Measurement of the $B^0 \rightarrow \pi^{-}\ell^{+}\nu$ and $B^+ \rightarrow \pi^{0}\ell^{+}\nu$ Branching Fractions and Determination of $|V_{ub}|$ in $\Upsilon(4S) \rightarrow B\bar{B}$ Events Tagged by a Fully Reconstructed $B$ Meson

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Abstract

We report preliminary measurements of the charmless exclusive semileptonic branching fractions of the $B^0 \rightarrow \pi^{-}\ell^{+}\nu$ and $B^+ \rightarrow \pi^{0}\ell^{+}\nu$ decays, based on 211 fb$^{-1}$ of data collected at the $\Upsilon(4S)$ resonance by the B\textsc{abar} detector. In events in which the decay of one $B$ meson to a hadronic final state is fully reconstructed, the semileptonic decay of the second $B$ meson is identified by the detection of a charged lepton and a pion. We measure the partial branching fractions for $B^0 \rightarrow \pi^{-}\ell^{+}\nu$ and $B^+ \rightarrow \pi^{0}\ell^{+}\nu$ in three regions of the invariant mass squared of the lepton pair, and we obtain the total branching fractions $\mathcal{B}(B^0 \rightarrow \pi^{-}\ell^{+}\nu) = (1.14 \pm 0.27_{\text{stat}} \pm 0.17_{\text{syst}}) \times 10^{-4}$ and $\mathcal{B}(B^+ \rightarrow \pi^{0}\ell^{+}\nu) = (0.86 \pm 0.22_{\text{stat}} \pm 0.11_{\text{syst}}) \times 10^{-4}$. Using isospin symmetry, we measure the combined total branching fraction $\mathcal{B}(B^0 \rightarrow \pi^{-}\ell^{+}\nu) = (1.28 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}}) \times 10^{-4}$. Theoretical predictions of the form-factor are used to determine the magnitude of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}| = (3.7 \pm 0.3_{\text{stat}} \pm 0.2_{\text{syst}+0.8_{FF}}^{-0.5_{FF}}) \times 10^{-3}$, where the last uncertainty is due to the form-factor normalization.

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1 Introduction

Precise measurements of the Cabibbo-Kobayashi-Maskawa matrix element $V_{ub}$ can be employed to test the consistency of the Standard Model description of $CP$ violation. $|V_{ub}|$ can be extracted from charmless semileptonic $B$ decays, such as $B \to \pi \ell \nu$ and $B \to \rho \ell \nu$, allowing for more stringent kinematic constraints and better background suppression than possible with inclusive measurements.

However the determination of $|V_{ub}|$ from exclusive decays is complicated by the presence of the strong interaction between the quarks in the initial and the final states. In the case of $B \to \pi \ell \nu$ decays the dynamics are described by a single form-factor $f(q^2)$ that depends on the square of the $B \to \pi$ momentum transfer. The shape of the form-factors can in principle be measured, while for their normalization we have to rely on theoretical predictions [1, 2, 3]. The measurements presented here rely on these theoretical predictions for both the shape and the normalization.

We present a measurement of the branching fractions of the exclusive charmless semileptonic decays $B^0 \to \pi^- \ell^+ \nu$ and $B^+ \to \pi^0 \ell^+ \nu$. The analysis is based on a sample of $B\bar{B}$ events produced at the $\Upsilon(4S)$ resonance that are tagged by a fully reconstructed hadronic decay. A semileptonic decay of the recoiling $B$ meson is identified by the presence of a charged lepton. The charmless mesons in the semileptonic decay are reconstructed and the missing mass is calculated assuming that the pion and the charged lepton are the only particles present in the recoil except for the undetected neutrino. Since the momentum of the tagging $B$ meson is measured, a transformation to the rest frame of the recoiling $B$ meson can be applied. The full reconstruction approach provides also a very clean sample of $B\bar{B}$ events, determines the flavor of the reconstructed $B$ meson and separates $B^0$ and $B^+$ decays.

Exclusive charmless semileptonic $B$ decays have been previously measured by the CLEO [4], Belle [5] and BaBar [6, 7, 8] collaborations. The approach presented in this paper results in small backgrounds in the whole $q^2$ spectrum with very loose kinematic selection criteria. We present here the partial branching fractions in three $q^2$ regions, $q^2 < 8 \text{GeV}^2/c^4$, $8 < q^2 < 16 \text{GeV}^2/c^4$, and $q^2 > 16 \text{GeV}^2/c^4$, the measurement of the total branching fraction, and the measurement of $|V_{ub}|$.

2 Data Sample

The preliminary results are based on a data sample of about 233 million $B\bar{B}$ pairs, corresponding to 211 fb$^{-1}$, collected with the BABAR detector [9] at the SLAC PEP-II asymmetric-energy $e^+e^-$ collider [10], operating at the $\Upsilon(4S)$ resonance. A Monte Carlo (MC) simulation of the BABAR detector based on Geant4 [11] has been used to optimize the selection criteria and to determine the signal efficiencies and background distributions.

