Photon counts statistics of squeezed and multi-mode thermal states of light on multiplexed on-off detectors

Radosław Chrapkiewicz

1Institute of Experimental Physics, Faculty of Physics, University of Warsaw, ul. Hoża 69, Warsaw, Poland

(Dated: April 21, 2015)

Photon number resolving detectors can be highly useful for studying the statistics of multi-photon quantum states of light. In this work we study the counts statistics of different states of light measured on multiplexed on-off detectors. We put special emphasis on artificial nonclassical features of the statistics obtained. We show new ways to derive analytical formulas for counts statistics and their moments. Using our approach we are the first to derive statistics moments for multi-mode thermal states measured on multiplexed on-off detectors. We use them to determine empirical Mandel parameters and recently proposed subbinomial parameters suitable for tests of nonclassicality of the measured states. Additionally, we investigate subpoissonian and superbunching properties of the two-mode squeezed state measured on a pair of multiplexed detectors and we present results of the Fano factor and second-order correlation function for these states.

OCIS codes: (270.5290) Photon statistics; (040.5570) Quantum detectors; (270.6570) Squeezed states.

http://dx.doi.org/10.1364/XX.99.099999

1. Introduction

Multi-photon states of light are highly applicable in precise quantum metrology [1–3]. Among these states, the N00N states [4, 5] and the squeezed states [6] are of the particular importance. The latter type of states have recently been successfully applied, for instance in increasing the precision of gravitational interferometer [7] or in low-noise quantum imaging [8].

Recent advances in technology allow for measurement of photon statistics of light with photon number resolving (PNR) detectors. They have lived to see many implementations among which the most popular are multiplexed on-off detectors based on the photon chopping concept [9]. They have been manufactured as fiber loop detectors [10–12], multi-pixel photon counters (MPPC) [13] or as single photon-sensitive cameras [14, 15]. Other types of detectors also in use include calorimetric transition-edge sensors [16] and hybrid photo-detectors [17–19]. The multiplexed on-off detectors have clear advantages such as a fast response and a relatively easy construction, since many of them are based on fast avalanche photodiodes.

Up till now there have been a number of successful experiments using PNR detectors, thus developing further the knowledge on quantum states of light and its sources [17–21]. Many other experiments may be enhanced by the use of PNR detectors, for example, the observation of macro-micro entanglement [22] or quantum imaging [8].

Here in this paper we will focus on the counting properties of the multiplexed on-off detectors, which are the type of PNR detectors most often used. These detectors alter the counts statistics as compared to the photon statistics of light illuminating a detector [9, 23–25]. A modification in counts statistics often leads to seemingly nonclassical properties of measured light [21, 26]; therefore, an appropriate interpretation of the counts statistics is indispensable.

In particular when it comes to an experiment, adequate criteria of nonclassicality based on empirical counts statistics have to be applied [27–29]. Criteria of nonclassicality for counts typically allow us to determine qualitatively whether one is dealing with a quantum state or a classical one. In order to better differentiate between these states, the counts statistics models are indispensable. In many cases the measured state can be identified only based on the analysis of the mean and the variance of counts.

In this paper we put a special emphasis on the multimode thermal states of light for which we are the first to derive the analytical model of counts statistics. Currently there is a great interest in the community to observe multimode light, not only generated in the spontaneous parametric down-conversion, but also in atomic systems including room-temperature alkali metals vapors [30–33]. These experiments could be naturally reimplemented into PNR regime with the use of single-
saturates for large
model Eq. 1 has to include photon losses
where
the same quantum efficiency
are very similar, thus principally they will have the
cal model for an idealized detector subject to certain as-
36, 37 Including all imperfections in a general form to
such as Bayessian approach to treat the cross-talk [13,
tors will be excluded from our model.

