Weyl geometric gravity and “breaking” of electroweak symmetry

Erhard Scholz

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Abstract
A Weyl geometric scale covariant approach to gravity due to Omote, Dirac, and Utiyama (1971ff) is reconsidered. It can be extended to the electroweak sector of elementary particle fields, taking into account their basic scaling freedom. Already Cheng (1988) indicated that electroweak symmetry breaking, usually attributed to the Higgs field with a boson expected at $0.1 - 0.3 \, TeV$, may be due to a coupling between Weyl geometric gravity and electroweak interactions. Weyl geometry seems to be well suited for treating questions of elementary particle physics, which relate to scale invariance and its “breaking”. This setting suggests the existence of a scalar field boson at the surprisingly low energy of $\sim 1 \, eV$. That may appear unlikely; but, as a payoff, the acquirement of mass arises as a result of coupling to gravity in agreement with the understanding of mass as the gravitational charge of fields.

1 Introduction
In the 1970s M. Omote, R. Utiyama, P.A.M. Dirac and others took up the idea of Jordan-Brans-Dicke theory to study gravitational Lagrangians linear in scalar curvature $R$ but coupled to a scalar field $\phi$ and studied it in the framework of Weyl geometry (Omote 1971, Omote 1974, Dirac 1973, Utiyama 1975a, Utiyama 1975b). During the following roughly two decades, different views of how mass might be “generated” due to a link between Weyl geometric gravity and symmetry breaking at the electroweak energy level were developed (Smolin 1979, Cheng 1988, Drechsler/Tann 1999, Drechsler 1999). Cheng indicated that the scalar field of gravity could well play a Higgs-like role, once it was extended to the weak isospin sector. He was not the only one to explore such a connection. In Brans-Dicke theory such a link was looked for in cosmology, the interaction zone of the “colliding
fields” gravity and elementary particle physics (Kaiser 2007, Kaiser 2006). Few authors drew upon the resources of Weyl geometry and the accordingly modified theory of gravity. W. Drechsler and H. Tann did so, but preferred to model the origin of mass by introducing a Lagrangian term which breaks scale symmetry explicitly.

For about ten years the topic seemed to be put aside, because of different theoretical interests in the mainstream of elementary particle physics and of gravitation theory. Only recently the discussion was reopened by proposals which foresee a Higgs-like role for the weak isospin extended scalar field of Weyl geometry, by introducing a second (real) scalar field $\sigma$ transforming under rescaling with the same weight $-1$ as $\phi$ (Nishino/Rajpoot 2004, Nishino/Rajpoot 2007b). After the first year of taking data at the LHC, it may be the right time to reconsider the topic of a possible connection between electroweak symmetry “breaking” and gravity from a Weyl geometric perspective. That is the goal of this article.

In order to make it essentially self-contained, and because of different notational conventions in the literature, basic definitions and properties of the Weyl geometric generalization of Riemannian geometry are recalled. A short discussion follows of how Weyl scaling is to be understood as a localized version of classical scaling in the sense of (Barenblatt 2003) (section 2.1).

A Weyl geometric version of gravity, slightly differing from Brans-Dicke theory by its explicit usage of the Weylian scale connection, but otherwise in strong formal analogy to the latter, was introduced and studied by M. Omote, R. Utiyama, P.A.M. Dirac and others in the 1970s. Its main structural ingredient is the Weylian scale connection $\varphi$ and its curvature $f = d\varphi$. Assuming a dynamical role for scale curvature $f$ leads to a mass term close to the Planck scale. That was observed by R. Utiyama, taken up in 1988 by Cheng. It was also seen, apparently independently, by L. Smolin in 1979. In the result, the dynamical aspect of scale symmetry is “broken” shortly below the Planck energy scale, and the Weylian scale connection loses the character of a field in its own right considerably below it. Geometrically, an integrable version of Weyl geometry remains at the laboratory level and for astronomically/astrophysically directly observable scales (section 2.2).

The scalar field $\phi$ of scale weight $-1$ ensures that this does not automatically imply a reduction of the geometric structure to Riemannian geometry, and of gravitation to Einstein gravity, although it is formally possible to choose the scale gauge such that Riemannian part of the metric alone survives and the scale connection vanishes (Riemann gauge). But scaling freedom of the theory and, with it, scale covariance of quantities remain; scale invariant expressions can be derived from them. Riemannian geometry expresses scale invariant quantities correctly only in the case of a trivial scalar field which is constant in Riemann gauge. A comparison with Einstein gravity is possible in different gauges, in particular those corresponding to “Jordan frame” or “Einstein frame” in Brans-Dicke theory (sections 2.3, 2.4).
The next passage contains a short exposition of how Weyl geometric gravity can be extended to the electroweak sector of elementary particle fields, taking into account their basic scaling freedom (section 3.1). The possibility, and naturality, of this extension is one of the reasons why one should not give up scaling freedom of geometry too early and without good physical reason. It should not come as too great a surprise, if the conceptual advantages of Weylian geometry over Riemannian one (even in the integrable version of Weyl structures) turns out to be of physical importance. Scale invariance and its “breaking” (more pragmatically, its reduction) is an essential ingredient of elementary particle physics. Weyl geometry is ideally suited to the purpose of giving such questions a clear conceptual and mathematical foundation, at least on the classical level. In this respect it agrees with the intention of some recent studies in conformal field theory (Gover e.a. 2009, Shaukat/Waldron 2010).

Sections 3.2 and 3.3 discuss what happens if the ordinary Higgs mechanism (in its non-quantized form) is transferred from special relativistic fields to Weyl geometrically “curved” spaces. That leads naturally to the question whether, from the present point of view, “gravity can do” what is usually ascribed to the Higgs “mechanism” (section 3.4). The answer indicated by Cheng, later roughly reiterated by van der Bij (van der Bij 1994), is that theoretically it can. This article investigates how that can be done and what it would mean. A most surprising aspect of the answer is that the scalar field boson, if any, would have classically negligible mass far below the threshold of present collider experiments, even after quantum corrections.

The core of this analysis is the biquadratic potential $R|\phi|^2 + \lambda_4|\phi|^4$ of the Weyl geometric scalar field, given by its coupling to scalar curvature $R$ (negative in the signature chosen here) and the quartic scale invariant term with coefficient $\lambda_4$ substituting the old “cosmological constant” term of Einstein gravity. The consequences of the potential for long range gravity are quantitatively tiny on solar system level, but of structural importance for cosmology. A most crucial difference to present standard cosmology is that, in the Weyl geometric framework, scalar curvature induced from the warp function of Friedman-Lemaitre cosmology becomes part of the field theoretic structure of the gravitational vacuum. Thus not only $\lambda_4$ but also $R$ induced from seemingly cosmological properties of large scale solutions should be present in empty space regions, otherwise well approximated by the classical Schwarzschild solution, and be attributed to the gravitational vacuum (section 4.2).

Short commentaries of different views of electroweak “symmetry breaking” and on the still wide open question of the relationship between gravity and the quantum round off the last section of the article (sections 4.1, 4.3).

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\[\text{1Later van der Bij changed his mind.}\]
2 Weyl geometric gravity (Omote-Dirac approach)

2.1 Weyl geometry/Weyl structures

Weyl geometry presupposes a (differentiable) manifold with a Weylian metric arising from a generalization of the Riemannian case. This generalization may be motivated by a conceptual analysis of what a differential geometrical structure should be able to express in agreement with field theoretical principles, and what it should not (no direct comparison of distant measurements). From a purely structural point of view, the definition of a Weylian metric may be streamlined by the more recent concept of a Weyl structure which, however, is not further considered here.

Weylian metric, Weyl scaling. Weyl introduced his “purely infinitesimal” generalization of Riemannian geometry as an attempt to find deeper roots for Einstein’s theory of gravity and to unify it with electromagnetism (Weyl 1918c, Weyl 1918b, Weyl 1918a). Weyl’s geometrical innovation was taken up in physics (Pauli 1921), not only in gravity (Eddington 1923, Bergmann 1942, Dirac 1973, Israelit 1999, Adler/Bazin/Schiffer 1975, Blagojević 2002), but also in classical (unified) field theory (Vizgin 1994). More recently, a handful of authors have considered it as a geometrical framework for exploring connections between electroweak theory and gravity (Smolin 1979, Cheng 1988, Drechsler 1999, Drechsler/Tann 1999, Nishino/Rajpoot 2004). In mathematics, Weyl geometry fared less favourably. In the first decades after its invention few differential geometers took it seriously. Only with the rise of modern differential geometric methods after the 1950s, it started to attract geometers’ interest (Folland 1970). Recently, Weylian scale gauge has been taken up and developed not only for the Riemannian case, but also for complex (hermitian and Kähler) and quaternionic geometry, under the heading of Weyl structures (Higa 1993, Gauduchon 1995, Calderbank 2000, Ornea 2001). As background literature physicists may like to use (Adler/Bazin/Schiffer 1975, chap. 15.2) and (Blagojević 2002, chap. 4) and the classics (Weyl 1918a, Eddington 1923, Bergmann 1942); more mathematically inclined readers may prefer (Weyl 1918c, Folland 1970, Higa 1993). As conventions and notations differ in the literature, we give a short account of basic concepts of Weyl geometry as used here in a fairly informal way.

Weyl geometry works in a Weylian manifold \((M, [g, \varphi])\), or above it, as far as fibre bundle constructions are concerned. \(M\) denotes a differentiable manifold, here usually of dimension \(n := \dim M = 4\). \([g, \varphi]\), the Weylian metric, is an equivalence class of pairs, each one consisting of a (pseudo-) Riemannian metric \(g\), the Riemannian component of the metric, and a real valued differentiable 1-form \(\varphi\), the scale or length connection. Locally \(g\) and \(\varphi\) are represented by \((g_{\mu\nu}) = g_{\mu\nu} dx^\mu dx^\nu\), respectively \((\varphi_\mu) = \varphi_\mu dx^\mu\). An

\(^2\)Needless to say that the literature citations are far from complete. They are intended to give an exemplary orientation on the topics mentioned.
equivalence between pairs \((g, \varphi) \sim (\tilde{g}, \tilde{\varphi})\) is given by conformally rescaling the Riemannian component of the metric
\[
\tilde{g} = \Omega^2 g,
\]
\(\Omega > 0\) a nowhere vanishing real valued function on \(M\). Simultaneously the length connection has to be \textit{gauge transformed} as part of the same procedure
\[
\tilde{\varphi} = \varphi - d \log \Omega.
\]

Choosing a representative \((g, \varphi)\) of the equivalence class \([g, \varphi]\), globally or locally, means to “gauge” the metric mathematically. Physically, such a choice expresses a locally dependent introduction of length units \(e_L\).

We thus adopt a primarily \textit{passive} interpretation of Weyl scaling (transformation of units), because material structures usually do not allow active up- or downscaling, at least not without further additional considerations (Barenblatt 2003). Historically, the revival of Weyl geometry in physics of the late 1960s and early 1970s was triggered by experimental indications of an approximative active scaling symmetry of deep inelastic scattering, i.e., form factors remained nearly invariant under rising energies. Further refinement of experimental knowledge has shown that, also in high energy physics, active scaling symmetry is broken and only of rough approximative validity. Nevertheless, in order to understand what is “broken” and how, respectively why, we need a deeper understanding of the underlying structure linking geometry and field theory. For that a mathematical framework designed to incorporate scale symmetry is useful. In this sense the reasons leading to the revival of Weyl geometry in physics continue to be valid, although in a different overarching perspective, more modest than in the 1960s. A similar motivation seems to lie at the basis of a recent research program studying the relation between Weyl’s gauge invariance, conformal geometry and mass generation using the symbolic tools of “tractors”, an extension of the tensor concept adapted to conformal rescaling (Gover e.a. 2009, Shaukat 2010, Shaukat/Waldron 2010). This approach differs from the Weyl geometric one by using a peculiar kind of covariant gradient operator (“Thomas \(D\)-operator”) which is no covariant derivative but can be “employed to a similar effect” (Gover e.a. 2009, 428).

3The Thomas \(D\)-operator is defined by means of the Levi-Civita derivative of any of the scaled metrics of the conformal class. It operates on quantities which transform according unified rules under scale transformations (tractors). It serves the purpose of unifying calculations in a conformal structure. In this way it differs from Weyl geometry proper.
energy $e_E$ become $e_T := c^{-1} e_L$, $e_E := \hbar e_T^{-1}$ etc.\textsuperscript{4} The total procedure, based on local scaling of length (or energy) units and a global convention for the numerical values of $c$ and $\hbar$, will be called \textit{Weyl scaling} of physical units.

