Mathematics Communication: Translation of Elementary Students' Idea

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Abstract: Communication is an essential ability for every human being. In this study, the focus of communication was on students’ mathematical communication. Mathematical communication is a person's ability to express their thoughts and is responsible for listening, interpreting, asking, and interpreting ideas from one to another to solve problems. This study aimed to describe the mathematical communication of Sombron Elementary School students in solving mathematical problems. This type of research is qualitative descriptive research. The research subjects were 23 students in class VI Sombron Elementary School. Three issues were selected for more in-depth analysis because they represented three categories: high, medium, and low ability students. Based on the subjects' works, the students could not communicate their mathematical ideas well yet. The most common communication error was a mistake in changing the word problem into a settlement model form. Based on the results and discussions that have been carried out with three different subjects, high ability student (ST), moderate ability student (SS), and low ability student (SR) of Sombron Elementary School at class VI, it can be concluded that ST and SS understand the meaning of the problem given and they know what was asked from the issue, but they did not use a correct symbol while writing their idea. ST and SS also made some mistakes in using mathematics symbols.

INTRODUCTION

Mathematics has a role as a symbolic language that can be used to communicate appropriately. Mathematics is not just a tool for thinking, but mathematics is also a communication tool between students and teachers [1][2]. Wilkinson argues that mathematical communication is a person's ability to express their thoughts and is responsible for listening, interpreting, asking, and interpreting ideas from one to another to solve problems both in the discussion and in-class [3][4]. By communicating, students can improve their vocabularies, develop speaking skills, write ideas systematically, and have better learning abilities.

Mathematics has some abstract ideas and concepts. Through communication, abstract ideas become objects that can be reflected, refined, discussed, and developed [5][6]. The communication process also helps build the meaning and permanence of ideas and publicize or explain concepts [6][7]. So, communication is an essential part of mathematics and mathematics education. Communication is a way of sharing ideas and clarifying understanding.
NCTM suggests that mathematical communication standards emphasize mathematics learning on students' abilities in the following matters [7].

1. Organize and combine their mathematical thinking through communication
2. Communicate their mathematical thinking logically and clearly to his friends, teachers, and others,
3. Analyzing and evaluating mathematical thinking and strategies used by others,
4. Using mathematical language to express mathematical ideas correctly.

Thoughts and abilities about students' mathematics skills are challenged during the learning process, so communication is essential for students conveying their thinking results verbally or in written form [3]. Students' mathematical communication makes the teacher understand their students' abilities to interpret and express students' understanding of the concepts they already learned [9]. It is expected to be used for all levels.

At least two crucial reasons that make communication in learning mathematics need to be improved among students — first, mathematics as a language. Mathematics is also an invaluable tool for communicating various ideas, which means that it is a valuable tool to share ideas precisely and accurately. Second, mathematics learning as a social activity. In mathematics learning, the interaction between students and teacher-student communication is an essential part of "nurturing children's mathematical potential." However, the students' mathematical communication skills have not received attention in learning enough [10].

In learning mathematics practice, teachers often try to make students answer some questions correctly without asking students for their answer reasons and not asking them to communicate their thoughts and ideas [11]. As a result, students are rarely argued in learning mathematics, and they will feel strange to talk about mathematics. The teacher loses interest in giving questions that require strong reasoning in solving the problem [11][12]. Simultaneously, items that require a strong rationale can lead to mathematical communication between students and teachers. Conversely, when procedural questions are given, students will easily imitate the steps taught not to communicate much. Aini stated that to bring up mathematical communication, items that need to be made to increase students' HOTS need critical and creative thinking [14].

Some researchers examined mathematical communication skills. However, they mostly researched on how to improve mathematical communication skills—as done by Rahmadhani [15], Fahradina [16], and Ariani [17]—. In contrast, it is necessary to conduct in-depth research regarding students' mathematical communication skills based on their mathematics level. It needs to be done because most of the classes in Indonesia group the students heterogeneously. So we can see a big picture of students' mathematical communication skills in a class, which is expected to help the teacher take some necessary actions.

METHOD

This study was a qualitative-descriptive study because it described the students' mathematical communication errors in solving word problems. This type of descriptive research aims to define a variable, symptom, or situation as it is. Besides, according to Creswell [18],
qualitative research is a research procedure that produces descriptive data in written or oral words from people's observed behavior.

