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**Numerable open covers and representability of topological stacks.**  
(English)  
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Summary: We prove that the class of numerable open covers of topological spaces is the smallest class that contains covers with pairwise disjoint elements and numerable covers with two elements, closed under composition and coarsening of covers. We apply this result to establish an analogue of the Brown-Gersten property for numerable open covers of topological spaces: a simplicial presheaf on the site of topological spaces satisfies the homotopy descent property for all numerable open covers if and only if it satisfies it for numerable covers with two elements and covers with pairwise disjoint elements. We also prove a strengthening of these results for manifolds, ensuring that covers with two elements can be taken to have a specific simple form. We apply these results to deduce a representability criterion for stacks on topological spaces similar to arXiv:1912.10544. We also use these results to establish new simple criteria for chain complexes of sheaves of abelian groups to satisfy the homotopy descent property. This article is available at arXiv:2203.03120v2.

MSC:

- 54-XX General topology
- 55-XX Algebraic topology
- 57-XX Manifolds and cell complexes

Keywords:

- simplicial presheaves; numerable covers

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