Jet quenching and effects of non-Gaussian transverse-momentum broadening on di-jet observables

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Abstract

We describe production of jet pairs in ultrarelativistic nuclear collisions within a framework that relies on binary collisions of incoming partons described with transverse-momentum broadening and an in-medium propagation of jet particles and uses stochastic transverse forces as well as medium-induced radiation. We find that the resulting di-jet observables feature the behaviour that deviates from that of jet-pairs which undergo transverse-momentum broadening following the Gaussian distribution.

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1 Introduction

A prominent feature of high-energy hadronic collisions is the abundant jet production which is a manifestation of the underlying QCD dynamics. Jets are loosely defined as collimated sprays of particles that act as proxies for the properties of highly virtual partons, quarks and gluons, that participate in the hard scattering. Events where two jets approximately balance their momenta give an additional handle on probing how initial-state processes and their associated parton distribution functions affect the properties of the final-state jets. It is important to point out that such vacuum effects lead to an appreciable azimuthal decorrelation as well as an imbalance of the transverse momentum of the leading and sub-leading, recoiling, jets.

Jet production in ultrarelativistic nucleus–nucleus collisions has a prominent role in probing the properties of the hot and dense nuclear matter formed in these events [1–3]. This leads to the suppression, or quenching, of high-$p_T$ hadron and inclusive jet spectra observed both at $\sqrt{s_{NN}} = 200$GeV collisions at RHIC and $\sqrt{s_{NN}} = 5$ TeV collisions at LHC (for a review see [4]). It was early established experimentally that the modifications arose due to final-state interactions. This lead to the theoretical development by Baier–Dokshitzer–Mueller–Peigne–Schiff and Zakharov for the in-medium stimulated (bremsstrahlung) emissions that typically is referred to as the BDMPS-Z formalism [5–8]. Such emissions are responsible for transporting energy rapidly away from the jet axis to large angles [10, 11]. For high-$p_T$ jets, the total energy loss depends on the fragmentation properties of the jet [12, 13] (for the Monte Carlo implementation of this results see Ref. [14]).

In the BDMPS-Z formalism, the medium affects the jet propagation and radiation via transverse momentum exchanges. Typical interactions are described by a diffusion constant $\hat{q}$. In a hot quark–gluon plasma (QGP) it is sensitive to its collective energy density. However, rare hard transverse kicks from the medium are also expected. These are manifestations of the quasi-particles of the hot and dense matter and affect both the spectrum of radiated gluons [15–21] as well as the distribution of particles in transverse momentum space [22,23]. An important question is whether jet observables are sensitive to such interactions, especially those that are sensitive to recoils. For example, the final-state interactions would lead to the gradual decorrelation of jets that originally were created from a vacuum $2 \rightarrow 2$ matrix element [24], see also [25, 26]. In addition, one needs to account for the initial state, i.e. evolution of the system that leads to hard scattering. It is therefore pertinent to further investigate how the details of the initial state affect the properties of the final state. In particular, how the transverse momentum dependence of the partons initiating the hard collision affect the azimuthal-angle decorrelations of the final-state jets or the $A_J$ observable, or dependence of $R_{AA}$ on the final-state transverse momentum. In approaches that account also for jet quenching the early stage of heavy-ion (HI) collisions is usually described by the collinear factorisation where parton densities obey the DGLAP evolution equation and the initial-state partons are on mass-shell. Consequently, the final-state partons are essentially produced back to back. To account for a non-vanishing transverse momentum imbalance of the final-state partons, one often uses parton showers via the application of Monte Carlo generators, see e.g. [27].

In this paper we propose an approach based on the $k_T$-factorisation, accounting for the longitudinal and transverse momentum dependence on the initial-state partons, with the evolution in terms of the

\footnote{For an equivalent treatment within the thermal field theory see Ref. [9].}
rate equation based on the final-state jet-plasma interaction. Such an approach allows for a detailed study of the influence of kinematics of the initial state on the properties of final-state system. We limit ourselves to study observables produced in central rapidity region collisions (the forward case was addressed by two of us in [24]) and, for now, do not account for the initial-state saturation effects [28–30]. In order to calculate the full spectrum, one has to account for both the quark and gluon degrees of freedom in the hard matrix elements as well as in the initial- and final-state fragmentation processes. In the present study we account only for gluons. This is because, to our knowledge, the appropriate rate equation accounting for the transverse momentum dependence of quarks has not been formulated yet.

In order to describe the propagation of partons produced in hard collisions inside a hot quark–gluon plasma, we have applied the recently developed code MINCAS [31] that solves the rate equation describing the rescattering and radiation of a hard parton. For a similar approach see [16, 17].

