Secure Quantum Bit Commitment Using Unstable Particles

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Using unstable particles which decay by emitting neutrinos, we propose a quantum bit commitment protocol that is humanly impossible to break. Neutrinos carry away quantum information, but their interaction with matter is so weak that it would take an astronomically-sized machine just to catch them, not to mention performing controlled unitary operations on them. As a result quantum information is lost, and cheating is not possible even if the participants had access to the most powerful quantum computers that could ever be built. Therefore, for all practical purposes, our new protocol is as good as unconditionally secure.

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Bit commitment is a simple cryptographic protocol involving two parties, customarily named Alice and Bob. Alice commits to Bob a secret bit \( b \in \{0, 1\} \) that is to be revealed at some later time. In order to ensure Bob that she will keep her commitment, Alice provides Bob with a piece of evidence with which he can verify her honesty when she unveils. The security of bit commitment is an important issue because it can be used to implement other more complicated cryptographic protocols [1].

A bit commitment protocol is secure if it satisfies the following two conditions. (1) Concealing: Bob cannot determine the value of \( b \) before Alice unveils it; (2) Binding: Alice cannot change \( b \) without Bob’s knowledge. Furthermore, if the protocol remains secure even if Alice and Bob had capabilities limited only by the laws of nature (this is sometimes referred to as the parties having unlimited computational power), then it is said to be unconditionally secure.

Consider a simple example. Alice writes down her bit \( b \) on a piece of paper and locks it in a box, which she gives to Bob as evidence of her commitment. She unveils by announcing the value of \( b \) and giving the key to Bob for verification. This protocol seems secure because Alice cannot change the bit without access to the box, and Bob cannot open the box without the key. However as with other classical cryptographic schemes, it is not unconditionally secure, because, e.g., Bob’s ability to open the box by himself is not in violation of any natural laws. By introducing quantum mechanics into the bit commitment game, one hopes to achieve unconditional security which is guaranteed by the laws of nature. In a quantum bit commitment (QBC) protocol, Alice and Bob execute a series of quantum and classical operations, which results in a quantum state with density matrix \( \rho_B^{(b)} \) in Bob’s hand. If

\[
\rho_B^{(0)} = \rho_B^{(1)},
\]

then the protocol is perfect concealing, and Bob is not able to extract any information about the value of \( b \) from \( \rho_B^{(b)} \). That means \( b \) is encoded in the representation of \( \rho_B^{(b)} \). In the unveiling phase, Alice is required to specify the representation so that Bob can check if she is honest.

It is generally accepted that unconditionally secure quantum bit commitment is ruled out as a matter of principle. This is due to a 1997 no-go theorem [2, 3] which states that, if Alice and Bob have access to quantum computers, then no QBC protocol can be concealing and binding at the same time. Furthermore, it has been shown recently that this is the case even if Bob employs secret parameters unknown to Alice [4, 5].

Given the fact that unconditionally secure QBC is ruled out in theory, it does not follow, however, that all protocols are breakable within human capabilities. This is relevant because QBC is a cryptographic task meant to be implemented in the real world; so if the security of a protocol is humanly impossible to break, then practically it is as good as unconditionally secure, even though it is not in the mathematical sense. The purpose of this paper is to show that the laws of physics permit a level of security which is not jeopardized by even the most powerful possible quantum computers.

Before proceeding further, we briefly review the original arguments leading to the no-go result for the perfect concealing case [2, 3]. (For the near-perfect case where \( \rho_B^{(0)} \approx \rho_B^{(1)} \), see Refs. [2, 3, 4] for more details.) The crucial ingredient is the observation that the whole commitment process, which may involve any num-
ber of rounds of quantum and classical exchanges between Alice and Bob, can be represented by an unitary transformation \( U^{(b)}_{AB} \) on some initial pure state \( |\phi^{(b)}_{AB}\rangle \). Therefore at the end of the commitment phase, there exists a pure state

\[
|\Psi^{(b)}_{AB}\rangle = U^{(b)}_{AB}|\phi^{(b)}_{AB}\rangle
\]

(2)

in the combined Hilbert space \( H_A \otimes H_B \) of Alice and Bob, instead of just a mixed state \( \rho^B_B \) in \( H_B \). \( |\Psi^{(b)}_{AB}\rangle \) is called a quantum purification of \( \rho^B_B \), such that

\[
\text{Tr}_A |\Psi^{(b)}_{AB}\rangle \langle \Psi^{(b)}_{AB}| = \rho^B_B,
\]

(3)

where the trace is over Alice’s share of the state. In this approach, all undisclosed parameters are left undetermined at the quantum level. Note that the implementation of \( U^{(b)}_{AB} \) in general requires Alice and Bob to have access to quantum computers, which is consistent with the assumption that they have unlimited computational power.

