Electroweak properties of kaons in a nuclear medium

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Kaon electroweak properties in symmetric nuclear matter are studied in the Nambu–Jona-Lasinio model using the proper-time regularization. The valence quark properties in symmetric nuclear matter are calculated in the quark-meson coupling model, and they are used as inputs for studying the in-medium kaon properties in the NJL model. We evaluate the kaon decay constant, kaon-quark coupling constant, and $K^+$ electromagnetic form factor by two different approaches. Namely, by two different ways of calculating the in-medium constituent quark masses of the light quarks. We predict that, in both approaches, the kaon decay constant and kaon-quark coupling constant decrease as nuclear density increases, while the $K^+$ charge radius increases by 20-25% at normal nuclear density.

I. INTRODUCTION

Kaons play special roles in strong and electroweak interaction phenomena, and in low-energy hadronic reactions associated with quantum chromodynamics (QCD) at low energies [1–14]. They appear as Nambu-Goldstone bosons like pions, but show the flavor SU(3) symmetry breaking by the heavier strange quark mass. Thus, kaons can provide us with useful information on chiral symmetry and its restoration, SU(3) flavor breaking effect as well as the nonperturbative bound state nature of QCD. The internal structure of kaons can be explored by the electromagnetic form factors (EMFFs), which reflect the underlying quark-gluon dynamics [15, 16]. Many attempts have been made to understand the internal structure of kaons using various approaches, e.g., the Dyson-Schwinger equation (DSE), Nambu–Jona-Lasinio (NJL) model, chiral perturbation theory (ChPT), and instanton vacuum-based models [15–29]. Experimentally, $K^-$ meson EMFFs are poorly known, except for the $K^-\to\pi^0\pi^0$ region ($Q^2<0.2$ GeV$^2$) [30], where $Q^2=-q^2$ and $q$ is the four-momentum transfer. Recently, $K^+$ EMFF was extracted from the kaon electroproduction data at JLab up to $Q^2=2.07$ GeV$^2$ [31], but the data uncertainties are still large. We await the new coming data with better precision up to $Q^2\approx5.5$ GeV$^2$.

On the other hand, theoretical studies for $K^-$ meson properties have been made mostly in vacuum, but not in medium. For example, kaon properties and $K^+$ EMFF in symmetric nuclear matter were studied only recently in Ref. [32] based on a light-front constituent quark model. There, the in-medium valence light-quark properties were calculated in the quark-meson coupling (QMC) model, and they were used as inputs to study the in-medium kaon properties. Although this study provides us with some insights on the kaon properties in a nuclear medium, more complete studies are necessary for the following reasons; (i) the constituent quark masses in vacuum are the input parameters in Ref. [32], but it is preferable to calculate them dynamically, (ii) the vacuum in the light-front approach is generally believed to be "trivial", and there is no clearly defined quark chiral condensates in the light-front approach. Thus, the model does not have the dynamical chiral symmetry breaking mechanism nor does the model have direct connection with the emergence of (pseudo-)Goldstone bosons such as kaons, and (iii) the in-medium kaon decay constant as well as the kaon-quark coupling constant are assumed to be density-independent. The purpose of the present work is to improve further the work of Ref. [32]. We extend the approach used for studying the pion properties in symmetric nuclear matter in Ref. [33], and study the in-medium kaon properties using the Nambu–Jona-Lasinio (NJL) model [34, 35], supplemented by the QMC model inputs. The NJL model is a powerful chiral effective quark theory of low-energy QCD. Importantly, the model describes the dynamical chiral symmetry breaking, the origin of the pseudoscalar Goldstone bosons such as pions and kaons. Furthermore, the model satisfies the chiral limit as QCD dictates. Thus, the NJL model, which has several improved aspects as addressed above, is suitable to study the in-medium kaon properties.

The experimental evidences, such as the EMC effect [36, 37] and the observed modifications of bound proton EMFFs at JLab [38], suggest that the internal structure of hadrons would be modified in a nuclear medium. The phenomena of in-medium modifications of hadron properties [39–50] are tightly connected with partial restoration of chiral symmetry [51–54]. The order parameters of chiral symmetry in QCD are the light-quark chiral condensates, and their changes are expected to be one of the most important driving forces for the change of hadron properties in a nuclear medium.

Spontaneous breaking of chiral symmetry generates the nonet of massless pseudoscalar Goldstone bosons. But the explicit breaking of U(1) axial symmetry selectively shifts up the $\eta^0$-singlet mass, leaving the SU(3) flavor octet of pions, $K$-mesons, and $\eta^8$ to be massless. Then the explicit chiral
symmetry breaking by non-vanishing current quark masses leads to the experimentally observed low-lying pseudoscalar meson mass spectrum [55, 56]. Since the chiral symmetry has such a big impact on the low-lying pseudoscalar meson mass spectrum with its explicit breaking, partial restoration of chiral symmetry in a strongly interacting medium is inevitable to study the change of hadron properties in a nuclear medium. Studying of kaon property changes in medium allows us to explore the nature of the bound state with the light and strange quarks in different environments from vacuum.

In the present work, we study the spacelike \( K^+ \)-meson EMFF, kaon leptonic (weak) decay constant, and kaon-quark coupling constant in symmetric nuclear matter using the NJL model. The NJL model has been very successful in studying pseudoscalar meson properties [55–57]. Recently, the model was applied to study the \( K^+ \) EMFF in vacuum [25, 28, 58], kaon valence quark distributions in medium [59], and in-medium \( \pi^+ \) EMFF [33]. In these works the in-medium valence light-quark properties were calculated in the QMC model, and they were used as inputs for studying the in-medium pion and kaon properties in the NJL model. Extending the works of Refs. [28, 33], we study here the electroweak properties of kaons in symmetric nuclear matter using the NJL model.

