Symmetry breaking and cosmic acceleration in scalar field models

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Abstract

We propose a new scenario for the onset of acceleration of our Universe based on symmetry breaking in a scalar field dark energy model. In this model, when dark matter density becomes less than a critical value, the effective shape of the potential is changed and, unlike the symmetron and hybrid quintessence models, the quintessence field climbs up along its own potential. This procedure establishes the positivity of the potential required for the Universe to accelerate. In addition, we show that by choosing an appropriate interaction between dark sectors there is the possibility that the scalar field resides in a new vacuum giving rise to a positive cosmological constant which is responsible for a permanent late time acceleration.

1 Introduction

To explain the positive acceleration of the Universe in the present era many models have been introduced. The most simple and natural candidate for dark energy is a cosmological constant which was first introduced by Einstein to oppose gravitational attraction resulting in a static universe. This idea was abandoned after the Hubble discovery but resurfaced again later to describe the present acceleration of the Universe [1]. The cosmological constant compatible with observation is nearly 120 orders of magnitude less than what is obtained by considering vacuum energy in the standard particle physics framework with a cutoff at the planck scale. Hence it is reasonable to consider it as a constant of nature [2]. In the cosmological constant model dark energy density is constant and the equation of state parameter of dark energy is always -1. This situation changes when dynamical dark energies are employed.

A natural and simple model for dynamical dark energy is the scalar field model [3]. In this type of models, the acceleration of the Universe depends on the potential considered for the scalar field. This is similar to scalar field models of the early universe when inflation occurs during slow roll for nearly flat potentials or during rapid oscillations for specific power law potentials [4]. Note that, due to the presence of dark and ordinary matter at late times the problem is not as straightforward as in the inflationary epoch [5]. An important specification of scalar field models is their usability in the phenomenon of symmetry breaking in particle physics, where the initial symmetric state becomes unstable and the system rolls down to a new vacuum, breaking the initial symmetry [6]. Through this formalism and in the context of dark energy models, when the system leaves its false vacuum and settles down to a true vacuum, the cosmological constant is reduced. This formalism with fine tuning

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of the initial conditions can be considered as a bridge connecting the large value of the cosmological constant in the early universe to its small value at late times \cite{2}.

In recent years some attempts have been made to relate the onset of the acceleration of the Universe to symmetry breaking in scalar field models. Inspired by \cite{7} where the symmetry breaking were used to end the slow roll inflation, a hybrid quintessence model was introduced in \cite{8}. In this model beside the dark energy scalar field, another scalar field is present whose evolution causes the symmetry to brake. In this framework, the Universe may experience an acceleration while the scalar field is rolling down the total potential and also when it settles to the new minimum, provided that an appropriate potential is chosen. The same model was used in \cite{9} to describe the phantom divide line crossing. In \cite{10}, due to the special form of the coupling of matter and scalar field dark energy, an effective potential has been obtained such that when the matter energy density becomes less than a critical value (determined by the parameters of the model) the symmetry is broken; the scalar fields rolls down towards the minimum of the effective potential and acceleration begins. In this model, dubbed symmetron, the matter density plays the role of the trigger field in the hybrid model. The same model has also been used to study the onset of inflation \cite{11}. In these models, the fields rolls down along its own potential as well as the effective potential at the same time. So if initially (before the symmetry breaking) the field resides at the extremum of the unbroken effective potential, the symmetry breaking which decreases the value of the potential and increases the kinetic energy, not only is not in favor of the positive acceleration but is against it. This issue will be explained more in the second section. The acceleration obtained via numerical methods in these models arises from a positive constant term which was initially included (implicitly or explicitly) in the potential and has nothing to do with the symmetry breaking. Indeed in these types of models the symmetry breaking may be rather used to end the acceleration or the inflation \cite{7}.

In this paper we introduce a new proposal for the onset of acceleration based on symmetry breaking in a scalar field dark energy model in a spatially flat Friedman Robertson Walker space time. In our model, after the effective potential shape is changed, contrary to symmetron and hybrid models, the quintessence climbs over its own potential. This procedure may establish the positivity of the potential which is necessary for the acceleration of the Universe.

