Hydrostatic equilibrium of causally consistent and dynamically stable neutron star models

P. S. Negi*

Department of Physics, Kumaun University, Nainital – 263 002, India

Accepted 2008 April 17. Received 2008 April 15; in original form 2008 January 12

ABSTRACT

We show that the mass–radius (M–R) relation corresponding to the stiffest equation of state (EOS) does not provide the necessary and sufficient condition of dynamical stability for equilibrium configurations, because such configurations cannot satisfy the ‘compatibility criterion’. In this regard, we construct sequences composed of core–envelope models such that, like the central condition belonging to the stiffest EOS, each member of these sequences satisfies the extreme case of the causality condition, \( v = c = 1 \), at the centre. We thereafter show that the M–R relation corresponding to the said core–envelope model sequences can provide the necessary and sufficient condition of dynamical stability only when the ‘compatibility criterion’ for these sequences is ‘appropriately’ satisfied. However, the ‘compatibility criterion’ can remain satisfied even when the M–R relation does not provide the necessary and sufficient condition of dynamical stability for the equilibrium configurations. In continuation of the results of a previous study, these results explicitly show that the ‘compatibility criterion’ independently provides, in general, the necessary and sufficient condition of hydrostatic equilibrium for any regular sequence. In addition to its fundamental result, this study can explain simultaneously the higher and the lower values of the glitch healing parameter observed for the Crab-like and Vela-like pulsars respectively, on the basis of the starquake model of glitch generation.

Key words: dense matter – equation of state – stars: neutron – pulsars: individual: Crab – pulsars: individual: Vela.

1 INTRODUCTION

Characteristics of super-dense objects such as neutron stars (NSs) are based on calculations of the equation of state (EOS) for matter at very high densities. However, nuclear interactions beyond a density of \( \sim 10^{14} \text{ g cm}^{-3} \) are empirically not well known (Dolan 1992), and all known EOSs far beyond this density range are only extrapolations of empirical results. Various EOSs based on theoretical manipulations are available in the literature (Arnett & Bowers 1977). Because the status of the EOS of nuclear matter cannot be established empirically beyond a certain density range, one can apply physical constraints to obtain an upper bound on the NS mass. Brecher & Caporaso (1976) assumed that the speed of sound in the nuclear matter equals the speed of light and obtained a value of 4.8 \( M_\odot \) as an upper limit on the NS mass. However, the matter described by this stiffest EOS, \( dP/\partial E = 1 \) (geometrized units) or \( P = E - E_s \) (where \( P \) is the pressure, \( E \) is the energy-density, and \( E_s = 2 \times 10^{14} \text{ g cm}^{-3} \) represents the surface density), has a super-dense self-bound state at the surface of the configuration where the pressure is vanishingly small, which represents an ‘abnormal state of matter’ (Lee 1975; Haensel & Zdunik 1989). This ‘abnormality’ can be specified as (i) the pressure vanishes at average nuclear densities, and (ii) the speed of sound is equal to that of light even when the pressure is vanishingly small. This ‘abnormality’ can be removed if we ensure continuity of pressure, density and both of the metric parameters at the boundary of the structure (Negi & Durgapal 2000).

Prior to Brecher & Caporaso’s work, Rhoades & Ruffini (1974), without going into the details of the nuclear interactions, assumed that, beyond a certain density of \( 4.6 \times 10^{14} \text{ g cm}^{-3} \) (the range of densities where no extrapolated EOS is known), the EOS in the core is given by the criterion that the speed of sound attains the speed of light; that is, \( dP/\partial E = 1 \). They matched the core to an envelope with the BPS (Baym, Pethick & Sutherland 1971) EOS and obtained an upper limit for the neutron star mass of 3.2 \( M_\odot \). Hartle (1978) emphasized that the maximum masses of NSs obtained in this manner involve a scale factor, \( k = [E_m/10^{14} \text{ g cm}^{-3}]^{-1/2} \), such that the matching density, \( E_m \), plays a sensitive role in obtaining an upper bound on NS masses. Usually, \( k \) is taken to be equal to or greater than one in conventional models. For densities less than \( E_m \), the matter composing the object is assumed to be known and unique. That is, the EOSs of the envelopes of these stars are chosen so that the ‘abnormalities’ in the sense mentioned above are removed. Friedman & Ipser (1987) calculated the masses of NSs...
for various values of the matching density using the BPS and NV (Negele & Vautherin 1973) EOSs, and concluded that the EOS chosen for the envelope does not make any significant difference to the results, because in each case the mass in the envelope turns out to be insignificant compared with that of the core containing the stiffest material.

In order to implement this ‘insignificant’ mass of the envelope component, Crawford & Demiański (2003) recently constructed NS models for seven representative EOSs of dense nuclear matter by covering a range of NS masses. They computed the ‘fractional moment of inertia’ of the core component, which is defined as the glitch healing parameter, \( Q \), in the starquake mechanism of glitch generation as

\[
Q = \frac{I_{\text{core}}}{I_{\text{total}}},
\]

where \( I_{\text{core}} \) represents the moment of inertia of the entire configuration. Their study shows that the much larger values of \( Q \) (>0.7) for the Crab pulsar are fulfilled by all the six EOSs (out of seven corresponding to the case of the Vela pulsar, their models predict to central energy-density\[\equiv\rho_{\text{central}}\]), the compactness ratio \( u \), in the starquake mechanism of glitch generation as

\[
0.2 < u < 0.5 \text{M}_\odot,
\]

corresponding to the case of the Vela pulsar, their models predict a NS mass \(<0.5 \text{M}_\odot\), which is too low compared with the ‘realistic’ NS mass range 1.4 \pm 0.2 \text{M}_\odot. By contrast, for the much lower values of \( Q \leq 0.2 \) corresponding to the case of the Vela pulsar, their models predict a NS mass \(<0.5 \text{M}_\odot\), which is too low compared with the ‘realistic’ NS mass range. Thus, their study concludes that ‘starquake’ is a feasible mechanism for glitch generation in Crab-like pulsars (corresponding to larger values of \( Q \)), and that ‘vortex unpinning’, another mechanism of glitch generation, is suitable for Vela-like pulsars (corresponding to lower values of \( Q \)). However, it seems quite surprising that, if the internal structure of NSs is supposed to be described by the same two-component conventional model, different kinds of glitch mechanisms are required to explain a glitch!

The purpose of this study is to provide an insight into a fundamental ‘theorem’ concerning the hydrostatic equilibrium of NS models (which was lacking in earlier studies) and to show that, as soon as this theorem is implemented in an appropriate conventional NS sequence, the various shortcomings of the conventional NS models (as mentioned above) can be resolved.

