Heavy-Quark Correlations in Photon-Hadron Collisions

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Abstract

We describe a next-to-leading-order calculation of the fully exclusive parton cross section at next-to-leading order for the photoproduction of heavy quarks. We use our result to compute quantities of interest for current fixed-target experiments. We discuss heavy-quark total cross sections, distributions, and correlations.
1. Introduction

Heavy-quark photoproduction is a phenomenon of considerable interest. It is closely related to the hadroproduction phenomenon, but it is also considerably simpler, since the incoming photon is a much better understood object than an incoming hadron. Aside from being a good testing ground of our understanding of perturbative QCD, it is also a probe of the structure of the target hadron. In fact, it has been often pointed out that heavy-quark photoproduction is a viable way to measure the gluon structure function in the proton \[1\].

Radiative corrections to the single-inclusive photoproduction of heavy quarks have been first computed in ref. \[2\]. The recent work of ref. \[3\] has confirmed the first computation, thus making the photoproduction cross section up to order \(O(\alpha_{em}\alpha_s^2)\) a well-established result. From the next-to-leading-order computations the following facts have emerged. First of all, the photoproduction cross section receives more moderate next-to-leading corrections than the hadroproduction case. This result has improved the consistency of the data on charm production with the theoretical computation. In fact, before the radiative corrections were known, it was difficult to accommodate the experimentally observed hadroproduction and photoproduction cross sections with the same value of the charm quark mass, the first one requiring much smaller masses.

A large amount of experimental information is available on photoproduction of heavy flavours \[4\]. Comparison between theory and experiments has not gone much further than the total cross section. This is a consequence of the fact that only charm production data have been available, and that the single-inclusive charm spectrum is strongly modified by non-perturbative effects. There is reasonable hope that, by looking at more exclusive distributions, we could learn more from photoproduction results. Modern fixed-target photoproduction experiments have the capability to study correlations between the heavy quark and antiquark. Furthermore, at the \(ep\) collider HERA, a large charm and bottom cross section is expected. It is clear, therefore, that in order to make progress in the physics of heavy-quark production, an exclusive next-to-leading-order calculation of the photoproduction cross section is needed. This may turn out to be useful both in charm production at fixed-target experiments and at HERA, and in bottom production at HERA. Since higher-order
corrections are moderate even in the charm case, it is possible that certain charm
distributions may be used for QCD studies.

In this paper we describe a next-to-leading-order computation of the doubly differ-
ential cross section for the photoproduction of heavy-quark pairs. This computation
follows closely the analogous work of refs. [5] and [6] for hadroproduction of heavy
quarks. Our result is implemented in the form of a “parton” event generator, which
can be used to compute any distribution accurate to the next-to-leading order in the
strong coupling constant. The problems arising from soft and collinear divergences
are dealt with by generating appropriate sequences of correlated events, in such a way
that the cancellation of collinear and soft singularities takes place for any well-defined
physical distribution (i.e. distributions that are insensitive to soft and collinear emis-
sion). The advantage of this method (developed for the first time in ref. [7]) is that
it does not require any artificial regularization of the cross section for producing the
quark-antiquark pair plus a light parton. A detailed description of this method is
given in ref. [8]. In what follows we will describe the photoproduction calculation,
with some emphasis on the differences with the hadroproduction case.

This paper is organized as follows. In Section 2 we give a general description of
the calculation. Some subtleties arise in the photoproduction calculation, which have
to do with factorization scale choices. We discuss these problems in Section 3. Some
phenomenological applications of our result have already been given in ref. [8], where
a particular doubly differential cross section (of interest to the extraction of the gluon
density from heavy-quark photoproduction data) is studied. In this work, we limit
ourselves to the study of fixed-target photoproduction. More detailed studies of heavy
quark production at HERA will be given in future works [9]. In Section 4 we discuss
the total cross section, and in Section 5 we discuss the differential distributions in
fixed-target experiments.

2. Description of the calculation

The partonic subprocesses relevant for heavy-quark photoproduction at order
$\alpha_{em}\alpha_s^2$ are the two-body process

\[ \gamma g \rightarrow Q\bar{Q} \]  \hspace{1cm} (2.1)
and the three-body processes

\[ \gamma g \rightarrow Q\bar{Q}g \]
\[ \gamma q \rightarrow Q\bar{Q}q. \]  

(2.2)

We will describe the two-body process in terms of the quantities

\[ s = (p_1 + p_2)^2 \]
\[ t = (p_1 - k_1)^2 - m^2 = (p_2 - k_2)^2 - m^2 \]
\[ u = (p_1 - k_2)^2 - m^2 = (p_2 - k_1)^2 - m^2, \]  

(2.3)

where \( p_1 \) is the photon momentum, \( p_2 \) is the gluon momentum, and \( k_1, k_2 \) are the momenta of the heavy quark and antiquark, respectively. We have \( p_1^2 = p_2^2 = 0 \) and \( k_1^2 = k_2^2 = m^2 \), where \( m \) is the mass of the heavy quarks, and \( s + t + u = 0 \) (notice that the definition of \( t \) and \( u \) is not the conventional one).

We will use dimensional regularization to deal with the divergences appearing in intermediate steps of the calculation. For this reason, we will need the expressions of phase spaces in \( d = 4 - 2\epsilon \) dimensions. The two-body phase space is given by

\[ d\Phi_2 = \frac{2^{2\epsilon}}{\Gamma(1 - \epsilon)} \left( \frac{4\pi}{s} \right)^{\epsilon} \frac{1}{16\pi} \beta^{1-2\epsilon} \sin^{-2\epsilon} \theta_1 d\cos \theta_1, \]  

(2.4)

where \( \beta = \sqrt{1 - \rho} \), \( \rho = 4m^2/s \) and \( \theta_1 \) is the angle between \( \vec{p}_1 \) and \( \vec{k}_1 \) in the centre-of-mass system of the incoming partons. Therefore,

\[ t = -\frac{s}{2}(1 - \beta \cos \theta_1). \]  

(2.5)

The three-body processes are characterized by five independent scalar quantities:

\[ s = (p_1 + p_2)^2 \]
\[ t_k = (p_1 - k)^2 \]
\[ u_k = (p_2 - k)^2 \]
\[ q_1 = (p_1 - k_1)^2 - m^2 \]
\[ q_2 = (p_2 - k_2)^2 - m^2 \]  

(2.6)
where $p_1$ is the photon momentum, $p_2$ is the momentum of the incoming parton, $k_1$ and $k_2$ are the momenta of the heavy quark and antiquark, respectively, and $k$ is the momentum of the emitted light parton. We will often use the variable $s_2$, the invariant mass of the heavy quark-antiquark pair, which is related to our independent invariants through

$$s_2 = (k_1 + k_2)^2 = s + t_k + u_k. \quad (2.7)$$

It will be convenient to introduce variables $x$ and $y$, where $x = s_2/s$ and $y$ is the cosine of the angle between $\vec{p}_1$ and $\vec{k}$ in the centre-of-mass system of the incoming partons. We have

$$\rho \leq x \leq 1, \quad -1 \leq y \leq 1 \quad (2.8)$$

and

$$t_k = -\frac{s}{2}(1 - x)(1 - y), \quad u_k = -\frac{s}{2}(1 - x)(1 + y). \quad (2.9)$$

In the centre-of-mass frame of the $Q\bar{Q}$ system, our four-momenta are given by

$$p_1 = p_1^0 (1, 0, 0, 1) \quad p_2 = p_2^0 (1, 0, \sin \psi, \cos \psi) \quad k = k^0 (1, 0, \sin \psi', \cos \psi')$$

$$k_1 = \frac{\sqrt{s_2}}{2} (1, \beta_x \sin \theta_2 \sin \theta_1, \beta_x \cos \theta_2 \sin \theta_1, \beta_x \cos \theta_1)$$

$$k_2 = \frac{\sqrt{s_2}}{2} (1, -\beta_x \sin \theta_2 \sin \theta_1, -\beta_x \cos \theta_2 \sin \theta_1, -\beta_x \cos \theta_1), \quad (2.10)$$

where

$$p_1^0 = \frac{s + t_k}{2\sqrt{s_2}}, \quad p_2^0 = \frac{s + u_k}{2\sqrt{s_2}} \quad k^0 = -\frac{t_k + u_k}{2\sqrt{s_2}}$$

$$\cos \psi = 1 - \frac{s}{2p_1^0 p_2^0}, \quad \sin \psi > 0$$

$$\cos \psi' = 1 + \frac{t_k}{2p_1^0 k^0}, \quad \sin \psi' > 0$$

$$\beta_x = \sqrt{1 - \frac{4m^2}{s\rho}}. \quad (2.11)$$
The two remaining independent invariants \( q_1, q_2 \) are given by

\[
q_1 = -\frac{s + t_k}{2}(1 - \beta_x \cos \theta_1)
\]

\[
q_2 = -\frac{s + u_k}{2}(1 + \beta_x \cos \theta_2 \sin \theta_1 \sin \psi + \beta_x \cos \theta_1 \cos \psi).
\] (2.12)

Now all invariants are expressed in terms of \( x, y, \theta_1, \theta_2 \) and \( s \) through eqs. (2.9), (2.11) and (2.12).

The three-body phase space in terms of the variables \( x, y, \theta_1, \theta_2 \) is given by

\[
d\Phi_3 = H N d\Phi_2^{(x)} \frac{s^{1-\epsilon}}{2\pi}(1-x)^{1-2\epsilon}(1-y^2)^{-\epsilon} dy \sin^{-2\epsilon} \theta_2 d\theta_2,
\] (2.13)

where

\[
H = \frac{\Gamma(1-\epsilon)}{\Gamma(1+\epsilon)\Gamma(1-2\epsilon)} = 1 - \frac{\pi^2}{3} \epsilon^2 + \mathcal{O}(\epsilon^3)
\] (2.14)

\[
N = \frac{(4\pi)^\epsilon}{(4\pi)^2} \Gamma(1+\epsilon)
\] (2.15)

and

\[
d\Phi_2^{(x)} = \frac{2^{2\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{4\pi}{sx}\right)^\epsilon \frac{1}{16\pi} \beta_x^{1-2\epsilon} \sin^{-2\epsilon} \theta_1 d \cos \theta_1 dx.
\] (2.16)

Both \( \theta_1 \) and \( \theta_2 \) range between 0 and \( \pi \).

