Abstract. We study $J/\psi$ production in $pp$ collisions at $\sqrt{s} = 1.96$ and 7 TeV using the Colour-Singlet Model (CSM), including next-to-leading order (NLO) corrections and dominant $\alpha_s^5$ contributions (NNLO*). We find that the CSM reproduces the existing data if the upper range of the NNLO* is near the actual –but presently unknown – NNLO. The direct yield polarisation for the NLO and NNLO* is increasingly longitudinal in the helicity frame when $P_T$ gets larger. Contrary to what is sometimes claimed in the literature, the prompt $J/\psi$ yield polarisation in the CSM is compatible with the experimental data from the CDF collaboration, when one combines the direct yield with a data-driven range for the polarisation of $J/\psi$ from $\chi_c$.

1. Introduction

The numerous quarkonium-production puzzles at hadron colliders were attributed not too long ago to non-perturbative effects associated with channels in which the heavy quark-pair is produced in a colour-octet state [1]. $\alpha_s^4$ and $\alpha_s^5$ corrections to the CSM [2] are now widely recognised as essential to understand the $P_T$ spectrum of $J/\psi$ and $\Upsilon$ produced in high-energy hadron collisions [3, 4, 5, 6, 7]. This calls for a factorised description of high-$P_T J/\psi$ beyond leading power [8]. The effect of QCD corrections is also manifest in the polarisation predictions. While the $J/\psi$ and $\Upsilon$ produced inclusively or in association with a photon are predicted to be transversely polarised at LO, it has been found that their polarisation at NLO is increasingly longitudinal when $P_T$ gets larger [5, 6, 9]. In recent works [10, 11], we have also shown that the CSM alone is sufficient to account for the magnitude of $d\sigma/dy$ at RHIC, Tevatron and LHC energies.

We evaluate here the $P_T$ dependence of the $J/\psi$ yield and its polarisation at Tevatron and LHC energies. We describe the procedure used to obtain a first evaluation of some dominant contributions at $\alpha_s^5$ (NNLO*) in addition to the yield at NLO (up to $\alpha_s^4$). We then compare available data from the Tevatron and the LHC with our results: the direct yields differential in $P_T$ along with the polarisation vs $P_T$ for the prompt yield using an essentially data-driven estimation of the polarisation for $J/\psi$ from $\chi_c$. 

$J/\psi$ production at $\sqrt{s} = 1.96$ and 7 TeV: Color-Singlet Model, NNLO* and polarisation

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2. Cross-section

For the NLO cross section, we use the partonic matrix elements of [3]. In order to investigate the expected impact of NNLO QCD corrections for increasing $P_T$, we also present the NLO results plus the real-emission contributions at $\alpha_s^3$ evaluated along the lines of [6], referred to as NNLO*. At $\alpha_s^3$, the last kinematically-enhanced topologies open up, with a $P_T^{-4}$ fall off of $d\sigma/dP_T^2$. The procedure used here for the NNLO* is exactly that of [6]: the real-emission contributions at $\alpha_s^3$ are evaluated using MADONIA [12] by imposing a lower bound on the invariant-mass squared of any light partons ($s_{ij}$). The dependence on this cut should decrease for larger $P_T$ since no collinear or soft divergences can appear there for the new channels opening up at $\alpha_s^3$ with a leading-$P_T$ behaviour, i.e. the ones which interest us. For other channels, whose Born contribution is at $\alpha_s^5$ or $\alpha_s^3$, the cut would produce logarithms of $s_{ij}/s_{ij}^{\text{min}}$. These are not necessarily small, but they are expected to be factorised over their corresponding Born contribution, which scales as $P_T^{-8}$ or $P_T^{-6}$. They are thus suppressed by at least two powers of $P_T$ with respect of the leading-$P_T$ contributions ($P_T^{-4}$). The sensitivity on $s_{ij}^{\text{min}}$ is expected to be small at large $P_T$.

![Figure 1](image1)

Figure 1: $d\sigma/dP_T \times \text{Br}$ for direct $J/\psi$ production from NLO and NNLO* CS contributions at $\sqrt{s} = 1.96$ TeV (left) and at $\sqrt{s} = 7$ TeV for central (middle) and forward (right) rapidities. These are compared to the CDF [14], ATLAS, CMS and LHCb data [15, 16, 17] multiplied by a constant direct fraction from CDF [18]. See text for details on theoretical-error bands.

Our results are shown on Fig. 1. The CSM is very close to the existing data, if the upper range of the NNLO* is a relevant evaluation of the NNLO. The uncertainty bands at NLO are obtained from the combined variations of the charm-quark mass ($m_c = 1.5 \pm 0.1$ GeV), the factorisation $\mu_F$ and the renormalisation $\mu_R$ scales chosen in the couples $((0.75, 0.75); (1, 1); (1, 2); (2, 2)) \times m_T$ with $m_T^2 = 4m_Q^2 + P_T^2$. The band for the NNLO* is obtained using a combined variation of $m_c$, $0.5m_T < \mu_R = \mu_F < 2m_T$ and $2.25 < s_{ij}^{\text{min}} < 9.00$ GeV$^2$. We have used the NLO set CTEQ6_M [13] and have taken $|R_{J/\psi}(0)|^2 = 1.01$ GeV$^3$ and Br($J/\psi \to \ell^+\ell^-$) = 0.0594.

