ROTATION OF COSMIC VOIDS AND VOID SPIN STATISTICS

JOUNGHUN LEE AND DAESEONG PARK

Department of Physics and Astronomy, Frontier Physics Research Division (FPRD), Seoul National University, Seoul 151-747, South Korea; jounghun@astro.snu.ac.kr, pds2001@astro.snu.ac.kr

Received 2006 June 20; accepted 2006 July 25

ABSTRACT

We present a theoretical study of void spins and their correlation properties. The concept of the spin angular momentum for an unbound void is introduced to quantify the effect of the tidal field on the distribution of matter that makes up the void. Both the analytical and numerical approaches are used for our study. Analytically, we adopt the linear tidal-torque model to evaluate the void spin-spin and spin-density correlations, assuming that a void forms in the initial region where the inertia momentum and the tidal shear tensors are maximally uncorrelated with each other. Numerically, we use the Millennium Run galaxy catalog to find voids and calculate their spin statistics. The numerical results turn out to be in excellent agreement with the analytic predictions, both of which consistently show that there are strong spatial alignments between the spin axes of neighboring voids and strong antialignments between the void spin axes and the directions to the nearest voids. We expect that our work will provide a deeper insight into the origin and properties of voids and the large-scale structure.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION

Recent galaxy redshift surveys (e.g., 2dFGRs; Colless et al. 2001) have allowed us to map the universe on the largest scale ever and systematically study the detailed properties of cosmic large-scale structures. It is confirmed by these surveys that the galaxies in the universe are not distributed evenly, but develop a pattern of networks connected by filaments and sheets, which in turn are separated by immense voids.

Voids are defined observationally as large regions with very low galaxy number density and are shown to occupy approximately 40% of the cosmic volume (Hoyle & Vogeley 2004). The striking vastness of the observed void volumes inspired a flurry of research on the origin and properties of cosmic voids (El-Ad & Piran 1997; Schmidt et al. 2001; Peebles 2001; Mathis & White 2002; Benson et al. 2003; Sheth & van de Weygaert 2004; Hoyle & Vogeley 2004; Rojas et al. 2004; Colberg et al. 2005; Hoyle et al. 2005; Gottlöber et al. 2003; Shandarin et al. 2006). It is now generally believed that voids originate from the local minima of the primordial density fluctuations and expand faster than the rest of the universe.

The very fact that voids are extremely underdense and expand faster has led many authors to assume somewhat naively that the shapes of voids should be spherical. In fact, previous analytical and numerical works on voids are based on this assumption (e.g., Dubinski et al. 1993; Van de Weygaert & Van Kampen 1993; Friedmann & Piran 2001; Sheth & van de Weygaert 2004).

Very recently, however, Shandarin et al. (2006) systematically investigated the shapes of voids by applying the excursion set approach to the density field in N-body simulations and demonstrated that the intrinsic shapes of voids are far from spherical symmetry. They claimed that voids are most vulnerable to external shears due to their low density, so that their shapes are disturbed to be nonspherical, in spite of their faster expansion.

Although Shandarin et al. (2006) focused on the shapes and morphology of voids, we note here that if the tidal shear plays a significant role in the case of voids, then it should not only cause the shapes of voids to be nonspherical, but also cause the voids to acquire spin angular momentum. Our goal here is to construct an analytic model for the spin angular momentum of voids originating from the tidal shear effect and test the analytic predictions against the data from the recent Millennium Run simulation.

The organization of this paper is as follows. In § 2, the analytic models for the void spin angular momentum and the void spin correlation properties are presented. In § 3, the Millennium Run galaxy catalog is analyzed to determine void spin statistics, and the numerical results are compared with the analytic predictions. In § 4, the achievements and caveats of our work are discussed, and a final conclusion is drawn.

2. THE ANALYTIC MODEL

Let us consider a Lagrangian region in the linear smoothed density field. According to the linear tidal-torque theory (Doroshkevich 1970; White 1984), the spin angular momentum of matter that makes up this region, \( J \equiv \langle I_\ell \rangle \), is proportional to the tensor product as

\[
J_\ell \propto \epsilon_{ijk} T_{\mu \nu} I_{\mu \nu},
\]

where \( I \equiv \langle I_\ell \rangle \) is the inertia tensor of the region, and \( T \equiv \langle T_{\mu \nu} \rangle \) is the local tidal tensor at the center of the mass of the region defined as the second derivative of the perturbation potential, \( T_{\mu \nu} \equiv \partial_\mu \partial_\nu \Phi \).