The paper is organised as follows. In Section 2 we present the theoretical framework of our study. In Section 3 numerical results of our Monte Carlo simulations are presented and discussed. Section 4 concludes this work. Finally, in Appendix A we briefly describe the algorithm used in the numerical simulations.

2 Theoretical framework

We factorise the production of a pair of gluon jets in nuclear collisions into the production of a pair of gluons and their subsequent in-medium evolution into gluon jets. The first step is described as the production of two gluons $G_1$ and $G_2$ via the hard collision of two gluons $G_A$ and $G_B$ which stem from two nuclei $N_A$ and $N_B$. The second step is given by the processes of scattering and medium-induced radiation that lead to the fragmentation of $G_1$ ($G_2$) into a jet $j_1$ ($j_2$). Thus, the total process can be summarised as

$$N_A + N_B \rightarrow G_A + G_B + X \rightarrow G_1 + G_2 + X \rightarrow j_1 + j_2 + X,$$

where $X$ is the production of additional particles which are not used in our descriptions of (di)jet observables. The entire process is depicted schematically in Fig. 1. This sections proceeds by first detailing the hard process $N_1 + N_2 \rightarrow G_1 + G_2 + X$, then the in-medium propagation $G_i \rightarrow j_i$ ($i = 1, 2$), followed by a short description of the modelling of the medium.

2.1 Transverse-momentum-dependent factorisation of hard processes

In the $k_T$-factorisation the initial state process reads

$$N_A(P_1) + N_B(P_2) \rightarrow G_A(k_1) + G_B(k_2) + X \rightarrow G_1(q_1) + G_2(q_2) + X,$$

where the momenta $k_1$ and $k_2$ have components transverse to that of the incoming nuclei, an essential property for the description of the di-jet observables presented below,

$$k_1 = x_1 P_1 + k_{1T}, \quad k_2 = x_2 P_2 + k_{2T}.$$
Figure 1: Gluon-jet production via two scattering gluons in hard nuclear collisions: colliding nuclei (horizontal ellipse) with the momenta $P_1$ and $P_2$ yield incoming gluons (with the momenta $k_1$ and $k_2$) which interact in a hard scattering process (vertical ellipse) and yield two gluons (with the momenta $q_1$ and $q_2$), which are subject to in-medium scattering (gluon interaction with •), while simultaneously fragmenting into jets denoted by the purple colour.

The momentum fractions $x_i$ and transverse momenta $k_{iT}$ follow the transverse momentum distributions for gluons in both of the colliding nuclei given at a certain factorisation scale $\mu_F$.

Thus, the $k_T$-factorisation formula for the parton-level differential cross section $\sigma_{pp}$ at the tree-level for the $gg$-pair production reads

$$d\sigma_{pp} \over dy_1dy_2d^2q_{1T}d^2q_{2T} = \int {d^2k_{1T}d^2k_{2T} \over \pi \pi} {1 \over 16\pi^2(x_1x_2s)^{\gamma} \left| M_{\text{off-shell}}^{g^*g^*\rightarrow g\bar{g}} \right|^2} \times \delta^2\left( \vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_{1T} - \vec{q}_{2T} \right) F_g(x_1,k_{1T}^2,\mu_F^2) F_g(x_2,k_{2T}^2,\mu_F^2),$$

where $M_{\text{off-shell}}^{g^*g^*\rightarrow g\bar{g}}$ is the off-shell matrix element for the hard subprocess and $F(x_i,k_{iT}^2,\mu_F^2)$ is the Transverse-Momentum-Dependent (TMD) gluon density [32]. For the purpose of the paper we use the Martin–Ryskin–Watt (MRW) [33] gluon densities obtained from the CT10NLO parton distribution functions (PDFs) via the application of a Sudakov form-factor . The momentum fractions $x_i$, the rapidities $y_i$ and the transverse momenta $q_{iT}$ of the outgoing particles are related to one another as

$$x_1 = \frac{q_{1T}}{\sqrt{s}} \exp(y_1) + \frac{q_{2T}}{\sqrt{s}} \exp(y_2), \quad x_2 = \frac{q_{1T}}{\sqrt{s}} \exp(-y_1) + \frac{q_{2T}}{\sqrt{s}} \exp(-y_2).$$

For the numerical calculation of the hard-process cross sections we rely on the KATIE framework [35].

