Spectral Properties of Interstellar Turbulence via Velocity Channel Analysis

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Abstract. In this presentation we review the link between the statistics of intensity fluctuations in spectral line data cubes with underlying statistical properties of turbulence in the interstellar medium. Both the formalism of Velocity Channel Analysis for optically thin lines and its extension to the lines with self-absorption is described. We demonstrate that by observing optically thin lines from cold gas in sufficiently narrow (thin) velocity channels one may recover the scaling of the stochastic velocities from turbulent cascade, in particular, Kolmogorov velocities give \( K^{-2.7} \) contribution to the intensity power spectrum. Synthetically increasing the channel thickness separates out the underlying density inhomogeneities of the gas. Effects of self absorption, on the other hand, retain the velocity signature even for integrated lines. As a result, intensity fluctuations tend to show universal but featureless scaling of the power \( \propto K^{-3} \) over the range of scales.

INTRODUCTION

There is little doubt that interstellar medium is turbulent (see reviews \[1,2,3\]). Turbulence proved to be ubiquitous in molecular clouds [4], diffuse ionized [5] and neutral [6] media. Recent years were marked by a substantial progress in theoretical and numerical description of both incompressible and compressible Magneto-Hydrodynamical (MHD) turbulent cascade [7,8,9,10,11,12]. MHD turbulence controls many essential astrophysical processes including star formation, transport and acceleration of cosmic rays, of heat and mass (see [13,14,15,16,10]).

The progress in testing and advancing our theoretical understanding of these astrophysical processes rely on recovering the actual parameters of turbulent cascade from observational data.

For this, the statistical approach is useful (see \[17,18\]). In fact, the turbulent medium is, generally, a non-uniform distribution of matter with stochastic, random, density \( \rho(x) \), that moves according to stochastic velocity field \( u(x) \). Recovering the scaling of statistical descriptors for stochastic density and velocity will allow us to get insight into mechanisms, responsible for turbulent driving. Among the two, the most direct information about the turbulence is contained in the statistics of the velocity field, while the statistics of density fluctuations usually provides complimentary indirect way of testing turbulence.

A wealth of observational information is contained in spectral or channel maps of the emission lines produced by gas tracing the turbulent cascade. Both inhomogeneous distribution of the emitting matter and its motions result in small scale fluctuations of the emission intensity observed at a given velocity. The problem is to relate the
two dimensional statistics of intensity maps to underlying three dimensional properties of the turbulence and disentangle the effects of random motions from the effects of density fluctuations. This problem is addressed in the velocity channel analysis (VCA) \[19, 20\] which we review here. VCA provides theoretical framework for quantitative interpretation of observational data in terms of turbulent properties.

The observational data has been obtained for the variety of lines, mapping different regions of our galaxy (or its satellites) and different physical conditions - 21 cm diffused HI in outer parts of the Galactic disk \[21\], towards the Galactic center \[22\] and in Small Magellanic Cloud \[23\], CO lines in molecular clouds \[24\], \(H_\alpha\) lines in Reynolds layer \[25\]. In all the cases the reported index of the power spectrum of turbulence is close to \(n \approx -2.7\). This is really remarkable given that these results are obtained for quite different observables and that \(n = -2.7\) does not correspond to Kolmogorov \[26\] cascade, the one case when some universality may be expected. To complicate the matter, some lines are optically thin, given view for the full depth of the emitters along the line of sight, while for the other (definitely for CO, but also for HI viewed towards the Galactic center) self absorption is important. Here, using the formalism of VCA, we will demonstrate how the universal power spectral scaling in the data may be explained.

STATISTICAL DESCRIPTION OF THE TURBULENCE

Stochastic density and velocity fields

The main feature of the stochastic random fields arising as a result of turbulent processes is that they are correlated. The central object which describes their properties is the correlation function \(\xi(r)\) (or where more suitable - the structure function \(d(r)\)). Related and often less limited descriptor is the power spectrum \(P(k)\), which is a Fourier transform of the correlation (structure) functions.

