Fuzzyfication of supplier–retailer inventory coordination with credit term for deteriorating item with time-quadratic demand and partial backlogging in all cycles

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Abstract. A finite planning horizon supply chain model is discussed for materials substances such as metals, ceramics, or plastics manufactured which is deteriorating in nature. Parameter such as holding cost and all the other cost are fuzzyfied. Triangular fuzzy numbers are used for the fuzzyfication of parameters. Defuzzyfication of the model is done using total -integral value. We also have used Graded Mean Representation method to defuzzify the model. Later the comparison between the two defuzzyfied model is presented and conclusion is drawn.

Keywords: Inventory, Triangular fuzzy number, Green supply chain management.

1. Introduction

There are several products which can be remanufactured such as electronic goods, plastic materials, furniture, etc. As discussed by [15] green supply chain management is the desperate need of the 21st century. Industries are quickly moving to sustainable, reliable environment-friendly methods of supply chain management. A good supply chain policy would be to have a positive impact on Ecology without compromising quality was stated by [15]. There are different ways suppliers and retailers cooperate with each other such as by sharing detailed information of the storage of inventory and thereby increasing their profit and profit sharing [16]. For greening of the model remanufacturing was introduced by [16] after different levels in the process of finding remanufacturable/repairable products. [5] studied remanufacturing in a supply chain model for a short life cycle product. The behaviour of their model including different process such as JIT delivery and remanufacturing was explained with a diagramatic approach. Optimal values using the hessian matrix were derived.

Taking into account the importance of rework, minimal repair [8] discussed the economic production quantity for their model.

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Many retail giant has adopted the concept of the green supply chain by recycling of the plastic, repairing of the electronic goods thereby reducing of the scrap or plastic waste [13]. Also, [13] found that retailer profits with regular investment in green operations. Recently, [14] considered Weibull deteriorated items in the greening of the supply chain by remanufacturing.

To fuzzify the model, [17] took order and shortage as triangular fuzzy numbers. Authors using triangular fuzzy numbers for parameters value are [18, 3, 4, 2, 1, 12] and others.

[9] discussed a bifuzzy model and have used λ integral value for crisp representation. An economic production model with two parameter weibull deterioration using λ integral value for crisp representation [12] analysed a model using fuzzyfication by triangular fuzzy number.

A fuzzy inventory model three parameter weibull distribution pattern along with the permissible delay in payment introducing credit period [10] found optimum solution defuzzifying using total λ integral value for symmetric triangular fuzzy parameters.

Graded Mean Representation method is used by [11] to defuzzify the total cost function and find its minimum value where the parameters are pentagonal fuzzy numbers. Graded Mean Representation along with Signed Distance and Centroid methods were used to defuzzify the total cost function by [6]. Our present model thus fills the gap for the fuzzy green inventory model by remanufacturing in a finite planning horizon. The paper is further divided into 4 more sections. Section 2 is for assumptions and notations. Section 3 is for the conceptualization of the proposed model. The last section 5 comprises of conclusions.

2. Assumptions and notations
The initial demand($\tilde{a}$), holding cost($\tilde{H}_o$), cost of lost sale($\tilde{L}_o$) and shortage cost($\tilde{S}_o$) are considered to be a triangular fuzzy number.

Except for the fact that the total cost is fuzzified, the present model is an extension of [14] therefore assumptions and notations are same as [14]. The present model is fuzzified using λ integral value and Graded mean representation method.

3. Proposed model
The model is represented by the following equation:-

\[
\frac{dI_{i+1}^D(t)}{dt} + (\theta_1 + \theta_2) I_{i+1}^D(t) = -f(t), \{i=1, 2 \ldots n_2^D\}
\]  

Figure 1. Graphical representation of Inventory Model
Supplier’s fuzzy total cost after coordination with the supplier = $TC^C_s(t_j, s_j, n^2_C) =$

$$\sum_{j=1}^{n^2_C} \{S_s * n + P_s * (I_{o3} + S_j) + (T_c + p + DsAsm * p + Rem * p + \tilde{H}o * p) * I_{o3}\} + \tilde{TC}^C_r(t_j, s_j, n^2_C) - \tilde{TC}^C_r(t_{o3}, s_{o3}, n_{o3})$$

