A Framework of Performance Analysis for Distributed Antenna Systems Based on Random Matrix Theory

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Abstract—Future communications systems will definitely be built on green infrastructures. To realize such a goal, recently a new network infrastructure named cloud radio access network (C-RAN) is proposed by China Mobile to enhance network coverage and save energy simultaneously. In C-RANs, to order to save more energy the radio front ends are separated from the colocated baseband units and distributively located in physical positions. C-RAN can be recognized as a variant of distributed antenna systems (DASs). In this paper we analyze the performance of C-RANS using random matrix theory. Due to the fact that the antennas are distributed geographically instead of being installed nearby, the variances of the entries in the considered channel matrix are different from each other. To the best of the authors’ knowledge, the work on random matrices with different variances is largely open, which is of great importance for DASs. In our work, some fundamental results on the eigenvalue distributions of the random matrices with different variances are derived first. Then based on these fundamental conclusions the outage probability of the considered DAS is derived. Finally, the accuracy of our analytical results is assessed by some numerical results.

Index Terms—C-RAN, distributed antenna systems, random matrix theory.

I. INTRODUCTION

In order to meet the demands from surging wireless data services, it is undoubted that future wireless systems should have high spectrum efficiency. On the other hand, energy efficiency has attracted a lot of attention recently [1–3]. A green communication system is much more preferable in practice. As a result, these two contradict performance metrics simultaneously become to be the most important goals of the future wireless communication designs. In order to realize these two goals both transmission technologies and network architectures should evolve.

In order to realize high quality of wireless communications with high spectrum efficiency, various transmission technologies have been introduced such as relay-based or cognitive radio-based technologies [4–11]. Generally speaking, to achieve high spectrum efficiency, the distances between mobile terminals and wired networks (such as core networks) should be deduced, as it can achieve high frequency reuse factor and effective interference control. In other words, the coverage area of a base station should become smaller and smaller, and the number of base stations will greatly increase. Simply increasing the number of base stations without developing the traditional cellular network architecture is far from desirable due to the lack of energy efficiency and high operation expenditure.

It is well-known that in distributed antenna systems (DASs), antennas are separately distributed and connected to the central unit via a high-bandwidth low-delayed channels such as optical fibers [12–14]. As a result DAS can provide an open and flexible infrastructure for the whole wireless systems. Based on this open and flexible infrastructure of DAS, recently a new network architecture named cloud radio access network (C-RAN) [15] is proposed by China Mobile the biggest communications operator in China. That technology aims at improving the coverage and reducing the energy cost toward Green communications. In C-RAN, the baseband unit (BBUs) are installed in a centralized pool and the radio elements are installed distributively.

C-RANs can achieve high spectrum efficiency and energy efficiency at the same time. In C-RANs as the BBUs are collocated, the operation expenditure will be significantly reduced. For example, BBUs can share a common air-condition. On the other hand, remote radio heads (RRHs) can cooperate to enhance network coverage and achieve high network capacity [16, 17]. Meanwhile, One RRH can be equipped with one antenna or multiple antennas. It has been shown that for the same number of antennas, single antenna case is better than that equipped with multiple antennas due to macrodiversity [18].

Acting as an open architecture for future wireless communications DASs have attracted a lot of attention from both academic and industrial societies. Diversity and multiplexing gains for DASs are investigated in [19]. Some theoretical limits of cellular networks with distributed antennas have also been discussed in [18].
DASs with linear relaying operations for amplify-and-forward MIMO relaying systems are considered in detail in [20]. Moreover, the resource allocation for DASs has been investigated in [21, 22]. Distributed organization of DASs and antenna placement have been discussed in [23].

In most of the existing work on DASs, the individual power constraints at each antenna act as the most distinct difference from the existing work for MIMO systems with sum power constraints. It should be pointed out that the most important fact for DASs is that the different positions of antenna elements such as RRHs can introduce macrodiversity. To clearly reveal the performance nature of the considered DASs, the position differences of RRHs should be investigated carefully. This is the motivation of our work. Recently, random matrix theory is accepted as a powerful mathematical tool to analyze the performance of MIMO systems. There is a rich body of publications on the performance analysis for the communications and signal processing theories using random matrix theory [24–27]. It is well-established that using random matrix theory, many important performance bound of different MIMO systems can be rigorously derived.

In this paper, we concentrate our attention to the performance analysis of a DAS using random matrix theory. Notice that the main difference between DAS and a traditional MIMO system is that the antennas of base station is largely separately. As a result, it means that for DASs the covariances of the elements of the involved channel matrix will be different from each other instead of being equivalent with each other such as in the traditional MIMO systems. In our work, we build a fundamental framework for the ordered eigenvalue distribution of such kind of random matrices. Based on this result, the outage probability of the considered DAS over Raleigh fading channels is analyzed. In addition, the place effect and direction effect are also investigated. Finally, the numerical results assess our performance analysis.

**Notation:** The following notations are used throughout this paper. Boldface lowercase letters denote vectors, while boldface uppercase letters denote matrices. The notations $\mathbf{Z}^T$, $\mathbf{Z}^*$ and $\mathbf{Z}^H$ denote the transpose, conjugate and conjugate transpose of the matrix $\mathbf{Z}$, respectively and $\text{Tr}(\mathbf{Z})$ is the trace of the matrix $\mathbf{Z}$. The symbol $\mathbf{I}_M$ denotes an $M \times M$ identity matrix, while $\mathbf{0}_{M \times N}$ denotes an $M \times N$ all zero matrix. The symbol $\mathbb{E}$ denotes statistical expectation operation.

II. SYSTEM MODEL

In this paper, we consider a distributed antenna system (DAS) system in which there is one source with $T$ transmit antennas and one destination with $R$ receive antennas. It should be highlighted that for DASs in downlink the source is a set of single-antenna RRHs. On the other hand, in uplink the destination is a set of single-antenna RRHs. It is also assumed that perfect channel state information (CSI) is available among all the wireless terminals. In the future wireless systems, high frequency band e.g., 60GHz will be exploited to support high data rates in the order of Gbps. In high frequency band, single data stream is more practical for multiple antenna array due to physical limitations. Thus, the received signal at the destination can be formulated as

$$y = \frac{P}{T} \bar{\mathbf{H}} \mathbf{w} x + \mathbf{n},$$

where $y \in \mathbb{C}^{R \times 1}$ is the received signal at the destination, $\bar{\mathbf{H}}$ is the channel matrix between the RRHs and the destination. In addition, $\mathbf{w} \in \mathbb{C}^{T \times 1}$ denotes the beam-forming vector at the source, which has a normalized norm i.e., $\mathbb{E} \left[ \| \mathbf{w} \|^2 \right] = 1$. The scalar variable $x$ is the desired signal for the destination with $\mathbb{E} \left[ |x|^2 \right] = 1$, and $\mathbf{n} \sim \mathcal{CN}(0, \mathbf{I}_{N_j})$ is the noise term that is Gaussian distributed with zero mean and covariance matrix of $\mathbb{E}[\mathbf{n} \mathbf{n}^H] = N_0 \mathbf{I}$. By the way, we denote by $P$ the maximum total transmit power.