We shall restrict our discussion to the case of the turbulence that is statistically homogeneous and isotropic in three dimensional position space \[17\], \(^1\) in which case correlation functions depend only on separation distance \(r\) between two positions in space. In particular, for the density field \(\rho(x)\) we define the correlation function

\[
\xi(r) = \xi(r) = \langle \rho(x)\rho(x+r) \rangle.
\]

\(^1\) In the presence of magnetic field MHD turbulence becomes axisymmetric in the system of reference related to the local direction of magnetic field. The Goldreich & Sridhar \[7\] model of incompressible turbulence prescribes the Kolmogorov scaling of mixing motions perpendicular to magnetic field lines with \(D(r) \propto r^{2/3}\) and a different scaling along the magnetic field. Further research in \[8, 9\] has shown that the basic features of the Goldreich-Sridhar turbulence carry over for Alfvénic perturbations to the compressible regime. Observations, however, are usually unable to identify the local orientation of magnetic field and deal with the magnetic field projection integrated over the line of sight. As the result the locally defined perpendicular and parallel directions are mixed together in the process of observations \[10\]. There is some residual anisotropy, but this anisotropy is scale-independent and is determined by the rate of the meandering of the large scale field. So from the observational point of view the picture of the isotropic turbulence remains to some extend applicable. The spectra of intensity fluctuations obtained from the Goldreich-Sridhar turbulence and the isotropic turbulence are similar.
or, alternatively, the structure function

\[ d(r) = \langle (\rho(x+r) - \rho(x))^2 \rangle \ . \tag{2} \]

Statistical descriptors can often be assumed to have power-law dependence on scale \( \propto r^{-\gamma} \), where \( \gamma \) can be both positive and negative. The correlation function \( \xi(r) \) is an appropriate choice when stochastic inhomogeneities have more power on small scales, which corresponds to \( \gamma > 0 \), while when the power is concentrated on large scales and \( \gamma < 0 \), the structure function should be used. With this substitution in mind both cases can be treated similarly in the most interesting regime of small \( r \).

For power law correlations the power spectrum \( P(k) = \int d^N r e^{i k r} \xi(r) \) also has power law form \( P(k) \propto k^n \), where \( n = \gamma - N \). It is important to keep in mind that the relation between spectral index \( n \) and correlation scaling \( \gamma \) involves the dimensionality of space \( N \) and is different for three dimensional fields and two dimensional maps.

To be exact, in the case of density it is more appropriate to assume a power law scaling for the density fluctuations around the mean \( \delta \rho = \rho - \bar{\rho} \) rather than for the density field itself, so that

\[ \xi(r) = \bar{\rho}^2 \left[ 1 + \left( \frac{r}{r_0} \right)^{-\gamma} \right] . \tag{3} \]

For example, in Kolomogorov turbulence the medium is incompressible, however the passive tracers of the flow, which we may observe as emitters develop passive scalar density inhomogeneities which scale as \( r^{2/3} \), i.e \( \gamma = -2/3 \) in our notations.

Whether the medium is incompressible or not, the stochastic motions are the essence of turbulence. An isotropic random velocity field \( \mathbf{u}(x) \) is fully described by the structure tensor \( \langle \Delta u_i \Delta u_j \rangle \), which can be expressed via longitudinal \( D_{LL} \) and transverse \( D_{NN} \) components [17]

\[ \langle \Delta u_i \Delta u_j \rangle = (D_{LL}(r) - D_{NN}(r)) \frac{r_i r_j}{r^2} + D_{NN}(r) \delta_{ik} \ , \tag{4} \]

where \( \delta_{ik} \) equals 1 for \( i = k \) and zero otherwise. We need only projection of the velocity structure function onto the line of sight, which we identify with \( z \) direction \( D_z(r) \equiv \langle \Delta u_i \Delta u_j \rangle \hat{z} \hat{z}_j \). For the power-law regime

\[ D_z(r) \sim C r^m \ . \tag{5} \]

In Kolmogorov turbulence velocity scales the same way, \( m = 2/3 \), as the density of the passive scalar. Thus the Kolmogorov spectrum index is \(-11/3\). In turbulence literature the energy spectrum \( E(k) = 4\pi k^2 P(k) \) is usually used. In this notation the Kolmogorov spectrum is \( E(k) \sim k^{-5/3} \), often referred to as \(-5/3\) law.

\[ ^2 \text{There is a residual dependence of } D_z \text{ on the angle } \cos \theta = \mathbf{r} \cdot \hat{z} \text{ which differs for solenoidal and potential flows, see [19, 20]. VCA formalism is shown to be insensitive to it, so an important problem of separating solenoidal and potential components of the velocity has to be addressed by combination of techniques, possibly by combining VCA and velocity centroids [27].} \]
Statistical description in Position-Position-Velocity space

