Aharonov-Bohm phase operations on a double-barrier nanoring charge qubit

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We present a scheme for charge qubit implementation in a double-barrier nanoring. The logical states of the qubit are encoded in the spatial wavefunctions of the two lowest energy states of the system. The Aharonov-Bohm phase introduced by magnetic flux, instead of tunable tunnelings, along with electric fields can be used for implementing the quantum gate operations. During the operations, the external fields should be switched smoothly enough to avoid the errors caused by the transition to higher-lying states. The structure and field effects on the validity of the qubit are also studied.

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I. INTRODUCTION

Solid state systems seem to be good candidates for quantum computing implementations. In some schemes, qubits can be encoded in nuclear (or electron) spin states. Although spin states have relatively long decoherence times (\(\sim ms\)), spin operations are rather slow processes. Besides, the single-spin measurement are still a great challenge. Many recent researches focus on charge-based quantum computing technology, in which qubits are encoded in the charge degrees of freedom. Charge-based qubits have shorter decoherence times (\(\sim ns\) for GaAs and maybe much longer in some other materials) than their spin-based counterparts. Yet all quantum gates can be done in very short times (\(\sim ps\)), which are well below the decoherence times. The initialization and readout of charge qubits have been proposed.

For charge-qubit implementation, a typical scheme is a coupled quantum dots (CQDs) system with an electron tunneling back and forth. It can be easily scaled up based on the staggered CPHASE or CNOT configuration. Such scheme, however, is based on the variability of the tunneling. In many physical quantum systems, the handle on the tunneling is limited or impossible. Then the implementation of full single-qubit manipulation requires tunable external magnetic fields and the architecture of the system must be elaborately designed.

Nowadays, benefiting from new fabrication and experiment techniques we can fabricate ringlike quantum dot, namely nanoring, with various materials, such as InAs/GaAs, Si, SiGe, and so on. Its ringlike geometry is suitable for observing Aharonov-Bohm (AB) effect. It has been shown that additional structures, such as two barriers or impurities can bring unique electronic and transport characters to the system. There are two important modes of AB oscillations in double-barrier nanoring, named X and O modes. The ground state entanglement with environment and the persistent current oscillations are also widely studied. To a certain extent, such a system can be viewed as CQDs with a multiply connected domain. Then it may serve as a charge qubit and facilitate the quantum operation by changing AB phase caused by the magnetic flux.

Employing the external fields to modify the wavefunctions of an electron in nanostructures is the foundation of solid quantum computation. So in this work, we will study the evolutions of the wavefunctions and demonstrate the validity of the charge qubit based on the double-barrier nanoring. Full single-qubit operations can be carried out by electric fields and magnetic flux. The remainder of this paper is organized as follows. The descriptions of model Hamiltonian and the calculation method for the evolution of states are presented in Sec.II. Main results and discussions are given in Sec.III, followed by a summary in Sec.IV.

II. MODEL SYSTEM

The model Hamiltonian for an electron in a two-dimensional ring with two identically sectorial barriers, subjected to a magnetic flux \(\phi\) and an in-plane electric field \(F\) applied along the axis \(\theta = 0\), as shown in Fig.1, is written as

\[
H = \left( -i\nabla + \frac{\phi}{r} \right)^2 + V_c + V_g + F \cdot r
\]  

(1)

where \(r_a\) and \(r_b\) are , respectively, the inner and outer radii. \(V_c\) is the hard wall potential which is 0 in the ring and infinite elsewhere. \(V_g\) are the barriers in the ring whose height are \(V_0\) in the barriers and 0 elsewhere. The width of the barrier is selected quite small to ensure that the wave function has maximally one angular node inside each barrier. Here we use the effective atomic units, in which the effective Rydberg \(Ry^* = m_e^*e^4/2\hbar^2 (4\pi\varepsilon_0\varepsilon_r)^2\), the effective Bohr radius \(\alpha_B^* = 4\pi\varepsilon_0\varepsilon_r\hbar^2/m_e^*e^2\) and \(\phi_0 = 2\pi\hbar c/e\) are taken to be the energy, length, and magnetic flux units, respectively. The units for the electric field is \(F_0 = Ry^*/e\alpha_B^*\). For InAs/GaAs materials, for example, \(Ry^* = 5.8\) meV, \(\alpha_B^* = 10\) nm and \(F_0 = 5.8\) KV/cm. \(\phi_0\) included by a 1D ring with a radius of \(10\) nm corresponds to the magnetic field \(13.18\) T.

The eigenenergies \(E_n\) and corresponding eigenstates \(\psi_n\) of the double-barrier nanoring were computed by di-
rect diagonalization of the Hamiltonian in a basis set of 59 eigenstates of the same nanoring without a barrier. Since the nanoring is divided into two segments by the barriers, the electron states localized in right and left segments can be respectively regarded as the logical state $|0\rangle$ and $|1\rangle$, as will be shown below.

