Conformal Phase Transition, $\beta$-Function, and Infrared Dynamics in QCD

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Abstract

The dynamics in QCD with different number of fermion flavors $N_f$ is discussed. The emphasis is on the description of the conformal phase transition with respect to $N_f$ separating a phase with no confinement and chiral symmetry breaking and a phase with confinement and with chiral symmetry breaking.

1 Introduction

The infrared dynamics of QCD can be viewed only dimly via presently available tools. Because of that, there may be surprises, as it has already happened with our understanding of nonperturbative dynamics in $N=1$ supersymmetric QCD [1]. In particular, those studies showed that the conventional wisdom which has accepted that the asymptotic freedom implies confinement is not always true.

Recently, there has been considerable interest in the existence of a nontrivial conformal dynamics in 3+1 dimensional non-supersymmetric vector like gauge theories, with a relatively large number of fermion flavors $N_f$. The roots of this problem go back to a work of Banks and Zaks [9] who were first to discuss the consequences of the existence of an infrared-stable fixed point $\alpha = \alpha^*$ for $N_f > N_0^f$ in vector-like gauge theories. The value $N_0^f$ depends on the gauge group: in the case of SU(3) gauge group, $N_0^f = 8$ in the two-loop approximation.

A new insight in this problem [2, 3] has been, on the one hand, connected with using the results of the analysis of the Schwinger-Dyson (SD) equations describing chiral symmetry breaking in QCD (for a review, see Refs. [10, 11]) and, on the other hand, with the discovery of the conformal window in $N = 1$ supersymmetric QCD [1].
In particular, Appelquist, Terning, and Wijewardhana\(^2\) suggested that, in the case of the gauge group SU\((N_c)\), the critical value \(N_f^c \simeq 4N_c\) separates a phase with no confinement and chiral symmetry breaking \((N_f > N_f^c)\) and a phase with confinement and with chiral symmetry breaking \((N_f < N_f^c)\). The basic point for this suggestion was the observation that at \(N_f > N_f^c\) the value of the infrared fixed point \(\alpha^*\) is smaller than a critical value \(\alpha_{cr} \simeq \frac{2N_c}{N_c^2 - 1} \pi\), presumably needed to generate the chiral condensate\(^{10, 11}\).

The authors of Ref.\(^2\) considered only the case when the running coupling constant \(\alpha(\mu)\) is less than the fixed point \(\alpha^*\). In this case the dynamics is asymptotically free (at short distances) both at \(N_f < N_f^c\) and \(N_f^c < N_f < N_f^{**} \equiv \frac{11N_c}{2}\).

Yamawaki and the author\(^3\) analyzed the dynamics in the whole \((\alpha, N_f)\) plane and suggested the \((\alpha, N_f)\)-phase diagram of the SU\((N_c)\) theory (see Fig. 1 below).\(^4\) In particular, it was pointed out that one can get an interesting non-asymptotically free dynamics when the bare coupling constant \(\alpha^{(0)}\) is larger than \(\alpha^*\), though not very large.

The dynamics with \(\alpha^{(0)} > \alpha^*\) admits a continuum limit and is interesting in itself. Also, its better understanding can be important for establishing the conformal window in lattice computer simulations of the SU\((N_c)\) theory with such large values of \(N_f\). In order to illustrate this, let us consider the following example. For \(N_c = 3\) and \(N_f = 16\), the value of the infrared fixed point \(\alpha^*\) is small: \(\alpha^* \simeq 0.04\) (see below). To reach the asymptotically free phase, one needs to take the bare coupling \(\alpha^{(0)}\) less than this value of \(\alpha^*\). However, because of large finite size effects, the lattice computer simulations of the SU\((3)\) theory with such a small \(\alpha^{(0)}\) would be unreliable. Therefore, in this case, it is necessary to consider the dynamics with \(\alpha(\mu) > \alpha^*\).

The existence of the phase transition with respect to \(N_f\) in QCD raises the following question: what are the infrared properties of the QCD \(\beta\) function for different \(N_f\) and how the \(\beta\) function structure reflects the existence of this phase transition? I will discuss those issues at the end of my talk but first I will discuss the dynamics in the conformal window of QCD in detail. In particular, I will consider the spectrum of low energy excitations in that dynamics in the presence of a bare fermion mass\(^{12}\). We will see that in this case, unlike the familiar QCD with a small \(N_f\) (\(N_f = 2\) or 3), glueballs are much lighter than bound states composed of fermions, if the value of the infrared fixed point is not too large. Another characteristic point is a strong (and simple) dependence of the masses of all the colorless bound states on the bare fermion mass, even if the latter is tiny.