One does not observe the gas distribution in the real space galactic coordinates $xyz$ where the 3D vector $x$ is defined. Rather, intensity of the emission in a given spectral line is defined in Position-Position-Velocity (PPV) cubes towards some direction on the sky and at a given line-of-sight velocity $v$. In the plane parallel approximation the direction on the sky is identified with $xy$ plane where the 2D spatial vector $X$ is defined, so that the coordinates of PPV cubes available through observations are $(X,v)$. The relation between the real space and PPV descriptions is defined by a map $(X,z) \rightarrow (X,v)$.

The central object for our study is a turbulent cloud in PPV coordinates. The number of particles per unit volume $dX$ $dv$ is given by the PPV density $\rho_s(X,v)$, which statistical properties depend on the density of gas in real galactic coordinates, but also on velocity distribution of gas particles. Henceforth, we use the subscript $s$ to distinguish the quantities in $(X,v)$ coordinates from those in $(X,z)$ coordinates. We shall always assume 2D statistical homogeneity and isotropy of $\rho_s(X,v)$ in $X$-direction over the image of a cloud. However, homogeneity along the velocity direction can only be assumed after additional considerations, if at all. Naturally, there is no symmetry between $v$ and $X$.

In [20] we discussed in detail the derivation of the correlation function in PPV space

$$
\xi_s(R,v_1,v_2) = \langle \rho_s(R_1,v_1)\rho_s(R_2,v_2) \rangle, \quad R = |R_1 - R_2|
$$

in a general case of matter concentrated in a finite cloud and possibly subjected to coherent non-random flow, for example rotation. The main result is that for small scales the complications arising from the finiteness of the matter distribution or its non-random motion can be neglected and

$$
\xi_s(R,v) \sim \int_{-\infty}^{\infty} dz \frac{\xi(r)}{[D_z(r)+2\beta]^{1/2}} \exp \left[ -\frac{v^2}{2(D_z(r)+2\beta)} \right], \quad v = v_1 - v_2
$$

where $\beta = k_B T / m$ is related to the thermal velocity of atoms. How small should the separation $R$ be? In case of a cloud of size $S$, obviously we need $R \ll S$ to neglect boundary effects. In case of the coherent flows, it is the magnitude of the shear introduced by the flow relative to the magnitude of the stochastic motions that is important. Exact criterion is $R \ll [f^2 C]^{-1/2}$ [19], if we parameterize the line-of-sight variation of the coherent velocity as $v_{coh} = f^{-1}z$. In case of our Galaxy disk rotation, $f^{-1} \approx 14$ km/s/kpc, while relative turbulent motions reach 30 km/s at few tens of parsecs separation. In [19] we have concluded that for scales less than 100 pc or perhaps even a bit more, the rotation of the disk is not important. This argument, on the other side, demonstrates that HI velocity is a poor indicator of the real line-of-sight position in PPV cubes.

The equation (7) is an important and rather universal result. However, it contains several approximations which entered the derivation and have to be kept in mind:

- We have assumed that the underlying turbulent velocity obeys Gaussian statistics.
- No such assumptions have been made for the density, however.

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3 All velocities are the line-of-sight velocities, we omit any special notation to denote z-component.
• Perhaps most importantly, we have assumed that the velocity and density are uncorrelated. It is definitely true if they are taken at the same point due to vector nature of velocity field, but needs not be accurate at finite separations. An example of a process which may produce correlations is the self-gravity of the gas, which will point some velocity vectors towards overdensities. This assumption was tested numerically in [28, 29]. Correlations have to be quite high to change our results, and that level of correlation has not been observed.

• At the final stage, we assume homogeneity along the velocity dimension as well as in two spatial directions on the sky. For this to be applicable in practice, one may have to subtract the average spectral line profile and work with fluctuations of intensity.

• The formalism, leading to the equation (7) has to be extended if the temperature of the gas varies from point to point and the turbulence is near subsonic.

