Research Article

Steady-State Analysis and Comparison of Control Strategies for PMSM

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Abstract

Permanent Magnet Synchronous Motor (PMSM) has been considered as the best choice for numerous applications. To make PMSM a high performance drive, effective control system is required. Vector control is accepted widely due to its decoupling effect but it is not the only performance requirement. Additional control methods such as constant torque angle control (CTAC), optimum torque per ampere control (OTPAC), unity power factor control (UPFC), constant mutual flux linkages control (CMFLC), and angle control of air gap flux and current phasor (ACAGF) can also be implemented. This paper therefore presents some important control strategies for PMSM along with merits and limitations which provide a wide variety of control choices in many applications. The performance characteristics for each strategy under steady state are modelled and simulated in MATLAB environment. Based on the simulation results, a conclusion is drawn that OTPAC is superior in normalized torque per unit normalized stator current ($\frac{T_{e}}{i_{s}}$) ratio whereas UPFC yields very low $\frac{T_{e}}{i_{s}}$ ratio. In addition, performances of these control strategies are compared, which is a key to select optimum strategy depending on requirements. Based on the comparative study, it can be concluded that CMFLC is superior to CTAC, ACAGF, OTPAC, and UPFC. Hence, it can be a good control strategy to consider.

1. Introduction

Recently, PMSM drive has emerged as a top competitor amongst AC drives for industrial servo drives, hybrid electric vehicles, and other applications due to features like high speed, low power waste, large starting torque, high power factor, and high efficiency [1–4]. Also control of PMSM is comparatively simpler than that of induction motor and high performance of PMSM can be achieved by means of vector control as it provides decoupled control of torque and flux [5, 6]. But decoupled control of torque and flux is not only the performance requirement for PMSM drive [7]. Therefore, in this paper, different control strategies such as constant torque angle control, optimum torque per ampere control, unity power factor control, constant mutual air gap flux linkages control, and angle control of air gap flux and current phasor are considered in detail for the variable speed motor drive. For the speeds lower than base speed, the control strategies for PMSM are constant torque angle control, optimum torque per ampere control, unity power factor control, constant mutual air gap flux linkages control, and maximum efficiency control, while, for the speeds higher than base speed, control strategies are six-step voltage and constant back emf [8]. The comprehensive analysis of control strategies for the speeds lower than base speeds is made and compared in this paper. With the help of phasor diagrams, this paper analyses the characteristics of both surface and interior mounted permanent magnet motors. Each of these control strategies has its own merits and limitations. For example, the constant torque angle control forces the electromagnetic torque to be proportional to the stator current magnitude but results in low power factor, optimum torque per ampere current control strategy provides maximum electromagnetic torque for a given stator current, a unity
power factor control strategy optimizes the volt ampere (VA) requirement of the system, and a constant mutual air gap flux linkages control limits the flux linkage of the air gap equal to rotor permanent magnet flux linkage which helps to avoid the saturation of core. Similarly, a maximum efficiency control reduces the net loss in the motor and is appropriate for applications where saving the energy is important [8]. A detail analysis and comparison of these control strategies have been made so as to choose the control strategy that optimizes the operation of a particular speed control system.

This paper is organized in the following manner: Section 1 begins by providing a brief introduction about PMSMs and a study of different existing control strategies. In Section 2, the dynamic model and decoupled control of PMSM are explained shortly. Section 3 presents control strategies for PMSM drive. In Section 4, simulation results are presented to verify the unique feature and capability of the control strategies introduced in the paper emphasizing their merits. The comparison of control strategies based on current, voltage, VA rating, and power factor requirement of the system, and a constant mutual air gap flux linkages control optimizes the volt ampere (VA) requirement of the system.

