Eight types of physical "arrows" distinguished by Newtonian space-time symmetry.

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The paper draws the attention to the spatiotemporal symmetry of various vector-like physical quantities. The symmetry is specified by their invariance under the action of symmetry operations of the Opechowski nonrelativistic space-time rotation group $O(3) \otimes \{1, 1'\} = O'(3)$, where $1'$ is time-reversal operation. It is argued that along with the canonical polar vector, there are another 7 symmetrically distinct classes of stationary physical quantities, which can be – and often are – denoted as standard three-components vectors, even though they do not transform as a static polar vector under all operations of $O'(3)$. The octet of symmetrically distinct "directional quantities" can be exemplified by: two kinds of polar vectors (electric dipole moment $P$ and magnetic toroidal moment $T$), two kinds of axial vectors (magnetization $M$ and electric toroidal moment $G$), two kinds of chiral "bi-directors" $C$ and $F$ (associated with the so-called true and false chirality, resp.) and still another two achiral "bi-directors" $N$ and $L$, transforming as the nematic liquid crystal order parameter and as the antiferromagnetic order parameter of the hematite crystal $\alpha$-Fe$_2$O$_3$, respectively.

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Physical quantities defined by a magnitude and an oriented axis in 3D space are often represented by three-component Euclidean vectors. Frequently, polar and axial (or pseudo-) vectors are distinguished, depending on whether they change their sense or not, respectively, upon the operation of spatial inversion (parity operation $\bar{1}$.\footnote{For classification of temporal processes or magnetic phenomena of vectorial nature, the action of the time-inversion operator ($1'$) can be used. For example, magnetization $M$ and magnetic field vector $H$ are "time-odd axial" vectors (preserved by $1$ operation but changing their sign under the $1'$ operation), electric polarization $P$ or electric field $E$ are "time-even polar" vectors, while other quantities like velocity $v$ or toroidal moment $T$ are "time-odd polar" vectors.\footnote{The two inversion operations generate an Abelian group of 4 elements $\{1, 1', I', I\}$ and 4 one-dimensional irreducible representations.}} For classification of temporal processes or magnetic phenomena of vectorial nature, the action of the time-inversion operator ($1'$) can be used. For example, magnetization $M$ and magnetic field vector $H$ are "time-odd axial" vectors (preserved by $1$ operation but changing their sign under the $1'$ operation), electric polarization $P$ or electric field $E$ are "time-even polar" vectors, while other quantities like velocity $v$ or toroidal moment $T$ are "time-odd polar" vectors.\footnote{The two inversion operations generate an Abelian group of 4 elements $\{1, 1', I', I\}$ and 4 one-dimensional irreducible representations; the symmetry operations this group allows to classify these vectors into 4 categories (see Table I).} The two inversion operations generate an Abelian group of 4 elements $\{1, 1', I', I\}$ and 4 one-dimensional irreducible representations; the symmetry operations this group allows to classify these vectors into 4 categories (see Table I).

The aim of this paper is to emphasize that there are another four types of quantities, which are also defined by a magnitude, an axis and a geometrical sign, and which are also often associated by 3D vectors, but which possess a different spatio-temporal symmetry than the examples given in Table I. We are going to specify here all 8 types of "directional quantities" (i) by describing their transformation properties under the action of the elements of the Opechowski general space-time rotation group $O(3) \otimes \{1, 1'\} = O'(3)$, (ii) by enumerating the associated limiting groups defining their symmetry invariance, (iii) by providing several examples to each case. We shall also briefly discuss possibilities and difficulties with introduction of formal algebraic manipulations. Simultaneous considerations about all 8 different types of such directional quantities can be useful in various areas of physics.