The correlation in PPV space reflects, as expected, both underlying inhomogeneities of the matter through $\xi(r)$ and its velocity through $D_x(r)$. Important feature of the PPV space is that $\rho_s(X, v)$ exhibits fluctuations even if the flow is incompressible and no density fluctuations are present. Indeed, when one substitutes the expanded expression (3), $\xi(r) = \hat{\rho}^2 + \hat{\rho}^2 (r_0/r)^7$, into eq (7), both terms give rise to non-trivial contributions to $\xi_s(R, v)$.

FROM THE LINE INTENSITY FLUCTUATIONS IN VELOCITY CHANNELS TO THE CHARACTERISTICS OF A TURBULENT CASCADE

Intensity $I_v(X)$ in the spectral line measured at the velocity $v$ in the direction $X$ is obtained by integrating the standard equation of radiative transfer (Spitzer 1978)

$$dI_v = -g_v I_v dz + j_v dz$$

along the line of sight. In the case of self-adsorbing emission in spectral lines the coefficients are proportional to first power of density:

$$g_v(z) = \alpha(z) \rho(z) \phi_v(z)$$
$$j_v(z) = \epsilon \rho(z) \phi_v(z)$$

where $\phi_v(z)$ describes the velocity distribution of atoms at the position $z$ along the line of sight, and all quantities have implicit dependence on the sky direction $X$. Obvious relation $\int dz \rho(z) \phi_v(z) = \rho_s(X, v)$ links us to the formalism of PPV density.

A solution of the radiative transfer equation if no external illumination is present and absorption coefficient $\alpha$ can be taken as constant (the latter is the essence of the Sobolev approximation) can be written in a compact form [20]

$$I_v(X) = \frac{\epsilon}{\alpha} \left[ 1 - e^{-\alpha \rho_s(X, v)} \right] .$$
In the case of vanishing absorption, the intensity is given by the linear term in the expansion of the exponent in eq. (10)

\[ I_\nu(X) = \varepsilon \rho_s(X, \nu) \]  

and reflects the PPV density of the emitters. If, however, the absorption is strong, the intensity of the emission is saturated at the value \( \varepsilon / \alpha \) wherever \( \rho_s(X, \nu) \gg 1 / \alpha \).

Identification of the low contrast residual fluctuations may be difficult in practice. In observations we register the intensity \( I_C(X) \), integrated over velocity channel

\[ I_C(X) = \int_{-\infty}^{\infty} dv W(v - v^*) I_\nu(X) \]  

which width and shape is described by the window function \( W(v) \). The properties of the window function are restricted, first of all, by the experimental setup. For instance for CO lines studies [30, 24] integration is typically being performed over the whole line, \( W(v) = 1 \). On the other hand, measurements in 21 cm HI line are performed in velocity slices of PPV data cube (channel maps), which corresponds to \( W(v - v^*) \) strongly peaked at a particular velocity \( v^* \).

When velocity channels are narrow we have an opportunity to synthetically increase the width of the channels by combining the data from the adjacent ones. Within the VCA technique we point out that by varying the width of synthetic velocity channels one can separate statistics of turbulent velocities and density inhomogeneities of emitting medium. The minimal width of the velocity channel is determined by the resolution of an instrument. However, thermal motions of the gas lead to the smearing of intensity fluctuations similar in effect to finite resolution. Qualitatively, one can think of the minimal width of the channels available for VCA as given by a convolution of instrumental and thermal effects.

Main statistical quantity that we measure from two dimensional intensity maps \( I(X) \) is the structure (correlation) function

\[ \mathcal{D}(R) \equiv \left\langle [I_C(X_1) - I_C(X_2)]^2 \right\rangle, \quad R = X_1 - X_2, \]  

\[ \Xi(R) \equiv \left\langle I_C(X_1)I_C(X_2) \right\rangle, \]  

(or, alternatively, 2D power spectrum). Here I shall proceed to describe how underlying three dimensional statistics are reflected in this function.

