Unparticle-Higgs Mixing: 
MSW Resonances, See-saw Mechanism and Spinodal Instabilities

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Abstract

Motivated by slow roll inflationary cosmology we study a scalar unparticle weakly coupled to a Higgs field in the broken symmetry phase. The mixing between the unparticle and the Higgs field results in a see-saw type matrix and the mixing angles feature a Mikheyev-Smirnov-Wolfenstein (MSW) effect as a consequence of the unparticle field being non-canonical. We find two (MSW) resonances for small and large space-like momenta. The unparticle-like mode features a nearly flat potential with spinodal instabilities and a large expectation value. An effective potential for the unparticle-like field is generated from the Higgs potential, but with couplings suppressed by a large power of the small see-saw ratio. The dispersion relation for the Higgs-like mode features an imaginary part even at “tree level” as a consequence of the fact that the unparticle field describes a multiparticle continuum. Mixed unparticle-Higgs propagators reveal the possibility of oscillations, albeit with short coherence lengths. The results are generalized to the case in which the unparticle features a mass gap, in which case a low energy MSW resonance may occur for light-like momenta depending on the scales. Unparticle-Higgs mixing leads to an effective unparticle potential of the new inflation form. Slow roll variables are suppressed by see-saw ratios and the anomalous dimensions and favor a red spectrum of scalar perturbations consistent with Cosmic Microwave Background (CMB) data.

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I. INTRODUCTION

As evidence for physics beyond the Standard Model accumulates, exploration of extensions is becoming more focused by the possibility of constraining them with forthcoming collider experiments. A recent proposal by Georgi [1] suggests that a conformal sector with a non-trivial infrared fixed point coupled to the standard model might be a possible extension with a wealth of phenomenological consequences, some of which may be tested at the Large Hadron Collider [2, 3, 4]. Early work by Banks and Zaks [5] provides a realization of a conformal sector emerging from a renormalization flow towards the infrared below an energy scale $\Lambda$ through dimensional transmutation, and supersymmetric QCD may play a similar role [6]. Below this scale there emerges an effective interpolating field, the unparticle field, that features an anomalous scaling dimension [1].

Recently various studies recognized important phenomenological [1, 2, 7] (for important caveats see [8]), astrophysical [9, 10, 11] and cosmological [12, 13, 14, 15, 16, 17, 18] consequences of unparticles, including Hawking radiation into unparticles [19] and potentially relevant phenomenology in CP-violation [20], flavor physics [21] and low energy parity violation [22].

A deconstruction program describes the unparticle from the coupling of a particle to a tower of a continuum of excitations [23]. Although this is an interesting interpretation of unparticles, the physics of anomalous dimensions arising from the exchange of massless (conformal) excitations is an ubiquitous feature in critical phenomena, a field that was already well developed before the advent of unparticle physics. Anomalous dimensions in the fermion propagators in gauge theories had been understood in the mid 40’s-50’s [24, 25], where multiple emissions and absorptions of massless quanta leads to anomalous dimensions. This is also well known within the context of QCD [26].

Critical phenomena associated with second order phase transitions provide a natural realization of unparticle physics. Indeed, a scalar order parameter, such as the magnetization in a three dimensional Heisenberg ferromagnet, features anomalous scaling dimensions at a critical point. That the multiple exchange of massless excitations leads to anomalous scaling exponents at critical points has been known since the 70’s and well understood via renormalization group or large N resummations by the 80’s [27, 28]. Scale invariance appears as a consequence of renormalization group flow towards an infrared fixed point and
the correlation functions of the order parameter scale with anomalous scaling dimensions. This, of course was the original motivation behind the Banks-Zaks suggestion within the context of non-abelian gauge theories \[5\]. In critical phenomena, integrating out degrees of freedom below a cutoff scale (in condensed matter systems determined by the lattice spacing) down to a renormalization scale \( \Lambda \) yields an effective field that describes long-wavelength phenomena below this scale. The effective potential of the unparticle field is the Landau-Ginzburg free energy, a functional of the order parameter, whose second derivative with respect to the (scalar) order parameter (the unparticle field) vanishes at the critical point; the effective potential becomes flat, reflecting the underlying scale invariance emerging at the infrared fixed point \[27, 28\].

All of these predate the “deconstruction” interpretation, in some cases by decades. Although the practitioners of “unparticle-physics” in the literature may prefer the “deconstruction” interpretation, we would like to emphasize that unparticle fields are ubiquitous in critical phenomena, and that the “deconstruction” interpretation is one, but by no means the only one.

In this article, we are motivated to study unparticle physics by the similarity between the effective field theory of single field slow roll inflation as a paradigm for cosmological inflation whose predictions are in remarkable agreement with WMAP data \[29, 30\], and a nearly critical theory. On the one hand, single field, slow roll inflation is based on the dynamics of a scalar field, the inflaton, whose evolution is determined by a fairly flat potential. The power spectrum of inflaton fluctuations is nearly Gaussian and scale invariant \[31\], and non-linear couplings are small. In ref. \[32\] it was argued that an effective field theory description a \( \text{là} \) Landau-Ginzburg provides a compelling description of single field slow roll inflation in which the hierarchy of slow roll parameters emerges as a systematic expansion in the number of e-folds and the non-linear couplings emerge as see-saw like ratios of two widely different scales, the Hubble scale during inflation and the Planck scale. The nearly Gaussian and scale invariant spectrum of fluctuations, the flatness of the potential, necessary to allow at least 60 e-folds, with the concomitant smallness of the mass of the inflaton field, and the smallness of the relevant couplings all suggest that perhaps a hidden scale (or conformal) invariant sector is underlying the successful paradigm of slow roll inflation (see also \[33\]).

In ref. \[32\], this observation led to the suggestion that perhaps, slow roll inflation is described by an effective field theory near a low energy fixed point, thus unparticle physics
may provide a framework for inflationary cosmology. Indeed, recently in [34] some of the roles of unparticles in inflationary cosmology were studied, along with the intriguing possibility that the unparticle field itself may be the inflaton.

On the other hand, however, inflation cannot be described by an exactly scale invariant theory: the power spectrum of inflaton fluctuations is not exactly scale invariant, and the inflaton potential cannot be completely flat, since inflation must end, eventually merging with standard Hot Big Bang cosmology, which in turn implies that the inflaton mass cannot vanish, thus preventing the inflaton from being described by a conformally invariant unparticle field [34].

