Abstract In order to probe a possible time variation of the fine-structure constant ($\alpha$), we propose a new method based on Strong Gravitational Lensing and Type Ia Supernovae observations. By considering a class of runaway dilaton models, where $\Delta \alpha = -\gamma \ln (1 + z)$, we obtain constraints on $\Delta \alpha$ at the level $\gamma \sim 10^{-2}$ ($\gamma$ captures the physical properties of the model). Since the data set covers the redshift range $0.075 \leq z \leq 2.2649$, the constraints derived here provide independent bounds on a possible time variation of $\alpha$ at low, intermediate and high redshifts.

1 Introduction

The hypothesis of large numbers (HLN), proposed a long time ago by Paul Dirac [1], has opened many possible approaches associated with a variation of the constants of nature. For example, an early investigation addressed a possible variation of the gravitational constant ($G$), but as the main result, this temporal dependence of $G$ was ruled out by [2] years after. Recently, the HLN has gained a lot of attention with the experimental advance. In this concern, Dirac’s hypothesis has been tested in many physical contexts, e.g., by using geological evidence, no variation in $G$ was either found today [3] or at recombination was marginally preferred over a no-variation model [19]. By using the physics of the cosmic microwave background (CMB), some researchers have used CMB anisotropies measurements to test models with varying $\alpha$. For example, from the Planck satellite data [20,21], experiments of South Pole Telescope [22,23] and Atacama Cosmology Telescope [24], some authors obtained that the difference between the $\alpha$ today and at recombination was $\Delta \alpha/\alpha \leq 7.3 \times 10^{-3}$ at 68% of Confidence Level [25–34]. However, this limit obtained from the CMB physics is inferred considering a specific cosmological model (flat $\Lambda$CDM), and being weakened by opening up the parameter space to variations of the number of relativistic species or the helium abundance. (see e.g. [35] and references therein).

A possible time variation of the fine structure constant during the Big Bang nucleosynthesis (BBN) [36] is also explored. Moreover, in the context of a supermassive black hole in the Galactic Center with a high gravitational poten-
tial, it is used late-type evolved giant stars data from the S-star [37]. Recently, the Ref. [38] revisited the framework where the cosmological constant, \( \Lambda \), is \( \Lambda \propto \alpha^{-6} \) (the so-called \( \Lambda(\alpha)CDM \) models). Using cosmological observations present in CAMB and CosmoMC packages and 313 data points from the absorption systems in the spectra of distant quasars, constraints on two specific \( \Lambda(\alpha)CDM \) models with one and two model parameters were performed. The authors found that the model parameters are constrained to be around \( 10^{-4} \), very similar to the results discussed by [39] but more accurately. However, the authors of the Ref. [40] showed that fitting turbulent models necessarily generate or enhance model non-uniqueness, adding a substantial additional random uncertainty to \( \Delta\alpha/\alpha \).

Particularly, the low-energy string theory models predict the existence of a scalar field called dilaton, a spin-2 graviton scalar partner [17,41,42]. In this scenario, the runaway of the dilaton towards strong coupling can lead to temporal variations of \( \alpha \). However, the runaway dilaton and chameleon models have not been completely ruled out by the experiments that test violations on the weak equivalence principle [17,43–45]. Constraints on the Runaway Dilaton Model by using Galaxy clusters measurements have been proposed to probe a possible time variation in \( \alpha \) (see the Ref. [46]). In [47], for instance, is introduced a method capable of probing a possible time variation in \( \alpha \) by using galaxy cluster (GC) gas mass fraction measurements only. Constraints on \( \Delta\alpha/\alpha \) achieved precision at the level \( \sim 10^{-2} \) (1\( \sigma \) c.l.). Using the angular diameter distance of GC and luminosity distance of type Ia supernovae, a possible temporal variation in \( \alpha \) was also investigated, obtaining \( \sim 10^{-2} \) at 1\( \sigma \) c.l. [48]. Several other tests capable of probing \( \alpha \) with galaxy cluster data have been emerging since then (see, for example, [49,50] and references therein).

In this work, by assuming a flat universe, it is discussed for the first time the strong gravitational lensing (SGL) role on a possible temporal variation of the fine-structure constant. The proposed method is performed by using combined SGL systems and Type Ia Supernovae (SNe Ia). For that purpose, we use 92 pair of observations (SGL-SNe Ia) covering the redshift ranges \( 0.075 \leq z_i \leq 0.722 \) and \( 0.2551 \leq z_s \leq 2.2649 \). These data shall be considered to limit the \( \gamma \) parameter, that is, considering runaway dilaton models. The approach developed here offers new limits on the \( \gamma \) parameter using observations in higher redshifts than those from galaxy clusters (\( z \approx 1 \)).

