SNAP Near Infrared Detectors

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ABSTRACT

The SuperNova/Acceleration Probe (SNAP) will measure precisely the cosmological expansion history over both the acceleration and deceleration epochs and thereby constrain the nature of the dark energy that dominates our universe today. The SNAP focal plane contains equal areas of optical CCDs and NIR sensors and an integral field spectrograph. Having over 150 million pixels and a field-of-view of 0.34 square degrees, the SNAP NIR system will be the largest yet constructed. With sensitivity in the range 0.9–1.7 \( \mu \text{m} \), it will detect Type Ia supernovae between \( z = 1 \) and 1.7 and will provide follow-up precision photometry for all supernovae. HgCdTe technology, with a cut-off tuned to 1.7 \( \mu \text{m} \), will permit passive cooling at 140 K while maintaining noise below zodiacal levels. By dithering to remove the effects of intrapixel variations and by careful attention to other instrumental effects, we expect to control relative photometric accuracy below a few hundredths of a magnitude. Because SNAP continuously revisits the same fields we will be able to achieve outstanding statistical precision on the

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photometry of reference stars in these fields, allowing precise monitoring of our detectors. The capabilities of
the NIR system for broadening the science reach of SNAP are discussed.

**Keywords:** Cosmology, Supernovae, Dark Energy, Near Infrared, Photometry, HgCdTe

1. **INTRODUCTION**

Near infrared observations from space are essential to identify the nature of the dark energy responsible for the
acceleration of the universe that we see today. For redshifts of order one or larger the universe was dominated
by matter and underwent deceleration, while for redshifts of order one or less the universe has been dominated
by dark energy resulting in acceleration. Whereas measurements at low redshift provide the best determination
of the dark energy content of our universe today, measurements at high redshift provide the best opportunity
to distinguish among dark energy models and to control systematic errors. The SuperNova/Acceleration Probe
(SNAP)\(^1\) will obtain precision calibrated light curves and spectra for over 2500 Type Ia supernovae at redshifts
from 0.1 to 1.7 to determine the nature of the dark energy.

For supernovae at redshifts above one, the rest-frame visible emission at 500 nm is shifted beyond the long-
wavelength cutoff of optical CCD detectors and into the near infrared. Supernova (SN) photometry must be
performed at a common rest-frame (blue) wavelength if the supernovae are to be used as standard candles; hence,
NIR imaging at wavelengths of 1.0–1.7 \(\mu\)m is a critical aspect of the mission. Indeed the NIR system on SNAP
provides all the restframe optical photometry above \(z = 1.2\). NIR photometry is also essential to the diagnosis
and correction of potential systematic errors caused by gray dust, SN evolution and SN magnification. As an
integral part of the SNAP observation program, NIR observations will allow photometric redshift determination
for all SNAP galaxies, providing a trigger for rejecting high-\(z\) Type II supernovae. The unique wide-field SNAP
NIR system also permits a significant expansion of the science reach of SNAP beyond the core mission.

Extensive mission simulations have shown that a wide-field NIR camera integrated with optical CCDs into a
single focal plane provides the optimal configuration to extract the most cosmological information. In particular,
the measurement of broadband colors for every supernova provides the necessary leverage to determine both
the equation of state \(w = p/\rho\) of the universe and its time variation with sufficient precision to constrain dark
energy models. As presently conceived, the NIR detectors for SNAP constitute fully \(\frac{1}{2}\) of the focal plane with
every supernova observed in every color from 350 to 1700 nm.

2. **IDENTIFYING THE NATURE OF DARK ENERGY**

SNAP will use measurements of Type Ia supernovae as a probe of the nature of dark energy. The essential SNe
measurement for this purpose is a precise comparison of luminosity distance to redshift from \(z = 0\) to \(z = 1.7\).

