Research Article
Tsallisian Gravity and Cosmology

Kavoos Abbasi and Shirvan Gharaati

Department of Physics, College of Sciences, Yasouj University, Yasouj 75918-74831, Iran

Correspondence should be addressed to Kavoos Abbasi; kabbasi@yu.ac.ir

Received 4 August 2020; Revised 9 September 2020; Accepted 25 September 2020; Published 7 October 2020

Academic Editor: Hooman Moradpour

Copyright © 2020 Kavoos Abbasi and Shirvan Gharaati. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this paper, we adopt the Verlinde hypothesis on the origin of gravity as the consequence of the tendency of systems to increase their entropy and employ the Tsallis statistics. Thereinafter, modifications to the Newtonian second law of motion, its gravity, and radial velocity profile are studied. In addition, and in a classical framework, the corresponding cosmology and also its ability in describing the inflationary phases are investigated.

1. Introduction

The study of the relation between thermodynamics and gravity has a long history [1–7]. On the one hand, Gibbs shows that gravitational systems are not extensive [1], a conclusion in agreement with the Bekenstein entropy of black holes [2], which is a nonextensive entropy. On the other, it seems that all gravitational systems satisfy the Bekenstein entropy bound expressed as [8]

$$S_{BE} = \frac{A c^3}{4 G \hbar},$$  \hspace{1cm} (1)

where $A = 4 \pi R^2$ and $R$ denote the area of the system boundary and its radius, respectively, and $k_B$ (Boltzmann constant).

Using this entropy and Clausius relation, one can show that the Einstein gravitational field is in fact a thermodynamic equation of state [9]. This amazing result is valid in various gravitational and cosmological setups which lead to notable predictions about the behavior of cosmos and gravitational systems [10–30]. Motivated by the Gibbs work [1], the nonextensivity of the Bekenstein entropy, and based on the long-range nature of gravity [31], recently, the use of nonextensive statistical mechanics (based on possible generalizations of Gibbs entropy) has been proposed to model and study some phenomena such as the cosmic evolution [32–39], black holes [40–49], and Jeans mass [50, 51].

In order to find the probable thermodynamic aspects of gravity, Verlinde describes it as the implication of the tendency of systems to increase their entropy [52], an astonishing approach which attracts investigators to itself [53–65]. In the framework of generalized entropies, the Verlinde hypothesis leads to significant implications on the cosmic evolution [35, 66–68], Newtonian gravity [69], Jeans mass (as a stability criterion) [70], and also gravitational systems [71–76]. Indeed, the differences between generalized entropies and the Bekenstein entropy, originated from the nonextensive viewpoint, can (i) describe the universe inflationary phases [32–34, 39], (ii) relate Padmanabhan emergent gravity scenario to the Verlinde hypothesis [32], and (iii) propose an origin for the MOND theory [69].

Based on the Verlinde hypothesis [52], the entropy change of a system increases as

$$\Delta S = 2 \pi \frac{mc}{\hbar} \Delta x,$$  \hspace{1cm} (2)

when the test mass $m$ has distance $\Delta x = \hbar mc / \lambda_c$ (reduced Compton wavelength) with respect to the holographic screen (boundary of system). This screen consists of $N$ degrees of freedom calculated by

$$N = \frac{A c^3}{G \hbar},$$  \hspace{1cm} (3)
in agreement with Eq. (1) and thus $S_{BE} = N/4$ [2]. Following [55, 56], we assume $\Delta x = \eta \lambda$, from now, and use the Unruh temperature [7]

$$ T = \frac{\hbar a}{2\pi c}, \quad (4) $$

to get [55, 56]

$$ F = T \frac{\Delta S}{\Delta x} = T \frac{dS}{dA} \Delta x = ma, \quad (5) $$
as the net force that source $M$ applies to particle $m$, which finally brings it acceleration $a$. Indeed, this result is available if $\eta = 1/8\pi$ leading to $\Delta x = \lambda / 8\pi$, to get Eq. (5). Now, combining $A = 4\pi R^2$ and Eq. (3) with

$$ E = \frac{1}{2} NT = Mc^2, \quad (6) $$

and using Eq. (5), one easily reaches at Newtonian gravity

$$ a = G \frac{M}{R^2}. \quad (7) $$

It is also useful to mention that it seems there is a deep connection between generalized entropies and quantum gravity scenarios, and indeed, quantum aspects of gravity may also be considered as another motivation for considering generalized entropies [77, 78]. Tsallis entropy is one of the generalized entropy measures which leads to acceptable generalized entropies [77, 78]. Tsallis entropy is one of the multifractal structure of horizon in quantum gravity [78] and modifies Eq. (1) as $S \sim A^\delta$ ($\delta$ is a free unknown parameter [77]).

