Spontaneous CP Violation in next to minimal renormalizable SUSY SO(10)

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Abstract

The minimal renormalizable SUSY SO(10) model (MSGUT), is a very compact and predictive theory. It was very popular till one realized that it cannot account for the masses of the neutrinos. The best cure to this problem is to add the $120$ Higgs representation, the “next to minimal” version, sometimes called “new minimal susy GUT” (NMSGUT). To reduce the number of free parameters, it was suggested in recent papers to use only real parameters in the superpotential and induce CP violation via complex VEVs. This is what one usually calls spontaneous CP violation. The number of free parameters turned out, then, to be even smaller than in the original minimal model and good fits to all known masses and mixings were obtained.

Out of those papers, only that of Aulakh and Garg discusses how CP is spontaneously violated. Some heavy MSSM singlet VEVs generate a phase at high scale and CP violation is carried down to the CKM matrix by the mixing of the scalar MSSM doublets. They study the model in great detail and give a large set of solutions. As a proof of principle, two of the solutions are shown to induce realistic phenomenological fits. It is not clear, however, how the right physical solution is obtained. The aim of this paper is to present a scenario how this can be done. I study the way solutions for spontaneous CP violation affect the scalar potential. The one that gives the lowest minimum of the potential, in terms of a given set of parameters, is the right physical one. In the way of doing so, I will prove that complex MSSM singlet VEVs lead actually to lower minima than the real (CP conserving) ones. This proves that CP is spontaneously violated in this model.

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1 Introduction

SO(10) is the minimal GUT gauge group that involves naturally light massive neutrinos through the seesaw mechanism. Its supersymmetric version, SUSY SO(10), stabilizes the hierarchy and has R-Parity (matter parity) as a gauge symmetry. In the renormalizable case R-Parity survives all symmetry breaking. Renormalizability requires high Higgs representations, e.g. at least one \( \mathbf{126} \) Higgs. Therefore, the gauge coupling becomes “strong” (Landau pole) just above the GUT scale. Allowing for non-renormalizable contributions (suppressed by \( 1/M_{\text{Planck}} \) ) one can get along with smaller representations. (E.g. \( \mathbf{16} \times \mathbf{16} \approx \mathbf{126} \) can play the role of \( \mathbf{126} \) in SO(10)). On the other hand, renormalizable models require less ad hoc assumptions and fewer parameters than non-renormalizable ones. Their lightest SUSY particle is stable (LPS - a good dark matter candidate). Such models do not involve uncertain effects of gravitational interactions. The fact that a Landau pole is not far from the GUT scale is not a serious problem here, because one does not use the physics above it.

Quite a few papers discuss renormalizable SO(10) models. The main attention was devoted recently to the minimal renormalizable SUSY SO(10) (MSSO(10)). This version is very compact and predictive and several groups studied the model in great detail. One finds that the requirement that SUSY remains unbroken at high energies allows one to calculate the gauge symmetry breaking. This is related to the fermionic masses and mixing through the fact that only two Higgs doublets remain light in the minimal supersymmetric standard model (MSSM). The bi-doublet Higgs of the MSSM are linear mixtures of all the original scalar doublets and the mixing parameters depend on the way the gauge symmetry is broken. Nice fits to the fermionic masses and mixing are obtained, except for the absolute masses of the neutrinos. This is because gauge unification and other reasons oblige the masses of the right handed (RH) neutrinos to lie not far from the GUT scale. This leads in the seesaw mechanism to too small masses of the neutrinos.

Recently suggested solutions involve adding the \( D(120) \) Higgs representation, adding type II seesaw, considering possible contribution from soft SUSY breaking terms or adding warped extra dimensions. Adding \( D(120) \) Higgs is the only suggestion that is discussed in great detail. The idea here is that when the \( H(10) \) and \( D(120) \) Higgs representations dominate the contributions to the fermionic masses, the Yukawa couplings of \( \Sigma(\mathbf{126}) \) can be smaller and hence acquire smaller RH neutrino masses. This gives larger neutrino masses and the right scale for leptogenesis.

However, the generic fits involve many parameters. To reduce the number of free parameters, it was suggested recently to add the requirement that CP violation should be generated spontaneously. This means practically that all parameters in the superpotential are real and CP violation is induced by complex VEVs. In this way the number of free parameters is even smaller than in MSGUT. Actually, spontaneous CP violation (SCPV) was already applied to SUSY SO(10) several years ago.
Yet, the paper of Grimus and Kühböck\[13\] studies only generic fits to the fermionic masses and mixing using complex VEVs, without explaining how the SCPV was generated. Aulakh and Garg\[16\], on the other hand, discuss in detail the gauge symmetry breaking in the NMSGUT into the effective MSSM (by fine tuning to keep a single pair of Higgs doublets light). They use an analytic expression for all heavy MSSM singlet VEVs as solutions of a cubic equation in a single variable “x”. The Higgs fraction parameters which are determined by the fine tuning condition are also functions of x. They prove then that the only way to have non trivial phases in the CKM matrix, for real superpotential parameters, is for x to be complex. Therefore, any complex solution of the cubic equation leads to spontaneous CP violation. There are, however, many such solutions. As an example, two solutions are shown by the authors to give realistic fits to the fermionic masses and mixings. Which one of those solutions is the correct physical one? Obviously, the solution that leads to the lowest minimum of the scalar potential.