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$^2$In the current study, to have a clear picture of broadening due to the non-Gaussian effects, we use just proton TMDs both for the proton–proton and nucleus–nucleus (A–A) collisions, but in the future we plan to use nTMDs for the A–A case [34].
2.2 In-medium evolution

In this subsection we describe the processes of the jet evolution in the medium, 

\[ G_i(q_i) \rightarrow j_i(p_i) \quad i = 1, 2. \]

(5)

The momentum of the jets is modified by interactions with the medium. We have \( p = l + q \), where the \( l \) is the change of the jet transverse momentum and \( p^+ = \bar{x}q^+ \) is the change of its longitudinal momentum due to kicks from the medium and from the medium-induced branchings\(^3\). To account for quenching, the cross section \( \sigma_{pp} \) for the production of the vacuum jets should be convoluted with the fragmentation function \( D(\bar{x}, l, \tau) \) for the jets in the medium. Thus, for the inclusive production of individual jets, the cross-section \( d\sigma_{AA}/d\Omega_p \) for emission of jets into the phase-space element \( d\Omega_p = dp_+ dp \) can be written as

\[
\frac{d\sigma_{AA}}{d\Omega_p} = \int d\Omega_q \int d^2l \int_0^1 \frac{d\bar{x}}{\bar{x}} \delta(p^+ - \bar{x}q^+) \delta^{(2)}(p - l - q) D(\bar{x}, l, \tau(q^+)) \frac{d\sigma_{pp}}{d\Omega_q} \\
= \int d^2q \int_0^1 \frac{d\bar{x}}{\bar{x}^2} D(\bar{x}, p - q, \tau(p^+/\bar{x})) \frac{d\sigma_{pp}}{dq^+ d^2q} \bigg|_{q^+=p^+/\bar{x}},
\]

where \( d\Omega_q = dq^+ d^2q \), \( \tau(q^+) = \bar{a}\sqrt{q^+/q^+}L \), and

\[
D(\bar{x}, l, \tau) \equiv \frac{dN}{d\bar{x}d^2l}.
\]

(6)

For the production of di-jets, the differential cross-section for the emission of two jets into the phase-space elements \( d\Omega_{p_1} \) and \( d\Omega_{p_2} \) can be written as

\[
\frac{d\sigma_{AA}}{d\Omega_{p_1} d\Omega_{p_2}} = \int d\Omega_{q_1} d\Omega_{q_2} \int d^2l_1 \int d^2l_2 \int_0^1 \frac{d\bar{x}_1}{\bar{x}_1} \delta(p_1^+ - \bar{x}_1 q_1^+) \int_0^1 \frac{d\bar{x}_2}{\bar{x}_2} \delta(p_2^+ - \bar{x}_2 q_2^+) \\
\delta^{(2)}(p_1 - l_1 - q_1) \delta^{(2)}(p_2 - l_2 - q_2) D(\bar{x}_1, l_1, \tau(q_1^+)) D(\bar{x}_2, l_2, \tau(q_2^+)) \frac{d\sigma_{pp}}{d\Omega_{q_1} d\Omega_{q_2}} \\
= \int d^2q_1 \int d^2q_2 \int_0^1 \frac{d\bar{x}_1}{\bar{x}_1^2} \int_0^1 \frac{d\bar{x}_2}{\bar{x}_2} D(\bar{x}_1, p_1 - q_1, \tau(p_1^+/\bar{x}_1)) D(\bar{x}_2, p_2 - q_2, \tau(p_2^+/\bar{x}_2)) \frac{d\sigma_{pp}}{dq_1^+ dq_2^+ d^2q_1 d^2q_2} \bigg|_{q_1^+=p_1^+ / \bar{x}_1, q_2^+=p_2^+ / \bar{x}_2}
\]

(8)

where it is assumed implicitly that the fragmentation processes of the jet 1 and the jet 2 factorise from the hard scattering process as well as from one another.

The evolution equation for the gluon transverse-momentum-dependent distribution \( D(\bar{x}, l, \tau) \) in the dense medium, obtained under the assumption that the momentum transfer in the kernel is small,

\(^3\)Throughout this article, we use the following notation: The index ‘T’ denotes the momentum components transverse to the beam axis of nuclear collisions, while symbols in bold-face represent the momentum components transverse to the jet-axis of one of the produced jets. The exception to this convention is the broadening of momenta transverse to the jet-axis which we call the “\(k_T\)-broadening”, in order to be in agreement with many papers written on this subject.
reads \cite{36}

\[
\frac{\partial}{\partial t} D(\tilde{x}, l, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[ \frac{1}{z^2} \sqrt{\frac{z}{\tilde{x}}} D\left(\frac{\tilde{x}}{z}, l, t\right) \theta(z - \tilde{x}) \frac{z}{\sqrt{\tilde{x}}} - \frac{z}{\sqrt{\tilde{x}}} D(\tilde{x}, l, t) \right]
+ \int \frac{d^2 q}{(2\pi)^2} C(q) D(\tilde{x}, l - q, t),
\]  

