New Results on Octupole Collectivity

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Abstract. Octupole correlations play an important role in determining the level structure of nuclei throughout the periodic chart. Microscopically, octupole correlations are the result of the long-range, octupole-octupole interaction between nucleons occupying pairs of orbitals which differ in both orbital and total angular momentum by 3 units. Two distinct collective modes have long been identified: octupole vibration and octupole deformation. Vibrations are characterized by low-lying negative-parity states which decay to the yrast band via enhanced E1 transitions. If static octupole deformation is present, the yrast spectrum of an even-even nucleus is characterized by interweaving positive- and negative-parity states. The asymmetric shape results in a strong electric dipole moment. The enhanced electric dipole, $E1$, transitions between positive- and negative-parity states compete in deformed nuclei with electric quadrupole transitions, $E2$, which are characteristic of rotational bands. Evidence for static octupole deformation has only been found in selective areas of the nuclear chart, when orbitals which differ by $\Delta j, \Delta l = 3$ approach the Fermi level for both protons and neutrons. Most of these examples are located in the $A \sim 222$ and $A \sim 146$ regions. The orbitals responsible for this enhanced octupole collectivity are summarized in figure 1. Two excellent review articles describing both experimental observables and theoretical descriptions of octupole collectivity can be found in references [1] and [2].

In this paper, emphasis is placed on new results from the actinide region [3,4], where negative-parity structures appear to evolve with increasing angular momentum from an octupole vibration into the rotation of a static octupole deformed shape. Additionally, newly observed rotational structures built on an excited $0^+$ state have been tentatively associated with a double-octupole phonon excitation.
in $^{238}$U and $^{240}$Pu. These newly observed properties can be described well by a model based on the concept of rotational-aligned octupole phonon condensation [5].

2. Experimental Techniques
Both the $^{A\sim 222}$ and $^{A\sim 144}$ regions are difficult to study experimentally. For the $^{A\sim 222}$ region, the difficulty lies in the fact that fission dominates the cross sections associated with fusion evaporation. In the pioneering work by Ward et al., it was shown that actinide nuclei could be studied to high-spin using inelastic excitation of the target ($^{238}$U) with a heavy projectile ($^{209}$Bi) [6]. The large cross-sections for inelastic excitation and the relatively small cross section for fission, enabled numerous rotational bands in $^{238}$U to be identified up to $I\sim 30\hbar$. Many of the rotational bands identified in $^{238}$U were associated with vibrational excitations. While the work of ref. [6] only looked at states associated with the excitation of the target, Cocks et al. demonstrated that multi-nucleon transfer could also be used to populate high-spin states in actinide nuclei [7]. Utilizing a $^{232}$Th target and a $^{136}$Xe beam, ref. [7] was able to study both the yrast and lowest-lying negative-parity bands in even-even Ra and Th nuclei to $I>20\hbar$, using the Gammasphere detector array [8]. As a consequence, the evolution of octupole collectivity in these $^{A\sim 222}$ nuclei was characterized to relatively high spins for the first time. Most recently, Reviol et al. have shown that fusion evaporation reactions can be used to study high-spin states in neutron-deficient actinide nuclei using a recoil detector, Hercules [9], in conjunction with Gammasphere in order to discriminate between reactions resulting in fission from those resulting in fusion evaporation [10]. These studies have been able to characterize octupole collectivity in $^{220}$Th where the quadrupole deformation is quite small due to the proximity to the $N=126$ shell gap.

The $^{A\sim 144}$ octupole region poses quite a different challenge to study experimentally. The Ba-Xe nuclei of interest are neutron rich and cannot be made in fusion evaporation reactions using stable beam-target combinations. These nuclei have been traditionally studied to moderate spins following the spontaneous fission of $^{252}$Cf and $^{244}$Cm [11]. In the coming year, study of re-accelerated neutron-rich Ba and Xe nuclei will be possible at the ATLAS facility at Argonne National Laboratory, once the CARIBU upgrade is completed. Using multi-step Coulomb excitation, the measurement of $B(E1)$ and $B(E3)$ matrix elements associated with octupole states in $^{144}$Ba and neighboring nuclei will provide critical information to enhance our understanding of octupole collectivity in this region.
3. Aligned Phonon Condensation

