Two-Impurity Kondo Effect in Double-Quantum-Dot Systems: Effect of Interdot Kinetic Exchange Coupling

Watari Izumida
Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan

Osamu Sakai
Department of Physics, Tokyo Metropolitan University, Tokyo 192-0397, Japan

Tunneling conductance through two quantum dots, which are connected in series to left and right leads, is calculated by using the numerical renormalization group method. As the hopping between the dots increases from very small value, the following states continuously appear: (i) Kondo singlet state of each dot with its adjacent-site lead, (ii) singlet state between the local spins on the dots, and (iii) double occupancy in the bonding orbital of the two dots. The conductance shows peaks at the transition regions between these states, especially, the peak at the boundary between (i) and (ii) has the unitary limit value of $2e^2/h$ because of coherent connection through the lead-dot-dot-lead. For the strongly correlated cases, the characteristic energy scale of the coherent peak shows anomalous decrease relating to the quantum critical temperature known for the two-impurity Kondo effect. The two dots system gives the new realization of the two-impurity Kondo problem.

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I. INTRODUCTION

Dilute magnetic impurities in metal bring the single-impurity Kondo effect [12]. The anti-ferron magnetic coupling between spins on impurities $J$, such as the RKKY interaction, would compete with the Kondo effect. To study such competition effect, the two impurities in metal have been studied extensively [3,4]. If the Kondo binding energy is much larger than $J$, each local spin on the magnetic impurity forms the Kondo singlet state with the conduction electrons. On the other hand for $T_K < J$, the two local spins form the local spin singlet state. From the numerical renormalization group (NRG) calculation, Jones et al. had pointed out that the transition between the two states occurs as a quantum critical phenomenon [3]. However, the advanced investigation made clear that the critical transition is an antiferromagnetic coupling of the model neglecting the parity splitting terms such as the d-d hopping term between the impurity atoms [3]. In this paper we will investigate the effect of the d-d hopping term in double-quantum-dot (DQD) systems in detail, and give the new realization to the two-impurity Kondo problem.

It might be difficult to observe the two-impurity Kondo effect in metal systems as pointed out by previous studies, because the alloy contains many types of impurity pairs, and because the coupling between impurities is weak in each material. Recently, the Kondo effect is observed in the single quantum dot system [3-12]. The experimental data show good agreement with the results of numerical calculation based on the single-impurity Anderson model [12]. These works demonstrated that the quantum dot systems are suitable for sensitive experiment of the Kondo problem. On the DQD system, each dot corresponds to an impurity atom, and the coupling between the dots can be changed freely by applying the split gate voltage between the dots [23]. It would be expected that we can investigate the two-impurity Kondo system actually in the DQD systems.

For the DQD systems, which the two dots are connected to the left and the right lead in series as "lead-dot-dot-lead", there are some theoretical works including the Kondo effect [23-27]. We have reported the large enhancement of the tunneling conductance through the two dots when the condition $T_{LR} > T_K$ holds, by using the NRG calculation [23]. Here $T_{LR} = 4t^2/U$ is the anti-ferron magnetic kinetic exchange coupling between the two dots, $t$ is the hopping between the two dots, $U$ is the Coulomb repulsion on the dot, and $T_K$ is the Kondo temperature at $t = 0$. We note that the anti-ferron magnetic coupling is the inevitable effect due to the kinetic process and the Coulomb repulsion on the dot $U$. There are investigations with the slave boson mean field theory (SBMFT). Aono et al. had already studied the same model of us, however they could not find the relation $T_{LR} > T_K$ on the peak of the conductance pointed out by us because the SBMFT cannot treat the kinetic exchange process properly [24]. Georges et al. introduced the anti-ferron magnetic coupling $J$ between the two dots by antiferromagnetic coupling of the model, and discussed the effect related to the critical transition on the conductance by using the SBMFT [24]. However, the introduction of the antiferromagnetic coupling $J$ in the model and the calculation of the conductance within the SBMFT framework bring some questions as follows: (a) Does the effect of the anti-ferron magnetic coupling pointed by Georges et al. actually appear in the DQD system?, because the hopping itself breaks the quantum critical transition of the Kondo effect. If it appears, however, (b) how the conductance of the kinetic exchange coupling that would cause the critical transition and the parity splitting that suppress the critical transition, compe...
And then, (c) how they appear in the conductance? Since the SBMFT could not treat the kinetic exchange process properly, this approximation for the two-impurity Kondo problem like the DQD systems seems to be unfavorable. The reliable calculation is necessary for such a sensitive problem.

