Neutrino Oscillations in Dark Matter

Ki-Young Choi,1 Eung Jin Chun,2 and Jongkuk Kim2

1Department of Physics, BK21 Physics Research Division, Institute of Basic Science, Sungkyunkwan University, 16419 Korea
2Korea Institute for Advanced Study, Seoul 02455, Korea

We study neutrino oscillations in a medium of dark matter which generalizes the standard matter effect. A general formula is derived to describe the effect of various mediums and their mediators to neutrinos. Neutrinos and anti-neutrinos receive opposite contributions from asymmetric distribution of (dark) matter and anti-matter, and thus it could appear in precision measurements of neutrino or anti-neutrino oscillations. Furthermore, neutrino oscillations can occur from the symmetric dark matter effect even for massless neutrinos.

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Introduction: When neutrinos propagate in matter, neutrino oscillations are affected by their coherent forward elastic scattering in which the matter remains unchanged and its effect is described by an effective (matter) potential [1]. This can lead to a dramatic impact in neutrino oscillation, with a resonance enhancement of the effective mixing parameter [2], which is recognized as the MSW effect confirmed as the source of the solar neutrino deficit.

Recently various medium effects have been considered extensively to study its impact on neutrino oscillations or fit experimental data better in a medium of dark matter (DM) [3–21], or dark energy [22–27]. However, there has not been a systematic study of the general medium effect on neutrino oscillations.

In this article, we derive a general formula describing the medium effect which can be applied to various dark matter models with different mediators to neutrinos. The medium effect includes modifications of the neutrino mass and potential which are different for neutrinos and anti-neutrinos in a medium of asymmetric dark matter. Remarkably, the neutrino oscillation phenomena can arise solely from the symmetric medium effect in a certain model parameter space.

![Feynman diagrams for the scattering of neutrino with electron or positron through the W boson exchange in SM](image)

FIG. 1: Feynman diagrams for the scattering of neutrino with electron or positron through the W boson exchange in SM.

Standard matter effect in a medium of electrons and positrons: Let us begin by revisiting the standard calculation of the matter effect driven by the charged current interaction of the Standard Model (SM). For this, we consider neutrino/anti-neutrino propagation in a general background densities of electrons Ne and positrons Nē. Calculating the coherent forward scattering for the u and s channel processes in Fig. 1 one can find the generalized matter potential:

$$V_{ν,\bar{ν}}^{SM} = \sqrt{2}G_F(N_e + N_{\bar{e}})\frac{\pm \epsilon m_W^4 - 2m_W^2m_eE_\nu}{m_W^2 - 4m_e^2E_\nu^2},$$

where $G_F \equiv g^2/(4\sqrt{2}m_W^2)$ is the Fermi constant, and $\epsilon \equiv (N_e - N_{\bar{e}})/(N_e + N_{\bar{e}})$ describes the asymmetry of electron and positron distributions. Note that it reduces to the Wolfenstein potential $\sqrt{2}G_FN_e$ for neutrinos and anti-neutrinos, respectively in the ordinary situation: $\epsilon = 1$ ($N_{\bar{e}} = 0$) and $m_W^2 \gg 2m_eE_\nu$. Furthermore, we notice that the matter potential at high energy takes the form:

$$V_{ν,\bar{ν}}^{SM} \approx \sqrt{2}G_Fm_W^2(N_e + N_{\bar{e}})/2m_eE_\nu,$$

which mimics the standard oscillation parameter $\Delta m^2/2E_\nu$ acting in the same way for neutrinos and anti-neutrinos. The basic formula Eq. (1) already describes main features of a more general medium effect, which will be described below.

Variant models of medium and mediator: Staying close to the Standard Model case, we can consider a model of (Dirac) fermionic dark matter $f_i$ and dark photon $X$ as its messenger to neutrinos:

$$L_{int} = g_{αi}f_μ^iP_Lν_αX^μ + h.c.$$  (3)

Introducing the flavor-dependent couplings, we will get a flavor-dependent medium potential generalized from Eq. (1) with $m_e = m_f$, and $m_W = m_X$.

The second model consists of fermionic dark matter $f$ with Dirac mass $m_f$ and bosonic messenger $φ_i$ with mass $m_φ$:

$$L_{int} = g_{αi}f_PLν_αφ_i^μ + h.c.$$  (4)
where we introduced different flavors in the mediator \( \phi_i \) instead of the dark matter \( f \) for simplicity. This model also leads to the same kind of medium potential with \( m_s = m_f \) and \( m_W = m_{\phi_i} \).

