Towards Online Optimization for Power Grids

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Abstract
In this note, we discuss potential advantages in extending distributed optimization frameworks to enhance support for power grid operators managing an influx of online sequential decisions. First, we review the state-of-the-art distributed optimization frameworks for electric power systems, and explain how distributed algorithms deliver scalable solutions. Next, we introduce key concepts and paradigms for online optimization, and present a distributed online optimization framework highlighting important performance characteristics. Finally, we discuss the connection and difference between offline and online distributed optimization, showcasing the suitability of such optimization techniques for power grid applications.

Keywords: Online optimization, Distribution networks, Distributed Optimization

1 Distributed Optimization for Power Grids

1.1 The evolving power grid
The rise of distributed and renewable energy resources including wind and solar backed by energy storage technologies, is accelerating the evolution of the electric power grid [30]. The evolution is supported by recent advances in communication [15], sensor technologies [31], data processing [1], and operational technologies [7, 9, 10, 32], enabling prosumers to generate and deliver surplus renewable energy back to the power grid [4]. As the complexity of the evolving power grid continues to increase, with renewable technologies becoming increasingly distributed and spatially diverse, scalable approaches are needed to manage electricity flows to and from millions of energy prosumers. Distributed optimization frameworks that support scalability in managing renewable energy flows in transmission [22] and distributions networks [18, 28], that additionally supporting solutions to integrate energy storage [21, 29], including electrical vehicles [13, 25, 26], potentially enhance both the operation and resilience of the evolving power grid.

1.2 Distributed optimization frameworks
The key promise in distributed optimization is a dramatic improvement in scalability to accommodate data and decisions scattered in physically decentralized locations. See [23] for an in-depth survey.

Example 1. (Economic Power Dispatch [9]) Consider $N$ generators indexed in $V = \{1, \ldots, N\}$. At a fixed time there is a total power demand $P$ that needs to be met by these $N$ generators. Let generator $i$ be allocated a power $x_i \in [x_i^{\min}, x_i^{\max}]$, leading to a cost $\ell_i(x_i)$. The economic power dispatch problem:

$$\min_{x} \sum_{i=1}^{N} \ell_i(x_i) \tag{1}$$

s.t. $x_i^{\min} \leq x_i \leq x_i^{\max}$, $i = 1, \ldots, N$

$x_1 + \cdots + x_N = P$.

Here $\ell_i(\cdot)$ is a function mapping from $\mathbb{R}^{\geq 0}$ to $\mathbb{R}^{\geq 0}$. An optimal decision on the $x_i$ for all $i$ should minimize the total generation cost.

Example 2. (Optimal Power Flow [10]) Consider an electrical network with $N$ nodes indexed in $V = \{1, \ldots, N\}$. Let $v_i \in \mathbb{C}$ and $i_i \in \mathbb{C}$ be the voltage and inflow current at node $i$. The network structure is captured by an admittance matrix $A \in \mathbb{C}^{N \times N}$. Then $x_i := \text{Re}(v_i i_i^\dagger)$ defines the active power at node $i$, where $^\dagger$ is the complex conjugate. Let $\ell_i(x_i)$ denote the cost associated with the power at node $i$. An optimal power flow problem is given in the following form:

$$\min_{x} \sum_{i=1}^{N} \ell_i(x_i) \tag{2}$$

s.t. $x_i = \text{Re}(v_i i_i^\dagger)$, $i = 1, \ldots, N$

$v_i i_i^\dagger = v_i \sum_{j=1}^{N} A_{ij}^\dagger v_j$, $i = 1, \ldots, N$. 
In centralized optimization for (1) and (2), each local cost function \( f_i(\cdot) \) and local parameter such as \( x_i^{\min}, x_i^{\max}, A_{ij} \) needs to be sent a central coordinator; the coordinator solves the respective problem (1) or (2) and sends the optimal decisions for \( x_i \) to each agent. In distributed optimization for (1) and (2), there is an underlying communication graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) over which agents share their decisions, and computations are carried out locally in parallel at each individual agent based on the local cost functions and parameters. The distributed computing architecture naturally allows scalability; the absence of a central coordinator improves resilience since failures at the coordinator have system-level impact while failures at individual agents harm the system-level performance at a limited level.

1.3 Distributed optimization algorithms

There are many algorithms for distributed optimization. In power systems, the Alternating Direction Method of Multipliers (ADMM) has been popular.

