INTERVAL VALUED FUZZY MODELING AND INDIRECT ADAPTIVE CONTROL OF QUADROTOR

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ABSTRACT

In this paper, a combination of fuzzy clustering estimation and sliding mode control is used to control a quadrotor system, whose mathematical model is complex and has unknown elements, including structure, parameters, and so on. In addition, they may be affected by external environmental disturbances. At first, the nonlinear unknown part of the system is estimated by a fuzzy model, A new method is presented for constructing a Takagi-Sugeno (TS) interval-valued fuzzy model (IVFM) based on input-output data of the identified system. Following the construction of the fuzzy model that estimates the unknown part of the quadrotor system, a control and on-line adjusting of the fuzzy modeled part of dynamics is used. In this step, the system model will be estimated in adaptive form so that the dynamic equations can be used in sliding mode control. Finally, the proposed technique is applied, and the simulation results are presented to show the effectiveness of this approach in controlling the quadrotor with unknown nonlinear dynamics.

Keywords: Interval values Modeling · fuzzy clustering · fuzzy control · Indirect adaptive control · quadrotor control · sliding mode control · type-2 fuzzy sets.

1 Introduction

The quadrotor unmanned aerial vehicle (UAV) is a typical second-order, nonlinear, strongly coupled dynamic system. An adaptive control scheme is significant for unmanned quadrotor helicopters, and this has received attention in the literature. External disturbances can have a impact on the stability and reliability of these systems. Therefore, it is very important to study the effects of system disturbances and design control strategies that address actuator faults and disturbances. To date, many methods and strategies have been presented in the literature [H.Mo and Farid, 2019]. Authors presents PD control as well as a fuzzy adaptive PD control scheme in which the controller’s gains are automatically adjusted in relation to the error signal rate [Gao et al., 2015]. The fuzzy adaptive PD controller is quicker at achieving the desired response as compared to the other one. The authors of [Rabhi et al., 2011] used a robust fuzzy control approach to stabilize a quadrotor’s attitude angles. They used the parallel distributed compensation technique to design the fuzzy controller. High-order sliding mode observer was designed to provide state information...

*Use footnote for providing further information about author (webpage, alternative address)—not for acknowledging funding agencies.
required for fuzzy controller design. The study of Niroumand and Seyedsajadi [2013], integrated integral backstepping with fuzzy logic to improve the flight controller’s performance. Simulation results show superior performances for the fuzzy-integral backstepping approach over conventional integral backstepping. Fuzzy based sliding mode control for quadrotor UAV has recently been reported in [Pazooki and Mazinan [2017] and control structure is augmented via a genetic algorithm for optimization purposes. This approach can be further investigated using second order sliding mode control to avoid chattering. In [UI Amin and Kamsin [2017], NRB and E-RBF are used collectively to approximate the unknown dynamics of the quadrotor. A study Dierks and Jagannathan [2010], addresses online learning of the dynamics of a quadrotor nonlinear model through neural networks. Online learning is normally preferred over offline learning for the reason that quadrotors may fly in a dynamic environment where different worse case perturbations may occur, and such operating scenarios are hard to generate offline. Also, this study conveys the idea of using NNs to control the six degrees of freedom of a quadrotor along with designing an ANN observer to estimate states of the system. A neural network is used to approximate unmodeled dynamics during flight in [Dierks and Jagannathan [2009], and interconnection errors in flight formation are investigated. In [Li and Zheng [2016] the position controller considered as nonlinear system and approximated using radial basis function neural networks. This study is unique in a sense that a complete controller is approximated via adaptive rule and trained online. This neuronal network-based controller provides adaptation for parametric uncertainties along with other external disturbances. In [Lee [2021] authors proposed a nonlinear-sliding-surface-based SMC for tracking and stabilization with an SMO. To improve the performance of the quadrotor system, they remodeled the augmented quadrotor dynamics and estimated the unmeasured states and their derivatives through the sliding mode observer. To improve the SMC control performance, a nonlinear sliding surface is used to reduce the convergence time with a modified deceleration curve. The authors proved the performance of a tracking system. Based on the above research analysis, we propose an adaptive control strategy, and design an adaptive controller for a quadrotor UAVs in the presence of external and internal disturbances. The introduced adaptive control strategy is based on fuzzy logic control theory and adaptive sliding mode theory. A new method for constructing a Takagi-Sugeno (TS) type-2 fuzzy model offline, based on input-output data, is proposed, and the obtained fuzzy model of the dynamic is used to build the model-based control of the system, with an adaptive control strategy scheme that compensates the effects of the disturbance and an unmodeled dynamics control system. The system’s global asymptotic stability is validated by the Lyapunov function. The effectiveness and feasibility of the proposed method are demonstrated by simulation studies of the unmanned vehicle. The rest of the paper is organized as follows. Section 2 describes the proposed method of IVFM construction from input-output datasets of the system. Section 3 presents the application of the proposed method to build the IVFM of the quadrotor. Section 4 presents the proposed control strategy based on fuzzy logic control and adaptive sliding mode for the quadrotor UAV under external disturbances and parameter changes. The stability analysis of the new methodology is validated by the Lyapunov function. The results of simulation studies that demonstrate the performance of the proposed adaptive fuzzy schemes are presented in Section 5. Finally, general conclusions and future work plans are provided.

2 Fuzzy modeling

The proposed method for constructing a Takagi-Sugeno (TS) type-2 fuzzy model, based on the input-output data of the identified system is constructed in three steps: (1) structure identification by fuzzy clustering; (2) envelope detection; and (3) parameter identification. In the structure identification phase, a clustering method based on the Gustafson-Kessel algorithm (GKCA) is used in order to detect the linear subsystems of the whole nonlinear system Babuska et al. [1998] (local linearization). Then, an envelope detection algorithm (EDA) based on the derivative concept is proposed to estimate both the upper and lower membership functions of the type-2 fuzzy membership function (T2MF) defined point-wise. In the parameter identification step, the least squares algorithm is applied to compute the best parameter values for the premises (Gaussians) and the consequences (straight lines) parameters.

2.1 Fuzzy Clustering overview

Clustering is the partitioning of data into subsets or groups based on similarities between the data. The main potential of clustering is to detect the underlying structure in data, not only for classification and pattern recognition, but for model reduction and optimization. To detect clusters of different geometrical shapes in a single data set, we used the Fuzzy Gustafson Kessel Clustering (FGK), which extends the standard fuzzy c-means algorithm by employing an adaptive distance norm Babuska and Verbruggen [1997], Gustafson and Kessel [1979], Filho and Serra [2016]. Each cluster has its own norm-inducing matrix, $A_i$, which yields the inner-product norm Eqn. (1) shown below:

$$D_{ik}^2 = (x_k - v_i)^T A_i (x_k - v_i),$$

(1)
where \(1 \leq i \leq c, 1 \leq k \leq N\). The matrices \(A_i\) are used as optimization variables in the \(C - \text{means}\) functional, which will allow to each cluster to adapt the distance norm to the local topological structure of the data. Let \(A\) denote a \(c\)-tuple of the norm-inducing matrices: \(A = (A_1, A_2, ..., A_c)\). The objective functional of the GK algorithm is defined by:

\[
J(X, U, V, A) = \sum_{i=1}^{c} \sum_{k=1}^{N} (\mu_{ik})^m D^2_{ikA}
\]

where \(N\) the number of data points, \(c\) the number of clusters, \(x_k\) the \(k^{th}\) data point, \(v_i\) the \(i^{th}\) cluster center, \(\mu_{ik}\) the degree of membership of the \(k^{th}\) data in the \(i^{th}\) cluster, \(A_i\) the norm-inducing matrix of the \(i^{th}\) cluster and \(m\) is a weighting exponent that determines the fuzziness of the resulting clusters (typically \(m = 2\)). In our paper, the GKCA is applied in order to obtain the fuzzy partition matrix \(U = [\mu_{ik}]_{c \times N}\), with \(\mu_{ik} \in [0,1]\) a membership degree.

