ABSTRACT

We discuss the concept of handedness applied to $\tau$ production in $e^+e^-$ annihilations with $\tau$ decaying into $a_1\nu_\tau \rightarrow 3\pi\nu_\tau$. The $a_1 \rightarrow 3\pi$ decay is particularly interesting since it (together with $h_1 \rightarrow 3\pi$) is the lightest hadronic decay in which the helicity of the initial state can be determined from the distribution of the final particle momenta. Thus it is the first example of a helicity self-analysing strong decay, as $\Lambda \rightarrow N\pi$ is for weak decays.

Apart from providing a new method for determining the $\tau$ polarization from the parity conserving part in the weak decay, this reaction provides a very instructive and useful demonstration of the handedness asymmetry as a measure of parton polarization. Thus we believe that experience gained from analyses of handedness in $\tau \rightarrow 3\pi\nu_\tau$ decay will be very useful when looking for the similar asymmetry of jet handedness which can measure quark or gluon polarization.

1. Introduction

This talk is based on recent work together with A. Efremov and L. Mankiewicz [1]. Previously we introduced the concept of jet handedness [2] (see also Ref. [3] - [5]), which will hopefully provide a measurable asymmetry for determining experimentally the polarization of the initial quark or gluon. There, parity conservation of strong interactions requires that one has at least three particles (spinless or spin-averaged) in the final state in order to have a correlation in the decay distribution with the initial helicity. Namely, from three particle momenta one can construct a pseudovector $n_\mu = \epsilon_{\mu\nu\rho\sigma}k_1^\nu k_2^\rho k_3^\sigma$ which gives, when contracted with the initial polarization pseudovector, a scalar component in the strong process. Thus the average of the vector $n_\mu$ can give information on the initial polarization, provided the correlation (or analysing power $\alpha$) is large enough to be measurable. We called this quantity the handedness vector ($\vec{H} = \frac{2}{3}(\alpha\langle \vec{n} \rangle)$).

Because of the complicated process of jet fragmentation much of the correlation (i.e. the analysing power $\alpha$) between polarization and jet handedness is...
expected to be washed away when averaging over phase space and/or summing over different choices of the final three momenta $k_i$. Therefore, we find it useful and instructive to look at the simpler situation of particle decay, where one can even calculate the expected handedness and analysing power $\alpha$.

Looking at the lightest hadronic decays the process $a_1 \to 3\pi$ (together with $h_1 \to 3\pi$) is unique in the sense that it is the lightest strong decay where one can measure a handedness, which correlates with the helicity of the initial state. Thus this strong decay is similar to the well-known weak decay $\Lambda \to \pi N$ in the sense that it is "self-analysing" as to polarization.

One might first think that the $3\pi$ decay of a vector meson ($\omega, \phi, J/\psi$) could also be self-analysing, since we do have three final-state particles. However, these decays do not fulfil a second requirement: that we must have at least two amplitudes which depend differently on the initial polarization, since it is the interference between the two amplitudes which gives the helicity-dependent term. For $\omega \to 3\pi$ we have only one amplitude $\propto \epsilon_{\mu\nu\rho\sigma} k_1^\nu k_2^\rho k_3^\sigma$, and therefore we can only measure the symmetric (tensor) part of the spin-density matrix ($\rho_{ij} \propto \text{Re} \epsilon_i \epsilon_j^*$), where $\epsilon_i$ is the polarization vector of the helicity $i$ state. Note that it is the antisymmetric part of $\rho_{ij} \propto \text{Im} \epsilon_i \epsilon_j^*$ which is sensitive to the helicity or circular polarization vector $P_\mu = i \epsilon_{\mu\nu\rho\sigma} p_\nu \epsilon_\rho \epsilon_\sigma^* / m$. The latter is the true Pauli-Lubanski pseudovector for angular momentum of spin 1.

We shall consider $a_1$ produced in $e^+e^-$ annihilations to $\tau^+\tau^-$ with $\tau \to a_1\nu_\tau$. Since the $\tau$ is polarized (on the average $\approx 14\%$) at the $Z^0$, owing to the weak interaction, this polarization is measurable through the one it induces on the $a_1$. Therefore, a detailed study of the polarization asymmetries in the reaction $\tau \to a_1\nu_\tau \to 3\pi\nu_\tau$ is of experimental interest. However, the $a_1$ is polarized also by the parity violation in the decay $\tau \to a_1\nu_\tau$, even if the $\tau$ is unpolarized. This was studied by Kühn and Wagner [6] and by Feindt [7], and was used by the ARGUS group [8] to measure the helicity of the $\nu_\tau$ to be left-handed, as expected in the standard model for a sequential third-generation $\tau$ neutrino.