The second one has recently been calculated in [49] by relying on statistical properties of degrees of freedom distributed on the holographic screen. The result is compatible with a detailed study in the framework of quantum gravity [47]. This case proposes an exponential relation between the horizon entropy and its surface, and we will focus on it in this paper. In the next section, modifications to the Newtonian second law of motion and also Newtonian gravity is derived by using the Tsallis entropy. Its implications on the radial velocity are also addressed. In the third section, after evaluating the Tsallis modification to the gravitational potential, we adopt the approach of paper [79] and find out the corresponding Friedmann first equation in a classical way in which a test mass is located on the edge of the universe, namely apparent horizon [79]. The possibility of obtaining an accelerated universe is also debated in this section. A summary of the work is presented in the last section.

2. Tsallis Gravity and Dynamics

Employing the Tsallis statistics, it has been recently shown that Eq. (1) is modified as [49]

$$ S_q^T = \frac{1}{1-q} \left[ \exp \left( (1-q) S_{BE} \right) - 1 \right], \quad (8) $$
in full agreement with quantum gravity calculations [47]. Here, $q$ is a free parameter evaluated from other parts of physics and also observations, and Eq. (1) is recovered when $q = 1$ [31, 47, 49]. In the nonextensive scenarios, Eq. (6) takes the form [35, 80]

$$ E = \frac{1}{5 - 3q} NT = Mc^2, \quad (9) $$

which approaches Eq. (1) at the appropriate limit of $q = 1$.

Now, following the recipe which led to Eq. (5), one can use Eq. (8) to find

$$ F^T = T \frac{dS_q^T}{dA} \frac{\Delta A}{\Delta x} = ma \exp \left( \frac{\delta (2 + 3\delta) M^2 c^3}{2\hbar} \right), \quad (10) $$

where $\delta = 1 - q$ is the Tsallis second law of motion. Clearly, Eq. (5) is recovered whenever $\delta = 0$, and therefore, this approach claims the net force $F^T$ that source $M$ applies to $m$ depends on $M$. In order to obtain the above result, we used $S_{BE} = N/4$ [2], and $N = ((5 - 3q)M^2c^3)/T$. Of course, since the relation $F = ma$ works very well (classical regime), one can deduce that $\delta$ is very close to 0 meaning that the exponential factor may have nonsensible effects in the classical regime.

The modified form of Eq. (7), called Tsallis gravity, is also obtained as

$$ a^T = G_q M \frac{L_p^2}{L} \exp \left( \frac{\delta R_0^2}{R^2} \right), \quad (11) $$

where $R_0^2 = G\hbar c^3 \pi = l_p^2/\pi$, $l_p$ denotes the Planck length, and $G_q = ((5 - 3q)/2)G$ in full agreement with [35]. In order to have a comparison between the Tsallis second law of motion and also the Tsallis gravity and those of Newton, let us write

$$ \frac{F^T}{F} = \exp \left( \frac{d}{a} \right), \quad (12) $$

$$ \frac{a^T}{a} = (5 - 3q) \exp \left( \frac{l}{R^2} \right), $$

where $d = (2 + 3\delta) M^2 c^3)/2\hbar$ and $l = \delta R_0^2$. As a crucial point, one should note that, for an event, the sign of $a$ and $a^T$ should be the same (the predictions of different theories about the value of accelerations should address the same motion meaning that both of $a^T$ and $a$ should have the same sign). It leads to this limitation $q < 5/3$ meaning that $\delta > -2/5$. Thus, $l$ and $d$ can be negative.

Now, let us compare Eq. (11) with the results of [55] and [56] where authors employ different entropies in the
framework of the Verlinde theory and address two modifications for the Newtonian gravity. Unlike Eq. (11) of [11], the modified gravity obtained in [55] (Eq. (17)) diverges at large distances ($R \gg 1$). Of course, both of them claim that the gravitational force between the source $M$ and test particle $m$ can vanish for some points on their interface line, a property incompatible with the Newtonian gravity and experience. From Eq. (11), one can easily see that the obtained gravitational force does not diverge at large distances where it will be ignorable. Thus, it seems that this equation is a more reliable modification to the Newtonian gravity compared with those of [55, 56].