### 2.2 Proposed envelope detection and estimation algorithm

The individual partitioning subsets \(\mu_{ik}\) projected on the regressor \(X\) will define directly fuzzy regions, in which the data can be reasonably approximated by linear sub-models defined by clusters. Projection data can also make the fuzzy system easy to read by humans.

#### 2.2.1 Detection of lower and upper premise MF

In order to detect the fuzzy premise MFs (lower MF \(f\) and the upper MF \(\bar{f}\)) we propose in what follows an approach called for Envelope Detection Algorithm (EDA) based on derivative concept. The algorithm is described in the following algorithm:

1) First, we interpolate the points linearly (dotted curve in Fig.1(a). between all segments \([y(i), y(i+1)]\) where \(i\) represents the data indices.

The third-level of heading follows the style of the second-level heading. 2) We compute the slopes between each successive data points \([y(i), y(i+1)]\) as:

\[
d(i+1) = \frac{y(i+1) - y(i)}{x(i+1) - x(i)}
\]

3) Two cases can arise:

- When the slope \(d\) in interval \([x(i), x(i+1)]\) is positive, then the point is considered as higher membership point and memorized with its indices as \((I_h, y_h(I_h))\), where \(I_h = i + 1\) represents the indices of the corresponding higher membership point \(y_h\) s.t. \(y_h(I_h) = y(i+1)\).

- Otherwise, when the slope \(d\) in interval \([x(i), x(i+1)]\) is negative, then the point is considered as lower membership point and memorized with its indices as \((I_L, y_L(I_L))\), where \(I_L = i + 1\) represents the indices of the corresponding lower membership point \(y_L\) s.t. \(y_L(I_L) = y(i+1)\).

By applying our method cited above, we show in Fig.1 the shape of the obtained upper and lower envelopes, which represent the upper and lower type-2 MFs. The details of this technique are given in EDA Algorithm [Bouhentala et al. 2017]. This algorithm doesn’t ensure the separation of envelopes; to do that, we can apply the same algorithm again for both the upper and lower sets. When we apply this algorithm to the initial set of points, we get two sets: \(f\) and \(\bar{f}\). Next, we use the same algorithm for both sets \(\{f_{\text{and} \bar{f}}\}\) to get another four sets. The proposed algorithm can be presented as the algorithm described here:

1. Applying \(ADE(\text{initial set})\) gives two sets \((f, \bar{f})\)
2. make \(i = j = 1\)
3. \((f_j, \bar{f}_i) = ADE(\text{ initial set })\)
4. Calculate the upper set \(\bar{f}\)
   (a) Make \(i = i + 1\)
   (b) \((f_i, \bar{f}_i) = ADE(\bar{f})\)
(c) If go to step (a) else next:
(d) $\bar{f} = f_i$

5. Calculate the lower set
   (a) Make $j = j + 1$
   (b) $(f_j, \bar{f}_j) = ADE(f)$
   (c) If $j \neq j_{\text{max}}$ go to step (a) else next:
   (d) $f = f_j$

Look to the next example where the initial function is a Gaussian with added noise to create a cloud to its shape, in
the Fig. 2 where the first ADE is applied and its corresponding fitting result as shown in the figure (b) there are many
2.3 Constructing fuzzy models from partitions

At this step, we have two data sets of points: data set 1 represents the data of $f_L$ and data set 2 represents the data of $f_U$. Gaussian membership functions will be used in this investigation for the premise MF; thus, data sets 1 and 2 will be fitted to Gaussian models, yielding the lower and upper Gaussian membership functions $\bar{f}$ and $f$. Until now, we have obtained, by using the proposed method, type-2 fuzzy Gaussian membership functions for the premises. The global model can be conveniently represented as a set of affine Takagi-Sugeno (TS) rules $R_i$ as follows:

$$\text{if } x \text{is } \tilde{A}_i \text{then } y_i = a_i x + b_i \text{ for } i = 1, 2, ..., c$$  \hspace{1cm} (4)

where $x$ is the input, and $y$ is the output, $\tilde{A}_i = [f_L, \bar{f}_i]$ the $i^{th}$ obtained antecedent type-2 fuzzy MF. The consequent parameters $a_i$ and $b_i$ are estimated from the data using least square method.

3 Fuzzy modeling of unknown dynamics of quadrotor

In the following application, we will use the method presented above to model the unknown functions using input-output data, and then we will use those initial models to build the adaptive control.

3.1 Dynamic model of Quadrotor

Let’s consider the system in Eqn. (5) as presented in Lee [2021], Bouabdallah [2007] in which the state representation $x = (\varphi, \dot{\varphi}, \theta, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z})$ is the state vector and $U = [U_o \ U_q \ U_q \ U_z]$ is the control vector. The transformation matrix between the rate of change of the orientation angles $(\dot{\varphi}, \dot{\theta}, \dot{\psi})$ and the body angular velocities $(p, q, r)$ can be considered as unity matrix if the perturbations from hover flight are small. Then, one can write $(\dot{\varphi}, \dot{\theta}, \dot{\psi}) \approx (p, q, r)$ Bouabdallah [2007]. Each axis can be approximately decoupled and controlled independently when the angular
The model Eqn. (5) can be decomposed into three subsystems:

\[
\begin{align*}
\dot{x} &= \begin{pmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi} \\
\dot{u}_x \\
\dot{v}_x \\
\dot{w}_x
\end{pmatrix} = \begin{pmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi} \\
\dot{u}_x \\
\dot{v}_x \\
\dot{w}_x
\end{pmatrix}
\end{align*}
\]

\[a_1 = (I_{yy} - I_{zz})/I_{xx} \quad b_1 = l/I_{xx}
\]
\[a_2 = J_x/I_{xx} \quad b_2 = l/I_{yy}
\]
\[a_3 = (I_{zz} - I_{xx})/I_{yy} \quad b_3 = 1/I_{zz}
\]
\[a_4 = J_x/I_{yy} \quad u_x = (c_1 \phi - s_1 \psi)
\]
\[a_5 = (I_{xx} - I_{yy})/I_{zz} \quad u_y = (c_2 \phi - s_2 \psi)
\]

The inputs of each axis can be composed of the speed of the propeller and described as Bouabdallah [2007]:

\[
U_\phi = (\Omega_1 - \Omega_3) \quad U_\theta = (\Omega_4 - \Omega_2) \quad U_z = (\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4)
\]

