Bootstrap quark model and spectra masses, electromagnetic properties of low-lying hadrons

S.M. Gerasyuta, and S. Bielefeld

Dept. of Physics, Technical Academy, St.Petersburg,
Max-Planck-Institut für Kernphysik, Heidelberg

Abstract

The paper is devoted to the construction of low-energy quark scattering amplitudes. Bootstrap quark model allows to describe the light and heavy hadron spectroscopy based on three principles: unitarity, analicity and crossing symmetry. The resulting quark interaction appeared to be effectively short-range. This interaction is determined mainly by the exchange in the gluon channel: the constituent-gluon mass appeared to not be small. Our calculation indicates an important role of an interaction which is induced by instantons. Such an interaction is necessary for deriving both the pion mass and η-η' mass splitting. However, due to the rules of 1/N_c-expansion this interaction influences slightly the other channels while in the η-η' channel it provides the correct values of masses and gives the angle of η_1 - η_8 mixture close to that of the quark model. We discuss the possibility of constructing the amplitudes which take into account the quark confinement.
1 Introduction

The quark phenomenology is based on the hypothesis of the quasinuclear hadron structure: the hadron consists of two (meson) or three (baryon) spatially separated dressed quarks like the nucleus consists of nucleons. A dressed quark of the quasinuclear model should be a quark-gluon cluster consisting of the valence quark, sea quarks and anti-quarks. The small size of the dressed quark may be connected with the small range of gluon interaction. One should try to describe soft processes operating with dressed quarks as quasi-particles. The project is devoted to the construction of low-energy quark scattering amplitudes. The iteration bootstrap procedure includes two types of point-like input interactions, namely the four-fermion interaction with quantum numbers of the gluon and four-fermion interaction induced by instantons (the exchange of white states with $J^P=0^\pm$). Calculations performed in the framework of this model argue in favor of the quasinuclear quark structure of hadrons. The resulting quark interaction appeared to be effectively short-range. This interaction is determined mainly by the exchange in the gluon channel: the calculated constituent gluon mass $M_G=0.67$ GeV to be not small. The creation of mesons (pion included) is mainly due to the gluon exchange. The mass values of the lowest mesons ($J^P=0^{-+},1^{--},0^{++}$) and their quark content are obtained. In the color channel $\bar{3}_c$ the bound states of the scalar diquarks with the masses $m_{ud}=0.72$ GeV and $m_{us}=m_{ds}=0.86$ GeV are obtained. The dispersion relation technique allows us to calculate the form factors of light mesons using quark amplitudes in corresponding channels. The calculated values of the charge radii squared of pion and Kaon are in good agreement with data. The calculated values of the charge radii squared of u, d, s quarks are equal: $r_u^2=2.06$ GeV$^{-2}$, $r_d^2=1.75$ GeV$^{-2}$, $r_s^2=1.03$ GeV$^{-2}$. The charge radii of non-strange and strange scalar diquarks are calculated: $\langle r_{ud}^2 \rangle^{1/2}=0.55$ fm, $\langle r_{us}^2 \rangle^{1/2}=0.65$ fm, $\langle r_{ds}^2 \rangle^{1/2}=0.5$ fm.

In this consideration one has neglected the forces of confinement. This is due to the fact that the model is actually based on the assumption that the confinement radius is much larger than the radius of dressed quarks, and also larger than the radius of the forces responsible for the existence of the low-lying hadrons. In the case of the orbital excited mesons: P-, D-, F-wave mesons one can not neglect the confinement potential. Therefore the main idea is the modification of precise theory by help of changing the interaction of quarks and anti-quarks at large range and do not charge the interaction at small range. Therefore one constructs the approximate model, in which there are free quarks, but the theory of quark-anti-quark scattering uses only the necessary part of the confinement potential. This idea can be used in different approach. In our case the confinement potential is imitated by the simple increasing of constituent quark masses. It allows us to construct the P-, D-, F-wave meson amplitudes and calculate the mass spectrum, which is in good agreement with data. We calculate the behavior of meson Regge trajectories for the low-energy
region, which is based on the principles of multi-color QCD. In the framework of an approach developed previously for light quarks one calculates the scattering amplitudes of heavy quarks $q\bar{Q} - q\bar{Q}$ and $Q\bar{Q} - Q\bar{Q}$ ($q=u, d, s; Q=c, b$). The interaction of heavy quarks is described by forces corresponding of exchange of light white and colored mesons. The main role in the formation of the spectrum of heavy mesons is played by the forces that correspond to exchange of a massive gluon. It is necessary to take into account the renormalization of the quark amplitudes for light quarks, which is determined by the rescattering of the heavy quarks. We obtained the masses of lowest multiplets of $c$- and $b$-mesons with quantum numbers $J^{PC} = 0^{-+}, 1^{--}, 0^{++}$.

Then we use a quark interaction potential, which is obtained from the nonrelativistic limit of relativistic quark amplitudes of the Bootstrap quark model. But the quark amplitudes obtained in the Bootstrap procedure depend not only on the square $t$ of the momentum transfer, but also on the energy variable $s$. Therefore, a literal transition to nonrelativistic potentials is not possible: these amplitudes rather correspond to quasipotentials. If the energy $s = s_0$ is fixed, and already at fixed energy the dependence on the momentum transfer is considered to be a potential nature.

