SERF: Towards better training of deep neural networks using log-Softplus ERror activation Function

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Abstract
Activation functions play a pivotal role in determining the training dynamics and neural network performance. The widely adopted activation function ReLU despite being simple and effective has few disadvantages including the Dying ReLU problem. In order to tackle such problems, we propose a novel activation function called Serf which is self-regularized and non-monotonic in nature. Like Mish, Serf also belongs to the Swish family of functions. Based on several experiments on computer vision (image classification and object detection) and natural language processing (machine translation, sentiment classification and multimodal entailment) tasks with different state-of-the-art architectures, it is observed that Serf vastly outperforms ReLU (baseline) and other activation functions including both Swish and Mish, with a markedly bigger margin on deeper architectures. Ablation studies further demonstrate that Serf based architectures perform better than those of Swish and Mish in varying scenarios, validating the effectiveness and compatibility of Serf with varying depth, complexity, optimizers, learning rates, batch sizes, initializers and dropout rates. Finally, we investigate the mathematical relation between Swish and Serf, thereby showing the impact of the pre-conditioner function ingrained in the first derivative of Serf which provides a regularization effect making gradients smoother and optimization faster.

Related Work
One of the mostly used activation functions is Rectified Linear Unit (ReLU) (2010). Originally proposed for Restricted Boltzmann Machines, this activation function gained prominence because of its simplicity and effectiveness and even-
tually replaced the sigmoid and tanh units. Despite being computationally efficient, it is not entirely devoid of shortcomings. In order to address these, Leaky ReLU (LReLU) was introduced which replaced the constant zero portion of the ReLU function with a linear function thereby 'leaking' some information (2013). LReLU showed superior performance compared to ReLU, and the performance was further enhanced when the slope of the negative part was learnt as an extra parameter using Parametric ReLU (PReLU) (2015). However, lower boundedness is important in order to render strong regularization effects which was absent in both LReLU and PReLU. Furthermore, similar to ReLU, they are also not differentiable.

Keeping these aspects in mind, researchers proposed activation functions like Exponential Linear Units (ELU) (2015) and Scaled Exponential Linear Units (SELU) (2017). ELU and SELU possess better convergence characteristics along with a saturation plateau in its negative region. However, these activation functions have found to be incompatible with Batch Normalization (BN) (2015).

Finally, using self-gating property Swish was proposed which addressed the aforementioned drawbacks to a greater extent abreast demonstrating superior results compared to the previous established activation functions (2017). Belonging to the same class as Swish, another activation function called Mish was proposed which performed equally well or better than Swish in most of the computer vision tasks (2019). Our proposed activation function, Serf, is also inspired from the self-gating mechanism and thus belongs to the Swish-like class of functions. It has been shown experimentally that our proposed Serf outperforms other activation functions in a variety of computer vision and natural language processing tasks.

Serf

Motivation

Activation functions introduce non-linearity in the neural networks and they play a very important role in the overall performance of a network. ReLU has been the most widely used activation function in neural networks. However, it suffers from several disadvantages, the most noticeable one being the dying ReLU phenomenon. This problem ensued from the missing negative part in the ReLU activation function which restrains the negative values to zero. At the same time, ReLU is not continuously differentiable. Furthermore, ReLU is a non-negative function. This creates a non-zero mean problem where the mean activation larger than zero. Such an issue is not desirable for network convergence (2015).

In order to address these aforesaid problems to some extent, in the recent past several new activation functions emerged including leaky ReLU, ELU, Swish, etc. Swish is seemingly an ideal candidate for an activation function with properties including non-monotonicity and ability to preserve small negative weights abreast maintaining a smooth profile. Similar to Swish, activation functions like GELU (2016) has gained popularity especially in the transformer based architectures used both in the fields of Computer Vision (ViT (2020) and MLP Mixer (2021)), as well as Natural Language Processing (GPT-2 (2019) and GPT-3 (2020)). Another activation function which rose to prominence due to its performance in state-of-the-art classification and object detection tasks, is Mish. Mish has its roots in Swish and was developed by methodical analysis over the attributes that led to the efficacy of Swish.