It has been shown that Type Ia supernovae have very uniform peak \(B\)-band brightness, but only when
they are corrected for a stretch factor which describes the relation between absolute brightness and explosion
duration.\(^2\) To fully standardize the peak brightness of a supernova, a variety of additional observations must be
made. The color throughout the light curve provides constraints on both host galaxy and Milky Way extinction.
The nature of the host galaxy and the location of the supernova within it may provide important additional
information. Finally, a spectrum of the supernova obtained near maximum light will allow identification of the
explosion as a Type Ia as well as of secondary supernova characteristics.

To construct the desired SNe Hubble diagram, restframe \(B\) and \(V\) lightcurves, restframe \(B\) and \(V\) galaxy
images, and an accurate host redshift are required. At \(z = 1.7\), restframe \(B\) and \(V\) have shifted beyond
the sensitivity of CCDs, well into the NIR. Observations in the NIR thus are crucial to the core project of
SNAP. Without them SNAP cannot directly compare restframe peak brightnesses of SNe across a wide range
of redshifts.

At low redshift two independent measurements\(^2,3\) have determined that the fraction of the critical density
consisting of dark energy, \(\Omega_A\), is nonzero and possibly of order unity. Measurement of small scale fluctuations
in the cosmic microwave background radiation subsequently have determined that the universe is nearly flat.\(^4\)
This implies then that \(\Omega_A \sim \frac{1}{4}\) and \(\Omega_M \sim \frac{1}{4}\). Observations of galaxy clustering\(^5\) have shown that the fraction
of the critical density consisting of matter is $\Omega_M = 0.3$, consistent with the results obtained from the supernova measurements. SNAP will obtain a magnitude-redshift (Hubble) relation containing over 2000 Type Ia supernovae, extending to $z = 1.7$ (see Fig. 1). Cosmological parameters can be extracted from this Hubble diagram, including a precise value for the dark energy equation of state (predicted to be $w = -1$ for a cosmological constant) and constraints on its generically expected time variation $w' \equiv dw/dz$. Figure 2 shows the shift in magnitude relative to a universe with $\Omega_\Lambda = 0.7$ and $w = -0.7$ for flat universes with slightly different values of $w, w'$, and $\Omega_M$ varied one at a time. As can be seen, the greatest sensitivity to cosmological parameters is obtained with high redshift measurements in the NIR.

NIR observations will also allow us to control various systematic errors. While restframe $B$ and $V$ photometry provide some constraint on the impact of ordinary dust on SNe measurements, they may not provide sufficient information to constrain “gray” dust. Truly gray dust is undetectable by color measurement, but physically motivated models of dust are actually only “grayer” dust; they still affect light of different wavelengths differently. The best opportunity to detect such gray dust is to observe each supernova over the broadest possible wavelength range. NIR observations extend the wavelength range over which each supernova will be imaged by more than a factor of two, providing a better chance to constrain a wider variety of dust models.

Supernova evolution is an important systematic that is controlled primarily through spectroscopic observations. Nearby supernovae span the entire range of metallicity and other SN parameters that will be observed for more distant supernovae. It is unlikely that population changes will go unnoticed by SNAP. However, Type Ia SNe simulations have shown that restframe $V - R$ photometry may be less sensitive to changes in metallicity than $B - V$ and thus NIR measurements provide an important cross check on this systematic. Restframe R measurements are possible up to $z = 1.3$ with the SNAP NIR detectors.

A significant factor in operation of the SNAP satellite is the supernova trigger. While the full survey area will be regularly imaged in all colors, so that every supernova will have full photometric coverage, Type Ia supernovae must be identified at the earliest possible time for targeting of spectroscopic observations. This trigger can be substantially enhanced by prior knowledge of host galaxy redshifts. These can be estimated quite accurately from high S/N measurements of broadband colors. This is especially simple when the available colors

![Figure 1](image1.png) **Figure 1.** Simulation of the Hubble diagram that will be obtained by SNAP containing over 2000 Type Ia supernovae spanning both the acceleration and deceleration epochs of the universe.

