On the value of $\alpha_s$ from the analysis of the SLAC/BCDMS deep inelastic scattering data.

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We performed NLO QCD analysis of the nonsinglet part of the combined SLAC/BCDMS data on $F_2$ with the extraction of $\alpha_s$ and high twist contribution. It was shown that the value of $\alpha_s$ obtained in the analysis is sensitive to the statistical inference procedures dealing with systematic errors on the data. The fit with the complete account of point-to-point correlations of the data gave the value of $\alpha_s(M_Z) = 0.1180 \pm 0.0017$ (68\% C.L.), to be compared with the previously reported value of $\alpha_s(M_Z) = 0.113 \pm 0.003$ (99\% C.L.). This new value of $\alpha_s$ is compatible with the LEP measurements and the world average. The high twist contribution being strongly anti-correlated with the value of $\alpha_s$ became lower than it was previously reported.

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I. INTRODUCTION

It is well known that the value of the strong coupling constant $\alpha_s(M_Z)$ measured at LEP is larger than the value of $\alpha_s(M_Z)$ obtained from the evolution of results of the analysis of the combined SLAC/BCDMS DIS data on proton and deuterium [2] laying at lower $Q^2$ [1]. This discrepancy caused a lot of discussions (see e.g. [3]) and is often attributed to the existence of new fundamental particles, which can change the dependence of $\alpha_s$ on $Q^2$. Meanwhile, the value of $\alpha_s$ from [2] is strongly correlated with the value of simultaneously fitted high twist (HT) contribution. This correlation is inevitable if one does not make a sufficient $Q^2$ cut of data, otherwise the power corrections can essentially, if not completely, imitate the logarithmic scaling violation [4]. The separation of the power and logarithmic behavior is complicated in the case of SLAC/BCDMS data analysis because these data do not practically overlap and exhibit significant discrepancies in the vicinity of the overlapping regions. To achieve a satisfactory description of the data, one is to invent a method to interpret these discrepancies, which is obviously cannot be done without some adoptions. The larger is the correlation of the fitted parameters the more sensitive their values are to the perturbations of other inputs to the fit and hence any adoption made in the analysis should be accurately clarified.

The analysis [2] is not absolutely rigorous in the points concerning the inference of systematic errors. The number of independent systematic errors for the combined SLAC/BCDMS data set is about 40 and the authors of [2] combined most of them in a quadrature claiming that this would not distort the results. In the present work we investigated the effect of this adoption on the bias of the fitted parameters.

II. THE DATA AND THEIR SYSTEMATIC ERRORS

We analysed essentially the same data set [2] as in [2] with the minor differences:

- we used the data on cross sections separated by the beam energies instead of merged data on $F_2$. For the SLAC data we withdraw the merging errors in this way. The BCDMS data within this approach were reduced to the value of $R = \sigma_L/\sigma_T$ [2] common to the SLAC data.

- we imposed the more stringent cut $x \geq 0.3$ to prevent additional uncertainties due to a poorly known gluon distribution. This cut leaves the data which can be in good approximation described by the pure nonsinglet structure functions, which essentially reduces the number of the fitted parameters. At the same time the value of $\alpha_s$ in the fit to the combined SLAC/BCDMS data is basically determined by the high-x points and we did not loose statistical significance of the analysis as one can see from the final results. The cut $x \leq 0.75$ coinciding with [2] and rejecting the region where the binding effects in deuterium can be important was also imposed in the analysis. The $Q^2$ range of the data left after the cut is $1 \text{ GeV}^2 < Q^2 < 230 \text{ GeV}^2$.

The number of data points (NDP) and the number of independent systematic errors (NSE) for each experiment used in our analysis are presented in Table I. The systematic errors on the BCDMS data are presented by the following independent sources: calibration of the measurement of the incident and scattered muon energy, resolution
of the spectrometer, detector and trigger inefficiencies, relative normalization of data from internal and external targets, general normalization and relative normalization uncertainties between the data set taken at different beam energies. (The latest were ascribed to the data at beam energies of 100, 120 and 280 GeV while the data at 200 GeV were considered as the reference ones.) In the analysis \[2\] the systematic errors from the first three sources were combined in quadrature into a single error called a “main systematic error” and the data points were shifted by the value proportional to this combination while the proportionality coefficient was determined from $\chi^2$ minimization.

The general normalization was also considered as a free parameter and then the value of normalization uncertainty presented in the source paper \[6\] was not explicitly accounted for. The rest systematic errors were considered as uncorrelated and were combined in quadrature with statistical errors.

The correlated systematic errors on the SLAC data arose due to: background contamination, spectrometer acceptance uncertainties and radiative corrections uncertainties. In addition, as far the older SLAC data were normalized to the data from the E-140 experiment, there are two more systematic errors on them: target dependent and target independent relative normalization uncertainties. (The data from E-140 experiment have only one additional absolute normalization error). In the analysis \[2\] all these errors were combined in quadrature with statistical ones.

