SiGe quantum wells with oscillating Ge concentrations for quantum dot qubits

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Large-scale arrays of quantum-dot spin qubits in Si/SiGe quantum wells require large or tunable energy splittings of the valley states associated with degenerate conduction band minima. Existing proposals to deterministically enhance the valley splitting rely on sharp interfaces or modifications in the quantum well barriers that can be difficult to grow. Here, we propose and demonstrate a new heterostructure, the “Wiggle Well,” whose key feature—Ge concentration oscillations—is realized inside the quantum well, rather than outside, and does not require atomic-scale precision. Experimentally, we observe large and widely tunable valley splittings from 54 to 239 \(\mu\)eV in a single-electron dot. Using tight-binding calculations, we make quantitative predictions for the valley splitting in several different Wiggle Well heterostructures. Our results point to the Wiggle Well as a robust method for enhancing the valley splitting in future qubit devices.

Quantum dots formed in silicon-germanium heterostructures are promising candidates for quantum computing, but the degeneracy of the two conduction band minima (or “valleys”) in silicon quantum wells can pose a challenge for forming qubits [1–6]. In such structures, the energy splitting between the valley states, \(E_v\), is typically tens to a few hundred \(\mu\)eV and can vary widely due to heterostructure design and unintentional defects [7–19]. The small size and intrinsic variability of \(E_v\) has motivated several schemes for modifying or tuning its value. An ambitious scheme to engineer the quantum well barriers, layer-by-layer, has been proposed to increase \(E_v\) [20, 21]. Simpler heterostructure modifications have already been implemented in the laboratory. For example, including additional germanium at the quantum well interface was not found to significantly impact \(E_v\) [13], while a single spike in germanium concentration within the quantum well was found, theoretically and experimentally, to cause an approximate doubling of \(E_v\) [18]. Even more practically, \(E_v\) can be tuned after device fabrication by changing the applied vertical electric field [14, 22, 23] or the lateral dot position [9, 15, 17], though such tunability tends to be modest in a typical qubit operating range.

Here, we report theory and experiment on a novel Si/SiGe heterostructure, the Wiggle Well, which has an oscillating concentration of germanium inside the quantum well. The wavevector is specially chosen to couple the conduction-band valleys in silicon, thereby increasing \(E_v\). This wavevector can be chosen either to couple valleys within a single Brillouin zone or between zones. We measure a quantum dot device fabricated on a Wiggle Well heterostructure grown by chemical vapor deposition (CVD) with Ge concentrations oscillating between 0% and 9%, with wavelength of 1.8 nm, corresponding to the shortest interzone coupling wavevector. The valley splitting is measured using pulsed-gate spectroscopy in a singly occupied quantum dot, obtaining results that are both large and tunable in the range of 54-239 \(\mu\)eV. We employ an effective mass method to treat Ge concentration variations in the virtual crystal approximation (EMVC method) to obtain an approximate picture of \(E_v\) as a function of the oscillation wavelength. We also perform tight-binding simulations of disordered heterostructures using NEMO-3D [24], which qualitatively validates our understanding from the EMVC theory and quantitatively incorporates the effects of both strain and

![Figure 1](https://example.com/fig1.png)

**FIG. 1.** The Wiggle Well. (a) Schematic of the Wiggle Well heterostructure, showing Ge oscillations throughout the quantum well. The darker regions have higher Ge concentration. (b) Plot of Ge concentration versus position in a quantum well. The darker regions have higher Ge concentration. (c) Prediction of the EMVC method for the contribution to the valley splitting \(E_v\) versus \(q\) due to a sinusoidal Ge concentration in the quantum well, for \(n_{Ge} = 5\%\) (orange), 10\% (green), 15\% (red), and 20\% (purple), and a vertical electric field of 8.5 MV/m. The inset shows two neighboring Brillouin zones in the silicon conduction band with constant energy surfaces around the valley minima shown in blue. The peaks at \(q \approx 3.5\) nm\(^{-1}\) arise from Umklapp coupling between the \(z\) valleys in neighboring Brillouin zones, and the peaks at \(q \approx 20\) nm\(^{-1}\) arise from coupling between \(z\) valleys within a single zone. The peak maxima at \(q \approx 20\) nm\(^{-1}\) lie between 5-23 meV and are shown on a different scale in the Supplemental Materials.
random-alloy disorder. These theoretical methods are also used to make predictions about a number of additional heterostructures with varying germanium oscillation wavelengths and amplitudes.

