On propagation of photon in a medium

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Abstract

The equations for variation of the Stokes parameters and intensity of photons propagating in a medium, whose optical properties may be described by the permittivity tensor, are derived. Classification of different cases of photon propagation is suggested on a basis of these equations.

1 Introduction

It is well known that polarized photons propagating in an optically active medium change initial polarization state [1, 2]. It has important meaning in a lot of fields of physics. We have pointed to some existing problems:

a) propagation effects in a magnetosphere of pulsars [3, 4];

b) results of the circular polarization measurements of extragalactic radio sources [5] and [6] (in our Galaxy);

c) cosmic birefringence (see [7] and literature therein).

d) problems of propagation x-rays [8] and high energy $\gamma$-quanta [9, 10] in single crystals and creation of the polarimeters for hard photon beams;

e) experimental observation of the birefringence in an electromagnetic vacuum [11, 12];

f) investigations of propagation of $\gamma$-quanta in laser waves [13, 14] with the aim of polarization control on future $\gamma\gamma$-colliders.

Besides, the possibility of propagation polarization phenomena of the radiation emitted from quark-gluon plasma must not be ruled out.

Various phenomena (birefringence, Faraday rotation and etc.) of the visible light are well known and described in literature [1, 2]. In [2] the detailed consideration of these processes was carry out in anisotropic and gyrotrropic media. However, the description of propagation was realized on a basis of the polarization states, whereas representation in terms of density matrices is more adequate in some cases. A parallel with well known ones, the new cases of propagation of photons were investigated in recent papers [13, 14, 15]. In this connection the problem of classification of these cases and its completeness arises.

On the other hand, the description of the optical properties of a medium with the help of the permittivity tensor is the universal and traditional method. This method may be used in wide energy ranges of propagating photons. The proof of validity of the method for

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\(\gamma\)-quanta and samples of its using one can find in [16, 17, 18]. Using of permittivity tensor presumes the linear response of media on perturbations. It is a very good assumption for many cases (however, see [19]).

The aims of this paper are: firstly, the derivation of equations, describing all the cases of propagation of photons in media, whose optical properties may be described with the help of the permittivity tensor and, secondly, creation on a basis of these equations classification of the various cases of photon propagation. Our consideration is based on some suggestions such as smallness of the permittivity tensor components in compare to unit and straight line motion of the photons. It is not the most general case, of course, but the most interesting one for practical applications.

It should be noted, that the problems of propagation of the photon beam were considered in some papers [20, 21, 22]. For example, in [22] the equations of the general type for propagation process were obtained. However, it seems that their using in specific applications is difficulty, because the physical nature of the parameters in the equations is unknown.

## 2 Equations

We write the equations of the electromagnetic field in a medium in the following form [1, 2]:

\[
\begin{align*}
\text{rot}\mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}, \\
\text{div}\mathbf{D} &= 0, \\
\text{rot}\mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \\
\text{div}\mathbf{B} &= 0, \\
\end{align*}
\]

(1)

where \(\mathbf{E}\) is the intensity of electric field and \(\mathbf{D}\) and \(\mathbf{B}\) are the electric and magnetic induction vectors, \(t\) is the time, \(c\) is the speed of light. All the properties of the medium are reflected in the relation between \(\mathbf{B}, \mathbf{E}\) and \(\mathbf{D}\). Eqs.(1) would suffice to describe the propagation of photons in a medium and such a property as the intensity of magnetic field is not needed [1, 2]. We represent the relation between \(\mathbf{D}\) and \(\mathbf{E}\) in the form

\[
D_i(\omega) = \varepsilon_{ij} E_j(\omega), \quad (i, j = 1, 2, 3),
\]

(2)

where \(\varepsilon_{ij} = \varepsilon_{ij}' + i\varepsilon_{ij}''\) is the complex permittivity tensor and \(\omega\) is the frequency of the photon beam propagating in the medium.

