A Reliable Solution of Nonlinear Time Dependent Fractional Model of Ebola Virus Disease with Arbitrary Order Derivative in Liouville–Caputo Sense

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Abstract
In this article, the analysis of an arbitrary order Ebola virus disease model is conducted to find out its reliable solution. The fractional derivative is taken in Liouville–Caputo sense. The solution of this nonlinear model is achieved using fractional variational iteration scheme. The convergence analysis of the obtained solution is also presented which confirms that it is positive, bounded and convergent. The outcomes are discussed with figures explaining variation of susceptible, infected, recovered population and number of disease induced deaths with time. The negligible error in successive iterations of various population shows the competency of the presented scheme. The results endorse that FVIM is extremely effective, powerful and easy in usage.

Keywords Fractional differential equations (FDE) · Ebola disease model · Liouville–Caputo fractional derivative · Fractional variational iteration scheme (FVIM)

List of Symbols

\( t \quad \text{Time variable} \\
\beta \quad \text{Order of the fractional derivative} \\
\mathbb{N} \quad \text{Set of natural numbers} \\
\lambda \quad \text{Lagrangian multiplier} \\
\tilde{S}_n \quad \text{Restricted variation} \\
l \quad \text{Tolerance limits} \\
\mathbb{R} \quad \text{Set of real numbers} \\
P_i ; 1 \leq i \leq 4 \quad \text{Operators from Banach space to Banach space} \\
BS \quad \text{Banach Space}

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Introduction

Ebola is a fatal and complex pathogen for a human being. It attacks healthy cells of the body of host and replicates itself in it [1, 2]. It is a rare virus that causes bleeding [3–8]. In 1976, it was first detected in two outbreaks, one in South Sudan, and the other in Congo near the river Ebola. In spite of wide search, its reservoir could not be recognized but it may contain bats [1]. The 2014–2016 outbreak of Ebola in West Africa was the biggest since 1976. After starting in Guinea, it moved to Liberia and Sierra Leone. The recent outbreak of Ebola virus disease (EVD) is found in the Republic of Guinea on 14 February 2021. However, on 19 June 2021, the outbreak was declared over. There were 16 confirmed and 7 probable cases, of whom 12 people died [9].

This virus transmits through physical connection with secretions, tissues, body fluids or semen of infected ones. Its early symptom are headache, fever, sore throat, joint ache, muscle ache, weakness etc. In the later stage, the symptom are vomiting, diarrhea, bleeding, stomach pain and organ failure. Its incubation period is between 2–21 days and infection period is about 4–10 days. Its mortality rate ranges from 25 to 90% [10]. Despite high mortality rate, some patients survive and few develop long-lasting manifestations. It is seen that the individual genetic differences have a major role in the deaths by EVD [11]. In 2014, the average incubation time was 11.4 days. The average period from the beginning of symptom to hospitalization is 5 ± 4.7 days. The average period to death after hospitalization was 4.2 ± 6.4 days [12]. Ebola virus remains in semen for a long time in survivors with a risk of transmission [13]. The antivirals like Favipiravir is found to be effective in delaying survival and dropping viral load. The study of Ebola virus is done in [14–16]. The transmission EVD Dynamics is studied by various researchers in [17–19].

Mathematical investigation is done to state several phenomena of biological [20–22], medical [23] and physical [24–28] significance. The fractional derivatives [29–34] are used to describe the real-life problems. The arbitrary order of derivative is an index of memory [35]. For details of work on the fractional EVD models (see, for details, [36–43]). Fractional calculus [44] is a latent tool in electromagnetic theory [45], bio-engineering [46], plasma physics [47], control theory [48], neurophysiology [49], electric technology [50], visco-elasticity [51], disease dynamics [52–54] and many others. Over the years, the researchers have worked on various techniques for the solution of nonlinear FDE such as homotopy perturbation method [55], homotopy analysis method [56], Adomian decomposition method [57], fractional order homotopy perturbation method [58], variational iteration scheme and its modifications [59–66] etc.

However, the consistency in the solution of schemes [67] is vital. Most of the FDE have no exact solution, therefore approximation by FVIM is needed. He [68] established VIM for the solution of nonlinear equations and then applied to FDE. Highly nonlinear equations cannot successfully be solved by general methods. FVIM [69–72] is remarkably competent to address the challenge. It offers solution in an infinite convergent series. It entails no discretization and linearization. In FVIM, initial solution is picked with unknown parameters as searched solutions. It reduces computation significantly and has meticulousness in finding Lagrange’s multiplier.