![Figure 2](image2.png) **Figure 2.** The shift in magnitude, relative to a cosmology with $\Omega_\Lambda = 0.7$ and $w = -0.7$ for three different flat cosmological models. Note that high-redshift measurements in the near infrared are essential to distinguish among models of dark energy causing the acceleration of the universe.
span the restframe 4000 Å break. Since this break appears at 0.8 µm at redshift 1, and 1.2 µm at redshift 2, NIR information is essential for accurate measurement of photometric redshifts in this range. With broadband colors from 350 to 1700 nm, SNAP can measure photometric redshifts for galaxies from $z = 0$ to $z = 3.2$. Knowing the host galaxy redshift in advance helps to eliminate Type II supernovae and other transients from the sample targeted for resource-intensive spectroscopic observations.

To complement its supernova cosmology observations, SNAP will conduct a wider area weak lensing survey. These weak lensing observations provide important independent measurements and complementary determinations of the dark matter and dark energy content of the universe. They will substantially enhance SNAP’s ability to constrain the nature of dark energy. SNAP weak lensing observations benefit enormously from the high spatial resolution, the accurate photometric redshifts, and the very high surface density of resolved galaxies available in these deep observations. SNAP NIR observations enable the photometric redshift measurements which allow SNAP lensing to track the evolution of the matter power spectrum. This substantially tightens the lensing constraints on the equation of state.

3. SECONDARY SCIENCE MADE POSSIBLE BY NIR OBSERVATIONS

NIR observations made from the ground are hindered by the brightness and opacity of the night sky. As a result, space-based NIR observations augment even the most powerful ground-based instruments in important ways. We outline here a few areas in which SNAP NIR survey observations will impact our understanding of the universe. SNAP will conduct two primary surveys, a $\sim 15$ square degree ultradeep ($m_{AB} \sim 30$ for point sources) supernova survey, and a $\sim 300$ square degree shallower ($m_{AB} \sim 27.8$ for point sources) weak lensing survey.

Galaxy evolution and clustering: Within the 15 square degree supernova survey area, SNAP will make accurate photometric redshift measurements for at least $5 \times 10^7$ galaxies from redshift 0 to 3.5. This data set, which will include morphological information for every object, will provide a unique opportunity to study the evolution of galaxies through more than 90% of the age of the universe. The utility of this data set for galaxy evolution is hinted at by the flood of galaxy evolution papers based on the Hubble Deep Fields.

Identification of high redshift galaxy clusters: Galaxy clusters, the most massive bound objects in the universe, provide important probes of our understanding of structure formation. Constraining their formation and evolution is an important observational goal for the coming decade. Recent advances have overcome earlier limitations of optically selected cluster samples, essentially by using photometric redshift information to eliminate projection effects. The SNAP surveys will provide detailed information on roughly 15,000 galaxy clusters with masses above $5 \times 10^{13} M_\odot$.

A census of R-, I-, z-, and J-band dropout galaxies: Photometric redshifts can be estimated from the 4000 Å break for galaxies out to about $z = 3.2$. For galaxies at still higher redshift, the simplest indicator is the Lyman break. For SNAP, the Lyman break enters the optical imager around redshift 3. In principle it can be followed using SNAP data beyond redshift 10, allowing identification of extremely high redshift galaxies.

Mapping the quasar luminosity function to $z = 10$: Quasars are identified in multi-color imaging surveys by their non-stellar colors. This method has been shown by the Sloan Digital Sky Survey to be extremely effective at identifying quasars to redshift 6 and beyond. SDSS quasar discovery is limited to redshift 6 by the CCD sensitivity cutoff at 1.0 µm$^2$ (see Fig. 3). The most distant SDSS quasar, at redshift 6.28, has a $z$-band magnitude of 20. By probing to wavelengths 1.7 times greater, and to depths 9 magnitudes fainter, SNAP will be able to detect quasars beyond redshift 10, and to probe the quasar luminosity function to 100 times fainter than the brightest quasars.