### 3.2 Interval values Fuzzy modeling

The model Eqn. (5) can be decomposed into three subsystems:

\[
\begin{align*}
\dot{x}_1 &= f_\phi(\dot{\phi}, \dot{\psi}) + g_1 U_1 \\
\dot{x}_2 &= f_\theta(\dot{\phi}, \dot{\psi}) + g_2 U_2 \\
\dot{x}_3 &= f_\psi(\dot{\phi}, \dot{\psi}) + g_3 U_3
\end{align*}
\]

where \( f_\phi(\dot{\phi}, \dot{\psi}, \Omega_r) = \dot{\phi} \phi a_1 + \dot{\phi} a_2 \Omega_r + \theta f_\phi \) and \( f_\theta(\dot{\phi}, \dot{\psi}, \Omega_r) = \dot{\theta} \phi a_3 + \dot{\theta} a_4 \Omega_r \). The parameters \( g_1, g_2, g_3 \) and the functions \( f_\phi, f_\theta \) and \( f_\psi \) are considered as unknown functions, and they will be estimated as follow:

the first step is to identify the functions \( f_i(x, \dot{\psi}, \Omega_r) \) and \( g_i \) where \( \{i = \phi, \theta, \psi\} \) using clustering. \( \dot{f}_i \) can be written as an TS approximation of the form IVFM Eqn. (4):

\[
\begin{align*}
f_\phi(x) &= \hat{f}_\phi(x, x_{12}, \Omega_r, \theta f_\phi) + \varepsilon = \theta f_\phi + e \varphi f_\phi(x) \\
f_\theta(x) &= \hat{f}_\theta(x, x_{12}, \Omega_r, \theta f_\theta) + \varepsilon = \theta f_\theta + e \varphi f_\theta(x) \\
f_\psi(x) &= \hat{f}_\psi(x, x_{12}, \theta f_\psi) + \varepsilon = \theta f_\psi + e \varphi f_\psi(x)
\end{align*}
\]

### Table 1: Parameters of the Quadrotor

| \( I_{xx} \) | \( I_{yy} \) | \( I_{zz} \) | \( m \) | \( l \) |
|---------------|---------------|---------------|-------|-------|
| 0.09 kg.m²   | 0.18 kg.m²   | 0.18 kg.m²   | 1.2 kg| 0.23 m|
| 6N.m.s² | 9.81 m.s² | 32 kg.m | 11.4 | 1.32 e-3 kg.m² |
where $\theta_f_i = [w_i^T, w_r^T]^T$ and $\psi_f_i(x, \dot{x}) = \frac{1}{2} [\xi_{i1}^T(x, \dot{x}), \xi_{i2}^T(x, \dot{x})]^T$.

To collect input-output data, we generate two sets of data, one for identification by clustering, and the second for validation. We start with a sequence of 4000 samples (2000 for identification and 2000 for validation) The sampling time is $T = 0.01s$ with normal noise (05\% of amplitude) is added to every output. The obtained data is treated using EDA in order to construct the membership functions as presented in Fig. 4. The membership functions type-1 and type-2 obtained with simulation data for $f_\phi(x, \Omega)$, $f_\theta(x, \Omega)$ and $f_\psi(x, \Omega)$ are depicted in The figures (Fig. 5, 7), respectively.
Figure 6: Membership Functions of $\hat{f}_\theta(x)$

Figure 7: Membership Functions of $\hat{f}_\psi(z)$
Table 2: Comparison between RMSE of T1FM and IVFM

|                  | T1FM | IVFM  |
|------------------|------|-------|
| $f_1(x, \Omega)$ | 1.5958 | 1.4016 |
| $f_8(x, x_{12}, \theta_{f_8})$ | 1.1717 | 1.1009 |
| $f_9(x, \Omega)$ | 0.2072 | 0.1930 |

The validation of fuzzy models obtained (the fuzzy model type-2 ; IFVS and the fuzzy model type-1 ; T1FS) as shown in Fig. 8 and the Table 2 show that our proposed technique (IVFM) is more close to real functions of system compared to the type-1 model.

4 Indirect Adaptive control

The initial model of the antecedent part in Eqn. (4) plays an important role in the structure of the system. The previous clustering algorithm is used to build the initial structure of the unknown part of a nonlinear system with a fuzzy model. The laws of Lyapunov are then applied to adjust the identified model while maintaining system stability.

Let $e(t) = x_m(t) - x(t)$ the error of tracking, $K = [k_0, 1]$ a vector of parameters of conception and the equation of sliding is given by:

$$e_s(t) = e + k_0 \dot{e}$$

where $e_s(t) = (e_{x}, e_{y}, e_{z}, e_{\phi}, e_{\theta}, e_{\psi})$ represent the sliding surface.

4.1 Design of fuzzy adaptive SMC control law

The approximated functions $f(x)$ and $g(x)$ with the universe approximator flous, take the following forms: $\hat{f}(x, \theta) = \theta_f^T \varphi_f(x)$ and $\hat{g}(x, \theta) = \theta_g^T \varphi_g(x)$ With $\varphi_f$ and $\varphi_g$ are the basis of fuzzy vector functions, supposed convenablement fixed by the fuzzy identification presented in 3.2 and are the vectors of paramètres ajustables supposed définis respectively in the domaine $\Omega_f$ and $\Omega_g$. We défin the sub-set $S_x \subseteq \mathbb{R}^n$ as a space in which the trajectories of states $x$ stay inside it in the closed loop. The functions $f(x)$ and $g(x)$ are written as follow:

$$\dot{\hat{f}}(x, \theta_f) = f^*(x, \theta_f^*) + w_f(x)$$
$$\dot{\hat{g}}(x, \theta_g) = g^*(x, \theta_g^*) + w_g(x)$$
with $w_f$ and $w_g$ represent the errors of approximation. $f^*(x, \theta)$ and $g^*(x, \theta)$ are respectively the optimal parameters of $\hat{f}(x)$ and $\hat{g}(x)$. The values of parameters $\theta_f$ and $\theta_g$ minimize respectively the errors of approximation $w_f(x)$ and $w_g(x)$. Those optimal parameters satisfy:

$$
\theta_f^* = \arg \min_{\theta_f \in \Omega_f} \left( \sup_{x \in S_x} \left| \theta_f^T \varphi_f(x) - f(x) \right| \right)
$$

$$
\theta_g^* = \arg \min_{\theta_g \in \Omega_g} \left( \sup_{x \in S_x} \left| \theta_g^T \varphi_g(x) - g(x) \right| \right)
$$

(11)
We suppose that $|w_f(x)| \leq W_f$ and $|w_g(x)| \leq W_g$ where $W_f(x)$ and $W_g(x)$ are the known upper bound of approximation error. Both functions can be choose arbitrarily small. The parameters of errors are:

\[
\begin{align*}
\hat{\theta}_f(t) & = \theta_f(t) - \theta_f^* \\
\hat{\theta}_g(t) & = \theta_g(t) - \theta_g^*
\end{align*}
\]  
\hspace{1cm} (12)

Considering the following proposed indirecte fuzzy control

Both terms $u_{eq}$ and $u_s$ will be calculated with details in the next sections. In the following we note IVFC (Interval value fuzzy control) and IVFM (Interval value fuzzy Model) for the control based on type-2 fuzzy system, in other hand we note T1FC and T1FM for the model based on type-1 fuzzy and Type-2 fuzzy system respectively.

