Robust Image Registration via Empirical Mode Decomposition

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Abstract

Spatially varying intensity noise is a common source of distortion in images. Bias field noise is one example of such distortion that is often present in the magnetic resonance (MR) images. In this paper, we first show that empirical mode decomposition (EMD) can considerably reduce the bias field noise in the MR images. Then, we propose two hierarchical multi-resolution EMD-based algorithms for robust registration of images in the presence of spatially varying noise. One algorithm (LR-EMD) is based on registering EMD feature-maps of both floating and reference images in various resolution levels. In the second algorithm (AFR-EMD), we first extract an average feature-map based on EMD from both floating and reference images. Then, we use a simple hierarchical multi-resolution algorithm based on downsampling to register the average feature-maps. Both algorithms achieve lower error rate and higher convergence percentage compared to the intensity-based hierarchical registration. Specifically, using mutual information as the similarity measure, AFR-EMD achieves 42% lower error rate in intensity and 52% lower error rate in transformation compared to intensity-based hierarchical registration. For LR-EMD, the error rate is 32% lower for the intensity and 41% lower for the transformation.

1 Introduction

Accurate registration and alignment of two images has been a challenging problem in a wide variety of applications such as medical image processing [1], remote sensing [2] and computer vision [3]. Particularly, registration of the medical images has been widely used in tumor localization and targeting [4], organ growth studies [5] and brain atlas reconstruction [6]. The registration algorithms can be categorized in two general classes [7]. One is based on the intensity of images. In these approaches, a similarity measure is defined two quantify the similarity of both floating (or moving) and reference (or target) images. Then an optimization process identifies the optimal map of the floating image with highest similarity to the reference image. In the second class of registration algorithms, a set of features such as landmarks [8], histogram of intensity [9] or responses to Gabor [10] and Alpha stable filters [11][12] are first extracted from both floating and reference images. Then two images are aligned through the mapping of these features. Similarity measure between two images is a crucial component of intensity-based image registration. A wide variety of measures such as sum-of-squared-differences (SSD), correlation coefficient (CC) or mutual information (MI) [13][14] or have been used in the registration process. However, these measures are not robust to spatially varying intensity noise. Bias field noise that is
often present in the magnetic resonance (MR) images are one example of the spatially varying noise [15].

In general, two approaches have been studied to overcome the bias-field noise effect in image registration. One approach is focused on defining robust similarity measures. Residual complexity (RC) [16], rank-induced [17, 18] and sparsity-based [19, 20] similarity measures are two examples. Second approach is based on reducing noise effect and registering the denoised images [21, 22]. In this paper, we concentrate on the later approach by examining the ability of a state-of-the-art hierarchical signal decomposition techniques in removing spatially varying noise in MR images. Specifically, we propose to use empirical mode decomposition (EMD) to overcome the effect of bias field noise in MR images. EMD-based algorithms have been previously studied for the basic image registration task [23, 24]. However, to our knowledge, our paper is the first study on the EMD-based image registration in the presence of spatially varying noise.

We introduce two novel feature-based registration algorithms based on EMD of the images [25]. EMD decomposes any n-dimensional signal into Intrinsic Mode Functions (IMFs) and residual components. In section 3, We first show that EMD can considerably reduce bias field noise in images by removing the spatially varying distortion in IMFs and absorbing them in residual components. Then, we propose two multi-resolution [26] registration algorithms based on EMD. In one algorithm (LR-EMD), we hierarchically register EMD feature-maps of both floating and reference images in various resolution levels (scales). In the second algorithm (AFR-EMD), we first extract an average feature-map (average of IMFs) based on EMD from both floating and reference images. Then, we use a simple hierarchical multi-resolution algorithm based on downsampling to register the average feature-maps. The registration performance and comparisons to benchmark algorithms has been discussed in section 4.

2 Empirical mode decomposition

Decomposing nonlinear and non-stationary signals into their intrinsic modes has been a challenging task in signal processing. Most of these decomposition are based on time-frequency transformations such as Short-Time Fourier Transform [27] and wavelet transform that expands the signal into a set of basis functions. Unlike these transformations, algorithms based on empirical mode decomposition (EMD) [25, 28] are able to decompose the linear or nonlinear signal into a set of functions defined by the signal itself [29]. EMD consists of a set of spatial and temporal processes that decomposes signal into Intrinsic Mode Functions (IMFs). Each IMF is an oscillating mode of the signal. In contrast with harmonic modes, oscillating modes have time-variant amplitude and frequency. EMD has been widely used in applications of signal decomposition such as prediction [30], classification [31] and denoising [32].

