Coolsing a nanomechanical resonator by a triple quantum dot

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received 13 May 2011; accepted in final form 7 July 2011
published online 3 August 2011

PACS 03.65.Ta – Foundations of quantum mechanics; measurement theory
PACS 42.50.Gy – Effects of atomic coherence on propagation, absorption, and amplification of light; electromagnetically induced transparency and absorption
PACS 73.21.La – Quantum dots

Abstract – We propose an approach for achieving ground-state cooling of a nanomechanical resonator (NAMR) capacitively coupled to a triple quantum dot (TQD). This TQD is an electronic analog of a three-level atom in Λ configuration which allows an electron to enter it via lower-energy states and to exit only from a higher-energy state. By tuning the degeneracy of the two lower-energy states in the TQD, an electron can be trapped in a dark state caused by destructive quantum interference between the two tunneling pathways to the higher-energy state. Therefore, ground-state cooling of an NAMR can be achieved when electrons absorb readily and repeatedly energy quanta from the NAMR for excitations.

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Introduction. – Nanomechanical resonators (NAMRs) with high resonance frequencies and large quality factors are currently attracting considerable attention owing to their wide range of potential applications (see, e.g., [1,2]). Moreover, quantized NAMRs are potentially useful for quantum information processing. For example, quantized motion of buckling nanoscale bars has been proposed for qubit implementation [3] and also for creation of quantum entanglement [4–6]. However, the dynamics of the NAMRs must approach the quantum regime, which is usually difficult to arrive at due to interactions with other components as well as the environments.

One way to achieve the quantum regime for an NAMR is to increase the resonance frequency so that an energy quantum of the NAMR is larger than the thermal energy. Recently, NAMRs based on metallic beams [7] and carbon nanotubes [8] with resonance frequencies about several hundred megahertz have been developed. However, a temperature lower than 10 mK (below the typical dilution refrigerator temperature) is required for an NAMR with a frequency of 200 MHz to operate in the quantum regime. Therefore, one still needs to cool the NAMR further, for instance via coupling to an optical or an on-chip electronic system. Various experiments on the cooling of a single NAMR via radiation pressure or dynamical backaction have been reported (see, e.g., [9–15]). Other cooling mechanisms based on a Cooper pair box [16] or a three-level flux qubit [17] with periodic resonant coupling have also been theoretically proposed. In these approaches, a strong resonant coupling between the NAMR and the qubit is required for the NAMR to achieve its ground-state cooling.

In the weak coupling regime, a conventional method for cooling an NAMR is the sideband cooling approach (see, e.g., refs. [18–23]), where an NAMR is usually coupled to a two-level system (TLS) with the two states being electronic states in quantum dots [18–20], photonic states in a cavity [21–23], or charge states in superconducting circuits [24]. In order to achieve ground-state cooling, the resolved-sideband cooling condition $\omega_m \gg \Gamma$ (with $\omega_m$ denoting the oscillating frequency of the NAMR and $\Gamma$ the decay rate of the TLS) must be followed to selectivity drive the lowest sideband of the TLS. However, the frequency of a typical NAMR is about 100 MHz [7,8] which is in general of the same order of the decay rate of the two-level system, indicating that the resolved-sideband cooling condition is not easy to be fully fulfilled.

In this letter, we propose a different approach to cool an NAMR, in the non–resolved-sideband cooling regime, via quantum interference in a capacitively coupled triple quantum dot (TQD) (see fig. 1(a)). Here we focus on the strong Coulomb-blockade regime with at most one electron being allowed in the TQD at one time. The TQD acts.
The unperturbed Hamiltonian $H_0$ is defined as

$$H_0 = H_{\text{TQD}} + H_{\text{leads}} + H_R + H_{\text{ph}},$$

with

$$H_{\text{TQD}} = -\Delta_1 a_1^+ a_1 - \Delta_2 a_2^+ a_2 + (\Omega_1 a_1^+ a_3 + \Omega_2 a_2^+ a_3 + \text{H.c.}),$$

$$H_{\text{leads}} = \sum_{i,k} E_{ik} c_{ik}^+ c_{ik},$$

$$H_R = \omega_m b_i^+ b_i,$$

$$H_{\text{ph}} = \sum_q \omega_q b_q^+ b_q$$

are Hamiltonians of the TQD, the electrodes, the NAMR and the thermal bath, respectively. We have put $\hbar = 1$.

