Leptogenesis and Reheating in Complex Hybrid Inflation

Carlos Martínez-Prieto,1 David Delepine,2 and L. Arturo Ureña-López2

1Instituto de Física y Matemáticas de la Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Cd. Universitaria, A.P. 2-33, 58040, Morelia, Michoacán, México.
2Departamento de Física, DCI, Campus León, Universidad de Guanajuato, C.P. 37150, León, Guanajuato, México.

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We study the transformation into a baryon asymmetry of a charge initially stored in a complex (waterfall) scalar field at the end of a hybrid inflation phase as described in Ref. [1]. The waterfall field is coupled to right-handed neutrinos, and is also responsible for their Majorana masses. The charge is finally transferred to the leptons of the Standard Model through the decay of the right-handed neutrinos without introducing new CP violating interactions. Other needed processes, like the decay of the inflaton field and the reheating of the Universe are also discussed in detail.

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INTRODUCTION

We have shown in a recent paper[1] that a complex hybrid inflation model can generate a charge asymmetry that may be further transferred into a baryon charge, then providing a possible solution to the baryogenesis problem of Cosmology[2]. The scalar field potential associated of the complex hybrid inflation model is

\[ V(\phi, a) = \frac{1}{4\lambda^2} (M^2 - \lambda^2 |a|^2)^2 + \left( \frac{m^2}{2} + \frac{g^2}{2} |a|^2 \right) \phi^2 + \frac{\delta^2}{4} a^2 \phi^2 + \text{c.c.} \]  

(1)

where \( \phi \) is the inflaton field and \( a \) is the (complex) waterfall field. Notice that there is an explicit term that violates the \( U(1) \) global symmetry. The needed charge is generated during inflation and is associated to the charge of the waterfall field \( a \) that puts an end to inflation.

The aim of this paper is to discuss the transfer of the \( a \)-charge to fermionic matter after the end of inflation. The task is not a simple one, there may be some transfer processes during the reheating phase of the Universe that can wash out the generated charge.

To fix ideas, we shall work on a leptogenesis model in which the \( a \)-charge is transferred to Standard Model particles through interactions between the waterfall field \( a \) and a right-handed neutrino \( N_R \). The interaction Lagrangian reads

\[ \mathcal{L} = h_Y \bar{\ell}_L \Phi N_R + h_2 \bar{N}_R^c N_R a + \text{h.c.} \]  

(2)

where \( \Phi \) is the Higgs doublet, \( \ell_L \) is the leptonic doublet, and \( h_Y \) is the usual Yukawa coupling.

The interaction Lagrangian (2) is inspired in the majoron model and in the standard leptogenesis scenario[3, 4, 2, 4]. Our model is then composed of just one family of leptons that contains one leptonic doublet and one right handed neutrino.

A summary of the paper is as follows. In Sec. we briefly review the inflationary dynamics of the complex hybrid inflation model as presented in Ref.[1]. In Sec. we study the post-inflationary dynamics of the different fields involved in the model, and focus our attention in the stages of preheating and reheating that may appear. Sec. is entirely devoted to the study of the Boltzmann equations in order to estimate the amount of the \( a \)-charge that is finally converted into a useful baryon charge. Finally, conclusions are presented in Sec.

INFLATIONARY DYNAMICS

The model is given by the potential (1), where \( g \) and \( \lambda \) are real constants, and \( \delta \) is a complex parameter; for \( \delta = 0 \) we recover the standard hybrid inflation model[7]. The \( \delta \)-term violates the \( U(1) \) symmetry associated to the complex field \( a \). However, the potential (1) is \( CP \) conserving as the phase of complex parameter \( \delta \) can be removed through a phase redefinition of the \( a \)-field.

The scalar potential has a local maximum at \( \phi = |a| = 0 \) with a height given by \( V(0, 0) = M^4 / (4\lambda^2) \) that corresponds to a false vacuum. The true vacuum of the system corresponds to the global minimum located at \( \phi = 0 \) and \( \lambda |a| / M = 1 \); this true vacuum is degenerate. The \( U(1) \) charge density at any time is given by \( n_a = n_{a_r} \bar{a}_r - n_{a_l} \bar{a}_l \) where the \( r \) and \( i \) refers to the real and imaginary components of the \( a \)-field.

The constant term in the potential (1) is initially the dominant one, which is usually dubbed as false vacuum inflation[5]. In the regime of slow-roll the scale factor grows exponentially with time, \( R(t) = R_{end} \exp[H_0(t - t_{end})] \), whereas the evolution of the inflaton field is given by \( \phi(t) = \phi_{end} \exp[(m^2 / 3H_0)(t_{end} - t)] \), where \( H_0 \) is the (almost constant) Hubble parameter during inflation.