This work is organized as follows: in Sect. 2 we shall discuss the theoretical model used to describe \( \Delta\alpha/\alpha \). In Sect. 3 we describe the method developed to probe a time variation of \( \alpha \). In Sect. 4 the data set to be used in our analyses, while Sect. 5 shows the results. Finally, in Sect. 6, the conclusions of this paper are presented.

2 Theoretical framework

In the modified gravity theories associated to a scalar field with non-minimal multiplicative coupling to the usual electromagnetic Lagrangian, the entire electromagnetic sector is changed (see details in [51,52]). Actually, such a non-minimal coupling is motivated by several alternative theories, as the low-energy action of string theories, in the context of axions, generalized chameleons, etc. In this kind of theory, a variation of \( \alpha \) can arise either from a varying \( \mu_0 \) (vacuum permeability) or a variation of the charge of the elementary particles. Both interpretations lead to the same modified expression of the fine structure constant [5,53,54].

In this paper, we focus on the runaway dilaton model [41,42,51]. The idea behind this model is to exploit the string-loop modifications of the four-dimensional effective low-energy action, where the Lagrangian is given by:

\[
\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{8\pi G} (\nabla\phi)^2 - \frac{1}{4} B_F(\phi) F^2 + \cdots ,
\]

(1)

here, \( R \) is the Ricci scalar, \( \phi \) is the scalar field named dilaton, \( G \) is the gravitational constant, \( F \) is the usual electromagnetic tensor, and \( B_F \) is the gauge coupling function. From this action, the corresponding Friedmann equation and the motion equation for the dilaton field are given, respectively, by:

\[
H^2 = 8\pi G \rho + \frac{\rho}{3 + (1 + z) \frac{d\phi}{dz}}
\]

(2)

and

\[
(1 + z) \frac{d^2\phi}{dz^2} + \left[ 1 - \frac{8\pi G}{2H^2} (\rho - p) \right] (1 + z) \frac{d\phi}{dz}
\]

\[
= - \frac{8\pi G}{2H^2} \sum_i \beta_i(\phi)(\rho_i - 3p_i),
\]

(3)

where \( H \) is the Hubble parameter concerning the components of the universe and dilaton field, the total energy density and the pressure are, respectively, \( \rho = \sum_i \rho_i \) and \( p = \sum_i p_i \), except the corresponding part of \( \phi \). The \( \beta_i \) are the couplings of \( \phi \) with each component of matter \( i \). However, the relevant parameter of the runaway dilaton model to study a possible time variation of \( \alpha \) is the coupling of \( \phi \) to the hadronic matter. The central hypothesis is that all gauge fields couple to the same \( B_F \). From Eq. (1), it is possible one obtains \( \alpha \propto B_F^{-1}(\phi) \) (see [46] and references therein). Thus, it follows:

\[
\frac{\Delta\alpha}{\alpha} = \frac{1}{40} \beta_{had,0} \left[ 1 - e^{-\phi(z) - \phi_0} \right],
\]

(4)

where \( \beta_{had,0} \) is the current value of the coupling between the dilaton and hadronic matter and

\[
\beta_{had}(\phi) \sim 40 \frac{\partial}{\partial\phi} B_F^{-1}(\phi) \sim 1 - B_F e^{-c\phi},
\]

(5)
where \( c \) and \( b_F \) are constant free parameters.

As we are interested in a possible time evolution of dilaton up to \( z \approx 2.26 \), an acceptable approximation to the field evolution is given by \( \phi \sim \phi_0 + \phi'_0 \ln a \), where \( a \) is the cosmic scale factor [46]. Thus, one may obtain:

\[
\frac{\Delta a}{a} \approx - \frac{1}{40} \beta_{\text{had},0} \Phi'_0 \ln (1 + z) \approx - \gamma \ln (1 + z), \tag{6}
\]

where \( \Phi'_0 \equiv \frac{\partial \phi}{\partial a} \) at the present time, and \( \gamma \equiv \frac{1}{40} \beta_{\text{had},0} \Phi'_0 \).

This equation\(^1\) is that one we will use to compare the model predictions with combined SGL and SNe Ia data.

### 3 Methodology

Strong gravitational Lensing systems, one of the predictions of GR [55], have recently become a powerful astrophysical tool. They can investigate gravitational and cosmological theories, measure various cosmological parameters, and investigate fundamental physics. For example, time-delay measurements of gravitational lensings can be used to measure the Hubble constant [56], and the cosmic diameter distance relation (CDDR) [57]. Other statistical properties of SGL can restrict the deceleration parameter of the universe [58], spacetime curvature [59,60], also departures of CDDR [61,62], and others. It is a purely gravitational phenomenon occurring when the source (\( s \)), lens (\( l \)), and observer (\( o \)) are at the same signal line forming a structured ring called the Einstein radius (\( \theta_E \)) [65]. In the cosmological scenario, a lens can be a foreground galaxy or cluster of galaxies positioned between a source–Quasar, where the multiple-image separation from the source only depends on the lens and source angular diameter distances.

