EPR Correlations as an Angular Hanbury-Brown—Twiss Effect

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It is shown that EPR correlations are the angular analogue to the Hanbury-Brown—Twiss effect. As insight provided by this model, it is seen that, the analysis of the EPR experiment requires conditional probabilities which do not admit the derivation of Bell inequalities.

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Bell’s Theorems purport to prove that an objective, local hidden-variable extension of Quantum Mechanics (QM) is impossible. This result has been called “beautiful” and the century’s most significant discovery. For some, however, this result is a symptom of error or misunderstanding.

Of course, a theorem does not establish a universal, unrestricted truth; it only tests symbolic manipulations, that is mathematics, for consistency, given an hypothesis. A search for error in Bell’s analysis, therefore, is nothing but a critical review of its hypothesis. The obdurate realist, who wishes to challenge Bell’s conclusion, has only two options: QM must be wrong (perhaps incomplete or otherwise defective on the margins) or, the hypothesis contains error.

Within QM, all that is needed to obtain the expressions relevant to the Einstein, Rosen and Podolsky (EPR) experiment, as modified by Bohm (EPRB), at the heart of Bell’s analysis, is a superposition state; the rest follows from simple geometric transformations. Indeed, exactly this feature of QM has been questioned, starting with Furry. He suggested that for macroscopic distances, a superposition state converts to a mixed state.

The second option, seeking error in the hypothesis of what should constitute an objective local extension of QM, is likewise lean on possibilities. The hypothesis Bell used was scarcely more than the assertion that the coincidence intensity for the EPRB experiment is to be given by:

$$P(a, b) = \int I_A(a, \lambda)I_B(b, \lambda)\rho(\lambda)d\lambda,$$  \hspace{1cm} (1)

where notation and content are taken from Bell with the modification that $I_A$ stands for the count rate, or photoemission probability, at measuring station $A$, etc. 

For ideal photodetectors, this count rate is proportional to the impinging field intensity; i.e., to the square of the field strength.

It is the purpose here to analyze just these assumptions, in particular the second, and to show that in fact application of the principles underlying the Hanbury-Brown—Twiss Effect, permits an objective, local interpretation of the EPR correlations.

The application of Furry’s proposal to the EPRB experiment, proceeds as follows. It is assumed that the source emits classical electromagnetic radiation polarized in a particular but random direction. It is taken that in each arm of the setup, this radiation is to be directed through a polarizer and then detected using a photodetector which obeys the square law; i.e., it emits photoelectrons in proportion to the square of the intensity of the absorbed radiation. That is, the probability of emission of a photoelectron in each arm of an EPR experiment is $\langle E \cos(\theta) \rangle^2$, where $\theta$ is the angle between the polarization direction of the signal and the axis of the polarizer used in the detector.

Finally, the total coincidence rate is obtained by averaging over many pairs of signals, each with its own randomly given polarization angle $\theta$, that is

$$\frac{1}{2\pi} \int_0^{2\pi} [\cos(\theta) \cos(\theta - \phi)]^2 d\theta = 1/4 + 1/8\cos(2\phi). \hspace{1cm} (2)$$

To convert this intensity to a probability it must be re-cast as the ratio of a coincidence rate divided by the total count rate. For ideal detectors, the total number of detections is linearly proportional to the sum of the field intensities at both detectors, that is: $2\langle I_0 \rangle I(\theta)d\theta = 1$. This model was examined as a semiclassical EPR variant in Ref. [3].

This expression seems perfectly rational and, as the resulting correlation is $\cos(2\phi)/2$, it does not violate a Bell Inequality. It would resolve the conundrums evoked by Bell’s Theorems were it to agree with experiment. However, this result has a nonzero minimum, whereas the QM equivalent, $\cos^2(\phi)/2$, does go to zero and this difference has been observed and reported in Ref. [3]. Eq. (2) does not conform to Nature.

