Identification of the nonlinear characteristics of rubber bearings in model-free base-isolated buildings using partial measurements of seismic responses

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Abstract
Rubber-bearing isolation is one of the most successfully and widely used isolation technologies to provide lateral flexibility and energy dissipation capacity for reducing structural vibration and protecting the superstructure from damage. The seismic performance of the base-isolated structures partly depends on the nonlinear characteristics of the base isolation system. However, it is hard to establish proper mathematical models for the nonlinear hysteretic behaviors of base isolation due to the complexities of nonlinearities. Consequently, it is strongly desired to develop model-free methodologies for the nonlinear hysteretic performance identification with no assumption on the nonlinear hysteretic models of base isolation. In this paper, a novel method is proposed for this purpose. Firstly, the base isolation is in the linear state when the structure is under the weak earthquake, the restoring force is only provided by linear stiffness and viscous damping of base isolation, and the structural physical parameters can be estimated based on the extended Kalman filter approach. Then, the base isolation is in the nonlinear state when the structure is under the strong earthquake. The nonlinear hysteretic restoring forces from base isolation are treated as “unknown fictitious inputs” to the corresponding structural systems without base isolation. The generalized Kalman filter with unknown input algorithm is adopted for the simultaneous identification of the corresponding structural systems and the hysteretic restoring force of base isolation using only partial structural responses. No information about the structure is needed, and the responses at the location of the base isolation are not required, the proposed method is capable of identifying nonlinear characteristics of base isolation by the direct use of partial structural dynamic response. To validate the performances of the proposed method, some numerical simulation examples of identifying nonlinear hysteretic restoring forces of base isolation in different models are used.

Keywords
The nonlinear hysteretic restoring force, base isolation, model-free, identification of nonlinear system, extended Kalman filter, generalized Kalman filter with unknown input

Introduction
With the gradual establishment of earthquake engineering theory and the further test of the actual earthquake on structural engineering, base isolation has become one of the effective and widely applied methods to reduce structural vibration and damage of major infrastructure and engineering structures by serious earthquakes.1–5
In many isolation structures, rubber-bearing isolation is one of the most successfully and widely used isolation technologies. It is used to set up the flexible rubber-bearing isolation system at the bottom of the superstructure to provide lateral flexibility and energy dissipation capacity for protecting the superstructure from damage.

The seismic performance of the base-isolated structures partly depends on the characteristics of the base isolation system. The force–displacement relation of the typical rubber-bearing isolation is nonlinear. Due to the complexities of the excitation, and the inherent dynamics characteristics of restoring force of the base isolation systems, the response of base-isolated structures subject to strong earthquakes often experiences excursion into the inelastic range. Therefore, in designing base-isolated structures, the nonlinear hysteretic restoring force model of the base isolation system is frequently used to predict structural response and to evaluate structural safety.

The nonlinear hysteretic performance of the base-isolated structure has been taken seriously by researchers. Several mathematical models have been proposed in the previous literature for describing the nonlinear behavior of base isolation. Chang constructed the analysis model of laminated rubber bearing using a stiffness summation procedure from Haringx’s theory. Furukawa et al. produced an identification of a base-isolated structure using the prediction error method system identification technique in conjunction with nonlinear state-space models. Using a variety of nonlinear restoring force models and bidirectional recorded seismic responses, several identification are conducted to evaluate the accuracy of the selected models. Bhuiyan et al. proposed an elasto-viscoplastic rheology model of high damping rubber bearings for seismic analysis based on the mechanical behavior of high damping rubber bearings. In this model, the Maxwell model is extended by adding a nonlinear elastic spring and an elasto-plastic model (spring-slider) in parallel. Yin et al. analyzed the nonlinear dynamic behavior of the isolation bearing using the simplified Bouc–Wen model and obtained the parameters of the isolation bearing model by applying the sequential nonlinear least-squares method. They also proposed the adaptive extended Kalman filter approach to identify the parameters and track the parametric variation of rubber-bearing isolated structure online and to identify the displacements of every story using the measured acceleration responses.

The abovementioned method for identification of nonlinear characteristics of rubber-bearing isolator required the assumptions of the proper mathematic models for the rubber bearings. However, it is still a difficult and challenging task to establish a proper mathematical model for a rubber-bearing base isolation system due to the complexity of nonlinear material of rubber bearing and the limited available data in experiments.

