Polaritonic crystal formed of a tunnel-coupled microcavity array and an ensemble of quantum dots

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Abstract. Propagation of polariton excitations in a defect-containing one-dimensional lattice of microcavities with embedded ultracold atomic nanoclusters (quantum dots) is being considered. The virtual crystal approximation is used to study the properties of electromagnetic excitation spectrum resulting from random variations of the atomic subsystem composition and positions of micropores, as well as from a homogeneous elastic deformation of the considered one-dimensional structure. The group velocity dependence of polariton excitations on structural defect concentration and on deformation parameter is being numerically modeled.

1. Introduction
Some important features of photonic band-gap structures, which have been drawing an extensive attention [1] are related to the so-called ‘slow’ light, which is one of the promising fundamental physical phenomena that can be exploited in designing various quantum optical storage devices. Its correct understanding permits to effectively reduce the group velocity of electromagnetic excitation in coupled-resonator optical waveguides [2, 3] as well as in various types of solid-state semiconductor multilayer structures [4]. The key role in reducing the group velocity in these systems is played by the so-called bright and dark polaritons, which are linear superposition of photon states of the external electromagnetic field and macroscopic (coherent) perturbations of the two-level atomic medium.

In atomic systems, the lifetime of polaritons is limited by the lifetime of excited atoms and is usually of a nanoscale order [5]. At the present level of advancement of nanotechnologies and nanophotonics it is possible to study the "slow" light as well as phase transitions of polaritons by fabricating arrays of coupled microcavities with embedded two-level atoms [6-8]. Technologically, creation of data structures can be based on photonic crystals containing defects formed of microcavities doped with such atoms [9].

In order to gain theoretical understanding of the above systems, in the present paper we propose and investigate a modeling polaritonic crystal constituted by atomic nanoclusters (quantum dots) weakly interacting with the localized electromagnetic field in an array of tunnel-coupled microcavities. A remarkable feature of such a structure is the possibility of localization of polaritons, which is similar to the nonlinear optical phenomenon of light localization in photonic crystals (see e.g. [9]) as well as to exciton localization in quasi-periodic structures in solid state physics [10].

Basing on the previously developed description of ideal photonic structures [11], here we consider one such particular non-ideal system, namely a polaritonic crystal with an atomic subsystem formed of impurity atomic clusters. Several of our previous works have been devoted to designing microcavity structures where modulation of the dispersion of photon modes is achieved by introduction of...
appropriate defects [12, 13]. In technical applications, structural defects in supercrystals play a minor role as compared to temporary defects introduced by application of external fields and/or elastic strains. The present work analyzes in detail the effect of a uniform elastic strain on a one-dimensional microcavity array with embedded quantum dots. Such a system combines the advantages of an extreme optical non-linearity stemming from the coupling of quantum dots to photonic modes and a high sensitivity of optical eigen-modes to the applied strains. In other words, we dwell on a particular realization of a topologically ordered microcavity (resonator) system, which can have promising applications in optical integrated circuits.

2. Theoretical background

Basing on the approach developed in Refs. [12, 13], let us first consider the dispersion of optical eigen modes in the most general case of a microcavity supercrystal composed of $S$ sublattices. Each of tunnel-coupled microcavities is assumed to confine a single optical mode. The cavity photon Hamiltonian $\hat{H}(\hat{\epsilon})$ is dependent on the deformation tensor $\hat{\epsilon}$ that is sensitive to the applied strain.

Adapting the Heitler-London approximation, the Hamiltonian $\hat{H}(\hat{\epsilon})$ can be written as (see e.g. Ref. [14] for further details):

$$\hat{H}(\hat{\epsilon}) = \sum_{\alpha,\beta} D_{\alpha\beta}^{11} \hat{\omega}_{\alpha\beta}^0 \hat{\Phi}_{\alpha\beta}^+ \hat{\Phi}_{\alpha\beta},$$