We consider here only monochromatic photon beam, which propagates in defined direction. In this case conveniently to use in calculations the inverse \(\eta_{\alpha\beta}\) of the tensor \(\varepsilon_{ij}\). In Cartesian coordinate system with one axis oriented along the direction of photon motion this tensor can be represented as two-dimensial tensor \((\alpha = 1, 2, \beta = 1, 2)\) because of perpendicularity of \(\mathbf{D}\) to the photon wave vector. The obtained below equations are written in this coordinate system.

In our consideration we assume that the photon beam move along straight line. It means that in transverse directions (relative to motion of the beam) a medium is uniform ([1]). In the most of practical cases it is true with very high accuracy.

Knowing the tensor \(\eta_{\alpha\beta}\), one can find the refractive indices of propagating photons [2, 14, 15]:

\[
\tilde{n}^{-2} = \left(\eta_{11} + \eta_{22}\right)/2 \pm \sqrt{(\eta_{11} - \eta_{22})^2/4 + \eta_{12}\eta_{21}},
\]

(3)
Thus, in the general case the photon beam propagates through the medium as the superposition of two electromagnetic waves with different refractive indices. In the general case the refractive indices are complex values.

One can describe the polarization state either (of the two) wave by the use of the Stokes parameters $X_i, Y_i$ ($i = 1 - 3$) ($X_i$-values correspond to one wave and $Y_i$ to another). These parameters are determined by the following relations \[14, 15\]

\begin{align*}
X_1, Y_1 &= \frac{\kappa + \kappa^*}{1 + \kappa \kappa^*}, \\
X_2, Y_2 &= \frac{i(\kappa - \kappa^*)}{1 + \kappa \kappa^*}, \\
X_3, Y_3 &= \frac{\kappa \kappa^* - 1}{1 + \kappa \kappa^*}. \\
\end{align*}

where $\kappa$ is the ratio of components of electric induction vector $D(D_1, D_2, 0)$:

\[ \frac{D_1}{D_2} = \kappa = \frac{\bar{n} - \eta_{22}}{\eta_{21}} = \frac{|D_1|}{|D_2|} e^{i\delta}, \]

where $\kappa$ calculated with refractive index $\bar{n}_1$, $(\bar{n}_2)$ corresponds to $X_i (Y_i)$-parameters (denote the $\kappa$-values correspondingly as $\kappa_1$ and $\kappa_2$) and $\delta$ is the phase shift between $D_1$ and $D_2$.

This ratio $\kappa$ can be reduced to zero or to the form $\kappa = i\rho$ (since $|D_1||D_2|\sin\delta = b_1b_2$, where $b_1$ and $b_2$ are the semiaxes of the ellipse and $|\rho| = b_1/b_2$) by the rotation of the coordinate system around the photon wave vector (the third axis is constantly aligned with the wave vector). The first case corresponds to the propagation of a linearly polarized wave and the second case corresponds to an elliptically polarized wave; in addition, $\rho > 0$ ($\rho < 0$) corresponds to left (right) - hand polarization of $\gamma$-quanta.

From our consideration follows the important conclusion that these two waves (named as electromagnetic normal waves \[2\]) with determined refractive indices and polarization states are the eigenfunctions of problem and any monochromatic electromagnetic wave propagating in the medium in defined direction is superposition of these two waves.

One can obtain the following relations:

\[ \kappa_1 \kappa_2 = -\frac{\eta_{12}}{\eta_{21}}, \]

In the case of symmetric $\eta_{\alpha\beta}$-tensor, the product $\kappa_1 \kappa_2 = -1$ and $X_1 = -Y_1, X_2 = Y_2, X_3 = -Y_3$. Another important case take a place when $\eta_{12} + \eta_{21} = 0$ (but $\eta_{12} \neq 0$) and then $\kappa_1 \kappa_2 = 1, X_1 = -Y_1, X_2 = Y_2, X_3 = -Y_3$. However, in general the absolute values of circular and linear polarizations for both the normal waves are not equal in between (i.e. $|X_2| \neq |Y_2|, X_1^2 + X_3^2 \neq Y_1^2 + Y_3^2$).

