DISTRIBUTED DYNAMIC PLATOONS CONTROL AND JUNCTION CROSSING OPTIMIZATION FOR MIXED TRAFFIC FLOWS IN SMART CITIES- PART I. FUNDAMENTALS, THEORETICAL AND AUTOMATIC DECISION FRAMEWORK

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August 31, 2022

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ABSTRACT

This article studies the problems of distributed dynamic platoons control and smart junction crossing optimization for a mixed traffic flow with connected automated vehicles(CAVs) and social human-driven vehicles(HDVVs) in a smart city. The goal of this two-part article is to provide an automatic decision framework to ensure the safe and efficient cruising and crossing junctions with multiple dynamic and resilient platoons CAVs, while against the social driving behaviors(SDBs) of the HDVs considered as surrounding vehicles(SVs) and unknown traffic lights connected by Cellular-Vehicle-to-X (C-V2X) infrastructure. We shall show that despite the nonlinearity, non-smoothness, and uncertainty of mixed traffic flows in smart cities, the solutions of safe, efficient, and fuel economic platoon CAVs control for the above problems can be solved by our proposed automatic decision and smart crossing assistant system(ADSCAS) that includes four decision stages and six cruising states characterized within this article. More precisely, in part I, we provide a dynamic platoon management strategy to determine the platoon size with respect to the SDBs of the preceding HDVs and upcoming traffic lights, where the dynamic platoon size is composed of minimizing safe distances and nonlinear functions of the platoon’s cruising velocity. We also present the decision and switching scheme for a finite state machine of platoons in the ADSCAS, the design of the reference trajectory planning, and the solution of the fuel economic optimization problem. In part II, we develop a distributed and resilient nonlinear platoon space error strategy that can ensure a subsequent safe cruising and junction crossing, upon which a distributed cooperative control protocol is designed and a finite time robust cooperative trajectory tracking optimization algorithm is proposed to ensure the fast formation of the platoon within a finite time horizon. Once the proper length of the platoon that is determined at the planning moment by the ADSCAS characterized in part I is formed, we propose a method to estimate the leader CAV’s acceleration adjustment by developing distributed observers that ensure a safe junction crossing of the entire CAV platoon, taking into account the SDBs of the SVs, the dynamics of the follower CAVs and an upcoming traffic signal schedule while minimizing the overall platoon fuel consumption. Extensive simulation case studies are conducted to demonstrate the performance of the proposed ADSCAS in various conditions for mixed traffic flows and different junctions.

Index Terms—Connected automated vehicles (CAVs); mixed traffic flow; cooperative driving; social driving behaviors; optimization; distributed control; nonlinear car-following model; formation control.

I. Introduction

TODAY, smart mobility is the intelligent ability to move from one location to another efficiently and quickly, which is a key function of a livable and sustainable smart city. An efficient and green people-mover and automatic decision traffic-responding systems are the core competitiveness of a smart city that has been drawing worldwide attention. In anticipation of travel demand growth and demographic changes, it is vital to develop an integrated and sustainable cooperative driving system that can guide the vehicles to meet the diverse needs of traffic management to provide feasible options for enabling users to better drive and monitor transportation conditions. To this dream, the key lies in our ability to harness the capabilities of information communication technologies such as the Cellular-Vehicle-to-X (C-V2X) techniques, take resilience into traffic planning and management, and develop greener traveling modes to respond to people of all ages groups in a smart city. Despite being an economical and green way to get around, connected automated vehicles (CAVs) can offer better driving quality and safety levels, especially for aging and unable drivers [1], [2]. However, with all advanced traffic light optimizations and managements [3], [4], fundamentally, we are still facing the main challenges of how to ensure safety, comfort, and affordability in terms of travel cost, CO2 emissions, environmental footprint, and social impact. Specifically, the European Commission has presented its strategic long-term vision [5] for a prosperous, modern, competitive, and climate-neutral economy by 2050. With this target, the EU has already updated its ambitious goals for 2030 and now aims to reduce greenhouse gas emissions by at least 40% below 1990 levels, and improve energy efficiency by 32.5%, where the road transport emissions contributed almost 74%. In order to cope with these limitations, an effective method to reduce fuel consumption and, consequently, reduce greenhouse gas emissions in a smart city is vehicle platoons, where a group of vehicles is managed by a short inter-vehicular distance to reduce the overall aerodynamic drag which is related to fuel consumption, resulting in a scaled-up solution of reducing greenhouse gas emissions that can manage multiple platoons simultaneously in a large network.

Notably, some experimental results have shown that platooning can bring large effects on fuel efficiency, which can significantly reduce fuel consumption. For example, the experiments estimate a fuel-saving of 4 − 7% for platoons of heavy-duty vehicles [6], [7], which is a fundamental tradeoff between the aerodynamic drag reduction and the increase in braking to maintain a platoon. An analysis of key performance indicators including gear changes, average speed, and instantaneous fuel consumption was shown that the cooperative eco-driving can achieve 10% fuel savings [8]. Specifically, several realistic scenarios suggest a possible fuel-saving of up to 12% for the follower CAVs [9]. This clearly shows that platooning into land-based transportation is crucial to carbonizing the European and global economies. However,
the critical limitations that fuel-saving experiments still faces are summarized below:

(1) The majority of the research efforts on a fuel-efficient platoon rely on perfect environmental conditions, e.g., the smooth movement for flat roads, and the ideal performance without the effects from the surrounding vehicles(SVs) like human-driven vehicles(HDVs).

(2) The fuel savings experimental tests are only applicable for highways with free cruising environments which do not take the impact of traffic lights in an urban environment into account.

In reality, platoons are expected to drive over urban roads where the dynamic platoon management and external traffic lights shall have a large influence on their behaviors and fuel consumption. In addition, it is important to note that different social driving behaviors(SDBs) will lead to different energy consumption impacts. For example, the research has shown that when AVs are programmed to drive more aggressively, the subsequent emission factors could be reduced by up to 26% on the expressway, while cautiously programmed AVs could deteriorate traffic performance and lead to a 35% increase in emissions [10], which arises a cooperative driving issue for mixed traffic flows in a smart city, e.g., how to develop the proper cooperative driving strategies with the advanced infrastructure information to obtain a maximum benefit of urban emission and reduction performance. Therefore, in this article, we address how to explore a distributed control and optimization strategy to improve cooperative driving techniques in consideration of the SDBs of the HDVs, which is essentially a cooperative control mechanism, to enhance the safety, comfort, and affordability of daily travels in a large complex urban network with human factors.

In general, there are two classes of scenarios for mixed traffic flows- signalized and non-signalized intersections. For the non-signalized scenario, the aim is to focus on platoon maneuvers to ensure driving safety and increase traffic throughput. For example, the cut-in/cut-out maneuvers were studied in [11], which only considers the influence of cut-in/cut-out for the last vehicle as an HDV. A hierarchical and implementation-ready cooperative driving framework with mixed traffic flows was addressed in [12], where a state transition diagram was designed for different modes of CAVs. A distributed control framework for mixed urban and freeway traffic networks was proposed in [13], where the mixed traffic network is established by a multi-agent network including urban agents and freeway agents and each agent solves its optimization problem independently with local information and transmitted information to seek the Nash equilibrium. Furthermore, a mixed traffic flow in which some cars are under car-following control and others are under bilateral control was explored in [14], which provides a necessary stability condition for the mixed traffic flow. The second method is to address the signalized intersection for mixed traffic flows to ensure safe crossing, which is closer to the interest of this article. For example, the design of a real-time cooperative eco-driving strategy for a group of vehicles with mixed AVs and HDVs was developed in [15], where the AVs are considered for the leaders of the mixed traffic flow with the aim of minimizing the total fuel consumption of the whole platoon without sacrificing the travel time of the leaders. A similar setting can also be found from [16], [17]. Notably, in all the above frameworks, the CAVs are considered as the leader to provide suggestion commands to all the follower HDVs to improve the group’s fuel economy, where the suggested velocity commands are held constant for a predefined time to allow the follower HDVs to reach the suggested velocity. However, this is a very ideal environmental condition where the influence of SVs on the real CAV traveling is ignored, in which the SVs will sacrifice the travel time of the leaders so that the crossing junction task may not be finished in the current windows of traffic lights. Notably, HDVs are usually uncontrollable due to the complex traffic conditions, which makes it difficult for a group of CAVs to maintain the desired platoon performance. In addition, the ultimate efficiency of the platoon is bounded by many uncontrollable factors, i.e., driving sensitivities, SDBs of the SVs, and road conditions, in which the SDBs of the HDVs will significantly affect the fuel efficiency of the dynamic platoon management of CAVs in a smart city. Generally, the maximum fuel economy is achieved when vehicles drive at a constant cruising velocity with a smooth movement [18]. However, the SDBs of the HDVs may interrupt the constant movement of the platoon from time to time in different cruising states, making it important to derive a solution that should provide a distributed dynamic platoon management to respond to the SDBs of the HDVs while maintaining the platoon performance and ensuring safe junction crossing. Although social driving behaviors (SDBs) for a single HDV have been studied in [10], [19] based on the social preferences of SVs, to the best of our knowledge, there have been no directed reports on the distributed dynamic platoon management and smart junction crossing optimization with the response of the SDBs of the HDVs in a smart city. Therefore, it is necessary to explore the interaction between CAVs and SDBs of the HDVs to analyze how the HDVs affect the smooth moving performance of dynamic platoon CAVs in different cruising states for the mixed traffic flows to mitigate traffic congestion and enhance cruising and junction crossing safety.

Therefore, the aim of this two-part article is, for the first time, to propose a distributed dynamic platoon management and smart junction crossing optimization framework for the mixed traffic flows to respond to the possible SDBs of the HDVs and upcoming traffic lights while ensuring platoon safety and junction crossing effectiveness. Some fundamental and yet challenging problems are addressed:

(1) What and why a distributed and resilient platoon space error strategy can be used against the SDBs of SVs by exploring a like-driver natural perception scheme;

(2) When and how to determine how many CAVs should be formed as an individual platoon to accomplish the smart junction crossing with respect to the mixed traffic flows and unknown upcoming traffic lights;

(3) How to design the reference trajectory, optimize the platoon forming time, and determine the economic acceleration adjustment scheme for multiple platoons to ensure smart crossing process(SCPs) in a smart city.

To solve these problems, our method is carried out in a general dynamic platoon management and automatic decision framework, where a full-process cooperative driving decision from free-
cruising to smart junction crossing is developed and does not necessarily implement additional strictness decisions. To this end, we first consider that all the SDBs can be classified into two groups being rude and courteous, respectively, due to the fact that humans prioritize social interactions over their own goals when making driving decisions, and then make interpretable decisions to platoon control to respond to the SDBs of HDVs. Furthermore, a general method for dynamic platoon size estimation and an automatic decision framework of smart junction crossing are considered respectively, where four cruising stages and six driving states are defined such that all the trajectories of the platoons are restricted in our proposed automatic decision framework by considering the SDBs of the HDVs and the upcoming traffic lights. One of our main results is to show that despite the nonlinear and nonsmooth character of complex traffic dynamics, the distributed dynamic platoon management and decisions are comprised of the reference trajectories planning and economic acceleration adjusting that can be solved by a robust cooperative trajectory tracking optimization problem within a finite time.

Our contributions to this two-part article can be summarized as follows.

1) In part I, we consider the distributed cooperative driving framework on dynamic platoon management and junction crossing and provide an automatic decision and smart crossing assistant system (ADSCAS), which can, in general, describe all the platoons cruising states and smart junction crossing procedures under the influence of SDBs of the preceding HDVs. In particular, we show the fundamental definition of critical stages in different platoon states, while a finite state machine is designed to determine the driving states of the platoons according to the traveling position with respect to the HDVs and neighboring junctions. Furthermore, we show the dynamic platoon size estimation and reference trajectory planning for the leader CAV according to the finite state machine of platoon CAVs with respect to the possible SDBs of the HDVs and upcoming traffic lights under the safety requirements. Finally, the economic acceleration adjusting scheme is developed during the different adjusting periods to ensure fuel economy and driving comfortability.

2) In part II, we consider the distributed and resilient design of dynamic platoon control by developing a fully distributed variable time headway and velocity-dependent nonlinear cooperative car-following model, which can simulate the realistic driving decision while increasing traffic throughput and reducing energy consumption. We further show that despite the nonsmooth characteristics of these dynamics under different driving states which are exactly determined in part I, the stability analysis will be given by the solutions of robust cooperative trajectory tracking optimization problems for a heterogeneous traffic flow dynamic within a finite time horizon. These results are used to formulate corresponding platoon controllers, and various examples and case studies are discussed.

The part I of this article is organized as follows. Section II introduces the problem formulation and driver models. Section III provides the smart crossing produces and dynamic platoon size determination method under the influence of the SDBs of the HDVs. Section IV proposes a finite-state machine model to describe the platoon management strategy. Section V presents the reference trajectory planning and economic acceleration adjusting scheme, which is also a competitive solution for a single CAV eco-control in a smart city. This is a technical result that will be used in part II to formulate the platoon controller and analyze the traffic flow stability. Section VI gives a conclusion to this part.

