Self-Dual Conformal Supergravity and the Hamiltonian Formulation

Guoying Chee  
China Center of Advanced Science and Technology (World Laboratory)  
P. O. Box 8730, Beijing 100080, China  
Department of Physics, Normal University of Liaoning, Dalian 116029, China  
Email address: ggyq@mail.dlptt.ln.cn

Yanhua Jia  
Department of Applied Mechanics, Beijing Institute of Technology, Beijing 100081, China

In terms of Dirac matrices the self-dual and anti-self-dual decomposition of a conformal supergravity is given and a self-dual conformal supergravity theory is developed as a connection dynamic theory in which the basic dynamic variables include the self-dual spin connection i.e. the Ashtekar connection rather than the triad. The Hamiltonian formulation and the constraints are obtained by using the Dirac-Bergmann algorithm.

PACS numbers: 04.20.Cv, 04.20.Fy, 04.65.+e

I. INTRODUCTION

Among the various approaches to the construction of a unified model for the fundamental interactions including gravity many attempts have been made to write down gravity as a Yang-mills type gauge theory where the basic dynamical object is a connection one-form associated with some group. In this approach the metric (the tetrad) and the Lorentz connection are identified as different components of a connection one-form. A famous example is the MacDowell-Mansouri gravitational formalism [1] which mimics, as much as possible, the Yang-Mills type gauge theory in four space-time dimensions and has been successfully applied to construct different supergravity theories [2].

In 1986, a somewhat different, but nonetheless related approach was initiated [3] with introducing the new variables in general relativity which can be thought of as a Yang-Mills connection one form on a spacelike hypersurface. much of the success associated with the new variables appears to be intimately related to their character as gauge fields. Not long after Ashtekar’s results, Jacobson was able to formulate supergravity in the new variables [4]. Further, Capovilla, Jacobson, and dell developed a pure-connection theory of gravity, i.e., a formulation of general relativity without metric [5]. On the other hand, as early as 1974, Chern and Simons constructed a pure-connection theory of gravity [6].

Recently, several authors [7-11] proposed a self-dual generalization of the MacDowell-Mansouri formalism which includes the Ashtekar-Jacobson theory as well as Yang-Mills theory starting from the (anti-) de Sitter group. Beside the de Sitter or Poincare supergravity there is another class of supergravity i.e., the conformal supergravity. And it is conformal supergravity that provides a true unification of gravity and gauge fields. By gauging SU(2,2|1) group and imposing some constraints on curvature a simple conformal supergravity has been developed by Nieuwenhuizen et al [2, 12-14]. However, in this theory tetrad rather than connection was taken to be a basic dynamical variable in the second-order formalism. Therefore, it is not a connection dynamical but a geometrodynamical theory in a sense. On the other hand, the Lagrangian in this theory is quadratic in the curvature and then is different from the Einstein-Hilbert Lagrangian. It is reasonable to expect that one of the basic dynamical variable be the connection instead of the tetrad. In this paper we show that this is the case. A self-dual conformal supergravity is developed and its Hamiltonian formulation is obtained. In Sec. 2 we
start by recalling the conformal superalgebra su(2,2|1) and then define the dual of a element of su(2,2|1), its self-dual and anti-self-dual part using the Dirac matrix \( \gamma_5 \). The Lagrangian of the conformal supergravity is constructed in Sec. 3 and then the decomposition into self-dual and anti-self-dual is given in Sec. 4, a self-dual conformal supergravity is obtained. In Sec. 5 its Hamiltonian formulation is investigated and the structure of the constraints is discussed. In the appendix we list the Poison brackets of the constraints. The complicated structure of these Poison brackets makes the classification of the constraints impossible. The Dirac brackets, however, permit us to get rid of the second class constraints. We obtain a constrained Hamiltonian system. The action is first order in the time derivatives and the Hamiltonian results to be a linear combination of the constraints.

II. THE CONFORMAL SUPeralgebra SU(2,2|1)

The conformal superalgebra su(2,2|1) is given by \[ \{ \gamma_I, \gamma_J \} = 2\eta_{IJ}, \{ \gamma_I, \gamma_5 \} = 0, \gamma_5^2 = -1, \] (1)

and

\[
\gamma^{IJ} = \frac{1}{2}(\gamma^{I} \gamma^{J} - \gamma^{J} \gamma^{I}).
\] (2)

To fulfill these relations we can choose the matrix representations of the Bose basis

\[
P_I = -\frac{1}{2}\gamma_I(1 + i\gamma_5), \quad K_I = \frac{1}{2}\gamma_I(1 - i\gamma_5),
\]

\[
M_{IJ} = \frac{1}{2}\gamma_{IJ}, \quad D = \frac{i}{2}\gamma_5, \quad A = -\frac{i}{4}I,
\]

and the Majorana spinor representations of the Fermi basis

\[ Q^\alpha = \left( \frac{Q^A}{Q_A^A} \right), \text{ and } S^\alpha = \left( \frac{S^A}{S_A^A} \right). \] (3)

In this paper we adopt the following index notation: \( I, J, K, L, \ldots \) are group indices; \( \alpha, \beta, \ldots \) are Majorana spinor indices; \( \mu, \nu, \rho, \ldots \) are spacetime indices ; \( i, j, k, \ldots \) are spatial indices, \( A, B, \ldots \) and \( A', B', \ldots \) are used to denote SL(2C) spinor indices.

Using the identities

\[ \epsilon^{IJKL}\gamma_{IJ} = 2\gamma_5\gamma^{KL}, \text{ and } \epsilon^{IJKL}\gamma_I\gamma_J\gamma_K = -6\gamma_5\gamma^L, \] (4)

2
for any element $O$ of $SU(2,2|1)$, we can define its dual by
\[ O^\ast = \gamma_5 O \] (7)
then the self-dual and the anti-self-dual parts of $O$ are given by, respectively,
\[ O^+ = \frac{1}{2} (O - iO^\ast) = \frac{1}{2} (1 - i\gamma_5)O, \]
\[ O^- = \frac{1}{2} (O + iO^\ast) = \frac{1}{2} (1 + i\gamma_5)O. \]

