Quantum Mechanics and Superconductivity in a Magnetic Field

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The influence of a magnetic field on superconductivity is usually described either phenomenologically, using Ginzburg-Landau theory, or semiclassically using Gor’kov theory. In this article we discuss the influence of magnetic fields on the mean-field theory of the superconducting instability from a completely quantum mechanical point of view. The suppression of superconductivity by an external magnetic field is seen in this more physically direct picture to be due to the impossibility, in quantum mechanics, of precisely specifying both the center-of-mass state of a pair and the individual electron kinetic energies. We also discuss the possibility of novel aspects of superconductivity at extremely strong magnetic fields where recent work has shown that the transition temperature may be enhanced rather than suppressed by a magnetic field and where a quantum treatment is essential.
I. INTRODUCTION

The relationship between superconductivity and magnetic fields is both of practical importance in the design of superconducting devices and of fundamental importance to the superconductivity phenomenon. In the absence of an external magnetic field superconductivity is associated with the pairing of time-reversed electron states. As we discuss in detail below, magnetic fields break time-reversal-invariance symmetry and frustrate this pairing. For sufficiently weak external magnetic fields superconductors prefer to completely expel any external magnetic flux (the Meissner effect) in order to avoid this frustration. At stronger magnetic fields type-II superconductors, which are used in the construction of superconducting magnets, can form a mixed state in which superconductivity coexists with magnetic flux. Superconductivity in the mixed state is usually described in terms of Ginzburg-Landau theory \[26\] which predicts a decrease in the temperature to which superconductivity can survive \(T_c\) proportional to the external magnetic field strength. For sufficiently weak external fields and temperatures close to \(T_c\) Ginzburg-Landau theory was derived microscopically by Gor’kov \[12\]. Gor’kov’s theory has a wider range of validity than Ginzburg-Landau theory and predicts \[13,14,1\] that \(T_c\) decreases monotonically with increasing magnetic field and is eventually driven to zero. However Gor’kov’s theory treats the magnetic field in a semi-classical approximation which is not valid when the temperature is sufficiently low and the disorder is sufficiently weak that the Landau quantization of motion in planes perpendicular to the field direction becomes important. In the past few years, following seminal work by Rasolt, Tešanović and collaborators, it has been realized \[24,2\] that (at least within the standard mean-field-theory known to be accurate at weak magnetic fields) superconductivity can survive to arbitrarily strong magnetic fields once Landau quantization is accounted for. In this article we discuss the superconducting instability in a magnetic field from a completely quantum-mechanical point of view. We explain how the results of Ginzburg-Landau theory and Gor’kov theory can be understood in terms of the microscopic quantum mechanics of charged particles in a magnetic field and why Gor’kov theory can fail at sufficiently strong
II. $T_c$ AT ZERO MAGNETIC FIELD

It is useful to begin by discussing the familiar implicit equation for $T_c$ in the absence of a magnetic field: \[ 1 = \frac{V}{\Omega} \sum_{\vec{k},\vec{k}'} \left[ \frac{1 - f(\epsilon_{\vec{k}}) - f(\epsilon_{\vec{k}'})}{\epsilon_{\vec{k}} + \epsilon_{\vec{k}'}} \right] \delta_{\vec{k}+\vec{k}',\vec{p}} \] (2.1)

(Energies are measured from the chemical potential $\nu$ and $\Omega$ is the volume of three-dimensional systems or the area of two dimensional systems. The Fermi energy, $\epsilon_F = mV_F^2/2$ is the zero temperature limit of $\mu$.) This equation is for the usual BCS model with attractive interactions of constant strength $V$. All the discussion in this article will be in terms of this simple model. The prime on the sum over wavevectors denotes the usual separable high energy cutoff requiring both electron energies to be within $E^+$ of the Fermi level. The numerator of the factor in square brackets in Eq. (2.1) expresses through the Fermi occupation numbers the requirement that the pairing come either from electrons outside the Fermi sea, as in the Cooper problem, or from holes inside the Fermi sea. Note that this factor vanishes at finite temperature, and even at $T = 0$ for $\vec{P} \neq 0$, when $\epsilon_{\vec{k}} + \epsilon_{\vec{k}'}$ is near zero. In a superconductor a bound state occurs for the relative motion of electrons in a Cooper pair and the temperature at which the bound state first occurs, $T_c$, depends on the center-of-mass (COM) momentum of the pair, $\vec{P}$, as we discuss in the following paragraph.

