CALIBRATED ULTRA FAST IMAGE SIMULATIONS FOR THE DARK ENERGY SURVEY

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Received 2015 April 5; accepted 2015 November 24; published 2016 January 19

ABSTRACT

Image simulations are becoming increasingly important in understanding the measurement process of the shapes of galaxies for weak lensing and the associated systematic effects. For this purpose we present the first implementation of the Monte Carlo Control Loops (MCCL), a coherent framework for studying systematic effects in weak lensing. It allows us to model and calibrate the shear measurement process using image simulations from the Ultra Fast Image Generator (UFig) and the image analysis software SExtractor. We apply this framework to a subset of the data taken during the Science Verification period (SV) of the Dark Energy Survey (DES). We calibrate the UFig simulations to be statistically consistent with one of the SV images, which covers ~0.5 square degrees. We then perform tolerance analyses by perturbing six simulation parameters and study their impact on the shear measurement at the one-point level. This allows us to determine the relative importance of different parameters. For spatially constant systematic errors and point-spread function, the calibration of the simulation reaches the weak lensing precision needed for the DES SV survey area. Furthermore, we find a sensitivity of the shear measurement to the intrinsic ellipticity distribution, and an interplay between the magnitude-size and the pixel value diagnostics in constraining the noise model. This work is the first application of the MCCL framework to data and shows how it can be used to methodically study the impact of systematics on the cosmic shear measurement.

Key words: gravitational lensing: weak – methods: numerical – methods: statistical – surveys

1. INTRODUCTION

Within the last decades our picture of the universe has changed dramatically with the discovery of its accelerating expansion attributed to a mysterious dark energy. Together with dark matter they make up the dark sector of the universe. The introduction of dark energy has led to the establishment of the ΛCDM-model as the current cosmological standard model. The model agrees well with observations from different cosmological probes (e.g., Hinshaw et al. 2013; Planck Collaboration et al. 2014). Nonetheless, understanding the nature of the dark sector is one of cosmology’s most pressing challenges.

Weak gravitational lensing (for reviews see Refregier 2003; Hoekstra & Jain 2008) is a distortion effect of galaxy shapes due to structures along the line of sight. As it is a gravitational effect, both dark and baryonic matter cause a deflection of light. Furthermore, dark energy affects the weak gravitational lensing effect by changing cosmological distances and the growth of structures. Weak gravitational lensing has a large potential to shed light on the mystery of the dark sector on its own and in combination with other cosmological probes such as SNe Ia, Baryon Acoustic Oscillations, and galaxy clusters (Albrecht et al. 2006). However, the induced distortions on galaxy shapes are weak (~1%). In order for weak lensing to reach its full potential as a cosmological probe, the systematic errors of the measurement need to be sub-dominant to the statistical uncertainties of large future data sets. Therefore, galaxy shapes need to be measured to a very high accuracy (e.g., Huterer et al. 2006; Amara & Refregier 2008, henceforth AR08).

Several large wide-field imaging surveys started taking data recently including the Dark Energy Survey3 (DES), the Kilo Degree Survey4 (KiDS), and Hyper Suprime-Cam5 (HSC).

Others such as Euclid6, the Large Synoptic Survey Telescope7 (LSST), and the Wide-Field Infrared Survey Telescope8 will start taking data in the coming years. In this work, we will test our presented method on publicly available data from the Science Verification (SV) phase of DES.

Many shape-measurement algorithms have been developed over the past two decades (for an overview see, e.g., Mandelbaum et al. 2015, and references therein). Image simulations play an important role for calibrating and validating several of these methods. To test the performance of various shear measurement codes on simulated images, public challenges like the Shear Testing Programs (STEP) (Heymans et al. 2006; Massey et al. 2007) and the GReat Accuray Testing (Bridle et al. 2009; Kitching et al. 2012; Mandelbaum et al. 2014) were established. Valuable insight into the measurement process could be gained and significant progress was made. Nonetheless, these challenges reaffirmed that a careful and rigorous treatment of systematic errors is essential to weak lensing as a cosmological probe.

Recently, a novel shear measurement method, the Monte Carlo Control Loops (MCCL; Refregier & Amara 2014, henceforth RA14), was presented. While the focus of the community has been to develop general shear measurement methods and use image simulations to calibrate or validate these methods, the MCCL approaches the problem differently by putting most of the effort on the simulation and calibration process. Image simulations are generated and tuned to be in statistical agreement with the data. They are then used to calibrate the shear measurement method and

http://www.darkenergysurvey.org/
http://kids.strw.leidenuniv.nl/
http://www.naoj.org/Projects/HSC/index.html
http://sci.esa.int/euclid/
http://www.lsst.org/lsst/
http://wfirst.gsfc.nasa.gov/
test the stability of the calibration to yield a robust measurement.

In this paper, we present an initial implementation of the MCCL approach at the one-point statistics level. We use image simulations generated by the Ultra Fast Image Generator (UFig; Bergé et al. 2013, henceforth B13). These pass through the same lensing measurement pipeline as the data, where we use the standard image analysis software SExtractor (henceforth SE; Bertin & Arnouts 1996), to forward model the measurement process. Although the shape measurement tool used in this work is simple, we rely on the MCCL feedback mechanisms to calibrate the measurement. The MCCL framework dynamically modifies the lensing pipeline and aims to provide a shear measurement with systematic errors smaller than the statistical errors for the survey being considered. Thus, not only can the shear measurement be calibrated with this approach, the nature of the pipeline allows us to test the robustness of the calibration and hence inform us whether a better shape measurement method is needed.

This paper is organized as follows. In Section 2, we explain the main concept and the requirements for using the MCCL framework to tackle the shear measurement problem. In Section 3 we give a description of the DES SV data. The main features of UFig are described in Section 4. We focus especially on the properties of the simulated galaxies, the point-spread function (PSF), the noise, and the shear field. In Section 5 we present the MCCL framework and its implementation. We show in Section 6 different diagnostics of our calibrated image simulations for DES. Furthermore, the results of our first tentative analysis of the robustness of the shear measurement calibration are presented. We conclude in Section 7.

