Resolving Electron Mass Inconsistency Using Negative Mass

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Abstract

In a previous publication, the author discussed the electron mass and charge inconsistencies resulting from classical models. A model was proposed using classical equations and two opposite charges to resolve the charge inconsistency. The model proposed in that article is modified herein using classical equations to define a model that also resolves the mass inconsistency. The positive mass of the outer shell of the electron core is replaced with a negative mass. The small negatively-charged core at the center still has positive mass.

Keywords

Classical Electron Model, Electron Radius, Electron Magnetic Dipole Moment, Electron Spin Angular Momentum, Negative Mass, Electron Mass Inconsistency, Electron Charge Inconsistency, Particle Physics

1. Introduction

Reference [1] addresses a great inconsistency between the measured electron spin magnet dipole moment and the moment calculated from spinning the electron charge. A similar inconsistency exits between the spin angular momentum derived from quantum theory [1] and the classical momentum calculated from spinning the electron mass [1]. The author proposed in [1] a classical model for resolving the magnetic dipole moment inconsistency. That same model is extended herein using classical physics equations to resolve the spin angular momentum inconsistency. The modified model has the following additional features:

- core having a negative mass outer shell and a positive mass inner core at the center;
- resolve the inconsistency between the spin angular momentum $S$ and the momentum calculated from spinning the electron mass;
no tensile or compressive forces on the core material for a ring shape;

- radius close to the classical radius;

- no intrinsic or induced electric dipole moment.

Except where otherwise noted, all constants and equations in this article are expressed in cgs units.

The shape of the electron in the model will first be assumed to be a ring. A spherical shape will be considered later.

### Table Electron Constants

| constant                  | symbol | value [cgs] [2]                  |
|---------------------------|--------|---------------------------------|
| charge                    | $q$    | $-4.8032 \times 10^{-10}$       |
| mass                      | $m$    | $9.1094 \times 10^{-28}$        |
| classical radius          | $R$    | $2.82 \times 10^{-13}$          |
| spin angular momentum     | $S$    | $9.1329 \times 10^{-28}$        |
| magnetic dipole moment    | $M$    | $-9.284764 \times 10^{-21}$     |
| Planck’s constant         | $h$    | $6.6261 \times 10^{-27}$        |
| speed of light            | $c$    | $2.99792458 \times 10^{10}$     |

### 2. Mass Inconsistency

#### 2.1. Background

As discussed in [1], electron mass calculated from the spin angular momentum $S$ using classical equations is more than 100 times greater than experimentally observed mass. To resolve the mass inconsistency, the classical model of the electron is shown to require a spin rotation speed much greater than the speed of light or a radius much greater than the classical radius. As shown in the following proposal, the mass inconsistency can be resolved without excessive rotation speeds or radii by introducing a negative mass into the electron model.

#### 2.2. Negative Mass Proposal

Reference [1] addresses the inconsistency between the electron charge $q$ and the charge derived from the spin magnetic moment $M$. The model for the electron resolved the inconsistency by introducing into the model a charge of polarity opposite to the polarity of $q$. Similarly, it is proposed in this article to introduce into the model a mass $m^-$ having a negative mass to resolve the mass inconsistency.

The existence of negative mass is controversial. Numerous papers, for example [3], have been written which support the notion that there can be both positive and negative masses, just as there are positive and negative charges. The introduction of a negative mass into the electron model would certainly be helpful in resolving the mass inconsistency, discussed above.

An interesting property of negative mass is that the mass moves in a direction opposite to that of an applied force. From $F = ma$, a positive force $F$ applied to
a negative mass $m$ will cause a negative acceleration $a$. Therefore, negative mass will react to a positive force as if it were a negative force.

The core of the electron in the model is comprised of two parts:
- outer shell having a charge $q^+$ and a negative mass $m^-$
- central core having a charge $q^-$ and a positive mass $m^+$

The distribution of $q^-$ within the central core is unimportant as long as it appears to be located at the center of the outer shell.

