On the statistical nature of fatigue crack-growth through Monte Carlo simulations and experimental data

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Abstract. Understanding factors that contribute to scatter in crack-growth rates data is fundamental to reduce risk of unexpected structural failures. This document discusses the statistical nature of material scattering observed in fatigue crack propagation tests through a comparison between Monte Carlo simulations and experimental test data. The case study consists of constant amplitude tests on central cracked 2024-T3 aluminium alloy plate specimens loaded in pure Mode-I with stress ratio R equals to 0.2. It was found that results of simulations matched the experimental fatigue lives in terms of both mean value and dispersion around it. Based on these outcomes, the crack-growth behaviour can be described in a statistical manner with the expectation to reduce the inaccuracy present in life predictions.

1. Introduction
Life prediction of structures undergoing variable loads necessitates deep knowledge about the fatigue phenomenon. As it is known today, the metal fatigue process is complex, still not fully understood, with many variables which can influence the life of a structure, such as: material, loading, geometric characteristics, pre-existing defects, etc. [1-4]. Quantifying and controlling these variables is essential to enhance the competitiveness in designing fatigue-safe products, e.g. for turbine engines, railway axles, etc. [1, 3, 5-6]. Engineering components generally fail due to irreversible micro-plastic deformation from which cracks are nucleated, generally from notches or at critical locations [6-8]. Considering the inhomogeneous microstructure and non-uniform mechanical properties of metal materials, crack tips and defects may encounter diversiform microstructures, in turn demonstrating variable strength against Fatigue Crack-Growth (FCG), as if “the cracks would pass through a different material possessing different properties” [9]. As a consequence, it can be stated that FCG has inevitably stochastic characteristics, at least from a macroscopic point of view.

According to numerous researches [2-3, 9-13], scatter of material data seems to represent the most relevant contribute to the variability of life predictions of engineering structures. This investigation developed statistical concepts to FCG behaviour by considering only the material scattering as source of variability for the definition of the lifespan distribution for a given case study, i.e. a plate specimen made of AA2024-T3, loaded in pure Mode-I, see Figure 1. More in detail, this document discusses the statistical nature of material scattering observed in fatigue crack propagation tests performed in [9, 13], through a comparison between Monte Carlo simulations and the literature test data. The document also attempted to describe crack-growth behaviour in a statistical manner with the expectation that this
statistical description could be able to reduce the large amount of error currently present in numerous life prediction methods.

**Figure 1.** Case study description.

### 2. Materials and Methods

The study of fatigue crack propagation behaviour has been widely conducted for some time in an effort to understand metal fatigue more fully. To this aim, experimental tests on specimens are generally used in a scale approach from small uniaxial laboratory samples to larger and complex multiaxially loaded mechanical components. The information obtained from crack propagation studies on specimens is then used in estimating the fatigue life of structures and components. The current investigation has the main purpose of applying statistical concepts and theory to the study of fatigue crack propagation behaviour for laboratory tests on specimens. This experimental campaign was carried out in [9, 13] and was here briefly recalled (Figure 1). Although [9, 13] do not represent recent researches, these were considered as the perfect benchmark where to cross-validate the up-to-date stochastic framework. In particular, the experimental results of [9, 13] in terms of Crack-Growth Rates (CGRs), i.e. \( \frac{da}{dN} \) vs. \( \Delta K \) (see Figure 2), were here used to perform life predictions for the same geometry, material and loading conditions in a probabilistic way through Monte Carlo simulations. Such a case study sounded perfect to this aim because consisting of 68 replica tests in which only the source of variability coming from material properties could be identified. Therefore, this represented the perfect benchmark through which derive quantifications based only on the variability of material data.

The experimental campaign consisted of 68 identical constant amplitude tests on central cracked AA2024-T3 aluminium alloy plate specimens loaded sinusoidally (load amplitude \( P \)) in pure Mode-I with stress ratio \( R=0.2 \). A rigorous pre-cracking phase was carried out for all the 68 replica tests so as to obtain a precise half crack length of 9 mm from which start the cycle counting. The aim of this experimental campaign was the same of the current investigation, namely to investigate the material scattering and how it can affect the fatigue crack propagation process.
Figure 2. $\frac{da}{dN}$ vs. $\Delta K$ experimental data. [9, 13]

Material data of Figure 2 are generally adopted in order to calibrate a FCG law useful to perform life estimates of the cracked components [14-16]. A Paris-like FCG law, based on Equations 1-3, was used in this work and implemented in a stochastic way. This law was selected among those available in literature since material data of Figure 2 presented a mostly perfectly linear tendency. More sophisticated FCG law were instead adopted in [3, 11] through a framework similar to that proposed here.

Thanks to the stochastic implementation of the FCG law, no material parameters needed to be directly calibrated through CGR test data. The two main parameters $C$ and $m$ of Equation 1 were derived through Monte Carlo simulations as detailed in the followings, see Figure 3. The plane stress/strain constraint factor $\alpha$ was kept as fixed to 1.5, as suggested in [3] with reference to relatively low-strength materials. $\sigma/\sigma_0$ ratio was calculated according to the applied load $P$ ($\sigma = P/Wt$) and the flow stress $\sigma_0$ of the material (439 MPa). $W$ is the width of the plate (i.e. 152.4 mm), $t$ is the thickness of the plate (i.e. 2.54 mm), whereas $a$ is the continuously increasing half crack length (varying from 9 to 49.8 mm, as for the experimental campaign). More details about how to calibrate these parameters can be found in [3].

