Chiral anomaly role in $\pi_1(1600) \to \pi \eta'$

FRANCESCO GIACOSA

Institute of Physics, Jan Kochanowski University, Kielce, Poland

The ground-state (lightest) hybrid nonet with exotic quantum numbers $J^{PC} = 1^{--}$ and the nonet of their chiral partners with $J^{PC} = 1^{+-}$ build a homochiral multiplet involving left- and right-handed currents, which under chiral transformation change just as (axial-)vector mesons. Masses and interactions of hybrids can be obtained in the context of the extended Linear Sigma Model. Here, we concentrate on the decays of hybrids into two pseudoscalar mesons, such as $\eta \pi$ and $\eta' \pi$ modes. Indeed, $\pi_1(1400) \to \pi \eta$ and $\pi_1(1600) \to \pi \eta'$ have been seen in experiments. Assuming that $\pi_1(1400)$ and $\pi_1(1600)$ correspond to the same state $\pi_{1\text{hyb}}^b$, we show that these decays (and similar ones) follow from a chirally symmetric interaction term that breaks explicitly the axial anomaly. In this respect, these decays would be an additional manifestation of the axial (or chiral) anomaly in the mesonic sector.

1. Introduction

Hybrids are unconventional mesons made of a quark-antiquark pair and a constituent gluon. While they are predicted in many different approaches to QCD, see e.g. Ref. [1,2], they could not be yet confirmed experimentally (even if a substantial experimental effort is ongoing [3,4]). Quite remarkably, in the PDG [5] the two enigmatic and very broad resonances $\pi_1(1400)$ and $\pi_1(1600)$ have exotic quantum numbers $J^{PC} = 1^{--}$ (impossible for ordinary $\bar{q}q$ pairs). The state $\pi_1(1400)$ has been seen in the decay channels $\rho \pi$ and $\eta \pi$, while the state $\pi_1(1600)$ in the channels $b_1(1270)\pi$, $f_1(1270)\pi$, and $\eta' \pi$. Recently, the COMPASS experiment [6] confirmed the existence of $\pi_1(1600)$, in particular by studying the decay $\pi_1(1600) \to \eta' \pi$. As we shall discuss below, this decay is one of the main subject of this work.

In the framework of lattice QCD exotic mesons with $J^{PC} = 1^{--}$ are the lightest hybrid mesons and have a mass of about 1.6 GeV [6,7]. Yet, only a single $\pi_1$ state is expected; it is then hard to accommodate both $\pi_1(1400)$ and $\pi_1(1600)$. In Ref. [8] it was argued that the states $\pi_1(1400)$ and $\pi_1(1600)$ correspond to the very same pole in the complex energy plane,
and hence to a single state, whose mass is close to $\pi_1(1600)$. Within this scenario, there is no conflict with lattice and model predictions about the exotics and, as a consequence, the unique $\pi^\text{hyb}_1$ resonance -to be identified with $\pi_1(1600)$- decays both into $\eta\pi$ and $\eta'\pi$.

The first immediate question concerning hybrids is the following: if the $\pi_1(1600)$ is the lightest hybrid meson with isospin 1, where are the other members of the multiplet? Namely, hybrids form nonets just as regular states. As a consequence, one expects two states with $I = 0$, denoted as $\eta^\text{hyb}_{1,N}$ (predominantly nonstrange) and $\eta^\text{hyb}_{1,S}$ (predominantly strange)) as well as four states with $I = 1/2$ collectively denoted as $K^\text{hyb}_1$. Their mass should be also close to 1.6 GeV.

Moreover, chiral symmetry also implies that chiral partners of this low-lying hybrid nonet should exist. The corresponding quantum numbers are $J^{PC} = 1^{++}$ (just as pseudovector mesons, hence we deal with cryptoexotic states). Their mass should be at about 2 GeV (or heavier).

An attempt to answer these questions was recently presented in Ref. [9]. The ground-state hybrids and their chiral partners form a chiral multiplet which can be coupled to ordinary $\bar{q}q$ states as (pseudo)scalar and (axial-)vector mesons. This is achieved in the framework of a well established chiral model of QCD, the so-called extended Linear Sigma Model (eLSM). In this model, symmetries of QCD (and their violations) are implemented at the level of composite hadrons. As shown in Ref. [10], a fit using masses and decays of various mesons up to 1.8 GeV shows a good agreement with experimental data. In addition, also baryonic d.o.f. [11, 12], various glue-balls [13], extensions to excited states [14] as well as studies at nonzero temperature and densities [15] have been developed.