In order to demonstrate the implementation of quantum gate operations, we need to calculate the evolutions of the states when $F$ and $\phi$ are changing. Assume that the Hamiltonian is the sum of $H$ and $H'(t)$, where $H'(t)$ is the change of the Hamiltonian from time $t = 0$, the time evolution of a state

$$
\psi(t) = \sum_k C_k(t) \exp \left( -\frac{i}{\hbar} E_k t \right) \psi_k
$$

is given by

$$
\frac{dC_n(t)}{dt} = \sum_k C_k(t) \langle \psi_n | H'(t) | \psi_k \rangle \exp \left[ \frac{i}{\hbar} (E_n - E_k) t \right]
$$

This ordinary differential equation set were solved by Runge-Kutta method. Then we can explore the external field and structure effects in implementations of different gate operations.

III. ANALYSIS

For the nanoring with a hard-wall potential, we can respectively define the radius $R$ and the width $W$ as the average value of inner and outer radii and the difference between them.

A. Energy spectra and qubit definition

In our scheme of charge qubit, the logical states are encoded in the wavefunctions of the first two states of the double-barrier nanoring. Due to the AB effect, it can be seen from the energy spectra in Fig. 2(a) that the ground state ($\psi_1$) and first excited state ($\psi_2$) of a parallel double-barrier nanoring (the angle between two barriers is equal to $\pi$) are degenerate when $\phi = 0.5\phi_0$ and $F = 0$. These two occasionally degenerate states are just X-type states which had been discussed in previous work. The virtue of this condition is that the qubit can be frozen in any linear superposition in its qubit space to avoid unnecessary evolutions in idle time. This request can be also satisfied by applying high enough barriers, just like the situation in CQDs system. It is convenient to take the normalized sum and difference of the two states ($\psi_1 \pm \psi_2$)/$\sqrt{2}$ as logical states $|0\rangle$ and $|1\rangle$, respectively. Within such choice, the electron of state $|0\rangle$ ($|1\rangle$) is almost completely localized in the right (left) segment of the ring. The energy spectra for a nonparallel double-barrier nanoring are shown in Fig. 2(b). The first two energy levels can be still tuned to degenerate by $F$ with $\phi = 0.5\phi_0$. So the qubit states can be always defined properly to avoid needless evolutions. Due to the inherent ringlike geometry of the system, in our charge qubit scheme it is much easier to implement AB phase operation than that in CQDs. In the following of the paper, we will focus on a parallel double-barrier nanoring with a initial magnetic flux $\phi = 0.5\phi_0$. The initialization of qubit states to the state $|0\rangle$ can be realized by applying an appropriate electric field along the axis $\theta = 0$. The readout can be implemented by a single-electron transistor (SET) or quantum point contact (QPC) detector. Because the double-barrier nanoring can be also viewed as a new kind of CQDs, the qubit based on it can be scaled up by present CPHASE configuration scheme.

B. State evolutions with $F$ and $\phi$

With logical states $|0\rangle$ and $|1\rangle$ defined, we can calculate the evolutions of the wavefunctions and qubit states with
FIG. 3: (Color online) The evolutions of wavefunctions during the Z-gate operations by applying \( F \) with \( V_0 \to \infty \) and \( \phi = 0 \) (a) or finite \( V_0 \) and \( \phi = 0.5\phi_0 \) (b), the NOT-gate operations by changing \( V_0 \) with \( \phi = 0 \) (c) or changing \( \phi \) with fixed finite \( V_0 \) (d). The real and imaginary parts and the modulus of the wavefunctions are plotted in three rows respectively. Different line styles represent the quantities at different time. The black solid lines and red dotted lines represent the quantities at the starting and ending time, respectively. The vertical dashed lines indicate the positions of the two barriers.

FIG. 4: (Color online) Evolutions of the qubit states with in-plane electric field pulse (a), magnetic flux pulse (b) and both fields together (c). Black solid, red dashed and green dotted lines correspond to the square root of probability density of the state occupying \( |0\rangle \), \( |1\rangle \) and the phase difference, respectively. The trajectories of Bloch vectors corresponding the evolutions in (a)∼(c) are shown in (d) noted as Z-, NOT- and H-gate, respectively.

the change of external parameters by employing Runge-Kutta method. We will present the results concentrating on the characters of evolutions in this subsection. The quantitative analyses are left to next subsection.