Taking inspiration from the development of Mish, we propose an activation function called Serf. Serf is defined as:

\[ f(x) = x \text{erf}(\ln(1 + e^x)) \]  \hspace{1cm} (1)

Properties

Serf is bounded below and unbounded above. Serf is smooth, non-monotonic and differentiable. It also preserves small portion of negative weights. Serf is inspired by Swish and Mish where the self-gating property has been used to multiply the output of a non-linear function of an input with the same non-modulated input. Self-gating is advantageous because it requires only a single-scalar input, whilst normal gating requires multiple two-scalar inputs (2017).

- **Upper unboundedness**: Activation functions like tanh and sigmoid have upper bounds. So, initialization should happen in the linear regime of these activation functions. Such a property is not desirable since it leads to saturation while training due to near-zero gradients (2010). ReLU being unbounded above attempted to avoid the saturation problem. This is a crucial attribute which can be noticed in all the successors of the ReLU function like leakyReLU, GELU, Swish, Mish, etc. Serf also possesses this feature, with its positive side as an approximate linear function of the input (see Figure 1). This makes Serf a good candidate for an activation function.

- **Lower boundedness**: Activation functions must possess lower bounds in order to provide strong regularization effects. However, in the ReLU activation function a neuron receiving negative input will always output zero eventually becoming dead or inactive and hence useless. This is referred to as the dying ReLU phenomenon (2019) (2013). It usually happens when the learning rate is high or if there is a large negative bias. By preserving a small portion of negative information, Serf mitigates the aforementioned problem furthermore resulting in better expressivity and improved gradient flow. The negative bound for Serf is approximately 0.3484 (see Figure 1).

- **Differentiability**: Unlike ReLU, Serf is continuously differentiable. This is beneficial owing to the fact that it avoids singularities and any concomitant ill-effects during gradient-based optimization.

- **Preconditioner**: Serf is closely related to Swish which can be noticed in its first derivative. The first derivative of Serf is given as:

\[ f'(x) = \frac{2}{\sqrt{\pi}} e^{-\ln((1 + e^x))^2} x \sigma(x) + \frac{f(x)}{x} \]

\[ = p(x) \text{swish}(x) + \frac{f(x)}{x} \]  \hspace{1cm} (2)
Here, $\sigma$ is the sigmoid function and $p(x)$ is a preconditioner function. Preconditioners make the gradients smoother and have been previously used extensively in optimization problems. The inverse of a symmetric positive definite matrix has been used as a preconditioner in case of gradient descent. Application of such preconditioners makes the objective function smoother thereby increasing the rate of convergence \cite{1986}. Therefore, the strong regularization effect contributed by such preconditioner in case of Serf makes the gradients smoother and optimization faster, thereby outperforming Swish as can be noticed in the experiments.

Mish also has a preconditioner which makes it perform better than Swish. The difference between Mish and Swish is that in Serf we used the error function ($\text{erf}$) whereas in Mish $\tanh$ function is used. Serf, however, outperforms Mish in most experiments (see Experiments and Ablations). We speculate that Serf’s preconditioner function renders better regularization effects than that of Mish.

- **Smoothness**: Smooth loss landscapes indicates easier optimization with less local optima and hence better generalization minimizing influence of initializations and learning rates. The output landscapes of a randomly chosen 6-layered neural network with ReLU and Serf activation functions has been shown in Figure 2. It is to be noted that output landscape is indicative of the loss landscape. We randomly initialize a 6-layered neural network, where we pass the $x$ and $y$ coordinates of each point in a grid as input, and plot the scalar network output for each grid point. For ReLU activation function, the output landscape of the neural network has sharp transitions in contrast to that of Serf. This conforms to the enhanced performance of Serf as compared to ReLU.

### Experiments

In this section we will demonstrate the performance of our proposed activation function, Serf when used in different state-of-the-art architectures on image, sequence and graph datasets for disparate tasks. We will also present results from ablation studies done on MNIST and CIFAR-10 datasets. Overall, our proposed activation function, Serf outperformed its contemporaries in most of the tasks.

### Ablations

Model hyper-parameters play an important role in the training and optimization processes of a neural network thus having direct consequences in the generalizability of a network. Such hyper-parameters include network depths, network widths, type of weight initializations, dropout rates, batch sizes, learning rates, and optimizers. Here we analyze and compare the impacts of different hyper-parameters on our chosen networks with three different activation functions namely Swish, Mish, and Serf. We have used MNIST and CIFAR-10 datasets for this purpose.

