Extra-symmetric Born–Infeld theory of electroweak and gravitational fields

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Abstract

This paper suggests a generalization of the Born–Infeld action (1932) for the case of electroweak and gravitational fields. Basic notions one deals with are Dirac matrices, $\gamma_a$, and dimensionless covariant derivatives, $\pi_a = -i\ell\nabla_a$, given in spinorial and scalar representations. The action contains a characteristic length $\ell$ (which is of order of magnitude of Planck’s length), as a parameter and possesses an extra symmetry with respect to transformations of the Lorentz group imposed on pairs $(\gamma_a, \pi_a)$. It’s shown that parameter of the Lorentz group is associated with a constant value of the electroweak potential at spatial infinity.

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I. INTRODUCTION

The presently accepted view of the standard model, and of general relativity, is that these provide the leading terms in effective field theories [5].

It is due to superstring theories (e.g., [1,2]), that interest in the Born–Infeld (BI) field theory [3], initially formulated for electromagnetic fields, was revived.

Nonlinear modifications of standard gravitational and electromagnetic theories were studied with different corrections, for example, the coupling of BI electromagnetic action to Einstein–Hilbert action was considered by Wirschins et al. [6] and by Ayon-Beato and Garcia [4,5], and the nonlinear in curvature tensor gravitational actions of BI type were investigated by Feigenbaum et al. [10] and myself [11].

In general, actions are constructed from invariants of symmetry group. The higher the symmetry—the more restricted the class of invariants that enters the action. Ultimately, incorporating a complete symmetry group (not known yet), one would, probably, obtain a unique lagrangian function.

The point of this paper is to bring additional attention to the BI theory by reformulating it in terms of Dirac matrices, $\gamma_a$, and dimensionless covariant derivatives, $\pi_a$, given in spinorial and scalar representation. All interactions are concealed in connections, associated with $\pi_a$. Such reformulation allows one to postulate additional (to general covariance, gauge invariance, and invariance under unitary matrix transformations) symmetry of the action by making it invariant under the Lorentz transformations imposed on pairs $(\gamma_a, \pi_a)$. This in turn allows (1) a decrease of ambiguity in the selection of invariants entering the action, and (2) obtaining invariants, which, being decomposed into rows in powers of parameter $\ell$, allow coexistence of actions for electroweak, gravitational, and scalar fields as leading terms of the row.

Thus, the following basic notions are introduced. (α) Dirac matrices, (with dimensions $N \times N$), $\gamma_a$, submitted to relations,

$$\gamma(a \gamma b) = g_{ab} \hat{1}, \tag{1.1}$$

where $g_{ab}$ is the metric tensor; (β) dimensionless operators,

$$\pi_a = -i\ell \nabla_a \tag{1.2},$$

where $\nabla_a$ is a covariant derivative and $\ell$ is fundamental constant. Action of (1.2) on spinors, scalars, or Dirac matrices is specified in the following text. For spinors, for example, one obtains, $\pi_a \Psi = -i\ell (\partial_a \Psi - \Gamma_a \Psi)$. By a postulate, the action for fields is form-invariant with respect to substitutions, $\gamma_a \mapsto \gamma_a'$, and $\pi_a \mapsto \pi_a'$, where

$$\gamma_a' = \cosh \theta \gamma_a + \sinh \theta \pi_a, \tag{1.3}$$

$$\pi_a' = \sinh \theta \gamma_a + \cosh \theta \pi_a, \tag{1.4}$$

and $\theta$ doesn’t depend on coordinates.

* It follows that $\ell$ is of order of magnitude of the Planck’s length.
II. OBJECTS OF THE THEORY

1. One may introduce a dimensionless curvature tensor,
\[ \rho_{ab} = 2\pi\left(\partial_{[a}\Gamma_{b]} - \Gamma_{[a}\Gamma_{b]}\right), \]
(2.1)
where \(\Gamma_a\) are connections, given in spinorial or scalar representation. One assumes, that the Christoffel symbol, \(\Gamma^c_{ab}\), associated with the covariant derivative is symmetric in the lower indices. Operator \(\rho_{ab}\) is matrix \(N \times N\).

2. Define a tensorial operator,
\[ \phi_{ab} = \gamma_{[a} \pi_{b]} . \]
(2.2)

3. Define a scalar density,
\[ \phi = \frac{1}{5!N} e^{abcd} e^{efgh} \text{Tr} \{\phi_{ae} \phi_{bf} \phi_{cg} \phi_{dh}\} , \]
(2.3)
where \(e^{abcd}\) is the absolute antisymmetric symbol, \(e^{0123} = 1\).