Now we consider the case of photon propagation in the medium, whose optical properties described by the tensor $\eta_{\alpha\beta}$ at condition $\eta_{12} + \eta_{21} = 0$ (but $\eta_{12} \neq 0$). This case was investigated in \[14\]. The variations of intensity and Stokes parameters one can write in the following form:

\[ J_\gamma(x) = J_1(x) + J_2(x) + 2J_3(x), \]

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The differential form of these equations:

\[ \xi_1(x) = (X_1 J_1(x) + Y_1 J_2(x) + p_1 J_3(x))/J_1(x), \]  
\[ \xi_2(x) = (X_2 J_1(x) + Y_2 J_2(x) + p_2 J_4(x))/J_1(x), \]  
\[ \xi_3(x) = (X_3 J_1(x) + Y_3 J_2(x) + p_3 J_4(x))/J_1(x), \]

where \( J_1, J_2, J_3, J_4 \) are the intensity and Stokes parameters of the photon beam on the distance equal to \( x \). The partial intensities \( J_i(x), (i = 1 - 4) \) have the following form:

\[ J_1(x) = J_1(0) \exp(-2 \text{Im}(\tilde{n}_1)\omega x/c), \]
\[ J_2(x) = J_2(0) \exp(-2 \text{Im}(\tilde{n}_2)\omega x/c), \]
\[ J_3(x) = \exp(-\text{Im}(\tilde{n}_1 + \tilde{n}_2)\omega x/c) \{ J_3(0) \cos(\text{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c) - J_4(0) \sin(\text{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c) \}, \]
\[ J_4(x) = \exp(-\text{Im}(\tilde{n}_1 + \tilde{n}_2)\omega x/c) \{ J_3(0) \sin(\text{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c) + J_4(0) \cos(\text{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c) \}. \]

The initial partial intensities are defined from the following relations:

\[ J_1(0) = \frac{1 - X_1 \xi_1(0) + X_2 \xi_2(0) + X_3 \xi_3(0)}{2(X_2^2 + X_3^2)}, \]
\[ J_2(0) = \frac{1 - X_1 \xi_1(0) - X_2 \xi_2(0) - X_3 \xi_3(0)}{2(X_2^2 + X_3^2)}, \]
\[ J_3(0) = \frac{X_1 \xi_1(0) - X_2^2}{2(X_2^2 + X_3^2)}, \]
\[ J_4(0) = \frac{X_1 (X_2 \xi_3(0) - X_3 \xi_2(0)))}{2(X_2^2 + X_3^2)}. \]

The relations between \( X_i \) and \( Y_i \) values were used, because of this the \( Y_i \)-values are absent in Eqs.\((18)-(21)\). Besides, we assume that \( J_\gamma(0) = 1 \). Now one can obtain the differential form of these equations:

\[ \frac{c}{\omega} \frac{dJ_\gamma}{dx} = -J_\gamma \{ -\mathcal{G} - \mathcal{C} \xi_2 - \mathcal{B} \xi_3 \}, \]
\[ \frac{c}{\omega} \frac{d\xi_1}{dx} = -\mathcal{C} \xi_1 \xi_2 - \mathcal{B} \xi_1 \xi_3 - \mathcal{A} \xi_2 - \mathcal{D} \xi_3, \]
\[ \frac{c}{\omega} \frac{d\xi_2}{dx} = \mathcal{C} (1 - \xi_2^2) - \mathcal{B} \xi_2 \xi_3 + \mathcal{A} \xi_1, \]
\[ \frac{c}{\omega} \frac{d\xi_3}{dx} = \mathcal{B} (1 - \xi_3^2) - \mathcal{C} \xi_2 \xi_3 + \mathcal{D} \xi_1. \]

where

\[ \mathcal{A} = (\eta_{11}' - \eta_{22}')/2, \]
\[ \mathcal{B} = (\eta_{11}'' - \eta_{22}'')/2, \]
\[ \mathcal{C} = (\eta_{12}' - \eta_{21}')/2, \]
\[ \mathcal{D} = (\eta_{12}'' - \eta_{21}'')/2, \]
\[ \mathcal{G} = -(\tilde{n}_1'' + \tilde{n}_2'') = (\eta_{11}'' + \eta_{22}'')/2. \]
It should be noted, that these equations are true only at $|\delta_{\alpha\beta} - \eta_{\alpha\beta}| \ll 1$, where $\delta_{\alpha\beta}$ is the Kroneker $\delta$-function. This is due to the fact that the intensity of photon beam is calculated approximately ($J \sim DD^*$).