In this paper, we present the detailed investigation of the Kondo effect in the DQD system. The numerical calculation is performed by using the NRG method. This numerical method is known to be a reliable one for the two-impurity Kondo problem [24, 25, 26]. We calculate the tunneling conductance through the two dots. We note that some preliminary results were presented at SCES98 [27], and one of the central results was presented at LT22 [28].

We find that the following states continuously appear when the hopping between the two dots increases from very small value; (i) Kondo singlet state \( \Delta t U, T_{K}^{0} X \), (ii) singlet state between local spins on the dots \( t U, J_{LR}^{0} T_{K}^{0} X \), and (iii) double occupancy in the bonding orbital of the two dots \( t > U \). The conductance shows peaks at the transition regions between these states. The main peak, the boundary between (i) and (ii) with the condition \( J_{LR}^{0} T_{K}^{0} X \) has a unitary limit value of \( 2e^{2}/h \) because of coherent connection through the lead-dot-dot-sample. Especially for the strongly correlated cases, the width of the main peak becomes very narrow and the characteristic temperature of the peak is largely suppressed compared with the Kondo temperature of the single dot system \( T_{K}^{0} X \). These anomalies of the main peak closely relate to the quantum critical phenomenon in the two-impurity Kondo problem. The quantitative calculation in this paper gives the new realization for the two-impurity Kondo problem, and suggests the possibility of the systematics study of the anomalous two-impurity Kondo effect in the DQD system.

The formulation is presented in II. The numerical results are presented in III. The summary and discussion are given in IV.

II. FORMULATION

We investigate the following model Hamiltonian for the DQD system, so that the two dots are connected to the left lead and the right lead in series:

\[
H = H_{1} + \frac{1}{2}V_{L}d_{L}^{\dagger}d_{L} + \frac{1}{2}V_{R}d_{R}^{\dagger}d_{R} + h
\]

\[
H_{1} = \sum_{k,k'} \epsilon_{k}c_{k}^{\dagger}c_{k} + \sum_{q} \epsilon_{q}c_{q}^{\dagger}c_{q}
\]

\[
H_{d} = \sum_{n} n_{n,LR} + \sum_{n} n_{n,RR}
\]

\[
\begin{align*}
X & = d_{L}^{\dagger}d_{L} + d_{R}^{\dagger}d_{R} + h
\end{align*}
\]

\[
+ U_{L}n_{d,LR} + U_{R}n_{d,RR} + n_{d,LR}n_{d,RR}
\]

\[
H_{1,d} = V_{L}d_{L}^{\dagger}c_{k} + V_{R}d_{R}^{\dagger}c_{q} + h
\]

\[
X
\]

\[
\frac{2e^{2}}{h} \int G_{0}(0) - G_{0}(0) f
\]

\[
\text{G}
\]

\[
\text{G}
\]

\[
\text{G}
\]
\[ \frac{2\epsilon^2}{h} \left( 1 + (t^e)^2 \right)^2. \]  

(5)

We have used the relation, \( G_0 = z_p = (\frac{\epsilon}{\rho} + i \frac{\epsilon}{\rho}) \), \( z_p = \frac{\epsilon}{\rho} \), \( (p = e \rho) \) at \( T = 0 \). Here the \( s \) x \( s \) denote the even and odd parity orbitals in the two dots. We note that the even orbital is the bonding orbital, and the odd orbital is the anti-bonding orbital. We now consider the case of \( m_{\alpha i} + m_{\alpha i} = 2 \), then \( t^e \cdot e = e \cdot 0 \), \( e = e \cdot 0 \). Here \( t^e \) is the effective hopping between the dots, and \( e \) is the effective hybridization strength between the leads and the dots. At \( T = 0 \), we calculate the effective parameters from the analysis of the CM chart of the renormalized energy level structure in the NRG calculation, and then calculate the conductance from eq. 5.

### III. NUMERICAL RESULTS

In numerical calculation we choose the half of the band width as an energy unit. The Coulomb repulsion is set at \( U = 0 \) throughout this paper. We calculate the conductance as a function of the hopping \( t \) for various hybridization strength \( U \) as noted in previously, \( t \) and \( U \) can be changed by applying the split gate voltage between the dots, between the dots and the leads, respectively. The gate voltage on the dots is set at \( U = 0 \), then the DQD is in the half-filled case, i.e., each dot contains one electron.