We consider a spatially oscillating germanium concentration of the form \( \frac{1}{2} n_{\text{Ge}}[1 - \cos(qz)] \), as illustrated in Fig. 1(a). Here, \( z \) is the heterostructure growth direction, \( n_{\text{Ge}} \) is the average Ge concentration in the well, and \( q \) is the wavevector corresponding to wavelength \( \lambda = 2\pi/q \), as indicated in Fig. 1(b). The wavevector \( q \) can be chosen to greatly enhance the valley splitting. For any Si/SiGe quantum well, the energies of the valley states split in the presence of a sharp interface. The Wiggle Well experiences an additional contribution to \( E_{\text{v}} \), due to the oscillating Ge concentration, which gives rise to a potential energy term in the Hamiltonian of the form \( V_{\text{osc}}(z) \propto [1 - \cos(qz)] \). The electron wavefunction also oscillates as \( \phi_{\pm}(z) \propto \exp(\pm i k_0 z) \), resulting in interference. Here, \( k_0 \) is the location of the conduction band minimum in the first Brillouin zone [3]. Since \( k_0 \) occurs near the zone boundary, the wavefunction oscillations are very short-wavelength. Hence, for long-wavelength concentration oscillations, with \( q \approx 0 \), \( V_{\text{osc}} \) has almost no capacity to couple valleys, due to negative interference. However, for large \( q \) values on the order of \( k_0 \), positive interference can occur, as we now describe.

The EMVC method allows us to build intuition about the underlying mechanisms that enhance valley splitting by computing the valley splitting in the Wiggle Well. Figure 1(c) shows the Wiggle Well contribution to the valley splitting \( E_{\text{v}} \), calculated using the EMVC method for several values of \( n_{\text{Ge}} \), as a function of the wavevector \( q \) of the oscillations. We observe that the valley splitting is enhanced at specific germanium oscillation wavevectors. The wavevector \( q \approx 3.5 \, \text{nm}^{-1} \), corresponding to \( \lambda_{\text{long}} = 1.8 \, \text{nm} \), describes coupling between valleys in two neighboring Brillouin zones, as indicated by arrows in the inset. A much larger enhancement of the valley splitting can be achieved for the wavevector \( q \approx 20 \, \text{nm}^{-1} \), corresponding to the much shorter wavelength, \( \lambda_{\text{short}} = 0.32 \, \text{nm} \), which describes coupling between the z-valley states within a single Brillouin zone, also shown with arrows. Thus, choosing the oscillation wavelength \( \lambda = 2\pi/q \) with care enables the generation of a wavevector in the potential that couples valley minima either between or within Brillouin zones. Note that the small feature observed at \( q \approx 10 \, \text{nm}^{-1} \) in Fig. 1(c) is likely a harmonic, arising from finite-size effects in the calculation, while at small \( q \), there are additional features associated with the details of the barrier interface.

Figure 2(a) shows a scanning transmission electron micrograph of a Wiggle Well heterostructure grown by chemical vapor deposition (CVD), demonstrating an oscillating concentration of germanium with \( \lambda \approx 1.7 \, \text{nm} \), as described in Methods. Based on this result, the growth parameters were adjusted slightly to achieve the desired \( \lambda_{\text{long}} \) oscillation period, with an estimated \( n_{\text{Ge}} = 4.5\% \). The closest match to this value in Fig. 1(c) (orange curve) suggests a valley splitting enhancement of about 63 eV due to these oscillations. Hall bar devices were fabricated on the heterostructure and measured at a temperature of \( \sim 2 \, \text{K} \), revealing mobilities in the range of \( 1-3 \times 10^4 \, \text{cm}^2\text{V}^{-1}\text{s}^{-1} \) for an electron density range of \( 2-6 \times 10^{11} \, \text{cm}^{-2} \). (See Supplemental Materials.)
To define quantum dots, atomic layer deposition was used to deposit a 5 nm layer of aluminum oxide. Electron beam lithography was used to pattern three layers of overlapping aluminum gates isolated from one another by the plasma-ash enhanced self-oxidation of the aluminum metal, following the procedure described in Ref. [25]. (See Methods.) Figure 2(b) shows a false-colored scanning electron micrograph of a quantum dot device lithographically identical to the one measured. The left half of the device was used for the measurements described below, with a double quantum dot formed in the lower channel and a charge sensing dot formed in the upper channel. Figure 2(c) shows a stability diagram of the double dot, where the absolute number of electrons can be determined by counting the number of lines crossed in the color plot. All measurements were performed using the last (leftmost) electron transition in this figure, near the magenta star, in a dilution refrigerator with a base temperature below 50 mK.

