Lepton flavor-changing processes in $R$-parity violating MSSM: $Z \to \ell_i \bar{\ell}_j$ and $\gamma\gamma \to \ell_i \bar{\ell}_j$ under new bounds from $\ell_i \to \ell_j \gamma$

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Abstract

We examine the lepton flavor-changing processes in $R$-parity violating MSSM. First, we update the constraints on the relevant $R$-violating couplings by using the latest data on the rare decays $\ell_i \to \ell_j \gamma$. We find that the updated constraints are much stronger than the old ones from rare $Z$-decays at LEP. Then we calculate the processes $Z \to \ell_i \bar{\ell}_j$ and $\gamma\gamma \to \ell_i \bar{\ell}_j$. We find that with the updated constraints the $R$-violating couplings can still enhance the rates of these processes to the sensitivity of GigaZ and photon-photon collision options of the ILC.

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I. INTRODUCTION

The minimal supersymmetric model (MSSM) is a popular extension of the Standard Model (SM). In this model the invariance of $R$-parity, defined by $R = (-1)^{2S + 3B + L}$ for a field with spin $S$, baryon-number $B$ and lepton-number $L$, is often imposed on the Lagrangian in order to maintain the separate conservation of baryon-number and lepton-number. Although $R$-parity plays a crucial role in the phenomenology of the MSSM (e.g., forbid proton decay and ensure a perfect candidate for cosmic dark matter), it is, however, not dictated by any fundamental principle such as gauge invariance and there is no compelling theoretical motivation for it. The most general superpotential of the MSSM consistent with the SM gauge symmetry and supersymmetry contains $R$-violating interactions which are given by

$$W_R = \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k + \frac{1}{2} \lambda''_{ijkl} \epsilon^{abcd} U_a L_i H_2,$$

where $i, j, k$ are generation indices, $c$ denotes charge conjugation, $a, b$ and $d$ are the color indices with $\epsilon^{abcd}$ being the total antisymmetric tensor, $H_2$ is the Higgs-doublet chiral superfield, and $L_i(Q_i)$ and $E_i(U_i, D_i)$ are the left-handed lepton (quark) doublet and right-handed lepton (quark) singlet chiral superfields. The dimensionless coefficients $\lambda_{ijk}$ (antisymmetric in $i$ and $j$) and $\lambda'_{ijk}$ in the superpotential are $L$-violating couplings, while $\lambda''_{ijk}$ (antisymmetric in $j$ and $k$) are $B$-violating couplings. So far both theorists and experimentalists have intensively studied the phenomenology of $R$-parity breaking supersymmetry in various processes [2, 3] and obtained some bounds [4].

The lepton flavor-changing (LFC) processes, which have been searched in various experiments [5–7], are a sensitive probe for new physics because they are extremely suppressed in the SM but can be greatly enhanced in new physics models like supersymmetry [8]. In $R$-parity breaking supersymmetry, these rare processes may receive exceedingly large enhancement since both $\lambda$ and $\lambda'$ couplings can make contributions. Such enhancement was considered in the decays $l_i \rightarrow l_j \gamma$ [9] and $Z \rightarrow \ell_i \ell_j$ [10], the $\mu - e$ conversion in nuclei [11], and the di-lepton productions $p\bar{p}/pp \rightarrow e^+e^- + X$ [12] and $e^+e^- \rightarrow e^+\mu^-$ [13].

Since the GigaZ and photon-photon collision options of the ILC can precisely measure the LFC processes $Z \rightarrow \ell_i \ell_j$ and $\gamma \gamma \rightarrow \ell_i \ell_j$ ($i \neq j$ and $\ell_i = e, \mu, \tau$), we in this work study these processes in $R$-violating MSSM. Noting that the experimental upper bounds on the LFC $\tau$-decays became more stringent recently [6], we will first check the constraints on the relevant $R$-violating couplings from the latest measurement of $\ell_i \rightarrow \ell_j \gamma$. Then, with the
updated bounds on the relevant $R$-violating couplings, we calculate $Z \to \ell_i \bar{\ell}_j$ and $\gamma \gamma \to \ell_i \bar{\ell}_j$ to figure out if they can reach the sensitivity of the GigaZ and photon-photon collision options of the ILC.

The paper is organized as follows. In Sec. II we describe the calculations for $\ell_i \to \ell_j \gamma$, $Z \to \ell_i \bar{\ell}_j$ and $\gamma \gamma \to \ell_i \bar{\ell}_j$. In Sec. III we present some numerical results and discussions. Finally, a conclusion is drawn in Sec. IV.