We investigated S-wave baryons with quantum numbers $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ consisting of $u$ and $d$ quarks. The masses and wave function of the baryons were obtained from the solution of the nonrelativistic Faddeev equations for the three-quark system with the diquark potentials $V_{0^+}$ and $V_{1^+}$. One obtained the electric and magnetic form factors of the nucleons. The values of the masses of the nucleons and the $\Delta$ isobar agree well with the experimental values, then we have a good agreement with experiment for the magnetic moments $\mu_p = 2.79(2.79)$ and $\mu_n = -1.86(-1.91)$. The charge radius of the proton has been to be almost a factor of two smaller than experimental value $\langle R_p^2 \rangle^{1/2} = 0.4(0.8) \text{ fm}$, charge radius of the neutron has been found to be zero. This calculation has not new parameters besides the Bootstrap quark model results. The behavior of additional linear potential are investigated and do not essentially change the results. In our paper a relativistically generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting particles. The mass spectrum of S-wave baryons $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ including $u$, $d$, $s$ quarks was calculated by a method based on isolating of the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered the approximation, which corresponds to taking into account two-body and triangle singularities, and define all the smooth functions of the sub-energy variables (as compared with the singular part of the amplitude) in the middle point of physical region of Dalitz-plot, then the problem reduces to one of solving simple algebraic system equations. The masses of the baryons of the two lowest multiplets with $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ are calculated and found to be in a good agreement with the experimental results. The behavior of the electromagnetic form factors
of nucleons and hyperons in the region of low and intermediate momentum transfers $Q^2 < 1.5 \text{ GeV}^2$ is determined. The calculated value (without new parameters) of the charge radius of the proton was found to be $\langle R^2_p \rangle^{1/2} = 0.4 \text{ fm}$, hyperons $\langle R^2_{\Sigma^+} \rangle^{1/2} = 0.43 \text{ fm}$, $\langle R^2_{\Sigma^-} \rangle^{1/2} = 0.39 \text{ fm}$, $\langle R^2_{\Xi^-} \rangle^{1/2} = 0.38 \text{ fm}$. The charge radius of the neutron, and other noncharge hyperons were found to be practically equal to zero. In the framework of dispersion relation technique the relativistic Faddeev equations for charmed baryons are found. The approximate solutions of the relativistic three-particle problem based on the extraction of leading singularities of amplitudes are obtained. The mass values of lowest charmed baryon multiplets $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ are calculated. The behavior of electromagnetic form factors of charmed baryons ($J^P = \frac{1}{2}^+$) in the region of low and intermediate momentum transfers $Q^2 < 1.5 \text{ GeV}^2$ is determined. The calculated value (without new parameters) of the charge radius of baryons like $\Sigma^+_c$, $\Lambda^+_c$ and so on $(0.12 - 0.20) \text{ fm}$. The charge radii of noncharge charmed baryons are equal to zero. We calculated the multiplets masses of radial excited baryon, included the Roper resonance, with quantum number $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ at $N = 2, 56^*$. The problem of confinement is solved analogous to the case of excited mesons. Finally, a practical treatment of relativistic two- and three-hadron quark systems have been developed. The theory is based on the three principles of unitarity, analyticity and crossing symmetry. These principles are applied to the two-body subenergy channels. The scattering quark amplitudes allow to calculate the spectrum and electromagnetic properties of hadrons. In soft processes, where small momentum transfers are essential, the perturbative QCD technique is not applicable. In this case the way of phenomenology should include quark model results as well. Quark models make it possible to describe the qualitative properties of QCD in the nonperturbative region. In the framework of quark models important information is obtained on the properties of light mesons [1]- [9] and baryons [10]- [16]. The reason for the successful use of quark potential models is connected with a successful choice of the effective parameters: the mass of the dressed quarks, the characteristics of the confinement potential, and the coupling constant $\alpha_s$. In quark models, which describe rather well the masses and static properties of light hadrons, the masses of the quarks usually have the same values for both mesons and baryons. However, this is achieved at the expense of some difference in the characteristic of the confinement potential. It should be borne in mind that for a fixed hadron mass the masses of dressed quarks which enter into the composition of the hadron will become smaller when the slope of the confinement potential increases or its radius decreases. Therefore, conversely one can change the masses of the dressed quarks when going from the spectrum of s-wave mesons to s-wave baryons, while keeping the characteristic of the confinement potential unchanged. The mass spectrum of charmed and beauty mesons is subject of investigation of many theoretical papers, based on various approaches. Calculation in potential models are widespread [17]- [22].
The unquestionable advantage of the potential approach to the description of heavy mesons is its simplicity and transparency. It makes it possible to calculate (in the framework of the adopted model of the potential) the positions of the levels, the widths of radiative transitions between levels, and the widths of annihilation decays. Thus, in fact it succeeds in determining of behavior of the wave function of the system in a fairly wide range of distances, which is very important for an understanding of the dynamics of the interaction of the quarks. A serious difficulty of the potential approach is the fact that it cannot be fully substantiated in the framework of QCD. The method of QCD sum rules has been successfully and to determine the masses of the light and heavy mesons [23, 24]. In this approximation the vacuum condensates are considered as phenomenological parameters determined from the experimental data or from self-consistency of the sum rules. The most serious difficulty for the application of the sum-rule method to bottomonium (for instance) is the problem of taking into account relativistic corrections. So far there has been no such calculation.

2 Iteration method in the Bootstrap procedure

The project is devoted to the construction of low-energy quark scattering amplitudes. The amplitudes of dressed quarks are calculated in the framework of the dispersion technique with the help of the iteration Bootstrap procedure. The theory is based on three principles of unitarity, analyticity and crossing symmetry which are applied to the three-body sub-energy channels. The iteration Bootstrap procedure for quark-quark and quark-anti-quark amplitudes was made with the four-fermion interaction as an input:

\[ g_\nu(\bar{q}\gamma_\mu\vec{\lambda}q)(\bar{q}\gamma_\mu\vec{\lambda}q) \]  

where \( \vec{\lambda} \) are the Gell-Mann matrices. The point-like structure of this interaction is motivated by the idea of the two characteristic sizes in the hadron. On the other hand, the applicability of (1) is verified by the success of the De-Rujula-Georgi-Glashow quark model [1], where only the short-range part of the Breit potential connected with the gluon exchange is responsible for the mass splitting in hadron multiplets. However, the correct description of low-lying meson masses is impossible using the effective gluon exchange interaction (1) only. The large value of the \( \eta - \eta' \) mass splitting and the small pion values indicate the existence of an additional interaction. Such an additional four-fermion interaction can be induced by instantons. This interaction should also be a short-range one because of the small radius of the effective gluon interaction. We have included this type of interaction in the iteration Bootstrap calculation. The scheme of the iteration Bootstrap procedure suggested in this
Fig. 1. Iteration of Bootstrap procedure