Strictly speaking, equation (1) gives a valid first-order approximation to the spin angular momentum only when the Lagrangian regions correspond to bound halos. However, to quantify the effect of tidal fields on voids, we extend the validity of the linear tidal-torque theory to the case of underdense protovoids and define the spin angular momentum of a void by equation (1).

Since a void is an unbound system, its spin angular momentum has a meaning distinct from that of a bound halo, although defined in a similar way. Basically, the spin angular momentum of a void is introduced to quantify how a protovoid region reacts to the effect of the local tidal force. The local tidal field will generate nonradial motion of the material in the protovoid region, leading the distribution of matter in the void to deviate from spherical symmetry. The magnitude of the void spin angular momentum quantifies the degree of this deviation, and its direction indicates the direction along which the maximum deviation occurs.
It has been shown by \(N\)-body simulations that in the overdense protohalo regions, the principal axes of \(T\) are in fact strongly correlated with those of \(\mathbf{T}\) (Lee & Pen 2000; Porciani et al. 2002). Very recently, Lee (2006) has shown analytically that the degree of the misalignment between the principal axes of \(I\) and \(\mathbf{T}\) decreases monotonically with the linear density of the region. In other words, the more underdense a region is, the more misaligned the principal axes of \(I\) and \(\mathbf{T}\) are.

Given these numerical and analytical findings, we propose a new hypothesis that a void forms from the initial underdense region, where the principal axes of \(I\) and \(\mathbf{T}\) are maximally misaligned with each other. This hypothesis combined with equation (1) leads us to expect that the alignment between the void spin axes and the principal axes of the local tidal tensor should be strongest for the protovoid regions.

Lee & Pen (2000) showed that the alignment between the spin axis of a Lagrangian region and the intermediate principal axes of the local tidal tensor can be best quantified by calculating the expectation value of the unit spin given the tidal tensor, and they derived an approximate formula for it. Their original formula is characterized by a correlation parameter, \(c\), in the range of \([0, 1]\), which was introduced to represent the degree of the alignment \(^1\) between the spin axes and the principal axes of the tidal tensors. The case of maximum alignment corresponds to \(c = 1\), while the no-alignment case corresponds to \(c = 0\). For the case of halo spins, they found that \(c \approx 0.3\), empirically. In accordance with our hypothesis, \(c = 1\) for void spins.

Now, with the value of \(c\) set at unity, the formula of Lee & Pen (2000) is rewritten for the case of void spins as

\[
\langle \tilde{j}_x, \tilde{j}_y | \hat{T} \rangle = \left( \frac{1}{3} + \frac{1}{5} \right) \delta_{ij} - \frac{3}{5} \hat{T}_{ik} \hat{T}_{kj},
\]

where \(\tilde{j} \equiv j/|j|\) and \(\hat{T} \equiv T/|T|\), with \(\hat{T}_{ij} \equiv T_{ij} - \text{Tr}(T) \delta_{ij}/3\). Equation (2) indicates that the spin axes of voids are spatially correlated due to the spatial correlation of the tidal field. The alignments between the spin axes of neighbor void spin can be quantified by the two-point spin-spin correlation function, which was introduced by Pen et al. (2000) for the case of dark halos:

\[
\eta_V(r) = \langle |\tilde{j}(x) \cdot \tilde{j}(x + r)|^2 \rangle - 1/3.
\]

For the case of power-law power spectrum, the void spin-spin correlation function, \(\eta_V(r)\), can be approximated (Pen et al. 2000) as

\[
\eta_V(r) \approx \frac{3}{50} \xi_{\delta}^2(r) / \xi_{\delta}^2(0).
\]