The aim of this paper is to suggest a scenario how this can be done. The idea is to write the potential in terms of the scalar doublets and the heavy MSSM singlet VEVs. Then some VEVs are given a phase and one looks for the lowest minimum of the potential with respect to these phases. One can then fix x and the corresponding Higgs fraction parameters “α\text{u,d}_i”, which dictate the CKM matrix. In the way of doing all this, I prove that complex MSSM singlet VEVs lead to lower minima than the real (CP conserving) ones. This proves that CP is spontaneously violated in the model.

The plan of the paper is as follows: In Sector 2, I will give an introduction to SCPV. Sector 3 will present SUSY SO(10) and in particular the NMSGUT. Then in Sector 4 the SCPV in NMSGUT will be discussed in detail using an example. How CP violation is spontaneously generated at the high scale and carried down to low energies will be summarized in Sector 5. The conclusions come in Sector 6.

2 Spontaneous CP violation

There are three manifestations of CP violation in Nature:

1) Fermi scale CP violation as is observed in the K and B decays\[17\]. This violation is induced predominantly by a complex mixing matrix of the quarks (CKM).

2) The cosmological matter antimatter asymmetry (BAU) is an indication for high scale CP violation\[18\]. In particular, it’s most popular explanation via leptogenesis\[19\] requires CP breaking decays of the heavy right-handed (RH) neutrinos.

3) The strong CP problem called also the QCD Θ problem\[20\] lies in the non-observation of CP breaking in the strong interactions while there is an observed CP violation in the interaction of quarks.
It is still not clear if there is one origin to those CP breaking manifestations. What is the nature of the violation of CP? Is it intrinsic in terms of complex Yukawa couplings or due to spontaneous generation of phases in the Higgs VEVs?

Spontaneous violation of CP \[21\] is more difficult to realize, but has advantages with respect to the intrinsic ones:

1) It is more elegant and involves less parameters. The intrinsic breaking becomes quite arbitrary in the framework of SUSY and GUT theories.

2) It solves the SUSY CP violation problem (too many potentially complex parameters) as all parameters are real.

3) It leads to the vanishing of $\Theta_{QCD}$ (but not ArgDetM) at the tree level. This can be used as a first step towards solving the CP problem by adding extra symmetries and exotic quarks \[22\][\[23\][\[24\].

For good recent discussion of spontaneous CP violation, with many references, see Branco and Mohapatra\[25\].

It is preferable to break CP at a high scale. This is what we need for the BAU. Especially, if this is due to leptogenesis i.e. CP violating decays of heavy neutrinos, it is mandatory. This is also needed to cure the domain wall problem \[26\].

Also, SCPV cannot take place in the standard model (SM) because of gauge invariance. Additional Higgs bosons must be considered and those lead generally to flavor changing neutral currents. The best way to avoid these is to make the additional scalars heavy\[25\].

As a warm-up simple example for SCPV at the GUT scale let me present a possible SCPV in the renormalizable non-SUSY SO(10).

SO(10) fermions are in three $16$ representations: $\Psi_i(16)$.

$$16 \times 16 = (10 + 126)_S + 120_{AS} . \quad (1)$$

Hence, only $H(10)$, $\bar{\Sigma}(126)$ and $D(120)$ can contribute directly to Yukawa couplings and fermion masses. Additional Higgs representations are needed for the gauge symmetry breaking.

One and only one VEV $\bar{\Delta} =< \bar{\Sigma}(1,1,0) >$ can give a mass to the RH neutrinos via

$$Y^i_{\ell j} \nu^i_R \bar{\nu}^j_R \quad (2)$$

and so induces the seesaw mechanism. It breaks also B-L and $\text{SO}(10) \rightarrow \text{SU}(5)$.

To generate SCPV in conventional SO(10) one can use the fact that $\bar{\Sigma}(126)$ is the only relevant complex Higgs representation. Its other special property is that $(\Sigma)^i_s$ is invariant in $\text{SO}(10)\[27\]$. This allows for a SCPV at the high scale, using the scalar
potential \[28\]:

\[ V = V_0 + \lambda_1 (H)^2_s + (\Sigma)^2_s + \lambda_2 (\Sigma)^4_s + (\Sigma^*)^4_s \, . \] (3)

Inserting the Ansatz VEVs

\[ < H(1, 2, -1/2) > = \frac{v}{\sqrt{2}} \quad \Sigma = e^{i\alpha} \] (4)

in the neutral components, the phase dependent part of the scalar potential reads

\[ V(v, \sigma, \alpha) = A \cos(2\alpha) + B \cos(4\alpha) \, . \] (5)

For \( B \) positive and \( |A| < 4B \) the absolute minimum of the potential requires

\[ \alpha = \frac{1}{2} \arccos \left( \frac{A}{4B} \right) \, . \] (6)

This ensures the spontaneous breaking of CP\[24\].

It is not possible to realize the above scenario in renormalizable SUSY theories, as \( \Phi^4 \) cannot be generated from the superpotential in this case. A different approach is needed as will be presented later.

3 The minimal renormalizable SUSY \( SO(10) \) and next to minimal one

Renormalizable SUSY \( SO(10) \) models were studied in many papers \[3\][4][5][6][7]. In particular the so-called \emph{minimal renormalizable SUSY \( SO(10) \) model} (MSGUT)\[8] became very popular recently\[9][10] due to its simplicity, predictability and automatic \( R \)-parity invariance (i.e. a dark matter candidate).

It includes the following Higgs representations

\[ H(10), \quad \Phi(210), \quad \Sigma(126) \oplus \sum(\overline{126}) \, . \] (7)

Both \( \Sigma \) and \( \overline{\Sigma} \) are required to avoid high scale SUSY breaking (\( D \)-flatness) and \( \Phi(210) \) is needed for the gauge breaking.