### 2 REMOVAL OF ABNORMALITIES FROM THE STIFFEST EOS, AND BOUNDARY CONDITIONS FOR NEUTRON STAR MODELS COMPATIBLE WITH HYDROSTATIC EQUILIBRIUM

In order to construct such an appropriate sequence of NS models, consistent with causality and dynamical stability, we offer here an entirely different approach to the whole problem, which not only will remove the ‘abnormalities’ of the stiffest EOS (as discussed earlier in detail in Section 1) but also can ensure the necessary and sufficient condition of hydrostatic equilibrium for the resulting configuration. As we will show later in Section 3, the \( M-R \) relations corresponding to the configurations (i) governed by the pure stiffest EOS, and (ii) resulting from the removal of ‘abnormalities’ from the stiffest EOS (core–envelope models) do not provide the necessary and sufficient condition of dynamical stability unless the ‘compatibility criterion’ (Negi & Durgapal 2001) is ‘appropriately’ satisfied. This ‘compatibility criterion’ states that: ‘for each and every assigned value of \( \sigma \equiv (P_{\text{central}}/E_{\text{central}}) \equiv \text{ratio of central pressure to central energy-density} \), the compactness ratio \( u \equiv (M/R) \) of the entire configuration should not exceed the compactness ratio, \( u_{\text{abs}} \), of the corresponding homogeneous density sphere (that is, \( u \leq u_{\text{abs}} \)).

The present approach is based on a ‘theorem’ applicable to a wide range of conventional NS sequences (including the core–envelope models), provided that every member of such a sequence satisfies the condition \( \left( \frac{dp}{dE} \right)_h = 1 \) (here and elsewhere in the paper, the subscript ‘0’ represents the value of the respective quantity at the centre of the configuration). This theorem asserts that, in order to ensure the necessary and sufficient condition of dynamical stability for the mass, the maximum stable value of \( u \) (corresponding to the case of the first maximum among masses in the mass–radius \((M–R)\) relation) and the corresponding central value of the ‘local’ adiabatic index \((\Gamma_{1})_{h} \equiv [\langle P_{0} + E_{0} \rangle / (P_{0})] \left( \frac{dp}{dE} \right)_{h} \) of such sequences must satisfy the inequalities \( 0.2 < u_{\text{abs}} \sim 0.3406 \) and \((\Gamma_{1})_{h} \sim 2.5946 \), respectively (Negi 2007). In addition to the result of a previous study regarding the ‘compatibility criterion’, we show in the present study that ‘the \( M-R \) relation (or the mass–central density relation) corresponding to an equilibrium sequence provides the necessary and sufficient condition of dynamical stability only when the ‘compatibility criterion’ for the equilibrium sequence is also satisfied. However, the fulfilment of the ‘compatibility criterion’ does not depend on the fulfilment of the necessary and sufficient condition of dynamical stability provided by the \( M-R \) relation’. The proof of this statement would explicitly show that the fulfilment of the ‘compatibility criterion’ independently provides, in general, the necessary and sufficient condition of hydrostatic equilibrium for any sequence composed of regular configurations, as the first sentence of this statement has already been proved in a previous study for equilibrium sequences of the type mentioned above (Negi 2007).

In order to provide a proof of the last statement, the present paper deals with the construction of a four-step method as follows.

**Step (1).** We construct the NS models corresponding to the pure stiffest EOS, \( dp/dE = 1 \), and show that the \( M-R \) relation corresponding to this sequence does not provide the necessary and sufficient condition of dynamical stability, as the ‘compatibility criterion’ cannot be fulfilled by the equilibrium configurations corresponding to this ‘abnormal’ EOS.

**Step (2).** We construct the NS sequences composed of core–envelope models by considering the stiffest EOS, \( dp/dE = 1 \), in the core and the EOS of the classical polytrope, \( d \ln P/d \ln \rho = \Gamma_{1} \) (where \( \rho \) is the density of the rest-mass and \( \Gamma_{1} \) is the constant adiabatic index; see, for example, Tooper 1965), in the envelope (so that each member of these sequences satisfies the extreme case of the causality condition, \( v = c = 1 \), at the centre) for an ‘arbitrarily’ assigned value of the pressure to density ratio, \( (P_{h}/E_{h}) \) (say), at the core–envelope boundary. Although this procedure removes the ‘abnormalities’ from the stiffest EOS, we show in Section 3 that the \( M-R \) relations corresponding to NS models with an envelope with \( \Gamma_{1} = 5/3 \) or 2 do not provide the necessary and sufficient condition of dynamical stability, since the ‘compatibility criterion’ remains unsatisfied by various stable (as well as unstable) models corresponding to the said sequences.

**Step (3).** In this step we reconstruct the NS sequence composed of core–envelope models as described in step (2) for the same boundary value of \( (P_{h}/E_{h}) \). However, instead of the constant \( \Gamma_{1} = 5/3 \) or 2, the constant \( \Gamma_{1} = 4/3 \) is used for the envelope. We show that, although the ‘compatibility criterion’ is satisfied by all stable (as well as unstable) models comprising the sequence, the \( M-R \) relation still does not provide the necessary and sufficient condition of dynamical stability, since the ‘compatibility criterion’ is not ‘appropriately’ satisfied by the equilibrium configurations.
Step (4). We reconstruct the NS sequences described in steps (2) and (3), respectively, for an ‘appropriate’ boundary value of \((P_b/E_b) = (P_b/E_b)_2\) (say). Since for this particular value of \((P_b/E_b)\) the ‘compatibility criterion’ turns out to be ‘appropriately’ satisfied by all equilibrium sequences corresponding to an envelope with \(\Gamma_1 = 4/3, 5/3\) or \(2\), it follows that the \(M–R\) relation does provide the necessary and sufficient condition of dynamical stability for the said sequences. [The value \((P_b/E_b)\) corresponds to the minimum value of \((P_b/E_b)\) for which the \(M–R\) relation fulfils the necessary and sufficient condition of dynamical stability for the equilibrium sequence corresponding to NS models with an envelope \(\Gamma_1 = 4/3, 5/3\) or \(2\), respectively, as well as fulfilling the necessary and sufficient condition of dynamical stability provided by the \(M–R\) relation. This condition is termed the ‘appropriate’ fulfilment of the ‘compatibility criterion’.

The methodology regarding the construction of the various models mentioned in steps (1)–(4) above and the important outcomes emerging therefrom are discussed in the following section.