Thus a mechanism that would lead to a small breaking of conformal invariance of the unparticle sector is sought. In ref. [6], it was recognized that if a scalar unparticle field couples to a Higgs field, a non-vanishing expectation value of the Higgs field leads to the explicit breakdown of the conformal symmetry in the unparticle sector. Although slow roll inflation is not exactly scale invariant, it is nearly so, so that the explicit breaking of scale invariance must be such so as to lead to a nearly flat inflaton potential, leading in turn to a nearly scale invariant spectrum of fluctuations [31]. This reasoning suggests that we should study the coupling between the unparticle and the Higgs field of the see-saw form, just as in the case of neutrino mixing [35, 36, 37] (see [38] for some work on this topic, and [39] where the unparticle and Higgs particle are taken as composites).

In this article, we focus on studying in detail the mixing between a scalar unparticle and a Higgs field in a spontaneously broken phase in flat space time as a prelude to dealing with inflationary cosmology. In particular we address the following issues.

- What are the consequences of mixing fields of different scaling dimensions? More specifically, using the language of neutrino mixing, how do we compute the mixing angles and “mass eigenstates”? Although unparticle-Higgs mixing has been studied in the literature [4, 6], to the best of our knowledge these issues have not been addressed (although see the first reference in [38]).

- Consider a see-saw mass matrix between two canonical scalar fields, one massless and one massive, namely

\[
\begin{pmatrix}
0 & m^2 \\
m^2 & M^2
\end{pmatrix}.
\] (I.1)
For $M^2 \gg m^2$, it follows that there is one eigenvalue $\sim M^2$ corresponding to the massive scalar and another eigenvalue $\sim -m^4/M^2$ corresponding to the lighter state. However, this latter eigenvalue describes an instability. Since a canonical scalar field is a special case of unparticle, it is natural to ask the questions if and how the fact that the unparticle field has non-canonical scaling dimension alters the see-saw mechanism and whether there is an instability as in the case of canonical fields.

It is interesting to note that in the absence of underlying symmetries, the vanishing matrix element corresponding to the lighter scalar would be modified by radiative corrections. However, it is precisely the underlying conformal symmetry in the unparticle sector that guarantees the vanishing of that matrix element.

What we find in this study is quite interesting. First we see that the mixing between the unparticle and the Higgs field enjoys a number of similarities with the MSW phenomenon of neutrino mixing in a medium [35, 36, 37, 40], namely the mixing angle depends on the energy. This is a direct consequence of the non-canonical nature of the unparticle fields, with the hidden sector that lends the multiparticle nature to the unparticle interpolating field acting as a “medium.” We find the possibility of two MSW resonances one at low and one at high energy.

We also show that the combined unparticle-Higgs system exhibits spinodal instability as well as a nearly flat potential. The propagator of the diagonal field closest to the unparticle field exhibits a pole for space-like momenta (in Minkowski space-time) which is exactly the signal of spinodal instability. For small unparticle-Higgs mixing we show that this instability implies that the field corresponding to the unparticle develops a large expectation value and its potential is nearly flat. This instability is a remnant of the instability described by the see-saw matrix (1.1). The unparticle-like field develops a potential with self-interaction which is suppressed by a high power of the see-saw ratio $m/M$.

Even when the unparticle-Higgs mixing is described by a linear coupling between them, namely $\propto UH$, the propagator of the Higgs-like field features a complex pole, the imaginary part (in Minkowski space-time) of which describes the decay of the Higgs-like degree of freedom into unparticle-like degrees of freedom. The fact that this decay can happen even for linear coupling reflects the fact that the unparticle field describes multi-particle states and the Higgs couples to a continuum described by the spectral density of the unparticle.
We generalize the above results for the case when symmetry breaking in the Higgs sector induces a mass gap in the unparticle sector.

We find that unparticle-Higgs coupling leads to an unparticle effective potential of the new inflation form, with coefficients that are suppressed by the see-saw ratios and further suppression from the anomalous dimensions for the resulting slow-roll parameters.

II. UNPARTICLE-HIGGS MIXING

The unparticle field describes a low energy conformal (or rather, a scale invariant) sector [1, 2, 5]. The unparticle field scales with an anomalous scale dimension that can be interpreted as a non-integral number of “invisible” particles [1]. This situation is akin to that of a scalar order parameter at a non-trivial (Wilson-Fisher) infrared fixed point in critical phenomena [27, 28]. At this critical point this sector becomes scale invariant and correlation functions of the unparticle field scale with anomalous dimensions. The anomalous scaling dimension reflects the nature of the multiparticle intermediate states and the unparticle propagator features a dispersive representation with a spectral density that features anomalous scaling exponents and describes branch cut singularities for time-like momenta.

We consider the following Euclidean-space Lagrangian for unparticle-Higgs mixing

\[ L = \int d^4x \int d^4y \left[ \frac{1}{2} U(x) F(x-y) U(y) + \Phi(x)(-\Box) \Phi(y) \right] + \int d^4x \left[ g \Lambda U(x) \Phi^2(x) + V(\Phi) \right], \]

where \( \Lambda \) has dimensions of mass (or momentum) and is a scale that characterizes the unparticle-field [1, 2, 5]. The unparticle field \( U \) emerges as a composite interpolating field that describes the infrared fixed point below this scale [1, 2, 5], and \( g \ll 1 \) is a dimensionless coupling.

The scalar potential \( V(\Phi) \) features a symmetry breaking minimum at \( \varphi \), therefore we write

\[ \Phi = \varphi + H, \]

and in order to study the mixing between the unparticle and the Higgs sector we keep only up to quadratic terms in \( U \) and \( H \) in the above Lagrangian,

\[ L_{(2)} = \int d^4x \int d^4y \left[ \frac{1}{2} U(x) F(x-y) U(y) + H(x) D(x-y) H(y) \right] + \int d^4x \left[ h U(x) + m^2 U(x) H(x) \right], \]
where

\[ h = g\Lambda \varphi^2 \; ; \quad m^2 = 2g\Lambda \varphi. \quad (II.4) \]

The vacuum expectation value of the Higgs field breaks explicitly scale invariance in the unparticle sector [6].

