Design of Fractional-order Sobel Filters for Edge Detections

Ibtisam Edress¹, Emad A. Al-Sabawi¹ and Majid Dherar Younus¹

¹Computer and Information Engineering Department, College of Electronics Engineering, Ninevah University

Email: ibtisamedrees3@gmail.com

Abstract. Most image processing applications use edge detection to extract information as a preliminary step to object segmentation and feature extraction. Edge accuracy is one of the edge detector challenges; The current work presents the design of the fractional-order Sobel filters based on Yi_Fei-1 to Yi_Fei-5. A comparison among the proposed filters has been implemented for edge detection based on a supervised assessment, using mean square error, misclassification error, and symmetric distance. Many images with their ground truths have been used in the evaluation. Results showed that the fractional-order of Sobel-based Yi_Fei-2 has the best edge map among the proposed filters.

1. Introduction

Image edge detection plays an important role as one of the precursory standards in pattern recognition applications and computer vision [1]. Edge detection considers one of the vital research works and a very key step towards identification, segmentation, and object recognition which can occur when edges of an object are detected [2].

An edge is well-known as a discontinuity; that may be represented as texture, color, or gray level of the image. Detection of edges tends to extract vital features about the objects of the image, like illumination, reflectivity, and geometry [3]. For example, the pixel's gray-level which value is identical to the other around pixel’s gray-level; there is maybe not an edge at that position. Actually, if neighbors of a pixel have widely varying gray levels, the pixel possibly an edge pixel [4].

Many operators that have been used to extract edges from the images, the classical approaches are grouped as gradient-based techniques and Gaussian-based techniques, which adopted the first and second derivatives to detect the edges in an image. But, thicker edges usually result from the first-order derivative techniques which cause a lack of image details. The second-order derivative techniques have a strong response to fine specifics, but high noise sensitivity [5].

To reduce this disadvantage, fractional calculus has been involved in edge detection techniques. Due to the frequency characteristic of the fractional-order derivative, it can nonlinearly preserve more low-frequency outline characteristics in flat areas; save the aloft frequency marginal function wherever the grayscale varies frequently, and also improve mid-frequency texture specifics [6]. Caputo, Riemann-Liouville (R-L), and Grünwald-Letnikov (G-L) are the most common definitions of fractional-order derivative [7][8]. Considering these benefits, Yi. Pu et al. [9] proposed six fractional differential masks and presented the structures and parameters of all masks individually in the eight directions.

Experiments showed that, for a digital image which rich in detail, the capability of fractional differential-based approaches seemed obvious better than classical methodologies. Dan, J. Wu and Y. Yang [8] proposed a novel fractional-order gradient detector to extract features from the structure of a medical image. They were generalizing Sobel operators depending on the GL definition of non-integer order derivative. Experiments appeared that the modified Sobel operator yields well observable effects.

In this work, we generalize the classical Sobel [10], using the five fractional-order operators from the six in [9]. The proposed operators are evaluated based on objective metrics to get the minimum difference between the ground truth images and the candidate.
2. Edge Detection using Fractional Derivative of Sobel

Let us consider \( I(x, y) \) as a digital image, whereas \((x, y)\) denote spatial coordinates. The magnitude of the image gradient is well-defined as:

\[
G = \sqrt{(Gx^2 + Gy^2)}
\]

(1)

Where \(Gy\) and \(Gx\) are components of image gradient in \(y\) and \(x\) directions respectively.

\[
G_y = \frac{\partial I(x,y)}{\partial y}
\]

(2)

\[
G_x = \frac{\partial I(x,y)}{\partial x}
\]

(3)

The image \(I(x, y)\) is filtering using Sobel kernels \(Kx\)and \(Ky\). \(Gx\) and \(Gy\) are obtained as shown in equation (4).

\[
G_x = I \cdot K_x , \quad G_y = I \cdot K_y
\]

