In this paper, we prove that the set of sentences parsed by Marcus' parser constitutes a context-free language. The proof is carried out by constructing a deterministic pushdown automaton that recognizes those strings of terminals that are parsed successfully by the Marcus parser.

1. Introduction

While Marcus [4] does not use phrase structure rules as base grammar in his parser, he points out some correspondence between the use of a base rule and the way packets are activated to parse a construct. Charniak [2] has also assumed some phrase structure base grammar in implementing a Marcus style parser that handles ungrammatical situations. However, neither has suggested a type for such a grammar or the language accepted by the parser. Berwick [1] relates Marcus' parser to $LR(k,0)$ context-free grammars. Similarly, in [5] and [6] we have related this parser to $LRRL(k)$ grammars. Inevitably, these raise the question of whether the string set parsed by Marcus' parser is a context-free language.

In this paper, we provide the answer for the above question by showing formally that the set of sentences accepted by Marcus' parser constitutes a context-free language. Our proof is based on simulating a simplified version of the parser by a pushdown automaton. Then some modifications of the PDA are suggested in order to ascertain that Marcus' parser, regardless of the structures it puts on the input sentences, accepts a context-free set of sentences. Furthermore, since the resulting PDA is a deterministic one, it confirms the determinism of the language parsed by this parser. Such a proof also provides a justification for assuming a context-free underlying grammar in automatic generation of Marcus type parsers as discussed in [5] and [6].

2. Assumption of a finite size buffer

Marcus' parser employs two data structures: a pushdown stack which holds the constructs yet to be completed, and a finite size buffer which holds the lookaheads. The lookaheads are completed constructs as well as bare terminals. Various operations are used to manipulate these data structures. An "attention shift" operation moves a window of size $k+3$ to a given position on the buffer. This occurs in parsing some constructs, e.g., some NP's, in particular when a buffer node other than the first indicates start of an NP. "Restore buffer" restores the window to its previous position before the last "attention shift". Marcus suggests that the movements of the window can be achieved by employing a stack of displacements from the beginning of the buffer, and in general he suggests that the buffer could be unbounded on the right. But in practice, he notes that he has not found a need for more than five cells, and PARSIFAL does not use a stack to implement the window or virtual buffer.

A comment regarding an infinite buffer is in place here. An unbounded buffer would yield a parser with two stacks. Generally, such parsers characterize context-sensitive languages and are equivalent to linear bounded automata. They have also been used for parsing some context-free languages. In this role they may hide the non-determinism of a context-free language by storing an unbounded number of lookaheads. For example, $LR$-regular [3], $BCP(m,n)$, $LR(k,m)$ and $FSPA(k)$ parsers [8] are such parsers. Furthermore, basing parsing decisions on the whole left contexts and $k$ lookaheads in them has often resulted in defining classes of context-free (context-sensitive) grammars with undecidable membership. $LR$-regular, $LR(k,\omega)$ and $FSPA(k)$ are such classes. The class of $GLRRL(k)$ grammars with unbounded buffer (defined in [5]) seems to be the known exception in this category that has decidable membership. Walters [9] considers context-sensitive grammars with deterministic two-stack parsers and shows the undecidability of the membership problem for the class of such grammars.

In this paper we assume that the buffer in a Marcus style parser can only be of a finite size $b$ (e.g., $b=5$ in Marcus' parser). The limitation on the size of the buffer has two important consequences. First, it allows a proof for the context-freeness of the language to be given in terms of a PDA. Second, it facilitates the design of an effective algorithm for automatic generation of a parser. (However, we should add that: 1- some Marcus style parsers that use an unbounded buffer in a constrained way, e.g., by restricting the window to the $k$ rightmost elements of the buffer, are equivalent to pushdown automata. 2- Marcus style parsers with unbounded buffer, similar to $GLRRL$ parsers, can still be constructed for those languages which are known to be context-free.)
3. Simplified parser

A few restrictions on Marcus' parser will prove to be convenient in outlining a proof for the context-freeness of the language accepted by it.

