Development and research of the rangefinder of the information and measurement system of air traffic control based on data from on-board sensors of the aircraft

A P Pudovkin¹, Yu N Panasyuk¹, M P Belyaev², S N Danilov¹, S P Moskvitin¹, L G Varepo³, I V Nagornova⁴

¹Tambov State Technical University, 106, Sovetskaya Str., Tambov, 392000, Russia
²Military Training and Scientific Center of the Air Force the Zhukovsky - Gagarin Air Force Academy, 54A, St.Bolshievik str., Voronezh, 394064, Russia
³Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia
⁴Moscow Polytechnic University, 38, B. Semenovskya str., Moscow, 107023, Russia

E-mail: appudovkin.tstu@mail.ru, larisavarepo@yandex.ru

Abstract The question of improving the standard deviation of the phase coordinate error, stability, and noise immunity of the rangefinder for information and measurement system of the automated air traffic control of aircraft is considered. Rangefinder state and observation models which consider kinematic and dynamic characteristics of air crafts have been proposed. A rangefinder algorithm has been developed and investigated through simulation modelling on a computer with research results obtained.

1. Introduction

The successful resolution of the challenges facing aviation and the further improvement of the efficiency of its application in the national economy and for other purposes is now unthinkable without the widespread introduction of air traffic control (ATC) information and measurement systems (IMS) for various purposes. The growth in the number of air crafts and their use has increased the air traffic load. This creates the possibility of dangerous aircraft convergence in flight operations. The problem of accuracy, stability, and interference immunity of ATC IMS depends on the aircraft trajectory in the airfield area. The trajectory of the aircraft depends on their manoeuvres (banking, turnaround, zoom climb, etc.). For such airborne manoeuvres, existing ATC IMSs using Kalman rangefinder tracking filters drastically increase range estimation errors, which can lead to rangefinder filter divergence [1]. The increase in range estimation error during aircraft manoeuvring results from the fact that manoeuvring aircraft changes its dynamic and kinematic parameters, having complex time dependencies. The current state and observation models of the ATC IMS rangefinder do not use information about the dynamic and kinematic parameters of the aircraft. Current state and observation models of the ATC IMS rangefinder use constant speed or constant acceleration hypotheses for aircraft. Range estimation errors affect aircraft traffic characteristics, in particular, air traffic capacity and safety [1]. To improve the performance of aircraft movement, the preferred option is to develop an ATC IMS rangefinder to estimate the aircraft range using information from its dynamic and kinematic characteristics. Information about the dynamic and kinematic characteristics of the aircraft can be obtained through an aircraft telecommunication system (discrete-address data communication system, where an individual-address request is used) [1]. The discrete-address data communication system
enables the transmission of aircraft flight parameters on its dynamic and kinematic characteristics. Information on the dynamic and kinematic characteristics of the aircraft will allow a more accurate selection of the rangefinder system model tailored to aircraft manoeuvres, compared to existing system models.

2. Selection and justification of rangefinder models

The development of a rangefinder model to derive its algorithms for tracking aircraft trajectories using a radar station (radar) involves the description of processes and systems in space of state [1]. The evolution of state vector $X$, in the development of trajectory tracking algorithms in radar, is represented by a model of aircraft motion. A model of aircraft movement is defined as mathematical dependencies approximating the evolution of phase coordinates and characterizing the location and movement of the aircraft centre of mass in the polar coordinate system with the origin of coordinates in the phase centre of the radar antenna, with a phased array supposed to be used as such. The trajectory state vector must provide the estimation of all phase coordinates necessary to solve the problem of aircraft control in flight, as well as their stable high-precision tracking: range $R$, velocity $V$ and closure acceleration $a$, as well as angular coordinates by azimuth $\phi_a$ and elevation angle $\phi_h$, angular velocity of movement by azimuth $\omega_a$ and elevation angle $\omega_h$ of the sight lines, and angular accelerations by azimuth $\alpha_a$ and elevation angle $\alpha_h$. The state models describing the evolutions of the phase coordinates of the relative aircraft motion, as well as the evolution of the initial model parameters are continuous functions of time. At the same time, radar measurement models are discrete. The discreteness of the information arrival in automatic trajectory tracking stems from the principle of space surveillance itself, the principle of obtaining primary measurements in a pulse radar. It should be noted that the movement of the aircraft is driven by the thrust of the engines, which varies randomly during flight, and is thus a random process. During the flight, the aircraft is affected by lift, gravity, drag and wind. The direction of air movement in flight can vary randomly due to atmospheric turbulence. The trajectory of the aircraft also depends on the aircraft's manoeuvre. For the banking manoeuvre, Fig. 1 shows the time dependence of the aircraft range along the line of sight.