As the RRHs are separately distributed, in downlink the signals transmitted from different RRHs will go through different large scale fading. Similarly, in uplink different RRHs will receive the signals experiencing different large scale fading. Denoting the $\{i, j\}$th complex entry of $\bar{\mathbf{H}}$ as $\bar{h}_{i,j}$, which describes the channel gain between the $j$th transmit antenna and the $i$th receive antenna, it means in uplink for different $i$’s, $\bar{h}_{i,j}$’s will have different variances while in downlink for different $j$’s, $\bar{h}_{i,j}$’s will have different variances. For convenience, a unified discussion is given in the following.

Firstly, we denote by $\bar{h}_j$ the $j$th column of $\bar{\mathbf{H}}$. In order to take into account the different large scale fading, $\bar{h}_j$ can be expressed as

$$\bar{h}_j = \Psi_j \bar{h}_j, \quad j \in [1, T],$$

where the entries in the vector $\bar{h}_j$ are independent and identically distributed (i.i.d) zero mean complex Gaussian random variables. Furthermore, the $R \times R$ diagonal matrix $\Psi_j = \text{diag} \left\{ \frac{1}{\sqrt{D_{1,i,j}}, \frac{1}{\sqrt{D_{2,i,j}}, \ldots, \frac{1}{\sqrt{D_{R,i,j}}}} \right\}$ makes $\bar{h}_{i,j}$’s have different variances. The path loss is characterized by $D_{v,i,j}$ where $D_{i,j}$ is the distance between the $j$th transmit antenna and the $i$th receive antenna and $v$ is the path-loss exponent with typical values ranging from 2-6. For the case of Rayleigh fading, it implies that $\mathbb{E} \left[ \bar{h}_j \right] = 0$ and the covariance matrix equals

$$\Sigma_j = \mathbb{E} \left\{ \bar{h}_j \bar{h}_j^H \right\} = \Psi_j \Psi_j^H = \text{diag} \left\{ \frac{1}{D_{v,1,i,j}^2}, \frac{1}{D_{v,2,i,j}^2}, \ldots, \frac{1}{D_{v,R,i,j}^2} \right\}, \quad j \in [1, T].$$

At the destination for multi-antenna array with single data stream maximum ratio combining (MRC) is usually
chosen as an efficient method to recover signal. Then we have the following equation
\[
 z = \mathbf{w}^\dagger \mathbf{H}^\dagger \mathbf{y} = \sqrt{\frac{P}{T}} \mathbf{w}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{w} x + \mathbf{w}^\dagger \mathbf{H}^\dagger \mathbf{n},
\]
(4)

based on which the instantaneous output signal to noise ratio (SNR) can be directly derived to be
\[
 \gamma = \gamma \mathbf{w}_\text{opt}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{w}_\text{opt} = \gamma \lambda_{\text{max}},
\]
(5)

where \( \lambda_{\text{max}} \) is maximum eigenvalue of \( \mathbf{H}^\dagger \mathbf{H} \). It is obvious that the performance of DAS BF depends directly on the statistical properties of \( \lambda_{\text{max}} \) which will be derived in the following section.

III. New Results of Random Matrix Theory

In this section, we derive the exact closed-form expression for the ordered eigenvalues of a Wishart matrix with each entry having different variance. Let \( m = \max \{ R, T \} \) and \( n = \min \{ R, T \} \). It is well-known that the statistical distributions of the nonzero eigenvalues of \( \mathbf{H}^\dagger \mathbf{H} \) and \( \mathbf{H} \mathbf{H}^\dagger \) are equivalent with each other [24]. Based on this fact, we first define a matrix as follows
\[
 \mathbf{W} = \begin{cases} 
 \mathbf{H}^\dagger \mathbf{H}, & \text{if } R \leq T, \\
 \mathbf{H} \mathbf{H}^\dagger, & \text{if } R \geq T.
\end{cases}
\]
(7)

If the elements in \( \mathbf{H} \) are i.i.d. zero-mean complex Gaussian variables, i.e., \( \Sigma_1 = \Sigma_2 = \cdots = \Sigma_T = \mathbf{I} \), \( \mathbf{W} \) is called a central Wishart matrix. However, in the DASs we have a general case of \( \Sigma_1 \neq \Sigma_2 \neq \cdots \neq \Sigma_T \neq \mathbf{I} \), and in this case \( \mathbf{W} \) is a unequal-covariance Wishart matrix which is the focus of this section. The main contribution of this section is to derive the closed-form expressions for the joint probability density function (PDF) of the ordered eigenvalues and the cumulative distribution function (CDF) of the maximum eigenvalues, which are of great importance for performance analysis for DASs.

A. Joint PDF of the Ordered Eigenvalues

Before giving the joint PDF of the ordered eigenvalues of the unequal-covariance Wishart matrix, some auxiliary variables are first defined. With a vector argument \( a = [a_1, a_2, \ldots, a_L] \), the matrix \( \mathbf{V}_1(\mathbf{x}) \) is generated according to the following equation
\[
 \mathbf{V}_1(\mathbf{a}) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\
 a_1 & a_2 & \cdots & a_L \\
 \vdots & \vdots & \ddots & \vdots \\
 a_1^{L-1} & a_2^{L-1} & \cdots & a_L^{L-1} \end{bmatrix},
\]
(8)

and a matrix \( \mathbf{G}(\mathbf{x}, \mathbf{\sigma}) \) is also defined as the following matrix
\[
 \mathbf{G}(\mathbf{x}, \mathbf{\sigma}) = \begin{bmatrix} 
 e^{-\frac{x_1 \sigma_1}{\sigma_1^2}} & \frac{1}{(-\sigma_1)^{m-2}} & \frac{1}{(-\sigma_1)^{m-3}} & \cdots & 1 \\
 e^{-\frac{x_2 \sigma_2}{\sigma_2^2}} & \frac{1}{(-\sigma_2)^{m-3}} & \frac{1}{(-\sigma_2)^{m-4}} & \cdots & \vdots \\
 \vdots & \vdots & \ddots & \ddots & \vdots \\
 e^{-\frac{x_m \sigma_m}{\sigma_m^2}} & \frac{1}{(-\sigma_m)^{m-4}} & \frac{1}{(-\sigma_m)^{m-5}} & \cdots & 1 
\end{bmatrix}.
\]
(9)