The partial amplitudes are calculated through the dispersion technique. The zero approximation in Fig. 1d is drawn as the sum of diagrams shown in Fig. 1a, b, c etc. The amplitude of the first approximation (Fig. 1h) is obtained when the zero approximation amplitude is taken as an interaction like Eq.(1), i.e. the interaction in the first approximation is determined by the exchange diagrams of the zero approximation. The first approximation amplitude is the sum of diagrams shown in Figs. 1e, f, g etc. The use of the first approximation amplitude as an “interaction force” provides us with the second approximation, and so on. We consider the rescattering amplitudes in three sub-energy channels u, s, t. We construct scattering amplitudes of the dressed quarks of three flavors (u, d, s) the poles of these amplitudes determine the masses of the light mesons [25,26]. The masses of the constituent quarks u and d are of the order of 300-400 MeV; strange quark is 100-150 MeV heavier. The constituent quark is a color triplet and quark amplitudes obey the global color symmetry. For the $0^+$ multiplet the discrepancy between calculated and observed values of masses more than for $0^-$ and $1^-$. It is possible that this is due to the admixture of the glueball states as $q\bar{q}q\bar{q}$-states in the scalar mesons. In all the versions of our calculus there is a bound state in the gluon channel with the mass of order of 0.7 GeV. Its quark wave function is

$$\frac{(1.06(u\bar{u} + d\bar{d}) + 0.87 \ s\bar{s})}{\sqrt{3}};$$

hence with a good accuracy it is a singlet of the flavor $SU(3)_f$-group. This bound state should be identified as a constituent gluon. In the diquark channel there are bound states with $c = \bar{3}$ and $J^P = 0^+$. The dispersion relation technique allows us to calculate the form factors of
\[
\begin{array}{cccc}
J^{PC} = 0^{+} & J^{PC} = 1^{--} & J^{PC} = 0^{++} \\
\pi & 0.14 (0.14) & \rho & 0.77 (0.77) & a_0 & 0.78 (0.98) \\
\eta & 0.48 (0.55) & \omega & 0.77 (0.78) & f_0 & 0.87 (0.98) \\
K & 0.50 (0.50) & K^* & 0.89 (0.89) & K^*_0 & 0.88 (1.35) \\
\eta' & 0.96 (0.96) & \phi & 1.00 (1.02) & f_0' & 1.16 (1.30) \\
\theta = -24^\circ & \theta = 30^\circ & \theta = -82^\circ \\
\end{array}
\]

Table 1

Masses of the lowest meson multiplets (GeV) - Experimental values of mesons are in parenthesis. Parameters of the model: gluon constant \( g_V = 0.226 \), cut-off \( \Lambda_q = 17.3 \), instanton constants \( g_I = -0.081 \), \( g_S = 0.55 \), \( m_u = 0.385 \), \( m_s = 0.501 \) GeV.


calculated values of both pion and Kaon charge radii are equal

\[
\langle R_{\pi \pm}^2 \rangle^{1/2} = 0.76 \text{ fm} \quad \langle R_{K^\pm}^2 \rangle^{1/2} = 0.69 \text{ fm} \quad R_{K^0}^2 = -1.1 \text{ GeV}^{-2} \quad (2)
\]

For the interaction with the photon current dressed-quark radii are determined by the behavior of quark amplitudes in the channel \( 1^- \) near \( t = 0 \). The calculated values of the charge radii are equal for u, d, s quarks:

\[
\langle r_{ud}^2 \rangle^{1/2} = 0.55 \text{ fm} \quad \langle r_{us}^2 \rangle^{1/2} = 0.65 \text{ fm} \quad \langle r_{ds}^2 \rangle^{1/2} = 0.5 \text{ fm} \quad (3)
\]

Parameters of this model are: cut-off energy \( \Lambda = 17.3 \), gluon constant \( g_V = 0.226 \), four-fermion instanton interaction \( g_I = -0.081 \), interaction with exchange of white isosinglet \( g_S = 0.55 \), quark masses \( m = 0.385 \) GeV, \( m_s = 0.501 \) GeV, see Table 1.

3 P-, D-, F-wave mesons in the Bootstrap method

One of the most surprising, and so far unexplained, dynamical features of the spectroscopy of light quarks is the existence of five almost completely filled nonets in the quark-anti-quark \( q\bar{q} \)-system, predicted in the nonrelativistic quark model: two S-wave and three P-wave nonets. In the well established \( J^{PC} = 1^{--} \) and \( 2^{++} \) nonets ideal mixing of the \( SU(3)_f \) singlet and isoscalar components of the octet is realized, and this implies the suppression of the transition \( (u\bar{u} + d\bar{d})/\sqrt{2} \leftrightarrow s\bar{s} \) in these channels (the Okubo-Zweig-Iitzuka-rule). We should say that the simplest nonrelativistic quark model predicts ideal nonets. The status of the axial with \( J^{PC} = 1^{++} \) and \( 1^{+-} \) is not as clear as the status of the tensor mesons. Apparently, the \( 1^{++} \) nonet is also an ideal nonet. The \( 1^{+-} \) nonet is not yet completely filled, with eight of the ten
members established. The strange meson $K_1^{*}$, belonging to the multiplet with $J^{PC} = 1^{++}$, is observed in a mixture with the meson $K_1$ of the multiplet with $J^{PC} = 1^{-+}$. Experimentally, the $K_1^{*}$ meson is observed with the main decay channel $Kp$, and $K_1$ is observed with the main channel $K^{*}\pi$.

The pseudoscalar $0^{-+}$ nonet is not ideal. Nowadays, the reason for this is understood. It is connected with the solution of the U(1)-problem in QCD, and with instanton contributions [28]-[31]. At present the properties of the scalar mesons with $J^{PC} = 0^{++}$ are among the most obscure aspects of the spectroscopy of light mesons. Although the resonances $a_0$, $f_0$, $K_0^{*}$ and $\tilde{f}_0$ can be considered well established, their interpretation in the language of the quark model is not clear. The superficially natural assumption that they form a nonet of $q\bar{q}$ states cannot be reconciled with their masses. This may be due to the existence of an appreciable admixture of glueball states or multiquark states in the scalar mesons [32,33]. In this section the Bootstrap quark model technique allows us to calculate the spectra mass of P-, D-, F-mesons. The Regge trajectories for the low-energy region, which describe the mesonic resonances with orbital numbers $L = 0, 1, 2, 3$ are constructed.

The technique of carrying out the Bootstrap procedure of excited mesonic states is analogous to one of low-lying mesons. However, the forces of the interaction between quarks are effectively of short range. The effective short-range nature is a consequence of two circumstances. Exchange of a gluon state is most important in the interaction of the quarks. A constituent gluon is a fairly massive particle (700 MeV), and this is one of the reasons for the short-range nature. The other circumstance is that the quark amplitude depend relatively weak on the energy in the region $t < 0$. Since the values of the amplitudes of the crossed channels for $t < 0$ are precisely the interaction forces, relatively small variation of quark amplitude in the region $t < 0$ lead to an extra decrease of the range of interaction of the quarks. This is connected with the assumption that the confinement radius is much larger than the radius of the forces responsible for the existence of the low-lying hadrons, i.e. the confinement forces can be neglected. But when one researches the excited states the confinement potential can not be neglected. The main idea is the modification of precise theory by help of changing the interaction of quarks and anti-quarks at large range and do not change the interaction at small range. Therefore one constructs the approximate model in which there are free quarks, but the low-lying spectrum of mesons is former.