![Figure 1. The time dependence of the range of the aircraft along the line of sight for the banking manoeuvre](image)

The following state models are used in existing rangefinders:

- rangefinder state model, which uses the hypothesis of aircraft moving at constant speed:
  \[
  \begin{align*}
  R(k) &= R(k-1) + V(k-1)T; \\
  V(k) &= V(k-1),
  \end{align*}
  \]

- rangefinder state model, which uses the hypothesis of aircraft moving with constant acceleration:
\[ R(k) = R(k-1) + V(k-1)T + 0.5a(k-1)T^2, \]
\[ V(k) = V(k-1) + a(k-1)T; \]
\[ a(k) = a(k-1), \]
\( \) (2)

- Range finder state model, where acceleration through a stationary process is applied to all possible aircraft trajectories:
\[ R(k) = R(k-1) + V(k-1)T + 0.5a(k-1)T^2; \]
\[ V(k) = V(k-1) + a(k-1)T; \]
\[ a(k) = (1 - \alpha T)a(k-1) + \xi_a(k-1). \]
\( \) (3)

The observation (measurement) equation for existing range finders is
\[ R_i(k) = R(k) + \xi_R(k). \]
(4)

In expressions (1) – (4), \( T \) is the sampling interval; \( k \) is the time-discrete number; \( \alpha \) is the manoeuvre time constant, which is chosen for all possible aircraft trajectories; \( \xi_a(k-1) \) is the Gaussian atmospheric turbulence noise, which is known a priori and has a noise disturbance dispersion \( P_a \); \( \xi_a(k-1) \) is the Gaussian range measurement noise with known dispersion \( P_R \).

In the range finder model, equation of state (1) describes aircraft movement with constant speed, while in observation equation (4), only the range is measured; therefore, range estimate for banking manoeuvre (Fig. 1) by Kalman filter (developed based on range finder models (1) and (4)) will have large range estimation errors throughout the aircraft flight (\( k = 0...600 \)), especially at \( k = 250...320 \) and \( k = 450...550 \), where the range derivatives \( R \) change sharply.

In the range finder model, equation of state (2) describes the aircraft movement with constant acceleration, while observation equation (4) measures the range only; therefore, the estimation of range change for the banking manoeuvre (Fig. 1) by the Kalman filter (developed based on range finder models (2) and (4)) will have small range estimate errors at \( k = 0...250 \), \( k = 320...450 \) and \( 450...600 \) (range derivatives \( R \) change slightly), and at \( k = 250...320 \) and \( k = 450...550 \) there will be large range estimation errors because range derivatives change significantly at these segments.

In the range finder model, equation of state (3) describes the aircraft motion with acceleration (probabilities of all possible trajectories are included), while in observation equation (4), only the range is measured; therefore, the range change is estimated for the banking manoeuvre (Fig. 1), then the generated Kalman filter is optimal for a set of trajectories and non-optimal for a single trajectory (developed using range finder models (2) and (4)). At \( k = 250...320 \) and \( k = 450...550 \) (Fig. 1), there will be significant errors where acceleration changes according to a non-linear law. Studies have shown that acceptable range, speed and acceleration estimates for this model are available if the aircraft travels at a constant speed or a uniform acceleration. When aircraft manoeuvres, the standard deviation of the range estimation error for model (3) and (4) increases by 2...3 times, which is unacceptable in ATC IMS for ensuring the given capacity and safety of air traffic. An aircraft manoeuvre for a radar station is a non-stationary process, as the trajectory consists of segments where the acceleration varies depending on the type and stage of the manoeuvre. The Singer model is statistical for different types of aircraft trajectories, so it is difficult to describe the law of changes in acceleration for a given trajectory precisely enough. To improve ATC IMS for a given air traffic capacity and safety, a state model must describe the change in acceleration during each manoeuvre with sufficient precision, or for the observation equation to measure the current aircraft acceleration with high accuracy.

