Higgs Dynamics during Inflation

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ABSTRACT: We investigate inflationary Higgs dynamics and constraints on the Standard Model parameters assuming the Higgs potential, computed to next-to-next leading order precision, is not significantly affected by new physics. For a high inflationary scale $H \sim 10^{14}$ GeV suggested by BICEP2, we show that the Higgs is a light field subject to fluctuations which affect its dynamics in a stochastic way. Starting from its inflationary value the Higgs must be able to relax to the Standard Model vacuum well before the electroweak scale. We find that this is consistent with the high inflationary scale only if the top mass $m_t$ is significantly below the best fit value. The region within $2\sigma$ errors of the measured $m_t$, the Higgs mass $m_h$ and the strong coupling $\alpha_s$ and consistent with inflation covers approximately the interval $m_t \lesssim 171.8 \text{ GeV} + 0.538(m_h - 125.5 \text{ GeV})$ with $125.4 \text{ GeV} \lesssim m_h \lesssim 126.3 \text{ GeV}$. If the low top mass region could be definitively ruled out, the observed high inflationary scale alone, if confirmed, would seem to imply new physics necessarily modifying the Standard Model Higgs potential below the inflationary scale.
1. Introduction

With the confirmed discovery of the Standard Model (SM) Higgs boson at LHC [1], it is now apposite to study in detail the evolution of the Standard Model Higgs during inflation. Here we do not adopt any particular inflationary model but merely assume that there is a period of superluminal expansion with a very slowly changing Hubble rate $H$. Intriguingly, as has been much discussed lately, the Higgs field could be the inflation [2, 3, 4, 5], albeit at the expense of an abnormally large non-minimal coupling to gravity. However, if confirmed, the detection of primordial gravitational waves by BICEP2 determines the inflationary energy scale to be $\rho^{1/4} \sim 10^{16}$ GeV at the horizon crossing of the observable patch, together with a tensor-to-scalar ratio $r = 0.20^{+0.07}_{-0.05}$ [6], which appears to be at odds with Higgs inflation, see however [7, 8, 9]. Here we do not consider this or any other modified SM scenario but rather investigate SM Higgs dynamics assuming its couplings are not significantly affected by whatever the new physics is driving inflation.

The starting point for our analysis is the next-to-next to leading order expression for the SM effective Higgs potential. As is well known, at high Higgs field values the SM potential typically becomes unstable as one moves beyond a critical field value $h = h_c$ and there is a local maximum located at $h_{\text{max}} < h_c$. Both the point of instability $h_c$ and $h_{\text{max}}$ are very sensitive to the SM parameter values as measured at the electroweak scale [10, 11, 12]; for the best fit SM parameters $h_c \sim 10^{10}$ GeV. However, the instability can be pushed up to $10^{16}$ GeV and beyond by lowering the top mass value. Consistency of the setup of course requires the Higgs potential to
be stable at the inflationary scale implied by BICEP2 [6]. In particular, as pointed out already in [13] and later discussed in [14], the inflationary fluctuations should not push the Higgs field over the local maximum $h_{\text{max}}$ and into the false vacuum during the last 60 e-folds of inflation, corresponding to the observable universe. In other words: we require that at the end of inflation we find ourselves in the region of field space from where the SM vacuum can dynamically be reached. Imposing this constraint we identify the region in the space of the top mass, the Higgs mass, and strong coupling, where the SM Higgs potential remains compatible with the measured inflationary scale of $\rho^{1/4} \sim 10^{16}$ GeV.

Given the generic form of the Higgs potential the question then is: how does the Higgs field evolve during inflation? The answer very much depends on whether the Higgs is a light field or not, but also on the initial field value. Our starting point is that well before the electroweak symmetry breaking takes place, the Higgs field must find itself far away from the instability and close to the low-energy vacuum $h = \nu \simeq 246$ GeV. If at the onset of inflation, the Higgs is on the wrong side of the local maximum at $h_{\text{max}}$, it must tunnel during inflation to the other side unless the false vacuum is lifted by thermal corrections after the end of inflation. Since the SM expression of the Higgs potential much beyond $h_{\text{max}}$ must be modified by unknown new physics, we cannot assign a model-independent probability measure for such a tunneling event. However, since tunneling rates depend exponentially on the differences of the free energies, if tunneling from $h \gg h_{\text{max}}$ takes place, afterwards the most probable field value is $h_{\text{max}}$, the local maximum. Tunneling could take place any time before the end of inflation, and of course, the initial field value could also be $h \ll h_{\text{max}}$ simply by chance.