Additionally, based on the diagonal matrices \( \mathbf{\Psi}_i \)’s a diagonal matrix \( \Sigma \) with larger dimension is defined as
\[
 \Sigma = \begin{cases} 
 \text{diag} \{ \mathbf{\Psi}_1 \mathbf{\Psi}_1^\dagger, \ldots, \mathbf{\Psi}_T \mathbf{\Psi}_T^\dagger \}, & \text{if } R \leq T, \\
 \text{diag} \{ \mathbf{\Psi}_1 \mathbf{\Psi}_1^\dagger, \ldots, \mathbf{\Psi}_R \mathbf{\Psi}_R^\dagger \}, & \text{if } R \geq T,
\end{cases}
\]
(10)

where \( \mathbf{\Psi}_i = \text{diag} \{ \frac{1}{\sigma_1^i}, \frac{1}{\sigma_2^i}, \ldots, \frac{1}{\sigma_m^i} \} \), \( i \in [1, R] \). Then denoting \( \sigma_i \) as the \( i \)th largest eigenvalue of \( \Sigma \) and constructing a vector \( \mathbf{\sigma} = [\sigma_1, \sigma_2, \ldots, \sigma_m] \), a Vandermonde matrix \( \mathbf{V}_2(\mathbf{\sigma}) \) is given as follows
\[
 \mathbf{V}_2(\mathbf{\sigma}) = \mathbf{V}_1(\begin{bmatrix} 1 \\ \sigma_1^{-1} \\ \sigma_2^{-1} \\
 \vdots \end{bmatrix}) = \begin{bmatrix} \sigma_1^{-1} \\ \sigma_2^{-1} \\
 \vdots \end{bmatrix}.
\]
(11)

Based on the previous definitions, we have the following theorem about the joint PDF of the ordered eigenvalues of unequal-covariance Wishart matrix.

Theorem 1: Denoting \( \lambda_i \) as the \( i \)th largest eigenvalue of \( \mathbf{W} \), the joint PDF of the ordered eigenvalues \( \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n] \) is given by
\[
 f_{\lambda}(x_1, x_2, \ldots, x_n) = K_\Sigma \left( \frac{\mathbf{G}(\mathbf{x}, \mathbf{\sigma})}{\sum_{i=1}^n x_i} \right) \prod_{i=1}^n \prod_{i<j}^n (x_i - x_j)^2,
\]
(12)

where \( K_\Sigma \) is a normalizing constant given by
\[
 K_\Sigma = \frac{\Gamma(mn)}{\Gamma(m) \Gamma(n) |\Sigma|^{\frac{1}{2}}}.
\]
(13)

with \( \Gamma(\cdot) \) denoting the Gamma function and
\[
 \Gamma_s(t) = \prod_{i=1}^s \Gamma(t - i + 1).
\]
(14)

Proof: See Appendix I.

It can be seen that the result in Theorem 1 is analytically closed-formed and it involves no complex special functions. Thus, Theorem 1 is very useful for
the performance analysis of MIMO systems and other derivations of the statistic functions with respect to an unequal-covariance Wishart matrix.

B. The CDF of the Largest Eigenvalues

In the above subsection, we have derived the joint PDF of the ordered eigenvalues of an unequal-covariance Wishart matrix. Employing Theorem 1, in this part we will derive another important statistic function that is the CDF of the maximum eigenvalues of an unequal-covariance Wishart matrix. This result will be used for deriving the outage probability and symbol error rate (SER) of over Rayleigh fading channels. Before the derivation of the CDF of the largest eigenvalues, the following Lemma is given first, which is the basis for the following derivations.

**Lemma 1:** If \( \sum_{i=1}^{n} v_i > \mu \), \( D = \{0 \leq x_1, x_2, \cdots, x_n \leq \} \), we have the following equalities

\[
I (x; b, \mu, v_1, v_2, \ldots, v_n) = \int \cdots \int_{D} e^{-b \sum_{i=1}^{n} x_i} \prod_{i=1}^{n} x_i^{v_i-1} d\mathbf{x} = \frac{\prod_{i=1}^{n} \Gamma (v_i)}{\Gamma (\mu)} b^\mu \left( \mu, \sum_{i=1}^{n} v_i - \mu \right) + (-1)^{v_1} \sum_{(k_1, k_2, \ldots, k_n)} B \left( \mu, \sum_{i=1}^{n} v_i - \mu \right) \times \Psi \left( \mu, \mu + 1 - \sum_{i=1}^{n} g_2 (v_i, k_i) \right) \times \prod_{i=1}^{n} g_1 (v_i, k_i, b, x),
\]

where \( B (\cdot, \cdot) \) is the beta functions and \( \Psi (\cdot, \cdot) \) is the confluent hypergeometric function. \( \{k_1, k_2, \ldots, k_n\} = \{0 \leq k_1 \leq v_1, 0 \leq k_2 \leq v_2, \ldots, 0 \leq k_n \leq v_n\} \) except \( k_1 = k_2 = \ldots = k_n = 0 \). In addition, the functions \( g_1 (\cdot), g_2 (\cdot), g_3 (\cdot) \) are defined as

\[
g_1 (v_i, k_i, b, x) = \begin{cases} \frac{(1)^{v_i-1} \Gamma (v_i)}{\Gamma (v_i + 1 - k_i)} & k_i = 0, \\ \frac{(1)^{v_i-1} \Gamma (v_i)}{\Gamma (v_i + 1 - k_i)} & \text{others,} \end{cases} \quad g_2 (v_i, k_i) = \begin{cases} v_i & k_i = 0, \\ k_i & \text{others,} \end{cases} \quad g_3 (k_i) = \begin{cases} 0 & k_i = 0, \\ 1 & \text{others.} \end{cases}
\]

Proof: See Appendix II.

Based on Lemma 1 and employing order statistics theory, the CDF of the maximum eigenvalues of the unequal-covariance Wishart matrix can be derived to be

\[
F_{\lambda_{\text{max}}} (x) = P (x \geq x_1, x_2, \ldots, x_n) = \int_{D_{\text{ord}}} f_\lambda (x_1, x_2, \ldots, x_n) d\mathbf{x} = \frac{1}{\Gamma (n + 1)} \int_{D} f_\lambda (x_1, x_2, \ldots, x_n) d\mathbf{x},
\]

where \( D_{\text{ord}} = \{x \geq x_1 \geq x_2 \geq \cdots \geq x_n \geq 0\} \). Applying the Laplace expansion, \( F_{\lambda_{\text{max}}} (x) \) can be expressed as (21) on the top of the next page, where \( \kappa = \kappa_1, \kappa_2, \ldots, \kappa_{mn} \) is a permutation of the integers 1, 2, \ldots, mn. Symbols \( \delta = \delta_1, \delta_2, \ldots, \delta_n \) and \( \eta = \eta_1, \eta_2, \ldots, \eta_n \) are two irrelevant permutations of the integers 1, 2, \ldots, n. The sums are over all permutations, and \( \text{sgn} (\kappa) \), \( \text{sgn} (\delta) \) and \( \text{sgn} (\eta) \) denote the sign of the permutations.

