General Static Spherical Solutions of $d$-dimensional Charged Dilaton Gravity Theories

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Abstract

We get the general static, spherically symmetric solutions of the $d$-dimensional Einstein-Maxwell-Dilaton theories by dimensionally reducing them to a class of 2-dimensional dilaton gravity theories. By studying the symmetries of the actions for the static equations of motion, we find field redefinitions that nearly reduce these theories to the $d$-dimensional Einstein-Maxwell-Scalar theories, and therefore enable us to get the exact solutions. We do not make any assumption about the asymptotic space-time structure. As a result, our 4-dimensional solutions contain the asymptotically flat Garfinkle-Horowitz-Strominger (GHS) solutions and the non-asymptotically flat Chan-Horne-Mann (CHM) solutions. Besides, we find some new solutions with a finite range of allowed radius of the transversal sphere. These results generalize to an arbitrary space-time dimension $d$ ($d > 3$).
I. INTRODUCTION

The low energy effective action of the string theory in $d$-dimensional target space contains the dilaton field $f$, the metric tensor $g_{\mu\nu}^{(d)}$ and a $U(1)$ gauge field, among other possible fields. The coupling of the dilaton field to the $U(1)$ gauge field makes this theory different from the Einstein-Maxwell theory. Specifically, this coupling is present in the action

$$I = \int d^d x \sqrt{g^{(d)}} \left( R - \frac{1}{2} g^{(d)\alpha\beta} \partial_\alpha f \partial_\beta f + \frac{1}{4} e^{\chi f} F^2 \right)$$

when the value of the constant $\chi$ is non-zero. Here $F^2$ is the square of the curvature 2-form of the $U(1)$ gauge field. The usual $d$-dimensional Einstein-Maxwell-Scalar theories correspond to (1) with $\chi = 0$. The $d$-dimensional target space effective action, derived from the condition of vanishing beta-function from heterotic string theory, reduces to the same action with non-vanishing $\chi$ if we set the 3-form field to be zero, after a scale transformation of the metric.

The action (1) provides us with a relatively simple model through which we can learn the implications of the string theory on the gravitational physics. The dilaton field $f$, which can either result from the conventional Kaluza-Klein theory or from the string theory, drastically modifies the classical and quantum dynamics of the space-time. Gibbons first obtained exact black hole solutions of the action (1) in the 4-dimensional space-time [2]. Then, the further generalizations into the $d$-dimensional case were achieved by Gibbons and Maeda (GM) [3]. As the interest in the string theory rises, the 4-dimensional black hole solutions were rediscovered by Garfinkle, Horowitz, and Strominger (GHS) in the context of the low energy effective theory of the string theory [4]. Furthermore, general static, spherically symmetric and asymptotically flat solutions were obtained in Ref. [5] for the 4-dimensional theory and in Ref. [6] for $d$-dimensional theory. These works imply the uniqueness of the GHS black holes (or GM black holes in $d$-dimensions) in the sense that asymptotically flat solutions other than the GHS (or GM) black holes contain naked singularities.

Recently, Chan, Horne and Mann (CHM) [7] made an interesting point by obtaining
some new static and spherically symmetric black hole solutions that do not satisfy asymptotic flatness condition, via a special ansatz. Considering these developments, it is of some interest to classify all the possible solutions of (1) without assuming the asymptotic flatness (including the most important \( d = 4 \) case). In this paper, we achieve this task by obtaining the general static spherically symmetric solutions of (1) in a local space-time region.

The assumption of the spherical symmetry is essential in our approach since we can then dimensionally reduce our \( d \)-dimensional system to 2-dimensional one. Thus, what we specifically solve in this paper is the following 2-dimensional action \([8]\) \([9]\).

\[
I = \int d^2x \sqrt{-g} e^{-2\phi} [R + \gamma g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + \mu e^{2\lambda \phi} - \frac{1}{2} g^{\alpha\beta} \partial_\alpha f \partial_\beta f + \frac{1}{4} e^{\phi + \chi f} F^2],
\]

where \( R \) denotes the 2-dimensional scalar curvature and \( F \), the curvature 2-form for an Abelian gauge field. The fields \( \phi \) and \( f \) represent a (2-dimensional) dilaton field and a massless scalar field, respectively. The parameters \( \gamma, \mu, \lambda, \epsilon \) and \( \chi \) are assumed to be arbitrary real numbers satisfying the condition \( 2 - \lambda - 2\gamma/2 + \epsilon/2 = 0 \). Of course, the action (4) is of interest in itself as a class of exactly solvable 2-dimensional dilaton gravity theories coupled with a U(1) gauge field and a scalar field \([8]\). In our context, however, its importance comes from the fact that (4) is the spherically symmetric reduction of (1) as reported, for example, in \([10]\). We can relate the 2-dimensional dilaton field \( \phi \) to the geometric radius of each \( (d-2) \)-dimensional sphere in \( d \)-dimensional spherically symmetric space-time. Thus, we write the spherically symmetric \( d \)-dimensional metric as the sum of longitudinal part and transversal angular part,

\[
ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta - \exp \left( -\frac{4}{d-2} \phi \right) d\Omega,
\]

where \( d\Omega \) is the metric of a sphere \( S^{d-2} \) with the unit radius and we use \( (+ - \cdots -) \) metric signature. The spherically symmetric reduction of the action (1) becomes Eq. (2) with \( \gamma = 4(d-3)/(d-2), \lambda = 2/(d-2), \epsilon = 0, \) and the parameter \( \mu \leq 0, \) which depends on

\[\text{\footnotesize\footnote{The meaning of this condition will be explained in Sec. II.}}\]
the area of $(d - 2)$-dimensional sphere after the $(d - 2)$-dimensional angular integration. Thus, the $d$-dimensional dilaton field $f$ becomes the scalar field of the 2-dimensional theory. From the point of view of the 2-dimensional dilaton gravity theories, the theories in our consideration continuously change from the $d = 4$ Einstein-Maxwell-Dilaton theory to the Callan-Giddings-Harvey-Strominger (CGHS) model as we change the value of the real parameter $d$ from 4 to the positive infinity \[11\].

