthogonal STBC in [21], the destination can cancel the symbols of Sources 2 to $J$ for each Alamouti system. Denote $B^*$ as the IC matrix for system *, ($* = +, -$), which can be obtained similarly following the iterative IC process in Subsection 4.1.1. Let $G_1^* = \left[ G_1_{11}^* \cdots G_1_{1N}^* \right]^T$ and $y^* = \left[ y_{11}^* \cdots y_{1N}^* \right]^T$. The resulting system equation for Source 1’s information after the IC can be expressed as

\[
\begin{bmatrix} B^+ y^+ \\ B^- y^- \end{bmatrix} = \sqrt{P} c_2 \begin{bmatrix} B^+ G_1^+ & 0 \\ 0 & B^- G_1^- \end{bmatrix} \begin{bmatrix} s_1^{(1)} + s_4^{(1)} \\ s_3^{(1)} - s_2^{(1)} \end{bmatrix} + c_2 \begin{bmatrix} B^+ G_1^+ & 0 \\ 0 & B^- G_1^- \end{bmatrix} \begin{bmatrix} v^{(1)+} \\ v^{(1)-} \end{bmatrix} + \begin{bmatrix} B^+ 0 \\ 0 B^- \end{bmatrix} w, \quad (23)
\]

where $w = [w_1^{1+} \cdots w_1^{1-} w_1^{-} \cdots w_1^{-}]^T$. $H$ and $n$ denote the $4(N-J+1) \times 4$ equivalent channel matrix
and the $4(N - J + 1) \times 1$ equivalent noise vector, respectively. From (23), it can be shown that two procedures of pair-wise ML decoding are sufficient to decode the four symbols of Source 1.

4.1.3 Concurrent $R \rightarrow D$-IC$_D$ for general $1_J \times M_1 \times N_1$ MARNs

For general $1_J \times M_1 \times N_1$ MARNs, each source transmits a vector of $2^n$ symbols in TDMA during the first step, where $2^n$ is the minimum number that is no less than $\lceil \frac{M}{J} \rceil$. The relay constructs one $2^n \times 2^n$ DSTC using the quasi-orthogonal STBCs with ABBA structure for each source [25][26]. During the second step, the first $\lfloor \frac{M}{J} \rfloor$ columns of each DSTC are forwarded using $\lfloor \frac{M}{J} \rfloor$ antennas of the relay. All DSTCs are concurrently forwarded to the destination. The destination separates the equivalent system into $2^n-1$ Alamouti systems [16], then decouples information of each source by IC [21], after which, decodes each source’s information independently.

4.2 Diversity Gain Analysis

In this subsection, we analyze the achievable diversity gain of Concurrent $R \rightarrow D$-IC$_D$. As discussed in Subsection [3.2.2] the diversity gain can be calculated using the outage probability of the instantaneous normalized receive SNR as in (15). To help the presentation, we use an equivalent representation of (15). We say that an instantaneous normalized receive SNR $\gamma$ provides a diversity gain of $d$ if $P(\gamma < \epsilon) = \alpha_1 \epsilon^d + o(\epsilon^d)$ with $\alpha_1$ independent of $\epsilon$. To calculate the diversity gain of Concurrent $R \rightarrow D$-IC$_D$, the following lemma is used [16].

**Lemma 1.** Let $\gamma_1, \gamma_2, \ldots, \gamma_k, \gamma_g$ be $k+1$ instantaneous normalized receive SNRs. $\gamma_g$ is independent of $\gamma_n$ for $n = 1, 2, \ldots, k$. $\gamma_g$ provides a diversity gain of $d_1$; $\sum_{n=1,k} \gamma_n$ provides a diversity gain of $d_2$. If $\gamma = \sum_{n=1,k} \frac{\gamma_n \gamma_g}{\gamma_n + \gamma_g}$, $\gamma$ provides a diversity gain of $\min\{d_1, d_2\}$.

Here is the theorem on the diversity gain of Concurrent $R \rightarrow D$-IC$_D$.

**Theorem 2.** In $1_J \times M_1 \times N_1$ MARNs, Concurrent $R \rightarrow D$-IC$_D$ achieves a diversity gain of $\min\{M, \lfloor \frac{M}{J} \rfloor(N - J + 1)\}$.