We are now ready to compute the cross section for the real emission processes of eq. (2.2). The technique is the same as that used in ref. [5]. We begin with the subprocess \( \gamma g \rightarrow Q\bar{Q}g \). The cross section (in \( d \) space-time dimensions) is given by

\[
d\sigma_{\gamma g}^{(r)} = \mathcal{M}_{\gamma g}^{(r)}(s, t_k, u_k, q_1, q_2) d\Phi_3
\] (2.17)

\[
\mathcal{M}_{\gamma g}^{(r)}(s, t_k, u_k, q_1, q_2) = \frac{1}{2s} \frac{1}{[2(1-\epsilon)]^2 (N_c^2 - 1)} \sum_{\text{spin, color}} |A_{\gamma g}^{(r)}|^2,
\] (2.18)

where \( A_{\gamma g}^{(r)} \) is the invariant amplitude. The invariant cross section \( \mathcal{M}_{\gamma g}^{(r)} \) has singularities in \( t_k = 0 \) and \( u_k = 0 \), corresponding to soft \( (x = 1) \) and collinear \( (y = -1) \) gluon emission. No collinear emission from the photon line takes place at this order for the \( \gamma g \rightarrow Q\bar{Q}g \) subprocess, and therefore \( \mathcal{M}_{\gamma g}^{(r)} \) is regular at \( y = 1 \). It can be shown that the leading soft singularity behaves like \( 1/(1-x)^2 \), and that no double poles appear.
in $t_k$ and $u_k$. Therefore the function
\[ f_{\gamma g}(x, y, \theta_1, \theta_2) = 4t_k u_k \mathcal{M}^{(r)}_{\gamma g}(s, t_k, u_k, q_1, q_2) \] (2.19)
is regular for $y = -1$ and $x = 1$ (the dependence of $f_{\gamma g}$ upon $s/m^2$ is not explicitly shown). Using eqs. (2.9) we get
\[ M^{(r)}_{\gamma g}(s, t_k, u_k, q_1, q_2) = f_{\gamma g}(x, y, \theta_1, \theta_2) s^2 (1 - x)^2 (1 - y^2). \] (2.20)
The three-body contribution to our cross section, including the phase space, is then given by
\[ d\sigma^{(r)}_{\gamma g} = H N d\Phi_2^{(x)} \frac{s^{-1-\epsilon}}{2\pi} dy \sin^{-2\epsilon} \theta_2 d\theta_2 (1 - x)^{-1-2\epsilon} (1 - y^2)^{-1-\epsilon} f_{\gamma g}(x, y, \theta_1, \theta_2). \] (2.21)
We can now use the following expansions, valid for small $\epsilon$
\begin{align*}
(1 - x)^{-1-2\epsilon} &= -\tilde{\beta}^{-4\epsilon} \delta(1 - x) + \left( \frac{1}{1 - x} \right) \tilde{\rho} - 2\epsilon \left( \frac{\log(1 - x)}{1 - x} \right) \tilde{\rho} \\
+ &\mathcal{O}(\epsilon^2) \quad (2.22) \\
(1 - y^2)^{-1-\epsilon} &= -[\delta(1 + y) + \delta(1 - y)] \frac{(2\omega)^{-\epsilon}}{2\epsilon} \\
+ &\frac{1}{2} \left[ \left( \frac{1}{1 - y} \right) \omega + \left( \frac{1}{1 + y} \right) \omega \right] + \mathcal{O}(\epsilon), \quad (2.23)
\end{align*}
where the distributions in round brackets are defined according to the prescriptions
\begin{align*}
\int_{\tilde{\rho}}^1 h(x) \left( \frac{1}{1 - x} \right) \tilde{\rho} dx &= \int_{\tilde{\rho}}^1 h(x) - h(1) \frac{1}{1 - x} dx \\
\int_{\tilde{\rho}}^1 h(x) \left( \frac{\log(1 - x)}{1 - x} \right) \tilde{\rho} dx &= \int_{\tilde{\rho}}^1 [h(x) - h(1)] \frac{\log(1 - x)}{1 - x} dx \\
\int_{1-\omega}^1 h(y) \left( \frac{1}{1 - y} \right) \omega dy &= \int_{1-\omega}^1 h(y) - h(1) \frac{1}{1 - y} dy \\
\int_{-1}^{-1+\omega} h(y) \left( \frac{1}{1 + y} \right) \omega dy &= \int_{-1}^{-1+\omega} h(y) - h(-1) \frac{1}{1 + y} dy, \quad (2.24)
\end{align*}
for any test function $h(x)$. We define $\tilde{\beta} = \sqrt{1 - \tilde{\rho}}$. The parameters $\tilde{\rho}$ and $\omega$ should
be chosen within the ranges
\[ \rho \leq \tilde{\rho} < 1, \quad 0 < \omega \leq 2. \] (2.25)

The final results will not depend on the particular values chosen for \( \tilde{\rho} \) and \( \omega \), but different choices can lead to better convergence in the numerical programs, as discussed in ref. [5]. We obtain

\[
d\sigma^{(r)}_{\gamma g} = d\sigma^{(s)}_{\gamma g} + H N d\Phi_2^{(x)} s^{-1-\epsilon} \frac{dy}{2\pi} \sin^{-2\epsilon} \theta_2 d\theta_2
\times \left[ \left( \frac{1}{1-x} \right) - 2\epsilon \left( \frac{\log(1-x)}{1-x} \right) \right] \left( 1 - y^2 \right)^{-1-\epsilon} f_{\gamma g}(x, y, \theta_1, \theta_2), \tag{2.26}
\]

where

\[
d\sigma^{(s)}_{\gamma g} = H N d\Phi_2^{(x)} s^{-1-\epsilon} \frac{dy}{2\pi} \sin^{-2\epsilon} \theta_2 d\theta_2
\times \left[ -\tilde{\beta}^{-4\epsilon} \frac{\delta(1-x)}{2\epsilon} \right] \left( 1 - y^2 \right)^{-1-\epsilon} f_{\gamma g}(x, y, \theta_1, \theta_2). \tag{2.27}
\]

The details of the calculation of the soft component of the cross section, \( d\sigma^{(s)}_{\gamma g} \), are given in Appendix A. Equation (2.27) can be explicitly integrated over \( x, y \) and \( \theta_2 \) to obtain

\[
d\sigma^{(s)}_{\gamma g} = -H N d\Phi_2^{(x)} \frac{1}{4\pi\epsilon} s^{-1-\epsilon} \tilde{\beta}^{-4\epsilon} f^{(s)}_{\gamma g}(\theta_1), \tag{2.28}
\]

where the function \( f^{(s)}(\theta_1) \) is given in Appendix A.

We now expand \( (1 - y^2)^{-1-\epsilon} \) in the second term of eq. (2.26), observing that we only need the expansion up to order \( \epsilon^0 \). As noticed above, \( \mathcal{M}^{(r)}_{\gamma g} \) is regular at \( y = 1 \), and therefore the term proportional to \( \delta(1 - y) \) gives no contribution. We get

\[
d\sigma^{(r)}_{\gamma g} = d\sigma^{(s)}_{\gamma g} + d\sigma^{(c-)}_{\gamma g} + d\sigma^{(f)}_{\gamma g}, \tag{2.29}
\]

where

\[
d\sigma^{(c-)}_{\gamma g} = N d\Phi_2^{(x)} s^{-1-\epsilon} \frac{dy}{2\pi} \sin^{-2\epsilon} \theta_2 d\theta_2 \left[ \left( \frac{1}{1-x} \right) - 2\epsilon \left( \frac{\log(1-x)}{1-x} \right) \right] \left( 1 - y^2 \right)^{-1-\epsilon} f_{\gamma g}(x, y, \theta_1, \theta_2)
\times \left[ -\frac{(2\omega)^{-\epsilon}}{2\epsilon} \delta(1 + y) \right] f_{\gamma g}(x, y, \theta_1, \theta_2) \tag{2.30}
\]
and

\[ d\sigma^{(f)}_{\gamma g} = N s^{-1} \frac{1}{64\pi^2} \beta_x d \cos \theta_1 d \theta_2 dy dx \]
\[ \times \left( \frac{1}{1-x} \right) \rho \left( \frac{1}{1-y} \right) \left( 1 + \frac{1}{1+y} \right) f_{\gamma g}(x, y, \theta_1, \theta_2). \]  

(2.31)

The technique for the evaluation of the collinear limit of the invariant cross section is described in detail in ref. [5]. The result in our case is

\[ d\sigma^{(c-)}_{\gamma g} = -N s^{-1-\epsilon} \left( \frac{2}{\omega} \right)^\epsilon d\Phi_2 \left[ \left( \frac{1}{1-x} \right) \rho \left( \frac{1}{1-y} \right) \left( 1 + \frac{1}{1+y} \right) \right] f^{(c-)}_{\gamma g}(x, \theta_1), \]  

(2.32)

where

\[ f^{(c-)}_{\gamma g}(x, \theta_1) = 64\pi C_A \alpha_s^{(b)} s \left( 1 - x \right) \left[ \frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right] \mathcal{M}^{(b)}_{\gamma g}(s, q_1). \]  

(2.33)

Here \( \alpha_s^{(b)} = \alpha_s \mu^{2\epsilon} \) is the dimensionful coupling constant in \( d \) dimensions (the suffix \( b \) stands for bare), and \( \mathcal{M}^{(b)}_{\gamma g}(s, t) \) is the invariant cross section for \( \gamma g \to Q\bar{Q} \) at the Born level,

\[ d\sigma^{(b)}_{\gamma g} = \mathcal{M}^{(b)}_{\gamma g}(s, t) d\Phi_2. \]  

(2.34)

The explicit expression for \( \mathcal{M}^{(b)}_{\gamma g} \) is given in Appendix A. The term in the square bracket in eq. (2.33) is, up to a factor \( 2C_A \), the gluon-gluon Altarelli-Parisi splitting function in \( d \) dimensions for \( x < 1 \), and the Born cross section is taken in \( d \) dimensions.

