Theoretical prediction of punching shear capacity of flat slabs without shear reinforcement strengthened by concrete topping

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Abstract. The main reasons for strengthening flat slabs are the change of the use of a building, increase in the value of loads, degradation of the concrete cover layer, or insufficient reinforcement. This paper is focused on the assessment of punching shear capacity of the strengthened flat slabs without shear reinforcement. One of the possibilities how to enhance punching shear capacity is the addition of reinforced concrete topping. The main goal of this paper is to compare the possibilities for calculation of the increase in the punching shear capacity by investigation of the influence of different thicknesses of concrete toppings and different reinforcement ratio. A reference specimen is represented by a fragment of a flat slab with the thickness of \( h = 200 \text{ mm} \) supported by circular column with the diameter of 250 mm. Three different thicknesses (50 mm, 100 mm, 150 mm) of concrete toppings were considered together with three different reinforcement ratios for each thickness of concrete overlay. Theoretical predictions of the punching shear resistance of flat slabs were evaluated by design guidelines according to the relevant standards: Eurocode 2 (EN 1992-1-1), Model Code 2010 and draft of the second generation of Eurocode 2 (prEN 1992-1-1). The differences in the influence of reinforcement ratio are significant. In Model Code 2010 the reinforcement ratio in concrete topping was considered in equation of moment of resistance. This is unlike in both of the mentioned Eurocodes, where the reinforcement ratio was assumed as a geometric average value of the original reinforcement ratio in the slab before strengthening and of the reinforcement ratio of concrete topping. All the predicted theoretical calculations are based on the perfect connection and bond between the original and new layer of concrete. These predictions should be verified by experimental investigation, which is going to be prepared shortly. By the additional increase in the thickness of concrete topping or in the amount of added reinforcement the attention should be payed to the limitation of the punching shear resistance by the value of the maximum punching shear resistance in the compression concrete strut.

1. Introduction
The failures of flat slabs often occur in existing buildings. The causes of these failures can be mistakes in design of reinforcement of the ceiling slab, mechanical damage, degradation of the concrete cover layer, or change of the use of a building, which is the most common problem when renovating existing buildings. In this case, the resistance of the ceiling slab is insufficient and needs to be increased by strengthening.

The most common ways, how to increase in shear resistance of the flat slabs are the increase in thickness of a slab around columns by concrete or steel column heads, the increase in size of the...
columns, the addition of different types of transverse reinforcement or the increase in the thickness of a slab by concrete topping.

My research is focused on the assessment of punching shear capacity of the strengthened flat slabs without shear reinforcement. One of the possibilities how to enhance punching shear capacity is the addition of reinforced concrete topping. Main advantages of this method of strengthening are: good accessibility to the upper face and easy implementation. Disadvantages of this method of strengthening are: necessary shear connection between the flat slab and the concrete overlay, the possibility of use for flat slabs that are not accessible from upper surface e.g. garage slabs and reduction of the ceiling height.

2. Calculation of shear resistance
The theoretical study consisted of examining the specimen represented by a fragment of a flat slab 2.5m x 2.5m wide with the thickness of $h = 200$ mm. Concrete topping was considered with various thicknesses - 50 mm, 100 mm and 150 mm. The fragment of the flat slab is supported by a circular column with a diameter of 250 mm.

The shear resistance of the reinforced slab without shear reinforcement with concrete overlay was calculated according to three different design models. The first design model considered in this study was Eurocode 2 [1], which is the current valid code for the design of reinforced concrete structures in Europe. The second calculation model was Model Code 2010 [2] [3], which takes into account rotations in the slab and is based on Critical Shear Crack Theory [4]. The last used design model was the latest version of the new generation of Eurocode 2 [5], which is based on the Model Code 2010. The aim of this paper is to compare the results of individual design approaches.

2.1. Eurocode 2
According to Eurocode 2 the punching shear resistance without shear reinforcement of the flat slab was calculated according to the equation (Eq.1)

$$ V_{Rd,c} = C_{Rd,c} \cdot k_h \cdot [100 \cdot \rho_1 \cdot f_{ck} / u_1 \cdot d]^{1/3} $$

where:
- $C_{Rd,c}$ empirical factor, for concrete partial safety factor $\gamma_C = 1 \rightarrow C_{Rd,c} = 1.8$MPa
- $k_h$ size effect factor
- $\rho_1$ flexural reinforcement ratio, geometric mean value of ratios in orthogonal directions $\rho_{lx}, \rho_{ly}$
- $f_{ck}$ characteristic cylinder compressive strength of concrete
- $u_1$ length of basic control perimeter at the distance $2d$ from the column face
- $d$ effective depth, the average value of the effective depths in two orthogonal directions $dx, dy$

