$J^{PC} = 1^{++}$ heavy hybrid masses from QCD sum-rules

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Abstract

QCD Laplace sum-rules are used to calculate axial vector ($J^{PC} = 1^{++}$) charmonium and bottomonium hybrid masses. Previous sum-rule studies of axial vector heavy quark hybrids did not include the dimension-six gluon condensate, which has been shown to be important in the $1^{--}$ and $0^{++}$ channels. An updated analysis of axial vector heavy quark hybrids is performed, including the effects of the dimension-six gluon condensate, yielding mass predictions of 5.13 GeV for hybrid charmonium and 11.32 GeV for hybrid bottomonium. The charmonium hybrid mass prediction disfavours a hybrid interpretation of the $X(3872)$, if it has $J^{PC} = 1^{++}$, in agreement with the findings of other theoretical approaches. It is noted that QCD sum-rule results for the $1^{--}$, $0^{++}$ and $1^{++}$ channels are in qualitative agreement with the charmonium hybrid multiplet structure observed in recent lattice calculations.

Keywords: QCD sum-rules, heavy quark hybrids

1. Introduction

Hybrids are mesons that include explicit gluonic degrees of freedom. These can have non-exotic $J^{PC}$ and hence may coexist with heavy quarkonia. The numerous charmonium-like and bottomonium-like “XYZ” states discovered since 2003 have inspired the search for hybrids within the charmonium and bottomonium sectors $^{[2–5]}$.

In Ref. $[7]$ QCD Laplace sum-rules were used to perform mass predictions for axial vector ($J^{PC} = 1^{++}$) charmonium and bottomonium hybrids. The flux tube model predicts the lightest charmonium hybrids at 4.1-4.2 GeV $^{[8]}$. Lattice QCD $^{[9–11]}$ yields quenched predictions of about 4.0 GeV for the lightest charmonium hybrids, and unquenched predictions of approximately 4.4 GeV for $1^{++}$ charmonium hybrids in particular. Refs. $^{[12–14]}$ comprise the first studies of heavy quark hybrids using QCD sum-rules. Multiple $J^{PC}$ including $1^{++}$ were examined; however, many of the resulting sum-rules exhibited instabilities, leading to unreliable mass predictions. Refs. $^{[15, 16]}$ re-examined the $1^{--}$ and $0^{++}$ channels respectively, finding that the dimension-six gluon condensate which was not included in Refs. $^{[12–14]}$ stabilizes the sum-rules in these channels. Motivated by these results, we have investigated the effects of the dimension-six gluon condensate for axial vector heavy quark hybrids using QCD Laplace sum-rules $^{[7]}$. The resulting mass predictions are discussed with regard to the nature of the $X(3872)$ and in relation to the charmonium hybrid multiplet structure suggested by recent lattice calculations $^{[11]}$.

2. Laplace Sum-Rules for Axial Vector Heavy Quark Hybrids

The correlation function used to study axial vector ($J^{PC} = 1^{++}$) heavy quark hybrids is given by

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq\cdot x} \left\langle 0 \left| T \left[ j_\mu(x) j_\nu(0) \right] \right| 0 \right\rangle$$  \hspace{0.5cm} (1)$$

$$j_\mu = \frac{\bar{Q} Q}{2} \gamma_\mu + \bar{Q} i \gamma_5 G_{\mu\nu} \gamma^\nu \gamma_\lambda \gamma_\sigma \gamma_\tau \frac{1}{\sqrt{2}} \epsilon_{\mu\nu\rho\sigma} G_{\rho\sigma}$$  \hspace{0.5cm} (2)$$

with $Q$ representing a heavy quark field $^{[12]}$. The transverse part $\Pi_V$ of (1) couples to $1^{++}$ states

$$\Pi_{\mu\nu}(q) = \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi_V(q^2) + \frac{q_\mu q_\nu}{q^2} \Pi_S(q^2)$$  \hspace{0.5cm} (3)$$

In Refs. $^{[12, 13]}$, the perturbative and gluon condensate $\langle \alpha G^2 \rangle = \langle \alpha G_{\mu\nu} G^{\mu\nu} \rangle$ contributions to the imaginary part of $\Pi_V(q^2)$ were calculated to leading order. The Feynman diagrams for these are represented in Fig. $^{[1]}$. 

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The full expression for the hadronic spectral function $V$ is

$$V(q^2) = \frac{g^3}{152\pi^2} \left[ \frac{3(17z - 9)}{z - 1} - \frac{3(17 - 46z + 27z^2)}{(z - 1)^2} + \frac{2(2 - 9z + 6z^2)}{(z - 1)^2} \right] zF_1 \left( 1, \frac{5}{2}; z \right).$$

The imaginary part of $V$ is

$$\text{Im}\Pi^G_G(q^2) = \frac{g^3}{384\pi} \frac{\sqrt{z - 1}}{\sqrt{z}} \left[ \frac{2(1 - 3z)}{z - 1} + \frac{(2 - 9z + 6z^2)}{(z - 1)^2} \right], \quad z > 1,$$

which is singular at $z = 1$. This poses a problem since the sum-rules will involve integrating [7] from $z = 1$. Below it will be shown how this difficulty can be overcome.