We are thus led to study the dynamics starting from arbitrary initial values in the range $h \leq h_{\text{max}}$. We find that the SM Higgs is either a light field to start with or becomes light after at most a few e-folds, and its energy density is small compared to the inflationary scale. As its contribution to the total energy density is tiny, the Higgs condensate (zero mode) field acquires nearly scale invariant fluctuations on superhorizon scales. We find that quantum fluctuations dominate over the classical motion close to the maximum $h_{\text{max}}$ as well as in the asymptotic regime $h \ll h_{\text{max}}$. In the asymptotic quantum regime the mean field fluctuates and is random walking while local perturbations are also being generated. The typical values of the Higgs condensate after the end of inflation, together with its fluctuations, are directly determined by the inflationary scale. If the mechanism for generating curvature perturbation is sensitive to the Higgs value, for example through a modulation of the inflaton decay rate \cite{15, 16}, the Higgs fluctuations could leave an imprint in the primordial metric perturbations. In this case the transition from classical Higgs dynamics to the quantum regime could also generate characteristic features in the primordial perturbations, provided the transition occurs when observable scales are crossing the horizon.
The paper is organized as follows. In section 2 we review the form of the radiatively corrected SM Higgs potential and derive consistency conditions for a high scale inflation with the SM Higgs as a spectator field during inflation. In section 3 we present a detailed analysis of the dynamics of the SM Higgs during inflation. Finally, in section 4 we summarize the results and discuss possible consequences of SM modifications.

2. Standard Model Higgs and high scale inflation

For large field values $h \gg v \simeq 246$ GeV the radiatively corrected effective potential of the Standard Model Higgs can be expressed in the form

$$V(h) = \frac{\lambda(h)}{4} h^4.$$  \hspace{1cm} (2.1)

The running of $\lambda(h)$ has been computed explicitly to next-to-next to leading order precision $[10, 11, 12]$. Throughout this work we will refer to the effective potential evaluated in the $\overline{MS}$ renormalization scheme, and all the couplings and field values are given within this scheme. As these are not directly physical, a different choice of the renormalization scheme would give different numbers to be associated with the same physical quantities.

The self coupling $\lambda(h)$ is determined from its $\beta$-function

$$\beta_{\lambda} = \frac{d\lambda}{d\ln h},$$  \hspace{1cm} (2.2)

which together with the $\beta$-functions of the other couplings forms a set of coupled differential equations. At one-loop level the dominant contribution would read $\beta_{\lambda}^{(1)} = 12\lambda^2 + 6y_t^2\lambda - 3y_t^4$, where $y_t$ is the top quark Yukawa coupling. At higher orders one also has to account for the coupling to gluons. To solve for the coupling $\lambda(h)$, we will employ the next-to-next to leading order code available at [17], which is based on $[11, 12]$. For the best fit values of the electroweak scale Higgs mass $m_h = 125.7$ GeV, top mass $m_t = 173.1$ GeV and the strong coupling constant $\alpha_s = 0.1184$, the Higgs potential takes the form shown in Figure 1. The local maximum $V'(h_{\text{max}}) = 0$ occurs at

$$\lambda(h_{\text{max}}) + \frac{\beta(h_{\text{max}})}{4} = 0.$$  \hspace{1cm} (2.3)

The maximum eventually vanishes if the Higgs mass is sufficiently increased or the top mass decreased. Similarly, increasing the Higgs mass moves the instability scale towards higher field values while increasing the top mass works in the opposite direction.