The excited-state spectrum of a singly occupied quantum dot was measured using pulsed-gate spectroscopy [17, 26–30], as shown in Fig. 2(d). Here, the differential conductance of the charge sensor is plotted as a function of the dc voltage on gate P1 vs. the amplitude of the square-wave pulse applied to P1. The data show a sudden change of color when the rate at which electrons enter or leave the dot changes significantly, allowing us to estimate the excited-state energies (see Methods). Figure 2(e) shows in blue the averaged result of 16 individual P1 voltage scans obtained with a 16 mV square-wave amplitude. The green curve is a numerical derivative of the blue curve with respect to $V_{P1}$. Here, the voltage differences corresponding to the valley splitting $E_v$ and the orbital splitting $E_{orb}$, are labeled with arrows. The dips in the differentiated signal are fit to extract the voltage splittings, using the methods described in Ref. [17], and then converted into energy splittings using the appropriate lever arm (see Supplemental Materials), yielding a valley splitting of 164±3 µeV for this particular device tuning.

To develop an understanding of how the germanium concentration oscillations affect the valley splitting, we make use of our ability to change the quantum dot’s shape and position in-situ through changes in gate voltages. Importantly, such changes in size and shape can be made while keeping the electron occupation constant. First, we shift the dot’s lateral position by changing the voltages on the screening gates S1 and S2 asymmetrically [17]. Because germanium atoms sit at discrete locations, the concentration oscillations are not identical at all locations in the quantum well; instead, each physical location represents a random instance, which only follows a smooth sine wave pattern when averaged over a wide region. Since the dot is finite in size, changes in position therefore cause it to sample local fluctuations of the Ge concentration. Moving the dot in this way also modifies the size and shape of the electron probability distribution in the plane of the quantum well.

The orbital and valley splittings resulting from these two different tuning schemes are shown in Fig. 3(a). Both tuning schemes yield a large change in the orbital splitting $E_{orb}$, as shown in the inset to Fig. 3(a), because both change the size and shape of the quantum dot. The valley splitting shows markedly different behavior in the two cases. The first tuning scheme, which moves the dot laterally to sample different realizations of the Wiggle Well oscillations, yields a large change in the measured valley splitting of nearly 200 µeV. The second approach, which does not move the quantum dot, results in a much smaller change in the valley splitting. This large difference in behavior is demonstrated most obviously by the linear fits to the data, which we will compare below to numerical calculations of the valley splitting for many different atomistic realizations of the Wiggle Well. While tunable valley splittings (and closely related singlet-triplet splittings) of Si/SiGe quantum dots have recently been achieved by changing gate voltages [9, 14, 15, 17, 18], the observed range of behavior has been modest: for example, 15% tunability with a maximum of $E_v = 213$ µeV [15] or 140% tunability with a maximum of $E_v = 87$ µeV [17]. Here in contrast, we report a striking $>440\%$ tunability with a maximum of $E_v = 239$ µeV.

