The ion-acoustic turbulence in the skin layer of the inductively coupled plasma

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The theory of the nonmodal ion-acoustic instability in the skin layer of the inductively coupled plasma (ICP) is developed. This instability has a time-dependent growth rate and is driven by the current formed in the skin layer by the accelerated motion of electrons relative to ions under the action of the ponderomotive force. We found that the development of the ion-acoustic turbulence (IAT) in the skin layer and the scattering of electrons by IAT are primary channels of the nonlinear absorption of the RF energy in the skin layer.

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I. INTRODUCTION

The inductively coupled plasma (ICP) is a widespread and dominant plasma source for materials processing applications and operated in the low gas pressure regime of a few mTorr range. In this regime, the electron mean-free-path is comparable to the characteristic size of plasma devices, and the electron collision frequency with neutrals is much less than the driving frequency, \( \omega_0 \), of the applied electromagnetic (EM) wave. The absorption of the RF energy by electrons in this regime occurs due to the breaking of the phase coherence between the velocity of the individual thermal electron and the spatially nonuniform EM wave with frequency \( \omega \) much less than the electron plasma frequency \( \omega_p e \). The RF electric field in ICPs with such a frequency is non-propagating and localized mainly in the skin layer near the plasma boundary adjacent to the RF antenna, where the main interaction of EM fields with plasma electrons takes place. This phenomenon involving the spatially inhomogeneous RF fields and nonlocal electron kinetics is well-known in plasma physics as the Landau damping effect. This regime of the ICP operation is also named as the regime of the anomalous skin effect.

It is well known, however, that the relative motion of electrons and ions in a plasma under the action of EM waves is the source of the development of numerous parametric and current driven instabilities.9,10 The development of these instabilities is an alternative channel of the absorption of the EM wave and heating of plasma components, which is not included in the linear theory of the anomalous skin effect. It was found that the uniformly accelerated motion of electrons occurs in the skin layer, which stems from the ponderomotive force formed by the coupled action of the electric and the magnetic component of EM waves.9 The kinetic stability theory of the plasma with accelerated electrons11 found that the electrons accelerated in skin layer triggers the current-driven instabilities. The analytical treatment of the instabilities driven by a current with spatially inhomogeneous or time-dependent current velocity can’t be investigated by employing the normal mode analysis, which assumes that the plasma perturbations have a structure \( \sim \exp (i \mathbf{k} r - \omega t) \) of a plane wave.

We employed the nonmodal approach,12 which starts with the transformation of the position \( \mathbf{r} \) and the velocity \( \mathbf{v} \) variables of the Vlasov equation for the distribution function \( F_\alpha (\mathbf{v}, \mathbf{r}, t) \) of species \( \alpha \) (ions or electrons) determined in the laboratory frame to the variables \( \mathbf{r}_\alpha \) and \( \mathbf{v}_\alpha \) determined in the frame moving with spatially inhomogeneous time-dependent velocity \( \mathbf{V}_\alpha (\mathbf{r}, t) \). In our approach, this spatially inhomogeneous velocity is determined by the Euler equation for the ideal fluid of particles species \( \alpha \) immersed in the spatially inhomogeneous non-stationary EM field. The Vlasov equation in the moving coordinates \( \mathbf{r}_\alpha \) and \( \mathbf{v}_\alpha \) has the same form as that in a plasma without an external EM field at a finite time interval during which a particle does not move into appreciably different regions of the EM field. The solution of this Euler equation was derived in our previous study13 for the case of the high frequency RF wave, for which the force by the RF electric field acting on electrons in the skin layer prevails over the Lorentz force by the RF magnetic field. It was found that the electron velocity \( \mathbf{V}_e \)
in this case is equal to the electron accelerated velocity under the action of the ponderomotive force.

For the frequency range of the applied RF wave corresponding to the classical skin effect\textsuperscript{[3,4]}, it was revealed\textsuperscript{[2]} that the accelerated electrons leave the skin layer in a short time for the strong RF field for which the ponderomotive current velocity $U$ is larger than the electron thermal velocity $v_{Te}$. This time appears to be insufficient for the development of the Buneman instability in the skin layer, which excites when the current velocity is larger than the electron thermal velocity. It was also found that electrons accelerated in the skin layer can trigger the ion-acoustic (IA) instability in the bulk plasma past the skin layer when the quasi-steady electron current velocity exceeds the IA velocity $v_s = (T_e/m_e)^{1/2}$ in this region\textsuperscript{[3,4]}.

In this paper, we apply our nonmodal approach to the theory of the IA instability driven by the weak RF field, for which the accelerated electron current velocity in the skin layer is less than the electron thermal velocity, but exceeds the IA velocity. Under this condition, the accelerated electrons can develop the IA instability in the skin layer. In Sec. II, we present basic transformations of the system of the Vlasov-Poisson equations, employed in the developed theory. We introduce a more general and simple solution of the Euler equation, presented in Appendix, for the electron velocity $V_e(r,t)$ in the decaying EM field without an initial assumption of the stronger electric force than the Lorentz force acting on electrons in the skin layer. In Sec. III, we present the theory of the IA instability in the skin layer driven by the accelerated electrons. The nonlinear theory of the IA turbulence in the skin layer is presented in Sec. IV followed by conclusions in Sec. V.