The charge $q$ of the electron is
\[
q = q^+ + q^-
\]
and the mass $m$ of the electron is
\[
m = m^+ + m^-.
\]

Aside from its charge $q^-$, the nature of the central core was not discussed in [1]. It is proposed to have a positive mass $m^+$. The radius of the central core must be small enough such that its spin angular momentum is negligible compared with the spin angular momentum $S$ of the outer shell. Spin angular momentum of a mass $m$ at a radius $r$ can be written as
\[
S = \frac{2\pi mr^2}{T},
\]
where $T$ = period of rotation.

The masses $m^+$ of the outer shell and $m^+$ of the central core have nearly the same absolute values. Therefore, if the radius $r$ of the central core were to be 0.01$R$, or 1% of the outer shell radius $R$, the spin angular momentum of the central core will be only 0.01% of the value for the outer shell and a negligible contributor to the total spin angular momentum $S$.

The negative mass $m^-$ may be distributed along radials within the outer shell of the core, but appears to be located at a center of mass located in a ring of radius $R_m$ to produce the spin angular momentum $S$. As will be seen below, the calculated value of $R_m$ has a value close to but less than the predicted electron radius $R_q$. The outer ring spin angular momentum is
\[
S = \frac{2\pi m^- R_m^2}{T}.
\]
The minus sign in the expression cancels the minus sign in the value for $m^-$, so $S$ has a positive value. In [1], the spin was reversed to create a negative magnetic moment $M$ from a spinning positive charge. That same reversal creates a positive spin angular momentum $S$ from a spinning negative mass $m^-$. Reference [1] assumes that the modeled electron is spinning less than but very close to the speed of light $c$. This assumption was made during the resolution of the charge inconsistency. Therefore,
\[
T = \frac{2\pi R_q}{c},
\]
where $R_q$ is the electron radius to be calculated for the model.
The rotation speed of the outer shell center of mass is

\[ v = \frac{2\pi R_m}{T} = \left( \frac{R_m}{R_q} \right) c \]

The centrifugal force on \( m^- \) is

\[ m^- \frac{v^2}{R_m} = m^- R_m \left( \frac{c}{R_q} \right)^2. \]

The sign convention for forces in this article is: positive forces are repulsive and directed away from the center; negative forces are attractive and directed toward the center. The centrifugal force in this case is an attractive force because the mass \( m^- \) of the outer shell is negative.

### 3. Electric Forces

The electron modeled in [1] is a free electron, and the influence of external forces was not considered. The application of an external electric field will create forces on the two charges \( q^+ \) and \( q^- \) that tends to push them apart. Unless physically restrained, a displacement of \( q^+ \) from the center of \( q^- \) will result, creating an induced electric dipole moment. Reference [4] reports an experiment where the upper limit of the electric dipole moment was measured. If a dipole moment exists at all, it is very small, much smaller than the model in [1] with unrestrained charges would predict. Therefore, a fundamental assumption for the model proposed herein is that the electric dipole moment induced by an external electric field is zero.

Although the outer shell and central core of the model herein could be physically bound to each other, that is not a requirement. The two charges need not be physically constrained. Consequently, when an external electric field \( E \) is applied, these two components of the core must accelerate together at exactly the same rate to keep the core intact. Therefore, the accelerations are

\[ \frac{qE}{m} = \frac{q^+ E}{m^+} = \frac{q^- E}{m^-}. \]

The charge and mass relationships derived from the accelerations are

\[ \frac{m^-}{m} = \frac{q^+}{q} \quad \text{and} \quad q^+ = \left( \frac{m^-}{m} \right) q \]

The mutually repulsive force upon each increment of \( q^+ \) due to all of the other increments of \( q^- \) actually acts as a negative attractive force on the outer shell, because the mass of the shell is negative. The apparent force on all increments is

\[ -\left( \frac{q^+}{r} \right)^2 = -\left( \frac{m^-}{m} \right)^2 \left( \frac{q}{r} \right)^2 \]
where \( r \) is the radius of charge shell \( q' \). The attractive force between \( q' \) and \( q \) acts as a repulsive positive force upon the outer shell. The force is

\[
-\left(\frac{q'q}{r^2}\right) = -\left(\frac{m^+}{m}\right)\left(\frac{q-q'}{r^2}\right)q = -\left(\frac{m^+}{m}\right)\left(\frac{q}{r}\right)^2 \left[1 - \left(\frac{m^+}{m}\right)\right].
\]

