Maintaining cooperation in complex social dilemmas using deep reinforcement learning

Adam Lerer\textsuperscript{*}  
Facebook AI Research

Alexander Peysakhovich\textsuperscript{*}  
Facebook AI Research

Abstract

In social dilemmas individuals face a temptation to increase their payoffs in the short run at a cost to the long run total welfare. Much is known about how cooperation can be stabilized in the simplest of such settings: repeated Prisoner’s Dilemma games. However, there is relatively little work on generalizing these insights to more complex situations. Solving this problem is extremely important if we wish to construct artificially intelligent agents that interact with each other and humans in real world scenarios. We show how to use modern reinforcement learning methods to generalize a highly successful Prisoner’s Dilemma strategy: tit-for-tat. We construct agents that act in ways that are simple to understand, nice (begin by cooperating), provokable (try to avoid being exploited), and forgiving (following a bad turn try to return to mutual cooperation). We show both theoretically and experimentally that such agents can maintain cooperation in more complex environments. We also show that such agents teach simple reactive agents to be cooperative. In contrast, we show that standard ‘reactive self-play’ paradigms, which have made great strides in zero-sum games, may not be enough for constructing agents that maintain cooperation in positive-sum interactions.

1 Introduction

A key component of human sociality is the ability to maintain long run cooperative relationships with others when both partners face short term incentives to defect. In such situations humans often use the ‘shadow of the future’ - rewarding a partner’s cooperation today with our cooperation tomorrow - as a way to solve these social dilemmas (Axelrod, 2006; Nowak, 2006; Fudenberg & Maskin, 1986). Here we investigate how to construct agents that can maintain cooperation in this way while avoiding being exploited by cheaters.

Recent work in deep reinforcement learning (RL) has made great strides towards artificial agents that can learn to achieve goals in complex environments. However, the biggest successes in this literature have come in environments that are either single-agent (e.g. Atari Mnih et al. (2015)), zero-sum\textsuperscript{2} or coordination games without a temptation to defect\textsuperscript{3}.

Outside of deep RL there is a large body of theoretical (Axelrod, 2006; Fudenberg & Maskin, 1986), simulation-based (Nowak, 2006; Macy & Flache, 2002) and experimental (Bo, 2005; Fudenberg et al., 2012) work studying the emergence of bilateral cooperation. A weakness of this research program is

\textsuperscript{*}Equal contribution. Author order was determined via random.org.

\textsuperscript{2}Zero sum games are those where for one player to win, another player must lose. These include Backgammon (Tesauro, 1995), Go (Silver et al., 2016), first person shooter games (Kempka et al., 2016; Wu & Tian, 2016), poker (Heinrich & Silver, 2016) and StarCraft (Ontanón et al., 2013; Foerster et al., 2017; Usunier et al., 2016).

\textsuperscript{3}This class of games are games where both players win or lose together. It includes team-based coordination games (Lowe et al., 2017), sports like soccer (Riedmüller et al., 2009) as well as the growing RL-based literature on language emergence in games (Lazaridou et al., 2017; Das et al., 2017; Evtimova et al., 2017; Havrylov & Titov, 2017; Jorge et al., 2016).

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that it almost exclusively studies simplified matrix-based games such as the two action, stochastically repeated Prisoner’s Dilemma (PD).\footnote{Some modern work has gone beyond the simple PD setup. \cite{Kleiman-Weiner2016} propose a model that infers the intention of other agents and decides whether to cooperate. Constructing this model uses some similar techniques to what we propose but the research is primarily concerned with modeling what humans actually do rather than constructing artificial agents or formally proving the properties of certain strategies. \cite{Leibo2017,Perolat2017} study standard RL agents in more complex social dilemmas but are primarily concerned with describing how incentives affect the emergence of cooperative or non-cooperative behavior. There has also been recent interest in natural language bargaining which is far less structured than a PD but also not zero-sum \cite{Lewis2017}.}

Our key contribution is to take the game theoretic insights gleaned from the simple PD and combine them with tools from deep reinforcement learning to construct agents that are capable of solving arbitrary bilateral social dilemmas via the shadow of the future. We do this by generalizing a simple yet successful strategy for stochastically repeated PD play: tit-for-tat \cite{Axelrod2006}.

Tit-for-tat (TFT) works by copying the prior behavior of their partner. If a partner cooperated last period, TFT cooperates, otherwise it defects. TFT has many desirable properties: it is clear (the partner understands it), nice (it begins by cooperating), provokable (it cannot be exploited by a pure defector), and forgiving (following a bad turn in the relationship TFT returns to cooperation if the partner returns to cooperation).

A central feature of TFT is that if one agent can credibly commit to TFT then for their partner the strategy of always cooperating dominates all other policy choices. This property is important if we are going to be designing artificial agents that will interact with others (including humans) in real situations. If the artificial agent is a form of TFT then there is a clear dominant strategy for their partner: there is no gain from coming up with ways to cheat the system, just be nice and everyone will get ahead.

Implementing a TFT-like strategy is non-trivial for general stochastic games. For example, consider what will be our workhorse situation - the Coin game. In the Coin game two players move on a 2D board. Players each have one of two colors and coins, which also have one of two colors, appear on the board periodically. A player receives a reward of 1 for collecting (moving over) any coin. However, if a player picks up a coin of the other players’ color, the other player loses 2 points. The strategy which maximizes total payoff is for each player to only pick up coins of their own color; however each player is tempted to pick up the coins of the other color. Implementation of TFT in this environment is tantamount to maintaining a deal of ‘I won’t touch your coins if you don’t touch mine.’ Note that an agent which commits to behave only in a pro-social way (ie. always only match their own color) can be exploited by an unscrupulous cheater.

It is easy for a human to look at the game and recognize the pro-social and selfish strategies as well as come up with a mutually beneficial and stable policy. Humans are good at navigating real world social dilemmas using social heuristics that have been tuned via evolution and social learning \cite{Tomasello2009,Hoffman2015,Peykakhovich2015,Rand2014}. Our goal will be to give our artificial agents simple versions of these heuristics.

We observe that the naive strategy of training purely reactive policies for the agents by reinforcement learning does not converge to socially optimal outcomes in the Coin game. This can be explained by two factors. First, in order to maintain cooperation from a selfish agent, there must be consequences tomorrow for defection today. In Markov games, there is not always sufficient information in the state today to sufficiently reveal past behavior. Therefore, there may not be Nash equilibria which achieve socially optimal payoffs. This can be solved via brute force by simply appending to the state a history of all past states and actions. However, this means the state and policy spaces will grow exponentially in memory size.