(i) Prohibition of features:
Marcus allows syntactic nodes to have features containing the grammatical properties of the constituents that they represent. For implementation purposes, the type of a node is also considered as a feature. However, here a distinction will be made between this feature and others. We consider the type of a node and the node itself to convey the same concept (i.e., a non-terminal symbol). Any other feature is disallowed. In Marcus' parser, the binding of traces is also implemented through the use of features. A trace is a null deriving non-terminal (e.g., an NP) that has a feature pointing to another node, i.e., the binding of the trace. We should stress at the outset that Marcus' parser outputs the annotated surface structure of an utterance and traces are intended to be used by the semantic component to recover the underlying predicate/argument structure of the utterance. Therefore one could put aside the issue of trace registers without affecting any argument that deals with the strings accepted by the parser, i.e., frontiers of surface structures. We will reintroduce the features in the generalized form of PDA for the completeness of the simulation.

(ii) Non-accessibility of the parse tree:
Although most of the information about the left context is captured through the use of the packeting mechanism in Marcus' parser, he nevertheless allows limited access to the nodes of the partial parse tree (besides the current active node) in the action parts of the grammar rules. In some rules, after the initial pattern matches, conditional clauses test for some property of the parse tree. These tests are limited to the left daughters of the current active node and the last cyclic node (NP or S) on the stack and its descendants. It is plausible to eliminate tree accessibility entirely through adding new packets and/or simple flags. In the simplified parser, access to the partial parse tree is disallowed. However, by modifying the stack symbols of the PDA we will later show that the proof of context-freeness carries over to the general parser (that tests limited nodes of parse tree).

(iii) Atomic actions:
Action segments in Marcus' grammar rules may contain a series of basic operations. To simplify the simulation, we assume that in the simplified parser actions are atomic. Breakdown of a compound action into atomic actions can be achieved by keeping the first operation in the original rule and introducing new singleton packets containing a default pattern and a remaining operation in the action part. These packets will successively deactivate themselves and activate the next packet much like "run <rule> next's in PIDGIN. The last packet will activate the first if the original rule leaves the packet still active. Therefore in the simplified parser action segments are of the following forms:

1. (1) Activate packets1; [deactivate packets2].
2. (2) Deactivate packets1; [activate packets2].
3. (3) Attach ith; [deactivate packets1]; [activate packets2].
4. (4) [Deactivate packets1]; create node; activate packets2.
5. (5) [Deactivate packets1]; catch node; activate packets2. 1
6. (6) Drop; [deactivate packets1]; [activate packets2].
7. (7) Drop into buffer; [deactivate packets1]; [activate packets2].
8. (8) Attention shift (to ith cell); [deactivate packets1]; [activate packets2].
9. (9) Restore buffer; [deactivate packets1]; [activate packets2].

Note that forward attention shift has no explicit command in Marcus' rules. An "AS" prefix in the name of a rule implies the operation. Backward window move has an explicit command "restore buffer". The square brackets in the above forms indicate optional parts. Feature assignment operations are ignored for the obvious reason.

4. Simulation of the simplified parser

In this section we construct a PDA equivalent to the simplified parser. This PDA recognizes the same string set that is accepted by the parser. Roughly, the states of the PDA are symbolized by the contents of the parser's buffer, and its stack symbols are ordered pairs consisting of a non-terminal symbol (i.e., a stack symbol of the parser) and a set of packets associated with that symbol.

