Analytic method of structural analysis of modular buildings

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Abstract. The paper presents an analytical method of modular buildings calculation. It focuses upon basic design assumptions that are accepted for modular buildings calculation. The authors introduce their classification of loads and impacts, which are basic for modular buildings. They also describe a method of determining inertial forces from seismic action designed on the basis of a cantilever analytical model. Analytical and numerical methods for determining the forces in the elements of modular units are further compared. It is revealed that the analytical method strongly agrees with FEM (that is the finite elements method) with the error being 2-6%. The analytical method is sufficiently accurate for engineering calculations.

1. Introduction
Building construction by using a modular technology is presently considered as one of the most progressive directions in civil engineering [1-6]. Such buildings have a number of advantages compared to the classical technology of construction: low labour intensity of construction, high speed of installation, high quality of modules, etc. These advantages are especially important for the construction of buildings in inaccessible regions with extreme climatic conditions, e.g. areas of oil and gas fields development.

Despite the active implementation of modular buildings in Russia, there is no regulatory framework for this type of structures design. This is primarily due to the lack of analytical methods for calculating modular buildings. Currently, the finite element method (FEM) is used for the calculation of buildings and structures. This method makes it possible to calculate the forces in any building structures. However, FEM has a disadvantage associated with the need to know the exact geometry and cross-sections of structural elements, while there is no direct connection between them. When using FEM, the calculation of a private scheme is performed every time, so it is difficult to identify general patterns in the work of forces modular buildings.

This paper presents an analytical method of modular buildings calculation that does not require major calculation schemes in software systems using FEM. This method makes it possible to quickly calculate the forces in the elements of modular units.

2. Materials and methods
A single modular unit is a system of two horizontal rectangular frames joined together by four corner posts. All elements are rigidly connected to each other, which ensures stability of its geometrical shape. Slabs are formed by beams, which are hinged to the elements of horizontal frames. Insulation is laid between the beams. The beams are sheathed with galvanized steel or cement-chipboard. Figure 1 shows one modular unit design scheme.
There are certain assumptions accepted in determining the forces and natural vibration frequency of modular buildings. They are as follows:

1. The coupling of all elements of the unit (posts and horizontal frames) with each other is accepted as rigid.
2. The coupling of blocks with foundations is considered to be hinge-fixed at the corners of the blocks.
3. The coupling of the blocks with each other is accepted as hinged at the corner points of the horizontal frames.
4. The floor slab (the roof slab) is not deformed in its plane.

The following types of loads can be applied to a modular building:
- dead load, \( P_g \);
- snow load, \( P_s \);
- static wind load, \( P_{ws} \);
- wind pulsation, \( P_{wp} \);
- imposed load, \( P_p \);
- seismic load, \( P_{sc} \).

All these loads can be divided into three groups according to the nature of their impact:
- uniform distributed vertical load on the cross-beams (dead load, snow load and imposed load);
- uniform distributed horizontal load on the posts (static wind load);
- concentrated horizontal forces in the level of the cross-beam (inertial forces from wind pulsation and seismic load).

The forces from the static vertical load (dead load, snow load and imposed load) are calculated by construction mechanics methods or by means of calculation tables [7]. The calculation of natural oscillation frequencies, inertial forces from wind pulsation, as well as determination of forces from wind load is given in Papers [8, 9].

In determining the seismic load on modular buildings, a cantilever model with masses concentrated in the slabs can be adopted. This scheme corresponds to the calculated dynamic model presented in the Russian Building Design Standards for seismic areas (Figure 3).
The calculated seismic load applied to Point \( k \) and corresponding to \( i \)-form of natural vibrations is determined by Formula 1:

\[
S_{ik}^j = K_0 \cdot K_1 \cdot S_{0ik}^j,
\]

where \( K_0, K_1 \) are the coefficients determined by SP 14.13330.2018 depending on the purpose of the structure and the allowable damage;

\( S_{0ik} \) is the seismic load value for \( i \)-form of the structure natural vibrations, assuming elastic deformation of the system:

\[
S_{0ik}^j = m_k^j \cdot A \cdot \beta \cdot \psi \cdot \eta_{ik},
\]