For studying the in-medium kaon properties microscopically, we need the valence quark properties in a nuclear medium. These are calculated by the QMC model, and used as inputs for studying the in-medium kaons properties. For this purpose, we adopt two approaches in this work. The first approach is that, the in-medium constituent quark masses are calculated by the QMC model [43], which will be denoted by “QMC-based approach”. The second approach is that, the in-medium constituent quark masses are calculated by solving the NJL model gap equations [60] using the in-medium “current quark properties” calculated by the QMC model as inputs. This will be denoted by “NJL-based approach”. Although the two approaches yield different in-medium constituent quark masses of the light quarks, they lead to very similar predictions for the \( K^+ \) EMFF and its charge radius in symmetric nuclear matter.

This paper is organized as follows. In Sec. II, we present the in-medium constituent valence-quark and kaon properties based on the two approaches, the QMC-based and NJL-based approaches. In section III we present how the in-medium \( K^+ \) EMFF in the NJL model is calculated, while the numerical results are given in Sec. IV. We summarize and conclude in section V.

II. IN-MEDIUM KAON PROPERTIES

In this section we discuss the details of the quark and kaon properties in symmetric nuclear matter with both the QMC-based and NJL-based approaches. To study the \( K^+ \) EMFF in the NJL model, we need the in-medium valence quark properties as inputs calculated in the QMC model. There are two ways of adapting the inputs. It is concerned about how the in-medium current quark properties are calculated and used. Namely, we first evaluate the in-medium “current quark” properties in the QMC model and then calculate the in-medium dynamical (constituent) quark masses in the NJL model using the QMC model inputs. This approach is called the NJL-based approach. A similar approach was adopted in the study of the in-medium pion properties in Ref. [33]. The second approach is called the QMC-based approach, uses the constituent quark mass values in vacuum determined in the NJL model, and the values are plugged in the QMC model to calculate the density dependence of the constituent quark masses.

A. In-medium quark properties in the QMC model

We briefly review the in-medium properties of the valence light and strange quarks in the QMC model [61]. The QMC model has been successfully applied for many topics of nuclear and hadronic physics. (See, for example, Refs. [43, 62], and references therein.) It has also been used to study the medium modifications of the nucleon weak and electromagnetic form factors for the neutrino scattering in dense matter [63, 64]. Recently, the model has been further applied for studying the in-medium properties of low-lying strange, charm, and bottom baryons [65].

In the QMC model medium effects arise from self-consistent exchange of the Lorentz-scalar (\( \sigma \)) and Lorentz-vector (\( \omega, \rho \)) meson fields that couple directly only to the light quarks confined in hadrons. The physics behind this is that the light-quark chiral condensates in medium are more sensitively change than those of the strange and heavier quarks. Here, we work with symmetric nuclear matter in its rest frame in the Hartree mean field approximation. (For the Fock term effects in the QMC model, see Ref. [66].)

Effective Lagrangian for symmetric nuclear matter in the QMC model is given by [67, 68]

\[
\mathcal{L}_{\text{QMC}} = \bar{\psi}_N \left[ i \gamma \cdot \partial - M_N'(\sigma) - g_{\omega} \omega^\mu \gamma_\mu \right] \psi_N + \mathcal{L}_\text{meson},
\]

where \( \psi_N, \sigma, \) and \( \omega \) are respectively the nucleon, scalar meson \( \sigma, \) and vector meson \( \omega \) fields. Note that the isospin-dependent \( \rho \)-meson field is absent, since we use the Hartree approximation for symmetric nuclear matter (isospin saturated), and the \( \rho \) mean field vanishes. The effective nucleon mass \( M_N'(\sigma) \) is defined by

\[
M_N'(\sigma) = M_N - g_{\sigma}(\sigma)\sigma.
\]

Here, \( g_{\sigma}(\sigma) \) and \( g_{\omega} \) are the \( \sigma \)-dependent nucleon-\( \sigma \) coupling strength and the nucleon-\( \omega \) coupling constant, respectively. We define \( g_{\sigma}'(\sigma) = g_{\sigma}(\sigma = 0) \) for later convenience. The mesonic Lagrangian density \( \mathcal{L}_\text{meson} \) in Eq. (1) is given by

\[
\mathcal{L}_\text{meson} = \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{2} \partial_\mu \omega_\nu \left( \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \right)
\]

\[+ \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu. \quad (3)
\]

In the Hartree approximation for symmetric nuclear matter, the nucleon Fermi momentum \( k_F \) is related with the baryon
(\rho_B) and scalar (\rho_s) densities respectively,
\[
\rho_B = \frac{4}{(2\pi)^3} \int dk \theta (k_F - |k|) = \frac{4k_F^3}{3\pi^2},
\]
\[
\rho_s = \frac{4}{(2\pi)^3} \int dk \theta (k_F - |k|) \frac{M_N^*(\sigma)}{\sqrt{M_N^{*2}(\sigma) + k^2}}.
\]  

In the QMC model [43, 67, 68], nuclear matter is treated as the collection of nonoverlapping nucleon-MIT bags [69]. The Dirac equations for the quarks and antiquarks in a hadron bag are given by
\[
\left[ i\gamma \cdot \partial_x - (m_l - V_{\omega l}^0) \pm \gamma^0 \left( \frac{1}{2} V_{\rho l}^0 + V_{\rho l} \right) \right] \left( \begin{array}{c} \psi_l(x) \\ \psi_{\bar{l}}(x) \end{array} \right) = 0,
\]
\[
\left[ i\gamma \cdot \partial_x - (m_s - V_{\omega s}^0) \pm \gamma^0 \left( \frac{1}{2} V_{\rho s}^0 + V_{\rho s} \right) \right] \left( \begin{array}{c} \psi_s(x) \\ \psi_{\bar{s}}(x) \end{array} \right) = 0,
\]
\[
\left[ i\gamma \cdot \partial_x - m_s \right] \left( \begin{array}{c} \psi_s(x) \\ \psi_{\bar{s}}(x) \end{array} \right) = 0.
\]

where \( l \) stands for “light”, i.e., \( l = u \) or \( d \), and the effective light-quark mass \( m_l^* \) is defined as
\[
m_l^* \equiv m_l - V_{\omega l}^0.
\]

Here we assume SU(2) symmetry, \( m_l = m_u = m_d \) for the valence quark masses throughout this study (thus \( m_l^* = m_d^* \)), and \( m_s \) is the strange valence quark mass in vacuum, and \(-V_{\omega l}^0\) is the Lorentz scalar potential, which couples only to the light quarks. The strange quark is decoupled from the scalar and vector potentials so that its effective mass does not change in a nuclear medium, \( m_s^* = m_s \).

