Computational scheme for ab-initio predictions of chemical compositions interfaces realized by deposition growth

Jochen Rohrer and Per Hyldgaard

BioNano Systems Laboratory, Department of Microtechnology, MC2, Chalmers University of Technology, SE-412 96 Gothenburg

We present a novel computational scheme to predict chemical compositions at interfaces as they emerge in a growth process. The scheme uses the Gibbs free energy of reaction associated with the formation of interfaces with a specific composition as predictor for their prevalence. It explicitly accounts for the growth conditions by rate-equation modeling of the deposition environment. The Bell-Evans-Polanyi principle motivates our emphasis on an effective nonequilibrium thermodynamic description inspired by chemical reaction theory. We illustrate the scheme by characterizing the interface between TiC and alumina. Equilibrium thermodynamics favors a nonbinding interface, being in conflict with the wear-resistant nature of TiC/alumina multilayer coatings. Our novel scheme predicts that deposition of a strongly adhering interface is favored under realistic conditions.

I. INTRODUCTION

Density functional theory calculations are today routinely applied to characterize structural and electronic properties of condensed matter systems. They serve as an important complement to and extension of experimental methods. Atomic (and electronic) structure and chemical composition are inseparably interwoven. Information about chemical composition at, for example, interfaces (including surfaces) is therefore of great importance for reliability of such calculations.

Traditional ab initio atomistic thermodynamics methods aim at describing and predicting compositions at oxide surfaces. These schemes assume that equilibrium is established between the oxide and surrounding O$_2$. This criterion is often justified for oxide surfaces that are in direct contact with an O-rich environment. For (solid-solid) interfaces, the situation is more complicated. Interfaces are typically exposed to the surrounding environment only at the moment of creation. Furthermore, oxides are seldomly grown directly from O$_2$, and it is not clear what gas(es) they are in (dynamic) equilibrium with during deposition, if at all. A generalization of the ab initio atomistic thermodynamics scheme from surfaces to interfaces can therefore be problematic. In fact, we have shown that the equilibrium configuration at the interface between TiC and alumina predicted by such a generalized scheme does not describe the wear-resistance of TiC/alumina multilayer coatings. We have attributed this inconsistency to the fact that the scheme does not account for the actual growth conditions. This may be a serious shortcoming also for other interfaces realized by deposition growth.

This paper presents a computational scheme that explicitly accounts for deposition conditions. At the same time no a priori equilibrium assumptions are introduced. The scheme is therefore capable to predict chemical compositions at interfaces (including surfaces) as they arise in a deposition process. While kinetic accounts are generally required to describe the full nonequilibrium nature of growth, the Bell-Evans-Polanyi (BEP) principle motivates an effective thermodynamic description inspired by chemical reaction theory.

The key elements of our scheme are the use of Gibbs free energies of reaction as a predictor for the chemical composition and a modeling of the deposition conditions in terms of rate equations describing the deposition environment. Our approach extends the method of ab initio thermodynamics of deposition growth for surfaces to interface modeling. Applying the scheme to the TiC/alumina interface in a chemical vapor deposition (CVD) environment we predict the formation of strongly adhering structures in agreement with the wear-resistance of CVD TiC/alumina multilayers.

The paper is organized as follows. Section II motivates the use of the Gibbs free energy of reaction as a predictor of chemical composition. Section II contains the details of our modeling. Results are presented in Sec. IV and Sec. V contains our conclusions.

II. PREDICTOR FOR AS-GROWN CHEMICAL COMPOSITION

We use the Gibbs free energies of reaction $G_r$ as a predictor for the prevalent chemical composition at surfaces and interfaces realized by deposition growth. We justify the use of $G_r$ as predictor using chemical reaction theory and the BEP principle. A description of nonequilibrium growth generally requires a kinetic description. The prevalent surface or interface composition will in principle depend on the reaction barriers for all relevant processes. However, the BEP principle motivates our emphasis on an effective nonequilibrium thermodynamic approach. The BEP principle states that the more exothermic a reac-
tion is, the smaller is the associated reaction barrier. The nonequilibrium system will therefore continuously evolve along paths that release the largest Gibbs free energy of reaction. In some cases we can even obtain quantitative predictions for the prevalence of competing chemical compositions without having to calculate specific rates.