The method we use in section II to obtain the exact solutions originates from \[12\]. It was applied to $d$-dimensional Einstein-Maxwell-Scalar theories in \[13\]. Just as in case of the Einstein-Maxwell-Scalar theories where $\chi = 0$, the integration of the coupled second order differential equations to obtain the first order system is possible through the identification of 4 underlying symmetries of our action (2), since we have 4 fields to solve in conformal gauge that we adopt in this paper. By studying these symmetries, we find a set of field redefinitions, which nearly reduces the $\chi \neq 0$ theories to the $\chi = 0$ theories. These field redefinitions are essentially the dressing of the 2-dimensional dilaton field and the conformal factor of the metric with the factor $\exp(\chi / (2a)hf)$ where $h$ is the scaling dimension of each field and $a \equiv 2 - \lambda - \gamma/4$. They almost reduce the original equations of motion to the ones with $\chi = 0$ with accompanying transformations of the Noether charges. We can directly solve them to get the closed form expressions for our fields. As a by-product of our method, the relationship between the $\chi = 0$ theories and $\chi \neq 0$ theories becomes apparent, thereby telling us the influence of the dilaton field on the space-time geometry.

The novel point in our approach is the lack of the assumptions about the global structure of the space-time. As shown in section III, our results in 4-dimensional case contain the asymptotically flat GHS black holes, as well as the CHM black holes where the space-time is not asymptotically flat. If the dilaton charge changes slightly from the value needed to give a black hole solution, naked singularities form near the would-be position of the black hole horizon. This behavior is consistent with the no-hair property of a generic black hole. It is found that the existence of the CHM solutions depends on the fact that $\chi \neq 0$. If $\chi = 0$, the analog of the CHM solutions has the global structure of $\mathcal{M}_2 \times S^2$, where the
transversal sphere has a constant radius. This structure is quite different from the CHM solutions and the 4-dimensional spherically symmetric Minkowski space-time. Some new solutions in this paper, though we believe they are unphysical in the end, include the space-time geometries with the finite range of the radius of the transversal sphere, which have other peculiar structures. All the generic features of the 4-dimensional results generalize to an arbitrary space-time dimension $d$ where $d > 3$.

II. SYMMETRIES AND DERIVATION OF SOLUTIONS

In this section, we present a method for the derivation of the general static solutions of the action in our consideration. We start by reviewing the static equations of motion we should solve. The crucial next step in our method is the construction of the Noether charges and understanding them. Once the symmetries are properly understood, it is pretty straightforward to obtain the general static solutions in a closed form.

A. Static Equations of Motion

The analysis in this paper will be performed under the choice of a conformal gauge, where the metric tensor is given by

$$g_{+-} = -e^{2\rho + \gamma \phi / 2}, \quad g_{--} = g_{++} = 0.$$  

We require the negative signature for a space-like coordinate and the positive signature for a time-like coordinate. The $\phi$ is included deliberately in the conformal factor to cancel the kinetic term for the dilaton field in Eq. (2) up to total derivative terms. The original action Eq. (2) then simplifies, modulo total derivative terms, to

$$I = \int dx^+ dx^- (4\Omega \partial_+ \partial_- \rho + \frac{\mu}{2} e^{2\rho} \Omega^{1-\gamma/4} + \Omega \partial_+ f \partial_- f - e^{\chi f - 2\rho} \Omega^{1+\gamma/4 - \epsilon/2} F_{+-}^2),$$

where we introduce $\Omega = \exp(-2\phi)$ and the curvature of the $U(1)$ gauge field is given by $F_{+-} = \partial_- A_+ - \partial_+ A_-$. In obtaining the solutions, we should supplement the equations of motion from Eq. (3) with gauge constraints resulting from the choice of conformal gauge,
\[ \frac{\delta I}{\delta g^{\pm \pm}} = 0, \]  
(4)

where the functional derivative of \( I \) in Eq. (2) is taken. Explicitly, they are written as

\[ \partial_+^2 \Omega - 2 \partial_+ \rho \partial_+ \Omega + \frac{1}{2} \Omega (\partial_{\pm} f)^2 = 0. \]  
(5)

The equations of motion from Eq. (3) are

\[ \partial_+ \partial_- \Omega + \frac{\mu}{4} e^{2\rho} \Omega^{1-\lambda-\gamma/4} + \frac{1}{2} e^{\chi_f} \Omega^{1+\gamma/4-\epsilon/2} F^{2-}_+ = 0, \]  
(6)

\[ \partial_+ \partial_- \rho + \frac{\mu}{8} (1 - \lambda - \gamma/4) e^{2\rho} \Omega^{-\lambda-\gamma/4} + \frac{1}{4} \partial_{\pm} \partial_{\pm} f = 0, \]  
(7)

along with the equations for the massless scalar field,

\[ (\partial_+ \partial_- f + \partial_- \partial_+ f) + 2 \Omega \partial_+ \partial_- f + \chi e^{\chi_f} \Omega^{1+\gamma/4-\epsilon/2} F^{2-}_+ = 0 \]  
(8)

and for the gauge fields,

\[ \partial_- (e^{\chi_f} \Omega^{1+\gamma/4-\epsilon/2} F^+_-) = 0, \]  
(9)

\[ \partial_+ (e^{\chi_f} \Omega^{1+\gamma/4-\epsilon/2} F^-_+) = 0. \]  
(10)

The equations for the Abelian gauge field are immediately integrated to give

\[ F_{-+} = e^{-\chi_f+2\rho} \Omega^{-1-\gamma/4+\epsilon/2} Q, \]  
(11)

where \( Q \) is a constant. The absence of the emission of physical photons in \( s \)-wave Maxwell theory, due to the transversal polarization of physical photons, is responsible for the particularly simple dynamics of the \( U(1) \) gauge field.