The properties of the model are dictated by the superpotential. This involves all possible renormalizable products of the superfields

\[ W = M_\Phi \Phi^2 + \lambda_\Phi \Phi^3 + M_\Sigma \Sigma \overline{\Sigma} + \lambda_\Sigma \Phi \Sigma \overline{\Sigma} + M_H H^2 + \Phi H (\kappa \Sigma + \overline{\kappa} \overline{\Sigma}) + \Psi_i (Y_{ij}^{(10)} H + Y_{ij}^{(126)} \overline{\Sigma}) \Psi_j \] (8)
The symmetry breaking goes in two steps

\[ SU\, SY\, SO(10) \xrightarrow{\text{strong gauge breaking}} MSSM \xrightarrow{\text{SUSY breaking}} SM \]  \hspace{1cm} (9)

The \( F \) and \( D \)-terms must vanish during the strong gauge breaking to avoid high scale SUSY breakdown ("\( F,D \) flatness").

**D-flatness:** only \( \Sigma, \bar{\Sigma} \) are relevant, therefore

\[ |\Delta| = |\bar{\Delta}|. \]  \hspace{1cm} (10)

The situation with \( F\)-flatness is more complicated. The strong breaking is dictated by the VEVs that are SM singlets. Those are, in the \( SU_C(4) \times SU_L(2) \times SU_R(2) \) notation:

\[ \phi_1 = < \Phi(1,1,1) >, \quad \phi_2 = < \Phi(15,1,1) >, \quad \phi_3 = < \Phi(15,1,3) >, \]
\[ \Delta = < \Sigma(10,1,3) >, \quad \bar{\Delta} = < \bar{\Sigma}(10,1,3) >. \]

The strong breaking superpotential in terms of those VEVs is then\(^1\)

\[ W_H = M_\phi (\phi_1^2 + \phi_2^2 + \phi_3^2) + \frac{\lambda_\phi}{\sqrt{2}} \left( \frac{1}{3} \phi_1^2 + \frac{1}{3} \phi_1 \phi_2 \phi_3 \right) + M_W \Delta \bar{\Delta} + \frac{\lambda_\psi}{10} \Delta \bar{\Delta} \left( \frac{1}{\sqrt{6}} \phi_1 + \frac{1}{\sqrt{2}} \phi_2 + \phi_3 \right). \]  \hspace{1cm} (11)

\[ \frac{\partial W_H}{\partial v_i} = 0 \] gives a set of equations. Their solutions dictate the details of the strong symmetry breaking. \(^4\)\(^10\)

One tunes the parameters such that the breaking

\[ SU\, SY\, SO(10) \rightarrow MSSM \]

will be achieved\(^9\)\(^10\).

The MSSM vacuum is fixed then by one parameter \( x \), the solution of the cubic equation\(^4\):

\[ 8x^3 - 15x^2 + 14x - 3 = -\frac{\lambda_\psi M_\Sigma}{\lambda_\Sigma M_\phi} (1-x)^2. \]  \hspace{1cm} (12)

The high scale VEVs are then given as a function of \( x \):

\[ \Phi_1 = -\frac{2 \sqrt{3} M_\phi}{\lambda_\phi} \frac{x(1-5x^2)}{(1-x)^2}, \quad \Phi_2 = -\frac{2 \sqrt{18} M_\phi}{\lambda_\phi} \frac{(1-2x-x^2)}{(1-x)^2}, \quad \Phi_3 = \frac{12 M_\phi}{\lambda_\phi} x, \]
\[ \Delta \bar{\Delta} = \frac{240 M_\phi^2}{\lambda_\phi \lambda_\Sigma} \frac{(1-3x)(1+x^2)}{(1-x)^4}. \]

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\(^{1}\) Using the notation of Ref. \(^10\).
SUSY is broken by the soft SUSY breaking terms. The gauge MSSM breaking is induced by the VEVs of the SM doublet φ_u,d(1, 2, ±1/2) components of the Higgs representations.

The mass matrices of the higgsinos and Higgs scalars are:

\[ M^u_{ij} = \left[ \frac{\partial^2 W}{\partial \phi_i^u \partial \phi_j^u} \right]_{\phi_i = <\phi_i>} \quad M^d_{ij} = \left[ \frac{\partial^2 W}{\partial \phi_i^d \partial \phi_j^d} \right]_{\phi_i = <\phi_i>} \]  \quad (14)

The requirement

\[ \det(M^u_{ij}) \approx 0 \quad \det(M^d_{ij}) \approx 0 \]  \quad (15)

leaves only two light combinations of doublet components and those play the role of the bi-doublets h_u, h_d of the MSSM. (This also is discussed in detail in the papers of [9][10].)

However, as was explained in the introduction, the minimal model MSGUT cannot account for the right neutrino masses. I will study therefore its minimal extension NMSGUT. One adds here the Higgs representation \(D(120)\) that couples antisymmetrically to the fermions. The fermionic mass matrices can then be formally written as follows:

\[ M^i = Y^i_{10} H + Y^i_{126} \Sigma + Y^i_{120} D \]  \quad (16)

in terms of the Yukawa matrices, where \(i = (u, d, e, \nu_D)\).