The organization of the paper is as follows: In the second section we briefly review two models which claim to have associated acceleration to symmetry breaking; the hybrid and symmetron models. In section three we introduce a new proposal for late time acceleration based on symmetry breaking where the scalar field climbs over its own potential while descending down along the effective potential after the potential shape is changed into the form dictated by symmetry breaking. To do so, we need to consider an appropriate interaction between the dark sectors which is not linear in terms of dark matter energy density. In this context, two frameworks are presented, one in which the form of the interaction between dark sectors is borrowed from the scalar-tensor theories but employs two dark energy scalar fields giving rise to a transient acceleration and the other where we try to obtain a permanent acceleration arisen from symmetry breaking by employing appropriate interactions (not derived from an action) between dark sectors of the Universe. In this case the Universe tends to a de Sitter space time with positive acceleration. We also examine the stability of the model and illustrate our results via numerical depictions.

In our study, by “symmetry breaking,” we generally mean the procedure where the potential gets the shape of a “symmetry breaking potential” even though the field has not yet been settled at the new minimum.

We use the units $\hbar = c = 1$. 

2
A brief review and analysis of models ascribing acceleration to symmetry breaking

In a spatially flat Friedman Robertson Walker space-time, filled (nearly) with dark matter \( \rho \) and dark energy scalar fields \( \phi_i \) with potentials \( V(\phi_i) \), the Friedmann equations are

\[
H^2 = \frac{1}{3M_P^2} \left( \frac{1}{2} \sum_i \dot{\phi}_i^2 + V(\phi_i) + \rho \right),
\]

\[
\dot{H} = -\frac{1}{2M_P^2} \left( \sum_i \phi_i^2 + \rho \right),
\]

where in terms of the scale factor \( a(t) \), the Hubble parameter is given by \( H = \frac{\dot{a}(t)}{a(t)} \). A dot indicates time derivative and \( M_P = 2.4 \times 10^{18}\text{GeV} \) is the reduced Planck mass. The (positive) acceleration of the Universe is specified by \( \ddot{a} > 0 \) which, by using \( \dot{H} + H^2 = \frac{\ddot{a}}{a^2} \), leads to

\[
\dot{H} + H^2 = \frac{1}{6M_P^2} \left( -2 \sum_i \dot{\phi}_i^2 + 2V(\phi_i) - \rho \right) > 0.
\]

This can be rewritten in terms of the deceleration parameter \( q \) as

\[
q = -\frac{\dot{H} + H^2}{H^2} = \frac{1}{2} (1 + 3\omega) < 0,
\]

where \( \omega \) is the equation of state parameter of the Universe. From \( [8] \), it is clear that to have a positive acceleration we need a positive potential which drives acceleration

\[
V(\phi_i) > \sum_i \dot{\phi}_i^2 + \frac{\rho}{2}.
\]

In this section we briefly analyze two models introduced previously in the literature to attribute the acceleration to the symmetry breaking of the (effective) potential.

### 2.1 Hybrid quintessence

Based on the “hybrid inflation” model introduced in \( [7] \), the Authors of \( [8] \) suggested a hybrid quintessence model to study the present acceleration of the Universe. In this model an additional scalar field \( \psi \) is employed to trigger the slow roll of the quintessence field \( \phi \) via symmetry breaking at late times. The potential is taken as

\[
V(\phi, \psi) = \beta \phi^4 + \alpha \phi^2 + h \psi^2 \phi^2 + \lambda \psi^4 + \mu \psi^2,
\]

where \( \alpha, \beta, h, \lambda \) and \( \mu \) are real constants satisfying \( \{ \beta > 0, \lambda > 0, \mu < 0, \alpha > 0, h < 0 \} \) and \( \{ h^2 - 4\lambda \beta < 0, h\mu - 2\alpha \lambda > 0, h\alpha - 2\beta \mu > 0 \} \) \( [8] \). When \( \psi^2 < \psi_c^2 \), where \( \psi_c^2 = -\frac{\mu}{h} \), the effective squared mass of \( \phi \) is positive and this field is settled down to \( \phi = 0 \). But when \( \psi \) becomes greater than the critical value, \( \psi^2 > \psi_c^2 \), the squared effective mass of \( \phi \) becomes negative. \( \phi = 0 \) as the local maximum of the potential becomes an unstable state, hence \( \phi \) rolls down the potential, leading to the present acceleration of the Universe as claimed in the scenario described in \( [8] \). Finally these fields settle down to the minimum of the potential and acquire their final values \( \psi_f, \phi_f \). Using \( \psi_f^2 = -\frac{\mu + h\phi_f^2}{2\lambda} \), and \( \phi_f^2 = -\frac{\alpha + h\psi_f^2}{2\beta} \), we find

