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Sensitivity limits comparison of surface resistance measurements based on dielectric loaded resonators

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Abstract. We present a study of the uncertainties on the measures of the surface resistance, obtainable with different measurement methods based on the use of dielectric-loaded resonators. We consider the so-called round robin rotation method versus the single measurement technique and we evaluate the respective uncertainties. This comparison is done in the case of samples with surface resistance near that of the resonant cavity walls. We establish the limits of application of the round robin technique versus those of the single measurement.

1. Introduction
The surface resistance $R_s$ is the physical quantity of interest for the study of dissipation phenomena of materials involved in microwave applications. Hence, the experimental study of $R_s$ is of great relevance and the reduction of the measures uncertainties $u(R_s)$ is necessary to achieve higher precision. The uncertainty $u(R_s)$ depends on the chosen measurement method. In this paper, we analyze different techniques of comparing small variations on the $R_s$ of different metallic samples.

To reach a satisfactory sensitivity, one often makes use of dielectric-loaded cylindrical resonators (see Figure 1) [1]. Noticeably, for some applications this method is nowadays considered as a de facto standard for the surface resistance measurements [2]. We present here a study on the $R_s$ uncertainties obtainable by different strategies of the so-called end-wall replacement method, where the sample under study replaces one base of the resonator [1].

We compare the single measure method (SM), where one sample at a time is placed in the resonant cavity [1], and the round robin method (RR) [3–5], where at the same time both bases of the resonator are replaced with samples which are swapped following all the possible permutations in subsequent measurements. We consider the specific case of three samples. The uncertainties of SM have already been studied when applied to superconductors [6]. We extend those results to room temperature measurements and we compare our extended results to the performances of the RR method.

2. Method and mathematical determination of the uncertainties
The structure of the dielectric-loaded cylindrical resonator is sketched in Figure 1. It is a cylindrical cavity where a dielectric rod, with high permittivity and low losses, is loaded inside. The dielectric rod is useful to focus the electromagnetic field near the $z$-axis of the resonator, hence to reduce the losses on the lateral wall of the resonator and/or to allow the measurements of samples smaller than the bases of the resonator. A sapphire single crystal, at room temperature,
is the best choice due to its electromagnetic characteristics: low loss tangent \((\tan \delta \sim 8 \cdot 10^{-6})\) and high, anisotropic, relative permittivity \((\varepsilon_{\parallel} = 11.5, \varepsilon_{\perp} = 9.3)\) [7][8]. However the anisotropy must be well controlled to avoid unexpected responses of the system [9]. When replacing one or both bases of the resonator with the samples under study, the measured resonant curve is perturbed by the surface resistance of the sample. The more the samples are conductive, the higher the quality factor \(Q\) of the resonator due to the reduction of the conduction losses on the bases of the cavity. \(Q\) is related to all the dissipation sources [1] through the expression:

\[
\frac{1}{Q} = \sum_k \frac{R_{s,k}}{G_k} + \eta \tan \delta \equiv \sum_k R_{s,k} A_k + l_d \equiv l, \tag{1}
\]

where \(R_{s,k}\) is the surface resistance of the \(k\)-th dissipative surface inside the cavity and \(A_k = G_k^{-1}\) is the inverse of the respective geometrical factor \(G_k\). Finally \(l_d = \eta \tan \delta\) where \(\eta \sim 1\) is the dielectric filling factor, \(\tan \delta\) is the loss tangent of the dielectric used in the resonator and \(l = Q^{-1}\) is the inverse of the measured quality factor \(Q\).

It is useful to represent the set of \(m\) measurements in matrix form. We consider a resonating cavity entirely made of the same material (as normally used in these measurement systems, usually Copper) with surface resistance \(R_0\), and \(n - 1\) samples with surface resistances \(\{R_1, R_2, ..., R_{n-1}\}\). Hence, we define the \(n\)-dimensional vector \(R = \{R_1, R_2, ..., R_{n-1}, R_0\}\). Then we define the \(m\)-dimensional vectors \(l = \{l_1, l_2, ..., l_m\}\) and \(l_d = \{l_d, l_d, ..., l_d\}\). \(l\) is formed by the inverses of the \(Q\) measured through the \(m\) measurements. Finally the matrix \(A = \{a_{i,j}\}\), with \(i = 1, 2, ..., m\) and \(j = 1, 2, ..., n\), where the element \(a_{i,j}\) represents the inverse of the geometrical factor of the surface with surface resistance \(R_j\) at the \(i\)-th measure. Then (1) becomes \(l = A \cdot R + l_d\). The number of measurements \(m\) and of unknowns \(n\) must be the same \((m = n)\) so that the system of equations is determined. Hence \(A\) becomes a square matrix and the system has the following solution: \(R = A^{-1} \cdot (l - l_d)\). The uncertainties \(u(R_j)\) on the measured \(R_j\) are calculated with the well known formula [10]:

\[
u(R) = \sum_{i,j} \left( \frac{\partial R}{\partial a_{i,j}} u(a_{i,j}) \right)^2 + \sum_i \left( \frac{\partial R}{\partial l_i} u(l_i) \right)^2 + \left( \frac{\partial R}{\partial l_d} u(l_d) \right)^2, \tag{2}
\]

where the squares are element-wise operators. We work under the assumption that all the quantities are statistically independent; the geometrical factors \(a_{i,j}\) are physical quantities associated to the different surfaces, the \(l_i\) are the independently measured quantities and \(l_d\) depends only on the dielectric used.

