A Neurofuzzy Approach to Modeling Longitudinal Driving Behavior and Driving Task Complexity

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Technological innovations can be assumed to have made the driving task more complex. It is, however, not yet clear to what extent this complexity leads to changes in longitudinal driving behavior. Furthermore, it remains to be seen how these adaptation effects can best be modeled mathematically. In order to determine the effect of complexity on empirical longitudinal driving behavior we performed a driving simulator experiment with a repeated measures design. Through this experiment we established that complexity of the driving task leads to substantial changes in speed and spacing. In order to provide insight into how complexity is actually related to changes in longitudinal driving behavior we introduce a new theoretical framework based on the Task-Capability-Interface model. Finally in this paper we take some first steps towards modeling of adaptation effects in longitudinal driving behavior in relation to complexity of the driving task through the introduction of a new neurofuzzy car-following model and based on the proposed theoretical framework. In this paper we show that this model yields a relatively good prediction of longitudinal driving behavior in case of driving conditions with differing complexity. The paper finishes with a discussion section and recommendations for future research.

1. Introduction

Technological innovations have increased the amount of information provided by road side and in-vehicle information systems dramatically. Systems such as adaptive cruise control (ACC), navigation systems, smart phones, in-vehicle and road side traffic information systems, and automatic lane control (ALC) have shifted drivers from being a controller of the driving task towards being a manager of information while driving. This shift in the role of the driver can be assumed to have made the driving task more complex.

This shift in the complexity of the driving task can be assumed to have a substantial influence on longitudinal driving behavior (i.e., driving behavior in the same lane) with presumably an influence on traffic safety. In this sense, the most compelling evidence of the influence of driving task complexity on traffic safety may stem from crash statistics. Violanti and Marshall [1] compared 100 randomly selected US drivers involved in a crash over the last two years with another 100 drivers who were not involved in crashes. The results indicated a risk ratio of 5.6 : 1 for drivers who talk more than 50 minutes per month on mobile phones. Furthermore, in Redelmeier and Tibshirani [2] 70 drivers who were mobile phone users and were involved in crashes with substantial property damage were studied. Redelmeier and Tibshirani reported the risk of a crash occurring being 4.3 times higher when using a mobile phone.

In addition, Tijerina et al. [3] investigated operating four commercial navigation systems while driving on an oval test track with traffic. In their experiment 16 drivers were involved. From this study it followed that when operating these devices drivers are often not looking at the road, and...
accident risk increased substantially. An important question is, however, to what extent complexity of the driving task actually influences empirical longitudinal driving behavior. Therefore, the first research question of this paper is “to what extent does complexity of the driving task influence empirical longitudinal driving behavior, represented by changes in speed $v$ and following distance $s$?”

In order to answer this first research question we performed a driving simulator experiment with a repeated measures design among 25 participants. In a control condition, normal driving conditions were simulated, while in the experimental condition complexity was added to the driving task through the introduction of narrow lanes with road side concrete barriers. Using the obtained driving simulator data we statistically analyzed speed $v$ and spacing $s$, providing us with an indication of adaptation effects in longitudinal driving behavior following a change in the complexity of the driving conditions.

In the influence of complexity of the driving task on longitudinal driving behavior, human factors may be assumed to play a substantial role. Examples of likely candidates in this context are mental workload (e.g., [4]), situational awareness [5], and static driver characteristics (e.g., age and driving experience). Drivers may, for example, be distracted by the information which is provided to them or distracted due to the need to operate the system and consequently pay less attention to what the lead vehicles are doing with an influence on their driving behavior (see, e.g., [6]). It is, however, not yet clear how these human factors are actually related to the adaptation effects in longitudinal driving behavior in relation to changes in the complexity of the driving task due to the previously mentioned shift in the role of the driver. In this sense in this paper, we introduce a theoretical framework based on the Task-Capability-Interface model by Fuller [7]. In this theoretical framework adaptation effects in longitudinal driving behavior, consisting of compensation and performance effects, come forth from interactions between driver capability and task demands.

Furthermore, it is not yet clear how the possible adaptation effects in longitudinal driving behavior following a change in the complexity of the driving task should be modeled mathematically. Modeling of driving behavior (i.e., car-following) is of high importance, as these mathematical models form the core of microscopic simulation models. These models are used to ex ante determine the influence of for example information systems and new vehicular technology on traffic flow operations, safety, and emissions. However, current mathematical models of longitudinal driving behavior insufficiently incorporate human factors. As we argued in the aforementioned that human factors may be assumed to play a substantial role in the occurrence of changes in longitudinal driving behavior in relation to complexity of the driving task, we conjecture that current mathematical models of longitudinal driving behavior are fundamentally inadequate to model these effects. Therefore, the second research question is “in which way can the influence of complexity of the driving task on longitudinal driving behavior best be modeled mathematically?”

In order to answer the second research question, we also take a first step towards modeling of the influence of complexity of the driving task on longitudinal driving behavior. We aimed at developing a new model based on a neurofuzzy logic modeling approach including human factors with structure and parametric learning using data derived from the driving simulator experiment. In this sense we determined the optimal model complexity and trained this model with optimal complexity. Finally we compared the predictions of the model with actual data.

The objective of this paper is therefore to experimentally determine the influence of complexity of the driving task on empirical longitudinal driving behavior as well as to develop and test a new mathematical model aimed at modeling the influence of complexity of the driving task on longitudinal driving behavior.