Our goal is to investigate an efficient solution of time fractional Ebola virus model by FVIM. This paper is planned as below. In Sect. 1, there is introduction. In Sect. 2, the definition along with properties of some fractional operators are specified. In Sect. 3, the fractional EVD model is discussed. In Sect. 4, the idea of FVIM is discussed with analysis
of convergence and solution by FVIM is also found. In Sect. 5, we elaborate results and afterwards conclusion is found.

**Preliminaries**

We give certain properties with definition of Liouville–Caputo operator (see, for details, [73–76]).

**Definition 2.1:** Liouville–Caputo $\alpha$-order derivative is defined as:

$$
LC D^{\alpha}_{a+} \varphi(\eta) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{\eta} (\eta - \zeta)^{n-\alpha-1} D^{n}_{\zeta} \varphi(\zeta) d\zeta, \quad (n-1 < \alpha \leq n; n \in \mathbb{N}),
$$

where, $D^{n}_{\eta} := \frac{d^{n}}{d\eta^{n}} (n \in \mathbb{N}_{0} := \mathbb{N} \cup \{0\})$.

If $\alpha$ is positive integer, it turns into ordinary derivative,

$$
LC D^{\alpha}_{a+} = D^{\alpha}_{\eta} (\alpha \in \mathbb{N}).
$$

**Definition 2.2:** Liouville–Caputo $\alpha$-order derivative ($\alpha > 0$) on space $\mathbb{R} = (-\infty, \infty)$ is:

$$
LC D^{\alpha}_{-\infty+} \varphi(\zeta) = \frac{1}{\Gamma(n-\alpha)} \int_{-\infty}^{\zeta} (\zeta - \varsigma)^{n-\alpha-1} D^{n}_{\varsigma} \varphi(\varsigma) d\varsigma, \quad (n-1 < \alpha \leq n; n \in \mathbb{N}).
$$

a. $I^{\zeta}_{t} h(x, t) = \frac{1}{\Gamma(\zeta)} \int_{0}^{t} (t-s)^{\zeta-1} h(x, s) ds$; $\zeta, t > 0$.

b. $D^{\psi}_{t} V(x, \tau) = I_{\tau}^{m-\nu} \frac{d^{m}}{d\tau^{m}} V(x, \tau), m - 1 < \nu \leq m$.

c. $D^{\zeta}_{t} I^{\zeta}_{t} h(t) = h(t), m - 1 < \zeta \leq m, m \in \mathbb{N}$.

d. $D^{\zeta}_{t} D^{\zeta}_{t} h(t) = h(t) - \sum_{k=1}^{m-1} h^{(k)}(0^{+}) \frac{\zeta^{k}}{k!}, m - 1 < \zeta \leq m, m \in \mathbb{N}$.

e. $I^{\beta}_{t} t^{\alpha} = \frac{\Gamma(\alpha+1)}{\Gamma(\beta+\alpha+1)} t^{\beta+\alpha}$.

f. $D^{\beta}_{t} t^{\alpha} = \frac{\Gamma(\alpha+1)}{\Gamma(\beta+\alpha+1)} t^{\alpha-\beta}$.

**Model Description**

Atangana and Goufo [19] developed a model describing the spread of Ebola fever in the West African countries. Following this, we have reconstructed a nonlinear time dependent EVD model to include the features using memory effect. Let $N$ be the total population of West African countries, $S$ be the susceptible, $I$ be the infected, $R$ be the number of recovery and $D$ be the number of disease induced deaths due to EVD in these countries. We consider that the people get susceptible at rate $s$, infect at rate $i$, recover at rate $r$ and the disease induced death rate is $d$. The death rate due to other reasons is assumed as $\alpha$.

