Disentangling and broadcasting an entangled state simultaneously by asymmetric cloning

Yafei Yu, Jian Feng, Xiaoqing Zhou, Mingsheng Zhan

State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics,
Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences,
Institute of Optical Communication, Liaocheng University, Liaocheng, 252059, Shandong, PR China

We construct a quantum machine which, by using asymmetric cloner, deals with disentangling and broadcasting entanglement in a single unitary evolution. The attainable maximum value of the scaling parameter $s$ for disentangling is identical to that obtained in previous works. The fidelity of the cloning state with respect to the input entangled state is state-dependent.

PACS number(s): 03.67.HK

I. INTRODUCTION

Quantum entanglement plays an important role in quantum information field. And the manipulation of entanglement, such as purification, broadcasting, is an intriguing issue in the research for entanglement. Recently disentanglement attracts a lot of attention. Disentanglement is a process in which an initial entangled state of composite system can be transformed into separable state with unaffectioning the reduced density matrices of subsystems. However like other "no-going" theorems (e.g., the no-cloning theorem, no-deleting theorem, no-broadcasting theorem), the perfect disentanglement also is prohibited by elementary rules of quantum mechanics. But dropping the constraint that the reduced density matrices of subsystems is perfectly unaffected, the approximate disentanglement can be realized by local operation, for example, by local cloning and by teleportation via separable channels. In other words, for a two-qubit entangled state, the transformation can be achieved:

$$\rho^{\text{ent}} \rightarrow \rho^{\text{disent}}$$

together with

$$\text{Tr}_j(\rho^{\text{disent}}) = s_i \text{Tr}_j(\rho^{\text{ent}}) + \left(\frac{1 - s_i}{2}\right)I, i \neq j; i, j = 1, 2$$

for all $\rho^{\text{ent}}$, where $s_i$ ($0 < s_i < 1$ for $i = 1, 2$) is a scaling parameter independent of $\rho^{\text{ent}}$, standing as a measure of closeness of the $i$th reduced density matrix before and after the transformation. The attainable values of $s_1$ and $s_2$ satisfies the inequality $s_1 s_2 \leq \frac{1}{3}$. More explicitly, if only one party undergoes local operation, i.e., $s_1 = 1$ (or $s_2 = 1$), the maximum value of $s_2$ (or $s_1$) is $\frac{1}{\sqrt{3}}$; if both two parties undergo the same local operation separately, the maximum value of $s_2$ (or $s_1$) is $\frac{1}{\sqrt{3}}$.

On the other hand, the connection between copying (cloning and broadcasting) and disentanglement is noted. It is found that the conditions of perfect disentanglement into product states corresponds to cloning of one of the subsystems while the conditions of perfect disentanglement into separable states corresponds to broadcasting of one of the subsystems. And comparing the schemes of disentanglement and broadcasting entanglement by local symmetric cloning, one can notice that for disentanglement the reduction factor $\eta$ describing the quality of symmetric cloner satisfies $\eta \leq \frac{1}{\sqrt{3}}$, whereas for broadcasting it needs $\eta \geq \frac{1}{\sqrt{3}}$ ($\eta = \frac{2}{3}$ for the optimal quantum cloner). In the meantime, disentanglement erases the quantum correlation (inseparability) between subsystems as broadcasting needs to retain the quantum correlation between subsystems, except that in both process the output reduced density matrix of each subsystem is as close as possible to the corresponding input reduced density matrix. There is an intuitive understanding that the disentangling state of an entangled state can be viewed as a separable copy of it. This suggests that it is possible to look for a unified way of dealing with broadcasting and disentangling an entangled state simultaneously.
In this short note, we substitute asymmetric (isotropic) cloner for the symmetric (isotropic) cloner in the schemes of broadcasting entanglement and disentanglement by local cloning. Therefore a quantum machine is constructed, which can implement the broadcasting and disentanglement of an entangled state in a single unitary evolution. In Section II we first discuss the mechanics of the quantum machine in the method similar to entanglement splitting, then consider its functions in the framework of entanglement broadcasting. Finally, the conclusions is given in Section III.