II. THE NONMODAL APPROACH TO THE THEORY OF THE INSTABILITIES DRIVEN BY THE ACCELERATED CURRENT IN THE SKIN LAYER

In this paper, we consider the effect of the relative motion of plasma species on the development of the short-scale electrostatic perturbations in the skin layer under the condition of the classical skin effect. The skin effect is classified as classical or normal when the frequency $\omega_0$ of the RF field belongs to the frequency range\textsuperscript{[3,4]}

$$\frac{\omega_{pe}v_{Te}}{c} \ll \omega_0 \ll \omega_{pe},$$

where $\omega_{pe}$ is the electron plasma frequency, and $v_{Te}$ is the electron thermal velocity.

We consider a plasma occupying region $z \geq 0$. The RF antenna which launches the RF wave with a frequency $\omega_0$ is assumed to exist to the left of the plasma boundary $z = 0$. In the frequency range of Eq. (1), the electric, $E_0$, and the magnetic, $B_0$ fields of the RF wave are exponentially decaying with $z$ and sinusoidally varying with time,

$$E_0(z,t) = e_yE_0y e^{-\kappa z} \sin \omega_0 t,$$

and

$$B_0(z,t) = e_xE_0y e^{-\kappa z} \cos \omega_0 t,$$

where $E_0$ and $B_0$ satisfy the Faraday's law, $\partial E_0/\partial z = \partial B_0/c \partial t$, and $\kappa^{-1} = L_s$ is the skin depth for the classical skin effect\textsuperscript{[3,4]},

$$L_s = \frac{c}{\omega_{pe}}.$$

Our theory bases on the Vlasov equations for the velocity distribution functions $F_\alpha$ of species $\alpha = e$ for electrons and $\alpha = i$ for ions,

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha} \left( E_0(z,t) + \frac{1}{c} \mathbf{v} \times \mathbf{B}_0(z,t) \right) - \nabla \varphi(r,t) \cdot \frac{\partial F_\alpha}{\partial \mathbf{v}} = 0,$$

and the Poisson equation for the electrostatic potential $\varphi(r,t)$ of the electrostatic plasma perturbations,

$$\nabla^2 \varphi(r,t) = -4\pi \sum_{\alpha = e,i} e_\alpha \int f_\alpha(\mathbf{v},r,t) \, d\mathbf{v}. \quad (6)$$

Here, $f_\alpha$ is the perturbation of the equilibrium distribution function $F_\alpha$. That is to say, $F_\alpha = F_{\alpha0} + f_\alpha$.

In this paper, we employ the nonmodal approach to the solution of the Vlasov equation (5) with RF fields (2) and (3), developed in Ref. 8. The first step in this approach is the transformation of the velocity $\mathbf{v}$ and the position $\mathbf{r}$ coordinates determined in the laboratory frame in Eq. (6) to the coordinates $\mathbf{v}_0$ and $\mathbf{r}_0$,

$$\mathbf{v}_\alpha = \mathbf{v} - \mathbf{V}_\alpha(r,t),$$

$$\mathbf{r}_\alpha = \mathbf{r} - \mathbf{R}_\alpha(r,t) = \mathbf{r} - \int_0^t \mathbf{V}_\alpha(r,t) \, dt_1,$$

determined in the moving frame of references with a velocity $\mathbf{V}_\alpha(r,t)$. With new coordinates, the Vlasov equation for electrons becomes

$$\frac{\partial F_e(\mathbf{v}_e, \mathbf{r}_e, t)}{\partial t} + \mathbf{v}_e \cdot \frac{\partial F_e}{\partial \mathbf{r}_e} - v_{ej} \int_{t_0}^t \frac{\partial V_{ek}(r,t_1)}{\partial r_j} \, dt_1 \frac{\partial F_e}{\partial r_{ek}}$$

$$- v_{ej} \int_{t_0}^t \frac{\partial V_{ek}(r,t_1)}{\partial r_j} \, dt_1 \frac{\partial F_e}{\partial r_{ek}} + \frac{e}{m_e} \left( \nabla \varphi(r,t) - \frac{1}{c} \left[ \mathbf{v}_e \times \mathbf{B}_0(z,t) \right] \right) \cdot \frac{\partial F_e}{\partial \mathbf{v}_e}$$

$$- \left\{ \frac{\partial V_e(z,t)}{\partial t} + V_{ez}(z,t) \frac{\partial V_e(z,t)}{\partial z} \right\}$$

$$+ \frac{e}{m_e} \left( E_0y(z,t) + \frac{1}{c} \left[ \mathbf{V}_e(z,t) \times \mathbf{B}_0(z,t) \right] \right) \cdot \frac{\partial F_e(\mathbf{v}_e, \mathbf{r}_e, t)}{\partial \mathbf{v}_e} = 0.$$

(8)
Velocity $V_e(z,t)$ in our approach is determined by the equation vanishing the expression in braces in Eq. (8). This equation is the Euler equation for the velocity of the ideal fluid in the EM field. For the electric field $E$ and the magnetic field $B$, the equations for $V_{ey}(z,t)$ and $V_{ez}(z,t)$ are

$$
\frac{\partial V_{ey}(z,t)}{\partial t} + V_{ez}(z,t) \frac{\partial V_{ey}(z,t)}{\partial z} = -\frac{e}{m_e} \left( E_{by}(z,t) + \frac{1}{c} V_{ez}(z,t) B_{ox}(z,t) \right),
$$

(9)

$$
\frac{\partial V_{ez}(z,t)}{\partial t} + V_{ez}(z,t) \frac{\partial V_{ez}(z,t)}{\partial z} = \frac{e}{m_e} V_{ey}(z,t) B_{ox}(z,t).
$$

(10)

In Appendix, we present the detailed solution of these equations for $V_{ey}(z,t)$ and $V_{ez}(z,t)$.