3 Overview of Analysis Method

The analysis strategy follows closely that of Ref. [12]. The starting point is the selection of a sample of events in which the hadronic decay of one of the two $B$ mesons ($B_{\text{reco}}$) is fully reconstructed. About 1000 different $B \to D Y$ decay chains are selected, where $D$ refers to a charm meson and $Y$ represents a collection of hadrons with total charge $\pm 1$ and composed of $n_1 \pi^\pm + n_2 K^\pm + n_3 K^0 + n_4 \pi^0$, with $n_1 + n_2 < 6$, $n_3 < 3$ and $n_4 < 3$.

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4 Charge-conjugate modes are implied throughout this paper, unless explicitly stated otherwise.
We reconstruct $D^- \rightarrow K^+\pi^-\pi^-$, $K^+\pi^-\pi^0$, $K^0\pi^-$, $K^0\pi^-\pi^0$, $K^0\pi^-\pi^+ + D^+ \rightarrow \bar{D}^0\pi^-$; $D^0 \rightarrow K^+\pi^0$, $K^+\pi^-\pi^+ + K^0\pi^-\pi^0$, and $D^+ \rightarrow \bar{D}^0\pi$, $D^0\gamma$. Then we use $D^-$ and $D^*$ ($D^0$ and $D^*$) decays as a “seed” to reconstruct $B^0$ ($B^+$) decays. Overall, we correctly reconstruct one $B$ candidate in 0.3% (0.5%) of the $B^{0}\bar{B}^0$ ($B^+\bar{B}^-$) events. The kinematic consistency of a $B_{\text{reco}}$ candidate with a $B$ meson decay is checked using two variables: the beam-energy substituted mass $m_{\text{ES}} = \sqrt{s/4 - \vec{p}_B^2}$ and the energy difference $\Delta E = E_B - \sqrt{s/2}$. Here $\sqrt{s}$ refers to the total energy in the $\Upsilon(4S)$ center of mass frame, and $\vec{p}_B$ and $E_B$ denote the momentum and energy of the $B_{\text{reco}}$ candidate in the same frame. For signal events the $m_{\text{ES}}$ and $\Delta E$ distributions peak at the $B$ meson mass and at zero, respectively.

The combinatorial background from $B\bar{B}$ events and $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) production, in the $B_{\text{reco}}$ sample, is subtracted by performing an unbinned likelihood fit to the $m_{\text{ES}}$ distribution, using the following threshold function [13]

$$\frac{dN}{dm_{\text{ES}}} = N \cdot m_{\text{ES}} \cdot \sqrt{1-x^2} \cdot \exp \left(-\xi \cdot (1-x^2)\right)$$

(1)

for the background (where $x = m_{\text{ES}}/m_{\text{max}}$ and $m_{\text{max}}$ is the endpoint of the curve) and a Gaussian function corrected for radiation losses [14] peaked at the $B$ meson mass for the signal (Fig. 1).

Figure 1: Fit to the $m_{\text{ES}}$ distribution for events with a fully reconstructed $B^0$ (left) or $B^+$ (right) decay, after the request of a prompt energetic lepton (top) and after all selection criteria for $B \rightarrow \pi\ell\nu$ decays (bottom). The fitted curve (black line) to the data points (black dots) is the sum of a radiation loss corrected gaussian and a threshold function described by Eq. 1 (dashed line).
Once a $B_{\text{reco}}$ meson has been reconstructed in the event, the selection of the other $B$ meson ($B_{\text{sig}}$) decay follows in two steps. First, a “semileptonic selection” is applied to obtain a sample of semileptonic decays. Then, a refined selection is applied to build a sample of $B \to \pi \ell \nu$ decays, as described in Sec. 4. In order to minimize the systematic uncertainties, the exclusive branching fractions are measured relative to the inclusive semileptonic one.

The semileptonic selection is based essentially on the presence of a charged (electron or muon) lepton with its momentum $p_{\ell}^{\text{cut}}$ in the $B_{\text{sig}}$ rest frame greater than a given $p_{\ell}^{\text{cut}}$. The number of $B \to X\ell \nu$ events, $N_{\text{sl}}^{\text{meas}}$, and the number of remaining background events, $BG_{\text{sl}}$, peaking at the $B$ mass in the $m_{ES}$ distribution and estimated with the Monte Carlo simulation, are related to the true number of semileptonic decays $N_{\text{sl}}^{\text{true}}$ as

$$N_{\text{sl}}^{\text{meas}} - BG_{\text{sl}} = \epsilon_{t}^{\text{sl}} \epsilon_{\ell}^{\text{sl}} N_{\text{sl}}^{\text{true}}. \tag{2}$$

Here $\epsilon_{t}^{\text{sl}}$ refers to the efficiency for selecting a lepton from a semileptonic $B$ decay with a momentum above $p_{\ell}^{\text{cut}}$ in an event with a hadronic $B$ decay, reconstructed with tag efficiency $\epsilon_{t}^{\text{sl}}$. The background normalization is taken from the Monte Carlo scaled to the luminosity of the data sample.

