Embedding QM into an objective framework

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Abstract

An elementary model is given which shows how an objective (hence local and noncontextual) picture of the microworld can be constructed without conflicting with quantum mechanics (QM). This contradicts known no-go theorems, which however do not hold in the model, and supplies some suggestions for a broader theory in which QM can be embedded.

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According to the standard (Copenhagen) interpretation, quantum mechanics (QM) is nonobjective, which can be briefly expressed by saying that “a measurement does not, in general, reveal a preexisting value of the measured property”\(^{(1)}\). Though accepted by generations of physicists, nonobjectivity implies a number of nonintuitive consequences and puzzling paradoxes, which spread out from QM to all theories based on it. However, it is strongly supported by a number of arguments, among which Bell-Kochen-Specker’s\(^{(2),(3)}\) and Bell’s\(^{(4),(5)}\). These are usually seen as no-go theorems which show that a noncontextual and local (hence objective) picture of the microworld cannot be consistent with QM, so that one must come to terms with the paradoxes following from nonobjectivity.

The elementary set-theoretical model provided here aims to show that the above conclusion can be circumvented without altering the formalism and the minimal (statistical) interpretation of QM. The resulting objective picture, despite its simplicity, has some deep theoretical implications, that will be briefly expounded and commented on at the end.
To begin with, let us accept the standard notion of state of a physical system $S$ as a class of physically equivalent preparing devices.\(^{(6)}\) Furthermore, let us call physical object any individual sample $x$ of $S$ obtained by activating a preparing device, and say that $x$ is in the state $S$ (at time $t$) if the device $\pi$ preparing $x$ (at $t$) belongs to $S$. Whenever $S$ is a microscopic physical system, let us introduce a set $E$ of microscopic physical properties $f$, $g$, ..., characterizing $S$, which play the role of theoretical entities. For every physical object $x$, every property $f \in E$ is associated with $x$ in a dichotomic way, so that one briefly says that every $f \in E$ either is possessed or it is not possessed by $x$. The set $F_0$ of all macroscopic properties is then introduced as in standard QM, that is, it is defined as the set of all pairs of the form $(A_0, \Delta)$, where $A_0$ is an observable (that is, a class of physically equivalent measuring apparatuses) with spectrum $\Lambda_0$, and $\Delta$ a Borel set on the real line (for every observable $A_0$, different sets containing the same subset of $\Lambda_0$ obviously define physically equivalent properties). Yet, every observable $A_0$ is obtained from a suitable observable $A$ of standard QM by adding to the spectrum $\Lambda$ of $A$ a further outcome $a_0$ that does not belong to $\Lambda$, called the no-registration outcome of $A_0$ (note that such an outcome can be introduced also within the standard quantum measurement theory, but it plays here a different theoretical role), so that $\Lambda_0 = \Lambda \cup \{a_0\}$. The set $E$ of all microscopic properties is then assumed to be in one-to-one correspondence with the subset $F \subseteq F_0$ of all macroscopic properties of the form $F = (A_0, \Delta)$, where $A_0$ is an observable and $a_0 \notin \Delta$.

Basing on the above definitions and assumptions, one can provide the following description of the measurement process. Whenever a physical object $x$ is prepared by a given device $\pi$ in a state $S$, and $A_0$ is measured by means of a suitable apparatus, the set of microscopic properties possessed by $x$ produces a probability (which is either 0 or 1 if the model is deterministic) that the apparatus does not react, so that the outcome $a_0$ may be obtained. In this case, $x$ is not detected and one cannot get any explicit information about the microscopic physical properties possessed by $x$. If, on the contrary, the apparatus reacts, an outcome different from $a_0$, say $a$, is obtained, and one is informed that $x$ possesses all microscopic properties associated with macroscopic properties of the form $F = (A_0, \Delta)$, where $\Delta$ is a Borel set such that $a_0 \notin \Delta$ and $a \in \Delta$ (for the sake of brevity we also say that $x$ possesses all macroscopic properties as $F$ in this case).

In order to place properly quantum probability within the above intuitive picture, let us consider a preparing device $\pi \in S$ that is activated repeatedly.
In this case a (finite) set $S$ of physical objects in the state $S$ is prepared. Then, let us partition $S$ into subsets $S^{(1)}$, $S^{(2)}$, ..., $S^{(n)}$, such that in each subset all objects possess the same microscopic properties, and assume that a measurement of an observable $A_0$ is done on every object. Furthermore, let us introduce the following symbols.

- $N$: number of physical objects in $S$.
- $N_0$: number of physical objects in $S$ that are not detected.
- $N^{(i)}$: number of physical objects in $S^{(i)}$.
- $N_0^{(i)}$: number of physical objects in $S^{(i)}$ that are not detected.
- $N_F^{(i)}$: number of physical objects in $S^{(i)}$ that possess the macroscopic property $F = (A_0, \Delta)$ corresponding to the microscopic property $f$.