2. MCCL AND THE SHEAR MEASUREMENT PROBLEM

The main goal of this paper is to tackle the weak lensing shear measurement problem using the MCCL approach proposed by RA14. In this section we elaborate on the main concepts behind the MCCL framework and how that translates into the specific implementations (see Section 5) carried out in this paper.

A key concept is that each of the Control Loops (CL) needs to be specifically “controlled” by certain criteria, or targets. For example, in our first CL, we define criteria within which we view the simulations and the data to be statistically consistent. Specifically, we compare the one- and two-dimensional distributions or diagnostics (e.g., the magnitude-size two-dimensional distribution of all detected objects) between the simulations and the data.

There is an overall target that controls the entire MCCL framework as well, and it is naturally tied to the science goals. In our case, the target is to produce measurements of shear one-point statistics, i.e., the average galaxy ellipticities as a function of shear, that are accurate within statistical errors of the DES data set of interest.

We choose to set the main target of this paper using results from AR08. First, we parameterize our measured shear to be to first order linearly related to the true underlying shear via

\[
\gamma_i^\text{obs} = (1 + m_i) \gamma_i^\text{true} + c_i + N,
\]

where \(\gamma_i^\text{true}\) is the true and \(\gamma_i^\text{obs}\) the estimated shear, \(m_i\) and \(c_i\) are the multiplicative and additive biases. \(N\) is a noise term that becomes negligible when averaging over a large number of galaxy ellipticities to measure shear. Then, according to AR08, in order for the shear measurement not to be systematics-dominated, we would require \(m_i \lesssim 0.025\) and \(c_i \lesssim 1.65 \cdot 10^{-3}\) for a DES SV-like 200 deg\(^2\) survey, and \(m_i \lesssim 0.005\) and \(c_i \lesssim 0.75 \cdot 10^{-3}\) for the full 5000 deg\(^2\) DES survey, where we set \(n_g \approx 7\) galaxies per arcmin\(^2\) and \(z_m \approx 0.7\). While these upper limits were derived for two-point statistics, they also place requirements on shear one-point distributions. Namely, they set the targets that the absolute means of \(m_i\) and \(c_i\) must stay below the limits stated. This can be thought of as the requirements for the case of spatially constant systematic effects.

While the method itself is general, a second key concept is that the MCCL framework derived in this work depends on the specific measurement and the survey. The targets are set by the problem of interest to perform the measurement on a given data set, and the CLs are designed to achieve these targets. This suggests that conclusions drawn from applying the MCCL approach should not be readily applied to different problems and data sets. For example, in this paper our goal is the measurement of shear one-point functions. Therefore, the results presented in this work are not appropriate to answer questions regarding two-point measurements of shear (e.g., spatial correlation of the shear measurements). A new MCCL framework with different target values and diagnostics will need to be designed for each particular question.

Another important aspect is that in the case where the targets are not met, the degree of complexity of the MCCL framework needs to be increased. This can be achieved by (i) including additional diagnostics to improve the characterization of the data and (ii) improving or replacing the shape measurement method and its calibration. For this reason it is preferable to begin the process with simple diagnostic and shear estimation tools, since the MCCL mechanism will add complexity when needed.

3. THE DARK ENERGY SURVEY

DES is a wide-field optical imaging survey that will cover 5000 deg\(^2\) in the southern sky during its 5 years of operation and will record information of over 300 million galaxies. The survey area overlaps with other surveys such as the South Pole Telescope (SPT) and the Visible and Infrared Survey Telescope for Astronomy (VISTA) Hemisphere Survey (VHS). Focusing on SNe Ia, Baryon Acoustic Oscillations, galaxy clusters, and weak gravitational lensing as main cosmological probes, DES aims to study the nature of dark energy. The instrument achieved first light on 2012 September 12 and the main science survey officially started on 2013 August 12.

Images are taken with the Dark Energy Camera (DECam; Flaugher et al. 2015), designed specifically for DES. The camera is mounted on the Blanco-4 m telescope at Cerro Tololo Inter-American Observatory in Chile. DECam has 0\(')27/pixel resolution. Good seeing on this site ranges between 0\(')7 and 1\(')1.

In this work, we will test our method using a fraction of the SV-AI release, which covers \(\sim 200\) deg\(^2\) in total. Single exposures that were stacked to coadded images, whose raw

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9 [http://pole.uchicago.edu/](http://pole.uchicago.edu/)

10 [http://www.vista-vhs.org/](http://www.vista-vhs.org/)
data are publicly available, were processed by the DES Data Management pipeline version “SVA1” (B. Yanny et al. 2015, in preparation). The images were taken during the SV period, which lasted between 2012 November and 2013 February. For this work, we selected images covering \( \sim 50 \text{ deg}^2 \) in the SPT-E field that are free of significant image artifacts. We demonstrate our MCCL method on one image with an area of \( \sim 0.5 \text{ deg}^2 \), DES0441-4414, while using the rest of this SPT-E subsample to derive the statistical errors. The area is sufficiently large and contains enough stars and galaxies for the simulations to be calibrated to this image.

4. ULTRA FAST IMAGE GENERATOR (UFIG)

In this paper we analyze images simulated with UFIG. The image generation process consists of two steps. First, galaxy and star catalogs are generated. Then, the catalogs are turned directly into a coadded image without simulating individual exposures. A brief overview of the properties of the UFIG-generated galaxy catalogs, the PSF and noise models, and the shear field is given below, while a full description can be found in B13. Note that some of the models used in B13 are not fully realistic (e.g., modeling the galaxies with single-Sérsic profiles, or the PSF as constant, elliptical Moffat profiles Moffat 1969), but they provide a good starting point. The output from our MCCL framework would inform us if more sophisticated models are needed to describe the data.

