Bridging the pseudo-Dirac dark matter and radiative neutrino mass in a singlet doublet scenario

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Abstract

We examine simple extension of the standard model with a pair of fermions, one singlet and a doublet, in a common thread linking the dark matter problem with the smallness of neutrino masses associated with several exciting features. In the presence of a small bare Majorana mass term, the singlet fermion brings in a pseudo-Dirac dark matter capable of evading the strong spin-independent direct detection bound by suppressing the dark matter annihilation processes mediated by the neutral current. In consequence, the allowed range of mixing angle between the doublet and the singlet fermions gets enhanced substantially. Presence of the same mass term in association of singlet scalars also elevates tiny but nonzero masses radiatively for light Majorana neutrino satisfying observed oscillation data.

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I. INTRODUCTION

We now boast a remarkably successful and precisely validated Standard Model (SM) of particle physics, scalar sector of which lately being examined at the Large Hadron Collider (LHC) [1, 2]. In spite of that, many of the experimentally observed phenomena of the Universe still lacking any amicable and well-accepted explanation within this framework. One of the major mysteries of the present Universe is the fundamental nature of dark matter which has long been inferred from different celestial and cosmological observations and estimated as accounts for nearly 26% of the total energy density of the Universe. None from the trunk of SM particles owns the appropriate properties which are necessarily required to constitute a suitable cold dark matter (DM) candidate. Plausible origin of tiny but non-zero neutrino mass, which also unequivocally established in different solar, atmospheric and reactor neutrino oscillation experiments, remains another long-standing puzzle. Besides, questions surrounding naturalness issue, baryogenesis and dark energy persist. Supersymmetry [3] seems to have the ability to answer many of these unresolved questions. However, lack of any clinching evidence of supersymmetry yet in LHC encourages us to build an alternative scenario beyond the Standard Model (BSM) to explain the observed anomalies consists of dark and neutrino sectors. Although numerous proposals exist, a concrete theoretical construction of new sector that attempts to address these seemingly unrelated issues in a minimalistic manner should earn attention.

In this paper, we study a simple extension of Standard Model, which offers a common origin for pseudo-Dirac dark matter interaction with the visible sector and radiative generation of neutrino mass. To look for a particle DM candidate, several dedicated direct search experiments namely XENON 1T [4, 5], Panda-X [6] etc. are ongoing. However, so far, we have not found any positive signature of DM. This hints at the possibility of DM interaction with the visible sector is weaker than the current precision of the measurements. The singlet doublet fermionic dark matter scenario is studied extensively [7–37], and it falls within the weakly interacting massive particle (WIMP) paradigm. There are two neutral fermion states in this setup which mix with each other and the lightest one is identified as the DM candidate. The mixing angle depends on the coupling strength of the singlet and doublet fermion with the SM Higgs. The magnitude of this mixing angle determines whether the DM is singlet like or doublet dominated. In singlet doublet model DM candidate can be probed
at direct search experiments through its interaction with nucleon mediated by the SM Higgs and the neutral gauge boson. However, the null results at direct search experiments restrict the range of the mixing angle below $\lesssim 0.06$ [7], making the DM almost purely singlet dominated. It was proposed in Ref. [38] and later on in Ref. [39] that an impurity in the form of a small Majorana mass term in the Lagrangian for the singlet fermion would split the DM eigenstate into two non-degenerate Majorana states. In the small Majorana mass limit, the splitting does not make any difference to the relic abundance analysis, however making a vital portal to direct detection of the pseudo-Dirac DM candidate [38].

We make use of the same Majorana mass term for the singlet fermion in generating the low energy neutrino mass radiatively [40, 41]. The present mechanism of neutrino mass generation is also familiar as the scotogenic inverse seesaw scheme. In the process, we extend the minimal version of the singlet doublet DM framework with multiple copies of a real scalar singlet fields $1$. These additional scalar fields can couple with the SM leptons and the doublet fermion through lepton number violating vertices. Thus in the radiative one-loop level DM particles and the singlet scalars take part in the generation of neutrino masses. As a result, the eigenvalues of the SM neutrinos are determined by the masses of DM sector particles, scalar singlets and the Majorana mass parameter of the singlet fermion. More importantly, the Majorana nature of the SM neutrino is solely determined by the introduced Majorana mass term for the singlet fermion, which also helps in successfully evading the spin-independent (SI) constraints in dark matter. Thus the DM sector and the neutrino mass parameters are strongly correlated in the present set up which we are going to explore in detail.