Second, even when there is sufficient information in the state to maintain a socially optimal payoff, there is no guarantee that standard RL methods will converge to these strategies. Even in the simple repeated PD the folk theorem states that there can be equilibrium trajectories which involve cooperation, equilibrium trajectories of pure defection and even equilibrium trajectories which only cooperate on time periods which are factorizable into small primes \cite{Fudenberg1986}. Indeed, we show that in a simple one-memory repeated PD where players can condition on last period behavior, reactive RL converges almost exclusively to defection. Intuitively this is because
cooperative strategies are complex (and only pay off if the other person also chooses a complex strategy) whereas defecting strategies are simple. This simplicity creates larger basins of attraction and thus prevents the evolution of cooperation (Leibo et al., 2017).

Our approach sidesteps these issues by reducing an arbitrary social dilemma to a PD-like representation. We do this by first reducing the policy space to just a few potential Markov policies. We then use sufficient summary statistics of past behavior to create a meta-policy which chooses which of the reduced policies to follow at any time. We construct a generalization of TFT for the theoretical case where the agent has access to the complete value function of the game under any state and set of policies. We show analytically that this generalization can maintain cooperation with selfish or pro-social partners. We call this generalization Markov TFT (mTFT).

In practice, agents do not have access to the true value functions. Instead, we often have access to the game to learn approximate value functions via RL methods. These value functions are not general, rather they are conditioned on a policy or set of policies. To deal with this we propose an approximate version of mTFT (amTFT) which retains most of its incentive properties but is tractable to compute.

amTFT is constructed as follows: at training time the agent receives the game and uses standard self-play to compute two sets of Markov policies (maps from current game state to action). The first set of policies is consistent with all players acting in a way to maximize the sum of rewards (we call these the Cooperation policies) and one which is consistent with everyone selfishly optimizing as if the game were a one-player Markov Decision problem (we call these the Selfish policies because they treat the other agent as a part of the environment rather than as something that can be reasoned with).

At test time, when faced with an actual partner, the amTFT agent keeps track of whether their partner’s behavior is consistent with Cooperative (C) or Selfish (D) policies. If the partner is behaving according to C, the agent responds by also behaving according to C. If the partner behaves according to D, the agent responds by behaving according to D for long enough such that any gains the partner has gotten from trying to cheat are erased. We use learned value functions and Monte Carlo policy rollout to implement the computations required for this switching. We discuss conditions on the stochastic game under which the approximate version maintains the incentive properties of the full mTFT.

2 Formalizing Cooperation

We now turn to formalizing the discussion above. We begin by a brief, and no means complete, introduction to the basics of cooperation in Prisoner’s Dilemma (PD) games. The familiar reader can feel free to skip this subsection.

2.1 Cooperation in the Repeated Prisoner’s Dilemma

The PD is the workhorse game of the cooperation literature. The simplest PD has 2 players each of which has two strategies, cooperate (C) or defect (D). If both players choose C then they both receive a payoff of 1, if both choose D, they both receive a payoff of 0. If one chooses C and the other chooses D the cooperator receives the sucker payoff $-s$ and the defector receives the payoff $1 + w$. This is illustrated in the payoff matrix below, with each element of each tuple representing the rewards accruing to the row and column players respectively.

|       | Player Y |
|-------|----------|
|       | C        | D        |
| Player X | (1, 1)   | (-s, 1 + w) |
| C       | (1 + w, -s) | (0, 0) |

The dominant strategy is D (meaning the only Nash equilibrium of the one-shot game is for both to defect) while the socially optimal strategy is for both to cooperate. Cooperation can be maintained if

\[ s, w > 0 \text{ and } s > w \] otherwise the fair optimal strategy is to cycle between one player cooperating and the other defecting forever.
the PD is repeated an infinite number of times and the players discount future payoffs with rate $\delta$. An alternative interpretation of these conditions is that $1 - \delta$ can represent the probability that the game continues another period.

It is straightforward to see that cooperation can be maintained with the following strategy if $\delta$ is large enough:

**Definition 1** A Grim trigger strategy is one which cooperates in the first round and in any round where the opponent has always cooperated before. If the opponent has ever defected, Grim trigger defects.

Both players playing Grim trigger is a perfect Nash equilibrium in the repeated game - after any possible history, neither player can deviate from the Grim strategy to increase their own payoff, and, on the trajectory of play Grim will indeed implement cooperation. However, note that any slight amount of noise will inevitably destabilize Grim.

A well studied strategy is tit-for-tat (TFT Axelrod (2006)). TFT works by copying the last period play of their partner. Thus, if a partner cooperated last period, TFT cooperates, otherwise it defects. TFT is simple but came to prominence when Robert Axelrod invited game theorists to submit strategies for the repeated PD that could be implemented using computer programs. These strategies were then played against themselves and others. TFT won the tournament and has been studied relentlessly ever since. TFT has many desirable properties: it is clear (the partner understands it), nice (it begins by cooperating), provokable (it cannot be exploited by a pure defector), and forgiving (following a bad turn in the relationship TFT returns to cooperation if the partner returns to cooperation). TFT has the following, very useful, incentive property. We report the well known result here:

**Theorem 1** There exists $\hat{\delta}$ such that if $\delta \in (\hat{\delta}, 1)$ then if one’s partner is using TFT, the best response is to always cooperate.

This result is well known and we relegate its proof to the Appendix. Note that this best-response theorem implies both individuals choosing TFT is not a subgame perfect equilibrium (Fudenberg & Tirole 1991). For example, if two TFT players find themselves in a state where one defected last period and the other cooperated they will cycle between the $(C, D)$ and $(D, C)$ states forever (and one of the TFT players can do better by deviating to pure cooperation at that point). There is much work in expanding TFT or finding other equally simple strategies to deal with this issue (Nowak & Sigmund 1993; Fudenberg et al. 2012). Nevertheless, TFT is a simple starting point for strategies that maintain cooperation in more complex environments.

### 2.2 Reactive Learning and the PD

The PD gives us a simple environment to discuss the obstacles to using reactive RL to train cooperative agents. In the introduction we touched on several reasons why standard RL may not be able to maintain cooperation in Markov games. This intuition can be demonstrated in the $k$-memory PD where agents play the PD according to the payoff matrix described above. However, they can condition their action choice at each time on the last $k$ time steps of play.

Note that if $k$ is 0 then there is insufficient information in the states that agents observe to be able to maintain cooperation. As we raise $k$ we get $4^k$ possible states and thus also an exponentially increasing set of policies.