The general static solutions under a particular choice of the conformal gauge can be found by assuming that the metric \( \rho \), the scalar field \( f \), and \( \Omega \) depend only on a single
space-like coordinate \( x = x^+ + x^- \). Then, Eq. (11) implies the curvature 2-form \( F_{++} \) also depends only on \( x \). Therefore, from here on, we introduce a field \( A(x) \) that depends only on \( x \) and satisfies

\[
F_{++} = \frac{dA}{dx}.
\]

In fact, the choice of the vector potential \( A_\pm = \pm A/2 \), gives the same expression for the curvature 2-form. The original equations of motion, Eqs. (6)–(10), then consistently reduce to a system of the following coupled second order ordinary differential equations (ODE’s).

\[
\Omega'' + \frac{\mu}{4} e^{2\rho} \Omega^{1-\gamma/4} + \frac{1}{2} e^{\gamma f-2\rho} \Omega^{1+\gamma/4-\epsilon/2} A'^2 = 0,
\]

(12)

\[
\rho'' + \frac{\mu}{8} (1 - \lambda - \frac{\gamma}{4}) e^{2\rho} \Omega^{-\lambda-\gamma/4} + \frac{1}{4} f'^2 - \frac{1}{4} (1 + \frac{\gamma}{4} - \frac{\epsilon}{2}) e^{\gamma f-2\rho} \Omega^{1+\gamma/4-\epsilon/2} A'^2 = 0,
\]

(13)

\[
\Omega f'' + \Omega' f' + \frac{\chi}{2} e^{\gamma f-2\rho} \Omega^{1+\gamma/4-\epsilon/2} A'^2 = 0,
\]

(14)

\[
\frac{d}{dx} \left( e^{\gamma f-2\rho} \Omega^{1+\gamma/4-\epsilon/2} A' \right) = 0,
\]

(15)

where the prime represents taking a derivative with respect to \( x \). The general solutions of the above ODE’s are the same as the general static solutions of the original action under a particular choice of the conformal coordinates. They can be derived from the action

\[
I = \int dx [\Omega' \rho' - \frac{\mu}{8} e^{2\rho} \Omega^{1-\gamma/4} - \frac{1}{4} \Omega f'^2 + \frac{1}{4} e^{\gamma f-2\rho} \Omega^{1+\gamma/4-\epsilon/2} A'^2].
\]

(16)

The gauge constraints reduce to

\[
\Omega'' - 2\rho' \Omega' + \frac{1}{2} \Omega f'^2 = 0,
\]

(17)

if we restrict our attention to static solutions. Eqs. (10) and (17) are the starting point for our further considerations on general static solutions.
B. Symmetries, Noether Charges and Field Redefinitions

We observe the following four continuous symmetries of the action Eq. (16)

\( (a) \quad f \rightarrow f + \alpha, \quad A \rightarrow A e^{-\chi \alpha / 2}, \)

\( (b) \quad A \rightarrow A + \alpha, \)

\( (c) \quad x \rightarrow x + \alpha, \)

\( (d) \quad x \rightarrow e^{\alpha x}, \quad \Omega \rightarrow e^{\alpha \Omega}, \quad e^{2\rho} \rightarrow e^{-(2-\lambda-\gamma/4)\alpha} e^{2\rho}, \quad A \rightarrow e^{-(1-\lambda/2-\epsilon/4)\alpha} A, \)

where \( \alpha \) is an arbitrary real parameter of each transformation. The Noether charges for these symmetries are constructed as

\[
f_0 = \Omega f' + \frac{\chi}{2} e^{xf-2\rho} \Omega^{1+\gamma/4-\epsilon/2} AA',
\]

\[
Q = e^{xf-2\rho} \Omega^{1+\gamma/4-\epsilon/2} A',
\]

\[
c_0 = \Omega' \rho' - \frac{1}{4} \Omega f'^2 + \frac{1}{4} e^{xf-2\rho} \Omega^{1+\gamma/4-\epsilon/2} A'^2 + \frac{\mu}{8} e^{2\rho} \Omega^{1-\lambda-\gamma/4},
\]

\[
s + c_0 x = -\frac{1}{2} (2 - \lambda - \frac{\gamma}{4}) \Omega' + \rho' \Omega - \frac{1}{2} (1 - \frac{\lambda}{2} - \frac{\epsilon}{4}) e^{xf-2\rho} \Omega^{1+\gamma/4-\epsilon/2} AA',
\]

respectively. The gauge constraint, Eq. (17), implies \( c_0 = 0. \)

We compare these symmetries to the symmetries in Ref. [13] where one considers the case of \( \chi = 0. \) The symmetries \( (b), (c) \) and \( (d) \) are exactly the same in both cases. The symmetry \( (b) \) results from the trivial dynamics of the \( U(1) \) gauge field in \( s \)-wave theories. The symmetries \( (c) \) and \( (d) \) are remnants of the underlying classical conformal symmetries of the original action (2). The only difference between our case and Ref. [13] lies in the symmetry \( (a) \). Due to the coupling of \( f \) to the gauge field (the \( \exp(\chi f) \) factor in Eq. (16)), the translation of the \( f \) field should be compensated by an additional scale transformation of \( A \), to make the action Eq. (16) invariant.

\[2 \text{In [13], we chose different conformal coordinates. Our choice in this paper is related to the one in [13] via a conformal transform } x^\pm \rightarrow \ln x^\pm. \]
The symmetry \((d)\) defines the scaling dimension of each field. From here on, we will impose a condition \(a \equiv 2 - \lambda - \gamma/4 = 1 - \lambda/2 - \epsilon/4 \neq 0\), which enables us to straightforwardly solve the equations of motion to get the solutions in a closed form. This condition means the scaling dimension of the \(A\) field and that of the \(\exp(2\rho)\) are the same. We note all \(s\)-wave reduction of the \(d\)-dimensional Einstein-Maxwell-Dilaton theories satisfies this condition and gives the value \(a = (d - 3)/(d - 2)\).

The difference between the \(\chi = 0\) case and the non-zero \(\chi\) case is the additional scale transformation of the \(A\) field in the symmetry \((a)\). However, we can adjoin \((d)\) to \((a)\) to make \(A\) invariant. Thus, we consider such an adjoined symmetry

\[(a)' \ x \rightarrow e^{-\chi \alpha/(2a)} x, \quad \Omega \rightarrow e^{-\chi \alpha/(2a)} \Omega, \quad e^{2\rho} \rightarrow e^{\chi \alpha/2} e^{2\rho}, \quad A \rightarrow A, \quad f \rightarrow f + \alpha, \quad (22)\]

where we made a certain \((d)\) transformation following the \((a)\) transformation to make \(A\) invariant. Now as far as \(f\) and \(A\) fields are concerned, the transformations in \((a)'\) are independent of the value of \(\chi\). For other fields, we introduce the following field redefinition

\[
\bar{\Omega} = \exp(\frac{\chi}{2a} f) \Omega, \\
e^{2\bar{\rho}} = \exp(-\frac{\chi}{2} f) e^{2\rho}, \\
d\bar{x} = \exp(\frac{\chi}{2a} f) dx. \quad (23)
\]

Then, written in terms of these redefined fields, we have

\[(a)' \ \bar{x} \rightarrow \bar{x}, \quad \bar{\Omega} \rightarrow \bar{\Omega}, \quad e^{2\bar{\rho}} \rightarrow e^{2\bar{\rho}}, \quad A \rightarrow A, \quad f \rightarrow f + \alpha \quad (24)\]

for the transformation \((a)'\). The symmetry \((b)\) obviously survives the field redefinition.