The EMVC calculations presented in Fig. 1 provide intuition about how oscillating germanium concentrations affect the valley splitting: wave vectors describing the germanium-induced oscillating potential in the quantum well connect valley minima within or between Brillouin zones, as determined by the wavelength of the oscillations. However, such calculations do not provide information about the effect of different atomistic realizations of these oscillations. Moreover, from Fig. 3(a), it is clear that the variations in $E_v$ due to atomistic randomness can be even larger than its mean value. To study the deterministic enhancement of the valley splitting arising from the Wiggle Well, combined with the presence of individual germanium atoms in the quantum well, we therefore perform atomistic tight-binding simulations in NEMO-3D using a 20-band $sp^3d^5s^*$ strain-dependent model [24]. The quantum well concentration profile of Fig. 1(b) is used to construct a heterostructure atom-by-atom, where the probability that an atom is Ge is given by the average Ge concentration at that atom’s layer. For all simulations, we assume a typical electric field of 8.5 MV/m.

Figure 3(b) shows the results of these simulations, for the two experimental scenarios shown in Fig. 3(a): in
the left-hand panel, the position of the dot changes as its size and shape are varied; on the right, only the size and shape are varied. In both cases, the orbital energy splitting changes from $m_1\omega^2(x^2 + y^2)$, where $m_1 = 0.19m_0$ is the transverse effective mass. Each curve reports the change in valley splitting $\Delta E_v$ across this range of energy splittings, for a different atomistic realization. The left-hand panel confirms that a wide range of valley splittings may be accessed by moving the dot; the experimental slope found for this tuning method (shown by the dashed line) lies within the range of simulation results. The NEMO-3D results in the right-hand panel show a much narrower range of changes in valley splittings, consistent with the experimental observations shown in blue in Fig. 3(a) (dashed line). These results highlight the ability of random-alloy disorder to affect valley splitting in this system, as compared to the more deterministic concentration oscillations, and the ability of a moving dot to sample these fluctuations.

We now use NEMO-3D tight-binding calculations to make quantitative predictions about valley splitting in other Wiggle Well structures. The top panel in Fig. 3(c) reports results for long-wavelength Wiggle Wells ($\lambda_{\text{long}}=1.8$ nm) with average Ge concentrations of 5%, 10%, 15%, and 20%. Here, each distribution shows the results of 40 simulations with different realizations of alloy disorder. The bottom panel reports results for short-wavelength Wiggle Wells ($\lambda_{\text{short}}=0.32$ nm) with average Ge concentrations of 0.5%, 1%, and 1.5%. In this case, results are shown for 20 random-alloy realizations. For all simulations shown in Fig. 3(c), we assume an orbital excitation energy of $h\omega = 2$ meV. For the long-period Wiggle Well, we see that the effects of alloy disorder are relatively large compared to the deterministic enhancement of the valley splitting caused by Ge oscillations, as indicated by the large spread in results. We also note that the 5% amplitude NEMO-3D results in the top panel are consistent with the experimental valley splittings shown in Fig. 3(a). For the short-period Wiggle Well, NEMO-3D predicts very large boosts in the deterministic contribution to the valley splittings, even for low-amplitude Wiggle Well oscillations.

In conclusion, we have introduced a new type of silicon/silicon-germanium heterostructure with a periodically oscillating concentration of germanium within the quantum well. Using effective mass theory, we showed that the Wiggle Well can induce couplings between the z-valley states, both within a Brillouin zone and between neighboring zones, thereby enhancing the valley splitting. We reported the growth of such a heterostructure with a Ge oscillation period of 1.8 nm within the quantum well, which showed mobility large enough, and corre-
sponding disorder small enough, to form stable and con-
trollable gate-defined quantum dots. Pulsed-gate spec-
troscopy revealed large valley splittings that were widely tuneable through changes in gate voltages. Tight-binding
simulations were used to validate the understanding of
the experiment and to make predictions about how alloy
disorder and structural changes (e.g., in the amplitude
and wavelength of the germanium oscillations) can be
expected to influence the valley splitting.

Note added: we are aware of related work studying the
atomistic role of Ge on valley splitting [31].