We know that for $t \geq 1$ with large $\delta$ there exist strategies which maintain cooperation. However, it is not obvious that reactive RL agents would find them. We can consider the simplest possible case, letting $t = 1$ and let the initial state be $(C, C)$ (this is the most optimistic possible setup). We consider RL agents that use policy gradient optimized using Adam (Kingma & Ba 2014) and learning rate .01 to learn policies from states (last period actions) to behavior. Note that the one-memory policy space contains TFT (copy the opponent’s action), Grim Trigger (cooperate if last period was $(C, C)$ and

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*Note that finite repetition (with known termination time) cannot sustain cooperation since by backward induction both players will defect at the last period which means both should defect at the period before which means both should defect the period before that and so on. While it is known that humans do not perform backward induction there is strong evidence that when finitely repeated games are played repeatedly (that is, each super-game is played with new partners) cooperation does unravel S. Selten & A. Stoecker (1986).*
defect otherwise) and the modified Win-Stay-Lose-Shift (cooperate after \((C, C)\) or \((D, D)\), defect otherwise).\cite{Nowak1993,Fudenberg2012}.

We run 100 replicates and for parameter settings \(w = 0.5\) and \(s = -1\) we see that 0 of 100 dyads discover mutual cooperation (Figure 1 panel 1). We sweep over parameter values of \(s\) and \(w\) each with 100 replicates and again find that when settings are such that the game is indeed a PD \((w > 0\) and \(s < 0\)) policy gradient agents almost never converge to cooperate in the last 100 rounds (Figure 1 panel 2). Note that cooperation only robustly occurs when it is a dominant strategy for both players (and thus the game is no longer a social dilemma). Thus, even in the most basic situation reactive RL seems to be insufficient to maintain cooperation straight out of the box.

![Figure 1: Results from training one-memory strategies using policy gradient in the repeated Prisoner’s Dilemma. We see that over 100 dyads all of them converge to mutual defection with our default parameters (left). We see that this is true for any set of parameters \((w > 0, s < 0)\) that make the game a social dilemma (right).](image_url)

3 Markov Games

We now move to extending the ideas developed above to more complex games. We can extend standard game theoretic definitions to the Markov Decision Problem (MDP) setups as follows.\cite{Shapley1953}

**Definition 2 (Shapley\cite{1953})** A (2-player) Markov game consists of

- A set of states \(S = \{s_1, \ldots, s_n\}\)
- A set of actions for each player \(A^1 = \{a^1_1, \ldots, a^1_k\} \), \(A^2 = \{a^2_1, \ldots, a^2_k\} \)
- A transition function \(\tau : S \times A^1 \times A^2 \rightarrow \Delta(S)\) which tells us the probability distribution on the next state as a function of current state and actions
- A reward function for each player \(R_i : S \times A^1 \times A^2 \rightarrow \mathbb{R}\) which tells us the utility that player gains from a state, action tuple

A policy for each player is a map \(\pi^i : S \rightarrow A^i\). Note that here we abuse notation and suppress the stochastic nature of transition functions (so there are lots of places where there should be an \(E\)) and we assume deterministic policies. It is easy to extend the discussion below to stochastic ones but only at the cost of additional notation.

\footnote{Note that this is different from evolutionary game theoretic results on the emergence of cooperation \cite{Nowak2006}. Those results show that indeed cooperation can robustly emerge in these kinds of strategy spaces under evolutionary processes. Those results differ because they rely on the following argument: suppose we have a population of defectors. This can be invaded by mutants of TFT because TFT can try cooperation in the first round. If it is matched with a defector, it loses once but it then defects for the rest of the time, if it is matched with another TFT then they cooperate for a long time. Thus, for sufficiently long games the risk of one round of loss is far smaller than the potential fitness gain of meeting another mutant. Thus TFT can eventually gain a foothold. It is clear why in learning scenarios such arguments cannot apply.}

\footnote{In the game theory literature such games are called stochastic games \cite{Shapley1953,Dutta1995} while in the RL literature they are called Markov games. In this paper we use the Markov game terminology.}
**Definition 3** A value function for a player $i$ inputs a state and a pair of policies $V^i(s, \pi^1, \pi^2)$ and gives the expected discounted reward to that player from starting in state $s$. We assume agents discount the future with rate $\delta$ which we subsume into the value function.

**Definition 4** A policy for agent $j$ denoted $\pi^j$ is a best response to a policy $\pi^i$ for agent $i$ if for any $\pi^j$ and any $s$ we have

$$V^j(s, \pi^i, \pi^j) \geq V^j(s, \pi^i, \pi^j').$$

We denote the set of best responses to $\pi^i$ as $BR_j(\pi^i)$.

Extending the standard definition of a Nash equilibrium to this game is:

**Definition 5 (Dutta (1995))** A Markov equilibrium is a pair of Markov policies $(\pi^1, \pi^2)$ such that $\pi^1 \in BR^1(\pi^2)$ and $\pi^2 \in BR^2(\pi^1)$.

We now extend to non-zero sum settings by considering social optima:

**Definition 6** Socially optimal Markov policies $(\pi^1_C, \pi^2_C)$ are those which, starting from any state $s$, maximize

$$V^1(s, \pi^1, \pi^2) + V^2(s, \pi^1, \pi^2).$$

For simplicity we assume that the game in question has a single pair of socially optimal Markov policies. When there is a multiplicity of such policies, coordination problems can arise [Kleiman-Weiner et al., 2016]. Solving these coordination problems (eg. by allowing communication within each time step) is a task beyond the scope of this paper.

The conditions for socially optimal policies to be Markov equilibria are well known in the stochastic games literature (Dutta, 1995). It amounts to requiring that, if players face a short term incentive to stray from the policy, action histories can somehow be baked into the state (for example, if an action causes the destruction of something in the game environment). Note, however, that if players are only able to condition on states in the MDP we cannot implement socially optimal outcomes as equilibria in general.

We move to constructing agents that can maintain cooperation. We first consider expansions of the Markov state (for each player) which we term memory augmentations. We look for memory augmentations that are a) general to any Markov game, b) simple and c) give the agent enough information to play strategies that can maintain cooperation. We formalize this as follows:

**Definition 7** A simple memory augmentation to a Markov game is a function

$$M^i: \mathbb{R}^2 \times S \times A^1 \times A^2 \rightarrow \mathbb{R}^2.$$

This allows our agents to choose policies as functions given by

$$\pi^i: S \times M^i \rightarrow A^i.$$

We call a tuple $(M^i, \pi^i)$ an augmented Markov policy.

