Black hole gas in the early universe

Mónica Borunda, Manuel Masip

1CAFPE and Departamento de Física Teórica y del Cosmos
Universidad de Granada, E-18071 Granada, Spain
mborunda@ugr.es, masip@ugr.es

Abstract

We consider the early universe at temperatures close to the fundamental scale of gravity ($M_D \ll M_{\text{Planck}}$) in models with extra dimensions. At such temperatures a small fraction of particles will experience transplanckian collisions that may result in microscopic black holes (BHs). BHs colder than the environment will gain mass, and as they grow their temperature drops further. We study the dynamics of a system (a black hole gas) defined by radiation at a given temperature coupled to a distribution of BHs of different mass. Our analysis includes the production of BHs in photon-photon collisions, BH evaporation, the absorption of radiation, collisions of two BHs to give a larger one, and the effects of the expansion. We show that the system may follow two different generic paths depending on the initial temperature of the plasma.
1 Introduction

Large (ADD) \(^{[1]}\) or warped (RS) \(^{[2]}\) extra dimensions open the possibility that the fundamental scale of gravity \(M_D\) is much lower than \(M_P \approx 10^{19}\) GeV. This could imply that the transplanckian regime \(^{[3]}\) is at accessible energies. Collisions in that regime are very different from what we have experienced so far in particle colliders: due to its spin 2 the graviton becomes strongly coupled and dominates at distances that increase with the center of mass energy \(\sqrt{s}\). In particular, at small impact parameters one expects that gravity bounds the system and the two particles collapse into a microscopic black hole (BH) of mass around \(\sqrt{s}\). Such BH would evaporate \(^{[4]}\) very fast (the time scale is set by \(1/M_D\)) into final states of high multiplicity, a possible LHC signature that has been extensively discussed in the literature \(^{[5]}\).

These collisions, however, may have also occurred in the early universe if the temperature was ever close to \(M_P\). Consider a period of inflation produced by a field on the brane, followed by reheating into 4-dimensional (4-dim) species. If the reheating temperature is \(T_{rh} \geq 0.1 M_P\), particles in the tail of the Boltzmann distribution may collide with enough energy to form BHs. Hot BHs will evaporate, but if a mini-BH is colder than the environment it absorbs more than it emits \(^{[6]}\), growing and becoming colder, which in turn increases its absorption rate (see below). The fact that heavier BHs live longer distinguishes them from massive particles or string excitations possibly produced at these temperatures, since the lifetime of the latter is inversely proportional to their mass. As a consequence, one expects that BHs are a critical ingredient at temperatures near the fundamental scale. Notice that these BHs are not the primordial ones formed from the gravitational collapse of density fluctuations \(^{[7]}\) at lower temperature \(T\).

Although BHs would also be present in a 4-dim universe at \(T \sim M_P\), they are most relevant in TeV gravity models. The reason is that the expansion of the universe is a long-distance process, so its rate is dictated by \(M_P\) (not \(M_D\)) also in models of TeV gravity. If the bulk is basically empty \(^{[11]}\) or thin (see below), then the expansion at \(T \leq M_D\) will be negligible in terms of the fundamental time scale \(1/M_D\), and there is plenty of time for collisions producing BHs to occur and for the BHs to grow. In contrast, in a 4dim universe the expansion rate at \(T \approx M_P\) is of order \(H^{-1} \approx 1/M_P\), the temperature of the universe drops before BHs have grown, and once it goes below \(T_{BH}\) they evaporate in the same time scale.

In this article we explore the implications of having initial temperatures near the fundamental scale of gravity. First we define a consistent set up for TeV gravity. Then we study the behaviour of a single BH inside a thermal bath in an expanding universe. Finally we
consider the generic case, a black hole gas, with radiation coupled to a distribution of BHs of different mass. We find remarkable that the effect of these mini-BHs in the early universe has been almost completely overlooked in the literature (the only analysis that we have found is given by Conley and Wizansky in [14]), although many authors have considered temperatures close (and above) the Planck scale (see [15] and references therein).

