Freezeout of resonances and nuclear fragments at RHIC

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Abstract. We quantify the conditions at which “composites”, the resonances and bound states $d, He^3$ are produced at RHIC. Using Hubble-like model for late stages, one can analytically solve the rate equations and also calculate the relevant optical depth factors. We calculate also the modification of $\rho$ mass and width, and predict a radiacal shape change of $\sigma$.

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1. Introduction

This talks is based on two papers, one with G.Brown [1] and one ongoing work with P.Kolb [2], who will report a part of it related with late time evolution here. Their common goal is to understand what are the conditions which determine the timing of production of observable resonances, and estimate at what conditions this happens. We have emphasized observable above because there are many resonances produced in the system but unobserved in the final invariant mass spectra because of re-scattering of their decay products. The development of simple analytic model of the kinetics of resonance production/absorption, as well as evaluation of the “optical depth” integral.

An old but still important idea is “matter modification” of hadrons, providing the experimental test of the conditions in question. As such we discuss modification of the mass and shape of two classic $\pi\pi$ resonances, $\rho$ and $\sigma$. Recent STAR observations of these effects are reproduced (for $\rho$) and predicted (for $\sigma$).

The issue of production of nuclear fragments is also an old one, at RHIC reduced to $d, He^3$ and their antiparticles. In literature those are studied either by statistical or coalescence models, which left open many important issues. First of all, like resonances the observable fragments must escape all interactions with any particles in order to survive at the end. The second point is that this process is production-rate-limited, thus it can lead to non-thermal quasi-equilibria. Furthermore, new element is the consistent evaluation of the 2-to-1 production rate itself in a recent
work by Ioffe et al \cite{Ioffe}. 

2. The optical depth factor 

Let me start with the simplest pedagogical points about the observability of resonances and fragments, or “composites” as we may call them collectively, for brevity. 

In the next section we will discuss rate equations which can be solved and determine the number of composites \( N(t) \) at time \( t \). The “observability condition” of a resonance can be written as 

\[
\nu_{\text{visible}}(t) = \Gamma N(t) \exp \left( - \int_t^\infty \nu(t') dt' \right)
\]  

where the the l.h.s. is the production rate of visible resonances, \( \Gamma \) is the resonance decay width and the exponent is the optical depth factor containing integrated \( \nu(t) \), the combined scattering rate for all decay products. The \( N(t) \) decreases with time due to expansion and cooling, while the exponent changes from 0 at early time to 1 at late times: so the product naturally has a maximum at the time \( t_m \) such that 

\[
\frac{1}{N(t_m)} \frac{dN(t_m)}{dt} + \nu(t_m) = 0
\]  

This condition means that for observable resonances the freezeout condition is different from that for stable particles and reads: the rate of their number change is equal to the absorption rate of all the decay products. For example, for \( \rho \) and \( \sigma \) we should not know their scattering rates but just that of two pions. Since for short-lived \( \rho \) and \( \sigma \) the first factor is close to overall expansion rate of matter at late time which follows from Hubble-like late-time regime \( d \log N(t_m)/dt \approx 3/t \), and the second is the same, we conclude that ‘visible” \( \rho \) and \( \sigma \) are produced at the same time. 

The formation rate for a fragment made of \( A \) nucleons is made by some coalescence, and such rate is obviously proportional to nucleon density to that power, \( \sim (n_N(t))^A \). After it is produced, however, it still has very small probability to survive. Assuming that the destruction rate for a fragment \( \nu_A \approx A \nu_N \), where \( \nu_N \) is a scattering rate for one nucleon, one finds that for \( A \)-fragment the time distribution is approximately the \( A \)-th power of the same universal function 

\[
n_{\text{fragments}}(t) \sim \left[ n_N(t) \exp \left( - \int_t^\infty \nu_N(t') dt' \right) \right]^A
\]  

So, the maximum of production of any visible fragment happens at the same time for all \( A \). Furthermore, the width of the distribution over production time decreases as \( A \) grows, as \( 1/\sqrt{A} \).
3. Solving the rate equations

The equations themselves describing dynamics of resonances are well known, generically they contain the sink (the decay) and the source terms

$$\frac{\partial n(t, \vec{r})}{\partial t} = -\Gamma n(t, \vec{r}) + S(n_i)$$  \hspace{1cm} (4)

(where for expanding source the time should be understood as proper time in the rest frame of all volume elements.) In many papers in literature (e.g. (3)) the source term is ignored citing “instantaneous hadronization”, but (especially for resonances we consider) it is not true: in fact the primary generation of resonances die out long before the “observable” ones are born.

We use an approximate power fit of the source time dependence \( \int d^3r S = \Gamma N_0 \left( \frac{t_0}{t} \right)^P \). Its power can be related to fireball expansion. If the volume \( V(t) \sim 1/n(t) \sim t^a \) the integrated source is proportional to \( V(t)[n_\pi(t)]^N \) where \( N \) is the multiplicity in resonance decay (\( N = 2 \) for \( \sigma, \rho \)). The pion number is “chemically frozen”, \( N_\pi = V(t)n_\pi(t) = \text{const}(t) \) from which we conclude that the source term power is \( P = (N - 1)a \) (for \( \sigma, \rho \) and other binary resonances \( P = a ) \). An example of such equation solved is shown in Fig.1.

