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Approximate Capacity of the Gaussian Interference Channel with Noisy Channel-Output Feedback

Victor Quintero, Samir M. Perlaza, Iñaki Esnaola and Jean-Marie Gorce

Abstract—In this paper, an achievability region and a converse region for the two-user Gaussian interference channel with noisy channel-output feedback (G-IC-NOF) are presented. The achievability region is obtained using a random coding argument and three well-known techniques: rate splitting, superposition coding and backward decoding. The converse region is obtained using some of the existing perfect-output feedback outer-bounds as well as a set of new outer-bounds that are obtained by using genie-aided models of the original G-IC-NOF. Finally, it is shown that the achievability region and the converse region approximate the capacity region of the G-IC-NOF to within a constant gap in bits per channel use.

Index Terms—Capacity, Interference Channel, Noisy Channel-Output Feedback.

I. NOTATION

Throughout this paper, $(\cdot)^+$ denotes the positive part operator, i.e., $(\cdot)^+ = \max(\cdot, 0)$ and $E_x[\cdot]$ denotes the expectation with respect to the distribution of the random variable $X$. The logarithm function is assumed to be base 2.

II. SYSTEM MODEL

Consider the two-user G-IC-NOF in Figure 1. Transmitter $i$, with $i \in \{1, 2\}$, communicates with receiver $i$ subject to the interference produced by transmitter $j$, with $j \in \{1, 2\} \setminus \{i\}$. There are two independent and uniformly distributed messages, $W_i \in \mathcal{W}_i$, with $\mathcal{W}_i = \{1, 2, \ldots, 2^{N_{R_i}}\}$, where $N$ denotes the block-length in channel uses and $R_i$ is the transmission rate in bits per channel use. At each block, transmitter $i$ sends the codeword $X_i = (X_{i,1}, X_{i,2}, \ldots, X_{i,N})^T \in \mathcal{X}_i^N$, where $\mathcal{X}_i$ and $\mathcal{X}_j^N$ are respectively the channel-input alphabet and the codebook of transmitter $i$.

The channel coefficient from transmitter $j$ to receiver $i$ is denoted by $h_{ij}$; the channel coefficient from transmitter $i$ to receiver $i$ is denoted by $h_{ii}$; and the channel coefficient from channel-output $i$ to transmitter $i$ is denoted by $h_{i,j}$. All channel coefficients are assumed to be non-negative real numbers. At a given channel use $n \in \{1, 2, \ldots, N\}$, the channel output at receiver $i$ is denoted by $Y_{i,n}$. During channel use $n$, the input-output relation of the channel model is given by

$$Y_{i,n} = h_{ii}X_{i,n} + h_{ij}X_{j,n} + \bar{Z}_{i,n},$$  \hspace{1cm} (1)

The components of the input vector $X_i$ are real numbers subject to an average power constraint:

$$\frac{1}{N} \sum_{n=1}^{N} E(X_{i,n}^2) \leq 1,$$  \hspace{1cm} (4)

where the expectation is taken over the joint distribution of the message indexes $W_1$, $W_2$, and the noise terms, i.e., $\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_1$, and $\bar{Z}_2$. The dependence of $X_{i,n}$ on $W_1$, $W_2$, and the previously observed noise realizations is due to the effect of feedback as shown in (2) and (3).
Assume that during a given communication, $T$ blocks are transmitted. Hence, the decoder of receiver $i$ is defined by a deterministic function $\psi_i : \mathbb{R}_i^{NT} \to \mathbb{W}_i^T$. At the end of the communication, receiver $i$ uses the vector $(\mathbb{Y}_{i,1}, \mathbb{Y}_{i,2}, \ldots, \mathbb{Y}_{i,NT})$ to obtain an estimate of the message indices

$$\left(\hat{W}_i^{(1)}, \hat{W}_i^{(2)}, \ldots, \hat{W}_i^{(T)}\right) = \psi_i(\mathbb{Y}_{i,1}, \mathbb{Y}_{i,2}, \ldots, \mathbb{Y}_{i,NT}),$$

(5)

where $\hat{W}_i^{(t)}$ is an estimate of the message index sent during block $t \in \{1,2,\ldots,T\}$. The decoding error probability in the two-user G-IC-NOF during block $t$ of a codebook of block-length $N$, denoted by $P_e^t(N)$, is given by

$$P_e^t(N) = \max\left(\Pr[\hat{W}_1^{(t)} \neq W_1^{(t)}], \Pr[\hat{W}_2^{(t)} \neq W_2^{(t)}]\right).$$

(6)

The description of an achievable rate pair $(R_1,R_2) \in \mathbb{R}_+^2$ is given below.