2 Consistent TeV gravity models

There are two generic frameworks that may imply unsuppressed gravity at $M_D \approx 1$ TeV (or at any scale $M_D < M_P$). In the first one (ADD), $n$ compact dimensions of length $L < 1$ mm introduce a large number of KK excitations of the graviton. These gravitons, of effective 4-dim mass proportional to $m_c \equiv L^{-1}$, couple very weakly ($\approx \sqrt{s}/M_P$) to ordinary matter. However, the large number of effective gravitons $\approx (\sqrt{s}/m_c)^n$ involved in a collision gives an amplitude of order one,

$$\frac{s}{M_P^2} \times \left(\frac{\sqrt{s}}{m_c}\right)^n \approx 1 ,$$

at $\sqrt{s} = M_D$ if

$$m_c = M_D \left(\frac{M_D}{M_P}\right)^{2/n} .$$

In contrast, in the second scenario (RS) the KK excitations of the graviton have unsuppressed couplings to matter but large masses, right below $M_D$, so a few KK modes suffice to define an order one gravitational interaction at that scale.

The ADD set up has the basic cosmological problem pointed out in [12]. Essentially, at temperatures $T \gg m_c$ KK gravitons will be abundantly produced in annihilations of brane particles, due to the large multiplicity of final states. If the initial temperature is large these gravitons will change the expansion rate at the time of primordial nucleosynthesis. Even if the initial temperature is as low as 1 MeV, their late decay will distort the diffuse gamma ray background in an unacceptable way. Obviously, an initial temperature close to $M_D$ would bring too many massive gravitons.

One solution would be to consider RS models of TeV gravity. There, at $T \approx M_D$ bulk and brane species have similar abundances, but massive gravitons will decay fast once $T < m_c \approx 0.1 M_D$, returning all the energy to the brane and defining acceptable 4-dim cosmological models.

Even within the ADD framework, however, we can consider hybrid models where the connection between $m_c$ and $M_D$ is not the one given in Eq. (2). This can be obtained, for
example, with a warp factor \[^{[16]}\]. The effect would be to *push* the KK modes towards the 4dim brane, reducing the effective compact volume to \( V \approx (1/m_c)^n \) while increasing their coupling to matter, \[
\frac{s}{M_P^2} \rightarrow \frac{s}{M_D^2} \left( \frac{m_c}{M_D} \right)^n .
\] (3)

In this way, a smaller number of KK modes will imply an order one gravitational interaction at the same scale \( \sqrt{s} = M_D \). If the free parameter \( m_c \) takes the value in Eq. (2) we recover ADD, whereas for \( m_c \) approaching \( M_D \) we obtain RS. At distances below \( 1/m_c \) gravity would be higher dimensional (similar to ADD) whereas at larger distances it becomes 4-dimensional (like in the usual RS scenario).

In this framework the KK modes of the graviton are not produced at temperatures \( T < m_c \). Therefore, \( m_c > 10 \text{ MeV} \) avoids astrophysical bounds \[^{[17]}\] for any number \( n \) of extra dimensions \[^{[16]}\] and \( M_D = 1 \text{ TeV} \). On the other hand, as these KK gravitons have stronger couplings to matter, they can decay much faster than in the usual ADD model or decouple at temperatures below their mass. By changing \( m_c \) (with \( M_D \) fixed) it seems easy to obtain models with no gravitons at the time of primordial nucleosynthesis that are consistent with all cosmological observations.

3 **Single black hole in a thermal bath**

Let us consider the hybrid framework described in the previous section with \( n \) extra dimensions, a fundamental scale \( M_D \), and an independent compactification mass \( m_c \geq 1 \text{ GeV} \). Gravity at energies above \( M_D \) and distances smaller than \( 1/m_c \) is strongly coupled and higher dimensional (just like in ADD), so the radius and temperature of a BH of mass \( M \) are

\[
r_H = \frac{a_n}{M_D} \left( \frac{M}{M_D} \right)^{1/(n+1)} ; \quad T_{BH} = \frac{n + 1}{4\pi r_H} ,
\]

(4)

with

\[
a_n = \left( \frac{2^n n^{(n-3)/2} \Gamma \left( \frac{n+3}{2} \right)}{n + 2} \right)^{1/(n+1)} .
\]

(5)

Once a BH reaches a radius \( r_H = 1/m_c \) and fills up the whole compact space its (4-dim) size will not keep growing, since all the KK gravitons but the zero mode provide short distance interactions. The radius should start growing significantly only when the usual 4-dim horizon (produced by the massless graviton) is of order \( 1/m_c \), i.e., for BH masses \( M \approx M_P^2/m_c \). Above this threshold BHs are basically 4-dim.
We will assume that the particles in the brane and the bulk (with $g_*$ and $g_b$ degrees of freedom, respectively) are initially in thermal equilibrium at a temperature $T = T_0$. Notice that reheating in just the brane after a period of inflation would not justify an empty bulk, as in a time of order $M_D^2/T^3$ most of the energy would escape into KK modes. Here, however, the bulk may be much thinner and emptier than in the usual ADD model, implying a slow expansion in terms of the fundamental time $1/M_D$ (see below).