1.3.1 The ADMM. Given an optimization problem

\[
\begin{align*}
\min_{x,z} & \ f(x) + g(z) \\
\text{s.t.} & \ Ax + Bz = c, \\
& \ x \in \mathbb{R}^n, z \in \mathbb{R}^m.
\end{align*}
\]

(3)

ADMM proceeds by first defining the augmented Lagrangian

\[
L_{\alpha}(x,z,y) = f(x) + g(z) + y^T (Ax + Bz - c) + \frac{\alpha}{2} \|Ax + Bz - c\|^2
\]

with dual variable \( y \). Then the algorithm runs recursively, where in each round there are updates in the decision variable for \( x,z,y \) that are arranged sequentially.

### ADMM Algorithm [1]

Define an initial point \((x^{(0)}, z^{(0)}, y^{(0)})\), smoothing parameter \( \alpha \), and iteration limit \( n \).

For \( k = 0, \ldots, n \) DO

(i) Update the first variable as

\[
\hat{x}^{(k+1)} = \arg \min_{x \in \mathbb{R}^n} L_{\alpha}(x,z^{(k)}, y^{(k)});
\]

(ii) Update the second variable as

\[
\hat{z}^{(k+1)} = \arg \min_{z \in \mathbb{R}^m} L_{\alpha}(\hat{x}^{(k+1)}, z, y^{(k)});
\]

(iii) Update the dual variable as

\[
y^{(k+1)} = y^{(k)} - \alpha(A\hat{x}^{(k+1)} + B\hat{z}^{(k+1)} - c).
\]

1.3.2 Distributed ADMM. The original ADMM algorithm was proposed in the 1970s [12, 14], and regained its popularity in recent years due to its suitability for large-scale distributed computing problems [3]. If we write \( f(x) = \sum_{i=1}^N f_i(x_i) \) for the cost functions in (1) and (2) with \( x = (x_1, \ldots, x_N) \), and suitable auxiliary decision variables \( z \) from the constraints, problems in the form of (1) and (2) can be written in the standard ADMM form (3). For example, the problem \( (1) \) with \( x_i^{\min} = -\infty \) and \( x_i^{\max} = \infty \) can be written as (Chapter 7, [3])

\[
\begin{align*}
\min_{x,z} & \ f(x) + gp \left( \sum_{i=1}^N z_i \right) \\
\text{s.t.} & \ x_i - z_i = 0, \ i = 1, \ldots, N,
\end{align*}
\]

where \( gp(a) = 1 \) if \( a = P \) and \( gp(a) = +\infty \) otherwise. Then, due to the separable nature of the function \( f \) and the constraints in (4), the resulting ADMM algorithm can be naturally decomposed into parallel computations at the agents along each primal variable \( x_i \) and dual variable \( y_i \) for Step (i) and Step (iii). The Step (ii) of the ADMM algorithm relies on all \( x_i \) and \( y_i \) for \( i = 1, \ldots, N \), and can be implemented in a distributed fashion with the help of the communication graph \( \mathcal{G} \).

1.3.3 Alternatives to ADMM. There are many other distributed optimization methods aside from ADMM. Some popular algorithms based on the augmented Lagrangian technique are analytical target cascading [8], auxiliary problem principle [5], dual decomposition method [3], and proximal message passing [19]. One can also move away from the augmented Lagrangian, and use optimality condition decomposition [6], consensus methods, or distributed algorithms developed from subgradient methods [24], and dynamic programming [2].

2 Online Convex Optimization

2.1 Online optimization

The online optimization paradigm applies a robust optimization perspective for sequentially arriving data and costs that are too complex to be efficiently modeled. With its roots in classical ideas of sequential decisions in multi-armed bandit problems from the 1930s, online optimization has recently emerged as a prominent tool in machine learning, solving problems ranging from recommender systems to spam filtering [16, 27]. Online optimization portrays decisions for optimizing time-varying cost functions as a feedback process, where one learns from experience as time evolves. Performance is considered with respect to a static optimal decision taken in hindsight. Formally, the procedure of online optimization may be described as a game between a learner and an adversary played across a finite time horizon \( t = 1, \ldots, T \).
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Online Optimization Paradigm [27]

For \( t = 1, \ldots, T \), DO

(i) The adversary selects a cost function \( \ell_t(\cdot) : \mathcal{X} \subseteq \mathbb{R}^d \rightarrow \mathbb{R} \) and keeps it to itself;

(ii) The learner makes a decision \( x_t \in \mathcal{X} \);

(iii) The learner suffers a loss \( \ell_t(x_t) \), and receives the cost function \( \ell_t(\cdot) \) (full information), or just value of the loss \( \ell_t(x_t) \) (bandit information).

