Do we understand excited 0\(^+\) states in nuclei?

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Abstract. Excited 0\(^+\) states are the least understood of any low-energy degree of freedom in nuclei. Some examples of excited 0\(^+\) states that are reasonably well understood are followed by details, from recent experimental work, of excited 0\(^+\) states that are causing major reassessment of some models.

1. Introduction

Excited 0\(^+\) states can arise in nuclei in association with the nucleon pairing degree of freedom and in model spaces with collective shape degrees of freedom. Models of pairing in nuclei are by now developed to the point that there is a wide consensus regarding the basic physics issues. Models of collective shape degrees of freedom are much further from a consensus on the basic physics issues. Collectivity in nuclei has been one of the major topics of nuclear structure research for sixty years. From Bohr’s initial work [1], Bohr and Mottelson [2] working with their many collaborators at the Niels Bohr Institute in Copenhagen, extensively explored the case for low-energy collective motion in nuclei dominated by quadrupole shapes. This developed the Bohr model into a more unified perspective (with coupling of nucleon degrees of freedom) that has come to be known as the Bohr-Mottelson model. Finer details of the Bohr model were developed by Greiner and the Frankfurt School (see [3]).

Three further lines of development were pursued beyond the early work in Copenhagen and Frankfurt. The first attempted to arrive at the parameters of the Bohr-Mottelson-Frankfurt model by mapping from many-nucleon degrees of freedom: this approach was initiated by Kumar and Baranger [4]; and activity along this line continues intensively. The second adopted a boson approximation: the many boson model approaches are reviewed in Klein and Marshalek [5]. The most notable of the boson-based models is the interacting boson model (IBM) of Arima and Iachello [6]; and activity using this model remains intensive. The IBM has particularly introduced many practitioners of nuclear structure research, especially experimentalists, to the language of dynamical groups and spectrum generating algebras. The third approached the successes of the Bohr model from the fundamental view that it must be a submodel of the nuclear shell model. Building on work of Elliott [7] and Weaver, Biedenharn and Cusson [8], Rosensteel and Rowe [9] showed that the solution to this problem resided in the symplectic group, Sp(3,R).

This rich arsenal of collective models has provided the language with which theorists and experimentalists have expressed their ideas and discoveries for sixty years. But, do data point towards a preference between these models? The best-established mode of quadrupole collectivity in nuclei is

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(near-rigid) quadrupole shapes and their rotational degrees of freedom. These can be fitted phenomenologically with high precision using the symmetric and asymmetric top limits of the Bohr model and their low-order corrections resulting from band mixing. (For recent critical assessments of “best case” axially symmetric and asymmetric rotors, see Kulp et al. [10] and Allmond et al. [11], respectively.) The symplectic model offers two dynamical insights into nuclear rotation that go beyond phenomenology: first it predicts that vorticity must be present in rotating nuclei, second it accommodates the Elliott [SU(3)] model. While the existence of vorticity in rotating nuclei remains an open question, SU(3) models of nuclear rotation have been thoroughly developed by Rowe and coworkers (see [12] and references therein) and Draayer and coworkers (see, e.g., [13]) and are strongly supported by data (see, e.g., [14]). Most notably, these descriptions show that it is not necessary to use effective E2 charges.

Beyond the basic rotational degrees of freedom exhibited by many nuclei, the most visible and widespread low-energy degrees of freedom in even-even nuclei are states with spin-parity \(0^+\). There appear to be nearly as many theories purporting to describe these states as there are experimentally well-characterized examples. Low-lying excited \(0^+\) states appear to arise in a variety of ways in even-even nuclei with little evident commonality in their structure. In the following, a brief summary of well-characterized excited \(0^+\) states is given. This is followed by details, based on very recent (and ongoing) programs of experimental study, of two major redirections of interpretation that are necessitated: (1) It is questionable as to whether nuclei possess low-energy quadrupole vibrational degrees of freedom. (2) It is likely that a unified view of excited \(0^+\) states in nuclei based on pair excitations across shell and subshell gaps is supported.