II. CALCULATIONS

In terms of the four-component Dirac notation, the Lagrangian of the $L$-violating interaction is given by (in our calculations we take the presence of $\lambda'_{ijk}$ as an example)

$$\mathcal{L}_\lambda = -\lambda'_{ijk} \left[ \bar{\nu}_i L \bar{d}_k R \bar{d}_j L + \bar{d}_k R \nu^c_L \left( (\bar{d}_k R)^* (\nu^c_L) \right) d^j_L \right. \\
- \bar{d}_k R \nu^c_L - \bar{u}^c_L d^j_R - \left. (\bar{d}_k R)^* (\bar{u}^c_L) \right] + \text{h.c.} \quad (2)$$

The LFC interactions $\ell_i \bar{\ell}_j V (V = \gamma, Z)$ are induced at loop level by exchanging a squark $\tilde{u}^c_L$ or $\tilde{d}^k_R$, which is shown in Fig.1.

FIG. 1: Feynman diagrams for $\ell_i - \ell_j$ transition induced by the $L$-violating couplings at one-loop level.

For the decays $\ell_i \to \ell_j \gamma$ we take $\mu \to e\gamma$ as an example to show the analytic results. The gauge invariant amplitude of $\mu \to e\gamma$ is given by

$$M(\mu \to e\gamma) = 2A \bar{u}(p_e) P_R(2 e \cdot p_\mu - m_\mu \cdot q) u(p_\mu), \quad (3)$$
FIG. 2: Feynman diagrams for $\gamma\gamma \rightarrow \ell_i \bar{\ell}_j$ induced by the $L$-violating couplings at one-loop level.

The effective $\gamma - \ell_i - \ell_j$ vertex in (a,b) is defined in Fig. 1.

where $A$ is given by (assuming the degeneracy for squark masses)

$$A = \frac{i e \lambda'_{1jk} \lambda'_{2jk} m_{\mu}}{16\pi^2} \left[ f_1 \left( \frac{m_{d_k}^2}{m_{\tilde{q}}^2} \right) + f_2 \left( \frac{m_{u_j}^2}{m_{\tilde{q}}^2} \right) \right]$$

(4)

with

$$f_1(x) = \frac{1}{8(x-1)^3} \left[ \frac{2}{3} \left( 2x^2 + 5x - 1 - \frac{6x^2 \ln x}{x-1} \right) - \frac{1}{3} \left( x^2 - 5x - 2 + \frac{6x \ln x}{x-1} \right) \right],$$

(5)

$$f_2(x) = \frac{1}{8(x-1)^3} \left[ \frac{1}{3} \left( 2x^2 + 5x - 1 - \frac{6x^2 \ln x}{x-1} \right) - \frac{2}{3} \left( x^2 - 5x - 2 + \frac{6x \ln x}{x-1} \right) \right].$$

(6)

The decay branching ratio reads

$$BR(\mu \rightarrow e\gamma) = \frac{48\pi}{G_F m_{\mu}^2} |A|^2.$$  

(7)

For the decays $Z \rightarrow \ell_i \bar{\ell}_j$ we calculate the decay rates numerically by using the effective vertex presented in Appendix A. Note that according to the effective vertex method [14], the external legs of the effective vertex can be on-shell or off-shell and thus the vertex can be used in any relevant process. The expression in Eqs.(3-6) can be obtained from the effective vertex in Appendix A by putting both leptons on shell.

For the process $\gamma\gamma \rightarrow \ell_i \bar{\ell}_j$, besides Fig.2 (a,b) induced by the effective vertex given in Appendix A, more diagrams shown in Fig.2 (c-i) also come into play. The analytic...
expressions of the amplitudes of these diagrams are given in Appendix B. These amplitudes contain the Passarino-Veltman one-loop functions, which are calculated by using LoopTools [26]. We checked that the amplitudes have gauge invariance and the ultraviolet divergence cancelled.

Since the photon beams in $\gamma\gamma$ collision are generated by the backward Compton scattering of the incident electron- and the laser-beam, the events number is obtained by convoluting the cross section of $\gamma\gamma$ collision with the photon beam luminosity distribution:

$$N_{\gamma\gamma \rightarrow \ell_i\bar{\ell}_j} = \int d\sqrt{s_{\gamma\gamma}} \frac{dL_{\gamma\gamma}}{d\sqrt{s_{\gamma\gamma}}} \hat{\sigma}_{\gamma\gamma \rightarrow \ell_i\bar{\ell}_j}(s_{\gamma\gamma}) \equiv L_{e^+e^-} \sigma_{\gamma\gamma \rightarrow \ell_i\bar{\ell}_j}(s)$$