4.1.1 Terme of equivalence control $u_{eq}$

Take $\bar{e}_s(t) = k_d\cdot e^{(d-1)}(t) + \cdots + k_0 \dot{e}(t)$ in order to have $\bar{e}_s(t) = \dot{e}_s(t) - e^{(d)}(t)$. The certainty equivalence control term [Sastry and Isidori 1989] become as:

\[
u(t) = \frac{-1}{\ddot{g}(x)} (\ddot{f}(x) + \nu(t))
\]  
\hspace{1cm} (13)

where $\nu(t) = y_d^{(d)} + \gamma e_s + \bar{e}_s$ and $\gamma > 0$ is a design parameter. The $d$th derivative of the output error $e^{(d)} = y_d^{(d)} - y^{(d)}$ with $u = u_{eq} + u_s$, hence :

\[
\begin{align*}
\dot{e}^{(d)}(x) & = y_d^{(d)}(x) - f(x) - g(x)u(t) \\
& = y_d^{(d)}(x) - f(x) - \frac{g(x)}{\ddot{g}(x)}(-\dot{f}(x) + \nu(t)) - (g(x))u_s
\end{align*}
\]  
\hspace{1cm} (14)

Note that the two first terms of the Eqn. (14):

\[
\begin{align*}
y_d^{(d)} - f(x) & = y_d^{(d)} - \dot{f}(x) - f(x) + \ddot{f}(x) \\
& = (-\dot{f}(x) + \nu(t)) - f(x) + \ddot{f}(x) - \gamma e_s - \bar{e}_s
\end{align*}
\]  
\hspace{1cm} (15)

We substitute Eqn. (15) in Eqn. (14), leads to :

\[
\begin{align*}
\dot{e}^{(d)}(x) & = (\ddot{f}(x) - f(x)) + (\dot{g}(x) - g(x))u_{eq} \\
& - \gamma e_s - \bar{e}_s - g(x)u_s
\end{align*}
\]  
\hspace{1cm} (16)

And with $\bar{e}_s = \dot{e}_s - e^{(d)}$, leads to :

\[
\dot{e}_s + \gamma e_s = (\ddot{f}(x) - f(x)) + (\dot{g}(x) - g(x)) u_{eq} - (g(x)) u_s
\]

4.1.2 law of parameters adaption

Consider the lyapunov candidate function:

\[
V_i = \frac{1}{2} \dot{e}_{s}^2 + \frac{1}{2\eta_f} \ddot{\theta}_f^T \dot{\theta}_f + \frac{1}{2\eta_g} \ddot{\theta}_g^T \dot{\theta}_g
\]  
\hspace{1cm} (17)

Where $\eta_f > 0$ and $\eta_g > 0$ are the parameters of conception. The derivative of Lyapunov function Eqn. (17), gives us:

\[
\begin{align*}
\dot{V}_i & = e_s \left(-\gamma e_s + (\dot{f} - f) + (\dot{g} - g)u_{eq} - (g)u_s \right) \\
& + \frac{1}{\eta_f} \ddot{\theta}_f^T \dot{\theta}_f + \frac{1}{\eta_g} \ddot{\theta}_g^T \dot{\theta}_g
\end{align*}
\]  
\hspace{1cm} (18)

In other hand ,

\[
\ddot{f} - f = \theta_f^T \phi_f(x) - \theta_f^T \bar{g} \phi_f(x) - w_f(x)
\]  
\hspace{1cm} (19)
We choose the following adaptation laws:

\[
\dot{\theta}_f(t) = -\eta_f \varphi_f(x)e_s
\]
\[
\dot{\theta}_g(t) = -\eta_g \varphi_g(x)e_s u_{eq}
\]  

where \(\varphi_f(x) = \frac{1}{2}(\xi_f^T + \xi_r^T)\) and \(\varphi_g(x) = \frac{1}{2}(\xi_g^T + \xi_r^T)\), the speed of adaptation related to the choice of the values of \(\eta_f\) and \(\eta_g\). We suppose that the ideal parameters are constants \(\dot{\theta}_f = \hat{\dot{\theta}}_f\) and \(\dot{\theta}_g = \hat{\theta}_g\). With those conditions we find that:

\[
\dot{V}_i = -\gamma e_s^2 + (-w_f(x) - w_g(x)u_{eq} - g(x)u_s) e_s
\]

### 4.1.3 Projection Modification to Parameter’s Update Laws

The laws of adaptation in the equations (18–19) don’t guarantee that \(\theta_f \in \Omega_f\) and \(\theta_g \in \Omega_g\). We must use the projection to ensure that (e.g. to guarantee that \(\hat{\theta}_f \in \Omega_f\) and \(\hat{\theta}_g \in \Omega_g\). We suppose in particular that the \(\theta_f\) and \(\theta_g\) is in the known interval: \(\theta_f \in [\theta_f^{\text{min}}, \theta_f^{\text{max}}]\) and \(\theta_g \in [\theta_g^{\text{min}}, \theta_g^{\text{max}}]\) and we want that and , we define as a point in the acceptable region. We can resume this algorithm by using the following rule: if \(\theta_f(t) \notin [\theta_f^{\text{min}}, \theta_f^{\text{max}}]\) and \(\theta_g(t) \notin [\theta_g^{\text{min}}, \theta_g^{\text{max}}]\) then \(\dot{\theta}_f(t) = 0\) else \(\dot{\theta}_f(t) = -\eta_f \varphi_f u_{eq}\).

In the stability analysis, this projection modification to the update laws will always result in a parameter estimation error that will decrease \(V_i\) at least as much as if the projection were not used; hence, the right-hand side of Eqn. (22) will overbound the \(V_i\) that would result if projection is used. For this reason, we conclude that:

\[
\dot{V}_i \leq -\gamma e_s^2 + (-w_f(x) - w_g(x)u_{eq} - g(x)u_s) e_s
\]

### 4.1.4 Sliding Mode Control Term

To ensure that Equation Eqn. (23) is less than or equal to zero, we choose:

\[u_s = \eta(x) \text{sgn}(e_s) = \frac{1}{g_0} (W_f(x) + W_g(x)|u_{eq}|) \text{sgn}(e_s)\]