II. DISENTANGLING AND BROADCASTING BY ASYMMETRIC CLONING

In this section we will analyze the actions of the quantum machine which deals with disentangling and broadcasting an entangled state simultaneously. The mechanics of the quantum machine is discussed in the method similar to entanglement splitting. Then the functions of the quantum is considered in the framework of entanglement broadcasting. That is, the discussion is divided into two cases, using the cloner on only one party and on both parties separately. For simplicity, the discussion focuses on the case of qubit, and only 1→2 asymmetric (isotropic) cloner is exploited.

The actions of asymmetric cloner in general is described by a Pauli channel. But, for convenience, here the actions of asymmetric cloner is specified by a particular unitary transformation on the state \( |\psi\rangle \) of the cloner. The asymmetric cloning transformation outputs two copies with different fidelities for all input states. And the distribution of information at the outputs of the cloner is controlled via changing the value of \( p \) which satisfies the no-cloning inequality deduced from Eq. [6] in [17]. Therefore the distribution of information at the outputs of the cloner is controlled via changing the value of \( p \). In the following this family of asymmetric cloners will be used to obtain a disentangling state and a broadcasting state of an entangled state simultaneously.

A. only copying one qubit asymmetrically

We first discuss why the asymmetric cloner can be used to disentangle and broadcast entanglement. The discussion is in the method similar to entanglement splitting, i.e., only one qubit is copied by asymmetric cloning.

Suppose qubits \( a_I \) and \( a_{II} \) share an entangled state

\[
|\chi\rangle = \alpha |00\rangle_{a_Ia_{II}} + \beta |11\rangle_{a_Ia_{II}},
\]

where \( \alpha \) and \( \beta \) are defined as before. Let the asymmetric cloner copy one qubit (say, qubit \( a_{II} \)) as doing in entanglement splitting and output two copies on the qubits \( a_{II} \) and \( b_{II} \). Then the original state \( |\chi\rangle \) is splitted into two branches. To
the purpose of dealing with disentanglement and entanglement broadcasting simultaneously, the quantum correlation in one branch is erased whereas that in another needs to be partially retained. The state of each branch can be described as:

\[
\rho_{a_{1}a_{II}}^{\text{out}} = \frac{1 + p^2}{N} \alpha^2 \left| 00 \right> \left< 00 \right| + \frac{1 + p^2}{N} \beta^2 \left| 11 \right> \left< 11 \right| + \frac{2}{N} \alpha^2 \left| 01 \right> \left< 01 \right| \\
+ \frac{2}{N} \beta^2 \left| 10 \right> \left< 10 \right| + 2 \frac{p}{N} \alpha \beta \left| 00 \right> \left< 11 \right| + 2 \frac{p}{N} \alpha \beta \left| 11 \right> \left< 00 \right| \tag{9}
\]

\[
\rho_{a_{II}b_{1}}^{\text{out}} = \frac{1 + p^2}{N} \alpha^2 \left| 00 \right> \left< 00 \right| + \frac{1 + p^2}{N} \beta^2 \left| 11 \right> \left< 11 \right| + \frac{2}{N} \alpha^2 \left| 01 \right> \left< 01 \right| \\
+ \frac{2}{N} \beta^2 \left| 10 \right> \left< 10 \right| + 2 \frac{p}{N} \alpha \beta \left| 00 \right> \left< 11 \right| + 2 \frac{p}{N} \alpha \beta \left| 11 \right> \left< 00 \right| \tag{10}
\]