2. Dynamic Model and Decoupled Control of PMSM

In general, the dynamic equations of $d$- and $q$-axes stator voltages of a PMSM in rotor reference frame are [9]

\[
\begin{align*}
\dot{v}_{ds} &= (R_s + L_d p) \dot{i}_{ds} - \omega_r L_q i_{qs}, \\
\dot{v}_{qs} &= (R_s + L_q p) \dot{i}_{qs} + \omega_r (L_d i_{ds} + \lambda_{ad}).
\end{align*}
\]

(1)

(2)

The stator voltage phasor magnitude is given by

\[
V_s = \sqrt{(v_{ds}^2) + (v_{qs}^2)}.
\]

(3)

The phase voltages in $a$-$b$-$c$ frame are obtained from the above $d$-$q$ voltages by using the inverse Park transformation as defined in the following:

\[
\begin{bmatrix}
\dot{v}_{as} \\
\dot{v}_{bs} \\
\dot{v}_{cs}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_r & \sin \theta_r & 1 \\
\cos \left( \theta_r - \frac{2\pi}{3} \right) & \sin \left( \theta_r - \frac{2\pi}{3} \right) & 1 \\
\cos \left( \theta_r + \frac{2\pi}{3} \right) & \sin \left( \theta_r + \frac{2\pi}{3} \right) & 1
\end{bmatrix}
\begin{bmatrix}
\dot{i}_{qs} \\
\dot{i}_{ds} \\
i_0
\end{bmatrix}.
\]

(4)

Similarly, the relationship between $d$-$q$-$o$ and $a$-$b$-$c$ currents is obtained through the Park transformation as defined in the following:

\[
\begin{bmatrix}
\dot{i}_{qp} \\
\dot{i}_{qs} \\
i_0
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_r & \cos \left( \theta_r - \frac{2\pi}{3} \right) & \cos \left( \theta_r + \frac{2\pi}{3} \right) \\
\sin \theta_r & \sin \left( \theta_r - \frac{2\pi}{3} \right) & \sin \left( \theta_r + \frac{2\pi}{3} \right) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs}
\end{bmatrix}.
\]

(5)

In order to achieve linear transformation in modeling, analysis, and simulations, the power input to the three-phase machine has to be equal to the power input to the two-phase machine.

The $d$- and $q$-axes currents in the rotor frame of reference are obtained as [10]

\[
\begin{bmatrix}
i_{ds} \\
i_{qs}
\end{bmatrix} =
\begin{bmatrix}
i_f \\
i_T
\end{bmatrix} =
\begin{bmatrix}
\cos \delta \\
\sin \delta
\end{bmatrix}
\begin{bmatrix}
i_f \\
i_T
\end{bmatrix},
\]

(6)

where “$i_f$” is the flux producing and “$i_T$” is the torque producing component.

Electromagnetic torque is the most important variable as it determines the rotor position and speed. The expression for the electromagnetic torque developed by the machine can be obtained from the input power and other quantities as given in the following [11]:

\[
T_e = \frac{3}{2} \frac{P}{2} \lambda_{ad} + (L_d - L_q) i_{ds} i_{qs}.
\]

(7)

By substituting the value of $i_{ds}$ and $i_{qs}$ from (6), (7) can be expressed as

\[
T_e = \frac{3}{2} \frac{P}{2} \lambda_{ad} + (L_d - L_q) i_i \cos \delta i_i \sin \delta.
\]

(8)

From (8), it can be seen that the air gap torque is the sum of reluctance torque ($T_{rel}$) and synchronous torque ($T_{syc}$). From the loci (refer to Figure 1), it is observed that the peak of air gap torque ($T_{rel}$) occurs at an angle between 90° and 180° and reduces between 0° and 90°. Hence, the preferred angle is 90° < $\delta$ < 180° [10].

3. Control Strategies for PMSM

The most commonly used five different control strategies applicable to PM synchronous machines are discussed in this section:

(1) Constant torque angle control (CTAC).

(2) Optimum torque per ampere control (OTPAC).

(3) Unity power factor control (UPFC).