Recently, Frauendorf has proposed an alternative explanation for the alternating parity sequences traditionally associated with static octupole deformation. Details of this description can be found in ref. [5]. This description begins by pointing out that the total energy of a nucleus in an harmonic $n$-phonon state with frequency $\Omega_3$, rotating with angular velocity $\omega$, is

$$E_n = \hbar \Omega_3 (n + 1/2) + \omega^2 \Omega/2,$$

which is the sum of the rotational and vibrational energies. By transforming this energy into the rotating frame (Routhian) and recognizing that the lowest energy for an $n$-phonon state occurs when all the phonons are aligned with the rotational axis, the energy relative to the 0-phonon excitation for the yrast band of an $n$-phonon state can be written as

$$E_n'(\omega) - E_0'(\omega) = n\hbar \Omega_3 - n\omega,$$

where $i$ is the angular momentum carried by one phonon. This equation is linear in $\omega$ with the initial excitation energy given by $n\hbar \Omega_2$ and an alignment of $ni$. All $n$-phonon states will cross the 0-phonon state at a critical rotational frequency $\omega_c = \Omega_3/i$, where the $n$-phonon states form a condensate.

In the laboratory frame, the consequences on the energy spectrum for the lowest lying positive- and negative-parity states is as follows: in the region of spin where the 1-phonon band crosses the 0-phonon band, the positive- and negative-parity sequences appear to be interweaved. As the 1-phonon states begin to separate from the 0-phonon states, the two-phonon band approaches the one-phonon band, thus extending the appearance that the positive- and negative-parity states are interweaved. Depending on the anharmonicity of the vibration, this sequence of band crossings extends over an extensive region of spin. It is seen as an oscillation of the energy differences between the positive- and negative yrast levels. In ref. [5], it was suggested that the behavior of the yrast positive- and negative-parity states in $^{220}$Ra is consistent with the description of aligned multi-phonon states. If this description is correct, one would expect to observe $n>1$ phonon excitations, but at the time ref. [5] was published, no reports of $n>1$ octupole phonon states in nuclei reported to possess static octupole deformation were available.

4. New Results on $^{240}$Pu and $^{238}$U

With the siting of Gammasphere at the ATLAS facility in 1998, Wiedenhoever et al. performed a study of high-spin states in $^{238,244}$Pu isotopes following inelastic excitation of isotopically enriched Pu targets with a $^{208}$Pb beam [12,13]. This measurement allowed both the yrast and lowest-lying negative-parity bands in a series of Pu isotopes to be studied. A summary of these results is presented in figure 2 where the single-particle alignments, $i_x$, are plotted as a function of the rotational frequency, $\hbar \omega = E_\gamma/2$. The striking feature of these plots is the absence of the first band crossing in $^{238,239,240}$Pu. This absence is not understood in the context of the cranked shell model assuming only moderate quadrupole deformation. In addition, for $^{239,240}$Pu the negative-parity bands begin to become interweaved with the positive-parity yrast levels at the highest observed spins, suggesting the onset of octupole deformation. While the observation of possible octupole deformation at $N=146$ was somewhat surprising, Sheline and Riley have suggested, following an analysis of $\alpha$-decay hindrance factors and an examination of the excitation energies of the lowest lying negative-parity states, that the U and Pu isotopes around $N=146$ might exhibit enhanced octupole collectivity [14]. In ref. [12], it was suggested that the absence of the first band crossing in these select Pu isotopes could be a consequence of rotationally-induced octupole deformation.
Figure 2. Single-particle alignments as a function of rotational frequency for rotational bands in $^{238,244}\text{Pu}$ (taken from ref. [12]), where $i_x = I(\omega) - I(\omega)_{ref}$. Open circles represent the yrast bands, solid circles represent bands built on the lowest lying octupole vibrational state.

