Correlation functions for a di-neutron condensate in asymmetric nuclear matter

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Recent calculations with an effective isospin dependent contact interaction show the possibility of the crossover from superfluidity of neutron Cooper pairs in $^1S_0$ pairing channel to Bose-Einstein condensation (BEC) of di-neutron bound states in dilute nuclear matter. The density and spin correlation functions are calculated for a di-neutron condensate in asymmetric nuclear matter with the aim to find the possible features of the BCS-BEC crossover. It is shown that the zero-momentum transfer spin correlation function satisfies the sum rule at zero temperature. In symmetric nuclear matter, the density correlation function changes sign at low momentum transfer across the BCS-BEC transition and this feature can be considered as a signature of the crossover. At finite isospin asymmetry, this criterion gives too large value for the critical asymmetry $\alpha^c_2 \sim 0.9$, at which the BEC state is quenched. Therefore, it can be trusted for the description of the density-driven BCS-BEC crossover of neutron pairs only at small isospin asymmetry. This result generalizes the conclusion of the study in Phys. Rev. Lett. 95, 090402 (2005), where the change of sign of the density correlation function at low momentum transfer in two-component quantum fermionic atomic gas with the balanced populations of fermions of different species was considered as an unambiguous signature of the BCS-BEC transition.

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I. INTRODUCTION

Few-body correlations play an important role in low-density nuclear systems such as expanding nuclear matter in heavy ion collisions, the tails of nuclear density distributions in exotic nuclei, or nuclear matter in the crust of a neutron star. Feasible low-density phenomena include $\alpha$-particle condensation \cite{1}, the formation of two-body (deuteron) \cite{2} and three-body (triton) \cite{3} bound states, and their possible coexistence/competition \cite{4}. Of special interest also is the crossover from superfluidity of nucleon Cooper pairs to Bose-Einstein condensation (BEC) of two-body bound states in dilute nuclear matter. The transition from large overlapping Cooper pairs to tightly bound pairs of fermions can be described on the basis of BCS theory, if the effects of fluctuations are disregarded. First, this transition was studied in excitonic semiconductors \cite{5}, ordinary superconductors \cite{6} and in an attractive Fermi gas \cite{7}. Although the BCS and BEC limits are physically quite different, the crossover between them was found to be smooth within the BCS theory. In nuclear matter, the BCS-BEC crossover is usually mentioned in respect to the formation of Bose-Einstein condensate of deuterons at low densities, which is caused by strong neutron-proton ($np$) pair correlations in the $^3S_1-^3D_1$ pairing channel \cite{8, 9}. During this transition the chemical potential changes sign at certain critical density (Mott transition), approaching half of the deuteron binding energy at ultra low densities. Despite the fact that in the BCS regime even small isospin asymmetry effectively destroys a $np$ condensate \cite{8, 9, 10}, at low densities Bose-Einstein condensate of deuterons is weakly affected by an additional gas of free neutrons even at very large asymmetries \cite{11, 12}. There is a rather general reason to expect that di-neutron correlations in low-density nuclear matter should be of significance as well. As is known, the bare nucleon-nucleon interaction in the $^1S_0$ channel leads to a virtual state around zero energy characterized by the large negative scattering length $a_{nn} \approx -18.5$ fm. This implies a very strong attraction between two neutrons in the spin singlet state that may lead to the formation of compact bound pairs of neutrons in low-density nuclear systems. Recent theoretical studies confirm this suggestion. In particular, the spatial structure of the neutron Cooper pair in low-density nuclear and neutron matter was analyzed with the use of the bare G3RS force and the effective D1 Gogny interaction \cite{13}. It was shown that the size of the Cooper pair becomes smaller than the average inter-neutron distance at a wide range of neutron densities $\rho_n/\rho_0 \sim 10^{-3} - 10^{-1}$, independently of the type of the interaction ($\rho_0 = 0.16$ fm$^{-3}$ being the nuclear saturation density). The neutron pair wave function demonstrates the BCS-BEC crossover behavior, being spatially extended and of the oscillatory type at moderate densities and possessing the well localized single peak at low densities. The spatial structure of the wave function of two neutrons was studied also in the halo nucleus $^{11}$Li \cite{14}. The behavior of the neutron pair at different densities was simulated by calculating the pair wave function at several distances between the core nucleus and the center of mass of the two valence neutrons. Analogously to the case of infinite nuclear matter, the same BCS- and BEC-like structures in the pair wave function were found, corresponding to the normal and low density regions, respectively. In Ref. \cite{15}, the di-neutron correlations were studied on the base of a new effective density dependent contact pairing interaction. An important feature is that this contact interaction contains the isospin dependence