Defining an effective pairing density-of-states by
\[
\nu^p(\epsilon : \vec{P}, T) \equiv \frac{1}{\Omega} \sum_{\vec{k}} [1 - f(\epsilon_{\vec{k}}) - f(\epsilon_{\vec{P} - \vec{k}})] \delta(\epsilon - \epsilon_{\vec{k}} - \epsilon_{\vec{P} - \vec{k}}),
\] (2.2)

the $T_c$ equation can be rewritten in the form
\[
1 = V \int_{-\infty}^{\infty} \nu^p(\epsilon : \vec{P}, T) / \epsilon.
\] (2.3)

At $\vec{P} = 0$ and $T = 0$ \[ \nu^p(\epsilon) = \theta(2E^+ - |\epsilon|)(\epsilon/|\epsilon|)\nu(2\epsilon)/2 \] where $\nu(\epsilon)$ is the single-electron density of states per spin. At finite $\vec{P}$ and $T$ $\nu^p(\epsilon : \vec{P}, T)$ is reduced toward zero for
\[ |\epsilon| \ll E^- \equiv \text{sup}(k_B T, V_F P) \] because of the combination of Fermi factors appearing in but otherwise is nearly constant. (Here \( V_F \) is the Fermi velocity. Low energy pairs tend to be composed of states on opposite sides of the Fermi energy for \( \vec{P} \neq 0 \). See Fig.(1). The pairing densities-of-states for two-dimensions (2D) and three-dimensions (3D) are reduced as a consequence when \(|(\epsilon - 2\epsilon_F)/V_F P| < 1\). For \( T = 0 \) the reduction in \( \nu^p \) is illustrated in Fig.(2). For \(|(\epsilon - 2\epsilon_F)/V_F P| < 1\)

\[ \nu^p(\epsilon : \vec{P}) = \frac{\nu(2\epsilon)}{2} \left[ \frac{\epsilon - 2\epsilon_F}{V_F P} \right] \] (2.4)

for 3D and

\[ \nu^p(\epsilon : \vec{P}) = \frac{\nu(2\epsilon)}{2} \left[ 1 - \frac{2}{\pi} \cos^{-1}\left(\frac{\epsilon - 2\epsilon_F}{V_F P}\right) \right] \] (2.5)

for 2D. For weak coupling \((E^- \ll E^+)\) the \( T_c \) equation reduces to \( E^- \sim E^+ \exp(-1/\lambda) \) which will have no solution once \( V_F P \) exceeds \( \sim k_B T_c(\vec{P} = 0) \). \( \lambda \equiv V\nu(0) \). It is assumed that \( \nu(\epsilon) \) is nearly constant over the energy range \( E^+ \). Later we will relate this result for the dependence of \( T_c \) on the COM momentum of the Cooper pair directly to the dependence of \( T_c \) on an external magnetic field.

### III. Pair States in a Magnetic Field

Note that the \( T_c \) equation depends both on the COM state of the pair and, through the Pauli-exclusion-principle requirements expressed by Fermi factors, on the states of the individual electrons making up the pair. The states of a pair of electrons may be described either in terms of COM and relative motion states or in terms of the individual electron states. In the absence of a magnetic field this connection is trivial. To describe superconductivity in a magnetic field quantum mechanically we must start by discussing the relationship between these two descriptions in a magnetic field. The Hamiltonian \( \tilde{H} \) for two non-interacting electrons, \( h \), is

\[ h = \frac{1}{2m} (-i\hbar \nabla_1 + \frac{e}{c} \vec{A}(\vec{r}_1))^2 + \frac{1}{2m} (-i\hbar \nabla_2 + \frac{e}{c} \vec{A}(\vec{r}_2))^2 \] (3.1)
or

\[ h = \frac{1}{2M} (-i\hbar \nabla_R + \frac{2e}{c} \tilde{A}(\tilde{R}))^2 + \frac{1}{2\mu} (-i\hbar \nabla_F + \frac{e}{2c} \tilde{A}(\tilde{r}))^2. \]  