It seems then natural to use the eLSM to evaluate decays of hybrids, especially in order to find out which decays are favoured and which ones are suppressed. Moreover, various ratios among decays represent clear predictions of the approach. As shown in Ref. [2] one of the dominant decays is found to be $\pi_1(1600) \rightarrow b_1(1230)\pi$, in agreement with lattice [6] and with other model predictions [16].

In this work, we concentrate on the decay into $\pi_1(1600)$ into $\eta\pi$ and $\eta'\pi$. Quite interestingly, this decay turns out to be possible only through an interaction term that breaks the axial symmetry. The importance of this so-called chiral (or axial) anomaly is well appreciated for the masses and mixing of the mesons $\eta$ and $\eta'$ [18]. Yet, this anomaly can affect also other parts of the hadronic spectrum, as recently discussed in Refs. [12, 17]. Along this line, we show that the axial anomaly can be also responsible for the decays of hybrids into $\eta\pi$ and $\eta'\pi$. 
2. Fields and model

Here, we briefly review the model presented in Ref. [4] and its results.

First, we recall the (pseudo)scalar sector. The $3 \times 3$ matrix $P$ contains the light pseudoscalar nonet $\{\pi, K, \eta, \eta'\}$ with quantum numbers $J^{PC} = 0^−$. At a fundamental level, it is made of quark-antiquark elements given by $P_{ij} = 2^{−1/2}\bar{q}_j\gamma^5q_i$ with $i, j = u, d, s$. The matrix $S$, whose $q\bar{q}$ elements are the scalar currents $S_{ij} = 2^{−1/2}\bar{q}_jq_i$, contains the scalar fields $\{a_0(1450), K_0^0(1430), \sigma_N \approx f_0(1370), \sigma_S \approx f_0(1710)\}$ with $J^{PC} = 0^+$. The scalar and pseudoscalar matrices are combined into the matrix $\Phi = S + iP$, which under chiral transformations $U_L(3) \times U_R(3)$ changes as $\Phi \rightarrow U_L\Phi U_R^\dagger$ ($U_L$ and $U_R$ being $3 \times 3$ unitary matrices). Under parity: $\Phi \rightarrow \Phi^\dagger$ and under charge conjugation (denoted as $C$): $\Phi \rightarrow \Phi^C$. Moreover: $\Phi^\mu \rightarrow \Phi_\mu^C$ under parity and $\Phi^\mu \rightarrow -\Phi^\mu$ under $C$.

Next, we consider pseudovector and excited vector states. They follow from the (pseudo)scalar currents upon introducing a derivative in between the quarks. The nonet $B^\mu$ with elements $B^\mu_{ij} = 2^{−1/2}\bar{q}_j\gamma^\mu\partial^\mu q_i$ has quantum numbers $J^{PC} = 1^−$ and describes the fields $\{b_1(1230), K_1(1270)/K_1(1400), h_1(1170), h_1(1380)\}$, see Ref. [9] for details. This nonet, together with the nonet of orbitally excited vector mesons $V^\mu_{E,ij} = 2^{−1/2}\bar{q}_j\gamma^\mu q_i$ involving the resonances $\{\rho(1700), K^*(1680), \omega(1650), \phi(1320)\}$, builds the chiral multiplet, $\tilde{\Phi}^\mu = V^\mu_E - iB^\mu$, which transforms just as (pseudo)scalar fields under chiral transformations: $\tilde{\Phi}^\mu \rightarrow U_L\tilde{\Phi}^\mu U_R^\dagger$. This is a consequence of the fact that the derivative does not modify the chiral properties; in general, multiplets transforming in this way are called heterochiral [17]. Moreover: $\tilde{\Phi}^\mu \rightarrow \tilde{\Phi}_\mu^C$ under parity and $\tilde{\Phi}^\mu \rightarrow -\tilde{\Phi}^\mu$ under $C$.

We turn to (axial-)vector states. The matrix $V^\mu$, with $V^\mu_{ij} = 2^{−1/2}\bar{q}_j\gamma^\mu q_i$, carries the vector mesons $\{\rho(770), K^*(892), \omega(782), \phi(1020)\}$ with $J^{PC} = 1^−$. Analogously, the matrix $A^\mu$, with $A^\mu_{ij} = 2^{−1/2}\bar{q}_j\gamma^\mu q_i$, contains the axial-vector mesons $\{a_1(1230), K_1(1270)/K_1(1400), f_1(1285), f_1(1420)\}$ with $J^{PC} = 1^+$. These matrices are combined into the right- and left-handed combinations $R^\mu = V^\mu - A^\mu$ and $L^\mu = V^\mu + A^\mu$ which under chiral transformation behave as $R^\mu \rightarrow U_RR^\mu U_R^\dagger$ and $L^\mu \rightarrow U_LL^\mu U_L^\dagger$, thus in an utterly different way w.r.t. to (pseudo)scalars. We refer to them as an homochiral multiplet [17]. Under parity: $R^\mu \rightarrow L^\mu$ and $L^\mu \rightarrow R^\mu$; under $C$: $R^\mu \rightarrow -L^\mu$ and $L^\mu \rightarrow -R^\mu$.