Above the instability scale $h > h_c$ the SM vacuum would no longer be the global minimum and new physics must be evoked to restore its stability. Although not
Figure 1: The effective potential of the Higgs field, \( V = \frac{1}{4} \lambda(h) h^4 \), for the best fit SM parameters \( m_t = 173.1 \text{ GeV}, \alpha_S = 0.1184 \) and \( m_h = 125.7 \text{ GeV} \). Here \( h_{\text{max}} \simeq 2.1 \times 10^{10} \text{ GeV} \) and \( h_c \simeq 2.8 \times 10^{10} \text{ GeV} \).

required by theoretical consistency, new physics could of course appear already at much lower energies. Here we concentrate on the Higgs dynamics within the SM in the stable regime \( h < h_c \) assuming the new physics does not significantly affect the potential in this regime.

2.1 Conditions for consistency

Without a large non-minimal coupling to gravity, \( \xi h^2 R \) with \( \xi \gg 1 \), the SM Higgs potential (2.1) in general is not flat enough to support slow roll inflation with at least the required \( N_{\text{CMB}} \sim 60 \) e-folds. For a very fine-tuned choice of the SM parameters the potential develops a saddle point or a shallow false minimum. In the vicinity of this point Higgs inflation could occur for more moderate values of the non-minimal coupling and even yield the measured tensor-to-scalar ratio \( r \approx 0.2 \) [6].

In the pure SM case the Higgs potential is however too steep to support inflation, and the same holds true even if small modifications, such as a small non-minimal coupling \( \xi \lesssim 1 \), are added to the model. Therefore, if the SM Higgs potential is not strongly modified at the inflationary scale, inflation should be driven by new physics. The Higgs energy density must then be subdominant in order not to spoil the inflationary epoch. This implies that the allowed range of Higgs values and SM parameters is constrained by

\[
V_{\text{SM}}^{1/4}(h) \ll \left( 3 M_P^2 H^2 \right)^{1/4} \approx 1.6 \times 10^{16} \text{ GeV} .
\]  

Here we have used the BICEP2 detection of tensor-to-scalar ratio \( r \approx 0.2 \) [6] to fix the inflationary scale \( \rho_{\text{inf}}^{1/4} \approx 1.6 \times 10^{16} \text{ GeV} \).
From whatever value the Higgs field initially takes within the allowed range, it should relax close to the SM vacuum \( h = \nu \simeq 246 \text{ GeV} \) well before the electroweak symmetry breaking crossover. If the Higgs field finds itself beyond the local maximum \( h > h_{\text{max}} \) at some point during inflation, it should tunnel to the regime \( h < h_{\text{max}} \) and stay there. As we will show in the next section, the SM Higgs is a light field for \( h < h_{\text{max}} \) and subject to fluctuations generated by the inflationary expansion. If the energy density stored in the fluctuations is higher than the height of the Higgs potential at \( h_{\text{max}} \), the fluctuations will generically push the Higgs back to the regime \( h > h_{\text{max}} \), which is incompatible with the observed universe. Requiring the SM vacuum to be stable against inflationary fluctuations then yields another constraint between SM parameters and the inflationary scale \([13, 14]\).

Taking into account that the kinetic energy of the fluctuations is \( \rho_{\text{kin}}(h) \sim H^4 \) and using the inflationary scale \( H \simeq 10^{14} \text{ GeV} \) implied by BICEP2 the stability condition against inflationary fluctuations is given by

\[
V^{1/4}(h_{\text{max}}) \gtrsim 10^{14} \text{GeV} . \tag{2.5}
\]

It is readily seen that the Higgs energy density is negligible (2.4) whenever the above inequality holds. The condition (2.5) places a direct constraint on the SM parameters, \( m_h, m_t \) and \( \alpha_s \), which determine the scale \( V(h_{\text{max}}) \). Therefore, if no new physics modifies the SM up to the inflationary scale \( \rho_{\text{inf}} \sim 10^{16} \text{ GeV} \), the SM parameters should lie within the regime depicted in Figure 2 where the inequality (2.5) is satisfied. For parameter values outside this regime, in particular for the best fit SM parameter values, the inflationary fluctuations would rapidly push the Higgs field into the false vacuum \( h > h_{\text{max}} \), suggesting that new physics is required to modify the Higgs potential and make it stable against inflationary fluctuations.