Since the integral of (21) is similar to the integral of Lemma 1, we finally have the expression of \( F_{\lambda_{\text{max}}} (x) \) as (22) on the top of the next page, where the definition of \( I (\cdot, \cdot, \cdot, \cdots) \) is given in (15). In this section, all the expressions are closed-formed with simple functions, which can be easily applied to the performance analysis of the MIMO systems with distributed antennas. As there is little work on the random matrix theory for the random matrices with elements having different variances, the authors believe that the proposed results in this paper will be useful for the further research in the related area.

IV. PERFORMANCE ANALYSIS OF DAS SYSTEMS

In the previous section, the closed-form expressions for the ordered eigenvalues of the unequal-covariance Wishart matrix has been derived for the perspective of random matrix theory. These results are the theoretical basis to analysis the performance of DAS in this section. The outage probability and the average SER of over Rayleigh fading channels will be investigated in detail.

A. Outage Probability

The outage probability is an important quality of service measure, defined as the probability that the received SNR drops below an acceptable SNR threshold \( \gamma_{\text{th}} \). The outage probability can be computed directly from the CDF \( F_{\lambda_{\text{max}}} (x) \) based on (6) and (22), which is equivalent to

\[
P_{\text{out}} (\gamma_{\text{th}}) = \Pr (\gamma \leq \gamma_{\text{th}}) = F_{\lambda_{\text{max}}} (\gamma_{\text{th}}) \).
\]

This is a new exact outage probability expression that is valid for arbitrary number of the transmit antennas and receive antennas. It is worth noting that (23) can be calculated in closed form, involving only finite summations of exponentials, powers, Bessel functions, and confluent hypergeometric functions.
\[ F_{\lambda_{\text{max}}} (x) \]

\[ = \frac{K \Sigma}{\Gamma(n + 1)} \sum_{\kappa} \frac{\text{sgn}(\kappa) e^{-\frac{x}{\kappa}} \sum_{i=1}^{\kappa} x_i^{mn} \left( \prod_{i=2}^{\kappa} -\sigma_{\kappa_i} \right)^{mn-1}}{\Gamma(n + 1)} x_i^{m-n} \left[ \sum_{\delta} \text{sgn}(\delta) \prod_{i=1}^{\delta} x_i^{\delta_i-1} \right] \left[ \sum_{\eta} \text{sgn}(\eta) \prod_{i=1}^{\eta} x_i^{\eta_i-1} \right] dx \]

\[ = \frac{K \Sigma}{\Gamma(n + 1)} \sum_{\kappa} \frac{\text{sgn}(\kappa) \sum_{\delta} \text{sgn}(\delta) \sum_{\eta} \text{sgn}(\eta) \prod_{i=2}^{\kappa} (-\sigma_{\kappa_i})^{mn-1}}{\Gamma(n + 1)} \int \left( \prod_{i=1}^{n} x_i^{m-n+\delta_i+\eta_i-2} \right) dx \] (21)

\[ F_{\lambda_{\text{max}}} (x) \]

\[ = \frac{K \Sigma}{\Gamma(n + 1)} \sum_{\kappa} \frac{\text{sgn}(\kappa) \sum_{\delta} \text{sgn}(\delta) \sum_{\eta} \text{sgn}(\eta) \prod_{i=2}^{\kappa} (-\sigma_{\kappa_i})^{mn-1}}{\Gamma(n + 1)} \times I \left( x; 1, mn - 1, m - n + \delta_1 + \eta_1 - 1, m - n + \delta_2 + \eta_2 - 1, \cdots, m - n + \delta_n + \eta_n - 1 \right) \] (22)

B. SER Analysis

In this subsection, the closed-form expressions for the average SER of DAS will be derived step by step. Our results can be applied for all general modulation formats that have an SER expression of the form

\[ P_s = \mathbb{E}_{\gamma} \left[ \alpha Q \left( \sqrt{2/\beta \gamma} \right) \right] \], (24)

where \( Q(\cdot) \) is the Gaussian Q-function, and \( \alpha \) and \( \beta \) are modulation-specific constants. Such modulation formats include binary phase-shift keying (BPSK) (\( \alpha = 1, \beta = 1 \)); binary frequency-shift keying (BFSK) with orthogonal signaling (\( \alpha = 1, \beta = 0.5 \)); M-ary pulse amplitude modulation (PAM) (\( \alpha = (M - 1)/M, \beta = 3/(M^2 - 1) \)). Employing the results from [25], (24) can be expressed in terms of the CDF of the output SNR as follows:

\[ P_s = \frac{\alpha \sqrt{\beta}}{2\sqrt{\pi}} \int_0^\infty e^{-\beta u} F_{\lambda_{\text{max}}} \left( u/\gamma \right) du \]. (25)

Substituting (15) and (22) into (24), after some simplification, we have the equation (26) on the top of the next page, where

\[ c_1 = mn - 1, c_2 = mn - \sum_{i=1}^{\kappa} g_2 (v_i, k_i), c_3 = \left( \sum_{i=1}^{\kappa} g_3 (k_i) \right) / (\gamma \sigma_{\kappa_1}), c_4 = \left( \sum_{i=1}^{\kappa} g_2 (v_i, k_i) \right) - 1/2, c_5 = \sum_{i=1}^{\kappa} g_3 (k_i) / (\gamma \sigma_{\kappa_1}) + \beta \]

and \( v_i = m - n + \delta_i + \eta_i - 1, i \in [1, n] \) with \( \Xi_1, \Xi_2 \) and \( \Xi_3 \) defined as

\[ \Xi_1 = \frac{\sum_{i=1}^{\kappa} (v_i)}{\Gamma(\mu)} B \left( \mu - \sum_{i=1}^{\kappa} v_i, mn - 1 \right), \]

\[ \Xi_2 = \frac{(-1)^{\sum_{i=1}^{\kappa} v_i-n}}{\sum_{i=1}^{\kappa} g_2 (v_i, k_i)}, \]

\[ \Xi_3 = \frac{(-1)^{v_i-1+g_3(k_i)} \Gamma(v_i)}{\Gamma(v_i + 1 - g_2 (v_i, k_i))}. \] (29)

Using the expressions of [28, 3.361.2], \( I_5 \) equals to

\[ I_5 = \sqrt{\frac{\pi}{\beta}}. \] (30)

Applying \( \Psi (a, b; x) = x^{-\frac{1}{2}} e^{\frac{x}{2}} W_{\frac{1}{2}a, \frac{1}{2}b} (x) \) with \( W_{\cdot, \cdot} (\cdot) \) denoting the Whittaker function, \( I_6 \) can be derived as

\[ I_6 = c_3^{-c_2/2} \int_0^\infty u^{c_4 - \frac{c_2}{2}} e^{\frac{c_2}{2} - c_3 u} W_{c_5 - c_1, c_5 + 1} (c_3 u) du \]

\[ = c_3^{-c_2/2} B \left( c_5 - c_3, 2, 2 \right) \Gamma (a - p + 1, 2), \] (31)

where \( a, b, p, q \) is applied in the last step with \( B (\cdot ; \cdot, \cdot) \) defined as

\[ B (x; s, a, p, q, c) \]

\[ = \frac{\Gamma (a + q + \frac{3}{2} \Gamma (a - q + \frac{3}{2})) e^{\gamma^2 (s + \frac{1}{2})} (s + \frac{1}{2})^{-a-q-\frac{3}{2}}}{\Gamma (a - p + 2)} \times 2 F_1 \left( a+q+\frac{3}{2}, q-p+\frac{1}{2}, a-p+2; \frac{2s-c}{2s+c} \right), \] (32)

where \( 2 F_1 (\cdot ; \cdot ; \cdot) \) is the generalized hypergeometric function.