### III. FITTED FORMULA

The QCD input leading twist (LT) structure functions of proton and neutron were parametrized at the starting value of $Q_0^2 = 9\text{GeV}^2$ as follows:

$$F_2^p(x, Q_0) = A_p x^{a_p} (1 - x)^{b_p} \frac{2}{N_p}$$

$$F_2^n(x, Q_0) = A_n x^{a_n} (1 - x)^{b_n} \frac{1}{N_n}.$$ 

Here conventional normalization factors $N_p$ and $N_n$ are

$$N_{p,n} = \int_0^1 dx x^{a_{p,n} - 1} (1 - x)^{b_{p,n}}.$$ 

These distributions were evolved through the region of $Q^2$ occupied by the data in NLO QCD approximation within $\overline{MS}$ factorization scheme \[7\] with the help of the code used earlier \[8\]. The $Q^2$ dependence of $\alpha_s$ was calculated as the numerical solution of the equation

$$\frac{1}{\alpha_s(Q)} - \frac{1}{\alpha_s(M_Z)} = \frac{\beta_0}{2\pi} \ln \left( \frac{Q}{M_Z} \right) + \beta \ln \left[ \frac{\beta + 1/\alpha_s(Q)}{\beta + 1/\alpha_s(M_Z)} \right],$$

where

$$\beta_0 = 11 - \frac{2}{3} n_f, \quad \beta = \frac{2\pi \beta_0}{51 - \frac{19}{3} n_f}$$

and the number of the active fermions $n_f$ was changed at the values of $Q$ equal to quark masses keeping the continuity of $\alpha_s$. The final formula for structure function used in the fit with account of twist-4 contribution was choose the same as in \[2\]:

$$F_2^{(P,D), HT} = F_2^{(P,D), LT} \left[ 1 + \frac{h^{(P,D)}(x)}{Q^2} \right],$$

where $F_2^{(P,D), LT}$ are the leading twist terms with account of the target mass correction \[8\]. The functions $h^{(P,D)}(x)$ were parametrized in the model independent way: their values at $x = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$ were fitted, between these points the functions were linearly interpolated. As we mentioned before, we used the common value of $R$ \[9\] for all the data including BCDMS ones.

\[1\]We checked that extra polynomial-type factors do not improve the quality of the fits.
IV. RESULTS

A. BCDMS reanalysis

At the first stage of our analysis we used the inference procedures analogous to [2]. The parameters were evaluated through minimization of the functional

$$\chi^2 = \sum_{K,i} \left[ \frac{(f_i - \lambda_K \Delta y_i) / \xi_K - y_i}{\sigma_i^2} \right]^2,$$  \hspace{1cm} (4.1)

where $K$ runs through the data subsets obtained by separation of all analysed data on experiments and targets; $i$ - through data points within these subsets. The other notations are: $y_i$ - the measurements, $\sigma_i$ - the statistical errors, combined with some systematic errors as described above, $f_i$ - theoretical model prediction depending on the fitted parameters, $\Delta y_i$ - the “main systematic error” on the BCDMS data, $\lambda_K$ and $\xi_K$ are fixed at 0. and 1., correspondingly for the SLAC experiments and are the fitted parameters for BCDMS. For the test purposes we fitted formula with the parameters $\xi$ and $\lambda$ fixed at their values as given in [2]. The obtained results are presented in column 1 of Table II and on Fig. 1. The values of HT coefficients obtained in the analysis [2] are also presented on Fig. 1. As far the errors quoted for them in [2] correspond to the change of $\chi^2$ equal to 9., their pictured errors are scaled by the factor of 1/3 to provide a meaningful comparison with our figures. One can see that they coincide within the statistical fluctuations.

The next step was to release these parameters (the results are presented in column 2 of Table II). We can note that for this fit the BCDMS data are renormalized slightly smaller. As a consequence, the value of $\alpha_s$, which exhibits negative correlation with this normalization factor became slightly less than that in [2]. In this connection note that one could suppose the dependence of the normalization factor on the $x$-cut because the $x$-shape of the BCDMS data does not match the SLAC data very well (in particular, it was pointed in [11]). The errors of the $\alpha_s$ value increased two times comparing with the first fit. This is in accordance with the above observation, that $\alpha_s$ is strongly correlated with the normalization factors for the BCDMS data - releasing the latest we allowed more room for the $\alpha_s$ variation.