Recently, researchers have tried to study the identification of nonlinear systems without mathematical models. Masri and Caughey presented a restoring force surface method, constructing the nonlinear system restoring force surface with state displacement and velocity, which solved the identification problem of nonlinear parameters of single degree of freedom dynamic system. They also proposed a nonlinear parametric reduced-order model, which is completely based on the measurement data of structure dynamic responses. Liu et al. proposed a model-free structural nonlinear restoring force approximated by a power series polynomial and identify structural nonlinear restoring forces and structural systems based on the unscented Kalman filter. Lei and He treated the nonlinear effect of rubber bearing as “fictitious loading” on the linear building under severe earthquake. The proposed new algorithm is based on the sequential Kalman estimator for the structural responses and the least-squares estimation of the “fictitious loading” to identify the nonlinear force of rubber-bearing isolator. But the limitation of this method is that the horizontal acceleration response of the isolation layer is required. Therefore, identification of the nonlinear characteristics of rubber-bearing isolator without mathematical models using only partial measurements of structural responses is still an important but challenging task.

In this paper, a novel two-step approach is proposed to identify the nonlinearities of model-free rubber-bearing isolator in structures using only partial measurements of structural dynamic responses. Firstly, the rubber-bearing isolators are in the linear state when the structure is under weak earthquake, the restoring force is only provided by linear stiffness and viscous damping of rubber-bearing isolator, the structural physical parameters can be estimated based on the extended Kalman filter (EKF) approach. Then, the rubber-bearing isolators are in the nonlinear state when the structure is under strong earthquake. The nonlinear hysteretic restoring forces from rubber-bearing isolator are treated as “unknown fictitious inputs” to the corresponding structural systems without base isolation. The generalized Kalman filter with unknown input (GKF-UI) algorithm is adopted for the simultaneous identification of the corresponding structural systems and the hysteretic restoring force of rubber-bearing isolator using only partial structural responses. No information about the structure is needed, and the responses at the location of the rubber-bearing isolator are not required, the proposed method is capable of identifying nonlinear characteristics of base isolation by the direct use of partial structural dynamic response measurements. To validate the performances of the proposed method, some numerical simulation examples of identifying hysteretic restoring forces of base isolation in different models are used. Simulation results
demonstrate that the proposed algorithms are capable of identifying the nonlinear properties of rubber-bearing isolator with good accuracy.

Identification algorithm
For an \( n \)-storied shear building with a rubber-bearing isolation system subject to earthquake ground motion as shown by Figure 1, the equation of relative motion with respect to the ground can be written as

\[
M \ddot{x}(t) + C \dot{x}(t) + R[x(t), \dot{x}(t), z(t)] = -MI \ddot{x}_g(t)
\]

where \( x, \dot{x}, \) and \( \ddot{x} \) are the \( n + 1 \)-th dimensional vectors of displacement, velocity, and acceleration response of the building relative to the ground motion, respectively, \( \ddot{x}_g \) is the ground acceleration motion, \( M, C \) are the mass, damping matrix of the whole structure system, respectively, \( I \) is an \( n + 1 \)-dimensional unit vector, \( R[x(t), \dot{x}(t), z(t)] \) is the nonlinear restoring force of the rubber-bearing base isolator, the \( n + 1 \)-dimensional column vector, which is composed of elastic restoring force and plastic restoring force, i.e.

\[
R[x(t), \dot{x}(t), z(t)] = Kx(t) + H_b f_b
\]

where \( Kx(t) \) is the elastic restoring force and \( K \) is the stiffness matrix of the whole structure system. \( H_b f_b \) is the plastic restoring force and \( H_b = [1, 0, \cdots, 0]^T \) is an \( n + 1 \)-th dimensional vector presenting the location of isolation system at the base of the building.

Identification of the structures in linear state
When the structure is under weak earthquake, the base isolation is in the linear state. Thus, the nonlinear plastic restoring force \( f_b \) is zero, the nonlinear restoring force of the rubber-bearing base isolator \( R[x(t), \dot{x}(t), z(t)] = Kx(t) \). Then, the equation of motion equation (1) becomes

\[
M \ddot{x}_l(t) + C \dot{x}_l(t) + Kx_l(t) = -MI \ddot{x}_g(t)
\]

where \( \ddot{x}_l, \dot{x}_l, \) and \( x_l \) are the vectors of displacement, velocity, and acceleration response of the structure in linear state relative to the ground motion, respectively.

The extended state vector is defined as

\[
Z = \begin{bmatrix} x_l^T & \dot{x}_l^T & \theta_l^T \end{bmatrix}^T
\]

Figure 1. Schematic diagram of base-isolated structure.
where $\theta = \{ k_b, k_1, k_2, \ldots, k_n, c_b, c_1, c_2, \ldots, c_n \}^T$, $k_b$ and $c_b$ are the initial stiffness and viscosity damping coefficient of base isolator, respectively, $k_i$ ($i = 1 \cdots n$) and $c_i$ ($i = 1 \cdots n$) are the stiffness and damping coefficient of superstructure.