(9)

where

\[
\mathcal{K}(z) = \frac{[f(z)]^{5/2}}{[z(1-z)]^{3/2}}, \quad f(z) = 1 - z + z^2, \quad 0 \leq z \leq 1,
\]

(10)

is the \(z\)-kernel function, and

\[
\frac{1}{t^*} = \frac{\bar{\alpha}}{\tau_{br}(E)} = \bar{\alpha} \sqrt{\frac{\hat{q}}{E}}, \quad \bar{\alpha} = \frac{\alpha_s N_c}{\pi},
\]

(11)

where \(t^*\) is the stopping time, i.e. the time at which the energy of an incoming parton has been radiated off in form of soft gluons, \(E\) is the energy of the incoming parton, \(z\) – its longitudinal momentum fraction, \(\hat{q}\) – the quenching parameter, \(\alpha_s\) – the QCD coupling constant and \(N_c\) – the number of colours.

The collision kernel \(C(q)\) is given by

\[
C(q) = w(q) - \delta(q) \int d^2 q' w(q').
\]

(12)

Here we consider a situation where the quark–gluon plasma equilibrates and the transverse-momentum distribution of medium particles assumes the form \cite{37}

\[
w(q) = \frac{16\pi^2 \alpha_s^2 N_c n}{q^2 (q^2 + m_D^2)},
\]

(13)

where \(m_D\) is the Debye mass of the medium quasi-particles. In the following we consider the expression of Eq. (13) inside the collision kernel \(C(q)\).

The above integral equations can be formally solved by iteration. Denoting \(\tau = t/t^*\), we get \cite{31}

\[
D(\tilde{x}, l, \tau) = \int_0^1 d\tilde{x}_0 \int d^2 l_0 D(\tilde{x}_0, l_0, \tau_0) \left\{ e^{-\Psi(\tilde{x}_0)}(\tau - \tau_0) \delta(\tilde{x} - \tilde{x}_0) \delta(l - l_0) \right. \\
+ \sum_{n=1}^\infty \prod_{i=1}^n \left[ \int_{\tau_i}^\tau d\tau_i \int_0^1 dz_i \int d^2 q_i G(z_i, q_i) e^{-\Psi(\tilde{x}_i) - (\tau - \tau_{i-1})} \right] \\
\times \left. e^{-\Psi(\tilde{x}_n)}(\tau - \tau_n) \delta(\tilde{x} - \tilde{x}_n) \delta(l - l_n) \right\},
\]

(14)
where
\[ \Psi(\tilde{x}) = \Phi(\tilde{x}) + W, \tag{15} \]
\[ \Phi(\tilde{x}) = \frac{1}{\sqrt{\tilde{x}}} \int_0^{1-\epsilon} dz z K(z), \tag{16} \]
\[ W = t^* \int_{|q|>q_{\text{min}}} d^2q \frac{w(q)}{(2\pi)^2}, \tag{17} \]
\[ G(z, q) = \sqrt{\frac{z}{\tilde{x}}} z K(z) \theta(1-\epsilon-z) \delta(q) + t^* \frac{w(q)}{(2\pi)^2} \theta(|q|-q_{\text{min}}) \delta(1-z), \tag{18} \]
and
\[ \tilde{x}_n = z_n \tilde{x}_{n-1}, \quad l_n = z_n l_{n-1} + q_n, \tag{19} \]
with \( \tilde{x}_0 \) and \( l_0 \) being some initial values of \( \tilde{x} \) and \( l \) at the initial evolution time \( \tau_0 \), given by the distribution \( D(\tilde{x}_0, l_0, \tau_0) \).

Furthermore, after integration of Eq. (9) over the transverse momentum \( l \) one obtains the evolution equation for the gluon energy density [36]:
\[ \frac{\partial}{\partial t} D(\tilde{x}, t) = \frac{1}{t^*} \int_0^1 dz K(z) \left[ \sqrt{\frac{z}{\tilde{x}}} D \left( \frac{\tilde{x}}{z}, t \right) \theta(z-\tilde{x}) - \frac{z}{\sqrt{\tilde{x}}} D(\tilde{x}, t) \right], \tag{20} \]
where \( D(\tilde{x}, t) \equiv \int d^2l D(\tilde{x}, l, t) \). The iterative solution of this equation reads [31]
\[ D(\tilde{x}, \tau) = \int_0^1 d\tilde{x}_0 D(\tilde{x}_0, \tau_0) \left\{ e^{-\Phi(\tilde{x}_0)(\tau-\tau_0)} \delta(\tilde{x}-\tilde{x}_0) + \sum_{n=1}^{\infty} \prod_{i=1}^n \left[ \int_{\tau_i}^\tau d\tau_i \int_0^1 dz_i \sqrt{\frac{z_i}{\tilde{x}_i}} z_i K(z_i) \theta(1-\epsilon-z_i) e^{-\Phi(\tilde{x}_{i-1})(\tau_i-\tau_{i-1})} \right] \times e^{-\Phi(\tilde{x}_n)(\tau-\tau_n)} \delta(\tilde{x}-\tilde{x}_n) \right\}. \tag{21} \]

