MODIFIED JOSEPHSON RELATION

Jan Koláček, Pavel Lipavský

Institute of Physics, Academy of Sciences, Cukrovarnická 10, 16253 Prague 6, Czech Republic

For type II superconductors, Josephson has shown that vortices moving with velocity \( \mathbf{v}_L \) create an effective electric field \( \mathbf{E}' = -\mathbf{v}_L \times \mathbf{B}_V \). By definition the effective electric field is gradient of the electrochemical potential, what is the quantity corresponding to voltage observed with the use of Ohmic contacts. It relates to the true electric field \( \mathbf{E} \) via the local chemical potential \( \mu \) as \( \mathbf{E}' = \mathbf{E} - \nabla \mu/e \). We argue that at low temperatures the true electric field in the bulk can be approximated by a modified Josephson relation \( \mathbf{E} = (\mathbf{v}_S - \mathbf{v}_L) \times \mathbf{B}_V \), where \( \mathbf{v}_S \) is the condensate velocity.

Keywords: superconductors; electric field; vortex dynamics.

1. Introduction

For type II superconductors with moving vortices Josephson derived the formula

\[
\mathbf{E}' \equiv -\nabla(\varphi + \mu/e) - \partial \mathbf{A}/\partial t = -\mathbf{v}_L \times \mathbf{B}_V.
\] (1)

The effective electric field \( \mathbf{E}' \) is a gauge-invariant generalization of the gradient of the electrochemical potential \( \varphi + \mu/e \), with \( \mu \) being the local chemical potential,\(^a\) \( \mathbf{v}_L \) is the velocity of the vortex lattice and \( \mathbf{B}_V = nV\phi_0 \) is the averaged magnetic field generated by vortices distributed with the density \( n_V \).

The Josephson relation (1) proved to be useful in situations when one evaluates the difference between electrochemical potentials as measured by Ohmic contacts. On the other hand it is not explaining the voltage measured by a contactless capacitive pickup\(^2,3\). It also cannot be used for interpretation of the FIR data,\(^4\) because electric field of the laser light has to be matched with the true electric field \( \mathbf{E} = -\nabla \varphi - \partial \mathbf{A}/\partial t \), not with its effective counterpart \( \mathbf{E}' \).

Vector potential is the linear and electrostatic potential is the quadratic function of the local condensate velocity\(^5\), so in principle one can evaluate the electric field directly from its definition. Such approach would require a microscopic picture of the moving Abrikosov vortex lattice what represents a demanding numerical task.

\(^a\)Here we are using the nowadays commonly used convention in which the so called electrochemical potential is identical with the Gibbs chemical potential \( \mu_{\text{Gibbs}} = \mu + e\varphi \). With a good approximation, the electrostatic potential \( \varphi \) represents the long-range interaction while the chemical potential \( \mu \) is a local functional of the density, temperature, BCS gap, and so on. Josephson in his original paper\(^1\) uses the term chemical potential for the Gibbs chemical potential.
We argue that at low temperatures, when the contribution of normal electrons can be neglected, the true electric field can be approximated as

$$\mathbf{E} = (\mathbf{v}_S - \mathbf{v}_L) \times \mathbf{B}_V,$$

(2)

where \(\mathbf{v}_S\) is the mean velocity of condensate obtained by averaging over the elementary cell of the Abrikosov vortex lattice.

2. Electric field in type I superconductor

It was theoretically predicted\(^5\) and experimentally confirmed\(^2,3\) that superconducting current generates transversal electric field. In the planar geometry, this field can be derived from the Newton equation \(m\mathbf{v}_S = e\mathbf{E} + e\mathbf{v}_S \times \mathbf{B}\). Since the acceleration is zero, one finds \(\mathbf{E} = -\mathbf{v}_S \times \mathbf{B}\). From the London condition \(\nabla \times \mathbf{v}_S = -e\mathbf{B}/m\) solved by \(m\mathbf{v}_S = -e\mathbf{A}\) one can derive a more general formula \(e\varphi = -\frac{1}{2}mv_S^2\), which applies also for curved streamlines with nonzero centripetal acceleration and for time dependent vector potentials. Electric field is acting also on the crystal lattice and by this way the Lorentz force is mediated to it.