With the usual definition

\[ \frac{1}{\epsilon} = \frac{1}{\epsilon} - \gamma_E + \log(4\pi), \]  

(2.35)

we can rewrite eq. (2.32) as

\[ d\sigma^{(c-)}_{\gamma g} = -s^{-\epsilon} \left( \frac{2}{\omega} \right)^\epsilon C_A \alpha_s^{(b)} \pi \left[ \left( \frac{1}{1-x} \right) \rho \left( \frac{1}{1-y} \right) \left( 1 + \frac{1}{1+y} \right) \right] \left[ x + \frac{(1-x)^2}{x} + x(1-x)^2 \right] \mathcal{M}^{(b)}_{\gamma g}(s, q_1) d\Phi_2^{(x)}. \]  

(2.36)

We see that the \( 1/\epsilon \) divergence in the collinear term assumes the form dictated by the factorization theorem. According to this factorization theorem, any partonic
The cross section can be written as

\[ d\sigma_{ij}(p_1, p_2) = \sum_{kl} \int d\bar{\sigma}_{kl}(x_1 p_1, x_2 p_2) \Gamma_{ki}(x_1) \Gamma_{lj}(x_2) dx_1 dx_2, \]  

(2.37)

where

\[ \Gamma_{ij}(x) = \delta_{ij} \delta(1 - x) - \frac{1}{\varepsilon} \frac{\alpha_s}{2\pi} P_{ij}(x) + \frac{\alpha_s}{2\pi} K_{ij}(x) + \mathcal{O}(\alpha_s^2) \]  

(2.38)

and \( d\bar{\sigma} \) is free of singularities as \( \varepsilon \) goes to zero. The collinear factors \( \Gamma_{ij}(x) \) are usually reabsorbed into the hadronic structure functions, and only the quantities \( d\bar{\sigma}_{kl} \) will enter the physical cross section. The functions \( P_{ij}(x) \) are the Altarelli-Parisi kernels. The functions \( K_{ij}(x) \) in eq. (2.38) are completely arbitrary, different choices corresponding to different subtraction schemes. The choice \( K_{ij}(x) = 0 \), to which we stick in the following, corresponds to the \( \overline{\text{MS}} \) subtraction scheme \([11]\).

Expanding eq. (2.37) order by order in perturbation theory, we find for our case

\[ d\bar{\sigma}_{\gamma g}(p_1, p_2) = d\sigma_{\gamma g}(p_1, p_2) + \frac{1}{\varepsilon} \frac{\alpha_s}{2\pi} P_{gg}(x) \mathcal{M}_{\gamma g}^{(b)}(x, q_1) d\Phi_2^{(x)}, \]  

(2.39)

where

\[
P_{gg}(x) = 2C_A \left[ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + 2\pi b_0 \delta(1-x)
\]

\[
= 2C_A \left[ \frac{x}{(1-x)_{\bar{\rho}}} + \frac{1-x}{x} + x(1-x) \right] + (2\pi b_0 + 4C_A \log 3) \delta(1-x),
\]  

(2.40)

and

\[ b_0 = \frac{11C_A - 4T_F n_f}{12\pi}. \]  

(2.41)

Here \( n_f \) is the number of light flavours, and for \( N_C = 3 \) we have

\[ C_A = 3, \quad T_F = \frac{1}{2}. \]  

(2.42)

The final expression for the short-distance cross section, after subtraction of the collinear divergences, eq. (2.39), becomes

\[ d\bar{\sigma}_{\gamma g} = d\bar{\sigma}_{\gamma g}^{(b)} + d\bar{\sigma}_{\gamma g}^{(c^e)} + d\bar{\sigma}_{\gamma g}^{(s)} + d\bar{\sigma}_{\gamma q}^{(v)} + d\bar{\sigma}_{\gamma g}^{(f)}, \]  

(2.43)
where \( d\sigma_{\gamma g}^{(b)} \) and \( d\sigma_{\gamma g}^{(f)} \) are as given in eqs. (2.34) and (2.31), respectively,

\[
d\hat{\sigma}^{(c-)}_{\gamma g} = \frac{C_A\alpha_s}{\pi} \left[ \left( \log \frac{s}{\mu_F^2} + \log \frac{\omega}{2} \right) \left( \frac{1}{1 - x} \right) + 2 \left( \frac{\log(1 - x)}{1 - x} \right) \right] \times \left[ x + \frac{(1 - x)^2}{x} + x(1 - x)^2 \right] \mathcal{M}_{\gamma g}^{(b)}(xs, q_1) d\Phi^2 \tag{2.44}
\]

\[
d\hat{\sigma}^{(s)}_{\gamma g} = d\sigma_{\gamma g}^{(s)} + \frac{C_A\alpha_s}{\pi} \frac{1}{\epsilon} \left( 2\pi b_0 + 4C_A \log \tilde{\beta} \right) \cdot \mathcal{M}_{\gamma g}^{(b)}(s, t) d\Phi_2. \tag{2.45}
\]

The scale \( \mu \), appearing explicitly in the expression of \( \alpha_s^{(b)} \), has been set equal to a scale \( \mu_F \) characteristic of the subtraction of the singularity due to collinear emission from the incoming gluon, while \( \alpha_s \) is taken everywhere at the renormalization scale \( \mu_R \) (the problem of scale definitions and choices will be discussed extensively in the next section). The remaining singularities in \( d\hat{\sigma}^{(s)}_{\gamma g} \) are cancelled by the singularities in the virtual contribution to the cross section \( d\sigma_{\gamma g}^{(v)} \). Therefore the quantity

\[
d\sigma_{\gamma g}^{(sv)} = d\hat{\sigma}^{(s)}_{\gamma g} + d\sigma_{\gamma g}^{(v)} \tag{2.46}
\]

is finite as \( \epsilon \to 0 \), and so is the full expression for \( d\hat{\sigma}_{\gamma g} \).

We now turn to the other three-body subprocess present at the \( \alpha_{em}\alpha_s^2 \) level, namely \( \gamma q \to Q\bar{Q}q \). In this case, collinear emission takes place both from the photon and from the incoming light quark. On the other hand, there is no order \( \alpha_{em}\alpha_s \) contribution to heavy-quark pair production via \( \gamma q \) fusion. Therefore, no soft singularity is expected. We explicitly checked that this is indeed the case. The three-body cross section is given by

\[
d\sigma_{\gamma q}^{(r)} = \mathcal{M}_{\gamma q}^{(r)}(s, t_k, u_k, q_1, q_2) d\Phi_3 \tag{2.47}
\]

\[
\mathcal{M}_{\gamma q}^{(r)}(s, t_k, u_k, q_1, q_2) = \frac{1}{2s} \frac{1}{2(1 - \epsilon)N_C} \sum_{\text{spin,color}} |\mathcal{A}_{\gamma q}^{(r)}|^2, \tag{2.48}
\]

where \( \mathcal{A}_{\gamma q}^{(r)} \) is the invariant amplitude. The invariant cross section \( \mathcal{M}_{\gamma q}^{(r)} \) has singularities for \( t_k = 0 \), corresponding to collinear \( (y = 1) \) light-quark emission from the photon, or \( u_k = 0 \), corresponding to collinear \( (y = -1) \) light-quark emission from the incoming quark. The function

\[
f_{\gamma q}(x, y, \theta_1, \theta_2) = 4t_ku_k\mathcal{M}_{\gamma q}^{(r)}(s, t_k, u_k, q_1, q_2) \tag{2.49}
\]
is therefore regular for \( y = \pm 1 \), and vanishes as \( (1 - x)^2 \) for \( x \to 1 \). For this reason, the three-body cross section can be rewritten as

\[
\frac{d\sigma_{yq}}{d\gamma_q} = H N \Phi_2 \frac{s^{1-\epsilon}}{2\pi} dy \sin^{-2\epsilon} \theta_2 d\theta_2
\times \left[ \frac{1}{1-x} - 2\epsilon \left( \frac{\log(1-x)}{1-x} \right) \right] (1-y^2)^{-1-\epsilon} f_{\gamma q}(x, y, \theta_1, \theta_2). \tag{2.50}
\]

Expanding \( (1-y^2)^{-1-\epsilon} \) in eq. (2.50) we get

\[
d\sigma_{yq}^{(r)} = d\sigma_{yq}^{(c_+) +} + d\sigma_{yq}^{(c_-) -} + d\sigma_{yq}^{(f)}, \tag{2.51}
\]

where

\[
d\sigma_{yq}^{(c\pm)} = N \Phi_2 \frac{s^{1-\epsilon}}{2\pi} dy \sin^{-2\epsilon} \theta_2 d\theta_2 \left[ \frac{1}{1-x} - 2\epsilon \left( \frac{\log(1-x)}{1-x} \right) \right]
\times \left[ -\frac{(2\omega)^{-\epsilon}}{2\epsilon} \delta(1 \mp y) \right] f_{\gamma q}(x, y, \theta_1, \theta_2) \tag{2.52}
\]

and

\[
d\sigma_{yq}^{(f)} = N \frac{s^{1-\epsilon}}{64\pi^2} \beta_x d \cos \theta_1 d\theta_2 dy dx 
\times \left[ \frac{1}{1-x} - \frac{1}{1-y} \right] \left[ \frac{1}{1-y} \right] \omega \left[ \frac{1}{1+y} \right] \omega f_{\gamma q}(x, y, \theta_1, \theta_2). \tag{2.53}
\]

Performing the \( y \) and \( \theta_2 \) integrations in eq. (2.52) we obtain

\[
d\sigma_{yq}^{(c\pm)} = -N \frac{s^{1-\epsilon}}{4\epsilon} \left( \frac{2}{\omega} \right)^\epsilon \Phi_2^{(x)} \left[ \frac{1}{1-x} - 2\epsilon \left( \frac{\log(1-x)}{1-x} \right) \right] f_{\gamma q}^{(c\pm)}(x, \theta_1). \tag{2.54}
\]

