Improved resolution for separation between acoustical transmitters with their locations using Eigenvector algorithm

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Abstract. Through the mathematical model that is derived to localize and separate acoustic sources. We used two methods and compared the results of them: classical method (Fourier transform), with the newer method (Eigenvector) or what represents (super resolution). The results of the comparison revealed the superiority of (Eigenvector) over (Fourier transform) in several aspects, the most important of which are contiguous sources in addition to the location of the sources. The study relied on a set of different criteria directly related to the problem.

1. Introduction
There is a problem facing many applications such as wireless communication, estimating the direction of arrival, etc., which is the estimation of frequencies, angles, locations, with their magnitudes. In order to obtain the data, a synthetic process of aperture scanning of the acoustic receiver was used, which collects a set of data. The non-parametric Fourier transform technique (FT), also known as (traditional spectral estimation) technique, is considered one of the effective techniques in the calculation. Nevertheless, it has some flaws, the most important of which are: the side lobes that may overcome a weak, desired, transmitted signal. Moreover, the side lobes seem to be an additional, undesired, transmitting signals, where it may cause a confusion in discriminating the acoustical sources, as well as the accuracy as it is low resolution achievement when compared with other modern methods [1]. To solve this problem, it requires more data, and for this we need a "long slot" to overcome such spectral leakage. Resolution is the ability of a spectral method to separate the points that make up the object, that is the scene of transmitters. The main goal of the paper is to compare the traditional (Fourier transform) low-resolution method with the modern (super resolution) method (Eigenvector method). Eigenvector based method has an important role in solving the side-lobes problem as well as it has a high resolution [1] The process of localizing the sources along with their separation process has two steps:

1. The recording of the data in the form of amplitude and phase samples of acoustic waves, and then,
2. Process the data and applying one of the spectral estimation methods, either the Fourier transform or the Eigenvector one.

Eigenvector method has different applications [2, 3], as high-resolution technique. The contribution of this paper is the application of EV method for discriminating the small separation between the transmitting sources and their locations with high resolution. Other relating work can be found in [4,5].

2. Field analysis at the Fresnel region.
The problem to be solved is represented by two sound sources, or we can call them (object), which have a field distribution \( \Psi \). Therefore, \( S(X) \) represents the spread of this field towards the registration axis by [6 -8]
\[ S(X) = \frac{B}{Z^2} \int_{\mathcal{P}} d(\mathcal{P}) \exp(jK\mathcal{R}(\mathcal{P}, X)) d\mathcal{P} \]  \hspace{1cm} (1)

The previous equation is very similar to the (Fresnel principle). Where \( K \) represents the wave number, and \( B \) can be considered as constant. \( Z \) indicates the distance between the object plane and the recording plane. There is a distance connecting a certain point on the recording axis \( X \) and another point on the object \( \mathcal{P} \), which is \( \mathcal{R} \) that can be calculated through

\[ \mathcal{R}(\mathcal{P}, X) = \sqrt{Z^2 + (X - \mathcal{P})^2} \]  \hspace{1cm} (2)

When we used approximation, we get

\[ ((X - \mathcal{P})^2/Z^2) \ll 1 \]  \hspace{1cm} (3)

The \( \mathcal{R}(\mathcal{P}, X) \) is given as

\[ \mathcal{R}(\mathcal{P}, X) = Z + \frac{(X - \mathcal{P})^2}{2Z} - \frac{2}{8Z^3} - \frac{4}{2Z^5} + \cdots \cdots \cdots \cdots \cdots \cdots \cdots \]  \hspace{1cm} (4)

and since we are in the Fresnel region, we will only use two terms from the equation (4) for \( \mathcal{R} \), i.e.

\[ \mathcal{R}(\mathcal{P}, X) = Z + \frac{X^2}{2Z} + \frac{\mathcal{P}^2}{2Z} - \frac{\mathcal{P}X}{Z} \]  \hspace{1cm} (5)

Substituting equation (5) in equation (1) produce

\[ S(X) = B_1 \exp\left(\frac{jKX^2}{2Z}\right) \int_{\mathcal{P}} d(\mathcal{P}) \exp\left(\frac{jKP^2}{2Z}\right) \exp(-jKX \cdot \frac{P}{Z}) d\mathcal{P} \]  \hspace{1cm} (6)

\( B_1 \): a composite constant producing from substituting equation (5) in equation (1).

3. Eigenvector Method principle

In fact, Eigen-vector (EV) method, can be considered as a method derived from the multiple signal classification (MUSIC) method and among several methods, the method Eigen-vector can estimate complex exponential frequencies in a noise. The EV method estimates the exponential frequencies from the peaks of the Eigen spectrum. Whereas, the basis of its action depends on the processes of the autocorrelation matrix \( Q_s \) of the observed data [9, 10].

Assuming that the signal contains of \( g \) exponential of a complex form and \( b \) denotes the number of recorded samples, provided that \( b \) is subject to \( b > g + 1 \) then the (EV) will estimate the exponential frequencies from the peaks of the eigen spectrum[11].

\[ \hat{p}_{\text{eig}}(e^{i\omega}) = \frac{1}{(\sum_{l=g+1}^{b} \frac{1}{|\mathcal{H}_l\mathcal{V}_l|^2})} \]  \hspace{1cm} (7)

where the eigenvectors \( \mathcal{V}_l \), \( l = 1, 2 \ldots \) associated with the eigenvalue \( \lambda_l \) equations (8)

\[ \mathcal{e} = [1, e^{i\omega}, e^{i2\omega}, \ldots , e^{i(q-1)\omega}] \]  \hspace{1cm} (8)

Where \( \mathcal{H}_l \) denotes the conjugate transpose [12].
4. Sources localization

Multiplication of the recorded data $S(X)$ by the quadratic phase factor $\left(\frac{KX^2}{2Z}\right)$ and then applying one of the spectral resolution methods: either a Fourier transform method, or a super resolution (Eigenvector) method.