The assignment of values of \(\Gamma_1\) in the range \((4/3)\) to \(2\) in the envelope of NS models discussed under steps (2) to (4) above follows from the fact that \((4/3)\) represents the EOS of ultrarelativistic degenerate electrons and non-relativistic nuclei (Chandrasekhar 1935) or of a relativistic degenerate neutron gas (Zeldovich Ú Novikov 1978). \((5/3)\) corresponds to the well-known EOS of a non-relativistic degenerate neutron gas (Oppenheimer µ Volkoff 1939), and \(\Gamma_1 = 2\) represents the case of extreme relativistic baryons interacting through the vector meson field (Zeldovich 1962). [A value of \(\Gamma_1 \geq 2\) is also possible for some EOSs; see, for example, Malone, Johnson µ Bethe (1975) or Clark, Heintzmann µ Grewing (1971). However, the results obtained in this paper are not affected by choosing \(\Gamma_1 > 2\).] The various values of \(\Gamma_1\), chosen in this range \((4/3 \leq \Gamma_1 \leq 2)\) can cover almost all of the nuclear EOSs discussed in the literature (and, therefore, can cover almost the full range of nuclear densities that might be applicable for the envelope region as well), and, in our opinion, it is more appropriate to choose an average (constant) value of \(\Gamma_1\) for a conventional NS model than to go into details of the density range below \(E_b\) specified by different EOSs [for example BPS, NV, or FPS (Lorenz, Ravenhall µ Pethick 1993)] that are frequently used by authors in conventional models of NSs in spite of their uncertainty (Friedman µ Ipser 1987; Dolan 1992), as mentioned above. The EOS of the classical polytropic for the said \(\Gamma_1\) values considered in the present study not only will simplify the procedure but will also provide insight into the suitability of the EOS for the envelope region, as this study yields the important finding that stable sequences of NS models terminate at the same value of the maximum mass, independent of the EOS of the envelope.

3 METHODOLOGY AND DISCUSSION

The metric for spherically symmetric and static configurations can be written in the following form (remembering that we are using geometrized units, i.e. \(G = c = 1\), where \(G\) and \(c\) represent, respectively, the universal constant of gravitation and the speed of light in a vacuum):

\[
ds^2 = e^{\nu}dr^2 - e^{\lambda}d\rho^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2,
\]

where \(\nu\) and \(\lambda\) are functions of \(r\) alone. The Oppenheimer–Volkoff (O–V) equations (Oppenheimer µ Volkoff 1939), resulting from Einstein’s field equations, for systems with isotropic pressure \(P\) and energy-density \(E\), can be written as

\[
P' = -(P + E)[4\pi P r^3 + m]/r(r - 2m),
\]

\[
\nu'/2 = -P'(P + E),
\]

\[
m'(r) = 4\pi Er^2, \]

where \(m(r) = \int_0^r 4\pi Er^2dr\) is the mass contained within the radius \(r\), and the prime denotes the radial derivative.

In order to solve equations (3)–(5) for the various models considered in the present study, we consider the stiffest EOS

\[
P = E - E_s
\]

for the entire configuration (or for the core region), and the EOS of the classical polytrope

\[
dlnP/dlnP = \Gamma \]

for the envelope region. For core–envelope models, the fractional moment of inertia (given by equation 1) can be calculated using an approximate but very precise empirical formula based on 30 theoretical EOSs of dense nuclear matter. For NS models, the formula yields the following form (Bejger µ Haensel 2002):

\[
I \simeq \frac{2}{9}(1 + 5x)M R^2, \quad x > 0.1,
\]

where \(x\) is the compactness parameter measured in units of \([M/\odot]/[\mathrm{km}]\), i.e.

\[
x = \frac{M/R}{M/\odot/\mathrm{km}} = \frac{u}{1.477}.
\]

In accordance with the four-step method detailed above, we construct models in the following manner.

Models mentioned in step (1). The coupled equations (3)–(5) are solved for the EOS given by equation (6) until the pressure vanishes at the surface of the configuration for a fiduciary choice of \(E_s = 2.7 \times 10^{14} \, \mathrm{g \, cm}^{-3}\), the nuclear saturation density. At the surface, \(r = R\), we obtain \(P = 0, E = E_s, m_r(r = R) = M, \) and \(e_r = e_s = (1 - 2M/R)/(1 - 2\eta)\). The results of the calculations are shown in Table 1, and the \(M–R\) diagram is presented in Fig. 1. It follows from Table 1 that, along the stable branch of the sequence, the maximum value of mass corresponds to the maximum ‘stable’ value of \(\eta_{\text{max}} \simeq 0.3539\) and the corresponding ‘local’ value of \((\Gamma_r)_{\eta} \simeq 2.4990\). Although this value of \((\Gamma_r)_{\eta}\) turns out to be consistent with that of the absolute upper bound on \((\Gamma_r)_{\eta}\), namely \((\Gamma_r)_{\eta, \text{max}, \text{abs}} \simeq 2.5946\), the maximum ‘stable’ value of \(\eta_{\text{max}} \simeq 0.3539\) is found to be inconsistent with that of the absolute upper bound on \(\eta_{\text{max}, \text{abs}} \simeq 0.3406\). Thus, the configuration turns out to be inconsistent with corollary 1 of Theorem 2 (Negi 2007). It follows, therefore, that the \(M–R\) relation corresponding to the stiffest EOS does not provide a necessary and sufficient condition for dynamical stability for the equilibrium configurations. As the total mass ‘\(M\)’ that appears here does not fulfil the definition of the ‘actual mass’ that should be present in the exterior Schwarzschild solution (Negi 2004, 2006), the equilibrium sequence corresponding to the pure stiffest EOS does not, therefore, even fulfil the necessary condition of hydrostatic equilibrium (Negi 2007); as a result, the ‘compatibility criterion’ cannot be satisfied by such configurations. This is also evident from a comparison of columns 2 and 6 of Table 1, which show that, for each assigned value of \((P_b/E_b)\), \(u_{\text{eff}} > u_b\).
Models mentioned in step (2). The coupled equations (3)–(5) are solved by considering equation (6) in the core (0 ≤ r ≤ b) and equation (7) in the envelope (b ≤ r ≤ R) for the constant values of Γ₁ = 5/3 and 2, respectively, until the boundary conditions P = E = 0, m(r = R) = M, and e' = e'' = (1 − 2M/R) = (1 − 2u) are reached at the surface, r = R, of the configuration. Equation (8) is also used, together with equations (3)–(5), to calculate the fractional moment of inertia given by equation (1) and the moment of inertia of the entire core–envelope model considered in the present section. The calculations are performed for a fiduciary choice of the boundary density, E₀ = 2.7 × 10¹⁴ g cm⁻³, and for an ‘arbitrarily’ assigned value of the ratio of pressure to energy-density at the core–envelope boundary, namely (P₀/E₀) = (P₀/E₀)₁ = 1.0645 × 10⁻². The results of the calculations are summarized in Tables 2 and 3 (indicated with a superscript ‘b’ for the Γ₁ = 5/3, and ‘c’ for the Γ₁ = 2 envelope models), and the M–R–diagram is shown in Fig. 2 (with the label ‘b’ for the Γ₁ = 5/3 and ‘c’ for the Γ₁ = 2 envelope model). Although such core–envelope models appropriately fulfill the definition of the ‘actual mass’ appearing in the exterior Schwarzchild solution (Negi 2007), the choice of (P₀/E₀) = (P₀/E₀)₁ yields sequences (corresponding to NS models with an envelope Γ₁ = 5/3 and 2 respectively) that fulfill only the necessary (but not sufficient) condition of hydrostatic equilibrium, as the ‘compatibility criterion’ remains unsatisfied by such sequences, as shown in Table 3. From Table 2, we find that, at the maximum mass values along the stable branch of the sequences, the sequence composed of NS models with an envelope Γ₁ = 5/3 yields uₘₐₓ ≤ 0.3493 and the corresponding value of (Γ₁b)ₗ₀ ≤ 2.4984, whereas the sequence composed of NS models with an envelope Γ₁ = 2 yields uₘₐₓ ≤ 0.3509 and the corresponding value of (Γ₁c)ₗ₀ ≤ 2.4984. Both pairs of these values are, however, found to be inconsistent with that of the pair of absolute upper bounds uₘₐₓ,ₙₐₐₙ(b) (≤ 0.3406) and (Γ₁b)ₗₒₙₐₐₙ(b) (≤ 2.5946). It follows that the M–R relation corresponding to the sequences (composed of NS models with an envelope with Γ₁ = 5/3 or 2) does not provide the necessary and sufficient condition of dynamical stability, as shown in Fig. 2.

