Pointing error compensation of electro-optical detection systems using Gaussian process regression

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Abstract. Pointing accuracy is an important indicator for electro-optical detection systems, as it significantly affects the system performance. However, as a result of misalignment, nonperpendicularity in the manufacturing and assembly processes, as well as the sensor errors such as camera distortion and angular sensor error, the pointing accuracy is significantly affected. These errors should be compensated before using the system. Parametric models are firstly proposed to compensate for the errors, whilst the semi-parametric models with the nonlinearity added are also put forward. Both methods should analyse the parametric part first, which is a complicated and inaccurate process. This paper presents a nonparametric model, without any prior information about mechanical dimensions, etc. It depends only on the test data. Gaussian Process regression is used to represent the relationship between data and predict the compensated output. The test results have shown that the regression variances have decreased by more than an order of magnitude, and the means have also been significantly reduced, with the pointing error well improved. The nonparametric model based on Gaussian Process is thus demonstrated to be an effective and powerful tool for the pointing error compensation.

Keywords: Gaussian process regression / pointing error / nonparametric model / EODS

1 Introduction

Electro-optical detection systems (EODSs) have been widely used to collect targets location information with visible and infrared cameras in many applications, such as vehicles, ships, aircrafts, and spacecraft. It always contains a biaxial mechanical structure, the camera is fixed on the inner frame. With the two axial motor rotation, the camera can search and track the target in a certain angle range. As the pointing accuracy significantly affects the target tracking and location, it is necessary to obtain the pointing direction of line of sight (LOS) accurately [1,2]. The pointing error can be approximately divided into two categories, the first is the mechanical error, which is caused by misalignment, nonperpendicularity, etc. in the manufacturing and assembly processes. The second is the sensor errors, including camera error and angular sensor error. A minor bias of LOS will result in significantly affected. These errors should be compensated before using the system. Parametric models are firstly proposed to compensate for the errors, whilst the semi-parametric models with the nonlinearity added are also put forward. Both methods should analyse the parametric part first, which is a complicated and inaccurate process. This paper presents a nonparametric model, without any prior information about mechanical dimensions, etc. It depends only on the test data. Gaussian Process regression is used to represent the relationship between data and predict the compensated output. The test results have shown that the regression variances have decreased by more than an order of magnitude, and the means have also been significantly reduced, with the pointing error well improved. The nonparametric model based on Gaussian Process is thus demonstrated to be an effective and powerful tool for the pointing error compensation.

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model. Semiparametric models add extra nonlinear error factors, model accuracy is more accurate. Reference [9] firstly obtained the integrant error model and then applied semiparametric compensation model to improve the pointing accuracy of the EODS. In reference [10], a telescope’s kinematics model was established based on the Denavit-Hartenberg convention, mechanical errors were analysed, and a semi-parametric model was established for pointing error compensation. Reference [11] analysed, and a semi-parametric model was established for the underlying functional mapping $f(X)$.

$Y = f(X) + e, e \sim N(0, \sigma^2 I)$. (4)

where $\sigma^2$ is the variance of the noise, $I$ is the identity matrix. Equivalently, the noise model can also be denoted as [17]

$p(Y|f) = N(f, \sigma^2 I)$. (5)

The main regression task is to estimate the mapping function $f(X)$ from the training data $X$ and $Y$. The primary objective is to give the optimal estimate $Y*$ from the test input vectors, $X* = [x_{*1}, x_{*2}, ..., x_{*m}]$.

A GP defines a probability distribution on functions $p(f)$, which can be used as a Bayesian prior for the regression estimate, and Bayesian inference is used to make predictions from data as shown in (6) [17]

$p(f|X) = \frac{p(X|f)p(f)}{p(X)}$. (6)

We normally assume the zero mean GP prior on $f$ satisfies

$p(f(X)) = N(0, K)$. (7)

where $K$ is the covariance of $X$. The marginal likelihood can be obtained by integrating over the unobserved function $f$ [17],

$p(Y|X) = \int p(Y|f, X)p(f|X)df = N(0, K + \sigma^2 I)$. (8)

As the mean is assumed zero, the significant factor affecting the regression estimate result is the covariance function. A commonly used form is the ‘squared exponential’, shown in (9) [17]

$k(x, x') = \sigma^2 \exp\left(-\frac{(x - x')^2}{2l^2}\right)$. (9)

where $\sigma^2$ is the maximum allowance variance, $l$ is the length factor. Then the covariance $K$ can be computed as