where

$$D_{\alpha\beta}^{11} = \hbar \omega_{\alpha\beta}^0 + V_{\alpha\beta},$$

$$D_{\alpha\beta}^{22} = \hbar \omega_{\alpha\beta}^1 - A_{\alpha\beta}.$$

In Eqs. (1, 2) $\omega_{\alpha\beta}$ is the frequency of the photonic mode localized in the $n\alpha$-th lattice site (microcavity), $\hat{\Phi}_{\alpha\beta}^+$ and $\hat{\Phi}_{\alpha\beta}$ are bosonic creation and annihilation operators for this mode written in the node representation, $\hbar \omega_{\alpha\beta}^0$ is excitation energy of the quantum dot in the $n\alpha$-th lattice site, $\hat{B}_{\alpha\beta}^+$ and $\hat{B}_{\alpha\beta}$ are creation and annihilation operators of quantum dot excitons, $A_{\alpha\beta} = \hat{\omega}_{\alpha\beta}^1$ is the matrix of resonance interaction, which describes an overlap between optical fields of resonators in the $n\alpha$-th and $m\beta$-th lattice sites and hence defines the jump probability of the corresponding electromagnetic excitation, $V_{13} = \hat{\Phi}_{\alpha\beta}^+ \hat{\Phi}_{\alpha\beta}$ is the matrix of resonance interaction between quantum dots embedded in the $n\alpha$-th and $m\beta$-th lattice sites, $g_{\alpha\beta}(\hat{\epsilon})$ is the matrix of resonance interaction between quantum dots embedded in the $n\alpha$-th lattice site and electromagnetic field localized at the same site. Values 1 and 2 of indices $\lambda, \sigma$ indicate, respectively, the presence or absence of quantum dots in corresponding cavities.

In the right-hand side expression of Eq. (1) (summation over $k$) matrices $D_{\alpha\beta}^{11}(\hat{\epsilon})$ and $\hat{\Phi}_{\alpha\beta}^+ (\hat{\epsilon})$ have the forms

$$D_{\alpha\beta}^{11}(\hat{\epsilon}) = \sum_{\alpha,\beta} D_{\alpha\beta}^{11} \hat{\omega}_{\alpha\beta}^0 \hat{\Phi}_{\alpha\beta}^+ \hat{\Phi}_{\alpha\beta} \exp \left[ i k \cdot (\hat{r}_{\alpha\beta} - \hat{r}_{\mu\beta}) \right],$$

and

$$\hat{\Phi}_{\alpha\beta}^+ = \frac{1}{\sqrt{N}} \sum_{\alpha\beta} \hat{\Phi}_{\alpha\beta}^+ \exp \left[ -i k \cdot \hat{r}_{\alpha\beta} \right] \quad (N \text{ is the number of elementary cells in the lattice}).$$

Such representation of matrices is possible due to preservation of the translation invariance of the system under the uniform strain. Let us note that the wave vector $\hat{k}$, which characterizes eigenstates of electromagnetic excitations, ranges within the first supercrystal Brillouin zone, whose boundaries are in their turn functions of strain through the tensor $\hat{\epsilon}$.
Eigenvalues of the Hamiltonian (1) are found by its diagonalization through the Bogolyubov-Tyablikov transformation [14]. This yields the following equation for elementary excitation spectrum $\Omega(k, \varepsilon)$:

$$\det \left[ D_{\alpha\beta}(k, \varepsilon) - \hbar \Omega(k, \varepsilon) \delta_{\alpha\beta} \delta_{\beta\varepsilon} \right] = 0$$

and then we obtain the relation for the group velocity $V(k, \varepsilon)$ of polariton excitations propagation in the system under study: $V(k) = \frac{\partial \Omega(k)}{\partial k}$.

Below, on the basis of the above theory, the features of the dependence of the group velocity of elementary electromagnetic excitations on the concentration of structural defects and homogeneous deformation in a specific non-ideal porous 1D structure are investigated.

3. Results and discussion

As an example, let’s consider polaritons in a one-sublattice quantum-dot-containing chain of unevenly spaced microcavities under a uniform elastic deformation (the corresponding component of the tensor $\varepsilon$ is $\varepsilon$). We consider the array of identical cavities with randomly embedded quantum dots of two types, whose concentrations are, correspondingly, $C^{(1)}_c$ and $C^{(2)}_c$. It is assumed, in addition, that microcavities are unevenly spaced; namely that $C^{(1)}_t$ neighboring pairs of cavities are separated by distance $a_1(\varepsilon)$ and the remaining $C^{(2)}_t$ pairs are separated by distance $a_2(\varepsilon)$, figure 1.

![Figure 1. Schematic representation of an averaged non-ideal two-sublattice 1D array of micropores, the first sublattice of which contains the same quantum dots.](image)