The simple memory augmentation is itself Markov and thus can be thought of as appending a two dimensional real valued vector to the state vector (the strategies below can be implemented with a single scalar but we keep 2 to make intuition simple and notation clear). We can think of this state, for each agent, as summarizing the current state of the relationship. This allows agents to condition their actions on both the state of the environment and a summary of past behavior. We extend the definition of a Markov equilibrium to using some augmented Markov policies.

**Definition 8** Given two policies and a starting state $s$ and policies $\pi^1, \pi^2$ a trajectory is the set of actions and states that result from starting in that state and having both individuals follow those policies. A cooperative trajectory is one where at each state both individuals play according to socially optimal Markov policies.

**Definition 9** Given a pair of policies $\pi^1, \pi^2$ and a state $s$, agent $i$ and action $a$ for that agent the $Q$ function is defined as

$$Q^i(s, a, \pi^1, \pi^2) = R^i(s, a, \pi^1) + \delta V^i(\tau(s, a, \pi^1(s)), \pi^1, \pi^2).$$

$Q$ represents the expected reward gained from choosing $a$ now and then continuing according to $\pi^i$. 
Definition 10  Fix agent $i$ and the policy of the other agent. Given policy $\pi_i^a$ and state $q'$ define a one shot deviation at $s$ as behaving according to $\pi_i^a$ everywhere except choosing $a'$ at state $s$. We say that a one shot deviation is $q$-profitable if
\[
Q^i(s, a', \pi^1, \pi^2) = Q^i(s, \pi^1(s), \pi^1, \pi^2) + q.
\]

Definition 11  We say that a trajectory $t$ starting at $s$ is implementable if there exists augmented Markov policies which are an equilibrium and generate the trajectory $t$.

Theorem 2  The cooperative trajectory $t$ resulting from policies $(\pi_M^1, \pi_M^2)$ starting from state $s$ in a Markov game is implementable if for either agent $i$ and any point on the cooperative trajectory and any one-shot deviation $a'$ which is $q$-profitable we have that there exists a Markov equilibrium $(\pi_M^1, \pi_M^2)$ starting from the next state $\tau(s, a')$ such that
\[
\delta V^i(\tau(s, a', \pi_C^1(s)), \pi_M^1, \pi_M^2) + q < V^i(s, \pi_M^1, \pi_M^2).
\]

The Grim equilibrium proceeds as follows. Both agents play the cooperative policies. If either agent deviates, they move to playing some other Markov equilibrium (with worse payoffs for both than $\pi_C, \pi_C$) starting from the next state. The memory augmentation function here is trivial: $M^i$ takes the value 1 if everything has proceeded according to the prescribed path and 0 otherwise. The prescribed augmented strategies implement a Grim trigger.

As with the PD, Grim is very fragile to noise either in action implementation ('trembling hands') or in any form of policy/value function approximation. In particular, there can be many states where continuation values from any action are very close or even identical. However, $\pi_C$ prescribes one trajectory exactly. This means that just a slight bit of noise in our function approximation can cause enough joint misunderstandings such that the trigger is activated and cooperation ends.

Grim’s lack of robustness leads to a practical question: can we use the Grim strategies and construct a TFT-like strategy which, though it may not be a symmetric equilibrium (for the same reasons as TFT), will have the good robustness properties if one agent can commit to it?

3.1 Oracle Markov TFT

For illustration, suppose that agent 1 has access to an oracle with infinite compute as well as access to the true value functions of the game as well as the associated $Q$ functions for each player given by $Q^1, Q^2$. In this case we define Markov tit-for-tat (mTFT) as the following construction. We use the oracle and value functions to construct Markov policies $(\pi^C_1, \pi^C_2)$ and $(\pi^1_M, \pi^2_M)$ which are the cooperative and worst case Markov equilibrium policies respectively.

We make one additional restriction on the payoffs which generalizes a property of the PD: if one agent is cooperating and one agent is behaving according to $\pi_M$ then the cooperating agent is worse off than when both cooperate. Formally, this is:

Definition 12  Say that $\pi_M$ withholds cooperation if for either agent $i$ and any state $s$ if $\pi_i = \pi^1_C$ and $\pi_i = \pi^i_M$ then
\[
V^i(s, \pi^1_C, \pi^1_M) < V^i(s, \pi^i_C, \pi^i_M).
\]

The mTFT agent keeps a counter $b$ which they initialize at 0. While the counter is at 0 the agent behaves according to $\pi_C$ (we refer to this as the $C$ phase). While the counter is above 0 the agent acts according to $\pi_M$ and decrements the counter by one every time step (we refer to this as the $D$ or punishment phase). If mTFT’s partner deviates from $\pi_C$ to some action $a'$ at any point, the mTFT agent adds $k(s, a', b)$ to the counter. $k$ is chosen using the partners’ $Q$ function to make sure that whatever the partner gained from choosing $a'$ instead of the $\pi_C$ action is erased by an additional $k$ time steps of the mTFT agent choosing according to $\pi_M$ and the partner continuing according to $\pi_C$.

mTFT allows us to preserve the incentive properties of the Grim Trigger above without the associated fragility – a single mistake in Grim Trigger ends cooperation forever whereas mTFT always gives a

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The careful reader will notice that the construction can be used to implement any trajectory that yields payoffs above some ‘min-max’ payoffs. This is explored in the game theory literature on folk theorems ([Fudenberg & Maskin 1986]), however is beyond the scope of this work.
way to repair any deviations in finite time. In addition, mTFT is robust to degenerate equilibria, that is, games which include multiple policies which implement socially optimal payoffs. Note that even if the mTFT agent is already playing according to $\pi_M$ deviations from $\pi_C$ still cause increments of the counter. The full pseudocode for the mTFT meta-policy is in algorithm 1.

Algorithm 1 Oracle Markov Tit For Tat (for agent 1)

| Input: Oracle value functions and $k^1(\cdot)$ function |
| $b \leftarrow 0$ |
| while Game do |
| if $b = 0$ then |
| Choose $a^1 = \pi^1_C(s)$ |
| if $b > 0$ then |
| Choose $a^1 = \pi^1_M(s)$ |
| $b = b - 1$ |
| if $a^2 \neq \pi_C(s)$ then |
| $b = b + k^1(s,a^2,b)$ |

Like regular TFT, Markov TFT has good properties. In particular, it returns to cooperation after a mistake as long as its partner continues to cooperate. Note that this implies that we can construct games where TFT does not incentivize a partner to always cooperate, but it does maintain cooperation on the equilibrium path (even for arbitrarily small discount rates). A simple such example is given by the following game:

| Player X | C | D |
|------|----|----|
| C | $(.5^t, .5^t)$ | $(-10, 1.5 \times .5^t)$ |
| D | $(1.5 \times .5^t, -10)$ | $(0, 0)$ |

Player Y

If the discount factor $\delta$ is equal to 1 then the payoffs from continuing down the cooperate path at any $t$ are

$$\frac{.5^t}{1-\delta} = 2 \times .5^t.$$  

Notice that if one’s partner is Grim Trigger then choosing to Defect today gives a gain of $1.5 \times 5^t$ but incurs a future loss of $2 \times .5^{t+1}$, therefore Grim Trigger (and thus mTFT) can maintain cooperation on the cooperative path. However, suppose that both players have defected last period and one’s partner is mTFT. The only way to return to cooperation is to take the hit of one turn of $(D, C)$, however the cost of $-10$ far outweighs the future benefits. Therefore, mTFT/Grim Trigger can maintain cooperation on the cooperative trajectory but mTFT cannot make it advantageous to always cooperate (conditional on any state) in this game.