Bidimensional empirical mode decomposition [33] is particularly our focus in this paper, since it deals with two-dimensional signals such as images. Each IMF in the bidimensional empirical mode decomposition could be extracted from the image using a sifting process. Let \( \mathbb{I} \) denote the image, \( \text{IMF}_i \) and \( \text{RES}_i \), denote the \( i^{th} \) IMF and residual, respectively. Here \( i \) represents the level or scale of the decomposition. For simplicity, let’s assume that the residual at level 0 is the original image. That is:

\[
\text{RES}_0 = \mathbb{I}.
\]

To find the IMF in level \( i \), we repeat the following iterations. We first identify all the local minimum and maximum of the \( \text{RES}_{i-1} \). Then we generate the bounding maximum and minimum
envelopes, $E_{\text{max}}$ and $E_{\text{min}}$ using an interpolation or a surface fitting techniques. The IMF in this iteration is equal to $\text{RES}_{i-1}$ subtracted by the mean of maximum and minimum envelope:

$$\text{IMF}_i = \text{RES}_{i-1} - \frac{E_{\text{max}} + E_{\text{min}}}{2}$$

We repeat this process until a convergence condition is met. At this point, the residual is defined as:

$$\text{RES}_i = \text{RES}_{i-1} - \text{IMF}_i$$

First IMF has the highest frequency and the next IMFs have lower frequency contents, respectively. The extracted IMFs have a number of crucial characteristics. First, the number of zero-crossing points is equal or one less than the number of extrema in each IMF. Second, each IMF has one minimum with zero value. Finally, mean value of envelope of local maximum points and local minimum points is equal to zero, in each point of IMF.

One of the limitations of the empirical mode decomposition is in processing narrow-band signals [34]. EMD has shown to have limited accuracy and efficiency when decomposing a signal with concentrated energy in a narrow frequency band. In this case, the decomposition cannot accurately extract the intrinsic modes of the signal. In this paper, we discuss biomedical images and particularly MRI that does not have narrow-band frequency component.
3 Bias field noise and empirical mode decomposition

Spatially varying noise or bias field is a common source of noise in MR images. Bias field can potentially cause the registration process to diverge, particularly, when similarity measure is not robust to bias field. SSD, CC and MI are some of these similarity measures. Even RC similarity measure is not completely successful in reducing bias field noise. In this section, we investigate the ability of EMD in reducing and removing this noise.

EMD practically decomposes signal into components with different frequency bands. Extracted IMFs in each level (scale) of the EMD are the signal component with a particular frequency band. The remaining components of the signal are characterized by the residual in that level. Considering the sifting process, each IMF is the signal subtracted by the average of the minimum and maximum envelopes. In two-dimensional space, a bias field noise is a local spatial variation in signal. The average of minimum and maximum envelopes of the signal is aware of this spatially varying noise, therefore, by subtracting this noise from the signal, IMF is noise free.

Bias field noise can be represented by a mixture of two-dimensional Gaussian functions (kernels) \( [16] \). We distort image \( I(x, y) \) with bias field noise according to the following equation:

\[
I(x, y) = I(x, y) + \frac{1}{K} \sum_{k=1}^{K} e^{-||[(x, y) - \mu_k]|^2/2\sigma^2}
\]  

where \( K \) is the number of Gaussian functions. Figure 1-A shows a sample Gaussian kernel. We manually add one Gaussian kernel \( (K = 1) \) with random mean to both floating and reference MR
Algorithm 1 LR-EMD: Image Registration using EMD levels

Input: $I_{fl}$ floating image, $I_{ref}$, reference image, $n$ number of IMFs or EMD levels, SIM Similarity measure

Extract $n$ IMFs of both $I_{fl}$ and $I_{ref}$,

Initialize registration with unity transform $f(x) = x$

for $i = 1$ to $n$ do

Register the $i$th IMF of $I_{fl}$ to the $i$th IMF of $I_{ref}$ based on SIM Similarity measure and find transform $f(x)$

Initialize transform for next level of IMF with $f(x)$

end for

images. A sample pair of corrupted images are shown in Figure 1-B. The standard deviation of the Gaussian function is set to be $\frac{1}{16}$ of image width. To investigate the ability of EMD in reducing bias field noise, we decompose both floating and reference images corrupted by our bias field noise to IMFs and residuals. Figure 2 illustrates the set of IMFs and residuals for three levels for both images. The Gaussian kernel is explained by residuals and IMFs are noise free. This visualization shows that EMD can successfully separate noise from the intrinsic components of the image.