In section IIIA and IIIB we present the numerical results at zero temperature \( T = 0 \), and in section IIIC we present the results at finite temperatures.

#### A. Conductance in the strongly correlated case

First we present the conductance in the strongly correlated case with the hybridization strength satisfying \( U = 15 \times 10^3 \), \( U = 15 \times 10^2 \), i.e., \( U = 15 \times 10^3 \)

We show the conductance at \( T = 0 \) as a function of the hopping \( t \) in Fig. 1. (The occupation number and the phase shift are also shown in Fig. 1.) There are two peaks in the conductance, the large peak near \( t = 10^3 \), and the small peak near \( t = 10^2 \). (Hereafter we call the large peak as the main peak.' Why these peaks appear? In the later paragraph we will analyze various quantities for the parameter cases showing the peaks.

The density of states on the even orbital of the two dots, \( \rho(0) \), for several cases is shown in Fig. 2. The density \( \rho(0) \) for \( m_{\alpha i} + m_{\alpha i} = 2 \), where \( \rho(0) \) is the density of states on the odd orbital.

At \( t = 0 \), there is the Kondo peak on the Fermi energy. (Fermi energy corresponds to \( ! = 0 \).) Naturally, this Kondo peak is caused by the Kondo singlet states between the left lead and the left dot, and between the right lead and the right dot. As \( t \) increases to \( t = 5 \times 10^2 \), the conductance has the main peak and the strength of \( \rho(0) \) becomes half of that at \( t = 0 \). In this region we consider that the Kondo effect with the spins on the orbitals extending the two dots, the even and the odd orbitals, occurs. As \( t \) still increases, the conductance decreases rapidly, and the strength of \( \rho(0) \) is largely suppressed as shown at \( t = 10 \times 10^3 \). This suppression means the disappearance of the Kondo coupling.
between the leads and the dots, the conductance has the small peak at $t = 2*10^2$. At $t = 2.5*10^2$, the density of states on the even and odd orbitals have peaks at $t \approx 10^1$, respectively, from Fig. 2. At the same time the occupation numbers begin to change as shown in Fig. 1. In the case of $t = 2.5*10^2$, the occupation numbers of the even and odd orbitals at $t > 1.5*10^0$ are almost same with each other, $n_{\sigma,1} \approx n_{\sigma,1}'$. For $t > 1*10^1$, the two electrons occupy the even orbital. The border between them is at $U = 2.5*10^2$.

Above analysis implies the following scenario. In the case of $U = 4 (= 2.5*10^2)$, the hopping term causes the anti-ferromagnetic kinetic exchange coupling, $J_{LR}$. For smaller hopping case with $J_{LR} < T_K$, there are the Kondo singlet states between the left lead and the dot, and between the right lead and the dot each other. At $t$ increases and then $J_{LR} > T_K$, the two local spins on each dot form the local singlet state. At the transition region between two states we have a main peak with the unitarity limit value of $2e^2/h$ in the conductance. This will indicate that the leads and the dots are coherent connected by the even and the odd orbital states. When $t$ becomes still large and the condition $t > U = 4$ holds, the local spins do not appear, instead, the two electrons occupy the even orbital. The small peak of the conductance reflects the transition of the electronic states in the DQD.

![FIG. 3. Effective parameter $t^\sigma$, $t^0$ of the xed point non-interacting Anderson Hamiltonian, given by the analysis of the ow chart of the renormalized energy level structure in the NRG calculation. $U = 1.5*10^{-3}$.
](image)

Here we show the effective parameters $t^\sigma$ and $t^0$ as a function of $t$ in Fig. 3. We note that the effective parameters have been used already for the calculation of the conductance shown in Fig. 1. In $t < 10^4$ case, the effective parameters behave as $t^\sigma = t^0 = 0$. Then the conductance coincides with the non-interacting one when one substitutes the effective parameters into eq. (4). At $t$ increases, the condition near $J_{LR} > T_K$, $t^\sigma$ once decreases, and it has local minimum, and then it increases. At the same time the slope of $t^\sigma$ once decreases and then increases. When $t^\sigma = t^0$ coincides with each other, the conductance has a peak at $t = 5*10^1$, i.e. $J_{LR} > T_K$. At $t$ increases slightly beyond this point, the conductance sharply decreases because $t^\sigma$ decreases to the minimum even though $t^0$ increases. Here we stress that the relation $J_{LR} > T_K$ holds when $t^\sigma = t^0$, and at the same time $t^0$ becomes very small in the transition region. When $t$ increases further, the ratio $t^\sigma = t^0$ increases gradually in the region $t < U = 4 (= 2.5*10^2)$. At $t = U = 4$, the ratio $t^\sigma = t^0$ begins to decreases and then increases. Therefore the conductance shows a broad peak near the region $t = U = 4$. For $t > U = 4$ case, the effective parameters behave $t^\sigma = t^0 = 0$. We note that the conductance has the expression of the non-interacting one itself in $t > U = 4$ region.

