Strong Gravitational Lensing Parameter Estimation with Vision Transformer

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Abstract. Quantifying the parameters and corresponding uncertainties of hundreds of strongly lensed quasar systems holds the key to resolving one of the most important scientific questions: the Hubble constant ($H_0$) tension. The commonly used Markov chain Monte Carlo (MCMC) method has been too time-consuming to achieve this goal, yet recent work has shown that convolution neural networks (CNNs) can be an alternative with seven orders of magnitude improvement in speed. With 31,200 simulated strongly lensed quasar images, we explore the usage of Vision Transformer (ViT) for simulated strong gravitational lensing for the first time. We show that ViT could reach competitive results compared with CNNs, and is specifically good at some lensing parameters, including the most important mass-related parameters such as the center of lens $\theta_1$ and $\theta_2$, the ellipticities $e_1$ and $e_2$, and the radial power-law slope $\gamma'$. With this promising preliminary result, we believe the ViT (or attention-based) network architecture can be an important tool for strong lensing science for the next generation of surveys. The open source of our code and data is in https://github.com/kuanweih/strong_lensing_vit_resnet.

1 Introduction

The discovery of the accelerated expansion of the Universe [1,2] and observations of the Cosmic Microwave Background (CMB; e.g., [3]) established the standard cosmological paradigm: the so-called $\Lambda$ cold dark matter (CDM) model, where $\Lambda$ represents a constant dark energy density. Intriguingly the recent direct 1.7% $H_0$ measurements from Type Ia supernovae (SNe), calibrated by the traditional Cepheid distance ladder ($H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$; SH0ES collaboration [4]), show a 4.2σ tension with the Planck results ($H_0 = 67.4 \pm 0.5$ km s$^{-1}$ Mpc$^{-1}$ [5]). However, a recent measurement of $H_0$ from SNe Ia calibrated by the Tip of the Red Giant Branch ($H_0 = 69.8 \pm 0.8$(stat) $\pm 1.7$(sys) km s$^{-1}$ Mpc$^{-1}$; CCHP
collaboration [6]) agrees with both the Planck and SH0ES results. The spread in these results, whether due to systematic effects or not, clearly demonstrates that it is crucial to reveal unknown systematics through different methodology.

Strongly lensed quasar system provides such a technique to constrain \( H_0 \) at low redshift that is completely independent of the traditional distance ladder approach (e.g., [7,8,9]). When a quasar is strongly lensed by a foreground galaxy, its multiple images have light curves that are offset by a well-defined time delay, which depends on the mass profile of the lens and cosmological distances to the galaxy and the quasar [10]. However, the bottleneck of using strongly lensed quasar systems is the expensive cost of computational resources and man power. With commonly used Markov chain Monte Carlo (MCMC) procedure, modeling single strongly lensed quasar system requires experienced modelers with a few months effort in order to obtain robust uncertainty estimations and up to years to check the systematics (e.g., [11,12,13,14,15,16]). This is infeasible as \( \sim 2600 \) of such systems with well-measured time delays are expected to be discovered in the upcoming survey with the Large Synoptic Survey Telescope [17,18].

Fig. 1. Left panel: simulated strong lensing imaging with real point spread functions (top two: space-based telescope images; bottom: ground-based adaptive-optics images). Each image contains the lensing galaxy in the middle, the multiple-lensed quasar images, and the lensed background host galaxies (arc). Right panel: Vision Transformer attention map: the overall average attentions are focusing on the strong lens system. Each individual head is paying attention to different subjects such as attention heads #2 are focusing the center of lens, heads #1 and #3 are looking into particular lensed quasars, and heads #4 are dealing with the arc.
Deep learning provides a workaround for the time-consuming lens modeling task by directly mapping the underlying relationships between the input lensing images and the corresponding lensing parameters and their uncertainties. Hezaveh et al. [19] and Perreault Levasseur et al. [20] first demonstrated that convolution neural networks (CNNs) can be an alternative to the maximum likelihood procedures with seven orders of magnitude improvement in speed.

