New Mathematical Model Based on Affine Transformation for Remote Sensing Image with High Resolution

ZHANG Jianqing  ZHANG Zuxun

1 Introduction

The rapid development of the economy and the society requires the suitable three-dimensional representation for districts, states and the earth. The digital elevation model (DEM) and georeferenced remote sensing images with 1-meter resolution are the base. Recently, the most effective means is remote sensing technology. That is, the spatial information of the earth is captured from the images taken by various sensors installed in the space vehicles. A series of lunch plans for acquiring the remote sensing images with high resolution have been executed. Thus, it becomes possible that this basic spatial information can be obtained using the remote sensing images with high resolution at a low cost.

In order to process the captured remote sensing image as georeferenced image, the first step is calculating the parameters of image position and orientation. After the parameters have been computed, the original image can be rectified precisely based on the corresponding DEM, so that the image becomes georeferenced. However, because the photographic station of the remote sensing images with high resolution is very high, and the photographic viewing angle is very small, there is very strong relativity between their traditional position and orientation parameters. Because of the very strong relativity of the image parameters, the calculation of the image parameters is not solved perfectly all the time. Although many algorithms for overcoming the strong relativity, such as grouping iteration, combining relative items, etc., have been proposed, the reasonable solution can not be obtained in some cases. Up to now, the algorithm, which is used relatively frequently, is the fitting on
the basis of reasonable polynomials, proposed by Kratky.[1,2] It is an approximate method. If the better result is desired, the polynomials with higher order should be applied, therefore, more control points should be needed. Even though, the reasonable solution can not be obtained sometimes.

Okamoto[3,4,5,6] had proposed a model based on affine projection. Susumu Hattori[7] and Tetsu Ono[8] had further investigated and used the model. Under the hypothesis that the central projection is approximately the same as the parallel projection in the case of the small viewing angle, the calculation of position and orientation parameters using the affine model can overcome the strong relativity between their position and orientation parameters of SPOT images, it is effective in the case of the maps on smaller scale from SPOT images, which demands the lower precision, but it is an approximate method nonetheless. The image with higher resolution is incompletely the same as the SPOT's. Its viewing angle is quite smaller, and the relativity of the parameters must be quite stronger. It is necessary to investigate whether the method will be suitable for the case of the maps on larger scale from the images with about 1-meter resolution, which demands the higher precision. Therefore, searching a strict mathematical model of the calculation for the position and orientation parameters of the remote sensing image with higher resolution, and solving the problem completely are quite important for its application.

The mathematical model of the calculation for the position and orientation parameters of the remote sensing image with higher resolution, proposed in this paper, adopts the method with three steps of transformations. Recently, the remote sensing image with higher resolution is similar to SPOT image, which is imaged by pushbrooming ahead with the linear array CCD. That is, it is central projection in the scanning direction, and parallel projection in the flight direction. The first step of the mathematical model is reducing the three dimensional space to the image space by the similar transformation. Then, the small space is project ed to the level plane which passes the center of the image plane by parallel rays (Affine transformation). Finally, the level image is transformed to the original declining image. Every step of the new method is strict, and the map function of each transformation is the first order polynomials. The final calculation of the parameters is for the linear equations with good status. As a result, the problem of the relativity of image parameter calculation is solved completely.

2 Imaging geometry of parallel ray projection

As shown in Fig. 1, the intersection line of the ground level plane and the plane consisting of principle ray $S_0X_0$ and straight-line $x_0x'$ passed by push-broom linear array is $X_0X'$. The perpendicular of $X_0X'$, which passes $S$, crosses $X_0O$ at $O$. In coordinate system $O-XYZ$, $OY'$ is $X$-axes, and $OS$ is $Z$-axes.

![Fig. 1 Imaging geometry of parallel ray projection](image)

The straight line, which passes point $O$ and parallels the motion direction of CCD linear array, is $Y$-axes (it is possible that the $Y$-axes is not perpendicular to plane $O-XZ$). The coordinates of the point $O$ in ground coordinate system $O_xX_yX_z$ are $(X_0Y_0Z_0)$. Then, the relationship between the coordinates of a ground point $P$ in two coordinate systems $(X, Y, Z)$ and $(X_g, Y_g, X_g)$ is a three-dimensional affine transformation:

$$\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = \begin{pmatrix}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{pmatrix} \begin{pmatrix}
X_g - X_0 \\
Y_g - Y_0 \\
Z_g - Z_0
\end{pmatrix}$$ (1)
If let \( P_x \) parallel the principle ray \( Sx_0 \), and the surface profile of the ground is projected to the image plane \( x'y_0 \) in the same direction of \( Sx_0 \), then

\[
x' - x_0 = (X_0 - X_1) \cos \alpha = (X + Z \sin \alpha - X_1) \cos \alpha \tag{2}
\]

\[
y' - y_0 = Y \tag{3}
\]

From Eqs. (1), (2) and (3), we have

\[
x' - x_0 = (r_{11}(X' - X_0) + r_{12}(Y' - Y_0) + r_{13}(Z' - Z_0)) \sin \alpha - X_1 \cos \alpha \tag{4}
\]

\[
y' - y_0 = r_{21}(X' - X_0) + r_{22}(Y' - Y_0) + r_{23}(Z' - Z_0) \tag{5}
\]

namely,

\[
x' = a_0 + a_1 X + a_2 Y + a_3 Z \tag{6}
\]

\[
y' = b_0 + b_1 X + b_2 Y + b_3 Z \tag{7}
\]

Eq. (6) and Eq. (7) show that imaging by parallel ray projection is the affine transformation from three-dimensional to two-dimensional.