**Optically thin lines**

From the point of view of VCA approach, the most informative data comes from optically thin lines, such as, for example HI 21 cm. In the limit when the absorption can be neglected line intensity contains direct information about PPV density of emitters (11) and the correlation function is simply

\[ \mathcal{D}(R) = \varepsilon^2 \int dv_1 W(v_1) \int dv_2 W(v_2) [d_s(R, v_1, v_2) - d_s(0, v_1, v_2)] \]  

\[ \Xi(R) = \left\langle I_C(X_1)I_C(X_2) \right\rangle. \]
\[ \Xi(R) = \varepsilon^2 \int dv_1 W(v_1) \int dv_2 W(v_2) \xi_s(R, v_1, v_2). \quad (16) \]

Let us assume that after subtracting the mean line profile, the fluctuations of PPV density are statistically homogeneous not only in \( \mathbf{X} \), but in velocity direction as well\(^4\). We shall write out in detail the correlation function as leading to more compact expressions, but present the final results in terms of more appropriate \( \vartheta(R) \). From the eq. (7), in the expanded form,

\[ \Xi(R) \propto \varepsilon^2 \int_{-\infty}^{\infty} dz \xi(r) D_z(r)^{-1/2} \int dv W^2_e(v) \exp \left[ -\frac{v^2}{2D_z(r)} \right] , \quad (17) \]

where \( v \) is the difference in velocities between two emitters, \( v = v_1 - v_2, v_+ = v_1 + v_2 \) and the effective channel window \( W^2_e(v) \) is a square of the experimental window convolved with the gaussian that describes the thermal velocities

\[ W^2_e(v) = \frac{1}{\sqrt{4\pi\beta}} \int dy \exp \left[ -\frac{(v-y)^2}{4\beta} \right] \int dv_+ W_C(v_+ - y)W_C(v_+ + y). \]

For convenience let us parameterize the effective window with a gaussian shape with the width \( \Delta V \), \( W^2_e = \Delta V^{-1} \exp[-v^2/(2\Delta V^2)] \). The exact shape, of course, depends on the properties of the instrument.

Equation (17) provides the main theoretical foundation for VCA of optically thin lines. It describes two fundamental regimes:

**thick channel** The effective channel width \( \Delta V \) is larger than the turbulent velocities at the scale \( R \), \( \Delta V \gg D_z(R)^{1/2} \). In this case we collect inside our channel essentially all emitters, however scattered along the velocity axis they are by the turbulence. Essentially, \( W^2_e \approx 1 \) and

\[ \Xi(R) \propto \int_{-\infty}^{\infty} dz \xi(r) . \quad (18) \]

Information about the turbulent velocities is erased, but one can recover information about density inhomogeneities.

**thin channel**, \( \Delta V \ll D_z(R)^{1/2} \). Turbulent velocity differences are important, many emitter pairs close along the line of sight acquire disparate velocities and do not contribute to the correlation of sources restricted to the narrow velocity channel. The amplitude of correlation decreases, but the most important outcome is that this decrease is scale dependent and we have a change in scaling law which reflects the underlying velocity statistics. Indeed, in this case

\[ \Xi(R) \propto W^2_e(0) \int_{-\infty}^{\infty} dz \xi(r) D_z(r)^{-1/2} . \quad (19) \]

\(^4\) For an infinite emitting medium homogeneous turbulence produces a homogeneous image in the velocity space. For a finite emitting cloud the PPV image is approximately homogeneous over velocity separations much less than the Doppler line-width. If we use the structure function, we do not need to explicitly worry about subtracting the mean line profile, since its integrand does not depend on it.
TABLE 1. Scaling of emission intensity in optically thin lines

|                  | $\mathcal{D}_v(R)$ | $\mathcal{D}_\rho(R)$ | $P_v(K)$ | $P_\rho(K)$ |
|------------------|---------------------|------------------------|----------|-------------|
| Thin channel, $\Delta V^2 \ll CR^m$ | $\propto R^{1-m/2}$ | $\propto R^{1-\gamma-m/2}$ | $\propto K^{-3+m/2}$ | $\propto K^{n+m/2}$ |
| Thick channel, $\Delta V^2 \gg CR^m$ | 0 | $\propto R^{1-\gamma}$ | 0 | $K^n$ |

Interpretation of thick channel data contains no surprises, reflecting projected density inhomogeneities of the gas. However if measurements are done in a sufficiently thin channel (and the criterion depends on scale between two lines of sight), the fluctuations of intensity may come from two sources. Recall that $\xi(r) \propto 1 + (r/r_0)^{-\gamma}$, therefore

$$\Xi(R) \propto \int_{-\infty}^{\infty} dz D_z(r)^{-1/2} + \int_{-\infty}^{\infty} dz (r/r_0)^{-\gamma} D_z(r)^{-1/2} \propto 1 + \Xi_v(R) + \Xi_\rho(R) \ . \ (20)$$

The first term describes the intensity fluctuations arising just from turbulent motions of emitters. The second is the velocity modified scaling of the underlying density fluctuations. In Table 1 we summarize the predicted scaling for the optically thin emission intensity in the channel maps, giving both the structure functions, and the correspondent two dimensional power spectra $P_v(K) \propto \int dR \Xi_v(R) e^{iKR}$ and $P_\rho(K) \propto \int dR \Xi_\rho(R) e^{iKR}$.

