GENERALIZED FRACTIONAL OPERATOR REPRESENTATIONS OF JACOBI TYPE ORTHOGONAL POLYNOMIALS.

K.S. NISAR

Abstract. The aim of this paper is to apply generalized operators of fractional integration and differentiation involving Appell’s function $F_3(z)$ due to Marichev-Saigo-Maeda (MSM), to the Jacobi type orthogonal polynomials. The results are expressed in terms of generalized hypergeometric function. Some of the interesting special cases of the main results also established.

1. Introduction

The familiar generalized hypergeometric function $pF_q$ is defined as follows (see, [2, 16]):

$$pF_q\left(\begin{array}{c} (a_p); \\ (c_q); \end{array} x \right) = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p} (a_j)_n}{\prod_{j=1}^{q} (c_j)_n} \frac{x^n}{n!},$$

$(p \leq q, x \in \mathbb{C}; p = q + 1, |x| < 1)$,

where $(a)_n$ denoted by Pochhammer symbol given by

$$(a)_n = a(a+1)_{n-1}; (a)_0 = 1.$$

In particular, if $p = 2$ and $q = 1$, (1.1) reduced to Gaussian hypergeometric function,

$$2F_1(a, b; c; x) = \sum_{k=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n} \frac{x^n}{n!},$$

(1.2)

The polynomial $M^{(p,q)}_n(x)$ is the solution of the differential equation

$$x(x+1)y''_n(x) + (2-p)x + (1+q)y'_n(x) - n(n-1+p)y_n(x) = 0,$$

(1.3)

is defined by

$$M^{(p,q)}_n(x) = (-1)^n n! \sum_{k=0}^{n} \frac{p - n - 1}{k} \frac{q + n}{n - k} (-x)^k.$$

(1.4)

These polynomials are orthogonal on $[0, \infty]$ with respect to the weight function $w_{p,q}(x) = x^q (1 + x)^{-(p+q)}$ if and only if $p > 2n + 1$ and $q > -1$.

Key words and phrases. MSM fractional integral and differential operators, Generalized hypergeometric function, Jacobi type polynomials.

AMS 2010 Subject Classification: 26A33, 33C05, 33C20, 33E20.
The relation between $M^{(\rho,\varsigma)}_n(x)$ and $M^{(\delta,\nu,\mu,\varsigma)}_n(x)$ can be expressed as:

$$M^{(\rho,\varsigma)}_n(x) = (-1)^n n! \binom{q+n}{n} \, _2F_1(-n, n+1-p; q+1; -x).$$  \hspace{1cm} (1.5)

The relation between Jacobi polynomial $P^{(\alpha,\beta)}_n(x)$ and $M^{(\delta,\nu,\mu,\varsigma)}_n(x)$ is given by

$$M^{(\alpha,\beta)}_n(x) = (-1)^n n! P^{(\alpha,-\nu-q)}_n(2x+1),$$  \hspace{1cm} (1.6)

which can be expressed as

$$P^{(\alpha,-\nu-q)}_n(x) = \frac{(-1)^n}{n!} M^{(-\nu-q,\nu)}_n \left( \frac{x-1}{2} \right).$$  \hspace{1cm} (1.7)

For more details about Jacobi polynomials and related results, one can refer \cite{8, 9, 10, 11}.

Then the generalized fractional integral operators involving the Appell functions $F_3$ are defined for $\delta, \delta', \mu, \mu', \epsilon \in \mathbb{C}$ with $\Re(\epsilon) > 0$ and $x \in \mathbb{R}^+$ as follows:

$$\left( I^{\delta,\delta',\mu,\mu',\epsilon}_0 f \right)(x) = \frac{x^{-\delta}}{\Gamma(\epsilon)} \int_0^x (x-t)^{-1} t^{-\delta} F_3 \left( \delta, \delta', \mu, \mu'; \epsilon; 1 - \frac{t}{x}, 1 - \frac{x}{t} \right) f(t) \, dt,$$  \hspace{1cm} (1.8)

and

$$\left( I^{\delta,\delta',\mu,\mu',\epsilon}_\infty f \right)(x) = \frac{x^{-\delta}}{\Gamma(\epsilon)} \int_x^\infty (t-x)^{-1} t^{-\delta} F_3 \left( \delta, \delta', \mu, \mu'; \epsilon; 1 - \frac{t}{x}, 1 - \frac{x}{t} \right) f(t) \, dt.$$  \hspace{1cm} (1.9)

The integral operators of the types (1.8) and (1.9) have been introduced by Marichev \cite{12} and later extended and studied by Sagio and Maeda \cite{17}. Recently, many researchers (see, \cite{17, 14, 15}) have studied the image formulas for MSM fractional integral operators involving various special functions.

