WHAT THE $K_{e4}$ DECAY TELLS US ON A VALUE OF
$\sigma$-PARTICLE MASS AND ON A NATURE OF THE SPIN-1
MESONS
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Abstract

The data on the form factors of the $K_{e4}$ decay permit to fix a
value of the non-specified in the theory parameter entering in the
lagrangian of the system of $0^+$ and $0^-$ mesons. Then, for a mass of
the lightest $\sigma$-meson we find: $m_\sigma=663$ MeV. As for a nature of the
spin-1 mesons, also contributing into $K_{e4}$ form factors, the data do
not allow to interpret them as the gauge bosons of the chiral theory.

1 Introduction

In present paper we argue in favour of the chiral theory incorporating not
only the pseudoscalar mesons, but also the scalar ones. A reason is that
appearing as the intermediate particles in the pions interaction, the last
allow to understand and explain the dependence of the processes with pions
of energy. Besides, the low-energy scalar resonances exist and their specific
properties get an explanation in such a theory.

The data on the form factors of the $K_{e4}$ decay obtained in [1], where
400,000 events of the decay $K^+ \to \pi^+ \pi^- e^+ \nu$ had been observed, will give us
a possibility to explain the role of the scalar mesons in this decay and, in
addition, to get an answer on a nature of the spin-1 mesons, also contributing
into $K_{e4}$ decay.

In QCD, the spinless flavoured objects are $\bar{q}_R t^a q_l$, and their Hermitian
conjugate. These objects incorporate the parts with the opposite parity. To
reproduce this property in the lagrangian of the real particles, it must be
written in terms of the matrix

$$ U = (\sigma_a + i \pi_a) t_a $$

where $t_0 = \sqrt{\frac{3}{2}} T_1, t_{1,...,8} = \sqrt{\frac{1}{2}} \lambda_{1,...,8}$, and $\sigma_a, \pi_a$ are the nonets of the scalar
and the pseudoscalar mesons. This idea is not original and there had been a

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number of interesting models [2], [3], but we pass right away to the final form of the lagrangian for the spinless fields containing all forms of breakdown of chiral symmetry [4], [5]:

\[
L_{\text{meson}} = \frac{1}{2} Tr \left( \partial_\mu U \partial_\mu U^+ \right) - c \text{Tr} \left( UU^+ - A^2 \eta_0 \right)^2 - c \xi \left( \text{Tr} \left( UU^+ - A^2 \eta_0 \right) \right)^2 + \frac{F_\pi}{\sqrt{2}} \text{Tr} \left\{ M \left( U + U^+ \right) \right\} + \frac{\Delta m^2_p F^2}{12} \cdot \left\{ \frac{1}{2} \text{Tr} \left( \ln \frac{U}{U^+} \right) \right\}^2.
\] (2)

Here the parameter \( A \) characterises a strength of spontaneous breakdown of the symmetry between the scalars and pseudoscalars, the matrix \( M \) that in the limit \( m_u = m_d \) has the form

\[
M = \sqrt{\frac{1}{3} (2m_K^2 + m_\pi^2)} t_0 - 2 \sqrt{\frac{2}{3} (m_K^2 - m_\pi^2)} t_8 \] (3)

produces the hard breaking \( SU(3) \) symmetry and the last term and the term proportional to the parameter \( \xi \) lower the \( U(3)_L \otimes U(3)_R \) symmetry down to \( SU(3)_L \otimes SU(3)_R \) symmetry due to mixing of \( \pi_0 \) and \( \sigma_0 \) state with the glueball states \( \left( \alpha_\pi C^a_{\mu\nu} \tilde{C}^a_{\mu\nu} \right) \) and \( \left( \alpha_\pi C^a_{\mu\nu} C^a_{\mu\nu} \right) \) respectively. The values of the constants \( \Delta m_P \) and \( \xi \) are not specified in the theory; they can be determined by fitting the theoretical predictions to the experimental data. As for the masses of the scalar mesons, one finds from (2):

\[
m_{\sigma_\pi}^2 - \mu^2 = (m_K^2 - \mu^2)[(R - 1)(2R - 1)]^{-1} \equiv \Lambda^2, \] (4)

\[
m_{\sigma_K}^2 - \mu^2 = \Lambda^2 (2R - 1)R \] (5)

where \( \mu^2 = m_\pi^2 \) and

\[
R = \frac{F_K}{F_\pi} \] (6)

The parameter \( R \) can be determined from \( K, \pi \to \mu\nu \) decays, or from the relation (4) where the isotriplet scalar meson \( \sigma_\pi \) is identified with the resonance \( a_0(980) \). In the last case we find for the mean value of \( R \):

\[
R = 1.176. \] (7)

This value is a little smaller (by 1 percent) than the one following from the \( K, \pi \to \mu\nu \) decays. But we shall use just the magnitude (7) for the self-consistency of the further results.

