Constrained Superconducting Membranes

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We present a geometrical canonical description for superconducting membranes. We consider a general action which includes a general class of superconducting extended objects (strings and domain walls). The description is inspired in the ADM framework of general relativity but, instead of the standard canonical variables a different kind of phase space is considered. The Poisson algebra of the constraints and the counting of degrees of freedom is performed. The new description is illustrated considering a superconducting domain wall on a curved background spacetime.

98.80.Cq, 98.80Hw, 11.27+d

I. INTRODUCTION

Recently a number of outstanding problems in physics are currently being solved using geometrical techniques involving the machinery of differential geometry. In this way, superconducting strings moving on a curved background spacetime have also been the subject of intense research because it believed that they appear in the early Universe as topological defects [1]. However, domain walls can exhibit a superconducting character, too. Superconducting domain walls appear in supersymmetry and grand unified theories [2]. In addition, domain walls transform in superconducting membranes in a similar way like in Witten’s superconducting string [3].

Among the most important aspects around extended objects is the question of its dynamical behavior. For example, in the case of circular strings, the dynamics shows us when the system exhibit string collapse from radial configuration or not. A convenient way to deal with the last question lies in the Hamiltonian formulation since it cast out a form of the effective potential for the relativistic membrane. It could help us to separate the equilibrium configurations from the unequilibrium (collapsing) ones [4]. In the Hamiltonian context, superconducting strings evolving in a curved background spacetime have been explored in [4-6].

The purpose of this article is to study the Hamiltonian formulation for superconducting membranes in a geometric language without specification of any special membrane configuration and particular background geometry. We get a truly canonical formulation for this system which involves the full machinery of an Hamiltonian formulation, i.e., the specification of an appropriate set of phase space variables, of a Poisson bracket structure and a Hamiltonian function. This description possess a rich geometric content. To our knowledge this geometrical canonical analysis for superconducting membranes has not been considered before. Another goal of the work, and important application, is the preparation of the classical theory in a suitable form for its canonical quantization. In fact, the investigation of the structure of classical dynamics of totally constrained systems will shed a good light on how to find a consistent quantum formulation of them. Quantum mechanics aspects in superconducting strings have been reported in [7,8].

The paper is organized as follows: In Sec. II we develop the mathematical issues needed for the ADM fashion in order to achieve the Hamiltonian formulation. In Sec. III we perform the Hamiltonian formulation of our system. The variation of the phase space constraints and their physical meaning is done in Sec. IV. The constraint algebra is presented in Sec. V. Finally, in Sec. VI, we treat an specific example that illustrates our results.

II. MATHEMATICAL TOOLS

We consider a relativistic membrane of dimension d (for strings d = 1 and for domain walls d = 2). Its worldsheet, m, of dimension d+1 is an oriented timelike surface embedded in an arbitrary fixed 4-dimensional background spacetime \( \{ M, g_{\mu \nu} \} \). Following the ADM treatment of canonical general relativity [9], we assume that \( m \) has the topology \( \Sigma \times R \) such that we can consider a foliation of the worldsheet \( \{ m, \gamma_{ab} \} \) into spacelike hypersurfaces of dimension \( d, \Sigma_{t} \) defined by the constant value of a certain scalar function \( t \). Each leaf of the foliation represents the system at an instant of time, and each one is diffeomorphic to each other. The hypersurface \( \Sigma_{t} \) may be described by the embedding of \( \Sigma_{t} \) into \( m \)
\[ \xi^a = X^a(u^A), \]  

where \( u^A \) are local coordinates on \( \Sigma_t \) and \( \xi^a \) are local coordinates on \( m \) \((a, b = 0, \ldots, d+1)\) and \((A, B = 1, \ldots, d)\). Locally, one can think of a decomposition of the worldsheet embedding functions \( \chi^\mu \) into a timelike coordinate \( t \), and spacelike coordinates \( X^a \) where \( \chi^\mu \) are the embedding functions of \( m \) on \( M \).