The concealing condition, Eq. (1), together with Schmidt decomposition theorem \( \{e_i^A\}, \{f_i^A\}, \{e_j^B\}, \) implies that \( |\Psi^{(0)}_{AB}\rangle \) and \( |\Psi^{(1)}_{AB}\rangle \) can be written as

\[
|\Psi^{(0)}_{AB}\rangle = \sum_i \sqrt{\lambda_i^1} |e_i^A\rangle \otimes |\phi_i^B\rangle,
\]

(4)

\[
|\Psi^{(1)}_{AB}\rangle = \sum_i \sqrt{\lambda_i^1} |f_i^A\rangle \otimes |\psi_i^B\rangle,
\]

(5)

where \( \{e_i^A\}, \{f_i^A\}, \{e_j^B\} \) are orthonormal bases in \( H_A \) and \( H_B \) as indicated. Notice that \( |\Psi^{(0)}_{AB}\rangle \) and \( |\Psi^{(1)}_{AB}\rangle \) are identical except for the bases \( \{e_i^A\} \) and \( \{f_i^A\} \), which are related by an unitary operator \( U_A \):

\[
|f_i^A\rangle = U_A|e_i^A\rangle.
\]

(6)

Hence we also have

\[
|\Psi^{(1)}_{AB}\rangle = U_A|\Psi^{(0)}_{AB}\rangle.
\]

(7)

It is important to note that \( U_A \) acts on \( H_A \) only so that Alice can implement it without Bob’s help. It then follows that she can cheat with the following sure-win strategy (called EPR attack). Alice always commits to \( b = 0 \) in the beginning. Later on if she wants to keep her initial commitment, she simply follows the protocol honestly to the end. Otherwise if she wants to switch to \( b = 1 \) instead, she only needs to apply \( U_A \) to the qubits in her control, and then proceeds as if she had committed to \( b = 1 \) in the first place. Bob would conclude that Alice is honest in either case, because his density matrix \( \rho^B_B \) is not affected by the transformation \( U_A \). Therefore, if a QBC protocol is concealing, it cannot be binding at the same time.

Notice that, in the impossibility proof outlined above, it is implicitly assumed that Alice can maintain full control over her share of the pure state \( |\Psi^{(b)}_{AB}\rangle \) indefinitely after the end of the commitment phase. This is however not possible if the protocol involves unstable particles which can carry quantum information only for a finite period of time. Consider, for example, the neutron \( (n) \) which decays spontaneously via weak interaction \( (\beta-\text{decay}) \) into a proton \( (p) \), an electron \( (e) \), and an anti-electron neutrino \( (\bar{\nu}_e) \),

\[
n \rightarrow p + e + \bar{\nu}_e,
\]

(8)

with a mean lifetime of \( \tau_n = 885.7 \) seconds \( \mathbb{[8]} \). If Alice is required to take certain action on a neutron, it is very unlikely that she could maintain full control over the resulting state for a period much longer than a few \( \tau_n \)’s.

One might argue that, by coherent manipulation of the decay products, it is still possible to control the spin of the neutron after it decays. This is in principle true. However to do so, one must be able to preserve the coherence between the decay products and the rest of the system for an indefinite length of time, which is practically impossible. The reason is that the wave functions of the light particles \( (e \text{ and } \bar{\nu}_e) \) propagate outward in all directions at near light-speed \( c \), so that the volume containing the decay fragments increases with time as \( (ct)^3 \), which would soon enclose the entire earth. Moreover there will be numerous neutrons decaying into the same volume, and one would have to be able to identify and manipulate the wave functions originating from a single neutron, without disturbing the others. On top of this, an even more serious problem is that the (anti-)neutrino participates in weak interactions only. Its interaction with matter is so weak that a “neutrino passing through the entire earth has less than one chance in a thousand billion of being stopped by terrestrial matter” \( \mathbb{[9]} \). That means, on the one hand, the earth is not likely to cause decoherence to the anti-neutrino. On the other hand, one would need a detector a thousand billion times the size of the earth just to catch a particular neutrino, not to mention a machine to perform controlled unitary transformations on it. And there are additional complications, \( \mathbb{e.g.} \), neutrinos change identities due to flavor oscillations \( \mathbb{[10]} \). Certainly, by measuring the momenta of the electron and the proton, one could determine the momentum and spin direction of \( \bar{\nu}_e \), without actually detecting \( \bar{\nu}_e \) itself. However this operation is neither controlled nor unitary, and hence is not useful to the cheating party. From the above discussion, we conclude that, for all practi-
the security parameter, and quantum computers are available or not. Let the laws of physics, independent of whether we shall see, the security of this protocol is guaranteed out loss of generality, we shall take \( J \) with a mean lifetime \( \tau \), and the Cobalt-60 nucleus (\( ^{60}\text{Co} \)) are also unstable against \( \beta \)-decay with mean lifetimes of \( 2.2 \times 10^{-6} \) second [8] and 5.3 years [13] respectively.