Next, one builds objects analogous to the (axial-)vector fields in the hybrid sector, upon including the gluon field. To this end, we consider the objects

$$\Pi^{hgb,\mu} = 2^{−1/2}\bar{q}_jG^{\mu\nu}\gamma^\nu q_i$$

and

$$E^{hgb,\mu}_{ij} = 2^{−1/2}\bar{q}_jG^{\mu\nu}\gamma^\nu\gamma^5 q_i$$

(1)
which besides the standard quark-antiquark pair involve also explicitly the gluon field strength tensor $G_{\mu\nu}$, being responsible for the switch of the $C$-parity. As a consequence, the nonet $\Pi_{\text{hyb},\mu}$ has exotic quantum numbers $J^P C = 1^- +$; the corresponding nonet is denoted as $\{\pi_1(1600), K_1(?), \eta_1(?)\}$, where at present only the isovector member can be assigned to a physical resonance. The nonet $B_{\text{hyb},\mu}$ with $J^{PC} = 1^{+-}$ contains $\{b_1(?)$, $K_{1,\beta}(?), h_1(?)\}$, for which there are not yet experimental candidates. These two nonets are grouped into the right-handed and left-handed currents $R_{\text{hyb},\mu} = \Pi_{\text{hyb},\mu} - B_{\text{hyb},\mu}$ and $L_{\text{hyb},\mu} = \Pi_{\text{hyb},\mu} + B_{\text{hyb},\mu}$, which transform as $R_{\text{hyb},\mu} \to U_R R_{\text{hyb},\mu} U_R^\dagger$ and $L_{\text{hyb},\mu} \to U_L L_{\text{hyb},\mu} U_L^\dagger$ (just as (axial-)vectors). Moreover: $R_{\text{hyb},\mu} \to U_L L_{\text{hyb},\mu}$ and $L_{\text{hyb},\mu} \to U_R R_{\text{hyb},\mu}$ under parity and $R_{\text{hyb},\mu} \to L_{\text{hyb},\mu}$ and $L_{\text{hyb},\mu} \to R_{\text{hyb},\mu}$ under $C$. (For a list of other possible multiplets together with their homo/heterochirality, see the classification of Ref. [17].)

In Ref. [9] a Lagrangian that couples the hybrid multiplet to conventional mesons is presented. Both masses and decays of hybrids can be described. For instance, a term proportional to $\text{Tr} \left( L_{\mu}^{\text{hyb}} \Phi U_{\text{hyb},\mu} \Phi^\dagger \right)$ is present. It is invariant under $U_L(3) \times U_R(3)$ as well as parity and $C$, as one can verify by using the transformations above; it is important since it generates the mass difference between the hybrids with $J^{PC} = 1^{+-}$ and with $J^{PC} = 1^{--}$. Namely, one finds $m_{b_{\text{hyb}}^1}^2 - m_{\pi_{\text{hyb}}^1}^2 \propto \phi_N^2$, where $\phi_N$ is the chiral condensate emerging from the spontaneous symmetry breaking of chiral symmetry. Thus, just as for ordinary mesons, also for hybrids the mass splitting between chiral partners is generated by the chiral condensate. Finally, one gets $m_{\pi_{\text{hyb}}^1} \simeq m_{\pi_{\text{hyb}}^1} \simeq 1660$ MeV, $m_{K_{\text{hyb}}^1} \simeq 1707$ MeV, and $m_{\eta_{\text{hyb}}^1} \simeq 1751$ MeV for the lighter $1^{+-}$ nonet, and $m_{b_{\text{hyb}}^1,1,N,B} \simeq m_{b_{\text{hyb}}^1} \simeq 2000$ MeV, $m_{K_{\text{hyb}}^1,1,B} \simeq 2063$ MeV, $m_{h_{\text{hyb}}^1,1,N,B} \simeq 2126$ MeV for the heavier $1^{+-}$ nonet.

