Correlation functions of the global \( E_8 \) symmetry currents in the Heterotic 5-brane theory.

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We consider the 5-brane placed at one end of the world in the Heterotic \( E_8 \times E_8 \) theory. The low energy theory is a 6 dimensional \((1, 0)\) superconformal theory with \( E_8 \) as a global symmetry. We calculate the two-point correlator of the \( E_8 \) current in 6 dimensions and in 4 dimensions after compactification on \( T^2 \). This correlator is derived in 3 different ways: From field theory, from 11 dimensional supergravity and from F-theory.

This paper is a written version of the talk given at the 1998 Trieste Conference on Superfivebranes and Physics in 5+1 Dimensions. The talk presented the work of \( \text{[1]} \) where a more elaborate discussion can be found.

1 Introduction

In the past two years, many examples of nontrivial IR fixed points in various dimensions have emerged. Some of the most exciting ones are the 5+1D chiral theories. The first of such theories with \( \mathcal{N} = (2, 0) \) SUSY has been discovered in \( \text{[2]} \) as a sector of type-IIB compactified on an \( A_1 \) singularity. A dual realization was found in \( \text{[3]} \) as the low-energy description of two 5-branes of M-theory. Another theory of this kind arises as an M-theory 5-brane approaches the 9-brane \( \text{[4]} \). When the distance between the 5-brane and 9-brane is zero, the low-energy is described by a nontrivial 5+1D fixed point. This theory is chiral with \( \mathcal{N} = (1, 0) \) SUSY and a global \( E_8 \) symmetry. In \( \text{[5]} \) more examples of \( \mathcal{N} = (1, 0) \) theories have been given. We will use the terminology of \( \text{[5]} \) and call the \( E_8 \) theory \( V_1 \). Many other 5+1D theories have been recently constructed in \( \text{[6]} \).

String theory is a powerful tool to study such theories. The idea is to identify a dual description such that quantum corrections of the original theory appear at the classical level of the dual \( \text{[7]} \). The toroidal compactification of the \( \mathcal{N} = (1, 0) \) 6D theory (and hence \( 4D \mathcal{N} = 2 \) QCD) can be studied using the brane-probe technique discovered in \( \text{[8]} \). The world-volume theory on a brane probe in a heterotic string vacuum (which is quantum mechanically corrected) is mapped by duality to a world-volume theory on a brane inside a curved background which is not quantum mechanically corrected. This allows one to
determine the low-energy behavior in 4D. At the origin of the moduli space one obtains an IR fixed point with $E_8$ global symmetry.

The purpose of the present work is to extract information about the local operators of such theories. The $E_8$ theory $V_1$ has a local $E_8$ current $j^a_\mu(x)$ ($a = 1 \ldots 248$ and $\mu = 0 \ldots 5$). We will be interested in the correlator $\langle j^a_\mu(x)j^b_\nu(0) \rangle$. The strategy will be to couple the theory to a weakly coupled $E_8$ gauge theory and calculate the effect of $V_1$ on the $E_8$ coupling constant. We will study the question both for the 5+1D theory and for the 3+1D conformal theories. We will present three methods for evaluating the correlator. The first method is purely field-theoretic and applies to the 3+1D theories. Deforming the theory with a relevant operator one can flow to the IR where a field-theoretic description of $SU(2)$ or $U(1)$ with several quarks can be found. This will allow us to determine the correlator as a function on the moduli space. From this function we can deduce the high-energy behavior of the correlator and find out how many copies of the $E_8$ theory can be gauged with an $E_8$ SYM before breaking asymptotic freedom.

The other two methods for determining the correlators involve M-theory and F-theory. The gravitational field of a 5-brane of M-theory which is close to a 9-brane changes the local metric on the 9-brane. After compactification on a large $K3$ this implies that the volume of the $K3$ at the position of the 9-brane is affected by the distance from the 5-brane (sec 3). This can be interpreted as a dependence of the $E_8$ coupling constant on the VEV which specifies the position of the 5-brane. From this fact we can extract the current correlator. The third method involves the F-theory realization of the $E_8$ theory. The $V_1$ theory is obtained in F-theory compactifications on a 3-fold by blowing up a point in the (two complex dimensional) base. By studying the effect of the size of the blow-up on the size of the 7-brane locus we can again determine the dependence of the $E_8$ coupling constant on the VEV.