The energy of state $|3\rangle$ is chosen as the zero-energy point and $-\Delta_{1(2)}$ is the energy of state $|1\rangle(2)$ relative to state $|3\rangle$. $a_i^+$ creates an electron in the $i$-th dot ($i = 1, 2, 3$) and $c_{ik}^+$ ($c_{ik}$) is the creation (annihilation) operator of an electron with momentum $k$ in the $i$-th electrode. The phonon operators $b_i^+$ and $b_i$ create and annihilate an excitation of frequency $\omega_m$ in the NAMR, respectively. The thermal bath is modeled as a bosonic bath with $b_q^+$ ($b_q$) being the bosonic creation (annihilation) operator at frequency $\omega_q$.

The electromagnetic coupling between the NAMR and dots 1 and 3 of the TQD is given by

$$H_{\text{int}} = -(g_3 a_3^+ a_3 + g_1 a_1^+ a_1)(b_i^+ + b_i),$$

with a coupling strength $g_i = \eta_i \omega_m$ ($i = 1, 3$). For simplicity we consider $g_3 = g_1 = g$ (or $\eta_3 = \eta_1 = \eta$). For a typical electromechanical coupling, $\eta \sim 0.1$ (see, e.g., ref. [31]).

The tunneling coupling between the TQD and the electrodes is

$$H_T = \sum_{i,k} (\Omega_{ik} a_i^+ c_{ik} + \text{H.c.}),$$

where $\Omega_{ik}$ characterizes the coupling strength between the $i$-th dot and the associated electrode via a tunnel barrier. Moreover, the coupling of the NAMR to the outside thermal bath is characterized by

$$H_{\text{ep}} = \sum_q \Omega_q (b_q^+ b_q + \text{H.c.}),$$

with $\Omega_q$ being the coupling strength.

**Quantum dynamics of the triple quantum dot.**

**Analogy between a TQD and a Λ-type three-level atom.**

We now show that in the absence of the NAMR, the TQD system is analogous to a typical Λ-type three-level atom in the presence of two classical electromagnetic fields (see fig. 1(b)). The Hamiltonian of the field-driven Λ-type three-level system can be written as $H_\Lambda = \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2 + \omega_3 a_3^+ a_3 + \Omega_0 \cos(\omega t) (a_1^+ a_3 + a_2^+ a_1) + \Omega_0 \cos(\omega t) \times (a_3^+ a_3 + a_2^+ a_2)$, where $\omega_i$ ($i = 1, 2$ or 3) is the energy of the $i$-th state in the three-level system; $\omega_a$ and $\omega_b$ are the
frequencies of the two driving fields with $\Omega_a$ and $\Omega_b$ being the corresponding driving strengths. In order to eliminate the corresponding driving strengths. In order to eliminate the time dependence of the Hamiltonian, we transform the system into a rotating frame defined by $U_R = e^{i\mathcal{H}t}$ with $R = \omega_a a_1^+ a_1 + \omega_b a_2^+ a_2 - \omega_3 (a_1^+ a_2 + a_1 a_2^+)$. Within the rotating-wave approximation, the Hamiltonian becomes

$$
\hat{H}_A = -\Delta_1 a_1^+ a_1 - \Delta_2 a_2^+ a_2 + \Omega_1 (a_1^+ a_3 + a_3^+ a_1) + \Omega_2 (a_2^+ a_3 + a_3^+ a_2),
$$

where $\Delta_1 = \omega_3 - \omega_1 - \omega_3$ and $\Delta_2 = \omega_3 - \omega_2 - \omega_3$ are the frequency detunings while $\Omega_1 = \Omega_{a2}/2$ and $\Omega_2 = \Omega_{b2}/2$ are the effective driving strengths of the two fields. It is now clear that the Hamiltonian in eq. (9) is formally equivalent to that of the TQD described above (i.e., eq. (2)).

