Comment on “Bell inequalities and quantum mechanics” by J. H. Eberly

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Abstract

Errors in Eberly’s derivation of several Bell inequalities are pointed out: (1) it is based on an equation that is incorrect; (2) it uses neither two-particle states nor locality to derive Bell’s inequalities and; (3) it does not use entanglement to obtain violations of Bell’s inequalities. Even leading and outstanding physicists – as certainly is the case of Prof. Eberly – sometimes make elementary mistakes, and this by no means diminishes the importance of their scientific contribution. In general, this is a consequence of an excessive attachment to an idea (nowadays it has become fashionable to be against realism). This shows the importance of trying to keep an open mind.

In an article on Bell inequalities and quantum mechanics [1], Eberly uses an arrangement of optical loops to derive several Bell inequalities. As we will see, his reasoning is incorrect, and his conclusion, according to which “there really isn’t a sound when a tree falls if there is no way to record it,” is unfounded [2].

Eberly’s set-up uses “five analyzer loops, two to the left and three to the right of a photon source.” Each loop is constituted by “a pair of birefringent crystals arranged with an air gap between them, and cut and positioned in such a way that a light beam entering the first crystal is divided into orthogonally polarized components that travel separately across the air gap and are then recombined into the original beam by the second crystal.” The source emits polarization entangled photons in the state $(1/\sqrt{2})(|x\rangle|y\rangle−|y\rangle|x\rangle) = (1/\sqrt{2})(|\theta\rangle|\bar{\theta}\rangle−|\bar{\theta}\rangle|\theta\rangle)$, where the bar “is used to denote orthogonal complement. For example, $\bar{\theta} = \theta \pm 90^\circ$, and all angles are measured from the $x$ axis.” The experiment takes place in three stages. In stage 1 “The experimenter records the fraction of times a photon is detected on the right, given the detection of a $y$-polarized photon on the left. This fraction will be designated as $f(x, \phi)$ to indicate that the right-moving photon was originally $x$-polarized but was detected as $\phi$-polarized (necessarily so, because the $\bar{\phi}$ channel was blocked).” I will designate this fraction as $f_1(x, \phi)$, to make it clear that it is recorded in stage 1. In stage 2 “The experimenter records the fraction of times a photon is detected on the right, given the detection of the $x$-polarized photon on the left. This fraction will be designated as $f(y, \theta)$ to indicate that the right-moving photon was originally $y$-polarized but was detected as $\theta$-polarized.
I will designate this fraction as \( f_2(y, \theta) \), to make it clear that it is recorded in stage 2. In stage 3 “The experimenter records the fraction of times a photon is detected on the right, given the detection of the \( \theta \)-polarized photon on the left. This fraction will be designated as \( f(\theta, \phi) \) to indicate that the right-moving photon was originally \( \theta \)-polarized but was detected as \( \phi \)-polarized (necessarily so, because the \( \phi \) channel was blocked).” I will designate this fraction as \( f_3(\theta, \phi) \), to make it clear that it is recorded in stage 3.

According to Eberly, in stage 1 it appears to be obviously true that “Because we do not ask which of the \( \theta \) or \( \theta \) channels any of those photons went through in traversing the intermediate loop, we can decompose \( f(x, \phi) \) to include both possibilities, which we indicate by writing \( f(x, \phi) = f(x, \theta, \phi) + f(x, \bar{\theta}, \phi) \).” But Eberly thinks this is actually a mistake that leads to the puzzling features of the first Bell inequality he derives. However, his reasoning is incorrect. To verify this, the above expression can be rewritten as

\[
f_1(x, \phi) = f_1(x, \theta, \phi) + f_1(x, \bar{\theta}, \phi).
\] (1)

Applying the same “mistaken” reasoning to stage 2, Eberly decomposes \( f(y, \theta) \) by writing \( f(y, \theta) = f(y, \theta, \phi) + f(y, \theta, \bar{\phi}) \). This can be rewritten as

\[
f_2(y, \theta) = f_2(y, \theta, \phi) + f_2(y, \theta, \bar{\phi}).
\] (2)

Applying the “mistaken” reasoning to stage 3, Eberly decomposes \( f(\theta, \phi) \) by writing \( f(\theta, \phi) = f(x, \theta, \phi) + f(y, \theta, \phi) \), which can be rewritten as

\[
f_3(\theta, \phi) = f_3(x, \theta, \phi) + f_3(y, \theta, \phi).
\] (3)

Following his line of reasoning, Eberly writes: “By direct addition of the numbers of photons in the categories described, we see that

\[
f(x, \phi) + f(y, \theta) = f(x, \theta, \phi) + f(x, \bar{\theta}, \phi) + f(y, \theta, \phi) + f(y, \theta, \bar{\phi}).
\]

(4)

It is simple to observe that among the terms on the right-hand side of Eq. (4), we find both \( f(x, \theta, \phi) \) and \( f(y, \theta, \phi) \), and the sum of them is \( f(\theta, \phi) \). That is, another way to write Eq. (4) is

\[
f(x, \phi) + f(y, \theta) = f(\theta, \phi) + f(x, \bar{\theta}, \phi) + f(y, \theta, \bar{\phi}).
\]

(5)

If we drop the two final terms (both are positive or zero fractions), we obtain the following inequality:

\[
f(x, \phi) + f(y, \theta) \geq f(\theta, \phi),
\]

(6)

which is an example of what is called a Bell inequality, after the physicist John Bell, who first studied their consequences in quantum physics in the mid-1960s.”

