The Octet Meson and Octet Baryon Interaction with strangeness and the \( \Lambda(1405) \)

Jun He\textsuperscript{1,3,4} and Pei-Liang Lü\textsuperscript{1,2}

\textsuperscript{1} Theoretical Physics Division, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
\textsuperscript{2} University of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{3} Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China
\textsuperscript{4} State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

junhe@impcas.ac.cn

The octet meson and baryon interaction with strangeness \( S = -1 \) is studied fully relativistically with chiral Lagrangian. In this work, a Bethe-Salpeter equation approach with spectator quasipotential approximation is applied to study the reactions \( K^- p \rightarrow MB \) with \( MB = K^- p, K^0 n, \pi^- \Sigma^+, \pi^0 \Sigma^0, \pi^+ \Sigma^- \) and \( \eta \Lambda \) with all possible partial waves and theoretical results are comparable with experimental data. It is found that the Weinberg-Tomozawa potential derived from the lowest order chiral Lagrangian only provides the contributions from partial waves with spin-parities \( J^P = 1/2^+ \) and \( 1/2^- \). Two-pole structure of the \( \Lambda(1405) \) is confirmed with poles at \( 1383 \pm 99 \)i and \( 1423 \pm 14 \)i MeV. The lower and higher poles originate from \( \Sigma \pi \) interaction as a resonance and \( \bar{K}N \) interaction as a bound state, respectively.

Keywords: Octet meson and baryon interaction; \( \Lambda(1405) \); Bethe-Salpeter equation

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1. Introduction

Among the observed excited hyperons, the \( \Lambda(1405) \) with spin parity \( J^P = 1/2^- \) and isospin \( I = 0 \) attracts peculiar attention.\textsuperscript{3} The existence of \( \Lambda(1405) \) was theoretically predicted in 1959 by Dalitz and Tuan,\textsuperscript{2,3} and confirmed in the experiment as early as 1961 in the \( \pi \Sigma \) invariant mass spectrum.\textsuperscript{4} After a half century, \( \Lambda(1405) \) has been well established experimentally and is currently listed as a four-star resonance in the table of the Particle Data Group (PDG)\.\textsuperscript{5} However, there is still no universal agreement on the nature of this state.

As the first excited \( \Lambda \) resonance, the \( \Lambda(1405) \) should be assigned as a P-wave baryon in the constituent quark model.\textsuperscript{3,6} However, although the constituent quark model provides a general scheme to describe the non-strange low-mass baryons, it
has difficulty in computing the Λ(1405) mass. The theoretical mass, about 1500 MeV, is about one hundred MeV higher than the observed one. It is even lower than the mass of its nonstrange counterpart \( N(1535) \), which is very strange because in the constituent quark model strange quark is about one third heavier than nonstrange quarks. Due to such difficulties it is popular to adopt an interpretation beyond the three quark picture for the \( \Lambda(1405) \) in the literature.

Within the framework of the constituent quark model, a proposal was made to solve the reverse mass order problem for \( \Lambda(1405) \) and \( N(1535) \) by introducing five-quark component. Besides nonstrange quarks, the suggested five-quark component in a \( N(1535) \) carries a strange-quark pair while in a \( \Lambda(1405) \) the five-quark component carries only one strange quark, which naturally leads to a conclusion in such picture that the \( N(1535) \) is heavier than the \( \Lambda(1405) \). A more popular interpretation of the \( \Lambda(1405) \) is dynamically generated state with a two-pole structure within chiral unitary approach which combines the low energy interaction governed by chiral symmetry and the unitarity condition for the coupled-channel scattering amplitude. Another theory describes the state as a quasibound state of \( N\bar{K} \) embedded in a \( \Sigma\pi \) continuum with a one-pole structure. A recent lattice QCD investigation also supports that the structure of the \( \Lambda(1405) \) is dominated by a molecular bound state of an anti-kaon and a nucleon.