\[ a_{nn} \approx -18.5 \text{ fm} \]
and was adjusted to reproduce the pairing gap in symmetric nuclear matter and neutron matter, obtained in the microscopic G-matrix calculations with the realistic Argonne $v_{18}$ nucleon-nucleon (NN) potential \cite{14}. This study shows that for the screened pairing interaction a di-neutron BEC state is formed in symmetric nuclear matter at neutron densities $\rho_n/\rho_0 \sim 10^{-3}$ and there is no evidence to the appearance of the BEC state in neutron matter.

Since a two-neutron system in free space has a virtual state around zero energy, then the medium polarization effects should play the major role in the possible formation of the di-neutron BEC state in low-density nuclear systems. However, the influence of the nucleon medium on the di-neutron correlations seems to be different in neutron matter and in symmetric nuclear matter. Most of the recent quantum many-body calculations, e.g., based on the Brueckner theory \cite{16,17}, or renormalization group approach \cite{18}, show the reduction of the pairing gap in neutron matter by a factor of 2-3 due to the screening effect of the pairing interaction. The only exception is the variational quantum Monte Carlo calculations of Ref. \cite{19}, showing a small influence of the medium polarization on the neutron $^1S_0$ pairing. At the same time, calculations based on the Brueckner theory show that in symmetric nuclear matter the medium polarization of the interaction enhances the di-neutron pairing correlations due to the proton particle-hole excitations \cite{16}. The same evidence in favor of the medium enhancement of the neutron $^1S_0$ pairing was obtained for finite nuclei, where the pairing interaction becomes more attractive because of the surface vibrations \cite{20,21,22}. Thus, one can conclude that the conditions for the formation of a di-neutron BEC state are more favorable in a nuclear environment rather than in neutron matter.

The recent upsurge of interest to the BEC-BCS crossover is caused by finding the BCS pairing in ultracold quantum atomic gases \cite{23,24,25}. Ultracold atomic gases provide an experimental play-ground for testing pairing phenomena due to the possibility to control the inter-atomic interactions via a magnetically-tuned Feschbach resonance. In fact, as one goes through the resonance starting from the negative s-wave scattering length and passing to its positive values, the Cooper pairs of two atoms in two hyperfine states shrink in size and go over continuously into the Bose-Einstein condensate of diatomic molecules. In this respect, an important question arises as to what are the qualitative features allowing one to distinguish between the BCS and BEC states.

In the low density limit, when the interaction is characterized by a single parameter, the s-wave scattering length $a$, the $^1S_0$ pairing can be described on the base of the so-called regularized gap equation \cite{26}. In this model, the strength of the interaction is controlled only by the dimensionless parameter $1/k_Fa$. The range $1/k_Fa \ll -1$ corresponds to the weak coupling BCS regime while the range $1/k_Fa \gg 1$ is related to the strong coupling BEC regime. The boundaries of the crossover region between the BCS and BEC states, according to Ref. \cite{28}, are determined by the values $1/k_Fa = \pm 1$. The advantage of this regularized model is that all integrals in the gap equation and the particle number equation can be expressed in terms of some special functions and, hence, the model is easily solvable. Then all quantities of interest can be calculated, in particular, at the boundaries $1/k_Fa = \pm 1$ of the crossover region and these values can be used as the reference values for the study of the BCS-BEC transition in a specific Fermi system at low density.

