Updated Limits on TeV–Scale Gravity from Absence of Neutrino Cosmic Ray Showers Mediated by Black Holes

Luis A. Anchordoqui,1 Jonathan L. Feng,2 Haim Goldberg,1,3 and Alfred D. Shapere4

1Department of Physics, Northeastern University, Boston, MA 02115
2Department of Physics and Astronomy, University of California, Irvine, CA 92697
3Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139
4Department of Physics, University of Kentucky, Lexington, KY 40506

Abstract

We revise existing limits on the $D$-dimensional Planck scale $M_D$ from the nonobservation of microscopic black holes produced by high energy cosmic neutrinos in scenarios with $D = 4+n$ large extra dimensions. Previous studies have neglected the energy radiated in gravitational waves by the multipole moments of the incoming shock waves. We include the effects of energy loss, as well as form factors for black hole production and recent null results from cosmic ray detectors. For $n \geq 5$, we obtain $M_D > 1.0 - 1.4$ TeV. These bounds are among the most stringent and conservative to date.

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I. INTRODUCTION

Forthcoming colliders \cite{1}, cosmic ray observatories \cite{2, 3, 4}, neutrino telescopes \cite{5}, and space-based experiments \cite{6} will be able to observe black holes (BHs) if the fundamental scale of gravity is sufficiently close to 1 TeV \cite{7}. Observations of highly characteristic BH events at any of these facilities could conceivably provide the first evidence for the existence of extra dimensions and make possible the direct study of strong quantum gravity effects and strings. On the other hand, the lack of such events in any experiments to date leads to lower limits on the scale of higher-dimensional gravity \cite{8}.

To make useful predictions about higher-dimensional gravity based on observations of such events, or their absence, a quantitative understanding of the process of BH production in high-energy collisions is required. An intuitive picture of this process is provided by a simple model known as Thorne’s hoop conjecture \cite{9}, according to which a BH forms in a two-particle collision when and only when the impact parameter is smaller than the radius \( r_s \) of a Schwarzschild BH of mass equal to the total center-of-mass energy \( E_{CM} \). The hoop conjecture thus predicts a total cross section for BH production equal to the area subtended by a “hoop” of radius \( r_s \):

\[
\sigma_{BH}^{\text{hoop}} = \pi r_s^2 (E_{CM}).
\]  

(1)

Up to now, all studies of BH production in TeV-scale gravity have been based on this rather heuristic cross section, and have thus been subject to substantial theoretical uncertainties.

Relatively recently, significant progress has been made in determining the cross section for BH production. Early analytic calculations in four dimensions \cite{10, 11} for head-on collisions illustrated the process of horizon formation and found that the mass of the final BH was about 84% of the initial center-of-mass energy. These calculations were extended to nonzero impact parameter by Eardley and Giddings \cite{12}, who analytically derived a lower bound on the total cross section of approximately 65% of Eq. (1). Relatively recently, a calculation of the cross section in higher dimensions was performed by Yoshino and Nambu using numerical techniques \cite{13}. In addition, these authors observed significant reductions in the mass of the final-state black hole as a function both of impact parameter and dimension.

If TeV-scale gravity is realized in nature, then the first observational evidence for it will likely come from BH-mediated neutrino cosmic ray showers. Ultra-high energy cosmic rays hit the Earth with collision center-of-mass energies ranging up to roughly \( 10^5 \) GeV. QCD cross sections dominate over the BH production cross section by a factor of roughly \( 10^9 \). Thus, black holes produced by hadronic cosmic rays are effectively unobservable. This is not the case for incoming neutrinos, whose cross section for producing black holes can be orders of magnitude larger than SM cross sections, but much less than hadronic \cite{2}. As a consequence, neutrinos interact with roughly equal probability at any point in the atmosphere, and the light descendants of the black hole may initiate quasi-horizontal showers in the volume of air immediately above the detector. Because of these considerations the atmosphere provides a buffer against contamination by mismeasured hadrons, allowing a good characterization of BH-induced showers when the BH entropy \( S \gg 1 \) \cite{3}. Additionally, neutrinos that traverse the atmosphere unscathed may produce black holes through interactions in the ice or water \cite{5}. Because the BH production cross section is suppressed by a power of the fundamental Planck scale \( M_D \) (approaching \( M_D^2 \) for large numbers of extra dimensions), the absence of neutrino showers mediated by black holes implies lower bounds on \( M_D \).

