General aspects of speed increaser gearboxes

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Abstract. This study aims to present the general aspects of the speed increaser gearboxes. The second part of this study will focus on the identification of the possibility to replace the special materials used for gearing execution of the speed increaser gearboxes with materials having inferior mechanical characteristics. Gearing execution from 18CrNiMo7-6 material is expensive in terms of price and applied execution technology. Also to obtain the mechanical characteristics for these materials is necessary to apply some special expensive heat treatments as carburizing heat treatment. We will replace this material with C45 steel for gear and pinion execution, we will apply an induction hardening heat treatment for getting a surface teeth hardness HRC 45 an for the increase the safety for tooth root stress coefficient we will apply a supplementary operation consisting in a tooth root shoot peening.

1. Introduction
The gears represent mechanical transmissions used in many applications due to the fact that they have high efficiencies and could cover a large scale of the speed and powers. Because the gears are executed from alloy special steel with specific heat treatments, these transmissions have small dimensions [1].

The speed increaser gearboxes are mechanical transmissions with subunit ratio. Depending on the imposed total ratio, the speed multipliers could be constructed in one or more stages.

The gears of the speed increaser gearboxes can be with cylindrical teeth or cylindrical helical teeth. We recommend cylindrical teeth when the peripheral speeds are low and the axial forces in the sleeve bearing are not accepted. When the peripheral speed are high (over 12m/s) [2] and the function is silent, we recommend using gears with cylindrical helical teeth.

Speed increaser gearboxes used in the Kaplan turbines are constructed with cylindrical helical teeth, the multipliers sleeve bearings are dimensioned so that to take the forces from the cylindrical gearing and the axial forces generated by the turbine.

2. Short description of the gears system
The gears system is classified according to the following criteria [3], [4]:

I. Axes position: A) Gears with parallel axes Figure 1.a; B) Gears with crossed axes Figure 1.d; C) Gears with intersected axes Figure 1e-f;
II. Teeth form: A) Gears with cylindrical teeth; B) Gears with cylindrical helical teeth; C) Gears with teeth in V,W,Z form; D) Gears with bevel helical teeth;

III. Relative position of the meshing surfaces: A) External meshing; B) Internal meshing

IV. Direction of rotation: A) In the same direction; B) In opposite direction;

Ratio: A) Constant ratio; B) Variable ratio;

Size of the ratio: A) Ratio \(\geq 1\)-gearbox; B) Ratio \(\leq 1\)-speed multipliers

Teeth profile: A) Evolving profile; B) Archimedean spiral for worm gear; C) Cycloidal profile; D) Straight line just for rack

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3. The geometric elements of the speed increaser gearboxes. The meshing low

The meshing low, also known as Willis theory, status the condition that has to be fulfilled by the profile curves of the teeth in contact so that the moment can be transmitted at a constant transmission ratio [3].

\[
i_{12} = \frac{n_1}{n_2} = \frac{\omega_1}{\omega_2}
\]  

(1)

In order that two gears wheels to transmit the rotation moment at a constant transmission ratio it is necessary that the teeth profiles to be generated in such manner that their common normal straight line in the contact points to pass through a fixed point C (pitch point) (Figure 2) from the centre line. The conjugate profiles are those to satisfy the meshing low [3], [5].
Figure 2. The geometric elements of the speed increaser gearboxes

Involute (also known as evolvent) is a curve obtained from another given curve by attaching an imaginary taut string to the given curve and tracing its free end as it is wound onto that given curve [6].

Figure 3. The geometric elements of the speed increaser gearboxes

The pressure angle $\alpha$ is the angle between the line of pressure (which is normal for the tooth surface) and the plane tangent to the pitch surface. The involute equation is [3]:

$$\phi = \arcsin \frac{r}{R}$$
\[ \text{Inv} \alpha = \tan \alpha - \alpha \]  
\[ r = \frac{r_p}{\cos \alpha} \]  

4. The forces acting in the cylindrical helical teeth of the speed increaser gearboxes. The stresses appearing in the cylindrical helical teeth of the speed increaser gearboxes

The normal force \((F_n)\) which appears in gearing, decomposes in two components: the tangential force \((F_t)\) and radial force \((F_r)\). The tangential force introduce tooth root maximum stretching stresses in point A and maximum compression stresses in point B, and the radial force introduce the compression stresses in the ABC plan \([1]\).

So, adding these stresses result that the stretching stresses in point A have a maximum value of \(\sigma_i - \sigma_c2\) and in point B the compression stresses have a maximum value of \(\sigma_c1 + \sigma_c2\) \([1]\).

The tooth crack, as the result of the fatigue bending and shock stresses, appears due to the stretching stresses that occur around point A \([2], [7]\).

We observe that an increase of the compression stress around this point would lead to decrease of the stretching stress. This can be done applying a supplementary operation consisting in a shoot peening in zone point A \([2], [7]\).