As $c$ and $\hbar$ are dimensionally independent physical constants, they can be given arbitrary numerical values $|c|, |\hbar|$ also in the Weyl geometric context, like in “classical”, i.e. global, dimensional analysis (Barenblatt 2003). Typical examples for such global conventions are, e.g.,

(1) $|c| = |\hbar| := 1$ (“natural units”),
(2) or $|c| := 2.99792458 \cdot 10^{10}, |\hbar| := 6.5821220 \cdot 10^{-16}$ (comparable to the values in the cgs system).

In both examples no numerical value of the gravitational constant has been prescribed; attempting to do so would break Weyl scaling freedom/symmetry. Therefore the “natural” units (1) are not identical to Planck units and the system (2) is not identical with cgs units. An appropriate specification of length/time or energy units in addition to (2) leads to the latter (cgs)\textsuperscript{4}.

In the physics literature, the scale connection $\varphi = (\varphi_\mu)$ is often called “the Weyl (co-)vector field”. This is a misleading terminology; one has to keep in mind that $\varphi$ transforms as a connection with values in the (multiplicatively trivial) Lie algebra $\mathbb{R}$ of the localized group $(\mathbb{R}^+, \cdot)$, rather than as a covector field in the proper sense.

In general, we assume Lorentz signature of the Riemannian part of the metric, $\text{sig}\, g = (1,3) = (+\ldots)$. Geometrical investigations in general relativity often prefer signature $(3,1) = (\ldots++)$. For translation between the signature conventions we use the factor $\epsilon_{\text{sig}} = +1$ for $\text{sig}\, (g) = (1,3)$, and $\epsilon_{\text{sig}} = -1$ otherwise.

**Important properties (theorems)** A direct comparison of lengths of vectors $\xi, \eta$ is meaningful only if they are “attached” to the same point of the manifold, $\xi, \eta \in T_p M$; similarly for other physical quantities which are affected by local choice of scale, like energy, momentum etc. These will be called \textit{scale covariant} quantities (affected by the choice of the Riemannian component of the Weylian metric) or \textit{Weyl fields}. If a (scalar, vector, tensor spinor) field $X$ on $M$ is affected by rescaling under $\Omega$ according to (1) such that $X \mapsto \tilde{X} = \Omega^k X$ with $k \in \mathbb{R}$, then $w(X) := k$ is the Weyl or scale \textit{weight} of $X$.

Comparison of scale covariant quantities $X(p), X(q)$ at different points $p \neq q$ is meaningful only after “transport of length standards”, resulting in multiplication of $X(p)$ by $\lambda(\gamma)^w(X)$, where $\lambda(\gamma)$ is a “scale transfer” function

\textsuperscript{4}Of course, in place of a the local choice of length units also energy units might be chosen locally as the basic physical quantity. From a phenomenological point of view, the scale connection $\varphi$ then represents an “energy connection” rather than a “length connection” like in Weyl’s original view. Mathematically both views are equivalent.

\textsuperscript{5}In or context, a natural choice in scalar field gauge is (40).
arising from integration of the scale connection along a path \( \gamma \) between \( p \) and \( q \)

\[
\lambda(\gamma) = e^{\int_{\gamma} \varphi'(\gamma)}.
\]

Important theorems of Weyl geometry are:

- Like in the Riemannian case, a Weylian manifold \((M, [g, \varphi])\) possesses a uniquely determined compatible affine (i.e., torsion free) connection \( \Gamma = (\Gamma_{\nu\lambda}^\mu) \), the Weyl geometric Levi-Civita connection with corresponding covariant derivative \( \nabla = \nabla_\Gamma \), compatible with the Weylian metric\( ^6 \)
- “Uniquely determined” means that \( \Gamma \) and \( \nabla = \nabla_\Gamma \) are independent of scale gauge; in particular, \( \nabla \) is a “scale invariant” covariant derivative.
- Thus also the Riemannian curvature \( \text{Riem} = (R_{\lambda\rho\nu}^\mu) \) of \( \Gamma \) and its Ricci curvature \( \text{Ric} = (R_{\mu\nu}) \) are scale invariant, i.e., independent of the choice of units. Scalar curvature \( R = R_{\lambda}^{\lambda} \), on the other hand, is only scale covariant and of weight \( w(R) = -2 \).
- The Weyl geometric Levi-Civita connection \( \Gamma \) can be expressed in terms of the Levi-Civita connection \( g\Gamma \) of the Riemannian component of any gauge \((g, \varphi)\) and of the corresponding scale connection \( \varphi \) by

\[
\Gamma_{\nu\lambda}^\mu = g_{\nu\lambda} \Gamma_{\nu\lambda}^\mu + \delta_{\nu}^{\mu} \varphi_{\lambda} - g_{\varphi} \varphi_{\lambda} - g_{\nu\lambda} \varphi^\mu.
\]  \hspace{1cm} (3)

- Similar reductions exist for the Weyl geometric curvatures \( \text{Riem}, \text{Ric}, R \)\( ^7 \)

They can be expressed, in any gauge, by corresponding curvatures of the Riemannian component \( g\Gamma \) and expressions in the scale connection \( \varphi R \) etc. In particular \( \text{Ric} = g\text{Ric} + \varphi \text{Ric}, R = gR + \varphi R \) with

\[
\varphi \text{Ric}_{\mu\nu} = (n - 2)(\varphi_{\mu} \varphi_{\nu} - g_{\nabla_{(\mu} \varphi_{\nu)\varphi}}) - g_{\mu\nu}(n - 2)\varphi_{\lambda} \varphi^\lambda + g_{\nabla_{\varphi}} \varphi_{\lambda} \varphi^\lambda
\]  \hspace{1cm} \text{for } n = 4

\[
\varphi R = -(n - 1)(n - 2)\varphi_{\lambda} \varphi^\lambda - 2(n - 1)g_{\nabla_{\varphi} \varphi^\lambda}.
\]  \hspace{1cm} \text{for } n = 4

Here \( g_{\nabla} \) denotes the covariant derivative with respect to the Riemannian component only, \( g_{\nabla} = \nabla_\Gamma \).

- Scale curvature, i.e. the curvature \( f \) of the scale connection, is \( f = d\varphi \).

Note language: scale curvature \( \neq \) scalar curvature (“Ricci scalar”).

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\( ^6 \)Compatibility with the metric may be expressed by the demand that parallel transport of a vector \( X(p) \) along a path \( \gamma \) from \( p \) to \( q \) to \( X(q) \) leads to consistency with length transfer: \(|X(q)| = \lambda(\gamma)|X(p)|\).

\( ^7 \)Here the sign convention of differential geometry is used, \( \text{Riem}(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z \) in Ricci calculus \( R_{\mu\lambda\kappa\nu} = \partial_{\kappa} \Gamma_{\mu\lambda}^{\nu} - \partial_{\lambda} \Gamma_{\mu\kappa}^{\nu} + (\Gamma_{\mu\rho}^{\nu} \Gamma_{\rho\kappa}^{\lambda} - \Gamma_{\mu\kappa}^{\nu} \Gamma_{\rho\lambda}^{\rho}) \). Contraction convention for \( \text{Ric} \) is \( \text{Ric}_{\mu\nu} = R_{\mu\nu} = R_{\mu\lambda\nu} \).
• If \( f = 0 \), there exist a local choice of units s.th. \( \tilde{\varphi} = 0 \). In this case we have, in simply connected regions, a locally integrable Weyl geometry (IWG). Then the scale choice \((\tilde{g}, 0)\) defines a Riemannian metric. It is called Riemann gauge of the IWG.

• Generally, the covariant derivative \( \nabla X \) of a scale covariant quantity \( X \) is, itself, not scale covariant. Therefore a scale covariant derivative (sometimes also called “Weyl covariant derivative”) \( D \) is defined by

\[
DX := \nabla X + w(X) \varphi \otimes X
\]

\( DX \) is again scale covariant (a Weyl field) and \( w(DX) = w(X) \).

• Example: \( \nabla g \) is not scale covariant, but \( Dg \) is. Moreover

\[
Dg = \nabla g + 2\varphi \otimes g = 0 ;
\]

i.e., \( g \) is scale covariantly constant. Compare: In Riemannian geometry the metric is covariantly constant, \( \tilde{\nabla} g = 0 \).

• In the literature of theoretical physics \( \nabla \) is sometimes considered as an indicator of “non-metricity” of the scale covariant derivative; sometimes it is called, more generously, “semi-metricity”. From a Weyl geometric point of view, the scale covariant derivative is just “metrical”, \( Dg = 0 \).

Comparison of conventions In the physics literature (and in Weyl’s writings) the weights are often half of ours, because the scaling condition is written as \( \tilde{\gamma} = \Omega g \) rather than \( \tilde{\gamma} = \Omega^2 g \). Moreover, in most of the physics literature (not so in most of Weyl’s writings) the sign convention for the scale connection \( \varphi_{\text{lit}} \) is inverse to ours (and Weyl’s generic choice); both together result in \( \varphi_{\text{lit}} = -2\varphi \). Thus the formulas \( \text{(2)} \) ff. appear in a slightly different form, \( \text{(2)} \) itself becomes, e.g., \( \varphi_{\text{lit}} = \varphi_{\text{lit}} + \frac{4}{\sqrt{2}} \log \Omega_{\text{lit}} \) etc. (\( \varphi \) our convention, \( \varphi_{\text{lit}} \) the other one). The reason for our choice of sign can be found at the end of footnote \( \text{6} \) (in addition to keeping to Weyl’s generic convention); the weights are oriented at lengths as reference quantities \( \text{6} \).

2.2 Weyl geometric gravity, Omote-Utiyama-Dirac approach

Weyl geometry is a modest modification of Riemannian geometry; therefore it allows for natural generalizations of Einstein gravity. Its basic principle
is scale invariance of the underlying Lagrange density $\mathcal{L} = L \sqrt{|\det g|}$. As $\sqrt{|\det g|}$ is of weight 4, only Lagrange functions $L$ of weight $w(L) = -4$ are to be considered, in particular for the gravitational term $L_G$. Weyl explored Lagrangians $L_G$ quadratic in the curvature. Coupling of curvature with a scalar field broadens the possibilities. The first author to apply such a Brans-Dicke like modified Hilbert action in Weyl geometry was M. Omote (Omote 1971). A little later, papers by Utiyama and Dirac followed (Dirac 1973, Utiyama 1975a, Utiyama 1975b). For the sake of conciseness, I shall call that approach Weyl-Omote-Dirac gravity (WOD). For an excellent survey see (Blagojević 2002, chap. 4).

**Lagrangians of Weyl-Omote-Dirac gravity (WOD)** The action of a Weyl geometric real or complex scalar field $\phi$ of weight $-1$ contains the kinetic Lagrangian density $L_\phi = \gamma L_\phi \sqrt{|\det g|}$ with constant $\gamma$ and

$$L_\phi = \frac{1}{2} D_\nu \phi^* D^\nu \phi,$$

(8)

where $D$ is the scale covariant derivative. For a scalar field, $D_\nu \phi = \partial_\nu \phi - \varphi_\nu \phi$. $\phi^*$ denotes complex conjugation (later, in extensions, it will denote dualization in complex vector spaces or an even more general conjugation).

The gravitational Lagrangian density $L_{G\phi} = L_{G\phi} \sqrt{|\det g|}$ couples scalar curvature in a Brans-Dicke like manner to the Weyl geometric scalar field $\phi$. Bringing its own coupling factor $\gamma$ in order of magnitude into agreement with the one of the scalar curvature (by putting $\gamma = \xi^2$), we can write for $\text{sig}(g) = (1,3)$

$$L_{G\phi} = \frac{\xi^2}{\hbar c} (-\alpha |\phi|^2 R + \frac{1}{2} D_\nu \phi^* D^\nu \phi - V_4(\phi))$$

(9)

$\alpha$ is a coupling constant, regulating relative strength of scalar field kinetic term and the modified Hilbert term; $\xi^2$ (and with it $\gamma$) is a squared “hierarchy factor” (see section 3, equ. (11)), $|\phi|^2 = \phi^* \phi$. For $\alpha = -\frac{1}{\sqrt{12}}$ one gets Penrose’s conformal coupling in $\dim M = 4$. Many authors choose this value, among them Dirac (Dirac 1973) and his followers, e.g. (Israelit 1999, Drechsler/Tann 1999, Drechsler 1999). In our context, that is an unnecessary restriction, as far as $L_{G\phi}$ is concerned. Weyl geometric scalar curvature $R$ and the scale covariant derivatives $D$ assure weights $-4$ for all terms of $L_{G\phi}$ anyhow, and thus scale invariance of $L_{G\phi}$. We may just as well leave $\alpha$ open, at least for the moment. In doing so, we follow (Omote 1971, Omote 1974),

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10Utiyama knew Omote’s paper and cited it, while Dirac was silent about it. I owe the hint at Omote’s work to F. Hehl.