The constraint (2.5) can be satisfied within the SM only if the top mass is significantly below the best fit value. From the figure, one finds that the region still consistent with inflation and within 2σ errors of the measured SM parameters is given by

\[
m_t \lesssim 171.8 \text{ GeV} + 0.538(m_h - 125.5 \text{ GeV}) \qquad 125.4 \text{ GeV} \lesssim m_h \lesssim 126.3 \text{ GeV} . \tag{2.6}
\]

One might ask how the constraint (2.5) changes if the Standard Model is slightly modified. After all, new physics is in any case needed above the instability scale of the SM and we have also explicitly assumed that inflation is driven by fields beyond SM which inevitably couple to SM at least through gravitational interactions. As long as the new fields coupled to Higgs can be integrated out during inflation, the changes of the Higgs potential at the inflationary scale can be encoded into a change in its coupling \( \lambda \rightarrow \lambda + \delta \lambda \). For a modified SM we can then schematically write a stability condition against inflationary fluctuations analogous to (2.5)

\[
\frac{V_{\text{SM+mod}}(h_{\text{max}})}{H^4} = \frac{V_{\text{SM}}(h_{\text{max}})}{H^4} \left( 1 + \frac{\delta \lambda}{\lambda} \right) \gtrsim 1 . \tag{2.7}
\]
Figure 2: The dots depict the top mass $m_t$ and Higgs mass $m_h$ region consistent with the observed SM vacuum, pure SM during inflation and the inflationary scale $H = 1.2 \times 10^{14}$ GeV as implied by the BICEP2 detection. We have marginalized over the strong coupling constant $\alpha_s = 0.1184 \pm 0.0007$ [18], the results marginalized over the observational $1\sigma$, $2\sigma$ and $3\sigma$ regimes are depicted respectively by large, medium and small dots. The contours in the figure depict the $1\sigma$, $2\sigma$ and $3\sigma$ regions of $m_t = (173.1 \pm 0.7)\text{GeV}$ [10, 19] and $m_h = (125.7 \pm 0.4)\text{GeV}$ [1, 20].

Therefore, we find that the condition (2.5) is not significantly affected unless the modifications are sizeable $|\delta \lambda| \gg \lambda$.

Let us also note in passing that the new physics might be such that thermal effects after the end of inflation would lift the false vacuum. In this case it would be possible to reach the SM vacuum even if inflationary fluctuations were to push the Higgs field over the local maximum into the regime $h > h_{\text{max}}$. Making more quantitative statements about such a case would however require the specification of the unknown nature of new physics. In what follows we will therefore stick to the condition (2.5) that is valid within the SM.

3. Generation and dynamics of a Higgs condensate

Having specified the very generic condition (2.5) for the consistency of the SM Higgs with the measured inflationary scale $\rho_{\text{inf}}^{1/4} \sim 10^{16}$ GeV, we now move on to study the Higgs dynamics during inflation in more detail.

As discussed above, if the Higgs finds itself in a false vacuum in the regime $h > h_{\text{max}}$ at some point during inflation, it should tunnel to the regime $h < h_{\text{max}}$ before the end of inflation. Here we assume the SM potential beyond the instability scale $h > h_c > h_{\text{max}}$ is stabilized by some new physics which does not affect the
potential below $h_{\text{max}}$. The tunneling probability is maximized for a process that leaves the Higgs field at $h = h_{\text{max}}$ with a zero kinetic energy. Denoting the difference between the false vacuum and the local maximum as $\Delta V = V(h_{\text{max}}) - V(h_{\text{false}})$ (we assume $V(h_{\text{false}}) > 0$) the tunneling rate can be estimated by (see e.g. [21])

$$\Gamma/H \sim \exp\left(-\frac{8\pi^2 \Delta V}{3H^4}\right). \quad (3.1)$$

Here we have neglected the change of the Hubble scale $H$ as the Higgs contribution to the total energy density is required to be negligible (2.4). While the tunneling rate is suppressed by the condition (2.5), unless the false vacuum would be very shallow, it could be compensated by a very long period of inflation before the onset of the observable e-folds. So if inflation lasts sufficiently long the Higgs could initially start from the false vacuum. Tunneling could of course take place also during the observable e-foldings but the probability for this process is suppressed by the limited number of available e-folds, $N_{\text{CMB}} \sim 60$.