The MCCL approach typically requires the simulation and analysis of many thousands of images (e.g., about 80,000 images were simulated in this work). Thus, speed is crucial. In order not to be dominated by the image generation, its speed needs to be at least comparable to the analysis. Due to several computational optimizations in the image generation process, UFIG is several orders of magnitude faster than the publicly available image simulators SkyMaker (Bertin 2009) and Simage (Dobke et al. 2010) (see B13). Furthermore, we perform a comparison with GalSim11 (Rowe et al. 2015). In comparable configurations typical for the images simulated in this paper, UFIG generates an image in about 50 s on a modern computer using a single core, which is comparable to executing SE, while GalSim generates an image in about 30 minutes.

A key property of UFIG is its flexibility in adjusting to different telescope setups. In this paper we choose to model r-band coadded images taken by DECam, but it is straightforward to simulate images from other wide-field imaging surveys.

4.1. Galaxies

A galaxy is simulated in UFIG by sampling the galaxy’s light distribution photon-by-photon, and is then placed with a uniform probability on the image. The following quantities define a galaxy in the simulations:

1. \((x, y)\): Position within the image;
2. \(\text{mag}\): Magnitude;
3. \(r_{50}\): Intrinsic half-light radius;
4. \(n\): Sérsic index;
5. \((e_1, e_2)\): Intrinsic ellipticity components;
6. \((\gamma_1, \gamma_2)\): Reduced shear components (see 4.5);
7. Local PSF model (see 4.3).

Due to the finite number of photons sampled, the simulated galaxy images naturally include Poisson noise. PSF convolution in this approach is simply a displacement of the photons drawn from a probability distribution in the shape of the PSF (see Section 4.3). We model the galaxy with a single-Sérsic profile for the radial distribution to which we apply a distortion to generate the apparent ellipticity. This profile is, for efficiency reasons only, sampled up to several multiples of the intrinsic radii, depending on the Sérsic index of the profile.

The radial profile of each galaxy is defined by the Sérsic index, the intrinsic magnitude, and the intrinsic size. The latter two quantities are non-trivially correlated. To be able to vary the size distribution and the correlation between the quantities while keeping the intrinsic magnitude distribution approximately fixed, we choose the following parametrization. We describe this distribution in a space, where magnitudes and sizes are approximately uncorrelated (\(\text{mag}_r, \log r_{50,r}\)). This space is related to the magnitude and size plane (\(\text{mag}_r, \log r_{50,p}\)) through a rotation by an angle \(\theta\) around a pivotal point (\(\text{mag}_p, \log r_{50,p}\)), i.e.,

\[
\begin{pmatrix}
\text{mag}_{r,i} \\
\log r_{50,i}
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\text{mag}_r \\
\log r_{50,r}
\end{pmatrix} + \begin{pmatrix}
\text{mag}_p \\
\log r_{50,p}
\end{pmatrix}.
\]

(2)

We parameterize the distribution of rotated galaxy intrinsic half-light radii \(r_{50,r}\) with a log-normal distribution with rms dispersion \(\sigma\). The distribution of rotated magnitudes \(\text{mag}_r\) is approximated by the distribution of intrinsic magnitudes \(\text{mag}_p\) shifted by \(\text{mag}_r\). This is a good approximation for the small values of the rotation angle \(\theta\) we find in Section 6.1. The intrinsic magnitude distribution was compiled by B13 from different ground- and space-based surveys. For this parametric fit data from the surveys VIRMOS Descartes (McCracken et al. 2003), COSMOS (Capak et al. 2007), and SXDS (Furusawa et al. 2008), and the Herschel Telescope and Hubble Deep fields (Metcalfe et al. 2001) were used.

The two parameters \(\theta\) and \(\sigma\), the compiled magnitude distribution, and the pivotal point uniquely describe the two-dimensional distribution in the magnitude-size plane for our modeled galaxy sample. Lastly, the Sérsic index distribution was derived by fitting single-Sérsic profiles to different galaxy samples in B13.

We use the following definition of the complex ellipticity to describe the intrinsic ellipticity of the galaxy (see, e.g., Rhodes et al. 2000)

\[
e = e_1 + ie_2 = \frac{l_{11} - l_{22} + 2il_{12}}{l_{11} + l_{22}},
\]

(3)

where \(l_{ij}\) are the unweighted quadrupole moments of the galaxy’s light profile and \(e_1, e_2\) are the two components of the ellipticity. In this paper, we sample \(e_1\) and \(e_2\) separately from normal distributions with mean zero and rms dispersion \(e_{1,rms}\) and \(e_{2,rms}\).

4.2. Stars

Since stars are typically brighter than galaxies, it is optimal to simulate them pixel-by-pixel rather than photon-by-photon. They are therefore simulated directly on the image pixel grid and also placed on the image with a uniform probability. To simulate a star, the following quantities need to be set:
1. (x, y): Position within the image  
2. mag: Magnitude  
3. Local PSF model (see 4.3).

The profile is given by the PSF integrated within each pixel of the image grid (see Section 4.3). Poisson noise is included by drawing a value from the corresponding Poisson distribution in every pixel.

To simulate a star, only a magnitude needs to be drawn besides the position. We sample a cumulated magnitude distribution derived from the stellar population synthesis model Besançon (Robin et al. 2003). For this work, we choose a typical stellar density for most of the DES SV region and the image considered (DES0441-4414). In case the resulting intensity in a pixel is larger than DECam’s saturation threshold, bleeding trails are modeled.

4.3. PSF

In this initial implementation of the MCCCL framework we choose as a baseline for future work a spatially constant, elliptical Moffat profile to describe the PSF. An ellipticity \( e_1, e_2, \text{PSF} \) is applied to the circular Moffat profile given by

\[
I(r) = \frac{I_0}{\left(1 + \left(\frac{r}{\alpha}\right)^2\right)^\beta},
\]

where the scale parameter \( \alpha \) is related to the seeing. The defining parameters of the Moffat profile are the seeing, the exponent \( \beta \), and the ellipticities \( e_1, \text{PSF} \) and \( e_2, \text{PSF} \). We find that the radial profile of stars in coadded DES SV images roughly follow a Moffat distribution with some variation in the parameters \( \alpha \) and \( \beta \).