Rates of change in susceptible and infected with time $t$ are,

$$
D^{\beta}_{t} S(t) = -i S(t) I(t) + s R(t) - \alpha N,
$$

$$
D^{\beta}_{t} I(t) = i S(t) I(t) - (d + r) I(t),
$$
Here, $iS(t)I(t)$ are the people removed from the susceptible. Rates of change in the recovered and the number of deaths with time are,

\[
D_t^\beta R(t) = rI(t) - sR(t),
D_t^\beta D(t) = dI(t) + \alpha N,
\]

Hence the EVD model is written as,

\[
\begin{aligned}
D_t^\beta S(t) &= -iS(t)I(t) + sR(t) - \alpha N, \\
D_t^\beta I(t) &= iS(t)I(t) - (d + r)I(t), \\
D_t^\beta R(t) &= rI(t) - sR(t), \\
D_t^\beta D(t) &= dI(t) + \alpha N.
\end{aligned}
\]

with conditions $S(0) = S_0$, $I(0) = I_0$, $D(0) = D_0$, $R(0) = R_0$ and $0 < \beta \leq 1$. The property of non-locality is main use of working with FDE. It means that the next state of system is dependent on prior states. Therefore, these models adhere to reality. The time dependent fractional derivative submits memory modulation [77] of a system. Clearly, the presented EVD model is impacted by memory in time that confirms aptness of fractional order modeling. Therefore, a systematic and exhaustive study of Eq. (1) is important. Equation (1) is nonlinear and it is visibly tough to find its exact solution. Therefore, it turned into our motivation to solve Eq. (1) by FVIM because it gives reliable results for nonlinear equations without finding special polynomials.

**FVIM for the Fractional Ebola Virus Disease Model**

Consider a fractional EVD model in Eq. (1). The correctional functionals for Eq. (1) are,

\[
\begin{aligned}
S_{n+1}(t) &= S_n(t) + \lambda \int_0^t \left( \frac{d^\beta S_n(\tau)}{d\tau^\beta} + i \tilde{S}_n \tilde{I}_n - s \tilde{R}_n + \alpha N \right) (d\tau)^\beta, \\
I_{n+1}(t) &= I_n(t) + \lambda \int_0^t \left( \frac{d^\beta I_n(\tau)}{d\tau^\beta} - i \tilde{S}_n \tilde{I}_n + (d + r) \tilde{I}_n \right) (d\tau)^\beta, \\
R_{n+1}(t) &= R_n(t) + \lambda \int_0^t \left( \frac{d^\beta R_n(\tau)}{d\tau^\beta} - r \tilde{I}_n + s \tilde{R}_n \right) (d\tau)^\beta, \\
D_{n+1}(t) &= D_n(t) + \lambda \int_0^t \left( \frac{d^\beta D_n(\tau)}{d\tau^\beta} - d \tilde{I}_n - \alpha N \right) (d\tau)^\beta.
\end{aligned}
\]

Here, $\lambda$ is Lagrangian multiplier.

By variational theory,

\[
\frac{d^\beta \lambda}{d\tau^\beta} \bigg|_{\tau=t} = 0, \quad \text{and} \quad 1 + \lambda \bigg|_{\tau=t} = 0.
\]

We instantly get,

\[
\lambda = -1.
\]
From Eq. (2),

\[
S_{n+1}(t) = S_n(t) - \int_0^t \left( \frac{d^\beta S_n(\tau)}{d\tau^\beta} + i S_n(t) I_n(t) - s R_n(t) + \alpha N \right) (d\tau)^\beta,
\]

\[
I_{n+1}(t) = I_n(t) - \int_0^t \left( \frac{d^\beta I_n(\tau)}{d\tau^\beta} - i S_n(t) I_n(t) + (d + r) I_n(t) \right) (d\tau)^\beta,
\]

\[
R_{n+1}(t) = R_n(t) - \int_0^t \left( \frac{d^\beta R_n(\tau)}{d\tau^\beta} - r I_n(t) + s R_n(t) \right) (d\tau)^\beta,
\]

\[
D_{n+1}(t) = D_n(t) - \int_0^t \left( \frac{d^\beta D_n(\tau)}{d\tau^\beta} - d I_n(t) - \alpha N \right) (d\tau)^\beta.
\]

Consecutive approximations \( S_n(t), I_n(t), R_n(t) \) and \( D_n(t) \); \( n \geq 0 \) are hence calculated.