Such a viewpoint was adopted in Refs. \cite{13,15} for the study of a possible BCS-BEC crossover of neutron pairs in nuclear matter. There were considered a few probable characteristics of the crossover, such as the ratios $\xi_{rms}/d_n$, $\Delta_n/\varepsilon_{Fn}$ \cite{13,15}, $\mu_n/\varepsilon_{Fn}$ \cite{15}, and the probability $P(d_n)$ for the pair neutrons to come close to each other within the relative distance $d_n$ \cite{13,15} ($\xi_{rms}$, $d_n$, $\Delta_n$, $\varepsilon_{Fn}$ and $\mu_n$ being the r.m.s. radius of the Cooper pair, the mean average distance between neutrons, the neutron energy gap, the neutron Fermi kinetic energy, and the effective neutron chemical potential, respectively). It was shown that these criteria are closely related to each other and give approximately the same density range $\rho_n/\rho_0 \sim 10^{-4} - 10^{-1}$ for the domain, where the neutron pairs are strongly spatially correlated \cite{13}. Besides, in terms of these criteria the BCS-BEC crossover was found for a di-neutron condensate in symmetric nuclear matter with the density depending contact interaction at neutron densities $\rho_n/\rho_0 \sim 10^{-3}$ \cite{15}.

However, it is necessary to note that in the case of nuclear matter the assumption of the dilute gas limit may be justified only at very low densities $\rho_n/\rho_0 \lesssim 10^{-5}$ \cite{17}. Hence, the use of the regularized model with the contact interaction for getting the reference values of various criteria can lead only to the qualitative results at the above densities.

Thus, it is desirable to approach the problem from the different point of view and to look for some other possible characteristics for identifying the BCS and BEC states. In this respect, using the analogies with other low-density Fermi systems, such as, e.g., ultracold atomic gases, could be expedient. To that end, let us mention the idea to utilize the density-density correlations in the image of an expanding gas cloud to detect superfluid states in the system of fermionic atoms released from the trap \cite{27}. This idea was explored in Ref. \cite{28} to study the alterations in the density-density correlations through the BCS-BEC crossover. It was learned that the density correlation function of two-component ultracold fermionic atomic gas with singlet pairing of fermions changes sign at low momentum transfer across the BCS-BEC transition. This feature was considered as an unambiguous signature of the crossover. Here we would like to check this criterion with respect to the BCS-BEC crossover in a di-neutron condensate of nuclear matter. Besides, the calculations of Ref. \cite{28}, performed for the balanced populations of fermions of two species, will be extended to unbalanced populations of unlike fermions (for neutron-
In the limit of vanishing density, \( f_2 \) over into the Schrödinger equation for a di-neutron bound potential \( \mu \) should be solved self-consistently with the equation for the neutron chemical potential \( \mu \), which is isotropic and can be incorporated into the effective boundary between the BCS and BEC states occurs when the chemical potential reaches the zero value. Just this feature was considered as an indication of the BCS-BEC crossover in the earlier researches on the BCS-BEC transition in nuclear matter \([2, 3]\).

### II. BASIC EQUATIONS

The equation for the neutron energy gap in the \( ^1S_0 \) pairing channel reads \([29]\)

\[
\Delta_n(k) = -\frac{1}{V} \sum_{k'} \frac{E_n(k')}{2E_n(k')} \tan \left( \frac{E_n(k')}{2T} \right) \tag{1}
\]

where

\[
E_n(k) = \sqrt{\xi_n^2(k) + \Delta_n^2(k)}, \quad \xi_n(k) = \varepsilon_n(k) - \mu_n,
\]

\( \varepsilon_n(k) \) and \( \mu_n \) being the neutron single particle energy and neutron chemical potential, respectively. Further, the single particle energy will be taken in the form corresponding to the Skyrme effective interaction, \( \varepsilon_n = \hbar^2 k^2/2m_n + U_n \), with \( m_n \) and \( U_n \) being the neutron effective mass and neutron single particle potential, respectively. The quadratic on momentum term in the neutron single particle potential is already included into the kinetic energy term and the remaining part \( U_n \) is momentum independent and can be incorporated into the effective neutron chemical potential \( \mu_n \equiv \mu_n - U_n \). Eq. (1) should be solved self-consistently with the equation for the neutron particle number density \( \varrho_n \),

\[
\varrho_n = \frac{2}{V} \sum_k \frac{1}{2} \left( 1 - \frac{\xi_n(k)}{E_n(k')} \right) \tan \left( \frac{E_n(k')}{2T} \right) \equiv \frac{2}{V} \sum_k f_k. \tag{2}
\]

