On the Determination of Spin–Dependent Parton Distributions in Semi–Inclusive Deep Inelastic Scattering

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Abstract

New polarized fragmentation functions are introduced and justified, in addition to those conventional ones assumed to be independent of the helicity of the parent parton. It is demonstrated that due to our present ignorance concerning these new parton–spin dependent leading–twist fragmentation functions, it is impossible to utilize current experiments on spin–dependent semi–inclusive deep inelastic lepton nucleon scattering to disentangle the separate polarized parton distributions.
Semi–inclusive deep inelastic scattering (SIDIS) in, say, $ep \to ehX$ reactions depends on the parton distributions in the proton, $f(x, Q^2) = u, \bar{u}, d, \bar{d}, \ldots$, as well as on their fragmentation functions $D^h_f (z, Q^2)$ into the (unpolarized) hadron $h (= \pi, K$ dominantly). A common assumption [1] concerning the fragmentation functions is their mere dependence on $f$ irrespective of its origin. This is the basis underlying the factorized structure of SIDIS cross sections which in leading order (LO) of perturbative QCD are:

$$\frac{d\sigma^h}{dx \ dy \ dz} = \frac{2\pi\alpha^2}{Q^2} \frac{1 + (1 - y)^2}{y} F^h_1 (x, z, Q^2)$$

(1)

for the unpolarized SIDIS process $eN \to ehX$, and

$$\frac{d\Delta\sigma^h}{dx \ dy \ dz} = \frac{2\pi\alpha^2}{Q^2} (2 - y) 2g^h_1 (x, z, Q^2)$$

(2)

for the polarized SIDIS process $e\bar{N} \to ehX$, with $x, y, z$ the common scaling variables and $Q^2 = xy s$. The factorized structure is expressed in LO via

$$2F^h_1 (x, z, Q^2) = \sum_{f=q,\bar{q}} e_f^2 f(x, Q^2) D^h_f (z, Q^2)$$

(3)

$$2g^h_1 (x, z, Q^2) = \sum_{f=-q,\bar{q}} e_f^2 \Delta f(x, Q^2) D^h_f (z, Q^2)$$

(4)

with $f = f_+ + f_-$ and $\Delta f = f_+ - f_-$ are the usual unpolarized and polarized parton distributions of the nucleon and $D^h_f$ their common fragmentation functions into $h = \pi, K, \ldots$. Considering, for example, a nucleon with helicity $+\frac{1}{2}$, its partons with positive and negative helicities are described by $f_\pm$. The spin–independent and spin–dependent $ep \to ehX$ SIDIS cross sections $\sigma$ and $\Delta\sigma$ are defined in terms of cross sections of definite positive and negative helicities of the initial electron and nucleon, $\sigma_{\lambda_e,\lambda_N}$, according to $4\sigma = \sigma_{++} + \sigma_{+-} + \sigma_{-+} + \sigma_{--} = 2(\sigma_{++} + \sigma_{+-})$ and $4\Delta\sigma = \sigma_{++} - \sigma_{+-} + \sigma_{-+} - \sigma_{--} = 2(\sigma_{++} - \sigma_{+-})$, respectively, where $\sigma_{\lambda_e,\lambda_N} = \sigma_{-\lambda_e,-\lambda_N}$ due to parity conservation of the strong (QCD) interactions.

These standard results rely on the assumption that the fragmentation function $D^h_f$ is independent of the helicities of the fragmenting partons, $f_\pm$, i.e., that the hadronization
process is the same for \( f_+ \) and \( f_- \). This is obviously only correct as long as one considers a single quark (parton) fragmenting into hadrons independently of the remnant ‘spectator’ core (in this case parity conservation gives \( D^h_{f_+} = D^h_{f_-} \)) which is a mere approximation and needs not necessarily hold true in general. Indeed, the hadronization process is due to the separation of two colored objects, the struck (anti)quark and the ‘spectator’ core, and quark–antiquark pair creations in the vacuum are then generated by the increasing potential energy of these separating colored objects as illustrated in Fig. 1 where the helicities of the struck quark, the core and the initial nucleon are specified explicitly. The process \( \gamma^* N \to \pi N X \) is represented by Fig. 1(a) which corresponds to the fragmentation function \( D^{\pi^+}_{q_+}(z, Q^2) \), while Fig. 1(b) represents a further possible process \( \gamma^* N \to \pi \Delta X \) in this channel which corresponds to the fragmentation function \( D^{\pi^-}_{q_-}(z, Q^2) \). It is conceivable that the possible additional occurrence of the heavier \( \Delta \) resonances in Fig. 1(b) results in \( D^{\pi^+}_{q_+}(z, Q^2) \neq D^{\pi^-}_{q_-}(z, Q^2) \), i.e. \( \Delta D^{\pi}_{q}(z, Q^2) \equiv D^{\pi^+}_{q_+} - D^{\pi^-}_{q_-} \neq 0 \). Although such effects may be relevant at any value of \( Q^2 \), they are particularly expected in the soft non–perturbative low–\( Q^2 \) region, \( Q^2 < Q^2_0 = O(1 \text{ GeV}^2) \), where the available phase space \( W^2 = Q^2(1/x - 1) + M_N^2 \) is limited, inducing the boundary conditions \( \Delta D^h_f(z, Q^2_0) \neq 0 \). Clearly, due to our present inability to calculate non–perturbative fragmentation effects, the magnitude of \( \Delta D^h_f(z, Q^2) \) cannot be predicted but has to be determined experimentally.