Since the original action and the field redefinition does not have explicit dependence on \(x\), the symmetry \((c)\) remains intact. The symmetry \((d)\) still holds even if we change \((\rho, \Omega)\) into \((\bar{\rho}, \bar{\Omega})\), since the \(f\) field does not change under the symmetry \((d)\). Thus, in terms of the redefined fields, the symmetries \((a)', (b), (c)\) and \((d)\) do not depend on the value of \(\chi\), and look identical to the symmetries of Ref. [13] where one considers the \(\chi = 0\) case. Thus, a natural expectation is that if we use the redefined fields to solve the equations of motion, the analysis will be similar to the \(\chi = 0\) case.
C. Explicit Solutions

In terms of redefined fields, we rewrite Eqs. (18) and (19) as

\[ f_0 = \bar{\Omega} \dot{f} + \frac{1}{2} \chi QA, \]  
(25)

\[ Q = e^{-2\rho_1 \bar{\Omega} + 1} \dot{A}, \]  
(26)

where the overdot represents the differentiation with respect to \( \bar{x} \). We notice that, unlike Eq. (19), the \( \chi \) dependence is absent in Eq. (26). After the field redefinition and using Eq. (25), Eq. (21) becomes

\[ \bar{s} = -\frac{a}{2} \dot{\Omega} + \frac{\dot{\rho}}{\bar{\Omega}} - \frac{a}{2} \dot{Q} A. \]  
(27)

All the \( \chi \) dependence can be absorbed into the redefinition of the parameters given by

\[ \bar{s} = s - \frac{\chi}{2} f_0, \quad \bar{Q} = \frac{a + \chi^2/2}{a} Q. \]

If we set \( \chi = 0 \), there is no change other than the disappearance of the overbars in Eq. (27).

In our further consideration, we only consider the case when \( Q \neq 0 \). By this assumption, we exclude the case when the \( A \) field becomes degenerate, being a strict constant. Then, we combine Eqs. (26) and (27) to get

\[ \frac{d}{d\bar{x}}(Q e^{2\rho_1 \bar{\Omega} - a}) = (2 \bar{s} + a \dot{Q} A) \dot{A}, \]

which can be integrated to yield

\[ Q e^{2\rho_1 \bar{\Omega} - a} = 2 \bar{s} A + \frac{a}{2} \dot{Q} A^2 + c \equiv P(A), \]  
(28)

where \( c \) is the constant of integration. After the field redefinition and upon using Eqs. (25), (20) and (28), we rewrite Eq. (20) as

\[ \frac{d}{d\bar{x}}(Q e^{2\rho_1 \bar{\Omega} - a}) = (2 \bar{s} + a \dot{Q} A) \dot{A}, \]

which can be integrated to yield

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(28)

where \( c \) is the constant of integration. After the field redefinition and upon using Eqs. (25), (20) and (28), we rewrite Eq. (20) as
\[ 0 = \ddot{\Omega} + \frac{\mu}{8}e^{2\bar{\phi}}\bar{\Omega}^{a-1} - \frac{1}{4} \frac{f_0^2}{\bar{\Omega}} - \frac{1}{4} \bar{Q} \dot{A}, \]  

(29)

where we introduced

\[ f_0^2 = a - \frac{\chi^2/2}{a} f_0^2 + \frac{2\chi}{a} s f_0 + \frac{\chi^2}{2a} Q c. \]

For \( \chi = 0 \), we get \( f_0^2 = f_0^2 \). As before, the disappearance of the overbars is the only change in Eq. (29) as we set \( \chi = 0 \). As expected from the previous discussions, the expressions for the Noether charges, Eqs. (26), (27), and (29), become independent of \( \chi \), modulo the redefinitions of the parameters and fields.

Although the Noether charge expression for the symmetry \( (a) \), Eq. (25), still contains \( \chi \) dependence, it is simple enough for us to give an exact analytic treatment. Specifically, from Eqs. (25), (26) and (28), we can determine \( f \) via

\[ f = \frac{f_0 - \chi Q A/2}{P(A)} \dot{A}, \]  

(30)

which, upon integration, becomes

\[ f(A) = \frac{a f_0 + \chi s}{a + \chi^2/2} I(A) - \frac{\chi/2}{a + \chi^2/2} \ln |P(A)| + f_1, \]  

(31)

where \( f_1 \) is the constant of integration and we introduce \( I(A) = \int P(A) dA \).

Using Eqs. (26) and (28), we can rewrite Eq. (29) as

\[ 0 = 4a \left( \frac{d\bar{\phi}}{dA} \right)^2 - \frac{2}{P(A)} \frac{dP(A)}{dA} \frac{d\bar{\phi}}{dA} - \frac{f_0^2}{2P^2(A)} + 2\bar{Q} + \frac{\mu e^{-4a\bar{\phi}}}{4P(A)}, \]  

(32)

where we introduced \( \bar{\phi} \) via \( \bar{\Omega} = (a/(a + \chi^2/2))^{1/(2a)} \exp(-2\bar{\phi}) \). By differentiating the above equation with respect to \( A \), we have

\[ 0 = \left[ \frac{1}{P} \frac{dP(A)}{dA} - 4a \frac{d\bar{\phi}}{dA} \right] \left[ \frac{d^2 \bar{\phi}}{dA^2} + 2a \left( \frac{d\bar{\phi}}{dA} \right)^2 - \frac{f_0^2}{4P^2} \right]. \]  

(33)

Eqs. (32) and (33) are exactly the same as Eqs. (31) and (32) of Ref. [13] where one considers the \( \chi = 0 \) case, except the overbars on the variables. Thus, we can follow that paper to obtain the exact solutions. The form of the solutions depends on a parameter \( D \) defined by
\[ D^2 \equiv 4s^2 - 2aQc + 2af_0^2 = 4s^2 - 2aQc + 2af_0^2. \]

Here one interesting observation is about the second identity. We can easily check that the \( \chi \) dependence in \( D \) drops out as the identity shows.

