Cosmological dark matter amplification through dark torsion

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Abstract
A cosmological approach based on considering a cosmic background with non-zero torsion is shown in order to give an option of explaining a possible phantom evolution, not ruled out according to the current observational data. We revise some aspects of the formal schemes on torsion and, according them, we develop a formalism which can be an interesting alternative for exploring Cosmology.

Keywords: spin tensor, dark matter, torsion

1. Introduction
The current knowledge of the nature of dark matter is scarce. However, the cumulative evidence seems to favor the scenario of dark matter as a non-interacting form of matter instead of some modified gravity theory. It is true in particular when considering phenomena as cluster collisions (e.g., references \[1, 2\]).

Our ignorance of dark matter physical behavior implies a lack of knowledge of its features as a source of gravity. In particular, it is unknown whether or not the spin tensor of dark matter vanishes. The issue is relevant since a non-vanishing spin tensor is a torsion source, and torsion requires to go beyond general relativity (GR). The closest framework to GR is the Einstein–Cartan–Sciama–Kibble (ECSK) theory of gravity. Some reviews of ECSK and other torsional theories can be found in References \[3–16\]. There are many other alternatives theories of gravity with torsion, but ECSK is probably the simplest one that includes spinning matter and torsion.

The ignorance regarding the physical nature of dark matter is in sharp contrast with the knowledge of the standard model matter’s behavior. For instance, from the Yang–Mills (YM)
Lagrangian
\[ \mathcal{L}_{\text{YM}} = -\frac{1}{4} F_{\mu\nu}^{A} F^{B\mu\nu} \text{tr}(T_{A}T_{B}), \] (1)

it is straightforward to see that YM bosons’ spin tensor vanishes. Therefore, YM bosons are not a source of torsion, and there is no YM bosons-torsion interaction. In contrast, the standard model fermions have a non-vanishing spin tensor, and therefore they are a source of torsion. They should interact with torsion, but the effect is so feeble that it is hard to foresee any particle-physics experiment capable of detecting it (see chapter 8.4 of reference [17] and references [18–20]).

It is important to stress that with spin, we refer only to intrinsic quantum mechanical spin, and we should not confuse this with the angular momentum density. This natural confusion has already led to some mistakes in the literature, see reference [21]. In the context of ECSK, torsion does not interact with matter at a classical level, and neither does it with electromagnetic phenomena [19]. For instance, regardless of background torsion, classical point particles should follow torsionless geodesics, and electromagnetic waves should travel through torsionless null geodesics. To standard model matter, torsion is ‘dark’. Perhaps the only realistic way of detecting torsion could be through precise measurement of gravitational waves’ polarization [24–26].

Even further, due to interaction and decoherence, standard model baryons are highly localized, and they form astrophysical structures. In the context of ECSK theory, torsion cannot propagate in a vacuum (in glaring contrast to the behavior of Riemannian curvature). Therefore, given both the granular nature of the baryonic matter in the Universe’s current epoch and that torsion vanishes in the vacuum, the effective spin tensor of baryonic matter should vanish in cosmological scales. In other words, it seems unrealistic (see reference [27]) to consider standard model baryons as a spin fluid on a cosmological scale: the effective spin tensor of a gas of galaxies vanishes in long scales.

The concept of Cartan length [28] quantifies the intuitions of last paragraph. In ECSK theory, the spin tensor and torsion become relevant only when the average distance between particles is comparable to the Cartan length for those particles. For standard model fermions, the Cartan length ranges between \(10^{-29}\) and \(10^{-25}\) m, i.e., an extremely tight packaging of particles seems a necessary condition to have a relevant torsion. The Cartan length is several orders smaller than any concept of particle ‘size’, as the Compton wavelength.

Thus, the spin tensor of the standard model particles is a relevant source of torsion in two situations. The first is in a Universe filled with a high-density fermion plasma at very early epochs. The second is in the high-density fermion plasma evolving into a black hole singularity. In the first scenario, torsion reproduces inflation behavior without an inflaton field [29–36]. In this sense, torsional models seem a significant contender to standard inflation.

The second scenario is particularly interesting since torsion makes both situations the same. Torsion creates a black hole cosmology or ‘Universe in a black hole’ scenario, where the singularity softens [6, 36–39] and gives rise to a new Universe [29–31]. The idea of black holes producing new universes is much older [40–46], but torsion provides a mechanism for it to happen.

The situation is very different for dark matter. Lacking a specific dark matter Lagrangian, it is impossible to calculate its spin tensor, torsion, and Cartan length. However, the lack of dark

\[ \text{It would be possible only if we introduce much more dramatic changes in the theory, see for instance reference [22].} \]

\[ \text{There may be other tests [23], but they are of local nature. Gravitational waves could provide a test over cosmological distances.} \]
matter interaction with standard model particles and its incapability to create structures lead to the conjecture that the decoherence effects could be feeble for dark matter. If it is so, we can expect dark matter wave functions to be extremely wide, even cosmological-scale-wide. This dark matter picture fits well its distribution in the wide halos and interconnected filaments of the cosmic web.

For non-interacting dark matter particles with broad wave functions extending through the known interconnected filament structure, it seems plausible to expect that their Cartan length could correspond to a vast, cosmological distance ($> 100$ Mpc). It would be the exact opposite of strongly-interacting standard model fermions with a tiny Cartan length.