**Proof.** See Appendix B.

Intuitively, the result in Theorem 2 can be explained as follows. From the protocol design, since sources are assigned to orthogonal channels in the first step of transmission, the maximum diversity gain that can be achieved in the source-relay link is $M$. For the second step, each source is allocated $\lfloor \frac{M}{J} \rfloor$ antennas of the relay. Then, the transmit diversity gain is $\lfloor \frac{M}{J} \rfloor$. Similar to MAC systems, the destination uses $J - 1$ antennas to cancel the symbols of interfering sources and obtains the full receive diversity gain $N - J + 1$ at remaining antennas. Thus, the maximum achievable diversity gain in the relay-destination link is $\lfloor \frac{M}{J} \rfloor(N - J + 1)$. The overall
diversity gain of Concurrent_{R→D}-IC_D is thus upperbounded by the minimum of the two values, and Theorem 2 shows that Concurrent_{R→D}-IC_D achieves this upperbound. When $M = 2J$ and $M = 4J$, the following corollary is obtained for special cases of Theorem 2.

Corollary 2. In the $1_J \times (2J)_1 \times N_1$ MARN, Concurrent_{R→D}-IC_D achieves a diversity gain of $2 \min\{J, N - J + 1\}$; In the $1_J \times (4J)_1 \times N_1$ MARN, Concurrent_{R→D}-IC_D achieves a diversity gain of $4 \min\{J, N - J + 1\}$.

4.3 Discussion

In this subsection, we discuss several properties of Concurrent_{R→D}-IC_D, including the constraint on the number of sources, the condition to achieve the int-free diversity gain, and the symbol rate. The comparison of Concurrent_{R→D}-IC_D with other linear schemes is also presented. Finally, we provide an application in the interference relay network.

First, we discuss the constraint on the number of sources for Concurrent_{R→D}-IC_D. Since in this protocol, each source is allocated a different set of relay antennas for the concurrent transmission in the relay-destination link, the number of relay antennas needs to be no less than the number of sources, i.e., $M \geq J$. At the destination, to fully decouple signals of different sources, at least $J - 1$ antennas are required to cancel the symbols of $J - 1$ sources. In other words, $N \geq J$. Therefore, $J \leq \min\{M, N\}$ is required. This condition is the same as that for Concurrent_{S→R→D}-IC_D as discussed in Subsection 3.3. To guarantee this condition, user admission control in the upper layer is needed.

Next, we show that Concurrent_{R→D}-IC_D has the potential to achieve the int-free diversity gain. Recall that the int-free diversity gain is defined as the maximum achievable diversity gain when there is no interference in both links. For $1_J \times M_1 \times N_1$ MARNs, the int-free diversity gain is $\min\{M, MN\} = M$, achievable by TDMA_{S→R→D} [18]. Theorem 2 indicates that when

$$N \geq \frac{M}{\lfloor \frac{M}{J} \rfloor} + J - 1,$$

Concurrent_{R→D}-IC_D achieves the int-free diversity gain of $M$. Eq. (24) is called the int-free condition. For networks satisfying the int-free condition, Concurrent_{R→D}-IC_D allows multi-source transmission in the relay-destination link without sacrificing the diversity gain. If $M$ is a multiple of $J$, this condition can be further simplified as $N \geq 2J - 1$. Examples of networks satisfying the int-free condition are: $1_2 \times 2_1 \times 3_1, 1_2 \times 4_1 \times 3_1, 1_3 \times 3_1 \times 5_1$, and $1_3 \times 6_1 \times 5_1$ MARNs.

In what follows, we discuss the symbol rate of Concurrent_{R→D}-IC_D. In $1_J \times M_1 \times N_1$ MARNs, for each source to transmit a vector of $2^n$ symbols ($2^n$ is the minimum number no less than $\lfloor \frac{M}{J} \rfloor$), $2^nJ$ channel uses are needed in the first link and $2^n$ channel uses are needed in the second link. Thus, $2^n$ symbols from each
source are transmitted using $2^n(J + 1)$ channel uses from end to end. The symbol rate can thus be calculated as $R = \frac{2^n}{2^n(J + 1)} = \frac{1}{1+J}$ symbols/source/channel use.