### III. CONFORMAL SUPERGRAVITY

Introducing the $su(2,2|1)$ algebra valued connection one-form
\[ \Gamma = \omega + e + f + b + A + \psi + \phi \]
\[ = \frac{1}{2} \omega^{IJ} \otimes M_{IJ} + e^I \otimes P_I + f^I \otimes K_I + b \otimes D + \]
\[ a \otimes A + \psi^\alpha \otimes Q_\alpha + \phi^\alpha \otimes S_\alpha, \] (8)
and its curvature
\[ \Omega = D\Gamma = d\Gamma + \frac{1}{2} [\Gamma, \Gamma], \] (9)
we can compute
\[ \Omega = \Omega(M) + \Omega(P) + \Omega(K) + \Omega(D) + \Omega(A) + \Omega(Q) + \Omega(S) \]
\[ = \frac{1}{2} \Omega^{IJ}(M) \otimes M_{IJ} + \Omega^I(P) \otimes P_I + \Omega^I(K) \otimes K_I + \Omega(D) \otimes D \]
\[ + \Omega(A) \otimes A + \Omega^\alpha(Q) \otimes Q_\alpha + \Omega^\alpha(S) \otimes S_\alpha, \] (10)
where
\[ \Omega^{IJ}(M) = d\omega^{IJ} + \omega^{IK} \wedge \omega_{KJ} - 4(e^I \wedge f^J - e^J \wedge f^I) \]
\[ + \frac{1}{4\sqrt{2}} \bar{\psi} \wedge \gamma^I \psi \]
\[ - 2e^I \wedge b, \]
\[ \Omega^I(P) = de^I + \omega^{IJ} \wedge e_J - \frac{1}{4\sqrt{2}} \bar{\psi} \wedge \gamma^I \psi \]
\[ - 2e^I \wedge b, \]
\[ \Omega^I(K) = df^I + \omega^{IJ} \wedge f_J + \frac{1}{4\sqrt{2}} \bar{\phi} \wedge \gamma^I \phi \]
\[ + 2f^I \wedge b, \]
\[ \Omega(D) = db - 2e^I \wedge f_I + \frac{1}{4\sqrt{2}} \bar{\phi} \wedge \phi, \]
\[ \Omega(A) = da + \frac{1}{4\sqrt{2}} \bar{\psi} \wedge \gamma_5 \phi, \]
\[ \Omega(Q) = d\psi + \frac{1}{4} \omega^{IJ} \wedge \gamma_{IJ} \psi + b \wedge \psi + 3a \wedge \gamma_5 \psi + \sqrt{2} \gamma_I e^I \wedge \phi, \]
\[ \Omega(S) = d\phi + \frac{1}{4} \omega^{IJ} \wedge \gamma_{IJ} \phi - b \wedge \phi - 3a \wedge \gamma_5 \phi - \sqrt{2} \gamma_I f^I \wedge \psi. \] (11)

For a gauge theory its Lagrangian can be chosen among the four types
\[ \langle \Omega \wedge \Omega^\ast, \rangle, \langle \Omega \wedge \Omega, \rangle, \langle \Omega^\ast \wedge \Omega, \rangle, \text{and} \langle *\Omega \wedge \Omega^\ast, \rangle \]
where $*\Omega$ denotes the usual Hodge dual of $\Omega$ with respect to the spacetime metric and $\langle, \rangle$ is the Killing inner product defined in the superalgebra $su(2,2|1)$. In the bosonic sector
\[ \langle O, O' \rangle = \text{Tr}(OO'), \]

and in the fermionic sector

\[ \langle O, O' \rangle = \overline{O}O', \]

where \( \overline{O} \) is the Dirac conjugation of \( O \). Using (11) and (12) we can compute, for example,

\[ \langle \Omega \wedge \Omega^* \rangle = \langle \Omega(M) \wedge \Omega(M)^* \rangle + \langle \Omega(D) \wedge \Omega(A)^* \rangle + \langle \Omega(A) \wedge \Omega(D)^* \rangle, \tag{12} \]

where

\begin{align*}
\langle \Omega(M) \wedge \Omega(M)^* \rangle &= -\frac{1}{4} \varepsilon^{IJKL} \langle R(\omega)_{IJ} \wedge R(\omega)_{KL} \rangle - 8 \varepsilon^{IJKL} e_I \wedge e_K \wedge f_L - \frac{1}{16} \varepsilon^{IJKL} \overline{\psi} \gamma_{IJ} \phi \wedge \overline{\psi} \gamma_{KL} \phi, \tag{13} \\
\langle \Omega(D) \wedge \Omega(A)^* \rangle + \langle \Omega(A) \wedge \Omega(D)^* \rangle &= 2 da \wedge db + \frac{i}{2} (\psi_A \wedge \phi^A + \overline{\psi}_A' \wedge \overline{\phi}'_A) \wedge db \\
&- da \wedge (4e^I \wedge f_I - \frac{1}{2} \psi_A \wedge \phi^A + \frac{1}{2} \overline{\psi}'_A \wedge \overline{\phi}'_A) \\
&- ie^I \wedge f_I \wedge (\psi_A \wedge \phi^A + \overline{\psi}'_A \wedge \overline{\phi}'_A) \\
&+ \frac{i}{8} (\psi_A \wedge \phi^A \wedge \psi_B \wedge \phi^B - \overline{\psi}_A' \wedge \overline{\phi}'_A \wedge \overline{\psi}_B' \wedge \overline{\phi}'_B). \tag{14} 
\end{align*}

It is notable that the property

\[ \overline{\psi} \phi = -\overline{\phi} \psi \tag{15} \]

leads to

\[ \langle \Omega(Q) \wedge \Omega(S)^* \rangle = - \langle \Omega(S) \wedge \Omega(Q)^* \rangle \tag{16} \]

and then there are no dynamical terms of the Fermi fields \( \psi \) and \( \phi \) in the Lagrangian, which is different from the Lagrangian given by Nieuwenhuizen [6–8]:

\[ \mathcal{L} = 4 \langle \Omega(M) \wedge \Omega(M)^* \rangle - 32 \langle \Omega(D) \wedge \Omega(A)^* \rangle + \langle \Omega(Q) \wedge \Omega(S)^* \rangle \tag{17} \]

It is notable that the Lagrangian (13) is obtained without using the constraints on curvature which is indispensable for the Nieuwenhuizen approach.

