Tunneling between two Luttinger liquids with long range interaction

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(Received 31 October 1996, to be published in Physical Review B)

The non linear charge transfer through a tunnel junction between two Luttinger systems is studied for repulsive, finite range interaction between electrons on the same, \( V_{11} \), and on different, \( V_{12} \), sides of the junction. Features of the Coulomb blockade effect are observed if \( V_{12} = 0 \). We predict a novel interaction induced enhancement of the current if \( V_{12} > 0 \). When \( V_{12} = V_{11} \), the current is suppressed at small bias, but the "charging energy", obtained from the asymptotic behavior at high bias voltage, vanishes.

PACS numbers: 73.40.Gk, 72.10.Bg, 72.10.-d

Due to the repulsive interaction between the electrons, tunneling through mesoscopic tunnel junctions is suppressed for voltages \( U < U_C \equiv \varepsilon/2C \), and temperatures \( T < T_C \equiv E_C/k_B \) \((k_B \) Boltzmann constant, \( \varepsilon \) elementary charge, \( C \) capacitance). The quantity \( E_C \equiv eU_C \) is called charging energy, the suppression of the transport Coulomb blockade.

In the semi-phenomenological theory, the tunnel junction is replaced by a capacitance and a tunnel resistance. An "external" impedance \( Z(\omega) \) represents the coupling of the tunneling particles to a reservoir of dissipative degrees of freedom. When \( Z(0) \equiv R = 0 \), the current-voltage characteristic \( I(U) \) is linear. For \( R \neq 0 \), the current is depleted for \( U \ll U_C \). When \( U_C \ll U \rightarrow \infty \) \( I \propto U - U_C \). The shift \( U_C \) is an important feature of the Coulomb blockade phenomenon for \( R \rightarrow \infty \) which is also found in experiments.

In the quantum mechanical theory, the Hamiltonian of the system, even using the Luttinger approximation, becomes non linear. For zero range repulsive interaction between the electrons, it was shown by using renormalization group arguments and by conformal field theory that an infinitesimally small scattering barrier in a one dimensional (1D) Luttinger liquid becomes completely insulating at zero temperature. The current is suppressed at small voltage, \( I(U) \propto U^{2/\gamma - 1} \) (interaction parameter \( g < 1 \) for repulsive interaction). This result was also found to the order \( \Delta^2 \) for the model of two Luttinger systems with \( N \gg 1 \) transport channels connected by a tunnel junction described by a transmission probability \( \Delta \). An effective interaction parameter \( g(N) \) was identified which tends to unity for \( N \rightarrow \infty \).

In the present paper, we study a tunnel junction connecting two 1D Luttinger systems with finite range interaction. The correlations between the particles located at different sides of the junction are taken into account. The current is calculated in terms of the density-density correlation function. We recover the above mentioned depletion of the current at small voltages, independent of the relative strength of the interaction between electrons on the same, \( V_{11} \) and on different sides, \( V_{12} \), of the junction. However, the latter turns out to determine the behavior at higher bias voltages: for \( V_{12} = 0 \) the "Coulomb blockade" is found. For \( V_{12} > 0 \), we find that the depletion of the current at high voltage is reduced. When \( V_{12} = V_{11} \), the current approaches asymptotically the non interacting, linear limit. As a consequence, a charging energy can only be obtained for an interaction potential with a finite, non zero range, when \( V_{12} < V_{11} \). For \( V_{12} > V_{11} \), though the interaction is assumed to be repulsive, the current is larger than without interaction.