The system of SGL depends on a model for mass distribution. On the assumption of the singular isothermal sphere (SIS) model, the Einstein radius \( \theta_E \) is given by [55]

\[
\theta_E = \frac{4\pi D_{\text{Al}} \sigma_{\text{SIS}}^2}{D_A \left(c^2\right)}, \tag{7}
\]

where \( D_{\text{Al}} \) is the angular diameter distance of the lens to the source, \( D_A \) the angular diameter distance of the observer to the source, \( c \) the speed of light, and \( \sigma_{\text{SIS}} \) the velocity dispersion caused by the lens mass distribution. It is important to note here that \( \sigma_{\text{SIS}} \) is not exactly equal to the observed stellar velocity dispersion (\( \sigma_0 \)) due to a strong indication, via X-ray observations, that dark matter halos are dynamically hotter than luminous stars [66–69]. Taking this fact into account, we introduce a purely phenomenological free parameter: \( f_e \), where \( \sigma_{\text{SIS}}^2 = (f_e)^2 \sigma_0^2 \), with \( 0.8 < f_e < \sqrt{1.2} \) (see [70]). As it is largely known, the \( f_e \) parameter accounts not only for systematic errors caused by taking the observed stellar velocity dispersion as \( \sigma_{\text{SIS}} \), but it also accounts for deviation of the real mass density profile from the SIS. Moreover, the effects of secondary lenses (mainly nearby galaxies) and line-of-sight contamination are also quantified by this factor (see also [71]).

The method developed by the Ref. [62] provided a robust test for CDDR using SGL systems and SNe Ia. The procedure is based on Eq. (7) for lenses and an observational quantity defined by

\[
D \equiv \frac{D_{\text{Al}}}{D_A} = \frac{\theta_E c_s^2}{4\pi \sigma_{\text{SIS}}^2}, \tag{8}
\]

where \( c_s \) is the speed of light measured between the source and us. However, such method did not take into consideration any possible variation of the fine structure constant on SGL observations. Here, we extend the method and investigate both effects of varying \( \alpha \) and deviation of CDDR via SGL and SNe Ia observations. Thus, according to the definition of the fine structure constant (\( \alpha_s = \alpha^2/hc_s \)) the Eq. (8) is rewritten by:

\[
D \equiv \frac{D_{\text{Al}}}{D_A} = \frac{e^4 \theta_E}{\hbar^2 \alpha_s^2 4\pi \sigma_{\text{SIS}}^2}. \tag{9}
\]

On the other hand, assuming a flat universe with the comoving distance between the lens and the observer being \( r_{ls} = r_s - r_l \), and using the relations \( r_s = (1 + z_s)D_{Ai} \), \( r_l = (1 + z_l)D_{Ai} \), \( r_{ls} = (1 + z_s)D_{Al} \), it is possible to obtain

\[
D = 1 - \frac{(1 + z_l)}{(1 + z_s)} \frac{D_{Ai}}{D_{Al}}. \tag{10}
\]

Considering a possible deviation of CDDR by \( D_{Al} = D_{Li}/\eta(z_l)/(1 + z_l)^2 \), we obtain:

\[
D = 1 - \frac{(1 + z_s)}{(1 + z_l)} \frac{D_{Li} \eta(z_s)}{D_{Li} \eta(z_l)}. \tag{11}
\]

where \( D_{Li} \) and \( D_{Li} \) are the luminosity distances to lens and source, respectively, and \( \eta(z_l) \) captures any deviation of CDDR.

As mentioned before, it was shown in Refs. [51,52] that for the class of theories obeying the Eq. (1), a variation of \( \alpha \) necessarily leads to a violation of CDDR, and both changes are intimately and unequivocally related to each other by:

\[
\frac{\Delta \alpha}{\alpha} = \frac{\alpha(z) - \alpha_0}{\alpha_0} = \eta^2(z) - 1. \tag{12}
\]
Considering $\alpha(z) = \alpha_0 \phi(z)$, where $\alpha_0$ is the current value of the fine-structure constant, and $\phi(z)$ is a scalar field that controls a variation of $\alpha$, the Eq. (12) gives $\phi(z) = \eta^2(z)$. Thus, the Eqs. (9) and (11) shall be rewritten, respectively, by:

$$D = \frac{e^4 \theta_E}{4\pi \alpha_0^2 h^2 \sigma_{\text{SIS}}^2} \phi^{-2}(z_s) = D_0 \phi^{-2}(z_s)$$

