Wind load effect in topology optimization problems

R Zakhama\textsuperscript{1,2,3}, M M Abdalla\textsuperscript{1}, Z Gürdal\textsuperscript{1} and H Smaoui\textsuperscript{2,3}

\textsuperscript{1} Aerospace Structures, Technical University of Delft, Delft Kluyverweg 1 2629, The Netherlands
\textsuperscript{2} Department of Civil Engineering, National School of Engineering at Tunis, Belvédère 37 1002, Tunisia
\textsuperscript{3} LASMAP, Polytechnic School of Tunisia, La marsa 2078, Tunisia

E-mail: r.zakhama@tudelft.nl

Abstract. Topology optimization of two dimensional structures subject to dead and wind loading is considered. The wind loading is introduced into the formulation by using standard expressions for the drag force. A design problem formulation is constructed for minimum compliance design subject to a volume constraint and using the popular SIMP material model. The method of moving asymptotes (MMA) is used to solve the optimization problem. The MMA is modified by including line search and changing the formula for the update of asymptotes. In order to obtain black/white design, intermediate density values, which are used as design variables, are controlled by imposing an explicit constraint. Numerical examples demonstrate that the proposed formulation is successful in incorporating the effect of wind loading into the topology optimization problem.

1. Introduction

Research into the optimization of the topology of continuum structures (Bendsøe and Kikuchi [1]) is well established and continues to attract the attention of researchers and software companies in various engineering fields. However, few researches take into account the effect of design dependent loads on the topology optimization problem. This class of problems is present in different engineering domains: the fluid pressure loading on structures in hydrostatic problems, snow loading in civil engineering structures, and wind and hydrodynamic loading on off shore structures. In these problems, the location, direction, and the magnitude of loading is dependent on the design of the structure and cannot be prescribed a priori.

In one of the early papers on design dependent loads Hammer and Olhoff [2] considered the problem of topology optimization of elastic continuum structures under static pressure loading. The authors of this paper introduced a curve of iso-volumetric density to simulate the variable loading surface. In a follow on paper by Du and Olhoff [5, 6], the use of a parameterized iso-volumetric density curve to represent the variable loading surface was extended. The authors used a modified isoline technique to make the process more robust for the loading surface identification.

Also one of the early works in this area is the paper by Fuchs and Moses [3] where a line of action is used to prescribe the allowable moving (transmissible) load while keeping the total magnitude constant. The load magnitude of each line of action is given but its location along that line is a part of the optimization process. Later Fuchs and Shemesh [7] improved the earlier work [3] by introducing a low modulus material in fluid regions and enforcing smooth transition.
between the solid and fluid regions. The new formulation resulted in improved performance in the design of hydraulically loaded structures such as dams.

Another contribution of the design-dependent loading application for the topology optimization problem is attributed to Chen and Kikuchi [4]. In this work, the authors proposed an approach to simulate the design-dependent loads by fictitious thermal loads. The topology optimization problem is transformed from a two-phase to a three-phase material distribution problem within the design domain in which the solid, void, and hydrostatic fluid phases are optimally distributed. More recently, Sigmund and Clausen [8] suggested a new way to solve pressure load problems in topology optimization by using a mixed displacement-pressure formulation for the underlying finite element problem.

Of the main ideas described above not all of them are applicable to the wind loading problem which is addressed in this paper. For example, wind loading cannot be represented using the method of transmissible loads especially in three dimensions since no wind loading is applied when there is no surface obstructing the wind. Although the iso-volumetric surface method can be applied, it is interesting to develop a direct density based method that does not involve explicit construction of the loading surfaces.

In this paper, wind loading is introduced into the topology optimization formulation for two dimensional problems by using the standard formula for drag forces. An optimization problem is constructed as a compliance minimization problem and is solved by the modified Method of Moving Asymptotes (MMA) (Svanberg [9] 1). The lower asymptote is modified according to the continuous optimality criteria interpreted as local Kuhn-Tucker conditions (Abdalla and Gürdal [10]). Due to the non-monotonous convergence of compliance, the coefficients of the approximation of the objective function suggested by Svanberg [9] are also modified. Besides, the line search method (Zillober [11]) is used to satisfy the constraints and the decreasing of the objective function. Finally, an explicit function is introduced as a constraint to obtain a black/white design.