The full reconstruction of the $B_{\text{reco}}$ meson allows for a precise determination of the neutrino four-momentum $p_{\nu}$, estimated from the missing four-momentum in the event:

$$p_{\nu} = p_{\text{miss}} = p_{T(4S)} - p_{B_{\text{reco}}} - p_{\pi} - p_{\ell}, \tag{3}$$

where all momenta are measured in the laboratory frame: $p_{T(4S)}$ is the sum of the four-momenta of the colliding beams, $p_{B_{\text{reco}}}$ is the measured four-momentum of the $B_{\text{reco}}$, $p_{\pi}$ is the measured four-momentum of the $\pi^-$ or $\pi^0$ and $p_{\ell}$ is the measured four-momentum of the lepton. For signal events the only missing particle should be a single undetected neutrino, while for background events the missing momentum and energy in the event are due to other undetected or poorly measured particles. Then, in signal events the resulting missing mass, defined as $m_{\text{miss}}^{2} = p_{\text{miss}}^{2}$, peaks at zero and for background events it tends to have larger values, and allows for a powerful discrimination of signal and background.

The number of $B \to \pi \ell \nu$ events after the combinatoric subtraction in a given $q^2$ range, $N_{\text{excl}}^{\text{meas}}$, and the number of peaking background events, $BG_{\text{excl}}$, are related to the true number of $B \to \pi \ell \nu$ events $N_{\text{excl}}^{\text{true}}$ as

$$N_{\text{excl}}^{\text{meas}} - BG_{\text{excl}} = \epsilon_{\text{sel}}^{\text{excl}} \epsilon_{t}^{\text{excl}} \epsilon_{\ell}^{\text{excl}} N_{\text{excl}}^{\text{true}}, \tag{4}$$

where the signal efficiency $\epsilon_{\text{sel}}^{\text{excl}}$ accounts for all selection criteria applied to the sample after the requirement of a charged lepton with momentum $p_{\ell}^{\text{cut}} > p_{\ell}^{\text{cut}}$. The ratio between the partial branching fractions of the signal in a particular $q^2$ region and the branching fraction of $B \to X\ell \nu$ decays is

$$R_{\text{excl}}^{\text{sl}} = \frac{\Delta B(B \to \pi \ell \nu)}{B(B \to X\ell \nu)} = \frac{N_{\text{true}}^{\text{excl}}}{N_{\text{true}}^{\text{sl}}} = \frac{(N_{\text{meas}}^{\text{excl}} - BG_{\text{excl}})/(\epsilon_{\text{sel}}^{\text{excl}})}{(N_{\text{meas}}^{\text{sl}} - BG_{\text{sl}})/(\epsilon_{\text{sel}}^{\text{sl}})} \times \frac{\epsilon_{t}^{\text{excl}} \epsilon_{\ell}^{\text{excl}}}{\epsilon_{t}^{\text{sl}} \epsilon_{\ell}^{\text{sl}}}. \tag{5}$$

The ratio of efficiencies for $B \to X\ell \nu$ and signal events in Eq. 4 is expected to be close to, but not equal to unity. Due to the difference in multiplicity and the different lepton momentum spectra, we expect the tag efficiencies $\epsilon_{t}^{\text{sl,excl}}$ and the charged lepton efficiencies $\epsilon_{\ell}^{\text{sl,excl}}$ to be slightly different for the two classes of events. Finally using the semileptonic branching ratio $B(B \to X\ell \nu) = (10.73 \pm 0.28)\%$ [15] and the ratio of the $B^{0}$ and $B^{+}$ lifetimes $\tau_{B^{+}}/\tau_{B^{0}} = 1.086 \pm 0.017$ [15], the partial branching ratios $\Delta B(B^{0} \to \pi^{-}\ell^{+}\nu)$ and $\Delta B(B^{+} \to \pi^{0}\ell^{+}\nu)$ can be derived.
4 Event Reconstruction and Selection

On the recoil of a fully reconstructed $B$, the $B \to \pi \ell \nu$ decay of $B_{\text{sig}}$ is constructed by combining a pion with a charged lepton from only the tracks and neutral clusters which do not contribute to the $B_{\text{reco}}$. Electron candidates are identified by a likelihood-based algorithm, while the muon identification is based on a tight selection. Remaining tracks are assumed to be pions if they are not identified as either a muon or an electron.