This talk is based on papers\(^{12, 13}\).

\(^1\)This phase diagram is essentially different from the original Banks-Zaks diagram\(^9\). For details, see Sec.VII in Ref.\(^3\)
2 Dynamics in the Conformal Window in QCD like Theories

I begin by recalling the basic facts concerning the two-loop $\beta$ function in an $\text{SU}(N_c)$ theory. The $\beta$ function is

$$\beta = -b\alpha^2 - c\alpha^3$$

(1)

with

$$b = \frac{1}{6\pi}(11N_c - 2N_f),$$

(2)

$$c = \frac{1}{24\pi^2}(34N_c^2 - 10N_cN_f - 3\frac{N_c^2 - 1}{N_c}N_f).$$

(3)

While these two coefficients are invariant under change of a renormalization scheme, the higher-order coefficients are scheme dependent. Actually, there is a renormalization scheme in which the two-loop $\beta$ function is (perturbatively) exact [15]. We will use such a renormalization scheme.

If $b > 0$ ($N_f < N_f^{**} \equiv \frac{11N_c}{2}$) and $c < 0$, the $\beta$ function has a zero, corresponding to a infrared-stable fixed point, at

$$\alpha = \alpha^* = -\frac{b}{c}.$$ 

(4)

When $N_f$ is close to $N_f^{**}$, the value of $\alpha^*$ is small. For example, from Eqs. (2), (3), and (4), one gets $\alpha^* \simeq 0.04, 0.14, 0.28, \text{and } 0.47$ for $N_c=3$ and $N_f=16, 15, 14, \text{and } 13$, respectively.

The value of $\alpha^*$ becomes equal to $\alpha_{cr} = \frac{2N_c - 2}{N_c - 3}$ at $N_f$ close to $N_f \simeq 4N_c$. And the fixed point disappears at the value $N_f = N_f^*$, when the coefficient $c$ becomes positive ($N_f^*$ is $N_f^* \simeq 8.05$ for $N_c=3$).

The $\beta$ function (1) leads to the following solution for the running coupling:

$$b \log \left( \frac{q}{\mu} \right) = \frac{1}{\alpha(q)} - \frac{1}{\alpha(\mu)} - \frac{1}{\alpha^*} \log \left( \frac{\alpha(q)(\alpha(\mu) - \alpha^*)}{\alpha(\mu)(\alpha(q) - \alpha^*)} \right).$$

(5)

We emphasize that this solution is valid both for $\alpha(\mu) < \alpha^*$ and $\alpha(\mu) > \alpha^*$.

Let us first consider the case with $\alpha(\mu) < \alpha^*$. It is convenient to introduce the parameter [1]

$$\Lambda = \mu \exp \left[ -\frac{1}{b\alpha^*} \log \left( \frac{\alpha^* - \alpha(\mu)}{\alpha(\mu)} \right) - \frac{1}{b\alpha(\mu)} \right].$$

(6)

Then, Eqs. (6) and (3) imply that

$$\frac{1}{\alpha(q)} = b \log \left( \frac{q}{\Lambda} \right) + \frac{1}{\alpha^*} \log \left( \frac{\alpha(q)}{\alpha^* - \alpha(q)} \right).$$

(7)
Taking $q = \Lambda$, we find that

$$\frac{\alpha^*}{1 + e^{-1}} \simeq 0.73\alpha^* < \alpha(\Lambda) < \alpha^*.$$  \hspace{1cm} (8)

One may think that $\Lambda$ plays here the same role as $\Lambda_{QCD}$ in the confinement phase. However, as we will see, its physical meaning is somewhat different.

Eq. (8) implies that

$$\alpha(q) \simeq \frac{1}{b \log \frac{\Lambda}{q}}$$  \hspace{1cm} (9)

for $q >> \Lambda$ (the usual behavior in asymptotically free theories), and

$$\alpha(q) \simeq \frac{\alpha^*}{1 + e^{-1}\left(\frac{q}{\Lambda}\right)^{b\alpha^*}}$$  \hspace{1cm} (10)

for $q << \Lambda$, governed by the infrared fixed point $\alpha^*$.