**IV. SELF-DUAL CONFORMAL SUPERGRAVITY**

Using the definition of the self-dual and the anti-self-dual introduced in Sec. 2, the connection \( \Gamma \) can be decomposed into two parts:

\[ \Gamma = \Gamma^+ + \Gamma^-, \]

4
where
\[ \Gamma^\pm = \omega^\pm + \mathbf{e}^\pm + \mathbf{f}^\pm + \mathbf{b}^\pm + A^\pm + \psi^\pm + \phi^\pm, \tag{18} \]
and so can the curvature
\[ \Omega = \Omega^+ + \Omega^-, \tag{19} \]
where
\[ \Omega^\pm = \Omega^\pm(M) + \Omega^\pm(P) + \Omega^\pm(K) + \Omega^\pm(D) + \Omega^\pm(A) + \Omega^\pm(Q) + \Omega^\pm(S). \tag{20} \]

Since
\[ \langle \Omega \wedge \Omega^* \rangle = \langle \Omega \wedge \Omega^* \rangle^+ + \langle \Omega \wedge \Omega^* \rangle^-, \tag{21} \]
where
\[ \langle \Omega \wedge \Omega^* \rangle^+ = \langle \Omega^+ \wedge \Omega^+ \rangle = i \langle \Omega^+ \wedge \Omega^+ \rangle, \]
\[ \langle \Omega \wedge \Omega^* \rangle^- = \langle \Omega^- \wedge \Omega^- \rangle = -i \langle \Omega^- \wedge \Omega^- \rangle, \tag{22} \]
and \( \langle \Omega \wedge \Omega^* \rangle \) does not include dynamical terms of the fields \( \psi \) and \( \phi \), we choose the self-dual part of the Nieuwenhuizen Lagrangian
\[ \mathcal{L} = 4 \langle \Omega(M) \wedge \Omega(M)^* \rangle^+ - 32 \langle \Omega(D) \wedge \Omega(A)^* \rangle^+ + 8 \langle \Omega(Q) \wedge \Omega(S)^* \rangle \tag{23} \]
to be the Lagrangian of the self-dual conformal supergravity theory instead of \( \langle \Omega \wedge \Omega^* \rangle \).

In order to obtain the explicit expression of \( \mathcal{L} \) we use the matrix representation of the superalgebra \( \text{su}(2,2|1) \). In the chiral representation of the Dirac matrices we have
\[ \gamma^I = \sqrt{2} \begin{bmatrix} 0 & \sigma^{IAA'} \\ (\sigma^A_{AA'})^I & 0 \end{bmatrix}, \]
\[ \gamma^{IJ} = \begin{bmatrix} \sigma^{IAA'} & \sigma^{J BA'} - \sigma^{J AA'} \sigma^I_{BA'} \\ \sigma^I_{AA'} & -\sigma^I_{AA'} \sigma^J_{AB'} & \sigma^I_{AA'} \sigma^J_{AB'} \end{bmatrix}, \]
\[ = \begin{bmatrix} \gamma^{+IJ}_A & 0 \\ 0 & \gamma^{-IJ}_{A'B'} \end{bmatrix}, \]
\[ A = -\frac{i}{4} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, \text{ and } D = -\frac{1}{2} \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}. \tag{24} \]

In this representation the spin connection \( \omega \) and its curvature \( R(\omega) = d\omega + \frac{1}{2} [\omega, \omega] \) have the two component spinor forms
\[ \omega = \begin{bmatrix} \omega^+_{AB} & 0 \\ 0 & \omega^-_{A'B'} \end{bmatrix}, \]
and
\[ R(\omega) = \begin{bmatrix} R^+_{AB} & 0 \\ 0 & R^-_{A'B'} \end{bmatrix} \tag{25} \]
where $$\omega^+ A_B = \frac{1}{2} \omega^{IJ} \gamma_{IJ} A_B$$ and $$R^+ A_B(\omega) = \frac{1}{2} R(\omega)^{IJ} \gamma_{IJ} A_B$$ are the self-dual parts of $$\omega$$ and $$R(\omega)$$ respectively. From (25) we get $$\gamma^{IJC} \gamma_{IJ} A_B = 4 e^{CA} \epsilon_{DB} - 4 \delta^C_B \delta_A^D$$, and $$\gamma^{IJC} \gamma_{IJ} A_B = 0$$. Then we can obtain

$$4 \langle \Omega(M) \wedge (\Omega(M))^+ \rangle^+ = i \{ 4 B \cdot B - 4 \overrightarrow{C} \cdot \overrightarrow{C} + 32 (e^A e^{BB'} f_{AB}' - e^{AA'} e^{BB'} f_{AB}') + 2 \psi^A \phi_B \psi_B \phi_A \} \sigma A x,$$

(26)

where $$\sigma = \text{det}(\sigma_{AB})$$, $$f^{AA'} = f^{I} \sigma_{AB}$$.

$$B \cdot B = \frac{1}{16} R_{AB'CD'} E^F_{CF} D^F_{CH} D^H_{DH'},$$

$$\overrightarrow{C} \cdot \overrightarrow{C} = \frac{1}{16} R_{EAB'CD'} E^F_{CF} D^F_{CH} D^H_{DH'},$$

(27)

and the spacetime indices $$\mu, \nu, \ldots$$ have been transformed to spinor indices $$AB', CD', \ldots$$ using the formula, for example

$$V_{AB'} = V^{\mu} e_{I} \sigma_{AB}.$$

(28)

From the matrix expression of $$D$$ and $$A$$ we see that

$$\Omega^+(A) = \Omega^{-}(A), \quad \Omega^+(D) = -\Omega^{-}(D),$$

and then

$$\langle \Omega(D) \wedge (\Omega(A))^+ \rangle^+ = \langle \Omega(D) \wedge (\Omega(A))^+ \rangle^{-} = \frac{1}{2} \langle \Omega(D) \wedge (\Omega(A))^+ \rangle^-. $$

(29)

Using (12) and (25) and (31) we have

$$-32 \langle \Omega(D) \wedge (\Omega(A))^+ \rangle^+ = -16 da \wedge db - 8i \psi_A \wedge \phi^A \wedge db + da \wedge (32 e^I \wedge f_{I} - 8 \psi_A \wedge \phi^A) + 16i \psi_A \wedge \phi^A \wedge e^I \wedge f_{I} - 2i \psi_A \wedge \phi^A \wedge \psi_B \wedge \phi^B.$$

(30)

From (12), (11), (5), (20) and (8) we obtain

$$8i \Omega(Q) \wedge (\Omega(S))^+ = 8i D \psi_A \wedge D \phi^A - 8(3a - ib) \wedge (D \psi_A \wedge \phi_A - D \phi^A \wedge \psi_A) + 16(3a - ib) \wedge (\psi^A \wedge f_{AA'} \wedge \overrightarrow{\psi'} - \phi^A \wedge \sigma_{AA'} \wedge \overrightarrow{\phi'}) - 16i (D \psi^A \wedge f_{AA'} \wedge \overrightarrow{\psi'} + D \phi^A \wedge \sigma_{AA'} \wedge \overrightarrow{\phi'}) - 32i \overrightarrow{\phi'} \wedge \sigma_{AA'} e^A_B \psi_B \psi_{B'}.$$

