Assessment of Residual Stresses in a T-joint Weld by Combined Experimental/Theoretical Approach

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Abstract. Residual stresses are common in metal structures, essentially influencing their mechanical behaviour. We consider a combined experimental/theoretical approach to residual stresses. The theoretical basis of analysis is provided by the recently developed \textit{F}_0-approach, operating with explicit relation between load-free and stress-free configurations. The titanium alloy Ti-6Al-4V is modelled with the multiplicative decomposition of the deformation gradient into the elastic and the plastic parts. Isotropic hyperelastic relations between stresses and elastic strains are assumed. The weak invariance of the material model allows for incorporation of residual stresses without additional numerical costs. In order to demonstrate the new experimental/theoretical approach to residual stresses, experimentally measured stresses are extrapolated from the surface inside the welded T-joint. The robustness of the stress extrapolation procedure is confirmed on synthetic experimental data.

1. Introduction

Residual stresses may significantly affect behaviour of welded metal structures [5, 3]. Direct thermo-mechanical simulation of the welding process is conventionally used to predict residual stresses and to analyse their impact. However, that approach becomes numerically expensive if the after-weld behaviour is supposed to be repeatedly simulated (e.g., for optimisation of post-weld heat treatment [16]). Additionally, the prediction of residual stresses can be highly inaccurate [14, 8] due to interaction of complex mechanical phenomena.

We improve the numerical efficiency with a special \textit{F}_0-approach, which allows a relatively simple restart for a number of cases [12, 13]. A combined experimental/theoretical approach to residual stresses was presented earlier [13] to improve the accuracy of predictions. This approach benefits from the strong points of experimental and theoretical procedures. Importantly, it also allows to extrapolate the stresses inside the analysed structure, starting from measurement results on the surface. Such surface data are usually available due to non-destructive experimental measurements [8, 4].

The material model of elasto-viscoplasticity with the multiplicative decomposition of the deformation gradient is utilised in the study. This special structure leads to various advantages: the constitutive equations are thermodynamically consistent, free from parasitic shear stress oscillations, objective under superimposed rigid body motion, and w-invariant. It is shown that the original material model can be exploited even after restart from the new configuration due to w-invariance. Owing to w-invariance, the numerical efficacy of integration schemes remains the same upon the introduction of residual stresses [13].

A demonstration problem of surface weld on a single plate was considered in the previous work [13], whilst an applied problem of a T-joint weld is under focus in the current study. In addition to that, the stability of the approach with respect to the unavoidable experimental measurement errors is briefly examined.
2. Material model

2.1. Stresses, strains and evolution equations

The mechanical behaviour of the material is described by the model of Simo and Miehe [9] which will be represented on the reference configuration. Let \( \mathbf{F} \) be the deformation gradient. The right Cauchy-Green tensor is then denoted as \( \mathbf{C} = \mathbf{F}^T \mathbf{F} \). Assuming multiplicative decomposition of the deformation gradient into the elastic and the plastic parts \( \mathbf{F} = \mathbf{F}_e \mathbf{F}_p \), the plastic right Cauchy-Green tensor is introduced: \( \mathbf{C}_p := \mathbf{F}_p^T \mathbf{F}_p \).

The material model used in the study corresponds to one already employed in [13]. With the hyperelastic response given by the assumption of Hartmann and Neff [2] for the volumetric part and the neo-Hookean model for the isochoric part, Colemann-Noll procedure allows to compute the second Piola-Kirchhoff stress as

\[
\mathbf{T} = 0.1 \, k(\theta) \left( \left( \sqrt{\det \mathbf{C}} / f_\theta(\theta) \right)^5 - \left( \sqrt{\det \mathbf{C}} / f_\theta(\theta) \right)^5 \right) \mathbf{C}^{-1} + \mu(\theta) \mathbf{C}^{-1} \left( \mathbf{C}_p \right)^{-1} \mathbf{D}.
\]

Here \( \theta \) is the current temperature; \( k(\theta) \) and \( \mu(\theta) \) are the bulk and shear moduli; \( f_\theta(\theta) \) is the temperature-related volume change; \( \mathbf{A} = (\det \mathbf{A})^{-1/3} \mathbf{A} \) is the unimodular part of a tensor and \( \mathbf{A}^0 = \mathbf{A} - \frac{2}{3} \mathbf{A} \) is the deviatoric part of the tensor.