In the next section we provide a brief state of the art. In this section we present an overview of the available research on the influence of the complexity of the driving task on empirical longitudinal driving behavior. This section is followed by the introduction of a theoretical framework relating complexity of the driving task to longitudinal driving behavior based on the Task-Capability-Interface model by Fuller [7], followed by a discussion of mathematical modeling of longitudinal driving behavior in relation to complexity of the driving task.

The state of the art is followed by a presentation of the research method. In this section we provide an introduction to the driving simulator used in this study, followed by a description of the driving environment developed for this experiment, the characteristics of the participants, and the data analysis method used to statistically determine the effect of the complexity of driving conditions on empirical longitudinal driving behavior. In the following section we present the results with regard to the influence of driving task complexity on adaptation effects in empirical longitudinal driving behavior.

In the following section we describe the proposed approach to modeling the influence of the complexity of the driving task on longitudinal driving behavior, that is, the neurofuzzy modeling approach. Here, we describe the results of the structure and parametric learning, the training of the model, and validation of the model through a comparison of the predicted behavior versus the actual longitudinal driving behavior in relation to complexity of the driving task. This paper concludes with a discussion section in which conclusions are drawn from the results and recommendations for future research are provided.

2. State of the Art

2.1. Adaptation Effects in Empirical Longitudinal Driving Behavior in relation to Complexity of the Driving Task

Adaptation effects in longitudinal driving behavior due to complexity may be assumed to be governed by compensation and performance effects. Compensation effects entail the assumption that drivers regulate their driving behavior in order to compensate for any reduction in attention to the
driving task. Very little research has been performed on compensation effects in relation to complexity of the driving task, as research is mainly focused on performance effects. However, research strongly suggests that drivers do engage in a range of conscious adaptations in their driving behavior in order to maintain a certain level of risk [8]. Numerous studies have shown that, at the operational level [9], drivers aim at reducing their risk level during the use of in-vehicle technology or due to external circumstances (e.g., adverse weather conditions). Compensation effects in longitudinal driving behavior may consist of speed reductions [8, 10, 11] and changes in the distance to the lead vehicle [12, 13].

In Haigney et al. [8] it was shown that mean speed decreased while participants were conversing on a mobile telephone. More recent research carried out in a driving simulator by Rakauskas et al. [11] also found that drivers’ mean speed decreased and their speed variability increased while carrying out a conversation on a mobile phone. The aforementioned speed reductions could be the result of a modification of performance goals of drivers and the acceptance of a suboptimal level of performance. Finally in Strayer et al. [13] it was shown that conversing on a hands-free mobile phone while driving led to an increase in the distance to the lead vehicle.

Besides compensation effects due to changes in the complexity of the driving task, adaptation effects in driving behavior are also governed by performance effects (i.e., the quality of conducting the driving task). For example, in Brookhuis et al. [4] it was shown that significant effects were found in telephone conversations while driving on reareview mirror checking, the adaptation of speed to the speed of the lead vehicle, and braking in reaction to decelerations of the lead vehicle. In Makishita and Matsunaga [14] an experiment was performed in which reactions of drivers in various age groups were examined in order to assess the influence of driving task complexity. In their research experiments were performed on a simulated street in order to identify drivers with large reaction times and drivers whose reaction times are strongly affected by driving task complexity. The results show that a secondary task (mental calculations) increased average reaction times for all age groups. This secondary task increased the differences between age groups and individuals and increased differences in the drivers’ individual performance. Reaction times especially of elderly drivers were affected substantially.

From the aforementioned it can be concluded that complexity of the driving task (e.g., through the addition of a secondary task) has a substantial influence on driving behavior through compensation effects and performance effects. However, as was mentioned before, research especially on conscious compensation effects following a change in the complexity of driving conditions is scarce and focuses mainly on mobile telephone conversations. It is therefore crucial to gain more insight into the influence of complexity of driving conditions on empirical longitudinal driving behavior. Furthermore, from the available research it does not become clear how these compensation and performance effects in driving behavior are actually related to complexity of the driving task. To this end in the next section we introduce a new theoretical framework based on the Task-Capability-Interface model by Fuller [7].

2.2. Introducing a Theoretical Framework Behavioral Adaptation to Changes in Complexity of Driving Conditions. In the previous section we discussed the available research on changes in driving behavior following a change in the complexity of the driving task. It was concluded that it may be assumed that complexity of the driving task has a substantial influence on longitudinal driving behavior, although research is quite scarce. The aforementioned, however, does not yet inform us how these changes in driving behavior are actually related to complexity of the driving task. To this end in this section we introduce a new theoretical framework.

In the proposed theoretical framework based on the Task-Capability-Interface model by Fuller [7], external circumstances (such as road design, weather interactions with other vehicles, roadside traffic management measures, and in-car technology) determine the complexity of the driving task. Complexity may have an influence on driver capability, moderated by driver characteristics, as well as on task demands. Therefore, in the proposed theoretical framework driving task difficulty comes forth from a dynamic interface between the demands of the driving task and the capability of the driver (see Figure 1).

In this context, Fuller [7] mentions that driver capabilities are restricted by biological personal characteristics of the driver as well as by driving experience. However, these capabilities due to biological personal characteristics (e.g., age, gender, and ethnicity) and driving experience alone do not determine the total temporal capabilities of the driver, as more dynamic variables play a substantial role as well. An example of a dynamic driver characteristic is activation level. Activation level has been shown to have a substantial influence on driving behavior. For example, in Matthews et al. [16] the hypothesis was tested whether activation level is associated with driving performance. Eighty young adult participants performed a simulated test drive concurrently with a reasoning task. The data indicated that performance was characterized by adaptive mobilization of effort in order to meet the changing task demands. Drivers with a high activation level adapted to high levels of demand fairly efficiently, but were at risk of performance reduction when the task required little effort.