Since then, other works adopt CNN for strong lensing science related inference [21,22,23,24,25,26,27,28,29,30].

In this work, instead of using traditional CNN-based models, we explore the attention-based Vision Transformer (ViT, [31,32]) that has been shown to be more robust compared with CNN-based models [33]. Furthermore, ViT retains more spatial information than ResNet [34] and hence is perfectly suitable for the strong lensing imaging as the quasar configuration and the spatially extended background lensed galaxy provide rich information on the foreground mass distribution (see Figure 1).

2 Data and Models

In Section 2.1, we describe the strong lensing simulation for generating the datasets in this work. In Section 2.2, we describe the deep learning models we use to train on the simulated dataset for strong lensing parameters and uncertainty estimations.

2.1 Simulation and Datasets

Simulating strong lensing imaging requires four major components: the mass distribution of the lensing galaxy, the source light distribution, the lens light distribution, and the point spread function (PSF), which convolves images depending on the atmosphere distortion and telescope structures. We use the lenstronomy package [35,36] to generate 31,200 strong lensing images with the corresponding lensing parameters for our imaging multi-regression task. For the mass distribution, we adapt commonly used (e.g., [37,15]) elliptically symmetric power-law distributions [38] to model the dimensionless surface mass density of lens galaxies,

\[
\kappa_{pl}(\theta_1, \theta_2) = \frac{3 - \gamma'}{1 + q} \left( \frac{\theta_E}{\sqrt{\theta_1^2 + \theta_2^2/q^2}} \right)^{\gamma' - 1},
\]

where \(\gamma'\) is the radial power-law slope (\(\gamma' = 2\) corresponding to isothermal), \(\theta_E\) is the Einstein radius, and \(q\) is the axis ratio of the elliptical isodensity contour. The light distribution of the lens galaxy and source galaxy are described by elliptical S{\'e}rsic profile,

\[
I_S(\theta_1, \theta_2) = I_s \exp \left[ -k \left( \frac{\sqrt{\theta_1^2 + \theta_2^2/q^2}}{R_{\text{eff}}} \right)^{1/n_{\text{sersic}}} - 1 \right],
\]
where $I_s$ is the amplitude, $k$ is a constant such that $R_{\text{eff}}$ is the effective radius, $q_L$ is the minor-to-major axis ratio, and $n_{\text{Sérsic}}$ is the Sérsic index [39]. For the PSFs, we use six different PSF structures including three real Hubble space telescope PSFs generated by Tinytim [40] and corrected by the real HST imaging [15], and three adaptive-optics (AO) PSFs reconstructed from ground-based Keck AO imaging [41,42,43]. Three example images are shown in Figure 1.

We split the whole simulated dataset of 31,200 images into a training set of 27,000 images, a validation set of 3,000 images, and a test set of 1,200 images. We rescale each image as $3 \times 224 \times 224$ and normalize pixel values in each color channel by the mean $[0.485, 0.456, 0.406]$ and the standard deviation $[0.229, 0.224, 0.225]$ of the datasets. Each image has eight target variables to be predicted in this task: the Einstein radius $\theta_E$, the ellipticities $e_1$ and $e_2$, the radial power-law slope $\gamma'$, the coordinates of mass center $\theta_1$ and $\theta_2$, the effective radius $R_{\text{eff}}$, and the Sérsic index $n_{\text{Sérsic}}$.

### 2.2 Models

We use the Vision Transformer (ViT) as the main model for our image multi-regression task of strong lensing parameter estimations. Inspired by the original Transformer models [31] for natural language processing tasks, Google Research proposed the ViT models [32] for computer vision tasks. In this paper, we leverage the base-sized ViT model (ViT-Base), which was pre-trained on the ImageNet-21k dataset and fine-tuned on the ImageNet 2012 dataset [44].

Taking advantage of the transfer learning concept, we start with the pre-trained ViT-Base model downloaded from the module of HUGGINGFACE’s TRANSFORMERS [45], and replace the last layer with a fully connected layer whose number of outputs matches the number of target variables in our regression tasks. The ViT model we use thus has 85,814,036 trainable parameters, patch size of 16, depth of 12, and 12 attention heads.

