Affecting of the Long-Term Deformation to the Stability of RC Frame-Bracing Structural Systems under Special Accidental Impacts

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Abstract. The paper is devoted to investigation of the loading duration affecting to stability of structural elements of reinforced concrete building frames at post-critical states caused by the sudden removal of one of the load-bearing constructions. Modelling of strength and deformation characteristics of reinforced concrete under long-term loading was performed using the theory of plasticity of concrete and reinforced concrete proposed by G.A. Geniev. Following to this theory, the rheological model of long-term strength of concrete was presented by two elements connected in series, one of which characterizes the process of short-term loading, and the second viscous element corresponds to the Kelvin-Voigt model and takes into account the processes of long-term deforming. It was established that at long-term loads acting to operated reinforced concrete structural system the value of critical force corresponding to loss of stability of the structural element decreases. This fact should be taken into account when assessing the resistance of such building frames to progressive collapse under special accidental impacts caused by removal one of load-bearing elements of a structural system.

1. Introduction
Since 70-s of the last century sustainable growth of a number of papers devoted to problems of protection of buildings and structures against progressive collapse is observed in scientific literature. Accumulated currently experience and knowledge in this field allowed introducing in economically leading countries the regulatory documents [1-7], determining requirements to design of a defense of buildings and structures under special accidental impacts caused by possible sudden failure of a load bearing construction. However a number of theoretical problems still requires advanced investigation. Among such problem there is a problem of stability of operated reinforced concrete frames under hazardous actions.

Stability of structural systems made of a composite material such as reinforced concrete is linked with changes of deformation properties due to physical nonlinearity [8], developing in time creep [8-11] and nonstationary processes of resistance to environment actions [12-14]. In this regard the article is devoted to investigation of affecting of long-term processes to stability of structural elements at post-critical states caused by sudden failure of a load bearing construction of a building.
2. Models and methods
When solving the stability problems of reinforced concrete frames we accepted elastic-plastic model of construction material, assuming that features of deforming and cracks' formation can be described by a single mathematical model based on idealized stress-strain diagrams of a material. Taking in account features deforming features of structures made of an elastic-plastic material the stability equation of an element of a system, in accordance with [15], takes the form:

\[ EJ(z)\frac{d^2\varepsilon}{dt^2} + N\varepsilon = 0 , \]  

(1)

where \( E \) is an initial deformation modulus, \( \nu \) is a deflection in arbitrary section of a rod, \( N \) – axial force, \( J_2 \) – second moment of area of “the second calculation section”. As the “the second calculation section” in accordance with [15] we mean such a reduced section, the general geometric parameters of which is calculated using tangential modulus in each point of real deformed cross section:

\[ J_z = J_o - A_0a_0^2 , \]  

(2)

where

\[ \theta = E_v/E , \quad A_0 = \int_0^\theta d\varepsilon , \quad S_0 = \int_0^\theta \varepsilon dA , \quad a_0 = S_0/A_0 , \quad J_o = \int_0^\theta \varepsilon dA . \]  

(3)

In the formulas (2) and (3) \( E_v \) – tangential modulus, \( \theta \) – relative tangential modulus, \( A_0 \) – area of “the second calculation section”, \( S_0 \) – static moment of “the second calculation section”, \( a_0 \) – distance from the gravity center of “the second calculation section” to gravity center of a real section, \( J_0 \) – second moment of area of “the second calculation section” around the axis plotted through the gravity center of a real section, \( J_z \) - second moment of area of “the second calculation section” around the axis plotted through the gravity center of “the second calculation section”.

It should be noted that “the second calculation section” characterizes resistance of a bar to deflection from reached equilibrium state under infinitely small external actions.

Modeling of changes of strength and deformation properties of a power resistance of reinforced concrete under serial static-dynamic loading of structural elements of a building frame is based on theory of plasticity proposed by G.A. Geniev [16]. Assumed that structural system designed such that it elements are stable under design combination of loads during normal operation conditions. As deformation rheological model of static-dynamic deforming we use model that consists of serially connected elements: quasi-static - 0 characterized by sectorial modulus \( E_{sec,0} \) calculated taking in account creep deformations; and elastic-plastic element - 1, presented by Kelvin-Voigt model (figure 1, a):

\[ \Delta e = \Delta\sigma\left(1-e^{-\frac{t}{\tau}}\right)/E, \]  

(4)

\( \omega = E_{sec,0}/K , \)

where \( E_{sec,0} \) – sectorial modulus at normal operation (figure 1, c), \( K \) – modulus of viscous resistance of an element, \( t \) – duration of dynamic impact to elements of a structural system at sudden structural transformation, \( \Delta\sigma \) and \( \Delta e \) – increment of stresses and strains in a section of structural element under additional dynamic impact.