Tables 2 and 3 and Fig. 3 show that the range of the fractional moment of inertia, 0.652 ≤ Q ≤ 0.948, corresponding to the ‘stable’ models (with an envelope Γ₁ = 5/3 and 2 respectively) possesses NS masses in the range 1.35 M⊙ ≤ M ≤ 4.1 M⊙. This feature is consistent with those of the conventional models discussed in the literature and can explain only the higher values of the glitch healing parameter observed for the Crab-like pulsars. However, for the minimum weighted mean value of Q ≈ 0.7 corresponding to the Crab pulsar, we obtain from Fig. 3 the minimum masses Mₘₐₓ ≥ 1.6 M⊙ and Mₚ ≥ 1.44 M⊙ for the Crab pulsar.

Models mentioned in step (3). In order to construct core–envelope models corresponding to constant Γ₁ = 4/3 for the envelope, we use equation (7) for the envelope (b ≤ r ≤ R) and solve equations (6) for the core (0 ≤ r ≤ b) and equation (8) for the same boundary conditions as mentioned in the last subsection (step 2) for the surface and the core–envelope boundary, namely E₀ = 2.7 × 10¹⁴ g cm⁻³, and (P₀/E₀) = (P₀/E₀)₁ = 1.0645 × 10⁻², respectively. The various parameters obtained for this model are indicated with the superscript ‘a’ in Tables 2 and 3, and the M–R diagram (marked with label ‘a’) is presented in Fig. 2. Table 3 shows that, although the ‘compatibility criterion’ is satisfied by all members of the sequence corresponding to NS models with an envelope Γ₁ = 4/3 for the choice of the boundary condition (P₀/E₀) = (P₀/E₀)₁, this choice does not appear to be ‘appropriate’ because, at the maximum value of mass along the stable branch of the sequence in the M–R relation, the model yields the

| (P₀/E₀) | u₀ | (Γ₁)₀ | (M/Mₐ) | R(km) | uₘₐₓ | Q | Γ₁ | m(r = R) | M | E₀ | P₀/E₀ | P₀/E₀,ₙₐₐₙ(b) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0.10142 | 0.14343 | 10.8600 | 1.27922 | 12.94001 | 0.14601 | 3 |  3 | 0.69032 | 0.36037 | 2.24128 | 4.07480 | 17.00784 | 0.35608 |
| 0.13324 | 0.17226 | 8.50526 | 1.67675 | 14.09223 | 0.17574 | 3 |  3 | 0.24024 | 0.24025 | 5.16250 | 2.71665 | 16.27046 | 0.24661 |
| 0.16786 | 0.19835 | 6.95734 | 2.06245 | 15.01761 | 0.20284 | 3 |  3 | 0.21115 | 0.22511 | 5.73597 | 2.47752 | 15.85387 | 0.23081 |
| 0.19416 | 0.21528 | 6.15039 | 2.32350 | 15.56089 | 0.22054 | 3 |  3 | 0.20111 | 0.21528 | 7.53957 | 2.77520 | 16.15387 | 0.23801 |
| 0.21115 | 0.22025 | 5.16250 | 2.71665 | 16.27046 | 0.24661 | 3 |  3 | 0.20111 | 0.21528 | 7.53957 | 2.77520 | 16.15387 | 0.23801 |
| 0.20111 | 0.21528 | 6.05039 | 2.32350 | 15.56089 | 0.22054 | 3 |  3 | 0.20111 | 0.21528 | 7.53957 | 2.77520 | 16.15387 | 0.23801 |
| 0.20111 | 0.21528 | 7.53957 | 2.77520 | 16.15387 | 0.23801 | 3 |  3 | 0.20111 | 0.21528 | 7.53957 | 2.77520 | 16.15387 | 0.23801 |

Figure 1. The mass–radius diagram of the models corresponding to the stiffest EOS, P = E − E₁ (as discussed in the text: step 1, Sections 2 and 3) for an assigned value of the density E₁ = 2.7 × 10¹⁴ g cm⁻³ at the surface. The models do not fulfill the ‘compatibility criterion’, as shown in Table 1. Furthermore, the mass–radius diagram does not provide the necessary and sufficient condition of dynamical stability because the inequalities (Γ₁)₀ ≤ 2.5946 and uₘₐₓ ≤ 0.3406 are not fulfilled simultaneously at the maximum value of mass, as shown in Table 1.
maximum value of $u (u_{\text{max}}) \simeq 0.3444$ and the corresponding value of \((\Gamma_1)_{b} \simeq 2.4984\), as shown in Table 2. These values, however, are inconsistent with the pair of absolute upper bounds $u_{\text{max}, \text{abs}} \simeq 0.3406$ and \((\Gamma_1)_{b, \text{max}, \text{abs}} \simeq 2.5946\), respectively. It follows from this result that the necessary and sufficient condition of dynamical stability (provided by the $M$–$R$ relation) may remain unsatisfied even when all members of the sequence do satisfy the ‘compatibility criterion’.

Tables 2 and 3 and Fig. 3 also indicate that the range of the fractional moment of inertia, $i_{\text{max}} \simeq 0.558 \leq i \leq 0.922$, for the ‘stable’ sequence corresponding to the $\Gamma_1 = 4/3$ envelope model provides the mass limit of NS models in the range $1.36 \leq M_{\odot} \leq 4.1 \leq 1.0645$ $M_{\odot}$. These higher values of the glitch heating parameter are consistent only with those of the Crab-like pulsars, as discussed in a previous subsection (step 2). However, for the minimum value of $Q \simeq 0.7$, Fig. 3 yields the minimum mass $M_{\odot} \simeq 2.03$ $M_{\odot}$ for the Crab pulsar.

