Model of rainwater runoff formation on the surface of complex topography

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Abstract. The article discusses the issues of water disposal, with curvilinear coatings. The tasks of calculating rainfall at industrial sites and urban areas of complex topography arise in the process of modern urban construction. Modern requirements for design quality necessitate accurate calculations of rainfall, location, size and number of drainage devices. The article points out the existence of a gap between the practice of calculating rainfall runoff and the theory of mathematical modeling of runoff. Also, modern mathematical models of the formation of rainfall are analyzed. A runoff model is proposed that provides a compatible consideration of the relationship between flow rates and water volumes in the area with the runoff genetic formula. The catchment area is torn by a network of lines of greatest slope and isochrones. For each elementary site, the ratio of the water balance is determined. The continuity equation for the catchment is obtained. For the numerical implementation of the runoff model, a difference scheme is used. Specific programs are provided for the developed algorithms. A computer program in the MatLab language has been developed for the proposed runoff model. The program has been tested at specific facilities.

1. Introduction
One of the main tasks in the complex of engineering preparation of urban areas is the drainage of rainwater. The ever-increasing requirements for design quality necessitate more accurate calculations of rainwater discharge on the surface, knowledge of the exact flow distribution, location, size and number of drainage devices, considering the geometry of the surface.

The drainage issues arising from the lack of a methodology for calculating rainwater flow on curved surfaces, on industrial sites and urban areas of complex topography are the result of the hydrological aspects of urbanization associated with modern urban construction and development, which necessitate the study of rainwater flow formation processes. Most notably, urbanization affects maximum water consumption, volume and shape of rainfall floods, as runoff from built-up areas differs sharply in quantity and quality from that from natural catchments. [1,2,3,4,5]

In practice, rainfall runoff is calculated on the assumption of water movement along the lines of the largest surface slope, and the lines are conditionally assumed to be straight lines, the surface is replaced by a plane with a slope equal to the average slope of the surface slope. As a rule, the expenditure based on this method is overestimated, which leads to an unjustified increase in the cross-sections of drainage devices. [6,7,8]
Known studies of rainwater flow formation in urbanized areas tend to focus on identifying empirical dependencies between some of the flow characteristics for a particular area.

The existing gap between the practice of rainwater discharge calculations and the theory of mathematical flow simulation is primarily due to the existence of building codes based on empirical formulas from many years ago.

However, the mathematical models of runoff formation in an urbanized area proposed by some researchers are entirely unacceptable for the practice of calculations because of their complexity and a large number of assumptions leading to distortion of maximum flows. [1,2]

Flow modelling on a homogeneous unfiltered surface (covering, urban area, industrial sites of complex shape) has its peculiarities and requires a new approach, the leading factor of flow, in this case, is the surface geometry.

Mathematical models of runoff are usually understood as mathematical and logical relations with the help of which quantitative associations between flow characteristics and characteristics of flow-forming factors are established. The definition considered is very general and includes both "information models", which do not contain any a priori information and are derived from the processing of specific observations, and "gnoseological models", which are built on understanding the physical patterns. [9,10]

In the first works on mathematical modeling of effluent formation processes on the watershed, the desire for a detailed description of elementary processes of effluent formation on the watershed was observed. Such models include a large number of parameters, which, as a rule, are physically interpreted, but they cannot be measured directly, and their definition is now an almost insoluble task. At the same time, attempts have been made to construct models in a purely formal way, based on the general theory of dynamic systems (the "black box" method) and very general concepts of the physical laws of flow formation. Both of these approaches do not allow to build a reliable model used for flow calculations in real catchments. The main reason for this is the separation of the process of selecting the model structure and determining its parameters. As a result, the models of the first type had a purely descriptive character and allowed only qualitative estimates for real catchments, or numerical experiments by changing their parameters to clarify the mechanism of physical processes occurring in the catchment. The models of the second type, using the developed mathematical apparatus of the system theory, well-matched input and output parameters, often remaining unsuitable for new implementations. [9,10,11,12,13,14]

In another formulation, the watershed is considered as a dynamic system with concentrated parameters, in which, generally speaking, both parameters and structure are unknown, but the class of equations in which the model structure is searched for, and on the values of parameters are imposed restrictions. Such limitations, which reduce the uncertainty of structure and parameters, are the general physical concepts of the processes of flow formation, empirical dependence, available additional observations.