The paper is organised as follows. In Section II we present the structure of our model, which is primarily an extended form of the singlet doublet model. We describe the field content, their interactions and insertion of additional Majorana term. In section III we discuss the consequence of our model in dark matter phenomenology. We examine the properties of our pseudo-Dirac dark matter candidate and how it extends its model parameter space evading the spin-independent direct detection limits. In Section IV we explain the mechanism of radiative generation of neutrino mass and look at the parameter space where oscillation data can be satisfied simultaneously along with the dark matter constraints and

1 A similar exercise on the radiative generation of neutrino mass within the singlet doublet DM framework is performed in Ref. [30] except having a pure Majorana type DM.
BSM and SM Fields | $SU(3)_C \times SU(2)_L \times U(1)_Y \equiv \mathcal{G}$ | $Z_2$
---|---|---
$\Psi \equiv \begin{pmatrix} \psi^0 \\ \psi^- \end{pmatrix}$ | 1 | 2 | $-\frac{1}{2}$ | $-$
$\chi$ | 1 | 1 | 0 | $-$
$\phi_i \ (i = 1, 2, 3)$ | 1 | 1 | 0 | $-$
$\ell_L \equiv \begin{pmatrix} \nu_e \\ \ell \end{pmatrix}$ | 1 | 2 | $-\frac{1}{2}$ | $+$
$H \equiv \begin{pmatrix} \frac{1}{\sqrt{2}} w^+ \\ \frac{1}{\sqrt{2}}(v + h + iz) \end{pmatrix}$ | 1 | 2 | $\frac{1}{2}$ | $+$

TABLE I. Field contents and charge assignments under the SM gauge symmetry and additional $Z_2$.

relic. Finally, we conclude highlighting features of our study in Section V.

II. THE MODEL

We extend the SM model particle sector by one $SU(2)_L$ doublet fermion ($\Psi$) and one gauge singlet fermion ($\chi$). In addition, we also include three copies of a real scalar singlet field ($\phi_{1,2,3}$). The BSM fields are charged under an additional $Z_2$ symmetry while SM fields transform trivially under this additionally imposed $Z_2$ (see Table I). The BSM fields do not carry any lepton numbers. The Lagrangian of the scalar sector is given by

$$L_{\text{scalar}} = |D^\mu H|^2 + \frac{1}{2}(\partial_\mu \phi)^2 - V(H, \phi), \quad (1)$$

where,

$$D^\mu = \partial^\mu - ig \frac{\sigma^a}{2} W^{a\mu} - ig' Y \frac{1}{2} B^\mu, \quad (2)$$

with $g$ and $g'$ being the $SU(2)_L$ and the $U(1)_Y$ gauge couplings respectively. The scalar potential $V(H, \phi)$ takes the following form

$$V(H, \phi_i) = -\mu_H^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2 + \mu_{ij}^2 \phi_i \phi_j + \frac{\lambda_i}{4!} \phi_i^4 + \frac{\lambda_{ij}}{2} \phi_i \phi_j (H^\dagger H). \quad (3)$$
The vacuum expectation values (vev) of both the scalars \( H \) and \( \phi_i \)'s after minimising the scalar potential in the limit \( \mu_H^2, \mu_{ij}^2 > 0 \) are obtained as,

\[
\langle H \rangle = v, \quad \langle \phi_i \rangle = 0.
\] (4)

For simplification, we take \( \mu_{ij}^2 \) diagonal parameterised as \( \text{Diag}(M_{\phi_1}^2, M_{\phi_2}^2, M_{\phi_3}^2) \). Since \( \langle \phi_i \rangle = 0 \), \( \mathcal{Z}_2 \) remains unbroken. The Lagrangian for the fermionic sector (consistent with the charge assignments) is written as:

\[
\mathcal{L} = \mathcal{L}_f + \mathcal{L}_Y,
\] (5)

where,

\[
\mathcal{L}_f = i \bar{\Psi} \gamma_\mu D^\mu \Psi + i \bar{\chi} \gamma_\mu \partial^\mu \chi - M_\Psi \bar{\Psi} \Psi - M_\chi \bar{\chi} \chi - \frac{m_{\chi L}}{2} \bar{\chi} P_L \chi - \frac{m_{\chi R}}{2} \bar{\chi} P_R \chi,
\] (6)

and

\[
\mathcal{L}_Y = Y \bar{\Psi} \tilde{H} \chi + h_{ij} \bar{\ell}_i \Psi \phi_j + h.c..
\] (7)