(3.2)

Here we have assumed a gauge where the vector potential is linear in the coordinates, \( M = 2m \), \( \mu = m/2 \), \( \tilde{R} = (\tilde{r}_1 + \tilde{r}_2)/2 \) and \( \tilde{r} = \tilde{r}_1 - \tilde{r}_2 \). Notice that the charge appearing in the center of mass term is \( 2e \) while the charge appearing in the relative motion term is \( e/2 \) so that both relative and center of mass kinetic energies (KE’s) are quantized in the same units as for individual electrons, \( \hbar \omega_c = eB/mc \). (The individual electron eigenvalues measured from the chemical potential are \( \epsilon_N = \hbar \omega_c(N + 1/2) - \mu \equiv \hbar \omega_c(N - N_F) \). \( N_F \) is the Landau level index at the Fermi level.) In the Landau gauge the eigenfunctions for individual electrons are well known:

\[ \psi_{N,X}(\tilde{r}_i) = \exp(-iXy_i/\ell^2)\phi_N((x_i - X)) \sqrt{L_y} \]  

(3.3)

where \( L_y \) is the length of the system in the \( y \) direction, \( \ell \equiv (\hbar c/eB)^{1/2} \) is the magnetic length, and \( \phi_N(x) \) is a one-dimensional harmonic oscillator eigenstate for mass \( m^* \) and frequency \( \omega_c \). The expressions for the center-of-mass and relative eigenstates, \( \psi^R \) and \( \psi^r \), are identical except that the characteristic lengths are scaled to account for the changes of charge and mass. (The effective magnetic lengths are \( \ell^R = \ell/\sqrt{2} \) and \( \ell^r = \sqrt{2} \ell \) for the center-of-mass and relative eigenstates respectively.)

In the lowest Landau level \( \phi_{N=0}(x) \sim \exp(-x^2/4\ell^2) \) so that

\[ \psi_{0,X+Y/2}(\tilde{r}_1)\psi_{0,X-Y/2}(\tilde{r}_2) = \psi^R_{0,X}(\tilde{R})\psi^r_{0,Y}(\tilde{r}). \]  

(3.4)

The relationship is easily generalized to higher Landau levels by writing the Hamiltonian in terms of ladder operators.

\[ h = \hbar \omega_c(a_1^\dagger a_1 + a_2^\dagger a_2 + 1) = \hbar \omega_c(a_R^\dagger a_R + a_r^\dagger a_r + 1) \]  

(3.5)

and noting that \( a_R = (a_1 + a_2)/\sqrt{2} \), and \( a_r = (a_1 - a_2)/\sqrt{2} \). Here \( a_j = \frac{\ell}{\sqrt{2\hbar}}(\pi_{xj} - i\pi_{yj}) \), and \( \tilde{\pi}_j = -i\hbar \nabla_j + \frac{e}{c} \tilde{A}_j \). It follows that
\[ \psi_{N,X+Y/2}(\vec{r}_1)\psi_{M,X-Y/2}(\vec{r}_2) = \sum_{j=0}^{N+M} B_{j}^{N,M} \psi_{j,X}^{R}(\vec{R})\psi_{N+M-j,Y}^{T}(\vec{r}) \] (3.6)

where

\[ B_{j}^{N,M} = \left( \frac{j!(N+M-j)!N!M!}{2^{N+M}} \right)^{1/2} \sum_{m=0}^{j} \frac{(-)^{M-m}}{(j-m)!(N+m-j)!(M-m)!} \] (3.7)

Note that both left and right hand sides of Eq.(3.6) are manifestly eigenstates of \( \hbar \) with eigenvalue \( \hbar \omega_c(N+M+1) \). The coefficients \( B_{j}^{N,M} \) give the amplitude for having KE \( \hbar \omega_c(j+1/2) \) in the center-of-mass motion (and \( \hbar \omega_c(N+M-j+1/2) \) in the relative motion) when the individual particles have definite KE's \( \hbar \omega_c(N+1/2) \) and \( \hbar \omega_c(M+1/2) \).