3.1 LR-EMD: Image registration algorithm based on EMD levels

We take advantage of the ability of EMD in removing bias field noise effect and propose a hierarchical multi-resolution algorithm for image registration based on EMD levels in different scales. We call this algorithm Level-based Registration using EMD or LR-EMD. Algorithm 1 shows the details of LR-EMD. We first find IMFs for both floating and reference images. These IMFs are the feature-maps of each image in different resolutions. Then, we start with the most coarse resolution and estimate the transform from floating image IMF to the reference image IMF. This map forms the initial transformation for the next level of IMF registration (with higher resolution). We continue this process to the finest level IMF. We use free form deformation to estimate the transform, however, other techniques are also applicable. In each registration level, any similarity measure can be used to achieve an accurate registration. In this paper, we have use SSD, CC, RC and MI as similarity measures.

LR-EMD takes advantage of IMFs in different levels (scales) and registers two images in a hierarchical order. Therefore, it benefits from the information stored in each level of IMFs during the process of the registration. The registration at each resolution level of IMFs provides an initial transformation for the registration in the next level. In principle, the hierarchical order increases the robustness of LR-EMD against local minimum in the optimization process.

3.2 AFR-EMD: Image registration algorithm based on average EMD feature-maps

We also propose a registration algorithm based on the average of EMD-based features. This algorithm is called Average Feature-map Registration using EMD or AFR-EMD. Algorithm 2 presents the details of AFR-EMD. We first construct a single feature-map of each of the floating and reference images by taking average of IMFs from all of the EMD levels (scales). Then, we register two average feature-maps based on a pre-defined similarity measure. To take advantage of hierarchical registration, we downsample the feature-map and perform a level-by-level registration. In this paper, we have studied SSD, CC, RC and MI similarity measures in all of our experiments with AFR-EMD. Free form deformation is used here to estimate the transform.
Algorithm 2 AFR-EMD: Image registration based on Average EMD feature-maps

| Input: | $I_{fl}$ floating image, $I_{ref}$, reference image, $n$ number of IMFs or EMD levels, SIM Similarity measure |
|---|---|
| | Extract $n$ IMFs for both $I_{fl}$ and $I_{ref}$ |
| | Find $F_{fl}$ and $F_{ref}$, the average of IMFs for both $I_{fl}$ and $I_{ref}$ |
| | Initialize registration with unity transform $f(x) = x$ |
| for $i = 1$ to $n$ do |
| | $\triangleright$ where $i = 1$ is the coarse-grained level and $i = n$ is the fine-grained level |
| | Compute $F_{fl}^i$ and $F_{ref}^i$, the downsampled versions of $I_{fl}$ and $I_{ref}$ with respect to scale level $i$ |
| | Register $F_{fl}^i$ to $F_{ref}^i$ based on SIM Similarity measure and find transform $f(x)$ |
| | Initialize transform for next level of registration with $f(x)$ |
| end for |

Taking average of the IMFs is a form of feature reduction, therefore, it benefits through save in computational cost and increasing the interpretability of feature-maps. However, since we enjoy a hierarchical registration based on downsampling in AFR-EMD, the gain in computational cost is not considerable compared to LR-EMD. The average of IMFs is a denoised and normalized feature-map. We will study the ability of this feature-map in robust registration of images in the presence of bias field noise.

4 Results

We have evaluated our proposed registration algorithms on two magnetic resonance (MR) image datasets. The BrainWeb dataset [36] contains simulated brain MR volumes from several protocols, including T1-weighted (MR-T1), T2-weighted (MR-T2), and proton density (MR-PD). Here, we study a two-dimensional slice of MR-T1 (218 181). The intensities are normalized between 0 and 1. The second dataset used in this paper is Internet Brain Segmentation Repository (IBSR). IBSR consists of real and normal MR images provided by Massachusetts General Hospital (MGH) in association with Harvard Medical School. Again, we have studied a two-dimensional slice of the data.