where $dL_{\gamma\gamma}/d\sqrt{s_{\gamma\gamma}}$ is the photon-beam luminosity distribution and $\sigma_{\gamma\gamma \rightarrow \ell_i\bar{\ell}_j}(s)$ ($s$ is the squared center-of-mass energy of $e^+e^-$ collision) is defined as the effective cross section of $\gamma\gamma \rightarrow \ell_i\bar{\ell}_j$. In optimum case, it can be written as [15]

$$\sigma_{\gamma\gamma \rightarrow \ell_i\bar{\ell}_j}(s) = \frac{1}{\sqrt{a}} \int_{x_{\text{max}}}^{x_{\text{max}}} 2zdz \hat{\sigma}_{\gamma\gamma \rightarrow \ell_i\bar{\ell}_j}(s_{\gamma\gamma} = z^2s) \int_{z_{\text{max}}/x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x} F_{\gamma/e}(x) F_{\gamma/e}(\frac{z^2}{x})$$

where $F_{\gamma/e}$ denotes the energy spectrum of the back-scattered photon for the unpolarized initial electron and laser photon beams given by

$$F_{\gamma/e}(x) = \frac{1}{D(\xi)} \left[ 1 - x + \frac{1}{1 - x} - \frac{4x}{\xi(1 - x)} + \frac{4x^2}{\xi^2(1 - x)^2} \right]$$

with

$$D(\xi) = (1 - \frac{4}{\xi} - \frac{8}{\xi^2}) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2}.$$

Here $\xi = 4E_eE_0/m_e^2$ ($E_e$ is the incident electron energy and $E_0$ is the initial laser photon energy) and $x = E/E_0$ with $E$ being the energy of the scattered photon moving along the initial electron direction.

III. NUMERICAL RESULTS AND DISCUSSIONS

In our calculations we take the SM parameters as [16]

$$m_\mu = 0.106 \text{ GeV}, \ m_\tau = 1.777 \text{ GeV}, \ m_b = 4.2 \text{ GeV}, \ \alpha = 1/137, \ \sin^2 \theta_W = 0.223$$

The top quark mass is taken as the new CDF value $m_t = 172.3 \text{ GeV}$ [17]. The relevant SUSY parameters in our calculations are the masses of squarks as well as the $R$-parity violating couplings listed in Table I. The strongest bound on squark mass is from the Tevatron
experiment. For example, from the search of the inclusive production of squark and gluino in $R$-conserving minimal supergravity model with $A_0 = 0$, $\mu < 0$ and $\tan \beta = 5$, the CDF gives a bound of 392 GeV at the 95% C.L. for degenerate gluinos and squarks [18]. However, this bound may be not applicable to the $R$-violating scenario because the SUSY signal in case of $R$-violation is very different from the $R$-conserving case. The most robust bounds on sparticle masses come from the LEP results, which give a bound of about 100 GeV on squark or slepton mass [19]. In our numerical calculations, we assume the presence of the minimal number of $R$-violating couplings, i.e., for each process only the two relevant couplings (not summed over the family indices) are assumed to be present.

For $\ell_i \rightarrow \ell_j \gamma$, the latest experimental data is [7]

$$BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11},$$

$$BR(\tau \rightarrow e\gamma) < 1.1 \times 10^{-7},$$

$$BR(\tau \rightarrow \mu\gamma) < 4.5 \times 10^{-8}.$$  

(13)

(14)

(15)

We use these data to update the bounds on the relevant $L$-violating couplings. The new bounds are compared with the old ones in Table I for $m_\tilde{q} = 100$ GeV (here we take squark mass of 100 GeV for illustration and for heavier squarks the bounds on the $L$-violating couplings will become weak, as will be shown later). We can see that the new bounds are much stronger than the old ones. Since the bounds on $\lambda'_{i33} \lambda'_{j33} (i \neq j)$ are weakest, we only consider the contribution of $\lambda'_{i33} \lambda'_{j33}$ in our following numerical calculations.