The coefficients appearing in the unitary transformation between the two sets of two-particle eigenstates, \( \{B_{j}^{N,M}\} \) will play a central role in the discussion below. Note that the transformation is block-diagonal with no mixing between eigenstates of different total kinetic energy. The completeness of either set of eigenstates implies the following identities:

\[ \sum_{N=0}^{K} B_{j,K-N}^{N,K-N} B_{j}^{N,K-N} = \delta_{j,j'} \] (3.8)

\[ \sum_{j=0}^{K} B_{j,K-N'}^{N',K-N} B_{j}^{N,K-N} = \delta_{N',N} \] (3.9)

Since the center of mass kinetic energy does not commute with the individual particle kinetic energies the center of mass is necessarily uncertain if the individual particle states are known precisely. Conversely, for given center-of-mass and relative state kinetic energies the individual particle kinetic energies are necessarily uncertain. Given \( j \) and the relative motion eigenstate, or equivalently \( j \) and the total kinetic energy index \( K \), \( |B_{j}^{N,K-N}|^2 \) gives the normalized probability distribution for the individual electron states with the same total kinetic energy. Explicit expressions for small \( j \) are easily obtained from Eq.(3.7):

\[ |B_{0}^{N,K-N}|^2 = \frac{1}{2^K} \binom{K}{N} \] (3.10)

\[ |B_{1}^{N,K-N}|^2 = \frac{1}{2^K} \frac{K \cdot (K-2N)^2}{N} \] (3.11)
\[ |B_{2}^{N,K-N}|^2 = \frac{1}{2K} \binom{K}{N} \frac{((K-2N)^2 - K)^2}{2K(K-1)}. \] (3.12)

For \( K \gg |k|, (k \equiv N - M) \) it can be shown \cite{8} that

\[ B_{j}^{(K+k)/2,(K-k)/2} \sim \left( \frac{2}{K\pi} \right)^{1/4} \left( \frac{1}{2^{j!}} \right)^{1/2} H_j(k/\sqrt{2K}) \exp(-k^2/4K) \] (3.13)

where \( H_j \) is a Hermite polynomial.

\[ \text{IV. } T_c \text{ IN A MAGNETIC FIELD} \]

The implicit \( T_c \) equation in a magnetic field \cite{10} is completely analogous \cite{6} to the \( B = 0 \) equation (Eq.( 2.1)) cited at the beginning of this article:

\[ 1 = \frac{V}{4\pi\ell^2} \sum_{N,M}^{I} \left[ \frac{1 - f(\epsilon_N) - f(\epsilon_M)}{\epsilon_N + \epsilon_M} \right]|B_{j}^{N,M}|^2 \] (4.1)

As in the \( B = 0 \) case \( T_c \) depends on the Cooper pair state. As we discussed previously the superconducting \( T_c \) decreases with \( |\vec{P}| \) for \( B = 0 \). For \( B \neq 0 \), \( T_c \) is independent of the guiding center quantum number \( X \) for the Cooper pair. The fact that instabilities occur simultaneously in a macroscopic number of channels is responsible for the dimensional reduction \cite{7} which causes superconducting fluctuations to be qualitatively altered by a magnetic field. The superconducting instability still depends, however, on the Landau level index of the Cooper pair. We first examine the weak field limit where \( k_B T \gg \hbar \omega_c \). In this limit the sums over Landau levels may be replaced by integrals and Eq.( 4.1) becomes

\[ 1 = \lambda \int_{2N_F}^{K^+} \frac{dK}{K - 2N_F} \int_{0}^{\infty} dk [1 - f(\epsilon_{(K+k)/2}) - f(\epsilon_{(K-k)/2})]|B_{j}^{(K+k)/2,(K-k)/2}|^2. \] (4.2)