(4) Constant mutual air gap flux linkages control (CMFLC).
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(5) Angle control of air gap flux and current phasors (ACAGF).

Such control strategies are important as they provide a wide variation of control choices in many applications. To obtain better performance, these control strategies are analyzed and derived step by step for the steady-state operations only where the state rate of change of current is zero.

3.1. Constant Torque Angle Control. In this control strategy, the torque angle (δ) which is the angle between the rotor field and current phasor is maintained at 90°. Hence, by maintaining δ = 90° from (6), we can conclude that the flux producing component is equal to 0 and the torque producing component is equal to the supply current making PMSM operate like a separately excited DC machine [7]. Therefore, this strategy is also referred to as zero direct axis current (ZDAC) control. The main advantage of ZDAC control strategy is that it gives the simplest and easiest control for PMSM.

Hence, the following relevant equations hold for this strategy is derived in the following.

Since \( f_{d} = 0 \), from (6) and (7), the electromagnetic torque becomes

\[
T_{e} = \frac{3}{2} \frac{p}{2} \lambda_{ad} f_{q} = \frac{3}{2} \frac{p}{2} \lambda_{ad} i_{s}.
\]

From (1) and (2), the steady-state \( d \)- and \( q \)-axes stator voltages are

\[
\begin{align*}
\nu_{d} &= -\omega_{s} L_{q} i_{s}, \\
\nu_{q} &= (R_{s} + L_{q} p) i_{s} + \omega_{s} \lambda_{ad}.
\end{align*}
\]

3.2. Optimum Torque per Ampere Current Control. This strategy is one of the most widely used control strategies for PMSM [12–16]. Application of the optimum torque per ampere (OTPA) control strategy ensures maximum torque for a minimum possible value of current which in turn minimizes the ohmic losses [17–24]. The mathematical model of this strategy is developed as follows. Consider the electromagnetic torque equation of PMSM given in (8):

\[
T_{e} = \frac{3}{2} \frac{p}{2} \left[ \lambda_{ad} (L_{d} - L_{q}) i_{s} \sin \delta - L_{d} i_{s}^{2} \cos \delta \right],
\]

(11)

The normalized torque expression can be obtained as

\[
T_{en} = \frac{T_{e}}{T_{b}} = \frac{1}{\lambda_{ad} b} \left[ \sin \delta + \frac{1}{2} (L_{dn} - L_{qn}) i_{sn} \sin 2\delta \right].
\]

(12)

Let \( \lambda_{ad} = I_{b} L_{b}, i_{sn} = i_{s} / I_{b}, L_{dn} = L_{d} / L_{b}, \) and \( L_{qn} = L_{q} / L_{b}. \)

Rewrite (12) as follows:

\[
T_{en} = i_{sn} \left[ \sin \delta + \frac{1}{2} (L_{dn} - L_{qn}) i_{sn} \sin 2\delta \right].
\]

(13)

From (13), the torque per unit stator current is defined as

\[
\frac{T_{en}}{i_{sn}} = \left[ \sin \delta + \frac{1}{2} (L_{dn} - L_{qn}) i_{sn} \sin 2\delta \right] = 0.
\]

(14)

The torque angle where the PMSM produces maximum torque per unit stator current is obtained by differentiating (14) with respect to \( \delta \) and equating it to zero; that is, the following equation should be satisfied [8]:

\[
\frac{d}{d\delta} \left[ \sin \delta + \frac{1}{2} (L_{dn} - L_{qn}) i_{sn} \sin 2\delta \right] = 0.
\]

(15)

The solution of the above equation gives

\[
\left\{ \cos \delta + \frac{1}{2} (L_{dn} - L_{qn}) i_{sn} \right\} = 0.
\]

(16)

Using the double-angle identities, \( \cos(2\delta) = 2\cos^{2}(\delta) - 1 \) in (16) can be rewritten as

\[
\left\{ \cos \delta + \frac{1}{2} (L_{dn} - L_{qn}) i_{sn} \right\} = 0.
\]

(17)

Solving (17) for \( \delta \) gives

\[
\delta = \cos^{-1} \left\{ \frac{-1}{4 (L_{dn} - L_{qn}) i_{sn}} \right\} \pm \frac{1}{2} \left\{ \frac{1}{4 (L_{dn} - L_{qn}) i_{sn}} \right\}^{2}.
\]

(18)
In (18), $90^\circ < \delta < 180^\circ$ so as to minimize field in the air; hence, only positive sign is considered [7].