**Definition 1 (Achievable Rate Pairs):** A rate pair $(R_1,R_2) \in \mathbb{R}_+^2$ is achievable if there exists at least one pair of codebooks $X_1^N$ and $X_2^N$ with codewords of length $N$, and the corresponding encoding functions $f_1^{(1)}, \ldots, f_1^{(N)}$ and $f_2^{(1)}, \ldots, f_2^{(N)}$ such that the decoding error probability $P_e^t(N)$ can be made arbitrarily small by letting the block-length $N$ grow to infinity, for all blocks $t \in \{1,2,\ldots,T\}$.

The two-user G-IC-NOF in Figure 1 can be fully described by six parameters: $\text{SNR}_{i1}$, $\hat{\text{SNR}}_{i1}$, and $\text{INR}_{ij}$, with $i \in \{1,2\}$ and $j \in \{1,2\} \setminus \{i\}$, which are defined as follows:

$$\hat{\text{SNR}}_{i1} = \frac{\hat{h}_{i1}^2}{\text{INR}_{ij}},$$

(7)

$$\text{INR}_{ij} = h_{ij}^2,$$

(8)

$$\text{SNR}_{i1} = \frac{h_{i1}^2}{\hat{h}_{i1}^2 + 2 h_{i1} h_{ij} + h_{ij}^2 + 1}.$$

(9)

### III. MAIN RESULTS

This section introduces an achievable region (Theorem 1) and a converse region (Theorem 2), denoted by $\mathcal{C}_{G-IC-NOF}^A$ and $\mathcal{C}_{G-IC-NOF}^C$ respectively, for the two-user G-IC-NOF with fixed parameters $\text{SNR}_{i1}$, $\text{SNR}_{i2}$, $\text{INR}_{12}$, $\text{INR}_{21}$, $\text{SNR}_{i1}$, and $\hat{\text{SNR}}_{i2}$. In general, the capacity region of a given multi-user channel is said to be approximated to within a constant gap according to the following definition.

**Definition 2 (Approximation to within $\xi$ units):** A closed and convex set $\mathcal{T} \subset \mathbb{R}_+^m$ is approximated to within $\xi$ units by the sets $\mathcal{T}_L$ and $\mathcal{T}_U$ if $\mathcal{T}_L \supseteq \mathcal{T} \supseteq \mathcal{T}_U$ and for all $t = (t_1, \ldots, t_m) \in \mathcal{T}$ then $(t_1 - \xi, \ldots, t_m - \xi) \in \mathcal{T}_L$.

Denote by $\mathcal{C}_{GIC-NOF}$ the capacity region of the 2-user G-IC-NOF. The achievable region $\mathcal{C}_{G-IC-NOF}^A$ and the converse region $\mathcal{C}_{G-IC-NOF}^C$ approximate the capacity region $\mathcal{C}_{GIC-NOF}$ to within 4.4 bits per channel use (Theorem 3).

### A. An Achievable Region for the Two-User G-IC-NOF

The description of the achievable region $\mathcal{C}_{G-IC-NOF}^A$ is presented using the constants $a_{1,i}$; the functions $a_{2,i} : [0,1] \to \mathbb{R}_+$, $a_{1,i} : [0,1]^2 \to \mathbb{R}_+$, with $l \in \{0,1\}$; and $a_{2,j} : [0,1]^3 \to \mathbb{R}_+$, which are defined as follows, for all $i \in \{1,2\}$, with $j \in \{1,2\} \setminus \{i\}$:

$$a_{1,i} = \frac{1}{2} \log \left( \frac{2 + \text{SNR}_{i1}}{\text{INR}_{ij}} \right) - \frac{1}{2},$$