Another important determinant influencing driver capability is distraction. It can be assumed that in case of distraction (e.g., due to mobile telephone conversations while driving) driver capability will be reduced [4]. It can be assumed that especially driver distraction plays a substantial role in adaptation effects in driving behavior in relation to complexity of the driving task due to the shift in the role of the driver.

As is the case with driver capabilities, driving task demands are also related to a multitude of elements [7]. However, important elements in task demands are the elements over which the driver of the vehicle has direct
control. These conscious actions of the driver are in the ensuing referred to as compensation effects. Here, speed of the vehicle is clearly the most significant element: the faster a driver is moving, the less time is available to perceive stimuli, process information, and make decisions. As Taylor [17] regards the driving task as self-paced, driving task demand is in a fundamental way under the control of the driver through speed selection.

When drivers fail the driving task, a loss of control can be observed as a consequence. Thus, in essence, task difficulty is inversely proportional to the difference between task demand and the capability of the driver. According to Fuller [7], at the threshold where task demand begins to exceed the capability of the driver, a fragmented degradation of the driving task is to be expected. Fuller [7] continues by stating that with a static level of capability, any event that increases task demand will therefore reduce this critical difference, increase task difficulty, and potentially influence driving task performance.

In sum, changes in driver capability or task demands related to complexity (due to, e.g., in-vehicle systems or roadside traffic management measures) are expected to lead to compensatory changes in driving behavior (see also [4]). When these compensatory reductions are insufficient in order to balance task demand with driver capability, driving performance will suffer (performance effects).

2.3. Mathematical Models of Longitudinal Driving Behavior in relation to Complexity of the Driving Task. In the previous sections we discussed the available knowledge on the influence of complexity of the driving task on longitudinal driving behavior and proposed a new theoretical framework relating changes in driving behavior to complexity. This does not, however, yet inform us how adaptation effects
in longitudinal driving behavior due to changes in the complexity of the driving task can best be modeled. To this end in the present section a brief overview is provided on current mathematical models of longitudinal driving behavior in relation to complexity of the driving task. In this context we will start with discussing a few often used models in which we will show that current models insufficiently incorporate human factors.

The GHR model [18] is perhaps the most well-known model of longitudinal driving behavior and dates from the late fifties and early sixties. The model is expressed in the following equation:

\[ a(t) = c m(t) \left( \frac{\Delta v(t - \tau)}{\Delta x(t - \tau)} \right). \]  

(1)

In (1) \( a \) is the acceleration of a vehicle implemented at time \( t \) and is proportional to the speed \( v \), relative speed \( \Delta v \) (speed difference with the lead vehicle), and relative distance to the lead vehicle \( \Delta x \) (distance headway) assessed at an earlier time \( t - \tau \). In this equation \( \tau \) represents the reaction time of the driver. Furthermore, in this equation \( m, l \) and \( c \) are the parameters to be determined. As acceleration \( a \) is dependent on relative speed \( \Delta v \) and relative distance \( \Delta x \) this model can be qualified as a stimulus-response model.

From this model it can clearly be observed that human factors are to a very limited degree incorporated. The only human factor that is incorporated in the GHR model [18] is a finite reaction time.

An alternative approach to car-following modeling was taken by Treiber et al. [19]. Their Intelligent Driver Model (IDM) was developed as the models developed up to this point had unrealistically small acceleration and deceleration times (e.g., in case of Bando et al. [20]) and because the more high fidelity models like the Wiedemann model [21] have too many parameters. Furthermore, Treiber et al. [19] conjectured that most models do not adequately incorporate traffic flow phenomena, such as traffic instabilities and hysteresis.

Acceleration in the IDM [19] is a continuous function incorporating different driving models for all speeds in freeway traffic as well as city traffic [22]. Besides the following distance \( \Delta x \) and speed \( v \) the IDM [19] also takes relative speed \( \Delta v \) into account. The IDM acceleration is given by

\[ a = a_{\text{max}} \left[ 1 - \left( \frac{v}{v_0} \right)^\delta - \left( \frac{s^*(v, \Delta v)}{\Delta x} \right)^2 \right], \]

(2)

\[ s^*(v, \Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{a_{\text{max}} b_{\text{max}}}}. \]

The expression combines a free flow acceleration regime \( a[1 - (v/v_0)^\delta] \) with a deceleration strategy \( -a(s^*/\Delta x)^2 \). The latter becomes relevant when the distance to the lead vehicle \( \Delta x \) is not significantly larger than the desired distance to the lead vehicle \( s^* \). The free flow acceleration is characterized by free speed \( v_0 \), maximum acceleration \( a_{\text{max}} \), and the component \( \delta \). The component \( \delta \) characterizes how acceleration decreases with speed.

The desired distance to the lead vehicle \( s^* \) is composed of a minimal stopping distance (jam distance) \( s_0 \), and a speed dependent distance \( vT \). This corresponds to following the lead vehicle with a constant desired time headway \( T \) and a dynamic contribution which is only active in nonstationary traffic conditions. This implements an “intelligent” driving behavior that, in normal situations, limits braking decelerations to the maximum deceleration \( b_{\text{max}} \) [22].

