THEORETICAL ESTIMATES OF TWO-POINT SHEAR CORRELATION FUNCTIONS USING TANGLED MAGNETIC FIELDS

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ABSTRACT

The existence of primordial magnetic fields can induce matter perturbations with additional power at small scales as compared to the usual ΛCDM model. We study its implication within the context of a two-point shear correlation function from gravitational lensing. We show that a primordial magnetic field can leave its imprints on the shear correlation function at angular scales \( \lesssim \) a few arcminutes. The results are compared with CFHTLS data, which yield some of the strongest known constraints on the parameters (strength and spectral index) of the primordial magnetic field. We also discuss the possibility of detecting sub-nano Gauss fields using future missions such as SNAP.

Key words: cosmology: observations – cosmology: theory – gravitational lensing: weak – magnetic fields

Online-only material: color figures

1. INTRODUCTION

In recent years, weak gravitational lensing has proved to be one of best probes of the matter power spectrum of the universe. In particular, this method can reliably estimate the matter power spectrum at small scales that are not directly accessible to other methods, e.g., galaxy surveys (for details and further references see, e.g., Munshi et al. 2008; Hoekstra & Jain 2008; Refregier 2003; Bartelmann & Schneider 2001).

Magnetic fields play an important role in the many areas of astrophysics and are ubiquitously seen in the universe. They have been observed in galaxies and clusters of galaxies with coherence lengths up to \( \simeq 10–100 \) kpc (for a review see, e.g., Widrow 2002). There is also evidence of coherent magnetic fields up to supercluster scales (Kim et al. 1989). Still little is known about the origin of cosmic magnetic fields and their role in the evolutionary history of the universe. These fields may have originated from dynamo amplification of very tiny seed magnetic fields \( \simeq 10^{-20} \) G (e.g., Parker 1979; Zeldovich et al. 1983; Ruzmaikin et al. 1988). It has been shown that dynamo mechanism can amplify fields to significant values in collapsing objects at high redshifts (Ryu et al. 2008; Schleicher et al. 2010; Arshakian et al. 2009; de Souza & Opher 2010; Federrath et al. 2011a, 2011b; Schober et al. 2011). It is also possible that much larger primordial magnetic fields \( \simeq 10^{-9} \) G were generated during the inflationary phase (Turner & Widrow 1988; Ratra 1992) and the large-scale magnetic fields observed today are the relics of these fields. In the latter case, of interest to us in this paper, a magnetic field starts with a large value in the intergalactic medium, while in the former case large magnetic fields are confined to bound objects.

While the presence of primordial magnetic fields has the potential to explain the observed magnetic fields coherent at a range of scales in the present universe, such fields also leave detectable signatures in important observables at cosmological scales in the universe.

The impact of large-scale primordial magnetic fields on CMBR temperature and polarization anisotropies has been studied in detail (e.g., Subramanian & Barrow 1998b, 2002; Seshadri & Subramanian 2001, 2009; Mack et al. 2002; Lewis 2004; Gopal & Sethi 2005; Tashiro & Sugiyama 2006; Sethi & Subramanian 2005, 2009; Sethi et al. 2008, 2010; Kahnashvili & Ratra 2005; Giovannini & Kunze 2008; Yamazaki et al. 2008). More recently, lower bounds \( \simeq 10^{-15} \) G on the strength of magnetic fields have been obtained based on observations of high-energy y-ray photons (e.g., Dolag 2010; Neronov & Vovk 2010; Tavecchio et al. 2010; Taylor et al. 2011).

Wasserman (1978) showed that primordial magnetic fields can induce density perturbations in the post-recombination universe. Further work along these lines have investigated the impact of this effect for the formation of first structures, reionization of the universe, and the signal from the redshifted H I line from the epoch of reionization (e.g., Kim et al. 1996; Gopal & Sethi 2003; Sethi & Subramanian 2005, 2009; Tashiro & Sugiyama 2006; Schleicher et al. 2009). The matter power spectrum induced by primordial magnetic fields can dominate the matter power spectrum of the standard ΛCDM model at small scales. Weak gravitational lensing can directly probe this difference and therefore reveal the presence of primordial fields or put additional constraint on their strength.