From Hamiltonian (1) it can be imagined that changing the magnetic flux, the height of the two barriers and the electric field all can achieve some evolutions of wavefunctions. In Fig.3(a) and (b) we illustrate the evolutions of wavefunctions when applying an in-plane electric field along \( \theta = 0 \) from \( t = 0 \). The positions of the barriers are indicated by the vertical dashed lines. The initial state are both \( \psi_1 \), although the wavefunctions are not same because \( V_g \) is infinite and \( \phi = 0 \) in Fig.3(a) but \( \phi = 0.5\phi_0 \) and \( V_g \) is finite in Fig.3(b). Both conditions make the states \( \psi_1 \) and \( \psi_2 \) degenerate. After some time the final states can be both \( \psi_2 \), although their wavefunctions are not equal either. Recalling the definition of logical states of the qubit, the initial and final states are just \( (|0\rangle + |1\rangle)/\sqrt{2} \) and \( (|0\rangle - |1\rangle)/\sqrt{2} \), respectively. It can be seen from the third row of Fig.3(a) and (b) that the modulus of the wavefunctions are unchanged during the evolutions. This evolution is just the single-qubit gate operation namely the Z-gate, which will be discussed in the following section.

One goal of the paper is to identify the similar effects of changing AB phase and the barrier heights on the qubit operations. Thus we verify the evolutions of wavefunctions when changing the barrier heights and the magnetic flux from \( t = 0 \). The results are shown in Fig.3(c) and (d).
The initial states are both logical state $|0\rangle$ with different wavefunctions, because there is no flux in Fig.3(a) but 0.5$\phi_0$ flux before $t = 0$ in Fig.3(b). It can be found that the final states are both $|1\rangle$. It means that the two methods can implement similar quantum operations on the logical states of the qubit. From Fig.2 it can be seen that the crossing of the first two states becomes anticing when $\phi \neq 0.5\phi_0$ which means that the two states can mix up. Similar with varying the barrier height, the change of the flux determines the admixture level. So in the following, we will adopt the magnetic flux, instead of the barrier height, and the electric field to explore the evolutions of the qubit states.

The first case which we concern is still the electric field. An in-plane electric field applied along the axis $\theta = 0$ from $t = 0$ make the energies of one electron localized in the left and right segments unequal. As shown in Fig.3(b) and Fig.4(a), the electric field will not change the probability of the states projecting to $|0\rangle$ or $|1\rangle$. It can only bring a phase difference between the two components. The trajectory of Bloch vector correspond to such an evolution is a rotation around z-axis of Bloch sphere.

The situation is different when the magnetic flux changes. Because we have assumed that there is an always-on magnetic flux $\phi = 0.5\phi_0$, we will decrease this flux from $t = 0$ and the initial state is chosen to be $|0\rangle$ without electric fields. Then the degeneracy of $|0\rangle$ and $|1\rangle$ is removed, and the mixture of the two states forms a cycle trajectory of Bloch vector around x-axis of Bloch sphere. From Fig.4(b) we can see that the variation of the flux changes the probability of each component. It also changes the sign of the phase difference when Bloch vector rotates half a cycle.

By applying both the electric field and magnetic flux pulse, we can implement arbitrary rotation of Bloch vector. In Fig.4(c), we plot one of such rotations which can be used for implementing the Hadamard gate.

C. Characters of quantum operations

Through the above analysis, it is straightforward to implement the single-qubit quantum operations by applying the electric field and changing the magnetic flux.

The first important gate operation is the Z-gate which implements the transform from $\alpha|0\rangle + \beta|1\rangle$ to $\alpha|0\rangle - \beta|1\rangle$, where $\alpha$ and $\beta$ are arbitrary constants. Applying an electric field and controlling the pulse duration to make the angle of rotation equal to $\pi$ just achieves a Z-gate operation. As shown in the left column of Fig.5, for the nanoribbon subjected to a magnetic flux $\phi = 0.5\phi_0$ with $R = 1.2a_B^*$, $W = 0.4a_B^*$ and $V_0 = 90Ry^*$, the time for completing a Z-gate operation is 4.66ps if the amplitude of the electric pulse is 0.05$F_0$. This operation time is much smaller than the typical charge decoherence time for GaAs.

In order to implement full single-qubit operations, we still need an operation which can make Bloch vector rotate around another axis other than the z-axis. This operation can be achieved by changing the magnetic flux as discussed in the previous subsection. In the middle column of Fig.5, we have shown such operation by changing the flux from 0.5$\phi_0$ to 0.445$\phi_0$ from $t = 0$. The electric field is set to zero. When the rotation angle of Bloch vector is again equal to $\pi$, we can get the NOT-gate which changes the state $|0\rangle$ to $|1\rangle$ and vice versa. The time for the NOT operation is 37.23ps which is a little longer than the Z-gate but still much smaller than the decoherence time.
With these two operations, we can implement arbitrary single-qubit operations. For example, the Hadamard gate which may be the most important single-qubit gate in quantum computation can be achieved by accurately control the external fields and the operation time as shown in the right column of Fig.5. With the pulses $F = 0.00563F_0$ and $\phi = 0.445\phi_0$, the time for a Hadamard operation is $26.65\text{ps}$.