Concurrent$_{R→D}$-IC$_D$ fits the linear framework in which the relay linearly transforms its received signals to generate output signals without decoding and the destination decouples signals from different sources to separately decode each source’s information. In what follows, we compare the diversity gain, symbol rate, and CSI requirements at the relay of Concurrent$_{R→D}$-IC$_D$ with the two existing linear schemes, TDMA$_{S→R→D}$ and Concurrent$_{S→R}$-IC$_R$, as well as Concurrent$_{S→R→D}$-IC$_D$. The results are shown in Table 1. For MARNs satisfying the int-free condition, Concurrent$_{R→D}$-IC$_D$ and TDMA$_{S→R→D}$ achieve the maximum int-free diversity gain, higher than that of Concurrent$_{S→R}$-IC$_R$. Both Concurrent$_{R→D}$-IC$_D$ and Concurrent$_{S→R}$-IC$_R$ achieve higher symbol rate compared to TDMA$_{S→R→D}$. Thus, Concurrent$_{R→D}$-IC$_D$ outperforms TDMA$_{S→R→D}$ in terms of symbol rate and exceeds Concurrent$_{S→R}$-IC$_R$ in terms of diversity gain. For Concurrent$_{R→D}$-IC$_D$ and Concurrent$_{S→R}$-IC$_R$, the relay needs to know its channels with all sources, which can be obtained by training. For MARNs not satisfying the int-free condition, Concurrent$_{R→D}$-IC$_D$ may not achieve the int-free diversity. For the two proposed protocols, each has its advantage over the other: Concurrent$_{R→D}$-IC$_D$ achieves a higher diversity gain, whereas Concurrent$_{S→R→D}$-IC$_D$ has a higher symbol rate.

Concurrent$_{R→D}$-IC$_D$ can also be applied to the interference relay network with one multi-antenna relay and several multi-antenna destinations. The relay processes its received signals in the same way as that in the MARN. Each destination cancels the information of undesired sources and decodes the information of its interest as long as the numbers of antennas at each destination and the relay are no less than the number of sources.

5 Numerical Results

In this section, we present simulated BERs of Concurrent$_{S→R→D}$-IC$_D$ and Concurrent$_{R→D}$-IC$_D$ and compare with the BERs of other existing schemes with similar complexities and CSI requirements. Since the average power constraints at all nodes are equal to $P$ and noises are normalized, the average transmit SNR at each node is $P$. For all figures, the horizontal axis represents the average transmit SNR, measured in dB; the vertical axis represents the BER.

In Fig. 4 the BER of the first proposed scheme, Concurrent$_{S→R→D}$-IC$_D$, is demonstrated for 6 MARNs: $1_2 \times 2_1 \times 2_1$, $1_2 \times 2_1 \times 3_1$, $1_2 \times 2_1 \times 4_1$, $1_2 \times 4_1 \times 2_1$, $1_2 \times 4_1 \times 3_1$, $1_2 \times 4_1 \times 4_1$, and $1_3 \times 4_1 \times 3_1$. BPSK modulation is used. Fig. 4 shows that the scheme achieves a diversity gain of 1 in the $1_2 \times 2_1 \times 2_1$, $1_2 \times 2_1 \times 3_1$, and $1_2 \times 2_1 \times 4_1$ MARNs. Additional array gain can be achieved when the number of destination antennas is increased. In the $1_3 \times 4_1 \times 3_1$ MARN, the diversity gain is 2; while in the $1_2 \times 4_1 \times 2_1$, $1_2 \times 4_1 \times 3_1$, $1_2 \times 4_1 \times 4_1$,
and $1_2 \times 4_1 \times 4_1$ MARNs, the diversity gain is slightly less than 3. This is because of the $\log P$ factor in the error rate formula [17]. As $P$ increases, the diversity gain approaches 3. These results justify the validity of the diversity upperbound presented in Theorem 1 and show the achievability of the upperbound for these network scenarios. Comparing the results for the $1_3 \times 4_1 \times 3_1$ and $1_2 \times 2_1 \times 3_1$ MARNs, we can see that the number of sources that a MARN can accommodate and the diversity gain of a MARN can be improved simultaneously by increasing the number of relay antennas. From the results for the $1_2 \times 4_1 \times 3_1$ and $1_3 \times 4_1 \times 3_1$ MARNs, we conclude that with a fixed number of relay antennas, the diversity gain decreases as the number of sources in the network increases.