II. Problem Formulation

We first consider a cooperative driving and distributed dynamic platoon formulation framework for a mixed traffic flow of CAVs and the SDBs of the HDVs on urban roads as illustrated in Fig. 1. The HDVs will be the preceding vehicles of the group of CAVs, which are driven by individuals driving experiences and affected by the traffic lights, while the platoon CAVs travel with a specific cooperative driving technique to be feasible within a suitable C-V2X infrastructure that can provide the traffic light schedule information. The goal is to develop the ADSCAS to solve the problems of dynamic platoon forming and smart junction crossing of multiple groups of CAVs, taking the traffic signal schedule, intelligent driving HDVs, driver’s reaction-time delay, and platoon fuel economy into account, to reduce the RED light idling and ensure cruising safety. The system formulation and cooperative driving model will be given in the following, respectively.

A. Systems formulation

Unlike existing platoon control techniques for highways, urban traffic signals and SDBs of the HDVs force the platoons to accelerate and decelerate frequently to ensure driving safety, making it a paramount need for a fuel-efficient platoon control that can ensure a safe junction crossing for multiple groups of platoon CAVs with a fuel economy. In addition, different from the existing platoon formulation [20], [21] which only considers a predefined and fixed inter-space safe distance, the distributed dynamic platoon control and optimization in this framework will involve a time-varying nonlinear inter-space policy and the uncontrollable HDVs, where we aim to develop a full distributed dynamic finite-time platoon management strategy to ensure a large external-space safe distance between the platoon CAVs and HDVs while having a greater velocity to improve the traffic throughput.

To formulate the above issues in Fig. 1 the following statements and constraints are considered:

Statement 1 (Driving modes):

- [1] There are two driving modes for the HDVs containing the Intelligent driving and Intelligent crossing, and four modes for the platoon CAVs including the Cruising, Following control, Platoon forming control, and Smart crossing, respectively; and
- [2] The considered HDVs are moving in front of the platoon of CAVs on the Target-lane.

Statement 2 (Systems parameters):

- [3] The vehicle types for the CAVs and the HDVs are identical;
The platoon size of CA Vs for the Platoon forming control procedures (SCPs) contains four stages: The smart crossing procedures (SCP5).

Fig. 1. The arrangement of platoons in car-following and smart junction crossing environments.

Statement 3 (Mixed traffic flows):

- Each platoon contains one Leader CAV, one Lag CAV, and some Follower CAVs. For each platoon, the Leader CAV will make the driving decision and the reference trajectory planning to ensure safe driving. The Follower CAVs will drive by their observer information from the Leader CAV and their cooperative controllers. The Lag CAV is the last Follower CAV whose traveling trajectory will be affected by the platoon size and the position of the HDVs.
- For each platoon, the number of the Follower CAVs can be free when the CAVs travel on the Cruising and the Following control; and
- The number of the Follower CAVs for each platoon will be dynamically affected by the driving states of the HDVs and a traffic light schedule when the CAVs travel on the Platoon forming control and the Smart crossing.

Statement 4 (Vehicle roles): All the HDVs of SVs that may interact closely with the platoon CAVs can be divided into the Front-vehicles and Target-vehicles according to the relative travel distance with respect to the platoon CAVs. The Target-vehicles is the last Front-vehicle whose traveling trajectory will directly affect the platoon size of CAVs for the Platoon forming control and the Smart crossing.

Statement 5 (Smart crossing procedures): The smart crossing procedures (SCP5) contains four stages: Stage I, Stage II, Stage III, and Stage IV, and three types of zones: Planning zone, Optimization zone and Intersection zone, respectively.

- In the Stage I, all the platoon CAVs will approach the Planning zone and the Leader CAV observes the trajectory of the Target-vehicles and estimates the upcoming traffic lights to determine the platoon size so that the specified numbers of CAVs can be formed as a new platoon within a finite time horizon, where the rest number of the CAVs will generate another new platoon to accomplish the SCPs in the next traffic lights;
- In the Stage II, all the new platoons of CAVs will reach the Optimization zone, where the optimal velocity of the Leader CAV will be re-planned and adjusted to ensure that all the Follower CAVs will cross the junction with the fuel economy in the GREEN phase;
- In the Stage III, all the new platoons of CAVs will leave the Optimization zone and the optimal velocity of the Leader CAV will be readjusted to ensure safe cruising by considering the Target-vehicles;
- In the Stage IV, all the new platoons of CAVs will depart the Planning zone and the tracking control for all the multiple platoons that complete the junction crossing and the traffic flow stability shall be considered.

Statement 6 (Space policies): There are three different types of safe distances: a smaller inter-space distance $D_{p,s}$, a larger external-space distance $D_{p,c}$, and a larger platoon-space distance $D_{p,p+1}$.

- $D_{p,s}$ is a time-varying desired safe distance between two consecutive CAVs in the specific platoon $p$ which will be dynamically designed to respond to the upcoming traffic lights and maintain safe cruising distance with respect to the Target vehicles;
- $D_{p,c}$ is the safe traveled distance between the platoon $p$ and the HDVs which will be constructed by a function associated with the longitudinal distance and velocity for the SVs.
- $D_{p,p+1}$ is the desired safe distance between two adjacent platoons to ensure traffic flow stability.

Constraint 7 (Cruising constraints): The velocity and the acceleration of all the vehicles are bounded by the following constraints

\[
\begin{align*}
    v_{p_{\min}} & \leq v_{p_i}(t) \leq v_{p_{\max}}, \\
    a_{p_{\min}} & \leq a_{p_i}(t) \leq a_{p_{\max}}, \\
    v_{h_{\min}} & \leq v_{h_i}(t) \leq v_{h_{\max}}, \\
    a_{h_{\min}} & \leq a_{h_i}(t) \leq a_{h_{\max}},
\end{align*}
\]
where \( v_{p,i}(t) \) and \( a_{p,i}(t) \) are the velocities and accelerations of the CAV \( i \) for the \( p \)-th platoon, and \( v_{h,i}(t) \) and \( a_{h,i}(t) \) are the velocities and accelerations of the HDV \( i \), \( v_{p,\text{min}} \), \( v_{p,\text{max}} \), \( a_{p,\text{min}} \) and \( a_{p,\text{max}} \) are the minimum and maximum speed and acceleration limits for all the CAVs, and \( v_{h,\text{min}} \), \( v_{h,\text{max}} \), \( a_{h,\text{min}} \) and \( a_{h,\text{max}} \) are the minimum and maximum speed and acceleration limits for the HDVs, respectively. In general, the considered velocity limit in (1) is smaller than that in (3) to ensure driving safety in a mixed traffic environment.

The basic idea of this article is to develop the ADSCAS framework that contains the dynamic platoon formulation and smart junction crossing with the fuel economy to reduce the RED light idling and enhance cruising safety for mixed traffic flows under the upcoming traffic lights. Firstly, an automatic driving machine is proposed to determine the cooperative driving states of the platoon CAVs and the specified reference trajectory planning for the Leader CAV. To this end, it is assumed that the upcoming traffic lights and the trajectory of the SDBs of the HDVs can be detected in the Stage I by using C-V2X infrastructure. To ensure driving safety and cross the intersection smoothly within the GREEN phase, the platoon CAVs will yield to the possible priorities of the HDVs by using the automatic driving machine to switch to proper modes, where the Leader CAV plays the key role to merge the observations for the SVs and estimate the upcoming traffic lights to determine the feasible platoon size and implement the driving state determination and dynamic platoon management decision. Then, the fuel economic crossing problem will be solved in the Stage II, where the specified reference information for the Leader CAV will be replanned within a regulation time interval and the determined Follower CAVs can achieve the tracking control to guarantee that the formed platoon CVs can accomplish the smart junction crossing within the rest time of the current GREEN window, where the rest of the Follower CAVs will be formed as a new platoon to accomplish the SCPs in the next traffic lights. Next, all the platoon tracking safety problems will be addressed in the Stage III, where the specified reference information for the Leader CAV will be adjusted to maintain the larger external-space distance to the preceding Target vehicles. Last, the traffic flow stability problem will be investigated in the Stage IV, where multiple platoons tracking control problems will be solved. By developing the distributed control and optimization strategy, the platoon CAVs will track the reference trajectory planning and follow the ADSCAS to reduce the influence of the SDBs on the HDVs and improve the traffic throughput.

Remark 8: Generally, the frequent platoon forming and splitting for multiple groups of CAVs are not applicable on the highway due to the requirements of traffic flow stability and safety. Specifically, frequent adjusting for all the CAVs will cause excessive fuel consumption and thus limit the goal of reducing energy consumption. However, this characteristic is consistent with the driving conditions in an urban traffic control due to the fact that the smoothly cruising for the platoon CAVs cannot be held on complex urban roads, where uncertain traffic lights will force all SVs to have to stop and go, resulting in the frequent operations for all the CAVs to check the safety and tracking performance. Ideally, after a few smart crossing junctions, the speed and spacing of most platoon CAVs and HDVs will be the same as expected by using our developed approaches so that the separation and merging processes require only a small amount of CAVs with a fine tuning to avoid red lights idling, resulting in approximate free-signal traffic co-driving scenarios. This is also the application potential of this study to actual traffic control.

Remark 9: Notably, if there are no HDVs, the cooperative driving and dynamic platoon formulation framework will be reduced as the State I, State II, and State IV, and the cooperative driving and smart crossing problem for a single platoon will be simplified as platoon formulation and fuel economic crossing problem, as made in [22]. Furthermore, if there are no HDVs and traffic lights, the cooperative driving and dynamic platoon formulation framework will be reduced as the State I and State IV, and the cooperative driving for multiple platoons will additionally involve the platoon formulation and traffic flow stability problem, as studied in [23]–[27]. It is noted that different from all the above works, a fully distributed variable time headway and velocity-dependent nonlinear cooperative car-following model will be developed in the current work, making a more flexible platoon space scheme only using the local information. In reality, for the smart crossing issues, the SDBs of the HDVs as SVs are very important factors, which are neither unavoidable nor can be ignored as distractions because their uncontrollable state will affect the platoon size and fuel efficiency such that the above approaches cannot be applicable anymore, which motivates the current study.

B. Cooperative driving model

So far, the optimal velocity model (OVM) [28]–[30] and intelligent driver model (IDM) [31] have been widely addressed in traffic flow analysis. In this work, for CAVs, the combination of the consensus-based platoon model (CBPM) and the OVM model is developed to describe cooperative driving and perception; the IDM model is used to simulate the SDBs of the HDVs.

1) CBPM-OVM model

For all platoons, the dynamic of each CAV is inherently nonlinear, which is affected by the engine, brake system, aerodynamics drag, rolling resistance, and gravitational force, etc. [32]. Some reasonable assumptions, i.e., i) the powertrain dynamics are lumped into a first-order inertial transfer function; ii) the vehicle body is considered to be rigid and symmetric; iii) the influence of pitch and yaw motions is neglected; and iv) the driving and braking torques are integrated into one control input, were first proposed in [24], [33] to obtain the following concise vehicle model

\[
q_{p,i}(t) = v_{p,i}(t),
\]

\[
\dot{v}_{p,i}(t) = \frac{1}{M} \left( \zeta_T R_{p,i}(t) - C_A v_{p,i}^2(t) - M g f \right),
\]

\[
\tau R_{p,i}(t) + R_{p,i}(t) = R_{p,i,\text{des}}(t)
\]

where \( q_{p,i}(t) \) and \( v_{p,i}(t) \) are the position and velocity of the \( i \)-th CAV for the \( p \)-th platoon, respectively, \( M \) is the vehicle mass, \( C_A \) is the coefficient of aerodynamic drag, \( g \) is the gravity constant, \( f \) is the coefficient of rolling resistance, \( \tau \) is the reaction time delay of vehicle longitudinal dynamics, \( R_w \) is the tire radius, and \( \zeta_T \) is the mechanical efficiency of the driveline. \( R_{p,i}(t) \) is the actual driving and braking torque, \( R_{p,i,\text{des}}(t) \) is the desired driving...
or braking torque which is associated with the control input of the feedback linearization law as shown below

\[ R_{p,i,\text{des}}(t) = \frac{1}{\zeta_T} (CA_{v_{p,i}}(t)(2\tau_{p,i}(t)) + Mg_f + Mu_{p,i}(t))R_w, \]  

(6)

where \( u_{p,i}(t) \) is the control input. Then, we introduce the following first-order longitudinal dynamics of the acceleration for a group of follower CAVs to describe the reaction-time delay response of the changes of dynamics of preceding CAVs

\[ \tau a_{p,i}(t) + a_{p,i}(t) = u_{p,i}(t) + y_{p,i}(t) + g_{p,i}(t), i \in \mathbb{F}, p \in \mathbb{M}, \]  