The third model which is of our particular interest has complex bosonic dark matter \( \phi \) with mass \( m_{\phi} \) and fermionic messenger \( f_i \) with Dirac mass \( m_f \):

\[
\mathcal{L}_{\text{int}} = g_{\alpha i} f_i P_L \nu_{\alpha} \phi^* + h.c. \tag{(5)}
\]

which generates the medium potential as well as corrections to the neutrino mass as we will see later.

For all the cases, we will use the unified notations of \( m_{\text{DM}} \) for the dark matter mass, \( \rho_{\text{DM}} = m_{\text{DM}} (N_{\text{DM}} + N_{\text{DM}}^*) \) for the total dark matter energy density, and

\[
e \equiv \frac{N_{\text{DM}} - N_{\text{DM}}^*}{N_{\text{DM}} + N_{\text{DM}}^*} \tag{(6)}
\]

for the asymmetry between the dark matter and anti-dark matter number densities.

**General formulation:** Neutrino/anti-neutrino propagation in a medium can be described by the following minimal form of the equations of motion in the momentum space:

\[
(\not{p} - \Sigma) u_L = (M^\dagger + \Sigma_0) u_R, \tag{(7)}
\]

\[
(\not{p} - \Sigma) u_R = (M + \Sigma_0) u_L,
\]

where \( M \) is the symmetric (Majorana) neutrino mass matrix; \( \Sigma \equiv \Sigma_\mu \gamma^\mu, \bar{\Sigma} \equiv \Sigma_\mu \gamma^\mu, \Sigma_0, \) and \( \Sigma_0 \) are corrections coming from the effect of coherent forward scattering of neutrinos/anti-neutrinos within medium. Here \( \Sigma_\mu, \bar{\Sigma}_\mu \) are hermitian matrices. Note that we used \( u_L \) and \( u_R \) to represent the neutrino and anti-neutrino state, respectively. Equivalently one may use \( v_L \) for the anti-neutrino state using the relation: \( \bar{u}_R = -\bar{v}_L^T C \) and \( u_R = C \bar{v}_L^T \) where \( C = -i \gamma^2 \gamma^0 \) is the charge-conjugation matrix.

In a Lorenz invariant medium, \( \Sigma \) and \( \bar{\Sigma} \) can be expressed by

\[
\Sigma = \phi \Sigma_1 + \bar{k} \Sigma_2; \quad \bar{\Sigma} = \phi \bar{\Sigma}_1 + \bar{k} \bar{\Sigma}_2, \tag{(8)}
\]

where \( \bar{k} \) is the energy-momentum of the dark matter which we will take \( (k_0, \bar{k}) = (k_0, 0) \) corresponding to averaging over random momentum distribution, and \( k_0 \) becomes the dark matter mass \( m_{\text{DM}} \) in the non-relativistic medium. The scalar terms \( \Sigma_0/\Sigma_0^* \) appear in some situations \([8, 10, 12, 13, 18]\), which will not be discussed further in this article.

Recall that the SM matter effect contributes to the vector current terms \( \Sigma_2/\Sigma_2^* \). Similar terms are generated in the models in Eqs. \([3, 11, 15]\) and thus a medium potential similar to the standard matter potential Eq. \((1)\) is produced. On the other hand, the correction to the neutrino kinetic term, \( \Sigma_1/\Sigma_1^* \), arises only in Eq. \((5)\).

**FIG. 2:** Feynman diagrams for the scattering of neutrino and complex scalar dark matter mediated by a fermion in the scenario of Eq. \((5)\).

The canonical basis of the kinetic term can be recovered by the transformation

\[
u_L \simeq \left(1 + \frac{\Sigma_1}{2}\right) \bar{u}_L, \tag{(9)}
\]

\[
u_R \simeq \left(1 + \frac{\Sigma_1}{2}\right) \bar{u}_R,
\]

in the leading order of \( \Sigma/\Sigma \). This leads to the medium-dressed neutrino mass matrix

\[
\tilde{M} \simeq \left(1 + \frac{\Sigma_1}{2}\right) M \left(1 + \frac{\Sigma_1}{2}\right), \tag{(10)}
\]

and thus we obtain

\[
(\not{p} - \bar{k} \Sigma_2) \bar{u}_L = \tilde{M}^\dagger \bar{u}_R, \tag{(11)}
\]

\[
(\not{p} - \bar{k} \bar{\Sigma}_2) \bar{u}_R = \tilde{M} \bar{u}_L.
\]

This takes the same form as in the case of the SM matter effect and thus one obtains neutrino/anti-neutrino propagation Hamiltonians

\[
H_\nu = E_\nu + \tilde{M}^\dagger \tilde{M} + \frac{k^0}{2} \Sigma_2, \tag{(12)}
\]

\[
H_\bar{\nu} = E_\bar{\nu} + \tilde{M} \tilde{M}^\dagger + \frac{k^0}{2} \bar{\Sigma}_2, \tag{(13)}
\]

in the ultra-relativistic limit: \(|\bar{p}_\nu| \simeq E_\nu|\).