It is easy to see that if the plasma temperature $T$ and $1/r_H$ are larger than $m_c$ a BH at rest in the brane will change its mass according to

$$
\frac{dM}{dt} \approx \sigma_4 A_4 (T^4 - T_{BH}^4) + \sigma_{4+n} A_{4+n} (T^{4+n}_B - T_{BH}^{4+n}),
$$

where $\sigma_4 = g_*/120$, $c_n$ is an order 1 coefficient that depends on the geometry of the compact space, and we have neglected gray-body factors. The expression above assumes a thermalized photon (in the brane) and graviton (in the bulk) plasma, so it requires that changes occur slowly. In particular, if the BH grows very fast one has to make sure that the gain in mass is always smaller than the total energy of the plasma in causal contact with the hole:

$$
\frac{dM}{dt} < \frac{4\pi^3 t^2}{30} T^4 \left( g_* + g_b c_n \frac{T^n}{m_c^n} \right),
$$

where we have assumed $t > 1/m_c$.

As the BH grows it enters a new phase when its mass reaches

$$
M_1 \approx M_D \left( \frac{M_D}{a_n m_c} \right)^{n+1},
$$

which corresponds to a radius $r_H = 1/m_c$ filling up the whole extra volume. For $M > M_1$ the BH keeps gaining mass as far as the plasma temperature $T$ is above $T_{BH} \approx m_c$. However, as explained above, KK fields do not reach distances beyond $r_H \approx 1/m_c$, so the BH radius (and its temperature) will stay basically constant.

Finally, masses above $M_2 \approx M_p^2/m_c$ turn the BH into a purely 4 dimensional object: its radius

$$
r_H = \frac{2M}{M_p^2}
$$

grows larger than $1/r_c$ and the BH becomes too cold to emit massive gravitons. In this regime, if $T$ is larger than $m_c$ the BH changes its mass according to

$$
\frac{dM}{dt} \approx \frac{\pi}{480} \left( g_* + g_b c_n \frac{T^n}{m_c^n} \right) \frac{T^4}{T_{BH}^2},
$$

5
whereas at lower plasma temperature it goes as
\[
\frac{dM}{dt} \approx \frac{\pi}{480} g_* \left( \frac{T^4}{T_{BH}^4} - T_{BH}^2 \right).
\] (11)

Eq. (10) implies that lighter (hotter) BHs evaporate with an approximate lifetime
\[
\tau \approx \frac{1}{M_D} \left( \frac{M}{M_D} \right)^{\frac{4+n}{1+n}},
\] (12)
and that BHs colder than the plasma will gain mass: heat flows from the hot plasma to the cold BH, but the effect is to cool the BH further and increase \( T - T_{BH} \). This is, indeed, a very peculiar two-component thermodynamical system.

On the other hand, the expansion of the universe is also affected by the presence of bulk species. We will assume that the extra dimensions are frozen (do not expand) and will integrate the matter content in the bulk. The large values of \( m_c \) that we will consider imply large enough couplings with brane photons, so that the (equilibrium) abundance of KK modes of mass larger than the plasma temperature will be negligible. Therefore, at \( T > m_c \) we have a universe with a radiation density
\[
\rho_{rad} \approx \frac{\pi^2}{30} T^4 \left( g_* + g_b c_n \frac{T^n}{m_c^n} \right),
\] (13)
and an expansion rate
\[
\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3} \rho_{rad},
\] (14)
\( i.e., \rho_{rad} = \rho_{rad0} \left( \frac{R_0}{R} \right)^4 \). Notice that the second term in Eq. (13) will slow down the change in the plasma temperature \( T(t) \) due to the expansion. In particular, if the bulk energy dominates then \( T \propto t^{2/(4+n)} \) for times larger than a Hubble time. At temperatures below \( m_c \) this term vanishes exponentially and all the bulk energy is transferred to the brane.