Let us turn to a less familiar case with $\alpha(\mu) > \alpha^*$. One still can use Eq. (5). Introduce now the parameter $\tilde{\Lambda}$ as

$$\tilde{\Lambda} \equiv \mu \exp \left[ -\frac{1}{b\alpha^*} \log \left( \frac{\alpha(\mu) - \alpha^*}{\alpha(\mu)} \right) - \frac{1}{b\alpha(\mu)} \right]$$  \hspace{1cm} (11)

(compare with Eq. (6)). Then, Eqs. (5) and (11) imply

$$\frac{1}{\alpha(q)} = b \log \frac{q}{\tilde{\Lambda}} + \frac{1}{\alpha^*} \log \left( \frac{\alpha(q)}{\alpha(q) - \alpha^*} \right).$$  \hspace{1cm} (12)

What is the meaning of $\tilde{\Lambda}$? It is a Landau pole at which $\alpha(q)|_{q=\tilde{\Lambda}} = \infty$. Indeed, taking $q = \tilde{\Lambda}$ in Eq. (12), one gets

$$\frac{1}{\alpha(\tilde{\Lambda})} = \frac{1}{\alpha^*} \log \frac{\alpha(\tilde{\Lambda})}{\alpha(\tilde{\Lambda}) - \alpha^*}.$$  \hspace{1cm} (13)

The only solution of this equation is $\alpha(\tilde{\Lambda}) = \infty$.

The presence of the Landau pole implies that the dynamics is not asymptotically free. To get a more insight in this dynamics, let us introduce an ultraviolet cutoff $M$ with the bare coupling constant $\alpha^{(0)} \equiv \alpha(q)|_{q=M}$. Now all momenta $q$ satisfying $q \leq M$.

Eq. (12) implies that at finite $\alpha^{(0)} = \alpha(M)$, the cutoff $M$ is less than $\tilde{\Lambda}$, with $\alpha(\tilde{\Lambda}) = \infty$. Therefore the Landau pole is unreachable in the theory with cutoff $M$ and with $\alpha^{(0)} < \infty$. Still one can use $\tilde{\Lambda}$ (11) for a convenient parametrization of the running coupling $\alpha(q)$ (see Eq. (12)). However, one should remember that momenta $q$ satisfy $q \leq M < \tilde{\Lambda}$.

Eq. (12) implies that

$$\alpha^2(q) \simeq \frac{\alpha^*}{2b \log \frac{q}{\tilde{\Lambda}}}$$  \hspace{1cm} (14)
for $\alpha(q) >> \alpha^*$, and

$$\alpha(q) \simeq \frac{\alpha^*}{1 - e^{-\frac{\alpha}{\Lambda^*} b\alpha^*}}$$  \hspace{1cm} (15)$$

when $\alpha(q)$ is close to $\alpha^*$, i.e. when $\alpha(q) - \alpha^* << \alpha^*$. Thus, now $\alpha(q)$ approaches the fixed point $\alpha^*$ from above (compare with Eq.(10)). And, in general, Eq.(11) implies that $\alpha(q)$ monotonically decrease with $q$, from $\alpha(q) = \alpha(0)$ at $q = M$ to $\alpha(q) = \alpha^*$ at $q = 0$.

Does a meaningful continuum limit exist in this case? The answer is of course "yes". As it follows from Eq.(12), when $M$ (and therefore $\tilde{\Lambda}$) goes to infinity, and the bare coupling $\alpha(0) > \alpha^*$ is arbitrary but fixed, $\alpha(q)$ is equal to the fixed value, $\alpha(q) = \alpha^*$, for all $q < \infty$. Therefore it is a non-trivial conformal field theory.

So far we considered the solution for $\alpha(q)$ connected with the perturbative (and perturbatively exact in the 't Hooft renormalization scheme [15]) $\beta$ function (1). However, unlike ultraviolet stable fixed points, defining dynamics at high momenta, infrared-stable fixed points (defining dynamics at low momenta) are very sensitive to nonperturbative dynamics leading to the generation of particle masses. For example, if fermions acquire a dynamical mass, they decouple from the infrared dynamics, and therefore the perturbative infrared fixed point (4) will disappear.

The phase diagram in the $(\alpha(0), N_f)$-plane in this theory was suggested in Ref.[3]. It is shown in Fig. 1. The left-hand portion of the curve in this figure coincides with the line of the infrared-stable fixed points $\alpha^*(N_f)$ in Eq.(2). It separates two symmetric phases, $S_1$ and $S_2$, with $\alpha(0) < \alpha^*$ and $\alpha(0) > \alpha^*$, respectively. Its lower end is $N_f = N_f^c$ (with $N_f^c \simeq 4N_c$ if $\alpha_c \simeq \frac{2N_c}{N_c^2 - 1}$): at $N_f^c < N_f < N_f^c$ the infrared fixed point is washed out by generating a dynamical fermion mass.

