Connectivity optimization in interference-limited random DSA networks

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Abstract. Network connectivity in dynamic spectrum access (DSA) networks has been well studied—most of which are under the unit disk model. However, the disk model does not capture the primary-secondary and secondary-secondary interference; hence signal to interference and noise ratio (SINR) based models are more appropriate. Moreover, in the SINR regime, there is no unique way to characterize connectivity and hence its maximization becomes even more challenging.

In this paper, we develop the long eluding network connectivity objective function which we use to build three connectivity optimization techniques each of which targets a particular network setup. The proposed techniques are: i) Fittest deployment density, ii) Fittest receive-only ratio, and iii) Fittest TDMA slotting.

To develop the aforementioned objective function, we start by addressing the lack of any relation between deployment density and network connectivity in interference-limited DSA networks. Next, percolation theory in conjunction with the Boolean model are utilized to develop such a relationship between the density of the percolation visible nodes and the network connectivity in interference limited environments. Finally, we use that relation to build the objective function for connectivity maximization along three optimization techniques. Theoretical findings are validated by simulating networks under various scenarios. Results provide a blueprint to establish and maximize connectivity using physical layer parameters (density, coverage radius, etc.) which can be used in conjunction with higher layer techniques. Also, tackling the connectivity problem at the physical layer relieves the other higher layers like the MAC layer from excess signalling and complex protocol designs.

1. Introduction

In a random dynamic spectrum access (DSA) network, secondary users/nodes (non-license holders) are capable of opportunistically accessing primary nodes’/users’ channels (license holders). It should be noted that, a secondary node can only communicate with other near-by nodes due to the secondary nodes’ limited transmission range [1]. Network connectivity is defined as the build-up of a giant component that spans the majority of the network’s nodes [2]. It is to be noted that network connectivity is different from the pair-wise connectivity which aims at maximizing the number of transmitter-receiver pairs that can achieve successful transmissions. It is also different from k-connectivity[3]. As far as DSA networks are concerned, achieving and maintaining a connected network is a tedious task because of the primary user’s restraints [4]. In order to get the best network services, the goal should not only be to attain connectivity but also to maximize it.
1.1 Connectivity and network connection models
For a continuum network i.e., on $\mathbb{R}^2$, the formation of communication links is governed by the network connection model. For continuum wireless network, one of following two models is usually used. The first of which is the Boolean model (disk model); while the 2nd is the random Boolean model. Fixed range nodes are used in the Boolean model, while in the latter, the nodes use random ranges [5]. However, in both cases the nodes are connected if they are within each other’s range.
In an interference limited system, neither the Boolean nor the random Boolean models can be used. Thus, a model that accounts for the noise, interference, and the signal should be used i.e., a model that addresses the SINR values [6]. In the SINR model, a link is formed across two neighboring nodes when the received signal power at the receiver is above a certain threshold i.e., the signal at the receiver can be correctly decoded.

1.2 Connectivity and percolation theory
The application of any connection model to a network results in a graph whose biggest component represents the network’s connectivity; the size of which depends on various factors like: deployment density, network size, transmission range of nodes, etc. Hence, it is of utter most importance to provide a unified measure for size/breadth of the biggest component.
Percolation theory has proven to be a powerful tool for characterizing connectivity as it analyzes the creation of connected batches of nodes (clusters) in a graph [7]. Theoretically, percolation refers to the formation/emergence of the infinite component also known as the infinite cluster [8]. As in connectivity of continuum networks, the percolation depends on their connection model. The authors in [7] were the first to work on continuum networks with Boolean connection model as they found their percolation condition. They showed that the network percolates as long as the deployment density ($\lambda_s$) is more than the critical density ($\lambda_{sc}$) i.e., $\lambda_s > \lambda_{sc}$.

1.3 The challenge of connectivity maximization
Connectivity maximization refers to connecting the maximal number of nodes i.e., maximizing the size of the connected component. Under the Boolean model, connectivity maximization is a straight forward process where connectivity has been shown to be monotonic with deployment density as well as the coverage radius [7]. i.e., the more the deployed nodes the bigger the connected component.
However, under the SINR model, that is not the case due to mutual interference—the connectivity and the deployment density have a non-linear relation. Where a sparse network results from deploying a few nodes while a large number of deployed ones results in increased interference rendering the nodes unable to communicate. Thus the biggest hurdle in connectivity maximization under the SINR model is to formally find the objective function i.e., how to define connectivity in terms of the network and radio parameters. Without such an objective function, no optimizations can be performed.