In this initial implementation we choose for simplicity to fit a spatially invariant PSF to the image of interest (DES0441-4414, see Section 3) in a pre-calibration step. We use in this work a PSF of FWHM 1.0, ellipticity \( e_1, \text{PSF} = 0.035 \), \( e_2, \text{PSF} = 0.02 \), and \( \beta = 3.5 \) to match the mean PSF of this coadded image. Note that this PSF size is slightly larger than the projected median seeing of the main survey coadded image. Note that this PSF size is slightly larger than the DES SV data set.

As shear measurement is more challenging with larger PSF sizes, we expect our MCCCL framework to produce results similar or better on an image with better seeing conditions.

4.4. Noise

The background model in UFIG consists of two different components. First, we simulate galaxies down to magnitudes \( r \sim 29 \). Since most of these faint galaxies are not detected, every image contains correlated noise arising from a large number of unresolved, faint galaxies. These galaxies, furthermore, skew the measurements of various properties of detected objects due to blending. In particular, this effect can lead to percent-level changes in the calibration of the shape measurement (Hoekstra et al. 2015).

Second, we add a Gaussian background noise centered around 0 with a constant rms dispersion \( \sigma_N \) across the image. This should capture noise induced by emission from the sky, and noise induced by the data processing. We perform Lanczos resampling (Duchon 1979) with a kernel of width five pixels and a half-a-pixel offset on the simulated pixel grid. This allows us to mimic correlated noise in real images, while bypassing the expensive simulation and data reduction of raw images (see B13).

4.5. Shear field

We employ the following shear conventions in UFIG and throughout this paper (Bartelmann & Schneider 2001; Rhodes et al. 2001)

\[
\gamma_1 = \frac{1}{2} \left( \frac{\partial_1^2 - \partial_2^2}{\partial_2} \right) \Psi \quad \text{and} \quad \gamma_2 = \partial_1 \partial_2 \Psi,
\]

where \( \Psi \) is the projected lensing potential.

To be close to real surveys, we use a \( \Lambda \)CDM shear power spectrum and model the shear field as a Gaussian random field. These Gaussian random fields are simulated with Lang & Poitthoff (2011)’s fast algorithm. Throughout the paper, we use \( H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1} \), \( \Omega_m = 0.3 \), \( \Omega_\Lambda = 0.7 \), \( \sigma_8 = 0.8 \).

5. METHOD

The MCCCL framework is designed to validate the shear measurement process on simulated images and to test its robustness. RA14 identified three key iterative steps in the shear measurement process, which are labeled as CL, each with a distinct goal.

The first step (CL1) is designed to find a fiducial configuration of simulation parameters such that the simulations agree with the data. In order to quantify the level of agreement, this step relies on defining a set of diagnostics and metric targets. The next step (CL2) is to calibrate the shear measurement at this fiducial point. The final and computationally most demanding step (CL3) aims to explore the robustness of the calibration scheme from CL2 within the parameter space volume for which data and simulations are in good agreement. This scheme ensures that the systematic errors induced by uncertainties in the calibration of the measurement are subdominant to the statistical errors. Should the results of CL3 show that the employed calibration scheme is not robust enough over all parameter space allowed by CL1, then the whole MCCCL framework needs to be applied again with more stringent diagnostic requirements and possibly additional diagnostics.

It is clear now that in order to assess the robustness of this calibration scheme the generation and analysis of many tens or even hundreds of thousands of images are required. From a computational viewpoint this is only feasible if every step is very fast. This echoes our statement in Section 4 on the importance of using UFIG as our main image simulation tool.

The detailed implementation of each of the CLs is presented below.

5.1. Control Loop 1

To make statements about the consistency of the data and simulations output, we analyze the following three distributions in this first implementation:

1. Histogram of pixel values in ADUs (Figure 2): 1D

This is a valuable diagnostic to test the background properties of the image by comparing the peak of the distribution in the sky-subtracted images. Furthermore, it allows us to test the magnitude zero point of the image. Different magnitude zero points shift the tail of the large
pixel values vertically, as they affect the number of pixels with small respectively large pixel values.

2. Binned magnitude versus size-plane (Figure 3): 2D

This diagnostic probes the magnitude and size distributions of identified objects in the images and their correlation. We use the SE columns MAG_BEST for the magnitude and FLUX_RADIUS for the size.

3. Binned $e_1$ versus $e_2$-plane in three different magnitude bins (Figure 4): 2D

This tests the ellipticity distribution of identified objects. We estimate the ellipticity using weighted quadrupole moments and Equation (3). We note that the ellipticities are prior to PSF-correction. Furthermore, the ellipticity distributions are truncated (see Table 1) due to a slight excess at large $e_1$-values of a few tens of objects in the data, which all either contain saturated pixels or are close to bleeding trails. The objects are split into three different magnitude bins, each containing a similar number of objects. This allows us to probe the ellipticity distribution in each magnitude bin individually. With the brightest bin being the least affected by the effects of the PSF and noise bias (e.g., Kacprzak et al. 2012; Refregier et al. 2012), different intrinsic ellipticity distributions can be distinguished. The bin containing the faintest objects, whose shape of the distribution is affected the most by the PSF and noise, on the other hand allows us primarily to test the properties of the PSF. Furthermore, the signal-to-noise ratio ($S/N$) of objects can be constrained, as the noise biases the ellipticity measurement depending on the $S/N$.