Let N be the set of non-terminal symbols, and Σ be the set of terminal symbols of the parser. We assume the top S node, i.e., the root of a parse tree, is denoted by S0, a distinct element of N. We also assume that a final packet is added to the PIDGIN grammar. When the parsing of a sentence is completed, the activation of this packet will cause the root node S0 to be dropped into the buffer, rather than being left on the stack. Furthermore, let P denote the set of all packets of rules, and 2P the powerset of P, and let Pγ1,Pγ2,... be elements of 2P. When a set of packets P is active, the pattern segments of the rules in these packets are compared with the current active node and contents of the virtual buffer (the window). Then the action segment of a rule with highest priority that matches is executed. In effect the operation of the parser can be characterized by a partial function M from active packets, current active node and contents of the window into atomic actions, i.e.,

M: 2PγN(1)γV(k) → ACTIONS

1 "Catch" is used as a short notation for "create and attach".

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where $V = N \cup \Sigma$, $V(k)= V_0+V_1+\ldots+V_k$ and ACTIONS is the set of atomic actions (1) - (9) discussed in the previous section.

Now we can construct the equivalent PDA $A=(Q,\Sigma,\Gamma,\delta,q_0,Z,e)$ in the following way.

$\Sigma$ is the set of input symbols of $A$, is the set of terminal symbols in the simplified parser.

$\Gamma$ is the set of stack symbols $[X,P]$, where $X \in \Sigma$ is a non-terminal symbol of the parser and $P$ is a set of packets.

$Q$ is the set of states of the PDA, each of the form $<P_1,P_2,\text{buffer}>$, where $P_1$ and $P_2$ are sets of packets. In general $P_1$ and $P_2$ are empty sets except for those states that represent dropping of a current active node in the parser. $P_1$ is the set of packets to be activated explicitly after the drop operation, and $P_2$ is the set of those packets that are deactivated. "buffer" is a string in $\{0,1\}^k\cup\{\lambda\}$, such that $|\text{buffer}|=k$. The last vertical bar in "buffer" denotes the position of the current window in the parser and those on the left indicate former window positions.

$q_0$ is the initial state $=<\lambda,\lambda>$, where $\lambda$ denotes the null string.

$f$ is the final state $=<\lambda,\lambda>$. This state corresponds to the outcome of an activation of the final packet in the parser. In this way, i.e., by dropping $\Sigma$ node into the buffer, we can show the acceptance of a sentence simultaneously by empty stack and by final state.

$Z_e$ is the start symbol $=[S_e,P_e]$, where $P_e$ is the set of initial packets, e.g., ISS-Start, C-Pool in Marcus' parser.

$\delta$ is the move function of the PDA, defined in the following way:

Let $P$ denote a set of active packets, $X$ an active node and $W_1W_2\ldots W_n$, $n \leq k$, the content of a window. Let $\alpha|W_1W_2\ldots W_n\beta$ be a string (representing the buffer) such that: $\alpha \in (\{0,1\}^k\cup\{\lambda\})^*$ and $\beta \in \Sigma^*$, where $\text{Length}(\alpha|W_1W_2\ldots W_n\beta)\leq k$, and $\alpha'$ is the string $\alpha$ in which vertical bars are erased.

\textbf{Non-}\textbf{\textlambda{}}-moves: The non-\textlambda{}-moves of the PDA $A$ correspond to bringing the input tokens into the buffer for examination by the parser. In Marcus' parser input tokens come to the attention of parser as they are needed. Therefore, we can assume that when a rule tests the contents of $n$ cells of the window and there are fewer tokens in the buffer, terminal symbols will be brought into the buffer. More specifically, if $M(P,X,W_1W_2\ldots W_n)$ has a defined value (i.e., $P$ contains a packet with a rule that has pattern segment $[X][W_1]\ldots [W_n]$), then 

\[ \delta(<\lambda,\lambda>|W_1\ldots W_n\beta) = (<\lambda,\lambda>|W_1\ldots W_j\lambda|W_{j+1}\ldots W_n\beta) \text{ for all } \alpha \text{ and for } j = 0, \ldots, n-1 \text{ and } W_{j+1} \in \Sigma. \]

\textbf{\textlambda{}}-moves: By \textlambda{}-moves, the PDA mimics the actions of the parser on successful matches. Thus the $\delta$-function on input corresponding to each individual atomic action is determined according to one of the following cases.