In this article, we bring up to date existing limits \cite{8, 14} on \( M_D \) from the nonobservation of BHs at cosmic neutrino detection experiments. Besides incorporating the cross section and
energy loss results of Yoshino and Nambu, we also make use of updated parton distribution functions and recently available cosmic ray data.

II. ENERGY LOSS IN BLACK HOLE CREATION

Previous calculations of the cross section for producing a BH have neglected energy loss in the creation of a BH, assuming that the mass of the created black hole $M_{\text{BH}}$ was identical to the incoming parton center-of-mass energy $\sqrt{s}$. However, recent work\textsuperscript{13} has shown that the energy lost to gravitational radiation is not negligible, and in fact is large for larger $n$ and for large impact parameters. The trapped mass (called $M_{\text{A.H.}}$ in Ref.\textsuperscript{13}, and which we continue to call $M_{\text{BH}}$\textsuperscript{15}), is given by

$$M_{\text{BH}}(z) = y(z)\sqrt{s},$$

(2)

where the inelasticity $y$ is a function of $z \equiv b/b_{\text{max}}$. Here $b$ is the impact parameter and

$$b_{\text{max}} = \sqrt{F(n)} r_s(\sqrt{s}, n, M_D)$$

(3)

is the maximum impact parameter for collapse, where

$$r_s(\sqrt{s}, n, M_D) = \frac{1}{M_D} \left[ \frac{\sqrt{s}}{M_D} \right]^{\frac{1}{1+n}} \left[ \frac{2^{n-3} \pi^{(n+3)/2}}{n+2} \right]^{\frac{1}{1+n}}$$

(4)

is the radius of a Schwarzschild BH in $(4+n)$ dimensions\textsuperscript{16}, and $F(n)$ is the form factor explicitly given in Ref.\textsuperscript{13}.

This complicates the parton model calculation, since the production of a BH of mass $M_{\text{BH}}$ requires that $\hat{s}$ be $M_{\text{BH}}^2/y^2(z)$, thus requiring the lower cutoff on parton momentum fraction to be a function of impact parameter\textsuperscript{17}. In what follows we take the $\nu N$ cross section as an impact parameter-weighted average over parton cross sections, with the lower parton fractional momentum cutoff determined by the requirement $M_{\text{BH}}^{\text{min}} = x_{\text{min}} M_D$\textsuperscript{18}. This gives a lower bound $x_{\text{min}} M_D^2/[y^2(z) s]$ on the parton momentum fraction $x$. With this in mind, the $\nu N \rightarrow \text{BH}$ cross section is

$$\sigma(E_\nu, x_{\text{min}}, n, M_D) \equiv \int_0^1 2z \, dz \int_{(x_{\text{min}} M_D)^2}^{(r_s^{-1})^2} \, dx \, F(n) \, \pi r_s^2(\sqrt{s}, n, M_D) \sum_i f_i(x, Q),$$

(5)

where $x_{\text{min}}$ is determined by the requirement that the BH have at least an approximate semi-classical description, $\hat{s} = 2x_m E_\nu$, $i$ labels parton species, and the $f_i(x, Q)$ are parton distribution functions (pdfs)\textsuperscript{8}.