We see that:

\[
\dot{V}_i \leq -\gamma e_s^2 + (-w_f(x) + w_g(x)u_{eq}) e_s \leq (|w_f(x)| + |w_g(x)u_{eq}|) |e_s| \text{ with } \frac{g(x)}{g_0} \geq 1 \text{ hence:}
\]

\[
\dot{V}_i \leq -\gamma e_s^2 + (|w_f(x)| + |w_g(x)u_{eq}|) |e_s| - e_s \frac{g(x)}{g_0} (W_f(x) + W_g(x)|u_{eq}|) \text{sgn}(e_s)
\]

\[
\dot{V}_i \leq -\gamma e_s^2 + |w_f(x)||e_s| + |w_g(x)u_{eq}| |e_s| - e_s \text{sgn}(e_s) W_f(x) - e_s \text{sgn}(e_s) W_g(x)|u_{eq}|
\]

where \(|e_s| = e_s \text{sgn}(e_s), \forall e_s \neq 0\) and \(|w_g(x)| \leq W_g(x)\) then:

\[
|w_f(x)||e_s| - e_s \text{sgn}(e_s) W_f(x) = |e_s| (|w_f(x)| - W_f(x)) \leq 0
\]

\[
|w_g(x)u_{eq}| |e_s| - e_s \text{sgn}(e_s) W_g(x)|u_{eq}| = |e_s| (|w_g(x)u_{eq}| - W_g(x)|u_{eq}|) \leq 0
\]

\[
\dot{V}_i \leq -\gamma e_s^2
\]

because \(\gamma e_s^2 \geq 0\), this shows that \(\dot{V}_i\) which is a measure of the tracking error and parameter estimation error, is a nonincreasing function of time. Notice that \(\gamma > 0\) has an influence on how fast \(\dot{V}_i \to 0\). By picking \(\gamma\) larger you will often get faster convergence of the tracking error.
Due to the hypothesis considered (that the reference signals are bounded, \( x \) is measurable, \( d = n \), and the projection ensures that the \( u_{eq} \) term is well-defined), the following statements hold:

1. The system output \( y, \hat{y}, ... , y^{(d-1)} \) are bounded
2. The control signal \( u, u_{eq} \) et \( u_s \) are bounded.
3. The parameters \( \theta_f(t) \) and \( \theta_g(t) \) are bounded.
4. The magnitude of the output error decreases at least asymptotically to zero (\( \lim_{t \to \infty} e(t) = 0 \))

To prove this, Note that since \( V_i \) is positive and \( V_i \leq -\gamma e_x^2 \). We know that \( e_x, \theta_f, \theta_g \) are bounded. Since \( e_x \) is bounded and \( y, \hat{y}, ... , y^{(d-1)} \) are bounded. Hence, \( x \) are bounded. Hence, \( f(x), \hat{f}(x), y(x) \) and \( \hat{y}(x) \) are bounded. Since \( x \) is bounded and \( \hat{y}(x) \geq \theta_0 \), \( u_{eq} \) and \( u_s \) and hence \( u \), are bounded.

Next, note that:

\[
\int_0^\infty \gamma e_x^2 dt \leq - \int V_i dt = V_i(0) - V_i(\infty)
\]

This establishes that \( e_x \in L_2 \) on \( L_2 = \{ z(t) : \int_0^\infty z(t)^2 dt < \infty \} \) since \( V_i(0) \) and \( V_i(\infty) \) are bounded. Note that via the equation (15), \( e_x \) is bounded. Hence, because \( \dot{e}_x \) and \( e_x \) are bounded and \( e_x \in L_2 \), and via the barbarat’s lemma we have \( \lim_{t \to \infty} e_x(t) = 0 \). Hence \( \lim_{t \to \infty} e(t) = 0 \).

It is possible to reduce the chattering that can be result in the sliding mode term of control, we introduce \( e_x = (e_{x_f}, e_{x_g}, e_{x_\theta}, e_{x_{eq}}, e_{x_s}) \) we can use the smooth function \( \text{sgn}(e_x) \) Slotine and Coetssee [1986]. In this case, however, you only get convergence to an neighborhood of \( e_x = 0 \). To prove the stability, we define the error \( e_x \).

\[
e_x = e_x - \varepsilon \text{sat}(\frac{e_x}{\varepsilon})
\]

where

\[
\text{sat}(x) = \begin{cases} 
1 & 1 \leq x \\
\frac{1}{2} & -1 < x < 1 \\
-1 & x \leq -1
\end{cases}
\]

where \( e_x \) represent the distance between \( e_x \) and the desired “boundary layer,” so the \( e_x = 0 \) when \( e_x \) is inside the layer. The equivalent control is defined to be:

\[
u_{eq} = -\frac{1}{\gamma(e_x)}(\hat{f}(x) + \nu_x(t)) \text{ with } \nu_x(t) = y_d(n) + \gamma e_x + \bar{e}_x
\]

we consider the following lyapunov function:

\[
V_i = \frac{1}{2} e_x^2 + \frac{1}{2\eta_f} \hat{\theta}_f^T \hat{\theta}_f + \frac{1}{2\eta_g} \hat{\theta}_g^T \hat{\theta}_g
\]

And the Adaptation laws became:

\[
\dot{\theta}_f(t) = -\eta_f \varphi_f(x)e_x
\]

\[
\dot{\theta}_g(t) = -\eta_g \varphi_g(x)e_x u_{eq}
\]

with those laws the derived lyapunov functions:

\[
\dot{V}_i \leq -\gamma e_x^2 + (|w_f(x)| + |w_g(x)u_{eq}|) |e_x| - g(x)u_s e_x
\]

the terme \( u_s \) in Eqn. (30) become a control with smooth action.

\[
u_s = \frac{1}{\theta_0} (W_f(x) + W_g(x)|u_{eq}|) \text{sat}(\frac{e_x}{\varepsilon})
\]

with \( e_x \text{sat}(\frac{e_x}{\varepsilon}) = |e_x| \), we can proof that \( \dot{V}_i \leq -\gamma e_x^2 \), that’s make \( |e_x| \) asymptotically stable, in addition \( e_x \) will converge to the \( \varepsilon \)-neighbor of \( e_x = 0 \), and \( e \) will converge to 0.
4.2 Control of position and attitude of Quadrotor

**Proposition 1**: for the conception of attitude controller described by the following sub-systems \(\ref{eqn:6}, \ref{eqn:7}\) and \(\ref{eqn:8}\), we propose a sliding mode controller for every subsystem \(i\) where \(i = \{\phi, \theta, \psi\}\) as follow:

\[
U_i = u_{eqi} + u_{si}
\]

where :

\[
u_i = -\frac{1}{g_i(x)} \left( \dot{f}_t(x, \varphi_i, \theta_i) + \ddot{y}_d + \gamma e_x + \overline{e}_s \right)
\]

\[
u_i = \frac{1}{g_i(x)} (W_f(x) + W_g(x)|u_{eqi}|) \text{sat}(\frac{e_{si}}{\varepsilon})
\]

where \(\overline{e}_s = k_i e_s, y_d = \{\phi_d, \varphi_d, \theta_d\}\), \(d = 2\), \(W_f(x)\) and \(W_g(x)\) are the errors bounds of the approximation of \(f_i\) and \(g_i\) respectively, with \(g_0\) is the minimum value of \(b_i\) the control input Eqn. \(\ref{eqn:31}\) ensures the asymptotic convergence of \(\varphi, \theta\) and \(\psi\) to \(\phi_d, \theta_d, \psi_d\) respectively.