The figure 1 shows a general plausibility argument for dark matter as a source of torsion in ECSK theory. Baryons are ‘clumpy’, and dark matter distributes in the halos and interconnected filaments of the cosmic web, figure 1(a). For this reason, the torsion created by baryonic matter cannot propagate (short Cartan length, figure 1(b). In contrast, if dark matter particles have a long Cartan length, they would source torsion along the cosmic web’s filaments. The torsion propagating in one single filament does not need to be homogenous nor isotropic, figure 1(c) and (d).

Therefore, if dark matter has a nonvanishing spin tensor, and considering the effective impact of torsion riding on the widespread dark matter distribution, it seems natural to expect nonvanishing torsion in cosmological scales. This collective effect at cosmological scales has to respect the Copernicus symmetries, figure 1(e).

The torsion created through this mechanism would be as dark as its source: standard model matter would not interact with it. When moving these ‘dark torsion’ terms to the right-hand side of the field equations, they behave just as an extra (and dark) source of standard torsionless Riemannian curvature.

From an observational point of view, it is possible to measure only the Riemannian curvature and not the torsion. Therefore, in this scenario, the observed gravitational dark matter effects correspond to the ones created by the ‘bare’ dark matter plus the torsional ‘dark dress’ it creates through its spin tensor.

The current article explores how ‘dark torsion’ could amplify the effects of a small amount of dark matter in a cosmological setting. Given the disparity between the amount of dark matter and standard model baryons in the Universe, a mechanism as this one may seem of interest. The section 2 briefly reviews ECSK gravity and shows how torsion amplifies ‘bare dark matter’, creating a higher effective energy density (or ‘dressed’ density). This total torsion-dressed density would correspond to the observed dark matter density instead of the bare piece. In

![Figure 1.](image-url)
the case of standard model fermions, the canonical approach describes their spin tensor as a Weyssenhof fluid. However, given the lack of dark matter self-interaction, this Ansatz does not seem correct. In the section 2.2, we offer a different Ansatz for the spin tensor of dark matter using symmetry and dimensional analysis arguments. The section 3 use the generalized Friedmann equations to analyze the cosmological consequences of torsion and its dark matter amplification effect. The section 4 studies the thermodynamical effects of the torsional dress of dark matter. Finally, in section 5, we present some conclusions and possible further works.

2. Dark matter and dark torsion

There are many works in the context of cosmology using alternative theories of gravity involving a nonvanishing torsion [29, 47–53]. The present work focuses on the most straightforward approach, i.e., ECSK theory. It is also the closest to standard GR. The idea is to taste some of the consequences of nonvanishing spin tensor for dark matter in the most straightforward context before considering more exotic approaches.

Let us consider a four-dimensional spacetime with \((-, +, +, +)\) signature described by the Einstein–Cartan geometry, i.e., the metric \(g_{\mu\nu}\) and the connection \(\Gamma^\lambda_{\mu\nu}\) are independent degrees of freedom. The ECSK action principle corresponds to

\[
S = \int \sqrt{|g|} d^4x (\mathcal{L}_G + \mathcal{L}_b + \mathcal{L}_{\text{DM}}),
\]

where we are using units \(c = 8\pi G = k_B = 1\). In equation (2), \(\mathcal{L}_b\) stands for the Lagrangian for baryonic matter, and \(\mathcal{L}_{\text{DM}}\) corresponds to an unknown Lagrangian for dark matter. The gravity Lagrangian \(\mathcal{L}_G\) corresponds to the standard Einstein–Hilbert term a la Palatini, i.e., without imposing the torsionless condition (and therefore with the metric and the connection as independent degrees of freedom),

\[
\mathcal{L}_G (g, \Gamma, \partial \Gamma) = \frac{1}{2} R (g, \Gamma, \partial \Gamma) - \Lambda.
\]

Here \(R = g^{\sigma\nu} R'_{\sigma\mu\nu}\) is the Ricci scalar’s generalization constructed from the generalized Riemann tensor (or Lorentz curvature)

\[
R'_{\sigma\mu\nu} = \partial_\sigma \Gamma^\nu_{\mu\sigma} - \partial_\mu \Gamma^\nu_{\sigma\nu} + \Gamma^\rho_{\mu\sigma} \Gamma^\lambda_{\rho\nu} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma},
\]

where \(\Gamma^\rho_{\sigma\nu}\) is a general connection (not necessarily the Christoffel one).

The action principle, equation (2), may seem general. However, it is fair to remark that the Lagrangian choice equation (2) assumes the minimal coupling between dark matter, baryons, and gravity, and it does not include torsional terms as the Holst term. Axion-like nonminimal couplings with gravitational terms are sources of torsion [25, 26]. It is also possible to include nonminimal torsion-matter couplings [18, 19]. Therefore it is worth remembering that equation (2) corresponds to the simplest ECSK case, and there are many other more exotic choices.