We consider the Hamiltonian \( H = H_0 + H_I + H_U \). The unperturbed part, \( H_0 \), describes two separate spinless Luttinger systems, 1 and 2. They are assumed to extend from \(-L\) to 0 and from 0 to \( L \) (\( L \rightarrow \infty \), respectively). The tunnel junction (at \( x = 0 \)) is represented by \( H_I \), and \( H_U \) is the energy contributed by the external voltage. The interaction energy is

\[
H_{\text{int}} = \frac{1}{2} \int_{-L}^{L} dx dy \rho(x)\rho(y) \times V(|x-y|) [\lambda_{11} \Theta(xy) + \lambda_{12} \Theta(-xy)] .
\]

The interaction potential \( V(|x-y|) \), is assumed to introduce a length scale, say \( \alpha^{-1} \). For a screened potential, \( V(x) \propto \exp(-|x|/\ell) \), \( \alpha^{-1} = \ell \). For a Coulomb potential in 1D, \( V(x) \propto (x^2 + d^2)^{-1/2} \), \( \alpha^{-1} = d \). The (real and positive) parameters \( \lambda_{11} \) and \( \lambda_{12} \) are introduced in order to vary the strengths of \( V_{11} \equiv \lambda_{11} V \) and \( V_{12} \equiv \lambda_{12} V \) separately. The density operator is \( \rho(x) \equiv \rho^{(1)}(x) \Theta(x) + \rho^{(2)}(x) \Theta(-x) \) (\( \Theta(x) \) Heavyside function).

A crucial point is that the boundary conditions are such that the original Fermion fields vanish at \( x = \pm L \). This implies that the corresponding left and right moving parts are not independent but \( \Psi_R^{(j)}(x) = -\Psi_L^{(j)}(-x) \), \( \Psi_R^{(j)}(x + 2L) = \Psi_L^{(j)}(x) \), and either one of the two alone suffices to describe the system. Then, neglecting 2kF-scattering, \( \rho^{(j)}(x) = \rho_R^{(j)}(x) + \rho_L^{(j)}(-x) \), \( j = 1, 2 \), such that the Fourier transform of the interaction Hamiltonian contains terms that are non diagonal in the wave
numbers. We will see below that these lead to considerable complications in the calculation of the non linear current-voltage relation and the charging energy. The tunneling Hamiltonian is given by $H_t \equiv H_t^+ + H_t^- \equiv L \Delta (\Psi_R^{(2)}(0)|\Psi_R^{(1)}(0)\rangle + \text{h.c.}$). The energy associated with the external voltage is $H_U \equiv -e \int_0^L dx \, U(x)\rho(x)$. We assume that the bias voltage $U(x)$ drops across the tunnel contact only $\Omega^{(2)}$.

The average of the current operator $I \equiv ie[H_\gamma - H_t^+]$ is calculated from the backward and forward scattering rates $\gamma^\pm = \int_{-\infty}^{\infty} dt \langle H_t^\pm(t)H_t^+(0)\rangle$. The average can be performed by using the phase fields $\Phi^{(j)}(x,t)$ that are related to the Fermi fields as usual $12$.

$$I(U) = e [\gamma^+ - \gamma^-] = \frac{ie\Delta^2}{2} \int_{-\infty}^{\infty} dt \sin(eUt) e^{-W(t)} \tag{2}$$

where

$$W(t) \equiv -(\delta \Phi(t)\delta \Phi(0) + (\delta \Phi(0))^2) \equiv \int_0^\infty d\omega \frac{J(\omega)}{\omega^2} \times \left[ (1 - \cos(\omega t)) \coth \frac{\beta \omega}{2} + i \sin(\omega t) \right], \tag{3}$$

with $\delta \Phi(t) = \Phi^{(2)}(0,t) - \Phi^{(1)}(0,t)$ and the spectral correlation function $J(\omega)$. This shows that the non linear transport characteristic of the tunnel barrier is related to local current fluctuations represented by the phase fields $\Phi^{(j)}$, i.e. the dynamic and not the static properties of the system. This will be seen below in more detail, when we calculate the charging energy in terms of the interaction.