Models mentioned in step (4). We reconstruct the NS sequences (described of core−envelope models) described in the last two subsections (steps 2 and 3 respectively), but for an ‘appropriate’ value of \((P_0/E_0) = (P_0/E_0) = 4.694 \times 10^{-2}\). The results of the calculations are summarized in Tables 4 and 5, and the $M$–$R$ diagram is shown in Fig. 4. The superscripts ‘a’, ‘b’ and ‘c’, which appear in connection with various parameters in these tables, represent models with an envelope with $\Gamma_1 = 4/3, 5/3$ and 2, respectively. The $M$–$R$ diagram is labelled in a similar fashion. The value \((P_0/E_0)\), in fact, represents the minimum value of \((P_0/E_0)\) for which all the sequences, corresponding to NS models with an envelope with $\Gamma_1 = 4/3, 5/3$ or 2, fulfil the necessary and sufficient condition of dynamical stability provided by the $M$–$R$ relation. Because the value \((P_0/E_0)\), it follows that the ‘compatibility criterion’ would automatically be satisfied by all the sequences corresponding to NS models with an envelope with $\Gamma_1 = 4/3, 5/3$ or 2. This is evident from Tables 4 and 5, respectively. A comparison of column 2 with columns 3, 4 and 5 of Table 5 shows that the ‘compatibility criterion’ is ‘appropriately’ satisfied by all the sequences. Table 4 shows that, at the maximum values of mass along the stable branch of the sequences, the models corresponding to an envelope with $\Gamma_1 = 4/3, 5/3$ or 2 yield the maximum values of $u (u_{\text{max}}) \simeq 0.3404, 0.3324$ or 0.3404, respectively, for the same value of \((\Gamma_1)_{b} \simeq 2.4955\). Evidently, these values are consistent with those of the absolute upper bounds $u_{\text{max}, \text{abs}} \simeq 0.3406$ and \((\Gamma_1)_{b, \text{max}, \text{abs}} \simeq 2.5946\), respectively.

Tables 4 and 5 and Fig. 5 show that the ranges of the fractional moment of inertia, $0.069 \leq i \leq 0.779$, obtained for the stable sequences corresponding to the $\Gamma_1 = 5/3$ and 2 envelope models have
masses in the range $1.48 \, \text{M}_\odot \lesssim M \lesssim 4.2 \, \text{M}_\odot$. Obviously, these ranges are capable of fulfilling the higher as well as the lower values of the glitch healing parameter, corresponding to the cases of the Crab-like and the Vela-like pulsars, respectively. For the maximum weighted mean value of $Q \simeq 0.2$ corresponding to the Vela pulsar, Fig. 5 yields the maximum masses $M^p \simeq 2.06 \, \text{M}_\odot$ and $M^v \simeq 1.85 \, \text{M}_\odot$ for the Vela pulsar, whereas for the minimum weighted mean value of $Q \simeq 0.7$, the models yield the minimum masses $M^p \simeq 4.1 \, \text{M}_\odot$ and $M^v \simeq 3.98 \, \text{M}_\odot$ for the Crab pulsar. It also follows from Tables 4 and 5 and Fig. 5 that for the stable range of the fractional moment of inertia $0.016 \leq Q \leq 0.655$, corresponding to the $\Gamma_1 = 4/3$ envelope models, the NSs can have masses in the range $1.79 \, \text{M}_\odot \lesssim M \lesssim 4.2 \, \text{M}_\odot$. For the maximum weighted mean value of $Q \simeq 0.2$, Fig. 5 yields the maximum mass $M^p \simeq 2.67 \, \text{M}_\odot$ for the Vela pulsar; however, the minimum weighted mean value $Q \simeq 0.7$ corresponds to the minimum mass $M^v \simeq 3.98 \, \text{M}_\odot$ for the Crab pulsar, which lies on the unstable branch of Fig. 5.

The important conclusions emerging from the four-step method and discussion presented above can be summarized as follows.

(i) Steps (1), (2) and (4) show that the $M$–$R$ relation provides the necessary and sufficient condition of dynamical stability only when the ‘compatibility criterion’ for the equilibrium configurations is satisfied. If the ‘compatibility criterion’ is not satisfied, the $M$–$R$ relation cannot provide the necessary and sufficient condition of dynamical stability.

(ii) The inclusion of step (3), however, modifies the above conclusion in the following way. The $M$–$R$ relation provides the necessary and sufficient condition of dynamical stability only when the ‘compatibility criterion’ is ‘appropriately’ satisfied. Thus, the necessary and sufficient condition provided by the $M$–$R$ relation is totally dependent on the ‘appropriate’ fulfilment of the ‘compatibility criterion’. On the other hand, the fulfilment of the ‘compatibility criterion’ is quite independent of the fulfilment of the necessary and sufficient condition of dynamical stability provided by the $M$–$R$ relation, because the ‘compatibility criterion’ can be satisfied even without fulfilment of the necessary and sufficient condition of dynamical stability provided by the $M$–$R$ relation. Because the fulfilment of the ‘compatibility criterion’ alone is a measure of the fulfilment of the necessary and sufficient condition of hydrostatic equilibrium for any static and spherical configuration (Negi 2007), it follows from the above discussion that the fulfilment of the ‘compatibility criterion’ alone independently provides, in general, the necessary and sufficient condition of hydrostatic equilibrium.
of dynamical stability. This is because at the maximum value of mass along the $M-R$ relation the pair of the maximum ‘stable’ value of compactness, $u_{\text{max}} \simeq 0.3539$, and the corresponding central value of the ‘local’ adiabatic index, $(\Gamma_1)_{\text{b}} \simeq 2.4990$, is inconsistent with that of the pair of absolute values, $u_{\text{max}, \text{abs}} \simeq 0.3406$ and $(\Gamma_1)_{\text{b, max, abs}} \simeq 2.5946$, and the pair of absolute values is in agreement with the structure of general relativity, causality, and dynamical stability. The reason behind this inconsistency lies in the fact that the ‘compatibility criterion’ (Negi & Durgapal 2001) cannot be fulfilled by any of a sequence, composed of regular configurations corresponding to a single EOS with finite (non-zero) values of surface and central density (Negi 2004, 2006).

We have further constructed sequences composed of core–envelope models such that, like the central condition of the stiffest EOS, each member of these sequences satisfies the extreme case of the causality condition, $v = c = 1$, at the centre. This was achieved by considering the stiffest EOS in the core and a polytropic equation with the constant adiabatic index $\Gamma_1 = [\ln P/\ln \rho]$ in the envelope and investigating the resulting configurations on the basis of the ‘compatibility criterion’ and the $M-R$ relation for the values $\Gamma_1 = 4/3, 5/3$ and 2. Together with the finding corresponding to the case of the stiffest EOS mentioned above, the investigation of the said sequences of the core–envelope models explicitly shows that the $M-R$ relation corresponding to the said sequences can provide the necessary and sufficient condition of dynamical stability only when the ‘compatibility criterion’ for the sequences is ‘appropriately’ satisfied. However, the fulfilment of the ‘compatibility criterion’ can remain satisfied even when the $M-R$ relation does not provide the necessary and sufficient condition of dynamical stability for the equilibrium configurations.