Alongside the ViT model, we also train a ResNet152 model [46] for the same task as a comparison between ViT and the classic benchmark CNN-based model. We leverage the pre-trained ResNet152 model from the TORCHVISION package [47] and modify the last layer accordingly for our multi-regression purpose.

For regression tasks, the log-likelihood can be written as a Gaussian log-likelihood [48]. Thus for our task of $K$ targets, we use the negative log likelihood as the loss function [20]:

$$\text{Loss}_n = -\mathcal{L} (y_n, \hat{y}_n, \hat{s}_n)$$

$$= \frac{1}{2} \left( \sum_{k=1}^{K} e^{-\hat{s}_{n,k}} \| y_{n,k} - \hat{y}_{n,k} \|^2 + \hat{s}_{n,k} + \ln 2\pi \right)$$

(3)

where $(y_n, \hat{y}_n, \hat{s}_n)$ are the (target, parameter estimation, uncertainty estimation) for the $n$th sample, and $(y_{n,k}, \hat{y}_{n,k}, \hat{s}_{n,k})$ are the (target, parameter estimation, uncertainty estimation) for the $n$-th sample of the $k$-th target. We note that in practice, working with the log-variance $\hat{s}_n = \ln \hat{\sigma}_n^2$ instead of the variance $\hat{\sigma}_n^2$.
improves numerical stability and avoids potential division by zero during the training process [49]. Choosing this loss function instead of the commonly used mean squared error results in the uncertainty prediction as well as the parameter prediction, which provides more statistical information than point-estimation-only predictions.

It is worth noting that we apply dropout before every hidden layers for both models with dropout rate of 0.1 to approximate Bayesian networks for the uncertainty estimate, but not for the attention layers in the ViT model. This is to include the "epistemic" uncertainties in neural networks by leaving dropout on when making predictions, together with the "aleatoric" uncertainties described by $\hat{\sigma}_n^2$ to account for intrinsic noise from the data. We refer readers to [48,20] for detailed discussion and derivation of the uncertainties.

Using the training set of 27,000 images, we train our ViT-Base and ResNet152 models with the loss function in Equation (3), the Adam optimizer [50] with 0.001 for the initial learning rate, the batch size of 20. Based on the validation set of 3,000 images, we evaluate the model predictions by the mean squared error across all 8 target variables to determine the best models. We then report the performance of the best ViT and ResNet models according to the test set of 1,200 images in Section 3.

3 Results

In this section, we present the performance of the best ViT and ResNet models on the test set of 1,200 images regards of our image multi-regression task of the strong lensing parameter and uncertainty estimation. Following the procedure in [20], for each model, we execute the prediction on the test set for 1000 times with dropout on to catch the epistemic uncertainty of the model. For each parameter prediction $\hat{y}_n$ and uncertainty prediction $\hat{\sigma}_n$ amongst the 1000 predictions, we draw a random number from a Gaussian distribution $N(\hat{y}_n, \hat{\sigma}_n)$ as the prediction of the parameter. Therefore for each test sample, we have 1000 predicted parameters so that we take the mean and standard deviation as the final parameter and uncertainty predictions respectively.

The overall root mean square errors (RMSEs) for the lensing parameter estimation are 0.1232 for our best ViT-Base model and 0.1476 for our best ResNet152 model. The individual RMSEs for each target variable are summarized in Table 1. Our models indicate that except for the Einstein radius $\theta_E$, the attention-based model ViT-Base outperforms the CNN-based model ResNet152 for all the other parameters in this image multi-regression task. Despite a higher RMSE of the Einstein radius for our best ViT than that of our best ResNet, it still reaches the benchmark precision of about 0.03 arcsec [19].