Equations 1-3 were directly inserted in the MATLAB programming platform [17], in which a vast series of $(C, m)$ sets were randomly sampled within the grid shown in Figure 3 (top left graph). A CGR curve was drawn for each set and compared with all the test data (in Figure 2). Each CGR curve was then judged as acceptable only if the related root mean square error was lower than a cut-off threshold (set to 2%). CGR curves that insufficiently correlated with test data were immediately discarded, whereas the “valid” ones were used to compute a fatigue life prediction according to Equation 4 [18], with this latter used to calculate the $K$ values along the crack advancing.

These Monte Carlo simulations were performed until that a total of 1e5 CGR good curves (and corresponding life predictions) were derived (~30 min of run on a regular laptop).

$$\frac{da}{dN} = C \left[ \left( 1 - \frac{f}{1 - R} \right) \Delta K \right]^m$$

(1)
Figure 3. Monte Carlo simulation scheme.

3. Results and discussion
Ideally, it is desirable that the estimated life will exactly predict the actual life. Unfortunately, there are many variables which influence predictions and some of them are still not well understood. One of the most important of these variables is how well the FCG laws (Paris, NASGRO, …) calibrated onto experimental data actually represent the observed crack propagation behaviour.

Thanks to the Monte Carlo framework here adopted (Figure 3), the authentic distributions of the main FCG law parameters $C$ and $m$ can be derived. These are arranged in the correlation plot reported in Figure 4. A first key aspect to notice is that these parameters have rather large ranges of equally acceptable values. Namely, parameter $m$ accepts a range of variability from 2.25 to 3.5, whereas $\log_{10} C$ can vary from -8.9 to -10.3 (thus $C$ can vary from $1.3e-9$ to $5e-11$). However, their variation is
not arbitrary since $\log_{10} C$ and $m$ are mostly linearly correlated. This has been here assessed with reference to an aluminium alloy but was also observed in [3, 11] for a high-strength steel and already discussed from more theoretical standpoints, see e.g. [19-20].

Figure 5 reports all the CGR curves that were obtained from the Monte Carlo simulations. All these CGR curves returned a root mean square error lower than 2% and, therefore, they can be all equally considered as “good-fit” for the test data of Figure 2. The corresponding life predictions are reported in Figures 6-7 where it can be noticed a very good agreement between the simulation results and the outcomes measured experimentally in [9, 13].

![Figure 4](image.png)

**Figure 4.** Correlation plot for FCG law parameters $C$ and $m$.

![Figure 5](image.png)

**Figure 5.** Simulated CGR curves.
In Figure 5, the CGR curve that returned the lowest root mean square error against the test data (i.e. the “best-fit” curve) is also highlighted, whereas the related predictions are also highlighted in Figures 6-7. “Best-fit” outcomes reported in Figures 4-7 represent what can be obtained by using a traditional deterministic approach. Although perfectly predicting the mean value of the lifespan distribution, a “best-fit” deterministic calculation cannot add any substantial information to the prediction itself, e.g. the likelihood to reach the predicted life, the likelihood of failure for a given number of cycles, etc. Furthermore, also the conservativeness or the non-conservativeness of this prediction cannot be guaranteed a priori [3, 11]. On the contrary, the stochastic approach here described and used to account for the material scattering properly captured both the mean value of experimental lives and its variance.

A further observation can be made with reference to Figure 8, in which three sets of equally acceptable FCG laws and corresponding life predictions were extracted (data sets shown in Figure 4). It is worth noting that high values of parameter C, associated with low values of parameter m (required to reduce the steepness of the curve so as to fit test data), produce a high \(da/dN\) in the initial part of propagation, in turn returning a reduced life prediction. Finally, it can be noticed that, although all these three lines well fit the test data, the variability of their life predictions can be non-negligible, returning a prediction for “Data C” more than 50% higher than that of “Data A” (310k vs. 200k cycles respectively). Due care has to be made when defining the FCG law parameters to perform fatigue crack-growth predictions, especially if considering that the results presented here refer to laboratory specimens, thus mostly taking into account of material scattering only, whereas numerous further contributors of uncertainty and variability have to be added when working on real structures.

![Figure 6. Comparison of experimental and simulated \(a\) vs. \(N\) curves.](image-url)
Figure 7. Histogram of the experimental and simulated life predictions.

Figure 8. Comparison of three CGR data sets and corresponding predictions (data sets highlighted in Figure 4).

4. Conclusions
The main purpose of this work was to apply statistical concepts to the study of fatigue crack propagation behaviour. In doing this, this study involved the determination of the effects of the material properties variability on the life prediction for plate specimens made of AA2024-T3 loaded in Mode-I.

Monte Carlo simulations were run to derive the distributions of the two main FCG law parameters ($C$ and $m$) and a vast series of CGR curves were drawn and used to predict the life of the specimens, together with an analytical equation used to calculate the $K$ values at various crack lengths.

A very good match was achieved between experimental and simulated life predictions in terms of both mean value and dispersion around it. This demonstrated that the Monte Carlo simulations well captured the ranges of variability of parameters $C$ and $m$ of the FCG law. As a consequence, the whole stochastic framework demonstrated to correctly capture the scattering observed in experimental data. The study attempted to describe crack-growth behaviour in a statistical manner with the expectation that this stochastic framework can be used to reduce the uncertainty of life predictions by providing a solid range of variability to the predictions.
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