Research on the Directionality of Difference Frequency Sound Field Based on Circular Ultrasonic Array

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Abstract. In a closed environment of underground space, noise pollution is serious, and sound waves are required to have strong directionality, providing people with a relatively independent and quiet working space. In order to obtain a high directivity sound source, a method combining difference frequency sound field with the circular array of transducers is proposed. Firstly, the advantage and disadvantage of the sound field generated by difference frequency mode and the sound field radiated by direct vibration in conventional mode are demonstrated. Secondly, based on the principle of sound field radiation, the directivity function of differential frequency sound field of circular piston transducer with multi-circumference arrays is derived. Finally, the influence parameters of directivity of differential frequency sound field are determined by three-dimensional and two-dimensional directivity diagrams, which can provide reference for the design of acoustic directional transducer array.

1. Introduction

In our daily life, sound waves exist in every corner of the space around us. Useful sound waves and noise mix with each other, which seriously affects the quality and efficiency of people's work. In recent years, audio-oriented technology has achieved great development [1]. Its working principle is to use the characteristics of high directivity of the ultrasonic wave to modulate the audible sound onto the ultrasonic wave, and use the nonlinear interaction between the ultrasonic wave and the air to generate a highly directional self-demodulating audible sound, thereby realizing the directional propagation of the audible sound wave [2-3]. In practical applications, in order to improve the acoustic emission power and optimize the propagation direction, the acoustic directional propagating loudspeakers are mostly designed using ultrasonic transducer arrays [4-5].

In order to further improve the directivity of acoustic field, this paper proposes a method combining the acoustic field generated by the difference frequency mode of acoustic parametric array with the circular array of transducers. Considering the fact that the vibration area of a single ultrasonic transducer used for arrays may affect the directivity of the array, based on the directivity function of a single ultrasonic transducer with a certain vibration area, the directivity function of a single circular array and a multi-circular array are further deduced. The influence of array element radius and circumference radius on the directivity of sound field is studied by computer simulation, which provides a basis for the design of high directivity transducer array.
2. Sections, sub-sections and sub-subsections

2.1. Directivity function of parametric emission array

Generally, low-frequency sound wave has a longer propagation distance than high-frequency sound wave because the absorption rate of sound wave in the air increases with the increase of the frequency of sound wave. However, if the low-frequency sound wave wants to form a high-directivity beam, the size of the transducer needs to be large enough. It is generally about 20 times the wavelength, so it is difficult to solve in linear acoustics. However, according to the nonlinear acoustic theory proposed by Helmholtz \cite{6} and the parametric acoustic array theory proposed by Westervelt \cite{7}, the acoustic parametric array can transmit two high-frequency sound waves of \( f_1 \) and \( f_2 \) respectively through small-sized transducers. The low frequency difference frequency \( f_1 - f_2 \) and other high frequency components are generated by the nonlinear action between the sound waves as follows:

\[
\rho(\omega_1) + \rho(\omega_2) \rightarrow \rho(\omega_1) + \rho(\omega_2) + \rho(\omega_1 - \omega_2) + \rho(\omega_1 + \omega_2) + \rho(2\omega_1) + \rho(2\omega_2)
\]

(1)

The high-frequency components are quickly attenuated, leaving only the difference-frequency waves to continue to propagate in the air. It can be seen that the difference frequency signal in the audible range can be made directional by the acoustic parametric array.

According to the principle of acoustic parametric array and the definition of directivity function of transducer, the following directivity functions are obtained:

\[
D(\theta, \omega_d) = \frac{\alpha_1 + \alpha_2 - \alpha_d}{\sqrt{\left(\alpha_1 + \alpha_2 - \alpha_d \cos \theta\right)^2 + \left(2\omega_d \sin \theta\right)^2}}
\]

(2)

\( D(\theta, \omega_d) \) is a directivity function when the virtual source is viewed as a line source with a small aperture along the axis. It was first derived by Westervelt in 1960 \cite{6}. In the above formulas, \( \alpha_1 \) and \( \alpha_2 \) are the absorption coefficients of two fundamental waves in the atmosphere, and \( \alpha_d \) is the absorption coefficients of differential frequency waves, representing the wavenumber difference of two fundamental waves. \( k_d = k_1 - k_2 \), \( k_d \) represents the wavenumber difference of the two columns of fundamental waves.