The acceleration of the range finder aircraft during manoeuvring depends on kinematic characteristics: trajectory inclination angle \( \theta \), trajectory rotation angle \( \varphi \), speed roll angle \( \gamma \), azimuth \( \varphi_a \), elevation angle \( \varphi_e \), and dynamic characteristics: longitudinal overload \( n_y \), normal overload \( n_x \), lateral overload \( n_z \).
The acceleration of the rangefinder aircraft is obtained as follows. It is known that the acceleration vector of the aircraft is determined by

\[ \ddot{a} = \ddot{n}g \]  

(5)

In expression (5) \( \ddot{n} \) is the aircraft overload vector; \( g \) is the gravitational acceleration.

Formula (5) shows that the change in the acceleration vector is proportional to the change in the overload vector. For the rangefinder, it is necessary to know the acceleration along the line of sight (the direction between the radar and the aircraft). To find the line-of-sight acceleration, the projection of the overload vector onto the line-of-sight axis must be found. To find the overload vector projection on the line-of-sight axes, one must select the coordinate system on the aircraft and the information source of the aircraft’s trajectory, i.e. the radar. The common coordinate system for the aircraft and radar is the Earth’s normal rectangular coordinate system \( OX_gY_gZ_g \) (Figure 2), with axis \( OY_g \) pointing up in a straight line that coincides with the direction of gravitational force \( G \) (the direction of the gravitational force is independent of the space position of the aircraft and always points vertically down), axis \( OX_g \) is perpendicular to axis \( OY_g \) (for the aircraft axis \( OX_g \) points north), axis \( OZ_g \) lies in the horizontal plane and completes the coordinate system to the right. To determine the aircraft flight trajectory parameters, in particular the aircraft acceleration, a trajectory rectangular coordinate system \( OX_tY_tZ_t \) must be selected on the aircraft, which is movable relative to coordinate system \( OX_gY_gZ_g \) (Figure 2). Axis \( OX_t \) coincides with the direction of the aircraft velocity vector (the aircraft velocity vector coincides with the aircraft tangent trajectory). Axis \( OY_t \) is perpendicular to \( OX_t \) and lies in the vertical plane. Axis \( OZ_t \) lies in the horizontal plane and forms the right-hand coordinate system \( OX_tY_tZ_t \).

![Figure 2. Determining the relationship between the normal, trajectory, and radial coordinate systems](image)

The overload vector \( n \) is related to the trajectory rectangular coordinate system \( OX_tY_tZ_t \) via speed roll angle \( \gamma_t \) (Figure 2). Normal overload vector \( n_x \) coincides with axis \( OX_t \), longitudinal overload vector \( n_y \) differs from axis \( OY_t \) by speed roll angle \( \gamma_t \), longitudinal overload vector \( n_z \) differs from axis \( OZ_t \) also by speed roll angle \( \gamma_t \).

Trajectory rectangular coordinate system \( OX_tY_tZ_t \) is related to coordinate system \( OX_gY_gZ_g \) via the inclination angle of the trajectory \( \theta \) and the rotation angle of the trajectory \( \phi \). The relationship between the trajectory and normal rectangular coordinate systems is shown in Table 1.

