Evaluation of the Work of a Single-Span Beam in the Cracked Section - Calculation Example

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Abstract. In Poland, in the last 20 years there have been large and often far-reaching changes in designing of construction works, including changes in normative regulations. Polish standards were gradually adapted to European eurocodes obligatory in European Union Member States, to be eventually replaced by publication of an identical text or by endorsement. Currently valid in Poland for the design of reinforced concrete structures Eurocode 2, part 1-1 was approved by the Polish Committee for Standardization in 2008 and eventually replaced the conflicting national standards in 2010. According to the information contained in the general provisions, it is intended to prove compliance of buildings and engineering structures with the requirements of Council Directive 89/106/EEC concerning, among other things, resistance and stability, applicable to new, designed structures. Part 1-1 EC2 concerns the design of reinforced concrete structures with the exception of plain surface bars previously used quite commonly in structural elements working under average values of loads: floor slabs or secondary beams for example. On the other hand, in point 7.3.4 of the mentioned code, containing the rules of calculations of crack width, there is a coefficient $k_1$ taking account of the bond properties of the bonded reinforcement, permitting both ribbed and with an effectively plain surface bars. Therefore, in the presented article it was made an attempt to analytically assess the crack phenomenon of the beam with plain surface reinforcement and compare this with the results of experimental research. The calculations were made for a single-span beam with one-side tensile reinforcement only, loaded with two concentrated forces of equal values, placed symmetrically in relation to the half of the span. The presented assessment includes of the cracking moment, calculations of the spacing of cracks and the width of cracks at various load levels. The obtained results were compared with the results of experimental investigations of two beams made of the same concrete and the same rebars.

1. Introduction

Laboratory investigations show that the crack width in reinforced concrete members depends on many factors, among them rheological phenomena, type and duration of load action, reasons for deformation, distribution of reinforcement and its degree [1], shear reinforcement [2], and also adhesion [3]. Access to research means that the calculation methods are still being improved. This is also followed by the normative regulations [4]. Changes in standards also apply to materials and their properties.
In Poland, in the last 20 years there have been large and often far-reaching changes in designing of construction works, including changes in normative regulations. The last amendment of the regulations, consisting in the final entry into use of Eurocode 2, part 1-1 [5] and replacing by it other conflicting national standards took place in 2010. This standard, differing in many points more or less than the standard PN-B-03264: 2002 [6] adapting Polish provisions to the regulations in force in the European Union at the time, contains a provision significantly limiting the possibilities of its application only to new, designed structures. In addition, in point 1.1.2. the use of smooth bars was excluded, which is reflected in the table C.1 in Annex C. Properties of reinforcement. The lower limit of the characteristic yield point of $f_y$ up to 400 MPa does not include previously commonly used smooth steel grades in Poland as well as some types of ribbed steel. On the other hand, in point 7.3.4. the above mentioned standard has a coefficient $k_p$, depending on the reinforcement, permitting both ribbed and smooth rebars. This prompted the authors of the article to perform calculations of the limit state of cracking according to PN-EN 1992-1-1: 2008 [5] for a beam with smooth bars and an average reinforcement ratio despite the limitations of the standard discussed above, discussing them and comparing results with experimental results. These activities are based on the similarities between the procedures contained in PN-EN 1992-1-1: 2008 [5] and PN-B-03264: 2002 [6] based on the earlier version of the Eurocode and permitting the use of smooth rebar.