These diagnostics are chosen to break the degeneracies between the simulation parameters we vary (e.g., between parameters describing the size and magnitude distributions of galaxies; see the Appendix). They are refined iteratively to meet the requirements to pass $CL3$ (see Section 5.3). The first diagnostic uses information in the individual pixels of the images, the latter two use SE estimators. A comparison of the performance of certain SE estimators on these images is shown in (e.g., Bertin & Arnouts 1996; Chang et al. 2015, and references therein).

To assess how likely it is that two different distributions of data and simulations could be different realizations of the same underlying model, we use a $\chi^2$-method. We apply appropriate cuts to the three diagnostic distributions and bin them. The cuts and binning scheme are chosen empirically. By varying the binning scheme, the number, and the widths of the bins, we have checked that we recover similar results. Binning the data allows us to compute $\chi^2$ for each diagnostic and combine them by adding them up. For a number of different diagnostic
distributions \#Diag, the total \( \chi^2_{\text{red}} \) for two binned data sets of different sizes is given by (e.g., Press et al. 2002)

\[
\chi^2_{\text{red}} = \frac{1}{\sum_i \sum_j N_i} \sum_i \sum_j \left( \frac{\sqrt{f_i \sigma_{ij}} - \sqrt{g_i \sigma_{ij}}}{\sigma_{d,ij} + \sigma_{s,ij}} \right)^2.
\]

Here, for the \( i \)th diagnostic distribution of the real (simulated) image, \( f_i \) (\( g_i \)) is the number of counts in the \( j \)th bin, \( N_i \) bins is the number of bins for this diagnostic with counts \( f_i \) above a certain threshold, and \( f_i \) (\( g_i \)) is the sum of all counts in those bins. \( \sigma_{d,ij} \) and \( \sigma_{s,ij} \) are the standard deviations of the data and the simulation for the \( i \)th diagnostic distribution within the \( j \)th bin. These dispersions are estimated in the data and the simulations by applying the same binning scheme to a sample of images. For each diagnostic distribution, the variances of the resulting distributions within each bin can be estimated. For the data, the sample of images consists of the subsample described in Section 3. For the simulations, 100 different random realizations of the same simulation configuration are generated. For this \( \chi^2 \)-method the variables in each bin should follow a Gaussian distribution. We therefore only include bins with at least 50 objects (about 36,000 objects are detected in the real image). We find this to be a good approximation.

We minimize \( \chi^2_{\text{red}} \) to find a fiducial configuration. For this first implementation, we choose to vary six simulation parameters to generate new samples describing the galaxy population, the image properties, and the noise level: the magnitude zero point of the image \( \text{mag}_0 \), the rms of the log-normal size distribution \( \sigma_\text{size} \) (Section 4.1), the rotation angle \( \theta \) between the magnitude-size plane, and the plane where the quantities are approximately uncorrelated (Section 4.1), the rms of the Gaussian background noise \( \sigma_N \) (Section 4.4), and the rms of the Gaussian distributions for the ellipticities \( \epsilon_1, \epsilon_2 \) (Equation (3)). These six parameters are not constrained by fits performed in B13. For each configuration an image is simulated and the \( \chi^2_{\text{red}} \)-value is computed (Equation (6)).

The \( \chi^2_{\text{red}} \) minimization procedure is designed to find a sensible parameter regime in a small number of iterations. It consists of two steps: first, we sample the parameter space coarsely and identify the region in the parameter space where the minimum \( \chi^2_{\text{red}} \) is located. Then, by successive one-dimensional minimizations we find the minimum \( \chi^2_{\text{red}} \). We vary each parameter while holding the others fixed and compute the new \( \chi^2_{\text{red}} \) values. The specific parameter value that minimizes \( \chi^2_{\text{red}} \) defines a new configuration. We repeat this step iteratively until it converges (typically \( \lesssim 10 \) steps). The final result of the iteration is the fiducial configuration that is analyzed in the subsequent CLs.

![Figure 3. Distribution of r-band magnitudes (MAG_BEST) and the sizes in pixels (FLUX_RADIUS) of objects identified by SE. Isodensity contours of the number of objects track the shape of the distribution. Red is the DES0441-4414 and blue is a simulated UFIG image with the fiducial configuration after CL1. Histograms on the right and the bottom show the projected distributions in different size and magnitude bins. The black marks denote the difference between the red and blue histograms in every bin.](image-url)
If Equation (6) can be applied, i.e., the quantities in each bin of the diagnostic distributions are Gaussian distributed, then confidence limits on the parameters can be computed. In this case, for a model with six degrees of freedom the 95% confidence limits are given by (e.g., Chernick & Friis 2003)

$$\Delta \chi^2_{\text{red}} = \frac{1}{\sum_i N_i} \cdot 12.59.$$  \hspace{1cm} (7)

This gives for every parameter a range of values for which data and simulations are statistically consistent (see the Appendix).

5.2. Control Loop 2

The task of CL2 is to calibrate the shear measurement by comparing input and estimated shear signal on simulated data. We choose as a starting point a simple, effective calibration prescription for the whole galaxy population and increase the complexity as required (see Section 2). CL3 (see Section 5.3) has the task to assess the robustness of this calibration scheme and whether it satisfies the overall targets (see Section 2).

The image that we use to illustrate the MCCL framework is part of the DES SV-A1 release. In order to not be limited by the precision of the calibration of the shear measurement, the uncertainty of the latter needs to be sub-dominant to the statistical errors of the considered data set (see Section 2).

Table 1

| Diagnostic | Number of Bins | Cuts |
|------------|----------------|------|
| Histogram of pixel values | 64 | $-50 \text{ ADUs} \leq \text{Pixel value} \leq 250 \text{ ADUs} $ |
| MAG_BEST versus FLUX_RADIUS | $16 \times 16$ | $14 \leq \text{MAG}_\text{BEST} \leq 28$, $0 \leq \text{FLUX}_\text{RADIUS} \leq 6$ |
| $e_1$ versus $e_2$ | $3 \times 8 \times 8^a$ | $-0.4 \leq e_i \leq 0.4$ |

$^a$ Note. The $e_1$-$e_2$ distribution is analyzed in three individual magnitude bins.
We apply a S/N-cut of 15 on detected galaxies, where we define the S/N as SE’s FLUX_BEST/FLUXERR_BEST, and a size-cut of 1.2 times the PSF size using SE’s FLUX_RADIUS measurement. This allows us to select galaxies large and bright enough for calibration, and leads to a number density of about 5.0 galaxies per arcmin² for the DES SV image considered in this work.

