Auctioning Incentive Contracts

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This paper draws a remarkably simple bridge between auction theory and incentive theory. It considers the auctioning of an indivisible project among several firms. The firms have private information about their future cost at the bidding stage, and the selected firm ex post invests in cost reduction. We show that (1) the optimal auction can be implemented by a dominant strategy auction that uses information about both the first bid and the second bid; (2) the winner faces a (linear) incentive contract; (3) the fixed transfer to the winner decreases with his announced expected cost and increases with the second lowest announced expected cost; and (4) the share of cost overruns borne by the winner decreases with the winner’s announced expected cost.

I. Introduction

In Laffont and Tirole (1986) we studied the optimal contract between a government and a firm realizing a project for the government. In our model both the firm and the government are risk neutral, but the government faces a double asymmetry of information, not knowing the exact productivity parameter of the firm and not observing its effort level. With socially costly transfers the optimal strategy for a
utilitarian government is to offer a menu of so-called incentive contracts each composed of a fixed payment, a function of the announced cost, and a linear sharing of overruns, that is, of the difference between ex post observed costs and announced costs. The optimal contract trades off the truthful elicitation of information about productivity (which would lead to a cost-plus contract) and the ex post inducement of an appropriate level of effort (which would lead to a fixed-price contract).

When several firms are possible candidates to realize the project, it has been argued by Demsetz (1968) that competitive bids should be elicited. In this paper we extend our analysis to the case of multiple firms. However, the model is meant to formalize a one-shot project, and we leave aside the questions raised by the auctioning of an activity repeated over time\(^1\) (see Williamson 1976; Laffont and Tirole 1987).

In Section II, we set up a natural monopoly model; \(n\) firms can realize a project that has a fixed large value for the government. Under complete information, the government would select the most efficient firm and impose an optimal level of effort. Section III characterizes the optimal Bayesian auction for a utilitarian government under asymmetric information. The best auction awards the project to the firm that announces the smallest expected cost. The type of contract between the government and the selected firm is then similar to the one derived in the case of a single firm. It can be written as the sum of a fixed payment function of the announced cost and of a linear sharing of overruns; the coefficient characterizing this sharing rule is a function of the announced cost. This dichotomy property is the main result of this paper, which can then rely on Laffont and Tirole (1986) for a detailed study of the optimal contract. Section IV constructs a dominant strategy auction that implements the optimum, and Section V presents conclusions.

Before presenting the formal analysis, we give a heuristic explanation of the dichotomy property. For this it is useful to recall the intuition for the case of a single bidder. Suppose that the firm has cost \(C = \beta - e + \epsilon\), where \(\beta\) is the firm’s cost parameter, \(e\) is the effort of its manager, and \(\epsilon\) is a cost disturbance. The government observes \(C\) but not \(\beta\) and \(e\), which are private information to the manager. The government has beliefs \(f(\beta)\) with cumulative distribution function \(F(\beta)\) on \([\underline{\beta}, \bar{\beta}]\) about the firm’s cost parameter. Finally, let \(\psi(e)\) denote the increasing and convex (monetary) disutility of effort of the manager and \(U(\beta)\) the manager’s rent in the firm (where the outside opportunity is normalized to zero). This rent decreases with \(\beta\) at rate \(\psi'(e(\beta))\) because the manager can always exert effort \(e(\beta) = d\beta\) when

\(^1\) In Laffont and Tirole (1985) we study dynamic contracting but with a single firm.
the cost parameter is $\beta - d\beta$ and yield the same (stochastic) cost structure that would obtain were the parameter $\beta$. Thus he saves a disutility of effort $\psi'(e(\beta))d\beta$. This implies that the manager's rent (which is supposed to be costly to the government) is entirely determined by the effort function: $dU/d\beta = -\psi'(e(\beta))$ with $U(\bar{\beta}) = 0$. Now suppose that the government changes the incentive scheme slightly so as to increase the effort $e(\beta)$ of type $\beta$ by a unit amount. The social gain of doing so is proportional to $[1 - \psi'(e(\beta))]f(\beta)$ (because the cost saving is equal to unity, and the cost reduction as well as the marginal disutility of effort applies to a fraction proportional to the density at $\beta$). On the other hand, with probability $F(\beta)$, the cost parameter is lower than $\beta$ and the manager's rent is increased by $\psi''(e(\beta))$. Hence, for the optimal incentive scheme, $[1 - \psi'(e(\beta))]f(\beta) = kF(\beta)\psi''(e(\beta))$, where $k$ is a proportionality coefficient that depends on the social cost of transfers from the government to the firm. This means that the effort and therefore the slope of the incentive scheme that induces it are determined by the "hazard rate" $F(\beta)/f(\beta)$.

Next consider an auction. Let $\beta$ denote the winner's cost parameter. Bidding changes the government's opportunity cost of controlling the winner. The government could replace the winner by the second most efficient firm, with cost parameter $\beta^* > \beta$. Everything is as if the winner were regulated under a smaller extent of asymmetric information, with the government knowing that it has a parameter in ($\bar{\beta}$, $\beta^*$) instead of ($\beta$, $\bar{\beta}$). But note that the hazard rate is invariant to upward truncations of the distribution. Hence there is no gain from changing the slope of the incentive scheme faced by the winner. Bidding only reduces the transfer.