The explicit form of \( f_{\gamma q}^{(c\pm)}(x, \theta_1) \) can be obtained following ref. 3. The results are

\[
f_{\gamma q}^{(c+)}(x, \theta_1) = 32\pi\alpha_e e_q^2 s(1-x) \mathcal{P}_{\gamma q}(x) \mathcal{M}_{q\bar{q}}^{(b)}(s, t), \tag{2.55}
\]

where \( e_q \) is the charge of the emitted light quark in electron charge units, \( \mathcal{M}_{q\bar{q}}^{(b)}(s, t) \) is the lowest-order invariant cross section for \( q\bar{q} \to \bar{Q}Q \),

\[
d\sigma_{qq}^{(b)} = \mathcal{M}_{q\bar{q}}^{(b)}(s, t)d\Phi_2, \tag{2.56}
\]
and
\[
P_{q\gamma}(x) = N_C \left[ x^2 + (1 - x)^2 - 2x(1 - x)\epsilon \right]
\] (2.57)
is the Altarelli-Parisi splitting function in \(4 - 2\epsilon\) dimensions entering the probability of finding a quark in a photon, which is clearly equal to the splitting function of a quark in a gluon, up to a colour factor \(T_F/N_C\), due to the \(\lambda\) matrix in the \(gq\bar{q}\) vertex. For \(f_{\gamma q}^{(c-)}\) we obtain
\[
f_{\gamma q}^{(c-)}(x, \theta_1) = 32\pi\alpha_s s(1 - x)P_{gq}(x)M_{\gamma q}^{(b)}(xs, q_1),
\] (2.58)
where
\[
P_{gq}(x) = \frac{C_F}{x} \frac{1 + (1 - x)^2 - \epsilon x^2}{x}
\] (2.59)
is the Altarelli-Parisi splitting function of a gluon into a quark in \(4 - 2\epsilon\) dimensions. The subtraction of collinear singularities takes place as discussed for the previous case. We just give our final formulae,
\[
d\hat{\sigma}_{\gamma q} = d\hat{\sigma}_{\gamma q}^{(c+)} + d\hat{\sigma}_{\gamma q}^{(c-)} + d\sigma_{\gamma q}^{(f)},
\] (2.60)
where \(d\sigma_{\gamma q}^{(f)}\) is given in eq. (2.53),
\[
d\hat{\sigma}_{\gamma q}^{(c+)} = \frac{N_C\alpha_{em}e^2}{2\pi} M_{\gamma g}^{(b)}(xs, q_2)d\Phi_2^{(x)}
\times \left[ 2x(1 - x) + (x^2 + (1 - x)^2) \left( \log \frac{s}{\mu_{\gamma}^2} + \log \frac{\omega}{2} + 2\log(1 - x) \right) \right]
\] (2.61)
\[
d\hat{\sigma}_{\gamma q}^{(c-)} = \frac{C_F\alpha_S}{2\pi} M_{\gamma g}^{(b)}(xs, q_1)d\Phi_2^{(x)}
\times \left[ x + \frac{1 + (1 - x)^2}{x} \left( \log \frac{s}{\mu_{\gamma}^2} + \log \frac{\omega}{2} + 2\log(1 - x) \right) \right].
\] (2.62)
Notice that collinear emission from the photon is characterized by a scale \(\mu_{\gamma}\) which is \textit{a priori} different from the hadronic factorization scale \(\mu_F\). The quantities \(d\sigma_{\gamma g}^{(f)}\) in eq. (2.43) and \(d\sigma_{\gamma q}^{(f)}\) in eq. (2.60) can be found in the literature\cite{12}, and we did not need to explicitly evaluate them. The quantity \(d\sigma_{\gamma g}^{(v)}\) in eq. (2.43) was obtained from the authors of ref. \cite{2}.

The analytical results presented here are implemented as a parton event generator, written in FORTRAN. The interested reader can obtain the code from the authors.
3. Scales in the photoproduction process

Heavy-quark photoproduction differs from hadroproduction in the treatment of collinear singularities. In fact, when a light parton is collinearly emitted by the incoming photon, the subtracted term is a signal from the non-perturbative region where the photon splits into quarks and gluons before interacting with the partons in the hadronic target. This fact is taken into account by inserting in the photon-hadron cross section a contribution in which the photon is formally treated as a hadron (the so-called hadronic or resolved photon component, to distinguish it from the point-like or pure-photon component, in which the photon directly couples with the partons of the hadronic target). The photon structure functions will also depend upon the momentum scale \( \mu_\gamma \) at which the collinear singularities of the photon leg are subtracted. Neither the point-like nor the hadronic components are separately independent of \( \mu_\gamma \), because the subtracted term in the point-like component is responsible for the redefinition of the photon structure functions in the hadronic component.

Let us consider first the heavy-quark production process of an on-shell photon colliding with a hadron \( H \) at centre-of-mass energy \( \sqrt{S} \). In order to clarify the rôle of the various scale dependences in the process, we write the \( \mathcal{O}(\alpha_{\text{em}} \alpha_s^2) \) cross section in the following form

\[
\sigma_{Q\bar{Q}}^{(\gamma H)}(s) = \sum_i \int dx f_i^{(H)}(x, \mu_F) \hat{\sigma}_{\gamma i}(xS, \alpha_s \mu_R, \mu_R, \mu_F, \mu_\gamma) \\
+ \sum_{ij} \int dx_1 dx_2 f_i^{(\gamma)}(x_1, \mu_\gamma, \mu'_F) f_j^{(H)}(x_2, \mu'_F) \hat{\sigma}_{ij}(x_1 x_2 S, \alpha_s \mu'_R, \mu'_R, \mu'_F) \\
+ \mathcal{O}(\alpha_{\text{em}} \alpha_s^3) \tag{3.1}
\]

with

\[
\hat{\sigma}_{\gamma i}(s, \alpha_s(\mu_R), \mu_R, \mu_F, \mu_\gamma) = \alpha_{\text{em}} \alpha_s(\mu_R) \tilde{\sigma}_{\gamma i}^{(0)}(s) + \alpha_{\text{em}} \alpha_s^2(\mu_R) \tilde{\sigma}_{\gamma i}^{(1)}(s, \mu_R, \mu_F, \mu_\gamma) \\
\hat{\sigma}_{ij}(s, \alpha_s(\mu_R), \mu_R, \mu_F) = \alpha_s^2(\mu_R) \tilde{\sigma}_{ij}^{(0)}(s) + \alpha_s^3(\mu_R) \tilde{\sigma}_{ij}^{(1)}(s, \mu_R, \mu_F, \mu_\gamma). \tag{3.2}
\]

Here \( \mu_R \) and \( \mu'_R \) are renormalization scales, \( \mu_F \) and \( \mu'_F \) are factorization scales for collinear singularities arising from strong interactions, and \( \mu_\gamma \) is a factorization scale for collinear singularities arising from the electromagnetic vertex. If one wanted to extend eq. (B.11) to even higher orders, one should also include an explicit dependence
of the structure functions upon the renormalization scale. At the order we are considering, the renormalization scale in the structure functions can be kept equal to the factorization scale, as is usually done. The left-hand side is independent of all the scales up to terms of order $\alpha_{em}\alpha_s^3$, provided the parton density functions obey the appropriate evolution equations. The hadronic and photonic parton densities obey the usual Altarelli-Parisi equations in $\mu_F$. In addition, the photonic parton densities have also an inhomogeneous evolution in $\mu_\gamma$, which, at the leading order, is given by

$$\frac{\partial f_i^{(\gamma)}(x, \mu_F, \mu_\gamma)}{\partial \log \mu_\gamma^2} = \frac{\alpha_{em}}{2\pi} e_i^2 \left[ x^2 + (1-x)^2 \right] + O(\alpha_s), \quad (3.3)$$

where $e_i$ is the charge of the parton $i$ in electron charge units. The compensation of the scale dependence takes place in the following way. The $\mu_R$ scale dependence is compensated in the expressions for the partonic cross sections: the scale dependence of $\alpha_s$ in the Born term $\hat{\sigma}_{gi}^{(0)}$ is compensated by the explicit scale dependence of the next-to-leading term $\hat{\sigma}_{gi}^{(1)}$. A similar cancellation occurs in $\hat{\sigma}_{ij}$. The dependence upon $\mu_F (\mu'_F)$ cancels between the explicit dependence in the next-to-leading order term and the dependence in the structure functions convoluted with the Born terms. This holds independently for the two terms of eq. (3.1). The dependence upon $\mu_\gamma$ cancels between the explicit dependence in the next-to-leading order component of the first term of eq. (3.1) and the $\mu_\gamma$ dependence of $f_i^{(\gamma)}$, as given in eq. (3.3), multiplied by the Born level partonic cross section. In the commonly-used photon density parametrizations, $\mu_\gamma$ is usually kept equal to $\mu_F$, so that the term given in eq. (3.3) becomes a correction to the usual Altarelli-Parisi equation (the so-called inhomogeneous term). Therefore, in our calculation, we use for consistency $\mu'_F = \mu_\gamma$. In some cases, the inclusion of the hadronic component gives only a small effect, and will be neglected. In these cases we have chosen $\mu_\gamma = 1$ GeV, which amounts to setting the photon structure function to zero at a scale of the order of a typical hadron mass. We have found that varying $\mu_\gamma$ between 0.1 and 5 GeV does not affect the results in a noticeable way.

4. Total cross sections

We begin our phenomenological study with the total cross sections for charm and bottom production. We will concentrate here on the analysis of the dependence of the total cross sections on the input parameters of the calculation, namely the choice
of parton distribution functions (PDFs), the choice of quark mass and the choice of
the values of the scales discussed in Section 3. A discussion of the ranges of masses
and scales within which to explore the dependence of the cross sections can be found
in ref. [6].

The target will always be an isosinglet nucleon, \( N = \frac{(p + n)}{2} \), and unless other-
wise stated we will use the parton distribution set MRSD0[13].

The default values of the charm and bottom mass will be 1.5 and 4.75 GeV
respectively, and the default choices for \( \mu_F \) and \( \mu_R \) will be:

\[
\mu_R = m_c, \quad \mu_F = 2m_c
\]

(4.1)

for charm and

\[
\mu_R = \mu_F = m_b
\]

(4.2)

for bottom. The asymmetry in the default choice for the charm is related to the scale
threshold below which PDFs extrapolations are not available, as explained in detail
in ref. [6]. As for \( \mu_\gamma \), we fix as a default \( \mu_\gamma = 1 \) GeV, as explained in Section 3.

As an illustration of the reliability of the theoretical prediction we present in fig. 1
(fig. 2) the leading and next-to-leading results for the total charm (bottom) cross
section. The bands in figs. 1 and 2 are obtained by varying only the renormalization
and factorization scales, everything else being kept fixed.

Figures 1 and 2 deserve some comments. First of all, the scale uncertainty associ-
ated with the charm production cross section is significantly smaller at the next-to-
leading order for beam energies above 200 GeV, while below 200 GeV the uncertainties
at leading and next-to-leading order are similar. The residual uncertainty is much
smaller than in the case of hadroproduction for comparable beam energies. The con-
tribution of the hadronic component of the photon is always smaller than 5% of the
total cross section for current energies. Uncertainties coming from the determination
of the photon structure functions are therefore negligible with respect to others.