The antisymmetric part of the connection \(\Gamma^\lambda_{\mu\nu}\) defines the torsion tensor as

\[
T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}.
\]
The difference between the general connection $\Gamma^{\lambda}_{\mu\nu}$ and the canonical Christoffel connection
$\Gamma^{\lambda}_{\mu\nu} = (1/2) g^{\lambda\sigma} \left( \partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu} \right)$ defines the contorsion tensor $K^{\lambda}_{\mu\nu}$.
\[
\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\mu\nu} = K^{\lambda}_{\mu\nu}. \tag{6}
\]
Contorsion and torsion are related through
\[
K_{\mu\nu\lambda} = \frac{1}{2} \left( T_{\nu\mu\lambda} - T_{\mu\nu\lambda} + T_{\lambda\mu\nu} \right). \tag{7}
\]
It is possible to decompose the generalized curvature in terms of the contorsion as
\[
R^{\alpha\beta}_{\mu\nu} = \tilde{R}^{\alpha\beta}_{\mu\nu} + \nabla_\mu K^{\alpha\beta}_{\nu} - \nabla_\nu K^{\alpha\beta}_{\mu} + K^{\alpha}_{\lambda\mu} K^{\beta\lambda}_{\nu} - K^{\alpha}_{\lambda\nu} K^{\beta\lambda}_{\mu}, \tag{8}
\]
where $\tilde{R}^{\alpha\beta}_{\mu\nu}$ is the canonical torsionless Riemann tensor in terms of the Christoffel connection $\Gamma^{\lambda}_{\mu\nu}$, and $\nabla_\mu$ is the standard torsionless covariant derivative in terms of $\Gamma^{\lambda}_{\mu\nu}$.

The metric equations of motion correspond to
\[
R^+_{\mu\nu} - \frac{1}{2} R_{\mu\nu} R + \Lambda g_{\mu\nu} = T^{(b)}_{\mu\nu} + T^{(DM)}_{\mu\nu}, \tag{9}
\]
where $R^+_{\mu\nu}$ is the symmetric part of the generalized Ricci tensor and $T^{(b)}_{\mu\nu}$ and $T^{(DM)}_{\mu\nu}$ are the stress-energy tensors associated with $L_b$ and $L_{DM}$. The affine equations of motion are given by
\[
T_{\lambda\mu\nu} = g_{\lambda\rho} T^{\rho}_{\mu\nu} + g_{\lambda\sigma} T^{\sigma}_{\rho\mu\nu} = \sigma^{(b)}_{\lambda\rho\mu\nu} + \sigma^{(DM)}_{\lambda\rho\mu\nu}, \tag{10}
\]
where $\sigma^{(b)}_{\lambda\rho\mu\nu}$ and $\sigma^{(DM)}_{\lambda\rho\mu\nu}$ are the spin tensors
associate with $L_b$ and $L_{DM}$.

Let us end this brief review of ECSK pointing that in general $\nabla_\mu \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \neq 0$, and therefore the right-hand side of equation (9) is no longer ‘conserved’. To write down a genuine conservation law, let us use equation (8) to move all the torsional terms to the right-hand side
\[
\dot{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \dot{R} + \Lambda g_{\mu\nu} = T^{(b)}_{\mu\nu} + T^{(DM)}_{\mu\nu} + T^{(T)}_{\mu\nu}. \tag{11}
\]
Here $\dot{R}_{\mu\nu}$ stands for standard torsionless Ricci tensor, and $T^{(T)}_{\mu\nu}$ is the effective stress-energy tensor for torsion. It corresponds to
\[
T^{(T)}_{\mu\nu} = g_{\mu\nu} \left( \nabla_\alpha K^{\alpha\rho}_{\mu} + \frac{1}{2} \left[ K^{\alpha}_{\lambda\rho} K^{\lambda\rho}_{\mu} - K^{\alpha}_{\lambda\rho} K^{\lambda\rho}_{\nu} \right] \right) + \frac{1}{2} \left( \nabla_\alpha K^{\alpha}_{\lambda\mu} + \nabla_\mu K^{\alpha}_{\lambda\nu} + K^{\alpha}_{\lambda\rho} K^{\lambda\rho}_{\nu} \right) + K^{\alpha}_{\lambda\nu} K^{\lambda\rho}_{\mu} - \left[ \nabla_\lambda + K^{\alpha}_{\lambda\nu} \left( K^{\lambda}_{\mu\rho} + K^{\lambda}_{\nu\rho} \right) \right], \tag{12}
\]

\textit{Footnotes:}
\footnotetext[6]{It seems there is no agreement in the literature on the name of this tensor. Some authors call it ‘contortion’, while others use ‘contorsion’. We have chosen to use the latter one because it sounds closer to torsion. The word ‘contortion’ may also be confused with a twisting motion.}
\footnotetext[7]{In the case of non-vanishing torsion, the generalized Ricci tensor has an antisymmetric part given by $R^+_{\mu\nu} = \frac{1}{2} \left( R_{\mu\nu} - R_{\nu\mu} \right) = \frac{1}{2} \left( \nabla_\nu T^{\rho}_{\mu\nu} + \nabla_\mu T^{\rho}_{\lambda\nu} - \nabla_\rho T^{\lambda}_{\mu\nu} \right) + \frac{1}{2} \left( T^{\rho}_{\mu\lambda} T^{\lambda}_{\nu\rho} + T^{\rho}_{\lambda\nu} T^{\lambda}_{\rho\mu} - T^{\rho}_{\nu\mu} T^{\lambda}_{\rho\lambda} \right).$}
\footnotetext[8]{The spin tensor is the variation of the matter Lagrangian with respect to the connection, in the same way as the stress-energy tensor is the variation of the matter Lagrangian with respect to the metric. The spin tensor of classical matter (e.g., dust) vanishes, but the spin tensor of a fermionic particle does not. For instance, the spin tensor of an electron is proportional to its axial current.}
Doing this, the ‘conservation law’ takes the form
\[ \nabla_{\mu} \left( T^{(b)}_{\mu\nu} + T^{(DM)}_{\mu\nu} + T^{(T)}_{\mu\nu} \right) = 0. \] (13)