Alternatively, it is useful to consider the direct embedding, via map composition, of \( \Sigma_t \) into the background spacetime \( \{M, g_{\mu\nu}\} \),

\[ x^\mu = X^\mu(u^A) = \chi^\mu(\xi^a(u^A)), \]  

where \( e^\mu_a \) are the tangent vectors to \( m \) associated with the embedding \( \chi^\mu \). The \( d \) tangent vectors to \( \Sigma_t \) are defined with

\[ e^\mu_a = \gamma_a^B \eta^a_{AB}, \]  

so that the positive-definite metric induced on \( \Sigma_t \) is

\[ h_{AB} = \gamma_a^B \eta^a_{AB} = g_{\mu\nu} e^\mu_a e^\nu_b, \]  

where \( \eta^a_{AB} \) denotes the tangent vectors to \( \Sigma_t \) associated with the embedding (1). The unit, future-directed, timelike normal to \( \Sigma_t \), \( \eta^a \), is defined, up to a sign, with

\[ \gamma_{ab} \eta^a \eta^b = -1. \]  

In order to describe the evolution of the leaves of the foliation, we define a worldsheet time vector field with

\[ t^a = N \eta^a + N^A e^a_A, \]  

where \( N \) is called the lapse function, and \( N^A \) the shift vector. The deformation vector of a spacelike hypersurface in spacetime is simply the push-forward of \( t^a \), \( \dot{X}^\mu := \dot{t}^a e^\mu_a \). A basic step in the canonical analysis of the most important physical theories is the decomposition of the several geometric quantities involved in the theory in the normal to and tangential parts to an embedded hypersurface \( \Sigma_t \). Thus, the worldsheat metric, \( \gamma_{ab} \), can be decomposed in the standard ADM fashion as

\[ \gamma_{ab} = \left( \begin{array}{cc} -N^2 + N^A N_A & N_A \\ N_A & h_{AB} \end{array} \right). \]  

and for the inverse

\[ \gamma^{ab} = \frac{1}{N^2} \left( \begin{array}{c} -1 \\ N^A (h_{AB} N^2 - N^A N^B) \end{array} \right). \]  

Note that it follows from the expression (7) that

\[ \det(\gamma_{ab}) = -N^2 \det(h_{AB}). \]  

Furthermore, we denote with \( D_A \) the (torsionless) covariant derivative compatible with \( h_{AB} \). For a more general treatment about this kind of geometrical decomposition in extended objects see the reference [10].

### III. HAMILTONIAN FORMALISM

We consider superconducting membranes described by the generic effective action

\[ S = \int_m \sqrt{-\gamma} \mathcal{L}(\omega) \]  

where \( \mathcal{L}(\omega) \) is a function depending of the internal and external fields acting on the membrane through \( \omega = \gamma^{ab} (\nabla_a \phi + A_{\mu} e^{\mu}_a)(\nabla_b \phi + A_{\mu} e^{\mu}_b) \), which is the worldsheat projection of the gauge covariant derivative of a worldsheat scalar field \( \phi \) and \( A_{\mu} \) is the background electromagnetic potential. Note that it has absorbed the worldsheat differential \( d^{d+1} \xi \).
into the integral sign. Similarly, we will do the same for integrals over \( \Sigma_t \). The ADM decomposition of the action:\[ L[X, \dot{X}; \phi, \dot{\phi}] = \int_{\Sigma_t} N \sqrt{h} \mathcal{L}(\omega). \]

From now on, we should understand \( \mathcal{L} \) as in the ADM fashion. Besides, the ADM decomposition of \( \omega \) is given by:

\[
\omega = -\frac{1}{N^2} [L_t \dot{\phi} - N^A \bar{D}_A \dot{\phi} + (t \cdot A)]^2 + \ddot{\omega},
\]

where we have defined the \( \Sigma_t \)-projection of the gauge covariant derivative of the worldsheet scalar field \( \phi \), i.e., \( \bar{D}_B \phi \) with \( A_B = A_\mu \epsilon^\mu B \) and, \( L_t \phi \) denotes the Lie derivative of \( \phi \) along the deformation vector field \( t^\mu \) and \( \ddot{\omega} = h^{AB} \bar{D}_A \phi \bar{D}_B \phi \) is the \( \Sigma_t \)-projection of \( \omega \).

The momenta are defined as:

\[
\pi = \frac{\delta L}{\delta \dot{\phi}} = -2\sqrt{h}(\ddot{\omega} - \omega)^{1/2} \frac{d\mathcal{L}}{d\omega},
\]

\[
P_\mu = \frac{\delta L}{\delta \dot{x}^\mu} = \left[ -\sqrt{h}\mathcal{L}(\omega) + (\ddot{\omega} - \omega)^{1/2} \pi \right] \eta_\mu - \pi \bar{D}_A \phi \epsilon_\mu^A + \pi A_\mu.
\]

The phase space for our superconducting membrane is naturally associated with the geometry of \( \Sigma_t \), \( \Gamma := \{ \chi^\mu, P_\mu; \phi, \pi \} \). In order to get more simplicity we define the kinetic momentum \( \Upsilon_\mu \) as follows, \( \Upsilon_\mu := P_\mu - \pi A_\mu \).