A good step towards the incorporation of human factors in car-following models was taken by Tampere et al. [23] as in their macroscopic flow model finite reaction times, anticipation and driving style variations (i.e., attention level) were incorporated. In their gas-kinetic model the general law for the conservation of probability was transformed through use of the method of moments. Through this method the following macroscopic traffic flow model was derived:

\[ \frac{\partial k}{\partial t} + \frac{\partial kV}{\partial x} = \left( \frac{dk}{dt} \right)_{\text{event}}, \]

(3)

\[ \frac{\partial kV}{\partial t} + \frac{\partial (kV^2 + k\theta)}{\partial x} = k \left( \frac{dv}{dt} \right) + \left( \frac{dkV}{dt} \right)_{\text{event}}. \]

In these equations \( k \) denotes the density, \( V \) denotes the speed, and \( \theta \) denotes the distribution of speed. Driving style variations were implemented by characterizing an individual's state not only by the individual speed \( v \) and the distance to the lead vehicle, but also by the attention level \( a \). Again the authors use the method of moments in order to obtain the speed dynamic equation [23]:

\[ \frac{\partial A}{\partial t} + V \frac{\partial A}{\partial x} = \left( \frac{da}{dt} \right)_{v,a} + \frac{1}{k} \int_v a \left( \frac{dp}{dt} \right)_{\text{event}} dv \cdot da - A \left( \frac{dk}{dt} \right)_{\text{event}}. \]

(4)

In (4) the first term on the right represents the effect of driver induced changes in the attention level \( A \). The second term represents the effect due to events in the flow while the last term represents the redistribution of the total attention level \( A \) over the population \( k \) in case the density does not remain constant [23].

From this section it can be concluded that in its current form, current mathematical models such as the GHR model [18] and the IDM [19] insufficiently incorporate human factors (e.g., attention level, mental workload, and personal characteristics of drivers) and can therefore assumed to be less adequate in describing the effect of complexity of the driving task on longitudinal driving behavior and traffic flow operations. These elements are, to some extent, incorporated in the macroscopic traffic flow model by Tampere et al. [23]. As this is a macroscopic model, individual changes in driving behavior can be less adequately observed. Furthermore, this model lacks a thorough theoretical framework.

In Hoogendoorn et al. [24] we therefore proposed a new car-following model based on the theoretical framework discussed in this section. Besides the theoretical framework, this model was based on the previously discussed Intelligent Driver Model [19]. In the proposed model it was assumed that compensation effects and performance effects...
in longitudinal driving behavior following a change in the complexity of the driving task come forth from the difference between task driver capability and task demands at a certain time instant. The difference between task demands and driver capability $m_d(t)$ at time $t$ is formulated as follows:

$$m_d(t) = m_t(t) - m_c(t). \quad (5)$$

In this equation $m_c(t)$ represents driver capability, while $m_t(t)$ represents task demands of a driver at time $t$. In the proposed model it was assumed that $-1 < m_d(t) < 1$. A driver will try to minimize the difference between driver capability and task demands, by exerting influence of those elements in driving behavior over which he has direct control (compensation effects). We assumed in the proposed model that a driver has direct control over maximum acceleration $a_{\text{max}}$, maximum deceleration $b_{\text{max}}$, free speed $v_0$, and desired time headway $T$ (see also [25]). This is mathematically formulated as follows:

$$a(t) = \left(\left(-m_d(t)^3 a_{\text{max}}\right) + a_{\text{max}}\right) \times \left[1 - \left(\frac{\nu(t)}{\left(-m_d(t)^3 v_0\right) + v_0}\right)\right]^\delta - \left(\frac{s^*(\nu(t), \Delta \nu(t))}{\Delta x(t)}\right)^2,$$

$$s^*(\nu(t), \Delta \nu(t)) = s_0 + \nu(t)\left(\left(m_d(t)^3 T\right) + T\right) + \frac{\nu(t) \Delta \nu(t)}{2\sqrt{\left(\left(-m_d(t)^3 a_{\text{max}}\right) + a_{\text{max}}\right)\left(\left(-m_d(t)^3 b_{\text{max}}\right) + b_{\text{max}}\right)}}. \quad (6)$$

In these equations it was shown that the contribution of maximum acceleration $a_{\text{max}}$, maximum deceleration $b_{\text{max}}$, free speed $v_0$, and desired time headway $T$ to the IDM acceleration $a$ is dependent on the difference between task demands and driver capability $m_d$. The effect of $m_d$ on $a_{\text{max}}$, $b_{\text{max}}$, $v_0$, and $T$ is assumed to be cubic.

Besides these conscious compensation effects, the adaptation of the Task-Capability-Interface model [7] also assumed that when a driver is unable to resolve the imbalance between driver capability and task demands, performance effects will occur. It is assumed that the difference between driver capability and task demands is related to performance effects with an inverted U-shaped function (see also [44]):

$$m_p(t) = -\left(\alpha m_d^2 + \beta m_d + \gamma\right). \quad (7)$$

In this equation $m_p$ represents the performance effects in driving behavior due to complexity of the driving task at time $t$. Furthermore, in the equation $\alpha$, $\beta$ and $\gamma$ are parameters.