(7)

where \( a_{p,i}(t) \) is the acceleration of the \( i \)-th CAV for the \( p \)-th platoon, \( \mathbb{F} = \{1, \ldots, N\} \) and \( \mathbb{M} = \{1, \ldots, m\} \) are two index sets, \( u_{p,i}(t) \) is the cooperative control input, and \( y_{p,i}(t) \) is a car-following model which represents the optimal speed for CAVs with an adjustable sensitivity, and \( g_{p,i}(t) \) is a difference velocity model which reflects the difference error between the current CAV \( i \) and its neighboring CAVs for the \( p \)-th platoon. The cooperative control input \( u_{p,i}(t) \), car-following model \( y_{p,i}(t) \), and difference velocity model \( g_{p,i}(t) \) will construct a like-driver natural perception scheme to simulate the driver decision-making in different driving environments. The vehicle communication can be described by graph theory. Please see [34]–[37] for more details. For the sake of platoon control, the following third-order model is proposed to represent the dynamics of the \( i \)-th Follower CAV in the \( p \)-th platoon

\[ \dot{q}_{p,i}(t) = v_{p,i}(t), \]

\[ \dot{v}_{p,i}(t) = a_{p,i}(t), \]

\[ \tau a_{p,i}(t) + a_{p,i}(t) = u_{p,i}(t) + y_{p,i}(t) + g_{p,i}(t), i \in \mathbb{F}, p \in \mathbb{M}, \]  

(8)

where

\[ y_{p,i}(t) = \dot{a}^{P} \sum_{j=0}^{N} \alpha_{ij}^{P} \left[ V_{p,i}(y_{p,ij}(t)) - v_{p,i}(t) \right], \]

\[ g_{p,i}(t) = \dot{a}^{P} \sum_{j=0}^{N} \alpha_{ij}^{P} (v_{p,j}(t) - v_{p,i}(t)), i \in \mathbb{F}, p \in \mathbb{M}, \]  

(9)

where \( \dot{a}^{P} \) and \( \alpha^{P} \) are the sensitivity constants, \( \alpha_{ij}^{P} \) is the adjacency weight of the inter-vehicle communication which will be defined later, and \( V_{p,i}(y_{p,ij}(t)) \) is the nonlinear reaction function, also named as “Optimal Velocity Function (OVF)” [28]–[30], to capture the interactions between CAVs \( i \) and \( j \) in the \( p \)-th platoon, as shown below

\[ V_{p,i}(y_{p,ij}(t)) = D_1 + D_2 \tanh \left( D_3 \left( y_{p,ij}(t) - D_4 \right) \right), \]  

(10)

where \( D_1, D_2, D_3, D_4 \) are four positive constants. The average bumper-to-bumper distance \( y_{p,ij}(t) \) between CAVs \( i \) and \( j \) can be given by:

\[ y_{p,ij}(t) = (q_{p,j}(t) - q_{p,i}(t)) / (j - i), i, j \in \mathbb{F}, p \in \mathbb{M}. \]  

(11)

This paper aims to study how distributed control and optimization methods can ensure smart junction crossing and efficient platoon management to reduce the influence of different SDBs for the HDVs, which requires that the dynamics of the Leader CAV can be programmable based on the perception information to observe the trajectory of HDVs. To this end, the following dynamics of the Leader CAV are introduced

\[ \dot{q}_{p,0}(t) = v_{p,0}(t), \]

\[ \dot{v}_{p,0}(t) = a_{p,0}(t), \]

\[ \tau a_{p,0}(t) + a_{p,0}(t) = f(AutoC(t), u_{p,0}(t)), p \in \mathbb{M}, \]  

(12)

where \( q_{p,0}(t), v_{p,0}(t) \) and \( a_{p,0}(t) \) are the position, velocity, and acceleration of the Leader CAV \( 0 \) in the \( p \)-th platoon, respectively, \( f(AutoC(t), u_{p,0}(t)) \) is a nonlinear regular function including a finite state machine function \( AutoC(t) \) and a planning demand \( u_{p,0}(t) \) which represents the braking or driving dynamics of the Leader CAV to describe the emergent acceleration, deceleration or constant speed to respond to the traffic lights and the SDBs of the HDVs.

Remark 10: Notably, \( AutoC(t) \) is a decision function that is based on the observation and prediction information to determine the cooperative driving states of the platoon under the ADSCAS. Then, the planning demand \( u_{p,0}(t) \) will be designed to dynamically adjust the platoon gap to accomplish the safe cruising and smart crossing mission to respond to the underlying SDBs of the HDVs.

2) Extended IDM model

The IDM model can describe car-following behaviors of manual driving accurately. Specifically, the acceleration assumed in the IDM is a continuous function of the velocity \( v_{h,i}(t) \), the gap \( s_{h,i}(t) \) between the \( i \)-th Target vehicle and the preceding vehicle, \( i \in \mathbb{N} \), and the velocity difference (approach rate) \( \Delta v_{h,i}(t) \) to the preceding vehicle [31], [38], which is expressed as

\[ \dot{v}_{h,i}(t) = a_{h,\max} \left| 1 - \left( \frac{v_{h,i}(t)}{v_f} \right)^\delta - \frac{s^*(v_{h,i}(t), \Delta v_{h,i}(t))}{s_{h,i}(t)} \right|^2. \]  

(13)

This expression is an interpolation of the tendency to accelerate with

\[ a_f(v_{h,i}(t)) = a_{h,\max} \left| 1 - \left( \frac{v_{h,i}(t)}{v_f} \right)^\delta \right|, \]  

(14)

on a free road and the tendency to brake with deceleration

\[ -b_f(s_{h,i}(t), v_{h,i}(t), \Delta v_{h,i}(t)) = -a_{h,\max} \frac{s^*(v_{h,i}(t), \Delta v_{h,i}(t))}{s_{h,i}(t)}, \]  

(15)

when the HDV \( i \) comes too close to the vehicle in front. The deceleration term depends on the ratio between the “desired minimum gap” \( s^* \) and the actual gap \( s_{h,i}(t) \), where the desired gap varies dynamically with the velocity and the approach rate, which is defined as follows:

\[ s^*(v_{h,i}(t), \Delta v_{h,i}(t)) = s_0 + s_1 \sqrt{\frac{v_{h,i}(t)}{v_f} + T_i v_{h,i}(t) + \frac{v_{h,i}(t) \Delta v_{h,i}(t)}{2a_{h,\max} a_f}}, \]  

(16)

where \( \delta > 0 \) is the acceleration component, \( s_0 \) is the minimum safety distance and \( s_1 \) is the jam distance, \( T_i \) is the safety time headway, \( v_f \) and \( v_f \) are the desired velocity and deceleration. The
actual gap $s_{h,i}(t)$ is defined as follows:

$$s_{h,i}(t) = h_i(t) - l_c,$$

where $h_i(t)$ is the space headway between the preceding and the Target vehicle and $l_c$ is the longitudinal length of the vehicle.

Due to the reaction time delays, we here consider an extended IDM model (EIDM) as follows:

$$ \dot{q}_{h,i}(t) = v_{h,i}, \\
\dot{v}_{h,i}(t) = a_{h,i}(t), \\
\tau_h \dot{a}_{h,i}(t) + a_{h,i}(t) = a_{h,\text{max}} \left[ 1 - \frac{v_{h,i}(t)}{v_f} \right] \delta - \left( s^*(v_{h,i}(t), \Delta v_{h,i}(t)) \right) \frac{\gamma}{s_{h,i}(t)}.$$

Remark 11: It is noted that the traditional IDM models \cite{31}, \cite{38} consider an ideal vehicle dynamic, which requires that the stimuli from the preceding vehicle can be directly and indiscriminately transformed to the subject vehicles. However, for the car-following behaviors of manual driving, the reaction-time delay cannot be ignored. Motivated by this, we here propose a more accurately third-order model to describe the driving dynamics of the $i$-th HDV, as shown in equation (18). Notably, for the third-order systems \cite{8} and \cite{12}, the traffic flow stability analysis of multiple platoons for the existing methods \cite{25}, \cite{26} is quite cumbersome, whereas the stability analysis conditions are hard to derive so which will be a big challenge. In this paper, we address the traffic flow stability of the entire mixed platoon system consisting of the CAV and its preceding HDVs and aim to improve the platoon performance via a distributed control and optimization framework for the multiple groups of the CAVs only.

III. Smart crossing produces and platoon size determination

In a congested urban traffic scenario, the gaps between two successive SVs are small. Therefore, the trajectory of the HDVs will directly affect the traffic capacity of the follower CAVs in signalized intersections. Specifically, the HDVs often exhibit some SDBs at intersections, such as fast crossing with high acceleration and so on. Such SDBs of the HDVs will limit the platoon size of CAVs in the SCPs. In what follows, we will introduce the critical moments of SCPs, the classification of SCIBs, and the platoon size determination methods to formulate the problem of SCPs.

A. Definitions for SCPs

In order to analyze the dynamic interaction among the platoon CAVs and the preceding HDVs over the whole SCPs, the following four critical moments (as shown in Fig. [2]) are firstly defined:

**Definition 12 (Planning moment):** Let $t_p$ be the Planning moment when the left front tire of the Leader CAV enters the communication range of the Planning zone. Then, the platoon CAVs will be in the STATE I, where the Leader CAV employs the observation to determine the platoon size and the specified numbers of the Follower CAVs track their Leader CAV to form a new platoon.

**Definition 13 (Optimization moment):** Let $t_o$ be the Optimization moment when the left front tire of the Leader CAV approaches the communication range of the Optimization zone. Then, the specified platoon CAVs will be in the STATE II, where the ADSCAS provides the optimal crossing velocity for the Leader CAV to ensure that the specified Follower CAVs can cross the junction economically and efficiently within the GREEN phase.

**Definition 14 (Safe-check moment):** Let $t_s$ be the Safe-check moment when the left front tire of the Leader CAV departs from the communication range of the Optimization zone to enter the Planning zone in the downstream. Then, the specified CAVs will be in the STATE III, where the optimal velocity of the Leader CAV should be redesigned to ensure safe cruising with respect to the Target-vehicles.

**Definition 15 (Platoons merging moment):** Let $t_{ps}$ be the Platoons merging moment that refers to the endpoint of the Planning zone in the downstream when the left front tire of the Leader CAV leaves the communication range of the Planning zone. Then, the specified CAVs will be in the STATE IV, where the multiple platoons formulations will be implemented.

Although there exist many SVs in congested traffic, the purpose of this paper is to reduce the influence of SDBs on the HDVs by developing the reference trajectory planning for the Leader CAV to adjust the nonlinear time-varying inter-space policy among the Follower CAVs and determine the proper platoon size such that the specified platoon can smoothly cross the signalized intersections with fuel economy. Notably, the HDVs as the preceding vehicles are driven by individuals driving experiences, where the social preferences, e.g., rude and courteous SDBs of the HDVs as shown in Fig. [3] will be specifically considered in the platoon control and driving state decision, which will affect the smooth moving of a group of CAVs in the signalized intersections owing to the limited traffic light windows. In the following, two types of SDBs for SCPs are respectively defined:

**Definition 16 (Rude SDBs for SCPs):** The HDVs behave rudely and compete for the right of way with the other SVs to implement the smart crossing issues which are driven by the Rude EIDM model, as shown in \cite{8} (a), where aggressive or near-collision behaviors will arise in this process and the HDVs traveling at a high acceleration compete with respect to the other SVs for crossing the junction within the GREEN phase of the traffic lights. For this case, the Rude SDBs of HDVs will create a larger external-space distance to the following platoon to ensure safety, which will greatly reduce the specified numbers of platoon CAVs allowed to pass safely and smoothly due to the limited traffic light timing.

**Definition 17 (Courteous SDBs for SCPs):** The HDVs behave courteously and give way to the other SVs driving by the Courteous EIDM model, as shown in \cite{3} (b), where aggressive or near-collision behaviors will not occur during this process and the HDVs travel at a low speed with respect to the other SVs for crossing the junction within the GREEN phase of the traffic lights.
The definition of four critical moments can depict Remark 18: allowed to pass smoothly under the limited traffic timing. For this case, the Courteous SDBs of HDVs will create a smaller Fig. 3. Social driving behaviors (SDBs).

Fig. 2. Critical moments in SCPs.

ever, the Safe-check moment CA Vs and the HDVs approach the signalized intersections. How-

Remark 18: The definition of four critical moments can depict almost all SCPs in the mixed traffic environment when the platoon CAVs and the HDVs approach the signalized intersections. However, the Safe-check moment and Platoons merging moment could happen at the same time when there are no HDVs. In this case, the traffic flow stability for multiple platoons will be implemented in the Safe-check moment directly to improve the traffic throughput.

Remark 19: Under the above two SDBs, we would like to consider that the platoon shall decelerate to adjust the safe inter-space distance to avoid collision with the rude HDVs or accelerate to surpass the likely courteous HDVs, which will lead to the problems of the dynamic platoon size determination, the automatic driving decision, the reference trajectory planning for the Leader CAV, and dynamic platoon merging and management of platoons in a finite time.