![Figure 4. The forces in the cylindrical helical teeth](image-url)
During the meshing the teeth are exposed to dynamic stressing [1].

**Figure 5.** Tooth root stress schema

**Figure 6.** The geometric elements of the speed increaser gearboxes
The static stresses increase with following coefficients:
- \( K_A \) – transverse load factor,
- \( K_{F\beta} \) – face load factor at tooth root,
- \( K_V \) – dynamic factor,
- \( K_{Fa} \) – transverse load factor at tooth root.

Taking into account all these coefficients according to ISO 6336-3:2006 results that the tooth root stress is:

\[
\sigma_F = \sigma_{Fo} \times K_A \times K_V \times K_{F\beta} \times K_{Fa}
\]  
(4)

where:
- \( \sigma_{Fo} \) – nominal tooth root stress in static conditions;

Considering:

\[
\sigma_{Fo} = \frac{F_t}{b m_n} \times Y_F \times Y_S \times Y_{\beta} \times Y_B \times Y_{DT}
\]  
(5)

Results:

\[
\sigma_F = \frac{F_t}{b m_n} \times Y_F \times Y_S \times Y_{\beta} \times Y_B \times Y_{DT} \times K_A \times K_V \times K_{F\beta} \times K_{Fa}
\]  
(5)

where:
- \( Y_F \) – tooth form factor,
- \( Y_S \) – stress correction factor,
- \( Y_{\beta} \) – helical load factor,
- \( Y_B \) – gear rim factor,
- \( Y_{DT} \) – deep tooth factor,
- \( F_t \) – nominal circumferential force at pitch circle,
- \( b \) – face width,
- \( m_n \) – normal module.

The safety for tooth root stress coefficient according to ISO 6336-3:2006 is:

\[
\sigma_F = \frac{\sigma_{FG}}{\sigma_{F1}} \geq S_{f \text{ min}}
\]  
(6)

where:
- \( \sigma_{FG} \) – limit strength toot root;

According to ISO 6336-3:2006:

\[
\sigma_{FP} = \frac{\sigma_{F \text{lim}} \times Y_{ST} \times Y_{NT}}{S_{f \text{ min}}} \times Y_{\text{relT}} \times Y_{\text{relT}} \times Y_s
\]  
(7)

Considering:

\[
\sigma_{FE} = \sigma_{F \text{lim}} \times Y_{ST}
\]  
(8)

Result:

\[
\sigma_{FP} = \frac{\sigma_{FE} \times Y_{NT}}{S_{f \text{ min}}} \times Y_{\text{relT}} \times Y_{\text{relT}} \times Y_s
\]  
(9)
where:
- \( S_{\text{min}} \) – required safety,
- \( \sigma_T \) – tooth root stress,
- \( Y_{\text{eelT}} \) – support factor,
- \( Y_{\text{RefT}} \) – surface factor,
- \( Y_x \) – size coefficient,
- \( Y_{ST} \) – stress correction factor.

5. Materials used for gear construction of the speed increaser gearboxes

Depending on the mechanical stresses that result from the calculation, the materials used in RRR company for the execution of the gears and pinions are:

- Steel for quenching and tempering: I) 42CrMo4 according to EN 10083-3; II) 30CrNiMo8 according to EN 10083-1; III) 30MoCNi20 according to STAS 791 [8].
- Case hardening steel: 18CrNiMo7-6 according to EN 10084 [9].

Carburized and heat treated this steel develops: I) Surface hardness- HRC 61; II) Tensile strength - \( R_m=1010-1300 \, [\text{N/mm}^2] \); III ) Yield point - \( R_p=815-1050 \, [\text{N/mm}^2] \).

For my application I will use C45 steel for gear and pinion execution with the following mechanical properties in the quenched and tempered conditions.

- Tensile strength - \( R_{m\text{min}}=630 \, [\text{N/mm}^2] \);
- Yield point - \( R_{p\text{min}}=370 \, [\text{N/mm}^2] \).

We will apply an induction hardening heat treatment for getting a surface teeth hardness HRC 57.

To increase the safety for tooth root stress coefficient, we will apply a supplementary operation consisting in a shot peening.

6. Choose of the constructive solution for speed increaser gearbox type AV 11.25. Geometric gears calculation of the speed increaser gearbox

Choosing the constructive variant was made starting the following input data: power \( P = 34.50 \, (\text{kW}) \); input speed \( n_1=250 \, (\text{rpm}) \); output speed \( n_2 = 1000 \, (\text{rpm}) \); application factor \( KA = 1 \);

The main geometrical elements of this speed increaser gearbox are [10]: center distance \( a = 112.500 \, (\text{mm}) \); centre distance tolerance ISO 286:2010 measure js7; normal module \( m_n = 2.0000 \, (\text{mm}) \); pressure angle at normal section \( \alpha_n = 20.0000 \, (\text{°}) \); helix angle at reference circle \( \beta = 10.0000 \, (\text{°}) \); number of teeth \( z_1= 22 \) and \( z_2 = 88 \); face width \( b_1= 47.50 \, (\text{mm}) \) and \( b_2 = 45.00 \, (\text{mm}) \); accuracy grade [Q-ISO 1328:1995], [6].