The rest of this paper is organized as follows. Formulation of the standard topology optimization is given in Section 2. Introduction of the wind loading in the topology optimization formulation is described in Section 3. In Section 4 sensitivity analysis formulation is presented. Modification to the MMA and the implementation of the line search in the optimization process are described in Section 5. An explicit constraint is defined in Section 6 to lead the topology to a black/white design. Numerical examples are presented in Section 7 followed by conclusions in Section 8.

2. Standard topology optimization formulation:
The standard objective of a topology optimization problem is to find the optimal material distribution within a given domain. In this paper, the topology optimization problem is posed following the commonly used minimum compliance formulation with a material volume constraint:

$$\min_{\rho} W(u^*) = F \cdot u^*$$
subject to
$$V = \int_{\Omega} \rho \, d\Omega \leq V^*$$
$$0 \leq \rho \leq 1,$$

where $W$ is the compliance of the structure which can be written either as the work done by the external forces or as twice the total elastic energy at equilibrium. $F$ is the external force vector,

1 Prof. Svanberg also provided us with the computer code which is sincerely acknowledged
and \( u^* \) is the displacement field of the domain \( \Omega \) at equilibrium. The local density distribution of material \( \rho \) of the discretized model is chosen as a design variable, where \( \rho \) is a small number set as a lower bound on \( \rho \) to avoid numerical instability. \( V^* \) is the limited volume allowed within the domain of the structure.

In order to achieve a black/white topology designs, the well recognized SIMP approach (Solid Isotropic Material with Penalization, Rozvany et al. [12]) is selected to define the local material stiffness \( E \) as,

\[
E = \overline{\rho}^p E_0, \tag{2}
\]

where \( E_0 \) is the base elasticity modulus, and \( p \) is a penalization parameter \( p \geq 3 \). In the present formulation the local density measures \( \rho_i \) of the discretized structure are associated with the nodes of the finite element mesh. The element densities \( \overline{\rho} \) are obtained by an average compliance interpolation [10] for the four nodes surrounding the element and given by,

\[
\frac{1}{\overline{\rho}^p} = \frac{1}{\rho_i^p} \tag{3}
\]

Using this scheme checkerboard patterns are suppressed automatically during the optimization process.

3. Topology design with wind loading

Wind typically blows in many directions with varying speeds. In order to model a realistic wind behavior acting on a structure, multiple load cases are needed to be taken into account in the topology optimization process. In this paper only one load case is considered; with the incoming wind being uni-directional, say from left to right (see Figure 1(a)). This assumption remains valid in three dimensions if we consider only two perpendicular loading axes.

![Wind direction and peak function](image)

**Figure 1.** Wind direction and peak function.

Wind loading is included in the formulation using a direct density based method that does not involve explicit construction of the loading surfaces. The wind load is applied to each node of the discretized domain using the standard formula for drag forces and introducing a loading function \( g \),

\[
F_C = C_D Q_\infty A g, \tag{4}
\]

where \( Q_\infty \) is the incoming wind dynamic pressure, \( C_D \) is the drag coefficient, \( A \) is the area of the element side facing the wind, and \( g \) is a peak function interpolation [13] given by,

\[
g = e^{-\frac{(1-(\rho_C-\rho_N))^2}{2 \sigma^2}}, \tag{5}
\]
where $\rho_C$ is the density of the node $C$ and $\rho_N$ is the density of the node neighbor in the direction opposite to wind direction ($\rho_N = \rho_L$ when the incoming wind is from left to right), and $\sigma$ is a standard deviation parameter. The force $F_C$ is applied at every node of the domain. However, the use of a peak function interpolation for $g$ (see Figure 1(b)) ensures that the wind loading is applied only at nodes where a large change in density occurs indicating a structural surface, and that the loading is unidirectional. The load is applied only when the node $C$ is in a solid region and the node $N$ is in void. In the present formulation, we are interested in truss like structures. Thus, we make the assumption that no wind shadowing is present.