Neutral pions are reconstructed by using pairs of photons with an energy greater than 80 MeV in the laboratory. Moreover the energy of the most energetic photon used to reconstruct the $\pi^0$ is required to be greater than 300 MeV in the $B_{\text{sig}}$ rest frame, to reject combinatorial background.

In the semileptonic selection, we require:

- the presence of a fully reconstructed $B_{\text{reco}}$, neutral for $B^0 \to \pi^- \ell^+ \nu$ and charged for $B^+ \to \pi^0 \ell^+ \nu$.
- one lepton, either an electron with $p_{el}^* > 0.5 \text{ GeV/c}$ or a muon with $p_{\mu}^* > 0.8 \text{ GeV/c}$, originating from the $B_{\text{sig}}$.
- correlation between the lepton charge and $B_{\text{reco}}$ flavor for $B^+ \to \pi^0 \ell^+ \nu$. In unmixed signal events $Q_{b(\text{reco})}Q_\ell < 0$, where $Q_{b(\text{reco})}$ is the charge of the $b$-quark inside the $B_{\text{reco}}$ and $Q_\ell$ is the charge of the signal lepton. In mixed $B^0$ decays or in some background events $Q_{b(\text{reco})}Q_\ell > 0$. For $B^+ \to \pi^0 \ell^+ \nu$ decays, $Q_{b(\text{reco})}Q_\ell < 0$ is therefore required. If the reconstructed $B$ is neutral, both events with $Q_{b(\text{reco})}Q_\ell < 0$ (right sign, “rs”) and $Q_{b(\text{reco})}Q_\ell > 0$ (wrong sign, “ws”) are accepted and the sample is subsequently corrected for the effects of $B\bar{B}$ mixing:

$$N_B = \frac{1 - \chi_d}{1 - 2 \chi_d} N_{rs} - \frac{\chi_d}{1 - 2 \chi_d} N_{ws}$$

where $\chi_d = 0.186 \pm 0.004$ \cite{15} is the mixing parameter.

To select the decay modes of interest, the following additional selection criteria are applied:

- event charge conservation: $Q_{\text{tot}} = Q_{B_{\text{reco}}} + Q_{B_{\text{sig}}} = 0$. This condition rejects preferentially $b \to c\ell \nu$ events, since their higher charge multiplicity leads to higher loss of charged tracks.
- a decay-mode dependent cut on $m_{\text{miss}}^2$.
- the only tracks allowed to be present in the recoil are the charged lepton and the charged pion in the case of $B^0 \to \pi^- \ell^+ \nu$ decay.
- a $J/\psi$ veto for the $B^0 \to \pi^- \ell^+ \nu$ mode. $J/\psi \to \ell^+ \ell^-$ decays introduce a background due to the mis-identification of a lepton as a pion. To remove this background, the lepton mass hypothesis is applied to the charged pion and the invariant mass $m_{\pi\ell}$ of the lepton-$\pi$ pair is requested to be outside the range $3.08 < m_{\pi\ell} < 3.12 \text{ GeV/c}^2$.
- in the $B^0 \to \pi^- \ell^+ \nu$ channel the residual photon energy $E_{\text{neutral}}$ is required to be smaller than 0.45 GeV to reject $B^0 \to \rho^- \ell^+ \nu$ events which constitute the main $b \to u\ell\nu$ background.
- the $\pi^0$ reconstructed mass is required to satisfy $110 < m_{\gamma\gamma} < 160 \text{ MeV/c}^2$.
The selection criteria described above have been optimized by minimizing the statistical error on $R_{\text{excl/sl}}$ and are summarized in Tab. 1. When more than one $B \to \pi \ell \nu$ candidate is reconstructed in the same event, the one with $m_{\text{miss}}^2$ closest to zero is chosen. The selection efficiencies $e_{\text{sel}}^{\text{excl}}$ as estimated from the Monte Carlo simulation are reported in Tab. 2 and 3. The number of events after the semileptonic selection and after all analysis cuts are obtained with the $m_{\text{ES}}$ fit described in Section 3. The fit results on data are shown in Fig. 1.

Table 1: Summary of event selection for $B^0 \to \pi^- \ell^+ \nu$ and $B^+ \to \pi^0 \ell^+ \nu$.

| Selection               | $B^0 \to \pi^- \ell^+ \nu$                                                  | $B^+ \to \pi^0 \ell^+ \nu$                                                                 |
|-------------------------|------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| Lepton momentum         | $p_{\ell}^* > 0.5 \text{ GeV}/c, p_{\mu}^* > 0.8 \text{ GeV}/c,$            | mixing correction                                                                  |
| Number of leptons       | $N_{\text{lepton}} = 1,$                                                     | $-0.5 < m_{\text{miss}}^2 < 0.7 \text{ GeV}^2/c^4$                                       |
| Charge conservation     | $Q_{\text{tot}} = 0,$                                                        | $110 < m_{\gamma \gamma} < 160 \text{ MeV}/c^2$                                        |
| Number of tracks        | no additional charged tracks                                                 | $-$                                                                                     |
| Charge correlation      | $Q_{b(\text{reco})}Q_{\ell} < 0$                                            | $-$                                                                                     |
| Missing mass squared    | $|m_{\text{miss}}^2| < 0.3 \text{ GeV}^2/c^4$                                 | $-$                                                                                     |
| $\pi^0$ mass            | $-$                                                                           | $-$                                                                                     |
| Neutral energy          | $E_{\text{neutral}} < 0.45 \text{ GeV}$                                     | $-$                                                                                     |
| Lepton-$\pi$ mass      | $|m_{\pi \ell} - m_{J/\psi}| > 0.02 \text{ GeV}/c^2$                       | $-$                                                                                     |