![Figure 7](image1.png)  **Figure 7.** The geometric elements of the speed increaser gearboxes

![Figure 8](image2.png)  **Figure 8.** Engagement between two gears

Materials used for pinion and gear wheel are [11]:

A) Gear 1: C45 , Through hardened steel, flame/induction hardened according to ISO 6336-5 Figure 11/12 (MQ)
B) Gear 2: C45 Through hardened steel, flame/induction hardened according to ISO 6336-5
Figure 11/12 (MQ)
C) Surface hardness HRC 57
The geometric gears calculation was made using the Kisssoft gear software and was made according to DIN 3960:1987, [12].

Table 1. Tooth root strength calculation

|                      | Gear 1 | Gear 2 |
|----------------------|--------|--------|
| Tooth form factor [YF]| 1.79   | 1.53   |
| Stress correction factor [YS]| 1.81   | 2.15   |
| Working angle [αFen] (°) | 21.05 | 21.20 |
| Bending lever arm [hf] (mm) | 2.48   | 2.84   |
| Tooth thickness at root [sFn] (mm) | 4.07   | 4.07   |
| Tooth root radius [roF] (mm) | 1.06   | 0.80   |
| Contact ratio factor [Yeps] | 1.000  |        |
| Helical load factor [Ybet] | 0.917  |        |
| Deep tooth factor [YDT] |        | 1.000  |
| Gear rim factor [YB] | 1.000  | 1.000  |
| Effective facewidth [beff] (mm) | 47.50 | 45.00  |
| Nominal stress at tooth root [σF0] (N/mm²) | 461.84 | 492.75 |
| Tooth root stress [σF] (N/mm²) | 599.17 | 639.26 |

Permissible bending stress at root of Test-gear

|                      | Gear 1 | Gear 2 |
|----------------------|--------|--------|
| Support factor [YdrelT] | 0.994  | 1.004  |
| Surface factor [YRrelT] | 0.957  | 0.957  |
| Size coefficient (Tooth root) [YX] | 1.000 | 1.000 |
| Finite life factor [YNT] | 1.000  | 1.000  |
| Alternating bending coefficient [YM] | 1.000 | 1.000 |
| Technology factor [YT] | 1.200  | 1.200  |
| Stress correction factor [Yst] |        | 2.00   |
| Yst*σFlim [σFE] (N/mm²) | 740.00 | 740.00 |
| Permissible tooth root stress (N/mm²) | 563.13 | 568.69 |
| Limit strength tooth root [σFG] | 844.69 | 853.03 |
| Required safety [SFmin] | 1.50   | 1.50   |
| Safety for Tooth root stress | 1.41   | 1.33   |

Table 2. Tooth root stresses and safeties taking into account the pretension force

|                      | Gear 1 | Gear 2 |
|----------------------|--------|--------|
| Pretension [σP] (N/mm²) | 45     | 85     |
| Limit strength for tooth root with pretension [σFG'] (N/mm²) | 898.99 | 956.92 |
| Safety for Tooth root stress [SF=σFG'/σF] | 1.50   | 1.050  |
| Pretension [σP] (N/mm²) | 95     | 140    |
| Limit strength for tooth root with pretension [σFG'] (N/mm²) | 959.32 | 1023.64 |
| Safety for Tooth root stress [SF=σFG'/σF] | 1.60   | 1.60   |
| Pretension [σP] (N/mm²) | 145    | 190    |
| Limit strength for tooth root with pretension [σFG'] (N/mm²) | 1019.66 | 1084.57 |
| Safety for Tooth root stress [SF=σFG'/σF] | 1.7    | 1.7    |
We notice that without a pretention at tooth root, the safety coefficients for tooth root stress are smaller than required safety coefficients.

7. Conclusion
The speed increaser gearboxes are mechanical transmission with subunit ratio. The pinions and the gears of the speed increaser gearboxes are executed from case hardening steel. Carburised and heat treated these materials develop high values for surface hardness and for the tensile strength. We will replace these materials with C45 steel. Applying an induction heat treatment we will get a teeth surface hardness, HRC 57.

To increase the safety factor for tooth root stress we will apply a shoot peening process. This process will be performed by bombarding the tooth root teeth of the gears with small spherical media.

By this process we will introduce the residual compressive stresses in the superficial layer of the tooth root and will improve the bending fatigue strength of the gears.

In conclusion, taking into consideration that applying an induction hardening heat treatment, we will get a surface teeth hardness HRC 57 and applying a soot peening process for growing the safety factor for the tooth root stress it is possible to replace the special materials like 18CrNiMo7-6 or 16MnCr5 with normal steel like C45.

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