The choice of the momentum transfer $Q$ is governed by considering the time or distance scale probed by the interaction. Roughly speaking, the formation of a well-defined horizon occurs when the colliding particles are at a distance $\sim r_s$ apart. This has led to the advocacy of the choice $Q \sim r_s^{-1}$\textsuperscript{14}, which has the advantage of a sensible limit at very high energies. However, the dual resonance picture of string theory would suggest a choice $Q \sim \sqrt{s}$. Fortunately, as noted in Refs.\textsuperscript{8, 14}, the BH production cross section is largely insensitive to the details of the choice of $Q$. In what follows we use the CTEQ6M pdfs\textsuperscript{19} with $Q = \min\{r_s^{-1}, 10 \text{ TeV}\}$.
FIG. 1: Cross sections $\sigma(\nu N \rightarrow BH)$ for $n = 1, \ldots, 7$ from below, assuming $M_D = 1$ TeV and $x_{\text{min}} = 1$. Energy loss has been included according to Eq. (5). The SM cross section $\sigma(\nu N \rightarrow \ell X)$ is indicated by the dotted line.

In Fig. 1 we show the BH production cross section for $n = 1, \ldots, 7$ extra dimensions with energy loss incorporated as given in Eq. (5). The rapid rise in cross section is pushed to higher $E_\nu$ than in the case with energy loss neglected. However, the cross sections are still well above the SM cross section at $E_\nu \sim 10^8$ GeV and above, where, as we will see, the cosmogenic neutrino flux is large.

III. COSMIC NEUTRINO DETECTORS

Energy loss also impacts event rates at cosmic neutrino detectors, not only because the cross section is modified, but also because the apertures of cosmic neutrino detectors are functions of shower energy. Let $N_A$ be Avogadro’s number, $A(y E_\nu)$ the neutrino aperture of a given experiment for shower energy $y E_\nu$, and $T$ be the experiment’s running time. The number of neutrino showers mediated by BHs is then

$$\mathcal{N}(x_{\text{min}}, n, M_D) = N_A T \int dE_\nu \int_0^1 2z \, dz \int_{(x_{\text{min}} M_D)^2}^{y E_\nu} \frac{d\Phi}{dE_\nu} \, A(y E_\nu) \times F(n) \pi r_s^2(\sqrt{s}, n, M_D) \sum_i f_i(x, Q),$$

where $d\Phi/dE_\nu$ is the diffuse flux of cosmic neutrinos hitting the Earth.

There are several techniques employed in detecting neutrino showers [20]. The most commonly used method involves giant arrays of particle counters that sample the lateral and temporal density profiles of the muon and electromagnetic components of the shower front. Another well-established method involves measurement of the air shower evolution — its growth and subsequent attenuation — as it develops by sensing the fluorescence light
produced via interactions of the charged particles in the atmosphere. A third method exploits naturally occurring large volume Čerenkov radiators such as deep water or ice. Especially useful at relevant center-of-mass energies for BH production is emission of Čerenkov radiation at radio frequencies. For fluorescence data, a direct measurement of the depth of shower maximum $X_{\text{max}}$ and the shape of the longitudinal profile provide sensitive diagnostics in discriminating between neutrino and hadron showers. In the case of surface arrays, the composition information is extracted from a number of shower characteristics which reflect the depth of shower maximum and the ratio of muon to electromagnetic content of the shower.

The AGASA Collaboration \cite{21} reports no significant enhancement of deeply-developing shower rates given the detector’s resolution. Specifically, there is only 1 event observed, consistent with the expected background of 1.72 from hadronic cosmic rays. For details, see Ref. \cite{8}.

The Fly’s Eye detector ceased operation in July 1992 after a life of 11 years. It was designed to collect the atmospheric nitrogen fluorescence light produced by air shower particles on moonless nights without cloud cover, achieving an overall duty cycle of $\approx 10\%$. The experiment recorded more than 5000 events, but no unusual deeply developing showers have been found \cite{22}.