To illustrate the orders of magnitude involved, let us discuss a toy model with \( M_D = 1 \) TeV, \( n = 1 \) (\( c_1 = 0.3 \)), and just photons (\( g_* = 2 \)) and gravitons (\( g_b = 5 \)) at \( T_0 = 100 \) GeV. A BH of mass \( M < M_{\text{crit}} = 12 \) TeV would be hotter than the environment, and it would evaporate in a time of order \( \tau \approx 10^{-3} \) GeV\(^{-1} = 6.5 \times 10^{-28} \) s. The Hubble time of a universe at this temperature is
\[
H^{-1} = \frac{R}{\dot{R}} = 1.4 \times 10^{14} \text{ GeV}^{-1} = 9.2 \times 10^{-9} \text{ s}.
\] (15)
A BH of initial mass \( M_0 = 100 \) TeV will have a starting temperature of 8.7 GeV. In a time of order 17 GeV\(^{-1} \) its mass is already around \( 4.7 \times 10^7 \) GeV and its radius as large as the size
of the extra dimension (0.1 GeV$^{-1}$). Then the BH keeps growing at approximately constant rate, since its size and temperature ($T_{BH} \approx 1.6$ GeV) change little with the mass. In a Hubble time the BH reaches a mass $M \approx 5.7 \times 10^{19}$ GeV. At later times the expansion cools the plasma, which slows the growth of the BH. Its maximum mass, $M \approx 2 \times 10^{21}$ GeV, is achieved when $T = T_{BH}$ at times of order $t \approx 10^{18}$ GeV$^{-1}$. Finally, the BH will evaporate after $\tau \approx 10^{22}$ GeV$^{-1} \approx 1$ s.

In Fig. 1 we plot this case together with the mass evolution for a larger initial temperature, $T_0 = 200$ GeV. In this second case the expansion rate is faster (the Hubble time $H^{-1} \approx 2.6 \times 10^{13}$ GeV$^{-1}$ is shorter), but the higher radiation density provides larger BH masses. Lower (higher) values of $m_c$ ($M_D$) would also imply larger BH masses.

In none of the two cases described above the BH becomes purely 4 dimensional, i.e., with a mass $M > 10^{37}$ GeV. These large values of $M$ can be obtained increasing the number $n$ of extra dimensions and/or the ratio $M_D/m_c$. A BH radius larger than $1/m_c$, however, would not stop its growth, on the contrary, it would enhance the absorption rate of radiation by the BH. What stops the growth of a BH in any TeV-gravity framework is just the drop in the temperature of the plasma due to the expansion.

It is also important to emphasize that the qualitative features of the process described above do not depend much on the details of the compactification (the length or the shape)
of the extra dimensions. They just depend on the fact that the fundamental scale of gravity $M_D$ is low and that the initial temperature of the radiation is close to it. In particular, the sequence of events would be similar in RS models, although there $m_c$ is close to $M_D$ and the BHs are smaller ($r_H \approx 1/m_c$, and the absorption rate grows with the BH area).

Finally, notice that the usual cosmological constraints [12, 13] on the initial (reheating) temperature of the early universe or the astrophysical bounds on $M_D$ are actually a probe of $1/L = m_c$, so they are avoided if $m_c$ is above 1 GeV.

4 Dynamics of a black hole gas

Let us now deduce the equations that describe the production and evolution of BHs inside a thermal bath of temperature $T$. If $T < M_D < M$ the BHs will be produced in collisions between radiation in the high-energy tail of the Boltzmann distribution $f_\gamma(\vec{k})$. Once formed these BHs will be non-relativistic, with a kinetic energy $K \approx T$ negligible versus the mass $M$ and a velocity $v \approx \sqrt{2T/M}$. We will then assume that the two components of the system are well described by the temperature $T(t)$ of the plasma and the distribution $f(M, t)$ expressing the number of BHs of mass $M$ per unit mass and volume at time $t$. The total energy density is

$$\rho(t) = \rho_{\text{rad}}(t) + \rho_{\text{BH}}(t) \approx \frac{\pi^2}{30} T^4 \left( g_* + g_b c_n \frac{T^n}{m_c^n} \right) + \int dM M f(M, t). \quad (16)$$

We identify the following processes changing the number density of BHs of mass $M$.