Let us introduce the neutron anomalous distribution function \([29]\)

\[
\psi(k) = \langle a^+_n,k k \rho_n(k) \rangle = \Delta_n(k) / 2E_n(k') \tan \left( \frac{E_n(k')}{2T} \right),
\]

where \( \langle ... \rangle \equiv \text{Tr}... \), \( \rho \) being the density matrix of the system. Then, using Eq. (2), one can represent Eq. (1) for the energy gap in the form

\[
\frac{\hbar^2 k^2}{m_n} \psi(k) + \left( 1 - 2f_k \right) \sum_{k'} V(k,k') \psi(k') = 2\mu_n \psi(k). \tag{3}
\]

In the limit of vanishing density, \( f_k \to 0 \), Eq. (3) goes over into the Schrödinger equation for a di-neutron bound state. The corresponding energy eigenvalue is equal to \( 2\mu_n \). The change of sign of the effective neutron chemical potential \( \mu_n \) under decreasing density of nuclear matter signals the transition from the regime of large overlapping neutron Cooper pairs to the regime of non-overlapping di-neutron bound states.

Let us consider the two-neutron density correlation function

\[
\mathcal{D}(x,x') = \langle \hat{\Delta}n(x) \Delta\hat{n}(x') \rangle, \quad \Delta\hat{n}(x) = \hat{n}(x) - \hat{n}, \tag{4}
\]

\[
\hat{n}(x) = \frac{1}{V} \sum_{\sigma k k'} e^{i(k'-k)\cdot x} a^+_{n,k\sigma} a_{n,k'\sigma},
\]

\[
\hat{n} = \frac{1}{V} \sum_k \bar{a} a_{n,k\sigma} a_{n,k'\sigma}.
\]

Its general structure in the spatially uniform and isotropic case reads \([30]\)

\[
\mathcal{D}(x,x') = \varrho_n \delta(r) + \varrho_n \mathcal{D}(r), \quad r = x - x'. \tag{5}
\]

The function \( \mathcal{D}(r) \) is called the density correlation function as well. We will just be interested in the behavior of the function \( \mathcal{D}(r) \). Calculating the trace in Eq. (4) and going to the Fourier representation

\[
\mathcal{D}(q) = \int \frac{dr \, e^{iqr} \mathcal{D}(r)}{} \tag{6}
\]

one can get

\[
\mathcal{D}(q) = I_\psi(q) - I_f(q).
\]

where

\[
I_f(q) = \frac{2}{\pi^2} \varrho_n \int_0^\infty dr \, r^2 j_0(qr) \left[ \int_0^\infty dk \, k^2 f_j(k) j_0(kr) \right]^2,
\]

\[
I_\psi(q) = \frac{2}{\pi^2} \varrho_n \int_0^\infty dr \, r^2 j_0(qr) \left[ \int_0^\infty dk \, k^2 \psi_j(k) j_0(kr) \right]^2.
\]

Here \( j_0 \) is the spherical Bessel function of the first kind of order zero. The functions \( I_f \) and \( I_\psi \) represent the normal and anomalous contributions to the neutron density correlation function.

Analogously, we can consider the two-neutron spin correlation function

\[
S_{\mu\nu}(x,x') = \langle \Delta\hat{s}_\mu(x) \Delta\hat{s}_\nu(x') \rangle, \quad \Delta\hat{s}_\mu(x) = \hat{s}_\mu(x) - \hat{s}_\mu,
\]

\[
\hat{s}_\mu(x) = \frac{1}{2V} \sum_{\sigma \sigma' k k'} e^{i(k'-k)\cdot x} a^+_{n,k\sigma} a_{n,k'\sigma} (\sigma \sigma' a_{n,k'\sigma}^+ a_{n,k\sigma},
\]

\[
\hat{s}_\mu = \frac{1}{2V} \sum_{\sigma \sigma' k} \bar{a} a_{n,k\sigma} (\sigma \sigma' a_{n,k'\sigma}^+ a_{n,k\sigma},
\]

where \( \sigma, \sigma' \) are the Pauli matrices. The general structure of the neutron spin correlation function for the spin unpolarized case is

\[
S_{\mu\nu}(x,x') = \frac{\varrho_n}{4} \delta_{\mu\nu} \delta(r) + \varrho_n S_{\mu\nu}(r). \tag{8}
\]

