ENERGY AND COVERAGE EFFICIENCY TRADE-OFF IN 5G SMALL CELL NETWORKS

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Abstract

When small cells are densely deployed in the fifth generation (5G) cellular networks, the base stations (BSs) switch-off strategy is an effective approach for saving energy consumption considering changes of traffic load. In general, the loss of coverage efficiency is an inevitable cost for cellular networks adopting BSs switch-off strategies. Based on the BSs switch-off strategy, an optimized energy density efficiency of hard core point process (HCPP) small cell networks is proposed to trade off the energy and coverage efficiency. Simulation results imply that the minimum active BS distance used for the BSs switch-off strategy is recommended as 150 meters to achieve a tradeoff between energy and coverage efficiency in 5G small cell networks.

Index Terms

Small cells, energy efficiency, coverage probability, base station switch-off.
I. INTRODUCTION

With the development of key technologies for the fifth generation (5G) mobile communication systems, such as the massive multi-input multi-output (MIMO) and millimeter wave technologies, small cell networks are emerging as a reasonable solution for 5G cellular networks [1]. To meet the requirement of seamless coverage, a large number of small cell BSs has to be densely deployed in 5G cellular networks [2]. On the other hand, the energy efficiency of 5G small cell backhaul networks need to be optimized to satisfy the requirement of future 5G green networks [3]. Since the 80% energy of cellular networks is consumed at base stations (BSs), BSs switch-off strategies are effective approaches to saving energy in 5G small cell networks [4], [5]. However, the loss of coverage efficiency is an inevitable cost for 5G small cell networks when a part of small cell BSs are switched off. Therefore, it is an important challenge to trade off the energy and coverage efficiency in 5G small cell networks.

Some studies have been carried out for BSs switch-off strategies of long term evolution-advanced (LTE-A) cellular networks [6]–[8]. Considering the overlapping coverage of small cells, a cooperative sleep-mode strategy was proposed to optimize the energy efficiency of heterogeneous mobile networks [6]. To minimize the total power consumption of cellular networks while satisfying the user quality of service (QoS) determined by each target data rate, a dual decomposition technique was proposed for jointly optimizing the power and carrier allocation of downlinks as well as the selection of switch-off small cell BSs [7]. To overcome issues caused by the user mobility, an algorithm combining the BS sleeping scheme and the dynamic clustering scheme was developed to improve the energy efficiency of LTE-A cellular networks adopting the coordinated multi-point (CoMP) technology [8]. The traffic load is an important factor for BSs switch-off strategies in cellular networks. Considering the traffic load of small cells and different power consumption profiles of multi-type small cells, an energy efficiency aware sleeping strategy was proposed to save the power of heterogeneous networks [5]. To track the time-varying traffic demand, an adaptive algorithm was developed to dynamically select the appropriate mapping matrix (MM) from a predefined MM set by taking into account the network QoS requirements [9]. A maximum likelihood estimator of the traffic density was obtained based on a rigorous analysis of the joint distribution of the number of vehicles in each hop of vehicular networks [10]. Based on the coverage overlaps of BSs and the traffic intensity in densely deployed cellular
networks, an energy efficiency algorithm was developed by dynamically adjusting the working models (active or sleeping) of BSs according to the traffic variation with respect to the certain blocking probability requirement [11]. By formulating the probability of energy saving as a constrained graphic game where each BS acts as a game player with the constraint of traffic load, the neighboring BS switching strategy was designed to maximize the energy saving of cellular networks while guaranteeing users’ minimal service requirements [12]. A cluster-based approach was proposed to optimize the energy efficiency of small cell networks, where the clusters use an opportunistic BS sleep-wake switching mechanism to strike a balance between the delay and energy consumption [13]. Utilizing the Markov chain method, a sleep mode with dual-thresholds was developed to minimize the network energy consumption while avoiding the frequently switching on/off BSs in small cell networks [14].

Considering the random distribution characteristic of BSs, the BSs switch-off strategies of random cellular networks were investigated in [15]–[17]. Two sleep mode strategies, i.e., random and strategic sleeping modes, were proposed and analyzed for the energy efficiency of random cellular networks [15]. Based on stochastic geometry-based heterogeneous cellular network model, the coverage probability, average achievable rate and energy efficiency were derived for heterogeneous K-tier wireless networks with different sleep modes for small cells [16]. Moreover, the trade-off between the energy efficiency and success probability was investigated for small cell networks with different sleeping policies [17].

Above studies of BS switch-off strategies are based on the traditional Poisson point processes (PPP) random cellular networks. However, adjacent BSs could be infinitely closed in PPP random cellular network. This assumption conduces to a result that the interference will approach infinite in PPP random cellular networks. Moreover, the distribution of BSs is not a PPP model when some BS have been switched off in PPP random cellular networks. As a consequence, there exist issues to analyze the performance of random cellular networks adopting BSs switch-off strategies based on traditional PPP models. In this paper, we introduce the hard core point process (HCPP) to model the BSs distribution of random cellular networks adopting BSs switch-off strategies. The minimum active BS distance is a critical metric for BSs switch-off strategies in 5G small cell networks. By formulating the energy density efficiency of HCPP small cell networks, an optimized minimum active BS distance is obtained to trade off the energy and coverage efficiency in 5G small cell networks. The contributions of this paper are summarized as follows:
1) The HCPP is introduced to model the random cellular networks with BSs switch-off strategies. Moreover, the energy and coverage efficiency of HCPP and PPP random cellular networks are compared each other in this paper.

2) An optimized energy density efficiency of HCPP small cell networks with coverage probability constraints is proposed to trade off the energy coverage and efficiency in 5G small cell networks.

3) Considering the BSs switch-off strategy, simulation results imply that the minimum active BS distance should be configured as 150 meters to trade off the energy coverage and efficiency in 5G small cell networks.

The remainder of this paper is organized as follows. Section II introduces the system model. The interference models of HCPP and PPP random MIMO cellular networks are presented in Section III. The energy efficiency of HCPP and PPP random MIMO cellular networks is proposed in Section IV. The coverage probability of HCPP and PPP random MIMO cellular networks is analyzed in Section V. Considering the BSs switch-off strategy, the optimal energy efficiency of random cellular networks with coverage probability constraints is formulated and simulated in Section VI. Finally, conclusions are drawn in Section VII.

II. SYSTEM MODEL

To save the energy consumption of small cell networks in low traffic load scenarios, the BSs switch-off strategy is one of main approaches to improve the energy efficiency of small cell networks. In this case, a part of adjacent small cell BSs are selected to switch off to guarantee that the coverage efficient of small cell networks has not obviously reduced. Therefore, the minimum active BS distance is a critical matric for optimizing BSs switch-off strategies in small cell networks.

