Modules Over Color Hom-Poisson Algebras

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Abstract

In this paper we introduce color Hom-Poisson algebras and show that every color Hom-associative algebra has a non-commutative Hom-Poisson algebra structure in which the Hom-Poisson bracket is the commutator bracket. Then we show that color Poisson algebras (respectively morphism of color Poisson algebras) turn to color Hom-Poisson algebras (respectively morphism of Color Hom-Poisson algebras) by twisting the color Poisson structure. Next we prove that modules over color Hom–associative algebras extend to modules over the color Hom-Lie algebras L(A), where L(A) is the color Hom-Lie algebra associated to the color Hom-associative algebra A. Moreover, by twisting a color Hom-Poisson module structure map by a color Hom-Poisson algebra endomorphism, we get another one.

Keywords: Color hom-associative algebras; Color hom-Lie algebras; Homomorphism; Formal deformation; Hom-modules; Modules over color Hom-Lie algebras; Modules over color Hom-Poisson algebras

Introduction

Color Hom-Poisson algebras are generalizations of Hom-Poisson algebras introduced in [1], where they emerged naturally in the study of 1-parameter formal deformations of commutative Hom-associative algebras. Color Hom-Poisson algebras generalizate, on the one hand, color Hom-associative [2,3] and color Hom-Lie algebras [2,3] which have been recently investigated by various authors. On the other hand, they generalize Hom-Lie superalgebras [4]. These structures are well-known to physicists and to mathematicians studying differential geometry and homotopy theory. The cohomology theory of Lie superalgebras [5] has been generalized to the cohomology of Hom-Lie superalgebras in [6]. A cohomology theory of Lie algebras was introduced and investigated in [7], and the representations of color Lie algebras were explicitly described in [8]. Modules over Poisson algebras receive various definitions [9,10] we will use theorems introduced in [9]. The aim of this paper is to study color Hom-Poisson algebras and modules over color Hom-Poisson algebras. The paper is organized as follows. In section 4, we recall some basic notions related to color Hom-associative algebras and color Hom-Lie algebras. In section 5, we define color Hom-Poisson algebras and point out that any color Hom-associative algebra one can associate a color Hom-Poisson algebra. Next, starting from a color Poisson algebra and color Poisson algebra morphism we get another one by twisting the associative product and Lie bracket. In section 6, we introduce modules over color Hom-Lie algebras and prove that starting from a color Hom-Poisson module we get another one by twisting the module structure map by a color Hom-Poisson algebra endomorphism. All vector spaces considered are supposed to be over fields of characteristics different from 2.

Preliminaries

Let G be an abelian group. A vector space V is said to be a G-graded if, there exist a family \((V_a)_{a \in G}\) of vector subspaces of V such that

\[ V = \bigoplus_{a \in G} V_a \]

An element \(v \in V\) is said to be homogeneous of degree \(a \in G\) if \(v \in V_a\). We denote \(H(V)\) the set of all homogeneous elements in \(V\). Let \(V = \bigoplus_{a \in G} V_a\) and \(V' = \bigoplus_{a \in G} V'_a\) be two G-graded vector spaces. A linear mapping \(f : V \to V'\) is said to be homogeneous of degree b if \(f(V_a) \subseteq V'_{a+b}\) for all \(a \in G\).

If \(f\) is homogeneous of degree zero i.e. \((f(V_a) \subseteq V'_{a})\) holds for any \(a \in G\) then \(f\) is said to be even.

An algebra \((A, \mu, \varepsilon)\) is said to be G-graded if its underlying vector space is G-graded i.e. \(A = \bigoplus_{a \in G} A_a\) and if furthermore \(\mu(A_a, A_b) \subseteq A_{a+b}\) for all \(a, b \in G\).

Let \(A'\) be another G-graded algebra. A morphism \(f : A \to A'\) of G-graded algebras is by definition an algebra morphism from A to \(A'\) which is, in addition an even mapping.