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METHODS

Theory. Valley splitting occurs when a sharp perturbation $V(z)$ couples the wavefunctions corresponding to the two lowest valley states. The unperturbed wavefunctions are given by

$$\phi_{\pm}(\vec{r}) = \psi(z) e^{\pm i k_0 z} \sum_{\vec{K}} c_{\pm}(\vec{K}) e^{i \vec{K} \cdot \vec{r}},$$

where $\psi$ is an envelope function, $\vec{K} = \pm (0,0,k_0)$ are the locations of the valleys in the first Brillouin zone, with $k_0 = 0.84(2\pi/a_0)$, $a_0 = 0.543$ nm is the cubic lattice constant, and $\vec{K}$ are reciprocal lattice vectors. As explained in the Supplemental Materials, the coefficients $c_{\pm}(\vec{K})$ are Fourier expansion coefficients of the cell-periodic part of the Bloch functions, determined by using the results of density functional theory for silicon [32] and accounting for the effects of the added germanium using the virtual crystal approximation. The valley splitting is then given by

$$E_v = 2 \left| \langle \phi_+ | V(z) | \phi_- \rangle \right| = 2 \sqrt{\sum_{\vec{K},\vec{K}'} c_{+}(\vec{K}) c_{-}(\vec{K}') \delta_{K_x,K'_x} \delta_{K_y,K'_y} I(K_z - K'_z)},$$

For the Wiggle Well, $E_v$ is dominated by the Ge concentration oscillations:

$$I(K_z - K'_z) = \int |\psi(z)|^2 e^{iQz} \frac{n_G V_0}{2} [1 - \cos(qz)] dz,$$

where $Q = K_z - K'_z - 2k_0$ and the integral is taken over the quantum well. The magnitude of $V_0$ is discussed in the Supplemental Materials. As in the NEMO-3D simulations, we compute the envelope function $\psi(z)$ assuming a typical vertical electric field of 8.5 MV/m. For the orbital ground state, $|\psi(z)|^2$ is smooth, with a single peak; its Fourier transform therefore has a single peak at zero wavevector. Hence, $I(K_z - K'_z)$ is strongly peaked when

$$q = \pm Q = \pm (K_z - K'_z - 2k_0).$$

Because of the sum over reciprocal lattice vectors in Eq. (2), $E_v(q)$ is enhanced whenever the condition $K_z - K'_z = \pm (q \pm 2k_0)$ is satisfied.

Heterostructure Growth. The measured heterostructure is grown on a linearly graded SiGe alloy with a final 2 μm layer of Si$_{0.705}$Ge$_{0.295}$. Prior to heterostructure growth, the SiGe substrate is cleaned and prepared as described in Ref. 13. The substrate is loaded into the growth chamber and flash heated to 825 °C while silane
and germane are flowing. The temperature is lowered to 600 °C, at which point a 550 nm 29.5% Ge alloy layer is grown. For the quantum well, the growth begins with a 10 second pulse of pure silane gas at 90 sccm. Then, 90 sccm of silane and 4.88 sccm of germane are introduced for 10.63 seconds followed by 10 seconds of pure silane. This SiGe–Si pulse sequence is repeated a total of 5 times. The pulse times are tuned to achieve a period of 1.8 nm and a peak Ge concentration of 9%. After the quantum well, a 60 nm Si$_{0.705}$Ge$_{0.295}$ spacer is grown and the heterostructure is capped with a thin 1 nm layer of pure silicon.

**Pulsed-Gate Spectroscopy** Pulsed-gate spectroscopy is used to measure the valley and orbital splitting of a singly-occupied quantum dot. A square wave voltage is applied to the plunger gate of a dot at a frequency comparable to the tunnel rate to the electron reservoir. The charge sensor current is measured with a lock-in amplifier referenced to the fundamental frequency of the square wave. When the dc voltage of the gate is swept over the dot transition, the electron is loaded and unloaded into the dot as the dot’s chemical potential, split by the square wave, straddles the fermi level of the reservoir. As the amplitude is increased, additional states such as the excited valley state and excited orbital state can be loaded during the high voltage period of the wave, modifying the tunnel rate into the dot. These changes in tunnel rate lead to a changing lock-in response. These changes can be seen in Fig. 2(d).

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**COMPETING INTERESTS**

DES, MF, RJ, SNC, and MAE are inventors on U.S. Patent No. 11,133,388, which pertains to a class of heterostructures of which that studied here is an example.