1.4 Our contributions
In this paper, we maximize the network connectivity under the SINR model in terms of the network and radio parameters (i.e., the deployment density, coverage radius and other transmit-receive attributes) using concepts from percolation theory. Using the concept of effective density (density of percolation visible nodes), we establish and prove a relation between the network parameters and connectivity in Proposition 1.
Next, we use the relationship for connectivity maximization and set the basis for connectivity maximization in Proposition 2 where we develop three connectivity maximization techniques in terms of: i) deployment density: Which targets networks in the planning phase; ii) receiver-only ratio: Which targets networks that are already deployed and requires no coordination between the nodes; and iii) time slotting: (TDMA) which also targets already deployed networks; however, it requires some
synchronization but provides guaranteed channel access for each node. We validate the theoretical findings via extensive simulations that consider various network scenarios. Results provide insights on how to maximize network connectivity via controlling the physical layer parameters (density, coverage radius, etc.). This is contrary to the traditional approach of burdening and compromising the efficiency of medium access control and routing protocols [9], [10], [11].

As for the organization of the paper: The system model is to be presented next. Followed by the objective function for connectivity maximization. Later, the findings are applied to solve the connectivity maximization problem using 3 distinct techniques. Then, the results are illustrated. Finally, the paper is concluded.

2. System Model
A DSA network with secondary users that coincide with primary users are considered. The generation and distribution of secondary nodes (transceivers) is done via a Poisson point process of λs (density) nodes/unit-area. All the secondary users are assumed to have a maximal coverage radius of rs to be attained with the least amount of interference. Each node also has an interference radius ri. With ri > rs. ri denotes the farthest distance from which a transceiver can sense/receive interference from other secondary transmitter-nodes. Also every transceiver/receiver has β which is the receive-threshold as well as γ which is the noise cancellation factor, also referred to as the processing gain. Gains from the various transmitter-receiver interference cancellation and coding techniques is abstracted in γ. A neighbour: is defined as any node/transceiver that is within rs from another transceiver.

The primary users, also transceivers, are generated by a Poisson point process whose density is λp users/unit area. Protection radius of rp, is considered for each primary user following the literature of primary-compliance. A primary transmitter is assumed to have Pp of transmission power. As in in [12], [13], [14], zero-interference tolerance is used within rp. This means, all secondary users stop transmission once they detect a primary user within rp. To keep the results generic as in [15], [12], [12][13] the secondary transceivers apply no power control. They simply follow a simple rule where they can transmit with Ps if they is no primary user with distance rp from them.

2.1 Signal to interference and noise ratio
SINRj,i is proportion between the received signal power and the interference and noise power at receiver i from its transmitter j. It is to be mentioned that under the SINR connection model, node j has a maximum coverage radius rs, however an edge between j to i (dij < rs) is formed if and only if:

\[
\text{SINR}_{j,i} = \frac{P_j}{\sum \frac{\alpha P_z}{d_{j,i}^{\alpha}} + \sum \frac{P_z}{d_{z,i}^{\alpha}} + N_0} \geq \beta
\]  

where \(d_{i,j}\) is the Euclidean distance from node i to node j, and \(\alpha\) refers to the path-loss exponent.

Finally, \(N_0\) represents the Gaussian noise.

2.2 Objective function for network connectivity
In this section, we develop the objective function to maximize the connectivity of interference limited networks by coupling ideas from percolation in the Boolean model with the concept of effective density.
2.3 Effective density

The authors in [4] found that although the network nodes were being deployed with a density of $\lambda_s$ nodes per unit area, due to interference and primary activity some nodes were rendered invisible with the thinning probability $P_{\text{thin}}$. Poisson thinning [16]. This reduced the density of the nodes which participated towards the network connectivity from $\lambda_s$ to $P_{\text{thin}} \times \lambda_s$. It was also shown that although the nodes were deployed with coverage radius of $r_s$, that was reduced to $\overline{r}_s$ i.e., the coverage shrank due to interference. From a connectivity perspective, the shrinkage also reduced the deployment density $\lambda_s$ by a factor of $(\overline{r}_s/r_s)^2$.