Here we also adopt the virtual crystal approximation [15, 16] based on the diagonalization of the averaged Hamiltonian (1). The corresponding procedure yields a system of uniform linear equations, whose solvability condition is given by:

$$\left\| \left( \begin{array}{cc} \hbar \left\{ \omega_n^\varepsilon (\varepsilon) \right\}_c + \left\{ V(k, \varepsilon) \right\}_{c,t} - \hbar \Omega(k, \varepsilon) & \left\{ g_n(\varepsilon) \right\}_t \\hbar \omega^\varepsilon(\varepsilon) - \left\{ A(k, \varepsilon) \right\}_{t} - \hbar \Omega(k, \varepsilon) \end{array} \right\| = 0, \quad (4)$$

where $\left\{ \omega_n^\varepsilon \right\}_c = \sum_{r=1}^{2} \omega_r^\varepsilon C_r^\varepsilon$, $\left\{ g_n \right\}_c = g^{(1)}_c C^{(1)}_c + g^{(2)}_c C^{(2)}_c$, (it is implied that $C^{(1)}_c + C^{(2)}_c = 1$, and hence $C^{(1)}_c = 1 - C^{(2)}_c \equiv C_c$);
\[
\langle V(k) \rangle_{C,T} = \sum_{V,\mu} V^\mu(k,\{C_T\},\varepsilon) C^\mu C^\varepsilon, \quad V^{\mu\nu}(k,\{C_T\},\varepsilon) = \sum_m \langle V^\mu_m(e) \rangle_T \exp[i k r_m(\{C_T\},\varepsilon)].
\]

Similarly, \( A(k,\{C_T\},\varepsilon) = \sum_m \langle A_m(e) \rangle_T \exp[i k r_m(\{C_T\},\varepsilon)] \),

where \( r_m(\{C_T\},\varepsilon) = d(\{C_T\},\varepsilon)(n-m) \), \( (C_T^{(1)} + C_T^{(2)} = 1, C_T^{(1)} = 1-C_T^{(2)} \equiv C_T) \).

Angular brackets in (4) denote the procedure of configuration averaging of the microcavity array over all possible positions of cavities (index “T”) and compositions of quantum dots (index “C”).

\( d(\{C_T\},\varepsilon) \) is the period of the “virtual” one-dimensional microcavity lattice obtained by averaging \( r_m(\{C_T\},\varepsilon) = C_T^{(1)} a_1(\varepsilon) + C_T^{(2)} a_2(\varepsilon) \).

Within the nearest-neighbor approximation, the quantities \( V(k,\{C_T\},\varepsilon) \), \( A(k,\{C_T\},\varepsilon) \) can be found as:

\[
\begin{align*}
V^\mu(\{C_T\},\varepsilon) &= \frac{1}{2} \begin{bmatrix} V^\mu[d(\{C_T\},\varepsilon)] \end{bmatrix} \\
A^\mu(\{C_T\},\varepsilon) &= \frac{1}{2} \begin{bmatrix} A^\mu[d(\{C_T\},\varepsilon)] \end{bmatrix} \cos(kd[\{C_T\},\varepsilon]) \end{align*}
\]

It follows from (4) that the dispersion relation \( \Omega(k,\{C_T\},\varepsilon) \) of polariton modes is defined by frequency characteristics of the cavities and the dots as well as by the explicit form of expressions \( A(k,\{C_T\},\varepsilon) \) and \( V^{\mu\nu}(k,\{C_T\},\varepsilon) \). In the framework of our model, the functions \( A[d(\{C_T\},\varepsilon)] \) and \( V^\mu[d(\{C_T\},\varepsilon)] \) of the strain degree and the defect concentrations are assumed (for \( a_2(\varepsilon) > a_1(\varepsilon) \) ) to be equal to:

\[
\begin{align*}
V^\mu[d(\{C_T\},\varepsilon)] &= \frac{1}{2} \begin{bmatrix} V^\mu[d(\{C_T\},\varepsilon)] \end{bmatrix} \\
A[d(\{C_T\},\varepsilon)] &= \frac{1}{2} \begin{bmatrix} A[d(\{C_T\},\varepsilon)] \end{bmatrix} \exp\left[- \frac{-d(\{C_T\},\varepsilon)}{a_1(\varepsilon)} \right] \frac{a_1(\varepsilon)}{a_1(\varepsilon)}
\end{align*}
\]

\( a_1 \mid_{\varepsilon=0} \equiv a_1, \quad a_2 \mid_{\varepsilon=0} \equiv a_2 \). Quantities \( A(a_1), V^\mu(a_1) \) characterize an overlap of optical fields of neighboring cavities and an interaction between neighboring quantum dots in a one-dimensional lattice with period \( a_1 \), respectively. Such a lattice is chosen to be a reference one for the subsequent variation of distances between resonators.