For isotropic expansion of the yield surface in the stress space, isotropic hardening \( R \in \mathbb{R} \) is employed:

\[
R = \gamma(\theta)s,
\]

where \( \gamma(\theta) \) is the hardening temperature-dependent modulus and \( s \) is the plastic arc-length. In order to describe a viscoplastic behaviour allowing for stress states beyond the elastic domain we use the norm \( \mathfrak{g} \) of the deviatoric part of the Kirchhoff stress and the viscous overstress \( f \):

\[
\mathfrak{g} = \left\| \left( \mathbf{C}_p \right)^{1/2} \right\|^{0.5}, \quad f = \mathfrak{g} - (2/3)^{0.5}(K(\theta) + R).
\]

Here, \( K(\theta) \) is the uniaxial quasi-static temperature-dependent yield stress.

Perzyna’s law of viscoplasticity allows to estimate the plastic strain rate \( \lambda_p = 1/\eta \left( f / f_\phi \right)^m \), where \( (x) := \max(x, 0), \eta \) is the fixed viscosity, \( m \) is the stress exponent and \( k_\phi = 1 \) MPa. Evolution equations on the reference configuration are written as

\[
\dot{\mathbf{C}}_p = 2 \lambda_p / \mathfrak{g} \left( \mathbf{C}_p \right)^{1/2} \mathbf{D}, \quad \dot{s} = (2/3)^{0.5} \lambda_p.
\]

Note that the plastic flow remains incompressible \( (\det \mathbf{C}_p) = 1 \). Finally, to close the system of constitutive equations we include initial conditions \( \mathbf{C}_p|_{t=0} = \mathbf{C}_p^0, s|_{t=0} = s^0 \).

2.2. Material constants

For demonstration purposes, we consider the alloy Ti-6Al-4V. Material properties are described assuming \( \theta = 20 \) °C as a reference temperature. The temperature-induced volume change of material is then described with the non-dimensional quantity \( \text{Vol} \) given by (cf. [6, 15])

\[
\text{Vol}(\theta) = \begin{cases} 
1 + 10^{-5}(0.85(\theta - 20) + 10^{-4}(\theta - 20)^2), & \text{if } \theta < 770 \, ^\circ\text{C}, \\
1 + 10^{-5}(693.75 + \theta - 20 - 750), & \text{if } \theta \geq 770 \, ^\circ\text{C}.
\end{cases}
\]

For the initial temperature \( \theta_0 \) of the material point, the volume change relative to the initial state is computed as \( f_0(\theta) = \text{Vol}(\theta) / \text{Vol}(\theta_0) \). Bulk and shear moduli are given by the standard relations

\[
k(\theta) = E(\theta) / (3 - 6\nu(\theta)), \quad \mu(\theta) = E(\theta) / (2 + 2\nu(\theta)).
\]

The elasticity modulus (in MPa) is an explicit function of temperature (cf. [15])

\[
E(\theta) = \begin{cases} 
1.2 \cdot 10^5 - (1.2 \cdot 10^5 - E_{1300})\theta / 1300, & \text{if } \theta < 1300 \, ^\circ\text{C}, \\
E_{1300} - (E_{1300} - 100)(\theta - 1300)/1200, & \text{if } \theta \geq 1300 \, ^\circ\text{C},
\end{cases}
\]
where $E_{1300}$ is the elasticity modulus at 1300 °C. The dependence of the Poisson ratio is described by (cf. [1])

$$
\nu(\theta) = \begin{cases} 
0.34 + 0.06 \theta / 1700, & \text{if } \theta < 1700 \, ^\circ \text{C}, \\
0.4 + 0.09(\theta - 1700) / 200, & \text{if } 1700 \, ^\circ \text{C} \leq \theta < 1900 \, ^\circ \text{C}, \\
0.49, & \text{if } \theta \geq 1900 \, ^\circ \text{C}.
\end{cases}
$$

(8)

The temperature-dependent yield stress (in MPa) is computed through (cf. [15])

$$
K(\theta) = \begin{cases} 
850 - 0.8\theta, & \text{if } \theta < 1000 \, ^\circ \text{C}, \\
50, & \text{if } \theta \geq 1000 \, ^\circ \text{C}.
\end{cases}
$$

(9)

The isotropic hardening is assumed to be proportional to the yield stress:

$$
\gamma(\theta) = 5000K(\theta)/K(20) \, \text{MPa}.
$$

(10)