Finally, substituting (30) and (31) into (26) yields the desired closed-form SER expression as (33) on the top of the next page. We also point out that Eq. (33) can
be calculated in closed-form, via many useful softwares, such as Matlab. All the expressions in this section can be used to predict the error performance of DAS systems.

V. NUMERICAL RESULTS

In this section, the numerical results are presented to demonstrate the accuracy of our theoretical results. Moreover, the impacts of different parameters e.g., the number of RRHs, the location of RRHs, etc., on the outage probability and SER are also investigated. For the sake of convenience, we only simulate a general case in which there are \( R \) largely separated receive antennas and \( T \) largely separated transmit antennas. This case includes both uplink and downlink of C-RAN as its special cases. For the locations of antennas or RRHs, without lost of generality, it is assumed that \( D_{i+1,j} = D_{i,j} + d \), \( D_{i,j+1} = D_{i,j} + R \cdot d \) to simplify the simulations, where \( d \) denotes how the antennas are separated from each other. The average value of \( D_{i,j} \) is \( 1 \) for \( i \in [1, R], j \in [1, T] \), which ensures that the average distance between the transmit antennas and receive antennas remains the same when the number of antennas changes. Furthermore, our simulations concentrate on a practical urban scenario with the path loss exponent \( \alpha = 3.5 \) [29]. In the following figures, the exact outage probability is evaluated by substituting (15), (22), into (23). On the other hand, the exact SER is evaluated by substituting (32) into (33).

Fig. 2 shows the outage probability of DAS in the case of \( d = 0.1 \) with different numbers of RRHs. It can be seen that the “Analytical” curves perfectly agree with the Monte Carlo simulated curves. It can be observed that the outage probability decreases with the number of antennas. For example, at the outage probability of \( 10^{-5} \), increasing \( T \) from 2 to 3 brings an outage probability advantage of 5.8dB, while increasing \( T \) from 3 to 4 brings an outage probability advantage of 4dB. The SER of DAS is shown in Fig. 3 with the same number of RRHs and location configurations as the setting in Fig. 2. The “analytical” curves agree precisely with the Monte Carlo simulated results. We can see that the SER performance is significantly improved as the number of antennas are increased. For example, at the SER of \( 10^{-5} \), increasing \( T \) from 2 to 4 brings an SER advantage of 8.3dB.

To reveal the impact of different locations, in the following the case of \( T = 2 \) and \( R = 2 \) is considered. It should be highlighted that in this case \( d \to 0 \) represents the traditional MIMO systems in which the antennas of base station are close to each other. Fig. 4 shows the outage probability of DAS with various locations. It can be observed that the outage probability decreases with the value of \( d \), which indicates that DAS can improve the outage probability of the system compared to the traditional MIMO system. We note that the \( d = 0.2 \) case has an outage probability advantage of 1dB over traditional MIMO system at the outage probability of \( 10^{-5} \).

Fig. 5 depicts the SER of DASs with different values of \( d \), in which the number of RRHs and location configurations are the same as that in Fig. 4. It can also be observed that the SER decreases with the value of \( d \), which demonstrates that the SER of the system can also be improved by DAS compared to the traditional MIMO system. For example, at the SER of \( 10^{-5} \), the \( d = 0.2 \) case has an outage probability advantage of 1.5dB over traditional MIMO system.

The impact of angles \( \theta_1 \) and \( \theta_2 \) with \( r = 0.3 \) is depicted in Fig. 6. In practice, this case corresponds to a very simple DAS with only two separated transmit antennas and two separated receive antennas. Fig. 7 shows the Outage probability of the considered DAS with various angles. It can be seen that for a consistent \( \theta_1 \), the optimal value of \( \theta_2 \) is \( \theta_2 = 0 \). Moreover, the optimal value of \( \theta_1 \) is \( \theta_1 = 0 \) for a consistent \( \theta_2 \). In sum, the optimal angles in this scenario are \( \theta_1 = 0 \) and \( \theta_2 = 0 \), which indicates that the optimal location is that all the
VI. CONCLUSIONS

In this paper, the performance of a DAS was analyzed based on random matrix theory. The main difference distinguishing our work from the existing work is that the covariances of the elements of the considered random matrix care different. It corresponds to the fact the antennas of the base station are distributed geographically in the coverage instead of being installed nearby each other. We first derived some fundamental results on the eigenvalue distributions of the random matrices with each element having different variance from the perspective of random matrix theory. Then based on these results, the outage probability of the considered DAS has been derived. Finally, the simulation results demonstrated the accuracy of our results.

APPENDIX A
PROOF OF THEOREM 1

An important lemma will be given first in the following, which is much useful for deriving the ordered eigenvalues of the unequal-covariance Wishart matrix.

Lemma 2: Consider a generic matrix \( A(m \times n) = \{a_{i,j}\}_{i=1,2,...,m, j=1,2,...,n} \) with \( m \leq n \), we denote the ordered eigenvalues of \( AA^\dagger \) as \( w = [w_1, w_2, \ldots, w_m] \), with \( w_1 \geq w_2 \geq \cdots \geq w_m \). The ordered eigenvalues of \( \text{Vec}(A)\text{Vec}(A)^\dagger \) are denoted as \( \varphi = [\varphi_1, \varphi_2, \ldots, \varphi_{mn}] \), with \( \varphi_1 \geq \varphi_2 \geq \cdots \geq \varphi_{mn} \). We have the following relationship:

\[
\varphi_1 = \sum_{i=1}^{n} w_j, \tag{34}
\]

\[
\varphi_2 = \varphi_3 = \cdots = \varphi_{mn} = 0. \tag{35}
\]

Proof: It is well-known that for any given two matrices \( A(m \times n) \) and \( B(n \times m) \) with \( m \leq n \), the \( (n \times n) \) matrix \( BA \) has the same \( m \) eigenvalues as the \( (m \times m) \) matrix \( AB \), together with other \( n - m \) eigenvalues identically equal to zero [30, p. 53]. As a scalar, \( \text{Vec}(A)^\dagger \text{Vec}(A) \) has only one non-zero eigenvalues, which indicates that \( \text{Vec}(A)^\dagger \text{Vec}(A) \) has \( mn - 1 \) zero eigenvalues and (35) is proved.