The differential form of equations for the general case of $\eta_{\alpha\beta}$-tensor may be obtained by the similar direct calculations. However, another method is more conveniently to use for it. Knowing the relations for Stokes parameters in an anisotropic medium [10, 15], one can find the differential equations for this case. They have a similar form as Eqs.(22)-(25). Combining these equations and Eqs.(22)-(25) we get

\[
\frac{c}{\omega} \frac{dJ_1}{dx} = -J_1( -G - F \xi_1 - C \xi_2 - B \xi_3 ),
\]
\[
\frac{c}{\omega} \frac{d\xi_1}{dx} = F(1 - \xi_1^2) - C \xi_1 \xi_2 - B \xi_1 \xi_3 - A \xi_2 - D \xi_3,
\]
\[
\frac{c}{\omega} \frac{d\xi_2}{dx} = C(1 - \xi_2^2) - F \xi_1 \xi_2 - B \xi_2 \xi_3 + A \xi_1 - E \xi_3,
\]
\[
\frac{c}{\omega} \frac{d\xi_3}{dx} = B(1 - \xi_3^2) - F \xi_1 \xi_3 - C \xi_2 \xi_3 + D \xi_1 + E \xi_2,
\]

where

\[
E = (\eta'_{12} + \eta'_{21})/2,
\]
\[
F = (\eta''_{12} + \eta''_{21})/2,
\]

and $A, B, C, D, G$ are the same as in Eqs.(26)-(30).

It is clear that these equations are required and they describe the propagation process in the general case. It obvious from following:

1) the form of relations for intensity and Stokes parameters for the general case is similar to Eqs.(9)-(21). The main difference is contained that $J_1(0), J_2(0), J_3(0), J_4(0)$-values have another mathematical representation. However, it is significant that these values are linear functions of Stokes parameters;

2) the derivatives of Stokes parameters in Eqs.(32)-(34) are equal to zero when $\xi_1 = X_1, Y_1, \xi_2 = X_2, Y_2$ and $\xi_3 = X_3, Y_3$. This fact supports the choice of $A, ..., F$- parameters (see Eqs.(26)-(29) and (35)-(36));

3) Eq.(31) for intensity was obtained in [14] for the general case.

Note that $(-G - F \xi_1 - C \xi_2 - B \xi_3) \geq 0$ for any real medium.

The obtained here equations describe the variations of intensity and Stokes parameters for photon beam propagating in the medium, whose optical properties may be represented by the use of $\eta_{\alpha\beta}$-tensor. Besides, one can get the following equation for variations of polarization $P = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$ of propagating photon:

\[
\frac{c}{2\omega} \frac{dP^2}{dx} = (1 - P^2)( -F \xi_1 - C \xi_2 - B \xi_3 ).
\]
Obtained in previous section equations describe all the cases of propagation of photons in a medium. The difference between various cases is determined by the kind of $\eta_{\alpha\beta}$-tensor. Here we consider briefly all the possible cases of the photon beam propagation in media.

First and foremost it should be noted that the maximal number of values, which determine the variations of photon state at its propagation in a medium, is equal to seven. However, the $G$-value is responsible only for the intensity of photon beam (see Eqs. (31)-(34)). By this means only six (or less) values ($A, \ldots, F$) is required for description of the variations of photon polarization state [22]. On the other hand, one can make the $E$-value (or $F$-value) equal to zero by rotation of the coordinate system around the wave vector of photon. It is simplify Eqs. (31)-(34), but does not reduce the number of parameters (in this case the angle of rotation is such a parameter). For further consideration we select coordinate system in which $E = 0$.