Cases (1) and (2):

If $M(P,X,W_1W_2\ldots W_n) = "\text{activate} P_1; \text{deactivate} P_2"$ (or "\text{deactivate} P_2; \text{activate} P_1"), then 

\[ \delta(<\lambda,\lambda>|W_1\ldots W_n\beta) = <\alpha,\lambda>|W_1\ldots W_n\beta, \lambda, [X,P] \text{ for all } \alpha \text{ and } \beta. \]

Case (3):

If $M(P,X,W_1W_2\ldots W_n) = "\text{attach} i\text{th} (\text{normally} i \text{is} 1); \text{deactivate} P_1; \text{activate} P_2", \text{then} \delta(<\lambda,\lambda>|W_1\ldots W_n\beta) = <\alpha,\lambda>|W_1\ldots W_{i-1}\lambda W_{i+1}\ldots W_n\beta, [X,P \cup P_2]P_1 \text{ for all } \alpha \text{ and } \beta. \]

Cases (4) and (5):

If $M(P,X,W_1W_2\ldots W_n) = "\text{deactivate} P_1; \text{create}/\text{attach} Y; \text{activate} P_2", \text{then} \delta(<\lambda,\lambda>|W_1\ldots W_n\beta) = <\alpha,\lambda>|W_1\ldots W_n\beta, \lambda, [X,P_2]P_1 \text{ for all } \alpha \text{ and } \beta. \]

Case (6):

If $M(P,X,W_1W_2\ldots W_n) = "\text{drop} \text{into buffer}; \text{deactivate} P_1; \text{activate} P_2", \text{then} \delta(<\lambda,\lambda>|W_1\ldots W_n\beta) = <\alpha,\lambda>|W_1\ldots W_n\beta, \lambda, [X,P_2]P_1 \text{ for all } \alpha \text{ and } \beta. \]

Case (7):

If $M(P,X,W_1W_2\ldots W_n) = "\text{shift} \text{attention to} i\text{th cell}; \text{deactivate} P_1; \text{activate} P_2", \text{then} \delta(<\lambda,\lambda>|W_1\ldots W_n\beta) = <\alpha,\lambda>|W_1\ldots W_n\beta, \lambda, [X,P \cup P_2]P_1 \text{ for all } \alpha \text{ and } \beta. \]

Case (8):

If $M(P,X,W_1W_2\ldots W_n) = "\text{restore} \text{buffer}; \text{deactivate} P_1; \text{activate} P_2", \text{then} \delta(<\lambda,\lambda>|W_1\ldots W_n\beta) = <\alpha,\lambda>|W_1\ldots W_n\beta, \lambda, [X,P \cup P_2]P_1 \text{ for all } \alpha \text{ and } \beta. \]

Now from the construction of the PDA, it is obvious that $A$ accepts those strings of terminals that are parsed successfully by the simplified parser. The reader may note that the value of $\delta$ is undefined for the cases in which $M(P,X,W_1W_2\ldots W_n)$ has multiple values. This accounts for the fact that Marcus' parser behaves in a deterministic way. Furthermore, many of the states of $A$ are unreachable. This is due to the way we constructed the PDA, in which we considered activation of every subset of $P$ with any active node.
and any lookahead window.

5. Simulation of the general parser

It is possible to lift the restrictions on the simplified parser by modifying the PDA. Here, we describe how Marcus' parser can be simulated by a generalized form of the PDA.

(i) Non-atomic actions:
The behaviour of the parser with non-atomic actions can be described in terms of $M'$ and $M^*$, a sequence of compositions of $M$, which in turn can be specified by a sequence $\delta'$ in $\delta^*$. 