We follow Rhodes et al. (2001) to first order to estimate the galaxy shear

\[ \gamma = \frac{e'}{2 - \langle |e|^2 \rangle}, \]

where

\[ e' = \frac{J_{11}' - J_{22}'}{J_{11}' + J_{22}'} \]

is the lensed ellipticity, and \( e \) is the unlensed one. In the weak lensing limit we can approximate

\[ \langle |e|^2 \rangle \approx \langle |e'|^2 \rangle. \]

We use SE’s X2WIN_IMAGE, Y2WIN_IMAGE, and XYWIN_IMAGE to measure the weighted quadrupole moments of the PSF-convolved image \( \hat{J}_{ij} \). To linear order and ignoring weight function terms, the PSF can approximately be corrected for using

\[ J_{ij}' = \hat{J}_{ij} - P_{ij}, \]

where \( P_{ij} \) is the mean of the weighted quadrupole moments of the stars. We use \( J_{ij}' = \hat{J}_{ij} \) in Equation (9) when PSF correction is not applied.

The galaxies are then binned in input shear signal and the mean estimated shear is computed in every bin. We calibrate the shear measurement to first order by fitting and applying a linear correction

\[ \gamma_i = \alpha_i \gamma_{\text{in},i} + \beta_i, \]

where \( \gamma_{\text{in},i} \) is the input shear and the subscript \( i \) denotes the two shear components, \( \alpha_i \) and \( \beta_i \) are calibration factors of the shape measurement method being considered.

5.3. Control Loop 3

From CL1 the ranges of parameter values for which data and simulations are statistically consistent are known, and thus we can test the robustness of the calibration schemes for shear measurements for different configurations in this parameter space volume (CL3.1). We vary each parameter in a range slightly larger than that allowed by the data, while keeping the other parameters fixed. On this new locations in parameter space, we explore how much the calibration parameters (\( \alpha \) and \( \beta \); see Equation (12)) change relative to the calibration for the fiducial configuration resulting from applying CL1. This uncertainty in the shear calibration corresponds to the systematic error we expect in the shear measurement.

Uncertainties in the calibration parameters \( \alpha_i \) and \( \beta_i \) correspond to multiplicative and additive biases \( m \) and \( c \) (see Equation (1)). With the assumption of spatially constant systematic effects (see Section 2) the corresponding errors in the calibration of the shear measurement method are computed by evaluating

\[ m_i = \frac{\Delta \alpha_i}{\alpha_i} \quad \text{and} \quad c_i = \frac{\Delta \beta_i}{\beta_i}, \]

where \( \Delta \alpha_i \) and \( \Delta \beta_i \) are the changes relative to the fiducial calibration parameters. We require \( m_i \) and \( c_i \) to meet the targets set in Section 2, otherwise the diagnostics themselves need to be refined and additional tests could be required (CL3.2), affecting all the previous loops.

6. RESULTS

6.1. Control Loop 1

Excerpts of the DES0441-4414 image and the UFic image simulated with the fiducial configuration are displayed in Figure 1. They appear similar visually. For a quantitative comparison, Figures 2–4 show the diagnostic plots for the DES image and the UFic image. Appropriate cuts and binning schemes are applied to these distributions to focus on the regions where most objects lie (see Table 1). The combined \( \chi^2_{\text{red}} \) of the individual values for each diagnostic have a value of 1.06. Thus, the fiducial configuration we find is a good fit to the data in the chosen diagnostics. To avoid combining very different \( \chi^2_{\text{red}} \) values, we assure that the individual \( \chi^2_{\text{red}} \) values are also close to 1. For the fiducial configuration, the individual ones for each diagnostic are within \( |\chi^2_{\text{red}} - 1| < 0.4 \) (see the Appendix).

Figure 2 shows the histograms of pixel values for all the pixels in both images (solid). The overall behavior agrees well (\( \chi^2_{\text{red}} \approx 1.38 \)). The histograms agree well around the peak, with the distribution of the pixels in the UFic image being slightly broader. The pixels are furthermore divided using SE’s segmentation map into two sets to allow us to understand differences and similarities better. One set contains all the pixels associated with identified objects (dashed), and the other those associated with the background (dotted). The histograms of pixels associated with objects agree well (\( \chi^2_{\text{red}} \approx 1.10 \)). However we observe a low-level discrepancy in the background pixel histograms at high pixel values. These pixels with high pixel values are found around the brightest galaxies in the image, which are few in number. For efficiency reasons, UFic only samples galaxies up to several multiples, depending on the galaxies’ Sérsic indices, of the intrinsic radii. Thus, the edges of these bright galaxies are sharper in the simulated image than in the real image. However, since the number of background pixels with values of \( >30 \) ADUs is small compared to the total, this discrepancy in the background pixels does not affect the value of \( \chi^2_{\text{red}} \) significantly.

Figure 3 shows the magnitude–size plane of all objects identified by SE in both the simulation and the data. Overall, the distributions resemble each other qualitatively and quantitatively (\( \chi^2_{\text{red}} \approx 1.26 \)). In particular, the main bulk of the galaxy distributions, the location of the stellar loci, and the saturation turnoffs all agree well. Some slight differences can be noted however. The dispersion around the stellar loci is
smaller in the UFor image, which is, due to our simple PSF model, constant in size. Furthermore, the shapes of the density contour lines and the magnitude limits are slightly different. We believe that changes in the galaxy model would improve this.