Fig. 5 exhibits the BER of the second proposed scheme Concurrent$_{R \rightarrow D}$-IC$_D$ in 8 MARNs: $1_2 \times 2_1 \times 2_1$, $1_2 \times 2_1 \times 3_1$, $1_2 \times 2_1 \times 4_1$, $1_3 \times 3_1 \times 3_1$, $1_3 \times 3_1 \times 5_1$, $1_2 \times 4_1 \times 2_1$, $1_2 \times 4_1 \times 3_1$, and $1_2 \times 8_1 \times 2_1$. In all scenarios, BPSK modulation is used. For MARNs with parameters $1_2 \times 2_1 \times 2_1$, $1_3 \times 3_1 \times 3_1$, $1_2 \times 4_1 \times 2_1$, and $1_2 \times 8_1 \times 2_1$, Concurrent$_{R \rightarrow D}$-IC$_D$ achieves the diversity gains of 1, 1, 2, and 4, respectively. Note that the int-free diversity gains in these networks are 2, 3, 4, and 8, respectively. Concurrent$_{R \rightarrow D}$-IC$_D$ does not achieve the int-free diversity gain for these scenarios. For MARNs with parameters $1_2 \times 2_1 \times 3_1$, $1_2 \times 2_1 \times 4_1$, $1_3 \times 3_1 \times 5_1$, and $1_2 \times 4_1 \times 3_1$, Concurrent$_{R \rightarrow D}$-IC$_D$ achieves the diversity gains of 2, 2, 3, and 4, respectively, which are the int-free diversity gains. The parameters of these four networks satisfy the int-free condition given in Eq. (24). The simulation results for these eight networks justify our diversity result in Theorem 2.

In the following, we compare the proposed Concurrent$_{S \rightarrow R \rightarrow D}$-IC$_D$ (Scheme 1) and Concurrent$_{R \rightarrow D}$-IC$_D$ (Scheme 2) with other schemes: Concurrent$_{S \rightarrow R}$-IC$_R$ (Scheme 3), TDMA$_{S \rightarrow R \rightarrow D}$ (Scheme 4), Concurrent$_{R \rightarrow D}$-IC$_D$ (Scheme 5), and Concurrent$_{S \rightarrow R \rightarrow D}$ (Scheme 6). Schemes 3 and 4 are introduced in Section 1. To compare our methods with schemes having decoding at the relay, Scheme 5 is introduced. It is similar to Scheme 2 except that the relay conducts the ML decoding based on the soft estimate in (16). After that, symbols are re-encoded and forwarded to the destination using the same constellation. Scheme 6 is similar to Scheme 1, but in Scheme 6 the destination jointly decodes all sources’ information without IC. Note that Schemes 1, 2, 3, 4 satisfy the constraints of the linear framework, but Schemes 5 and 6 do not fit the linear framework and have higher complexity than the other four schemes. For fair comparison in the numerical experiments, we fix the bit rate to be 1 bit/source/channel use regardless of the scheme and plot the BERs of the schemes as a function of the average transmit SNR. Thus, QPSK, 8PSK, 8PSK, 16PSK, 8PSK, and QPSK are used for Schemes 1, 2, 3, 4, 5, and 6, respectively.

Figs. 6, 7, and 8 show BERs of these schemes in the $1_2 \times 2_1 \times 2_1$, $1_2 \times 2_1 \times 3_1$, $1_2 \times 4_1 \times 3_1$ MARNs, respectively. We compare the BERs of the four linear schemes. We first look at the $1_2 \times 2_1 \times 2_1$ MARN whose BERs are shown in Fig. 6. Only Scheme 4 achieves the maximum int-free diversity, thus it has the best performance at high SNR (26 dB and up). The other three schemes have a diversity gain of 1. Scheme 3 has
the lowest BER for SNR less than 26 dB, because of its high signal to interference-plus-noise ratio (SINR) at the destination. Thus, for the $1_2 \times 2_1 \times 2_1$ MARN, the proposed schemes, Schemes 1 and 2, are inferior in BER. The next is the $1_2 \times 2_1 \times 3_1$ MARN. We can see from Fig. 7 that only Schemes 2 and 4 achieve the maximum int-free diversity, while Scheme 2 has lower BER than the other schemes for all the simulated SNR values. Its advantage over Scheme 4 is about 5 dB. This is because Scheme 2 has a higher symbol rate. For the same bit rate, it can use a smaller constellation, which provides higher array gain. Thus, for the $1_2 \times 2_1 \times 3_1$, Scheme 2 is the best. Finally, in the $1_2 \times 4_1 \times 3_1$ MARN shown in Fig. 8 Scheme 2 has the highest diversity gain. When SNR is higher than 23 dB, Scheme 2 has the lowest BER. Scheme 1 outperforms the other three in the SNR regime from 17 to 23 dB, because it has the highest symbol rate and uses the smallest constellation to achieve the same bit rate. When the SNR is smaller than 17 dB, Scheme 3 has the lowest BER. Therefore, for the $1_2 \times 4_1 \times 3_1$ MARN, our proposed two schemes have lower BER compared to the existing schemes when SNR is higher than 17 dB. We can conclude from the three experiments that the relative quality of the four schemes depends on the network parameters and SNR range. The proposed Scheme 1, Concurrents$_{\rightarrow R \rightarrow D}$-IC$_D$, is expected to have good reliability in the low to moderate SNR range, as observed in Fig. 8. The proposed Scheme 2, Concurrent$_{R \rightarrow D}$-IC$_D$, is expected to have good reliability for MARNs whose relay-destination link is much stronger than the source-relay link (e.g., the $1_2 \times 2_1 \times 3_1$ MARN). These are due to the nature of the design explained in Section 3 and Section 4.

