Joint Design of Hybrid Beamforming and Phase Shifts in RIS-Aided mmWave Communication Systems

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Abstract—This paper considers a reconfigurable intelligent surface (RIS)-aided millimeter wave (mmWave) downlink communication system where hybrid analog-digital beamforming is employed at the base station (BS). We formulate a power minimization problem by jointly optimizing hybrid beamforming at the BS and the response matrix at the RIS, under signal-to-interference-plus-noise ratio (SINR) constraints. The problem is highly challenging due to the non-convex SINR constraints as well as the non-convex unit-modulus constraints for both the phase shifts at the RIS and the analog beamforming at the BS. A penalty-based algorithm in conjunction with the manifold optimization technique is proposed to handle the problem, followed by an individual optimization method with much lower complexity. Simulation results show that the proposed algorithm outperforms the state-of-art algorithm. Results also show that the joint optimization of RIS response matrix and BS hybrid beamforming is much superior to individual optimization.

I. INTRODUCTION

Reconfigurable Intelligent Surfaces (RISs) have emerged as a new technique to enhance wireless communications by manipulating the radio propagation environment. An RIS is an artificial meta-surface consisting of a large number of passive reflection elements that can be programmed to control the phase of the incident electromagnetic waves [1]. It is appealing for communications as it can create passive beamforming (BF) towards desired receivers without radio frequency (RF) components. Compared to traditional active multi-input multi-output (MIMO) relaying, RISs are more cost-effective and do not cause any noticeable processing delay.

RISs bring a new degree of freedom to the optimization of BF design. The work [2] studies the joint optimization of active and passive BF in an RIS-aided multi-user system for transmit power minimization under signal-to-interference-plus-noise ratio (SINR) constraints. In [3], the joint optimization of active and passive BF is investigated for weighted-sum-rate (WSR) maximization under transmit power constraints. The work [4] considers the sum-rate maximization problem when only a limited number of discrete phase shifts can be realized by the RIS. Note that in all these works on joint active-passive BF design, the active BF part is fully digital as in most of the MIMO BF literature, which requires each antenna to be connected to one RF chain.

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modulus constraints. Unlike the conventional semidefinite relaxation (SDR) method, we adopt a manifold optimization technique to handle these unit-modulus constraints. Overall, we propose a two-layer penalty-based algorithm in conjunction with the Riemannian manifold optimization to find a stationary solution to the original problem. Simulation results show that the proposed penalty-based algorithm outperforms the traditional SDR-based optimization algorithm. Results also show that the proposed hybrid beamforming at the BS can perform closely to the fully digital beamforming.

II. System Model and Problem Formulation

A. System Model

As shown in Fig. 1 we consider an RIS-aided downlink mmWave system where one BS, equipped with M antennas, communicates with K single-antenna users via the help of one RIS with F unit cells. The BS employs the sub-connected hybrid A/D beamforming structure with N RF chains, each connected to D = M/N antennas. Let s_j denote the information signal intended to user j, for j ∈ {1, 2, ..., K}. It is assumed to be independent to each other and satisfies E[|s_j|^2] = 1. Each of these signals is first weighted by a digital beamforming vector, denoted as w_j ∈ ℂ^N×1. These weighted signal vectors are summed together and each entry is sent to an RF chain, then multiplied by an analog beamforming vector, denoted as v_n ∈ ℂ^{D×1}, for n ∈ N = {1, 2, ..., N}. Each entry of v_n, denoted as v_n,d, ∀d ∈ D = {1, 2, ..., D} is a phase shifter, i.e., |v_n,d| = 1. The overall analog beamforming matrix can be represented as V = diag{v_1, ..., v_N} ∈ ℂ^{M×N}. At the RIS, let F = {1, 2, ..., F} denote the set of total RIS unit cells, and define a diagonal matrix Θ = diag{b_1, b_2, ..., b_F} as the response-coefficient matrix, where b_j = e^j2πφ, ∀φ ∈ [0, 2π) being the phase shift of the jth unit cell. The total transmit power of the BS is given by

$$P_{\text{total}} = \sum_{k=1}^{K} ||Vw_k||^2 = D \sum_{k=1}^{K} ||w_k||^2.$$  \hspace{1cm} (1)

We assume the BS-user link is blocked, and thus the direct path can be ignored. The channel state information (CSI) of all links is assumed to be perfectly known at the BS and all the channels experience quasi-static flat-fading.