In this paper, we attempt to constrain primordial magnetic fields within the framework of the two-point shear correlation function induced by gravitational lensing, including the contribution of matter perturbations induced by these magnetic fields. We compare our results with the Canada–France–Hawaii Telescope Legacy Survey (CFHTLS) Wide data (Fu et al. 2008).

Throughout the paper, we used a flat \((k = 0)\) ΛCDM universe with \( \Omega_m = 0.24, \Omega_b = 0.044, h = 0.73, \) and \( \sigma_8 = 0.77.\)

2. MATTER POWER SPECTRUM

Tangled magnetic fields can be characterized by a power-law power spectrum: \( M(k) = A k^n.\) In the pre-recombination era, the magnetic fields are dissipated at scales below a scale corresponding to \( k_{\text{max}} \simeq 200 \times (10^{-9} \text{G}/B_{\text{eff}})^{-2} \) (e.g., Jedamzik et al. 1998; Subramanian & Barrow 1998a). Here \( B_{\text{eff}} \) is the rms at this cutoff scale for a given value of the spectral index, \( n.\) Another possible normalization, commonly used in the literature, is the value of rms at \( k = 1 \text{Mpc}^{-1}.\) These two normalizations are related as \( B_{\text{eff}} = B_0 (k_{\text{max}})^{(n+3)/2}.\) It is possible to present results using either of the pair \( (B_{\text{eff}}, n) \) or \( (B_0, n).\)
Tangled magnetic fields induce large matter perturbations in the post-recombination era which grow by gravitational collapse. The matter power spectrum of these perturbations is given by \( P(k) \propto k^{2n_+} \), for \( n < -1.5 \), the range of spectral indices we consider here (Wasserman 1978; Kim et al. 1996; Gopal & Sethi 2003).

The magnetic-field-induced matter power spectrum is cut off at the magnetic field Jeans’ wave number: \( k_J \simeq 15(10^{-9} \text{G}/B_{\text{eff}})^{-1} \) (e.g., Kim et al. 1996; Kahniashvili et al. 2010). The dissipation of tangled magnetic fields in the post-recombination era also results in an increase in the thermal Jeans’ length (Sethi & Subramanian 2005; Sethi et al. 2008). For most of the range of magnetic field strengths considered here, the scales corresponding to \( k_J \) generally exceed or are comparable to the thermal Jeans’ length (Figure 4 of Sethi et al. 2008).

For our computation, we need to know the time evolution of the matter power spectrum induced by tangled magnetic fields. It can be shown that the dominant growing mode in this case has the same time dependence as the ΛCDM model (see, e.g., Gopal & Sethi 2003 and references therein).

3. WEAK LENSING AND COSMIC SHEAR

The cosmic shear power spectrum, \( P_\kappa(\ell) \), or the lensing convergence power spectrum, \( P_\kappa \), is measure of the projection of matter power spectrum, \( P_\delta \), and is given by the following expression (Bartelmann & Schneider 2001):

\[
P_\kappa(\ell) = \frac{9}{4} \Omega_m^2 \left( \frac{H_0}{c} \right)^4 \int_0^{\chi_{\text{lim}}} a^2(\chi) P_\delta \left( \frac{\ell}{f_K(\chi)} \right) \times \left[ \int_\chi^{\chi_{\text{lim}}} d\chi n(\chi') f_K(\chi') \left( \frac{f_K(\chi')}{f_K(\chi)} \right)^2 \right]^2,
\]

where \( \chi \) is the comoving distance along the light ray and \( \chi_{\text{lim}} \) is the limiting comoving distance of the survey; \( f_K(\chi) \) is the comoving angular diameter distance; for a spatially flat (\( K = 0 \)) universe \( f_K(\chi) \) is numerically equal to \( \chi \) and the expression for \( \chi \) in the flat universe is given as follows:

\[
\chi(z) = \frac{c}{H_0} \int_0^z (\Omega_m (1+z)^3 + \Omega_\Lambda)^{-1/2} dz,
\]

where \( n(z) \) is the redshift distribution of the sources and \( \ell \) is the modulus of a two-dimensional wave vector perpendicular to the line of sight. \( P_\delta \) is the matter power spectrum. In this paper, we use tangled magnetic power spectrum as \( P_\delta \) to compute the shear power spectrum for the magnetic cases.