Studies of GRBs to very high redshift: Current evidence suggests that gamma-ray bursts are associated with the collapse of massive stars which live short lives and die where they are born. As a result, GRBs may trace the cosmic star formation rate. If so, there should be GRBs essentially coincident with the epoch of formation of the first stars. The most distant GRB known occurred at redshift of 3.4. SNAP will be able to identify GRB afterglows, and the orphan afterglows predicted by some models of beaming in GRBs to $z = 10$. Such orphan afterglows may even be detected as backgrounds to the SNe search.
Figure 3. This figure, from Ref. 7, shows a VLT/FORS2 spectrum of the $z = 6.28$ quasar SDSS J1030+0524 in the observed frame. The bottom panel shows a gray-scale representation of the sky-subtracted two-dimensional spectrum plotted on the same wavelength scale. Note the apparent Gunn-Peterson trough; a complete absence of flux from 8450 to 8710 Å.

Probing the structure of reionization: The universe became neutral at the time of recombination, around $z = 1000$, and the thermal radiation from that epoch travels to us undisturbed as the cosmic microwave background radiation. The lack of a Gunn-Peterson effect in the spectra of most quasars demonstrates that the universe was reionized at some time between $z = 1000$ and $z = 6$. The source of the ionizing radiation is the subject of substantial speculation. The recent discovery of an apparent Gunn-Peterson trough in the most distant $z > 6$ SDSS quasar spectra may provide the first glimpse of the epoch of reionization. By identifying many quasars and galaxies to $z = 10$, SNAP will set the stage for mapping the epoch of reionization in unprecedented detail. In combination with ground based and NGST spectroscopy, it will enable measurements of the proximity effect and studies of the spatial structure of reionization.

Discovering the coolest nearby objects: Most of the applications above stress the ability of SNAP NIR observations to probe restframe UV and optical emission in objects at high redshift. These NIR observations can also be used to probe very sensitively the restframe NIR emission from nearby objects. Of particular interest would be a census of low-mass L and T stars and brown dwarfs throughout the Milky Way disk (see Fig. 4).

Gravitational lensing measurements: The high spatial resolution of SNAP NIR observations will enable the discovery of a large number of new strong lenses. The NIR observations, which are much less sensitive to dust extinction within the lens galaxy, are especially important in this regard. In weak lensing measurements, SNAP spatial resolution and NIR sensitivity will allow the use of a huge number of faint, high redshift background galaxies. Because these source galaxies must be resolved to be useful for lensing measurements, and because galaxies fainter than $R=25$ have half-light radii less than 0.2′′, these measurements are impossible from the ground. With these galaxies, it will be possible to extend weak lensing studies to lower mass objects, and to study lens objects beyond $z = 1$.

Discovery of outer solar system objects: SNAP time series data will provide an excellent probe of faint, red objects in the Kuiper belt and beyond. A 2–3 month SNAP survey would detect 10–50,000 Kuiper belt objects down to the size of the Halley’s nucleus.

Identification of targets for spectroscopic instruments: In the most general sense, a large area, space based, NIR survey will provide important input for the spectroscopic capabilities of the new extremely large ground
based telescopes and for the NGST. By identifying quasars, galaxies, and GRBs to high redshift, SNAP will set the table for OWL, CELT, and NGST in very much the same way that the Palomar Sky Survey supported spectroscopy at 4m telescopes, and the SDSS supports spectroscopy for 8–10m telescopes.

Figure 4. This figure, from Ref. 8, shows optical/NIR spectra for five stars ranging from a late L dwarf at the top through a series of T dwarves. The nearly complete absence of optical flux from these very cool stars, along with their strong NIR emission, is apparent.