Besides the operation time, there is still another important index of quantum gate. We know that there is a small fraction of probability ($P$) which is lost from the first two states to the higher-lying ones during every gate operation. This phenomenon will result in errors in quantum computation and may also be considered as a decoherence source. Then this probability must be regarded as an important judgement of the validity of the qubit scheme.

In Fig.5 we have shown two kinds of probability loss corresponding to different forms of the external field pulses. It can be seen that the value of $P$ can be less than $0.1\%$ if the pulse has a smooth rising or trailing edge. Although such an error rate is still greater than the threshold ($10^{-4}$ for an estimated value widely accepted at present) for fault-tolerance quantum computation, it is indeed small enough for coherence quantum operations in single-qubit experiments. If the changes of external field parameters are instantaneous, $P$ will be much larger and have severe vibration for both electric fields and magnetic flux. This also coincides with the idea that the evolution of the qubit states should be adiabatic during the quantum operations. A smooth change of the fields ensures such premise.

### D. Structure and field effects

![FIG. 6: (Color online) (a) Error probability ($\Delta$) as functions of $V_0$ with different $R$. The curves correspond to $R = 1.1a_B^*$ to $R = 2.0a_B^*$ from the top down. (b) Maximal probability loss ($P_{\text{max}}$) and operation time ($t_{\text{NOT}}$) as functions of the change of magnetic flux ($\Delta \phi$) with three different $V_0$ (plotted with different line styles) during the NOT-gate operation. The $V_0$-dependence of $P$ is shown in the inset. In all the cases, $W = 0.4a_B^*$.

In this section, we will discuss the structure and field effects on the validity of our qubit and corresponding operations.

First of all, we have chosen the two localized states of the electron in a double-barrier nanoring as the logical states $|0\rangle$ and $|1\rangle$. However, because of the finite height of the two barriers, this two states are not completely localized in one segment of the ring. The state mainly localized in one segment indeed has a probability ($\Delta$) expanding to another segment. This probability leads to errors in readout process and must be limited in a tiny value. It can be convinced from Fig.6(a) that the probability can be depressed by choosing appropriate $V_0$ for different $R$. And the larger $R$ is, the lower $V_0$ is needed.

Second, we have seen that changing the magnetic flux can implement the NOT-gate operation. But its operation time is longer than the Z-gate which is implemented by applying a small electric field. Of course, this operation time is related to the amplitude of the change of the flux. It can be seen in Fig.6(b) that increasing the change of the flux can apparently speed the operation. But on the other hand, it also increases the probability loss during the operation. So in our calculation we have chosen the field parameters carefully to ensure both a short operation time and a small $P$. It is worthwhile to indicate that the value of $V_0$ can also affect the operation time and $P$. Lower barriers can speed the operation and decrease $P$. But in fact it must be selected high enough to avoid the increase of $\Delta$.

![FIG. 7: (Color online) Operation time for the NOT-gate as a function of $R$ and $V_0$. An example of appropriate choice of the structural parameters is illustrated by the green line and its projection to the $R - t_{\text{NOT}}$ plane is plotted in the inset.

Finally, we present $R$ and $V_0$-dependence of the operation time for the NOT-gate in Fig.7. It can be found intuitively that large radius $R$ and high barriers $V_0$ will dramatically increase the operation time. An appropri-
ate choice of the structural parameters is illustrated by the green line in Fig. 7 where small rings correspond to high barriers and large rings correspond to low barriers. Such selections can take into account both the speed of the operation and tolerable errors.

IV. SUMMARY

A new scheme of qubit based on one electron’s charge degree of freedom in a double-barrier nanoring is presented. Because the first two states of the system are degenerate when \( \phi = 0.5 \phi_0 \), the logical states can be frozen in any linear superposition in the qubit space to avoid unnecessary evolutions. The electric fields can implement the z-axis gate operations of the qubit. By virtue of the ringlike geometry of the system, the x-axis operations can be achieved by AB effect of the magnetic flux. As a result, full qubit operations can be implemented even if the barrier height is kept constant. The structure and field effects are important for the validity of the qubit. The external field pulse should also have an appropriate rising or trailing edge to decrease the transition of the electron to higher-lying states. The radius and barrier height should be selected appropriately to speed the operations and depress the errors. These results will be helpful in understanding the evolutions of wavefunctions during the quantum operations and useful for future implementation of qubits in solid system.

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