Mean Field Effects In The Quark-Gluon Plasma

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Abstract

A transport model based on the mean free path approach for an interacting meson system at finite temperatures is discussed. A transition to a quark gluon plasma is included within the framework of the MIT bag model. The results obtained compare very well with Lattice QCD calculations when we include a mean field in the QGP phase due to the Debye color screening. In particular the cross over to the QGP at about 175 MeV temperature is nicely reproduced. We also discuss a possible scenario for hadronization which is especially important for temperatures below the QGP phase transition.

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Numerical calculations of a system of hadrons and the transition to the quark-gluon plasma (QGP) at finite temperatures and zero baryonic density within a Lattice QCD (LQCD) approach are feasible nowadays thanks to very performing computers\cite{1}. Up to date LQCD results suggest that there is a cross over from a meson system to a QGP at a temperature of about 175 MeV \cite{1}. These features could be experimentally confirmed or rejected in relativistic heavy ion collisions (RHIC). In particular high temperatures and very small baryonic densities can be obtained at the very high beam energies reached at the Brookhaven National Laboratory (BNL) and soon at the Large Hadron Collider (LHC) at CERN. Because of the high complexity of the problem, models are needed that take into account the relevant physics at such high energy densities, plus the possibility that the system is out of equilibrium during the collisions. Of course exact microscopic simulations for out of equilibrium-finite systems are out of reach at present. On the other hand transport approaches \cite{2, 3} have been very useful in the past in describing many features of lower energies heavy ion collisions. Generalizations to relativistic energies of low energy heavy ion collisions \cite{3, 4, 5, 6} (known as Boltzmann Uehling Uehlenbeck (BUU), Vlasov(VUU)/ Landau (LV)) have been proposed. The method we discuss in this work is known as Boltzmann Nordheim Vlasov (BNV) approach at low energies\cite{7, 8, 9}. It is based on the concept of the mean free path approach\cite{7}. In a previous paper we studied the equilibrium features of our model, i.e. the equation of state (EOS) effectively implemented in the model. To this purpose, in our calculations pions are enclosed in a box with periodic boundary conditions to simulate an infinite system. The pions can collide elastically or inelastically according to the elementary cross sections which are parametrized from available data. If we restrict our calculations to an hadron system only and we include all the measured resonances (\(\rho\), \(\omega\) etc.) we obtain the so called Hagedorn limiting temperature, i.e. when we increase the energy density of the system we do not obtain an increase of the temperature as well because higher mass resonances are excited thus reducing the available kinetic energy \cite{9}. We can easily include the possibility of a QGP using the bag model \cite{10, 11}. In fact, for each elementary hadron-hadron collision, we can calculate the local energy density and the pressure. If such quantities overcome the Bag pressure and energy density, then \(n_q = n_{\bar{q}}\) quarks and antiquarks and \(n_g\) gluons are created. The number of quarks and gluons are calculated assuming local thermal equilibrium. In this way we can simulate a hadron gas and its transition to the QGP. In \cite{9}, we discussed the cases of \(N_f = 0, 2\), where \(N_f\) is the...
number of flavors. In this paper we generalize our results to 3 flavors, and compare to LQCD results. We also discuss the possibility the quarks could recombine and gluons decay into two quarks during the dynamical process. We show that in order to have a reasonable description of hadronization the local quark (gluon) density must not exceed a critical density calculated from the MIT bag model. If the local density is larger than such a value (which is equivalent to having a high temperature or energy density) the quarks cannot recombine to form hadrons (or the gluons decay into $q\bar{q}$ pair). From a comparison of our results to LQCD we realize that in the QGP phase our $\epsilon/T^4$ is higher than the value suggested by lattice calculation and indeed it approaches the Stefan-Boltzmann limit as it should be. Knowing that there is Debye color screening in the QGP phase we calculate the corresponding mean field and adjust the parameters to reproduce lattice data.

Quantum statistics (i.e. Pauli and Bose statistics) are included similarly to [12] for Bose and [13] for Fermi statistics [9].

I. FORMALISM

The mean free path method discussed above has been studied in detail at low energies and it has been shown to solve the Boltzmann eq. in the cases where an analytical solution is known [7]. We have generalized the approach to keep into account relativistic effects. The particles move on straight lines during collisions since we have not implemented any force yet. For short we name the method proposed as Relativistic Boltzmann equation (ReB). In order to test our approach, we have discussed also some simple cases where analytical solutions are known to verify the sensitivity of our numerical approximations [7, 8, 9].