4. Define a tensor of the second rank,
\[ \chi_{ab} = \overline{\varphi} \{\gamma_{[a} \pi_{b]} - \pi_{[a} \pi_{b]}\} \varphi = \overline{\varphi} g_{ab} - \overline{\varphi} \pi_{(a} \pi_{b)} \varphi . \]
(2.4)
The quadruplets of scalars \(\overline{\varphi}, \varphi\), are introduced. One assumes that \(\gamma_a\) are Dirac matrices \(4 \times 4\).

Scalar density, reducing to \(\sqrt{-g}\) in the limit \(\ell \to 0\), is \(\sqrt{-\phi}\). Objects, defined in (2.2), (2.3), and (2.4) are form-invariant with respect to substitutions (1.3) and (1.4).

One may prove a useful formula,
\[ e^{abcd} e^{efgh} \gamma_{[a} \gamma_{e]} \gamma_{[b} \gamma_{f]} \gamma_{[c} \gamma_{g]} = 10 \ g_{\gamma^{[h}} \gamma^{d]} , \]
(2.5)
following from (1.4).

III. ACTION FOR FIELDS

Consider a system including electron, neutrino, electroweak and gravitational fields, together with some scalar fields. Introduce two spinorial bases. First, L-basis, is represented by octets \((N = N_L = 8)\),
\[ L_{el} = [\nu_{el}, e_{el}]^T ; \overline{L}_{el} = [\nu_{el}, \pi_{el}] . \]
(3.1)
Here \(e_L\) and \(\nu_L\) are 4-spinors for the left-handed electron and neutrino, respectively; subscript \(el\) reminds that respective spinors belong to electron’s flavor. The second, R-basis, is represented by quadruplets, \((N = N_R = 4)\), \(e_R\) and \(\pi_R\), corresponding to right-handed electron. Introduce two sets of quadruplets of scalar fields. First one is made of scalars, \(\overline{\varphi}, \varphi\), and the second one is made of tensors \(\chi_{ab}\),

\[ \chi_{ab} = \overline{\varphi} \{\gamma_{[a} \pi_{b]} - \pi_{[a} \pi_{b]}\} \varphi = \overline{\varphi} g_{ab} - \overline{\varphi} \pi_{(a} \pi_{b)} \varphi , \]

Here \(\gamma^a = g^{ab} \gamma_b\), where \(g^{ab} g_{cb} = \delta^a_c\).
\[ \Phi = [u, u', v, v']^T = [U, V]^T; \]
\[ \Phi^\dagger = [u^\dagger, u'^\dagger, v^\dagger, v'^\dagger] = [\overline{U}, \overline{V}]. \]

The second one is made of scalars, \( \Pi \), and \( \Pi \). One may specify action of \( \Pi \) on these fields.

\[ \pi_a L_{el} = -i\ell \left( \partial_a + \frac{i}{2} g' \sigma_k \otimes \hat{1}_4 W^k_a + \frac{i}{2} g'' Y_L \otimes \hat{1}_4 B_a - \hat{1}_2 \otimes \Gamma_a \right) L_{el}, \]
\[ \pi_a e_R = -i\ell \left( \partial_a + \frac{i}{2} g'' Y_R B_a - \Gamma_a \right) e_R, \]
\[ \pi_a U = -i\ell \left( \partial_a + \frac{i}{2} g' \sigma_k W^k_a + \frac{i}{2} g'' Y_L W_a \right) U, \]
\[ \pi_a V = -i\ell \left( \partial_a + \frac{i}{2} g' \sigma_k W^k_a + \frac{i}{2} g'' Y_W B_a \right) V, \]
\[ \pi_a \Pi = -i\ell \partial_a \Pi, \]

where \( B_a \) and \( W^k_a \) \((k = 1, 2, 3)\) are potentials of electroweak fields; \( Y_L = diag(-1, -1), Y_R = -2, \) and \( Y_W = diag(1, 1) \) are hypercharge operators; \( \sigma_k \) \((k = 1, 2, 3)\) are standard \( 2 \times 2 \) Pauli matrices; \( g' \) and \( g'' \) are interaction constants. Connections \( \Gamma_a \) \((matrices 4 \times 4)\) pertain to gravitational fields only. \( A \otimes B \) denotes the Kronecker product of matrices \( A \) and \( B \). Dirac matrices, \( \gamma_a \), have the following structure. In L-basis,