The rest of Eq. (8) contains only the spatial derivatives of $V_e(z,t)$. Therefore, Eq. (8) in convected variables $v_e$ and $r_e$ has a form as in plasma without an EM field for the spatially uniform time-dependent EM field (so-called dipole approximation). For the case of the spatially inhomogeneous EM field, Eq. (8) becomes very suitable for the investigation of the short scale perturbations with a wavelength less than the spatial inhomogeneity of the EM field. For the considered problem of the skin layer stability, the solution for $F_e(v_e, r_e, t)$ may be derived in the form of power series in the small parameter $\kappa \delta r_e \ll 1$, where $\delta r_e$ is the amplitude of the displacement of electron in the RF field. With velocities $V_{ey}(z,t)$ and $V_{ez}(z,t)$, determined by Eqs. (14) and (15), the Vlasov equation becomes

$$
\frac{\partial F_e}{\partial t} + v_e \left[ \frac{\omega_{ce}}{\omega_0} \sin \omega_0 t e^{-\frac{t}{\tau}} \right] \frac{\partial F_e}{\partial \omega_e} + v_e \frac{\omega_{ce}}{\omega_0} \sin \omega_0 t e^{-\frac{t}{\tau}} \frac{\partial F_e}{\partial \omega_e}
$$

$$
+ \frac{\partial F_e}{\partial r_e} + \frac{\partial F_e}{\partial r_e} = \frac{e}{m_e} \nabla \varphi(r,t) \cdot \frac{\partial F_e}{\partial v_e} = 0.
$$

(11)

In this equation, $\omega_{ce}$ is the electron cyclotron frequency formed by the magnetic field of the RF wave, which is determined by Eq. (14). Equation (11) is applicable for the treatment of the processes in the skin layer for $\kappa z_e \ll 1$ as well as those outside the skin layer for $\kappa z_e > 1$, which occur during a time limited by the interval $\omega_0^{-1} < t < \omega_{ce}^{-1}$. For the real experimental conditions, this time interval is wide enough. For example, for a plasma with electron density $n_{de} = 10^{11}$ cm$^{-3}$ in the electric field $E_{by} = 1$ V/cm at $z_e = 0$ with a frequency $\omega_0 = 10^{-2} \omega_{pe} = 1.7 \times 10^8$ s$^{-1}$, and a skin depth $L_s = \kappa^{-1} = 1.7$ cm, the electron cyclotron frequency $\omega_{ce}$ is equal to $6 \times 10^6 \approx 3.5 \times 10^{-2} \omega_0$. The presence of the small parameter $\omega_{ce}/\omega_0 \ll 1$ in Eq. (11) gives a possibility to simplify Eq. (11) greatly.

### III. ION-ACOUSTIC INSTABILITY OF THE SKIN LAYER DRIVEN BY THE ACCELERATED ELECTRONS

In this section, we present the theory of the ion-acoustic instability in the skin layer, driven by the accelerated electrons. The growth rate $\gamma$ of this instability, presented below by Eq. (20), is of the order as $\gamma \sim kv_s(m_e/m_i)^{1/2}$. The IA instability can develop and saturate in the skin layer when this growth rate is larger than $\omega_{ci}$. This occurs for the short wavelength perturbations with $k v_e > v_{Te} / v_{ri}$. For the argon plasma, $(m_A/m_e)^{1/2} = 278$, with an argon ion temperature $T_i \approx 0.026$ eV and an electron temperature $T_e = 2$ eV the wave number $k$ of the unstable IA perturbations should be larger than $10^2$ cm$^{-1}$. For the time $t \sim \gamma^{-1} < \omega_{ce}^{-1}$, Eq. (11) has a form as in the uniform steady plasma for the electron distribution function $F_e$ in the electron moving frame and for $F_i$ in the ion frame, which in this problem almost coincides with a laboratory frame. Therefore, we select functions $F_{e0}(v_e)$ and $F_{i0}(v_i)$ as the Maxwellian distributions in the convective coordinates,

$$
F_{e0}(v_{ey}, v_{ez}) = \frac{n_{de}}{2\pi v_{Te}^2} \exp \left[ -\frac{v_{ey}^2 + v_{ez}^2}{2v_{Te}^2} \right].
$$

(12)