2.2. Directivity simulation of difference frequency sound field

According to formula (2), It is not difficult to see that the directivity of point virtual source difference frequency sound field is related not only to the absorption coefficient of two fundamental waves in the atmosphere, but also to the wavenumber difference of two fundamental waves. The absorption coefficient of sound wave in the atmosphere is determined by the frequency of sound wave and the atmospheric condition. The wave number difference is affected by the frequency of difference frequency wave. Therefore, under the same atmospheric conditions, the directivity is determined by the frequency of fundamental and differential frequencies.

Fig. 1 shows a fixed fundamental frequency of 40 kHz, and a differential frequency of 1 kHz, 2 kHz, 3 kHz and 4 kHz, respectively. With the increase of the frequency of the difference frequency wave, the directivity of the difference frequency sound field becomes better and better. Fig 2 shows the fixed difference frequency frequency of 2 kHz, the fundamental frequency is 32, 30 kHz; 42, 40 kHz; 52, 50 kHz.
kHz; 62, 60 kHz. As can be seen from the figure, as the fundamental frequency increases, the directivity of difference frequency sound field becomes worse and worse.

3. Directivity Function of Differential Frequency Acoustic Field of Circular Array
As shown in Fig.3, the transducer array is placed in the xOy plane, the center of the array is the coordinate origin, and the N circular piston transducers with radius R are arranged circumferentially in the xOy plane. The distance from the center of the array element to the origin of the coordinate is \( r_i \), the angle with the x-axis is \( \phi_i \), the unit vector of the sound ray direction is \( e \), the angle with the z-axis is \( \theta \), and the angle between the projection in the xOy plane with the x-axis is \( \phi \).

![Circular piston circulator coordinate chart](image)

According to the Bring product principle, the directivity function of the circular array of circular piston transducers is:

\[
D(\theta, \phi, w) = D_1(\theta, \phi, w) \cdot D_2(\theta, \phi, w)
\]

(3)

\( D_2 \) is a directivity function of a single circular piston transducer:

\[
D(\theta, \phi, w) = \frac{2 J_1(kR \sin \theta)}{kR \sin \theta}
\]

(4)

\( D_1 \) is the circular array directivity function composed of N point sound sources. Here, each array element is regarded as a point sound source. As shown in FIG. 4, the vector of the i-th point sound source is \( r_i \) then

\[
r_i = x_i \hat{i} + y_i \hat{j} = r_i \cos \phi_i \hat{i} + r_i \sin \phi_i \hat{j}
\]

(5)

\[
e = e_x \hat{i} + e_y \hat{j} + e_z \hat{k}
\]

(6)

In Formula (6): \( e_x = \sin \theta \cos \phi \), \( e_y = \sin \theta \sin \phi \), \( e_z = \cos \theta \)

Then the phase difference \( \Delta \phi_i \) of the i-th array element relative to the origin O is:

\[
\Delta \phi_i = k r_i \cdot e = k r_i (\sin \theta \cos \phi \cos \phi_i + \sin \theta \sin \phi \sin \phi_i) = k r_i \sin \theta \cos (\phi - \phi_i)
\]

(7)

The discrete array directivity function according to the arbitrary distribution of N point sources is:

\[
D(\theta, \phi, \omega) = \frac{1}{N} \sum_{i=1}^{N} e^{-j \Delta \phi_i}
\]

(8)

Substituting (7) into (8) yields a directivity function for a circular array of point sources:

\[
D_1(\theta, \phi, \omega) = \frac{1}{N} \sum_{i=1}^{N} e^{-j k r_i \sin \theta \cos (\phi - \phi_i)}
\]

(9)

\( \phi = \frac{2 \pi}{N} (i - 1) + \Delta \phi_x, \Delta \phi_y \) is the angle between the first point source on the circumference counterclockwise and the positive direction of the x-axis.

According to the Bring product principle, the directivity function of a circular array of circular piston transducers is:

\[
D(\theta, \phi, \omega) = \frac{1}{N} \sum_{i=1}^{N} e^{-j k r_i \sin \theta \cos (\phi - \phi_i)} \cdot 2 \frac{J_1(kR \sin \theta)}{kR \sin \theta}
\]

(10)

According to the principle of addition, the M circular array directivity functions are:
4. Circular Array Difference Frequency Sound Field Directivity Simulation Analysis