We keep a small Majorana mass \( (m_{\chi L,R} \ll M_\chi) \) term for the \( \chi \) field in Eq. (6). In this particular set up the lightest neutral fermion is a viable dark matter candidate which has a pseudo-Dirac nature provided a tiny \( m_{\chi L,R} \) exists. The choice of this non-vanishing \( m_{\chi L,R} \) is kept from the necessity of evading strong spin-independent dark matter direct detection bound. As we will see later that this term is also helpful in generating light neutrino mass radiatively. The first term in Eq. (7) provides the interaction of DM with the SM particles mediated through the Higgs. While the second term in Eq. (7) violates the lepton number explicitly. This kind of lepton number violation could trigger a thermal or non-thermal leptogenesis (baryogenesis) in the early Universe, provided sufficient CP asymmetry is generated [42].

### III. DARK MATTER

The different variants of singlet doublet fermion dark matter are extensively studied in the literature [7–30] over the years. Here we go through the DM phenomenology in

\[\text{Consideration of complex scalar singlets instead of real ones would lead to the conservation of the lepton number [30].}\]
FIG. 1. Region of parameter space allowed from both the relic density and direct detection bounds are shown in a plane of dark matter mass $M_{\xi_1}$ and mixing angle $\sin \theta$, in the limit Majorana mass $m_{\chi_{L,R}} = 0$. Different colors are for different values of mass gap $\Delta M = (M_{\xi_2} - M_{\xi_1})$ allowed here. In this scenario, upper limit in $\sin \theta$ is strongly constrained from direct detection bounds which gradually relaxed with higher dark matter mass and thus a lower cross section.

brief. In the present study, we consider $m_\phi \gg m_\psi, m_{\chi_{L,R}}$ such that the role $\phi$ fields in DM phenomenology is minimal. The Dirac mass matrix for the neutral DM sector after the spontaneous breakdown of the electroweak symmetry is obtained as (in $m_{\chi_{L,R}} \to 0$ limit),

$$
\mathcal{M}_D = \begin{pmatrix} M_\psi & M_D \\ M_D & M_\chi \end{pmatrix},
$$

(8)

where we define $M_D = \frac{Y_v}{\sqrt{2}}$. Therefore, we are left with two neutral Dirac particles which we identify as $(\xi_1, \xi_2)$. The mass eigenvalues of $(\xi_1, \xi_2)$ are given by,

$$
M_{\xi_1} \approx M_\chi - \frac{M_D^2}{M_\psi - M_\chi}
$$

(9)

$$
M_{\xi_2} \approx M_\psi + \frac{M_D^2}{M_\psi - M_\chi}
$$

(10)

Therefore, the lightest state is $\xi_1$, which we identify as our DM candidate. The DM stability is achieved by the unbroken $Z_2$ symmetry. The mixing between two flavor states, i.e. neutral
part of the doublet ($\psi^0$) and the singlet field ($\chi$) is parameterised by $\theta$ as

$$\sin 2\theta \simeq \frac{2Yv}{\Delta M},$$

where $\Delta M = M_{\xi_2} - M_{\xi_1} \approx M_{\Psi} - M_{\chi}$ in the small $Y$ limit. In small mixing case, $\xi_1$ can be identified with the singlet $\chi$. The DM phenomenology is mainly controlled by the following independent parameters.

$$\{M_{\Psi}, M_{\chi}, \theta\}.$$  (12)

The DM would have both annihilation and coannihilation channels to SM particles, including the gauge bosons [19, 23]. It turns out that the coannihilation channels play the dominant role in determining the relic abundance for pure singlet doublet fermion DM since the annihilation processes are velocity suppressed. The DM can be searched directly through its spin-independent scattering with nucleon mediated by both SM Higgs and Z boson. In Fig. 1 we show the observed relic abundance by Planck 2018 [43] and spin-independent direct detection bounds (from XENON 1T [5]) satisfied region in $\sin \theta - M_{\xi_1}$ plane for different values of $M_{\xi_2}$ in the absence of the Majorana mass term ($m_{\chi_{L,R}}$). We have used Micromega 4.3.5 [44] package for the numerical analysis. It is observed that the relic abundance is satisfied for a particular $M_{\xi_1}$ when $\Delta M = M_{\xi_2} - M_{\xi_1}$ is small. This means the coannihilation processes are dominant compared to the annihilation processes in determining the observed relic abundance. One important point to note is that the required amount of $\Delta M$ increases with the DM mass for any fixed value of $\sin \theta$. Fig. 1 also evinces strong constraint on $\sin \theta \lesssim 0.06$ primarily from the direct detection bounds, which gradually relaxed with higher dark matter masses because of a lower cross section. Finally, it keeps the DM framework alive from spin-independent direct detection bound.