A precursor to our paper is McAfee and McMillan (1986) (see also Fishe and McAfee 1983). Their paper looks at the trade-off among giving the agent incentives for cost reduction, stimulating bidding competition, and sharing risk. They restrict the contract to be linear in observed cost and in the successful bidder's bid ("first-bid auction"). Our paper shows that, under risk neutrality, the optimal contract can indeed be taken to be linear in observed costs. However, the coefficient of risk sharing should decrease with the successful bidder's bid.

Riordan and Sappington (1985) consider a framework complementary to ours. Bidders submit bids and then learn their true marginal cost. Each firm's private information at the bidding stage is a signal correlated with final cost. Thus Riordan and Sappington are interested in repeated adverse selection as opposed to ex ante adverse selection and ex post moral hazard. In the independent signals case they consider two classes of auctions. One is a first-price auction, in which the successful bidder's allocation depends only on his bid, not on the other bidder's bid. The other is a second-price auction, in
which the winner gets the terms corresponding to the second bid, independently of the value of the first bid. Riordan and Sappington show that the revenue equivalence theorem (equivalence between the first- and second-bid auctions) does not hold.

II. The Model

We assume that \( n \) firms can participate in the auction. Each firm \( i, i = 1, \ldots, n, \) is able to realize the indivisible project for a cost:

\[
C^i = \beta^i - e^i, \tag{1}
\]

where \( e^i \) is manager \( i \)'s ex post effort level and \( \beta^i \) is firm \( i \)'s efficiency parameter. We will later show that adding a forecast or auditing error (so that \( C^i = \beta^i - e^i + \epsilon^i \)) does not affect our conclusions.

The efficiency parameters \( (\beta^i) \) are drawn independently from the same distribution with a cumulative distribution function \( F(\cdot) \) on the interval \([\beta, \overline{\beta}]\) and a differentiable density function \( f \) that is bounded below by a strictly positive number on \([\beta, \overline{\beta}]\). Moreover, \( F(\cdot) \) is common knowledge and satisfies the monotone hazard rate property.

**Assumption 1.** \( F/f \) is nondecreasing.

Assumption 1 facilitates the analysis by ensuring that the optimal incentive scheme in the one-firm problem fully separates the potential types of the firm (no bunching).

Manager \( i, i = 1, \ldots, n, \) has the utility function

\[
t^i = \psi(e^i), \tag{2}
\]

where \( t^i \) is the net (i.e., in addition to cost) monetary transfer that he receives from the government and \( \psi(e^i) \) is his disutility of effort, \( \psi' > 0, \psi'' > 0 \). Moreover, we postulate the following assumption.

**Assumption 2.** \( \psi'' \geq 0 \).

Assumption 2 facilitates the analysis by ensuring that in the one-firm problem there is no need for using stochastic incentive schemes (and similarly in the \( n \)-firm case; see App. A).

Let \( S \) be the social utility of the project. Under complete information the government (regulator) selects the firm with the lowest parameter \( \beta \), say firm \( i \), and makes a transfer only to that firm. The gross payment made by the government to firm \( i \) is \( t^i + C^i \). The social cost of one unit of money is \( 1 + \lambda, \lambda > 0 \), so that the social net utility of the project for a utilitarian government is

\[
S - (1 + \lambda)(t^i + C^i) = S - \lambda U^i - (1 + \lambda)[C^i + \psi(e^i)], \tag{3}
\]

where \( U^i \) denotes firm \( i \)'s utility level.

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2 If assumption 1 is not satisfied, the methods described in Guesnerie and Laffont (1984) must be used.
Procurement in which the principal is a private firm would lead to a different objective function of the type profits minus transfers. None of our results would be affected; the important feature is that the principal dislikes transfers.

Each firm has outside opportunities (its individual rationality level) normalized to zero. Under complete information, the optimal level of effort of the firm selected should be determined by $\psi'(e^x) = 1$ (marginal disutility of effort equals marginal cost savings), and the net transfer to that firm should be $\psi(e^x)$.

However, the regulator observes neither $\beta^i$ nor $e^i$. Ex post, he observes the realized cost of the firm that has been chosen to carry out the project. To select a firm and regulate it, he organizes an auction as explained below.