(13)

and

$$D = 1 - \frac{(1 + z_s) D_{L_s} \phi^{1/2}(z_s)}{(1 + z_l) D_{L_l} \phi^{1/2}(z_l)}$$

(14)

where $D_0 \equiv e^4 \theta_E/4\pi \alpha_0^2 h^2 \sigma_{\text{SIS}}^2$ (if $\Delta \alpha/\alpha = 0$, so $\phi(z) = 1$ and $D = D_0$). Therefore, combining Eqs. (13) and (14), it is possible to obtain

$$D_0 = \phi^2(z_s) \left[ 1 - \frac{(1 + z_s) D_{L_s} \phi^{1/2}(z_s)}{(1 + z_l) D_{L_l} \phi^{1/2}(z_l)} \right].$$

(15)

Note that if $\Delta \alpha/\alpha = 0$, the quantity $D < 1$, which means that systems with $D > 1$ has no physical meaning.

4 Samples

4.1 Type Ia Supernovae

Now, let us consider the pair of luminosity distances for each SGL system, which we obtain from the SNe Ia sample called Pantheon [72]. It is worth mentioning that Pantheon is the most recent wide refined sample of SNe Ia observations found in the literature, consisting of 1049 spectroscopically confirmed SNe Ia and covers a redshift range of $0.01 \leq z \leq 2.3$. The sample of $D_L$ is constructed from the apparent magnitude ($m_b$) of the Pantheon catalog by considering $M_b = -19.23 \pm 0.04$ (absolute magnitude) by the relation

$$D_L = 10^{(m_b - M_b - 25)/5} \text{Mpc}.$$  

(16)

However, to perform the appropriate tests, we must use SNe Ia at the same (or approximately) redshift of the lens-source of each system. Thus, we make a selection of SNe Ia according to the criterion: $|z_l - z_{\text{SNe}}| \leq 0.005$ and $|z_l - z_{\text{SNe}}| \leq 0.005$. Then, we perform the weighted average for each system by [62] (see Fig. 1):

$$\bar{D}_L = \frac{\sum_i D_{L_i}/\sigma_{D_{L_i}}^2}{\sum_i 1/\sigma_{D_{L_i}}^2},$$

(17)

$$\sigma_{\bar{D}_L}^2 = \frac{1}{\sum_i 1/\sigma_{D_{L_i}}^2}.$$  

(18)

It is important to stress that the influence of a possible variation of $\alpha$ on SNe Ia observations has been discussed in literature (see [73] and references therein). Briefly, the peak luminosities of SNe Ia depend on $\alpha$ and a variation of this constant directly translates into a distortion of the apparent magnitude. In other words, the distance modulus is modified. However, the analyses of the Ref. [73] concluded that at $3\sigma$, the parameters of the SNe Ia data used (JLA and Union2.1 compilations) are consistent with a null variation of $\alpha$.

4.2 SGL systems

We consider a specific catalog containing 158 confirmed sources of strong gravitational lensing by [74]. This compilation includes 118 SGL systems identical to the compilation of [55]. The SGL were obtained from SLOAN Lens ACS, BOSS Emission-line Lens Survey (BELLS), and Strong Legacy Survey SL2S, along with 40 new systems recently discovered by SLACS and pre-selected by [75] (see Table I in [74]).

However, studies using lensing systems have shown that the pure SIS model may not be an accurate representation of the lens mass distribution when $\sigma_0 < 250 \text{ km/s}$, for which non-physical values of the quantity $D_0$ are usually found ($D_0 > 1$). In [74] is also mentioned the need for attention when the SIS model is used as a reference since the impact caused on the density profile can cause deviations on the observed stellar velocity dispersion ($\sigma_0$). For this reason, by excluding non-physical measurements of $D_0$, and the system J0850-0347 [74], our sample finishes with 140 measurements of $D_0$. We also consider a general approach to describe the lensing systems: the one with spherically symmetric mass distribution in lensing galaxies in favor of power-law index $\gamma$.