The corresponding fractional differential operators have their respective forms:

$$\left( D^{\delta,\delta',\mu,\mu',\epsilon}_0 f \right)(x) = \frac{d}{dx} \left( \frac{[\Re(\epsilon)]+1}{\Gamma(\epsilon)} \left[ I^{\delta,\delta',\mu,\mu',\epsilon}_0 \right] \left( \frac{d}{dx} \right)^{[\Re(\epsilon)]+1} \right) f(x),$$  \hspace{1cm} (1.10)

and

$$\left( D^{\delta,\delta',\mu,\mu',\epsilon}_\infty f \right)(x) = \frac{d}{dx} \left( \frac{[\Re(\epsilon)]+1}{\Gamma(\epsilon)} \left[ I^{\delta,\delta',\mu,\mu',\epsilon}_\infty \right] \left( \frac{d}{dx} \right)^{[\Re(\epsilon)]+1} \right) f(x).$$  \hspace{1cm} (1.11)

Here, we recall the following results (see \cite{17}):

**Lemma 1.1.** Let $\delta, \delta', \mu, \mu', \epsilon, \tau \in \mathbb{C}$ be such that $\Re(\epsilon) > 0$ and

$$\Re(\tau) > \max\{0, \Re(\delta - \delta' - \mu - \epsilon), \Re(\delta' - \mu')\}.$$

then there exists the relation

$$\left( I^{\delta,\delta',\mu,\mu',\epsilon}_0 \right)(x) = \frac{\Gamma(\tau + \epsilon - \delta - \delta' - \mu) \Gamma(\tau + \mu' - \delta') \Gamma(\tau + \epsilon - \delta' - \mu)}{\Gamma(\tau + \mu) \Gamma(\tau + \epsilon - \delta - \mu) \Gamma(\tau + \epsilon - \delta' - \mu)} x^{\tau - \delta - \delta' - \epsilon - 1}$$  \hspace{1cm} (1.12)
**Lemma 1.2.** Let $\delta, \delta', \mu, \mu', \epsilon, \tau \in \mathbb{C}$ such that $\Re(\epsilon) > 0$ and 
\[ \Re(\tau) > \max\{\Re(\mu), \Re(-\delta - \delta' + \epsilon), \Re(-\delta - \mu' + \epsilon)\}, \]
then
\[ \left( I_{0+}^{\delta, \delta', \mu, \mu', \epsilon} t^{-\tau} \right) (x) = \frac{\Gamma (-\mu + \tau) \Gamma (\delta + \delta' - \epsilon + \tau) \Gamma (\delta + \mu' - \epsilon + \tau)}{\Gamma (\tau) \Gamma (\delta - \mu + \tau) \Gamma (\delta + \delta' + \mu - \epsilon + \tau)} x^{-\delta - \delta' + \epsilon - \tau}. \]

Also, we need the following lemmas [3]:

**Lemma 1.3.** Let $\delta, \mu, \epsilon \in \mathbb{C}$ be such that $\Re(\delta) > 0, \Re(\tau) > \max[0, \Re(\mu - \epsilon)]$ then
\[ \left( I_{0+}^{\delta, \mu, \epsilon} t^{-\tau} \right) (x) = \frac{\Gamma (\tau) \Gamma (\tau + \epsilon - \mu)}{\Gamma (\tau - \mu) \Gamma (\tau + \epsilon + \delta)} x^{\tau - \mu - 1}. \] (1.13)

In particular,
\[ \left( I_{0+}^{\delta} t^{-\tau} \right) (x) = \frac{\Gamma (\tau)}{\Gamma (\tau + \delta)} x^{\tau + \delta - 1}, \Re(\delta) > 0, \Re(\tau) > 0, \] (1.14)
\[ \left( I_{t, \epsilon}^{\delta} t^{-\tau} \right) (x) = \frac{\Gamma (\tau + \epsilon)}{\Gamma (\tau + \delta + \epsilon)} x^{\tau + \delta - 1}, \Re(\delta) > 0, \Re(\tau) > \Re(\epsilon). \] (1.15)

