$e^{-} e^{-}$ Collisions Mediated by Composite Neutrinos

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Abstract

The lepton number violating process, $e^{-} e^{-} \rightarrow W^{-} W^{-}$ has been widely discussed in the Majorana neutrino exchange mechanism. Here, we discuss this process in a composite neutrino model where excited Majorana neutrinos are exchanged. We found several qualitatively different features from the neutrino exchange case: (1) The longitudinally polarized $W$’s are not produced, (2) the neutrinoless double beta decay does not constrain much and a much larger cross section is expected, and (3) CP violating phases may be explored because all excited neutrinos are heavy so that large mixings among them are expected.

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1 Introduction

Rizzo[1] was the first to analyze the $e^-e^- \rightarrow W^-W^-$ scattering mediated by Majorana neutrinos and a triplet Higgs boson in both an extended standard model and the left-right symmetric model. The interest lies in the fact that if this process is observed, one can say that at least the electron neutrino is a massive Majorana neutrino by using a similar argument to the one made for the neutrinoless double beta ($\beta\beta_{0\nu}$) decay[2]. Later, many authors[3],[4] have paid much attention to this process mediated by heavy neutrinos, and the feasibility of observing it by the future TeV linear colliders was discussed. However, Belanger et al.[4] have argued that it may not be possible to observe this scattering with the next linear collider (NLC) of $\sqrt{s} = 1$TeV and an energy of at least 4TeV is needed to observe this. This unpleasant result is due to the constraint from neutrinoless double beta decay.

In this paper, we analyze this process in a composite model. In this scenario, the scattering occurs by the exchange of excited neutrinos. We found qualitatively different features from the Majorana neutrino exchange case: (1) The longitudinally polarized $W$’s are not produced in this mechanism, while the production of longitudinally polarized $W$’s is the dominant mode in the neutrino exchange case. (2) A relatively large cross section is expected in comparison with the tiny one in the neutrino exchange case where the the neutrinoless double beta ($\beta\beta_{0\nu}$) decay gives a severe constraint[4]. The ($\beta\beta_{0\nu}$) decay is considered in a composite neutrino model by Takasugi[5], and Panella et al.[6], where the ($\beta\beta_{0\nu}$) decay takes place by the exchange of excited neutrinos. In this paper, we show that the constraint form the ($\beta\beta_{0\nu}$) decay does give only a mild constraint on the cross section. (3) Large mixings of electron to heavy excited neutrinos are expected because all excited neutrinos are heavy, whereas for the heavy neutrino mediated case, mixings of electron to heavy neutrinos are tiny. Thus, there could be a good chance to explore the CP violation phases in Majorana neutrino system.

In Sec. 2, we explain the ($\beta\beta_{0\nu}$) decay constraint and give the cross section formula. In Sec. 3, the numerical analysis is given. The CP violation in heavy neutrino system is
discussed in Sec.4. Summary is presented in Sec.5.

2 The cross section

A. Interaction

Excited neutrinos couple to the ground state leptons by the dimension five magnetic coupling[7]. This interaction is expressed as[8]

\[ L_{\text{int}} = g \frac{\lambda_W}{m_{\nu^*}} \bar{\nu}_e \sigma^{\mu\nu} (\eta_L^* R + \eta_R^* L) \nu_e^* \partial_\mu W^- + \text{h.c.}, \]  

(1)

where \( \nu^* \) is a heavy excited electron neutrino, \( L = (1 - \gamma_5)/2 \), \( R = (1 + \gamma_5)/2 \), and \( m_{\nu^*} \) is the mass dimension which is of order of the mass of \( \nu^* \), i.e., \( m_* \). This interaction may arise from an \( SU(2) \times U(1) \)-invariant higher-dimensional interaction[7],[8]. Normalization parameters \( \eta_L \) and \( \eta_R \) are given by \( (\eta_L, \eta_R) = (1, 0) \) or \( (0, 1) \) to respect the chirality conservation. We consider the mixing among excited neutrino. The excited electron neutrino \( \nu_e^* \) may be expressed by a superposition of mass eigenstate Majorana neutrinos \( N_j^* \) with the mass \( m_j \) as

\[ \nu_e^* = \sum_j U_{ej} N_j^*. \]  

(2)

Extensive search for \( \nu^* \) have been made by accelerator experiments[9] and it has been found that \( m_{\nu^*} > 91 \text{ GeV} \) by assuming that \( \lambda_Z > 1 \), which is the coupling for \( \nu^* \rightarrow \nu Z \) decay similarly defined to \( \lambda_W \). Hereafter, we assume that the excited neutrino mass is much larger than the W boson mass, i.e., \( m_j \gg m_W \).