11Change of signature flips all signs (take care for $V_4$ !), variation leads to the same dynamical equations.

12Considering the (abelian) Yang-Mills action $-\frac{1}{4} f_{\mu \nu} f^{\mu \nu}$ of $\phi$, a reasonable although not compelling motif for this choice becomes visible; see below.
presented also in (Blagojević 2002, 102), and (with the restriction $\alpha = 1$) in (Utiyama 1975a, Utiyama 1975a, Smolin 1979).  

$V_4$ is a quartic polynomial written in a form such that it changes sign with signature change

$$V_4(\phi) = \epsilon_{\text{sig}} \sum_{j=1}^{4} \lambda_j |\phi|^j.$$  

Scale invariance demands $\lambda_j = 0$ for $j = 1, 2, 3$; but some authors decide in favor of an explicitly symmetry breaking term $\lambda_2 \neq 0$ (Drechsler/Tann 1999) or introduce a second (real) scalar field $\sigma$ of weight $-1$, setting $\lambda_2 = \sigma^2$ (Nishino/Rajpoot 2004). We consider this artificial and try to avoid it, keeping the simplest possible scale invariant Lagrangian, as long as we are not compelled to give it up by problems of empirical adequacy. That reduces the potential to a quartic monomial,

$$V_4(\phi) = \epsilon_{\text{sig}} \lambda_4 |\phi|^4.$$  

$V_4$ plays the formal role of a scale invariant “cosmological” term. Its energy tensor will be absorbed by the energy tensor of the scalar field more broadly (see below, equ. (17)).

Assuming a dynamical role for the Weylian scale connection, a Maxwell-like Yang-Mills action $L_\phi = L_\phi \sqrt{|\det g|}$ has to be added with

$$L_\phi = -\frac{\hbar c^4}{4} f_{\mu\nu} f^{\mu\nu}.$$  

Disassembling the modified Hilbert term according to (5) and the kinetic term of $\phi$, leads to a Proca mass-like term for the scale connection $\xi^2 \left(\frac{\hbar c^4}{16\pi G} \right) \phi_{\mu} \phi^{\mu}$, the coefficient of which should be $= \frac{1}{16\pi G}$ in the GRT “limit”. The field is massless iff $\alpha = -\frac{1}{12}$. In all other cases the scale connection acquires a huge mass. The Proca mass factor turns out to be $\frac{(m^2 c^2)^2}{\hbar c^4} = \frac{\epsilon^4}{16\pi G}$ and thus close to the Planck scale (Smolin 1979, Cheng 1988):

$$m^2_\phi = \frac{\hbar c}{8\pi G} = \frac{1}{8\pi} m^2_{Pl}$$  

This surprising effect has been seen and discussed by Utiyama, Smolin and later again by Cheng. Probably Dirac chose the value of $\alpha = -\frac{1}{12}$ deliberately (using our notational conventions) in order to avoid such mass effects

\footnote{Because of his interest in quantization, Smolin used a systematcially broader range of all the possible scale invariant curvature terms, including those of second order. Utiyama was aware of Omote’s work and quoted it. Smolin did not; neither did he quote Dirac.}

\footnote{A similar explicit breaking by a quadratic term was already foreseen in a setting of conformal coupling between scalar curvature and the scalar field by (Deser 1970).}

\footnote{A misplaced sign choice led Utiyama even to assume a “tachyonic” Planck mass. That was corrected by (Hayashi/Kugo 1979).}
of the scale connection. Here we are not interested in the details of this discussion. The common structural features in the papers of the authors mentioned are characterized by the combined Lagrangians (9) + (11).

The second long range interaction field, classical electromagnetism with 4-potential \( iA = i(A_\mu) \), has the well known scale invariant (in flat space even conformally invariant) Lagrangian

\[
L_{em} = -\frac{[hc]}{4}F_{\mu\nu}F^{\mu\nu},
\]

with \( F^{(em)} = (F_{\mu\nu}) = dA \).

Lagrangians of classical matter fields \( L_m \) may be adapted to a scale covariant form, by modifying their action densities as demanded by dimensional analysis and principles of Weyl scaling of units, e.g. like in (Scholz 2009). An extension to fermionic fields \( L_\Psi \) and the electroweak sector of bosonic interactions \( L_{ew} \) of the standard model of particle physics will be discussed in section 3.

**Arguments for the integrable version, IWOD**  
As the range of the field strength of the scale connection is restricted to the order of the Planck length \( l_{Pl} \) (if it represents a dynamical entity at all), \( \phi \) will not be able to display any observable curvature effects far above \( l_{Pl} \). For all “practical purposes”, far below the Planck energy, respectively far above the Planck length, we can safely consider \( f \) and the Lagrangian (8) as negligible. With the Lagrangian (9) alone we effectively work in *integrable Weyl-Omote-Dirac (IWOD) geometry*. Only coming close to the Planck scale, one has to consider the full Lagrangian (9) + (11) and “full” Weyl geometry with non-vanishing scale curvature \( f \).

These structural properties seem to indicate that the scale symmetry of integrable WOD gravity may display a non-dynamical residuum of a truly dynamical symmetry which becomes effective close to the Planck scale. If this is the case, it may turn out that \( \phi \) can be considered an order parameter of some condensate after “spontaneous breaking of dynamical scale symmetry” close to the Planck scale. Given our present ignorance of Planck scale physics, it would be premature, however, to delve deeper into such speculative waters here. For our purpose, we can perfectly well work in the mathematical structure of integrable Weyl geometry, independent of further ontological interpretations.

**Dynamical equations, in particular Einstein equation**  
Assuming a total Weyl geometric Lagrangian of the form

\[
\mathcal{L}_{G\phi} + \mathcal{L}_M + \mathcal{L}_{Int}
\]

He did not mention that, but rather talked about getting “the simplest equation for the vacuum”. Moreover, he sticked to the interpretation of \( \phi \) as electromagnetic potential, long before given up by Weyl (Dirac 1973, 410).
with classical or field theoretic matter terms indexed by $M$ (or, respectively $\Psi$) and interaction terms indexed by $\text{Int}$ (electromagnetic or standard model interactions), dynamical equations are derived as usual by variation $\delta v$ with respect to the dynamical variables $v$. In particular, variation with respect to the Riemannian component of the metric $g$ leads to a Weyl geometric version of the Einstein equation.

Varying the modified Hilbert term in $L_{G\phi}$ with respect to $g$ leads to an expression similar to the classical one plus an additional term $A$,

$$\sqrt{|\det g|}^{-1} \delta_g R = |\phi|^2 (Ric - \frac{R}{2} g) + A$$ \hspace{1cm} (14)

with $A_{\mu\nu} = D_\lambda D^\lambda |\phi|^2 g_{\mu\nu} - D_{(\mu} D_{\nu)} |\phi|^2$ (Tann 1998, (372)), (Blagojević 2002, (4.47)). $A$ has the form of an additional contribution to the dynamical energy tensor of the scalar field $T_\phi$, like for any matter or interaction field $X$,

$$T(X) = 2\sqrt{|\det g|}^{-1} \delta_g L_X,$$ \hspace{1cm} (15)

$$\sqrt{|\det g|}^{-1} \delta_g L_\phi = \frac{\xi^2}{2} (D_{(\mu} \phi^* D_{\nu)} \phi - \frac{1}{2} D_\lambda \phi^* D^\lambda \phi g_{\mu\nu}).$$

With $A$ and the contribution from the $V_4$ term, $|\det g|^{-1} \delta_g (-V_4) = \frac{V_4}{4} g$, the Weyl geometric Einstein equation in the massless case $L_M + L_{\text{Int}} = 0$, becomes:

$$\text{Ric} - \frac{R}{2} g = \Theta(\phi) = \Theta^{(I)} + \Theta^{(II)}$$ \hspace{1cm} (16)

$$\Theta^{(I)} = |\phi|^{-2} (\frac{1}{2\alpha} V_4 - \frac{1}{4\alpha} D_\lambda \phi^* D^\lambda \phi - D_\lambda D^\lambda |\phi|^2) g$$ \hspace{1cm} (17)

$$\Theta^{(II)}_{\mu\nu} = |\phi|^{-2} \left( \frac{1}{2\alpha} D_{(\mu} \phi^* D_{\nu)} \phi + D_{(\mu} D_{\nu)} |\phi|^2 \right)$$

with Weyl geometric curvatures $\text{Ric}, R$ and scale covariant derivatives $D_\mu$. Terms without factor $\alpha^{-1}$ are due to the “improvement” $A$.

$\Theta^{(I)}$ is proportional to the Riemannian component of the metric, i.e., of the form $\Lambda g$ with $\Lambda = |\phi|^{-2} (\frac{1}{2\alpha} V_4 - \frac{1}{4\alpha} D_\lambda \phi^* D^\lambda \phi - D_\lambda D^\lambda |\phi|^2)$ (in any scale gauge). In this sense, it looks like the vacuum or “dark energy” tensor $\Lambda g$ of the received approach. Note, however, that in our case the negative pressure $-\Lambda$ is no constant, independent of the matter content of the universe, but dynamically determined by the scalar field. The latter is, in turn, related via

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17 See, among others, (Tann 1998, Drechsler/Tann 1999, Blagojević 2002)

18 It has been noted by different researchers that $A$ contributes to the energy tensor of the scalar field in exactly the same way as the “improvement” term of Callan, Coleman and Jackiw (Tann 1998, Drechsler/Tann 1999), (Blagojević 2002, 96ff.).
curvature to matter by the scalar field equation (below) and the potential condition (section 3, equ. 38).

\[ \Theta^{(II)} \]

is an additional energy-stress-momentum contribution of \( \phi \), beyond the metric proportional \( \Theta^{(I)} \). It may be worthwhile to investigate whether there are cosmologically relevant solutions in which the matter content is dominated by \( \Theta^{(II)}_{00} \); if so, that might open new vistas on the “dark matter” problem.

If massive fields or non-gravitational interactions are present, \( \mathcal{L}_M + \mathcal{L}_{\text{Int}} \neq 0 \), the energy momentum tensors \( T^{(X)} \), \( X = M, \text{Int} \), appear as additional terms on the right hand side of the Einstein equation with massive source:

\[
Ric - \frac{R}{2} g = \Theta^{(\phi)} + \frac{[\hbar c]}{2\alpha \xi^2 |\phi|^2} (T^{(M)} + T^{(\text{Int})}) \tag{18}
\]

Variation with respect to \( \phi^* \) leads to the scalar field equation:

\[
D_\nu D^\nu \phi = -(2\alpha R + 4\epsilon_{\text{sig}} \lambda_4 |\phi|^2) \phi + \text{terms in } \delta_\phi L_M. \tag{19}
\]

If the Higgs-like potential condition for the ground state of the scalar field, discussed in the next section is realistic, the scalar field equation governs only the classical field fluctuations about the ground state.

For \( \alpha \neq -\frac{1}{12} \) the expectation of a massive boson near the Planck scale reduces the range of the field strength \( f \), and the effective curvature effects of the scale connection \( \varphi \), to Planck scale physics. At larger distances \( \varphi \) is effectively integrable and thus reduced to its purely geometrical role as part of the metrical structure of IWG. It establishes consistency under scale transformations, comparable to the correction terms in \( \partial \Omega \) of the affine connection in Jordan-Brans-Dicke (JBD) theory.

That is obviously different for the scalar field with its kinetic term and its coupling to gravity. In the next subsection it will become clear that in certain scale gauges, the dynamical properties of the scalar field \( \phi \) may be expressed by the length connection \( \varphi \). But even then the dynamical properties of both are due to the scalar field rather than to the scale connection as such, which can be scaled away.