The waterfall field is trapped in the false vacuum \( a = 0 \), but when the inflaton field passes through the value \( \phi_- = M / \sqrt{g^2 - \delta} \) the imaginary component of the waterfall field \( a \) presents a tachyonic instability[5], and falls down towards its true vacuum value. Likewise, when the inflaton field passes through the value \( \phi_+ = M / \sqrt{g^2 + \delta} \)
the real component \(a_r\) becomes unstable and moves too to its true vacuum value; it is at this point that inflation ends.

The asymmetric evolution of the components of the waterfall field generates a dynamical \(CP\) violating phase during the phase transition at the end of inflation, and produces an asymmetry in the charge of the \(a\) field. 

Taking into account different observational constraints, it was shown that the appropriate values of the parameters correspond to the case \(\lambda^2 \gg g^2 \gg \delta\). Hereafter, we will consider these to be the right case for the parameters of our model.

**POST-INFLATIONARY DYNAMICS AND REHEATING**

We begin at the end of the inflation once the inflaton field passes through the second critical point \(\phi_+\). The dynamics afterward depends upon the values of the different parameters in the model, but it very much resembles that of typical hybrid inflation coupled to a third massless scalar field. We shall follow the calculations presented in Refs. 9, 10, 11, where more details can be found.

**Dynamics of preheating**

First, we revisit the critical points of the potential 11; they are to be found from the equations

\[
\begin{align*}
\frac{\partial V}{\partial \phi} &= \left[ m^2 + g^2|a|^2 + \delta(a_r^2 - a_i^2) \right] \phi = 0, \quad (3a) \\
\frac{\partial V}{\partial a_r} &= \left[ -M^2 + \lambda^2|a|^2 + \phi^2(g^2 + \delta) \right] a_r = 0, \quad (3b) \\
\frac{\partial V}{\partial a_i} &= \left[ -M^2 + \lambda^2|a|^2 + \phi^2(g^2 - \delta) \right] a_i = 0. \quad (3c)
\end{align*}
\]

As we mentioned before, the critical points exist only for \(\phi = 0\); they are the origin of coordinates, \((\phi = 0, |a| = 0)\), and the degenerate circle on the complex plane, \((\phi = 0, |a| = M/\lambda)\).

However, it is very instructive to consider the position of the critical points on the complex plane \((a_r, a_i)\) for non-zero values of the inflation field. We notice that the location of the critical points changes with time as shown in Fig. 1. There are 5 critical points after passing through the first instability point \(\phi_-\), but only two of them correspond to minima of the scalar potential; in general, the two minima are located along the real axis, \(a_r = 0\). The critical values of the imaginary part of the waterfall field are determined from the ellipse equation

\[
\lambda^2 a_i^2 + (g^2 - \delta) a_r^2 = M^2. \quad (4)
\]

On the other hand, we should take into account the effective masses of the different fields; they are given by

\[
\begin{align*}
m^2_{\phi} &= \frac{\partial^2 V}{\partial \phi^2} = m^2 + g^2|a|^2 + \delta(a_r^2 - a_i^2), \quad (5a) \\
m^2_{a_r} &= \frac{\partial^2 V}{\partial a_r^2} = -M^2 + \lambda^2|a|^2 + 2\lambda^2 a_i^2 + (g^2 + \delta)\phi^2, \quad (5b) \\
m^2_{a_i} &= \frac{\partial^2 V}{\partial a_i^2} = -M^2 + \lambda^2|a|^2 + 2\lambda^2 a_r^2 + (g^2 - \delta)\phi^2. \quad (5c)
\end{align*}
\]

The values of the masses at the effective minima can be rewritten as

\[
\begin{align*}
m^2_{\phi} &= m^2 + \frac{(g^2 - \delta)}{\lambda^2} M^2 \left( 1 - \frac{\phi^2}{\phi_c^2} \right), \quad (6) \\
m^2_{a_r} &= 2\delta \phi^2 = \frac{2\delta}{g^2 + \delta} \left( \frac{\phi^2}{\phi_c^2} \right), \quad (7) \\
m^2_{a_i} &= 2M^2 \left( 1 - \frac{\phi^2}{\phi_c^2} \right). \quad (8)
\end{align*}
\]

where \(\phi_-\) is the first instability point at the end of inflation. We notice that the field \(a_r\) is practically massless, and in the limit \(\phi \to 0\) we find \(m_{a_r} \to m_{\phi} \equiv gM/\lambda\), \(m_{a_i} \to 0\), and \(m_{a_r} \to m_{a_i} \equiv \sqrt{2}M\).