The different magnitude limits and the discrepancies in the background-only histograms of pixel values call for more noise in the simulations. Increasing the width of the Gaussian background-only histograms of pixel values call for more noise improve this.

We believe that changes in the galaxy model would improve the agreement between the data and the simulation. In RA14 unknown systematics or effects not yet included in the simulations may affect the shear measurement. However, the MCCL approach provides a framework for testing aspects of the measurement process that are in doubt. The PSF-uncorrected shape measurement does not perform as well as the PSF-corrected one, and lies slightly outside the tolerance band in some parameters. To make statements about whether the calibration scheme is robust enough for a 5-year DES-like 5000 deg²-survey in the parameters varied, a larger area needs to be simulated to increase the accuracy of the calibration. Furthermore, as described in Section 2, achieving this new target requires refinements on the MCCL framework.

Figures 7 and 8 show the resulting additive biases. For the parameters considered, both shape measures already even satisfy the requirements for a full DES-like survey with 5 years worth of images.

We find in this first tolerance analysis that the calibration of the shear measurement seems to depend sensitively on the intrinsic ellipticity distribution. The general trend of this effect is consistent with previous findings (Viola et al. 2014; Hoekstra et al. 2015). In particular, this dependences can be characterized by linear relations with negative slopes as well. While there is not a significant additive bias due to an uncertainty in $e_{1,\text{rms}}$ and $e_{2,\text{rms}}$, the ellipticity distribution needs to be taken special care of such that no significant multiplicative bias is induced. The diagnostics likely need to be refined further to reduce this residual systematic effect such that stricter targets can be met in further MCCL analyses.

### 7. CONCLUSION

We have presented an initial implementation of the MCCL (Refregier & Amara 2014), a novel approach for weak lensing shear measurements. The method contains a set of three CL applied to data and image simulations to forward-model the shear measurement process. They are designed specifically to calibrate the shear measurement and test its robustness with the goal of reaching a certain sensitivity. The requirements in this paper are chosen such that the lensing measurement, assuming spatially invariant systematic errors, on the final data set of a DES- and also DES SV-like imaging survey is not limited by systematic errors, i.e., the systematic error of the measurement is smaller than the statistical error.

The MCCL approach provides a consistent way of analyzing systematic errors in the measurement. It allows us to probe
potential sources of error for their effect on the measurement, e.g., noise bias \citep{Kacprzak2012, Refregier2012} and model bias \citep{Kacprzak2014}, provided that they are included in the simulations. However, the simulation and analysis of a large number of images is essential in this approach. To not be limited computationally, every step in the CLs needs to be fast, especially the generation of images. This led to the development of the UFIG \citep{Berge2013}, whose speed is comparable to executing SExtractor \citep{Bertin1996}, the image analysis tool used in this paper.

We illustrate this first implementation of the MCCL framework using an image taken during the SV phase of DES. For this purpose, we choose a spatially invariant PSF model, vary six simulation parameters, and consider only one-point shear measurements. With these assumptions, we find that the image calibration achieves multiplicative and additive biases within the needed weak lensing precision for a DES SV-like \citep[200 deg$^2$]{2016ApJ...821...25B} survey, assuming them to be spatially invariant. We also find with the tolerance analysis that the shear measurement is very sensitive to the intrinsic ellipticity distribution. Furthermore, we find an interplay between the magnitude-size and the histogram of pixel values diagnostics in fitting the noise level to the image. To accommodate both diagnostics, an extension of the Gaussian noise model will be implemented in future work.

To achieve our goal of not being systematics-limited when measuring shear on a 5000 deg$^2$ DES-like survey, several features in the MCCL framework need to be refined. First, we will incorporate more realistic instrument and noise models in the image simulations. Next, we will extend this framework to include two-point functions in the analysis. In addition, we are planning to test the effects of a spatially varying PSF and other PSF models. We will also explore the effect of more complex galaxy models and non-uniform distributions of galaxies on the calibration of the shear measurement. And finally, a more rigorous tolerance analysis varying more simulation parameters is required.

The results we present in this work require the simulation of about 40,000 deg$^2$ of images. From a simple extrapolation of this figure, the computational resources needed for the full 5-year DES data appear large. However, several improvements on the framework can readily result in significant speed-ups. For example, better sampling strategies can in principle speed up the tolerance analysis in CL3.1 by orders of magnitude, which is by far the most computationally expensive step in this work. Furthermore, improvements in the diagnostics used in CL1 will increase the discriminatory power between different simulation configurations, reducing further the parameter space one needs to sample, and thus the computational time.

All these improvements will pave the way to exploit the full potential of weak lensing through the understanding of systematic effects within the MCCL framework.

The authors would like to thank Sarah Bridle, Tomasz Kacprzak, David Bacon, Matthew Becker, Barnaby Rowe, Gary Bernstein, and the other members of the DES team.
Figure 6. Uncertainty in the shear calibration leading to a multiplicative bias in the measurement of $\gamma_2$. Similar to Figure 5.