A. Overview of HCPP

Before the BSs switch-off strategy is adopted, in this paper the BSs distribution of random small cell networks is assumed to to be governed by a PPP, i.e., PPP random small cell networks. After the BSs switch-off strategy is adopted, i.e., a part of BSs are switched off based on the minimum active BS distance, the BSs distribution of random small cell networks is governed by a HCPP in this study [18]. One of characteristics in HCPP is that there exists a minimum
distance $\delta$ among points of HCPP. The Matern hard-core process of Type II, which represents a special case of HCPP, is essentially a stationary PPP $\Pi_{PPP}$, i.e., the Poisson point process of intensity $\lambda_p$, to which a dependent thinning is applied. Similarly, the BSs switch-off process in PPP random small cell networks is regarded as a dependent thinning process in PPP. Moreover, the thinned process $\Pi_{HCPP}$, i.e., the Matern hard-core process is defined by [19]

$$\Pi_{HCPP} = \{ x \in \Pi_{PPP} : \Phi(x) < \Phi(x^*) \text{ for all } x^* \text{ in } \Pi_{PPP} \cap d(x, \delta) \}.$$  \hspace{1cm} (1)

The points of PPP are marked with random numbers uniformly distributed in $[0,1]$ independently. The dependent thinning retains the point $x$ of PPP with mark $\Phi(x)$ if the disk $d(x, \delta)$ contains no points of PPP with marks smaller than $\Phi(x)$, where $d(x, \delta)$ is a disk region with central point $x$ and the radius $\delta$ [19].

Assume that both BSs and user equipments (UEs) are randomly located in the infinite plan $\mathbb{R}^2$. Besides, UEs’ motions are isotropic and relatively slow, such that during an observation period, e.g., a time slot, the relative position of BSs and UEs are assumed to be stationary. When the BSs switch-off strategy is triggered in small cell networks, the distribution of BSs is assumed to be governed by a thinned process $\Pi_{HCPP}$ applied to a stationary PPP $\Pi_{PPP}$ of intensity $\lambda_p$. Moreover, the minimum BS distance is assumed as $\delta$. Locations of BSs are denoted as $\prod_{BS} = \{ x_{BS_i} : i = 1, 2, 3, \ldots \}$, where $x_{BS_i}$ is the two-dimensional Cartesian coordinates, denoting the location of the $i$th BS $BS_i$. The distribution of UEs is assumed to be governed by a PPP with intensity $\lambda_M$. The distribution of BSs is illustrated in Fig. 1. Fig. 1(a) is the BSs distribution without the BSs switch-off strategy in small cell networks, where the BSs distribution is governed by a PPP. Fig. 1(b) is the BSs distribution with the BSs switch-off strategy in small cell networks, where the BSs distribution is governed by a HCPP and the minimum BS distance is configured as $\delta = 120$ meters.

B. Signal Model

In this paper we focus on the downlinks of small cell networks. Every small cell includes one BS equipped with $M$ antennas and $K$ UEs equipped with $N$ antennas. Considering that a typical 5G transmission technology, i.e., the massive MIMO technology is adopted at BSs, the number of antennas at the BS is obviously larger than the number of antennas at the UE, i.e. $N = 1$. In this case, the channel model between the desired BS and UEs is expressed by [20]

$$H = GD^{\frac{1}{2}}.$$  \hspace{1cm} (2)
where $G$ is a $M \times K$ matrix expressing the fast fading coefficients between the BS and $K$ UEs, elements of which are independent identical distribution (i.i.d.) complex Gaussian random variables. The element $[G]_{mk} = g_{mk}$ located at the $m$ row and the $k$ column of matrix $G$ is a random variable with the unit variance and the zero mean, which is the fast fading coefficient between the $m$–th antenna at the desired BS and the $k$–th UE. $D$ is a $K \times K$ diagonal matrix and the diagonal element of $D$, i.e., $[D]_{kk} = \beta_k$ is the large scale fading coefficient between the desired BS and the $k$–th UE. Moreover, the large scale fading coefficient is extended by $\beta_k = \frac{w_k}{r_k^\alpha}$, where $w_k$ is the shadowing effect in wireless channels and is calculated $w_k = e^{s/10}$, $s \sim N(0, \sigma_s^2)$, i.e., the random variable $s$ is governed by the Gaussian distribution with the zero mean and the variance $\sigma_s^2$ \[21\], $r_k$ is the distance between the desired BS and the $k$–th UE, $\alpha$ is the path loss coefficient. The received signal vector $y_i$ of $K$ UEs in the $i$–th small cell is expressed as \[22\]

$$
    y_i = H_{ii}^T x_i + \sum_{u \neq i, BS_u \in \Pi_{BS}} H_{ui}^T x_u + n,
$$

$$
    x_i = \sqrt{P_f} F_i s_i,
$$

$$
    x_u = \sqrt{P_f} F_u s_u,
$$

where $H_{ii}^T$ is the channel matrix between the desired BS $BS_i$ and the $K$ UEs in the $i$–th small cell, $x_i$ is the $M \times 1$ signal vector transmitted by the desired BS $BS_i$, $H_{ui}^T$ is the channel matrix between the interfering BS $BS_u$ and the $K$ UEs in the $i$–th small cell, $x_u$ is the signal vector transmitted by the interfering BS $BS_u$, $n$ is the $K \times 1$ noise vector at the UEs in the $i$–th small cell and the power of noise at all UEs is the same and equal to $\sigma_n^2$, i.e., the noise covariance matrix at the UE is $\mathbb{E}(nn^\dagger) = \sigma_n^2 I_{K \times K}$, where $I_{K \times K}$ is the $K \times K$ unit matrix. $F_i$ and $F_u$ are the $M \times K$ precoding matrix at the desired BS $BS_i$ and the interfering BS $BS_u$, respectively. $\sqrt{P_f} s_i$ and $\sqrt{P_f} s_u$ are the $K \times 1$ original signal vectors without precoding at the desired BS $BS_i$ and the interfering BS $BS_u$, respectively, where $P_f$ is the signal transmission power, $s_i \sim \mathcal{CN}(0, 1)$ and $s_u \sim \mathcal{CN}(0, 1)$ are the independent data stream at the desired BS $BS_i$ and the interfering BS $BS_u$, respectively.

Assumed that the pilot signals in different small cells are occupied the same frequency and the pilot signals of $K$ UEs are orthonormal each other in a small cell, i.e., the pilot pollution is
only occurred at inter-cells and ignored at intra-cell. In this case, the estimated channel matrix between the desired BS and the $K$ UEs in the $i$–th small cell is expressed by

$$
\hat{H}_{ii} = \sqrt{P_p} \left( H_{ii} + \sum_{u \neq i, BS \in \Pi BS} H_{iu} \right),
$$

(4)

where $H_{ii}$ is the $M \times K$ channel matrix between the desired BS and the $K$ UEs in the $i$–th small cell, $H_{iu}$ is the $M \times K$ channel matrix between the desired BS and the $K$ UEs in the $u$–th small cell, $P_p$ is the pilot signal power.