**Lemma 1.4.** Let $\delta, \mu, \epsilon \in \mathbb{C}$ be such that $\Re(\delta) > 0, \Re(\tau) < 1 + \min[\Re(\mu), \Re(\epsilon)]$ then
\[ \left( I_{1-}^{\delta, \mu, \epsilon} t^{-\tau} \right) (x) = \frac{\Gamma (\mu - \tau + 1) \Gamma (\epsilon - \tau + 1)}{\Gamma (1 - \tau) \Gamma (\delta + \mu + \epsilon - \tau + 1)} x^{\tau - \mu - 1}. \] (1.16)

In particular,
\[ \left( I_{1-}^{\delta} t^{-\tau} \right) (x) = \frac{\Gamma (1 - \delta - \tau)}{\Gamma (1 - \tau)} x^{\tau + \delta - 1}, 0 < \Re(\delta) < 1 - \Re(\tau), \] (1.17)
\[ \left( K_{c, \epsilon}^{\tau} t^{-\tau} \right) (x) = \frac{\Gamma (\epsilon - \tau + 1)}{\Gamma (\delta + \tau + 1)} x^{\tau - 1}, \Re(\tau) < 1 + \Re(\epsilon). \] (1.18)

The main aim of this paper is to apply the generalized operators of fractional calculus for the Jacobi type orthogonal polynomials in order to get certain new image formulas. The basic definitions and results of fractional calculus, one may refer to [5, 6, 13, 18].

2. Fractional Integrals of Jacobi Type Orthogonal Polynomials

In this section, we derive the following theorems,

**Theorem 1.** Let $\delta, \delta', \mu, \mu', \epsilon, \tau \in \mathbb{C}$ be such that $\Re(\epsilon) > 0$ and
\[ \Re(\tau) > \max\{0, \Re(\delta - \delta' - \mu - \epsilon), \Re(\delta' - \mu')\}. \]
then
\[ \left( I_{0+}^{\delta, \delta', \mu, \mu', \epsilon} M_{n}^{\mu, \nu} (t) \right) (x) \]
On interchanging the integration and summation, we obtain

\[
\times 5F_4 \left[ \sum_{k=0}^{\infty} \binom{n-p-1}{k} \binom{q+n-1}{n-k} (-t)^k \right].
\]

Proof. Applying the definition of \(M_n^{(p,q)}(t)\) given in (1.4) and denote the left hand side by \(I_1\), we get

\[
I_1 = \left( \sum_{k=0}^{\infty} \binom{n-p-1}{k} \binom{q+n-1}{n-k} (-t)^k \right) (x),
\]

On interchanging the integration and summation, we obtain

\[
I_1 = (-1)^n n! \sum_{k=0}^{\infty} \frac{\Gamma(p-n)}{k! \Gamma(p-n-k)} \frac{\Gamma(q+n+1)(-1)^k}{\Gamma(q+n+1)(q+n+1)} \left( \sum_{k=0}^{\infty} \binom{n-p-1}{k} \binom{q+n-1}{n-k} (-t)^k \right) (x),
\]

Now, for any \(k = 0, 1, 2, \ldots\) Since

\[
\Re(\tau + k) \geq \Re(\tau) > \max \left[ 0, \Re(\delta - \delta' - \mu - \epsilon), \Re(\delta' - \mu') \right],
\]

and applying Lemma [1.1] and after some calculations, we get

\[
I_1 = \sum_{k=0}^{\infty} (-1)^n \frac{\Gamma(q+n+1)(n-p)k(-n)k(-1)^k}{k! \Gamma(q+1+k)}
\]

\[
\times \frac{\Gamma(\tau + k) \Gamma(\tau + k + \delta - \delta' - \mu) \Gamma(\tau + k + \mu' - \delta')}{\Gamma(\tau + k + \mu') \Gamma(\tau + k + \epsilon - \delta - \delta') \Gamma(\tau + k + \epsilon - \delta' - \mu)}
\]

\[
\times x^{\tau + k - \delta - \delta' + \epsilon - 1}.
\]

In view of (1.1) and using the well known relation

\[
\Gamma(x+k) = (x)_k \Gamma(x),
\]

we arrive at desired result.