III. The Optimal Bayesian Auction

Firms bid simultaneously by announcing efficiency parameters $\hat{\beta} = (\hat{\beta}^1, \ldots, \hat{\beta}^n)$. Let $x^i(\hat{\beta})$ be the probability that firm $i$ is selected to carry out the project. We must have

$$\sum_{i=1}^{n} x^i(\hat{\beta}) \leq 1 \quad \text{for any } \hat{\beta},$$

$$x^i(\hat{\beta}) \geq 0, \quad i = 1, \ldots, n, \text{ for any } \hat{\beta}. \quad (4)$$

A. Firms' Bidding Behavior

Since both the government and managers are risk neutral, there is no need to consider random transfers. Let $t^i(\hat{\beta})$ be manager $i$'s expected transfer as a function of the announced bids. Ex ante, manager $i$'s expected utility is

$$E_{\hat{\beta}^{-i}} [t^i(\hat{\beta}) - x^i(\hat{\beta})\psi(e^x)], \quad (6)$$

where $\hat{\beta}^{-i} = (\hat{\beta}^1, \ldots, \hat{\beta}^{i-1}, \hat{\beta}^{i+1}, \ldots, \hat{\beta}^n)$. The ex post observability of cost enables us to rewrite (6) as

$$E_{\hat{\beta}^{-i}} \{t^i(\hat{\beta}) - x^i(\hat{\beta})\psi[\beta^i - C^i(\hat{\beta})]\}, \quad (7)$$

where $C^i(\hat{\beta})$ is the cost level that the government requires the firm to reach given announcements $\hat{\beta}$.

We look for mechanisms $(x^i(\hat{\beta}), C^i(\hat{\beta}), t^i(\hat{\beta}))$ that induce a truth-
telling Bayesian Nash equilibrium. A necessary condition for truth telling is, at $\hat{\beta}^i = \beta^i$,

$$
\frac{d}{d\hat{\beta}^i} E_{\hat{\beta}^i} t'(\hat{\beta}) = \frac{d}{d\beta^i} E_{\beta^i} \{x'(\hat{\beta})\psi[\beta^i - C'(\hat{\beta})]\}
$$

(8)
differentiable almost everywhere for $i = 1, \ldots, n$. (Differentiability of these expectations almost everywhere results from their monotonicity, which in turn stems from incentive compatibility; see App. B.)

Moreover, if the following condition (9) is satisfied, (8) can be shown to be sufficient (see App. B):

$$
\frac{dC^i}{d\beta^i} \geq 0, \quad \frac{d}{d\beta^i} E_{\beta^i} x'(\beta) \leq 0.
$$

(9)

In what follows we first ignore the second-order conditions (9) and check later that they are indeed satisfied at the optimum.

Let $U^i(\beta^i)$ be manager $i$’s expected utility level when telling the truth:

$$
U^i(\beta^i) = E_{\beta^i} \{t^i(\beta) - x^i(\beta)\psi[\beta^i - C^i(\beta)]\}.
$$

(10)

From (7) and (8) we have

$$
\dot{U}^i(\beta^i) = -E_{\beta^i} \{x'(\beta)\psi'[\beta^i - C^i(\beta)]\}.
$$

(11)

B. Optimal Auction

From (11) we see that $U^i$ is nonincreasing in $\beta^i$ so that manager $i$’s individual rationality level is satisfied if it is satisfied at $\beta^i = \bar{\beta}$. As the government’s objective function is decreasing in $U^i$, it will be tight at $\bar{\beta}$, that is,

$$
U'(\bar{\beta}) = 0, \quad i = 1, \ldots, n.
$$

(12)

Suppose for simplicity that $S$ is so large that even a firm with type $\bar{\beta}$ is worth selecting.

The maximand of a utilitarian government is

$$
\left(\sum_{i=1}^n x^i\right)S - (1 + \lambda) \sum_{i=1}^n t^i - (1 + \lambda) \sum_{i=1}^n x^iC^i + \sum_{i=1}^n (t^i - x^i\psi')
$$

$$
= \left(\sum_{i=1}^n x^i\right)S - \lambda \sum_{i=1}^n U^i - (1 + \lambda) \sum_{i=1}^n x^i(C^i + \psi').
$$

(13)
The first term on the left-hand side of (13) is the expected surplus from the project, the second the social cost of transfers, the third the social cost of the expected product cost (the sum of these three terms is equal to the net expected consumer surplus), and the fourth the expected sum of utilities of the firm’s managers.

The government’s optimization problem under incomplete information is then

$$\max_{(x'(\cdot),C'(\cdot),U''(\cdot))} E \left[ \sum_{i=1}^{n} x'(\beta) \right] S - \lambda \sum_{i=1}^{n} U'(\beta')$$

$$- (1 + \lambda) \sum_{i=1}^{n} x'(\beta) \{ C'(\beta) + \psi[\beta' - C'(\beta)] \}$$

subject to

$$\hat{U}'(\beta') = - E_{\beta'} \{ x'(\beta) \psi'[\beta' - C'(\beta)] \}$$

differentiable almost everywhere, $i = 1, \ldots, n$,

$$U'(\beta) = 0, \quad i = 1, \ldots, n,$$

$$- \sum_{i=1}^{n} x'(\beta) + 1 \geq 0 \quad \text{for any } \beta,$$

$$x'(\beta) \geq 0 \quad \text{for any } \beta, \quad i = 1, \ldots, n.$$  

Without loss of generality we can assume that $\psi(0) = 0$ and $\psi'(0) = 0$, otherwise in the next proofs write $\psi(x) = \psi(0) + \hat{\psi}(x)$ with $\hat{\psi}(0) = 0$, $\psi'(x) = \psi'(0) + \hat{\psi}'(x)$ with $\hat{\psi}'(0) = 0$.