In the case of the model in Eq. \((5)\), the calculation of the \( s \) and \( u \) channel diagrams in Fig. 2 for the forward elastic scattering gives

\[
\Sigma_1 \simeq \lambda^T \rho_{\text{DM}} \frac{2 m_{\text{DM}} E_\nu - m_X^2}{m_{\text{DM}}^2 m_X^2 - 4 m_{\text{DM}}^2 E_\nu^2} \tag{(14)}
\]

\[
\Sigma_2 \simeq \lambda^T \rho_{\text{DM}} \frac{2 m_{\text{DM}}}{m_{\text{DM}}^2 m_X^2 - 4 m_{\text{DM}}^2 E_\nu^2} \tag{(15)}
\]

where the coupling matrix \( \lambda \) is defined by \( \lambda_{\alpha \beta} = g_{\alpha i} g_{\beta i}/2 \) for the transition \( \nu_\beta \to \nu_\alpha \) and the same mass \( m_X \) is assumed for the mediators \( f_i \) and \( m_{\text{DM}} = m_{\phi} \) is the dark matter mass. From the expression \((14)\), one can find a remarkable property that the medium mass matrix \( \tilde{M} \) becomes symmetric only for the symmetric dark matter
Like the medium effect to the mass matrix, the medium  

needs to be distinguished from the theory with CPT vi-

dence \cite{28–32}.

This formula is applicable to the all three scenarios with  
different candidates of dark matter and mediators. No-

More specifically, when $|\varepsilon|m_X^2 \gg 2m_{DM}E_\nu$, we get the  

matter potential

\begin{equation}
V_{\nu,\bar{\nu}}^{\text{DM}} \approx \pm \varepsilon \frac{\lambda^{(T)}}{4} \frac{\rho_{DM}}{m_{DM}E_\nu^{\text{peak}}}. \tag{19}
\end{equation}

Given the masses as chosen above, the conventional  

bounds on non-standard interactions (NSI) can be ap-

plied to each component of $\varepsilon \lambda^{(T)}$. Normalizing $V_{\alpha \beta}^{\text{DM}}$ by  

\begin{equation}
\varepsilon_{\alpha \beta} \approx 0.01\lambda^{(T)} \varepsilon \frac{20\text{meV}}{m_{DM}} \frac{1\text{TeV}}{E_\nu^{\text{peak}}} \left(\frac{\rho_{DM}}{0.3\text{GeVcm}^{-3}}\right), \tag{20}
\end{equation}

taking $N_\varepsilon \approx 1.3 \times 10^{24}/\text{cm}^3$ for the earth mantle density. The  

values of $\varepsilon$ are constrained to be smaller than around 0.1 or 0.01 \cite{33–36}, which  
is larger or smaller than the reference neutrino energy $E_\nu^{\text{ref}}$ and also whether the medium potential at high  

energy is larger or smaller than $\Delta m^2/2E_\nu$, that is,

\begin{equation}
m_{DM}^2 > \frac{\lambda \rho_{DM}}{2|\Delta m^2|}, \quad \text{or} \quad m_{DM}^2 < \frac{\lambda \rho_{DM}}{2|\Delta m^2|}. \tag{18}
\end{equation}

In the region 1 and 2, the medium potential is sub-

dominant and may give small modification to the stan-

standard oscillation, or a peculiar feature can show up at the  

energy of the peak. In the region 3 and 4, if $E_\nu^{\text{peak}}$ is in  
the range of 1 MeV –100 GeV, there would appear a high  
distortion in various standard neutrino oscillation data and  
thus it is strongly disfavored. In region 4, there is no  
signals at low energy data, however the future experi-
iments of neutrino oscillation at high energy can probe  
this region. The boundary of the regions 1 and 3 (the  
dashed line) with $E_\nu^{\text{peak}} \ll 1\text{MeV}$ is of particular inter-

est as the medium potential mimics the SM mass term  
and can explain the neutrino oscillation data even with  

massless neutrinos.