- BH production in photon-photon collisions. Photons will collide with a cross section $\sigma \approx \pi r_H^2$ to form a BH of mass $M \approx \sqrt{s}$:

$$\left( \frac{\partial f(M, t)}{\partial t} \right)_{\gamma\gamma \rightarrow M} = \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{f_\gamma(\vec{k}_1)f_\gamma(\vec{k}_2)}{\sigma(M) \vert \vec{v}_1 - \vec{v}_2 \vert} \delta(\sqrt{(k_1^\mu + k_2^\mu)^2} - M), \quad (17)$$

with $v_i = 1$. If $T \ll M$ this expression can be approximated in terms of a modified Bessel function,

$$\left( \frac{\partial f(M, t)}{\partial t} \right)_{\gamma\gamma \rightarrow M} \approx \frac{g_*^2 a_n^2}{16\pi^3} T M^2 \left( \frac{M}{M_D} \right)^{\frac{4+2n}{1+n}} K_1(M/T). \quad (18)$$

To simplify our analysis we will neglect BH production in collisions of bulk particles. Although KK modes dominate the energy density, their cross section to form a BH is smaller (their wave function is diluted within the bulk), so this would be an order one contribution.
• The collision of two BHs, of mass $M_1$ and $M_2$, to form a BH of mass $M = M_1 + M_2$:

$$\left(\frac{\partial f(M, t)}{\partial t}\right)_{M_1M_2 \rightarrow M} = \int dM_1 dM_2 \ f(M_1, t) f(M_2, t) \ \sigma(M_1, M_2) \ v_{12} \ \delta(M_1 + M_2 - M) \ ,$$

(19)

where the BH velocity is $v_i = \sqrt{2T/M_i}$ and $v_{12} = \langle |\vec{v}_1 - \vec{v}_2| \rangle$. We will take a BH–BH cross section of

$$\sigma(M_1, M_2) = \pi (r_{H1} + r_{H2})^2 \ .$$

(20)

Notice also that for a minimum BH mass of $M_D$, this contribution to $f(M, t)$ is nonzero only if $M \geq 2M_D$.

• A BH of mass $M$ may collide with any other BH, which would reduce $f(M, t)$:

$$\left(\frac{\partial f(M, t)}{\partial t}\right)_{MM_1 \rightarrow M_2} = - \int dM_1 \ f(M, t) f(M_1, t) \ \sigma(M, M_1) \ v_{01} \ ,$$

(21)

• We can describe the absorption and emission of radiation using $dM/dt$ in (6). The BHs of mass $M$ will have a mass $M + dM$ at $t + dt$, i.e.,

$$f(M, t) = f(M + dM, t + dt) \ .$$

(22)

This implies

$$\left(\frac{\partial f(M, t)}{\partial t}\right)_{abs/em} = \frac{\partial f(M, t)}{\partial M} \ \frac{dM}{dt} \ .$$

(23)

• Finally, we have to add the effect of the expansion. The 4-dim scale factor grows according to

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3} (\rho_{rad} + \rho_{BH}) \ .$$

(24)

This dilutes the number of BHs at a rate

$$\left(\frac{\partial f(M, t)}{\partial t}\right)_{exp} = -3 \ f(M, t) \frac{\dot{R}}{R} \ .$$

(25)

The total change in $f(M, t)$ per unit time will result from the addition of the 5 contributions above. This fixes $\dot{\rho}_{BH}$:

$$\dot{\rho}_{BH} = \int dM \ M \ \frac{\partial f(M, t)}{\partial t} \ .$$

(26)

On the other hand, to obtain the change in $T$ (or $\rho_{rad}(T)$) we impose energy conservation,

$$d(\rho_{rad} R^3) + d(\rho_{BH} R^3) = -\frac{1}{3} \rho_{rad} dR^3 \ ,$$

(27)
where we have neglected the pressure of the BHs (they behave like non-relativistic matter). The equation above implies
\[
\dot{\rho}_{\text{rad}} = -4\rho_{\text{rad}} \frac{\dot{R}}{R} - \dot{\rho}_{\text{BH}} - 3\rho_{\text{BH}} \frac{\dot{R}}{R}.
\] (28)