B. Platoon size determination

To ensure a smart crossing junction for a specified platoon, it is important to determine the platoon formulation process in the State I and ensure that sufficient remaining time of the GREEN phase of the traffic light timing is left for the SCPs in the State II. Due to the limited traffic timing and the uncertain HDVs in front, the longer the platoon, the longer the time is required to form, which will shorten the time for the SCPs, making the platoon size compete with the safe crossing issues. However, if the specified platoon is too short, then the benefit of using multiple platoons to improve traffic throughput may disappear. Thus, we need to properly determine the platoon size in the State I, or equivalently, the total time that we may spend in the platoon forming stage, which leads to the following problem:

Problem 20 (Platoon size determination problem): Given the CBPM-OVM model (8) and (12) and the Extended IDM model (13), assuming the given nonlinear time-varying safety distance $D_{p,s}$ and $D_{p,c}$, an expected cruising speed $v_{p,0}^{\text{plan}}$ of the $p$-th platoon, the available cruising speeds of the SDBs of the preceding HDVs, and a cycle-based traffic signal schedule, determine the maximum allowable platoon size that ensures safe SCPs for all the platoon CAVs within an earliest GREEN period.

Remark 21: The platoon size determination problem was first proposed in our previous work [22], where the fixed safety distance among the platoon CAVs and the pure CAVs control were considered. However, the dynamic platoon gap and the SDBs of HDVs will affect the length of the platoon allowed to cross the junction, which will arise a challenge for determining the platoon size of mixed traffic flows in the SCPs.

At the moment of SCPs, when the Leader CAV enters the concerned link, we expect to quickly determine how many CAVs should be placed as a new platoon with respect to the SDBs of the HDVs and the traffic lights schedule in the State I, to ensure that the specified platoon will pass through the current or the next junction safely, i.e., the tail of the Lag CAV must clear the junction in the upcoming GREEN periods. To answer this question, we first need to know the maximum number of CAVs in the mixed traffic flow that could be allowed to form the specified platoon at the Planning zone. Similarly to [22], by assuming that the platoon has been formed at $t$ which will be determined later, in an ideal situation, all CAVs in the specified platoon must have the same cruising speed to implement the SCPs and follow the possible SDBs of the HDVs if the Leader CAV maintains a safe cruising speed movement. Notably, under the influence of the possible SDBs of the preceding HDVs, the typical length of each link cannot be too long such that each platoon can pass through the junction either in the current GREEN period or in the next GREEN period at the latest; otherwise, all the platoons will have to cruise with a very low speed that is almost equivalent to a full stop, causing more fuel consumption later when the platoon restarts again. With this consideration, we make the following assumption:

Assumption 22: The traffic signal schedule and traveling states such as the position, speed, and acceleration of the Target vehicle are known to the Leader CAV due to C-V2X and any CAV in a link appearing in the current cycle must be able to pass the downstream junction no later than the end of the next cycle.

With this assumption, the traveling distance of SCPs for the Leader CAV started at the Planning moment can be defined as $Q_{p,0}(t)$, as shown in Fig 4, which satisfies the following constraint

$$Q_{p,0}(t) \leq D_{p,s}(t) + Q_{h,i}(t) + l_c + Z_l,$$

(19)
where \( Q_{p,0}(t) \) is the travelling distance of the Target vehicle and \( D_{p,c}(t) \) is the larger external-space distance between the Leader CAV and the Target vehicle, \( Z_i \) is the length of the Intersection zone, and the external-space distance is designed by

\[
D_{p,c}(t) = D_{p,c,0} + \zeta^* \cdot v_{h,i}^*(t),
\]

where \( D_{p,c,0} \) is the minimum safe distance, \( \zeta^* \) is a velocity factor, \( v_{h,i}^*(t) \) is the velocity of the HDVs to be constructed by

\[
\{ v_{h,i}^*(t), \zeta^* \} = \begin{cases} [v_{h,i}^*(t), \zeta^*], & \text{Rude SDBs;} \\ [v_{h,i}^*(t), \zeta^*], & \text{Courteous SDBs}. \end{cases}
\]

Notably, the different SDBs of the HDVs will cause the heterogeneous \( Q_{p,0}(t) \), where the specified platoon may not clear the junction in the current GREEN phase \( t_{g1} \) which will be determined later. Therefore, letting \( l_{Z1}, l_{Z2}, l_{Z3}, \) and \( l_{Z4} \) be the distance between the start point and the endpoint of the Planning zone and the distance between the start points of the Optimization zone and the Intersection zone at the upstream, and the distance between the endpoints of the intersection zone and the Optimization zone and the distance between the start point and the endpoint of the Optimization zone at the downstream, respectively, we have the following conditions

\[
\begin{align*}
Q_{p,0}^*(t_{g1}) &\leq l_{Z2} + Z_i + l_c, \text{Case-I;} \\
Q_{p,0}^*(t_{g1}) &> l_{Z2} + Z_i + l_c, \text{Case-II}, 
\end{align*}
\]

where Case-I implies that the Leader CAV of the specified platoon can finish the SCPs in the current GREEN Phase and Case-II is for the next GREEN Phase.

Now, we consider how to determine a target velocity \( v_{p,0}^*(t) \) for the Leader CAV in the Optimization zone for a specified junction \( l \) to estimate the maximum allowable platoon size. Let \( \sigma \in \{0, 1\} \) be the traffic signal, whose value is 1 for the GREEN light and 0 for the RED light. Then we have

\[
v_{p,0}^*(t) = \begin{cases} Q_{p,0}^*(t_{g1}) - t - \rho, & \text{if } \sigma = 1 \land \text{Case-I} \land \frac{Q_{p,0}^*(t_{g1})}{K_w(t_{cycle} - t - \rho)} \leq v_{p,\max}; \\
Q_{p,0}^*(t_{g1}) - (K_w + 1)(t_{cycle} - t - \rho), & \text{otherwise,}
\end{cases}
\]

\[
\sigma = \begin{cases} 1, & \text{if } 0 \leq \text{mod} \left( \frac{t}{t_{cycle}} \right) \leq t_g; \\
0, & \text{if } t_g < \text{mod} \left( \frac{t}{t_{cycle}} \right) \leq t_{cycle},
\end{cases}
\]

Subject to \( v_{p,\min} \leq v_{p,0}^*(t) \leq v_{p,\max} \),

\[
t_{cycle} = t_r + t_g,
\]

\[
K_w = \left[ \frac{t}{t_{cycle}} \right] + 1,
\]

where \( t_r, t_g \) and \( t_{cycle} \) are the RED and GREEN light periods and total cycle duration, respectively, \( K_w \) is an integer describing the traffic light cycle number, the function \( \text{mod}(\cdot) \) is a modulo function, and \( \rho > 0 \) is a chosen small regulation parameter that ensures the platoon to reach \( v_{p,0}^*(t) \) during the period of \( [t, t + \rho] \) to ensure a smooth junction crossing by the end of the GREEN period for the \( p \)-th platoon. In (23), it is considered that there are two phases in each fixed cycle, that is, GREEN and RED phases, where GREEN always appears before RED.

**Remark 23:** Although we have a fixed cycle traffic signal scheme, the splitting ratio of GREEN and RED phases in each cycle may change over time, depending on actual traffic demands. Without loss of generality, we consider that the ratio is known in this paper. With this setup, the first item of (23) captures the scenario of the Leader CAV of the \( p \)-th platoon passing through the intersection in the remaining GREEN period of the current cycle, if the constraints \( \frac{Q_{p,0}^*(t_{g1})}{K_w(t_{cycle} - t - \rho)} \leq v_{p,\max} \) can be satisfied for the SDBs of the Target Vehicles; otherwise, the Leader CAV will try to pass through the intersection in the GREEN period of the next cycle to estimate the maximum allowable platoon size, as shown in the second item of (23). The last two items of (23) provide the constraints on traffic signal indexes so that \( K_w \) is increased by 1 at \( k = K_w t_{cycle} \).

Define the platoon length as the number of CAVs for a specific platoon \( p \). Then, we present the following result to estimate the platoon size to ensure SCPs.

**Theorem 24:** Given a fixed cycle traffic signal schedule with a fixed phase sequence of GREEN and RED, the platoon can pass through the intersection at the earliest GREEN period with a constant velocity starting at a time instant \( t \), if the maximum allowable length \( N \) of the platoon satisfies the following conditions

\[
N = \begin{cases} \min \{ N_1, N_2 \}, & \text{if the Target vehicle is Rude SDB,} \\
\min \{ N_3, N_4 \}, & \text{otherwise},
\end{cases}
\]

\[
\begin{align*}
N_1 &= L_{z,2} + Z_i + l_c, \\
N_2 &= L_{z,2} + Z_i + l_c + \frac{Q_{p,0}^*(t_{g1}) - (K_w + 1)(t_{cycle} - t - \rho)}{K_w}, \\
N_3 &= L_{z,2} + Z_i + l_c + \frac{Q_{p,0}^*(t_{g1}) - (K_w + 1)(t_{cycle} - t - \rho)}{K_w} + \rho, \\
N_4 &= L_{z,2} + Z_i + l_c + \frac{Q_{p,0}^*(t_{g1}) - (K_w + 1)(t_{cycle} - t - \rho)}{K_w} + \rho + \rho,
\end{align*}
\]
where
\[ \hat{N}_1(t) = \begin{cases} 
\frac{v_{p,\text{max}} \times t_1 - Q^r_{p,0}(t_1) + D_{p,s}(v_{p,0}^{\text{plan}}, t_1)}{D_{p,s}(v_{p,0}^{\text{plan}}, t_1) + l_c}, & \text{if } \sigma = 1 \land \text{Case-I} \land v_{h,s}(t) \leq v_{h,\text{max}} \land Q^r_{p,0}(t_1) + N_1 \times (D_{p,s}(v_{p,0}^{\text{plan}}, t_1) + l_c) - D_{p,s}(v_{p,0}^{\text{plan}}, t_1) \\
\leq v_{p,\text{max}}; & + \infty, \text{ otherwise,} 
\end{cases} \]

\[ \hat{N}_2(t) = \begin{cases} 
\frac{v_{p,\text{max}} \times t_2 - Q^r_{p,0}(t_2) + D_{p,s}(v_{p,0}^{\text{plan}}, t_2)}{D_{p,s}(v_{p,0}^{\text{plan}}, t_2) + l_c}, & \text{if } \sigma = 1 \land \text{Case-I} \land v_{h,s}(t) \leq v_{h,\text{max}} \land Q^r_{p,0}(t_2) + N_3 \times (D_{p,s}(v_{p,0}^{\text{plan}}, t_1) + l_c) - D_{p,s}(v_{p,0}^{\text{plan}}, t_1) \\
\leq v_{p,\text{max}}; & + \infty, \text{ otherwise,} 
\end{cases} \]

\[ \hat{N}_3(t) = \begin{cases} 
\frac{v_{p,\text{max}} \times t_2 - Q^r_{p,0}(t_2) + D_{p,s}(v_{p,0}^{\text{plan}}, t_2)}{D_{p,s}(v_{p,0}^{\text{plan}}, t_2) + l_c}, & \text{if } \sigma = 1 \land \text{Case-I} \land v_{h,s}(t) \leq v_{h,\text{max}} \land Q^r_{p,0}(t_2) + N_3 \times (D_{p,s}(v_{p,0}^{\text{plan}}, t_1) + l_c) - D_{p,s}(v_{p,0}^{\text{plan}}, t_1) \\
\leq v_{p,\text{max}}; & + \infty, \text{ otherwise,} 
\end{cases} \]

\[ t_1 = K_\text{cycle} \times t_{\text{cycle}} - t_r - t - \rho, \]

\[ t_2 = (K_\text{cycle} + 1) \times t_{\text{cycle}} - t_r - t - \rho, \]

where \( v_{h,\text{max}} \) and \( \bar{v}_{h,\text{max}} \) are two minimum speed limits for the rude and courteous SDBs of the HDVs.

**Proof:** To obtain the stability condition, we consider the following four cases in urban roads. For each case, we mainly analyze the allowable length of the platoon that can ensure all the CAVs pass through the intersection under the upcoming traffic signals.

**Case 1:** When the Target vehicle is rude and all the CAVs enter the **Planning zone**, the upcoming traffic signal is just changed from the RED light to GREEN light, and the platoon can just pass through the intersection with the current GREEN window \( t_{g1} \), i.e., Case-I, where \( Q^r_{p,0}(t_1) \leq l_{c,2} + Z_l + l_c \). According to the target’s velocity functions (23) and (3), the velocity of the Leader CAV and the Target vehicle satisfy the following constraints

\[ v_{\text{target}}(t) \leq v_{p,\text{max}}, v_{h,s}(t) \leq v_{h,\text{max}}, \]  

and the velocity of the last vehicle satisfies

\[ v_{\text{last}}(t) = Q^r_{p,0}(t_1) + N_1 \times (D_{p,s}(v_{p,0}^{\text{plan}}, t_1) + l_c) - D_{p,s}(v_{p,0}^{\text{plan}}, t_1), \]

\[ \leq v_{p,\text{max}}, t_{g1} = K_\text{cycle} \times t_{\text{cycle}} - t_r - t - \rho \]  

For an already formed platoon, we have \( v_{p,0}(t) = v_{\text{last}}(t) \). It is assumed that the velocity of the Target vehicle under the Rude SDBs is \( v_{h,s}(t) = v_{h,\text{max}} \). Therefore, we have

\[ N_1 \leq v_{p,\text{max}} \times t_{g1} - Q^r_{p,0}(t_1) + D_{p,s}(v_{p,0}^{\text{plan}}, t_1) \]

\[ \frac{D_{p,s}(v_{p,0}^{\text{plan}}, t_1)}{D_{p,s}(v_{p,0}^{\text{plan}}, t_1) + l_c} \]  

Then, the length of the platoon should be \( \bar{N}_1 = |N_1| \) to ensure the platoon with a more efficient passing speed.