### 2.1. FLRW evolution in ECSK theory

As mentioned in section 1, the baryons’ spin tensor \( \sigma^{(b)}_{\lambda\mu\nu} \) and its torsion could have been relevant under the extremely high fermion densities of the very early Universe [29]. However, in current times \( \sigma^{(b)}_{\lambda\mu\nu} = 0 \) should be an excellent approximation to describe cosmic evolution.

Considering \( \sigma^{(b)}_{\lambda\mu\nu} = 0 \) and tracing the affine equation of motion (10), it is clear that
\[ T_{\lambda\mu\nu} = \sigma^{(DM)}_{\lambda\mu\nu} + \frac{1}{2} \left( g_{\lambda\nu} \sigma^{\mu\rho\nu}_{\rho\mu} - g_{\lambda\mu} \sigma^{\rho\nu\rho}_{\nu\rho} \right). \] (14)

It means that torsion vanishes in the absence of dark matter, as it must be in ECSK theory. The equation (9) takes the form
\[ \dot{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \dot{R} + \Lambda g_{\mu\nu} = T^{(b)}_{\mu\nu} + T^{(eff-DM)}_{\mu\nu}, \] (15)

where the effective dark matter stress-energy tensor \( T^{(eff-DM)}_{\mu\nu} = T^{(DM)}_{\mu\nu} + T^{(T)}_{\mu\nu} \) causes dark matter’s observed effects. Here \( T^{(DM)}_{\mu\nu} \) corresponds to the bare dark matter stress-energy tensor, and torsion amplifies its importance through the effective tensor \( T^{(T)}_{\mu\nu} \) from equation (12).

Current observations can only detect the Riemannian gravity piece of the geometry. It means that they would be sensitive only to the combined or dressed effect of \( T^{(eff-DM)}_{\mu\nu} = T^{(DM)}_{\mu\nu} + T^{(T)}_{\mu\nu} \), and they will not distinguish bare dark matter from torsion. Torsion should be as dark as its source. Perhaps, only a careful measurement of the propagation of gravitational waves’ polarization [25] could distinguish bare dark matter from its ‘torsional dress’.

The lack of precise knowledge of the nature of dark matter creates what may seem like an insurmountable problem when trying to model its spin tensor. There are some usual Ansätze for the spin tensor, as the Weyssenhof fluid [54, 55]. However, given our ignorance of dark matter physics, any Ansatz for \( \sigma^{(DM)}_{\lambda\mu\nu} \) may seem excessive. The problem is that since we do not have any information on the spin tensor of dark matter \( \sigma^{(DM)}_{\lambda\mu\nu} \), we cannot use the field equation (14). Without this field equation, we do not have information on torsion, and it seems impossible to solve the system.

In what follows, we treat this problem in a cosmological setting. The general idea is that whatever dark matter is, it is possible to use symmetry arguments and dimensional analysis to arrive at a general Ansatz for \( \sigma^{(DM)}_{\lambda\mu\nu} \) in cosmological scales. Using this Ansatz, it becomes possible to study the torsion’s effects created by dark matter in cosmic evolution.

Let us start by considering the canonical FLRW metric with a homogeneous, isotropic, and Riemannian-flat spatial section
\[ ds^2 = -dt^2 + a^2(t) \left( dx^2 + dy^2 + dz^2 \right). \] (16)

Despite not knowing the Lagrangian \( \mathcal{L}_{DM} \), we know that the only stress-energy tensor compatible with the cosmological symmetries \( \mathcal{L}_{\xi} T^{(DM)}_{\mu\nu} = 0 \) is the canonical
\[ T^{(DM)}_{\mu\nu} = \left( \rho_{DM} + p_{DM} \right) U_{\mu} U_{\nu} + p_{DM} g_{\mu\nu}, \] (17)

where \( \rho_{DM} \) and \( p_{DM} \) are the dark matter density and pressure. With the spin tensor, it is possible to do the same. The most general spatially isotropic and homogeneous spin tensor \( \mathcal{L}_{\xi} \sigma^{(DM)}_{\lambda\mu\nu} = 0 \)
for dark matter must have the ‘Cartan staircase’ form
\[
\sigma^{(DM)}_{\lambda\mu\nu} = -2 \left( g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda} \right) h^\rho(t) - 2 \sqrt{|g|} \epsilon_{\lambda\mu\nu\rho} f^\rho(t),
\]
(18)
with the 4-vectors \( h^\rho(t) \) and \( f^\rho(t) \) having the forms
\[
h^\rho(t) = -h(t) U^\rho,
\]
\[
f^\rho(t) = -f(t) U^\rho.
\]
In terms of components,
\[
\sigma^{(DM)}_{0\mu} = 0,
\]
(19)
\[
\sigma^{(DM)}_{ij} = 2 g_{ij} h(t),
\]
(20)
\[
\sigma^{(DM)}_{ijk} = 2 \sqrt{|g|} \epsilon_{ijk} f(t),
\]
(21)
where \( i, j, k = 1, 2, 3 \) and \( \lambda, \mu, \nu = 0, 1, 2, 3 \). From equations (19)–(21), it is already clear that dark matter spatial isotropy and homogeneity are not fully compatible with usual models as the Weyssenhoff spin fluid.