For the isosinglet scalar mesons, the mass formulae are more complicated than those in (4), (5), since \( \sigma_0 \) is mixed with the scalar gluonium as well as
with $\sigma_8$ owing to breakdown of $SU(3)$ symmetry. As a result, the physical isosinglet scalar mesons are

$$\sigma' = \sigma_0 \cos \theta_S + \sigma_8 \sin \theta_S$$
$$\sigma_\eta = -\sigma_0 \sin \theta_S + \sigma_8 \cos \theta_S.$$  \tag{8}

For these states we obtain:

$$m_{\sigma'}^2 - \mu^2 = \Lambda^2 \{1 + 2R(R-1)(\cos \theta_S - \sqrt{2} \sin \theta_S)^2 + \frac{1}{3} \xi[(2R+1)\cos \theta_S - 2\sqrt{2}(R-1)\sin \theta_S]^2\},$$
$$m_\eta^2 - \mu^2 = \Lambda^2 \{1 + 2R(R-1)(\sin \theta_S + \sqrt{2} \cos \theta_S)^2 + \frac{1}{3} \xi[(2R+1)\sin \theta_S + 2\sqrt{2}(R-1)\cos \theta_S]^2\}.$$  \tag{9}

The coupling constants of $\sigma'_\eta$ and $\sigma_\eta$ to the pseudoscalar mesons also depend on the parameter $\xi$. The expressions for them are adduced in Appendix. The mixing angle $\theta_S$ is connected with the parameters $R$ and $\xi$ by the relation

$$\theta_S = \frac{1}{2} \arctan \left\{ \frac{1 + \xi(2R+1)(3R)^{-1}}{2\sqrt{2} - \xi(2R+1)^2[6R(R-1)]^{-1}[1 - 8(R-1)^2(2R+1)^{-2}]} \right\}$$  \tag{10}

As it follows from (9) and Appendix, at $\xi = 0$ only the one $\sigma'_\eta$ interacts with two pions and its mass coincides with a mass of the $\sigma_\pi$ meson that is identified with the resonance $a_0$ with the mass 980 MeV. But the numerous searches for $\sigma$ meson have shown that a mass of the lightest $\sigma$ meson has to be smaller than 700 MeV. \[6\]. In our theory the last condition can be fulfilled, if the parameter $\xi$ is negative, but $|\xi| \leq \frac{1}{3}$. This limitation is connected with a requirement of positiveness of energy in the theory. As the parameter $\xi$ enters into the amplitudes originated by isosinglet $\sigma$ mesons, to extract its magnitude from the experimental data, we need to pick out just this part from the data. In particular, to exclude from the data a contribution originated by the spin-1 mesons. However, a value of their contribution into considered here processes depends on their nature.

For a long time, the spin-1 mesons were interpreted as the gauge bosons of the local chiral symmetry \[7\]. But an availability of a mass, does not permit to be sure that the successful predictions being fair in such a theory in limit $m_{V,A} = 0$, remain to be fair at $m_{V,A} \neq 0$ \[8\]. This idea led to reconsideration of a nature of the spin-1 mesons \[8\]- \[12\].

In \[9\], it was shown that a treatment of the spin-1 mesons as the divergences of the corresponding antisymmetric tensor fields permits to consider
them, on a level with the spinless mesons, as the members of the \((3, \bar{3}) + (\bar{3}, 3)\) representation of the \(SU(3) \otimes SU(3)\) chiral group.

In [10], [11], it was stressed that "there is no proof for the existence of dynamical gauge bosons of local chiral symmetry in QCD" and "there is nothing special about vector and axial-vector mesons compared to scalar, pseudoscalar or any other meson resonances".

In the paper [12], the idea that spin-1 mesons are the gauge bosons was rejected proceeding from a principle of necessity of one-to-one coincidence of the general properties between the quark constructions in the quark-gluon space and the physical particles in real world. The quark combinations

\[
\bar{q}\gamma_\mu t_a q, \quad \bar{q}\gamma_\mu\gamma_5 t_a q
\]  

associated in the gauge theory with the spin-1 particles, do not satisfy the subsidiary condition \(\partial_\mu V_\mu(x) = 0\) and \(\partial_\mu A_\mu(x) = 0\) eliminating the scalar components in the real spin-1 objects. Interpretation of the spin-1 fields as the divergences of the antisymmetric tensor fields liberates one from a necessity to put on the subsidiary condition "by hands". However, the examples of the evident discrepancy between the values of the observed effects and their values calculated in the framework of gauge theory, were not presented in [8]-[12]. Such an example will be considered here.