(ii) Accessibility of descendants of current active node, and current cyclic node:

What parts of the partial parse tree are accessible in Marcus' parser seems to be a moot point. Marcus [4] states

"the parser can modify or directly examine exactly two nodes in the active node stack... the current active node and $S$ or NP node closest to the bottom of stack... called the dominating cyclic node... or... current cyclic node... The parser is also free to examine the descendants of these two nodes... although the parser cannot modify them. It does this by specifying the exact path to the descendant it wishes to examine."

The problem is that whether by descendants of these two nodes, one means the immediate daughters, or descendants at arbitrary levels. It seems plausible that accessibility of immediate descendants is sufficient. To explore this idea, we need to examine the reason behind partial tree accesses in Marcus' parser. It could be argued that tree accessibility serves two purposes:

1. Examining what daughters are attached to the current active node considerably reduces the number of packet rules one needs to write.
2. Examining the current cyclic node and its daughters serves the purpose of binding traces. Since transformations are applied in each transformational cycle to a single cyclic node, it seems unnecessary to examine descendants of a cyclic node at arbitrarily lower levels.

If Marcus' parser indeed accesses only the immediate daughters (a brief examination of the sample grammar [4] does not seem to contradict this), then the accessible part of the a parse tree can be represented by a pair of nodes and their daughters. Moreover, the set of such pairs of height-one trees are finite in a grammar. Furthermore, if we extend the access to the descendants of these two nodes down to a finite fixed depth (which, in fact seems to have a supporting evidence from X theory and C-command), we will still be able to represent the accessible parts of parse trees with a finite set of finite sequences of fixed height trees.

A second interpretation of Marcus' statement is that descendants of the current cyclic node and current active node at arbitrarily lower levels are accessible to the parser. However, in the presence of non-cyclic recursive constructs, the notion of giving an exact path to a descendant of the current active or current cyclic node would be inconceivable; in fact one can argue that in such a situation parsing cannot be achieved through a finite number of rule packets. The reader is reminded here that PIDGIN (unlike most programming languages) does not have iterative or recursive constructs to test the conditions that are needed under the latter interpretation.

Thus, a meaningful assumption in the second case is to consider every recursive node to be cyclic, and to limit accessibility to the subtree dominated by the current cyclic node in which branches are pruned at the lower cyclic nodes. In general, we may also include cyclic nodes at fixed recursion depths, but again branches of a cyclic node beyond that must be pruned. In this manner, we end up with a finite number of finite sequences (hereafter called forests) of finite trees representing the accessible segments of partial parse trees.

Our conclusion is that at each stage of parsing the accessible segment of a parse tree, regardless of how we interpret Marcus' statement, can be represented by a forest of trees that belong to a finite set $T_{N,h}$. $T_{N,h}$ denotes the set of all trees with non-terminal roots and of a maximum height $h$. In the general case, this information is in the form of a forest, rather than a pair of trees, because we also need to account for the unattached subtrees that reside in the buffer and may become an accessible part of an active node in the future. Obviously, these subtrees will be pruned to a maximum height $h-1$. Hence, the operation of the parser can be characterized by the partial function $M$ from active packets, subtrees rooted at current active and cyclic nodes, and contents of the window into compound actions, i.e.,

$$M: P \rightarrow T_{N,h} \cup (\lambda)M(T_{C,h} \cup (\lambda)M(T_{N,h-1} \cup \Sigma)^{(k)})$$

where $T_{C,h}$ is the subset of $T_{N,h}$ consisting of the trees with cyclic roots.

In the PDA simulating the general parser, the set of stack symbols $\Gamma$ would be the set of triples $[T_Y, T_X, P]$, where $T_Y$ and $T_X$ are the subtrees rooted at current cyclic node $Y$ and current active node $X$, and $P$ is the set of packets associated with $X$. The states of this PDA will be of the form $<X,Y,P,buffer>$. The last three elements are the same as before, except that the buffer may now contain subtrees belonging to $T_{N,h-1}$. (Note that in the simple case, when $h=1$, $T_{N,h-1}=N$). The first entry is usually $\lambda$ except that when the current active node $X$ is dropped, this element is changed to $T_X$. The subtree $T_X$ is the tree dominated by $X$, i.e., $T_X$ pruned to the height $h-1$.