At this point, we start to notice how different it is to model a high-density fermionic plasma and non-interacting dark matter as sources of torsion. A high-density fermionic plasma in the early Universe is well modeled as a Weyssenhoff spin fluid because their strong interactions create a rapidly changing spin tensor in short scales. It respects the Copernican principle because only the average of these local spin anisotropies matters in longer cosmological scales. The same arguments do not seem to hold for a non-interacting dark matter fluid extended over cosmological distances in the current epoch. That is why the full Ansatz equation (18) for the spin tensor of dark matter may be a far better choice than the standard Weyssenhoff spin fluid.

The spin tensor, torsion, and contorsion are all algebraically related through equations (14) and (7). From them, it is straightforward to conclude that whatever \( h(t) \) and \( f(t) \) are
\[
T_{\lambda\mu\nu} = \left( g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda} \right) h^\rho(t) - 2 \sqrt{|g|} \epsilon_{\lambda\mu\nu\rho} f^\rho(t),
\]
(22)
\[
K_{\mu\nu\lambda} = \left( g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda} \right) h^\rho(t) + \sqrt{|g|} f^\rho(t) \epsilon_{\rho\mu\nu\lambda},
\]
(23)
i.e., the same (still unknown) functions \( h(t) \) and \( f(t) \) describe torsion and contorsion.

From the expressions (16), (22) and (23), it is possible to calculate the Lorentz curvature components (8). To focus only in the behavior of dark matter and the torsion it can produce, let us consider a simplified model with negligible baryons and no cosmological constant (as we shall see, torsion is more than enough to accelerate the Universe expansion). In this case, the field equation (9) lead to the generalized Friedmann relations
\[
3 \left[ (H + h)^2 - f^2 \right] = \rho_{DM},
\]
(24)
\[
2 \left( \dot{H} + \dot{h} \right) + (3H + h)(H + h) - f^2 = -\rho_{DM},
\]
(25)
where the dot denotes the usual derivative with respect to comovil cosmic time.

To solve the equations (24) and (25), we need to know the dependence of \( f \) and \( h \) on other physical variables, like dark matter density and pressure. The next section shows how to find an Ansatz for these ‘torsional equations of state’, and to solve the system.

\[9\] There are hypothetical dark matter candidates that can produce this kind of spin tensor. They are the dark spinors/Elko spinors, see references [56–60]. However, it is not clear that these spinors are the only viable alternative. Thus, we will not choose any fundamental particle model. Instead, we will use only dimensional arguments to keep the discussion as general as possible.
2.2. Torsional dressing of dark matter

Instead of making some standard conjecture (e.g., Weyssenhoff fluid, Frenkel condition, Tulczyjew condition, among others) on the physical nature of the dark matter spin tensor $\sigma^{(DM)}_{\lambda\mu\nu}$, we adopted a more straightforward approach. We may not understand $\sigma^{(DM)}_{\lambda\mu\nu}$ from first principles, but we have some clues about its form. On the one hand, replacing equation (23) in equation (12), we can get an effective stress-energy tensor $T^{(T)}_{\mu\nu}$ in terms of $f$ and $h$. Using dimensional analysis on it, it is clear that torsion components scale with the dark matter density as

$$f \sim \sqrt{\text{energy density}},$$  
(26) $$h \sim \sqrt{\text{energy density}}.$$  
(27)

On the other hand, it is clear that in a dark matter vacuum ($\rho_{DM} = 0$), the associated spin tensor vanishes and $h(t) = f(t) = 0$. Similarly, it seems reasonable to expect $h(t)$ and $f(t)$ to grow for higher values of $\rho_{DM}$. Thus, it seems natural to propose an Ansätze of ‘torsiotropic relations’ between $h(t)$, $f(t)$, and the dark matter energy density $\rho_{DM}$ of the form

$$f \sim \sqrt{\rho_{DM}},$$  
(28) $$h \sim \sqrt{\rho_{DM}}.$$  
(29)

Of course, much more complex relationships are possible, but these seem to be the simplest torsional equations of state. Let us consider a standard barotropic relation for the dark matter pressure $p_{DM} = \omega_{DM} \rho_{DM}$ and let us write the torsiotropic Ansatz for $f$ as

$$f = \alpha_f \sqrt{\rho_{DM}/3},$$  
(30)

where $\alpha_f$ is a constant. In terms of $\alpha_f$, it proves practical to define the ‘semi-dressed’ dark matter energy density and pressure

$$\rho_f = \rho_{DM} + 3f^2 = \left(1 + \alpha_f^2\right) \rho_{DM},$$  
(31) $$p_f = p_{DM} - f^2 = \left(\omega_{DM} - \frac{1}{3} \alpha_f^2\right) \rho_{DM}.$$  
(32)