The cosmological shear field induced by density perturbations is a curl-free quantity and is denoted as an E-type field. One can decompose the observed shear signal into \( E \) (non-rotational) and \( B \) (rotational) components. Detection of non-zero B-modes indicates a non-gravitational contribution to the shear field, which might be caused by systematic contamination to the lensing signal.\(^1\)

These decomposed shear correlation functions can be expressed as

\[
\xi_{E,B}(\theta) = \frac{\xi_E(\theta) \pm \xi_B(\theta)}{2},
\]

where \( \xi' \) is given by

\[
\xi'(\theta) = \xi_+(\theta) + \int_0^\infty \frac{d\theta}{\theta} \xi_-(\theta) \left( 4-12 \left( \frac{\theta}{\theta_0} \right)^2 \right),
\]

where \( \xi_+ \) and \( \xi_- \) are two-point shear correlation functions that are related to the matter power spectrum according to the following relation:

\[
\xi_{\pm}(\theta) = \frac{1}{2\pi} \int_0^\infty d\ell \ell P_\kappa(\ell) J_{0,4}(\ell \theta),
\]

where \( \theta \) is the angular separation between the galaxy pairs and \( J_{0,4} \) are Bessel functions of the first kind.

4. SHEAR POWER SPECTRUM FROM TANGLED MAGNETIC FIELD POWER SPECTRUM

We use the tangled magnetic field matter power spectrum \( P_\delta \) to compute the shear power spectrum \( P_\kappa(\ell) \), which in turn is used to calculate \( \xi_+, \xi_-, \xi_E, \) and \( \xi_B \) using Equations (3)–(5). We have used the same source redshift distribution as Fu et al. (2008):

\[
n(z) = A \frac{a^2 + z^{ab}}{z^b + c}; \quad A = \left( \int_0^{z_{\text{max}}} \frac{z^{a} + z^{ab}}{z^b + c} dz \right)^{-1},
\]

where \( z_{\text{max}} = 6 \). Values of the parameters \( a, b, c, \) and \( A \) are also taken from Fu et al. (2008). Values of these parameters as quoted in the paper are \( a = 0.612 \pm 0.043; b = 8.125 \pm 0.871; c = 0.620 \pm 0.065, \) and \( A = 1.555 \). To evaluate the integral (1) we changed the variable from \( \chi \) to \( z \) using Equation (2):

\[
P_\kappa(\ell) = \frac{9}{4} \Omega_m^2 \left( \frac{H_0}{c} \right)^4 \int_0^{z_{\text{lim}}} \frac{dz}{a^4(z)} P_\delta(k, z) \times \left[ \int_z^{z_{\text{lim}}} \frac{dz'}{a^4(z')} \frac{\chi(\ell \theta)}{\chi(\ell \theta')} \right]^2,
\]

where \( k = \ell/\chi(z) \). Again \( P_\delta(k, z) \) can be written as

\[
P_\delta(k, z) = P_\delta(k) \times D^2(z),
\]

where \( D(z) \) is the growth factor, which, as noted above, is the same as for the flat ΛCDM mode and is given by (Peebles 1993)

\[
D(z) = \frac{5\Omega_m}{2} \frac{1 + z}{\Omega_m (1 + z)^3 + \Omega_\Lambda}^{1/2} \int_z^\infty \frac{dz'}{z'} (\Omega_m (1 + z')^3 + \Omega_\Lambda)^{1/2}.
\]

We used \( z_{\text{lim}} = 2.5 \) for our calculations, just as in Fu et al. (2008) did.

For comparison, we also compute all the relevant quantities for the linear and nonlinear ΛCDM models. For ΛCDM linear power spectrum we used \( P(k, z) = A k^2 D(z) D^2(z) \), where the transfer function \( T(k) \) is given by Bond & Efstathiou (1984). For nonlinear ΛCDM we followed the prescription given in Peacock & Dodds (1996).