The Gibbs free energy of reaction is defined as the gain in Gibbs free energy in a chemical reaction. For a general reaction, according to chemical reaction theory, $G_r$ is related to the forward ($f$) and backward ($b$) reaction rates $\Gamma_f = k_f P_r^{\nu_f}$ and $\Gamma_b = k_b P_r^{\nu_b}$ via

$$-\beta G_r = \ln \Gamma_f / \Gamma_b$$

(1)

Here $\beta$ is the inverse temperature in units of energy, $k_{(f,b)}$ is forward and backward rate constant, $[P_r]$, $(P_p)$ is the concentration of the $r$-th ($p$-th) reactant R (product P), and $\nu_{(p,r)}$ is the corresponding stoichiometric coefficient. The key observation is that the ratio of forward and backward rates in a given chemical process is entirely given by the nonequilibrium Gibbs free energy of reaction. This quantity can be evaluated without knowing the kinetic details of the process.

To illustrate the approach, we first discuss the problem of surface termination in deposition growth, seeking to predict which surface (A terminated or B terminated) will emerge when growing a binary material AB, see Fig. 1. We consider two coupled reactions $I$ and $II$ that change the surface termination from A to B and vice versa,

$$I : \text{B} + \text{React}_I \rightleftharpoons \text{A} + \text{Prod}_I$$

$$II : \text{A} + \text{React}_II \rightleftharpoons \text{B} + \text{Prod}_II.$$  

(2a, b)

Here, React$_{I,II}$ and Prod$_{I,II}$ collectively denote the reactants and products in the two reactions. The forward and backward rates are the same as in (1) but have an additional index to differentiate between the two reactions. Taking the difference between the free energies of reaction in $I$ and $II$, we find after exponentiation and use of the geometric mean $\langle x, y \rangle_{gm} = (xy)^{-1/2}$

$$\exp \left(-\beta [G_r^I - G_r^{II}] \right) = \frac{\Gamma_f^I \Gamma_b^{II}}{\Gamma_f^{II} \Gamma_b^I} \left[ \frac{\Gamma_f^I \Gamma_b^{II}}{(\Gamma_b^I \Gamma_f^{II})_{gm}} \right]^2.$$  

(3)

The reactions in (2) can also be described by the rate equations,

$$\partial_t P_A = -(\Gamma_f^I + \Gamma_f^{II}) P_A + (\Gamma_f^I + \Gamma_f^{II}) P_B$$

$$\partial_t P_B = (\Gamma_f^I + \Gamma_f^{II}) P_A - (\Gamma_f^I + \Gamma_f^{II}) P_B.$$  

(4a, b)

In particular, if the reaction that creates an A-terminated surface has lower $G_r$ than the reaction that creates a B-terminated surface, here $G_r^I < G_r^{II}$, an A-terminated surface is more likely. Away from dynamic equilibrium, this relation is no longer exact. However, comparison of $G_r$ for different terminations still indicates which termination is most prevalent, since the geometric and arithmetic means [Eqs. (3) and (4)] are approximately equal, Ref. [8].

By analogy to the case of surface terminations, we can also use differences in $G_r$, see [9], as measure to predict chemical compositions at interfaces formed by deposition growth (and assumed to have retained the structure specified by the deposition environment). Considering related interface configurations of similar thickness but different compositions, the one with lowest (most negative) Gibbs free energy of reaction is most likely to describe the nature of the interface as it is formed by deposition growth. We note that $G_r$ will in general decrease if the reaction is favorable) as the number of constituents in the film describing the interface increases by any integer multiple of the bulk stoichiometry. The predictor is useful when comparing films of similar thickness. Again it is possible to obtain quantitative predictions without having to consider kinetic details of the deposition, see also Ref. [11].