The configurations of the medium potential \cite{10} are  
presented in the small boxes of Fig. 3. One can con-

sider four different regions depending on whether $E_\nu^{\text{peak}}$  
defined by

\begin{equation}
E_\nu^{\text{peak}} = \frac{m_X^2}{2m_{DM}}. \tag{17}
\end{equation}

\footnote{The sign of the anti-neutrino potential was opposite to ours. But the authors agreed with our result in a private communication.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The schematic plot for the shape of the medium potential for different regimes of $m_{DM}$ and $E_\nu^{\text{peak}}$. In the small boxes, we show the absolute value of the medium potential with respect to the neutrino energy. The solid (dashed) red line is for asymmetric (symmetric) distribution of DM. For comparison, $\Delta m^2/2E_\nu$ is shown by the black solid line. The black dotted vertical line denotes the reference neutrino energy scale $E_\nu^{\text{ref}}$ in a certain experiment of interest.}
\end{figure}
which behaves same as the standard neutrino masses and mixing explaining the observed neutrino oscillations for \( m_X \ll 200\text{eV} \sqrt{(m_{DM}/20\text{meV})(E_\nu/1\text{MeV})} \). Therefore, the experimental data can be fitted by the couplings given by

\[
\lambda = \frac{2m_{\tilde{D}}^2 U^* \text{diag}(\Delta m^2) U^T}{\rho_{DM}},
\]

\[
\simeq \begin{pmatrix}
0.026 & 0.091 & 0.085 \\
0.091 & 0.381 & 0.408 \\
0.085 & 0.408 & 0.478
\end{pmatrix}
\begin{pmatrix}
m_{DM} \\
20\text{meV} \\
\rho_{DM}
\end{pmatrix}^2 \begin{pmatrix}
0.3\text{GeV cm}^{-3}
\end{pmatrix},
\]

where \( \text{diag}(\Delta m^2)_{ii} = m_i^2 - m_1^2 \) and \( U \) is the neutrino mixing matrix assuming the normal hierarchy and vanishing CP phases. In this model of “dark matter assisted neutrino oscillation”, the leading correction with non-vanishing \( \epsilon \) may help to fit better the observations of neutrinos from Sun and the reactor anti-neutrino experiments [21, 37]. It would be interesting to observe the oscillation of high energy neutrino which might be affected by this medium effect [38–44]. In the future, the measurement of the absolute value of neutrino mass may rule out this model.

When the observed neutrino oscillations come from the standard neutrino mass matrix, the medium corrections are constrained. That is, requiring the medium corrections less than about 1% for the region 1 or 3, we find

\[
m_{DM} \gtrsim 200\text{meV} |\lambda|^{1/2},
\]

\[
m_{DM} \gtrsim 0.3\text{meV} |\epsilon|^{1/3} |\lambda|^{1/3} \left( \frac{1\text{MeV}}{E_\nu} \right)^{1/3},
\]

from \( \Sigma_2 \) and \( \Sigma_1 \), respectively.

**Apparent CPT violation**: The Lagrangian itself is CPT invariant, but the medium of asymmetric DM is not. Thus, the effective neutrino mixing and mass-squared differences are modified in a different way for neutrinos and antineutrinos. Such a CPT violation may appear in precision measurement of neutrino oscillations, or is highly constrained by the present data, particularly if the peak energy resides between 1 MeV and 100 GeV where the standard neutrino oscillation has been confirmed.

**Two-flavor oscillation**: To see the medium effect in more detail, let us consider the two-flavor \((\nu_\mu - \nu_\tau)\) oscillation described by the effective Hamiltonian:

\[
\mathcal{H}_M = \mathcal{H}_{\text{vac}} + \begin{pmatrix}
V_{\mu\mu} & V_{\mu\tau} \\
V_{\tau\mu} & V_{\tau\tau}
\end{pmatrix},
\]

where \( \mathcal{H}_{\text{vac}} \) is for the oscillation in the standard model

\[
\mathcal{H}_{\text{vac}} = \frac{\Delta m^2}{4E} \begin{pmatrix}
-\cos 2\theta & \sin 2\theta \\
\sin 2\theta & \cos 2\theta
\end{pmatrix}.
\]

Up to the diagonal term proportional to the identity matrix, which is irrelevant to the oscillation, \( \mathcal{H}_M \) can be rewritten as

\[
\mathcal{H}_M = \frac{\Delta m^2}{4E} \begin{pmatrix}
-(\cos 2\theta - x) & \sin 2\theta + y \\
\sin 2\theta + y & \cos 2\theta - x
\end{pmatrix},
\]

where

\[
x \equiv \frac{(V_{\mu\mu} - V_{\tau\tau})/2}{\Delta m^2/4E}, \quad y \equiv \frac{V_{\mu\tau}}{\Delta m^2/4E}.
\]