Equation (3) can be written into the following extended state equation for the extended state vector

$$
\dot{Z} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_l \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} M^{-1} \left( -MI\dot{y}_g(t) - C\dot{x}_l(t) - Kx_l(t) \right) \\ 0 \end{bmatrix} = g(Z, \dot{x}_g) + w(t)
$$

(5)

where $w(t)$ is the model noise (uncertainty) with zero mean and a covariance matrix $Q(t)$.

Usually, limited acceleration sensors are installed in the structure, so the observation equation by limited accelerometers installed in the structure at time $t = (k + 1)\Delta t$ can be written as

$$
y_{k+1} = h(Z_{k+1}, \dot{x}_{g,k+1}) + v_{k+1}
$$

(6)

where $y_{k+1}$ is the measured response vector and $D_{acc}$ is the matrix associated with the locations of accelerometers. $v_{k+1}$ is the measured noise vector, which is assumed a Gaussian white noise vector with zero mean and a covariance matrix $R_{k+1}$.

Then, the structural stiffness parameters, damping coefficient, and the linear stiffness of base isolation can be estimated based on the EKF approach as follows.

Let $\hat{Z}_{k|k}$ and $\hat{Z}_{k+1|k}$ be the estimates of $Z_k$ and $Z_{k+1}$ given the observations ($z_1, z_2, \ldots, z_k$), respectively, equations (5) and (6) can be linearized at $\hat{Z}_{k|k}$ and $\hat{Z}_{k+1|k}$ by Taylor series expansion to the first order as follows

$$
g(Z, \dot{x}_g) \approx g(\hat{Z}_{k|k}, \dot{x}_{g,k}) + G_{k|k}(Z - \hat{Z}_{k|k}); \quad G_{k|k} = \frac{\partial g(Z, \dot{x}_g)}{\partial Z} \bigg|_{Z = \hat{Z}_{k|k}}
$$

(8)

$$
h(Z_{k+1}, \dot{x}_{g,k+1}) \approx h\left(\hat{Z}_{k+1|k}, \dot{x}_{g,k+1}\right) + H_{k+1|k}(Z_{k+1} - \hat{Z}_{k+1|k}); \quad H_{k+1|k} = \frac{\partial h(Z, \dot{x}_g)}{\partial Z} \bigg|_{Z = \hat{Z}_{k+1|k}}
$$

(9)

The extended Kalman filter mainly consists of the two procedures: the time update (prediction) and measurement update (correction).

The first procedure is the time update (prediction), where

$$
\hat{Z}_{k+1|k} = \hat{Z}_{k|k} + \int_{k\Delta t}^{(k+1)\Delta t} g(\hat{Z}_{\ell|k}, \dot{x}_g)\text{d}\ell
$$

(10)

and the prediction error of $\hat{Z}_{k+1|k}$ is $\hat{e}_{k+1|k} = Z_{k+1} - \hat{Z}_{k+1|k}$ with the prediction error covariance matrix $P_{k+1|k} = E[\hat{e}_{k+1|k} \hat{e}_{k+1|k}^T]$.

Based on equations (5)-(8) and (10), it can be derived that

$$
P_{k+1|k} = \Phi_{k|k} \hat{P}_{k|k} \Phi_{k|k}^T + Q_{k}
$$

(11)

where $\Phi_{k|k} = I_{2n+1} + \Delta t G_{k|k}$, $\hat{P}_{k|k} = E[\hat{e}_{k|k} \hat{e}_{k|k}^T]$, $I_{2n+1}$ is a unit matrix of dimension $2n + 1$.

The second process of the EKF is the measurement update (correction) procedure, where

$$
Z_{k+1|k+1} = \hat{Z}_{k+1|k} + K_{k+1} \left[ y_{k+1} - h\left(\hat{Z}_{k+1|k}, \hat{x}_{g,k+1}\right) \right]
$$

(12)
where $\hat{Z}_{k+1\mid k+1}$ is the estimate of $Z_{k+1}$ given the observations $(y_1, y_2, \ldots, y_{k+1})$ and $K_{k+1}$ is the Kalman gain matrix, which is derived as

$$K_{k+1} = \hat{P}_{k+1\mid k}H_{k+1\mid k}^T \left( H_{k+1\mid k} \hat{P}_{k+1\mid k}H_{k+1\mid k}^T + R_{k+1} \right)^{-1} \tag{13}$$

Then, the error for the measurement updated $\hat{Z}_{k+1\mid k+1}$ is derived from equations (6)–(9) and equations (11) and (12) as

$$\hat{e}_{k+1\mid k+1} = Z_{k+1} - \hat{Z}_{k+1\mid k+1} = \hat{e}_{k+1\mid k+1} + K_{k+1} \left[ y_{k+1} - h(\hat{Z}_{k+1\mid k}, \hat{x}_{k+1}) \right]$$

$$= (I_{2n+l} - K_{k+1}H_{k+1\mid k}) \hat{e}_{k+1\mid k} + K_{k+1}v_{k+1} \tag{14}$$