In continuation of the results of a previous study (Negi 2007), these results explicitly show that the ‘compatibility criterion’ independently provides, in general, the necessary and sufficient condition of hydrostatic equilibrium for any regular sequence.

The causal sequences of NS models constructed in the present study (which fulfil the necessary and sufficient condition of hydrostatic equilibrium and dynamical stability simultaneously) terminate at the same value of maximum mass, $M_{\text{max}} \approx 4.2 \, M_\odot$ (for $E_b = 2.7 \times 10^{14} \, \text{g cm}^{-3}$), independent of the EOS of the envelope; however, the maximum compactness, $u_{\text{max}} \simeq 0.3404$, yields for the sequence corresponding to the $\Gamma_1 = 2$ envelope models. Although for the same value of transition density $E_b = 2.7 \times 10^{14} \, \text{g cm}^{-3}$ at the core–envelope boundary, this upper bound on NS mass is found to be fully consistent with those of models formulated with advanced nuclear theory (Kalogera & Baym 1996), the $\Gamma_1 = 2$ envelope model indicates the appropriateness of the (average) value of $\Gamma_1$ for the entire envelope.

Besides its fundamental result, this study underlines the importance of the applicability of the ‘compatibility criterion’ to conventional models of NSs. We find that when the ‘compatibility criterion’ is not satisfied (for models corresponding to an envelope with $\Gamma_1 = 5/3$ and 2, if the ratio of pressure to energy-density at the core–envelope boundary, $P_r/E_b$, is ‘arbitrarily’ assigned to be about $1.0645 \times 10^{-2}$) or is not ‘appropriately’ satisfied (for models corresponding to an envelope with $\Gamma_1 = 4/3$, if $P_r/E_b \approx 1.0645 \times 10^{-2}$) by the sequences, the corresponding range of the glitch healing parameter turns out to be $0.558 \leq Q \leq 0.948$. This feature is consistent with other conventional NS models discussed in the literature and can explain only the higher values of $Q$ on the basis of the starquake model of glitch generation for Crab-like pulsars. For the minimum weighted mean value of $Q \simeq 0.7$, our models (corresponding to an envelope with $\Gamma_1 = 2, 5/3$ or 4/3) yield the minimum masses

**Figure 2.** The mass–radius diagram of the models as discussed in the text (steps 2 and 3, Sections 2 and 3) for an assigned value of matching density $E = E_b = 2.7 \times 10^{14} \, \text{g cm}^{-3}$ at the core–envelope boundary. The labels a, b and c represent the models for an envelope with $\Gamma_1 = 4/3, 5/3$ and 2, respectively. The value of the ratio of pressure to energy-density, $P_b/E_b = (P_b/E_b)_0 = 1.0645 \times 10^{-2}$, at the core–envelope boundary is assigned in such a manner that the ‘compatibility criterion’ is satisfied by all models corresponding to an envelope with $\Gamma_1 = 4/3$, whereas it remains unsatisfied by various models corresponding to an envelope with $\Gamma_1 = 5/3$ or 2, as shown in Table 3. Since the ‘compatibility criterion’ has not been satisfied ‘appropriately’ by the models corresponding to an envelope with $\Gamma_1 = 4/3$ (and has not been satisfied by various models corresponding to an envelope with $\Gamma_1 = 5/3$ or 2), the mass–radius relation does not provide the necessary and sufficient condition of dynamical stability for any sequence, as shown in Table 2.

**Figure 3.** Fractional moment of inertia $Q(\equiv l_{\text{int}}/l_{\text{total}})$ versus total mass $M$ for the models presented in Tables 2 and 3 and shown in Fig. 2. The labels a, b and c denote the models for an envelope with $\Gamma_1 = 4/3, 5/3$ and 2, respectively.

4 RESULTS AND CONCLUSIONS

We have investigated the fact that the $M-R$ relation corresponding to the stiffest EOS does not provide the necessary and sufficient condition of dynamical stability. This is because at the maximum value of mass along the $M-R$ relation the pair of the maximum ‘stable’ value of compactness, $u_{\text{max}} \simeq 0.3539$, and the corresponding central value of the ‘local’ adiabatic index, $(\Gamma_1)_{\text{b}} \simeq 2.4990$, is inconsistent with that of the pair of absolute values, $u_{\text{max}, \text{abs}} \simeq 0.3406$ and $(\Gamma_1)_{\text{b, max, abs}} \simeq 2.5946$, and the pair of absolute values is in agreement with the structure of general relativity, causality, and dynamical stability. The reason behind this inconsistency lies in the fact that the ‘compatibility criterion’ (Negi & Durgapal 2001) cannot be fulfilled by any of a sequence, composed of regular configurations corresponding to a single EOS with finite (non-zero) values of surface and central density (Negi 2004, 2006).

We have further constructed sequences composed of core–envelope models such that, like the central condition of the stiffest EOS, each member of these sequences satisfies the extreme case of the causality condition, $v = c = 1$, at the centre. This was achieved by considering the stiffest EOS in the core and a polytropic equation with the constant adiabatic index $\Gamma_1 = [\ln P/\ln \rho]$ in the envelope and investigating the resulting configurations on the basis of the ‘compatibility criterion’ and the $M-R$ relation for the values $\Gamma_1 = 4/3, 5/3$ and 2. Together with the finding corresponding to the case of the stiffest EOS mentioned above, the investigation of the said sequences of the core–envelope models explicitly shows that the $M-R$ relation corresponding to the said sequences can provide the necessary and sufficient condition of dynamical stability only when the ‘compatibility criterion’ for the sequences is ‘appropriately’ satisfied. However, the fulfilment of the ‘compatibility criterion’ can remain satisfied even when the $M-R$ relation does not provide the necessary and sufficient condition of dynamical stability for the equilibrium configurations.

In continuation of the results of a previous study (Negi 2007), these results explicitly show that the ‘compatibility criterion’ independently provides, in general, the necessary and sufficient condition of hydrostatic equilibrium for any regular sequence.