As displayed in Ref. 8, the single manifestation of the RF wave on the plasma in this case is the accelerated motion of the electrons relative to the practically unmovable ions. The equation for the perturbed electrostatic potential $\varphi(k, t)$, Fourier-transformed in the ion (laboratory) frame, is determined by the Fourier transformation of the Poisson equation in the laboratory frame. This equation for the time $t \gg \omega_0^{-1}$ has a form

$$
\varphi(k, t) = 1 \int_{t_0}^{t} dt_1 \varphi(k, t_1) \frac{d}{dt_1} \left( e^{-\frac{k^2 v_{be}^2}{4}(t-t_1)^2} \right)
$$

$$
+ \frac{1}{k^2 \lambda_{Di}^2} \int_{t_0}^{t} dt_1 \varphi(k, t_1) e^{-\frac{k^2 v_{ae}^2}{4}(t-t_1)^2}
$$

$$
\times \frac{d}{dt_1} \left( e^{-\frac{k^2 v_{be}^2}{4}(t-t_1)^2} \right) = 0,
$$

(13)

where $\lambda_{Di}(e)$ is the ion (electron) Debye length, and $a_e = \omega_{ce}^2 / 2k$. For the adiabatic electrons, the approximation $e^{-\frac{k^2 v_{be}^2}{4}(t-t_1)^2} \approx e^{-k^2 v_{be}^2 (t-t_1)^2}$ may be used for the most fast varying function in the electron term. For $T_e \gg T_i$, the electron term which contains the nonmodal time dependence is much less than the ion one. Therefore, we are looking for the solution to Eq. (13) in the form

$$
\varphi(k, t) = \varphi(k) e^{-i\omega(t)t},
$$

(14)
where \( \omega (k,t) \) slowly changes on the time scale \( \sim \omega^{-1} \). Then, the partial integration of Eq. (13) with \( \varphi (k,t) \) in a form of Eq. (14) gives the following equation for \( \omega (k,t) \),

\[
\varepsilon (k,t) \equiv 1 + \frac{1}{k^2 \lambda_D^2} \left( 1 + i \sqrt{\frac{\pi}{2}} z_1 (t) W (z_1 (t)) \right) + \frac{1}{k^2 \lambda_D^2} \left( 1 + i \sqrt{\frac{\pi}{2}} z_2 (t) W (z_2 (t)) \right) = Q (k,t,t_0), \tag{15}
\]

where \( W(z) = e^{-z^2} \left( 1 + (2i/\sqrt{\pi}) \int_0^z e^{i^2 t} \, dt \right) \) is the complex error function, \( z_1 (t) = \omega (t) / Kv_T, \) and \( z_2 (t) = (\omega (t) - k_z a_{te} t) / Kv_T. \) The function \( Q(k,t,t_0) \),

\[
Q(k,t,t_0) = \frac{1}{k^2 \lambda_D^2} e^{i \omega (t)(t-t_0)} - \frac{1}{k^2 \lambda_D^2} e^{i \omega (t-k_z a_{te} t)(t-t_0)} + \frac{1}{k^2 \lambda_D^2} e^{i \omega (t-k_z a_{te} t)(t-t_0)} - \frac{1}{k^2 \lambda_D^2} e^{i \omega (t-k_z a_{te} t)(t-t_0)}, \tag{16}
\]
determines the input from the \( t = t_0 \) limit of the integration of Eq. (21) by parts. For the unstable solutions of Eq. (15) for \( \omega (t) \), function \( Q(k,t,t_0) \) is exponentially small and may be neglected. The solution of Eq. (15) for the IA instability is

\[
\omega (t) = \omega_s (k) + i \gamma (k,t). \tag{17}
\]

Here, the IA frequency \( \omega_s (k) \) is determined as

\[
\omega_s^2 (k) = \frac{k^2 v_s^2}{1 + k^2 \lambda_D^2}, \tag{18}
\]

where \( v_s = (T_e / m_i)^{1/2} \) is the ion sound velocity. The growth/damping rate of the IA instability is \( \gamma (k,t) = \gamma_c (k,t) + \gamma_i (k), \) where

\[
\gamma_i (k) = \frac{\omega_s (k)}{1 + k^2 \lambda_D^2} \sqrt{\frac{8}{T_e T_i}} \left( \frac{T_e}{T_i} \right)^{3/2} \times \exp \left( \frac{T_e}{2 T_i (1 + k^2 \lambda_D^2)} \right), \tag{19}
\]

stems from the ion Landau damping of the IA waves, and

\[
\gamma_c (k,t) = \sqrt{\frac{\pi m_e}{8 m_i}} \frac{\omega_s (k)}{1 + k^2 \lambda_D^2} \left( \frac{k_z a_{te} t}{\omega_s (k)} - 1 \right)
= \gamma_s (k,t) \left( \frac{k_z a_{te} t}{\omega_s (k)} - 1 \right). \tag{20}
\]

For ICP plasmas with \( T_e \gg T_i, \gamma_i (k) \) is negligible small. The growth rate \( \gamma_c (k,t) \) corresponds to the initial linear stage of the IA instability development, at which electrons are uniformly accelerated under the action of the homogeneous electric field. In our case, electron acceleration occurs under the action of the effective electric field

\[
E_{\text{eff}} = \frac{1}{2} m_e \frac{e k}{\omega_0^2} e^{-2 \kappa_{ze}} = E_{0y} \frac{\omega_{ce}}{2 \omega_0} e^{-2 \kappa_{ze}}, \tag{21}
\]

formed by the ponderomotive force. \( E_{\text{eff}} = 1.75 \times 10^{-2} \) V/cm for the numerical sample considered in this paper with \( E_{0y} = 1 \) V/cm at \( z_e = 0, \omega_0 = 10^{-2} \omega_{pe} = 1.7 \times 10^8 \) s\(^{-1}\). \( L_s = k = 1.7 \) cm, \( \omega_{ce} = 6 \times 10^6 \) s\(^{-1}\) \( \approx 3.5 \times 10^{-2} \omega_0. \)