Finally, the expression for torque angle is given as

$$\delta = \cos^{-1} \left\{ \frac{-1}{4 (L_{dn} - L_{qn}) i_{sn}} + \frac{1}{2} + \frac{1}{4 (L_{dn} - L_{qn}) i_{sn}} \right\}^2. \tag{19}$$

3.3. Unity Power Factor Control. Power factor can be defined as the cosine of the phase angle between voltage and current as given in the following:

$$\text{p.f.} = \cos \phi, \tag{20}$$

where p.f. is the power factor and “$\phi$” denotes the angle between voltage and current. In some applications, the main goal is to have a unity power factor during the operation of motor [25–27]. Unity power factor control implies the volt ampere (VA) requirement of the inverter can be reduced by maintaining the power factor at unity [28]. The performance equations in this strategy are derived and given below.

In UPF control strategy, the phase angle has to be zero which implies the following relationship:

$$\tan \delta = \frac{v_{qs}}{v_{ds}} = \frac{v_{qs}}{v_{ds}}. \tag{21}$$

Substituting (1), (2), and (6) into (21) results in

$$\tan \delta = \frac{(R_s + L_{q} p) i_{q} + \omega_r (L_{d} i_{d} + \lambda_{af})}{(R_s + L_{q} p) i_{q} - \omega_r L_{q} i_{q}}, \tag{22}$$

$$\tan \delta = \frac{(R_s + L_{q} p) i_{q} \sin \delta + \omega_r (L_{d} i_{d} \cos \delta + \lambda_{af})}{(R_s + L_{q} p) i_{q} \cos \delta - \omega_r L_{q} i_{q} \sin \delta},$$

$$\tan \delta = \frac{R_s i_{d} \sin \delta + \omega_r L_{q} i_{d} \cos \delta + \omega_r \lambda_{af}}{R_s i_{d} \cos \delta - \omega_r L_{q} i_{d} \sin \delta},$$

$$\sin \delta = \frac{1 + L_{dn} i_{sn} \cos \delta + (R_{mn} i_{sn}/(\omega_r)) \sin \delta}{R_{mn} i_{sn} \cos \delta/(\omega_r - L_{qn} i_{sn} \sin \delta).}$$

Solving for $\delta$,

$$\delta = \cos^{-1} \left\{ \frac{-1 + \sqrt{1 - 4L_{qn} i_{sn} (L_{dn} - L_{qn})}}{2i_{sn} (L_{dn} - L_{qn})} \right\}. \tag{23}$$

From (23), it is evident that $\delta$ is independent of rotor speed. Positive sign in (23) and $(L_{ds} < L_{qn})$ should be considered so as to utilize maximum possible torque under UPF control strategy [29].

3.4. Constant Mutual Flux Linkages Control. In constant mutual flux linkage control (CMFLC), the mutual flux linkages are maintained constant and usually set equal to rotor flux linkages. The reason behind this is that machine is protected against magnetic saturation [30]. Limiting the mutual flux linkages, the stator voltage requirement can be kept consciously low. This is the main advantage of CMFL strategy. In addition, for the speeds higher than base speed, this strategy provides flux weakening as compared to the other schemes that are limited for operation at speeds lower than the base speed [31]. In this case, the magnitude of mutual flux linkage is expressed as follows:

$$\lambda_m = \sqrt{(\lambda_{af} + \lambda_{af}^q)^2 + (L_{q} i_{q} \sin \delta)^2}. \tag{24}$$

