Research Article

The Stability Analysis and Transmission Dynamics of the SIR Model with Nonlinear Recovery and Incidence Rates

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In the present paper, the SIR model with nonlinear recovery and Monod type equation as incidence rates is proposed and analyzed. The expression for basic reproduction number is obtained which plays a main role in the stability of disease-free and endemic equilibria. The nonstandard finite difference (NSFD) scheme is constructed for the model and the denominator function is chosen such that the suggested scheme ensures solutions boundedness. It is shown that the NSFD scheme does not depend on the step size and gives better results in all respects. To prove the local stability of disease-free equilibrium point, the Jacobian method is used; however, Schur–Cohn conditions are applied to discuss the local stability of the endemic equilibrium point for the discrete NSFD scheme. The Enatsu criterion and Lyapunov function are employed to prove the global stability of disease-free and endemic equilibria. Numerical simulations are also presented to discuss the advantages of NSFD scheme as well as to strengthen the theoretical results. Numerical simulations specify that the NSFD scheme preserves the important properties of the continuous model. Consequently, they can produce estimates which are entirely according to the solutions of the model.

1. Introduction

Infectious diseases are key troubles for human beings [1, 2]. They can cause disability, mortality as well as produce economic and social problems for society. Infectious diseases are disorders which are happen by organisms such as fungi, parasites, viruses, or bacteria. Several organisms exist inside and on our bodies. Some infectious diseases can be transmitted from person to person, and some are passed by insects or other animals. Symptoms and signs differ depending on the organism causing the infection. Mild infections may respond to rest and home medication, while some dangerous diseases might require hospitalization. Malaria, measles, pneumonia, diarrheal diseases (cholera), HIV, tuberculosis (TB), and more recently COVID-19 are the major deadly infectious diseases [3, 4].

Infectious diseases create a constant threat to the health of individual and public. TB is one of the dangerous infectious diseases that cause many deaths globally. In underdeveloped countries, conventional methods are utilized in diagnosing TB [5–7]. COVID-19 is an infection that can cause lung problems such as pneumonia and, in the most rigorous cases, acute respiratory syndrome. The COVID-19 is an infectious sickness that can be spread through aerosols of droplets and by direct contact with the people [8–14]. This disease has extended rapidly over the world after it was displayed to have a more significant level of irresistible and
pandemic threat than SARS. Because of the sharp increment level of spread, the WHO articulated the COVID-19 outbreak as a pandemic on 11 March 2019. The indications of COVID-19 are cough, fever, exhaustion, sputum, cerebral pain, lymphopenia, and run dyspnoea [15]. In serious cases, COVID-19 can cause pneumonia and even death [16]. To stop the spreading of COVID-19, different governments estimated different preventive measures like wearing a mask, remaining six feet social distance, washing hands regularly, and staying away from sick individuals. Vaccines also serve a serious role to prevent people from infectious diseases and contribute toward controlling the spread of the disease. The vaccinated people also need to be attentive of the additional defensive behaviors required to manage the disease.

The mathematical modeling is a helpful device to concentrate on the procedure that how an irresistible infection can reach out into a population. The researchers are using fractional order [17, 18] as well as integer order [19–24] mathematical models to discuss epidemic infectious diseases; however, mathematical models of integer order are among the most studied problems in the world. The disparate kinds of recovery and incidence rates play a significant role to examine the dynamical behavior of epidemic disease models. Many researchers have used these saturated rates in their works [19–30]. For instance, the standard bilinear incidence rates $\beta SI$ have been regularly utilized in epidemiological models [19–23]. In Ref. [24], the SEIR epidemic model with nonlinear incidence rate, vaccination, and quarantine strategies is provided. Yusuf and Benyah [25] have presented and broke down a discrete SIR epidemic model with nonlinear recovery rates. Anderson and May [26] offered the nonlinear incidence rate $aSI/1 + aI$ which got drenched due to a cluster of infective persons at a high level of disease. One more nonlinear incidence rate $aSI/1 + aS + a_2I$ is extensively presented in Refs. [27–29]. In Ref. [30], the author modified the above incidence rate to nonlinear recovery rate $aSI/1 + aS + a_2I^2$. The nonlinear recovery rate can display rich dynamics such as saddle-node, backward, Hopf, and Bogdanov–Takens bifurcations.