**Case 2:** When the Target vehicle is rude and all the CAVs enter the **Planning zone**, the upcoming traffic signal has changed from RED to GREEN, and the platoon cannot pass through the intersection at the current GREEN window \( t_{g1} \), i.e., Case-II, where \( Q^r_{p,0}(t_1) > l_{c,2} + Z_l + l_c \), but the specific platoon can pass through at the next GREEN window \( t_{g2} \). According to the second item of the target’s velocity function (27), the velocity of the last vehicle will satisfy

\[ v_{\text{last}}(t) = Q^r_{p,0}(t_2) + N_2 \times (D_{p,s}(v_{p,0}^{\text{plan}}, t_2) + l_c) - D_{p,s}(v_{p,0}^{\text{plan}}, t_2) \]

\[ \leq v_{p,\text{max}}, t_{g2} = (K_\text{cycle} + 1) \times t_{\text{cycle}} - t_r - t - \rho \]  

Since all the CAVs will follow the leader’s velocity in a platoon, that is, \( v_{\text{last}}(t) = v_{0}(t) \). Thus, it follows from (28) that

\[ N_2 \leq v_{p,\text{max}} \times t_{g2} - Q^r_{p,0}(t_2) + D_{p,s}(v_{p,0}^{\text{plan}}, t_2) \]

\[ \frac{D_{p,s}(v_{p,0}^{\text{plan}}, t_2) + l_c}{D_{p,s}(v_{p,0}^{\text{plan}}, t_2) + l_c} \]  

Thus, the length of the platoon should be chosen as \( \bar{N}_2 = |N_2| \) to pass through the intersection.

**Case 3:** When the Target vehicle is courteous and all the CAVs enter the **Planning zone**, the upcoming traffic signal is just changed from the RED light to GREEN light, and the platoon can just pass through the intersection with the current GREEN window \( t_{g1} \), i.e., Case-I, where \( Q^r_{p,0}(t_1) \leq l_{c,2} + Z_l + l_c \). According to the target’s velocity functions (23) and (3), the velocities of the Leader CAV and the Target vehicle satisfy the following constraints

\[ v_{\text{target}}(t) \leq v_{p,\text{max}}, v_{h,s}(t) \leq v_{h,\text{max}}, \]  

where \( v_{h,\text{max}} \) is the maximum traveling velocity for the courteous SDBs of the HDVs. Therefore, it follows from (26) that

\[ N_3 \leq v_{p,\text{max}} \times t_{g1} - Q^r_{p,0}(t_1) + D_{p,s}(v_{p,0}^{\text{plan}}, t_1) \]

\[ \frac{D_{p,s}(v_{p,0}^{\text{plan}}, t_1) + l_c}{D_{p,s}(v_{p,0}^{\text{plan}}, t_1) + l_c} \]  

Then, the length of the platoon should be \( \bar{N}_3 = |N_3| \) to ensure the platoon with a more efficient passing speed.

**Case 4:** When the Target vehicle is courteous and all the CAVs enter the **Planning zone**, the upcoming traffic signal has changed from RED to GREEN, and the platoon cannot pass through the intersection at the current GREEN window \( t_{g1} \), i.e., Case-II, where \( Q^r_{p,0}(t_1) > l_{c,2} + Z_l + l_c \), but the specific platoon can pass through at the next GREEN window \( t_{g2} \). According to the
second item of the target’s velocity functions \((25)\) and \((3)\), it follows from \((29)\)

\[
N_4 \leq \frac{v_{p,\text{max}} \times t_2 - Q_{p,0}(t_2) + \frac{D_{p,s}(v_{p,0},t_2)}{D_{p,s}(v_{\text{plan}},t_2)} + l_c}{Q_{p,0}(t_2)} = D_{p,c} + \frac{Q_{p,0}(t_2)}{Q_{p,0}(t_2)} + l_c + Z_l.
\]

Thus, the length of the platoon should be chosen as \(\hat{N}_4 = \lceil N_4 \rceil\) to pass through the intersection.

Since either one of the cases could happen in reality, to ensure the earliest junction crossing, as required by Problem \((20)\) whenever the platoon can pass through the junction in the current cycle, \(\hat{N}_1\) must be chosen. By Assumption \((22)\) we know that \(\hat{N}_2 < +\infty\). Whenever \(\hat{N}_1 < +\infty\), we have \(\hat{N}_1 \leq \hat{N}_2\) for the rude SDBs.

With the conditions \((21), (22), (27), \) and \((29)\), we have

\[
\left( l_{z_2} - Q_{h,1}(t_2) - D_{p,c,0} \right) < \frac{Q_{p,0}(t_2)}{Q_{p,0}(t_2)} + l_c + Z_l.
\]

Similarly, we have \(\hat{N}_3 \leq \hat{N}_4\) for the courteous SDBs. With the conditions \((21), (22), (31), \) and \((32)\), we have

\[
\left( l_{z_2} - Q_{h,1}(t_2) - D_{p,c,0} \right) < \frac{Q_{p,0}(t_2)}{Q_{p,0}(t_2)} + l_c + Z_l.
\]

Thus, the maximum allowable length of the platoon, ensuring the earliest junction crossing, should be chosen as \(\hat{N} = \min\{N_1, N_2\}\) when the \(\text{Target vehicle}\) is rude SDBs; \(\hat{N} = \min\{N_3, N_4\}\) otherwise.

With this platoon length under consideration, the formed platoon intends to maintain the velocity \(v_{p,0}(t)\) to cross the \(\text{Planning zone}\) after a given time instant \(t\) whose value shall be determined in the next section. In reality, if the whole platoon cannot pass the junction at the current \(\text{GREEN}\) window, the platoon will be split to ensure some CAVs can cross safely, where the split platoon size can also use the condition \((23)\) to estimate. Then, the rest of CAVs will maintain a new velocity \(v_{p,0}(t)\) by using the second item of the condition \((23)\) and the relative distance \(Q_{p,0}(t_1) + \hat{N} \times (D_{p,s} + l_c) - D_{p,s}\) to cross the \(\text{Planning zone}\). It is noted that the design of the target velocity of \((23)\) only considers what velocity the \(\text{Leader CAV}\) can pass through the intersection by ignoring the influence of the neighboring vehicles, which will be optimized in the \(\text{optimization zone}\) to ensure all the CAVs can accomplish the SCPs under the SDBs of HDVs. In the following, we shall design a finite state machine to determine the driving state of the platoons and develop an economic crossing strategy to accomplish SCPs.

\[\text{IV. Finite state machine of platoon CAVs under SCPs}\]

By defining the front bumper of the \(\text{Leader CAV}\) as a reference, all the roles of the SVs will be determined as the \(\text{Front-vehicles}\) and \(\text{Rear-vehicles}\) according to the relative positions of HDVs, where the roles of the SVs are dynamically changed due to uncontrolled trajectories of the HDVs. Since the \(\text{Leader CAV}\) may have many HDVs ahead, we define the \(\text{Target-vehicles}\) as the last the \(\text{Front-vehicles}\) such that the SCPs of the platoon CAVs only need to take the single \(\text{Target-vehicles}\) into account. Therefore, this paper aims to design an efficient platoon decision and control strategy to reduce the influence of the SDBs of the \(\text{Target vehicles}\) to achieve the SCPs with fuel economy and crossing safety. Considering different SDBs of HDVs and crossing scenarios, four classes of platoon crossing states are defined to construct the ADSCAS as shown in Fig. \[\text{5}\]

\[\text{Definition 25 (STATE 1-Free-cruising): Define the end point of the Intersection zone for the l-th intersection as} \ Z_{l,0}. \text{ If the traveling distance with respect to the platoon CAVs is longer than a given safe cruising distance, the traveling behaviors of the platoon CAVs will not be affected by any rude or courteous HDVs and the upcoming traffic lights. In this case, the platoon CAVs will be in the STATE 1-Free-cruising, i.e., AutoC(t) = 0. The free-cruising distance of the upstream is given by:} \]

\[
\hat{D} = \frac{\|q_p,i(t) - Z_{l,0}\| > D_c,} \]

\[
\hat{D} = \frac{\|q_p,i(t) - q_{h,1}(t)\| > D_{p,h},} \]

\[
\{\langle D, \hat{D} \rangle | \text{Case-III} \land \text{Case-IV}\}, \]

\[
\text{Case-III} := \{\langle D, \hat{D} \rangle | D > D_{p,c} > D > D_c \}
\]

\[
\text{Case-IV} := \{\langle D, \hat{D} \rangle | (l_{z_1} + l_{z_2} + Z_l) < (l_{z_1} + l_{z_2} + Z_l) \}
\]

\[
D_c = d_{c,0} + \tilde{\beta}_1 \cdot v_{p,0}(t),
\]

\[
D_{p,h} = d_{h,0} + \tilde{\beta}_2 \cdot v_{h,1}(t),
\]

\[\text{where} \ D_c \ \text{is designed by the combination of the safe cruising distance and velocity,} \ D_{c,0} \ \text{is the large safe distance,} \ D_{p,c} \ \text{is the specified cruising external-space distance,} \ v_{p,0}(t) \ \text{and} \ v_{h,1}(t) \ \text{are the velocities of the Leader CAV and Target vehicle,} \ \tilde{\beta}_1 \ \text{and} \ \tilde{\beta}_2 \ \text{are the velocity factors of cruising distance,} \ D_{p,h} \ \text{is designed by the velocity-dependent spacing policy,} \ D_{h,0} \ \text{is the standstill safe distance,} \ \tilde{\gamma}_1 \ \text{and} \ \tilde{\gamma}_2 \ \text{are the constant time headways,} \ \tilde{\beta}_1 \ \text{and} \ \tilde{\beta}_2 \ \text{are two scalars to measure the traveling distance. Specifically, Case-III and Case-IV represent the larger spacing between the Leader CAV and Target vehicle, where the Case-III implies that the Leader CAV and the Target vehicle are not in the same stream; Case-IV otherwise.} \]

\[\text{Definition 26 (STATE 2-Tracking-cruising): For any rude or courteous HDV, if the distance with respect to the platoon CAVs satisfies a given safe cruising distance, the traveling behaviors of the platoon CAVs will not be affected by the traffic light but only the HDV. Then, the platoon CAVs will be in the STATE 2-Tracking-cruising, i.e., AutoC(t) = 1. In this case, the platoon will adjust the cooperative control strategy to maintain a safe cruising distance with respect to the HDVs. The specified cruising distance of the upstream is given by:} \]

\[
\hat{D} > D_c,
\]

\[
\hat{D} > D_{p,h},
\]

\[
\{\langle D, \hat{D} \rangle | \text{Case-V}\}, \]

\[
\text{Case-V} := \{\langle D, \hat{D} \rangle | D_{h,0} < D_{p,h} < D < D_{p,c} < (l_{z_1} + l_{z_2} + Z_l) \}
\]

\[
< D_c < \hat{D} < \tilde{\beta}_2(l_{z_1} + l_{z_2} + Z_l),
\]

\[\text{where} \ D_c \ \text{is designed by the combination of the safe cruising distance and velocity,} \ D_{c,0} \ \text{is the large safe distance,} \ D_{p,c} \ \text{is the specified cruising external-space distance,} \ v_{p,0}(t) \ \text{and} \ v_{h,1}(t) \ \text{are the velocities of the Leader CAV and Target vehicle,} \ \tilde{\beta}_1 \ \text{and} \ \tilde{\beta}_2 \ \text{are the velocity factors of cruising distance,} \ D_{p,h} \ \text{is designed by the velocity-dependent spacing policy,} \ D_{h,0} \ \text{is the standstill safe distance,} \ \tilde{\gamma}_1 \ \text{and} \ \tilde{\gamma}_2 \ \text{are the constant time headways,} \ \tilde{\beta}_1 \ \text{and} \ \tilde{\beta}_2 \ \text{are two scalars to measure the traveling distance. Specifically, Case-III and Case-IV represent the larger spacing between the Leader CAV and Target vehicle, where the Case-III implies that the Leader CAV and the Target vehicle are not in the same stream; Case-IV otherwise.} \]
where $D_{h,0}$ is the standstill safe distance and Case-V represents that the Leader CAV will approach the Target vehicle in the same stream but both the vehicles are still far from the junction.