Now we focus on the trace of \( \text{Vec}(A)^\dagger \text{Vec}(A) \), which can be expressed as

\[
\text{tr} \left( \text{Vec}(A)^\dagger \text{Vec}(A) \right) = \text{tr} \left( \text{Vec}(A)^\dagger \text{Vec}(A) \right) = \text{Vec}(A)^\dagger \text{Vec}(A) = \sum_{i=1}^{m} \sum_{j=1}^{n} |a_{i,j}|^2 = \text{tr} \left( AA^\dagger \right). \tag{36}
\]

It is obvious that for any square matrix, the trace equals to the sum of the eigenvalues, which indicates that (36) can be rewritten as

\[
\sum_{i=1}^{mn} \varphi_i = \sum_{j=1}^{m} w_j, \tag{37}
\]

and substituting (36) into (37) gives (34).

In the following, Theorem 1 will be proved case by case for \( R \leq T \) and \( R \geq T \).

A. The \( R \leq T \) Case

Without loss of generality, the case \( R \leq T \) is investigated first. According to [31], the PDF of a Gaussian random vector is given by

\[
f_{\tilde{h}_j}(x) = \frac{1}{(2\pi)^R |\Sigma_j|} \exp \left( -\frac{1}{2} x^\dagger \Sigma_j^{-1} x \right). \tag{38}
\]

Since the columns of \( \tilde{H} \) are independent, we can obtain the joint PDF of multiple Gaussian random vectors through multiplying the PDFs of single Gaussian random vector together, which is

\[
f_{\tilde{h}_1, \tilde{h}_2, \ldots, \tilde{h}_T}(\tilde{h}_1, \tilde{h}_2, \ldots, \tilde{h}_T) = \prod_{j=1}^{T} f_{\tilde{h}_j}(\tilde{h}_j) = \frac{1}{(2\pi)^{RT} |\Sigma|} \exp \left( -\frac{1}{2} \sum_{j=1}^{T} \tilde{h}_j^\dagger \Sigma_j^{-1} \tilde{h}_j \right). \tag{39}
\]

Using the vectorization, we can rewrite exponential term in the above expression as

\[
\sum_{j=1}^{T} \tilde{h}_j^\dagger \Sigma_j^{-1} \tilde{h}_j = \left[ \begin{array}{c} \tilde{h}_1^\dagger \\ \tilde{h}_2^\dagger \\ \vdots \\ \tilde{h}_T^\dagger \end{array} \right] \left[ \begin{array}{c} \Sigma_1^{-1} \\ \Sigma_2^{-1} \\ \vdots \\ \Sigma_T^{-1} \end{array} \right] \left[ \begin{array}{c} \tilde{h}_1 \\ \tilde{h}_2 \\ \vdots \\ \tilde{h}_T \end{array} \right] = \text{Vec} \left( \tilde{H} \right)^\dagger \Sigma^{-1} \text{Vec} \left( \tilde{H} \right), \tag{40}
\]

where \( \Sigma \) is defined in (10). As \( \sum_{j=1}^{T} \tilde{h}_j^\dagger \Sigma_j^{-1} \tilde{h}_j \) is a scalar, the above expression (40) can be rewritten as

\[
\sum_{j=1}^{T} \tilde{h}_j^\dagger \Sigma_j^{-1} \tilde{h}_j = \text{tr} \left( \text{Vec} \left( \tilde{H} \right)^\dagger \Sigma^{-1} \text{Vec} \left( \tilde{H} \right) \right) = \text{tr} \left( \Sigma^{-1} \text{Vec} \left( \tilde{H} \right) \text{Vec} \left( \tilde{H} \right)^\dagger \right). \tag{41}
\]

Substituting (41) into (39), we now reformulate the joint PDF of multiple Gaussian random vectors as

\[
f_{\tilde{h}_1, \tilde{h}_2, \ldots, \tilde{h}_T}(\tilde{h}_1, \tilde{h}_2, \ldots, \tilde{h}_T) = \frac{1}{(2\pi)^{RT} |\Sigma|} \exp \left( -\text{tr} \left( \Sigma^{-1} \text{Vec} \left( \tilde{H} \right) \text{Vec} \left( \tilde{H} \right)^\dagger \right) \right). \tag{42}
\]
Similar to the derivations in [32], the joint PDF of the ordered eigenvalues \(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n\) of \(\mathbf{W}\) can be derived as

\[
f_{\mathbf{X}}(x_1, x_2, \ldots, x_n) = K|\Sigma|^{-1} \tilde{F}_0 \left( -\Sigma^{-1}, \text{Vec}(\mathbf{H}) \text{Vec}(\mathbf{H}^\dagger) \right) \times |\mathbf{W}|^{m-n} \prod_{i<j} (x_i - x_j)^2,
\]

(43)

where \(K\) is a normalizing constant given by

\[
K = \frac{1}{\Gamma_n(m) \Gamma_n(n)},
\]

(44)

and \(\tilde{F}_0(\mathbf{A}, \mathbf{B})\) denotes the hypergeometric function of Hermitian matrix arguments, whose definition is given in [32, eq. (88)] in terms of a series involving zonal polynomials.

Let \(\xi = [\xi_1, \xi_2, \ldots, \xi_{mn}]\), with \(\xi_1 \geq \xi_2 \geq \cdots \geq \xi_{mn}\) denoting the ordered eigenvalues of \(\text{Vec}(\mathbf{H}) \text{Vec}(\mathbf{H}^\dagger)\). Using [24, eq. (13)], we have

\[
\tilde{F}_0 \left( -\Sigma^{-1}, \text{Vec}(\mathbf{H}) \text{Vec}(\mathbf{H}^\dagger) \right) = L_{mn} \frac{|\mathbf{E}(\xi, \sigma)|}{|\mathbf{V}_1(\xi)||\mathbf{V}_2(\sigma)|},
\]

(45)

where \(L_{mn}\) is a constant defined as

\[
L_{mn} = \prod_{j=1}^{mn} \Gamma(j),
\]

(46)

and \(\mathbf{E}(\xi, \sigma)\) is defined by

\[
\mathbf{E}(\xi, \sigma) = \begin{bmatrix}
  e^{-\xi_1} & e^{-\xi_2} & \cdots & e^{-\xi_{mn}} \\
  e^{-\xi_1} & e^{-\xi_2} & \cdots & e^{-\xi_{mn}} \\
  \vdots & \vdots & \ddots & \vdots \\
  e^{-\xi_1} & e^{-\xi_2} & \cdots & e^{-\xi_{mn}}
\end{bmatrix},
\]

(47)

with \(\mathbf{V}_1(\cdot), \mathbf{V}_2(\cdot)\) and \(\sigma\) defined in (11) and (8).

