Matrix method of reflecting activity in the digital twin of the social system

Матричный способ отражения деятельности в цифровом двойнике социальной системы

Abstract

The paper considers the approach to solving the problem of exact reflection of processes taking place in the company in the digital twin of the social system – not only technological and production ones but also the processes of interaction between subjects. The approach presented is the development aimed at the creation of the digital twin of the comprehensive mathematical model of the social system functioning in the active environment. Due to the presentation of agents’ actions as the transformation act of the resource base controlled by them, there appeared an opportunity to use multidimensional matrixes reflecting the phase transition of the social system resource base for solving the problem of process fixation. Combined with the calculation, the probabilities of a human to perform certain conditioned matrix actions reflecting resource transformations allow the digital twin to forecast the activity results, calculate deviations from the target trajectory of the system motion and calculate the required control actions. The novelty lies in the activity representation as a multidimensional matrix. As an example, the paper considers the use of three-dimensional matrix but the possible need in using matrixes of larger dimensionality is pointed out.

Аннотация

В статье рассматривается подход к решению задачи точного отражения в цифровом двойнике социальной системы процессов, происходящих в компании – не только технологических и производственных, но и процессов взаимодействия субъектов. Представленный подход является развитием для целей создания цифрового двойника комплексной математической модели социальной системы, функционирующей в активной среде. За счёт представления действий агентов как акта преобразования подконтрольной им ресурсной базы появилась возможность для решения задачи фиксации процессов использовать многомерные матрицы, отражающие фазовые переходы ресурсной базы социальной системы. Вместе с расчётом вероятности совершения человеком определённых обусловленных действий матрицы, отражающие ресурсные преобразования, позволяют цифровому двойнику прогнозировать результаты деятельности, рассчитывать отклонения от заданной траектории движения системы и рассчитывать необходимые управляющие воздействия. Новизна заключается в представлении деятельности в виде многомерной матрицы. Для примера в статье рассматривается использование трёхмерной матрицы, но отмечается возможная необходимость использования матриц большей размерности.

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Keywords: digital twin of company, digital twin of social system, comprehensive mathematical agent-based model of social system, simulation model of social system, economy digitalization, digital transformation, resource and functional approach, active system.

Introduction

The need in economy digitalization, computerization of company activities is undeniable. And we already have examples of production computerization demonstrating the possibility and reasonability of such systems. Thus, as early as in 2015 Changying Precision Technology opened a completely computerized factory without physical presence of a human (The world of technology, 2015). But the today’s examples are mainly the ones of production activity computerization. Besides, the management and activity arrangement are not properly computerized – the existing products only fragmentarily solve this problem.

For management computerization it is necessary to have a model adequately reflecting the management object dynamics. But for proper system dynamics modeling it is required to have the possibility to fix and calculate all phase transitions of the system. For the social system these are the state changes caused by the participants’ actions, including the change in the state of interacting subjects. As discussed earlier in (Samosudov, 2022), a social system state is described by the aggregate of phase variables, which can be grouped as follows:

- Phase variables of the system state connected with the participants of the system corporate relations: number of participants, their behavior, available resources, needs, vision of their state, and state of social and economic space (SES).
- Phase variables connected with the social system resource base: types and amount of resources available in the system (obtained from the agents), distribution of the resources among the agents (social localization of resources).
- Phase variables connected with institutional environment: structure and parameters of social institutions, including external and internal, formal and informal institutions that define the rules of interaction of agents.
- Phase variables connected with the activity of the system and its participants: flows of resources and messages, transmission channels used, array of agents obtaining resources and messages from the system.

In their previous papers the authors described the key fixation aspects of institutional environment parameters, calculation of the influence of information flows on the agents’ behavior, formalization of the social system agents’ activeness in a digital twin. In this paper we deal with the issue of formalization (fixation) of the resource base dynamics occurring as a result of process implementation in a company (performance of certain actions by the agents).

The task of process registration in a digital twin (DT) comes down to the registration of all existing phase transitions (changes in the values of phase variables).

In this paper we set the goal to find the way to fix changes in the system resource base, caused by certain actions performed by the agents, in DT.

Literature overview

To develop complex software products for computerizing the activity of economic systems, the works in the field of creating DT and activity computerization means based on them seem perspective (Lee at al., 2015; Brenner & Hummel, 2017; Söderberg at al., 2017; Uhleman at al., 2017; Asimov at al., 2018; Bolton at al., 2018; Tao at al., 2018; Kurganova at al., 2019; Jones at al., 2020; Barkalov at al., 2021; Budiardjo & Migliori, 2021; Traoré, 2021; Becker & Pentland, 2022; Hamzaoui & Julian, 2022; Korovin, 2022; Strielkowski at al., 2022). As pointed out in (Kurganova at al., 2019) "when DTs are developed for newly
created productions, they get the opportunity … to reveal possible risks and defects, to correct the project through its operation simulation. DT of the existing production allows working through the introduction or change in the technological processes without actual interference with the operation”. So, there emerges an opportunity to decrease the number of errors in the process of activity arrangement. But the perspectives to use DT specifically for managing social systems look even more attractive.