Thus, we are in a position when velocity information can be obtained from thin channel data, while increase of channel thickness will determine the density distribution. If the density scaling is steep, $n < -3$, as is the case of emitters, behaving as passive scalars in incompressible Kolmogorov cascade, thin channel measurements are velocity dominated and exhibit spectra $\propto K^{-3+m/2} \sim K^{-2.7}$ if the stochastic velocities are Kolmogorov, $m = 2/3$.

It is a question whether one can obtain thin channels in a particular observations. The criterion is scale dependent, for sufficiently small separation between the lines of sight, the effective width of the channel is always large, but the needed larger separations may be more difficult to measure. Thermal effects also increase $\Delta V$. Thus, observing in thin channels may require special design and be achievable only for cold gas. However, as we have shown in [19], the HI observations, in particular [21, 23], are, effectively, the thin channel measurements. Thus we may interpret the observed slope of $-2.7$ as a signature of the Kolmogorov stochastic velocities, but we cannot exclude an alternative that the underlying density inhomogeneities have enhanced small scale power and $n \approx -3$.

Effects of self-absorption on spectral line statistics

Relation to the underlying turbulence of the intensity in spectral lines that exhibit self-absorption is much more involved [20]. If the absorption is strong, the intensity of the

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5 this term contains constant which corresponds just to mean intensity in the map. It has to be regularized out to make sense of the integral, which is done automatically if structure functions are used.
FIGURE 1. Structure function of intensity fluctuations from Kolmogorov turbulent motion in case of weak self-absorption in the spectral line.

emission (10) is saturated at the value \( \varepsilon / \alpha \) wherever \( \rho_s(X,v) \gg 1/\alpha \). Identification of the low contrast residual fluctuations may be difficult in practice.

However the study of such lines is crucial, in particular for understanding molecular clouds. To make a progress, we look for the regimes when intensity spectra can still exhibit scaling response to the power-law turbulent cascade. In [20] we have found that in the case of weak but not zero absorption, the universal scaling solution \( D(R) \propto R \) (this corresponds to \( K^{-3} \) for the power spectrum) arise over the range of intermediate scales.

Let us return to the structure function \( D(R) \). One can always consider sufficiently small scales such \( \alpha^2 \left[ d_s(R,v) - d_s(0,v) \right] \ll 1 \). If this holds, we argue in [20] that

\[
D(R) \propto \int dv \ W_e^2(v) \ e^{-\frac{\alpha^2}{2}d_s(0,v)} \left[ d_s(R,v) - d_s(0,v) \right], \tag{21}
\]

provides a good (although not fully rigorous) approximation to intensity structure function. Although this expression looks similar to eq (16) for the optically thin case, the conditions for its validity are more relaxed than a limit \( \alpha \to 0 \).

The most important effect induced by absorption is retained. It is an additional exponential down-weighting of the contribution from the points with large velocity separation \( v \) in a manner which itself depends on the turbulence statistics. This has an effect of the stochastic velocities having an impact even on the fully integrated over the frequency lines.

Here we limit ourselves to the short discussion of this effect, considering density inhomogeneities to be subdominant in PPV statistics. For more general issues see [20]. Numerical calculations of the \( D(R) \) for Kolmogorov underlying velocity given in Figure II show that at large scales \( R \) the effects of absorption become highly nonlinear (indeed, the approximation (21) breaks down), correlations are erased and the structure function flattens. At shorter scales, where one may perhaps expect the regime of absorption behaving like a thin velocity channel, the structure function exhibits the
TABLE 2.  Scalings of structure functions of intensity fluctuations arising from velocity fluctuations for the power-law underlying 3D velocity statistics. In the strong absorption regime $\mathcal{D}(R)$ does not follow a simple power-law. The regime of thin channel is not realized for $m \geq 2/3$, including the Kolmogorov case.

