Near ground results of the CO-SLIDAR $C_n^2$ profiler

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Abstract. CO-SLIDAR jointly uses the slopes and scintillation indices from a double source recorded with a Shack-Hartmann (SH) wavefront sensor which leads to a robust $C_n^2$ profile restoration. This technique coupled to a 0.35-m telescope in the mid-IR has provided horizontal $C_n^2$ profiles over 2.7 km with a 300 m resolution. The SH wavefront sensor data revealed a very good fit of the spatial spectra of the phase to the Kolmogorov model, and of the scintillation distributions to the small perturbation regime as well. Quantitatively the inversion of the measurement covariances allows us to retrieve the $C_n^2$ profile with small error bars and stable structure. These $C_n^2$ distributed estimations are comparable with the averaged $C_n^2$ measurements from scintillometers set in parallel.

1. Introduction

The development of $C_n^2$ profile measurement is flourishing in the astronomy domain. In the past years, several techniques have been developed, based on the analysis of the beam coming from a single or binary star. These techniques consist in analysing the fluctuations of intensity (scintillation) or those of the wavefront angle of arrival. To measure the $C_n^2$ profile in the whole atmospheric column, ONERA developed the CO-SLIDAR technique based on a Shack-Hartmann (SH) wavefront sensor [1]. The joint use of correlations of slopes and of scintillation, both measured on a binary source with a Shack-Hartmann, allows CO-SLIDAR to combine sensitivity to both low (close) and high altitude (far) layers.

In astronomical instrumentation, the knowledge of the $C_n^2$ profile is a key element to design and to evaluate the performance of future wide-field adaptive optics systems (WFAO) [2]. In this perspective, recent CO-SLIDAR experiments in the visible at the Observatoire de la Côte d’Azur (France) have measured $C_n^2$ profiles (with error bars) over 17 km of atmosphere with a resolution of 600 m. These $C_n^2$ profiles were compared successfully to those calculated from the re-analysis of meteorological data [3]. The CO-SLIDAR technique is of course applicable to oblique or horizontal paths provided one stays in the Rytov regime. In practice, limitations appear for horizontal 100-m paths in the surface layer where turbulence intensity is the highest, which leads to a saturation effect of scintillation indices in the visible range and thus prevents the inversion of the $C_n^2$ distribution. Longer distances are possible using the CO-SLIDAR in the thermal infrared domain for km paths. Thus our IR Shack-Hartmann [4] was used to provide distributed $C_n^2$ values along a horizontal path near the Earth surface. These $C_n^2$ horizontal profiles can be further used to estimate sensible and latent heat surface fluxes at parcel scale which is relevant for climate, hydrological, and agronomics studies.

As the CO-SLIDAR principle and the on-sky results obtained at Observatoire de la Côte d’Azur have been presented in [3], the proceeding will focus on the near ground results.
2. Multi-instrument campaign for near ground $C_n^2$ profiles

A field experiment named AMOSC was conducted at the Centre de Recherche Atmosphérique (CRA) in Lannemezan (France) to first optically calibrate CO-SLIDAR horizontal $C_n^2$ profiles. This experiment was conducted in fall 2012 (ONERA with French labs LTHE, INRA, CESBIO). The SH ($\lambda = 3.8 \mu m$) was installed on the CRA roof, aiming at a vertical double halogen source placed 2.7 km apart in the church tower of the Campistrous village. In addition, 3 scintillometers (using a diode at $\lambda = 0.9 \mu m$ as a source) were co-aligned with this line of sight, that was 22 meters in average above ground (see Fig. 1). The CO-SLIDAR profiles were to be compared with the $C_n^2$ values estimated from the scintillometers. Each one had been configured with a specific configuration of apertures - with their own weighting function - to highlight different parts of the path. We got finally 21 days of SH data synchronized with the scintillometers and micro-meteorological data. Data processing is still in progress and we present first encouraging results in this paper.

Figure 1. Terrain profile (black) and line of sight (dashed). Model of atmospheric boundary layer $C_n^2 (h)$ profile (red) according to $C_n^2 (h_0 = 10m) \times h^{-4/3}$, corresponding to diurnal conditions (unstable), with $h$ being the height of the line of sight. Derived turbulence parameters are given at $\lambda = 3.8 \mu m$.