2.1. Velocity Profile. For a circular motion at radius $r$ with velocity $v$, and thus acceleration $(v^2/r)$ obeying Eq. (12), one reaches

$$v = \sqrt{\frac{G_M}{r}} \exp \left( \frac{l}{2r^2} \right),$$

which implies that we should have $q < 5/3$ to get real values of velocity.

On the other hand, if one assumes the mass $m$ in the gravitational field of source $M$ feels the force $G M m r^2$, then using (10), we can write

$$G M m r^2 = F^T,$$

yields

$$G M r = v^2 \exp \left( \frac{dr}{v^2} \right),$$

for $a \equiv (v^2/r)$, finally leading to

$$v^2 = \frac{G m}{r} - dr$$

if we expand $v^2/dr$ as $1 + (dr/v^2)$. For a constant $d$, this approximation is valid when radial acceleration $(v^2/r)$ is small. Indeed, in this manner, the $dr$ term leads to an increase in the velocity of particle $m$, compared with the Newtonian case for which $v^2 \equiv (G m / r)$, if $d < 0$.

3. A Tsallis Cosmology

In order to find the Friedmann first equation corresponding to the obtained Tsallis gravity, we follow the classical viewpoint fully described in [79]. The series expansion $\exp \left( l/r^2 \right) = \sum_{n=0}^{\infty} F_n r^{2n}$ leads to

$$\int \frac{\exp \left( l/r^2 \right)}{r^2} dr = \sum_{n=0}^{\infty} \frac{l^n}{n! r^{2n+2}} dr = -\frac{1}{R} \sum_{n=0}^{\infty} \frac{l^n}{n! (2n+1) R^{2n}},$$

combined with Eq. (11) to help us in calculating Tsallis gravitational potential as

$$\phi(r) = -\frac{G M}{r} \sum_{n=0}^{\infty} \frac{l^n}{n! (2n+1) r^{2n}}.$$  

Considering a test particle on the edge of a flat FRW universe, and following the recipe of [79], this equation leads to

$$H^2 = \frac{8\pi G}{3} \rho \sum_{n=0}^{\infty} \frac{l^n H^{2n}}{n! (2n+1)},$$

in which $\rho$ is the cosmic fluid density and $H$ denotes the Hubble parameter, and we used the fact that the apparent horizon is located at $r = 1/H$. Moreover, the standard Friedmann first equation [79] is recovered at the desired limit of $q = 1$ (or equally, $\delta = 0(\|l = 0\)$).

3.1. Accelerated Universe. Bearing the fact that the Hubble parameter decreases during the cosmic evolution in mind, rewriting Eq. (19) as

$$\frac{H^2}{\sum_{n=0}^{\infty} \left( l/n! (2n+1) \right)} = \frac{8\pi G}{3} \rho$$

and keeping terms up to the $H^4$ term in LHS (the first corrective term to the standard cosmology ($H^2 = (8\pi G/3) \rho$) due to Tsallis gravitational potential), one easily reaches at

$$H^2 = \frac{3}{2l} \left( 1 \pm \sqrt{1 - \frac{32\pi G l}{9} \rho} \right).$$

In order to have real solutions for $H^2$, this equation claims that there is a maximum bound on the density of cosmic fluid as $\rho_{\text{max}} = 9/32\pi G l$, at which the universe feels a de-Sitter phase with $H = \sqrt{3/2l}$ when $l > 0$. As the universe expands, $\rho$ decreases, and when $\rho = 0$, the positive branch experiences again the primary de-Sitter phase ($H = \sqrt{3/l}$ for $l > 0$), but forever, while the universe expansion rate vanishes for the negative solution. In fact, the vacuum solution ($\rho = 0$) of the above Friedmann first equation is an inflationary universe for the positive branch, and a Minkowski universe for the negative branch.

4. Summary

In the framework of the Verlinde hypothesis on the origin of gravity, we employed the recently proposed Tsallis entropy [47, 49] to find its implications on the Newtonian dynamics (second law of motion) and gravity. The velocity profile in a circular motion has also been analyzed. Finally, adopting the classical approach to get the Friedmann first equation described in [79], the corresponding cosmology was achieved after finding the Tsallis gravitational potential. The obtained modified Friedmann first equation (20) includes a complex function of $H$.  