Neutrinoless double beta decay mediated by excited neutrinos in composite models has been examined by Takasugi[5] and Panella et al.[6]. By comparing the theoretical calculation with the Heidelberg-Moscow data[10] for \(^{76}\text{Ge} \) decay, \( T_{1/2}^{0\nu} > 1.2 \cdot 10^{25} \text{yr}(90\% \text{C.L.}) \), the following constraint on the coupling parameter is given

\[ \left( \frac{\lambda_W}{\hat{m}_{\nu^*}} \right)^2 \sum_j U_{ej}^2 \left( \frac{\hat{m}_j + 2}{(\hat{m}_j + 1)^2} - \frac{0.129}{\hat{m}_j} \right) < 1.4 \times 10^{-2}, \]  

(3)

where the hatted quantities are dimensionless ones scaled by \( m_W \); \( \hat{m}_{\nu^*} \equiv m_{\nu^*}/m_W \) and \( \hat{m}_j \equiv m_j/m_W \). Fig.1 shows the upperlimit of \( \lambda_W/\hat{m}_{\nu^*} \) as a function of \( m_1 \) which is
denoted by \( m_* \), by assuming \( U_{e1} = \delta_{j1} \). This situation is reasonable because all excited neutrinos are heavy so that some mixings \( U_{ej} \) must be large. From this figure, we see that \( \lambda_W/\hat m_{\nu_*} < 0.33 \) for \( m_* > 0.5 \text{TeV} \) and the bound becomes less severe as the excited neutrino mass increases.

B. Cross section

From the interaction in Eq.(1), the invariant amplitude for the \( e^-e^- \rightarrow W^-W^- \) occurring via the t- and u-channel exchange of excited neutrinos is given by

\[
m_{fi} = \left( g \frac{\lambda_W}{m_{\nu_*}} \right)^2 \sum_j m_j U_{ej}^2 \left\{ \frac{k_{1\mu}k_{2\rho}(k_1)\epsilon_\sigma(k_2)}{(k_1 - p_1)^2 - m_j^*} + \frac{k_{1\mu}k_{2\rho}(k_1)\epsilon_\sigma(k_2)}{(k_1 - p_1)^2 - m_j^*} \right\}
\]

\[
\times \bar u(p_1)\sigma^{\mu\nu}\sigma^{\rho\sigma}(\eta^2_L + \eta^2_R)u(p_2)
\] (4)

where \( u^C \) is the charge conjugation of \( u \). From the structure of the above amplitude, one finds that the longitudinally polarized \( W \) \((\epsilon_\mu(k) \simeq k_\mu)\) can not be produced in the excited neutrino exchange mechanism. In other words, the \( \alpha \) part of

\[
\sum_\lambda \epsilon^{(\lambda)}_\mu(k_1)\epsilon^{(\lambda)*}_\mu(k_1) = -g_{\mu\nu'} + \alpha k_{1\mu}k_{1\nu'}
\] (5)

does not contribute to the spin sum of \( |m_{fi}|^2 \). In contrast, the longitudinally polarized \( W \) production is the dominant contribution in the heavy neutrino exchange mechanism for \( \sqrt{s} >> m_W \).