From the discussion above, we see that as the system approaches the true vacuum of the system the 3-fields system we are dealing with can be matched to the standard theory of hybrid inflation plus the addition of a massless field. That is, our system \((\phi, a_r, a_i)\) behaves as in the theory of preheating in hybrid inflation presented in Ref. 11, in which the three fields in interaction are \((\phi, \sigma, \chi)\) being \(\chi\) the massless field. Because of this similarity, our next calculations follow those presented in Ref. 11.

For the dynamics after inflation, two different regimes have been identified. For reasons explained before, we shall be interested in the regime corresponding to \(\lambda^2 \gg g^2\). The field \(a_r\) behaves as a massless field, and we expect it to evolve adiabatically following the instantaneous position of its critical point at \(a_r = 0\).

As for the other fields, \(\phi\) and \(a_i\), they also evolve adiabatically along the ellipse 11; the field \(a_i\) does it with negligible oscillations and then most of the energy is stored in the oscillations of the inflaton field.

Next step is to study the production of \(\phi\) and \(a_i\)-particles during the so-called preheating stage which happens during the oscillating phase of the fields; for this we need to write the evolution equations of the quantum fluctuations \(\delta \phi_k, \delta a_{r,k},\) and \(\delta a_{i,k}\) in linear perturbation theory.

It is better to define new variables \(\varphi = R^{3/2}\delta \phi, \psi = R^{3/2}\delta a_i,\) and \(\eta = R^{3/2}\delta a_r\); thus, the production rate of particles of the different fields is determined by the Mathieu equations

\[
\begin{align*}
\varphi''_k + \left[ A_{\varphi}(k) - 2q_{\varphi}(2\varphi) \right] \varphi_k &= 0, \quad (9a) \\
\psi''_k + \left[ A_{\psi}(k) - 2q_{\psi}(2\psi) \right] \psi_k &= 0, \quad (9b) \\
\eta''_k + \left[ A_{\eta}(k) - 2q_{\eta}(2\eta) \right] \eta_k &= 0, \quad (9c)
\end{align*}
\]
FIG. 1: Different snapshots of the potential as projected on the complex plane of the waterfall field \( a \) and for different values of the inflaton field \( \phi \). There is only one minimum during inflation, but two minima appear after the crossing of the first instability point \( \phi^- \); the full restoration of the \( U(1) \) symmetry is almost complete well after the end of inflation.

where a prime denotes derivative with respect to \( z = m_\phi t \). The production of particles can be specially efficient if \( A(k) \ll 2q \) and \( q \geq 1/4 \), and then it is necessary to study each case separately.

\( \phi \)-particles. The Mathieu parameters are

\[
A_\phi(k) = \frac{k^2}{R^2 m_\phi} + 1 + 2q_\phi, \quad q_\phi = \frac{\Phi^2(t)}{4}, \quad (10)
\]

where \( \Phi^2(t) = 2(\phi^2/\phi_\text{m}^2) \propto 1/t^2 \) is the averaged squared amplitude of the inflaton oscillations around the minimum of the potential, and \( R(t) \) is the scale factor of the Universe.

For the regime \( \lambda^2 \gg g^2 \), it is possible that the inflation field performs large-amplitude oscillations after inflation, and then initially \( q_\phi \simeq 1/4 \); this may indicate a successful scenario for the production of particles. However, careful studies show that the small oscillations of the field \( a_i \) prevent the existence of an explosive production of \( \phi \)-particles \[1\].

\( a_i \)-particles. The Mathieu parameters are

\[
A_{a_i}(k) = \frac{k^2}{R^2 m_{\phi}} + \frac{2\lambda^2}{g^2} + 2q_{a_i}, \quad q_{a_i} = \frac{2\lambda^2 \Phi^2(t)}{g^2} \quad (11)
\]

Even though \( q_{a_i} > 1/4 \) initially because of the regime
\( \lambda^2 \gg g^2 \); this also means that \( A_{a_i} \gg q_{a_i} \). Then, our system is far above the resonance band for the production of particles; thus, no explosive production of \( a_i \)-particles is expected.

\( a_i \)-particles. The Mathieu parameters are

\[
A_{a_i}(k) = \frac{k^2}{R^2 \bar{m}_\phi} + 2q_{a_i}, \quad q_{a_i} = \frac{24 M^2 \Phi^2(t)}{g^2 \bar{m}_\phi^2} \frac{4}{4}.
\]

Initially, we would find that \( q_{a_r} \approx \lambda^2 \delta/g^4 \ll 1 \); thus, we again find that we cannot expect an explosive production of \( a_i \)-particles.