$K = \begin{bmatrix}
  k(x_1, x_1) & \cdots & k(x_1, x_n) \\
  \vdots & \ddots & \vdots \\
  k(x_n, x_1) & \cdots & k(x_n, x_n)
\end{bmatrix}$. (10)
2.2 Prediction

Considering the test input, $X_*$, we have the covariance matrices of $X_*$ to $X_*$ and $X_*$ to $X$.

$$K_{**} = \begin{bmatrix} k(x_{s1}, x_{s1}) & \cdots & k(x_{s1}, x_{sm}) \\ \vdots & \ddots & \vdots \\ k(x_{sm}, x_{s1}) & \cdots & k(x_{sm}, x_{sm}) \end{bmatrix}$$ (11)

$$K_* = \begin{bmatrix} k(x_{s1}, x_1) & \cdots & k(x_{s1}, x_n) \\ \vdots & \ddots & \vdots \\ k(x_{sm}, x_1) & \cdots & k(x_{sm}, x_n) \end{bmatrix}.$$ (12)

As the premise, we assumed the data complying with a multivariate Gaussian distribution, the multivariate distribution with additive independent identically distributed noise is presented as [17]

$$\begin{bmatrix} Y \\ Y_* \end{bmatrix} \sim \begin{bmatrix} K + \sigma^2 I & K_*^T \\ K_* & K_{**} \end{bmatrix}$$ (13)

where $K^T$ is the transposition of $K_*$. Then the conditional distribution of $Y_*$ given $Y$ is [17]

$$Y_* | Y \sim N(K_* (K + \sigma^2 I)^{-1} Y, K_{**} - K_* (K + \sigma^2 I)^{-1} K^T_*).$$ (14)

The optimal estimate for the output is the mean in (14)

$$\bar{Y}_* = K_* (K + \sigma^2 I)^{-1} Y$$ (15)

and its uncertainty is the variance in (14)

$$\text{var}(Y_*) = K_{**} - K_* (K + \sigma^2 I)^{-1} K^T_*.$$ (16)

2.3 Parameter selection

Take the squared exponential covariance function as an example, in order to ensure GP regression to be a practical tool in pointing error compensation, we have to select proper $\sigma_f$ and $l$ for (9) to obtain the best regression. We define the hyper-parameters of the covariance function as

$$w = [\sigma_f, l].$$ (17)

According to Bayes’ theorem in (6), to obtain the maximum posteriori estimate of $w$, $p(w|X, Y)$, we should maximize the $p(Y|X, w)$, as obtained in (8). To simplify the computation, the log marginal likelihood is often used [17]

$$\log p(Y|X, w) = -\frac{1}{2} Y^T (K + \sigma^2 I)^{-1} Y - \frac{1}{2} \log|K| + \sigma^2 I - \frac{n}{2} \log(2\pi).$$ (18)

3 Model selection

During the covariance computation, there are plenty of possible covariance functions to choose from, including squared exponential, polynomial, neural network, etc., each has a number of undetermined hyper-parameters. Choosing proper covariance functions for a particular application is vital to the regression. A complex covariance function with many undetermined parameters needs a huge amount of test data, and it is difficult to converge to the optimal solution. According to the experimental data and the complexity of EODS, we chose to employ the squared exponential covariance function, which is universal and easily convergent. The general form [17] is shown as:

$$k(x, x') = \sigma_f^2 \exp \left(-\frac{1}{2} (x - x')^T M (x - x') \right)$$ (19)

where the matrix $M$ may be one of the following forms

$$M_1 = l^{-2} I, \quad M_2 = \text{diag}(l)^{-2}, \quad M_3 = \text{diag}(l)^{-2} + \Lambda \Lambda^T$$ (20)

where $l$ is a vector of positive values $l = l_1, l_2 \ldots l_D; \Lambda$ is a $D \times k(k < D)$ matrix.

In this paper, as the inputs are two dimensional, we applied the first form in (20) to estimate the overall trend of the pointing error, and utilized the second to remedy the differences of each dimension. The final covariance function is shown in (21)

$$k(x, x') = \sigma_f^2 \exp \left(-\frac{(x - x')^T (x - x')}{2l^2} \right) + \sigma^2 I$$ (21)

where $l_i = \text{diag}(l_{i1}, l_{i2})$. 

Fig. 1. Test apparatus of the EODS.
Here, it should be noted that “nonparametric” is raised corresponding to “parametric” and “semiparametric”. “Parametric” model contains determined error sources and their propagation models, and the compensation is targeted. “Semiparametric” model includes both “parametric” part mentioned above, and “nonparametric” part denoting the nonlinear error sources, which cannot be expressed by specific formulas. “Nonparametric” in this paper means there are no determined error sources in the compensation model, and the error is compensated as a whole.