In the experiment with the PNR detector, we have
3. Counts statistics on a single detector

In general we can find average values of polynomial
functions \( \chi(k) \) of counts:

\[
\langle \chi(k) \rangle = \sum_{k=0}^{N} \chi(k) \frac{N!}{k!} \sum_{n=k}^{\infty} \frac{1}{N^n} S(n,k)f_n
\]  

To evaluate Eq. (3) we can use two useful properties of Stirling number of the second kind [32]:

\[
\sum_{n=k}^{\infty} S(n,k)x^n = \frac{1}{k!}(e^x - 1)^k
\]

\[
\sum_{n=k}^{\infty} S(n,k)x^n = \frac{(-1)^k}{(1 - \frac{1}{x})^k}
\]
Figure 2. (a) Mean and variance of counts for coherent state on a detector with $N = 4$. (b) Measured $Q_F$ parameter is artificially nonclassical contrary to $Q_B = Q_M = 0$.

where $(\cdot)_n$ denotes the Pochhammer symbol: $(y)_n = \frac{\Gamma(y+n)}{\Gamma(y)}$.

These two properties facilitate the determination of means and variances for states of light frequently used in experiments. For instance, using the property Eq. (4) one can find the mean $\langle k \rangle$ and the variance $\langle (\Delta k)^2 \rangle$ of counts for the coherent state $f_{\text{coh}} = (\frac{n}{m})^n e^{-\langle n \rangle}$:

$$\langle k \rangle = N(1 - e^{-\langle n \rangle}/N)$$  \hspace{1cm} (6)

$$\langle (\Delta k)^2 \rangle = N(1 - e^{-\langle n \rangle}/N)e^{-\langle n \rangle}/N$$  \hspace{1cm} (7)

Here we see that the detector reduces both the mean and the variance of counts as compared with the values for the photon statistics Fig. 2 (a).

This reduction leads to the seemingly nonclassical properties of the measured light. One of nonclassicality criteria for single-mode light is the negativity of the Mandel parameter $Q_M = \langle (\Delta n)^2 \rangle / \langle n \rangle - 1$. The parameter can also be evaluated for counts statistics $Q_F = \langle (\Delta k)^2 \rangle / \langle k \rangle - 1$, here for coherent states being always negative $Q_F = e^{-\langle n \rangle}/N - 1 < 0$ (Fig. 2 (b)).

Recently proposed modified criterion of nonclassicality is based on the subbinomial parameter $\Gamma$:

$$Q_B = N \frac{\langle (\Delta k)^2 \rangle}{\langle k \rangle (N-\langle k \rangle)} - 1,$$

which can be readily associated with the single mode thermal state of the mean $\langle n \rangle$ for which $a = (1 + \langle n \rangle)^{-1}$ and $b = \langle n \rangle/(1 + \langle n \rangle)$.

Then, using the property Eq. (3), we find the average of any function of counts for given statistics $f_n$:

$$\langle \chi(k) \rangle_f = a \sum_{k=0}^{N} \chi(k) \binom{N}{k} \frac{(-1)^k}{(1-N/b)^k}.$$  \hspace{1cm} (9)

We focus particularly on the first and second moment of the counts statistics:

$$\langle k \rangle_f = \frac{abN}{(b-1)(b(N-1)-N)}$$  \hspace{1cm} (10)

$$\langle k^2 \rangle_f = \frac{ab(b+1)N^2}{(b-1)(b(N-2)-N)(b+N-bN)}.$$  \hspace{1cm} (11)

We are also able to find moments of counts distribution for another type of statistics yielded from Eq. (3), in particular:

$$g_n = a \frac{(n+m)!}{n!} b^{n+m},$$  \hspace{1cm} (12)

which can be expressed as:

$$g_n = a \partial_{b,m} b^{n+m} \partial_{b,m} f_n.$$  \hspace{1cm} (13)

Note that $g_n$ can be readily associated with the multi-mode thermal state with an average number of photons $\langle n \rangle$ and the number of modes $\mathcal{M}$ is described by the statistics $g_n$ Eq. (12) for $a = \frac{1}{\prod_{\mathcal{M}} \binom{m}{\langle n \rangle + \mathcal{M}}}$, $b = \langle n \rangle / (\mathcal{M} + \langle n \rangle)$ and $m = \mathcal{M} - 1$.