At finite temperatures the presence of quasiparticles must be also taken into account. When flowing, normal electrons dissipate energy and therefore, in spite of the presence of an electric field, the average velocity of the normal state fluid \(\mathbf{v}_N\) is zero at equilibrium. This is explained by the interaction between the superconducting and the normal state fluid, known as quasiparticle screening\(^6\). Due to it the electrostatic Bernoulli potential is reduced by the share of the condensate so that it reads \(e\varphi = -n_S/\left(\frac{1}{2}m^*v_S^2\right)\). Arguments explaining electric field in superconductors can be found in the BCS theory\(^7\) and the extended Ginzburg-Landau theory\(^8\), too. The fact that electric field generated by surface currents mediate Lorentz force to the crystal lattice follows from the Budd-Vannimenus (BV) theorem\(^9\). For simplicity in the following we restrict ourselves to low temperatures, where interaction with quasiparticles can be neglected.

3. Electric field in type II superconductor

Superconducting current acts on the vortices by the Magnus force, which (supposing that vortices are aligned parallel to the \(z\)-axis) reads\(^10\)

$$\mathbf{F}_V^M = \frac{n_S h}{2} (\mathbf{v}_S - \mathbf{v}_L) \times \mathbf{z}.$$

(3)

It follows from the third Newton law that with the Magnus force acting on vortices also a reaction force felt by the superconducting particles \(\mathbf{F}_M^S = -\frac{n_S}{n_S} \mathbf{F}_V^M = -e (\mathbf{v}_S - \mathbf{v}_L) \times \mathbf{B}_V\) must be introduced.

In steady state in which vortices and superconducting fluid move without acceleration, the total forces acting on them have to vanish. The superconducting particles feel the effect of electric field, Lorentz force (interaction with external magnetic field
$\mathbf{B}_{\text{ext}}$ penetrating into the superconductor) and the reaction of Magnus force. From the Newton equation of motion

$$m \ddot{v}_S = e \mathbf{E} + e v_S \times \mathbf{B}_{\text{ext}} - e (v_S - v_L) \times \mathbf{B}_V$$

one gets

$$\mathbf{E} = -v_S \times \mathbf{B}_{\text{ext}} + (v_S - v_L) \times \mathbf{B}_V.$$  \hspace{1cm} (4)

In the bulk, where external magnetic field is screened off, the electric field can be approximated by modified Josephson relation (2).

From this it follows, that the true electric field is nonzero even if vortices are pinned. It is understandable. The sum of Magnus forces acting on the individual vortices is equal to the Lorentz force, which the current feels in magnetic field $\mathbf{B}_V$ created by the vortex lattice. Nevertheless, the Magnus force represents the interaction of the superconducting fluid with vortices, and not an interaction with the external magnetic field $\mathbf{B}_{\text{ext}}$. Pinned vortices can remain in the superconductor even if external magnetic field is switched off and in this case the total force acting on the crystal lattice is zero. Vortices are not moving, as they are kept by the pinning force. The reaction of the Magnus force felt by the superfluid is balanced by electric field (2) and it also balances pinning force felt by the crystal lattice. If external magnetic field is present, according to the BV theorem, electric field induced by the surface currents mediates Lorentz force on the crystal lattice.

4. Conclusion

Effective electric field in type II superconductor is given by Josephson relation (1). At low temperatures in steady state the true electric field in the bulk can be approximated by the modified Josephson relation (2). In the presence of transport current, the Lorentz force acting on the superconducting wire in external magnetic field is mediated by electric field generated by the surface currents.

Acknowledgments

This work was supported by MŠMT program Kontakt ME 601 and GAČR 202000643, GAAV A1010312 grants. The European ESF program VORTEX is also acknowledged.

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