The causal sequences of NS models constructed in the present study (which fulfil the necessary and sufficient condition of hydrostatic equilibrium and dynamical stability simultaneously) terminate at the same value of maximum mass, $M_{\text{max}} \approx 4.2 \, M_\odot$ (for $E_b = 2.7 \times 10^{14} \, \text{g cm}^{-3}$), independent of the EOS of the envelope; however, the maximum compactness, $u_{\text{max}} \simeq 0.3404$, yields for the sequence corresponding to the $\Gamma_1 = 2$ envelope models. Although for the same value of transition density $E_b = 2.7 \times 10^{14} \, \text{g cm}^{-3}$ at the core–envelope boundary, this upper bound on NS mass is found to be fully consistent with those of models formulated with advanced nuclear theory (Kalogera & Baym 1996), the $\Gamma_1 = 2$ envelope model indicates the appropriateness of the (average) value of $\Gamma_1$ for the entire envelope.

Besides its fundamental result, this study underlines the importance of the applicability of the ‘compatibility criterion’ to conventional models of NSs. We find that when the ‘compatibility criterion’ is not satisfied (for models corresponding to an envelope with $\Gamma_1 = 5/3$ and 2, if the ratio of pressure to energy-density at the core–envelope boundary, $P_r/E_b$, is ‘arbitrarily’ assigned to be about $1.0645 \times 10^{-2}$) or is not ‘appropriately’ satisfied (for models corresponding to an envelope with $\Gamma_1 = 4/3$, if $P_r/E_b \approx 1.0645 \times 10^{-2}$) by the sequences, the corresponding range of the glitch healing parameter turns out to be $0.558 \leq Q \leq 0.948$. This feature is consistent with other conventional NS models discussed in the literature and can explain only the higher values of $Q$ on the basis of the starquake model of glitch generation for Crab-like pulsars. For the minimum weighted mean value of $Q \simeq 0.7$, our models (corresponding to an envelope with $\Gamma_1 = 2, 5/3$ or 4/3) yield the minimum masses
Table 4. Mass (M), size (R), compactness ratio (u ≡ M/R), and the ‘local’ value of the adiabatic index at the centre, (Γ₁)₀, for various values of the ratio of central pressure to central energy-density, P₀/E₀, for the core–envelope models discussed in the text (step 4, Section 3). Various parameters are obtained by assigning a fiduciary value of E₀ = 2.7 × 10¹⁹ g cm⁻³ and for an ‘appropriate’ value of the ratio of pressure to energy-density, P₀/E₀ = (P₀/E₀)₀ ≡ 4.694 × 10⁻¹⁰, at the core–envelope boundary. The superscripts a, b, and c, which appear with various parameters, represent the models corresponding to an envelope with Γ₁ = 4/3, 5/3 and 2, respectively. The values in italics correspond to the limiting case up to which the configurations remain pulsationally stable. It is seen that the M–R relation does provide the necessary and sufficient condition of dynamical stability for all of the sequences, as all of the sequences satisfy both of the inequalities, namely uₘₐₓ ≤ 0.3460 and (Γ₁)₀ ≤ 2.5946, simultaneously at the maximum value of mass. Because all members of the sequences corresponding to an envelope with Γ₁ = 4/3, 5/3 and 2 fulfil the ‘compatibility criterion’, u ≤ uₙₙ (where uₙₙ represents the compactness ratio of a homogeneous-density sphere for the corresponding value of P₀/E₀), together with fulfilling the necessary and sufficient condition of dynamical stability provided by the M–R relation, it follows that the assigned value of P₀/E₀ = (P₀/E₀)₀ ≡ 4.694 × 10⁻¹² corresponds to the minimum value for which the ‘compatibility criterion’ is ‘appropriately’ satisfied by all the sequences corresponding to NS models with an envelope with Γ₁ = 4/3, 5/3 and 2, respectively, as discussed in step 4 of Section 3.

| (P₀/E₀)₀ | (Γ₁)₀ | (M/₁₀⁶M⊙) | R (km) | u | (M/₁₀⁶M⊙) | R (km) | u | (M/₁₀⁶M⊙) | R (km) | u |
|----------|-------|------------|-------|---|------------|-------|---|------------|-------|---|

Mₖ ≃ 1.44 M⊙, M₀ ≃ 1.6 M⊙ and Mₘ ≃ 2.03 M⊙ for the Crab pulsar. Among these values, the first two values are comparable with those of the minimum values 1.35 M⊙ and 1.65 M⊙ obtained by Crawford & Demianski (2003) using the GWM (Glendenning, Weber & Moszkowski 1992) and HKP (Haensel, Kutschera & Proszynski 1981) EOS's. By contrast, corresponding to Q ≃ 0.7, the other five EOSs of the dense nuclear matter considered by Crawford & Demianski (2003) yield the minimum mass M < 1 M⊙ for the Crab pulsar (see, for example, Crawford & Demianski 2003, and references therein). However, for the maximum mean weight value of Q ≃ 0.2 corresponding to the Vela pulsar, our models yield an unrealistically small mass value for the Vela pulsar, as do the other models discussed in the literature (Crawford & Demianski 2003).

However, as soon as the ‘compatibility criterion’ is ‘appropriately’ satisfied by all the models corresponding to an envelope with Γ₁ = 4/3, 5/3 or 2, respectively (that is, if the ratio of pressure to energy-density at the core–envelope boundary, P₀/E₀, is set — and not ‘arbitrarily’ assigned — to be about 4.694 × 10⁻¹² for all the sequences), the corresponding range of the glitch healing parameter becomes 0.016 ≲ Q ≲ 0.779. This range, however, can explain both values (the higher as well as the lower) of Q on the basis of the starquake model of glitch generation for the Crab-like, as well as for the Vela-like pulsars. The sequences corresponding to an envelope with Γ₁ = 5/3 and 2 yield the maximum masses M₀ ≃ 2.06 M⊙, Mₘ ≃ 1.85 M⊙ for the Vela pulsar (Q ≃ 0.2) and the minimum masses Mₖ ≃ 4.1 M⊙, Mₘ ≃ 3.98 M⊙ for the Crab (Q ≃ 0.7) pulsar. The sequence corresponding to an envelope with Γ₁ = 4/3, however, yields the maximum mass Mₘ ≃ 2.67 M⊙ for the Vela pulsar (Q ≃ 0.2) but it does not provide a minimum stable mass for the Crab pulsar as soon as the constraint of Q > 0.7 is imposed.

