Creation of cosmological magnetic fields in a bouncing cosmology

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Abstract. We show (in a completely analytical and exact manner) that an efficient magnetic field amplification method is operative during the bounce in a time-dependent gauge coupling model. The cosmological magnetic fields so generated have particular spectral features, and may be observed by future cosmic microwave background measurements and by direct cluster measurements.

Keywords: string theory and cosmology, quantum field theory on curved space, physics of the early universe

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Introduction

The origin, evolution, and structure of large-scale magnetic fields are among the most important open issues in astrophysics and cosmology. The magnetic field present in galaxies is typically of the order of a few $\mu$G, and is coherent on the galactic scale. In the case of clusters, the observed field is of the order of the $\mu$G and is correlated over 10–100 kpc. The standard mechanism for accounting for these fields is the dynamo [1], which amplifies small seed fields to the above-mentioned values [2]. Three different origins for the seeds that fuel the dynamo have been discussed. They can be called astrophysical, cosmological, and primordial. The most popular astrophysical mechanism for the generation of these pre-galactic magnetic fields is the Biermann battery [3], based on the conversion of the kinetic energy of the turbulent motion of the conductive interstellar medium into magnetic energy. The battery works for the generation of seeds to be amplified by the dynamo in galaxies, but can hardly account for the fields in clusters [4]–[6]. Other astrophysical mechanisms (involving for instance starbursts, jet-lobe radio sources, or accretion discs of black holes in AGNs [7]) seem to need pre-existing magnetic fields.

The cosmological mechanism is based on the generation of magnetic fields from cosmological perturbations [8]. The idea is that due to collision effects, there is a difference between the velocities of protons and electrons around and after the decoupling time, which leads to an electric current that generates a magnetic field. By this mechanism, magnetic fields of $\approx 10^{-18}$ G at the 1 Mpc scale and $\approx 10^{-14}$ G at the 10 kpc scale can be generated [9].

The current prevalent view is that the magnetic fields observed in galaxies and clusters have a primordial origin (see [10] for a review). The processes that may account for this origin can be divided in two types: causal (those in which the seeds are produced at a given time inside the horizon, like QCD and EW phase transitions [10]), and inflationary (where correlations are produced outside the horizon [11]). The former presents problems due to the fact that the coherence length of the fields so generated is much smaller than what is needed as input seed fields for a galactic dynamo [6], while in the latter, vacuum fluctuations of the electromagnetic field are ‘stretched’ by
the evolution of the background geometry to super-horizon scales, and they could appear today as large-scale magnetic fields. However, since Maxwell’s equations are conformally invariant in the FRW background, the amplification of the vacuum fluctuations (which amounts to particle production) via inflation can work only if conformal invariance is broken at some stage of the evolution of the universe. There are several ways in which this invariance can be broken [12]: non-minimal coupling between gravitation and the electromagnetic field [11,13,14], quantum anomaly of the trace of the stress-energy tensor of electrodynamics [15], coupling of the EM field to a charged scalar field [11,16], exponential coupling between a scalar (whose potential drives inflation) and the EM field [17], and a non-zero mass for the photon [18]. Yet another possibility is dilaton electrodynamics [19], in which there is a scalar field (the dilaton) which couples exponentially to an Abelian gauge field. In this model the inflationary expansion is driven not by the potential but by the kinetic term of the scalar field. The exponential coupling is naturally implemented in the low energy limit in string theory [20], and in Weyl integrable spacetime (WIST) [21], and can be viewed also as a time dependence of the coupling constant, an idea considered first by Dirac [22]. This avenue has been pursued by Giovannini in a series of articles [23]. More recently, a model with both a dilaton and an inflaton was analysed in [24], and later generalized to a non-commutative spacetime [25]. In all these articles, different aspects of photon production have been analysed. We studied in [26] the features of the creation of photons in a non-singular exact solution of Einstein’s gravity coupled to the dilaton, in which the passage through the bounce is described by the equations of the model without resorting to unknown (Planck-scale) physics. The conformal symmetry was broken through the exponential coupling of the dilaton to the EM field. Using the formalism of the squeezed states [27], the number of created photons in this model was obtained through numerical calculation in [26]. In this paper we shall re-address the creation of photons in the above-mentioned model by a different quantization technique (i.e. canonical quantization). The advantage in this case is that the results are obtained in a completely analytic manner. Moreover, we present here a complete analysis of the produced magnetic field spectrum. We shall find an exact solution for the mode equation of the vector potential $A_\mu$, and from it an exact expression for the spectrum of the magnetic field will follow. We shall see that the requirement that the magnitude of the seed fields produced by the bounce are compatible with observation can be fulfilled by a large range of values of the parameters of the model.