5. Results and conclusions

To solve the research problem, computer programs were used. Two sources acoustic transmission were used as (two points). This was done by placing the first source at a lateral distance $P1=2$ cm from the assumed main axis, from which the future acoustic transducer starts. After this step, we placed the second transmitter source at a sideways distance $P2$ from the main axis, after which we took a set of samples from the received wave from the two sources at the receiving (recording) axis, according to the synthetic aperture scanning and across samples ($N_o$). Where $Z$ represents the axial distance between the receiving axis and the transmitter axis. As for the wavelength it is $\lambda$, there is a sampling interval, which is $\Delta X$. Represents the difference between the two sources location $\Delta P$, which means that it is equal to the difference as shown ($P_2- P1$).

A set of different values were used for each of $Z$, $\Delta X$, $\lambda$ and $P2$ in light of this, we obtained a set of results by applying two methods to estimate the spectrum: Fourier transform (FT) and method Eigenvector (EV).

Where the minimum separation between sources (accuracy) is given by the following equations[13]

$$6_1 = \frac{\Lambda Z}{a_0}$$  \hspace{1cm} (9)

$a_0$: it is half of the aperture length.

$$a_0 = \frac{(N_o - 1)}{2} \Delta X$$  \hspace{1cm} (10)

(P.E.) represents the percentage of error of $\Delta P$, since it is equal to ($P2- P1$) and we assumed that this ratio is acceptable at (P. E< 20%).

5.1. Results without noise data

Figure 1 shows the results (implementation) of the two methods when applied. According to equations (9) and (10), $6_1 > 4.4$ cm and thus (FT) was not able to separate the two sources (two objects), rather it appeared as a single source (object), especially for the values. Less than $6_1 = 5$, and this is in contrast to what happened with the (EV) method where it was able to separate the two sources at $P2 = 3$ cm, i.e., $\Delta P = 1$ cm, not only that, but the percentage of error (PE) compared to (FT) method. Much less, as shown in Figure 1.
Figure 1. Two points without noise ($Z=25, N_o=10, \Delta X=0.5, \lambda=0.4, \varnothing_1=2\text{cm}$).

The results of the figure 2, show the implementation of the two methods when applied. According to equations (9) and (10), $\delta_1 > 2.8$ cm and thus (FT) was not able to separate the two sources (two objects), rather it appeared as a single source (object), especially for the values. Less than $\delta_1 = 4$, and this is in contrast to what happened with the (EV) method where it was able to separate the two sources at $\varnothing_2 = 3$ cm, i.e., $\Delta \varnothing = 1$ cm, not only that, but the percentage of error (PE) compared to (FT) method) Much less, as shown in figure 2.

Figure 2. Two points without noise ($Z=25, N_o=15, \Delta X=0.5, \lambda=0.4, \varnothing_1=2\text{cm}$).

Figure 3 shows the results (implementation) of the two methods when applied. According to equations (9) and (10), $\delta_1 > 3.4$ cm and thus (FT) was not able to separate the two sources (two objects), rather it appeared as a single source (object), especially for the values. Less than $4$ cm, and this is in contrast to what happened with the (EV) method where it was able to separate the two sources at $\varnothing_2 = 3$ cm, i.e., $\Delta \varnothing = 1$ cm, not only that, but the percentage of error (PE) compared to (FT) method. Much less.
5.2. Results with noise data

In Figure 4 we see the performance of the two methods with adding noise, where the results show the method of (EV) the appearance of the two peaks of the two acoustics sources and clearly with $\Delta \Psi = 2$, while the results of applying the method of (FT) do not show the two sources, but they appear as one source at $\sigma_1 > 5.71$.

**Figure 3.** Two points without noise ($Z = 30, N_0 = 15, \Delta X = 1, \lambda = 0.8, \Psi_1 = 2\text{cm}$).

**Figure 4.** Two points with noise ($Z = 25, \Psi_1 = 2\text{cm}; \Psi_2 = 4; N_0 = 15; \Delta X = 0.5; \lambda = 0.8$).

In Figure 5 we see the performance of the two methods with adding noise, where the results show the method of (EV) the appearance of the two peaks of the two acoustics sources and clearly with $\Delta \Psi = 1$,.
while the results of applying the method of (FT) do not show the two sources, but they appear as one source at $\delta_1 > 2.8$.

![Eigenvector method](image1)

![Fourier transform method](image2)

**Figure 5.** Two points with noise ($Z=25; \varphi_1=2; \varphi_2=3; N_o=15; \Delta X=0.5; \lambda=0.4$).

6. Conclusions
Through our apparent results, it is possible to prove the effectiveness of the method (EV) in detecting sources close to each other when the values of $\Delta \varphi$ are small, with high accuracy and clarity, and not only that, but the error rate. P.E. Is small and this is the opposite of what was observed in the method (FT) where it failed to discover sources close to each other, and in addition, the error rate P.E was higher than the method (EV). In addition, one of the most important conclusions that must be pointed out is accuracy, as it was noted in figure 1, that the difference between $\varphi_1$ and $\varphi_2$ in (EV) is 1 while in (FT) it is equal to 4, according to the laws of precision that clarify the preference for (EV). When observing the error rate shown in the graphs (percentage difference), we will notice a difference between (EV) and (FT), as the (FT) score most of the time is much greater than what is found in (EV).

7. References
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