(We’ve noted that \( \nu(0) = 1/(2\pi\ell^2\hbar\omega_c) \) and \( K^+ \) is the maximum kinetic energy index allowed by the high energy cutoff.) To understand why superconductivity is suppressed by weak magnetic fields it is sufficient to consider the \( T = 0 \) limit. The Landau levels with indices \( (K+k)/2 \) and \( (K-k)/2 \) are on the same side of the Fermi level and can contribute to the pairing only if \( |k| < |K - 2N_F| \). (See Fig.( 3).) For a given center-of-mass index \( j \) of the Cooper pair and a given total kinetic energy the probability of finding both members of a
Cooper pair on the same side of the Fermi energy ($\epsilon_F \equiv \mu(T = 0)$), is necessarily less than one. In Fig. (4) we plot

$$P_j(K) \equiv \sum_k [1 - \theta(2N_F - K - k) - \theta(2N_F - K + k)]|B_j^{(K+k)/2,(K-k)/2}|^2$$

(4.3)

for $j = 0$ and $N_F = 12.5$ against $K$. From Eq. (3.13) we see that most of the contribution to $P_j(K)$ comes from $|k| \sim \sqrt{(j + 1/2)K}$. The logarithmic divergence of the integral over $K$ in Eq. (4.2) which guarantees a solution is therefore cutoff since $P_j(K)$ will fall to zero for $|K - 2N_F| \sim \sqrt{(2j + 1)N_F}$. It follows that solutions at $T = 0$ exist only if

$$\hbar \omega_c < \sim (k_B T_c)^2/(2j + 1)\epsilon_F.$$  

(4.4)

The superconducting instability is suppressed most weakly for Cooper pairs with $j = 0$, i.e. for COM in the lowest Landau level, in agreement with Ginzburg-Landau theory.

At zero magnetic field the superconducting instability occurs first for COM momentum $\vec{P} = 0$; the pairing of time-reversed single-particle states guarantees that all pairs are allowed by the Pauli exclusion principle at $T = 0$ even if their energies are very close to the Fermi energy. In a magnetic field time-reversal-symmetry is broken so that time reversed pairs of single particle states no longer exist. The kinetic energy eigenstates in a magnetic field are usefully thought of as having a definite magnitude of momentum corresponding to the quantized kinetic energy but completely uncertain direction of momentum since they are executing circular orbits. For definite COM and relative kinetic energies, $\epsilon_R$ and $\epsilon_r$, the mean-square difference in individual electrons kinetic energies is

$$\langle(\epsilon_1 - \epsilon_2)^2\rangle_\theta = 2\epsilon_R\epsilon_r$$

(4.5)

The average here is over the angle between the COM and relative momenta which is completely uncertain in a magnetic field. This classical root-mean-square energy difference agrees with the energy width of the quantum mechanical distribution function discussed above. When the mean energy of the pair is within this width of the Fermi energy contributions to pair formation are suppressed by the Pauli exclusion principle. For $\epsilon_R = \hbar \omega_c(j + 1/2) \ll \epsilon_r \sim 2\epsilon_F$ the resulting low-energy cutoff is
\[ E^- \approx 2\sqrt{\hbar \omega_c (j + 1/2) \epsilon_F} = V_F P_j. \quad (4.6) \]

In Eq. (4.6) \( \hbar P_j^2 / 4m = \hbar \omega_c (j + 1/2) \) so that \( P_j \) is the ‘quantized’ magnitude of the COM momentum. We see from this discussion that pairing in COM Landau level \( j \) in a magnetic field is very similar to pairing at COM momentum \( P_j \) in the absence of a magnetic field.