(10a)

$$a_{2,i}(\rho) = \frac{1}{2} \log \left( b_{1,i}(\rho) + 1 \right) - \frac{1}{2},$$

(10b)

$$a_{3,i}(\rho,\mu) = \frac{1}{2} \log \left( \frac{\text{SNR}_{i1} b_{2,i}(\rho) + 2 + b_{1,i}(1) + 1}{\text{INR}_{ij}((1-\mu)b_{2,i}(\rho) + 2) + b_{1,i}(1)+1} \right),$$

(10c)

$$a_{4,i}(\rho,\mu) = \frac{1}{2} \log \left( (1-\mu)b_{2,i}(\rho) + 2 \right) - \frac{1}{2},$$

(10d)

$$a_{5,i}(\rho,\mu) = \frac{1}{2} \log \left( 2 + \frac{\text{SNR}_{i1} b_{2,i}(\rho) + \text{SNR}_{i1} b_{2,i}(\rho)}{\text{INR}_{ij}((1-\mu)b_{2,i}(\rho) + 2) + b_{1,i}(1)+1} \right),$$

(10e)

$$a_{6,i}(\rho,\mu) = \frac{1}{2} \log \left( \frac{\text{SNR}_{i1} b_{2,i}(\rho) + \text{SNR}_{i1} b_{2,i}(\rho) + 1}{\text{INR}_{ij}((1-\mu)b_{2,i}(\rho) + 2) + b_{1,i}(1)+1} \right),$$

(10f)

$$a_{7,i}(\rho,\mu,\mu) = \frac{1}{2} \log \left( \frac{\text{SNR}_{i1} b_{2,i}(\rho) + \text{SNR}_{i1} b_{2,i}(\rho) + 1}{\text{INR}_{ij}((1-\mu)b_{2,i}(\rho) + 2) + b_{1,i}(1)+1} \right),$$

(10g)

where the functions $b_{1,i} : [0,1] \to \mathbb{R}_+$, with $(l, i) \in \{1,2\}^2$ are defined as follows:

$$b_{1,i}(\rho) = \text{SNR}_{i1} + 2 \rho \sqrt{\text{SNR}_{i1} \text{INR}_{ij} + \text{INR}_{ij}}$$

(11a)

$$b_{2,i}(\rho) = \left( 1 - \rho \right) \text{INR}_{ij} - 1,$$

(11b)

with $j \in \{1,2\} \setminus \{i\}$.

Note that the functions in (10) and (11) depend on $\text{SNR}_{i1}$, $\text{SNR}_{i2}$, $\text{INR}_{12}$, $\text{INR}_{21}$, $\hat{\text{SNR}}_{i1}$, and $\hat{\text{SNR}}_{i2}$, however as these parameters are fixed in this analysis, this dependence is not emphasized in the definition of these functions. Finally, using this notation, Theorem 1 is presented on the next page.

**Proof:** The proof of Theorem 1 is presented in [1].

### B. Comments on the Achievability

The achievable region is obtained using a random coding argument and combining three classical tools: rate splitting, superposition coding, and backward decoding. This coding scheme is described in [1] and it is specially designed for the two-user IC-NOF. Consequently, only the strictly needed number of superposition code-layers is used. Other achievable schemes, as reported in [2], can also be obtained as special cases of the more general scheme presented in [3]. However, in this more general case, the resulting code for the IC-NOF contains a handful of unnecessary superposing code-layers, which complicates the error probability analysis.

### C. A Converse Region for the Two-User G-IC-NOF

The description of the converse region $\mathcal{C}_{G-IC-NOF}^C$ is determined by the ratios $\frac{\text{SNR}_{i1}}{\text{INR}_{ij}}$, and $\frac{\text{SNR}_{i2}}{\text{INR}_{ij}}$, for all $i \in \{1,2\}$, with $j \in \{1,2\} \setminus \{i\}$. All relevant scenarios regarding these ratios
Theorem 1: The capacity region $C_{\text{GIC-NOF}}$ contains the region $C_{\text{G-IC-NOF}}$ given by the closure of the set of all possible non-negative achievable rate pairs $(R_1, R_2)$ that satisfy