An alternative possibility to account for the normalization error of the data is to introduce the correlation matrix

$$C_{ij} = \delta_{ij} \sigma_i \sigma_j + f_i f_j s_K^2$$

into the minimized functional in the following way:

$$\chi^2 = \sum_{K,i,j} \left[ (f_i - \lambda_K \Delta y_i) - y_i \right] E_{ij} \left[ (f_j - \lambda_K \Delta y_j) - y_j \right],$$  \hspace{1cm} (4.2)

where $s_K$ is the data normalization uncertainty for each target as it is estimated by the experimentalists and $E_{ij}$ is the inverse of $C_{ij}$; $j$ runs through the data points of each data subset, $\delta_{ij}$ is the Kronecker symbol and the other notations are the same as in (4.1). This approach is natural if one considers a systematic error as a random variable, i.e. within the Bayesian approach (see more in [10] on this scope). The fit within this approach is, in principle, more stable comparing with the renormalization approach (4.1) because in (4.2) the normalization parameter variation is limited by the scale of $s$. In our particular case this is not so important as far one can see from Table II, that the normalization factors for the BCDMS data are anyway within their normalization systematic error (3%). This anticipation is supported by the results of the fit within the approach (4.2) which are also presented in Table II (column 3). Analogously the fitted parameters should not be sensitive to the the stabilization term $(\xi - 1)^2 / s^2$ added to functional (3.3) in [2] as far this term corresponds effectively to the additional measurement of $\xi$ with the average of 1. and the error of $s$; the weighted averaging of this measurement with the value of $\xi$ from Table II cannot evidently change the latest one.

To proceed with the implementation of Bayesian approach for the treatment of systematic errors, we minimized the functional

$$\chi^2 = \sum_{K,i,j} (f_i - y_i) E_{ij} (f_j - y_j),$$  \hspace{1cm} (4.3)

where $E_{ij}$ is the inverse of the correlation matrix

$$C_{ij} = \delta_{ij} \sigma_i \sigma_j + f_i f_j (s_i^K \cdot s_j^K).$$
and each 4-component vector $s^K_i$ includes the normalization uncertainty as well as the three systematic errors which were initially combined into the “main systematic error” of the BCDMS data. The most interesting difference of this fit results (presented in Table II, column 4) from the previous fits is the increase of $\alpha_s$. The value of $\alpha_s$ is strongly anticorrelated with the HT contribution at high $x$ and naturally the last-named decreases correspondingly. The effect is of the order of one standard deviation (as could be anticipated because the value of $\lambda$ is of the order of 1. when it is released in the fit), with the tendency to decrease the discrepancy with the LEP data. Alongside one can observe the decrease of $\chi^2$, which is connected with the fact that in the earlier fits main systematic errors were, as a whole, underestimated when combined in quadrature.

An additional improvement is to account, within this approach, for two more BCDMS systematic errors, which were not included in the “main systematics”: The errors due to detector and trigger inefficiencies. The results of this fit are presented in column 5 of Table II. Again we can see the enlargement of $\alpha_s$ value and the correlated decrease of the HT contribution, although not so large as in the case of the re-account of “main systematics”.

The next step of our analysis was to re-account the errors corresponding to the uncertainty in the relative normalizations of the data subsets for different energies. The results are presented in column 6 of Table II. The value of $\alpha_s$ again increased and the effect is even more pronounced than in the case with the re-account of “main systematics”. This is not surprising because as was stated by the BCDMS collaboration itself the uncertainty in the relative normalizations have the most effect on the error of $\alpha_s$.

Our final exercise with the BCDMS data concerns the correlation of systematic errors on the data from the proton and deuterium targets. The authors note that this correlation is large, but do not quantify it. To investigate the scale of this correlation effect, we performed one more fit assuming the total correlation (column 7 of Table II). The parameter estimates for real proton/deuterium correlation lie between the values from column 6 and 7, more close to 7 and we again observe the increase of $\alpha_s$.

Summarizing, we can conclude that a complete account of point-to-point correlations due to systematic errors on the BCDMS data in the combined SLAC/BCDMS analysis cancels the discrepancy with the LEP results. The effect of $\alpha_s$ increase comparing with the previous analysis arises mainly due to re-account of “main systematics” and the errors due to relative normalizations of the data taken at different energies.

**B. SLAC reanalysis**

For the completeness we accounted for the point-to-point correlation of the SLAC data too. At first we proceeded with the systematic errors on the E-140 data only. The results of the fit are presented in column 1 of Table III and do not essentially differ from the previous fit. As mentioned above the older SLAC data were renormalized to the data from E-140 experiment. Due to the absence of E-140 proton data the renormalization of proton data subsets was performed using “bridging” through the E-49B experiment, which introduced additional uncertainties. As far we used more of the proton data in the analysis, we preferred to perform the independent renormalization. Then, we removed from the systematic errors on the older SLAC data the relative normalization uncertainties which arose due to their renormalization to E-140 and introduced the fitted normalization parameters for each experiment and target into the functional (4.3):

$$\chi^2 = \sum_{K,j} (f_i/\xi_K - y_i)E_{ij}(f_j/\xi_K - y_j),$$

where $\xi_K$ are fixed at 1. for the BCDMS and E-140 data subsets. The results of this fit are presented in Table III, column 2. One can see that our renormalization factors are, as a whole, compatible with 1. within the errors, although there is some tendency to shift proton data up comparing with.