The above discussion explains from a quantum mechanical point of view the familiar suppression of superconductivity by a magnetic field in the weak field regime where the discretization of allowed kinetic energies transverse to the magnetic field is washed out either by temperature of disorder. In clean 2D systems the Landau level structure becomes important in the thermally averaged density of states for \( \hbar \omega_c > \sim k_B T \); in 3D systems the free motion along the magnetic field partly obscures the Landau level structure and the strong field regime is reached only for \( \hbar \omega_c > \sim \sqrt{N_F k_B T} \). In the strong field regime the density-of-states has strong peaks and the chemical potential tends to be pinned to these peaks. It is these density of states peaks which can reverse the decrease of \( T_c \) with field and lead to a peculiar regime where \( T_c \) increases with field. As the strong field limit is approached the Landau level at the Fermi energy contributes more strongly to the sum in Eq. (4.1). One immediate effect apparent even at comparatively weak fields is the decrease in \( T_c \) for \( j \) odd. (For COM \( j \) odd the probability of pairs occupying the same Landau level is zero.) Magnetoooscillations in \( T_c \), and in all properties of the mixed state of the superconductor occur as Landau levels pass through the Fermi level. These oscillations have been observed experimentally and are not yet understood in complete detail.

At extremely strong fields a regime can be reached where only electrons in the Landau level at the Fermi energy contribute importantly to the pairing. In this limit (for 2D systems) \( T_c \) reaches a maximum when the Landau level is half full and Eq. (4.1) reduces to

\[ T_{cj} = \frac{\hbar \omega_c \lambda}{8} |B_j^{N_F,N_F}|^2 \quad (4.7) \]

Note that \( T_{cj} \) is proportional to magnetic field strength. In the extreme quantum limit all electrons are in the lowest Landau level and \( N_F = 0 \). Since the maximum value of \( j \) is
it happens that superconductivity occurs in the $j = 0$ channel just as in the weak magnetic field limit. This similarity in the nature of the superconducting order in the weak and infinitely strong field regimes suggests that no novel behavior can occur at intermediate fields. The suggestion is misleading as we can see by looking at the case where $N_F \neq 0$. The maximum COM kinetic energy channel for the Cooper pair is $2N_F$ and pairing can occur in any even $j$ channel. From the expression for $B^{N_F,N_F}_j$ we find that in this case $T_c$ tends to be larger for $j$ close to either its minimum or maximum values and is always equal for $j = 0$ and $j = 2N_F$. (See Table I) This result can be understood by calculating the probability that two electrons of the same energy $\epsilon_F$ but with completely uncertain relative orientation of momentum will have a given COM kinetic energy, $\epsilon_R$. Averaging over angles it is easy to show that

$$P(\epsilon_R) = \frac{1}{\pi} (\epsilon_R(2\epsilon_F - \epsilon_R))^{-1/2}$$

which is peaked near the minimum and maximum possible values for $\epsilon_R$. Thus in the extremely strong field regime there is the possibility of unusual superconducting states in which Cooper pairs are in states with elevated kinetic energies. In mean-field theory the vortex-lattice state is found [21] to have $j > 0$ and to have associated unusual properties including the possibility of having several vortices per period of the lattice.

V. CONCLUDING REMARKS

In this article we have discussed how the suppression of superconductivity by a magnetic field can be understood completely microscopically in terms of the quantum mechanics of pairs of particles in a magnetic field. The results obtained in this way are equivalent to those obtained by Ginzburg-Landau theory and Gor’kov theory in their ranges of validity. The suppression is related to the quantum uncertainty in the kinetic energies of the individual electrons making up a Cooper pair of definite COM kinetic energy. We have also discussed how the suppression can be overcome by the enhancement of the density of states near
the Fermi energy which occurs for sufficiently strong magnetic fields in clean samples and explained why the Cooper pair wavefunction can be unusual in this regime. Ginzburg-Landau theory is not valid in the regime of strong-field superconductivity except for the case where pairing occurs entirely within the $N = 0$ Landau level.