2. Excited \(0^+\) states that are well understood

The \(0^+\) excited state with the most enduring interpretation is the first excited state in \(^{16}\)O which was proposed by Morinaga [15] to be a proton-pair-neutron-pair excitation across the N, Z = 8 closed shells with associated deformation resulting from the interaction between protons and neutrons. This led to a similar interpretation of excited \(0^+\) states in \(^{40}\)Ca and a plausible continuation in \(^{56}\)Ni (see [16,17,18] for reviews of these developments). Another doubly closed-shell nucleus that exhibits an excited \(0^+\) state, which is reliably interpreted, occurs in \(^{208}\)Pb [19]. The state results from a pair excitation across the N = 126 shell gap. However, this is not a deformed structure: this points to the interaction between active protons and neutrons being important for deformation.

In singly closed-shell nuclei the widespread occurrence of deformed bands built on low-lying excited \(0^+\) states has emerged to become a cornerstone of nuclear structure under the title of shape coexistence. These structures involve pair excitations across closed shells and occur lowest in energy near mid-shell for the other kind of nucleon with a “parabolic” energy trend as a function of open-shell nucleon number. Such structures persist in nuclei adjacent to closed shells. Shape coexistence has been reviewed, most recently, by Heyde and Wood [18].

Two types of low-lying excited \(0^+\) state that have been widely accepted among nuclear structure physicists are the two-phonon \(0^+\) state of the harmonic quadrupole vibrator model applied to nuclei believed to be spherical and the one-phonon beta vibrational state of the quadrupole deformed models applied to nuclei believed to be deformed. However, detailed investigations of some of the best candidate nuclei for exhibiting these modes show that vibrational behavior is untenable: this is taken up in the next section.

3. Excited \(0^+\) states that necessitate a reinterpretation

Extensive investigation of \(^{110,112,114,116}\)Cd (see [20]) has revealed that these nuclei, far from being “text-book” cases of near harmonic spherical vibrators, exhibit serious disagreement with expected multi-phonon patterns of low-energy excitation. This is summarized in Fig. 1. The experimental work needed to explore collective structures at the level exhibited by Fig. 1 is highly demanding on experimental technique. Lifetimes of some of the levels are in the femtosecond range and need to be determined by Doppler shifted \(\gamma\)-ray energies following inelastic neutron scattering (see, e.g., [21]).
Important collective transitions are often low energy and compete with high-energy (non-collective) decay branches: this incurs the $(E\gamma)^5$ factor for E2 transition rates which results in the $\gamma$-rays corresponding to the transitions of interest having decay branches $< 10^{-3}$ of the strongest decay branches (see, e.g., [22]).

The state in $^{110,112,114,116}$Cd which has popularly been considered to be a two-phonon $0^+$ state is systematically observed to decay with a B(E2) less than 1% of the strength expected for a harmonic quadrupole vibrational two-phonon state. Attempts have been made to rationalize this with detailed IBM model calculations that take into account the mixing between the candidate vibrational states and the deformed intruder states (see [23] for the most recent investigation of this approach). However, such calculations require the fine-tuning of a number of model parameters that vary from nucleus to nucleus: this casts doubt on the validity of such a description.

The question arises as to the nature of the purported $0^+$ two-phonon state. It is plausibly a proton-pair excitation involving an energy gap between the $p_{1/2}$ and $g_{9/2}$ proton subshells. However, this idea will need exploration via one- and two-proton transfer reaction spectroscopy. It should be noted that this $0^+$ state has an associated $2^+$ state with a B(E2) that is comparable to the ground-state collective structure (and a similar energy spacing). Thus, the structure would differ in pair structure, rather than deformation, relative to the ground state.