Basic symmetry argument. These 8 symmetrically different species are resumed pictographically in Fig. 1. Why do we have just 8 of such quantities? Let us consider any stationary physical quantity $X$ (attached to a physical object), which simultaneously defines a two-valued, geometry-related sign, a nonnegative magnitude and a unique 1D linear subspace of 3D Euclidean space (an axis of this quantity), but nothing more. Since the quantity $X$ defines a unique axis in the space, the symme-

| $1$ | $1'$ | $I'$ | $I$ |
|---|---|---|---|
| $1$ | $1$ | $1$ | electric toroidal moment $G$ |
| $1$ | $-1$ | $1$ | electric dipole moment $P$ |
| $1$ | $-1$ | $-1$ | magnetic dipole moment $M$ |
| $1$ | $-1$ | $1$ | (magnetic) toroidal moment $T$ |

TABLE I: Action of space ($\bar{1}$) and time ($1'$) inversion operations on selected examples of vectorial quantities; $1$ stands for the invariance, $-1$ stands for the sign reversal.\footnote{TABLE I: Action of space ($\bar{1}$) and time ($1'$) inversion operations on selected examples of vectorial quantities; $1$ stands for the invariance, $-1$ stands for the sign reversal.}
try of $X$ can be classified by those $O(3),I'$ group operations that leave this axis invariant. Such operations form an infinite subgroup of $O(3),I'$ that could be denoted as the $\infty/mm.1'(D_{\infty h})$ group. Moreover, it is natural to postulate that the magnitude of $X$ ($|X| \geq 0$) does not change under the operations of $O(3), I'$ group. This implies that transformation properties of $X$ can be fully defined by specifying how its sign is changed when elements of $\infty/mm.1'$ are applied to it. Since we restrict ourselves only to the quantities for which the sign of $X$ can have only one of the two possible values, the symmetry operation can either preserve the sign or change it to the opposite one. In other words, the action of the associated $\infty/mm.1'$ group operations consist in multiplication of the geometrical sign of $X$ either by 1 or by -1. In terms of theory of groups, this implies that $X$ transforms as one-dimensional (necessarily irreducible) representation of the associated $\infty/mm.1'$ group. It is known that the $\infty/mm$ ($D_{\infty h}$) group has 4 distinct one-dimensional irreducible representations so the $\infty/mm.1'(D_{\infty h} \otimes 1,1')$ one has twice as much of them. Therefore, the physical quantities defined by a sign, a magnitude and an axis can be classified in 8 symmetrically different categories.

**Classification by irreducible representations and basic examples.** The list of all one-dimensional irreducible representations of the $\infty/mm.1'$ group is given in Table III. First column gives the irreducible representations label following the convention used e.g. in Refs. [4, 12] resp., the last column contains a letter symbol used in Table III and in Fig. 1. Remaining columns in the table are associated with the classes of symmetry elements of the $\infty/mm.1'$ group. There are various physical quantities having the listed transformation properties. For example, polarization ($P$) and magnetization ($M$) transform as $A_{1v}(\Sigma_g^\pm)$ and $m_{A_{2g}}(\Sigma_g^\pm)$ irreducible representations, resp. The symbol $T$ invokes the often discussed toroidisation or toroidal moment, [17–21] even though there are many other more frequently used quantities that also transform as the $A_{1v}(\Sigma_g^\pm)$ representation, such as electric current, momentum or velocity of a particle, vector potential or the Poynting vector $S = E \times H$. It is clear from Table III that this "magnetic" toroidal moment $T$ has a different symmetry than the "electric" toroidal moment $G$, the latter exploited e.g. for characterization of electric polarization vortex states of small ferroelectric particles [22–24] or poloidal spin currents [25]. Recently, spontaneous magnetic toroidization has been found e.g. in Ba$_2$CoGe$_2$O$_7$ crystal [20], the G-type distortion has been identified e.g. in the "ferroaxial" structures of CaMn$_2$O$_{12}$ and RbFe(MoO$_4$)$_2$ crystals [27–29].