We have restricted our attention here to aspects which follow directly from the quantum mechanics of pairs of charged particles in a magnetic field and the reader should be aware that many other issues arise, some parasitically, especially when considering superconductivity in extremely strong magnetic fields. For example, in our discussion we have, for the sake of definiteness, taken the electron $g$–factor to be zero; a non-zero g-factor will affect results at strong fields [24]. For the sake of our discussion here we have also assumed that the standard mean-field theory of superconductivity which leads to the expressions for $T_c$ we have employed and which is known to be reliable at weak fields can also be used in the strong field regime. It is certain [2] that this is not entirely correct especially in the case of 2D [3] case although we believe that the considerations discussed here are still essential for the physics in that regime.

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REFERENCES

[1] Non-monotonic behavior is possible in special circumstances when the electron g-factor is different from zero.

[2] For a recent review see Mark Rasolt and Zlatko Tešanovic, Rev. Mod. Phys. 64, 709 (1992).

[3] A.H. MacDonald, H. Akera, and M.R. Norman, unpublished.

[4] M.R. Norman, H. Akera, and A.H. MacDonald, Physica C 196, 43 (1992).

[5] J.E. Graebner and M. Robbins, Phys. Rev. Lett. 36, 422 (1976).

[6] For a detailed derivation of this result see A.H. MacDonald, H. Akera, and M.R. Norman, Phys. Rev. B 45, 10147 (1992).

[7] G. Bergmann, Z. Phys. 225, 430 (1969); P.A. Lee and S.R. Shenoy, Phys. Rev. Lett. 28, 1025 (1972); D.J. Thouless, Phys. Rev. Lett. 34, 946 (1975); E. Br’ezin, D.R. Nelson, and A. Thiaville, Phys. Rev. B 31, 7124 (1985); E. Br’ezin, A. Fujita, and S. Hikami, Phys. Rev. Lett. 65, 1949 (1990).

[8] A.H. MacDonald, unpublished.

[9] Motion parallel to the field direction is unaffected a magnetic field so we restrict our attention to planes perpendicular to the field direction.

[10] This is the mean-field $T_c$ equation for two spatial dimensions. In three-dimensions (3D) it is necessary to integrate over momenta along the field direction in addition to summing over Landau level indices. For notational simplicity we restrict our attention here to the 2D case. In 3D $T_c$ depends on the Cooper pair momentum along the field direction as well as on the Landau level of the Cooper pair.

[11] Our discussion is readily adapted to the case of strong-coupling superconductors.

[12] L.P. Gor’kov, Zh. Eksp. Teor. Fiz., 34, 735 (1958).
[13] L.P. Gor’kov, Zh. Eksp. Teor. Fiz., 37, 833 (1959).

[14] E. Helfand and N.R. Werthamer, Phys. Rev., 147, 288 (1966); N.R. Werthamer, E. Helfand, and P.C. Hohenberg, Phys Rev., 147, 295 (1966).

[15] T.T.M. Palstra, B. Battlogg, L.F. Schneemeyer, R.B. vanDover, and J.V. Waszcak, Phys. Rev. B 38, 5102, 1988.

[16] J. Yu, S. Massidda, A.J. Freeman, and D.D. Koelling, Phys. Let. 122A, 203, (1987); W.E. Pickett, R.E. Cohen, and H. Krakauer, Phys. Rev. B 42 8764, 1990.

[17] G. Kido, K. Komorita, H. Katayama-Yoshida, T. Takahashi, Y. Kitaoka, K. Ishida, and T. Yoshitomi, Proceedings of the Third International Symposium on Superconductivity, (Springer-Verlag, Tokyo, 1991): J.L. Smith, C.M. Fowler, B.L. Freeman, W.L. Hults, J.C. King, and F.M. Mueller, ibid. C.G. Olson, R. Liu, D.W. Lynch, R.S. List, A.J. Arko, B.W. Veal, Y.C. Chang, P.Z. Jiang, and A.P. Paulikus, Phys. Rev. B, 42, 381 (1990).

[18] A.K. Rajagopal and R. Vasudevan, Phys. Rev. B 44, 2807 (1991).

[19] A.H. MacDonald, unpublished.

[20] This form depends on the observation that the scattering amplitude does not depend on the center of mass guiding center which is conserved in a scattering event.