5. RESULTS

In Figure 1, we show the tangled magnetic field matter power spectra for a range of spectral indices \( n \) and magnetic field strengths \( B_0 \) at \( z = 0 \). The matter power spectra are

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\(^1\) The presence of primordial magnetic fields will also generate the B-modes of the shear power spectrum. Both the vector and tensor modes generated by magnetic fields could source these modes. Vector modes are more likely to play a more dominant role at angular scales of interest to us in the paper. We hope to explore this possibility in a future work.
plotted for $k < k_J$; a sharp cutoff below this scale is assumed for our computation. For comparison, we have also displayed the linear and nonlinear ΛCDM matter power spectra (the nonlinear power spectrum is obtained following the method introduced by Peacock & Dodds 1996). The figure shows that the magnetic-field-induced matter power spectra can dominate over the ΛCDM case at small scales. Possible implications of this excess have already been studied for early formation of structures, reionization, and the HI signal from the epoch of reionization (Sethi & Subramanian 2005, 2009; Tashiro & Sugiyama 2006; Schliecher et al. 2009; Sethi et al. 2010). Here we explore the observational signatures of this excess in the weak-lensing data.

In Figure 2, we show the shear power spectra for magnetic and non-magnetic cases. The blue and red curves present the shear power spectrum for ΛCDM linear and nonlinear matter power spectra, respectively. The dashed blue curve shows the shear power spectrum for the tangled magnetic field power spectrum ($B_{\text{eff}} = 3.0$ nG and $n = -2.9$). In this figure we can see the impact of additional power in the tangled magnetic-field-induced matter power spectrum as an enhancement in the shear power spectrum on angular scales $\approx 1\arcmin$.

The peak of the matter power spectra of both the ΛCDM model and the magnetic-field-induced matter power spectra is also seen in the shear power spectra. The ratio of angular scales at the peak of the two cases corresponds to the ratio of these peaks of the matter power spectra: $k_{\text{eq}}/k_J$. In the ΛCDM model the power at small scales falls as $k^{-3}$, while $k_J$ imposes a sharp cutoff in the magnetic case. In both the cases, there is power at angular scales smaller than the peak of the matter power spectra. But the sharp cutoff in the matter power spectrum at $k > k_J$ results in a steeper drop in shear power spectra as compared to the ΛCDM case. This cutoff ensures that the magnetic-field-induced effects dominate the shear power spectrum for only a small range of angular scales.

In Figure 3, the two-point shear correlation functions $\xi_E$ and $\xi_B$ are shown for magnetic and non-magnetic cases. As noted in the previous section, we use the parameters found in Fu et al. (2008) for all our computation, which allows us to directly compare our results with their data, as shown in Figure 3.

For detailed comparison with the data of Fu et al. (2008), we calculated $\chi^2$ including the effect of both the ΛCDM (nonlinear model with the best-fit parameters as obtained by Fu et al. 2008) and the magnetic-field-induced signal. We fitted the sum of these two signals ($\xi_E^B + \xi_E^{\Lambda \text{CDM}}$) against the CFHTLS data to obtain limits on the magnetic field strength $B_0$ and the spectral index $n$. As seen in Figure 3, the magnetic-field-induced signal dominates the data for only a small range of angular scales below a few arcminutes. However, this can put stringent constraints on
the magnetic field model. Our best-fit values are $B_0 = 1.5\, \text{nG}$ and $n = -2.96$. In Figure 4, we show the allowed contours of these parameters for a range of $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$. It should be noted that $B_0 = 0$ is an acceptable fit to the data because we fix the best-fit parameters obtained by Fu et al. (2008).

6. DISCUSSION

Primordial magnetic fields leave their signatures in a host of observables in the universe. Their impacts on CMBR temperature and polarization anisotropies have been extensively studied. Yamazaki et al. (2010) compute the allowed region in the $(B_0, n)$ plane by comparing the predictions of primordial magnetic field models with existing CMBR observations. Other constraints come from early formation of structures, Faraday rotation of CMBR polarization (e.g., Kahniashvili et al. 2010), and reionization in the presence of magnetic fields (Schleicher & Miniati 2011).

In addition to the upper bounds on the magnetic field strength obtained by these observables, recent results suggest that there might be a lower bound of $\gtrsim 10^{-13}\, \text{G}$ on the magnetic field strength (e.g., Dolag 2010; Nerionov & Vovk 2010; Tavecchio et al. 2010; Taylor et al. 2011). This would suggest that the magnetic fields lie in the range $10^{-15} < B_0 < 10^{-9}\, \text{G}$. This range is still too large for a precise determination of the magnetic field strength.