Integrating performance effects into the Intelligent Driver Model [19] yields the following equations:

$$a(t) = \left(1 - m_p(t)\right)\left(\left(-m_d(t)^3 a_{\text{max}}\right) + a_{\text{max}}\right) \times \left[1 - \left(\frac{\nu(t)}{\left(-m_d(t)^3 v_0\right) + v_0}\right)\right]^\delta - \left(\frac{s^*(\nu(t), \Delta \nu(t))}{\Delta x(t)}\right)^2,$$

$$s^*(\nu(t), \Delta \nu(t)) = s_0 + \nu(t)\left(\left(m_d(t)^3 T\right) + T\right) + \frac{\nu(t) \Delta \nu(t)}{2\sqrt{\left(\left(-m_d(t)^3 a_{\text{max}}\right) + a_{\text{max}}\right)\left(\left(-m_d(t)^3 b_{\text{max}}\right) + b_{\text{max}}\right)}}. \quad (8)$$

From Hoogendoorn et al. [24] it followed that the model showed quite well the influence of the difference between task demands and driver capability on longitudinal driving behavior and that the model was also able to provide a relatively adequate explanation for the so-called capacity funnel phenomenon as well as the influence of an optimal amount of information provision and information overload on driving behavior and macroscopic traffic flow operations. However, one of the main problems is to actually estimate changes in task demands and driver capability following a change in the complexity of the driving task. For instance, how does using a navigation system influence the balance between task demands and driver capability.

To determine these relationships in this paper we propose a new car-following model based on a neurofuzzy network approach. We chose this approach as a neurofuzzy network modeling approach allows for learning of the model structure and enables the establishment of relationships between stimuli and output variables. However, before presenting this model more insight is needed into the influence of complexity on empirical driving behavior. To this end in the following section the research method of the driving simulator experiment is presented.

### 3. Research Method

In the previous section we discussed the available research on changes in driving behavior following a change in the complexity of the driving task and proposed a new theoretical framework aimed at relating complexity to changes in driving behavior. The introduction of the theoretical framework was followed by a discussion of the available mathematical models of driving behavior in relation to complexity. We concluded that human factors are not incorporated in these models and can therefore be assumed to be fundamentally inadequate in describing adaptation effects following a change in the complexity of the driving task.

However, in order to be able to propose an adequate mathematical model of longitudinal driving behavior in relation to complexity of the driving task, more insight
was needed into the extent to which complexity of the driving task actually influences empirical longitudinal driving behavior. To this end in the present section we present the research method. In this sense we present the experimental design, followed by an introduction into the used driving simulator and a presentation of the developed driving environment. This section is followed by a description of the research sample and the data analysis method.

3.1. Experimental Design. All participants participated in the experimental condition as well as in the control condition, rendering up a complete within-subjects design. Adaptation effects in longitudinal driving behavior, represented by adaptations in speed $v$ and spacing $s$, were measured through registered behavior in the driving simulator at a sampling rate of 10 samples per second during both conditions.

3.2. The Driving Simulator and Driving Environment. The fixed-base driving simulator consists of three screens placed at an angle of 120 degrees, a driver’s seat mockup, and hardware and software interfacing of this mockup to a central computer system. From the driver’s seat the view consists of a projection of 210 degrees horizontally and 45 degrees vertically. The software was developed by ST Software.

For the purpose of the experiment, a driving environment was developed consisting of three segments. The first segment consisted of a short test drive through a suburban area to accustom participants to driving in a driving simulator and also to investigate whether the participants were prone to simulator sickness.

The other two segments were used in the experiment. These test trials took place on a virtual motorway with three lanes in the same direction. The length of the three segments combined was 9.45 km. In the control condition normal driving conditions with a medium density were simulated, while in the experimental condition narrow lanes and roadside concrete barriers were applied aimed at increasing the complexity of the driving task (Figure 2). Traffic with a medium intensity was simulated. The behavior of the other vehicles was derived from a pilot study and consisted of larger values of spacing $s$ as well as a reduction in speed $v$.

3.3. Participants and Data Analysis Method. The research population consisted of 25 employees and students of Delft University of Technology (16 male and 9 female participants). The age of the participants varied from 22 to 54 years with a mean age of 29.68 years (SD = 6.93). Driving experience varied from 1 to 35 years with a mean of 9.6 years (SD = 7.50).

Adaptation effects were analyzed through a comparison of the indicators of longitudinal driving behavior (i.e., speed $v$ and spacing $s$) between the control and the experimental condition using a paired samples $t$-test with a significance level of 0.05.

4. Results of Driving Task Complexity and Empirical Longitudinal Driving Behavior

In the previous section we presented the research method aimed at establishing the influence of complexity of the driving task (i.e., concrete barriers with narrow lanes) on empirical longitudinal driving behavior. In this context, the first research question was “to what extent does complexity of the driving task influence empirical longitudinal driving behavior, represented by changes in speed $v$ and spacing $s$?”

From the paired samples $t$-test it followed that a significant reduction in mean speed in the experimental condition ($M = 77.80, SD = 12.64$) compared to the control condition ($M = 100.84, SD = 16.12$) could be observed, $t(24) = 4.23, P < .05$. As an illustration individual speeds and mean speeds along with the standard deviations are displayed in Figure 3.

Spacing $s$ also showed a significant difference between the control and the experimental condition. In the control condition mean spacing amounted to 15.44 m (SD = 27.04), while in the experimental condition it amounted to 39.94 m (SD = 31.44). The difference between the two conditions was significant, as $t(24) = 9.88, P < .05$.

This leads to the conclusion that a change in the complexity of the driving task (i.e., narrow lanes and roadside concrete barriers) leads to substantial and significant compensation effects in longitudinal driving behavior, represented by a significant reduction in speed $v$ and spacing $s$. 