Thus, for TFT to maintain cooperation everywhere requires a simple condition. Anywhere off the cooperative trajectory a partner faces a question of paying a cost now (behaving according to $\pi_C$ instead of $\pi_M$ for $k$ periods) but receiving

$$\delta^k E_{\nu^t}(V(s', \pi_C, \pi_C))$$

upon returning to cooperation as a benefit.

3.2 Approximate Markov TFT

In most applications of RL, we do not have access to the true value functions of the game. Rather, we have access to the environment itself and then learn policies as well as their associated value and $Q$ functions. We can use RL methods to construct the components required for mTFT by approximating the cooperative and punishment policies.

To construct approximations of the required policies we use self-play and two reward schedules: selfish and cooperative. In the selfish reward schedule each agent $i$ treats the other agent just as a part
of their environment and tries to maximize their own reward. This is the standard way of training RL in the various multiagent situations described in the introduction of this paper.

In this paper we are agnostic to which learning algorithm is used to compute the pieces of the mTFT meta-policy. To simplify things, in this section we assume that RL training converges and we call the converged policies under the self-play reward schedule $π_D^*$ and the associated $Q$ function approximations $Q_{D,D}$.

Note that convergence with this training implies that $π_D^*$ is the best response of agent 1 if agent 2 behaves according to $π_D^*$, and vice-versa. Thus, they form a Markov equilibrium (up to function approximation).

We also train policies and value functions using reward schedule $C$. Here each learning agent gets rewards both from their own payoff and the rewards the other agent receives. That is, we modify the reward function so that it is

$$R^C_i(s, a_1, a_2) = R_1(s, a_1, a_2) + R_2(s, a_1, a_2).$$

We call the converged policy and value function approximations $π^*_C$ and $Q^*_C$. Note that here convergence guarantees are much easier to get – we can think about this training schedule is that one super-agent controls both players and tries to optimize for a single scalar reward. From the point of view of the super-agent the environment is again a stationary MDP and thus we can get standard approximation and convergence guarantees.

Unfortunately, the constructed policies are not enough to construct the mTFT meta-policy in general. This is because RL methods typically do not learn the full value functions for the game, rather they learn the value function(s) conditional on a particular set of policies. These value functions may not have enough information to implement the mTFT strategy.

The reason mTFT cannot be implemented with the learned functions alone is that the mTFT agent may see their partner deviate from $π_C$ in a $D$ phase. When this happens, the mTFT agent needs to compute an additional $k$ time steps to add to the current $D$ phase. However, in order to compute what this $k$ should be the mTFT agent needs access to not just the value functions associated with $π_C$ and $π_D$ but also the value functions associated with the policy given by “behave according to $π_D$ for $k$ periods and $π_C$ afterwards” (we denote this as $π_{D,k}^C$). These value functions can be computed using standard methods, but it becomes quite cumbersome.

If we restrict ourselves to a subset of games which generalize another property of the Prisoner’s Dilemma, we can use a much simpler training procedure and meta-policy while keeping approximately preserving mTFT’s incentive properties.

**Definition 13** We say a game is $π_D$ dominant (for player 2) if for any $k$, any state $s$, and any policy $π_A$ we have

$$V^2(s, \hat{π}_D^1, \hat{π}_D^2, \hat{π}_A) \geq V^2(s, \hat{π}_D, \hat{π}_A).$$

While we know that $π_D^*$ is a best response to $π_D^*$, the $π_D$ dominance assumption says that even in a truncated game where $(\hat{π}_D^1, \hat{π}_D^2)$ starts after $k$ turns the best response property of $π_D$ to itself holds. Many cooperative settings of interest are those where individuals pay personal costs to give benefits to others. Here $π_D$ represents choosing not to pay the cost at a given time step and often our dominance requirement holds. Also note that if, as in the PD, there is a single best way to extract value from a partner, $\hat{π}_D$ dominance holds trivially.

When $π_D$ is dominant we can use a much simpler construction which only requires learning $π_D^1$ and $π_D^2$, and their associated value and $Q$ functions. The key idea of the construction will be that due to the dominance of $π_D$ we can use the continuation values implied by $π_D$ to bound the length of a $D$ phase following a deviation from $π_C$. We call this simplified algorithm approximate mTFT (amTFT).

---

10We note that in general versions of this it is difficult to prove convergence results as well as stabilize the training procedure [Fudenberg & Levine 1998]. This is because if both agents are learning the environment from the perspective of each agent is not stationary. In our environment we find that standard tools appear to work though in more complex environments we may need more complex training procedures to construct these policies (cf. [Foerster et al. 2017]).

11We note that this is learnable with the following algorithm: first learn $π_D$ and $π_C$, then train a new RL agent on episodes where their partner plays according to $π_{D,C}$ where $k$ is observed. This will allow us to approximate the best response policies to $π_{D,C}$ which will then give us what we need to compute the responses to deviations from $π_C$ in the $D$ phase that incentivize full cooperation.
With the value functions and policies in hand from the procedure above, we can construct an amTFT meta-policy. For the purposes of this construction, we consider agent 1 as the amTFT agent (but everything is symmetric). The amTFT agent augments the Markov state with a vector \((W_t, b_t)\) which both start at 0. We use the algorithm below to combine pieces computed above into the approximate mTFT strategy.

The amTFT agent sees the action \(a'\) of their partner at time \(t\) and approximates the gain from this deviation as
\[
D_t = \hat{Q}^2_{CC}(s, a^2_t) - \hat{Q}^2_{CC}(s, \pi^2_C(s)).
\]
This is approximate total change in (both present and future) value that the partner receives from choosing action \(a^2_t\) instead of what is prescribed by \(\pi^2_C\).