When the second factor in Eq. (33) vanishes, we have

\[ e^{-4a\phi} = \frac{8D^2\bar{Q}}{-\mu} \frac{c_1 e^{DI}}{P[c_1 e^{DI} - 2a]^2} \]  

(34)

for \( D^2 > 0 \),

\[ e^{-4a\phi} = \frac{4\bar{Q}}{-\mu a} \frac{1}{P(I + c_1)^2} \]  

(35)

for \( D^2 = 0 \) and

\[ e^{-4a\phi} = \frac{\bar{D}^2\bar{Q}}{-\mu a \sin^2(\bar{D}I/2 + c_1)} \]  

(36)

for \( D^2 = -\bar{D}^2 < 0 \), where \( c_1 \) is the constant of integration. If \( P(A) = 0 \) has two real roots \( A = \bar{A}_- \) and \( A = -\bar{A}_+ \), we have

\[ e^{DI} = \left| \frac{A - \bar{A}_-}{A + \bar{A}_+} \right|^{D/\sqrt{4s^2 - 4aQc}} \]

and

\[ P(A) = \frac{a}{2} \bar{Q}(A + \bar{A}_+)(A - \bar{A}_-). \]

There are two other expressions for \( \exp(DI) \) depending on the sign of \( 4s^2 - 2aQc \); when \( P(A) \) has a double root and when \( P(A) \) has two complex roots, respectively.

When the first factor in Eq. (33) vanishes, we have

\[ e^{-4a\phi} = \frac{D^2\bar{Q}}{\mu a} \frac{1}{P}. \]  

(37)

This solutions does not contain a further constant of integration.

Since \( Q \neq 0 \), we find \( A \) as a function of \( x \) by plugging Eq. (28) into Eq. (26),

\[ x - x_0 = \int \frac{\bar{Q}(A)}{P(A)} dA, \]  

(38)

where \( x_0 \) is the constant of integration. Eqs. (28), (31), (34) - (37) and (38) represent explicit expressions for the general solutions in terms of the redefined fields.
III. ASPECTS OF GENERAL STATIC SOLUTIONS

Our derivation of the general static solutions does not sensitively depend on the value of the parameter $a$, as long as the restriction $a > 0$ holds. Thus, we will give a unified description of the properties of the general static solutions for an arbitrary space-time dimension $d$, which is larger than 3. When interpreted as the dimensionally reduced 2-d dilaton gravity theories, the parameter $d$ can even be considered as a continuous parameter. Being the general solutions, our results are somewhat complicated and the solution space has a quite rich structure. As a result, we will first concentrate on a class of solutions near to the GHS solutions and CHM solutions in $d$-dimensional space-time with an arbitrary dilaton coupling constant $\chi$. Then, we will discuss the solutions presented in Eq. (37).

A. GHS and CHM Solutions in $d$ dimensions

The assumption of the asymptotic flatness plays an important role in many literature concerning the derivation of the static solutions in $d$-dimensional Einstein-Maxwell-Dilaton theories with an arbitrary or a fixed value of $\chi$ [2] [3] [4] [5] [6]. These kinds of investigation lead to the GHS solutions (or Gibbons-Maeda solutions) in $d$-dimensional space-time. However, the discovery of CHM black holes [7] shows there are some interesting non-asymptotically flat solutions. One of the main purposes of this subsection is to illuminate the relationship between the CHM and the GHS solutions, since our solutions include both of them and show how they are continuously connected.

Throughout the presentation in this subsection, we attempt to show the relationship between the $\chi = 0$ solutions and the $\chi \neq 0$ solutions. In case of $\chi = 0$ theories, the no-hair theorem prevents the non-zero value of the scalar charge. It is possible to verify this theorem by showing that any non-zero value of the scalar charge produces a naked singularity. We find there is essentially the same phenomenon in case of $\chi \neq 0$.

Among the general static solutions we obtained in Sec. II, the solutions described by
Eq. \((34)\) depend on 7 independent parameters, \(f_0, f_1, Q, c, s, c_1,\) and \(\bar{x}_0,\) satisfying \(D^2 > 0.\) We note each parameter with overbar reduces to the one without overbar if we set \(\chi = 0.\) We expect 8 constants of motion from solving the second order differential equations for 4 fields. One of them is set to zero by the gauge constraint for the choice of a conformal gauge. In order to make contact with aforementioned theories, we further impose two conditions, \(Q \neq 0\) and \(4s^2 - 2a\bar{Q}c > 0,\) for the constants of motion. Under these conditions, from Eq. \((28)\) we have

\[
2e^{2\rho}\Omega^{-a} = (a + \chi^2/2)e^{\chi f}(A + \bar{A}_+)(A - \bar{A}_-)
\]

where we introduced

\[
\bar{A}_\pm = (\sqrt{4s^2 - 2a\bar{Q}c} \pm 2\bar{s})/(a\bar{Q}).
\]

Here, \(e^{2\rho}\Omega^{-a}\) corresponds to the conformal factor of the longitudinal part of the metric. For the dilaton field, Eq. \((31)\) yields

\[
f(A) = f_+ \ln |A + \bar{A}_+| - f_- \ln |A - \bar{A}_-| - \frac{\chi/2}{a + \chi^2/2} \ln |a\bar{Q}/2| + f_1,
\]

where we defined two parameters \(f_\pm\) via

\[
f_\pm = (f_0 \pm \chi Q \bar{A}_\pm/2)/\sqrt{4s^2 - 2a\bar{Q}c}.
\]

The \(\Omega\) field can be obtained from Eq. \((34)\) as follows.

\[
\Omega^{2a} = C \frac{|A + \bar{A}_+|^{h_+} |A - \bar{A}_-|^{h_-}}{(c_1|A - \epsilon A_\epsilon|^{\alpha} - 2a|A + \epsilon A_\epsilon|^{\alpha})^2}
\]

where \(\alpha = |D|/\sqrt{4s^2 - 2a\bar{Q}c} > 0,\) \(\epsilon = D/|D|,\) \(h_\pm = \alpha - 1 \pm \chi f_\pm\) and

\[
C = \frac{8D^2c_1|Q|}{-\mu} \frac{e^{-\chi f_1}}{|a\bar{Q}/2|^{\alpha/(a+\chi^2/2)}}.
\]