(31)

Equation (24) with (27), (32) and (33) gives the Lagrangian for a self-dual conformal supergravity.
V. HAMILTONIAN FORMULATION

Following standard methods [15,16] a 3+1 decomposition of the Lagrangian can be carried out to pass on to the Hamiltonian framework. In this decomposition the tetrad variables $\sigma_{i}^{AA'}$ are split into $\sigma_{0}^{AA'}$ and $\sigma_{i}^{AA'}$ ( $i$, $j$, $\ldots = 1$, 2, 3). The spatial spinor-valued forms $\sigma_{i}^{AA'}$ determine the spatial metric $q_{ij} = -tr\sigma_{i}\sigma_{j}$ on a surface $\Sigma_{t}$ with $t = \text{const}$. The spinor version $n^{AA'}$ of the unit timelike future directed normal $n^{\mu}$ to $\Sigma_{t}$ can be used together with the $\sigma_{i}^{AA'}$ to make a basis for the space of spinors with one unprimed and one primed index. It is determined by the $\sigma_{i}^{AA'}$ through the conditions $n_{AA'}\sigma_{i}^{AA'} = 0$, $n_{AA'}n^{AA'} = -1$. The remaining variables $\sigma_{0}^{AA'}$ can be expanded out as

$$\sigma_{0}^{AA'} = Nn^{AA'} + N^{i}\sigma_{i}^{AA'},$$

(32)

where $N$ and $N^{i}$ are the lapse and shift, respectively. Similarly the other forms, e. g. the $\psi_{\mu}^{A}$ are split in to $\psi_{A}$, and $\psi_{A}$ and their conjugates. Then a 3+1 decomposition of the Lagrangian can be computed:

$$\mathcal{L} = \tilde{p}_{A}^{B}(\omega)\tilde{A}_{B}^{A} + \tilde{p}(a)\tilde{A}_{B}^{A} + \tilde{p}(b)\tilde{A}_{B}^{A} + \tilde{\psi}^{A}\tilde{\pi}_{A}(q) + \tilde{\psi}_{A}\tilde{\pi}_{A}(s) - \sigma_{0}^{AA'}\tilde{H}_{AA'}(c) - f_{0}^{AA'}\tilde{H}_{AA'}(f) - \omega_{0}^{A}\tilde{J}_{A} - a_{0}\tilde{H}(a) - b_{0}\tilde{H}(b) - \psi_{0}\tilde{A}_{A}(q) - \varphi_{0}^{A}\tilde{S}_{A}(s) - \tilde{\psi}_{A}\tilde{S}_{A}(q) - \tilde{\varphi}_{A}\tilde{S}_{A}(q),$$

(33)

where

$$\tilde{p}_{A}^{B}(\omega) = 4i\tilde{\eta}^{jk}D_{j}\omega_{k}^{B},$$

$$\tilde{p}(a) = -8\tilde{\eta}^{jk}(2\partial_{j}b_{k} + 4f_{jk} + \psi_{kA}\varphi_{A}),$$

$$\tilde{p}(b) = -8\tilde{\eta}^{jk}(2\partial_{j}a_{k} + i\psi_{kA}\varphi_{A}),$$

$$\tilde{\pi}_{A}(q) = -8\tilde{\eta}^{jk}[iD_{j}\varphi_{A} + (3a_{j} - ib_{j})\varphi_{kA} + 2if_{jAA}\varphi_{kA}'],$$

$$\tilde{\pi}_{A}(s) = -8\tilde{\eta}^{jk}[iD_{j}\psi_{kA} + (3a_{j} - ib_{j})\psi_{kA} + 2i\sigma_{jAA}\varphi_{A}],$$

(34)

and

$$\tilde{H}_{AA'}(c) = 64i\tilde{\eta}^{jk}f_{BA'}(f_{j}^{BB'}\sigma_{kA'B'} - \sigma_{j}^{BB'}f_{kAB'}) + 2f_{iAA'}\tilde{p}(b) + 2\tilde{\pi}_{A}(q)\tilde{\varphi}_{A},$$

$$\tilde{H}_{AA'}(f) = 64i\tilde{\eta}^{jk}\sigma_{BA'}(\sigma_{j}^{BB'}f_{kAB'} - f_{j}^{BB'}\sigma_{kAB'}) - 2\sigma_{AA'}\tilde{p}(b) + 2\tilde{\pi}_{A}(s)\tilde{\varphi}_{A'},$$

$$\tilde{J}_{A} = D_{i}\tilde{p}_{A}^{i}(\omega),$$

$$\tilde{H}(b) = 2[\tilde{\pi}_{A}^{A}(q)\tilde{\psi}_{A}^{A} - \tilde{\pi}_{A}(s)\tilde{\varphi}_{A}'] + 16\tilde{\eta}^{jk}(2\partial_{j}f_{ik} + \psi_{A}\psi_{kA'\tilde{\varphi}_{A}' - \varphi_{A}\sigma_{jAA'}\tilde{\varphi}_{kA'}},$$

$$\tilde{S}_{A}^{A}(q) = -D_{i}\tilde{\pi}_{A}(q) - i(3a_{i} - ib_{i})\tilde{\pi}_{A}(q) + \frac{1}{2}\tilde{\pi}_{A}(q)\tilde{p}(b) + i\tilde{p}(a)]$$

$$-16\tilde{\eta}^{ijk}\tilde{\varphi}_{A}\psi_{jA'}^{B}\varphi_{kB},$$

$$\tilde{S}_{A}^{A}(s) = -D_{i}\tilde{\pi}_{A}(s) - i(3a_{i} - ib_{i})\tilde{\pi}_{A}(s) - \frac{1}{2}\tilde{\pi}_{A}(q)\tilde{p}(b) + i\tilde{p}(a)]$$

$$-16\tilde{\eta}^{ijk}\tilde{\psi}_{A}\psi_{jA'}^{B}\varphi_{kB},$$

$$\tilde{S}_{A}^{A}(q) = 2f_{iA'}\tilde{\pi}_{A}^{A}(q),$$

$$\tilde{S}_{A}^{A}(s) = 2\sigma_{iA'\tilde{\pi}_{A}^{A}(q)}.$$

(35)

Here we use $\eta^{ij}$ to denote the Levi-Civita tensor density on $\Sigma_{t}$ and the tilde "over a tensor density to indicate its weight +1. The meaning of all terms in (35) will be clear in the following.
To pass on to the Hamiltonian formulation we have to use the Legendre transformation and the Dirac-Bergmann algorithm [17,18]. Calculating the canonical momenta conjugate to all the field variables gives primary constraints.

