Using the Program "MatLab" to Solve the Problems of Mechanics: Determining the Calculated Values of Internal Forces in a Statically Indeterminate Continuous Beam from the Actions of Constant Stationary and Temporary Mobile Loads by the Matrix Method

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Abstract. This article discusses the using of the MATLAB program to solve problems of mechanics. Analyzing the calculated values of internal forces in a statically indeterminate continuous beam from the actions of constant quiescent and temporary moving loads by the matrix method. The main algorithms for solving problems using MatLab program and graphical design of tasks are given.

1. Introduction

The calculated values of internal forces in the cross-sections of the elements under the action of a constant quiescent and temporary moving loads are determined by the formula:

\[ S_k^{\text{max}} = S_k^{\text{const}} + \sum S_k^{\text{temp}} > 0 \]
\[ S_k^{\text{min}} = S_k^{\text{const}} + \sum S_k^{\text{temp}} < 0 \]

and from these calculated values, their enveloping diagrams are constructed.

When calculating a statically indeterminate continuous beam, these formulas will have the form:

\[ M_k^{\text{max}} = M_k^{\text{const}} + \sum M_k^{\text{temp}} > 0 \]
\[ M_k^{\text{min}} = M_k^{\text{const}} + \sum M_k^{\text{temp}} < 0 \]

In the analytical calculation of a continuous beam, the values of the support moments \( M_i \) from the action of a constant quiescent load are determined from the solution of the system of equations of three moments:

\[ M_{i-1} \cdot l'_{i-1} + 2 \cdot M_i \cdot (l'_i + l'_{i+1}) + M_{i+1} \cdot l'_{i+1} = -6 \cdot \left( \frac{R_i \cdot a_i}{l_i^2} \cdot l'_i + \frac{R_{i+1} \cdot b_{i+1}}{l_{i+1}^2} \cdot l'_{i+1} \right) \]

The values of the bending moments in any cross-section of each span of the beam’s main system are determined by:
and from these calculated values the diagram of the bending moments from the action of a constant quiescent load is constructed.

The calculation from the action of a temporary quiescent load, which can have any position with any discontinuities, is made by the method of focal relations. The focal relations of the beams on each span are determined by:

\[ k_i = 2 + \frac{l_i-1}{l_i} \left( 2 - \frac{1}{k_{i-1}} \right) \]  - left focal relations

\[ k'_i = 2 + \frac{l_i+1}{l_i} \left( 2 - \frac{1}{k_{i+1}} \right) \]  - right focal relations

The support moments in each loaded span of the beam from the action of the temporary moving load are determined by:

\[ M_{i\text{left}} = M_{i-1} - \frac{6 \cdot \Omega_{pi}}{l_i^2} \cdot \frac{b_i \cdot k'_i - a_i}{k'_i \cdot k_i - 1} \]

\[ M_{i\text{right}} = M_i - \frac{6 \cdot \Omega_{pi}}{l_i^2} \cdot \frac{a_i \cdot k'_i - b_i}{k'_i \cdot k_i - 1} \]

The support moments in the unloaded spans are determined by:

\[ M_{i-1} = \frac{M_i}{k_i} \]

\[ M_{i+1} = \frac{M_i}{k_{i+1}} \]

The calculation of continuous beams using this method is quite laborious. Therefore, the solution of such problems is proposed by the matrix method, using the Maxwell-Mohr’s integral in solving the Simpson method.

2. Methodology

When solving a statically indeterminate continuous beam, the basic system and the resolving equations of calculation are assumed to be the same as the method of equations of three moments. The basic system is obtained by introducing complete hinges in all the support sections of the beam and replacing them with the support moments \( M_i \).