In sharp contrast to the view of classical optimization (i.e. a classical learner), where the loss function \( \ell_t(\cdot) \) is revealed before the learner attempts to minimize it, online optimization acknowledges the difficulty in knowing \( \ell_t(\cdot) \) or even a model of it before decisions are made. The information that the learner receives about \( \ell_t(\cdot) \) may be the whole function, a scenario referred to as full information; or the learner only experiences losses at selected decisions, and in this case, we talk about bandit information. The loss functions \( \ell_t(\cdot) \) are generally assumed to be arbitrary (but chosen from a given function class). Hence, it is impossible for the learner to infer \( \ell_t(\cdot) \) before the decisions are made. As a result, it is sensible for the learner to identify \( x_1, \ldots, x_T \in \mathcal{X} \) so that regret, i.e.,

\[
\text{Reg}(T) := \sum_{t=1}^T \ell_t(x_t) - \min_{x \in \mathcal{X}} \sum_{t=1}^T \ell_t(x)
\]

is minimized. From the definition, \( \min_{x \in \mathcal{X}} \sum_{t=1}^T \ell_t(x) \) is the minimal accumulative loss of an oracle making a static decision to whom all \( \ell_t(\cdot) \) are known before \( t = 1 \). Therefore, \( \text{Reg}(T) \) represents the difference between the actual accumulative loss experienced by the learner compared to that of such an oracle, i.e., the regret.

2.2 Impact of feedback
Let \( \mathcal{X} \subseteq \mathbb{R}^d \) be a compact convex set containing the origin, for which \( \mathbb{P}_\mathcal{X} \) is the projection onto \( \mathcal{X} \). A simple yet effective algorithm for the online learner is gradient descent implemented sequentially. The standard online gradient descent algorithm for solving the online optimization problem with full information is described below where \( \alpha_t \) is the stepsize.

Online Gradient Descent: Full Information Feedback [16]

For \( t = 1, \ldots, T \), DO

(i) The adversary selects a cost function \( \ell_t(\cdot) : \mathcal{X} \subseteq \mathbb{R}^d \rightarrow \mathbb{R} \) and keeps it to itself;

(ii) The learner makes a decision \( x_t = \mathbb{P}_\mathcal{X}(x_{t-1} - \alpha_t \nabla \ell_{t-1}(x_{t-1})) \);

(iii) The learner suffers a loss \( \ell_t(x_t) \), and receives \( \ell_t(\cdot) \).

With bandit information, the learner only experiences losses and the loss function \( \ell_t(\cdot) \) (and its gradient) is still unknown. Denote \( \mathcal{K}_d = \{ x : \frac{1}{2} x^T x \in \mathcal{X} \} \). Let \( \mathbb{S} \) be the unit sphere in \( \mathbb{R}^d \) under standard Euclidean norm. Then one can build unbiased gradient estimates from experienced losses to replace the true gradients in online gradient descent, leading to the following online bandit optimization algorithm.

Online Bandit Optimization: Bandit Information Feedback [16, 17]

Initialize \( y_0 = 0 \). For \( t = 1, \ldots, T \), the learner DO

(i) Draw \( u_t \in \mathbb{S} \) uniformly at random.

(ii) Play \( x_t = y_t + \delta u_t \); receive loss \( \ell_t(x_t) \).

(iii) Build gradient estimate \( g_t = d\ell_t(x_t)u_t/\delta \); Update \( y_{t+1} = \mathbb{P}_\mathcal{X,\delta}(x_t - \eta g_t) \).

It is taking place that in both (5) and (6), feedback is taking place. The promise of these online optimization algorithms lies in the fact that, when the stepsizes are selected as some suitable learning rates, the algorithms will produce sublinear regrets \( \lim_{T \rightarrow \infty} \text{Reg}(T)/T = 0 \), for a suitably regular classes of convex cost functions. This is a strong testimony to the performance of true learning during the sequential decisions. The regret averaged over time is close to zero for sufficiently long time horizon: it is as if all the \( \ell_t(\cdot) \) are known before the whole play starts and the learner decides to play a static optimal decision. With careful classification of the function classes for the loss functions, refined upper bounds on \( \text{Reg}(T) \) can be established at \( O(\log T) \), \( O(\sqrt{T}) \), \( O(T^{3/4}) \), etc [16].