4. NEAR INFRARED IMAGER CONCEPT

The NIR system for SNAP is an integral part of the overall focal plane. In the baseline concept, thirty-six 2k \times 2k HgCdTe imaging sensors with a pixel pitch of 18 \mu m will be placed in four 3 \times 3 arrangements symmetric with the CCD placement as shown in Fig. 5. Table 1 lists the current performance specifications for the SNAP NIR system. With over 150,000 pixels and a plate scale of 0.17'' per pixel, the NIR system will have a field of view of 0.34 square degrees. The HAWII-2RG Focal Plane Array (FPA) manufactured by Rockwell Scientific Corporation (RSC) is well matched to our needs. These devices exhibit low read noise and dark current while providing excellent quantum efficiency (typically 50%-80% over the wavelength interval 1.0–1.7 \mu m). A cutoff wavelength of \lambda_{c} = 1.7 \mu m is ideally suited to our mission since it renders our focal plane effectively blind to the thermal radiation from the warm telescope and allows passive cooling of the focal plane to 140 K. Three special filters fixed above the FPAs in the pattern shown will provide overlapping red-shifted B-band coverage from 0.9–1.7 \mu m. As SNAP repeatedly steps across its target fields in the north and south ecliptic poles, every supernova will be seen in every filter in both the visible and NIR. Because of their larger linear size, each NIR filter will be visited with twice the exposure time of the visible filters. This, combined with the time-dilated light curve, will ensure that Type Ia supernovæ out to \( z = 1.7 \) will be detected with a S/N > 3 at least 2 magnitudes below peak brightness.
Figure 5. The SNAP near infrared imager concept: The imager contains optical and IR sensors integrated on a common focal plane. A total of thirty-six, $2k \times 2k$, 18 $\mu$m detectors will be used to cover the 900 nm to 1700 nm range.

| PARAMETER            | SPECIFICATION | REASONING                                      |
|----------------------|---------------|------------------------------------------------|
| Field of View        | $\sim 0.3$ square degrees | match CCD FoV to observe                      |
| Plate Scale          | 0.17 arcsec/pixel | every SN in every color                       |
| Wavelength Coverage  | 0.9 $\mu$m - 1.7 $\mu$m | to observe restframe $V$ band out to $z = 1.7$ |
| Read Noise           | $5 e^- (w/multiple reads)$ | Assures that photometry is                     |
| Dark Current         | $< 0.1 e^-$/pixel/sec | zodiacal light limited                        |
| Quantum Efficiency   | $> 60\%$     | to achieve adequate S/N at $z = 1.7$ within time constraints |
| Filters              | three special filters | to obtain redshifted $B$ bands over 0.9 to 1.7 $\mu$m |

The HAWAII-2RG manufactured by RSC is currently the largest available NIR device produced by Molecular Beam Epitaxy (MBE). MBE allows precise control of the HgCdTe deposition which should result in improved quantum efficiency and more uniform pixel response. The latter is important for achieving precise SN photometry. In the near future, competitive FPAs are expected to become available that can meet the SNAP performance specifications.

The read noise specification of $5e^-$ is approximately half the background zodiacal light in the SNAP fields for nominal SNAP exposure times. This specification will likely be the hardest to achieve. It assumes that the intrinsic read noise for a single read will be $10e^-$ and that 4 reads at the beginning and end of an exposure will reduce this by a factor of $\sqrt{4}$. This noise performance has been achieved for HAWAII multiplexers mated to HgCdTe material with a cut-off of 2.5 $\mu$m or greater. Recently excess read noise of $25e^-$ or more, associated with 1.7 $\mu$m material hybridized to HAWAII multiplexers, has been reported by the Wide Field Camera 3 (WFC3) group. Efforts are underway at RSC to solve this problem and first results are expected within the year. Depending on the outcome, the SNAP readout strategy may need to be modified to accommodate read noise in excess of expectations.