The scalar and vector mean fields in symmetric nuclear matter are defined, respectively, by the mean expectation values,
\[
V_{\omega l}^0 \equiv g_{\omega l}^0 \sigma = g_{\omega l}^0 \sigma, \quad V_{\rho l}^0 \equiv g_{\rho l}^0 \omega = g_{\omega l}^0 \partial_\mu \partial^\mu (\omega^0),
\]
where the light-quark-meson coupling constants, \( g_{\omega l}^0 \) and \( g_{\rho l}^0 \), are defined later through Eq. (7). The eigenenergies in units of \( 1/R_h^* \) with the bag radius of the hadron \( h \), are given by
\[
\left( \begin{array}{c} \epsilon_l \\ \epsilon_{\bar{l}} \end{array} \right) = \Omega_l^* \pm R_h^* \left( \frac{1}{2} V_{\rho l} + V_{\omega l}^0 \right),
\]
\[
\left( \begin{array}{c} \epsilon_s \\ \epsilon_{\bar{s}} \end{array} \right) = \Omega_s^* \pm R_h^* \left( \frac{1}{2} V_{\rho s} + V_{\omega s}^0 \right),
\]
\[
\epsilon_s = \epsilon_{\bar{s}} = \Omega_s^*.
\]

where
\[
\Omega_l^* = \Omega_s^* = \left[ \frac{1}{2} \left( R_h^* m_l^* \right)^2 \right]^{1/2},
\]
\[
m_l^* = m_l - V_{\omega l}^0 = m_l - g_{\omega l}^0 \sigma,
\]
\[
\Omega_l^* = \Omega_s^* = \left[ \frac{1}{2} \left( R_h^* m_s^* \right)^2 \right]^{1/2}.
\]

The effective mass of hadron \( h \) in nuclear medium \( m_h^* \), which will be shown to be Lorentz scalar, is calculated as
\[
m_h^* = \sum_{j=l,s,\bar{s}} \frac{n_j \Omega_j^* - z_h}{R_h^*} + \frac{4}{3} \pi R_h^3 B,
\]  

and the in-medium bag radius \( R_h^* \) is determined by the stability condition for a given baryon density self-consistently,
\[
\frac{dm_h^*}{dR_h^*} = 0.
\]

In Eq. (10) \( z_h \) is assumed to be density independent. It is related with the sum of the center-of-mass and gluon fluctuation corrections [67], which is determined by the hadron mass in vacuum. The bag constant \( B \) is fixed by the inputs for the nucleon in vacuum, namely, \( R_N = 0.8 \) fm with \( M_N = 939 \) MeV, for a chosen value of \( m_l \).

The ground state wave function of a quark in hadron \( h \) satisfies the boundary condition at the bag surface,
\[
j_0(x) = \beta_q j_1(x)
\]

with \( q = l (= u, d) \) or \( s \), and \( j_0 \) and \( j_1 \) are the spherical Bessel functions, and
\[
\beta_q = \sqrt{\Omega_q^* + m_q R_h^*}.
\]

The scalar \( \sigma \) and vector \( \omega \) meson fields at the nucleon level are related as
\[
\omega = g_\omega \rho_B, \quad \frac{\sigma}{m_\omega} = \frac{4g_\omega^N C_N(\sigma)}{(2\pi)^3 m_\sigma^2} \int d^4k \theta(k_F - |k|) \frac{M_N^*(\sigma)}{\sqrt{M_N^{*2}(\sigma) + k^2}},
\]

where \( C_N(\sigma) \) is defined by
\[
C_N(\sigma) = -\frac{1}{8} \frac{g_\omega^N}{m_\sigma} \left( \frac{\partial M_N^*(\sigma)}{\partial \sigma} \right),
\]

with \( C_N(\sigma) = 1 \) for the pointlike nucleon case such as in Quantum Hadrodynamics (QHD) [70, 71]. The \( \sigma \)-dependent coupling \( g_\omega(\sigma) \) or \( C_N(\sigma) \) is the origin of the novel saturation properties in the QMC model, and the quark dynamics is included in the effective nucleon mass \( M_N^*(\sigma) \) via a self-consistent manner as in Eqs. (10) [with \( m_h^* \to M_N^* \)] and (14). By solving the self-consistent equation for the \( \sigma \) mean field Eq. (14), the total energy per nucleon is calculated by
\[
E_{\text{tot}}/A = \frac{4}{\rho_B} \int d^4k \theta(k_F - |k|) \sqrt{M_N^{*2}(\sigma) + k^2}
\]

\[
+ \frac{m_\sigma^2 c^2}{2 \rho_B} + \frac{g_\omega^2 \rho_B}{2 m_\omega^2}.
\]