It is obviously that medium is transparent if $\frac{dJ}{dx} = 0$. This condition take a place at $G = 0, F = 0, C = 0, B = 0$. The excellent example of transparent medium is the monochromatic laser wave of not high intensity. The beam of $\gamma$-quanta propagates in its practically without interactions if the energy of the beam is less then the threshold of electron-positron pair production. Taking into account the importance of this case for practice, we write the solution of Eqs. (31)-(34) for transparent medium in the following form:

$$\xi_1(x) = \xi_1(0) \cos \phi + (\xi_2(0)X_3 - \xi_3(0)X_2) \sin \phi,$$

$$\xi_2(x) = -\xi_1(0)X_3 \sin \phi + X_2^2 \xi_2(0) + X_2X_3\xi_3(0) + (\xi_2(0)X_3^2 - \xi_3(0)X_2X_3) \cos \phi,$$

$$\xi_3(0) = \xi_1(0)X_2 \sin \phi + X_2X_3\xi_2(0) + X_3^2 \xi_3(0) + (\xi_3(0)X_2^2 - \xi_2(0)X_2X_3) \cos \phi,$$

where $\phi = \text{Re}(\vec{n}_1 - \vec{n}_2)\omega x/c$ and $X_2, X_3$ are calculated with the help of Eqs. (31)-(34). Specifically, these equations describe the well-known cases of the light propagation as birefringence ($X_2 = 0, X_3 = \pm 1$) and Faraday rotation ($X_2 = \pm 1, X_3 = 0$). It easy to see that $P = \text{const}$ in the transparent medium. Note that the absolute transparent medium is only physical abstraction [1].

In general any tensor of second rank may be decomposed on the two parts: symmetric and antisymmetric. The antisymmetric part is dual to some axial vector, named as the vector of gyration. When the vector of gyration is equal to zero, the medium is named as anisotropic one. It means that the optical properties of anisotropic medium are determined by a symmetric $\eta_{\alpha\beta}$-tensor, or, in other words, in the case under consideration the following relations are true: $C = 0, D = 0$. We remind that $E = 0$ due to the choice of the coordinate system. The anisotropic media may be classified by the help of the two types of $\eta_{\alpha\beta}$-tensor. In the first case $F = 0$, and $F \neq 0$ in the second case. The linearly polarized monochromatic laser wave is a simple example of the anisotropic medium of the first type [14]. In this case the normal electromagnetic waves are linearly polarized along the principal axis of the $\eta_{\alpha\beta}$-tensor and result of propagation of the initially linearly polarized photon beam in this medium is the effect of birefringence. This effect applies to transparent $B = 0$ and absorbed $B \neq 0$ media. However, in absorbed media of considered type [23] the initially unpolarized propagating photon beam becomes linearly polarized (at $x \to \infty$).
The first letter(s) in the second column is the name of case, the letter after comma denotes the transparent (T) or absorbent (D) medium. Here I is an isotropic medium, A₁, A₂ are anisotropic media of the first and second type, G is the gyrotropic medium. E = 0 due to the choice of the coordinate system. Symbol ± means that one of the two signs is used.

| N | Case          | Zero val. | Nonzero val. | Pol. states of normal waves |
|---|---------------|-----------|--------------|-----------------------------|
| 1 | I, T          | A, B, C, D, F | -            | Elliptically degenerated    |
| 2 | A₁, T         | B, C, D, F  | A            | X₃ = ±1, Y₃ = −X₃           |
| 3 | A₁, D         | C, D, F     | A, B         | X₃ = ±1, Y₃ = −X₃           |
| 4 | A₂, D         | C, D        | A, B, F      | Y₂ = X₂ ≠ 0, Y₁ = −X₁, Y₃ = −X₃ |
| 5 | G, T          | A, B, C, F  | D            | X₂ = ±1, Y₂ = −X₂           |
| 6 | G, D          | A, B, F     | C, D         | X₂ = ±1, Y₂ = −X₂           |
| 7 | G + A₁, T     | B, C, F     | A, D         | X₁ = Y₁ = 0, Y₂ = −X₂ ≠ 0, Y₃ = −X₃ |
| 8 | G + A₁, D     | F           | A, B, C, D   | Y₂ = −X₂ ≠ 0, Y₁ = X₁, Y₃ = −X₃ |
| 9 | G + A₂, D     | -           | A, B, C, D, F | |X₂| ≠ |Y₂|, X₁² + X₃² ≠ Y₁² + Y₃² |

Table 1: The different cases of propagation of photons in media.