3. Basic relations of $F_0$-approach

3.1. Transformation between configuration

Let the reference configuration before solidification be $\bar{R}^{bs}$. The solidification process takes place in the time interval $t_0 \leq t \leq t_{\text{solid}}$. The load-free configuration after solidification is $\bar{R}^{sf}$, which is transformed to the current configuration $K$ by the deformation gradient $F^s$. Likewise, assume that $\bar{R}^{sf}$ is the local configuration in the stress-free state and $F^s$ is the corresponding deformation gradient: $F^s = F^sF_0^{-1}$. Let $\bar{K}^s$ be the $\bar{R}^{sf}$ configuration with cancelled volume changes. Thus, configuration $\bar{K}^s$ is transformed to $\bar{R}^{sf}$ by $\varphi^1$, where $\varphi = [f_s/\det F_0]^{1/2}$ and $f_s = f_0(\theta(t_{\text{solid}}))/f_0(\theta(t_0))$ (for details see [13]).

Owing to the w-invariance of the material model, the constitutive equations are transferred from $\bar{R}^{bs}$ to $\bar{R}^{sf}$ without any changes. Following [13] the transformation rules for initial conditions, the second Piola-Kirchhoff stress operating on $\bar{R}^{sf}$ and the tangent operator are given by

$$
C^s|_{\text{solid}} = \bar{F}^T_{0} F_0, \quad \bar{T}^{sf} = \varphi^{-1} \bar{K}^s, \quad \partial \bar{T}^{sf} / \partial C^s = \varphi \partial \bar{K}^s / \partial C^s.
$$

(11)

Thus, in order to restart the simulation at $t = t_{\text{solid}}$ from the new reference configuration $\bar{R}^{sf}$ one needs only the fields $F_0$, $J_s$ and $s$. Upon the introduction of pre-stresses, the complexity of numerical algorithms does not increase, cf. [13]. The material model and the $F_0$-approach are implemented into the commercial finite element code MSC.MARC using Hypela2 interface.

3.2. Parametrisation of the $F_0$-field

The numerical simulations of residual stresses are typically highly inaccurate due to aberrations like ill-defined boundary conditions and complex micro-structural evolutions [1, 7]. Therefore, new simplified approaches with increased accuracy are urgently required. The main idea in this study is to avoid modelling of the fully-coupled thermo-mechanical problem by setting an approximate temperature field within the analysed structure. Meanwhile, a pair of parameters are assumed to be free letting them to control the temperature evolution and hence the $F_0$-field. High accuracy of residual stresses’ prediction in that case becomes possible due to the combined experimental/theoretical approach [13].

We assume in the current study that the process of welding is idealised such that the hot metal filler is instantly applied to cold base metal. Specifically, the cooling of the filler is described by

$$
\theta_{\text{filler}}(t) = 20 + 2400 \cdot 2^{-0.1t}.
$$

(12)

In this formula the time is measured in seconds. A two-step process of heating followed by cooling is assumed in the base metal:

$$
\begin{align*}
\theta_{\text{base}}(t) &= 20 + \frac{t}{t_1} \cdot R(d) \cdot C_0 \cdot (\theta_{\text{filler}}(t_1) - 20), \quad \text{if } t \leq t_1, \\
\theta_{\text{base}}(t) &= 20 + (\theta_{\text{base}}(t_1, d) - 20) \cdot 2^{-0.1(t-t_1)}, \quad \text{if } t > t_1.
\end{align*}
$$

(13)

Here, $t_1 = 3 \, s$, $C_0$ is a non-dimensional coefficient defining the dependence between filler's and base’s temperatures; $R(d) = \max(0, (d_{\text{max}} - d)/d_{\text{max}})$ with $d_{\text{max}}$ being a maximum expected heating distance (in mm); $d$ is the distance between filler and the current point (in mm). In summary, the unknown free variables chosen for parametrisation of the $F_0$-field are $C_0$, $d_{\text{max}}$ and $E_{1300}$. 


4. Demonstration problem of the T-joint

4.1. Problem statement
Let us consider residual stresses in the one-sided T-joint after tungsten inert gas welding. Both plates and filler are made of Ti-6Al-4V alloy. The left edge of the horizontal plate is fixed in space; plane strain is assumed, meaning that all displacements in Z-direction are zero. The dimensions of the structure are shown in figure 1(a). Longitudinal to the weld (in Z-direction) and lateral to the weld (in X- or Y-direction) stresses are measured on the surface of plates as also shown in figure 1(a). For the FEM simulation twenty-node hexagonal elements with quadratic approximation of geometry and displacements are used with a total number of 1728 elements. The FEM model is shown in figure 1(b).