It is apparent that the number $N_F^{(i)}$ either coincides with $N^{(i)} - N_0^{(i)}$ or with 0. The former case occurs whenever $f$ is possessed by the objects in $S^{(i)}$, since all objects that are detected then yield outcome in $\Delta$. The latter case occurs whenever $f$ is not possessed by the objects in $S^{(i)}$, since all objects that are detected then yield outcome different from $A_0$ but outside $\Delta$. In both cases one generally gets $N^{(i)} - N_0^{(i)} \neq 0$ (even if $N^{(i)} - N_0^{(i)} = 0$ may also occur, in particular in a deterministic model), so that the following equation holds:

$$
\frac{N_F^{(i)}}{N^{(i)}} = \frac{N^{(i)} - N_0^{(i)}}{N^{(i)}} \frac{N_F^{(i)}}{N^{(i)} - N_0^{(i)}} .
$$  \hspace{1cm} (1)

The term on the left in Eq. (1) represents the frequency of objects possessing the property $F$ in $S^{(i)}$, the first term on the right the frequency of objects in $S^{(i)}$ that are detected, the second term (which either is 1 or 0) the frequency of objects that possess the property $F$ in the subset of all objects in $S^{(i)}$ that are detected.

The frequency of objects in $S$ that possess the property $F$ is given by

$$
\frac{1}{N} \sum_i N_F^{(i)} = \frac{N - N_0}{N} \left( \sum_i \frac{N_F^{(i)}}{N - N_0} \right) .
$$  \hspace{1cm} (2)

Let us assume now that all frequencies converge in the large number limit, so that they can be substituted by probabilities, and that these probabilities do not depend on the choice of the preparing device $\pi$ in $S$. Hence, if one considers the large number limit of Eq. (1), one gets

$$
\mathcal{P}_S^{(i)}(F) = \mathcal{P}_S^{(i)\pi}(F) \mathcal{P}_S^{(i)}(F) ,
$$  \hspace{1cm} (3)
where $P^{(i)t}(F)$ is the overall probability that a physical object $x$ possessing the microscopic properties that characterize $S^{(i)}$ also possess the property $F$, $P^{(i)d}(F)$ is the probability that $x$ be detected when $F$ is measured on it, $P^{(i)}_{S}(F)$ (which either is 0 or 1) is the probability that $x$ possess the property $F$ when detected. Moreover, the large number limit of Eq. (2) yields

$$P^{t}_{S}(F) = P^{d}_{S}(F)P_{S}(F) ,$$

where $P^{t}_{S}(F)$ is the overall probability that a physical object $x$ in a state $S$ possess the property $F$, $P^{d}_{S}(F)$ is the probability that $x$ be detected when $F$ is measured on it, $P_{S}(F)$ is the probability that $x$ possess the property $F$ when detected. It is then reasonable to identify $P_{S}(F)$ with the quantum probability that a physical object in the state $S$ possess the property $F$, so that $P_{S}(F)$ can be evaluated by following the rules of standard QM, hence in particular representing any state $S$ by means of a trace class operator on a Hilbert space $H$ associated with $S$ and any macroscopic property that corresponds to a microscopic property by means of a projection operator on $H$. Thus, one need not modifying the formalism and the statistical interpretation of standard QM.

As anticipated at the beginning, however, the set-theoretical model illustrated above provides an objective (hence local and noncontextual) picture of the microworld which is consistent with QM. Indeed, for every physical object $x$ in the state $S$, every macroscopic property of the form $F = (A_{0}, \Delta)$ (where $a_{0}$ may belong or not to $\Delta$) either is possessed or is not possessed by $x$, and the probability that it is possessed/not possessed is determined by the microscopic properties possessed by $x$, which do not depend on the measuring apparatus (hence microscopic properties play in the model a role similar to states in objective local theories\textsuperscript{(7)}). This violates standard expectations and can be explained as follows.

The Hilbert space formalism of standard QM does not associate any mathematical object with the microscopic properties $f$, $g$, ... Furthermore, projection operators represent only macroscopic properties of the form $(A_{0}, \Delta)$, where $A_{0}$ is an observable and $a_{0} \notin \Delta$, so that the mathematical representation of the entities appearing in the model is only partial. Hence, every QM law stated by means of the standard formalism necessarily relates (possibly probabilistically) only entities that are mathematically represented, that is, states and macroscopic properties of the form specified above. If the law is interpreted in the observative language of the theory, it may undergo a
process of empirical verification, and one can classify it as *empirical*. Yet, because of the above remarks, such a law refers only to objects that are detected; moreover, it can be actually verified only in those physical situations in which the verification procedure does not lead to simultaneous measurements of noncommensurable observables. If these restrictive conditions are satisfied, the relations among macroscopic properties established by the law match analogous relations among the corresponding microscopic properties. If, on the contrary, the restrictive conditions are not satisfied, one can neither assert the validity of the relations predicted by the law among macroscopic properties, nor transfer these relations to microscopic properties. As an example, think of a physical object $x$ on which an observable $A_0$ is measured, obtaining outcome $a_0$. In this case, no macroscopic property of the form $(A_0, \Delta)$, with $a_0 \notin \Delta$, is possessed by $x$, hence no non-trivial relation among properties of this form holds, and no relation among the microscopic properties possessed by $x$ can be inferred from quantum laws (but the model predicts that microscopic properties must be such that the probability of the $a_0$ outcome is not 0).