The action (10) is invariant under reparameterization of the worldsheet so, we should expect to have phase space constraints that generate this gauge freedom. Indeed, from the definition of the momenta we have the constraints:

\[
C_0 = g^{\mu \nu} \Upsilon_\mu \Upsilon_\nu + h \left( \mathcal{L}(\omega) + \frac{\pi^2}{2 h (d\mathcal{L}/d\omega)} \right)^2 - \pi^2 \left( \omega + \frac{\pi^2}{4 h (d\mathcal{L}/d\omega)^2} \right),
\]

\[
C_A = \Upsilon_\mu \epsilon_\mu^A + \pi \bar{D}_A \phi.
\]

The former, or scalar constraint, is the generalization to superconducting membranes of the scalar constraint for a parametrized relativistic particle in external electromagnetic field. The latter, or vector constraint, is universal for all reparametrizations invariant actions of first order in derivatives of the embedding functions. In fact, (14) is that depends from a particular form of the Lagrangian. It should be noted that for a superconducting cosmic string in a stationary background the constraint (15) reduce to eq. (24) in [4], hence our result is more general. Observe that the constraints (13) and (16) are valid for higher dimensions, \( d > 2 \), but unfortunately, nowadays there is not an effective theory describing such higher extended objects, at the most, strings and domain walls.

The constraints perform a double duty. Their first job is to restrict the possible values of the phase space variables, i.e., they tell us that there is a redundancy in the characterization of a field configuration in terms of phase space points. The second job of the constraints is that, before they are set to zero, they generate dynamics. They are the generators of the canonical evolution of the system.

In order to obtain the geometric information encoded in the constraints (13) and (16), we recast them as functionals on \( \Gamma \) (called constraint functions). To do this, we smear them with test fields \( \lambda \) and \( \lambda^A \), defining a scalar constraint function:

\[
S_\lambda[X, \phi, P, \pi] = \int_{\Sigma_t} \lambda \left[ g^{\mu \nu} \Upsilon_\mu \Upsilon_\nu + h \left( \mathcal{L}(\omega) + \frac{\pi^2}{2 h (d\mathcal{L}/d\omega)} \right)^2 - \pi^2 \left( \omega + \frac{\pi^2}{4 h (d\mathcal{L}/d\omega)^2} \right) \right],
\]

and a vector constraint function:

\[
V_\lambda[X, \phi, P, \pi] = \int_{\Sigma_t} \lambda^A \left( \Upsilon_\mu \epsilon_\mu^A + \pi \bar{D}_A \phi \right).
\]

The smearing field \( \lambda \) must be a scalar density of weight minus one since the scalar constraint function should be well defined. Using Hamilton equations, we can show how \( \{ \lambda, \lambda^A \} \) are related to the lapse function and the shift vector.

Note that our Hamiltonian vanishes, \( H = 0 \). This was to be expected from reparameterization invariance. However, according to the standard Dirac treatment of constrained systems, the Hamiltonian vanishes only weakly [11]. It is a linear combination of the constraints, \( H[X, \phi, P, \pi] = \int_{\Sigma_t} (\lambda C_0 + \lambda^A C_A) \).
IV. CONSTRAINTS

For purposes of reach a true canonical description of the superconducting membranes, in the following subsections we discuss the variation of the phase space constraints. The variation of the action with respect to the phase space variables contains the variations of each constraint function which contribute a surface term. They are neglected for lack of simplicity.

A. Vector Constraint

The Hamiltonians vectors fields generated by (17) and (18) should correspond to evolution along the vectors fields $N^\eta$ and $N^A$, respectively. To evaluate the Poisson brackets of the constraints and to determine the motions they generate on phase space we compute, first, the infinitesimal canonical tranformations generated by the vector constraint $V_\lambda$. Modulo some boundary conditions we get the functional derivatives

\[ \frac{\delta V_\lambda}{\delta X^\mu} = -D_A (\lambda^A p_\mu) = -L_\lambda X^\mu \]
\[ \frac{\delta V_\lambda}{\delta p_\mu} = \lambda^A \epsilon^\mu_A = L_\lambda X^\mu \]
\[ \frac{\delta V_\lambda}{\delta \phi} = -D_A (\lambda^A \pi) = -L_\lambda \pi \]
\[ \frac{\delta V_\lambda}{\delta \pi} = \lambda^A D_A \phi = L_\lambda \phi , \]

where $L_\lambda$ denotes the Lie derivative along the vector field $\lambda^A$. The Hamiltonian vector field generated by $V_\lambda$ is

\[ W_{V_\lambda} = \int_{\Sigma} \left[ (L_\lambda X^\mu) \frac{\delta}{\delta X^\mu} + (L_\lambda \phi) \frac{\delta}{\delta \phi} + (L_\lambda \pi) \frac{\delta}{\delta \pi} \right] . \]