The paper is organized as follows. In section (2) we calculate the current-current correlators in 3+1D using field theory arguments and we argue that 10 copies of the $E_8$ theory can be coupled to a gauge field. In section (3) we study the effect of a 5-brane on the volume of a 9-brane in M-theory and deduce the correlator from this setting. In section (4) we present the F-theory derivation. In section (5) we conclude.

2 Field theory derivation of the 4 dimensional correlator

In this section we will derive the form of the $E_8$ current-current correlator for the $E_8$ conformal theory and as a result we will argue that in 4D one can couple up to 10 copies of the $E_8$ theory to a $\mathcal{N} = 2$ $E_8$ Yang-Mills gauge theory.
We start with the $E_8$ conformal theory in 4 dimensions whose Seiberg-Witten curve is given by

$$y^2 = x^3 + u^5,$$

(1)

$u$ parameterizing the moduli space of the Coulomb branch. We are looking for an expression of the form

$$\langle j^a_\mu(x) j^b_\nu(y) \rangle = \frac{1}{g^2} \eta^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \delta^{ab} f(q^2, u, \Lambda), \quad a, b = 1 \ldots 248,$$

(2)

where $q$ is the momentum and $\Lambda$ is some fixed UV-cutoff. This UV-cutoff is not physical. It is just an artifact of the Fourier transform. The space-time correlator $\langle j^a_\mu(x) j^b_\nu(y) \rangle$ does not require a cutoff.

To determine the form of $f$ in (2) for $q^2 = 0$ we can couple the $E_8$ SCFT to a weakly coupled $E_8$ gauge field and ask how the $E_8$ coupling constant changes as a function of $u$. When the $E_8$ coupling constant is very small the coupling does not change the curve (1) by much. For a generic value of $u$ the massless modes of the $E_8$ SCFT are neutral under the global $E_8$ and the charged matter has a typical energy of order $u^{1/6}$. The $\langle jj \rangle$ correlator will modify the low energy $E_8$ coupling constant to the form

$$\frac{1}{g(u)^2} = \frac{1}{g_0^2} + f(q^2 = 0, u, \Lambda),$$

where $g_0$ is the bare coupling constant. On the other hand, standard renormalization arguments require that it should be possible to re-absorb the $\Lambda$ dependence in the bare coupling constant. Thus, dimensional analysis restricts the form of $f(0, u, \Lambda)$ to

$$f(0, u, \Lambda) = c \log \left( \frac{\Lambda}{|u|^{1/6}} \right).$$

(3)

To determine $c$ we deform the theory by adding a relevant operator to its (unknown) Lagrangian such as to break the global $E_8$ symmetry down to $D_4$ ($SO(8)$) by putting Wilson lines on the torus. The advantage is that the $D_4$ conformal fixed point can be analyzed in standard field-theory. It is the IR free theory of $SU(2)$ coupled to 4 massless quarks.

The global $E_8$ of the original theory has been broken by the operators to a global $SO(8)$. For the $SO(8)$ theory we can ask what is

$$\langle j^A_\mu(q) j^B_\nu(-q) \rangle, \quad A, B = 1 \ldots 28,$$

(4)

where $A, B$ are $SO(8)$ indices. The point is now that we can calculate this correlator for the $SO(8)$ theory from field theory. The relevant field theory is
SU(2) gauge theory with 4 quarks. From this correlator we can extract the original $E_8$ correlator. The details can be found in 1. We conclude that for the $E_8$ theory

$$\langle j^a_\mu(q) j^b_\nu(-q) \rangle = -\frac{3C(\text{fund.})}{4\pi^2} \delta^{ab}(q^2 \eta_{\mu\nu} - q_\mu q_\nu) \log \left| \frac{(\Lambda \lambda^2)^{1/3}}{u^{1/6}} \right|. \quad (5)$$

This means that the value of $c$ in 3 is

$$c = -3 \frac{C(\text{fund. of } SO(8))}{4\pi^2} = - \frac{1}{10} \times \frac{C_2(E_8)}{4\pi^2}.$$  

This value of $c$ implies that 10 copies of the $E_8$ SCFT can be coupled to an $E_8$ SYM.

### 3 Derivation from M-theory

The system of the (1, 0) $E_8$ theory ($V_1$) coupled to $E_8$ SYM can be realized in M-theory as a 5-brane which is close to the 9-brane. The modes of the $V_1$ theory come from the 5-brane bulk and from membranes stretched between the 5-brane and 9-brane while the $E_8$ SYM comes from the 9-brane bulk. Let us compactify on $K3 \times T^2$.