The bilinear incidence rates do not take into account the impact of the preventive measures such as mask-wearing, quarantine, and isolation, which play an important role to manage the spread of an infectious disease. Continuous compression and impact techniques also make the feedback of the incidence rate more leisurely compared to the standard bilinear structure $aSI$. Therefore, it is crucial to take into account the influence of preventive measures as well as preventive steady decline on the transmission of infectious diseases. In order to address the above problems, the author [31] plan to alter the SIR model by taking the nonlinear Monod type equation as incidence rate to study the effect of preventive reduction on the transmission of infectious diseases. The reason behind considering Monod type equation as incidence rate was to explore the effect of intervention decrease on the spread of infectious disease. The Monod equation as incidence suggests that the incidence rate is low for little quantities of infected individuals due to rigorous intervention; however, it increases as the number of infected individuals increases until it becomes autonomous of the diseased subpopulation. The author explored the essential analytical results, containing the stability of disease–free and disease endemic equilibria for the continuous model. The aim of the present paper is to use a more advanced NSFD scheme to verify different characteristic of the model to display its sustainability and biological vitality. The purpose is to develop policies for preventing or regulating disease transmission among individuals and to better understand disease dynamics. The NSFD scheme constructed for the model is dynamically reliable with the original system for any step size. Our theoretical and numerical outcomes show that the NSFD scheme conserves the necessary qualitative properties of the continuous model. Therefore, this scheme is not only reasonable for the model but also the outcomes acquired through this scheme are extremely proficient and precise.

The rest of paper is arranged as follows: In Section 2, the SIR system is presented and the associated parameters are explained. In the same section, the expression for reproduction number, and disease-free and disease endemic equilibria are obtained for the system. The NSFD scheme is established for the system in Section 3. The local asymptotic stability (LAS) of disease–free and disease endemic equilibria for the discrete model obtained by the NSFD scheme are proved by the Jacobian method and Schur–Cohn conditions; however, the Enatsu criterion and Lyapunov function are employed to discuss the global asymptotic stability (GAS). Some important conclusions are given in the final section.

2. Mathematical Model and Equilibria

2.1. System of Differential Equations. The dynamical system [31] with nonlinear recovery and Monod type equation as incidence rates including three differential equations is given as follows. The total population $N(t)$ is distributed into three classes: susceptible $S(t)$, infected $I(t)$, and recovered $R(t)$ where $N(t) = S(t) + I(t) + R(t)$. The detailed description of the model can be seen in Figure 1.

\[
\frac{dS}{dt} = C - \frac{aIS}{u + I} - \tau S, \\
\frac{dI}{dt} = \frac{aIS}{u + I} - \left(\gamma_0 + (\gamma_1 - \gamma_0) \frac{\theta}{\theta + 1}\right)I - (\delta + \tau)I,\\
\frac{dR}{dt} = \left(\gamma_0 + (\gamma_1 - \gamma_0) \frac{\theta}{\theta + 1}\right)I - \tau R.
\]

The information about the variables and parameters used in model (1) is given in the following table (see Table 1).

\[
\frac{dS}{dt} = C - \frac{aIS}{u + I} - \tau S, \\
\frac{dI}{dt} = \frac{aIS}{u + I} - \left(\gamma_0 + (\gamma_1 - \gamma_0) \frac{\theta}{\theta + 1}\right)I - (\delta + \tau)I.
\]
The first two equations of the given model
\[
\frac{dS}{dt} = -\frac{aIS}{u + I} S
\]
\[
\frac{dI}{dt} = \frac{aIS}{u + I} - (\gamma_0 + (y_1 - y_0)\frac{\theta}{\theta + 1}) I - (\delta + \tau) I
\]
\[
\frac{dR}{dt} = \tau \delta S
\]