In (24), the magnitude of mutual flux linkage is kept constant and equal to $\lambda_{af}$. Also, substituting (6) into (24) gives

$$\lambda_{af} = \sqrt{(\lambda_{af} + L_{d} i_{d} \cos \delta)^2 + (L_{q} i_{q} \sin \delta)^2}, \tag{25}$$

$$\lambda_{af}^q = (\lambda_{af} + L_{d} i_{d} \cos \delta)^2 + (L_{q} i_{q} \sin \delta)^2. \tag{26}$$

Using the formulae, $a^2 + b^2 = (a + b)^2 - 2ab$ in (26) can be rewritten as

$$= \left[ (\lambda_{af} + L_{d} i_{d} \cos \delta + (L_{q} i_{q} \sin \delta) \right]^2 - 2(\lambda_{af} + L_{d} i_{d} \cos \delta)^2 + L_{d} i_{d} \cos \delta)^2 + (L_{q} i_{q} \sin \delta)^2 \right]$$

$$- 2(\lambda_{af} + L_{d} i_{d} \cos \delta)^2 (L_{q} i_{q} \sin \delta)^2. \tag{27}$$

Using trigonometric-Pythagorean identities, that is, $\cos^2 \delta + \sin^2 \delta = 1$, the above equation can be rewritten as

$$2\lambda_{af} L_{d} i_{d} \cos \delta + (L_{d} i_{d} \cos \delta)^2 + (L_{q} i_{q} \sin \delta)^2 = 0. \tag{28}$$

In order to determine the magnitude of $\delta$, two different cases arise depending upon the saliency ratio, that is, $L_{q}/L_{d}$.

Case 1 (for surface mounted PMSM $L_{q}/L_{d} = 1$). Solving (28) for $\delta$ yields

$$\delta = \cos^{-1} \left\{ \frac{-L_{d} i_{d}}{2 \lambda_{af}} \right\}. \tag{29}$$

In normalized form, the torque angle $\delta$ is derived as

$$\delta = \cos^{-1} \left\{ \frac{-i_{d} L_{d}}{2 L_{b} L_{b}} \right\} = \cos^{-1} \left\{ \frac{i_{m} L_{dn}}{2 \lambda_{af}} \right\}, \tag{30}$$

where $\lambda_{af} = I_{b} L_{b}$. 
Case 2 (for interior mounted PMSM $L_q/L_d \neq 1$). Solving (28) for $\delta$ yields

$$\delta = \cos^{-1} \left[ \frac{1}{L_{dn} i_{sn}} \left[ 1 - \left( \frac{L_q}{L_d} \right)^2 \right] \right] \pm \sqrt{\frac{1}{L_{dn} \left[ 1 - \left( \frac{L_q}{L_d} \right)^2 \right] i_{sn}^2 - \frac{1}{1 - \left( \frac{L_q}{L_d} \right)^2}}}. \quad (31)$$

For CMFLC strategy, $\delta$ has to be greater than 90°. The CMFLC is preferred over UPF control strategy as it provides significant torque [32].

3.5. Angle Control of Air Gap Flux and Current Phasors. In this strategy, the air gap torque expression may be derived as follows.

Consider (7)

$$T_e = \frac{3}{2} \frac{P}{2} \left[ \lambda_{af}' q_s + (L_d - L_q) i_{ds}' i_{qg}' \right]. \quad (32)$$

The above equation can be written in the following form:

$$T_e = \frac{3}{2} \frac{P}{2} \left[ \lambda_{af}' q_s + (L_d - L_q) i_{ds}' i_{qg}' \right]. \quad (33)$$

Rearrange (33) as follows:

$$T_e = \frac{3}{2} \frac{P}{2} \left[ (\lambda_{af} + L_d i_{ds}) i_{qg}' - L_q i_{ds} i_{qg}' \right]. \quad (34)$$