The coupling constants \( g_\omega^N = g_\omega(\sigma = 0) \) and \( g_\omega = g_\omega^N \) in Eq. (16) (\( g_\omega^N \) is implicit) are determined to reproduce the
binding energy of symmetric nuclear matter 15.7 MeV at the saturation density \(\rho_0 = 0.15 \text{ fm}^{-3}\), and they are, respectively, \(s_0^N = 3g_{\pi N}^l S_N(\sigma = 0)\) and \(g_{\omega N}^l = 3g_{\omega N}^l\). In this study, we will use \(m_l = 16.4\) and 400 MeV, and thus we give the explicit values of the coupling constants for both the quark mass values. Namely, \(g_{\pi N}^l = 5.6251\) [47009] for \(m_l = 16.4\) [400] MeV, where \(S_N(\sigma)\) is defined through \([44, 61]\)

\[
\frac{dM_N^*(\sigma)}{d\sigma} = -3g_{\pi N}^l \int \frac{d^3 y}{V} \bar{\psi}(y) \gamma_\mu \gamma_5 \psi(y) = -3g_{\pi N}^l S_N(\sigma) = -\frac{d}{d\sigma} [g_{\pi N}^l(\sigma)\sigma],
\]

which gives \(S_N(\sigma = 0) = 0.4899\) [0.6950] for \(m_l = 16.4\) [400] MeV and \(R_N = 0.8\) fm with \(\psi_l\) being the lowest mode bag wave function in medium. Note that, the right hand side of Eq. (17) is the quark scalar charge, which is Lorentz scalar, and thus the left-hand-side of Eq. (17) is Lorentz scalar, and thus \(M_N^*(\sigma)\) as well. These relations show that the in-medium quark dynamics is explicitly included in the QMC model. Since the light quarks in any hadrons should generally feel the same scalar and vector mean fields as those in the nucleon (after one chooses a fixed light-quark mass value in vacuum), we can systematically study the hadron properties in medium without introducing any new coupling constants for the \(\sigma\) and \(\omega\) mean fields for different hadrons.

### B. Kaon properties in the NJL model

Based on the SU(3) NJL model in vacuum described in Ref. [28], we explore the in-medium properties of dynamical quark and kaon properties in this section. The medium effect is implemented through the in-medium potentials for the valence quarks generated by the QMC model. The dynamical quark mass in medium, \(M_q^*(\sigma = u, d, s)\), in the NJL model with the proper-time regularization scheme is given by [33]

\[
M_q^* = m_q^0 + \frac{3G_\pi M_q^*}{\pi^2} \int_{4\Lambda_{\text{UV}}^2}^{\infty} \frac{d\tau}{\tau^2} e^{-\tau M_q^0},
\]

where \(M_q^*\) and \(m_q^0\) are the in-medium dynamical and current quark masses, respectively. \(G_\pi\) is the four-fermion coupling constant, which is taken the same as in vacuum, and \(\Lambda_{\text{UV}}\) is the ultraviolet cutoff. For the infrared cutoff \(\Lambda_{IR}\), we take \(\Lambda_{IR} = 1/\Lambda_{\text{IR}}^2 = +\infty\) in nuclear medium. (This change in \(\Lambda_{IR}\) from that in Ref. [28] gives negligible difference in results.) Note that the expression of \(M_q^*\) is different from that in Refs. [72, 73], since the information of “nucleon” Fermi momentum and nuclear matter saturation properties are included in \(m_q^0\) (and \(\gamma_l\)) calculated in the QMC model, and we should drop the density dependent term.

The in-medium dressed quark propagators in Eq. (18) are given by

\[
S_q^*(k^*) = \frac{k^* + M_q^*}{(k^*)^2 - (M_q^*)^2 + i\epsilon},
\]

\[
S_q^*(k^*) = S_q(k) = \frac{k + M_q^*}{k^2 - M_q^2 + i\epsilon},
\]

where the quantities with the asterisk above denote those in medium, and medium effects enter in the light-quark mass \(M_q^*\) and momentum \(k^*\) by \(k^* = k + V\) due to the vector mean field \(V = (V_\mu, 0)\). The modification of the space component of the light-quark momentum \(k^\mu\) is neglected, since it is known very small [66].

In Eq. (18) the vector potential entering in the light-quark propagator was eliminated by the shift of the integral variable [74]. Note that, the in-medium strange quark propagator is the same as that in vacuum, since the strange quark is decoupled from the scalar and vector mean fields in the QMC model, and the same assumption is adopted. Namely, \(M_q^* = M_s\) is assumed also in the present NJL model calculation.

The description of kaon as the dressed quark-antiquark bound state is obtained by solving the Bethe-Salpeter equation (BSE) in the random phase approximation (RPA). The solution to the BSE in each meson channel is given by the two-body scattering \(t\)-matrix, that depends on the interaction channel. We also introduce below the reduced \(t\)-matrix for vector mesons, since it will be used later in Eqs. (33) and (34). The reduced \(t\)-matrices in the \(K\)-meson and vector meson \(V\) take the forms [28],

\[
\tau_K^*(p^*) = \frac{-2iG_\rho}{1 + 2G_\rho \Pi_K^*(p^{*2})},
\]

\[
\tau_V^{\mu\nu}(p^*) = \frac{-2iG_\rho}{1 + 2G_\rho \Pi_V^*(p^{*2})} \left(g^{\mu\nu} + 2G_\rho \Pi_V'(p^{*2}) P^{\mu\nu} p^{\nu}/p^{*2}\right),
\]

where \(G_\rho\) is the four-fermion coupling constant for \(\rho\) channel, and the bubble diagrams in nuclear medium give

\[
\Pi_K^*(p^{*2}) = 6i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}_D \left[ \gamma_5 S_q^*(k^*) \gamma_5 S_q^*(k^* + p^*) \right],
\]

\[
\Pi_V^{qq}(p^{*2}) P^{\mu\nu}_T = 6i \int \frac{d^4 k}{(2\pi)^4} \text{Tr}_D \left[ \gamma_\mu S_q^*(k^*) \gamma_\nu S_q^*(k^* + p^*) \right],
\]

with \(\Pi_V^\mu = \Pi_V^{uu}, \Pi_V^{qd} = \Pi_V^{sd}\). The trace is only for the Dirac indices, and \(P^{\mu\nu}_T = g^{\mu\nu} - p^{\mu} p^{\nu}/p^{*2}\).