In the chiral unitary approach, an on-shell factorization is adopted. In such approach, in potential both baryon and meson are put on shell, which makes it possible to factorize the potential out of the integral. It leaves a loop function where a sum over all intermediate (off-shell) states is performed. After such factorization the Bethe-Salpeter equation (BSE) becomes an algebra equation instead of an integral equation. In this work, we follow another way to solve the Beth-Salpeter equation. In the literature, a BSE approach with a spectator quasipotential approximation, which is fully relativistic, has been developed to study the deuteron and extended to study the hadronic molecular state recently, such as the \( Y(4274) \), the \( \Sigma_{c}(3250) \) and the \( N(1875) \) in Ref. 22. In this work, we will study octet meson and baryon interaction with strangeness \( S = -1 \) and the relevant \( \Lambda(1405) \) in the BSE approach with the quasipotential approximation.

The paper is organized as follows. In next section, the potential is derived with the help of the chiral Lagrangian. In section 3 we develop a theoretical frame to study the octet meson and baryon interaction by solving the coupled-channel BSE with the quasipotential approximation. The numerical results are given in Section 4. In the last section, a summary is given.

2. The chiral Lagrangian and potential

The potential from the chiral Lagrangian has been well discussed in the literature. At the lowest order in the chiral expansion, besides the Weinberg-
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Tomozawa term there are also the $s$ and $u$-channel diagrams involving the coupling of the meson-baryon channel to an intermediate baryon state. However, the contribution of these terms is very moderate. Although they were shown to be helpful in reproducing more physical values of the subtracting constants in some cases, they do not influence significantly the quality of the fits. We will neglect these terms in the present study. And from a recent study the next-to-leading order is only sensitive to the $\bar{K}N \rightarrow K\Xi$ reaction, which is not focused on in this work. Hence, only the Weinberg-Tomozawa term will be included in the current work.

The lowest order chiral Lagrangian, coupling the octet of pseudoscalar mesons to the octet of $1/2^+$ baryons, is written as

$$L^{(B)}_1 = \frac{1}{4f^2} \langle \bar{B}i\gamma^\mu [\mathbb{P}\partial_\mu \mathbb{P} - \partial_\mu \mathbb{P}]B \rangle,$$

where $B$ and $\mathbb{P}$ stand for baryon and pseudoscalar meson fields, and the constant $f$ is the meson decay constant. The potential is easily evaluated and given by

$$iV_{\lambda\lambda'}^{kl} = -C_{kl} \frac{1}{4f^2} \bar{u}(p')\gamma^\mu u(p)(k_\mu + k'_\mu),$$

where $u(p')$ and $\bar{u}(p)$ are the Dirac spinors and $k^{(\ell)}$ and $p^{(\ell)}$ are the momenta of the outgoing (incoming) meson and baryon. The coefficient $C_{kl}$ for channel $l$ to channel $k$ is fixed by chiral symmetry, and the explicit value was given in Ref. 11.

In this work the $K^-p \rightarrow MB$ reactions will be investigated and ten interaction channels $K^-p$, $\bar{K}^0n$, $\Sigma^+\pi^-$, $\Sigma^-\pi^+$, $\Sigma^0\pi^0$, $\Lambda\eta$, $\Lambda\pi^0$, $\Sigma^0\eta$, $K^+\Xi^-$ and $K^-\Xi^+$ will be included in a coupled-channel calculation. The $\Lambda(1405)$ is an isoscalar state, so the isospin zero state will be constructed as in Ref. 11 to study the relation between the interactions and the $\Lambda(1405)$, in which four interaction channels $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$ and $K\Xi$ are included. The corresponding coefficient $D_{kl}$ for a channel with fixed isospin was also given in Ref. 11.