**Definition 27 (STATE 3-Platoon dynamic forming):** The distance of the specific HDV compared with the Leader CAV in the upstream satisfies

$$
D_m < \bar{D} \leq D_c,
$$

$$
D > D_{p,h},
$$

$$
\{(D, \bar{D})|\text{Case-VI} \land \text{Case-VII} \land \text{Case-VIII}\},
$$

**Case-VI:**

$$
D_m < \bar{D} < D_c \geq \bar{D}
$$

$$
\geq \hat{\beta}_1(l_{z,1} + l_{z,2} + Z_l)
$$

$$
\geq (l_{z,1} + l_{z,2} + Z_l) > D_m \geq \hat{\beta}_3(l_{z,2} + Z_l)
$$

$$
> l_{z,2} + Z_l\},
$$

**Case-VII:**

$$
\{(D, \bar{D})|l_{z,2} + Z_l < \hat{\beta}_3(l_{z,2} + Z_l) < D_m
$$

$$
< (l_{z,1} + Z_l) \leq \hat{\beta}_4(l_{z,1} + Z_l) \leq D_{p,c} < D_{p,h}
$$

$$
< D \leq (l_{z,1} + l_{z,2} + Z_l)
$$

$$
\leq \bar{D} \leq D_c \leq \hat{\beta}_2(l_{z,1} + l_{z,2} + Z_l)\},
$$

**Case-VIII:**

$$
\{(D, \bar{D})|l_{z,2} + Z_l < \hat{\beta}_3(l_{z,2} + Z_l) < D_m
$$

$$
< D_{h,0} < D_{p,h} < D < D_{p,c} < (l_{z,1} + Z_l)
$$

$$
\leq \hat{\beta}_4(l_{z,1} + Z_l) < (l_{z,1} + l_{z,2} + Z_l) \leq \bar{D}
$$

$$
\leq D_c \leq \hat{\beta}_2(l_{z,1} + l_{z,2} + Z_l)\},
$$

$$
D_m = D_{m,0} + \hat{\gamma}_3 \cdot v_{p,0}(t),
$$

where $D_{m,0}$ is the basic safe distance, $(\hat{\gamma}_3$ is the constant time headway, and $\hat{\beta}_1$ and $\hat{\beta}_2$ are two scalars to measure the traveling distance. Then, the Leader CAV will determine the platoon size and the platoon will be in the STATE 3-Platoon dynamic forming, i.e., $\text{AutoC}(t) = 2$. In this case, the platoon will form in a finite time to ensure the safe crossing, where Case-VI and Case-VIII represent the pure platoon CAVs forming issue in the Planning zone(Case-VI) represents that the SVs are not in the same stream; Case-VIII otherwise), and Case-VIII implies that the platoon CAVs forming will be affected by the preceding Target vehicle.

**Definition 28 (STATE 4-Smart crossing control):** For any rude or courteous Target vehicle, the platoon will accelerate to switch into the STATE 4-Smart crossing control, i.e., $\text{AutoC}(t) = 3$, following a specified optimal crossing speed to pass the junction, when the distance between the platoon CAVs and the Target vehicle in the upstream is within the following range

$$
D_n < \bar{D} \leq D_m,
$$

$$
D > D_{p,h},
$$

$$
\{(D, \bar{D})|\text{Case-IX} \land \text{Case-X}\},
$$

**Case-IX:**

$$\{(D, \bar{D})|D > D_{p,h} > D_m \geq \hat{\beta}_3(l_{z,2} + Z_l)
$$

$$
\geq l_{z,2} + Z_l + l_c \geq \bar{D} > D_n \geq \hat{\beta}_5(l_{z,2} + Z_l) > 0\},
$$

**Case-X:**

$$\{(D, \bar{D})|0 < \hat{\beta}_5(l_{z,2}) < D_n < \bar{D} \leq D_m < D_{h,0}
$$

$$
< D_{p,h} < D < D_{p,c} \leq l_{z,2} + Z_l + l_c \leq \hat{\beta}_3(l_{z,2} + Z_l)\},
$$

$$
D_n = D_{n,0} + \hat{\gamma}_4 \cdot v_{p,0}(t),
$$

where $D_{n,0}$ is the basic safe distance, $\hat{\gamma}_4$ is the constant time headway, and $\hat{\beta}_5$ is a scalar to measure the traveling distance. Case-IX represents the pure platoon CAVs crossing issue in the specified junction under the Optimization zone and Case-X implies that the platoon crossing will be affected by the preceding Target vehicle.
**Definition 29 (STATE 5-Safe-check control):** When the platoon finishes the SCPs, the platoon will accelerate to switch into the **STATE 5-Safe-check control**, i.e., AutoC\(t\) = 4, following a specified cruising speed to ensure safety with respect to the rude or courteous **Target vehicle**, if the distance between the platoon CAVs and the **Target vehicle** in the downstream is within the following range

\[
D_o \leq \bar{D} < D_p, \\
D > D_{p,h}, \\
\{(D, \bar{D})|\text{Case-XI} \land \text{Case-XII}\},
\]

**Case-XI** := \{(D, \bar{D})|D > D_{p,h} > \hat{\beta}_9(l_{z,3}) > l_{z,3} + l_c \}

\[
> (D_p \geq \bar{D} \geq D_o \geq 0),
\]

**Case-XII** := \{(D, \bar{D})|0 \leq D_o \leq \bar{D} < D_{h,0} < D_{p,h} < D < D_{p,c} \}

\[
< D_p \leq l_{z,3} + l_c \leq \hat{\beta}_9(l_{z,3}),
\]

\[
D_o = D_{o,0} + \hat{\gamma}_5 \cdot v_{p,0}(t), \\
D_p = D_{p,0} + \hat{\gamma}_6 \cdot v_{p,0}(t),
\]

(37)

where \(D_{o,0}\) and \(D_{p,0}\) are the basic safe distances, \(\hat{\gamma}_5\) and \(\hat{\gamma}_6\) are the constant time headways, and \(\hat{\beta}_9\) is a scalar to measure the traveling distance, **Case-XI** represents the pure platoon CAVs traveling on the **Optimization zone** and **Case-XII** implies that the platoon safety will be checked by concerning with the preceding **Target vehicle**.

**Definition 30 (STATE 6-Traffic flow control):** If the **Leader CAV** and the **Target vehicle** drive into the following range, the multiple platoons will switch into the **STATE 6-Traffic flow control**, i.e., AutoC\(t\) = 5, following a proper traveling speed ensure the traffic flow stability

\[
D_p \leq \bar{D} < D_q, \\
D > D_{p,h}, \\
\{(D, \bar{D})|\text{Case-XIII} \land \text{Case-XIV}\},
\]

**Case-XIII** := \{(D, \bar{D})|D > D_{p,h} > \hat{\beta}_7(l_{z,3} + l_{z,4}) \}

\[
> l_{z,3} + l_{z,4} + l_c > D_q > \bar{D} \geq D_p \\
> \hat{\beta}_8(l_{z,3}) > l_{z,3} + l_c \}
\]

**Case-XIV** := \{(D, \bar{D})|l_{z,3} \leq D_p \leq \bar{D} < D_{h,0} < D_{p,h} \}

\[
< D < D_{p,c} < D_q \leq l_{z,3} + l_{z,4} + l_c \\
\leq \hat{\beta}_8(l_{z,3} + l_{z,4}),
\]

\[
D_q = D_{q,0} + \hat{\gamma}_7 \cdot v_{p,0}(t),
\]

(38)

where \(D_{q,0}\) is the basic safe distance, \(\hat{\gamma}_7\) and \(\hat{\gamma}_8\) are the constant time headways, and \(\hat{\beta}_7\) is a scalar to measure the traveling distance, **Case-XIII** represents the pure platoon CAVs traveling on the **Planning zone** which will be formed as a traffic flow and **Case-XIV** implies mixed traffic flows and the traffic flow stability will be affected by concerning with the preceding **Target vehicle**.

**Remark 31:** Notably, determining the traveling state of the platoon CAVs is mainly based on the traveling distance between the platoon CAVs, the HDVs, and the boundary line of the **intersection zone**. By combining the relative traveling distances and the weighted velocity errors, the finite state machine decision algorithm for the ADSCAS can provide an automatic decision-making framework to determine the roles of SVs and the relationship between the HDVs and the platoon CAVs so that the cooperative driving and optimization for the platoon CAVs can be respectively implemented for the specified driving scenarios under different SDBs of the HDVs. The detailed strategy transition of a finite state machine with six states for the ADSCAS is shown in the above Fig. 5, where the CAVs control, platoon forming, and smart crossing are depicted in detail. It is also noted that the automatic decision framework could be simplified for the platoon tracking control as made in \([16, 17]\), where all the HDVs track a CAV to cross the junction. However, the applicability of the above methods will be reduced in the real traffic environment because the **Leader CAV** is only affected by the traffic lights such that they are essentially a single CAV crossing problem. To solve this problem, we provide the ADSCAS framework to address the influence of different SDBs for the platoon CAVs on the SCPs.

V. Reference trajectory planning and economic acceleration adjusting scheme

A. Reference trajectory planning

Based on the above finite state machine of platoon CAVs with respect to the possible SDBs of the HDVs, the reference trajectory of the **Leader CAV** for the \(p\)-th platoon will be planned by developing the following reference velocity design criterions for the ADSCAS to keep a larger external-space distance from the SDBs of the HDVs and the preceding platoons while adjust the inter-space distance to ensure safe cruising.

**Criterion 32 (C-I):** In the **STATE 1-Free-cruising**, the velocity of the platoon CAVs will not be affected by the HDVs since the **Case-III** and **Case-IV** represent the larger spacing between the **Leader CAV** and **Target vehicle**. Then, the reference trajectory planning of the **Leader CAV** will be determined by the desired cruising velocity \(v_{p,ref}^1(t)\)

\[
v_{p,ref}^1(t) = v_{p,c}(t),
\]

Subject to : **Case-III \land Case-IV**, (39)

where \(v_{p,c}(t)\) is the free cruising velocity for the **Leader CAV** in the \(p\)-th platoon by using the C-V2X infrastructure according to the schedule demands and the preceding platoons.

**Criterion 33 (C-II):** In the **STATE 2-Tracking-cruising**, the platoon CAVs will approach the **Target vehicle** but still far away from the junctions due to the **Case-V**. Then, the reference trajectory planning of the **Leader CAV** is to expect to maintain a safe external-space distance \(D_f\) with respect to the **Target vehicle** shown below

\[
D_f \geq D_{f,0} + \mu_1 v_{h,1,s}(t),
\]

where \(D_{f,0}\) is a basic following distance, and \(\mu_1\) is the velocity factor of the desired following distance. Then, the reference velocity \(v_{p,ref}^2(t)\) is planned as follows:

\[
v_{p,ref}^2(t) = v_{h,1,s}(t) + \hat{\delta}_1(q_{p,0}(t) - q_{h,1}(t) - D_f),
\]

Subject to : **Case-V**, (41)
where \( q_{h,i}(t) \) and \( q_{p,0}(t) \) are the positions of the Target vehicle and the Leader CAV, respectively, \( \delta_2 > 0 \) is a velocity adjustment factor.

**Criterion 34 (C-III):** In the STATE 3-Platoon dynamic forming, the CAVs will approach the Planning zone at the Planning moment \( t_p \), where the platoon forming will not be affected by the Target vehicle for the Case-VI and Case-VII to represent the pure platoon CAVs forming issue in the Planning zone. In both cases, the reference velocity \( v_{p,ref}^{31} \) is planned as follows:

\[
v_{p,ref}^{31}(t) = \begin{cases} 
\frac{t_{\sigma} + Z_l + l_c}{K_w t_{cycle} - t_r - t - \rho}, & \text{Case-I}, \\
\frac{t_{\sigma} + Z_l + l_c}{(K_w + 1)t_{cycle} - t_r - t - \rho}, & \text{Case-II}, 
\end{cases}
\]

Subject to: Case-VI \& Case-VII,

\[
(v_{p,ref}^{31}(t) + (K_w t_{cycle} - t_r - t - \rho)) - q_{p,0}(t_p) \leq l_{z,1} + l_{z,2} + Z_l + l_c, \\
\text{where the constraints } v_{p,ref}^{31}(t) \text{ and } v_{p,ref}^{31}(t) \text{ represent the prediction displacement for the Case-I and Case-II, respectively.}
\]

For the Case-VIII, the platoon CAVs forming will be affected by the preceding Target vehicle. To avoid a collision, the displacement of the Leader CAV should satisfy the following condition

\[
t^*_p = (l_{z,1} + Z_l)/v_{h,i}(t), \\
q_{h,i}(t^*_p) - q_{p,0}(t_p) \geq \Delta_g, \\
\Delta_g = q_{h,i}(t^*_p) - q_{h,i}(t_p) + q_{h,i}(t_p) - q_{p,0}(t_p), \\
\text{Subject to: Case-VIII},
\]

where \( t^*_p \) is the predicted Optimization moment with the velocity \( v_{h,i}(t) \) and \( q_{h,i}(t^*_p) \) is the displacement under the time interval \( t^*_p \). It is assumed that the SVs are planned to move by a uniform acceleration. Then, the reference velocity \( v_{p,ref}^{32} \) is planned as:

\[
v_{p,ref}^{32}(t) = v_{p,0}(t) + \frac{D_g - D_t - D_b}{t^*_p - (t - t_p)}, \\
\text{where } D_t \text{ is the distance between the Leader CAV and the preceding Target vehicle, and } D_b \text{ is a safe buffer for the safe reference treatment.}
\]

where \( D_{b0} \) is a minimum safe distance and \( \mu_2 \) is the velocity factor of the smaller following distance.