The in-medium meson mass is defined by the pole in the corresponding \(t\)-matrix as in the vacuum case,

\[
1 + 2G_\pi \Pi_K^*(p^{*2} = m_K^2) = 0,
\]

where the similar conditions determine other meson masses in
medium. This leads to the in-medium kaon mass,
\[
m_K^2 = \left( \frac{m^*_q}{M^*_q} + \frac{m^*_l}{M^*_l} \right) \frac{1}{G_{\pi N}(m_K^0)} + (M^*_K - M^*_l)^2,
\]
where \( m^*_q = m^*_u = m^*_d \) and \( I_{is}(p^2) \) is defined by,
\[
I_{ab}(p^2) = \frac{3}{\pi^2} \int_0^1 dz \int_0^\infty \frac{d\tau}{\tau} \times e^{-r[-(1-\tau)p^2 + 2pV_0(1-\tau) - (1-\tau)\sigma - (1-\tau)V_0^2 + x_M^2 + (1-\tau)M^2_l]},
\]
(24)
This demonstrates the Goldstone boson nature of the kaon in the chiral limit. The residue at the pole in the \( \bar{q} q \) \( t \)-matrices defines the kaon-quark coupling constant \( g_{KKqq}^{t} \) in medium:
\[
(g_{KKqq}^t)^2 = -\left. \frac{\partial \Pi_{K}^{t}(p^2)}{\partial p^2} \right|_{p^2 = m_K^2}.
\]
(25)

Following Refs. [28, 58, 63], we have chosen \( \Lambda_{IR} = 240 \) MeV, which is of the order of \( \Lambda_{QCD} \), and \( M_l = 400 \) MeV in vacuum. These values were fixed to give \( m_\pi = 140 \) MeV and \( m_K = 495 \) MeV, together with the pion decay constant \( f_\pi = 93 \) MeV. Furthermore, this gives \( \Lambda_{UV} = 645 \) MeV, \( g_\rho = 19.0 \) GeV\(^{-2}\). Fit to the physical masses of the vector mesons, \( m_\rho = 770 \) MeV and \( m_\omega = 782 \) MeV, gives \( G_\rho = 11.0 \) GeV\(^{-2}\), and \( G_\omega = 10.4 \) GeV\(^{-2}\), respectively. These parameters are used in the present study, except for \( (1/\Lambda_{IR}^2) \rightarrow \infty \) in medium.

C. QMC-based and NJL-based approaches

As mentioned already, we adopt two approaches to estimate the in-medium constituent quark masses. In the NJL-based approach we use the in-medium “current quark” properties evaluated by the QMC model, and calculate the in-medium dynamical (constituent) quark masses in the NJL model. While in the QMC-based approach, we use the vacuum constituent quark mass values fixed by the fit in the NJL model, and the QMC model calculates the density dependence of the constituent quark masses. Note that, the in-medium kaon properties including effective kaon mass \( m^*_K \) and quark EMFFs, are all calculated in the NJL model for both approaches.

We first discuss the parameters of the NJL-based approach. Following Ref. [33], we adopt \( m_l = m_u = m_d = 16.4 \) MeV and \( m_s = 356 \) MeV, for the vacuum current quark mass values. These values are optimally fixed by the NJL model in Refs. [16, 28]. The other parameters in the QMC model are fixed by the nucleon mass \( M_N = 939 \) MeV, nucleon bag radius \( R_N = 0.8 \) fm, and the nuclear matter binding energy per nucleon of 15.7 MeV at the saturation density \( \rho_0 = 0.15 \) fm\(^{-3}\). For the QMC-based approach, the NJL-model vacuum constituent quark mass values are used as inputs to calculate the density dependence of the constituent quark masses in the QMC model. In this case we use \( M_l = M_u = M_d = 400 \) MeV and \( M_s = 611 \) MeV. They are summarized in Table I.

In Fig. 1 we compare the in-medium constituent quark mass \( M_l^* \) in the NJL-based approach (solid line) with \( m_l = 16.4 \) MeV, and \( m_l^* \) in the QMC-based approach (dashed line) with \( m_l = 400 \) MeV.

![FIG. 1. In-medium constituent quark mass \( M_l^* \) in the NJL-based approach (solid line) with \( m_l = 16.4 \) MeV, and \( m_l^* \) in the QMC-based approach (dashed line) with \( m_l = 400 \) MeV.](image-url)

In Tables III and IV we list the density dependence of the light-quark dynamical quark mass, effective kaon mass, and kaon-quark coupling constant \( g_{Kqq}^t \) decrease in medium as nuclear density increases. The in-medium kaon leptonic de-
TABLE III. Constituent quark mass of the light quark and kaon properties in symmetric nuclear matter calculated in the NJL-based approach \((m_l = 16.4\) MeV). The masses and decay constant are given in units of GeV while the coupling constant \(g_{Kqq}^*\) is dimensionless.

| \(\rho_B/\rho_0\) | \(M_l^*\) | \(m_K^*\) | \(f_K^*\) | \(g_{Kqq}^*\) |
|-------------------|---------|---------|---------|---------|
| 0.00              | 0.400   | 0.495   | 0.091   | 4.570   |
| 0.25              | 0.370   | 0.465   | 0.091   | 4.536   |
| 0.50              | 0.339   | 0.437   | 0.090   | 4.495   |
| 0.75              | 0.307   | 0.411   | 0.089   | 4.455   |
| 1.00              | 0.270   | 0.386   | 0.088   | 4.408   |
| 1.25              | 0.207   | 0.359   | 0.084   | 4.332   |

In this section, we present the calculation of the \(K^+\) EMFF in symmetric nuclear matter following Ref. [28]. Here, the in-medium light-quark propagators appearing in the Bethe-Salpeter equation are modified (see Eq. (19)).

The electromagnetic interaction with the quark is obtained by the minimal substitution: \(i\partial \rightarrow i\partial - \vec{Q} A_{\mu} \gamma^\mu\), where \(A_{\mu}\) is photon field, \(e\) is the positron charge and \(\vec{Q} = \text{diag}[e_u, e_d, e_s] = \frac{2}{3}(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8)\) is the quark charge operator with \(e_u, d, s\) being the respective quark charges.