The numerical calculations were carried out for the following modeling values of parameters. The frequency of cavity-localized resonance photonic modes was put equal to \( \omega^p = 2\pi \times 203\,\text{THz} \approx 1280\cdot10^2\,\text{Hz} \); the two types of quantum dots were assumed to be characterized by exciton resonance frequencies \( \omega^e = 2\pi \times 191\,\text{THz} \approx 1200\cdot10^2\,\text{Hz} \) and \( \omega^e_2 = 2\pi \times 202\,\text{THz} \approx 1269\cdot10^2\,\text{Hz} \), whereas \( A/2\hbar = 8\cdot10^{13}\,\text{Hz} \), \( V^{11}/2\hbar = 1\cdot10^{13}\,\text{Hz} \), \( V^{22}/\hbar = 3\cdot10^{13}\,\text{Hz} \), \( V^{12} \approx V^{21} = 6\cdot10^{13}\,\text{Hz} \), \( g^{(1)}/\hbar = 5\cdot10^{12}\,\text{Hz} \), \( g^{(2)}/\hbar = 1.5\cdot10^{12}\,\text{Hz} \) (within the adopted approximation the magnitude of resonance interaction of a quantum dot with electromagnetic field localized at the same cavity is independent of deformation \( \varepsilon \)). The lattice periods were put equal to \( a_1 = 3\cdot10^{-6}\,\text{m} \) and \( a_2 = 7\cdot10^{-6}\,\text{m} \).
The two dispersion branches $\Omega_\pm(k, C_c, C_T, \varepsilon)$ of the considered collective excitations in the microcavity array are plotted in Figures 2a,b for several values of $C_c$ and $C_T$. Let us remind that $k$ ranges between $\frac{-\pi}{a_2(\varepsilon) + C_T[a_1(\varepsilon) - a_2(\varepsilon)]} \leq k \leq \frac{\pi}{a_2(\varepsilon) + C_T[a_1(\varepsilon) - a_2(\varepsilon)]}$, whereas $C_T$ ranges between 0 to 1.

**Figure 2.** Dispersion relations $\Omega_\pm(k, C_c, C_T, \varepsilon)$ of polaritonic excitations in a one-sublattice quantum-dot-containing chain of unevenly spaced microcavities plotted for different values of dot concentrations $C_c, C_T$ and deformation parameter $\varepsilon$.

It should be noted that the shape of the dispersion curve in Figure 2a indicates the existence of Bose-Einstein exciton condensate (BEEC), where energy minima occur for a number of states with non-zero $k$'s (in addition to those with $k = 0$).

Careful examination of plots $\Omega_\pm(k, C_c, C_T, \varepsilon)$ depicted in Figures 2a, b shows that the zero group velocities $V_\pm(k, C_c, C_T, \varepsilon)$ at non-zero $k$'s correspond to maxima of function $\Omega_\pm(k, C_c, C_T, \varepsilon)$, throughout the whole range of concentration and deformation values (unlike for $\Omega_\pm(k, C_c, C_T, \varepsilon)$ and in contrast to systems without deformation considered in [12]), and hence do not represent stable BEEC states. The dependence of polariton excitation group velocity $V_\pm(k, C_c, C_T, \varepsilon)$ in the considered structure on defect concentrations $C_c, C_T$ and on deformation parameter $\varepsilon$ is illustrated in Figure 3. The dependencies $k(C_c, C_T, \varepsilon)$ following from the condition $V_\pm(k, C_c, C_T, \varepsilon) = 0$ are indicated in figure 4. In Figure 4 we have indicated the critical values $C_{c0}, C_{T0}$ of structural defects concentrations $C_c, C_T$ which demarcate the transition from case a) to case b) in Figure 2.
Figure 3. Polariton excitation group velocities $V_\pm(k, C_c, C_T, \epsilon)$ in the considered non-ideal structure.

Figure 4. Dependencies $k(C_c, C_T, \epsilon)$ resulting from the condition $V_s(k, C_c, C_T, \epsilon) = 0$ with indicated critical values of structural defects concentrations $C_{c0}, C_{T0}$. 
4. Conclusion
Theoretical investigation into photonic band structure of non-ideal lattices comprised of tunnel-coupled microcavities with embedded quantum dots shows that subjecting these systems to controllable elastic strains as well as introduction of structural defects provide effective tools for altering their eigen-mode structure and optical properties. Elastic strains and photonic crystal structure disorder have a direct effect on the magnitude of the group velocity of elementary excitations in the specified systems. This is illustrated, in particular, by the theoretical result that the slow light mode formation can be efficiently controlled by an externally applied strain. The presented results of numerical simulations contribute to modeling a new class of functional materials, namely photonic crystalline systems constituted of coupled microcavities, whose capabilities include controllable propagation of electromagnetic excitations. The obtained conclusions pave the way to applications of irregular microcavity arrays in optical integrated circuits as well as in classical and/or quantum optical switches.

5. References
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