1. Field equations

A time-dependent gauge coupling is a generic feature of four-dimensional theories obtained by compactification of a more general theory such as Kaluza–Klein theory [28] and string theory [29]. Time-dependent gauge couplings are also present in WIST [21]. In all these cases, the action can be conveniently written in the form

$$S = \frac{1}{2} \int d^4x \sqrt{-g} f(\omega) F_{\alpha\beta} F^{\alpha\beta},$$

(1)

Two other interesting features of the solution representing this model are that it goes automatically into a radiation regime after a short time, and it displays a constant value for the dilaton after entering the radiation era.
where \( \omega \) is the dilaton (in the case of string theory) or the scalar field associated to Weyl geometry (in the case of WIST) and \( F_{\alpha \beta} \) is an Abelian field. The function \( f(\omega) \) will be set in the following to \( e^{-2\omega} \), which corresponds to the case of string theory and WIST.

From now on we shall work in conformal time, with the FRW metric given by

\[
d s^2 = a(\eta)^2 \left[ d\eta^2 - \gamma_{ij}(\vec{x}) \, dx^i \, dx^j \right],
\]

where \( \gamma_{ij} \) is the metric of the hypersurface \( \eta = \text{constant} \), and \( i = 1, 2, 3 \).

Using the definition of the canonically normalized vector potential \( A_{\mu} \equiv e^{-\omega} A_{\mu} \), and the radiation gauge, defined by \( A_0 = 0 \), \( \vec{\nabla} \cdot \vec{A} = 0 \), the Lagrangian density in terms of \( A_{\alpha} \) can be obtained from equation (1):

\[
\mathcal{L} = \frac{1}{2} \left[ (A_{\alpha}')^2 + (\omega'^2 - \omega^2) A_{\alpha}^2 - (\partial_\alpha A_{\alpha})^2 \right],
\]

where the prime indicates derivative w.r.t. conformal time \( \eta \). The equation of motion that follows from this Lagrangian is

\[
\Box A_i + (\omega'' - \omega^2) A_i = 0. \tag{2}
\]

Since we are dealing with vector quantities we shall use a vector basis \( P^i \) defined on a hypersurface \( \eta = \text{constant} \) by the following relations [30]

\[
P^i = P^i(x^i), \quad \gamma^{ij} \nabla_i \nabla_j P^i = -m^2 P^i, \quad \gamma^{ij} \nabla_i P_j = 0.
\]

The eigenvalue \( m \) denotes the wavenumber of the corresponding vector eigenfunction of the Laplacian operator on the spatially homogeneous hypersurfaces. The spectrum of eigenvalues depends on the 3-curvature \( \epsilon \), and is given by

\[
m^2 = s^2 + 2, \quad 0 < s < \infty, \quad \text{for } \epsilon = -1,
\]

\[
m = s, \quad 0 < s < \infty, \quad \text{for } \epsilon = 0,
\]

\[
m^2 = s^2 - 2, \quad s = 2, 3, \ldots, \quad \text{for } \epsilon = 1.
\]

In terms of the basis \( P^i \) the vector potential can be expanded as follows:

\[
A^i(\eta, \vec{x}) = (2\pi)^{-3} \sum_{\alpha, \sigma} \int d^3 m \, A^{(\sigma)}_{\alpha \sigma}(\eta) \, P^{(\sigma) i}_{\alpha \sigma}(\vec{x}).
\]