\( \tilde{S}_n, \tilde{I}_n, \tilde{R}_n \) and \( \tilde{D}_n \) are the restricted variations so, \( \delta \tilde{S}_n = 0, \delta \tilde{I}_n = 0, \delta \tilde{R}_n = 0, \delta \tilde{D}_n = 0. \)

We get \( S_{n+1}(t), I_{n+1}(t), R_{n+1}(t) \) and \( D_{n+1}(t), n \geq 0. \)

Solution is,

\[
\begin{align*}
S(t) &= \lim_{n \to \infty} S_n(t), \\
I(t) &= \lim_{n \to \infty} I_n(t), \\
R(t) &= \lim_{n \to \infty} R_n(t), \\
D(t) &= \lim_{n \to \infty} D_n(t).
\end{align*}
\]  

(5)

and

**FVIM Procedure**

**Step I.** find \( S_0 = S(0), I_0 = I(0), R_0 = R(0), D_0 = D(0) \), set \( n = 0 \);

**Step II.** use \( S_n, I_n, R_n \) and \( D_n \) to get \( S_{n+1}, I_{n+1}, R_{n+1} \) and \( D_{n+1} \) from Eq. (4);

**Step III.** define \( S_n := S_{n+1}, I_n := I_{n+1}, R_n := R_{n+1} \) and \( D_n := D_{n+1} \)

**Step IV.** if \( \max\{|S_{n+1} - S_n| < Tol, \max|I_{n+1} - I_n| < Tol, \max|R_{n+1} - R_n| < Tol \) and max \( |D_{n+1} - D_n| < Tol \) stop, otherwise continue;

**Step V.** set \( S_{n+1} := S_n, I_{n+1} := I_n, R_{n+1} := R_n \) and \( D_{n+1} := D_n \);

**Step VI.** Set \( n = n + 1 \), return to step II;

**Convergence Analysis of FVIM**

Consider operators \( P_1, P_2, P_3 \) and \( P_4 \):

\[
\begin{align*}
P_1 &= \int_0^t (-1) \left( D^\beta S_n(\tau) + i S_n(\tau) I_n(\tau) - s R_n(\tau) + \alpha N \right) (d\tau)^\beta, \\
P_2 &= \int_0^t (-1) \left( D^\beta I_n(\tau) - i S_n(\tau) I_n(\tau) + (d + r) I_n(\tau) \right) (d\tau)^\beta, \\
P_3 &= \int_0^t (-1) \left( D^\beta R_n(\tau) - r I_n(\tau) + s R_n(\tau) \right) (d\tau)^\beta, \\
P_4 &= \int_0^t (-1) \left( D^\beta D_n(\tau) - d I_n(\tau) - \alpha N \right) (d\tau)^\beta.
\end{align*}
\]  

(6)
also, state the components \( v_{k(i)}, k = 0, 1, \ldots; 1 \leq i \leq 4 \),

\[
S(t) = \lim_{n \to \infty} S_n(t) = \sum_{k=0}^{\infty} v_{k(1)}, \quad I(t) = \lim_{n \to \infty} I_n(t) = \sum_{k=0}^{\infty} v_{k(2)}, \quad R(t) = \lim_{n \to \infty} R_n(t) = \sum_{k=0}^{\infty} v_{k(3)}, \quad D(t) = \lim_{n \to \infty} D_n(t) = \sum_{k=0}^{\infty} v_{k(4)}. \tag{7}
\]

Equation (7) quickly converges [78].

**Theorem 1** [79]: Let \( P_1, P_2, P_3 \) and \( P_4 \) be the operators from Banach space BS to BS. Equation (7) converge [80, 81] if \( 0 < \xi < 1 \) exists in such a way that,

\[
\|P_i[v_{0(i)} + v_{1(i)} + v_{2(i)} + \cdots + v_{k(i)}]\| \leq \xi \|P_i[v_{0(i)} + v_{1(i)} + v_{2(i)} + \cdots + v_{k(i)}]\|, \quad 1 \leq i \leq 4,
\]

(i.e. \( v_{k+1} \leq \xi v_k \), \( \forall k \in \mathbb{N} \cup \{0\} \)).