### MNIST

- **Dense Units**: The number of dense units refers to the number of neurons present in a dense layer. In this case we have used a 4 layered architecture with one dense layer followed by a Batch Normalization Layer and SGD \cite{1951} as an optimizer. We observe that as the number of dense units increases, the model complexity increases and Serf outperforms Swish and Mish (Fig 3). This suggests that Serf works well with complex models. This has also been noticed in other experiments.
**Figure 3:** Ablations for MNIST dataset. Top: Testing Accuracies vs Dense Units (Left), Dropout Rates (Middle) and Initializers (Right) for Swish, Mish and Serf. Bottom: Testing Accuracies vs Learning Rates (Left), Optimizers (Middle) and Number of Layers (Right) for Swish, Mish and Serf.

- **Dropout Rates:** As the dropout rate increases, the overall performance for all three activation functions drop, however, the performance degradation for Serf is relatively lesser as compared to Swish and Mish (Fig 3).

- **Initializers:** The performance of Serf is better than both Swish and Mish in all except for random uniform initialization (Fig 3). This suggests that Serf is a better candidate compared to its contemporaries.

- **Learning Rates:** With varying learning rates, Serf performs better than both Swish and Mish (Fig 3). Particularly, with higher learning rates, the degradation is quite pronounced in Swish and not that much in Mish and Serf. We have used SGD [1951] as an optimizer in this case.

- **Optimizers:** In this case, with varying optimizers, the overall performance of Serf is equal or marginally better than Swish and Mish (Fig 3). Performance drop can be noticed for all three activation functions in case of Adam optimizer [2014].

- **Number of layers:** In this case, each dense layer was followed by a Batch Normalization layer. As the number of dense layers increases, i.e., the depth of the network increases, models become complex and optimization becomes difficult. The degradation in the performances for all the three different activation functions conforms the aforementioned fact. However, Serf maintained a significantly higher accuracy as compared to Swish and Mish (Fig 3). This makes Serf a suitable candidate for large and complex networks.

**CIFAR-10** We have used a ResNet-18 model with a dense layer and a classification head in tandem. The results are obtained with training the model without pre-trained weights over multiple runs for 20 epochs, which gives a decent convergence point.

- **Batch Size:** We observed that with decreasing training batch sizes, the performance for all the competing activation functions drop (Fig 4), however, Serf holds the better position out of all three over all the batch sizes. Adam [2014] is used as the optimizer here.

- **Optimizers:** In this case, with varying optimizers, the overall performance of Serf is equal or marginally better than Swish and Mish (Fig 4). Performance drop can be noticed for all three activation functions in case of SGD optimizer [1951].

- **Learning Rates:** Serf is observed to be performing better or equal to both Mish and Swish on all of the learning rates evaluated barring 0.1 where a steeper drop is observed in Serf compared to the other two activation functions (Fig 4). Adam [2014] is used as the optimizer here.
Image Classification

For image classification we have considered different standard architectures applied on two datasets namely CIFAR-10 and CIFAR-100.

CIFAR-10/100: We have considered different deep learning architectures (for CIFAR-10: SqueezeNet (2016), Resnet-50 (2016a), WideResnet-50-2 (2016), ShuffleNet-v2 (2018), ResNeXt-50 (2017), Inception-v3 (2016), DenseNet-121 (2017), MobileNet-v2 (2018) and EfficientNet-B0 (2019); for CIFAR-100: Resnet-164 (2016b), WideResnet-28-10 (2016), DenseNet-40-12 (2017), Inception-v3 (2016)) with three disparate activation functions, namely, ReLU (baseline), Mish and Serf (proposed). This has been done for image classification task on CIFAR-10 and CIFAR-100 datasets where for each network we have only changed the activation functions and have kept every other parameter constant for fair comparisons. Tables 1 and 2 show that Serf consistently outperformed both ReLU and Mish activation functions across all the architectures used in the experiment (1% to 2% improvement over baseline ReLU) for both CIFAR-10 and CIFAR-100 datasets, thereby suggesting Serf to be the best candidate activation function for CIFAR-10/100 image classification tasks.