\(D(120)\) does not involve MSSM singlets, hence it does not take part in the strong gauge breaking. I.e., the equations of the F,D-flatness are exactly as in the minimal model[16]. \(D(120)\) contributes, however, new terms to the superpotential:

\[ W_D = \frac{M_\Omega}{2} D^2 + \lambda_1 D H \Phi + \lambda_2 D D \Phi + D \Phi (\lambda \Sigma + \bar{\lambda} \bar{\Sigma}) + \Psi_i (Y^i_{120} D) \Psi_j. \]

The MSSM relevant part of the superpotential includes the SM doublets:

\[ \phi^u = <\Phi(1, 2, 1/2)> \quad \phi^d = <\Phi(1, 2, -1/2)> \]
\[ H^u = <H(1, 2, 1/2)> \quad H^d = <H(1, 2, -1/2)> \]
\[ \Delta^u = <\Sigma(1, 2, 1/2)> \quad \Delta^d = <\Sigma(1, 2, -1/2)> \]
\[ \bar{\Delta}^u = <\bar{\Sigma}(1, 2, 1/2)> \quad \bar{\Delta}^d = <\bar{\Sigma}(1, 2, -1/2)> \]
\[ D^u_1 = <D(1, 2, 1/2)> \quad D^d_1 = <D(1, 2, -1/2)> \]
\[ D^u_{15} = <D(1, 2, 1/2)> \quad D^d_{15} = <D(1, 2, -1/2)> \]  \quad (17)

Note that \(D(120)\) involves two kind of contributions under \(SU_C^c(4) \times SU_L(2) \times SU_R(2)\)

\[ D(120): D^{u,d}_{1}(1, 2, 2) \quad D^{u,d}_{15}(15, 2, 2). \]
The scalar doublet mass matrix is now $6 \times 6$.

The VEVs of these doublets are linear combinations of its physical eigenvectors. Using the fine tuning requirement (14), only the MSSM Higgs doublets $h^u, h^d$ remain light

\begin{align*}
\phi^{u,d} &= \alpha_{\phi}^{u,d} h^{u,d} + \text{heavy (decoupled)} \\
H^{u,d} &= \alpha_{H}^{u,d} h^{u,d} + \text{heavy (decoupled)} \\
\Delta^{u,d} &= \alpha_{\Delta}^{u,d} h^{u,d} + \text{heavy (decoupled)} \\
\bar{\Delta}^{u,d} &= \alpha_{\bar{\Delta}}^{u,d} \bar{h}^{u,d} + \text{heavy (decoupled)}
\end{align*}

Here $\alpha_{i}^{u,d}$ are the Higgs fractions, given in explicit complicate expressions in the paper of Aulakh and Garg[16]. They play a crucial role in dictating the CKM matrix. The $\alpha_{i}^{u,d}$ are a function of $x$ and it was shown by Aulakh and Garg that for real values of $x$ the CKM matrix remain real. Hence, for real values of the superpotential $x$ must be complex to have CP violation.

4 The spontaneous CP violation in NMSGUT

Let us assume that all parameters of the superpotential as well as those of the soft SUSY breaking terms are real. CP will be violated spontaneously if certain VEVs generate a phase. In other words, the scalar potential will have a minimum with non-trivial phases. As was explained in Sec. 2, we would like the phases to be generated for the heavy VEVs and if possible also for $\bar{\Delta}$, in order to have naturally leptogenesis. In terms of eq. (13) it is evident that complex MSSM singlet VEVs require complex $x$. There are obviously a large set of such complex solutions of eq. (12). Two solutions are actually used in ref. [16] as a basis for realistic generic fits for the fermionic masses and mixing. (One of them corresponds to that of Grimus and Küböck[13]). The authors emphasize themselves, however, “that these fits are significant purely as proof of principle”. So, which SCPVing solution is the right physical one? Clearly the solution that leads to the lowest minimum of the scalar potential.

Let me present in the following a scenario how this can be done.
The part of the effective superpotential (after D- and F-flatness are taken into account) that involves the coupling of the MSSM singlets to the doublets is as follows:

\[
W_{\text{eff}} =
\frac{\lambda}{\sqrt{2}}(\Phi^u \Delta^d \Delta + \Phi^d \Delta^u \Delta) \quad - \quad \frac{\kappa}{\sqrt{2}} \Phi^d H^u \Delta - \frac{\epsilon}{\sqrt{2}} \Phi^u H^d \Delta
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_1 (\Delta^d H^u - \Delta^u H^d)
+ \frac{\lambda u}{\sqrt{2} \sqrt{2}} \Phi_2 (\Delta^d H^u - \Delta^u H^d)
- \frac{\lambda d}{\sqrt{2} \sqrt{2}} \Phi_3 (\Delta^d H^u + \Delta^u H^d)
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_{15} (D_i^u H^d + D_i^d H^u)
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_3 (D_i^u H^d + D_i^d H^u)
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_3 (D_i^d H^u - D_i^u H^d)
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_3 (D_i^d H^u - D_i^u H^d)
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_{15} D_i^u D_i^d \Phi_2
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_{15} D_i^u D_i^d \Phi_2
- \frac{1}{\sqrt{2} \sqrt{2}} (\lambda D_i^u \Phi^d \Delta + \tilde{\lambda} D_i^d \Phi^u \Delta)
- \frac{1}{\sqrt{2} \sqrt{2}} (\lambda D_i^u \Phi^d \Delta + \tilde{\lambda} D_i^d \Phi^u \Delta)
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_3 (\Delta^d \Delta^u - \Delta^u \Delta^d)
+ \frac{\lambda}{\sqrt{2} \sqrt{2}} \Phi_3 (\Delta^d \Delta^u - \Delta^u \Delta^d)
\]

Here the conventions of ref.\[10\] are used.