[21] H.Akera, A.H. MacDoanld, S.M. Girvin, and M. Norman Phys. Rev. Lett. 67, 2375 (1991).

[22] See for example Chapter 7 in J.R. Schrieffer Theory of Superconductivity (Benjamin, Reading, 1964) and Superconductivity, edited by R.D. Parks (Dekker, New York, 1969).

[23] Both disorder and non-zero g-factors tend to diminish the importance of quantum corrections. For a more detailed discussion see M.R. Norman, H. Akera, and A.H. MacDonald, in Physical Phenomena at High Magnetic Fields (Addison- Wesley, Reading, 1991).
[24] M. Rasolt, Phys. Rev. Lett. 58, 1482 (1987); Zlatko Tešanović and M. Rasolt, Phys. Rev. B 39, 2718 (1989); Zlatko Tešanović, M. Rasolt, and L. Xing, Phys. Rev. Lett. 63, 2425 (1989); Zlatko Tešanović, M. Rasolt, and Lei Xing, Phys. Rev. B 43, 288 (1991); M.R. Norman, Phys. Rev. B 42, 6762 (1990); C.T. Rieck, K. Scharnberg, and R.A. Klemm, Physica C 170, 195 (1990).

[25] A.K. Rajagopal and R. Vasudevan, Phys. Lett. 23, 539 (1966); L.W. Gruenberg and L. Gunther, Phys. Rev. 176, 606 (1968).

[26] See for example, P. G. de Gennes, Superconductivity in Metals and Alloys (Benjamin, New York, 1966); Michael Tinkham Introduction to Superconductivity (McGraw-Hill, New York, 1975).
FIGURES

FIG. 1. Pauli blocking of low-energy pair states at finite center-of-mass momentum, $\vec{P}$, of the pair. In terms of the relative momentum of the pair the Fermi surfaces for the two electrons composing the pair are displaced by $\vec{P}$. The pair must be composed of unoccupied electron states or hole states. At zero temperature the allowed values of relative momentum are either inside both Fermi surfaces or outside both Fermi surfaces. The shaded regions where the energies are close to the Fermi energy are forbidden.

FIG. 2. Pairing density of states at finite COM momentum for three-dimensions (solid line) and two-dimensions (dashed line).

FIG. 3. Probability of having individual electron kinetic energies $\hbar \omega_c(K + k + 1)/2$ and $\hbar \omega_c(K - k + 1)/2$ given center-of-mass kinetic energies $\hbar \omega_c(j + 1/2)$ and total kinetic energies $\hbar \omega_c(K + 1/2)$. The probabilities are represented by the vertical lines at even integer values of $k$. ($k$ must be even when $K$ is even and odd when $K$ is odd.) The results shown here are for $K = 30$ and $j = 0$. For $N_F = 12.5$, i.e. for the first 12 Landau levels occupied, the two single-particle states are both occupied or both empty only for $k = 0$, $k = \pm 2$ and $k = \pm 4$. Larger values of $k$, for which the probability is indicated by a dashed line, are Pauli blocked and cannot contribute to pairing in a $j = 0$ center-of-mass state. For this case the probability that the two single-particle states will be on the same side of the Fermi energy is $P_0(K = 30) = 0.6384$. The solid line which envelopes the probabilities is the large $K$ expression Eq.(3.13).

FIG. 4. $P_j$ against $K$ for $j = 0$ and $N_F = 12.5$, i.e. for the first twelve Landau levels occupied. For the case $K = 30$ $P_j$ is given by the sum of the probabilities indicated by the solid lines in Fig.(3).
TABLES

TABLE I. \(|B_{j}^{N_F,N_F}|^2\) for \(N_F = 0, 1, 2, 3\).

| \(j = 0\) | \(j = 2\) | \(j = 4\) | \(j = 6\) |
|-----------|-----------|-----------|-----------|
| 1         | 0         | 0         | 0         |
| 1/2       | 1/2       | 0         | 0         |
| 3/8       | 1/4       | 3/8       | 0         |
| 15/48     | 3/16      | 3/16      | 15/48     |