We note that the Lagrangian density (II.1) can be extended to include a term of the form \( U^2(x)\Phi^2(x) \) which would result in an explicit mass term \( M_U^2 U^2(x) \) for the unparticle field when the Higgs acquires an expectation value. This explicit mass term, which obviously breaks conformal symmetry, moves the unparticle threshold in the spectral density to \( M_U^2 \). Indeed the same massless “hidden sector” that gives the unparticle its multiparticle nature yields a spectral density that now features a threshold at the value of the conformal breaking mass \( M_U^2 \) [6]. We will consider this case explicitly in section (V).

The linear term in \( U \) in (II.3) leads to a tadpole contribution to the unparticle field that requires renormalization [4]. In what follows we will neglect this term and focus on the quadratic form to study the mixing.

The Euclidean space-time Fourier transforms of the non-local kernels \( F \) and \( D \), denoted by \( \mathcal{F}(p) \) and \( \mathcal{D}(p) \), respectively, are given by [1, 6]

\[ \mathcal{F}(p) = p^2 \left( \frac{p^2}{\Lambda^2} \right)^{-\eta}, \quad (II.5) \]
\[ \mathcal{D}(p) = p^2 + M_H^2, \quad (II.6) \]

where \( 0 \leq \eta < 1 \).

We will consider weak unparticle-Higgs mixing and that the unparticle scale \( \Lambda \gg \varphi \) [1, 2]. These result in the following hierarchy of mass scales:

\[ m \ll M_H \ll \Lambda. \quad (II.7) \]

For \( M_H \gg m \) the mixing matrix will be of the see-saw form which is our primary interest for inflationary cosmology as discussed above. Furthermore, consistent with the unparticle interpretation, the symmetry breaking scale of the Higgs sector must be below the unparticle scale so that the interpolating unparticle field is a suitable description of the hidden sector in the broken symmetry phase.

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1 The anomalous dimension \( \eta \) defined in this manner is twice the critical exponent for the two point correlation function in a critical theory [27].
In absence of unparticle-Higgs interaction, the Euclidean propagator for the unparticle field
\[ G_U(p) = \mathcal{F}^{-1}(p) = \frac{1}{p^2 \left( \frac{p^2}{\Lambda^2} \right)^{-\eta}} \quad \text{(II.8)} \]
is normalized so that
\[ G_U(\Lambda) = \frac{1}{\Lambda^2} \quad \text{(II.9)} \]
as is usually done in critical phenomena. This normalization fixes the wave function renormalization constant at the scale \( \Lambda \) [27]. Alternatively the field can be normalized so that
\[ dG_U^{-1}(p^2) dp^2 \bigg|_{p^2=\Lambda^2} = 1 \]
which differs from the previous normalization by a finite wavefunction renormalization constant for \( \eta < 1 \). This field normalization differs from the usually adopted one in the literature [1]. We note that the unparticle field \( U \) normalized in this manner features \textit{engineering} mass dimension 1 just like an ordinary scalar field, but \textit{scaling} mass dimension \( 1 - \eta \) as befits a conformal field with anomalous dimension \( \eta \). Therefore the \textit{engineering} mass-dimensions of \( h \) and \( m \) are 3 and 1, respectively. It is convenient to pass on to Fourier transforms (in Euclidean space-time) introducing the Fourier transforms of the fields as \( \tilde{U}, \tilde{H} \) respectively in terms of which the quadratic part of the action (II.3) becomes
\[ S_{(2)} = \int d^4 p \frac{1}{2} \left( \tilde{U}(-p) \tilde{H}(-p) \right) \left( \begin{array}{cc} \mathcal{F}(p) & m^2 \\ m^2 & \mathcal{D}(p) \end{array} \right) \left( \begin{array}{c} \tilde{U}(p) \\ \tilde{H}(p) \end{array} \right). \quad \text{(II.10)} \]
The Lagrangian is diagonalized in the basis of “mass” eigenstates (borrowing from the language of neutrino mixing) \( \Psi \) and \( \chi \), related to the unparticle and Higgs fields as
\[ \begin{pmatrix} \tilde{U}(p) \\ \tilde{H}(p) \end{pmatrix} = \begin{pmatrix} C(p) & S(p) \\ -S(p) & C(p) \end{pmatrix} \begin{pmatrix} \Psi(p) \\ \chi(p) \end{pmatrix} \quad \text{(II.11)} \]
where
\[ C(p) = \frac{1}{\sqrt{2}} \left[ 1 + \frac{\mathcal{D}(p) - \mathcal{F}(p)}{\left( \mathcal{D}(p) - \mathcal{F}(p) \right)^2 + 4m^4} \right]^{\frac{1}{2}} \quad \text{(II.12)} \]
\[ S(p) = \frac{1}{\sqrt{2}} \left[ 1 - \frac{\mathcal{D}(p) - \mathcal{F}(p)}{\left( \mathcal{D}(p) - \mathcal{F}(p) \right)^2 + 4m^4} \right]^{\frac{1}{2}} \quad \text{(II.13)} \]
are effectively the cosine \( C(p) \) and sine \( S(p) \) of the “mixing angle” between the unparticle and Higgs fields.
In terms of “mass eigenstate” fields Ψ, χ, we can write the action as

\[ S = \int d^4p \frac{1}{2} \left( \begin{array}{c} \Psi(-p) \\ \chi(-p) \end{array} \right) \left( \begin{array}{cc} G_\Psi^{-1}(p) & 0 \\ 0 & G_\chi^{-1}(p) \end{array} \right) \left( \begin{array}{c} \Psi(p) \\ \chi(p) \end{array} \right), \]  

where

\[ G_\Psi^{-1}(p) = \frac{1}{2} \left( \mathcal{F}(p) + \mathcal{D}(p) - \left[ (\mathcal{D}(p) - \mathcal{F}(p))^2 + 4m^4 \right]^{\frac{1}{2}} \right), \]  

\[ G_\chi^{-1}(p) = \frac{1}{2} \left( \mathcal{F}(p) + \mathcal{D}(p) + \left[ (\mathcal{D}(p) - \mathcal{F}(p))^2 + 4m^4 \right]^{\frac{1}{2}} \right). \]