In consequence, all the considerations above strongly suggest that the dynamics of the complex hybrid inflation model is (almost) entirely described by the classical evolution of the inflaton and waterfall fields. In other words, there are not important preheating processes for the production of particles after inflation, and we can say that all fields evolve coherently in very good approximation.

Reheating

Now that we have established the coherence of the inflaton oscillations, we must consider the reheating of the Universe after inflation; that is, the transfer of the inflaton’s energy into relativistic degrees of freedom in its initial phase of rapid oscillations.

The (perturbative) theory of reheating says that the inflaton can decay into other (whether scalar or fermionic) degrees of freedom if its mass is larger than those of the products. In the case of interest here, \( \lambda^2 \gg g^2 \), the effective mass of the \( a_i \)-field is \( m_{a_i} \approx \sqrt{2} M \), whereas the effective mass of the inflaton field is \( m_{\phi} \approx gM/\lambda \ll m_{a_i} \); hence, the decay of \( \phi \)-particles into \( a_i \)-particles is kinematically forbidden.

On the other hand, the decay of \( \phi \)-particles into \( a_r \)-particles is possible because the latter field is effectively massless. Because the minimum of the potential is at \( \phi = 0 \), an estimate of the decay width \( \Gamma \) is

\[
\Gamma(\phi \rightarrow a_r a_r) \sim \frac{(g^4 + \delta^2) \phi^2}{8\pi \bar{m}_\phi} \Phi^2 \sim g^2 \lambda \Phi^2.
\]

For that, we assume that the interaction part of the potential has a term of the form

\[
\mathcal{L}_{\phi\psi} = h_1 \bar{\psi} \psi \phi,
\]

where \( h_1 \) is the interaction constant and is small enough to avoid large loop corrections to the inflaton potential. The corresponding rate of decay is

\[
\Gamma_\phi = \frac{h_1^2 m_{\phi}}{8\pi \lambda} \sim \frac{h_1^2 g M}{8\pi \lambda}.
\]

Following standard calculations, the inflaton field decays completely and the reheating temperature is estimated to be

\[
T_{reh} \approx 0.1 \sqrt{\Gamma_{\phi} m_{Pl}} \approx 0.1 h_1 \sqrt{(g/\lambda)} M_{Pl}.
\]

Asymmetric charge after reheating

After inflation and during the rapid oscillations of the inflaton field, the Boltzmann equation for the charge of the waterfall field \( n_a \) after inflation is

\[
(R^3 n_a) = -2\delta R^3 \phi a_r a_i.
\]

The classical evolution then points out that the source term on the r.h.s. is negligible because of the smallness of the asymmetry parameter \( \delta \) and of the field \( a_r \). Therefore, we conclude that there is not further production of a charge asymmetry after the end of inflation. The charge of the \( a \)-field is then conserved and dilutes at the usual rate \( n_a \sim R^{-3} \).

The final charge stored in the waterfall field can be estimated from the solutions of the equations of motion after passing by through the instability points, see Fig. 1. The details of the calculation can be found in [1], and the final result is

\[
|n_a| \sim \frac{M^2 m^2}{54 \pi^2 H_0} x^2 e^{-x^2},
\]

where variable \( x \) is defined as

\[
x = \frac{3 \pi^{3/2} M^5}{2 \lambda^2 g \bar{m}^5} \approx 2.93 \times 10^{-2} \frac{\lambda}{g^2},
\]

with \( \bar{M} \equiv M/m_{Pl} \) and \( \bar{m} \equiv m/m_{Pl} \) are the Planck normalized values of the waterfall and inflaton mass terms, respectively.

The asymmetry in Eq. (18) is not the one we need to transfer to leptons, but rather we need to calculate the quantity \( |n_a|/s_{reh} \), where \( s_{reh} = (2\pi^2/45)q_{reh} T_{reh}^3 \) is the entropy density generated during the reheating process; here, \( q_{reh} \approx 10^2 \) represents the entropic degrees of freedom at the reheating temperature \( T_{reh} \).
After a straightforward calculation taking Eqs. (16) and (18) we find
\[ \frac{|n_a|}{s_{reh}} \simeq 0.01 \frac{\lambda^{3/2} M^{7/2}}{g'^{3/2} h_1^3}. \] (20)

Eq. (20) differs from Eq. (15) in Ref.[1] because in the latter we assumed prompt reheating after inflation. Also, for the preferred values of the parameters of the model the exponential term is of the order of 0.1.