**Criterion 35 (C-IV):** In the STATE 4-Smart crossing control, the platoon will approach the Optimization zone at the Optimization moment \( t_o \), where Case-IX represents the pure platoon CAVs crossing issue in the specified junction under the Optimization zone. In this case, the reference velocity \( v_{p,ref}^{51} \) is planned as follows:

\[
v_{p,ref}^{51}(t) = \begin{cases} 
\frac{l_{z,2} + Z_l + l_c}{K_w t_{cycle} - t_r - \rho}, & \text{if } \sigma = 1 \text{ \& Case-I}, \\
\frac{l_{z,2} + Z_l + l_c}{(K_w + 1)t_{cycle} - t_r - \rho}, & \text{otherwise,}
\end{cases}
\]

\[
\sigma = \begin{cases} 
1, & \text{if } 0 \leq \text{mod} (\frac{t_p}{t_{cycle}}) \leq t_g, \\
0, & \text{otherwise,}
\end{cases}
\]

Subject to: Case-IX,

\[
(v_{p,ref}^t(t) + (K_w t_{cycle} - t_r - t - \rho)) - q_{p,0}(t_o) \geq l_{z,2} + Z_l + l_c, \\
\text{where } t_r, t_g \text{ and } t_{cycle} \text{ are the RED and GREEN light periods and total cycle duration, respectively, } K_w \text{ is an integer describing the traffic light cycle number, the function mod}(\cdot) \text{ is a modulo function, and } \rho > 0 \text{ is a chosen small regulation parameter that ensures the platoon to reach } v_{p,ref}^{51}(t) \text{ during the period of } [t, t + \rho] \text{ to ensure a smooth transition.}
\]

For the Case-X, the platoon will leave the Optimization zone. In this case, the reference velocity \( v_{p,ref}^{42} \) is planned as follows:

\[
v_{p,ref}^{42}(t) = \text{target}(t),
\]

Subject to: Case-X,

\[
q_{h,i}(t_o) - q_{p,0}(t_o) < l_{z,2} + Z_l, \\
q_{h,i}(t^*_o) - q_{h,i}(t_o) \leq l_{z,1} + l_{z,2} + Z_l, \\
\text{where } v_{target}(t) \text{ is a designed cruising speed for the Leader CAV in the Optimization zone which ensures that the platoon CAVs will pass through the intersection in the GREEN period of the current or the next cycle. The detailed design is given in [23].}
\]

**Criterion 36 (C-V):** In the STATE 5-Safe-check control, the platoon will leave the Optimization zone and drive into the Planning zone at the Safe-check moment \( t_s \) in the downstream, where Case-XI represents the pure platoon CAVs traveling on the Optimization zone. In this case, the reference velocity \( v_{p,ref}^{51} \) is planned as follows:

\[
v_{p,ref}^{51}(t) = v_{p,ref}^{51}(t),
\]

Subject to: Case-XI,

For the Case-XII, the platoon safety will be checked by concerning with the preceding Target vehicle. To avoid the collision,
the Leader CAV should maintain a safe external-space distance $D_s$ with respect to the Target vehicle which is given below

$$D_s \geq D_{s0} + \mu_3 v_{h,i}^s(t),$$

where $D_{s0}$ is a basic following distance, and $\mu_3$ is the velocity factor. Then, the reference velocity $v_{ref}^p(t)$ is planned as follows:

$$v_{ref}^p(t) = v_{h,i}^s(t) + \delta_2 (q_{p,0}(t) - q_{h,i}(t) - D_s),$$

Subject to: Case-XIII,

where $q_{h,i}(t)$ and $q_{p,0}(t)$ are the positions of the Target vehicle and the Leader CAV, respectively, and $\delta_2 > 0$ is a velocity adjustment factor.

Criterion 37 (C-VI): In the STATE 6-Traffic flow control, the platoon will leave the Planning zone at the Platoons merging moment $t_{ps}$ in the downstream, where Case-XIII represents the pure platoon CAVs traveling on the Planning zone. To realize the traffic flow stability, the Leader CAV will maintain a larger platoon-space distance $D_{p,p+1}$ with respect to the preceding platoon described as

$$D_{p,p+1} = D_S + N^{p+1} (D_{s0} + h_{p+1,0}v_{p+1,0}(t)),$$

where $D_S$ is the basic inter-platoon spacing, $N^{p+1}$ is the number of CAVs for the $(p+1)$-th platoon, $h_{p+1,0}$ is the minimum headway for the stable platoon $p$, $v_{p+1,0}(t)$ is the velocity of the Leader CAV for the $p+1$-th platoon. In this case, the reference velocity $v_{ref}^{p+1}(t)$ is planned as follows:

$$v_{ref}^{p+1}(t) = v_{p+1,0}(t) + \delta_3 (q_{p,0}(t) - q_{p+1,0}(t) - D_{p,p+1}),$$

Subject to: Case-XIII,

where $q_{p+1,0}(t)$ and $v_{p+1,0}(t)$ are the displacement and velocity of the Leader CAV of the $p+1$-th platoon, respectively, and $\delta_3 > 0$ is a velocity adjustment factor.

For the Case-XII, the traffic flow stability will be affected by the preceding Target vehicle. To avoid the collision, the displacement of the Leader CAV should satisfy the following condition

$$t_{ps}^* = l_{z,A}/v_{h,i}^s(t),$$

$$q_{p+1,0}(t_{ps}) - q_{h,i}(t_{ps}) = D_h,$$

$$D_h = D_{p,p+1} + q_{h,i}(t_{ps}) - q_{h,i}(t_{ps}) + q_{h,i}(t_{ps}) - q_{p,0}(t_{ps}),$$

Subject to: Case-XIV $\land q_{h,i}(t_{ps}) - q_{p,0}(t_{ps}) < l_{z,A},$

where $t_{ps}^*$ is the predicted Platoons merging moment with the velocity $v_{h,i}^s(t)$ and $q_{h,i}(t_{ps})$ is the displacement under the time interval $t_{ps}^*$. It is assumed that the SVs are planned to move by a uniform acceleration. Then, the reference velocity $v_{ref}^{p+1}(t)$ is planned as:

$$v_{ref}^{p+1}(t) = v_{p,0}(t) + \frac{D_h - D_{p,p+1} - D_t - D_o}{t_{ps}^* - (t - t_o)},$$

where $D_t$ is the distance between the Leader CAV and the preceding Target vehicle, and $D_o$ is a safe buffer for the safe reference

$$D_o = D_{s0} + \mu_3 v_{h,i}^s(t),$$

where $D_{s0}$ is the basic safe distance, and $\mu_3$ is the velocity factor of the following control distance.

### B. Economic acceleration adjusting scheme

The aim here is to provide the acceleration planning for the Leader CAV under C-I - C-VI and the cooperative controller development for the Follower CAVs will be presented in part II [39] to implement the safe cruising and SCPs. It is noted that the velocity adjusting of the Leader CAV shall consider the platoon economy, safety, and comfortability. Therefore, the finite state machine function $AutoC(t)$ in the ADSCAS can be designed by

$$AutoC(t) = \begin{cases} 
S0 : & C-I, Case-III \land Case-IV, \\
S1 : & C-I \Rightarrow C-II, Case-V, \\
S2 : & C-II \Rightarrow C-III, Case-VI \land Case-VII \land Case-VIII, \\
S3 : & C-III \Rightarrow C-IV, Case-IX \land Case-X, \\
S4 : & C-IV \Rightarrow C-V, Case-XI \land Case-XII, \\
S5 : & C-V \Rightarrow C-VI, Case-XIII \land Case-XIV, 
\end{cases}$$

where “⇒” represent the switching scheme.

Notably, if the platoon is traveling with a constant velocity $v_{p,c}$ under C-I, the acceleration of the Leader CAV will be zero. Our aim is to adjust the platoon’s velocity for a given time domain $[t^-, t^+]$ to ensure that the platoon can reach the planning demands $AutoC(t)$ to accomplish the SCPs of multiple platoons and ensure driving safety and economy. In general, we can use the maximum acceleration to adjust the traveling velocity to reach the reference velocity under C-II - C-VI. However, it may cause a risk to driving safety due to the existing possible SDBs of the HDVs. Therefore, we will try to design an optimal regulation time $T$, where $T \leq t_{++}$ with $t_{++} = t^+ - t^-$ being a time interval between any two former and later switching moments, which are associated with the reference velocity adjustments $v_{ref}^{fam}(t)$ and $v_{ref}^{int}(t)$ and the stability of the platoon with the reference trajectory under C-II - C-IV within a finite time. Notably, $T$ is free for the Cruising state because the Leader CAV will maintain the constant cruising velocity under C-I. For other cases, since the Leader CAV adjusts the status first under C-II - C-VI and then all the Follower vehicles track the preceding CAVs, the option of the finite time $T$ will be the key factor to ensure the safety, economy, and comfortability of the platoon tracking and the SCPs with respect to the underlying SDBs of the HDVs, which will be designed specifically in the below.

To solve the above problem, let us first introduce a lemma, which will be used frequently throughout the following analysis.

**Lemma 38:** Consider the vehicle dynamic [12] with the initial conditions $q_{p,0}(t^-)$, $v_{p,0}(t^-)$, and $a_{p,0}(t^-)$. If the control input
Then, with the differential equation (58), we finally obtain

\[ q_{p,0}(t^+) = q_{p,0}(t^-) + v_{p,0}(t^-)(t - t^-) + \frac{1}{6(1 + \tau)} u_{p,0}(t - t^-)^3, \]

(65)

The energy consumption is then calculated as

\[ J^u = \int_{t^-}^{t^+} (a_{p,0}(t))^2 dt \]
\[ = \int_{t^-}^{t^+} \left( \frac{1}{1 + \tau} u_{p,0}(t - t^-) \right)^2 dt \]
\[ = \frac{1}{3(1 + \tau)^2} u_{p,0}(t - t^-)^3. \]

(66)

This completes the proof.

To determine the optimal and economic adjusting of the Leader CAV under C- II - C-VI, the following optimization problem is first investigated, which ignores the influence of the small differences in the vehicle cruising speed on energy consumption in the congested traffic environment.

**Problem 39 (Fuel economic optimization problem):**

\[ \min J^u = \int_{t^-}^{t^+} (a_{p,0}(t))^2 dt, \]

(67)

Subject to \( \dot{v}_{p,0} = v_{p,0} \) for the Problem 39 will be given

\[ \dot{v}_{p,0}(t) = \frac{1}{1 + \tau} u_{p,0}(t - t^-), \]

(68)

where \( v_{p,0} \) for the Leader CAV reference velocities, and the function \( J^u \) captures the energy consumption.

We can derive the necessary conditions for the solution to Problem 39 which are summarized in the following theorem.

**Theorem 40:** There exists two positive constants \( \rho^{**} \) and \( T^* \) with respect to the acceleration and velocity limits within the period \([t^*, t^* + \rho]\). If \( \rho^{**} \leq T^* \), the minimum fuel consumption \( J^u \) and the function of \( a_{p,0}(t) \) for the Problem 39 will be optimally determined.

**Proof:** The fuel economic optimization problem can be rewritten in the following form

\[ \min J_a = \int_0^\rho (a_{p,0}(t))^2 dt, \]

(69)

Subject to \( \dot{v}_{p,ref} = v_{p,ref} \) for the \( v_{p,ref} \) reference velocities, where \( \rho \) is a constant. This is a standard variational problem with an equality constraint. We can easily check that the optimal solution is

\[ a_{p,0}(t) = \frac{v_{p,ref} - \rho}{v_{p,ref}}, \]

(70)
optimal regulation domain. The resulting fuel consumption is

\[
J^*_a = \int_0^\infty (a_\delta(\tau))^2 d\tau = \frac{\left(v_{p,ref}^\text{lat}(t) - v_{p,ref}^\text{for}(t)\right)^2}{\rho},
\]

which means the larger the value \(\rho\), the smaller fuel consumption \(J^*_a\) in (61). Notably, a smaller \(\rho\) in (66) will result in a smaller acceleration \(a_{p,0}^\text{lat}(t)\) because the \(v_{p,ref}^\text{lat}(t)\) and \(v_{p,ref}^\text{for}(t)\) are the known vehicle reference velocities, which will affect the item \(u_{p,0}(t-t^-)\) such that the item \(J^*_a\) will also be smaller, as shown in (61) and (66).

