Study on the Influence of the Eigen Factor on Slip Effects of Composite Beams

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Abstract. In service time, the slip-induced deflection must be considered. It is influenced by many factors. Most of them can be combined by the eigen factor, which represents the property of structure itself, including parameters of cross-section and member. The classical differential equation is used to make a parameter analysis. The results show that in the range of practice, the influences of the eigen factor are critical and nonlinear, and the stiffness reduction ratio in buildings is large, about 0.8 in the example.

1. Introduction
It is important to consider the interface slip in the deflection calculation of composite beams [1-7]. It is dependent on many factors like loading, boundary condition, material, shape and size of section. Among them, the eigen factor ωL (a dimensionless quantity) combining the main properties of cross-section and member arises the most interest of research [8-10]. By the exact solutions, the influences of ωL on the slip effects of composite beams in buildings will be discussed in the following text.

2. Definition of the eigen factor
The eigen factor ωL is the product of the length of beam (L) and parameter ω (in Equation 1), the unit of ω is m⁻¹, and therefore ωL is dimensionless. The equation is the classical second-order differential equation [11] with the internal axial force (Ni) as the independent variable, for composite beams subjected to the pure bending moment (M).

\[ N_i'' - \omega^2 N_i = -\frac{kd}{E_c I_c + E_s I_s} M, \quad \omega^2 = k \left( \frac{d^2}{E_c I_c + E_s I_s} + \frac{1}{E_c A_c} + \frac{1}{E_s A_s} \right) \]  (1)

where k is the shear stiffness of interface, d is the distance between the center of gravity of the two individual sections (concrete slab and steel beam), E, A and I are the elastic modular, axial and flexural stiffness respectively, the subscribe "c" and "s" denote "concrete" and "steel" respectively.
The coefficient $\beta$ is introduced to indicate the composite action between the two individual sections, 

$$\beta = \frac{EAd^2}{EI}$$

where $EA$ is the axial series stiffness of the two individual sections, 

$$\frac{1}{EA} = \frac{1}{E_A} + \frac{1}{E_A}$$

and $EI$ is the flexural stiffness in the case of no connector that is 

$$EI = E_1I_1 + E_sI_s.$$ 

Then the eigen factor can be rewritten as 

$$\omega L = \sqrt{\left(1 + \beta\right) \frac{kL^2}{EA}}$$

The magnitude of $\omega L$ is between 0 and $\infty$ in the case of elastic connector, because interface stiffness $k$ is equal to 0 and $\infty$ in the cases of no and rigid connector respectively. Nevertheless, in practice, the actual magnitude of $\omega L$ is within a small range (like 3.82–10.82 in Table 1).

It is worth mentioning that $\omega L$ is a synthetic factor, it has no relationship with loading and boundary condition, and only relates with properties of structure itself, like material, length of beam, size and shape of section. In addition, the main factor of $\omega L$ is $k$ because the other factors should always obey some other their own design routines, like the span-depth ratio requirement. And, $k$ is mainly dependent on the shear stiffness and spacing of connector.

3. Parameter analysis

The section and deflection of a simply-supported composite beam subjected to the uniformly distributed load is shown in Figure 1, where the stiffness of interface is assumed as the independent quantity, which results the change of $\omega L$.

Figure 1 shows it is almost equal to the condition of no and rigid shear connector for $\omega L = 0.2$ and 35 respectively. The influences of $\omega L$ on the deflection is nonlinear and it is more significant for small $\omega L$. The trend will be more clear if the deflection and internal axial force in mid-span are calculated as shown in Figure 2.
Deflection and internal axial force in mid-span

Figure 2 shows that the influences are almost bilinear. The behavior of slip effects is almost close to that of rigid connector after $\omega L$ reaches 20. In other words, it is useless for slip control by increasing the shear stiffness of connector or decreasing the spacing between connectors after the limiting value is reached. In practice, the range of $\omega L$ is normally from 5 to 8 [10], therefore $\omega L$ is the critical factor of slip effects because the slope of the line is very large in this range.

Then considering the condition of the elastic to the rigid shear connector, the deflection magnification and effective stiffness deduction are calculated in Figure 3 and they are reciprocal. It can be observed that the ratios are inconstant and large, which means that slip effects must be considered in the service-time design.

In addition, the virtual effective stiffness method is often used for the convenience that it represents the equivalent stiffness after the consideration of interface slip. If the effective stiffness can be obtained, the slip effects can be directly calculated instead of the complicate solving procedure of the differential equation. The effective stiffness is reversely obtained.

Therefore, the effective stiffness and its composition of the composite beam in mid-span are analysed and the results are shown in Table 1, which also includes the two idea states (rigid and no shear connector). The subscribe "eff" denotes "effective".

Table 1. Bending stiffness composition in mid-span.

| Spacing (cm) | $\omega L$ | $EI + EA_{\text{eff}} d^2$ | $EA_{\text{eff}} d^2$ | $\beta_{\text{eff}}$ | $EA_{\text{eff}}$ | $EI + EA_{\text{eff}}^2$ | Ratio $\frac{1}{\beta_{\text{eff}}} / \frac{1}{\beta_{\text{eff}}}$ |
|--------------|------------|-----------------------------|------------------------|----------------------|-------------------|--------------------------|--------------------------------------------------|
| No           | 0          | 2.28                        | 0                      | 0                    | 0                 | 2.28                     | /                                                |
In table 1, the single line uniform layout for headed stud connectors is assumed with the shear stiffness 6700N/cm. The longitudinal centre-to-centre spacing of shear connectors is given from 10 to 80cm according to Eurocode 4 [12].

The results show that the shear connector plays an important role for the increase of the effective stiffness is close to the double. For connector spacing in buildings like 20cm, the slip-induced reduction of stiffness is about 0.8, and the reduction may reach 0.6 for large spacing like 70cm. It is considerable and the reduction is significantly associated with the spacing of shear connectors.

4. Conclusions
The dimensionless quantity (the eigen factor $\omega L$) is the combination of the structural properties of composite beams. It is the critical factor for slip calculation. In practice, the range of $\omega L$ is not large. Slip effects can be considered by the virtual equivalent slip-transformed stiffness (effective stiffness), which is easily understood by structural designers. The effective stiffness is reduced by slip significantly, like 0.8 in this paper. For more general conditions of step changes spacing of shear connectors, the reduction must be further studied.

Acknowledgments
This research was financially supported by National Natural Science Foundation of China (Grant No. 51868034), Yunnan Provincial Department of Education Science Research Fund (Grant No. 2017ZZX089) and Research Projects of Kunming University (Grant No. YJL16010).

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