### III. MSW EFFECT: RESONANCES, MIXING AND OSCILLATIONS

The similarity with the MSW effect of neutrinos in a medium [40] can be established by defining a “self-energy” for the unparticle field

\[ \Sigma_U(p) = p^2 \left[ \frac{p^2}{\Lambda^2} \right]^{-\eta} - 1, \]  

which for \( \eta \ll 1 \) is reminiscent of the one-loop self energy of a scalar order parameter at an infrared non-trivial critical point renormalized at a scale \( \Lambda \), namely \( \Sigma_U(p) \simeq -\eta p^2 \ln(p^2/\Lambda^2) \) [27]. The action (II.10) can be written as

\[ S = \int d^4p \frac{1}{2} \left( \tilde{U}(-p) \quad \tilde{H}(-p) \right) \left[ \left( p^2 + \frac{M_H^2}{2} \right) \mathbf{I} + \frac{1}{2} \left[ M_H^4 + 4m^4 \right]^{\frac{1}{2}} \left( \begin{array}{cc} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{array} \right) \right] \left( \begin{array}{c} \Sigma_U(p) \\ 0 \\ 0 \end{array} \right), \]  

where \( \mathbf{I} \) is the identity 2 × 2 matrix and

\[ \cos(2\theta) = \frac{M_H^2}{\left[ M_H^4 + 4m^4 \right]^{\frac{1}{2}}}, \]

\[ \sin(2\theta) = \frac{2m^2}{\left[ M_H^4 + 4m^4 \right]^{\frac{1}{2}}}. \]
This action can be written in terms of a “mixing angle in the medium” $\theta_m(p)$ as follows

$$L = \int d^4p \frac{1}{2} \left( \bar{U}(-p) \tilde{H}(-p) \right) \times$$

$$\left[ \left( p^2 + \frac{M_H^2}{2} + \frac{\Sigma_U(p)}{2} \right) I + \frac{1}{2} \left( (\Sigma_U(p) - M_H^2)^2 + 4m^4 \right)^{\frac{1}{2}} \left( \begin{array}{cc} -\cos(2\theta_m(p)) & \sin(2\theta_m(p)) \\ \sin(2\theta_m(p)) & \cos(2\theta_m(p)) \end{array} \right) \right]$$

$$\times \left( \frac{\bar{U}(p)}{\tilde{H}(p)} \right),$$

(III.5)

with

$$\cos(2\theta_m(p)) = \frac{M_H^2 - \Sigma_U(p)}{\left[ (\Sigma_U(p) - M_H^2)^2 + 4m^4 \right]^{\frac{1}{2}}},$$

(III.6)

$$\sin(2\theta_m(p)) = \frac{2m^2}{\left[ (\Sigma_U(p) - M_H^2)^2 + 4m^4 \right]^{\frac{1}{2}}},$$

(III.7)

This form makes manifest that the multiparticle contribution that defines the unparticle can be associated with a “medium effect”, namely the degrees of freedom that have been integrated out leading to the anomalous scaling dimension of the unparticle field, and leads to a MSW phenomenon [40], namely a dependence of the mixing angle on the energy (here the Euclidean momentum). From the expressions (III.6,III.7) it is clear that $C(p)$ and $S(p)$ of (II.12,II.13) are identical to $\cos(\theta_m(p))$ and $\sin(\theta_m(p))$, respectively.

There is an MSW resonance phenomenon when the condition

$$\Sigma_U(p) = M_H^2$$

(III.8)

is fulfilled. It is convenient to define

$$x \equiv \frac{p^2}{\Lambda^2}$$

(III.9)

in terms of which the resonance condition becomes

$$x \left[ x^{-\eta} - 1 \right] = \frac{M_H^2}{\Lambda^2}.$$ 

(III.10)

The function $x(x^{-\eta} - 1)$ is depicted in fig. 1. From this figure it becomes clear that there are two MSW resonances whenever

$$\frac{M_H^2}{\Lambda^2} < \eta \left( 1 - \eta \right)^{\frac{1-\eta}{\eta}}.$$

(III.11)
Consider the case $\Lambda^2 \gg M_H^2$ such that the condition for MSW resonances (III.11) is fulfilled: there is a low energy resonance ($p^2 \ll \Lambda^2$) at

$$p^2 \simeq M_H^2 \left( \frac{M_H^2}{\Lambda^2} \right)^{1/\eta}$$

(III.12)

and a high energy resonance ($p^2 \sim \Lambda^2$) at

$$p^2 \simeq \Lambda^2 \left[ 1 - \frac{1}{2\eta} \frac{M_H^2}{\Lambda^2} \right].$$

(III.13)

Upon analytically continuing from Euclidean to Minkowski momenta $p^2 \to -(p_0^2 - \vec{p}^2) - i0^+$ we note that the low energy resonance occurs near the light cone but for slightly space-like momenta (in the limit $M_H^2 \ll \Lambda^2$) whereas the high energy resonance occurs at large space-like momenta.

A. Mixing and oscillations

Unlike the case of neutrinos wherein a single particle Fock representation is a suitable description of the quantum mechanics of mixing and oscillations, the fact that the unparticle field is an effective field that describes multiparticle states prevents a similar analogy. However, we can learn about mixing and oscillation phenomena by studying correlation...
functions of the unparticle and Higgs fields. This is best achieved by introducing sources $J_U, \Phi$ conjugate to the respective fields and a generating functional $Z[J_U, J_\Phi]$ that yields the correlation functions through functional derivatives. It is straightforward to obtain the generating functional by inverting the quadratic form in the action (II.10), we find

$$\ln Z[J_U, J_\Phi] = \frac{1}{2} \int d^4 p \left( J_U(-p) J_\Phi(-p) \right) \left( G_\Psi(p) G_\chi(p) \left( D(p) - m^2 \right) \right) \left( J_U(p) \right),$$

(III.14)

where the Green’s functions of “mass eigenstates” $G_{\Psi,\chi}(p)$ are given by eqns. (II.15,II.16). For convenience of notation, we introduce

$$\alpha(p) = \frac{1}{2} \left( \mathcal{F}(p) + D(p) \right) = p^2 + \frac{M_H^2}{2} + \frac{\Sigma_U(p)}{2},$$

(III.15)

$$\beta(p) = \frac{1}{2} \left[ \left( \mathcal{F}(p) - D(p) \right)^2 + 4m^4 \right]^{\frac{1}{2}} = \frac{1}{2} \left[ \left( \Sigma_U(p) - M_H^2 \right)^2 + 4m^4 \right]^{\frac{1}{2}},$$