4.1. Sound field generation by difference frequency method and conventional method
Comparing the sound field produced by the same size circular piston transducer by differential frequency method and the direct vibration radiation sound field by conventional method. According to the Bring product principle, the directivity function of the circular piston circulator to generate the difference frequency sound field is:

\[
D(\theta, \varphi, \omega) = \sum_{j=1}^{N} \sum_{i=1}^{M} \frac{1}{2} e^{-jk_r \sin \theta \cos (\varphi - \varphi_j)} \left| 2 \frac{J_1(R_k \sin \theta)}{R_k \sin \theta} \right|}
\]

(11)

4.2. Directivity simulation of difference frequency sound field
In practical application, in order to further directivity of strong acoustic wave, acoustic directional transmitter usually adopts the form of transducer array. Next, the influence of circular piston transducer array on directivity of differential frequency acoustic field is explored. According to the Bring product principle, the directivity function of the difference frequency sound field generated by the circular array of the circular piston transducer is as follows:

\[
D(\theta, \varphi, \omega) = \left\{ \frac{\alpha}{1 + \left(2k_r \sin \theta / \alpha \right)^2} \right\} \left| 2 \frac{J_1(R_k \sin \theta)}{R_k \sin \theta} \right| \sum_{j=1}^{N} \sum_{i=1}^{M} \frac{1}{2} e^{-jk_r \sin \theta \cos (\varphi - \varphi_j)}
\]

(13)
As shown in Fig. 6, 29 circular piston transducers with radius $R$ are array elements, 4 concentric circles are evenly distributed in the xOy plane ($M=4$). The fixed fundamental frequency is 40 kHz ($\alpha=1.3$), and the difference frequency $f_d$ is 4 kHz. The influence of the element radius and the circumferential radius on the directivity of the difference frequency sound field is analyzed.

4.2.1. Influence of array element radius on the directivity of the difference frequency sound field
When the circumferential radius $r_j$ is fixed to 0, 40, 80, 160 mm, the radius $R$ of the array element is changed to 5 mm, 10 mm, 20 mm respectively, and the xOz plane is oriented surface, then the directivity of differential frequency sound field of circular array is shown in Fig. 7.

![Fig. 7 Influence of array element radius on the directivity of the difference frequency sound field](image)

It can be seen from Fig. 8 that merely increasing the radius of the array element does not affect the main lobe of the difference frequency sound field, but the amplitude of the side lobe and the grating lobe is suppressed to some extent.

4.2.2. Influence of circumferential radius on the directivity of the difference frequency sound field
Since the array element spacing will cause the entire array size to change, $R=5$ mm. The following two cases are analyzed:

One case is to keep the number of circumferential $M=4$ unchanged and change the size of the circumferential radius ($M=4, r_j=10,15,20$ mm). The directivity of sound field is shown in Fig. 8.

Another case is to keep the array size unchanged ($M=4, r_j=20; M=3, r_j=30; M=2, r_j=60$). The directivity of sound field is shown in Fig. 9.

![Fig. 8 The size of the array remains unchanged](image)
![Fig. 9 The size of the array changed](image)
As can be seen from Fig. 9, when the number of circumferential rings is constant, the radius of the circumference is increased, the size of the entire array is also increased, the directivity of the main beam is improved, but the amplitude of the side lobes is also increased. Fig. 10 is to keep the size of the array unchanged, increase the radius of the circle, the main lobe is slightly narrower, but the change is not very obvious, but the amplitude of the flap is increased significantly. It is indicated that a smaller circumferential radius is advantageous for suppressing the amplitude of the side lobes, and an increase in the size of the array can improve the directivity of the main beam.

5. Conclusion
This paper combines the acoustic field generated by the difference frequency mode of the acoustic parametric array with the circular array of the transducer. By deducing the directivity function of the difference frequency acoustic field of the circular array of the circular piston transducer, and discussing the influencing factors of the directivity of the acoustic field, the following conclusions are drawn:

(1) Under the same size, the ultrasonic differential frequency method produces sound field directivity far superior to the sound field directivity of conventional speakers, and can effectively suppress the appearance of side lobes. In addition, increasing the frequency of the difference frequency and reducing the frequency of the fundamental wave can effectively improve the directivity of the difference frequency sound field.

(2) For the difference frequency sound field generated by the circular array of transducers, first, the directivity of the main beam of the difference frequency sound field is mainly related to the size of the entire transducer array and the frequency of the difference frequency wave. Secondly, expanding the radius of the element and reducing the radius of the circle are beneficial to suppress the amplitude of the side lobes.

Acknowledgments
The authors would like to thank the project supported by the National Natural Science Foundation of China (Grant No. 61573073).

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