4. Sensitivity analysis

The sensitivity of the compliance with respect to the design variables, which are the node densities in this case, is obtained from,

$$\frac{dW}{d\rho} = \frac{dF}{d\rho} u + F^T \frac{du}{d\rho}. \quad (6)$$

The sensitivity of the displacement in the above equation can be obtained by taking derivative of the equilibrium equation with respect to the node densities and it is expressed as,

$$\frac{du}{d\rho} = K^{-1} \left( \frac{dK}{d\rho} u - \frac{dF}{d\rho} \right), \quad (7)$$

where $K$ is the global stiffness matrix. Thus, the sensitivity of the compliance is given by,

$$\frac{dW}{d\rho} = 2u^T \frac{dF}{d\rho} - u^T \frac{dK}{d\rho} u. \quad (8)$$

As can be observed from equation (8), the sensitivity of the compliance is decomposed into two terms; the first term is due to the dependence of external forces on the design variables, and the second term is due to the dependence of the stiffness on the design variables. We denote these two terms by $\psi$ and $\phi$, respectively. However, external forces applied to a given structure can be decomposed into two parts, which are the static(dead) loads and the incoming wind forces. Dead loads are assumed to be independent of the structural configurations. Thus, the expression of $\psi$ for each node $C$ depends only on wind loading and simplifies to,

$$\psi_C = \frac{C_D Q_0 A}{\sigma^2} \left\{ \left[ 1 - (\rho_C - \rho_L) e^{-\frac{(1-(\rho_C - \rho_L))^2}{2 \sigma^2}} \right] u_C - \left[ 1 - (\rho_R - \rho_C) e^{-\frac{(1-(\rho_R - \rho_C))^2}{2 \sigma^2}} \right] u_R \right\}, \quad (9)$$

where the subscripts $L$ and $R$ indicate the node points to the left and right of the node point $C$ and the wind direction is assumed to be from left to right. Following the same notation, $u_C$ and $u_R$ are the horizontal displacement at the node point $C$ and its right neighbor, respectively.

5. MMA modifications and line search implementation

The optimization problem is solved by the Method of Moving Asymptotes (MMA) \cite{9}. According to \cite{9}, the compliance is approximated as,

$$W^{(k)}(\rho) = r_0^{(k)} + \sum_{j=1}^{n} \left( \frac{p_{0j}^{(k)}}{U_j^{(k)} - \rho_j} + \frac{q_{0j}^{(k)}}{\rho_j - L_j^{(k)}} \right), \quad (10)$$

where $p_{0j}^{(k)}$ and $q_{0j}^{(k)}$ are determined by a first-order approximation of the first Kuhn-Tucker condition, and $L_j^{(k)}$ and $U_j^{(k)}$ are the lower and upper asymptotes respectively.
It is observed that in some cases that the optimization process tends to oscillate when the MMA solver tries to update the low asymptotes $L_j^{(k)}$ and the upper asymptotes $U_j^{(k)}$. The convergence is stabilized by modifying the lower asymptote according to the continuous optimality criteria interpreted as local Kuhn-Tucker conditions [10]. The update of the upper asymptote is kept the same as suggested by Svanberg [9].

By using the continuous optimality criteria interpreted as local Kuhn-Tucker conditions the objective function is approximated as,

$$ W^{(k)} = \sum_{j=1}^{n} \phi_j^{(k)} \left( \rho_j^{(k)} \right)^p, $$

This approximation can be convexified by linearizing the stiffness coefficient $\rho_j^P$ around the most recent design point to obtain,

$$ W^{(k)} = \sum_{j=1}^{n} \frac{q_{0j}^{(k)}}{\rho_j^{(k)} - L_j}, $$

where $L_j = \left( 1 - \frac{1}{p} \right) \rho_j^{(k-1)}$ is the modified formula of the lower asymptote.