Notice that if the radiation and the BHs where decoupled, then one would have \(d(\rho_{\text{BH}} R^3) = 0\), i.e., \(\dot{\rho}_{\text{rad}} = -4\rho_{\text{rad}} (\dot{R}/R)\). This, however, is not the case since there is energy exchange between the two components. In particular, a variation in \(\rho_{\text{rad}}\) changes \(T\), implying a change in the absorption/emission rate of the BHs and in \(\rho_{\text{BH}}\). It is easy to see that
\[
d(\rho_{\text{BH}} R^3) = R^3 \alpha \, d\rho_{\text{rad}},
\] (29)
where we define \(\alpha\) as
\[
\alpha \equiv \left(\frac{\partial \rho_{\text{BH}}}{\partial \rho_{\text{rad}}}\right)_{R=\text{cons}}.
\] (30)
Substituting this equation in (27) we obtain
\[
R^3 (1 + \alpha) d\rho_{\text{rad}} = -\frac{4}{3} \rho_{\text{rad}} dR^3
\] (31)

The equation above expresses that BH evaporation can slow down the cooling of the radiation due to the expansion: as the universe expands, \(T\) drops, this leaves some BHs hotter than the plasma, so they evaporate and reheat the environment. Taking an interval where \(\alpha\) is constant, eq. (31) can be integrated to
\[
\rho_{\text{rad}} \approx \rho_{\text{rad}0} \left(\frac{R_0}{R}\right)^{\frac{4}{1+\alpha}}.
\] (32)
If \(\alpha \gg 1\) BH evaporation would stop the change in the density and the temperature of the radiation due to the expansion.

5 Black-hole dominated plasma

We will now apply these equations to an initial configuration with only radiation (no BHs) at a given temperature \(T(0) = T_0\). We find two generic scenarios depending on the value of \(T_0\). Values closer to \(M_D\) produce a larger number of BHs, that grow and absorb all the radiation before a Hubble time. Lower values of \(T_0\) imply a smaller number of BHs, that grow at basically constant temperature up to times of order \(H^{-1}\). In this section we will focus on the first case. We will describe the sequence of events using the toy model with \(n = 1\), \(g_* = 2\), \(g_b = 5\), \(m_c = 10\) GeV, \(M_D = 1\) TeV and an initial temperature of \(T_0 = 200\)
Figure 2: Distribution $f(M,t)$ for $t_i = 0.1, 1$ GeV$^{-1}$ and $T_0 = 200$ GeV. Dashes describe the evolution neglecting BH–BH collisions.

GeV. The critical BH mass (corresponding to $T_{BH} = T_0$) is $M_{crit} = 3.0$ TeV. In Fig. 2 we plot the distribution $f(M,t)$ for two values of $t$. We have included the distributions with and without the effect of collisions of two BHs to form a larger one.

We can distinguish four phases in the evolution of the BH gas.

1. In a first phase BH’s of $M > M_{crit}$ are produced at constant rate (see Fig. 3) and absorb radiation of temperature $T \approx T_0$. As the number of BHs grows (see Fig. 2), collisions between two BHs to produce a larger one become important. This reduces the number of BHs and increases their average mass (in Fig. 3). At times around $t \approx 2$ GeV$^{-1}$ BHs start dominating the energy density and the temperature of the radiation drops (see Fig. 4).

2. The drop in $T$ stops the production of BHs in photon-photon collisions. In addition, the lighter BHs become hotter than the plasma, so they evaporate and feed $\rho_{rad}$. The evaporation reduces the number of BHs per unit volume, but the average BH mass grows: there is a continuous transfer of energy from the radiation and from the lighter (evaporating) BHs to the larger BHs of lower temperature. Once the BHs get a mass around $5 \times 10^7$ GeV their radius stops growing.

3. At $t \approx 10^4$ GeV$^{-1}$ the temperature of the radiation and of the BHs is similar, around
1.6 GeV. The (slow) mass growth of the heavier BHs is compensated by the decay of the lighter ones, with the radiation temperature basically constant. In this phase the energy density $\rho = \rho_{\text{rad}} + \rho_{\text{BH}}$ is matter (BH) dominated.

4. At times of order $H^{-1} \approx 2.6 \times 10^{13}$ GeV$^{-1}$ the expansion cools the radiation. The BHs, of mass around $10^9$ GeV, decay fast ($\tau \approx 4 \times 10^9$ GeV$^{-1}$) and the universe becomes radiation dominated. The lightest KK modes also decay fast ($\tau_{KK} \approx M_D^{2+n}/m_c^{3+n} \approx 10^5$ GeV$^{-1}$), so only 4dim photons survive below $T \approx 1$ GeV.