By applying the Peres-Horodeck theorem [18,19] to test the inseparability of \(\rho_{a_{1}a_{II}}^{\text{out}}\) and \(\rho_{a_{II}b_{1}}^{\text{out}}\), it turns out that if we require that \(\rho_{a_{1}a_{II}}^{\text{out}}\) is inseparable while \(\rho_{a_{II}b_{1}}^{\text{out}}\) is separable for any input pure two-qubit entangled state, the values of \(p\) must satisfy \(\sqrt{3} - 1 \leq p \leq 1\). That is, \(\rho_{a_{1}a_{II}}^{\text{out}}\) is a copying state of the initial entangled state \(|\chi\rangle\) while \(\rho_{a_{II}b_{1}}^{\text{out}}\) is a disentangling state of it when the parameter \(p\) specifying the asymmetric cloning takes value between \(\sqrt{3} - 1\) and 1. When \(\alpha = \beta = \frac{1}{\sqrt{2}}\), two output density matrices can be rewritten as:

\[
\rho_{a_{1}a_{II}}^{\text{out}} = \frac{1}{2} \left( \frac{1 + p^2}{N} \left| 00 \right> \left< 00 \right| + \frac{1 + p^2}{N} \left| 11 \right> \left< 11 \right| + \frac{2}{N} \left| 01 \right> \left< 01 \right| + \frac{2}{N} \left| 10 \right> \left< 10 \right| + 2 \frac{p}{N} \left| 00 \right> \left< 11 \right| + 2 \frac{p}{N} \left| 11 \right> \left< 00 \right| \right) \tag{11}
\]

\[
\rho_{a_{II}b_{1}}^{\text{out}} = \frac{1}{2} \left( \frac{1 + p^2}{N} \left| 00 \right> \left< 00 \right| + \frac{1 + p^2}{N} \left| 11 \right> \left< 11 \right| + \frac{2}{N} \left| 01 \right> \left< 01 \right| + \frac{2}{N} \left| 10 \right> \left< 10 \right| + 2 \frac{p}{N} \left| 00 \right> \left< 11 \right| + 2 \frac{p}{N} \left| 11 \right> \left< 00 \right| \right) \tag{12}
\]

According to the viewpoint in [16,17], it is clear that the qubits \(a_{II}\) and \(b_{II}\) emerge from depolarizing channels of probability \(P = 3 \frac{(1-p)^2}{2N}\) and \(P' = 3 \frac{p^2}{2N}\), respectively. Hence the variation of the parameter \(p\) changes the capacities of two quantum channels such that the quantum correlation of the initial entangled state \(|\chi\rangle\) is filtered in the branch \(a_{II}b_{II}\) but partially transferred to the branch \(a_{1}a_{II}\). And it is noted that the clone of \(\sqrt{3} - 1 \leq p \leq 1\) corresponds to the case of \(x \geq \frac{1}{\sqrt{3}}\) in Fig.2 in Ref. [17].

Hence the flow of information in the asymmetric cloning is controlled by varying the value of \(p\) of the cloner. Because of this ability, disentangling and broadcasting an entangled state is possible to be achieved in a single unitary evolution.

**B. copying both qubits separately**

In the previous subsection we have discussed the mechanics of the quantum machine which does with disentangling and broadcasting an entangled state in a single unitary evolution, and checked the features of the asymmetric cloner employed. In this subsection we consider how the quantum machine achieves the goal of combining disentanglement and broadcasting in a united way. The goal is achieved by copying both qubits separately.

Now both two qubits in the state \(|\chi\rangle\) are cloned according to the transformation defined by Eqs. (3) and (4) separately. Then two copies \(\rho_{a_{1}a_{II}}^{\text{out}}\) and \(\rho_{b_{1}b_{II}}^{\text{out}}\) of the entangled state \(|\chi\rangle\) are produced. What we want to do is to obtain a disentangling state and a copying state of the state \(|\chi\rangle\). We check the inseparability of two copies \(\rho_{a_{1}a_{II}}^{\text{out}}\) and \(\rho_{b_{1}b_{II}}^{\text{out}}\). The output density matrices \(\rho_{a_{1}a_{II}}^{\text{out}}\) and \(\rho_{b_{1}b_{II}}^{\text{out}}\) are given by