3. Strict geometric model

3.1 Relationship of image coordinates \((x, y)\) and space coordinates \((X_g, Y_g, Z_g)\)

Within the plane \( XOZ \), perpendicular from \( x_0 \) to \( OZ \) crosses \( OZ \) at \( O' \). Let \( m = SO/\overline{SO'} \). Centering at \( S \), the real surface model is reduced by \( m \) times by similar transformation (Fig. 2), and \((x'', y'')\) are the coordinates of the image by parallel projection. Thus

\[
x'' - x_0 = \frac{(x' - x_0)}{m} \cos \alpha \tag{8}
\]

\[
y'' - y_0 = \frac{(y' - y_0)}{m} = y - y_0 \tag{9}
\]

\[
z = \frac{Z}{m} \tag{10}
\]

If \( f \) is the principal distance, \( a \) is the side watch angle, and \( x \) and \( y \) are the coordinates of image, from Fig. 2 the equations are acquired:

\[
\frac{BC}{B'C'} = \frac{AB}{A'B'} = \frac{SA}{SA'}
\]

That is,

\[
\frac{(x'' - x_0)}{x - x_0} = \frac{f - \frac{Z}{m \cos \alpha}}{f - (x - x_0) \tan \alpha} = \frac{f - \frac{Z}{m \cos \alpha}}{f - (x - x_0) \tan \alpha}
\]

Fig. 2 Imaging profile

From Eqs. (4), (5), (8) and (9), we have

\[
f - \frac{Z}{m \cos \alpha} (x - x_0) = a_0 + a_1 X + a_2 Y + a_3 Z \tag{10}
\]

\[
(y - y_0) = b_0 + b_1 X + b_2 Y + b_3 Z \tag{11}
\]

Eqs. (10) and (11) express the strict mathematical relationship of the image coordinates \((x, y)\) and the space coordinates \((X_g, Y_g, Z_g)\).

3.2 Relationship of space coordinates \((X_g, Y_g, Z_g)\) with left and right image coordinates \((x_l, y_l)\) and \((x_r, y_r)\)

Let subscripts \( l \) and \( r \) denote the elements of the left and right images, respectively, four linear equations can be acquired from Eqs. (10) and (11) and the coordinates \((x_l, y_l)\) and \((x_r, y_r)\) of left and right images:
Eqs. (12) to (15) are the strict mathematical relationship of the space coordinates \((X_g, Y_g, Z_g)\) with the left and right images' coordinates \((x_l, y_l)\) and \((x_r, y_r)\).

### 4 Calculation strategy

#### 4.1 Calculation of parameters

Because \(a\) in the left of Eq. (10) is unknown, the equation is not linear. The calculation procedure is iterative based on the linearization. For simplifying, let \(x\) denote \(x - x_0\), \(y\) denote \(y - y_0\), \(X\) denote \(X_g\), \(Y\) denote \(Y_g\) and \(Z\) denote \(Z_g\) in the next part of this paper. The Eq. (10) is linearized as the following error equation:

\[
a_0 + X_0 + Y_0 + Z_0 \frac{Z_i \sin a}{m (f - x_i \tan a \cos^2 a)} \frac{Z_i (f - Z_i (m \cos a))}{(f - x_i \tan a \cos^2 a)} \frac{Z_i \sin a}{m (f - x_i \tan a \cos^2 a)} = 0
\]

Using error equation (16) and more than five control points, \(a, a_0, a_1, a_2, a_3\), whose initial values are 0, can be solved iteratively.

From Eq. (11), the linear equation can be acquired:

\[
b_0 + Xb_0 + Yb_1 + Zb_2 = y_0 - y_i = 0
\]

By using Eq. (17), \(b_0, b_1, b_2\) and \(b_3\) can be solved directly without the iteration.

#### 4.2 Calculation of image coordinates \((x, y)\) from space coordinates \((X, Y, Z)\)

After \(a, a_0, a_1, a_2, a_3, b_0, b_1, b_2\) and \(b_3\) are computed by using control points, following equations can be adopted for calculating the image coordinates \((x, y)\) from space coordinates \((X, Y, Z)\).

\[
x = (a_0 + a_1 X + a_2 Y + a_3 Z) \frac{f - x \tan a}{m \cos a} = 0
\]

4.3 Calculation of space coordinates \((X, Y, Z)\) from the left and right images coordinates \((x_l, y_l)\) and \((x_r, y_r)\)

Algorithm 1: From Eq. (12) to Eq. (15), the linear equations can be acquired:

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} + \frac{x_l}{m \cos \alpha_i (f - x_l \tan \alpha_i)} \\
b_{11} & b_{12} & b_{13} \\
a_{21} & a_{22} & a_{23} + \frac{x_r}{m \cos \alpha_i (f - x_r \tan \alpha_r)} \\
b_{21} & b_{22} & b_{23}
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
Z
\end{bmatrix}
= \begin{bmatrix}
a_0 \\
y - y_0 \\
Y - y_i
\end{bmatrix}
\]

or denoted as \(AX = L\), and then the resolution is \(X = (A^T A)^{-1} A^T L\).

