We are concerned here with the curious history of Wetzel’s problem: If \( \{f_\alpha\} \) is a family of distinct analytic functions (on some fixed domain) such that for each \( z \) the set of values \( \{f_\alpha(z)\} \) is countable, is the family itself countable?

In September 1963, Paul Erdős submitted to the Michigan Mathematics Journal a stunning solution to Wetzel’s problem (Figure 1). He proved that an affirmative answer is equivalent to the negation of the Continuum Hypothesis. Erdős ends in an understated manner: “Paul Cohen’s recent proof of the independence of the continuum hypothesis gives this problem some added interest.” Together, these results render Wetzel’s problem undecidable in ZFC.

Erdős had a knack for solving “innocent-looking problems whose solutions shed light on the shape of the mathematical landscape” [7, p. 2]. In this case, the landscape he revealed was one of underground tunnels, surprising links, and glittering mysteries. However, our interest lies not with the solution itself, but rather on the story of how Erdős encountered Wetzel’s problem in the first place.

Our first exposure to Wetzel’s problem was Proofs from THE BOOK by Aigner and Ziegler. “Paul Erdős liked to talk about The Book,” they write, “in which God maintains the perfect proofs for mathematical theorems, following the dictum

\[\text{Figure 1. The first few paragraphs of Erdős' paper [6].}\]
of G.H. Hardy that there is no permanent place for ugly mathematics. Erdős also said that you need not believe in God but, as a mathematician, you should believe in The Book” [1]. Aigner and Ziegler asked Erdős to assemble a moderate approximation of The Book; included in it was Erdős’ answer to Wetzel’s problem [1, p. 102-6].

Regarding the origins of the problem, Erdős simply asserted that Wetzel posed the question in the Ann Arbor Problem Book in December, 1962. Ziegler referred us to several mathematicians who might be the Wetzel in question. At last we contacted John E. Wetzel (Professor Emeritus at the University of Illinois, Urbana-Champaign), who confirmed that the problem was indeed his.

John Wetzel, born on March 6, 1932 in Hammond, Indiana, earned a B.S. in mathematics and physics from Purdue University in 1954 and went on to study mathematics at Stanford University (see Figure 2). While studying spaces of harmonic functions on Riemann surfaces under Halsey Royden, he posed the following question in his dissertation:

Let $V$ be a collection of harmonic functions on a Riemann surface $R$ such that for each point $p$ of $R$ the set $V_p = \{ v(p) : v \in V \}$ is countable. Must $V$ then be countable? [9, p. 98]

With regards to the origin of the question, Wetzel explained that Royden had asked him to investigate a specific conjecture:

I thought about it for a while and eventually showed that what he conjectured was, in fact, not true. I remember reporting to him, thinking that all I had to do was write my work up in good form and I’d be finished with my dissertation; and I remember clearly Royden’s reaction that my result would make up perhaps a third of an acceptable dissertation. The question might have had its genesis in the subsequent confusion. [8]

Wetzel left Stanford in 1961 to become an Instructor at UIUC, having not yet finished his dissertation. He married Rebecca Sprunger in September, 1962 and completed the writing of his dissertation in 1963. During this time, Paul Erdős visited the Illinois campus with his mother (Erdős often brought his mother with him as he traveled from campus to campus [7, p. 7]).

During one of his casual contacts with Erdős, Wetzel recounts,

I mentioned the question to him, rather timidly, if memory serves. He thought about it briefly and said it was interesting – and that was the extent of my mathematical contact with him. I don’t think he ever told me that he had
settled the matter, but every time he visited the campus in the next few years he always asked me if I had any new interesting questions [8].

Upon learning (probably not before 1966, Wetzel said) about Erdős’ proof, Wetzel wrote to Royden that “Erdős has showed that the answer to a question I asked in my dissertation is closely tied to the continuum hypothesis! So once again a natural analysis question has grown horns” [8].