\[
\rho_{a_{1}a_{II}}^{\text{out}} = \frac{(1 + p^2)^2}{N^2} \alpha^2 \left| 00 \right> \left< 00 \right| + \frac{(1 + p^2)^2}{N^2} \beta^2 \left| 11 \right> \left< 11 \right| + \frac{4}{N^2} \alpha^2 \left| 01 \right> \left< 01 \right| + \frac{4}{N^2} \beta^2 \left| 10 \right> \left< 10 \right| + 4 \frac{p}{N^2} \alpha \beta \left| 00 \right> \left< 11 \right| + 4 \frac{p}{N^2} \alpha \beta \left| 11 \right> \left< 00 \right| \tag{13}
\]
\[
\rho_{b_{11}}^{\text{out}} = \left( \frac{(1+q^2)^2}{N^2} - \frac{p^2}{N^2} \right) |00\rangle \langle 00| + \left( \frac{(1+q^2)^2}{N^2} - \frac{p^2}{N^2} \right) |11\rangle \langle 11| \\
+ \frac{4sq\alpha\beta}{N^2} |00\rangle \langle 11| + \frac{4sq\alpha\beta}{N^2} |11\rangle \langle 00|. 
\]

(14)

Again, it follows from Peres-Horodecki theorem that if \( \frac{1-\sqrt{3}+\sqrt{3}}{2} \leq p \leq 1 \), \( \rho_{a_{11}a_{11}}^{\text{out}} \) is inseparable for

\[
\frac{1}{2} - \frac{1}{4} - \frac{(1+p^2)(1-p^2)^2}{4p^2} \leq \alpha^2 \leq \frac{1}{2} + \frac{1}{4} - \frac{(1+p^2)(1-p^2)^2}{4p^2},
\]

however \( \rho_{b_{11}}^{\text{out}} \) is separable for any values of \( \alpha^2 \). So by choosing appropriate value of \( p \) of the asymmetric cloner, the copying state \( \rho_{a_{11}a_{11}}^{\text{out}} \) and the disentangling state \( \rho_{b_{11}}^{\text{out}} \) can be obtained in a single evolution.

For the copying state \( \rho_{a_{11}a_{11}}^{\text{out}} \), the fidelity with respect to the original entangled state \( |\chi\rangle \) is examined. The fidelity is defined as

\[
F = \langle \chi | \rho_{a_{11}a_{11}}^{\text{out}} | \chi \rangle = \left( 1 + \frac{p^2}{N^2} - \frac{8pq^2}{N^2} |\alpha^2| \right)^2.
\]

(16)

Obviously, the fidelity \( F \) is dependent on the input entangled state \( |\chi\rangle \). For the disentangling state \( \rho_{b_{11}b_{11}}^{\text{out}} \), the factors \( s_i \) of qubits \( b_I \) and \( b_{II} \) are inspected.

\[
\rho_{b_I}^{\text{out}} = Tr_{b_{II}}(\rho_{b_{11}b_{11}}^{\text{out}}) = \frac{2q}{N} (\alpha^2 |0\rangle \langle 0| + \beta^2 |1\rangle \langle 1|) + \frac{p^2}{N} (|0\rangle \langle 0| + |1\rangle \langle 1|)
\]

\[
= \frac{2q}{N} Tr_{a_{II}}(|\chi\rangle \langle \chi|) + \frac{p^2}{N} I,
\]

(17)

\[
\rho_{b_{II}}^{\text{out}} = \frac{2q}{N} Tr_{a_I}(|\chi\rangle \langle \chi|) + \frac{p^2}{N} I.
\]

(18)

It follows that \( s_{b_I} = s_{b_{II}} = \frac{2(1-p)}{N} \) (p=1-q). In the range of \( \frac{1-\sqrt{3}+\sqrt{3}}{2} \leq p \leq 1 \), \( s_{b_I} = s_{b_{II}} \leq \frac{1}{\sqrt{N}} \). So the maximum value of closeness which can be achieved by this process is \( \frac{1}{\sqrt{3}} \) as in \( [13] \). Of course, the maximum value \( \frac{1}{3} \) of \( s \) can be achieved in the quantum machine by copying only one qubit.