The current performance specification for the NIR detector dark current of $< 0.1 e^-$/sec/pix is approximately half the background zodiacal light for a nominal 300 sec exposure. The dark current depends sensitively on
\( \lambda_c \) and the detector temperature. Dark currents as low as \( 0.02 \, e^-/sec/pix \) at 150 K have been achieved with 1.7 \( \mu m \) devices.\(^{10}\) It is reasonable to expect that the dark current should decrease by a factor 6–7 for the SNAP operating temperature of 140 K. Thus we are confident that the SNAP dark current specifications are achievable and the margin on this specification may help us accommodate excess read noise.

The SNAP NIR quantum efficiency specification of > 60% results from the need to detect Type Ia supernovae at 2 magnitudes below peak luminosity out to \( z = 1.7 \) within the time constraints of the SNAP observation plan. Fabrication of WFC3 devices by RSC has led to a substantial improvement in quantum efficiency over the wavelength range 1.0–1.7 \( \mu m \). Quantum efficiencies as large as 85% from 1.4–1.6 \( \mu m \) falling to 56% at 1.0 \( \mu m \) have been reported.\(^{10}\) The WFC3 development should naturally provide the SNAP quantum efficiency solution.

Sensitivity variations on the scale of a single pixel can significantly reduce photometric accuracy of undersampled images. For SNAP, diffraction dominates the point-spread function at all NIR wavelengths. At 1 \( \mu m \) the Airy disk is Nyquist undersampled by a factor of three for a pixel size of 18 \( \mu m \). Intrapixel variations for PACE HgCdTe detectors have been measured by Gert Finger (ESO)\(^{11}\) and found to be quite large. The intrapixel response has not been measured yet for MBE HgCdTe devices. These should have a more uniform pixel response because the internal fields can be tuned to collect charge more efficiently from the edges of the pixels. Simulations\(^{12}\) show that a simple \( 2 \times 2 \) dither pattern can reduce this component of the photometry error to below 1\% for all wavelengths and for reasonable levels of intrapixel variation.

A R&D program has begun to determine the science driven requirements for the SNAP NIR system. These requirements will be used to establish SNAP science-grade specifications for the NIR FPAs. Measurements focusing on read noise, intrapixel variation, dark current and quantum efficiency will be made with the goal of demonstrating a SNAP science-grade prototype meeting all SNAP science-grade specifications within two years.

5. CONCLUSION

Near infrared observations in space are essential to understanding the nature of the dark energy that is causing the expansion of the universe to accelerate. As an integral part of this core mission, SNAP will collect a rich source of survey data of unprecedented quality and scope, permitting detailed studies of weak lensing and many secondary science objectives. The SNAP NIR system will be the largest wide-field instrument ever deployed in space and will provide important input for targeting future large ground and space-based telescopes. The NIR system concept that has been developed for SNAP is capable of meeting this broad range of science goals. A R&D program is under way to firmly establish the science driven requirements for the NIR system and to demonstrate that detectors can be obtained that meet these requirements.

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REFERENCES

1. G. Aldering et al. Proc. SPIE 4835-21, 2002.
2. S. Perlmutter et al. Astrophysical Journal 517, pp. 565–586, 1999.
3. A. Riess et al. Astronomical Journal 116, pp. 1009–1038, 1998.
4. A. Jaffe et al. Phys. Rev. Lett. 86, pp. 3475–3479, 2001.
5. N. Bahcall et al. Physics Reports 333, pp. 233–244, 2000.
6. D. Huterer Phys. Rev. D. 65(063001), 2002.
7. L. Pentericci et al. Astronomical Journal 123, pp. 2151–2158, 2002.
8. S. Leggett et al. Astrophysical Journal Letters 536, pp. L35–L38, 2000.
9. D. Hall et al. Proc. SPIE 4008, pp. 1268–1279, 2000.
10. B. Hill private communication.
11. G. Finger private communication.
12. G. Bernstein PASP 114, pp. 98–111, 2001.