**Theorem 2** [79]: Let Eq. (7) converge to \( S(t), I(t), R(t) \) and \( D(t) \). If \( \sum_{k=0}^{q} v_{k(1)} \) is used as approximation to \( S(t) \) of Eq. (1), maximum error \( E_q(t) \) is,

\[
E_q(t) \leq \frac{1}{1 - \xi} q^{q + 1} \|v_0\|,
\]

If \( \forall i \in \mathbb{N} \cup \{0\} \), we define,

\[
\chi = \begin{cases} 
\frac{\|v_{i+1}\|}{\|v_i\|}, & \|v_i\| \neq 0 \\
0, & \|v_i\| = 0.
\end{cases}
\]

then \( \sum_{k=0}^{\infty} v_{k(i)}, 1 \leq i \leq 4 \) of Eq. (7) converge if \( 0 \leq \chi_i < 1 \).

From Theorem 2,

\[
\left\| S(t) - \sum_{k=0}^{\infty} v_{k(1)} \right\| \leq \frac{1}{1 - \chi} q^{q + 1} \|v_0\|, \quad \text{where} \quad \chi = \max\{\chi_i, i = 0, 1, 2, \ldots, q\}.
\]

**Implementation of FVIM**

By Eq. (4),

\[
S_{n+1}(t) = S_n(t) - \int_0^t \left( \frac{d^\beta S_n(\tau)}{d\tau^\beta} + i S_n I_n - s R_n + \alpha N \right) (d\tau)^\beta,
\]

\[
I_{n+1}(t) = I_n(t) - \int_0^t \left( \frac{d^\beta I_n(\tau)}{d\tau^\beta} - i S_n I_n + (d + r) I_n \right) (d\tau)^\beta,
\]
\[ R_{n+1}(t) = R_n(t) - \int_0^t \left( \frac{d^\beta R_n(\tau)}{d\tau^\beta} - r I_n + s R_n \right)(d\tau)^\beta, \]
\[ D_{n+1}(t) = D_n(t) - \int_0^t \left( \frac{d^\beta D_n(\tau)}{d\tau^\beta} - d I_n - \alpha N \right)(d\tau)^\beta. \]

Placing \( n = 0, 1, \ldots \) in Eq. (4),
\[ S_1(t) = S_0 - (i S_0 I_0 - s R_0 + \alpha N) \frac{t^\beta}{\Gamma(1 + \beta)} = S_0 - \rho \frac{t^\beta}{\Gamma(1 + \beta)}, \]
\[ I_1(t) = I_0 + (i S_0 I_0 - d I_0 - r I_0) \frac{t^\beta}{\Gamma(1 + \beta)} = I_0 - \xi \frac{t^\beta}{\Gamma(1 + \beta)}, \]
\[ R_1(t) = R_0 + (r I_0 - s R_0) \frac{t^\beta}{\Gamma(1 + \beta)} = R_0 + \gamma \frac{t^\beta}{\Gamma(1 + \beta)}, \]
\[ D_1(t) = D_0 + (d I_0 + \alpha N) \frac{t^\beta}{\Gamma(1 + \beta)} = D_0 + \eta \frac{t^\beta}{\Gamma(1 + \beta)}, \]
\[ S_2(t) = S_0 - \rho \frac{t^\beta}{\Gamma(1 + \beta)} + \left\{ i(\rho I_0 - \xi S_0) + s \chi \right\} \frac{t^{\beta+1}}{\Gamma(1 + 2\beta)} + i \rho \xi \frac{t^3}{\Gamma(1 + \beta)(1 + 2\beta)} + \frac{(1 + 2\beta)t^3}{\Gamma(1 + 3\beta)}, \]
\[ I_2(t) = I_0 + \xi \frac{t^\beta}{\Gamma(1 + \beta)} + \left\{ i(\xi S_0 - \rho I_0) - (d + r)\xi \right\} \frac{t^{\beta+1}}{\Gamma(1 + 2\beta)} - i \rho \xi \frac{t^3}{(1 + \beta)(1 + 2\beta)} - \frac{(1 + 2\beta)t^3}{(1 + \beta)(1 + 3\beta)}, \]
\[ R_2(t) = R_0 + \gamma \frac{t^\beta}{\Gamma(1 + \beta)} + (r \xi - s \chi) \frac{t^2}{\Gamma(1 + 2\beta)}, \]
\[ D_2(t) = D_0 + \eta \frac{t^\beta}{\Gamma(1 + \beta)} + d \eta \frac{t^2}{\Gamma(1 + 2\beta)}, \]
and so on.