We now show that program (14) can be simplified by considering functions $C'(\beta)$ that are functions of $\beta^i$ only.

**Lemma 1.** Under assumption 2, at the optimum, $C^{*i}(\beta)$ is a function of $\beta^i$ only, $i = 1, \ldots, n$.

The proof of lemma 1 is provided in Appendix C. The intuition is that making $C^{*i}(\beta)$ depend on announcements other than $\beta^i$, in the absence of correlation between the parameters, amounts to a stochastic incentive scheme. Such a stochastic incentive scheme has two drawbacks. First, the manager, whose disutility of effort function is convex, dislikes randomness in effort. So the objective function (14) is decreased. Second, if $\psi'' \geq 0$, $\psi'$ is convex and the slope of the manager’s rent as a function of the firm’s efficiency (given by [15]) increases.

For given optimal $x^{*i}(\cdot)$ and therefore given $X^{*i}(\cdot)$, where $X^{*i}(\beta') = E_{\beta'} \{ x^{*i}(\beta) \}$, the optimization with respect to $C'(\beta')$ can be decom-
posed into \( n \) programs of the type

\[
\max_{\beta} \int_{\beta} \left( -\lambda U'(\beta^i) - (1 + \lambda)X'(\beta^i)[C'(\beta^i) + \psi[\beta^i - C'(\beta^i)]] \right) f(\beta^i) d\beta^i
\]

subject to

\[
\dot{U}^i(\beta^i) = -X'(\beta^i)\psi'[\beta^i - C'(\beta^i)],
\]

\[
U^i(\beta) = 0.
\]

When \( U^i \) is considered as the state variable and \( C^i \) as the control variable, the Hamiltonian of this program is

\[
H^i = (-\lambda U'(\beta^i) - (1 + \lambda)X'(\beta^i)[C'(\beta^i) + \psi[\beta^i - C'(\beta^i)]] f(\beta^i) + \mu^i(\beta^i)[-X'(\beta^i)\psi'[\beta^i - C'(\beta^i)].
\]

The Pontryagin principle gives

\[
\dot{\mu}^i(\beta^i) = \lambda f(\beta^i),
\]

\[
(1 + \lambda)(1 - \psi'[\beta^i - C'(\beta^i)])\mu^i(\beta^i) = \mu^i(\beta^i)\psi'[\beta^i - C'(\beta^i)],
\]

\[
\mu^i(\beta) = 0.
\]

Integrating (23) and using the transversality condition (25), we get

\[
\mu^i(\beta^i) = \lambda F(\beta^i).
\]

The optimal cost function \( C^*_i(\beta^i) \) is therefore determined by

\[
(1 + \lambda)[1 - \psi'[\beta^i - C^*_i(\beta^i)] = \lambda \frac{F(\beta^i)}{f(\beta^i)} \psi'[\beta^i - C^*_i(\beta^i)].
\]

For an intuitive derivation of (27), see the Introduction.

We then substitute the \( C^*_i(\beta^i) \) into (14) to solve for the optimal \( x'(\beta) \).

Integrating (20), we can write the Lagrangian as

\[
E_\beta \left[ \sum_{i=1}^{n} x^i(\beta) \right] S - \lambda \sum_{i=1}^{n} \int_{\beta} X'(\beta^i)\psi'[\beta^i - C^*_i(\beta^i)] d\beta^i
\]

\[
- (1 + \lambda) \sum_{i=1}^{n} x^i(\beta)[C^*_i(\beta^i) + \psi[\beta^i - C^*_i(\beta^i)]]
\]

\[
+ E_\beta \gamma(\beta) \left[ 1 - \sum_{i=1}^{n} x^i(\beta) \right] + E_\beta v'(\beta)x'(\beta),
\]

where \( \gamma(\beta)f(\beta^1) \ldots f(\beta^n) \) is the multiplier associated with the constraint \( 1 - \sum_{i=1}^{n} x^i(\beta) \geq 0 \) and \( v'(\beta)f(\beta^1) \ldots f(\beta^n) \) is the multiplier associated with \( x'(\beta) \geq 0 \).
When we integrate with respect to \( \beta^{-i} \), the coefficient of \( X'(\beta^i) \) is then

\[
f(\beta')S - \lambda F(\beta')\psi'[\beta^i - C'^i(\beta^i)] - (1 + \lambda)
\times \{C'^i(\beta^i) + \psi[\beta^i - C'^i(\beta^i)]\} f(\beta') - f(\beta') \left[ E_{\beta^{-i}} \gamma(\beta) - E_{\beta^{-i}} \nu'(\beta) \right].
\]

The function \( x'(\beta) \) can equal one only if \( \nu'(\beta) = 0 \) by complementary slackness. For \( i \) to be selected we must have, dividing (28) by \( f(\beta') \),

\[
S - (1 + \lambda)[C'^i(\beta^i) + \psi[\beta^i - C'^i(\beta^i)]]
- \lambda \frac{F'(\beta')}{f(\beta')} \psi'[\beta^i - C'^i(\beta^i)] \geq E_{\beta^{-i}} \gamma(\beta) \geq 0.
\]