Calculating the trace in Eq. (7), for the Fourier transform of the spin correlation function one gets

\[
S_{\mu\nu}(q) = S(q) \delta_{\mu\nu}, \quad S(q) = -\frac{1}{4} (I_f(q) + I_\psi(q)). \tag{9}
\]
At zero momentum transfer, the normal $I_f$ and anomalous $I_\psi$ contributions read

$$I_f(q = 0) = \frac{1}{\pi^2 q n} \int_0^\infty dk k^2 f^2(k), \quad (10)$$

$$I_\psi(q = 0) = \frac{1}{\pi^2 q n} \int_0^\infty dk k^2 \psi^2(k). \quad (11)$$

At zero temperature, the zero-momentum transfer neutron spin correlation function satisfies the sum rule

$$S(q = 0) = -\frac{1}{4} I_f(0) + I_\psi(0) = -\frac{1}{4}. \quad (12)$$

**III. $^1S_0$ Neutron Energy Gap at Zero Temperature**

Further we will take the pairing interaction in the $^1S_0$ neutron pairing channel in the form of the effective density dependent contact interaction

$$V(r_1, r_2) = v_0 g(g_n, g_p) \delta(r_1 - r_2). \quad (13)$$

In the gap equation, the interaction (12) should be supplemented with the cut-off energy $E_c$. The interaction strength $v_0$ is determined in such a way that the interaction (12) reproduces the low-energy neutron-neutron scattering phase shifts obtained with the Argonne $v_{18}$ potential. The isospin dependent factor $g$ was constructed in Ref. [15] and reads

$$g = g_1 + g_2,$$

$$g_1 = 1 - (1 - \alpha) \frac{\eta_n}{\eta_p} \beta_n^s \left( \frac{\eta_p}{\eta_n} \right) \beta_n^p,$$

$$g_2 = \eta_n \left[ 1 + e^{\frac{k_F n - 1.4}{\eta_p 0.5}} \right]^{-1} - \left[ 1 + e^{\frac{k_F n - 0.1}{\eta_p 0.5}} \right]^{-1}. \quad (14)$$

Here $\alpha = (g_n - g_p)/g$ is the isospin asymmetry parameter, $k_{Fn} = (3\pi^2 g_n/2)^{2/3}$ is the neutron Fermi momentum, $\beta_n^s, \beta_n^p, \eta_n, \eta_p$ are the fitting parameters, adjusted to reproduce the pairing gap in symmetric nuclear matter and neutron matter obtained in the G-matrix calculations with the Argonne $v_{18}$ potential with account of the medium polarization effects [16]. The term $g_1$ contains the density dependence in a power law form as it was introduced in the contact interaction of Ref. [31]. Besides, the term $g_1$ contains the isospin dependence, built such as to reproduce the position and the magnitude of the maximum of the energy gap in both symmetric nuclear matter and neutron matter. The term $g_2$ is necessary to improve the results of the fitting procedure for the energy gap at the neutron Fermi momenta $k_{Fn} > 1$ fm$^{-1}$. Concerning the term $g_2$, let us make one more comment. In the original article [15], this term is written as $g_2 = g_2(k_F)$, where the quantity $k_F$ is not explicitly determined. There are two possibilities: (1) $g_2 = g_2(k_F)$, where $k_F$ is parametrized in terms of the total density $k_F = (3\pi^2 g/2)^{2/3}$, and (2) $g_2 = g_2(k_{Fn})$ with $k_{Fn}$ being the neutron Fermi momentum. There will be no difference between these two parametrizations for symmetric nuclear matter; however, the difference occurs for isospin asymmetric matter and will be most serious for neutron matter. The calculations of the energy gap in neutron matter show that the results of the fitting procedure, presented in Ref. [15], correspond to the case $g_2 = g_2(k_{Fn})$. This point was confirmed in the communication [32]. Therefore, the term $g_2$ is, in fact, isovector and not isoscalar, as it is called in Ref. [15]. Since the microscopically calculated neutron energy gap in symmetric nuclear matter and neutron matter is well fitted with $g_2 = g_2(k_{Fn})$, one can use this form also for calculations at intermediate asymmetries $0 < \alpha < 1$. Besides, if to apply the isospin dependent contact interaction (12), (13) for the calculation of the pairing gaps in finite nuclei [33], one has to change $\alpha \rightarrow -\alpha$ in the term $g_1$ and to use $g_2 = g_2(k_{Fp}) (k_{Fp}$ being the proton Fermi momentum) in the proton $^1S_0$ pairing channel.