Interestingly enough, the charge asymmetry in the waterfall field only depends upon the waterfall mass term and any appearance of the inflaton mass is naturally taken out of the final expression.

**LEPTOGENESIS**

We now proceed to study the decay of the waterfall field into a right-handed neutrino, for which we propose an interaction term in the Lagrangian of the form
\[ \mathcal{L}_{aN_R} = h_2 \bar{N}_R^c N_R a. \] (21)
whose corresponding decay width is \( \Gamma_a = h_2^2 M/8 \pi \).

We shall impose the condition \( \Gamma_a > \Gamma_c \) to assure that the reheating process is finished well before the a-charge is transferred to the right handed neutrino. Explicitly, the condition is \( h_2 < \sqrt{g'/\lambda} h_1 \).

When the a-particles begin to disintegrate rapidly at a time \( t_a = \Gamma_a^{-1} \), we have a Universe dominated by relativistic fermions and the dominant particle processes are CP conserving. These are: \( a \longrightarrow N_R + N_R \) and \( a^c \longrightarrow N_R^c + N_R^c \), with decay width \( \Gamma_a \); \( N_R \leftrightarrow \Phi + \ell_L \), and \( N_R^c \rightarrow \Phi^c + \ell_L^c \), with decay width \( \Gamma_D \); and \( N_R \leftrightarrow N_R^c \) with decay width \( \Gamma_M \).

The \( \Gamma_M \)-term appears because of the presence of the non-zero vev of the waterfall field in the Yukawa coupling with the right-handed neutrino; it is an interaction term that converts the right handed neutrino into its own antineutrino.

The decay rates corresponding to the right-handed neutrinos and the leptons are explicitly given by
\[ \Gamma_D = \frac{\tilde{m}_1 M_N^2}{8\pi v^2}, \quad \Gamma_M = M_N/8\pi, \] (22)
where \( \tilde{m}_1 \) is the effective light neutrino mass if the origin of neutrino masses comes from the usual see-saw mechanism. Taking the experimental limit on neutrino masses, one finds that \( \tilde{m}_1 \sim 10^{-10} \text{ GeV} \) for \( v = 174 \text{ GeV} \[[12].\]

It should be stressed out that in our model there is no CP violation in the decay of the right handed neutrino; its interaction with the waterfall field serves only for the transfer of a leptonic number from the a-charge to the leptons of the Standard Model. However, the mass term of the right-handed neutrino indeed violates leptonic number, and it acts as a suppression term of the leptonic charge. This shall be explained in the next section.

**Boltzmann equations**

Let us write and study the Boltzmann equations for the different species in our model. Because of the CPT theorem we have \( \Gamma(X \rightarrow Y) = \Gamma(X^c \rightarrow Y^c) \) for each decay; besides, for each one of the species we define the quantity \( Y = n/s \), which is the number of particles in a comoving volume. The Boltzmann equations are then the usual ones of the specialized literature\[4, 14, 15].

For the waterfall field \( a \), we write
\[
\frac{dY_a}{dz} = -\frac{z}{sH(z=1)} \left[ \frac{Y_a}{Y_{aq}} \gamma(a \rightarrow N_R + N_R) - \frac{Y_{N_R} Y_{N_R}}{Y_{aq} Y_{aq}} \gamma(N_R + N_R \rightarrow a) \right. \\
+ \frac{Y_a}{Y_{aq}} \gamma(a \rightarrow \phi + \phi) + \frac{Y_a}{Y_{aq}} \gamma(a \rightarrow \phi + \phi) \right], \tag{23a}
\]
\[
\frac{dY_{a^c}}{dz} = -\frac{z}{sH(z=1)} \left[ \frac{Y_{a^c}}{Y_{a^c q}} \gamma(a^c \rightarrow N_R^c + N_R^c) - \frac{Y_{N_R^c} Y_{N_R^c}}{Y_{a^c q} Y_{a^c q}} \gamma(N_R^c + N_R^c \rightarrow a^c) \right. \\
+ \frac{Y_{a^c}}{Y_{a^c q}} \gamma(a^c \rightarrow \phi + \phi) + \frac{Y_{a^c}}{Y_{a^c q}} \gamma(a^c \rightarrow \phi + \phi) \right], \tag{23b}
\]
where \( z = m_{N_R}/T, \ H(z=1) \) is the Hubble parameter at \( T = M_{N_R}, \ Y_a = n_a/s, \ Y_{a^c} = n_{a^c}/s, \ Y_{N_R} = n_{N_R}/s, \) and \( Y_{N_R^c} = n_{N_R^c}/s; \) in all terms, \( s \) denotes the entropy density. Notice that we are assuming that the relevant
mass scale for the Boltzmann equations is the mass of the right-handed neutrino. For the right-handed neutrino one has