We note once again that \(\mu < 0\) for the spherically symmetric \(d\)-dimensional Einstein-Maxwell-Dilaton theories when \(d > 3.\) The relation between the \(A\) field and the coordinate \(x\) is given by Eq.\((38)\) as
\[ dx = \frac{\Omega}{P(A)} dA. \] (42)

For our further consideration, we set \( \epsilon = + \) and \( c_1 > 0 \). We also consider the case when \( Q > 0 \) and \( 0 < k \leq 1 \) where \( k = (2a/c_1)^{1/\alpha} \). This gives an inequality \( \bar{A}_- > -\bar{A}_+ \). From Eq. (41), the physical range of \( A \) is restricted to satisfy \( A \geq \bar{A}_- \) or \( A \leq -\bar{A}_+ \) in order to make the left hand side positive semidefinite.

Our solutions are defined on a local space-time region. The natural places to regard as boundaries of a space-time region are \( \Omega = 0 \), where the transversal sphere collapse to a point, and \( \Omega = \infty \), which corresponds to the spatial infinity. Thus, we start from finding the zeros and infinities of \( \Omega \) given in Eq. (41). If \( \chi = 0 \), we immediately see \( h_- = h_+ = 0 \) if and only if \( \bar{f}_0 = 0 \), and both of \( h_\pm \) are positive-definite for other values of \( f_0 \). The similar story holds in case of \( \chi \neq 0 \). We have an identity

\[ h_+ h_- = \frac{a}{a + \chi^2/2} \frac{|D|}{\sqrt{4s^2 - 2aQc}} - 1 \geq 0. \]

We observe \( h_- h_+ = 0 \) if and only if \( \bar{f}_0 = 0 \). If \( \bar{f}_0 \neq 0 \), the product should be larger than zero. Furthermore, if \( \bar{f}_0 = 0 \), we have \( \{ h_+, h_- \} = \{ 0, \chi^2/(a + \chi^2/2) \} \) and, as a result, one of \( h_\pm \) vanishes and the other becomes larger than zero for a non-zero value of \( \chi \). Since \( h_\pm \) are continuous functions of \( \bar{f}_0 \), we infer that \( h_\pm \) are positive definite if \( \bar{f}_0 \neq 0 \). Given this information it is easy to see the zeros and infinities of \( \Omega \) given in Eq. (41).

For \( 0 < k < 1 \), \( \Omega \) becomes infinity when the denominator of Eq. (41) vanishes. Specifically, there are two values of \( A \), \( A = \bar{A}_I^L = (\bar{A}_- - k(-\bar{A}_+))/(1 - k) \) and \( A = \bar{A}_I^U = (\bar{A}_- + k(-\bar{A}_+))/(1 + k) \), for which the condition is met. The value \( \bar{A}_I^U \) lies between \( -\bar{A}_+ \) and \( \bar{A}_- \), and satisfies \( (A_- - \bar{A}_I^L) = k(\bar{A}_I^U - (-\bar{A}_+)) \). The value \( \bar{A}_I^L \) is larger than the both of \( \bar{A}_- \) and \( -\bar{A}_+ \), and satisfies \( (\bar{A}_I^L - \bar{A}_-) = k(\bar{A}_I^L - (-\bar{A}_+)) \).

The structure of the zeros depends on whether \( \bar{f}_0 = 0 \) or not. For \( \bar{f}_0 = 0 \) case, we consider \( h_- = 0 \) and \( h_+ = \chi^2/(a + \chi^2/2) \), thereby resulting \( \alpha = 1 \). Then there are three zeros of \( \Omega \) at \( A = -\infty \), \( A = -\bar{A}_+ \) and \( A = \infty \). It turns out that the GHS (or GM) black holes outside the outer horizon belong to this case. The range of \( A \) that should make a
invertible map between $A$ and $\Omega$ is chosen to be $[\bar{A}_-, \bar{A}_\infty]$. The limit $A = \bar{A}_-$ is special since for this value of $A$ and under the above conditions, we find

$$\partial_+ \Omega = \frac{d\Omega}{dx} = \frac{\bar{Q}}{4} [h_-(A + \bar{A}_+) + h_+(A - \bar{A}_-)]$$

$$- 2\alpha \frac{|A + \bar{A}_+|^\alpha (A - \bar{A}_-) - k^\alpha |A - \bar{A}_-|^\alpha (A + \bar{A}_+)}{|A + \bar{A}_+|^\alpha - k^\alpha |A - \bar{A}_-|^\alpha}$$

(43)

becomes zero, signaling the existence of the apparent horizon, which becomes the event horizon when considering static solutions. Indeed, for $\bar{A}_- < A < \bar{A}_\infty$, $\partial_+ \Omega$ monotonically increases. In terms of the geometric gauge, the solutions can be rewritten as

$$ds^2 = (1 - \frac{r_+}{r})(1 - \frac{r_-}{r})^{-\frac{a-\chi^2/2}{a+\chi^2/2}} dt^2$$

$$- \frac{1}{-\mu a} (1 - \frac{r_+}{r})^{-1}(1 - \frac{r_-}{r})^{-\frac{a-\chi^2(1/2)}{a+\chi^2/2}} r_+ \frac{2-4dr^2 - r_+^{-\frac{1-a}{a}} (1 - \frac{r_-}{r})^{\frac{1-a}{a}} \frac{\chi^2}{a+\chi^2/2}d\Omega}$$

(44)

for the metric and

$$e^{\chi f} = e^{\chi f_\infty} (1 - \frac{r_-}{r})^{-\chi^2/(a+\chi^2/2)}$$

(45)

for the dilaton field. Here the parameters $r_+, r_-$ and $f_\infty$, the value of the dilaton field at the spatial infinity, are related to the black hole mass $M$, the electric charge $Q_E$ and the dilaton charge $Q_D$ as follows.

$$2M = \frac{a - \chi^2/2}{a + \chi^2/2} r_+ - r_- , \quad Q_E = \frac{r_+ + r_- e^{-\chi f_\infty}}{2a + \chi^2} (1/2) , \quad Q_D = \frac{\chi^2}{a + \chi^2/2} r_-$$