Note, that \(W_{\text{eff}}\) gives also the the mass matrix of the doublets\[16\] \[20\]. This mass matrix is fine tuned, see eq.(15). The effect of this fine tuning to the MSSM will be taken into account, as in eq. (18), using the Higgs fractions. The MSSM singlet VEVs are not affected. \(W_{\text{eff}}\) is not the only source of the scalar potential, other MSSM effective terms must be obviously added. Let us, however, discuss first the part derived from \(W_{\text{eff}}\). To prove SCPV, one must show that complex VEVs lead to a minimum of the scalar potential. In this case the Higgs fractions \(\alpha_i^u,d\) are also complex. Yet, the different phases are correlated in view of their \(x\) dependence. Hence, to prove that CP is spontaneously violated, and to find out what are the physical solutions of eq. (12), it is enough to show that two of the phases lead to a minimum.

The effective scalar potential of \(W_{\text{eff}}\) is as follows

\[
V_{\text{eff}}(\lambda_0, \lambda_\Sigma, \kappa, \tilde{\kappa}, \lambda_i, \lambda, \tilde{\lambda}, V_i) = \sum \left| \frac{\partial W_{\text{eff}}}{\partial V_i} \right|^2 V_i. \quad (20)
\]

Here \(V_i\) stand vor the different VEVs and fields. This is a long and complicated expression. The corresponding derivatives can be found in Appendix I. Note that instead of writing explicitly the derivative with respect to e.g. \(\alpha_i^u\), we use the derivatives with respect to \(\Phi^u = \alpha_i^u h^u\) etc.
Looking at the derivatives of $W_{\text{eff}}$ in Appendix I, one sees that the singlet VEVs appear always linearly. The most general phase dependence of the singlet VEVs looks then as follows:

$$\frac{\partial W_{\text{eff}}}{\partial V_k}(\phi) = A^k + \sum B^k_j e^{i\phi_j}. \quad (21)$$

Where $A^k$ and $B^k$ are combinations of real coupling constants, real VEVs and the $\alpha_i$. We disregard here the phases of the $\alpha_i$, as they are anyhow correlated with the other phases.

Hence,

$$V_{\text{eff}} = \sum_k \left| \frac{\partial W_{\text{eff}}}{\partial V_k} \right|^2 V_k = \sum_k \left| A^k + \sum_j B^k_j e^{i\phi_j} \right|^2 = \sum_{k,j} (A^{k^2} + B^{k^2}_j + 2A^k B^k_j \cos \phi_j + 2 \sum_{\ell \neq j} B^k_{\ell} B^k_j \cos (\phi_\ell - \phi_j)). \quad (22)$$

SCPV means here that some phases appear in the minimum of the scalar potential for a finite range of the parameters.

Let us look for special cases.

If only one VEV has a phase, the trivial solution $\phi = 0, \pi$ results.

The simplest possibility is that two VEVs generate a phase. Let us stick to this possibility for simplicity (a generalization is straightforward).

Which phases should be involved?

$\bar{\Delta}$ is a most wishful candidate. Its phase will induce CP violation in the RH neutrinos decay, as is needed for leptogenesis (BAU). One cannot have both $\bar{\Delta}$ and $\Delta$ as candidates, no derivative involves both of them (Appendix I).

So the simplest possibility is that $\bar{\Delta}$ and one of the $\Phi_i$ will generate a phase $(\bar{\delta}, \phi)$ spontaneously.

One obtains in this case the following generic scalar potential:

$$V(\bar{\delta}, \phi) = S + R \cos \bar{\delta} + Q \cos \phi + T \cos (\bar{\delta} - \phi). \quad (23)$$

Where,

$$S = \sum (A^2_k + B^2_{\bar{\Delta}_k} + B^2_{\Phi_k}), \quad R = 2 \sum A_k B_{\bar{\Delta}_k},$$

$$Q = 2 \sum A_k B_{\Phi_k}, \quad T = 2 \sum B_{\bar{\Delta}_k} B_{\Phi_k}.$$  

$S, Q, R$ and $T$ are combinations of coupling constants, VEVs and $\alpha_i$. They depend on the $\Phi_i$ one takes. They can be fixed in principle by the phenomenological fits and certain simplifying assumptions.

Now, to look for spontaneous generation of CP violation, we have to show that there is a minimum of the scalar potential in a certain range of parameters, with non-trivial values of the phases. (For the minimalization conditions for two variables see Appendix II.)
\[
\frac{\partial V}{\partial \bar{\delta}} = -R \sin \bar{\delta} - T \sin (\bar{\delta} - \phi) = 0 \\
\frac{\partial V}{\partial \phi} = -Q \sin \phi - T \sin (\bar{\delta} - \phi) = 0
\] (24)

Solving the equations, one obtains

\[
\sin \phi = -\frac{R}{Q} \sin \bar{\delta} \quad \cos \bar{\delta} = \frac{TQ}{2} \left( \frac{1}{R^2} - \frac{1}{Q^2} - \frac{1}{T^2} \right).
\] (25)

The second derivatives are

\[
\frac{\partial^2 V}{\partial \bar{\delta}^2} = -R \cos \bar{\delta} - T \cos (\bar{\delta} - \phi) \\
\frac{\partial^2 V}{\partial \phi^2} = -Q \cos \phi - T \cos (\bar{\delta} - \phi) \\
\frac{\partial^2 V}{\partial \bar{\delta} \partial \phi} = T \cos (\bar{\delta} - \phi).
\] (26)