From our assumptions, the Hamiltonian is concave; that is, \( (1 + \lambda)\psi''f + \lambda F\psi'' \geq 0 \). Then, differentiating (27), we check that assumption 1 is sufficient for \( C'^i \) to be nondecreasing in \( \beta^i \). Consequently, using assumption 1 again, we see that the left-hand side of (29) is a nondecreasing function of \( \beta^i \). Therefore, we must choose \( x'^i(\beta) \) as

\[
x'^i(\beta) = 1 \quad \text{if} \quad \beta^i < \min_{k \neq i} \beta^k,
\]

\[
x'^i(\beta) = 0 \quad \text{if} \quad \beta^i > \min_{k \neq i} \beta^k.
\]

Hence, the \( x'^i(\cdot) \) and \( X'^i(\beta') \) are nonincreasing almost everywhere and \( C'^i(\beta') \) are nondecreasing in \( \beta^i \) almost everywhere. The neglected second-order conditions (9) are therefore satisfied. We have obtained the following theorem.

**Theorem 1.** Under assumptions 1 and 2, an (interior) optimal auction awards the contract to the firm announcing the lowest cost parameter. The cost level (and consequently the effort level) required from the manager selected is a solution of (27). The transfer to the firm must be a solution of (8) and (12).

This result deserves some comments. The optimal auction is deterministic. The level of effort determined by equation (27) is below the optimal level and is decreasing in the cost parameter. Actually, the effort level is identical to the one we obtained with only one firm (Laffont and Tirole 1986). Competition for the best firm amounts to a truncation of the interval \( (\beta, \beta) \) to \( (\beta, \beta^*) \), where \( \beta^* \) is the second-lowest bid. As \( n \) grows, the winner's productivity parameter \( \beta^i \) becomes close in probability to \( \beta \), \( F(\beta') \) becomes close to zero, and the effort level is close to the optimal one. Competition asymptotically
solves the moral hazard problem by solving the adverse selection problem (this extreme result clearly relies on risk neutrality).¹

IV. Implementation by a Dominant Strategy Auction

From (10) the system of transfers of the optimal Bayesian auction is such that

\[ t^{*i}(\beta') = E_{\beta'} t^{*i}(\beta) = U^{*i}(\beta') + X^{*i}(\beta')\psi(\beta' - C^{*i}(\beta')). \]  

(31)

Using (11) and (12) we have

\[ t^{*i}(\beta') = \int_{\beta'}^{\bar{\beta}} X^{*i}(\tilde{\beta}')\psi(\tilde{\beta}' - C^{*i}(\tilde{\beta}'))d\tilde{\beta}' \]

\[ + X^{*i}(\beta')\psi(\beta' - C^{*i}(\beta')). \]

(32)

We now construct a dominant strategy auction of the Vickrey type that implements the same cost function (and effort function) and also selects the most efficient firm. Let

\[ l'(\beta) = \psi(\beta' - C^{*i}(\beta')) + \int_{\beta'}^{\bar{\beta}} \psi'(\tilde{\beta}' - C^{*i}(\tilde{\beta}'))d\tilde{\beta}' \]

(33)

if \( \beta' = \min_k \beta^k \) with \( \beta^j = \min_{k \neq i} \beta^k \),

\[ l'(\beta) = 0 \quad \text{otherwise.} \]

(34)

When firm \( i \) wins the auction, its transfer is equal to the individually rational transfer plus the rent the firm gets when the distribution is truncated at \( \beta' \). Therefore, we are back to the monopoly case, except that the truncation point \( \beta' \) is random. But for any truncation point, truth telling is optimal in the monopoly case. Therefore, truth telling is a dominant strategy (see App. D).

If \( \beta' = \bar{\beta} \), from (33), \( U'(\bar{\beta}) = 0 \) for any \( \beta' \) and therefore also in expectation. The dominant strategy auction is individually rational. Using the distribution of the second-order statistics, we can easily check that \( E_{\beta'} l'(\beta) = t^{*i}(\beta') \). Therefore, the dominant strategy auction costs the same expected transfers as the optimal Bayesian auction.

Remark 1.—Equation (33) enables us to compute the gain in ex-

¹ If there is a loss involved in the ex post observability of costs, for \( n \) large enough, it will be better to use a fixed-price contract rather than a contract depending on ex post cost, which is very close to a fixed-price contract.