Let us now turn to the calculation of the neutron energy gap in the $^1S_0$ pairing channel that is the necessary preliminary step before the calculation of the correlation functions. The zero temperature gap equation with the effective contact interaction (12) reads

$$\Delta_n = -v_0 g \int_{k_- < k < k_+} \frac{d^3k}{(2\pi)^3}\frac{\Delta_n}{2E_n(k)}.$$

In the gap equation, the cut-off momenta $k_{\pm}$ are defined from the requirement that the quasiparticle energy should be less than the cut-off energy $E_c$, $E_n(k) < E_c$. If $k_-$ becomes imaginary, one should set $k_- \equiv 0$. Just this choice of the cut-off parameters is often used in the HFB calculations. Eq. (14) should be solved self-consistently with Eq. (2) for the neutron particle number density. In numerical calculations, we use the screened-II interaction of Ref. [15] with the parameters given in Table 1. The single particle energy $\epsilon_n(k)$ is taken for the SLy4 Skyrme effective interaction.

In Fig. 1a, the neutron energy gap is plotted as a function of the neutron Fermi momentum at zero temperature and different isospin asymmetries. With increasing asymmetry, the magnitude of the energy gap decreases and the position of its maximum is shifted to the larger neutron Fermi momentum. In Fig. 1b, the ratio of the effective neutron chemical potential $\mu_n$ to the neutron Fermi kinetic energy $\epsilon_{F_n} = \hbar^2 k_{Fn}^2/2m_n$ is depicted as

| $v_0$ (MeV fm$^3$) | $E_c$ (MeV) | $\eta_n$ | $\beta_n$ | $\eta_p$ | $\beta_p$ |
|-----------------|-------------|----------|---------|--------|--------|
| -542            | 40          | 0.8      | 1.80    | 0.27   | 1.61   | 0.122  |
FIG. 1: (Color online) (a) The zero temperature neutron energy gap and (b) the ratio $\mu_n/\varepsilon_n$ of the effective neutron chemical potential to the neutron Fermi kinetic energy as functions of the neutron Fermi momentum at different isospin asymmetries for the screened-II contact interaction.

a function of the neutron Fermi momentum. This ratio is indicative of the importance of the di-neutron correlations, since in the absence of pairing correlations it is equal to unity, otherwise it is less than unity. It is seen that the di-neutron correlations are most strong in symmetric nuclear matter and with increasing isospin asymmetry their strength is diminished. In accordance with Eq. (3), we will identify the BCS and BEC states of the pair condensate by the positive (BCS regime) and negative (BEC regime) values of the effective chemical potential $\mu_n$. For symmetric nuclear matter, a di-neutron BEC state is realized at the finite density range $0.09 \text{ fm}^{-1} < k_{F_n} < 0.25 \text{ fm}^{-1}$ and surrounded on both sides by the BCS state. For neutron matter, the self-consistent equations have no at all solutions with the negative chemical potential $\mu_n$ and the BEC state is not formed. For asymmetric nuclear matter, the BEC state is quite sensitive to the isospin asymmetry and is quenched at the critical value $\alpha_c \approx 0.26$. Note that, as was clarified in Ref. [16], the medium polarization acts differently on the di-neutron correlations at low-density region, increasing the neutron energy gap in symmetric nuclear matter and decreasing it in neutron matter.

