A Method for Vibration Optimization of Damped Blade Parameters by Golden Section

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Abstract. A vibration optimization method of damped blade based on golden section is proposed in this paper. The optimal normal loads and damper positions can be determined fast and reliably. Specifically, this method is applied to optimize vibration security of an actual turbine blade stage. By using Mindlin friction model, a numerical analysis of dynamic characteristics is performed. The gap of shrouds is taken as the optimal object and the value is determined finally. The results indicate that, the optimum value is obtained as 1.60 mm. The frequency avoidance ratio of the damped blade is sufficient for security. Meanwhile, the response amplitudes are decreased significantly.

1. Introduction
With the continual increase of both capacities and operating parameters of the unit, the safety and reliability requirements of the steam turbine continue to increase. As one of the key components of the steam turbine, the blade has an important task for converting the thermal energy of the working fluid into mechanical energy. Its reliability directly impact on the safe operation of the turbine unit. The blade, working in a very harsh environment, always operates under large local stress, vibration fatigue and other issues [1, 2]. It is of great significance to improve the design and manufacture of steam turbine blades and optimize the vibration performance.

The dry friction structure, which can effectively suppress the vibration of the blade, has a wide application prospect. The optimization study of friction structure is an important topic in the research of damped blades. Researches in this area can be traced back to 1980s. Griffin et al. [3,4] constructed a single-degree-of-freedom lumped mass model of aero-engine blade with a Blade-to-Ground (B-G) damping structure and finally optimized the vibration. Panning et al. [5] optimized the damping structure of damped blade with B-B and B-G damper to obtain the best damping effect to minimize blade vibration response. Deü et al. [6] optimized the blade-wheel damping structure with electronic components. Hajžman [7] established a damping blade optimization method for different excitation forces with the damped shroud of the blade as the optimization object, which provides the reference for the design of the damped blade. Yonghui Xie et al. [8] combined the three-dimensional model of damped blade with mathematical optimization algorithm to establish the optimization method and optimized the vibration of various damped blades. The results above show that the response level and stress level of optimized blade are obviously improved.

For the study of vibration reduction by means of dry friction forces, the contributions made by researchers are far from enough. On one hand, the morphology and structure of the dry friction structure are diverse, but the research methods mentioned in the literatures are mainly only appropriate for a
specific type of damper, which is not universal. Since the optimization parameters include structure size, normal load, contact gap, damper position, etc., there is still a lot of work need to be done in the vibration optimization study of damper. On the other hand, a variety of numerical methods and optimization algorithms are used in the optimization analysis of the vibration of the damped blades, but part of common numerical methods have problems such as slow and unstable. There are still many specific studies of damping structures subjected to dry friction that have not been addressed.

In this paper, a new optimization method of the vibration response of damped blade is proposed based on golden section method. Then, by using the proposed method, the optimal shroud gap of finite element model of an actual turbine blade is analyzed by programming APDL. Research results are of great reference value for improving the damper design and the operation safety of turbine blades.

2. Vibration optimization method

2.1. Overall vibration level index

When blades are excited, the forced responses at different locations along the blade height vary considerably. In order to facilitate the judgment, this paper takes a weighted average of the resonance responses at different positions as a measurement index of overall vibration level. Specifically, we take n points along the blade height in terms of certain spacing requirements, and assume that the resonance response of i point subjected to friction is \( a_i \) and its resonance response at free blade is \( a_{fi} \). Thus, the weight \( w \) and the weighted average \( s \) are given by the following equations:

\[
w = a_i / \sum_{i=1}^{n} a_{fi}
\]

\[
s = \sum_{i=1}^{n} (a_i w) = \sum_{i=1}^{n} (a_i^2) / \sum_{i=1}^{n} a_{fi}
\]

As the selection of \( n \) points is not being stifled by many restrictions, which means that we can take the vibration level of all key points into account in accordance with the actual need. The ratio of \( a_i \) to \( \sum a_{fi} \) is taken as the weight, and the overall index is positively correlated with the square of vibration level of each point. It can be seen that the weighted average has fully taken into account the contribution of the larger vibration response to the overall response level.

2.2. Vibration optimization method

For a determined damper position and a certain exciting force, there is an optimal normal load to reduce the first order bending response of the blade minimally. In addition, the damper position also affects the total response, which means there is one damper position along the height of the blade to make the resonance response to be minimum. The golden section method, the classical algorithm in the optimization calculation, is suitable for finding the extreme value in response curve with single peak. The algorithm is simple and reliable, with uniform convergence speed and better effect. Hence, a vibration optimization method to obtain the optimal parameter (normal load or damper position) is proposed. Figure 1 shows the optimization iterative process of the vibration parameter.