In terms of $\rho_f$ and $p_f$, the equations (24) and (25) take the more straightforward form

$$3(H + h)^2 = \rho_f,$$  
(33) $$2 \left(\dot{H} + \dot{h}\right) + (3H + h)(H + h) = -p_f,$$  
(34)

where the ‘semi-dressed’ pressure $p_f$ obeys the barotropic relation $p_f = \omega_f \rho_f$ and

$$\omega_f = \frac{\omega_{DM} - \alpha_f^2/3}{1 + \alpha_f^2}.$$  
(35)

In short, the $f$-component of the spin tensor has the effect of replacing the original ‘bare’ dark matter density $\rho_{DM}$ by an amplified ‘semi-dressed’ energy density $\rho_f$ equation (31). Similarly, a smaller ‘semi-dressed’ $p_f$ pressure, equation (32) replaces $p_{DM}$. It is worth to notice that in the case of cold dark matter $\omega_{DM} = 0$, it leads us to an effective negative pressure $-1/3 < \omega_f \leq 0$. 


This way, torsion can easily produce an effective ‘non-particle’ negative pressure $p_f$ from canonical cold dark matter $\omega_{\text{DM}} = 0$. From equations (33) and (34), we may feel compelled to define a generalized Hubble parameter $H + h$. However, it is essential to remember that our observations describe the behavior of classical particles (i.e., galaxies). Classical particles are sensitive only to the Riemannian piece of the geometry and oblivious to torsion, and therefore observations measure $H$ and not $H + h$. Thus, it is convenient to write down the equations (33) and (34) as
\begin{align}
3H^2 &= \rho_f + \rho_h, \\
2\dot{H} + 3H^2 &= - \left( p_f + p_h \right),
\end{align}
where $\rho_h$ and $p_h$ are the effective density and pressure originated when moving all the $h(t)$-terms to the right-hand side of the field equations
\begin{align}
\rho_h &= -3 \left( h + 2H \right) h, \\
p_h &= h^2 + 4Hh + 2\dot{h}.
\end{align}
The dressed density and pressure
\begin{align}
\rho_{\text{dressed}} &= \rho_f + \rho_h, \\
p_{\text{dressed}} &= p_f + p_h,
\end{align}
decribe the total dark matter and torsion combined effect.

Using the relation (29), we propose the ‘torsiopic’ Ansatz
\begin{equation}
h = \alpha_h \sqrt{1 + \alpha_f^2 \rho_{\text{DM}}} = \alpha_h \sqrt{\rho_f},
\end{equation}
and from here, the behavior of the dark matter spin tensor, and its torsion becomes more apparent. The two functions $f$ and $h$ parametrize the dark matter spin tensor, and they create an effective ‘torsional dress’ for $\rho_{\text{DM}}$ and $p_{\text{DM}}$. For instance, a small $\rho_{\text{DM}}$ may be amplified for torsion and create a much bigger $\rho_{\text{dressed}} = \rho_f + \rho_h$. Current observations would measure the effective $\rho_{\text{dressed}}$ and not the original dark matter density $\rho_{\text{DM}}$. The torsional-dressed density $\rho_{\text{dressed}} = \rho_f + \rho_h$ has more complicated behavior than standard density. Replacing equation (33) in equation (34), it is possible to prove that the two dark matter-torsion modes, the $f$-dressed density $\rho_f$ and the $h$-dressed density $\rho_h$, interchange energy among them
\begin{align}
\dot{\rho}_f + 3H \left( \rho_f + p_f \right) &= -Q, \\
\dot{\rho}_h + 3H \left( \rho_h + p_h \right) &= Q,
\end{align}
where
\begin{equation}
Q = \left(1 + 3\omega_f \right) h \rho_f.
\end{equation}

Therefore, the effective density $\rho_{\text{dressed}} = \rho_f + \rho_h$ obeys the canonical conservation relation
\begin{equation}
\dot{\rho}_{\text{dressed}} + 3H \left( \rho_{\text{dressed}} + p_{\text{dressed}} \right) = 0,
\end{equation}
with $p_{\text{dressed}}$ obeying a non-trivial equation of state $p_{\text{dressed}} = p_{\text{dressed}} \left( \rho_f, \rho_h \right)$.

The next section analyses the phenomenology of this system for some relevant particular cases.
3. Cosmological consequences of dark torsion

The two torsional modes, $h$ and $f$, create very distinctive phenomenology in the context of cosmic evolution. The simplest case is $h = \alpha h = 0$. This case is the closest to standard $\Lambda$CDM dynamics, but with $\rho_f$ playing the role of $\rho_{DM}$. The only difference with the standard $\Lambda$CDM torsionless case is that for cold dark matter $\omega_{DM} = 0$, the effective barotropic constant

$$\omega_f = \left(\omega_{DM} - \alpha^2 f / 3\right) / \left(1 + \alpha^2 f\right)$$

has the allowed range $-1/3 < \omega_f \leq 0$. Since $\rho_f = (1 + \alpha^2 f) \rho_{DM}$, it means that for large values of $\alpha_f$ a small quantity of dark matter can get significantly amplified.

The reference [61] studied this case in depth. This simple case is remarkable because it provides a natural solution to the Hubble parameter tension problem while keeping dynamics as close as possible to standard $\Lambda$CDM.

In the current article, let us focus our attention on the $h \neq 0$ case. To focus our attention on dark matter-torsion behavior, let us keep out of the game, for now, the baryons and the cosmological constant.