Let us note that $G$ and $M$ are symmetric with respect to the perpendicular mirror plane operation $m_{\perp}$ and $P$ and $T$ are symmetric with respect to the parallel mirror plane $m_{||}$. Thus, none of these quantities is chiral [30]. In fact, only two irreducible representations from the Table III fulfill the group theoretical condition of a chiral object (absence of improper rotation symmetry, such as center of inversion or mirror planes [31]: $A_{2u}$ and $m_{A_{2u}}$. They are naturally suitable for representation of chiral directional quantities, as their geometrical sign can reflect the sign of their enantiomorphism. For example, a helix might be characterized by its axis, the magnitude (given by the pitch of the helix) and a geometrical sign, indicating whether the helix is right-handed or left-handed. Such a chiral quantity $C$ transforms as $A_{2u}$ irreducible representation. As a beautiful example of $m_{A_{2u}}$ quantity ($F$) can be taken the antiferromagnetic order parameter of the linear-magnetoelastic chromite crystal Cr$_2$O$_3$, [31, 32] This latter kind of chirality, reversible upon time reversal, is sometimes called "false chirality" [30, 34].

Finally, there are also two irreducible representations symmetric with respect to both $m_{||}$ and $m_{\perp}$ (L and N). The time-odd variant (L) can be used to describe another type of directional antiferromagnetic order parameter, e.g. in the hematite crystal α-Fe$_2$O$_3$, [32]. The fully sym-

| irr. repr. | $E$ | $I'$ | $m_{||}$ | $2_1$ | $1'$ | $m_{\perp}$ | $2'_1$ | symbol |
|-----------|----|------|---------|------|------|---------|------|--------|
| $\infty$  | 1  | 1   | 1       | 1    | 1    | 1       | 1    | G      |
| 2         | 1  | -1  | -1      | -1   | -1   | -1      | -1   | P      |
| $\infty'$ | 1  | 1   | 1       | 1    | 1    | 1       | 1    | M      |
| $m_{\perp}$ | 1  | -1  | -1      | -1   | -1   | -1      | -1   | T      |
| $m_{A_{2g}}(\Sigma_g^\pm)$ | 1  | -1  | -1      | -1   | -1   | -1      | -1   | F      |

**TABLE III:** List of 8 symmetrically distinct "arrow" quantities and their transformation under three independent operations $\infty/mm.1'(D_{\infty h})$ group attached to the axis. ($m_{||}$ stands for any mirror plane operation parallel to the axis.)
metric (\(A_1\)) representation is perhaps the most singular one. It can be associated with the so-called director, exploited in the theory of liquid crystals to characterize the spontaneously parallel spatial orientation of rod-like molecules in nematic phases\(^{[55]}\). In this particular case there is no reason to define its geometrical sign. However, there are other \(N\)-like quantities that do have a sign. For example, a consistently defined Frank vector of a wedge disclination\(^{[54,58]}\) should allow to distinguish whether the disclination can be formed by removing or inserting material body adjacent to the plane of the cut. At the same time, this disclination itself is invariant against all operations of the \(\infty/mnm.1'(D_{\infty h})\) group.\(^{[56]}\)