And now we can see where Eberly’s mistake lies. Eq. (4) can be rewritten as

\[
f_1(x, \phi) + f_2(y, \theta) = f_1(x, \theta, \phi) + f_1(x, \bar{\theta}, \phi) + f_2(y, \theta, \phi) + f_2(y, \theta, \bar{\phi}),
\]

(7)

and there is no justification to assume that \( f_1(x, \theta, \phi) + f_2(y, \theta, \phi) = f_3(x, \theta, \phi) + f_3(y, \theta, \phi) \), as Eberly did, where \( f_1(x, \theta, \phi) \) correspond to right-moving
photons that were originally x-polarized, \(f_2(y, \theta, \phi)\) to right-moving photons that were originally y-polarized, and \(f_3(x, \theta, \phi)\) and \(f_3(y, \theta, \phi)\) to right-moving photons that were originally \(\theta\)-polarized. "A variety of inequalities that are similar to Eq. (6) [Eq. (3) in the original text]" derived by Eberly are based on the same incorrect reasoning.

It is also interesting to observe that Eberly’s argument actually involves only single particle states. We can simply ignore the two analyzer loops to the left and send right-moving x-polarized photons in stage 1, right-moving y-polarized photons in stage 2, and right-moving \(\theta\)-polarized photons in stage 3. This makes it clear that no authentic Bell inequality can be obtained in this way, since Bell’s argument requires two-particle states and the locality assumption. Moreover, although we don’t need entanglement to derive Bell’s inequalities – as correctly emphasized by Eberly – entanglement is an essential ingredient in obtaining violations of Bell inequalities.

I would like to add that, in principle, Eberly’s experiment is also valid for material particles with spin [4] (we only have to use Stern-Gerlach apparatuses to split the beams and magnetic fields to recombine them), and that it can be explained by Bohmian mechanics: with no need, however, it is important to stress, of taking into consideration Bohmian mechanics’ nonlocal features [5].

Just for the sake of completeness, I will introduce a realistic model, based on the pilot wave interpretation [5,6], that explicitly demonstrates the incorrectness of Eberly’s standpoint. In (1) we have \(f_1(x, \theta, \phi) = \cos^2 \theta \cos^2 \phi\), where \(\cos^2 \theta\) is the probability of having the photon follow the \(\theta\) channel and an empty wave follow the \(\overline{\theta}\) channel, and \(\cos^2 \phi\) is the probability of having the photon follow the \(\phi\) channel. Similarly, \(f_1(x, \overline{\theta}, \phi) = \sin^2 \theta \cos^2 \phi\), where \(\sin^2 \theta\) is the probability of having the photon follow the \(\overline{\theta}\) channel. Applying the same reasoning, in (2) we have \(f_2(y, \theta, \phi) = \sin^2 \theta \cos^2(\phi - \theta)\) and \(f_2(y, \theta, \overline{\theta}) = \sin^2 \theta \sin^2(\phi - \theta)\), and in (3) we have \(f_3(x, \theta, \phi) = \cos^2 \theta \cos^2(\phi - \theta)\) and \(f_3(y, \theta, \phi) = \sin^2 \theta \cos^2(\phi - \theta)\) [7].

References

[1] J. H. Eberly, “Bell inequalities and quantum mechanics,” Am. J. Phys. 70 (3), 276-279 (2002).

[2] The complete passage reads: “But apart from all its striking calculational successes, quantum theory brings more to physics, namely a world view that can be quite uncomfortable. To adopt an extreme phrasing, there really isn’t a sound when a tree falls if there is no way to record it.”

[3] P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, “New high-intensity source of polarization-entangled photon pairs,” Phys. Rev. Lett. 75, 4337-4341 (1995).

[4] D. Bohm, Quantum Theory (Prentice-Hall, 1951).

[5] The interested reader can consult D. Bohm and B. J. Hiley, The Undivided Universe (Routledge, 1993).

[6] It might be argued that the experiment described by Wang, Zou, and Mandel in Phys. Rev. Lett. 66, 1111-1114 (1991) invalidates the pilot wave interpre-
tation; however, as explicitly stated in their article, their experiment only
tests an idea proposed by Croca, Garuccio, Lepore, and Moreira. Similarly,
the experiment described by Zou, Grayson, Wang, and Mandel in *Phys. Rev. Lett.* 68, 3667-3669 (1992), as explicitly stated by the authors, only tests a
version of the pilot wave proposed by Selleri and Croca. But what is impor-
tant here is that the pilot wave interpretation can consistently be applied
to Eberly’s experiment, making it evident, by a concrete example, that it
admits a local realistic interpretation.

[7] Please, note that these are general conditions that a local model should fulfil, assuming ideal conditions (e.g., ideal detectors, and so on). They make it clear that – contrary to his claims – Eberly’s experiment can be interpreted from a local realistic standpoint.