\[
\begin{align*}
\Phi^{0}_{AA'}(e) &= \bar{p}^{0}_{AA'}(e) = 0, \\
\Phi^{i}_{AA'}(e) &= \bar{p}^{i}_{AA'}(e) = 0, \\
\Phi^{0}_{AA'}(f) &= \bar{p}^{0}_{AA'}(f) = 0, \\
\Phi^{i}_{AA'}(f) &= \bar{p}^{i}_{AA'}(f) = 0, \\
\Phi^{0}_{A}(\omega) &= \bar{p}^{0}_{A}(\omega) = 0, \\
\Phi^{i}_{A}(\omega) &= \bar{p}^{i}_{A}(\omega) = \bar{p}^{j}_{A}(\omega) - 4i \eta_{ijk} R_{ikA}^{j}(\omega) \approx 0, \\
\Phi^{0}_{A}(a) &= \bar{p}^{0}_{A}(a) = 0, \\
\Phi^{i}_{A}(a) &= \bar{p}^{i}_{A}(a) + \eta_{ijk} (16 \partial_{j}b_{k} + 32 f_{jk} + 8 \psi_{jA}\varphi^{A}k) \approx 0, \\
\Phi^{0}_{A}(q) &= \bar{p}^{0}_{A}(q) = 0, \\
\Phi^{i}_{A}(q) &= \bar{p}^{i}_{A}(q) + \eta_{ijk} [8i D_{j}\varphi_{k}A - 8\varphi_{jA}(3a_{k} - ib_{k}) + 16f_{jA}\varphi_{k}^{A}] \approx 0, \\
\Phi^{0}_{A}(s) &= \bar{p}^{0}_{A}(s) = 0, \\
\Phi^{i}_{A}(s) &= \bar{p}^{i}_{A}(s) + \eta_{ijk} [8i D_{j}\psi_{k}A - 8\psi_{jA}(3a_{k} - ib_{k}) + 16\sigma_{jA}\varphi_{k}^{A}] \approx 0, \\
\Phi^{0}_{A}(\zeta) &= \bar{p}^{0}_{A}(\zeta) = 0, \\
\Phi^{i}_{A}(\zeta) &= \bar{p}^{i}_{A}(\zeta) = \bar{p}^{i}_{A}(\zeta) = 0, \\
\Phi^{0}_{A}(\bar{\zeta}) &= \bar{p}^{0}_{A}(\bar{\zeta}) = 0, \\
\Phi^{i}_{A}(\bar{\zeta}) &= \bar{p}^{i}_{A}(\bar{\zeta}) = \bar{p}^{i}_{A}(\bar{\zeta}) = 0.
\end{align*}
\] (36)

The basic canonical variables in the theory can then be reduced to \(\omega^{A}_{B}, a_{i}, b_{j}, \psi^{A}, \varphi_{A}^{i}\) and their conjugate momenta \(\bar{p}^{A}_{B}(\omega), \bar{p}^{i}_{A}(a), \bar{p}^{i}_{A}(q), \) and \(\bar{p}^{i}_{A}(s)\). The \(\omega^{A}_{B}\) is just the Ashtekar connection. The canonical momentum conjugate to \(\omega^{A}_{B}\), however, is not the \(\bar{p}^{i}_{A}\) but the \(\bar{p}^{i}_{A}(\omega) = 4i \eta^{ijk} D_{j}\omega_{k}^{B}A\) being different from the Ashtekar theory. The remaining variables \(\sigma^{AA'}, f^{AA'}, \varphi^{AA'}, \psi^{AA'}, \bar{\psi}^{AA'}, \bar{\varphi}^{AA'}, \bar{\bar{\varphi}}^{AA'}\) play the role of Lagrange multipliers. The \(\sigma^{AA'}, f^{AA'}\) are neither dynamical variables nor Lagrange multipliers. The canonical Hamiltonian is

\[
H_{c} = \int_{\Sigma} \sigma^{0}_{AA'} \bar{H}_{AA'}(e) + f^{0}_{AA'} \bar{H}_{AA'}(f) + \omega^{A}_{B} \bar{\omega}_{A}^{B} + a_{0} \bar{H}(a) + b_{0} \bar{H}(b) + \psi^{A}_{0} \bar{S}_{A}(q) + \varphi^{A}_{0} \bar{S}_{A}(s) + \bar{\psi}^{A}_{0} \bar{S}_{A}(\zeta) + \bar{\varphi}^{A}_{0} \bar{S}_{A}(\bar{\zeta}).
\] (37)

Using \(H_{c}\) and the linear combination of the primary constraints with arbitrary function coefficients we can construct the primary (or total) Hamiltonian. Then the consistency conditions i.e. the requirements of preserving constraints under time evolution lead to secondary constraints

\[
\begin{align*}
\bar{H}_{AA'}(e) &= 0, \\
\bar{H}_{AA'}(f) &= 0, \\
\bar{J}_{A}^{B} &= 0, \\
\bar{H}(a) &= 0, \\
\bar{H}(b) &= 0, \\
\bar{S}_{A}(q) &= 0, \\
\bar{S}_{A}(s) &= 0, \\
\bar{S}_{A}(\zeta) &= 0, \\
\bar{S}_{A}(\bar{\zeta}) &= 0.
\end{align*}
\] (38)

which are the generators of the superconformal group SU(2,2|1). In order to classify the constraints (36) and (38) we have to compute Poisson brackets between each pair of them. The complicated results which are given in the appendix make this classification very difficult. However using Dirac brackets instead of Poisson brackets one finds that all the constraints are first class and the constraints (38) are the generators of the superconformal group SU(2,2|1).

In summary, we have given a Hamiltonian formulation of the self-dual conformal supergravity which is a constrained Hamiltonian system. The Lagrangian (33) is first order in the time derivatives and the Hamiltonian (37) results to be a linear combination of the constraints. This is a theory of connection dynamics in which one of the basic dynamical variables is the self-dual spin connection (i.e. the Ashtekar connection) \(\omega^{A}_{B}\) rather than the triad \(\sigma^{AB}\). Unfortunately, the Dirac bracket structure is very involved in our case, and we were not able to compute it explicitly.
VI. APPENDIX

In order to classify the constraints we compute the Poisson brackets between them according to the method given by Casalbuoni [19] the nonvanishing Poisson brackets are listed here.