This expression is valid in the cases \( \epsilon = -1, 0 \). For \( \epsilon = 1 \), the integral must be replaced by a sum. The index \( \alpha = 1, 2 \) describes the two transverse degrees of freedom. We shall work with an expansion in terms of travelling waves, hence \( \sigma \) takes the values ‘+’ or ‘−’ according to

\[
A^i(\eta, \vec{x}) = (2\pi)^{-3} \sum_{\alpha} \int d^3 m \, (A^{(+)}_{\alpha \sigma}(\eta) \, P^{(+)}_{\alpha \sigma}(\vec{x})) + (A^{(-)}_{\alpha \sigma}(\eta) \, P^{(-)}_{\alpha \sigma}(\vec{x})).
\]

In the case of a 3-space with zero curvature, the \( P^i \) are such that \( P^{(+)}_{\alpha \sigma}(\vec{x}) = \delta_{\alpha \sigma} e^{\pm i \vec{m} \cdot \vec{x}} \), with \( \vec{e}^\alpha \cdot \vec{m} = 0, \vec{e}^\alpha \cdot \vec{e}^\sigma = \delta_{\alpha \sigma} \). Substituting this expansion in equation (2) we obtain the equation that governs the evolution of each of the polarization modes of \( A^i \):

\[
A^{(+)}_{\alpha \sigma}(\eta) + [m^2 - V(\eta)] \, A^{(+)}_{\alpha \sigma}(\eta) = 0,
\]

and a similar equation for \( A^{(-)}_{\alpha \sigma}(\eta) \), where \( V(\eta) = -\omega'' + \omega'^2 \).
The vector potential can be quantized following standard procedures, by transforming the mode functions into operators [31]:

\[ \hat{A}_i(\eta, \vec{x}) = \sum_\alpha \int \frac{d^3m}{(2\pi)^{3/2}} \left[ \hat{a}_{\vec{m} \alpha} A_m(\eta) P^{(+)}_{\vec{m} \alpha}(\vec{x}) + \hat{a}^{\dagger}_{\vec{m} \alpha} A_m(\eta)^* P^{(-)}_{\vec{m} \alpha}(\vec{x}) \right], \]

\[ \hat{\Pi}_i(\eta, \vec{x}) = \sum_\alpha \int \frac{d^3m}{(2\pi)^{3/2}} \left[ \hat{a}_{\vec{m} \alpha} \Pi_m(\eta) P^{(+)}_{\vec{m} \alpha}(\vec{x}) + \hat{a}^{\dagger}_{\vec{m} \alpha} \Pi_m(\eta)^* P^{(-)}_{\vec{m} \alpha}(\vec{x}) \right]. \]

Here

\[ \Pi_m(\eta) = A'_m(\eta), \quad [\hat{a}_{\vec{m} \alpha}, \hat{a}^{\dagger}_{\vec{m} \alpha}] = \delta_{\alpha \beta} \delta^{(3)}(\vec{m} - \vec{p}), \]

and the modes \( A_m \) obey equation (3).

2. The background

As discussed in [32], there are non-singular solutions in the theory of WIST that describe a FRW geometry plus a scalar field, as well as in string theory [33]. Let us briefly review these solutions. The EOM for gravitation plus scalar field written in conformal time are [32]

\[ a'^2 + \epsilon a^2 + \frac{\lambda^2}{6} (\omega')^2 = 0, \]

\[ \omega' = \gamma a^{-2}, \]

where \( \gamma = \text{constant} \), and \( \lambda^2 \) is the coupling constant of the scalar field to gravity. Notice that the equation of state of the scalar field is given by \( \rho_\omega = p_\omega \), where

\[ \rho_\omega = -\frac{\lambda^2}{2} \left( \frac{\omega'}{a} \right)^2. \]