The amTFT agent accumulates the total payoff balance of their partner as
\[
W_t = W_{t-1} + D_t.
\]
If \(W_t\) is below a fixed threshold \(T\) the amTFT agent chooses actions according to \(\pi_C\). If \(W_t\) crosses the threshold the mTFT agent computes the expected value to their partner of the policy pair \((\pi_{D_k, C}, \pi^2_{D_k, C})\) starting at \(s\). To do this, we can, for each \(k\) simply perform Monte Carlo policy rollouts to estimate the value playing \(\pi_D\) for any \(k\) periods followed by the continuation value from the resulting state implied by the learned \(\hat{V}_{CC}\).

Note that because the game is \(\pi_D\) dominant this is an upper bound on the partners’ payoff if the amTFT agent commits to playing according to \(\pi_{D_k, C}\).

The amTFT agent uses this rollout strategy to compute the minimal \(k\) such that the loss to the deviator from \(\pi_{D_k, C}\) is greater than \(\alpha W^1_t\) where \(\alpha > 1\) allows us to extend the \(D\) phase to account for approximation errors in our calculation of \(Q\) and \(\pi\). If our approximations are good, it is now no longer worth it for the amTFT’s partner to try to cheat while on the \(\pi_C\) path. The pseudocode for the meta-policy is given in algorithm 2:

**Algorithm 2** Approximate Markov Tit For Tat (for Agent 1)

```plaintext
Input: \(\hat{\pi}_C, \pi_D\) and their \(\hat{Q}; \alpha, T\)

\(b \leftarrow 0, W \leftarrow 0\)

while Game do
  \(D \leftarrow \hat{Q}^2_{CC}(s, a^2) - \hat{Q}^2_{CC}(s, \pi^2_C(s))\)

  if \(b = 0\) then
    Choose \(a \leftarrow \hat{\pi}^1_C(s)\)
    \(W = W + D\)
  
  if \(b > 0\) then
    Choose \(a \leftarrow \hat{\pi}^1_D(s)\)
    \(b = b - 1\)

  if \(W > T\) then
    Compute \(k(s)\) via batched policy rollouts and \(\hat{Q}^2\)
    \(b = k(s)\)
    \(W = 0\)
```

The hyperparameters \(T\) and \(\alpha\) trade off robustness to approximation error and noise. As we raise \(T\) we allow for more approximation error in the calculation of the debit. However, this creates opportunities for particularly canny partners to try to cheat the agent. Raising \(\alpha\) makes the cost of defection higher (since it raises the loss from entering the \(D\) phase) but itself has an efficiency cost, sending \(\alpha = \infty\) and \(T = \epsilon\) gives us back the Grim Trigger strategy. Tuning these parameters is important but an analytical characterization is beyond the scope of this paper.

We note that amTFT has one main difference from the mTFT above: when standard mTFT is acting according to \(\pi_D\) it still expects the partner to play according to \(\pi_C\). On the other hand the approximate version we present does not. This means that the approximate algorithm does not make cooperation a strictly dominant strategy (it is better for the other agent to play \(\pi_D\) during the \(D\) phase).

This means that the likelihood of two amTFT agents getting locked in cycles of cooperation and defection is reduced. In this way, the approximate algorithm is more similar to the strategy of Win-
Stay-Lose-Shift in classical repeated Prisoner’s Dilemma \cite{Nowak1993}. Additionally, the approximate algorithm saves on training complexity since we no longer need to compute best responses to $\pi_{D_tC}$.

4 Experiments

4.1 Coin Game Setup

We implement amTFT in a simple situation - the Coin game. In the Coin game two players move on a $5 \times 5$ board. The game has a small probability of ending every time step, we set this so the average game length is 500 time steps. Coins of different colors appear on the board periodically, and a player receives a reward of 1 for collecting (moving over) any coin. However, if a player picks up a coin of the other players’ color, the other player loses 2 points. The payoff for each agent at the end of each game is just their own point total. The strategy which maximizes total payoff is for each player to only pick up coins of their own color; however each player is tempted to pick up the coins of the other color. The key challenge for an artificial agent here is to learn what it means to ‘cooperate’ and what it means to ‘defect’. Figure 2 shows this graphically.

Figure 2: An example of the 5x5 Coin Game which we use to illustrate the mTFT meta-policy. In a Markov game, unlike in the simple PD, ‘cooperate’ and ‘defect’ are no longer single actions but rather full policies. Constructing an artificial agent that can cooperate effectively is thus to learn what these policies look like and, knowing this, to be able to recognize and disincentivize ‘Defect’ behavior from their partner.

We are interested in constructing general strategies which scale beyond tabular games so we approximate the policy and value function with a convolutional neural network (see Appendix for full architecture).

We perform both Selfish (self play with reactive agents receiving own rewards) and Cooperative (self play with both agents receiving sum of rewards). We perform 100 replicates of each training type. Figure 3 shows that Selfish training leads to suboptimal behavior while Cooperative training does find policies that implement socially optimal outcomes. Total reward (which we call efficiency) is near 0 when both agents play according to $\hat{\pi}_D$ even though the number of total number of coins being collected in each game is almost identical. This is because $\hat{\pi}_D$ agents converge to picking up coins of all colors while social $\hat{\pi}_C$ agents learn to only pick up matching coins.

\footnote{amTFT does not correspond to a generalization of WSLS either since WSLS only goes back to cooperation after both agents have chosen $D$ once whereas amTFT does not require any particular behavior from their partner during any $D$ phase.}
Figure 3: In the Coin game training RL agents which gain utility from their own reward and treat their partner as a part of the environment (Selfish) leads to inefficient outcomes as agents scramble to pick up all the coins they can. However, training agents with social preferences finds policies which cooperate to pick up only coins of their own color. Lines are smoothed at the 1000 episode level and reflect means averaged over 100 replicates, error bars reflect standard deviations calculated from independent replicates.

4.2 Tournament Details

As with the origins of TFT we evaluate the performance of various Markov social dilemma strategies in a tournament. We then construct matchups between augmented memory strategies by drawing two random policies from the pool of 100 Selfish and 100 Cooperative policies we have trained and giving them meta-strategies from the set

1. Always use $\pi_C$ - we refer to this as the Pro Social strategy
2. Always use $\pi_D$ - we refer to this as the Selfish strategy
3. Approximate Markov Tit-for-Tat (amTFT)
4. Markov Grim Trigger
5. Strict Grim Trigger

We construct Strict Grim Trigger by using the algorithm described in the Markov Games section: Strict Grim deviates to $\hat{\pi}_D$ whenever its partner strays from the action prescribed by $\hat{\pi}_C$. We construct Markov Grim Trigger by using the amTFT algorithm and setting the punishment multiplier $\alpha$ to $\infty$. For amTFT we set the threshold $T$ to 1 and multiplier $\alpha$ to 4.