Advances in High Energy Physics
Since the Hubble parameter decreases during the cosmic evolution, and because the standard Friedmann first equation \((H^2 = 8\pi G/3\rho)\) has notable achievements, we only focused on the first corrective term due to the Tsallis gravitational potential (i.e., we only hold terms up to \(H^4\) in writing Eq. (21)). We saw that, in some situations and depending on the value of \(\delta\), the resulting equation addresses the (anti) de-Sitter universes with \(H = \sqrt{3}\pi/2\delta P\) and \(H = \sqrt{3}\pi/8\delta P\), depending on \(\rho\). It also admits an upper bound on the energy density of cosmic fluid of order of \(((l_P^2 G^{-1})/((2 + 3\delta)l)) \sim (10^{31})/((2 + 3\delta)l))\). We also obtained that there are two branches for the assumed approximation. Whenever \(\rho = 0\), the positive branch, depending on the value of \(\delta\), guides us to an eternal (anti) de-Sitter phase, and the negative branch addresses a Minkowskian fate for the universe.

Data Availability

There is no data used in this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

We are grateful to the anonymous reviewer for worthy hints and constructive comments.

References

[1] J. W. Gibbs, *Statistical Mechanics*, Mathematical Physics, Charles Scribner’s Sons, New York, NY, 1902.
[2] J. D. Bekenstein, “Black holes and entropy,” *Physical Review D*, vol. 7, no. 8, pp. 2333–2346, 1973.
[3] S. W. Hawking, “Particle creation by black holes,” *Communications In Mathematical Physics*, vol. 43, no. 3, pp. 199–220, 1975.
[4] S. W. Hawking, “Black hole explosions?,” *Nature*, vol. 248, no. 5443, pp. 30–31, 1974.
[5] J. M. Bardeen, B. Carter, and S. W. Hawking, “The four laws of black hole mechanics,” *Communications in Mathematical Physics*, vol. 31, no. 2, pp. 161–170, 1973.
[6] P. C. W. Davies, “Scalar production in Schwarzschild and Rindler metrics,” *Journal of Physics A: Mathematical and General*, vol. 8, no. 4, pp. 609–616, 1975.
[7] W. G. Unruh, “Notes on black-hole evaporation,” *Physical Review D*, vol. 14, no. 4, pp. 870–892, 1976.
[8] M. Srednicki, “Entropy and area,” *Physical Review Letters*, vol. 71, no. 5, pp. 666–669, 1993.
[9] T. Jacobson, “Thermodynamics of Spacetime: the Einstein equation of state,” *Physical Review Letters*, vol. 75, no. 7, pp. 1260–1263, 1995.
[10] T. Padmanabhan, “Classical and quantum thermodynamics of horizons in spherically symmetric spacetimes,” *Classical and Quantum Gravity*, vol. 19, no. 21, pp. 5387–5408, 2002.
[11] T. Padmanabhan, “Gravity and the thermodynamics of horizons,” *Physics Reports*, vol. 406, no. 2, pp. 49–125, 2005.
[12] C. Eling, R. Guendens, and T. Jacobson, “Nonequilibrium thermodynamics of spacetime,” *Physical Review Letters*, vol. 96, no. 12, p. 121301, 2006.
[13] M. Akbar and R. G. Cai, “Friedmann equations of FRW universe in scalar–tensor gravity, f(R) gravity and first law of thermodynamics,” *Physics Letters B*, vol. 635, no. 1, pp. 7–10, 2006.
[14] M. Akbar and R. G. Cai, “Thermodynamic behavior of field equations for f(R) gravity,” *Physics Letters B*, vol. 648, no. 2-3, pp. 243–248, 2007.
[15] M. Akbar and R. G. Cai, “Thermodynamic behavior of the Friedmann equation at the apparent horizon of the FRW universe,” *Physical Review D*, vol. 75, no. 8, 2007.
[16] R. G. Cai and L. M. Cao, “Unified first law and the thermodynamics of the apparent horizon in the FRW universe,” *Physical Review D*, vol. 75, no. 6, 2007.
[17] R. G. Cai and L. M. Cao, “Thermodynamics of apparent horizon in brane world scenario,” *Nuclear Physics B*, vol. 785, no. 1-2, pp. 135–148, 2007.
[18] A. Sheykhi, B. Wang, and R. G. Cai, “Thermodynamical properties of apparent horizon in warped DGP braneworld,” *Nuclear Physics B*, vol. 779, no. 1-2, pp. 1–12, 2007.
[19] A. Sheykhi, B. Wang, and R. G. Cai, “Deep connection between thermodynamics and gravity in Gauss-Bonnet braneworlds,” *Physical Review D*, vol. 76, no. 2, 2007.
[20] T. Padmanabhan, “Thermodynamical aspects of gravity: new insights,” *Reports on Progress in Physics*, vol. 73, no. 4, 2010.
[21] H. Moradpour, A. Sheykhi, N. Riazi, and B. Wang, “Necessity of Dark Energy from Thermodynamic Arguments,” *Advances in High Energy Physics*, vol. 2014, Article ID 718583, 9 pages, 2014.
[22] H. Moradpour and N. Riazi, “Thermodynamic equilibrium and rise of complexity in an accelerated universe,” *International Journal of Theoretical Physics*, vol. 55, no. 1, pp. 268–277, 2016.
[23] H. Moradpour and R. Dehghani, “Thermodynamical Study of FRW Universe in Quasi-Topological Theory,” *Advances in High Energy Physics*, vol. 2016, 7248510 pages, 2016.
[24] H. Moradpour and J. G. Salako, “Thermodynamic Analysis of the Static Spherically Symmetric Field Equations in Rastall Theory,” *Advances in High Energy Physics*, vol. 2016, Article ID 3492796, 5 pages, 2016.
[25] H. Moradpour and S. Nasirimoghdam, “Thermodynamic Motivations of Spherically Symmetric Static Metrics,” *Romanian Journal of Physics*, vol. 61, no. 9-10, p. 1453, 2016.
[26] H. Moradpour, N. Sadeghnezhad, S. Ghaffari, and A. Jahan, “Thermodynamic Analysis of Gravitational Field Equations in Lyra Manifold,” *Advances in High Energy Physics*, vol. 2017, Article ID 9687976, 6 pages, 2017.
[27] H. Moradpour, R. C. Nunes, E. M. C. Abreu, and J. A. Neto, “A note on the relations between thermodynamics, energy definitions and Friedmann equations,” *Modern Physics Letters A*, vol. 32, no. 13, p. 1750078, 2017.
[28] H. Moradpour, J. P. M. Graça, I. P. Lobo, and I. G. Salako, “Energy Definition and Dark Energy: A Thermodynamic Analysis,” *Advances in High Energy Physics*, vol. 2018, 8 pages, 2018.
[29] K. Bamba, A. Jawad, S. Rafique, and H. Moradpour, “Thermodynamics in Rastall gravity with entropy corrections,” *European Physical Journal C: Particles and Fields*, vol. 78, no. 12, 2018.
[30] H. Moradpour and M. Valipour, "Generalized Misner–Sharp energy in generalized Rastall theory," Journal de Physique, vol. 98, no. 9, pp. 853–856, 2020.