Where, $TC^C_s(t_j, s_j, n^2_C)$ = Total cost of the retailer during a planning horizon = Purchase cost + Holding cost + Deterioration cost + Shortage cost + cost of Lost sale + Screening Cost =

$$\sum_{j=1}^{n^2_C} \{P_s [I_{o3} + S_j] + \tilde{H}o \left[ \int_{t_j}^{t_j} j_{1j}(t) dt + \int_{t_j}^{t_j} j_{2j}(t) dt + \int_{t_j}^{t_j} j_{3j}(t) dt \right] + Dc_{o3} \beta \left[ \int_{t_j}^{t_j} t^{\beta-1} j_{1j}(t) dt + \int_{t_j}^{t_j} t^{\beta-1} j_{2j}(t) dt + \int_{t_j}^{t_j} t^{\beta-1} j_{3j}(t) dt \right] + \tilde{S}_o \int_{s_j}^{s_j} \frac{(t_j - t) (\tilde{a} + bt + ct^2)}{1 + \delta (t_j - t)} dt + \tilde{L}_o \int_{s_j}^{s_j} \frac{\delta (t_j - t) (\tilde{a} + bt + ct^2)}{1 + \delta (t_j - t)} dt + S_c * \tilde{r}_{o3} \}$$

4. Numerical Example

Same numerical example’s solution is discussed as in [14] for the comparison without changing the value of parameters except that $\tilde{a}$, $H_0$, $L_0$, $S_0$ are fuzzyfied and there fuzzy triangular values are $\tilde{a} = (1500 - \psi_1, 1500, 1500 + \psi_1)$, $H_0 = (1 - \psi_2, 1, 1 + \psi_2)$, $L_0 = (600 - \psi_3, 600, 600 + \psi_3)$, $S_0 = (290 - \psi_4, 290, 290 + \psi_4)$ where $\psi_1 = 0.02, \psi_2 = 0.002, \psi_3 = 0.02, \psi_4 = 0.02$.

Table 1 gives for different number of replenishment cycles and 5 different values of $\tilde{a}$ the total cost of retailer with optimal cost of supplier in a decentralized case. Convexity for retailers total cost when $\tilde{a} = (1500 - \psi_1, 1500, 1500 + \psi_1)$ can be seen from 2. Table 2 shows its optimal schedule for optimal replenishment cycles ie 3 and 5 different values of $\tilde{a}$. Table 3 shows the obtained total cost of supplier in a centralized case for different number of replenishment cycles and 5 different values of $\tilde{a}$. Convexity for suppliers total cost when $\tilde{a} = (1500 - \psi_1, 1500, 1500 + \psi_1)$ can be seen from 3. Table 4 shows the optimal schedule for optimal replenishment cycles ie 2 and 5 different values of $\tilde{a}$ when coordination exist. Table 5 gives defuzzy total improved cost in case of existing coordination by with $\lambda$–integral value. Table 6 defuzzy percentage improved cost in case of existing coordination with $\lambda$–integral value. Similarly Table 7 gives defuzzy total improved cost in case of existing coordination by with graded mean representation method and table 8 shows defuzzy percentage improved cost in case of existing coordination with graded mean representation method along with graphical representation of convexity in graph 4 and 5.
Figure 2. Convex graph for total cost of retailer when $\tilde{a} = (1500 - \psi_1, 1500, 1500 + \psi_1)$ with $\lambda$–integral value
Table 1. For different number of replenishment cycles and 5 different values of \( \tilde{a} \) the total cost of retailer with optimal cost of supplier in a decentralized case

| \( \tilde{a} \) | \( n_1 \) | \( T_{C^D} \) | \( T_{C^D} \) |
|----------------|--------|----------------|----------------|
| \( \tilde{a} \) | 1      | 2              | 3              | 4              | 5      | 6      | 7      | 8      |
| 1499.99        | 52674.7| 48809.6        | 48652.3        | 49934.9        | 52141.3| 55095.8| 58724.9| 62993.2|
| 1499.995       | 52675.8| 48810.2        | 48652.7        | 49935.1        | 52141.3| 55095.8| 58724.8| 62993.1|
| 1500           | 52676.9| 48810.9        | 48653.1        | 49935.2        | 52141.4| 55095.7| 58724.7| 62993. |
| 1500.005       | 52678.1| 48811.5        | 48653.4        | 49935.4        | 52141.4| 55095.7| 58724.6| 62992.8|
| 1500.01        | 52679.2| 48812.2        | 48653.8        | 49935.6        | 52141.5| 55095.7| 58724.5| 62992.7|