The strong upper bound on $\sin \theta$ can be alleviated by taking the presence of $m_{\chi_{L,R}}$ into account. The tiny nature of $m_{\chi_{L,R}}$ makes $\xi_1$ pseudo-Dirac. In the limit $m \to 0$ where we define $m = (m_{\chi_L} + m_{\chi_R})/2$, the Majorana eigenstates of $\xi_1$ (i.e. $\zeta_1$, $\zeta_2$) become degenerate.

The presence of a non-zero $m_{\chi_{L,R}}$ breaks this degeneracy, and we can still write

$$\zeta_1 \simeq \frac{i}{\sqrt{2}}(\xi_1 - \xi_1^c),$$

$$\zeta_2 \simeq \frac{1}{\sqrt{2}}(\xi_1 + \xi_1^c).$$

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FIG. 2. Mass spectrum of the dark sector, showing the lightest pseudo-Dirac mode as dark matter and other heavy BSM fermions and scalars. Generation of large mass difference ($\Delta M$) and small mass gap ($m$) discussed at the text expressed at the zeroth order of $\delta r$. Scalars are assumed to be heavier in this study.

in the pseudo-Dirac limit $m \ll M_{\zeta_1}, M_{\zeta_2}$ where $M_{\zeta_1,\zeta_2} \simeq M_{\xi_1} \mp m$. Similarly, the state $\xi_2$ is split into $\zeta_3$ and $\zeta_4$. Hence we will have four neutral pseudo-Dirac mass eigenstates in the DM sector. The complete mass spectrum of the neutral dark sector particles is displayed in Fig. 2. The mass of the charged fermion $\psi^-$ lies in between $\zeta_3$ and $\zeta_2$ as followed from Eq. (9). The pseudo-Dirac nature of the eigenstates forbid the interaction of DM ($\zeta_1$) with the neutral current mediated by SM $Z$ boson at zeroth order of $\delta r \simeq (m_{\chi_L} - m_{\chi_R})/m_{\xi_1}$. Thus the pseudo-Dirac DM could have the potential to escape the SI direct search bound. Although at next to leading order, the DM still possesses non-vanishing interaction with $Z$ boson depending on the magnitude of $\delta r$. This will be analysed in the next paragraph. It is important to note that the $m$ can not be arbitrarily small since there exists a possibility of the lighter state $\zeta_1$ to scatter inelastically with the nucleon to produce heavier state $\zeta_2$ [45–47]. It imposes some sort of lower bound on $m \gtrsim \mathcal{O}(1)$ KeV [45–47] in order to switch off such kind of interaction. However, the presence of a vertex like $\bar{\zeta}_1 \gamma^\mu \zeta_2$ can give rise to huge $Z$ mediated s-channel coannihilation cross section of the DM with the next to lightest state (NLSP) [46] in the above mentioned limiting value of $m$. This cross section would have a suppression factor of $\sin^4 \theta$. In spite of this, for moderate values of $\sin \theta$, the cross section can
FIG. 3. Region of parameter space allowed from both the relic density and direct detection bounds are shown in a plane of dark matter mass $M_{\xi_1}$ and mixing angle $\sin \theta$, in case of a nonzero but small Majorana mass $m_{\chi L,R}$ insertion. Different colors are for different values of mass gap $\Delta M = (M_{\xi_2} - M_{\xi_1})$ allowed here. It is instructive to compare this present plot with Fig.1. Unlike the previous $m_{\chi L,R} = 0$ case (denoted by black dotted line here), upper limit from direct detection is much relaxed and barely constrained in this scenario. The present upper limit in $\sin \theta$ is primarily constrained from the relic density criteria and (unlike the previous case) constrain is being stronger at higher dark matter mass.