3. CO-SLIDAR principle: direct problem, inversion and error bars

We currently only exploit correlations of $x$-slopes, of $y$-slopes and of scintillation to speed up the computing time since coupling between $x$ and $y$-slopes are weaker. Correlations are averaged over all pairs of subapertures with given separation and represented as auto- or cross-correlation map as it relates to one or two sources. Then, one pixel of these six maps is stacked into a vector $C_{\text{mes}}$. The latter is related to the discretized $C_n^2$ profile at different distances $z$, denoted $C_{n}^{z}$, by the following linear relationship:

$$C_{\text{mes}} = MC_{n}^z + C_{d} + u. \quad (1)$$

$M$ is the matrix of the weighting functions for slopes and for scintillation [3] derived here for a spherical wave since sources are at a finite distance. Because of detection noises, the pseudo-measurements $C_{\text{mes}}$ are biased with the averaged correlations of these noises $C_{d}$. As we estimate the correlations from a finite number of frames, $u$ represents a Gaussian convergence noise.

The $C_n^2$ profile is retrieved by minimizing the Maximum A Posteriori (MAP) metric composed of the Maximum Likelihood criterion with a spike preserving regularization $J_p (C_n^2)$ (white L1L2 [5]) assuming independent layers, which is relevant for horizontal profiles:

$$J_{\text{MAP}} (C_n^2) = (C_{\text{mes}} - C_{d} - MC_{n}^2)^T C_{\text{conv}}^{-1} (C_{\text{mes}} - C_{d} - MC_{n}^2) + J_p (C_n^2). \quad (2)$$
The covariance matrix of the convergence noise $C_{\text{conv}} = \langle uu^\top \rangle$, is approximated with the pseudo-measurements correlations. In addition, the diagonal values of matrix $(M^T C_{\text{conv}}^{-1} M)^{-1}$ represent an upper bound of the variances of the $C_n^2$ estimates [3], from which $3\sigma$ error bars on the reconstructed profile are readily computed.

4. Resolution in distance of the horizontal $C_n^2$ profiles
Since our IR Shack-Hartmann looks at a double source with an angular separation $\theta = 60$ arc second and has 5 subapertures in its pupil diameter $D$, we can retrieve at least 5 layers up to the maximal distance of triangulation $H_{\text{max}} = D/\theta = 1.3 \text{ km}$ thanks to the cross-correlations of slopes and of scintillation. Beyond the triangulation range $H_{\text{max}}$ we will also estimate the profile from the auto-correlations of scintillation averaged on every single source and involving several subaperture baselines sampling the structure of the farthest weighting functions of scintillation (Cf. Fig. 2). Note the FWHM\(^1\) of the 3 functions at $z = 1.7, 2.0$ and $2.3 \text{ km}$ span over 2 subapertures allowing to exploit their mutual scintillation. In the following we will estimate the $C_n^2$ profiles on a grid of 8 layers regularly spaced along the path.

![Figure 2. Cross sections of the scintillation weighting functions varying the distance $z$ to the SH.](image)

5. Shack-Hartmann data quality check
Before performing inversion of (1) to estimate the $C_n^2$ profile from the slopes and scintillations, we need to check if the data respect the model assumptions, namely Kolmogorov and the weak perturbation regime. To do so, we extract slopes and scintillation from the SH image spots ($25 \times 25$ pixels) in moving windows of $7 \times 8$ pixels, and we check the turbulence statistics from the two kinds of data. From the slopes we compute the Zernike coefficient variances. It shows a Kolmogorov turbulence without noticeable outer scale effect (see Fig. 3). We validate the weak perturbation regime using intensity fluctuations, by fitting a log-normal distribution (see Fig 4) and estimating the Rytov variance. The latter was found to be far below 0.3, validating the hypothesis of weak perturbations at $\lambda = 3.8 \mu\text{m}$. Derived log-amplitude from intensity is in excellent agreement with the Gaussian law. The correlation maps in Fig. 5 present the expected patterns: The cross-correlation map of scintillation (Fig. 5a) and that of the slopes (Fig. 5d) shows peaks of correlation in the direction of the source alignment, representing the turbulent layers' signatures up to $H_{\text{max}}$. Last but not least the auto-correlation map of scintillation (Fig. 5b) covers 2 subapertures, and this signal is useful to estimate the $C_n^2$ values beyond $H_{\text{max}}$. All these verifications confirm the data consistency with the model assumptions.