DT is based on the imitation model defining the data form and structure, which allows calculating the object dynamics in its functioning environment. As applicable to a company, this is the simulation of social system motion in SES. As indicated in (Petrov, 2018, p. 58), “… simulation modeling … is defined as a numerical research method of complex systems whose elements are described by heterogeneous mathematical apparatus and combined by a linking model”. Simulation models are often realized as hardware-software complexes. At the same time, “traditional simulation modeling methods consider employees …, suppliers, clients, products, projects, etc. as an arithmetic mean or as passive resources” (Tsenina, 2017, pp. 367-368). Therefore, agent models are more and more often used to model social systems (Churyukin, 2009; Petrov, 2018; Tsenina, 2017) – they allow modeling the social system dynamics through the simulation of agents’ interaction.

In many cases, speaking of DT the authors of the papers mean DT of technical systems, separate objects, parts, units, production lines, etc. But DTs of social systems, in particular, have not been spread properly yet. This is partly caused, as indicated before, by the fact that DT requires the availability of the simulation model properly reflecting the system dynamics. At the same time, “Among the management tasks in social and economic, organizational, political and other spheres, … the complex problems aimed at changing the state of things in general in the desired way are the most complicated ones. In this case, the whole problem region, considered as the dynamic situation consisting of the aggregate of heterogeneous interacting factors, is the management object. … When trying to use information technologies to solve such problems, as a rule, one has to come across the fact that in contrast to the majority of technical systems, the control object (i.e. situation) has not only been formalized but is also weakly structured” (Kuznetsov, n.d.).

Sometimes the company processes are considered from the position of PDCA cycle (see, for example, (Manakhova at al., 2022)). But this only allows understanding the process general logics and does not allow tracing all phase transitions taking place during the process realization. The schemes are traditionally used to describe business processes. Such notations as IDEF, BPMN, eEPC, etc. are frequently used to form the schemes. But the notations are only the rule of the process schematic depiction. The schemes, as a rule, do not reflect all necessary information about the process, do not provide sufficiently detailed description to have the possibility to exactly define deviations of the actual process from the calculated one when designing the social system. Therefore, they cannot be used for creating DT since they do not allow fixing the process with the sufficient accuracy. They can be quite reasonably used when designing the processes for initial elaboration but in DT it is necessary to fix the system “passing” through the process with rather high accuracy providing the system dynamics tracing (i.e. the change in its resource base), and the abovementioned notations do not provide this information. Although, notation IDEF allows describing a process in more detail, it does not provide the sufficient detailing for the sake of the process reflection in DT. The matrix methods of reflecting the company dynamics are used in some papers (Kukharenko at al., 2015), but this is more appropriate for reflecting the company functioning indices in the matrix form but not the processes themselves as they are. Network models are also used (Dorrer at al., 2020; Dorrer, 2021). However, to create DT the fixation of the system phase transitions is particularly required but not of the results, which are the consequence of changes in the system.

Method

To complete the task of creating DT, we use the mathematical modeling method. The social system moving in SES serves as the modeling object.

The following entities are used in the model (Samosudov, 2022):

- Invariants and conditional invariants (unchangeable within the problem being solved): a priori existing entities, which the agents can exchange (transferable resources); messages as the aggregate of information initial elements (signs, other information elements identified by a human with the help of own sensory organs); transmission channels structuring the SES.
• Variative entities depending on SES point, in which they are considered: values of the resources; content and sense of messages emerging in individual subspace when interpreting the messages based on the individual alphabet.

• Service entities used to calculate the system dynamics: agents’ actions, institutional environment, behavior vector – matrix with the dimensions $1 \times n$, every element of which defines the probability of a human to perform a certain conditioned action.

The model considers the agents’ activeness not only as the ability to make decisions based on own interests but also as an opportunity to act upon other agents to change own situation in compliance with own interests (system activeness).

The system motion is calculated through the calculation of changes in phase variables as a result of agents’ interactions. At the same time, the value of variative entities is calculated based on the information on the values of invariants through the calculation of gradients of resource and information flows within SES point. This allows taking into account the resource value relativity, sense of messages and other factors in the calculations, as well as calculating the forecast of SES dynamics to form managerial and marketing actions.