When the matched filter precoding scheme is adopted at BSs [23], the precoding matrices $F_i$ and $F_u$ used at the desired BS and the interfering BS are given by

$$
F_i = \hat{H}_{ii}^* = \sqrt{P_p} \left( H_{ii} + \sum_{u \neq i, BS \in \Pi BS} H_{iu} \right)^*,
$$

(5a)

$$
F_u = \hat{H}_{ui}^* = \sqrt{P_p} \left( H_{uu} + \sum_{i \neq u, BS \in \Pi BS} H_{ui} \right)^*,
$$

(5b)

where $(\cdot)^*$ is the conjugation operation at the matrix. Substitute (5a) and (5b) into (3b) and (3c), the received signal vector $y_i$ of $K$ UEs in the $i$–th small cell is derived by

$$
y_i = \sqrt{P_f P_p} H_{ii}^T \left( H_{ii} + \sum_{u' \neq i, BS \in \Pi BS} H_{iu'} \right)^* s_i \\
+ \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi BS} H_{ui}^T \left( H_{uu} + \sum_{i' \neq u, BS \in \Pi BS} H_{ui'} \right)^* s_u + n \\
= \sqrt{P_f P_p} H_{ii}^T H_{ii}^* s_i + \sqrt{P_f P_p} \sum_{u' \neq i, BS \in \Pi BS} H_{iu'}^* s_i + \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi BS} H_{ui}^T H_{uu}^* s_u \\
+ \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi BS} \sum_{i' \neq u, BS \in \Pi BS} H_{ui}^T H_{ui'}^* s_u + n \\
= \sqrt{P_f P_p} H_{ii}^T (H_{ii}^T)^\dagger s_i + \sqrt{P_f P_p} \sum_{u' \neq i, BS \in \Pi BS} (H_{iu'}^T)^\dagger s_i + \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi BS} H_{ui}^T (H_{uu}^T)^\dagger s_u \\
+ \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi BS} \sum_{i' \neq u, BS \in \Pi BS} (H_{ui'}^T)^\dagger s_u + n
$$

(6)
where $(\cdot)^{T}$ is the transposition operation at the matrix, $(\cdot)^{\dagger}$ is the conjugation transposition operation at the matrix. When $M$ approaches infinite and $M \gg K$, the following results are derived by [22]

$$\frac{1}{M} \mathbf{H}_{ii}^{T} (\mathbf{H}_{ii}^{T})^{\dagger} \to \mathbf{D}_{ii}, \quad (7)$$

$$\frac{1}{M} \sum_{i' \neq u, BS} \mathbf{H}_{ui}^{T} (\mathbf{H}_{ui'}^{T})^{\dagger} \to \mathbf{D}_{ui}, \quad (8)$$

When massive MIMO antennas are assumed to be equipped at BSs, the fast fading effect and the noise in wireless channels can be ignored based on the results in [22]. As a consequence, (6) can be simplified by

$$y_i \to M \sqrt{P_f P_p} \mathbf{D}_{ii} \mathbf{s}_i + M \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi_{BS}} \mathbf{D}_{ui} \mathbf{s}_u, \text{ as } M \to \infty, \quad (9)$$

where $\mathbf{D}_{ii}$ and $\mathbf{D}_{ui}$ are $K \times K$ diagonal matrices and the diagonal elements of $\mathbf{D}_{ii}$ and $\mathbf{D}_{ui}$ are denoted by $[\mathbf{D}_{ii}]_{kk} = \beta_{iki}$ and $[\mathbf{D}_{ui}]_{kk} = \beta_{uki}$, which are the large scale fading coefficients over wireless channels. Moreover, the large scale fading coefficients are calculated by $\beta_{uki} = \frac{w_{uki} \alpha_{uki}}{r_{uki}}$ and $\beta_{iki} = \frac{w_{iki} \alpha_{iki}}{r_{iki}}$, where $w_{uki}$ and $w_{iki}$ are shadowing effects over wireless channels, $r_{uki}$ is the distance between the interfering BS $BS_u$ and the $k$–th UE in the $i$–th small cell, and $r_{iki}$ is the distance between the desired BS $BS_i$ and the $k$–th UE in the $i$–th small cell, $\alpha$ is the path loss coefficient over wireless channels. Furthermore, the received signal at the $k$–th UE in the $i$–th small cell is expressed by

$$y_{ik} = M \sqrt{P_f P_p} \beta_{uki} s_{ik} + M \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi_{BS}} \beta_{uki} s_{uk}, \quad (10)$$

where $s_{uk}$ is the $k$–th element in the vector $\mathbf{s}_u$ and is the independent data stream transmitted for the $k$–th UE. Hence, the interference power received at the $k$–th UE in the $i$–th small cell is derived by

$$I_{ik} = \left( M \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi_{BS}} \beta_{uki} s_{uk} \right)^{\dagger} \left( M \sqrt{P_f P_p} \sum_{u \neq i, BS \in \Pi_{BS}} \beta_{uki} s_{uk} \right)$$

$$= M^2 P_f P_p \sum_{u \neq i, BS \in \Pi_{BS}} \beta_{uki}^2 (s_{uk})^{\dagger} s_{uk}$$

$$= M^2 P_f P_p \sum_{u \neq i, BS \in \Pi_{BS}} \beta_{uki}^2 \quad (11)$$
III. INTERFERENCE MODELS OF HCPP AND PPP SMALL CELL NETWORKS

A. Interference Model of HCPP Small Cell Networks

For a Poisson point distribution, the mean of point number in a circle with radius $\delta$ is assumed to be $\lambda_p \pi \delta^2$. Based on the results in [24], the first moment of HCPP is expressed by

$$\zeta^{(1)} = \lambda_p \int_0^1 e^{-\lambda_p \pi \delta^2 t} \, dt = \frac{1 - e^{-\lambda_p \pi \delta^2}}{\pi \delta^2}. \quad (12)$$

Utilizing (12), the probability that there is a point in the infinitesimal small region $dx$ is denoted as $\zeta^{(1)}(dx)$ for HCPP small cell networks. Moreover, the second moment of HCPP is expressed by

$$\zeta^{(2)}(r) = \lambda_p^2 \varphi(r), \quad (13a)$$

$$\varphi(r) = \int_0^1 e^{-\lambda_p t_1 \pi \delta^2} \int_0^{t_1} e^{-\lambda_p t_2 [V_\delta(r) - \pi \delta^2]} \, dt_2 \, dt_1 + \int_0^1 e^{-\lambda_p t_2 \pi \delta^2} \int_0^{t_2} e^{-\lambda_p t_1 [V_\delta(r) - \pi \delta^2]} \, dt_1 \, dt_2,$$

$$= \left\{ \begin{array}{ll}
\frac{2V_\delta(r) [1 - e^{-\lambda_p \pi \delta^2}] - 2\pi \delta^2 (1 - e^{-\lambda_p V_\delta(r)})}{\lambda_p^2 \pi \delta^2 V_\delta(r) [V_\delta(r) - \pi \delta^2]}, & r > \delta \\
0, & r \leq \delta
\end{array} \right., \quad (13b)$$