² Note that as \( n \) is large, \( \beta' \) is close to \( \bar{\beta}' \) in probability and the transfer converges to the disutility of the "second-lowest bidder."
expected transfer from having an auction. Consider the following thought experiment: Fix the distribution of the winner's cost parameter, and consider the two situations in which the winner must bid against other bidders or is regulated as a monopoly. The gain in transfer from the auction is equal to

\[ G = \psi[\beta' - C^*(\beta')] + \int_{\beta'}^{\beta'} \psi'[\tilde{\beta}' - C^*(\tilde{\beta}')] d\tilde{\beta}' - \tilde{\tau}(\beta) \]

\[ = \int_{\beta'}^{\beta'} \psi'[\tilde{\beta}' - C^*(\tilde{\beta}')] d\tilde{\beta}'. \]

The expected gain is thus \( E_{\beta'} G \). The second-order statistic in the sample of \( n \) parameters, \( \beta' \), has distribution given by

\[ \text{prob}(\beta' \leq x) = 1 - [1 - F(x)]^n - nF(x)[1 - F(x)]^{n-1}. \]

For instance, for a uniform distribution on \((\beta, \bar{\beta})\) and quadratic disutility of effort \( \psi(e) = e^2/2 \), it can be shown that

\[ E_{\beta'} G = \Delta \beta - \frac{2\Delta \beta}{n + 1} - \frac{\lambda}{2(1 + \lambda)} \Delta \beta^2 \left[ 1 - \frac{6}{(n + 1)(n + 2)} \right], \]

where \( \Delta \beta \equiv \bar{\beta} - \beta \).

Remark 2.—An alternative, two-stage way of implementing the optimal allocation is to ask the firms how much they are willing to pay to be regulated as a monopolist. In the second stage the winner is asked to reveal its \( \beta \) (or, alternatively, it is reimbursed as a function of observed cost). To see that this procedure is equivalent to the dominant strategy auction above, suppose for instance that the first stage is a Vickrey auction. A firm with type \( \beta \) is willing to pay

\[ \int_{\beta}^{\beta'} \psi'[\tilde{\beta}' - C^*(\tilde{\beta}')] d\tilde{\beta}' \]

to become a monopolist. This number is therefore its bid in the Vickrey auction. The first-stage price to be paid by the winner is the second bid, that is,

\[ \int_{\beta}^{\beta'} \psi'[\tilde{\beta}' - C^*(\tilde{\beta}')] d\tilde{\beta}', \]

which yields the same overall transfer as (33).

Remark 3.—McAfee and McMillan (1985) have independently extended the Laffont-Tirole (1986) model to competitive bidding. They focus on the equivalent of a first-bid auction, in which the incentive scheme of the winner depends only on its bid. They show that the effort level is identical to the monopoly one. Together with our optimality and dominant strategy results, their result demonstrates the analogue of the revenue equivalence theorem (which says that for
auctions on a good rather than a contract, the optimum can be implemented by an auction using only information about the high bid as well as by a dominant strategy auction using information about the first and second bids).

V. Optimality of Linear Contracts

Implicit in the approach with mechanisms that we have followed is the fact that if the selected firm’s ex post cost differs from $C^*(\beta^i)$, it incurs an infinite penalty. However, this mechanism is not robust to the introduction of a random disturbance $\epsilon^i$ (uncorrelated with $\beta^i$ into the cost function $C' = \beta^i - \epsilon^i + \epsilon^i$).

We will now rewrite the transfer function in a form that is closer to actual practice and is robust to cost disturbances. Let

$$s'(\beta, C) = i(\beta) + K(\beta^i)[C^*(\beta^i) - C],$$

where $K(\beta^i) = \psi[\beta^i - C^*(\beta^i)] \in [0, 1]$, noting that the optimal cost function $C^*$ is independent of $i$.

It is easy to check that (35) still induces the appropriate ex post effort level and truth telling. The transfer is now decomposed into a transfer $l'(\beta)$ computed at the time of the auction and into a sharing of overruns (meaningful when costs are random) determined by the coefficient $K(\beta^i)$.

The auction selects the best firm on the market and awards the winner an incentive contract to induce a second-best level of effort. Thus we generalize the result in Laffont and Tirole (1986) that the optimal allocation can be implemented by offering the firm(s) a menu of linear contracts.

In particular, the fixed transfer $l^i$ decreases with the winner’s bid ($\beta^i$) and increases with the second bid ($\beta^j$). The slope of the incentive scheme $K$ decreases with the winner’s bid. Thus the contract moves toward a fixed-price contract when the winning bid decreases.

Finally, we note that firm $i$’s expected cost, if it is chosen, is an increasing function of $\beta^i$. This implies that the optimal auction can have the firms announce expected cost $C^ai$. Equation (35) can then be rewritten as

$$s^i(C^a, C) = i(C^ai, C^aj) + K(C^ai)(C^ai - C),$$

where $C^ai \leq C^aj \leq C^ak$ for all $k \neq i, j$.

VI. Conclusion

This paper draws a remarkably simple bridge between auction theory and incentive theory. The optimal auction truncates the uncertainty
range for the successful bidder’s intrinsic cost from \([\beta, \bar{\beta}]\) to \([\beta', \bar{\beta}']\), where \(\beta^j\) is the second bidder’s intrinsic cost. The principal offers the successful bidder the optimal incentive contract for a monopolist with unknown cost in \([\beta, \bar{\beta}']\). Previous work then implies that the successful bidder faces a linear cost reimbursement contract, the slope of which depends only on his bid, and not on the other bidder’s bids.