The standards mentioned above, which for simplicity will be referred to as PN-EN and PN-B, provide the following formulas for calculating the crack width:

\[
\begin{align*}
    \text{PN-EN} & \quad w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) \\
    \text{PN-B} & \quad w_k = \beta \cdot s_{rm} \cdot \varepsilon_{sm}
\end{align*}
\]

where:

\begin{itemize}
    \item $w_k$ – the maximum crack width,
    \item $s_{r,max}$ – the maximum crack spacing,
    \item $s_{sm}$ – the average crack spacing,
    \item $\beta$ - the coefficient expressing the ratio of the calculated crack width to the average width,
    \item $\varepsilon_{cm}$ - the mean strain in the concrete between cracks,
    \item $\varepsilon_{sm}$ - the mean strain in the reinforcement between cracks, taking into account the effects of tension stiffening.
\end{itemize}

Considering the significance of $\beta$ coefficient, it is stated that in both cases one of the factors taken into account is the maximum crack spacing. In addition, in the derivation of the formulas in [7], it is shown that the strain $\varepsilon_{sm}$ in the formula (2) is the difference between the average strain reinforcement and medium deformation of concrete in the section between two cracks. As it turns out, in principle both formulas are the same.

The cracks spacing is calculated in PN-EN from one of the two formulas chosen according to the distance between bars. In this work the formula (7.11) was used, and with PN-B the formula (113):

\[
\begin{align*}
    \text{PN-EN} & \quad s_{r,max} = 3,4 \cdot c + k_1 \cdot k_2 \cdot 0,425 \cdot \frac{\phi}{\rho_{p,eff}} \\
    \text{PN-B} & \quad s_{rm} = 50 + 0,25 \cdot k_1 \cdot k_2 \cdot \frac{\phi}{\rho_r}
\end{align*}
\]

where:

\begin{itemize}
    \item $c$ – the cover,
    \item $\rho_{p,eff}, \rho_r$ – the effective reinforcement ratio,
    \item $k_1$ – the coefficient which takes account of the bond properties,
    \item $k_2$ – the coefficient which takes account of the distribution of strain.
\end{itemize}

In both of the above formulas, the spacing of cracks depends on the bond conditions of concrete to reinforcing steel and the value of coefficient $k_1$ is equal to 0.8 for ribbed bars and 1.6 for smooth ones.
in both standards. This means that the influence of bond on cracking phenomenon is the same in both standards.

In the light of the arguments given, the PN-EN standard seems to be adapted to calculate the scratching of elements reinforced with plain rebar.

2. Description of reinforced concrete beams and their loading

The calculations of cracking were made for a beam with the same characteristic as two laboratory members had. They were made with the same batch of concrete and the same batch of reinforcement. The beams had a rectangular cross-section of 0.12 × 0.30 m and a total length of 3 m. They were supported on pins at a distance of 0.175 m from the ends, that gave an effective length of 2.65 m. Beams were loaded with two equal concentrated forces moved away from the supports by 0.825 m. This resulted in a zone of permanent bending moment, with a length of 1 m.

The average compressive strength of concrete tested on cubes of 150mm was f_c = 18.34 MPa, and the average modulus of elasticity E_c = 24.68 GPa. The tests were carried out on the 28th day after concreting of the samples.

The calculated average tensile strength of concrete was f_{ct} = 1.063 MPa.

The reinforcement was made of steel with an average yield limit of f_y = 325.2 MPa. Reinforcement was made up of 3 bars φ16 and compression reinforcement of 2 bars φ8. The thickness of the cover was c = 15 mm. Stirrups were made of φ6 bars. A spacing of 0.1 m between the support and the point of load and 0.225 m in the central zone was used.

The beams were loaded with forces of 10 to 60 kN with an increase of 10 kN, followed by a complete unloading, and then forces applied again 60 kN and the load continued to 80 kN also with a the increase of 10 kN.

3. Calculations and test results

3.1. Calculations of cracks according to PN-EN

Calculations of cracking moment were made taking into account both tensile and compression reinforcement. Its value was M_{cr} = 3.30 kNm. Calculations of cracks width were made for values of bending moments for which appearing cracks were registered and their widths were measured. Detailed results of calculations are presented in table 1.