Here \(\xi_{\delta}\) is the correlation function of the linear density field smoothed on a typical Lagrangian scale of voids, defined as

\[
x_{\delta}(r) \equiv \int_{-\infty}^{\infty} \Delta^2(k) \frac{\sin kr}{kr} W^2(kR_v) d \ln k,
\]

where \(\Delta^2(k)\) is the dimensionless power spectrum, and \(W(kR_v)\) is the top-hat spherical filter of scale radius, \(R_v\). Since voids and galaxy clusters can be thought of as the counterparts that are supposed to form at the local minima and maxima of the initial density field, respectively, we assume that the typical Lagrangian scale of voids is the same as that of galaxy clusters, setting the value of \(R_v\) at \(8 \, h^{-1} \, \text{Mpc}\). Since the concordance \(\Lambda\)CDM cosmology is well represented by a power-law power spectrum \(\Delta^2(k) \propto k\) on the scale of \(8 \, h^{-1} \, \text{Mpc}\), one can regard equation (3) as a valid approximation.

It is worth mentioning here that the misalignment between the principal axes of \(\mathbf{T}\) and \(\hat{T}\) (i.e., the value of the correlation parameter \(c = 1\)) is characteristic for those underdense regions in the initial density field that will eventually become voids. Although the majority of the initial underdense regions will evolve into voids, some underdense regions could merge into overdense regions and collapse, rather than become voids (Colberg et al. 2005).

Lee & Pen (2001) showed both analytically and numerically that for the case of dark halos, the separation vectors between the neighbor halos, \(r \equiv (r_i)\), are aligned with the major axes of the tidal tensors. To quantify the degree of this alignment, Lee & Pen (2001) proposed the following formula:

\[
\langle \hat{r}_x \hat{r}_y | \hat{T} \rangle = \frac{1}{3} \delta_{ij} - b \left( \frac{1}{3} \delta_{ij} - \hat{T}_{ik} \hat{T}_{kj} \right),
\]

where \(\hat{r} \equiv r/|r|\), and \(b\) is a correlation parameter in the range of \([0, 1]\). The negative sign in front of the parameter \(b\) indicates that \(\hat{r}\) is aligned with the major principal axis of \(\mathbf{T}\), unlike \(J\). The value of \(b\) was empirically determined to be approximately 0.3 for the case of halos (Lee & Pen 2001).

Similarly, we expect that the separation vectors between the neighbor voids are aligned with the minor axes of the tidal tensors. Then equation (6) should also be true for the case of voids. We assume further that \(b\) has the same value of 0.3.

Now that the spin axes of voids are aligned with the intermediate axes of the local tidal tensors (eq. [2]), while the separation vectors to the nearest voids are aligned with the minor axes of the local tidal tensors, one can expect that the spin axes of voids should be antialigned with the separation vectors to the nearest voids.

The expected spin-direction antialignment can be quantified by the correlation function (Lee & Pen 2001)

\[
\omega_V(r) = \langle |\tilde{j} \cdot \hat{r}|^2 \rangle - 1/3.
\]

It was found by Lee & Pen (2001) that for the case of small-scale galactic halos, equation (8) can be approximated as \(\omega_V \approx 0\), where \(R\) is the typical Lagrangian size of a galactic halo, and \(R' \equiv R + r\). This approximation was made under the assumption that the tidal field does not vary significantly within the Lagrangian distance \(r\). For large-scale voids, however, this assumption may not be valid, since within the void separation the tidal field is likely to vary significantly. Taking into account the spatial variance of the tidal field, we find that for the case of voids, the spin-direction correlation function is better approximated as

\[
\omega_V(r) \approx -\frac{3}{100} \xi_{\delta}^2(r) / \xi_{\delta}^2(0).
\]

3. RESULTS FROM SIMULATIONS

3.1. Identifying Voids

To test the analytic predictions made in § 2, we use the Millennium Run galaxy catalog carried out by the Virgo Consortium (Springel et al. 2005). The catalog consists of a total of

\(^1\) In the original formula, Lee & Pen (2000) used a rescaled correlation parameter, \(a\), which is nothing but \(3/5\) times \(c\).
8,964,936 galaxies at redshift $z = 0$ in a periodic box of linear size $500 \, h^{-1} \, \text{Mpc}$, with information on various galaxy properties such as position, velocity, mass, magnitude, and star formation rate.