Generally, the spectral function can be decomposed

$$J(\omega) \equiv J_{11}(\omega) + J_{22}(\omega) + J_{12}(\omega) + J_{21}(\omega). \tag{4}$$

It is determined by the excitation spectrum of the Bosonic bulk modes, $\omega(q) = v_F|q|/g(q)$ with $g^{-2}(q) = 1 + \lambda_{11}\widetilde{V}(q)/\pi e v_F$. For small wave numbers $q$, $\omega(q) = v_F|q|/g$ such that $J(\omega \to 0)$ is given by the enhancement of the Fermi velocity $v_F$ with the interaction parameter, $g \equiv g(0)$, that contains only $\lambda_{11}$, $J(\omega) = 2\omega/g + \mathcal{O}(\omega^3/\alpha^2)$. The first two terms in eq. $4$ represent the contributions of the separate left and right Luttinger systems, and are given by the expectation values $\langle \Phi^{(j)}(0,t)\Phi^{(j)}(0,0)\rangle$. That they dominate the current at small voltages independent of $\alpha$ and $\lambda_{12}$, is reflected in the behavior of $J$ for small $q$.

The off-diagonal terms in eq. $4$ are due to the average $\langle \Phi^{(1)}(0,t)\Phi^{(2)}(0,0)\rangle$. They are typical vertex contributions to the average $\langle H_t(t)H_t(0)\rangle$, and describe correlations between the charge fluctuations on both sides of the junction. They influence in a characteristic way the spectral function at high frequencies,

$$J(\omega \to \infty) = 2\omega \left[ 1 - \frac{1}{2\pi} (\lambda_{11}(\omega\nu_c) + 2\lambda_{12}\nu_c) \right], \tag{5}$$

where $\nu_c \equiv \tilde{V}(q = \omega/v_F)$.

Since the interaction range is assumed to be finite in position space, the second term on the right hand side of eq. $4$ vanishes for $\omega \to \infty$ such that only the contribution of the free, non interacting electrons survives, namely $J(\omega) = 2\omega$. The presence of the term $\propto \lambda_{12}$ on the right hand side reflects the above mentioned fluctuation correlations. We will see below in detail that the latter counteract the suppression of the current induced by the coupling to the bulk modes and eventually lead to a vanishing of the charging energy if $\lambda_{11} = \lambda_{12}$. If $\lambda_{12} > \lambda_{11}$ the current at high bias voltage becomes even enhanced above the value without interaction. The behavior of the current-voltage characteristic at zero temperature is shown schematically in Fig. $4$.

![FIG. 1. The zero temperature current-voltage characteristic, schematically, of a tunnel junction (tunneling resistance $R_t$) connecting two semi infinite Luttinger liquids for various ratios of the interaction strengths between electrons on different, $\lambda_{12}$, and the same, $\lambda_{11}$, sides of the junction. Full line: $\lambda_{12}/\lambda_{11} = 0$; short dashes: $\lambda_{12}/\lambda_{11} < 1$; dots: $\lambda_{12}/\lambda_{11} > 1$; long dashes: no interaction.]

The connection with the earlier, semi classical results $4$ is established by rewriting $I(U)$ as

$$I(U) = \frac{1}{eR_t} \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} dE' f(E)[1 - f(E')] \times P(E + U - E'), \tag{6}$$

with the Fermi function $f(E)$ and

$$P(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{iEt} e^{-[W(t) - W_{\gamma = \gamma}(t)]}. \tag{7}$$

Only in the semi classical approach, the latter function plays the role of a probability density for a bulk excitation of energy $E$. In the present microscopic model, it can become negative. This is a result of the extraction of the Fermi factor $f(1 - f)$ in the integrand in eq. $6$ which is artificial, though formally correct, for the present interacting system. The tunnel resistance $R_t \equiv 2\omega^2/\epsilon e^2 \Delta^2 \pi$ contains the cutoff frequency $\omega_c$ which serves to regularize the Luttinger model.

The result for zero temperature,
\[ I(U) = \frac{1}{eR_t} \int_{0}^{eU} dE(eU - E)P(E), \]  

\[ \text{can be used to identify the parameters in the semi classical approach in terms of the microscopic model. The function that corresponds to the impedance in the semi classical theory is } Z(\omega) \equiv J(\omega)/\omega - 2. \]  