2 The amplitude of the \(K^+ \rightarrow \pi^+\pi^-e^+\nu\) decay

The matrix element of this decay has the following form:

\[
M = (2\pi)^4 \delta^4(k - p - p' - p_e - p_\nu)(8E_K E_\pi E_{\pi'})^{-1/2} \times \\
(G_F V_{us}/\sqrt{2}m_K) \times \\
< \pi^+(p)\pi^-(p')|A_\mu^{(K)} + V_\mu^{(K)}|K^+(k) > \times \\
\bar{\nu}(p_\nu)\gamma_\mu(1 + \gamma_5)e(p_e). 
\]  

where \(m_K\) is a mass of \(K\) meson and

\[
< \pi^+(p)\pi^-(p')|A_\mu^{(K)}|K^+(k) > = f_1(p + p')_\mu + f_2(p - p')_\mu + f_3(k - p - p')_\mu, 
\]  

\[
< \pi^+(p)\pi^-(p')|V_\mu^{(K)}|K^+(k) > = \frac{if_1}{m_K^2}\epsilon_{\mu\rho\sigma\tau}k_\rho p_\sigma p'_\tau. 
\]
A probability of the $K_{e4}$ decay practically completely is determined by contribution of the terms proportional to $f_1$ and $f_2$ in (13). The part proportional to $f_3$ is very small because of proportionality to $m_e/m_K$. The part proportional to $f_4$ gives a contribution of order of 0.04%, and can be neglected in view of 2% uncertainty of the measured up to now probability itself. So that, we shall consider in the present paper the structure of $f_1$ and $f_2$ only.

The current algebra methods and PCAC allow to evaluate the magnitudes of $f_1$ and $f_2$ in the soft-pion limit, that is, at $(p + p')^2 = \mu^2$ [13]. More simply, their values can be found in a theory with the non-linear realization of chiral symmetry [14]. At $(p + p')^2 = \mu^2$

$$f_1 = f_2 = -m_K/(\sqrt{2}F_\pi), \quad F_\pi = 93 \text{ MeV} \quad (15)$$

At the constant $f_{1,2}$, a probability of the $K^+ \rightarrow \pi^+\pi^-e^+\nu$ decay is [15]

$$w^{th} = \frac{G_F^2 V_{us}^2 m_K^3}{2^{14}3\pi^5}(f_1^2 \cdot 3.923 \cdot 10^{-3} + f_2^2 \cdot 7.728 \cdot 10^{-4}). \quad (16)$$

At the magnitudes (15)

$$w^{th} = 1.357 \cdot 10^3 s^{-1} \quad (17)$$

This value is more than two times smaller of the experimental one:

$$w^{exp} = 3.302(1 \pm 0.022) \cdot 10^3 s^{-1} \quad (18)$$

It means that a dependence of $Q^2$, at least $f_1$, is significant. But the experimental data show that in the region $4\mu^2 \leq Q^2 \leq m_K^2$ the form factors change very slowly. Consequently, a growth of the form factors must occur in the non-physical region $\mu^2 \leq Q^2 \leq 4\mu^2$. A fast increase of $f_1(Q^2)$ is possible, if the pair $\pi\pi$ appears from decay of the intermediate $\sigma$ meson [5]. Then a presence of the structure $(m_\sigma^2 - Q^2)^{-1}$ in the expression of $f_1(Q^2)$ can ensure a desirable magnitude of $f_1(Q^2) = 4\mu^2$ at the corresponding value of $m_\sigma$. In the point $Q^2 = 4\mu^2$ a fast growth $f_1$ converts into very slow increase at $Q^2 > 4\mu^2$. The mechanisms originating this effect are the following. The first - an appearance of the imaginary part in the propagator of $\sigma$ meson [5]. The second - an appearance of the form factor in the vertex $\sigma\pi\pi$ decreasing its value with a growth of $Q^2$ [12]. And the last - a rescattering of the final pions [16]. The detailed study of these effects is supposed to present elsewhere. Such a study is particularly desirable in view of the observed in [1]
deviation of the form factor $f_1(Q^2)$ from the expected behavior. At present, a value of mass of the $\sigma$ meson, identified with the resonance $f_0(500)$, is very uncertain.\cite{17}. Our approach will permit to fix a value of $m_\sigma$ with a good accuracy.