Recently, data from an upscaled version of the Fly’s Eye experiment have become available \cite{23}. The effective aperture of the High Resolution Fly’s Eye detector is on average about 6 times the Fly’s Eye aperture, with a threshold around $10^8$ GeV. The instrument includes two sites (HiRes I and II) located 12.6 km apart. Each site consists of a large number (22 at HiRes I and 42 at HiRes II) of telescope units pointing at different parts of the sky. Between November 1999 and September 2001, 1198 events were recorded with at least one reconstructed energy greater than $10^8.7$ GeV \cite{23}. Because of bad weather conditions, 272 events were discarded from the sample. None of the 723 events that survived all of the cuts required for stereo-mode triggering has $X_{\text{max}} > 1200$ g/cm$^2$. Additionally, there are no events detected in monocular mode with $X_{\text{max}} > 1500$ g/cm$^2$. In the spirit of Ref. \cite{24}, we parametrize the HiRes aperture for deeply ($X_{\text{max}} > 1500$ g/cm$^2$) developing showers by

$$A(E_\nu) = 1.8 \left\{ 2.7 + \log \left( \frac{E_\nu}{10^{10} \text{ GeV}} \right) \right\}^2 - 0.5 \right\} \text{ km}^3 \text{ sr}. \quad (7)$$

We note that there is an additional small contribution to the HiRes exposure in the energy range $10^8 - 10^9$ GeV from data collected during 2878 hours livetime \cite{25}.

The Radio Ice Čerenkov Experiment (RICE) is designed to detect the radio frequency Čerenkov radiation produced by neutrino-induced showers in ice \cite{26}. Specifically, the electromagnetic channel of the shower produces a radio pulse with a duration of a few nanoseconds and with power concentrated around the Čerenkov angle. Several radio antennae positioned in the ice allow for reconstruction of the interaction vertex. For primary energies above $10^9$ GeV, the Landau Pomeranchuk-Migdal (LPM) effect \cite{27} leads to a significant suppression of the Bethe-Heitler cross sections for the pair production and Bremsstrahlung processes in dense materials, and thus dramatically changes the character of the development of electromagnetic showers.

Almost instantaneously after its formation, the TeV-scale BH decays \cite{28}, predominantly through radiation of standard model (SM) particles \cite{29}. About 75% of the BH energy is carried off by quarks and gluons and roughly a third of this energy goes into the electromagnetic channel via $\pi^0$ decay. Only about 5% of particles directly emitted from the BH
FIG. 2: The monotonically rising curves are the exposures as functions of shower energy for AGASA (solid), Fly’s Eye (dotted), HiRes (short dash), and RICE (long dash). The remaining solid curve, with a peak around $10^{8.5}$ GeV, is the cosmogenic neutrino flux. 

$(\nu'$s, $\tau'$s, $\mu'$s) do not partake in the shower. The rest of the energy eventually devolves into secondary electromagnetic cascades with particle energies below that for which the LPM effect is important [30]. As a conservative estimate we model the aperture for neutrino showers mediated by BHs using the hadronic effective volume reported by the RICE Collaboration [31] with average inelasticity $\langle y \rangle = 0.8$. This estimate is supported by the fact that BH-induced showers mimic SM neutral current events, characterized by hadronic dominated showers with no leading charged lepton. A more rigorous analysis of the BH acceptance at the RICE facility is underway [32].

The RICE detector comprises 16 dipole radio receivers installed in the holes drilled for the AMANDA experiment in the Antarctic ice. Four transmitter antennae are also deployed in the ice for calibration purposes. The trigger requires a 4-fold coincidence within a 1.2 $\mu$s time window, and various cuts are applied to reject thermal and anthropogenic backgrounds. For example, shower vertices are reconstructed using a $\chi^2$ fit to the signal arrival times, and the resulting fits are required to be of sufficient quality and to indicate vertices at least 50 m below the ice’s surface. During a 1 month run in August of 2000 with a livetime of 333.3 hours, a total of 22 events passed all the automated cuts. These events were then scanned for quality and the RICE Collaboration concluded that there are no events consistent with neutrino sources [31].