First, a three-dimensional finite element model of the blade is devised. With boundary conditions and exciting force applied, an initial parameter boundary \( a \) and \( b \) (\( a \) is smaller than the optimum value, while \( b \) is larger) are determined based on known variables. According to the golden method shown in Figure 2 (\( \lambda = 0.618 \)), the parameter \( a_1 \) and \( a_2 \) are obtained. In combination with friction model and harmonic balance method, the optimal value of the vibration response of damped blade is obtained. The frequency response values of the blade under \( a, b, a_1 \) and \( a_2 \) are obtained, respectively. Furthermore, the vibration weighted average \( s(a) \), \( s(b) \), \( s(a_1) \)and \( s(a_2) \) of the blade at the selected position are calculated according to Eq.(2). If \( s(a_1) > s(a_2) \), then \( a = a_1, a_1 = a_2, a_2 = a + \lambda \times (b-a) \). While if \( s(a_1) < s(a_2) \), then \( b = a_2, a_2 = a_1, a_1 = b - \lambda \times (b-a) \). And then the overall vibration levels of damped blade under the new \( a, b, a_1 \) and \( a_2 \) are calculated. The iterative process is repeated until the
convergence precision $\varepsilon$ is reached. In this paper, $\varepsilon$ is set as 1%. Thus, the optimal parameter (normal load or damper position) is obtained.

![Diagram](image)

**Figure 1.** Optimization process of vibration parameter

**Figure 2.** Iteration diagram of parameter

3. Application

Furthermore, this method is used to optimize the vibration safety of actual turbine blade with damped shroud. The gap of shroud contact is taken as the optimization variable. Then, the vibration characteristic analysis of the actual blade is carried out.

3.1. Numerical model

Figure 3 shows the 3-D finite element model of whole circle turbine blades subjected to dry friction. Due to simple harmonic excitation, whole blade structure is need for numerical calculation. This periodic structure consists of 60 blades with shroud and the corresponding disk. The total number of elements and nodes are 2110560 and 2377156. In the working condition of 3000 rpm, spring damping elements are established on the adjacent blade contact surfaces to simulate the friction contact. Axial and tangential displacement constraints are applied on the inlet end of the rotor. The axial displacement coupling and the tangential displacement constraint are applied on the outlet end. It should be noted that, the distribution of gas exciting force applied on the blade surface is obtained by CFD numerical analysis. Table 1 shows the material properties of the blade and disk, respectively.
Figure 3. The finite element model of whole blades and disk

Table 1. Material properties of the blade and disk

| Items                        | Blade  | Disk   |
|------------------------------|--------|--------|
| Density(kg/m³)               | 7810   | 7890   |
| Elasticity modulus(GPa)      | 202.39 | 206.97 |
| Poisson ratio                | 0.3    | 0.3    |

3.2. Friction model

The motion equation of the damped system subjected to simple harmonic excitation can be expressed as:

\[ M\ddot{X} + C\dot{X} + KX = F + G \]  \hspace{1cm} (3)

where \( X \) is displacement vector, \( M \) is mass matrix, \( C \) is damping matrix, \( K \) is stiffness matrix, \( F \) is excitation force vector acting on blade disk system, and \( G \) is dry friction vector of contact surface.

Mindlin model is chosen as the friction model to friction damping characteristics between the contact surfaces accurately, as shown in Figure 4. The hysteresis curve of the tangential friction force \( f \) of the contact surface changes with the relative motion displacement \( u \). When relative displacement amplitude \( A \leq 1.5A_0 \), the contact surface does not occur the overall slip, shown as the hysteresis curve in Figure 4(a). On the contrary, when relative motion amplitude \( A > 1.5A_0 \), the contact surface slips as a whole, as shown in Figure 4(b).

\[ K_d \approx \frac{4E}{2(1+\mu)(2-\mu)} \sqrt{\frac{3NR(1-\mu^2)}{2E}} \]  \hspace{1cm} (4)

where \( K_d \) is tangential stiffness, \( E \) is elasticity modulus of material, \( \mu \) is Poisson's ratio, \( N \) is the normal load of contact surfaces, and \( R \) is spherical radius.
Friction interface can be simplified as a massless spring damping system. Thus, the friction force can be expressed by sum of elastic force and damping force

\[ f = K_{eq}A\cos \theta - C_{eq}\omega A\sin \theta \]  

where \( \omega \) is vibration frequency, \( \theta \) is the phase of relative motion displacement, \( K_{eq} \) is equivalent stiffness coefficient, and \( C_{eq} \) is equivalent damping coefficient. Assuming relative displacement can be written as \( A\cos \theta \), \( K_{eq} \) and \( C_{eq} \) between contact surfaces can be written as

\[ K_{eq} = \frac{1}{\pi A} \int_{0}^{2\pi} f(A, \theta) \cos \theta d\theta \]  

\[ C_{eq} = -\frac{1}{\omega \pi A} \int_{0}^{2\pi} f(A, \theta) \sin \theta d\theta \]  

The equivalent stiffness coefficient \( K_{eq} \) and the equivalent damping coefficient \( C_{eq} \) are averaged to that of each damping matrix element, respectively. Thus, the friction contacts are connected in parallel by a plurality of spring-damp elements. The initial relative displacement between the contact is reasonably assumed before iterations. According to the optimization iterative process in Figure 1, the convergence of the solver could be addressed.