Technically, the optimal auction can be viewed by each bidder as the optimal monopoly contract with a random upward truncation point. This trivially implies that the auction is a dominant strategy auction. Our dichotomy—the winner’s fixed transfer depends on the first and the second bids, and the slope of his incentive scheme depends only on the first bid—comes from the fact that the ex post incentive constraint is downward binding while bidding leads to an upward truncation of the conditional distribution. Finally, we should note that the optimal allocation can be implemented by a dominant strategy auction that uses information about the first and second bids.

It would be desirable to develop the testable implications of our model. While it may be difficult to observe or find proxies for some variables (such as preferences), some others may be available, such as the number of bidders, the distribution of bids, or proxies for the degree of asymmetric information. For instance, our model indicates that, ceteris paribus, the contract resembles more a fixed-price contract, the higher the number of bidders and the lower the degree of asymmetric information. Of course, the number of bidders is not always exogenous. If it depends on a fixed cost of obtaining a private estimate of \(\beta\), the previous conclusion carries over. But it also may increase with the uncertainty (because the firm’s rent is higher, the higher the asymmetry of information). Then the degree of uncertainty directly increases the probability that a contract resembles a cost-plus one but indirectly reduces the latter probability through the effect on the number of bidders. Although a careful analysis of its empirical implications is outside the scope of this paper, we feel that the simplicity of our model would warrant such investigations.

Appendix A

Nonoptimality of Random Incentive Schemes

In the one-firm problem, the optimization program of the government is

\[
\max_{C(\cdot), U(\cdot)} \int_\beta^\bar{\beta} (S - \lambda U(\beta) - (1 + \lambda)\psi(\beta - C(\beta)))f(\beta) d\beta
\]

(A1)

\[6\] It would also be useful to introduce risk aversion. For the monopoly model, see Laffont and Tirole (1986, sec. 6) and Baron and Besanko (1985).
subject to
\[ \dot{U}(\beta) = -\psi'[\beta - C(\beta)], \]  
\[ U(\beta) = 0. \]  

Suppose that the optimal control \( C^*(\beta) \) is random. We show that replacing it by \( EC^*(\beta) \) improves the program, a contradiction.

From Jensen’s inequality and \( \psi'' > 0 \),
\[-E\psi[\beta - C^*(\beta)] < -\psi[\beta - EC^*(\beta)].\]

The maximand can be replaced by the larger quantity
\[ \int_{\beta} (S - \lambda U(\beta) - (1 + \lambda)[EC^*(\beta) + \psi[\beta - EC^*(\beta)]]f(\beta)d\beta. \]

If \( C^*(\beta) \) is random, the constraint (A2) must be written as
\[ \dot{U}(\beta) = -E\psi'[\beta - C^*(\beta)]. \]

From Jensen’s inequality and \( \psi'' \geq 0 \),
\[-E\psi'[\beta - C^*(\beta)] \leq -\psi[\beta - EC^*(\beta)].\]

As the maximand is decreasing in \( U \), the value of the program is increasing if we replace \(-E\psi'[\beta - C^*(\beta)]\) by \(-\psi[\beta - EC^*(\beta)]\).

The same property holds in the competitive case. Q.E.D.

Appendix B

Characterization of Truth Telling

To minimize technicalities, we assume that the allocation is almost everywhere differentiable. It can be shown that it is indeed the case. For truth telling to be a best Bayesian Nash strategy, \( \beta' \) must be the best answer for firm \( i \) when it assumes that the other firms are truthful, that is, must be the solution of
\[
\begin{align*}
\max_{\beta'} U(\beta', x', C', t') &= E_{\beta'} \{t'(\beta', \beta^-) \} \\
&- x'(\beta', \beta^-)\psi[\beta' - C'(\beta', \beta^-)].
\end{align*}
\]

The objective function satisfies the conditions \( CS_-, CS_+ \) of Guesnerie and Laffont (1984), that is,
\[
\begin{align*}
\frac{\partial}{\partial \beta'} \left( \frac{\partial U}{\partial x'} \right) &= -E_{\beta^-} \psi' < 0, \\
\frac{\partial}{\partial \beta'} \left( \frac{\partial U}{\partial C'} \right) &= E_{\beta^-} x'\psi'' > 0.
\end{align*}
\]

Transfers must satisfy the first-order condition of incentive compatibility:
\[
\frac{\partial}{\partial \beta'} E_{\beta^-} \{t'(\beta)\} = \frac{\partial}{\partial \beta'} E_{\beta^-} \{x'(\beta)\psi[\beta' - C'(\beta)]\}. \tag{B1}
\]

If \( C' \) is nondecreasing in \( \beta' \) almost everywhere and if \( E_{\beta^-} x'(\beta) \) is nonincreasing in \( \beta' \) almost everywhere, then following the proof of theorem 2 in Gues-
nerie and Laffont (1984), we show that the functions $x'(\beta)$ and $C'(\beta)$ can be implemented by transfer solutions of (B1).