The covariance matrix for error $\hat{e}_{k+1\mid k+1}$ is defined as $\hat{P}_{k+1\mid k+1} = E[\hat{e}_{k+1\mid k+1} \hat{e}_{k+1\mid k+1}^T]$. From equation (11), it is known that

$$\hat{P}_{k+1\mid k+1} = (I_{2n+l} - K_{k+1\mid k+1}H_{k+1\mid k}) \hat{P}_{k+1\mid k} (I_{2n+l} - K_{k+1\mid k+1}H_{k+1\mid k})^T + K_{k+1\mid k+1}R_{k+1\mid k+1} \tag{15}$$

Thus, the extended state vector $Z = \{x^T, \dot{x}^T, \theta^T\}$, including the structural stiffness parameters $K$, damping $C$, the linear stiffness $k_b$, and damping $c_b$ of base isolation can be estimated based on the EKF approach.

It is worth noting that only the identification result of structural stiffness and damping parameters are used in the second step to identify the nonlinear restoring force of the base isolation without mathematical model.

**Identification of nonlinear property of base isolation**

When the structure is subjected to strong earthquake, the base isolation is in nonlinear state, i.e., $f_h \neq 0$. Thus, the nonlinear restoring force $R[x(t), \dot{x}(t), z(t)] = Kx(t) + H_h f_k$. The nonlinear plastic restoring force $f_k$ is treated as “unknown fictitious inputs” to the corresponding structural systems. Then, the motion equation (1) can be rewritten as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -MI\ddot{x}_g(t) - H_h f_k$$

$$\tag{16}$$

Thus, the recent GKF-UI algorithm is adopted for the real-time identification of the “unknown fictitious inputs” with only partial measurements of structural responses.

**A brief review of recent GKF-UI.** The equation of motion of $n$-dimensional linear structure under unknown external excitation is as follows

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Df(t) + Ef''(t)$$

$$\tag{17}$$

where $f(t)$ is the known external excitation vector with influence matrix $D$, $f''(t)$ is the unknown external excitation vector with influence matrix $E$.

In order to overcome the limitations of requiring the measurement of the response at the location of external excitation, it is assumed that the excitation to be identified is linear in the sampling interval using discrete method of first-order hold sampling instead of zero-order hold sampling.

$$f''(t) = f''_k + \frac{f''_{k+1} - f''_k}{\Delta t} (t - k\Delta t); \quad k\Delta t \leq t \leq (k + 1)\Delta t$$

$$\tag{18}$$

where $f''_k$ and $f''_{k+1}$ are the unknown external excitation vector at time $t = k\Delta t$ and $t = (k + 1)\Delta t$ with $\Delta t$ being the sampling time step. The state equation of the system in the discrete form can be expressed as

$$X_{k+1} = A_k X_k + D_k f'_k + B_k f''_k + G_{k+1} f''_{k+1} + w_k$$

$$\tag{19}$$

where $X_{k+1}$ and $X_k$ are the state vector at time $t = (k + 1)\Delta t$ and $t = k\Delta t$. $A_k$ is the state transformation matrix. $f_k$ is the known external excitation vector at time $t = k\Delta t$ with influence matrix $D_k$. $B_k$ and $G_{k+1}$ are the influence
matrix of unknown external excitation vector at time $t = k\Delta t$ and $t = (k + 1)\Delta t$, and $w_k$ is the model noise (uncertainty) with zero mean and a covariance matrix $Q_k$.

The discrete observation equation by limited accelerometers installed in the structure at time $t = (k + 1)\Delta t$ can be written as

$$Y_{k+1} = C_{k+1}X_{k+1} + H_{k+1}^e\hat{f}_{k+1} + \hat{H}_{k+1}^u + v_{k+1}$$  \hspace{1cm} (20)

where $C_{k+1}$ is known measurement matrices associated with structural state, $H_{k+1}^e$ and $H_{k+1}^u$ are known measurement matrices associated with known external force vectors and unknown external force vectors at time $t = (k + 1)\Delta t$, respectively. When the response at location of unknown external force is measured, $H_{k+1}^u \neq 0$, otherwise $H_{k+1}^u = 0$.

The predicted $X_{k+1}$ in the time update procedure is derived as

$$\hat{X}_{k+1|k} = A_k\hat{X}_{k|k} + D_kf_k + B_k\hat{f}_k^u + G_k\hat{f}_{k+1|k+1}$$  \hspace{1cm} (21)

where $\hat{X}_{k+1|k}$ and $\hat{X}_{k|k}$ denote the predicted $X_{k+1}$ and estimated $X_k$ at time $t = k\Delta t$, $f_k$ is the known excitations at time $t = k\Delta t$, and $\hat{f}_k^u$ and $\hat{f}_{k+1|k+1}$ are the estimated unknown excitations at time $t = k\Delta t$ and $t = (k + 1)\Delta t$.