\( m_k^j \) is the mass of the building related to Point \( k \);

\( A \) is an acceleration value, depending on the calculated seismic intensity;

\( \beta \) is a dynamic response factor, depending on the category of soil and the period of natural oscillations;

\( K_\psi \) is a coefficient that takes into account the ability of buildings and structures to dissipate energy;

\( \eta_{ik} \) is a coefficient depending on the building or structure oscillations shape in \( i \)-form, from the nodal point of the load application and the direction of the seismic load.

According to the adopted cantilever scheme, under the translational seismic load \( \eta_{ik} \)-coefficient is determined by Formula 3:

\[
\eta_{ik} = \frac{X_i(x_k) \cdot \sum_{j=1}^{n} m_j \cdot X_j(x_j)}{\sum_{j=1}^{n} m_j \cdot X_j^2(x_j)}
\]

where \( X_i(x_k) \) and \( X_j(x_j) \) are the displacement of the building with its own fluctuations in \( i \)-form at the considered point \( k \) and at all points \( j \);

\( m_j \) is the mass of the building assigned to Node \( j \).

The described methodology, with account of Papers [8, 9], allows calculating forces in the elements of modular buildings in an analytical form without drawing up complex design models while using complicated software.

3. Analytical and numerical methods comparison

To confirm the correctness of the method, the forces determined analytically are compared to the forces calculated by FEM. Lira Soft 2013 was used for the finite element method calculations. The calculation model represented a spatial bar scheme. A universal spatial type was assigned for the elements. Corner posts and cross-beams were divided into 8 finite elements to obtain more accurate deformation patterns. The horizontal rigidity of the slabs was modeled by rods having the type of a finite element of a spatial truss. The coupling of modular units with each other was modeled by...
combining the displacements of angular points in three linear directions. The load was applied to the longitudinal cross-beams in the form of a evenly distributed load per unit of length.

The building is a three-storey structure, with 7×3 units and 3×6×3.3(h) modules. The moment of corner posts inertia $I_1$ is 236.3 cm$^4$, the moment of cross-beams $I_2$ is 1090 cm$^4$. The dead load on the lower frame $p_1$ is 120 kg/m$^2$ and on the top frame $p_2$ is 75 kg/m$^2$. The imposed load $p$ is 195 kg/m$^2$. The snow load $S$ is 280 kg/m$^2$, which corresponds to Snow Region IV. The wind load $w$ corresponds to Wind Region III. It is 38 kg/m$^2$ with its terrain type adopted as "A". The area seismic intensity is taken as 7 points, its soil type as II; the coefficient, taking into account the permissible damage $K_1$ equals 0.25; the coefficient, taking into account the ability of buildings to dissipate energy $K_0$ is 1.3; the coefficient $K_0$ is 1.