The received signal of user k can be represented as

$$y_k = h_k^H \Theta G V \sum_{j=1}^{K} w_j s_j + n_k, \forall k \in K,$$  \hspace{1cm} (2)

where G ∈ ℂ^{F×M} is the channel matrix from the BS to the RIS, h_k^H ∈ ℂ^{1×F} is the channel vector from the RIS to user k, and n_k ∼ CN(0, σ_n^2) is the additive white Gaussian noise at the receiver of user k.

The received SINR of user k can be expressed as

$$\text{SINR}_k = \frac{|h_k^H \Theta G V w_k|^2}{\sum_{j\neq k} |h_k^H \Theta G V w_j|^2 + \sigma_n^2}, \forall k \in K.$$  \hspace{1cm} (3)

B. mmWave Channel Model

We adopt the widely used Saleh-Valenzuela channel model \[8\] for mmWave communications. Specifically, the channel matrix between BS and RIS can be written as

$$G = \sqrt{\frac{MV}{N_{c1}N_{ray1}}} \sum_{i_1=1}^{N_{cl1}} \sum_{i_2=1}^{N_{cl2}} \alpha_{i_1,i_2} a_R(\phi_{1i_1}, \delta_{1i_1}) a_B(\phi_{2i_2}, \delta_{2i_2}) H.$$  \hspace{1cm} (4)

Here, N_{cl1} denotes the number of scattering clusters, N_{ray1} denotes the number of rays in each cluster, α_{i_1,i_2} denotes the channel coefficient of the i_2th ray in the i_1th propagation cluster. Moreover, a_R(ϕ_{1i_1}, δ_{1i_1}) and a_B(ϕ_{2i_2}, δ_{2i_2}) represent the receive array response vectors of the RIS and the transmit array response vectors of the BS respectively, where ϕ_{1i_1}, \delta_{1i_1} and ϕ_{2i_2}, \delta_{2i_2} represent azimuth and elevation angles of arriving at the RIS (or departing from the BS).

The channel vector between the RIS and the k-th user can be represented as

$$h_k^H = \sqrt{\frac{F}{N_{c2}N_{ray2}}} \sum_{i_2=1}^{N_{cl2}} \sum_{i_1=1}^{N_{cl1}} \beta_{i_2,i_1} a_R(\phi_{1i_2}, \delta_{1i_2}) a_B(\phi_{2i_1}, \delta_{2i_1}) H.$$  \hspace{1cm} (5)

Here, N_{cl2}, N_{ray2}, \beta_{i_2,i_1}, \phi_{2i_1}, \delta_{2i_1} and \phi_{1i_2}, \delta_{1i_2} are defined in the same way as above.

We consider the uniform planar array (UPA) structure at both BS and RIS. The array response vector can be denoted as

$$a_z(ϕ, δ) = \frac{1}{\sqrt{A_1A_2}} [1, ..., e^{j2π(d_1o \sin ϕ \sin δ + p \cos δ)},$$

$$..., e^{j2π(d_1o(A_1-1) \sin ϕ \sin δ + (A_2-1) \cos δ)}]^T,$$  \hspace{1cm} (6)

where z ∈ {R, B}, λ is the signal wavelength, d_1 is the antenna or unit cell spacing which is assumed to be half wavelength distance, 0 ≤ o < A_1 and 0 ≤ p < A_2, A_1 and A_2 represent the number of rows and columns of the UPA in the 2D plane, respectively.

C. Problem Formulation

We aim to minimize the transmit power by jointly optimizing the digital beamforming matrix \[W = [w_1, ..., w_K] \in ℂ^{N×K}\] and the analog beamforming matrix \[V\] at the BS, as well as the overall response-coefficient matrix \[Θ\] at the RIS, subject to a minimum SINR constraint for each user. Thus, the optimization problem can be formulated as

$$P_0 : \min_{(V, W, Θ)} \left\{ D \sum_{k=1}^{K} ||w_k||^2 \right\} \hspace{1cm} (7a)$$

s.t.

$$\text{SINR}_k \geq \gamma_k, \forall k \in K,$$  \hspace{1cm} (7b)

$$|v_{n,d}| = 1, \forall n \in N, \forall d \in D,$$  \hspace{1cm} (7c)

$$|b_f| = 1, \forall f \in F,$$  \hspace{1cm} (7d)

where \[γ_k > 0\] is the minimum SINR requirement of user k.

