The impact of communications channel bandwidth on the accuracy of pulse-width signal transmission

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Abstract. Pulse-width modulation (PWM) signals used in various areas are sent to the receiver through a communications channel that distorts their waveform due to the limitations of the frequency range. It is not always possible to reduce additive (fluctuation) noises that are also present within the PWM signal to negligible levels. Limiting the range of frequencies transmitted over a communications channel results in both the deterioration of PWM signal front slopes and the changes in the spectral specifications of the fluctuation noise. The simulation of pulse signal formation helped identify a correlation between the pulse front slope and the number of harmonic components transmitted over the communications channel. Through the analysis, we established a correlation between pulse time and the additive noise parameters along with the bandwidth of the real communications channel. These calculations might be useful for problems where it is necessary to formulate the requirements for the communications channel transmitting the PWM signal.

1. Introduction

A pulse-width-modulated (PWM) signal is a crucial means of data transfer in telecommunications, engine power control systems, information and measurement systems, and voltage control systems [1-3].

Increasing the accuracy of PWM signal front and tail formation helps reduce data transfer losses, yet, in practice, there are factors shifting the front and the tail that cannot be completely eliminated. Areas where fluctuation noises distorting the impulse waveform occur include:

- semiconductor circuits with high switching frequency [4];
- signals transmitted by implanted pacemakers [5];
- coaxial cables used for transmitting high-voltage impulses [6];
- power source management in industrial electronics [7-8];
- direct current pulse converters [9];
- Gaussian impulse formation in radiation detectors [10];
- testing materials for dynamic stress and deformation [11];
- forming optical impulses to be used in laser devices [12-13];
- data transfer through optic fiber channels [14-17].
Besides, communications channels transmitting PWM signals cannot be completely free from frequency range limitations for the signals they transmit [18]. The data signal and the additive noise are filtered at the output of the real communications channel. This impacts the shifts of trigger instants for impulse formation relative to the true positions. The goal of this research is to identify dependencies for the distribution of trigger instants for PWM signal formation and the communications channel bandwidth.

2. Methodology
PWM signals are transmitted over large distances through communications channels that distort their waveform due to the limitation of the frequency range of the transmitted signals. Changing impulse amplitude or front time results in the error in time span allocation by the thresholder of the secondary meter. Let us review the correlation of this error and the harmonic content, start and end reporting limits, PWM signal amplitude, and length.

The frequency range of the communications channel (CC) is characterized by the amplitude and frequency parameters (AFP) and the phase-frequency response (PFR) with respective equations $C(\omega) = U_2(\omega)/U_1(\omega)$ and $\phi(\omega) = \varphi_2(\omega) - \varphi_1(\omega)$, where $U_1(\omega)$ and $U_2(\omega)$ are sine voltage amplitudes at the CC input and output and $\varphi_1(\omega)$ and $\varphi_2(\omega)$ are input and output voltage phases.

Since the signal reduction by the linear system (i.e. scaling) can be compensated by the amplification and does not impact the waveform of the output signal, we will use normalized AFP from now on.

$$
C(\omega) = \frac{U_2(\omega)}{U_1(\omega)}
$$

(1)

Non-distorted impulse transmission over CC requires that its AFP defined by the lower $\Omega_L$ and upper $\Omega_U$ limit frequencies be ideal (rectangular) across the entire frequency range (i.e. $\Omega_L \rightarrow 0$, $\Omega_U \rightarrow \infty$), while the PFR shall be linear.

$$
\phi(\omega) = \tau_0 \omega + k_1 \pi,
$$

(2)

where $k_1$ is some factor, $k_1 = 0, 1, 2, \ldots$; $\tau_0$ is the value defining the delay time for the input signal of a constant waveform.

Let us review the impact of frequency range limitation on the length of fronts and output impulses, provided that expression (2) is valid and CC input impulses have constant repetition frequency and length.

Let us consider a periodic rectangular impulse with an amplitude of $U_{01}$, a length of $\tau_p$, and an interval of $T_0$ as a Fourier series

$$
U_1(t) = U_{01}\left(\frac{\tau_p}{T_0} + \sum_{n=1}^{\infty} 2 \frac{\sin n\pi \tau_p/T_0}{n\pi} \cos 2\pi n \frac{t}{T_0}\right)
$$

(3)

Considering formula (1), we will obtain the following rectangular AFP signal at the CC output

$$
a_2(t) = \frac{U_2(t)}{U_{02}} = \frac{\tau_p}{T_0} + \sum_{n=1}^{k} 2 \frac{\sin n\pi \tau_p/T_0}{n\pi} \cos 2\pi n \frac{t}{T_0},
$$

(4)

where $a_2(t)$ is the normalized output signal; $k$ is the number of harmonics transmitted over the CC.