In this case, the resolving equations will have the form:

\[ \theta_i = \theta_{i-1} + \theta_{i+1} = 0 \]

The solution of these equations is carried out by the Maxwell-Mohr’s integral

\[ \theta_i = \sum_{0}^{l_i} \frac{M_i \cdot M_{xp}}{EI} \, dz \]

When composing matrices, these integral equations are solved by Simpson’s numerical method

\[ \theta_i = \sum_{0}^{l_i} \frac{M_i \cdot M_{xp}}{EI} \, dz = \sum_{0}^{l_i} \left[ \frac{l}{6EI} (M_{i}^h \cdot M_{xp}^h + 4M^e_i \cdot M_{xp}^e + M_{i}^k \cdot M_{xp}^k) \right]_n \]

Then we obtain the matrix equation

\[ L_{\bar{M}_x} = L_{\bar{M}_p} + L_{\bar{m}} \cdot \left[ \left( I_{m}^T \cdot D_g \cdot L_{m}^T \right)^{-1} \cdot \left( I_{m}^T \cdot D_g \cdot L_{m}^T \right) \right] \overset{\text{max}}{\rightarrow} \left[ \bar{M}_{Mx}^{\text{pct}} \vdots \bar{M}_{Mx}^{\text{p2}} \vdots \bar{M}_{Mx}^{\text{p3n}} \right] \]

\[ = \left[ \bar{M}_{Mx}^{\text{max}} \bar{M}_{Mx}^{\text{min}} \right] \]

Where:

\[ L_{\bar{M}_p} = \left[ \bar{M}_{Mx}^{\text{pct}} \vdots \bar{M}_{Mx}^{\text{p2}} \vdots \bar{M}_{Mx}^{\text{p3n}} \right] - \text{matrix obtained from vectors of bending moments} \]
from actions of constant and temporary loads, which occupies each span of the main beam system separately;
$L_{m} = \begin{bmatrix} L_{m1} & L_{m2} & ... & L_{mn} \end{bmatrix}$ – matrix composed of vectors of bending moments obtained from the action of single moments in the main beam system;
$D_{x}$ – quasi-diagonal stiffness matrix of a beam;
$\begin{bmatrix} M_{x}^{\text{const1}} & M_{x}^{\text{temp2}} & ... & M_{x}^{\text{tempn}} \end{bmatrix}$ – matrix from the vectors of the final diagrams of the bending moments from the actions of constant quiescent and temporary moving loads;
$\begin{bmatrix} M_{x}^{\text{max}} & M_{x}^{\text{min}} \end{bmatrix}$ – matrix from the vectors of the calculated values of the bending moments, on which the enveloping (calculated) diagram $M_{x}$ is constructed.

From the values of the support moments for each individual loading, the values of the shearing forces in any cross-section of the beam of the main system are obtained from

$Q_{x}(z) = Q_{x}^{0}(z) - M_{x-1} \frac{1}{l_{i}} + M_{x} \frac{1}{l_{j}}$

and we obtain an analogous matrix of the calculated values of shearing forces

$L_{Q_{y}} = \begin{bmatrix} Q_{y}^{\text{const}} & Q_{y}^{\text{temp1}} & Q_{y}^{\text{temp2}} & ... & Q_{y}^{\text{tempn}} \end{bmatrix} = \begin{bmatrix} Q_{y}^{\text{max}} & Q_{y}^{\text{min}} \end{bmatrix}$

The calculated values of the support reactions will be from the excision of the support nodes

$R_{i} = Q_{yi-1}^{\text{min}} + Q_{yi+1}^{\text{max}}$

Example: When the main loads on the beam are represented: a constant load with intensity $q_{\text{const}} = 4$ kN/m, temporary moving loads with intensity $q_{\text{temp}} = q_{\text{eq}} = 6$ kN/m and a beam with spans $l_{1} = 4$ m, $l_{2} = 10$ m, $l_{3} = 6$ m, $l_{4} = 8$ m, stiffness $EI_{x} = \text{constant}$.

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**Figure 1.** Initial scheme

**Figure 2.** Diagrams from a single loading
Figure 3. Diagrams of bending moments from the actions of constant and moving loads when each span is loaded separately of the main beam system.