Two different measures are used in this paper to compare the accuracy of registration. First measure is the transformation root mean square error (RMSE) between the true and estimated transformations. We call this measure T-RMSE throughout this paper. T-RMSE is equal to:

$$T\text{-RMSE} = \sqrt{\frac{1}{N}||T_{true} - T_{estimated}||^2}$$

Where $N$ is the number of pixels. Second measure is RMSE between intensities of the reference and the registered images. We call this measure I-RMSE. I-RMSE is equal to:

$$I\text{-RMSE} = \sqrt{\frac{1}{N}||I_{reference} - I_{registered}||^2}$$

The mean and standard deviation (SD) of two error measures are calculated by 15 runs of registration in all the experiments. At each run, the intensity distortion and the geometric transform parameters were randomly reinitialized.
Figure 3: Registration performance when there is no bias field. Mean and variance of T-RMSE and I-RMSE for images in BrainWeb dataset are shown in top and middle panels. Bottom panel compares the running time for all methods.

4.1 BrainWeb dataset

We use the simulated MR images in BrainWeb dataset to evaluate the performance of our registration algorithms. Each image is geometrically distorted using a random perturbation to generate the floating image. Specifically, a $14 \times 14$ uniform grid is randomly perturbed (by a uniform distribution on $[-6, 6]$) and used as the FFD grid of the floating image. Then, the distorted image is registered back to the original image using our algorithms and benchmark intensity based algorithm. For each algorithm, four registration processes are studied each employing one of the SSD, CC, RC or MI similarity measures. In all of the cases, FFD is exploited as the model of geometric transform. FFD is used with three hierarchical levels of B-spline control points. We have set the levels of IMFs, $n$, to 3 in order to have a fair comparison with the intensity-based registration. Iterative Gradient descent has been used to optimize the transformation parameters. The accuracy is evaluated using both T-RMSE and I-RMSE.

We start with registering images without any bias field noise in section 4.1.1. Then, we manually add spatially varying distortions or bias field noise to both floating and reference images. The noise is generated using mixture of Gaussian functions and is added to the images according to Equation 1. The mean of the Gaussian functions are selected at random. The standard deviation is set to $\frac{1}{16}$ of image width.

4.1.1 No bias field case

Figure 3 shows the registration performance when there is no bias field. Mean and variance of T-RMSE and I-RMSE for each of SSD, CC, RC and MI similarity measures are shown separately in the top and middle panels. The registration performances for LR-EMD, AFR-EMD and benchmark intensity based registration are compared in each panel. The registration process for all
Figure 4: The convergence rate for baseline hierarchical registration with downsampling and our proposed algorithms based on EMD. Each panel corresponds to one similarity measure (SSD, CC, RC and MI). Here, the convergence is defined as T-RMSE error lower than 4 pixels.

images have converged in this setting. AFR-EMD have similar performance to the intensity-based registration when there is no bias field noise. LR-EMD has slightly higher error, particularly when RC has been used as similarity measure. SSD and CC have higher average error compared to RC and MI (approximately 25% higher in both T-RMSE ans I-RMSE). Generally, MI achieves the highest accuracy and fastest convergence time among the similarity measures. The bottom panel in Figure 3 compares the running time for all methods. Comparing the registration algorithms, there is no significant difference between running times. Among the similarity measures, RC is the slowest similarity measure. In general, our results show that EMD-based registration algorithms perform similar to the intensity-based method when none of the images are corrupted by spatially varying noise.

4.1.2 Convergence in the presence of bias field

Next, we investigate the registration of MR images in the presence of bias field. We are particularly interested in this scenario since intensity-based registration algorithms tend to fail in converging to a valid solution when image has spatially varying noise. Additionally, when converging, the accuracy is lower compared to registration without any bias field noise. To study the convergence of registration algorithms in the presence of bias field, we have manually added Gaussian functions to both floating and reference images according to Equation 1. Then, we have performed image registration using LR-EMD, AFR-EMD and intensity-based algorithm. Figure 4 shows the convergence rate for each of the four similarity measures (SSD, CC, RC and MI). Here, the convergence is
defined as T-RMSE error lower than 4 pixels. The number of Gaussian kernels are varied between $K = 0$ (no bias field) and $K = 4$ (Four Gaussian kernels in each floating and reference images). For each image in the BrainWeb dataset, the registration process is repeated for 15 random selection of Gaussian mean and the percentage of convergence is reported in this figure. For all four similarity measures, adding one Gaussian function degrades the convergence rate (red curves in Figure 4). This confirms that intensity-based registration is not robust to the bias field noise. Particularly, SSD never converged with one Gaussian kernel. RC is the most robust similarity measure for regular hierarchical registration with 80% convergence rate in the presence of one Gaussian kernel and 100% for two to four Gaussian kernels. Our proposed algorithms based on EMD improves the convergence rate for almost all of the cases. Using MI or CC as the similarity measure, both LR-EMD and AFR-EMD converge 100% of the times while intensity-based registration converges in 20% to 60% of the times for MI and 25% to 80% of the times for CC. For SSD, both LR-EMD and AFR-EMD have higher convergence rate compared to benchmark registration for one to three Gaussian kernels. When four Gaussian kernels are present, LR-EMD has lower convergence rate compared to other two algorithms. This is mainly because of low SNR in image. Our results confirm the that RC similarity measure is more robust to the bias field noise compared to other similarity measures.