The combined effects of node invisibility and coverage shrinkage resulted in a reduced density of the percolation visible nodes which is referred to as the effective density ($\lambda_{\text{eff}}$) and is given by:

$$\lambda_{\text{eff}} = \lambda_s (1 - P_{\text{thin}}) \times \left(\frac{\overline{r}_s(N)}{r_s}\right)^2$$  \hspace{1cm} (2)

where $P_{\text{thin}}$ accounts for all the invisible nodes due to interference and primary presence. For complete analysis of $P_{\text{thin}}$ and its sub-probabilities refer to [4].

2.4 Node visibility and connectivity

Up to this point $\lambda_{\text{eff}}$ has been characterized in terms of $\lambda_s$ and other network parameters (coverage radius, receiver sensitivity, primary density and coverage, etc.), however that relation still does not link $\lambda_s$ or $\lambda_{\text{eff}}$ to the network connectivity. We show it in the next propositions:

**Proposition 1**: There exists a monotonic relation between the network connectivity and the effective density.

**Proof**: The proof is done by induction. Under the Boolean model, since all the $\lambda_s$ deployed nodes per unit area are possible candidates for percolation, they are percolation visible nodes whose density is $\lambda_{\text{eff}}$. This means, the deployment density is the same as the effective density ($\lambda_s = \lambda_{\text{eff}}$). We know that, under the Boolean model, there exists a monotonic relation between the network connectivity and the deployment density [7]. Thus, it can be stated, there exists a monotonic relation between the network connectivity and the density of the percolation visible nodes i.e., the effective density $\lambda_{\text{eff}}$.

**Proposition 2**: Under the SINR model, $\lambda_s$ controls connectivity maximization indirectly via $\lambda_{\text{eff}}$.

**Proof**: The proof will be done by induction as well. From Proposition 1 we showed the monotonic relation between $\lambda_{\text{eff}}$ and the network connectivity. In Equation (2), we showed $\lambda_{\text{eff}}$ as a function of $\lambda_s$. It follows that using $\lambda_s$ to maximize $\lambda_{\text{eff}}$ also maximizes the network connectivity. Thus the connectivity is maximized indirectly using $\lambda_s$ through $\lambda_{\text{eff}}$.

It follows that the objective function to maximize the network connectivity in terms of $\lambda_s$ can be written as:

$$\text{Maximize for } \lambda_s: \left(\frac{\overline{r}_s(N)}{r_s}\right)^2 \times \lambda_s (1 - P_{\text{thin}})^2 \text{ with } \lambda_s \in \mathbb{R}$$  \hspace{1cm} (3)

3. Connectivity Optimization Techniques

With the objective function given in Equation (3), we propose three connectivity maximization techniques: i) fittest deployment density: we find $\lambda_s$ that yields the maximum effective density, ii) fittest ‘receive-only’ ratio: where we find the fraction of nodes to be placed in ‘receive-only’ mode, and iii) fittest TDMA slotting: we find the best number of TDMA slots, in terms of connectivity maximization, such that only a subset (portion) of nodes are ON in each time slot.

To the best of our knowledge, we are the first to build a connectivity objective function and propose such maximization techniques for DSA networks under the SINR model.
3.1 Fittest deployment density
This technique seeks to find the value of $\lambda_s$ that results in the maximum network connectivity. We refer to such a value as the fittest deployment density ($\lambda_{sfit}$). Such technique suits the planning and pre-deployment phases when the decision is to be made about what value of $\lambda_s$ should be used to maximize the network connectivity. Prior to this technique, connectivity maximization involved arbitrarily changing the deployment density and observing the resultant connectivity. $\lambda_{sfit}$ can be written as:

$$\lambda_{sfit} = \left( \frac{\bar{r}_s(N)}{r_s} \right)^2 \times \arg \max_{\lambda_s} \lambda_s (1 - P_{\text{thin}})^1 \text{ with } \lambda_s \in \mathbb{R} \tag{4}$$

3.2 Fittest receive-only ratio
This technique can be used in the planning phase as well as for networks that are already deployed. Under the SINR model, if the nodes transmit simultaneously, they will interfere and jam each other. Thus reducing both of their coverage and the network. It makes sense to prevent a certain fraction of nodes to transmit i.e., put a fraction $\tau$ of the nodes into receive-only mode. This results in $\lambda_s \times \tau$ nodes in receive-only mode along $\lambda_s \times (1 - \tau)$ simultaneously transmitting nodes.

Choosing the subset of nodes that need to be in receive-only-mode can be done in two ways.