Thus one obtains the usual mixing angle and mass-squared difference in the medium given by

\[
\sin^2 2\theta_M = \frac{(\sin 2\theta + y)^2}{(\cos 2\theta - x)^2 + (\sin 2\theta + y)^2},
\]

\[
\Delta m^2_M = \Delta m^2 \sqrt{(\cos 2\theta - x)^2 + (\sin 2\theta + y)^2},
\]

which gives the transition probability in the medium:

\[
P_M(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta_M \sin^2 \left( \frac{\Delta m^2_M L}{4E} \right).
\]

In Fig. 4 we show the change of \( \sin^2 2\theta_M \) for neutrinos in terms of \( E_\nu \) taking only one non-vanishing coupling \( \lambda = \lambda_{\mu\mu} \), and the masses of \( m_{DM} = 30\lambda^{12} \text{meV} \) and \( m_X = 7.7\lambda^{14} \text{keV} \). The solid, dashed, and dash-dotted lines correspond to the DM asymmetry \( \epsilon = -1, 0 \), and 1, respectively. Here we used \( \Delta m^2 = 2.56 \times 10^{-3} \text{eV}^2 \).

\[
\mathcal{H}_{\text{DM}} = \lambda_{\mu\mu} \text{diag}(\Delta m^2)_{ii} \left( \frac{1\text{MeV}}{E_\nu} \right)^{1/3},
\]

\[
\Delta m^2_{\text{DM}} = \frac{\lambda_{\mu\mu}^2}{\rho_{DM}} \left( \frac{1\text{MeV}}{E_\nu} \right)^{1/3},
\]

\[
\sin^2 2\theta_{\text{DM}} = \frac{(\sin 2\theta + y)^2}{(\cos 2\theta - x)^2 + (\sin 2\theta + y)^2},
\]

\[
\Delta m^2_{\text{DM}} = \Delta m^2 \sqrt{(\cos 2\theta - x)^2 + (\sin 2\theta + y)^2},
\]

where

\[
x \equiv \frac{(V_{\mu\mu} - V_{\tau\tau})/2}{\Delta m^2/4E}, \quad y \equiv \frac{V_{\mu\tau}}{\Delta m^2/4E}.
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which gives the transition probability in the medium:

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P_M(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta_M \sin^2 \left( \frac{\Delta m^2_M L}{4E} \right).
\]
for small couplings $|\lambda| \ll 1$. Around the peak energy, the
difference is amplified for non-zero DM asymmetry, while
the effect becomes moderate away from the peak energy.
The grey region is excluded by the measurement of the
$\Delta m^2$ difference between neutrinos and anti-neutrinos [29]

\[
|\Delta m^2_{21} - \Delta m^2_{31}| < 4.7 \times 10^{-5} \text{ eV}^2,
\]
\[
|\Delta m^2_{31} - \Delta m^2_{31}| < 3.7 \times 10^{-4} \text{ eV}^2,
\]
where the second bound is applied in the plot.

In Fig. 6 we show the same plot as Fig. 4 but with $E_{\nu}^{\text{peak}} = 1 \text{ TeV}$:
the solid and dashed lines are for $\lambda_{\mu\mu} \neq 0$ with $x \rightarrow 10$, $y = 0$, and $\lambda_{\mu\mu} = \lambda_{\nu\nu} \neq 0$ with $x \rightarrow 10$, $y \rightarrow 10$ at high energy limit,
respectively.

\[\Delta m^2 = 2.56 \times 10^{-3} \text{ eV}^2,\]
\[m_x = 7.7 \times 10^4 \text{ keV},\]
\[m_{DM} = 3.2 \times 10^6 \text{ meV}.\]

In the former case, the medium potential mimics
the standard oscillation parameters and thus solar and atmospheric
neutrino data might be accounted for even with massless neutrinos.
This “dark matter assisted neutrino oscillation” could be a good alternative to the standard oscillation paradigm if the absolute neutrino mass measured in neutrinoless beta decay, single beta decay or
cosmological observations turns out to be unexpectedly small [43].
In the latter case, ultra-high energy neutrino oscillations are described by the symmetric medium effect, and thus could be totally different from the standard neutrino oscillations which have been confirmed by various experiments at lower energies.

Our formulation brings many interesting questions:
what will be the implications to the standard neutrino
oscillations; how our medium parameters are constrained
by various cosmological and astrophysical observations; and how a low-energy scenario for the dark sector coupling to neutrinos can arise from a UV-completed theory [46].

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