In the framework of this model we can construct the theory of quark-anti-quark scattering using only the necessary part of the confinement potential. In our case the confinement potential is imitated by the simple increasing of constituent quark masses. The shift of quark mass (parameter $\Lambda$) effectively takes into account the changing of the confinement potential. We have shown that inclusion of only gluon exchange indeed does not lead to the appearance of bound states corresponding to the mesons of the P-, D-, F-waves. The using of mass shift $\Lambda_f$ is possible to obtain the mass spectra of P-, D-, F-mesons (Tables 2, 3, 4) [34,35]. In Table 5 we use the results of Bootstrap quark model
with u, d, s–quarks.

In the recent papers [36,37] the high energy asymptotic of multicolor QCD is considered, that allows to construct the Bethe-Salpeter equation for the n reggeized gluons. Then the author obtained the Pomeron trajectory as the bound state of two reggeized gluons. Therefore it is important to obtain the meson Regge trajectories of glueballs.

4 Heavy mesons ($J^{PC} = 0^{+-}, 1^{--}, 0^{++}$)

We consider the spectrum of the lowest $Q\bar{Q}$ and $Q\bar{q}$ mesons constructed from light ($q=u, d, s$) and heavy ($Q=c, b, t$) quarks. The main role in the formation
Regge trajectories $\alpha(t)$

| $\alpha_{\rho,\omega}$ | 0.5 (0.5) | 0.9 (0.9) |
| $\alpha_{K^*}$         | 0.4 (0.4) | 0.8 (0.8) |
| $\alpha_{\varphi}$    | 0.2 (0.1) | 0.8 (0.9) |
| $\alpha_{\pi}$        | 0. (0.)   | 0.8 (0.8) |
| $\alpha_K$            | -0.3 (-0.3) | 0.7 (0.7) |
| $\alpha_{\eta}$       | -0.2 (-0.2) | 0.8 (0.8) |
| $\alpha_{\varphi}'$   | -0.6 (-)  | 0.8 (-)   |
| $\alpha_{a_0,f_0}$    | -0.3 (-0.5) | 0.6 (0.6) |
| $\alpha_{K^*_0}$      | -0.4 (-)  | 0.6 (-)   |
| $\alpha_{f_0}$        | -0.6 (-)  | 0.6 (-)   |
| $\alpha_{a_1,f_1}$    | -0.4 (-)  | 0.8 (-)   |
| $\alpha_{K_1}$        | -0.5 (-0.5) | 0.7 (0.7) |
| $\alpha_{f_1}$        | -0.6 (-)  | 0.7 (-)   |

Table 5
Regge trajectories $q\bar{q}$ mesons – $\alpha(t) = \alpha_0 + \alpha't$; here, we use the results of Bootstrap quark model with u, d, s-quarks. Experimental results are in parenthesis.

of the spectrum of heavy mesons is played by the forces that correspond to exchange of a massive gluon. Owing to the point-like nature of the interaction of heavy quarks, their contribution leads only to numerical renormalization of the loop-diagrams of light quarks. Therefore, the introduced vertex function of the interaction of heavy quarks can effectively take into account his renormalization. The parameters $\alpha_Q$ and $\alpha_{Qq}$ of the interaction of heavy quarks and cut-off parameters $\Lambda_Q$ and $\Lambda_{Qq}$ are given by a definite set of experimental values of $Q\bar{Q}$ and $Q\bar{q}$-mesons.

We have obtained the masses of mesons with c and b quarks ($J^{PC} = 0^{\pm}, 0^{++}, 1^{--}$). In the calculation we have used the parameters $\alpha_Q$ and $\alpha_{Qq}$, and also the parameter $\Lambda_Q$, the cut-off in the scattering amplitude of heavy quarks. The cut-off in the scattering amplitude of heavy quarks with light quarks has been chosen in the following form: $\Lambda_{Qq} = \frac{1}{4}(\sqrt{\Lambda_Q} + \sqrt{\Lambda_q})^2$. The value $\Lambda_q = 17.3$ and masses $m_{u,d} = 385$ MeV, $m_s = 501$ MeV. The parameters $\alpha_Q$, $\alpha_{Qq}$ and $\Lambda_q$ have been determined for the c quark from the experimental values of the masses of the meson $D$, $D^*$, and $J/\Psi$, and for the b quark from the masses of the mesons $B$, $B^*$, and $\Upsilon$. The masses of the heavy quarks are also parameters of the model and have been determined from the best description of the masses of the $\eta_c$ for the c quark and $B_s$ for the b quark.

The results of the calculation of the meson mass spectrum are given in Tables 6 and 7 [38]. Fig. 3 shows the approximation curve of the variation of the
Fig. 2. (a) Diagrams which determine the interaction amplitudes of heavy quarks, (b) renormalization of the vertex of the interaction of light quarks due to the contribution of heavy quarks.

vertex function in (a) and that of the cutoff in (b) with increasing mass [38].

|      | M(0−+)   | M(1−−)   | M(0+++) |
|------|----------|----------|---------|
| D    | 1.867 (1.867) | 2.010 (2.010) | cū 2.119 (-) |
| Ds   | 2.010 (1.971) | 2.120 (2.113) | cū 2.300 (-) |
| ηc   | 2.955 (2.980) | J/Ψ 3.097 (3.097) | χ0 3.453 (3.415) |

Table 6
Masses of the lowest states of charmonium and of states with open charm (GeV)
- Note: the parameters of the model are: Λc = 5.94, αqc = 6.56, αc = 5.41 and m_c = 1.645 GeV.- In parenthesis we give the experimental numbers.

|      | M(0−+)   | M(1−−)   | M(0+++) |
|------|----------|----------|---------|
| B    | 5.270 (5.270) | 5.320 (5.320) | bū 5.486 (-) |
| Bs   | 5.375 (5.340) | 5.425 (5.390) | bū 5.652 (-) |
| būc  | 6.085 (-)   | 6.320 (-) | būc 6.735 (-) |
| būb  | 9.340 (-)   | Υ 9.460 (-) | būb 10.070 (-) |

Table 7
Masses of the lowest states of bottomonium and of states with open bottom (GeV)
- Note: the parameters of the model are: Λb = 4.91, αqb = 4.08, αcb = 2.44, αb = 1.12 and m_b = 4.940 GeV.- In parenthesis we give the experimental numbers.