III. MODELING

A. Materials Background

We illustrate our computational scheme by studying the interface composition between TiC and alumina.
supply gas
\[ H_2 \]
\[ CO_2 \]
\[ AlCl_3 \]
CVD chamber
\[ \text{constant } T, \text{ constant } p \]
\[ H_2 + CO_2 \rightarrow H_2O + CO \]
\[ 2AlCl_3 + 3H_2O \rightarrow Al_2O_3 + 6HCl \]
gas exhaust

FIG. 2: Illustration of chemical vapor deposition of alumina. A H₂-AlCl₃-CO₂ gas mixture is supplied to a chamber at rate \( R_S \). The chamber is kept at fixed temperature \( T \) and fixed total pressure \( p \). The gases react to simultaneously form water and alumina. Reaction products and unused reactants are exhausted at rate \( R_E \).

FIG. 3: Exfoliating (left panel) and wear-resistant alumina (right panel) films on TiC. Color coding: Ti = dark green, Al = blue, O = red.

TiC/alumina multilayers are commonly used as wear-resistant coating on cemented-carbide cutting tools [6]. They are routinely fabricated by chemical vapor deposition (CVD).

Figure 2 illustrates the experimental setup for CVD of alumina [12]. A H₂-AlCl₃-CO₂ supply gas mixture is injected into a hot chamber which is kept at a fixed temperature and a fixed total pressure. The CVD process proceeds in two steps which, however, take place in parallel. Water forms at an (unknown) rate \( R_{H_2O} \) according to

\[
H_2 + CO_2 \xrightarrow{R_{H_2O}} H_2O + CO. \tag{7}
\]

Alumina is deposited at an (unknown) rate \( R_{Al_2O_3} \) according to

\[
2AlCl_3 + 3H_2O \xrightarrow{R_{Al_2O_3}} Al_2O_3 + 6HCl. \tag{8}
\]

B. Ab initio structure search for thin alumina films and TiC/alumina interface models

In Refs. [5, 13, 14] we have (in collaboration with others) presented a computational strategy for identifying energetically favorable geometries of thin-film alumina on a TiC substrate. We have constructed alumina candidate configurations from a pool of structural motifs as they exist in stable and metastable bulk alumina phases. The motifs are characterized by stacking of the O layers and by the coordination of the Al ions. We have considered alumina films of thicknesses up to four O layers. In addition, we allowed for off-stoichiometries in these films.

Each candidate configuration was structurally optimized by ab initio total-energy and force calculations and subsequent relaxation (using standard quasi-Newton and conjugate gradient algorithms). For a given thickness and stoichiometric composition, the optimal geometry was identified as the one with lowest total energy after structural relaxations.

Figure 3 details the atomic structure of two optimized alumina films with a thickness of four O layers and different stoichiometric compositions. These films are referred to as exfoliating (left) and wear-resistant (right) films. These terms refer to the adhesion properties of the two films (exfoliating = no or very weak adhesion, wear-resistant = strong adhesion). Details concerning adhesion properties of these films are given elsewhere [11]. For other thicknesses, the films possess similar structural (and adhesion) characteristics. Focusing on the films in Fig. 3 therefore does not pose a restriction of generality on the presented discussion of the TiC/alumina interface.

The results of our (total-energy) structure search (for each class of AlₓMᵧOₙ films) can be combined with the equilibrium thermodynamics of Refs. [2, 4]. We find that this approach erroneously identifies the exfoliating film as thermodynamically stable [2]. This incorrect equilibrium prediction applies to a wide range of temperatures and O₂ pressures and is in apparent conflict with wear-resistance of TiC/alumina multilayer coatings [2]. Below we show that a more physical account of alumina growth emerges with our new nonequilibrium thermodynamics theory.

C. Formation of excess atoms and free energies of reaction

The films shown in Fig. 3 can be divided into a stoichiometric part and an excess part. For a general AlₓMᵧOₙ film, the number of stoichiometric units in the stoichiometric part of the film is \( n_{\text{alumina}} = \min ([M/2], [N/3]) \), where \([x]\) is the largest integer smaller or equal to \( x \). The corresponding number of excess Al and O atoms is \( \Delta n_{\text{Al}} = N - 2n_{\text{alumina}} \) and \( \Delta n_{\text{O}} = M - 3n_{\text{alumina}} \).