With the modifications discussed so far the MMA solver could not always converge monotonously and diverged in some cases, especially for strong wind loading. In order to improve the performance, the calculation of the the coefficients $p_{0j}^{(k)}$ and $q_{0j}^{(k)}$ defined in [9] is modified by considering the sensitivity contributions from the $\phi$ and $\psi$ terms independently,

$$ p_{0j}^{(k)} = \begin{cases} 
(U_j^{(k)} - \rho_j^{(k)})^2 \psi_j, & \psi_j > 0 \\
0, & \psi_j \leq 0 
\end{cases} $$

$$ q_{0j}^{(k)} = (\rho_j^{(k)} - L_j^{(k)})^2 \phi_j + \begin{cases} 
0, & \psi_j \geq 0 \\
-(\rho_j^{(k)} - L_j^{(k)})^2 \psi_j & \psi_j < 0. 
\end{cases} $$

Moreover, it is also observed that the objective function is not always decreased after an MMA step and that the constraints are violated in some cases. In an earlier paper, Zillober [11] demonstrated that by adding a line search in the optimization process, the behavior of the MMA method is stabilized. In this paper, line search is used to ensure the satisfaction of constraints and the decrease of the value of the objective function. Let the solution produced by MMA after the $k^{th}$ iteration be denoted by $\rho^*$. A solution is sought of the form,

$$ \left( \rho^{(k+1)} \right) = \left( \rho^* - \rho^{(k)} \right) \alpha + \rho^{(k)}, $$

where $\alpha \in [0,1]$ is a parameter that needs to be determined to satisfy the constraints or to assure the decrease of the objective function. In order to find the parameter $\alpha$, the constraints and the objective function are approximated along the search direction using cubic polynomials. The value of $\alpha$ is obtained by solving the one-dimensional optimization problem obtained from the original problem by using the approximate cubic polynomials for the objective and the constraints and $\alpha$ as the independent variable.

6. Explicit constraint

Preliminary results of a numerical example of an off-shore wind turbine support structure problem demonstrates that the resulting topology design to be a grey design (see Figure 2(a)) as opposed to an expected black/white one. The explanation for this problem is found to be related to the distribution of the wind loads for each node for the problem as shown in Figure.
2(b). Each column in the figure is a histogram of the magnitude of the loads on nodes along that vertical set of nodes. Note that the wind loads are distributed throughout the domain, but have significant magnitude only for a few nodes at the bottom of the structure. But, the distribution of the wind loads must be uniform on a surface facing the wind, which is not the case. In order to get black/white design, an explicit constraint (Borrvall et al [14]) was added to the problem (1) to control the intermediate densities. The explicit constraint is defined as,

$$m(\rho) = \int_{\Omega} (1 - \rho)(\rho - \rho)d\Omega \leq \epsilon_p.$$  \hspace{1cm} (16)

In its numerical implementation, we propose starting without imposing the explicit constraint (16) to find an initial (grey) design. The value of the constraint function for this design is denoted by $\epsilon_p^{(0)}$. A black/white design is forced by successively reducing $\epsilon_p$ according to the formula,

$$\epsilon_p^{(k)} = \epsilon_p^{(k-1)}(1 - \beta).$$  \hspace{1cm} (17)

The process continues until the solver fails to converge for the updated value of $\epsilon_p$.

![Figure 2](image_url)

(a) Topology with wind.  
(b) Wind load distribution for each node of the discretized domain.

**Figure 2.** Off-shore wind turbine problem without explicit constraint.

### 7. Numerical examples

In order to evaluate the influence of including wind loading in the topology optimization formulation, a numerical example modelling an off-shore wind turbine support structure is considered. The problem is modelled in the simplest way as shown in Figure 3(a). The hub and rotor weight $P_{WR}$, and the rotor thrust $F_W$ are considered as dead forces with values of 2.135 MN and 0.155 MN, respectively. A rectangular domain is selected to model the chosen problem with dimensions of 320 m in height, 32 m in width and 32 m in thickness. The structure is clamped at the bottom. In this problem, the incoming wind is considered from left to right with a wind speed of 30 m/s. The material used is steel with a Young’s modulus $E = 200000$ MPa, and Poisson ratio $\nu = 0.3$. The penalization parameter is set to 3, the volume fraction is set to be 0.5, and a lower bound $\rho = 10^{-3}$ is adopted for the density. The value of $2\sigma^2$ of the peak function is fixed at 0.1 and the value of $\beta$ to 0.05.