(46)

These relations are derived from the asymptotic behavior of each field. In terms of our constants of motion, we have the expressions

$$r_+ = 2a \sqrt{\frac{e^{-\chi f_\infty}}{-\mu c_1(a + \chi^2/2)}} Q , \quad r_- = \sqrt{\frac{c_1 e^{-\chi f_\infty}}{-\mu(a + \chi^2/2)}} Q$$

and

$$e^{\chi f_\infty} = e^{\chi f_1} \left( \frac{c_1}{c_1 - 2a \sqrt{4s^2 - 2aQc}} \right)^{-\chi^2/(a+\chi^2/2)}.$$
These results reproduce the GHS black hole \[1\] for \(d = 4\), i.e., \(a = 1/2\) and \(\mu = -2\), and, for \(d > 4\), become the GM black holes \[3\]. Physically distinctive solutions are parameterized by 4 parameters that represent the black hole mass, the electric charge, the dilaton charge, and the value of the dilaton at the spatial infinity. Moreover, there is a relation that gives the dilaton charge in terms of the electric charge, the black hole mass and the value of the dilaton field at the spatial infinity, resulting from \(\bar{f}_0 = 0\). This reproduces Eq. (10) of \[4\], which plays the equivalent role. The original parameter space is 7 dimensional, but the degrees of freedom in choice of the reference time, the reference scale, and in the addition of an arbitrary real number to the \(A\) field (we recall \(dA/dx\) is related to the electric field) produce 3 dimensional orbit. The quotient space of the original parameter space by this orbit results the 4 dimensional parameter space. The condition \(\bar{f}_0 = 0\) selects the 3 dimensional subspace of independent parameters immersed in the 4 dimensional space. It is interesting to note that the no-hair theorem in case of \(\chi = 0\) prohibits the black hole from carrying the scalar charge, i.e., \(f_0 = 0\) for the black hole solutions, while we have \(\bar{f}_0 = 0\) for \(\chi \neq 0\).

For \(\bar{f}_0 \neq 0\), we have 4 zeros of \(\Omega\) at \(A = -\infty\), \(A = -\bar{A}_+, A = \bar{A}_-\) and \(A = \infty\) since \(h_\pm > 0\). The positions of the infinities of \(\Omega\) are the same as the \(\bar{f}_0 = 0\) case. We consider what happens to the solutions for the same range of \(A\) that reproduces the GHS (or GM) solutions, \([\bar{A}_-, \bar{A}_{\infty}^L]\). In this case, \(A = \bar{A}_-\), the would-be position for the black hole horizon just becomes the point \(\Omega = 0\). Furthermore, we have \(\partial_+ \Omega = h_- \sqrt{4s^2 - 2aQc/(2a)} > 0\) for \(A = \bar{A}_-\) from Eq. (33) and, as \(A\) increases up to \(A = \bar{A}_{\infty}^L\), the function \(\partial_+ \Omega\) monotonically increases. Thus, at \(A = \bar{A}_-\), we have a naked singularity in contrary to the case when \(\bar{f}_0 = 0\). This behavior becomes more understandable if we compute

\[
\frac{f_+ f_-}{a + \chi^2/2} = \frac{\bar{f}_0^2}{(4s^2 - 2aQc)}.
\]

Thus, if \(\bar{f}_0 \neq 0\), both of \(f_\pm\) becomes non-zero. This implies the dilaton field \(f\) diverges logarithmically at \(A = \bar{A}_-\), as can be explicitly seen from Eq. (30). In other words, if the dilaton field \(f\) carries the dilaton charge other than the one that satisfies \(\bar{f}_0 = 0\), the dilaton field diverges logarithmically near the position of the would-be horizon. The stress-energy
induced by this divergence cuts the space-time off at that position and makes the would-be horizon the point of a naked singularity. If $\chi = 0$, we go back to the usual story of the No-scalar-hair theorem for the charged black holes. If $\chi \neq 0$, we have a similar story but $\bar{f}_0 = 0$ gives non-vanishing value of $f_0$.

Our next concern is the case when $k = 1$. As before we first study the case when $\bar{f}_0 = 0$ and choose to consider $h_- = 0$ and $h_+ = \chi^2/(a + \chi^2/2)$. The constant $\alpha$ is again 1 for $\bar{f}_0 = 0$. The most distinctive change now from the case $0 < k < 1$ is the location of the infinities of the $\Omega$ field. The $\bar{A}_\infty^I$ is now the average value of the $\bar{A}_-$ and $-\bar{A}_+$. However, $\bar{A}_\infty^I$ gets pushed to the infinity. Thus, $\Omega$ becomes infinity at $A = -\infty$, $A = (\bar{A}_- - \bar{A}_+)/2$ and $A = \infty$. The zeros of $\Omega$ is located at $A = -\bar{A}_+$. From this consideration, it is clear that the range we should choose to get the analog of the GHS black holes is $[\bar{A}_-, +\infty)$. Eq. (43) once again shows $\partial_+ \Omega$ is zero at $A = \bar{A}_-$ and increases monotonically from there as $A$ increases. Thus, $A = \bar{A}_-$ can play the role of the black hole horizon. By introducing the geometric gauge, our solutions in this region can be written as

$$d s^2 = U(r) d t^2 - U^{-1}(r) d r^2 - r^{2N} d \Omega$$

for the metric

$$e^{\chi f} = \frac{2 Q^2}{(d-2)(d-3)} \frac{a + \chi^2/2}{a} r^{-2N(d-3)}$$

for the dilaton field. Here we use the formula $\mu = -(d-2)(d-3)$ and assume $\chi \neq 0$. The function $U(r)$ is given by

$$U(r) = \frac{\chi^4}{4 N^2(a + \chi^2/2)^2} r^{2(1-N)} - \frac{4 M}{(d-2) N} r^{(1-\chi^2/(2a))(1-N)},$$

and the number $N$ satisfies $N = \chi^2/(2(d-3)a + \chi^2)$. In terms of our constants of motion, the parameter $M$ is given by

$$M = \frac{\chi(s \chi + a f_0)}{2a(a + \chi^2/2)}.$$

We reproduce the CHM black holes by transforming $f$ and $\chi$ via $f \to f \sqrt{8/(d-2)}$, $\chi \to \chi \sqrt{2/(d-2)}$. 