Both Eqs. (9) and (20) are solved numerically within the MINCAS framework with the use of dedicated Markov Chain Monte Carlo (MCMC) algorithms [31]. In this article, we generally evolve the gluon di-jets following the \( k_T \)-dependent evolution equation (9) and compare the results to the di-jet evolution using Eq. (20) combined with the Gaussian \( k_T \)-broadening, in order to study the effects of the non-Gaussian \( k_T \)-broadening.

### 2.3 Medium model

While numerous approaches exist that describe the evolution of the QGP-medium with time (see Refs. [38, 39] and references therein), we use the temperature-dependence on time that follows from the Bjorken-model [40] due to reasons of simplicity. There, the medium temperature depends by a power law on the proper time \( t \) since the creation of the medium at time \( t_0 \):
\[ T(t) = T_0 \left( \frac{t_0}{t} \right)^3 \quad \text{with } T_0 \equiv T(t_0). \tag{22} \]
This model has the advantage that it provides a simple description of the temperature profile in the medium that depends only on two parameters. On the other hand, the temperature dependencies of the medium-properties that are necessary to describe the jet evolution following Eqs. (9) and (20) can be obtained by phenomenological considerations. Works of the jet-collaboration [41] obtained the temperature dependence of the transport parameter $\hat{q}$ as

$$\hat{q}(T) = c_q T^3.$$

The number of scattering centres can be estimated by assuming a medium consisting of fermions and bosons at the thermal equilibrium, i.e. by assuming the Fermi–Dirac/Bose–Einstein distributions for the densities of quarks, anti-quarks and gluons, $n_q$, $n_{\bar{q}}$, and $n_g$, respectively. As can be shown, cf. e.g. Eq. (3.14) in [42], the Taylor-expansion in $T$ yields the number densities as the cubic power of $T$ at the lowest orders in $T$, so that one can write

$$n(T) = n_q + n_{\bar{q}} + n_g = c_n T^3.$$

For the Debye-mass $m_D$ we assumed that $m_D \propto g T$ which is consistent with findings of the hard-thermal-loop (HTL) approach. In particular, following [43], we use the relation

$$m_D^2 = \left( \frac{N_C}{3} + \frac{N_F}{6} \right) g^2 T^2.$$

With respect to Ref. [31], MINCAS itself has not been modified, therefore the program uses constant values for the transport parameter $\hat{q}$, the number densities $n$, and the Debye mass $m_D$. To this end, we use their average values, assuming that the jets evolve within the medium from the time-scale $t_0$ to the timescale $t_L$, i.e.

$$\langle \hat{q} \rangle = \frac{c_q}{t_L - t_0} \int_{t_0}^{t_L} d\tau T_0 \frac{t_0}{\tau} = \frac{c_q T_0^3 \ln (t_L/t_0)}{t_L/t_0 - 1},$$

$$\langle n \rangle = \frac{c_n T_0^3 \ln (t_L/t_0)}{t_L/t_0 - 1},$$

$$\langle m_D \rangle = \sqrt{ \left( \frac{N_C}{3} + \frac{N_F}{6} \right) \frac{4\pi^2}{N_C} \frac{3}{2} \frac{\alpha}{2} \left( \frac{t_L}{t_0} \right)^{2/3} - 1} T_0.$$