\[ \text{For small bias, } eU \ll \alpha e_F, \text{ only the behavior of } P(E) \text{ for small energies is important. It is determined by the diagonal contributions in eq. (8) and always positive. The electrons loose energy via dissipation into the bulk modes. We find } I(U) \approx (U/R_t)(eU/\alpha e_F)^2/\omega - 2, \text{ which yields the dissipative resistance } R = Z(0) = 2(\gamma^{-1} - 1) \text{ (} R_K = h/e^2 \text{ von Klitzing constant). This shows that the } \text{elementary excitations of the Luttinger system play here the role of the dissipative Bosonic degrees of freedom of the "electromagnetic environment".} \]

\[ \text{At high bias voltage, } eU > \alpha e_F, \text{ } J_{12} \text{ and } J_{21} \text{ become dominant. They counteract the two dissipative diagonal terms } J_{11} \text{ and } J_{22}. \text{ While the latter always suppress, the former tend to } \text{transport} \text{. Formally, this can be seen from } P(E \to \infty). \text{ We start by calculating the second derivative of the current-voltage relation, eq. (8)} I''(U) = eP(eU)/R_t. \text{ For very large } U, \text{ } P(eU) = Z(eU)/eU. \text{ If we make the (not very restrictive) assumption that for large wave numbers } \hat{V}(q) \propto q^{-2}, \text{ we have } \pi Z(\omega \to \infty) = 2V_U(\lambda_{11}/2 - \lambda_{12}) \text{ which changes sign when } \lambda_{12} > \lambda_{11}/2. \text{ When the coupling between electrons left and right of the junction exceeds a certain strength, the curvature of the current-voltage curve changes from positive to negative. This occurs for voltages that correspond to the intrinsic length scale } \alpha^{-1}. \]

\[ \text{For } U \gg \alpha e_F, \text{ only short times contribute to the integrand of eq. (8), the current, eq. (8), becomes } I(U) = R_t^{-1}(U - E_C/e) + O(U^{-1}) \text{ with the charging energy } \]

\[ E_C = \int_{0}^{\infty} d\omega Z(\omega) = V(x = 0)(\lambda_{11} - \lambda_{12}). \]

\[ \text{The derivation of this result is far from trivial. The starting point is the relation between } W(t), \text{ eq. (8)}, \text{ and the fluctuation correlation function } G_{kk'}(t) \]

\[ F(t) \equiv -i\Theta(t) \langle [\delta \Phi(0, t), \delta \Phi(0, 0)] \rangle \]

\[ \equiv \frac{4\pi}{\mathcal{L}} \sum_{kk', \lambda > 0} \frac{1}{kk'} G_{kk'}(t). \]

\[ \text{In } W(t), \text{ the average } \langle [\delta \Phi(0, t), \delta \Phi(0, 0)] \rangle \text{ enters. It can be expressed via the dissipation-fluctuation theorem in terms of the imaginary part of the Fourier transform of } F(t). \text{ This leads eventually to the identity} \]

\[ J(\omega) = -\frac{1}{\pi} \omega^2 \mathcal{I}m \tilde{F}(\omega). \]

\[ \text{The set of equations of motion for the correlation function can be closed for our model. The resulting linear integral equation is used to expand } \hat{G}_{kk'}(\omega) \text{ in powers of } \hat{G}_k^{(0)}(\omega)B_{kk'}(\omega), \text{ where } \hat{G}_k^{(0)} \equiv \omega^2/(\omega^2 - \omega_k^2) \text{ is the unperturbed correlation function. It contains the excitation spectrum of the bulk modes. The function } B_{kk'} \text{ contains the off-diagonal part of the interaction, } \]

\[ \mathcal{V}(k, k') \equiv -\frac{1}{\pi} \int_{-\infty}^{\infty} dq (q^2 - k^2)(q^2 - k'^2)^{-1}. \]

\[ \text{By performing first the } \omega \text{-integration (cf. eq. (8)) and then the summations over the wave numbers, one can show that in the integral over the imaginary part of } \mathcal{F} \text{ only the term linear in } B_{kk'} \text{ contributes. All higher order contributions vanish, due to exact sum rules. The calculation leads eventually to} \]

\[ E_C = -2 \int_{0}^{\infty} d\omega \left[ \frac{2}{\mathcal{L}} \sum_{kk'} \frac{\omega \mathcal{I}m \hat{G}_{kk'}(\omega)}{kk'} + 1 \right] \]