The relative exposures for the different experiments are given in Fig. 2. For details on the apertures of AGASA and Fly’s Eye, the reader is referred to our previous paper [14]. All in all, there is only 1 event observed with an expected background of 1.72 from hadronic cosmic rays, leading to a 95% CL limit of 3.5 BH events [33].

To derive the bounds on $M_D$, we use the “guaranteed” flux of cosmogenic neutrinos arising from the decay of $\pi^\pm$ produced in collisions of ultra-high energy protons with the cosmic
FIG. 3: 95% CL lower limits on the fundamental Planck scale as a function of $x_{\text{min}}$ for $n = 1, \ldots, 7$ extra dimensions (from below).

microwave background. As in our previous analyses, we conservatively adopt the estimates of Protheroe and Johnson [34] with nucleon source spectrum scaling as $d\Phi_N/dE \propto E^{-2}$ and extending up to the cutoff energy $10^{12.5} \text{ GeV}$. The total ultra-high energy cosmogenic neutrino flux is also shown in Fig. 2.

IV. BOUNDS

In Fig. 3 we show 95% CL lower bounds on $M_D$ as derived from Eq. (6) using the exposures and the cosmogenic neutrino flux given in Fig. 2 requiring $N < 3.5$ events to be observed in cosmic neutrino data samples [35]. The BH entropy is a measure of the validity of the semi-classical approximation. For $x_{\text{min}} \gtrsim 3$ and $n \geq 5$, the entropy

$$S = \frac{4\pi M_{\text{BH}} r_s(M_{\text{BH}})}{n + 2} \gtrsim 10,$$

yielding small thermal fluctuations in the emission process [36]. Hence, for $x_{\text{min}} \gtrsim 3$ and $n \geq 5$, strong quantum gravity effects may be safely neglected. Moreover, gravitational effects due to brane back-reaction are expected to be insignificant for $M_{\text{BH}}$ well beyond the brane tension, which is presumably of the order of $M_D$. The uncertainty illustrated in Fig. 3 associated with $x_{\text{min}}$ only concerns BH production and is highly insensitive to decay characteristics [37]. This is because the signal in neutrino detection experiments relies only on the existence of visible decay products. Whatever happens around $x_{\text{min}} \approx 1$, it seems quite reasonable to expect that BHs or their Planckian progenitors will cascade decay on the brane.

String theory provides a more complete picture of the decay for $M_{\text{BH}}$ close to $M_D$, which may further justify setting $x_{\text{min}} = 1$. (Such arguments do not address the issue of brane
back-reaction, however.) In string theory, the ultimate fate of the black hole is determined by the string/BH correspondence principle [38]: when the Schwarzschild radius of the black hole shrinks to the fundamental string length $\ell_s \gg \ell_D$, where $\ell_D$ is the fundamental $(4+n)$-dimensional Planck length, an adiabatic transition occurs to a massive superstring mode. Subsequent energy loss continues as thermal radiation at the unchanging Hagedorn temperature [39]. The continuity of the cross section at the correspondence point, parametrically in both the energy and the string coupling, provides independent support for this picture [40]. Thus, the cross sections given in Fig. 11 can be thought of as lower bounds on $\sigma$ as $M_{\text{BH}}$ approaches $M_D$ [41].

V. SUMMARY

Incorporation of the results of Ref. [13] has eliminated many of the sources of uncertainty enumerated in Ref. [8] and recapitulated in Ref. [42]. In Ref. [8] we identified two sources of uncertainty that could reduce the total cross section: the reduction of the mass of the final-state BH relative to the initial center-of-mass energy, and expectations for a reduced cross section at nonzero center-of-mass angular momentum.\(^1\) On the other hand, we pointed out that the classical photon capture cross section and nonrelativistic estimates suggest a possible enhancement to the naïve geometric cross section $\pi r_s^2$ by a factor of 2 or more. (The claim of [42] that this upside uncertainty casts doubt on the program of setting limits on $M_D$ from nonobservation of BHs is mistaken.) Thus we concluded that, in the absence of a better quantitative understanding of the process of BH formation, the naïve geometric cross section provided a reasonable estimate.