Fig. 2 shows the evolution of the neutron normal $f(k)$ and anomalous $\psi(k)$ distribution functions with density in symmetric nuclear matter. In the weak coupling BCS regime ($k_{F_n} = 0.8 \text{ fm}^{-1}$), the normal distribution function resembles the Fermi step distribution function with the edge near the neutron Fermi momentum. With decreasing density, the step-like structure is gradually washed out and in the region of the strong coupling BEC regime ($k_{F_n} = 0.2 \text{ fm}^{-1}$) the normal distribution function smoothly changes with the momentum. However, under further decreasing density, we go back to the BCS regime ($k_{F_n} = 0.02 \text{ fm}^{-1}$) and the step-like structure becomes more and more pronounced. The anomalous distribution function $\psi(k)$ in the BCS regime has a well developed peak, indicating that pairing exists only near the Fermi surface. In the strong coupling BEC regime $\psi(k)$
exhibits a smooth variation typical of the wave function of a bound state. At ultra-low densities, the spike in the momentum dependence of the anomalous distribution function appears again.

IV. DENSITY AND SPIN CORRELATION FUNCTIONS FOR A DI-NEUTRON CONDENSATE AT ZERO TEMPERATURE

A. Symmetric nuclear matter

After the neutron energy gap and the effective neutron chemical potential have been found, the density $D(q)$ and spin $S(q)$ correlation functions can be obtained directly from Eqs. (6) and (9). Fig. 3 shows the density correlation function as a function of the momentum transfer for a set of the neutron Fermi momenta in symmetric nuclear matter. Let us begin with the consideration of the BCS regime at the moderate densities ($k_{Fn} = 0.8 \text{fm}^{-1}$), where at low momentum transfer $D(q)$ is negative. Under decreasing density ($k_{Fn} = 0.6, 0.4 \text{fm}^{-1}$), the zero-momentum transfer density correlation function $D(q = 0)$ at certain neutron Fermi momentum changes sign from negative to positive. In the BEC regime ($k_{Fn} = 0.2 \text{fm}^{-1}$), the correlation function $D(q)$ is everywhere positive. If to decrease density further ($k_{Fn} = 0.07, 0.02 \text{fm}^{-1}$), the zero-momentum transfer value $D(q = 0)$ changes sign again, now from positive to negative, and in the ultra-low density BCS regime the function $D(q)$ has the negative sign at low momentum transfer as it was at the moderate densities. The change of sign of the density correlation function at low momentum transfer across the BCS-BEC transition region was considered in Ref. [28] as a signature of the crossover. We see that this feature is qualitatively well reproduced for a di-neutron condensate in symmetric nuclear matter. At the same time, the spin correlation function, as can be seen from Fig. 3, fluently evolves across

FIG. 3: (Color online) (a) Density $D(q)$ and (b) spin $S(q)$ correlation functions as functions of the momentum transfer at different neutron Fermi momenta in symmetric nuclear matter.

FIG. 4: (Color online) Zero-momentum transfer density correlation function $D(q = 0)$ and its normal $I_p(q = 0)$ and anomalous $I_v(q = 0)$ contributions as functions of the neutron Fermi momentum in symmetric nuclear matter.
the BCS-BEC transition region without any qualitative change. In order to establish the boundaries of the density interval for the BEC state on the basis of the formulated criterion, let us consider the density correlation function at zero momentum transfer, depicted in Fig. 3. Under decreasing density, the density correlation function $D(q = 0)$ twice changes sign, first, at the point of the BCS-to-BEC transition and then at the point of the reverse BEC-to-BCS crossover, that corresponds to the earlier consideration. The BEC state is realized in the range $0.03 \text{fm}^{-1} < k_{Fn} < 0.5 \text{fm}^{-1}$ and this interval is wider than that determined on the basis of the negative values of the effective neutron chemical potential.