It is critical to point out that the model operates the whole variety of resources, which can be used in the social system activity. The rules of quantitative estimation of resources of different nature were defined for this (for more detail, see (Samosudov, 2019)). From the point of applied realization, the list of the model resources is defined by the social system analysis to reveal the resources used or by the activity design (for newly created systems).

**Results**

Taking the foregoing into account, under the company DT we understand the computer software providing the fixation and processing of the sufficient amount of data to trace the situation change in the company when modeling different actions onto it – controlling, disturbing, etc. Such data complex should reflect all essential cause-and-effect relations, contain the necessary and sufficient data set, which allows simulating the social system dynamics in SES.

From the realization point, DT is a set of program modules fixing data in the required format, processing them in a certain way and thus providing the possibility to model the activity, imitate the dynamics of the social system state, calculate its behavior and properties, consequences of one or another effect. To be used in the management system, it should provide the possibility to perform the following actions:

• Fix the state of companies and environment, in which it is functioning, as a set of values of phase variables (system and environment parameters).

• Fix the change in the company state if any phenomena occur, circumstances change, any actions are performed by the participants.

• Show (calculate) how the social system state and properties change with the change in the values of one or several parameters.

**Formalization of processes in DT**

The social system activity means the actions of participants of corporative relations. If the social system is considered within SES and its motion in SES, the system process change with time is conditioned by the actions of all participants (agents), with whom the system is interacting: company employees, partners, clients, competitors, participants of corporative relations of these agents, etc. But for the purposes of our paper and specific task of modeling the particular company we can significantly limit the number of participants and consider the actions of only this limited number. At the same time, it should be pointed out that the approach presented is easily scaled and allows formalizing the activity of any social system.

As indicated before (Samosudov, 2022), the set of actions $O_{j}(t)$ performed by $j$th agent at moment $t$ is defined by the current value of its behavior vector $B_{j}(t)$ representing matrix $1 \times n$, every element of which defines the probability of a human to perform the corresponding conditioned action:

$$ O_{j}(t) = B_{j}(t) $$
In turn, the agent’s behavior vector value depends on information $\tilde{I}_j(t)$ obtained at the moment of making decision on performing an action based on deciphering the message received from the environment:

$$B_j(t) = B_j(t_0) + \Delta B_k \left( \tilde{I}_j(t) \right) = B_j(t_0) + \left( \int_{t_0}^{t} \text{div} B_j^1 \left( \tilde{I}_j(t) \right) dt, ..., \int_{t_0}^{t} \text{div} B_j^n \left( \tilde{I}_j(t) \right) dt \right)$$

(2)

At the same time, all information obtained is the result of interpretation of all messages $M_j^k$ based on alphabet $M_j$ received by $j$th agent from all active $k$ participants. The interpretation result are subjective evaluations of stimuli $s_n(o_s)$ (obtained resources), limitations $l_m(o_l)$ (lost resources) and probability of obtaining $p(s_n)$ or losing $p(l_m)$ resources caused by the performance of certain actions.

$$\tilde{I}_j(t) = \varphi \left( \sum_k M_j^k(t), M_j \right) = \{s_n(o_s), l_m(o_l), p(s_n), p(l_m)\}$$

(3)

In the activity process the initial resource base is successively transformed into the result: final resource or aggregate of resources. Moreover, the action is the act of transforming the resource base controlled by the action subject into the action result. And, consequently, the action result can be a message, resources transferred to someone, new resource or aggregate of resources.

It is crucially important to indicate that to reach the required accuracy of activity comprehension, it is necessary to consider, be able to identify and have the possibility to measure all types of resources (material, informational, social, spatial, intellectual, time), be able to identify and fix phase transitions of the resource base.

The arbitrary action $o_z$ of the activity subject is formalized by two matrices: the matrix of resources controlled by the participant at the moment of performing actions and matrix of the resultant distribution of the resources in the system corresponding to this action.

$$\begin{pmatrix} r_1 & r_2 & \cdots & r_n \end{pmatrix} \xrightarrow{o_z} \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{j1} & \cdots & r_{jn} \end{pmatrix}$$

(4)

But only the action of one participant is formalized in this way. If we consider the aggregate of participants performing actions in the social system, the formalization of the $z$th action $o_z$ performed by the $i$th participant will look as follows:

$$\begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{in} & \cdots & r_{mn} \end{pmatrix} \xrightarrow{o_z} \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{j1} & \cdots & r_{jn} \end{pmatrix}$$

(5)

The number of matrix columns is defined by the number of resource types used in the activity, the number of lines – by the number of participants of the activity (economic agents participating in the activity).
As shown before (Samosudov, 2019), the resource base undergoes the following changes in the activity process:

- Resources appear and disappear (they are spent).
- Resources change the spatial-time and social localization.
- Resources transfer from the passive form into the active one and back.