For what concerns decays, four terms can be built [9]. The first two terms fulfills chiral and dilatation invariance and are expected to be dominant: the first terms is responsible for e.g. $\pi_1 \to b_1(1230)\pi$, the second for e.g. $b_{1,\eta_{1}} \to \pi\pi\eta$ and $\pi_{1,\eta_{1}} \to \pi\rho\eta$. The third term breaks dilatation invariance and gives rise to $\pi_{1,\eta_{1}} \to \rho\pi$ and $\pi_{1,\eta_{1}} \to K^*K$. Finally, the fourth one generates $\pi_{1,\eta_{1}} \to \eta\pi$ and $\pi_{1,\eta_{1}} \to \eta\pi$. This decay, even if small, is important since the final state is easily measurable. We study it studied in more details in the next section.
3. Anomalous decays of hybrids

The interaction term that describes \( \pi_{\text{hyb}}^1 \to \eta \pi \) and \( \pi_{\text{hyb}}^1 \to \eta' \pi \) cannot be constructed by a term that fulfills \( U_L(3) \times U_R(3) \). Yet, one can construct it by breaking the \( U_A(1) \) symmetry, and thus implementing the chiral anomaly. The corresponding Lagrangian

\[
\mathcal{L}_{\text{eLSM-anomaly}} = \beta_{\text{hyb}} \left( \det \Phi - \det \Phi^\dagger \right) \text{Tr} \left( I_{\mu}^{\text{hyb}} \left( \partial^\mu \Phi \Phi^\dagger - \Phi \partial^\mu \Phi^\dagger \right) - R_{\mu}^{\text{hyb}} \text{(h.c.)} \right)
\]

(2)

fulfills \( SU_L(3) \times SU_R(3) \) and also parity and \( C \). Since it involves the determinant, it explicitly breaks \( U_A(1) \). Using \( \det \Phi - \det \Phi^\dagger \propto \eta_0 + ... \) (\( \eta_0 \) is the flavor-singlet combination) [12], one obtains \( \mathcal{L}_{\text{eLSM-anomaly}} \propto \eta_0 \text{Tr}(P_{\mu}^{\text{hyb}} \partial^\mu P) + ... \). Then, decays of the type \( \pi_{\text{hyb}}^1 \to \eta \pi \) and \( \pi_{\text{hyb}}^1 \to \eta' \pi \) emerge. Quite importantly, one can predict the ratio \( \frac{\Gamma_{\pi_{\text{hyb}}^1 \to \eta' \pi}}{\Gamma_{\pi_{\text{hyb}}^1 \to \eta \pi}} \simeq 12.7 \), showing that the \( \eta' \pi \) channel is favoured. In Eq. (3) we present the results of the ratios for all the nonet members.

\[
\begin{align*}
\frac{\Gamma_{\pi_{\text{hyb}}^1 \to \eta' \pi}}{\Gamma_{\pi_{\text{hyb}}^1 \to \eta \pi}} & = 12.7 , \\
\frac{\Gamma_{K_{\text{hyb}}^1 \to K \eta'}}{\Gamma_{\pi_{\text{hyb}}^1 \to \pi \eta}} & = 0.69 , \\
\frac{\Gamma_{K_{\text{hyb}}^1 \to K \eta'}}{\Gamma_{\pi_{\text{hyb}}^1 \to \pi \eta}} & = 5.3 , \\
\frac{\Gamma_{\eta_{1,S} \to \eta' \eta'}}{\Gamma_{\pi_{\text{hyb}}^1 \to \pi \eta}} & = 2.2 , \\
\frac{\Gamma_{\eta_{1,S} \to \eta' \eta'}}{\Gamma_{\pi_{\text{hyb}}^1 \to \pi \eta}} & = 1.57 .
\end{align*}
\]

(3)

4. Conclusions

The identification of \( \pi_1(1600) \) as an exotic hybrid state implies that a full nonet of hybrids as well as a nonet of chiral partners should exist. We have investigated the chiral properties of hybrids and coupled them to ordinary \( \bar{q}q \) states in order to evaluate masses and decays. In particular, we have concentrated on the decays into two pseudoscalar mesons, such as \( \pi_{\text{hyb}}^1 \to \eta' \pi \) and \( \pi_{\text{hyb}}^1 \to \eta \pi \). These decays are a consequence of an interaction term that breaks axial symmetry and thus may represent an interesting additional manifestation of chiral anomaly in the mesonic sector. The ratios reported in Eq. (3) can be verified/falsified in ongoing and future experiments.

Acknowledgments: The author thanks W. Eshraim, D. Parganlija, and C. Fischer for cooperation leading to Ref. [9]. Moreover, the author acknowledges support from the Polish National Science Centre NCN through the OPUS projects no. 2019/33/B/ST2/00613 and no. 2018/29/B/ST2/02576.
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