3. Case studies and simulations

We examine the RR and the SM with three samples under study to find which one guarantees smaller uncertainties.
The resistances vector is \( \mathbf{R} = \{ R_1, R_2, R_3, R_0 \} \). The different placements of the samples are represented by the \( \mathbf{A} \) matrix. It is possible to conceptually break up the cylindrical resonator in the two bases, with geometrical factors \( A_{b1} \) and \( A_{b2} \), and the side wall represented by \( A_0 \). We consider the case when the samples entirely cover the bases of the resonator. In this case the only independent geometrical factors are \( A_{b1} \), \( A_{b2} \) and \( A_0 \), hence the derivatives of the first term of (2) are \( \frac{\partial}{\partial A_{b1}}, \frac{\partial}{\partial A_{b2}} \) and \( \frac{\partial}{\partial A_0} \). We define the matrices \( \mathbf{A}_{SM \; \text{and}} \mathbf{A}_{RR} \) for the SM and the RR techniques, respectively:

\[
\mathbf{A}_{SM} = \begin{bmatrix}
A_{b1} & 0 & 0 & A_0 + A_{b2} \\
0 & A_{b1} & 0 & A_0 + A_{b2} \\
0 & 0 & A_{b1} & A_0 + A_{b2} \\
0 & 0 & 0 & A_0 + A_{b1} + A_{b2}
\end{bmatrix}
\]
\[
\mathbf{A}_{RR} = \begin{bmatrix}
A_{b1} & A_{b2} & 0 & A_0 \\
A_{b1} & 0 & A_{b2} & A_0 \\
0 & A_{b1} & A_{b2} & A_0 \\
0 & 0 & 0 & A_0 + A_{b1} + A_{b2}
\end{bmatrix}
\]

We now give some quantitative estimates of the uncertainties of the SM and RR methods. To this aim, we refer to the specific cylindrical copper (Cu) sapphire loaded resonator described in [11, 12] and excited in the TE\(_{011}\) mode. Due to the symmetry of the resonator, we note that \( A_{b1} = A_{b2} \equiv A_b \) (see Figure 1). We have calculated \( A_b = 3.70 \times 10^{-3} \; \Omega^{-1} \), \( A_0 = 2.00 \times 10^{-5} \; \Omega^{-1} \), \( u(A) = 0.01 \), \( u(A_0) = 0.01 \; A_0 \). The Cu (wall) surface resistance is \( R_0 \sim R_{Cu} = 3.15 \times 10^{-2} \; \Omega \) at \( f = 1.30 \times 10^{10} \) Hz and room temperature, and \( l_d \sim 8.00 \times 10^{-6} \). We take \( u(Q)/Q = u(l)/l = 0.01 \) as suggested in [6] to be a common relative uncertainty achievable value.

![Figure 2](image1.png)

**Figure 2.** The relative uncertainty on \( R_1 \) in RR (a) and SM (b) as a function of \( R_1 \) and \( u(Q)/Q \) taking the other resistances fixed (\( R_0 = R_2 = R_3 = R_{Cu} = 3.15 \times 10^{-2} \; \Omega \)). For \( u(Q)/Q = 0.01 \) and \( R_1 = R_{Cu} = 3.15 \cdot 10^{-2} \; \Omega, u(R_1)_{RR}/R_1 \sim 2.0\% \) and \( u(R_1)_{SM}/R_1 \sim 2.4\% \).

We have calculated the relative uncertainties on the resistance \( R_1 \), with the RR \( (u(R_1))_{RR}/R_1 \) and the SM \( (u(R_1))_{SM}/R_1 \), when \( R_{Cu} < R_1 < 3R_{Cu} \). Figure 2 shows the dependence of the obtained results by the expected uncertainties on \( Q \) [6]. In both cases \( u(R_s) \) is strongly sensitive to \( u(Q) \). The RR method proves better than SM, in particular at low \( R_1 \) values and small \( u(Q)/Q \).

For the perspective use of different dielectrics [14–16] in Figure 3 we compare the uncertainties of the two measurement techniques as a function of \( l_d \) and of the surface resistance of the samples \( R_{\text{samples}} \). We examine the case of three samples with the same \( R_s \). If the dielectric losses are negligible the SM guarantees smaller uncertainties than the RR only if \( R_0 < 0.6R_{\text{samples}} \). When \( R_{\text{samples}} \) become smaller, and/or the dielectric losses increase, the RR guarantees the smallest uncertainties.
4. Conclusions

We analyzed the uncertainties achievable with the SM and the RR methods using a standard measurement technique based on a dielectric loaded copper resonator, in the demanding case of samples with $R_s$ near that of the cavity wall. We have shown that in this particular case, and with standard measurement setup, RR allows to distinguish copper samples with differences of $R_s$ of 2.0% (Figure 2). In the same conditions with the SM, $u(R_s)_{SM}/R_s = 2.4\%$. For the analyzed case the calculated $u(R_s)_{RR}$ is the sensitivity limit on the $R_s$ measure, in the X-band [17], at room temperature, with the actual technology and the known measurement techniques.

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