Create m fail:

\[ l_1 = 4; l_2 = 10; l_3 = 6; l_4 = 8; \]
\[ p = 4; q = 6; E = 2 \times 10^8; I_x = 2000 \times 10^{-8}; \]
\[ \mathbf{L}_m = \begin{bmatrix} 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 & 0 \end{bmatrix}; \]
\[ \mathbf{L}_{mp} = \begin{bmatrix} 0 & p \times l_1^2 / 8 & 0 & p \times l_2^2 / 8 & 0 & p \times l_3^2 / 8 & 0 & q \times l_1^2 / 8 & 0 & 0 & 0 & 0 \end{bmatrix}; \]
\[ \mathbf{D} = \begin{bmatrix} 1 & 0 & 0; 0 & 4 & 0; 0 & 0 & 1 \end{bmatrix}; \]
\[ \mathbf{O} = \begin{bmatrix} 0 & 0 & 0; 0 & 0 & 0; 0 & 0 & 0 \end{bmatrix}; \]
\[ \mathbf{D} = \mathbf{D}(1/(6 \times E \times I_x)) \mathbf{D}; \]
\[ \mathbf{D} = \mathbf{D}(2/(6 \times E \times I_x)) \mathbf{D}; \]
\[ \mathbf{D} = \mathbf{D}(3/(6 \times E \times I_x)) \mathbf{D}; \]
\[ \mathbf{D} = \mathbf{D}(4/(6 \times E \times I_x)) \mathbf{D}; \]
\[ \mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0; 0 & 1 & 0 & 0; 0 & 0 & 1 \end{bmatrix}; \]
\[ \mathbf{L}_{mp} = \begin{bmatrix} 0 & p \times l_1^2 / 8 & 0 & p \times l_2^2 / 8 & 0 & p \times l_3^2 / 8 & 0 & q \times l_1^2 / 8 & 0 & 0 & 0 \end{bmatrix}; \]
\[ \mathbf{L}_{mx} = \mathbf{L}_{mp} + \mathbf{L}_m \times \mathbf{D} = \mathbf{L}_m \times \mathbf{D}; \]
\[ \mathbf{L}_{mx} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -101.9211 & 10.0602 & -20.4417 & 1.6748 & -1.0827 \end{bmatrix}; \]
\[ \mathbf{L}_{mx} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -219.8421 & -3.8797 & -40.8835 & 3.3496 & -2.1654 \end{bmatrix}; \]
-219.8421 -3.8797 -40.8835 3.3496 -2.1654
194.6579 -1.3083 36.7951 -3.0147 1.9489
-190.8421 1.2632 -35.5263 -9.3789 6.0632
-190.8421 1.2632 -35.5263 -9.3789 6.0632
-69.9737 0.4962 -13.9568 17.5297 -11.3323
14.8947 -0.2707 7.6128 -9.5617 -28.7278
14.8947 -0.2707 7.6128 -9.5617 -28.7278
39.4474 -0.1353 3.8064 -4.7808 33.6361

\[ L_{max} = \]
\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
-90.1861 & -123.4455 & & & & \\
-216.4925 & -266.7707 & & & & \\
-216.4925 & -266.7707 & & & & \\
233.4019 & 190.3350 & & & & \\
-183.5158 & -235.7474 & & & & \\
-183.5158 & -235.7474 & & & & \\
-51.9477 & -95.2628 & & & & \\
22.5075 & -23.6654 & & & & \\
22.5075 & -23.6654 & & & & \\
76.8898 & 34.5312 & & & & \\
0 & 0 & & & & \\
\end{array}
\]

\[ \]

**Figure 4.** Enveloping (calculated) diagram $M_x$

### 3. Conclusion

From the example you can see the solution of the technical problem with the help of the MatLab program are simplified. The practical significance of the research is that the application of the program "MATLAB" for the solution of problems of mechanics", with the use of information technologies, is realized in the practice of professional training of bachelors of a technical university (engineering-technical institute of the North-Eastern Federal University).

### References

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