The (second) Yang-Mills equation for \( \varphi \) with curvature \( f = d\varphi = 0 \),

\[
\ast d*f = J,
\]

19 Compare, among others, (Drechsler/Tann 1999, Blagojević 2002).

20 This difference is analogous to Riemann’s observation, in the third part of his inaugural lecture, that space curvature may effectively vanish on large scales, while there may be strong curvature effects in the small. He warned that one ought to be cautious in drawing conclusions from astronomical evidence for vanishing sectional curvature: “...we cannot draw conclusions from metric relations of the great, to those of the infinitely small; (...) the curvature at each point may have an arbitrary value in three directions, provided that the total curvature of every measurable portion of space does not differ sensibly from zero” (Riemann 1854/1873, 68).
becomes trivial sufficiently above the Planck scale. Varying with regard to $\varphi$ and discussing Yang-Mills theory in IWOD gravity would turn an idle wheel. It may become important for physics at the Planck scale only.

2.3 Scale invariant observables, scalar field gauge

Freedom of scale choice seems to introduce an arbitrary element in the determination of metrical (scale covariant) quantities $X$. For a meaningful comparison of measured, or calculated, quantities, it is necessary to extract some scale invariant magnitude $\hat{X}$ of any scale covariant $X$. In geometry this is an old problem, solved already in antiquity and later taken up by Descartes, through the introduction of proportion theory and proportions of geometrical quantities. In this way “real quantities” of extended geometrical magnitudes were made independent of the (subjective) choice of units. The same idea works, generalized, in Weyl geometry for any local physical quantity underlying Weyl scaling. One only needs to form proportions of the correct weight with some scale covariant reference quantity. In WOD theory, the norm of the scalar field $|\varphi|$ is a natural candidate for a universally accessible reference quantity.

Some authors therefore even declare $\varphi$ to be a “measure field” (Omote 1974, Utiyama 1975b, Israelit 1999).

That leads to an obvious method for extracting a scale invariant magnitude $\hat{X}$ from a scale covariant local quantity $X$ of weight $w := w(X)$. One just has to consider the proportion with the appropriately weighted power of $|\varphi|$ at any point $p$:

\[
\hat{X}(p) := \frac{X(p)}{|\varphi|^{-w}(p)} = X(p) |\varphi|^w(p)
\]  (20)

(the negative sign in the exponent of the denominator arises because we work with length, not energy, weights). By definition $\hat{X}$ is scale invariant.

An experimentally observable magnitude $X^{(exp)}$ corresponding to a magnitude $X$ of the theory is measured by a system of appropriate material devices and scientific practices. The devices, e.g. radiotelescopes, or the detectors of the LHC, are lawfully constituted by classical systems (probably environmentally decohered from quantum states) and are operated by a complex of technical practices, on which several levels of data selection and evaluation are superimposed, based on well defined theoretical practices. The corresponding observable magnitude $\hat{X}(p)$ of the theory should basically be proportional to $X^{(exp)}$, up to a global scaling factor depending on unit choice $const_u$:

\[
\hat{X}(p) = const_u X^{(exp)}
\]  (21)

---

21 (Euclides 1925, Bos 2001)

22 Weyl applied basically the same idea with respect to scalar curvature as reference quantity for his gravitation theory (second order in $R$) (Weyl 1923, 298f.).
For any nowhere vanishing scale covariant quantity $Y$ of weight $w(Y) \neq 0$ there is a scale gauge in which $|Y| = \text{const}$; just set $\Omega = |Y|^{\frac{1}{w(Y)}}$. In his early theory, Weyl proposed to consider in a Weylian manifold with nowhere vanishing scalar curvature the scale gauge in which $R = \text{const}$ (Weyl 1923, 298f.). It will be called Weyl gauge.\(^{23}\)

In WOD gravity another specification appears much more natural, a scale gauge in which the scalar field is normed to a constant. We call it scalar field gauge. Starting from any gauge $(g, \phi)$ in which the scalar field may assume the value $\phi$, the scalar field gauge $(\tilde{g}, \tilde{\phi})$ is given by

$$\tilde{g} = \Omega^2 g \quad \text{with} \quad \Omega := \text{const} |\phi|.$$ \(^{(22)}\)

In it, the invariant magnitudes are directly expressed (up to a global constant factor) by the scale covariant magnitude itself. A scale covariant magnitude $X$ given in scalar field gauge will be denoted accordingly by $\tilde{X}$, a dotted equality $\doteq$ expresses relations which hold in a specific gauge only (preferably scalar field gauge). Thus $\tilde{X} \doteq \beta \tilde{X}$, up to a constant factor $\beta$.

### 2.4 Two transitions to Einstein gravity

There are two natural transitions to Einstein gravity.

- In IWOD gravity the scale connection can be integrated away. By geometrical default, one may thus like to choose Riemann gauge. Then the geometry looks like usual, but the gravitational coefficient $\frac{[\hbar c]}{2\alpha \xi^2 |\phi|^2}$ in \((18)\) is no longer a constant but depends on the norm of the scalar field (comparable to “Jordan frame” in JBD theory). Moreover, the scale invariant magnitudes $\tilde{X}$ can be calculated only if the norm of the scalar field is known!

- If one takes the methodological principle of scale invariance seriously, the choice of scalar field gauge is much more compelling (“Einstein frame” in JBD theory). Then the scale invariant magnitudes $\tilde{X}$ are read off directly from the value of the quantity in this gauge, $\tilde{X} \sim \tilde{X}$. Because of \((18)\) the condition

$$\frac{[\hbar c]}{\alpha \xi^2 |\phi|^2} \doteq \frac{16\pi G}{|c|^2},$$

with Newton constant $G$, \(^{(23)}\)

fixes the global constant of scalar field gauge naturally. Then $\tilde{\phi} \neq 0$, if the scalar field itself is not already trivial, i.e., constant in Riemann gauge. IWG shows different features from Riemannian geometry!

In the first case (Riemann gauge), Einstein gravity arises as special case for a constant scalar field, in the second one (scalar field gauge) for a vanishing scale connection. In both case coherence is established if $\frac{[\hbar c]}{2\alpha \xi^2 |\phi|^2} = \frac{8\pi G}{|c|^2}$.\(^{24}\)

\(^{23}\)Not to be confused with “Weyl gauge” in electromagnetic theory.

\(^{24}\)Both scale gauges are closely related in WOD gravity; see section 3.3.
3 Extension to the electroweak sector

3.1 Standard model fields in Weyl geometry

The Lagrangians of the present standard theory of elementary particles are scale invariant over Minkowski spacetime. As far as quasi-classical fields are concerned, i.e. before “second” quantization, they can be generalized straightforwardly to scale covariant “curved metrics”. In particular they can be imported to Weyl geometry and combined with WOD gravity without major conceptual problems.

Some technical work has to be done mathematically. First of all, the Dirac operator has to be imported to the underlying “curved” metric. In the Riemannian case it is well known how this is done (Weyl 1929, Drechsler/Mayer 1977) (Frankel 1997, chap. 19). The construction can be adapted to the Weyl geometric case (see below). As a mathematical condition, we have to assume both space and time orientability of \( M \). Moreover, \( M \) has to be a spin manifold, i.e. it has to admit a principal \( SL(2, \mathbb{C}) \) fibre bundle globally (e.g. \( H_2(M, \mathbb{Z}_2) = 0 \)), otherwise Dirac operators exist only locally (Frankel 1997, 515ff.). If \( M \) is spin, we can reduce the structure group of the tangent bundle \( TM \) to \( SL(2, \mathbb{C}) \). For any representation \( \rho \) of \( SL(2, \mathbb{C}) \) (e.g. the Dirac spinor representation \( \rho_D : SL(2, \mathbb{C}) \rightarrow GL(4, \mathbb{C}) \)) the associated bundle can be constructed, the sections of these serve as spinorial “wave functions” \( \psi \). (Local) trivializations of the spinor bundle go in hand (are associated) to those of the tangent bundle, arising from a specification of local orthonormal tetrads.

The Levi-Civita connection \( \Gamma \) can be reduced to \( SO(1, 3) \); let us call it \( \omega \). Then it has values in \( so(1, 3) \), given by coefficients \( (\omega^j_l) \) with regard to vector fields \( u = u^j e_j \) developed with reference to orthonormal tetrads \( e_j \) and their duals \( e^j \) \((0 \leq j \leq 3)\).

For \( M \) spin, \( \omega \) can be lifted coherently to a spinor connection \( \tilde{\Gamma} \) with values in the Lie algebra of the universal covering \( SL(2, \mathbb{C}) \) of \( SO(1, 3) \). With (generalized) Dirac matrices \( \gamma^i \) (in case of \( \rho = \rho_D \) the ordinary Dirac matrices), \( \gamma_i = \eta_{ij} \gamma^j \) and \( \gamma^\mu = e^\mu_j \gamma^j \) etc., the Lie algebra of \( \rho(SL(2, \mathbb{C})) \) has generators \( \frac{i}{8} \gamma^i \gamma^j \) (Frankel 1997, (19.55)).

The covariant derivative of \( \psi \) with respect to the spinor connection \( \tilde{\Gamma} \) becomes (still in the Riemannian context)

\[
(\nabla_{\tilde{\Gamma}} \psi)_l = \partial_l \psi + \tilde{\Gamma}_l \psi = \partial_l \psi + \frac{i}{8} \omega^{jk}_l \gamma_j \gamma_k \psi ,
\]

\(25\) (Cheng 1988, Drechsler/Tann 1999, Drechsler 1999, Nishino/Rajpoot 2004) and (Blagajević 2002, chaps. 4, 8.1, 8.4).

\(26\) Latin indices indicate coefficients with regard to the orthotetrad, Greek ones with regard to coordinate derivatives. Partial derivation with regard to the vector field defined by \( e_j \) will be denoted by \( \partial_j := e_j \), the Minkowski metric (with regard to tetrad coefficients) by \( \eta = (\eta_{ij}) \).
and the purely gravitational Dirac operator (with vanishing scale and internal connection) is
\[ \hat{\psi}_{\mu} = i\hbar \gamma^j (\nabla^j_{\mu} \psi)_{\mu}. \] (25)

Secondly the structure group of the theory has to be extended in order to account for the “internal” dynamical degrees of freedom. The electroweak (ew) interaction is characterized by dynamical symmetries in \( G_{\text{ew}} = SU(2) \times U(1) \), constituted by the weak isospin group \( SU(2) \) and the hypercharge group \( U(1) \). Chromodynamics (cd) presupposes \( G_{\text{cd}} = SU(3) \) as structure group of “local” physical automorphisms.

Mathematically, the extension proceeds by introducing trivial principal fibre bundles with regard to the structure groups \( G_{\text{ew}} \) and \( G_{\text{cd}} \), and by tensoring the Dirac spinor bundles with the appropriate representation spaces of the dynamical groups. For \( G_{\text{cd}} \) the “colour” representation space is isomorphic to \( \mathbb{C}^3 \), whereas for \( G_{\text{ew}} \) the representation spaces are the weak isospin spaces well known from the standard model of elementary particle physics:

- Isospin \( I = \frac{1}{2} \), representation spaces isomorphic to \( \mathbb{C}^2 \) for the chirally “left”-handed spinors \( \psi_L \) of three generations with basis respectively \((\nu_g, e_g)\) for the leptons \( (Y = \frac{1}{2}) \) and \((u_g, d_g)\) for quark flavors \( (Y = \frac{1}{6}) \), \( g = 1, 2, 3 \).
- Isospin \( I = 0 \), representation spaces isomorphic \( \mathbb{C} \) for the chirally “right”-handed spinors \( \psi_R \), basis respectively \((e^R_g)\), perhaps also \((\nu^R_g)\), and \((u^R_g, d^R_g)\), with properly adapted hypercharge \( (Y = -1 \text{ for } (e^R_g) \) etc.).

The sections in the tensorized bundles (the “full” wave functions) may be denoted by \( \Psi = \psi \otimes \psi_{\text{int}} \) with \( \psi \) the Dirac spinor contribution, \( \psi_{\text{int}} \) the representation space of the internal dynamics. The values of \( \psi_{\text{int}} \) lie in representation spaces of \( G_{\text{ew}} \times G_{\text{cd}} \).