Definition

Let G be an abelian group. A map \(\varepsilon : G \times G \to K^*\) is called a skew-symmetric bicharacter on G if the following identities hold,

1. \(\varepsilon(a, b) = \varepsilon(b, a) = 1\)
2. \(\varepsilon(a, b + c) = \varepsilon(a, b) \varepsilon(a, c)\)
3. \(\varepsilon(a + b, c) = \varepsilon(a, c) \varepsilon(b, c)\)

Remark that \(\varepsilon(0, a) = \varepsilon(a, 0) = 1\) for all \(a \in G\).

Where, 0 is the identity of G. If x and y are two homogeneous elements of degree a and b respectively and \(\varepsilon\) is a skew-symmetric bicharacter, then we shorten the notation by writing \(\varepsilon(x, y)\) instead of \(\varepsilon(a, b)\).

Definition

A color Hom-associative algebra is a quadruple \((A, \mu, \varepsilon, a)\) consisting of a G-graded vector space A, an even bilinear map \(\mu : A \times A \to A\) and an even linear map such \(a : A \to A\) that

\[ a \mu = \mu a \]

\[ a(a(x) y) = \varepsilon(a, b) a(x) \varepsilon(a, b) y \]

\[ a(x y) = \varepsilon(a, b) a(x) \varepsilon(a, b) y \]

\[ a(x) a(y) = \varepsilon(a, b) a(x) \varepsilon(a, b) y \]

\[ a(a(x)) = \varepsilon(a, b) a(x) \varepsilon(a, b) \]

\[ a(a(x) y) = \varepsilon(a, b) a(x) \varepsilon(a, b) y \]

\[ a(x y) = \varepsilon(a, b) a(x) \varepsilon(a, b) y \]

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\[ a(a(x) y) = \varepsilon(a, b) a(x) \varepsilon(a, b) y \]

\[ a(x y) = \varepsilon(a, b) a(x) \varepsilon(a, b) y \]

\[ a(a(x)) = \varepsilon(a, b) a(x) \varepsilon(a, b) \]
where we have, an even bilinear bracket (7)

\( l \) any is said to be a morphism of color such that (i.e \( a \) bicharacter, and be an even algebra and (Multiplicativity)                                (4)

(8) for all \( \text{when } A \text{ is a color-Hom-associative algebra.} \)

means cyclic summation.

is a \( is a color Hom-Lie algebra, denoted (6)

\( \text{is a color Hom-Lie algebra, } (A, \mu, \varepsilon, \alpha) \) then \( \) it can be written equivalently

\( \text{as } x \in A. \)

Lemma

((17)) Let \( (A, \mu, \varepsilon) \) be a color associative algebra and \( \alpha \) an even linear map \( f: A \to A' \) is said to be a \( \text{morphism of color Hom-associative algebras if } \) \( f \circ \alpha = \alpha' \circ f \)

For all \( x, y \in A. \)

Definition

((17)) A color Hom-Lie algebra is a quadruple \((A, \{.,.\}, \varepsilon, \alpha)\)

\( \text{is also a morphism of color Hom-associative algebras.} \)

Lemma

((17)) Let \((A, \mu, \varepsilon, \alpha)\) be a color Hom-associative algebra and \( \alpha \) an even linear map \( f: A \to A' \) such that \( f \circ \alpha = \alpha' \circ f \) and \( f(\mu(x, y)) = \mu'(f(x), f(y)) \)

\( \text{for all } x, y \in A. \)

Theorem

According to Lemma 4.2, it remains to prove the color Hom-Leibniz identity 7. For any \( x, y, z \in H(A) \)

\[ \{\alpha(x), \mu(y, z)\} = \mu(\alpha(x), \mu(y, z)) - \mu(\mu(y, z), \alpha(x)) \]

\( \text{which color Hom-associative product is replaced by the color Hom-Lie bracket. Examples of color Hom-Lie algebras are provided in [2,3].} \)

The following lemma connects color Hom-associative algebras to color Hom-Lie algebras.

Lemma

((17)) Let \((A, \mu, \varepsilon, \alpha)\) be a color Hom-associative algebra.

Then \((A, \{.,.\} = \mu - \varepsilon(\mu), \varepsilon, \alpha)\) is a color Hom-Lie algebra, denoted by \( L(A) \).