Our model is actually based on the assumption that the confinement radius is much larger than the radius of dressed quarks, and also larger than the
radius of forces responsible for the existence of the low-lying hadrons. In the discussion of heavy mesons we have neglected the forces on confinement. However, estimates show that for meson of type $\bar{q}t$ the confinement radius becomes comparable with the radius of gluon exchange.

5 Bootstrap quark potential and nonrelativistic Faddeev equations.

Electromagnetic properties of non-strange baryons

With a sufficient number of parameters the potential models give a good description of the spectrum of hadrons. However, a serious difficulty for potential scattering is that its characteristics cannot be calculated directly in the framework of QCD. If the gluon field is unable to follow a change of the quark system, there is retardation, and, as a consequence, the interaction acquires a non-potential nature. Under these conditions a successful description of hadron in the framework of the potential approach seems highly nontrivial. It is probable that the above-mentioned non-potential nature shows up only at intermediate distances. At short distances the Coulomb-like one-gluon exchange potential works, and at large distances a linear potential is applicable. In a purely potential approach the non-potential nature of the interaction at intermediate distances effectively reduces to a redefinition of the parameters of the potential, which in this approach are not calculated, but fitted for optimal agreement with the experimental data.

We use a quark interaction potential, which is obtained from the nonrelativistic limit of relativistic quark amplitudes of the Bootstrap quark model. The main contribution describing the interaction in the diquark channel (the states

Fig. 3. Approximation curve of the variation vertex function (a), and the cut-off (b) with increasing mass [38].
Fig. 4. (a) Quark interaction potential in the state with $J^P = 0^+$; three versions (I)-(III) of the choice of the confinement potential $r_0 = 0.6, 1.0, 1.3$ fm, respectively. - (b) Quark interaction potential in state $J^P = 1^+$. $J^P = 0^+, 1^+$) is determined by gluon exchange in the t channel:

$$ (\bar{q}_c \lambda \gamma_\mu q) A(s, t)(\bar{q}_c \lambda \gamma_\mu q) ,$$

(4)

where $A(s, t)$ quark amplitude determines the strength of the interaction of the quarks in the gluon channel. The mass $M_G$ of the constituent gluon determines the region in which the gluon has a short-range nature and has the consequence that the gluon field can adjust to a change of the quark system and that non-potential effects are successfully included. However, the quark amplitudes obtained in the Bootstrap procedure depend not only on the square $t$ of the momentum transfer, but also on the energy variable $s$. Therefore, a literal transition to nonrelativistic potentials is not possible: these amplitudes rather correspond to quasi-potentials. Then in transforming from Bootstrap quark amplitudes to quark potential, the energy $s = s_0$ is fixed, and already at fixed energy the dependence on the momentum transfer is considered to be of a potential nature. Fixing of the energy $s$ requires the introduction of a momentum cut-off parameter $\Lambda_\phi$ in the Fourier transformation. This parameter is chosen in such a way that the spectrum of low-lying mesons and scalar diquarks calculated in the framework of the Bootstrap procedure is reproduced. In our case we obtained $\Lambda_\phi = 5.55 \, m$, where $m$ is the mass of a constituent quark. The explicit form of the potentials of the quark interaction in a color-antisymmetric state with quantum numbers $J^P = 0^+, 1^+$ for non-strange quarks is given in Fig.4(a) and (b) [39].

For the confinement potential we have chosen the usual linear potential with a slope determined by the angle $\alpha$. We have considered three cases, in which
the confinement potential has been added to the Bootstrap potential from the distances \( r_0 = 0.6, 1.0, 1.3 \text{ fm} \). The inclusion of such a potential shifts the spectrum of low-lying mesons. In this case the nature of the change of the spectrum is similar to the change of masses of the mesons and scalar diquarks due to a change of the constituent quark masses. Therefore, by inclusion of the confinement potential we have changed the masses of the quarks so as to reproduce the spectrum of low-lying mesons. In this case the Bootstrap potential was altered in the following way, the scale of the abscissa was changed in proportion to \( \sim 1/m \), and the scale of the ordinate in proportion to \( \sim m \). In contrast to most quark potentials, the Bootstrap potential is finite at \( r = 0 \) because of the cut-off in the energy introduced in the calculation of Bootstrap quark amplitudes. Moreover, the momentum cut-off \( \Lambda_\phi \) leads to small oscillations of the potential at distances \( \sim 1.5 \text{ fm} \), which are important for excited states and do not have a significant effect on the spectrum of low-lying baryons.

We have investigated S-wave baryons with the quantum numbers \( J^P = \frac{1}{2}^+, \frac{3}{2}^+ \) consisting of u and d quarks. The masses and wave functions of the baryons were obtained from the solution of the Faddeev equations for the three-quark system with the diquark potentials \( V_0 \) and \( V_1 \). Three-particle forces were not included in this case. The results of the calculations are given in Table 8.

| \( r_0 \) | \( m \) | \( \alpha \) | \( m_N \) | \( m_\Lambda \) | \( \langle R_{pp}^2 \rangle^{1/2} \) | \( R_{pp}^2 \) | \( \mu_p \) | \( \mu_n \) | \( (v/c)_N^2 \) | \( (v/c)_\Lambda^2 \) |
|---------|---------|---------|--------|---------|----------------|---------|--------|--------|----------------|---------|
| \( \text{fm} \) | \( \text{MeV} \) | \( \text{GeV}^2 \) | \( \text{MeV} \) | \( \text{MeV} \) | \( \text{fm} \) | \( \text{fm}^2 \) | \( \text{fm} \) | \( \text{fm} \) | \( \text{fm} \) | \( \text{fm} \) |
| 0.6     | 343     | 0.048   | 932    | 1236    | 0.37           | 2.73    | -1.82  | 1.3    | 1.1           |
| 1.0     | 336     | 0.138   | 944    | 1222    | 0.39           | 2.79    | -1.86  | 1.2    | 1.0           |
| 1.3     | 334     | 0.194   | 938    | 1190    | 0.41           | 2.81    | -1.87  | 1.2    | 0.96          |

Table 8
Structure parameters of baryons for (I)-(III) versions of the potential, Fig.4.