**Proof**: similarly, to sections \(\ref{sec:4.1.1}\) and \(\ref{sec:4.1.4}\), we take \(n = 2\), the equations Eqn. \(\ref{eqn:13}\) and Eqn. \(\ref{eqn:30}\) became:

\[
U_i = \frac{1}{b_i} \left( -\dot{f}_x(x, \varphi_i, \theta_i) + \gamma e_x + \overline{e}_s + \ddot{y}_d \right)
\]

\[
+ \frac{1}{g_i(x)} (W_f(x) + W_g(x)|u_{eqi}|) \text{sat}(\frac{e_{si}}{\varepsilon})
\]

using the equations in Eqn. \(\ref{eqn:29}\), the adaptations laws can be written as follow:

\[
\dot{\varphi}_i = -\frac{\eta_{i\varphi}}{g_i} \varphi_i(x) e_x
\]

\[
\dot{\theta}_i = -\frac{\eta_{i\theta}}{g_i} e_x u_{eqi}
\]

Notice that the functions \(g_i\) in Eqn. \(\ref{eqn:33}\) is considered as a constants that’s why it is approximated with a constant and not a fuzzy function, so we take \(\varphi_{f_i} = 1\).

**Proposition 2**: for the altitude controller, the subsystem that governs the movement \(z(t)\) is:

\[
\begin{bmatrix}
\dot{x}_7 \\
\dot{x}_8
\end{bmatrix} = 
\begin{bmatrix}
\dot{z} \\
U g_z U z
\end{bmatrix}
\]

where \(U = \cos \phi \cos \theta\), \(g_z = \frac{1}{m}\) and \(f_z\) is the gravity \(g\). The sliding mode control Eqn. \(\ref{eqn:34}\) ensure the convergence of \(z(t)\) toward the consign \(z_d\)

\[
U_z = \frac{1}{U g_z} (-\dot{f}_z(x) + \ddot{z}_d + \gamma e_z + \overline{e}_s)
\]

\[
+ \frac{1}{g_i(x)} (W_f(x) / U + W_g(x)|u_{eqi}|) \text{sat}(\frac{e_{si}}{\varepsilon})
\]

Where \(W_f(x)\) and \(W_g(x)\) are the bounded limit of error of the approximation of \(f_z\) and \(g_z\) respectively.

**Proof**: following the same procedure in the sections \(\ref{sec:4.1.1}\) and \(\ref{sec:4.1.4}\) considering \(\ddot{z} = f_4(x) - U g_z U z(t)\) where , the term of equivalent control is defined as:

\[
u_{z} = -\frac{1}{g_z} \left( \dot{f}_z(x) |\theta f_z| + \ddot{z}_d + \gamma e_z + \overline{e}_s \right)
\]

where \(\nu(t) = \ddot{z}_d + \gamma e_z + \overline{e}_s\) and \(\gamma > 0\)

The 2\textsuperscript{nd} derivative of the output error, along with the equation Eqn. \(\ref{eqn:14}\) we can write:

\[
\ddot{e} = \ddot{z}_d - f_z(x) - \frac{\nu}{g_z} \left( -\dot{f}_z(x) + \nu(t) \right) - U g_z u_s
\]
Take the two first terms as follow:
\[
\ddot{z}_d - f_z(x) = \left( -\dot{f}_z(x) + \nu(t) \right) - f_z(x) + \dot{f}_z(x) - \gamma e_s - \dot{e}_s
\]
we substitute both terms in the equation Eqn. \([35]\), using \(e_s = \dot{e}_s - e^{(d)}\) we find:
\[
\dot{e}_s + \gamma e_s = (\dot{f}_z - f_z) + (\dot{g}_z - g_z)Ue_{eq} - Ug u_s
\]
The candidate function of Lyapunov \(V_1\):
\[
V_1 = \frac{1}{2} e_s^2 + \frac{1}{2\eta_f} \hat{\theta}_{f_2}^T \hat{\theta}_f + \frac{1}{2\eta_g} \hat{\theta}_{g_2}^T \hat{\theta}_g
\]
where \(\eta_f > 0\) and \(\eta_f > 0\) are the adaptation gain. the derivative of \(V_i\) gives us:
\[
\dot{V}_1 = \frac{1}{2\eta_f} \hat{\theta}_{f_2}^T \hat{\theta}_f f_{f_2} + U \hat{\theta}_{g_2}^T \hat{\theta}_g u_{eq} - U w g u_{eq} - U g u_s e_s
\]
\[
- \gamma e_s^2 + \frac{1}{\eta_f} \hat{\theta}_{f_2}^T \hat{\theta}_f f_{f_2} + \frac{1}{\eta_g} \hat{\theta}_{g_2}^T \hat{\theta}_g f_{g_2} \tag{36}
\]
Suppose that those ideal parameters are constants \(\hat{\theta}_f = \hat{\theta}_f\) and \(\hat{\theta}_g = \hat{\theta}_g\), and the chosen adaptation law is:
\[
\hat{\theta}_f (t) = -\eta_f \varphi f(x) e_s, \\
\hat{\theta}_g (t) = -\eta_g \varphi g(x) e_s u_{eq} (\cos \phi \cos \theta)
\]
That’s lead to: \(\frac{1}{\eta_f} \hat{\theta}_{f_2}^T \hat{\theta}_f g_{f_2} = -\hat{\theta}_{g}^T \varphi g(x) e_s u_{eq}\) and \(\frac{1}{\eta_f} \hat{\theta}_{f_2}^T \hat{\theta}_f f_{f_2} = -\hat{\theta}_{f}^T \varphi f(x) e_s\), so the equation Eqn. \([36]\) can be written as:
\[
\dot{V}_1 = -\gamma e_s^2 + (-w_f(x) - U w g(x) u_{eq} - U g(x) u_s) e_s
\]
To analyze the stability, we conclude that:
\[
\dot{V}_1 \leq -\gamma e_s^2 + (-w_f(x) - U w g(x) u_{eq} - U g(x) u_s) e_s \tag{37}
\]
To ensure that the equation Eqn. \([37]\) must be less than or equal to zero, we choose:
\[
u_s = \frac{1}{\eta_0} \left( \frac{W_{f_2}(x)}{U_1} + W_{g_2}(x) |u_{eq}| \right) \text{sat}\left(\frac{e_s}{\tau}\right) = \eta_s \text{sat}\left(\frac{e_s}{\tau}\right)
\]
We note that: \(- (w_f(x) + w_g(x) u_{eq}) e_s \leq (|w_f(x)| + |w_g(x) u_{eq}|) |e_s|\), so:
\[
\dot{V}_1 = -\gamma e_s^2 + (-w_f(x) - U w g(x) u_{eq}) e_s \\
- U \frac{\nu_s}{\eta_0} \left( \frac{W_f(x)}{U} + W_{g_2}(x) |u_{eq}| \right) \text{sat}\left(\frac{e_s}{\tau}\right) e_s \tag{38}
\]
\[
\dot{V}_1 \leq -\gamma e_s^2 + (|w_f(x)| + |U w g(x) u_{eq}|) |e_s| \\
- U e_s |g_2(x)| \left( \frac{1}{\eta_0} \left( \frac{W_f(x)}{U} + W_{g_2}(x) |u_{eq}| \right) \text{sat}\left(\frac{e_s}{\tau}\right) \right) \tag{39}
\]
Now, we consider the last term of the equation Eqn. \([39]\) and because \(\frac{q_2(x)}{\eta_0} \geq 1\) so :
\[
\dot{V}_1 = -\gamma e_s^2 + |w_f(x)| |e_s| - e_s \text{sat}\left(\frac{e_s}{\tau}\right) W_f, (x) \\
+ |U w g(x) u_{eq}| |e_s| - U e_s |g_2(x)| \text{sat}\left(\frac{e_s}{\tau}\right) W_{g_2}(x) u_{eq} \tag{40}
\]
with $|e_z| = e_z \text{sat}(e_z / \varepsilon)$, $|w_f(x)| \leq W_f(x)$ and $|w_g(x)| \leq W_g(x)$ so:

$$|w_{f_z}(x)| |e_z| - e_z \text{sat}(\frac{\varepsilon e_z}{\varepsilon}) W_f(x)$$

$$= |e_z| |w_{f_z}(x)| - W_{f_z}(x) \leq 0$$

(41)

$$|Uw_{g_z}(x)u_{eq_z}| |e_z| - U e_z \text{sat}(\frac{\varepsilon e_z}{\varepsilon}) W_{g_z}(x)|u_{eq_z}| =$$

$$\begin{cases} 
U |e_z| (|w_{g_z} u_{eq_z}| - W_{g_z} |u_{eq_z}|) \leq 0 & \text{if } U \geq 0 \\
|U| |e_z| (|w_{g_z} u_{eq_z}| + W_{g_z} |u_{eq_z}|) \leq 0 & \text{if } U < 0
\end{cases}$$

(42)

$$\dot{V}_z = - \gamma e_z^2 + |e_z| (|w_{f_z}(x)| - W_{f_z}(x))$$

$$+ |u_{eq_z}| |e_z| (|Uw_{g_z}(x)| - U W_{g_z}(x))$$

(43)

Since $\gamma e_z^2 \geq 0$ this shows that $V_z$ is a non-increasing function of time.

**Proposition 3:** from the Eqn. (5), the subsystems that govern the dynamics of position $(x, y)$ is given by:

$$\begin{pmatrix}
\dot{x}_9 \\
\dot{x}_{10}
\end{pmatrix} = \begin{pmatrix}
\dot{x} \\
u_x \frac{1}{m} U_z
\end{pmatrix}$$

(44)

$$\begin{pmatrix}
\dot{x}_{11} \\
\dot{x}_{12}
\end{pmatrix} = \begin{pmatrix}
\dot{y} \\
u_y \frac{1}{m} U_z
\end{pmatrix}$$

(45)

The following proposed indirect fuzzy adaptative control $u_x$ and $u_y$ Eqns. (46):

$$u_x = \frac{\tilde{m}}{U_z} (x_d^{(n)} + \gamma e_x + \bar{e}_x) + \eta_x(x) \text{sat}(\frac{e_x}{\varepsilon})$$

(46)

$$u_y = \frac{\tilde{m}}{U_z} (y_d^{(n)} + \gamma e_y + \bar{e}_y) + \eta_y(x) \text{sat}(\frac{e_y}{\varepsilon})$$

Ensure the stability toward the desired trajectory asymptotically.

Where $\eta_x(x) = \frac{1}{g_{x_0}} (+W_{g_x}(x)|u_{eq}|)$ and $\eta_y(x) = \frac{1}{g_{y_0}} (+W_{g_y}(x)|u_{eq}|)$ with $W_{g_x}(x)$ and $W_{g_y}(x)$ are the known bounds of approximation errors of functions: $g_x$ and $g_y$ respectively.

**proof:** for the subsystem that govern the variable $x$, we have:

$$\hat{\varphi}_g = \frac{1}{m} \rightarrow u_x \frac{1}{m} U_1 = u_x \hat{g}_x U_z$$

and by the choice of the candidate function of Lyapunov $V_i$, where $i = \{x, y\}$:

$$V_i = \frac{1}{2} e_z^2 + \frac{1}{2} \theta_i \dot{\varphi}_g \dot{\varphi}_g + \frac{1}{2} \tilde{\theta}_g \tilde{\varphi}_g$$

(47)

its derivative is given by:

$$\dot{V}_i = \left(U_z \tilde{\varphi}'_g \varphi_g u_{eq_z} - U_z w_{g_z}(x) u_{eq_z} - U_1 (g_z(x) u_{eq_z} \right) e_z$$

$$+ \frac{1}{\eta_g} \tilde{\theta}'_g \varphi_g \dot{\varphi}_g - \gamma e_z^2$$

(48)

We take the following adaptation law: $\dot{\varphi}_g(t) = -\eta_g \varphi_g(x) e_z u_{eq} U_z$

If the parameter $g_i$ is constant, so the adaptation law for the position controller is:

$$\dot{V}_i = \left(U_z \tilde{\varphi}'_g \varphi_g u_{eq_z} - U_1 w_{g_z}(x) u_{eq_z} - U_z (g_z(x) u_{eq_z} \right) e_z$$

$$- \gamma e_z^2 - U_z \tilde{\varphi}_g \varphi_g (x) e_z u_{eq_z}$$

(48)

$$\Rightarrow \dot{V}_i \leq -\gamma e_z^2 + (-U_z w_{g_z}(x) u_{eq} - U_z (g_z(x) u_{eq_z} e_z$$
To ensure that the equation Eqn. (48) being less or equal to zero, we choose $u_s$ as follow:

$$u_{s_x} = \frac{g_i}{g_{0_i}} (W_{g_i}(x) | u_{eqi}|) \text{sat}(e_{s_x})$$

We constate that : $- (w_g(x) | u_{eqi}|) e_x \leq (w_g(x) | u_{eqi}|) |e_x|$ so :

$$\dot{V}_i = \left(-U_1 w_g(x) u_{eqi} - U_2 \left(\frac{g_i}{g_{0_i}} \right) (W_{g_i}(x) | u_{eqi}|) \text{sat}(e_s/e) \right) e_x$$

$$- \gamma e_x^2 + u_{s_x} - U_2 e_x \left(\frac{g_i}{g_{0_i}} \right) (W_{g_i}(x) | u_{eqi}|) \text{sat}(e_{s_x}) \left(\frac{e_s}{e} \right)$$

$$\dot{V}_i \leq -\gamma e_x^2 + (U_2 w_g(x) u_{eqi} ) |e_x|$$

$$\leq -U_2 e_x (U_2 (w_g(x)|u_{eqi}|) \text{sat}(e_{s_x}) \left(\frac{e_s}{e} \right))$$

(49)

With : $(\frac{g_i}{g_{0_i}}) \geq 1, U_2 \geq 0, |w_{f_i}(x)| \leq W_{f_i}(x)$ and $|w_{g_i}(x)| \leq W_{g_i}(x)$ then $\forall e_x \neq 0$ we have

$$\dot{V}_i =$$

$$- \gamma e_x^2 + |U_2 w_g(x) u_{eqi} | |e_x| - U_2 e_x \text{sat}(e_{s_x}) W_{g_i}(x) | u_{eqi} |$$

$$\leq U_2 |e_x| (|w_{g_i}(x)| u_{eqi}| - W_{g_i}(x) | u_{eqi} |) \leq 0$$

$$\dot{V}_i \leq -\gamma e_x^2$$

since $\gamma e_x^2 \geq 0$, this shows that $V_i$ is a nonincreasing function of time.