We have examined and found that keeping $m \sim \mathcal{O}(1)$ GeV effectively prevents the $Z$ mediated s-channel coannihilation of the DM with the NLSP [47] even with moderate values of $\sin \theta$. A similar result is obtained in Ref. [38, 48]. At linear order in $\delta_r$, a direct search of pseudo-Dirac dark matter through $Z$-mediation is still possible which we discuss below.

The vector operator for the SI direct search process mediated by $Z$ boson will be modified to

$$\mathcal{L} \supset \alpha (\bar{\zeta}_1 \gamma^\mu \zeta_1) (\bar{q} \gamma_\mu q),$$

with $\alpha = \frac{4g^2 \delta_r \sin^2 \theta}{m_Z^2 \cos^2 \theta_W}$, $C_V^\theta = \alpha' C_V^0$ and $g$ as the $SU(2)_L$ gauge coupling constant. Note that, at zeroth order in $\delta_r$, vector boson interaction of dark matter would vanish, and only the Higgs mediated processes would contribute to the direct search. Considering DM mass larger than
the nucleon mass, the spin-independent direct detection cross section per nucleon is obtained as [7, 9]

\[ \sigma_{\text{SI}} \simeq \frac{a}{\pi} \frac{M_{\chi}^2 m_N^2 \alpha^2}{(M_{\xi_1} + m_N)^2 A^2} \left[ Z C_V^p + (A - Z) C_V^n \right]^2, \]  

(16)

where \( m_N = 940 \text{ MeV} \), the nucleon mass, \( \theta_W \) is the Weinberg angle and \( C_V^p = \frac{1}{2} (1 - 4 \sin^2 \theta_W) \), \( C_V^n = -\frac{1}{2} \). It is clear from the smallness of the term \( (1 - 4 \sin^2 \theta_W) \) that, the DM particle rarely talks to protons, and hence the SI cross section mainly depends on the DM interaction with neutrons. For Dirac fermion \( a = 1 \) [49], while for Majorana \( a = \frac{1}{4} \) [49]. From the above relation, one can extract \( \delta_r \) as follows,

\[ \delta_r = 1.07 \times 10^{19} \left( \frac{\sigma_{\text{SI}}}{\text{cm}^2} \right)^{1/2} \left( \frac{1}{\sin^2 \theta} \right). \]  

(17)

Now to evade direct search constraints for the DM mass \( \gtrsim 100 \text{ GeV} \), it is sufficient to have \( \sigma_{\text{SI}} \lesssim 10^{-47} \text{ cm}^2 \). Imposing this bound in Eq. (17), we can report an upper bound on the difference of Majorana mass parameters \( m_{\chi_L} - m_{\chi_R} \) which is,

\[ m_{\chi_L} - m_{\chi_R} \lesssim 3.4 \times 10^{-5} \frac{M_{\xi_1}}{\sin^2 \theta}. \]  

(18)

The above bound turns out to be strongest for smaller \( M_{\xi_1} \) and larger \( \sin \theta \). For the present analysis, where we accommodate a WIMP like candidate with mass \( \mathcal{O}(100) \text{ GeV} \) and \( \sin \theta \lesssim 0.3 \). This automatically sets the bound as follows

\[ m_{\chi_L} - m_{\chi_R} \lesssim 13.5 \text{ MeV}. \]  

(19)

Taking the contribution of the Z mediated interaction of the DM with nucleon of the order of \( \mathcal{O}(10^{-47}) \text{ cm}^2 \) and considering \( m_{\chi_L} \simeq m_{\chi_R} = 1 \text{ GeV} \), we have plotted the relic abundance and direct search allowed points on \( \sin \theta - M_{\xi_1} \) plane in Fig. 3. Different colors are presented for different values of mass gap \( \Delta M = (M_{\xi_2} - M_{\xi_1}) \) allowed here. It is instructive to compare this present plot with Fig. 1. Unlike the previous \( m_{\chi_{L,R}} = 0 \) case (upper constraint limit of which is illustrated by a black dotted line in current plot), here upper limit from direct detection is much relaxed and barely constrains this scenario. In fact, the present upper limit in \( \sin \theta \) is primarily constrained from the relic density criteria, and unlike the previous case, the constraint is being stronger at higher dark matter mass. From this analysis, it is clear that the earlier obtained limit on \( \sin \theta \) got relaxed at a considerably good amount.
FIG. 4. Generation of neutrino mass radiatively at one loop level getting contributions from tiny Majorana mass term inserted in the dark sector along with the heavy singlet scalars.