**Classification in terms of symmetry invariance groups.** Table II defines fully transformation properties of various uniaxial quantities discussed above. For many purposes, it is enough to consider only those symmetry operations, which leave the quantity invariant.\(^{[10,11,39]}\) Such operations form infinite subgroups of the \(\infty/mnm.1'\) group. They are listed for each irreducible representation in Table II. The content of these invariance groups can be easily figured out from the pictographic symbols in Fig. 1. In addition, each pictograph shows a segment indicating the magnitude of the quantity and an arrow associated with its geometric sign (see Figs. 1 and 2). Arrows in pictographs drawn by dashed lines should be considered as indicating a stationary current or motion (time inversion operation does change their sense). This is the case of time-odd quantities (\(L, M, T, F\)). In contrast, the arrows in pictographs drawn by full lines should be considered as time-irreversible (time inversion operation does not change them, as they have a grey-group\(^{[12]}\) symmetry). These pictographs stands for the time-even quantities \(N, G, P, C\). Let us note that the \(P, T, N, L\) quantities, symmetric with respect to the parallel mirror plane operation \(m_{\parallel}\), have arrows only in the radial direction, while \(m_{\perp}\)-antisymmetric quantities, \(G, M, C, F\), have all only tangential arrows (bend arrows should be understood as drawn on a visible curved surface of a coaxial circular cylinder.) One can also easily distinguish the single-arrow pictographs of \(2_\perp\)-antisymmetric quantities \(G, P, M, T\) (proper vectors) from all the double-arrow graphical symbols standing for \(2_\perp\)-invariant quantities \(N, C, L, F\), which we call here as bi-directors.

**Meaning of the geometric parity signs, bi-directors.** The fact that the parity sign can be represented in this way emphasizes its geometrical nature. Obviously, the *strict meaning of the parity sign* relies on some convention, too. For example, the vector of electric dipole moment is taken as pointing towards the center of the positive charge (and not the opposite), the arrow associated with the velocity of a particle is drawn towards its future position (and not the opposite), the sense of the electric current refers normally to the velocity of the positive charges, and the arrow in the pictograph standing for magnetic dipole moment is that of the equivalent positive stationary electric current circulating around the indicated axis.

| \(\sigma\) | \(1\) | \(1'\) | examples |
|---|---|---|---|
| \(G, G, T, T, P, P, M, M, \nabla P\) | mass, charge |
| \(\epsilon\) | \(-1\) | \(1\) | \(P, G, T, M\) |
| \(\tau\) | \(-1\) | \(1\) | \(M, G, T, P\) | time |
| \(\mu\) | \(-1\) | \(1\) | \(T, G, M, P\) | magnetic monopole |

**TABLE IV:** Four scalar types\(^{[4]}\) specified according to their invariance under space-inversion and time-reversal operations (time-even scalar \(\sigma\), time-even pseudoscalar \(\epsilon\), time-odd scalar \(\tau\) and time-odd pseudoscalar \(\mu\)).

Another set of conventions is needed to facilitate the *algebraic representation* of such quantities. Typically, a polar vector is represented by three coordinates defined by its scalar-product projections to an oriented set of three orthonormal basis vectors. It is so practical that we tend to represent all other quantities in a similar way.

In case of "true" vector quantities (those of Table II), such algebraic representation is usually defined through the time derivatives and vectorial products or equivalent rules. In fact, this representation justify the common usage of the simple \(P\)-arrow pictograph for all other vector quantities of the Table II. For example, magnetic moment \(\mathbf{m}\) of a current turn is defined as a vector perpendicular to the turn and directed so that the current observed from the end of vector \(\mathbf{m}\) envelops the turn counterclockwise.\(^{[10]}\) Therefore, the pictograph for \(\mathbf{M}\) (as well as for \(\mathbf{G}\) and \(\mathbf{T}\)) can be formally replaced by that of \(\mathbf{P}\), even though these quantities actually do have a different symmetry (in fact, Fig. 1 could conveniently serve as the replacement table). Moreover, this algebraic representation allows to calculate any scalar and vectorial products in the usual way. Interestingly, vectorial products of true vectors are true vectors and scalar products of true vectors transforms as one of the four possible scalar species\(^{[4]}\) (time-even scalar \(\sigma\), time-even pseudoscalar \(\epsilon\), time-odd scalar \(\tau\) and time-odd pseudoscalar \(\mu\), see Table IV).