The conditions for an extremum (Appendix II) require

\[
F \equiv (R \cos \bar{\delta} + T \cos (\bar{\delta} - \phi))(Q \cos \phi + T \cos (\bar{\delta} - \phi)) - T^2 \cos^2 (\bar{\delta} - \phi) > 0
\] (27)

Using the above solutions (23), one obtains

\[
F = R^2 \sin^2 \bar{\delta} > 0
\]

so that we have an extremum, \textit{independent of the explicit expressions for }S, R, Q, T. To have a minimum one needs also

\[
G \equiv -R \cos \bar{\delta} - T \cos (\bar{\delta} - \phi) > 0.
\]

In terms of the solutions (25) it requires

\[
G = \frac{TR}{Q} > 0.
\]

Hence, we have a non-trivial minimum for the range \(TR/Q > 0\). This means that \(\bar{\Delta}\) and one of the \(\Phi_i\) generate spontaneously phases at the high scale, for the range \(\frac{TR}{Q} > 0\). \(CP\) is therefore violated at high energies. The values of the phases are given in eq. (25).

It is important to note that

\[
\bar{\delta} = \phi = 0
\]

cannot be taken into account because it is a maximum. Therefore, \(CP\) \textit{must be violated spontaneously in the NMSGUT model.}
The explicit expressions for $R$, $Q$ and $T$ depend on what $\Phi_i$ we choose, and they are generally very complicated combinations of coupling constants, VEVs and the $\alpha_i$. It is useful to take $\bar{\Delta}$ and $\Phi_3$ as the corresponding VEVs.

Once phases of the complex singlets are known, one can, in principle, use $\Phi_3$ in eq. (13) to fix the value of $x$ and hence the Higgs fraction parameters $\alpha_i^{u,d}$ as well. Explicit expressions for the $\alpha_i$ as a complicate functions of $x$ are given in ref. [16]. Those parameters dictate then the CP violating CKM matrix as will be explained in the next section.

Now, we did not consider the other contributions to the scalar potential, and in particular the effective MSSM Higgs potential and the soft SUSY breaking terms. Those contributions, however, do not involve the singlet heavy VEVs. Hence, they can at most add a small contribution to the $A^k$ as they involve only low energy VEVs. The fact that the expression (23) mixes large values with small ones does not matter, because the phases are defined by ratios. The terms with heavy VEVs will decouple in the MSSM limit. Note that, it is not surprising that high energy terms are involved in the scalar potential that dictates the phases. Also in ref. [16] the phases are generated at the high scale breaking.

5 CP violation at low energies

We have seen that the scalar potential of the NMSGUT triggers SCPV at the high scale and the violation is carried down to low energies via the $\alpha_i^{u,d}$. At low energies, when the heavy fields decouple, the fine tuning condition (14) leads to the MSSM. The effective MSSM superpotential, involves then the light Higgs fields, $h^u$ and $h^d$ with their Higgs fraction parameters $\alpha_i^{u,d}$, as is given by eq. (18). The Yukawa terms dictate the fermionic mass matrices of equ.(15) as follows:

$$
M_u = (\alpha^u_H Y_{10} + \alpha^u_\Sigma Y_{\Sigma} + \alpha^u_{D_1} + \alpha^u_{D_{15}})Y_{120}v^u
$$

$$
M_d = (\alpha^d_H Y_{10} + \alpha^d_\Sigma Y_{\Sigma} + \alpha^d_{D_1} + \alpha^d_{D_{15}})Y_{120}v^d
$$

$$
M_e = (\alpha^d_H Y_{10} - 3\alpha^d_\Sigma Y_{\Sigma} + \alpha^d_{D_1} - 3\alpha^e_{D_{15}})Y_{120}v^d
$$

$$
M^{\nu}_{\nu} = (\alpha^u_H Y_{10} - 3\alpha^u_\Sigma Y_{\Sigma} + \alpha^u_{D_1} - 3\alpha^u_{D_{15}})Y_{120}v^u.
$$

Where $Y_i$ are the corresponding Yukawa matrices and $<h^u>=v^u$, $<h^d>=v^d$.

Complex $\alpha_i^{u,d}$ induce complex mass matrices. Hence, we obtain a CP violating CKM matrix.
6 Conclusions

Spontaneous CP violation has many advantages on the intrinsic breaking and is more natural. Nevertheless, SCPV has been rarely used in GUTs.

Recently two groups applied spontaneous CP violation in the renormalizable NMSGUT to reduce the number of free parameters.\(^2\)

One of those paper by Aulakh and Garg\(^{[16]}\) showes that SCPV is actually posible in NMSGUT. They proved that there are complex solutions to the GUT scale cubic equation for \(x\), and those lead to CP violation in the CKM matrix, via the the complex Higgs fractions \(a_i^{u,d}\). It is not clear, however, what is the right physical solution.

I have proven that CP is really violated in NMSGUT, by showing that the minimum of the scalar potential violates CP, in terms of MSSM singlet VEVs with very specific phases. The phases that minimize the scalar potential can be used therefore to dictate the physical \(x\). The corresponding Higgs fractions alows one then to get the physical complex CKM matrix. One of the complex VEVs is that of the \(\{126\}\), hence the needed high scale CPV for leptogenesis is also suppressed. Also, the spontaneous breaking is at the high scale, FCNCs and domain walls are avoided. The minimalization of the scalar potential completes therefore the program of Aulakh and Garg.