(III.16)

in terms of which $G^{-1}_\Psi(p) = \alpha(p) + \beta(p)$, $G^{-1}_\chi(p) = \alpha(p) - \beta(p)$. The diagonal and off-diagonal correlation functions in the unparticle-Higgs basis are given by

$$\langle U(p) U(-p) \rangle = \frac{D(p)}{\alpha^2(p) - \beta^2(p)},$$

(III.17)

$$\langle \Phi(p) \Phi(-p) \rangle = \frac{\mathcal{F}(p)}{\alpha^2(p) - \beta^2(p)},$$

(III.18)

$$\langle U(p) \Phi(-p) \rangle = -\frac{\beta(p)}{\alpha^2(p) - \beta^2(p)} \sin(2\theta_m(p)) = \frac{1}{2} \sin(2\theta_m(p)) \left[ \frac{1}{G_\chi(p)} - \frac{1}{G_\Psi(p)} \right].$$

(III.19)

The off-diagonal propagator (III.19) is exactly of the same form of the mixed propagators for two neutrinos in the flavor basis (II) where $G^{-1}_{\Psi,\chi}$ correspond to the propagators of mass eigenstates. In the case of neutrino mixing, the transition probability emerges directly from the off-diagonal correlation function as follows. Analytically continuing to Minkowski spacetime, each propagator has a simple pole at the values of $p_0 = E_1(\vec{p}), E_2(\vec{p})$ respectively. Then the time evolution of the off-diagonal correlator is obtained by performing an inverse Fourier transform in time, which in turn yields the time dependence for the off-diagonal correlator

$$\propto \sin(2\theta_m(p)) \left[ e^{i E_1(\vec{p}) t} - e^{i E_2(\vec{p}) t} \right].$$

The transition probability is obtained from the absolute value squared of the off-diagonal correlator (II) $P(t) \propto \sin^2(2\theta_m(p)) \sin^2 \left[ (E_1(\vec{p}) - E_2(\vec{p})) t / 2 \right]$. 

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Hence the off-diagonal correlation function (III.19) describes the generalization of mixing and oscillations for unparticle-Higgs mixing. For weak unparticle-Higgs coupling, we expect the propagators to feature singularities near those corresponding to the unparticle cut beginning at \( p^2 = 0 \) and Higgs pole at \( p^2 = M_H^2 \) respectively in Minkowski space-time. Therefore, unlike the case of almost degenerate neutrinos, the unparticle-Higgs oscillation will not be coherent over long space-time intervals. Instead, dephasing occurs on space-time scales of the order of the Compton wavelength of the Higgs-like mode \( \sim \frac{1}{M_H} \) and is further suppressed by the decay of this mode (see below).

IV. SINGULARITY STRUCTURE: POLES AND CUTS

The singularity structure of the propagators for the “mass eigenstates” is obtained from the conditions

\[
G^{-1}_\Psi(p) = 0, \quad G^{-1}_\chi(p) = 0.
\]

(IV.1)

Both conditions can be combined into

\[
\mathcal{F}(p) D(p) = m^4.
\]

(IV.2)

For \( m^2 \ll M_H^2 \ll \Lambda^2 \) we expect to find singularities near the Higgs “mass shell” \( p^2 \sim -M_H^2 \) and near the beginning of the massless threshold for the unparticle \( p^2 \sim 0 \).

- Higgs-like pole: To find the position of the singularity near the Higgs mass shell we write

\[
p^2 + M_H^2 \equiv M_H^2 \Delta ; \quad \Delta \ll 1
\]

(IV.3)

The condition (IV.2) yields

\[
\Delta \simeq \frac{m^4}{M_H^4} \left( \frac{-M_H^2}{\Lambda^2} \right)^\eta + \mathcal{O}\left( \frac{m^8}{M_H^8} \left( \frac{-M_H^4}{\Lambda^4} \right)^\eta \right).
\]

(IV.4)

Upon analytic continuation to Minkowski space time \( p_E^2 \to -p_M^2 ; M_H^2 \to M_H^2 - i0^+ \) the pole becomes complex with an imaginary part

\[
\Gamma_H \simeq \frac{m^4}{2 M_H^4} \left( \frac{M_H^2}{\Lambda^2} \right)^\eta \sin(\pi \eta).
\]

(IV.5)
This is a complex pole of $G_\chi(p)$ given by (II.15). From the expressions (II.12,II.13) with $p^2 \sim -M_H^2$, $D(p) \sim 0$, and $q^2(p) \gg m^4$, it follows that $C(p) \sim 1$, $S(p) \sim 0$ and the complex pole in $G_\chi(p)$ describes a Higgs-like unstable particle. What is remarkable in this result is the fact that the unparticle-Higgs coupling is linear in both fields, and the decay width emerges at “tree level”. This is in marked contrast with the decay rate from non-linear couplings studied in ref. [4]. If the unparticle field were canonical, such coupling would not result in an imaginary part in the propagator of the Higgs-like mode at tree level. However, the unparticle field is an interpolating field that describes a multiparticle composite with a continuum spectral density. Since the threshold begins at $p^2 = 0$, it follows that upon coupling the fields, the Higgs-pole on the positive real axis in the $p^2$ plane (in Minkowski space time) is actually embedded in the continuum of states described by the unparticle field, resulting in the motion of this pole off the physical sheet into a second or higher Riemann sheet. The imaginary part just describes the decay of the Higgs at “tree” level.

We note that the real part of the pole receives a finite mass renormalization, and the real part is light-like, therefore far away from the MSW resonance region which occurs for space-like momenta.

• Near unparticle threshold: The singularity near the unparticle threshold at $p^2 = 0$ can be found by setting $p^2 = 0$ in $D(p)$ and corresponds to a singularity in $G_\psi(p)$ given by eqn. (II.15).

Defining $x = p^2/\Lambda^2 \ll 1$ the condition (IV.2) yields

$$x^{1-\eta} \simeq \frac{m^4}{M_H^2 \Lambda^2} \Rightarrow p^2 \simeq \frac{m^4}{M_H^2} \left( \frac{m^4}{M_H^2 \Lambda^2} \right)^{\eta/(1-\eta)}$$

(IV.6)

For $\eta = 0$ one recovers the negative eigenvalue $-m^4/M_H^2$ of the see-saw mass matrix (I.1). Furthermore it is clear that although this pole is space like, it is well below the position of the low energy MSW resonance (III.12).