Two remarks are here in order. First, this is a toy model with only photons, gravitons, and BHs. In a more complete set up one should include baryons at $T$ below 0.1 GeV (two types of matter, baryons and BHs, could coexist in models with lower values of $m_c/M_D$). Second, this generic high $T$ case, with $M_D > 5$ TeV to avoid bounds from colliders and the inclusion of all the light standard model species (not just photons) at each temperature, could define a realistic set up. In these hybrid models it seems natural to obtain a plasma dominated by the standard particles at $T \approx m_c \gg \Lambda_{QCD}$. The predictions for primordial nucleosynthesis would then be consistent with observations.
Figure 4: Temperature $T$ of the radiation as a function of $t$. At larger values of $t$ (up to a Hubble time $\approx 10^{13}$ GeV$^{-1}$) $T \approx 1.6$ GeV.

6 Radiation dominated plasma

Let us now discuss the scenario with a low initial temperature and $\rho_{BH} \ll \rho_{rad}$ at any $t$. We take $n = 1$, $g_\ast = 2$, $g_b = 5$, $m_c = 10$ GeV, $M_D = 1$ TeV and $T_0 = 100$ GeV. The critical BH mass (corresponding to $T_{BH} = T_0$) is $M_{\text{crit}} = 12$ TeV, higher than in the previous case. As a consequence, the production rate of BHs colder than the plasma is much smaller. We can separate three phases in the evolution of this BH gas.

1. At times below $H^{-1} = 1.4 \times 10^{14}$ GeV$^{-1}$ BHs are produced at constant rate in photon-photon collisions, with $T \approx T_0$. The number of BHs is so small that BH collisions can be neglected. All these BHs grow like the one discussed in Section 2.

2. When the expansion cools the photons, BH production drops exponentially, whereas BH growth slows down. In Fig. 5 we plot the BH distribution $f(M, t)$ after one and four Hubble times. The BHs reach a maximum mass around $M = 10^{21}$ GeV (there is just a 10% mass difference between 99% of the BHs) and $T_{BH} \approx 1.6$ GeV. This universe is always radiation dominated. At $t \approx 10^{18}$ GeV$^{-1}$ the photon gas becomes colder than the BHs.

3. In the last phase all the BHs evaporate in a time scale $\tau \approx 10^{22}$ GeV$^{-1}$.
This generic scenario is in principle consistent with primordial nucleosynthesis, since BHs are just a small fraction of the total energy density and do not change the expansion rate. In the particular case that we have discussed they decay when the plasma temperature is $T \approx 0.01$ GeV, but increasing the ratio $M_D/m_c$ one can obtain BHs that become 4 dimensional and with a much longer lifetime. Their late decay could introduce distortions in the diffuse gamma ray background. In addition, in a more complete set up including baryons and structure formation they might work as seeds for macroscopic (primordial) BHs.

7 Summary and discussion

A transplanckian regime at accessible energies would have new and peculiar implications in collider physics and cosmology. What makes this regime special is that as the energy grows, softer physics dominate. For example, the production of a regular heavy particle at the LHC would provide an event with very energetic (hard) jets from its decay. In contrast, the production of a mini-BH would be seen as a high multiplicity event, with dozens of less energetic (soft) jets. Analogously, the production of massive particles or mini-BHs in the early universe would have very different consequences. The heavier the elementary particle, the shorter its lifetime, which tends to decouple these particles from low temperatures. For mini-BHs is just the opposite, heavier BHs are colder and live longer. In addition, if an

---

Figure 5: Distribution $f(M, t)$ for $t_1 = H^{-1} = 1.4 \times 10^{14}$ GeV$^{-1}$ and $t_2 = 4H^{-1}$ with $T_0 = 100$ GeV.
approximate symmetry makes a massive relic long lived, its late decay will produce very energetic particles. BHs would imply a much softer spectrum of secondaries [19], with different cosmological consequences.

In this paper we have analyzed the dynamics of a two-component gas with photons and BHs in an expanding universe. The system is characterized by the temperature $T(t)$ of the radiation and the distribution $f(M,t)$ of BHs. Our equations take into account BH production, absorption of photons, BH evaporation, and collisions of two BHs. We have discussed the two generic scenarios that may result from an initial temperature close to the fundamental scale of gravity $M_D$. In the first scenario BHs empty of radiation the universe and dominate $\rho$ before a Hubble time, whereas in the second case there are few BHs that grow at constant $T$ up to $t \approx H^{-1}$.

