The use of interpolation methods for modelling multifactor processes based on an experiment planning matrix

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Abstract. This article is dedicated to the development of experimental data processing tools based on the geometric theory of multidimensional interpolation. At the same time, a scientific problem was solved, which is dedicated to the analytical determination of models of multifactor processes while maintaining the existing approach to experiment planning. This makes it possible not only to use interpolation methods for the mathematical description of new experiments, but also to use the experimental data obtained earlier to improve their mathematical interpretation. The paper provides two examples that confirm the effectiveness of the proposed approach to constructing models of multivariate processes using multivariate interpolation methods. The first of the above examples contains two geometric models of the stress-strain state of metal polyhedral bent struts, which are presented in the form of ruled response hypersurfaces passing through 8 predetermined points in the 4-dimensional space. The second one is dedicated to the construction of a 2-factor process using a 2-parameter parabolic geometric interpolant with subsequent optimization by methods of mathematical analysis of a function of two variables. As a result, the optimization of the aerated concrete manufacturing process was carried out to achieve the maximum values of the ultimate strength in compression after heat and moisture treatment.

1. Introduction

It is difficult to find such a branch of knowledge accumulated by mankind that would not rely on experimental data. The experiment is the basic tool for understanding the laws of nature and the universe. Therefore, it is not surprising that conducting an experiment is an important part of any research in a wide variety of branches of science and technology. With the growth of the computational capabilities of modern computer systems, more and more often a real experiment is replaced by a computational one, which significantly saves the researcher's time and budget, but generates new problems related to the reliability of the results obtained. Therefore, the share of such experiments is not yet high in comparison with real experiments. The most important task after the experiment is to analyse the results obtained, most often based on modelling the experimental results. Based on this, already at the stage of preparing the experiment, special planning matrices are formed, which in the future will provide convenient use of mathematical modelling methods. The most popular of them was regression analysis [1-3], based on the approximation of the initial array of experimental data using analytical dependencies presented as a response function. The use of planning matrices for conducting an experiment is presented in GOST 24026-80 [4], which also widely presents theories of regression analysis and mathematical statistics. However, with a small number of observations, more effective methods for modelling multivariate processes are not approximation, which includes regression analysis, but interpolation methods. An example of the effective use of methods based on the geometric theory of multidimensional interpolation [5-7] was presented in [8-10]. Then the
problem arises of creating a mathematical apparatus for multidimensional interpolation, which would use the same initial data (which are experiment planning matrices) as regression analysis to build models of multivariate processes and phenomena.

It should be noted that the advantage of using interpolation methods in comparison with approximation methods lies in a more accurate mathematical description of experimental data, in which a complete functional dependence is established. However, as mentioned above, the most effective application of these methods is to simulate experimental data with a small array of observations. The required number of experiments (observations) to compile the experiment planning matrix is determined by the following relationship:

\[ n = m^k, \]

where \( m \) is the number of levels of factors variation;

\( k \) is the number of factors.

Given the high cost of conducting a large number of experiments, planning matrices are widely used in engineering practice. They include the number of levels of variation within 2-3 with the number of factors in the range of 2-4. We consider some of them as an example.

2. Geometric modelling of the stress-strain state of metal polyhedral bent struts using a ruled response hypersurface

As a first example, we will consider the construction of a geometric model of the stress-strain state (SSS) of metal polyhedral bent struts (PBS) using multidimensional interpolation. The analysis of the design features of metal PBS and the operating conditions of such structures under load, carried out in [11], showed that the main factors affecting the stress-strain state of the struts are the variability of their design parameters, such as:

– rack wall thickness, \( t_w \) (factor \( x_1 \));
– diameter in the butt of the rack, \( d_b \) (factor \( x_2 \));
– load application mark or overlap of the uprights in the telescopic joint, \( h_{Np} \) (factor \( x_3 \)).

The number of levels of variation of factors in [11] was adopted 2. Considering expression (1), 8 experiments were carried out. As a result, the following experiment planning matrix was obtained to analyse the influence of the above 3 factors on the meridional tensile stresses in the support zone of the strut \( \sigma_y \) and displacement of the strut top \( f_t \) (table 1).