This pole on the positive real axis in Euclidean $p^2$ describes an instability since upon analytic continuation back to Minkowski space-time $p^2 \rightarrow -p^2 = -(\omega^2 - k^2)$ corresponding to frequencies

$$\omega(k) = \sqrt{k^2 - M^2} \quad ; \quad M^2 = m^4 \left( \frac{m^4}{M_H^2 \Lambda^2} \right)^{\eta/(1-\eta)}$$

(IV.7)
These become imaginary in the band of spinodally unstable modes \[42\]

\[0 < k^2 \leq \mathcal{M}^2. \quad (IV.8)\]

corresponding to spinodal instabilities \[42\].

Thus we see that the instability obtained from the see-saw mass matrix (I.1) is reproduced but with a coefficient that depends on the ratio of scales and the anomalous dimension. For this pole the mixing angles \(C(p) \sim 1, \ S(p) \sim 0\) and the \(\Psi\) field is identified with an unparticle-like mode, the pole for \(p^2 < 0\) in Minkowski is on the opposite side of the branch cut singularity for \(p^2 > 0\) and is isolated from the unparticle continuum.

The spinodal instabilities signal that the “potential” associated with the \(\Psi\) (unparticle-like) mode features a minimum away from the origin and the instabilities reflect the “rolling” of the expectation value of the \(\Psi\) field towards the minimum \[43\].

The unparticle-Higgs mixing leads to a potential for the unparticle-like field \(\Psi\). Consider the Higgs self-interaction \(\lambda H^4\) from (II.11)

\[
\lambda H^4 = \lambda \left(C(p)\chi(p) - S(p)\Psi(p)\right)^4 \approx \lambda S^4(p)\Psi^4(p), \quad (IV.9)
\]

where we have focused on the direction along which \(\chi = 0\). For the unstable pole we can set \(p \approx 0\) in the expression for the mixing angle (II.13 or III.7) and we find

\[
S(p) \sim \theta_m(p \approx 0) \approx \frac{m^2}{M_H^2}, \quad (IV.10)
\]

combining this result with eqn. (IV.7) leads to the effective unparticle potential for the \(\Psi\) mode

\[
\mathcal{V}(\Psi) \approx \mathcal{V}(0) - \frac{\mathcal{M}^2}{2} \Psi^2 + \lambda \frac{m^8}{M_H^8} \Psi^4. \quad (IV.11)
\]

which is of the form describing new inflation. Thus we see that the effective self-coupling for the \(\Psi\) mode (unparticle-like) is a see-saw ratio of scales, which for \(m \ll M_H\) consistent with small breaking of conformal invariance, entails that \(\mathcal{V}(\Psi)\) is very shallow. Even for \(\lambda \sim 1\), the quartic self-coupling for the unparticle field is a large power of the see-saw ratio \(m/M_H\) very similarly to the effective field theory approach to slow roll inflation discussed in ref. \[32\].
The expectation value of $\Psi$ is obtained by balancing the quadratic term whose coefficient is determined by the unstable pole (IV.6) and the quartic term (IV.11); we find

$$\langle \Psi \rangle \simeq \frac{M_H^3}{\lambda m^2} \left( \frac{m^2}{M_H\Lambda} \right)^{\eta/(1-\eta)}.$$  \hspace{1cm} (IV.12)

It is interesting to note that the spinodal instabilities exist independently of the specific form of the Higgs potential. Even if we replace the Higgs and its $\lambda H^4$ self-interaction with a scalar field with no potential, these instabilities persist. In this latter case, the unparticle field will acquire a potential by the virtue of the Coleman-Weinberg mechanism \[44\] and the instabilities reflect the rolling toward the minimum of this potential.

V. UNPARTICLE MASS GAP

In the study presented in the previous section, we have neglected a mass gap for the unparticle field.

As mentioned in section (II), a term $U^2\Phi^2$ in the unparticle-Higgs Lagrangian yields an explicit mass term for the unparticle field, which manifestly breaks conformal invariance. The emission and absorption of massless excitations of the “hidden” conformal sector that leads to the multiparticle nature of unparticles, results in that this mass term moves the threshold away from $p^2 = 0$ to $p^2 = M_U^2$ in the complex $p^2$ plane.

In ref. \[6\], a simple manner to introduce a mass scale to break conformal invariance in the unparticle sector was introduced by modifying the spectral representation of the unparticle propagator. Such a modification was also used in the study in ref. \[4\] where it is argued that the mass gap in the spectral representation arises from unparticle-Higgs coupling. The introduction of an unparticle mass gap $M_U$ results in a spectral density that features a branch cut beginning at a threshold $p^2 = -M_U^2$ in Euclidean momentum. The spectral density featuring a branch discontinuity with threshold $M_U^2$ resulting in anomalous scaling dimensions is reminiscent of the Bloch-Nordsieck \[24\] resummation of the emission and absorption of nearly collinear (soft) photons in quantum electrodynamics \[25\] where the fermion propagator acquires an anomalous dimension from infrared threshold divergences \[25\].

In ref. \[46\] it was shown that the renormalization group resummation of the infrared divergences arising from the emission and absorption of soft massless quanta by a massive
field is equivalent to the Bloch-Nordsieck resummation and yields precisely a spectral density
with a branch cut beginning at a threshold given by the mass of the particle that emits and
absorbs the soft quanta.

Motivated by these cases and following refs. [4, 6], we include an unparticle mass gap $M_U$
by modifying the $\mathcal{F}(p)$ in eqn. (II.5) to

$$\mathcal{F}(p) = (p^2 + M_U^2) \left[ \frac{p^2 + M_U^2}{\Lambda^2} \right]^{-\eta}.$$ (V.1)

This is the type of inverse propagator obtained by a renormalization group or alternatively a
Bloch-Nordsieck resummation of infrared divergences arising from emission and absorption
of soft massless quanta [46]. Consistent with a small breaking of conformal invariance, we
focus on the case in which $M_U \ll M_H \ll \Lambda$.