Table 2. Optimal schedule for optimal replenishment cycles ie 3 and 5 different values of \( \tilde{a} \)

| \( \tilde{a} \) | \( s_1 \) | \( s_2 \) | \( s_3 \) | \( s_4 \) | \( \tilde{a} \) | \( t_1 \) | \( t_2 \) | \( t_3 \) |
|----------------|--------|--------|--------|--------|----------------|--------|--------|--------|
| 1499.99        | 0      | 0.823114| 1.97992| 4.     | 1499.99        | 0.000208968| 0.823715| 1.98039|
| 1499.995       | 0      | 0.823577| 1.98016| 4.     | 1499.995       | 0.000209054| 0.823878| 1.98064|
| 1500           | 0      | 0.823741| 1.98041| 4.     | 1500           | 0.000209141| 0.824042| 1.98088|
| 1500.005       | 0      | 0.823904| 1.98066| 4.     | 1500.005       | 0.000209227| 0.824205| 1.98113|
| 1500.01        | 0      | 0.824067| 1.9809 | 4.     | 1500.01        | 0.000209314| 0.824369| 1.98137|
Table 3. In a centralized case For different number of replenishment cycles and 5 different values of \( \tilde{a} \) the total cost of supplier

| \( \tilde{a} \rightarrow n_1 \) | 1       | 2       | 3       | 4       |
|------------------------------|---------|---------|---------|---------|
| 1499.99                      | 33589.1 | 23742.6 | 27719.8 | 38861.4 |
| 1499.955                     | 33604.1 | 23743.6 | 27721.2 | 38863.1 |
| 1500                        | 33604.9 | 23744.5 | 27722.6 | 38864.6 |
| 1500.005                     | 33605.8 | 23745.5 | 27724.1 | 38866.2 |
| 1500.01                      | 33606.6 | 23746.4 | 27725.4 | 38867.7 |

Figure 3. Convex graph of total cost of supplier when \( \tilde{a} = (1500 - \psi_1, 1500, 1500 + \psi_1) \) with \( \lambda \)-integral value

Table 4. Optimal schedule for optimal replenishment cycles ie 2 and 5 different values of \( \tilde{a} \)

| \( \tilde{a} \) | \( s_1 \) | \( s_2 \) | \( s_3 \) | \( \tilde{a} \) | \( t_1 \) | \( t_2 \) |
|--------------|---------|---------|---------|--------------|---------|---------|
| 1499.99      | 0       | 2.15319 | 4.      | 1499.99      | 0.00120329 | 2.15445 |
| 1499.995     | 0       | 2.15317 | 4.      | 1499.995     | 0.00120347 | 2.15442 |
| 1500         | 0       | 2.15314 | 4.      | 1500         | 0.00120364 | 2.1544   |
| 1500.005     | 0       | 2.15312 | 4.      | 1500.005     | 0.00120381 | 2.15437 |
| 1500.01      | 0       | 2.15309 | 4.      | 1500.01      | 0.00120398 | 2.15435 |
Table 5. Defuzzy total improved cost in case of existing coordination with $\lambda$–integral value

| $\tilde{a}$ | $\tilde{H}_0$ | $\tilde{L}_0$ | $\tilde{S}_0$ | $\tilde{T}_C^{DO}$ | $T_C^{DO}$ | $n_1^{DO}$ | $\tilde{Q}^{DO}$ | $n_2^{CO}$ | $\tilde{Q}^{CO}$ | $T_C^{COP}$ | $T_C^{COP}$ |
|-----------|-------------|-------------|-------------|----------------|-----------|-----------|-------------|-----------|-------------|-------------|-------------|
| 1499.99   | 0.999      | 599.99     | 289.99     | 48652.3       | 35585.9   | 3         | 6734.9     | 2         | 6737.09     | 37353.4     | 27078.7     |
| 1500.0    | 0.9995     | 599.995    | 289.995    | 48652.7       | 35584.7   | 3         | 6734.92    | 2         | 6737.11     | 37351.3     | 27079.8     |
| 1500.0    | 1.00       | 600.0      | 290.        | 48653.1       | 35583.4   | 3         | 6734.94    | 2         | 6737.13     | 37349.3     | 27081.0     |
| 1500.01   | 1.0005     | 600.005    | 290.005    | 48653.4       | 35582.2   | 3         | 6734.96    | 2         | 6737.15     | 37347.2     | 27082.2     |
| 1500.01   | 1.001      | 600.01     | 290.01     | 48653.8       | 35580.9   | 3         | 6734.98    | 2         | 6737.17     | 37345.1     | 27083.3     |