Appendix C

Proof of Lemma 1

Suppose that $C^k_i$ is not a function only of $\beta'$. We show that the optimal value of program (14) can be increased, a contradiction. Denote $X'(\beta') \equiv E_{\beta'} \{x'(\beta')\}$ and choose $C'(\beta')$ such that

$$C'(\beta') = E_{\beta' \mid x'(\beta') = 1} C'(\beta). \quad \text{(C1)}$$

Observe that from the linearity of the program in $x'(\beta)$, the optimum can be chosen so that the $x'(\beta)$ are zeros and ones. Then, since $\psi(0) = 0$,

$$E_{\beta' \mid x'(\beta') = 1} \{x'(\beta)\psi[\beta' - C'(\beta)]\} = X'(\beta') E_{\beta' \mid x'(\beta') = 1} \psi[\beta' - C'(\beta)]. \quad \text{(C2)}$$

Since $\psi' > 0$ by Jensen’s inequality, the expression in (C2) exceeds

$$X'(\beta') \psi \left[ E_{\beta' \mid x'(\beta') = 1} \{\beta' - C'(\beta)\} \right] = X'(\beta') \psi[\beta' - C'(\beta')]. \quad \text{(C3)}$$

The maximand of (14) can therefore be replaced by the larger quantity

$$E \left( \sum_{i=1}^{n} x^i \right) S - \sum_{i=1}^{n} E_{\beta' \mid x'(\beta') = 1} \{\lambda U'(\beta') + (1 - \lambda)X'(\beta')\{C'(\beta') + \psi[\beta' - C'(\beta')]\} \}. \quad \text{(C4)}$$

Moreover, the constraints can be relaxed. Since $\psi'(0) = 0$ and, by assumption 2, $\psi'' \geq 0$, we have similarly

$$E_{\beta' \mid x'(\beta') = 1} \{x'(\beta)\psi[\beta' - C'(\beta)]\} = X'(\beta') \psi[\beta' - C'(\beta')]. \quad \text{(C5)}$$

Therefore, since the objective function is decreasing in $U'$, the constraints are relaxed if we replace (15) by

$$U'(\beta') = -X'(\beta') \psi[\beta' - C'(\beta')]. \quad \text{(C6)}$$

Appendix D

Implementation in Dominant Strategy

Let us check that indeed truth telling is a dominant strategy. Suppose first that, by announcing the truth, manager $i$ wins the auction. Since the auction is individually rational and losers get nothing, he cannot gain by lying and losing the auction. What about an answer $\tilde{\beta}' \neq \beta'$ but such that $\tilde{\beta}' < \min_{k \neq i} \beta^k$?

Manager $i$'s program is

$$\max_{\beta' \leq \min_{k \neq i} \beta^k} \{ \tilde{u}'(\tilde{\beta}', \beta' - i) - \psi[\beta' - C^k(\tilde{\beta}')] \}. \quad \text{(D1)}$$
The first-order condition for an interior solution is

$$\psi'[\tilde{\beta}^i - C^*(\tilde{\beta}^i)] \left[ 1 - \frac{dC^*}{d\beta^i} (\tilde{\beta}^i) \right] - \psi' [\beta^i - C^*(\beta^i)]$$

$$+ \psi' [\beta^i - C^*(\beta^i)] \frac{dC^*}{d\beta^i} (\beta^i) = 0.$$  \hspace{1cm} (D2)

Hence $\tilde{\beta}^i = \beta^i$ is the only solution of the first-order condition. Moreover, the second-order condition at $\tilde{\beta}^i = \beta^i$ is satisfied since $dC^*/d\beta^i \geq 0$.

Finally, observe that a loser of the auction does not want to bid less than his true parameter. If he still loses the auction, he gains nothing. If he wins the auction, his transfer does not compensate him for his increased effort level.

Let $\beta^i$ be his true parameter and let his announcement $\tilde{\beta}^i$ be smaller than the smallest announcement $\beta^i$:

$$U^i(\tilde{\beta}^i) = \psi[\tilde{\beta}^i - C^*(\tilde{\beta}^i)] + \int_{\tilde{\beta}^i}^{\beta^i} \psi'[\tilde{\beta}^i - C^*(\tilde{\beta}^i)]d\tilde{\beta}^i$$

$$- \psi[\beta^i - \tilde{\beta}^i + \tilde{\beta}^i - C^*(\tilde{\beta}^i)]$$

$$\leq - (\beta^i - \tilde{\beta}^i)\psi'[\tilde{\beta}^i - C^*(\tilde{\beta}^i)]$$

$$+ \int_{\tilde{\beta}^i}^{\beta^i} \psi'[\tilde{\beta}^i - C^*(\tilde{\beta}^i)]d\tilde{\beta}^i$$

$$\leq - (\beta^i - \tilde{\beta}^i)\psi'[\tilde{\beta}^i - C^*(\tilde{\beta}^i)]$$

$$+ (\beta^i - \tilde{\beta}^i) \psi'[\tilde{\beta}^i - C^*(\tilde{\beta}^i)]$$

as (from [33])

$$\frac{dC^*}{d\beta^i} = 1 - \frac{dC^*}{d\beta^i} \leq 0.$$  

Since $\beta^i \geq \tilde{\beta}^i$, $U^i(\tilde{\beta}^i) \leq 0$.

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