\[
\psi_f^2 = -\frac{h\alpha - 2\beta \mu}{h^2 - 4\lambda \beta},
\]

\[
\phi_f^2 = -\frac{2\alpha \lambda + \mu h}{h^2 - 4\lambda \beta}.
\]
Note that
\[ V_f(\phi, \psi) = \frac{\beta \mu^2 + \alpha^2 \lambda - \alpha \mu h}{h^2 - 4 \beta \lambda}, \tag{8} \]
is negative. At the critical value \( \psi_c \), the potential is \( V_c = -\frac{\mu \alpha}{h} + \frac{\lambda \alpha^2}{h^2} \), which can be shown to be greater than \( V_f \). So the potential decreases after symmetry breaking as is expected because the fields roll down to the minimum of the potential. The symmetry breaking thus reduces the potential and, if the fields were initially at rest, it is seen that the symmetry breaking is not in favor of acceleration, see (3). As the final potential is also negative, to obtain a positive potential required for late time acceleration we must add a positive cosmological constant \( V_0 > 0 \) to the potential; \( V \to V + V_0 \). However the acceleration in this model, if happens, as was shown numerically in [8], is driven by the cosmological constant \( V_0 \) which is not arisen from the symmetry breaking. This is similar to the hybrid inflation model where the acceleration is driven by the vacuum energy density [7].

### 2.2 The symmetron model

Another model used to relate symmetry breaking to the present acceleration of the Universe is the symmetron model presented in [10]. This model uses the scalar tensor action [12]
\[
S = \int d^4x \sqrt{-g} \left( \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) + S_m[\tilde{g}^{\mu\nu}, \psi^i], \tag{9} \]
where \( \phi \) is a scalar field and \( S_m \) is the action for the \( \psi^i \) which can represent dark matter and other species [12]. The coupling between the scalar field and each ingredient, \( \psi^i \), is realized via \( \tilde{g}^{\mu\nu} = A_i^2(\phi) g^{\mu\nu} \), where \( A_i(\phi) \) is a positive function. As we are interested only in cosmic acceleration resulting from interaction between dark sectors, we assume that \( A_i \) is nontrivial \( A_i \neq 1 \) only for dark species and do not consider ordinary matter and its possible coupling to dark energy. The scalar field equation of motion is derived from \( \delta S / \delta \phi = 0 \)
\[ \ddot{\phi} + 3H \dot{\phi} + V_{,\phi} + A^{-1} A_{,\phi} \rho_E = 0. \tag{10} \]
The continuity equation for cold dark matter which we consider as a perfect fluid is
\[ \dot{\rho}_E + 3H \rho_E = A^{-1} A_{,\phi} \rho_E \dot{\phi}, \tag{11} \]
where \( \rho_E \) is dark matter energy density in the Einstein frame. In terms of re-scaled energy density \( \rho \) defined by \( \rho_E = A \rho \), the above equations may be expressed in a more simple form
\[ \ddot{\phi} + 3H \dot{\phi} + V_{,\phi} + A_{,\phi} \rho = 0, \]
\[ \dot{\rho} + 3H \rho = 0. \tag{12} \]
Note that \( \rho \) is not the physical density, rather a mathematical object to simplify our equations. The physical density appearing in the Friedmann equations (1), (2) and (3) is \( \rho_E = A \rho \) [10]. The first equation in (12) implies that one may consider a “\( \rho \) dependent” effective potential, \( V^{eff} = V_{,\phi} + A_{,\phi} \rho \), for \( \phi \). The Friedmann equation obtained from Einstein equations are
\[
H^2 = \frac{1}{3M_P^2} \left( \frac{1}{2} \dot{\phi}^2 + V + A \rho \right),
\]
\[ \dot{H} = -\frac{1}{2M_P^2} \left( \dot{\phi}^2 + A \rho \right). \tag{13} \]
Now, with these preliminaries let us study how the model works. It is clear from (12) that \( \rho \) is a decreasing function of time. One can arrange the model such that when \( \rho < \rho_c \), the symmetry is broken and the scalar field gains a negative squared mass so that the system becomes unstable and
the field slowly rolls down the potential giving rise to the present acceleration. Somehow, this is similar to the scenario in the previous subsection; $\rho$ plays the role of $\psi$ and its evolution causes symmetry breaking to occur. The acceleration happens when

$$\dot{H} + H^2 = \frac{1}{6M_P^2} \left(-2\dot{\phi}^2 + 2V(\phi) - A(\phi)\rho\right) > 0.$$  \hspace{1cm} (14)