Table 6. Defuzzy percentage improved cost in case of existing coordination with $\lambda$–integral value

| $\tilde{a}$ | $\tilde{H}_0$ | $\tilde{L}_0$ | $\tilde{S}_0$ | Systems improved cost | Percentage improvement in retailer’s cost | Percentage improvement in supplier’s cost |
|-----------|-------------|-------------|-------------|-----------------------|------------------------------------------|------------------------------------------|
| 1499.99   | 0.999      | 599.99     | 289.99     | 19806.2               | 44.3425                                  | 44.3425                                  |
| 1500.0    | 0.9995     | 599.995    | 289.995    | 19806.2               | 44.3405                                  | 44.3405                                  |
| 1500.0    | 1.00       | 600.0      | 290.        | 19806.2               | 44.3386                                  | 44.3386                                  |
| 1500.01   | 1.0005     | 600.005    | 290.005    | 19806.3               | 44.3366                                  | 44.3366                                  |
| 1500.01   | 1.001      | 600.01     | 290.01     | 19806.3               | 44.3346                                  | 44.3346                                  |
Figure 4. Convex graph for total cost of retailer when \( \tilde{a} = (1500 - \psi_1, 1500, 1500 + \psi_1) \) with Graded Mean Representation method

Table 7. Defuzzy total improved cost in case of existing coordination by with Graded Mean representation method

| \( \tilde{a} \) | \( \tilde{H}_o \) | \( \tilde{L}_o \) | \( \tilde{S}_o \) | \( \tilde{T}_{C^D_1} \) | \( \tilde{T}_{C^D_2} \) | \( n_1^{DO} \) | \( \tilde{Q}_{DO} \) | \( n_2^{CO} \) | \( \tilde{Q}^{CO} \) | \( \tilde{T}_{C^C_1} \) | \( \tilde{T}_{C^C_2} \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1470 | 0.98 | 588.2 | 284.2 | 47827.3 | 35326.8 | 3 | 6614.74 | 2 | 6616.83 | 37024.9 | 26531.9 |
| 1485 | 0.99 | 594.1 | 287.1 | 48240.1 | 35455.2 | 3 | 6674.84 | 2 | 6676.98 | 37187.1 | 26806.3 |
| 1515 | 1.01 | 606.2 | 292.9 | 49066.4 | 35711.5 | 3 | 6795.04 | 2 | 6797.28 | 37511.5 | 27356.1 |
| 1530 | 1.02 | 612.2 | 295.8 | 49480.1 | 35839.5 | 3 | 6855.13 | 2 | 6857.43 | 37673.8 | 27631.7 |
| 1500 | 1 | 600.1 | 290 | 48653.1 | 35583.4 | 3 | 6734.94 | 2 | 6737.13 | 37349.3 | 27081. |

5. Conclusion
Crisp solution for a fuzzy green supply chain model given by [14] is obtained and shown in tables 5, 6, 7 and 8 by \( \lambda \)–integral value and graded mean representation method. When compared [14]
Figure 5. Convex graph of total cost of supplier when $\tilde{a} = (1500 - \psi_1, 1500, 1500 + \psi_1)$ with Graded Mean Representation method

Table 8. Defuzzy total improved cost in case of existing coordination with Graded Mean representation method

| $\tilde{a}$ | $\tilde{H}_0$ | $\tilde{L}_0$ | $\tilde{S}_0$ | Systems improved cost | Percentage improvement in retailer’s cost | Percentage improvement in supplier’s cost |
|------------|--------------|--------------|--------------|----------------------|------------------------------------------|------------------------------------------|
| 1470.0     | 0.98         | 588.0        | 284.2        | 19597.3              | 44.5256                                  | 44.5256                                  |
| 1485.0     | 0.99         | 594.0        | 287.1        | 19701.9              | 44.4315                                  | 44.4315                                  |
| 1515.0     | 1.01         | 606.0        | 292.9        | 19910.3              | 44.2467                                  | 44.2467                                  |
| 1530.0     | 1.02         | 612.0        | 295.8        | 20014.2              | 44.156                                   | 44.156                                   |
| 1500.0     | 1.00         | 600.0        | 290.0        | 19806.2              | 44.3386                                  | 44.3386                                  |

it has been observed that the value of the total cost and the profit percentage differs with that of the present model. The fact that uncertainty looms and fuzzy parameters cannot be avoided [11] besides our present model justifies the same. Therefore for the generalization, fuzzy parameters cannot be ignored as shown in our present model. The present model can be extended by taking
all the parameters as fuzzy or considering multi item as in [7].

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