On temperature determining by maximum spectral radiation isothermal system of opaque bodies

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Abstract. It is shown that, for the temperature determination of the isothermal opaque bodies system, one can use the same techniques those are used to determine the temperature of a free-radiating opaque body. For an isothermal system of opaque bodies, a relationship was obtained, which relates the wavelength of maximum spectral radiation, the spectral effective emissivity and the thermodynamic temperature. In particular, this relationship can be used as an additional condition to determine the thermodynamic temperature of the isothermal system of bodies by the thermal radiation spectrum of the target surface, when the effective emissivity of the radiation source is unknown.

1. Introduction
The thermodynamic (true) temperature $T$ is an important parameter of state of matter [1]. The source of thermal emission is the target (target surface) for which the temperature is to be measured. In determining this temperature by radiation thermometry methods, it is commonly assumed that the target surface radiates freely [2]. As is known, the radiation thermometer records the effective spectral intensity $I_{ef}(\lambda, M, s_M)$, which is the sum of the spectral intensity $I_{em}(\lambda, M, s_M)$ of emitted radiation and the spectral intensity $I_{ref}(\lambda, M, s_M)$ of reflected radiation at point of $M$, in the direction of $s_M$ and fixed wavelength of $\lambda$, i.e.

$$I_{ef}(\lambda, M, s_M) = I_{em}(\lambda, M, s_M) + I_{ref}(\lambda, M, s_M).$$

(1)

In this paper, it is shown that the methods presented in [3] can be extended to an isothermal system consisting of $n$ opaque bodies. It is assumed that all bodies have the same temperature $T$.

For this purpose, resolvent angular factors (radiative exchange factor [4]) offered in [5, 6] are used. A detailed derivation of resolvent angular factors for diffusely reflecting and radiating bodies is presented in [7]. For an isothermal system of opaque bodies, the relationship is obtained, which relates the wavelength of maximum spectral radiation, the spectral effective emissivity and thermodynamic temperature of the target surface. In the particular case of black or gray radiation, this relationship is identical to the Wien displacement law.

2. Basic relationships
Previously [8, 9], it was demonstrated that, for free-radiating body, one can obtain the relationship similar to the Wien displacement law for blackbody spectral radiation maximum. In
the present paper, it is shown that a similar relationship can also be obtained for an isothermal system of \( n \) bodies separated by a transparent medium.

Before deduction of the temperature calculation relationship on an intensity maximum, one should accept following assumptions:

(i) isothermal system consists of \( n \) radiating opaque isothermal bodies with the same temperature and optical properties;
(ii) in the selected spectral range, the spectral emissivity \( \epsilon \) of material is a continuous and monotonic function on \( \lambda \);
(iii) in the selected spectral range, the spectral distribution of \( I_{ef}(\lambda) \) has only a single maximum.

In this case, radiative exchange is convenient to consider between \( n \) surfaces of these bodies. Let the surface element \( dF_{Mi} \) belongs to the \( i \)-th body. Then according to [6] (1) can be written in more detail \((M_i \in F_i, N_j \in F_j, i = 1, \ldots, n)\)

\[
I_{ef}(\lambda, M_i, s_{Mi}) = I_{em}(\lambda, M_i, s_{Mi}) + \sum_{j=1}^{n} \int_{F_j} \Omega(\lambda, M_i, N_j, s_{NjM_i}, s_{Mi}) I_0(\lambda, N_j) dF_{Nj}, \tag{2}
\]

where \( I_0(\lambda, N_j) \) is the spectral blackbody intensity; \( I_{ef}(\lambda, M_i, s_{Mi}) \) is the effective spectral intensity in direction of \( s_{Mi} \); \( I_{em}(\lambda, M_i, s_{Mi}) \) is the emitted spectral intensity in direction of \( s_{Mi} \); \( \Omega(\lambda, M_i, N_j, s_{NjM_i}, s_{Mi})dF_{Nj} \) is the resolvent angular factors that characterizes radiation exchange between two surface elements \( dF_{Mi} \) and \( dF_{Nj} \).

For an isothermal system, temperature \( T_{Mi} = T_{Nj} = T \). Therefore, \( I_0(\lambda, N_j) \equiv I_0(\lambda, M_i) \equiv I_0(\lambda, T) \). If divide (2) termwise by \( I_0(\lambda, T) \), one can write:

\[
\epsilon_{ef}(\lambda, M_i, s_{Mi}) = \epsilon(\lambda, M_i, s_{Mi}) + \sum_{j=1}^{n} \int_{F_j} \Omega(\lambda, M_i, N_j, s_{NjM_i}, s_{Mi}) dF_{Nj}, \tag{3}
\]

where \( \epsilon_{ef}(\lambda, M_i, s_{Mi}) \) is the effective spectral emissivity in point \( M_i \) in direction \( s_{Mi} \); \( \epsilon(\lambda, M_i, s_{Mi}) \) is the emitted spectral emissivity in point \( M_i \) in direction \( s_{Mi} \).