![Figure 1](image1.png)

**Fig. 1.** (a) The time dependence of the \( \rho \)-meson density, starting from chemical equilibrium at \( t = 5 \) and \( 10 \, \text{fm} \). Plotting its ratio to the pion density in (b) one observes the transition from hydro to free streaming regimes and that the \( \rho \) density decreases power-like rather than exponentially, because of the source term.

We then evaluate the optical factors for pions and nucleons, using realistic re-scattering rates, with chemically frozen composition, using papers by Hung and myself [5] and by Tomasik and Wiedemann [8]. Example for the final time distributions for visible \( \rho \) and \( d \) is shown in Fig.2. Note that both distribution have maxima we discussed above, and that the “visible” \( d \) are indeed produced very
late. This is our main point: we are speaking about a very dilute matter, after the freezeout of all the basic ingredients of the fireball.

Fig. 2. The time dependence of the visible $\rho$-meson (a) and deuteron (b) production, for the $r = 0$ point in central AuAu collisions at RHIC.

4. The resonance modification

It has been argued over the years that in matter the resonances should be modified, with shifted mass, increased width and even significantly changed shape. With the very late stages of RHIC collisions relevant, we can now access very dilute matter in which those effects must be calculable in the lowest order of the density, providing a benchmark test to all such discussions. In such case hadron modification is expressed in terms of their forward scattering amplitude $M_{ij}(t = 0, s)$. Note that the scattering amplitude is complex, and that this approach gives both the real and imaginary part of the dispersion law modification, also known as the optical potential.

There are two major theoretical approaches to the issue discussed in literature, to be called an $s$–channel and a $t$–channel one. The former approach assumes that the scattering amplitude is dominated by $s$-channel resonances which are known to decay into the $i + j$ channel. For most mesons such as $\pi, \omega, \rho, K$ in a gas made of pions such calculation has been made e.g. in [6] related with such $\pi \rho$ resonances as $a_1$ or $N \rho$ resonance $N^+(1520)$ [4] for $\rho$ at SPS. Note that the signs of the effects are opposite in those examples, as seen from the following table:

The majority of the particles in the matter are Goldstone bosons $\pi, K, \eta$ which do not interact at small momenta. However attraction between other particles is there. Using a simple expression for the mass shift one gets $\delta m_N^N \approx -28\text{MeV}$ due to all $\bar{B} + B$. An additional shift $\delta m_\rho^\omega \approx -10\text{MeV}$ comes from scalar exchanges between $\rho$ and all other vector mesons $\rho, \omega, K^*$. The main difference between the
two mechanisms of the mass shift discussed above is that the t-channel attraction
is not associated with the broadening, while the s-channel resonances increase the
width by about 50 MeV. On the other hand, there is a “kinematic” effect working
to the opposite direction. The negative mass shift discussed above automatically
reduces the width, both because of the reduced phase space and also due to the
power of p in the P-wave matrix element. The magnitude of this effect for the
predicted mass shift is

\[
\delta \Gamma = 3 \frac{\delta m}{m} \Gamma \approx -50 \text{ MeV} \quad (5)
\]

So, inside the accuracy these two effects cancel each other.

The invariant mass distributions in pp and mid-central AuAu of the \( \pi^+ \pi^- \)
system, with a transverse momentum cut 0.2 < \( p_t \) < 0.9 GeV have been measured
by STAR \cite{9}, see also the C.Markert’s talk here. I would not have time here to
discuss the shift in pp (see \cite{1}), and I only comment that in AuAu the \( \rho \) peak is
found to be shifted by additional \( \sim -40 \text{ MeV} \) in mass, but the width is the same.
This agrees well with estimates above.

The same approach should of course be applied to many other resonances.
For more narrow resonances, like \( K^* \), we expect smaller shifts, while for wider
resonances like \( \sigma \) we predicted a complete change of shape. At small freezeout \( T \approx
100 \text{ MeV} \) sigma was predicted to be deformed into a much more narrow structure
at mass of about 400 MeV, see figures in \cite{1}. Exactly such a peak has been seen
by STAR\footnote{I have seen it first at this workshop, on the day after my talk, shown by Gary Westfall in a plot of the balance function as a function of \( q_{inv} \) with a peak he said “nobody understands”. It was rather good timing between the prediction and its experimental confirmation.}. Another confirmation of very late freezeout and low \( T \), the \( \sigma/\rho \) ratio
strongly grows toward central collisions. We are waiting for quantitative analysis
of these data with great interest.

5. Fragment coalescence

The issue of coalescence, such as \( p + n \rightarrow d \), was discussed in many papers over
the years, and authors struggled with the question how to calculate its rate. In
particular, it is clear that when the level crosses zero the wave function at the
origin vanishes, and so the production should do so too. And if the production rate is small compared to two other relevant rates, $\nu_{abs}$ and $dlogN/dt$, there is never thermal equilibrium and one should not use statistical models.

Significant progress has been made in recent paper by Ioffe et al [4] who have pointed out how to use consistently the in-matter widths of all particles and obtain the production rate. We are now incorporating it into the picture of expansion and the optical depths discussed above, and hope to get quantitative results for $\bar{d}, d$ spectra soon.

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