**Dark state in the TQD.** The existence of a dark state in a $\Lambda$-type three-level atom in quantum optics is able to suppress absorption or emission when the lower-energy states become degenerate, i.e., $\Delta_1 = \Delta_2$. We demonstrate below that a similar dark state also exists in the TQD and can be used for the ground-state cooling of the NAMR.

After tracing over the degrees of freedom of the electrodes, quantum dynamics of the TQD in the absence of the NAMR is described by

$$
\dot{\rho}_d = -i[H_{\text{TQD}}, \rho_d] + \Gamma_d \rho_d + \gamma \rho_d,
$$

where $\rho_d$ is the reduced density matrix of the TQD and $\Gamma_d$ is the rate for electrons tunneling into or out of the $i$-th dot. The notation $\mathcal{D}$ for any operator $A$ is given by $\mathcal{D}[A] \rho = A \rho A^\dagger - \frac{1}{2} (A^\dagger A \rho + \rho A^\dagger A)$.

**Quantum dynamics of coupled NAMR-TQD system.** Rather than analyzing directly the energy exchange between the NAMR and the TQD which involves tedious algebra, we apply a canonical transform $U = e^{iS}$ on the whole system, where $S = \eta (a_3^+ a_3 + a_1^+ a_1) (b - b^\dagger)$. The transformed Hamiltonian is given by

$$
H = U H_{\text{total}} U^\dagger
= H_{\text{leads}} + H_{pb} + \omega_m b^\dagger b - \Delta_1 a_1^+ a_1 - \Delta_2 a_2^+ a_2 + \Omega_1 (a_1^+ a_1 + a_2^+ a_2) \Omega_2 (a_1 a_1^+ + a_2 a_2^+) + \frac{\Omega_3}{2} (a_1^+ a_1 + a_2^+ a_2) B + \text{H.c.}
+ \sum_q \{ \Omega_q b_q^\dagger (b + \eta (a_3 a_3 + a_1 a_1)) + \text{H.c.} \}
+ \sum_k [\Omega_{1k} a_1^+ c_k B^\dagger + \Omega_{2k} a_2^+ c_k B^\dagger + \text{H.c.}],
$$

where we have neglected a small level shift of $\eta^2 \omega_m$ to the states $|1\rangle$ and $|3\rangle$ and defined $B = \exp[-\eta (b - b^\dagger)]$. To describe the quantum dynamics of the coupled NAMR-TQD system, we have derived a master equation (under the Born-Markov approximation) by tracing over the degrees of freedom of both the electrodes and the thermal bath. Up to second order in $\eta$, the master equation can be written as

$$
\frac{d\rho}{dt} = -i[\omega_m (b^\dagger b), \rho] - i[H_{\text{TQD}}, \rho] - i[V (b^\dagger - b), \rho]
- i[V^\dagger (b - b^\dagger)^2, \rho] + \mathbb{L}_T \rho + \mathbb{L}_D \rho,
$$

where $\mathbb{L}_T$ and $\mathbb{L}_D$ denote the Lindblad operators for the system and bath, respectively.
where

\[ V = \eta \Omega_2 (a_1 \rho a_2 - a_2 \rho a_1), \quad V' = \frac{\eta}{2} \Omega_2 (a_1 \rho a_3 + a_3 \rho a_1), \]

\[ \mathcal{L}_T \rho = \mathcal{L}^{(1)}_T \rho + \mathcal{L}^{(2)}_T \rho, \]