The Ann Arbor Problem Book that Erdős mentions is most likely the Math Club Book, which has its own fiery history. In the words of Peter Duren (Professor Emeritus at the University of Michigan):

> When I came to Ann Arbor in 1962, I learned of a local tradition called “Math Club,” an informal gathering of faculty and graduate students which met in the evening every month. A speaker was announced in advance, but the main attraction (in my view) was the series of 5-minute talks, unannounced and often spontaneous, that preceded the announced lecture. There people would tell their colleagues about neat mathematics they had come across, or raise questions, whatever they thought would be of interest. Afterwards each speaker was invited to write a short summary of his presentation in a book maintained for that purpose. The Secretary of the Math Club acted as guardian of the book, and both locals and visitors were invited to look through it. Unfortunately, the book was lost around 1965, on the streets of Chicago. The man then serving as Secretary of the Math Club had carried the book (or books) with him when he drove to Chicago and had left it in his car overnight. Someone broke into the car and set it on fire, and the Math Club book was lost (among other items, including the car). The Math Club continued to meet, probably with a new book, but attendance gradually declined and the meetings were discontinued around 1980, as I recall. What Paul Erdős called the Ann Arbor Problem Book must have been the Math Club book. But his reference can’t be checked, since the entries for 1962 no longer exist [5].

However, we now know that Erdős erred in his citation since Wetzel declared “I have never visited the University of Michigan; I’ve never even been to Ann Arbor” [8]. Given Erdős’ unique manner of doing mathematics and his myriad colleagues, such a slip-up is understandable. Duren wrote:

> I vividly recall him asking people (including me) at math conferences, “Where are you located?”, which was the polite way of asking, “Who are you? I know I’ve met you somewhere.” This was only natural, since he traveled so much and met so many mathematicians. It’s easy to imagine that he didn’t remember correctly where he had seen the problem [5].

After much investigation, we are now able to trace the actual evolution from the original question sparked by Wetzel’s dissertation to the problem treated by Erdős in his paper.

Within the UIUC mathematics library is a volume of particular importance for us. Wetzel explained:

> The library maintains a bound volume of blank pages called the Boneyard Book, in which faculty and visiting faculty are encouraged to write problems, including whatever supporting information or commentary they care to include, and faculty looking for interesting problems can browse in it for inspiration [8].

Senior Library Specialist Margaret Lewis recovered the relevant page of the Boneyard Book (see Figure 3), despite the fact that the volume had been collecting dust in the UIUC archives for decades. Duren recounted that during the early 1960s,
Erdős travelled frequently between the University of Michigan and the University of Illinois Urbana-Champaign, so conflating the two schools’ problem books would have been a natural mistake.

However, the Boneyard Book led to many more questions. There are clearly four scribes who contributed to the page, so our next task was to identify the mathematicians involved. While the final entry of the page certainly boasts Erdős’ distinctive penmanship, the remaining three entries required further scrutiny.

Wetzel explained that the first entry, though attributed to him, was not written by him: “I haven’t a clue who wrote the problem in our Boneyard Book and wrote my name next to it. I thought for a few moments that it might have been Ranga Rao, with whom I shared an office during my (and his) first year at Illinois, 1961-62. But his handwriting was recognizably ‘Indian English penmanship,’ and I think him unlikely” [8]. We therefore decided to investigate the second and third entries before tackling the first.

Jane Bergman, secretary of the chair of the UIUC math department, suggested that the Dixon mentioned in the Boneyard Book was Robert (“Bob”) Dan Dixon:

> I was able to find out that Mr. Robert Dan Dixon received his Ph.D. from Ohio State University in 1962 in the same year he was hired here as an instructor
in Mathematics. In 1964 he was recommended for promotion to assistant professor, but there is no result in his file of that recommendation. I would guess that he moved on at that point, but there is nothing in his file to support my guess. He was born in 1936. I hope you find this helpful.

A colleague of Wetzel’s, George Robert (“Bob”) Blakley, had more information about Dixon. Bob Blakley and Bob Dixon both arrived at UIUC in September 1962 as new Ph.D.s and both left in 1964. Blakley, in personal correspondence with the authors and Wetzel, wrote:

Bob [Dixon] went to the nascent Wright State University in the Dayton area as a founding father. First he founded the math department and headed it. Then he founded the computer science department and headed it. . . Late in the last century he retired from WSU, covered with glory. . . He still manages to fleece me regularly and disreputably in the most varied sorts of bets. But I think he has given up 100 mile bike rides.