\footnote{It deviates by more than $5\sigma$ from all the considered models.}
where \( \rho \propto r^{-\gamma} \) (PLAW). This kind of model is essential since several recent studies have shown that slopes of density profiles of individual galaxies show a non-negligible scatter from the SIS model [66–69]. Under this assumption, the quantity \( D_0 \) of Eq. (13) shall be rewritten by:

\[
D_0 = \frac{e^{4\gamma E}}{\alpha_0^2 h^2 4\pi \sigma_{ap}^2} \left( \frac{\theta_{ap}}{\theta_E} \right)^{2-\gamma} f^{-1}(\gamma),
\]

(19)

where \( \sigma_{ap} \) is stellar velocity dispersion inside an aperture of size \( \theta_{ap} \), \( \gamma \) the power-law index (if \( \gamma = 2 \), Eq. (19) resumes the SIS model), and

\[
f(\gamma) = \frac{(5-2\gamma)(1-\gamma)}{\sqrt{\pi}(3-\gamma)} \frac{\Gamma(\gamma-1)}{\Gamma(\gamma-3/2)} \left( \frac{\Gamma(\gamma/2-1/2)}{\Gamma(\gamma/2)} \right)^2.
\]

(20)

In this paper, the factor \( \gamma \) is approached as a free parameter. The uncertainty related to Eq. (19) is given by:

\[
\sigma_{D_0} = D_0 \left[ 4 \left( \frac{\sigma_{\sigma_E}}{\sigma_{ap}} \right)^2 + (1-\gamma)^2 \left( \frac{\sigma_{\gamma}}{\theta_E} \right)^2 \right]^{1/2}.
\]

(21)

Following the approach taken by [79], Einstein’s radius uncertainties follows \( \sigma_{\gamma} = 0.05 \theta_E \) (5% for all systems).

As mentioned before, our sample consists of 140 SGL systems covering a wide range of redshift. However, not all the SGL systems have the corresponding pair of luminosity distances via SNe Ia that obey the previous criteria. We ended up with 92 pairs of observations (SGL-SNe Ia) for our analyses by also excluding these systems.

5 Analysis and discussions

We used Markov Chain Monte Carlo (MCMC) methods to calculate the posterior probability distribution functions (pdf) of free parameters [80]. For SIS model, the free parameter space is \( \Theta = (\gamma, f_E) \), and for PLAW model is \( \Theta = (\gamma, \gamma) \).

Thus, the likelihood distribution function is given by:

\[
L(Data|\Theta) = \prod \frac{1}{\sqrt{2\pi} \sigma_{\mu}} \exp\left( -\frac{1}{2} \chi^2 \right),
\]

(22)

where

\[
\chi^2 = \left( \frac{D_0 - \zeta}{\sigma_T} \right)^2,
\]

(23)

\[
\zeta = \phi^2(z_s) \left[ 1 - \frac{(1 + z_s)D_{Li} \phi^{1/2}(z_s)}{(1 + z)L_{Li} \phi^{1/2}(z_l)} \right],
\]

(24)

\[
\sigma_T = (\sigma_{D_0}^2 + \sigma_{\zeta}^2)^{1/2},
\]

(25)

\[
\sigma_{\zeta}^2 = \frac{\phi^2(z_s)}{\phi(z_l)} \left( \frac{1 + z_s}{1 + z_l} \right) \left( \frac{D_{Li}}{D_{Li}} \right)^2 K,
\]

(26)

\[
K = \left\{ \left( \frac{\sigma_{D_{Li}}}{D_{Li}} \right)^2 + \left( \frac{\sigma_{D_{Li}}}{D_{Li}} \right)^2 \right\}^{1/2}
\]

(27)

the associated errors. As mentioned before, \( \phi(z_s) = 1 - \gamma \ln(1 + z_s) \) and \( \phi(z_l) = 1 - \gamma \ln(1 + z_l) \), where \( \gamma \) is the parameter to be constrained. The pdf posteriori is proportional to the product between the likelihood and the prior, that is,

\[
P(\Theta|Data) \propto L(Data|\Theta) \times P_0(\Theta).
\]

(28)

In our analysis, we assume flat priors: \(-1.0 \leq \gamma \leq +1.0; 1.5 \leq \gamma \leq 2.5; 0.8 \leq f_E \leq 1.2\).

Our main results are:

- For the SIS model: \( \gamma = 0.04^{+0.05}_{-0.06} \), and \( f_E = 1.02^{+0.02}_{-0.02} \), where \( \chi^2_{\text{red}} \approx 1.57 \) (1\sigma).
- For the PLAW model: \( \gamma = -0.03^{+0.02}_{-0.02} \), and \( \gamma = 1.99^{+0.04}_{-0.04} \), where \( \chi^2_{\text{red}} \approx 1.58 \) (1\sigma).

As one may see, the PLAW model agrees to the SIS model within 2\sigma c.l.