Here, \( \rho = (i S_0 I_0 - s R_0 + \alpha N), \ \xi = (i S_0 I_0 - (d + r)I_0), \ \chi = (r I_0 - s R_0), \ \) and \( \eta = (d I_0 + \alpha N). \)

Similarly, we can find the ensuing terms. Consequently, we get the solution as given in Eq. (5).

From Eqs. (6) and (7), the iterations for Eq. (1) are,
\[ v_{0(1)} = S_0, \quad v_{0(2)} = I_0, \quad v_{0(3)} = R_0, \quad v_{0(4)} = D_0, \]
\[ v_{1(1)} = \frac{-\rho t^\beta}{\Gamma(1 + \beta)}, \quad v_{1(2)} = \frac{-\xi t^\beta}{\Gamma(1 + \beta)}, \quad v_{1(3)} = \frac{\chi t^\beta}{\Gamma(1 + \beta)}, \text{ and } v_{1(4)} = \frac{\eta t^\beta}{\Gamma(1 + \beta)}, \]
and so on.

If we compute \( \chi_i \), we get,
\[ \chi_i = \frac{\| u_{i+1(1)} \|}{\| u_{i(1)} \|} < 1, \]
\[ \chi_i' = \frac{\| u_{i+1(2)} \|}{\| u_{i(2)} \|} < 1, \]
Fig. 1 Variation of a Susceptible $S(t)$, b Infected $I(t)$, c Recovered $R(t)$, d Death $D(t)$ population with time $t$ at diverse values of order $\beta$ of fractional derivative

Table 1 Value of parameters for simulation [19]

| S. no | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|-------|----|----|----|----|----|----|----|----|----|----|
| Parameters | $N$ | $\alpha$ | $r$ | $i$ | $s$ | $d$ | $S_0$ | $I_0$ | $R_0$ | $D_0$ |
| Values    | 1000 | 0.01 | 0.4 | 0.01 | 0.02 | 0.6 | 900 | 10 | 0 | 0 |
\[
\begin{align*}
\chi''_i &= \left\| v_{i+1}(3) \right\| \left\| v_i(3) \right\| < 1, \quad \text{and} \\
\chi'''_i &= \left\| v_{i+1}(4) \right\| \left\| v_i(4) \right\| < 1,
\end{align*}
\]

when for instance, \( i > 1 \) and \( 0 < \beta \leq 1 \).

It approves that the solution of Eq. (1) by FVIM is bounded, convergent and positive.

### Numerical Results and Discussion

Simulations are done for \( S, I, R \) and \( D \) at distinct fractional values of \( \beta \) and \( \beta = 1 \) using Maple package. The FVIM solution of Eq. (1) is achieved using value of parameters given in Table 1. Figure 1a depicts the behavior of susceptibles with time for distinct \( \beta \). It shows that the susceptibles increase with increase in value of the arbitrary order \( \beta \). Figure 1b portrays the behavior of infected ones with time for distinct \( \beta \). In it, the infected increase with decrease in \( \beta \). Fig. 1c illustrates the behavior of recovery with time for separate \( \beta \). In it, this number decreases as we increase the value of \( \beta \). Fig. 1d shows behavior of number of deaths with time for diverse \( \beta \). In it, \( D \) increases with decreasing values of \( \beta \). In Table 2 the absolute error between the successive iterations of susceptibles decrease sharply at the orders \( \beta = 0.5 \) and 1 of the fractional derivative. In Table 3 the absolute error between the consecutive approximations of infected also decrease sharply at orders \( \beta = 0.5 \) and 1. In Table 4, the absolute error between the succeeding estimates of recovered reduce at orders \( \beta = 0.5 \) and 1 while in Table 5 the absolute error between the subsequent iterates of disease induced deaths also decrease at the orders \( \beta = 0.5 \) and 1.