This is consistent with the geometrical interpretation of the vector constraint $V_\lambda$ as the phase space generator of diffeomorphisms on the surface $\Sigma$. Acting $W_{V_\lambda}$ on each one of the phase space variables we get

\[ X^\mu \rightarrow X^\mu + \epsilon L_\lambda X^\mu \]
\[ \phi \rightarrow \phi + \epsilon L_\lambda \phi \]
\[ \pi \rightarrow \pi + \epsilon L_\lambda \pi , \]

which is the motion on $\Gamma$ generated by $V_\lambda$. Here $\epsilon$ is an infinitesimal parameter.

B. Scalar Constraint

We turn now to the scalar constraint. The computation of its Hamiltonian vector field is more complicated due to the several variations involved. In a similar way to vector constraint, from the equation (17), after tedious algebra we get the functional derivatives

\[ \frac{\delta S_\lambda}{\delta X^\mu} = -2 \lambda (p^\nu - \pi A^\nu) \pi A_{\nu , \mu} + 2 \lambda h L^+ (\omega) L (\omega) K^I m^J \eta_{IJ} \]
\[ - D_A (2 \lambda h L^+ (\omega) L (\omega)) \epsilon^\mu_A - 4 \lambda h L (\omega) \frac{dL}{dw} \tilde{D}_A \phi B A^B \]
\[ + D^A (4 \lambda h L (\omega) \frac{dL}{dw} \tilde{D}_A \phi B A^B) \epsilon^B - D_A (4 \lambda h L^+ (\omega) \frac{dL}{dw} h^{AB} \tilde{D}_B \phi A^B) \]
\[ + 4 \lambda h L (\omega) \frac{dL}{dw} h^{AB} \tilde{D}_A \phi B A^B \]
\[ \frac{\delta S_\lambda}{\delta \phi} = -D_A (4 \lambda h L^+ (\omega) \frac{dL}{dw} h^{AB} \tilde{D}_B \phi) \]
\[ \frac{\delta S_\lambda}{\delta p_\mu} = 2 \lambda (p^\nu - \pi A^\nu) + 2 \lambda h^{AB} \pi \tilde{D}_A \phi e^\mu_B \]
\[ \frac{\delta S_\lambda}{\delta \pi} = -2 \lambda (p^\nu - \pi A^\nu) A_\mu + \lambda \frac{L^+ (\omega) \pi}{dL/d\omega} - 2 \lambda h^{AB} \pi \tilde{D}_A \phi A_B , \]
where we have defined the quantity $\mathcal{L}^+(\omega) := \mathcal{L}(\omega) + \frac{\dot{\omega}^2}{\mathcal{L}(\omega)}$; $K_{AB}^I$ denotes the extrinsic curvature of $\Sigma$ associated with the embedding $\mathcal{I}$, $m^{\mu I} = \{n^\mu, n^{\mu I}\}$ is the complete orthonormal basis associated with $\mathcal{I}$, and $\eta_{IJ}$ is the Minkowski metric with signature $(-, +, ...)$, $I = 0, i$ [12]. The corresponding Hamiltonian vector field is given by

$$W_{\mathcal{S}_\lambda} = \int_{\Sigma_t} \frac{\delta \mathcal{S}_\lambda}{\delta \mu} \frac{\delta}{\delta X^\mu} + \frac{\delta \mathcal{S}_\lambda}{\delta \phi} \frac{\delta}{\delta \phi} - \frac{\delta \mathcal{S}_\lambda}{\delta X^\mu} \frac{\delta}{\delta X^\mu} \frac{\delta}{\delta \phi} - \frac{\delta \mathcal{S}_\lambda}{\delta \phi} \frac{\delta}{\delta \phi} ,$$