The values of the masses of the nucleons and the \( \Lambda \) isobar agree well with the experimental values. In this case the mass of the constituent u and d quarks is 336 MeV (version (II),Fig.4), which gives good agreement with experiment for the magnetic moments \( \mu_p = 2.79(2.79), \mu_n = -1.86(-1.91) \). The charge radius of the proton has been found to be a factor of two smaller than the experimental value, \( \langle R_{pp}^2 \rangle^{1/2} = 0.4 \text{ fm} \), \( (v/c)_N = 0.71 \text{ fm} \).

The aim of this calculation is to investigate the possibility of describing the lowest three-quark systems by means of a Bootstrap potential, using the non-relativistic Faddeev equations [40]. The results of the calculations show that the spin-spin splitting of the levels \( J^P = \frac{1}{2}^+, \frac{3}{2}^+ \), which depends on the wave function of the diquark at \( r = 0 \), is reproduced rather well. Our values of the mass of the constituent quarks are close to the standard values. Therefore, the magnetic moment of the nucleons and the \( \Lambda \) isobar agree with the experimental values. The parameter \( v/c \) shows that we are dealing with a relativistic
system of three particles. Therefore, the next step must be to calculate the characteristics of baryons in relativistic treatment.

6 Relativistic three-quark equations and spectroscopy of low-lying baryons

In the framework relativistic approach of three-hadron system one can consider the pair interaction between the particles [41]-[43]. There are three isobar channels, each of which consists of a two-particle isobar and the third particle. The presence of the isobar representation together with the condition of unitarity in the pair energies and analyticity leads to a system of integral equations in a single variable. Their solution makes it possible to describe the interaction of the produced particles in three-hadron systems.

In our consideration we construct a relativistic generalization of the three-particle Faddeev equations in the form of dispersion relation in the pair energy of the two interacting particles. By the method of extraction of the leading singularities of the amplitude we calculate the mass spectrum of S-wave baryons, the multiplets $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$, and we obtained the electric form factors of nucleons and hyperons at low and intermediate $Q^2$.

![Fig. 5. Diagrams corresponding to: (a) production of three quarks, (b)-(f) successive pair interactions.](image)

The diagrams in Fig.5 can be grouped according to which of the three quark pairs undergoes the last interaction, i.e., the total amplitude can be represented graphically as a sum of diagrams, as shown in Fig.6.

Using the diagrams in Fig.5 it is very easy to write down a graphical equation.
Fig. 6. Graphical representation of the equations for the amplitude interaction of three particles.

for the amplitude interaction of three particles. Then to write down a concrete equation we must specify the amplitude of the pair interaction of the quarks. For this we shall use the results of the Bootstrap quark model \[26\] and construct the amplitude of the interaction of two quarks in the states \(J^P = 0^+, 1^+\).

The construction of the approximate solution of Fig.6 is based on the extraction of the leading singularities in the neighborhood of \(s_{ik} = 4m^2\). The structure of the singularities of amplitudes with a different number of rescatterings (Fig.5) is the following. The strongest singularities in \(s_{ik}\) arise from pair rescatterings of quarks \(i\) and \(k\): a square-root singularity corresponding to a threshold and pole singularities corresponding to bound states (on the first sheet in the case of real bound states, and on the second sheet in the case of virtual bound states). The diagrams of Fig.1 (b) and (c) have only these two-particle singularities. In addition to two-particle singularities the diagrams of Fig.1 (d) and (e) have their own specific triangle singularities. The diagram of Fig.1 (f) describes a larger number of three-particle singularities. In addition to singularities of the triangle type it contains other weaker singularities. Such a classification of singularities makes it possible to search for an approximate solution of Eq.(Fig.6), taking into account a definite number of leading singularities and neglecting the weaker ones.

We use the approximation in which the singularity corresponding to a single interaction of all three particles, the triangle singularity, is taken into account. If we choose the approximation in which two-particle and triangle singularities are taken into account, and if all functions which depend on the pair energies will be determined at the central point of the physical region of the Dalitz plot, the problem of solving the system of integral equations reduces to one of solving simple algebraic equations \([44,45]\). In Table 9 we give the calculated
masses of S-wave baryons.

|       | $m(J^P = \frac{1}{2}^+)$ | $m(J^P = \frac{3}{2}^+)$ |
|-------|--------------------------|--------------------------|
| N     | 0.940 (0.940)            | 1.232 (1.232)            |
| Λ     | 1.098 (1.116)            | 1.377 (1.385)            |
| Σ     | 1.193 (1.193)            | 1.524 (1.530)            |
| Θ     | 1.325 (1.315)            | 1.672 (1.672)            |

Table 9

Baryon masses $m(J^P)$ (GeV) - Note: the parameters of the model are as follows: the cut-off $\lambda_1 = 12.2$, the cut-off strange diquarks with $J^P = 0^+$ $\lambda_0 = 9.7$, the vertex functions for $J^P = 1^+$ and $J^P = 0^+$ diquarks, respectively ($g_1 = 0.540$, $g_0 = 0.702$).

The calculated vertex functions $g_1$ and $g_2$ turned out to be close to vertex function of diquarks in the Bootstrap procedure for S-wave mesons.

In our approximate solution of the three-particle equations the conditions of analyticity, unitarity and relativistic invariance of the constructed three-particle amplitudes are satisfied. This approximation is similar to the Bootstrap procedure for S-wave mesons.

The dispersion technique makes it possible to determine the form factors of composite particles (in our case, of baryons). On the one hand, the technique of dispersion integration is relativistically invariant and not related to the consideration of any specific coordinate system. On the other hand, there are no problems with the appearance of extra states, since in the dispersion relations the contributions of intermediate states are under control.

The behavior of electric form factor of proton as a function of the transverse momentum is shown in Fig.7. The form factor of non-strange quark will be assumed to be the same for u and d quarks: $f_q(q^2) = \exp(q^2/\Lambda_q)$ with $\Lambda_q = 3$ GeV$^2$ and $f_s(q^2) = \exp(q^2/\Lambda_s)$ with $\Lambda_s = 5$ GeV$^2$ for s-quarks. We can not use new parameters as compared to Bootstrap procedure.