As mentioned by [74], it is necessary to add 12.22% of intrinsic error associated to \( D_0 \) measurement. As the random variation in galaxy morphology is almost Gaussian, the authors of Ref. [74] found that an additional error term of about 12.22% is necessary to have 68.3% of the observations lie within 1\sigma of the best-fit \( \omega \)CDM model, which is smaller than 20% scatter as suggested by [55]. Moreover, this procedure makes \( D \) more homogeneous for the lensing sample located at different redshifts. Therefore, our main results are:

- For the SIS model: \( \gamma = 0.04^{+0.07}_{-0.08} \), and \( f_E = 1.02^{+0.03}_{-0.03} \), where \( \chi^2_{\text{red}} \approx 0.91 \) (see Fig. 2).
- For the PLAW model: \( \gamma = -0.03^{+0.03}_{-0.04} \), and \( \gamma = 1.97^{+0.05}_{-0.05} \), where \( \chi^2_{\text{red}} \approx 0.91 \) (see Fig. 3).

Table 1 shows the bounds on \( \gamma \) derived in this paper, along with other recent constraints obtained from galaxy clusters and SNe Ia observations. As one may see, our results are in full agreement with the previous ones from galaxy clusters plus SNe Ia analyses.

6 Conclusions

The search for a possible temporal or/spatial variation of the fundamental constants of nature has received significant
interest in the last decades, given the improvement in astrophysics’ observational data. In this paper, a new technique was proposed to investigate a possible time variation of the fine structure constant, such as $\alpha(z) = \alpha_0 \phi(z)$, with data at high redshifts by using recent measurements of SGL systems and SNe Ia observations. A possible time variation of $\alpha$ in a class of runaway dilaton models, with $\phi(z) = 1 - \gamma \ln(1+z)$, was investigated.

As we have already discussed, considering the SIS model to describe the mass distribution in lensing galaxies, we obtained: $\gamma = +0.04^{+0.05}_{-0.06}$ and $f_e = 1.02^{+0.02}_{-0.02}$. By considering the PLAW model, we obtained: $\gamma = -0.03^{+0.02}_{-0.02}$ and $\Upsilon = 1.97^{+0.04}_{-0.04}$. By adding $\alpha_{int} \approx 12.22\%$ of intrinsic error, we obtain: for the SIS model $\gamma = 0.04^{+0.07}_{-0.08}$ and $f_e = 1.02^{+0.03}_{-0.03}$ and for the PLAW model $\gamma = -0.03^{+0.03}_{-0.04}$ and $\Upsilon = 1.97^{+0.05}_{-0.05}$, both in $1\sigma$ of confidence level. These results are in full agreement with the standard cosmology. Although SGL systems data are not competitive with the limits imposed by quasar absorption systems, the constraints imposed in this paper provide new and independent limits on a possible time variation of the fine structure constant.

Finally, as an interesting extension of the present work, one may check the consequences of relaxing the rigid assumption that the stellar luminosity and total mass distributions follow the same power law [82,83]. Moreover, the well-known Mass-sheet degeneracy (see [84] and references

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**Table 1** A summary of current constraints on a possible time evolution of $\alpha$ for a class of runaway dilaton models ($\Delta \alpha/\alpha = -\gamma \ln (1+z)$) by using galaxy cluster observations and SNe Ia measurements. The symbol * denotes galaxy cluster data.

| Data set | Profile | $\gamma$ |
|----------|---------|----------|
| Gas Mass Fractions* [47] | Non-Isothermal double $\beta$-Model | $+0.065 \pm 0.095$ |
| Angular Diameter Distance* plus SNe Ia [48] | Isothermal Elliptical $\beta$-Model | $-0.037 \pm 0.157$ |
| Gas Mass Fractions* plus SNe Ia [49] | Universal Pressure Profile | $+0.008 \pm 0.035$ |
| Gas Mass Fractions* plus SNe Ia [49] | Virialized ideal gas | $+0.018 \pm 0.032$ |
| Gas Mass Fractions* plus SNe Ia [49] | Non-thermal Pressure and Adiabatic Model | $+0.010 \pm 0.030$ |
| Gas Mass Fractions* plus SNe Ia [49] | Mass Dynamical Estimate from Galaxy Velocity Dispersions | $+0.030 \pm 0.033$ |
| $Y_{SZ} D^2 L / Y_X$ scaling-relation* [81] | Universal pressure profile | $-0.15 \pm 0.10$ |
| This work–SGL | SIS Model | $+0.04^{+0.07}_{-0.08}$ |
| This work–SGL | PLAW Model | $-0.03^{+0.03}_{-0.04}$ |
therein) in the gravitational lens system and its effect on our results also could be explored further.