The matrix element of the in-medium electromagnetic current for the \(K^+\)-meson (will be denoted simply as kaon hereafter, otherwise stated) is written as

\[
J^{\mu}(p^\prime, p^\prime) = (p^{\prime\mu} + p^{\prime\mu}) F^*_{K}(Q^2),
\]

where \(p\) and \(p'\) are the initial and final four-momenta of the kaon, respectively, with \(q^2 = (p^{\prime\mu} - p^{\mu})^2 = -Q^2\) and \(F^*_K(Q^2)\) denotes the in-medium kaon EMFF.

As the case in vacuum, the in-medium kaon EMFF in the NJL model is calculated by the sum of the two Feynman dia-

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**TABLE II.** In-medium light-quark chiral condensates, \(-\langle \bar{q}q \rangle^{1/3}_{\text{in}} = -\langle \bar{q}q \rangle^{1/3}_{\text{vac}}\) (GeV), calculated in the two approaches.

| \(\rho_B/\rho_0\) | NJL-based approach | QMC-based approach |
|-------------------|-------------------|-------------------|
| 0.00              | \(0.171\)         | \(0.171\)         |
| 0.25              | \(0.167\)         | \(0.165\)         |
| 0.50              | \(0.162\)         | \(0.158\)         |
| 0.75              | \(0.156\)         | \(0.151\)         |
| 1.00              | \(0.149\)         | \(0.144\)         |
| 1.25              | \(0.136\)         | \(0.137\)         |

...as nuclear density increases, but the reduction rate is smaller than that of pion [33], which is consistent with the result in Ref. [75].

![Figure 3](image_url) **FIG. 3.** Effective nucleon mass \(M_N^*\) for symmetric nuclear matter calculated in the QMC model, corresponding to the NJL-based approach (solid line), and for the QMC-based approach (dotted line).
grams depicted in Fig. 5, which give [28]

\[
j^\mu_1(K)(p^\alpha, p^\nu) = i g^2_{Kqq} \int \frac{d^4k}{(2\pi)^4} \times \text{Tr}\{\gamma_5 \lambda_\alpha S^\nu_\gamma(p^\nu + k^\nu) \hat{Q} \gamma^\mu S^\alpha_\gamma(p^\alpha + k^\alpha) \gamma_5 \lambda_\alpha S^\gamma_\gamma(k^\gamma)\},
\]

where the trace is over the Dirac, color, and flavor indices. The index \( \alpha \) labels the state and \( \lambda_\alpha \) are the corresponding flavor matrices. In flavor space the in-medium quark propagator is defined by \( S^\alpha_\gamma(k^\gamma) = \text{diag}[S^\alpha_\gamma(k^\gamma), S^\gamma_\gamma(k^\gamma), S^\gamma_\gamma(k^\gamma) = S_\gamma(k)] \) (see Eq. (19)).

We begin with the relation between the in-medium quark bare EMFFs and kaon EMFF:

\[
F^{s(\text{bare})}_{K^+}(Q^2) = e_u f^{ks}_{K^+}(Q^2) - e_s f^{ks}_{K^+}(Q^2).
\]

In the expression \( f^{s(\text{bare})}_{K^+}(Q^2) \) the first superscript \( a \) indicates the struck quark by the photon, and the second superscript \( b \) indicates the spectator, and explicit expression is given by

\[
f^{s(\text{bare})}_{K^+}(Q^2) = \frac{3g_{Kqq}^2}{4\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} \exp(-\tau[M^2_a + x(1-x)Q^2]) \int_0^{1-x} dx' \int dz \int d\tau \tau x[(x+z)m_{K^\tau}^2 + (M_{b^\tau}^2 - M_{b^\tau}^2) (x+z) + 2M_{b^\tau}^2(M_{b^\tau}^2 - M_{b^\tau}^2)] \times \exp(-\tau[(x+z)(x+z-1)+2M_{b^\tau}^2 + (1-x-z)M_{b^\tau}^2 + xzQ^2]).
\]

These results are denoted as “bare”, because the quark-photon vertex is elementary, i.e., \( V^{\mu(\text{bare})}_{q\gamma} = \hat{Q} \gamma^\mu \), with \( \hat{Q} \) being the quark-charge operator. We note that these expressions satisfy charge conservation exactly.

The quark sector EMFFs in medium for a hadron \( \alpha \) can be defined similarly to those in vacuum [28]:

\[
F^{\alpha}_{\alpha}(Q^2) = e_u F^{uu}_{\alpha}(Q^2) + e_d F^{ud}_{\alpha}(Q^2) + e_s F^{us}_{\alpha}(Q^2) + \cdots.
\]

The “bare” quark form factor in medium \( F^{s(\text{bare})}_{\alpha}(Q^2) \) in a pseudoscalar meson can easily be related with Eq. (29).

Generally, the quark-photon vertex is not elementary \( (\hat{Q} \gamma^\mu) \)
but dressed. With the dressing, the quark-photon vertex is
given by the inhomogeneous Bethe-Salpeter equation as
illustrated in Fig. 6. For denoting the “dressing”, we replace
by (q, u, d, s) → (Q, U, D, S) in the rest of this section. With
the NJL-model interaction kernel, the solution to the dressed
quark-photon vertex for a flavor Q quark has the form:

\[ \Lambda_{\gamma Q}^{\mu}(p^\star, p^\prime) = e_q \gamma^\mu + \left( \gamma^\mu - \frac{p^\prime \cdot p^\star}{q^2} \right) F_Q^{\star}(Q^2) \]

\[ \rightarrow \gamma^\mu F_{1_Q}^{\star}(Q^2). \] (32)