$$V_\delta(r) = \left\{ \begin{array}{ll}
2\pi \delta^2 - 2\delta^2 \arccos\left(\frac{r}{\delta}\right) + r \sqrt{\delta^2 - \frac{r^2}{4}}, & 2\delta > r > 0 \\
2\pi \delta^2, & r \geq 2\delta
\end{array} \right., \quad (13c)$$

where $r$ is the distance between two points which are located in the infinitesimally small regions, i.e., $dx_1$ and $dx_2$. Based on (13a), the probability that the distance between two small regions $dx_1$ and $dx_2$ equals $r$ is denoted by $\zeta^{(2)}(r) dx_1 dx_2$ for HCPP small cell networks. The location vector of the desired BS $BS_i$ is denoted as $x_{BS_i}$, the distance vector between the desired BS $BS_i$ and the $k-th$ UE $UE_{ik}$ in the $i-th$ small cell is denoted as $x_{int}$. The location vector of the interfering BS $BS_u$ is denoted as $x_{BS_u}$. For HCPP cellular networks, the distance between the interfering BS $BS_u$ and the $k-th$ UE $UE_{ik}$ in the $i-th$ small cell is denoted as $r_{uki} = ||x_{BS_u} - x_{BS_i} - x_{int}||$. Substituting $\beta_{uki} = \frac{w_{uki}}{r_{uki}}$ into (11), the interference power received at the $k-th$ UE in the $i-th$ small cell of HCPP small cell networks is derived by

$$I_{ik}^{HCPP} = \sum_{u \neq i, BS_u \in \Pi_{BS}} \frac{M^2 P_f P_p w_{uki}^2}{||x_{BS_u} - x_{BS_i} - x_{int}||^{2\alpha}}. \quad (14)$$
Assumed that every UE is associated with the closest BS in the plane $\mathbb{R}^2$. The total interference power of HCPP small cell networks is expressed by
\[
\sum_{BS_i \in \Pi_{BS}} I_{ik}^{HCPP} = \sum_{BS_i \in \Pi_{BS}} \sum_{u \neq i, BS_u \in \Pi_{BS}} \frac{M^2 P_f P_p w_{uki}^2}{||x_{BS_u} - x_{BS_i} - x_{int}||^{2\alpha}}.
\]  
(15)

Utilizing the second moment of HCPP in (13a), the expectation of the total interference power of HCPP small cell networks is derived by
\[
\mathbb{E}\left[\sum_{BS_i \in \Pi_{BS}} I_{ik}^{HCPP}\right] = \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \mathbb{E}_{w_{uki}} \left[\frac{M^2 P_f P_p w_{uki}^2}{||x_1 - x_2 - x_{int}||^{2\alpha}}\right] \zeta^{(2)}(||x_1 - x_2||) dx_1 dx_2,
\]  
(16)

where $\mathbb{E}(\cdot)$ is the expectation operation. Based on the first moment of HCPP in (12), the average BS number of HCPP small cell networks is $\int \zeta^{(1)}(x) dx$ in the plane $\mathbb{R}^2$. As a result, the average interference of the $k$-th UE $UE_{ik}$ in the $i$-th small cell of HCPP small cell networks is derived by
\[
I_{ik}^{HCPP}_{avg} (||x_{int}||) = \frac{1}{\zeta^{(1)}} \int_{\mathbb{R}^2} \mathbb{E}_{w_{uki}} \left[\frac{M^2 P_f P_p w_{uki}^2}{||x + x_{int}||^{2\alpha}}\right] \zeta^{(2)}(||x||) dx.
\]  
(17)

B. Interference Model of PPP Small Cell Networks

The distribution of BSs is assumed to be governed by a PPP in small cell network before the BS switch-off strategy is adopted. In this case, the interference power received at the $k$-th UE in the $i$-th small cell of PPP small cell networks can be expressed by (11). To obtain the PDF of the interference power in PPP small cell networks, let
\[
\mathcal{J}_{ik} = \frac{1}{M^2 P_f P_p} I_{ik}^{PPP} = \sum_{u \neq i, BS_u \in \Pi_{BS}} \beta_{uki}^2 = \sum_{u \neq i, BS_u \in \Pi_{BS}} \frac{w_{uki}^2}{r_{2\alpha}^2}.
\]  
(18)

The PDF and characteristic function of $w_{uki}^2$ are denoted as $f_Q(q)$ and $\phi_Q(w)$, respectively. Based on the Campbell theory and the results in [25], the characteristic function of $\mathcal{J}_{ik}$ is derived by
\[
\phi_{\mathcal{J}_{ik}}(\omega) = \mathbb{E}\left\{e^{i\omega \cdot \mathcal{J}_{ik}}\right\}
\]  
= \exp \left\{-2\pi \lambda_{inf} \int \int \left(1 - e^{i\omega \cdot \frac{r}{r_{2\alpha}}}\right) f_Q(q) dq dr\right\},
\]  
(19a)
with

\[
\delta_{\mathcal{I}_{ik}} = \lambda_{\text{inf}} \pi \Gamma(2 - 1/\alpha) \cos(\frac{\pi}{2\alpha}) \mathbb{E}(w_{uki}^{2/\alpha}),
\]

(19b) \[
\mathbb{E}(w_{uki}^{2/\alpha}) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} w_{uki}^{2\alpha \alpha - 1} e^{-(\ln w_{uki})^2 / 2\sigma^2} dw_{uki},
\]

(19c)

where \(\text{sign}(\omega)\) is a sign function, \(\lambda_{\text{inf}}\) is the density of interfering BSs in PPP small cell networks, \(0 < \lambda_{\text{inf}} \leq \lambda_p\), \(\Gamma(\cdot)\) is a Gamma function, \(\sigma = \frac{\sigma_0}{10}\). When the inverse Fourier transform is operated at (19a), we can get the PDF of \(\mathcal{I}_{ik}\) and denote it as \(f_{\mathcal{I}_{ik}}(x)\). Thus, the PDF of the interference power received at the \(k\)-th UE in the \(i\)-th small cell of PPP small cell networks is expressed by

\[
f_{I_{ik}}^{P P P}(x) = \frac{f_{\mathcal{I}_{ik}} \left( \frac{x}{M^2 P_f P_p} \right)}{M^2 P_f P_p}.
\]

(20)

IV. ENERGY EFFICIENCY OF HCPP AND PPP SMALL CELL NETWORKS

Based on (10) and (11), the signal-to-interference-noise-ratio (SINR) received at the \(k\)-th UE in the \(i\)-th small cell of random small cell networks is expressed by

\[
\text{SINR}_{ik} = \frac{(M \sqrt{P_f P_p} \beta_{iki})^\dagger (M \sqrt{P_f P_p} \beta_{iki})}{I_{ik}} = \frac{M^2 P_f P_p \beta_{iki}^2}{M^2 P_f P_p \sum_{u \neq i, BS_u \in \Pi_{BS}} \beta_{uki}^2}.
\]

(21)