The problem is non-convex due to the non-convex SINR constraints (7b) and the unit-modulus constraints (7c), (7d).
A commonly used approach to solve this type of optimization problems approximately is to apply the block coordinate descent (BCD) techniques in conjunction with the SDR method. More specifically, the digital beamforming matrix $\mathbf{W}$, the analog beamforming matrix $\mathbf{V}$, and the RIS coefficient matrix $\mathbf{\Theta}$ are updated in an alternating manner in each iteration. The sub-problem of finding $\mathbf{W}$ can be solved by second-order cone program (SOCP) method, and both the sub-problems of finding $\mathbf{V}$ and finding $\mathbf{\Theta}$ can be solved by SDR. However, the solution obtained by SDR is not guaranteed to be rank-one and additional randomization approach is needed. In addition, when the number of users is close to the number of RF chains at the BS, the randomization procedure may fail to find a feasible solution.

### III. Penalty-based Joint Optimization Algorithm

In this section, we propose a two-layer penalty-based algorithm for the considered problem $\mathcal{P}_0$. The BCD method is adopted in the inner layer to solve a penalized problem and the penalty factor is updated in the outer layer until converge. Specifically, we firstly introduce auxiliary variables $\{t_{k,j}\}$ to represent $\mathbf{h}_k^H \mathbf{\Theta} \mathbf{G} \mathbf{V} \mathbf{w}_j$ such that variables $\mathbf{W}, \mathbf{V}$ and $\mathbf{\Theta}$ can be decoupled. Then, the non-convex constraints (7) can be equivalently written as

\[
\frac{|t_{k,j}|^2}{\sum_{j \neq k} |t_{k,j}|^2 + \sigma_k^2} \geq \gamma_k, \forall k \in \mathcal{K}, \tag{8a}
\]
\[
t_{k,j} = \mathbf{h}_k^H \mathbf{\Theta} \mathbf{G} \mathbf{V} \mathbf{w}_j, \forall k, j \in \mathcal{K}. \tag{8b}
\]

Then, the equality constraints (8d) is relaxed and added to the objective function as a penalty term. Thereby, the original problem $\mathcal{P}_0$ can be converted to

\[
\mathcal{P}_1 : \min_{\mathbf{V}, \mathbf{W}, \mathbf{\Theta}, \{t_{k,j}\}} \quad \sum_{k=1}^{K} \|\mathbf{w}_k\|^2 + \frac{\rho}{2} \sum_{j=1}^{K} |\mathbf{h}_k^H \mathbf{\Theta} \mathbf{G} \mathbf{V} \mathbf{w}_j - t_{k,j}|^2 \tag{9}
\]
\[
s.t. \quad \mathbf{8a}, \mathbf{8c}, \mathbf{8d}, \tag{9b}
\]

where $\rho > 0$ is the penalty factor. The choice of $\rho$ is crucial to balance the original objective function and the equality constraints. It is seen that the objective function in $\mathcal{P}_1$ is dominated by the penalty term when $\rho$ is large enough and consequently, the equality constraints (8d) can be well met by the solution. Thus, we can start with a small value of $\rho$ to get a good start point, and then by gradually increasing $\rho$, a high precision solution can be obtained. Similar approach is adopted in [9].

#### A. Inner Layer: BCD Algorithm for Solving Problem $\mathcal{P}_1$

For any given $\rho$, the problem $\mathcal{P}_1$ is non-convex but all the optimization variables $\{\mathbf{V}, \mathbf{W}, \mathbf{\Theta}, \{t_{k,j}\}\}$ are decoupled in the constraints. We therefore adopt BCD method to optimize each of them alternately while keeping the others fixed.