| M [kNm] | \( \sigma_s \) [MPa] | \( \sigma_{cr} \frac{E_m}{E_m} \times 10^{-4} \) | \( s_{t,\text{max}} \) [mm] | \( w_k \) [mm·10^{-2}] |
|--------|-----------------|-----------------|-----------------|-----------------|
| 13.13  | 90.67           | 3.91            | 113             | 4.4             |
| 17.50  | 120.89          | 5.40            | 113             | 6.1             |
| 21.88  | 151.12          | 6.93            | 113             | 7.9             |
| 26.25  | 181.35          | 8.44            | 113             | 9.6             |
| 26.25\*| 181.35          | 8.44            | 113             | 9.6             |
| 30.63  | 211.57          | 9.96            | 113             | 11.3            |
| 35.00  | 241.79          | 11.47           | 113             | 13.0            |

\* after unloading.

The factor k_s is equal to 0.6 as for short-term load. The depth of the compression zone was constant, of the value of x = 0.111 m.

3.2. Tests results

The tests were carried out the 28th (beam B1) and the 29th (beam B2) day after concreting the beams. During the tests, the values of the load, the order of the appearance of cracks and their widths were measured at the level of reinforcement. Cracks were registered on both sides of the beam. In the tables below cracks are distinguished as left-sided and right-sided.
First cracks were registered at the bending moment of a similar value, i.e. of 12.5 kNm for the B1 beam and 13 kNm for the B2 beam, which gives an average bending moment equal to 12.75 kNm. The widths of the cracks are listed in table 2. for the beam B1 and in table 3. for the beam B2. Tables contain measuring results only for cracks that appeared in the zone of constant bending moment. The cracks numbering on the left (L) and on the right (R) is independent.

| M side | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 12 | 13 | 16 | 17 | 20 | 21 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| 13.13  | L | 10| 7 | - | - | 5 | 5 | 5 | 2 | -  | -  | -  | -  | -  | -  | -  |
|        | R | 5 | - | 5 | 6 | - | - | - | - | -  | -  | -  | -  | -  | -  |
| 17.50  | L | 50| 10| - | - | 5 | 5 | 10| 5 | -  | -  | -  | -  | -  | -  | -  |
|        | R | 10| - | 5 | 8 | - | - | - | - | -  | -  | -  | -  | -  | -  |
| 21.88  | L | 10| 10| - | - | 5 | 10| 5 | - | 4  | -  | -  | -  | -  | -  | -  |
|        | R | 10| - | 10| 10| - | - | - | - | -  | -  | -  | -  | -  | -  |
| 26.25  | L | 10| 10| - | - | 7 | 10| 12| 10 | 7  | -  | -  | -  | -  | -  | -  |
|        | R | 9 | - | 10| 15| - | - | - | 3 | 6  | -  | 4  | -  | -  | -  | -  |
| 26.25a | L | 10| 10| - | - | 8 | 10| 12| 10 | -  | 3  | -  | -  | -  | -  | -  |
|        | R | 5 | - | 5 | 15| - | - | - | 3 | 12 | -  | 4  | -  | -  | -  | -  |
| 30.63  | L | 8 | 10| - | - | 10| 12| 10 | 8  | 5  | 5  | 2  | -  | -  | -  | -  |
|        | R | 10| - | 10| 15| - | - | - | 10| 10 | -  | 5  | 5  | -  | -  | -  |
| 35.00  | L | 15| 20| - | - | 10| 17| 20| 15 | 7  | 5  | 7  | 5  | -  | -  | -  |
|        | R | 10| - | 10| 17| - | - | - | 10| 9  | 5  | 5  | -  | -  | -  | -  |

* after unloading.