From equations (4) we get

\[ a'^2 = -\epsilon a^2 - \frac{a_0^4}{a^2}, \]

where we have defined \( a_0^2 = \frac{\lambda \gamma}{\sqrt{6}} \). Equation (5) shows that only solutions with \( \epsilon = -1 \) are possible. Hence, from now on we shall restrict to the negative curvature case, for which equation (5) can be easily integrated. The result of the integration for the scale factor is\(^5\)

\[ a(\eta) = a_0 \sqrt{\cosh(2\eta + \delta)}. \]

Equation (4) yields

\[ \omega(\eta) = \pm \frac{\sqrt{6}}{2\lambda} \arctan(e^{2\eta + \delta}) + \frac{4\pi}{\sqrt{6} \lambda}, \]

where \( \delta \) is an integration constant. Notice that the model here presented has two independent constants. The plots for these functions are given in figure 1. The scale factor displays a bounce, produced by the violation of the strong energy condition by the scalar field [34].

\(^5\)The expression for the scale factor in terms of the cosmological time was given in [32], although not in a closed form.
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Figure 1. Plot of the scale factor and $\omega$ as a function of the conformal time for $\lambda = a_0 = 1$.

Figure 2. Plot of $V(\eta)$ for $\lambda = 0.1, 0.3, 0.5$.

3. Mode equation and spectrum

In terms of the field given in equation (7), the potential takes the form

$$V(\eta) = \frac{2\sigma \sinh(2\eta) + \sigma^2}{\cosh^2(2\eta)},$$

(where $\sigma \equiv \sqrt{6}/\lambda$) and its plot is given in figure 2. In order to calculate quantities of interest related to the magnetic field, we need to solve the mode equation (3), which in the case at hand is