Another notable feature of Fig. 3 is that for lighter DM, large mass splitting is allowed for higher values of $\sin \theta$. This follows from the fact that annihilation cross sector starts to play equivalent role as conannihilation at large $\sin \theta$. The above values of Majorana mass parameters will be used to evaluate the neutrino mass.

IV. NEUTRINO MASS

In the presence of the small Majorana mass term $(m_{\chi_{L,R}})$ of $\chi$ field and the lepton number violating operator in Eq. (7), it is possible to generate active neutrino mass radiatively at one loop as displayed in Fig. 4. It is worth mentioning that this type of mass generation scheme is known as one loop generation of inverse seesaw neutrino mass [50].

The neutrino mass takes the form as provided below [40, 41, 50],

$$m_{\nu_{ij}} = h_{ki}^T \Lambda_{kk} h_{kj},$$  \hspace{1cm} (20)

where, $\Lambda_{kk} = \Lambda_{kk}^L + \Lambda_{kk}^R$ with

$$\Lambda_{kk}^L = m_{\chi_L} \cos^2 \theta \sin^2 \theta \left[ \int \frac{d^4q}{(2\pi)^4} \left( q^2 - M_{\phi_k}^2 \right) (q^2 - M_{\xi_1}^2) \right] + \int \frac{d^4q}{(2\pi)^4} \left( q^2 - M_{\phi_k}^2 \right) (q^2 - M_{\xi_2}^2) \left[ - \int \frac{d^4q}{(2\pi)^4} \left( q^2 - M_{\phi_k}^2 \right) (q^2 - M_{\xi_1}^2)(q^2 - M_{\xi_2}^2) \right],$$  \hspace{1cm} (21)
where the Yukawa coupling as defined in Eq. (7). Each integral of the above two expressions for $\Lambda_{kk}$ can be decomposed as two 2-point Passarino-Veltman functions $[51, 52]$ as provided below:

$$
\Lambda^R_{kk} = \frac{1}{16\pi^2} m_{\chi_R}^2 \cos^2 \theta \sin^2 \theta \left[ \frac{M^2_{\xi_1}}{M^2_{\phi_k} - M^2_{\xi_1}} \left\{ B(0, M_{\xi_1}, M_{\phi_k}) - B(0, M_{\xi_1}, M_{\xi_1}) \right\} 
+ \frac{M^2_{\xi_2}}{M^2_{\phi_k} - M^2_{\xi_2}} \left\{ B(0, M_{\xi_2}, M_{\phi_k}) - B(0, M_{\xi_2}, M_{\xi_2}) \right\} 
- \frac{2M_{\xi_1}M_{\xi_2}}{M^2_{\xi_2} - M^2_{\xi_1}} \left\{ B(0, M_{\xi_2}, M_{\phi_k}) - B(0, M_{\xi_1}, M_{\phi_k}) \right\} \right],
$$

(23)

$$
\Lambda^L_{kk} = \frac{1}{16\pi^2} m_{\chi_L}^2 \cos^2 \theta \sin^2 \theta \left[ B(0, M_{\xi_1}, M_{\phi_k}) + \frac{M^2_{\xi_1}}{M^2_{\phi_k} - M^2_{\xi_1}} \left\{ B(0, M_{\xi_1}, M_{\phi_k}) - B(0, M_{\xi_1}, M_{\xi_1}) \right\} 
+ \frac{M^2_{\xi_2}}{M^2_{\phi_k} - M^2_{\xi_2}} \left\{ B(0, M_{\xi_2}, M_{\phi_k}) - B(0, M_{\xi_2}, M_{\xi_2}) \right\} 
- \frac{2M_{\xi_1}M_{\xi_2}}{M^2_{\xi_2} - M^2_{\xi_1}} \left\{ B(0, M_{\xi_2}, M_{\phi_k}) - B(0, M_{\xi_1}, M_{\phi_k}) \right\} \right],
$$

(24)

where $B(p, m_1, m_2)$ is defined as $[53]$,

$$
B(p, m_1, m_2) = \int_0^1 dx \left[ \frac{2}{\bar{\epsilon}} + \log \left( \frac{\mu^2}{m_1^2 x + m_2^2 (1 - x) - p^2 x (1 - x)} \right) \right],
$$

(25)

with $\bar{\epsilon} = \frac{2}{\bar{\epsilon}} - \gamma_E + \log(4\pi)$, $\epsilon = n - 4$ and $\gamma_E$ is the Euler-Mascheroni constant.