1) Optimize $\mathbf{W}$: When $\mathbf{V}, \mathbf{\Theta}$ and $\{t_{k,j}\}$ are fixed, problem $\mathcal{P}_1$ becomes a non-constraint convex optimization problem. Thus, the optimal $\mathbf{W}$ can be obtained by the first-order optimality condition, i.e.,

\[
\mathbf{w}_k = \rho \mathbf{A}^{-1} \sum_{j=1}^{K} \tilde{\mathbf{h}}_j^T t_{j,k}, \forall k \in \mathcal{K}, \tag{10}
\]

where $\tilde{\mathbf{h}}_j = \mathbf{h}_j^H \mathbf{\Theta} \mathbf{G} \mathbf{V}$ and $\mathbf{A} = 2D \mathbf{I}_N + \rho \sum_{j=1}^{K} \tilde{\mathbf{h}}_j^H \tilde{\mathbf{h}}_j$.

2) Optimize $\mathbf{\Theta}$: Let $\mathbf{b} = [b_1, b_2, \ldots, b_F]^H$. When other variables are fixed, problem $\mathcal{P}_1$ is reduced to (with constant terms ignored)

\[
\min_{\mathbf{b}} \quad \sum_{j=1}^{K} \sum_{k=1}^{K} |\mathbf{h}_k^H \mathbf{c}_{k,j} - t_{k,j}|^2 \tag{11a}
\]
\[
s.t. \quad |b_f| = 1, \forall f \in \mathcal{F}, \tag{11b}
\]

where $\mathbf{c}_{k,j} = \text{diag}(\mathbf{h}_k^H) \mathbf{G} \mathbf{V} \mathbf{w}_j \in \mathbb{C}^{F \times 1}$. Although the objective function is convex for $\mathbf{b}$, the problem (11) is still non-convex due to the unit-modulus constraints (11b). To handle this problem, one way is to alternately optimize the $F$ units one by one as in (11b). Although closed-form expression is available for each unit, this method is still inefficient since the unit number $F$ is usually very large. Another way is to adopt the SDR technique as in [2]. But its complexity is high and additional randomization procedure is needed. Note that the unit-modulus constraints (11b) form a complex circle manifold $\mathcal{M} = \{\mathbf{b} \in \mathbb{C}^F : |b_1| = \cdots = |b_F|\}$. Therefore, different from the above approaches, we adopt the manifold optimization technique to solve this problem efficiently and optimally. In specific, we adopt the Riemannian conjugate gradient (RGC) algorithm. The RGC algorithm is widely applied in hybrid beamforming design [11] and recently applied in RIS-aided systems as well [12, 13]. Each iteration of the RGC algorithm involves three key steps, namely, to compute Riemannian gradient, to find search direction and retraction.

The Riemannian gradient $\nabla_f f(\mathbf{b})$ of the function $f(\mathbf{b})$ is defined as the orthogonal projection of the Euclidean gradient $\nabla_e f(\mathbf{b})$ onto the tangent space $T_\mathbf{b}\mathcal{M}$ of the manifold $\mathcal{M}$, which can be expressed as

\[
T_\mathbf{b}\mathcal{M} = \{\mathbf{z} \in \mathcal{M} : \Re \{\mathbf{z} \odot \mathbf{b}^*\} = 0\}, \tag{12}
\]

where $\odot$ denotes the Hadamard product. The Euclidean gradient of $f(\mathbf{b})$ over $\mathbf{b}$ is given by

\[
\nabla_e f(\mathbf{b}) = 2 \sum_{j=1}^{K} \sum_{k=1}^{K} \mathbf{c}_{k,j}^H (\mathbf{c}_{k,j}^H \mathbf{b} - t_{k,j}^H). \tag{13}
\]

Then, the Riemannian gradient is given by

\[
\nabla_f f(\mathbf{b}) = \nabla_e f(\mathbf{b}) - \Re \{\nabla_e f(\mathbf{b}) \odot \mathbf{b}\} \odot \mathbf{b}. \tag{14}
\]

With the Riemannian gradient, we can update the search direction $\mathbf{d}$ by conjugate gradient method, i.e.,

\[
\mathbf{d} = - \nabla f(\mathbf{b}) + \lambda_1 \mathcal{T}(\mathbf{d}), \tag{15}
\]

where $\lambda_1$ is the update parameter, $\mathbf{d}$ is the previous search direction and $\mathcal{T}(\mathbf{d}) = \overline{\mathbf{d}} - \Re \{\overline{\mathbf{d}} \odot \mathbf{b}^*\} \odot \mathbf{b}$.
Since the updated point may leave the previous manifold space, a retraction operation RetrB is needed to project the point to the manifold itself:

$$\text{Retr}_B : b_f \leftarrow \frac{(b + \lambda_2 d_f)}{|(b + \lambda_2 d_f)|},$$

(16)

where $\lambda_2$ is the Armijo backtracking line search step size.

