Density of dark matter in Solar system and perihelion precession of planets

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Abstract

Direct model-independent relation between the secular perihelion precession of a planet and the density of dark matter \( \rho_{\text{dm}} \) at its orbit is indicated, and used to deduce upper limits on local values of \( \rho_{\text{dm}} \).
The analysis of the perihelion precession (though not of a planet, but of the asteroid Icarus) was for the first time used to obtain an upper limit on the dark matter density ($\rho_{\text{dm}}$) in the Solar system in [1]. This limit was on the level

$$\rho_{\text{dm}} < 10^{-16} \text{ g/cm}^3.$$  

(1)

Recently, precision EPM ephemerides were constructed in [2, 3]. The analysis of thus obtained possible corrections (i.e. of the deviations of the results of theoretical calculations from the observational data) to the secular perihelion precession $\delta \phi$ of three planets resulted in much stronger upper limit on the $\rho_{\text{dm}}$ in the Solar system on the level [4]

$$\rho_{\text{dm}} < 3 \times 10^{-19} \text{ g/cm}^3.$$  

(2)

More sophisticated analysis of the data and of possible effects performed in [5–7] has confirmed this result, at least qualitatively.

In the present note I wish to point out a simple relation pertinent to the problem, as well as a quite important physical conclusion following from this relation. The relation applies to any spherically symmetric, but otherwise arbitrary, $\rho_{\text{dm}}(r)$. If the orbit eccentricity $e$ for a planet is small, the relative shift of its perihelion per period can be written as

$$\frac{\delta \phi}{2\pi} = -\frac{2\pi \rho_{\text{dm}}(r) r^3}{M_{\odot}},$$  

(3)

Here $r$ is the radius of the (approximately circular) orbit, and $M_{\odot}$ is the mass of the Sun. Formula (3) is accurate up to a correction on the order of $e^2$. This correction is small for any planet of our Solar system, and can be safely ignored for the discussed problem, at least at the present level of accuracy.

Formula (3) can be derived, for instance, as follows. The perturbation of the gravitational potential for a planet of mass $m$ moving along a circular orbit of radius $r$ is

$$\delta U(r) = k m \int_{r_1}^{r} \frac{dr_1}{r_1^2} \mu(r_1),$$  

(4)

where

$$\mu(r) = 4\pi \int_0^{r_1} \rho(r_2) r_2^2 dr_2$$

is the total mass of dark matter inside a sphere of radius $r$, and $k$ is the Newton gravitational constant (as usual, the potential is defined up to a constant, so that the value of the lower integration limit in (4) is irrelevant). Then one computes the corrections, due to the perturbation $\delta U(r)$, to the frequency $\omega_{\phi}$ of rotation and to the frequency $\omega_r$ of radial oscillations. The difference between these corrections multiplied by the unperturbed period of rotation gives the perihelion shift.

Obviously, under the assumption of constant density $\rho_{\text{dm}}$ made in [4], equation (3) goes over into formula

$$\frac{\delta \phi}{2\pi} = -\frac{3}{2} \frac{\mu(r)}{M_{\odot}},$$  

(5)
derived therein; here and below $\mu(r)$ is the total mass of dark matter inside the sphere of radius $r$ (we have omitted in rhs of eq. 5 the factor $\sqrt{1-e^2}$ present in the corresponding formula of [4]). It should be mentioned here also that a formula equivalent to (3), but in the special case of constant $\rho dm$, was presented in [6] (also with the factor $\sqrt{1-e^2}$ in rhs). On the other hand, under the assumption

$$\rho_{dm} = \rho_0 (r/r_0)^{-\gamma}$$

made in [7], equation (3) is equivalent to relation

$$\frac{\delta \phi}{2\pi} = -\frac{3 - \gamma}{2} \frac{\mu(r)}{M_\odot}$$

from that reference.

Let us emphasize here the following. All equations, (3), (2), and (7), are valid only under the assumption, implicit or explicit, that the dark matter density $\rho_{dm}(r)$ is spherically-symmetric with the center coinciding with the Sun. Such a picture looks reasonable since the typical limits on dark matter density in the Solar system discussed in [4–7], are much higher than typical galactic values of $\rho_{dm}$.

Then, the assumption of constant $\rho dm$ density made in [4–6] in the analysis of perihelion rotation for inner planets, corresponds to the situation when $\rho_{dm}$ varies at the distances much larger than 1 astronomical unit (au). However, it is far from being clear what could be the true scale of $\rho_{dm}(r)$ variation. The same word of caution refers to the assumption (6) with $r_0 = 1$ au and $\gamma \sim 1$, so much the more when it is applied, in line with near planets, to Jupiter, Saturn, Uranus.

The special physical implication of formula (3) is as follows. The perihelion rotation is governed directly by a local dark matter property, i.e. by its density $\rho_{dm}(r)$ at the planet trajectory of radius $r$, but not by its global property, the total mass $\mu(r)$ of dark matter inside the sphere of radius $r$. Therefore, the analysis of the observational data for the secular perihelion precession of various planets results in direct, model-independent upper limits on the local $\rho dm$ at various distances from the Sun corresponding to the orbit radii. The results of this analysis are presented in Table 1. The upper limits in the last line of the Table are derived in an obvious simple-minded way from the numbers in the previous line.
### Table 1. Dark matter density at different distances from the Sun

|                     | Mercury | Earth   | Mars        |
|---------------------|---------|---------|-------------|
| Absolute perihelion shift, " per century | −0.0036 ± 0.0050 | −0.0002 ± 0.0004 | 0.0001 ± 0.0005 |
| Relative perihelion shift, \((\delta\phi/2\pi) \times 10^{11}\) | −0.67 ± 0.93 | −0.15 ± 0.31 | 0.14 ± 0.73 |
| Orbit radius, au     | 0.39    | 1.00    | 1.52        |
| Dark matter density, g/cm³ | \(1.1(1.5) \times 10^{-17}\) | \(1.4(3.0) \times 10^{-19}\) | \(< 0.4(2.0) \times 10^{-19}\) |
|                      | \(< 2.6 \times 10^{-17}\) | \(< 4.4 \times 10^{-19}\) | \(< 1.6 \times 10^{-19}\) |

The corresponding data from [3] on the perihelion precession of Venus (as well as the analogous information on far planets, Jupiter, Saturn, Uranus) are not considered here, since they are essentially less accurate.

Of course, the limits presented in Table 1 agree with those derived previously. But they are obtained without extra assumptions, in a model-independent way, and refer to various distances from the Sun.

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