From equations (2) and (3), one has

\[
I_{ef}(\lambda, M_i, s_{Mi}) = \epsilon_{ef}(\lambda, M_i, s_{Mi}) I_0(\lambda, T). \tag{4}
\]

According to (3) at \( T_{Mi} = T_{Nj} = T \) value \( 0 < \epsilon_{ef}(\lambda, M_i, s_{Mi}) < 1 \).

As is well known, for free-radiating element surface \( dF_{Mi} \),

\[
I_{em}(\lambda, M_i, s_{Mi}) = \epsilon_{em}(\lambda, M_i, s_{Mi}) I_0(\lambda, T). \tag{5}
\]

Formally, (4) is identical to (5), if in (4) the index of \( ef \) replaced by the index of \( em \). In principle, the relations (4) and (5) can be close together and from the physical point of view, for example, if the emitting surface roughness is significant, and therefore the surface radiates onto itself. In this case, for the analysis of radiative exchange, it is necessary to use relations (4), which take into account multiple reflections.

Thus, to determine the thermodynamic temperature \( T \) in an isothermal system of bodies, one can use the same methods as for the free-radiating element \( dF_{Mi} \) of surface \( F_i \).

It is demonstrated that, for the isothermal system of bodies, as well as for free-radiating body, can be obtained the relationship similar to the Wien displacement law for blackbody spectral radiation maximum. This approach is based on the use of information about peak-wavelength
$\lambda_{\text{max}}$ determined experimentally, when intensity $I_{\text{ef}} = I_{\text{ef}}(\lambda, M_i, s_M)$ = $\epsilon_{\text{ef}}(\lambda, M_i, s_M) I_0(\lambda, T)$ of $dF_{M_i}$ is maximal, and its derivative with respect to $\lambda$ equals zero. In this case, one has

$$(I_{\text{ef}})'_{\lambda=\lambda_{\text{max}}} = (\epsilon_{\text{ef}} I_0)'_{\lambda=\lambda_{\text{max}}} = 0. \quad (6)$$

If divide (6) termwise by $\epsilon_{\text{ef}} I_0$ at $\lambda = \lambda_{\text{max}}$, one can write:

$$(I_{\text{ef}})'_{\lambda=\lambda_{\text{max}}}/I_{\text{ef}}(\lambda_{\text{max}}, T) = (\ln I_{\text{ef}})'_{\lambda=\lambda_{\text{max}}} = (\ln \epsilon_{\text{ef}})'_{\lambda=\lambda_{\text{max}}} + (\ln I_0)'_{\lambda=\lambda_{\text{max}}} = 0. \quad (7)$$

Then, according to the Planck formula and expression $x = c_2/(\lambda T)$, one can write:

$$(\ln I_0)'_{\lambda} = [\ln(C_1/\pi) - 5 \ln \lambda - \ln(\exp x - 1)]'_{\lambda} = \left[\frac{x}{1 - \exp(-x)} - 5\right] \lambda^{-1}. \quad (8)$$

From equations (7) and (8), one has

$$\rho_{\text{ef}} + \frac{x}{1 - \exp(-x)} - 5 = 0, \quad (9)$$

where $\rho_{\text{ef}} = \lambda (\ln \epsilon_{\text{ef}})'_{\lambda} = \lambda (\epsilon_{\text{ef}})'_{\lambda}/\epsilon_{\text{ef}}$.

For black (or gray) radiation, $(\epsilon_{\text{ef}})'_{\lambda} \equiv 0$ and the relationship (9) reduces to the known equation for determination of the Wien displacement law constant. Equation (9) was written in the form convenient for determination of desired value $x$ by the method of simple iteration

$$x = (5 - \rho_{\text{ef}})[1 - \exp(-x)]. \quad (10)$$

The value of $\rho_{\text{ef}}$ is unknown. To determine the temperature $T$, dependence of $\epsilon_{\text{ef}}(\lambda)$ is proposed to use the method presented in [8]. Then the value of $\lambda_{\text{max}}$ can be calculated according to equation (10). On the other hand the value of $\lambda_{\text{max}}$ can be estimated on the basis of experimental data analysis. In this case, the equation (10) performs the role of an additional check the proposed approach. Expressions (4) and (5) have the same structure. Therefore, the analysis of the temperature uncertainty by means (10) can be carried out using freely-radiating body data [9].

3. Conclusion

It is shown that for the temperature determination of the isothermal opaque bodies system can be used the same techniques that are used to determine the temperature of a free-radiating opaque body.

The relationship (9), which relates the parameters $T$, $\epsilon_{\text{ef}}(\lambda_{\text{max}}, M_i, s_M)$ and $\lambda_{\text{max}}$ for any fixed values $M_i \in F_i$ and $s_M$ is found. In the relationship (9), each of these parameters can be determined, if known are the other two parameters. In particular, the relationship (9) can be used as an additional condition to determine the thermodynamic temperature of the isothermal cavity by the thermal radiation spectrum of the target surface when the effective emissivity of the radiation source is unknown.

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