In the following we restrict our considerations to the electroweak sector. It is the one which links most directly to gravitation. Chromodynamics can, in principle, be appended like in the flat (Minkowski) case, but subtleties like mixing of downlike quark states and of neutrinos, CP-violating phase etc. have to be taken into account.

The electroweak potential decomposes as \( W + B \) where

- \( W \) is a connection with values in \( su_2 \), \( W = (W_\mu) \), curvature \( F_W = dW + \sqrt{2g}[W, W] = (W_{\mu\nu}) \) globally in an \( ad \ SU_2 \)-bundle.\(^{28}\)

\(^{27}\)More precisely, chirality of the weak interaction presupposes a chiral decomposition of Dirac spinors: \( \psi_L = \frac{1}{2}(1 - \gamma_5)\psi, \psi_R = \frac{1}{2}(1 + \gamma_5)\psi, \gamma_5 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = i\gamma^0\gamma^1\gamma^2\gamma^3 \). The tensorial coupling of internal dynamics to the “external” degrees of freedom (encoded by the spinor spaces) differentiates between the two chiral components. See below.\(^{39}\)

\(^{28}\)With generators \( \tau_\alpha = \frac{1}{\sqrt{2}} \sigma_\alpha \) of \( su_2 \), \( (\alpha = 1, 2, 3) \), \( W_\mu = A^\alpha_\mu \tau_\alpha \) \( W = A^\alpha_\mu \tau_\alpha dx^\mu \).
and \( B \) a connection with values in \( u_1 \), \( B = (i B_\mu) \), curvature \( F_B = dB = (i B_{\mu\nu}) \).

The dynamical covariant derivative (ew sector only) of an isospinor field \( \psi_{I,Y} \) with representation characteristics (‘charges’) isospin \( I \) and hypercharge \( Y \) is

\[
(D_{\text{ew}} \psi_{I,Y})_\mu = (\partial_\mu + i g W_\mu + Y g' B_\mu) \psi_{I,Y}
\]

with coupling coefficients \( g, g' \) for the weak isospin and hypercharge interactions.

Finally, the whole construction can be adapted to the Weyl geometric case (Drechsler 1991, Drechsler 1999, Blagojević 2002). Obviously the orthonormal tetrads are of Weyl weight \( w(\epsilon_j) = -1 \), \( w(\eta) = w(\omega) = 0 \). Also the tetrad related Dirac matrices \( \gamma_j, \gamma_k \) have Weyl weight 0, while \( w(\gamma^\mu) = -1 \). Spinors are scale transformed by weight \( w(\psi) = -\frac{3}{2} \) in order to achieve a scale invariant Lagrangian. Of course also the weights of the internal group connections and their spinors remain unaffected by rescaling, \( w(W) = w(B) = 0 \) etc. (Blagojević 2002, 92ff).

The Weyl geometric scale covariant derivative \( \tilde\Gamma \), respectively the Dirac operator \( \partial \), of Dirac spinors becomes

\[
\begin{align*}
D_{\text{ew}} \psi &= \nabla_{\text{ew}} \Psi - \frac{3}{2} \varphi \otimes \Psi \\
\partial \Psi &= i h \gamma^\mu (D_{\text{ew}} \Psi)_\mu
\end{align*}
\]

The \textit{ew-dynamical scale covariant derivative} \( \tilde{D} \) is

\[
(\tilde{D} \Psi)_\mu = (\partial_\mu + (\nabla_{\text{ew}})_\mu - \frac{3}{2} \varphi_\mu + i g W_\mu + Y g' B_\mu) \psi_{I,Y}
\]

(Drechsler 1991, Nishino/Rajpoot 2004).

\[
F_W = W_\mu dx^\mu dx^\nu = A^a_{\mu\nu} \tau_a dx^\mu dx^\nu \text{ with } A^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu. \text{ This presupposes implicitly a gauge fixing (mathematically a standard trivialization) of the ew principal bundle and its isospinor adjoints such that the charge eigenstates lie “up” (1, 0) and “down” (0, 1).}
\]

\[
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu
\]

In the textbooks often the traditional value for hypercharge \( \tilde{Y} = 2Y \) is used, in order to safeguard the ‘historical’ Gell-Mann-Nishijima formula \( q = I_3 + \frac{\tilde{Y}}{2} \) for electric charge \( q \) \((I_3 3\text{-component of isospin})\). The hypercharge coupling factor in (26) then becomes \( \tilde{Y} \frac{g'}{\kappa} \).

Obviously in the convention chosen here, \( q = I_3 + Y \).
3.2 Extending the scalar field to the ew sector, Lagrangians

It seems natural to extend also the scalar field $\phi$ of Weyl-Omote-Dirac gravity by a weak isospin tensor factor. The tensorized scalar field will be denoted by $\Phi$. If its properties lead to empirically sound conclusions it represents a gravitationally coupled electroweak vacuum structure. Several authors have noticed that for weak isospin $I = \frac{1}{2}$ and hypercharge $Y = \frac{1}{2}$, $\Phi$ gets formal properties analogous to the Higgs field of the standard model (Cheng 1988, Drechsler/Tann 1999, Drechsler 1999). We follow these authors and assume the extended scalar field with values in the $(I = \frac{1}{2}, Y = \frac{1}{2})$ representation space of $G_{ew}$, isomorphic to $C^2$,

$$\Phi(x) = (\phi_1(x), \phi_2(x)),$$  \hspace{1cm} (28)

where the basis is chosen such that $\phi_1$ and $\phi_2$ characterize the electrically charged, respectively the neutral state ($q = I_3 + Y$).

Analogous to (27) its ew-dynamical scale covariant derivative is

$$\left(\partial_\mu - \varphi_\mu + \frac{1}{2}gW_\mu + \frac{1}{2}g'B_\mu\right) \Phi.$$  \hspace{1cm} (29)

To better account for the common factor $\xi^2$ for the extended Lagrangian like in (8) or (9), we shall use the notation

$$\tilde{D}_\mu \Phi := (\partial_\mu - \varphi_\mu + \frac{1}{2}\tilde{g}W_\mu + \frac{1}{2}\tilde{g}'B_\mu) \Phi.$$  \hspace{1cm} (29)

Then the Lagrangian of the extended scalar field $\Phi$ has a form like in (3), but now it contains a coupling between scalar field and ew gauge fields, due to the ew scale covariant derivative (29). With the exception of the Weylian scale connection term $\varphi_\mu$ and up to the factor $\xi^2$ (which cancels with the $\xi^{-1}$ in the couplings for terms quadratic in electroweak potentials), the Lagrangian (30) comes down essentially to that of the Higgs field in the standard model.

For the conjugate (respectively the adjoint) of spinor\footnote{Complex conjugation is denoted by $\psi \mapsto \bar{\psi}$ and transposition $\psi \mapsto ^t\psi$ (in both tensorial factors separately).} the usual convention

$$\Psi^* = \bar{\Psi} \gamma^0$$  \hspace{1cm} (31)

is applied. Using abbreviated notation $\Psi = \Psi^{(f,g,t)}$ with indexes $f = l, q$ for the family (lepton or quark), $g = 1, 2, 3$ for the generation, and $t = u, d$ for...
the type (“uplike”, “downlike” weak isospin \( I_3 \) eigenstates) \[ \text{fermionic Lagrangian} \quad L_\Psi = L_\Psi \sqrt{|\text{det} g|} \] is given by

\[
L_\Psi = \frac{i}{2}[\hbar c] \sum_{f,g} \sum_{J=L,R} \left( \Psi_J^* \gamma^\mu \tilde{D}^\mu \Psi_J - (\tilde{D}_\mu \Psi_J)^* \gamma^\mu \Psi_J \right) - \epsilon_{\text{sig}} \sum_{f,g} \left( y(\Psi_L^* \Phi \Psi_R + \tilde{\Psi}_R^* \Phi \Psi_L) + y'(\tilde{\Psi}_L^* \tilde{\Phi} \Psi_R + \tilde{\Psi}_R^* \tilde{\Phi} \Psi_L) \right),
\]

where \( \tilde{\Phi} = i\sigma_2 \Phi \), in ‘unitary’ gauge \( \tilde{\Phi} = (\phi_o \bar{0}) = (\phi_o \bar{0}) \).

The covariant derivative \( \tilde{D} \) of spinors is that of \( (27) \). \( \Psi_R \) runs through up and down states separately. For the quark family, \( \Psi \) denotes down types transformed to the mass eigenstates by the inverse of the (unitary) Cabbibo-Kobayashi-Maskawa (CKM) matrix, uplike types remain unchanged. For the lepton family, \( \Psi \) denotes Maki-Nakagawa-Sakate (MNS) matrix transforms of the uplike types (neutrino mixing), while the downlike types are untransformed. \( y \) and \( y' \) are Yukawa coupling coefficients for down and up states respectively, for each set of \( (f,g,t) \) one. It may be appropriate to conceive of the different generations \( g \) as excitation modes of spinor fields of given fermionic character \( f \) and type \( t \).

\[ \Psi_L, \Psi_R \] are “left” and “right”-handed editions of \( \Psi \),

\[ \gamma_5 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = i \gamma_0 \gamma^1 \gamma^2 \gamma^3 \]

The Yang-Mills action for the ew boson field \((W,B)\) (similar for the chromodynamic field) is as usual

\[
L_{\text{ew}} = \frac{1}{4}[\hbar c] \left( \text{tr}(W_{\mu \nu} W^{\mu \nu}) + (i B_{\mu \nu})(i B^{\mu \nu}) \right) \sqrt{|\text{det}, g|}
\]

Like usual, electrons are represented as “downlike”, neutrinos as “uplike”, i.e., \( \Psi_L^{1,1} = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \) etc. As an ad-hoc construction, neutrino masses are dealt with like the quark up states, i.e., right handed neutrinos are assumed.

Yukawa Lagrange terms are essentially the same as in (Nishino/Rajpoot 2004), with conjugate terms added (reality of Lagrangian) like in (Drechsler 1999). Compare the slightly different Lagrangian in (Meissner/Nicolai 2009).

Counting of parameters gives: number of Yukawa parameters \(|f| \cdot |g| \cdot |t| = 2 \cdot 3 \cdot 2 = 12\), 3 each for CKM and NNS matrices, 2 for CP-violating phase and QCD vacuum angle, 2+1 coupling coefficients for ew and cd interactions, two essential parameters for gravity \((\xi, \lambda_4)\) (perhaps three, depending on whether \( \alpha \) is considered as accidental or not), no separate Higgs coupling coefficients (see below); sum 25 (without \( \alpha \)). It is likely that the representation of neutrino masses is redundant.

Here, of course, 1 = \( \begin{pmatrix} 1 \\ 1 \end{pmatrix} \).
Interactions with the fermionic fields are built into the EW gauge components of \( \tilde{D}_\mu \) of (33).

In a full account, strong interactions have to be included in \( \tilde{D}\Psi \) by covariant derivative components with values in \( g_{cd} = \text{Lie}(G_{cd}) = su(3) \) and strong charges as coupling coefficients. Recent high precision lattice calculations show that about 99 percent of total energy/mass of the observable bound states of mesons and baryons is due to the dynamics of QCD (Dürr/Fodor e.a. 2008, Wilczek 2008). In regions of high matter density, their influence on the right hand side of the Einstein equation will be considerable.

Clearly, the EW covariant derivative terms of (30) lead to formal mass terms for the EW bosons,

\[
\frac{1}{4}\xi^2 g^2 |\Phi|^2 W_\mu W^\mu \quad \text{and} \quad \frac{1}{4}\xi' g'^2 |\Phi|^2 B_\mu B^\mu.
\] (34)

Assuming that a change of basis (Glashow-Weinberg ‘rotation’ about the weak mixing angle \( \Theta \)) in the Liealgebra \( g_{ew} = \text{Lie}(G_{ew}) \) transforms the generators \( W_0, W_1, W_2 \) of \( su(2) \) and \( B \) of \( u(1) \) into physical states \( W^\pm, Z^0 \), the mass terms of the latter become

\[
m^2_W = \frac{g^2}{4} |\Phi_o|^2, \quad m^2_Z = \frac{g^2}{4 \cos \Theta^2} |\Phi_o|^2,
\] (35)

with \( g = \xi \tilde{g}, g' = \xi \tilde{g}' \) and \( \cos \theta = g (g^2 + g'^2)^{-\frac{1}{2}} \) like in special relativistic field theory. This presupposes that the scalar field \( \Phi \) acquires a ground state \( |\Phi_o| \) like the one of the quartic “Mexican hat” potential in the known approach. In the next section we shall see that such a ground state arises naturally from the coupling between the scalar field and Weyl geometric gravity (WOD).