In the above expressions, \(\xi\) is unknown. Using Lemma 2, which is given in the beginning of this appendix, the relationship between \(\xi\) and \(\lambda\) holds:

\[
\xi_1 = \sum_{i=1}^{n} \lambda_i,
\]

(48)

\[
\xi_2 = \xi_3 = \cdots = \xi_{mn} = 0.
\]

(49)

Substituting (48) and (49) into (45), we have

\[
\lim_{\xi_2, \cdots, \xi_{mn} \to 0} \tilde{F}_0 \left( -\Sigma^{-1}, \text{Vec}(\mathbf{H}) \text{Vec}(\mathbf{H}^\dagger) \right) = \frac{L_n}{|\mathbf{V}_2(\sigma)|} \lim_{\xi_2, \cdots, \xi_{mn} \to 0} \frac{\mathbf{E}(\xi, \sigma)}{|\mathbf{V}_1(\xi)|}.
\]

(50)

Denoting \(f_i(\xi_j) = e^{-\xi_j/\sigma_i}\), to evaluate the above limits, we apply [33, Lemma 2] to obtain

\[
\mathcal{I} = \lim_{\xi_2 \to 0} \left[ \frac{1}{\xi_1 - \xi_2} \xi_1^{mn-2} \xi_2^{-2} \Gamma_{mn-2} (mn - 2) \right] \begin{bmatrix}
  f_1(\xi_1) & f_1(\xi_2) & f_1^{mn-2}(0) & \cdots & f_1(0)
\end{bmatrix} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  f_{mn}(\xi_1) & f_{mn}(\xi_2) & f_{mn}^{mn-2}(0) & \cdots & f_{mn}(0)
\end{bmatrix},
\]

(51)

We can now apply the Taylor expansion to the functions

\[
f_i(\xi_2) = \sum_{k=0}^{mn-2} \alpha_i(k) \xi_2^k + O(\xi_2^{mn-1}),
\]

(52)

where \(O(\xi_2^{mn-1})\) denotes the omitted terms of order \(\xi_2^{mn-1}\). As the determinant is an alternating multilinear function of the columns, using (52), Eq. (52) can be reformulated as (53) on the top of the next page.

In (53), the determinants for \(k = 0, 1, \ldots, mn-3\) are zero since there are coincident columns. Hence, in the limit for \(\xi_2 \to 0\), only the determinant for \(k = mn - 2\) remains and (53) can be simplified as

\[
\mathcal{I} = \frac{1}{\xi_1^{mn-1} \Gamma_{mn-1} (mn - 1)} \begin{bmatrix}
  \frac{\xi_2}{\xi_1} & \frac{1}{(-\sigma_1)^{mn-2}} & \cdots & 1 \\
  \vdots & \vdots & \ddots & \vdots \\
  \frac{\xi_2}{\xi_{mn}} & \frac{1}{(-\sigma_{mn})^{mn-2}} & \cdots & 1
\end{bmatrix},
\]

(54)

Then substituting (54) and (50) into (43) after a tedious but straightforward derivations we can achieve the final result.

**B. The \(R \geq T\) Case**

The derivation procedure for the case \(R \leq T\) is exactly the same as that for the case case \(R \geq T\) via a simple complex conjugate transpose on the channel matrix.

**APPENDIX B**

**PROOF OF LEMMA 1**

It is obvious that the integral in (15) is too complicated to be derived directly. Therefore, we embark on considering another most relevant integral first

\[
I_1 = \int \cdots \int_D \frac{1}{(a + \sum_{i=1}^{n} x_i)^{2}} dx,
\]

(55)

which is the same as the integral in (15) if \(a = 0\) and \(b_1 = b_2 = \cdots = b_n = b\). In order to derive the
\[
I_3 = \frac{1}{\xi_1^{mn-1} \Gamma_{mn-2} (mn-2)} \lim_{\xi_2 \to 0} \left( O (\xi_2) + \frac{1}{\xi_2^{mn-2}} \sum_{k=0}^{mn-2} c_k^2 k! \right) \begin{bmatrix}
\begin{array}{cccc}
 f_1 (\xi_1) & f_1^{(k)} (\xi_1) & f_1^{mn-3} (0) & \cdots & f_1^{(0)} (0) \\
 f_2 (\xi_2) & f_2^{(k)} (\xi_2) & f_2^{mn-3} (0) & \cdots & f_2^{(0)} (0) \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 f_{mn} (\xi_{mn}) & f_{mn}^{(k)} (\xi_{mn}) & f_{mn}^{mn-3} (0) & \cdots & f_{mn}^{(0)} (0)
\end{array}
\end{bmatrix}
\] (53)

new integral given by (55), the following property of integration operation will be exploited
\[
\frac{d^n}{dx^n} \int \cdots \int f (x) \, dx \cdots dx = f (x), \quad (56)
\]
based on which and taking \( a \) as the integration variable, the integral in (55) can be rewritten as
\[
I_1 = \int \cdots \int \frac{d^{\mu-1}}{d \mu^{\mu-1}} \cdot \frac{e^{-\sum b_i x_i} \prod x_i^{v_i-1}}{(a + \sum x_i)^n} \, da \cdots dx \quad (57)
\]
It is obvious that to calculate \( I_1 \) a \((\mu - 1)\)-fold integral with respect to \( a \) should be done. From (57), it can also be concluded that the remaining task becomes how to compute the term \( I_2 \). Taking a similar operation of \( a \) on \( b_1, b_2, \ldots, b_n \), \( I_2 \) is equivalent to
\[
I_2 = \int \cdots \int \frac{d^{v_1-1}}{d v_1^{v_1-1}} \cdot \frac{d^{v_n-1}}{d v_n^{v_n-1}} \cdot \frac{e^{-\sum b_i x_i} \prod x_i^{v_i-1}}{(a + \sum x_i)^n} \times \frac{e^{-\sum b_i x_i} \prod x_i^{v_i-1}}{(a + \sum x_i)^n} \, dx \cdot dx \quad (58)
\]
For the term \( I_3 \) in the previous equation we first calculate the integral of \( x_1 \) and have the following equation
\[
I_3 = \int_0^a \cdots \int_0^a e^{-\sum b_i x_i} \, dx_1 dx_2 \cdots dx_n
\quad (59)
\]
where \([28, 3.352.1]\) is used and \( F_1 (x, x_2, x_3, \ldots, x_n) \) equals to
\[
F_1 (x, x_2, x_3, \ldots, x_n) = e^{-b_1 (a + \sum x_i)} \, \left[ \text{Ei} (-b_1 (a + x + \sum x_i)) \right],
\quad (60)
\]
where \( \text{Ei} (\cdot) \) is the exponential integral function and has one transformation as \([28, 8.212.5]\)
\[
\text{Ei} (-\alpha \beta) = -e^{-a \beta} \int_0^\infty e^{t} dt.
\quad (62)
\]
Based on (62) and equality \( \beta = b_1, (60) \) can be expressed as
\[
F_1 (x, x_2, x_3, \ldots, x_n)
\quad (63)
\]
Substituting (63) into (59), \( I_3 \) can be derived to be (64) on the top of the next page.