| M side | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 11 | 12 | 13 | 14 | 17 |
|--------|---|---|---|---|---|---|---|---|----|----|----|----|----|
| 13.13  | L | 0 | - | - | - | - | - | - | -  | -  | -  | -  | -  |
|        | R | 0 | - | - | - | - | - | - | -  | -  | -  | -  | -  |
| 17.50  | L | 0 | 0 | 0 | 0 | 2 | 3 | - | -  | -  | -  | -  | -  |
|        | R | 0 | 5 | - | - | - | 5 | - | -  | -  | -  | -  | -  |
| 21.88  | L | 5 | 0 | 7 | 4 | 4 | 5 | - | -  | -  | -  | -  | -  |
|        | R | 7 | 2 | - | - | - | 7 | 6 | -  | -  | -  | -  | -  |
| 26.25  | L | 5 | 6 | 10| 9 | 6 | 6 | 0 | -  | -  | -  | -  | -  |
|        | R | 10| 5 | - | - | - | 10 | 7 | 5  | 10 | 5  | 10 | 5  |
| 26.25a | L | 6 | 0 | 5 | 5 | 4 | 5 | 3 | 0  | -  | -  | -  | -  |
|        | R | 5 | 5 | - | - | - | 10 | 7 | 5  | 12 | 9  | 10 | 6  |
| 30.63  | L | 7 | 4 | 5 | 5 | 5 | 5 | 7 | 5  | -  | -  | -  | -  |
|        | R | 7 | 1 | - | - | - | 10 | 7 | 8  | 12 | 8  | 12 | 1  |
| 35.00  | L | 8 | 7 | 5 | 6 | 8 | 6 | 7 | 5  | -  | -  | -  | -  |
|        | R | 5 | 2 | - | - | - | 10 | 7 | 8  | 12 | 10 | 20 | 8  |

* after unloading.

The "-" sign means no cracks, the "0" sign means the appearance of crack with an unmeasurable width.

The number of cracks throughout the crack phase was higher on the B1 beam. Also the widths of the average cracks were larger on this beam, which is clearly visible in table 4. It contains the widths and the spacing of the cracks representative for each beam and the values averaged for both beams.
together. Average values were calculated as weighted averages with the weight including the number of cracks on each beam.

In the beam B1, more than half of the cracks were formed before reaching the bending moment with a value of $M = 17.50$ kNm, and their widths were greater than the width of cracks formed later. It can be assumed in this case that the crack stabilization state occurred. In the B2 beam, 9 of 17 cracks were created before reaching the moment of the mentioned value, but after unloading and re-loading to $M = 26.25$ kNm, the largest widths reached the cracks created at the latest, i.e. directly before loading.

| Table 4. Average crack width and maximum spacing |
|---------------------|---------------------|---------------------|
| M [kNm] | w \[mm \times 10^{-2}\] | s │ w \[mm \times 10^{-2}\] | s │ w \[mm \times 10^{-2}\] | s │ Average \[mm\] |
|-----------|---------------------|-----|---------------------|-----|---------------------|-----|---------------------|
| 13.13     | 5.6                 | 182 | 0                   | 500 | 4.5                 | 267 |
| 17.50     | 7.6                 | 182 | 1.7                 | 182 | 4.6                 | 182 |
| 21.88     | 8.9                 | 167 | 4.7                 | 167 | 6.5                 | 160 |
| 26.25     | 8.7                 | 143 | 6.8                 | 111 | 7.9                 | 125 |
| 26.25*    | 8.2                 | 133 | 5.7                 | 105 | 6.8                 | 118 |
| 30.63     | 8.4                 | 111 | 6.3                 | 105 | 7.3                 | 108 |
| 35.00     | 10.7                | 100 | 7.9                 | 105 | 10.2                | 103 |

* after unloading.

In the case of both beams, it can be noticed that the width of some cracks after unloading and re-applying the load giving the bending moment of $M = 26.25$ kNm decreased, which is, among others, the effect of steel strain hardening. At the same time, the number of cracks increased. The larger average drop in the width of the cracks was recorded on the B2 beam. This effect is obviously also visible in the average crack width.

4. Discussions

The basis for the calculation of limit states according to EC2, and thus according to PN-EN, is the assumption about the linear elastic work of concrete and steel in both phase I and II. Consequently, in phase II, the shape of the stress mass in the compression zone is triangular. It is also assumed that the height of the compression zone is constant in this phase, depending on the area of reinforcement and geometry of the cross-section. The stress increments in the tensile reinforcement depend therefore exclusively and linearly on the load increments, and the stress value is underestimated due to overestimation of the internal force arm. In fact, at the advanced crack stage, the height of the compression zone decreases, and the stress figure describes the curvilinear function, which in the final stage before destruction takes the shape of a rectangle.