\[ \gamma_a = diag \left( \gamma_a, \gamma_a \right) = \hat{1}_2 \otimes \gamma_a, \]

where \( \gamma_a \) are Dirac matrices \( 4 \times 4 \). In R-basis, \( \gamma_a = \gamma_a \). To be more specific about gravitational sector, one may introduce 4 standard Dirac matrices \( 4 \times 4, \Delta_A, A = 0, 1, 2, 3: \)

\[ \Delta_0 = \begin{pmatrix} 0 & \hat{1}_2 \\ \hat{1}_2 & 0 \end{pmatrix}, \quad \Delta_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad k = 1, 2, 3. \]

Then, the following decompositions take place:

\[ \gamma_a = e_a^A \Delta_A, \]
\[ \Gamma_a = f_a^{AB} \Delta_a \Delta_B, \]

where \( e_a^A \) is a tetrad, and \( f_a^{AB} = f_a^{[AB]} \) are six gauge vector fields, associated with the Ricci rotation coefficients \( [2] \) in standard theory.\(^{[2]}\)

One may use the scalar fields \( \Phi, \Phi^\dagger \), implementing a symmetry breaking mechanism in order to generate masses for vector bosons and fermions. Thus, one assumes, \( \Phi(x^a) = \Phi_0 + \delta \Phi(x^a), \) (and an analogous expression for \( \Phi(x^a) \)).

Expanding the scalar density \( \sqrt{-\chi} \) in \( \ell \), one obtains, \( (\overline{\varphi} \varphi)^{-1} \sqrt{-\chi} = \sqrt{-g} \{ \overline{\varphi} \varphi - \frac{1}{2} \overline{\varphi} \pi^a \pi_a \varphi \} + O(\ell^4) \), where \( \pi^a = g^{ab} \pi_b, \) or after substitution of \( ([2], [3], [3], [3], [3], [3]) \) into \( (2.4) \).

\(^{[2]}\) Namely, \( f_a^{AB} = \frac{1}{4} e_b^B \nabla_a e^{AB} \), where \( \nabla_a \) is a covariant derivative, associated with metric \( g_{ab} = e_a^A e_b^B. \)

\(^{[3]}\) Here \( \chi = det |\chi_{ab}|. \)

** Here one distinguishes between \( \chi_\Phi \) constructed with \( \varphi = \Phi \) and \( \overline{\varphi} = \Phi^\dagger \), and \( \chi_\Pi \), constructed with \( \varphi = \Pi \) and \( \overline{\varphi} = \Pi. \)
\( (\Pi\Pi)^{-1}\sqrt{-\chi_{\Pi}} = \sqrt{-g}\Pi\Pi + \sqrt{-g}\frac{\ell^2}{2}\Pi g^{ab}\partial_a\partial_b\Pi + \cdots \) \tag{3.13} \\
\( (\Phi\Phi)^{-1}\sqrt{-\chi_{\Phi}} = \sqrt{-g}(p^2 + q^2) - \frac{\ell^2}{8}\sqrt{-g}g^2\left(W_a^1 - iW_a^2\right)\left(W_1^a + iW_2^a\right) - \sqrt{-g}(p^2 + q^2)\frac{\ell^2}{8}\left(g'W_3^a - g''B_a\right)\left(g'W_3^a - g''B_a\right) + \cdots \) \tag{3.14}

One assumes that scalars \( \Phi \) and \( \Phi \) have only the following components in equilibrium:

\[ \Phi_0 = [0, p, 0, q]^T; \quad \Phi_0 = [0, p, 0, q]. \] \tag{3.15}

Similarly, expanding (2.3) into row in powers of \( \ell \), and using (2.5), one obtains for two spinorial bases the following decompositions. For the L-basis,

\[
\sqrt{-\phi_L} = \sqrt{-g} + \frac{1}{48}\sqrt{-g}Tr\{\gamma^a\gamma^b\rho_{ab}\}
+ \frac{\ell^4}{320}\sqrt{-g}\left(g'^2W_k^aW_k^b + g''B^abB_{ab}\right) + O(\ell^4). 
\] \tag{3.16}