\begin{align}
R_1 &\leq \min \left\{ a_{2,1}(\rho), a_{6,1}(\rho, \mu_1) + a_{3,2}(\rho, \mu_1), a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{4,2}(\rho, \mu_1) \right\}, \\
R_2 &\leq \min \left\{ a_{2,2}(\rho), a_{3,1}(\rho, \mu_2) + a_{6,2}(\rho, \mu_2), a_{3,1}(\rho, \mu_2) + a_{4,1}(\rho, \mu_2) + a_{1,2} \right\}, \\
R_1 + R_2 &\leq \min \left\{ a_{2,1}(\rho) + a_{1,2}, a_{1,2} + a_{2,2}(\rho), a_{3,1}(\rho, \mu_2) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{7,2}(\rho, \mu_1, \mu_2), \right. \\
& \left. a_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{1,2}(\rho, \mu_1), a_{5,2}(\rho, \mu_1), a_{3,1}(\rho, \mu_2) + a_{7,1}(\rho, \mu_1, \mu_2) + a_{3,2}(\rho, \mu_1) + a_{1,2} \right\}, \\
2R_1 + R_2 &\leq \min \left\{ a_{2,1}(\rho) + a_{1,2}, a_{1,2} + a_{3,2}(\rho, \mu_1) + a_{7,2}(\rho, \mu_1, \mu_2), \right. \\
& \left. a_{3,1}(\rho, \mu_2) + a_{1,1} + a_{7,1}(\rho, \mu_1, \mu_2) + 2a_{3,2}(\rho, \mu_1) + a_{5,2}(\rho, \mu_1), a_{2,1}(\rho) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{5,2}(\rho, \mu_1) \right\}, \\
R_1 + 2R_2 &\leq \min \left\{ a_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{2,2}(\rho) + a_{1,2}, a_{3,1}(\rho, \mu_2) + a_{7,1}(\rho, \mu_1, \mu_2) + a_{2,2}(\rho) + a_{1,2}, \\
& 2a_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{3,2}(\rho, \mu_1) + a_{1,2} + a_{7,2}(\rho, \mu_1, \mu_2) \right\},
\end{align}

with $(\rho, \mu_1, \mu_2) \in \left[0, 1 - \max \left(\frac{1}{\ln12}, \frac{1}{\ln21}\right)\right] \times [0, 1] \times [0, 1].$

are described by two events denoted by $S_{l,1}$ and $S_{l,2}$, where $(l_1, l_2) \in \{1, \ldots, 5\}^2$. The events are defined as follows:

\begin{align}
S_{1,i}: \quad & \text{SNR}_{ij} < \min \{ \text{INR}_{ij}, \text{INR}_{ji} \}, \\
S_{2,i}: \quad & \text{INR}_{ij} \leq \text{SNR}_{ij} < \text{INR}_{ji}, \\
S_{3,i}: \quad & \text{INR}_{ij} \leq \text{SNR}_{ij} < \text{INR}_{ji}, \\
S_{4,i}: \quad & \max \{ \text{INR}_{ij}, \text{INR}_{ji} \} \leq \text{SNR}_{ij} < \text{INR}_{ij}, \text{INR}_{ji}, \\
S_{5,i}: \quad & \text{SNR}_{ij} \geq \max \{ \text{INR}_{ij}, \text{INR}_{ji}, \text{INR}_{ji}, \text{INR}_{ji} \}.
\end{align}