Note that the neutrino masses could also constrain the $\lambda'$ couplings, especially $\lambda'_{i33}$ [20]. But these constraints depend on more parameters in addition to the squark mass. For example, the one-loop $\lambda'$ contributions to the neutrino masses are sensitive to the left-right squark mixings and the two-loop contributions further involve the slepton mass. For small squark mixings with appropriate sign, there may exist a strong cancellation between one-loop and two-loop effects, and in this case, the constraints from the neutrino masses can be avoided. Since the aim of our study is the sensitivity of the LFC processes to $\lambda'$ couplings and the $\lambda'$ contributions to these LFC processes are irrelevant to the additional parameters involved in the contributions to the neutrino masses, in our analysis we did not consider such constraints from the neutrino masses.
TABLE I: Our new upper bounds on the $L$-violating couplings for $m_\tilde{q} = 100$ GeV from $\ell_i \rightarrow \ell_j \gamma$ data [7], in comparison with the old ones [4].

| couplings | New bounds     | Old bounds [4] |
|-----------|----------------|---------------|
| $\lambda_{111} \lambda_{211}^\prime$, $\lambda_{112} \lambda_{212}^\prime$ | $7.74 \times 10^{-5}$ | $5.7 \times 10^{-4}$ |
| $\lambda_{113} \lambda_{213}^\prime$ | $7.85 \times 10^{-5}$ | $5.7 \times 10^{-4}$ |
| $\lambda_{121} \lambda_{221}^\prime$, $\lambda_{122} \lambda_{222}^\prime$ | $7.78 \times 10^{-5}$ | $5.7 \times 10^{-4}$ |
| $\lambda_{123} \lambda_{223}$ | $7.89 \times 10^{-5}$ | $5.7 \times 10^{-4}$ |
| $\lambda_{131} \lambda_{231}^\prime$, $\lambda_{132} \lambda_{232}^\prime$ | $1.27 \times 10^{-3}$ | $7.7 \times 10^{-3}$ |
| $\lambda_{133} \lambda_{233}$ | $1.63 \times 10^{-3}$ | $1.0 \times 10^{-2}$ |
| $\lambda_{111} \lambda_{311}^\prime$, $\lambda_{112} \lambda_{312}^\prime$ | $5.54 \times 10^{-4}$ | $1.2 \times 10^{-2}$ |
| $\lambda_{113} \lambda_{313}$ | $5.56 \times 10^{-4}$ | $1.2 \times 10^{-2}$ |
| $\lambda_{121} \lambda_{321}^\prime$, $\lambda_{122} \lambda_{322}^\prime$ | $5.57 \times 10^{-4}$ | $1.2 \times 10^{-2}$ |
| $\lambda_{123} \lambda_{323}$ | $5.65 \times 10^{-4}$ | $1.2 \times 10^{-2}$ |
| $\lambda_{131} \lambda_{331}^\prime$, $\lambda_{132} \lambda_{332}^\prime$ | $9.06 \times 10^{-3}$ | $1.2 \times 10^{-2}$ |
| $\lambda_{133} \lambda_{333}$ | $1.17 \times 10^{-2}$ | $1.2 \times 10^{-2}$ |
| $\lambda_{211} \lambda_{311}^\prime$, $\lambda_{212} \lambda_{312}^\prime$ | $3.55 \times 10^{-4}$ | $10^{-2}$ |
| $\lambda_{213} \lambda_{313}$ | $3.60 \times 10^{-4}$ | $10^{-2}$ |
| $\lambda_{221} \lambda_{321}^\prime$, $\lambda_{222} \lambda_{322}^\prime$ | $3.56 \times 10^{-4}$ | $10^{-2}$ |
| $\lambda_{223} \lambda_{323}$ | $3.61 \times 10^{-4}$ | $10^{-2}$ |
| $\lambda_{231} \lambda_{331}^\prime$, $\lambda_{232} \lambda_{332}^\prime$ | $5.80 \times 10^{-3}$ | $10^{-2}$ |
| $\lambda_{233} \lambda_{333}$ | $7.48 \times 10^{-3}$ | $10^{-2}$ |

For $Z \rightarrow \ell_i \overline{\ell}_j$, the upper limits from LEP are [21, 22]

\[
BR(Z \rightarrow \mu e) < 1.7 \times 10^{-6}, \quad (16)
\]
\[
BR(Z \rightarrow \tau e) < 9.8 \times 10^{-6}, \quad (17)
\]
\[
BR(Z \rightarrow \tau \mu) < 1.2 \times 10^{-5}. \quad (18)
\]

The bounds from these LEP data are compared with the bounds from $\ell_i \rightarrow \ell_j \gamma$ in Fig.3. One can see that the upper bounds on the couplings from the LEP $Z$-decay data [21, 22] are weaker than the ones from $\ell_i \rightarrow \ell_j \gamma$ data [7]. Note that the bounds from the LEP $Z$-decay data were also studied in [10] and our results are consistent with theirs except that in [10]
FIG. 3: Various bounds on the $L$-violating couplings versus the squark mass. The solid, dashed and dotted curves are the bounds on $\lambda'_{133}\lambda'_{233}$, $\lambda'_{133}\lambda'_{333}$ and $\lambda'_{233}\lambda'_{333}$, respectively. Also shown are the 2σ sensitivity from $Z$-decays at GigaZ and the 3σ sensitivity from $\gamma\gamma \to e(\text{or } \mu)\tau$ at the ILC with center-of-mass energy of 500 GeV and a luminosity of $3.45 \times 10^2 fb^{-1}$.