If we set \(\delta', \mu' = 0, \delta = \delta + \mu, \mu = -\epsilon\) and \(\epsilon = \delta\) in theorem [1] then we get the following result,

**Corollary 2.1.** Let \(\delta, \mu, \epsilon \in \mathbb{C}\) such that \(\Re(\delta) > 0, \Re(\tau) > \max[0, \Re(\mu - \epsilon)]\), then

\[
\left( \sum_{k=0}^{\infty} \binom{n-p-1}{k} \binom{q+n-1}{n-k} (-t)^k \right) (x) = (-1)^n \frac{\Gamma(q+n+1)(n-p)k(-n)k(-1)^k}{k! \Gamma(q+1+k)}
\]

\[
\times 4F_3 \left[ \sum_{k=0}^{\infty} \binom{n-p-1}{k} \binom{q+n-1}{n-k} (-t)^k \right] (x).
\]
which is theorem 2.1 of [8].

Substituting $\mu = -\delta$ in corollary [2.1] and using (1.4), we get,

**Corollary 2.2.** Let $\delta, \tau \in \mathbb{C}$ such that $\Re(\delta) > 0, \Re(\tau) > 0$, then

\[
\left( I_{0^+}^t t^{-1} M_n^{(p, q)} (t) \right) (x) = \frac{(-1)^n \Gamma(1 + q + n) \Gamma(\tau)}{\Gamma(1 + q) \Gamma(\tau + \delta)} x^{\tau + \delta - 1} \times _3 F_2 \left[ \begin{array}{c} 1 + n - p, -n, \tau, \\ 1 + q, \tau + \delta, \end{array} \right] -x
\]

Substituting $\mu = 0$ in corollary [2.1] we have,

**Corollary 2.3.** Let $\delta, \epsilon, \tau \in \mathbb{C}$ such that $\Re(\delta) > 0, \Re(\tau) > \Re(\epsilon)$, then

\[
\left( I_{\epsilon, \delta}^t t^{-1} M_n^{(p, q)} (t) \right) (x) = \frac{(-1)^n \Gamma(1 + q + n) \Gamma(\tau + \epsilon)}{\Gamma(1 + q) \Gamma(\tau + \delta + \epsilon)} x^{\tau - 1} \times _3 F_2 \left[ \begin{array}{c} 1 + n - p, -n, \tau + \epsilon, \\ 1 + q, \tau + \delta + \epsilon, \end{array} \right] -x
\]

**Theorem 2.** Let $\delta, \delta', \mu, \mu', \epsilon, \tau \in \mathbb{C}$, and

\[
\Re(\epsilon) > 0, \Re(\tau) > \max\{\Re(\mu), \Re(-\delta - \delta' + \epsilon'), \Re(-\delta - \mu' + \epsilon)\},
\]

then

\[
\left( I_{\delta', \mu', \epsilon, \tau}^t t^{-1} M_n^{(p, q)} \left( \frac{1}{t} \right) \right) (x) = x^{\delta - \delta' + \epsilon - \tau} \frac{(-1)^n \Gamma(q + n + 1) \Gamma(\tau - \mu) \Gamma(\delta + \delta' - \epsilon + \tau) \Gamma(\delta + \mu' - \epsilon + \tau)}{\Gamma(\tau) \Gamma(q + 1) \Gamma(\delta - \mu + \tau) \Gamma(\delta + \delta' + \mu' - \epsilon + \tau)} \times _3 F_4 \left[ \begin{array}{c} 1 + n - p, -n, -\mu + \tau, \delta + \delta' - \epsilon + \tau, \delta + \mu' - \epsilon + \tau, \\ 1 + q, \tau, \delta - \mu + \tau, \delta + \delta' + \mu' - \epsilon + \tau, \end{array} \right] -\frac{1}{x}
\]

**Proof.** Applying the definition of $M_n^{(p, q)} (x)$ in (1.4) and denote the left hand side by $I_2$, we get

\[
I_2 = \left( I_{\delta', \mu', \epsilon, \tau}^t t^{-1} M_n^{(p, q)} \left( \frac{1}{t} \right) \right) (x)
\]

\[
= \left( I_{\delta', \mu', \epsilon, \tau}^t t^{-1} (-1)^n n! \sum_{k=0}^{\infty} \left( \begin{array}{c} p - n - 1 \\ k \end{array} \right) \left( \begin{array}{c} q + n \\ n - k \end{array} \right) \left( \frac{-1}{t} \right)^k \right) (x),
\]