The reduction of the variation band is even more pronounced in the case of bottom
production, once next-to-leading order corrections are included. This is what we
expect. For higher masses the value of \( \alpha_s \) is smaller, and the perturbative expansion
becomes more reliable. Observe that the size of the leading-order band for the bottom
cross section is not much smaller than the one for charm. This is due to the fact that,
as explained in ref. [6], we did not try to study the factorization scale dependence in
the case of charm production. The reader should therefore remember that a further
uncertainty should be added to the charm result, and that the band shown in fig. [1]
is only an underestimate of the uncertainties involved in the computation of charm
production cross sections.

We now turn to all other sources of uncertainties, such as the structure function
choice, the value of $\Lambda_{QCD}$, and the mass of the heavy quarks. In tables [1] and [2]
we give the cross sections for $\gamma$-nucleon collisions at various beam energies. The rates
were obtained using a reference scale $\mu_0 = m_c$ for charm and $\mu_0 = m_b$ for bottom. We
also show the effect of varying $m_c$ between 1.2 and 1.8 GeV, and $m_b$ between 4.5 and
5 GeV. The scale $\mu_R$ was varied between $\mu_0/2$ and $2\mu_0$. In the charm case, $\mu_F$ was
kept equal to $2\mu_0$, while for bottom we kept $\mu_F = \mu_R$. We verified that considering
independent variations for the factorization and renormalization scale does not lead
to a wider range in the bottom cross sections for the energies shown in the tables.

The tables are broken into three blocks, each corresponding to a different choice
of $\Lambda_{QCD}$ within the range allowed by the current uncertainties. The upper block
represents the default choice relative to the MRSD0 fit. The second and third blocks
correspond to the sets discussed in ref. [16] for the nucleon. The values of $\Lambda_4$ obtained
in the fits of ref. [16] range from 135 to 235 MeV, corresponding to a range for $\Lambda_5$
between 84 and 155 MeV. This range for $\Lambda_5$ is chosen because no good fit to deep
inelastic data is possible outside that range in the context of ref. [16] (i.e. with that
choice of structure function parametrization, etc.). We have chosen instead the wider
range $\Lambda_4 \leq 100 < \Lambda_4 < 300$ MeV, corresponding to $60 \leq \Lambda_5 < 204$ MeV. Therefore, in
order to take into account the full range of uncertainty associated with the value of
$\Lambda_{QCD}$, we were forced to account only partially for the correlation between $\Lambda_{QCD}$ and
the nucleon structure functions.

For the charm cross section, as can be seen, the value of the charm quark mass is
the major source of uncertainty. Differences between the extreme choices $m_c = 1.2$
and 1.8 GeV vary from a factor of 5 at 100 GeV to a factor of 3 at 400 GeV. Differences
due to the scale choice are of the order of a factor of 2 at low energy, and at most
50% at higher energy. A factor of 2 uncertainty also comes from the variation of $\Lambda_{QCD}$
within the chosen range.

We also explored independently the effect of varying $\mu_\gamma$. Differences are totally
negligible for values $0.1 \text{ GeV} < \mu_\gamma < 5 \text{ GeV}$, and are not included in the tables.

All of these uncertainty factors are systematically a factor of 2 or more smaller than in the case of hadroproduction. Notice however one pathology encountered when combining the most extreme choices of mass ($m_c = 1.2 \text{ GeV}$), scale ($\mu_R = m_c/2$) and $\Lambda_{QCD}$ (MRS235, $\Lambda_5 = 204 \text{ MeV}$): the cross section in this case decreases in the region $100 \text{ GeV} < E_\gamma < 600 \text{ GeV}$. This happens because for this particular choice of parameters the radiative corrections become negative in part of this energy range.

Low-energy measurements of charm photoproduction cross sections favour a mass value of approximately 1.5 GeV. Previous comparisons with theory, however, were made using a fixed value $\mu_R = 2m_c$. We expect that once the uncertainty on $\Lambda_{QCD}$ will be reduced, reliance on the next-to-leading-order calculation and the residual dependence on $\mu_R$ should allow a determination of $m_c$ to within 100–200 MeV.

The corresponding variations in the bottom case are smaller, in particular for energies sufficiently above the production threshold. At $E_\gamma = 100 \text{ GeV}$, where we observe the largest uncertainty in the perturbative calculation, we should also expect large non-perturbative effects and therefore the perturbative prediction is not fully reliable. Notice also from fig. 2 that the contribution of the hadronic component of the photon represents, at low energy, a significant fraction of the total cross section. The reason is that, close to the threshold, the photon-gluon fusion process is suppressed by the small gluon density at large $x$, while production via the hadronic component of the photon can proceed through a light valence-quark annihilation channel.

For completeness, we also give the contribution of the hadronic component of the photon in tables 3 and 4, evaluated using the photon structure function set ACFGPGmc of ref. [14] and the set LAC1 of ref. [15]. As can be seen, at current fixed-target energies, this contribution is small.

Our final prediction for the allowed range of charm and bottom production cross sections, including the full variation due to the scale choice, the value of $\Lambda_{QCD}$ and the nucleon structure functions is shown as a function of the beam energy in fig. 3, for $m_c = 1.2, 1.5$ and 1.8 GeV, and for $m_b = 4.5, 4.75, 5$ GeV. The small contribution of the hadronic component is not included in the figure.
\( m_c = 1.2 \text{ GeV} \) & \( m_c = 1.5 \text{ GeV} \) & \( m_c = 1.8 \text{ GeV} \) \\
\hline
\( E_b \mu_R = \) & \( m_c/2 \) & \( m_c \) & \( 2m_c \) & \( m_c/2 \) & \( m_c \) & \( 2m_c \) & \( m_c/2 \) & \( m_c \) & \( 2m_c \) \\
\hline
Nucleon PDF set MRSD0, \( \Lambda_5 = 140 \text{ MeV} \) \\
60 GeV & 1.442 & 0.937 & 0.715 & 0.533 & 0.357 & 0.275 & 0.203 & 0.138 & 0.107 \\
100 GeV & 1.673 & 1.205 & 0.952 & 0.746 & 0.537 & 0.427 & 0.341 & 0.248 & 0.198 \\
200 GeV & 1.826 & 1.506 & 1.238 & 0.979 & 0.776 & 0.640 & 0.525 & 0.414 & 0.342 \\
400 GeV & 1.906 & 1.753 & 1.481 & 1.160 & 0.997 & 0.844 & 0.691 & 0.582 & 0.495 \\
\hline
Nucleon PDF set MRS135, \( \Lambda_5 = 60 \text{ MeV} \) \\
60 GeV & 0.753 & 0.627 & 0.535 & 0.312 & 0.255 & 0.216 & 0.130 & 0.105 & 0.089 \\
100 GeV & 0.921 & 0.807 & 0.705 & 0.443 & 0.379 & 0.329 & 0.217 & 0.183 & 0.159 \\
200 GeV & 1.093 & 1.016 & 0.910 & 0.602 & 0.543 & 0.483 & 0.338 & 0.300 & 0.266 \\
400 GeV & 1.229 & 1.190 & 1.082 & 0.740 & 0.694 & 0.629 & 0.451 & 0.417 & 0.377 \\
\hline
Nucleon PDF set MRS235, \( \Lambda_5 = 204 \text{ MeV} \) \\
60 GeV & 2.794 & 1.280 & 0.886 & 0.874 & 0.467 & 0.332 & 0.304 & 0.176 & 0.127 \\
100 GeV & 3.012 & 1.631 & 1.179 & 1.183 & 0.700 & 0.518 & 0.502 & 0.314 & 0.236 \\
200 GeV & 2.923 & 2.015 & 1.530 & 1.474 & 1.002 & 0.776 & 0.751 & 0.522 & 0.409 \\
400 GeV & 2.714 & 2.329 & 1.830 & 1.663 & 1.280 & 1.024 & 0.956 & 0.731 & 0.592 \\
\hline

Table 1: Total charm cross sections (\( \mu b \)) in \( \gamma N \) collisions. point-like photon contribution. Nucleon PDF set as indicated.
\begin{align*}
E_b \mu = & \begin{array}{|c|c|c|c|c|c|c|c|}
\hline
m_{b} = 4.5 \text{ GeV} & m_{b} = 4.75 \text{ GeV} & m_{b} = 5 \text{ GeV} \\
\hline
m_b/2 & m_b & 2m_b & m_b/2 & m_b & 2m_b & m_b/2 & m_b & 2m_b \\
\hline
\end{array}
\end{align*}

| \text{Nucleon PDF set MRSD0, } \Lambda_5 = 140 \text{ MeV} | \text{Nucleon PDF set MRS135, } \Lambda_5 = 60 \text{ MeV} | \text{Nucleon PDF set MRS235, } \Lambda_5 = 204 \text{ MeV} |
|-----------------|-----------------|-----------------|
| \text{100 GeV} | \text{0.081} | \text{0.049} | \text{0.031} | \text{0.035} | \text{0.020} | \text{0.012} | \text{0.013} | \text{0.008} | \text{0.004} |
| \text{200 GeV} | \text{1.092} | \text{0.832} | \text{0.615} | \text{0.702} | \text{0.525} | \text{0.383} | \text{0.445} | \text{0.327} | \text{0.235} |
| \text{400 GeV} | \text{3.969} | \text{3.373} | \text{2.760} | \text{2.929} | \text{2.469} | \text{2.001} | \text{2.163} | \text{1.807} | \text{1.452} |

Table 2: Total bottom cross sections (\(\text{nb}\)) in \(\gamma N\) collisions. point-like photon contribution. Nucleon PDF set as indicated.

| \text{Nucleon PDF set ACFGP–mc} | \text{Photon PDF set LAC1} |
|-----------------|-----------------|
| \text{60 GeV} | \text{0.050} | \text{0.024} | \text{0.015} | \text{0.011} | \text{0.007} | \text{0.004} | \text{0.004} | \text{0.002} | \text{0.002} |
| \text{100 GeV} | \text{0.096} | \text{0.045} | \text{0.027} | \text{0.022} | \text{0.013} | \text{0.008} | \text{0.007} | \text{0.005} | \text{0.003} |
| \text{200 GeV} | \text{0.203} | \text{0.094} | \text{0.056} | \text{0.052} | \text{0.029} | \text{0.019} | \text{0.017} | \text{0.011} | \text{0.007} |
| \text{400 GeV} | \text{0.376} | \text{0.175} | \text{0.105} | \text{0.109} | \text{0.059} | \text{0.038} | \text{0.039} | \text{0.024} | \text{0.016} |

Table 3: Contribution of the hadronic component of the photon to the total charm cross sections (\(\mu b\)) in \(\gamma N\) collisions. Nucleon PDF set is MRSD0, photon PDF set as indicated.
In this section we present the results for the one- and two-particle differential distributions (the latter will also be referred to as correlations in the following) for charm and bottom production. Experiments can never obtain a monochromatic photon beam, even though the energy of the photon can often be measured event by event. In order to retain a large statistics, therefore, differential distributions are most often presented integrating over the full photon beam spectrum, rather than at a fixed value of $E_\gamma$. While the total cross section is a slowly varying function of $E_\gamma$, features of the distributions can be affected by the convolution over the photon beam energy spectrum. A meaningful study therefore requires a similar convolution to be performed within the theoretical calculation.