The possible sensitivity of GigaZ to the LFC decays of $Z$-boson could reach [23]

\begin{align}
BR(Z \to \mu e) & \sim 2.0 \times 10^{-9}, \\
BR(Z \to \tau e) & \sim \kappa \times 6.5 \times 10^{-8}, \\
BR(Z \to \tau \mu) & \sim \kappa \times 2.2 \times 10^{-8}
\end{align}

The sum over index $k$ is implied for $\lambda'_{i3k}\lambda'_{j3k}$ with $m_{\tilde{q}} = 200$ GeV.
with the factor $\kappa$ ranging from 0.2 to 1.0. In Fig. 3 we take $\kappa = 1.0$ to show the sensitivity. In contrast to the $R$-conserving case in which only $Z \rightarrow \mu \tau$ is accessible at the GigaZ [8], the $R$-violating couplings under the bound from $l_i \rightarrow l_j \gamma$ can still enhance all the channels $Z \rightarrow \ell_i \bar{\ell}_j$ to the sensitivity of the GigaZ. This implies that the GigaZ can further strengthen the bounds on $\lambda'_{i33} \lambda'_{j33}$ in case of un-observation. These bounds, unlike the constraints from neutrino masses which involve more parameters, are only dependent on the squark mass.

For the $\gamma \gamma$ collision results shown in Fig. 3, we fixed the parameters as $\xi = 4.8$, $D(\xi) = 1.83$ and $x_{max} = 0.83$ [15]. Since the $L$-violating couplings relevant to the process $\gamma \gamma \rightarrow e\bar{\tau}$ is stringently constrained by $\mu \rightarrow e\gamma$, we in Fig. 3 only show the results for the channels with a tau lepton in the final states, i.e., $\gamma \gamma \rightarrow e\bar{\ell}$, $\mu \bar{\ell}$. The background for $\gamma \gamma \rightarrow e\bar{\tau}$ comes from $\gamma \gamma \rightarrow \tau^+\tau^- \rightarrow \tau^- \nu_\ell \nu_\tau e^+$, $\gamma \gamma \rightarrow W^+W^- \rightarrow \tau^- \nu_\ell \nu_\tau e^+$ and $\gamma \gamma \rightarrow e^+e^-\tau^+\tau^-$, and we make kinematical cuts [13]: $|\cos \theta_\ell| < 0.9$ and $p_T^\ell > 20$ GeV ($\ell = e, \mu$), to enhance the ratio of signal to background. With these cuts, the background cross sections from $\gamma \gamma \rightarrow \tau^+\tau^- \rightarrow \tau^- \nu_\ell \nu_\tau e^+$, $\gamma \gamma \rightarrow W^+W^- \rightarrow \tau^- \nu_\ell \nu_\tau e^+$ and $\gamma \gamma \rightarrow e^+e^-\tau^+\tau^-$ at $\sqrt{s} = 500$ GeV are suppressed respectively to $9.7 \times 10^{-4}$ fb, $1.0 \times 10^{-1}$ fb and $2.4 \times 10^{-2}$ fb (see Table I of [13]). To get the $3\sigma$ observing sensitivity with $3.45 \times 10^2$ fb$^{-1}$ integrated luminosity [24], the production rates of $\gamma \gamma \rightarrow e\bar{\ell}, \mu \bar{\ell}$ after the cuts must be larger than $2.5 \times 10^{-2}$ fb [13]. We see from Fig. 3 that under the current bounds from $l_i \rightarrow l_j \gamma$, the $L$-violating couplings can still be large enough to enhance the productions $\gamma \gamma \rightarrow e\bar{\ell}, \mu \bar{\ell}$ to the $3\sigma$ sensitivity.

We also show the cross sections of $\gamma \gamma \rightarrow \ell_i \bar{\ell}_j$ as a function of center-of-mass energy $\sqrt{s}$ of the ILC in Fig.4. We see that with the increasing of the center-of-mass energy, the cross sections of these processes become smaller. Such a behavior is similar to the results in the R-conserving MSSM shown in [13].