In case of bi-directors, none of the SO(3) operations can reverse their geometrical sign. It indicates the fundamental difficulty with representation of bi-directors by three-component algebraic vectors. In fact, each of these bi-director quantities transforms as an "antitandem" arrangement of two vectors - as a couple ("dipole") of two opposite vectors \(X_1\) and \(X_2\) ((\(X_1 = -X_2\)) arranged on a common axis at some nonzero distance \(r_{21} = r_2 - r_1\)). Obviously, \(N\) transforms as an antitandem of two \(P\) vectors, \(C\) as a antitandem of two \(G\) vectors, \(L\) as a \(T\) vector antitandem, \(F\) as a \(M\) vector antitandem. Therefore, a bi-director can be represented by a simple "two-body" term \(a_{12} = X_2 - X_1\). Here it is assumed that the symmetry operations act both on the vectors and their position - operations that change \(r_{21}\) to the opposite are actually interchanging the sites 1 and 2. The geometrical parity sign of such antitandem quantities could be denoted as inward or outward, depending whether the vector \(X_2\) is parallel or antiparallel to the vector \(r_{21}\), and so its
evaluation actually requires to know two quantities at a time, $X_2$ and $r_{21}$. Having this in mind, a range of algebraic operations can be nevertheless easily extended to all the above vectors and bi-directors. For the sake of convenience, types of the quantities obtained as vectorial cross products or as multiplication by a scalar are given in Table V. Let us also note that from symmetry point of view, time derivative acts here as multiplication by the time-odd scalar $\tau$, so that e.g. the time derivative of the bi-director $L$ transforms as the bi-director $N$ and vice versa.

**Classification of axes and concluding remarks.** In general, a physical object may have a physical property transforming as one of the 8 discussed cases only if its symmetry invariance group is a subgroup of the corresponding limiting group. For example, macroscopic magnetization can exist only in crystals belonging to 31 different Heesch-Shubnikov point groups, that are subgroups of $\infty/mm\`$ group. If the axis of the limiting supergroup coincides with the symmetry axis of the object, it is often named according to the associated property (ferromagnetic axis, polar axis). Other axes could be similarly labeled as toroidal, truly-chiral, falsely-chiral, $G$-axis, fully-symmetric and so on.

The term vector is sometimes employed to describe phenomena that have a bi-director symmetry. For example, so-called Burgers vectors is widely used to characterize screw dislocations, which are obviously non-polar, truly chiral ($C$-type) objects. Similarly, the antiferromagnetic vector $[31-33]$ is often used to describe the falsely-chiral ($F$-type) antiferroelectric order. On the contrary, the so-called "chiral vector" or "vector chirality" is sometimes used to characterize cyclic spin arrangements on spin loops, for example in triangular antiferromagnetic lattices, even if the spin arrangement happens to have toroidal symmetry, which is "unidirectorial" but achiral (similarly as spin cycloids and Néel domain walls).[45]

Finally, it is well known that axial vector $G$ can be represented as a polar antisymmetric second-order tensor. The bi-director quantities can be also classified within the established tensorial calculus. They correspond to a special kind of second rank tensors, that has been once coined in Russian literature as the "simplest tensor" (i.e. symmetrical second rank tensor having in its canonical form only a single nonzero element).[3] In particular, $N$-type bi-director could be considered as dual to the simplest time-even polar tensor, $C$-type bi-director transforms as the simplest time-even axial tensor, $L$-type bi-director as the simplest time-odd polar tensor and $F$-type bi-director as the simplest time-odd axial tensor.

Nevertheless, we think that the unifying classification via irreducible representations of $\infty/m\`$ still provides a very practical concept, applicable in various areas of physics. In solid state physics at least, the simple perspective where vectors and bi-directors have equal legitimacy might be useful when dealing with problems where several such quantities are interacting, for classification of long-wavelength excitations or structural components of magnetoelectric multiferroic crystals, for description of macroscopic properties of chiral objects, or for classification of topological defects like domain walls, magnetic vortices or skyrmions. In fact, we would like to offer a more complete discussion of possible applications of this concept in future and so we would be grateful to learn about other cases where this perspective could bring some useful insight.

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