\[
\frac{dY_{N_R}}{dz} = -\frac{z}{sH(z = 1)} \left[ \frac{Y_{N_R} Y_{N_R}}{Y_{N_R}^{eq} N_R} \gamma(N_R + N_R \rightarrow a) - \frac{Y_a}{Y_{N_R}^{eq}} \gamma(a \rightarrow N_R + N_R) \right] \\
- \frac{z}{sH(z = 1)} \left[ \frac{Y_{N_R} Y_{N_R}}{Y_{N_R}^{eq} N_R} \gamma(N_R \rightarrow N_R^L) - \frac{Y_{N_R}^{eq}}{Y_{N_R}^{eq} N_R} \gamma(N_R \rightarrow N_R) \right], \tag{24a}
\]

\[
\frac{dY_{N_R^c}}{dz} = -\frac{z}{sH(z = 1)} \left[ \frac{Y_{N_R^c} Y_{N_R^c}}{Y_{N_R^c}^{eq} N_R^c} \gamma(N_R^c + N_R^c \rightarrow a^c) - \frac{Y_{a^c}}{Y_{N_R^c}^{eq}} \gamma(a^c \rightarrow N_R^c + N_R^c) \right] \\
- \frac{z}{sH(z = 1)} \left[ \frac{Y_{N_R^c}}{Y_{N_R^c}^{eq} N_R^c} \gamma(N_R^c \rightarrow N_R^c + l^c) - \frac{Y_{N_R^c}}{Y_{N_R^c}^{eq} N_R^c} \gamma(l^c \rightarrow N_R^c) \right], \tag{24b}
\]

whereas for the doublet we have

\[
\frac{dY_l}{dz} = \frac{z}{sH(z = 1)} \left( \frac{Y_{N_R} Y_{l}}{Y_{N_R}^{eq} N_R} \gamma(N_R \rightarrow l + \Phi) - \frac{Y_l Y_{N_R}}{Y_l^{eq} N_R} \gamma(l + \Phi \rightarrow N_R) \right), \tag{25a}
\]

\[
\frac{dY_{l^c}}{dz} = \frac{z}{sH(z = 1)} \left( \frac{Y_{N_R} Y_{l^c}}{Y_{N_R}^{eq} N_R} \gamma(N_R \rightarrow l^c + \Phi^c) - \frac{Y_{l^c} Y_{N_R}}{Y_{l^c}^{eq} N_R} \gamma(l^c + \Phi^c \rightarrow N_R^c) \right). \tag{25b}
\]

The terms \(\gamma(a \rightarrow Y)\) are defined as

\[
\gamma(a \rightarrow Y) = n_a^{eq} K_1(z) \Gamma(a \rightarrow Y), \tag{26}
\]

where \(K_1(z)\) and \(K_2(z)\) are the modified Bessel functions, and \(\Gamma\) is the usual decay width at zero temperature in the rest frame of the decaying particle. For two-body scattering we have

\[
\gamma(a + b \rightarrow Y) = n_a^{eq} n_b^{eq} \langle \sigma(a + b \rightarrow Y)|v| \rangle. \tag{27}
\]

and also

\[
\gamma(aX \rightarrow Y) = \int d\pi_a d\pi_X d\pi_Y (2\pi)^4 \delta^4(p_a + p_X - p_Y) f_a^{eq} f_X^{eq} f_Y^{eq} |M(aX \rightarrow Y)|^2, \quad f_a^{eq} = e^{-E_a/T}. \tag{28}
\]

Solving the Boltzmann Equations

In order to find semi-analytical solutions of the Boltzmann equations, we take into account only the dominant terms in each equation as described in Sec., and assume the standard condition of kinematic equilibrium for each species. The resulting equations are

\[
\frac{dY_a}{dz} = -\frac{\Gamma_a}{H(z = 1)} \frac{z K_1(z)}{K_2(z)} (Y_a - Y_a^{eq}), \tag{29a}
\]

\[
\frac{dY_{N_R}}{dz} = -\frac{\Gamma_a}{H(z = 1)} \frac{z K_1(z)}{K_2(z)} (Y_{N_R}^{eq} - Y_a) \\
- \frac{\Gamma_D}{H(z = 1)} \frac{z K_1(z)}{K_2(z)} (Y_{N_R} - Y_{N_R}^{eq}) \\
- \frac{\Gamma_M}{H(z = 1)} \frac{z K_1(z)}{K_2(z)} (Y_{N_R^c} - Y_{N_R^c}^{eq}), \tag{29b}
\]

\[
\frac{dY_l}{dz} = \frac{\Gamma_D}{H(z = 1)} \frac{z K_1(z)}{K_2(z)} (Y_{N_R} - Y_{N_R}^{eq}) \tag{29c}
\]
The equations for the antiparticles are exactly the same because in our model the CP symmetry is conserved in the true vacuum.