In what follows, we compare the proposed schemes with Schemes 5 and 6, which do not satisfy the linear constraints. We first compare Scheme 1 with Scheme 6. Note that the ML decoding of Scheme 1 is symbol-wise in the $1_2 \times 2_1 \times N_1$ MARN and pair-wise in the $1_2 \times 4_1 \times N_1$ MARN, while for Scheme 6, the destination needs to jointly decode four symbols in the $1_2 \times 2_1 \times N_1$ MARN and eight symbols in the $1_2 \times 4_1 \times N_1$ MARN. For networks with large $J$ and $M$, the decoding complexity of Scheme 6 is exponential in $JM$, thus becomes impractical. For Scheme 1, the decoding complexity is linear in $J$ and exponential in $M/2$, thus is much lower. Figs. 6, 7, and 8 show that this extra decoding complexity can improve both diversity gain and array gain. The diversity gain improvements are 1 in all three networks. For the $1_2 \times 4_1 \times 3_1$ MARN (Fig. 8), the array gain improvement is the smallest (about 3 dB at BER = $10^{-2}$) compared to the other two networks. Therefore, compared to Scheme 6, Scheme 1 is desired in large networks to trade performance degradation for lower complexity. Then, we compare Scheme 2 with Scheme 5 and see whether the extra decoding at the relay can provide better performance. For the three networks shown in Figs. 6, 7, and 8, Scheme 2 has the same diversity gain as Scheme 5. For the $1_2 \times 2_1 \times 2_1$ MARN (Fig. 6), Scheme 2 has approximately the same performance as Scheme 5 for all the simulated SNR values. For the $1_2 \times 2_1 \times 3_1$ MARN (Fig. 7), Scheme 2 has the same performance as Scheme 5 in the high SNR regime while is about 1 dB worse in the low to moderate SNRs. For the $1_2 \times 4_1 \times 3_1$ MARN (Fig. 8), Scheme 2 is approximately 2 dB worse for all SNRs. These
observations can be explained as follows. For the $1_2 \times 2_1 \times 2_1$ MARN, with Scheme 2, the BER of the network is mainly constrained by the second hop (with IC at the destination, the second hop has only a diversity gain of 1, but the first hop has a diversity gain of 2). The extra decoding at the relay only improves the performance in the first hop, does not help the overall performance much. For the $1_2 \times 2_1 \times 3_1$ and $1_2 \times 4_1 \times 3_1$ MARNs, with Scheme 2, the two links have similar qualities (both links have diversity gain 2 for the $1_2 \times 2_1 \times 3_1$ MARN and 4 for the $1_2 \times 4_1 \times 3_1$ MARN). The extra decoding complexity at the relay can provide a better performance.

6 Conclusions

This paper studies multi-source transmission schemes for $1_J \times M_1 \times N_1$ MARNs. For complexity considerations, a linear framework is introduced, where the relay conducts linear transformation without decoding and the destination decouples signals from different sources so that the decoding complexity is linear in the number of sources. We propose two protocols that use multi-antennas at the destination to resolve multi-source interference. The protocol of Concurrent$_{S \rightarrow R \rightarrow D}$-IC$_D$ allows concurrent transmission of information streams from multi-sources in both the source-relay link and the relay-destination link. The relay performs DSTC and does not require any CSI. The destination uses the multi-antenna IC technique to decouple signals from different sources. Concurrent$_{S \rightarrow R \rightarrow D}$-IC$_D$ achieves a symbol rate of $1/2$ symbols/source/channel use, but its diversity gain is shown to be upperbounded by $M - J + 1$. Thus, for this protocol, the diversity gain degradation is necessary to trade for symbol rate. To improve the diversity gain, we propose Concurrent$_{R \rightarrow D}$-IC$_D$, in which concurrent transmission is allowed in the relay-destination link but TDMA is used in the source-relay link. After receiving signals from the sources, the relay first conducts MRC to maximize the SNR of each source then concurrently transmits all sources’ information to the destination using DSTC. At the destination, IC is performed to decouple signals from different sources before decoding. Through analysis and simulations, it is shown that Concurrent$_{R \rightarrow D}$-IC$_D$ achieves a diversity gain of $\min \left\{ M, \left\lfloor \frac{M}{J} \right\rfloor (N - J + 1) \right\}$ with a symbol rate of $\frac{1}{J+1}$. When $N \geq 2J - 1$, Concurrent$_{R \rightarrow D}$-IC$_D$ achieves the same maximum int-free diversity gain of the network but with a higher symbol rate, compared to a full TDMA scheme.