The radar measures the coordinates and parameters of the aircraft, in particular the range to the aircraft along the line of sight (Figure 2). In this case, radial rectangular coordinate system \( OX_rY_rZ_r \) is used in the radar, which is movable relative to coordinate system \( OX_gY_gZ_g \) (Figure 2).
Table 1. Relationship between normal and trajectory rectangular coordinate systems

| Normal rectangular coordinate system | Trajectory rectangular coordinate system |
|-------------------------------------|------------------------------------------|
| $OX_g$                              | $cos \theta cos \varphi$                  |
| $OY_g$                              | $-sin \theta cos \varphi$                |
| $OZ_g$                              | $sin \varphi$                            |

Axis $OX_r$ coincides with the direction of the line of sight. Axis $OY_r$ is perpendicular to $OX_r$ and lies in the vertical plane. Axis $OZ_r$ lies in the horizontal plane and forms the right-hand coordinate system $OX_r,Y_r,Z_r$. Radial rectangular coordinate system $OX_r,Y_r,Z_r$ is related to coordinate system $OX_g,Y_g,Z_g$ via azimuth angle $\varphi_a$ and elevation angle $\varphi_h$. The relationship between the radial and normal rectangular coordinate systems is shown in Table 2.

Table 2. Relationships between radial and normal rectangular coordinate systems

| Normal rectangular coordinate system | Radial rectangular coordinate system |
|-------------------------------------|-------------------------------------|
| $OX_g$                              | $cos \varphi_a cos \varphi_h$       |
| $OY_g$                              | $sin \varphi_h$                     |
| $OZ_g$                              | $-sin \varphi_h cos \varphi_a$      |

The projection of overload vector $n_r$ on axis $OX_r$ of the radial coordinate system (line of sight) can be determined by the expression based on Figure 1 and Tables 1, 2.

\[
n_r = n_x \sin \varphi_h \sin \theta + n_x \cos \varphi_h \cos \theta \cos (\varphi_a - \varphi) + n_y \sin \varphi_a \sin \gamma_v \cos \varphi_h + \n_y \sin \varphi_h \cos \gamma_v \cos \theta - n_y \sin \theta \cos \varphi_a \sin \gamma_v \cos \varphi_h + \n_z \sin \varphi_h \sin \gamma_v + n_z \sin (\varphi_a - \varphi) cos \varphi_h \sin \gamma_v.
\]

Given expressions (5) and (6), the formula for the acceleration of the aircraft along the line of sight is derived with regard to the kinematic and dynamic characteristics of the aircraft

\[
a_r = g(n_x \sin \varphi_h \sin \theta + n_x \cos \varphi_h \cos \theta \cos (\varphi_a - \varphi) + n_y \sin \varphi_a \sin \gamma_v \cos \varphi_h + \n_y \sin \varphi_h \cos \gamma_v \cos \theta - n_y \sin \theta \cos \varphi_a \sin \gamma_v \cos \varphi_h + \n_z \sin \varphi_h \sin \gamma_v + n_z \sin (\varphi_a - \varphi) \cos \varphi_h \sin \gamma_v).
\]

When developing the rangefinder model, it is proposed to introduce measuring range $R$ and the acceleration of the sight line $a_i$ into observation equation (4), which can be derived indirectly from expression (7). Observation model state equation (3) remains unchanged. In this case, the rangefinder model will take the following form:

- rangefinder state equation:

\[
\begin{align*}
R(k) &= R(k-1) + V(k-1)T + 0.5a(k-1)T^2; \\
V(k) &= V(k-1) + a(k-1)T; \\
a(k) &= (1 - \alpha_T)a(k-1) + \varepsilon_a(k-1).
\end{align*}
\]
rangedefinder observation equation:

\[ R_{s}(k) = R(k) + \xi_{R}(k); \]
\[ a_{i}(k) = g(n_{x}(k)\sin \varphi_{h}(k)\sin \theta(k) + n_{y}(k)\cos \varphi_{h}(k)\cos \theta(k)\cos(\varphi_{a}(k) - \varphi(k)) + + n_{y}(k)\sin \varphi_{a}(k)\sin \gamma_{e}(k)\cos \varphi_{h}(k) + n_{x}(k)\sin \varphi_{h}(k)\cos \gamma_{e}(k)\cos \theta(k) - - n_{y}(k)\sin \theta(k)\cos \varphi_{a}(k)\cos \varphi_{h}(k)\cos \gamma_{e}(k) + + n_{x}(k)\sin(\varphi_{a}(k) - \varphi(k))\cos \varphi_{h}(k)\cos \gamma_{e}(k) + \xi_{a_{i}}(k). \] (9)

In equation (9), \( \xi_{a_{i}} \) is a Gaussian acceleration measurement noise with a known dispersion \( P_{a_{i}} \).