4.1.3 Registration performance in the presence of bias field

Next, we investigate the performance of the registration across the runs that have converged. First, we study the case that both reference and floating images are corrupted by one Gaussian function ($K = 1$ case). Top two panels in Figure 5 shows T-RMSE and I-RMSE for the converged registration processes. For SSD similarity measure, intensity-based registration has never converged, therefore, no corresponding error value is reported. For MI similarity measures, both LR-EMD and AFR-EMD are considerably more accurate compared to intensity-based registration (approximately 50% lower T-RMSE and 40% lower I-RMSE). This is particularly important since MI leads to the most accurate registration compared to other similarity measures. Additionally, registration processes based on MI are also generally faster (Figure 5 bottom panel). For CC similarity measure, both EMD-based registrations have slightly lower average error rate. Using RC as similarity measure, LR-EMD has slightly higher average error rate compared to two other methods. Bottom panel in Figure 5 compares the running time of registration procedures for benchmark and our proposed algorithms. CC similarity measure for intensity-based registration converges in a considerably lower time compared to our methods, however, its accuracy is not as high as EMD-based registrations.

Figure 6 shows the accuracy and running time for registrations in the presence of two Gaussian kernels in each of the floating and reference images. Two Gaussian kernels can model a more intense bias field noise. Using any of SSD, CC and MI similarity measures, registration methods based on EMD have higher accuracy both in T-RMSE and I-RMSE. For MI, EMD-based registrations have approximately 60% lower T-RMSE and 50% lower I-RMSE. For all similarity measures, LR-EMD is slightly more accurate compared to AFR-EMD. For the RC similarity measure, intensity-based registration has higher accuracy and lower running time. AFR-EMD with MI similarity measure achieves the highest accuracy among all of the methods and similarity measures. We have also studied the registration performance in the presence of three and four Gaussian kernels and reported the T-RMSE as well as I-RMSE and running time in Figures 7 and 8. Using SSD, CC or RC as similarity measure, EMD-based methods have approximately similar performance to the intensity-based registration. When using MI as similarity measure, EMD-based registrations achieves considerably higher accuracy (40% to 65%) both in T-RMSE and I-RMSE compared
Figure 5: Registration performance in the presence of one Gaussian kernel. Mean and variance of T-RMSE and I-RMSE for images in BrainWeb dataset are shown in top and middle panels. Bottom panel compares the running time for all methods.

Figure 6: Registration performance in the presence of two Gaussian kernels. Mean and variance of T-RMSE and I-RMSE for images in BrainWeb dataset are shown in top and middle panels. Bottom panel compares the running time for all methods.

to intensity-based registration. Again, this is particularly of our interest because MI achieves the most accurate and fastest registration compared to other three similarity measures.

A summary of the quantitative performance measures are presented in Table 1. The T-RMSE, I-RMSE and convergence percentage reported in this table are the average values across all the bias field noise situations ($0 \leq K \leq 4$). Using MI as similarity measure, AFR-EMD achieves 42% lower error rate in intensity and 52% lower error rate in transformation compared to intensity-based hierarchical registration. AFR-EMD with MI achieves 27% lower error rate in intensity and 21% lower error rate for transformation compared to intensity-based registration with RC similarity measure (which is robust to bias field noise). For LR-EMD, the error rate is 32% lower for intensity and 41% lower for transformation when using MI similarity measure.