To calculate the moments for given statistics $g_n$ we simply do the following:

$$\langle \chi(k) \rangle_g = \partial_{b,m} b^{n+m} \langle \chi(k) \rangle_f$$  \hspace{1cm} (14)

In this way, we can find the mean and the variance for a single-mode thermal state, where $a = (1 + \langle n \rangle)^{-1}$ and $b = \langle n \rangle/(1 + \langle n \rangle)$, similarly as in (27):

$$\langle k \rangle_{\text{Th,1}} = \frac{\langle n \rangle N}{\langle n \rangle + N}$$

$$\langle (\Delta k)^2 \rangle_{\text{Th,1}} = \frac{\langle n \rangle N^2 (\langle n \rangle N + \langle n \rangle + N)}{(\langle n \rangle + N)^2 (2\langle n \rangle + N)}.$$
example, we give analytical expressions for $\langle k \rangle_{\text{Th},M}$ for two-mode $M = 2$ thermal state:

$$\langle k \rangle_{\text{Th},M=2} = \frac{\langle n \rangle N (\langle n \rangle + 4N)}{\langle n \rangle + 2N^2}.$$

Further the second moment $\langle k^2 \rangle_{\text{Th},M=2}$ can be expressed analytically:

$$\langle k^2 \rangle_{\text{Th},M=2} = \frac{\langle n \rangle^2 N^2 (\langle n \rangle^3 + 6 \langle n \rangle^2 N + 3 \langle n \rangle N (2N+1) + 4N^2)}{\langle n \rangle^2 + N^2 (\langle n \rangle + 2N)^2}.$$

Analytical formulas for a higher number of modes can be readily found using any symbolic computation software and instead of presenting directly the formulas we gather the results for higher number of modes on plots.

Having found means and variances parameters, we can construct Mandel parameters for counts and compare them with theoretical values $Q_M = \langle n \rangle / \langle n \rangle$. In Fig. 3 we show the empirical $Q_F$ parameter for detectors of different $N$ for single- and multi-mode thermal state ($M = 4$). Depending on the mean number of input photons $\langle n \rangle$ and $N$, the measured $Q_F$ parameters appear to be nonclassical in certain regimes.

5. Counts properties of squeezed states

The above results can be measured in a single subsystem for single- and multi-mode squeezed states using a single detector. If we have two detectors, we can calculate the joint counts statistics $c_{k_1,k_2}$ related to the input statistics $f_{n_1,n_2}$ by relation:

$$c_{k_1,k_2} = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} p_N(k_1|n_1)p_N(k_2|n_2)f_{n_1,n_2}.$$

Calculations of the joint count statistics allow us to detect subpoissonian correlations between the two subsystems. A good measure to quantify these correlations is the Fano factor which, evaluated for counts, can be expressed as:

$$R = \frac{\langle \Delta(k_1 - k_2)^2 \rangle}{\langle k_1 \rangle + \langle k_2 \rangle},$$

which is $R \geq 1$ for all classical states.

Another commonly used parameter to characterize in experiments the two modes states of light is the second order correlation function, which evaluated on counts statistics can be expressed as:

$$g^{(2)} = \frac{\langle k_1 k_2 \rangle}{\langle k_1 \rangle \langle k_2 \rangle}.$$ 

It can be used to test if the investigated light demonstrate the superbunching properties i.e. for $g^{(2)} \geq 2$, which is characteristic for squeezed vacuum states but in principle may be also obtained using interference of thermal states.