(4)

\[
k_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 1 & 0 \end{bmatrix} \quad k_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}
\]

\[
g_x(x,y) = I(x + 1, y + 1) + 2I(x + 1, y) + (x + 1, y - 1) - I(x - 1, y + 1) - 2I(x - 1, y)
- I(x - 1, y - 1)
\]

(5)

\[
g_y(x,y) = I(x + 1, y + 1) + 2I(x, y + 1) + I(x - 1, y + 1) - I(x + 1, y - 1) - I(x - 1, y - 1)
\]

(6)

To get the fractional derivative of the Sobel operator, the five Yi_Fei operators illustrated in table 1[9] are used to generalize the classical one. The first operator was used in paper [11]. The fractional Sobel mask using operator 2 is illustrated in equations (8) and (9).

\[
D^p G_x = G_x^{1+p}
\]

(7)

\[
g_x^{1+p}(x,y) = \{ a_{+1}[I(x + 2, y + 1) + 2I(x + 2, y) + I(x + 2, y - 1) - I(x, y + 1) - 2I(x,y)
- I(x,y - 1)]
+ a_{0}[I(x + 1, y + 1) + 2I(x + 1, y) + I(x + 1, y - 1) - I(x - 1, y + 1)
- 2I(x - 1, y) - I(x - 1, y - 1)]
+ a_{-1}[I(x, y + 1) + 2I(x, y) + I(x, y - 1) - I(x - 2, y + 1) - 2I(x - 2, y)
- I(x - 2, y - 1)]
+ a_{-2}[I(x - 1, y + 1) + 2I(x - 1, y) + I(x - 1, y - 1) - I(x - 3, y + 1)
- 2I(x - 3, y) - I(x - 3, y - 1)]
+ a_{-3}[I(x - 2, y + 1) + 2I(x - 2, y) + I(x - 2, y - 1) - I(x - 4, y + 1)
- 2I(x - 4, y) - I(x - 4, y - 1)]\}
\]

(8)

\[
G_x^{1+p} = I \cdot M_{2x}, \quad G_y^{1+p} = I \cdot M_{2y}
\]

(9)

The mask using Yi_Fei-2 is given below:
\[
M_{2x} = \begin{bmatrix}
\left(\frac{p^2}{8} - \frac{3p^3 + p^4}{16}\right) & \left(\frac{p^2}{4} - \frac{3p^3}{8} + \frac{p^4}{8}\right) & \left(\frac{p^2}{8} - \frac{3p^3}{16} + \frac{p^4}{16}\right) \\
\left(\frac{5p}{4} + \frac{p^2}{8} + \frac{p^3}{4}\right) & \left(\frac{5p}{2} + \frac{p^2}{4} + \frac{p^3}{2}\right) & \left(\frac{5p}{4} + \frac{p^2}{8} + \frac{p^3}{4}\right) \\
\left(\frac{5p}{4} + \frac{p^2}{2} - \frac{p^3}{2}\right) & \left(\frac{5p}{2} + \frac{p^2}{4} - \frac{p^3}{2}\right) & \left(\frac{5p}{4} + \frac{p^2}{8} - \frac{p^3}{2}\right)
\end{bmatrix}
\]

\[M_{2y} = [M_{2x}]^T\]

Where \(\alpha = \Gamma(-p) \ast (-2 \ast p)\)

\[\text{Table 1. The five operators of Yi_Fei [9].} \]