The mass scale $\Lambda_{kk}$ is a function of DM mass, mixing angle $\theta$ and the masses of the scalar fields. The pseudo Dirac DM phenomenology restricts $\sin \theta$ for a particular DM mass in order to satisfy both relic and direct detection bound. Using that information one can estimate $\Lambda_{kk}$ for both higher and lower values of $\sin \theta$ for a particular DM mass. We use QCDloop $[52]$ to evaluate $\Lambda_{kk}$ numerically and which is found to be consistent with the analytical estimation of $\Lambda_{kk}$.
FIG. 5. (Upper plots) demonstrate the contours for $\Lambda_{11}$ for different values of $\Delta M$ in $\sin\theta - M_{\xi_1}$ plane. Similarly, (lower plots) demonstrate Contours for $\Lambda_{22}$.

In Fig. 5 (upper plots), we present the contours for $\Lambda_{11} = 10^5$ eV (left panel), $\Lambda_{11} = 10^{5.5}$ eV (right panel) considering several values of $\Delta M$ in the $\sin\theta - M_{\xi_1}$ plane. For this purpose, we fix $m_{\chi_{L,R}} = 1$ GeV and $M_{\phi_1}$ at $1.2 \times 10^3$ GeV. It is evident from this figure that, for a necessity of higher values of $\Lambda_{11}$ one has to go for larger $\sin\theta$ values. In Fig. 5 (lower plots), we present the contours for $\Lambda_{22} = 10^6$ eV (left panel), $\Lambda_{22} = 10^{6.5}$ eV (right panel) considering the set of earlier values of $\Delta M$ in the $\sin\theta - M_{\xi_1}$ plane. Here also we take $m_{\chi_{L,R}} = 1$ GeV and fix $M_{\phi_2}$ at $10^4$ GeV. One can draw a similar conclusion on the
| SL no. | $M_{\xi_1}$ (GeV) | $\Delta M$ (GeV) | $\sin \theta$ | $\Omega h^2$ | Log$_{10}[\sigma_{SI}/cm^2]$ | $\Lambda_{11}$ (eV) | $\Lambda_{22}$ (eV) | $\Lambda_{33}$ (eV) |
|--------|------------------|-----------------|-------------|-----------|-----------------|----------------|----------------|----------------|
| I      | 200              | 47              | 0.318       | 0.117     | -46.36          | 1.44 x 10$^7$ | 3.74 x 10$^7$ | 6.27 x 10$^7$ |
| II     | 800              | 103             | 0.073       | 0.121     | -48.24          | 2.2 x 10$^5$  | 2.49 x 10$^6$ | 5.30 x 10$^6$ |

TABLE II. Two sets of relic and direct search satisfied points and corresponding values of $\Lambda$ considering $m_{\chi_{L,R}} \sim 1$ GeV, scalar field masses, $M_{\phi_i} \sim \{1.2 \times 10^3, 10^4, 10^5\}$ (GeV) and the lightest active neutrino mass $m_{\nu}^{\text{lightest}} \sim 0.01$ eV. The points are also tested to satisfy Br($\mu \rightarrow e\gamma$) bound.

It is to note that, in order to make the three SM neutrinos massive one needs to take the presence of three scalars, although it is sufficient to have two scalars only for a scenario where one of the active neutrinos remains massless. In the presence of a third copy of the scalar, we would have evaluated the corresponding $\Lambda$ in a similar manner.

Once we construct the light neutrino mass matrix with the help of different $\Lambda_{ij}$s we can study the properties associated with neutrino mass. The obtained low energy neutrino mass matrix $m_{\nu_{ij}}$ thus constructed is diagonalized by the unitary matrix $U_{\nu}(U)$.

$$m_{\nu}^{\text{diag}} = U^T m_{\nu} U,$$  \hspace{1cm} (26)

We consider the charged lepton matrix to be diagonal in this model. In that case, we can identify $U$ as the standard $U_{\text{PMNS}}$ matrix [54] for lepton mixing.

