Neutrino-atom collisions

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Abstract. Neutrino-atom scattering provides a sensitive tool for probing nonstandard interactions of massive neutrinos in laboratory measurements. The ionization channel of this collision process plays an important role in experiments searching for neutrino magnetic moments. We discuss some theoretical aspects of atomic ionization by massive neutrinos. We also outline possible manifestations of neutrino electromagnetic properties in coherent elastic neutrino-nucleus scattering.

1. Introduction

The neutrino oscillations determined by many dedicated experiments (see the review articles [1, 2, 3, 4, 5, 6]) are generated by neutrino masses and mixing [7, 8, 9, 10]. Therefore, the Standard Model (SM) must be extended to account for the neutrino masses. Various extensions of the SM predict different properties for neutrinos [4, 11, 12]. In many such extensions, neutrinos acquire also electromagnetic properties through quantum loops’ effects, thus allowing interactions of neutrinos with electromagnetic fields and electromagnetic interactions of neutrinos with charged particles (see [13] for the most comprehensive review of neutrino electromagnetic properties and interactions).

The most well studied and understood among the neutrino electromagnetic characteristics are the dipole magnetic and electric moments. The diagonal magnetic and electric moments of a Dirac neutrino in the minimally-extended SM with right-handed neutrinos, derived for the first time in [14], are respectively

\[ \mu_{ii} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19} \mu_B \left( \frac{m_i}{1\text{eV}} \right), \quad e_{ii} = 0, \]  

(1)

where \( m_i \) is the neutrino mass and \( \mu_B \) is the Bohr magneton. According to (1), the value of the neutrino magnetic moment is very small. However, in many other theoretical frameworks (beyond the minimally-extended SM) the neutrino magnetic moment can reach values that are of interest for the next generation of terrestrial experiments and also accessible for astrophysical observations. The current best laboratory upper limit on a neutrino magnetic moment, \( \mu_\nu \leq 2.9 \times 10^{-11} \mu_B \) (90% CL), has been obtained by the GEMMA collaboration [15]. The best astrophysical limit, \( \mu_\nu \leq 3 \times 10^{-12} \mu_B \) (90% CL) [16], comes from the constraints on the
possible delay of helium ignition of a red giant star in globular clusters due to the cooling induced by the energy loss in the plasmon-decay process $\gamma^* \rightarrow \nu \bar{\nu}$.

The most sensitive and widely used method for the experimental investigation of the neutrino magnetic moment is provided by direct laboratory measurements of low-energy elastic scattering of neutrinos and antineutrinos with electrons in reactor, accelerator and solar experiments. Detailed descriptions of these experiments can be found in [13, 17, 18, 19, 20, 21]. The possibility for neutrino-electron elastic scattering due to the neutrino magnetic moment was first considered in [22] and the cross section of this process was calculated in [23] (for related short historical notes see [24]). In [25] the cross section of [23] was corrected and the antineutrino-electron cross section was considered in the context of the earlier experiments with reactor antineutrinos of [26, 27], which were aimed to reveal the effects of the neutrino magnetic moment. Discussions on the derivation of the cross section and on the optimal conditions for bounding the neutrino magnetic moment, as well as a collection of cross section formulae for elastic scattering of neutrinos (antineutrinos) on electrons, nucleons, and nuclei can be found in [24, 28].

In the above-mentioned experiments, the electrons are bound into atoms in the employed detectors and, hence, the elastic scattering of neutrinos and antineutrinos on these electrons can induce atomic ionization (see the review article [29] and references therein). With lowering the energy-transfer value an additional collision channel apart from ionization opens up, namely, the coherent elastic neutrino-nucleus scattering [30]. This particular channel has not been experimentally observed so far, but it is expected to be accessible, for example, in the reactor experiments when lowering the energy threshold of the employed Ge detectors down to several hundred eV [31, 32, 33]. Any deviation of the measured cross section from the very precisely known SM value [34] will provide a signature of the physics beyond the SM. Therefore, it is important to examine how the neutrino electromagnetic interactions can contribute to the coherent elastic neutrino-nucleus scattering.

The paper is organized as follows. Section 2 is devoted to scattering of neutrinos on free and atomic electrons due to neutrino magnetic moments. The role of the center-of-mass atomic motion in the processes of atomic ionization by neutrinos is also discussed. In section 3, we analyze how the neutrino magnetic moment, millicharge and charge radius can manifest themselves in neutrino-nucleus coherent scattering.