| Yi_FeiPU-1 | Yi_FeiPU-2 | Yi_FeiPU-3, p < 0 | Yi_FeiPU-4, p < 0 | Yi_FeiPU-5, 0 ≤ p < 1 |
|-------------|------------|------------------|------------------|----------------------|
| \(c_{-1}\)  | 0          | \(\frac{p^2}{4} - \frac{p^3}{8}\) | 0                | 0                    |
| \(c_0\)     | 1          | \(1 - \frac{p^2}{2} - \frac{p^3}{8}\) | \(\frac{1}{\Gamma(-p)(-2p)}\) | \(\frac{1}{\Gamma(-p)(p^2 - p)}\) | \(\frac{1}{\Gamma(2 - p)}\) |
| \(c_1\)     | \(-p\)     | \(-\frac{5p}{4} + \frac{p^2}{16} + \frac{p^4}{16}\) | \(\frac{2^p}{\Gamma(-p)(-2p)}\) | \(\frac{2^{1-p}}{\Gamma(-p)(p^2 - p)}\) | \(\frac{2^{1-p}}{\Gamma(2 - p)}\) |
| \(\vdots\)  | \(\vdots\) | \(\vdots\) | \(\vdots\) | \(\vdots\) | \(\vdots\) |
| \(c_t\)     | \(\frac{\Gamma(i - p)}{(i - 1)!\Gamma(-p)}\) | \(\frac{\Gamma(i - p - 1)}{(i - 1)!\Gamma(-p)}\) | \(\frac{(i - 1)^{2p} - p^2}{\Gamma(-2p)}\) | \(\frac{(i - 1)^{p} - (1 - i)^{p} - i^{p} - (1 - i)^{1-p}}{\Gamma(p^2 - p)}\) | \(\frac{(1 - v)^{p} - (1 - i)^{p} - (1 - i)^{1-p}}{\Gamma(2 - p)}\) |

The other masks using Yi_Fei-3 to five operators are as follow:

\[
M_{3x} = \begin{bmatrix}
\left(\frac{-2^{2p} + 3p}{\alpha}\right) & \left(\frac{-2^{1-2p} + 2 \ast 3p}{\alpha}\right) & \left(\frac{-2^{2p} + 3p}{\alpha}\right) \\
\left(\frac{1 - 2^{3p} + 2^{2p}}{\alpha}\right) & \left(\frac{2 - 2^{2} \ast 3p + 2^{1-2p}}{\alpha}\right) & \left(\frac{1 - 2^{3p} + 2^{2p}}{\alpha}\right) \\
\left(\frac{3^p - 2^p - 1}{\alpha}\right) & \left(\frac{2^2 \ast 3^p - 2^{1-2p}}{\alpha}\right) & \left(\frac{3^p - 2^p - 1}{\alpha}\right) \\
\left(\frac{-1 + 3^p - 2^p}{\alpha}\right) & \left(\frac{-2 + 2 \ast 3^p - 2^{1-2p}}{\alpha}\right) & \left(\frac{3^p - 2^p - 1}{\alpha}\right) \\
\left(\frac{2^p}{\alpha}\right) & \left(\frac{2^{1-p}}{\alpha}\right) & \left(\frac{2^p}{\alpha}\right) \\
\left(\frac{1}{\alpha}\right) & \left(\frac{2}{\alpha}\right) & \left(\frac{1}{\alpha}\right)
\end{bmatrix}
\]
Where \( \alpha = \Gamma(-p) * (p^2 - p) \), \( M_{3y} = [M_{3x}]^T \)

\[
M4_x = \begin{bmatrix}
\frac{\alpha (1 - p) * 2^{-2p} - 2^{-2p} + 3^1 p}{2^x p - 2^{-2p} + (1 - p) * 2^{-2p} - 1} & \frac{\alpha (1 - p) * 2^{-1 - 2p} - 2^{-3 - 2p} + 2 * 3^1 p}{2^x p - 2^{-2p} + (1 - p) * 2^{-2p} - 1} & \frac{\alpha (1 - p) * 2^{-2p} - 2^{-2p} + 3^1 p}{2^x p - 2^{-2p} + (1 - p) * 2^{-2p} - 1}
\end{bmatrix}
\]