Since the impulse tail under the assumptions mentioned is a mirror image of the front relative to the middle of the rectangular impulse, we may only study the tail and determine output signal values $a(t) = a_2(t)$ within $(\frac{\tau_p}{2} - \Delta \tau_p) \ldots (\frac{\tau_p}{2} + \Delta \tau_p)$. In that case, expression (4) is written as:

$$
a(t) = \frac{\tau_p}{T_0} + \sum_{n=1}^{k} 2 \frac{\sin n\pi \tau_p/T_0}{n\pi} \cos 2\pi n \left(\frac{\tau_p}{2T_0} + \frac{\Delta \tau_p}{T_0}\right)
$$

(5)

where $\frac{\Delta \tau_p}{T_0}$ is the transmission error normalized against the interval length under the specified front or tail level.
3. Simulation and approximations
Following expression (5), computer simulation was used to perform calculations for \( \frac{\tau_p}{T_0} = 0.1 \div 0.9 \) and \( k = 10 \div 500 \) with subsequent impulse tail approximation using the least-squares method \([19]\) within \( a = 0.2 \div 0.8 \) as a straight line. For the mentioned parameter change ranges, we can assume that the front and tail slope does not depend on the impulse length. Slope \( S \) is understood as a ratio of normalized transmission error \( \frac{\Delta \tau_p}{T_0} \) to deviation \( \Delta a \) of the actual reporting limit for length from the reference. The correlation between slope \( S = \frac{\Delta \tau_p}{T_0} / \Delta a \) and the number of harmonics transmitted over the CC is approximated using expression

\[
S = 0.515 \frac{1}{k} \tag{6}
\]

Minimum error \( \frac{\Delta \tau_p}{T_0} \) occurs when the tail front is set at \( a_f = 0.5 \), which can be viewed as the reporting limit for impulse length. Value \( \frac{\Delta \tau_p}{T_0} \) under changing \( a_f \) is determined using formula

\[
\frac{\Delta \tau_p}{T_0} = 0.515 \left( 0.5 - a_f \right)^{\frac{1}{k}} \tag{7}
\]

After the insertion of formula (6) into (7), we finally get

\[
\delta \tau_p = 0.515 \left( 0.5 - a_f \right)^{\frac{1}{k}} \tag{8}
\]

The relative change in impulse length due to the front or tail distortion can be determined by empirical dependency

\[
\delta \tau_p = 0.515 \frac{0.5 - a_f}{k \tau_p / T_0} \tag{9}
\]

Considering equation (8), we obtain the front and tail length within levels \( a = 0.2 \) and 0.8 equal to

\[
\tau_f = 0.39 \frac{T_0}{k} \tag{10}
\]

The error of normalized empirical dependencies does not exceed 5%. The same proportions can be obtained for a CC with a perfect AFP, provided that carrier frequency \( \omega_c \) is in the center of the bandwidth (i.e. \( \omega_c = 0.5(\Omega_U + \Omega_L) \)). In this case, formulae (6) – (10) are valid, and the number of envelope harmonics transmitted is determined as \( k = \text{ent} \left( \frac{0.5(\Omega_U + \Omega_L)}{\omega_0} \right) \). Thus, by \( a \) we understand carrier frequency envelope \( \omega_c \).

4. Mathematical model
The AFP of real CC can be approximated quite precisely through the expression

\[
C(\omega) = 1 - \left( \frac{\omega}{\Omega_U} \right)^p \tag{11}
\]

where \( p \) is the parameter characterizing the AFP waveform in the bandwidth; \( \Omega_U \) is the higher AFP frequency with \( C(\omega) = 0 \).

The respective fading of the CC is determined as follows:

\[
B(\omega) = \ln \frac{1}{C(\omega)} \tag{12}
\]

The waveform of \( C(\omega) \) and \( B(\omega) \) is shown in figure 1.
Figure 1. Diagrams $C(\omega/\Omega_U)$ and $B(\omega/\Omega_U)$ are shown in a) and b) respectively.