Blakley provided us with three possible ways to contact the elusive Dixon: two email addresses which may or may not have been current, along with his home address. Wetzel sent a letter out to all three addresses, and luckily one route was successful. Dixon responded:

I was there [UIUC] from the fall of 1962 until the spring of 1964. I remember Erdős visiting and may even have had some time with him but I don’t remember discussing this problem. The handwriting in the book is puzzling. The entry describing your problem is not familiar to me. The first entry that mentioned me could be by Bob [Blakley] as he generally printed. The second entry refers to me in third person but I could have written it. . . Although I can’t remember any details there is a bit of familiarity. I worked in complex analysis at the time and I had a very interesting course in graduate school that covered the relevant set theory. Lots of problems were thrown around in that group of young faculty.

Dixon elaborated on “that group of young faculty” known as the SixtyTwo Illini Hall Group. In 1962, UIUC hired twenty new mathematics faculty to add to their faculty of 100; Illini Hall was located across from the mathematics building. According to Dixon:

There was not a sense of privacy about the problems we were investigating. My own work was very specialized and detailed so I had no problems to share. Many others, like Jack Wetzel did have problems that they proposed, or pointed us to, that they were curious about or needed to solve to get on to the problem they really wanted to work on. We fell then into three overlapping groups, proposers, solvers, and brokers. I was in the solvers group but not particularly successful. Jack may have been in all three. Bob Blakley was in all three but was effective as a broker...There were many others who participated in this interchange but my memory of names is bad. That said it was an experience that had more to do with my career than my own doctoral research. I suspect that was true of several of the other SixtyTwo Illini Hall Group.

After examining a copy of the page in question, Blakley confirmed that he authored the entry in the Boneyard Book that reads, “Dixon has a short proof that the continuum hypothesis is true if there exists an uncountable family.” Blakley also remarked that he feels “rather strongly... that Dixon is the third scribe.” Given Erdős’s parenthetical remark in his paper that “I have been informed that R.D. Dixon proved the first part of the theorem last year” [6], it is likely that Robert Dan Dixon was indeed the scribe of the third entry. Since Dixon expressed
that he never spoke to Erdős about this problem during their overlapping time at UIUC, the information Erdős claimed was relayed to him almost certainly came from the Boneyard Book.

After immersion in the memories and details surrounding the problem, Wetzel recalled:

> I just remembered that I had given a faculty seminar and a departmental colloquium on the substance of my dissertation shortly after arriving at Illinois (even though the dissertation was not yet completely written), and that widened significantly the list of people who might have written the first entry in the Boneyard Book. [8]

A chance meeting between the first author and John P. D’Angelo (a Professor at UIUC) finally revealed the most likely candidate for the first scribe. D’Angelo was convinced that the handwriting was that of Lee A. Rubel:

> Rubel, who died in 1995, often contributed to the Boneyard Book. Furthermore, his many interests included the interplay between logic and function theory... Rubel would have been quite interested in this problem, and the handwriting is remarkably similar to that of notes he wrote to me around 1979-80. [3]

Of the possible candidates, Wetzel remarks:

> Lee certainly strikes me as the more likely... I truly don’t doubt that Rubel was the author, but I confess that I still find it a little surprising that he never mentioned it to me - admitting always the possibility that he did and I have forgotten. [8]

The final piece of evidence was a sample of Rubel’s handwriting, obtained by D’Angelo from the UIUC archives, which appears to validate this conclusion. In fact, D’Angelo tells us, Rubel was the creator of the Boneyard Book.

No story can ever have the entirety of its details pinned down. As Wetzel said: “It may require transfinite induction to bring this matter to a close.” However, we have identified with a high degree of certainty the trajectory of Wetzel’s question as it made its way to Erdős. It began in 1961, when Wetzel posed the original question (for harmonic functions on Riemann surfaces) in his evolving dissertation. When he arrived at UIUC in 1962, he gave a talk on his graduate research. Lee Rubel was almost certainly one of the attendees. Rubel wrote Wetzel’s question in the Boneyard Book in 1962, and Bob Blakley responded with an entry claiming that Bob Dixon had a proof, assuming the truth of the Continuum Hypothesis. Dixon crossed out Blakley’s entry and wrote (in third person), “He showed that if the Continuum Hypothesis is false, then each family is countable.” Dixon’s short proof was rediscovered and published by Erdős, who went on to prove that an affirmative answer to Wetzel’s problem is equivalent to the negation of the Continuum Hypothesis. Erdős’ Boneyard Book entry is likely from late 1962 or early 1963; his proof was submitted to the Michigan Mathematics Journal in September 1963, and finally published in 1964.

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