The final step of our analysis was the incorporation of the rest systematic errors into the correlation matrix. The results of this fit are presented in column 3, Table III. The value of $\alpha_s$ due to the last improvement remained unchanged, the main effect was a certain increase of $\chi^2$, while the statistical confidence of the fit remains good. This is readily understood because if one combines the correlated errors in quadratures, the $\chi^2$ is underestimated. In the final fit the relative normalization of SLAC data is in the range of few percent up comparing with the BCDMS data. In the global fits the SLAC data are often used as the reference ones and the BCDMS data are renormalized to them and usually are shifted down by few percent. Our renormalization scheme is in principle compatible with the commonly used one, except for the general normalization. This discrepancy cannot be clarified if one uses in the analysis only the data on DIS as far it is well known that they cannot define the absolute normalization parameters very well, moreover, we applied the cut on $x$ in the analysis. Anyway, it is obvious, that the ambiguity in the general absolute normalization cannot affect determination of a slope on $Q^2$ and, hence, change the value of $\alpha_s$. 

V. SUMMARY

The final value of $\alpha_s(M_Z)$ obtained in our analysis is presented in column 3 of Table III:

$$\alpha_s(M_Z) = 0.1180 \pm 0.0017 (\text{stat} + \text{syst}).$$

It is compatible with the values obtained in the LEP experiments [1] and in the analysis of CCFR data on $F_3$ [14], but is in certain contradiction with the results of [2]:

$$\alpha_s(M_Z) = 0.113 \pm 0.003 (\text{stat} + \text{syst}).$$

For a meaningful comparison it is worth to remind that in the last result the error corresponds to the change of $\chi^2 = 9$, i.e. three standard deviations and, consequently, the distance between our result and [2] is about 3 standard deviations.

The statistical confidence of our final fit ($\chi^2/NDP = 1179/1183$) is perfect, while in [2] $\chi^2/DOF = 599/687$ and hence the value of $\chi^2$ is by more than two standard deviations lower than its supposed mean. This is yet within possible statistical fluctuations, but nevertheless can signal about underestimation of $\chi^2$ due to the combining of systematic errors in quadrature.

In [2] the value of $\alpha_s(50 \text{ GeV}) = 0.180 \pm 0.008$ with the help of the approximate solution of (3.1)

$$\alpha_s(Q) = \frac{2\pi}{\beta_0 \ln(Q/\Lambda)} \left[ 1 - \frac{2\pi}{\beta_0 \beta} \ln(\ln(Q/\Lambda)) \right]$$

was transformed into the value of

$$\Lambda_{\overline{MS}}^{(4)} = 263 \pm 42 (\text{stat} + \text{syst}) \text{ MeV}.$$ 

Our value of $\alpha_s(50\text{GeV}) = 0.1935 \pm 0.0048$ can be analogously transformed into

$$\Lambda_{\overline{MS}}^{(4)} = 337 \pm 28 (\text{stat} + \text{syst}) \text{ MeV}.$$ 

The correlation matrix of the fitted parameters is presented in Table IV. One can see from the table that the correlation of $\alpha_s$ with the HT coefficients is rather large. This supports our initial statement that the separation of logarithmic and power effects in a scaling violation is unstable under various assumptions. Other effects, not taken into account before (e.g. nuclear effects in deuterium), should be investigated before one can elaborate reliable estimate of $\alpha_s$ from the analysis of these data.

As far the HT contribution and the value of $\alpha_s$ are strongly anticorrelated, the increase of $\alpha_s$, which we observed above, is accompanied by the decrease of HT [2]. The total effect on the HT magnitude is about a factor of 3/4, comparing with the results [2]. In this connection it is interesting to compare our results with the predictions of the infrared renormalon (IRR) model [12,13]. This model is known to reasonably reproduce the shape of HT contribution obtained in [2], but the absolute value prediction is about 2.5 times higher than the data (see [13]). The comparison of our results with the IRR model predictions is presented on Fig. 2. The model calculations were made in the nonsinglet approximation using the structure functions and the value of $\Lambda_{\overline{MS}}^{(4)}$ obtained in our analysis:

$$h(x) = \frac{A'_2}{F_{LT}^2(x, Q)} \left( \int_x^1 dz C_2(z) F_{LT}^{LT}(x/z, Q) \right)$$

$$C_2(z) = -\frac{4}{(1-z)_+} + 2(2 + z + 6z^2) - 9\delta(1-z) - \delta'(1-z)$$

$$A'_2 = -\frac{2C_F}{\beta_0} \left( \frac{\Lambda_{\overline{MS}}^{(4)}}{\Lambda_{\overline{MS}}^{(4)}} \right)^2 e^{-C},$$

2 We omitted the correlation coefficients corresponding to the normalization parameters of the SLAC data due to space limitation. The full correlation matrix can be obtained from the author on the request.