3. The algorithm of rangefinder operation

The development of rangefinder algorithms will be based on system model (8) and (9), which defines the state and observation vectors:

\[ x(k) = \begin{bmatrix} R(k) & V(k) & a(k) \end{bmatrix}^{T}; \]
\[ z_{i}(k) = \begin{bmatrix} R_{s}(k) & a_{i}(k) \end{bmatrix}^{T}. \] (10, 11)

To obtain the rangefinder algorithm, a generalised Kalman filtering algorithm \([2–7]\) will be used:

\[ x_{es}(k) = x_{e}(k) + K_{F}(k)[z_{i}(k) - H(k)x_{e}(k)]; \] (12)
\[ x_{e}(k) = F(k)x_{es}(k-1), \quad x_{e}(0) = x_{es}(0); \] (13)
\[ P_{e}(k) = F(k)\cdot P_{es}(k-1)\cdot F^{T}(k) + P_{e}(k), \quad P_{e}(0) = P_{es}(0); \] (14)
\[ K_{F}(k) = P_{e}(k)\cdot H^{T}(k)\left[H(k)\cdot P_{e}(k)\cdot H^{T}(k) + P_{i}(k)\right]^{-1}; \] (15)
\[ P_{es}(k) = P_{e}(k) - K_{F}(k)\cdot H(k)\cdot P_{i}(k). \] (16)

In equations (12) to (16), \( x_{es}(k) \) is state vector estimate (10); \( x_{e}(k) \) is the extrapolation of the state vector; \( K_{F}(k) \) is Kalman filter gain matrix; \( F(k) \) is transition matrix providing relationships between variables (10); \( P_{e}(k) \) is a posteriori filtering error matrix; \( P_{es}(k) \) is extrapolated (a priori) filtering error matrix; \( P_{i}(k) \) is measurement error matrix; \( P_{s}(k) \) is disturbance error matrix caused by airflow turbulence; \( H(k) \) is measurement matrix providing relationships between variables (10) and (11).

Using rangefinder state model (8), observation model (9), and generalized Kalman filtering algorithm (12)-(16), one obtains an algorithm for rangefinder operation that allows the estimation of range \( R_{es} \), speed \( V_{es} \), and acceleration \( a_{es} \) of the aircraft along the line of sight relative to the radar:

\[ R_{es}(k+1) = R_{e}(k+1) + K_{F_{11}}(k+1)\Delta R(k+1) + K_{F_{12}}(k+1)\Delta a(k+1); \] (17)
\[ V_{es}(k+1) = V_{e}(k+1) + K_{F_{21}}(k+1)\Delta R(k+1) + K_{F_{22}}(k+1)\Delta a(k+1); \] (18)
\[ a_{es}(k+1) = a_{e}(k+1) + K_{F_{31}}(k+1)\Delta R(k+1) + K_{F_{32}}(k+1)\Delta a(k+1); \] (19)
\[ R_{e}(k+1) = R_{es}(k) + V_{es}(k)T + 0.5a_{es}(k)T^{2}; \] (20)
\[ V_{e}(k+1) = V_{es}(k) + a_{es}(k)T; \] (21)
\[ a_{e}(k+1) = (1 - a_{e}T)a_{e}(k)+ \] (22)
\[ \Delta R(k+1) = R_{e}(k+1) - R_{e}(k+1); \] (23)
\[ \Delta a(k+1) = a_{e}(k+1) - a_{e}(k+1). \] (24)
4. Modelling and research of the rangefinder

To model the rangefinder on a computer using the simulation method, it is necessary to know the information about measured range values $R_i(k)$ and acceleration $a_i(k)$, obtained by the indirect measurement method. The measured values are to be processed using algorithm (17) – (24). The simulation of measured range value $R_i(k)$ is the change in the true range of the aircraft along the line of sight (Figure 2) and observation noise $\xi_R(k)$. The simulation of measured acceleration value $a_i(k)$ is the sum of the true line-of-sight acceleration value and observation noise $\xi_a(k)$. Observation noises $\xi_R(k)$ and $\xi_a(k)$ are simulated by a random number generator that changes according to the Gaussian law.