Voids are identified in the catalog by the void-finding algorithm described in Hoyle & Vogeley (2002, hereafter HV02), which was in fact first suggested by El-Ad & Piran (1997). To apply the HV02 algorithm to the Millennium Run catalog, we first determine the values of the key parameters, $l$ and $s_c$; $l$ represents the distance within which a field galaxy is required to not have any three neighbors, and $s_c$ represents a threshold for the minimum linear size of a void. For the Millennium Run catalog, the key parameter values we obtain are $l = 2.44 \, h^{-1} \, \text{Mpc}$ and $s_c = 6 \, h^{-1} \, \text{Mpc}$, respectively. For a detailed description of the procedures to identify voids from the Millennium Run catalog, see our companion paper (D. Park & J. Lee 2006, in preparation).

A total of 24,037 voids are found in the catalog. The volume $V$ of each void is measured by means of the Monte Carlo integration method described in HV02. The effective radius of each void is then determined as $R_e \equiv (3V/4\pi)^{1/3}$. The mean value of $R_e$ is found to be $R_e = 10.45 \, h^{-1} \, \text{Mpc}$.

The density of each void, $\delta_V$, is measured by counting the number of void galaxies; $\delta_V \equiv (n_V - \bar{n}_g)/\bar{n}_g$, where $n_V \equiv N/V$ is the number density of void galaxies, and $\bar{n}_g$ is the mean number density of the total catalog galaxies. Figure 1 plots the probability distribution of $\delta_V$. As can be seen, the voids found in the catalog are extremely underdense, with the mean density $\bar{\delta}_V = -0.92$.

### 3.2. Measuring the Void-Spin Angular Momentum

We first choose only those voids that contain more than 30 void galaxies. The total number of such voids is found to be 13,507. The specific spin angular momentum, $j$, of each selected void is determined as

$$j = \frac{1}{M_V} \sum_{\alpha} m_\alpha r_\alpha \times v_\alpha,$$

where $m_\alpha$ and $M_V \equiv \sum_{\alpha} m_\alpha$ are the mass of the $\alpha$th galaxy and the total mass of all galaxies in each void, respectively. The position and the velocity of the $\alpha$th void galaxy in the center of the
mass frame are represented by $r_s$ and $v_s$, respectively. We rescale this specific spin angular momentum to be dimensionless as

$$\tilde{j} = \frac{j}{\sqrt{2M \cdot R_e V}},$$

with $V^2 = GM_e/R_e$, which is analogous to the definition of the halo spin parameter (Bullock et al. 2001). Figure 2 illustrates a few examples of the voids identified from the catalog, with the directions of the void spin axes marked as short solid lines on the voids.

Figure 3 plots the distribution of $\tilde{j}$ for the voids from the catalog as histograms, which shows that $p(\tilde{j})$ has a lognormal shape. We fit the following lognormal formula to the histogram, adjusting the values of $\sigma_j$ and $\tilde{j}_0$:

$$p(\tilde{j}) = \frac{1}{j\sqrt{2\pi}\sigma_j} \exp \left[ -\frac{\ln^2(\tilde{j}/\tilde{j}_0)}{2\sigma_j^2} \right].$$

The best-fit values of $\sigma_j$ and $\tilde{j}_0$ are found to be 0.72 and 0.91, respectively. As can be seen, the value of $\tilde{j}$ spreads over a large range, revealing the strong shear effect on voids.

3.3. Determining the Void Spin-Spin and Spin-Density Correlations

We measure the cosine of the relative angle, $\cos \theta$, between the spin axes of each void pair in the catalog and determine the average of $\cos^2 \theta$ as a function of the separation distance between the two voids. For this, we also use only those voids that have more than 30 galaxies. The two-point spin-spin correlation function (eq. [3]) is then determined simply by subtracting one-third from the average, $\langle \cos^2 \theta \rangle$.