3 This effect was also recently observed in the analysis [17], where $\alpha_s(M_Z)$ was fixed at the value of 0.120.
where \( Q^2 = 9 \, \text{GeV}^2 \), \( C_F = 4/3 \), \( C = -5/3 \) and \( F_{LT}^2 \) does not include target mass corrections here. One can see the improved agreement of the IRR model predictions with the data at \( x = 0.5 - 0.7 \).

In conclusion, the separation of the logarithmic and power scaling violation effects in the analysis of deep inelastic scattering data is unstable due to a high correlation of these effects in the \( Q^2 \) region where they are both not small. The complete account of point-to-point correlations of the data lead to the shift in the value of \( \alpha_s \) by about 3 standard deviations comparing with the simplified statistical inference procedure. The HT contribution, which is strongly anti-correlated with \( \alpha_s \), decreases within this approach and becomes more compatible with the prediction of IRR model at moderate \( x \). Further investigation of a possible perturbation in the analysis of DIS data is needed before a reliable value of \( \alpha_s \) can be determined.

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FIG. 1. The high-twist contributions obtained in the fit with the functional (1) (full circles and lines). The results of the analysis [2] are presented for comparison (open circles).

FIG. 2. The high-twist contributions obtained in our final fit (full circles) and the results of the analysis [2] (open circles). The full curves represent the calculations on the IRR model.
TABLE I. The number of data points (NDP) and the number of independent systematic errors (NSE) for the analysed data sets.

| Experiment | NDP(proton) | NDP(deuterium) | NSE |
|------------|-------------|----------------|-----|
| BCDMS      | 223         | 162            | 9   |
| E-49A      | 47          | 47             | 5   |
| E-49B      | 109         | 102            | 5   |
| E-61       | 6           | 6              | 5   |
| E-87       | 90          | 90             | 5   |
| E-89A      | 66          | 59             | 5   |
| E-89B      | 70          | 59             | 5   |
| E-139      | –           | 16             | 5   |
| E-140      | –           | 31             | 4   |
| TOTAL      | 611         | 572            | 45  |

TABLE II. The results of the fits with the various approaches to the treatment of the BCDMS systematic errors. The parameters $\xi$ and $\lambda$ describe the renormalization and shift of the BCDMS data, $h_{3,4,5,6,7,8}$ are the fitted values of the HT contribution at $x = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$. For the description of the columns see the text.