2. Neutrino-electron elastic scattering

Let us consider the process

$$\nu_\ell + e^- \rightarrow \nu_{\ell'} + e^-,$$

(2)

where a neutrino with flavor $\ell = e, \mu, \tau$ and energy $E_\nu$ elastically scatters off a free electron (FE) at rest in the laboratory frame. Due to neutrino mixing, the final neutrino flavor $\ell'$ can be different from $\ell$. There are two observables: the kinetic energy $T_e$ of the recoil electron and the recoil angle $\chi$ with respect to the neutrino beam, which are related by

$$\cos \chi = \frac{E_\nu + m_e}{E_\nu} \left[ \frac{T_e}{T_e + 2m_e} \right]^{1/2}.$$

(3)

The electron kinetic energy is constrained from the energy-momentum conservation by

$$T_e \leq \frac{2E_\nu^2}{2E_\nu + m_e}.$$

(4)

Since, in the ultrarelativistic limit, the neutrino magnetic moment interaction changes the neutrino helicity and the SM weak interaction conserves the neutrino helicity, the two contributions add incoherently in the cross section, which can be written as [28]

$$\frac{d\sigma_{\nu e^-}}{dT_e} = \left( \frac{d\sigma_{\nu e^-}}{dT_e} \right)_{\text{SM}}^{\text{FE}} + \left( \frac{d\sigma_{\nu e^-}}{dT_e} \right)_{\text{mag}}^{\text{FE}}.$$

(5)
The weak-interaction cross section is given by
\[
\frac{d\sigma_{\nu e^-}}{dT_e}\text{SM} = \frac{G_F^2 m_e}{2\pi} \left\{ (g^\nu_A + g^\nu_A)^2 + (g^\nu_A - g^\nu_A)^2 \left(1 - \frac{T_e}{E^\nu}\right)^2 + \left[(g^\nu_A)^2 - (g^\nu_A)^2\right] \frac{m_e T_e}{E^\nu} \right\}, \tag{6}
\]
with the standard coupling constants \(g^\nu\) and \(g_A\) given by
\[
g^\nu_A = 2 \sin^2 \theta_W + 1/2, \quad g^\nu_A = 2 \sin^2 \theta_W - 1/2, \quad g^\nu_A = 1/2, \quad g^\nu_A = -1/2. \tag{7}
\]
For antineutrinos one must substitute \(g_A \rightarrow -g_A\).

The neutrino magnetic-moment contribution to the cross section is given by [28]
\[
\frac{d\sigma_{\nu e^-}}{dT_e}\text{mag} = \frac{\pi \alpha^2}{m^2_e} \left(\frac{1}{T_e} - \frac{1}{E^\nu}\right) \left(\frac{\mu_\nu}{\mu_B}\right)^2, \tag{8}
\]
where \(\mu_\nu\) is the effective magnetic moment [13]. It is traditionally called "magnetic moment", but it receives contributions from both the electric and magnetic dipole moments.

The two terms \((d\sigma_{\nu e^-}/dT_e)\text{SM}\) and \((d\sigma_{\nu e^-}/dT_e)\text{mag}\) exhibit quite different dependencies on the experimentally observable electron kinetic energy \(T_e\). One can see that small values of the neutrino magnetic moment can be probed by lowering the electron recoil-energy threshold. In fact, considering \(T_e \ll E^\nu\) in formulas (6) and (8), one can find that \((d\sigma/dT_e)\text{mag}\) exceeds \((d\sigma/dT_e)\text{SM}\) for
\[
T_e \lesssim \frac{\pi^2 \alpha^2}{G^2_F m^3_e} \left(\frac{\mu_\nu}{\mu_B}\right)^2. \tag{9}
\]

The current experiments with reactor antineutrinos have reached threshold values of \(T_e\) as low as few keV. These experiments are likely to further improve the sensitivity to low energy deposition in the detector. At low energies, however, one can expect a modification of the free-electron formulas (6) and (8) due to the binding of electrons in the germanium atoms, where, e.g., the energy of the \(K\alpha\) line, 9.89 keV, indicates that at least some of the atomic binding energies are comparable to the already relevant to the experiment values of \(T_e\). It was demonstrated [35, 36, 37, 38, 39] by means of analytical and numerical calculations that the atomic-binding effects are adequately described by the so-called stepping approximation introduced in [40] from interpretation of numerical data. According to the stepping approach,
\[
\frac{d\sigma_{\nu e^-}}{dT_e}\text{SM} = \frac{d\sigma_{\nu e^-}}{dT_e}\text{SM} \sum_j n_j \theta(T_e - I_j), \tag{10}
\]
\[
\frac{d\sigma_{\nu e^-}}{dT_e}\text{mag} = \frac{d\sigma_{\nu e^-}}{dT_e}\text{mag} \sum_j n_j \theta(T_e - I_j), \tag{11}
\]
where the \(j\) sum runs over all occupied atomic sublevels, with \(n_j\) and \(I_j\) being their occupations and ionization energies. Numerical calculations [41, 42] beyond the model of independent atomic electrons exhibit suppression of atomic factors relative to the stepping approximation when the energy-transfer value is close to the ionization threshold. This suppression can be explained by the electron-correlation effects [29].