\[ \mathcal{L}^{(1)}_T \rho = \Gamma D[a_1 b] \rho + \Gamma D[a_2 b] \rho + \Gamma D[a_3 b] \rho, \]

\[ \mathcal{L}^{(2)}_T \rho = -\eta^2 \Gamma D[a_1 b] \rho - \eta^2 \Gamma D[a_2 b] \rho + \eta^2 \Gamma D[a_3 b] \rho \]

\[ + \frac{\eta}{2} \int \left( [D(a_1 b)] \rho + [D(a_3 b)] \rho + [D(a_2 b)] \rho \right) \]

\[ + \eta^2 \Gamma(a_1 \rho b + \rho b a_1 + a_1 \rho b a_1 - a_1 \rho b a_1 + a_1 b a_1 + a_1 b a_3 + \rho a_1 b a_3 \]

\[ - a_1 \rho b a_3 - a_3 \rho a_1 b - b_1 \rho a_2 a_1 \rho a_1], \]

\[ \mathcal{L}_D \rho = \gamma_n (\omega_m + 1) [D(b) \rho + \gamma_n (\omega_m) D[b] \rho]. \]

Here \( \mathcal{L}_T \) and \( \mathcal{L}_D \) are Liouvillian operators: \( \mathcal{L}_T \rho \) accounts for the dissipation due to the electrodes and \( \mathcal{L}_D \rho \) the dissipation in the NAMR induced by the thermal bath. Also, \( \gamma \) denotes the decay rate of excitations in the NAMR induced by the thermal bath and \( n(\omega_m) \) is the average boson number at frequency \( \omega_m \) in the thermal bath. Usually, \( \gamma \ll \Gamma \) for an NAMR, so \( \mathcal{L}_D \rho \) is a higher-order term.

**Quantum dynamics of the NAMR.** In order to extract the dynamics of NAMR, we trace out the degrees of freedom of the thermal bath, which can be regarded as part of the environment experienced by the NAMR. This is achieved through adiabatical elimination [21,32] in the limit \( \gamma \ll g \ll \omega_m \), corresponding to a weak coupling of the NAMR with the TQD. At zeroth order in \( \eta \) in which NAMR and TQD are decoupled, the dynamics of the whole system is governed by

\[ \frac{d \rho}{dt} = \mathcal{L}_0 \rho, \quad (15) \]

where the Liouvillian operator \( \mathcal{L}_0 \) is defined by \( \mathcal{L}_0 \rho = -i \omega_m [b^\dagger b, \rho] - i [H_{\text{TQD}}, \rho] + \mathcal{L}^{(1)}_T \rho \). The steady-state solution to eq. (15) can be expanded [27,32] in the basis of \( \rho_{nm} = |n\rangle \langle n| \) \( \otimes \rho_{ss} \), with \( |n\rangle \) being eigenstates of the NAMR Hamiltonian (i.e., \( H_R |n\rangle = n \omega_m |n\rangle \)) and \( \rho_{ss} \) the density operator \( \rho_D \) of the TQD in the steady state (which is the solution to eq. (10) when \( d \rho_D / dt = 0 \)). Here \( \rho_{nm} \) are the eigenvectors of \( \mathcal{L}_0 \): \( \mathcal{L}_0 |n\rangle \langle n| \otimes \rho_{ss} = 0 \) \( (n, n' = 0, 1, \ldots) \). The eigenvalues \( \lambda_{nm} = -i(n - n') \omega_m \). In particular, for the zero eigenvalue \( \lambda_0 = 0 \) \( \rho_0 \), the eigenvectors are given by \( \mathcal{L}_0 |n\rangle \langle n| \otimes \rho_{ss} = 0 \) \( (n = 0, 1, \ldots)\), which are infinitesimally degenerate. The zeroth-order Liouvillian eigenstates with \( \lambda_0 = 0 \) are connected by the operators \( \mathcal{L}_1 \) \( (\mathcal{L}_T \rho = -i[V(b \rho - b^\dagger \rho)] + \mathcal{L}^{(2)}_T \rho + \mathcal{L}_D \rho) \) to the subspace associated with non-zero eigenvalues \( \lambda_k \neq 0 \) \( (k = 1, 2, \ldots) \). This is due to the coupling between the NAMR and TQD. In the regime \( g \ll \omega_m \) \( (i.e., n \ll 1) \), such coupling is weak and can be analyzed using perturbation theory and adiabatical elimination. We define a projection operator \( \mathcal{P} \) on the subspace with zero eigenvalue \( \lambda_0 = 0 \) of \( \mathcal{L}_0 \) according to