The nonvanishing Poisson brackets between the primary constraints are

\begin{align*}
\{\tilde{\Phi}_{iAA}(e), \tilde{\Phi}^j_A(a)\} &= 32 \int_{\Sigma_t} \tilde{\eta}^{jk} f_{kAA'}, \\
\{\tilde{\Phi}^j_{iAA}(e), \tilde{\Phi}^j_B(s)\} &= 16i \int_{\Sigma_t} \tilde{\eta}^{jk} \epsilon_{AB} \tilde{\Sigma}_{kA'}, \\
\{\tilde{\Phi}^i_{iAA}(f), \tilde{\Phi}^j_A(a)\} &= 32 \int_{\Sigma_t} \tilde{\eta}^{jk} \sigma_{kAA'}, \\
\{\tilde{\Phi}^i_{iAA}(f), \tilde{\Phi}^j_B(q)\} &= 16i \int_{\Sigma_t} \tilde{\eta}^{jk} \epsilon_{AB} \psi_{kA'}, \\
\{\tilde{\Phi}^i_{iA}(\omega), \Phi_A^j(q)\} &= 8i \int_{\Sigma_t} \tilde{\eta}^{jk} \delta_C^B \varphi_A, \\
\{\tilde{\Phi}^i_A(\omega), \Phi^j_A(s)\} &= 8i \int_{\Sigma_t} \tilde{\eta}^{jk} \delta_C^B \psi_A, \\
\{\tilde{\Phi}^i_A(q), \tilde{\Phi}^j_A(7)\} &= 16i \int_{\Sigma_t} \tilde{\eta}^{jk} f_{kAA'}, \\
\{\tilde{\Phi}^i_A(s), \tilde{\Phi}^j_A(\Sigma)\} &= 16i \int_{\Sigma_t} \tilde{\eta}^{jk} \sigma_{kAA'}, \\
\{\tilde{\Phi}^0_{iAA}(e), \tilde{\Phi}^0_{iA}(f)\} &= 0, \\
\{\tilde{\Phi}^0_{iAA}(e), \tilde{\Phi}^0_B(\omega)\} &= 0, \\
\{\tilde{\Phi}^0_{iA}(a), \tilde{\Phi}^0_C(b)\} &= 0, \\
\{\tilde{\Phi}^0_{iA}(q), \tilde{\Phi}^0_A(\mathcal{F})\} &= 0, \\
\{\tilde{\Phi}^0_A(s), \tilde{\Phi}^0_A(s')\} &= 0, \\
\{\tilde{\Phi}^0_A(\Sigma), \tilde{\Phi}^0_A(\Sigma')\} &= 0.
\end{align*}

The remaining Poisson brackets between the primary constraints vanish. One can find that the constraints \(\tilde{\Phi}^0_{iAA}(e), \tilde{\Phi}^0_{iA}(f)\), \(\tilde{\Phi}^0_B(\omega)\), \(\tilde{\Phi}^0_C(b)\), \(\tilde{\Phi}^0_A(q)\), \(\tilde{\Phi}^0_A(s)\), \(\tilde{\Phi}^0_A(s')\) are first class. In addition there are vanishing Poisson brackets

\begin{align*}
\{\sigma^j_A A^i \tilde{\Phi}^j_A(\omega), \tilde{\Phi}^j_A(\omega)\} &= 0, \\
\{\tilde{\Phi}^j_A(\omega), \tilde{\Phi}^j_A(\omega)\} &= 0, \\
\{\tilde{\Phi}^i_A(\omega), \tilde{\Phi}^j_A(\omega)\} &= 0, \\
\{\tilde{\Phi}^i_A(\omega), \tilde{\Phi}^j_A(\omega)\} &= 0.
\end{align*}

This means that there are more primary constraints which are first class.

The nonvanishing Poisson brackets between the primary constraints and the secondary constraints are