The disease-free and endemic equilibria can be found by solving the following equations:
\[
C - \frac{aIS}{u + I} - \tau S = 0,
\]
\[
\frac{aIS}{u + I} - \left(\gamma_0 + (y_1 - y_0)\frac{\theta}{\theta + 1}\right) I - (\delta + \tau) I = 0.
\]

To find the disease-free equilibrium (DFE) point, we take all other classes equal to zero except the susceptible class. Then, the DFE point \( E_0 \) becomes \( E_0 = (C/\tau, 0, 0) \). On the other hand, to find the disease endemic equilibrium (DEE) point, we simultaneously solve equation (3) for \( S \) and \( I \). If we denote DEE point by \( E^* = (S^*, I^*) \), then from equation (3) we get \( S^* (t) = [I^* \gamma_0 + \theta y_1 + \theta \delta + \delta I^* + \tau \theta + \tau I^*] u + I^* / (a(\theta + I^*)) \) and \( I^* (t) = C u - \tau u S^*/aS^* + \tau S^* - C \).

### 3. The NSFD Scheme for the Modified SIR Model

In 1994, the concept of NSFD was given by Mickens [33]. The NSFD schemes [34–38] is used to construct general method to find the numerical solution of ordinary and partial differential equations by generating discrete models. According to Shokri et al. [37], the exploration of NSFD schemes depend on two factors. First, how to estimate nonlinear terms in the most suitable way, and second is how to the discretized the derivative. One of the general methods for discretization is the forward finite difference approximation for derivative of the first order. In the standard form, the first order derivative \( dy/dx \) is represented as \( y(x + h) - y(x)/h \), where \( h \) represents step size. According to Mickens, this term can be expressed as \( y(x + h) - y(x)/\phi(h) \), where \( \phi(h) \) is an increasing continuous function known as denominator function. Different expressions for \( \phi(h) \) can be seen in Refs [37–39]; however, we will consider \( \phi(h) = \exp(\tau h) - 1/\tau \) in upcoming calculations. The quantity \( \phi(h) \) represents the time step size and should be nonnegative. We will show that NSFD is not only positive forever but converge quickly to DFE point for any step size. To discuss all the above properties, we first develop the NSFD scheme for system (1) in the following subsection.

#### 3.1. Construction of the Discrete NSFD Scheme

For model (1), we indicate \( S_{n+1}, I_{n+1}, \) and \( R_{n+1} \) as the numerical approximations of \( S(t), I(t), \) and \( R(t) \) at \( t = nh \), where \( n = 0, 1, 2, \ldots, \) and \( h \) denotes the time step size which should be nonnegative. Then, based on the above all information, we can construct the NSFD scheme for model (1) as follows:
\[
\frac{S_{n+1} - S_n}{\phi} = C - \frac{\alpha I_n S_{n+1}}{u + I_n} - \tau S_{n+1},
\]
\[
\frac{I_{n+1} - I_n}{\phi} = \frac{\alpha I_n S_{n+1}}{u + I_n} - \left(\gamma_0 + \left(\gamma_1 - \gamma_0\right) \frac{\theta}{\theta + I_n}\right) I_{n+1} - \left(\delta + \tau\right) I_{n+1},
\]
\[
\frac{R_{n+1} - R_n}{\phi} = \left(\gamma_0 + \left(\gamma_1 - \gamma_0\right) \frac{\theta}{\theta + I_n}\right) I_{n+1} - \tau R_{n+1}.
\]  

(7)

We assume that the initial values \(S_0, I_0,\) and \(R_0\) of the discrete NSF scheme (7) are nonnegative. The total population \(P_n\) from the discrete NSF scheme (8) satisfies
\[
P_n = S_n + I_n + R_n\] such that
\[
P_{n+1} - P_n \leq C - \tau P_{n+1} - \delta I_{n+1} \leq C - \tau P_{n+1}.
\]  

(8)

If we take \(\phi = \exp(\tau h) - 1/\tau\), then the solution of (8) satisfies
\[
P_n \leq \frac{C}{\tau} + \left(P_0 - \frac{C}{\tau}\right) \exp(-\tau h),
\]  