Now we shall focus on properties of joint counts statistics of the two-mode squeezed state of light:

$$|\psi\rangle = \sqrt{1 - |\zeta|^2} \sum_{n=0}^{\infty} \zeta^n |nn\rangle,$$

where $\zeta$ is the squeezing parameter. The Fano factor for such a state without losses equals $R = 0$, whereas the second order correlation function becomes always over two: $g^{(2)} = 1 + 1/|\zeta|^2 \geq 2$.

In Fig. 5 we show the effect of a finite number $N$ of component detectors and influence of losses on the joint counts statistics $c_{k_1,k_2}$ for $\zeta = 0.8$.

Both the finite number of detectors and the losses have influence on reducing the correlation between counts measured on two detectors. To better understand this mechanism, we found the Fano factors for different $N$, $\eta$ and $\zeta$. Fig. 6(a) presents pure influence of a finite $N$, without the losses.

For small squeezing parameters $\zeta$ high number of component detectors $N$ ensures a smaller Fano factor, whereas for high $\zeta$ the effect is opposite. This is the effect of artificial increasing of correlations due to the detector saturation. The saturation effects are even more
significant for the detector with finite quantum efficiency (Fig. 6 (b)), where \( \eta = 0.5 \). Here only a smaller number of detectors \( N \) can have an effect on reducing the Fano factor for each \( \zeta \).

In all these cases, the calculated counts statistics always remain nonclassical. In order to perform a reliable test of nonclassicality with no \textit{a priori} knowledge of the state of light, one can apply the recently proposed criteria [29].

It is also instructive to view how the second correlation function \( g^{(2)} \) is modified due to the limitations introduced by a finite number of component on-off detectors \( N \). It can be seen in Fig. 7 that the low number of component on-off detectors \( N \) decreases \( g^{(2)} \) by no more than 1. This means that limit between superbunched states and the bunched states \( (g^{(2)} = 2) \) will be exceeded for a sufficiently low \( N \) for squeezed states of the squeezing parameter higher than \( |\zeta|^2 > 1/2 \) as it is exemplified in Fig. 7 (b).

### 6. Summary

In this paper, we investigate the statistics for counts measured by a detector illuminated by squeezed or thermal states of light, the latter being a traced subsystem of the squeezed state.

In particular, we provide a universal manner of deriving statistics moments which we have used to determine analytical formulas for multi-mode thermal states. In this work we put special emphasis on the issues related to artificial nonclassicality of measured statistics in the context of Mandel and subbinomial parameters.

On the other hand, we show the influence of increasing and decreasing subpoissonian counts correlations for two-mode squeezed states of light measured on two multiplexed detectors. We show that the Fano factor always remains nonclassical. Moreover we provide the results for the second-order correlation function \( g^{(2)} \) which could drop below two in certain cases.

The results of the work can be used to identify the states of light measured by multiplexed on-off detectors, in particular to determine the number of modes in thermal states. The results can significantly contribute to designing experiments using multiplexed on-off detectors. They will play an increasingly important role in the future since the state of the art, photon number resolving detectors are becoming a very efficient tool for studying the multimode, multi-photon quantum states of light.

### Acknowledgments

I acknowledge Michal Parniak for the careful reading of the manuscript. The project was financed by the National Science Centre grant DEC-2013/09/N/ST2/02229.

### References

[1] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone. Quantum-enhanced measurements: beating the standard quantum limit. Science (New York, N.Y.), 306(5700):1330–6, 2004.
[2] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone. Advances in quantum metrology. *Nature Photonics*, 5(4):222–229, 2011.

[3] Rafal Demkowicz-Dobrzański, Jan Kolodynski, and Madalin Guta. The elusive Heisenberg limit in quantum-enhanced metrology. *Nature Communications*, 3:1063, 2012.

[4] M W Mitchell, J S Lundeen, and A M Steinberg. Super-resolving phase measurements with a multiphoton entangled state. *Nature*, 429(6988):161–4, 2004.

[5] Itai Afek, Oron Ambar, and Yaron Silberberg. High-NOON states by mixing quantum and classical light. *Science (New York, N.Y.)*, 328(5980):879–81, 2010.