The estimated $X_{k+1}$ in the measurement update procedure is derived as

$$\hat{X}_{k+1|k+1} = \hat{X}_{k+1|k} + K_{k+1}(Y_{k+1} - C_{k+1}\hat{X}_{k+1|k} - H_{k+1}^e\hat{f}_{k+1} - H_{k+1}^u\hat{f}_{k+1|k+1})$$  \hspace{1cm} (22)

where $\hat{X}_{k+1|k+1}$ is the estimated $X_{k+1}$ at time $t = (k + 1)\Delta t$ given the observations ($Y_1, Y_2, \ldots, Y_{k+1}$) and $K_{k+1}$ is the Kalman gain matrix, which is devised as

$$K_{k+1} = \hat{P}_{X_{k+1|k+1}}C_{k+1}^T(C_{k+1}\hat{P}_{X_{k+1|k+1}}C_{k+1}^T + R_k)^{-1}$$  \hspace{1cm} (23)

Under the condition that the number of response measurements is larger than the number of unknown external excitations, $\hat{f}_{k+1|k+1}$ can be estimated by minimizing the following error vector $\Delta_{k+1}$ as

$$\Delta_{k+1} = Y_{k+1} - C_{k+1}\hat{X}_{k+1|k+1} - H_{k+1}^e\hat{f}_{k+1} - H_{k+1}^u\hat{f}_{k+1|k+1}$$  \hspace{1cm} (24)

By inserting the expression of $\hat{X}_{k+1|k+1}$ in equation (22) into the above error vector, and based on the least-squares estimation, $\hat{f}_{k+1|k+1}$ is estimated as

$$\hat{f}_{k+1|k+1} = S_{k+1}[Y_{k+1} - C_{k+1}(A_k\hat{X}_{k|k} + B_k\hat{f}_k^u + D_kf_k) - H_{k+1}^e\hat{f}_k]$$  \hspace{1cm} (25)

where

$$S_{k+1} = (C_{k+1}G_{k+1} + H_{k+1}^e)^T[(C_{k+1}G_{k+1} + H_{k+1}^e)(C_{k+1}G_{k+1} + H_{k+1}^e)^T]^{-1}$$  \hspace{1cm} (26)

The error covariance matrix of $\hat{X}_{k+1|k+1}$ can be simplified as

$$\hat{P}_{X_{k+1|k+1}} = (I - C_{k+1}K_{k+1})\hat{P}_{X_{k+1|k}}$$  \hspace{1cm} (27)

The error covariance matrix of $\hat{f}_{k+1|k+1}$ can be expressed as

$$\hat{P}_{f_{k+1|k+1}} = M_kC_k[\hat{A}_k B_k] \begin{bmatrix} \hat{P}_{f_{k|k}} & \hat{P}_{f_{k}^Y} & \hat{P}_{f_{k}^T} & \hat{P}_{f_{k}^T} \end{bmatrix} \begin{bmatrix} \hat{A}_k^T & \hat{B}_k^T \end{bmatrix} (M_kC_k)^T + M_kR_k + M_kC_k Q_k (M_kC_k)^T$$  \hspace{1cm} (28)
The error covariance matrix of $\dot{X}_{k+1|k}$ can be derived as

$$
\hat{P}^X_{k+1|k} = (I - G_{k+1}M_{k+1}C_{k+1})[A_k \ B_k][P_{k|k} \ P_{k|k}^T][A_k^T \ B_k^T](I - G_{k+1}M_{k+1}C_{k+1})^T + G_{k+1}M_{k+1}R_{k+1|M_{k+1}}^T + (I - G_{k+1}M_{k+1}C_{k+1})Q_{k+1}(I - G_{k+1}M_{k+1}C_{k+1})^T
$$

(29)

The error covariance matrix $\hat{P}^{XY}_{k+1|k+1}$ can be derived as

$$
\hat{P}^{XY}_{k+1|k+1} = -(I - C_{k+1}K_{k+1})(I - G_{k+1}M_{k+1}C_{k+1})[A_k \ B_k][P_{k|k} \ P_{k|k}^T][A_k^T \ B_k^T] + (M_{k+1}C_{k+1}) + (I - G_{k+1}M_{k+1}C_{k+1})G_{k+1}M_{k+1} + K_{k+1}R_{k+1|M_{k+1}}^T

-I - C_{k+1}K_{k+1})(I - G_{k+1}M_{k+1}C_{k+1})Q_{k+1}(M_{k+1}C_{k+1})^T

\hat{P}^{FX}_{k+1|k+1} = \hat{P}^{FX}_{k+1|k+1}^T
$$

(30)

The GKF-UI algorithm is achievable for the station that the responses at location of unknown external force are not measured.