3) Optimize $\mathbf{V}$: Define $x \triangleq [v_1^T, v_2^T, \ldots, v_N^T]^T \in \mathbb{C}^{M \times 1}$, and $Z_j \triangleq \text{diag}(w_{j,1} I_D, \ldots, w_{j,N} I_D) \in \mathbb{C}^{M \times M}$, where $|x_m| = 1, \forall m \in \mathcal{M} \triangleq \{1, 2, \ldots, M\}$ and $w_{j,n}$ denotes the $n$-th entry of $w_j$. Then, we have $V w_j = Z_j x \in \mathbb{C}^{M \times 1}$. When other variables are fixed, problem $P_1$ is given by

$$\min \ x \ f(x) = \sum_{j=1}^{K} \sum_{k=1}^{K} |d_{k,j} x - t_{k,j}|^2$$

s.t. \quad |x(m)| = 1, \forall m \in \mathcal{M}.

(17a)

where $d_{k,j} = b^H \text{diag}(h_k^H) G Z_j \in \mathbb{C}^{1 \times M}$. Similar to Section III-A2, it can be effectively solved by the RCG algorithm and the details are skipped.

4) Optimize $\{t_{k,j}\}$: With other variables fixed, problem $P_1$ can be reduced to

$$\min_{\{t_{k,j}\}} \ \sum_{j=1}^{K} \sum_{k=1}^{K} |h_k^H \Theta G V w_j - t_{k,j}|^2$$

s.t. \quad \sum_{j \neq k} |t_{k,j}|^2 + \sigma_k^2 \leq \gamma_k, \forall k \in \mathcal{K}.

(18b)

The objective function is convex over $\{t_{k,j}\}$. Although constraints (18b) are still non-convex, they can be translated to the form of second-order cone, which can be effectively and optimally solved by SOCP method [14].

B. Outer Layer: Update Penalty factor

The penalty factor $\rho$ is initialized to be a small number to find a good start point, then gradually increased to tighten the penalty. Specifically,

$$\rho := \frac{\rho}{c}, 0 < c < 1,$$

(19)

where $c$ is a scaling parameter. A larger $c$ may lead to a more precise solution with higher running time.

C. Algorithm

The overall penalty-based algorithm is summarized in Algorithm 1. Define the stopping indicator $\xi$ as following

$$\xi \triangleq \max \ \{ |h_k^H \Theta G V w_j - t_{k,j}|^2, \forall k, j \in \mathcal{K} \}.$$  

(20)

When $\xi$ is below a pre-defined threshold $\epsilon_2 > 0$, the equality constraints (20) are considered to be satisfied and the proposed algorithm is terminated. Since we start with a small penalty and gradually increase its value, the objective value of problem $P_1$ is finally determined by the penalty part and the equality constraints are guaranteed to be satisfied. Note that, for any given $\rho$, problem $P_1$ is solved through the BCD method and each subproblem can obtain an optimal solution. Thus, Algorithm 1 is guaranteed to converge to a stationary point. The total complexity of Algorithm 1 is $O(I_O I_J (K N^3 + I_Q K^2 F + I_V K^2 M + K^7))$, where $I_O$, $I_J$, $I_Q$, and $I_V$ denote the iteration times of the outer loop, the inner loop, the inner RCG algorithm to update $\Theta$, and the inner RCG algorithm to update $\mathbf{V}$, respectively.

IV. INDIVIDUAL OPTIMIZATION

To reduce the complexity of solving problem $P_0$, we develop an individual optimization approach in this section, where the RIS response matrix $\Theta$, the analog beamformer $\mathbf{V}$, and the digital beamformer $\mathbf{W}$ are obtained sequentially without alternating optimization.