Achievements in this direction are related to the development of linear models (with concentrated parameters) of water flow over the catchment for the time constant parameters of the hydrological system. [1,2,16,17]

The development of methods for identification of dynamic systems and, in particular, the development of methods for optimization of multiparameter systems allowed further transition in the process of mathematical modeling of runoff from the models, which described only the process of runoff, to the models (conceptual), which take into account various elementary processes of runoff genesis: water absorption, surface retention, runoff on the surface and others.

Models of the third type - with the distributed parameters, unlike conceptual and models with the concentrated parameters, allow to consider spatial variability of a drain, however, thus the problem of definition of parameters becomes complicated. Although models of this type more accurately reproduce the physical regularities of flow formation, difficulties associated with their numerical implementation, restrained the development of these models. [1,2,3,4,5]
Researches of some scientists have proved that unaffected transfer of parameters of conceptual models defined for natural catchments into models of urbanized catchments, even if at the same time the features of built-up areas are taken into account, can lead to errors of maximum flow order. [1,2,18,19]

2. Methods
It seems clear that the set of tasks associated with determining surface rainwater discharge requires consideration corresponding to the first level of optimization, which is to select the best management idea. At the same time, methods of heuristic programming developed for the search of the best approach to the decision of many technical problems, in this case, do not lead to success. The reason is in incomparability of difficulties in the mathematical description of technical systems and complex natural phenomenon - rainwater flow formation. For the same reason, the engineering optimization problem - the choice of optimal sections and location of drainage devices on a complex surface - cannot be reduced to the problem of mathematical programming, successfully used in solving many engineering problems. Let us note the possible approaches to its solution:

1. Installation of specially organized experiments, aimed at the study of the formation of runoff on specific surfaces.
2. Unification and formalization of methods of disciplines connected with different aspects of the considered problem, which is a prerequisite for its automated solution.

Formalization of methods to describe the formation of rainwater runoff involves the creation of an abstract prototype of the flow on the surface, which requires the use of methods of geometry and the theory of mathematical modeling of runoff.

The following approaches are possible when building a flow formation model for urbanized areas:

1. Simplification of known physically most reasonable flow models.
2. Creation of a fundamentally new model that most fully takes into account the physical and geometric features of surfaces of coatings, industrial sites, urban areas.
3. Physically and theoretically grounded association of several known models into one that is convenient for mass calculations.

Among approaches to building a flow model on an urbanized territory, the third is the most acceptable, as simplification of the known models involves several assumptions leading, as practice shows, to the distortion of the final results, and the creation of a fundamentally new requires an accurate knowledge of all processes of flow genesis in the watershed under consideration.

It is necessary to note some conditions at the construction of a mathematical model of a rainwater runoff: 1) nonfiltering or close to them on physical properties of surfaces are subject to consideration; 2) the necessity of the account of climatic factors of the concrete area that is important for building designing.

The first condition limits the area of application of the mathematical model, but at the same time allows obtaining reliable results for the considered type of surfaces.

The proposed runoff model provides for the joint consideration of the dependence between the flow and volume of water on the area with the genetic formula of runoff. Since the leading flow component in the proposed model is a surface runoff, the greatest attention is paid to specifying the topographic characteristics of the catchment surface.

On the given surface the geometrical prototype of movement of rain water is set - the family of lines of the largest slope determining the direction of water flow, areas of thickening of the slope lines are marked. The catchment area is covered by a network of the largest gable and isochronous lines (Fig 1).
Surface sections bounded by watershed lines serve as elementary catchment areas, which in turn are divided into private areas with equal running times. According to the genetic formula of runoff, the amount of rainwater flowing to the structure is calculated as follows:

\[ Q(t) = \int_0^t \frac{df}{dt} (h - S)_{t-\tau} d\tau \]  \hfill (1)

Or in vector form \( Q = f \cdot h \), \( Q \) is a vector-column of elementary expenditure values, \( f \) and \( h \) are triangular matrices of elementary platform values and intensive water transfer. The scheme of flow formation, according to this model, is performed by the system:

\[
\begin{align*}
Q_1 &= f_1 h_1 \\
Q_2 &= f_2 h_1 + f_1 h_2 \\
Q_\tau &= f_\tau h_1 + f_{\tau-1} h_2 + \ldots + f_1 h_\tau
\end{align*}
\]

where \( \tau \) - the time for which the most distant drop runs to the sash; \( f_1, f_2, \ldots, f_\tau \) - the areas of private catchment areas, bounded by lines of equal run (isochrons).