**Identification of the nonlinear restoring forces of base isolation.** Based on equation (19), the motion equation (16) can be expressed as the state equation of the system in the discrete form

$$
X_{k+1} = A_kX_k + D_k\ddot{x}_{g,k} + B_kf_{h,k} + G_kf_{h,k+1} + w_k
$$

(32)

where $\ddot{x}_{g,k}$ is the earthquake excitation at time $t = k\Delta t$ with the influence matrix $D_k$, $f_{h,k}$ and $f_{h,k+1}$ are the nonlinear plastic restoring force of base isolation at time $t = k\Delta t$ and $t = (k + 1)\Delta t$, respectively, which is treated as “unknown fictitious inputs” to the corresponding structural systems. $B_k$ and $G_k$ are the influence matrix of “unknown fictitious inputs” at time $t = k\Delta t$ and $t = (k + 1)\Delta t$, respectively.

Only partial structural responses can be measured. Based on equation (20), the discrete form of the observation equation at time $t = (k + 1)\Delta t$ can be expressed as

$$
Y_{k+1} = C_{k+1}X_{k+1} + H_{k+1}X_{k+1}^* + H_{k+1}^*f_{h,k+1} + v_{k+1}
$$

(33)

When the response at location of “unknown fictitious inputs” is measured, $H_{k+1}^* \neq 0$, otherwise $H_{k+1}^* = 0$. In addition, since the seismic acceleration produce inertial forces at each floor in n-storied shear building, no matter acceleration responses of which floor are measured, the responses at location of the known external excitation are partially observed, so $H_{k+1}^* \neq 0$.

To circumvent the drift problem in the identification of displacement and nonlinear restoring force, partial measured strain is added. Data fusion of measured strain and acceleration responses are used in the observation equation, the measurement matrix $C_{k+1}$ is expressed as

$$
C_{k+1} = \begin{bmatrix} D_c & 0 \\ 0 & D_{acc} \end{bmatrix}\begin{bmatrix} P & 0 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}
$$

(34)

where $D_{acc}$ is the measurement matrix associate with the measured acceleration and $D_c$ is the measurement matrix associate with the measured strain. $P$ is derived from the relationship between strain and interlayer
displacement as

\[
P = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
-1 & 1 & \ddots & \\
0 & -1 & \ddots & 1 \\
\vdots & \ddots & \ddots & 1 \\
0 & \cdots & 0 & -1
\end{bmatrix}
\begin{aligned}
(6 - 12u/L) \cdot v/L^2
\end{aligned}
\]  

(35)

For column elements in shear building, the displacement vector of column element node \( \mathbf{d} = \{0 \ d_{c2} \ 0 \ 0 \ d_{c5} \ 0\}^T \), where \( d_{c2} \) and \( d_{c5} \) are the displacement of 2nd and 5th degree of freedom. The location of the strainometer at column element is \((u, v)\), \( L \) is the height of column. The displacement–strain relation can be expressed as follows

\[
v = (6 - 12u/L) \cdot v/L^2 \cdot (d_{c2} - d_{c5})
\]  

(36)

Then, the identified nonlinear plastic restoring forces of base isolation \( F_{id}^b \) can be estimated by GKF-UI algorithm with only partial measurements of structural responses. The identified displacement of base isolator respect to ground \( x_{id}^b(t) \) is also simultaneously estimated by GKF-UI algorithm. The identified linear stiffness \( k_{id}^b \) of base isolation is derived by EKF approach in the “Identification of the structures in linear state” section. The nonlinear restoring force of the rubber-bearing isolator is composed of elastic restoring force and plastic restoring force shown in equation (2). Hence the total nonlinear restoring force of the base isolator is identified as

\[
F_{id}^b(t) = -f_{id}^b(t) + k_{id}^b x_{id}^b(t)
\]  

(37)

**Numerical example**

A nine-storied shear building with rubber-bearing isolation is used as a numerical example to demonstrate and verify the proposed identification algorithm. The linear structure parameters are selected: \( m_i = 60 \text{ kg}, k_i = 1.2 \times 10^5 \text{ N/m} \) \((i = 1, 2, \ldots, n), n = 9\), viscous damping coefficient \( c_i = 1000 \text{ N} \cdot \text{s/m} \) \((i = 1, 2, \ldots, n)\).

The structural parameters of base isolator are selected: \( m_b = 65 \text{ kg}, k_b = 1.0 \times 10^5 \text{ N/m}, c_b = 800 \text{ N} \cdot \text{s/m} \).

Seismic ground motion is assumed as El-Centro earthquake with larger peak ground amplitudes, the excitation time was 31.2 s, and the sampling frequency was 1000 Hz.