If Furry’s Ansatz is to benefit a realist program, it must therefore be modified in some essential. The search need not be carried far. The radiation in an EPR experiment
emanates from a single source (which may comprise many microunits, atoms say) and is by assumption such that the twin emissions pairwise are not to carry off angular momentum. In the case of radiation, this effectively means that if one is polarized so as to pass a polarizer in any particular direction, the other must be blocked by a polarizer in this direction. Obviously, such emissions are not statistically independent in each arm. In turn, without statistical independence, the assumption of factorizability of the joint probability as employed in writing Eq. (1), is not admissible. Factorization is overly restrictive and not valid. This is quite reasonably so, as the question is: given that a particular result is obtained in one arm, — - absolute simultaneity is impossible — what are the probabilities of outcomes in the second arm? This calculation demands conditional probabilities and they are not factorizable.

This in no way, however, implies nonlocality; rather, it implies just statistical dependence; i.e., the probabilities of ‘Bertlmann’s socks.’ Nonlocality, taken as a violation of Einstein’s principle that all influences effective at a particular event (point) in Minkowski space, must originate at points in the past light cone of that event, is not violated. The correlation resulting from most forms (a realist holds: all forms) of statistical dependence is simply derived from a common cause. Of course, as factorizability always holds for the coincidence probability of statistically independent events, it inevitably implies no violation of locality. Indeed, such events have no cause-effect relationship. In summary, factorizable coincidences map onto but are not one-to-one on the set of all coincidence functions for events respecting locality.

Thus, as an alternate to the Furry inspired model described above, consider the following:

\[ P(a, b) = \frac{\langle E_A \cdot E_B E_B \cdot E_A \rangle}{\langle |E_A|^2 + |E_B|^2 \rangle}, \tag{5} \]

where the angle brackets indicate an ensemble average over all values of \( \theta \), the angle of attack of each separate signal, or, on an ergotic principle, over the random phases of the individual atomic sources. The dot product is with respect to the orthogonal set \{ \( \hat{x}, \hat{y} \) \}.

The numerator in Equation 3 is the probability of a coincidence count; as usual for ideal detectors, it is the product of the intensities of the separate signals, but in the form taught by coherence theory. Traditionally, intensity correlation calculations were based on the direct product of intensities, \((I_1 I_2)\), whereas coherence theory teaches that the correct form for this calculation is \((E_1 \cdot E_2 E_2 \cdot E_1)\). The effective difference is that the later form allows the phase to contribute to calculation. It is the information in the phase that is required to explain the Hanbury-Brown—Twiss effect as well as other coherence phenomena.

Note that all the information used in the calculation of the numerator of Equation 3 is equal to the total intensity of both signals in both detectors and is, therefore, proportional to the total photoelectron count, again, for ideal detectors. The ratio of the numerator to the denominator then is by definition the probability of coincidence counts.

Taking all the above into account, provides the following expression for the coincidence count rate:

\[ P(\hat{a}, \hat{b}) = \frac{\int_{0}^{2\pi}(\cos(\theta)\sin(\theta)-\sin(\theta)\cos(\theta-\phi))^2d\theta}{2\int_{0}^{2\pi}(\cos^2(\theta)+\sin^2(\theta))d\theta}. \tag{6} \]

Evaluated, this integral equals the QM result, Eq. (3):

\[ P(\hat{a}, \hat{b}) = \frac{1}{2}\sin^2(\phi). \tag{7} \]

This model, comprising non quantum components, is fully local in the Einsteinian sense; and, as it agrees with QM, it is in accord with those laboratory observations verifying QM. In essence it is, given the vector character of electromagnetic radiation, just the angular analogue of the Hanbury-Brown—Twiss Effect. It stands as a counterexample to Bell's conclusion.
Like QM, however, it violates Bell Inequalities. Such inequalities, however, are derived under the assumption that the relevant coincidence probabilities factor into two terms, which a second order coherence function does not in general allow. That is: Bell inequalities are not valid for all forms of fully local coincidences.