### 4.2 IBSR dataset

We have also studied the performance of LR-EMD and AFR-EMD for MR images in Internet Brain Segmentation Repository (IBSR) dataset [37]. IBSR contains real magnetic resonance images and it is not possible to manually distort these images. Therefore, we register IBSR images across subjects. Figure 8, left two images are two sample MR images from IBSR. The registered images using both LR-EMD, AFR-EMD as well as benchmark intensity-based registration are also shown in Figure 9. Here, MI is used as similarity measure. In general, all three registrations have visually similar performance. EMD-based registrations tend to have slightly more adaptation to the global stricture of the reference image compared to intensity-based registration. In the area identified by a red circle in Figure 9, the patterns are more curved and adapted to the reference image structure.
Figure 7: Registration performance in the presence of three Gaussian kernels. Mean and variance of T-RMSE and I-RMSE for images in BrainWeb dataset are shown in top and middle panels. Bottom panel compares the running time for all methods.

Figure 8: Registration performance in the presence of four Gaussian kernels. Mean and variance of T-RMSE and I-RMSE for images in BrainWeb dataset are shown in top and middle panels. Bottom panel compares the running time for all methods.

Table 1: Summary of the quantitative performance measures

| Similarity measure | Method         | Convergence percentage | T-RMSE         | I-RMSE         |
|--------------------|----------------|------------------------|----------------|----------------|
|                    | Intensity-based| 61.33%                 | 2.393 ± 0.375  | 0.075 ± 0.008  |
| SSD                | LR-EMD         | 80%                    | 2.088 ± 0.309  | 0.073 ± 0.008  |
|                    | AFR-EMD        | 90%                    | 1.927 ± 0.206  | 0.067 ± 0.007  |
| CC                 | Intensity-based| 76%                    | 2.167 ± 0.341  | 0.069 ± 0.007  |
|                    | LR-EMD         | 100%                   | 2.032 ± 0.323  | 0.069 ± 0.008  |
|                    | AFR-EMD        | 100%                   | 1.777 ± 0.213  | 0.062 ± 0.007  |
| RC                 | Intensity-based| 96%                    | 1.331 ± 0.142  | 0.048 ± 0.003  |
|                    | LR-EMD         | 97.33%                 | 1.544 ± 0.357  | 0.053 ± 0.009  |
|                    | AFR-EMD        | 100%                   | 1.217 ± 0.153  | 0.043 ± 0.003  |
| MI                 | Intensity-based| 53%                    | 2.205 ± 0.516  | 0.062 ± 0.009  |
|                    | LR-EMD         | 100%                   | 1.299 ± 0.325  | 0.042 ± 0.008  |
|                    | AFR-EMD        | 100%                   | 1.054 ± 0.266  | 0.035 ± 0.007  |

when using EMD-based registration algorithms.

To quantify the performance, we use the manual segmentation that accompanies the MR images and provided by the Center for Morphometric Analysis at Massachusetts General Hospital (MGH). The manual segmentations is prepared by the experts. Each of the three registration methods achieve an average of 73% overlap between the reference and registered images across all segmented areas. There is no spatially varying noise in IBSR images, therefore, these results are consistent with our observations in section 3.1.1.
5 Discussion and future works

Here, we focused on the application of empirical mode decomposition in the registration of single modal MR images. However, we believe that EMD has potentially a wide variety of applications such as multi-modal image registration, image fusion, and image denoising. Specifically, the multi-modal image registration has a great potential for more investigations because EMD-based features are relatively robust to the modality. IMFs are made of normalized minimum and maximum envelopes of intensity. Therefore, the effect of modality-related variabilities are much less in IMFS. The idea of using EMD for multi-modal registration has been previously discussed [24], however, a more careful investigation of the applications of EMD in multi-modal image registration such as CT-MR image registration is still necessary.

In this paper, our methods does not tackle the parametrization of transformation and optimization problems in the registration. We have used free form deformation to formulate the transformation, however, EMD embedded in other transformation techniques needs further investigation. In particular, the application of nonlinear transforms such as neural networks [35] and random forest [39] in EMD-based registration needs extensive study. We believe that compressed non-linear networks [40] together with EMD could achieve fast and accurate registration in the presence of bias field noise. One advantage of these models is that the transform is more interpretable [41,42]. Other nonlinear black-box modular techniques such as Hammerstein Wiener model have shown impressive performance for applications such as nonlinear mapping in biomedical signal processing [43,44] and computational Neuroscience [45]. Employing these models as a core transformation technique between two images and possible EMD-based boost remain for future work.

Finally, we believe that EMD-based registration techniques generalize well to other registration application. Remote sensing and satellite images, other medical applications such as retina and angiography image registration and computer vision applications such as stereo vision are a few examples. An extensive study on the performance of EMD-based registration for these applications is necessary in the future.

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