2.3 Distributed online optimization
In practice, the loss \( \ell_t(\cdot) \) might represent a system-level loss, scattered across a number of subsystems indexed in \( \mathcal{V} = \{ 1, \ldots, N \} \), such that \( \ell_t(\cdot) = \sum_{i=1}^N \ell_{ti}(\cdot) \). The overall system forms a network, where a directed graph \( \mathcal{G} = (\mathcal{V}, E) \) describes the communication structure of the network. Then the question arises on whether in the case that subsystems may only talk to their neighbors over the graph \( \mathcal{G} \), this will enable distributed online learning throughout the network. Following the distributed optimization framework, a distributed online optimization paradigm can be described as follows [35] (see also [17, 33, 34, 36]).

\footnote{For bandit feedback, the regret is technically \( \mathbb{E}\text{Reg}(T) \) where the expectation is taken over the randomness in the gradient estimate.}
**Distributed Online Convex Optimization** [35]

**Initialize** \( \mathcal{X} \) as a convex subset of \( \mathbb{R}^d \) defined by a family of inequalities: \( \mathcal{X} = \{ x \in \mathbb{R}^d | c_s(x) \leq 0, \ s = 1, \ldots, p \} \).

For \( t = 1, \ldots, T \), agents in \( \mathcal{V} \) DO

- Each agent \( i \in \mathcal{V} \) selects \( x_i(t) \in \mathcal{X} \), and a local adversary chooses \( \ell_i(x) : \mathbb{R}^d \to \mathbb{R} \) as a convex cost function;
- Each agent experiences a loss \( \ell_i(x_i(t)) \);
- The function \( \ell_i \) is revealed to agent \( i \); the decisions of the neighbors of the agent \( i \) are also revealed to \( i \), i.e., \( x_j(t) \) for \( j \in \mathcal{N}_i := \{(j, i) \in \mathcal{E}\} \).

The decision set \( \mathcal{X} \) implies that in each time-step each agent should be able to perform a projection onto \( \mathcal{X} \), which can be computationally expensive. Instead, one can only require that the constraints are satisfied in the long run, i.e., that \( \sum_{t=1}^{T} \sum_{j=1}^{N} c_i(x_i(t)) \leq 0 \). An effective distributed online learning algorithm, then should aim to minimize the accumulated system-wide loss. The system-level regret is defined as the worst possible regret for all agents:

\[
SReg(T) := \max_{i \in \mathcal{V}} \left[ \sum_{t=1}^{T} \sum_{j=1}^{N} \ell_{ij}(x_i(t)) - \sum_{t=1}^{T} \sum_{j=1}^{N} \ell_{ij}(x^*) \right]
\]

where \( x^* = \arg \min_{x \in \mathcal{X}} \sum_{t=1}^{T} \sum_{j=1}^{N} \ell_{ij}(x) \) is the system-level decision by a static optimal oracle. The performance of the algorithm is further characterized by the so-called cumulative absolute constraint violation defined by

\[
CACV(T) := \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{s=1}^{P} [c_s(x_i(t))]_+
\]

where \([a]_+ = \max\{0, a\} \).

### 2.4 Distributed online primal-dual gradient algorithm

In classical distributed optimization, it is popular to use a consensus algorithm as an information aggregation subroutine. Specifically, we may associate a doubly stochastic matrix \( A \) with the graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) such that \( A_{ij} > 0 \) if and only if \( (j, i) \in \mathcal{E} \). In general, for strongly connected graph \( \mathcal{G} \), one can always find such an \( A \).

**Example 3.** We illustrate the performance of the proposed algorithms using a simple experiment. Specifically, we consider a distributed online linear regression problem over a network, where \( \ell_{ij}(x) = (a_i(t)^T x - b_j(t))^2 / 2 \). The constraints are described as

\[
\begin{align*}
    c_m(x) &= L - x_m \leq 0, \ m = 1, \ldots, d, \\
    c_{d+m}(x) &= x_m - U \leq 0, \ m = 1, \ldots, d.
\end{align*}
\]

Every entry of \( a_i(t) \) and \( b_j(t) \) is generated uniformly at random within the interval \([-1, 1]\) and \([0, 1]\), respectively, independently for each time \( t = 1, \ldots, T \). Throughout the experiments, we implement the distributed online primal-dual gradient algorithms proposed in [35] with full information or bandit information feedback.

**System Setup.** The graph \( G \) is randomly generated and selected as depicted in Fig. 1. The weighting matrix associated with the network in Fig. 1 is generated according to the maximum-degree weights:

\[
A_{ij} = \begin{cases} 
    \frac{1}{1+d_{\max}}, & (j, i) \in \mathcal{E} \\
    1 - \frac{d_i}{\max d_{\max}}, & i = j \\
    0, & (j, i) \notin \mathcal{E}
\end{cases}
\]

where \( d_{\max} = \max_{v \in V} \{d_v\} \) is the maximum degree of \( G \) with \( d_i \) denoting the degree of node \( i \). We set the parameters as follows: \( N = 20, d = 2, L = -1/2, U = 1/2 \), and \( R_X = U \sqrt{d} \).