Varying with regard to \( \Psi^\dagger \) and \( \Psi \) leads to the Dirac equation and its adjoint, varying with regard to the EW potentials leads to the Yang-Mills equations. Variation with respect to \( \Phi^\dagger \) gives the scalar field equation. Because of the Yukawa-like contribution in (33), \( \Phi \) couples to fermionic matter fields.

### 3.3 Higgs potential condition, scalar field and Weyl gauge

The total scalar field action for \( \Phi \) contains quartic and quadratic terms from the modified Hilbert term in (9) and the quartic potential (10)

\[
\mathcal{L}_{\Phi,\text{tot}} = \frac{\xi^2}{\hbar c} \left( \frac{1}{2} \tilde{D}_\nu \Phi^\dagger \tilde{D}^\nu \Phi - V(\Phi) \right) \sqrt{|\det g|} \quad (36)
\]

\[
V(\Phi) = \epsilon_{\text{sig}} \lambda_4 |\Phi|^4 + \alpha R|\Phi|^2 \quad (37)
\]

With \( \text{sig}(g) = (1,3) \) and \( \epsilon_{\text{sig}} = +1 \), the scalar curvature \( R \) is < 0 for reasonable cosmological models, more precisely for all Robertson-Walker models with “expansion”, i.e., warp \( a' > 0 \), and spatial leaves of curvature \( \geq 0 \) or moderately negative (permitted amount depending on warp).
• For $\alpha > 0$ and $\lambda_4 > 0$ and $\text{sig} = (1, 3)$, the scalar field of WOD gravity, extended to the ew sector, has a natural “Mexican hat” potential given by gravitational, respectively cosmological coefficients: $V$ is bi-quadratic, $V(x) = a_4 x^4 + a_2 x^2$, with sign combination $(a_4 > 0, a_2 < 0)$, which assures stability of fluctuations about local minima given by $V'(x_o) = 0, x_o \neq 0$, i.e., $x_o^2 = -\frac{a_2}{2a_4}$.

• The calculation of the last item can be done at each spacetime point $(x_o \mapsto |\phi_o(p)|)$. Thus the potential acquires a minimum for $\phi_0$ with

$$\frac{|\Phi_o(p)|^2}{|\Phi_o(p)|^2} = -\frac{\alpha}{2} \epsilon_{\text{sig}} R(p) \lambda_4^{-1} \sim |R(p)| \lambda_4^{-1}.$$  

This relation will be called the potential condition for the scalar field. It holds in any scale gauge and does not presuppose a breaking of scale symmetry. This is an important difference of Weyl geometric gravity to Riemannian gravity.

• In the preferred scale choice (22), in particular, Weyl geometric scalar curvature $R$ is normed to a constant, if the potential condition is satisfied.

Equ. (38) shows that $|\Phi_o|^2$ is proportional to $R$, if $\Phi$ is subject to the potential condition $V(|\Phi_o|) = \text{min}$. In this way, the ground state of the scalar field, $\Phi_o$, adapts to Weyl geometric scalar curvature. Starting from the Lagrangian (36), the potential condition (38) implies that the scale gauge in which scalar curvature $R$ is constant is the same as the one in which the norm of the scalar field ground state $\phi := |\Phi_o|$ is constant. In other words, the potential condition ensures identity of scalar field gauge and Weyl gauge.

• In the sequel we shall assume that a ground state of the scalar field exists and is in the minimum of the potential, i.e. satisfies the potential condition (38).

The potential condition regulates the norm of the groundstate of the quasi-classical field $\Phi$ only. Even if one looks for a deeper, or more refined, physical understanding of it, the analogy to the standard model Higgs field is no longer helpful. The scalar field $\phi$ and its extension to the electroweak sector, $\Phi$, carry features of an order parameter which may express a possible underlying quantum reality close to the Planck scale only. It does not seem adequate to quantize the scalar field $\Phi$ as a whole. Far away from Planck

\footnote{Early in the 1970s “breaking of scale invariance” of a conformally invariant scalar coupled to gravity in a Brans-Dicke-like way has been studied by S. Deser. Deser introduced an explicit scale breaking term by quadratic term of the potential $\mu^2 \phi^2$ with a constant $\mu$ (Deser 1970, 252).}
scale physics, and as long as we do not know more about this level of reality, we should be content with analyzing the consequences of the structural properties of $\Phi$ assuming (38) for its ground state $\Phi_0$ (its normalization in the $I = \frac{1}{2}, Y = \frac{1}{2}$ representation of $SU(2)$ is discussed at the beginning of the next section).

This result is of considerable importance for Weyl geometric gravity. If Weyl-Omote-Dirac gravity holds, (9), and couples to the electroweak sector with minimal assumptions expressed in the Lagrangians (30), (33), the biquadratic potential for the scalar field (37) follows. The ground state of the scalar field then determines the scale invariant magnitudes of observables uniquely (up to a global constant) (20) and selects Weyl gauge as the one in which magnitudes are most directly read off. One even may, but need not, read this relation as a kind of “vindication” of Weyl’s claim that physical clocks calibrate “by adaptation” to a local field constellation in his discussion with Einstein about his early gauge theory (Weyl 1921a), (Weyl 1923, 298ff.).

Far away from dense mass concentrations, e.g. around ordinary stars and galaxies, empty space is usually modelled by the Schwarzschild solution of Einstein gravity. That remains a good approximation in Weyl geometry; but long-range effects of WOD gravity, usually considered as “cosmological”, have to be taken into account for a more precise determination of curvature. In particular in “empty” space regions of laboratory scale, $R$ cannot be assumed to be 0 in Weyl geometry, but has to be assumed to be of a “cosmological” value $R \sim H_1^2$. Here $H_1 = H_0 c^{-1} \sim 10^{-29} cm^{-1}$ is the observationally determined Hubble parameter at present time. The hypothesis of standard cosmology that the cosmological warp function (“space expansion”) is miraculously frozen inside galaxies (“Einstein-Strauss vacuoles” or other ad hoc modifications of Robertson-Walker solutions) looses any plausibility if considered from the Weyl geometric perspective (see section 4.2).

### 3.4 Can gravity do what the Higgs field is supposed to do? 38

By a point dependent gauge transformation in $SU(2)$, a gauge transformation in the isospin part of the $\text{ew}$ bundle, the ground state $\Phi_0$ can be normalized to the form

$$\Phi_0 \doteq (0, \phi_o), \quad \phi_o = |\Phi_0| \in \mathbb{R}. \quad (39)$$

---

37 Fluctuations $\chi$ about the ground state $\Phi = \Phi_0 + \chi$ can probably be dealt with as quasi-particles, comparable to phonons in solid state physics (section 4.3).

38 A similar question was asked by (Pawłowski 1990), there referring to a conformal theory of gravity.
$|\phi_o|$ is expected to correspond to the electroweak energy scale $v = \frac{2}{g_w} m_W c^2$.

This gauge fixing must be the same as in (28). In special relativistic ew theory this choice is called the unitary gauge of the scalar field. The isotropy group at each point is isomorphic to an $U(1) \subset G_{ew}$ and operates on the first isospinor component only (for the representation with $I = \frac{1}{2}, Y = \frac{1}{2}$). It encodes the electromagnetic gauge symmetry which allows for long-range electromagnetic (em) fields.

A classical derivation of the mass terms (35) is possible by developing (30) along the operations in direction of generators of the Lie algebra of $G_{ew}$ transversal to $U(1)_{em}$ (Bleecker 1981). In this respect, there are only minor formal differences to the usual Higgs field approach. “Ontologically” and, as we shall see, experimentally, the difference is considerable. In our approach, the scalar field expresses a $G_{ew}$-extended part of the gravitational structure; mass terms of the bosons (34) arise from coupling to gravitation, as it should be. In the standard view the nature of the Higgs field is wide open to physical speculation and philosophical controversy (Earman 2004, Lyre 2007, Smeenk 2006). Even in a cautious interpretation, the least one can say is that the extended Weyl geometric scalar field $\Phi$ expresses a connecting link between the electroweak sector and gravity via $\phi = |\Phi|$ and (9). But can a Weyl geometric Brans-Dicke like scalar field play a role usually ascribed to the Higgs field?

If so, the potential condition (38), the relation with the Newton constant (23) and the equality of $|\phi_o|$ with the experimentally determined electroweak energy scale

$$|\phi_o| = v \approx 246 \text{ GeV}, \quad \frac{v}{\hbar c} \approx 1.3 \cdot 10^{16} \text{ cm}^{-1}, \quad (40)$$

give detailed information on the coupling factor $\xi$ in (9),

$$\xi^2 = \frac{\hbar c |e|}{\alpha \alpha^2 16\pi G} = \frac{1}{16 \alpha \pi} \frac{E^2_{Pl}}{v^2}.$$  

Condition (40) specifies an energy, respectively length, unit in scalar field gauge. Together with the global convention (2) in section 2.1 it specifies cgs units in the Weyl geometric/gravity context uniquely.

---

39 $m_W = \text{mass of } W \text{ boson } \approx 80.42 \text{ GeV}$, $g_w = \frac{e}{\sin \theta_w} \approx 0.6295$, $g_e = e \approx 0.302$.  
40 Otherwise $\Phi_o$ would carry non-vanishing electrical charge and internal interactions.  
41 For the ‘left’ lepton representation with $I = \frac{1}{2}, Y = -\frac{1}{2}$ the electromagnetic group $U(1)_{em}$ operates on the second component only.  
42 Of course this is a specification on the level of theoretical principles. For metrological purposes a technical better controllable specification will be chosen, e.g., the one proposed for the new SI base units: The second, $s$, is set by “fixing the numerical value of the ground state hyperfine splitting frequency of the caesium 133 atom, at rest and at a temperature of 0 K, to be equal to exactly 9192631770” if expressed in $s^{-1}$ (BIPM 2010).
For $\alpha \sim 1$, $\xi$ turns out to be essentially the hierarchy factor between Planck and electroweak energy scales,

$$\xi = \frac{1}{\sqrt{16\pi}} \frac{E_{Pl}}{v} \sim 10^{16}. \quad (41)$$

Looked at it the other way round, $v$ is determined by the hierarchy factor $\xi$ in the Lagrangian (and $E_{Pl}$).

Similarly, (38) and (40) allow to determine the relative orders of magnitude of $\lambda_4$ and $R$. For cosmological curvature $R \sim -H_1^2 \sim 10^{-56} \text{cm}^{-2}$ the value of $\lambda_4$ turns out to be very small,

$$\lambda_4 \approx \frac{\alpha}{2} \frac{(hc)^2}{v^2} R \sim 10^{-88}. \quad (42)$$

Although at a first glance, this seems implausible because of its extreme smallness, a look at (9) shows that such an impression depends on comparing incomparables. Rather than comparing $\lambda$ with 1, the values of $R$ and $\lambda_4|\phi_o|^2$ have to be put side by side. (38) shows that this comparison fares quite well, $|R| \sim H_1^2 \sim 10^{-56} \text{[cm}^{-2}]$ (in cosmologically estimated values) and $\lambda_4|\phi_o|^2 \sim 10^{-88+32}$ [in cm$^{-2}$] by (40).

The last few paragraphs pursued a line of analysis. The “ontologically” more appropriate synthesis reverses the direction of the argument. We start from coefficients $\xi \sim 10^{16}$, $\lambda_4 \sim 10^{-88}$, find from cosmological observations $|R| \sim H_1^2 \sim 10^{-56} \text{[cm}^{-2}]$ and conclude from (38) $\phi_o^2 \sim hc|R|\lambda_4^{-1} \sim (100 \text{GeV})^2$ etc.

On the other hand, for fluctuations $\chi$ about the ground state, $\phi = \phi_o + \chi$, a classical mass term of the $\phi$-field may be developed from the seemingly “tachyonic” mass-like factor $\alpha R (R < 0)$ of the quadratic term in (37). Like in the ordinary “Higgs” model, and by the same calculation, the mass factor turns sign in such a development, due to the contribution of the quartic term, $\frac{1}{2}m_\chi^2 [\epsilon^4] = -[hc]^2 2\alpha R$. That results in

$$m_\chi^2 = 2[hc] \sqrt{-\alpha R} \sim 10^{-34} \text{eV} \quad \text{for } R \sim H_1^2, \alpha \sim 1. \quad (43)$$

That is a ridiculously small value. In fact it is the smallest amount of energy that might be considered meaningful in the cosmos.