Considering the massive MIMO technology adopting at BSs, the BS transmission power is derived by

\[
P_i = x_i^\dagger x_i
\]

\[
= \left[ \sqrt{P_p} \left( H_{ii} + \sum_{u' \neq i, BS_u \in \Pi_{BS}} H_{iu'} \right)^* s_i \right]^\dagger \left[ \sqrt{P_p} \left( H_{ii} + \sum_{u' \neq i, BS_u \in \Pi_{BS}} H_{iu'} \right)^* s_i \right]
\]

\[
= P_p s_i^\dagger \sum_{BS_u \in \Pi_{BS}} H_{iu}^* H_{iu} s_i
\]

\[
= MP_p \sum_{BS_u \in \Pi_{BS}} \sum_{k=1}^K s_{ik}^\dagger s_{ik} \beta_{uki}
\]

\[
= MP_p \sum_{BS_u \in \Pi_{BS}} \sum_{k=1}^K \beta_{uki}
\]

(22)
A. Energy Efficiency of HCPP Small Cell Networks

Based on (17), the average SINR of the $k$-th UE $UE_{ik}$ in the $i$-th small cell of HCPP small cell networks is derived by

$$SINR_{ik\_avg}^{HCPP} (\|x_{int}\|) = E \left( \frac{\sqrt{P_f P_p} \beta_{iki} s_{ik}}{H_{ik}^{HCPP}} \right)^{\dag} \left( \frac{\sqrt{P_f P_p} \beta_{iki} s_{ik}}{H_{ik}^{HCPP}} \right)$$

where, (a) according to the Jensen inequality \cite{26}, the function $\frac{1}{x}$ is easy proved as a concave. Using Jensen inequality and the mean of Chi-square distribution \cite{27}, an upper bound for the average available rate of the $k$-th UE $UE_{ik}$ in the $i$-th small cell of HCPP small cell networks is given by

$$R_{ik\_avg}^{HCPP} (\|x_{int}\|) \leq \log_2 \left\{ 1 + SINR_{ik\_avg}^{HCPP} \right\}$$

Moreover, the average sum available rate of the $i$-th small cell in HCPP small cell networks is calculated by

$$R_i^{HCPP} = \sum_{k=1}^{K} R_{ik\_avg}^{HCPP}.$$
The average BS transmission power for the $k$-th UE $U_{ik}$ in the $i$-th small cell of HCPP small cell networks is given by

$$\mathbb{E}(P_{ik}^{HCPP}) = M \rho \mathbb{E}(w_{uki}) \int_{\mathbb{R}^2} ||x + x_{int}||^{-\alpha} \zeta(2)(||x||) dx.$$  \hspace{1cm} (27)

Based on the decomposition of BS power consumption \cite{30}, a linear average BS power consumption model is simply presented as follows

$$\mathbb{E}(P_{BS}^{HCPP}) = \sum_{k=1}^{K} \left[ \frac{\mathbb{E}(P_{ik}^{HCPP})}{\eta} \right] + M P_{RF_{chain}} + P_{sta},$$  \hspace{1cm} (28)

where $\eta$ is the average efficiency of signal transmission circuits and $P_{RF_{chain}}$ is the power of radio frequency circuit consumed at an antenna, $P_{sta}$ is the BS operation power fixed as a constant. Furthermore, the average energy efficiency of HCPP small cell networks is derived by

$$EE^{HCPP} = \frac{\mathbb{E}(R_{ik}^{HCPP})}{\mathbb{E}(P_{BS}^{HCPP})} = \frac{\sum_{k=1}^{K} R_{ik}^{HCPP \text{ avg}}}{\sum_{k=1}^{K} \left[ \frac{\mathbb{E}(P_{ik}^{HCPP})}{\eta} \right]} + M P_{RF_{chain}} + P_{sta}.$$  \hspace{1cm} (29)

### B. Energy Efficiency of PPP Small Cell Networks

When the distribution of BSs is assumed to be governed by a PPP, the SINR received at the $k$-th UE in the $i$-th small cell of PPP small cell networks is expressed by $SINR_{ik} = \frac{\beta_{uki}}{||x_{int}||^{2\alpha} x_{ik}}$. Moreover, the available rate of the $k$-th UE in the $i$-th small cell of PPP small cell networks is given by $R_{ik}^{PPP} = \log_2 \left( 1 + \frac{w_{uki}^2}{||x_{int}||^{2\alpha} x_{ik}} \right)$. Based on (19), the average available rate of the $k$-th UE in the $i$-th small cell of PPP small cell networks is derived by

$$R_{ik \text{ avg}}^{PPP} (||x_{int}||) = \mathbb{E}(R_{ik}^{HCPP}) = \int_x \log_2 \left( 1 + \frac{\mathbb{E}(w_{uki}^2)}{||x_{int}||^{2\alpha} x} \right) f_{\xi ik}(x) dx.$$  \hspace{1cm} (30)

Based on (22), the BS transmission power for the $k$-th UE $U_{ik}$ in the $i$-th small cell of PPP small cell networks is derived by

$$P_{ik}^{PPP} = M \rho \sum_{BS_u \in \Pi BS} \beta_{uki}.$$  \hspace{1cm} (31)

Let $\varphi_{ik} = \sum_{BS_u \in \Pi BS} \beta_{uki} = \frac{P_{PPP}}{M \rho}$, the characteristic function of $\varphi_{ik}$ is derived by

$$\phi_{\varphi_{ik}}(\omega) = \exp \left\{ -\delta_{\varphi_{ik}} |\omega|^{2/\alpha} \left[ 1 - j \text{sign}(\omega) \tan \left( \frac{\omega}{\alpha} \right) \right] \right\},$$  \hspace{1cm} (32a)
with
\[ \delta \Psi_{ik} = \lambda \pi \Gamma(2 - 2/\alpha) \cos(\pi) \frac{\mathbb{E}(u_{uki}^{2/\alpha})}{1 - 2/\alpha}. \] (32b)
\[ \mathbb{E}(u_{uki}^{2/\alpha}) = \frac{1}{\sigma \sqrt{2\pi}} \int u_{uki}^{2/\alpha-1} e^{-(\ln w_{uki})^2/2\sigma^2} dw_{uki}. \] (32c)

When the inverse Fourier transform is operated at (32a), we can get the PDF of \( \Psi_{ik} \) and denote it as \( f_{\Psi_{ik}}(x) \). Thus, the average transmission power for the \( k - th \) UE \( U_{E_{ik}} \) in the \( i - th \) small cell of PPP small cell networks is given by
\[ P_{ik}^{PPP} = \mathbb{E}(P_{ik}^{PPP}) = \int_M P_{ik} f_{\Psi_{ik}}(x) dx. \] (33)

Based on (30) and (33), the average energy efficiency of PPP small cell networks is expressed by
\[ EE^{PPP} = \frac{\mathbb{E}(R_{ik}^{PPP})}{\mathbb{E}(R_{BS}^{PPP})} = \frac{\sum_{k=1}^{K} R_{ik}^{PPP}}{\sum_{k=1}^{K} \frac{\mathbb{E}(P_{ik}^{PPP})}{\eta}} + MP_{RF_{chain}} + P_{sta}. \] (34)