(9)

where \(P_0 = S_0 + I_0 + R_0\) for any \(h > 0\). When \(\delta = 0\), it can be expressed that the total population of the discrete NSF scheme (7) is precisely the same as that of model (1). The scheme (7) is implicit; however, it can be expressed explicitly as follows:
\[
S_{n+1} = \frac{S_n + \phi C}{1 + \phi \Psi_n},
\]
\[
I_{n+1} = \frac{I_n + \phi \Psi_n S_{n+1}}{1 + \phi \left(\gamma_0 + \left(\gamma_1 - \gamma_0\right) \frac{\theta}{\theta + I_n}\right) + (\delta + \tau)},
\]
\[
R_{n+1} = \frac{R_n + \phi \left(\gamma_0 + \left(\gamma_1 - \gamma_0\right) \frac{\theta}{\theta + I_n}\right) I_{n+1}}{1 + \phi},
\]  

(10)

where \(\Psi(z) = az/u + z\) and \(\Psi_n = \Psi(I_n)\). As all parameters in (10) are positive, therefore \(S_n \geq 0, I_n \geq 0, R_n \geq 0\) for all \(n > 0\) and for any \(h > 0\). These findings confirm that the discrete NSF scheme (10) maintains the nonnegative solutions for any step size \(h\).

It is clear that \(R_n\) is not included in first two equations of system (10). Therefore, for further investigation, we only take the following reduced model:
\[
S_{n+1} = \frac{S_n + \phi C}{1 + \phi \Psi_n},
\]
\[
I_{n+1} = \frac{I_n + \phi \Psi_n S_{n+1}}{1 + \phi \left(\gamma_0 + \left(\gamma_1 - \gamma_0\right) \frac{\theta}{\theta + I_n}\right) + (\delta + \tau)},
\]  

(11)

After simple computation, we can prove that the discrete NSF model (11) always has a unique DFE point \(E_0 = (C/\tau, 0)\) and DEE point \(E^* = (S^*, I^*)\). These DFE and DEE points, and their continuation situation is totally the same as the continuous model (1) no matter what is \(h\). In the upcoming subsection, we first examine the LAS of the above discussed equilibrium points.

3.2. Local Stability of the Discrete NSF Scheme. To investigate the LAS of DFE and DEE points, we consider
\[
S_{n+1} = \frac{S_n + \phi C}{1 + \phi \Psi_n + \tau} = F(S,I),
\]
\[
I_{n+1} = \frac{I_n + \phi \Psi_n S_{n+1}}{1 + \phi \left(\gamma_0 + \left(\gamma_1 - \gamma_0\right) \frac{\theta}{\theta + I_n}\right) + (\delta + \tau)} = G(S,I).
\]  

(12)

Theorem 1. If \(R_0 < 1\), then DFE point \(E_0\) of the NSF Scheme (12) is LAS for any step size \(h\).

Proof. We take the Jacobean matrix as
\[
J(S,I) = \begin{bmatrix}
\frac{\partial F}{\partial S} & \frac{\partial F}{\partial I} \\
\frac{\partial G}{\partial S} & \frac{\partial G}{\partial I}
\end{bmatrix}.
\]

(13)

After easy calculation and the putting \(E_0\), we get
\[
\frac{\partial F}{\partial S} = \frac{1}{1 + \phi \tau},
\]
\[
\frac{\partial F}{\partial I} = \frac{C \phi}{\tau(1 + \phi \tau)},
\]
\[
\frac{\partial G}{\partial S} = 0,
\]
\[
\frac{\partial G}{\partial I} = \frac{\tau + \phi C}{1 + \phi \left(\gamma_1 + (\delta + \tau)\right)}.
\]  

(14)

Therefore, the Jacobean matrix (13) becomes
\[
J(E_0) = \begin{bmatrix}
\frac{1}{1 + \phi \tau} & \frac{C \phi}{\tau(1 + \phi \tau)} \\
0 & \frac{1 + \phi C}{1 + \phi \left(\gamma_1 + (\delta + \tau)\right)}
\end{bmatrix}.
\]