In writing Eqs. (29) we made two further assumptions. First, that the decay of the waterfall field occurs out of equilibrium such that the inverse process $N_R \rightarrow a$ is kinematically forbidden because $M > M_{N_R}$. Second, the same reasoning applies for the inverse process $l + \Phi \rightarrow N_R$ and then the mass of the leptons should be smaller than that of the right-handed neutrino.

It should be noticed that the inverse processes involving the inflaton have also been neglected everywhere in the Boltzmann equations for the $a$-field (29) and (31). This approximation is well justified because we previously assumed that $\Gamma_a \ll \Gamma_\phi$ and then the inflaton field should have decayed completely at the time the leptogenesis process is taking place.

However, we should also prevent the creation of inflaton particles mediated by the decay rates

$$\Gamma(a \rightarrow \phi + \phi) \propto \frac{g^4 M}{64\pi \lambda^2} \ll \Gamma_a,$$  \hspace{1cm} (30a)

$$\Gamma(a + a \rightarrow \phi + \phi) \propto \frac{g^4 M}{32\pi} \ll \Gamma_a,$$ \hspace{1cm} (30b)

In order to prevent any wash-out of the $a$-asymmetry due to their interactions with the inflaton field it will suffice to impose the condition $g^2 \ll h_2$.

Let us define $\Delta_b = Y_b - Y_{b\bar{e}}$ for each species. The Boltzmann equation for the waterfall field simply reads

$$\frac{d\Delta_a}{dz} = -K_a \frac{z^2}{2 + z} \Delta_a.$$ \hspace{1cm} (31)

Its solution with an initial condition $\Delta_a(0) = \Delta_0$ is given by

$$\Delta_a(z) = \Delta_0 \exp\left(\frac{K_a z^2}{2 + z} - \frac{1}{4} \Gamma_a\right),$$ \hspace{1cm} (32)

where $K_a = \Gamma_a/H(z = 1)$.

Likewise, the Boltzmann equation for the right-handed neutrino is

$$\frac{d\Delta_{NR}}{dz} = -\frac{z^2}{2 + z} (K_a \Delta_a - K_{DM} \Delta_{NR}),$$ \hspace{1cm} (33)

where $K_{DM} = (\Gamma_D + 2 \Gamma_M)/H(z = 1)$. Up to quadratures, its solution under the initial condition $\Delta_{NR}(z = 0) = 0$ is

$$\Delta_{NR} = \frac{\exp\left(\frac{K_{DM} z^2}{2 + z} - \frac{1}{4} \Gamma_{DM}\right) \int_0^z K_a x^2}{1 + z/2} \Delta_a(x) \, dx.$$ \hspace{1cm} (34)

Finally, the equation for the leptonic doublet is

$$\frac{d\Delta_l}{dz} = -\frac{z^2}{2 + z} K_D \Delta_{NR},$$ \hspace{1cm} (35)

where $K_D = \Gamma_D/H(z = 1)$.

The resulting leptonic charge can be obtained from the integration of Eq. (35), but for that we need also the analytic solution of Eq. (34). Fortunately, that is not necessary because Eqs. (31), (33), and (35) can be combined together to get a single equation for the three abundances. After integration under the initial conditions $\Delta_l(z = 0) = 0$, it can be shown that

$$\Delta_l + \frac{K_D}{K_{DM}} (\Delta_{NR} + \Delta_a) = \frac{K_D}{K_{DM}} \Delta_0.$$ \hspace{1cm} (36)

Eq. (36) is the main result in our work and represents the evolution and transfer of the $a$-charge through the leptogenesis process.

### Lepton and baryon asymmetries

According to Eq. (32), not a piece of the waterfall field charge survives the transfer process and then $\Delta_a \rightarrow 0$ in the limit $z \rightarrow \infty$. The same will happen to the charge of the right-handed neutrino in the same limit, see Eq. (34). Therefore, the only surviving charge will be the leptonic and the final expression is

$$\Delta_l^\infty = \frac{K_D}{K_{DM}} \Delta_0 = \frac{\Delta_0}{1 + 2 \Gamma_M/\Gamma_D}.$$ \hspace{1cm} (37)

The lepton asymmetry (37) should be further converted into a baryon charge asymmetry [9, 10]. Essentially, one would have $\Delta_l \sim \Delta_B$, where the proportionality between the lepton and baryon asymmetries depends upon the particle contents of the model [17, 18, 19].