Definition of the move function for this PDA is very similar to the simplified case. For example, under the
(iii) Accommodation of features:

The features used in Marcus' parser are syntactic in nature and have finite domains. Therefore the set of attributed symbols in that parser constitutes a finite set. Hence syntactic features can be accommodated in the construction of the PDA by allowing complex non-terminal symbols, i.e., attributed symbols instead of simple ones.

Feature assignments can be simulated by replacing the top stack symbol in the PDA. For example, under our previous assumption that two height-one trees rooted at current active node and current cyclic node are accessible to the parser, the definition of \( \delta \) function would include the following statement among others:

If \( M(P,T_x,T_y,W_1...W_n) = "drop; deactivate P_3; activate P_2" \)

(\( \lambda(T_x,T_y,T_z) \))

then

\[
\delta(<\lambda,A,B,C,D,W_1...W_n>,\lambda(T_x,T_y,T_z,T_{a'})) = <\lambda,A,B,C,D,W_1...W_n,\lambda(T_x,T_y,T_z,T_{a'}),T_{a'}>.
\]

Furthermore, if \( \lambda(T_x,T_y,T_z) \) then

\[
\delta(<\lambda,A,B,C,D,W_1...W_n>,\lambda(T_x,T_y,T_z,T_{a'})) = <\lambda,A,B,C,D,W_1...W_n,\lambda(T_x,T_y,T_z,T_{a'}),T_{a'}>.
\]

Not only may the resulting structures be hardly usable by any reasonable semantic/pragmatics component, but also parsing would be inefficient because of the huge number of non-terminals and productions.

In automatic generation of Marcus-style parsers, one can assume either a context-free or a context-sensitive grammar (as a base grammar) which one feels is naturally suitable for describing surface structures. However, if one chooses a context-sensitive grammar then one needs to make sure that it only generates a context-free language (which is unsolvable in general). In [5] and [6], we have proposed a context-free base grammar which is augmented with syntactic features (e.g., person, tense, etc.) much like attributed grammars in compiler writing systems. An additional advantage with this scheme is that semantic features can also be added to the nodes without an extra effort. In this way one is also able to capture the context-sensitivity of a language.

6. Conclusions

We have shown that the information examined or modified during Marcus parsing (i.e., segments of partial parse trees, contents of the buffer and active packets) for a PIGDIN grammar is a finite set. By encoding this information in the stack symbols and the states of a deterministic pushdown automaton, we have shown that the resulting PDA is equivalent to the Marcus parser. In this way we have proved that the set of surface sentences accepted by this parser is a context-free set.

An important factor in this simulation has been the assumption that the buffer in a Marcus style parser is bounded. It is unlikely that all parsers with unbounded buffers written in
this style can be simulated by deterministic pushdown automata. Parsers with unbounded buffers (i.e., two-stack parsers) are used either for recognition of context-sensitive languages, or if they parse context-free languages, possibly to hide the non-determinism of a language by storing an unlimited number of lookaheads in the buffer. However, this does not mean that some Marcus-type parsers that use an unbounded buffer in a constrained way are not equivalent to pushdown automata. Shipman and Marcus [7] consider a model of Marcus' parser in which the active node stack and buffer are combined to give a single data structure that holds both complete and incomplete subtrees. The original stack nodes and their lookaheads alternately reside on this structure. Letting an unlimited number of completed constructs and bare terminals reside on the new structure is equivalent to having an unbounded buffer in the original model. Given the restriction that attachments and drops are always limited to the k+1 rightmost nodes of this data structure, it is possible to show that a parser in this model with an unbounded buffer still can be simulated with an ordinary pushdown automaton. (The equivalent condition in the original model is to restrict the window to the k rightmost elements of the buffer. However simulation of the single structure parser is much more straightforward.)

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