**Optimal Per-Node Assignment:** In this approach, each node is compared against all of its neighbors to check whether to allow it to transmit or not as well as coordinating with the other to-be-allowed to transmit nodes i.e., to be assigned as to-transmit to not. Clearly, this problem lies in the domain of graph coloring as NP-complete and is in a totally different field of research, outside the scope of our work.

**Probabilistic Assignment:** Unlike the previous approach which requires inter-nodes coordination, the nodes in this approach are totally independently and random in their decision where each node "flips a coin" and with probability $\tau$ it goes into the receive-only mode; while it goes into transmit mode with probability $1 - \tau$. This distributed and random mechanism makes the approach much suited for practical implementations. At this point, we define the fittest receive ratio, $\tau_{fit}$, as the optimum $\tau$ that yields the highest connectivity for a given $\lambda_s$.

As in finding the fittest deployment density where the connectivity was maximized at $\lambda_{sfit}$, we seek to reduce the number of transmitting nodes to $\lambda_{sfit}$ instead of $\lambda_s$ using:

$$\tau_{fit} = 1 - \frac{\lambda_{sfit}}{\lambda_s} \quad \forall \lambda_s > \lambda_{sfit} \tag{5}$$

Hence, the expected number of transmitting and receive-only nodes are $(1 - \tau_{fit}) \times \lambda_s \Rightarrow \lambda_{sfit} \text{ and } \tau_{fit} \times \lambda_s$, respectively.

3.3 Fittest TDMA slotting
Like the previous technique (receive-only), this approach is also used in the planning phase as well as for networks that are already deployed. However, the receive-only approach provided no channel access mechanism for the transmitting nodes i.e., the nodes were not provided with any guarantee on their chance-to-transmit. Thus, we seek a solution that provides some form of channel access scheduling while maximizing the network connectivity at the same time.

Traditionally medium access control (MAC) protocols are used for scheduling purposes—a survey of which can be found in [17]. Time Division Multiple Access (TDMA) is a time-proven approach that provides channel access for each node on regular basis, it has been used in [18], [19], [20]. The use of TDMA solves the ‘chance to transmit’ issue. However, we still have to find how many time-slots are needed to maximize the connectivity. Thus the dilemma is figuring out the optimum number of time-slots that maximizes the connectivity $t_{fit}$ and grouping our nodes into subsets where each group of nodes transmit within its time-slot. Again, there could be two approaches.
Optimal-Per-Node Assignment: This problem is similar to the optimal-per-node assignment method in the previous section and it also lies in the domain of graph colouring as NP-complete problem [21], hence outside the scope of our work.

Probabilistic Assignment: Under this paradigm, nodes will flip a coin and randomly select one of the available time-slots for their transmit i.e., (randomly choose a number between 1 and $t_{fit}$). Nodes which end-up transmitting in the same time-slot are logically in the same group/subset.

At this point, the question becomes: what value should be set for $t_{fit}$ i.e., how many time-slots should we have in a time-frame 2. To obtain the optimum value, we elaborate on our findings from the previous section where network connectivity was maximized by scaling-down the number of transmitting-nodes to an average of $\lambda_{sfit}$ node per unit area.

So, rather than having $\lambda_s$ nodes per unit area active simultaneously, lets have an average of $\lambda_{sfit}$ of active-transmitters in each time slot. This leads to breaking-up the deployed nodes into subsets with each group/subset has an average $\lambda_{sfit}$ nodes/unit-area given that each group is active within a particular time-slot. Putting it together:

$$t_{fit} = \left\lfloor \frac{\lambda_s}{1 \times \frac{1}{\lambda_{sfit}}} \right\rfloor \quad \forall \lambda_s > \lambda_{sfit}^{fit}$$

(6)

where $\lfloor \cdot \rfloor$ is the nearest integer.

4. Simulation and Analytical Results

To confirm our propositions and techniques, we run Linux based simulation experiments. In order to cancel-out any favoritism in our settings we consider two disparate networks both of which are Poisson distributed. Both networks are used to demonstrate the relation between deployment density, connectivity and optimization techniques. The results for first network is introduced below as “Network-A”, while the 2nd network will be addressed as “Network-B”.

4.1 Network-A

In this scenario, the deployment area is set to 500 $\times$ 500 with $r_s = 55$, $r_I = 65$ and $\gamma = 0.06$. Such network has $\lambda_{sc} = 70/250000$ (attained from Boolean Model). To better illustrate the relations between $\lambda_s$, $\lambda_{eff}$, and $|C|/n$, the network is first considered in the overlay mode with $\lambda_p = 0$ (no primaries). The resulting connectivity is shown in Figure 1 while the analytical plot of the effective density is shown in Figure 2.