(15)

The above matrix clearly gives the following eigenvalues:
\[
\lambda_1 = 1/1 + \phi \tau < 1 \quad \text{and} \quad \lambda_2 = 1 + \phi C/1 + \phi \gamma_1 + (\delta + \tau)\tau.
\]

The eigenvalue \(\lambda_1\) can also be expressed as \(\lambda_2 = \tau u + \phi a C/\alpha A (\phi^2 (\gamma_1 + (\delta + \tau) + \phi) R_0).\) Thus, it is clear that if \(R_0 < 1\) then \(\lambda_1 < 1\), while if \(R_0 > 1\), then \(\lambda_2 > 1\). This verifies that if \(R_0 < 1\), then DFE point \(E_0\) is LAS, and it is unstable if \(R_0 > 1\).

In the following, we give the statement of Schur–Cohn criterion [20, 40] which plays an important role in the investigation of LAS of DEE point \(E^*\).

Lemma 1. The quadratic equation \(\lambda^2 - T\lambda + D = 0\) roots satisfy \(|\lambda_i| < 1, i = 1, 2\) if and only if the below three conditions are fulfilled:
\[
1 \quad D < 1
\]
\[
2 \quad 1 + T + D > 0
\]
\[
3 \quad 1 - T + D > 0.
\]
Theorem 2. If $R_0 > 1$, then the DEE point $E^*$ of the NSFD scheme (12) is LAS for any step size $h$.

Proof. The Jacobian matrix of the discrete NSFD scheme (12) is

$$J(S, I) = \begin{bmatrix}
\frac{\partial F}{\partial S}(S, I) & \frac{\partial F}{\partial I}(S, I) \\
\frac{\partial G}{\partial S}(S, I) & \frac{\partial G}{\partial I}(S, I)
\end{bmatrix}.$$

(16)

By replacing $F$ and $G$, and then putting DEE point $E^*$ in equation (16), we get

$$J(E^*) = \begin{bmatrix}
\frac{1}{p} & \frac{s}{p^2} \\
r & 1 + s \\
\frac{r}{q} & \frac{1}{q}
\end{bmatrix}.$$

(17)

By the characteristic equation of $J(E^*)$ is

$$\lambda^2 - T\lambda + D = 0,$$

where $T = \text{Trace } (J(E^*)) = 1/p + 1 + s/q > 0$ and $D = p(1 + s) + rs/p^2q > 0$. Then, we get the following results:

1. When $p > 1$, we have

$$D = \frac{p(1 + s) + rs}{p^2q} < \frac{1 + s}{p} + \frac{rs}{p^2q} < 0.$$

(19)

2. It is clear that $1 + T + D > 0$.

3. Direct calculation presents that if $R_0 > 1$, then

$$1 - T + D = \left(1 + \phi\left(\Psi^* + \tau\right)\right) \cdot \left[1 - \frac{\phi S^\alpha + \phi^2 C\alpha}{u + I^*} + \phi\left(\Psi^* + \tau\right)\right] > 0.$$

(20)

Based on the Schur–Cohn criterion discussed in Lemma 1, we conclude that DEE point $E^*$ is LAS if $R_0 > 1$ for any step size $h$.

3.3. Global Stability of the Discrete NSFD Model. In the following, we now explore the GAS of DFE and DEE points by applying an appropriate Lyapunov function.

Theorem 3. If $R_0 \leq 1$, then DFE point $E_0$ of the NSFD scheme (12) is GAS, as shown in Figure 2(a)–2(d).

Proof. We can define the discrete Lyapunov function as follows:

$$U_n = \frac{1}{\phi} \left[ S_0 \theta \left( \frac{S_n}{S_0} \right) + I_n \right] + S_0 \Psi_n.$$

(21)
Figure 2: Numerical solutions of the SIR model (1) attained from NSFD scheme with (a) $h = 0.5$, (b) $h = 2$, (c) $h = 50$, (d) $h = 500$. Other parameters are set as $C = 2.5, \alpha = 0.01, \tau = 0.3, \delta = 0.00197, \theta = 0.00768, \gamma_0 = 0.00989, \gamma_1 = 0.000007, \mu = 0.7$.