**Note added**

After the manuscript was finished, I learned about a preprint by Malinský\(^{[29]}\). He also discusses the Higgs sector of the NMSGUT, but without restricting the parameters of the superpotential. He calculates the mass matrices in a way similar to Aulakh and Garg\(^{[16]}\), and finds explicitly the corresponding Higgs fractions. Malinský’s results involve many free parameters as SCPV is not assumed. On top of that, because he uses different methods and phase conventions it is difficult to compare his results with those of Aulakh and Garg for the special case of SCPV.

\(^2\)See also ref. \([7]\)
Appendix I: The derivatives of the superpotential

\[ \frac{\partial}{\partial \Delta} = \frac{\lambda_5}{10} \Phi^d \Delta^u - \frac{\kappa}{\sqrt{5}} \Phi^d H^d - \frac{\lambda}{2\sqrt{30}} (D_1^u \Phi^d + D_{15}^u \Phi^d) \]

\[ \frac{\partial}{\partial \Delta} = \frac{\lambda_5}{10} \Phi^u \Delta^d - \frac{\kappa}{\sqrt{5}} \Phi^u H^d - \frac{\lambda}{2\sqrt{30}} (D_1^d \Phi^u + D_{15}^d \Phi^u) \]

\[ \frac{\partial}{\partial \Delta^d} = \frac{\lambda_5}{10} \Phi^u \Delta^d + \frac{\lambda_5}{15\sqrt{2}} \Phi^u \Delta^d + \frac{\lambda_5}{30} \Phi^u \Delta^d + \frac{\kappa}{\sqrt{10}} \Phi^u H^d - \frac{\kappa}{2\sqrt{5}} \Phi^u H^d + \frac{\lambda}{4\sqrt{30}} \Phi^u D_{15}^d \]

\[ \frac{\partial}{\partial \Delta^u} = \frac{\lambda_5}{15\sqrt{2}} \Phi^u \Delta^d - \frac{\lambda_5}{30} \Phi^u \Delta^d - \frac{\kappa}{\sqrt{10}} \Phi^u H^d - \frac{\kappa}{2\sqrt{5}} \Phi^u H^d + \frac{\lambda}{4\sqrt{30}} \Phi^u D_{15}^d \]

\[ \frac{\partial}{\partial \Delta^u} = \frac{\lambda_5}{15\sqrt{2}} \Phi^d \Delta^d + \frac{\lambda_5}{30} \Phi^d \Delta^d + \frac{\kappa}{\sqrt{10}} \Phi^d H^d - \frac{\kappa}{2\sqrt{5}} \Phi^d H^d + \frac{\lambda}{4\sqrt{30}} \Phi^d D_{15}^d \]

\[ \frac{\partial}{\partial \Delta^d} = \frac{\lambda_5}{15\sqrt{2}} \Phi^d \Delta^d - \frac{\lambda_5}{30} \Phi^d \Delta^d - \frac{\kappa}{\sqrt{10}} \Phi^d H^d - \frac{\kappa}{2\sqrt{5}} \Phi^d H^d + \frac{\lambda}{4\sqrt{30}} \Phi^d D_{15}^d \]

\[ \frac{\partial}{\partial \Phi^d} = -\frac{\kappa}{\sqrt{5}} \Phi^d \Delta^d + \frac{\kappa}{\sqrt{10}} \Phi^d \Delta^d + \frac{\kappa}{\sqrt{10}} \Phi^d \Delta^d - \frac{\kappa}{2\sqrt{5}} \Phi^d \Delta^d \]

\[ \frac{\partial}{\partial \Phi^u} = -\frac{\kappa}{\sqrt{5}} \Phi^u \Delta^d - \frac{\kappa}{\sqrt{10}} \Phi^u \Delta^d - \frac{\kappa}{\sqrt{10}} \Phi^u \Delta^d - \frac{\kappa}{2\sqrt{5}} \Phi^u \Delta^d \]

\[ \frac{\partial}{\partial \Phi^u} = -\frac{\lambda}{2\sqrt{2}} \Phi^d D_{15}^d \]

\[ \frac{\partial}{\partial \Phi^d} = -\frac{\lambda}{2\sqrt{2}} \Phi^d D_{15}^d \]

\[ \frac{\partial}{\partial \Phi^u} = -\frac{\lambda}{2\sqrt{2}} \Phi^u D_{15}^d \]