The instability develops due to the inverse electron Landau damping of the IA waves at time \( t > t_{th} = \omega_s (k) / k_z a_{te}. \) At that time, the temporal evolution of the IA spectral energy density \( W(k,t) \),

\[
W(k,t) = k^2 |\varphi (k,t)|^2 \omega_s (k) \frac{\partial \varphi (k,t)}{\partial \omega_s (k)} \approx \frac{\omega_s^2}{\omega^2} k^2 \frac{1}{4 \pi} |\varphi (k,t)|^2, \tag{22}
\]

is determined by the equation

\[
\frac{\partial W (k,t)}{\partial t} = 2 \gamma (k,t) W (k,t) \tag{23}
\]

and grows with time as \( \sim \exp \left( \gamma_s k_z a_{te} (t-t_{th})^2 / \omega_s \right) \).

The acceleration of the electron current velocity and the growth of the IA spectral intensity occur during a limited time until the effect of the ion acoustic turbulence (IAT) on the electron current velocity becomes negligibly small. At longer times, the temporal evolution of the IAT is determined by the nonlinear interaction of the electrons and ions with random electric fields of the IAT.\(^{16}\)

IV. THE NONLINEAR EVOLUTION OF THE IA INSTABILITY AND THE ANOMALOUS HEATING OF ELECTRONS BY THE ION-ACOUSTIC TURBULENCE

Because the growth rate \( \gamma (k,t) \) and the growth rate of the conventional IA instability\(^{14}\) are much less than the IA frequency \( \omega_s \), the nonlinear IAT theory is based on the methods of the weak turbulence theory. The conventional IAT theory involves the theory of the quasilinear relaxation of the electrons on the IA pulsation jointly with the theory of the induced scattering of the IA waves by the ions.

The quasilinear equation for the ensemble averaged electron distribution function \( \bar{f}_e (v_e, r_e, t) \) in the electron frame is derived easily from Eq. (11) for time \( t < \omega_{ce}^{-1}. \) It is equal to

\[
\frac{\partial \bar{f}_e (v_e, r_e, t)}{\partial t} = \left\langle \nabla \varphi (r, t) \frac{\partial f_e (v_e, r_e, t)}{\partial v_e} \right\rangle, \tag{24}
\]

where the angle brackets \( \langle \ldots \rangle \) indicate the ensemble averaging of the expression in it. Employing the relation

\[
\varphi (k,t) = \varphi_e (k,t) e^{-k_z a_{te} t^2}, \tag{25}
\]