In the present calculations we enclose a pion gas in a box with periodic boundary conditions. The total energy of the system and the density is fixed for each calculation case according to the Bosonic nature of the system [9]. The temperature can be calculated at each time step (after equilibrium has been reached) from the momentum distribution of the particles. The pions can collide elastically or inelastically according to the measured data. Resonances are included and their decay is simulated as well.

We recall how we include a QGP in our approach. First, for a massless quark and gluon plasma in equilibrium the following relations hold for the pressure $P$, quark (antiquark and
FIG. 1: Energy density versus temperature for a QGP with $N_f = 0$ (full circles), 2 (asterisks) and 3 (full squares).

Quark and gluon density $n_q, (\bar{q}), (g)$ and energy density $\epsilon$ versus temperature $T$ \[1, 9, 10, 11\]:

$$P = g_{tot} \frac{\pi^2}{90} T^4; n_q = n_{\bar{q}} = 1.202 \frac{3g_q}{4\pi^2} T^3; n_g = 1.202 \frac{g_g}{\pi^2} T^3;$$

where $\epsilon = 3P$, $g_{tot} = 16 + \frac{31}{2} N_f$, $N_f$ is the number of flavors. In the MIT bag model \[10, 11\] quarks and gluons are confined in the bag if the pressure is less than the critical pressure $B$ that the bag can sustain. Thus from the previous equation we can assume that the quarks and gluons are liberated in a collision if the energy density is larger than $3B$. This gives a critical energy density $\epsilon_c = 3B = 0.71 GeV fm^{-3}$ using $B^{1/4} = 0.206 GeV$ \[10, 11\]. For each $h-h$ elementary collision we know the energy of the collision and the interaction volume from the distance between the colliding particles in their center of mass system. This distance is taken as the radius of a sphere enclosing the two colliding particles and subsequently the newly formed partons. Thus we can calculate the number of quarks, antiquarks and gluons as a function of the energy density liberated in the collision inverting equation (1). We stress that these relations are strictly valid in thermal equilibrium thus it is perfectly justified in this work since we are mainly discussing equilibrium features of our system. The partons are followed in time exactly like the hadrons solving the transport equation. In particular the partons can collide elastically with other partons and hadrons using a cross section of 1
mb. If in a collision between a parton and a hadron the local energy density is larger than the critical value, new partons are liberated from the hadron similarly to the mechanism discussed above. The possibility of collisions among partons and hadrons is necessary in order that the total system can reach thermal equilibrium. After a certain time $\tau$, the quarks can hadronize and the gluons can decay into a $q, \bar{q}$ pair which later can hadronize as well. In the next section we will discuss the case when the parameter $\tau$ is infinite i.e. once the partons are formed they do not hadronize. In the following section we will discuss a possible approach to hadronization.

II. EQUATION OF STATE INCLUDING A MEAN FIELD

The results of our calculation when including the QGP are displayed in fig. (1), for $N_f = 0, 2, 3$, where we have used 10 MeV mass for the $u, d$ quarks and 160 MeV for $s$ quarks. The relevant equation (1) has been generalized to the finite quark masses used here.

In the figure the two main features of the system are seen, i.e. at low temperature the ratio $\frac{\epsilon}{T^4}$ is less than 1 which is the value expected for a mixture of bosons with masses $[10, 11]$. On the other hand, the ratio is about 4, 12 and 16 as it should be for 0, 2 and 3 flavors quark system respectively. Such values are larger than the values obtained in LQCD calculations $[1]$. This is evidently due to the neglect of an interaction among partons which is clearly important in LQCD. On the other hand the good agreement with the expected high temperature limits strengthens our numerical calculations.

In our model there is not a phase transition but simply a crossover from an hadronic state at low T to a state of partons at high T. However, the crossover is rather visible and it should have some effect in experimental data at RHIC such as large fluctuations of D-mesons for instance $[14, 15]$. We obtain a cross over above 175 MeV temperature depending on the number of flavors. These features are at least in qualitative agreement with microscopic LQCD results $[1]$ which strengthens our model somewhat. In fact, since the EOS of the two systems is similar, they should give similar consequences when features of heavy ion collisions are investigated. We stress that our approach being a kinetic one could be easily extended and applied to relativistic heavy ion collisions which is the main purpose of this work, even though we will have to pay a particular attention to hadron-hadron phenomenology at higher momentum transfers. Thus within this model we could
study features of the collision with and without the phase transition and compare to data and/or be of guidance to more experimental investigations.