[6] D. F. Walls. Squeezed states of light. *Nature*, 306(5939):141–46, 1983.

[7] The LIGO Scientific Collaboration. A gravitational wave observatory operating beyond the quantum shot-noise limit. *Nature Physics*, 7(12):962–965, 2011.

[8] G. Brida, M. Genovese, and I. Ruo Berchera. Experimental realization of sub-shot-noise quantum imaging. *Nature Photon.*, 4(4):227–230, 2010.

[9] H. Paul, P. Türmä, T. Kiss, and I. Jex. Photon Chopping: New Way to Measure the Quantum State of Light. *Physical Review Letters*, 76(14):2464–2467, 1996.

[10] Konrad Banaszek and Ian A. Walmsley. Photonic counting with a loop detector. *Optics Letters*, 28(1):52, 2003.

[11] J. Rehacek, Z. Hradil, O. Haderka, J. Perina, and M. Hamar. Multiple-photon resolving fiber-loop detector. *Physical Review A*, 67(6):061801, 2003.

[12] Daryl Achilles, Christine Silberhorn, Cezary Sliwa, Konrad Banaszek, Ian A. Walmsley, Michael J. Fitch, Bryan C. Jacobs, Todd B. Pittman, and James D. Franson. Photon-number-resolving detection using time-multiplexing. *Journal of Modern Optics*, 51(9-10):1499–1515, 2004.

[13] I. Afek, A. Natan, O. Ambar, and Y. Silberberg. Quantum state measurements using multipixel photon detectors. *Physical Review A*, 79(4):043830, 2009.

[14] Ondrej Haderka, Jan Perina, and Martin Hamar. Direct measurement and reconstruction of nonclassical features of twin beams generated in spontaneous parametric down-conversion. *Physical Review A*, 71(3):4, 2005.

[15] Jean-Luc Blanchet, Fabrice Devaux, Luca Furfaro, and Eric Lantz. Measurement of Sub-Shot-Noise Correlations of Spatial Fluctuations in the Photon-Counting Regime. *Physical Review Letters*, 101(23):233604, 2008.

[16] Giorgio Brida, Luigi Ciavarella, Ivo Pietro Degiovanni, Marco Genovese, Lapo Lolli, Maria Griselda Mingolla, Fabrizio Piacentini, Mauro Rajteri, Emanuele Taralli, and Matteo G A Paris. Quantum characterization of superconducting photon counters. *New Journal of Physics*, 14(8):085001, 2012.

[17] Alessia Allevi, Maria Bondani, and Alessandra Andreoni. Photon-number correlations by photon-number resolving detectors. *Optics letters*, 35(10):1707–9, 2010.

[18] Marco Lamperti, Alessia Allevi, Maria Bondani, and Radek Machulka. Optimal sub-Poissonian light generation from twin beams by photon-number resolving detectors. *Journal of the Optical Society of America B*, 31(1):20–25, 2014.

[19] Alessia Allevi, Marco Lamperti, Maria Bondani, Jan Perina, Vaclav Michalek, Ondrej Haderka, and Radek Machulka. Characterizing the nonclassicality of mesoscopic optical twin-beam states. *Physical Review A*, 88(6):063807, 2013.

[20] Radek Machulka, Ondrej Haderka, Jan Perina, Marco Lamperti, Alessia Allevi, and Maria Bondani. Spatial properties of twin-beam correlations at low- to high-intensity transition. ArXiv: 1405.1190, 2014.

[21] Tim J. Bartley, Gaia Donati, Xian-Min Jin, Animesh Datta, Marco Barbieri, and Ian A. Walmsley. Direct Observation of Sub-Binomial Light. *Physical Review Letters*, 110(17):173602, 2013.

[22] A. I. Lvovsky, R. Ghobadi, A. Chandra, A. S. Prasad, and C. Simon. Observation of micro-macro entanglement of light. *Nature Physics*, 9(9):541–544, 2013.