The horizontal, $N_f = N_f^c$, line describes a phase transition between the symmetric phase $S_1$ and the phase with confinement and chiral symmetry breaking. As it was suggested in Refs.[2, 3], based on a similarity of this phase transition with that in quenched QED$_3$ [10, 11, 16] and in QED$_3$ [17], there is the following scaling law for $m_{\text{dyn}}^2$:

$$m_{\text{dyn}}^2 \sim \Lambda_{cr}^2 \exp \left( - \frac{C}{\sqrt{\frac{\alpha^*(N_f)}{\alpha_c} - 1}} \right)$$  \hspace{1cm} (16)$$

where the constant $C$ is of order one and $\Lambda_{cr}$ is a scale at which the running coupling is of order $\alpha_{cr}$.

It is a continuous phase transition with an essential singularity at $N_f = N_f^c$. The characteristic point of this phase transition is that the critical line $N_f = N_f^c$ separates phases with essentially different spectra of low energy excitations [2, 3, 8] and the different structure of the equation for the divergence of the
dilatation current (i.e. with essentially different realizations of the conformal symmetry) \[3\]. It was called the conformal phase transition in Ref.\[3\].

At present it is still unclear whether the phase transition on the line \(N_f = N_f^{cr}\) is indeed a continuous phase transition with an essential singularity or it is a first order phase transition \[4\, 7\]. However, anyway, the two properties (the abrupt change of the spectrum of excitations and the different structure of the equation for the divergence of the dilatation current in those two phases) have to take place.

At last, the right-hand portion of the curve on the diagram occurs because at large enough values of the bare coupling, spontaneous chiral symmetry breaking takes place for any number \(N_f\) of fermion flavors. This portion describes a phase transition called a bulk phase transition in the literature, and it is presumably a first order phase transition. The vertical line ends above \(N_f = 0\) since in pure gluodynamics there is apparently no phase transition between weak-coupling and strong-coupling phases.

Up to now we have considered the case of a chiral invariant action. But how will the dynamics change if a bare fermion mass term is added in the action? This question is in particular relevant for lattice computer simulations: for studying a chiral phase transition on a finite lattice, it is necessary to introduce a bare fermion mass. We will show that adding even an arbitrary small bare fermion mass results in a dramatic changing the dynamics both in the \(S_1\) and \(S_2\) phases.

Recall that in the case of confinement SU(\(N_c\)) theories, with a small, \(N_f < N_f^{cr}\), number of fermion flavors, the role of a bare fermion mass \(m(0)\) is minor if \(m(0) << \Lambda_{QCD}\) (where \(\Lambda_{QCD}\) is a confinement scale). The only relevant consequence is that massless Nambu-Goldstone pseudoscalars get a small mass (the PCAC dynamics).

The reason for that is the fact that the scale \(\Lambda_{QCD}\), connected with a scale anomaly, describes the breakdown of the conformal symmetry connected both with perturbative and nonperturbative dynamics: the running coupling and the formation of bound state. Certainly, a small bare mass \(m(0) << \Lambda_{QCD}\) is irrelevant for the dynamics of those bound states.

Now let us turn to the phase \(S_1\) and \(S_2\), with \(N_f > N_f^{cr}\). At finite \(\Lambda\) in \(S_1\) and \(\Lambda\) in \(S_2\), there is still conformal anomaly: because of the running of the effective coupling constant, the conformal symmetry is broken. It is restored only if \(\Lambda \to 0\) in \(S_1\) and \(\Lambda > M \to \infty\) in \(S_2\). However, the essential difference with respect to confinement theories is that both \(\Lambda\) and \(\Lambda\) have nothing with the dynamics forming bound states: since at \(N_f > N_f^{cr}\) the effective coupling

\[2\]The fact that spontaneous chiral symmetry breaking takes place for any number of fermion flavors, if \(\alpha(0)\) is large enough, is valid at least for lattice theories with Kogut-Susskind fermions. Notice however that since the bulk phase transition is a lattice artifact, the form of this portion of the curve can depend on the type of fermions used in simulations (for details, see Ref.\[3\]).
is relatively weak, it is impossible to form bound states from massless fermions and gluons (recall that the $S_1$ and $S_2$ phases are chiral invariant).