| № of test | Factor coded values | Natural values of factors | Response function values |
|-----------|---------------------|--------------------------|------------------------|
|           | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( t_{ws} \), mm | \( d_{as} \), mm | \( h_{Np} \), mm | \( \sigma_y \), MPa | \( f_t \), mm |
| 1         | +       | –       | –       | 6.3 | 345 | 6950 | 217.6 | 183.1 |
| 2         | –       | +       | –       | 5.7 | 355 | 6950 | 231.4 | 189.3 |
| 3         | –       | –       | +       | 5.7 | 345 | 7050 | 246.5 | 211.1 |
| 4         | +       | +       | +       | 6.3 | 355 | 7050 | 214.3 | 178.6 |
| 5         | –       | –       | –       | 5.7 | 345 | 6950 | 240.6 | 202.4 |
| 6         | +       | +       | –       | 6.3 | 355 | 6950 | 209.2 | 171.2 |
| 7         | –       | +       | +       | 6.3 | 345 | 7050 | 222.9 | 191  |
| 8         | –       | –       | +       | 5.7 | 355 | 7050 | 237  | 197.4 |

We present the results of the experiment (table 1) in the form of 8 points belonging to the 4-dimensional space. Since we have 2 response functions, we get 2 sets of 8 points with four coordinates each. We construct a 3-parameter hypersurface through these 8 points, in accordance with the following geometric scheme (Figure 1).
Figure 1. Geometric scheme for modelling a 3-parameter ruled response hypersurface

An analytical description of such a ruled hypersurface [12] can be represented as a sequence of point equations [13-14] of line segments:

\[
\begin{align*}
N_A &= A_1 \bar{u} + A_2 u \\
N_B &= B_1 \bar{u} + B_2 u \\
N_C &= C_1 \bar{u} + C_2 u \\
N_D &= D_1 \bar{u} + D_2 u \\
N_{AD} &= A_1 \bar{v} + N_p \bar{v} \\
N_{CD} &= C_1 \bar{v} + N_p \bar{v} \\
M &= N_{AD} \bar{w} + N_{CD} w
\end{align*}
\]  

(2)

where \( A_1, B_1, C_1, D_1 \) are original points (Fig. 1), whose coordinates correspond to the original data (table 1);

\( u, v, w \) are current parameters that change from 0 to 1;

\( \bar{u} = 1 - u, \bar{v} = 1 - v, \bar{w} = 1 - w \) are addition of current parameters \( u, v, w \) to 1.

After transformations (2), we obtain a unified point equation for the ruled response hypersurface passing through 8 initial points, which is a 3-parameter line geometric interpolant:

\[
M = A_1 \bar{u} \bar{v} \bar{w} + A_2 \bar{u} \bar{v} \bar{w} + B_1 \bar{u} \bar{v} \bar{w} + B_2 \bar{u} \bar{v} \bar{w} + C_1 \bar{u} \bar{v} \bar{w} + C_2 \bar{u} \bar{v} \bar{w} + D_1 \bar{u} \bar{v} \bar{w} + D_2 \bar{u} \bar{v} \bar{w}.
\]  

(3)

Using equation (3), considering the coordinate-wise calculation in natural values of the factors, we obtain 2 ruled response hypersurfaces belonging to the 4-dimensional space, which are described by the following systems of parametric equation:

\[
\begin{align*}
t_{cw} &= ((-2.4v + 1.2)v - 0.6 + 1.2w)u + (-0.6 + 1.2w)v - 0.6w + 6.3 \\
d_v &= 345 + 10u - 349.3uvw \\
h_{wp} &= 100v + 6950 \\
\sigma_v &= ((91.5w - 46)v + 13.8 - 45.2w)u + (28.9 - 46.6w)v + 23w + 217.6 \\
t_{cw} &= ((-2.4v + 1.2)v - 0.6 + 1.2w)u + (-0.6 + 1.2w)v - 0.6w + 6.3 \\
d_v &= 345 + 10u - 349.3uvw \\
h_{wp} &= 100v + 6950 \\
f_v &= ((76.3w - 38.7)v + 6.2 - 37.4w)u + (28 - 39.4w)v + 19.3w + 183.1
\end{align*}
\]  

(4)
It should be noted that, considering the linear relationship between the factors of influence and the parameters of the equations system (4), there is a possibility of passing to the equation in an explicit form. However, the use of parametric equations is preferable, since the initial data differ from each other by several orders of magnitude and the final equation in explicit form will be too cumbersome.

3. Geometric modelling of the strength characteristics of aerated concrete

The initial data for the second example were the studies given in [15]. In accordance with them, the number of factors was equal to 2, and the number of levels of variation of factors was 3. Considering expression (1), the number of necessary experiments of the planning matrix was taken as 9 (table 2).