Taking into account the initial asymmetry of the waterfall field, see Eq. (29), and also the values of the decay widths given in Eq. (22), the baryon number would be approximately given by

$$\Delta_B \simeq 0.01 \frac{\lambda^{1/2}}{g^{5/2} h_1^4} \frac{\tilde{M}^{7/2}}{m_1 M_{N_R}} \simeq \frac{h_1^{-3} \tilde{M}^{7/2}}{1 + 2/M_{N_R}},$$ \hspace{1cm} (38)

where we have defined the (dimensionless) mass parameter $\tilde{M}_{N_R} \equiv \tilde{m}_1 M_{N_R}/v^2$, and also considered that $\lambda = 1$ and $g \simeq 0.1$ in the very last equality.

Interestingly enough, the leptogenesis process results in a final baryon asymmetry that is just the original one stored initially in the waterfall field except for a term that involves the mass of the right-handed neutrino.

The baryon charge asymmetry of the Universe is known to be in the following range [13]

$$4 \times 10^{-11} \leq \Delta_B \equiv n_B/s \leq 1.4 \times 10^{-10}.$$ \hspace{1cm} (39)

In the case $\tilde{M}_{N_R} \geq 2$, we see that the value of the baryon asymmetry is solely provided by the mass value of the waterfall field, and then $\tilde{M} \simeq h_1^{6/7} \Delta_B^{2/7}$. Recalling
the constraint \( g^2 < h_2 < \sqrt{g/\lambda} h_1 \) and the one arising from cosmic strings \( M^2/\lambda^2 < 10^{-6} \), then we find

\[
0.03 \lesssim h_1 \lesssim 1. \tag{40}
\]

All the parameters appear to be tightly constrained. A simple possibility is just to set \( h_1 = 1 \), \( h_2 = 0.1 \), \( M_{N_R} = 1 \), and then \( M \simeq \Delta_B^{2/7} \simeq 7 \times 10^{-4} \). In other words, the mass scales would be \( M \simeq 7.8 \times 10^{15} \text{GeV} \) and \( M_{N_R} \simeq 3 \times 10^{14} \text{GeV} \). Of course, larger values for the mass of the right-handed neutrino can also be considered, but we prefer a scenario in which \( M > M_{N_R} \). The reheating temperature, according to Eq. (10) is estimated to be \( T_{\text{reh}} \simeq 9 \times 10^{15} \text{GeV} \).

The other case is to have \( M_{N_R} \ll 2 \), in which the mass parameter of the right-handed neutrino participates in the final value of the baryon asymmetry; actually, the baryon asymmetry would now read \( \Delta_B \simeq h_1^{-3} M^{7/2} M_{N_R} \).

It is clear that the appearance of the right-handed neutrino mass asks for larger values of \( M \) in order to accomplish the baryon constraint (39). But, as we have seen in the simple exercise above, the cosmic strings constraint does not support large values of \( M \), and then the case \( M_{N_R} \ll 2 \) seems to be the only one allowed.

**FINAL REMARKS**

We have studied the transfer of an initial \( a \)-charge asymmetry produced at the end of inflation and stored in a complex waterfall field into a lepton asymmetry. In order to transfer this initial charge asymmetry first into a lepton asymmetry, and then to a baryon asymmetry, the model should fulfill two conditions.

First, one needs a mechanism of efficient reheating to produce enough waterfall particles; second, the waterfall field must be coupled to leptons. In order to solve the reheating process, we had to introduce new fermions which couple only to the inflaton field. These fermions may be candidates for dark matter as we further assumed they should have very weak interactions with Standard Model particles. As for the second condition, we assumed that the same waterfall field is the origin of the Majorana masses for the right-handed neutrinos.

Different constraints entered into play at each one of the stages of the model, but we were able to show that the simplest realization does not entail unnatural values for the diverse parameters. For instance, it was not necessary to introduce new fields in the leptogenesis process apart from the usual ones in the literature, nor the coupling parameters were obliged to have embarrassingly small values.

Once the \( a \)-charge is transformed into a baryon asymmetry we do not expect further changes because Standard Model interactions preserve the \( B - L \)-quantum number. It is important to stress out that our proposal is different to other leptogenesis models because in our approach there is not need for \( C P \) violating phases in the leptonic sector.

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* Electronic address: carlosr@ifm.umich.mx
† Electronic address: delepine@fisica.ugto.mx
‡ Electronic address: lurena@fisica.ugto.mx

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