Due to the traffic lights, the function AutoC(t) can be divided into three sets, i.e., SET-0 := \{S0\}, SET-1 := \{S1, S4, S5\} and SET-2 := \{S2, S3\}, where SET-0 is free cruising, the acceleration adjusting for the Leader CAV in the SET-1 will not be affected by the traffic lights windows and SET-2 represents the acceleration adjusting to being bound by the traffic lights. Now, we introduce a displacement function \(\phi(p)\) for SET-1 and SET-2 as follows:

\[
\phi(p) = \begin{cases} 
D_c, S1 \wedge \text{Case-V}, \\
D_m, S2 \wedge \text{Case-VI} \wedge \text{Case-VII} \\
D_n, S3 \wedge \text{Case-IX} \wedge \text{Case-X}, \\
D_n, S4 \wedge \text{Case-XI} \wedge \text{Case-XII}, \\
D_p, S5 \wedge \text{Case-XIII} \wedge \text{Case-XIV}.
\end{cases}
\]

Notably, by the constraint (1) for the rude or courteous HDVs, \(T\) and \(\rho\) will be upper bounded by

\[
\begin{align*}
\frac{\phi(p)}{v_{p,max}^\text{lat}} & \leq T \leq \frac{\phi(p)}{v_{p,min}^\text{lat}} \\
\frac{v_{p,ref}^\text{lat}(t) - v_{p,ref}^\text{for}(t)}{a_{p,max}^\text{lat}} & \leq \rho \leq \frac{v_{p,ref}^\text{lat}(t) - v_{p,ref}^\text{for}(t)}{a_{p,min}^\text{lat}}.
\end{align*}
\]

Then, according to the conditions (70) and (72), the optimal values of \(\rho\) and \(T\) that lead to the minimum fuel consumption are

\[
\begin{align*}
T^* &= \frac{\phi(p)}{v_{p,min}^\text{lat}}, \\
\rho^* &= \frac{v_{p,ref}^\text{lat}(t) - v_{p,ref}^\text{for}(t)}{a_{p,min}^\text{lat}}.
\end{align*}
\]

Specifically, by constraints (26), (28), (31) and (32) for the rude or courteous HDVs, \(\rho\) under C-IV will also be upper bounded by

\[
\rho \leq K_w t_{\text{cycle}} - t_r - t^* - \frac{Q_{p,0}^\text{g}(t_{g1}) + N^* \times (D_{p,s}(v_{\text{plan}}, t_{g1}) + l_c) - D_{p,s}(v_{\text{plan}}, t_{g1})}{v_{p,max}}.
\]

or

\[
\rho \leq (K_w + 1) t_{\text{cycle}} - t_r - t^* - \frac{Q_{p,0}^\text{g}(t_{g2}) + N^* \times (D_{p,s}(v_{\text{plan}}, t_{g2}) + l_c) - D_{p,s}(v_{\text{plan}}, t_{g2})}{v_{p,max}}.
\]

where \(Q_{p,0}^\text{g}(t_{g})\) represents the travel distance of the Leader CAV with respect to the SDBs of the HDVs, which is defined in (72) and \(N^*\) is the optimized platoon size which will be determined later. The constraint for the parameter \(\rho\) depends on whether the platoon is able to cross the junction in the current cycle or the next one. Thus, according to (20) - (32), the optimal value of \(\rho\) that also leads to the minimum fuel consumption being

\[
\begin{align*}
\rho^* &= (K_w + 1) t_{\text{cycle}} - t_r - t^* - \frac{Q_{p,0}^\text{g}(t_{g1}) + N^* \times (D_{p,s}(v_{\text{plan}}, t_{g1}) + l_c) - D_{p,s}(v_{\text{plan}}, t_{g1})}{v_{p,max}} \\
\rho^{**} &= (K_w + 1) t_{\text{cycle}} - t_r - t^* - \frac{Q_{p,0}^\text{g}(t_{g2}) + N^* \times (D_{p,s}(v_{\text{plan}}, t_{g2}) + l_c) - D_{p,s}(v_{\text{plan}}, t_{g2})}{v_{p,max}}.
\end{align*}
\]

Based on (73), (76) and (77), the optimal values of \(\rho\) and \(T\) that lead to the minimum fuel consumption are

\[
\begin{align*}
T^* &= \frac{\phi(p)}{v_{p,min}^\text{lat}}, \\
\rho^{**} &= \text{min}\{\rho^*, \rho^{**}\}.
\end{align*}
\]

Then, for \(\rho^{**} \leq T^*\), the optimal solution is determined by

\[
\begin{align*}
a_{p,0}^\text{lat}(t) &= \frac{v_{p,ref}^\text{lat}(t) - v_{p,ref}^\text{for}(t)}{\rho^{**}},
\end{align*}
\]

that is, a constant acceleration will lead to the minimum fuel consumption within the acceleration period \([t^*, t^* + \rho]\) in the optimal regulation domain. The resulting fuel consumption is

\[
J^a = \frac{(v_{p,ref}^\text{lat}(t) - v_{p,ref}^\text{for}(t))^2}{\rho^{**}}.
\]

This completes the proof.

\[\square\]

Remark 41: To ensure a fuel economic junction crossing, the acceleration \(a_{p,0}^\text{lat}(t)\) of the Leader CAV, which determines the fuel consumption rate, and the acceleration period \(\rho\) need to be optimally determined. Strictly speaking, energy consumption is determined by both acceleration and cruising speed. But in urban environments, such as China and Singapore’s urban roads, vehicles’ cruising speed differences are so small that their impact on energy consumption can be negligible. Due to this reason, we here consider the optimization problem [39] Notably, the solving of fuel economic optimization problems should consider the platoon’s safety and comfortability, making the optimal solution completely different from the existing optimal control problem. For example, a decentralized optimal control framework was developed in [1], [40], to achieve a trade-off between minimizing
trip time and minimizing energy consumption. Although the derived speed can be efficiently computed and control the motion of an autonomous vehicle, the developed optimal speed profile is based on Pontryagin’s minimum principle, which may result in a bang-bang control for platoon CAVs to avoid the red light idling for the upcoming traffic lights. In this case, the frequent braking and acceleration for the platoon CAVs will generate traffic waves, causing traffic congestion and increasing the risk of accidents. To avoid this situation, an economic acceleration adjusting scheme for the platoon velocity profile will be developed to achieve a trade-off between platoon safety and avoiding unnecessary braking and acceleration while maintaining minimizing energy consumption.

Besides, for the case \( \rho^{***} > T^* \), the Leader CAV needs to use a constant acceleration \( \rho^{***} \) to reach the reference velocity \( v_{p,\text{ref}}(t) \) under C-II - C-IV as an economical acceleration adjusting with the sufficient regulation time \( \rho^{***} \) that satisfies

\[
\rho^{***} < \epsilon \rho^{***} \leq T^*,
\]

\[
0 < \epsilon \leq \frac{T^*}{\rho^{***}}, \quad \text{(81)}
\]

where \( \epsilon \) is a regulation parameter to value the performance among the driving safety, comfortability, and energy consumption, and it generally takes the value \( \frac{0.1 T^*}{\rho^{***}} \) for the comfort level of driving into account.

Notably, when the Leader CAV achieves the planning demands within the period \([t^*, t^* + \rho]\), all the Follower CAVs will need to form the platoon and ensure the safe crossing distance within a finite time \( T^{**} \). Therefore, the control horizon will need to be satisfied in the following constraints

\[
\begin{align*}
\rho^{***} &< T^{**} = \text{Free,} \quad \text{SET-0,} \\
\rho^{***} &< T^{**} = T^* - t^{**}, \quad \text{SET-1,} \\
\rho^{***} &< T^{**} = T^* - t^{**} < TT, \quad \text{SET-2,} \\
TT &= \begin{cases} t^*_{g1}, & \text{if } \sigma = 1 \land \text{Case-I} \land (\frac{Q_{p,0}(t^*_{g1})}{t^*_{g1}}) \leq v_{p,\max}; \\
\text{otherwise}, & \\
t^*_{g2} = \begin{cases} K_w t_{\text{cycle}} - t_r - t - \rho^{***}, & \text{if } \sigma = 1 \land \text{Case-I} \land (\frac{Q_{p,0}(t^*_{g1})}{t^*_{g1}}) \leq v_{p,\max}; \\
(K_w + 1) t_{\text{cycle}} - t_r - t - \rho^{***}, & \text{otherwise}, \end{cases} \\
t^*_{g1} = K_w t_{\text{cycle}} - t_r - t - \rho^{***}, \quad \text{SET-2,} \\
\end{cases}
\end{align*}
\]

\[
(82)
\]

where \( t^{**} \) and \( t^{***} \) are two small adjustment parameters to ensure the safe crossing, and the first case is for the STATE 1-Free-cruising, and the second case covers the STATE 2-Tracking-cruising, STATE 3-Platoon dynamic forming, STATE 4-Safe-check control, and STATE 6-Traffic flow control, and the third case is for the STATE 4-Smart crossing control within the upcoming traffic timing, respectively.

Therefore, the economic acceleration adjusting scheme is selected by a constant value during the adjustment period \([0, \rho^{****}]\) and zero in the non-adjustment period \([\rho^{****}, T^*] \). According to the dynamic \( (12) \), the acceleration profile of the Leader CAV under

\[
\text{C-II - C-IV can be designed by}
\]

\[
\begin{aligned}
& \frac{v_{p,0}(t)}{\rho^{***}}(1 + \tau) + \frac{\rho^{***}}{2},
\end{aligned}
\]

\[
\begin{cases}
\frac{v_{p,0}(t)}{\rho^{***}}(1 + \tau), \quad t \in [0, \rho^{****}], \\
0, \quad t \in (\rho^{****}, T^*],
\end{cases}
\]

\[
\frac{v_{p,0}(t)}{\rho^{***}}(1 + \tau), \quad t \in (\rho^{****}, T^*],
\]

\[
u_{p,0}(t) = \begin{cases}
(\frac{\rho^{***}}{\rho^{****}})^2, & \text{if } \sigma = 1 \land \text{Case-I} \land (\frac{Q_{p,0}(t^*_{g1})}{t^*_{g1}}) \leq v_{p,\max}; \\
(\frac{\rho^{***}}{\rho^{****}})^2, & \text{otherwise},
\end{cases}
\]

\[
t^*_{g1} = K_w t_{\text{cycle}} - t_r - t - \rho^{***}, \quad \text{SET-2,}
\]

\[
t^*_{g2} = (K_w + 1) t_{\text{cycle}} - t_r - t - \rho^{***},
\]

where \( \sigma \in \{0, 1\} \) is the traffic signal, \([0, \rho^{****}] \) is the optimal time interval of the acceleration regulation in the current traffic GREEN light window \([0, T^*]\), and \([\rho^{****}, T^*] \) is the optimal time interval of the acceleration regulation that stretched into the next traffic GREEN light \([0, T^*]\).

Based on the above procedures, the nonlinear decision function \( f(\text{AutoC}(t), u_{p,0}(t)) \) will be constructed by the finite state function \( \text{AutoC}(t) = \{S0, S1, S2, S3, S4, S5\} \), the reference velocity \( v_{\text{ref}}(t) \), and the economical acceleration adjusting \( (83) \) and \( u_{p,0}(t) \equiv 0 \) under C-I.

Remark 42: Ideally, the controller design aims to obtain the acceleration planning for the Leader CAV under C-I - C-VI and the cooperative controller development for the Follower CAVs to ensure the safety cruising and SCPs with the condition \( \rho^{***} = T^* \). However, due to the reaction-time delays both in the CBPM-OVM model and Extended IDM model, the velocity and displacement trajectory cannot be responded to by the acceleration adjusting \( u_{p,0}(t) \) timely. To solve this problem, we here introduce two small adjusting parameters to ensure a safe crossing. Notably, the proposed strategy can significantly improve the robustness and expandability of platoon CAV management when there is external interference in vehicle control, such as noise or uncertain input, thanks to the redundant control horizon.

Remark 43: Notably, due to different intersection widths and lane lengths, the maximum allowable length of platoon crossing in the optimization zone for different junctions may not be fixed. Therefore, asking the whole platoon to pass the junction at the next traffic lights will reduce the traffic throughput capacity, if there are only a few CAVs that cannot accomplish the junction crossing. Considering this situation, it will arise the problems of splitting and merging platoons, resulting in multiple platoons control(which will be solved in part II work). When the whole platoon cannot cross the specific junction at the current traffic light, the platoon will be split into several smaller platoons. For
this case, the first condition of (83) can ensure that the first split platoon can safely pass through the intersection, and the rest of the platoons will use the second condition of (83) to pass through the intersection. When all the platoons finish the junction crossing, which will be merged as a whole to cruise. That’s why we so-call multiple platoons control. In fact, whether or not to split the platoon is a compromise between safety and traffic throughput. For example, if only one or a few CAVs can pass through the junction at the current GREEN window, to ensure the performance and safety of the platoon, we prefer to wait for the next GREEN window for all CAVs to pass. If most of the current CAVs can pass, we would like to consider splitting the platoon to improve throughput. It is important to note that all decisions and the developed ADSCAS are applicable for both single platoon and multi-platoon with cruising and SCPs.

VI. Conclusion

In this article, the problems of distributed dynamic platoons control and junction crossing optimization are addressed for a mixed traffic flow with CAVs and social HDVs in a smart city. We have shown that despite the nonlinearity, non-smoothness, and uncertainty of mixed traffic flows in smart cities, the solutions of safe, efficient, and fuel economic platoon control for the above problems are solved by our proposed ADSCAS framework that can provide the reference trajectory planning and the solution of the fuel economic optimization problem to ensure the safe and efficient SCPs. The cooperative controller design and stability analysis will be further characterized in part II. Various examples and case studies will also be presented in part II.

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