As shown in [43], the cross sections (10) and (11) become suppressed and even vanish when \(T_e \rightarrow I_j\) due to atomic recoil. The following estimate for the energy range where the atomic-recoil effects are important was derived within the Thomas-Fermi model:
\[
T_e - I \lesssim 2 Z^{1/3} E_h \frac{m_e}{M_N},
\]
where \(E_h = \alpha^2 m_e = 27.2\) eV is the Hartree energy and \(M_N\) is the nuclear mass. For germanium \((Z = 32)\) one obtains \(T_e - I \lesssim 0.04\) eV. This energy scale appears to be insignificant for the experiments searching for magnetic moments of reactor antineutrinos [15, 17].
3. Neutrino-nucleus coherent scattering

Let us consider the case when an electron neutrino scatters on a spin-zero nucleus with even numbers of protons and neutrons, Z and N. The matrix element of this process, taking into account the neutrino electromagnetic properties, reads [44]

\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} \bar{u}(k') \gamma^\mu (1 - \gamma_5) u(k) C_V + \frac{4\pi Z e}{q^2} \left( e_{\nu e} + \frac{e}{6} q^2 \langle r^2_{\nu e} \rangle \right) \bar{u}(k') \gamma^\mu u(k) \\
- \frac{4\pi Z e \mu_{\nu e} \bar{u}(k') \sigma^{\mu\nu} q_{\nu} u(k) \right] j^{(N)}_\mu, \tag{12}
\]

where \( C_V = [Z(1 - 4 \sin^2 \theta_W) - N]/2 \), \( j^{(N)}_\mu \) are the neutrino millicharge and charge radius [13]. For neutrinos with energies of a few MeV the maximum momentum transfer squared \( |q^2|_{\text{max}} = 4E^2_\nu \) is still small compared to \( 1/R^2 \), where \( R \), the nucleus radius, is of the order of \( 10^{-2} - 10^{-1} \) MeV\(^{-1} \).

Therefore, the nuclear elastic form factor \( F(q^2) \) can be set equal to one. Using (12), we obtain the differential cross section in the nuclear-recoil energy transfer \( T_N \) as a sum of two components. The first component conserves the neutrino helicity and can be presented in the form

\[
\left( \frac{d\sigma_{\nu e}}{dT_N} \right)_{\text{SM}}^Q = \eta^2 \left( \frac{d\sigma_{\nu e}}{dT_N} \right)_{\text{SM}}^\text{ext}, \quad \eta = 1 - \sqrt{2\pi eZ} \left[ \frac{e_{\nu e}}{M_N T_N} + \frac{e}{3} \langle r^2_{\nu e} \rangle \right], \tag{13}
\]

where \( M_N \) is the nuclear mass, and

\[
\left( \frac{d\sigma_{\nu e}}{dT_N} \right)_{\text{SM}} = \frac{G^2_F}{\pi} M_N C_V^2 \left( 1 - \frac{T_N}{T^\text{max}_N} \right) \tag{14}
\]

is the SM cross section due to weak interaction [45], with

\[
T^\text{max}_N = \frac{2E^2_\nu}{2E_\nu + M_N}.
\]

The second, helicity-flipping component is due to the magnetic moment only and is given by [28]

\[
\left( \frac{d\sigma_{\nu e}}{dT_N} \right)_{\text{mag}} = 4\pi \alpha \mu_{\nu e}^2 \frac{Z^2}{T_N} \left( 1 - \frac{T_N}{E_\nu} + \frac{T^2_N}{4E^2_\nu} \right). \tag{15}
\]

Formulas (13) and (15) describe a deviation from the well-known SM value (14) due to neutrino electromagnetic interactions. Two important features should be noted. First, the contributions from the neutrino millicharge and charge radius interfere with that from the weak interaction, while the neutrino magnetic moment contributes separately. Second, the roles of the neutrino millicharge and magnetic moment grow with lowering the energy transfer \( T_N \), in particular, when \( T_N \to 0 \) the \( e_{\nu e} \) contribution behaves as \( \propto 1/T^2_N \) and the \( \mu_{\nu e} \) contribution as \( \propto 1/T_N \).

It can be noted that the characteristic energy scale where \( (d\sigma_{\nu e}/dT_N)_{\text{mag}} \) exceeds \( (d\sigma_{\nu e}/dT_N)_{\text{SM}} \),

\[
T_N \lesssim \frac{\pi^2 \alpha^2}{G^2_F M_N m^2_e} \left( \frac{Z}{C_V} \right)^2 \left( \frac{\mu_{\nu e}}{\mu_B} \right)^2,
\]

appears to be by orders of magnitude smaller when compared to that for the elastic neutrino-electron scattering (9).
Acknowledgments
We are thankful to Nicolao Fornengo and Carlo Giunti for the kind invitation to participate in the 14th International Conference on Topics in Astroparticle and Underground Physics. This work was supported by RFBR grant nos. 14-22-03043 ofi, 15-52-53112 GFEN_a, and 16-02-01023 A. One of the authors (K.A.K.) also acknowledges support from the RFBR under grant no. 14-01-00420 A.

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