Thus, to have an accelerated expansion it is necessary to have $V > \dot{\phi}^2 + \frac{4\rho}{2}$. Therefore, if $V(\phi)$ becomes negative for $\rho < \rho_c$ then acceleration does not occur.

As an example, take $A(\phi) = 1 + \frac{\phi^2}{2M^2}$ and $V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + V_0$, where $M$ and $\mu$ are real constants with mass dimension and $\lambda$ is a positive dimensionless real number \cite{10}

$$V_{\text{eff}} = \frac{1}{2} \left(\frac{\rho}{M^2} - \mu^2\right) \phi^2 + \lambda\phi^4 + V_0.$$  \hspace{1cm} (15)

When $\rho > M^2\mu^2$ the symmetry is restored and when $\rho < M^2\mu^2$ the field rolls down the potential from $\phi = 0$, see Fig. 1.

![Figure 1: A schematic illustration of $V(\phi)$ and $V_{\text{eff}}$ in terms of $\phi$ and $\rho$.](image)

Therefore $V(\phi)$, like the model in the previous subsection, decreases and can still be positive only when $V_0 > 0$. But if the field was first at $\phi = 0$, it seems that the symmetry breaking is not in favor of acceleration, see \cite{9}. A proposal, which was adopted and confirmed via numerical methods in \cite{10} (without a concrete analytical analysis), is to assume that the initial rapid oscillations of the scalar field about the minimum of the effective potential is converted to slow roll when $\rho < \rho_c$. During this phase, due to presence of the positive term $V_0$, the Universe experiences an accelerated expansion phase.

In the symmetron model proposed in \cite{10}, the symmetry breaking concept was used to explain both the screening effect and the onset of the present positive acceleration. But the appropriate symmetron mass, required to explain the screening effect, obtained from gravitational local tests was not consistent with the expected mass of the quintessence without some fine tuning. But from the above discussion, it seems that even though we relax the constraints corresponding to the screening effect and consider only dark sectors, the idea proposed in this model to relate the cosmic acceleration to symmetry breaking, like the hybrid quintessence model, is not adequate.

### 3 A new proposal for acceleration from symmetry breaking

In this part we present a new proposal for the onset of acceleration where, in contrast to models briefly reviewed in the previous subsection, the scalar field climbs up the potential when dark matter energy
density admits values less than a critical value (this is in favor of acceleration as can be seen from (3)). As we have shown, a necessary condition for acceleration is the positivity of the potential. We propose a mechanism in which the positivity of the potential, which is responsible for acceleration, is due to symmetry breaking; when $\rho > \rho_c$, the scalar field is settled down to the bottom of the effective potential and $V(\phi) \leq 0$. When $\rho < \rho_c$, the shape of the effective potential changes and the scalar field acquires a negative effective mass squared. Hence it rolls down the effective potential while climbing over its own potential, thus causing transition between deceleration and acceleration to occur, see Fig. 2.

Figure 2: A schematic illustration of $V(\phi)$, and $V^{\text{eff}}$ in terms of $\phi$ and $\rho$, in our proposed model.

To realize our plan we need the scalar field to climb $V$ while descending $V^{\text{eff}}$. Hence $V^{\text{eff}}$ and $V_\phi$ must have opposite signs, e.g. $V^{\text{eff}}_\phi < 0$, $V_\phi > 0$. To construct the effective potential we consider an interaction between dark sectors

$$\ddot{\phi} + 3H\dot{\phi} + V(\phi,\rho) = -h(\phi,\rho),$$

leading to

$$V^{\text{eff}} = V + h(\phi,\rho),$$

where $h$ is an analytical function such that the sign of $V^{\text{eff}}_\phi$ changes when $\rho$ decreases, giving rise to an effective negative mass squared causing the field to roll down along the effective potential while climbing its own potential. To see whether this model works in the simple case that $h(\phi,\rho)$ is a linear function of $\rho$, consider

$$V^{\text{eff}}_\phi(\phi,\rho) = V_\phi(\phi) + A_\phi(\phi)\rho.$$  \hspace{1cm} (18)