To construct a matchup between two strategies we construct agents and have them play a 1000 round iteration of the game. At test time we use a fixed (rather than random) length game because this allows us to compare payoffs more efficiently. We use 1000 replicates per strategy pair to compute the average expected payoff of each potential meta-strategy choice.

A key component of the amTFT strategy is the computation of the per period debit $D_t$. One way is to use the modeled $\hat{Q}$ directly. This has the issue that any approximation error in $\hat{Q}$ (in particular, bias) is accumulated across periods. A less biased but more computationally intensive estimator of $D$ is to use the average of Monte Carlo policy rollouts optionally followed by the estimated continuation value from $\hat{Q}$. This estimators are unbiased in the limit of large horizon and batch size although eliding the continuation value may require long rollouts to avoid horizon effects in some games. These approaches trade off between computational complexity and accuracy (long rollouts are costly but accurate, $\hat{Q}$ is cheap but likely inaccurate).
In the tournament here we use we use policy rollouts to approximate $Q$ and elide the continuation value. This estimator has good performance for the Coin game. We also found that it was possible in the Coin game to use $\hat{Q}$ directly to calculate debit without policy rollouts. However, we found that to get $Q$ accurate enough to work well we had to use a modified training procedure. The results of the tournament are seen in Figure 4. Selfish agents can exploit Cooperators players at huge gains. However, cooperation is a better strategy than selfishness when one has a Markov Grim or amTFT partner. Finally, in self pairing amTFT maintains more efficient levels of cooperation than Trigger strategies and can almost reach the level of pure pro-social agents. Strict Grim Trigger leads to bad outcomes. This last result comes from the fact that our agents each train their own approximations to the C and D strategies and Grim Trigger does not allow for agents to return to cooperation from a (real or perceived) deviation from C behavior.

|               | Pro-Social | Selfish | amTFT   | mGrim   | Strict Grim |
|---------------|------------|---------|---------|---------|-------------|
| Pro-Social    | 69, 69     | −56, 108| 66, 69  | 65, 69  | −56, 109    |
| Selfish       | 108, −56   | 2, 2    | 33, 14  | 8, −1   | 2, 2        |
| amTFT         | 69, 66     | −14, 33 | 63, 63  | 43, 57  | −14, 33     |
| mGrim         | 76, 45     | −1, 8   | 57, 43  | 48, 48  | 2, 4        |
| Strict Grim   | 109, −56   | 2, 2    | 33, −14 | 4, 2    | 2, 2        |

Figure 4: Payoff matrix from 1000 replicates of tournament between the generated policies. Rows and column indicate a policy choice (from our RL generated policies) for either player. Entries indicate payoffs to the row and column player respectively. Learned Cooperative and Selfish policies indeed form a Prisoner’s Dilemma. In the expanded game amTFT can maintain good payoffs while providing protection against Selfish agents. Strict Grim Trigger does poorly because it assumes any deviation from it’s preferred cooperative trajectory implies defection. Markov Grim Trigger slightly ameliorates this problem but is still worse than amTFT.

4.3 amTFT As Teacher

The tournament results show that amTFT is a good strategy to employ in a mixed environment which includes some cooperators, some tit-for-tat agents and some defectors. However, what happens when amTFT’s partner is themselves a learning agent?

To answer this question we consider what happens if we fix the one player (the Teacher) to use a fixed policy but let the other player be a selfish reactive deep RL agent (the Learner). Recall that when selfish RL agents played with each other, they converged to the Selfish ‘grab all coins’ strategy. We see, however that while Learners exploit fully cooperative teachers an amTFT teacher is indeed able to teach a learner good cooperative policies (Figure 5 shows 20 replicates of this training procedure).

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To learn $Q$ we tried the following procedure: after finishing Selfish and Cooperative training, we did a second step of training using a fixed (converged) $\tilde{\pi}_C$. In order to sample states off the path of $\tilde{\pi}_C$ during this step, the learner behaves according to a mixture of $\pi_C$, $\pi_D$, and random policies while the partner continues according to $\tilde{\pi}_C$. $\hat{Q}$ is updated via off-policy Bellman iteration: $\hat{Q}_{u+1}(s_t, a^*_t) = \hat{Q}_u(s_t, a^*_t) + \lambda \sum_{a \sim \pi_C} \hat{Q}_u(s_{t+1}, a)$. We found this modified procedure produced a $Q$ function that was good enough to maintain cooperation (though still not as efficient as rollouts). For more complex games, an important area for future work work is to develop methodologies to compute more accurate approximations of $Q$ or combine $Q$ with rollouts effectively.
Figure 5: When Selfish learners are paired with Selfish teachers they learn to also play selfishly. Similarly, when Selfish learners are paired with Pro-Social teachers the learners exploit them and achieve high rewards at the expense of the teacher. However, when selfish learners are paired with amTFT teachers they learn that cheating doesn’t pay and thus behave cooperatively. Lines are smoothed at the 1000 episode level and reflect means averaged over 20 replicates, error bars reflect standard deviations calculated from independent replicates.

5 Conclusion

Humans are remarkably adapted to solving bilateral social dilemmas using heuristic strategies such as tit-for-tat. We have focused on how to give artificial agents this capability. Our theoretical results show that simple memory augmented strategies are possible in principle and we have given an algorithm to compute the required components. Finally, we have shown that this approximate Markov tit-for-tat strategy has good properties in simple experiments.

There is much more ground to be explored both in theory and in implementation. In particular, we have used off the shelf algorithms (actor-critic) to compute ‘on trajectory’ policies and expensive Monte Carlo rollouts to compute the ‘off trajectory’ components of mTFT. Finding an efficient method for computing high accuracy approximations of \( Q \) is an important future direction for this work.

From a more theoretical side, rather than thinking about mTFT in terms of policies, we can think of it in terms of a value function. Under this interpretation mTFT is an artificial implementation of the theory of conditional cooperation (Fischbacher et al. 2001) combined with warm glow altruism (Andreoni 1990). That is, agents behave according to a policy where they put weight not just on their own utility but also on the utility of their partner. The weight on the partner’s utility depends on whether the agent has inferred that their partner is acting in a way that is consistent with pure selfishness or whether their partner seems to be exhibiting warm glow altruism themselves. We note that this is extremely consistent with literature on human cooperation - we feel good when ‘good people’ do well (and thus strive to help them) and neutral or angry when those that have wronged us get ahead (and often are willing to pay costs to ourselves to reduce their payoffs, Ouss & Peysakhovich 2015).