5 Measurement of Branching Fractions

In order to extract $|V_{ub}|$ from the $B \to \pi \ell \nu$ decay rate, a measurement of partial branching fractions for $B^0 \to \pi^- \ell^+ \nu$ and $B^+ \to \pi^0 \ell^+ \nu$ is performed in bins of the invariant-mass squared of the lepton-neutrino system, $q^2 = (p_{\Upsilon(4S)} - p_{\text{reco}} - p_\pi)^2$. We consider three $q^2$ bins: $q^2 < 8$, $8 < q^2 < 16$ and $q^2 > 16 \text{ GeV}^2/c^4$. Since the observed resolution on $q^2$ is very good (about 0.25 GeV$^2$/c$^4$ for $B^0 \to \pi^- \ell^+ \nu$ and 0.50 GeV$^2$/c$^4$ for $B^+ \to \pi^0 \ell^+ \nu$) compared to the width of the $q^2$ bins, the cross-feed among the different $q^2$ bins is small and it is considered as background. We neglect the correlation among the different $q^2$ bins introduced by this cross-feed. The peaking background surviving the selection is estimated with a Monte Carlo simulation and scaled to the data yields after the requirements on the charged lepton.

The resulting partial branching fractions and all the corresponding input measurements are shown in Tab. 2 and Tab. 3 for $B^0 \to \pi^- \ell^+ \nu$ and $B^+ \to \pi^0 \ell^+ \nu$ respectively. Fig. 2 and 3 show the data $m_{\text{miss}}^2$ and $\pi^0$ mass distributions, respectively, for events in the full $q^2$ range. Signal and background components from the Monte Carlo, scaled to the number of events passing the semileptonic selection, are also overlayed.

6 Systematic Uncertainties

Since the estimation of the systematic uncertainties is affected by low statistics in the Monte Carlo, they have been calculated in the whole $q^2$ range and assumed to be the same in each of the three $q^2$ bins except for the systematic uncertainties due to form-factors and to Monte Carlo statistics. These uncertainties have been assessed separately for each $q^2$ bin.
Figure 2: $m^2_{\text{miss}}$ distribution from data (dots) and signal and background contributions from Monte Carlo (histograms) for $B^0 \to \pi^- \ell^+ \nu$. The background components are scaled to the data yields after the requirements on the charged lepton.

Figure 3: $m_{\gamma\gamma}$ (left) and $m^2_{\text{miss}}$ (right) distributions from data (dots) and signal and background contributions from Monte Carlo (histograms) for $B^+ \to \pi^0 \ell^+ \nu$. The background components are scaled to the data yields after the requirements on the charged lepton.
Uncertainties related to the reconstruction of charged tracks are determined by removing randomly a fraction of tracks corresponding to the uncertainty in the track finding efficiency. The systematic error due to the reconstruction of neutral particles in the EMC is studied by varying a fraction of tracks corresponding to the uncertainty in the track finding efficiency. The reported errors are statistical only.

Table 2: Measurement of $R_{\text{excl/sl}}$ for $B^0 \rightarrow \pi^- \ell^+\nu$ in $q^2$ bins and corresponding inputs. The reported errors are statistical only.

| $q^2$ bin [GeV$^2$/c$^4$] | $N_{\text{meas}}$ | $BG_{\text{excl}}$ | $\epsilon_{\text{excl}}^{\text{meas}}$ | $N_{\text{meas}} - BG_{\text{en}}$ | $\epsilon_{\text{excl}}^{\text{en}}$ | $\Delta \frac{B(B^0 \rightarrow \pi^- \ell^+\nu)}{B(B^+ \rightarrow X\ell\nu)} \times 10^{-3}$ |
|--------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------------------------|
| $q^2 < 8$                | 6.9 ± 3.1       | 2.7 ± 1.6       | 0.70 ± 0.03     | 43500 ± 300     | 0.97 ± 0.08     | 0.14 ± 0.10                     |
| 8 < $q^2$ < 16           | 10.5 ± 3.9      | 1.1 ± 1.0       | 0.54 ± 0.03     | 43500 ± 300     | 0.86 ± 0.09     | 0.34 ± 0.14                     |
| $q^2 > 16$               | 18.7 ± 5.0      | 2.7 ± 1.6       | 0.57 ± 0.05     | 43500 ± 300     | 0.98 ± 0.15     | 0.63 ± 0.20                     |
| Total                    | 36.1 ± 7.1      | 6.5 ± 2.3       | -               | 43500 ± 300     | -               | 1.11 ± 0.25                     |