In the QBC protocol to be proposed below, we shall generically call the weakly decaying particle \( W \), which could be an elementary particle or atomic nucleus. The \( W \) carries spin \( J \neq 0 \), and it beta decays into a daughter particle \( w \),

\[
W \rightarrow w + e + \bar{\nu}_e, \tag{9}
\]

with a mean lifetime \( \tau_W \). For simplicity, and without loss of generality, we shall take \( J = 1/2 \). As we shall see, the security of this protocol is guaranteed by the laws of physics, independent of whether quantum computers are available or not. Let \( N \) be the security parameter, and

\[
| + \hat{\tau} \rangle = |0\rangle, \quad | - \hat{\tau} \rangle = |1\rangle, \tag{10}
\]

\[
| \pm \hat{\tau} \rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle). \tag{11}
\]

The new protocol is specified as follows.

**Commitment phase:**

1. Bob sends Alice an ordered sequence of \( N \) stable qubits, each drawn independently from the set

\[
\mathcal{B} = \{| + \hat{\tau} \rangle, | - \hat{\tau} \rangle, | + \hat{\tau} \rangle, | - \hat{\tau} \rangle\} \tag{12}
\]

with even probability.

2. To commit to \( b = 0 \), Alice keeps the stable qubits intact. For \( b = 1 \), she swaps the states of the stable qubits into \( N \) unstable \( W \)-states, and measures the momentum of the electron emitted from each \( W \) when it decays.

**Unveiling phase:**

1. Alice unveils the value of \( b \). For \( b = 0 \), she sends the \( N \) stable qubits back to Bob in the original order. For \( b = 1 \), she announces the electron data obtained previously from her measurements. To ensure the security of the protocol, unveiling should take place after a finite fraction of the \( W \)'s has theoretically decayed.

2. Bob verifies Alice's honesty. If \( b = 0 \), he checks if the states of the stable qubits are the same as before. If \( b = 1 \), he calculates the electron asymmetry using Alice's data as follows. Let \( \hat{e}_i \) be the polarization vector of the \( i \)-th \( W \), where

\[
\hat{e}_i \in \{ + \hat{z}, - \hat{z}, \hat{x}, \hat{x} \}. \tag{13}
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momentum data. It is important to note that these

\[
\rho_{B}^{(0)} = \rho_{B}^{(1)} = \rho_{B},
\]

and the protocol is perfectly concealing.

Next we prove that it is binding. As explained before, the quantum information carried by a W is practically lost after its decay. It follows that if Alice first commits to \( b = 1 \) and changes her mind after a finite fraction of the W’s has decayed, her chance of escaping Bob’s detection is exponentially small.

The question remains, if Alice first commits to \( b = 0 \), could she change to \( b = 1 \) without Bob’s knowledge? Obviously the only way to proceed is to swap the states of the stable particles into unstable W’s, and wait for them to decay. However she cannot postpone her decision until the very last moment, because the W’s take time to decay. Suppose Bob wants to bind Alice to her commitment for a period no shorter than \( T \), then the following arrangement is sufficient, though not unique. Bob instructs Alice to use a kind of unstable particles with mean lifetime \( T_w = 10^7 T \), and Alice unveils \( 2T \) after the conclusion of the commitment procedure. In this situation, if Alice commits to \( b = 1 \) at the beginning, then by the time she unveils the average number of W’s decayed is given by

\[
\delta N(2T) = N(1 - e^{-0.2}).
\]

However if she first commits to \( b = 0 \), and changes her mind at a time \( T \) before unveiling, then the number of recorded decay events would be smaller:

\[
\delta N(T) = N(1 - e^{-0.1}).
\]

That means, to unveil \( b = 1 \), Alice would have to artificially generate \( \delta N(2T) - \delta N(T) \approx N/10 \) electron momentum data. It is important to note that these artificial data contribute to the denominator but not the numerator of Eq. (15). As a result, Bob would obtain an asymmetry which is smaller than what it should be by a factor of

\[
F = \delta N(T)/\delta N(2T) \approx 1/2.
\]

In an ideal world where there are no systematic errors, and statistical errors can be made as small as desired, a 1/2 reduction in \( A(\theta, p) \) is a clean and clear signal of cheating by Alice. One can readily show that, for any typical \((\theta, p)\), the chance of obtaining the correct \( A(\theta, p) \) by statistical fluctuation is exponentially small for large \( N \). This concludes the proof that our new protocol is secure. We emphasize that it would remain secure even if Alice had access to the most powerful quantum computer that could ever be built.

In summary, we have constructed a QBC protocol where some of the particles involved are unstable. Unstable particles can carry quantum information only for a finite period of time, and this property turns out to be useful in constructing secure QBC protocols. The idea is that the spontaneous decaying of the unstable particles may render the associated quantum information uncontrollable. If so, then cheating by EPR attack becomes impossible. In the case of any weakly decaying particles emitting neutrinos, controlling the decay products in a coherent manner would require an astronomically-sized quantum computer operating on neutrinos, which is clearly beyond the human capability to build. Therefore, for all practical purposes, our protocol is as good as unconditionally secure.

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