Color Hom-Poisson algebras

Definition

A color Hom-Poisson algebra consists of a G-graded vector space \( A, \) a multiplication \( \mu : A \times A \to A, \) an even bilinear bracket \( \{.,.\} : A \times A \to A \) and an even linear map \( \alpha : A \to A \) such that

1. \((A, \mu, \varepsilon, \alpha)\) is a color Hom-associative algebra,
2. \((A, \{.,.\}, \varepsilon, \alpha)\) is a color Hom-Lie algebra,
3. the color Hom-Leibniz identity is satisfied i.e.

\[ \{\alpha(x), \mu(y, z)\} = \mu(\{x, y\}, \alpha(z)) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \] (7)

For \( x, y, z \in H(A) \) any

If in addition \( \mu \) is \( \varepsilon \) commutative, the color Hom-Poisson algebra \((A, \{.,.\}, \varepsilon, \alpha)\) is said to be a \( \varepsilon \) commutative color Hom-Poisson algebra.

The condition 7 expresses the compatibility between the color Hom-associative product \( \mu \) and the color Hom-Lie bracket \( \{.,.\} \) it can be written equivalently

\[ \{\mu(x, y), \alpha(z)\} = \mu(\alpha(x), \{y, z\}) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \] (8)

Remark

We recover Poisson algebras ([6, 5]) when \( \alpha = 1d \) and \( \varepsilon = 1 \)

We need the following lemma in Proposition 6.1.

Lemma

If \((A, \mu \{.,.\} \varepsilon, \alpha)\) is a \( \varepsilon \) commutative color Hom-Poisson algebra, then\( (A, - \mu \{.,.\} \varepsilon, \alpha)\) is also a \( \varepsilon \) commutative color Hom-Poisson algebra.

The following theorem is the color version of ([11], Proposition 4.6).

Theorem

Let \((A, \mu \{.,.\} \varepsilon, \alpha)\) be a color Hom-associative algebra.

Then \((A, \mu \{.,.\} = \mu - \varepsilon(\mu), \varepsilon, \alpha)\sqrt{\beta - \alpha} \) is a color Hom-Poisson algebra.

Proof:

According to Lemma 4.2, it remains to prove the color Hom-Leibniz identity 7. For any \( x, y, z \in H(A) \)

\[ \{\alpha(x), \mu(y, z)\} - \mu(\{x, y\}, \alpha(z)) - \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) = \]

\[ = \mu(\alpha(x), \mu(y, z)) - \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \]

\[ = \varepsilon(x, y)\mu(\{x, y\}, \alpha(z)) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \]

\[ = \mu(\alpha(x), \mu(y, z)) - \mu(\mu(y, z), \alpha(x)) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \]

\[ = \mu(\alpha(x), \mu(y, z)) - \mu(\mu(y, z), \alpha(x)) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \]

\[ = -as_{\alpha}(x, y)z - \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) = 0 \]

This finishes the proof.

Corollary

Let \((A, \mu \{.,.\} \varepsilon, \alpha)\) be a color associative algebra and \( \alpha \) an even color algebra endomorphism. Then \((A, \mu \{.,.\} \varepsilon, \alpha)\) where
\( \mu = \alpha \circ \mu \) is a color Hom-Poisson algebra.

**Proof**

The proof follows from Lemma 4.1 and Theorem 3.1.

**Lemma**

Let \((A, \mu, \ldots, \varepsilon, \alpha)\) be a color Poisson algebra and \(\alpha\) be an even color Poisson algebra endomorphism. Then \((A, \mu_\alpha, \ldots, \varepsilon, \alpha)\) is a color Hom-Poisson algebra.

**Proof**

By Lemma 4.1 and ([3], Example 1.2), we only need to prove the color Hom-Leibniz identity. For any \(x, y, z \in H(A)\),
\[
\mu(x, y, z) - \mu([x, y], z) - \mu(x, [y, z]) = \alpha^2(x, y) \mu(x, [y, z]) - \mu(\alpha^2(x, y), \alpha^2(x, z)) - \epsilon(x, y) \mu(\alpha^2(x, y), \alpha^2(x, z)) = 0.
\]
This completes the proof.