Table 3: Measurement of $R_{\text{excl/sl}}$ for $B^+ \rightarrow \pi^0 \ell^+\nu$ in $q^2$ bins and corresponding inputs. The reported errors are statistical only.

| $q^2$ bin [GeV$^2$/c$^4$] | $N_{\text{meas}}$ | $BG_{\text{excl}}$ | $\epsilon_{\text{excl}}^{\text{meas}}$ | $N_{\text{meas}} - BG_{\text{en}}$ | $\epsilon_{\text{excl}}^{\text{en}}$ | $\Delta \frac{B(B^+ \rightarrow \pi^0 \ell^+\nu)}{B(B^+ \rightarrow X\ell\nu)} \times 10^{-3}$ |
|--------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------------------------|
| $q^2 < 8$                | 7.7 ± 2.9       | 2.6 ± 1.6       | 0.44 ± 0.04     | 69600 ± 400     | 0.97 ± 0.12     | 0.16 ± 0.09                     |
| 8 < $q^2$ < 16           | 13.5 ± 4.0      | 2.9 ± 1.7       | 0.42 ± 0.04     | 69600 ± 400     | 1.01 ± 0.09     | 0.37 ± 0.14                     |
| $q^2 > 16$               | 12.9 ± 3.8      | 4.1 ± 2.0       | 0.37 ± 0.06     | 69600 ± 400     | 0.72 ± 0.13     | 0.24 ± 0.10                     |
| Total                    | 34.1 ± 6.2      | 9.6 ± 3.1       | -               | 69600 ± 400     | -               | 0.77 ± 0.19                     |

The impact of the charm semileptonic branching fractions has been estimated by varying each of the exclusive branching fractions within one standard deviation of the current world average [15]. The effects due to exclusive $B \rightarrow X_u \ell\nu$ decays have been evaluated by varying their branching fractions by 30% for $B \rightarrow \rho \ell\nu$, by 40% for $B \rightarrow \omega \ell\nu$ and by 100% for the remaining decay modes.

The use of different theoretical models changes the lepton spectrum shape for the signal and, as a consequence, affects the efficiencies $\epsilon_{\text{excl}}^{\ell}$, $\epsilon_{\ell}^{\text{en}}$, and $\epsilon_{\text{sel}}^{\ell}$. The Monte Carlo samples used in this analysis were generated using the ISGW2 model [16]. We reweighted the event distributions according to the recent calculations by Ball and Zwicky [1] based on light-cone sum rules (LCSR) since, among the calculations currently available, these calculations result in distributions that differ most from those predicted by ISGW2. We assign the variations with respect to the ISGW2 as systematic uncertainties. This contribution is small because the signal efficiencies for $B^0 \rightarrow \pi^- \ell^+\nu$ and $B^+ \rightarrow \pi^0 \ell^+\nu$ are largely independent of $q^2$. 

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A summary of the systematic uncertainties discussed above is shown in Tab. 4.

Table 4: Systematic uncertainties in the measurement of $R_{\text{excl/sl}}$.

| Relative Uncertainty on $R_{\text{excl/sl}}$(%) | $B^0 \rightarrow \pi^- \ell^+ \nu$ | $B^+ \rightarrow \pi^0 \ell^+ \nu$ |
|-----------------------------------------------|-------------------------------|-------------------------------|
| Track reconstruction efficiency               | 1.1                           | 1.4                           |
| Photon resolution, $\pi^0$ reconstruction     | 1.2                           | 3.7                           |
| Electron identification                       | 1.1                           | 1.1                           |
| Muon identification                            | 2.3                           | 2.3                           |
| $m_{\text{ES}}$ fit                            | 9.4                           | 5.0                           |
| Cross-feed $B^0 \leftrightarrow B^+$           | 0.7                           | 1.4                           |
| $B \rightarrow D\ell\nu X$ and $D$ branching fractions | 0.2                           | 2.6                           |
| $B \rightarrow X_{\ell} \nu$ branching fractions | 4.2                           | 1.7                           |
| Form-factor model dependence ($q^2 < 8$)       | 0.3                           | 0.3                           |
| Form-factor model dependence ($8 < q^2 < 16$)  | 0.2                           | 0.2                           |
| Form-factor model dependence ($q^2 > 16$)      | 0.1                           | 2.2                           |
| MC statistics ($q^2 < 8$)                      | 18.3                          | 19.8                          |
| MC statistics ($8 < q^2 < 16$)                 | 11.8                          | 14.7                          |
| MC statistics ($q^2 > 16$)                     | 17.6                          | 23.0                          |
| Total error ($q^2 < 8$)                       | 21.2                          | 21.2                          |
| Total error ($8 < q^2 < 16$)                  | 16.0                          | 16.7                          |
| Total error ($q^2 > 16$)                      | 20.6                          | 24.2                          |