We assume that the deposition of the stoichiometric parts of the films is described by (8). Excess O can be deposited as

\[
CO_2 \rightarrow O_{\text{exc}} + CO, \tag{9a}
\]
\[
H_2O \rightarrow O_{\text{exc}} + H_2, \tag{9b}
\]

depending on which reaction gives a lower free energy of reaction. (Excess Al is not considered here.) The free
energy of reaction is defined accordingly,
\[
G_i^{\text{AlMnO}_N} = G_{\text{TIC/AlMnO}_N} - G_{\text{TIC}} + n_{\text{AlMnO}_3} (6 \mu_{\text{HCl}} - 2 \mu_{\text{AlCl}_3} - 
+ \Delta n_{\text{O}} \max [\mu_{\text{CO}} - \mu_{\text{CO}_2}, \mu_{\text{H}_2} - \mu_{\text{H}_2\text{O}}]).
\] (10)

D. Approximations for evaluation of \(G_r\)

We make standard approximations for the free energy of solid and gaseous constituents. The free energy of the TiC/alumina systems is approximated by their DFT total energies, that is, \(G_{\text{solid}} \approx E_{\text{solid}}^\text{DFT}.\) Vibrational effects are not considered \([3, 8]\). For gaseous constituents, we employ the ideal-gas approximation,
\[
\mu_i(T, p_i) = \epsilon_i + \Delta_i^0(T) + k_B T \ln(p_i/p^0).
\] (11)

Here \(\epsilon_i\) is the DFT total energy \([13]\) of the gas phase species (molecule), and \(k_B\) is the Boltzmann constant. \(\Delta_i^0(T)\) is the temperature dependence of \(\mu_i\) at a fixed pressure \(p^0\), and available for \(p^0 = 1\) atm in thermochemical tables \([10]\).

E. Rate-equation description of CVD of alumina

The actual evaluation of the individual chemical potentials of the different gaseous constituents in the CVD chamber requires the specification of the associated partial pressures. In the actual fabrication process, these are, however, not directly controlled; only the temperature and the total pressure are controllable.

We describe the CVD process in terms of rate equations for the individual (ideal gas) pressures,
\[
\partial_t p_i \propto c_i R_S - \frac{p_i}{P} R_S + \nu_i^4 H_2O R_{\text{H}_2O} + \nu_i^{\text{AlMnO}_3} R_{\text{AlMnO}_3}. \tag{12}
\]

Here, \(p_i = p_i(t)\) is the pressure of chemical species \(i\) at time \(t\) inside the reaction chamber, \(P = p(t) = \sum p_i(t)\) is the corresponding total pressure, \(c_i\) is the concentration of the chemical species \(i\) in the supply gas, and \(\nu_i^4 H_2O\) and \(\nu_i^{\text{AlMnO}_3}\) are the stoichiometric coefficients\(^3\) of the chemical species \(i\) in reaction \((7)\) and \((8)\), respectively.

The rate at which the gas is supplied to (exhausted from) the chamber is \(R_S\) (\(R_E\)) and the reaction rates for water production and alumina deposition are \(R_{\text{H}_2O}\) and \(R_{\text{AlMnO}_3}\).

\footnote{In fact, we correct the total energies of the films by subtracting the strain energy of the stoichiometric part of the film, \(E_{\text{film}} \rightarrow E_{\text{film}} - n_{\text{AlMnO}_3} \Delta s_{\text{strain}}\), where \(\Delta s_{\text{strain}}\) is the is difference between the strained (due to the expansion to the TiC lattice in the interface plane) and the unstrained bulk alumina per stoichiometric unit.}

\footnote{Here we use the standard conventions that stoichiometric coefficients are counted negative if a species is consumed and positive if a species is produced in a reaction.}

FIG. 4: Gibbs free energies of reaction \(G_r\) for exfoliating and wear-resistant alumina film (see Fig. \(\text{IV}\) as functions of the scaled reaction rate \(r_{\text{H}_2O}\). The vertical line limits \(r_{\text{H}_2O}\) to the right and corresponds to dynamic equilibrium in \(\text{IV}\). For simplicity, we assume \(R_{\text{AlMnO}_3} = R_{\text{H}_2O}/3\). The difference in \(G_r\) is a predictor for the relative presence of exfoliating and wear-resistant films, see \(\text{IV}\).