We have also used two recent architectures namely MLP Mixer (2021) and Compact Convolutional Transformers (CCT) (2021). We have trained the MLP Mixer for 10 epochs only on CIFAR-10 training dataset, separately with two different activation functions (GELU and Serf) and tested the models on the CIFAR-10 test dataset. The Top-1 % and Top-5 % Accuracy values (see Table 5) suggest that Serf’s performance is comparable to GELU’s (baseline) performance for MLP Mixer. We have also trained CCT for 50 epochs on CIFAR-10 training dataset, separately with three different activation functions (ReLU, Mish and Serf) and tested the models on CIFAR-10 test dataset. Serf clearly outperforms ReLU (baseline) and Mish in this case (see Table 3). The results indicate that Serf is a better activation function for transformer based architectures.

| Methods       | ReLU | Mish | Serf (Ours) |
|---------------|------|------|-------------|
| SqueezeNet    | 84.14| 85.98| 86.32       |
| Resnet-50     | 86.54| 87.03| 88.07       |
| WideResnet-50-2| 86.39| 86.57| 86.73       |
| ShuffleNet-v2 | 83.93| 84.07| 84.55       |
| ResNeXt-50 (32×4d) | 87.25| 87.97| 88.49       |
| Inception-v3  | 90.93| 91.55| 92.89       |
| DenseNet-121  | 88.59| 89.05| 89.07       |
| MobileNet-v2  | 85.74| 86.39| 86.61       |
| EfficientNet-B0 (Swish) | 78.26| 78.02| 78.41       |

Table 1: Top-1 % Accuracy values of different state-of-the-art methods for different activation functions on CIFAR-10

Object Detection

Object detection is considered one of the important visual scene understanding tasks. In our case, we have considered Pascal VOC dataset for object detection task using YOLOv3 (2018) and tiny YOLOv3 frameworks. Leaky ReLU is intrinsic to the YOLOv3 framework. For fair comparisons we have only changed the activation function keeping other hyperparameters fixed as outlined in (2018). Mean Average Precision (MAP) scores in Table 4 clearly indicate that our proposed Serf outperforms the baseline leaky ReLU based architectures in the object detection task for Pascal VOC dataset.

Semi-supervised Node Classification

Following the implementations outline in (2016), we have considered 3 different datasets, namely CITIESEER, CORA and PUBMED for semi-supervised node classification using three disparate activation functions, namely, ReLU (baseline), Mish and Serf (proposed). All training parameters and hyper-parameters have been kept same as mentioned in (2016) for fair comparisons. Table 5 shows that Serf either performed equally well or better than both ReLU
| Methods               | ReLU | Mish | Serf (Ours) |
|-----------------------|------|------|-------------|
| ResNet-164            | 74.55| 75.02| 75.13       |
| WideResNet-28-10      | 76.32| 77.03| 77.54       |
| DenseNet-40-12        | 73.68| 73.91| 74.16       |
| Inception-v3          | 71.54| 72.38| 72.95       |

Table 2: Top-1 % Accuracy values of different state-of-the-art methods for different activation functions on CIFAR-100

| Activations         | Top-1 % Acc | Top-5 % Acc |
|---------------------|-------------|-------------|
| MLP-Mixer (GELU)    | 64.14       | 96.71       |
| MLP-Mixer (Serf)    | **64.36**   | 96.59       |
| CCT (ReLU)          | 79.05       | 97.72       |
| CCT (Mish)          | 80.02       | **98.70**   |
| CCT (Serf)          | **80.23**   | 98.65       |

Table 3: Top-1 and Top-5 % Accuracy values (classification) after 10 epochs of training MLP-Mixer for GELU (SOTA) and Serf activation functions on CIFAR-10 test dataset and 50 epochs of training Compact Convolutional Transformer (CCT) for ReLU, Mish and Serf functions on CIFAR-10 test dataset.

and Mish activation functions across the three different datasets, thereby indicating versatility of the proposed activation function.

**Machine Translation, Sentiment Classification & Multi-modal Entailment**

In this section, we have demonstrated the effectiveness of our proposed Serf activation function in Machine Translation and Sentiment Classification tasks. We have considered 3 different architectures and datasets.