3 A contribution of the intermediate scalar mesons into form factors $f_{1,2}$.

In our theory the axial currents are given by

$$A^i_\mu = d_{ijk} (\sigma_j \partial_\mu \pi_k - \pi_j \partial_\mu \sigma_k)$$  \hspace{1cm} (19)

where the $d_{ijk}$ coefficients are those defined by Gell-Mann and $d_{0jk} = \sqrt{2/3} \delta_{jk}$. The states $\sigma_0$ and $\sigma_8$ have the non-zero expectation values:

$$< \sigma_0 > = \frac{F_\pi}{\sqrt{6}} (2R + 1), \quad < \sigma_8 > = -\frac{2F_\pi}{\sqrt{3}} (R - 1).$$  \hspace{1cm} (20)

In the terms of the physical states (8), the axial strange current looks as the following:

$$A^{(k)}_\mu = \frac{i}{\sqrt{6}} \{ [2\sqrt{2} \cos \theta_S - \sin \theta_S] (\sigma_\eta K^-) - [2\sqrt{2} \sin \theta_S + \cos \theta_S] (\sigma_\eta K^-) \} \cdot (p_\sigma + p_K)_\mu - i\sigma K^0 \pi^- \cdot (p_\sigma K + p_\pi)_\mu.$$  \hspace{1cm} (21)

A contribution of $\sigma$ mesons into the amplitude (13) is represented by the matrix elements $< \pi^\pi|A^{(K)}_\mu|K^+ >$ and $< \pi^-|A^{(K)}_\mu|K_{0} >$. So that,

$$< \pi^+(p)\pi^-(p')|A^{(K)}_\mu|K^+(k) > = \frac{1}{\sqrt{6}} \left\{ \frac{2\sqrt{2} \cos \theta_S - \sin \theta_S}{m^2_{\sigma,\eta} - (p + p')^2} \cdot g_{\sigma_\eta \pi \pi} - \frac{2\sqrt{2} \sin \theta_S + \cos \theta_S}{m^2_{\sigma,\eta} - (p + p')^2} \cdot g_{\sigma_\eta \pi \pi} \right\} \cdot (k + p + p')_\mu - m_\sigma \sigma K^0 \cdot (k - p + p')_\mu.$$  \hspace{1cm} (22)

Therefore, the form factors $f^{(\sigma)}_{1,2}$ are:

$$f^{(\sigma)}_1 = m_K \left( \frac{2\sqrt{2} \cos \theta_S - \sin \theta_S}{m^2_{\sigma,\eta} - (p + p')^2} \cdot g_{\sigma_\eta \pi \pi} - \frac{2\sqrt{2} \sin \theta_S + \cos \theta_S}{m^2_{\sigma,\eta} - (p + p')^2} \cdot g_{\sigma_\eta \pi \pi} \right) - f^{(\sigma)}_2,$$  \hspace{1cm} (23)

$$f^{(\sigma)}_2 = \frac{m_K g_{\sigma_\eta \sigma K^0 \pi}}{m^2_{\sigma K} - (k - p)^2}.$$  \hspace{1cm} (24)
At \( p = 0 \), the values of \( f_{1,2}^{(\sigma)} \) coincide with those represented in (15). Consequently, our theory not only reproduces the results of the algebra of currents and PCAC, but in addition, permits to extrapolate them from the nonphysical point \((p + p')^2 = \mu^2\) to the physical region \((p + p')^2 \geq 4\mu^2\), where the experimental data exist. A dependence of the constants \( \theta_S, m_{\sigma_{q'}}, m_{\sigma_q} \) and their coupling constants on a value of the parameter \( \xi \) permits to extract its magnitude from the data on \( f_{1,2}^{(\sigma)} \). But before, it is necessary to pick out from the data the possible contributions of other scalar resonances and the contribution produced by the spin-1 mesons.