The normal $I_f(q = 0)$ and anomalous $I_\psi(q = 0)$ contributions to $D(q = 0)$, given by Eq. (10), are shown in the bottom panel of Fig. 3. This figure qualitatively explains why the density correlation function $D(q = 0)$ changes sign at the boundary between the BCS and BEC states. In the weak coupling BCS regime ($k_{Fn} = 0.8 \text{fm}^{-1}$), the anomalous distribution function $\psi(k)$ is sharply peaked near $k = k_{Fn}$, as shown in Fig. 2 and, hence, the anomalous contribution to $D(q = 0)$ is much smaller than the normal one. Therefore, $D(q = 0) < 0$ on the BCS side from the transition boundary. However, in the strong coupling BEC regime ($k_{Fn} = 0.2 \text{fm}^{-1}$), the normal distribution function $f(k)$ is to a considerable extent depleted and the anomalous contribution dominates over the normal one. Hence, $D(q = 0) > 0$ on the BEC side from the transition boundary, and at some critical density between the BCS and BEC regimes the density correlation function $D(q = 0)$ should change the sign. Note also that, according to the sum rule in Eq. (11), the normal and anomalous contributions satisfy the relationship $I_f(0) + I_\psi(0) = 1$.

As seen from the figure, the contribution of the anomalous part is the largest at the neutron Fermi momentum $k_{Fn} \sim 0.2 \text{fm}^{-1}$. At the corresponding densities $\rho_n/\rho_0 \sim 10^{-3}$ the BEC di-neutron state can be considered as well formed. Just at these densities the BEC of bound neutron pairs in symmetric nuclear matter was found in Ref. 12 with the help of other quantitative characteristics, mentioned in Introduction.

**B. Asymmetric nuclear matter**

Now we will present the results of the numerical determination of the correlation functions at finite isospin asymmetry. The evolution of the density correlation function $D(q)$ with the isospin asymmetry is shown in Fig. 5. If to identify the BCS-BEC transition by the change of sign of the density correlation function at low momentum transfer, it is seen that with increasing asymmetry the density interval, where the BEC state exists, is contracted. At strong enough isospin asymmetry only the BCS state is realized, as can be seen from the limiting case $\alpha = 1$, corresponding to neutron matter.

In order to find an estimate for the critical isospin asymmetry, at which the BEC state disappears, let us consider the zero-momentum transfer density correlation

![FIG. 5](image1.png)

**FIG. 5:** (Color online) Density correlation function $D(q)$ as a function of the momentum at different neutron Fermi momenta and isospin asymmetries.

![FIG. 6](image2.png)

**FIG. 6:** (Color online) Zero-momentum transfer density correlation function $D(q = 0)$ as a function of the neutron Fermi momentum at different isospin asymmetries.
In summary, the density and spin correlation functions have been calculated for a di-neutron condensate in asymmetric nuclear matter with the aim of finding the possible signatures of the BCS-BEC crossover. As the primary characteristic for identifying the transition, it is accepted that the qualitative boundary between the BCS and BEC states occurs when the effective neutron chemical potential reaches the zero value. It has been shown that the zero-momentum transfer spin correlation function satisfies the sum rule at zero temperature. In symmetric nuclear matter, the density correlation function changes sign at low momentum transfer across the BCS-BEC transition and this feature can be considered as a signature of the crossover. This result qualitatively agrees with that obtained for two-component ultracold fermionic atomic gas with equal densities of fermions of different species [28]. The self-consistent calculations of the $^1S_0$ energy gap in a di-neutron condensate and the effective neutron chemical potential $\mu_n$ show that the BEC state ($\mu_n < 0$) is quite sensitive to isospin asymmetry and does not appear at $\alpha > \alpha_c \approx 0.26$. This is a reflection of the fact that the medium polarization effects act differently on di-neutron correlations at small and strong isospin asymmetry, enhancing them in symmetric nuclear matter and suppressing them in neutron matter. At finite isospin asymmetry, the criterion, based on the change of sign of the density correlation function, gives too large value for the critical asymmetry $\alpha_c^d \sim 0.9$, at which the BEC state is quenched. Therefore, it can be trusted for the description of the density-driven BCS-BEC crossover of neutron pairs only at small isospin asymmetry. This result generalizes the conclusion of Ref. [28], in which the change of sign of the density correlation function at low momentum transfer in two-component quantum fermionic atomic gas with the balanced populations of fermions of different species was considered as an unambiguous signature of the BCS-BEC transition.

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