Finally we compare between the phase shift and the occupation number shown in Fig. 1. The phase shift of the odd orbital rapidly changes from $=2$ to 0 near $t < 5*10^1$, even though $n_{\sigma,1}$ is still small at $n_{\sigma,1} = 1$. From $U = 0.1$, Friedel's sum rule in each channel does not hold, as already pointed out previously [4]. It seems that this behavior is enhanced when the anti-ferromagnetic coupling between the two sites competes with the Kondo effect.

**B. From Weakly to Strongly correlated cases**

In this subsection we present the numerical results of the conductance for various $U$ cases with $15*10^{-2} < U < 6*10^{-2}$ (1.7 $< u < 6.8$). The hybridization strength is changed in $15*10^{-3} = 60*10^{-5}$, and the Coulomb repulsion is set at $U = 0.1$). We consider the scenario shown in the previous subsection that $J_{LR} > T_K$ holds at the main peak of the conductance with $t^\sigma = t^0 = 0$. We also demonstrate how the kinetic exchange process appear in the conductance for arbitrary $U$ cases.

The calculated conductance is shown in Fig. 3. The horizontal axis is the hopping parameter of the hybridization strength, $t\sigma$. From inset of the Fig. 1, the conductance almost overlaps on the non-interacting curve in the region $t < U = 4$. Exactly, these regions should be classified $J_{LR} > T_K$ and $t < U = 4$, respectively, from the analysis in the previous subsection. The conductance is very small in all these regions, however this uniform properties should be useful to arrange the experimental data under uncertain $U$ cases.
FIG. 4. Conductance as a function of $t$ at zero temperature, from the weakly to strongly correlated cases in $1.5 \times 10^{-2} = U = 6.0 \times 10^{-2}$. The broken line shows the conductance for the non-interacting ($U = 0$) case.

All curves have a main peak with strength $2e^2/h$. For weakly correlated cases of $U > 4 \times 10^{-2}$, the conductance almost coincides with the non-interacting one through all $t$ region. As $U$ increases, the main peak shifts to the small $t$ side, and the peak width becomes narrower.

We already found the relation $J_{LR}^0 = T_K^0$ at the main peak position for $U = 1.5 \times 10^{-2}$ case in (IIA). Here we show the ratio $J_{LR}^0/T_K^0$ at the main peak position for various $U$ cases in Table I. We can see the relation $J_{LR}^0 = T_K^0$ commonly. (The relation at the main peak, $J_{LR}^0 = T_K^0$, would be generalized to $E_B = (2t)^2 + (U=2)^2$ $U=2$ is the singlet binding energy between the two dots.)

From the analysis in the previous and present subsection we can conclude the following effect of the hopping term. For the small $t$ case with $J_{LR}^0 = T_K^0$, (i) The Kondo singlet state is formed on the left (right) dot with its adjacent-site lead. On the other hand for the large $t$ case with $J_{LR}^0 = T_K^0$, (ii) the local spins on each of the dots couple as the singlet state. In the intermediate region, the Kondo effect of the local spins on the orbitals extending on the two dots (i.e., even and odd orbitals) occurs. The main peak of the conductance appears around the boundary between (i) and (ii) reflecting the coherent connection of the leads and the dots. As $U$ increases, the Kondo temperature $T_K^0$ exponentially decreases, the condition $J_{LR}^0 = T_K^0$ holds at the smaller $t = \tau$, then the main peak shifts to the smaller $t = \tau$ side. At the same time, the width of the peak becomes extremely narrow compared with the decreasing of $T_K^0$. This fact has been already shown as the steep minimum of $G$ in Fig. 3.