For the R-basis,

\[
\sqrt{-\phi_R} = \sqrt{-g} + \frac{1}{48}\sqrt{-g}Tr\{\gamma^a\gamma^b\rho_{ab}\} + \frac{\ell^4}{80}\sqrt{-g}g''B^abB_{ab} + O(\ell^4),
\] \tag{3.17}

where \( \rho_{ab} \) is constructed via \( \Gamma_a \) as in (2.1). Only terms pertaining to known components of the standard model are extracted. One uses notations \( W_k^a = \partial_aW_k^b - \partial_bW_k^a - g'\epsilon_{lm}^k W_l^aW_m^b \) and \( B_{ab} = \partial_aB_b - \partial_bB_a \) for electroweak gauge fields.\(^{††}\) As it’s shown below, the second term in the r. h. s. of (3.16) and (3.17) corresponds to Einstein–Hilbert term.

As it follows from (2.2), (2.3), and (3.8),

\[
\sqrt{-\phi_{\Pi}} = \sqrt{-g},
\] \tag{3.18}

where \( \phi_{\Pi} \) denotes \( \phi \) (c.f. (2.3)) given in \( \Pi \)-representation.

Comparing (3.13), (3.14), (3.16), (3.17), and (3.18), one finds as the action for the fields,

\[
S_f = \int d\Omega \times \left[ \lambda_L\sqrt{-\phi_L} + \lambda_R\sqrt{-\phi_R} + \lambda_\Phi(\Phi\Phi)^{-1}\sqrt{-\chi_\Phi} + \lambda_\Pi(\Pi\Pi)^{-1}\sqrt{-\chi_\Pi} - \Upsilon\sqrt{-\phi_{\Pi}} \right],
\] \tag{3.19}

where one introduces the constants, \( \lambda_L, \lambda_R, \lambda_\Phi, \lambda_\Pi, \) and \( \Upsilon \) contains the potential, implementing symmetry-breaking mechanism:

\[
\Upsilon = \lambda_L + \lambda_R + \bar{V},
\] \tag{3.20}

\(^{††}\) Indices \( k, l, m \ldots \) are raised and lowered with \( \delta_{km} = \delta^{km} = diag(1, 1, 1) \).
where $\tilde{V} = \tilde{V}(\Phi, \Pi)$ denotes the ‘potential energy’. The action (3.19) together with the action for spinors (see (4.3) below) should be varied with respect to fields $e^A_a$, $f^{AB}_a$, $W^a_k$, $B_a$, $\Phi$, $\overline{\Phi}$, $\Pi$, $\overline{\Pi}$, $L_{el}$, $e_R$, and $e_R$.

From the structure of (2.4) and (3.19) it follows that the differential equations for scalar fields contain third order derivatives in corrections of order $\ell^4$.

Comparing the expansion of (3.19) in $\ell$ with the standard action, one obtains the following expressions for constants:

$$\ell^2 = \frac{10}{3\alpha} (1 + 2 \sin^2 \theta_W) L_p^2;$$  \tag{3.21}

$$\lambda_L = -\frac{6}{\pi} \frac{\sin^2 \theta_W c^3}{1 + 2 \sin^2 \theta_W k\ell^2};$$ \tag{3.22}

$$\lambda_R = -\frac{3}{2\pi} \frac{1 - 2 \sin^2 \theta_W c^3}{1 + 2 \sin^2 \theta_W k\ell^2};$$ \tag{3.23}

$$\lambda_\Phi = -\frac{1}{8\pi c\ell^2};$$ \tag{3.24}

$$\lambda_\Pi = -\frac{1}{8\pi c\ell^2};$$ \tag{3.25}

$$p^2 + q^2 = 4 \sin^2 \theta_W \frac{m_W^2 c^4}{e^2},$$ \tag{3.26}

where $\alpha$ is the fine structure constant for an electron, $\theta_W$ is Weinberg’s mixing angle $[11]$, $L_p$ is Planck’s length, and $m_W$ is vector bosons’ rest mass.