With type (11) AFP, expression (5) looks like

$$a(t) = \frac{\tau_p}{T_0} + \sum_{n=1}^{k} \left[1 - \left(\frac{n\omega_0}{\Omega_U}\right)^p\right] 2 \frac{\sin \pi n \tau_p}{\pi n} \cos 2\pi n \left(\frac{\tau_p}{2T_0} + \frac{\Delta \tau_p}{T_0}\right),$$  \hspace{1em} (13)

where $k = \text{ent} \left(\frac{\Omega_U}{\omega_0}\right)$.

We used formula (13) to calculate $\frac{\tau_p}{T_0} = 0.1 \div 0.9$, $p = 0.1 \div 10$, and $k = 10 \div 500$. They showed that, in the specified change range $\frac{\tau_p}{T_0}$, a significant proportion of energy in the frequency sector is redistributed. With the AFP waveform shown in (11), the impulse waveform is significantly distorted at the CC output. According to $\frac{\tau_p}{T_0}$, impulse amplitude and its lower limit change as well.

The amplitude, middle and lower impulse levels are determined by empirical dependencies obtained through the analysis of the results of calculations using formula (13):

$$a_{\text{max}} = 1 \left(1 - \frac{\tau_p}{T_0}\right) e^{-b_1p} ; a_{\text{av}} = 0.5 + \left(\frac{\tau_p}{T_0} - 0.5\right) e^{-b_2p} ; a_{\text{min}} = \frac{\tau_p}{T_0} e^{-b_3p}$$  \hspace{1em} (14)

where $b_1 = \left(1.1 + 0.7 \frac{\tau_p}{T_0}\right) \sqrt{k}$; $b_2 = \left(1.8 - 1.4 \left|0.5 - \frac{\tau_p}{T_0}\right|\right) \sqrt{k}$;

$$b_3 = \left(1.8 - 0.7 \frac{\tau_p}{T_0}\right) \sqrt{k}.$$  \hspace{1em} (15)
The section of the take within $a = (0.2 \div 0.8) (a_{\text{max}} \div a_{\text{min}})$ is approximated by a straight line. Having generalized the family of such dependencies obtained as a result of calculations, we get the following empirical formula:

$$S = \left(0.515 + \frac{0.64}{p}\right) \frac{1}{k}.$$  \hspace{1cm} (16)

Considering equation (8), we get

$$\frac{\Delta \tau_p}{T_0} = (a_{av} - a_f) \left(0.515 + \frac{0.64}{p}\right) \frac{1}{k \tau_p/T_0}$$  \hspace{1cm} (17)

and

$$\delta \tau_p = (a_{av} - a_f) \left(0.515 + \frac{0.64}{p}\right) \frac{1}{k \tau_p/T_0}$$  \hspace{1cm} (18)

where $a_{av}$ is the medium level of the impulse for $\frac{\tau_p}{T_0} = 0.5$ assumed as the design reporting limit; $a_f$ is the actual reporting limit for length.

The length of the front and tail within $a = (0.2 \div 0.8) (a_{\text{max}} \div a_{\text{min}})$ is determined by the expression

$$\tau_f = 0.6 (a_{\text{max}} - a_{\text{min}}) \left(0.515 + \frac{0.64}{p}\right) \frac{1}{k \tau_p/T_0}.$$  \hspace{1cm} (19)

The resulting absolute mean square error in the impulse length identification equals

$$\sigma_x = \sqrt{(\sigma_a^2 + \sigma_f^2 + \sigma_p^2 + \sigma_{\text{p}}^2 + \sigma_{\phi}^2)}.$$  \hspace{1cm} (20)

After the transition to relative errors, we get

$$\delta_x = \frac{\tau_p}{T_0} \sqrt{(\delta_a^2 + \delta_f^2 + \delta_p^2 + \delta_{\text{p}}^2 + \delta_{\phi}^2)}.$$  \hspace{1cm} (21)

where $\delta_a$ is the error caused by the reporting limit instability; $\delta_f$ is the error caused by the changes in impulse length; $\delta_p, \delta_{\text{p}}, \delta_{\phi}$ are errors caused by CC parameter changes depending on the operational conditions; $\delta_{\phi}$ is the error caused by PFR non-linearity. Consider that

$$\delta_a = \pm 2 \Delta a_f \left(0.515 + \frac{0.64}{p}\right) \frac{1}{k \tau_p/T_0}$$  \hspace{1cm} (22)

where $\Delta a_f$ is the instability of reporting limit for length;