This can also be shown as follows. As a matter of fact, the most general form for the coincidence count between stations \( A \) and \( B \) in the EPRB experiment is a function of three sets of variables: \( P(a, b, \lambda) \), where \( a, b \) are those variables that specify conditions at the measuring stations \( A \) and \( B \), and \( \lambda \) specifies all common causes pertaining to the generation of the two signals. The \( \lambda \), not being explicit in QM, have been denoted “hidden variables.” The coincidence count considered in QM is the marginal probability derived from the full coincidence probability by integration over \( \lambda \):

\[
P(a, b) = \int P(a, b, \lambda) d\lambda. \tag{8}
\]

Now, the identity from probability theory:

\[
P(a, b, \lambda) = P(\lambda)P(a|\lambda)P(b|a, \lambda), \tag{9}
\]

where \( P(x|y) \) is the conditional probability of \( x \) contingent on \( y \), exposes the intrinsic structure of such a coincidence. [4]

This form reduces to that of the integrand of Equation 1 when \( P(b|a, \lambda) = P(b|\lambda) \); i.e., when the events at \( A \) and \( B \) are statistically independent; in other words, when there is no relationship between them. This is fundamentally contrary to the structure of the EPRB experiment in which it is taken that the emissions are correlated. It is easy to verify that no derivation of a Bell inequality goes through using Equation 9. Thus, such inequalities do not pertain to correlated events.

Equation 9 does not imply that information is telegraphed from station \( A \) to station \( B \). It means only that the counts registered at both stations will exhibit correlations that will become evident when the data is brought together at a later time for comparison. Such a comparison can be made, naturally, only at a point in Minkowski space for which the the past light cone includes the measuring stations \( A \) and \( B \). Likewise, the correlations did not arise with the help of superluminal, or any other, communication. The structure yielding the correlations when the measuring stations are specified by \( a, b \), is built into these signals at their source which is in the past light cones of both stations. \( P(b|a, \lambda) \) being contingent on \( a \) is a realization not of communication between stations \( A \) and \( B \), but of correlations invested in the signals at the common source. For the EPRB experiment, clearly, there can be no coincident count when the polarizers are parallel regardless of the orientation of the signals (so long as they are orthogonal, as assumed in the first place). Thus, the dependence of the conditional probability is the consequence of the necessity of the detectors to be set so as to admit detection of the correlated characteristics of the signals, here orthogonal polarization.

Of additional interest is the fact that the new model moves the nonfactorizable structure from a superposition wave function to the form of the coincidence probability. This affords considerable simplification of discussions on the interpretation of QM. Superposition wave functions have been the source of much confusion, requiring as they do, “collapse” for ontological meaning.

In conclusion, using the correct classical-physics method to calculate a coincidence count in the EPRB experiment, yields the QM result and exposes an inappropriate assumption in the derivation of Bell inequalities. No error has been found in QM, rather, just the argument against an objective local extension of QM has been put aside. This is at no cost to any established theoretical or empirical result from QM. The fact that experiments to test Bell inequalities have virtually beyond all argument supported QM, do not by themselves imply that QM is nonlocal. They prove no more than that inequalities that should obtain for objective local extensions of QM, but were derived under a false premise in any case, are not valid. Indeed, we see, they can not be.

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[1] W. H. Furry, Phys. Rev. 49, 393 (1936).
[2] J. S. Bell, Speakable and unspeakable in quantum mechanics, (Cambridge University Press, Cambridge, 1987).
[3] J. F. Clauser, Phys. Rev. A 6 (1), 49 (1972). See also: A. Afriat and F. Selleri, The Einstein, Podolsky and Rosen Paradox, (Plenum Press, New York, 1999) for additional analysis of the Furry hypothesis, in particular further empirical evidence of its inadequacy.
[4] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics (Cambridge University Press, Cambridge, 1995).
[5] Bell’s comments on conditional probabilities, Ref. [1], p. 152, are inexplicably unfathomable and false. (One is tempted to say ‘silly.’) Contrary to his speculations, no manipulations of variables, hidden or overt, “would allow decoupling of the fluctuations” [of statistically dependent outcomes], and render the probability that Professor Bertelsmann is wearing two pink socks factorizable [and, therefore, equal to \( 1/2 \times 1/2 = 1/4 \)]. His statement that, when \( \lambda \) the common cause variable, is fixed, the probabilities become statistically independant, is true with respect to all other variables, but of no use. In order to recover the marginal probabilities for which the inequalities obtain, the ‘hidden’ variables, \( \lambda \), must be integrated out, and such an integration is meaningless if the \( \lambda \) are all fixed and can not roam as a dummy of integration.
[6] W. Feller, An Introduction to Probability Theory and Its Applications, (John Wiley & Sons, 1950) p. 116.