This study took place at grade VI in Sombron State Elementary School, located in Sombron, Kec. Loceret, Nganjuk Regency, East Java. The researcher gave tests to 23 students in class VI. The test instrument used was one item in this study. From 23 students' results, three results were selected and discussed further. The researcher made some code for each subject, as shown in Table 1, to make it easier to say the subject's name during the analysis process. The three subjects' works will be analyzed in-depth through an interview. Those subjects were chosen using a purposive sampling method. The purposive sampling method is a selection of issues by considering certain factors [19]. In this study, the factors to be considered were the subjects' mathematics ability level in their class and the complexity of the subjects' works.

Table 1. Subject's Code

| Subject's Code | Level Ability Subject's        |
|----------------|-------------------------------|
| ST             | Highly ability students       |
| SS             | Medium ability student        |
| SR             | Low ability student           |

Some indicators were used to analyze students' mathematical communication. The arrows were obtained by combining the mathematical communication standards described in NCTM [7] and Wilkinson [3], and they are showed in Table 2.

Table 2. Mathematical Communication Ability Indicators

| No | Mathematical Communication Ability Indicator | Indicator's Description                                                                 | Indicator's Code |
|----|---------------------------------------------|----------------------------------------------------------------------------------------|-----------------|
| 1  | Express mathematical ideas through oral, written, and describe visually. | Students can explain, write, make sketches or drawings related to mathematical ideas possessed to solve problems | K1              |
| 2  | Analyze and evaluate ideas a. both verbally and in writing | Students can understand the meaning of the problem   |
|    |                                             | b. Students can exchange ideas about the subject matter intended in the given problem  |
|    |                                             | c. Students can write down the information contained in the issue to clarify the question to solve the problem further |
|    |                                             | d. Students can provide reasons for a solution                                             |
| 3  | Use terms, language, symbols, and writing structure when modeling a problem | Students can say and write mathematical words, languages, or symbols correctly of the given problem |

The test item that was given to the subjects was: Didin and his brother play a snakes and ladders game. They take a turn to toss a dice. To get a ladder, Didin needs the dice to show an odd number of a prime number. What is the probability he gets a ladder?
RESULT & DISCUSSION

This part provides descriptive information about subjects' results and the analysis of their mathematics communication skill.

ST's Result

As showed in Figure 1, ST wrote what was known and asked by the problem. But ST wrote the things known incorrectly. ST did not understand the “n(s)” symbol, because he wrote $n(s) = 1, 2, 3, 4, 5, 6$ instead $n(s) = 6$. ST also could not make sketches or drawings related to the problem. ST's result did not fulfill the first indicator expressing mathematical ideas through oral, written, and visually illustrating (K1).

ST could not analyze and evaluate mathematical ideas in written form, but he could deliver his correct view orally (K2). It was shown by the false information which was written by ST. Although his idea was right, he did not execute his concept correctly. In the third indicator, which uses mathematical terms, language or symbols, and their structures to model mathematical situations or problems (K3), ST could not write the character correctly[3]. The following interview shows ST's thought circled in red in Figure 1.

M: What does "n(s)" mean?
ST: (silence) I'm not sure... sample space?
M: OK...
M: Then, what the meaning of this symbol (point ∩ symbol)?
ST: It is "or".
M: Are you sure?
ST: Yes, ma'am

From this conversation, it appears that ST still does not understand the concepts and symbols of mathematics. So, it can be concluded that ST has not fulfilled the three mathematical communication indicators, and ST's mathematical communication skill is still low.

Figure 1. Test Result of High Ability Students (ST)

It is different from the results of the study conducted by Ega[20] and Maulyda [20][21], which explains that students with high ability can carry out oral and written communication. Students' verbal communication provides explanations by connecting their experiences in everyday life. Students with high knowledge can explain the theoretical basis by what is asked and support the answers. In written communication, they can use the notation appropriately and know the meaning of the inscription used [5], explain what is known, what is sought, and what is
asked, write a structured settlement, and no leaps. Research conducted by María & Clara Jessica explains that high-ability students have several aspects of mathematical communication that are still difficult for other students, including compiling a story problem from a given picture and drawing a problem situation in a visual form[23].