For Machine Translation we have used a sequence-to-sequence Transformer (2017) Encoder-Decoder based model trained (for 20 epochs) on the the Multi30k dataset for German-English translation (2016). For comparison purposes, we have considered ReLU, GELU, Mish and Serf (proposed) and observed that Serf outperformed the remaining three activation functions as suggested by the BLEU scores shown in the Table 6.

For sentiment classification, we have considered two datasets, namely imdb movie review sentiment and Pol Emo 2.0 sentiment datasets. For the imdb movie review sentiment dataset (2011), we have considered: (i) a simple architecture consisting of a 1D conv net with a text embedding layer which we have trained using three different activation functions and noticed that Serf outperformed the other two activation functions (ReLU and Mish) suggesting that Serf also works well with simple architectures (see Table 7), and (ii) a 4-layer Transformer model which we have also trained for 20 epochs for each of the three activation functions respectively obtaining the best results for the proposed Serf function (see Table 7). For the Pol Emo 2.0 sentiment database (2019), we have trained a BERT based model (2018) for two different activation functions, Mish and Serf. The Precision, Recall and F1-scores suggest that Serf performed equally well or better than Mish for this task (see Table 8).

For multi-modal entailment task, we have used the multi-modal entailment database, recently introduced by the Google Research. We have used a smaller variant of the original BERT model. The code used for this purpose is available at. We have used two activation functions for comparison purposes: GELU and Serf. Table 9 shows the accuracies on test dataset averaged over 5 runs (each trained for 10 epochs). The accuracy values suggest that Serf performs marginally better than GELU in this case.

**Conclusions and Future Works**

In this paper, we have proposed a novel activation function called Serf. Serf has properties such as upper unboundedness, lower boundedness, non-monotonicity and smoothness which are the desired properties for an activation function. Serf has shown to be effective against different ablations compared to its contemporaries like Swish and Mish. Experimental results with different state-of-the-art architectures on varied datasets for disparate tasks including image classification, object detection, sentiment classification, machine translation and multi-modal entailment demonstrate that the proposed Serf outperforms the baseline ReLU performance universally.

1https://github.com/google-research-datasets/recognizing-multimodal-entailment
2https://github.com/sayakpaul/Multimodal-Entailment-Baseline
Table 6: BLEU scores of seq2seq Transformer model (after training for 20 epochs) for different activation functions on Multi30k test dataset.

| Scores   | ReLU  | GELU  | Mish  | Serf (Ours) |
|----------|-------|-------|-------|-------------|
| BLEU     | 35.55 | 35.62 | 35.36 | **36.06**   |

Table 7: Top-1 % Accuracy values of 1D conv net with a text embedding layer and 4-layer Transformer model for ReLU, Mish and Serf on imdb movie review sentiment dataset.

| Activation Function | Precision | Recall | F1-score |
|---------------------|-----------|--------|----------|
| Mish                | 0.8374    | 0.8329 | **0.8346** |
| Serf (Ours)         | **0.8377**| **0.8330** | 0.8342     |

Table 8: Precision, Recall and F1-scores for different activation functions for sentiment classification using BERT on Pol Emo 2.0 sentiment database.

| Metric             | GELU | Serf (Ours) |
|--------------------|------|-------------|
| Mean Accuracy      | 85.28| **85.42**   |

and as well as other activation functions like Swish, Mish and GELU. The results can be improved with desired hyper-parameters for Serf which can be obtained with a hyperparameter search.

Future works include: (1) the understanding of the importance and contribution of pre-conditioner as a regularizer and how modifying it can have an impact on the final results; this can lead to the development of more effective activation functions, (2) the development of Hard-Serf like Hard-Swish and Hard-Mish and compare its performance on different benchmark datasets and tasks, (3) the development and exploration of probabilistic version of Serf as shown in [2019], (4) the development of parameterized Serf like PReLU, and finally (5) comparison the performance of Serf with other contemporary activation functions for tasks such as image super-resolution, image reconstruction, etc. Overall, Serf is a simple, effective and versatile activation function which can be incorporated in any neural network for better training and performance gains.

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