$$\delta_f = \pm 2 \gamma e^{-1.8 + 1.4 \gamma} \left(0.515 + \frac{0.64}{p}\right) \frac{1}{k \tau_p/T_0}$$  \hspace{1cm} (23)

where $\gamma$ is the indicator determining the range of changes $\frac{\tau_p}{T_0}$

$$[\frac{\tau_p}{T_0} = (0.5 - \gamma) \div (0.5 + \gamma)].$$

$$\delta_p = \pm 2 (a_{av} - a_f) \frac{0.64}{k \tau_p/T_0} \cdot \frac{\Delta p}{p^2}$$  \hspace{1cm} (24)

$$\delta_{\text{p}} = \pm 2 (a_{av} - a_f) \left(0.515 + \frac{0.64}{p}\right) \frac{1}{k \tau_p/T_0} \cdot \frac{\Delta k}{k}$$  \hspace{1cm} (25)

where $\Delta p$ and $\Delta k$ are the instability of CC AFP, $\Delta k = \text{ent} \left(\frac{\Delta a}{\omega_0}\right)$.

5. Conclusions and discussion

In the case of real communication channels, the procedural error in formulae (14-25) helps assess the contribution of each of the components in the total error when assessing the impulse length. Limiting
the range of PWM signal frequencies transmitted over a communications channel results in both the deterioration of their front slopes and the changes in the spectral specifications of the fluctuation noise. For this case, let us determine the impulse length identification error. Let us assume that before the limitation of the frequency range, standard additive noise $\eta(t)$ has a constant spectral density $G_0$ along the entire frequency range. In this case, the square value of noise on the receiving side of the communications channel is determined as follows.

$$\sigma_\eta^2 = \int_0^\infty G_0 \cdot C^2(f)df$$  \hspace{1cm} (26)

where $C(f)$ is the amplitude and frequency parameters of the communications channel. For type (1) and (11) $C(f)$, the value of $\sigma_\eta^2$ is

$$\sigma_\eta^2 = \int_0^{f_p} G_0 df = G_0 f_p$$  \hspace{1cm} (27)

$$\sigma_\eta^2 = \int_0^{f_p} G_0 \left[1 - \left(\frac{f}{f_p}\right)^p\right]^2 df = G_0 f_p \frac{2p^2}{2p^2 + 3p + 1}$$  \hspace{1cm} (28)

Let us use the expression from [20]

$$\omega_1(t) = \frac{\gamma}{\sqrt{2\pi\sigma_\eta}} \exp \left( -\frac{1}{2} \frac{\gamma^2 t^2}{\sigma_\eta^2} \right),$$  \hspace{1cm} (29)

where $\omega_1(t)$ is the density of probabilities of the instances of the first exceedance of the amplified triggering.

Besides, taking into account formulae (27) and (28) and expressions (6) and (16), we get

$$\sigma_t = \sqrt{G_0 f_p} \frac{0.515}{U_m f_p} = 0.515 \frac{G_0}{U_m} \sqrt{f_p}$$  \hspace{1cm} (30)

$$\sigma_t' = \frac{2p^2 \sqrt{G_0 f_p} (0.515 - \frac{0.64}{p})}{(2p^2 + 3p + 1) U_m f_p} = \frac{1.03p^2 + 1.28p}{(2p^2 + 3p + 1) U_m} \sqrt{\frac{G_0}{f_p}}$$  \hspace{1cm} (31)

Since in real conditions $f_p \gg 1/\tau_s$ where $\tau_s$ is the PWM signal impulse length, and the correlation ratio for start and end formation errors $f(\tau_s) \ll 1$, the formulae for the calculation of the generalized impulse length identification error can be expressed as follows:

$$\sigma_s = \frac{0.72}{U_m} \sqrt{\frac{G_0}{f_p}}$$  \hspace{1cm} (32)

$$\sigma_s' = \frac{1.45p^2 + 1.8p}{(2p^2 + 3p + 1) U_m} \sqrt{\frac{G_0}{f_p}}$$  \hspace{1cm} (33)

The resulting formulae (32) and (33) can be conveniently used for the transmission of PWM signals with both ideal and real amplitude and frequency parameters used in the communications channel. These formulae show that the extension of communications channel bandwidth can reduce the impulse length identification error despite the simultaneous increase in the current fluctuation noise. The obtained expression allows us to do the following:

- formulate the exchange conditions for accuracy parameters, generator, and communications channel parameters, and develop requirements for the frequency range of a communications channel;
- facilitate the comparability of errors occurring during signal transformation.

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