How do our constraints (Figure 4) compare with the existing bounds on primordial magnetic fields? CMBR constraints (e.g., Figure 1 of Yamazaki et al. 2010) are stronger than our constraints for $n < -2.95$. For the entire range of spectral indices above this value, we obtain stronger upper limits on $B_0$. Our limits are comparable to bounds obtained from the formation of early structures, which also arise from excess power in the magnetic-field-induced matter power spectrum (e.g., Kahniashvili et al. 2010).

Can primordial magnetic fields be detected in the weak-lensing data? As seen in Figure 3, detection of excess power in the measurement of $\xi_E$ over what is expected for the $\Lambda$CDM model, constrained well from other observations, could be interpreted as contributions from primordial magnetic fields.

The present data are noisy at the scales at which magnetic fields begin to make significant contributions, at least partly owing to errors inherent in ground-based measurements of shear, e.g., correction for point-spread function, etc. (e.g., Figure 4 of Schrabback et al. 2010; a brief look at this figure might suggest that their measurements would already put stronger constraints on magnetic field strength than presented here). Proposed space missions such as SNAP are likely to greatly improve the errors on these measurements. A comparison of Figure 4 of the white paper on weak lensing with SNAP (Albert et al. 2005) with our Figure 3 suggests that SNAP would easily be able to probe sub-nano Gauss magnetic fields.

The magnetic field signal could be degenerated with the overall normalization of the $\Lambda$CDM model as measured by $\sigma_8$; the Wilkinson Microwave Anisotropy Probe (WMAP) seven-year data give $\sigma_8 = 0.801 \pm 0.030$ (Larson et al. 2011). WMAP results are in reasonable agreement with the value of $\sigma_8$ as inferred by the weak-lensing data. This error is not sufficient to mimic the much larger signal from magnetic field strengths considered in this paper (e.g., Figure 4 of Schrabback et al. 2010). However, a more careful analysis will be needed to distinguish the error in $\sigma_8$ from the sub-nano Gauss magnetic fields.

One uncertainty in our analysis is that the magnetic Jeans’ scale, unlike the thermal Jeans’ scale which is well defined in linear perturbation theory, is obtained within an approximation in which the back-reaction of the magnetic field on the matter is not exactly captured (e.g., Kim et al. 1996; Sethi & Subramanian 2005). Even though our results capture qualitatively the impact of such a scale, there could be more power on sub-Jeans’ scale which is lost owing to our approximation of the sharp $k$-cutoff. As noted in Section 2, the cutoff scale is the larger of the magnetic Jeans’ length and the thermal Jeans’ length. Magnetic field dissipation can raise the temperature of the medium to $\gtrsim 10^7\, \text{K}$, thereby raising thermal Jeans’ length of the medium (see Figure 4 of Sethi et al. 2008 for a comparison between the two scales for different magnetic field strengths). For $B_0 \gtrsim 10^{-7}\, \text{G}$, the magnetic Jeans’ scale is the larger of the two scales, as the maximum temperature of the medium reached owing to this process does not exceed $10^4\, \text{K}$. In the more general case this would also be true as photoionization of the medium by other sources, e.g., the sources which could have cause reionization of the universe at $z \sim 10$, also results in comparable temperatures. For magnetic field strengths smaller than those considered in the paper, the cutoff scale is likely to be determined by thermal Jeans’ scale, caused by the photoionization of the medium by sources other than the magnetic field dissipation. Our approximation allows us to identify important length and angular scales for our study (Figures 2 and 3). However, further work along these lines could extend our analysis by taking into account the physical effects of sub-magnetic Jeans’ scales.

The analysis of Ly$\alpha$ forest in the redshift range $2 < z < 4$ is another powerful probe of the matter power spectrum at small scales (e.g., Croft et al. 2002). Primordial magnetic fields can alter this interpretation in many ways: (1) more small-scale power owing to magnetic-field-induced matter power spectrum (Figure 1), (2) dissipation of magnetic fields can change the thermal state of Ly$\alpha$ clouds (e.g., Sethi et al. 2010; Sethi & Subramanian 2005), and (3) magnetic Jeans’ length can reduce the power at the smallest probable scale. We hope to undertake this study in a future work.

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