The combined internal electrical forces act as a repulsive positive force on the outer shell:

\[
-\left(\frac{m^+}{m}\right)\left(\frac{q}{r}\right)^2
\]

4. Internal Forces

The sum of the internal forces must be zero for the core to be stable: The sum of the centrifugal and electrical forces is

\[
m^+ R_m \left(\frac{c}{r}\right)^2 - \left(\frac{m^+}{m}\right)\left(\frac{q}{r}\right)^2 = 0
\]

The solution for \( R_m \) in the above equation is the radius of the effective mass ring of the outer shell:

\[
R_m = \left(\frac{q}{c}\right)^2 \frac{1}{m} = 2.82 \times 10^{-13}
\]

\( R_m \) is exactly equal to the classical radius of the electron, calculated in [5]. The radius \( R_q \) of the charge ring is calculated in the following:

\[
S = -m^+ \left(\frac{R_m^2}{r}\right)c = -\left(\frac{m^+}{m}\right)\left(\frac{q}{m}\right)^2 r = -\left(\frac{m^+}{m}\right)\left(\frac{q}{mc^3}\right)S
\]

The magnetic moment \( M \) for a spinning ring of charge \( q' \) and radius \( r \) is

\[
M = \frac{q^2}{2} \omega r^2 \quad [\text{MKS}] \quad M = \frac{q^2}{2c} \omega r^2 \quad [\text{cgs}]
\]

where \( \omega = \frac{2\pi}{T} \) and for the electron model, \( T = \) period of rotation = \( \frac{2\pi}{c} \). The magnet moment for the electron model is

\[
M = -\frac{q^2}{2} r = -\frac{1}{2} \left(\frac{m^+}{m}\right) q r, \quad \left(\frac{m^+}{m}\right) = -\left(\frac{2M}{qr}\right)
\]

(As explained in [1], the spin direction in the model is reversed such as to produce a negative spin magnet moment for a spinning positive charge.)

The radius \( R_q \) of the charge ring is the solution to

\[
r = \left(\frac{2M}{qr}\right) \left(\frac{q}{mc^3}\right).
\]

\[
R_q = \sqrt{\frac{2M}{ms} \left(\frac{q}{c}\right)^3} = 3.030 \times 10^{-13} = 1.07R
\]

The radius \( R_q \) of the charge ring is 7% greater than the radius \( R_m \) of the effective mass ring. \( R_q \) is considered to be the radius of the modeled electron.
The radius of the outer shell is stable. Since its mass is negative, any perturbation to the outer shell radius will cause the shell to react in the opposite direction, canceling out the perturbation. Also, the center of the outer shell is stable with respect to the inner core. A displacement of the center from the inner core center will increase the attractive force between the outer shell and inner core in a direction opposite to that of the displacement. The increase in attractive force on the negative mass of the outer shell will cause it to move such as to nullify the displacement and restore the net force on the outer shell to zero.

5. Internal Attributes

The two electron model radii, \( R_m \) and \( R_q \), have been calculated for a ring-shaped core, so other attributes can now be calculated.

\[
m^- = -\frac{2M}{qR_q} = -1.162 × 10^{-25} = -127.6m
\]

\[
m^+ = m - m^- = 1.171 × 10^{-25} = 128.6m
\]

\[
q' = \left( \frac{m^-}{m} \right) q = 6.129 × 10^{-8} = 127.6q
\]

\[
q^- = q - q' = -6.177 × 10^{-8} = -128.6q
\]

\[
v = 0.93c
\]