**Identification of nonlinear property of base isolation simulated by Bouc–Wen model**

It is assumed that the force-deformation relation of base isolation is modeled by Bouc–Wen nonlinear hysteretic model. Its nonlinear restoring force can be expressed as

\[
F_{BW}^b = x_b k_b + (1 - x_b) k_b z_{BW}^b
\]  

(38)

where \( x_b \) is the displacement of the base isolation system relative to ground, \( k_b \) is the initial stiffness of the base isolation, \( x_b \) is the ratio of post-yielding stiffness to the stiffness before yielding, and \( z_{BW}^b \) is the dimensionless hysteretic components satisfying a first-order differential equation

\[
\frac{dz_{BW}^b}{dt} = x_b \left\{ \beta_b \frac{dz_{BW}^b}{dt} |z_{BW}^b|^{n_b-1} \frac{dz_{BW}^b}{dt} + \gamma_b \frac{dz_{BW}^b}{dt} |z_{BW}^b|^{n_b}\right\}
\]  

(39)

\( \beta_b, \gamma_b, \) and \( n_b \) are the dimensionless parameters of Bouc–Wen model, which control the shape of the hysteresis loop, and \( \dot{x}_b \) is the velocity of base isolator relative to ground. The parameters of Bouc–Wen model are selected as follows: \( x_b = 0.1, \beta_b = 2000, \gamma_b = 2000, \) and \( n_b = 1.25. \)
Then, the total nonlinear restoring force of the base isolator is expressed as

\[ R[x(t), \dot{x}(t), z(t)] = Kx(t) + H_b(1 - \alpha_b)k_b(z_{BW} - x_b) \]  \hspace{1cm} (40)

where \( Kx(t) \) is elastic restoring force.

It worth noting that the structural stiffness parameters, damping parameters and Bouc–Wen nonlinear model of base isolator are only used to simulate the structural responses, which are unknown in the identification process.

In the numerical example, only five accelerometers are deployed on the building, i.e., the 1st, 3rd, 5th, 7th, and 9th floor acceleration responses are measured. The acceleration responses at the location of base isolator are not measured. To circumvent the drift problem in the identification of displacement and nonlinear restoring force, partial measured strain is added. In the numerical example, the measured strain at the 1st floor of the building is added. Data fusion of the measured strain and the above five accelerations are used in the observation equation. To consider the influence of measurement noise on the identification results, all the measured acceleration responses are simulated by superimposing the theoretically computed responses onto the corresponding stationary white noise with 2% noise-to-signal ratio in RMS.

**Identification results of linear system.** When the peak ground amplitude (PGA) of the earthquake is small (PGA = 0.10 g), the whole structures including base isolator are in linear state. Based on the identification algorithm in the “Identification of the structures in linear state” section, the linear structural parameters are identified and compared with their actual values in Table 1. From the comparison, it is shown that the proposed algorithm can identify linear structural parameters with good accuracy. These identified linear structural parameters will be used for the identification of nonlinear characteristic of base isolator.

**Identification results of nonlinear system.** When the building is subjected to strong earthquake with a large PGA value (PGA = 0.5 g), the base isolator is in the nonlinear state. In the numerical example, it is assumed that the nonlinear hysteretic restoring force of base isolator is modeled by Bouc–Wen model. However, this nonlinear model is only used in the numerical simulation of structural responses and not used in the identification procedure by the proposed algorithm. Based on the proposed algorithm in the “Identification of nonlinear property of base isolation” section, the nonlinear hysteretic restoring forces of base isolator can be identified.

The comparison of identified nonlinear hysteretic restoring forces of base isolator with actual restoring force are shown in Figures 2 and 3, the comparison of the identified relative displacement with actual value are shown in Figure 4. From the comparison above, the identification of nonlinear hysteretic restoring forces and displacement are good. Thus, it is shown that the proposed algorithm can identify the nonlinear hysteretic restoring forces of model-free base isolator with good accuracy.