Since the potential kernel will be used in the BSE, both initial and final particles are put off shell firstly. With such requirement the potential can be expressed explicitly as

$$iV_{\lambda\lambda'}^{kl} = -C_{kl} \frac{1}{4f^2} \bar{u}(p')\gamma^\mu u(p)(k_\mu + k'_\mu)$$

$$= \sqrt{p^2 + \tilde{M}} \sqrt{p'^2 + \tilde{M}'} \phi_\lambda \left\{ k^0 + k'^0 + [p^2(p'^0 + \tilde{M}') + p'^2(p^0 + \tilde{M})] \frac{1}{(p'^0 + \tilde{M}')(p^0 + \tilde{M})} \right\} \phi_{\lambda'},$$

where the $W$, $M^{(\ell)}$ and $p^{(\ell)}$ are total energy of system and the mass and momentum of the outgoing (incoming) baryons in the center of mass frame. The $\tilde{M} = \sqrt{p^2}$ with $p^2$ being the square of four momentum of the baryon. The $\phi_\lambda$ and $\phi_{\lambda'}$ for the
initial and final baryons with helicities $\lambda$ and $\lambda'$ are defined as
\begin{align}
\phi^+_{1/2} &= (\cos(\theta/2), \sin(\theta/2)e^{i\varphi}), \\
\phi^-_{-1/2} &= (-\sin(\theta/2)e^{-i\varphi}, \cos(\theta/2)),
\end{align}
where $\theta$ and $\varphi$ are angles of the baryon momentum.

To reach the popular form of potential as given in Ref. 13 where on-shell factorization was adopted, we do not need to adopt the onshellness of both baryon and meson in the potential. Here only the baryon is put on-shell, which means $\tilde{M} = M$ and $E = \sqrt{M^2 + p^2}$. The potential is rewritten as
\begin{align}
\mathcal{V}_{kl}^{\lambda\lambda'} &= -\frac{C_{kl}}{4f^2} \sqrt{\frac{E + M}{2M}} \sqrt{\frac{E' + M'}{2M}} \\
&\quad \cdot \phi_{\lambda}^\dagger \left[2W - M - M' + \frac{(2W + M + M')(p \cdot p' + i \sigma \cdot p \times p')}{(E + M)(E' + M')} \right] \phi_{\lambda'}.
\end{align}
The $\phi_\lambda$ used here is dependent on the angles $\theta$ and $\varphi$. Without loss of generality, the momenta can be chosen as $p' = (E', 0, 0, p)$ and $p = (E, p \sin \theta, 0, p \cos \theta)$ with $p = |p|$ in order to avoid confusion with the four-momentum $p$.

The potential can be rewritten further as
\begin{align}
\mathcal{V}_{kl}^{\lambda\lambda'} &= \frac{C_{kl}}{4f^2} \sqrt{\frac{E + M}{2M}} \sqrt{\frac{E' + M'}{2M}} \\
&\quad \cdot \left[2W - M - M' + \frac{(2W + M + M')pp'}{(E + M)(E' + M')} \right] d_{\lambda\lambda'}^{1/2}(\theta).
\end{align}
We will see soon that the potential contributes only to $J = 1/2$ partial waves because it is proportional to a Wigner $d$-matrix $d_{\lambda\lambda'}^{1/2}(\theta)$.

3. The coupled-channel BSE for scattering amplitude

In Ref. 21 the BSE of the vertex function are adopted to study the bound state. In this work, we deal with the BSE for the scattering amplitude $\mathcal{M}$ directly, which is written as
\begin{align}
\mathcal{M} &= \mathcal{V} + \mathcal{V}G\mathcal{M},
\end{align}
where the $\mathcal{V}$ is the potential kernel and the $G$ is the propagator for two constituent particles.

To avoid the difficulties in the numerical solution, the four-dimensional BSE is usually reduced to a three-dimensional equation. There exist many methods to make such three-dimensional reduction, which include the $K$ matrix approximation, the Blankenbecler-Sugar approximation and the covariant spectator theory. In this work, we adopt the covariant spectator theory.

With the help of onshellness of the heavier constituent 2, baryon, the propagator can be
\begin{align}
G(p, P) = -\frac{\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p)}{(p^2 - M^2)((P - p)^2 - m^2)} = \tilde{G}(p) \sum_\lambda u_\lambda(p)\bar{u}_\lambda(p),
\end{align}
where $u_{\lambda}(p)$ is the spinor with helicity $\lambda$, and $M$ and $m$ are the masses of baryon and meson. After spinor is multiplied on both sides, the BSE results

$$\mathcal{M}_{\lambda,\lambda'}^{kl} = \mathcal{V}_{\lambda,\lambda'}^{kl} + \sum_{n,\lambda''} V_{\lambda,\lambda''}^{kn} G_{0}^{n} \mathcal{M}_{\lambda'',\lambda'}^{nl},$$