A. RIS design

The equivalent channel between the BS and the $k$th user via the RIS can be represented as $h_k^H \Theta G$. To ensure the receive signal quality of each user, we aim to find the optimal RIS response matrix for maximizing the equivalent channel gain of the user who has the worst channel state, i.e.,

$$\max_{\Theta} \ \min_{k \in \mathcal{K}} |h_k^H \Theta G|^2$$

s.t. \quad |b_f| = 1, \forall f \in \mathcal{F}.

(21a)

(21b)

This problem can be effectively solved by SDR.

B. Analog BF design

Orthogonal matching pursuit (OMP) method is widely adopted to design the analog beamformer [8]. If the BS adopts the fully digital BF structure, the optimal digital BF under the zero-forcing (ZF) scheme is given by $F_{\text{opt}} = \hat{H}^\dagger \text{diag}(\sqrt{\gamma_1 \sigma_1^2}, \ldots, \sqrt{\gamma_K \sigma_K^2})$, where $\hat{H} = [ (h_1^H \Theta G)^T, \ldots, (h_K^H \Theta G)^T ]^T$ and $\dagger$ denotes the pseudo-inverse. Define the overlapping coefficient as $\mu$, and denote the codebook as $\mathbf{A} = [ a_B(\psi_1, \phi_1), \ldots, a_B(\psi_1, \phi_{N_2}), \ldots, a_B(\psi_{N_y}, \phi_{N_2}), \ldots, a_B(\psi_{N_y}, \phi_{N_z}) ]$, where $N_y$ and $N_z$ denote the horizontal and vertical length, $\psi_i = \frac{2\pi(i-1)}{N_y}$, $i = 1, 2, \ldots, N_y$ and $\phi_j = \frac{2\pi(j-1)}{N_z}$, $j = 1, 2, \ldots, N_z$, respectively. Then, we can use a selection ma-
trix $T \in \mathbb{R}^{N_t \times N} \times N$ to select proper columns. Specifically, 
the analog BF problem can be formulated as
\[
T^* = \arg \min_T \| F_{\text{opt}} - A_t T F_{B B} \|_F \quad (22a)
\]
s.t. \[
\| \text{diag} (T T^H) \|_0 = N, \quad (22b)
\]
where $I_t$ is a $M \times 1$ zero-vector with the entry from $(t-1)D+1$ to $tD$ being one; $\odot$ denotes the Hadamard product. Since 
the structure of analog BF is sub-connected, we use $I_t$ to modify the 
the codebook. Then, the OMP method can be applied to obtain the 
the optimal $T^*$. The analog BF can be recovered, i.e., $V = A_t T^*$.

C. Digital BF design

After obtaining the RIS phase shifts and the analog beamformer, we need to obtain the optimal digital BF vector by solving following problem,
\[
\min_W \sum_{k=1}^K \| W_k \|_2^2 \quad (23)
\]
s.t. \[
(7b)
\]
Note that it is the conventional power minimization problem in 
the multi-input single-output (MISO) downlink system, which can be 
effectively solved by SOCP method.

V. SIMULATION RESULTS

We consider a $6 \times 6$ UPA structure at the BS with a 
total of $M = 36$ antennas and $N = 6$ RF chains located 
at $(0 \text{ m, } 0 \text{ m})$. The RIS is located at $(d_{RI} \text{ m, } 10 \text{ m})$ and equipped with $F_1 \times F_2$ unit cells where $F_1 = 6$ and 
$F_2$ can vary. Users are uniformly and randomly distributed 
in a circle centered at $(100 \text{ m, } 0 \text{ m})$ with radius $5 \text{ m}$. 
As for the channel, we set $N_{d_1} = N_{d_2} = 2$ clusters, 
$N_{ray_1} = N_{ray_2} = 5$ rays; the complex gain $\alpha_{il}$ and $\beta_{il}$ follow the 
the complex Gaussian distribution $N(0, 10^{-0.1PL(d)})$, where 
$PL(d) = \varphi_a + 10 \log_{10}(d) \times \xi (dB)$ with $\xi \sim N(0, \sigma^2)$, 
$\varphi_a = 72.0, \varphi_b = 2.92, \sigma = 8.7$dB [15]. The auxiliary vari-
ables $\{t_{k,j}\}$ are initialized following $\mathcal{CN}(0, 1)$. The penalty 
the factor is initialized by $\rho = 10^{-3}$. Other system parameters are 
set as follows unless specified otherwise later: $K = 3, F_2 = 
6, d_{RI} = 50, c = 0.9, \epsilon_1 = 10^{-4}, \epsilon_2 = 10^{-7}, \gamma_k = 10$dB,
$\sigma_k^2 = -85$dBm, $\forall k \in K$. All simulation curves are averaged 
over 100 independent channel realizations.