The analysis presented in the previous sections can be followed by replacing the unparticle
self-energy (III.1) by

$$\Sigma_U(p) = (p^2 + M_U^2) \left[ \left( \frac{p^2 + M_U^2}{\Lambda^2} \right)^{-\eta} - 1 \right],$$ (V.2)

and the action (III.5) by

$$L = \int d^4p \, \frac{1}{2} \left( \tilde{U}(-p) \, \tilde{H}(-p) \right) \times$$

$$\left[ (p^2 + M_H^2 + M_U^2) + \frac{\Sigma_U(p)}{2} \right] I + \frac{1}{2} \left[ \left( \Sigma_U(p) + M_U^2 - M_H^2 \right)^2 + 4m^4 \right]^{1/2} \begin{pmatrix} -\cos(2\theta_m(p)) & \sin(2\theta_m(p)) \\ \sin(2\theta_m(p)) & \cos(2\theta_m(p)) \end{pmatrix}$$

$$\times \begin{pmatrix} \tilde{U}(p) \\ \tilde{H}(p) \end{pmatrix},$$ (V.3)

with the “in medium” mixing angles determined by

$$\cos(2\theta_m(p)) = \frac{M_H^2 - M_U^2 - \Sigma_U(p)}{\left[ \left( \Sigma_U(p) + M_U^2 - M_H^2 \right)^2 + 4m^4 \right]^{1/2}},$$ (V.4)

$$\sin(2\theta_m(p)) = \frac{2m^2}{\left[ \left( \Sigma_U(p) + M_U^2 - M_H^2 \right)^2 + 4m^4 \right]^{1/2}}.$$ (V.5)

The condition for an MSW resonance now becomes

$$\Sigma_U(p) = M_H^2 - M_U^2 \equiv \delta M^2.$$ (V.6)
Upon introducing the variable $x = (p^2 + M^2_U)/\Lambda^2$ this condition becomes
\[ x[x^{-\eta} - 1] = \frac{\delta M^2}{\Lambda^2}, \] (V.7)
which is again satisfied with two resonances for
\[ 0 < \frac{\delta M^2}{\Lambda^2} < \eta (1 - \eta)^{\frac{1-n}{\eta}}. \] (V.8)
For $\Lambda \gg M_H^2 \gg M_U^2$ the resonances occur for
\[ p^2 \simeq -M_U^2 + \delta M^2 \left[ \frac{\delta M^2}{\Lambda^2} \right]^{\frac{1-n}{\eta}}, \] (V.9)
and
\[ p^2 \simeq -M_U^2 + \Lambda^2 \left[ 1 - \frac{1}{2\eta} \frac{\delta M^2}{\Lambda^2} \right]. \] (V.10)
Upon analytically continuing from Euclidean to Minkowski momenta $p^2 \rightarrow -(p_0^2 - \vec{p}^2) - i0^+$ we note that whereas the high energy resonance (V.10) occurs for space-like momenta there is the tantalizing possibility that the low energy resonance (V.9) could occur at light-like momenta. Obviously whether or not this possibility is realized depends on the details of the scales.

The results for mixing and oscillations obtained in section (III A) remain the same with obvious modifications in the mixing angles and propagators.

A. Singularity structure

The singularities in the propagators lead to the same condition (IV.2) as for $M_U^2 = 0$.

- Higgs-like pole: we write again
\[ p^2 + M_H^2 \equiv M_H^2 \Delta; \quad \Delta \ll 1 \] (V.11)
and the condition (IV.2) now yields
\[ \Delta \simeq -\frac{m^4}{M_H^4 \delta M^2} \left( \frac{-\delta M^2}{\Lambda^2} \right)^{\eta}. \] (V.12)

Upon analytic continuation to Minkowski space time $p^2_E \rightarrow -p^2_M; M_H^2 \rightarrow M_H^2 - i0^+$ the pole becomes complex with an imaginary part
\[ \Gamma_H \simeq \frac{m^4}{2 M_H \delta M^2} \left( \frac{\delta M^2}{\Lambda^2} \right)^{\eta} \sin(\pi \eta) \Theta(M_H^2 - M_U^2). \] (V.13)
The \( \Theta(M_H^2 - M_U^2) \) in (V.13) results from the fact that now the continuum spectral weight for the unparticle has a threshold at \( M_U^2 \). Thus, if \( M_H < M_U \) the pole in the bare Higgs propagator along the real axis in the Minkowski \( p^2 \) plane, lies below the unparticle continuum and the Higgs-like particle is stable. For \( M_H > M_U \) this (bare) pole is in the unparticle continuum and moves off into an unphysical sheet indicating a decay width for the Higgs-like particle.

- Near unparticle threshold: Defining \( x = (p^2 + M_U^2)/\Lambda^2 \ll 1 \) the condition (IV.2) yields

\[
x^{1-\eta} = \frac{m^4}{\delta M^2 \Lambda^2} \Rightarrow p^2 \simeq \frac{m^4}{\delta M^2} \left( \frac{m^4}{\delta M^2 \Lambda^2} \right)^{\eta/(1-\eta)}. \tag{V.14}
\]

For \( M_H^2 > M_U^2 \) this pole along the real axis in Euclidean space, again indicates spinodal instabilities. The band of spinodally unstable wave vectors is the same as in eqn. (IV.8) but with

\[
M^2 = \frac{m^4}{\delta M^2} \left( \frac{m^4}{\delta M^2 \Lambda^2} \right)^{\eta/(1-\eta)}. \tag{V.15}
\]

If \( M_U^2 > M_H^2 \) the value of \( p^2 \) becomes complex, indicating the possibility of the decay of the unparticle-mode. While this possibility is interesting and deserves to be explored in its own right, we are primarily interested in the case \( M_H^2 >> M_U^2 \) because this case corresponds to the see-saw mechanism with two widely different mass scales that may be of relevance for inflationary cosmology.

Therefore we conclude that in the relevant case \( M_H^2 > M_U^2 \) the results obtained are a straightforward generalization of the case \( M_U^2 = 0 \) with the same features, namely, a band of spinodally unstable wavevectors, a shallow effective potential for the unparticle mode and a width for the Higgs-like particle indicating its decay into unparticle modes as a consequence of the multiparticle continuum described by them.

Furthermore the effective unparticle potential is of the same form as (IV.11) but with the replacement \( M_H^2 \rightarrow \delta M^2 \) with similar conclusions.