|   | $A_p$       | $a_p$       | $b_p$       | $A_m$ | $a_m$       | $b_m$       | $\alpha(MZ)$ | $\lambda_p$ | $\lambda_D$ | $\xi_P$ | $\xi_D$ | $h^D_3$       | $h^D_4$       | $h^D_5$       | $h^D_6$       | $h^D_7$       | $h^D_8$       | $h^D_9$       | $h^D_{10}$      | $h^D_{11}$      | $h^D_{12}$      | $h^D_{13}$      | $h^D_{14}$      | $h^D_{15}$      |
|---|-------------|-------------|-------------|-------|-------------|-------------|--------------|--------------|--------------|---------|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1 | 0.612 ± 0.028 | 0.579 ± 0.028 | 0.581 ± 0.028 | 0.523 ± 0.024 | 0.531 ± 0.024 | 0.531 ± 0.024 | 0.519 ± 0.022 | 0.95 ± 0.13 | 1.2         | 0.99    | 1.004  | 1.004 ± 0.0016 | 0.130 ± 0.016 | 0.099 ± 0.022 | 0.026 ± 0.022 | 0.15 ± 0.022 | 0.250 ± 0.037 | 0.481 ± 0.072 | 0.146 ± 0.13 | 0.230 ± 0.30 | 1.99 ± 0.30 | 1.66 ± 0.31 | 1.41 ± 0.30 |
| 2 | 0.642 ± 0.028 | 0.689 ± 0.032 | 0.685 ± 0.032 | 0.748 ± 0.033 | 0.736 ± 0.032 | 0.734 ± 0.032 | 0.748 ± 0.030 | 0.95 ± 0.13 | 1.2         | 0.99    | 1.004  | 1.004 ± 0.0016 | 0.130 ± 0.016 | 0.099 ± 0.022 | 0.026 ± 0.022 | 0.15 ± 0.022 | 0.250 ± 0.037 | 0.481 ± 0.072 | 0.146 ± 0.13 | 0.230 ± 0.30 | 1.99 ± 0.30 | 1.66 ± 0.31 | 1.41 ± 0.30 |
| 3 | 0.758 ± 0.029 | 3.675 ± 0.038 | 3.670 ± 0.038 | 3.702 ± 0.038 | 3.686 ± 0.037 | 3.670 ± 0.037 | 3.667 ± 0.035 | 0.95 ± 0.13 | 1.2         | 0.99    | 1.004  | 1.004 ± 0.0016 | 0.130 ± 0.016 | 0.099 ± 0.022 | 0.026 ± 0.022 | 0.15 ± 0.022 | 0.250 ± 0.037 | 0.481 ± 0.072 | 0.146 ± 0.13 | 0.230 ± 0.30 | 1.99 ± 0.30 | 1.66 ± 0.31 | 1.41 ± 0.30 |
| 4 | 4.0 ± 3.2     | 4.0 ± 3.5    | 3.7 ± 3.0    | 4.7 ± 4.8    | 3.4 ± 2.6    | 4.2 ± 3.8    | 4.7 ± 4.4    | 0.95 ± 0.13 | 1.2         | 0.99    | 1.004  | 1.004 ± 0.0016 | 0.130 ± 0.016 | 0.099 ± 0.022 | 0.026 ± 0.022 | 0.15 ± 0.022 | 0.250 ± 0.037 | 0.481 ± 0.072 | 0.146 ± 0.13 | 0.230 ± 0.30 | 1.99 ± 0.30 | 1.66 ± 0.31 | 1.41 ± 0.30 |
| 5 | 0.14 ± 0.11   | 0.14 ± 0.12  | 0.15 ± 0.12  | 0.12 ± 0.12  | 0.16 ± 0.12  | 0.13 ± 0.12  | 0.12 ± 0.11  | 0.95 ± 0.13 | 1.2         | 0.99    | 1.004  | 1.004 ± 0.0016 | 0.130 ± 0.016 | 0.099 ± 0.022 | 0.026 ± 0.022 | 0.15 ± 0.022 | 0.250 ± 0.037 | 0.481 ± 0.072 | 0.146 ± 0.13 | 0.230 ± 0.30 | 1.99 ± 0.30 | 1.66 ± 0.31 | 1.41 ± 0.30 |
| 6 | 3.52 ± 0.12   | 3.52 ± 0.14  | 3.54 ± 0.14  | 3.48 ± 0.14  | 3.52 ± 0.14  | 3.48 ± 0.14  | 3.51 ± 0.12  | 0.95 ± 0.13 | 1.2         | 0.99    | 1.004  | 1.004 ± 0.0016 | 0.130 ± 0.016 | 0.099 ± 0.022 | 0.026 ± 0.022 | 0.15 ± 0.022 | 0.250 ± 0.037 | 0.481 ± 0.072 | 0.146 ± 0.13 | 0.230 ± 0.30 | 1.99 ± 0.30 | 1.66 ± 0.31 | 1.41 ± 0.30 |
| 7 | 0.1141 ± 0.0007 | 0.1089 ± 0.0016 | 0.1093 ± 0.0016 | 0.1119 ± 0.0015 | 0.1140 ± 0.0017 | 0.1173 ± 0.0018 | 0.1188 ± 0.0018 | 1090.5 | 1067.5 | 1068.3 | 963.7 | 964.3 | 973.3 | 971.5 |
TABLE III. The results of the fits with various approaches to the treatment of the SLAC systematic errors. The parameters $\xi$ describe the renormalization of the SLAC data, $h_{3,4,5,6,7,8}$ are the fitted values of the HT contribution at $x = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$. For the description of the columns see the text.