(9)

where $k, l$ or $n$ is for different channel $MB = K^- p, \bar{K}^0 n, \pi^- \Sigma^+, \pi^0 \Sigma^0, \pi^+ \Sigma^-$ or $\eta \Lambda$ in this work and will be omitted if not necessary. The remnant of propagator $\tilde{G}(p)$ is written down in the center of mass frame where $P = (W, 0)$ is

$$\begin{align*}
\tilde{G}(p) &= 2\pi i \frac{\delta^{+}(p^2 - M^2)}{(P - p)^2 - m^2} = 2\pi i \frac{\delta^{+}(p^0 - E(p))}{2E(p)[(W - E(p))^2 - \omega(p)]} \\
&= 2\pi i \delta^{+}(p^0 - E(p)) G_{0}(p),
\end{align*}$$

(10)

To reduce the BSE to one-dimensional equation, we apply the partial wave expansion as

$$\begin{align*}
\mathcal{V}_{\lambda,\lambda'}(p, p') &= \sum_{J, R} \frac{\sqrt{2J + 1}}{4\pi} D_{\lambda R, \lambda'}(\varphi, \theta, 0) \mathcal{V}_{\lambda,\lambda'}^{J}(p, p') D_{\lambda' R, \lambda}(\varphi', \theta', 0), \\
\mathcal{V}_{\lambda,\lambda'}^{J}(p, p') &= 2\pi \int d\cos \theta d\lambda_{\lambda'}^{J}(\theta) \mathcal{V}_{\lambda,\lambda'}(p, p'),
\end{align*}$$

(11, 12)

where the momenta are chosen as $p' = (E', 0, 0, p)$, and $p = (E, p \sin \theta, 0, p \cos \theta)$. Combined with Eq. (6), one can find that the Weinberg-Tomozawa potential derived from the lowest order chiral Lagrangian only provides the contributions for partial waves with spin $J = 1/2$.

Now we have the partial wave BSE as,

$$\mathcal{M}_{\lambda,\lambda'}^{J}(p, p') = \mathcal{V}_{\lambda,\lambda'}^{J}(p, p') + \sum_{\lambda''} \int \frac{p''^2 dp''}{(2\pi)^3} \mathcal{V}_{\lambda'',\lambda'}^{J}(p, p'') G_{0}(p'') \mathcal{M}_{\lambda'',\lambda'}^{J}(p'', p').$$

(13)

The partial wave BSE with fixed parity is of a form,

$$\mathcal{M}_{\lambda,\lambda'}^{J \tilde{P}} = \mathcal{V}_{\lambda,\lambda'}^{J \tilde{P}} + \sum_{\lambda'' > 0} \mathcal{V}_{\lambda'',\lambda}^{J \tilde{P}} G_{0}^{\lambda''} \mathcal{M}_{\lambda'' \lambda}^{J \tilde{P}},$$

(14)

where the amplitude with fixed spin parity is defined as

$$\mathcal{M}_{\lambda,\lambda'}^{J \tilde{P}} = \mathcal{M}_{\lambda,\lambda'}^{J} + \eta \mathcal{M}_{\lambda,\lambda'}^{J},$$

(15)

where $\eta = PP_{1}P_{2}(-1)^{J_1 - J_1 - J_2}$ with $P$ and $P_{1,2}$ being the parities and $J$ and $J_{1,2}$ being the angular momenta for the system and particle 1 or 2. Here the sum extends only over positive $\lambda''$. The potential with fixed parity is

$$\mathcal{V}_{\lambda,\lambda'}^{J \tilde{P}} = \mathcal{V}_{\lambda,\lambda'}^{J} + \eta \mathcal{V}_{\lambda,\lambda'}^{J},$$