The higher mass values mentioned in the last paragraph for the Crab pulsar seem to be unlikely, because none of the observational and/or theoretical studies predict such higher mass values for the Crab pulsar. Thus, unlike the results of steps (2)–(3) (Section 3), which deal with the study of NS models without implementing the ‘appropriate’ fulfilment of the ‘compatibility criterion’, the implementation of the ‘appropriate’ fulfilment of the ‘compatibility criterion’ also reveals that, in order to construct a ‘realistic’ NS sequence composed of NS masses comparable with those of observations, we have to modify the value of the matching density at the core–envelope boundary. In view of the modern EOSs of dense nuclear matter, the upper bound on NS mass compatible with causality and dynamical stability can reach a value of up to 2.2 M⊙, since among the variety of modern EOSs discussed in the literature only the following EOSs yield the maximum mass of an NS model in excess of 2 M⊙: SLy (Douchin & Haensel 2001) EOS, Mₘₐₓ = 2.05 M⊙; BGN1 (Balberg & Gal 1997) EOS, Mₘₐₓ = 2.18 M⊙; and APR (Akmal, Pandharipande & Ravenhall 1998) EOS, Mₘₐₓ = 2.21 M⊙ (see, for
Table 5. Compactness ratio, $u = M/R$, and the fractional moment of inertia, $Q = I_{\text{core}}/I_{\text{total}}$, for various values of the ratio of pressure to energy-density, $P_b/E_b$, at the centre for the models presented in Table 4. For each value of $P_b/E_b$, the compactness ratio of a homogeneous-density sphere, $u_0$, is shown in column 2. Various parameters are obtained by assigning a fiduciary value of $E_b = 2.7 \times 10^{14}$ g cm$^{-3}$ and for an ‘appropriate’ value of the ratio of pressure to energy-density, $P_b/E_b = (P_b/E_b)_2 = 4.694 \times 10^{-2}$, at the core–envelope boundary. The superscripts a, b and c, which appear with various parameters, represent the models corresponding to an envelope with $\Gamma_1 = 4/3, 5/3$ and 2, respectively. The values in italics correspond to the limiting case up to which the configurations remain pulsationally stable. It is seen that, for an assigned value of $P_b/E_b$, the inequality $u \leq u_0$ is ‘appropriately’ satisfied by all members of the sequences corresponding to an envelope with $\Gamma_1 = 4/3, 5/3$ or 2. It follows from this table that the ‘appropriate’ choice of $P_b/E_b = 4.694 \times 10^{-2}$ in the present context, for example) can provide a suitable explanation for both values (the higher as well as the lower) of the glitch healing parameter $Q$ in the range $0.016 \leq Q \leq 0.779$ (in the present context, for example), on the basis of the starquake mechanism of glitch generation, as shown in Fig. 5.

| $(P_b/E_b)$ | $u_0$  | $u^a$ | $u^b$ | $u^c$ | $Q^a$  | $Q^b$  | $Q^c$  |
|------------|--------|-------|-------|-------|--------|--------|--------|
| 0.10052    | 0.14253| 0.06106| 0.11116| 0.12456| 0.01548| 0.06873| 0.09775|
| 0.11236    | 0.15394| 0.07042| 0.12154| 0.13537| 0.02739| 0.09920| 0.13475|
| 0.12029    | 0.16116| 0.07693| 0.12843| 0.14236| 0.03739| 0.12071| 0.15969|
| 0.12955    | 0.16918| 0.08466| 0.13630| 0.15034| 0.05089| 0.14626| 0.18860|
| 0.14549    | 0.18205| 0.09798| 0.14940| 0.16340| 0.07777| 0.19020| 0.23645|
| 0.15354    | 0.18814| 0.10461| 0.15572| 0.16969| 0.09261| 0.21182| 0.25950|
| 0.20061    | 0.21911| 0.14077| 0.18919| 0.20255| 0.18555| 0.32652| 0.37684|
| 0.23990    | 0.24008| 0.16698| 0.21268| 0.22536| 0.26028| 0.40470| 0.45366|
| 0.27901    | 0.25763| 0.18963| 0.23274| 0.24469| 0.32671| 0.46877| 0.51517|
| 0.34816    | 0.28259| 0.22255| 0.26156| 0.27253| 0.42326| 0.55590| 0.59803|
| 0.44840    | 0.30929| 0.25816| 0.29256| 0.30222| 0.52531| 0.64342| 0.67967|
| 0.48306    | 0.31666| 0.26795| 0.30094| 0.31024| 0.52823| 0.66607| 0.70070|
| 0.51773    | 0.32332| 0.27667| 0.30848| 0.31746| 0.57715| 0.68625| 0.71944|
| 0.54455    | 0.32803| 0.28273| 0.31373| 0.32251| 0.59401| 0.70020| 0.73250|
| 0.58709    | 0.33482| 0.29128| 0.32112| 0.32956| 0.61777| 0.71982| 0.75062|
| 0.63150    | 0.34115| 0.29895| 0.32778| 0.33589| 0.63925| 0.73770| 0.76712|
| 0.66866    | 0.34953| 0.30438| 0.33242| 0.34040| 0.65474| 0.75034| 0.77916|
| 0.68296    | 0.34766| 0.30623| 0.33404| 0.34194| 0.66008| 0.75491| 0.78336|
| 0.69990    | 0.34952| 0.30820| 0.33573| 0.34352| 0.66592| 0.75976| 0.78778|
| 0.72302    | 0.35220| 0.31076| 0.33793| 0.34567| 0.67364| 0.76625| 0.79403|
| 0.74010    | 0.35401| 0.31235| 0.33934| 0.34700| 0.67868| 0.77068| 0.79815|
| 0.77503    | 0.35751| 0.31501| 0.34164| 0.34914| 0.68772| 0.77855| 0.80543|
| 0.80605    | 0.36041| 0.31651| 0.34294| 0.35044| 0.69398| 0.78427| 0.81111|
| 0.83972    | 0.36336| 0.31711| 0.34347| 0.35091| 0.69887| 0.78917| 0.81587|

Figure 4. The mass–radius diagram of the models as discussed in the text (Step 4, Sections 2 and 3) for an assigned value of matching density $E = E_b = 2.7 \times 10^{14}$ g cm$^{-3}$ at the core–envelope boundary. The labels a, b and c represent the models for an envelope with $\Gamma_1 = 4/3, 5/3$ and 2, respectively. The value of the ratio of pressure to energy-density, $P_b/E_b = (P_b/E_b)_2 = 4.694 \times 10^{-2}$, at the core–envelope boundary is obtained in such a manner that the ‘compatibility criterion’ is ‘appropriately’ satisfied by all the models corresponding to an envelope with $\Gamma_1 = 4/3, 5/3$ and 2, respectively, as shown in Table 5. That is, the necessary and sufficient condition of dynamical stability provided by the mass–radius relation is satisfied together with the ‘compatibility criterion’, as shown in Table 4.

Figure 5. Fractional moment of inertia $Q = I_{\text{core}}/I_{\text{total}}$ versus total mass $M$ for the models presented in Tables 4 and 5 and shown in Fig. 4. The labels a, b and c denote the models for an envelope with $\Gamma_1 = 4/3, 5/3$ and 2 respectively.

example, Haensel, Potekhin & Yakovlev 2006). However, on the basis of other modern EOSs for NS matter, fitted to experimental nucleon–nucleon scattering data and the properties of light nuclei, Kalogera & Baym 1996 (and references therein) have shown that the
This paper has been typeset from a TeX file prepared by the author.