SmartGD: A Self-Challenging Generative Adversarial Network for Graph Drawing

Xiaoqi Wang, Kevin Yen, Yifan Hu and Han-Wei Shen

Abstract—A multitude of studies have been conducted on graph drawing, but many existing methods only focus on optimizing particular aesthetic aspects of graph layout. Given a graph, generating a good layout that satisfies certain human aesthetic preference remains a challenging task, especially if such preference cannot be expressed as a differentiable objective function. In this paper, we propose a student-teacher GAN based graph drawing framework, SmartGD, which learns to draw graphs like how humans learn to perform tasks. The student network in the SmartGD learns graph drawing by imitating good layout examples, while the teacher network in SmartGD is responsible for providing ratings regarding the goodness of the generated layouts. When there is a lack of concrete aesthetic criteria to specify what constitutes a good layout, the student network can learn from the good layout examples. On the other hand, when the goodness of a layout can be assessed by quantitative criteria (even if not differentiable), the student network can use it as a concrete goal to optimize the target aesthetics. To accomplish the goal, we propose a novel variant of GAN, self-challenging GAN, to learn the optimal layout distribution with respect to any aesthetic criterion, whether the criterion is differentiable or not. The proposed graph drawing framework can not only draw graphs in a similar style as the good layout examples but also optimize the graph layouts according to any given aesthetic criteria when available. Once the model is trained, it can be used to visualize arbitrary graphs according to the style of the example layouts or the chosen aesthetic criteria. The comprehensive experimental studies show that SmartGD significantly outperforms 12 benchmark methods according to the commonly agreed metrics. In addition, we conduct qualitative evaluations on visualizing large graphs with hundreds to thousands of nodes to show that the proposed algorithm works perfectly on large graphs as well.

Index Terms—Graph Visualization, Deep Learning, Generative Adversarial Networks, Aesthetics

1 INTRODUCTION

A graph is a non-Euclidean data structure widely used to model networks (e.g., social networks, transportation networks) in many different applications. Unlike text or image data, a graph cannot be directly visualized because the essence of the graph is the relationships between nodes represented by an adjacency matrix, which is not inherently spatial. Because graph visualization plays an important role in facilitating a deeper understanding of graph topology, researchers have tackled the problem of graph drawing for the past several decades. Generally speaking, the most common way to visualize a graph is a node-link diagram, where nodes are placed in the visual space as points, and a line is drawn between a pair of nodes whenever there is an edge between them. The positions of the nodes are usually computed by a graph drawing algorithm.

Generating a good graph layout, however, is a challenging task for three reasons. First, to determine an appropriate position for each graph node, both the local neighborhood information and the global graph structure need to be considered [54] but the global structural information is usually difficult to capture. Besides, to evaluate the goodness of a graph layout, there exist several aesthetic criteria [11, 48], but many graph drawing algorithms are only able to generate layouts with certain criteria optimized. One such example is the stress majorization algorithm [15] that focuses on optimizing stress. As a result, without in-depth knowledge about the underlying algorithms used in different methods, it is challenging to choose a method that suits the desired aesthetics. Lastly, for many existing layout methods, users often need to rely on a trial-and-error process to obtain a good layout [1, 8, 14, 25], which is tedious and has no guarantee to generate good results, especially for users who do not have a basic understanding of different parameters in the layout method.

In this paper, we propose an intelligent graph drawing framework, SmartGD, which can not only draw graphs in a similar style as a collection of good layout examples but also optimize the layouts based on any aesthetic criteria, without the need for trial-and-error. SmartGD consists of two components working together as a teacher and a student. The student component learns to draw graphs just like how humans learn from examples, by observing and imitating the world around them. In our case, the student in SmartGD learns graph drawing by analyzing and imitating good layout examples. For every attempt the student makes, the teacher component will compare its drawing with a good layout example and try to distinguish the student-generated layout from the good layout example. The goal of the student is to make the teacher believe that the student-generated layout is better than the good layout examples. Specifically, SmartGD is a Generative Adversarial Network (GAN) based deep learning model, where the generator (student) learns the generative layout distribution that is as close as possible to the distribution of the good layout collection. Therefore, the student-generated layouts are drawn in a similar style as the good layout collection.

Because the goodness of the layout examples represents a quality ceiling in the eye of the students, it in fact constrains the goodness of the layouts the student can possibly generate. That is to say, if the quality of good layout examples can be improved continuously, one can expect that the student will do even better because the quality bar of the layout continues to rise. To achieve this goal, we propose self-challenging GAN, a novel variant of GAN, where we continuously improve the quality of layout examples when a quantitative aesthetic criterion is specified to measure the layout quality. This improvement is accomplished by replacing those example layouts that are not as good as the layouts just being produced by the student, according to the given criterion. As a result, the student in self-challenging GAN can learn from ever-improving layout examples by continuously challenging and replacing the layouts generated by itself. Thus, self-challenging GAN allows the student to learn a optimal layout distribution such that the generated layouts can be optimized with respect to the given criterion. Thanks to the great flexibility of self-challenging GAN, this quantitative criterion can be any function that is either differentiable or non-differentiable, even though the self-challenging GAN method itself is gradient-based.