**Theorem**

Let \((A, \mu, \ldots, \varepsilon, \alpha)\) be a color Hom-Poisson algebra and \(\beta : A \to A\) be an even color Poisson algebra endomorphism. Then, \((A, \mu \oplus \beta, \ldots, \varepsilon, \alpha)\) is a color Hom-Poisson algebra.

Moreover, suppose that \((A', \mu', \ldots, \varepsilon', \alpha)\) is a color Poisson algebra and \((A, \mu, \ldots, \varepsilon, \alpha)\) is an even color Poisson algebra. If \(f : A \to A'\) is a color Poisson algebra morphism that satisfies \(f \circ \beta = \alpha \circ f\), then \(f : (A, \mu, \ldots, \varepsilon, \beta \circ \alpha) \to (A', \mu', \ldots, \varepsilon, \alpha)\) is a color Hom-Poisson algebra homomorphism.

**Proposition 5.1**

Let \((A, \mu, \ldots, \varepsilon, \alpha)\) be a color Poisson algebra and \(\alpha\) an even color Poisson algebra endomorphism of the form \(\alpha = \alpha_0 + \sum_{i \geq 0} t^i \alpha_i\) where \(\alpha_0\) are endomorphism of \(A\) (as color Poisson algebra), \(t\) is a parameter in \(K\) and \(k\) is an entiger. Let \(\mu, \alpha, \beta\) be an even color Poisson algebra endomorphism. Then \((A, \mu, \ldots, \varepsilon, \alpha)\) is a color Hom-Poisson algebra which is a deformation of the color Poisson algebra \((A, \mu, \ldots, \varepsilon, \alpha)\) viewed as a color Hom-Poisson algebra \((A, \mu, \ldots, \varepsilon, \Id)\).

**Definition 6.1**

Let \(G\) be an abelian group. A Hom-module is a pair \((M, \alpha_M)\) in \(M\) is a \(G\)-graded vector space and \(\alpha_M : M \to M\) is an even linear map.

**Definition 6.2**

Let \((A, \mu, \ldots, \varepsilon, \alpha)\) be a color Hom-associative algebra. An A-module is a Hom-module \((M, \alpha_M)\) together with a bilinear map \(\mu_M : A \otimes M \to M\) called structure map, such that
\[
\mu_M(A, M_M) \subseteq M_{\alpha_M}.
\]

Twisting a module structure map by an algebra endomorphism, we get another one as stated in the following Lemma.

**Lemma 6.1**

Let \((A, \mu, \ldots, \varepsilon, \alpha)\) be a color Hom-associative algebra and \(\alpha \) an A-module with structure map \(\mu : A \otimes M \to M\). Define the map
\[
\mu_M = \mu_M \circ (\alpha_M \otimes \Id_M) : A \otimes M \to M.
\]
Then \(\mu_M\) is the structure map of another A-module structure on \(M\).
Proof
The proof is similar to that of ([14], Lemma 4.5).

Definition 6.3

(33) Let \((L, [\cdot, \cdot], \varepsilon, \alpha)\) be a color Hom-Lie algebra and \((M, \alpha_M)\) a Hom-module. An L-module on M consists of a \(K\)-bilinear map \(\mu_L : A \otimes M \rightarrow M\) such that

\[
\begin{align*}
\mu_L(a, M) & \subseteq M_{ab} \quad (19) \\
\alpha_M(\mu_L(x, m)) & = \mu_L(\alpha_M(x), \mu_L(m)) \quad (20)
\end{align*}
\]

for any \(m \in H(M), x, y \in H(L)\).

Remark 6.1

When \(\alpha_M = \text{Id}_M\) and \(\alpha = \text{Id}_L\), we recover the definition of Lie modules ([15-17]).

The following statement is the Lie analogue of Lemma 6.1.