The results of the off-shore wind turbine problem were generated with a discretized grid of 15 x 141 nodes. The optimum designs are represented in Figure 3(b) and 3(c) for the case without and with considering wind loads, respectively. It can be observed that the topology with wind uses more elements to support the loads coming from the wind. In order to compare the compliance
between the two solutions, the design obtained considering wind loading is postprocessed to force exact black/white topology. The compliance calculated for the postprocessed design (see Figure 3(d)) is 0.347 MNm, while the compliance for the design in Figure 3(b) when both dead and wind loading are both applied is 3.144 MNm. Thus, including wind loading in the design formulation leads to 89% increase in structural stiffness for the same material volume.

Figure 4(a) shows the distribution of the wind loads for each node for the design obtained considering wind loading. It can be observed that the wind loads are uniformly distributed throughout the domain, except for the top part of the structure. This is due mainly to the strong wind loading used in this problem. Figure 4(b) shows the convergence history. The convergence is reasonably smooth with jumps in compliance corresponding to tightening tolerance on grey density. Although the introduction of the explicit constraint leads to increased number of iterations to convergence, the solution obtained is more physically meaningful.
8. Conclusions
A formulation for the inclusion of wind loading in the minimum compliance topology optimization problem has been proposed. The method does not require the explicit construction of loading surfaces. The MMA method has been modified and the line search has been added to the process to guarantee the global convergence of the topology optimization problem. An explicit constraint has been added into the topology optimization formulation to control the intermediate density values. Numerical examples demonstrate the effect of the wind loads on the optimized topologies. Taking the wind loads into account in the formulation gives stiffer designs with the same volume. Thus, the implementation of this method in the preliminary design phase can lead to significant reduction in structural weight which reflects on the total cost of off-shore wind farms especially given the rising steel prices.

References
[1] Bendsøe M and Kikuchi N 1988 Topology optimization of nonlinear structures Comput. Methods Appl. Mech. Engng. 71 197
[2] Hammer V B and Olhoff N 2000 Topology optimization of continuum structures subjected to pressure loading Struct. Multidisc. Optim. 19 85
[3] Fuchs M B and Moses E 2000 Optimal structural topologies with transmissible loads Struct. Multidisc. Optim. 19 263
[4] Chen B C and Kikuchi N 2001 Topology optimization with design-dependent loads Finite Elements in Analysis and Design 37 57
[5] Du J and Olhoff N 2004 Topological optimization of continuum structures with design-dependent surface loading - Part I: new computational approach for 2D problems Struct. Multidisc. Optim. 27 151
[6] Du J and Olhoff N 2004 Topological optimization of continuum structures with design-dependent surface loading - Part II: algorithm and examples for 3D problems Struct. Multidisc. Optim. 27 166
[7] Fuchs M B and Shemesh N N Y 2004 Density-based topological design of structures subjected to water pressure using a parametric loading surface Struct. Multidisc. Optim. 28 11
[8] Sigmund O and Clausen P M 2006 Topology optimization using a mixed formulation: An alternative way to solve pressure load problems Comput. Methods Appl. Mech. Engng. doi:10.1016/j.cma.2006.09.021
[9] Svanberg K 1987 The method of moving asymptotes - a new method for structural optimization Int. J. Num. Meth. Eng. 24 359
[10] Abdalla M M and Gürdal 2002 Structural design using optimality based cellular automata, Denver, Co AIAA-2002-1676, 43th AIAA/ASME/ AHS/ASC Structures, Structural Dynamics and material Conference 24 359
[11] Zillober C 1993 A globally convergent version of the method of moving asymptotes Struct. Optim. 6 166
[12] Rozvany G I N, Zhou M and Birker T 1992 Generalized shape optimization without homogenization Struct. Optim. 4 257
[13] Yin L and Ananthasuresh G K 2001 Topology optimization of compliant mechanisms with multiple materials using a peak function material interpolation scheme Struct. Multidisc. Optim. 23 49
[14] Borrvall T and Petersson J 2001 Topology optimization using regularized intermediate density control Comput. Methods Appl. Mech. Engng. 190 4911