Figure 4 plots the numerical result as solid squares with errors. The errors are calculated as the standard deviation of $\cos^2 \theta$ for the case of no correlation. The dotted line corresponds to the case of no alignment. As can be seen, there is a clear signal of alignment up to the distance of $20 \ h^{-1} \ \text{Mpc}$, which is almost an order of magnitude stronger compared with the case of halos. The analytic prediction (eq. [4]) is also plotted as a solid line for comparison. As can be seen, the analytic prediction is in excellent agreement with the numerical result. It is worth emphasizing that the analytic prediction is not a fitting model but is derived first from physical principles, under the assumption that voids form in the initial regions, where the tidal and inertia tensors are maximally uncorrelated with each other. The good agreement between the numerical result and the analytical prediction supports this idea.

In a similar manner, we also determine the void spin-direction correlation function (eq. [7]). Figure 5 plots the numerical result as solid squares with errors. As can be seen, there is a clear signal of antialignment between the void spin axes and the direction toward the nearest neighbor voids, which is also an order of magnitude stronger compared with the case of halos. The analytic prediction (eq. [8]) is also plotted as a solid line for comparison.

We also find a good agreement between the analytical and numerical results for the spin-direction correlation.

4. DISCUSSION AND CONCLUSION

In light of the work of Shandarin et al. (2006), who claimed that the tidal shear must play a strong role in case of voids, we have
proposed a scenario in which voids originate from the regions in the initial density field, whose inertia tensors are completely uncorrelated with the local tidal tensors. Since the alignment between the principal axes of the inertia and tidal tensors plays a role in reducing the tidal shear effect according to the linear tidal-torque theory, the hypothesis implies that voids should have very high spin angular momentum. We define the concept of the spin angular momentum for an unbound void to quantify the effect of the tidal field on the distribution of matter that makes up voids.

Our analytic model based on the linear tidal-torque theory predicts that there is a strong spatial alignment between the spin axes of neighbor voids and a strong antialignment between the spin axes of voids and the directions to the nearest voids.

Our predictions have been tested against the Millennium Run galaxy catalog, where 13,507 large voids with more than 30 galaxies are identified. We have numerically measured the distribution and correlation functions of void spins and found that the predictions of our analytic model are in excellent agreement with the numerical results.

The success of our work is, however, subject to a couple of caveats. The first caveat comes from our implicit assumption that the matter voids are the same as the galaxy voids. In our analytic model, we consider the matter voids. While in the numerical analysis, we deal with the galaxy voids. Although the two types of voids are not necessarily the same, they should at least be closely correlated with each other.

The second caveat lies in the fact that there is no standard way to identify voids. Unlike the case of halo finding, the use of different algorithms could produce different results for some properties of voids. In particular, the shapes and volumes of voids can differ significantly by the underlying criterion of the void-finding algorithm. If one uses an algorithm that regards a void as a spherical region of low density, then all voids found by that algorithm will be spherical. In spite of this ambiguity in finding voids, we expect that our results on the correlation properties of the void spin angular momentum would not be changed significantly by the different choice of a void finding algorithm, since the correlation properties are basically determined, not by the shapes of voids, but by the galaxies in voids.

Another issue we would like to discuss here is the possibility of numerically testing our key assumption that a void forms in the initial underdense region, where the misalignment between the principal axes of the inertia and the tidal tensors becomes maximum. Although Lee (2006) has already shown analytically that our assumption is true, it would be highly desirable to test it against numerical data from the Millennium Run simulation. For this, it would be necessary to find the Lagrangian positions of all the particles that make up each void and to determine the inertia momentum tensor of each Lagrangian protovoid region in the principal axis frame of the initial tidal field. As the initial density field of the Millennium Run simulation is not yet available to the public, the numerical test of our assumption is postponed. We hope to report the test results sometime in the future.

Finally, we conclude that our work on void spins will provide a deeper insight into the origin and properties of the large-scale structure of the universe.

We thank the anonymous referee for helpful suggestions. The Millennium Run simulation used in this paper was carried out by the Virgo Supercomputing Consortium at the Computing Center of the Max-Planck Society in Garching.² This work is supported by the research grant R01-2005-000-10610-0 from the Basic Research Program of the Korea Science and Engineering Foundation.

² The semianalytic galaxy catalogue is publicly available at http://www.mpa-garching.mpg.de/galform/agnpaper.