As a result, in the example presented at point 3.1, at a bending moment of $M = 35.00$ kNm close to the actual breaking moment stress in the reinforcement were calculated with a value definitely below the average yield point. The calculated height of the compression zone was 0.111 m, while in the given range of bending moments the actual mean height of the compression zone for both beams, given in table 4 took values from 0.138 to 0.152 m.

In the standard calculation procedure, it is assumed that with loads slightly exceeding the cracking load, scratching stabilizes. New cracks are not created, and only the deformation increases in both the cracks and between them. This assumption should be treated as a simplification of the complicated and still not fully understood process of cracking, because, as shown by the researched studies, one can observe the phenomenon of stabilizing the number of cracks, but at some stage of the load their number may increase. It may occur as a result of a new impulse, which may be - as in the case of presented studies - unloading. As is also clear from these tests, the new cracks may be narrower than before as on the B1 beam, or more wider as on the B2 beam.
Calculated crack widths vary from $4.4 \times 10^{-2}$ mm to $13 \times 10^{-2}$ mm, and mean for both beams from $4.5 \times 10^{-2}$ mm to $10.2 \times 10^{-2}$ mm as shown in table 4. Taking into account the imperfection of computational methods and the large spread of results obtained on the tested beams, the values given above are characterized by high compliance. Significant differences, especially directly after cracking, can be seen between the calculated constant spacing of cracks $s_{cm} = 0.113$ m and the average spacing of cracks on each beam, or finally averaged for both beams together. In the latter case, the spacings $s_r$ listed in table 4 are from 0.267 m to 0.103 m.

5. Conclusions

The following conclusions emerge from the presented analysis, the results of calculations and tests and their comparisons:

1. The computational cracking moment is several times smaller than the real one.
2. The estimation in the form of the calculated value of stresses in the tensile reinforcement in the crack phase is clearly understated.
3. The calculated height of the compression zone is significantly different from the experimental values.
4. The assumption about the stabilization of the development of cracks turned out to be wrong.
5. The estimation of the crack opening width is sufficiently consistent with the test results.

References

[1] A.W. Beeby, “The influence of parameter $\Phi/\rho_{eff}$ on crack widths”, Structural Concrete, Vol. 5, No.2, pp. 71-83, 2004.
[2] A. Pérez Caldentey, H. Corres Peiretti, A. Giraldo Soto, J. Peset Iribarren, “Cracking of RC members revisited: Influence of cover, $\Phi/\rho_s\rho_{eff}$ and stirrup spacing - An experimental and theoretical study”, Structural Concrete, Vol. 14, pp. 69-78, 2013.
[3] Y. Goto, “Cracks formed in concrete around deformed bars in concrete”, ACI Jl, Vol. 68, No. 2, pp. 244-251, 1971.
[4] P. Koteš and J. Vičan, “Reliability-based evaluation of existing concrete bridges in Slovakia according to Eurocodes”, 4th International fib Congress 2014: Improving Performance of Concrete Structures, FIB 2014 - Proceedings2014, pp. 227-229, 2014.
[5] Polish standard: PN-EN 1992-1-1:2008 “Eurokod 2. Projektowanie konstrukcji z betonu. Część 1-1: Reguły ogólne i reguły dla budynków”.
[6] Polish standard: PN-B-03264:2002 “Konstrukcje betonowe, żelbetowe i sprężone. Obliczenia statyczne i projektowanie”, PKN.
[7] Praca zbiorowa pod red. Michała Knauffa, “Podstawy projektowania konstrukcji żelbetowych i sprężonych według Eurokodu 2”, Tom 2, Warszawa 1997.