In this model element \( A_1 \) corresponds to pure elastic work of composite material, and \( B_1 \) - pure plastic one. However, due to the time scale of elements 0 and 1 has perfectly different order, it is more conveniently to carry out calculation of reinforced concrete against progressive collapse using multilevel calculation schemes and simulate long-term and dynamic processes separately.

Correctness of the proposed approach that based on using of multilevel schemes was proved by a number of investigation results, for example [17-19], which established that dynamic effect, caused by sudden structural transformation at removal of a load bearing structural element, comparatively quickly damps with the distance growing from the local damage zone.

At this approach, at the first stage, we carry out calculation of structural system using spatial scheme (primary calculation scheme of the first level) (figure 2, a) and linear formulation that decrease difficulty of calculation and make results more clear. Obtained results we use to choose of potentially
danger zones of possible local failure of considering structural system. In order to carry out advanced assessment of stress-strain state of a reinforced concrete element of a frame it should be considered substructure (a fragment of the building frame) placed in a zone of possible removal of a load-bearing construction (figure 2, b). Calculation results obtained for the first level scheme serve as source data for advanced analysis of deforming and failure of elements of this frame. At the same time in the primary calculation scheme of the second level structural elements should be divided through the length into a few bar elements, which should be simulated taking in account creep, geometric and physical nonlinearity. As geometric nonlinearity we mean affecting of deflection to efforts into elements of a system [8]. Result of such calculation is plot of stress-strain state, which is caused in sections of the calculation model by design combination of loads for special limit state [1] at the stage of normal operation (figure 2, c).

Figure 1. Rheological model of static-dynamic (a) and dynamic (b) resistance, diagram of static-dynamic deforming (c) of a reinforced concrete element under axial compression.

Determination of deformed state of the frame under special accidental impacts is based on quasi-static method [12, 20]. At the same time, we accept rigidities, sections' and nodes' coordinates to bar elements of the calculation model (figure 2, d) taking in account results of deformation calculation for the primary model of the second level. And instead loads acted during normal operational stage, we attach in the place of removed element only generalized effort, which is determined as superposition of efforts acted in removal element of structural system at normal operational stage and attached in opposite direction (figure 3) in accordance with condition of constancy of full deformation energy:

\[
\Phi(\varepsilon'_{n-1}) = \Phi(\varepsilon'_n) = \sigma'_n (\varepsilon'_n - \varepsilon'_s) \quad \sigma'_{n-1} = 2\sigma'_n - \sigma'_s \quad N'_{n-1} = 2N'_n - N'_s,
\]

where \(\sigma'_{n-1}, \sigma'_s, \sigma'_n, \varepsilon'_{n-1}, \varepsilon'_s, \varepsilon'_n, \varepsilon'_n\) – respectively stresses and strains in the n-1 system (with removed restraint) at dynamic (d) and static (s) loading and in the n system at static loading; \(\Phi(\varepsilon'_{n-1}), \Phi(\varepsilon'_n)\) – respectively potential energy of deformation at dynamic and static loading in the systems n-1 and n.
Figure 2. Multilevel calculation schemes for deformation analysis under special accidental impact: primary spatial scheme of the first level (a); primary scheme of the second level (fragment of the building frame) (b); scheme based on results of deformation calculation for primary scheme of the second level (c); secondary scheme of the second level (d).

Stiffness of the rod elements of the fragment of a structural system for long-term deforming stage can be calculated on the base of sectorial deformation modulus $E_{sec}$, obtained using one of the nonlinear calculation methods: the method of elastic variables (MEV) [21], the method of incremental loading (MIL) [22], or integral modulus proposed by V.M. Bondarenko [23]. Work [24] presents
accuracy comparison of MEV and MIL with results of accurate calculation at accepted hypotheses about small deformations and features of relation between moments and curvatures. It shows that difference is 10% for MEV and 2% for MIL at a limited number of iterations. Further we briefly present technique of the method of elastic variables for calculation against considering action type since this method is relatively easy and gives satisfactory results for such problem.

**Figure 3.** Schemes to determination of generalized effort in the place of removed restraint (structural element): stress-strain diagram for axially compressed concrete (a); generalized effort in the place of removed column of the building frame.

At the first stage we carry out calculation of a fragment of the structural system under design combination of loads at initial deformation modulus. Advanced analysis of stress-strain state of the cross sections for the first iteration is based on the obtained values of internal efforts:

$$
\varepsilon_i = \varepsilon_c(t) + \varepsilon_{ij},
$$

where $$\varepsilon_c(t)$$ – creep deformations calculated in accordance with regulatory documents [9] or calculation dependencies from scientific literature [8, 10, 11] for time moment $$t$$;

$$\varepsilon_{ij}$$ – deformations caused by conventionally “instantaneous” attaching of static load and calculated at respective iteration by formula:

$$
\varepsilon_i = \varepsilon_a \pm \varepsilon_c = N_i / (E_i A) \pm M_i h_i (2E_i J).
$$

In the formula (7) $$N_i$$ and $$M_i$$ – respectively axial force and bending moment calculated for i-th iteration in a rod element; $$A$$ and $$J$$ – respectively area and second moment of area; $$h$$ – height of the cross section of the element.