The isosinglet scalar meson \( f_0(980) \) does not enter into the nonet of \( \sigma_{0\ldots8} \) mesons. Its contribution into the amplitude (13) is represented by matrix element \(<\pi^+\pi^-|\sigma><\sigma|A_\mu^{(K)}|K^+>\). The Adler ”self-consistency” condition [18] requires that the vertex \( f_0\pi\pi \) would have the form \( f_0(Q)\partial_\mu \pi(p)\partial_\mu \pi(p') \), or the form \( f_0(Q)(Q^2 - \mu^2)\pi(p)\pi(p') \). Using the last form and taking into account that the axial current in the case of \( f_0 \) meson has the form

\[
(A_\mu^{(f_0)})^{(K)} = \frac{2}{\sqrt{3}}(f_0\partial_\mu K^+ - K^+ \partial_\mu f_0)
\]  

(25)

we find the following expression for the additional contribution of \( f_0 \) into the form factor \( f_1(Q^2) \):

\[
\Delta f_1^{(f_0)} = 4g_{f_0\pi^+\pi^-}m_K(Q^2 - \mu^2)[\sqrt{3}m_{f_0}^2(m_{f_0}^2 - Q^2)]^{-1}.
\]  

(26)

The constant \( g_{f_0} \) could be determined using the data on the width of \( \Gamma_{f_0}(980) \). However, this width is very uncertain: \( 40 \leq \Gamma_{f_0} \leq 100 \) MeV [17]. At \( \Gamma_{f_0} \approx 53 \) MeV

\[
\Delta f_1^{(f_0)}(4\mu^2) = 0.108.
\]  

(27)

This special choice will be cleared up later. But the others values of \( \Gamma_{f_0} \) in the above-mentioned interval are not forbidden too, because a value of \( \Delta f_1^{(f_0)}(4\mu^2) \) changes in this interval from 0.094 to 0.1485 that leads to variation of \( f_1(4\mu^2) < 1\% \) while its experimental value is known with 1.4\% precision only.
4 A contribution produced by the intermediate spin-1 mesons to the form factors $f_{1,2}$.

In a theory where the vector fields $V^a_{\mu}$ are the divergences of the antisymmetric tensor fields $V^a_{\mu\nu}$, the following relation takes place:

$$\partial_{\mu} V^a_{\mu} = MV^a_{\nu},$$

where $M$ is a mass of the vector field $V^a_{\nu}$.

The part of effective lagrangian describing the $V\pi\pi$ interaction is

$$L^{\text{eff}}(V\pi\pi) = -\frac{g^a}{M} V^a_{\mu\nu} f^{abc} D_\mu \pi^b D_\nu \pi^c.$$  \hspace{1cm} (29)

In the case of the $\rho^{(3)}\pi\pi$ interaction we obtain the result:

$$A(\rho^{(3)}(Q) \to \pi(p)\pi(p')) = \frac{Q^2}{M^2} \rho^{(3)}_{\mu} (Q^2) f^{3ab} \partial_{\mu} \pi^a(p)\pi^b(p'),$$

where $Q^2 < M^2$ it becomes smaller by $Q^2/M^2$ times than the result of the gauge theory. This property will play an important role further.

The expression for the free propagator of the field $V_{\mu\nu}$ is \cite{10}, \cite{11}:

$$<0|T\{V_{\mu\nu}(x), V_{\rho\sigma}(y)\}|0> = iM^{-2} \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \times [(g_{\mu\rho} g_{\nu\sigma} - g_{\nu\rho} g_{\mu\sigma})(M^2 - k^2) + g_{\mu\rho} k_{\nu} k_{\sigma} - g_{\mu\sigma} k_{\nu} k_{\rho} - g_{\nu\rho} k_{\mu} k_{\sigma} + g_{\nu\sigma} k_{\mu} k_{\rho}],$$

\hspace{1cm} (31)

The diagrams with the intermediate vector mesons bringing up a contribution into $K_{e4}$ decay are represented by the matrix elements $<\pi^- | A^{(K)}_{\mu} | K^{*0} > < K^{*0} | \pi^+ K^- >$ and $<\pi^+ \pi^- | \rho^0 > < \rho^0 | A^{(K)}_{\mu} | K^+ >$, where $K^{*0}$ is the strange vector meson with a mass $m = 896$ MeV and $\Gamma(K^{*0} \to K\pi) = 48.7 \pm 0.8$ MeV.