(16)

which is analogous to the scattering amplitude $\mathcal{M}_{\lambda,\lambda'}^{J \tilde{P}}$.
Before solving the one-dimensional partial wave BSE numerically, we need to deal with the pole in $G_0(p)$,

$$iM(p,p') = i\mathcal{V}(p,p') + \int_0^{p_{\text{max}}'} \frac{d^3p''}{(2\pi)^3} i\mathcal{V}(p,p'')G_0(p'')iM(p'',p')$$

$$- i\mathcal{V}(p,p')\left[\int_0^{p_{\text{max}}'} \frac{d^3p''}{(2\pi)^3} \frac{A(p'')}{p''^2 - p'^2} + i\frac{p''^2 \delta G_0(p'')}{8\pi^2}\right] \cdot iM(p'',p')\theta(s-m_1-m_2)\theta(p_{\text{max}}' - p''),$$  \hspace{1cm} (17)

where we use the denotations

$$A(p'') \equiv [p''^2(p''^2 - p_0'^2)G_0(p'')]_{p''\to p_0''} = -\frac{p_0''^2}{2W}$$

$$\delta G_0(p'')\delta(p'' - p_0'') \equiv \delta(G_0(p'')) = \frac{1}{4Wp_0''}\delta(p'' - p_0''),$$  \hspace{1cm} (18)

with $p_0'' = \frac{1}{2W}\sqrt{(W^2 - (M + m)^2)[W^2 - (M - m)^2]}$. Here a cut off of $p''$, $p_{\text{max}}''$, has been considered. The sum of second term and the first part of third term on the right side of Eq. (17) is the principle value which is calculated with the help of the relation

$$\mathcal{P} \int_0^\infty dp'' \frac{f(p)}{p''^2 - p_0''^2} = \int_0^\infty dp'' \frac{f(p) - f(p_0'')}{p''^2 - p_0''^2}$$  \hspace{1cm} (19)

The $\theta$ functions mean that the pole only appears if $s > m_1 + m_2$ and $p_{\text{max}}'' > p_0''$.

It is easy to see that the above equation can be related to the formalism used by Oset et al. if the potential kernel $\mathcal{V}$ is only dependent on $s$ and $G_0$ is chosen as the same one used in Ref. [13]. In the above derivation the cutoff regularization is adopted as in Ref. [14]. In this work we will adopt an exponential regularization by introducing a form factor in the propagator as

$$G_0(p) \to G_0(p) \left[e^{-(k^2 - m^2)^2/4\Lambda^2}\right]^2,$$  \hspace{1cm} (20)

and let $p_{\text{max}}'' \to \infty$. Here the baryon is not involved in the form factor due to its onshellness. The cut off $\Lambda$ plays an analogous role as the cut off $p_{\text{max}}''$.

In this work, the covariant spectator theory instead of the one-shell factorization is adopted, so the integrand of the integration about momentum includes not only the propagator but also the potential. To Solve the integral equation, we discrete the momenta $p$, $p'$ and $p''$ by the Gauss quadrature with weight $w(p_i)$ and have,

$$iM_{ik} = i\mathcal{V}_{ik} + \sum_{j=0}^N i\mathcal{V}_{ij}G_j iM_{jk},$$  \hspace{1cm} (21)

with the discretized propagator

$$G_{j>0} = \frac{w(p_j')p_{j'}^2}{(2\pi)^3}G_0(p_j'),$$

$$G_{j=0} = -\frac{ip_0''}{32\pi^2W} + \sum_j \left[\frac{w(p_j)}{(2\pi)^3}\frac{p_0''^2}{2W(p_j'^2 - p_0'^2)}\right].$$  \hspace{1cm} (22)
In numerical solution, $N$ should be large enough to produce stable result. In the current work $N = 50$ is chosen.

In the calculation of the cross section of certain reaction, the initial and final particles should be on-shell. The scattering amplitude is

$$\hat{M} = M_{00} = \sum_j [(1 - VG)^{-1}]_{0j} V_{j0}. \quad (23)$$

The total cross section can be written as

$$\sigma = \frac{1}{16\pi s} \frac{|p'|}{|p|} \sum_{J,\lambda,\lambda' \geq 0} \frac{2J + 1}{2} \left| \frac{\hat{M}_{J\lambda\lambda'}}{4\pi} \right|^2. \quad (24)$$

Note that the second sum extends only over positive $\lambda$ and $\lambda'$. Since there is no interference between the contributions from different partial waves, the total cross section can also be divided into partial-wave cross sections, allowing a direct access to the importance of the individual partial waves.