4.1. $^{240}\text{Pu}$

Recently, a new measurement was performed with Gammasphere using a $^{240}\text{Pu}$ target and a $^{208}\text{Pb}$ beam with $E_{\text{beam}} = 1300$ MeV in order to extend both the yrast and negative-parity bands to higher spins and to look for $E1$ transitions connecting positive- and negative-parity states at the highest observed spins. A partial level scheme resulting from this work is given in Figure 3. With regards to bands 1 and 2, two new states have been added to both bands. In addition, $\Delta I=1$, $E1$ transitions are now observed connecting positive- to negative- and negative- to positive-parity states at the highest spins where the states of the two bands are interweaved. Induced intrinsic dipole moments, $D_0$, have been deduced for the $25^+$, $26^+$ and $28^+$ states using the in-band and out-band branching ratios and assuming a constant electric quadrupole moment, $Q_0 = 11.6$ eb. For all three states, $D_0 > 0.2 e$ fm, and these values are similar to those reported for the $A\sim222$ Ra and Th isotopes, which are thought to exhibit characteristics best described by the presence of static octupole deformation [2]. Bands 1 and 2 develop these same features at the highest spin, and thus, a description of static octupole deformation induced by rotation for $I>20 \hbar$ in $^{240}\text{Pu}$ is consistent with the data.

However, the observation of a new rotational band built upon the previously identified $0^+$ state at 861 keV adds crucial information on octupole collectivity in $^{240}\text{Pu}$. As illustrated in the level scheme of Figure 3, this band decays exclusively to band 2 which exhibits strong octupole collectivity. In addition, the $0^+$ band head lies $\sim 1.5$ times higher in energy than the $1^+$ band head of band 2. These characteristics suggest that band 3 can be associated with a double-octupole phonon excitation at low spin and excitation energy.

In order to characterize these three bands as a function spin, both the single-particle alignments and the single-particle Routhians are given on the right side of Figure 3. The alignment plots for all bands indicate the absence of the first band crossing which is observed in nearly all other neighbouring nuclei to occur at $\hbar\omega \sim 0.25$ MeV (see Figure 2). Band 2 slowly gains alignment with increasing rotational frequency and levels off at $3\hbar$. This behaviour is consistent with the alignment of a single
octupole phonon with the rotational axis, indicating that band 1 retains its phonon character up to the highest observed spins. Band 3 rapidly gains $\sim 5\hbar$ beginning at $\hbar\omega \sim 0.1$ MeV. Two aligned octupole phonons are expected to have an alignment $i_x = 6\hbar$. However, as band 3 approaches band 1 in energy, states of the same spin and parity will mix. This mixing results in a slightly reduced alignment for band 3 and a small increase in alignment ($\sim 1\hbar$) for band 1. Thus, the alignment plots for all three bands are consistent with a description which associates bands 1, 2 and 3 with $0E$, $1E$ and $2E$-octupole phonon excitations up to $I \sim 30\hbar$. An examination of the Routhian plots is also instructive. Band 2 clearly crosses band 1 at $\hbar\omega \sim 0.3$ MeV, and an extrapolation of band 3 suggests it will encounter band 1 at $\hbar\omega \sim 0.35$ MeV. In the phonon condensation description proposed by Frauendorf [5], all aligned multi-phonon bands should condense (cross) at the same critical rotational frequency. However, this assumes harmonic vibrations which is not the case for $^{240}$Pu as evidenced by the fact that the proposed 2-phonon $0^+$ state is only 1.5 times higher in excitation energy than the 1-phonon one and not 2 times higher as expected for harmonic vibrations. As a consequence, the fact that bands 2 and 3 do not cross band 1 at the same rotational frequency is not at odds with the phonon condensation description. If

![Figure 3](image_url)

**Figure 3.** Left side: Partial level scheme for $^{240}$Pu as given in ref. [3]. Right side top: Single-particle alignment as a function of rotational frequency deduced for bands 1, 2 and 3 in $^{240}$Pu. The inset provides the relative alignment $\Delta i_x$ of band 2 with respect to band 1. Right side bottom: Single-particle Routhians as a function of angular momentum for states in bands 1, 2 and 3 of $^{240}$Pu. The inset gives the excitation energy ($E_x$) for the three bands as a function of spin ($I$).
there are unharmonicities, the 0-phonon state couples to the 2-phonon one, resulting in a repulsion between the levels. Such an avoided crossing looks like a delayed one, when only the low-frequency part is observed. Hence, all observables for the three bands in $^{240}\text{Pu}$ are consistent with the condensation interpretation. This is the first time, to our knowledge, that the 2-phonon excitation has been observed to such high spins, and its observation is critical in establishing whether or not the condensation description is valid for nuclei in the actinide region.