Algorithm 2: Combining Eq. (13) with Eq. (12), Eq. (15) with Eq. (14), expunging \(Y\), the 1-order equations with two unknowns \(X\) and \(Z\) are determined, from which \(X\) and \(Z\) can be solved. Then \(Y_1\) and \(Y_2\) can be computed out and \(Z\) from Eqs. (13) and (15). Therefore

\[
Y = (Y_1 + Y_2)/2
\]

In this way, \(Y_1 - Y_2\) can be used for evaluating the quality of the solution.

### 5 Experiments

By the mathematical model of remote sensing image with high resolution mentioned above, some experiments are carried on for more than ten pairs of SPOT images, some of which can not be processed to generate right result by old algorithms. Some pairs of the remote sensing images with 3-meter and 1-meter resolution, including IKONOS im-
ages, are also used in the experiment. Table 1 shows the RMSE of ground coordinates from the control points after parameter computation of stereo image pair. Table 2 shows the RMSE of ground coordinates from the control and check points after parameter computation of SPOT stereo image pair with 78 known points. The last line of Table 2 is the result with arbitrarily selected 10 points as control points and the other 68 points as check points, which indicates that the solution is very stable. All of the other experimental results are quite perfect as well. Thus, the validity of the new method has been verified.

| Table 1 | Experimental results of stereo image pairs |
|---------|----------------------------------------|
| Pixel ground Point X-rms/m | Y-rms/m | Z-rms/m |
| resolution/m number | |
| 10 21 14.091 | 13.980 | 4.760 |
| 10 12 7.340 | 5.084 | 5.843 |
| 10 78 9.081 | 12.212 | 5.247 |
| 10 9 10.283 | 14.742 | 8.676 |
| 10 9 0.742 | 0.804 | 0.985 |
| 10 5 0.999 | 3.624 | 0.436 |
| 10 5 0.171 | 0.032 | 0.086 |
| 3 15 4.912 | 7.388 | 0.798 |
| 1 12 0.317 | 0.367 | 0.808 |

| Table 2 | Experimental results of SPOT stereo image pairs with check points |
|---------|-------------------------------------|
| Number X-rms/m | Y-rms/m | Z-rms/m |
| Control 67 9.401 | 11.745 | 5.311 |
| point 49 9.537 | 10.898 | 5.378 |
| 30 9.456 | 10.148 | 4.172 |
| 10 5.487 | 5.856 | 3.989 |
| 11 7.745 | 15.282 | 4.753 |
| Check 29 11.242 | 15.397 | 6.374 |
| point 48 10.553 | 13.853 | 7.111 |
| 68 11.749 | 15.553 | 6.072 |

6 Conclusions

The mathematical model presented above is strict in theory, which needs no prior parameters of sensor trajectory except more than four control points. The experimental results show that the parameter computation of remote sensing image with high resolution is very stable. The problem of the relativity of image parameter calculation is solved completely by the mathematical model.

Acknowledgements

Thanks Gene Dial of SPACEIMAGE for providing the IKONOS data and materials.

References

1. Kratky V (1989a) Rigorous photogrammetric processing of SPOT images at CCM Canada. ISPRS Journal of Photogrammetry and Remote Sensing, 44:53-71
2. Kratky V (1989b) On-line aspects of stereophotogrammetric processing of SPOT images. PE & RS, 55(3):311-316
3. Okamoto A (1981) Orientation and construction of models, part III: mathematical basis of the orientation problem of one-dimensional central-perspective photographs. PE & RS, 47(12):1739-1752
4. Okamoto A (1988) Orientation theory of CCD line-scanner images. International Archives of ISPRS, 27(B3):609-617
5. Okamoto A, Akamatsu S (1992a) Orientation theory for satellite CCD line scanner imagery of mountainous terrain. International Archives of ISPRS, 29(B2):205-209
6. Okamoto A, Akamatsu S (1992b) Orientation theory for satellite CCD line scanner imagery of hilly terrain. International Archives of ISPRS, 29(B2):217-221
7. Hattori S, Ono T, Fraser C, et al. (2000) Orientation of high-resolution satellite images based on Affine projection. International Archives of ISPRS 2000 Congress, XXXIII(B3):359-366
8. Ono T, Hattori S, Hasagawa H, et al. (2000) Digital mapping using high resolution satellite imagery based on 2D Affine projection model. International Archives of ISPRS 2000 Congress, XXXIII(B3):672-677