18
The counting of the physical degrees of freedom is interesting in this case. The degrees of freedom in choice of the reference scale \[ \gamma \] the reference time, and in the translation of \( A \) reduce the physical degrees of freedom to 4. The further imposition of 2 conditions, \( \tilde{f}_0 = 0 \) and \( k = 1 \), leaves us the 2-dimensional space for the physically independent parameters. In fact, the CHM solutions have two independent parameters \( Q_E \) and \( M \), and the dilaton field \( f \) contains no further independent parameters. In case of GHS solutions, we are able to freely set the value of the dilaton field at the spatial infinity.

What happens to the CHM solutions when \( \tilde{f}_0 \neq 0 \) is similar to the case of generic black holes. For \( \tilde{f}_0 \neq 0 \), we find that \( \Omega = 0 \) at \( A = \bar{A}_- \) and \( \partial_+ \Omega \) is positive definite for \( A \geq \bar{A}_- \), since \( h_\pm > 0 \). Thus, if the CHM black holes try to carry other values of the dilaton charge than the one required by \( \tilde{f}_0 = 0 \), naked singularities occur. Thus, we see that the story similar to the no-hair theorem is also working here.

Another interesting issue is what happens to the CHM solutions when we set \( \chi = 0 \). In fact, in this case, the \( \Omega \) field is set to a fixed real value as can be seen from Eq. (41), since \( h_\pm = 0 \) and the denominator is a constant for \( A > \bar{A}_- \). Thus, the topology of the space-time is \( M_2 \times S^{d-2} \) where the radius of the transversal sphere is a constant. This solution is drastically different from the usual spherically symmetric 4-dimensional space-time, where the \( \Omega \) field is given by a non-constant function.

To summarize, we recover the GHS (or GM) black holes for \( 0 < k < 1 \) and the CHM black holes for \( k = 1 \). As the black holes try to carry the dilaton charge other than the value prescribed by \( \tilde{f}_0 = 0 \), naked singularities occur. In this sense, the GHS (or GM) black holes and the CHM black holes are unique solutions where the essential singularities are hidden inside the horizon.

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\[ ^4 \text{In \cite{7}, the degree of freedom in the choice of the reference scale is represented by the parameter } \gamma. \text{ In their Eq. (3.2), we can redefine } \gamma^2 M \rightarrow M \text{ and } t/\gamma^2 \rightarrow t. \text{ Then, the metric and the dilaton field depend only on two parameters } Q \text{ and } M. \]
B. Solutions (37)

The solutions given in Eq. (37) have an interesting space-time structure. From Eq. (28), we see that \( P(A)/\bar{Q} \) should be chosen to be positive semi-definite. From Eq. (37), this implies \( D^2 < 0 \) case gives physically acceptable solutions since we have \( \mu < 0 \) for the theories in our consideration. The sufficient but not necessary conditions to satisfy all the restrictions are to require \( D^2 < 0 \) and \( 4s^2 - 2a\bar{Q}c < 0 \). Due to the latter condition, we see that \( PA/\bar{Q} \) is positive definite and, from Eq. (37), the range of \( \Omega \) has a finite maximum value. Thus, we have solutions with a finite range of allowed radius of the transversal sphere. The metric looks particularly simple if we further impose \( af_0 + \chi s = 0 \). Then, we indeed find

\[
D^2 = \frac{a}{a + \chi^2/2}(4s^2 - 2a\bar{Q}c) < 0
\]

and

\[
\bar{f}_0^2 = -\frac{1}{2a} \frac{\chi^2/2}{a + \chi^2/2}(4s^2 - 2a\bar{Q}c) > 0
\]

as long as \( 4s^2 - 2a\bar{Q}c < 0 \). For simplicity, we choose \( \bar{f}_0 > 0 \). Then in terms of geometric gauge, the metric can be written as

\[
ds^2 = \frac{4af_0^2}{-\mu\chi^2} \frac{dt^2}{r^{2a/(1-a)}} - \frac{a + \chi^2/2}{-\mu(1-a)^2} \frac{dr^2}{(r^*/r)^{2(a+\chi^2/2)/a} - 1} - r^2 d\Omega \tag{49}
\]

and the dilaton field is given by

\[
f(r) = \frac{\chi}{1 - a} \ln r - \frac{\chi}{2a} \ln \left( \frac{4aQ\bar{f}_0^2}{-\mu\chi^2} \right) + \frac{a + \chi^2/2}{a} f_1. \tag{50}
\]

The constant \( r^* \) is given by

\[
r^* = \left( \frac{\chi^2Q}{2af_0^2} \right)^{\frac{a}{2a+\chi^2}} \sqrt{\frac{4aQ\bar{f}_0^2}{\chi^2} e^{-\chi f_1}}.
\]

That this solution has a finite range of allowed value of the transversal sphere is clear from the above expression for the metric.
We obtained general static solutions on a local space-time region that continuously include the asymptotically flat GHS (or GM) black holes and the non-asymptotically flat CHM black holes. We then demonstrated an analog of the no-hair theorem for each black hole solution. In this class of solutions, we mention that $k > 1$ solutions exactly reproduce the black hole solution with $1/k < 1$. This statement can be straightforwardly verified by setting the range of $A$ to be $(\bar{A}_\infty^I, -\bar{A}_+^I]$ and repeating the calculations in section IIIA. This shows a kind of duality in our solutions and, incidently, the CHM solutions correspond to the self-dual solution under this dual transformation. It will be interesting to see whether there is some underlying reason for this behavior.

We find, additionally, there are whole other class of space-time with unusual geometries such as Eq. (37). Even in Eq. (34), if we set $c_1 \leq 0$, there are solutions with finite range of $\Omega$ and asymptotically flat solutions with naked singularities. Although all of these new solutions have unphysical feature, it is still interesting to observe their unusual space-time geometries. It remains to be seen whether we can construct non-trivial global space-time structure that makes physical sense.

Our analysis in this paper is valid for $a > 0$ as can be seen from our field redefinitions. This technical point excludes the case of the (2+1)-dimensional gravity theories from our consideration. The (2+1)-dimensional gravity theories with a negative cosmological constant contain black holes and have many interesting features [14]. The treatment for this case in presence of the dilaton field is in progress.

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