### 3 Numerical results

#### 3.1 Jet-quenching and medium parameters

As outlined in Subsection 2.3, the effective model for the medium and jet-medium interactions depends on five parameters, which additionally yield three values that are currently used as parameters within MINCAS. The parameters need to be tuned by comparison to data. The most inclusive widely studied observable is the nuclear modification ratio $R_{AA} \equiv \sigma_{AA}/\sigma_{pp}$. Of the aforementioned five parameters, $c_q$ is fixed by phenomenological considerations described in [41]. Also $c_n$ is fixed, as it is
the first coefficient in the Taylor-series expansion of $n(T)$. Thus, there remain three free parameters: $t_0$, $t_L$ and $T_0$. We fix $t_0$ to 0.6 fm/c (we assume that the medium is thermalised at this time). Assuming that a medium of a diameter of the order of 10 fm is created and that most jets are created in the centre of the colliding particles and pass the medium with a velocity close to the speed of light (i.e. ultrarelativistic jet particles), we set the parameter $t_L$ to 5 fm/c. The temperature $T_0$ at $t_0$ is then varied in order to reproduce experimental data on the jet-quenching. First, Fig. 2 shows the experimental data from ATLAS on $R_{AA}$ for jets in the Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [44] and 5.02 TeV [45] in comparison with the results from jet simulations with our combination of the KATIE and MINCAS Monte Carlo generators. In order to reproduce the data, the value of $T_0$ was tuned to 400 MeV. The results for $R_{AA}$ relying on the gluon TMDs and the gluon PDFs both exhibit a considerable suppression, corresponding to the values of $R_{AA}$ between 0.4 and 0.6, and, in general, show a similar behaviour. However, it can be noted that at the same temperature scales, the suppression for the results based on the gluon PDFs are in general slightly more suppressed than that for the gluon TMDs. As can be seen, both for $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, the experimental data can be reproduced for most data-points within uncertainties by the results that rely on TMDs.

With tuning the temperature scales to the experimental results for $R_{AA}$, we have fixed all model parameters, summarised in Table 1.

| fixed   | free          | resulting        |
|---------|---------------|------------------|
| $c_q$   | 3.7           | $t_0$ 0.6 fm/c   | $\langle \hat{q} \rangle$ 0.54 GeV$^2$/fm |
| $c_n$   | 5.228         | $t_L$ 5 fm/c     | $\langle n \rangle$ 0.154 GeV$^3$ |
|         | $T_0$ 0.4 GeV | $\langle m_D \rangle$ 0.684 GeV |

Table 1: Parameters for the medium model: the parameters from theoretical/phenomenological considerations (left), the freely adjustable parameters (middle) and the resulting medium parameters used for MINCAS (right).

### 3.2 Di-jet results

The use of TMDs in the calculation of the hard-scattering process allows to study observables which show that the di-jet production is not back-to-back in the transverse plane. In our results, we compare the production of jet-pairs in hard collisions without and with further in-medium evolution. The former case, labelled as the “vacuum” case, corresponds to, e.g. the di-jet production in the proton–proton collisions and was obtained numerically by the use of KATIE alone. Thus, it already contains an asymmetry in the transverse momenta $k_T$ of the jets due to the use of TMDs instead of PDFs. The latter case, labelled as the “medium” case, may contain additional $k_T$-broadening effects of the jet-axes due to the jet–medium interactions. We have obtained the results for the “medium” case by propagating within MINCAS the gluons produced in the hard collisions by KATIE, where the gluon-fragmentation functions follow Eqs. (9) and (20). To further investigate the in-medium $k_T$-broadening we have simulated two different cases:

1. A case, where the jet in-medium fragmentation follows Eq. (9). Inside Eq. (9), $C(q)$ yields the broadening of momenta transverse to the jet-axis. We call this case the “non-Gaussian $k_T$-broadening".
Figure 2: The nuclear modification factor $R_{AA}$ as a function of the jet transverse momentum $p_T$ for the Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in the mid-forward $1.2 < |y| < 2.1$ region (upper plot) and for the Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV in the $|y| < 2.8$ region (lower plot), both obtained with the initial-state PDFs and TMDs as indicated in comparison to the LHC data taken from [44] and [45].
Figure 3: **Upper plot:** Azimuthal di-jet decorrelations for collisions at $\sqrt{s_{\text{NN}}} = 5.02\text{ TeV}$ between central and forward jets, as indicated. The results are obtained for the proton–proton collisions (vacuum – the dashed blue line) as well as for the Pb–Pb collisions with the $k_T$-broadening as in MINCAS (Non-Gaussian – the solid red line) and the Gaussian $k_T$-broadening (the dotted black line). **Lower plot:** The ratio of the above distributions for the non-Gaussian to Gaussian $k_T$-broadening.
2. A case, where the the loss of jet-momentum components along the jet-axis, \( \vec{q}_i \) \((i = 1, 2)\), follows Eq. (20). Subsequently, the transverse momentum component \( l_i \) \( \perp \vec{q}_i \) is selected (for each jet-momentum individually) from the Gaussian distribution. As it can be argued that the absolute value of the total transverse-momentum transfer is of the order of \( \sqrt{q_{tL}} \), \( ||l_i|| \) is selected from the Gaussian distribution

\[
P(||l_i||) = \frac{1}{\sqrt{2\pi q_{tL}}} \exp\left(-\frac{l_i^2}{2q_{tL}}\right).
\] (29)

The azimuthal angle of the outgoing momenta \( p_i \) with regard to \( q_i \) is selected randomly from a uniform distribution in the range from 0 to 2\( \pi \). We label the resulting set of jets with “Gaussian \( k_T \)-broadening”.