4 Test results

The data acquisition system contains a high precision turntable, an autocollimator and the EODS, as shown in Figure 1. The test angular range is $-20^\circ \sim 20^\circ$ for azimuth, and $-20^\circ \sim 10^\circ$ for elevation. The turntable generates precision rotatory angles, the EODS rotates in the opposite direction, and the autocollimator gives the pointing error readout.

After the systemic error of the test system due to misalignment is compensated for, we apply GP regression to estimate the pointing errors using Gaussian processes for machine learning toolbox [19]. The azimuth and elevation results are shown in Figures 2–4 and Tables 1 and 2.

Figure 2 presents the regression results for azimuth and elevation over the measuring domain, which shows apparent differences in the components of pointing error, azimuth error is more significant. Figure 3 gives a detailed description of the prediction results in azimuth and elevation for the training data. Combined with Table 1, both the mean and variance values are greatly reduced after compensation. Figure 4 shows the comparisons of the actual measuring results and GP prediction results. The variances in the azimuth and elevation axes have decreased from $0.0258 \, (^\circ)^2$ and $0.0017 \, (^\circ)^2$ to $0.0014 \, (^\circ)^2$ and $0.0010 \, (^\circ)^2$, respectively, improved by more than an order of magnitude, and the means are also significantly reduced. These all illustrate that the proposed nonparametric compensation model based on GP regression is effective and successful.
Among the three types of compensation models [4], both nonparametric and semiparametric models present a better performance than parametric model based on integrant errors. Although the semiparametric model should be theoretically more advantageous as a result of its applicability in linear and nonlinear problems, in this paper the nonparametric model based on GP regression achieved the same effect as the semiparametric model. On the other hand, without complicated modelling process for each integrant error source, the nonparametric is more convenient than the other two models. Hence, the nonparametric method is effective and recommendable.

5 Conclusions

Pointing accuracy of EODS significantly affects the target tracking and location, it is necessary to obtain the pointing direction of LOS accurately. As for misalignment, nonperpendicularity, etc. in the manufacturing and assembly processes, this paper established a nonparametric compensation model based on GP regression. Different from the parametric and semi-parametric models, which should analyse the parametric part based on the physical structure and error source, it is complicated and inaccurate. This paper only focused on the test data, and it realised pointing error compensation based on the Gaussian process regression. It firstly obtained the marginal likelihood of the training data, and then the prediction equations by means of choosing proper covariance functions. To obtain the optimal hyper parameters, the maximisation of the log marginal likelihood equation was performed. The hyper parameters were finally utilized in pointing error regression of the EODS. The test results demonstrated that this method is effective, the variances were reduced by more than one order of magnitude with the

**Table 1. Compensation results comparison.**

| Pointing error | Variance ($^\circ^2$) | Mean ($^\circ$) |
|----------------|-----------------------|----------------|
|                | Original | Compensated | Original | Compensated |
| Azimuth        | 0.0492   | 0.0007      | -0.1674  | -0.0002     |
|                | 0.0258   | 0.0014      | -0.0894  | -0.0185     |
| Elevation      | 0.0065   | 0.0005      | -0.0454  | -0.0004     |
|                | 0.0017   | 0.0010      | -0.0161  | -0.0122     |

**Table 2. Prediction results comparison among different models for test data.**

| Pointing error | Variance ($^\circ^2$) | Mean ($^\circ$) |
|----------------|-----------------------|----------------|
|                | Azimuth   | Elevation | Azimuth   | Elevation |
| Original       | 0.0258   | 0.0017    | -0.0894  | -0.0161   |
| Parametric model | 0.0042   | 0.0016    | 0.0017   | -0.0160   |
| Semiparametric model | 0.0014   | 0.0009    | -0.0147  | -0.0130   |
| Nonparametric model | 0.0014   | 0.0010    | -0.0185  | -0.0122   |

Among the three types of compensation models [4], both nonparametric and semiparametric models present a better performance than parametric model based on integrant errors. Although the semiparametric model should be theoretically more advantageous as a result of its applicability in linear and nonlinear problems, in this paper the nonparametric model based on GP regression achieved the same effect as the semiparametric model. On the other hand, without complicated modelling process for each integrant error source, the nonparametric is more convenient than the other two models. Hence, the nonparametric method is effective and recommendable.

**Fig. 4. Prediction results for the test data.**

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pointing accuracy significantly improved. It has been demonstrated that GP regression can be effectively and conveniently used as a powerful tool in pointing error compensations.

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