For Eurocode 2, the control perimeter $u_1$ is considered at the distance of $2d$ from the column face. In EC 2 the shear resistance of the slab with concrete topping can be taken into account only by modification of reinforcement ratio and increasing thickness of the slab considering the perfect shear connection between the original slab and the concrete overlay. The effective depth of a slab strengthened by concrete topping was determined as the geometric mean value of the effective depth of the original 200 mm thick flat slab and the effective depth of a slab strengthened by concrete topping taking into account the reinforcement area. The reinforcement ratio of strengthened slab was theoretically estimated as the sum of the reinforcement ratio of a slab 200 mm thick reinforced with $\phi 16/125$mm and the reinforcement ratio of strengthened slab including only upper reinforcement in concrete topping.
2.2. Model Code 2010
Model Code is based on Critical Shear Crack Theory, published in 1991 by Aurelio Muttoni and Joseph Schwartz. Punching shear resistance according to Model Code 2010 (Eq.2) is calculated as follows:

\[ V_{Rd, c} = k_{\psi} \cdot \frac{f_{ck} \cdot b_0 \cdot d_v}{\gamma_C} \]  

(2)

\[ k_{\psi} = \frac{1}{1.5 + 0.9 k_{dg} \cdot \psi} \leq 0.6 \]  

(3)

\[ \psi = 1.2 \cdot \frac{r_s}{d} \cdot \frac{f_{yd}}{E_s} \cdot \left( \frac{m_{Ed}}{m_{Rd}} \right)^{1.5} \]  

(4)

where
- \( k_{\psi} \) parameter which depends on the deformations (rotations) of the slab
- \( \gamma_C \) concrete partial safety factor, \( \gamma_C = 1 \)
- \( b_0 \) length of basic control perimeter at the distance 0.5\( d_v \) from the column face
- \( d_v \) shear-resisting effective depth, the average value of the shear-resisting effective depths in two orthogonal directions \( dx, dy \) (\( d_v = \bar{d} \))
- \( k_{dg} \) parameter which depends on maximum size of aggregate (\( d_g \)) and its properties
- \( \psi \) slab rotation
- \( r_s \) value that is approximated as 0.22 \( L_x \) or 0.22 \( L_y \) for the \( x \) and \( y \) directions, where \( L_x \) and \( L_y \) are spans between support which are considered as \( L_x = L_y = 6m \)
- \( m_{Ed} \) average bending moment acting in the support strip, approximated as \( m_{Ed} = \frac{1}{8} V_{Ed} \)
- \( m_{Rd} \) average resisting bending moment
- \( E_s \) Young’s modulus of elasticity of steel \( E_s = 200GPa \)

Punching shear resistance depends on parameter \( k_{\psi} \) (Eq.3), which includes rotations of the flat slab. Calculations of rotations (Eq.4) around the supported area are based on four levels of approximations. The first level of approximation is suitable for flat slab without significant redistribution of internal forces. The second level of approximation is used if a significant redistribution of bending moments is considered in the design. If the bending moment \( m_{Ed} \) and the resisting bending moment \( m_{Rd} \) are determined from a linear elastic calculation without the influence of cracks, the third level of approximation is considered. The fourth level of approximation is used in exceptional cases, when the calculation is based on a non-linear analysis of the structure taking into account cracking, tension-stiffening effects, yielding of the reinforcement or any other non-linear effects. Maximum punching shear resistance in this paper was calculated by the third level of approximation. Average resisting bending moment \( m_{Rd} \) was determined from equilibrium conditions between internal forces in the reinforcement of concrete overlay, the reinforcement in the original slab and compression force in concrete.

2.3. New generation of Eurocode 2
The latest draft of the new generation of Eurocode 2 is based on Model Code 2010. Calculation of punching shear resistance is represented by (Eq.5):

\[ V_{Rd, c} = \frac{0.6}{\gamma_v} k_{pb} \cdot \left[ 100 \cdot p_i \cdot f_{ck} \cdot \frac{d_{dg}}{d} \right]^{1/3} \cdot b_0 \cdot d \]  

(5)

where
- \( k_{pb} \) the punching shear gradient enhancement coefficient
- \( d_{dg} \) a coefficient that takes into account the type of concrete and its aggregate properties
- \( \gamma_v \) partial factor for shear and punching resistance, \( \gamma_v = 1.4 \)
The change from the original Eurocode 2 is in the control perimeter, in which the shear resistance is assessed. The control perimeter $b_0$ considers the faces of the circular column and the basic control perimeter $b_{0.5}$ considers the distance $0.5d_v$. For a slab directly supported by column, $d_v = d$ ($d_v$ is equal to the average value of the effective depths in two orthogonal directions $d_x$, $d_y$). As Eurocode 2, the punching shear resistance of the strengthened flat slab by concrete overlay depends on thickness of the strengthened slab and the reinforcement ratio.