|          | 1              | 2              | 3              |
|----------|----------------|----------------|----------------|
| $A_p$    | $0.527 \pm 0.022$ | $0.546 \pm 0.025$ | $0.516 \pm 0.022$ |
| $a_p$    | $0.738 \pm 0.030$ | $0.723 \pm 0.030$ | $0.765 \pm 0.028$ |
| $b_p$    | $3.656 \pm 0.035$ | $3.642 \pm 0.034$ | $3.692 \pm 0.032$ |
| $A_\alpha$ | $3.8 \pm 2.9$   | $4.9 \pm 4.4$   | $4.8 \pm 4.1$   |
| $a_\eta$ | $0.15 \pm 0.11$  | $0.12 \pm 0.10$  | $0.118 \pm 0.097$ |
| $b_\eta$ | $3.54 \pm 0.12$  | $3.51 \pm 0.12$  | $3.51 \pm 0.11$  |
| $\alpha_s(M_Z)$ | $0.1188 \pm 0.0018$ | $0.1183 \pm 0.0017$ | $0.1180 \pm 0.0017$ |
| $h_{3}^H$ | $-0.140 \pm 0.017$ | $-0.136 \pm 0.018$ | $-0.120 \pm 0.017$ |
| $h_{4}^H$ | $-0.069 \pm 0.026$ | $-0.052 \pm 0.027$ | $-0.046 \pm 0.025$ |
| $h_{5}^H$ | $0.031 \pm 0.046$ | $0.059 \pm 0.045$ | $0.059 \pm 0.043$ |
| $h_{6}^H$ | $0.341 \pm 0.083$ | $0.400 \pm 0.081$ | $0.392 \pm 0.076$ |
| $h_{7}^H$ | $0.72 \pm 0.14$ | $0.79 \pm 0.13$ | $0.82 \pm 0.13$ |
| $h_{8}^H$ | $1.38 \pm 0.30$ | $1.44 \pm 0.28$ | $1.54 \pm 0.25$ |
| $h_{3}^D$ | $-0.128 \pm 0.021$ | $-0.134 \pm 0.019$ | $-0.123 \pm 0.018$ |
| $h_{4}^D$ | $-0.005 \pm 0.029$ | $-0.007 \pm 0.027$ | $-0.003 \pm 0.026$ |
| $h_{5}^D$ | $0.145 \pm 0.049$ | $0.159 \pm 0.045$ | $0.162 \pm 0.043$ |
| $h_{6}^D$ | $0.442 \pm 0.084$ | $0.446 \pm 0.080$ | $0.439 \pm 0.076$ |
| $h_{7}^D$ | $0.79 \pm 0.13$ | $0.77 \pm 0.12$ | $0.79 \pm 0.12$ |
| $h_{8}^D$ | $1.93 \pm 0.31$ | $1.84 \pm 0.29$ | $1.87 \pm 0.26$ |
| $\xi_{P,49}$ | $-$ | $1.016 \pm 0.017$ | $1.016 \pm 0.018$ |
| $\xi_{D,49}$ | $-$ | $1.007 \pm 0.016$ | $1.006 \pm 0.017$ |
| $\xi_{P,49}$ | $-$ | $1.021 \pm 0.017$ | $1.028 \pm 0.018$ |
| $\xi_{D,49}$ | $-$ | $1.006 \pm 0.016$ | $1.012 \pm 0.017$ |
| $\xi_{P,61}$ | $-$ | $1.019 \pm 0.020$ | $1.021 \pm 0.021$ |
| $\xi_{D,61}$ | $-$ | $1.004 \pm 0.018$ | $1.004 \pm 0.019$ |
| $\xi_{P,87}$ | $-$ | $1.018 \pm 0.017$ | $1.025 \pm 0.017$ |
| $\xi_{D,87}$ | $-$ | $1.006 \pm 0.016$ | $1.012 \pm 0.017$ |
| $\xi_{P,89}$ | $-$ | $1.023 \pm 0.018$ | $1.028 \pm 0.021$ |
| $\xi_{D,89}$ | $-$ | $1.001 \pm 0.017$ | $1.004 \pm 0.021$ |
| $\xi_{P,89}$ | $-$ | $1.022 \pm 0.017$ | $1.022 \pm 0.017$ |
| $\xi_{D,89}$ | $-$ | $1.007 \pm 0.016$ | $1.007 \pm 0.017$ |
| $\xi_{D,139}$ | $-$ | $1.012 \pm 0.016$ | $1.009 \pm 0.017$ |
| $\chi^2$ | $971.8$ | $1040.8$ | $1178.9$ |
|        | $a_p$ | $b_p$ | $a_n$ | $b_n$ | $\alpha_s(M_Z)$ | $A_p$ | $A_n$ | $h_3^p$ | $h_4^p$ | $h_5^p$ | $h_6^p$ | $h_7^p$ | $h_8^p$ | $h_3^D$ | $h_4^D$ | $h_5^D$ | $h_6^D$ | $h_7^D$ | $h_8^D$ |
|--------|-------|-------|-------|-------|-----------------|-------|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $a_p$  | 1.00  | 0.93  | -0.50 | -0.45 | -0.07           | -0.92 | 0.50  | 0.61    | 0.11    | -0.10   | -0.09   | 0.12    | 0.34    | 0.01    | 0.08    | 0.05    | 0.05    | 0.08    | 0.11    |
| $b_p$  | 0.93  | 1.00  | -0.46 | -0.44 | -0.27           | -0.82 | 0.46  | 0.55    | 0.22    | 0.09    | 0.14    | 0.37    | 0.53    | 0.11    | 0.25    | 0.25    | 0.24    | 0.28    | 0.25    |
| $a_n$  | -0.50 | -0.46 | 1.00  | 0.96  | -0.04           | 0.46  | -0.99 | -0.28   | 0.01    | 0.12    | -0.12   | -0.14   | 0.56    | 0.04    | -0.07   | -0.04   | 0.11    | 0.28    |