Where \( \alpha = 1/\Gamma(2 - p) \), \( M_{4y} = [M_{4x}]^T \)

\[
M5_x = \begin{bmatrix}
\frac{\alpha (2^{-2p} - (1 - p) * 2^{-2p} - 3^1 p)}{2^x p - 2^{-2p} + (1 - p) * 2^{-2p} - 1} & \frac{\alpha (2^{-3p} - (1 - p) * 2^{-2p} - 2 * 3^1 p)}{2^x p - 2^{-2p} + (1 - p) * 2^{-2p} - 1} & \frac{\alpha (2^{-2p} - (1 - p) * 2^{-2p} - 3^1 p)}{2^x p - 2^{-2p} + (1 - p) * 2^{-2p} - 1}
\end{bmatrix}
\]

Where \( \alpha = \Gamma(3 - p) \), \( M_{5y} = [M_{5x}]^T \)

The threshold is based on the mean of gradient magnitude in equation (10).

\[
Threshold = Scale \cdot \bar{G} \quad (10)
\]

Where \( \text{Scale} \) is the average of \(|G|\). The pixel is considered as an edge

\[
\text{if}(Dr = |G| > Threshold).
\]

3. Numerical Result

Test images and their ground truth have been used in the evaluation process. Ground truth image is an edge map acquired from human judgment or synthetic data. Test images have been filtered using the proposed filters with orders from 0 to 1 in steps of 0.01. The mean square error, misclassification error, and symmetric distance are illustrated in equations (11),(12), and (13) respectively [12][13][14].

\[
MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (Gt_{i,j} - Dr_{i,j})^2 \quad (12)
\]

\[
ME = 1 - \frac{TN + TP}{TP + TN + FN + FP} \quad (13)
\]
Where
False Negative (FN), missing edge points of Dr: \( FN = |Gt \cap \neg Dr| \),
False Positive (FP), fake detected edges of Dr: \( FP = |\neg Gt \cap Dr| \),
True Negative (TN), mutual non-edge points: \( TN = |\neg Gt \cap \neg Dr| \),
True Positive (TP), mutual points of \( Gt \) and \( Dr \): \( TP = |Gt \cap Dr| \).

\[
SDM (Gt, Dr) = \frac{\sum_{p_x \in Dr} d_G^l(p_x) + \sum_{p_x \in Gt} d_D^l(p_x)}{|Gt \cup Dr|} \tag{14}
\]
Where \( l \in R^+ \), for a pixel \( p_x \) belonging to the detector result \( Dr \), \( D_G(p_x) \) is the minimal Euclidean distance between \( p_x \) and \( Gt \). If \( p_x \) belongs to the ground truth \( Gt \), \( D_D(p_x) \) is the minimal distance between \( p_x \) and \( Dr \).

Figure 1, Figure 2, and Figure 3 show the mean square error, misclassification error, and symmetric distance respectively for the classical Sobel (\( p=0 \)) and the five fractional-order Sobel filters. As shown in the figures, the fractional-order Sobel filter based Yi_Fei-2 has the minimum scores and the behaviour of the three figures approximately the same.

![Figure 1. Mean square error for the five fractional-order Sobel filters.](image1)

![Figure 2. Misclassification error for the five fractional-order Sobel filters.](image2)
Figure 3. Symmetric distance for the five fractional-order Sobel filters.

Figure 4. Edge detection using fractional Sobel filter based Yi_Fei-2 ($p=0.25$).
4. Conclusion
The fractional-order Sobel filters based on Yi_Fei-2, Yi_Fei-3, Yi_Fei-4, and Yi_Fei-5 operators have been designed. According to assessment which based on mean square error, misclassification error, and symmetric distance, the traditional Sobel filter (p=0) has better performance than the fractional-order Sobel filter based on Yi_Fei-4, and Yi_Fei-5. Whereas fractional order Sobel filter based Yi_Fei-2 has better performance than classical Sobel, fractional-order Sobel filter based Yi_Fei-1, and Yi_Fei-3 when the order of differentiaion is greater than 0 and less than 0.47.

5. References
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