between the Fourier transform \( \varphi_e (k,t) \) of the potential \( \varphi (r,t) \) over \( r \) and the Fourier transform \( \varphi_e (k,t) \) of the potential \( \varphi_e (r_e, t) \) over \( r_e \), we derive the quasilinear
equation
\[
\frac{\partial \bar{E}_r}{\partial t} = \pi \frac{e^2}{m_e} \int d\mathbf{k} k \frac{\partial}{\partial \nu_e} |\varphi(\mathbf{k})|^2 \\
\times \delta (\omega (\mathbf{k}, t) - \mathbf{k} \mathbf{v}_e - \mathbf{k}_z a_{ce} t) \mathbf{k} \frac{\partial \bar{E}_r}{\partial \nu_e},
\]
which determines the temporal evolution of the distribution function \( F_e (\mathbf{v}_e, r, t) \) of the accelerated electrons under the action of IAT. By multiplying \( \nu_e \) on Eq. (26) and integrating it over \( \mathbf{v}_e \), we derive the equation
\[
\frac{d \nu_e}{dt} = -\nu_e \nu_{ez} = -\frac{1}{n_{0e} m_e} \int d\mathbf{k} \frac{k_z}{\omega_s(k)} \nu_{e}(\mathbf{k}, t) W(\mathbf{k}, t),
\]
which determines slowing down of electrons due to their interactions with the IAT. The temporal evolution of \( \nu_{ez} \) depends on the temporal evolution of the growth rate \( \gamma (\mathbf{k}, t) \) resulted from the quasilinear distortion of the electron distribution function by the IAT and on the temporal evolution of the IAT spectrum \( W(\mathbf{k}, t) \) caused by the induced scattering of IA waves on ions. The theory of the IAT, which simultaneously takes into account both these processes, was developed in Ref.\textsuperscript{17}. It was found\textsuperscript{16} that the effects of these processes on the nonlinear evolution of IAT spectrum depends greatly on the value of the applied electric field\textsuperscript{16}. When the electric field \( E_{eff} \) is less than \( E_{nl} \), where
\[
E_{nl} = \frac{m_e v_s \omega_{pi} T_e}{6 \pi |e|} \frac{T_i}{T_e}, \tag{28}
\]
the quasilinear effects should be accounted for in the balance equation, which includes the growth rate of the IA instability and the nonlinear damping rate resulted from the induced scattering of IA waves on ions. For the numerical data used in this paper \( (T_e = 2 \text{ eV}, T_i = 0.026 \text{ eV}, n_{0e} = 10^{11} \text{ cm}^{-3}) \), \( E_{nl} = 0.033 \text{ V/cm} \) which is almost two times larger than \( E_{eff} = 1.75 \times 10^{-2} \text{ V/cm} \) of our sample. In this case, the level \( W = \int W(\mathbf{k}) d\mathbf{k} \) of the total energy density of the IAT determined by this process does not depend on the magnitude of the applied electric field \( E_0 \) and was estimated\textsuperscript{16} by the expression
\[
W = n_{0e} T_e \sim 0.1 \frac{\omega_{pi} \lambda_D^2}{\omega_{pe} \lambda_D^2}, \tag{29}
\]
where \( \omega_{pi} \) is the ion plasma frequency. In the approach employed in our paper, the ion dynamics in the skin layer is not affected by the EM field. Therefore, the theory of the induced scattering of the IA waves on ions developed in Ref. 14 is completely applicable to the IA instability driven by the ponderomotive current for the time \( t < \omega_{ce}^{-1} \), considered here. For the numerical parameters presented above the level \( W \) is estimated as
\[
W = n_{0e} T_e \sim 2.8 \times 10^{-2}. \tag{30}
\]
In estimate (30), it should be accounted for that the directed accelerated velocity \( V_{ae} (t) \) is slowed down to the almost threshold velocity of the order of IA velocity\textsuperscript{16} at the nonlinearly established steady state of the IAT. On this level, the effective electron collision frequency with the IAT, \( \nu_{eff} \), determined by Eq. (27), is estimated as
\[
\nu_{eff} \sim \frac{W}{n_{0e} m_e \nu_{ez}} \sim 2.8 \times 10^{-2} \omega_s \left( \frac{m_e}{m_e} \right)^{1/2}. \tag{31}
\]
Because the induced scattering of ions redistributes the spectral maximum of the IAT to the longer IA waves, we use a value \( k = 10^2 \text{ cm}^{-1} \) discussed above for the IA waves in the skin layer for the estimation of \( \nu_{eff} \). We found from Eq. (30) that the magnitude of \( \nu_{eff} \sim 1.7 \times 10^8 \text{ s}^{-1} \) for this case is of the order of the frequency \( \omega_0 = 1.8 \times 10^8 \text{ s}^{-1} \) used in our estimates. At this case of the weak electric field, \( E_{eff} < E_{nl} \), a quasi-stationary state for the IAT is established mainly due to the quasilinear relaxation of the electron distribution function. Ohm’s law for the electron current density \( j \), derived for this case\textsuperscript{15,16},
\[
j = 2.14 |e| n_{0e} v_s = \sigma_A E_{eff}, \tag{32}
\]
predicts the dependence of \( \sigma_A \sim E_{eff}^{-1} \) for the anomalous conductivity \( \sigma_A \). By multiplying \( m_e v_s^2 / 2 \) on Eq. (26) and integrating it over \( \mathbf{v}_e \), we derive the equation
\[
\frac{n_{e0} dT_e}{dT} = \int d\mathbf{k} \frac{k_z}{\omega_s(k)} \nu_{e}(\mathbf{k}, t) W(\mathbf{k}) \sim \gamma_s(k_0) \frac{W}{n_{0e} T_e}, \tag{33}
\]
which determines the turbulent heating rate \( \nu_{Te} \sim \gamma W/n_{0e} T_e \) of the electrons due to their interaction with the IAT. For the numerical data, used above, \( \nu_{Te} \sim 2.8 \times 10^{-2} \). When the applied electric field \( E_{eff} \) is above \( E_{nl} \), quasilinear effects are weak and the dominant nonlinear process is the induced scattering of IA waves on ions. In this case, the steady state level of the IAT\textsuperscript{16},
\[
\frac{W}{n_{0e} T_e} \sim 0.1 \frac{\omega_{pi} \lambda_D^2}{\omega_{pe} \lambda_D^2} \sqrt{\frac{E_{eff}}{E_{nl}}}, \tag{34}
\]
grows with \( E_{eff} \) growth. The Ohm’s law for electron current density \( j \) for this case,
\[
j = \frac{4.48}{\pi} |e| n_{0e} v_s \sqrt{\frac{E_{eff}}{E_{nl}}}, \tag{35}
\]
and the anomalous conductivity \( \sigma_A \)\textsuperscript{16,17}
\[
\sigma_A \simeq 0.4 \omega_{pe} \frac{\lambda_D^2}{\lambda_{Dr}} \left( \frac{8 \pi n_{0e} T_e}{E_{eff}^2} \right)^{1/4} \sim E_{eff}^{-1/2}. \tag{36}
\]
were derived in Ref. 14. The effective collision frequency \( \nu_{eff} \) corresponding to this case is given by the Sagdeev equation\textsuperscript{16},
\[
\nu_{eff} = 2.5 \times 10^{-2} \omega_{pi} \frac{U_0}{v_s} \frac{T_e}{T_i}, \tag{37}
\]
where $U_0$ is determined from Eq. (26). The regime with $E_{\text{eff}} > E_{\text{nl}}$ occurs for $E_{\text{by}} = 1.3$ V/cm with the same other parameters considered above. For this electric field $\omega_{ce}/\omega_0 = 4.6 \times 10^{-2}$ and $E_{\text{eff}} \approx 5.89 \times 10^{-2}$ V/cm $> E_{\text{nl}} = 3.3 \times 10^{-2}$ V/cm, that gives
\[ \frac{W}{n_0 e T_e} \sim 0.05, \]  
(38)