There is recent interest in building forward models of an agent’s environment (eg. intuitive physics Denil et al. 2016; Lerer et al. 2016). However, agents and objects are very different - in particular, agents have beliefs, desires and some form of optimization while objects follow simple fixed rules. An important future direction in multi-agent RL is to go beyond simple partner models such as those of mTFT and use inverse reinforcement learning (Abbeel & Ng. 2004; Ng et al. 2000) to learn more complex theories of other agents (eg. Baker et al. 2009; Kleiman-Weiner et al. 2016).
There is a growing literature on hybrid systems which include both human and artificial agents (Crandall et al., 2017; Shirado & Christakis, 2017). We have constructed an agent which knows how to deal with perfectly selfish agents and so many of our techniques have focused on making the policy robust to approximation error that allows a selfish partner to cheat. However, human preferences are far more complex and include considerations of fairness (Fehr & Gächter, 2000), altruistic cooperation (Peyssakhovich et al., 2014) and social norms (Bicchieri, 2005). One strength of the amTFT construction is that it will maintain cooperation not only with purely selfish agents but also with agents that do exhibit some of social preferences.

However, recent work has shown that incorporating psychologically realistic models of human learning and social interaction can help those interested in designing systems that lead to good outcomes (Erev & Roth, 1998; Fudenberg & Peyssakhovich, 2016). An important future direction in the design of artificial cooperative agents is to relax the purely self-interested partner assumption and human psychology into account. Indeed existing work has shown that humans tend to be honest, cooperative and forgiving in social dilemmas and this can be leveraged to maintain good social outcomes (Hauser et al., 2014; Arechar et al., 2016).

There is much modern progress in reinforcement learning in zero sum games. However, this progress has proven to be extremely computationally challenging. There is a strong amount of selection here, modern researchers study precisely the zero-sum games that are hard for humans to solve. Indeed, to be successful in a complex zero-sum game, one must outwit one’s partner. However, in the case of cooperative games, especially those where humans are involved we are interested not in complex strategies which probe the weaknesses of our partner but simple, understandable strategies that allow coordination and cooperation to flourish. We hope that our work contributes to this important endeavor.

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6 Appendix

6.1 Proof of Grim Trigger Theorems

6.1.1 Prisoner’s Dilemma

Proof 1 The one shot deviation principle Fudenberg & Tirole (1991) tells us that it is sufficient to show that there is no single state where the partner of TFT gains from deviating from the policy of always cooperating. If no such ‘one shot deviations’ exist then always cooperate is an optimal policy.

There are 5 states to consider. Suppose that the game has just started, the payoff to choosing always cooperate is $1 - \delta$, the payoff to defecting this period and cooperating forever after is $1 + w - \delta s + \frac{\delta^2}{1 - \delta}$ which, for $\delta$ sufficiently close to 1 is dominated by cooperation.

Suppose that last period we both chose C. This is exactly the same condition as the one above, thus cooperation following this state is the best choice. Similarly, if we chose C and the TFT agent chose D last period, we have exactly the same problem.

Suppose that last period the TFT partner chose D and we chose D. Then if we play always cooperate we are guaranteed a payoff of $-s + \frac{\delta}{1 - \delta}$. However, choosing defect this period (and cooperating after) gets us $0 + \delta(-s + \frac{\delta}{1 - \delta})$. Thus, if $\delta$ is large enough, we will choose to cooperate after D, D.

Finally, starting from our choice of D and the TFT partner’s choice of C gives the same trajectory as $(D, D)$ above.

This completes the proof.

6.1.2 Markov Games

Proof 2 Let agent i play the ‘Grim trigger’ strategy $\pi_i^{G}$, which starts by playing $\pi_i^{C}$ but switches to $\pi_i^{M}$ forever if its partner ever deviates from $\pi_1^{C}$. This can be implemented with a one-bit memory augmentation: $M_i$ takes the value 1 if there has been a deviation. During the phase where both players are playing $\pi_C$, the value function inequality provided implies that there are no profitable one-shot deviations, because any deviation would switch both players to the less profitable $\pi_M$ equilibrium. During the $\pi_M$ phase, there are no profitable one-shot deviations since $(\pi_M^1, \pi_M^2)$ is an equilibrium. By the Principle of Optimality, a strategy profile is an equilibrium if there are no one-shot deviations, therefore $(\pi_1^{G}, \pi_2^{G})$ is an equilibrium that produces $t$.

6.2 Coin Game CNN

We represent policies in the Coin Game using a convolutional neural network. The CNN is a stack of four subunits, each of which contains a strided convolutional layers with $3 \times 3$ kernel, batch normalization, and ReLU. This produces a 104-dimensional feature vector per frame. The network has three heads: a 4-dimensional output with softmax that computes the policy $\pi$; a 4-dimensional output that computes $\hat{Q}$, and a 1-dimensional output that computes the critic $\hat{V}$ (used only for training).

For the Coin game, there are four actions (up, down, left, right), and $S$ is represented as a $4 \times 5 \times 5$ binary tensor where the first two channels encode the location of the each agent and the other two channels encode the location(s) of the coins (if any exist). At each time step a coin is generated at a random location with a random color, with probability $0.01$.

We use a multi-layer convolutional neural network to jointly approximate $\pi$ and $\hat{Q}$. For this small game, a simpler model could be used, but this model generalizes directly to games with higher-dimensional 2D state spaces (e.g. environments with obstacles). For a given board size, the model has $\lceil \log(2) \rceil + 1$ repeated layers, each consisting of a 2D convolution with kernel size $3$, followed by batch normalization and ReLU. The first layer has stride 1, while the successive layers each have stride 2, which decreases the width and height from $k$ to $\lceil k/2 \rceil$ while doubling the number of channels. For the $5 \times 5$ board, channel sizes are 13, 26, 52, 104. The model has three heads: $\pi, \hat{V}, \hat{Q}$.

The model is updated episodically - at the end of every game we take full history for each agent and update 3 heads on the network and learning rate $\lambda$ with the following update equations:
1. Loss between $\hat{V}$ and the Bellman reward estimate given by

$$\hat{V}_{u+1}(s_t) = \hat{V}_u + \lambda(\delta\hat{V}_u(s_{t+1}) + r_t))$$

2. Policy gradient with respect to the advantage

$$\sum_k \gamma^k r'_{t_k} - \hat{V}(s_t)$$

We train with a learning rate of 0.001, continuation probability .998 (i.e. games last on average 500 steps), discount rate 0.98, and a batch size of 32. We train for a total of 40,000 games.