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References

1. P.A.M. Dirac, The cosmological constants. Nature 139, 323 (1937)
2. E. Teller, On the change of physical constants. Phys. Rev. 73, 801 (1948)
3. E.P. Bellinger, J. Christensen-Dalsgaard, Asteroseismic constraints on the cosmic-time variation of the gravitational constant from an ancient main-sequence star. Astrophys. J. 887, 1 (2019). arXiv:1909.06378
4. J. Muller, L. Biskupek, Variations of the gravitational constant from lunar laser ranging data. Class. Quantum Gravity 24, 4533 (2007). arXiv:gr-qc/0509114
5. J.P. Uzan, Varying constants, gravitation and cosmology. Living Rev. Relativ. 14, 2 (2011). arXiv:0905.5514
6. T. Chiba, The constancy of the constants of nature: updates. Prog. Theor. Phys. 126, 993 (2011). arXiv:1111.0092
7. A. Chodos, S.L. Detweiler, Where has the fifth-dimension gone? Phys. Rev. D 21, 2167 (1980)
8. E.W. Kolb, M.J. Perry, T.P. Walker, Time variation of fundamental constants, primordial nucleosynthesis and the size of extra dimensions. Phys. Rev. D 33, 869 (1986)
9. P. Nath, M. Yamaguchi, Effects of extra space-time dimensions on the Fermi constant. Phys. Rev. D 60, 116004 (1999)
10. J.D. Bekenstein, Fine structure constant: is it really a constant? Phys. Rev. D 25, 1527 (1982)
11. J.K. Webb, J.A. King, M.T. Murphy, V.V. Flambaum, R.F. Carswell, M.B. Bainbridge, Indications of a spatial variation of the fine structure constant. Phys. Rev. Lett. 107, 191101 (2011). arXiv:1008.3907
12. J.A. King, J.K. Webb, M.T. Murphy, V.V. Flambaum, R.F. Carswell, M.B. Bainbridge, M.R. Wilczynska, F.E. Koch, Spatial variation in the fine-structure constant—new results from VLT/UVES. Mon. Not. R. Astron. Soc. 422, 3370 (2012). arXiv:1202.4758
13. J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, J.D. Barrow, A search for time variation of the fine structure constant. Phys. Rev. Lett. 82, 884 (1999). arXiv:astro-ph/9803165
14. M.T. Murphy, J.K. Webb, V.V. Flambaum, Further evidence for a variable fine-structure constant from Keck/HIRES QSO absorption spectra. Mon. Not. R. Astron. Soc. 345, 609 (2003). arXiv:astro-ph/0306483
15. A. Songaila, L.L. Cowie, Constraining the variation of the fine structure constant with observations of narrow quasar absorption lines. Astrophys. J. 793, 103 (2014). arXiv:1406.3628
16. C.J.A.P. Martins, A.M.M. Pinho, Stability of fundamental couplings: a global analysis. Phys. Rev. D 95, 023008 (2017). arXiv:1701.08724
17. C.J.A.P. Martins, The status of varying constants: a review of the physics, searches and implications. arXiv:1709.02923
18. T. Damour, F. Dyson, The Oklo bound on the time variation of the fine structure constant revisited. Nucl. Phys. B 480, 37 (1996). arXiv:hep-ph/9606486
19. M.R. Wilczynska et al., Four direct measurements of the fine-structure constant 13 billion years ago (2020). arXiv:2003.07627
20. P.A.R. Ade et al. [Planck Collaboration], Planck 2015 results. XIII. Cosmological parameters. Astron. Astrophys. 594, A13 (2016). arXiv:1502.01589
21. N. Aghanim et al. [Planck Collaboration], “Planck 2018 results. VI. Cosmological parameters”, arXiv:1807.06209
22. K.T. Story et al., A measurement of the cosmic microwave background damping tail from the 2500-square-degree SPT-SZ survey. Astrophys. J. 779, 86 (2013). arXiv:1210.7231
23. B.A. Benson et al., [SPT-3G Collaboration], SPT-3G: a next-generation cosmic microwave background polarization experiment on the south pole telescope. Proc. SPIE Int. Soc. Opt. Eng. 9153, 91531P (2014). arXiv:1407.2973
24. T. Louis et al. [ACTPol Collaboration], The Atacama Cosmology Telescope: two-season ACTPol spectra and parameters. J. Cosmol. AP 1706, 031 (2017). arXiv:1610.02360
25. P.P. Avelino et al., Early universe constraints on a time varying fine structure constant. Phys. Rev. D 64, 103505 (2001). arXiv:astro-ph/0102144