4. Numerical results

In the current model there are two parameters to be determined, the chiral constant $f$ and the cut off $\Lambda$ in the exponential form factor. In this work, we will not make a fit of the experimental data, so the $f$ for all channels is chosen as the value used in Ref. $f = 1.15f_\pi$ with $f_\pi = 93$ MeV. As usual the cut off for channel $l$ can be written as $\Lambda_l = m_l + \alpha \Lambda_{QCD}$ with meson mass $m_l$, $\Lambda_{QCD} = 0.22$ GeV and a common $\alpha$ which is used for all channels.

4.1. The cross section for $K^-p \rightarrow MB$ reactions

To determine the only free parameter $\alpha$, the total cross section for the $k^-p \rightarrow MB$ reactions will be considered. Since there is only one free parameter, we do not fit the total cross section directly but find best value of $\alpha$ to reproduce the following threshold ratio,

$$\gamma = \frac{\Gamma_{K^-p\rightarrow\pi^+\Sigma^-}}{\Gamma_{K^-p\rightarrow\pi^-\Sigma^+}} = 2.36 \pm 0.04,$$

$$R_c = \frac{\Gamma_{K^-p\rightarrow\pi^0\Sigma^\mp}}{\Gamma_{K^-p\rightarrow inelastic}} = 0.664 \pm 0.011,$$

$$R_n = \frac{\Gamma_{K^-p\rightarrow\pi^0\Lambda}}{\Gamma_{K^-p\rightarrow neutral}} = 0.189 \pm 0.011. \quad (25)$$

The best value of $\alpha$ is found at 1.63, and the results of the ratios are listed in Table 1. The scattering length for $K^-p \rightarrow K^-p$ is also listed in the table. The results are comparable with the experimental values except that the real part of the scattering length $a_{K^-p}$ is larger than the experimental value as in Ref. $11$. 

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Table 1. The threshold ratios and the scattering length.

|                | $\gamma$ | $R_c$ | $R_n$ | $\alpha_{K^-p}$     |
|----------------|---------|-------|-------|---------------------|
| This work      | 2.36    | 0.619 | 0.221 | -1.10 + 0.78 i      |
| Ref. [11]      | 2.33    | 0.640 | 0.217 | -0.99 + 0.97 i      |
| exp. [33, 34]  | 2.36    | 0.664 | 0.189 | -0.66 + 0.81 i      |
| ±0.04          | ±0.011  | ±0.011| ±0.07 | (±0.15) i           |

Fig. 1. Total cross section for $K^- p \rightarrow MB$. The solid and dashed lines are for the results in this work and the results of Oset et al. [11]. The experimental data are from experiments [33-40].

With determined parameters, the total cross section for $K^- p \rightarrow MB$ reactions with $MB = K^- p$, $K^0 n$, $\pi^0 \Lambda$, $\pi^0 \Sigma^0$, $\pi^- \Sigma^+$ and $\pi^+ \Sigma^-$ is calculated and shown in Fig. 1.

The experimental data are well reproduced with the parameters determined by the threshold ratios and close to the results from the off-shell factorization [11]. The cross section is dominated by the contribution from the partial wave $J^P = 1/2^-$ and the contribution from the partial wave $J^P = 1/2^+$ is negligible and not presented explicitly in the figure. It suggests that the $P$-wave contribution is negligible compared with the $S$-wave contribution.
4.2. The poles of the scattering amplitude and the Λ(1405)

Now that the model is fixed by the experimental data of the cross sections, we will search the poles of the scattering amplitude, which are related to the Λ(1405), by extrapolation from the real axis to a complex plane. The pole can be searched by variation of $z$ to satisfy

$$|1 - V(z)G(z)| = 0,$$

(26)

where $z = E_R + i\Gamma/2$ equals to the meson-baryon energy $W$ at the real axis. Since $E = \sqrt{m^2 + p^2 + M^2 + p^2}$, the p-plane corresponds to two Reimann sheets for $E$. The bound state is located in the first Reimann sheet while the resonances located in the second Reimann sheet with $\text{Im}(p)<0$.