As $\rho$ is positive, $A_\phi$ and $V_\phi(\phi)$ must have opposite signs and so a decrease in $\rho$ causes $V^{\text{eff}}_\phi(\phi,\rho)$ to have the same sign as $V_\phi(\phi)$ which contradicts our assumption that $\text{sgn}\left(V^{\text{eff}}_\phi V_\phi\right) < 0$ for $\rho < \rho_c$. Hence the model (18) fails to satisfy our requirements. Next, let us try another ansatz where $\rho$ does not appear as a simple linear term in the effective potential. In the following we study two examples in this category.

### 3.1 A model with transient acceleration

To obtain a nonlinear interaction in terms of dark matter density via an action, we use the scalar-tensor action (9). However, as we have seen this action with a single scalar field does not satisfy our requirements. Hence in this part we assume that $\psi^i$ appearing in (9) consists of a cold dark matter perfect fluid and a new scalar field $\chi$ with potential $v(\chi)$. We also assume that $A(\phi)$ is just nontrivial, that is $A(\phi) \neq 1$ for dark sectors. We take $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$ for dark matter and $\chi$ (we want to relate
the cosmic acceleration to the interaction of dark sectors). Our action, after some manipulation, becomes
\[ S = \int \! d^4x \sqrt{-g} \left( \frac{M_P^2}{2} R - \frac{1}{2} g^\mu\nu \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{A^2(\phi)}{2} g^\mu\nu \partial_\mu \chi \partial_\nu \chi - A^4(\phi) v(\chi) \right) + S_m[\tilde{g}_\mu\nu, \rho]. \] (19)

The Hubble parameter satisfies the Friedmann equations
\[ H^2 = \frac{1}{3M_P^2} \left( \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} A^2 \chi^2 + V + A^4 v + A\rho \right), \] (20)
\[ \dot{H} = -\frac{1}{2M_P^2} \left( \dot{\phi}^2 + A^2 \chi^2 + A\rho \right). \] (21)

The equation of motion for the scalar field \( \phi \) is
\[ \ddot{\phi} + 3H \phi + \left( V_\phi + A,\phi (\rho - A^2 \chi^2 + 4A^3 v,\chi + \rho) \right) = 0, \] (22)
and \( \rho \) satisfies (12), whose solution is given by
\[ \rho(t) = \rho_0 a^{-3}. \] (23)

For \( \chi \) we obtain
\[ \ddot{\chi} + \left( 3H + 2 \left( \frac{\dot{\phi} A,\phi}{A} \right) \right) \dot{\chi} + A^2 v,\chi = 0. \] (24)

Hereinafter, for the sake of simplicity we ignore \( v \), and (21) becomes
\[ \ddot{\chi} + \frac{d \ln(a^3 A^2)}{dt} \dot{\chi} = 0, \] (25)
whose solution is given by
\[ \dot{\chi} = C \rho A^{-2}, \] (26)
where \( C \) is a numerical constant. Equations (22) and (26) then lead to
\[ \ddot{\phi} + 3H \phi + \left( V + \frac{C^2}{4} \rho^2 A^{-4} + A\rho \right),\phi = 0, \] (27)
so that we may define an effective potential for \( \phi \)
\[ V^{\ eff}_{\phi} = V + A\rho + \frac{C^2}{4} \rho^2 A^{-4}. \] (28)

The effective potential, unlike (18), is not a linear function of \( \rho \). Comparing with (17), we find that
\[ h(\phi, \rho) = A\rho + \frac{C^2}{4} \rho^2 A^{-4}. \] (29)