With such calculations now in hand [13] we have repeated our analysis and eliminated much of the uncertainty contained in our previous limits on $M_D$, as well as incorporating updated exposures from the HiRes and RICE facilities and updated pdfs. (Incidentally, using the new pdfs contributed a net difference of about 2% to our results, confirming our previous claim [8] that there is very little sensitivity to different choices of pdfs. Furthermore, the bulk of the sensitivity is for pdfs at $x > m_{\text{BH}}^2/(2m_N E_\nu) \sim 10^{-2}$ and large $Q$, where the pdfs are expected to be quite accurate.) In the course of our analysis we observed a competition of effects leading to corrections to our previous estimates: enhancement of the geometric cross section by form factors of up to 1.9 and enhancement of apertures from new cosmic ray data, but a simultaneous reduction in the rate of production of BHs of mass greater than $x_{\text{min}} M_D$, after taking energy losses into account. It turns out that the latter effect dominates and leads to a slight weakening of our limits on $M_D$. At the same time, our limits are now on a much firmer theoretical footing, and maximally conservative in all respects.

In Fig. 4 we compare the bounds derived in this article with existing limits on the fundamental scale of large extra dimensions. Tests of the gravitational inverse-square law at length scales well below 1 mm show no evidence for short range Yukawa interactions. For $n = 2$, this negative result can be translated into a 95% CL upper limit of 150 $\mu$m on the compactification radius of flat extra dimensions, or equivalently to a unification mass scale

\(^1\) In fact, a slight modification of our estimate for the modification of the cross section due to angular momentum effects has been used to give a surprisingly accurate postdiction of the higher-dimensional cross section for BH production [43], providing further evidence for the correctness of the Yoshino and Nambu calculations.
FIG. 4: Bounds on the fundamental Planck scale $M_D$ from tests of Newton’s law on sub-millimeter scales, bounds on supernova cooling and neutron star heating, dielectron and diphoton production at the Tevatron, and nonobservation of BH production by cosmic neutrinos. The uncertainty in the Tevatron bounds corresponds to the range of brane softening parameter $\in (M_D/2, M_D)$; for details see Ref. [8]. The range in the cosmic ray bounds is for $x_{\text{min}} = 1 - 3$.

$M_D > 1.8$ TeV [44]. In such toroidal compactifications the accessibility of towers of Kaluza-Klein gravitons may drastically affect the phenomenology of supernovae and neutron stars. For $n \leq 3$, anomalous cooling of supernovae due to bulk graviton emission and neutron star heating by decay of gravitationally trapped Kaluza-Klein modes provide limits on $M_D$ that greatly exceed 1 TeV [45].

For $n \geq 4$ the sensitivity of table-top experiments and astrophysical observations to TeV-scale gravity is largely reduced: already for $n = 4$ ($n = 5$) supernova cooling yields $M_D > 4.0$ TeV ($M_D > 0.8$ TeV) [46]. For $n \geq 5$, the best existing limits on TeV-scale gravity are from the absence of trans-Planckian signatures (BH/stringball production) in neutrino detection experiments discussed here, and from searches for sub-Planckian signatures (graviton emission and virtual graviton exchanges) at the Tevatron [47] and LEP [48]. For $n \geq 5$ we have derived conservative bounds incorporating the lower limits on the the mass trapped in the gravitational collapse. The resulting bounds, $M_D > 1.0 – 1.4$ TeV for $x_{\text{min}} = 1 – 3$, are competitive with those obtained in colliders and among the most stringent to date.

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