\(^1\) Full Width at Half Maximum.
6. Reconstruction of the horizontal $C_n^2$ profiles

We present on Fig. 6 one example of $C_n^2$ profile obtained from 1 minute SH data. The structure of the profile is clearly visible on the 8 reconstructed layers with small error bars (convergence noise). There are three significant layers with $C_n^2$ ranging $3 - 5 \times 10^{-14} \text{ m}^{-2/3}$. Two of them are placed at the sight line ends (CRA roof and the village) and one thick layer is located between 900 m and 1200 m. Very low $C_n^2$ values are observed elsewhere, especially the one almost null located at 1500 m. The horizontal lines show the integrated $C_n^2$ measured from the scintillometers (dashed) and synthesized (continuous) from the estimated $C_n^2$ profile. Their length illustrates the effective footprint of the scintillometers weighting functions.

CO-SLIDAR $C_n^2$ profiles are now compared with $C_n^2$ values integrated from the scintillometers. A good match is obtained in the first part of the path (magenta) but for the other two parts (black, blue) the trend is opposite between integrated $C_n^2$ synthesized with CO-SLIDAR and that measured from scintillometers. Even if the orders of magnitude are correct, these discrepancies require further investigations.

In any case, $r_0$ and scintillation indices from the CO-SLIDAR $C_n^2$ profiles fit well those measured directly for the sequence of a dozen minutes (see Figs. 7-8). A 4% error is obtained for $r_0$, while a 10% error is obtained for the scintillation. This means that unsteady conditions cannot be involved in the discrepancies mentioned. Finally, Fig. 9 shows a day and night turbulence variation as expected and a stable structure of the $C_n^2$ profiles.
Near ground CO-SLIDAR in the mid-IR using a 0.35-m telescope provides $C_n^2$ profiles over 2.7 km with 300 m resolution. The joint use of slopes and scintillation leads to robust $C_n^2$ profile restoration with small error bars, even beyond the triangulation range thanks to scintillation. $r_0$ and scintillation indices from the CO-SLIDAR $C_n^2$ profiles fit well those measured directly from the slopes and scintillation variances. $C_n^2$ CO-SLIDAR profiles have been compared with $C_n^2$ values integrated from the scintillometers, both showing a day and night turbulence variation. But reverse order between both distributions require further investigations. To sort it out we have planned inter-comparison campaigns on homogeneous and restrained heterogeneous surfaces. CO-SLIDAR development is set out to build a lighter device to bring it outdoors easily and estimate in addition to the $C_n^2$ profile the transverse wind profile, perpendicular to the sight line. The motivation of this work is to obtain inputs for sensible and latent heat flux calculation.

CO-SLIDAR can also provide relevant inputs for site characterization, for design and performance evaluation of WFAO ELT instruments that will include multi-WFS. The need of an external vertical $C_n^2$ distribution to calibrate the internal low resolution $C_n^2$ profile is envisaged. Position with respect to other astronomical $C_n^2$ profilers is yet to be analyzed during inter-comparison campaigns [6]. Anyway whatever the application whether it be water management in environment or astronomical instrumentation, CO-SLIDAR $C_n^2$ profiles can help the forecast of atmospheric surface parameters and optical turbulence.

Figure 6. Comparison between $C_n^2$ profiles and integrated $C_n^2$ values from scintillometers on 14/09/12.

Figure 7. Fried parameter variation along time. Figure 8. Scintillation index variation along time.

7. Conclusion and perspectives

Near ground CO-SLIDAR in the mid-IR using a 0.35-m telescope provides $C_n^2$ profiles over 2.7 km with 300 m resolution. The joint use of slopes and scintillation leads to robust $C_n^2$ profile restoration with small error bars, even beyond the triangulation range thanks to scintillation. $r_0$ and scintillation indices from the CO-SLIDAR $C_n^2$ profiles fit well those measured directly from the slopes and scintillation variances. $C_n^2$ CO-SLIDAR profiles have been compared with $C_n^2$ values integrated from the scintillometers, both showing a day and night turbulence variation. But reverse order between both distributions require further investigations. To sort it out we have planned inter-comparison campaigns on homogeneous and restrained heterogeneous surfaces. CO-SLIDAR development is set out to build a lighter device to bring it outdoors easily and estimate in addition to the $C_n^2$ profile the transverse wind profile, perpendicular to the sight line. The motivation of this work is to obtain inputs for sensible and latent heat flux calculation.

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Figure 9. Mean $C_n^2$ profiles averaged over 10 minutes (red) and their standard deviations (blue). Variation during afternoon, evening (upper graphs), night, morning (lower graphs) on consecutive and sunny dates.

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