\[ \mathcal{P} \rho = \mathcal{P}^{\text{NAMR}} \otimes \mathcal{P}^{\text{TQD}}, \quad (16) \]

where

\[ \mathcal{P}^{\text{NAMR}} X \equiv \sum_n |n\rangle \langle n| X |n\rangle, \]

\[ \mathcal{P}^{\text{TQD}} X \equiv \rho_{ss} T_{\text{TQD}} X. \]

Projection of the above master equation (13) gives the master equation for the reduced density matrix \( \mu \) of the NAMR, up to second order in \( \eta \),

\[ \mu = -i(\omega_m + \delta_m)[b^\dagger b, \mu] + \frac{1}{2} [\gamma_n (\omega_m + 1) + A_-(\omega_m)] \]

\[ \times [2b^\dagger b^\mu - (b^\dagger b \mu + b b^\mu)] + \frac{1}{2} [\gamma_n (\omega_m) + A_+(\omega_m))[2b^\dagger \mu b - (b^\dagger b \mu + b b^\mu)]] \]

(18)

Here \( \delta_m \) is the driving-induced shift of the NAMR frequency given by \( \delta_m = \text{Im} \{G(\omega_m) + G(-i \omega_m)\} \)

\[ G(s) = -\langle \dot{V}(s) V(0) \rangle \]

in eq. (18), \( A_+ \) and \( A_- \) are induced by the coupling to the TQD and are given by \( A_{\pm} = 2 \text{Re} \{G(\pm i \omega_m)\} + \eta^2 \Gamma (\rho_{55}^{\pm} + \rho_{33}^{\pm}) \)

\( \rho_{55}^{\pm} \) are the steady-state probabilities of the states \( |0\rangle \) (empty TQD) and \( |3\rangle \) (single electron in dot 3), respectively.

**Average phonon number.** From master equation (18), the evolution equation for the phonon number probability distribution \( p_n = \langle n|\mu|n\rangle \) of the NAMR is obtained as

\[ \frac{d p_n}{dt} = \{\gamma n (\omega_m + 1) + A_- \}[(n + 1)p_{n + 1} - np_n] \]

\[ + [\gamma n (\omega_m) + A_+][np_{n - 1} - (n + 1)p_n]. \quad (19) \]

Hence the evolution of the average phonon number, \( \langle n \rangle = \sum_n np_n \), in the NAMR is described by

\[ \frac{d \langle n \rangle}{dt} = - (\gamma + W) \langle n \rangle + \gamma n (\omega_m) + A_+, \quad (20) \]

where \( W = A_+ - A_- \). In order to cool the NAMR, one needs \( W > 0 \) \( (i.e., A_+ > A_-) \). Consequently, the steady-state average phonon number in the NAMR is

\[ n_{st} = \frac{\gamma n (\omega_m) + A_+}{\gamma + W}. \quad (21) \]

Here the term \( \gamma n (\omega_m) \) in the numerator is due to the thermal bath while \( A_+ \) results from the scattering processes by the TQD. We assume that the NAMR is initially at equilibrium with the thermal bath, so that its phonon number is initially \( n(\omega_m) \). In order to cool down the NAMR significantly, \( i.e., n_{st} \ll n(\omega_m) \), one needs a large cooling rate \( W \gg \gamma \). This indeed can be achieved using typical experimental parameters and will be shown below.