\[ \frac{\partial}{\partial \Phi^d} = -\frac{\lambda}{2\sqrt{2}} (D_1^d H^d + D_{15}^d H^d) + \frac{\lambda}{4\sqrt{15}} (D_{15}^d \Delta^d + D_{15}^d \Delta^u) + \frac{\lambda}{4\sqrt{15}} (D_{15}^d \Delta^u + D_{15}^d \Delta^d) \]
\[ \frac{\partial}{\partial \Phi_2} = \]
\[ \sqrt{\frac{2}{15}} (\tilde{\Delta}^u \Delta^d + \tilde{\Delta}^d \Delta^u) + \frac{\kappa}{\sqrt{10}} (\Delta^d H^u - \Delta^u H^d) \]
\[ + \frac{\kappa}{\sqrt{10}} (\Delta^d H^u - \Delta^u H^d) + \frac{\alpha_0}{6\sqrt{2}} \Phi^u \Phi^d + \frac{\sqrt{2 \lambda_2}}{9} D_{15}^u D_{15}^d \]
\[ \frac{\partial}{\partial \Phi_3} = \]
\[ - \frac{\kappa}{2\sqrt{5}} (\tilde{\Delta}^d H^u + \Delta^u H^d) - \frac{\kappa}{2\sqrt{5}} (\Delta^d H^u + \tilde{\Delta}^u H^d) \]
\[ - \frac{\lambda_1}{2\sqrt{2}} (D_{15}^u H^d + D_{15}^d H^u) + \frac{\lambda_1}{4\sqrt{30}} (D_{15}^u \tilde{\Delta}^u + D_{15}^d \tilde{\Delta}^d) \]
\[ + \frac{\lambda}{6\sqrt{30}} (D_{15}^u \Delta^d - D_{15}^d \Delta^u) + \frac{\lambda}{6\sqrt{30}} (D_{15}^u \tilde{\Delta}^u - D_{15}^d \tilde{\Delta}^d) \]
\[ + \frac{\lambda}{6\sqrt{30}} (D_{15}^u D_{15}^d + D_{15}^d D_{15}^u) + \frac{\lambda}{30} (\Delta^u \Delta^d - \Delta^d \Delta^u) \]
\[ \frac{\partial}{\partial d} = \]
\[ - \frac{\kappa}{\sqrt{5}} H^u \Delta + \frac{\lambda_4}{10} \tilde{\Delta}^u \Delta + \frac{\lambda_4}{6} \Phi^u (\frac{1}{\sqrt{2}} \Phi_2 + \frac{1}{2} \Phi_3) \]
\[ - \frac{\lambda}{2\sqrt{30}} (D_{15}^u \Delta + D_{15}^d \tilde{\Delta}) \]
\[ \frac{\partial}{\partial D_{15}^u} = \]
\[ \frac{\lambda}{10} \Delta^d \tilde{\Delta} - \frac{\kappa}{\sqrt{5}} H^d \tilde{\Delta} + \frac{\lambda_3}{6} \Phi^d (\frac{1}{\sqrt{2}} \Phi_2 + \frac{1}{2} \Phi_3) \]
\[ - \frac{\lambda}{2\sqrt{30}} (D_{15}^d + D_{15}^d) \tilde{\Delta} \]
\[ \frac{\partial}{\partial D_{15}^d} = \]
\[ - \frac{\lambda_3}{4\sqrt{30}} \Phi_3 \Delta^d - \frac{\lambda_3}{2} \Phi_1 H^d + \frac{\lambda_3}{4\sqrt{30}} \Phi_3 \tilde{\Delta}^u + \frac{\lambda_3}{6\sqrt{30}} \Phi_3 D_{15}^u \]
\[ - \frac{\lambda}{2\sqrt{30}} \Phi^d \tilde{\Delta} \]
\[ \frac{\partial}{\partial D_{15}^u} = \]
\[ - \frac{\lambda}{4\sqrt{30}} \Phi_3 \tilde{\Delta}^d - \frac{\lambda}{2} \Phi_1 H^d + \frac{\lambda}{4\sqrt{30}} \Phi_3 \tilde{\Delta}^u + \frac{\lambda}{6\sqrt{30}} \Phi_3 D_{15}^u \]
\[ - \frac{\lambda}{2\sqrt{30}} \Phi^d \Delta \]
\[ \frac{\partial}{\partial D_{15}^d} = \]
\[ - \frac{\kappa_2}{2\sqrt{2}} \Phi_3 H^u - \frac{\lambda}{6\sqrt{10}} \Phi_3 \tilde{\Delta}^u + \frac{\lambda}{4\sqrt{10}} \Phi_3 \tilde{\Delta}^u + \frac{\lambda}{6\sqrt{10}} \Phi_3 \tilde{\Delta}^u \]
\[ + \frac{\lambda}{4\sqrt{15}} \Phi_1 \tilde{\Delta}^u + \frac{\sqrt{2 \lambda_2}}{9} \Phi_2 D_{15}^u + \frac{\lambda}{6\sqrt{3}} \Phi_3 D_{15}^u - \frac{\lambda}{2\sqrt{10}} \Phi^u \tilde{\Delta} \]
\[ \frac{\partial}{\partial D_{15}^u} = \]
\[ - \frac{\kappa_2}{2\sqrt{2}} \Phi_3 H^d + \frac{\lambda}{6\sqrt{10}} \Phi_3 \tilde{\Delta}^d + \frac{\lambda}{4\sqrt{15}} \Phi_1 \tilde{\Delta}^d - \frac{\lambda}{6\sqrt{10}} \Phi_3 \tilde{\Delta}^d \]
\[ + \frac{\lambda}{4\sqrt{15}} \Phi_1 \tilde{\Delta}^d + \frac{\sqrt{2 \lambda_2}}{9} \Phi_2 D_{15}^d + \frac{\lambda}{6\sqrt{3}} \Phi_3 D_{15}^d - \frac{\lambda}{2\sqrt{10}} \Phi^d \Delta. \]
Appendix II: Minimum of a function with two variables

Conditions for an extremum:

\[ a) \frac{\partial \mathcal{F}(x_0, y_0)}{\partial x} = \frac{\partial \mathcal{F}(x_0, y_0)}{\partial y} = 0 \]

\[ b) (\frac{\partial^2 \mathcal{F}(x_0, y_0)}{\partial x^2}) (\frac{\partial^2 \mathcal{F}(x_0, y_0)}{\partial y^2}) - (\frac{\partial^2 \mathcal{F}(x_0, y_0)}{\partial x \partial y})^2 > 0. \]

\( \mathcal{F}(x_0, y_0) \) is a minimum if on top of a) and b)

\[ (\frac{\partial^2 \mathcal{F}(x_0, y_0)}{\partial x^2}) > 0. \]

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