The last term is related to the presence of the field \( \chi \). Now \( V^{\ eff}_{\phi} = V_\phi + A,\phi \rho - C^2 A^{-5} A,\phi \rho^2 \) and the signs of \( V^{\ eff}_{\phi} \) and \( V_\phi \) may become opposite, in a consistent way, by a decreasing \( \rho \). As a specific example, consider (note that the mass term, unlike the symmetron model, is positive)
\[ V = \frac{\mu}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + V_0; \ \lambda > 0, \ \mu \geq 0, \ V_0 \leq 0, \] (30)
and $A = A \left( \frac{\phi^2}{M^2} \right)$, where $\phi^2 \ll M^2$. With this potential form, we have no acceleration before the symmetry breaking. By expanding $A$ we obtain

$$A \left( \frac{\phi^2}{M^2} \right) = A(0) + A'(0) \frac{\phi^2}{M^2} + \mathcal{O} \left( \frac{\phi^4}{M^4} \right)$$

$$A^{-4} \left( \frac{\phi^2}{M^2} \right) = A^{-4}(0) - 4 \frac{A'(0)}{A^3(0)} \frac{\phi^2}{M^2} + \mathcal{O} \left( \frac{\phi^4}{M^4} \right).$$

(31)

Therefore (27) and (31) imply

$$\ddot{\phi} + 3H \dot{\phi} + \left( \frac{2C^2A'(0)}{M^2A^3(0)} \rho^2 + \frac{2A'(0)}{M^2} \rho + \mu \right) \phi + \lambda \phi^3 = 0,$$

(32)

where a prime denotes derivative with respect to $\frac{\phi^2}{M^2}$. The squared mass term

$$\mu_{eff}^2 = -\frac{2C^2A'(0)}{M^2A^3(0)} \rho^2 + \frac{2A'(0)}{M^2} \rho + \mu,$$

(33)

is a second order polynomial in terms of $\rho$ and its sign can change twice at most. We require that for large $\rho$ the sign is positive and when $\rho$ is less than a critical value, it becomes negative. Hence we choose $\frac{A'(0)}{A^3(0)} < 0$. To allow the mass term to change sign we must have $\frac{A'(0)}{A^3(0)} + \frac{2C^2A'(0)\mu}{A^3(0)} > 0$. When $\rho$ is greater than the larger root of the polynomial in the right hand side of (33), $\mu_{eff}^2$ is positive and we may assume that the scalar field is settled at the minimum of the effective potential, $\phi = 0$, where we have $V = V_0$. When $\rho$ enters the interval between the roots of (33), $\mu_{eff}^2$ becomes negative, $\phi = 0$ becomes unstable and the scalar field rolls down the effective potential while climbing over its own potential. So if initially the necessary condition for acceleration, i.e. $V > 0$, does not hold this condition may be satisfied afterwards.

As an illustration, taking $A(\phi) = \left( 1 + \frac{\phi^2}{M^2} \right)^{-1}$ and $V(\phi) = \frac{1}{2} \phi^4$ ($\mu = 0$), the deceleration parameter and potential are numerically shown in terms of the dimensionless time $\tau = H_0 t$ in Fig. 3 showing a deceleration to acceleration phase transition when the field climbs over its own potential ($H_0$ is the present time Hubble parameter).

![Figure 3](image_url)
It is worth noting that the acceleration in this model is transient; as $\rho$ reduces more and becomes equal to or less than the second root of (33), the symmetry is restored and ultimately the effective potential takes the same shape as the initial potential. So the scalar field rolls down the effective potential as well as its own potential and will settle down to its initial value $\phi = 0$, its own potential becomes $V_0$ which by definition is not positive and the acceleration will end.

### 3.2 A model with persistent acceleration

To shed light on how a persistent acceleration can be obtained via symmetry breaking, we take advantage of interaction between dark sectors such that when $\rho$ becomes less than a critical value, the sign of the scalar field effective mass changes. Here we abandon our previous approach and do not insist on the precise form of the interaction via metric modification in the dark matter sector. We work in the Einstein frame and hereafter $\rho$ denotes the usual dark matter energy density. We require that the interaction behaves as a mass term and its sign changes during $\rho$ evolution. The equations of motions for the scalar field dark energy interacting with dark matter via a source $f(\rho)B(\phi)$, where $f$ and $B$ are analytical functions (this can be considered as a generalization of interaction $B(\phi)\rho$ considered in the literature [13] and is a special case of (17)) are

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -f(\rho)B'(\phi),$$
$$\dot{\rho} + 3H\rho = f(\rho)B'(\phi)\dot{\phi}.$$  \hspace{1cm} (34)

The effective potential is $V^{eff} = V(\phi) + f(\rho)B(\phi)$.