The transition rates \( A_\pm \) under the condition \( \Delta_1 = \Delta_2 = \Delta \) appropriate for the dark state are found to be

\[ A_{\pm} = \frac{4 \eta^2 \Omega_2^2 \Omega_2^2}{\Omega^2} \frac{\omega_m^2 \Gamma}{4[\Omega^2 - \omega_m (\omega_m + \Delta)]^2 + \omega_m^2 \Gamma^2} + \eta^2 \Gamma (\rho_{55}^{at} + \rho_{33}^{at}). \quad (22) \]
To cool the NAMR, one needs $A_- > A_+$, which is fulfilled either when $\Delta > 0$ and $\Omega < \omega_m$, or when $\Delta < 0$ and $\Omega > \omega_m$. Assuming also $W \gg \gamma$, the steady-state average phonon number in the NAMR is approximately given by $n_{st} \approx \gamma n(\omega_m)/W + n_f$, where

$$n_f \equiv \frac{A_+}{W} = 4\left(\frac{\omega_m}{\Delta} + \frac{\omega_m^2}{\omega_m^2 - \omega_0^2} \right)^2 \frac{\omega_m^2 \Gamma^2}{16\Delta \omega_m^2 (\omega_m^2 - \omega_0^2)}.$$  

(23)

It is easy to see that $n_f$ reaches a minimum $n_f^{\text{min}} = (\Gamma/4\Delta)^2$, when the term inside the square brackets in the r.h.s. of eq. (23) becomes zero, i.e.,

$$\Omega^2 = \omega_m(\omega_m - \Delta),$$  

(24)

or $\omega_m = (\phi + \Delta)/2$. The optimal cooling condition in eq. (24) can be fulfilled by properly choosing the parameters $\Omega$, $\omega_m$, and $\Delta$. Then, the steady-state average phonon number in the NAMR can be much smaller than unity provided $\Delta \gg \Gamma$, implying that ground-state cooling of the NAMR is possible. The phonon number $n_f$ attainable according to eq. (23) is identical to the previous result for the cooling of trapped atoms via quantum interference [27]. However, solid-state cooling system proposed here has notable advantages such as easy fabrication on a single chip and high controllability. Specifically, all the relevant parameters (i.e., the detuning $\Delta$, the tunneling rate $\Gamma$ and the interdot coupling strengths $\Omega_1$ and $\Omega_2$) can be controlled by tuning the gate voltages in the TQD. Thus, the optimal cooling condition in eq. (24) can be conveniently fulfilled for a specified frequency $\omega_m$ of the NAMR.

The underlying physics of the optimal cooling condition in eq. (24) can be understood more intuitively in the eigenstate basis of the TQD. In the limit $\gamma \ll g \ll \omega_m$ considered here, the TQD arrives quickly at the dark state $|\rangle \rightarrow |\rangle$, compared with the dynamic time scale of the NAMR. Therefore, the TQD is practically always maintained in the dark state. The coupling between the NAMR and the TQD excites the electron to the state $|\rangle$ most readily when the frequency $\omega_m$ of the NAMR is equal to the transition frequency $(\phi + \Delta)/2$ between the states $|\rangle$ and $|\rangle$, i.e., $\omega_m = (\phi + \Delta)/2$. This corresponds to the transition $|\rangle, n \rangle \rightarrow |\rangle, n + 1 \rangle$. The excited electron subsequently tunnels to the right electrode, i.e., $|\rangle, n \rangle \rightarrow |\rangle, n + 1 \rangle \rightarrow |\rangle, n - 1 \rangle$. Meanwhile, another electron tunnels into the TQD, and then the dark state returns promptly, i.e., $|\rangle, n - 1 \rangle \rightarrow |\rangle, n - 1 \rangle$. This whole process extracts an energy quantum from the NAMR. Therefore, the NAMR will be cooled to the ground state when this cycle repeats, i.e., $|\rangle, n \rangle \rightarrow |\rangle, n + 1 \rangle \rightarrow |\rangle, n - 1 \rangle \rightarrow |\rangle, n - 1 \rangle \rightarrow \ldots$, as illustrated in fig. 2. Here we emphasize that the condition of resonance transition for TQD from the state $|\rangle$ to the state $|\rangle$ via the NAMR is equivalent to the optimal cooling condition in eq. (24). Although an electron may also relax from the dark state $|\rangle$ to the ground state $|g\rangle$ by releasing energy to the NAMR, this heating process of the NAMR is strongly suppressed because the frequency of the NAMR is off-resonant to the transition $|\rangle \rightarrow |g\rangle$ in the TQD.