Finally, we point out that the LFC processes can also put bounds on the products $\lambda'_{i31} \lambda'_{j31}$ and $\lambda'_{i32} \lambda'_{j32}$, and our numerical results indicate that such bounds are quite similar to those in Fig.3. We note that these bounds on $\lambda'_{i31} \lambda'_{j31}$ and $\lambda'_{i32} \lambda'_{j32}$ from $Z \rightarrow l_i l_j$ at GigaZ are generally stronger than those from the neutrino masses [20].

IV. CONCLUSION

We evaluated the lepton flavor-changing processes in $R$-parity violating MSSM. First, we used the latest data on the rare decays $\ell_i \rightarrow \ell_j \gamma$ to update the constraints on the relevant
$\gamma \gamma \rightarrow \ell_i \bar{\ell}_j$ as a function of center-of-mass energy $\sqrt{s}$. The couplings $\lambda'_{133} \lambda'_{233}$, $\lambda'_{133} \lambda'_{333}$ and $\lambda'_{233} \lambda'_{333}$ are fixed at their upper bounds at $M_{\tilde{q}} = 300$ GeV.

$R$-violating couplings. Then we calculated the processes $Z \rightarrow \ell_i \bar{\ell}_j$ and $\gamma \gamma \rightarrow \ell_i \bar{\ell}_j$. We found that with the updated constraints the $R$-violating couplings can still enhance the rates of these processes to the sensitivity of GigaZ and photon-photon collision options of the ILC. So, the GigaZ and photon-photon collision of the ILC can either observe these $\lambda'$-induced LFC processes or further strengthen the bounds on the $\lambda'$ couplings in case of un-observation.

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Appendix A: Expressions of effective vertex $\gamma(Z) - \ell_i - \ell_j$

Here we list the expression for the L-violating contributions to the effective vertex $\gamma(Z) - e - \mu$. Other effective vertices $\gamma(Z) - \mu - \tau$ and $\gamma(Z) - e - \tau$ are similar to $\gamma(Z) - e - \mu$ and can be obtained by replacing the corresponding momentum and mass. The the effective vertex $\gamma(Z) - e - \mu$ is given by

$$\Gamma^{\gamma(Z)e\mu}_\lambda = \Gamma^{(Z)e\mu}_\lambda (\bar{u}^L_j) + \Gamma^{(Z)e\mu}_\lambda (\bar{d}^R_k),$$  \hspace{1cm} (A1)

where the two terms on the right side denote the L-violating loop contributions by exchanging respectively the squarks $\tilde{u}^L_j$ and $\tilde{d}^R_k$, given by

$$\Gamma^{\gamma(e\mu)}(p_\mu, p_e)|_{\bar{u}^L_j} = ae \left\{ \frac{1}{3} \left[ C^2_{\alpha\beta} \gamma^\alpha \gamma^\beta - C^4_{\alpha}(p_\mu - p_e) \gamma^\alpha \gamma^\alpha \right] P_L + \frac{2}{3} \left[ 2 C^4_{\alpha\beta} \gamma^\alpha - C^4_{\alpha}(p_\mu + p_e) \gamma^\alpha \right] P_L - \frac{1}{m^2_\mu} \gamma^\beta \bar{\gamma}_\mu \gamma^\alpha B^3_\alpha P_L + \frac{1}{m^2_\mu} \left[ \gamma^\beta \bar{\gamma}_\mu \gamma^\alpha P_L + m_\mu \gamma^\alpha \gamma^\alpha P_R \right] B^3_\alpha - \frac{1}{3} m^2_0 C^4_{0}(\gamma^\alpha P_L) \right\}$$  \hspace{1cm} (A2)

$$\Gamma^{\gamma(e\mu)}(p_\mu, p_e)|_{\bar{d}^R_k} = be \left\{ \frac{2s^2_\mu}{3} \left[ C^2_{\alpha\beta} \gamma^\alpha \gamma^\beta - C^4_{\alpha}(p_\mu - p_e) \gamma^\alpha \gamma^\alpha \right] P_L - \frac{2}{3} \left[ 2 C^4_{\alpha\beta} \gamma^\alpha - C^4_{\alpha}(p_\mu + p_e) \gamma^\alpha \right] P_L - \frac{1}{m^2_\mu} \gamma^\beta \bar{\gamma}_\mu \gamma^\alpha B^3_\alpha P_L + \frac{1}{m^2_\mu} \left[ (1 - 2s^2_\mu) \gamma^\beta \bar{\gamma}_\mu \gamma^\alpha P_L - 2s^2_\mu m_\mu \gamma^\alpha \gamma^\alpha P_R \right] B^3_\alpha \right\}$$  \hspace{1cm} (A3)