We have chosen to perform our phenomenological study using the photon beam energy spectra of the two experiments NA14/2 at CERN\cite{18} and E687 at FNAL\cite{19}. The spectra were kindly provided to us by the two collaborations. Since the energy range of the two spectra are rather different, this choice can give an idea of the energy dependence of our results. The spectra are shown in fig. 4 and can be parametrized as follows:

$$\frac{1}{N} \frac{dN}{dx} = C \left[ x^{a_1} (1 - x)^{a_2} + R x^{b_1} (1 - x)^{b_2} \right], \quad (5.1)$$

Table 4: Contribution of the hadronic component of the photon to the total bottom cross sections (nb) in $\gamma N$ collisions. Nucleon PDF set is MRSD0, photon PDF set as indicated.
where $x = E_\gamma/E_{\text{max}}$, $E_{\text{min}} < E_\gamma < E_{\text{max}}$,

$$E_{\text{max}} = 440 \text{ GeV}, \quad E_{\text{min}} = 125 \text{ GeV}$$

$$a_1 = 0.2294, \quad a_2 = 2.9533$$

$$R = 7.5019 \times 10^6, \quad b_1 = 16.766, \quad b_2 = 11.190$$ \hspace{1cm} (5.2)

for E687, and

$$E_{\text{max}} = 400 \text{ GeV}, \quad E_{\text{min}} = 20 \text{ GeV}$$

$$a_1 = 2.5987, \quad a_2 = 4.7211$$

$$R = -1.1862, \quad b_1 = 2.7133, \quad b_2 = 4.9516$$ \hspace{1cm} (5.3)

for NA14/2. The constant $C$ is chosen to normalize the distributions to unity. The results we present are obtained by convoluting the theoretical distributions with the above beam shapes. We will refer to these, respectively, as NA14 beam and E687 beam.

 Needless to say, only a detailed simulation of the detector acceptances and efficiencies can allow a complete comparison of our results with the experimental findings. Therefore one should take the results presented here as indicative of the most relevant features of the next-to-leading-order calculation. Additional non-perturbative effects such as fragmentation and intrinsic momentum of the partons inside the hadrons will also be discussed at the end.

As in the case of the total cross sections, the distributions will be calculated using the parton distribution set MRSD0\cite{13} for the nucleon, unless otherwise stated. The default values of the masses will be $m_c = 1.5 \text{ GeV}$ and $m_b = 4.75 \text{ GeV}$. The default values of the factorization and renormalization scales $\mu_F$ and $\mu_R$ will be:

$$\mu_F = 2\mu_0, \quad \mu_R = \mu_0$$ \hspace{1cm} (5.4)

for charm, and

$$\mu_F = \mu_0, \quad \mu_R = \mu_0$$ \hspace{1cm} (5.5)

for bottom, where

$$\mu_0 = \sqrt{m^2 + p_T^2}$$ \hspace{1cm} (5.6)
for the one-particle distributions and

\[ \mu_0 = \sqrt{m^2 + \frac{p_T^2 + \bar{p}_T^2}{2}} \]  

(5.7)

for correlations. Here \( p_T \) and \( \bar{p}_T \) are the transverse momenta of the heavy quark and antiquark, respectively. In the case of correlations, a further ambiguity arises in the choice of the scale (described in detail in ref. [5]), which has to do with the freedom of choosing the same or different values of the scales for the three-body event and for the corresponding counter-events. The results presented here will always follow the approach of recomputing the scale for the counter-events.

### 5.1. Charm production

We begin with one-particle inclusive rates. On the left side of fig. 5 we show the inclusive \( p_T \) distribution of \( c \) quarks in the case of the E687 and NA14 photon beams. The solid lines represent the full next-to-leading-order result. The dots give the leading-order contribution rescaled by a constant factor. A slight stiffening of the \( p_T \) distribution after radiative corrections is observed.

On the right-hand side of fig. 5 we show the inclusive \( x_F \) distribution for the full next-to-leading-order calculation, superimposed onto the rescaled Born result. Our definition of \( x_F \) is

\[ x_F = \frac{2p_{CM}^\parallel}{E_{CM}} \]  

(5.8)

where CM refers to the centre of mass of the target and the tagged photon beam. In other words, this centre of mass has a boost with respect to the laboratory frame, which depends on the energy of the photon responsible for the interaction; \( p_{CM}^\parallel \) is the momentum projection on the beam direction in the centre-of-mass frame, and \( E_{CM} \) is the centre-of-mass total energy. The fraction of \( c \) quarks produced with positive \( x_F \) is larger than 90\%, due to the hardness of the photon probe. Notice however that next-to-leading-order corrections induce a softening of the distribution, due to processes where a photon splits into a light-quark pair and interacts with a light quark from the nucleon. We verified that the inclusive \( p_T \) spectrum does not change in shape if we restrict ourselves to quarks produced at positive \( x_F \), which is the region where data are usually collected by the experiments.
Next we consider $c\bar{c}$ correlations, starting from the invariant mass of the heavy quark pair, which is shown in fig. 6. As before, the continuous lines represent the full result of the next-to-leading-order calculation, while the dots are the rescaled Born results. The lower lines are obtained by imposing $x_F > 0$ for both quarks. Notice that while at the inclusive level most of the $c$'s have positive $x_F$, a large fraction of pairs with large invariant mass have either the $c$ or the $\bar{c}$ at $x_F < 0$, as one should expect.

The difference in rapidity between the quark and the antiquark and the $x_F$ of the pair are shown in figs. 7 and 8. A slight broadening at next-to-leading order is observed for the rapidity correlation, while a dramatic change is observed at next-to-leading order in the case of the pair $x_F$ distribution. This dramatic change can be traced back to the particular kinematics of heavy-quark photoproduction. In the Born approximation, the $x_F$ of the heavy-quark pair is simply related to its invariant mass

$$x_F^{QQ} = 1 - \frac{M_{QQ}^2}{S} \quad \text{(5.9)}$$

and is therefore peaked at $x_F^{QQ}$ near 1. At the next-to-leading level, it is the $x_F$ of the $Q\bar{Q}g$ system that will be peaked near 1. This means that the $x_F$ of the heavy-quark pair will be markedly degraded. The contribution of $\gamma q$ fusion is even softer, because of the contribution of the photon splitting into a light quark-antiquark pair.

We now consider correlations that are trivial at the leading order, namely the difference in azimuth and total transverse momentum of the quark pair, $p_{\ell \ell}^{\ell \ell}$. At leading order these distributions are delta functions centred respectively at $\Delta \phi = \pi$ and $p_{\ell \ell}^{\ell \ell} = 0$. Higher-order real corrections such as gluon radiation or gluon splitting processes smear them. We plot these distributions in fig. 9. Even after the inclusion of higher-order effects, the azimuthal correlation shows a strong peak at $\Delta \phi \approx \pi$. Likewise, the $p_{\ell \ell}^{\ell \ell}$ distribution is dominated by configurations with $p_{\ell \ell}^{\ell \ell}$ smaller than 1 GeV. In both cases, the tails are higher at the higher energy of the E687 experiment.

For all the previous distributions, changing the values of the charm mass and renormalization scale $\mu_R$ results in large differences in rates but small and easily predictable shape modifications. The pattern of these changes is similar to what is observed in the case of fixed-target hadroproduction.
5.2. Bottom production

The differential distributions for bottom production are shown in figs. 10 to 12. The cross section at NA14 is very small. We nevertheless include their distributions for completeness.

The $p_T^2$ (single-inclusive) and the $M_{Q\bar{Q}}$ distributions are broader than the corresponding distributions for charm production. They are, however, narrower than would be expected on the basis of simple scaling arguments. This is because $b$ production at fixed-target energies is still too close to the threshold, and thus constrained by phase-space effects (this can also be noticed from the strong energy dependence of the shape of the curves). For the same reason, the $\Delta y$ distribution is narrower than in the charm case.

The $p_T^2$ and the $\Delta\phi$ distributions are much narrower, as a consequence of the smaller value of $\alpha_s$ at the $b$ mass, and of the previously mentioned phase-space constraints. The $x_F$ distribution is softer for bottom than for charm, and in particular the fraction of $b$ quarks with negative $x_F$ is larger. This is because a harder parton from the nucleon is required to reach the energy threshold for the creation of a $b$ pair.

5.3. Higher-Order Corrections and Hadronization

The results discussed so far were obtained with a purely perturbative calculation limited to the next-to-leading order. In the case of charm quarks, the dependence of the cross section on the renormalization and factorization scales indicates that higher-order corrections might be large. Nevertheless, the stability of the shapes under inclusion of the next-to-leading-order terms suggests that no significant changes should be expected in the differential distributions when yet higher-order terms are included. This is not necessarily true of possible non-perturbative corrections, such as the intrinsic $p_T$ of the initial-state partons, hadronization and fragmentation. In particular, in the regions of phase space close to threshold, or for $p_T^{Q\bar{Q}} \to 0$, we should expect significant corrections.