From (24) and (6), the above expression can be written in the following form:

$$T_e = \frac{3}{2} \frac{P}{2} \left[ \lambda_{ds}' i_{qg}' - \lambda_{qs}' i_{ds}' \right] = \frac{3}{2} \frac{P}{2} \lambda_m i_s \sin \theta_{ms}. \quad (35)$$

where $\lambda_{ds}' = \lambda_m \cos \theta_{ms}$ and $\lambda_{qs}' = \lambda_m \sin \theta_{ms}$.

Also, angle between the air gap flux phasor and current is $\theta_{ms} = \delta - \theta_{\lambda}$.

The air gap flux of PMSM cannot be kept constant for all values of current. So the main concept of this strategy is to maintain $\theta_{ms}$ at 90° which is analogous to the control of separately excited DC machine [33]. This is the main advantage of this strategy as it permits a simple control without a position sensor. The drawback with this strategy is that it cannot be used in the applications where low/zero speed is required as the magnitude of induced emf is very low [10].

4. Simulation Studies and Discussion

The performance characteristics of PMSM under different control strategies for rated speed (1 p.u.) are realized in MATLAB environment. Simulation results for five control strategies under which PMSM is operating are presented ahead. The plotted variables are in normalized units (p.u.). The parameters and rating of PMSM used to plot the curves in the simulation are given in the Appendix. Also, all the chosen quantities such as power factor, stator voltage required, electromagnetic torque, apparent power, mutual flux linkage, and input power are plotted on the same scale.

4.1. Constant Torque Angle Control. The performance characteristics for this control strategy are shown in Figure 2. From Figure 2, it is observed that the power factor ($\cos \phi$) deteriorates as the stator current rises. The normalized stator voltage ($V_{sn}$) required to drive the motor in this control strategy is presented in the following figure. Under this control strategy, the PMSM is able to produce a torque up to 2 p.u. The torque versus stator current curve shows that the electromagnetic torque ($T_{em}$) is directly proportional to the magnitude of stator current which is analogous to DC motor. Also, from the normalized mutual flux linkage (MFLn) characteristics, it is seen that it cannot reduce below 1 p.u. but can vary from 1 p.u. to a point greater than 1 p.u. This is only possible till torque angle is kept constant at 90° [7]. Due to this, it is limited to the applications which do not require flux weakening operation. In addition, the apparent power (VA) is also plotted so as to evaluate the VA rating requirement of the inverter.

4.2. Optimum Torque per Unit Current Control. Figure 3 plots the optimum torque per ampere (OTPA) locus which appears...
like a hyperbola in the rotor $i_{dr}$ and $i_{qr}$ frame. For plotting the OTPA locus for different values of commanded torque, the $q$-axis current is calculated first. Then, from (7), it is observed that $d$-axis current is the function of $q$-axis current from which the $d$-axis current is determined. These minimum current points for a given torque when connected together make a hyperbola which is referred to as OTPA trajectory. In determining the curves of Figure 4, it has been assumed the difference ($L_{dn} - L_{qn}$) should be positive. The magnitude of $T_{en}$ is proportional to $i_{sn}$. The $T_{en}/i_{sn}$ envelope for this strategy is slightly higher than unity. The OTPAC strategy results in reasonable p.f. varying from unity to roughly 0.65.

4.3. Unity Power Factor Control. Figure 5 shows the performance characteristics with the UPF control strategy.

Power versus current envelope shows the real power at any value of stator current. At the beginning, $T_{en}$ increases with the increase in $i_{sn}$ and attains to its peak value $T_{en}(\text{max})$ at $i_{sn}(\text{max})$. Afterwards, if $i_{sn}$ is increased further beyond $i_{sn}(\text{max})$, $T_{en}(\text{max})$ decreases. Also, the magnitude of $V_{sn}$ is decreasing with increase in the value of $i_{sn}$. But from the plot of $T_{en}/i_{sn}$, it is seen that its value is less than 1, indicating that UPF control is not optimum in terms of torque generation as the maximum torque offered in this control is smaller when compared to other control methods. This feature is needed in applications demanding extended speed range.