| Scale Range | Intensity Scaling | Regime |
|-------------|------------------|--------|
| $R/S < \left( \frac{\beta}{D_z(S)} \right)^{1/m}$ | velocity effects erased | (subsonic regime) |
| $R/S \ll \left( \frac{\Delta v_{ab}^2}{D_z(S)} \right)^{1/m}$ | $\mathcal{D}(R) \propto R^{1+m/2}$ | (transition to thick slice) |
| $R/S < \left( \frac{\Delta v_{ab}^2}{D_z(S)} \right)^{1/m}$ | $\mathcal{D}(R) \propto R^1$ | (intermediate scaling) |
| $\left( \frac{\Delta v_{ab}^2}{D_z(S)} \right)^{1/m} < R/S < \left( \frac{\Delta v_{ab}^2}{D_z(S)} \right)^{2/(2-m)}$ | $\mathcal{D}(R) \propto R^{1-m/2}$ | (thin slice) |
| $\left( \frac{\Delta v_{ab}^2}{D_z(S)} \right)^{2/(2-m)} < R/S$ | not a power law | (strong absorption regime) |

$R^1$ slope instead, which we found to be independent on underlaying velocity scaling. At even shorter scales the absorption becomes negligible and the ordinary thick channel regime takes over.

What determines the scales of transition? We may think of absorption as providing a window with the width $\Delta v_{ab}$ defined by

$$\alpha^2 d_s(0, \Delta v_{ab}) = 1 \quad (22)$$

Only the matter with relative velocities less than $\Delta v_{ab}$ contribute to observed intensity in a correlated fashion. Smaller is the $\Delta v_{ab}$, stronger are the absorption effects. This critical relative velocity depends on the mean density in PPV space, the absorption coefficient, and the slope of the PPV structure function. How important the absorption is for intensity correlations at a scale $R$ depends on comparison between $\Delta v_{ab}$ and $rms$ turbulent velocity at this scale, $D_z(S)^{1/2} (R/S)^{m/2}$ where $S$ is the size of our cloud. If $\Delta v_{ab}$ is larger than $rms$ velocity differences between emitters, its effect is gradually diminished. This translates into absorption being less important on small scales and more important on large scales. A detailed account of different regimes and transition scales is given in Table 2.

In view of this analysis, CO measurements of [24] which show shallower, $-2.7$, slope instead of $-3$ may be taken to indicate that in molecular clouds the density inhomogeneities have enhanced small scale power with $n > -3$ and dominate the velocity contribution. However to make definitive statement one needs to make sure that the difference of 0.3 in the power spectrum can not arise from the noise in the data. $\delta n = 0.3$ or better seems to be an accuracy we need to have to distinguish between different theoretical possibilities discussed in this paper.

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6 The asymptotic analysis in [24] shows how this behavior arises
SUMMARY

We have reviewed the formalism of Velocity Channel Analysis for optically thin lines and its extension to the lines with self-absorption. We demonstrate that by observing optically thin lines from cold gas in sufficiently narrow (thin) velocity channels one may recover the scaling of the stochastic velocities from turbulent cascade, in particular, Kolmogorov velocities give $K^{-2.7}$ contribution to the intensity power spectrum. This matches the observational data in HI, both in our Galaxy and in Small Magellanic cloud. Synthetically increasing the channel thickness separates out the underlying density inhomogeneities of the gas. An attempt to apply this technique to HI lines in SMC indeed showed the expected steepening of the spectrum with the channel thickness.

Effects of self absorption, on the other hand, retain the velocity signature even for integrated lines. As a result, intensity fluctuations from velocity tend to show universal scaling of the power $\propto K^{-3}$ over the range of scales. Observed shallower spectra $n \approx -2.6$ in CO lines therefore may indicate that real density spectrum is also shallow $n \approx -2.6$ and dominates the signal.

Progress in interpretation of the data require the knowledge of the power spectrum index better than with an accuracy of 0.3 and developing of the accurate theoretical and numerical models to match such accuracy in different regimes.

ACKNOWLEDGMENTS

Research of AL is supported by the NSF Grant AST-0307869 and by NSF Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas, while research of DP is supported by the Natural Sciences and Engineering Research Council of Canada.

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