\[ \mathcal{I} = E_C^0 - \frac{2(\lambda_{11} + \lambda_{12})}{\pi^2} \int_{0}^{\infty} dk \, dk' \mathcal{V}(k, k'). \]

\[ \text{The first term in this expression is the result obtained without taking into account the off-diagonal interaction terms, eq. (8), } E_C^0 = 2\lambda_1 V(x = 0). \text{ The second can be further evaluated and gives } -\lambda_{11} \lambda_{12} V(x = 0) \text{ such that finally the above result eq. (8) is obtained.} \]

\[ \text{There are several comments which have to be made at this point. When the interaction between electrons left and right of the tunnel junction is omitted, } \lambda_{11} = 1, \lambda_{12} = 0 \text{ the preliminary result obtained previously [4] is recovered apart from a factor } 1/2 \text{ which is due to discarding the influence of the boundary conditions on the fields. The latter influence is negligible for small frequencies [14]. For high frequencies, the boundary conditions influence quantitatively the result for the charging energy, though the qualitative behavior with the strength and the range of the interaction is the same as before. When the interaction of electrons in the left and the right part of the system is switched on, } \lambda_{12} \neq 0, \text{ the charging energy is reduced as compared to the previous case. For a given voltage, the current is increased, due to the presence of the additional, (formally) repulsive electron-electron term. In principle, when the interaction between electrons in the left and the right part of the system dominates, } \lambda_{11} < \lambda_{12}, \text{ the current increases even to values above that of the non-interacting system, } I(U) > U/R_t. \text{ When the bias voltage is sufficiently high, net charges with opposite signs are introduced left and right of the tunnel junction. This} \]
is also suggested by the model of the "Landauer dipole" at an impurity in the presence of a stationary current. As a consequence, the net interaction between the left and right Luttinger liquids becomes attractive and tends to decrease the charge difference – the bias voltage – via additional (tunnel) current. This cannot happen in the semi classical model since there the quantum processes are introduced into the model "by hand", and not inherently incorporated into the theory.

Insisting nevertheless on assigning a capacitance to the tunnel junction leads to a dependence of the latter on the bias voltage due to the suppression of the current well below the value of the system without interaction. When increasing the bias voltage the correlations between the electrons on the left and the right of the junction increase the current, i.e. decrease the effective charging energy such that the "capacitance" would increase to infinity when \( \lambda_{12} \rightarrow \lambda_{11} \). This shows its dynamical origin.

A similar result is obtained when considering a potential barrier in a Luttinger liquid, where the interaction between electrons left and right of the barrier is taken into account automatically, \( \lambda_{11} = \lambda_{12} \).

Experimentally, it is very difficult to observe Coulomb blockade for a single tunnel junction between two metallic wires due to the presence of a large shunt capacitance. By adding more junctions in series, this shunt capacitance is reduced due to the small capacitances of the other junctions. Our above result offers a microscopic interpretation: adding junctions in series reduces the interaction between electrons in the left and right leads. As a consequence, the observed charging energy is increased.

A direct experimental test could be performed by using quantum wires based on semiconductor heterostructures and adjusting the electron density such that only one subband is occupied. In the presence of a gate across the region of the tunnel barrier, the interaction between the leads on the left and the right hand side of the barrier could be locally screened such that the charging energy should appear.

In summary, we obtained the non linear current-voltage characteristic for a model of two 1D quantum wires of interacting electrons connected by a tunnel junction. The features of the Coulomb blockade phenomenon were identified. The charging energy was determined as a function of the parameters of the interaction potential. We showed that it is crucial in this microscopic model that the interaction, besides being of finite range, must not be too strong between electrons on different sides of the junction in order to produce a charging energy.

Discussions with Detlef Heitmann, Wolfgang Hansen, Klaus von Klitzing and Leo Kouwenhoven concerning experiments are gratefully acknowledged. This work has been supported by the EU via HCM and TMR programmes, contracts CHRX-CT93 0136, CHRX-CT94 0464, FMRX-CT96 0042, and by the Deutsche For-