The effect that we are trying to study corresponds to the following question. The gravitational field of the 5-brane affects the metric at the position of the 9-brane. Thus, as we change the distance of the 5-brane from the 9-brane the volume of the $K3$ changes as a function of $x^4$. The volume of $K3 \times T^2$ is related to the 3+1D $E_8$ coupling constant. In field-theory, this is interpreted as a running of the $E_8$ coupling constant as a result of the change of the VEV of the $V_1$ theory.

We apply the general setting and formulae of 3 to the case where the distance of the 5-brane from the 9-brane is much smaller than the compactification scale of $K3 \times T^2$ and calculate the effect.

We must also mention that the after compactification of the system of a 5-brane and 9-brane on $S^1$ we get 4-branes near 8-branes. This setting has been studied in 3 in the context of brane probes, where a related effect is observed. The position of the probe affects the value of a classical field, in that case the dilaton, which is then re-interpreted as a 1-loop effect in field theory. In fact, the relation between the classical supergravity calculation and the 1-loop field-theory calculation follows from perturbative string-theory. The 1-loop result is a loop of DD strings connecting the 4-brane to the 8-brane while the classical supergravity result is the same diagram viewed from the closed string channel.
3.1 Geometrical setup and review

In this section we will examine the theory of a 5-brane in M-theory on $\mathbb{R}^{5,1} \times K3 \times S^1/\mathbb{Z}_2$ and review some relevant facts from $\cite{[1],[4]}$

The geometric setup is as follows. The coordinates $(x^1, x^2, ..., x^6)$ parameterize $\mathbb{R}^{5,1}$, $(x^7, x^8, x^9, x^{10})$ parameterize K3 and finally $x^{11}$ parameterizes $S^1/\mathbb{Z}_2$. All 5-branes have their world-volume along $\mathbb{R}^{5,1}$ and are located at a point in $K3 \times S^1/\mathbb{Z}_2$. All configurations will be defined on the whole $S^1$ and are symmetric under the $\mathbb{Z}_2$ (working “upstairs” – in the terminology of $\cite{[1]}$). This means, for example, that every time there is a 5-brane between the two fixed planes of the $\mathbb{Z}_2$ there is also a mirror 5-brane. There would be an equivalent formulation (“downstairs”) where configurations were only defined on the interval between the two “ends of the world”.

We know that M-theory on $\mathbb{R}^{5,1} \times S^1/\mathbb{Z}_2$ is heterotic $E_8 \times E_8$ with one $E_8$ theory living on each fixed plane of the $\mathbb{Z}_2$. If we compactify this theory on K3 we need to supply a total of 24 instantons and 5-branes. The theory we are interested in is a single 5-brane coupled to an $E_8$ gauge theory. To achieve this we need to have no instantons in one of the $E_8$ theories and one 5-brane close to this “end of the world”. The remaining 23 instantons and 5-branes must therefore be either instantons in the other “end of the world” or 5-branes in the bulk.

In the 6-dimensional description the distance of the 5-brane from the “end of the world,” $x$, is a modulus. The effective gauge coupling of the $E_8$ depends on $x$. From the 6-dimensional point of view certain degrees of freedom connected to the 5-brane act as matter coupled to the $E_8$ gauge field. Since the couplings and masses of this matter depend on $x$, the low energy effective $E_8$ gauge coupling, $g$, will depend on $x$. Here we will calculate the $x$-dependence of $g$ from M-theory or more precisely from 11-dimensional supergravity. For supergravity to be applicable all distances involved in the problem need to be much bigger than the 11-dimensional Planck scale. This means especially that $\text{Vol}(K3) \gg l^4_{\text{Planck}}$. Furthermore we are interested in the behaviour of the theory when it is close to the point with tensionless strings or equivalently with a zero size instanton, which is $x = 0$. To be in that situation we take $x \ll \text{vol}(K3)^{\frac{1}{4}}$. The $x$-dependence of the 6-dimensional gauge coupling $g$, comes about because the volume of the K3 at $X^{11} = 0$ depends on $x$.

To calculate $g$ we need to find the form of the metric as a function of $x$. The calculation of the metric and of $g$ is described in detail in $\cite{[1]}$. Here we will just state the result, which is

$$\frac{1}{g^2} = \frac{1}{g_0^2} - \frac{1}{8\pi^2} x T_2.$$ (6)
Here $T_2$ is the tension of the membrane in 11 dimensions. The expression $xT_2$ is the tension of the strings in the six-dimensional theory. This is because the membrane is stretched with one direction along the 11th direction and two directions along $\mathbb{R}^{5,1}$. Compactifying further down to 4 dimensions on a torus of area $A$ is straightforward

$$\frac{1}{g^2} = \frac{1}{g_0^2} - \frac{1}{8\pi^2} AxT_2. \quad (7)$$

This equation contains the needed information about the 4 dimensional theory. It tells how the gauge coupling in a $E_8$ gauge theory runs as a result of coupling to the 4 dimensional superconformal theory with $E_8$ global symmetry. In it is shown how this implies that 10 of these saturate the $\beta$-function in complete agreement with the field theory derivation.