Therefore the absence of a mass for fermions and gluons is a key point for not creating bound states in those phases. The situation changes dramatically if a bare fermion mass is introduced: indeed, even weak gauge, Coulomb-like, interactions can easily produce bound states composed of massive constituents, as it happens, for example, in QED, where electron-positron (positronium) bound states are present.

To be concrete, let us first consider the case when all fermions have the same bare mass $m^{(0)}$. It leads to a mass function $m(q^2) \equiv B(q^2)/A(q^2)$ in the fermion propagator $G(q) = (\hat{q}A(q^2) - B(q^2))^{-1}$. The current fermion mass $m$ is given by the relation

$$m(q^2)|_{q^2=m^2} = m.$$  \hfill (17)

For the clearest exposition, let us consider a particular theory with a finite cutoff $M$ and the bare coupling constant $\alpha^{(0)} = \alpha(q)|_{q=M}$ being not far away from the fixed point $\alpha^*$. Then, the mass function is changing in the "walking" regime \cite{18} with $\alpha(q^2) \simeq \alpha^*$. It is

$$m(q^2) \simeq m^{(0)} \left(\frac{M}{q}\right)^{\gamma_m},$$ \hfill (18)

where the anomalous dimension $\gamma_m \simeq 1 - (1 - \frac{\alpha^*}{\alpha_{cr}})^{1/2}$ \cite{10,11}. Eqs.\cite{17} and \cite{18} imply that

$$m \simeq m^{(0)} \left(\frac{M}{m^{(0)}}\right)^{\gamma_m/m}.$$ \hfill (19)

There are two main consequences of the presence of the bare mass:

(a) bound states, composed of fermions, occur in the spectrum of the theory. The mass of a $n$-body bound state is $M^{(n)} \simeq nm$;

(b) At momenta $q < m$, fermions and their bound states decouple. There is a pure SU($N_c$) Yang-Mills theory with confinement. Its spectrum contains glueballs.

To estimate glueball masses, notice that at momenta $q < m$, the running of the coupling is defined by the parameter $\tilde{b}$ of the Yang-Mills theory,

$$\tilde{b} = \frac{11}{6\pi} N_c.$$ \hfill (20)

Therefore the glueball masses $M_{gl}$ are of order

$$\Lambda_{YM} \simeq m \exp(-\frac{1}{b\alpha^*}).$$ \hfill (21)

For $N_c = 3$, we find from Eqs.\cite{2}, \cite{3}, and \cite{20} that $\exp(-\frac{1}{b\alpha^*})$ is $6 \times 10^{-7}$, $2 \times 10^{-2}$, $10^{-1}$, and $3 \times 10^{-1}$ for $N_f=16, 15, 14, \text{and} 13$, respectively. Therefore
at $N_f=16$, 15 and 14, the glueball masses are essentially lighter than the masses of the bound states composed of fermions. The situation is similar to that in confinement QCD with heavy quarks, $m \gg \Lambda_{QCD}$. However, there is now a new important point: in the conformal window, any value of $m(0)$ (and therefore $m$) is "heavy": the fermion mass $m$ sets a new scale in the theory, and the confinement scale $\Lambda_{YM}$ \cite{21} is less, and rather often much less, than this scale $m$.

This leads to a spectacular "experimental" signature of the conformal window in lattice computer simulations: glueball masses rapidly, as $(m(0))^{-\frac{1}{1+\gamma m}}$, decrease with the bare fermion mass $m(0)$ for all values of $m(0)$ less than cutoff $M$.

Few comments are in order:

1. The phases $S_1$ and $S_2$ have essentially the same long distance dynamics. They are distinguished only by their dynamics at short distances: while the dynamics of the phase $S_1$ is asymptotically free, that of the phase $S_2$ is not. In particular, when all fermions are massive (with the current mass $m$), the continuum limit $M \rightarrow \infty$ of the $S_2$-theory is a non-asymptotically free confinement theory. Its spectrum includes colorless bound states composed of fermions and gluons. For $q < m$ the running coupling $\alpha(q)$ is the same as in pure SU($N_c$) Yang-Mills theory, and for all $q > m$ $\alpha(q)$ is very close to $\alpha^*$ ("walking", actually, "standing" dynamics). For those values $N_f$ for which $\alpha^*$ is small (as $N_f=16$, 15 and 14 at $N_c=3$), glueballs are much lighter than the bound states composed of fermions. Notice that, unlike the case with $m = 0$, there exists an $S$-matrix in this theory.