Quantum corrections may raise this value considerably. A very rough heuristic first estimation of $\Delta m^2 \sim \lambda \Lambda^2$, with $\lambda := \xi^2 \lambda_4$ the full (classical) quartic coefficient and with an energy cutoff at the order of Planck energy.

\[\text{ Compton wave length of } m_\chi \text{ is at the order of magnitude of the Hubble length.}\]
$\Lambda \sim 10^{28}\, eV$, would raise the self energy of scalar field quantum fluctuations to

$$\Delta m^2 \sim 10^{-56\pm2\, 28\, eV^2} \sim 1\, eV^2.$$

This is still far outside the theoretical and experimental limits accepted in the standard model. In this view such small values are theoretically excluded because a Higgs mass below 100 GeV would lead to instabilities at high energies in the “early stage” of the universe (Espinosa e.a. 2008). Experimental exclusion below 114 GeV is inferred from accelerator experiments at CERN’s large electron collider LEP (ALEPH e.a. 2003).

These limits are set from inside the framework of the standard model. It seems, however, unlikely that collider experiments would be able to “see” extremely small mass values of scalar field excitations as above. The theoretical exclusion, on the other hand, depends on the assumption of the correctness of the evolutionary picture of the early universe. Its reliability may have to be reconsidered if modifications of the standard model become necessary. For the moment, however, the question in the title of this subsection has to be answered by a qualified negation: Gravity cannot do without basic modifications of the standard model what the usual Higgs is expected to do, although in principle it may well serve for the generation of mass terms of fermions and electroweak interaction bosons.

4 Resumé and discussion

4.1 Reduction of symmetry

All this appears surprising, in its closeness and contrast to the established perspective on ew symmetry breaking. Different authors have developed other views on this topic from the point of view of Weyl geometry. Already since the 1970s scale covariance of the theory and “breaking” of scale symmetry of the fundamental fields played a central role in considerations of what eventually became to be called the “Higgs mechanism” (Englert/Gunzig 1975, Smolin 1979, Cheng 1988). Cheng considered the Weyl geometric scalar field $\Phi$ as “Higgs field” and “broke” its scale covariance by setting its norm (considered to be the “expectation value” of a not yet existing quantum theory) to the electroweak scale $v$. He did not discuss conditions and reasons for that move. Neither the special role of the scalar field gauge for determining scale invariant magnitudes nor the potential condition came into sight; so his “symmetry breaking” must have looked arbitrary to readers not well acquainted with Weyl geometry. But he realized that “may form a basis for unification of the gravitational interaction with the electroweak interaction” (Cheng 1988, 2183).

For many researchers with a background in differential geometry of fibre bundles, the whole discussion on symmetry breaking in field physics remained dubious. A. Trautman gave a clear analysis that, from this point
of view, talking about “breaking” of a symmetry from $G$ to $G' \subset G$ ought to be reserved for cases where a reduction of the structure group in the fibre construction is possible. Then the gauge group can be reduced to a “localized” symmetry in $G'$ (values of the connection in $g' = \text{Lie} G'$). Geometrically that presupposes a condition on the holonomy group $H$ of the connections appearing in a dynamical theory (an integrability condition for the respective fields), $H \subset G'$. A special subcase is an explicit breaking of the symmetry by a Lagrangian with smaller symmetry (Trautman 1970).

In Trautman’s sense, “ew symmetry breaking” carries both aspects in a seemingly paradoxical way: no breaking (no reduction) on the ew distance scale itself, but breaking, in the sense of group reduction, on distance scales much larger. On larger scales, the effective curvature of the ew connections is negligible exactly because of the massiveness of the ew bosons, i.e., as a result of the Higgs procedure. On the ew distance scale, however, where the Higgs “mechanism” takes place, symmetry in the sense of Trautman is not broken. To the contrary, it is even important that it is not. The gauge group remains $G = \text{SU}(2) \times \text{U}(1)_Y$ after introducing the scalar field.

Of course, the symmetry of $\Phi$, in mathematical terminology the isotropy group $G_\Phi$ of $\Phi$, is smaller than $G$, here $G_\Phi \cong \text{U}(1)_{\text{em}}$. But the gauge group still has local values in $G$ and cannot be reduced, as long as the ew field strengths do not vanish. Clearly, this is not the case: no ew field strengths no ew bosons, let alone massive ones. Moreover, for the formalism of the ordinary “Higgs mechanism”, the transition to unitary gauge of the scalar field, $\Phi \equiv (0, \phi_0)$, is essential. Being able to do so presupposes that the full gauge group is still applicable. From the geometrical (fibre bundle) point of view, no reduction of the gauge symmetry can be diagnosed, but rather a “different realization” of it (Drechsler 1999). The procedure comes down to representing the gauge connection at the local values of $\Phi$ (operating in each infinitesimal neighbourhood of $x$ close to $\Phi(x)$).

Our view is different from Drechsler’s and Tann’s and from the classical Higgs mechanism, although it overlaps with both in certain aspects. The introduction of scale invariant observable magnitudes, including those for mass parameters, equ. (20), makes “breaking” of scale symmetry unnecessary on the quasi-classical level. On the classical level the induced preferred scale gauge, equ. (22), serves as a tool for simplified calculation only. Although it allows to read off (or to plug in) classical mass values directly, it does not express an explicit or spontaneous breaking assumed by Drechsler/Tann or in the usual Higgs mechanism. On the other hand, it would not be surprising if also in the Weyl geometric setting, like in other scale invariant approaches, the implementation of scale symmetry runs onto an anomaly during perturbative quantization, i.e. cannot be consistently expressed on the quantum level. Whether this has to be interpreted as an indicator of “spontaneous

\footnote{Cf. footnote 20 and the text above it.}
symmetry breaking” in the usual sense seems doubtful, considered for itself. Far below the Planck level (energetically), scale symmetry is no longer dynamical anyhow (section 2.3). Independent of any substantive realistic interpretation, a restriction to the preferred scale gauge (22) would just mean that quantization presupposes an appropriate scale fixing on the classical level. The question of possible “dynamical symmetry breaking” close to the Planck scale latter is a different question; it may be that here ’t Hooft’s idea of naturalness turns out to be a fruitful guide (’t Hooft 1980).

Recently several authors have started to reconsider scale covariant approaches to ew symmetry breaking, sometimes conformal, sometimes explicitly Weyl geometric (Nishino/Rajpoot 2004, Nishino/Rajpoot 2007b, Foot e.a. 2007a, Foot e.a. 2007b, Meissner/Nicolai 2009). Closest to our and Cheng’s view is the approach of Nishino/Rajpoot (Nishino/Rajpoot 2004, Nishino/Rajpoot 2007b). These authors, however, stay closer to the ordinary Higgs mechanism and allow a mass possibly observable in LHC experiments. They do so by introducing a second (real) scalar field $\sigma$, in addition to $\Phi$, also of weight $w(\sigma) = -1$, and arrive at a more flexible Lagrangian, with a scale invariant term of form $\mu|\sigma|^2|\Phi|^2$. That leads to an adaptable mass term $\mu|\sigma|^2$ for $\Phi$, far above the tiny value induced by the purely gravitational quadratic term considered here.

For the moment, i.e., as long as no definite experimental results on the Higgs boson mass are available, the most simple form of the Weyl geometric Lagrangian for the scalar field, like in (29), remains a possibility for giving mass to the ew bosons. Outright dismissing it because of its surprising effects would be unjustified and at least premature. One may even appreciate this approach, because it avoids the vexing problem of compensatory terms for the self energy corrections to the Higgs boson mass. It gives, at least, a conceptually pleasing and, if we are lucky, also a physically convincing account of the acquirement of mass by coupling to gravity. Of course, tracing a Higgs mass at the LHC scale would experimentally refute the approach given here.

If, on the other hand, the Higgs mass remains elusive to the LHC detectors, the simple Weyl geometric approach to gravity discussed here may show a well prepared path out of the otherwise expected paradoxes for the standard theory of elementary particles. It shows that we need not necessarily take refuge to models in “higher dimensions”, multiplying the number of “Higgs fields”, or other highly underdetermined escape routes of theory. We can just as well look for a better understanding of the relation between gravity and ew interactions.

46In the 1990s other authors continued the conformal approach, among them (Pawłowski/Rączka 1995a, Pawłowski/ Rączka 1995b).
47Nishino/Rajpoot compare their approach with others in (Nishino/Rajpoot 2007a).
4.2 Consequences for long range effects

If the scalar field plays a material role in gravity, and if the potential condition regulates its ground state, the consequences for considerations on large astrophysical scales, galaxies and above, and even for cosmology will be considerable. Not so, however, on smaller scales like the solar system, as the Schwarzschild solution is a degenerate vacuum solution of the WOD Einstein equations, with \( \phi = \text{const} \) in Riemann gauge and vanishing energy momentum tensor.

Homogeneous and isotropic cosmological models behave slightly differently in the Weyl geometric approach. Their geometry in a manifold \( M \approx I \times S^{(3)} \), with \( I \subset \mathbb{R} \) and \( S^{(3)} \) a threedimensional manifold, can be characterized by a classical Robertson Walker metric with spacelike folia of constant sectional curvature and metric \( a(\tau)d\sigma_{\kappa}^2 \), where \( d\sigma_{\kappa}^2 \) denotes the 3-dimensional metric of constant sectional curvature \( \kappa \) and \( a(\tau) \) a warp function (“expansion”) depending on the cosmological time parameter \( \tau \):

\[
\tilde{g} : \quad ds^2 = -d\tau^2 + a(\tau)^2d\sigma_{\kappa}^2 \tag{44}
\]

\[
d\sigma_{\kappa}^2 = \frac{dr^2}{1 - \kappa r^2} + r^2(d\Theta^2 + r^2 \sin^2 \Theta d\phi^2)
\]

Looking at it in the Weyl geometric paradigm means to consider \( \tilde{g} \) as Riemannian component of an IWG given by \( (M, [\tilde{g}, 0]) \).

If we assume a scalar field \( \tilde{\phi} \) obeying the potential condition, the scalar field gauge is different from Riemann gauge. In fact, it is equal to Weyl gauge, as we have noted in \( (38) \). In Riemann gauge \( (\tilde{g}, 0) \) we have

\[
\tilde{R} = g\tilde{R} = -6\left( \frac{a'^2}{a^2} + \frac{\kappa}{a^2} + \frac{a''}{a} \right). \tag{45}
\]

So roughly \( |\tilde{R}| \approx -6H^2 \), for \( \kappa \approx 0, |\frac{a'}{a}| \ll |a'| \), with \( H = \frac{a'(\tau_0)}{a(\tau_0)} \) at “present”.

Of course, the scaling condition for matter under expansion has to be related to scale invariant magnitudes, the “observables” of section 2.3. Thus different models from those of standard cosmology have to be taken into account. As an important consequence of Weyl geometry, redshift (cosmological and gravitational) is invariant under change of scale gauge (Scholz 2009).

In Weyl gauge, a considerable contribution to cosmological redshift will be due to the time component \( \tilde{\phi}_o \) of the scale connection \( \tilde{\phi} \). The same holds for scalar curvature \( \tilde{R} \). There is no reason, nor even any plausibility, to assume a vanishing scale connection near galaxies. This is an important

\[A^\text{48}\]A first exploration can be found in (Scholz 2009) (still without using the potential condition). Using the potential condition, the scalar field energy momentum tensor ensures dynamical consistency (equilibrium) for the static “Weyl universes” discussed in (Scholz 2009), if a homogeneously distributed mass density \( \mu \) is assumed, for parameters \( \alpha = 1, \varsigma = 3.5, \Omega_m \approx 2.83 \). This does not seem particularly convincing from the empirical point of view. Surely, dynamical considerations have to be added.
difference to the received view in which “expansion” of the spatial folia is usually thought to reach standstill close to galaxies. The Einstein-Strauss vacuoles which are often referred to in this respect, have to be considered an ad-hoc construction designed to bring about some kind of coherence between the usage of the Scharzschild solution close to stars and the expanding space model of cosmology.