26. C.J.A.P. Martins, A. Melchiorri, G. Rocha, R. Trotta, P.P. Avelino, P.T.P. Viana, Wmap constraints on varying alpha and the promise of reionization. Phys. Lett. B 585, 29 (2004). arXiv:astro-ph/0302295
27. G. Rocha, R. Trotta, C.J.A.P. Martins, A. Melchiorri, P.P. Avelino, R. Bean, P.T.P. Viana, Measuring alpha in the early universe: cmb polarization, reionization and the fisher matrix analysis. Mon. Not. R. Astron. Soc. 352, 20 (2004). arXiv:astro-ph/0309211
28. K. Ichikawa, T. Kanzaki, M. Kawasaki, CMBS constraints on the simultaneous variation of the fine structure constant and electron mass. Phys. Rev. D 74, 023515 (2006). arXiv:astro-ph/0602577
29. E. Menegoni, S. Galli, J.G. Bartlett, C.J.A.P. Martins, A. Melchiorri, New constraints on variations of the fine structure constant from CMB anisotropies. Phys. Rev. D 80, 087302 (2009). arXiv:0909.3584
30. S. Galli, M. Martinelli, A. Melchiorri, L. Pagano, B.D. Sherwin, D.N. Spergel, Constraining fundamental physics with future CMB experiments. Phys. Rev. D 82, 123504 (2010). arXiv:1005.3808
31. E. Menegoni, M. Archidiacono, E. Calabrese, S. Galli, C.J.A.P. Martins, A. Melchiorri, The fine structure constant and electron mass. Phys. Rev. D 78, 023515 (2008). arXiv:0802.0945
32. P.A.R. Ade et al. [Planck Collaboration], Planck intermediate results—XXIV. Constraints on variations in fundamental constants. Astron. Astrophys. 580, A22 (2015). arXiv:1406.7482
33. I. de Martino, C.J.A.P. Martins, H. Ebeling, D. Kocevski, Constraining spatial variations of the fine structure constant using clusters of galaxies and Planck data. Phys. Rev. D 94, 083008 (2016). arXiv:1605.03053
34. L. Hart, J. Chluba, New constraints on time-dependent variations of fundamental constants using Planck data. Mon. Not. R. Astron. Soc. 474, 1850 (2018). arXiv:1705.03925
74. K. Leaf, F. Melia, Model selection with strong-lensing systems. MNRAS 478, 4 (2018). arXiv:1805.08640
75. Y. Shu, J.R. Brownstein, A.S. Bolton, L.V.E. Koopmans, T. Treu, A.D. Montero-Dorta, M.W. Auger, O. Czoske, R. Gavazzi, P.J. Marshall, L.A. Moustakas, The Sloan Lens ACS Survey. XIII. Discovery of 40 new galaxy-scale strong lenses. ApJ 851, 1 (2017). arXiv:1711.00072
76. J.-Q. Xia, H. Yu, G.-J. Wang, S.-X. Tian, Z.-X. Li, S. Cao, Z.-H. Zhu, Revealing studies of the statistical property of a strong gravitational lens system and model-independent constraint on the curvature of the universe. Astrophys. J. 834, 1 (2017). arXiv:1611.04731
77. Z. Li, X. Ding, G.-J. Wang, K. Liao, Z.-H. Zhu, Curvature from strong gravitational lensing: a spatially closed universe or systematics? Astrophys. J. 854, 146 (2018). arXiv:1801.08001
78. X. Li, L. Tang, H.-N. Lin, Probing cosmic acceleration by strong gravitational lensing systems. Mon. Not. R. Astron. Soc. 484, 3 (2019). arXiv:1901.09144v1
79. C. Grillo, M. Lombardi, G. Bertin, Cosmological parameters from strong gravitational lensing and stellar dynamics in elliptical galaxies. Astron. Astrophys. 477, 397 (2008). arXiv:0711.0882
80. D. Foreman-Mackey, D.W. Hogg, D. Lang, J. Goodman, emcee: the MCMC Hammer. Publ. Astron. Soc. Pac. 125, 925 (2013). arXiv:1202.3665
81. L.R. Colaço, R.F.L. Holanda, R. Silva, J.S. Alcaniz, Galaxy clusters and a possible variation of the fine structure constant. JCAP 03, 014 (2019). arXiv:1901.10947
82. S. Cao, M. Biesiada, X. Zheng, Z.-H. Zhu, Testing the gas mass density profile of galaxy clusters with distance duality relation. Mon. Not. R. Astron. Soc. 457, 1 (2016). arXiv:1601.00409
83. J. Schwab, A.S. Bolton, S.A. Rappaport, Galaxy-scale strong lensing tests of gravity and geometric cosmology: constraints and systematic limitations. Astrophys. J. 708, 750–757 (2010). arXiv:0907.4992
84. S. Birrer, A. Amara, A. Refregier, The mass-sheet degeneracy and time-delay cosmography: analysis of the strong lens RXJ1131-1231. JCAP 08, 020 (2016). arXiv:1511.03662