Appendix

A Proof of Theorem [1]

To prove this theorem, it suffices to find an upperbound on the instantaneous normalized receive SNR. We first show the scenario of $J = 2$ and $M = 2$, then its generalization.

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We first show the case that $B$ Proof of Theorem 2

\begin{equation}
\gamma = (BG\hat{\Phi}_1^{(1)})^* R_n^{-1} (BG\hat{\Phi}_1^{(1)}) < \hat{\Phi}_1^{(1)}*G*B^*(BB^*)^{-1}BG\hat{\Phi}_1^{(1)} < \hat{\Phi}_1^{(1)}*G*\hat{G}\hat{\Phi}_1^{(1)},
\end{equation}

where $\hat{\Phi}_1^{(j)}$ denotes the $i$-th column of $P^{(j)}$. For the second inequality we have used the fact that $B^*(BB^*)^{-1}B \prec I_{2N}$. Since $\hat{f}_1^{(1)}$ is orthogonal to $\hat{f}_2^{(2)}$ from (13), the projection $\hat{\Phi}_1^{(1)}$ is equivalent to project $\hat{f}_1^{(1)}$ onto the null space of $\hat{f}_2^{(2)}$ only, i.e., $\hat{\Phi}_1^{(1)} = \Xi \hat{f}_1^{(1)}$, where $\Xi = \left( I_4 - \hat{f}_2^{(2)}\hat{f}_2^{(2)*}/\|\hat{f}_2^{(2)}\|^2 \right)$. Note that $G^*G \prec tr \left( G^*G \right) I_4$. The right-hand side (RHS) of (25) can be further upperbounded by

\begin{equation}
\gamma < tr (G^*G) \hat{\Phi}_1^{(1)} \Xi^* \Xi \hat{f}_1^{(1)} = 2 \sum_{\delta=1:2N} \left( |g_{1\delta}|^2 + |g_{2\delta}|^2 \right) \hat{f}_1^{(1)} \Xi \hat{f}_1^{(1)} = 2g(1)^*\Theta(1) f(1)
\end{equation}

where the first equality holds because $\Xi^* \Xi = \Xi$ from the definition of projection; $f^{(j)}$ is a $2 \times 1$ channel vector from Source $j$ to the relay, i.e., $f^{(j)} = \left[ f_1^{(j)} f_2^{(j)} \right]^t$; and $\Theta = I_2 - \frac{f_2^{(2)*} f_2^{(2)}}{\|f_2^{(2)}\|^2}$, a $2 \times 2$ projection matrix to the null space of $f^{(2)}$. Clearly, the random variable $g$ is Gamma distributed with degree $2N$. Next, we show that given $f^{(2)}$, $f$ is Gamma distributed with degree 1. Note that the two eigenvalues of $\Theta$ are 1 and 0. Then, $f^{(1)^*} \Theta f^{(1)} = f^{(1)^*} u_1 u_1^* f^{(1)}$, where $u_1$ is the eigenvector corresponding to the eigenvalue 1. Since $\Theta$ depends on $f^{(2)}$ only, $u_1$ is independent of $f^{(1)}$. Thus, given $f^{(2)}$, $u_1^* f^{(1)}$ is $CN(0, 1)$ distributed and $f$ is Gamma distributed with degree 1. The outage probability of $\gamma$ can be bounded as

\[
P(\gamma < \epsilon) = \mathbb{E}_{f^{(2)},g} \left[ P(\gamma < \epsilon | f^{(2)},g) \right] > \mathbb{E}_{f^{(2)},g} \left[ P \left( 2gf < \epsilon | f^{(2)},g \right) \right] = \mathbb{E}_{f^{(2)},g} \left[ P \left( f < \frac{\epsilon}{2g} | f^{(2)},g \right) \right]
\]

\[
= \mathbb{E}_{f^{(2)},g} \left[ \frac{\alpha}{2g} + o(\epsilon) \right] + o(\epsilon) = \frac{\alpha}{2^2} + o(\epsilon) = \frac{\alpha}{2} + o(\epsilon).
\]

where $\alpha$ is a constant independent of $\epsilon$. By (15), the diversity gain is upperbounded by one.