7 Results for the Branching Fractions and $|V_{ub}|$

The $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$ decay rates are related to $|V_{ub}|$ through a hadronic form-factor. With the assumption of massless leptons and isospin symmetry, the differential decay rates are given by:

$$\frac{d\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu)}{dq^2} = 2 \times \frac{d\Gamma(B^+ \rightarrow \pi^0 \ell^+ \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |f_+(q^2)|^2 p_{\pi}^3,$$

where $p_{\pi}$ is the momentum of the pion in the rest frame of the $B$ meson. The form-factor (FF) $f_+(q^2)$ has been calculated with different assumptions, which predict different $q^2$ spectra.

We derived the partial branching fractions using the inclusive semileptonic branching ratio $B(B \rightarrow X\ell\nu) = (10.73 \pm 0.28)\%$ and the ratio of the $B^0$ and $B^+$ lifetimes $\tau_{B^+}/\tau_{B^0} = 1.086 \pm 0.017$ [15]. The results for the partial branching fractions of $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$ are reported in Tab. 5. The partial and total rates for exclusive decays involving $\pi^+$ and $\pi^0$ can be constrained using isospin symmetry, $\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu) = 2 \times \Gamma(B^+ \rightarrow \pi^0 \ell^+ \nu)$, and the ratio of the $B^0$ and $B^+$ lifetimes. We average the six results for the partial branching fractions by following the prescription suggested by the Heavy Flavor Averaging Group in [17]. The measurements agree with each other with $\chi^2 = 3.8$ for 3 degrees of freedom. The results are reported in Tab. 5 and in Fig. 4, including a comparison with other recent BABAR measurements for partial branching ratios.
Figure 4: Partial branching ratio in $q^2$ bins for $B^0 \rightarrow \pi^- \ell^+ \nu$ (top left), $B^+ \rightarrow \pi^0 \ell^+ \nu$ (top right), combination (bottom left) and comparison of the combination with the previous BABAR measurements (bottom right): untagged analysis with $\nu$ reconstruction [6] and analysis with $D^* \ell \nu$ tag [7,8]. The error bars represent the sum in quadrature of statistical and systematic uncertainties.
Table 5: Measured partial branching fractions of the $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$ decays. The last column lists the combined measurement for $B^0 \rightarrow \pi^- \ell^+ \nu$ assuming isospin symmetry. The first error is statistical, the second one systematic.

| $q^2$ bin | $\Delta B(B^0 \rightarrow \pi^- \ell^+ \nu)$ [$\times 10^{-4}$] | $\Delta B(B^+ \rightarrow \pi^0 \ell^+ \nu)$ [$\times 10^{-4}$] | $\Delta B(B^0 \rightarrow \pi^- \ell^+ \nu)$ [$\times 10^{-4}$] |
|-----------|----------------|----------------|----------------|
| $q^2 < 8$ GeV$^2$/c$^4$ | $0.14 \pm 0.10 \pm 0.03$ | $0.18 \pm 0.10 \pm 0.04$ | $0.21 \pm 0.10 \pm 0.03$ |
| $8 < q^2 < 16$ GeV$^2$/c$^4$ | $0.35 \pm 0.15 \pm 0.06$ | $0.41 \pm 0.16 \pm 0.06$ | $0.48 \pm 0.14 \pm 0.06$ |
| $q^2 > 16$ GeV$^2$/c$^4$ | $0.65 \pm 0.20 \pm 0.12$ | $0.27 \pm 0.12 \pm 0.06$ | $0.59 \pm 0.15 \pm 0.10$ |
| Total | $1.14 \pm 0.27 \pm 0.17$ | $0.86 \pm 0.22 \pm 0.11$ | $1.28 \pm 0.23 \pm 0.16$ |

Integrating Eq. 7 in a given $q^2$ interval, $|V_{ub}|$ can be extracted from the measured partial branching fraction $\Delta B(B^0 \rightarrow \pi^- \ell^+ \nu)$ using the following expression:

$$|V_{ub}| = \sqrt{\frac{\Delta B(B^0 \rightarrow \pi^- \ell^+ \nu)}{\Delta \zeta \cdot T_{B^0}}},$$

(8)

where $\tau_{B^0} = 1.536 \pm 0.014 \text{ps}$ \cite{15} is the $B^0$ lifetime, $\Delta B(B^0 \rightarrow \pi^- \ell^+ \nu)$ is the combined partial branching fraction for a given $q^2$ interval and $\Delta \zeta$ is the predicted form-factor normalization for the same $q^2$ interval and is defined as:

$$\Delta \zeta = \frac{G_F^2}{24\pi^3} \int_{q^2_{\text{min}}}^{q^2_{\text{max}}} |f_(q^2)|^2 p_\pi dq^2.$$

(9)

Different theoretical calculations of $f_+(q^2)$ are available in the literature \cite{1, 2, 3}, which predict different $q^2$ spectra and are considered reliable only in different limited $q^2$ ranges.