Based on algorithm (17) – (24), the dependencies of the real standard deviations of range, speed and acceleration errors on time along the line of sight of the aircraft during the banking manoeuvre were investigated.

The real standard deviations of the error in range, speed and acceleration of the aircraft were estimated from 100 realisations using the formula

$$\sigma_{x_p}(k) = \sqrt{\frac{\sum_{j=1}^{N} [x(k) - x_{es_j}(k)]^2}{N-1}}. \quad (25)$$

In expression (25), $\sigma_{x_p}(k)$ is the mean square deviation of the error of the aircraft phase coordinates estimation (range, speed and acceleration of the aircraft); $x(k)$ is true values of phase coordinates ($R, V, a$) of aircraft; $x_{es_j}(k)$ are estimated values of phase coordinates: range $R_{es_j}(k+1)$, speed $V_{es_j}(k+1)$ and acceleration $a_{es_j}(k+1)$ of the aircraft in the $j$-th realization; $N$ is the number of realizations.

Simulation modelling and research of rangefinder algorithms were carried out for two cases. In the first case, the rangefinder algorithm based on the Singer model (3) and (4) is investigated. In the second case, the rangefinder algorithm derived in expressions (17) to (24) is investigated.

Figures 3 to 5 present time dependence graphs for the standard deviation of the Kalman filtering error estimate of range $R$, velocity $V$ and acceleration $a$ along the line of sight of the aircraft. Figures 3 to 5 show that the standard deviations of the estimation error of $R$, $V$ and $a$ for the Zinger rangefinder model (3) and (4) are larger (curve 1) compared to the developed rangefinder model (8) and (9), which uses information on the kinematic and dynamic parameters of the aircraft (curve 2).
The application of the developed rangefinder algorithms (17) to (24) will enable better performance of air traffic control tasks and increased aircraft capacity and safety.

5. Simulation results
The analysis of the rangefinder algorithm research results showed that the root mean square deviations in the estimation error of range $R$, velocity $V$ and acceleration $a$ of the aircraft are 2 to 3 times less if the Kalman filter uses dynamic and kinematic characteristics, as compared to the Zinger model filter. The reason for this is that the use of information on the dynamic and kinematic characteristics of the aircraft allows a more accurate description of the rangefinder model.

Thus, the developed algorithm of rangefinder operation with regard to on-board sensor data (kinematic and dynamic aircraft parameters) for range tracking of the aircraft trajectory improves accuracy characteristics (mean square error deviation) of the Kalman range tracking filter, and, as a result, increases the air traffic capacity and safety level of the air traffic.

References
[1] Glistin V N et al 2020 Journal of Physics: Conference Series 1441 012059
[2] Xi Chen, Xiao Wang, Jianhua Xuan 2012 Tracking Multiple Moving Objects Using Unscented Kalman Filtering Techniques International Conference on Engineering and Applied Science (ICEAS 2012) 1802 01235 [cs.CV]
[3] Meng W et al 2013 UKF-Based Iterative Channel Estimation Using Two-Dimensional Block Spread Coding for Uplink Transmission in Multicarrier CDMA Networks IEEE Transactions on Vehicular Technology 62 no 9 pp 4444-4457
[4] Yun S, Choi J, Yoo Y, Yun K, Choi J Y 2018 Action-Driven Visual Object Tracking With Deep Reinforcement Learning IEEE Transactions on Neural Networks and Learning Systems 29(6) pp 2239 –2252
[5] Blackman S 2004 Multiple Hypothesis Tracking for Multiple Target Tracking IEEE Aerospace and Electronic Systems magazine 19(1) pp 5–18 doi: 10.1109/MAES.2004.1263228
[6] Pudovkin A P, Panasyuk Yu N, Danilov S N, Moskvitin S P 2018 Journal of Physics: Conference Series 1015(3) 032111
[7] Pudovkin A P et al 2020 Journal of Physics: Conference Series 1546 012026