For a spherically-shaped core, \( R_m \) has the same value as for a ring-shaped core. However, the model for spin magnetic dipole moment \( M \) is different, and consequently the values for other attributes will be different also. The model for \( M \) is

\[
M = \frac{q}{3c} \omega r^2 \quad \text{[MKS]} \quad M = \frac{q}{3c} \omega r^2 \quad \text{[cgs]}
\]

The equations above for the ring-shaped core can be adjusted to those for a spherically-shaped core, by replacing “\( 2M \)” with “\( 3M \)”. The internal attributes for a spherically-shaped core are then

\[
R_q = \frac{3M}{mS} \left( \frac{q}{c} \right)^3 = 3.711 × 10^{-13} = 1.32R
\]

\[
m^- = -\frac{3M}{qR_q} = -1.424 × 10^{-25} = -156.3m
\]

\[
m^+ = m - m^- = 1.433 × 10^{-25} = 157.3m
\]

\[
q' = \left( \frac{m^-}{m} \right) q = 7.507 × 10^{-8} = 156.3q
\]

\[
q^- = q - q' = -7.555 × 10^{-8} = -157.3q
\]

\[
v = 0.76c
\]

6. Core Material

The model presented in [1] relied on an incompressible or compressible core
material to provide stability of the internal force balance. The model proposed herein does not for a ring-shaped core and for forces in the equatorial plane of a spherically-shaped core. However, for the later shape, core material is still required to provide a stable force balance along the spin axis, as detailed in [1].

7. Summary

A model of the electron has been proposed which has two opposite electrical charges and both positive and negative masses. The positive charge $q^+$ and negative mass $m^-$ reside on the outer shell of the electron. The negative charge $q^-$ and positive mass $m^+$ reside at the center of the electron. They have radii small enough so that the spinning negative charge does not significantly contribute to the net magnetic moment and the spinning positive mass does not significantly contribute to the spin angular momentum. The shape of the electron can be a ring, spherical, or a shape in between the two.

The intrinsic electric dipole moment of the modeled electron is zero. Also, there is no induced electric dipole moment when the electron is in an external electric field.

The internal attributes of the modeled electron are presented in the table below (Table 1). They are expressed as ratios to their corresponding external attributes.

The outer shell mass can be distributed along the radial direction. The mass can be thought of as being located along a circle having a radius equal to the “mass radius”. The mass radius calculated for the electron model has a value exactly equal to the classical electron radius and independent of the core shape.

The charge radius is the radius of the positive charge on the surface of the outer shell. Its value is somewhat greater than the mass radius. The charge radius is considered to be the radius of the modeled electron. For a ring-shaped core, the calculated radius is only 7% greater than the classical radius.

The speed of the outer shell surface at the equator is assumed to be very close

### Table 1. Internal/external attribute ratios.

| attribute                      | ring   | sphere  |
|-------------------------------|--------|---------|
| mass radius                   | 1.00   | 1.00    |
| classical electron radius     |        |         |
| charge radius                 | 1.07   | 1.32    |
| classical electron radius     |        |         |
| outer shell mass / electron mass | -127.6 | -156.3  |
| electron charge               |        |         |
| central core mass / electron mass | 128.6 | 157.3   |
| electron charge               |        |         |
| outer shell mass speed / speed of light | 0.93  | 0.76    |
| speed of light                |        |         |
to the speed of light, and has been set equal to the speed of light in the calculations. The speed of the center of mass at the mass radius is somewhat less than the speed of light.

Single-charge single-mass models of the electron have a large inconsistency between the spin angular momentum $S$ and the momentum calculated from spinning the electron mass at close to the speed of light. The proposed dual-charge dual-mass model eliminates this inconsistency with a radius very close to the classical radius and a rotation speed slightly less than the speed of light.

The single-charge electron model subjects the core to a very high tensile force. The net force on the negative-mass outer shell is zero, so there is no tensile force on the core. The outer shell is inherently stable, and for a ring-shaped core does not require any assumptions about the core material tensile strength or compressibility. There is no tensile force on the spherical core, but the compressive electrical force along the spin axis must still be physically balanced by the core material.

**Conflicts of Interest**

The author declares no conflicts of interest regarding the publication of this paper.

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