(26)

which is the generator of time evolution on the worldsheet $m$. The motion on $\Gamma$ generated by $\mathcal{S}_\lambda$ is obtained acting $W_{\mathcal{S}_\lambda}$ on each one of the phase space variables. In fact, this constraint is the hard part in the description. It generated diffeomorphisms out of the spatial hypersurface which can be considered as diffeomorphisms normal to $\Sigma_t$. For a spatial observer this is dynamics. Furthermore, it is instructive to make a naive analysis of the scalar constraint. There is a kinetic term of the form $g^{\mu \nu} \gamma_\mu \gamma_\nu$. The potential term corresponds to an effective potential for the system which could help us to separate the stable configurations from the unstable ones. For a circular cosmic string an effective potential was obtained by Larsen [4]. In Sec. VI we will treat the case of a charged spherically symmetric membrane.

V. CONSTRAINT ALGEBRA

If one is interested in the canonical quantization program we compute the algebra of constraints since that is an important point in the promotion of phase space variables to operators [11]. Furthermore, at classical level, constraint algebra show us how the initial value equations are preserved in time in the canonical language by the closure of the Poisson algebra of the constraints. The Poisson bracket (PB) between any two functionals $f$ and $g$ of $\Gamma$ will be given by

$$\{f, g\} = \int_{\Sigma_t} \frac{\delta f}{\delta P_\mu} \frac{\delta g}{\delta X^\mu} + \frac{\delta f}{\delta X^\mu} \frac{\delta g}{\delta \phi} - \frac{\delta f}{\delta \phi} \frac{\delta g}{\delta \phi} .$$

(27)

The Dirac algebra is given by

$$\{\mathcal{V}_X, \mathcal{V}_X\} = -\mathcal{V}_{[X,X]}$$

(28)

$$\{\mathcal{V}_X, \mathcal{S}_\lambda\} = -\mathcal{S}_{\mathcal{L}_{\lambda(2X)}}$$

(29)

$$\{\mathcal{S}_\lambda, \mathcal{S}_\lambda\} = -\mathcal{V}_{\lambda^2} ,$$

(30)

where

$$\lambda^A = 4\hbar \left( \mathcal{L}^+(\omega) \mathcal{L}(\omega) h^{AB} - 2 \mathcal{L}^-(\omega) \frac{d\mathcal{L}}{d\omega} h^{AC} h^{BD} \tilde{D}_{C\phi} \tilde{D}_{D\phi} \right) \left( \lambda \mathcal{D}_B \lambda' - \lambda' \mathcal{D}_B \lambda \right) ,$$

(31)

and we have defined the quantity $\mathcal{L}^- (\omega) := \mathcal{L}(\omega) - \frac{\dot{\omega}^2}{\mathcal{L}(\omega)}$. Equation (28) is the algebra of spatial diffeomorphisms generated by $\mathcal{V}_X$, and it is isomorphic to the Lie algebra of infinitesimal spatial diffeomorphisms. Furthermore, this algebra is the same one would expect in any theory with gauge invariance. The PB (29) shows how $\mathcal{S}_\lambda$ transforms under spatial diffeomorphisms. Finally, the crucial PB (30) means that two infinitesimal normal deformations on $\Sigma_t$, performed in an arbitrary order, end on the same final hypersurface but not on the same point on that hypersurface. Hence, the algebra closes and it is first-class in the Dirac terminology [11]. Remarks are in order: i) Note that the right-hand side of (30) involves structure functions which is a source of problems in any attempt to use the Dirac algebra in the canonical quantisation program because the PB algebra will go over to a commutator algebra which is not a Lie algebra. ii) Since Hamiltonian is a linear combination of the constraints themselves, the constraints are preserved in time. iii) According to [13], the explicit counting of degrees of freedom goes as follows: $2 \times$ (number of physical degrees of freedom) = (total number of canonical variables) $- 2 \times$ (number of first-class constraints). Hence, there are $N - d$ physical degrees of freedom. iv) The Hamilton equations of the system are computed from the functional derivatives listed before. Their form is large and unaesthetic and we only mention that they are equivalent to the conservation of current density, i.e., $\nabla_a J^a = 0$, and to the equation of motion of the system, namely, $T^{ab} K^I_{aI} = F_a J^a$. 5
VI. EXAMPLE