It thus seems that in addition to the distributions

\[
D^h_f(z, Q^2) \equiv D^h_{f_+}(z, Q^2) + D^h_{f_-}(z, Q^2),
\]

appearing in the common Eqs. (3) and (4), one should consider the effects due to a possible nonvanishing

\[
\Delta D^h_f(z, Q^2) \equiv D^h_{f_+}(z, Q^2) - D^h_{f_-}(z, Q^2).
\]

Notice that the discussion above, motivating \( \Delta D^h_f \neq 0 \), only serves as an illustration for possible non–perturbative helicity correlation effects which are neither due to a direct quark–core interaction nor due to higher–twist contributions: \( D^h_{f_{\pm}} \) in (5) and (6) are stan-
standard leading twist–two distributions obeying the usual leading twist evolution equations \( Q^2 \geq Q_0^2 \):

\[
\dot{D}^h_{f\pm}(z, Q^2) = \sum_{f'} \left[ P_{f'\pm f\pm} \otimes D^h_{f'\pm} + P_{f'\pm f\mp} \otimes D^h_{f'\mp} \right] \tag{7}
\]

where \( f, f' = q, \bar{q}, g \), \( \dot{D} \equiv dD/d\ln Q^2 \) and \( \otimes \) denotes the usual convolution integral.

Using parity conservation in QCD (\( P_{f'\pm} = P_{f'\mp} \)) and taking the difference of the two equations in (7) gives

\[
\Delta \dot{D}^h_{f\pm}(z, Q^2) = \sum_{f'} \int_z^1 dy \frac{dy}{y} \Delta P_{f'\pm f}(y, Q^2) \Delta D^h_{f'\pm} \left( \frac{z}{y}, Q^2 \right) \tag{8}
\]

for the evolution of the polarized fragmentation function \( \Delta D^h_{f\pm} \) in (6) where \( \Delta P_{f'\pm f} = P_{f'\pm f\pm} - P_{f'\pm f\mp} \) and in LO \( \Delta P_{f'\pm f}(y, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \Delta P_{f'\pm f}^{(0)}(y) \). The sum of the two evolution equations in (7) results in the well known evolution equations for the unpolarized (spin–averaged) fragmentation functions \( D^h_{f\pm} \) in (5) where \( \Delta P_{f'\pm f} \) in (8) is replaced by the unpolarized (spin–averaged) splitting functions \( P_{f'\pm f} = P_{f'\pm f\pm} + P_{f'\pm f\mp} \).

The contribution of these distributions to \( F_1 \) and \( g_1 \) may be inferred directly from Fig. 1. Inspection of this figure immediately implies that \( F_1(g_1) \) are obtained by summing (substracting) the contributions from Fig. 1(a) and 1(b) which are proportional to \( f_+ D^h_{f\pm} \) and \( f_- D^h_{f\mp} \), respectively, thus yielding

\[
2F^h_1(x, z, Q^2) = 2 \sum_{f=q, \bar{q}} e_f^2 \left[ f_+ D^h_{f\pm} + f_- D^h_{f\mp} \right] = \sum_{f=q, \bar{q}} e_f^2 \left[ f(x, Q^2) D^h_{f\pm}(z, Q^2) + \Delta f(x, Q^2) \Delta D^h_{f\pm}(z, Q^2) \right] \tag{9}
\]

\[
2g^h_1(x, z, Q^2) = 2 \sum_{f=q, \bar{q}} e_f^2 \left[ f_+ D^h_{f\pm} - f_- D^h_{f\mp} \right] = \sum_{f=q, \bar{q}} e_f^2 \left[ \Delta f(x, Q^2) D^h_{f\pm}(z, Q^2) + f(x, Q^2) \Delta D^h_{f\pm}(z, Q^2) \right] \tag{10}
\]

where the last equalities in (9) and (10) follow from (5) and (6), and the corresponding definitions \( \Delta f \equiv f_+ - f_- \), \( f \equiv f_+ + f_- \). These expressions reduce to (3) and (4) when
\( \Delta D^h_f = 0 \) as commonly assumed. The consequences of the new terms in (9) and (10), due to \( \Delta D^h_f \neq 0 \), are as follows:

(i) since \( |\Delta f \Delta D^h_f| \ll f D^h_f \), the commonly utilized Eq. (3) provides a good approximation for unpolarized SIDIS;

(ii) due to the possibility that \( |f \Delta D^h_f| \simeq |\Delta f D^h_f| \) in (9), the commonly utilized Eq. (4) may lead to misleading conclusions. Concerning the flavor structure of the polarized partons as extracted from current experiments on polarized \( \bar{c}N \) SIDIS [3, 4, 5, 6] which are analyzed according to (4) obtained under the popular simplifying assumption [7, 8, 9, 10, 11] that \( \Delta D^h_f = 0 \).