Lemma 6.2

Let \((L, [\cdot, \cdot], \varepsilon, \alpha)\) be a color Hom-Lie algebra and M an L-module with structure map \(\mu_L = L \otimes M \rightarrow M\). Define the map

\[
\mu_M = \mu_L \circ (\alpha^2 \otimes \text{Id}_M) : L \otimes M \rightarrow M
\]

Then \(\mu_M\) is the structure map of another L-module structure on M.

Proof

Equations 19 and 20 are proved as in Lemma 6.1. Now, we prove

21 for \(\mu_M\). For any \(x, y \in L, m \in M\)

\[
\begin{align*}
\mu_M(\{x, y\}, \alpha_M(m)) & = \mu_M(\{x, y\}, \mu_L(m)) \\
& = \mu_L(\{x, y\}, \mu_L(m)) - \varepsilon(x, y)\mu_L(\{x, y\}, \mu_L(m)) \\
& = \mu_L(\{x, y\}, \mu_L(m)) - \varepsilon(x, y)\mu_L(\{x, y\}, \mu_L(m)) \\
& = \mu_L(\{x, y\}, \mu_L(m)) - \varepsilon(x, y)\mu_L(\{x, y\}, \mu_L(m)) \\
& = \mu_L(\{x, y\}, \mu_L(m)) - \varepsilon(x, y)\mu_L(\{x, y\}, \mu_L(m)) \\
& = \mu_L(\{x, y\}, \mu_L(m)) - \varepsilon(x, y)\mu_L(\{x, y\}, \mu_L(m)).
\end{align*}
\]

Hence the conclusion holds.

The following result shows that A-modules extend to L(A)-modules with same module structure map.

Theorem 6.1

Let \((A, \mu, \varepsilon, \alpha)\) be a color Hom-associative algebra and \((M, \alpha_M)\) an A-module with structure map \(\mu_M\). Then, M is a L(A)-module with structure map \(\mu_M\).

Proof

In fact, it suffices to show the relation 21. For any \(x, y \in H(A), m \in H(M)\), we have

\[
\begin{align*}
\mu_M(\{x, y\}, \alpha_M(m)) & = \mu_M(\{x, y\}, \mu_M(m)) \\
& = \mu_M(\{x, y\}, \mu_M(m)) - \varepsilon(x, y)\mu_M(\{x, y\}, \mu_M(m)) \\
& = \mu_M(\{x, y\}, \mu_M(m)) - \varepsilon(x, y)\mu_M(\{x, y\}, \mu_M(m)) \\
& = \mu_M(\{x, y\}, \mu_M(m)) - \varepsilon(x, y)\mu_M(\{x, y\}, \mu_M(m)) \\
& = \mu_M(\{x, y\}, \mu_M(m)) - \varepsilon(x, y)\mu_M(\{x, y\}, \mu_M(m)).
\end{align*}
\]
A color Hom-Poisson module structure on M consists of two K-bilinear maps $\mu_M : A \otimes M \to M$ and $\lambda_M : A \otimes M \to M$ such that

(i) M is an A-module and an L-module,

(ii) And for any $x, y \in H(A), m \in H(M)$,

\[
\lambda_M(\alpha(x), \mu_M(y, m)) = \mu_M(\{x, y\}, \alpha_M(m)) + \varepsilon(x, y)\mu_M(\alpha(y), \lambda_M(x, m)), \tag{23}
\]

\[
\hat{\lambda}_M(\mu(x, y), \mu_M(m)) = \mu_M(\alpha(x), \hat{\lambda}_M(y, m)) + \varepsilon(x, y)\mu_M(\alpha(y), \hat{\lambda}_M(x, m)). \tag{24}
\]

When $\alpha = Id$, $\lambda_M = Id$, and $\varepsilon = 1$

We recover the definition of modules over Poisson algebras ([9]).

**Example 6.1**

(i) Any module over a $\varepsilon$ commutative color Hom-associative algebra (resp. color Hom-Lie algebra) can be seen as a module over an $\varepsilon$ commutative color Hom-Poisson algebra with the trivial color Hom-Lie bracket (resp. trivial color Hom-associative product).