For MARNs with general $J$ and $M$, the IC operation at the destination similarly creates a virtual ZF operation at the relay as discussed in Subsection 5.2.1. The virtual ZF matrix $\Phi$ nulls out $F^{(j)}$, $j = 2, \ldots, J$. Following a similar process, it can be shown that the instantaneous normalized receive SNR is upperbounded by a product of two parts as in (26): the first part depends on $g_{im}$ only, the second part is equal to $f^{(1)^*} \Theta f^{(1)}$ where $f^{(j)}$ is the $M \times 1$ channel vector from Source $j$ to the relay and $\Theta$ is a projection matrix onto the null spaces of $f^{(2)}$ to $f^{(J)}$. Similarly, the diversity gain can be shown to be no higher than $M - J + 1$.

### B Proof of Theorem 2

We first show the case that $M = 2J$, then $M = 4J$, followed by the general scenario. When $M = 2J$, the channel vector experienced by $s_1^{(1)}$ is $B g_1$ from (20), where $g_1$ denotes the first column of $G_1$. Note that the
noise covariance matrix is given in [21]. By the definition of instantaneous normalized receive SNR, we have

\[
\gamma = g_1^* B^* \left( \frac{c_1^2}{\sum |f_i^{(1)}|^2} B G_1 G_1^* B^* + B B^* \right)^{-1} B g_1
\]

(27)

\[
= g_1^* B^* \left( (B B^*)^{-1} - (B B^*)^{-1} B G_1 \left( \frac{\sum |f_i^{(1)}|^2}{c_1^2} I_2 + G_1^* (B B^*)^{-1} B G_1 \right) \right)^{-1} B g_1
\]

(28)

\[
= g_1^* B^* (B B^*)^{-1} B g_1 - \left( \frac{\sum |f_i^{(1)}|^2}{c_1^2} + g_1^* (B B^*)^{-1} B g_1 \right)^{-1} g_1^* B^* (B B^*)^{-1} B G_1 G_1^* B^* (B B^*)^{-1} B g_1
\]

(29)

\[
= g_1^* B^* (B B^*)^{-1} B g_1 - \left( \frac{\sum |f_i^{(1)}|^2}{c_1^2} + g_1^* (B B^*)^{-1} B g_1 \right)^{-1} \left( g_1^* B^* (B B^*)^{-1} B g_1 \right)^2.
\]

(30)

From (27) to (28), the matrix inversion lemma is applied. For (29), we use the fact that $G_1^* B^* (B B^*)^{-1} B G_1$ is a Hermitian matrix with Alamouti structure. Thus, $G_1^* B^* (B B^*)^{-1} B G_1$ is a $2 \times 2$ diagonal matrix whose diagonal entries are equal to $g_1^* B^* (B B^*)^{-1} B g_1$. Eq. (30) follows from (29) because the second entry of the vector $g_1^* B^* (B B^*)^{-1} B g_1$ is zero. Let $y = g_1^* B^* (B B^*)^{-1} B g_1$ and $x = \sum_{i=1,M} |f_i^{(1)}|^2$. We have

\[
\gamma = \frac{xy}{x + yc_1^2},
\]

(31)

which is a scaled harmonic mean of variables $x$ and $yc_1^2$. Since $x$ is the sum of $M$ independent random variables with exponential distribution, $x$ is Gamma distributed with degree $M$. Thus, $x$ has a diversity gain of $M$. For $yc_1^2$, when $P \gg 1$, $c_1 = \lim_{P \to \infty} \sqrt{\frac{P}{MP + M}} \approx \frac{1}{\sqrt{M}}$. From the iterative algorithm and (19), $B$ depends on $G_{jn}$ for $j = 2, \ldots, J$. On the other hand, $g_1$ only depends on $G_{1n}$. Thus, $B^* (B B^*)^{-1} B$ and $g_1$ are independent. Because $B$ zero-forces $G_2$ to $G_J$, it can be shown that $B^* (B B^*)^{-1} B$ is a projection matrix onto the null space of the subspace spanned by columns of $G_j$ for $j = 2, \ldots, J$. Then, $y$ is Gamma distributed with degree $2(N - J + 1)$, implying $c_1^2 y$ has diversity gain $2(N - J + 1)$. Let $k = 1$ in Lemma 1. The diversity gain of $\gamma$ is the smaller of the diversities of $x$ and $y$. Then, the achievable diversity gain of ConcurrentR-D-ICD is

\[
\min \{M, 2(N - J + 1)\} = 2 \min \{J, N - J + 1\}.
\]