It can be noted that although $\lambda_s$ is increased, the connectivity increases, maximizes, then decreases i.e., it shows the non-linear behaviour mentioned earlier. Which confirms that, under the SINR model, randomly increasing $\lambda_s$ is an impractical approach to increase network connectivity. We also note that the connectivity is established for particular values of $\lambda_s$ and it does not for the remaining ones. The analytical plot of $P_{thin}$ is shown in Figure 3. It shows $P_{thin}$ is inversely proportional to $\lambda_{eff}$ and $|C|/n$. The analytical plot for the coverage shrinkage is shown in Figure 4. It shows that as the number of interferes increase (due to increase in $\lambda_s$), $\bar{r}_s(N)$ decreases.

To demonstrate the impact of the primary users, we add primary users to the simulation with density $\lambda_p$ and $r_p = 55$. The attained connectivity is shown in Figure 5. The corresponding analytical plot of $\lambda_{eff}$ along $\lambda_s$-$\lambda_p$ values is shown in Figure 6. The introduction of the primary users rendered some of the secondary nodes invisible and hence reduced the secondary’s connectivity; however, the connectivity shows the same relation with $\lambda_{eff}$. Thus to maximize the connectivity it is better to choose $\lambda_s$-$\lambda_p$ values which increase $\lambda_{eff}$ instead of randomly increasing $\lambda_s$. 

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4.2 Network-B

The deployment area is set to 300 × 300. As for the secondary network, the deployment density \( \lambda_s \) will be varied accordingly. Meanwhile is \( r_s = 25, r_I = 35, \alpha = 2, \gamma = 0.08 \) and \( \beta = 1 \). This setup results in \( \lambda_c = \frac{151}{90000} \) i.e., it takes at least \( \frac{151}{90000} \) nodes/unit-area to percolate.

The attained connectivity is illustrated in Figure 7. It can be noted that as in Network-A, although \( \lambda_s \) is increased, the connectivity increases, maximizes, then decreases i.e., it shows the same behaviour. Which again confirms our conclusions that under the SINR model, randomly increasing \( \lambda_s \) is an impractical approach to increase network connectivity. We also note that the connectivity is established for particular values of \( \lambda_s \) and it does not for the remaining ones.

To explain this act, we turn-back to the analysis and check the plot figure for \( \lambda_{eff} \) against \( \lambda_s \) for \( \gamma = 0.08 \) which is shown in Figure 8. It shows that although the deployment density is increased, however the effective density increases only for particular \( \lambda_s \) values (up to 300/90000) and it decreases after that. The network starts sparse, thus adding nodes increases the number of percolation visible nodes and the connectivity. However, as more nodes are added, the additional nodes lead to higher levels of interference which increases Pthin as shown in Figure 9 and causes coverage radius shrinkage as shown in Figure 12. This eventually reduces the number of percolation visible nodes and subsequently the connectivity. Figs. 7 and 8, show the correspondence between \( |C|/n \) and \( \lambda_{eff} \) as they increase and decrease jointly for the same values of \( \lambda_s \). The figures also show that the simulated network percolates only for a portion of \( \lambda_s \) values which result in \( \lambda_{eff} > \lambda_{sc} \) i.e., \( \lambda_{eff} > \lambda_{sc} \). Referring back to Figs. 7, 8, and 9 it can be seen that how Pthin is inversely proportional to \( \lambda_{eff} \) and \( |C|/n \). Also, Figure 12 shows that \( \tilde{r}_s(N) \) decreases as the number of interferences increase (due to increase in \( \lambda_s \)). Thus for maximization purposes, we should not rely solely on increasing \( \lambda_s \) but instead to focus on increasing \( \lambda_{eff} \) i.e., emphasize the \( \lambda_s \) values which increase the number of visible nodes per unit area.

A network of primary users is added to the setup to show their effects on the secondary users. Thus primaries are incorporated to the simulation with density \( \lambda_p \) and \( r_p = 35 \). The resultant connectivity is shown in Figure 10, while the corresponding analytical plot of \( \lambda_{eff} \) against \( \lambda_s - \lambda_p \) values is shown in
Figure 11. The results match our analysis where existence of primary users reduced the number of percolation-visible secondary nodes which reduces the over-all secondary network connectivity. Overall, connectivity of secondary network still shows the same monotonic behaviour against $\lambda_{\text{eff}}$.