Resources in the social system can appear as a result of their transfer into the system by the activity participants, and as a result of the participants’ actions (processes). The change in the social localization assumes the transfer of the resources into ownership or use. Transition of material resources into the active form means the resource preparation for action (switching on the device, etc.). For informational resources this is the information taken in by a human, association of this information with own actions. In many cases this requires time and, perhaps, other resources. In general case, the action can be performed if the corresponding component of the behavior vector equals 1 and all resources are in the active form. Availability of the techniques for measuring all types of resources, for resource analysis and synthesis allows taking into account all phase transitions of the resource base.

The succession of actions (in the compact form of the matrix recording) can be formalized as follows:

\[
\begin{pmatrix}
    r_{jm}(t_0) \\
    r_{jm}(t_1) \\
    r_{jm}(t_2) \\
    \vdots \\
    r_{jm}(t_z)
\end{pmatrix}
\xrightarrow{\alpha_1} \xrightarrow{\alpha_2} \xrightarrow{\alpha_3} \xrightarrow{\alpha_4} \xrightarrow{\alpha_z}
\begin{pmatrix}
    r_{jm}(t_0) \\
    r_{jm}(t_1) \\
    r_{jm}(t_2) \\
    \vdots \\
    r_{jm}(t_z)
\end{pmatrix}
\] (6)

At the same time, each action requires certain time for its performance \(\Delta t(o_i)\):

\[
\begin{pmatrix}
    r_{jm}(t_0) \\
    r_{jm}(t_0)
\end{pmatrix}
\xrightarrow{\alpha_0} \begin{pmatrix}
    r_{jm}(t_0 = t_0 + \Delta t(o_i))
\end{pmatrix}
\] (7)

Taking into account that the time for performing different actions differs, in DT it is necessary to have the possibility to consider the influence of different actions on a certain value of the resource base at a certain time moment.

Let us assume that at moment \(t_1\) the first participant successively performs two actions each requiring time \(\Delta t\), and the second participant – one action requiring time \(2\Delta t\). Then the actions of the first participant will change the matrix at time moments \(t_2\) and \(t_3\) in compliance with (Eq. 6), and the actions of the second participant – at time moment \(t_3\). In other words, at each time moment the resource matrix of the social system reflects all resource transformations completed by this moment due to the agents’ actions.

The succession of matrices of the resource base transformation can be represented as three-dimensional matrix \((r_{jm}(t_z))\) whose layers reflect the resource base state at specific time moment differing from the previous one by \(\Delta t\). To improve the fixation accuracy of processes in DT, the discretization frequency of the process matrix should provide the formalization of rather fast processes taking place in the company.

**Discussion**

The developed approach allows fixing processes of the company functioning in DT of the social system as the succession of resource transformations reflected in the multidimensional matrix. This gives the possibility to solve the problems of activity formalizing, revealing or forecasting (calculating) errors, etc. If the fixation of phase transitions (actual changes in the resource base) is provided, it is possible to fix, as promptly as possible, all complex of processes of the social functional system and, using the digital twin calculation modules, to forecast the course of events and, if necessary, make the decisions on controlling actions; in particular, to automatically form recommendations for the manager and even form the controlling actions themselves.

Phase transitions can be fixed both by devices and based on the marks of the process participants. To minimize the possibility of distorting the information about the process, it is recommended to use the double-entry recording method – the mark is put by the person who completed the work and obtained the required result, but this mark should be confirmed by the person who uses the work result of the previous activity participant.
It should be also pointed out that the resources possess the attributes reflecting their current status that, possibly, defines the need in using them for fixing the state more often than four-dimensional matrixes. But actually this does not essentially change the model and realization of the social system digital twin.

To arrange the work, at the stage of designing the company digital twin it is necessary to define the list of resources required for the activity and list of actions with the resources. This is done in the course of activity design.

Conclusion

To create DT of the social system, it is critical to have the possibility to fix the activity resource trace – this is the most accurate method to fix processes. Namely these processes change the system. Therefore, it is rather important to fix the processes and calculate possible consequences of the existing changes in the resource base. The probability of actions performed by the agents is calculated based on the messages, fixed in DT database, transmitted to the participant, as well as the information about the resources obtained by him or her that is fixed in the matrix of resource transformations in compliance with (Eq. 2) as shown in (Samosudov, 2022).

At the same time, it is necessary to emphasize (point out) once again that the model requires accuracy in elaborating the system resource base, defining and recording all types of resources used in the activity.

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