**Table 1.** Comparison of the identification results of linear structural parameters with actual values.

| Structural stiffness (kN/m) | Actual values | Identified values | Error (%) | Structural damping (N·s/m) | Actual values | Identified values | Error (%) |
|-----------------------------|---------------|-------------------|-----------|----------------------------|---------------|-------------------|-----------|
| \( k_b \)                  | 100           | 100.00            | 0.00      | \( c_b \)                 | 800           | 809               | 1.18      |
| \( k_1 \)                  | 120           | 120.00            | 0.00      | \( c_1 \)                 | 1000          | 985               | -1.48     |
| \( k_2 \)                  | 120           | 120.06            | 0.05      | \( c_2 \)                 | 1000          | 1012              | 1.17      |
| \( k_3 \)                  | 120           | 119.92            | -0.06     | \( c_3 \)                 | 1000          | 986               | -1.42     |
| \( k_4 \)                  | 120           | 120.08            | 0.06      | \( c_4 \)                 | 1000          | 1008              | 0.79      |
| \( k_5 \)                  | 120           | 119.92            | -0.06     | \( c_5 \)                 | 1000          | 991               | -0.92     |
| \( k_6 \)                  | 120           | 120.04            | 0.04      | \( c_6 \)                 | 1000          | 1005              | 0.50      |
| \( k_7 \)                  | 120           | 119.95            | -0.04     | \( c_7 \)                 | 1000          | 992               | -0.77     |
| \( k_8 \)                  | 120           | 120.04            | 0.03      | \( c_8 \)                 | 1000          | 1006              | 0.59      |
| \( k_9 \)                  | 120           | 119.81            | -0.16     | \( c_9 \)                 | 1000          | 984               | -1.56     |
Identification of nonlinear property of base isolation simulated by bilinear model

It is assumed that the force-deformation relation of base isolation is modeled by bilinear hysteretic nonlinear model. Its nonlinear restoring force can be expressed as

$$R_{bilinear}^b = k_b x_b + k_z z_b$$  \hspace{1cm} (41)

**Figure 2.** Comparison of identified restoring force of base isolator with actual restoring force.

**Figure 3.** Comparison of time-history curve of identified restoring force of base isolator with actual restoring force.

**Figure 4.** Comparison of the identified relative displacement with actual value. (a) The isolation layer. (b) The sixth floor.

**Identification of nonlinear property of base isolation simulated by bilinear model**

It is assumed that the force-deformation relation of base isolation is modeled by bilinear hysteretic nonlinear model. Its nonlinear restoring force can be expressed as
The nonlinear hysteretic restoring force of base isolator is modeled by bilinear model which is only used in the
identification results of nonlinear system. For the identification of nonlinear characteristic of base isolator.
Identify linear structural parameters with good accuracy. These identified linear structural parameters will be used
pared with their actual values in Table 2. From the comparison, it is shown that the proposed algorithm can
"Identification of the structures in linear state" section, the linear structural parameters are identified and com-
whole structures including base isolator are in linear state. Based on the identification algorithm in the
Identification results of linear system.

When the peak ground amplitude of the earthquake is small (PGA = 0.10 g), the base isolator is in the nonlinear state. Based on the identification algorithm in the
Identification results of nonlinear system. When the building is subjected to strong earthquake with a large PGA value
(PGA = 0.5 g), the base isolator is in the nonlinear state. In the numerical example, it is assumed that the
nonlinear hysteretic restoring force of base isolator is modeled by bilinear model which is only used in the
numerical simulation of structural responses. Based on the proposed algorithm in the “Identification of nonlinear property of base isolation” section, the nonlinear hysteretic restoring forces of base isolator can be identified.

The comparison of identified nonlinear hysteretic restoring forces of base isolator with actual restoring force are shown in Figures 5 and 6, the comparison of the identified relative displacement with actual value are shown in Figure 7. From the comparison above, the identification of nonlinear hysteretic restoring forces and displacement are good. Thus, it is shown that the proposed algorithm can identify the nonlinear hysteretic restoring forces of model-free base isolator with good accuracy.

Figure 5. Comparison of identified restoring force of base isolator with actual restoring force.

Figure 6. Comparison of time-history curve of identified restoring force of base isolator with actual restoring force.

Figure 7. Comparison of the identified relative displacement with actual value. (a) The third floor. (b) The sixth floor.
Conclusions

In this paper, a new two-step approach is proposed to identify the nonlinearities of rubber bearings in model-free base-isolated buildings using only partial measurements of structural seismic responses. In the first step, the structural physical parameters can be estimated based on EKF approach when the structure is subject to weak earthquake and base isolators are in the linear state. In the second step, the nonlinear hysteretic restoring forces from base isolators, treated as “unknown fictitious inputs” to the corresponding structural systems without base isolators, can be simultaneously identified with corresponding structural systems by GKF-UI when the structure is subject to strong earthquake and base isolators are in the nonlinear state. To circumvent the drift problem in the identification of displacement and “unknown fictitious inputs,” data fusion of measured strain and acceleration responses are used in the observation equation. No information about the structure is needed, and no acceleration responses at base isolator are required, the proposed method is capable of identifying hysteretic restoring forces of base isolator by partial structural dynamic response measurements. Such an innovation approach can overcome the limitation of the previous method10 to observe the acceleration responses at location of base isolation. To validate the performances of the proposed method, two numerical simulation examples of identifying hysteretic restoring forces of base isolation in different models are used with good accuracy.

In this paper, the proposed method is applicable to the cases when earthquake is known. Further research on the identification of the characteristics of the base isolation system when earthquake is unknown should be studied. To further testify the identification algorithms, experimental tests should be conducted in a future work.

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