\begin{align*}
\{\tilde{\Phi}^j_{iAA}(e), \tilde{H}_{BB'}(f)\} &= -64i \int_{\Sigma_t} \tilde{\eta}^{jk}(f_{jAB'} f_{kBA'} + f_{j} C^A f_{kCB'} \epsilon_{AB}), \\
\{\tilde{\Phi}^j_{iAA}(e), \tilde{H}_{BB'}(f)\} &= \int_{\Sigma_t} 2\epsilon_{AB} \epsilon_{A'B'} \tilde{p}(b) + 64i \tilde{\eta}^{jk} \sigma_{jC'} f_{kBC'} - f_{j} C^A \sigma_{kBC'} + 64i \tilde{\eta}^{jk} \sigma_{jAB} f_{kBA'} + f_{j} C^A \sigma_{kCB'} \epsilon_{AB}, \\
\{\tilde{\Phi}^i_{iAA}(f), \tilde{H}(a)\} &= \int_{\Sigma_t} \tilde{\eta}^{jk} f_{kAA'} - 16 \varphi_A \tilde{\psi}_{kA'}, \\
\{\tilde{\Phi}^i_{iAA}(f), \tilde{H}(b)\} &= 16 \int_{\Sigma_t} \tilde{\eta}^{jk} \varphi_A \tilde{\psi}_{kA'}, \\
\{\tilde{\Phi}^i_{iAA}(f), \tilde{H}_{BB'}(f)\} &= \int_{\Sigma_t} 2\epsilon_{AB} \epsilon_{A'B'} \tilde{p}(b) + 64i \tilde{\eta}^{jk} \sigma_{jC'} f_{kBC'} - f_{j} C^A \sigma_{kBC'} + 64i \tilde{\eta}^{jk} \sigma_{jAB} f_{kBA'} + f_{j} C^A \sigma_{kCB'} \epsilon_{AB}, \\
\{\tilde{\Phi}^{i}_{iAA}(f), \tilde{H}_{BB'}(f)\} &= -64i \int_{\Sigma_t} \tilde{\eta}^{jk} (\sigma_{jAB'} \tilde{\sigma}_{kBA'} - \sigma_{jC'} \tilde{\sigma}_{kCB'} \epsilon_{AB}), \\
\{\tilde{\Phi}^{i}_{iAA}(f), \tilde{H}(a)\} &= \int_{\Sigma_t} \tilde{\eta}^{jk} f_{kAA'} + 16 \Psi_A \tilde{\psi}_{kA'}, \\
\{\tilde{\Phi}^{i}_{iAA}(f), \tilde{H}(b)\} &= -16i \int_{\Sigma_t} \tilde{\eta}^{jk} \varphi_A \tilde{\psi}_{kA'}, \\
\{\tilde{\Phi}^{i}_{iA}(\omega), J_{CD}^A\} &= 8i \int_{\Sigma_t} \tilde{\eta}^{jk} \delta_{C}^B \omega_{jE} \tilde{\omega}_{kCE} + \delta_{C}^{BD} \omega_{jA} \tilde{\omega}_{kE} + \int_{\Sigma_t} \delta_{C}^B p^E A^i (\omega), \\
\{\tilde{\Phi}^{i}_{iA}(\omega), S_{CD}(q)\} &= \int_{\Sigma_t} \delta_{C}^B \tilde{\omega}_{jA} (\omega),
\end{align*}
\( \{ \Phi^i_A B(\omega), \bar{S}_C(s) \} = \int_{\Sigma} \delta^B_i C \bar{\pi}^i_A(s), \)
\( \{ \Phi^i_A(a), \bar{S}_A(q) \} = \int_{\Sigma} 8i\eta^{jk} [\omega_{jAC} \omega_{kB} + (3ia_j + b_j) \varphi_{kA} + 3i\bar{\pi}^i_A(q), \)
\( \{ \Phi^i_A(a), \bar{S}_A(s) \} = \int_{\Sigma} -8i\eta^{jk} [\omega_{jA} \psi_{kB} + (3ia_j + b_j) \psi_{kA}] + 3i\bar{\pi}^i_A(s), \)
\( \{ \Phi^i_A(b), \bar{S}_A(q) \} = \int_{\Sigma} 8i\eta^{jk} [\omega_{jAB} \varphi_{kB} - (3ia_j + b_j) \varphi_{kA}] + \bar{\pi}^i_A(q), \)
\( \{ \Phi^i_A(b), \bar{S}_A(s) \} = \int_{\Sigma} -8i\eta^{jk} [\omega_{jA} \psi_{kB} - (3ia_j + b_j) \psi_{kA}] + \bar{\pi}^i_A(s), \)
\( \{ \Phi^i_A(q), \bar{H}_{BB}^A(e) \} = 16i \int_{\Sigma} \eta^{jk} \varphi_{jA} f_{kB} B, \)
\( \{ \Phi^i_A(q), \bar{H}_{BB}^B(f) \} = -16i \int_{\Sigma} \eta^{jk} \varphi_{jA} \sigma_{kB} B, \)
\( \{ \Phi^i_A(q), \bar{J}_A^B \} = 8i \int_{\Sigma} \eta^{jk} (\delta^B_i \omega_{jA} \varphi_{kB} - \omega_{jC} \varphi_{kA}), \)
\( \{ \Phi^i_A(q), \bar{H}(a) \} = \int_{\Sigma} 16i \eta^{jk} [\omega_{jAB} \varphi_{kB} + (3ia_j + b_j) \varphi_{kA}] - \sigma_{jAA} \bar{\pi}^i_A + 2i\bar{\pi}^i_A(q), \)
\( \{ \Phi^i_A(q), \bar{H}(b) \} = \int_{\Sigma} 16i \eta^{jk} [\omega_{jAB} \psi_{kB} + (3ia_j + b_j) \psi_{kA}] + \sigma_{jAA} \bar{\pi}^i_A + 2i\bar{\pi}^i_A(q), \)
\( \{ \Phi^i_A(s), \bar{H}_{BB}^B(e) \} = -16i \int_{\Sigma} \eta^{jk} \varphi_{jA} f_{kB} B, \)
\( \{ \Phi^i_A(s), \bar{H}_{BB}^B(f) \} = 16i \int_{\Sigma} \eta^{jk} \varphi_{jA} \sigma_{kB} B, \)
\( \{ \Phi^i_A(s), \bar{J}_A^B \} = 8i \int_{\Sigma} \eta^{jk} (\delta^B_i \omega_{jA} \psi_{kB} - \omega_{jC} \psi_{kA}), \)
\( \{ \Phi^i_A(s), \bar{H}(a) \} = \int_{\Sigma} 16i \eta^{jk} [\omega_{jAB} \varphi_{kB} + (3ia_j + b_j) \psi_{kA}] - \sigma_{jAA} \bar{\pi}^i_A + 2i\bar{\pi}^i_A(s), \)
\( \{ \Phi^i_A(s), \bar{H}(b) \} = \int_{\Sigma} 16i \eta^{jk} [\omega_{jAB} \psi_{kB} + (3ia_j + b_j) \psi_{kA}] - \sigma_{jAA} \bar{\pi}^i_A + 2i\bar{\pi}^i_A(s), \)
\( \{ \Phi^i_A(q), \bar{S}_B(q) \} = 8i \int_{\Sigma} \eta^{jk} \varphi_{jA} \varphi_{kB}, \)
\( \{ \Phi^i_A(q), \bar{S}_B(s) \} = -\int_{\Sigma} 8i\eta^{jk} [\omega_{jAC} \omega_{kB} C + 2\omega_{jAB}(3ia_k + b_k) - \varphi_{jA} \psi_{kB} + 2\epsilon_{ABA} \bar{\pi}^i_A C] + \frac{1}{2} \epsilon_{AB} [i\bar{\pi}(a) + \bar{\pi}(b)], \)
\( \{ \Phi^i_A(s), \bar{H}_{BB}^B(e) \} = -16i \int_{\Sigma} \eta^{jk} \varphi_{jA} \varphi_{kB}, \)
\( \{ \Phi^i_A(s), \bar{H}_{BB}^B(f) \} = 16i \int_{\Sigma} \eta^{jk} \varphi_{jA} \sigma_{kB} B, \)
\( \{ \Phi^i_A(s), \bar{J}_A^B \} = 8i \int_{\Sigma} \eta^{jk} (\delta^B_i \omega_{jA} \psi_{kB} - \omega_{jC} \psi_{kA}), \)
\( \{ \Phi^i_A(s), \bar{H}(a) \} = \int_{\Sigma} 16i \eta^{jk} [\omega_{jAB} \psi_{kB} + (3ia_j + b_j) \psi_{kA}] + \sigma_{jAA} \bar{\pi}^i_A - 2i\bar{\pi}^i_A(s), \)
\( \{ \Phi^i_A(s), \bar{H}(b) \} = \int_{\Sigma} 16i \eta^{jk} [\omega_{jAB} \psi_{kB} + (3ia_j + b_j) \psi_{kA}] - \sigma_{jAA} \bar{\pi}^i_A - 2i\bar{\pi}^i_A(s), \)
\( \{ \Phi^i_A(q), \bar{G}_B(q) \} = -\int_{\Sigma} 8i\eta^{jk} [\omega_{jAC} \omega_{kB} C + 2\omega_{jAB}(3ia_k + b_k) + \psi_{jA} \varphi_{kB} - 2\epsilon_{ABA} \bar{\pi}^i_A C] + \frac{1}{2} \epsilon_{AB} [i\bar{\pi}(a) + \bar{\pi}(b)], \)
\( \{ \Phi^i_A(q), \bar{G}_B(s) \} = 8i \int_{\Sigma} \eta^{jk} \varphi_{jA} \psi_{kB}, \)
\( \{ \Phi^i_A(f), \bar{H}_{BB}^B(e) \} = -2i \int_{\Sigma} \bar{\pi}^i_B(s) \epsilon_{A'B'}, \)
\( \{ \Phi^i_A(f), \bar{H}_{BB}^B(f) \} = -2i \int_{\Sigma} \bar{\pi}^i_B(q) \epsilon_{A'B'}. \)