Figure 7. Uncertainty in the shear calibration leading to an additive bias in the measurement of $\gamma_1$. Similar to Figure 5.
collaborations for useful discussion. This work was supported
in part by grants 200021_14944 and 200021_143906 from the
Swiss National Science Foundation. Funding for the DES
Projects has been provided by the U.S. Department of Energy,
the U.S. National Science Foundation, the Ministry of Science
and Education of Spain, the Science and Technology Facilities
Council of the United Kingdom, the Higher Education Funding
Council for England, the National Center for Supercomputing
Applications at the University of Illinois at Urbana-Champaign,
the Kavli Institute of Cosmological Physics at the University of
Chicago, Financiadora de Estudos e Projetos, Fundação Carlos
Chagas Filho de Amparo à Pesquisa do Estado do Rio de
Janeiro, Conselho Nacional de Desenvolvimento Científico e
Tecnológico and the Ministério da Ciência e Tecnologia, the
Deutsche Forschungsgemeinschaft, and the Collaborating
Institutions in the Dark Energy Survey. The Collaborating
Institutions are Argonne National Laboratory, the University
of California at Santa Cruz, the University of Cambridge, Centro
de Investigaciones Energeticas, Medioambientales y Tecnoló-
gicas-Madrid, the University of Chicago, University College
London, the DES-Brazil Consortium, the Eidgenössische
Technische Hochschule (ETH) Zürich, Fermi National Accele-
ration Laboratory, the University of Edinburgh, the University
of Illinois at Urbana-Champaign, the Institut de Ciencies de
l’Espai (IEEC/CSIC), the Institut de Fisica d’Altes Energies,
Lawrence Berkeley National Laboratory, the Ludwig-Max-
imilians Universitát and the associated Excellence Cluster
Universe, the University of Michigan, the National Optical
Astronomy Observatory, the University of Nottingham, The
Ohio State University, the University of Pennsylvania, the
University of Portsmouth, SLAC National Accelerator Labora-
tory, Stanford University, the University of Sussex, and Texas
A&M University. This paper is Fermilab publication Fermilab
PUB-15-441-AE and DES publication DES 2014-0038. This
paper has gone through internal review by the DES
collaboration.

APPENDIX

REDUCED $\chi^2$ AS A FUNCTION OF DIFFERENT
SIMULATION PARAMETERS

As described in Section 5.1, we search for the fiducial
simulation configuration by minimizing the $\chi^2_{\text{red}}$ defined in
Equation (6). In this appendix, we describe in detail the
minimization procedure and point out some features in the
resulting $\chi^2_{\text{red}}$ functions, which may indicate interesting
physical insights to the data.

For each of the six simulation parameters considered, we
systematically vary its value around some initial guess and
calculate $\chi^2_{\text{red}}$ while holding the other parameters fixed. The six
parameter values that yield the minimum $\chi^2_{\text{red}}$ are then used for
the next iteration and the process continues until it converges
about the minimum. In this final set of parameters, the $\chi^2_{\text{red}}$
values along each of the one-dimensional axes are shown in
Figure 9. The blue shaded bands, which correspond to the blue
bands in Figures 5–8, are the 95%-confidence limits for each of
the parameters (see Equation (7)). Table 3 lists the correspond-
ing parameter values in the plots.

To better understand how the various diagnostics affect the
resulting $\chi^2_{\text{red}}$-function, we split it up into the contributions of
each diagnostic. For the fiducial configuration, which is denoted by a star, we find $|\chi^2_{\text{red}} - 1| < 0.4$ for all the individual diagnostics and their combined sum. The total $\chi^2_{\text{red}}$ is well approximated by quadratic fits, though $\chi^2_{\text{red}}$ from individual diagnostics can show very different behaviors. Furthermore, the fiducial configuration is close to the minima of the quadratic fits.

We want to point out a few features in the individual subfigures. First, the magnitude-size plane (see Figure 3) and the ellipticity plane (see Figure 4) do not react to changes in the magnitude zero point of the image ($\text{mag}_0$). Only the histogram of pixel values (see Figure 2) responds to changes in $\text{mag}_0$, as the magnitude zero point affects the normalization of the pixel values.

Second, when varying the width of the Gaussian background $\sigma_N$, the histogram of pixel values and the magnitude-size plane respond in opposite directions. As described in Section 6.1, the fiducial model produces a slightly deeper image compared to data. Thus, the magnitude-size plane is pushing for a higher noise level. The histogram of pixel values, however, constrains the background peak in the sky-subtracted image and cannot accommodate larger $\sigma_N$ values. Therefore, we believe that including an additional Non-Gaussian noise term can reconcile the tension.

Third, the diagnostics are rather flat if $e_{1,\text{rms}}$ and $e_{2,\text{rms}}$ are varied. As a result, the 95%-confidence limits for this parameter are relatively wide. Another less noisy diagnostic to constrain the ellipticity distribution might shrink these confidence limits. Furthermore, we note the slight asymmetry between the allowed values and ranges for $e_{1,\text{rms}}$ and $e_{2,\text{rms}}$ to produce simulations consistent with the data. While these should be the same in principle, the orientation of the pixels and also effects like charge saturation tending to bleed in the readout direction have a preferential direction leading to slightly different constraints.

Fourth, the $\chi^2_{\text{red}}$ values of the ellipticity diagnostic are below 1 in the relevant parameter ranges. As the quotient in Equation (6) is dominated by the error estimated on the data $\sigma_{d,ij}$, this behavior of the ellipticity diagnostics suggests issues in estimating the scatter of the data set.

Finally, the magnitude-size plane is the most sensitive diagnostic to changes in the magnitude and size distribution.

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| Parameter | Fiducial Value | Central Value | 95% CL | Description |
|-----------|----------------|---------------|--------|-------------|
| $\text{mag}_0$ | 30.565 | 30.576 | ±0.055 | Magnitude zero point |
| $\sigma_N$ | 5.10 | 5.17 | ±0.34 | rms of the background noise |
| $e_{1,\text{rms}}$ | 0.383 | 0.396 | ±0.045 | rms of the intrinsic $e_1$ distribution |
| $e_{2,\text{rms}}$ | 0.407 | 0.387 | ±0.041 | rms of the intrinsic $e_2$ distribution |
| $\sigma$ | 0.2422 | 0.2381 | ±0.0390 | rms of the intrinsic log-normal size distribution |
| $\theta$ | 0.1382 | 0.1370 | ±0.0068 | Rotation angle to plane intrinsic magnitudes and sizes are uncorrelated |

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Table 3. Minimum and 95%-Confidence Limits of the Quadratic Fits (Figure 9) and Parameter Values for the Fiducial Configuration
parameters $\sigma$ and $\theta$ of the galaxy population. Hence, the combined $\chi^2_{red}$-curve is mostly driven by this diagnostic.

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