In our previous study of heavy-quark correlations in hadroproduction we explored these effects using different phenomenological models. In particular, we considered the parton shower Monte Carlo HERWIG to simulate both the backward
evolution of the initial state and the formation of charmed hadrons. We found that the backward evolution of initial-state gluons gave rise to an artificially large intrinsic $p_T$ of the gluons themselves ($\langle k_T \rangle = 1.7 \text{ GeV}$). This large intrinsic $p_T$ would significantly broaden the $\Delta \phi$ correlation, in addition to softening the inclusive $p_T$ distribution. We also found that, as expected, the colour singlet cluster formation and decay into hadrons leads to a large colour drag in the direction of the hadron beam. The combination of the intrinsic $p_T$ smearing, of the colour drag, and of the decay of unstable charmed hadrons, resulted in inclusive $p_T$ and $x_F$ distributions of $D$ mesons that are slightly harder than those resulting from the purely perturbative calculation. The $\Delta \phi$ correlations remain significantly broader than described by next-to-leading-order QCD. Both these results are supported by current experimental evidence[21].

In ref. [6] we were also able to parametrize the effect of the intrinsic $p_T$ by using a Gaussian smearing, while we argued that there is no solid basis for the application of the Peterson formalism[22] to describe the fragmentation in the large-$x_F$ region. In this section, we repeat the analysis in the case of heavy-quark photoproduction, for the case of charm production with the E687 photon spectrum.

Figure 13 shows a comparison of inclusive $p_T$ and $x_F$ distributions obtained from the perturbative calculation (solid lines), from HERWIG before hadronization (dotted lines), and from HERWIG after hadronization (dashed lines). In the case of the inclusive $p_T$ distribution we find that, as observed in hadroproduction, the effects of intrinsic $p_T$ and of hadronization tend to respectively harden and soften the distribution. The net result is a softening of the shape. This is due to the fact that only one initial-state parton (namely the gluon) can acquire a transverse $p_T$ in photoproduction, contrary to hadroproduction where we have two gluons in the initial state and the effect is enhanced.

The most dramatic effects are however observed in the inclusive $x_F$ distribution. Hadronization effects heavily suppress the production of charmed hadrons at large $x_F$, which was favoured at the purely perturbative level. This is perhaps surprising, because it would be expected that for heavy quarks produced in the photon fragmentation region we should be able to obtain the correct distribution by using the perturbative calculation, convoluted with the fragmentation function for the $c$-quark fragmenting into a $D$ meson. This is not necessarily true. As in the hadroproduction case, there are no theoretical reasons to support this possibility. The fragmentation
function formalism is in fact applicable only when the elementary production process takes place at energies much larger than the mass of the heavy quark. This is the case, for example, in the production of heavy flavoured mesons in $e^+e^-$ collisions, or in hadroproduction and photoproduction at high transverse momentum. The factorization theorem states that the same fragmentation function, evolved to the appropriate scale, and convoluted with the perturbative calculation of the partonic subprocess, should describe all these processes. The $x_F$ spectrum, instead, is not really characterized by a high-energy elementary process. The heavy-quark pair is produced with a relatively small invariant mass, and the large $x_F$ region is reached when the production angle in the heavy-quark centre of mass is small. Under these circumstances, non-perturbative effects (other than the fragmentation effects) could also take place. For example, the heavy quark could feel the dragging of the heavy antiquark, or of the beam remnants.

Figure 14 shows the HERWIG results for the charm pair $p_T^{QQ}$ and $\Delta \phi$ distributions. While the $\Delta \phi$ correlation is still significantly broader than that calculated at the perturbative level, the distribution is more peaked than the one evaluated from HERWIG in the hadroproduction case. This results from the smaller effect of intrinsic $p_T$ present in the photoproduction, as already mentioned above. Existing data [18, 19] agree qualitatively with this result.

In fig. 15, finally, we show the charm pair $p_T^{QQ}$ and $\Delta \phi$ distributions obtained by giving a random intrinsic transverse momentum to the incoming gluon, with a Gaussian distribution [6], for different values of $\langle p_T^2 \rangle$. As can be seen, the choice $\langle k_T^2 \rangle = 3 \text{ GeV}^2$ reproduces quite closely the HERWIG result for the $\Delta \phi$ correlation. As already discussed in ref. [6], we expect such a large intrinsic $p_T$ to be a pure artefact of the Monte Carlo. If the data were to confirm the existence of broad $\Delta \phi$ correlations as shown in fig. 14, it might be interesting to try to justify theoretically on a more solid basis the possible existence of such a large intrinsic $p_T$ for the gluons inside the hadron.

5.4. Effect of the photon hadronic component

As already seen in Section 4, the effect of the hadronic component on the total cross section, predicted using standard photon structure functions, is generally small. The
question now arises whether its effect on the shape of distributions is also negligible. We have studied this problem using the next-to-leading calculation of ref. [3], together with the parametrization of the photon structure function set ACFGPG-mc of ref. [4], and the set LAC1 of ref. [5], at a fixed photon energy of 230 GeV. We have found that the only important modifications occur in the \( x_F \) distribution for a single heavy quark, and for the \( x_F \) of the pair. In the inclusive \( x_F \) distribution for charm, we find that the hadronic component becomes comparable to the point-like term for \( x_F \approx -0.3 \), and for smaller \( x_F \) it remains of the same order. For \( b \) production, we find instead that the hadronic component becomes comparable to the point-like term for \( x_F \approx -0.7 \), a region in which the cross section is several orders of magnitude below the peak value.

For the \( x_F \) of the pair, the effect of the hadronic component is more pronounced. This is due to the fact that the point-like component is concentrated near \( x_Q^Q = 1 \). On the other hand the hadronic component is distributed in the central region (we find that for charm it peaks at \( x_F \approx -0.15 \) with the LAC1 set, and at \( x_F \approx 0 \) with the ACFGPG-mc set). Its contribution in this region is therefore of the magnitude given in tables 3 and 4. Observe, however, that (as discussed in the previous subsection) hadronization effects do spread out the \( x_F \) distribution of the pair. It is unlikely, therefore, that one can use these distributions to make statements about the hadronic component of the photon.

We conclude, therefore, that for all practical purposes, the hadronic component can be neglected altogether in the fixed-target experimental configurations of present interest.

6. Conclusions

We have performed a calculation of next-to-leading-order QCD corrections to heavy-quark production in photon-hadron collisions. Our calculation improves over previous results, in that it can be used to compute any distribution in the heavy quark and antiquark, and possibly in the extra jet variables.

We have presented a phenomenological study of total cross sections, single-inclusive distributions and correlations, for charm and bottom production at fixed-target energies.
A detailed study of all the theoretical uncertainties of the calculation, namely those due to an independent variation of heavy-quark mass, factorization and renormalization scales and $\Lambda_{QCD}$, has been performed for the total cross sections. We found that the next-to-leading contribution is less important here than in the hadroproduction case. Therefore, we expect the full result to be more reliable for photoproduction than for hadroproduction of heavy flavours.

For single and double differential distributions, in the case of charm production, we always find that non-perturbative effects could be important. For the $p_T$ distribution, a possible description of the non-perturbative effects is given via the introduction of an intrinsic transverse momentum for the incoming gluon, which tends to stiffen the $p_T$ distribution, together with a fragmentation function similar to the ones used in $e^+e^-$ physics. This seems to give a reasonable description of the $p_T$ and $\Delta\phi$ distributions in both photoproduction and hadroproduction.

In the case of the $x_F$ distribution, it is very difficult to describe the non-perturbative effects in a simple way. From Monte Carlo studies, we conclude that it is likely that colour-dragging effects prevail, thus making it difficult to give a homogeneous description of the hadroproduction and photoproduction $x_F$ distribution.
Soft limit of the real amplitude

In this appendix we present the calculation of the amplitude for the process $\gamma g \rightarrow Q\bar{Q}g$ in the limit when the momentum of the emitted gluon tends to zero. This is the only case of interest, because the analogous limit for the process $\gamma q \rightarrow Q\bar{Q}q$ gives a trivial result. Momenta and colour and polarization indices are assigned as follows:

$$\gamma(p_1, \mu) + g(p_2, \nu, a) \rightarrow Q(k_1, i) + \bar{Q}(k_2, j) + g(k, \rho, b).$$ (1)

The main difference with respect to the heavy-quark hadroproduction case is that the gluon cannot directly couple to the incoming photon. In principle we are then left with three potentially singular diagrams when $k \rightarrow 0$, namely the diagrams in which the outgoing gluon is emitted by the incoming parton or by the heavy quark or antiquark. We indicate the amplitude for the process $\gamma g \rightarrow Q\bar{Q}$ with

$$\bar{u}(k_1) A_{\mu\nu;ij}^a v(k_2),$$ (2)

where the momenta and indices are as in eq. (1). It then follows that the contribution of the diagram in which the gluon is emitted from the outgoing heavy quark is given by

$$\frac{g_s \lambda^b_{il}}{2} \bar{u}(k_1) \gamma_\rho \frac{\hat{k} + \hat{k}_1 + m}{(k + k_1)^2 - m^2} A_{\mu\nu;ij}^a v(k_2).$$ (3)

When the gluon is emitted from the antiquark we have instead

$$\frac{g_s \lambda^b_{lj}}{2} \bar{u}(k_1) A_{\mu\nu;il}^a \frac{\hat{k} + \hat{k}_2 + m}{(k + k_2)^2 - m^2} \gamma_\rho v(k_2).$$ (4)

Finally, when the gluon is emitted from the incoming gluon leg the contribution is

$$ig_s \frac{g^{\sigma\lambda}}{(p_2 - k)^2} G_{\nu\rho\sigma}^{abc} (p_2, -k, k - p_2) \bar{u}(k_1) A_{\mu\lambda;ij}^a v(k_2),$$ (5)

where

$$G_{\mu_1\mu_2\mu_3}^{abc}(q_1, q_2, q_3) = f^{abc} [g_{\mu_1\mu_2}(q_1 - q_2)_{\mu_3} + g_{\mu_2\mu_3}(q_2 - q_3)_{\mu_1} + g_{\mu_3\mu_1}(q_3 - q_1)_{\mu_2}]$$ (6)

is the quantity that appears in the QCD three-gluon vertex. We can now evaluate the $k \rightarrow 0$ limit in eqs. (3), (4) and (5). The soft limit of the three-body real
amplitude is

\[ A^{(r)}_{\gamma g} (k \to 0) = \frac{g_s \lambda_b^a}{2} k_{1\rho} \bar{u}(k_1) A^a_{\mu\nu;ij} v(k_2) - \frac{g_s \lambda_i^b}{2} k_{2\rho} \bar{u}(k_1) A^a_{\mu\nu;il} v(k_2) \]

\[ + ig_s f^{abc} \frac{p_{2\rho}}{k \cdot p_2} \bar{u}(k_1) A^c_{\mu\nu;ij} v(k_2). \]