The case of anisotropic medium of the second type take a place only for absorbed media. This case was considered in different media like single crystals [10], plasma [2], laser wave [15]. In this case the normal electromagnetic waves are elliptically polarized. It is interesting that X₂ = Y₂ for this waves. The initially unpolarized propagating in this medium photon beam becomes linearly and circularly polarized. The simplest sample of the anisotropic medium of the second type for high energy photons is a dichromatic laser wave. (The dichromatic wave is a superposition of the two linearly polarized laser waves with different frequencies moving in the same direction and, broadly speaking, nonzero angle direction of polarization of these wave.)

The case of isotropic medium is described by the following relations: A = 0, B = 0, C = 0, D = 0, F = 0. It is obviously that the initial polarization state of photon beam is conserved at its propagation in the isotropic medium.

Now we consider the medium with the nonzero gyration vector. There are five cases of propagation of the photon beams. The first and second cases are the pure (transparent and absorbent, correspondingly) gyrotropic medium: A = 0, B = 0, E = 0, F = 0 and one or both of the values C, D are not equal to zero. The normal electromagnetic waves are circularly polarized (X₂ = −Y₂). The Faraday rotation take a place if C = 0. The circularly polarized monochromatic wave is the sample of such a medium (C = 0, D ≠ 0 or both the values are not equal to zero) for propagating γ-beam with energy below (above) of threshold of pair production.

The third and fourth cases are the gyrotropic medium with anisotropy of the first type (B = 0, C = 0, F = 0 for transparent and F = 0 for absorbent media). These cases are described with the help of Eqs. (8)-(10) and Eqs. (1)-(12) for transparent and absorbent media, correspondingly. The elliptically polarized monochromatic laser wave is the sample of such a medium.

The last case is the gyrotropic medium with the anisotropy of the second type. The
normal electromagnetic waves for this medium have the different absolute values of circular and linear polarizations ($|X_2| \neq |Y_2|, X_1^2 + X_3^2 \neq Y_1^2 + Y_3^2$). The sample of this medium is bichromatic combination of the two elliptically polarized waves moving in the same direction with the arbitrary angle between the semiaxis of polarization ellipses of these laser waves (for propagating $\gamma$-beam above the threshold of pair production).

The considered in this section cases of propagation are represented in table 1. In this table we assume conventionally that medium is transparent (T) independently of the quantity of $G$-value. It allows to avoid the consideration of additional unimportant cases.

Note, that the problems of C, P, T symmetries at propagation of the photon beam were investigated in [24].

4 Discussion

Eqs. (31)-(34) describe all the cases of propagation of photons in uniform (the components of $\eta_{\alpha\beta}$-tensor are constant along the direction of photon beam motion at fixed $\omega$) media. These differential equations of the first order have enough a simple form and dependent on differences of the corresponding components of $\eta_{\alpha\beta}$-tensor. Besides, the equations were obtained at condition that $|\eta_{\alpha\beta} - \delta_{\alpha\beta}| \ll 1$. One can find the some solutions of Eqs. (31)-(34) depending on a kind of $\eta_{\alpha\beta}$-tensor [10, 14]. We do not know universal solution of these equations (the first integrals) describing all the cases of propagation.

Eqs. (31)-(34) one can use for calculations of the propagation of photon beams in not uniform media, when the components of the $\eta_{\alpha\beta}$-tensor are functions of coordinate along direction of the photon wave vector. The condition, when this approximation is warrant, is smallness of dimensionless parameters $c/(R\omega)$ relative to unit, where $R$ is the distance of constancy of tensor components. In other words, these components should be fixed on the distances of photon wave length.

Note, that in paper [13] the process of propagation of the high energy $\gamma$-quanta in the monochromatic laser wave of arbitrary polarization was considered. For this purpose the well known scattering amplitudes for elastic scattering light by light [16] were used. As a result the differential equations similar in form to Eqs. (22)-(25) were obtained. Comparison of Eqs. (22)-(25) and similar ones in [13] allows to make conclusion that in this case the components of $\eta_{\alpha\beta}$-tensor are linear combinations of the invariant helicity amplitudes for the forward light by light scattering.