In the proposed runoff model, in contrast to the isochronous method, the water balance ratio is determined for each elementary catchment area bounded by equal catchment lines:

\[
\frac{\Delta V}{\Delta t} = (Q_1 - Q_2) + q \Delta S
\]  \hfill (2)

where \( \Delta V \) - change in water volume at the section under consideration for the time \( \Delta t \); \( Q_1, Q_2 \) - water discharge at the section boundaries; \( q \) - water inflow intensity at the section; \( S \) - section area.

It is assumed that \( \Delta V = \Delta h^* \Delta S \), \( \Delta h \) - change in runoff layer depth for time \( \Delta t \). Then

\[
\frac{\Delta V}{\Delta t} = \frac{\Delta h \Delta S}{\Delta t} = U_1 h_1 - U_2 h_2 + q \Delta S
\]  \hfill (3)

where \( U_1, U_2 \) are flow rates and \( h_1, h_2 \) are flow depths at the boundaries of the design section. The water flow equation is recorded as follows:

\[ U = ah^{n-1} \]  \hfill (4)

where \( a = C \cdot i \), \( i \) is the slope of the section (in case of approximation of the surface by linear splines, the slope \( i \) is understood as the arithmetic mean of the slopes of the sections constituting the private
section); C is the roughness coefficient, n=3√2 is an empirical constant. The water balance equation (3) is given in the form:

$$\frac{\Delta h \Delta S}{\Delta t} = ah_1^{n-1} - ah_2^{n-1} + q\Delta S$$

(5)

When we go to the limit, we get the continuity equation for the catchment:

$$\frac{dh}{dt} = a \frac{dn}{dt} + q(S,t)$$

(6)

In the interpolation nodes, the distance between which depends on the terrain features, it is necessary to set the initial information, primarily the intensity of the precipitation. The task is to perform interpolation of precipitation from a system of regular to irregular points. Linear interpolation is used:

$$H(x,y) = H_{ij}(1 - \alpha)(1 - \beta) + H_{i+1,j}(1 - \alpha)\beta + H_{i,j+1}a\beta + H_{i+1,j+1}(1 - \beta)a$$

(7)

$$\alpha = x/h \quad \beta = y/h$$

h - gauging cell side;

$$H_{ij} = \frac{1}{n} - m$$ - the amount of precipitation in the nodes of the gauge network.

For the numerical implementation of the proposed flow model, a four-point implicit difference scheme is used:

$$\frac{h_{i}^{j+1} - h_{i}^{j-1} + h_{i-1}^{j} - h_{i+1}^{j}}{2\Delta t} + \frac{1}{\Delta S_j}C\sqrt{i}[(h_{i}^{j})^n - (h_{i-1}^{j})^n] - \frac{1}{4}(q_{i+1}^{j} - q_{i}^{j+1} + q_{i-1}^{j-1} + q_{i}^{j+1}) = 0$$

(8)

Unlike the known schemes in the proposed one, the step on one of the parameters is taken unevenly. Step ΔSj means the area j - that elementary area limited by lines of equal run-up. [15]

Numerical experiments have shown good convergence of the difference scheme (8) to the differential equation (6). The scheme (8) is stable at any ratio of Δt and ΔS.

3. Results

A computer program in the MatLab language has been developed for the proposed runoff model. The program has been tested in specific areas of the cities of Uzbekistan. The developed program can work both autonomously and as part of CAD.

4. Conclusions

The proposed model, unlike the known methods, has the following advantages:

1. Takes into account the geometric and climatic features of the catchment.
2. A computer program not only calculates rainfall, but also draws a network of slope lines and isochrones on the surface.
3. A built network of slope lines and isochrones is a prerequisite for the optimal placement of drainage devices.
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