Spin Fluctuation Induced Dephasing in a Mesoscopic Ring

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We investigate the persistent current in a hybrid Aharonov-Bohm ring - quantum dot system coupled to a reservoir which provides spin fluctuations. It is shown that the spin exchange interaction between the quantum dot and the reservoir induces dephasing in the absence of direct charge transfer. We demonstrate an anomalous nature of this spin-fluctuation induced dephasing which tends to enhance the persistent current. We explain our result in terms of the separation of the spin from the charge degree of freedom. The nature of the spin fluctuation induced dephasing is analyzed in detail.

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Persistent current (PC) in a mesoscopic Aharonov-Bohm (AB) ring is an ideal probe of quantum coherence of electron motion in the equilibrium state. Usually the PC is likely to be suppressed by various dephasing processes. The role of the intrinsic dephasing at low temperature has not been well understood till now\cite{5}. An alternative viewpoint to this is to introduce an artificial dephasor, in order to study the effect of decoherence in a controlled manner\cite{6}. A conceptually simple but instructive example for that purpose is an AB ring attached to an electron reservoir which exchanges charges with the ring\cite{3}. In the reservoir electrons are scattered inelastically and there is no phase coherence between electrons absorbed and those emitted by the reservoir. Therefore charge transfer between the ring and the reservoir diminishes the coherence and thus the AB oscillation. On the other hand, the effect of the spin exchange interactions on the PC has been attracting growing interest in recent years\cite{7,8,9}. It has been proposed that the spin fluctuation affects the PC in a drastically different manner compared to the case of charge fluctuation\cite{10,11}. Experimentally, the role of coherent spin fluctuation has been investigated by transport measurements using AB interferometer setup\cite{12,13,14}.

In this Letter, we address the effect of dephasing induced by the spin fluctuations. For this purpose we consider the geometry schematically drawn in Figure 1, where the spin fluctuation between the ring and the reservoir is mediated via antiferromagnetic exchange interactions with the quantum dot (QD), while direct charge transfer is prohibited by the Coulomb blockade. We find a counterintuitive result that the dephasing tends to enhance the PC rather than to reduce it in this geometry. We argue that this enhancement can be regarded as a signature of the separation of the spin degree of freedom from the charge one.

Our model is described by the Hamiltonian:

\begin{equation}
H = H_0 + H_R + T ,
\end{equation}

where \(H_0, H_R\) and \(T\) stand for the hybrid dot-ring system, reservoir, and tunneling between the QD and the reservoir, respectively. \(H_0\) is decomposed into the three parts as

\begin{equation}
H_0 = H_{QD} + H_{ring} + H_V ,
\end{equation}

where \(H_{QD}, H_{ring},\) and \(H_V\) correspond to the Hamiltonians describing the quantum dot, the AB ring, and the dot-ring hybridization, respectively:

\begin{align}
H_{QD} &= \sum_{\sigma} \varepsilon_d d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow , \quad (3a) \\
H_{ring} &= -t \sum_{j=1}^{N} \sum_\sigma \left( e^{i\varphi/N} c_{j\sigma}^\dagger c_{j+1\sigma} + \text{h.c.} \right) , \quad (3b) \\
H_V &= -t' \sum_\sigma \left( d_\sigma^\dagger c_{0\sigma} + c_{0\sigma}^\dagger d_\sigma \right) . \quad (3c)
\end{align}

Here, we describe the ring by using a tight-binding Hamiltonian with \(N\) lattice sites, and the QD by a single Anderson impurity. The single particle energy and the on-site Coulomb repulsion in the QD are represented by \(\varepsilon_d\) and \(U\), respectively. The phase \(\varphi\) in Eq. (3b) comes from the AB flux, and is defined by \(\varphi = 2\pi \Phi / \Phi_0\), where \(\Phi\) and \(\Phi_0\) are the external flux and the flux quantum \((= hc/e)\), respectively. Note that Eq. (3b) can be diagonalized and the corresponding eigenvalues are given by \(\varepsilon_m = -2t \cos \frac{\varphi}{N}(2\pi m - \varphi)\) \((m\) being an integer number). The reservoir is modeled by a Fermi sea of electrons with single particle energies \(\{E_k\}\) :

\begin{equation}
H_R = \sum_{k\sigma} E_k a_{k\sigma}^\dagger a_{k\sigma} ,
\end{equation}

Finally, coupling between the QD and the reservoir is given by a tunneling Hamiltonian

\begin{equation}
T = \sum_{k\sigma} \tau_k \left( d_{k\sigma}^\dagger c_{0\sigma} + \text{h.c.} \right) .
\end{equation}

The hopping strength of reservoir-dot and that of ring-dot are represented by \(\Gamma_R\) and \(\Gamma'\), respectively. \(\Gamma_R\), assumed to be constant at the energy interval of \(-D < \varepsilon < D\), is defined as

\begin{align}
\Gamma_R &= \sum_{k} \tau_k \left( d_{k\sigma}^\dagger c_{0\sigma} + \text{h.c.} \right) , \\
\Gamma' &= \sum_{k} \tau_k \left( d_{k\sigma}^\dagger c_{0\sigma} + \text{h.c.} \right) .
\end{align}
\[ \Gamma_R = \pi \rho_R(\varepsilon)|\tau(\varepsilon)|^2, \]  
\[ \Gamma' = \frac{t'^2}{2t}, \]  
and the level discreteness at the Fermi energy is given by
\[ \delta = \frac{2\pi}{N}. \]

Our study is restricted to the simplest half-filled case, which does not affect the result and conclusion we will draw.