For $M = 4J$, the instantaneous normalized receive SNR can be shown to be $\gamma = \text{tr} \left( H^* R_N^{-1} H \right)$ [15], where $H$ denotes the equivalent channel matrix and $R_N$ denotes the covariance matrix of $n$ in (23). After straightforward calculation, we have

\[
R_N = \text{diag} \left( \frac{2c_2^2}{\sum |f_i^{(1)}|^2} B^* G(1)^* + G(1)^* B^* + B^* B^* + \frac{2c_2^2}{\sum |f_i^{(1)}|^2} B^{-1} G(1)^* G(1)^* B^{-1} + B^{-1} B^{-1} \right).
\]

By similar calculation procedures from (27) to (30), it follows that

\[
\gamma = \frac{xy}{x + 2c_2^2 y} + \frac{xz}{x + 2c_2^2 z},
\]

(32)
where $x = \sum_{i=1}^{M} |f_i^{(1)}|^2$, $y = g_1^+ B^+ (B^{-} B^{-})^{-1} B^+ g_1^+$, and $z = g_1^- B^- (B^{-} B^{-})^{-1} B^- g_1^-$, with $g_1^+$ the first column of $G_1^+$ for $\star = +,-$. The random variable $x$ is Gamma distributed with degree $M$ and has a diversity gain of $M$. The random variables $y$ and $z$ are independent and both have a diversity gain of $2(N - J + 1)$. Then, $y + z$ has a diversity gain of $4(N - J + 1)$. Let $k = 2$ in Lemma 1. The achievable diversity gain of ConcurrentR→D-ICD is $\min\{M, 4(N - J + 1)\} = 4 \min\{J, N - J + 1\}$.

For MARNs with a general $M$, the relay encodes the information of one source using one quasi-orthogonal DSTC with ABBA structure [25, 26] and forwards each codeword by $\lfloor \frac{M J}{J} \rfloor$ of its antennas. The destination conducts the multi-user IC technique [21]. The proof for this general case is a straightforward extension of the proofs for the cases that $M = 2J$ and $M = 4J$. Thus, the diversity result in Theorem 2 follows.

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Table 1  The diversity gain and symbol rate performance for linear schemes. (The schemes marked with * are proposed in this paper.)

| Protocol                  | Diversity Gain                      | Symbol Rate             | Relay Backward CSI |
|---------------------------|-------------------------------------|-------------------------|-------------------|
| Concurrent$_{S\rightarrow R\rightarrow D}$-IC$_D$ * | $\leq M - J + 1$                    | $\frac{1}{2}$           | No                |
| Concurrent$_{R\rightarrow D}$-IC$_D$ *         | $\min\{M, \left\lfloor \frac{M}{J} \right\rfloor (N - J + 1)\}$ | $\frac{1}{J+1}$        | Yes               |
| Concurrent$_{S\rightarrow R}$-IC$_R$             | $M - J + 1$                         | $\frac{1}{J+1}$        | Yes               |
| TDMA$_{S\rightarrow R\rightarrow D}$              | $M$                                 | $\frac{1}{J+1}$        | No                |

Figure 1  System block diagram of Concurrent$_{S\rightarrow R\rightarrow D}$-IC$_D$.

Figure 2  Equivalent system of Concurrent$_{S\rightarrow R\rightarrow D}$-IC$_D$ with zero-forcing at the relay.

Figure 3  System block diagram of Concurrent$_{R\rightarrow D}$-IC$_D$. 
Figure 4  BER performance of Concurrent $S \rightarrow R \rightarrow D$-IC$_D$, using BPSK modulation.

Figure 5  BER performance of Concurrent $R \rightarrow D$-IC$_D$, using BPSK modulation.
Figure 6  Performance comparison in a $1_2 \times 2_1 \times 2_1$ MARN, 1 bit/source/channel use for all schemes.

Figure 7  Performance comparison in a $1_2 \times 2_1 \times 3_1$ MARN, 1 bit/source/channel use for all schemes.
Figure 8  Performance comparison in a $1_2 \times 4_1 \times 3_1$ MARN, 1 bit/source/channel use for all schemes.