Figure 2: Driving environment developed for the purpose of the experiment. On the left the control condition is displayed, while on the right the experimental condition is displayed.
5. Neurofuzzy Modeling of Longitudinal Driving Behavior in relation to Complexity of Driving Conditions

In the proposed theoretical framework we stated that complexity of the driving task has an influence on longitudinal driving behavior through task demands and driver capability. Indeed, it was established through the driving simulator experiment that complexity of the driving task has a substantial influence on empirical longitudinal driving behavior.

These adaptation effects in empirical longitudinal driving behavior due to a change in the complexity of the driving task reported in the previous section, however, do not inform us how these adaptation effects can best be modeled. In this sense the second research question was “in which way can the influence of complexity of the driving task on longitudinal driving behavior best be modeled mathematically?” In the state of the art we discussed several car-following models and concluded that these models, in general, can be assumed to be less adequate in describing longitudinal driving behavior in relation to complexity of driving conditions as they insufficiently incorporate human factors. In this sense we also briefly discussed the model introduced in Tampere et al. [23]. We argued that this model, as it is a macroscopic flow model, is less adequate in showing the influence of individual driving behavior and lacks a theoretical framework.

We also discussed the adaptation of the Intelligent Driver Model [19] as proposed in Hoogendoorn et al. [24]. Although this model shows relatively well the influence of the difference between task demands and driver capability as proposed in the theoretical framework, the main problem of this model is to actually estimate the influence of complexity of driving conditions on the difference between task demands and driver capability and ultimately longitudinal driving behavior. Therefore, in order to model adaptation effects in longitudinal driving behavior in relation to complexity of the driving task we propose a neurofuzzy modeling approach using a Takagi-Sugeno fuzzy architecture [26] and based on the proposed theoretical framework (difference in task demands and driver capability). In the next section we will start with an introduction into this approach.

5.1. An Introduction into Neurofuzzy Logic Modeling. A Takagi-Sugeno fuzzy inference system [26] consists basically of a set of $r$ rules, such as

$$
\text{if } x_1 \text{ is } A_1^1 \text{ and } x_2 \text{ is } A_2^1 \cdots \text{ and } x_n \text{ is } a_n^1 \\
\text{then } y_1 = f_1^1(x_1, x_2, \ldots, x_n) \\
\vdots \\
\text{if } x_1 \text{ is } A_1^r \text{ and } x_2 \text{ is } A_2^r \cdots \text{ and } x_n \text{ is } a_n^r \\
\text{then } y^r = f^r(x_1, x_2, \ldots, x_n).
$$

(9)

The antecedent of this equation is defined as a fuzzy AND proposition, where $A_j^i$ is a fuzzy set on the $j$th premise variable defined by the membership function $\mu_j^i : \mathbb{R}^n \rightarrow [0, 1]$. The consequent is a function $f^i$ with $i = 1, \ldots, r$ of the input vector $[x_1, x_2, \ldots, x_n]$. Through the fuzzy sets $A_j^i$ the input is divided into smaller regions where the mapping is approximated by the models $f^i$. A weighted mean is used to recombine all the local representations in a global approximation:

$$
y = \frac{\sum_{i=1}^{r} \mu^i y_1^i}{\sum \mu^i}. 
$$

(10)

In (10) $\mu^i$ represents the degree to which the $i$th rule is fulfilled. However, when the consequent is a linear model, for instance, in case of predicting accelerations, the system can
be used to return a local linear approximation of a generic point of the input domain. Suppose that we have the input \( \bar{x} = [\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n] \). In this case (10) will return a linear approximation of \( f_{\text{lin}}(\bar{x}) \):

\[
 f_{\text{lin}}(\bar{x}) = \frac{\sum_{i=1}^{n-1} \mu_i \left( \sum_{j=1}^{n} p_{ij} \bar{x}_j + p_{i0} \right)}{\sum \mu_i}. \tag{11}
\]

In the aforementioned traditional approach to fuzzy systems the membership functions and models are fixed according to prior knowledge (expert opinions). However, when this knowledge is (not yet) available, the components (given a certain data set) can be represented in a parametric form and the parameters are tuned through a learning procedure. In this case the fuzzy system turns into a neurofuzzy approximator [27]. In neurofuzzy systems, two types of learning are required, namely,

(i) structural learning;
(ii) parameter learning.

The first aims at finding a suitable number of rules and a proper partition of the input space (membership functions). Given an optimal structure, a neurofuzzy approximator searches for the optimal membership functions together with the optimal parameters of the consequent models.

5.2. Structure and Parameter Learning. We started with creating an input-output matrix with speed \( v \), following distance \( s \) and relative speed \( \Delta v \) as inputs of the system derived from the driving simulator data discussed previously in this paper. Additionally we added a variable to the input-output matrix representing the difference between task demands and driver capability \( C_o \) as input (as proposed in the previously discussed theoretical framework). In case of normal driving conditions a 0 was attributed to this variable while in case of a complex driving task (i.e., concrete barriers and narrow lanes) a 1 was attributed. This represents an increase in task demands following an increase in the complexity of the driving task (see also (5)). As output of the system we added acceleration \( a \) to the matrix. The input and output variables were all scaled between \(-1 \) and \(1\).

With regard to the structure of the neurofuzzy logic car-following models there may be a lot of different structure/parameter combinations which provide a feasible solution. We therefore aimed at finding the solution which provides the best performance in terms of generalization [28]. In the approach used in this paper we chose to use the number of rules as a measure of model complexity. To this end we adopted an incremental approach where different architectures with different levels of model complexity are assessed and cross-validated.