- In the Weyl geometric approach, seemingly cosmological effects represented in Riemann gauge by the warp function $a(\tau)$ in (44) and interpreted in terms of an expansion of space in the standard approach, turn out to be due to ordinary, though very weak, field theoretical properties. In scalar field gauge, they are mainly expressed by the scale connection.

- If this approach is realistic, the corresponding effects are not truly cosmological in nature. They are just so weak that they were first observed over very long range, i.e. on cosmological scales (in particular Hubble redshift with Hubble parameter $H$).

- There is no sound reason to assume that these effects are shielded away by some unknown “mechanism” close to galaxies. Rather we have to assume that they pervade empty space everywhere. In particular, the curvature contribution of the warp function to scalar curvature $R$ has to be considered as field theoretic property of the gravitational vacuum.

Thus the gravitational background should also be “felt” even in vacua underlying interactions of elementary particle fields, like in (38).

4.3 Remarks on the relation between gravity and QFT

Up to here, we have approached the interface between gravity and quantum field theory (QFT) only marginally, basically from the classical side. The question of how to integrate the two sister theories of the 20th century about spacetime and matter more deeply has not yet been dealt with. In the framework of our approach, a natural next step would be to investigate quantized versions of the quasi-classical (spinor and connection) fields. That might be done perturbatively and/or by adapting methods of axiomatic and algebraic quantum field theory to curved spaces. In both methods (and others) a systematic quantization of the gravitational field itself has, up to now, resisted all attempts to come to definite results.

In our context, it may be worthwhile to look at a somehow moderate quantization of gravity. Why should we assume that it is necessary to quantize the metrical structure as a whole on all levels of energy? Years ago,

\footnote{For a critical discussion of Einstein-Strauss vacuoles and other attempts to relate globally expanding cosmological models with local geometry close to stars see (Carrera/Giulini 2010).}
Flato, Simon, and Račka have started to formulate a generalization of the Wightman axioms to curved spaces, slightly different from the one used today in theories of QFT on “curved spaces” (Wald 1994, Bär/Fredenhagen 2009). They foresaw a quantization of the scaling degree of the metric and, by this, a weak form of backreaction of quantum matter on the metric, while leaving the underlying conformal structure unquantized (Flato/Simon 1972, Flato/Račka 1988). To my knowledge this approach was not pursued further. In our approach one might consider the Weyl geometric structure to be determined by the environmentally decohered classical matter systems, while only the scalar field degree of freedom of fluctuations about the ground state is subjected to quantization.

In WOD gravity the scalar field $\Phi$ is directly linked, via (22) and (20) to the scaling degree of freedom of the Weylian metric. Quantization of the scalar field therefore comes down essentially to quantizing the scaling degree of freedom of the Weylian metric. Quantum field theory of a scalar covariant field is technically much better accessible than quantization of the complete metric and should be tractable by perturbative methods, also on curved spaces. Moreover, the scalar field seems to represent a material aspect of the extended gravity structure.

There is much room for investigations between the electroweak energy level and the Planck scale. It may well be that a moderate quantization, concentrating on the scaling degree of freedom of the metric, suffices to understand many of the effects in the energy regime accessible during the next decades to laboratory devices (LHC etc.) and astrophysical observation (radio, X-ray telescopes etc.).

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References

Adler, Ronald; Bazin, Maurice; Schiffer Menahem. 1975. Introduction to General Relativity. New York etc.: Mc-Graw-Hill. 2nd edition.

ALEPH, DELPHI, L3 and OPAL collaborations. 2003. “Search for the standard model Higgs boson at LEP.” Physics Letters B 565:61–75. arXiv:hep-ex/0306033.

Bär, Christian; Fredenhagen, Klaus. 2009. Quantum Field Theory on Curved Spacetimes. Vol. 786 of Lecture Notes in Physics Berlin etc.: Springer.

Barenblatt, Grigory I. 2003. Scaling. Cambridge: University Press.
Bergmann, Peter G. 1942. *Introduction to the Theory of Relativity*. Englewood Cliffs: Prentice Hall. Reprint New York: Dover 1976.

Blagojević, Milutin. 2002. *Gravitation and Gauge Symmetries*. Bristol/Philadelphia: Institute of Physics Publishing.

Bleecker, David. 1981. *Gauge Theory and Variational Principles*. Reading, Mass.: Addison-Wesley.

Bos, Henk J. M. 2001. *Redefining Geometrical Exactness. Descartes’ Transformation of the Early Modern Concept of Construction*. Berlin etc.: Springer.

Bureau International des Poids et Mesures. 2010. “On the possible future revision of the International System of Units, the SI. Draft resolution A.” Convocation of the General Conference on Weights and Measures - 24th meeting (17-21 October 2011).

Calderbank, D; Pedersen, H. 2000. Einstein-Weyl geometry. In *Surveys in Differential Geometry. Essays on Einstein Manifolds*, ed. C. Le Brun; M. Wang. International Press pp. 387–423.

Carrera, Matteo; Giulini, Domenico. 2010. “Influence of global cosmological expansion on local dynamics and kinematics.” *Reviews of Modern Physics* 82:169–208.

Cheng, Hung. 1988. “Possible existence of Weyl’s vector meson.” *Physical Review Letters* 61:2182–2184.

Deser, Stanely 1970. “Scale invariance and gravitational coupling.” *Annals of Physics* 59:248ff.

Dicke, Robert H. 1962. “Mach’s principle and invariance under transformations of units.” *Physical Review* 125:2163–2167.

Dirac, Paul A.M. 1973. “Long range forces and broken symmetries.” *Proceedings Royal Society London A* 333:403–418.

Drechsler, Wolfgang. 1991. “Geometric formulation of gauge theories.” *Zeitschrift f. Naturforschung* 46a:645–654.

Drechsler, Wolfgang. 1999a. “Mass generation by Weyl symmetry breaking.” *Foundations of Physics* 29:1327–1369.

Drechsler, Wolfgang; Mayer, M.E. 1977. *Fibre Bundle Techniques in Gauge Theories. Lectures in Mathematical Physics at the University of Austin*. Vol. 67 of *Lecture Notes in Physics* Berlin etc.: Springer.

Drechsler, Wolfgang; Tann, Hanno. 1999b. “Broken Weyl invariance and the origin of mass.” *Foundations of Physics* 29(7):1023–1064. [arXiv:gr-qc/98020vv v1].

Diirr, S.; Fodor, Zoltan; Frison J. e.a. 2008. “Ab-initio determination of light hadron masses.” *Science* 322(5905):1224–1227.

Eaman, John. 2004. “Laws, symmetry, and symmetry breaking: invariance, conservation principles, and objectivity.” *Philosophy of Science* 71:1227–1241.

Eddington, Arthur S. 1923. *The Mathematical Theory of Relativity*. Cambridge: University Press.

Ehlers, Jürgen; Pirani, Felix; Schild, Alfred. 1972. The geometry of free fall and light propagation. In *General Relativity, Papers in Honour of J.L. Synge*, ed. Lochlann O’Raifertaigh. Oxford: Clarendon Press pp. 63–84.

Englert, François; Gunzig, Edgar; Truffin C.; Wnley P. 1975. “Conformal invariant relativity with dynamical symmetry breakdown.” *Physics Letters* 57 B:73–76.

Espinosa, J.R.; Giudice, G.F.; Riotto A. 2008. “Cosmological implications of the Higgs mass measurement.” *Journal of Cosmology and Astroparticle Physics* 0805(002):1–24.
Euclides. 1925. *The Thirteen Books of Euclid’s Elements* (translation Thomas Heath), 3 vols. Cambridge: Universitry Press. Reprint New York 1956 (Dover).

Flato, Moshé; Račka, Ryszard. 1988. “A possible gravitational origin of the Higgs field in the standard model.” *Physics Letters B* 208:110–114. Preprint, SISSA (Scuola Internazionale Superiore di Studi Avanzate), Trieste, 1987 107/87/EP.

Flato, Moshé; Simon, J. 1972. “Wightman formulation for the quantization of the gravitational field.” *Physical Review D* 5:332–341.

Folland, George B. 1970. “Weyl manifolds.” *Journal of Differential Geometry* 4:145–153.

Foot, Robert; Kobakhidze, Archil; McDonald Kristian; Volkas Raymond R. 2007a. “Neutrino mass in radiatively-broken scale-invariant models.” arXiv:0706.1829.

Foot, Robert; Kobakhidze, Archil; McDonald Kristian; Volkas Raymond R. 2007b. “A solution to the hierarchy problem from an almost decoupled hidden sector within a classically scale invariant theory.” arXiv:0709.2750.

Frankel, Theodore. 1997. *The Geometry of Physics*. Cambridge: University Press. 2004.

Gauduchon, P. 1995. “La 1-forme de torsion d’une variété hermitienne compacte.” *Journal für die reine und angewandte Mathematik* 469:1–50.

Gover, A. Rod; Shaukat, Abrar; Waldron Andrew. 2009. “Tractors, mass, and Weyl invariance.” *Nuclear Physics B* 812:424–455.

Hayashi, Kenji; Kugo, Taichiro. 1979. “Remarks on Weyl’s gauge field.” *Progress of Theoretical Physics* 61:334–346.

Higa, T. 1993. “Weyl manifolds and Einstein-Weyl manifolds.” *Commentarii Mathematici Sancti Pauli* 42:143–160.

’t Hooft, Gerard. 1980. Naturalness, chiral symmetry, and spontaneous symmetry breaking. In *Recent Developments in Gauge Theories. Proceedings of the NATO Advanced Study Institute. Cargèse Proceedings, August 26–Sep. 8, 1979*, ed. G. ’t Hooft e.a.. New York pp. 135–157.

Israelit, Mark. 1999. *The Weyl-Dirac Theory and Our Universe*. New York: Nova Science.

Jordan, Pascual. 1952. *Schwerkraft und Weltall*. Braunschweig: Vieweg. 2nd revised edition 1955.

Kaiser, David. 2006. “Whose mass is it anyway? Particle cosmology and the objects of a theory.” *Social Studies of Science* 36(4):533–564.

Kaiser, David. 2007. “When fields collide.” *Scientific American* pp. 62–69.

Lyre, Holger. 2007. “Does the Higgs mechanism exist?” *International Studies in the Philosophy of Science* .

Meissner, Krzysztof; Nicolai, Hermann. 2009. “Conformal symmetry and the standard model.” *Physical Review D* 80:86005–86012. arXiv:hep-th/0612165.

Nishino, Hitoshi; Rajpoot, Subhash. 2004. “Broken Scale Invariance in the Standard Model.” arXiv:hep-th/0403039.

Nishino, Hitoshi; Rajpoot, Subhash. 2007a. “Comment on shadow and non-shadow extensions of the standard model.” arXiv:hep-th/0702080.

Nishino, Hitoshi; Rajpoot, Subhash. 2007b. “Standard model and SU(5) GUT with local scale invariance and the Weylon.” *AIP Conference Proceedings* 881:82ff. arXiv:0805.0613.

Omore, M. 1971. “Scale transformations of the second kind and the Weyl space-time.” *Lett. Nuovo Cimento* 2(2):58–60.

Omore, M. 1974. “Remarks on the local-scale-invariant gravitational theory.” *Lett. Nuovo Cimento* 10(2):33–37.
Weyl, Hermann. 1921a. “Über die physikalischen Grundlagen der erweiterten Relativitätstheorie.” *Physikalische Zeitschrift* 22:473–480. In (Weyl 1968, II, 228–236).

Weyl, Hermann. 1921b. “Zur Infinitesimalgeometrie: Einordnung der projektiven und der konformen Auffassung.” *Nachrichten Göttinger Gesellschaft der Wissenschaften* pp. 99–112. In (Weyl 1968, II, 195–207) [43].

Weyl, Hermann. 1923. *Raum - Zeit -Materie*, 5. Auflage. Berlin: Springer.

Weyl, Hermann. 1929. “Elektron und Gravitation.” *Zeitschrift für Physik* 56:330–352. In (Weyl 1968, III, 245–267) [85]. English in (O’Raifeartaigh 1997, 121–144).

Weyl, Hermann. 1968. *Gesammelte Abhandlungen*, 4 vols. Ed. K. Chandrasekharan. Berlin etc.: Springer.

Wilczek, Frank. 2008. “Mass by numbers.” *Nature* 456:449–450.