We note that this narrowing closely relates to the quantum critical transition between the Kondo singlet state and the local singlet state in the two-site Kondo model [24]. The shifting and narrowing behaviors shown here are also pointed out with the SBM FT with artificial addition of the anti-ferromagnetic coupling between dots to the model [23]. However, the SBM FT calculation should be checked by the method treating the kinetic exchange term properly. As noted in the introduction, the hopping term causes two competing effects on the critical transition of the two-site Kondo systems. One is the kinetic exchange coupling $J_{LR}^0$, which causes the "critical" transition through the competition with the Kondo effect. Another is the parity splitting, which suppresses the "critical" transition. The calculation in this section is the most reliable quantitative results of the two-site Kondo problem in the DQD systems.

There is also another small peak (or shoulder) structure for the strongly correlated cases of $U < 2 \times 10^{-2}$ ($U > 5$) at larger $t$ side of the main peak. In the previous subsection, we found that the small peak appears around the boundary between (ii) and (iii). However for the weakly correlated cases, the small peak could not be recognized because the condition of the border (i)-(ii) and (ii)-(iii) could not be distinguished clearly.

| $U$ | 1.5 $10^{-2}$ | 2 $10^{-2}$ | 3 $10^{-2}$ | 4 $10^{-2}$ | 6 $10^{-2}$ |
|-----|---------------|-------------|-------------|-------------|-------------|
| $J_{LR}^0/T_K^0$ | 2.66 | 2.34 | 2.15 | 2.09 | 2.23 |
| $E_B/T_K$ | 2.66 | 2.34 | 2.14 | 2.04 | 2.05 |

**TABLE I.** Ratios $J_{LR}^0/T_K^0$ and $E_B/T_K$ at the main peak position of the conductance.

C. Temperature dependence of the conductance

In this subsection we present the conductance at finite temperature. We calculate the conductance at finite temperatures by using the following formula [24]:

$$G = \frac{2e^2}{h} \ln \left| \frac{P^{(0)}(t)}{P^{(0)}(t)} \right|$$

Here $P^{(0)}(t)$ is the current spectrum for the current operator $J$. N$_T$, N$_n$ written as follows:

$$P^{(0)}(t) = \frac{1}{Z} \sum_{\sigma} e^{i \theta_{\sigma}}$$

$$Z = \sum_{\sigma} e^{i \theta_{\sigma}}$$
where $N_+\,$ is the time derivative of the electron number in the left lead, $Z = \sum e^{-E_n} \,$ is the partition function of the system, and $\frac{1}{E_n}$ is the inverse of the temperature ($\frac{1}{T} = 1$).

First we show the conductance at various temperatures for $U = 1.5 \times 10^2$. As the temperature increases from $T = 10^8$, the height of the main peak gradually decreases. At the same time, the peak position shifts to the larger $t$. We note that $T = 10^8$ is much lower than $T_K$. ($T_K = 3780 \times 10^6$.)

To discuss the characteristic behaviors of the conductance in finite temperatures, we show the density of states $\epsilon_0(\epsilon)$ and the current spectrum (divided by $1$) $P^0(\epsilon)$ at $T = 5 \times 10^4$. In Fig. 6, as the temperature increases to $T = 6 \times 10^8$, $P^0(\epsilon)$ at $T = 0$ becomes 60% of $P^0(\epsilon)$ at $T = 0$. At the same time, $\epsilon_0(\epsilon)$ shows a small change around $10^7$. This means that the effect of the temperature on the conductance is rather drastic. Here we show two sorts of magnetic excitation spectra $\epsilon_{m}(\epsilon)$ and $\epsilon_{m}^{0}(\epsilon)$, where $\epsilon_{m}(\epsilon)$ is the imaginary part of the dynamical susceptibility of the uniform magnetic moment of local spins, $(S_{1,\mu} + S_{2,\mu})/Z$, and $\epsilon_{m}^{0}(\epsilon)$ is that of the anti-ferromagnetic moment, $(S_{1,\mu} - S_{2,\mu})/Z$, respectively. We show the two magnetic excitation spectra at $t = 0$ and $t = 5 \times 10^4$ in Fig. 7. At $t = 0$, the two spectra agree with each other. However, at $t = 5 \times 10^4$, $\epsilon_{m}^{0}(\epsilon)$ has the structure in lower energy region than $\epsilon_{m}(\epsilon)$. It seems that $P^0(\epsilon)$ at the main peak of the conductance is dominated by the actuation given by $\epsilon_{m}(\epsilon)$ from Fig. 6 and Fig. 7.