A. Convergence Performance

We show the stopping indicator (20) of the penalty-based 
algorithm in Fig. 2 and the average convergence of the penalty-
the algorithm in Fig. 3. These curves are plotted with 
the average plus and minus the standard deviation. It is 
observed that the stopping indicator can always meet the 
the predefined accuracy $10^{-7}$ after about 110 outer layer iterations 
in Fig. 2. Thus, solutions obtained by the Algorithm 1 satisfy 
all SINR constraints. Fig. 3 shows that the proposed algorithm 
converges after about 300 total iteration numbers, which means 
that the inner layer runs averagely 3 times.

B. Performance Comparison with other schemes

To demonstrate the efficiency of the proposed algorithms 
and to reveal some design insights, we compare the perfor-
mance of the following algorithms:

- Penalty-Manifold joint design with hybrid BF structure 
(Penalty-Manifold HB): This is the proposed Algorithm 1 for joint design of hybrid BF and RIS phase shifts.
- Penalty-Manifold joint design with fully digital BF structure 
(Penalty-Manifold FD): This is the proposed Algorithm 1 but changing the hybrid BF to fully digital BF at the BS. This is done by letting $D = 1$.
- Penalty-Manifold joint design with random $\Theta$ (Random $\Theta$): The phase shifts at the RIS are randomly selected to be feasible values. Then the hybrid beamforming matrices $\{W, V\}$ at the BS are obtained by using the penalty-
manifold joint algorithm as in Algorithm 1 where the update of $\Theta$ is skipped. This is to find out the significance of optimizing the phase shifts at the RIS.
- Penalty-Manifold joint design with SDR $\Theta$ (SDR $\Theta$): The phase shifts at the RIS are designed for maximizing the effective channel of the worse-cast user by using the SDR approach based on (21). Then the hybrid beamforming matrices $\{W, V\}$ at the BS are obtained by using the penalty-
manifold joint algorithm as in Algorithm 1 where the udpate of $\Theta$ is skipped. This is again to find out the significance of optimizing the phase shifts at the RIS.
- BCD-SDR joint design (BCD-SDR): The conventional BCD method in conjunction with the SDR method, as mentioned in the end of Section II-C.
- Individual design: the proposed individual design where RIS phase shifts, analog BF, and digital BF are optimized sequentially in Section IV.
since it only employs FD. Note that the hybrid BF has much lower hardware cost than the power consumed by Penalty-Manifold HB is about 2.5dB higher than the power consumed by Penalty-Manifold joint design. Further optimizing the hybrid BF at the BS can only bring marginal improvement. Last but not least, the power consumed by Penalty-Manifold HB is about 10dB power reduction can be obtained. These observations indicate that the design of RIS phase shifts plays the crucial role for performance optimization. Third, the individual design is about 2dB worse than the joint design with SDR $\Theta$. This suggests that, when the RIS response matrix is designed individually for maximizing the effective channel gain of the worse-case user, further optimizing the hybrid BF at the BS can only bring marginal improvement. Last but not least, the power consumed by Penalty-Manifold HB is about 2.5dB higher than the power consumed by Penalty-Manifold FD. Note that the hybrid BF has much lower hardware cost since it only employs $N = 6$ RF chains, while the fully digital BF has $M = 36$ RF chains.

The influence of the RIS element number, $F$, is considered in Fig. 5. When $F$ increases from 12 to 60, the transmit power drops about 15dB. Thus, we conclude that the RIS can greatly reduce the transmit power by installing a large number of elements.