![Figure 4. Left side: Single-particle alignment as a function of rotational frequency deduced for bands 1, 2 and 3 in $^{238}\text{U}$. Right side: Single-particle Routhians as a function of angular momentum for states in bands 1, 2 and 3 of $^{238}\text{U}$.](image)

4.2. $^{238}\text{U}$

New experimental results have also become available for $^{238}\text{U}$, and a candidate 2-phonon octupole band has been identified [4] in this nucleus as well. The decay out of the band is more fragmented than that observed for $^{240}\text{Pu}$ with the majority of the decay proceeding through the 1-phonon band while decays to both the yrast and higher, $K=1$, 1-phonon band are also observed. While further details concerning the level structure of $^{238}\text{U}$ can be found in ref. [4], only the alignment and Routhian plots are presented here. The alignments for bands 1 and 2 behave similarly to the corresponding bands in $^{240}\text{Pu}$ at low frequency, and band 2 does appear to align its octupole phonon with the rotational axis. For $\hbar\omega > 0.20 \text{ MeV}$, both bands undergo a rapid increase in alignment. For the yrast band, this corresponds to the alignment of a pair of $i_{13/2}$ protons as the yrast band transitions to a two-quasiparticle structure. Band 2 also undergoes a transitions to a two-quasiparticle structure, but in order to maintain the correct parity, the 1-phonon band is likely to be crossed by a band having the configuration $\pi(i_{13/2}\otimes h_{9/2})$. As a consequence of crossing with the two-quasiparticle bands, the phonon states are destroyed and condensation is never reached as illustrated in the Routhian plot of Figure 4.

With regards to band 3, its alignment behaviour is similar to that of the 2-phonon band in $^{240}\text{Pu}$ except at the highest rotational frequencies. In contrast to bands 1 and 2, band 3 appears to not be crossed by a two-quasiparticle excitation. Estimates of the excitation energy of the $\pi(i_{13/2})^2$ band in $^{238}\text{U}$ indicate that this two-quasiparticle excitation lies lower in excitation energy than band 3. As a consequence, band 3 is never crossed by this quasiparticle structure, and thus retains its 2-phonon character up to the highest observed spins. In contrast, the 2-phonon state in $^{240}\text{Pu}$ lies lower in excitation energy than the $\pi(i_{13/2})^2$ band. It has been suggested in both refs. [3] and [4] that the low excitation energy of the 2-phonon band is the cause for the absence of the expected $i_{13/2}$ proton band crossing in $^{240}\text{Pu}$. In other words, the 2-phonon and possibly higher multi-phonon states interact and repel the $\pi(i_{13/2})^2$ band as it approaches the yrast line. As a consequence, the expected band crossing is delayed to higher rotational frequencies. In contrast, the multi-phonon bands lie higher in energy in $^{238}\text{U}$ and are unable to prevent either the yrast or 1-phonon band from crossing with the two-
quasiparticle states. Thus, the only fundamental difference between the 1- and 2-phonon states observed in $^{238}$U and $^{240}$Pu appears to be their excitation energies relative to the $\pi(i_{13/2})^2$ band.

5. Summary
Over the last decade, new experimental results have been obtained in actinide nuclei and they have provided a better characterization of octupole collectivity at high spins. These new results have been obtained using different types of reactions including, inelastic excitation of the target with heavy beams, multi-nucleon transfer, and fusion evaporation. All these measurements utilize Gammasphere, and in the case of fusion evaporation, the recoil detector, Hercules. Recent experimental results show that strong octupole collectivity develops at high spin in $^{240}$Pu, and candidate double-octupole phonon bands have been observed in $^{240}$Pu and $^{238}$U. These observations are consistent with the concept of “octupole condensation” proposed in ref. [5]. This description appears to be applicable to many nuclei in this mass region and is an alternative description to the mean-field model which attempts to describe these nuclei in terms of a static octupole deformation.

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