SS' Result

SS did not write about what was asked by the problem (Figure 2). SS wrote what was known from the situation but made a symbol that represents two different concepts. In Figure 2, we can see that SS wrote $P_{ganjil}$ to represent a set and probability. He also made a mistake when used an equal sign. Figure 2 show that SS wrote $\{1, 3, 5\} = \frac{3}{6}$. From the test result, the researcher knows that SS did not understand the equal sign symbol. It means SS did not fulfill the third indicator (K3).

Through an interview, SS could explain the problem’s intention, although he could not write it correctly. So, SS did not fulfill the first indicators of mathematical communication. SS is unable to express mathematical ideas in writing (K1). Although his statement was correct, he did not execute his argument correctly, which mean he did not get the second indicator (K2). SS can represent the union symbol. But the steps of completion were not perfect. The following show SS thought through an interview.

M: What do these symbols represent? (Pointed at"$P_{ganjil}$ and"$P_{prima}$")
SS: The probability of odd number in dice ma'am (unsure)
M: You wrote $P_{ganjil} = \{1,3,5\}$. Is it means that $P_{ganjil}$ is a set?
SS: mmm ... I forgot, ma'am

![Figure 2. Test Result of Medium Ability Students (SS)](image)

It can be concluded that the SS has not fulfilled the three mathematical communication indicators K1, K2, and K3, and SS lacks in mathematical communication skills. This result is supported by Muqtada, Irawati, & Qohar [24], which explains how students communicate poorly and cannot carry out oral and written communication. Students cannot explain the method they were used, do not use the notation correctly, solve unstructured problems, and many jumps occur [7]. Research conducted by Tanjungpura [25] and Maulyda & Hidayati [26] explains that students with moderate abilities in some aspects of mathematical communication are still challenging to compile some story problems from the pictures given, word problem, and others.
SR's Result

SR wrote his idea to solve the problem, as shown in Figure 3. It can be seen that SR did not note what was known and asked from the situation. He also seemed not to understand the meaning of the problem. SR used a symbol "P" to represent a probability and set, but when the researcher asked further which symbol meant, he could not explain his result [27]. Based on mathematics communication indicators, SR did not express mathematical ideas through oral, written, and visual describing (K1). SR also could not analyze and evaluate mathematical concepts, both verbally and in writing (K2). In the end, SR could not use mathematical terms, language or symbols, and structures to model mathematical situations or problems (K3)[3]. The following explains SR thought process.

M: What is the purpose of this "P" symbol?
SR: "Opportunity" ma'am.
M: What opportunity?
SR: mmm ... mmm .. (student is silent).

![Figure 3. Test Result of Low Ability Students (SR)](image)

From the interview, it appears that SR did not understand probability and set concepts. He could not communicate his idea in both written and oral forms because he did not know the problem meaning and how to solve it. So it can be concluded that SR did not fulfill the three mathematical communication indicators. It can be explained by a study conducted by Muqatada [24], which demonstrates that students with low ability can not communicate orally or in writing form because they are not able to catch what is asked about the problem and students experience errors in the process. Sir & Delice explain that students' low-ability are the same as students' medium-ability; in some aspects of mathematical communication, they find it difficult to arrange a story problem from a given picture, draw a problem situation in the form of space objects, and draw and calculate the results of solving the problem [28].

CONCLUSION AND SUGGESTION

Based on the results and discussions that have been carried out with three different subjects—high ability student (ST), moderate ability student (SS), and low ability student (SR)—of Sombron Elementary School at class VI, it can be concluded that ST and SS understand the meaning of the problem given and they know what was asked from the issue, but they did not use a correct symbol while writing their idea. ST and SS also made some mistakes in using mathematics symbols.

SR did not fulfill all three mathematical communication indicators through oral, written, and describe visually; he could not analyze and evaluate ideas both verbally and in writing; and
could not use terms, language, symbols, and writing structure when modeling a problem. It means that SR lacks in mathematical communication skills. In general, three subjects in this research are not understood yet about symbols used in probability subject.

For further research, it is recommended to use more than one research instrument, such as some questionnaire tests or interviews. So, we can find out students' weaknesses or difficulties in communicating mathematics ideas. Finally, the teacher should choose learning models that can improve students' mathematics communication skills wisely.

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