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Manuscript received xx xxx, 201x; accepted xx xxx, 201x. Date of Publication xx xxx, 201x; date of current version xx xxx, 201x. For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org. Digital Object Identifier: xx.xxxx/TVCG.201x.xxxxxx
We conduct experiments on generating graph layouts with regard to four criteria including minimizing edge crossing, maximizing crossing angle, minimizing stress, and optimizing a combination of 7 different aesthetics. The effectiveness and efficiency of SmartGD are evaluated quantitatively and qualitatively against 12 popular graph drawing algorithms. The experimental results show that our method significantly outperforms all the 12 benchmarks, both quantitatively and qualitatively. Additionally, the generalizability of SmartGD is assessed on real-world graphs from SuiteSparse Matrix Collection [10] with hundreds to thousands of nodes, which proves that SmartGD have the capability of generating a good layout for unseen large graphs. In summary, the primary contributions of this work includes:

1. We propose the first graph drawing framework that can not only draw graphs in a similar style as a collection of good layout examples, but also generate optimal layout according to an aesthetic criterion, without the need for trial-and-error efforts.
2. We propose a novel variant of GAN, self-challenging GAN, to learn the generative layout distribution with respect to a specific quantitative criterion without the need for special accommodation for non-differentiable criteria.
3. We conducted extensive experimental studies to show that our model consistently performs well on a large set of unseen graphs, compared against 12 popular graph drawing methods.

## 2 RELATED WORK

### 2.1 Graph Drawing

Since 1963 [47], a multitude of graph drawing algorithms have been proposed [7, 22, 50]. In order to evaluate the goodness of graph layouts, several commonly agreed aesthetics criteria (e.g., number of edge crossing and node occlusion) are formulated by researchers because extensive user studies show that these criteria are highly correlated with human preference regarding graph layouts [11, 48]. To be specific, each aesthetic criterion emphasizes a single aspect of aesthetics and some criteria contradict each others [21]. Until now, there is no general agreement about which criterion is the most effective one to measure human preference. Besides, some of the aesthetic criteria are non-differentiable functions so the gradient-based optimization method cannot be directly applied without special accommodation.

In general, many graph drawing algorithms focus on optimizing one aesthetic criterion, given that each criterion measures one aesthetic aspect. Take three aesthetics: stress, crossing angle, and edge crossing as examples. Some researchers focus on minimizing the stress energy [15, 27], where the graph is regarded as a physical system. To maximize the crossing angle, a force-directed-based algorithm [2] and a heuristic-based algorithm [6] were proposed. To minimize the edge crossing in the graph layout, several algorithms [9, 43, 43] are proposed to approximately solve this NP-hard problem. Unlike these methods, SmartGD is flexible on optimizing any quantitative criteria regardless of the differentiability of these criteria.

### 2.2 Machine Learning Approaches for Graph Drawing

In recent years, as machine learning becomes more and more popular, some researches have been conducted on applying machine learning approaches to visualize the graphs [49]. Generally, these machine learning-based graph drawing methods can be classified into two categories: graph drawing with human interaction [4, 5, 36, 38, 41, 45] and graph drawing without human interaction [1, 18, 33, 46, 51, 52]. The methods in the first category keep the human in the loop of the learning procedure. The general idea is that the fitness function is automatically adjusted according to the human feedback collected from the user interface. In the second category, these methods learn to draw graphs without involving humans. For example, Kwon et al. [34] designs an auto-encoder-based deep learning model which can visualize a graph in various layouts. The limitation is that the model needs to be trained specifically for each graph with new training data. In the same year, Wang et al. propose a deep learning model called DeepDrawing [52], which learns to visualize graphs in a similar layout fashion as the training data. However, since DeepDrawing encodes the graph structure information using adjacency vector with fixed length k for each nodes, only the connectivity information between the current node and k other nodes is accessible to DeepDrawing. As a result, the global graph topology is unable to be captured by DeepDrawing so that it is difficult to draw unseen graphs which have different topological characteristic than the graphs in the training data. Later on, DeepGD [51], a Graph Neural Networks (GNN) based deep learning framework, is proposed to generate the optimal layout according to different aesthetic criteria. While the algorithm works for general graphs, its generalizability to non-differentiable criteria is limited. In another paper [1], Ahmed et al. proposed (SGD)^2. It utilizes stochastic gradient descent to optimize the graph layout with respect to the loss function composed of multiple aesthetics. Similar to DeepGD, it cannot be directly generalized to non-differentiable aesthetic criteria without special accommodation because it is also a gradient-based method. Their remedy is to approximate some non-differentiable criteria with their differentiable forms such that the stochastic gradient descent can be applied. Nonetheless, in order to generate a good layout using (SGD)^2, the hyper-parameters including learning rate, momentum rate, sample size, and weights factor for each aesthetic are needed to be carefully tuned.