**AUTHOR CONTRIBUTIONS**

The project was conceived by RJ, TM, SNC, MF, and MAE. TM and BH fabricated the device and TM performed all experiments, with help from BH, JPD, MAW, and MAE. The heterostructure was grown by DES, with advice from MGL. Theoretical calculations were performed by YF and ML, with advice from RR, SNC, MF, and RJ. TM, ML, SNC, MF, RJ, and MAE wrote the manuscript, with input from all the authors.

**ADDITIONAL INFORMATION**

**Supplementary Information** The online version contains supplementary material available at https://

**Correspondence and request for materials** should be addressed to M.A.E.
SUPPLEMENTARY MATERIALS

S1. DETAILS OF THE EFFECTIVE MASS VIRTUAL CRYSTAL CALCULATION OF THE WIGGLE WELL VALLEY SPLITTING

The effective mass virtual crystal (EMVC) approximation calculations of the valley splitting $E_v$ shown in Fig. 1(c) of the main text and Fig. S1 were performed as follows. The electron is confined in a harmonic potential in the lateral $x$ and $y$ directions, and by a barrier and an applied electric field in the $z$ direction. Averaging over the lateral directions gives a one-dimensional, two-component Schrödinger equation for the envelope functions $\phi_\pm(z)$ that appear in Eq. (2) of the main text. The equation uses the longitudinal effective mass, $m_l = 0.92 m_0$, for the kinetic energy term. The diagonal potential for the model is $V(z) = V_F(z) + V_B(z) + V_{osc}(z)$. The external electrostatic potential energy is given by $V_F(z) = -eFz$, where $F = 8.5$ MV/m. The barrier potential is $V_B(z) = B [1 + \tanh(z/w)]$ with the barrier height $B = 0.15$ eV and barrier width $w = 1$ nm. The Wiggle Well potential is $V_{osc}(z) = n_{Ge} V_0 [1 - \cos(qz)]/2$, where $V_0$ is the difference in site energies between the silicon-like conduction-band levels of Si and Ge. We take this as $V_0 = -1.53$ eV from Table I of Ref. [33]. The off-diagonal potential that connects $\phi_+(z)$ to $\phi_-(z)$ has the additional factor $\exp[\pm i(K_z - K'_z - 2k_0)z]$ with the contributions of the reciprocal lattice vectors weighted by the appropriate combinations of $c_\pm(K)$, the coefficients of the cell-periodic parts of the Bloch functions. These coefficients are given in Table I of Ref. [34] for bulk Si. Extinction effects in the Si lattice turn out to be important for the calculation of $E_v$ for the long-period Wiggle Well. In particular, for pure Si, $c_+(K) = 0$ when $K = (0, 0, 4\pi/a_0)$. This is the wavevector for the Umklapp process, which accounts for the fact that $E_v$ at $\lambda_{long}$ is much less than $E_v$ at $\lambda_{short}$. This means that $c_+(0, 0, 4\pi/a_0)$ must be recalculated when Ge is present. For $n_{Ge} \geq 5\%$, the $c_\pm(K)$ coefficients need to be modified. This is also done using a virtual crystal approximation in which only the six largest $c_\pm(K)$ coefficients are used (Y. Feng and R. Joynt, to be published). This leads to an underestimate for $E_v$ when $\lambda \approx \lambda_{short}$ and $n_{Ge} \geq 5\%$.

S2. FABRICATION DETAILS AND HALL MEASUREMENT

Hall bars and quantum dot devices were fabricated simultaneously on the same ~10 mm chip. A 15 nm layer of aluminum oxide gate dielectric is grown by atomic layer deposition (ALD) at 200 °C. This oxide is etched by dilute HF in a 30 μm square region centered around the dot region. Another 5 nm of aluminum oxide is then deposited. This results in 5 nm of deposited oxide over the dot region and 20 nm over the Hall bars. The chip then undergoes a 15 min, 450 °C forming gas anneal. The Hall bar gate metal is a bi-layer of titanium and palladium, patterned by photolithography. The quantum dot gate design has three layers of aluminum patterned by electron-beam-lithography. Each

![Graph](image_url)