Using the prediction of the mean values and the corresponding uncertainties on the strong lensing parameters, we randomly select 50 test samples to illustrate the predictions from our ViT and ResNet models in Figure 2. Overall, the ViT model outperforms the ResNet model for $\theta_1$, $\theta_2$, $e_1$, $e_2$, and $R_{\text{eff}}$ while maintaining a competitive performance for the other parameters. We note that
Table 1. Comparison of RMSE of the parameter predictions between ViT and ResNet.

| Target       | ViT   | ResNet |
|--------------|-------|--------|
| Overall      | 0.1232| 0.1476 |
| $\theta_E$ [arcsec] | 0.0302 | 0.0221 |
| $\gamma'$    | 0.0789 | 0.0816 |
| $\theta_1$ [arcsec] | 0.0033 | 0.0165 |
| $\theta_2$ [arcsec] | 0.0036 | 0.0169 |
| $c_1$        | 0.0278 | 0.0364 |
| $c_2$        | 0.0206 | 0.0347 |
| $R_{\text{eff}}$ [arcsec] | 0.0241 | 0.0487 |
| $n_{\text{nsr}}$ | 0.0790 | 0.0959 |

Fig. 2. Comparison of predicted parameters and 1-$\sigma$ uncertainties to the ground truths for all eight targets between our best Vision Transformer (ViT) model and our best ResNet model for 50 random chosen test samples. For each target, the ground truth and the model prediction are shown on the x-axis and y-axis respectively. The ViT model outperforms the ResNet model for $\theta_1$, $\theta_2$, $c_1$, $c_2$, and $R_{\text{eff}}$ while maintaining a competitive performance for the other parameters.
Fig. 3. Comparison of predicted parameters to ground truths for all eight targets between our best Vision Transformer (ViT) model and our best ResNet model. Each panel shows the histogram of the percentage errors between the prediction means and the ground truths of the 1200 test samples for each target. The ViT model outperforms the ResNet model for $\theta_1$, $\theta_2$, $e_1$, $e_2$, and $R_{\text{eff}}$ while maintaining a competitive performance for the other parameters.

both models cannot well capture the features of $n_{\text{e}}$. In Figure 3, we show the percentage error between the predictions and the ground truths for all 1200 test samples, supporting the statements above.

4 Conclusion

Strongly lensed quasar systems provide unique tools to resolve the recent 4-$\sigma$ tension between the direct measurements of the $H_0$ and the prediction from the standard cosmological model ($\Lambda$CDM model) [51,4,6,7,42]. One of the key requirements is the mass parameter estimations of hundreds of strong lensing systems in order to achieve statistically significant results [52]. While this challenge cannot be achieved by the traditional and time-consuming MCMC method, deep neural network models can be a perfect alternative technique to efficiently achieve this goal. For example, Hezaveh et al. [19] and Perreault Levasseur et al. [20] showed that CNN-based models could be used to estimate the values and the corresponding uncertainties of the parameters given the strong lensing images.

In this work, we explored the recent state-of-the-art ViT as it has the advantage of capturing long-range interaction of pixels compared to CNN-based models. As a supervised multi-regression task, we trained ViT-Base as well as ResNet152 for the parameter and uncertainty estimations, using the dataset of 31,200 strong lensing images.
We show that ViT could reach competitive results compared with CNNs, and is specifically good at some lensing parameters, including the most important mass-related parameters such as the center of lens $\theta_1$ and $\theta_2$, the ellipticities $e_1$ and $e_2$, and the radial power-law slope $\gamma'$. With this promising preliminary result, we believe the ViT (or attention-based) network architecture can be an important tool for strong lensing science for the next generation of surveys.

Note that the mass distribution of real lensing galaxies are much more complicated than the simple power-law model and hence can potentially affect the $H_0$ measurement [53,54,55,56,57,58,59,60,61,62,63]. This effect has been illustrated with cosmological hydrodynamic simulations [56,64]. In the future work, we plan to train the neural network to directly learn from realistic hydrodynamic simulations without the need of mass profile assumptions. This cannot be achieved by traditional MCMC method, while neural network is the only way to directly test this long-standing debate about the possible systematics regarding the degeneracy between lensing $H_0$ results and the mass profile assumptions. We plan to open-source our code and datasets.

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