To rule out the effect of charge fluctuation, we consider the parameter limit of \(-\varepsilon_d, \varepsilon_s + U > \Gamma' + \Gamma_R\). In the absence of the reservoir, the Bethe ansatz result [8] shows that the PC is not affected by the QD in the Kondo limit with \(\delta/T_K \rightarrow 0\), where \(T_K\) stands for the Kondo energy scale. For finite value of \(\delta/T_K\), the coupling of the ring to the QD linearly reduces the PC as a function of \(\delta/T_K\) for small \(\delta/T_K\), and induces a crossover from the continuum Kondo limit (\(\delta/T_K \ll 1\)) to an effectively decoupled ring-dot system (\(\delta/T_K \gg 1\)) [8].

To calculate the PC in the presence of the reservoir, we adopt the leading order \(1/\gamma\)-expansion with \(\gamma\) being the magnetic degeneracy. This approach was shown to describe well the essence of the Kondo correlation preserving the Fermi liquid properties [13]. In addition, for the ring-dot system without reservoir, this approximation reproduces the exact Bethe ansatz result for \(\delta/T_K \rightarrow 0\) [8]. Here we consider the infinite-\(U\) limit since the consideration of finite \(U\) and double occupancy in the QD does not provide any modification to the renormalized physical quantities [14]. Then the problem is reduced to solving the self-consistent equation:
\[ E_G' = \frac{\Gamma'}{\pi} \delta \sum_{m\sigma} \frac{f(\varepsilon_m)}{E_G' + \varepsilon_m - \varepsilon_d} + \sum_{k\sigma} \frac{f(E_k)|\pi_k|^2}{E_G' + E_k - \varepsilon_d}, \]  
where \(f(\varepsilon)\) is the Fermi distribution function. \(E_G' \equiv E_G - E_G\), where \(E_G\) is the energy of the ground state without tunneling. \(E_G\) corresponds to the ground state energy of the coupled system at zero temperature.

At zero temperature the 2nd term of Eq. (9) can be calculated analytically and the latter is rewritten as
\[ E_G' = 2 \frac{\Gamma'}{\pi} \delta \sum_{\varepsilon_m < 0} \frac{1}{E_G' + \varepsilon_m - \varepsilon_d} + \frac{2\Gamma_R}{\pi} \log \frac{\varepsilon_d - E_G'}{D}. \]

The PC is defined in terms of the phase sensitivity of the ground state energy as
\[ I(\varphi) = -\frac{e}{\hbar} \frac{\partial E_G}{\partial \varphi}. \]  
Combining Eq. (11) with Eq. (10), the PC can be expressed as the sum of two terms, \(I_{\text{ring}}\) and \(I_{\text{int}}\), originating from the ideal ring and from the interactions, respectively:
\[ I(\varphi) = I_{\text{ring}}(\varphi) + I_{\text{int}}(\varphi), \]  
\[ I_{\text{ring}}(\varphi) = 2 \sum_{\varepsilon_m < 0} I_m(\varphi), \]  
\[ I_{\text{int}}(\varphi) = -2 \frac{\Gamma'}{\pi} \delta \sum_{\varepsilon_m < 0} \frac{1}{E_G' + \varepsilon_m - \varepsilon_d} I_m(\varphi), \]
where
\[ I_m(\varphi) = -\frac{e}{\hbar} \frac{\partial \varepsilon_m}{\partial \varphi}. \]

is the contribution to the PC from the bare ring energy level \(\varepsilon_m\), and
\[ Z = \left( 1 + \frac{2\Gamma'}{\pi} \delta \sum_{\varepsilon_m < 0} \left( \frac{1}{E_G' + \varepsilon_m - \varepsilon_d} \right)^2 + \frac{2\Gamma_R}{\pi} \frac{1}{\varepsilon_d - E_G'} \right)^{-1} \]
corresponds to the renormalization constant for the ground state. Note the negative sign at the R.H.S. of Eq. (12d) which leads to the reduction of the PC for finite value of \(\delta/T_K\).