$$A_m(\eta)^\prime\prime + \left( m^2 - \frac{2\lambda\sqrt{6} \sinh(2\eta) + 6}{\lambda^2 \cosh^2(2\eta)} \right) A_m(\eta) = 0.$$
The general solution of this equation is given by
\[ A_m(\eta) = d_1 Z_1(\eta) + d_2 Z_2(\eta), \]
where
\[ Z_1(\eta) = e^{-\omega(\eta)} F(a, -a; c; y(\eta)), \]
\[ Z_2(\eta) = e^{-\omega(\eta)} y(\eta)^{1-c} F(1 + a - c, 1 - a - c; 2 - c; y(\eta)), \]
with \( \omega(\eta) \) given by equation (7) and
\[ a = \frac{im}{2}, \quad c = \frac{1 + i\sigma}{2}, \quad y = \frac{1 + i\sinh(2\eta)}{2}. \]
The coefficients in the superposition in \( A_m(\eta) \) have been determined in order to satisfy the following asymptotic conditions:
\[ A_m(\eta) = \frac{1}{\sqrt{2m}} e^{-im\eta} \equiv A_m^{(\text{in})}(\eta) \quad \text{for} \quad \eta \to -\infty \]
(which implies that \( |A_m(\eta)|^2 = (1/2m) \), and
\[ A_m(\eta) = \left\{ \frac{[\Gamma((1 - im)/2)]^2}{\Gamma((1 - im - i\sigma)/2)\Gamma((1 - im + i\sigma)/2)} \right\} \frac{e^{-im\eta}}{\sqrt{2m}} - \left[ \frac{\sinh(\pi\sigma/2)}{\cosh(\pi m/2)} \right] \frac{e^{im\eta}}{\sqrt{2m}} \equiv A_m^{(\text{out})}(\eta) \quad \text{for} \quad \eta \to \infty. \]
They are given by
\[ d_1 = \frac{\sqrt{2\pi} \exp(i\pi/4) \Gamma((1 + im + i\sigma)/2) \Gamma((1 + im - i\sigma)/2) \Gamma((1 - im + i\sigma)/2)}{\sqrt{m} \Gamma((1 + im)/2) \Gamma((1 - im)/2) \left[ \Gamma((1 + im + i\sigma)/2) \Gamma((1 - im - i\sigma)/2) + \Gamma((1 - im + i\sigma)/2) \Gamma((1 + im - i\sigma)/2) \right]}, \]
\[ d_2 = -\frac{\sqrt{2\pi} \exp(i\pi/4) \Gamma((1 + im + i\sigma)/2) \Gamma((1 + im - i\sigma)/2) \Gamma((1 - im - i\sigma)/2) \Gamma((1 - im + i\sigma)/2)}{\sqrt{2} \Gamma((3 - im)/2) \Gamma((1 + im)/2) \left[ \Gamma((1 + im + i\sigma)/2) \Gamma((1 - im - i\sigma)/2) + \Gamma((1 + im - i\sigma)/2) \Gamma((1 + im + i\sigma)/2) \right]}. \]
From them we can read out the Bogoliubov coefficients that relate the ingoing with the outgoing fields, respectively given by
\[ \hat{A}^{(\text{in})}(\vec{x}, \eta) = \int d^3m \sum_s [\hat{a}_{\vec{m}, s} A_m^{(\text{in})}(\eta) P_{\vec{m}, s}^i + \hat{a}_{\vec{m}, s}^\dagger A_m^{(\text{in})\ast}(\eta)^* P_{\vec{m}, s}], \]
\[ \hat{A}^{(\text{out})}(\vec{x}, \eta) = \int d^3m \sum_s [\hat{b}_{\vec{m}, s} A_m^{(\text{out})}(\eta) P_{\vec{m}, s}^i + \hat{b}_{\vec{m}, s}^\dagger A_m^{(\text{out})\ast}(\eta)^* P_{\vec{m}, s}]. \]
The operators \( \hat{a}_{\vec{m}, s} \) and \( \hat{b}_{\vec{m}, s} \) are related by the Bogoliubov coefficients \( \alpha_{ij} \) and \( \beta_{ij} \) as follows:
\[ \hat{b}_i = \sum_j (\alpha_{ij}\hat{a}_j + \beta_{ij}^* \hat{b}_j^\dagger). \]
In our case matrices \( \alpha_{ij} \) and \( \beta_{ij} \) are given by
\[ \alpha_{\vec{m}, \vec{n}} = \delta(\vec{j} - \vec{m}) c^+_m, \quad \beta_{\vec{m}, \vec{n}}^* = \delta(\vec{j} + \vec{m})(-1)^s c^{-s}_{m}, \]
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\[ c^+_m = \frac{[\Gamma((1 - im)/2)]^2}{\Gamma((1 - im - i\sigma)/2)\Gamma((1 - im + i\sigma)/2)}, \]

\[ c^-_m = -(-1)^\ast \frac{\sinh(\pi\sigma/2)}{\cosh(\pi m/2)} \]

(8)

with \(|c^+_m|^2 - |c^-_m|^2 = 1\). These coefficients yield the spectrum of the magnetic field through the expression

\[ \rho(m) = \frac{\hbar m^4 c}{a^4 \pi^2} \left(1 + 2|c^-_m|^2 - |c^-_m| \left( c^+_m e^{-2im\eta} + c^*_m e^{2im\eta} \right) \right), \]

which can be rewritten as

\[ \rho(m) = \frac{\hbar m^4 c}{a^4 \pi^2} \left(1 + 2|c^-_m|^2 - 2|c^-_m| |c^+_m| \left( \cos(2m\eta - \theta_m) \right) \right), \]

where \( \theta_m = \arctan(\text{Im}(c^+_m)/\text{Re}(c^+_m)) \). When \( \pi\sigma \gg 1 \) (that is, when \( \sqrt{6}\pi \gg \lambda \)), and using \(|c^+_m|^2 - |c^-_m|^2 = 1\) this expression becomes:

\[ \rho(m) \approx \frac{\hbar m^4 c}{a^4 \pi^2} \left\{ 1 + 4|c^-|^2 \sin^2 \left( m \eta - \frac{\theta_m}{2} \right) \right\} \]

(10)

plus corrections \( \mathcal{O}(1/|c^-|^2) \). This expression for the spectrum is valid for \( \eta \gg 1 \). When translated into cosmological time, it gives \( t \gg \alpha \times a_0 \), where \( \alpha \approx 10^2 \). Consequently, our result is valid for \( t = \text{today} \) if we choose a convenient value of \( a_0 \).