In order to illustrate the results reached before, we now analyse the case of a charged spherically symmetric membrane \((d = 2)\) described by the action (10), which evolves in a general spherically symmetric static background spacetime \((N = 4)\)

\[
ds^2 = -A(r) \, dt^2 + B(r) \, dr^2 + C(r) \, d\Omega^2
\]

where the functions \(A, B\) and \(C\) depend on the particular ambient spacetime. We take as ansatz for our membrane the choice \(\phi = \phi(t)\) and \(A_{\mu} = (A_0(r), 0, 0, 0)\). The physical consequence due to last is that membrane is charged only and no currents exist there. According to the embedding of \(m\) into \(M\), \(\chi_{\mu} = (t, r(t), \theta, \phi)\) the induced metric on the worldsheet is

\[
\gamma_{ab} = \begin{pmatrix}
-A + B \dot{r}^2 & 0 & 0 \\
0 & C & 0 \\
0 & 0 & C \sin^2 \theta
\end{pmatrix},
\]

(33)

It follows that \(\gamma = (-A + B \dot{r}^2) C^2 \sin^2 \theta\). Now, the embedding of \(\Sigma_t\) in \(M\), \(X_{\mu} = (t_0, r_0, \theta, \phi)\), bear the induced metric

\[
h_{AB} = \begin{pmatrix} C_0 & 0 \\ 0 & C_0 \sin^2 \theta \end{pmatrix},
\]

(34)

where \(C_0 = C(r_0)\), hopefully that \(C_0\) in this section not create confusion with the constraint (13). Observe that \(h = C_0^2 \sin^2 \theta\). From (12) we find

\[
\omega = \frac{(\dot{\phi} + A_0)^2}{-A + B \dot{r}^2}.
\]

(35)

According to the continuity equation \(\sqrt{-\gamma} \nabla_a J^a = \partial_a (\sqrt{-\gamma} \frac{d\mathcal{L}}{d\omega} \gamma^{ab} \nabla_b \phi) = 0\) we can rewrite the equation (33) as

\[
\omega = -\frac{W^2}{4(d\mathcal{L}/d\omega)^2 C^2},
\]

(36)

where \(W\) is an integration constant.

The full information developed leads to an expression for the effective potential

\[
V_{\text{eff}} = C^2 \sin^2 \theta \left(\mathcal{L}^+(\omega)\right)^2
\]

\[
= C^2 \sin^2 \theta \left(\mathcal{L}(\omega) + \frac{W^2}{2d\mathcal{L}/d\omega C^2}\right)^2,
\]

(37)

which is of the separable form of the kinetic term in (17). To see this, expanding the kinetic part in the scalar constraint (12) and adding to (37) we obtain a first integral of motion of the system

\[
\frac{B}{A} \dot{r}^2 = 1 - \frac{C^2}{(E + W A_0)^2} \left(\mathcal{L}^+(\omega)\right)^2,
\]

(38)

where \(E\) denotes the energy of the system. It is worthy to mention that until now the Lagrangian \(\mathcal{L}(\omega)\) as been treated as an arbitrary function of \(\omega\). The several cases of the ambient spacetime and models describing the superconducting extended objects is an easy task. For instance, in flat background spacetime we can obtain an equilibrium configuration with the Witten model, \(\mathcal{L}(\omega) = 1 + \omega/2\). This kind of model has been used in an equivalent formulation in terms of a gauge field over the worldsheet in [16]. Another interesting systems have been considered in the literature, for instance, the case of charged current-carrying circular string on Kerr background spacetime and flat spacetime [1,6].
VII. CONCLUSIONS

In this work we have achieved a canonical analysis for superconducting membranes in a geometrical way. We find that the Dirac algebra is closed and therefore the constraints are of first-class. The canonical quantization for the system is performed by considering the phase space variables as operators and by replacing the PB by commutators, where physical states will satisfy the condition $\hat{H}\Psi = 0$. Furthermore, our Hamiltonian constraint is a general expression obtained in a geometrical way without hide any information. We gave as example, a charged bubble embedded in an arbitrary spherically symmetric static background spacetime. The separability of the presented system is obtained thanks to the symmetry, but not the integrability. In the case of a black hole as background, one must be careful with $r$ bubble coordinate with respect to $r_+$ (event horizon), so as to $r > r_+$. For superconducting strings exist a dual formalism in terms of a scalar gauge independent function and its canonical formulation can be easily extended in a similar way. However, when we deal with the superconducting wall new structure appeared due to the presence of a vector field over the worldsheet. The study of this subject is under current investigation.

ACKNOWLEDGMENTS

We thank X. Martin, R. Capovilla, J. Guven and E. Ayón for fruitful discussions. We are also grateful to H. García-Compeán for reading the manuscript and useful comments. This work was partially supported by CONACyT and SNI México.

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