The initialization of the architecture is provided by a hyperellipsoidal fuzzy clustering procedure [29]. In [29] it is proposed to cluster the data in the input-output domain through which a set of hyperellipsoids is obtained. This set can be regarded as a coarse representation of the input-output mapping.

Methods for initializing the parameters of a neurofuzzy system were derived from the procedure described in [30]. In the present paper we used the eigenvectors of the scatter matrix to initialize the parameters of the consequent functions \( f \). Furthermore, we projected the cluster centers on the input domain to initialize the centers of the antecedents and adopted the scatter matrix in order to compute the width of the membership functions.

In the parametric estimation the best set of parameters was searched for by minimizing the sum-of-squares cost function \( J_M \) dependent solely on the training data set. As the model proposed in this paper is a linear model, the minimization procedure was decomposed into a least squares problem to estimate the linear parameters of the consequent models \( f \) [31] and a nonlinear minimization (Levenberg-Marquart) to find the parameters of the membership functions \( A_j \) [27] (see also (9)). In this context in this paper we used triangular-shaped membership functions of the antecedents. Mathematically, these membership functions can be formulated as follows:

\[
 \mu_i(x) = \prod_{j=1}^{n} \max(0, 1 - \frac{|x_j - c_{ij}|}{b_j^i}). \tag{12}
\]

The consequent model is mathematically formulated as follows:

\[
 y_i = \sum_{j=1}^{n} p_{ij} x_j + p_{i0}. \tag{13}
\]

We determined the best model structure (related to the number of rules in the model) by gradually increasing the number of local models. Next we compared the different model structures in relation to their performance \( J_{CV} \) using a K-fold cross-validation [32]. We used a high proportion of the training data to determine the structure and the used method provided us with reliable estimates of the performance in generalization.

In Figure 4 the cross-validation versus complexity diagram is displayed. As was previously stated we chose the model with the lowest cross-validation error.

This leads to the conclusion that 6 rules provide the best model complexity:

\[
 \begin{align*}
 \text{if } x_1 & = A_1^1 \text{ and } x_2 = A_1^2 \cdots \text{ and } x_n = A_1^n \\
 \text{then } & y^1 = f^1(x_1, x_2, \ldots, x_n) \\
 & \vdots \\
 \text{if } x_6 & = A_6^1 \text{ and } x_2 = A_6^2 \cdots \text{ and } x_n = A_6^n \\
 \text{then } & y^6 = f^6(x_1, x_2, \ldots, x_n).
\end{align*} \tag{14}
\]

As was mentioned before, with regard to the structure learning, we aimed at determining the membership functions, represented by the values of the centers \( c \) and the bases \( b_j \) (see (12)). In Tables 1 and 2 the values of the centers \( c \) and bases \( b_j \) are displayed, respectively. Note that \( i \) represents the number of the rule in the model, while \( j \) represents, respectively, the difference between task demands and driver
Research has shown that complexity may be assumed to lead to shift in the role of the driver. Following this work on behavioral adaptation following a change in the complexity of driving conditions was lacking. Finally it was not yet clear how the adaptation effects in driving behavior although research is scarce and mainly focused on the use of mobile telephones while driving. Furthermore, a thorough theoretical framework on behavioral adaptation following a change in the complexity of driving conditions was lacking. Finally it was not yet clear how the adaptation effects in driving behavior in relation to complexity of the driving task can best be modeled mathematically.

Due to the technological innovations the amount of information directed at road users has increased substantially leading to shift in the role of the driver. Following this development the driving task is becoming more complex. Research has shown that complexity may be assumed to lead to adaptation effects in driving behavior although research is scarce and mainly focused on the use of mobile telephones while driving. Furthermore, a thorough theoretical framework on behavioral adaptation following a change in the complexity of driving conditions was lacking. Finally it was not yet clear how the adaptation effects in driving behavior in relation to complexity of the driving task can best be modeled mathematically.

5.3. Neurofuzzy Prediction of Acceleration. The model structure and the value of the parameters, however, do not yet inform us to what extent this model actually provides a good prediction of accelerations in relation to complexity of the driving task. To this end we compared the accelerations of one driver given the established optimal complexity of the fuzzy architecture, the centers $c_j$ and bases $b_j$ of the membership functions, and the parameters $p_{ij}$ and $p_{i0}$ of the consequences to data from the driving simulator experiment.

In Figure 5 we provide two examples of the results for one individual driver. In the left graph the output of the fuzzy logic architecture is compared to the data from the driving simulator experiment under normal driving conditions while in the right graph the output is compared to the data from the driving simulator experiment under complex driving conditions. For convenience purposes in the graphs the scaled accelerations are shown. In the graphs the blue line represents the output accelerations from the fuzzy logic architecture, while the red line represents the scales’ accelerations from the driving simulator experiment.

From Figure 5 it can be observed that overall the output of the fuzzy logic architecture resembles the actual accelerations of the driving simulator experiment fairly good, although in some cases the output of the architecture is somewhat less extreme. From an independent samples t-test it followed that the difference between the predictions and the driving simulator data was not significant ($P > .05$).

In this section we proposed a new car-following model able to model the influence of complexity of the task on longitudinal driving behavior using a neurofuzzy architecture and based on the Task-Capability-Interface model by Fuller [7]. We determined the optimal model complexity and determined the values of the centers and bases of the membership functions as well as estimated the parameters for the consequences. Finally we compared the output of the model to the actual data. We showed that the model provides a relatively good representation of accelerations under normal and in case of a complex driving task.