The performance of the algorithm is averaged over 10 runs.

**Figure 1.** A randomly generated network of 20 nodes.

**Performance.** We run the algorithms and plot the Average System Regret (ASR, for short) defined as \( SReg(T)/T \) and the Average Constraint Violations (ACV, for short) defined as \( CACV(T)/T \), as a function of the time horizon \( T \) in Fig. 2. Clearly both the ASR and ACV converge to zero.
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3 Perspectives on Online Optimization for Power Grid

3.1 Offline vs. online optimization

For problems such as the economic dispatch and optimal power flow in Example 1 and Example 2 over a time horizon \( t = 1, \ldots, T \), there may be two paradigms.

Distributed (Repeated Offline) Optimization [DRO-O]. For each time \( t \), independently treat the corresponding problem (1) and (2); apply distributed optimization algorithms until suitable convergence is guaranteed for time \( t \); implement the optimal decision \( x^*_i(t) \) for \( i = 1, \ldots, N \) repeatedly at the respective time \( t \).

Distributed Online Optimization [DO-O]. Employ the Distributed Online Convex Optimization paradigm outlined in Section 2.3; apply distributed online optimization algorithms throughout the horizon \( t = 1, \ldots, T \); implement the decision \( x_i(t) \) for \( i = 1, \ldots, N \) sequentially for \( t = 1, \ldots, T \).

The potential in developing online optimization frameworks for problems in power grids has drawn attentions in the literature. Online convex optimization has been adopted in [38] for the control of distributed energy sources in the context of social welfare maximization. Under a similar social welfare maximization paradigm, [37] considers control of distributed energy sources with both continuous and discrete constraints. Moreover, [20] provides a unified framework for economic dispatch and unit commitment and proposes a centralized and distributed online convex optimization method for exploring such a framework.

Next, we would like to offer a few perspectives towards the strengths, challenges, and possible future direction for online optimization in power grids.

3.2 Perspectives between [DRO-O] and [DO-O]

First, it is worth mentioning that the key difference between [DRO-O] and [DO-O] goes far beyond the respective classes of algorithms. Underpinning the two frameworks are fundamentally different views about the system:

- In [DRO-O], the time-varying cost functions are known before decisions;
- In [DO-O], the time-varying cost functions are experienced after decisions.

As a result, conceptually the [DRO-O] algorithms are optimizers, while the [DO-O] algorithms are learners. Therefore, [DO-O] suits systems that are uncertain or unpredictable.

Next, the strength of [DO-O] lies in guaranteed sub-linear regret against adversaries. In practice, the adversaries represent the worst-case scenarios. Remarkably, the aforementioned regret bounds of orders \( O(\log T) \), \( O(\sqrt{T}) \), and \( O(T^{3/4}) \) may be valid even for feedback adversaries, where the cost function \( f_i(\cdot) \) depends on the past experiences. Moreover, the online decisions in [DO-O] tend to converge to a static optimal
decision with respect to the cumulative cost over the entire horizon, while decisions in [DRO-O] tend not to converge as they are tracking real-time optimal decisions for time-varying cost functions. This is shown in Example 3 where online decisions indeed are more stationary compared to repeated offline optimal decisions.

In the context of power grids, [DO-O] might be more suited in problems related to wind or solar energy grid-integration, and energy storage applications including residential batteries and EVs. Such applications involve significant uncertainty regarding the weather, network impedance and topology, real-time price volatility, and user preferences including when, where and for how long an EV will require charging. Importantly, for problems with known grid and user information, [DRO-O] is a more sensible choice as the performance of [DO-O] is much more conservative.

3.3 Future directions
Towards establishing practical online optimization frameworks for problems in power grid, there are a few possible directions. First, the notion of regret needs to be taken into account for online optimization of power grid problems. Existing regret bounds for online optimization are for classes of convex, smooth, or strongly convex functions, etc. Cost functions in power grid problems and constraints are certainly more structured (despite being unknown before decisions), and thus refined regret bounds might exist. Second, the uncertain nature of online optimization needs to be carefully matched to practice. The characterization of cost functions should also be evaluated in the power system context. Third, hybrid decision frameworks that combine the strength of [DRO-O] and [DO-O], where the information and uncertainty of the cost functions can be jointly treated, would be of significant value for power grid applications.

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