To minimize the theoretical error on $|V_{ub}|$ the range of $q^2$ therefore is chosen to correspond to the region where the form-factor normalization is considered more reliable: $q^2 < 16$ GeV$^2$/c$^4$ for LCSR and $q^2 > 16$ GeV$^2$/c$^4$ for unquenched lattice QCD (LQCD) calculations (HPQCD and FNAL in Tab. 6). The extrapolation of the form-factors to the full $q^2$ range allows for the extraction of $|V_{ub}|$ from the total branching fraction, but introduces additional uncertainties which must be taken into account. The calculation of the form-factor over the full $q^2$ range is done in Refs. \cite{1, 2, 3} using empirical functions and additional uncertainties are quoted for the extrapolation. Table 6 summarizes the values of $|V_{ub}|$ extracted from the measured partial and total branching fractions.

Instead of averaging results based on different theoretical calculations, we report the value of $|V_{ub}|$ obtained from the total branching fraction based on one of the LQCD calculations \cite{3},

$$|V_{ub}| = (3.7 \pm 0.3_{\text{stat}} \pm 0.2_{\text{syst}} \pm 0.8_{0.5FF} \times 10^{-3},$$

as a representative result, where the last error is due to the normalization of the form-factor. This result is intermediate between the results based on the other two calculations and includes the most conservative estimation of the theoretical uncertainty due to the form-factor normalization.
Table 6: Preliminary results of $|V_{ub}|$ extracted from the measured partial (first three rows) and total (last three rows) branching fractions and form-factor calculations.

| FF calculation | $q^2$ range | $\Delta \zeta$ (ps$^{-1}$) | $|V_{ub}|$ ($10^{-3}$) |
|----------------|-------------|-----------------------------|-------------------------|
| Ball-Zwicky    | $< 16$ GeV$^2/c^4$ | 5.44 ± 1.43 | 2.9 ± 0.5 stat ± 0.1 syst + $0.3_{-0.5}^{+0.8}$ FF |
| HPQCD          | $> 16$ GeV$^2/c^4$ | 1.29 ± 0.32 | 5.4 ± 0.7 stat ± 0.5 syst + $0.3_{-0.5}^{+0.8}$ FF |
| FNAL           | $> 16$ GeV$^2/c^4$ | 1.83 ± 0.50 | 4.6 ± 0.6 stat ± 0.4 syst + $0.8_{-0.5}^{+0.8}$ FF |

8 Conclusions

Using events tagged by a fully reconstructed hadronic decay of one $B$ meson, we have measured the total branching fractions for $B^0 \to \pi^- \ell^+ \nu$ and $B^+ \to \pi^0 \ell^+ \nu$ decays:

$$B(B^0 \to \pi^- \ell^+ \nu) = (1.14 \pm 0.27_{\text{stat}} \pm 0.17_{\text{syst}}) \times 10^{-4},$$
$$B(B^+ \to \pi^0 \ell^+ \nu) = (0.86 \pm 0.22_{\text{stat}} \pm 0.11_{\text{syst}}) \times 10^{-4}.$$

Combining the results assuming isospin symmetry we have extracted the total branching fraction:

$$B(B^0 \to \pi^- \ell^+ \nu) = (1.28 \pm 0.23_{\text{stat}} \pm 0.16_{\text{syst}}) \times 10^{-4},$$

and the partial branching fractions in three $q^2$ bins:

$$\Delta B(B \to \pi \ell \nu) = \begin{cases} 
(0.21 \pm 0.10_{\text{stat}} \pm 0.03_{\text{syst}}) \times 10^{-4} & q^2 < 8 \text{ GeV}^2/c^4, \\
(0.48 \pm 0.14_{\text{stat}} \pm 0.06_{\text{syst}}) \times 10^{-4} & 8 < q^2 < 16 \text{ GeV}^2/c^4, \\
(0.59 \pm 0.15_{\text{stat}} \pm 0.10_{\text{syst}}) \times 10^{-4} & q^2 > 16 \text{ GeV}^2/c^4. 
\end{cases}$$

From the measured partial branching fractions and the theoretical predictions for the form-factor normalization we obtained a preliminary determination of the CKM matrix element $|V_{ub}|$:

$$|V_{ub}| = (3.7 \pm 0.3_{\text{stat}} \pm 0.2_{\text{syst}}) \times 10^{-3},$$

where the last error is due to the form-factor normalization.

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