Figure 3: Continued.
where \( g(y) = y - 1 - \ln y \geq g(1) = 0 \). Depending on the first and second equations of system (7), from (21), we attain

\[
\Delta U_n = U_{n+1} - U_n = \frac{1}{\phi} \left[ S_0 g \left( \frac{S_{n+1}}{S_n} - \frac{S_{n}}{S_{n+1}} \right) - S_0 g \left( \frac{S_{n}}{S_0} \right) + (I_{n+1} - I_n) \right] + S_0 (\Psi_{n+1} - \Psi_n)
\]

\[
= \frac{1}{\phi} \left[ S_0 \left( \frac{S_{n+1} - S_n}{S_0} - S_0 \ln \frac{S_{n+1}}{S_n} \right) + (I_{n+1} - I_n) \right] + S_0 (\Psi_{n+1} - \Psi_n)
\]

(22)

By using Entasu et al.’s [41] criterion

\[
\ln S_2/S_1 \geq S_2 - S_1/S_2, \text{ we get}
\]

\[
\leq \frac{1}{\phi} \left[ (S_{n+1} - S_n) - S_0 \left( \frac{S_{n+1} - S_n}{S_{n+1}} \right) + (I_{n+1} - I_n) \right] + S_0 (\Psi_{n+1} - \Psi_n)
\]

\[
= \frac{1}{S_{n+1}} \left[ (S_{n+1} - S_n) \left( \frac{S_{n+1} - S_n}{\phi} \right) + (I_{n+1} - I_n) \right] + S_0 (\Psi_{n+1} - \Psi_n)
\]

\[
= \frac{1}{S_{n+1}} (S_{n+1} - S_n) (C - \Psi_n S_{n+1} - r S_{n+1}) + \Psi_n S_{n+1} - \left( y_0 + (y_1 - y_0) \frac{\theta}{\theta + 1} \right) I_{n+1} - (\delta + \tau) I_{n+1} + S_0 (\Psi_{n+1} - \Psi_n)
\]

(23)

\[
= -\frac{\tau}{S_{n+1}} (S_{n+1} - S_0)^2 - (y_1 + (\delta + \tau) I_{n+1} + \frac{Ca I_{n+1}}{\tau (u + I_n)}
\]

\[
= -\frac{\tau}{S_{n+1}} (S_{n+1} - S_0)^2 - (y_1 + (\delta + \tau) \left( 1 - \frac{Ca}{\tau (y_1 + \delta + \tau) \right)} I_{n+1}
\]

\[
= -\frac{\tau}{S_{n+1}} (S_{n+1} - S_0)^2 - (y_1 + (\delta + \tau) (1 - R_0) I_{n+1}
\]

Definitely, if \( R_0 \leq 1 \) then \( U_{n+1} - U_n \leq 0 \) for all \( n \geq 0 \). This presents that \( U_n \) is monotonic decreasing sequences. As \( U_n \geq 0 \), then \( \lim_{n \to \infty} U_n \geq 0 \) and \( \lim_{n \to \infty} U_{n+1} - U_n = 0 \)

Accordingly, we get \( \lim_{n \to \infty} S_{n+1} = S_0 \) and \( \lim_{n \to \infty} (\delta + \tau) I_{n+1} = 0 \). It is clear that if \( R_0 \leq 1 \), then \( \lim_{n \to \infty} I_{n+1} = 0 \). Hence, we conclude that \( E_0 \) is GAS.
Theorem 4. If $R_0 > 1$, then DEE point $E^*$ of the NSFD model (8) is GAS for any step size $h$, as shown in Figure 3(a)–3(d).

