Comment on ”Constraining a possible dependence of Newton’s constant on the Earth’s magnetic field”

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Abstract
Recently A. Rathke has argued that the KK$\psi$ model explanation of the discrepant measurements of Newton’s constant is already ruled out due to Eötvös experiments by several orders of magnitude. The structure of the action of the KK$\psi$ model is even qualified as inconsistent in the sense that it would yield a negative energy of the electromagnetic field. Here, I refute both claims and emphasize the possibility still open to reconcile the experimental bounds on the test of the weak equivalence principle (WEP) with scalar-tensor theories in general by some compensating mechanism.

1 Introduction
In a recent paper [1], A. Rathke has expressed some criticisms on the KK$\psi$ model [2] - [4] by performing a rough estimate of the metric perturbation of a single nucleus and the effective contribution to the gravitational mass it may involve. The author claimed that such analysis can be applied to generic theories with gravielectric coupling. Thus, he concluded that the bounds of WEP violations put by the Eötvös experiments rule out the explanation of the discrepant measurements of Newton’s constant by such couplings by several orders of magnitude. Furthermore, Rathke argued that the very structure of the action of the KK$\psi$ model is inconsistent in the sense that it would yield a negative energy of the electromagnetic field. Consequently, according to him, the action of the KK$\psi$ model is just a mixture of
contributions written in different conventions leading to the misinterpretation that
the Kaluza-Klein (KK) theory is classically unstable. However, both claims, as
stated yet, are incorrect in many respects. In the next section we recall the major
arguments we have put forward to conclude the instability of the genuine KK the-
ory. In Sec. 3, we discuss the validity of the procedure used by Rathke to make a
conclusion on WEP violation by the KK$\psi$ model. In Sec. 4, we present a new op-
portunity to prevent KK-like theories or scalar-tensor theories from any significant
WEP violation. A proof is given in the appendix.

2 Instability of the genuine 5D KK theory

Rathke argues that our claim of the instability of the genuine 5D KK theory at the
classical level is a misconception due to the conventions we have employed. Further,
he suggests to us to refer to Ref. [5] for a didactic derivation and discussion of the
KK action in various frames. In this respect, I reiterate that we have shown the
instability of the genuine KK theory independently of the frame (see [6]). In partic-
ular, this is obvious in Einstein frame (one passes from the Jordan frame to Einstein
frame by the conformal transformation $\hat{g}_{AB} \rightarrow \Phi^{-1/3} \hat{g}_{AB}$, $A, B = 0, 1, 2, 3, 4$ are 5D
labels, whereas $\alpha, \beta = 0, 1, 2, 3$ are 4D labels), as can be seen in the literature [7, 10]
- [12] and displayed below :

$$S_{KK} = - \int \sqrt{-g} \left( \frac{R}{\kappa^2} + \frac{1}{4} \Phi F^{\alpha\beta} F_{\alpha\beta} + \frac{1}{6\kappa^2\Phi^2} \partial^\alpha \Phi \partial_\alpha \Phi \right) d^4x$$

(1)

to be compared to

$$S = - \int \sqrt{-g} \left( \frac{R}{\kappa^2} + \frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} - \frac{1}{2} \partial^\alpha \phi \partial_\alpha \phi + U(\phi) + J(\phi) \right) d^4x,$$

(2)
in the case of a classical real scalar field, $\phi$, minimally coupled to gravity with
the potential $U = U(\phi)$ and the source term $J = J(x^\alpha)$, where we have set $\kappa^2 =
16\pi G/c^4$. As regards the featuring of a kinetic term for the 5D KK scalar field,
\( \Phi \), which is usually absent in the Jordan frame, the reader may refer, e. g., to [7] (Sec. 3.3, Eq.(9)) for opposing views. In any case, the absence of a kinetic term just makes the 5D KK theory with zero electromagnetic field equivalent to a Brans-Dicke theory with parameter \( \omega = 0 \). Now, as shown by Noerdlinger [8], stability of the Brans-Dicke action requires \( \omega > 0 \). The limiting case \( \omega = 0 \) leads to an unstable vacuum [9]. Thus, our conclusion on the instability of the genuine KK theory still holds both in the Einstein frame (whose action meets a common consensus) and in the Jordan frame even if the KK scalar field kinetic term is removed from the action.

3 A proof of WEP violation?

First, the definition of the (effective) gravitational mass given in [1] is wrong and quite confusing. The author is manifestly dealing with the gravitational self-energy of a nucleus and not the gravitational mass itself, this inconsistency appearing clearly in the lack of a second mass density and a double integral in Eqs.(41), (42) of [1] in contrast with Eq.(2) of ref. [13] (Sec. 2). Rathke does not seem to realise that the WEP violation occurs in scalar-tensor theories only because of the spatial variation of the binding energies of composit particles. Moreover, in the spirit of Eötvös experiments, generally one considers a WEP violation in the external gravitational field of a macroscopic body by comparing accelerations of two test bodies of various composition (e. g., the Earth and the Moon in the gravitational field of the Sun [13]. A reason for that is the weakness of gravity as compared to the strength of other fundamental interactions like the electromagnetic interaction. In fact, it seems that Rathke just tried to evaluate the effective gravitational self-force of a nucleus and its influence on the WEP violation. However, it is not so simple to compute the gravitational self-force since one should deal with gauge and regularization problems which are by far out of reach of a simple phenomenological approach (see [14]). Furthermore, referring to the recent approach known as the ”chameleon” cosmology,
scalar fields like those of the KK$\psi$ model can acquire a huge mass in region of high mean density, as it is indeed the case within a nucleus in contrast with the atmospheric density or vacuum in usual laboratory experiments [15] (see also [17] for stability considerations).