The calculated value of the charge radii of proton and hyperons was found to be

$$\langle R_{p}^2 \rangle^{1/2} = 0.4 \text{ fm (0.8 fm)}[46]$$
$$\langle R_{\Sigma^+}^2 \rangle^{1/2} = 0.43 \text{ fm} \quad \langle R_{\Sigma^-}^2 \rangle^{1/2} = 0.39 \text{ fm} \quad \langle R_{\Xi}^2 \rangle^{1/2} = 0.38 \text{ fm} \quad (5)$$

The charge radius of the neutron was found to be practically equal to zero. It is probable that only the inclusion of higher excitations can lead to a result close to the experimental value ($R_N^2 = -0.116$ fm$^2$ [46], $R_N^2 = -0.012$ fm$^2$ [47]).

One points out that the new parameters for the calculations of electric form factors of hyperons [48] did not use.
7 Charmed baryons \((J^P = \frac{1}{2}^+, \frac{3}{2}^+)\)

In the framework of dispersion relation technique the relativistic Faddeev equations for charmed baryons are found. The inclusion of relativistic effects in composite system is important in considering the quark structure of hadrons. We give the calculated masses of S-wave charmed baryon (Table 10) \([49,50]\) and consider a method of obtaining the electric form factors of charmed baryons with quantum numbers \(J^P = \frac{3}{2}^+\) \([51]\). The behavior of electromagnetic form factors of charmed baryons \((J^P = \frac{1}{2}^+, \frac{3}{2}^+)\) in the region of low and intermediate momentum transfer \(Q^2 < 1.5 \text{ GeV}^2\) \((Q^2 = -q^2)\); the points are the experimental data \([46]\).

![Electric form factor of the proton at small and intermediate momentum transfer](image)

**Fig. 7.** Electric form factor of the proton at small and intermediate momentum transfer \(Q^2 < 1.5 \text{ GeV}^2\) \((Q^2 = -q^2)\); the points are the experimental data \([46]\).

| quark content | \(J^P = \frac{1}{2}^+\) | M | \(J^P = \frac{3}{2}^+\) | M |
|---------------|---------------------|---|---------------------|---|
| udc           | \(\Lambda_c^+\)     | 2.284 (2.285) | -                   | - |
| uuc, udc, ddc | \(\Sigma_c^{++,-,0}\) | 2.458 (2.455) | \(\Sigma_c^{++,-,0}\) | 2.516 (2.519) |
| usc, dsc      | \(\Xi_c^{+,-0(A)}\) | 2.467 (2.467) | \(\Xi_c^{+,0}\)    | 2.725 (2.645) |
| usc, dsc      | \(\Xi_c^{+,0(S)}\)  | 2.565 (2.562) | -                   | - |
| ssc           | \(\Omega_s^{0}\)    | 2.806 (2.704) | \(\Omega_c^{0}\)   | 3.108 (-) |
| ccu, ccd      | \(\Omega_{ccq,+}^{++}\) | 3.527 (-) | \(\Omega_{qq,+}^{++}\) | 3.597 (-) |
| ccs           | \(\Omega_{ccs}^{+}\) | 3.598 (-) | \(\Omega_s^{+}\)   | 3.700 (-) |
| ccc           | -                   | -          | \(\Omega_c^{++}\)  | 4.972 (-) |

**Table 10**
Charmed baryon masses \(J^P = \frac{1}{2}^+, \frac{3}{2}^+\) (GeV) - in parenthesis the PDG data is presented \([52]\); Note: parameters of model: cut-offs \(\lambda_q = 10.7\), \(\lambda_c = 6.5\) for q- and c-quarks respectively, \(g_c = 0.857\) vertex function of charmed diquark. Masses quarks \(m = 0.495 \text{ GeV}\), \(m_s = 0.77 \text{ GeV}\), \(m_c = 1.655 \text{ GeV}\).

factors of charmed baryons \((J^P = \frac{1}{2}^+)\) in the region of low and intermed-
ate momentum transfers $Q^2 < 1.5$ GeV$^2$ is determined. The calculated value (without new parameters) of charge radii: $\Lambda^+, \Sigma^+_c, \Xi^{+(A)}, \Xi^{+(S)}$ are equal $(0.12 - 0.20)$ fm, for the neutral baryons the charge radii are equal to zero.

8 Conclusion

Calculations performed in the framework of Bootstrap quark model argue in favor of the quasinuclear quark structure of hadrons. The resulting quark interaction appeared to be effectively short-range. As was stated above, this interaction is determined mainly by the exchange in the gluon channel: the constituent-gluon mass appeared to be not small. Moreover, calculations reveal an additional reason for the short-rangeness of the interaction, connected with certain specific features of the quark-quark interaction. The value of the constituent gluon mass obtained in this model (700 MeV) seems to be rather reasonable: just this mass value is required by hadron phenomenology. The mass of the constituent gluon should be close to that of vector particles. This is a consequence of $1/N_c$-expansion [53,54].

Our calculation indicates an important role of interaction which is induced by instantons. Such an interaction is necessary for deriving both the pion mass and the $\eta - \eta'$ mass splitting. However, the relative contribution of the instanton-induced interaction is less than that with the gluon exchange, the ratio of forces about 1/4. However, due to the rules of $1/N_c$-expansion this interaction influence slightly the other channels while in the $\eta - \eta'$ channel it provides the correct values of masses and gives the angle of $\eta_1 - \eta_8$ mixture close to that of the quark model. Just the minor addition of “instanton” interaction produces the correct value of the pion mass.

In the framework of the proposed approximate method of solving the relativistic three-particle problem, we have obtained a satisfactory spectrum of S-wave baryons. In this case it is obvious that the interaction forces which determine the baryon spectrum are in fact the same as for diquarks in the Bootstrap quark model for S-wave mesons. On account of the rules of the $1/N_c$-expansion the diquark forces are determined by one-gluon exchange (the instanton corrections are small in this channel). One-gluon exchange corresponds to a chromomagnetic interaction, which is responsible for the spin-spin splitting.

The mass spectrum of charmed and beauty mesons and baryons is the subject of many investigations.

In the framework of dispersion relation technique we calculated the mass spectrum of charmed mesons and baryons. The calculations show that the non-strange diquarks are in fact the same in the ordinary and charmed baryons. The interaction of the heavy quarks is described by quark amplitudes corresponding to exchange of light white and colored mesons. The main role in
the formation of the spectrum of heavy mesons is played by the forces that correspond to exchange of a massive gluon. Therefore we can point out, that the considered Bootstrap quark model allows to describe the light and heavy hadron spectroscopy based on three principles: unitarity, analyticity and relativistic symmetry (crossing symmetry) in good agreement with experimental data.