We now formulate the QCD Laplace sum-rules [17, 18]. Using a resonance plus continuum model for the hadronic spectral function

$$\rho(t) = \rho_{\text{had}}(t) + \theta(t - s_0) \text{Im}\Pi^\text{QCD}(t),$$

the Laplace sum-rules are given by

$$L^\text{QCD}_z(\rho, s_0) = \frac{1}{\pi} \int_{s_0}^\infty r^2 \exp\{ - tr \} \text{Im}\Pi\rho(t) dt,$$

where $s_0$ is the hadronic threshold. The quantity on the left hand side of (9) is given by

$$L^\text{QCD}_z(\rho, s_0) = \frac{1}{\pi} \hat{B} \left[ (-1)^k Q^{2k} \text{Im}\Pi\rho(Q^2) \right] - \frac{1}{\pi} \int_{s_0}^\infty r^2 \exp\{ - tr \} \text{Im}\Pi\rho(t) dt,$$

where $Q^2 = -q^2$ and $s_0$ is the continuum threshold. $\hat{B}$ is the Borel transform, which is closely related to the inverse Laplace transform [19]. The singular terms in (6) that are irrelevant for (7) are, however, relevant for the inverse Laplace transform. It is the inclusion of these terms that allows the integration of (7) from $z = 1$ to be defined as a limiting procedure. Thus the imaginary part (7) is insufficient to formulate the sum-rules for $1^{++}$ hybrids, as found for $0^{++}$ hybrids [16].

From the results for the leading order perturbative (4),
which can be used to calculate the ground state mass in Table 1. To make mass predictions for axial vector heavy quark hybrids we utilize a single narrow resonance model

\[
\frac{1}{\pi} f^\text{had}(t) = f^2 \delta \left( t - M^2 \right) .
\]

Inserting (13) in (9) gives

\[
\mathcal{L}^{\text{QCD}}_0 (\tau, s_0) = f^2 M^{2k} \exp \left( -M^2 \tau \right),
\]

which can be used to calculate the ground state mass \(M\) via the ratio

\[
M^2 = \frac{\mathcal{L}^{\text{QCD}}_0 (\tau, s_0)}{\mathcal{L}^{\text{QCD}}_0 (\tau, s_0)} .
\]

Before using (15) to calculate the mass, the QCD input parameters must be specified. For the charmonium and bottomonium hybrid analyses we use one-loop \(\overline{\text{MS}}\) expressions for the coupling and quark masses:

\[
\alpha (\mu) = \frac{\alpha (M_z)}{1 + \frac{2\pi a_s M_z^2}{12\pi} \log \left( \frac{\mu^2}{M_z^2} \right)}, \quad m_c (\mu) = \overline{m}_c \left( \frac{\alpha (\mu)}{\alpha (\overline{m}_c)} \right)^\overline{\alpha} ;
\]

\[
\alpha (\mu) = \frac{\alpha (M_z)}{1 + \frac{2\pi a_s M_z^2}{12\pi} \log \left( \frac{\mu^2}{M_z^2} \right)}, \quad m_b (\mu) = \overline{m}_b \left( \frac{\alpha (\mu)}{\alpha (\overline{m}_b)} \right)^\overline{\alpha} .
\]

The numerical values of the QCD parameters are given in Table 1.

| \(\alpha (M_z)\) | \(|\overline{m}_c|\) | \(|\overline{m}_b|\) | \(\alpha (G^2)\) | \(\langle g^3 G^3 \rangle \) |
|-------------------|-----------------|-----------------|-----------------|-----------------|
| 0.33              | 2.43 GeV        | 4.18 GeV        | (7.5 \pm 2.0) \times 10^{-2} GeV^{-4} | (8.2 \pm 1.0) GeV^{-4} |
|                  | (1.28 \pm 0.02) GeV | 0.118           |                  |                  |
|                  | (4.17 \pm 0.02) GeV |                  |                  |                  |

The sum-rule window is determined following Ref. [17]. The values of \(\alpha (M_z)\) and \(\alpha (M_z)\) are taken from Ref. [23]. Numerical values of the condensates are taken from Ref. [23].

The continuum threshold \(s_0\) is optimized by first determining the smallest value of \(s_0\) for which the ratio (15) stabilizes (exhibits a minimum) within the respective sum-rule windows. Then the optimal value is fixed by the \(s_0\) which has the best fit to a constant within the sum-rule window. The mass prediction (15) is shown for hybrid charmonium in Fig. 3 and for hybrid bottomonium in Fig. 4.
The present result and those of Refs. [15, 16] seem to be in qualitative agreement with this multiplet structure, although the mass splittings are significantly larger than those of Ref. [11]. Future work to update remaining unstable sum-rule channels in Refs. [12, 14] to include the effects of $\langle g^3 G^3 \rangle$ would clarify the QCD sum-rule predictions for the spectrum of charmonium hybrids.

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