[31] M. Masi, "A step beyond Tsallis and Rényi entropies," Physics Letters A, vol. 338, no. 3-5, pp. 217–224, 2005.

[32] H. Moradpour, "Implications, consequences and interpretations of generalized entropy in the cosmological setups," International Journal of Theoretical Physics, vol. 55, no. 9, pp. 4176–4184, 2016.

[33] N. Komatsu, "Cosmological model from the holographic equipartition law with a modified Rényi entropy," European Physical Journal C: Particles and Fields, vol. 77, no. 4, p. 229, 2017.

[34] H. Moradpour, A. Bonilla, E. M. C. Abreu, and J. A. Neto, "Accelerated cosmos in a nonextensive setup," Physical Review D, vol. 96, no. 12, p. 123504, 2017.

[35] E. M. C. Abreu, J. A. Neto, A. C. R. Mendes, A. Bonilla, and R. M. de Paula, "Tsallis entropy, modified Newtonian accelerations and the Tully–Fisher relation," EPL, vol. 124, no. 3, p. 30005, 2018.

[36] M. Tavayef, A. Sheykhi, K. Bamba, and H. Moradpour, "Tsallis holographic dark energy," Physics Letters B, vol. 781, pp. 195–200, 2018.

[37] A. Sayahian Jahromi, S. A. Moosavi, H. Moradpour et al, "Generalized entropy formalism and a new holographic dark energy model," Physics Letters B, vol. 780, pp. 21–24, 2018.