4.2. Direct problem and synthetic experimental data
For demonstration of the applicability of experimental/theoretical approach to the T-joint we consider the following re-identification problem. First, we manually preset unknown parameters as \( P^\text{set} = [C_\theta, d_{\text{max}}, E_{1300}] \) with \( C_\theta = 0.9, \ d_{\text{max}} = 25 \text{ mm} \) and \( E_{1300} = 10000 \text{ MPa} \). These parameters are close to those from real applications [13]. Next, we carry out the FEM simulation of welding and save Cauchy stresses computed at measure points. In order to mimic the real experimental data, we generate synthetic results by spoiling simulated values with a stochastic noise:

\[
\mathbf{T}_{\text{ii}, j}^\text{synth} = \mathbf{T}_{\text{ii}, j}^\text{simulated} + N_j,
\]

(14)

where \( N_i \) are independent random variables with normal distribution of zero mean and standard deviation of 20 MPa; \( ii \in \{xx, yy, zz\} \). The generated synthetic data are shown by crosses in figure 2.

4.3. Inverse problem
To re-identify the simulation parameters, we build the following error functional:

\[
\phi(P) = \sum_{i=1}^{N} \left( \mathbf{T}_{\text{ii}, j}^\text{synth} - \mathbf{T}_{\text{ii}, j}^\text{num}(P) \right)^2 + \sum_{i=1}^{N} \left( \mathbf{T}_{\text{lat}, j}^\text{synth} - \mathbf{T}_{\text{lat}, j}^\text{num}(P) \right)^2,
\]

(15)

where \( P \) is the set of design parameters, \( N \) is the number of measure points, \( \mathbf{T}_{\text{ii}, j}^\text{num}(P) \) and \( \mathbf{T}_{\text{lat}, j}^\text{num}(P) \) are longitudinal and lateral to the weld Cauchy stresses at \( j \)-th point calculated with the set \( P \). The error functional is minimised with the robust gradient-free Nelder-Mead procedure. Each direct problem is solved in the FEM code MSC.MARC. An external optimisation module is coupled to the FEM code to enable the Nelder-Mead iteration process. Optimised set of parameters is \( C_\theta = 0.9279, \ d_{\text{max}} = 23.751 \text{ mm} \) and \( E_{1300} = 9934.85 \text{ MPa} \), which is close to the original set of parameters. Simulated stresses along the measurement lines are shown in figure 2. Additionally, the stress distribution of \( \mathbf{T}_{zz} \) in the area of weld is shown in figure 3 for simulations with original and re-identified parameters and the distribution of the maximum principal shear value of \( \mathbf{T} \) is shown in figure 4. As is seen, the results are in good agreement. The following correlation matrix is obtained for the identified parameters:
\[
\text{Corr} = \begin{pmatrix}
1.0 & 0.3184 & -0.0388 \\
0.3184 & 1.0 & -0.2109 \\
-0.0388 & -0.2109 & 1.0
\end{pmatrix}. \tag{16}
\]

The non-diagonal elements of \(\text{Corr}\) are separated from \(\pm 1\). Thus, the correlation lies in acceptable range. This means that the problem of parameter identification is well posed.

**Figure 2.** Residual stresses after welding along the measurement lines. Synthetic values are indicated as crosses; numerical results for re-identified parameters are shown with the solid lines. (a) top of horizontal plate; (b) left side of vertical plate; (c) bottom of horizontal plate.

**Figure 3.** Distribution of Cauchy stresses (in MPa) near the weld in Z-direction after the solidification. (a) simulation for the original set of parameters; (b) simulation for the re-identified parameters.
5. Conclusion
A geometrically exact approach to the modelling of elasto-viscoplastic material is considered. Residual stresses are accounted for with a special $F_0$-approach, developed in [12] and generalised in [13]. The material model of elasto-viscoplasticity is based on the multiplicative decomposition of deformation gradient and possesses $w$-invariance, which allows to introduce residual stresses by means of $F_0$-field without any additional complications in the numerical schemes. Moreover, the stresses and the tangent operator are calculated in a straightforward way after the change of the reference configuration.

One-sided T-joint welding of Ti-6Al-4V plates is simulated in a plane strain, the resulting residual stresses are used to acquire the synthetic data imitating real measurements. After that the re-identification problem is solved. Since the re-identified parameters are close to the original set, the identification problem is stable with respect to measurement errors.\(^3\) The robustness of the optimisation is also verified by the similarities of stress distributions. In conclusion, given the stability of stress extrapolation with respect to chosen parametrisation the combined experimental/theoretical approach to residual stresses is indeed a reliable practical tool.

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\(^3\) A more general error-sensitivity analysis is presented in [10, 11].
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