(ii) Any $\varepsilon$ commutative color Hom-Poisson algebra is a module over itself.

**Example 6.2**

Let $(V, \mu_V, \lambda_V, \alpha_V)$ and $(W, \mu_W, \lambda_W, \alpha_W)$ be two modules over the $\varepsilon$ Commutative color Hom-Poisson algebra $(A, \mu, \{.,.,\}, \varepsilon, \alpha)$. Then the direct product $M = V \times W$ is a module over $A$ with structure maps $\mu_M : A \otimes M \to M$, $\lambda_M : A \otimes M \to M$, and $\alpha_M : M \to M$.

Defined by

$$\mu_M(\alpha(x, v), w) = \mu_V(\alpha(x), v), \mu_M(\alpha(x, w), v) = \mu_W(\alpha(x), w), \lambda_M(\alpha(x, v), w) = \lambda_V(\alpha(x), v), \lambda_M(\alpha(x, w), v) = \lambda_W(\alpha(x), w)$$

for any $x \in H(A), v \in H(V)$ and $w \in H(W)$.

**Proposition 6.1**

If $(M, \mu_M, \lambda_M, \alpha_M)$ is a module over the $\varepsilon$ commutative color Hom-Poisson algebra $(A, \mu, \{.,.,\}, \varepsilon, \alpha)$ then $(M, -\mu_M, -\lambda_M, -\alpha_M)$ is also a module.

Over the $\varepsilon$ commutative color Hom-Poisson algebra $(A, -\mu, \{-.,.-\}, \varepsilon, \alpha)$

**Proof**

The proof comes from Definition 6.4 and Lemma 5.1.

**Theorem 6.2**

Let $(A, \mu, \{.,.,\}, \varepsilon, \alpha)$ be an $\varepsilon$ commutative color Hom-Poisson algebra and $(M, \mu_M, \lambda_M, \alpha_M)$ color Hom-Poisson module. Then

\[
\mu_{\hat{M}} = \mu_M \circ (\alpha^2 \otimes \text{Id}_M) : A \otimes M \to \hat{M}, \tag{25}
\]

\[
\hat{\lambda}_M = \hat{\lambda}_M \circ (\alpha^2 \otimes \text{Id}_M) : A \otimes M \to \hat{M}, \tag{26}
\]

Define another color Hom-Poisson module structure on $\hat{M}$.

**Proof**

We know that $\mu_{\hat{M}}$ is a structure of another $A$-module structure on $\hat{M}$ (Lemma 6.1). And $\hat{\lambda}_M$ is a structure of another $L$-module structure on $\hat{M}$ (Lemma 6.2). Show relations 23 and 24 for $\mu_{\hat{M}}$ and $\hat{\lambda}_M$. For all $x, y \in H(A)$ and $m \in H(M)$

\[
\hat{\lambda}_M(\alpha(x), \mu_{\hat{M}}(y, m)) = \mu_{\hat{M}}(\{x, y\}, \alpha_M(m)) + \varepsilon(x, y)\mu_{\hat{M}}(\alpha(y), \hat{\lambda}_M(x, m)), \tag{23}
\]

\[
\hat{\lambda}_M(\mu_{\hat{M}}(x, y), \mu_{\hat{M}}(m)) = \mu_{\hat{M}}(\alpha(x), \hat{\lambda}_M(y, m)) + \varepsilon(x, y)\mu_{\hat{M}}(\alpha(y), \hat{\lambda}_M(x, m)). \tag{24}
\]

When $\alpha = Id$, $\lambda_M = Id$, and $\varepsilon = 1$

We recover the definition of modules over Poisson algebras ([9]).

**Corollary 6.3**

Let $(A, \mu, \{.,.,\}, \varepsilon)$ be an $\varepsilon$ commutative color Poisson algebra and $(M, \mu_M, \lambda_M, \alpha_M)$ a module over the color Hom-Poisson algebra $(A, \mu, \{.,.,\}, \varepsilon, \alpha)$. Then $\mu_{\hat{M}}, \hat{\lambda}_M$ define another color Hom-Poisson module structure on $M$.

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