![Graph showing the energy per entropy density as a function of temperature for different number of flavors.](image)

**FIG. 2:** Same as in fig.1 but with the mean field included for each $N_f$ in the QGP phase.

In order to improve the results displayed in fig.(1) and obtain a better agreement to the LQCD calculations, we recall that in the QGP phase the linear potential which keeps the quarks confined disappears while the Coulomb-like term due to one gluon exchange is screened due to the color charges and similar to the Quantum Electrodynamics (QED) case. In particular it is easy to realize that in order to reduce the ratio $\varepsilon/T^4$ we need to introduce an attractive mean field $U_m$ which increases the kinetic energy of the partons, thus the temperature. We can find the value of $U_m$ for the different flavors by fitting to the LQCD results. In fig.(2) we plot the EOS obtained including the mean field. Clearly, the values of $\varepsilon/T^4$ at high T are in agreement with the LQCD results[1].

The resulting mean field $U_m$ is plotted in fig.(3) as function of T. A clear linear dependence on T is observed. We can easily understand this results in the Abelian limit. Consider for instance a quark located at $r=0$. This quark will interact with another quark or antiquark
FIG. 3: Mean field vs T for each $N_f$, symbols as in fig. (1). The best fit values to $U_m$ are given in the figure.

located at a distance r through a screened Coulomb potential\[11]\):

$$V(r) = -\frac{4}{3}\alpha_s e^{-\frac{m_d}{r}}$$

(2)

where $m_d$ is the screening mass\[11] and $\alpha_s$ is the strong coupling constant. The mean field can be easily obtained summing up over all possible charges and integrating over the total volume at constant density:

$$U_m = \int 4\pi r^2 \rho V(r) dr = -\frac{4\pi\alpha_s \rho}{m_d^2}$$

(3)

Comparing eq.(3) to the figure(3) one clearly finds that $m_d \propto T$, since the density $\rho \propto T^3$, which is a well known result\[11]. In particular knowing that in the Abelian approximation:

$$m_d^2(\text{Ab}) = \frac{g_q q^2 T^2}{6}$$

(4)

with the charge $q^2 = \frac{4}{3} 4\pi\alpha_s$. For $N_f > 0$, we can derive the value of $m_d$, eq.(3), using the fit values in fig.3 and we can compare the result to eq.(4) obtaining: $m_d = 1.9 m_d(\text{Ab})$, i.e.
our estimate based on a fit to LQCD results, is about a factor two larger than the Abelian limit, eq.(4). This difference could be expected since the gluons which have been neglected in the Abelian limit play a role as well and their dynamics is also modified by some force as we have seen in figs.(2) and (3) for $N_f = 0$.

III. A SIMPLE SCENARIO FOR HADRONIZATION

![Graph showing the number of hadrons versus time at two different energy densities.](image)

**FIG. 4:** Number of hadrons divided the total number of particles versus time at two different energy densities.

In the previous section we have discussed the EOS assuming that after the partons are created they do not recombine again to form new hadrons. Of course we expect that this is true for the box calculations at very high temperatures, i.e. in the QGP phase. However at temperatures near the cross over and even in the hadronic phase it might happen that because of fluctuations in a given region the energy density is larger than the critical value and partons can be created. Since the energy density is on average not so large there might be a possibility for partons to combine again. We would like to discuss here some possible ways
to implement this mechanism and check how the EOS is modified. Already in a previous work of simulations of heavy ion collisions we had introduced a finite lifetime $\tau$ for quarks to combine into hadrons. To be more precise we let the quarks created at a given time $t$ evolve to a time $t + \tau_q$ (calculated in the quarks rest frame) before they can hadronize, while gluons decay after a time $t + \tau_g$ into a $q\bar{q}$ pair which can eventually hadronize after a time $\tau_q$. For the hadronization we assume that quarks can combine to form resonances depending on their total energy. The use of resonances is important to conserve energy and total momentum. Before hadronization the partons move on straight line trajectories and they can only collide elastically. If we adopt this scenario we obtain a surprising (at first) result which is plotted in fig.(4). In the figure the total number of hadrons divided the total number of particles (hadrons plus partons) is plotted versus time for two different energy densities using a formation time of 1.0 fm/c for quarks and a similar value for the decay time of gluons into $q\bar{q}$. At these energy densities from the previous figure (1) we expect to be at very high temperature i.e. in the QGP phase. But as we see from the figure (4), no matter how large the energy density is the system hadronizes completely after a few hundreds of fm/c. Thus the EOS that we get with this assumption of a finite formation time is similar to the Hagedorn one. In fact, after the first collisions some partons are created which later on hadronize in a larger number of hadrons than initially. Because of the increase of the number of hadrons, the temperature decreases and eventually saturates to a value of the order of the pion mass (the Hagedorn temperature) even if we increase largely the energy densities.