To evaluate the impact of the BSs switch-off strategy on the energy efficiency of random small cell networks, the energy efficiency difference \( \Delta EE \) between HCPP and PPP small cell networks is derived by
\[ \Delta EE = EE^{HCPP} - EE^{PPP} = \frac{\sum_{k=1}^{K} R_{ik_{avg}}^{HCPP}}{\sum_{k=1}^{K} \frac{\mathbb{E}(P_{ik}^{HCPP})}{\eta}} + MP_{RF_{chain}} + P_{sta} - \frac{\sum_{k=1}^{K} R_{ik_{avg}}^{PPP}}{\sum_{k=1}^{K} \frac{\mathbb{E}(P_{ik}^{PPP})}{\eta}} + MP_{RF_{chain}} + P_{sta}. \] (35)

C. Performance Analysis

To validate the proposed energy efficiency difference, some performance analysis is simulated by numerical results in Fig. 2 to Fig. 4. Fig. 2 illustrates the energy efficiency difference with respect to the minimum active BS distance and BS density in random small cell networks with the BSs switch-off strategy. When the BS density \( \lambda_p \) is fixed, the energy efficiency difference increases with the increase of the minimum active BS distance in HCPP small cell networks. When the minimum active BS distance \( \delta \) is fixed, the energy efficiency difference decreases with the increase of the BS density in random small cell networks.
Fig. 3 shows the energy efficiency difference with respect to the active UEs number in a small cell and the minimum active BS distance in HCPP small cell networks. When the minimum active BS distance $\delta$ is fixed, the energy efficiency difference increases with the increase of the active UEs number in a small cell.

When the number of active UEs is configured as 10 in a small cell, Fig. 4 describes the energy efficiency difference with respect to the path loss coefficient and the minimum active BS distance in HCPP small cell networks. When the minimum active BS distance $\delta$ is fixed, the energy efficiency difference increases with the increase of the path loss coefficient in HCPP small cell networks.

V. COVERAGE EFFICIENCY OF HCPP AND PPP SMALL CELL NETWORKS

Considering the self-similarity characteristic of network traffic, the traffic at the UE is assumed to be governed by Pareto distributions [24]. Moreover, the PDF of traffic at the UE is expressed by

$$f_\rho(x) = \frac{\theta \rho_{\min}^\theta}{x^{\theta+1}}, x \geq \rho_{\min} > 0,$$

where $\theta \in (1, 2]$ is the heavy tail coefficient of Pareto distribution, $\rho_{\min}$ is the minimum transmission rate which satisfies the UE traffic requirement. Furthermore, the average traffic at the UE is given by

$$\mathbb{E}(\rho) = \frac{\theta \rho_{\min}}{\theta - 1}. \quad (37)$$

Without loss of generality, the coverage efficiency of small cell is defined by

$$F(T) = \mathbb{P}\left(R_{ik_{\text{avg}}} > \frac{\theta \rho_{\min}}{\theta - 1}\right) = \mathbb{P}\left(\log_2 (1 + \text{SINR}_{ik_{\text{avg}}} > \frac{\theta \rho_{\min}}{\theta - 1})\right).$$

$$= \mathbb{P}\left(\text{SINR}_{ik_{\text{avg}}} > 2^{\frac{\theta \rho_{\min}}{\theta - 1} - 1}\right). \quad (38)$$

A. Coverage Efficiency of HCPP Small Cell Networks

In this paper, the UE is assumed to be associated with the closest BS and the distance vector between the UE and the associated BS is denoted as $x_{\text{int}}$. Based on the result in [28], the PDF of the distance $||x_{\text{int}}||$ is approximated by

$$f_{||x_{\text{int}}||}(r, \text{HCPP}) \approx 2r \cdot \frac{1 - e^{-\lambda_p \pi \delta^2}}{\delta^2} \cdot e^{-\lambda \pi M(r, \delta)}, \quad (39)$$
where \( \lambda^* \) is the density of the corresponding Poisson process and the value of \( \lambda^* \) is the only one that guarantees that \( f_{\|x_{\text{int}}\|}(r, \text{HCPP}) \) is a proper PDF, i.e., it is the unique value \( \lambda^* \) such that \( \int_0^\infty 2r \cdot \frac{1-e^{-\lambda^* r^2}}{\delta^2} \cdot e^{-\lambda^* M(r, \delta)} \, dr = 1 \). \( \delta \) also donates the coverage radius of the BS and the distance between the UE and the associated BS is denoted as \( r \). The UE coverage area is configured as a circle area with the radius \( r \). When the UE is located at the edge of cell, the UE coverage area without overlapping with the BS coverage area is denoted as \( M(r, \delta) \), which is illustrated in Fig. 5. Moreover, \( M(r, \delta) \) is expressed by

\[
M(r, \delta) = \left\{ \begin{align*}
0, & \quad \frac{\delta}{2} \geq r > 0 \\
\pi r^2 - (2 \arcsin \frac{\delta}{2r} + \arccos \frac{\delta}{2r}) r^2 + \delta \sqrt{r^2 - \frac{\delta^2}{4}}, & \quad r > \frac{\delta}{2}.
\end{align*} \right.
\]  

(40)

From (23), the average SINR of the \( k-th \) UE \( UE_{ik} \) in the \( i-th \) small cell of HCPP small cell networks can be denoted as a function of \( \|x_{\text{int}}\| \), i.e., \( SINR_{ik_{\text{avg}}}^{\text{HCPP}}(\|x_{\text{int}}\|) \). Moreover, the inverse function of \( SINR_{ik_{\text{avg}}}^{\text{HCPP}}(\|x_{\text{int}}\|) \) is denoted as \( \Theta_{\text{HCPP}}^{\text{HCPP}}(SINR_{ik_{\text{avg}}}^{\text{HCPP}}(\|x_{\text{int}}\|)) \). Let \( \gamma^{\text{HCPP}} = SINR_{ik_{\text{avg}}}^{\text{HCPP}}, \) the PDF of \( SINR_{ik_{\text{avg}}}^{\text{HCPP}}(\|x_{\text{int}}\|) \) is expressed by

\[
\int_{SINR_{ik_{\text{avg}}}^{\text{HCPP}}(\gamma^{\text{HCPP}}, \text{HCPP})} f_{SINR_{ik_{\text{avg}}}^{\text{HCPP}}}(\gamma^{\text{HCPP}}, \text{HCPP}) = \int_{\|x_{\text{int}}\|} \left[ \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}, \text{HCPP}), \text{HCPP} \right] \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}) \, d\gamma^{\text{HCPP}}.
\]  

(41)

where \( \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma) \) is the derivative of \( \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma) \).