Multiple spectroscopic studies of $^{152}$Sm reveal collectivity that supports two shape coexisting structures that strongly mix [24]. The interpretation of the first excited $0^+$ state as a beta vibration is refuted by multi-step Coulomb excitation data: some details are shown in Fig. 2. If the $0^+$ state at 685 keV were a beta vibration, the Coulomb excitation path from this state would connect to a two phonon...
$K^{\pi} = 0^+$ band at an excitation energy of $\sim 1370$ keV (besides strong population of the rotational band built on the 685 keV state). Evidently, such an excitation is completely absent below an energy of $\sim 2400$ keV (deduced from the $\gamma$-ray line intensities and the energy range covered in the spectrum).

**Figure 2.** Response of the first excited $0^+$ state in the nucleus $^{152}$Sm to multi-step Coulomb excitation. The only state that is observed with strong population is the head of a $K^{\pi} = 2^+$ band at 1769 keV. (The population of the rotational band built on the first excited $0^+$ state is masked by a decay branching ratio of 99.7% out of the band at 811 keV.)

A number of other features are revealed in Fig. 2. Two $K^{\pi} = 2^+$ bands occur with band-head energy separations that are nearly identical to that of the ground and first-excited $K^{\pi} = 0^+$ band-head energy separations. A similar pattern is exhibited by two $K^{\pi} = 0^-$ bands in $^{152}$Sm [25]. The structure of $^{152}$Sm appears rather as a nucleus possessing a “double-vacuum” [24] (see also [26]). There is an excited $K^{\pi} = 0^+$ band built on a $0^+$ state at 1083 keV. However, this band has all of the properties of a “pairing isomer” [27, 28, 29]: such structures possess characteristic two-nucleon transfer strengths with an asymmetry between pick-up and stripping (see, e.g., [30, 31, 32]).

The evidence for shape coexistence comes from electric monopole (E0) transitions which systematically occur between the lowest $K^{\pi} = 0^+$ bands in $N = 90$ isotones [18]. The essential behavior is schematically depicted in Fig. 3. The underlying $0^+$ excitation is plausibly a consequence of a subshell energy gap at $Z = 64$. There is accumulating evidence for E0 transition strength between pairs of $K^{\pi} = 2^+$ bands in the $N = 90$ isotones [33].
Figure 3. Schematic view of intruder states and shape coexistence at N = 90. The nuclei with N < 90 are weakly deformed, those with N > 90 are strongly deformed, and those at N = 90 exhibit near-degenerate coexisting structures; these strongly mix giving rise to strong electric monopole (E0) transition strength.

The E0 transition strengths between the lowest two $K^\pi = 0^+$ bands are shown in Fig. 4. The occurrence of E0 transition strength in nuclei is a model-independent measure of the coexistence of configurations with different mean-square charge radii that mix [34, 35].

4. Conclusions

The nature of excited $0^+$ states in nuclei would appear to need further investigation. The historical interpretation of many of these states as resulting from vibrations of either spherical or deformed equilibrium shapes with a dominance of quadrupole shape degrees of freedom must be questioned. The above examples suggest that a unified view of excited $0^+$ states in nuclei may be achievable by considering pair excitations across subshell energy gaps; similar to the occurrence of pair excitations across major shell energy gaps which give rise to shape coexistence.
Strong mixing of coexisting shapes produces strong electric monopole (E0) transitions and identical bands. E0 strength is a function of mixing.

\[
\rho^2 \cdot 10^3
\]

| Energy (MeV) | E0 2 | 4 | 6 | 8 | 10 + | 12 + | 14 + |
|--------------|------|---|---|---|------|------|------|
| E(MeV)       | 0    | 1 | 2 | 3 | 4    | 5    | 6    |
|              | 152  | 154 | 156 |

**Figure 4.** Electric monopole transition strengths between the lowest two \(K^\pi = 0^+\) bands in the \(N = 90\) isotones. (The figure is similar to one shown in ref. [18].)

**Acknowledgements**

The author wishes to acknowledge collaborations with Mitch Allmond, Paul Garrett, Kris Heyde, David Kulp, and Ed Zganjar who have significantly contributed to the ideas presented in this work.

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