### 3.3. Initial results

The initial gap of the shroud contacts is 1.0 mm. Figure 5 shows the initial nature frequencies versus nodal diameter of the whole blades. It can be seen that, for 2 order vibration, each resonant frequency and exciting force line with same order do not intersect with each other. While for 1 order vibration, resonant frequencies of 7 diameter 1 order approaches the exciting force line. The frequency of the 7 diameter low-frequency gas exciting force is 350 Hz. The resonance frequency of 7 diameter 1 order of whole blades is 344.5 Hz. The deviation between these two is small, with the avoidance rate of 1.6%. This design do not meet the vibration safety requirements, with a danger of easily resonating.

![Figure 5. Initial nature frequencies versus nodal diameter](image)

![Figure 6. Shroud gap](image)

### 3.4. Optimization parameter

In this part, the shroud gap is taken as the optimization parameter, as shown in Figure 6. The vibration optimization method proposed is used to aim at the minimum vibration amplitude of 7 diameter 1 order. The APDL optimizer is programed and solved with software ANSYS. The initial parameter boundary \( a \) and \( b \) of the shroud contact gap are set to be 0 mm and 2 mm.

### 3.5. Results and discusses

Figure 7 shows the frequency response curves of the blade tip under the optimal and other typical shroud gaps. The optimal gap \( s = 1.60 \) mm is obtained finally. It can be seen that, with the increase of
the gap from 0.0 mm to 2.0 mm, the maximum resonance amplitude of the blade decreases first and then increases, which is very similar to the phenomenon in optimization of normal load. This is mainly due to that the shroud gap contributes to the normal load of the shroud contact under the impact of centrifugal force, as shown in Table 2. In addition, the gap also affects the geometric nonlinearity of the blade. By means of parametric modeling of the gap, the influence of the gap on the resonance amplitude can be observed intuitively.

Table 2 shows the modal damping ratio of a serious of shroud gaps at the working speed. When the shroud gap increases from 0 mm to 1.60 mm, the modal damping ratio increases gradually, yet the degree gradually reduces. However, when the gap increases from 1.60 mm to 2.00 mm, the modal damping ratio decreases rapidly. That is with the gradual decrease of normal load enforcing on the contact, the modal damping ratio shows the trend of first increasing and then decreasing. This is consistent with the previous numerical and experiment results of the test blade.

Table 2. Modal damping ratio of material under several gaps

| Shroud gap (mm) | 0    | 0.76 | 1.24 | 1.42 | 1.60 | 1.71 | 2.00 |
|-----------------|------|------|------|------|------|------|------|
| Normal load (N) | 2847 | 1794 | 1110 | 854  | 617  | 428  | 191  |
| Modal damping ratio | 0.013 | 0.014 | 0.016 | 0.018 | 0.019 | 0.008 | 0.006 |

The optimal nature frequencies of the whole blades under each nodal diameter are shown in Figure 8. With the optimal shroud gap, the avoidance rate of 6 diameter 1 order is 4.8% at the working speed, and -6.8% for the 7 diameter 1 order. In the engineering application, the avoidance rate should be beyond the range from -6% to 3%, which means the optimal vibration is regarded as safe. Thus, the optimized shroud gap meets the requirements of the avoidance ratio. Moreover, as the shroud gap changes from 1 mm to 1.60 mm, the decrease of the normal load on the contact results in the decrease of contact stiffness of the damping, which finally brings about the decrease of the resonance frequency. In conclusion, the security problem of an actual blade is addressed successfully by using the vibration optimization method proposed.

4. Conclusions
In this paper, a vibration optimization method is proposed to optimize the parameters of turbine blade. It can be applied to accurately and stably solve the optimal normal load and the damper position. Specifically, this method is applied for vibration optimization of actual damped blades. Under the initial design conditions, the resonant frequency avoidance rate of 7 nodal diameter 1 order of blade subjected to gas excitation force is only 1.6%, which brings security risks. After optimization, the shroud gap is determined to be 1.60 mm. The final avoidance rate meets the requirement for the vibration frequency above engineering application.
5. References

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