4.4. Constant Mutual Flux Linkages Control. The performance characteristics of constant mutual flux linkages control for surface mounted (SM) and interior mounted (IM) PMSM are shown in Figures 6 and 7, respectively. On limiting the magnitude of mutual flux linkage to the rotor permanent magnet flux, the torque producing capability of PMSM is also limited. For the SMPMSM, $V_{sn}$ is maintained approximately constant while, for IMPMSM, $V_{sn}$ increases with $i_{sn}$. Also, from Figures 6 and 7, it is observed from the characteristics of p.f. that it is near to unity up to 1 p.u. of $i_{sn}$. This indicates that the CMFLC is closer to unity power factor when compared with CTAC where the p.f. is near to unity up to 0.25 p.u. of $i_{sn}$. The ratio of normalized torque per unit to normalized stator current ($T_{en}/i_{sn}$) is decreasing but offers significant $T_{en}$ over a greater current range when compared to the UPFC strategy.
4.5. Angle Control of Air Gap Flux and Current Phasors. The performance characteristics of angle control of air gap flux and current phasors for PMSM are shown in Figure 8. The salient feature of this strategy is that VA requirement is low as MFLn is decreasing with the increase in magnitude of stator current. Decrease in MFLn with the increase in magnitude of stator current also limits the requirement of stator voltage ($V_{sn}$) [10]. The ratio $T_{en}/i_{sn}$ is less than 1, indicating that ACAGF control is not optimal in terms of torque generation. All these features and characteristics closely resemble the characteristics of unity power factor control strategy.

5. Comparison of Control Strategies

For constant torque angle control, optimum torque per ampere current control, unity power factor control, constant mutual air gap flux linkages control, and angle control of air gap flux and current phasors, the different quantities versus torque are plotted and realized in MATLAB environment to compare the performances of these control strategies. The following simulation results presented ahead give comparisons between these control strategies for the most important characteristics, that is, current, voltage, VA rating, and power factor requirement versus normalized torque for rated speed (1 p.u.). This study will help to select the optimal control strategy depending upon the requirements.

5.1. Current Requirement as a Function of Torque. The performance characteristics of current requirement for different control strategies as a function of torque are shown in Figure 9. It should be noted that the OTPAC needs the minimum possible value of current for a given value of torque when compared with CTAC, UPFC, CMFLC, and ACAGF as expected. However, for all these five strategies, there is no major difference for the requirement of current up to 1 p.u. of $T_{en}$. Furthermore, it can be observed from the plot of
UPFC that, for each value of torque, there exist two operating points. But the points with lower current requirement are considered rather than points with higher current due to the current limitations [29], though the UPF needs the maximum possible value of current for a given value of torque higher than 0.5 p.u. when compared with CTAC, OTPAC, CMFLC, and ACAGF.

5.2. Voltage Requirement as a Function of Torque. The performance characteristics of voltage requirement for different control strategies as a function of torque are shown in Figure 10.

It should be noted that the voltage requirement for CTAC strategy is the highest, whereas for both UPFC and ACAGF it is the lowest.

5.3. VA Rating Requirement as a Function of Torque. The performance characteristics of volt ampere requirement for different control strategies as a function of torque are shown in Figure 11.

The comparison clearly reveals that the volt ampere requirement for CTAC strategy is the highest whereas for both UPFC and ACAGF it is the lowest. This is because the current and voltage requirement for CTAC are the highest and VA is the product of both. Also, the volt ampere requirement for CMFLC strategy is the lowest when compared with CTAC and OTPAC. However, all strategies approximately require the same volt ampere up to 1 p.u. of $T_{en}$; after that, the requirements diverge significantly. Again, from the plot of UPFC, it can be observed that for each value of torque there exist two operating points for volt ampere requirement. But the points with lower VA requirement are considered due to the current limitations.