‡ We do not expect any further kinematical enhancement as regards the $P_T$ dependence when going further in the $\alpha_s$ expansion: $P_T^{-4}$ is the slowest possible fall-off. Above $\alpha_s^3$, usual expectations for the impact of QCD corrections would then hold. One would expect a $K$ factor multiplying the yield at NNLO accuracy, which would be independent of $P_T$ and of a similar size as those of other QCD processes. A further enhancement by an order of magnitude between the NNLO and N$^3$LO results would be quite worrisome.
3. Polarisation

The polarisation parameter $\alpha$ is extracted bin by bin in $y$ or $P_T$ from the normalised distribution of the polar angle $\theta$ between the $\ell^+$ direction in the $J/\psi$ rest frame and its direction in the laboratory frame, $I(\cos \theta) = \frac{3}{2(\alpha+3)}(1 + \alpha \cos^2 \theta)$. We thus work in the helicity frame. $\alpha$ is also related to a ratio of the polarised cross sections: $\alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$.

For the time being, there does not exist any measurement of direct $J/\psi$ polarisation, i.e. after the extraction of the $\chi_c$ feed-downs (up to 30-40 %) which may strongly impact on the observed values of $\alpha$. It is however possible to constrain its effects by using existing data on $\sigma_{\chi_c 1}/\sigma_{\chi_c 2}$ and by relying on $E_1$ dominance for the transition $\chi_c \to J/\psi + \gamma$.

Indeed, using $E_1$ dominance [19], one can obtain [21] a range of the yield of longitudinally (transversely) polarised $J/\psi$ in terms of simple relations involving the polarised $\chi_c$ yields. Allowing for extreme cases, these relations allow the yield from $\chi_c$ to be fully transversely polarised, while there is a minimal value of $\alpha$. Following the discussion of [21] and taking $R_{12} = \frac{\sigma_{\chi_c 1} \text{Br}(\chi_c 1 \to J/\psi)}{\sigma_{\chi_c 2} \text{Br}(\chi_c 2 \to J/\psi)} = 2.5 \pm 0.1[20]$, one obtains $\alpha_{\text{from } \chi_c}^{\text{min}} \approx -0.42$, rather different than -1.

![Figure 2: Comparison between the extrapolation of $\alpha$ for prompt $J/\psi$ in $pp$ at $\sqrt{s} = 1.96$ TeV (red band), the direct NLO $\alpha$ (gray line), the direct NNLO* $\alpha$ (thinner dark-red band) and the CDF data [22] for prompt $J/\psi$.](image)

Since 30% of the $J/\psi$ come from $\chi_c$ nearly independent of $P_T$ in the range considered here [18], we expect a partial contribution to the polarisation ranging from $0.3 \times (+1)$ to $0.3 \times (-0.42)$. Regarding the other 70%, one multiplies the result for the direct yields by 0.7, since the polarisation of $J/\psi$ from $\psi(2S)$ is expected to be identical to the direct one. Doing so, one obtains the extrapolation shown on Fig. 2. If the $J/\psi$ from $\chi_c$ yield is strongly transversely polarised (the upper limit), the polarisation of the prompt yield is in rather good agreement with the data.

4. Conclusion

We have evaluated the NLO and NNLO* $J/\psi$ yield at Tevatron and LHC energies. As found for $\Upsilon$ at the Tevatron [6] and for $J/\psi$ at RHIC [21], the upper bound of the CSM predictions is very close to the experimental data from CDF, ATLAS, CMS and LHCb. However, the
\( J/\psi \) production at \( \sqrt{s} = 1.96 \) and 7 TeV

NNLO* evaluation is not a complete NNLO calculation. It is affected by logs of an IR cut-off whose effect might not vanish as quickly as one has anticipated. It may very well be that the upper limit of the prediction –close to the data– accurately reproduces the complete NNLO yield, or that the lower limit –close to the NLO yield– reproduces the NNLO yield. If the upper limit of the NNLO* does indeed overestimate the NNLO, the CSM alone is likely insufficient to account for the data. Conversely, the CSM alone is enough and the colour-octet contributions are not required.

As regards polarisation, we have derived a range for the prompt yield polarisation. This range is affected by admittedly large theoretical uncertainties, but the upper edge –corresponding to a transversely polarised feed-down– is in rather good agreement with the data from the Tevatron. We recall that the trend for a longitudinally (direct) \( \psi(2S) \) yield [22] was also met by the NNLO* [7].

In conclusion, the CSM may very well provide a good description of \( J/\psi \) production in high-energy \( pp \) collisions, both in terms of the cross section and the yield polarisation.

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