The correspondent’s angles are given by the two following equations Eqn. (51):

$$\phi_d = \arcsin(u_x \sin \psi_d - u_y \cos \psi_d)$$

$$\theta_d = \arcsin\left(\frac{1}{\cos \phi}\right)(u_x \cos \psi_d - u_y \sin \psi_d)$$

(51)

for $\psi \equiv 0$ we can approximate the Eqn. (51) using the following approximations: $\cos \psi_d \approx 1$ and $\sin \psi_d \approx 0 \Rightarrow \phi_d = \arcsin(-u_y)$ and $\theta_d = \arcsin\left(\frac{1}{\cos \phi}u_x\right)$

5 Results and discussions

To verify the reliability and robustness of the controller with simulations of tracking in the presence of different disturbance levels, considering the results obtained using the proposed control in Section 4.2 and considering the various trajectories where we use a sampling step of 0.01 and $W_{f_i} = 0.1$ for $i = \{ \phi, \theta, \psi \}$ and $W_{g_j} = 0.1$ for $j = \{ \phi, \theta, ..., y, z \}$ and $\gamma = 10$ for attitude and altitude control, while $\gamma = 2$ and $K = [5, 1]$ for position control.

5.1 Simulation with Disturbance

5.1.1 Attitude Control:

this test is about applying our proposed control to quadrotor to study its behavior in addition to its performances. Take the following trajectory:

$$\begin{cases} 
\theta_d(t) = \sin(t); 
\varphi_d(t) = \sin(t + \pi); \\
\psi_d(t) = 0.2; 
z_d(t) = 1 
\end{cases} \quad \forall t$$

The desired and the real angles of output (in radians) are presented in Fig. [11] and the sliding errors are presented in Fig. [12]

For a complicated trajectory, we choose the following desired signal:
Figure 11: Responses of system with a sinusoidal trajectory

Figure 12: Sliding errors $e_s$
Interval Valued Fuzzy Modeling and Indirect Adaptive Control of Quadrotor

Figure 13: Responses of system with a sinusoidal trajectory

Figure 14: Adaptations of Parameters $\theta_{g\phi}$, $\theta_{g\theta}$ and $\theta_{g\psi}$ in function of time: a)T1FC b)IVFC

\[
[\theta_d(t), \varphi_d(t), z_d(t), \psi_d(t)] = \begin{cases} 
0, 0, 1, 0.2 & \text{if } t \in [10, 12] \\
\sin(t), \sin(t + \pi), 1, 0.2 & \text{otherwise} 
\end{cases} \tag{52}
\]

desired angles and real angles of the system’s output (in radians) are shown in Fig. 13.

The figures Fig. 14 and Fig. 15 depict the evolution of the fuzzy model’s adaptation parameters. It is clear that parameters converge rapidly to their stationary values. Those peaks are due to system perturbations.

We note that the system met the desired reference. The error of tracking in Fig. 13 and the Table 3 can be explained by the error between the estimated model and the real model. Anyway, the effect of this error is too small, and we note that the error obtained with this approach is minimal.
5.1.2 Position Control:

we consider the following trajectory:

\[ x_d(t) = \sin(t), \quad y_d(t) = \sin(t + \pi), \quad z_d(t) = 1, \quad \psi_d(t) = 0.2 \]

In the absence of any parametric variation in the case of position tracking, the results of simulation are almost identical and sufficiently acceptable Fig. [16]. Figure Fig. [17] depicts the angles that correspond to position responses.

5.2 Simulation with Perturbation

5.2.1 Attitude Control:

This test is about the application of the control law to a quadrotor with perturbations in the range of time \([12s, 14s]\). This test is carried out by varying \(\delta J\) and \(\delta J_r\) around the values of the rotor’s inertia \(J_r\) and the drone’s inertia \((I_{xx}, I_{yy}, I_{zz})\). The variation can be written as \(\hat{J}_r = J_r + \delta J_r\) and \(\hat{I} = I + \delta I\).

Figures (Fig. [18], [19] and [20]) show the results of tracking for the three tests mentioned above. In the case where we introduce the model disturbance, we note a deviation from the desired consign.

Note that in all simulations, the added perturbations are quickly compensated by the controller IVFC proposed with respect to the conventional type-1 controller. Note also that in Table 4 the rejection of perturbation is more effective, especially when the system has a high level of disturbance.

5.2.2 Position Control:

In this simulation, the model is submitted to an external disturbance on the angles \(\varphi_d\) and \(\theta_d\) that attack the position controller. This additive perturbation is chosen to be \(\delta \varphi = 0.25 rad\) in the range \([8s, 8.5s]\) and \(\delta \theta = 0.25 rad\) in the range \([9s, 9.5s]\), as shown in the Fig. [22]. The tracking response of position is shown in Fig. [23].

The results of Fig. [23] and the MSE Table 5 show the efficiencies and robustness of the proposed control.
Figure 16: response of system with sinusoidal trajectory

Figure 17: correspondent’s Angles to responses of position a) T1FC b) IVFC

Table 4: Comparison of MSE between T1 and IVFC

|       | test 1 | test 2 | test 3 |
|-------|--------|--------|--------|
| MSE   | IVFC   | T1FC   | IVFC   | T1FC   | IVFC   | T1FC   |
| \(\tilde{\theta}\) | 7.492  | 13.216 | 7.590  | 9.693  | 8.060  | 9.118  |
| \(\tilde{\beta}\)  | 17.548 | 49.829 | 17.310 | 32.508 | 17.295 | 31.033 |
| \(\tilde{\psi}\)   | 1.055  | 1.056  | 1.055  | 1.055  | 1.055  | 1.053  |
Figure 18: Adaptative Sliding mode control in presence of disturbance $\delta J_r = 10\% J_r$

Figure 19: Adaptative Sliding mode control in presence of disturbance $\delta J_r = 15\% J_r$
6 CONCLUSION

In this paper, we presented an algorithm to build an IVFM fuzzy model from input-output data for a nonlinear system. The proposed method for constructing the Takagi-Sugeno (TS) type-2 fuzzy model, based on input-output data of the identified dynamics, is constructed by identifying the structure using fuzzy clustering, then, the envelope detection, and finally the identification of parameters. The proposed method is used to model the unknown parts of the quadrotor dynamics, and the model is then used to design a fuzzy sliding mode adaptive controller in which the unknown dynamics are approximated by an IVFM as an initial model, and the model is then adjusted online using Lyapunov theory-based law. The compared results are presented to demonstrate the effectiveness of the proposed method. Many cases are considered for the sake of showing and testing the performance of tracking and robustness. Finally, the results show that the tracking error has converged despite internal and external disturbances.

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Figure 22: Angles correspondent with perturbation $\delta \varphi = \delta \theta = 0.25\text{rad}$

Figure 23: Response of system using sliding mode adaptive fuzzy model based
Table 5: Comparison of MSE between T1 and IVFC

|    | MSE  | T1FC  | IVFC  |
|----|------|-------|-------|
| $\bar{x}$ |  46.6442 | 39.7494 |
| $\bar{y}$ | 127.3148 | 123.8104 |
| $\bar{z}$ |  30.7955 | 30.7955 |

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