5.4. Variation of Power Factor Requirement as a Function of Torque. The performance characteristics of power factor requirement for different control strategies as a function of torque are shown in Figure 12. The UPFC strategy yields unity power factor whereas for CTAC strategy it falls rapidly roughly around 0.68 to 0.64 when compared with OTPAC and CMFLC as the torque increases. Power factor requirement for CMFLC and OTPAC is next to UPFC [29].

6. Conclusion

In this paper, different control strategies for PMSM are derived and presented in detail. The study based on the simulation results reveals that OTPAC is superior in $(T_{en}/i_{sn})$ ratio among the five different control strategies whereas the UPF control yields a very low $(T_{en}/i_{sn})$ ratio. Also, all the performance characteristics for each strategy shown above are compared. And the comparative analysis reveals that the main advantage with UPFC is the voltage requirement which is comparatively low but the drawback lies in torque production in the PMSM which is about 1.2 p.u. On comparing UPFC with CMFLC, it should be noted that the voltage requirement for CMFLC is next to UPFC but can produce much higher electromagnetic torque. Finally, from the above comparative study, it can be concluded that the CMFLC has better steady-state performance characteristics and it can be
a good control strategy to consider when compared to the OTPAC, CTAC, UPFC, and ACAGF.

Appendix

- $R_{sn} = 0.1729 \text{ p.u.}$
- $L_{dn} = 0.4347 \text{ p.u.}$
- $L_{qn} = 0.6986 \text{ p.u.}$
- $L_b = 0.0129 \text{ H}$
- $V_b = 97.138 \text{ V}$
- $I_b = 12 \text{ A}$
- $\omega_b = 628.6 \text{ rad/s}$
- $J = 0.0012 \text{ kg.m}^2$
- $B = 0.01 \text{ N.m.s/rad}$
- $P = 6$
- $T_b = 5.5631 \text{ N.m}$
- $V_{dc} = 285 \text{ V (bus voltage)}$
- Power = 3.5 kW.

Nomenclature

- $B$: Damping constant, (N/rad/s)
- $i_{ds}, i_{qr}$: $d$- and $q$-axes stator currents in rotor reference frame, (A)
- $i_{a}, i_{b}, i_{c}$: Stator current magnitude, (A)
- $I_b$: Base current, (A)
- $J$: Total moment of inertia, (kgm$^2$)
- $L_d, L_q$: Stator $d$- and $q$-axes self-inductances, (H)
- $L_{dn}, L_{qn}$: Normalized stator $d$- and $q$-axes self-inductances, (H)
- $L_b$: Base inductance, (H)
- $P$: Number of poles
- $R_s$: Stator resistance per phase, (Ω)
- $T_e$: Electromagnetic torque, (N-m)
- $T_{em}$: Normalized electromagnetic torque, (p.u.)
- $T_l$: Load torque, (N-m)
- $T_b$: Base torque, (N-m)
- $\delta$: Torque angle
- $\lambda_{af}$: Armature flux linkages, (V-s)
- $\lambda_m$: Mutual flux linkages, (V-s)
- $\rho$: Differential operator, $d/dt$
- $V_s$: Stator voltage phasor magnitude, (V)
- $V_{a}, V_{b}, V_{c}$: Input phase voltages, (V)
- $V_{ds}, V_{qs}$: $d$- and $q$-axes stator voltages in rotor reference frame, (V)
- $\theta_a$: Actual rotor position, (radians)
- $\theta_1$: Angle between the mutual flux linkages and the permanent magnet rotor flux linkage
- $\omega_r$: Electrical rotor speed, (rad/s)
- $\omega_b$: Base speed, (rad/s).
Conflict of Interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

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