To arrive at the final result, we have used that the
\[ p^\mu p^\prime / p^2 \]
term does not contribute to the electromagnetic current
by the current conservation. Note that this form clearly
satisfies the Ward-Takahashi identity \( q_\mu \Lambda_{\gamma Q}^{\mu}(p^\star, p^\prime) = \)
\[ e_Q \left[ S^{-1}(p^\star) - S^{-1}(p^\prime) \right]. \]

For the dressed quark form factors, we find
\[ F_{1i=U,D}(Q^2) = \frac{e_i}{1 + 2 G_p \Pi_{i}^{\mu\nu}(Q^2)}, \] (33)

\[ F_{1S}^{\star}(Q^2) = \frac{e_s}{1 + 2 G_p \Pi_{v}^{\mu\nu}(Q^2)}. \] (34)

where the explicit form of the in-medium bubble diagram is

\[ \Pi_{v}^{\mu\nu}(Q^2) = \frac{3 Q^2}{\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} x(1 - x) \times \exp \left\{ -\tau \left[ M_{q}^2 + x(1 - x) Q^2 \right] \right\}. \] (35)

As in the vacuum case [28], we neglect the quark flavor
mixing for the in-medium dressed quark form factors, as well
as the in-medium dressed quark masses. \(^1\)

The final expression for the in-medium K* EMFF with a
dressed quark-photon vertex is given by

\[ F_{K^*}(Q^2) = F_{1U}(Q^2) f_{K^*}(Q^2) - F_{1S}(Q^2) f_{K^*}^{\star}(Q^2), \] (36)

where the in-medium quark EMFFs are obtained by Eqs. (31)-(34). (See also Eq. (29)).

IV. NUMERICAL RESULTS

We present our results for the K*+ meson EMFF in the two
approaches: the QMC-based and the NJL-based. First, we dis-
cuss the results obtained by the QMC-based approach. Those

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\(^1\) In the limit \( Q^2 \rightarrow \infty \) these form factors reduce to the elementary quark
charges as expected from asymptotic freedom in QCD. For small \( Q^2 \) these
results are similar to the expectations from vector meson dominance, where
the u and d quarks are dressed by \( \rho \) and \( \omega \) mesons and the s quark by the
\( \phi \) meson. Note, the denominators in Eqs. (33) and (34) are the same as the
pole condition obtained by solving the Bethe-Salpeter equation in the \( \rho \),
\( \omega \) or \( \phi \) channels. Therefore, the dressed u and d quark form factors have
poles at \( Q^2 = -m_{\rho}^2, -m_{\omega}^2 \), while the dressed s quark form factor has a
pole at \( Q^2 = -m_{\phi}^2 \).

---

\(^2\) The charge radius of the K* meson is calculated using the relation:

\[ \langle r_K^2 \rangle = \frac{\int_0^\infty 2 F_K(Q^2) \frac{dQ^2}{Q^2}}{\int_0^\infty 2 F_K(Q^2) \frac{dQ^2}{Q^2}} |Q^2=\alpha|^2. \]
FIG. 7. $K^+$ EMFF in symmetric nuclear matter calculated by the QMC-based approach. (a) Total kaon EMFF, $F_{K}(Q^2)$ as a function of $Q^2$ for a few densities, (b) Up quark EMFF, $F_{K}^u(Q^2)$ as a function of $Q^2$ for a few densities, (c) Strange quark EMFF, $F_{K}^s(Q^2)$ as a function of $Q^2$ for a few densities, and (d) $Q^2 F_{K}(Q^2)$ as a function of $Q^2$ for a few densities. The lines are for $\rho_B/\rho_0 = 0.00$ (solid line), 0.50 (dash-dotted line), and 1.00 (dotted line), respectively.

the values of the $u$-quark charge radius as well as the kaon charge radius increase as nuclear density increases. At normal nuclear density, the kaon and the $u$-quark charge radii increase respectively about 20% and 25% relative to those corresponding in vacuum. For the $s$-quark, the value of the charge radius together with $\rho = \rho_0$ is from Ref. [77].

TABLE V. $K^+$ charge radius and quark charge radii calculated in the QMC-based approach. The results are calculated using the medium inputs from the QMC model generated for $m_u = 400$ MeV and $m_s = 611$ MeV, namely the density dependent $m_u^*$ together with $m_s^* = m_s$. All the charge radii are in units of fm. The empirical result in vacuum is from Ref. [77].

| $\rho_B/\rho_0$ | $r_K$ | $r_u$ | $r_s$ | $r^{\text{expt}}$ |
|-----------------|-------|-------|-------|-----------------|
| 0.00            | 0.59  | 0.65  | 0.44  | 0.56 ± 0.03     |
| 0.25            | 0.62  | 0.69  | 0.44  |                 |
| 0.50            | 0.65  | 0.73  | 0.44  |                 |
| 0.75            | 0.68  | 0.77  | 0.44  |                 |
| 1.00            | 0.71  | 0.81  | 0.44  |                 |
| 1.25            | 0.74  | 0.85  | 0.44  |                 |

TABLE VI. Same as Table V, but calculated in the NJL-based approach using the in-medium inputs from the QMC model generated for $m_u = 16.4$ MeV, and $m_s = 356$ MeV. These $m_u$ and $m_s$ yield the NJL model in-vacuum dynamical (constituent) quark masses $M_u = 400$ and $M_s = 611$ MeV, respectively. Using the QMC model generated $m_u^*$ from $m_u = 16.4$ MeV, the NJL model calculates the density dependent dynamical quark mass $M_u^*$, and uses this $M_u^*$ together with $M_s^* = M_s$.