| $b_n$  | -0.45 | -0.44 | 0.96  | 1.00  | -0.09           | 0.40  | -0.96 | -0.19   | 0.07    | 0.16    | 0.13    | 0.00    | 0.14    | 0.51    | 0.06    | -0.02   | 0.04    | 0.12    | 0.36    |
| $\alpha_s(M_Z)$ | -0.07 | -0.27 | -0.04 | -0.09 | 1.00            | 0.04  | 0.04  | -0.33   | -0.79   | -0.89  | -0.87   | -0.67   | -0.55   | -0.90   | -0.95   | -0.92   | -0.92   | -0.67   |
| $A_p$  | -0.92 | -0.82 | 0.46  | 0.40  | 0.04            | 1.00  | -0.46 | -0.58   | -0.11   | 0.11    | -0.11   | -0.07   | -0.28   | -0.01   | -0.06   | -0.04   | -0.03   | -0.05   | -0.08   |
| $A_n$  | 0.50  | 0.46  | -0.99 | -0.96 | 0.04            | -0.46 | 1.00  | 0.28    | -0.00   | -0.12   | -0.12   | 0.00    | 0.14    | -0.56   | -0.04   | 0.07    | 0.04    | -0.11   | -0.27   |
| $h_3^p$| 0.61  | 0.55  | -0.28 | -0.19 | -0.33           | -0.58 | 0.28  | 1.00    | 0.40    | 0.34    | 0.25    | 0.32    | 0.35    | 0.20    | 0.35    | 0.34    | 0.31    | 0.32    | 0.25    |
| $h_4^p$| 0.11  | 0.22  | 0.01  | 0.07  | -0.79           | -0.11 | 0.00  | 0.40    | 1.00    | 0.76    | 0.78    | 0.71    | 0.54    | 0.45    | 0.75    | 0.77    | 0.74    | 0.73    | 0.52    |
| $h_5^p$| -0.10 | 0.09  | 0.12  | 0.16  | -0.88           | 0.11  | -0.12 | 0.34    | 0.76    | 1.00    | 0.84    | 0.81    | 0.56    | 0.50    | 0.81    | 0.85    | 0.82    | 0.81    | 0.58    |
| $h_6^p$| -0.09 | 0.14  | 0.12  | 0.13  | -0.89           | 0.11  | -0.12 | 0.25    | 0.78    | 0.84    | 1.00    | 0.82    | 0.65    | 0.50    | 0.81    | 0.86    | 0.84    | 0.83    | 0.60    |
| $h_7^p$| 0.12  | 0.37  | -0.01 | 0.00  | -0.87           | -0.07 | 0.00  | 0.32    | 0.71    | 0.81    | 0.83    | 1.00    | 0.64    | 0.48    | 0.79    | 0.83    | 0.82    | 0.82    | 0.61    |
| $h_8^p$| 0.34  | 0.53  | -0.14 | -0.14 | -0.67           | -0.28 | 0.14  | 0.35    | 0.54    | 0.56    | 0.65    | 0.64    | 1.00    | 0.35    | 0.61    | 0.64    | 0.63    | 0.64    | 0.50    |
| $h_3^D$| 0.01  | 0.11  | 0.56  | 0.51  | -0.55           | -0.01 | 0.56  | 0.20    | 0.45    | 0.50    | 0.50    | 0.48    | 0.35    | 1.00    | 0.51    | 0.50    | 0.44    | 0.52    | 0.50    |
| $h_4^D$| 0.08  | 0.25  | 0.04  | 0.06  | -0.90           | -0.06 | -0.04 | 0.35    | 0.75    | 0.81    | 0.81    | 0.79    | 0.61    | 0.51    | 1.00    | 0.90    | 0.86    | 0.83    | 0.59    |
| $h_5^D$| 0.05  | 0.25  | -0.07 | -0.02 | -0.95           | -0.04 | 0.07  | 0.34    | 0.77    | 0.85    | 0.86    | 0.83    | 0.64    | 0.50    | 0.90    | 1.00    | 0.92    | 0.89    | 0.60    |
| $h_6^D$| 0.05  | 0.24  | -0.04 | 0.04  | -0.92           | -0.03 | 0.04  | 0.31    | 0.74    | 0.82    | 0.84    | 0.82    | 0.63    | 0.44    | 0.86    | 0.92    | 1.00    | 0.89    | 0.66    |
| $h_7^D$| 0.08  | 0.28  | 0.11  | 0.20  | -0.92           | -0.05 | -0.11 | 0.32    | 0.73    | 0.81    | 0.83    | 0.82    | 0.64    | 0.52    | 0.83    | 0.89    | 0.89    | 1.00    | 0.68    |
| $h_8^D$| 0.11  | 0.25  | 0.28  | 0.36  | -0.67           | -0.08 | -0.27 | 0.25    | 0.52    | 0.58    | 0.60    | 0.61    | 0.50    | 0.50    | 0.59    | 0.60    | 0.66    | 0.68    | 1.00    |
Fig. 1
Fig. 2