$U_0 = 2.57 v_s$ and $\nu_{\text{eff}} = 8.35 \times 10^8$ s$^{-1} > \omega_0 = 1.8 \times 10^8$ s$^{-1}$.

The derived estimates for $\nu_{\text{eff}}$ reveal that the effective electron collision frequency with the IAT pulsations is of the order of the RF driving frequency and is much larger than the electron-ion and electron-neutral collision frequencies in the mTorr range of gas pressure for the considered numerical parameters corresponding to the experimental conditions of ICP sources.

V. CONCLUSIONS

In this paper, we present the theory of IA instability of the skin layer of ICP sources driven by the accelerated electrons, which move relative to ions under the ponderomotive force. This theory reveals that on the linear stage of the IA instability, driven by the steady electric field, always develops as the nonmodal instability with the growth rate growing with time.

At the finite time interval $\delta t < \omega_{ce}^{-1}$, the analysis of the nonlinear stage of the IA instability driven by the ponderomotive force is similar to the analysis of the IA instability and the IAT driven by the steady electric field. The accelerated electron velocity in the steady electric field decelerates due to the scattering of electrons by the IAT. This velocity approaches a particular steady value or decelerates due to the scattering of electrons by the IAT. The development of the IAT in the skin layer and the effective field $E_{\text{eff}}$ determined by Eq. (21), which is strongly inhomogeneous across the skin layer in ICPs. It is found that the effective electron collision frequency $\nu_{\text{eff}}$ with the IAT is of the order of or is larger than $\omega_0$ at all considered regimes of the IAT evolution. The derived results prove that the development of the IAT in the skin layer and scattering of electrons by IAT are the primary channels of the nonlinear absorption of the RF wave energy in the skin layer. This result is also valid for the case of the strong RF field considered in Ref. 8, for which the oscillatory velocity in the skin layer is larger than the electron thermal velocity, and the IA instability develops in the bulk of plasma past the skin layer.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Appendix A: Solutions to Eqs. (9) and (10) for $V_{ey}$ and $V_{ez}$

Here, we present the solutions to Eqs. (9) and (10) for $V_{ey}$ and $V_{ez}$, alternative to those presented in Ref. 8. These solutions do not require the usually employed assumption in the calculation of the electron velocity in EM wave that the force by the RF electric field acting on electrons in the EM wave prevails over the Lorentz force by the RF magnetic field.

With new variables $z_e$ and $t'$ determined by the relations,
\[ z = z_e + \int_0^{t'} V_{ez}(z_e, t'_1) dt'_1, \quad t = t'. \]  
(A1)

Eqs. (9) and (10) becomes
\[ \frac{\partial V_{ey}(z_e, t')}{\partial t'} = -\kappa \frac{\omega_{ce}}{\kappa} e^{-\kappa t'} V_{ez}(z_e, t'_1) dt'_1 \sin \omega_0 t' \]
\[ -\omega_{ce} \cos \omega_0 t' e^{-\kappa t'} V_{ez}(z_e, t'_1) dt'_1, \]  
(A2)

\[ \frac{\partial V_{ez}(z_e, t')}{\partial t'} = \omega_{ce} e^{-\kappa t'} V_{ey}(z_e, t'_1) dt'_1 \times V_{ez}(z_e, t') \cos \omega_0 t', \]  
(A3)

where
\[ \omega_{ce} = \frac{e \kappa E_{by} e^{-\kappa z_e}}{m_e \omega_0} \]  
(A4)
is the electron cyclotron frequency formed by the RF magnetic field at $z_e$. Now we derive the approximate solution to Eqs. (A2) and (A3) for the finite time interval, at which
\[ \kappa \int_0^{t'} V_{ez}(z_e, t'_1) dt'_1 \ll 1. \]  
(A5)
In the zero approximation, Eqs. (A2) and (A3) are reduced to the ordinary differential equation

\[
\frac{dU_e}{dt'} - i\omega_e \cos \omega t U_e = -\frac{\omega_e}{\kappa} \sin \omega_0 t, \tag{A6}
\]

in which variable \( z_e \) becomes a parameter, and

\[
U_e = U_e(z_e, t) = V_{ey}(z_e, t) + iV_{ez}(z_e, t). \tag{A7}
\]