On interchanging the integration and summation, we obtain

\[
I_2 = (-1)^n n! \sum_{k=0}^{\infty} \frac{\Gamma(p - n)}{k! \Gamma(p - n - k)} \frac{\Gamma(q + n + 1) (-1)^k}{\Gamma(n - k + 1) \Gamma(q + k + 1)} \left( I_{\delta', \mu', \epsilon}^t t^{-k} \right) (x)
\]
Now, for any \( k = 0, 1, 2, \ldots \), since

\[
\Re(\tau + k) > \max \left[ \Re(\mu), \Re\left( -\delta - \delta' - \epsilon \right), \Re\left( -\delta - \mu + \epsilon \right) \right],
\]

and applying Lemma 1.2 and some simple calculations, we get

\[
I_2 = \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(q + n + 1)(n - p)k(-n)_k(-1)^k}{k!(q + 1 + k)} \frac{\Gamma(\tau + k)\Gamma(\tau + \delta + \delta' - \epsilon + k)}{\Gamma(\tau + k)\Gamma(\tau + \delta - \mu)\Gamma(\delta + \delta' + \mu' - \epsilon + \tau + k)} \times x^{-\tau - k - \delta - \delta' + \epsilon}.
\]

In view of (1.1) and (2.1), we reach the required result. \( \square \)

If we set \( \delta', \mu' = 0, \delta = \delta + \mu, \mu = -\epsilon \) and \( \epsilon = \delta \) in theorem 2.2 then we get the following result,

**Corollary 2.4.** Let \( \delta, \mu, \epsilon \in \mathbb{C} \) such that \( \Re(\delta) > 0, \Re(\tau) > 1 + \min[\Re(\mu), \Re(\epsilon)] \), then

\[
\left( I_{\tau}^{\delta, \mu} \right)^{-1} M_n^{(p, q)} \left( \frac{1}{t} \right) (x) = \frac{\Gamma(1 + q + n)\Gamma(\mu - \tau + 1)\Gamma(\epsilon - \tau + 1)}{\Gamma(1 + q)\Gamma(1 - \tau)\Gamma(\delta + \mu + \epsilon - \tau + 1)} x^{\tau - \mu - 1} \times_4 F_3 \left[ \begin{array}{c} 1 + n - p, -n, \mu - \tau + 1, \epsilon - \tau + 1, \\ 1 + q, 1 - \tau, \delta + \mu + \epsilon - \tau + 1, \end{array} \middle| -\frac{1}{x} \right]
\]

which is theorem 2.2 of \([5]\).

Substituting \( \mu = -\delta \) in corollary 2.4 and using (1.4), we get,

**Corollary 2.5.** Let \( \delta, \epsilon, \tau \in \mathbb{C} \) such that \( \Re(\delta) > 0, \Re(\tau) < 1 + \min[\Re(-\delta), \Re(\epsilon)] \) and \( \tau + \delta \neq 1, 2, \ldots \), then

\[
\left( I_{\tau}^{\delta} \right)^{-1} M_n^{(p, q)} \left( \frac{1}{t} \right) (x) = \frac{(-1)^n \Gamma(1 + q + n)\Gamma(-\delta + 1)}{\Gamma(1 + q)\Gamma(1 - \tau)} x^{\tau + \delta - 1} \times_3 F_2 \left[ \begin{array}{c} 1 + n - p, -n, -\delta - \tau + 1, \\ 1 + q, 1 - \tau, \end{array} \middle| -\frac{1}{x} \right]
\]

Substituting \( \mu = 0 \) in corollary 2.4 we have,

**Corollary 2.6.** Let \( \delta, \epsilon, \tau \in \mathbb{C} \) such that \( \Re(\delta) > 0, \Re(\tau) < 1 + \min[0, \Re(\epsilon)] \) and let \( \tau - \epsilon \neq 1, 2, \ldots \), then

\[
\left( K_{\epsilon, \delta}^{\tau} \right)^{-1} M_n^{(p, q)} \left( \frac{1}{t} \right) (x) = \frac{(-1)^n \Gamma(1 + q + n)\Gamma(\epsilon - \tau + 1)}{\Gamma(1 + q)\Gamma(\delta + \epsilon - \tau + 1)} x^{\tau - 1} \times_3 F_2 \left[ \begin{array}{c} 1 + n - p, -n, \epsilon - \tau + 1, \\ 1 + q, \delta + \epsilon - \tau + 1, \end{array} \middle| -\frac{1}{x} \right]
\]
3. Fractional differentials of Jacobi type polynomials

In this section devoted to derive the MSM fractional differentiation of \([14]\). We recall the following lemmas (see \([3]\)).