We first study the azimuthal decorrelation \( dN/d\Delta\Phi \), given as the number of jet-pairs, where \( \Delta\Phi \) is the difference in azimuthal angles of the jet-axes. The results are shown in Fig. 3 for di-jets, where one jet is emitted in the forward-rapidity region \((2 < y < 3)\), while the other one is emitted in the central-rapidity range \((-1 < y < 1)\). It can be seen that the production of jet-pairs is clearly suppressed in the medium as compared to the production of jets without the subsequent in-medium propagation. While, compared to the vacuum, the \( dN/d\Delta\Phi \) values in the medium are similarly suppressed, whether we assume the Gaussian or non-Gaussian \( k_T \)-broadening, small differences in the behaviour of both curves occur, which can be made more visible by normalising the curves for \( dN/d\Delta\Phi \) to the values at their respective maximums. These results are shown in Fig. 4. The case with the non-Gaussian \( k_T \)-broadening exhibits a clear broadening in \( \Delta\Phi \) as compared to the vacuum case, while the case with the Gaussian \( k_T \)-broadening mostly follows the behaviour of the vacuum case.

Figure 4: Same as in Fig. 3 (upper plot), but now normalised to the maximum of the distribution.
Finally, we calculate the distribution \( dN/dA_j \) over the di-jet asymmetry \( A_j \) defined as

\[
A_j = \frac{p_{Tc} - p_{Tf}}{p_{Tc} + p_{Tf}},
\]

where \( p_{Tc} \) is the transverse momentum of the jet in the direction of the central rapidity, while \( p_{Tf} \) is the transverse momentum of the other jet in the direction of the forward rapidity. The results for collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV are shown in Fig. 5. As can be seen, the distributions for the jets inside the medium are largely suppressed and as compared to the vacuum result the slope of the distribution for smaller values of \( A_j \) does not change much. This indicates that the back-to-back configuration in the medium is less probable than in the vacuum. On the other hand, the differences between Gaussian and non-Gaussian \( k_T \)-broadening are comparatively small for these jet energies. The curve that corresponds to the non-Gaussian \( k_T \)-broadening is slightly less suppressed and broader than that for the Gaussian \( k_T \)-broadening.

4 Conclusions and outlook

We have developed a Monte Carlo algorithm for ultrarelativistic nuclear collisions which combines both a hard-scattering process depending on transverse momenta of partons within nucleons and an in-medium evolution for jet-particles. In order to allow for a reasonable jet in-medium evolution, we describe the medium with a simplified effective model that relies merely on three parameters: the
time of the beginning of the jet in-medium evolution $t_0$, the time of its end $t_L$ and the temperature $T_0$ at the time $t_0$. This algorithm combines the previously developed frameworks of KATIE [35] and MINCAS [31], and so far is restricted to the production of gluons only.

With the corresponding tuning of $T_0$, it was possible to reproduce the experimental results for the jet nuclear modification factor $R_{AA}$ for the central Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV as well as at $\sqrt{s_{NN}} = 5.02$ TeV from the ATLAS Collaboration at the LHC. Using the algorithm in this calibration, we were able to make predictions for the di-jet asymmetries $A_j$ and the azimuthal di-jet decorrelations $dN/d\Delta\Phi$ in the proton–proton as well as Pb–Pb collisions, where we compared the results for the Gaussian $k_T$-broadening in the medium with that of the non-Gaussian $k_T$-broadening which follows from Eq. (9). As, in general, the di-jet production is suppressed in the medium, the distributions $dN/dA_j$ were considerably suppressed in the Pb–Pb collisions, however with much smaller differences between the Gaussian and non-Gaussian $k_T$-broadening, where the results indicate a broader $A_j$ distribution for the non-Gaussian case. Our main result is that for the azimuthal decorrelations we have found that the non-Gaussian $k_T$-broadening leads to a considerable broadening of the shape of $dN/d\Delta\Phi$ as compared to the Gaussian $k_T$-broadening (as well as to the case of proton–proton collisions alone). Such a behaviour also differs from the one for the Gaussian $k_T$-broadening, which – while considerably suppressed – does not exhibit a broader distribution over $\Delta\Phi$, similarly as the vacuum case.

In the future, we plan to extend our framework (at first in the Gaussian approximation as is done in [14]) to account for quarks which have been neglected in the current study. Furthermore, we plan to investigate the broadening due to multiple scatterings in the more forward rapidity region, which is advocated in Ref. [26]. However, this may require accounting for saturation effects [46] which together with the Sudakov effects act to generate considerable broadening in the final-state observables of the p–p and p–Pb collisions [47]. It will be interesting to see the combined effect of the two kinds of broadening.