The nonvanishing Poisson brackets between the secondary constraints are
\[ \{ \bar{H}_{AA}(c), \bar{S}_B(q) \} = 2 \int_{\Sigma} f_{iAA} \bar{\pi}^i_B(q), \]
\[ \{ \bar{H}_{AA}(c), \bar{S}_B(s) \} = 2 \int_{\Sigma} f_{iAA} \bar{\pi}^i_B(s), \]
\[ \{ \bar{H}_{AA}(f), \bar{S}_B(q) \} = -2 \int_{\Sigma} \sigma_{iAA} \bar{\pi}^i_B(q), \]
\[ \{ \bar{H}_{AA}(f), \bar{S}_B(s) \} = 2 \int_{\Sigma} \sigma_{iAA} \bar{\pi}^i_B(s), \]
\[ \{ \bar{J}_A^B, \bar{S}_C(q) \} = \int_{\Sigma} \delta^B_i \omega_{iC} D^i D(q) - \omega_{iC} B \bar{\pi}^i_A(q), \]
\[ \{ \bar{J}_A^B, \bar{S}_C(s) \} = \int_{\Sigma} \delta^B_i \omega_{iC} D^i D(s) - \omega_{iC} B \bar{\pi}^i_A(s), \]
\[ \{ \bar{H}(a), \bar{S}_A(q) \} = \int_{\Sigma} 2i\omega_{iA} \bar{\pi}^i_B(q) - 2(3ia_i + b_i) \bar{\pi}^i_A(q) - [\bar{\pi}(a) - i \bar{\pi}(b)] \varphi_{iA} - 16i \eta^{jk} \omega_{iA} B f_{jBB'} + (3ia_i + b_i) f_{jAB} \bar{\pi}_{kB'} + 32i \eta^{jk} \varphi_{iA} \psi_{jB} \varphi_{kB'}, \]
\[ \{ \bar{H}(a), \bar{S}_A(s) \} = \int_{\Sigma} -2i\omega_{iA} \bar{\pi}^i_B(s) + 2(3ia_i + b_i) \bar{\pi}^i_A(s) - [\bar{\pi}(a) - i \bar{\pi}(b)] \psi_{iA} - 16i \eta^{jk} \omega_{iA} B \sigma_{jBB'} + (3ia_i + b_i) \sigma_{jAB} \bar{\pi}_{kB'} - 96i \eta^{jk} \psi_{iA} \psi_{jB} \varphi_{kB'}, \]

(41)
\begin{align}
\{\bar{H}(a), \bar{S}_A(q)\} &= \int_\Sigma_i 4i f_i A^A [\pi^A_i A(s) - 32i^{\gamma j k} f_i A^A \sigma_{jA} B R_k B'], \\
\{\bar{H}(a), \bar{S}_A(s)\} &= \int_\Sigma_i -4i \sigma_{iA} B \pi^A_i A(q) + 32i^{\gamma j k} \sigma_{iA} B f_j AB \psi_k B', \\
\{\bar{H}(b), \bar{S}_A(q)\} &= \int_\Sigma_i 2\psi_i B [\pi^A_i B(q) + 2(3ia_i + b_i)\pi^A_i A(q) + [\bar{p}^B(a) + \bar{p}^B(b)]\bar{\varphi}_{iA} - \\
&\quad 16\eta^{i j k} [\omega_{iAB} B f_j AB - (3a_i - ib_i) f_j AB \psi_k B'] + 32\eta^{i j k} \bar{\varphi}_{iA} \psi_{iB}B', \\
\{\bar{H}(b), \bar{S}_A(s)\} &= \int_\Sigma_i -2\omega_{iA} B \pi^A_i B(s) - 2(3ia_i + b_i)\pi^A_i B(s) + [\bar{p}^B(a) + \bar{p}^B(b)]\bar{\psi}_{iA} - \\
&\quad 16\eta^{i j k} [\omega_{iAB} \sigma_{jB} B' - (3a_i - ib_i) \sigma_{jB} B' + 96\eta^{i j k} \bar{\psi}_{iA} \psi_{iB}B'], \\
\{\bar{S}_A(q), \bar{S}_A'(q)\} &= \int_\Sigma_i 4i f_i A^A [\bar{p}^B(a) + \bar{p}^B(b)] + 32i^{\gamma j k} f_i A^A \sigma_{jA} B \psi_k B', \\
\{\bar{S}_A(q), \bar{S}_A'(s)\} &= \int_\Sigma_i -4i \sigma_{iA} B \pi^A_i A(q) + 32i^{\gamma j k} \sigma_{iA} B f_j AB \psi_k B', \\
\{\bar{S}_A(s), \bar{S}_A'(q)\} &= \int_\Sigma_i \eta^{i j k} [f_i A^A \varphi_{jA} B + f_i A^A \bar{\varphi}_{jA} B], \\
\{\bar{S}_A(s), \bar{S}_A'(s)\} &= \int_\Sigma_i \eta^{i j k} [f_i A^A \bar{\varphi}_{jA} B + f_i A^A \varphi_{jA} B], \\
\{\bar{S}_A(s), \bar{S}_A'(q)\} &= - \int_\Sigma_i \sigma_{iA} A^A [\bar{p}^B(a) + \bar{p}^B(b)] + 32i^{\gamma j k} (\sigma_{iA} A^A \bar{\varphi}_{jA} B + \sigma_{iA} A^A \varphi_{jA} B).
\end{align}

It is very difficult to classify constraints using these Poisson brackets. Only two first class secondary constraints can be found out:

\[\sigma_{iA} A^A H_{AA}(e) + f_i A^A H_{AA}(f) + H(a) - i\bar{H}(a).\]
[18] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton, New Jersey, 1992).

[19] R. Casalbuoni, Nuovo Cimento, 33A, 115(1976).