4 A possibility of compensating WEP violation

Like any physical model, the KK$\psi$ model may finally turn out not to be viable. However, it is worth noticing that the problem raised by the discrepant G measurements still remains. Moreover, the correlation that we have established from experimental data between the geomagnetic field and the laboratory measurements of G at different locations on Earth is still unchallenged. On the other hand, we agree with Rathke that Eötvös-like experiment put very strong constraints on any violation of the universality of free fall (UFF). Nevertheless, this does not necessarily imply that large gravielectric couplings are inconsistent with known experimental bounds on the WEP, as we show below. Actually, the claim that KK-like theories and scalar-tensor theories in general should violate the WEP is not new at all. It goes back to Jordan and Dicke themselves [16]. Nevertheless, there still remains a possibility of a mechanism that could help such models or variants of such models to conciliate a large gravielectric coupling with the WEP. Thus, in the framework of his version of varying fine structure constant (which too couples the Maxwell invariant $F^{\alpha\beta} F_{\alpha\beta}$ to a scalar field), Bekenstein has suggested a cancellation mechanism in order to escape any WEP violation due to the different internal constitutions of objects [18]. However, his proposal was soon after strongly criticized by Damour [19]. Let us recall that there was a time when the quark model was thought to be ruled out by the Pauli principle (which is at least as fundamental as the WEP) because of the existence of the resonance $\Delta^{++}$. In that case, a symmetry, the color, was a solution. Keeping this in mind, let us explore in what follows a new possibility
which may conciliate the experimental bounds on WEP violation with the KK-like models (see [3]). Following Nordtvedt [20], a body whose mass depends on some parameter $\alpha$ which varies in space will be subject to a body-dependent acceleration

$$\delta a = - \frac{\partial \ln M}{\partial \ln \alpha} c^2 \vec{\nabla} \ln \alpha$$

which accounts for UFF violation. Hence, two different bodies, labelled $i$ and $j$, of different composition will fall in the Earth’s gravitational field $\vec{g}$ with a difference in accelerations whose magnitude equals

$$\left| \vec{a}_i - \vec{a}_j \right| = \left| \frac{\partial \ln (M_i/M_j)}{\partial \ln \alpha} \right| c^2 \left| \vec{\nabla} \ln \alpha \right| \frac{\vec{g}}{g}$$

Consequently, although the mass $M = M(\alpha)$ of a given body will depend on the variable parameter $\alpha = \alpha(\Phi(\vec{r}), \psi(\vec{r}))$, there will be no observed UFF violation for $M = M_0 K(\Phi, \psi)$, where $M_0$ is a positive constant and $K(\Phi, \psi)$ is a universal function which reduces to unity when the scalar fields are not excited. We show in what follows that the Higgs mechanism of dynamical generation of elementary particles’ mass may also allow the latter possibility. Indeed, let us consider the mass $M$ of a given atom on account of its composition, namely, $Z$ protons and $N$ neutrons in the nucleus, plus $Z$ surrounding electrons. It is written in the general form:

$$M = (2Z + N) m_u + (2N + Z) m_d + Z m_e - \frac{\Delta E}{c^2}$$

where

$$\Delta E = \alpha_{em}^2 F_{em}(Z, N) + \alpha_{s eff}^n F_s(Z, N) + \alpha_{w eff}^n F_w(Z, N)$$

is the internal binding energy which includes the electromagnetic interaction, the strong interaction and the weak interaction binding energies contributions, respectively. The subscripts $em$, $s$ and $w$ stand for the electromagnetic, strong and weak interactions, respectively; the quantities $\alpha_{em}, \alpha_{s eff}, \alpha_{w eff}$ are the corresponding relevant effective coupling constants. The exponents $n_s$ and $n_w$ need not to be specified for our purpose. Now, the masses $m_u, m_d$ and $m_e$ result dynamically from the
Yukawa coupling of the $u$–quark, the $d$–quark and the electron to the Higgs field. Now, in our framework, the Yukawa coupling constants $\{g^{(u)}, g^{(d)}, g^{(e)}\}$ of these elementary particles should be replaced with the effective quantities $\{g_{\text{eff}}^{(u)}, g_{\text{eff}}^{(d)}, g_{\text{eff}}^{(e)}\}$ which all depend on both scalar fields $\Phi$ and $\psi$ (see Appendix). Clearly, the only way to get rid of a composition-dependent ratio $M/M_0$ consists in relating the effective coupling constants to each other so that