### A Algorithm

Here we outline the algorithm that simulates a hard-scattering process that yields two hard gluons which propagate as leading jet-particles through a medium. From a technical point of view, this algorithm merges two Monte Carlo programs: KATIE and MINCAS.

First, KATIE is executed. The centre-of-mass energy per nucleon of the hard collision $\sqrt{s_{NN}}$ needs to be set (2.76 TeV and 5.02 TeV for this work) as well as constraints for the minimum values of the $p_T$ of outgoing particles after the hard collisions. In general, the phase-space boundaries for the hard process, simulated by KATIE, are set larger than those of the finally obtained set of particles (which involves MINCAS as well. Also the factorisation scale is set and the sets of TMDs/PDFs are specified. For this article we used as scale average of transverse momenta of di-jets.

Then, the following steps are repeated for each of the events stored in the output files of KATIE:

1. From the KATIE output files every event is read in individually. Essential for the algorithm are the outgoing-particle energy $E_i$ and three-momenta $\vec{q}_i$ (with $i = 1, 2$) as well as the weight $w$ of the event.
2. Polar coordinates for the outgoing-parton three-momenta $\vec{q}_i \equiv (q_{ix}, q_{iy}, q_{iz})$ \((i = 1, 2)\) are calculated as:

$$q_i = \sqrt{q_{ix}^2 + q_{iy}^2 + q_{iz}^2}, \quad (31)$$

$$\theta_i = \arccos \frac{q_{iz}}{q_i}, \quad (32)$$

$$\phi_i = \arctan \frac{q_{iy}}{q_{ix}}. \quad (33)$$

The outgoing particles in KATIE are on the mass-shell, i.e. $E_i = q_i$.

3. The following steps are performed for each of the two outgoing particles \(i = 1, 2\) individually:

(a) Initialise MINCAS for the particle \(i\). There, as a single parameter from KATIE, the particle energy \(E_i\) (before propagation through the medium) is passed in order to calculate \(t^*\) via Eq. (11)\(^4\).

(b) The generation of a MINCAS-event – the function MINCAS_GenEve is executed.

(c) MINCAS_GenEve yields the fraction $\bar{x}_i$ with regard to the light-cone energy in the jet-frame\(^5\)

$$\bar{q}_i^+ = 2E_i$$

as well as the momentum components \(l_{ix}\) and \(l_{iy}\) in the directions transverse to \(\bar{q}_i\). Furthermore, to each jet, a weight \(\omega_i\) is associated.

(d) The energy \(E_{p_i}\) and the momentum component \(p_{iz}\) after the in-medium propagation are obtained in the jet-frame as:

$$E_{p_i} = \bar{x}_i q_i^+ + \frac{l_i^2}{4\bar{x}_i q_i^+},$$

$$p_{iz} = \bar{x}_i q_i^+ - \frac{l_i^2}{4\bar{x}_i q_i^+}, \quad (34)$$

while the transverse components of $\vec{p}_i$ are given as $l_i = (l_{ix}, l_{iy})$.

(e) The new momentum $\vec{p}_i$ is rotated back into the LAB frame:

$$\begin{pmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{pmatrix}_{\text{lab}} = \begin{pmatrix} \cos \phi_i & -\sin \phi_i & 0 \\ \sin \phi_i & \cos \phi_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta_i & 0 & \sin \theta_i \\ 0 & 1 & 0 \\ -\sin \theta_i & 0 & \cos \theta_i \end{pmatrix} \begin{pmatrix} k_{ix} \\ k_{iy} \\ p_{iz} \end{pmatrix}_{\text{jet}}. \quad (35)$$

(f) Then, $p_{iT}$ and $y_i$ in the LAB frame are obtained as:

$$p_{iT} = \sqrt{p_{ix}^2 + p_{iy}^2}, \quad (36)$$

$$y_i = \frac{1}{2} \log \frac{E_{p_i} + p_{iz}}{E_{p_i} - p_{iz}}. \quad (37)$$

\(^4\)This required some small changes in the original function MINCAS_Init, which is now called MINCAS_Init_for_KaTie.

\(^5\)For this purpose, we define the jet-frame as the one obtained after the rotation of the coordinate system in the LAB frame, such that $\bar{q}_i$ is parallel to the $z$-axis of the new coordinate system.
4. Finally, the event is written as the following three lines into the output-file:

\[ E_1, q_{1x}, q_{1y}, q_{1z}, \]
\[ E_2, q_{2x}, q_{2y}, q_{2z}, \]  
\[ p_{1T}, y_1, \phi_1, x_1, p_{2T}, y_2, \phi_2, x_2, \omega_1, \omega_2, w. \]  

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