In the considered here problem, only the $W^\pm$ bosons play a role of the gauge ones. So that, the covariant derivative in (29) has the form \cite{19}

$$D_{\mu} \hat{\pi} = \partial_{\mu} \hat{\pi} + i[\hat{W}_{\mu}, \hat{\pi}]_+ - [\hat{W}_{\mu}, \hat{\pi}]_-.$$  \hspace{1cm} (32)

Retaining in the matrix $\hat{\sigma}$ only the non-zero vacuum expectations of the $\sigma$ fields, namely, taking

$$\langle \hat{\sigma} \rangle = \text{diagonal} \frac{F_\pi}{\sqrt{2}} \{1, 1, (2R - 1)\},$$  \hspace{1cm} (33)
we obtain for $D_\mu K^+$ in (29)

$$D_\mu K^+ = \partial_\mu K^+ - \sqrt{2} F_\pi RW^+_{\mu} .$$ (34)

Then, a calculation of the matrix elements $< \pi^- | A_\mu^{(K)} | K^{*0} > < K^{*0} | \pi^+ K^- >$ and $< \pi^+ \pi^- | \rho^0 > < \rho^0 | A_\mu^{(K)} | K^+ >$ fulfilled basing on Eqs. (29), (31) and (34) yields

$$\Delta f_1^{(K^*)} = -\frac{\sqrt{2} g_\pi^2 R F_\pi m_K}{M_{K^*}^2 - (k - p)^2} \cdot \left( \frac{kp' - 2pp'}{M_{K^*}^2} \right) .$$ (35)

And the following contribution into $f_2$:

$$\Delta f_2^{(K^*)} = -\frac{\sqrt{2} g_\pi^2 R F_\pi m_K}{M_{K^*}^2 - (k - p)^2} \cdot \left( \frac{kp' - 2pp'}{M_{K^*}^2} \right) .$$ (36)

$$\Delta f_2^{(\rho)} = -\frac{\sqrt{2} g_\pi^2 R F_\pi m_K}{M_{\rho}^2 - (p + p')^2} \cdot \left( \frac{kp + kp'}{M_{\rho}^2} \right) .$$ (37)

The well known existence of the $\rho$ meson dominance [20] implying inessential of the other intermediate states, permits us to restrict the further consideration of spin-1 meson contribution into the form factors $f_{1,2}$. The last multipliers in the above formulae arising in a theory with the vector hadrons as the divergences of the antisymmetric tensor fields, diminish a role of the vector hadrons in the low-energy meson processes.

In a theory including the mesons with the spin 0 and spin 1, the strange axial-vector current, besides the part, presented in (19), contains additional part

$$A_\mu^{(K)} = \sqrt{2} F_\pi m_\mu a_\mu^{(K)} - \sqrt{2} F_\pi D_\mu K$$ (38)

Then an influence of the intermediate axial-vector mesons is reduced to appearance in the right-hand part of the relations (35)-(37) of the additional multiplier

$$\left( 1 - \frac{\sqrt{2} F_\pi R m_\alpha}{m_\alpha^2 - (k - p - p')^2} \right) .$$ (39)

Limiting ourself by a consideration only the lightest axial-vector meson $K_1(1270)$ contribution into $f_{1,2}$, we come to the results

$$\Delta f_1^{(K^*)}(4\mu^2) = -0.109706,$$

$$\Delta f_2^{(K^*)}(4\mu^2) = -0.226025,$$

$$\Delta f_2^{(\rho)}(4\mu^2) = -1.023669.$$ (40)
In our theory, the parts of $f_{1,2}$ originated by the intermediate scalar mesons and by the spin-1 mesons turn out to be of the same (negative) signs. Then, comparing our results with the experimental values, we need to compare the absolute value of $f_{1,2}^{\text{theor}}$ with $f_{1,2}^{\text{exp}}$.

The absolute value of the form factor $f_{2}^{\text{theor}}(4\mu^2)$ that is a sum of the parts $f_{2}^{(\sigma)}(4\mu^2)$, $\Delta f_{2}^{(K^*)}(4\mu^2)$, $\Delta f_{2}^{(\rho)}(4\mu^2)$ is equal

$$|f_{2}^{\text{theor}}(4\mu^2)| = 4.6837. \quad (41)$$

This result practically coincides with the experimental value of

$$f_{2}^{\text{exp}}(4\mu^2) = 4.687(1 \pm 0.024) \quad (42)$$

obtained by extrapolation of the data to the point $(p + p')^2 = 4\mu^2$. (We use for extrapolation the fitting formula $f(4\mu^2) = f(q^2)(1 + \lambda \cdot q^2)^{-1}$, where $q^2 = (s_{\pi} - 4\mu^2)/4\mu^2$).