Since \( t \) is irrelevant with \( x_2, x_3, \ldots, x_n \), the order of calculating the above integral can be changed, and then the integral of \( x_2, x_3, \ldots, x_n \) will be derived before \( t \) and \( I_3 \) equals to (65) on the top of the next page.
Substituting (65) into (68) and then based on the expressions of $I_2$, $I_1$ becomes to be the following form

$$I_1 = \frac{(-1)^{\sum v_i - n + \mu - 1}}{\Gamma (\mu)} \frac{d^{\nu_1 - 1}}{db_1^{\nu_1 - 1}} \cdots \frac{d^{\nu_n - 1}}{db_n^{\nu_n - 1}} \times \int_0^\infty \frac{e^{-at}}{\prod_{i=2}^n (t + b_i)} \prod_{i=1}^n \left[ 1 - e^{-(t+b_i)x} \right] dt. \tag{66}$$

Notice that there is a $(\mu - 1)^{th}$ order derivation with respect to $a$ in the above expression (66). After taking this derivation, $I_1$ can be further simplified as follows

$$I_1 = \frac{(-1)^{\sum v_i - n}}{\Gamma (\mu)} \frac{d^{\nu_1 - 1}}{db_1^{\nu_1 - 1}} \cdots \frac{d^{\nu_n - 1}}{db_n^{\nu_n - 1}} \times \int_0^\infty \frac{t^{\mu-1} e^{-at}}{\prod_{i=1}^n (t + b_i)} \prod_{i=1}^n \left[ 1 - e^{-(t+b_i)x} \right] dt \tag{67}$$

where the order of calculating the above integral is changed in the last step. Now we only need to derive the specific formulation of $F_2 (x, b_i)$ which equals

$$F_2 (x, b_i) = \frac{d^{\nu_i - 1}}{db_i^{\nu_i - 1}} \left( \frac{1}{t + b_i} \right) - e^{-tx} \frac{d^{\nu_i - 1}}{db_i^{\nu_i - 1}} \left( \frac{e^{-b_i x}}{t + b_i} \right)$$

$$= (-1)^{\nu_i - 1} \frac{\Gamma (\nu_i)}{(t + b_i)^{\nu_i}} - \sum_{k_i=1}^{v_i} (-1)^{k_i - 1} x^{v_i - k_i} \Gamma (v_i) e^{-b_i x - tx}, \tag{68}$$

where the second equality is based on the following equation

$$\frac{d^{\nu_i - 1}}{db_i^{\nu_i - 1}} \left( \frac{e^{-b_i x}}{t + b_i} \right) = \sum_{k_i=1}^{v_i} (-1)^{k_i - 1} x^{v_i - k_i} \Gamma (v_i) e^{-b_i x}. \tag{69}$$

Substituting (68) into (67), $I_1$ is rewritten as

$$I_1 = \frac{(-1)^{\sum v_i - n}}{\Gamma (\mu)} \int_0^\infty \frac{t^{\mu-1} e^{-at}}{\prod_{i=1}^n (t + b_i)} \prod_{i=1}^n \left[ 1 - e^{-(t+b_i)x} \right] dt \times \prod_{i=1}^n \left[ \frac{d^{v_i - 1}}{db_i^{v_i - 1}} \left( \frac{1 - e^{-(t+b_i)x}}{t + b_i} \right) \right] \\ \hat{F}_2(x, b_i) \tag{67}$$

where the order of calculating the above integral is changed in the last step. Now we only need to derive

Taking $a = 0$ and $b_1 = b_2 = \cdots = b_n = b$ in (70), we
directly have the following equation
\[ I(x; b, \mu, v_1, v_2, \ldots, v_n) = I_1 |_{a=0, b_1=b_2=\cdots=b_n=b} \]
\[ = (\frac{1}{\Gamma(\mu)} \int_0^{\infty} t^{\mu-1} \prod_{i=1}^{n} \sum_{k_{l}+\cdots+k_{m}=0} g_1(v_{i}, k_i, b, x) \frac{e^{-g_3(k_i)x}}{(t+b)^{g_2(v_i, k_i)}} dt, \quad (71) \]

where \( g_1(\cdot, \cdot, \cdot), g_2(\cdot, \cdot) \) and \( g_3(\cdot) \) are defined in (17), (18) and (19), respectively. Spreading the product in the above integral, (71) is expressed as
\[ I(x; b, \mu, v_1, v_2, \ldots, v_n) = (\frac{1}{\Gamma(\mu)} \int_0^{\infty} t^{\mu-1} \prod_{i=1}^{n} g_1(v_{i}, k_i, b, x) \frac{e^{-g_3(k_i)x}}{(t+b)^{g_2(v_i, k_i)}} dt. \]

When \( k_1 = k_2 = \ldots = k_n = 0 \), using [34, 2.2.4.24], \( I_4 \) equals
\[ I_4 = \int_0^{\infty} \frac{t^{\mu-1} \prod_{i=1}^{n} g_2(v_i, k_i) dt}{(t+b)^{\sum_{i=1}^{n} v_i}} = b^{\mu-1} \prod_{i=1}^{n} v_i \left( \frac{\mu}{\mu-1} \right). \]

For the general case of \( k_1, k_2, \ldots, k_n \), using [34, 2.3.6.9], \( I_4 \) is given by
\[ I_4 = \Gamma(\mu) b^{\mu-1} \prod_{i=1}^{n} g_2(v_i, k_i) \times \Psi \left( \mu, \mu + 1 - \sum_{i=1}^{n} g_2(v_i, k_i); bx \sum_{i=1}^{n} g_3(k_i) \right). \]

Substituting (73) and (74) into (72), we finish the proof.

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Fig. 1. A C-RAN network.

Fig. 2. Outage probability of DAS with various number of RRHs and $d = 0.1$.

Fig. 3. SER of DAS with various number of RRHs and $d = 0.1$. 
Fig. 4. Outage probability of DAS with different locations in the case of $T = 2$ and $R = 2$.

Fig. 5. SER of DAS with different locations in the case of $T = 2$ and $R = 2$.

Fig. 6. System model of the case of $T = 2$ and $R = 2$.

Fig. 7. SER of DAS with different angles in the case of $T = 2$, $R = 2$ and $r = 0.3$. 