The cross section is calculated from Eq.(4) by neglecting the electron mass and the differential cross section for the unpolarized electron beam is given by using the invariant variables, \( s, t, \) and \( u \) as

\[
\frac{d\sigma}{d\cos\theta} = \frac{g^4}{64\pi m_W^2} \left( \frac{\lambda_W}{\hat m_{\nu_*}} \right)^4 \frac{1}{5} \sqrt{1 - \frac{4}{\hat s}} \sum_{i,j} \hat m_i\hat m_j U_{ei}^2 U_{ej}^2 f_{ij},
\] (6)

where

\[
f_{ij} = \frac{1}{(t - \hat m_i^2)(\hat t - \hat m_j^2)} \left[ (\hat s - 2)(\hat t - 1)^2 - (\hat t - 1)(\hat u - 1) + \frac{\hat s}{4} \right]
\]

\[
+ \frac{1}{(\hat u - \hat m_i^2)(\hat u - \hat m_j^2)} \left[ (\hat s - 2)(\hat u - 1)^2 - (\hat t - 1)(\hat u - 1) + \frac{\hat s}{4} \right]
\]

\[
+ \frac{1}{2} \left( \frac{1}{(t - \hat m_i^2)(\hat t - \hat m_j^2)} + \frac{1}{(\hat u - \hat m_i^2)(\hat t - \hat m_j^2)} \right)
\]
\[
\times \left( \hat{s}(\hat{s} - 2) - (\hat{s} - 2)[(\hat{t} - 1)^2 + (\hat{u} - 1)^2] + 2(\hat{t} - 1)(\hat{u} - 1) - \frac{3}{2} \hat{s} \right), \tag{7}
\]

and all hatted-variables are dimensionless ones scaled by W mass as \(\hat{m}_{\nu^*} = m_{\nu^*}/m_W, \hat{s} = s/m_W^2, \hat{m}_j = m_j/m_W\) and so on. The invariant variables satisfy the relation \(\hat{s} + \hat{t} + \hat{u} = 2\).

For \(\hat{s} \gg 1\) \((s \gg m^2_W)\), the cross section takes the simpler forms as

\[
\frac{d\sigma}{d\cos \theta} = \frac{g_4^4}{64\pi m^4_W} \left( \frac{\lambda_W}{\hat{m}_{\nu^*}} \right)^4 \left| \sum_j \hat{m}_j U^2_{e_j} \left( \frac{\hat{t}}{(\hat{t} - \hat{m}_j^2)} + \frac{\hat{u}}{(\hat{u} - \hat{m}_j^2)} \right) \right|^2
\]

\[
\rightarrow \frac{g_4^4}{16\pi m^2_W} \left( \frac{\lambda_W}{\hat{m}_{\nu^*}} \right)^4 \left| \sum_j \hat{m}_j U^2_{e_j} \right|^2 \quad \text{for } s \gg \hat{m}_j^2,
\]

\[
\rightarrow \frac{g_4^4}{64\pi m^2_W} \left( \frac{\lambda_W}{\hat{m}_{\nu^*}} \right)^4 \left| \sum_j \frac{U^2_{e_j}}{\hat{m}_j} \right|^2 \hat{s}^2 \quad \text{for } s \ll \hat{m}_j^2. \tag{8}
\]

The above cross section formula is similar to the one for the neutrino mediated case and violates the unitarity bound for \(s \to \infty\). This difficulty may be avoided by assuming that masses and mixing angles satisfy \(\sum m_j U^2_{e_j}/m_j = 0\), similar to the heavy neutrino exchange case[3],[4]. If polarized beams are used such as in \(e_L e_L\) scattering which occurs for \((\eta_L, \eta_R) = (1, 0)\) or \(e_R e_R\) for \((\eta_L, \eta_R) = (0, 1)\), the cross section should be multiplied by 4.

3 The expected cross section

Firstly, we examine the \((\beta\beta)_0\) decay constraint on the cross section for \(m_W << \sqrt{s} << m_j\). In this region of \(\sqrt{s}\), the \((\beta\beta)_0\) constraint in Eq.(3) takes a simpler form as \((\lambda_W/\hat{m}_{\nu^*})^2|\sum_j U^2_{e_j}/m_j| < 1.4 \times 10^{-2}\). Then, by combining this constraint and the formula in Eq.(8) for \(s << \hat{m}_j^2\), we find that the upper bound of the cross section for the unpolarized beam

\[
\frac{d\sigma}{d\cos \theta} < 11 \left( \frac{\sqrt{s}}{m_W} \right)^4 \text{ fb}. \tag{9}
\]