In a gauge theory, a magnitude of $f_{2}(4\mu^2)$ would be

$$|f_{2}(4\mu^2)| = 2.7087 \quad [21]. \quad (43)$$

This value is inadmissible. So that, a treatment of the spin-1 hadrons as the gauge bosons is unacceptable. Now, using the results (27), (35) and the numerical value of $f_{1}^{\text{exp}}$ extrapolated to the point $(p + p')^2 = 4\mu^2$

$$f_{1}^{\text{exp}}(4\mu^2) = 5.812(1 \pm 0.014) \quad (44)$$

we find a value of $f_{1}^{(\sigma)}(4\mu^2)$

$$f_{1}^{(\sigma)}(4\mu^2) = f_{1}^{\text{exp}}(4\mu^2) - |f_{1}^{(K^*)}(4\mu^2) + f_{1}^{(\rho)}(4\mu^2)| = 5.5943 \pm 0.081. \quad (45)$$

The form factor $|f_{1}^{(\sigma)}(4\mu^2)|$ in (23) acquires a very close value (namely, 5.5786) at

$$\xi = -0.216. \quad (46)$$

A knowledge of a value of $\xi$ permits us to fix the magnitudes of the constants $\theta_s, g_{s\varphi\pi\pi}, g_{s\omega\pi\pi}, m_{s\sigma}, m_{s'\sigma}$. Using the numerical values of these constants presented in Appendix, one may check that in the soft-pion approximation (at $p = 0$) a value of $f_{1}^{(\sigma)}$ in (23) coincides with a value of $f_{1}$ in (15).
5  The masses and the widths of the isosinglet scalar mesons

Using the formulae (9) and the expressions for the coupling constants \( g_{\sigma'\pi\pi} \) and \( g_{\sigma\pi\pi} \) presented in Appendix, we find:

\[
m_{\sigma'} = 663 \text{ MeV}, \quad \Gamma_{\sigma'\pi\pi} = 767 \text{ MeV}, \quad (47)
\]

\[
m_{\sigma} = 1369 \text{ MeV}, \quad \Gamma_{\sigma\pi\pi} = 679 \text{ MeV}. \quad (48)
\]

The above widths are calculated on the mass shell. At the smaller energies their dependence on \( Q^2 = (p + p')^2 \) is given by the expression

\[
\Gamma_{\sigma\pi\pi}(Q^2) = \frac{3g_{\sigma\pi\pi}^2\sqrt{1 - 4\mu^2/Q^2}}{32\pi Q} \quad (49)
\]

6  A probability of the decay \( K^+ \to \pi^+\pi^- e^+\nu \).

Its probability is defined by the integral

\[
w(K_{e4}) = \frac{G_F^2 V_{us}^2 m_K^5}{214 \cdot 3 \cdot \pi^5} \int_0^1 \left[ f_1^2(x) \Phi_1(x) + f_2^2(x) \Phi_2(x) + f_4^2 \frac{m_K^2}{m_K^2} \Phi_4(x) \right] dx \quad (50)
\]

where \( x = Q^2/m_K^2 \) and \( f_4 = \frac{\sqrt{m_K^2}}{8\pi^2 F_\pi} \) [21].

\[
\Phi_1(x) = (1 - 8x + 8x^3 - x^4 - 12x^2 \ln x) \left( 1 - \frac{4\mu^2}{m_K^2} \cdot x \right)^{1/2} \quad (51)
\]

\[
\Phi_2(x) = \frac{1}{3} \left( 1 + 72x^2 - 64x^3 - 9x^4 + 12(3x^2 + 4x^3) \ln x \right) \left( 1 - \frac{4\mu^2}{m_K^2} \cdot x \right)^{3/2} \quad (52)
\]

\[
\Phi_4(x) = \frac{2}{5} m_K^2 [(x + x^2)(1 - 8x + 8x^3 - x^4 - 12x^2 \ln x) - \\
\frac{4}{5} x(1 - x)^5] \left( 1 - \frac{4\mu^2}{m_K^2} \cdot x \right)^{3/2} \quad (53)
\]

Taking also into account the observed in [1], [22] a small growth of the form factors in the region \( 4\mu^2 \leq Q^2 \leq m_K^2 \) described by the linear fit

\[
f_j(q^2) = f_j(0)(1 + \lambda_j q^2), \quad q^2 = (Q^2 - 4\mu^2)/4\mu^2,
\]

\[\lambda_j = \frac{1}{2} \quad (54)
\]

\[\Phi_4(x) = \frac{2}{5} m_K^2 \left[ (x + x^2)(1 - 8x + 8x^3 - x^4 - 12x^2 \ln x) - \\
\frac{4}{5} x(1 - x)^5 \right] \left( 1 - \frac{4\mu^2}{m_K^2} \cdot x \right)^{3/2} \quad (53)
\]
that in terms of the variable \( x \) transforms into