3.1 Dynamics close to the local maximum

Using the next-to-next-to leading order result for the radiatively corrected SM Higgs potential, the effective Higgs mass in the regime $h < h_{\text{max}}$ can be computed. We find that the Higgs is always massless at the local maximum $m_h \ll H^2$ in the regime consistent with the inflationary scale $H \sim 10^{14}$ GeV implied by BICEP2 and less than $2\sigma$ away from the measured SM parameters. Allowing for deviations at $3\sigma$ level, we find that the Higgs could be massive at $h_{\text{max}}$ but even in this case it becomes massless within $N \lesssim 5$ e-folds. For all practical purposes we can therefore treat the SM Higgs as a light field after it has tunneled away from the false vacuum.

Immediately after the tunneling to $h_{\text{max}}$ the classical force vanishes as $V'(h_{\text{max}}) \sim 0$, and the light Higgs field then undergoes random walk in the vicinity of $h_{\text{max}}$. As the different regions of the $h = h_{\text{max}}$ bubble become stretched out of causal connection by the inflationary expansion the stochastic Higgs evolution away from $h_{\text{max}}$ in general differs from patch to patch. In each patch the stochastic epoch eventually ends when the field has drifted to the point where the classical force $V' = -3H\dot{h}$ equals the quantum source term $\delta h/\delta t \sim H^2/2\pi$

$$V'(h_{\text{cl}}) = -\frac{3H^3}{2\pi}. \quad (3.2)$$

We only concentrate on the patches where $h_{\text{cl}} < h_{\text{max}}$ and the classical drift in the regime $h < h_{\text{cl}}$ drives the Higgs field towards the SM vacuum. The other patches where $h > h_{\text{max}}$ will relax back to the false vacuum and cannot describe the observable universe unless the Higgs again tunnels to $h_{\text{max}}$ and ends up rolling away from the false vacuum.
While the duration of the stochastic epoch differs from patch to patch we may estimate its typical time scale by investigating the behaviour of the two-point function $\langle h^2 \rangle$. The probability distribution $P(h, t)$ for the Higgs field on superhorizon scales obeys the Fokker-Planck equation \[\frac{\partial \langle h^2 \rangle}{\partial t} = \frac{H^3}{4\pi^2} - \frac{2}{3H} \langle hV'(h) \rangle.\] \[(3.3)\]
The first term on the right hand side represents the contribution of quantum fluctuations, and the second term corresponds to the classical drift which starts to grow as the field moves away from the local maximum $V'(h_{\max}) = 0$. Expanding the potential up to second order in the displacement $h - h_{\max}$,

\[V(h) = V(h_{\max}) \left( 1 + \frac{3}{2} H^3 \eta_{\max} (h - h_{\max})^2 \right),\] \[(3.4)\]
we can solve equation (3.3) for the variance as

\[\langle (h - h_{\max})^2 \rangle = \frac{H^2}{8\pi^2 \eta_{\max}} (1 - \exp(-2\eta_{\max} N)) ,\] \[(3.5)\]
We solve for the number of e-folds by equating the root mean square of the variance to the limiting field value of the classical regime (3.2) $\sqrt{\langle (h - h_{\max})^2 \rangle} = h - h_{\cl}$. Thus we find an estimate for the typical duration of the stochastic epoch after the tunneling as

\[N_{\cl} = \ln \frac{2}{2 |\eta_{\max}|} .\] \[(3.6)\]
For the SM parameters consistent with the vacuum stability against inflationary fluctuations, depicted in Figure 2, we find $N_{\cl} \lesssim 20$.