Table 1. Analytical method and FEM comparison

| Load type          | Stress type | FEM  | Analytic | $\Delta$, % |
|--------------------|-------------|------|----------|-------------|
| Dead load          | $M_1$       | 2.60 | 2.663    | 2.37        |
|                    | $M_2$       | 2.04 | 2.091    | 2.44        |
|                    | $M_3$       | 2.60 | 2.633    | 1.25        |
|                    | $M_4$       | 2.04 | 2.091    | 2.44        |
|                    | $M_5$       | 2.92 | 2.972    | 1.75        |
|                    | $M_6$       | 5.34 | 5.437    | 1.78        |
|                    | $N$         | -20.50 | -20.95   | 2.15        |
|                    | $M_1$       | 1.59 | 1.632    | 2.57        |
|                    | $M_2$       | 5.06 | 5.195    | 2.6         |
|                    | $M_3$       | 13.50 | 13.705  | 1.5         |
|                    | $N$         | -12.40 | -12.60   | 1.59        |
| Imposed load       | $M_1$       | 1.11 | 1.137    | 2.37        |
|                    | $M_2$       | 3.53 | 3.618    | 2.43        |
|                    | $M_3$       | 9.38 | 9.545    | 1.73        |
|                    | $N$         | -17.20 | -17.55   | 1.99        |
| Static wind load "Y" | $M_1$   | 1.98 | 2.023    | 2.13        |
|                    | $M_2$       | 1.32 | 1.329    | 0.68        |
|                    | $M_3$       | 1.85 | 1.893    | 2.27        |
|                    | $M_4$       | 1.44 | 1.459    | 1.3         |
|                    | $N$         | -1.39 | -1.36    | -2.28       |
| Static wind load "X" | $M_1$   | 2.14 | 2.194    | 2.46        |
|                    | $M_2$       | 0.693 | 0.679    | -2.06       |
|                    | $M_3$       | 1.86 | 1.910    | 2.62        |
|                    | $M_4$       | 0.96 | 0.963    | 0.31        |
|                    | $N$         | -2.28 | -2.08    | -9.62       |
| Wind pulsation "Y" | $M_1$       | 1.23 | 1.397    | 11.95       |
|                    | $N$         | -1.16 | -1.302   | 10.91       |
| Wind pulsation "X" | $M_1$       | 1.05 | 1.220    | 13.93       |
|                    | $N$         | -2.00 | -2.28    | 12.13       |
| Seismic load "Y"  | $M_1$       | 1.61 | 1.511    | -6.55       |
|                    | $N$         | -1.52 | -1.40    | -8.49       |
| Seismic load "X"  | $M_1$       | 1.64 | 1.570    | -4.46       |
|                    | $N$         | -3.13 | -2.88    | -8.72       |

Table 1 compares the forces obtained by analytical and numerical methods for different loads. The comparison was carried out in the most loaded design sections: in corner posts and cross-beams connections and in the middle of cross-beams (Figure 2). Table 1 demonstrates moment values in their absolute (excluding marks).
Table 1 proves that the mechanical method of calculating the forces from static loads are of complete convergence with the results obtained by FEM. The error in determining the forces for static loads is approximately 2%. The forces calculated by different methods from the dynamic effects have a greater discrepancy, it is about 10%. This discrepancy can be explained by some damping of the spatial system in the calculation program while an elastic planar scheme is adopted in the analytical solution.

In order to estimate the contribution of the error in determining the forces from dynamic impacts to the total forces from load combinations, a number of calculations for different values of wind and seismic loads were carried out (Figure 4). Geometric dimensions of the building and static loads (dead load and imposed load) were left unchanged, the snow load was not taken into account. Calculated load combinations have the form:

\[ C_1 = P_g + P_p + 0.9 \cdot (P_{w_s} + P_{w_p}); \]  

\[ C_2 = 0.9 \cdot P_g + 0.5 \cdot P_p + P_{sc}, \]

where \( P_g \) is the dead load; 
\( P_p \) is the imposed load; 
\( P_{w_s} \) and \( P_{w_p} \) are static and pulsation wind loads, respectively; 
\( P_{sc} \) is seismic load.

Figures 4 and 5 show that the contribution of the wind pulsation load to the total moment can reach 20%, of seismic load - 60%. The error in determining the forces of these impacts is 10%. Multiplying the contribution by the error, we see that the wind pulsation adds 2% - error in the total value of the moment, and the seismic load adds 6%. This error in determining the total force shows good convergence of the methods. The analytical method is sufficiently accurate for engineering calculations.

![Figure 4](image_url)  
**Figure 4.** Moments in the bearing cross-section of the corner posts at different wind loads.
Figure 5. Moments in the bearing cross-section of the corner posts at different seismic loads.

4. Conclusions
The research yielded the following conclusions:
1. The cantilever analytical model provided for determining the inertial forces from seismic impact is correct for calculating modular buildings.
2. The error in determining the forces from static loads by the analytical method equals 2-3%.
3. The error of determination of forces from dynamic influences by the analytical method is about 10%. Taking into account their total contribution, the error in determining the total force is 2-6%.
4. The analytical method is sufficiently accurate for engineering calculations.

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