| No. of test | Electrostatic field strength, $E$, kW/cm | Duration of electrical treatment, $r$, min | Electrostatic field strength, $E$, kW/cm | Duration of electrical treatment, $r$, min | Compressive strength of aerated concrete samples after heat and moisture treatment, $R_{\text{comp}}$, MPa |
|-------------|----------------------------------------|------------------------------------------|----------------------------------------|------------------------------------------|--------------------------------------------------|
| 1           | -1                                     | -1                                       | 1.0                                    | 10                                       | 5.06                                             |
| 2           | 0                                      | -1                                       | 1.5                                    | 10                                       | 5.03                                             |
| 3           | +1                                     | -1                                       | 2.0                                    | 10                                       | 4.39                                             |
| 4           | -1                                     | 0                                        | 1.0                                    | 20                                       | 5.14                                             |
| 5           | 0                                      | 0                                        | 1.5                                    | 20                                       | 6.79                                             |
| 6           | +1                                     | 0                                        | 2.0                                    | 20                                       | 4.24                                             |
| 7           | -1                                     | +1                                       | 1.0                                    | 30                                       | 5.02                                             |
| 8           | 0                                      | +1                                       | 1.5                                    | 30                                       | 4.46                                             |
| 9           | +1                                     | +1                                       | 2.0                                    | 30                                       | 4.26                                             |

Considering the number of levels of factors variation, we will choose a parabolic function for interpolation. As a result, we obtain the following geometric scheme for modelling the response surface (Figure 2), passing through 9 predetermined points in advance, which is a 2-parameter parabolic geometric interpolant.

$$
M_x = A_x \bar{u} (1 - 2u) + 4A_x \bar{u}u + A_x u (2u - 1)
$$

$$
M_y = B_y \bar{u} (1 - 2u) + 4B_y \bar{u}u + B_y u (2u - 1)
$$

$$
M_z = B_z \bar{u} (1 - 2u) + 4B_z \bar{u}u + B_z u (2u - 1)
$$

$$
M = M_x \bar{v} (1 - 2v) + 4M_x \bar{v}v + M_x v (2v - 1)
$$

Figure 2. Geometric scheme for modelling a 2-parameter parabolic response surface

For the analytical determination of the required response surface, we use the point equation of a parabola passing through 3 predetermined points in advance. As a result, we obtain the following computational algorithm for modelling the response surface:
where $A_i, B_i, C_i$ are original points (Figure 2), whose coordinates correspond to the original data (table 2);

$M_A, M_B, M_C$ are guide lines of the response surface, each of which passes through 3 predetermined points in advance;

$M$ is the current point of the generatrix of the response surface;

$u, v$ are current parameters that change from 0 to 1;

$\overline{u} = 1 - u, \overline{v} = 1 - v$ are addition of current parameters $u, v$ to 1.

After performing the coordinate-wise calculation of the sequence of point equations (5), we obtain the system of parametric equations:

\[
\begin{cases}
    E = 1 + u \\
    \tau = 20v + 10 \\
    R_{\text{comp}} = (-1.22 - 30.66v + 32.6v^2)u^2 + (0.55 + 29.83v - 31.86v^2)u + 5.06 + 0.36v - 0.4v^2
\end{cases}
\]  

(6)

Considering the special properties of the curves passing through predetermined points, obtained on the basis of Bernstein polynomials, and the uniform distribution of the values of the factors, it is easy to pass to the equation of the model of strength characteristics of aerated concrete in an explicit form:

\[
R_{\text{comp}} = (22.26 - 3.16\tau + 0.08\tau^2)E^2 + (-66.85 + 9.41\tau - 0.24\tau^2)E + 49.37 - 6.21\tau + 0.16\tau^2.
\]  

(7)

A graphic visualization of the resulting process model is shown in the Figure 3.

Figure 3. Graphic visualization of the geometric model of the strength characteristics of aerated concrete

Figure 3 shows that the response function has a pronounced maximum. Based on this, we will perform the optimization of the process by methods of mathematical analysis of the function of two variables based on the obtained model (7), using the ultimate strength in compression of aerated concrete samples after heat and moisture treatment as an objective function. As a result, the maximum value of the ultimate compressive strength $R_{\text{comp}} = 6.82$ MPa is achieved at an electrostatic field strength $E = 1.45$ kW/cm and the duration of electrical treatment $\tau = 19.31$ min.

4. Conclusion
The article posed and successfully solved the problem of developing a mathematical apparatus for multidimensional interpolation, which uses the same experiment planning matrices as regression analysis. This approach allows not only to obtain a more accurate mathematical description of the previously obtained experimental data, but also to preserve the general approach to planning
experimental data when constructing new models of multifactor processes and phenomena. The above examples demonstrate the effectiveness of the proposed approach based on multivariate interpolation for modelling multivariate processes based on existing experiment planning matrices with a small amount of experimental data.

Another advantage of the proposed approach is obtaining ready-made point equations of the geometric model of the process. It is enough just to substitute the initial data in accordance with the initial matrix for planning the experiment to use these equations in the practice of modelling multifactorial processes.

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