The first factor in equation (10) is the vacuum contribution, which is always present, and generates a spectrum which is unobservably small\(^6\).

It follows from the spectrum that the amplification factor with respect to the conformal vacuum (subtracting the vacuum contribution coming from the first factor in equation (10)) peaks for the modes with momenta such that \( m \sim 1.31 \), and it is of the order,

\[ \frac{\rho_m}{(\rho_m)_{cl}} \sim \exp \left( \frac{\pi \sqrt{6}}{\lambda} \right), \]

(11)

which is exponentially large for \( \lambda \ll 1 \). Using the constants \( a_0 \) and \( \lambda \) (linked to \( \gamma \) through the relation \( a_0^2 = \lambda \gamma / \sqrt{6} \)), the conditions for the spectrum to be greatly amplified and to be valid today are

\[ a_0 \ll ct_r, \quad \lambda \ll 1, \]

where \( t_r \) is the time at which the scalar field is negligible, in such a way that the electromagnetic field is free again.

At a comoving scale of about 10 kpc the strength of conformal vacuum fluctuations is of the order \( 10^{-55} \) G. To get it to the strength required to seed the galactic dynamo, \( B_{\text{seed}} \sim 10^{-20} \) G, which is a conservative estimate, we get for the required amplification factor, \( \exp(\pi \sqrt{6}/\lambda) \sim 10^{35} \), or \( \lambda \approx 0.1 \). If one takes for the comoving scale the size of the

\(^6\) The question of why this contribution does not add to the stress-energy tensor at very small wavelengths is tantamount to the cosmological constant problem, which is beyond the scope of this manuscript.
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Universe, $4 \times 10^6$ kpc, the amplification factor becomes, $\exp(\pi \sqrt{6}/\lambda) \sim 10^{46}$ for which we find $\lambda \simeq 0.07$.

Note that the magnetic energy produced in our model can be calculated by taking the time average of equation (10) and integrating in $m$, yielding to a good approximation

$$\langle \rho_B \rangle \approx \rho_{\text{cf}}(m = 2/\pi) e^{\pi \sqrt{6}/\lambda},$$

where $\langle \rangle$ denotes time average. Since $\rho_{\text{cf}}(m = 2/\pi)$ is extremely small, by taking the values of $\lambda$ mentioned in the previous paragraph, our mechanism allows from ample amplification without being in conflict with both the bounds from nucleosynthesis [35] and the bounds coming from [36].

4. Discussion

We have shown that when the universe evolves through a bounce in a theory in which the EM field is coupled to a scalar field through an exponential (as is the case of string theory and WIST), EM fields of the magnitude needed for a seed can be generated for a large range of values of the parameters in the theory ($\lambda$ and $a_0$). The expression for the spectrum has been obtained in a complete analytical manner. Notice that the evolution of the geometry on this model is continuous, since the bounce is caused not by ‘new physics’ but by the scalar field.

As regards the matching of the bounce era onto radiation and matter era, since gauge fields are conformal and so is the bounce at asymptotically late times, the matching onto the radiation era solutions,

$$\frac{1}{\sqrt{2m}} e^{+im\eta} \quad \text{and} \quad \frac{1}{\sqrt{2m}} e^{-im\eta}$$

is trivial (there is no Bogoliubov mixing), which in turn implies that the spectrum calculated during the bounce remains unchanged during radiation and matter eras.

The model presented here shows that the mechanism for production of magnetic fields during the bounce may work, but we do no claim that it actually solves the problem. Some more work is needed in certain aspects. In particular, if we want to interpret the spectrum obtained here as that observed today, we need to assume that no further amplification occurs at the bounce–radiation transition. The incorporation of matter into the model should also be considered (perhaps along the lines of [26]). We hope to deal with these matters in a future publication.

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