Sectorial modulus for the following iteration is determined from relationship:

$$
E_i = \left[ E_{i-1}^c a + E_{i-1}^a c \right] \left[ 2(a + c) \right],
$$

where $$a$$ and $$c$$ – distance from the neutral axis to the most compressed fiber and the less compressed (the most stretched) fiber respectively (figure 4); $$E_{i-1}^c$$ – sectorial modulus of compressed zone for i-th calculation iteration; $$E_{i-1}^a$$ – the similar modulus for stretched zone:

$$
E_{i-1}^c = \sigma_c / \varepsilon_c, \quad E_{i-1}^a = \sigma_a / \varepsilon_a, \quad \sigma_a = E_i \varepsilon_a - H_i \varepsilon_c^*, \quad \sigma_c = E_i \varepsilon_c - H_i \varepsilon_a^*.
$$

In the (9) formulas for stresses in outer fibers $$\sigma_a$$ and $$\sigma_c$$ correspond to approximation of stress-strain dependencies by polynomials of second order; $$\varepsilon_a$$ and $$\varepsilon_c$$ – deformations in the most and the less compressed fibers; $$H_i$$ – parameter determined from condition of equality to zero of tangential modulus at stresses equal ultimate strength of concrete.
Iteration process should be prolonged until difference between stiffness of elements of the system for both serial iterations doesn't exceed allowable limits.

![Figure 4](image)

**Figure 4.** Diagrams of deformations and stresses in section of a compressed-bent rod element: when there is stretched zone (a); without stretched zone (b).

3. Results and Discussion

Let us write sectorial deformation modulus $E_{\sec,1}$ that corresponds to dynamic loading stage of viscoelastic element 1 taking in account (4) in the form:

$$E_{\sec} = \Delta \sigma / \Delta \varepsilon = E_{\sec,0} / (1 - e^{-\omega t}).$$

(10)

Deformed state of the system under special accidental impact is determined using sectorial modulus $E_{\sec,1}$ as it was mentioned above. Further in accordance with approach described in the paper [15] for calculation of viscoelastic bars, we write expression for stiffness of "the second calculation section" of rod elements in load plane (XOZ):

$$B_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} E_{\sec}(y) y^2 dy = \frac{b \cdot E_0}{(1 - e^{-\omega t})} \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(1 - \frac{H_1}{E_0} \left(\varepsilon - \frac{\varepsilon_0 - \varepsilon}{h} \left(y + \frac{h}{2}\right)\right)\right) y^2 dy.$$  

(11)

Here we accepted the following designations:

$E_{\sec}(y) = E_{\sec,0} / (1 - e^{-\omega t})$ – dynamic tangential modulus,

$E_{\sec} = \frac{d \sigma}{d \varepsilon} = E_0 - 2H_1 \varepsilon_0$ – tangential modulus for deformed state before dynamic loading;

$\varepsilon = \varepsilon_0 + \frac{\varepsilon_0 - \varepsilon}{h} \left(y + \frac{h}{2}\right)$ – fiber deformation (figure 4), obtained for deformed state of structural element taking in account dynamic increment of load.

Integral expression for bending stiffness out of load plane can be written in the form:

$$B_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} E_{\sec}(y) x^2 dy dx.$$  

Therefore, stability analysis of nonlinear deforming elements of reinforced concrete frame under special accidental impact transforms to linear calculation of stability of a frame fragment with step-variable stiffness through the length of the structural elements. These rigidities should be calculated using tangential modulus. The obtained from such calculation value of critical force should be compared with axial forces in corresponding elements for deformed state of the system under dynamic
loading caused by sudden failure a load bearing element of the structural system. The system is sustainable if the following condition is met:

\[ P_{cr,dy} > N_{dy} \]  

(12)

where \( P_{cr,dy} \) – critical force causing buckling of the system, the stiffness of which is determined on the basis of deformation calculation of structural system under accidental impact, \( N_{dy} \) - dynamic axial force in an element of considering structural system.

It should be noted that critical force cannot be calculated using right extrapolation in case if there is some reserve as it was able for pure elastic systems. This linked with feature of system made of viscoelastic material calculation scheme of which permanently changes during loading process. It is also relevant to note that value of critical force during long-term loading permanently decreases. That is caused by decreasing of tangential modulus with stress level growing in cross sections of structural elements and creep deformations development in time.

4. Conclusions

1. In the article it was investigated affecting of long-term loading of reinforced concrete structural systems to stability of its elements in post-critical states caused by sudden removal of a load bearing element of a frame.
2. It is established that value of critical force decreases during long-term loading of operated reinforced concrete structural system.
3. Presented calculation expressions can be applied for assessing stability of reinforced concrete structural systems under static-dynamic deforming caused by sudden accidental failure of a load bearing element of a system.

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