Figure 3 displays a contour plot of the steady-state average phonon number $n_{st}$ in the NAMR as a function of the normalized driving detuning $\Delta/\omega_m$ and the interdot coupling $\Omega/\omega_m$. The two solid curves correspond to $n_{st} = 0.05$ and 0.02. The black dashed line represents $\Omega^2 = \omega_m(\omega_m - \Delta)$, under which the NAMR can be optimally cooled. We have chosen $\Omega_1 = \Omega_2 = \Omega/\sqrt{2}$ and typical parameters $\omega_m = 2\pi \times 100$ MHz, $\Gamma = \omega_m$, $Q = 10^5$, $\eta = 0.1$, and $\eta(\omega_m) = 21$.
Furthermore, from eq. (22), we obtain a cooling rate $W \approx 2\pi \times 8.52$ MHz by using typical experimental parameters [$7,33,34$]: $\omega_m = 2\pi \times 100$ MHz, $\Delta = -2\pi \times 40$ GHz and $g = 2\pi \times 10$ MHz, as well as by choosing the interdot couplings $\Omega_1 = \Omega_2 \approx 2\pi \times 1.41$ GHz to fulfill the optimal cooling condition $\Omega^2 = \omega_m (\omega_m - \Delta)$. Considering an NAMR with a quality factor $Q = 10^5$ (see, e.g., ref. [8]), one has $\gamma = \omega_m / Q = 2\pi \times 1$ kHz. Therefore, appreciable cooling with $W \gg \gamma$ can be achieved. In this case, an NAMR can be cooled from, e.g., an initial temperature $T_m = 100$ mK, corresponding to $n(\omega_m) = 21$, down to $T = 0.8$ mK with $n_{st} = 0.0025$. Starting from an initial temperature $T_m$, the final temperature of the cooled NAMR should be bound by [35] $T^* = \frac{n_{st}}{21} T_m$. In the parameter regime we studied above, $T^* = 0.25$ mK, which is indeed lower than the achieved temperature $T = 0.8$ mK. Moreover, it should be noted that the resolved-sideband cooling condition $\omega_m \gg \Gamma$ is not required in our scheme. Finally, note that in addition to the electrodes coupled to the TQD, there are interconnected processes induced by other degrees of freedom (e.g., background charge fluctuations) in the environment. These processes will affect the dark state of the TQD and then limit the cooling efficiency. Thus, one needs to produce a high-quality TQD with good quantum coherence.

**Conclusion.** – In summary, we have proposed an approach for achieving the ground-state cooling of an NAMR. It is shown that a dark state, which is decoupled from the excited state, can appear in the TQD in the absence of the NAMR when its two lower-energy localized states become degenerate. With the NAMR capacitively coupled to the TQD and being in resonance with the transition between the dark state and the excited eigenstate in the TQD, we have shown that the ground-state cooling of an NAMR can be achieved in the non–resolved-sideband cooling regime.

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This work was supported by the National Basic Research Program of China Grant No. 2009CB929300, the National Natural Science Foundation of China Grant No. 10625416, and the Hong Kong GRF Grant No. 5009/08P.

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