Of particular importance is the fact that the conclusions [3, 4] concerning small \( \Delta \bar{q}(x, Q^2) \) distributions could be misleading in magnitude as well as in sign due to the neglect of the \( f \Delta D^h_f \) term in (10)! It is therefore questionable whether even qualitatively very different expectations for the flavor–broken polarized sea densities \( \Delta \bar{u} \) and \( \Delta \bar{d} \), for example, as arising from the relativistic field theoretic chiral–quark soliton model [12, 13] and phenomenological Pauli–blocking ideas [14, 15] or from conventional meson–cloud models [16], can be reliably tested by present conventionally analyzed SIDIS experiments [3, 4, 13, 17]. Our present ignorance concerning \( \Delta D^h_f \) in (10) hinders our ability to extract the desired information from these experiments. In particular it should be clear by now that the quark–core correlation effects may not only affect the size of the fragmentation functions \( \Delta D^h_f \) but also their flavor properties could be affected by these correlations; the flavor structure of \( \Delta D^h_f \) may thus differ from the flavor structure of the spin–averaged fragmentation functions \( D^h_f \).

Similar remarks hold for analyses in next–to–leading order (NLO) of QCD [5, 6, 9, 18, 19] where apart from the unknown polarized fragmentation functions \( \Delta D^h_{u,q} \) also the
gluonic one \( \Delta D^h_\bar{g} \) will enter in addition:

\[
2g^h_1(x, z, Q^2) = \sum_q e_q^2 \{ \Delta q(x, Q^2) D^h_q(z, Q^2) + q(x, Q^2) \Delta D^h_q(z, Q^2) \\
+ \frac{\alpha_s(Q^2)}{2\pi} \left[ \Delta q \otimes \Delta C^{N}_{qq} \otimes D^h_q + q \otimes \Delta C^{H}_{qq} \otimes \Delta D^h_q \\
+ \Delta q \otimes \Delta C^{N}_{gq} \otimes D^h_g + q \otimes \Delta C^{H}_{gq} \otimes \Delta D^h_g \\
+ \Delta g \otimes \Delta C^{N}_{qg} \otimes D^h_g + g \otimes \Delta C^{H}_{qg} \otimes \Delta D^h_g \right] \}
+ (q \rightarrow \bar{q})
\]  

utilizing the notation and results of [19] with \( \Delta C^i_{q\bar{q}} = \Delta C^i_{qq} \), \( \Delta C^i_{g\bar{q}} = \Delta C^i_{gq} \) and \( \Delta C^i_{qg} \). Furthermore, the fragmentation functions now obviously satisfy the NLO two–loop evolution equations which implies for Eq. (8) that

\[
\Delta P^f_{f} (y, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \Delta P_{f}^{(0)} (y) + \left[ \frac{\alpha_s(Q^2)}{2\pi} \right]^2 \Delta P_{f}^{(1)} (y)
\]

with \( \Delta P_{f}^{(1)} \) being the well known polarized two–loop splitting functions which can be found, for example, in [20].

It will not be easy in practice to establish the possible relevance and importance of the \( \Delta D^h_f \) contributions. It could be achieved, at least in principle, for example in SIDIS experiments by analyzing the produced hadrons \( h \) with different energy fractions \( z \), but at fixed Bjorken–\( x \), or applying different \( z \)-cuts in \( \int_{z_0}^{1} D^h_f(z, Q^2) \) when working with integrated purities [3, 11], e.g. \( z_0 = 0.1, 0.2, \) and \( 0.3 \) instead of a fixed \( z_0 = 0.2 \) employed at present. If the observed polarized parton distributions \( \Delta f(x, Q^2) \) remain insensitive to such variations, the separate \( D^h_{f+} \) and \( D^h_{f-} \) will be similar, i.e. \( \Delta D^h_f \simeq 0 \). Alternatively one has to resort to other processes in addition, as for example to polarized hadronic Drell–Yan dilepton production, \( \vec{p} \vec{p} \rightarrow \mu^+ \mu^- X \), or to prompt photon production, \( \vec{p} \vec{p} \rightarrow \gamma X \), for extracting \( \Delta q \) and \( \Delta \bar{q} \) in order to determine \( \Delta D^h_f \) in (10) from SIDIS data.
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Figure 1: Transition of a nucleon with helicity $\lambda N = +\frac{1}{2}$ into a leading quark and a 'spectator' core with helicities (a) $\lambda_q = +\frac{1}{2}$, $\lambda_c = 0$ and (b) $\lambda_q = -\frac{1}{2}$, $\lambda_c = 1$. The corresponding different core fragments may induce nonvanishing polarized fragmentation functions $\Delta D_q^h \equiv D_{q+}^h - D_{q-}^h$. 

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