[23] Hwang Lee, Ulvi Yurtsever, Pieter Kok, George M. Hockney, Christoph Adami, Samuel L. Braunstein, and Jonathan P. Dowling. Towards photostatistics from photon-number discriminating detectors. *Journal of Modern Optics*, 51(9-10):1517–1528, 2004.

[24] J. S. Lundeen, A. Feito, H. Coldenstrodt-Ronge, K. L. Pregnell, Ch. Silberhorn, T. C. Ralph, J. Eisert, M. B. Plenio, and I. A. Walmsley. Tomography of quantum detectors. *Nature Physics*, 5(1):27–30, 2008.

[25] J. Sperling, W. Vogel, and G. S. Agarwal. True photo-counting statistics of multiple on-off detectors. *Physical Review A*, 85(2):023820, 2012.

[26] Radoslaw Chrapkiewicz, Wojciech Wasilewski, and Konrad Banaszek. High-fidelity spatially resolved multiphoton counting for quantum imaging applications. ArXiv: 1405.4400, 2014.

[27] J. Sperling, W. Vogel, and G. S. Agarwal. Sub-Binomial Light. *Physical Review Letters*, 109(9):093601, 2012.

[28] T. Kiesel and W. Vogel. Complete nonclassicality test with a photon-number resolving detector. *Physical Review A*, 86:032119, 2012.

[29] J. Sperling, W. Vogel, and G. S. Agarwal. Correlation measurements with on-off detectors. *Physical Review A*, 88(4):043821, 2013.

[30] Vincent Boyer, Alberto M Marino, Raphael C Pooser, and Paul D Lett. Entangled images from four-wave mixing. *Science*, 321(5888):544–7, 2008.

[31] V Boyer, A Marino, and P Lett. Generation of spatially broadband twin beams for quantum imaging. *Phys. Rev. Lett.*, 100(14):143601, 2008.

[32] C F McCormick, V Boyer, E Arimondo, and P D Lett. Strong relative intensity squeezing by four-wave mixing in rubidium vapor. *Optics letters*, 32(2):178–80, 2007.

[33] Radoslaw Chrapkiewicz and Wojciech Wasilewski. Generation and delayed retrieval of spatially multimode Raman scattering in warm rubidium vapors. *Optics Express*, 20(28):29540, 2012.

[34] Michal Dabrowski, Radoslaw Chrapkiewicz, and Wojciech Wasilewski. Hamiltonian design in real-out from room-temperature Raman quantum memory. ArXiv: 1406.6489, 2014.

[35] Eric W. Weissstein. Stirling Number of the Second Kind. MathWorld–A Wolfram Web Resource.

[36] Dmitry A Kalashnikov, Si-Hui Tan, and Leonid A Krivitsky. Crosstalk calibration of multi-pixel photon counters using coherent states. *Optics express*, 20(5):5044–51, 2012.

[37] Dmitry A Kalashnikov, Si-Hui Tan, Timur Sh Ishkakov, Maria V Chekhova, and Leonid A Krivitsky. Measurement of two-mode squeezing with photon number resolving multipixel detectors. *Optics letters*, 37(14):2829–31, 2012.
[38] A Feito, J S Lundeen, H Coldenstrodt-Ronge, J Eisert, M B Plenio, and I a Walmsley. Measuring measurement: theory and practice. *New Journal of Physics*, 11(9):093038, 2009.

[39] Joseph W. Goodman. *Statistical Optics*. Wiley, 2000.

[40] T Sh Iskhakov, A M Pérez, K Yu Spasibko, M V Chekhova, and G Leuchs. Superbunched bright squeezed vacuum state. *Optics letters*, 37(11):1919–21, 2012.

[41] Peilong Hong, Jianbin Liu, and Guoquan Zhang. Two-photon superbunching of thermal light via multiple two-photon path interference. *Physical Review A*, 86(1):013807, 2012.