Figure 2 displays the effect of the coupling to the reservoir on the PC. The PC is plotted as a function of the dimensionless coupling strength \(\gamma\) defined by
\[ \gamma = \Gamma_R/\Gamma'. \]  
Note that the PC is a universal function of \(\gamma\), \(\varphi\), and \(\delta/T_K\), which does not depend on the parameter detail chosen here. \(T_K\) in the figure denotes the Kondo energy scale in the absence of the coupling to the reservoir (\(\Gamma_R = 0\)) for \(\delta \rightarrow 0\), (while \(T_K\) stands for the counterpart including the reservoir). First, one should recall that for \(\Gamma_R = 0\) the PC is reduced to \(I_{\text{ring}}\) in the continuum limit (\(\delta \gg T_K^0\)), and it is suppressed for finite value of \(\delta/T_K\) [8]. It will be natural to believe that the reservoir would reduce the AB oscillation since it is expected to play the same role as a charge reservoir which induces decoherence. However, the result is opposite to this simple expectation. Coupling to the reservoir enhances the PC as shown for several values of \(\delta/T_K\). As \(\gamma\) increases, the PC is enhanced and eventually it saturates to \(I_{\text{ring}}\) for sufficiently large \(\gamma\).

This anomalous result is interpreted as follows. Instead of reducing the PC, coupling to reservoir suppresses \(I_{\text{int}}\) only, the contribution originating from the spin exchange interactions. This makes net increase of the PC because
the direction $I_{int}$ is opposite to $I_{ring}$. This is a unique signature that the spin and the charge degrees of freedom are decoupled. That is, the reservoir degrades the coherence of spin degree of freedom ($I_{int}$) only, while it does not affect the charge one ($I_{ring}$).

To be more precise, the influence of reservoir on the system can be classified into two factors. (i) Increase of the Kondo binding energy: As the coupling turns on, the effective spin exchange interactions between the electrons in the QD and the conduction electrons increase. This results in the enhancement of the binding energy (or reducing the size of the Kondo screening cloud). (ii) Decoherence of electrons: The Kondo scattering provides effective charge flow between the ring and the reservoir. Since the electrons in the reservoir are scattered inelastically, no phase coherence exists between the electrons absorbed and those emitted by the reservoir. This feature has never been addressed before in the Kondo limit.

Both effects ((i),(ii)) are present in the result of Figure 2. The effect of enhanced Kondo binding energy is not a unique feature of our geometry. That is, the energy scale, as shown in Figure 3(a). The effect (i) is already included in the renormalized parameter $\delta/T_K$ with $\gamma$-dependent $T_K$. The PC displays a crossover at $\delta/T_K \sim 1$ from the Kondo limit ($\delta/T_K \ll 1$) to the effectively decoupled limit ($\delta/T_K \gg 1$), regardless of the coupling to the reservoir. For large $\delta/T_K$ the PC saturates to the value which corresponds to that of the ideal ring with one electron subtracted from the present system (denoted by the dotted line). This demonstrates effective decoupling of the ring from the rest part of the system.

Here we point out that the PC increases as $\gamma$ increases even after subtracting the effect of rescaled Kondo energy, as shown in Figure 3(a). This demonstrates the anomalous nature of the Kondo-assisted dephasing that enhances AB oscillation. That is, the dephasing influences only the interaction part of the current, $I_{int}$, through the spin-fluctuation channel, which results in a net increase of the PC. This property is analyzed in more detail in Figure 3(b). For small $\delta/T_K$, $I_{int}/I_{ring}$ shows $\gamma$-dependent linear behavior as $I_{int}/I_{ring} = -c(\gamma)\delta/T_K$, where $c(\gamma) > 0$. One can see that the slope $c(\gamma)$ decreases as $\gamma$ increases. The reduction of the slope as a function of $\gamma$ is the result of dephasing through spin exchange interactions.

To complete our discussion it is instructive define the coherence factor, $\eta$, associated with the spin-fluctuation-induced dephasing by the ratio

$$\eta = \frac{c(\gamma)}{c(0)}.$$  \hspace{1cm} (14)

The coherence factor (Figure 4) decreases monotonically as $\gamma$ increases. This is a manifestation of decoherence mediated by the spin fluctuations. This behavior is quite universal independent of any parameter detail.

In conclusion, we have investigated the effect of reservoir coupled to a composite AB ring - QD system on the PC. In the Coulomb blockade limit, spin fluctuations induce decoherence of the system in an anomalous way. The persistent current circulating the ring is enhanced due to the dephasing in the Kondo limit. We have argued that this enhancement is closely related to the separation of the spin from the charge degree of freedom.

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FIG. 1. Schematic diagram of the hybrid quantum dot - AB ring structure coupled to a reservoir.

FIG. 2. Persistent current as a function of the renormalized coupling strength of the reservoir to the QD ($\gamma$) for $\Gamma' = 0.125t$, $\varepsilon_d = -0.7t$, and $\varphi = 0.1\pi$ with several values of $\delta/T_K$.

FIG. 3. (a) Persistent current as a function of the renormalized level spacing ($\delta/T_K$) for three different values of $\gamma$. Other parameters are given the same as those in Figure 2. The dotted line indicates the PC of the ideal ring with one electron subtracted. (For $\varphi = 0.1\pi$, it corresponds to $\frac{1}{4}I_{\text{ring}}$.) (b) Interaction contribution to the persistent current for the same parameters.

FIG. 4. Coherence factor (defined in Eq. (14)) as a function of $\gamma$. 

\[ I/I_{\text{ring}} \]

\[ \text{Coherence factor}(\gamma) \]

\[ \frac{\gamma}{0.5} \]