$$\Gamma^{\gamma(e\mu)}(p_\mu, p_e)|_{\bar{u}^L_j} = ce \left\{ \frac{1}{3} \left[ C^2_{\alpha\beta} \gamma^\alpha \gamma^\beta - C^4_{\alpha}(p_\mu - p_e) \gamma^\alpha \gamma^\alpha \right] P_L + \frac{2}{3} \left[ 2 C^4_{\alpha\beta} \gamma^\alpha - C^4_{\alpha}(p_\mu + p_e) \gamma^\alpha \right] P_L - \frac{1}{m^2_\mu} \gamma^\beta \bar{\gamma}_\mu \gamma^\alpha B^3_\alpha P_L + \frac{1}{m^2_\mu} \left[ (1 - 2s^2_\mu) \gamma^\beta \bar{\gamma}_\mu \gamma^\alpha P_L - 2s^2_\mu m_\mu \gamma^\alpha \gamma^\alpha P_R \right] B^3_\alpha \right\}$$  \hspace{1cm} (A4)

$$\Gamma^{\gamma(e\mu)}(p_\mu, p_e)|_{\bar{d}^R_k} = de \left\{ \frac{2s^2_\mu}{3} \left[ C^2_{\alpha\beta} \gamma^\alpha \gamma^\beta - C^4_{\alpha}(p_\mu - p_e) \gamma^\alpha \gamma^\alpha \right] P_L + \frac{2}{3} \left[ 2 C^4_{\alpha\beta} \gamma^\alpha - C^4_{\alpha}(p_\mu + p_e) \gamma^\alpha \right] P_L - \frac{1}{m^2_\mu} \gamma^\beta \bar{\gamma}_\mu \gamma^\alpha B^3_\alpha P_L + \frac{1}{m^2_\mu} \left[ (1 - 2s^2_\mu) \gamma^\beta \bar{\gamma}_\mu \gamma^\alpha P_L - 2s^2_\mu m_\mu \gamma^\alpha \gamma^\alpha P_R \right] B^3_\alpha \right\}$$  \hspace{1cm} (A5)

with $a = \frac{i3\lambda_{1\mu}^{\alpha\beta}\lambda_{2j}^{\beta\gamma}}{16\pi^2}$, $b = \frac{i3\lambda_{1\mu}^{\alpha\beta}\lambda_{2j}^{\beta\gamma}}{16\pi^2}$ and $p_e$ and $p_\mu$ denoting respectively the momenta of the electron and muon. In the above expressions, the functions $B^3_\alpha$ and $C^4_{\alpha,\beta}$ are the Passarino-Veltman functions. For these loop functions, we adopt the definition in [25] and use LoopTools [26] in the calculations. The functional dependence of these loop functions is
given by

\begin{align}
C^1(-p_\mu, p_e, m^2_{d_k}, m^2_{u_j}, m^2_{d_k}), \quad & C^2(-p_e, p_e - p_\mu, m^2_{d_k}, m^2_{u_j}, m^2_{u_j}), \quad \text{(A6)} \\
C^3(-p_\mu, p_e, m^2_{d_k}, m^2_{u_j}, m^2_{u_j}), \quad & C^4(-p_e, p_e - p_\mu, m^2_{d_k}, m^2_{d_k}, m^2_{u_j}), \quad \text{(A7)} \\
C^5(-p_\mu, p_e, m^2_{d_k}, m^2_{u_j}, m^2_{u_j}), \quad & C^6(-p_e, p_e - p_\mu, m^2_{d_k}, m^2_{d_k}, m^2_{u_j}), \quad \text{(A8)} \\
C^7(-p_\mu, p_e, m^2_{d_k}, m^2_{d_k}, m^2_{u_j}), \quad & C^8(-p_e, p_e - p_\mu, m^2_{d_k}, m^2_{d_k}, m^2_{u_j}), \quad \text{(A9)} \\
B^1(-p_\mu, m^2_{d_k}, m^2_{u_j}), \quad & B^2(-p_e, m^2_{d_k}, m^2_{u_j}), \quad \text{(A10)} \\
B^3(-p_\mu, m^2_{d_k}, m^2_{d_k}), \quad & B^4(-p_e, m^2_{d_k}, m^2_{d_k}), \quad \text{(A11)} \\
B^5(-p_\mu, m^2_{d_k}, m^2_{d_k}), \quad & B^6(-p_e, m^2_{d_k}, m^2_{d_k}), \quad \text{(A12)} \\
B^7(-p_\mu, m^2_{d_k}, m^2_{d_k}), \quad & B^8(-p_e, m^2_{d_k}, m^2_{d_k}). \quad \text{(A13)}
\end{align}