Moreover it is worthwhile to notice that, the range of value of \( p \) in the case of copying two qubits separately is not simply equal to that in the case of copying only one qubit as a result of the lost of quantum information when the entanglement is broadcasted by local cloning.

Therefore by copying both two qubits asymmetrically it is possible to combine disentanglement and broadcasting of entanglement in a single unitary evolution. The fidelity of the output copying state with respect to the input entangled state is state-dependent. While the scaling parameter \( s \), which can be achieved by the proposed quantum machine, has the same range as in the work \( [13] \).

### III. CONCLUSION

We have proposed a quantum machine, which for an input entangled state produces a disentangling state and a copying state in a single unitary evolution. The machine is based on the asymmetric cloner. The flow of information in the cloning process is controlled by varying the parameter \( p \) so that the quantum entanglement is partially retained in one copy of the entangled state but erased in another. If using the 1→2 asymmetric teleporting in quantum machine, we can distantly send a copying state of the entangled state to a receiver and a disentangling state of it to another according to the requirement of information distribution.

To conclude, in this short note the disentanglement and broadcasting is combined in a single evolution. We hope that it is helpful for understanding entanglement and useful for further studying quantum information and quantum computation.

**Acknowledgements**
This work has been financially supported by the National Natural Science Foundation of China under the Grant No.10074072.

[1] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, W. K. Wootters, Phys. Rev. Lett. 76, 722 (1996); D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, Phys. Rev. Lett. 77, 2818 (1996).
[2] V. Buzek, V. Vedral, M. B. Plenio, P. L. Knight, and M. Hillery, Phys. Rev. A 55, 3327 (1997).
[3] S. Bandyopadhyay and G. Kar, Phys. Rev. A 60, 3296 (1999).
[4] D. R. Terno, Phys. Rev. A 59, 3320 (1999).
[5] T. Mor, Phys. Rev. Lett. 83, 1451 (1999).
[6] W. K. Wootters and W. H. Zurek, Nature (London) 299, 802 (1982).
[7] A. K. Pati and S. L. Braunstein, Nature (London) 404, 164 (2000).
[8] H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa, and B. Schumacher, Phys. Rev. Lett. 76, 2818 (1996).
[9] S. Ghosh, S. Bandyopadhyay, A. Roy, D. Sarkar, and G. Kar, Phys. Rev. A 61, 052301 (2000).
[10] S. Bandyopadhyay, G. Kar, and A. Roy, Phys. Lett. A 258, 205 (1999).
[11] S. Ghosh, G. Kar, A. Roy, D. Sarkar, and U. Sen, Phys. Rev. A 64, 042114 (2001).
[12] T. Mor and D. R. Terno, Phys. Rev. A 60, 4341 (1999).
[13] S. Bandyopadhyay, G. Kar, and A. Roy, Phys. Lett. A 258, 205 (1999).
[14] D. Bruss, D. P. Divincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin, Phys. Rev. A 57, 2368 (1998).
[15] D. Bruss, Phys. Rev. A 60, 4344 (1999).
[16] N. J. Cerf, Acta Phys. Slov. 48, 115 (1998).
[17] N. J. Cerf, Phys. Rev. Lett. 84, 4497 (2000).
[18] A. Peres, Phys. Rev. Lett. 77, 1413 (1996).
[19] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A 223, 1 (1996).
[20] V. Buzek, M. Hillery, and R. Bednik, Acta Phys. Slov. 48, 177 (1998).