Suppose that the NLC integrated luminosity is about 80fb\(^{-1}\) at \(\sqrt{s} = 1\text{TeV}[11]\). In contrast, the upper limit of the differential cross section given in Eq.(9) is about \(3 \times 10^5\)fb so that we can say that the cross section is essentially not constrained by the neutrinoless
double beta decay in comparison with the expected luminosity. Therefore, we have to set a more realistic condition. A reasonable guess is that the relative coupling strength is of order one, i.e., $\lambda_W = 1$ and examine the dependence of the cross section on masses of excited neutrinos and the composite scale. In the reminder of this section, we assume for simplicity that one of the mixing is dominant, i.e., $U_{ej} = \delta_{j1}$. We denote the mass $m_1$ as $m_*$ and express the composite scale as $\Lambda_C \equiv m_*/\sqrt{2}$.

In Fig. 2, the center of mass energy dependence of the cross section for various excited neutrino mass $m_*$ with $\Lambda_C=1\text{TeV}$ and $\lambda_W = 1$. For energies less than $m_*$, the cross section is a decreasing function of $m_*$. If the energy exceeds $m_*$, the cross section becomes an increasing function of $m_*$. Therefore, the cross over between two curves for $m_* = 1$ and 2TeV around $\sqrt{s} = 1.9\text{TeV}$ appears. These behaviors can be seen analytically from the cross section formula in Eq.(8). It should be noted however that these behaviours are valid for energies which do not exceed the excited neutrino mass much.

Next we consider how far we can explore $\Lambda_C$ and $m_*$ by using NLC with the luminosity about $80\text{fb}^{-1}$. We assume that the minimum cross section needed to be detected is $\sigma=0.1\text{fb}$, i.e., eight events in a year. Then, we estimated the size of $\Lambda_C/\lambda_W$ and $m_*$ which can be explored. The result is shown in Fig.3 where the lower region from the curve is the region which corresponds to $\sigma >0.1\text{fb}$. Roughly speaking, we can explore about 20TeV scale of $m_*$ for $\Lambda_C$ of about a few TeV by NLC.

4 CP violation

Once the $e^- + e^- \rightarrow W^- + W^-$ scattering is observed and the cross section is much greater than $5 \times 10^{-3}\text{fb}$ at $\sqrt{s}=1\text{TeV}$ which is the upper bound for the heavy neutrino exchange case[4], the production mechanism would be the excited neutrino exchange in a composite neutrino scenario. More decisively, if the produced $W$'s turn out to have only the transverse polarization, the composite neutrino scenario is the only candidate at present. If the scattering is found, CP violation will become one of the urgent subjects to examine. In below, we shall discuss how CP violation phases get in the cross section.
formula. The following discussion is valid both for heavy neutrino scenario and also for composite neutrino scenario.

In general, as discussed by Bilenky, Hosek and Petcov\cite{12}, and by Doi, Kotani, Nishiura, Okuda and Takasugi\cite{13}, there are extra CP violating phases in Majorana neutrino system and they contribute to the lepton number violation process, in addition to the KM like CP violation phase which is intrinsic to Dirac system. In particular, we can parameterize mixing matrix elements for three generation as\cite{13}

\[ U_{e1} = c_1 e^{i\alpha}, \quad U_{e2} = -s_1 c_3 e^{i(\alpha + \beta)}, \quad U_{e3} = -s_1 s_3 e^{i(\alpha + \gamma)}, \tag{10} \]

where \( s_i = \sin \theta_i \) and \( c_i = \cos \theta_i \), \( \theta_i \) is the rotational angle around the \( i \)th axis of flavor space, and \( \beta \) and \( \gamma \) are CP violation phases. Then, the cross section behaves as