**Lemma 3.1.** Let \(\delta, \delta', \mu, \mu', \epsilon, \tau \in \mathbb{C}\) such that
\[ \Re(\tau) > \max\{0, \Re(-\delta + \mu), \Re(-\delta - \delta' - \mu + \epsilon)\}. \]
Then
\[ \left(D_{0+}^{\delta, \delta', \mu, \mu', \epsilon, t, \tau} x \right) = \frac{\Gamma(\tau) \Gamma(\delta + \delta' + \mu' - \epsilon + \tau) \Gamma(\delta + \delta' + \mu - \epsilon + \tau)}{\Gamma(-\mu + \tau) \Gamma(\delta + \delta' - \epsilon + \tau) \Gamma(\delta + \mu' - \epsilon + \tau)} x^{\delta + \delta' - \epsilon + \tau - 1}. \] (3.1)

**Lemma 3.2.** Let \(\delta, \delta', \mu, \mu', \epsilon, \tau \in \mathbb{C}\) such that
\[ \Re(\tau) > \max\{0, \Re(-\mu'), \Re(\delta' + \mu - \epsilon), \Re(\delta + \delta' - \epsilon) + \Re(\epsilon) + 1\}. \]
Then the following formula holds true:
\[ \left(D_{0+}^{\delta, \delta', \mu, \mu', \epsilon, t, \tau} x \right) = \frac{\Gamma(\tau) \Gamma(-\delta' + \mu + \tau) \Gamma(-\delta - \mu + \tau)}{\Gamma(-\delta' - \mu + \epsilon + \tau)} x^{\delta + \delta' - \epsilon - \tau}. \] (3.2)

**Theorem 3.** Let \(\delta, \delta', \mu, \mu', \epsilon, \tau \in \mathbb{C}\) such that
\[ \Re(\tau + k) > \max\{0, \Re(-\delta + \mu), \Re(-\delta - \delta' - \mu + \epsilon)\}, \text{ then the following formula hold true:} \]
\[ \left(D_{0+}^{\delta, \delta', \mu, \mu', \epsilon, t, \tau} M_{n, q}(t) \right) (x) = \frac{(-1)^n \Gamma(q + n + 1) \Gamma(\tau + \delta - \mu) \Gamma(\tau + \delta + \delta' + \mu' - \epsilon + \tau)}{\Gamma(q + 1) \Gamma(\tau - \mu) \Gamma(\tau + \delta + \delta' - \epsilon) \Gamma(\tau + \delta + \mu' - \epsilon)} x^{\delta + \delta' - \epsilon + \tau - 1} \times F_4 \left[ \begin{array}{c} 1 + n - p, -n, \tau, -\mu + \delta + \tau, \delta + \delta' + \mu' - \epsilon + \tau, q + 1, -\mu' + \tau, \delta + \delta' - \epsilon + \tau, \delta + \mu' - \epsilon + \tau, \end{array} \right. - x \right], \]

**Theorem 4.** Let \(\delta, \delta', \mu, \mu', \epsilon, \tau \in \mathbb{C}\) such that
\[ \Re(\tau + k) > \max\{0, \Re(-\mu'), \Re(\delta' + \mu - \epsilon), \Re(\delta + \delta' - \epsilon) + \Re(\epsilon) + 1\}, \text{ then the following formula hold true:} \]
\[ \left(D_{0+}^{\delta, \delta', \mu, \mu', \epsilon, t, \tau} M_{n, q}(1/\tau) \right) (x) = \frac{(-1)^n \Gamma(q + n + 1) \Gamma(\mu' + \tau) \Gamma(-\delta - \delta' + \epsilon + \tau) \Gamma(-\delta' - \mu + \epsilon + \tau)}{\Gamma(q + 1) \Gamma(\tau) \Gamma(-\delta' + \mu + \tau) \Gamma(-\delta' - \mu + \epsilon + \tau)} x^{\delta + \delta' - \epsilon - \tau} \times F_4 \left[ \begin{array}{c} 1 + n - p, -n, \tau + \mu', \tau - \delta - \delta' + \epsilon, \tau - \delta' - \mu + \epsilon, \end{array} \right. \tau, \tau - \delta' + \mu', \tau + \epsilon - \delta - \delta' - \mu, q + 1, \left. \frac{1}{x} \right], \]
provided both the sides exists.