A unique property of the $\tau \to 3\pi\nu_\tau$ decay is the possibility to separate measurements of the effects due to the $\tau$ polarization, i.e. the effects of parity violation in the neutral current, from the effects of left-handedness of the $\nu_\tau$, i.e., the parity violation in the charged current. The latter have to be assumed in studies of $\tau$ polarization with other decay channels [9]–[12].

2. General analysis

Let us consider the matrix element for $\tau^-$ production in $e^+e^-$ collisions at $Z^0$ energy, followed by its decay into $\nu_\tau$ and $\pi^+\pi^-\pi^-$ through the $a_1$ intermediate state. This naturally factorizes into three processes, such that the first $e^+e^- \to \tau^+\tau^-$ is given by the cross section $\sigma(e^+e^- \to \tau^+\tau^-)$ of unpolarized $\tau$ and by the $\tau$ polarization $P_\tau$ along some spin quantization axis $S^\mu$.

Then, the $\tau \to a_1\nu_\tau$ decay can be represented by two lepton tensors, one $\tilde{W}^{\mu\nu}(\tau \to a_1\nu_\tau)$ for unpolarized $\tau$’s and another $W^{\sigma,\mu\nu}(\tau \to a_1\nu_\tau)$ which will
multiply \( P_\tau \). Finally the decay \( a_1 \to 3\pi \) is determined by a hadronic tensor \( H^{\mu\nu}(a_1 \to 3\pi) \). Thus the cross section for the whole process can be written in an intuitive form as proportional to

\[
\sigma \propto \sigma(e^+e^- \to \tau^+\tau^-)[W^{\mu\nu}(\tau \to a_1\nu_\tau) + P_\tau W^{\sigma,\mu\nu}(\tau \to a_1\nu_\tau)]H_{\mu\nu}(a_1 \to 3\pi) .
\]

(1)

Now it is important to realize that \( W, W^{\sigma}, H \) all have both a symmetric and an antisymmetric part. Denoting the symmetric part with a bar and the antisymmetric part with a hat we thus have

\[
W^{\mu\nu} = \bar{W}^{\mu\nu} + \hat{W}^{\mu\nu}, \quad W^{\sigma,\mu\nu} = \bar{W}^{\sigma,\mu\nu} + \hat{W}^{\sigma,\mu\nu}, \quad H^{\mu\nu} = \bar{H}^{\mu\nu} + \hat{H}^{\mu\nu} .
\]

Writing out the explicit expressions for these tensors \[1\] one sees that the symmetric \( \bar{W}^{\sigma,\mu\nu} \) is parity conserving (\( \propto G_A^2 + G_V^2 \)), and the antisymmetric \( \hat{W}^{\sigma,\mu\nu} \) is parity violating (\( \propto 2iG_A G_V \)), while the symmetric \( \bar{W}^{\sigma,\mu\nu} \) is parity violating and the antisymmetric \( \hat{W}^{\sigma,\mu\nu} \) is parity conserving. Finally the hadronic tensor \( H^{\mu\nu} \) is of course always parity conserving, but contains both a symmetric and an antisymmetric part.

The \( \tau \) polarization must be measured from \( W^{\sigma,\mu\nu} \), as is obvious from Eq. (1). Usually one does this from the symmetric and parity violating part \( (\bar{W}^{\sigma,\mu\nu}) \) whereby one contracts with \( \bar{H}^{\mu\nu} \). Then a two-body hadronic decay, in this case \( a_1 \to \rho\pi \) (i.e. one can integrate over the \( 3\pi \) Dalitz plot), is sufficient to measure \( P_\tau \).

For our purpose it is, however, the parity conserving part \( (\hat{W}^{\sigma,\mu\nu}) \) of the \( \tau \) decay which is of interest. In order to pick out this term one must contract with an antisymmetric hadronic tensor \( \hat{H}^{\mu\nu} \). Then, we can measure the \( \tau \) polarization in a new way, and find a term which can be present also in a purely strong process such as quark jet fragmentation.