\[
\frac{d\sigma}{d\cos \theta} \propto m_1^2 c_1^4 f_{11} + m_2^2 s_1^4 c_3^2 f_{22} + m_3^2 s_1^4 s_3^2 f_{33} + 2m_1^2 m_2^2 s_1^2 c_1^2 c_3^2 \cos 2\beta f_{12} \\
+ 2m_1^2 m_3^2 s_1^2 c_3^2 \cos 2\gamma f_{13} + 2m_2^2 m_3^2 s_1^2 s_3^2 c_3^2 \cos 2(\beta - \gamma) f_{23}, \tag{11} \]

where \( f_{ij} \) is defined in Eq.\( (7) \). Thus, two CP violating phases \( \beta \) and \( \gamma \) appear. Among two phases, one is the phase intrinsic to a Dirac system and the other is the one intrinsic to a Majorana system. The above is a general form which corresponds to a general neutrino mass matrix.

Here, we face to a problem of a constant behavior of the cross section as \( \sqrt{s} \to \infty \), similar to neutrino exchange case. The constant behavior means the violation of the unitarity. There must be some mechanism to restore the unitarity. For composite model, it is out of our scope to introduce some new interaction to remedy this defect and thus we assume that the excited neutrino mass matrix is arranged such that

\[ m_{\nu e e} = (UDU^T)_{\nu e e} = \sum_j m_j U_{e j}^2 = 0, \tag{12} \]

where \( D \) is a diagonal mass matrix. This condition is similar to the heavy neutrino scenario. Because of this constraint, only one CP violation phase appears for three generation case and no CP violation phase appears for two generation case. This may be seen by rewriting the constraint as

\[ m_1 c_1^2 + m_2 s_1^2 c_3^2 e^{2i\beta} + m_3 s_1^2 s_3^2 e^{2i\gamma} = 0. \tag{13} \]
For three generation case, $\beta$ and $\gamma$ are not independent each other. Two generation mixing may be derived by setting $s_3 = 0$. Then, Eq.(14) forces $\beta = 0$ or $\pi$ so that there appears no CP violation. This can be seen explicitly by using the mass matrix. We consider a symmetric matrix which respect the condition (13). We find that all phases can be absorbed by the phase matrix $P$ as

\[
\begin{pmatrix}
0 & ae^{i\alpha} \\
 ae^{i\alpha} & be^{i(\alpha+\beta)}
\end{pmatrix} = P \begin{pmatrix}
0 & a \\
a & b
\end{pmatrix} P,
\]

where $P = \text{diag}(e^{i(\alpha-\beta)/2}, e^{i(\alpha+\beta)/2})$. Then, the mixing matrix $U$ can be expressed as $U = P^\dagger O$, where $O$ is a orthogonal matrix. Then, no CP violation phase appears in the $e^- + e^- \rightarrow W^- + W^-$ reaction.

5 Summary

We have discussed the $e^- + e^- \rightarrow W^- + W^-$ reaction in the composite neutrino scheme. In the composite model, the excited neutrinos couples to the ground state electron and $W$. By using the gauge invariant interaction of dimension five, we analyzed this process by exchanging the excited neutrino. We found various new features which are different from the neutrino exchange case as discussed in the introduction. Since the $(\beta\beta)_{0\nu}$ decay does not constrain the cross section, it is worthwhile and interesting to explore this process in NLC with $\sqrt{s} = 1\text{TeV}$ and the luminosity of $80\text{fb}^{-1}$. If this reaction is observed, it is likely that neutrinos are composite and that there exist excited states. The decisive confirmation can be made by observing the polarization of $W$. In the composite scenario, longitudinally polarized $W$’s are not produced.
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Figure Captions:

Fig.1: The upper bound of the coupling strength of excited neutrinos and the ground state electron and $W$ imposed by the neutrinoless double beta decay. The area below the curve is the allowed region.

Fig.2: The center of mass energy $\sqrt{s}$ dependence of the cross section for various values of the excited neutrino mass $m_*$ with the composite scale $\Lambda_C = 1\text{TeV}$ with $\lambda_W = 1$.

Fig.3: The region of $\Lambda_C/\lambda_W$ and $m_*$ for which one can explore, where the cross section is fixed to be $0.1\text{fb}$. The below the curve is the region of parameters where $\sigma > 0.1\text{fb}$.