4 The 6D current-current correlator from F-theory

In this section, we use the duality between F theory on elliptic Calabi-Yau 3-folds and Heterotic String on $K3$ to compute the effective gauge coupling of heterotic string in six dimensions. We shall see that the result agrees completely with the corresponding M-theory calculation to first order. A second order effect which is suppressed by a factor of the volume of the K3 and by the length of $S^1/\mathbb{Z}_2$ in calculations in the previous section naturally emerges in the F-theory setting. In the limit in which we extract the correlator for $V_1$, i.e. taking the volume of K3 and the size of $S^1/\mathbb{Z}_2$ to infinity, this second order effect vanishes.

We start with $V_1$ and couple it to a 6D $E_8$ SYM theory. The gauge theory is defined with a UV cut-off, but this imposes no problem for us since all we need is the dependence of the IR coupling constant on the VEVs of the $V_1$ theory. To be precise, we take the $E_8$ UV cut-off to be $\Lambda$ and fix the $E_8$ coupling constant at $\Lambda$. The Coulomb branch of the $V_1$ theory has a single tensor multiplet. We denote the VEV of its scalar component by $\phi$. $\phi$ is the tension of the BPS string in $\mathbb{R}^{5,1}$. In M-theory $\phi = xT_2$. The mass scale of the $V_1$ theory is thus $\phi^{1/2}$. We would like to find the dependence of the IR $E_8$ coupling constant on $\phi$ when $\phi \ll \Lambda$. Heuristically speaking, the running $E_8$ coupling constant will receive contributions from loops of modes from $V_1$ of mass $\sim \phi^{1/2}$.

The set-up that we have just described arises in the heterotic string compactified on $K3$ with a small $E_8$ instanton. We take the $(0, 23)$ embedding with a single 5-brane in the bulk close to the 9-brane with unbroken $E_8$. The F-theory dual has a base $B$ which is the Hirzebruch surface $F_{11}$ with one point.
blown-up. \( F_n \) is a \( P^1 \) bundle over \( P^1 \). Let the area of the fiber \( P^1 \) in \( F_{11} \) (i.e. the Kähler class integrated over the fiber) be \( k_F \) and the area of zero section \( P^1 \) of the fibration be \( k_D \).

We blow-up a point in the zero section of the fibration of \( P^1 \) over \( P^1 \). There are 10 7-branes wrapping that zero-section and passing through a point of the exceptional divisor. These are responsible for the unbroken \( E_8 \) gauge group. Let \( k_E \) be the area of the exceptional divisor. The area of the above mentioned 7-brane locus (part with unbroken \( E_8 \)) is \( k_D \). The Kähler class is

\[
k = (k_F - k_E)E + k_F D + (k_D + k_E - nk_F)F
\]

where \( E, D, F \) are the cohomology classes of the exceptional divisor, base and fiber.

\[
E \cdot E = -1, \quad E \cdot D = F \cdot D = 1, \quad (8)
\]

\[
D \cdot D = n - 1, \quad E \cdot F = F \cdot F = 0. \quad (9)
\]

A 3-brane wrapping the exceptional divisor gives a BPS string in \( \mathbb{R}^{5,1} \) (corresponding to the membrane connecting the M-theory 5-brane to the end of the world). Its tension is given by integrating the D3-brane tension over \( E \).

Using \( 18 \)

\[
2 \kappa^2 \tau^2_p = 2\pi(4\pi^2 \alpha')^{3-p}
\]

the tension of the BPS string is simply

\[
\phi = \pi^{1/2} k_E
\]

in the units \( \kappa = 1 \). The volume of the whole base is

\[
V = \frac{1}{2} k \cdot k = k_F (k_D + k_E) - \frac{1}{2} k_E^2 - \frac{n}{2} k_F^2. \quad (11)
\]

This volume is the 6D inverse gravitational constant and we have to keep it fixed. Although the \( V_1 \) modes have an effect on the gravitational constant as well, by dimensional analysis, this effect is much smaller than \( \phi \) and behaves as \( \sim \phi^2 \). How should \( k_F \) depend on \( \phi \), in our setting? \( k_F \) measures the tension of 3-branes wrapped on \( F \). On the heterotic side, these are elementary strings which occupy a point on K3. Their tension is fixed in the heterotic picture. Thus \( k_F \) is independent of \( \phi \).