![Graph](image-url)
near $T_0^\text{F}$, J had been already pointed out [8]. As $t$ still increases, $T_{AF}$ rapidly increases. From same analysis for the other $= U$ cases, we confirm that the minimum of $T_{AF}$ appears for the strongly correlated cases of $= U < 2 \times 10^2$. We show $T_{AF}$ at the main peak position in Table II.

Here we again see the temperature dependence of the conductance shown in Fig. 8 with the characteristic temperature shown in Fig. 3. We can see that $T_{AF}$ characterizes the main peak of the conductance in this temperature. The peak decreases as the temperature increases near $T = 1 \times 10^3$ ($0.1 T_{AF} (t = 5 \times 10^4)$) in Fig. 8. As the temperature increases and reaches to $T = 1 \times 10^3$ ($0.1 T_{AF} (t = 5 \times 10^4)$), the strength of the main peak becomes almost zero. Next we see the temperature dependence of the small peak. The small peak near $U = 4 = 2.5 \times 10^2$ increases as the temperature increases to about $T = 1 \times 10^2$ ($0.2 T_{AF} (t = 5 \times 10^4)$). It seems that the characteristic temperature of the conductance near the small peak is $T_{M}$. From above it seems that the characteristic temperature of the conductance is $m \in (T_{AF}; T_{M})$ in all $t$ region.

Here we show the conductance from the weakly to strongly correlated cases at fixed temperatures. We show the conductance at $T = 1.6 \times 10^5$ and $T = 1.4 \times 10^4$ in Fig. 8. The main peak for the strongly correlated cases is sensitive to the temperature. Then the main peak of the conductance will shift to a smaller temperature with increasing peak height when the temperature decreases as seen from Fig. 8. This behavior will be observed as the split gate voltage is varied. We note that $T = 1.4 \times 10^4$ corresponds to 16mK, and $T = 1.6 \times 10^5$ corresponds to 1.6mK, for $U = 1.6$ eV system.

| $= U$       | $1.5 \times 10^2$ | $2 \times 10^2$ | $3 \times 10^2$ | $4 \times 10^2$ | $6 \times 10^2$ |
|------------|------------------|----------------|----------------|----------------|----------------|
| $T_{AF}$   | 2.55 $10^6$      | 4.47 $10^5$    | 2.81 $10^4$    | 3.02 $10^3$    | 3.22 $10^3$    |
| $T_{H}$    | 2.66 $10^6$      | 5.64 $10^5$    | 7.73 $10^4$    | 3.47 $10^3$    | 1.20 $10^2$    |
| ( $T_{M}$ )| 2.66 $10^6$      | 5.64 $10^5$    | 7.73 $10^4$    | 3.47 $10^3$    | 1.20 $10^2$    |

**TABLE II.** Characteristic energies at the main peak position.

**FIG. 8.** Two characteristic energies $T_{M}$ and $T_{AF}$.

From the comparison with the effective parameters in Fig. 9, the larger of the effective parameters, $m \in (T_{AF}; T_{M})$, and the smaller of the characteristic temperature, $m \in (T_{AF}; T_{M})$, almost coincide with each other in all $t$, $m \in (T_{AF}; T_{M})$.

**FIG. 9.** Conductance from weakly to strongly correlated cases at $T = 1.6 \times 10^5$ (left graph) and at $T = 1.4 \times 10^4$ (right graph).
Finally, we note the accuracy of the conductance calculated from eqs. (1)–(7) by using the NRG method. It is not so accurate at very high temperatures for the small peak. In the case of $t = 0$, two dots completely decouple, then the conductance should be zero. However, as shown in Fig. 14, the calculated conductance has a finite value in $5 \times 10^{-3} < T < 1$ and it has a maximum at $T = 0.05$($= U=2$). Thus the result at very high temperatures has ambiguities. This improper finite conductance would be caused by estimation of eq. (3) at $T$ instead of $T = 0$. The finite value in the current spectrum at $T = 0.05$ would re ect the largeness of the dynamical charge fluctuation in the dots.

\[ T \]

**Fig. 10.** Numerical results of the conductance for $t = 0$ case.

**IV. SUMMARY AND DISCUSSION**

We calculated the tunneling conductance through the two quantum dots that connected to the left lead and the right lead in series. We investigated the effect of the kinetic exchange coupling between the dots, and also the competition with the Kondo effect.