From equation (33), and using the shortcut notation $s_X = \text{sign}(X)$, we can obtain

$$H(t) = \left(\sqrt{\frac{1}{3} s_{H+h} + s_h} \right) |h(t)|.$$

Therefore the $\rho_h$ density corresponds to

$$\rho_h = 3 \left|\alpha_h\right| - \frac{2}{\sqrt{3}} s_h s_{H+h} \frac{h^2}{|\alpha_h|}.$$  

(48)

Let us consider the possible signs’ combinations.

(a) Case $s_h = -1$ and $s_{H+h} = +1$. It leads to

$$H(t) = \left(\sqrt{-\frac{1}{3} s_h s_{H+h}} + 1\right) |h(t)|,$$

and therefore, we have $|\rho_h| < 1$. The effective $h$-density corresponds to

$$\rho_h = 3 \left|\alpha_h\right| - \frac{2}{\sqrt{3}} \frac{h^2}{|\alpha_h|}.$$  

(50)

(b) Case $s_h = +1$ and $s_{H+h} = -1$. It leads to

$$H(t) = \left(1 - \sqrt{-\frac{1}{3} s_h s_{H+h}}\right) |h(t)|,$$

and therefore, $H > 0$ implies $|\alpha_h| > \sqrt{\frac{1}{3}}$ and $\frac{|\rho_h|}{\rho_{DM}} > 1$. The effective $h$-density corresponds to

$$\rho_h = 3 \left|\alpha_h\right| + \frac{2}{\sqrt{3}} \frac{h^2}{|\alpha_h|}.$$  

(51)

(c) Case $s_h = +1$ and $s_{H+h} = +1$. It leads to

$$H(t) = \left(\sqrt{\frac{1}{3} s_h s_{H+h} - 1}\right) |h(t)|,$$

and therefore, $H > 0$ implies $|\alpha_h| > \sqrt{\frac{1}{3}}$ and $\frac{|\rho_h|}{\rho_{DM}} > 1$. The effective $h$-density corresponds to

$$\rho_h = 3 \left|\alpha_h\right| - \frac{2}{\sqrt{3}} \frac{h^2}{|\alpha_h|}.$$  

(52)
and therefore, \( H > 0 \) implies \(|\alpha_h| < \sqrt{\frac{2}{3}}\) and \(\frac{|h|}{|\alpha_h|} > 1\). The effective \( h \)-density corresponds to

\[
\rho_h = 3 \left( |\alpha_h| - \frac{2}{\sqrt{3}} \right) \frac{h^2}{|\alpha_h|},
\]  

(54)

The first case leads to \( \rho_h > 0 \), and in the last two, we can have \( \rho_h < 0 \). Let us focus our attention on the first \( \rho_h > 0 \) case. After replacing \( H \) and \( h = \alpha_h \sqrt{\rho_f} \) in \( \dot{\rho}_f + 3H (1 + \omega_f) \rho_f = - (1 + 3\omega_f) h \rho_f \), we obtain the following solution for \( h \),

\[ h(t) = h_0 \frac{1}{1 - \frac{\Delta}{t_0 - t}}, \]  

(55)

with \( t_0 \) our current time, \( h_0 = h(t_0) \), and

\[
\Delta = \left[ |\alpha_h| - \frac{\sqrt{3}}{2} \left( 1 + \omega_f \right) \right] \frac{|h_0|}{|\alpha_h|}.
\]  

(56)

For the cold dark matter case \( \omega_{DM} = 0 \), we have \( \omega_f = -\alpha_f^2 / 3 (1 + \alpha_f^2) \), and therefore

\[
\Delta = \left[ |\alpha_h| - \frac{\sqrt{3}}{2} \left( 1 + 2\alpha_f^2 / 3 \right) \right] \frac{|h_0|}{|\alpha_h|}.
\]  

(57)

A remarkable case of this system is \( \Delta < 0 \), or

\[
|\alpha_h| < \frac{\sqrt{3}}{2} \left( \frac{1 + 2\alpha_f^2 / 3}{1 + \alpha_f^2} \right) < 1.
\]  

(58)

In this case, \( h(t) \) corresponds to

\[ h(t) = h_0 \frac{1}{|\Delta| t_0 - t}, \]  

(59)

with the singularity time \( t_s \) corresponding to

\[ t_s = t_0 + \frac{1}{|\Delta|}. \]  

(60)

This case corresponds to phantom evolution. At \( t = t_s \) all components explode: the Hubble parameter, the densities \( \rho_{DM}, \rho_f, \rho_{dressed} \) and \( Q \). It is a big rip singularity.

The \( Q \)-function becomes

\[
Q(t) = -\frac{1}{|\alpha_h|^2 \left( 1 + \alpha_f^2 \right)} |h|^3,
\]  

(61)

and therefore there is energy being transferred from the torsional \( h \)-mode to the \( f \)-mode.

Of course, the whole analysis is model-dependent. For instance, using a torsional model beyond the simple ECSK theory may lead to a different cosmic evolution. Nevertheless, it is essential to observe that in general torsion seems to lead to more acceleration, regardless of model details. For instance, a model similar to the one shown here but with a different choice of torsiotropic constants [61] could explain the Hubble parameter tension. The reference
considered $h \sim H$, leading to exponential expansion-type solutions, and reference [62] obtained quintessence cosmological solutions in a similar setting. Similar behavior have been observed in reference [63]. Even more, completely different torsional theories in the $f(T)$-family [64] also lead to phantom schemes. From this, we see that torsion-caused acceleration seems to be a robust feature across different theories.