Based on (38) and let \( T = 2 \frac{\delta_{\text{min}}}{\kappa} - 1 \), the coverage efficiency of HCPP small cell networks is expressed by

\[
F^{\text{HCPP}}(T) = \mathbb{P}(SINR_{ik_{\text{avg}}}^{\text{HCPP}} > T).
\]  

(42)

When (41) is substituted into (42), the coverage efficiency of HCPP small cell networks is extended by

\[
F^{\text{HCPP}}(T) = \int_{T}^{\infty} \int_{\|x_{\text{int}}\|} \left[ \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}, \text{HCPP}), \text{HCPP} \right] \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}) \, d\gamma^{\text{HCPP}}
\]

\[
= \int_{T}^{\infty} \left[ 2 \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}) \cdot \frac{1 - e^{-\lambda^* \pi \delta^2}}{\delta^2} \cdot e^{-\lambda^* M(\Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}), \delta)} \right] \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}) \, d\gamma^{\text{HCPP}}
\]

\[
= \int_{T}^{\infty} \left[ e^{-\lambda^* (\Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}))^2 - (2 \arcsin \frac{\delta}{2 \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}})} + \arccos \frac{\delta}{2 \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}})})} \cdot \frac{1 - e^{-\lambda^* \pi \delta^2}}{\delta^2} \right] \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}) \, d\gamma^{\text{HCPP}}
\]

\[
= \int_{T}^{\infty} \left[ e^{-\lambda^* (\Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}))^2 - (2 \arcsin \frac{\delta}{2 \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}})} + \arccos \frac{\delta}{2 \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}})})} \cdot \frac{1 - e^{-\lambda^* \pi \delta^2}}{\delta^2} \right] \Theta_{\text{HCPP}}^{\text{HCPP}}(\gamma^{\text{HCPP}}) \, d\gamma^{\text{HCPP}}
\]  

(43)
B. Coverage Efficiency of PPP Small Cell Networks

When the distribution of BSs is assumed to follow PPP, the PDF of the distance between the UE and the desired BS is given by

\[ f_{\|x\|} (r, \text{PPP}) = 2\pi r \lambda_p e^{-\lambda_p \pi r^2}. \]  

(44)

The average SINR of the \( k \)-th UE \( U_{ik} \) in the \( i \)-th small cell of PPP small cell networks is expressed by

\[ \text{SINR}_{ik_{avg}}^{\text{PPP}} (\|x_{\text{int}}\|) = \mathbb{E} \left( \frac{w_{ik}^2}{\|x_{\text{int}}\|^{2\alpha} f_{ik}} \right) = \mathbb{E} \left( \frac{w_{ik}^2}{\|x_{\text{int}}\|^{2\alpha}} \right) \int_0^\infty \frac{1}{x} f_{ik} (x) dx \]  

\[ = \frac{\mathbb{E} (w_{ik}^2)}{\|x_{\text{int}}\|^{2\alpha}} \int_0^\infty \frac{1}{x} f_{ik} (x) dx. \]  

(45)

Let \( \gamma^{\text{PPP}} = \text{SINR}_{ik_{avg}}^{\text{PPP}} \), the coverage efficiency of PPP small cell networks is derived by

\[ F_{\text{PPP}} (T) = \mathbb{P} \left( \text{SINR}_{ik_{avg}}^{\text{PPP}} > T \right) \]

\[ = \int_T^\infty f_{\text{SINR}_{ik_{avg}}^{\text{PPP}} (\gamma^{\text{PPP}}, \text{PPP})} d\gamma^{\text{PPP}}, \]  

(46a)

\[ = \int_T^\infty f_{\|x_{\text{int}}\|} \left[ \mathcal{G}^{\text{PPP}} (\gamma^{\text{PPP}}, \text{PPP}) \right] \left| \mathcal{G}'^{\text{PPP}} (\gamma^{\text{PPP}}) \right| d\gamma^{\text{PPP}}, \]

\[ = \int_T^\infty 2\pi \mathcal{G}^{\text{PPP}} (\gamma^{\text{PPP}}) \lambda_p e^{-\lambda_p \pi \mathcal{G}^{\text{PPP}} (\gamma^{\text{PPP}})^2} \mathcal{G}'^{\text{PPP}} (\gamma^{\text{PPP}}) d\gamma^{\text{PPP}}, \]

with

\[ \mathcal{G}^{\text{PPP}} (\gamma^{\text{PPP}}) = \left[ \mathbb{E} (w_{ik}^2) \right]^{\frac{1}{2\alpha}} \left[ \int_0^\infty \frac{1}{x} f_{ik} (x) dx \right]^{\frac{1}{2\alpha}}, \]  

(46b)

\[ \mathcal{G}'^{\text{PPP}} (\gamma^{\text{PPP}}) = -\frac{1}{2\alpha} (\gamma^{\text{PPP}})^{-\frac{1}{2\alpha}} \mathcal{G}^{\text{PPP}} (\gamma^{\text{PPP}}), \]  

(46c)

where \( \mathcal{G}^{\text{PPP}} (\gamma^{\text{PPP}}) \) is the inverse function of \( \text{SINR}_{ik_{avg}}^{\text{PPP}} (\|x_{\text{int}}\|) \), \( \mathcal{G}'^{\text{PPP}} (\gamma^{\text{PPP}}) \) is the derivative of \( \mathcal{G}^{\text{PPP}} (\gamma^{\text{PPP}}) \).

C. Numerical Results

To validate the proposed coverage efficiency of HCPP and PPP small cell networks, numerical simulations are analyzed in Fig. 6–Fig. 8. Fig. 6 illustrates the coverage efficiency with respect to the SINR threshold and two types of random small cell networks. When the type of random small
cell network is fixed, the coverage efficiency decreases with the increase of the SIR threshold. When the SINR threshold is fixed, the coverage efficiency of PPP small cell networks is larger than the coverage efficiency of HCPP small cell networks. When the SINR threshold is fixed, the coverage efficiency decreases with the increase of the minimum active BS distance in HCPP small cell networks.

Fig. 7 shows the coverage efficiency with respect to the path loss coefficients considering different types of random small cell networks. When the SINR threshold is fixed, Fig. 7(a) depicts that the coverage efficiency decreases with the increase of the path loss coefficient in PPP and HCPP small cell networks. Fig. 7(b) describes the difference of the coverage efficiency between PPP and HCPP small cell networks with respect to the SINR threshold and the path loss coefficient. When the path loss coefficient is fixed, the difference of the coverage efficiency between PPP and HCPP small cell networks decreases with the increase of the SINR threshold. When the SINR threshold is fixed, the difference of the coverage efficiency between PPP and HCPP small cell networks increases with the increase of the path loss coefficient.

Fig. 8 describes the coverage efficiency with respect to the intensity of BSs considering different types of random small cell networks. When the SINR threshold is fixed, Fig. 8(a) illustrates that the coverage efficiency decreases with the increase of the intensity of BSs. When the SINR threshold is fixed, Fig. 8(b) exhibits that the difference of the coverage efficiency between PPP and HCPP small cell networks decreases with the increase of the intensity of BSs.