In Fig. 2 we present the results for the partial wave $J^P = 1/2^-$. Two poles at $1383 + 99i$ and $1423 + 14i$ MeV are produced from four coupled channels $\bar{K}N$, $\pi\Sigma, \eta\Lambda$ and $K\Xi$. 

![Fig. 2. The $|1 - G(z)V(z)|$ for partial wave $J^P = 1/2^-$ in the complex energy plane and the $\pi\Sigma$ mass spectrum. The solid, dashed lines are for the result in this work and result by Oset and Ramos. The dashed line is covered by the solid line at some energies. The solid, histogram, dashed histograms and filled dot are for processes $K^- p \rightarrow \pi^+\pi^-\Sigma^\pm\pi^\mp$ and $\pi^- p \rightarrow K^0(\Sigma\pi)$ and $pp \rightarrow pK^+(\Lambda(1405) \rightarrow \pi^0(\Sigma^0 \rightarrow \gamma(\Lambda \rightarrow p\pi^-)))$. The theoretical results are normalized to the experiment with solid histogram.](image)
The invariant mass spectrum of the $\Sigma\pi$ channel is also presented and compared with the experimental results. The invariant mass distribution is given approximately as

$$\frac{d\sigma}{dW} = C|\hat{M}^J|^2\lambda_\pi^\frac{1}{2}(W^2, M^2, m^2)\lambda_\pi^\frac{1}{2}(\hat{W}^2, W^2, m_3^2)/W,$$

where $\hat{W}$ is the total energy of the process and $m_3$ is the third final particle in Ref. 41, and $M^J$ is the scattering amplitude for $\Sigma\pi \rightarrow \Sigma\pi$. The theoretical result is close to the experimental data and the peak for the $\Lambda(1405)$ is well reproduced. The results are similar to these of Oset and Ramos.11

Since the $\Lambda(1405)$ lies between the $\bar{K}N$ and $\pi\Sigma$ thresholds, it is natural to expect that the two poles originate in the attractive interactions of these channels. This picture can be verified by switching off other channels. In Fig. 3 we present the pole positions of the scattering amplitude in the complex energy plane with $\bar{K}N$ and $\pi\Sigma$ channels, respectively. The $\bar{K}N$ channel supports a bound state while a resonance is generated in the $\pi\Sigma$ channel as shown in Fig. 3.

![Fig. 3. The $|1-G(z)V(z)|$ for partial wave $J^P = 1/2^-$ in the complex energy plane with channel $\bar{K}N$ and $\pi\Sigma$, respectively.](image)

The scattering amplitude with $J^P = 1/2^-$ is a S-wave contribution. The potential also provides a P-wave contribution with $J^P = 1/2^+$ which is proportional to
The octet meson and octet baryon interaction with strangeness and its relation to the \( \Lambda(1405) \) are studied in a Bethe-Salpeter equation approach with quasipotential approximation. In this work the covariant spectator theory instead of the on-shell factorization are adopted and the integral equation about the helicity amplitude is solved after partial wave expansion. The total cross section for \( K^- p \rightarrow MB \) reactions with \( MB = K^- p, K^0 n, \pi^- \Sigma^+, \Sigma^0, \pi^+ \Sigma^- \) and \( \eta \Lambda \) are calculated with all possible partial waves and compared with the experimental data. Two poles at 1383 + 199i and 1423 + 14i MeV are produced, which originate from the \( \pi \Sigma \).
interaction as a resonance and the $K\bar{N}$ interaction as a bound state, respectively. In this work, only the Weinberg-Tomozawa term is considered in the calculation. The experimental data can be reproduced generally and close to the results in Ref. [11]. However, there still exist considerable discrepancies between the current model and experiment. To give a better description of the experimental data, a more comprehensive investigation, such as explicitly fitting and including $s$ and $u$ channels and NLO terms, in our approach is needed.

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