The actual time when the classical regime $h_{\cl}$ is reached differs from patch to patch and is fluctuating around the average $N_{\cl}$. If the currently observable scales exited the horizon well after this epoch the differences are unobservable as the subsequent classical evolution carries no memory of the stochastic epoch. On the other hand, if the observable scales were still inside the horizon when the field value $h_{\cl}$ was reached, the slight differences in the expansion history might generate non-trivial features in the spectrum of Higgs perturbations. If the fluctuations of the subdominant Higgs condensate are converted to primordial perturbations after the end of inflation this structure could be imprinted in the CMB perturbations.

3.2 Intermediate stage and asymptotic dynamics

The dynamics of the Higgs field in each Hubble patch becomes dominated by the classical drift $3H \dot{h} \simeq -V'$ when the field has rolled down to $h_{\cl}$ (3.2). As the field keeps rolling towards the minimum, the slope $V'(h)$ starts to decrease and
eventually the dynamics again becomes dominated by the stochastic quantum noise. This happens for $h < h_{\text{as}}$, where

$$V'(h_{\text{as}}) = -\frac{3H^3}{2\pi}, \quad h_{\text{as}} < h_{\text{cl}}. \quad (3.7)$$

For the SM parameter values in Figure 2 consistent with $H_{\text{int}} \sim 10^{14}$ GeV we find that the Higgs can stay at most $N_{\text{int}} \lesssim 70$ e-folds in the classical regime $h_{\text{as}} < h < h_{\text{cl}}$. As the field has rolled down to $h_{\text{as}}$ its motion becomes dominated by the quantum fluctuations. The transition from the classical to stochastic epoch could again leave observable imprints into the primordial perturbations sourced by the Higgs condensate provided the transition takes place when observable scales are leaving the inflationary horizon.

In the asymptotic stochastic regime $h < h_{\text{as}}$ the classical drift towards the Standard Model vacuum gets overwhelmed by the backreaction of the generated quantum fluctuations and field starts to undergo a random walk. At the onset of the stochastic epoch the probability distribution of the Higgs field over a horizon patch is peaked around $h_{\text{as}}$ and then starts to spread out and move towards the equilibrium distribution $[22]$

$$P(h) \simeq C \exp \left( -\frac{8\pi^2 V(h)}{3H^4} \right). \quad (3.8)$$

The stability condition (2.5) guarantees that the equilibrium probability to fluctuate to the regime of the false vacuum is heavily suppressed. We can then normalize the probability within the regime $|h| < |h_{\text{max}}|$ so that

$$C^{-1} = \int_{-h_{\text{max}}}^{h_{\text{max}}} dh P(h). \quad (3.9)$$

In the asymptotic regime $h < h_{\text{as}}$ the running of the coupling $\lambda(h)$ in (2.1) is a small effect and the Higgs potential is nearly quartic with $V \sim h^4$. For a quartic potential, the spreading of an initial probability distribution towards the equilibrium result (3.8) is characterized by a decoherence time, which in terms of e-folds has been found $[24]$ to be given by $N_{\text{dec}} \approx 6\lambda^{-1/2}$. Using this estimate for the SM Higgs we find $N_{\text{dec}} \lesssim 100$. Therefore, if there was at least $N_{\text{cl}} + N_{\text{int}} + N_{\text{dec}} = \mathcal{O}(200) \text{ e-folds}$ of inflation from the tunneling of the Higgs to $h_{\text{max}}$ until the horizon exit of the observable scales, the Higgs amplitude is controlled by the equilibrium distribution (3.8) at the time during which the observable CMB scales are leaving the horizon. An estimate of the typical Higgs value in our patch is then given by the root mean square $h = \sqrt{\langle h^2 \rangle}$ computed from (3.8). Treating the Higgs coupling $\lambda(h)$ as a constant one then finds $[25]$

$$h \simeq 0.4\lambda^{-1/4} H \sim 10^{14}\text{GeV} \quad (3.10)$$
for the inflationary scale $H \sim 10^{14}$ GeV implied by BICEP2 [6]. We have checked that including the running of the coupling $\lambda(h)$ in (3.8) does not significantly change the result within the parameter range consistent with the stability condition (2.5) as depicted in Figure 2.