Appendix B: Expressions of amplitudes for $\gamma\gamma \rightarrow \ell_i \ell_j$

The amplitudes of the diagrams in Fig.2(a-i) are given by

\begin{align}
M_{(a)}|_{a_j^I, d_k^R} &= \frac{\pi(e)(ie\gamma\lambda)}{(p_2 - p_\mu)} \Gamma^\gamma \rho_{\mu}(p_2 - p_\mu, p_e) |_{a_j^I, d_k^R} v(\mu) e_1 e_2 \quad \text{(B1)} \\
M_{(b)}|_{a_j^I, d_k^R} &= \frac{\pi(e) \Gamma^\gamma \rho_{\mu}(p_2 - p_\mu, p_e) |_{a_j^I, d_k^R}}{(p_2 - p_\mu)} (ie\gamma\rho) v(\mu) e_1 e_2 \quad \text{(B2)} \\
M_{(c)}|_{a_j^I} &= -\frac{i}{16\pi^2} \left(\frac{8}{9} e^2\lambda_{1jk}^\prime \lambda_{jk}^\prime \pi(e) C_{\alpha}^{0\gamma} P_L v(\mu) e_1 e_2 \right) \quad \text{(B3)} \\
M_{(d)}|_{a_j^I, d_k^R} &= \frac{i}{16\pi^2} \left(\frac{2}{9} e^2\lambda_{1jk}^\prime \lambda_{jk}^\prime \pi(e) C_{\alpha}^{0\gamma} P_L v(\mu) e_1 e_2 \right) \quad \text{(B4)} \\
M_{(e)}|_{a_j^I} &= \frac{i}{16\pi^2} \left(\frac{4}{9} e^2\lambda_{1jk}^\prime \lambda_{3jk}^\prime \pi(e) \right) \left\{ 4D_{\rho\alpha\lambda}^2 - 2D_{\rho\alpha\lambda}^2 (2p_e - p_\mu - p_1) \right\} \quad \text{(B5)} \\
M_{(f)}|_{a_j^I} &= \frac{i}{16\pi^2} \left(\frac{2}{9} e^2\lambda_{1jk}^\prime \lambda_{jk}^\prime \pi(e) \right) \left\{ 2D_{\rho\alpha\lambda}^2 - (2p_e - 2p_2) \right\} \quad \text{(B6)} \\
M_{(g)}|_{a_j^I} &= \frac{i}{16\pi^2} \left(\frac{2}{9} e^2\lambda_{1jk}^\prime \lambda_{jk}^\prime \pi(e) \right) \left\{ 2D_{\rho\alpha\lambda}^2 - (2p_e - p_1) \right\} \quad \text{(B7)}
\end{align}
Here the effective vertices appearing in Eqs. (B1) and (B2) are defined in Appendix A. The amplitudes for the diagrams with the two photons exchanged are not presented here, which can be obtained from the above corresponding amplitudes with replacement $p_1 \leftrightarrow p_2$ and $\epsilon_1 \leftrightarrow \epsilon_2$. The functional dependence of the Passarino-Veltman loop functions $C^i$ and $D^{i\alpha\alpha\beta\alpha\beta\gamma}$ is given by

$$C^9(-p_e, p_\mu + p_e, m_{d_k}^2, m_{u^e_j}^2, m_{\tilde{d}_k}^2), \quad C^{10}(-p_e, p_\mu + p_e, m_{u^j_j}^2, m_{d_k}^2, m_{\tilde{d}_k}^2)$$

$$D^1(p_2, p_1, -p_e, m_{d_k}^2, m_{u^e_j}^2, m_{\tilde{d}_k}^2), \quad D^2(-p_e, p_1, p_2, m_{d_k}^2, m_{u^j_j}^2, m_{\tilde{d}_k}^2)$$

$$D^3(-p_e, p_2, -p_\mu, m_{d_k}^2, m_{u^e_j}^2, m_{\tilde{d}_k}^2), \quad D^4(p_2, p_1, -p_e, m_{u^j_j}^2, m_{d_k}^2, m_{u^j_j}^2)$$

$$D^5(-p_e, p_1, p_2, m_{u^j_j}^2, m_{\tilde{d}_k}^2, m_{\tilde{d}_k}^2), \quad D^6(-p_e, p_2, -p_\mu, m_{u^j_j}^2, m_{\tilde{d}_k}^2, m_{u^j_j}^2)$$

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