We use the resulting steady-state pressures
\[
p_i = p \frac{c_i + r_{\text{H}_2O} \nu_i^4 H_2O + r_{\text{AlMnO}_3} \nu_i^{\text{AlMnO}_3}}{1 + r_{\text{AlMnO}_3}}, \tag{13}
\]
as input for the evaluation of the individual chemical potentials \([11]\). In \(\text{IV}\) we have introduced the scaled reaction rates \(r_{\text{H}_2O} = R_{\text{H}_2O}/R_S\) and \(r_{\text{AlMnO}_3} = R_{\text{AlMnO}_3}/R_S\).

F. Limits on reaction rates

The reaction rates \(R_{\text{H}_2O}\) and \(R_{\text{AlMnO}_3}\) cannot assume any arbitrary value. For example, we require that \(R_{\text{AlMnO}_3} \leq R_{\text{H}_2O}/3 = R_{\text{AlMnO}_3}^\text{max}\), since reaction \(\text{V}\) requires three units of \(H_2O\), while reaction \(\text{VII}\) produces only one. Additional constraints on this particular growth process and interface formation are discussed in Ref. \([11]\).

IV. RESULTS

In Fig. \(\text{IV}\) we plot the free energies of reaction for the two films shown in Fig. \(\text{III}\) as functions of the scaled reaction rate \(r_{\text{H}_2O}\). Deposition parameters (supply gas composition, total pressure, and deposition temperature) as specified in Ref. \([12]\) have been used. In this illustration of the approach we assume for simplicity \(R_{\text{AlMnO}_3} = R_{\text{H}_2O}/3\).
Furthermore, the scaled reaction rate $r_{H_2O}$ is limited to the right by the limit of dynamic equilibrium in \([7]\), indicated by the vertical line.

We find that, over the entire range of the reaction rate $r_{H_2O}$, it is more favorable to grow wear-resistant overlayers than exfoliating overlayers.\(^5\) Qualitatively, this result is independent of the choice of $R_{Al_2O_3}$. We emphasize that this prediction is consistent with industrial use of TiC/alumina as wear-resistant coating; predictions based on an adaption of equilibrium-thermodynamics analysis \(2-4\) to this system \(5\) are not.

V. SUMMARY & CONCLUSIONS

We have presented a novel computational scheme to predict chemical compositions at interfaces as they emerge in a growth process. The scheme uses the Gibbs free energy of reaction associated with the formation of interfaces with a specific composition as predictor for their prevalence. We explicitly account for the growth conditions by rate-equation modeling of the deposition environment.

An earlier study of TiC/alumina interfaces \(5\) documented that the predicted composition at this interface is in conflict with the wear-resistance of multilayered TiC/alumina when using an equilibrium-thermodynamics scheme \(2-4\). Our results demonstrate that a careful account of deposition conditions in inter-

\(^5\) We notice that there is a third stoichiometric composition (not discussed in this work) that may be stabilized if $r_{H_2O}$ is sufficiently far away from the dynamic equilibrium limit. The corre-

face modeling is crucial for understanding the adhesion at CVD TiC/alumina.

We expect that a similar analysis will be necessary also for other buried interfaces that form during a deposition process in an environment that strongly differs from ambient conditions. We emphasize the predictive power of the here-presented method, adding to the DFT-based toolbox for accelerating innovation \(17\). In principle, it allows for the determination of deposition conditions required to experimentally create interfaces with a predetermined compositions. We argue that combining the structure search method of Ref. \(5\) and the new \textit{ab initio} thermodynamics with, for example, a genetic algorithm \(18, 19\) presents a valuable tool for characterization of surface and interface growth.

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