Note that for all $i \in \{1, 2\}$, the events $S_{1,i}$, $S_{2,i}$, $S_{3,i}$, $S_{4,i}$, and $S_{5,i}$ are mutually exclusive. This observation shows that given any 4-tuple $(\text{SNR}_{1,1}, \text{SNR}_{1,2}, \text{INR}_{12}, \text{INR}_{21})$, there always exists one and only one pair of events $(S_{1,i}, S_{2,i})$, with $(l_1, l_2) \in \{1, \ldots, 5\}^2$, that identifies a unique scenario. Note also that the pairs of events $(S_{2,1}, S_{2,2})$ and $(S_{3,1}, S_{3,2})$ are not feasible. In view of this, twenty-three different scenarios can be identified using the events in (13). Once the exact scenario is identified, the converse region is described using the functions $\kappa_{1,i}: [0, 1] \to \mathbb{R}_+$, with $(l, i) \in \{1, \ldots, 3\} \times \{1, 2\}$; $\kappa_{1}: [0, 1] \to \mathbb{R}_+$, with $l \in \{4, 5\}$; $\kappa_{6,1}: [0, 1] \to \mathbb{R}_+$, with $l \in \{1, \ldots, 4\};$ and $\kappa_{7,ij}: [0, 1] \to \mathbb{R}_+$, with $(i, l) \in \{1, 2\}^2$. These functions are defined as follows for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$:

\begin{align}
\kappa_{1,1}(\rho) &= \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{1,2}(\rho) &= \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \right) + \frac{1}{2} \log \left( 1 + \frac{b_{4,1}(\rho)}{1 + b_{5,1}(\rho)} \right), \\
\kappa_{3,i}(\rho) &= \frac{1}{2} \log \left( \frac{\text{SNR}_{ij}(b_{4,1}(\rho) + b_{5,1}(\rho) + 1)}{b_{1,1}(\rho) + 1} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{4}(\rho) &= \frac{1}{2} \log \left( 1 + \frac{b_{4,1}(\rho)}{1 + b_{5,1}(\rho)} \right) + \frac{1}{2} \log \left( b_{1,2}(\rho) + 1 \right), \\
\kappa_{5}(\rho) &= \frac{1}{2} \log \left( 1 + \frac{b_{4,2}(\rho)}{1 + b_{5,1}(\rho)} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{6,1}(\rho) &= \frac{1}{2} \log \left( \frac{b_{5,2}(\rho) \text{SNR}_{2}}{b_{1,1}(\rho) + 1} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{6,2}(\rho) &= \frac{1}{2} \log \left( \frac{b_{5,2}(\rho) \text{SNR}_{2}}{b_{1,1}(\rho) + 1} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{6,3}(\rho) &= \frac{1}{2} \log \left( \frac{b_{5,1}(\rho) \text{SNR}_{1}}{b_{1,1}(\rho) + 1} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{6,4}(\rho) &= \frac{1}{2} \log \left( \frac{b_{5,2}(\rho) \text{SNR}_{2}}{b_{1,1}(\rho) + 1} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{7,1}(\rho) &= \frac{1}{2} \log \left( \frac{b_{5,1}(\rho) \text{SNR}_{1}}{b_{1,1}(\rho) + 1} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{7,2}(\rho) &= \frac{1}{2} \log \left( \frac{b_{5,1}(\rho) \text{SNR}_{1}}{b_{1,1}(\rho) + 1} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right).
\end{align}
\[ \kappa_{6,3}(\rho) = \frac{1}{2} \log \left( b_{6,1}(\rho) + b_{5,1}(\rho) \frac{\text{INR}_{21}}{\text{SNR}_1} \left( \frac{\text{SNR}_1}{b_{1,1}} + b_{3,1} \right) \right) \]

\[ - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) + \frac{1}{2} \log \left( 1 + b_{5,2}(\rho) \frac{\text{SNR}_2}{b_{1,2}(1) + 1} \right) \]

\[ + \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \frac{\text{INR}_{21}}{\text{SNR}_1} \left( \frac{\text{INR}_{21} + b_{3,2} \frac{\text{SNR}_1}{b_{1,1}(1) + 1} \right) \right) \]

\[ - \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \frac{\text{INR}_{21}}{\text{SNR}_1} \right) + \log(2\pi e), \]  

and

\[ \kappa_{6,4}(\rho) = \frac{1}{2} \log \left( b_{6,1}(\rho) + b_{5,1}(\rho) \frac{\text{INR}_{21}}{\text{SNR}_1} \left( \frac{\text{SNR}_1}{b_{1,1}} + b_{3,1} \right) \right) \]

\[ - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) \]

\[ + \frac{1}{2} \log \left( 1 + b_{5,2}(\rho) \frac{\text{SNR}_2}{b_{1,2}(1) + 1} \right) \]