To start with Eq. (20), one can get the light neutrino mass in terms of the Yukawa couplings $h_{ij}$ and the mass scale $\Lambda_{kk}$. The $h_{ij}$ which is present in Eq. (20) can be connected to the oscillation parameters with the help of Casas-Ibarra parameterization [55], which allows us to use a random complex orthogonal rotation matrix $R$. Using this parameterization, we can express the Yukawa coupling by the following equation [55].

$$h^T = D_{\sqrt{\Lambda_{-}}\tau} R D_{\sqrt{m_{\nu}^{\text{diag}}}} U^\dagger,$$  \hspace{1cm} (27)

where, $D_{\sqrt{m_{\nu}^{\text{diag}}}} = \text{Diag}(\sqrt{m_{\nu_1}}, \sqrt{m_{\nu_2}}, \sqrt{m_{\nu_3}})$, $D_{\sqrt{\Lambda_{-}}} = \text{Diag}(\sqrt{\Lambda_{11}}, \sqrt{\Lambda_{22}}, \sqrt{\Lambda_{33}})$.

The $R$ can be parameterised through three arbitrary mixing angles which we choose to be ($\frac{\pi}{4}, \frac{\pi}{5},$ and $\frac{\pi}{6}$). Now to have a numerical estimate of the Yukawa couplings $h_{ij}$, as stated earlier we consider $m_{\chi_{L,R}}$ at 1 GeV and scalar field masses at $\{1.2 \times 10^3, 10^4, 10^5\}$ GeV.
and make use of two sets of relic density and direct search satisfied points as tabulated in Table II. At the same time, we use best fit central values of the oscillation parameters to construct the $U_{\text{PMNS}}$ matrix and choose the normal hierarchy mass pattern [56] with the lightest active neutrino mass eigenvalue as 0.01 eV. In Table III we represent the Yukawa coupling matrices ($h$) using the above sets of benchmark points. So far, the analysis of neutrino part has been carried out by keeping $m_\chi$ fixed at 1 GeV. One can go for an even higher value of $m_\chi$ (also $\Lambda$) values, however, in such a scenario the order of the elements of the $h$ matrix will be reduced further as evident from Eq. (20).

It is expected that constraint on the model parameter, specifically $h_{ij}$ may arise from the lepton flavor–violating (LFV) decays of $\phi$ fields. The most stringent limit comes from the $\mu \to e\gamma$ decay process [57–59]. However, the Yukawa couplings being very small $\sim \mathcal{O}(10^{-5})$ as tabulated in Table III easily overcome the present experimental bound [60]. The pseudo-Dirac nature of dark matter is testable at colliders through displaced vertices [48]. A details study is required whether a relaxed $\sin \theta$ has some role to play in this regard. Constraints on the model parameter are under consideration [42].

V. CONCLUSION

In this work, we study a simple extension of the standard model, including a singlet doublet dark sector in the presence of a small Majorana mass term. As a consequence generated eigenstates deviate from Dirac nature, owing to a small mass splitting between pair of two
pseudo-Dirac states. Lightest of these pseudo-Dirac fermionic states, considered as dark matter, can evade the strong spin-independent direct detection constrain by suppressing the scattering of dark matter with nucleon through the Z-boson mediation. We explicitly demonstrate this significant weakening of the direct detection constraint on the singlet doublet mixing parameter while ensuring that such dark matter is still capable of satisfying the thermal relic fully.

The same Majorana mass term provides an elegant scope to generate neutrino mass radiatively at one loop, which requires an extension of the dark sector model with copies of real scalar singlet fields. Introduction of these additional scalars is also motivated by stabilizing the electroweak vacuum even in the presence of a large mixing angle. They also provide a source of lepton number violation, generating light Majorana neutrinos satisfying oscillation data fully. Hence this present scenario offers the potential existence of a pseudo-Dirac type dark matter in the same frame with light Majorana neutrinos. We obtain two different bounds on the left and right component of the newly introduced Majorana mass parameter, i.e. \((m_{\chi_L} + m_{\chi_R}) \gtrsim \mathcal{O}(1) \text{ GeV}\) and \((m_{\chi_L} - m_{\chi_R}) \lesssim \mathcal{O}(1) \text{ MeV}\), accounting for the correct order of active neutrino masses and oscillation data. We further demonstrate the dependence of these model parameters and reference benchmark points satisfying best fit central values of the oscillation parameters and consistent with the pseudo-Dirac dark matter constraints.

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\begin{footnotesize}
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