The above arguments point out that quantum laws must be handled with care within the model. In particular, consider the condition stated by Kochen and Specker (briefly, KS)\(^{(3)}\) as a basic premise for the Bell-KS theorem, that can be reformulated as follows.\(^{(1)}\)

If a set of mutually commuting observables $A$, $B$, $C$, ... satisfies a relation of the form $f(A, B, C, ...) = 0$ then the values $v(A)$, $v(B)$, $v(C)$, ... assigned to them in an individual system must also be related by

$$f(v(A), v(B), v(C), ...) = 0.$$  \hspace{1cm} (5)

In all proofs of the Bell-KS theorem, the law (5) is applied repeatedly, inserting in it different sets of mutually commuting observables, and there are observables belonging to different sets that do not commute. This implies that, if Eq. (5) is checked for a given choice of observables, checking it (on the same objects) for a different choice requires simultaneous measurements of noncommensurable observables. Hence, one cannot assert that all relations among macroscopic properties established by equations of the form (5) are bound to hold simultaneously, nor that they can be translated into relations among the corresponding microscopic properties. This invalidates the premises on which the Bell-KS theorem stands, which explains how the model can circumvent this theorem and provide a noncontextual picture of the microscopic world.
Similar reasonings apply if Bell’s inequalities are considered. These are usually maintained to show that QM is a nonlocal theory (Bell’s theorem). Let us refer, for the sake of simplicity, to the inequalities propounded by Clauser et al. in 1969 (CHSH’s inequalities).\(^5\) Then, the quantum inequalities corresponding to CHSH’s inequalities relate (dichotomic) observables, hence macroscopic properties, and can be checked. However, the check is not trivial, since the inequalities contain noncommeasurable observables, so that they can be checked only “by blocks”, that is, measuring different correlation functions on different sets of physical objects, all in the same state. But this procedure considers in every set only the objects that are actually detected, and the frequencies that are obtained must be interpreted in terms of probabilities of the form \(P_S(F)\) (see Eq. 4), that are related as in QM, not in terms of probabilities of microscopic properties that do not appear in the formalism of QM. On the other side CHSH’s inequalities can be interpreted in the model as relating correlation functions of microscopic properties possessed by the physical objects, hence need not coincide with the corresponding quantum inequalities. This illustrates how the model can circumvent the Bell theorem and provide a local picture of the microscopic world.

The following remarks point out some further features of the model.

(i) From the viewpoint of the model, QM is a theory that is incomplete in several senses (it does not provide the probabilities \(P^t_S(F)\) and \(P^d_S(F)\) in Eq. (4), it does not say anything about the distribution of microscopic properties on physical objects in a given state whenever the objects are not detected, etc.). This agrees with the conclusion of Einstein, Podolski and Rosen in their famous paper\(^8\), which was however discarded by most physicists in favor of the opposite thesis upheld by Bohr. The model thus shows that the EPR perspective was not necessarily inconsistent with QM. If this is accepted, a broader theory embodying QM can be envisaged, according to which the quantum probability \(P_S(F)\) is considered as a conditional rather than an absolute probability (see Eq. (4)). It is then interesting to note that Eq. (4) could also be obtained in the framework of a model which introduces \(P^d_S(F)\) as efficiency of a non-ideal measuring apparatus, as in some existing attempts of rescuing local realism by resorting to the low efficiencies of the apparatuses in the existing experiments that confirm quantum inequalities (see, e.g., Refs. 9 and 10). Yet, in a model of this kind the no-registration outcome occurs because of flaws of the measuring apparatus, hence \(P^d_S(F)\) is 1 if ideal observables are considered. In the model presented here, instead, the no-registration outcome may occur because of the microscopic properties
of the physical object. Hence, $\mathcal{P}_S^d(F)$ may be less than 1 also in the case of an ideal apparatus (indeed every $\pi$ in $S$ prepares objects which do not possess the same microscopic properties, and some objects may possess sets of properties that make the detection of them by any apparatus measuring $F$ possible but not certain, or even impossible).

(ii) The microscopic properties $f, g, \ldots$ are hidden parameters in the model, but are not hidden variables in the standard sense. Indeed, it has been shown above that they are not bound to make Eq. (5) valid in every physical situation, which instead is required as a basic condition in the standard definitions of hidden variables.\(^{(1),(3)}\) This explains why microscopic properties are not contextual, as standard hidden variables must be.

Finally, note that the model presented here can be placed within the broader context of the objective interpretation of QM propounded by the author (see, e.g., Refs. 11-15). However, it is sufficient by itself to open some new interesting possibilities, that are usually ignored (or maintained impossible) in the literature.

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