4. Thermodynamics

We inspect two thermodynamics aspects in the presence of torsion, adiabaticity, and dark matter temperature. We start with Gibb’s relation

$$T dS = d \left( \frac{\rho_{DM}}{n} \right) + p_{DM} d \left( \frac{1}{n} \right), \tag{62}$$

implying

$$nT \frac{dS}{dt} = - (\rho_{DM} + p_{DM}) \frac{\dot{n}}{n} + \dot{\rho}_{DM}. \tag{63}$$

where $T$ is the temperature, $S$ the entropy, and $n$ the number particle density. Using the integrability condition ($S$ is a function of state) we have $\partial^2 S/\partial T \partial n = \partial^2 S/\partial n \partial T$, and therefore

$$n \frac{\partial T}{\partial n} + (\rho_{DM} + p_{DM}) \frac{\partial T}{p_{DM}} = T \frac{\partial \rho_{DM}}{\partial \rho_{DM}}. \tag{64}$$

Since $\rho_f = \left(1 + \alpha_f^2\right) \rho_{DM}$ and $\rho_f$ satisfies equation (43), we have that the bare dark matter density obeys the ‘conservation’ law

$$\dot{\rho}_{DM} + 3H \left(1 + \omega_f\right) \rho_{DM} = - \frac{Q}{1 + \alpha_f^2}. \tag{65}$$

Besides this, making the hypothesis of $\dot{n} + 3Hn = 0$ (conservation of the number of dark matter particles) we have

$$nT \frac{dS}{dt} = 3H \left(p_{DM} - \omega_f \rho_{DM}\right) - \frac{Q}{1 + \alpha_f}. \tag{66}$$

Therefore, in the case of cold dark matter $\omega_{DM} = 0$ it implies that

$$nT \frac{dS}{dt} = \frac{1}{1 + \alpha_f} \left(H \alpha_f^2 \rho_{DM} - Q\right), \tag{67}$$

and there is no adiabaticity. Given that $Q < 0$, then $dS/dt > 0$, and the second law of thermodynamics is satisfied.

The $\Lambda$CDM [65] model has a high success describing cosmic evolution, but it is not free of well-known problems. It motivates the concept of dark energy as an alternative to $\Lambda$, and observations do not rule out dark matter-dark energy interaction [66]. In this interaction framework, the non-adiabaticity is manifest [67]. From the analysis above, it is clear that torsion could be the cause of this non-adiabaticity.

Using the integrability condition, the temperature can be obtained from

$$\frac{\dot{T}}{T} = - 3H \omega_{DM} \left(1 + \frac{(1 + \omega_{DM})(1 + \alpha_f^2)}{1 + \omega_{DM} + 2\alpha_f^2/3 + Q/3H \rho_{DM}}\right)^{-1}, \tag{68}$$

and there is no adiabaticity. Given that $Q < 0$, then $dS/dt > 0$, and the second law of thermodynamics is satisfied.
and it is clear that \( \omega_{\text{DM}} = 0 \) implies \( T = \text{const.} \) The fact that torsion does not change the dark matter temperature is consistent with standard isothermal dark matter evolution.

5. Conclusions

Given our lack of knowledge concerning the nature of dark matter, its possible features are worth exploring. In general, it is implicitly assumed a negligible spin tensor in dark matter models, although there is no observational evidence supporting this belief. In the current article, we have focused on the consequences of lifting this hypothesis.

The spin tensor is a source of torsion. Since torsion does not interact with standard model bosons, it behaves as an extra dark source of standard torsionless Riemann curvature. In this scenario, the observed dark matter phenomenology corresponds to the combined effect of ‘bare’ dark matter and the torsion it creates through its spin tensor. The sum of the Riemannian effects of dark matter and the dark torsion it creates what we call the ‘dressed’ dark matter density \( \rho_{\text{dressed}} = \rho_f + \rho_h \) and pressure. This concept is convenient from an algebraic point of view, and it also corresponds to the observed energy density.

Since we do not know the dark matter composition, we did not use any of the usual spin tensor Ansätze. Instead of this, we only used dimensional analysis and some simple ‘torsiotropic’ relations between the spin tensor components and the dark matter density.

To study the consequences of a nonvanishing spin tensor, we choose to work with ECSK. It is the simplest and closest alternative to standard GR with nonvanishing torsion. Despite this, the induced cosmological evolution is far from trivial. Even in this simple scenario, the spin tensor and torsion can produce a phantom scheme without phantom dark energy. Since current observations do not rule out this kind of cosmic evolution, it is a worth-considering alternative.

In cosmological scales, Copernican symmetries allow for only two torsional degrees of freedom, \( h \) and \( f \). We found that both modes must interchange energy between them. Since future observations of gravitational waves polarization could constrain torsion components [24–26], it is a behavior that could be measured in the future.

From a thermodynamic point of view, cosmic evolution turns out non-adiabatic. It contrasts with the adiabatic behavior of \( \Lambda \)CDM, but the second law of thermodynamics is satisfied nevertheless. Interestingly, dark matter evolution remains isothermal, even in the presence of torsion.

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