### IV. ACTION FOR SPINORS

The relevant tensors, invariant with respect to (1.3) and (1.4), are:

$$\phi^{dh} = \frac{4}{5!} e^{abcd} e^{efgh} \phi_{ae} \phi_{bf} \phi_{cg};$$ \tag{4.1}

$$\Sigma_{ab} = \gamma_{[a} \pi_{b]} - \pi_{[a} \gamma_{b]};$$ \tag{4.2}

Then, the action for fermions may be constructed as follows:

$$S_\Psi = C \int d\Omega \sqrt{-g} \left[ L_{el} \phi^{ab} \Sigma_{ab} L_{el} + \sqrt{-g} e_R \phi^{ab} \Sigma_{ab} e_R + \frac{1}{2} \lambda_{\Pi} L_{el} e_R \epsilon + \overline{\epsilon} R L_{el} \right].$$ \tag{4.3}

Operators (4.1) and (4.2) are given in respective spinorial bases. Expanding (1.3) in powers of $\ell$, one obtains:

$$S_\Psi = \frac{2C}{\hbar} \int d\Omega \sqrt{-g} \left[ L_{el} \gamma^a p_a L_{el} + \overline{\epsilon} R \gamma^a p_a e_R + \frac{\hbar r p}{2\ell} (\epsilon R e_R + \overline{\epsilon} e_R L_{el}) \right] + \cdots$$ \tag{4.4}

The momentum operator, $p_a = -i\hbar (\partial_a - \Gamma_a)$, is given in respective spinorial bases.
One should mention another possibility for the action (see [13]). Consider (3.19) with a corrected potential, $Υ \rightarrow Υ + s (L_{el} e_R U + U e_R L_{el})$, and a corrected tensor $\phi_{ab} \rightarrow \phi_{ab} + C^\alpha \Sigma_{ab} \psi \bar{\psi}$. This alternative for the action seems appealing, but the price to be paid is nonlinearity of the Dirac equations in spinorial fields.

V. GRAVITATIONAL SECTOR

It will be shown, that the gravitational part of the action (3.19) in vacuum (in the limit $\ell \rightarrow 0$) is equivalent to the Einstein–Hilbert action in general relativity. First, extract the action of the gravitational field from the expansion of (3.19) in powers of $\ell$:

$$S_g = \kappa^{-1} \int d\Omega \sqrt{-g} Tr \left\{ \gamma^a \gamma^b \rho_{ab} \right\} , \quad (5.1)$$

Varying (5.1) with respect to $\gamma_a$ and $\Gamma_a$, one obtains the equations,

$$\left[ \gamma^b, \rho_{ab} \right] - \frac{1}{4} \gamma_a Tr \left\{ \gamma^e \gamma^d \rho_{cd} \right\} = 0 , \quad (5.2)$$

$$\frac{1}{\sqrt{-g}} \partial_b \left( \sqrt{-g} \gamma^a \gamma^b \right) - \left[ \Gamma_b, \gamma^a \gamma^b \right] = 0 . \quad (5.3)$$

Equations (5.3) are equivalent to the following:

$$\left( \gamma^a \gamma^b \right) ;_b = 0 , \quad (5.4)$$

where the semicolon denotes covariant derivative. For the covariant derivative of $\gamma_a$ one obtains,

$$\gamma_{a;b} = \partial_b \gamma_a - \Gamma^c_{ab} \gamma_c - \left[ \Gamma_{b}, \gamma_a \right] , \quad (5.5)$$

where $\Gamma^c_{ab} = \Gamma^c_{(ab)}$ is the Christoffel symbol, associated with $g_{ab}$. From (5.4) follows (as one possible solution),

$$\gamma_{a;b} = 0 . \quad (5.6)$$

Applying the antisymmetrized product of covariant derivatives to $\gamma_b$, one obtains,

$$R^a_{bcd} \gamma_a = \frac{1}{\ell^2} \left[ \gamma_b, \rho_{cd} \right] , \quad (5.7)$$

where $R^a_{bcd}$ is the Riemann tensor,

$$R^a_{bcd} = 2 \left( \partial_{[c} \Gamma^a_{b]d} - \Gamma^c_{b[c} \Gamma^a_{d]} \right) . \quad (5.8)$$

‡‡ Here $\psi \bar{\psi}$ denotes an operator, opposite to a scalar $\bar{\psi} \psi$.