**FIG. S1.** The contribution to the valley splitting $E_v$ due to a sinusoidal Ge concentration in the quantum well as a function of the wavevector $q$, similar to Fig. 1(c) in the main text, with a larger scale (here) to show the peaks at $q \approx 20$ nm$^{-1}$. The average concentration $n_{Ge}$ of Ge in the quantum well of each curve is shown in the inset legend. The energy splittings listed in the inset are the maximum $E_v$ calculated for each concentration.
FIG. S3. Lever arms for voltage tunings. (a) Lever arms for S1 (yellow), S2 (green), and P1 (blue) of the ‘symmetric’ voltage tuning method described in the main text. (b) Lever arms for S1 (yellow), S2 (green), and P1 (blue) of the ‘asymmetric’ voltage tuning method described in the main text. In both plots, the shaded regions are ±5% around the average.

S3. GATE LEVER ARMS FOR DOT TUNING

The lever arm of the plunger gate P1 to the dot used for pulsed-gate spectroscopy is measured by thermally broadening the charge-sensed electron charging transition. The gate voltage is swept over the transition as the mixing chamber temperature is increased and the current through the charge sensor is fit to [35]

\[ I_{CS}(V) = A \tanh \left( \frac{\alpha(V - V_0)}{2k_B T_e} \right) + bV + I_0, \] (S1)

where \( k_B \) is Boltzmann’s constant, \( T_e \) is the electron temperature, \( \alpha \) is the lever arm, and \( A, b, V_0 \) and \( I_0 \) are additional fitting parameters. The term \( T_e/\alpha \) is extracted from each fit as a function of the mixing chamber temperature, \( T_{MC} \). This is fit to the phenomenological expression

\[ \frac{T_e}{\alpha} = \frac{1}{\alpha} \sqrt{T_{MC}^2 + T_{e0}^2}, \] (S2)

and we extract a lever arm and a base electron temperature \( T_{e0} \) when \( T_{MC} = 0 \).

For the ‘symmetric’ tuning method where both screening gates S1 and S2 are changed in the same voltage direction, the lever arm is measured at every other voltage tuning. For voltage tunings where the P1 lever arm is not explicitly measured, the average of the two nearest tunings is used. For the ‘asymmetric’ tuning method where S1 and S2 are changed in opposite directions, the lever arm is measured at every tuning. Relative lever arms between a screening gate layer is isolated by the self oxidation of the aluminum, enhanced by a 15 min downstream oxygen plasma ash. Figure S2 shows an optical image of the Hall bars measured and the transport mobility results of the measurements as a function of carrier density, measured at \( \sim 2 \) K.
FIG. S4. Illustration of (a) the long-period Wiggle Well, and (b) the short-period Wiggle Well, simulated using NEMO-3D. Black lines represent the ideal concentration profile, and red points represent the concentrations sampled at each layer. Both wells have a linearly graded interface concentration with width \( W = 1 \) nm. The difference in Ge concentration between the bulk (\( \rho_b \)) and the well (\( \rho_w \)) is \( \Delta \rho \), which is always fixed at 0.25 to prevent the wavefunction from spilling out of the quantum well. The concentration oscillation periods are \( \lambda = 1.8 \) nm in (a) and \( \lambda = 0.32 \) nm in (b). The amplitude of the concentration oscillation was adjusted, such that \( \rho_w = A \), where \( A \) is the oscillation amplitude.

gate and P1 are determined by measuring the slope of a transition line as both gate voltages are changed. Using the absolute lever arm of P1 and the relative lever arms for the screening gates, their absolute lever arms to the dot are calculated.

Figure S3 shows these lever arms for both the ‘symmetric’ and ‘asymmetric’ tuning methods. As shown, the lever arms for all three gates stays within 5% of the average value for most tunings. There is no noticeable difference in the lever arms between the tuning methods, despite the significant difference in valley splitting tuning. This may indicate that this method of tracking the lever arms is not a sensitive enough technique to measure the lateral movement expected in the ‘asymmetric’ tuning scheme.

S4. ADDITIONAL DETAILS OF NEMO SIMULATIONS

Figure S4 shows schematic illustrations of the Ge concentration profiles used to generate the lattice simulated in NEMO-3D. At a given layer, each atom in the lattice is assigned to be either Si or Ge, where the probability of choosing Ge is given by the average concentration in a given layer.