Acknowledgment:
The authors would like to thank Profs. V.A. Franke, Yu.V. Novozhilov, and H.C. Pauli. S.M. Gerasyuta thanks the Max-Planck Institut für Kernphysik in Heidelberg for the hospitality where a part of this work was completed.

References

[1] A.De Rujula, H. Georgi, S.L. Glashow, Phys. Rev. D\textbf{12}, 147 (1975).
[2] B.R. Martin and L.J. Reinders, Nucl. Phys. B \textbf{143}, 309 (1978).
[3] D.P Stanley and D. Robson, Phys. Rev. D \textbf{21}, 3180 (1980).
[4] T. Barnes, Z. Phys. C \textbf{11}, 135 (1981).
[5] I.M. Dremin and A.V. Leonidov, Teor. Mat. Fiz. \textbf{51}, 178 (1982).
[6] S. Ono and F. Schoberl, Phys. Lett. B \textbf{118}, 419 (1984).
[7] M. Frank and P. O'Donnell, Phys. Rev. D \textbf{29}, 921 (1984).
[8] J.B. Choi, Phys. Rev. D \textbf{31}, 201 (1985).
[9] S. Godfrey and N. Isgur, Phys. Rev. D \textbf{32}, 189 (1985).
[10] N. Isgur and G. Karl, Phys. Lett. B \textbf{72}, 109 (1977), Phys. Lett. B \textbf{74}, 353 (1978).
[11] L.A. Copley et.al., Phys. Rev. D \textbf{20}, 768 (1979).
[12] K.T. Chao et.al., Phys. Rev. D \textbf{23}, 155 (1981).
[13] C.P. Forsyth and R.E. Cutkosky, Z. Phys. C \textbf{18}, 219 (1983).
[14] A.M. Badalyan et.al., Yad. Fiz. \textbf{46}, 226 (1987), (Sov. I. Nucl. Phys. \textbf{46}, 139 (1987)).
[15] S. Capstick and N. Isgur, Phys. Rev. D \textbf{34}, 2809 (1986).
[16] A. Hey and R. Kelly, Phys. Rep. \textbf{96}, 71 (1983).
[17] A.B. Henriques, Z. Phys. C \textbf{11}, 31 (1981).
[18] A. Martin and J.M. Richard, Phys. Lett. B 128, 453 (1983).
[19] D.D. Brayshaw, Phys. Rev. D 36, 1465 (1987).
[20] S.M. Gupta et.al., Phys. Rev. D 34, 201 (1986).
[21] H.W. Crater and P.V. Alstine, Phys. Rev. D 37, 1982 (1988).
[22] L.J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rep. 127, 1 (1985).
[23] T.N. Aliev and M.A. Shifman, Phys. Lett. B 112, 401 (1982).
[24] M.A. Shifman, A.I. Vainstein, V.I. Zakharov, Nucl. Phys. B 147, 385 (1979).
[25] V.V. Anisovich, S.M. Gerasyuta, Yad. Fiz. 44, 174 (1986).
[26] V.V. Anisovich, S.M. Gerasyuta, A.V. Sarantsev, Int. J. Mod. Phys. A 6, 625 (1991).
[27] S.M. Gerasyuta, D.V. Ivanov, Vest. St. Petersburg University, Ser. 4, 113 (1997).
[28] E.V. Shuryak, Nucl. Phys. B 203, 93, 116, 140 (1982).
[29] D.I. Dyakonov and V.Y. Petrov, Nucl. Phys. B 245, 259 (1984).
[30] K.D. Carlitz and D.B. Creamer, Ann. Phys. (NY) 118, 249 (1979).
[31] R.G. Betman and L.V. Laperashvili, Yad. Fiz. 41, 463 (1985), (Sov. J. Nucl. Phys. 41, 295 (1985)).
[32] S.M. Gerasyuta and I.V. Kochkin, Z. Phys. C 74, 325 (1997).
[33] S.M. Gerasyuta and I.V. Kochkin, N. Cim. A 110, 1313 (1997).
[34] S.M. Gerasyuta and I.V Keltuyala, Sov. J. Part. Nucl. 54, 793 (1991), (Sov. J. Nucl. Phys. 54, 479 (1991)).
[35] S.M. Gerasyuta, Proceedings of Fock symposium, St. Petersburg, July 1998.
[36] L.N. Lipatov, Pis'ma v JETP 59, 571 (1994).
[37] L.N. Lipatov, Phys. Lett. B 309, 394 (1993).
[38] S.M. Gerasyuta and A.V. Sarantsev, Yad. Fiz. 52, 1483 (1990), (Sov. J. Nucl. Phys. 52, 937 (1990)).
[39] S.M. Gerasyuta et.al., Yad. Fiz. 53, 1397 (1991), (Sov. J. Nucl. Phys. 53, 864 (1991)).
[40] A.A. Kvitinski et.al., Yad. Fiz. 38, 702 (1986), (Sov. J. Nucl. Phys. 38, 419 (1986)).
[41] I.J.R. Aitchison, J. Phys. G 3, 121 (1977).
[42] J.J. Brehm, Ann. Phys. (NY) 108, 454 (1977).
[43] I.J.R. Aitchison and J.J. Brehm, Phys. Rev. D 20, 1113 (1979).

[44] S.M. Gerasyuta, Yad. Fiz. 55, 3030 (1992), (Sov. J. Nucl. Phys. 55, 1693 (1992)).

[45] S.M. Gerasyuta, Z. Phys. C 60, 683 (1993).

[46] M. Gourdin, Phys. Rep. 11, 29 (1974).

[47] W. Bartel et.al., Nucl. Phys. B 58, 429 (1979).

[48] S.M. Gerasyuta and D.V. Ivanov, Vest. St. Petersburg University, Ser. 4, N 2 (11), 3 (1996).

[49] S.M. Gerasyuta and D.V. Ivanov, Yad. Fiz. V. 62, 1693 (1999).

[50] S.M. Gerasyuta and D.V. Ivanov, N. Cim. A 112, 261 (1999).

[51] S.M. Gerasyuta, D.V. Ivanov, O.A. Tsykaluk, Yad. Fiz. (2000) in press.

[52] Particle Data Group (R.M. Barnet et.al.), Phys. Rev. D 54, 1 (1996).

[53] G. Hooft, Nucl. Phys. B 72, 461 (1974).

[54] G. Veneziano, Nucl. Phys. B 117, 519 (1976).