The propagating in an medium photon beam can be represented as a superposition of two normal electromagnetic waves with the different refractive indices. Because of this, in absorbent media one normal wave is absorbed to greater extent than other and after propagation of some distance only this wave would then be left behind.

On the other hand, every normal wave propagates in such a way that its polarization state is conserved. Setting $d\xi_1/dx = 0$, $d\xi_2/dx = 0$, $d\xi_3/dx = 0$ in Eqs. (31)-(34) we can find the Stokes parameters of these waves (see Eqs. (3)-(6)). However, in the case of transparent medium the equations have infinite set of solutions and the third equation is $\xi_1^2 + \xi_2^2 + \xi_3^2 = 1$.

Eq. (37) is the consequence of Eqs. (31)-(34). The total polarization $|P|$ is decreased or increased in the relation of sign of the right side of Eq. (37). In the case, when $B$, $C$, $F$ are
equal to zero $P = \text{const}$. On the other hand, it is easy to get for the absorbent medium:

$$P^2(x) = 1 - \frac{(1 - P_0^2) \exp(2G\omega x/c)}{J^2_\gamma(x)},$$

(41)

where $P_0 = P(0)$ is the initial total polarization. Note, that $J_\gamma(0) = 1$. Besides, the value $-G\omega/c$ is the inverse mean path of the unpolarized photons in the medium and may be measured in principle. The obtained simple relation is universal and may be useful for experimental determination of photon beam polarization.

We have considered the propagation of monochromatic photons in media. The absorption of photon means one of the two possibilities: a) scattering; b) vanishing. Compton effect and pair production are the samples of these possibilities for $\gamma$-quanta. As a result, the cascade process take a place in media. As a rule for description of cascade processes the kinetic equations are used [25]. The kernels of these equations are the differential probabilities (in particular, per unit length) of the possible elementary processes. From this point of view Eqs. (31)-(34) may be useful in similar calculations.

We understand that our classification (see Table 1) is a matter of convention to a degree. For instance, cases 5 and 6 from table 1 at very small $\mathcal{C}$-value give practically the same description of linearly polarized photon-beam propagation on short distances. However, even so, photon beam obtains a small value of circular polarization (in case 6). Note, that on infinity this beam becomes completely circularly polarized, whereas in case 5 beam remains always linearly polarized.

Besides, case 4 at $A = \pm F$ and $B = 0$ may be considered as special one. Under this condition the refractive indices of normal waves are equal in between and $X_2 = Y_2$ (see for detail [2]). However, we interpret its as limit of case 4.

It should be noted that the analytical solutions of Eqs. (31)-(34) are exist and their full number is less than the number of cases in Table 1. The universal solution for cases 1-3, 5-7 one can found in the appendix of paper [14]. The propagation in anisotropic medium of the second type (case 4) is considered in [10, 15]. Eqs. (9)-(12) describe the propagation process in the gyrotropic medium with the anisotropy of the first type.

5 Conclusions

The derived equations for intensity and Stokes parameters of the monochromatic photons propagating in an uniform medium, whose optical properties may be described by the permittivity tensor, is a central result. The equations have the simple form and the variation of the Stokes parameters is determined only by the differences of corresponding components of the inverse $\eta_{\alpha\beta}$ of the permittivity tensor. It is shown that the equations may be used under certain conditions for calculations of the photon beam propagation in not uniform media.

On the basic of these equations the classification of different cases of photon propagation are suggested.

The problem of generalization of Eqs. (31)-(34) for arbitrary quantities of permittivity tensor components is of interest and requires further consideration.

Our results may be useful for investigations in astrophysics, and in some areas of high energy and nuclear physics, and physics of condensed media.
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1 In English version is the translation error: in Eqs(21), (22), (26) instead the function $arctan(x)$ must be $arccot(x)$ In Russian version the function $arccot(x)$ is denoted by $arcctg(x)$ ($arccot(x) \equiv arcctg(x)$).
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