\[ + \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \frac{\text{INR}_{21}}{\text{SNR}_1} \left( \frac{\text{INR}_{21} + b_{3,2} \frac{\text{SNR}_1}{b_{1,1}(1) + 1} \right) \right) \]

\[ + \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \frac{\text{INR}_{21}}{\text{SNR}_1} \right) + \log(2\pi e), \]  

\[ \kappa_{7,1,2}(\rho) = \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{ij} \right) \]

\[ - \frac{1}{2} \log \left( 1 + b_{5,j}(\rho) + \frac{1}{2} \log \left( 1 + b_{4,i}(\rho) + b_{5,j}(\rho) \right) \right) \]

\[ + \frac{1}{2} \log \left( 1 + (1 - \rho^2) \frac{\text{INR}_{j,i}}{\text{SNR}_j} \left( \frac{\text{INR}_{j,i} + b_{3,j} \frac{\text{SNR}_j}{b_{1,j}(1) + 1} \right) \right) \]

\[ + \frac{1}{2} \log \left( b_{6,j}(\rho) + \frac{b_{5,j}(\rho) \frac{\text{INR}_{j,i}}{\text{SNR}_j}}{\text{INR}_{j,i} + b_{3,j}} \right) \]

\[ + 2 \log(2\pi e), \]  

where the functions \( b_{l,i} \), with \((l, i) \in \{1, 2\} \) are defined in (11); \( b_{3,i} \) are constants; and the functions \( b_{l,i} : [0, 1] \to \mathbb{R}_+ \), with \((l, i) \in \{4, 5, 6\} \times \{1, 2\}\) are defined as follows, with \( j \in \{1, 2\} \setminus \{i\} \):

\[ b_{3,i} = \text{SNR}_i - 2\sqrt{\text{SNR}_i \text{INR}_{j,i} + \text{INR}_{j,i}}, \]

\[ b_{4,i}(\rho) = \left( 1 - \rho^2 \right) \text{SNR}_i, \]

\[ b_{5,i}(\rho) = \left( 1 - \rho^2 \right) \text{INR}_{j,i}, \]

\[ b_{6,i}(\rho) = \text{SNR}_i + \text{INR}_{j,i} + 2\rho \sqrt{\text{INR}_{j,i} \left( \sqrt{\text{SNR}_i} - \sqrt{\text{SNR}_j} \right) \frac{\text{INR}_{j,i}}{\sqrt{\text{SNR}_i} - 2\sqrt{\text{SNR}_j}}. \]

Note that the functions in (14), (15), (16) and (17) depend on \( \text{SNR}_1, \text{SNR}_2, \text{INR}_{12}, \text{INR}_{21}, \text{SNR}_1 \), and \( \text{SNR}_2 \). However, these parameters are fixed in this analysis, and therefore, this dependence is not emphasized in the definition of these functions. Finally, using this notation, Theorem 2 is presented below.

**Theorem 2:** The capacity region \( C_{\text{GIC--NOF}} \) is contained within the region \( \overline{C}_{\text{G--IC-NOF}} \) given by the closure of the set of non-negative rate pairs \((R_1, R_2)\) that for all \( i \in \{1, 2\}\), with \( j \in \{1, 2\} \setminus \{i\} \) satisfy:

\[ R_i \leq \min \left( \kappa_{1,1}(\rho), \kappa_{2,1}(\rho) \right), \]

\[ R_i \leq \kappa_{3,i}(\rho), \]

\[ R_1 + R_2 \leq 2\kappa_{6}(\rho), \]

\[ R_1 + R_2 \leq \kappa_{7,1,2}(\rho). \]
The gap, denoted by the achievable region 4 approximation notion described in Definition 2.

E. A Gap Between the Achievable Region and the Converse Region

The outer bounds (18a) and (18c) correspond to the outer bounds for the case of perfect channel-output feedback [4]. The bounds (18b), (18d) and (18e) correspond to new outer bounds that generalize those presented in [2] for the two-user symmetric G-IC-NOF. These new outer-bounds were obtained using the genie-aided models shown in Figure 2.