(.7)

At the lowest order, the amplitude for the two-body process \( \gamma g \to Q \bar{Q} \) can be written as

\[ \bar{u}(k_1) A^a_{\mu\nu;ij} v(k_2) \equiv \lambda^a_{ij} B_{\mu\nu}. \]

(.8)

Using the identity

\[ 2i f^{abc} \lambda^c_{ij} = \left( \lambda^a \lambda^b \right)_{ij} - \left( \lambda^b \lambda^a \right)_{ij} \]

(.9)
eqq. (7) becomes

\[ A^{(r)}_{\gamma g} (k \to 0) = \frac{g_s}{2} \left[ \left( \lambda^b \lambda^a \right)_{ij} \left( \frac{k_{1\rho}}{k \cdot k_1} - \frac{p_{2\rho}}{k \cdot p_2} \right) - \left( \lambda^a \lambda^b \right)_{ij} \left( \frac{k_{2\rho}}{k \cdot k_2} - \frac{p_{2\rho}}{k \cdot p_2} \right) \right] B_{\mu\nu}. \]

(.10)

Squaring and summing over initial and final degrees of freedom we obtain

\[ |A^{(r)}_{\gamma g} (k \to 0)|^2 = \frac{g_s^2}{4} \left[ \text{Tr} \left( \lambda^b \lambda^a \lambda^a \lambda^b \right) \left( (k_1 k_1) - 2(p_2 k_1) - 2(p_2 k_2) + (k_2 k_2) \right) \right. \]

\[ \left. - 2 \text{Tr} \left( \lambda^b \lambda^a \lambda^b \lambda^a \right) \left( (k_1 k_2) - (p_2 k_1) - (p_2 k_2) \right) \right] \sum_{\text{spin}} B_{\mu\nu} \left( B^{\mu\nu} \right)^{\dagger}, \]

(.11)

where we introduced the eikonal factors defined by

\[ (vw) = \frac{v \cdot w}{v \cdot k \cdot w \cdot k}, \]

(.12)

and we used the masslessness of the incoming parton, \( p_2^2 = 0 \). Evaluating the traces

\[ \text{Tr} \left( \lambda^b \lambda^a \lambda^b \lambda^b \right) = 16 T_F D_A C_F, \quad \text{Tr} \left( \lambda^b \lambda^a \lambda^b \lambda^a \right) = 16 T_F D_A \left( C_F - \frac{1}{2} C_A \right), \]

(.13)
inserting the proper flux factor, and averaging, we finally have the squared three-body amplitude in the soft limit

\[
\mathcal{M}_{\gamma g}^{(s)} = g_s^2 \left[ C_F \left( (k_1 k_1) + (k_2 k_2) \right) - 2 \left( C_F - \frac{1}{2} C_A \right) (k_1 k_2) \right. \\
- C_A \left( (p_2 k_1) + (p_2 k_2) \right) \left. \mathcal{M}_{\gamma g}^{(b)} \right].
\]

(14)

For QCD with three colours \( T_F = 1/2, D_A = 8, C_F = 4/3 \) and \( C_A = 3 \). Here

\[
\mathcal{M}_{\gamma g}^{(b)} = \frac{1}{2s 4 D_A} \text{Tr} (\lambda^a \lambda^a) \sum_{\text{spin}} B_{\mu\nu} (B^{\mu\nu})^\dagger
\]

(15)

is the Born amplitude squared for the two-body process \( \gamma g \rightarrow Q\bar{Q} \). By direct calculation we obtain

\[
\mathcal{M}_{\gamma g}^{(b)} = \frac{e_Q^2 g_s^2 T_F}{s u t} \left[ (t^2 + u^2 - \epsilon s) (1 - \epsilon) + 4m^2 s \left( 1 - \frac{m^2 s}{u t} \right) \right],
\]

(16)

where \( e_Q \) is the heavy-quark charge. Equation (14) can now be integrated over three-body phase space. Using the notations of Section 2 we have

\[
f_{\gamma g}^{(s)} (\theta_1) = \int dx dy d\theta_2 \delta(1 - x) \left( 1 - y^2 \right)^{-1 - \epsilon} \sin^{-2\epsilon} \theta_2 [4t_k u_k \mathcal{M}_{\gamma g}]
\]

\[
= \int dx dy d\theta_2 \delta(1 - x) \left( 1 - y^2 \right)^{-1 - \epsilon} \sin^{-2\epsilon} \theta_2 \left[ 4t_k u_k \mathcal{M}_{\gamma g}^{(s)} \right].
\]

(17)

The only non-trivial point of the calculation is the integration of the eikonal factors

\[
I(\nu \nu) = \int dy d\theta_2 \left( 1 - y^2 \right)^{-1 - \epsilon} \sin^{-2\epsilon} \theta_2 \left[ 4t_k u_k (\nu \nu) \right]_{x=1}.
\]

(18)

The expression for \( f_{\gamma g}^{(s)} (\theta_1) \) is obtained by formal substitution of each \( \nu \nu \) eikonal factor in eq. (14) for the corresponding integral \( I(\nu \nu) \). We give here all the eikonal integrals, thus correcting some misprints of the analogous formulae in ref. [3].

\[
I_{(p_1 p_2)} = -8s \frac{\pi}{\epsilon}
\]

\[
I_{(p_1 k_1)} = 4s \pi \left[ -\frac{1}{\epsilon} + \log \frac{-t}{m^2} + \log \frac{-t}{s} - \frac{\epsilon}{2} \log^2 \frac{1 + \beta}{1 - \beta} \\
-2\epsilon \text{Li}_2 \left( 1 + \frac{2t}{s(1 - \beta)} \right) - 2\epsilon \text{Li}_2 \left( 1 + \frac{2t}{s(1 + \beta)} \right)
\]
\[-2\epsilon \log \frac{-2t}{s(1 + \beta)} \log \frac{-2t}{s(1 - \beta)}\]

\[I_{(p_2k_1)} = I_{(p_1k_1)}(t \rightarrow u)\]

\[I_{(p_1k_2)} = I_{(p_2k_1)}\]

\[I_{(p_2k_2)} = I_{(p_1k_1)}\]

\[I_{(k_1k_1)} = 8s\pi \left(1 + \frac{\epsilon}{\beta} \log \frac{1 + \beta}{1 - \beta}\right)\]

\[I_{(k_2k_2)} = I_{(k_1k_1)}\]

\[I_{(k_1k_2)} = 8s \left(1 - \frac{\rho}{2}\right) \frac{\pi}{\beta} \left[\log \frac{1 + \beta}{1 - \beta} + \epsilon \left(G_{2\beta} \frac{1 + \beta}{1 - \beta} - G_{2\beta} \frac{-2\beta}{1 - \beta}\right)\right]. \quad (19)\]

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**Figure Captions**

Fig. 1: Total cross section at leading and next-to-leading order for charm production in $\gamma N$ collisions as a function of the beam energy. We plot the range of variation for the scale changes indicated in the figure, for $m_c = 1.5$ GeV. MRSD0 parton distribution set. The dotted line represents the contribution from the hadronic component of the photon, evaluated using the set ACFGP-mc.

Fig. 2: Total cross section at leading and next-to-leading order for bottom production in $\gamma N$ collisions as a function of the beam energy. We plot the range of variation for the scale changes indicated in the figure, for $m_b = 4.75$ GeV. MRSD0 parton distribution set.

Fig. 3: Total cross section for charm and bottom production in $\gamma N$ collisions as a function of the beam energy. We plot the overall range of variation for changes in the parameters as discussed in the text, each band representing the result for a specified value of the quark mass.

Fig. 4: Photon beam energy spectra for NA14/2 and for E687.

Fig. 5: Charm inclusive $p_t$ and $x_F$ distributions in $\gamma N$ collisions with the E687 and NA14 photon beam energy spectrum.

Fig. 6: Invariant-mass distribution of charm pairs produced in $\gamma N$ collisions with the E687 and NA14 photon beam energy spectrum.

Fig. 7: Rapidity correlation and $x_F$ distribution for charm pairs produced in $\gamma N$ collisions with the NA14 photon beam energy spectrum. The lower curves are obtained requiring both quarks to have $x_F > 0$.

Fig. 8: Same as fig. 7, but for the E687 photon beam spectrum.
Fig. 9: Charm pair $p_T^{Q\bar{Q}}$ and azimuthal correlation in $\gamma N$ collisions with the E687 and NA14 photon beam spectrum.

Fig. 10: Bottom inclusive $p_T$ and $x_F$ distributions in $\gamma N$ collisions with the E687 and NA14 photon beam spectrum.

Fig. 11: $p_T^{Q\bar{Q}}$ and invariant-mass distributions of bottom pairs produced in $\gamma N$ collisions with the E687 and NA14 photon beam spectrum.

Fig. 12: Bottom pair correlations in $\gamma N$ collisions with the E687 (solid lines) and NA14 (dashed lines) photon beam energy spectrum: azimuthal correlations (left side), $x_F$ of the pair (right side) and $b\bar{b}$ rapidity difference (left inset).

Fig. 13: Comparison between HERWIG and the $O(\alpha_{em}\alpha_s^2)$ result (solid) for inclusive distributions of charm with the E687 photon beam. For HERWIG we plot the variables relative to the charm quark before hadronization (dotted line) and relative to stable charm hadrons (dashed line).

Fig. 14: Comparison between HERWIG and the $O(\alpha_{em}\alpha_s^2)$ result (solid) for inclusive $p_T^{Q\bar{Q}}$ and $\Delta\phi$ distributions of charm with the E687 photon beam. Different line patterns are explained in the previous figure's caption.

Fig. 15: Effect of a non-perturbative $p_T$ kick for the incoming parton in the nucleon, compared with the $O(\alpha_{em}\alpha_s^2)$ effect. The curves with a $p_T$ kick are obtained with the Born cross section with MRSD0 structure functions, supplemented by a random $p_T$ kick on the incoming parton. The NLO curves are obtained with the same structure functions and were rescaled to the same normalization as the other curves.