3. Results of the parametric study
The theoretical assumptions for this parametric study considered C30/37 concrete class with characteristic cylinder compressive strength $f_{ck} = 30$MPa. The flat slab with a thickness of $h_d=200$ mm had upper reinforcement $\phi 16/125$mm in both directions. The effective depth of the slab $d$ was calculated for the 200mm thick fragment for the diameter of reinforcement of $\phi = 16$mm and concrete cover $c_{nom} = 30$mm. Concrete overlays of thicknesses of 50mm, 100mm and 150mm, specifically, were reinforced with upper reinforcement $\phi_{o,1} = 8$mm, $\phi_{o,2} = 12$mm and $\phi_{o,3} = 16$mm in two orthogonal directions $x$ and $y$ in grid of 150mm. Verification consist of 9 calculations of shear resistance of slabs without shear reinforcement with different thicknesses of concrete toppings and different reinforcement ratio in them and one calculation for a fragment of thickness 200 mm, which was considered as the original structure.

| Concrete overlay thickness [m] | $h_d$ [m] | $\phi_{o/n}$ [mm/-] | Eurocode 2 $V_{Rd,c,EC2}$ [kN] | New gen. of EC2 $V_{Rd,c,EC2}$ [kN] | Model Code 2010 $V_{Rd,c,MC10}$ [kN] | Eurocode 2 $V_{Rd,max}$ [kN] |
|--------------------------------|------------|---------------------|-------------------------------|---------------------------------|--------------------------------|----------------------------|
| 0                              | 0.20       | -                   | 475.43                        | 486.85                          | 412.29                          | 765.96                     |
| 0.05                           | 0.25       | 8/150               | 564.20                        | 562.78                          | 456.47                          | 824.30                     |
| 0.10                           | 0.30       | 8/150               | 611.18                        | 598.54                          | 488.96                          | 867.19                     |
| 0.15                           | 0.35       | 8/150               | 661.12                        | 636.10                          | 522.04                          | 910.09                     |

| Concrete overlay thickness [m] | $h_d$ [m] | $\phi_{o/n}$ [mm/-] | Eurocode 2 $V_{Rd,c,EC2}$ [kN] | New gen. of EC2 $V_{Rd,c,EC2}$ [kN] | Model Code 2010 $V_{Rd,c,MC10}$ [kN] | Eurocode 2 $V_{Rd,max}$ [kN] |
|--------------------------------|------------|---------------------|-------------------------------|---------------------------------|--------------------------------|----------------------------|
| 0                              | 0.20       | -                   | 475.43                        | 486.85                          | 412.29                          | 765.96                     |
| 0.05                           | 0.25       | 12/150              | 639.30                        | 628.07                          | 490.93                          | 859.65                     |
| 0.10                           | 0.30       | 12/150              | 733.55                        | 697.68                          | 557.16                          | 939.04                     |
| 0.15                           | 0.35       | 12/150              | 831.40                        | 795.82                          | 625.50                          | 1018.44                    |

| Concrete overlay thickness [m] | $h_d$ [m] | $\phi_{o/n}$ [mm/-] | Eurocode 2 $V_{Rd,c,EC2}$ [kN] | New gen. of EC2 $V_{Rd,c,EC2}$ [kN] | Model Code 2010 $V_{Rd,c,MC10}$ [kN] | Eurocode 2 $V_{Rd,max}$ [kN] |
|--------------------------------|------------|---------------------|-------------------------------|---------------------------------|--------------------------------|----------------------------|
| 0                              | 0.20       | -                   | 475.43                        | 486.85                          | 412.29                          | 765.96                     |
| 0.05                           | 0.25       | 16/150              | 709.46                        | 691.36                          | 519.80                          | 879.03                     |
| 0.10                           | 0.30       | 16/150              | 854.00                        | 795.82                          | 627.30                          | 992.10                     |
| 0.15                           | 0.35       | 16/150              | 988.49                        | 907.78                          | 732.75                          | 1105.17                    |
Tables 1-3 represent the results of calculations based on the 3 mentioned standards for different thicknesses of concrete overlay and different reinforcements in it, for better clarity, the tables also include the overall considered thicknesses of the flat slab.

4. Conclusions
Based on the calculations, if the perfect connection and bond between the original and new layer of concrete is assumed, the punching shear resistance of the strengthened flat slab increases in each case. From the results of the parametric study, it can be observed that with each reinforcement, the increase of the resistance for concrete overlay 150mm thick is approximately 30% greater than for 50mm thickness. At the same time, a significant effect of the amount of reinforcement in the concrete overlay can be seen on increasing the durability of the original structure. While for concrete overlay 150mm thick, the overall increase in resistance is approximately 40% with the reinforcement diameter ϕ8mm, with the same thickness but with reinforcement ϕ16mm the increase of its original resistance is almost twofold.
The most stringent results are given by the design model Model Code 2010, which determines the rotations in the slab, while the new generation of Eurocode 2 allows to be more tolerant as the current valid design standard. In calculations based on EC2 and nEC2 it can be observed that in a certain point of increase the thickness of concrete topping can lead to reach the value of the maximum punching shear resistance in the compression concrete strut (Figure 1-3).

Acknowledgment(s)
This work was supported by the Scientific Grant Agency VEGA under the contract No. VEGA 1/0522/20.

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