2. In order to get the clearest exposition, we assumed such estimates as $N_f^{cr} \approx 4N_c$ for $N_f^{cr}$ and $\gamma_m = 1 - \sqrt{1 - \frac{\alpha^*}{\alpha_{cr}}}$ for the anomalous dimension $\gamma_m$. While the latter should be reasonable for $\alpha^* < \alpha_{cr}$ (and especially for $\alpha^* << \alpha_{cr}$) \cite{10} \cite{11}, the former is based on the assumption that $\alpha_{cr} \approx \frac{2N_c}{N_c^2 - \frac{\pi}{3}}$, which, though seems reasonable, might be crude for some values of $N_c$. It is clear however that the dynamical picture presented in this paper is essentially independent of those assumptions.

3. So far we have considered the case when all fermions have the same bare mass $m(0)$. The generalization to the case when different fermions may have different bare masses is evident.

4. Lattice computer simulations of the SU(3) theory with a relatively large number of $N_f$ \cite{19} \cite{20} indeed indicate on the existence of a symmetric phase. However, the value of the critical number $N_f^{cr}$ is different in different simulations: it varies from $N_f^{cr} = 6$ \cite{20} through $N_f^{cr} = 12$ \cite{19}.

I hope that the signature of the conformal window considered in this talk can be useful to settle this important issue.
3 Padé-Summation Approach to QCD $\beta$-function Infrared Properties

How does the structure of the QCD $\beta$ function reflect the existence of the phase transition with respect to $N_f$? This problem has been addressed in the work [13].

The impetus for that work was the structure of the $\beta$ function of the $SU(N_c)$ SUSY gluodynamics which is known exactly if no matter fields are present [21]:

$$\beta(x) = -\frac{3N_c x^2}{4} \left[ \frac{1}{1 - N_c x^2/2} \right]; \quad x \equiv \frac{\alpha}{\pi}. \quad (22)$$

This structure shows the following two noticeable features:

a) Eq. (22) implies that, besides the conventional asymptotically free phase with $x < 2/N_c$, there exists a strong ultraviolet phase in which the coupling $x$ is greater than the $\beta$-function pole at $x = 2/N_c$. In that phase, the value $x = \infty$ is an ultraviolet fixed point. As in the case of the infrared fixed point considered in the previous section, these two phases share common infrared properties. Because of that, the $\beta$-function pole is called an infrared attractor. Unlike theories with an infrared fixed point, theories with an infrared attractor correspond to the confinement phase.

In Ref. [13], we addressed whether Padé-summations of the $\overline{\text{MS}}$ QCD $\beta$ function for a given number of flavors exhibit an infrared fixed point, or alternatively, an infrared attractor. The main results are the following. Below an approximant-dependent flavor threshold ($6 \leq N_f \leq 8$), the Padé-summation $\beta$ functions incorporating $[2|1], [1|2], [2|2], [1|3], \text{ and } [3|1]$ approximants whose Maclaurin expansions match known higher-than-one-loop contributions to the $\beta$-function series always exhibit a positive pole prior to the occurrence of their first positive zero, precluding any identification of this first positive zero as an infrared fixed point. This result is shown to be true regardless of the magnitude of the presently-unknown five-loop $\beta$-function contribution explicitly appearing within Padé-summation $\beta$ functions incorporating $[2|2], [1|3], \text{ and } [3|1]$ approximants. Like in the case of supersymmetric gluodynamics, the pole in question suggests the occurrence of dynamics in which both a strong and an asymptotically free phase share a common infrared attractor. As $N_f$ increases above an approximant-dependent flavor threshold, Padé-summation $\beta$ functions exhibit dynamics controlled by an infrared fixed point. This fixed point decreases in magnitude with increasing flavor number.

Thus utilizing Padé-summation QCD $\beta$ functions, we obtain a good degree of agreement with infrared properties predicted [2, 3, 6] via the ’t Hooft renormalization scheme [15] in which the $\beta$ function is truncated subsequent to two-loop order. It is noticeable that the infrared structure of the $\beta$ function we obtained in QCD with a small number of flavors is similar to that of the $\beta$ function in
SUSY gluodynamics: there are strong arguments in the literature in the support of essentially the same mechanism of confinement in those theories.

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Figure 1: The phase diagram in an SU($N_c$) gauge model. The coupling constant $g^{(0)} = \sqrt{4\pi\alpha^{(0)}}$ and $S$ and $A$ denote symmetric and asymmetric phases, respectively.