$$\frac{g_{\text{eff}}^{(u)}}{g^{(u)}} = \frac{g_{\text{eff}}^{(d)}}{g^{(d)}} = \frac{g_{\text{eff}}^{(e)}}{g^{(e)}} = \left(\frac{\alpha_{w \text{ eff}}}{\alpha_w}\right)^{n_w} = \left(\frac{\alpha_{s \text{ eff}}}{\alpha_s}\right)^{n_s} = \left(\frac{\alpha_{\text{eff}}}{\alpha}\right)^2 = K(\Phi, \psi), \quad (7)$$

where the sets of quantities $\{g^{(u)}, g^{(d)}, g^{(e)}\}$ and $\{\alpha_w, \alpha_s, \alpha\}$ denote the Yukawa coupling constants and the fundamental interaction coupling constants, respectively, when both scalar fields are not excited. A connection between the ratios $\alpha_{w \text{ eff}}/\alpha_w, \alpha_{s \text{ eff}}/\alpha_s$ and $\alpha_{\text{eff}}/\alpha$ has already been put forward in [4] (see the discussion), on the basis of the fundamental interactions unification scheme. Usually, the phenomenological formula given in the textbooks reads

$$\Delta E = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N - Z)^2}{4A} - \delta A^{1/2}, \quad (8)$$

where $A = Z + N$ and the values of the parameters $a_v, a_s, a_c, a_a$ and $\delta$ depend on the range of masses for which they are optimized (see, e. g., [21]). Hence, according to our analysis, the parameters $a_v, a_s, a_c, a_a, \delta$ should be considered as effective parameters that scale as $K(\Phi, \psi)$. This is still consistent with the natural expectation that these parameters should depend on the fundamental interactions coupling constants.

5 Conclusion

The Kaluza-Klein theory is unstable even in the Jordan frame, irrespective of whether or not the kinetic term of the KK scalar field is included in the action functional. A. Rathke’s claims that the KK$\psi$ model is already ruled out because
of its gravielectric coupling is not yet founded, based solely on the naive model he has sketched. The definition of the gravitational energy which he has used is also incorrect. Hence, his conclusions are not based on serious proofs. Instead, it seems still possible to accommodate a gravielectric coupling with the UFF on account of the Higgs mechanism of dynamical mass generation.

6 Appendix

The Lagrangian density of a fermion (quark or lepton) of mass $m$ whose wave function is the spinor $\Psi$ reads

$$L = \frac{i}{2} [\bar{\Psi} \gamma^\alpha D_\alpha \Psi - (D_\alpha \bar{\Psi}) \gamma^\alpha \Psi] - mc \bar{\Psi} \Psi,$$

(9)

where $D_\alpha$ stands for the covariant derivative and hereafter $\hbar = 1$. Noether’s theorem yields the energy-momentum density (see, e. g., [22], Sec. I. 7, p. 49 and Sec. I. 8, p. 60)

$$T^{\alpha\beta} = \frac{i}{2} [\bar{\Psi} \gamma^\alpha D^\beta \Psi - (D^\beta \bar{\Psi}) \gamma^\alpha \Psi] + mc \bar{\Psi} \Psi g^{\alpha\beta},$$

(10)

on account of the Dirac equation. Then, it follows

$$T^\alpha_\alpha = 4mc \bar{\Psi} \Psi.$$

(11)

In the KK$\psi$ models (see [4], Sec. 2.2), fermions will generate the $\psi$-field through a source term of the form

$$J = \frac{8\pi G}{3c^4} g(\Phi, \psi) T^\alpha_\alpha = \frac{32\pi G}{3c^4} g(\Phi, \psi) mc \bar{\Psi} \Psi.$$

(12)

The total action reads

$$S = S_{KK4} + \frac{c^4}{4\pi G} \int \sqrt{-g} \Phi \left[ \frac{1}{2} \partial_\alpha \psi \partial^\alpha \psi - U - J\psi \right] d^4x + \int \sqrt{-g} L d^4x,$$

(13)
where $S_{KK,A}$ denotes the genuine 5D KK action after dimensional reduction. As can be seen, the relation (12) provides the spinor $\Psi$ with an effective mass $m_{eff} = m K(\Phi, \psi)$ which is expressed as

$$m_{eff} = m \left[ 1 + \frac{8}{3} g(\Phi, \psi) \right].$$

(14)

As one knows, the mass of an elementary fermion is derived from the Yukawa coupling of $\Psi$ to the Higgs field, $\phi$, by replacing the mechanical mass $m$ in Eq. (9) with $g^{(\Psi)} \phi$ (see [22], Sec. VIII. 3 and [23, 24]), where $g^{(\Psi)}$ is the Yukawa coupling constant of the fermion. Thus, Eq. (14) involves an effective Yukawa coupling constant given by

$$g_{eff}^{(\Psi)} = g^{(\Psi)} \left[ 1 + \frac{8}{3} g(\Phi, \psi) \right].$$

(15)

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