### Table 1: Results of estimation of $c_j$ for the membership functions of the antecedents.

| $c_j$ | $j = 1$ | $j = 2$ | $j = 3$ | $j = 4$ |
|-------|--------|--------|--------|--------|
| $i = 1$ | 0.2671 | 0.2970 | 0.1258 | -0.0586 |
| $i = 2$ | 0.0042 | 0.9617 | 0.2265 | -0.2185 |
| $i = 3$ | 0.0056 | 0.8374 | 0.2332 | 0.1199 |
| $i = 4$ | 0.0065 | 0.4219 | 0.8657 | 0.8456 |
| $i = 5$ | 0.0596 | 0.2392 | 0.0822 | -0.1867 |
| $i = 6$ | 0.0278 | 0.2524 | 0.0796 | -0.2273 |

### Table 2: Results of estimation of $b_j$ for the membership functions of the antecedents.

| $b_j$ | $j = 1$ | $j = 2$ | $j = 3$ | $j = 4$ |
|-------|--------|--------|--------|--------|
| $i = 1$ | 1.4656 | 1.3729 | 1.7483 | 1.9349 |
| $i = 2$ | 1.9915 | 1.8484 | 1.5469 | 2.2208 |
| $i = 3$ | 1.9887 | 1.5963 | 1.5334 | 2.2399 |
| $i = 4$ | 1.9869 | 0.9362 | 1.7228 | 3.6481 |
| $i = 5$ | 1.8806 | 1.4886 | 1.4330 | 2.1912 |
| $i = 6$ | 1.9443 | 1.4620 | 1.8406 | 2.2723 |

### Table 3: Results of estimation of the parameters $p_{ij}$ and $p_{i0}$ for the consequences.

| $p_{ij}$ | $j = 1$ | $j = 2$ | $j = 3$ | $j = 4$ | $p_{i0}$ |
|----------|--------|--------|--------|--------|---------|
| $i = 1$ | 0.0000 | 5.1062 | -22.2048 | 5.3034 | 4.9813 |
| $i = 2$ | 0.0000 | 16.0963 | -0.6176 | 2.5492 | -11.7322 |
| $i = 3$ | 0.8418 | 0.4348 | -0.3756 | 0.3211 | 0.8418 |
| $i = 4$ | 0.8095 | 2.7108 | -0.3812 | 0.0757 | 0.8095 |
| $i = 5$ | 0.0000 | -2.9923 | 20.9578 | -7.1156 | 2.4450 |
| $i = 6$ | 0.0000 | -1.5716 | 2.0211 | -0.6674 | 1.2252 |

capability $Co$, speed $v$, distance to the lead vehicle $s$, and relative speed $\Delta v$.

Using the data from the driving simulator experiment we therefore aimed at estimating the parameters $p_{ij}$ and $p_{i0}$ for the consequences. In Table 3 the results of the estimation of these parameters are displayed.

**Figure 4:** Cross-validation error versus complexity diagram. On the horizontal axis the number of rules is displayed, while on the vertical axis the cross-validation error is displayed.
To this end we conducted a driving simulator experiment aimed at simulating the influence of a change in the complexity of the driving task on empirical longitudinal driving behavior (i.e., driving behavior in the same lane). Through this experiment we established that complexity of the driving task has a significant influence on speed \( v \) and spacing \( s \). In our experiment speed \( v \) was significantly lower in the experimental conditions compared to the control condition, while spacing \( s \) was significantly higher in the experimental condition compared to the control condition.

In order to explain the adaptation effects in longitudinal driving behavior following a change in the complexity of the driving task we introduced a new theoretical framework based on the Task-Capability-Interface model by Fuller [7]. In this model adaptation effects in driving behavior come forth from the dynamic interface between task demands and driver capability following a change in the complexity of the driving task.

This framework was used as a basis for the proposed mathematical model of longitudinal driving behavior in relation to complexity. In the proposed mathematical model we used a neurofuzzy architecture with structure and parameter learning. We used this method as current mathematical models insufficiently incorporate human elements. The models which can be regarded as a good step towards the incorporation of human elements either lack a theoretical framework or are less adequate in determining the influence of the complexity of the driving task on task demands and driver capability (e.g., [24]).

In the context of the neurofuzzy approach we started with determining the optimal model complexity, after which we estimated the bases and centers of the membership functions of the antecedents and the parameters of the consequences. Finally we compared the proposed model to actual data derived from the driving simulator experiment. We showed that the accelerations under normal driving conditions as well as complex driving conditions resemble the actual accelerations fairly well.

From a human factors point of view the model is still quite limited as we only incorporated the difference between task demands and driver capabilities in the model. Also the difference between task demands and driver capabilities was fixed according to the conditions. We therefore recommend to extend the model through adding human factors, such as activation level, the level of distraction, age, and driving experience as input into the neurofuzzy architecture. This allows for a more adequate approximation of driver capability and task demands.

Furthermore, the model does not make an explicit distinction between compensation effects and performance effects following a change in the complexity of the driving task. Future research should therefore also focus on detailed analyses of driving behavior under different levels of driving task complexity. The results of these analyses can then be used as training data for the neurofuzzy logic car following model.

In this context, attention should be given to behavioral explanations of the rules incorporated in the model. This can be achieved through a more elaborate analysis of the model. Finally, a very limited dataset was used in order to determine the optimal model complexity and perform the estimations. In this sense we recommend to conduct future research in which a more elaborate data set is used, in which different kinds of complexity are incorporated.

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