As the hopping between the two dots increased, (i) Kondo singlet state, (ii) local spin singlet state, and (iii) molecular orbital like state with double occupancy in even state, continuously appeared. For $U > 0$ cases, the Kondo binding between the left (right) lead and the left (right) dot, $T_K$, and the anti-ferromagnetic kinetic exchange coupling between the two dots, $J_{KR}$, competed. The boundary between (i) and (ii) was characterized as $J_{LR}^0 = T_K^0$, and the tunneling conductance showed a peak. This peak had the unitarity limit value of $2e^2/h$ by exciting the coherent connection through the lead-dot-dot-lead. At $U = 4$ of the boundary between (i) and (iii), we had a small peak.

The system showed the strongly correlated behaviors for $U < 2 \times 10^2$ ($U = 5$) cases. The borders of (i)–(ii) $J_{LR}^0 = T_K^0$ and (ii)–(iii) ($t = U=4$) were clearly distinguished, then there were two peak structures in the conductance. Further on the width of the main peak becme steeply narrow. The characteristic temperature of the main peak was strongly reduced compared with the Kondo temperature of the single dot system $T_K^0$. These anomalous behaviors of the main peak related to the quantum critical transition of the two-in-purity Kondo problem studied in previous. Though the hopping term has competing effects on the critical transition of the two-in-purity Kondo system, generation of it through the kinetic exchange coupling and suppression of it due to the parity splitting, we found that we see the sign of the anomaly in the tunneling conductance.

The quantitative calculation shown in this paper gave the new realization for the two-in-purity Kondo problem. This investigation suggested the importance of the systematic study of the DQD system for the two-in-purity Kondo problem.

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[13] I. A. Eck and A. W. W. Ludwig: Phys. Rev. Lett. 68, 1046 (1992).
[14] I. A. Eck, A. W. W. Ludwig and B. A. Jones: Phys. Rev. B 52, 9528 (1995).
[15] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magruder, U. Meirav and M. A. Kastner: Nature 391, 156 (1998).
[16] Sara M. Cronenwett, Tjeerk H. Oosterkamp and Leo P. Kouwenhoven: Science 281, 540 (1998).
[17] D. Goldhaber-Gordon, J. Gores, M. A. Kastner, H. Shtrikman, D. Mahalu and U. Meirav: Phys. Rev. Lett. 81, 5225 (1998).
[18] J. Schmied, J. Weiss, K. Eberland and K. Klitzing: Physica B 256-258, 182 (1998).
[19] F. Simmel, R. H. Blick, J. P. Kotthaus, W. Wegscheider and M. Bichler: Phys. Rev. Lett. 83, 804 (1999).
[20] O. Sakai, W. Izumida and S. Suzuki: Proceedings of the 4th Int. Symposium on Advanced Physical Field (March, 1999, Tsukuba, Japan), 143; W. Izumida and O. Sakai: Physica B (in press, Proceedings of the International Conference on Strongly Correlated Electron Systems, August, 1999, Nagano, Japan); W. Izumida, O. Sakai and S. Suzuki: in preparation.
[21] For example of the recent experimental studies of the double dot systems, T. H. Oosterkamp, T. Fujisawa, W. G. van der Wiel, K. Ishibashi, R. V. Hijman, S. Tanoue and L. P. Kouwenhoven: Nature 395, 873 (1998); T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tanoue and L. P. Kouwenhoven: Science 282, 932 (1998).
[22] T. Ivanov: Eur. Phys. Lett. 40, 183 (1997).
[23] T. Pohjola, J. König, M. M. Salomaa, J. Schmied, H. Schoeller and Gerd. Schon: Eur. Phys. Lett. 40, 189 (1997).
[24] T. Ahonen, M. Eto and K. Kawamura: J. Phys. Soc. Jpn. 67, 1860 (1998).
[25] W. Izumida, O. Sakai and Y. Shimizu: Physica B 259-261, 215 (1999).
[26] A. Georges and Y. Meir: Phys. Rev. Lett. 82, 3508 (1999).
[27] W. Izumida and O. Sakai: Physica B (in press, Proceedings of the XXII International Conference on Low Temperature Physics, August, 1999, Espoo and Helsinki, Finland).
[28] W. Izumida, O. Sakai and Y. Shimizu: J. Phys. Soc. Jpn. 67, 2444 (1998).
[29] W. Izumida, O. Sakai and Y. Shimizu: J. Phys. Soc. Jpn. 66, 717 (1997).