\[
f_j(x) = f_j(4\mu^2)(1 - \lambda_j + \lambda_j \frac{m_K^2}{4\mu^2} x) \tag{54}
\]

and using the result \[1\]

\[
\lambda_{1,2} = 0.079 \pm 0.015 \tag{55}
\]

we obtain

\[
\int_{4\mu^2/m_K^2}^{1} f_1^2(x)\Phi_1(x)dx = f_1^2(4\mu^2)(4.164 \pm 0.048)10^{-3}, \tag{56}
\]

\[
\int_{4\mu^2/m_K^2}^{1} f_2^2(x)\Phi_2(x)dx = f_2^2(4\mu^2)(0.8401 \pm 0.0126)10^{-3}. \tag{57}
\]

\[
\int_{4\mu^2/m_K^2}^{1} f_4^2\Phi_4(x)dx = f_4^2 \cdot 0.8213 \cdot 10^{-5}. \tag{58}
\]

Then, using the fixed above magnitudes of \( f_{1,2}(4\mu^2) \), we come to the result:

\[
w(K_{e4})^{\text{theor}} = (3.2426 \pm 0.0781)10^3 s^{-1}. \tag{59}
\]

This value is in agreement with the observed result:

\[
w(K_{e4})^{\text{exp}} = (3.302 \pm 0.078)10^3 s^{-1} \tag{60}
\]

### 7 Conclusions

The \( K_{e4} \) decay turned out to be the perfectly suitable object for a verification of the ideas on a nature of the particles originating the low-energy processes with participation of the pseudoscalar mesons.

Thus, a considerable exceeding of the observed value of the form factor \( f_1 \) its value calculated in the soft-pion approximation, gets an explanation considering the structure of the diagrams in Fig.1, containing the scalar mesons.

A fascinating idea, expressed by fifty years ago, that the spin-1 hadrons are the gauge bosons of chiral theory, remains recognized up to now, in spite
of the appearing from time to time doubt in literature concerning its truth. The data on the form factors of $K_{e4}$ decay permit to check how fair this idea.

We have found that in the theory, where the spin-1 hadrons are considered as the gauge bosons, the form factor $f_2(4\mu^2)$ turns out to be too small.

On the contrary, a theory treating the spin-1 hadrons as the divergences of the antisymmetric tensor fields, gives the quite satisfactory results for $f_2^{\text{theor}}(4\mu^2)$ and for $f_1^{\text{theor}}(4\mu^2)$ also.
APPENDIX.
Here we present the formulae for the coupling constants of scalar mesons necessary for calculation of the amplitudes

\[ g_{\sigma \eta' \pi \pi} = -\frac{\Lambda^2}{\sqrt{3}F_\pi} \left\{ \sqrt{2}[1 + \xi(2R + 1)] \cos \theta_S + [1 - 4\xi(R - 1) \sin \theta_S] \right\}, \]

\[ g_{\sigma \eta \pi \pi} = -\frac{\Lambda^2}{\sqrt{3}F_\pi} \left\{ -\sqrt{2}[1 + \xi(2R + 1)] \sin \theta_S + [1 - 4\xi(R - 1)] \cos \theta_S \right\}, \]

\[ g_{\sigma K_0 K^*} = -\frac{\Lambda^2(2R - 1)}{\sqrt{2}F_\pi} \]

where \( \Lambda^2 = (m_K^2 - \mu^2)/(R - 1)(2R - 1) \). In the theory with the lagrangian (2), the following relation takes place:

\[ \frac{g_{\sigma \eta' \pi \pi}^2}{(m_{\sigma \eta'}^2 - \mu^2)^2} + \frac{g_{\sigma \eta \pi \pi}^2}{(m_{\sigma \eta}^2 - \mu^2)^2} = \frac{1}{F_\pi^2} \]

The formulae for the other coupling constants, the reader can find in [5] and in Appendix of the paper [24]. We present here also the numerical values of the above constants and the masses of \( \sigma \) mesons at \( \xi = -0.216 \).

\[ \theta_S = 18.83728995 \text{ degree} \]

\[ g_{\sigma \eta' \pi \pi} = 4.335480173 \text{ GeV}, \quad g_{\sigma \eta \pi \pi} = 5.639539623 \text{ GeV} \]

\[ m_{\sigma \eta'} = 0.663209405 \text{ GeV}, \quad m_{\sigma \eta} = 1.368980326 \text{ GeV}. \]
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