The solution to Eq. (A6) for the initial value \( U_e(z_e, t = 0) = 0 \),

\[
U_e(z_e, t) = -\frac{\omega_e}{\kappa} \exp \left( i\frac{\omega_e}{\omega_0} \sin \omega_0 t \right)
\times \int_0^t dt' \sin \omega_0 t' \exp \left( -i\frac{\omega_e}{\omega_0} \sin \omega_0 t' \right), \tag{A8}
\]

is simply presented in the explicit, however cumbersome, form for any values of the \( \omega_e/\omega_0 \) ratio. The focus of our paper is on the weak high-frequency RF field for which \( \omega_e/\omega_0 \ll 1 \). By using the approximation for the exponent in Eq. (A8),

\[
\exp \left( \pm i\frac{\omega_e}{\omega_0} \sin \omega_0 t \right) \approx 1 \pm i\frac{\omega_e}{\omega_0} \sin \omega_0 t \tag{A9}
\]

which is valid for any values of \( \omega_0 t \), we derive the approximate solution for \( U_e(z_e, t) \) from Eq. (A8) in the form

\[
U_e(z_e, t) = \frac{\omega_e}{\kappa} \cos \omega_0 t + \frac{\omega_e^2}{2\kappa} t + i\frac{\omega_e}{4\omega_0\kappa} \sin 2\omega_0 t. \tag{A10}
\]

Equation (A10) gives

\[
V_{ey}(z_e, t) = \text{Re} \ U_e(z_e, t) = \frac{\omega_e}{\kappa} \cos \omega_0 t \tag{A11}
\]

and

\[
V_{ez}(z_e, t) = \text{Im} \ U_e(z_e, t) = \frac{\omega_e^2}{2\kappa} t + \frac{\omega_e^2}{4\omega_0\kappa} \sin 2\omega_0 t. \tag{A12}
\]

Solutions (A11) and (A12) are identical to solutions for \( V_{ey} \) and \( V_{ez} \), derived in Ref. 8, where other procedure was developed by the iterative solution of Eqs. (9) and (10). It was based on the calculation of the ponderomotive motion of an electron in the spatially inhomogeneous electromagnetic field, assuming that the force by the RF electric field acting on electrons in the skin layer prevails over the Lorentz force by the RF magnetic field.

The procedure developed in Ref. 8 used small parameter \( \kappa \xi_e \ll 1 \) where \( \xi_e = eE_{0y}(z_e)/m_e\omega_0^2 \) is the amplitude of the displacement of an electron along the coordinate \( y \). This parameter is identically equal to \( \omega_e/\omega_0 \). For the time at which \( \omega_0 t \gg 1 \),

\[
V_{ez}(z_e, t) \approx \frac{\omega_e^2}{2\kappa} t \tag{A13}
\]

and

\[
-\kappa \int_0^{t'} V_{ez}(z_e, t') dt' = -\frac{1}{4} \omega_e^2 t^2. \tag{A14}
\]

It follows from Eq. (A5) that solutions (A11) and (A12) are valid for the time \( t < \omega_e^{-1} \).

By employing the method of successive approximations to the solution of the nonlinear Eqs. (A2) and (A3) with \( V_{ez}(z_e, t) \) determined by Eq. (A12) as the initial approximation, we obtain the following solutions to these equations for \( V_{ey}(z_e, t) \),

\[
V_{ey}(z_e, t) = \frac{\omega_e}{\kappa} e^{-\frac{1}{4} \omega_e^2 t^2} \cos \omega_0 t, \tag{A15}
\]

and for \( V_{ez}(z_e, t) \),

\[
V_{ez}(z_e, t) = \frac{\omega_e^2}{2\kappa} e^{-\frac{1}{4} \omega_e^2 t^2} \sin 2\omega_0 t + O \left( \frac{\omega_e^2}{\omega_0} \right), \tag{A16}
\]

which are valid for the time \( t \gg \omega_0^{-1} \). It follows from Eq. (A16) that the maximum of the accelerating velocity \( V_{ez}(z_e, t) \) attains for \( t_s = \sqrt{2}/\omega_e \) at which \( \omega_e^2 t_s^2 = 2 \). At the time \( t > 2t_s \), an electron which was in \( z_e = 0 \) at the time \( t = 0 \) will cover the distance of the order of the skin depth. Therefore, the time admissible for the development and saturation of any instability in the skin layer driven by the accelerated electron current in the case of the high operation frequency for which \( \omega_0 \gg \omega_e \) is limited by time \( t \lesssim t_s \sim \omega_e^{-1} \).

In the case of the low frequency \( \omega_0 \), large amplitude RF wave, the electron cyclotron frequency \( \omega_{ce} \) may be larger than \( \omega_0^{13,14} \). The approximate solution to Eq. (A8) can be derived in this case for the limited time interval \( t \ll \omega_0^{-1} \), by employing the simplest approximation \( \sin \omega_0 t \approx \omega_0 t \).

The condition (A15) with velocity \( V_{ez}(z_e, t) \) determined by Eq. (A19) is valid at time \( t \ll \omega_0^{-1} \). The
derived solution (A17) can be easily improved by using the expansion \( \sin \omega_0 t \approx \omega_0 t - \frac{1}{6}(\omega_0 t)^3 \). In this case, the solution for \( V_{ez}(z_e, t) \) becomes equal to

\[
V_{ez}(z_e, t) \approx -\frac{\omega_0}{\kappa} \left(1 - 3\frac{\omega_0^2}{\omega_{ce}^2}\right) \omega_0 t. \tag{A20}
\]

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