4. Discussion

We have considered the constraints imposed on the Standard Model by the assumption that up to the inflationary scale, the Higgs potential is at least approximately given by the pure SM prediction and not significantly affected by the field(s) driving inflation. These constraints are of cosmological nature and follow from the fact that during inflation, for all practical purposes the SM Higgs is a light field, which we have verified. Thus during inflation the Higgs field is subject to fluctuations: there will be local field perturbations, but in addition, also the mean field performs a random walk. If inflation lasts long enough, about 200 efolds, the mean field will have settled into its equilibrium distribution, that can be derived in the stochastic approach, by the horizon exit of observable scales. This will provide the initial condition for the Higgs condensate after inflation which is an integral part of the initial data for the subsequent hot big bang epoch.

For the best fit parameters and in the next-to-next leading order, the potential of the SM Higgs has a local maximum at large field values, $h_{\text{max}} \sim 10^{10}$ GeV. Beyond the maximum there is a false vacuum, which can be either stable or unstable. If unstable, it should be stabilized by new physics modifying the SM potential above scale of the local maximum. The basic assumption here is that new physics has no significant impact on the Higgs potential at field values below $h_{\text{max}}$.

Whatever the value the Higgs field had at the end of inflation, it should relax to the SM vacuum by the time the electroweak symmetry breaking takes place. Unless the false vacuum gets lifted by thermal corrections after inflation, or is extremely shallow, this requirement implies that the Higgs field at the end of inflation must be at or below the local maximum $h_{\text{max}}$ so that it can relax into the correct vacuum by classical dynamics. Here we have pointed out, see also [13, 14], that for the best fit values this requirement is in tension with the high inflationary scale inflationary scale $H_{\text{inf}} \sim 10^{14}$ GeV implied by the BICEP2 detection of gravitational waves.

During inflation the SM Higgs turns out to be effectively massless for field values below the local maximum. Hence the mean field acquires fluctuations proportional to the inflationary scale $\delta h \sim H_{\text{inf}} \sim 10^{14}$ GeV. Therefore, it is not enough that during inflation the Higgs is located below $h_{\text{max}}$ when the observable scales exit the horizon. This configuration has to be also stable against inflationary fluctuations, which could carry the mean field over into the false vacuum. We argue that the tunneling rate out of the false vacuum should be negligible over the observable e-folds. We then show that the condition for the stability is given by $V(h_{\text{max}}) \gtrsim H^4$, 

\[ V(h_{\text{max}}) \gtrsim H^4, \]
where $V(h_{\text{max}})$ is the potential energy at the local maximum. Computing $V(h_{\text{max}})$ in the next-to-next leading order, we find that the SM Higgs the stability is guaranteed only for a sufficiently low top mass with 2-3 $\sigma$ below the best fit value, depending on the measured values of $m_h$ and $\alpha_s$.

There may be particle physics reasons for extending the Standard Model, but if the still allowed parameter region depicted in Fig. 2 can be ruled out, the observed high inflationary scale alone would require new physics modifying the Higgs potential. The required modifications should be significant as moderate shifts $|\delta \lambda| \sim \lambda_{\text{SM}}$ of the effective Higgs coupling from its SM value at the inflationary scale would not affect the orders of magnitude in the stability condition $V(h_{\text{max}}) \gtrsim H^4$. Note that since $H \propto r^{1/2}$ our conclusion is also not sensitive to the exact value of the tensor-to-scalar ratio. Even if the observed tensor-to-scalar ratio would go significantly down from $r \sim 0.2$ the SM vacuum for the best fit parameter values would remain unstable against inflationary fluctuations.

We have also carefully investigated the Higgs dynamics during inflation for the SM parameters consistent with the stability condition $V(h_{\text{max}}) \gtrsim H^4$. We have argued that the transitions between classical and stochastic regimes in the Higgs dynamics could leave distinct imprints in the spectrum of Higgs fluctuations. If the transitions occur when the observable scales leave the horizon, and if the Higgs perturbations source either adiabatic or isocurvature metric fluctuations, these imprints could be observable in the CMB.

While the paper was in preparation, there appeared an article [26] which also discusses SM stability in the light of BICEP2, with which our results are in a qualitative agreement.

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