Now we come to the gauge coupling. To do this calculation it is convenient to imagine that \( E_8 \) is broken down to \( U(8) \subset E_8 \). The gauge kinetic term for
8 unwrapped 7-branes of the same type is
\[ \int \tau_7 \frac{(2\pi\alpha')^2}{4} \text{tr}_8 \{ F^2 \} d^8x. \]

We are working in the conventions
\[ \text{tr}\{ T^a T^b \} = \delta^{ab}, \quad a, b = 1 \ldots 248. \]

For the \( U(8) \) subgroup this means that
\[ \text{tr}_8 \{ T^a T^b \} = \frac{1}{2} \delta^{ab}. \]

This means that for a configuration of 10 7-branes forming an \( E_8 \) gauge theory the gauge kinetic term is:
\[ \frac{1}{8} \int (2\pi\alpha')^2 \tau_7 \left( \sum_{a=1}^{248} F^a F^a \right) d^8x. \]

From this we read off (in units where \( \kappa = 1 \))
\[ \frac{1}{4g^2} = \frac{1}{8} (2\pi\alpha')^2 \tau_7 = \frac{1}{32} \pi^{-3/2}. \]

Wrapping the 7-branes on \( D \) we get a 5+1D \( E_8 \) gauge theory with coupling constant
\[ \frac{1}{4g^2} = \frac{1}{32} \pi^{-3/2} k_D. \]

From \( [1] \) we find that when \( V \) and \( k_F \) are kept fixed and \( k_E = \pi^{-1/2} \phi \), the \( E_8 \) coupling constant is
\[ \frac{1}{g(\phi)^2} = \frac{1}{8} \pi^{-3/2} [(k_D + k_E) - k_E] = \frac{1}{(g_0)^2} - \frac{1}{8\pi^2} \phi. \quad (12) \]

We have used the fact that \( (k_D + k_E) \) is fixed to first order in \( \phi \) when \( V \) is fixed. The other two terms in \( V \) are higher order corrections dual to taking \( K3 \) and the distance between the ends of the world to be large in the M-theory calculations. Eqn.\[12\] describes the running of the \( E_8 \) coupling constant because of the coupling to \( V_1 \). This is in complete agreement with the result obtained from M-theory.
5 Discussion

We have found that for the 3+1D \( E_8 \) super-conformal theory with Seiberg-Witten curve

\[ y^2 = x^3 + u^5, \]

the 2-point \( E_8 \) current correlator on the Coulomb branch satisfies:

\[
\langle j^a_\mu(q)j^b_\nu(-q) \rangle = \begin{cases} 
\frac{C_2(E_8)}{40\pi^2} \delta^{ab}(q_\mu q_\nu - q^2 \eta_{\mu\nu}) \log \left( \frac{\Lambda}{|u|^{1/6}} \right) & \text{for } |q| \ll |u|^{1/6} \\
\frac{C_2(E_8)}{40\pi^2} \delta^{ab}(q_\mu q_\nu - q^2 \eta_{\mu\nu}) \log \left( \frac{\Lambda}{|q|} \right) & \text{for } |q| \gg |u|^{1/6}
\end{cases}
\]

where \( \Lambda \) is a UV cutoff which is an artifact of Fourier transforming.

We deduced that 10 copies of the \( E_8 \) theory can be coupled as “matter” to an \( \mathcal{N} = 2 \) \( E_8 \) SYM gauge field.

In 5+1D we found the expression for the low-energy limit of the 5+1D correlator of the \( \mathcal{N} = (1,0) \) \( E_8 \) theory on the Coulomb branch and away from the origin:

\[
\langle j^a_\mu(q)j^b_\nu(-q) \rangle = \frac{C_2(E_8)}{240\pi^2} \delta^{ab}(q^2 \eta_{\mu\nu} - q_\mu q_\nu)(\Lambda^2 - \phi) \quad \text{for } |q| \ll \phi.
\]

where \( \phi \) is the VEV of the scalar of the low-energy tensor multiplet.

It would be interesting to determine the correlator in the UV region \( |q| \gg |\phi| \) or, equivalently, at the fixed point \( \phi = 0 \). It seems that the methods presented in this paper are not powerful enough for that purpose.

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