VI. OPTIMAL ENERGY EFFICIENCY WITH COVERAGE EFFICIENCY CONSTRAINTS

Based on above results, the energy efficiency of random small cell networks can be improved and the coverage efficiency of random small cell networks have to be reduced when the BSs switch-off strategy is adopted. For the telecommunication providers, the coverage efficiency of small cell networks must keep a high level considering the user's experiences. Therefore, it is an important issue how to optimize the energy efficiency of small cell networks with a specified coverage efficiency constraint. In this paper, we try to adjust the minimum active BS distance to trade off the energy and coverage efficiency of small cell networks.

To investigate the energy efficiency of small cell networks with coverage efficiency constraints, the coverage density of HCPP small cell network is given by [29]

\[
F_d^{HCPP}(\zeta^{(1)}, T) \triangleq \zeta^{(1)} \cdot F^{HCPP}(T). 
\]
Furthermore, the energy density efficiency of HCPP small cell networks is expressed by
\[ EE_d^{HCPP}(\zeta^{(1)}, T) \triangleq EE^{HCPP} \cdot F_d^{HCPP}(\zeta^{(1)}, T). \] (48)

When the BS switch-off strategy is adopted for small cell networks, the optimized energy
density efficiency of HCPP small cell networks with coverage efficiency constraints \( T_{sh} \) is
formulated by
\[
\begin{align*}
\text{maximize } & \quad EE_d^{HCPP} = F^{HCPP}(T)\zeta^{(1)} \cdot \frac{E(B^{HCPP})}{E(P^{HCPP})} = \frac{F^{HCPP}(T)\zeta^{(1)} \cdot \sum_{k=1}^{K} R_{ik_{avg}}^{HCPP}}{\sum_{k=1}^{K} \left[ E(P^{HCPP}) \eta \right] + MP_{RF_{chain}} + P_{sta}}. \\
\text{subject to } & \quad F^{HCPP}(T) \geq T_{sh}.
\end{align*}
\] (49)

Since the expression of \( F^{HCPP}(T) \) in (43) is very complex, it is difficult to derive an analytical
solution for \( \delta \). As a consequence, we have to analyze the optimal minimum active BS distance
\( \delta \) by simulation results.

Fig. 9 expresses the coverage efficiency with respect to the minimum active BS distance \( \delta \) and
the SINR threshold in HCPP small cell networks. There exists a maximum coverage efficiency
in Fig. 9 when the minimum active BS distance is in the range of 40 to 50 meters. When the
SINR threshold and the coverage efficiency threshold are configured as -10 dB and 0.8, the
minimum active BS distance \( \delta \) is fallen into the range of 50 to 140 meters. When the SINR
threshold and the coverage efficiency threshold are configured as -5 dB and 0.78, the minimum
active BS distance \( \delta \) is fallen into the range of 45 to 100 meters. When the SINR threshold
and the coverage efficiency threshold are configured as 0 dB and 0.75, the minimum active BS
distance \( \delta \) is fallen into the range of 40 to 120 meters.

Based on results in Fig. 9, the coverage efficiency can be kept a high level, i.e., larger than
or equal to 0.75 when the minimum active BS distance \( \delta \) is ranged from 40 to 150 meters
considering different SINR thresholds. Furthermore, the energy density efficiency of HCPP small
cell networks with respect to the minimum active BS distance \( \delta \) considering different SINR
thresholds is illustrated in Fig. 10. When the minimum active BS distance is fixed, the energy
density efficiency of HCPP small cell networks decreases with the increase of the SINR threshold.
When the SINR threshold is fixed, the energy density efficiency of HCPP small cell networks
increases with the increase of the minimum active BS distance. Moreover, the energy density
efficiency of HCPP small cell networks approaches to the stationary value when the minimum active BS distance is larger than 150 meters.

Based on results in Fig. 9 and Fig. 10, the increasing of the minimum active BS distance $\delta$ conduces to the contradictory effect on the coverage efficiency and the energy density efficiency of HCPP small cell networks when the BSs switch-off strategy is adopted. Hence, the coverage efficiency with respect to the energy density efficiency considering different SINR thresholds is analyzed in Fig. 11. Fig. 11 shows that there exist obvious knee points of the coverage efficiency when the energy density efficiency is larger than 6 bits/Hz/Joule/$Km^2$. Based on simulation results, the minimum active BS distance corresponding knee points is 150 meters. Therefore, the optimal minimum active BS distance is recommended as 150 meters to trade off the energy and coverage efficiency of HCPP small cell networks.

VII. CONCLUSION

Considering the BSs switch-off strategy, the energy and coverage efficiency of HCPP and PPP small cell networks are analyzed and compared in this paper. Based on our results, there exists a trade-off between the energy and the coverage efficiency of HCPP small cell networks. Moreover, the optimized energy density efficiency of HCPP small cell networks with coverage efficiency constraints is presented. Simulation results imply that the optimal minimum active BS distance is 150 meters to trade off the energy and coverage efficiency of 5G small cell networks with the BSs switch-off strategy. For the future study, the requirement of user QoS could be considered to further optimize the energy and coverage efficiency of 5G small cell networks.

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Fig. 1. the BSs distribution in a small cell network with and without the BSs switch-off strategy. (a) The BSs distribution is governed by a PPP before a BS switch-off strategy is adopted in a small cell network. (b) The BSs distribution is governed by a HCPP after a BS switch-off strategy is adopted and the minimum active BS distance is configured as $\delta = 120$ meters in a small cell network.

Fig. 2. Energy efficiency difference with respect to the minimum active BS distance and BS density in random small cell networks with the BSs switch-off strategy.
Fig. 3. Energy efficiency difference with respect to the active UEs number in a small cell and the minimum active BS distance in HCPP small cell networks.

Fig. 4. Energy efficiency difference with respect to the path loss coefficient and the minimum active BS distance in HCPP small cell networks.
Fig. 5. UE coverage area without overlapping with the BS coverage area.

Fig. 6. Coverage efficiency with respect to the SINR threshold and two types of random small cell networks.
Fig. 7. Coverage efficiency with respect to the path loss coefficients considering different types of random small cell networks. (a) Coverage efficiency with respect to the path loss coefficients considering PPP and HCPP small cell networks. (b) Difference of the coverage efficiency between PPP and HCPP small cell networks with respect to the SINR threshold and the path loss coefficient.
Fig. 8. Coverage efficiency with respect to the intensity of BSs considering different types of random small cell networks. (a) Coverage efficiency with respect to the intensity of BSs considering PPP and HCPP small cell networks. (b) Difference of the coverage efficiency between PPP and HCPP small cell networks with respect to the SINR threshold and the intensity of BSs.
Fig. 9. Coverage efficiency with respect to the minimum active BS distance and the SINR threshold in HCPP small cell networks.

Fig. 10. Energy density efficiency of HCPP small cell networks with respect to the minimum active BS distance considering different SINR thresholds.
Fig. 11. Coverage efficiency with respect to the energy density efficiency considering different SINR thresholds in HCPP small cell networks.