§§ One should take for $\psi$ either $L_{el}$ for L-basis, or $e_R$, for R-basis, respectively.
(Note, that from (5.7) follows $Tr(\gamma_a) = 0$.) For the scalar curvature and Ricci tensor one obtains, respectively,

$$R = \frac{1}{2\ell^2} Tr \left\{ \gamma^a \gamma^b \rho_{ab} \right\}, \quad (5.9)$$

$$R_{bd} = \frac{1}{4\ell^2} Tr \left\{ \gamma^a \gamma^b, \rho_{ad} \right\}. \quad (5.10)$$

Multiplying (5.2) by $\gamma_c$ and taking the trace, one obtains the vacuum Einstein equations,

$$R_{ac} - \frac{1}{2} g_{ac} R = 0, \quad (5.11)$$

where definitions (5.10) and (5.9) are used for the Ricci tensor and scalar curvature in \( \rho - \gamma \) representation. Note, that on the classical level, when the matter lagrangian doesn’t depend on $\Gamma_a$, equations (5.11) are still valid, if one adds the stress-energy tensor for matter to the r. h. s. of (5.11). In [12] equations for vacuum gravitational field are obtained, and static spherically symmetric solutions are considered. It is shown that the exterior solution, corresponding to Schwarzschild solution at $r \to \infty$ does not possess a singularity at $r = 0$. Instead, there is a massive ball with finite density of order of magnitude of Planck’s density.

VI. MEANING OF PARAMETER $\theta$

From the structure of the action for electron (4.4) follow expressions for the correction to the energy due to the value of the potentials of electroweak fields at spatial infinity. Namely,

$$\Delta E_L = \frac{\hbar}{2} \left( g' W_0^3(\infty) + g'' B_0(\infty) \right) \int dV \bar{\tau}_L \gamma^0 e_L; \quad (6.1)$$

$$\Delta E_R = \hbar g'' B_0(\infty) \int dV \bar{\tau}_R \gamma^0 e_R. \quad (6.2)$$

One uses notations, $W_0^3(x^a) \to W_0^3(\infty)$, as $|x^a| \to \infty$, etc. From (1.3) and (1.4) follows,

$$\int \bar{T}_{el} \gamma^a L_{el} d\Sigma_a = \cosh \theta \int \bar{T}_{el} \gamma^a L_{el} d\Sigma_a + \sinh \theta \int \bar{T}_{el} \pi^a L_{el} d\Sigma_a, \quad (6.3)$$

$$\int \bar{T}_{el} \pi^a L_{el} d\Sigma_a = \sinh \theta \int \bar{T}_{el} \gamma^a L_{el} d\Sigma_a + \cosh \theta \int \bar{T}_{el} \pi^a L_{el} d\Sigma_a. \quad (6.4)$$

The indices are raised with $g^{ab}$. One integrates over an arbitrary hypersurface with $d\Sigma_a = \sqrt{-g} e_{abcd} dx^b dx^c dx^d$. One may denote energy, $E = \int \bar{T}_{el} p^0 L_{el} dV$, and electrical charge, $Q = \int \bar{T}_{L} \gamma^0 e_L dV$, where $dV \equiv d\Sigma_0$ is the element of spatial volume, and $p^a = \frac{\hbar}{\ell} \pi^a$ is the momentum operator. Selecting spatial volume as a hypersurface of integration, one obtains,

$$E' = \cosh \theta E + \frac{\hbar}{\ell} \sinh \theta Q; \quad (6.5)$$

$$Q' = \cosh \theta Q + \frac{\ell}{\hbar} \sinh \theta E. \quad (6.6)$$

Equations (6.3) and (6.4) represent transformations of energy and electrical charge, respectively. Suppose, that $E = 0$. Then, $\tanh \theta = \frac{\ell}{\hbar} \frac{E'}{Q'}$, which means that energy $E'$ is due as a whole to electrical charge $Q'$, placed in constant electroweak potential, as in (6.1). Thus,
\[ \tanh \theta = \frac{\ell}{2} \left( g'W_0^3(\infty) + g''B_0(\infty) \right). \tag{6.7} \]

Analogous considerations for \( e_R \) instead of \( L_{el} \) lead to the equation
\[ \tanh \theta = \ell g''B_0(\infty), \tag{6.8} \]
from which it follows that one should put \( g'W_0^3(\infty) = g''B_0(\infty) \)

Thus, one should associate the \( \theta \)-transformation with that of electroweak potentials at spatial infinity. From (6.7) and (6.8) it follows that \( \theta \propto \ell \), and neglecting terms of order \( \ell^2 \), one obtains \( Q' = Q \).

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*** One should use \( \sin \theta_W \frac{g'}{\sqrt{2}} = \cos \theta_W \frac{g''}{\sqrt{2}} = \frac{\xi}{\sqrt{2}c}. \)
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