Proof. To present sufficient conditions for the global stability of $E^*$, we use Enatsu et al.’s [41] criterion. For this, we define the discrete Lyapunov function as follows:

$$W_n = S^* g(S_n / S^*) + I^* g(I_n / I^*) + \phi \Psi^* S^* g(Y_n / \Psi^*),$$

where $\Psi^* = \Psi(Y^*)$ and $g(y) = y - 1 - \ln(y) \geq g(1) = 0$.

By replacing the first and second equations of (7) into (24), we acquire

$$\Delta W_n = \frac{\tau \phi S^*}{\chi_{n+1}} (Y_{n+1} - 1)^2 + h \Psi^* S^* \left(1 - \frac{1}{Y_{n+1}}\right) (1 - \xi_n Y_{n+1})$$

$$+ \phi \Psi^* S^* \left(1 - \frac{1}{Y_{n+1}}\right) (1 - \xi_n Y_{n+1} - \eta_{n+1}) + \phi \Psi^* S^* \left(1 - \frac{1}{Y_{n+1}}\right) \left(\frac{Y_{n+1} \xi_n}{\eta_{n+1}} - 1\right)$$

$$+ \phi \Psi^* S^* \left(1 - \frac{1}{Y_{n+1}}\right) \left(\frac{Y_{n+1} \xi_n}{\eta_{n+1}} - 1\right).$$

Let us denote $Y_n = S_n / S^*$, $\eta_n = I_n / I^*$, and $\xi_n = \Psi(Y_n \Psi^*)$, then (25) can be expressed as

$$\Delta W_n \leq \frac{-\tau \phi S^*}{\chi_{n+1}} (Y_{n+1} - 1)^2 + h \Psi^* S^* \left(1 - \frac{1}{Y_{n+1}}\right) (1 - \xi_n Y_{n+1})$$

$$+ \phi \Psi^* S^* \left(1 - \frac{1}{Y_{n+1}}\right) (1 - \xi_n Y_{n+1} - \eta_{n+1}) + \phi \Psi^* S^* \left(1 - \frac{1}{Y_{n+1}}\right) \left(\frac{Y_{n+1} \xi_n}{\eta_{n+1}} - 1\right)$$

Using the definition of $g(y)$, we get

$$g(\xi_{n+1}) - g(\eta_{n+1}) = \frac{\Psi_{n+1}}{\Psi^*} \left(\frac{I_{n+1}}{I^*} \frac{\Psi_{n+1}^*}{\Psi_{n+1}}\right) \leq \frac{\Psi_{n+1}}{\Psi^*} \left(\frac{I_{n+1}}{I^*} \frac{\Psi_{n+1}^*}{\Psi_{n+1}} - 1\right)$$

$$= -\alpha \left(I_{n+1} - I^*\right)^2 \frac{I^*}{I^* (1 + \alpha I^*)} (1 + \alpha I_{n+1}) \leq 0.$$
Hence, $\Delta W_n$ is a monotonic decreasing sequence. By applying the same techniques as those in Theorem 3, we can show that $\lim_{n \to \infty} (W_{n+1} - W_n) = 0$. Therefore, we get $\lim_{n \to \infty} S_{n+1} = S^*$ and $\lim_{n \to \infty} I_{n+1} = I^*$ for all $h$. Hence, the proof is completed.

4. Conclusions

In the present work, we have discussed the SIR epidemic model with nonlinear recovery and Monod type equation as incidence rates. We calculated the basic reproduction which plays an essential role in the investigation of local and global stability of DFE and DEE points. The NSFD scheme is constructed for the model which is not only unconditionally convergent but also gives more accurate results which are mathematically and biologically reasonable. By using different criteria and conditions, the LAS and GAS of both DFE and DEE points are proved for the NSFD scheme. It is confirmed that for all time step sizes, the discrete NSFD scheme is vigorously reliable with the related continuous model. The numerical simulations are important to study the complex dynamical behavior the nonlinear models [34–43]. Therefore, numerical simulations have also been presented to verify the sustainability of the theoretical results. The advantages of NSFD scheme are provided, which explains that the outcomes of NSFD scheme are qualitatively precise and efficient. In the same manner, the NSFD can be constructed and analyzed for other generalized epidemic models.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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