Ebola virus may be used for bio-terrorism so the research on producing effective vaccines is in progress. The detailed study of EVD model given by Eq. (1) is relevant. Prevention from EVD can be done by the early identification of infected and stopping its transmission. A good control of outbreak depends on applying a set of involvements, such as case management, observation and contact tracing, better laboratory facility, safe burials and social mobilization. Ebanga and Inmazeb are the two monoclonal antibodies which were approved for the treatment of EVD infection by the US Food and Drug Administration in the late 2020 [10]. A vaccine rVSVdeltaG-ZEBOV-GP is found to be safe and immunogenic till now but more

### Table 2 Error between consecutive iterations for \( S(t) \) at order \( \beta \)

| \( t \) | \( \beta = 1 \) | \( \beta = 0.50 \) |
|---|---|---|
| \( |S_1 - S_0| \) | \( |S_2 - S_1| \) | \( |S_1 - S_0| \) | \( |S_2 - S_1| \) |
| 0 | 0 | 0 | 0 | 0 |
| 0.005 | 0.50 | 0.00887066667 | 7.978845608 | 3.522509382 |
| 0.010 | 1.00 | 0.03546933333 | 11.28379167 | 7.022576161 |
| 0.015 | 1.50 | 0.07977600000 | 13.81976597 | 10.50803302 |
| 0.020 | 2.00 | 0.1417706667 | 15.95769121 | 13.98167506 |
| 0.025 | 2.50 | 0.2214333333 | 17.84124116 | 17.44511768 |
Table 3 Error between consecutive iterations for $I(t)$ at order $\beta$

| $t$     | $\beta = 1$ | $\beta = 0.50$ |
|---------|-------------|----------------|
|         | $|I_1 - I_0|$, $|I_2 - I_1|$ | $|I_1 - I_0|$, $|I_2 - I_1|$ |
| 0       | 0           | 0              |
| 0.05    | 4.00        | 0.1033208333   |
| 0.15    | 12.00       | 0.9898875000   |
| 0.25    | 20.00       | 2.916354167    |
| 0.35    | 28.00       | 6.042708333    |
| 0.45    | 36.00       | 10.5289750     |

Table 4 Error between consecutive iterations for $R(t)$ at order $\beta$

| $t$     | $\beta = 1$ | $\beta = 0.50$ |
|---------|-------------|----------------|
|         | $|R_1 - R_0|$, $|R_2 - R_1|$ | $|R_1 - R_0|$, $|R_2 - R_1|$ |
| 0       | 0           | 0              |
| 0.001   | 0.004       | 0.00001596     |
| 0.002   | 0.008       | 0.00006384     |
| 0.003   | 0.012       | 0.00014364     |
| 0.004   | 0.016       | 0.00025536     |
| 0.005   | 0.020       | 0.00039900     |

Table 5 Error between consecutive iterations for $D(t)$ at order $\beta$

| $t$     | $\beta = 1$ | $\beta = 0.50$ |
|---------|-------------|----------------|
|         | $|D_1 - D_0|$, $|D_2 - D_1|$ | $|D_1 - D_0|$, $|D_2 - D_1|$ |
| 0       | 0           | 0              |
| 0.2     | 3.20        | 0.1920         |
| 0.4     | 6.40        | 0.7680         |
| 0.6     | 9.60        | 1.7280         |
| 0.8     | 12.80       | 3.0720         |
| 1.0     | 16.00       | 4.8000         |

studies are required to stop new outbreaks. The present EVD model exposes a new feature of $\beta$ which was absent in integer order ($\beta = 1$) (Figs. 2, 3, 4 and 5).
Conclusion

Severe devastation of mankind was observed in Africa by EVD few years ago. A time-fractional EVD model of arbitrary order is examined with efficacious use of FVIM in finding its new reliable solution. FVIM scheme is capable of lessening the size of calculation. The results verify that FVIM scheme is competent even with low iterations. The convergence analysis shows that the FVIM solution is convergent, bounded, positive. Tables 2a – 2d
display that the absolute error between consecutive iterations is insignificant for $S$, $I$, $R$ and $D$. As a future work, certain hybrid methods using integral transforms and numerical methods can also be applied to solve the nonlinear fractional EVD models. A comparative study of results can be performed to obtain more accuracy. It is concluded that FVIM is highly efficient
and influential in finding the trustworthy solution to the non-integer order nonlinear models of extreme significance for society.

**Author's Contribution**  
VB investigated the results, validated the solution and was major contributor in writing manuscript. MG conceptualized, visualized the solution of fractional Ebola virus disease model and supervised the methodology used. Both the authors read the final manuscript and approved.

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**Declarations**

**Conflicts of interest**  
The authors have no conflicts of interest/competing interests to disclose.

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