4. Conclusion

The generalized fractional integration and differentiation of of Jacobi type orthogonal polynomials are derived in this paper. Many known results (see [8], [14]) reduced as the particular case of the main theorems. This concludes that the results obtained here are general in nature and can easily obtain various known results.

References

[1] D. Baleanu, D. Kumar and S.D. Purohit, Generalized fractional integrals of product of two $H$-functions and a general class of polynomials, Int. J. Comput. Math. 93(8) (2016), 1320–1329.
[2] C. Fox, The asymptotic expansion of generalized hypergeometric functions, Proc. London. Math. Soc. 27(4) (1928), 389–400.
[3] K.K. Kataria and P. Vellaisamy, The generalized $k$-Wright function and Marichev-Saigo-Maeda fractional operators, J. Analysis, 23 (2015), 7587.
[4] A.A. Kilbas and N. Sebastian, Generalized fractional integration of Bessel function of the first kind, Integral Transforms Spec. Funct. 19(12) (2008), 869–883.
[5] Y.C. Kim, K.S. Lee, and H.M. Srivastava, Some applications of fractional integral operators and Ruscheweyh derivatives, J. Math. Anal. Appl., 197(2) (1996), 505–517.
[6] V. Kiryakova, All the special functions are fractional differintegrals of elementary functions, J. Physics A: Math. Gen., 30(14) (1997), 5085–5103.
[7] D. Kumar, S.D. Purohit, J. Choi, Generalized fractional integrals involving product of multivariable $H$-function and a general class of polynomials, J. Nonlinear Sci. Appl. 9(1) (2016), 8–21.
[8] P. Malik and S. R. Mondal, Some composition formulas of Jacobi type orthogonal polynomials, Commun. Korean Math. Soc. 32 (3)(2017), 677–688
[9] P. Malik, S. R. Mondal, and A. Swaminathan, Riemann-Liouville Fractional calculus for generalized Bessel functions, Proceedings of the ASME 2011, International Design Engineering Technical Conferences and Computers and Information in Engineering Conference 3 (2011), 409418.
[10] P. Malik and A. Swaminathan, Derivatives of a finite class of orthogonal polynomials defined on positive real line related to F-Distribution, Comput. Math. Appl. 61(4), (2011), 1180–1189.
[11] P. Malik and A. Swaminathan, Derivatives of a finite class of orthogonal polynomials relate distribution, Appl. Math. Comput. 218(11) (2012), 6251–6262.
[12] O.I. Marichev, Volterra equation of Mellin convolution type with a Horn function in the kernel, Izvestiya Akademii Nauk BSSR. Seriya Fiziko-Matematicheskikh Nauk, 1 (1974), 128–129 (Russian).
[13] K.S. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential equations, John Wiley & Sons, New York, USA, 1993.
[14] S.R. Mondal and K.S. Nisar, Marichev-Saigo-Maeda fractional integration operators involving generalized Bessel functions, Math. Probl. Eng., 2014 (2014), Article ID 274093, 11 pp.
[15] S.D. Purohit, D.L. Suthar and S.L. Kalla, Marichev-Saigo-Maeda fractional integration operators of the Bessel function, Le Matematiche, 67(1) (2012), 21–32.
[16] E.D. Rainville, Special functions, Macmillan, New York, 1960.
[17] M. Saigo and N. Maeda, More generalization of fractional calculus, in Transform Methods & Special Functions, Varna ‘96, pp. 386–400, Bulgarian Academy of Sciences, Bulgaria, Sofia, 1998.

[18] H.M. Srivastava, S.-D. Lin, and P.-Y. Wang, Some fractional calculus results for the $H$-function associated with a class of Feynman integrals, Russ. J. Math. Phys., 13(1) (2006), 94–100.

K. S. NISAR: DEPARTMENT OF MATHEMATICS, COLLEGE OF ARTS AND SCIENCE-WADI ALDAWASEER, PRINCE SATTAM BIN ABDULAZIZ UNIVERSITY, ALKHARJ, RIYADH REGION 11991, SAUDI ARABIA

E-mail address: ksnisar1@gmail.com, n.sooppy@psau.edu.sa