4.3 Connectivity optimization
For each network we compare the analytical and simulation results for $\lambda_{\text{fit}}$, $\tau_{\text{fit}}$, and $t_{\text{fit}}$ and illustrate how our approaches and propositions have achieved and increased connectivity.

4.3.1 Fittest density
For network A, solving Equation (4) results in $\lambda_{\text{fit}} = 0.000868$; while (Figure 1) for the simulation illustrates that the maximal connectivity is achieved at $\lambda_{\text{fit}} = 0.0009$ nodes/unit-area.

For network B, solving Equation (4) numerically results in $\lambda_{\text{fit}} = 0.0033$; while (Figure 7) for the simulation illustrates that the maximal connectivity is achieved at $\lambda_{\text{fit}} = 0.0035$ nodes/unit-area.

If either of the networks is to be deployed, then the obvious choice is to deploy with $\lambda_{\text{fit}}$ as over deployment with ($\lambda_{\text{s}} > \lambda_{\text{fit}}$) and under-deployment ($\lambda_{\text{s}} < \lambda_{\text{fit}}$) both lead to disconnected networks (Figs. 1 and 7).

4.3.2 Fittest receive-only ratio
For network A, under $\lambda_{\text{s}} = 800/250000$, Figure 1 shows that the resulting connectivity is around 0.09 where the network is not connected. To attain connectivity, we use the probabilistic assignment approach. Hence, we use the analytical value that was found for $\lambda_{\text{fit}} = 0.000868$ in Equation (5) results in $\tau_{\text{fit}} = 0.72875$. To verify that finding, we simulate the network with $\tau = [0, 1]$ and show the resulting connectivity Fig. 13(a). Simulation illustrates that the highest connectivity is achieved at $\tau = 0.725$.

For network B under $\lambda_{\text{s}} = 500/90000$, Figure 7 shows that the corresponding connectivity is around 0.35 (i.e., 35%) and the network is not connected. To maximize the network’s connectivity, we use the probabilistic assignment approach. Substituting $\lambda_{\text{fit}} = 0.0033$ (from previous section) in Equation (5) results in $\tau_{\text{fit}} = 0.4$. To verify the analytical value of $\tau_{\text{fit}}$, we simulate the network with $\tau = [0, 1]$ and show the resulting connectivity in Fig. 13(b). The simulation result shows that the highest connectivity is attained at $\tau = 0.4$.

For both networks which were originally disconnected, the use of fittest receive ratio approach ($\tau_{\text{fit}}$) not only resulted in their connectedness but also maximized it.
4.3.3 Fittest TDMA slotting:
For network-A with $\lambda s = 800/250000$, Equation (6) yields $t_{\text{fit}} = \lceil 800/217 \rceil = 4$ time slots. To check the theoretical finding against the simulation, we try value for $t$ from $t=1$ till 9 time-slots. The resultant connectivity of which is illustrated in Fig. 14(a). The simulation affirms our theoretical solution as it shows connectivity maximization with 4 time-slots.

For network-B with $\lambda s = 500/90000$, Equation (6) results in $t_{\text{fit}} = \lceil 500/300 \rceil = 2$ time slots. We check the theoretical result against the simulation and run $t$ from $t=1$ till 9 time slots. The resultant connectivity of which is illustrated in Fig. 14(b). Again, the simulation matches our analytical result with 2 time-slots.

5. Conclusion
In this paper, we show how the traditional approach, followed under the Boolean model, of adding more nodes to increase the network connectivity does not work under the SINR model. Where in the former the connectivity follows a monotonic relation with $\lambda s$ while in the latter it follows a non-linear relation. We also show how randomly increasing $\lambda s$ does not work for connectivity maximization as it causes coverage shrinkage in one hand and increases the node-to-node interference in the other. Thus it becomes pivotal to find a relation between $\lambda s$ and the network connectivity under the SINR model as connectivity maximization is unattainable without such relation. We do so by linking the number of of the percolation visible nodes per unit area (visible-node density) and the network connectivity at first. Next, we formulated an objective function for connectivity maximization using the concept of effective density.
as an intermediary between $\lambda$s and the network connectivity. Finally, we develop 3 maximization strategies to increase and maximize the connectivity— all of which rely on physical layer parameters which can benefit the upper layers. The maximization techniques account for planning and operational phases of the networks (pre-deployment, post-deployment). They also account for the nodes’ channel access. All theoretical propositions are validated via extensive simulations.

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