We think that the work presented here is a necessary first step in the search for observable effects from these BHs. It could also lead us to a better understanding of the early universe at the highest temperatures.

Acknowledgments

We would like to thank Mar Bastero, Thomas Hahn and Iacopo Mastromatteo for useful discussions. This work has been supported by MEC of Spain (FIS2007-63364 and FPA2006-05294) and by Junta de Andalucía (FQM-101 and FQM-437). MB acknowledges a Juan de la Cierva fellowship from MEC of Spain.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429 (1998) 263 [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 436 (1998) 257 [arXiv:hep-ph/9804398].

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370 [arXiv:hep-ph/9905221].

[3] T. Banks and W. Fischler, “A model for high energy scattering in quantum gravity,” arXiv:hep-th/9906038; R. Emparan, Phys. Rev. D 64 (2001) 024025 [arXiv:hep-th/0104009]; S. B. Giddings and S. D. Thomas, Phys. Rev. D 65 (2002) 056010 [arXiv:hep-ph/0106219]; D. M. Eardley and S. B. Giddings, Phys. Rev. D 66
G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 630 (2002) 293 [arXiv:hep-ph/0112161].

S. W. Hawking, Nature 248 (1974) 30; S. W. Hawking, Commun. Math. Phys. 43 (1975) 199 [Erratum-ibid. 46 (1976) 206]; D. N. Page, Phys. Rev. D 13 (1976) 198.

S. Dimopoulos and G. L. Landsberg, Phys. Rev. Lett. 87 (2001) 161602 [arXiv:hep-ph/0106295].

A. S. Majumdar, Phys. Rev. Lett. 90 (2003) 031303 [arXiv:astro-ph/0208048]; R. Guedens, D. Clancy and A. R. Liddle, Phys. Rev. D 66 (2002) 083509 [arXiv:astro-ph/0208299].

B. J. Carr and S. W. Hawking, Mon. Not. Roy. Astron. Soc. 168 (1974) 399.

J. D. Barrow, E. J. Copeland and A. R. Liddle, Mon. Not. Roy. Astron. Soc. 253 (1991) 675; J. D. Barrow, E. J. Copeland and A. R. Liddle, Phys. Rev. D 46 (1992) 645.

P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B 441 (1998) 96 [arXiv:hep-th/9808138].

A. S. Majumdar and N. Mukherjee, Int. J. Mod. Phys. D 14 (2005) 1095 [arXiv:astro-ph/0503473].

N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D 59 (1999) 086004 [arXiv:hep-ph/9807344].

S. Hannestad, Phys. Rev. D 64 (2001) 023515 [arXiv:hep-ph/0102290]; L. J. Hall and D. Tucker-Smith, Phys. Rev. D 60 (1999) 085008 [arXiv:hep-ph/9904267].

G. D. Starkman, D. Stojkovic and M. Trodden, Phys. Rev. D 63 (2001) 103511 [arXiv:hep-th/0012226].

J. A. Conley and T. Wizansky, Phys. Rev. D 75 (2007) 044006 [arXiv:hep-ph/0611091].

R. H. Brandenberger and C. Vafa, Nucl. Phys. B 316 (1989) 391; M. Gasperini and G. Veneziano, Phys. Rept. 373 (2003) 1 [arXiv:hep-th/0207130]; B. A. Bassett, M. Borunda, M. Serone and S. Tsujikawa, Phys. Rev. D 67 (2003) 123506 [arXiv:hep-th/0301180]; M. Borunda and L. Boubekeur, JCAP 0610 (2006) 002 [arXiv:hep-th/0604055].

G. F. Giudice, T. Plehn and A. Strumia, Nucl. Phys. B 706 (2005) 455 [arXiv:hep-ph/0408320].
[17] S. Hannestad and G. G. Raffelt, Phys. Rev. D 67 (2003) 125008 [Erratum-ibid. D 69 (2004) 029901] [arXiv:hep-ph/0304029].

[18] D. N. Page, Phys. Rev. D 13 (1976) 198.

[19] P. Draggiotis, M. Masip and I. Mastromatteo, JCAP 0807 (2008) 014 [arXiv:0805.1344 [hep-ph]]; M. Masip and I. Mastromatteo, JCAP 0812 (2008) 003 [arXiv:0810.4468 [hep-ph]].