Our plan is as follows: we require $f(\rho)B(\phi)$ to behave as a mass-like term. We choose $f(\rho)$ as an analytic function whose sign changes when $\rho$ decreases. We assume that when $\rho > \rho_c$, the effective potential has a minimum at a constant $\phi^*$ which is the true vacuum of the system. At this point $V(\phi^*) \leq 0$ and so we have a deceleration phase. When the scalar field is settled down to this vacuum, $\rho$ continues to decrease. However, when $\rho < \rho_c$, the symmetry is broken and $\phi^*$ is no more the true vacuum and the scalar field rolls down the effective potential so $\dot{\phi}^2$ increases while $\rho$ continues to decrease. The increasing $\dot{\phi}^2$ makes the potential to become greater than zero which is necessary for acceleration. The scalar field eventually lies at a fixed point $\phi_c$ (the new minimum of the effective potential) whose own potential is positive, giving rise to a permanent acceleration; the Universe tends to a de Sitter space-time with a constant Hubble parameter

$$H^2 = \frac{1}{3M_P^2}V(\phi_c).$$ \hspace{1cm} (35)

To study the stability of this model let us write the equations of motion in the form of an autonomous system

$$\dot{u} = -3Hu - V,_\phi - f(\rho)B,_\phi,$$
$$\dot{\rho} = -3H\rho + f(\rho)B,_\rho u,$$
$$\dot{H} = -\frac{1}{2M_P^2}(u^2 + \rho),$$
$$\dot{\phi} = u.$$ \hspace{1cm} (36)

The critical points $\bar{\phi}, \bar{\rho}$ of this system are given by

$$\bar{u} = \dot{\phi}|_{\phi = \bar{\phi}} = 0, \quad \bar{\rho} = 0, \quad (V,_\phi + f(\rho)B,_\phi)|_{\rho = 0, \phi = \bar{\phi}} = 0.$$ \hspace{1cm} (37)

We consider small homogeneous variations about the critical points: $\delta \phi, \delta u, \delta H, \delta \rho$ in (36) to obtain

$$\frac{d}{dt} \begin{pmatrix} \delta u \\ \delta \rho \\ \delta H \\ \delta \phi \end{pmatrix} = \mathcal{M} \begin{pmatrix} \delta u \\ \delta \rho \\ \delta H \\ \delta \phi \end{pmatrix},$$ \hspace{1cm} (38)
where

$$
\mathcal{M} = \begin{pmatrix}
-3H & -\left(\frac{d}{d\phi} f\right) \frac{d}{d\phi} B & -3\rho - \frac{d^2}{d\phi^2} V - f \frac{d^2}{d\phi^2} B \\
\frac{d}{d\phi} B & -3H + \left(\frac{d}{d\phi} f\right) \left(\frac{d}{d\phi} B\right) u & -3\rho u - f \left(\frac{d^2}{d\phi^2} B\right) u \\
\frac{-M^2}{2} & -\frac{1}{2M^2} & 0 \\
0 & 0 & 0
\end{pmatrix}.
$$

(39)

The roots of the characteristic polynomial of \( \mathcal{M} \) are a simple \( x = 0 \), and \( x \) satisfies

$$
x^3 + 6H x^2 + (ff\phi B^2 \phi + 9H^2 + (V_{\phi\phi} + f B_{\phi\phi})) x + 3H (V_{\phi\phi} + f B_{\phi\phi}) = 0,
$$

(40)
evaluated at the critical points. The corresponding Routh table is

$$
\begin{bmatrix}
1 & 9H^2 + B^2 f f_{\rho} + V_{\phi\phi} + f B_{\phi\phi} & x^3 \\
6H & 3H (B_{\phi\phi} f + V_{\phi\phi}) & x^2 \\
9H^2 + B^2 f f_{\rho} + \frac{1}{2} (V_{\phi\phi} + f B_{\phi\phi}) & 0 & x \\
3H (B_{\phi\phi} f + V_{\phi\phi}) & 0 & 1
\end{bmatrix}.
$$

(41)

So the system is stable at a critical point determined by (37) provided that \( V_{\phi\phi}^{eff} = B_{\phi\phi} f + V_{\phi\phi} > 0 \), and \( 9H^2 + B^2 f f_{\rho} + \frac{1}{2} (V_{\phi\phi} + f B_{\phi\phi}) > 0 \) at this point.