| $\rho_B/\rho_0$ | $r_K$ | $r_u$ | $r_s$ | $r^{\text{expt}}$ |
|-----------------|-------|-------|-------|-----------------|
| 0.00            | 0.61  | 0.69  | 0.44  | 0.56 ± 0.03     |
| 0.25            | 0.62  | 0.69  | 0.44  |                 |
| 0.50            | 0.66  | 0.74  | 0.44  |                 |
| 0.75            | 0.69  | 0.79  | 0.44  |                 |
| 1.00            | 0.74  | 0.86  | 0.44  |                 |
| 1.25            | 0.74  | 0.85  | 0.44  |                 |

in medium is identical to that in vacuum, because the $s$-quark properties are not modified in medium in the present approach. Table VI shows that at normal nuclear density $\rho_0$, the $K^+$ and
\( \rho_B = 0.00 \rho_0 \)
\( \rho_B = 0.50 \rho_0 \)
\( \rho_B = 1.00 \rho_0 \)
(a)
0 1 2 3 4
\( Q^2 \) (GeV^2)
0
0.2
0.4
0.6
0.8
1
\( F_K(Q^2) \)
(b)
0 1 2 3 4
\( Q^2 \) (GeV^2)
0
0.2
0.4
0.6
0.8
1
\( -F_{\bar{s}K}(Q^2) \)
(d)
0 1 2 3 4
\( Q^2 \) (GeV^2)
0
0.2
0.4
0.6
0.8
1
\( Q^2 F_K(Q^2) \)

FIG. 8. Same as Fig. 7 but for the NJL-based approach. The lines are for \( \rho_B/\rho_0 = 0.00 \) (dashed line), 0.50 (dash-dotted line), and 1.00 (dotted line), respectively.

\( u \)-quark charge radii increase respectively about 17\% and 22\% relative to those corresponding in vacuum.

Overall we find the results calculated by both the QMC-based and NJL-based approaches show very similar behavior for the in-medium \( K^+ \) and quark EMFFs up to the nuclear density \( \rho_B \leq 1.25 \rho_0 \), where the in-medium constituent quark masses for the both approaches are almost the same in this region as in Fig. 1. Consequently, they give nearly the same results for the in-medium \( K^+ \) EMFF and quark charge radii in this density region, as can be seen in Tables. V and VI. The difference in the in-medium \( K^+ \) charge radii in the both approaches is about 2\% at normal nuclear density.

V. SUMMARY AND CONCLUSION

We have studied the kaon properties and the space-like \( K^+ \)-meson electromagnetic form factor (EMFF) in symmetric nuclear matter in the Nambu-Jona-Lasinio (NJL) model. We take into account the in-medium effects in the dressed quark-photon vertex. The in-medium constituent light-quark masses are calculated by the two different approaches to use in the NJL model. The results of the two different approaches are compared, and they turned out to give very similar predictions. Namely, the NJL-based approach that the in-medium dynamical light-quark mass \( m^*_q = m^*_q \) calculated by the NJL model using the in-medium input for the current-quark mass \( m^*_q \) (in vacuum \( M_1 = 16.4 \) MeV) generated by the quark-meson coupling model, yields nearly the same results as those calculated in the QMC-based approach, that uses the in-medium constituent quark mass \( m^*_q \) (in vacuum \( m_1 = 400 \) MeV) calculated in the QMC model. In both approaches, the strange quark constituent quark masses are kept the same as that in vacuum, \( M^*_s = M_1 = m_1 = m^*_s = 611 \) MeV. The feature of yielding nearly the similar predictions in the two approaches, holds up to around the normal nuclear densities.

The difference in the two approaches appears only in the higher density region (as shown in Fig. 1). At normal nuclear density, the values of the constituent light-quark mass \( m^*_q \) calculated in the QMC-based approach is smaller about 10\% than that of calculated in the NJL-based approach.

The NJL model has been used as a complementary model to the MIT bag model in the variants of the QMC model in Ref. [73]. With similar motivation, we have studied the in-medium kaon properties in the NJL model using the in-medium inputs calculated by the QMC model. These are either using the in-medium current quark properties for the NJL-based approach, or the in-medium constituent quark properties for the QMC-based approach as mentioned above.
FIG. 9. In-medium $K^+$ EMFF in the QMC-based and the NJL-based approaches, compared with the existing data in vacuum ($\rho_B/\rho_0 = 0.00$) at low $Q^2$. Experimental data are in vacuum are from Ref. [30].

By the NJL-based approach we have calculated the effective kaon mass $m_{K^*}^*$, kaon decay constant, and kaon-quark coupling constant in symmetric nuclear matter. We predict that, at normal nuclear density, the constituent quark mass, effective kaon mass, kaon-quark coupling constant, and kaon leptonic decay constant decrease respectively about 33%, 22%, 4%, and 3% relative to those corresponding in vacuum.

Alternatively, in the QMC-based approach using the effective constituent quark mass of the light quark $m_l^*$ directly calculated in the QMC model, we have calculated the same quantities using the NJL model as those calculated in the NJL-based approach. In this case we predict, at normal nuclear density, the constituent light-quark mass, effective kaon mass, kaon-quark coupling constant, and kaon lepton decay constant decrease respectively about 39%, 25%, and 4%, and 4% compared to those corresponding in vacuum.

Based on the in-medium kaon properties calculated in the both approaches, the QMC-based and the NJL-based, we have studied the $K^+$ EMFF in symmetric nuclear matter using the NJL model. We have found that the in-medium $s$-quark EMFF increases as nuclear density increases, which is unexpected. However, the increasing rate against the increase of nuclear density is very small. This leads to nearly unmodified in-medium $s$-quark charge radius relative to that in vacuum, irrespective of nuclear densities studied.

In contrast, the in-medium $u$-quark EMFF as well as that of $K^+$, decrease appreciably as nuclear density increases. This leads to a larger $u$-quark charge radius as well as that of $K^+$ in symmetric nuclear matter. At normal nuclear density, the $K^+$ charge radius $r_{K^+}$ increases about 20% relative to that in vacuum in the QMC-based approach, while it yields about 17% increase in the NJL-based approach. These results indicate that the valence $s$-quark gives a significant contribution for the $K^+$ EMFF in vacuum as well as in symmetric nuclear matter, for larger $Q^2$ and larger nuclear density $\rho_B$. Our predictions can be verified by experiments such as the Compressed Baryonic Matter (CBM) experiment at GSI [78].

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