VI. POTENTIAL CONSEQUENCES FOR SLOW ROLL INFLATION

Although the WMAP data [29, 30] rules out a purely quartic inflaton potential, a systematic analysis combined with Markov-chain Montecarlo of the available data from the CMB
and large scale structure shows that a new-inflation type potential with a non-vanishing mass term, just as that given by eqn. (IV.11) fits the data remarkably well [45]. Therefore identifying the unparticle with the inflaton field [34], with the effective inflaton potential given by (IV.11), we can advance some important preliminary consequences for slow roll inflation. Both lowest order slow roll parameters

\[
\epsilon_v = \frac{M_{Pl}^2}{2} \left( \frac{V'}{V} \right)^2; \quad \eta_v = \frac{M_{Pl}^2}{2} \frac{V''}{V}
\]  

(VI.1)

involve \( M^2 \) for slow roll initial conditions (\( \Psi \sim 0 \)). We see from (IV.7) that the slow roll parameters are determined \( M^2 \) (for a general dependence see [45]). This term is suppressed by the small see-saw ratio \( m^2/M_H^2 \), but also for a non-trivial unparticle anomalous dimension \( 0 < \eta < 1 \) it is further suppressed by the factor

\[
\left( \frac{m^2}{M_H \Lambda} \right)^{\eta/(1-\eta)},
\]

making the slow roll parameters smaller. To lowest order in slow roll parameters, the scalar index of the power spectrum is \( n_s = 1 + 2\eta_v - 6\epsilon_v \) [29, 30]. We note that whereas \( \epsilon_v \) is manifestly positive, the sign of \( \eta_v \) determines whether the power spectrum is red or blue tilted. The effective unparticle potential (IV.11) distinctly yields a red tilt which is consistent with the results from WMAP [29, 30].

We would like to emphasize that here, we have used slow roll parameters for a field with canonical kinetic term. This is done to give an indication that simply based on the potential, an unparticle field description of the inflaton can yield a nearly scale invariant power spectrum. However, as the kinetic term of the unparticle-like field is not canonical, a deeper understanding of the slow roll conditions in this case is required. This will be explored in a future work. It is conceivable that the next generation of CMB observations may yield information on the unparticle anomalous exponent.

VII. CONCLUSIONS

Motivated by the successful and predictive paradigm of slow roll inflation, we explored the possibility that unparticle physics may yield to an underlying understanding of the main features of slow roll inflation: a fairly flat potential, nearly Gaussian and scale invariant
spectrum of fluctuations. We identify these as hallmarks of a nearly critical theory. Thus, unparticle physics, describing a conformally invariant theory appears as a natural candidate for an underlying theory of slow roll inflation. However, conformal invariance must be explicitly broken but in a “small” manner, since inflation must end. Coupling unparticle and Higgs fields in a broken symmetry phase of the Higgs sector leads to an explicit breaking of conformal symmetry in the unparticle sector \[6\]. However, we seek a mechanism that yields small breaking of this symmetry in the form of a weak coupling between unparticle and Higgs sectors and a see-saw type mixing matrix between them. Thus we study a model of a scalar unparticle weakly coupled to a Higgs field in a broken symmetry phase. The expectation value of the Higgs leads to linear Higgs-unparticle coupling and a see-saw type mixing matrix. We find a wealth of phenomena possibly relevant to inflationary cosmology but also of intrinsic interest.

As a consequence of the unparticle field being defined by non-canonical scaling dimensions we find that the mixing angles depend on the momentum four vector leading to an MSW effect: the hidden sector that is integrated out to define the interpolating unparticle field acts as a medium. We find two MSW resonances, one at low and one at high energy respectively. For low momentum we find isolated poles in the unparticle-like mode away from the branch cuts that characterize the unparticle spectrum. These poles describe spinodal instabilities and indicate that the unparticle field acquires an expectation value. Indeed, the Higgs potential generates an effective potential for the unparticle-like field because of the mixing. We find that even for a strongly self-coupled Higgs sector, the self-couplings of the unparticle-like mode are suppressed by large powers of the see saw ratios. The instability and small self-couplings both entail a nearly flat potential for the unparticle-like field, a hallmark of slow roll inflation. We also find the remarkable result that the propagator for the Higgs-like mode features a complex pole, whose imaginary part determines the decay rate. We emphasize that this is a “tree-level” effect, \textit{not a result of non-linear couplings} and is a consequence of the continuum in the spectral representation of the unparticle field. The pole in the bare Higgs propagator becomes embedded in the continuum of the unparticle field upon their coupling, even for a linear coupling.

Unparticle-Higgs mixing also leads to oscillation phenomena just as in the case of neutrino mixing. However, because of the large difference in scales, oscillations decohere on short space-time scales of the order of the Compton wavelength of the Higgs particle.
The results obtained for a massless scale invariant unparticle field were then generalized to the case in which there is an unparticle mass gap $M_U$. A see-saw mechanism consistent with a slow roll picture requires that the unparticle mass gap be much smaller than the Higgs mass, in which case all of the results of the massless unparticle case translate with minor modifications to the case of a massive threshold for the unparticles.

A major result of this exploration is that an unparticle field weakly coupled to a Higgs particle yields a remarkable similarity to the main features of slow roll inflation and could possibly provide an underlying justification for the slow roll paradigm. Unparticle-Higgs see-saw type coupling yields an effective unparticle potential of the new inflation form with coefficients that are suppressed by see-saw ratios. Simply based on this potential, the further suppression by the anomalous dimension of the unparticle field might lead to smaller departures from scale invariance and to a red spectrum consistent with the WMAP results on the scalar spectral index.

Since the unparticle field features non-canonical kinetic terms, we envisage these models as possible alternatives to Dirac-Born-Infeld (DBI) inflationary proposals [47]. An important follow-up will be to understand the influence of the non-canonical kinetic terms for the unparticle-like field so as to establish a firmer correspondence with the dynamics of slow roll inflation. Non-canonical kinetic terms in other contexts have been studied in refs. [48] and we will explore these aspects along with loop corrections in future work.

We focused our study in Minkowski space time as a prelude to a more comprehensive exploration of inflationary cosmology. The results obtained here are most certainly encouraging and suggest that further exploration within the inflationary context is worthwhile.

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