Fig. 6 illustrates the influence of the RIS location. It is seen that as the RIS horizontal distance $d_{RIS}$ increases, the transmit power increases firstly, and reaches the peak at 50 m, then decreases. This can be explained that the received power through the reflection of the RIS in the far field is proportional to $\frac{1}{d_1^2 + d_2^2}$, where $d_1$ and $d_2$ denote the distance between the BS-RIS and RIS-user, respectively. It is found that the RIS can be located near the BS or users to save energy.

VI. CONCLUSION

This paper proposed a two layer penalty-based algorithm to solve the RIS-aided hybrid beamforming optimization problem in mmWave systems. In the inner layer, we alternately optimize the digital beamforming and analog beamforming at the BS and the response coefficient at the RIS. The outer layer updates the penalty factor to obtain a high precision solution. A low-complexity individual optimization method is also proposed. Extensive simulation results demonstrate that the proposed algorithm has a good performance and the RIS can significantly improve the energy efficiency.

REFERENCES

[1] T. Cui, D. Smith, and R. Liu, Metamaterials: Theory, Design, and Applications. Springer, 2010.

[2] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Trans. Wireless Commun., vol. 18, no. 11, pp. 5394–5409, 2019.

[3] H. Guo, Y.-C. Liang, J. Chen, and E. G. Larsson, “Weighted sum-rate optimization for intelligent reflecting surface enhanced wireless networks,” 2019. [Online]. Available: https://arxiv.org/abs/1905.07920

[4] B. Di, H. Zhang, L. Song, Y. Li, Z. Han, and H. V. Poor, “Hybrid beamforming for reconfigurable intelligent surface based multi-user communications: Achievable rates with limited discrete phase shifts,” 2019. [Online]. Available: https://arxiv.org/abs/1910.14328

[5] A. F. Molisch, V. V. Ratnam, S. Han, Z. Li, S. L. H. Nguyen, L. Li, and K. Haneda, “Hybrid beamforming for massive MIMO: A survey,” IEEE Commun. Mag., vol. 55, no. 9, pp. 134–141, 2017.

[6] Y. Xiu, J. Zhao, W. Sun, M. D. Renzo, G. Gui, Z. Zhang, and N. Wei, “Reconfigurable intelligent surfaces aided mmwave noma: Joint power allocation,phase shifts, and hybrid beamforming optimization,” 2020. [Online]. Available: https://arxiv.org/abs/2007.05873

[7] K. Ying, Z. Gao, S. Lyu, Y. Wu, H. Wang, and M. Alouini, “GMD-based hybrid beamforming for large reconfigurable intelligent surface assisted millimeter-wave massive MIMO,” IEEE Access, vol. 8, pp. 19530–19539, 2020.

[8] O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, “Spatially sparse precoding in millimeter wave MIMO systems,” IEEE Trans. Wireless Commun., vol. 13, no. 3, pp. 1499–1513, 2014.

[9] Q. Wu and R. Zhang, “Joint active and passive beamforming optimization for intelligent reflecting surface assisted SWIPT under QoS constraints,” 2019. [Online]. Available: https://arxiv.org/abs/1910.06220

[10] P.-A. Absil, R. Mahony, and R. Sepulchre, Optimization algorithms on matrix manifolds. Princeton University Press, 2009.

[11] X. Yu, J.-C. Shen, J. Zhang, and K. B. Letaief, “Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems,” IEEE J. Sel. Topics Signal Process., vol. 10, no. 3, pp. 485–500, 2016.

[12] X. Yu, D. Xu, and R. Schober, “MISO wireless communication systems via intelligent reflecting surfaces,” 2019. [Online]. Available: https://arxiv.org/abs/1904.12199
[13] H. Guo, Y.-C. Liang, J. Chen, and E. G. Larsson, “Weighted sum-rate maximization for reconfigurable intelligent surface aided wireless networks,” *IEEE Trans. Wireless Commun.*, vol. 19, no. 5, pp. 3064–3076, 2020.

[14] A. Wiesel, Y. C. Eldar, and S. Shamai, “Linear precoding via conic optimization for fixed MIMO receivers,” *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, 2006.

[15] M. R. Akdeniz, Y. Liu, M. K. Samimi, S. Sun, S. Rangan, T. S. Rappaport, and E. Erkip, “Millimeter wave channel modeling and cellular capacity evaluation,” *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1164–1179, 2014.