In order to overcome this problem we can get some more insight in the hadronization mechanism using the Bag model again. In the model the partons are confined when the density (local) is smaller than a critical value which can be obtained from eq.(1) for a fixed value of the bag constant and for each $N_f$ flavor. Thus at each time step and for each particle we check the local density and if such a density is smaller than the critical value, the $q\bar{q}$ pair hadronizes or a gluon will decay into a $q\bar{q}$.

The results of the calculations with mean field included are given in figure (5), due to the inclusion of the condition on the critical density the partons cannot recombine at high $T$ and the expected EOS is recovered. Actually, the use of a condition on the density makes the use of the lifetime somewhat unnecessary. In fact even if we put the recombination time equal to zero, the partons cannot hadronize if the density is too large.
FIG. 5: EOS including the mean field and with recombination times $\tau_q = 1.0 \text{ fm}/c$ and $\tau_g = 0.5 \text{ fm}/c$.

IV. CONCLUSIONS

In this work we have applied a recently introduced transport approach to study the equation of state of a meson gas and its possible transitions to a Quark Gluon Plasma. The model includes the possibility of resonance formation and decay. The possibility of a QGP may be included based on the MIT bag model. Quantum statistics can be easily included but we have seen that at the temperature discussed here those effects are negligible. The Hagedorn limiting temperature is recovered for a pure hadronic system. When including a QGP the obtained equation of state is in qualitative agreement with LQCD calculations. A better description could be obtained by introducing some interaction (attractive) among partons. We have shown that the mean field that we have derived to fit the LQCD results gives a Debye screening mass about a factor two larger than expected in the Abelian limit putting in evidence the role of the gluons in our model. We have also discussed a possible scenario for hadronization assuming that $q\bar{q}$ could combine into hadrons or gluons split into
$q\bar{q}$ pairs if the local density is smaller than a critical value derived in the Bag model. Because of the condition on the density the particles are relatively close enough in space thus it is easy to combine into resonances in order to conserve total energy and momentum of the system. This, we feel, will be especially important when dealing with finite systems in order to avoid having isolated quarks in phase space.

The final goal of this work is to apply the model to relativistic heavy ion collisions at RHIC and Cern energies. To this purpose we believe we have a better control now on the ingredients of the model such as the effectively implemented equation of state. Next we are going to concentrate on a good reproduction of the available data on elementary hadron-hadron collisions in order to be able to investigate heavy ion collisions. Our efforts in these directions will be discussed in following papers.

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[1] F. Kartsch, Nucl.Phys. A698, 199c(2002).
[2] K. Geiger and B. Müller, Nucl. Phys. B 369, 600 (1992); K. Geiger, Phys. Rep. 258, 237 (1995).
[3] S.Bass, et al., Prog.Part.Nucl.Phys. 41, 255 (1998).
[4] J. Cugnon, T. Mitzulani, J. Meulen, Nucl. Phys. A 352, 505 (1981).
[5] G. F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988).
[6] Ben-Hao Sa, Wang Rui-Hong, Zhang Xiao-Ze, Zheng Yu-Ming, and Lu Zhong-Dao, Phys. Rev. C 50, 2614 (1994).
[7] A. Bonasera, F. Gulminelli, and J. Molitoris, Phys. Rep. 243,1(1994); A.Bonasera, T.Maruyama Progr.Theor.Phys. 90(1993)12.
[8] D.M.Zhou, S.Terranova and A.Bonasera, nucl-th/0501083.
[9] Z.G.Tan, D.M.Zhou, S.Terranova and A.Bonasera, nucl-th/0606055, and proc. XXII Winter Workshop on Nuclear Dynamics, pag.31, W.Bauer et al. eds.(2006).
[10] L. P. Csernai, ”Introduction to relativistic heavy ion collisions”, John Wiley and Sons, 1994.
[11] C. Y. Wong, Introduction to High-Energy Heavy ion Collisions, World Scientific Co., Singa-
[12] G. M. Welke and G. F. Bertsch, Phys. Rev. C 45, 1403 (1992).

[13] Ben Hao Sa and A. Bonasera, Phys. Rev. C 70, 034904 (2004).

[14] S. Terranova, D. M. Zhou and A. Bonasera, nucl-th/0412031, EPJA26, 333 (2005).

[15] S. Terranova and A. Bonasera, Phys. Rev. C 70, 024906 (2004); T. Maruyama and T. Hatsuda, Phys. Rev. C 61, 62201(R) (2000).