[38] H. Moradpour, S. A. Moosavi, I. P. Lobo, J. P. M. Graça, A. Jawad, and I. G. Salako, "Thermodynamic approach to holographic dark energy and the Rényi entropy," European Physical Journal C: Particles and Fields, vol. 78, no. 10, p. 829, 2018.

[39] S. Ghaffari, A. H. Ziaie, V. B. Bezerra, and H. Moradpour, "Inflation in the Rényi cosmology," Modern Physics Letters A, vol. 35, no. 1, 2020.

[40] C. Tsallis and L. J. L. Cirto, "Black hole thermodynamical entropy," European Physical Journal C: Particles and Fields, vol. 73, no. 7, p. 2487, 2013.

[41] T. S. Biró and V. G. Czinner, "A q-parameter bound for particle spectra based on black hole thermodynamics with Rényi entropy," Physics Letters B, vol. 726, no. 4-5, pp. 861–865, 2013.

[42] A. Belin, A. Maloney, and S. Matsuura, "Holographic phases of Renyi entropies," Journal of High Energy Physics, vol. 2013, no. 12, 2013http://arxiv.org/abs/1306.2640.

[43] G. Czinnera and H. Iguchia, "Rényi entropy and the thermodynamic stability of black holes," Physics Letters B, vol. 752, pp. 306–310, 2016.

[44] V. G. Czinnera and H. Iguchia, "Thermodynamics, stability and Hawking–Page transition of Kerr black holes from Rényi statistics," European Physical Journal C: Particles and Fields, vol. 77, no. 12, p. 892, 2017.

[45] A. Bialas and W. Czyz, "Rényi entropies of a black hole from Hawking radiation," EPL (Europhysics Letters), vol. 83, 2008.

[46] S. Ghaaffari, A. H. Ziaie, H. Moradpour, F. Ashgharyan, F. Feleppa, and M. Tavayef, "Black hole thermodynamics in Sharma–Mittal generalized entropy formalism," General Relativity and Gravitation, vol. 51, no. 7, 2019.

[47] K. Mejhrat and S. E. Ennadifi, "Thermodynamics, stability and Hawking–Page transition of black holes from non-extensive statistical mechanics in quantum geometry," Physics Letters B, vol. 794, pp. 45–49, 2019.

[48] E. Abreu, J. A. Neto, E. M. Barboza Jr., A. C. Mendes, and B. B. Soares, "On the equipartition theorem and black holes non-gaussian entropies," 2020, http://arxiv.org/abs/2002.02435.

[49] H. Moradpour, A. H. Ziaie, and M. Kord Zangeneh, "Generalized entropies and corresponding holographic dark energy models," European Physical Journal C, vol. 80, p. 732, 2020, http://arxiv.org/abs/2005.06271.

[50] K. Ourabah, E. M. Barboza, E. M. C. Abreu, and J. A. Neto, "Superstatistics: Consequences on gravitation and cosmology," Physical Review D, vol. 100, no. 10, p. 103516, 2019.

[51] K. Ourabah, "Jeans instability in dark matter halos," Physica Scripta, vol. 95, no. 5, 2020.

[52] E. Verlinde, "On the origin of gravity and the laws of Newton," JHEP, vol. 4, p. 29, 2011.

[53] T. PADMANABHAN, "EQUIPARTITION OF ENERGY IN THE HORIZON DEGREES OF FREEDOM AND THE EMERGENCE OF GRAVITY," Modern Physics Letters A, vol. 25, no. 14, pp. 1129–1136, 2011.

[54] R. G. Cai, L. M. Gao, and N. Ohta, "Friedmann equations from entropic force," Physical Review D, vol. 81, no. 6, 2010.

[55] L. Modesto and A. Randonino, "Entropic corrections to Newton’s law," 2010, http://arxiv.org/abs/1003.1998.

[56] A. Sheykhi, "Entropic corrections to Friedmann equations," Physical Review D, vol. 81, no. 10, p. 104011, 2010.

[57] A. Kobakhidze, "Gravity is not an entropic force," Physical Review D, vol. 83, no. 2, 2011.

[58] S. Gao, "Is gravity an entropic force?," Entropy, vol. 13, no. 5, pp. 936–948, 2011.

[59] M. Chaichian, M. Oksanen, and A. Tureanu, "On gravity as an entropic force," Physics Letters B, vol. 702, no. 5, pp. 419–421, 2011.

[60] M. Visser, "Conservative entropic forces," Journal of High Energy Physics, vol. 2011, no. 10, p. 140, 2011.