D. Comments on the Converse Region

The outer bounds (18a) and (18c) correspond to the outer bounds for the case of perfect channel-output feedback [4]. The bounds (18b), (18d) and (18e) correspond to new outer bounds that generalize those presented in [2] for the two-user symmetric G-IC-NOF. These new outer-bounds were obtained using the genie-aided models shown in Figure 2.

The gap, denoted by δ, between the active region $\mathcal{C}_{G-IC-NOF}$ and the converse region $\overline{\mathcal{C}}_{G-IC-NOF}$ can be approximated (Definition 2) as follows:

$$\delta \leq \max \left( \delta_{R_1}, \frac{\delta_{R_2}}{2}, \frac{\delta_{R_3}}{3}, \frac{\delta_{R_4}}{3} \right),$$

(19)

where

$$\delta_{R_1} \triangleq \min \left( \kappa_4(\rho) - \kappa_3(\rho), \kappa_3(\rho), \kappa_2(\rho), \kappa_1(\rho) \right) - \min \left( a_{2.1}(\rho), a_{1.1}, a_{2.2}(\rho), a_{3.1}(\rho, \mu_2) + a_{3.2}(\rho, \mu_1), a_{1.1} + a_{3.2}(\rho, \mu_2) + a_{4.2}(\rho, \mu_1), a_{3.1}(\rho, \mu_2) + a_{4.2}(\rho, \mu_1) + a_{1.2} \right),$$

(20a)

$$\delta_{R_2} \triangleq \min \left( \kappa_4(\rho) + a_{2.1}(\rho), a_{1.1} + a_{3.2}(\rho, \mu_1) + a_{4.2}(\rho, \mu_1), a_{3.1}(\rho, \mu_2) + a_{4.2}(\rho, \mu_1) + a_{1.2} \right),$$

(20b)

$$\delta_{R_3} \triangleq \kappa_7(\rho) - \min \left( a_{2.1}(\rho), a_{1.1}, a_{3.2}(\rho, \mu_1), a_{4.2}(\rho, \mu_1), a_{1.1} + a_{3.2}(\rho, \mu_1), a_{4.2}(\rho, \mu_1) + a_{1.2}, a_{1.1}(\rho) + a_{1.1}, a_{3.1}(\rho, \mu_2) + a_{3.2}(\rho, \mu_2) + a_{1.2} \right),$$

(20c)

$$\delta_{R_4} \triangleq \kappa_7(\rho) - \min \left( a_{2.1}(\rho), a_{1.1}, a_{3.2}(\rho, \mu_1), a_{4.2}(\rho, \mu_1), a_{1.1} + a_{3.2}(\rho, \mu_1), a_{4.2}(\rho, \mu_1) + a_{1.2}, a_{1.1}(\rho) + a_{1.1}, a_{3.1}(\rho, \mu_2) + a_{3.2}(\rho, \mu_2) + a_{1.2} \right).$$

(20d)

Note that $\delta_{R_1}$ and $\delta_{R_2}$ represent the gap between the active achievable single-rate bound and the active converse single-rate bound; $\delta_{R_3}$ represents the gap between the active achievable sum-rate bound and the active converse sum-rate bound; and, $\delta_{R_3}$ and $\delta_{R_4}$ represent the gap between the active achievable weighted sum-rate bound and the active converse weighted sum-rate bound.

Finally, it is important to highlight that, as suggested in [2], [4], and [5], the gap between $\mathcal{C}_{G-IC-NOF}$ and $\mathcal{C}_{G-IC-NOF}$ can be calculated more precisely. However, the choice in (19) eases the calculations at the expense of less precision.

Figure 3 presents the exact gap existing between the achievable region $\mathcal{C}_{G-IC-NOF}$ and the converse region $\overline{\mathcal{C}}_{G-IC-NOF}$ for the case in which $\rho = 1$. The maximum gap is 1.1 bits per channel use and occurs when $\alpha = 1.05$ and $\beta = 1.2$.

IV. CONCLUSIONS

An achievable region and a converse region for the two-user G-IC-NOF have been introduced. It has been shown that these regions approximate the capacity region of the two-user G-IC-NOF to within 4.4 bits per channel use.

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