As an example we choose the potential as \( V = \frac{1}{2} \mu \phi^2 + \frac{1}{4} \phi^4 \), \( \lambda > 0 \), \( \mu > 0 \). So in order that \( f(\rho)B(\phi) \) behaves as a mass-like term we take \( B(\phi) \propto \phi^2 \). As was mentioned before we choose \( f(\rho) \) as an analytic function such that the effective mass term sign changes when \( \rho \) decreases. Our simplest choice is then \( B(\phi) f(\rho) = \frac{\rho^2}{2M^2} (\rho - \rho_c) \), where \( M \) and \( \rho_c \) are two constants with dimension of [mass] and [mass]^4 respectively and we take \( \rho_c > \mu M^2 \). Now let us study the behavior of the system. When \( \rho > \rho_c - \mu M^2 \), the effective potential has a minimum at \( \phi = 0 \) and \( \phi = 0 \) is a solution of the system (34). During the time when \( \phi = 0 \), \( \rho \) continues to decrease. When \( \rho < \rho_c - \mu M^2 \), \( \phi = 0 \) is longer the true vacuum and the scalar field rolls down the effective potential so \( \phi^2 \) increases. By increasing \( \phi^2 \) the potential becomes greater than zero which is necessary for acceleration. Eventually \( \phi^2 \) settles at \( \phi^2 = \frac{\rho_c - \mu M^2}{\lambda M^4} \) (the new minimum of the effective action), while \( \rho \to 0 \). Therefore the Universe tends to a de Sitter space-time with a constant Hubble parameter

$$
H^2 = \frac{1}{12M^2} \rho_c^2 - \mu^2 M^4/\lambda M^4.
$$

(42)

As an illustration, let us use the equations of motions rewritten in terms of dimensionless parameters defined previously

$$
d^2 \varphi/d\tau^2 + 3H d\varphi/d\tau + \tilde{\mu} \varphi + \tilde{\lambda} \varphi^3 = -\frac{1}{M^2} (\tilde{\rho} - \tilde{\rho}_c) \varphi,
$$

$$
d\tilde{\rho}/d\tau + 3H \tilde{\rho} = \frac{1}{M^2} (\tilde{\rho} - \tilde{\rho}_c) \varphi d\varphi/d\tau,
$$

$$
d\tilde{H}/d\tau = -\frac{1}{2} \left( \left( d\varphi/d\tau \right)^2 + \tilde{\rho} \right),
$$

(43)
to show the deceleration parameter in terms of dimensionless time. The deceleration and Hubble parameter are shown in Fig. 4 for \( \{\tilde{\rho}(0) = 12, \mu = 0, \lambda = 10^{10}, \tilde{\rho}_c = 11, \tilde{M} = 0.01, \varphi(0) = 0, \tilde{H}(0) = 2\} \), showing that the Universe enters an acceleration phase and finally becomes a de Sitter space-time.
Figure 4: The deceleration (dashed) and Hubble parameters (line) in terms of dimensionless time $\tau$ for $\dot{\rho}(0) = 12$, $\mu = 0$, $\lambda = 10^{10}$, $\dot{\rho}_c = 11$, $\varphi(0) = 0$, $\dot{H}(0) = 2$.

4 Summary

First, we briefly studied and reviewed the possible relation between the symmetry breaking and the onset of cosmic acceleration in two models proposed in the literature, i.e. hybrid and symmetron scalar field dark energy models. We delineated the role of the positive scalar field potential which drives the acceleration and showed that in these two models the symmetry breaking is not in favor of the positive acceleration. Then, in the third section, we tried to relate the positivity of the potential to the symmetry breaking of the effective potential constructed from the interaction between dark sectors. To do so, we required that the scalar field climbs over its own potential while moving down along the effective potential. We proved that this requirement cannot be satisfied in a model whose interaction is linear in terms of the matter density (like the symmetron model). To obtain a non linear interaction, we introduced and additional scalar field in the dark energy sector in a scalar tensor type action consisting only of dark species. For the sake of simplicity we took it without a potential but one may examine a general potential for the problem. It was shown and illustrated that in this framework a transient deceleration to acceleration phase transition may occur. In the second model, to obtain a permanent acceleration, we presented an appropriate interaction between the dark sector (which does not have its roots in a fundamental action). We showed that in this new model, the Universe may evolve from a deceleration phase to a de Sitter space-time through symmetry breaking. We verified the stability of the model and illustrated our results through a numerical example.

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