[61] J. W. Lee, "On the origin of entropic gravity and inertia," Foundations of Physics, vol. 42, no. 9, pp. 1153–1164, 2012.

[62] M. Chaichian, M. Oksanen, and A. Tureanu, "On entropic gravity: the entropy postulate, entropy content of screens and relation to quantum mechanics," Physics Letters B, vol. 712, no. 3, pp. 272–278, 2012.

[63] E. M. C. Abreu and J. A. Neto, "Considerations on gravity as an entropic force and entangled states," Physics Letters B, vol. 727, no. 4-5, pp. 524–526, 2013.

[64] A. Sheykhi, H. Moradpour, and N. Riazi, "Lovelock gravity from entropic force," General Relativity and Gravitation, vol. 45, no. 5, pp. 1033–1049, 2013.

[65] H. Moradpour and A. Sheykhi, "From the Komar mass and entropic force scenarios to the Einstein field equations on the Ads brane," International Journal of Theoretical Physics, vol. 55, no. 2, pp. 1145–1155, 2016.

[66] R. C. Nunes, E. M. Barboza Jr., E. M. C. Abreu, and J. A. Neto, "Probing the cosmological viability of non-gaussian statistics," Journal of Cosmology and Astroparticle Physics, vol. 2016, no. 8, p. 51, 2016.

[67] R. C. Nunes, H. Moradpour, E. M. Barboza Jr., E. M. Abreu, and J. A. Neto, "Entropic gravity from noncommutative black holes," International Journal of Geometric Methods in Modern Physics, vol. 15, 2018.

[68] H. Moradpour, A. Amiri, and A. Sheykhi, "Implications of maximum acceleration on dynamics," Iranian Journal of Science and Technology, Transactions A: Science, vol. 43, no. 3, pp. 1295–1301, 2019.
[69] H. Moradpour, A. Sheykhi, C. Corda, and I. G. Salako, “Implications of the generalized entropy formalisms on the Newtonian gravity and dynamics,” *Physics Letters B*, vol. 783, pp. 82–85, 2018.

[70] H. Moradpour, A. H. Ziaie, S. Ghaffari, and F. Feleppa, “The generalized and extended uncertainty principles and their implications on the Jeans mass,” *Monthly Notices of the Royal Astronomical Society: Letters*, vol. 488, no. 1, pp. L69–L74, 2019.

[71] J. A. Neto, “Nonhomogeneous cooling, entropic gravity and MOND theory,” *International Journal of Theoretical Physics*, vol. 50, no. 11, pp. 3552–3559, 2011.

[72] E. Dil, “q-Deformed Einstein equations,” *Canadian Journal of Physics*, vol. 93, no. 11, pp. 1274–1278, 2015.

[73] E. Dil, “Can quantum black holes be (q, p)-fermions?,” *International Journal of Modern Physics A: Particles and Fields; Gravitation; Cosmology; Nuclear Physics*, vol. 32, no. 15, p. 1750080, 2017.

[74] E. M. C. Abreu, J. A. Neto, A. C. R. Mendes, A. Bonilla, and R. M. de Paula, “Cosmological considerations in Kaniadakis statistics,” *EPL*, vol. 124, no. 3, p. 30003, 2018.

[75] M. Senay and S. Kibaroğlu, “q-deformed Einstein equations from entropic force,” *International Journal of Modern Physics A*, vol. 33, no. 36, 2018.

[76] S. Kibaroğlu and M. Senay, “Effects of bosonic and fermionic q-deformation on the entropic gravity,” *Modern Physics Letters A*, vol. 34, no. 31, 2019.

[77] H. Moradpour, C. Corda, A. H. Ziaie, and S. Ghaffari, “The extended uncertainty principle inspires the Rényi entropy,” *EPL*, vol. 127, no. 6, p. 60006, 2019.

[78] J. D. Barrow, “The Area of a Rough Black Hole,” 2020, http://arxiv.org/abs/2004.09444.

[79] M. K. Zangeneh, H. Moradpour, and N. Sadeghnezhad, “A note on cosmological features of modified Newtonian potentials,” *Modern Physics Letters A*, vol. 34, no. 21, 2019.

[80] A. R. Plastino and J. A. S. Lima, “Equipartition and virial theorems within general thermostatistical formalisms,” *Physics Letters A*, vol. 260, no. 1-2, pp. 46–54, 1999.