Explicitely these antisymmetric terms have the forms

\[
\hat{W}^{\sigma,\mu\nu}_D = i(G_A^2 + G_V^2) m_\tau \epsilon^{\mu\nu\alpha\beta} q_\alpha S_\beta ,
\]

(2)

\[
\hat{H}^{\mu\nu} = (BW) \left( K^{\nu}_{(2)} K^{\nu}_{(1)} - K^{\nu}_{(2)} K^{\nu}_{(1)} \right) \text{Im}(\rho_1^* \rho_2) ,
\]

(3)

where \( q \) is the neutrino momentum, BW represents the \( a_1 \) Breit-Wigner function (whose form is not important here), \( (\rho_1 \rho_2) \) is the interference term between the two crossing \( \rho \) bands in the Dalitz plot. Note that now one cannot integrate over the whole Dalitz plot since then the term from eq.(3) vanishes because of the antisymmetry! Finally, the hadronic current

\[
K^\mu_i = \left( g^{\mu\alpha} - \frac{k^\mu k^\alpha}{k^2} \right) T^{\alpha\beta}_{ij}(k_+ - k_i)^\beta F_{a_1}(k^2) ,
\]

(4)

depends only on the pion momenta, \( k_1, k_2, k_+ \) for the two negative pions and the positive pion respectively. The tensor \( T^{\alpha\beta}_{ij} \) contains, in principle, two forms
of $a_1\rho\pi$ coupling ($S$- and $D$-wave), but experimentally we know that it is almost 100% $S$-wave. Finally the $W \rightarrow a_1$ formfactor $F_{a_1}$ is irrelevant in our application here, but included in eq. (4) for completeness.

After contraction of the two antisymmetric terms one finds a term

$$\hat{M}_1 = \hat{W}^{\sigma, \mu\nu} H^{\mu\nu} = 2\epsilon_{\mu\nu\alpha\beta} K^{\mu}_{(2)} K^{\nu}_{(1)} S^{\alpha} q^{\beta} \text{Im}\rho_1^* \rho_2$$

$$\Rightarrow 6n^z \left| \vec{p} \right| \frac{m_T}{m_T} (p_0 - |\vec{p}| \cos^2 \Psi) \text{Im}\rho_1^* \rho_2 ,$$

where the second expression is obtained after the averaging over the unknown neutrino direction and $\Psi$ is the $\tau$ polar angle in the $a_1$ CM frame. The relative magnitude of this term to that of the spin averaged term gives us an asymmetry which is our handedness

$$\mathcal{H} = \frac{N(n^z(s_1 - s_2) > 0) - N(n^z(s_1 - s_2) < 0)}{N(n^z(s_1 - s_2) > 0) + N(n^z(s_1 - s_2) < 0)}$$

$$= \frac{\int d\Omega(BW)|\hat{M}_1|}{\int d\Omega(BW)\hat{M}_1} P_{\tau}(\Theta_a) = \bar{\alpha} P_{\tau}(\Theta_a) ,$$

where the first equation (7) is given by the the experimentally seen number of events of opposite longitudinal handedness, defined by the sign of $n^z(s_1 - s_2)$.

Here $n^z = \vec{e}_a \cdot (\vec{k}_1 \times \vec{k}_2)$ denotes the third component of the normal to the decay plane when $\vec{e}_a$ is the direction of the line of flight of the $a_1$ in the $a_1$ CM, and $s_1$, $s_2$ are the Dalitz plot invariants for the two $\rho$'s. The second equation (8) gives the corresponding theoretical expression which also defines the analysing power $\bar{\alpha}$ which now is a number which can be calculated theoretically, for the same cuts in phase space as the data, using the conventional $a_1 \rightarrow 3\pi$ decay model described above. At least in one point of phase space ($s_1 = m_\rho + m_\rho \Gamma_\rho$, $s_2 = m_\rho - m_\rho \Gamma_\rho$) we know $\alpha$ is nearly 100% . Thus the average $\bar{\alpha}$ cannot be too small and by clever cuts one can increase the analysing power.

3. Concluding remarks.

The most important result of this work is that one can measure the polarization of a particle in a parity conserving process from a three-body final state, and that the $\tau \rightarrow 3\pi\nu_\tau$ decay provides a nice example. Our analysis thus provides a new way to measure the $\tau$ polarization, and is a very instructive example for how to proceed in the more complicated process of quark jet fragmentation. The quark fragmentation analogy would be that part of the time the quark fragments into an $a_1$. The incoming quark replaces the $\tau$ of our example (which is so short lived that it leaves no track, i.e., the $\tau$ is almost as "invisible" as a quark). Three particles in the jet are assumed to be pions coming from the $a_1$ (or an $a_1$-like object) and the remaining particles of the jet replace the unseen neutrino. If the polarized quark ends up in the $a_1$, the $a_1$ should be polarized and, then the $a_1 \rightarrow 3\pi$ decay should reveal the quark polarization in the same way as in the
\( \tau \) decay example discussed above. Because of background and combinatorical problems it is difficult to calculate the analysing power, but provided it is big enough it should be measurable. Once it is determined for jets defined in a definite way, it could be used in other contexts for similar jets to determine the quark polarization from the jet handedness.

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