The SmartGD we proposed also falls into the second category but it is different than previous approaches. First, unlike DeepDrawing, both the local neighborhood information and the global graph structure will be captured by SmartGD. For example, SmartGD can appropriately draw grid graphs or star graphs, even though the training data only contains general graphs. Secondly, compared with DeepGD and (SGD)^2, SmartGD can optimize non-differentiable criteria without the need for special accommodation. Lastly, once trained, SmartGD can generate the optimal layouts for arbitrary graphs with regard to the desired aesthetics, without the need for trial-and-error efforts.

### 2.3 Generative Adversarial Networks

Generative Adversarial Networks (GAN) are designed to learn a generative distribution that can ultimately approximate the distribution of real data [3, 26, 28, 39]. In 2014, the first GAN [19] emerged to generate fake images which are superficially authentic. Inspired by the great success of generating fake images, GANs are adopted to tackle other problems such as text-to-image translation [40] and super-resolution [35]. Later on, a conditional version of GAN [39] is proposed to learn a conditional generative distribution, by conditioning on some additional information. In this work, by taking the advantage of conditional GAN, SmartGD also learns a generative layout distribution conditioned on the graph. Additionally, we propose a novel variant of GAN, self-challenging GAN, with the goal of generating superior layouts when the goodness of data examples can be assessed quantitatively.

It is worth mentioning that, there are many research conducted on the design of adversarial loss. For example, WGAN [3] is proposed to use the Wasserstein distance to estimate the distance between the generative distributions and distribution of real data to encourage faster convergence. RGAN [26], which estimates the relativistic difference between two distributions, is proposed to generate fake data with better quality than WGAN.

### 3 SmartGD

In this paper, we propose a general deep learning framework for graph drawing. Our proposed framework is applicable in two scenarios. When there is a lack of concrete aesthetic criteria to specify what constitutes a good layout, but good layout examples can be provided, SmartGD learns to draw graphs in a similar style as the examples; if there exist quantitative criteria to evaluate the goodness of a layout, SmartGD further optimizes the generated layouts with respect to the given aesthetic criteria.

#### 3.1 Problem Statement

Let \( G = (V, E) \) be a graph, where \( V \) is a set of \( N \) nodes, \( E \) is a set of \( M \) edges. A graph \( G \) can be represented by an adjacency matrix \( A \) where \( a_{ij} = 1 \) indicates there exists an edge between nodes \( i \) and \( j \), while \( a_{ij} = 0 \) otherwise. The graph layout is denoted as \( X \in \mathbb{R}^{N \times 2} \), where \( X \),
is a 2-dimensional position vector for node $i$. The good layout example and generated layout are represented by $X_r$ and $X_f$, respectively ("r" for real and "f" for fake). If there exist a quantitative criterion to evaluate the goodness of a layout, we denote the criterion function as $\lambda(X, G)$.

Our learning objective is determined based on whether the goodness of layouts can be assessed quantitatively. If the layout preference is too abstract to be described mathematically, SmartGD will attempt to draw the graph in a similar drawing style as the good layout examples. In other words, the student network in the SmartGD learns graph drawing by analyzing and imitating the good layout examples. In this scenario, the learning objective is to learn a generative layout distribution $Q(X|G)$ as close as possible to the distribution of good layout examples $P(X|G)$. This objective can be achieved by adopting the conditional RGAN, which is described in subsection 3.2.

However, if the goodness of a layout can be measured by an aesthetic criterion, the goal of the student network is not just to imitate the good layout examples. The reason is that the globally optimal visual layout examples with respect to this criterion are typically not available. Hence, solely imitating the good layout examples will in fact constrain the quality of student-generated layouts. In this case, SmartGD will attempt to enhance the quality of generated layouts continuously regenerating a desired aesthetic $\lambda$ during training so that the student network can continue to improve by learning from better layouts. Thus, the learned generative distribution $Q(X|G)$ takes the criterion $\lambda$ into account such that a layout $X_f$ drawn from this distribution is expected to achieve an optimal value of the criterion $\lambda(X_f, G)$. The purpose of this framework is to solve the learning problem without the globally optimal ground truth layouts. That is to say, in order to learn the globally optimal layout distribution $P(X|G)$, SmartGD does not require the globally optimal $X_f$ to be provided. This paves the way for the extraordinary flexibility of SmartGD since it can be easily applied to optimize any quantitative aesthetic. Nonetheless, this learning setting also make our problem even more challenging, which inspires us to propose a novel variant of GANs, self-challenging GAN, explained in detail in subsection 3.3.

### 3.2 Conditional RGAN

In order to draw graphs in a similar drawing style as the good layout examples, the conditional RGAN is adopted to achieve this goal. Specifically, the conditional GAN [39] allows us to learn the generative layout distribution conditioned on the graph; the RGAN [26] specifies the adversarial loss we use. By combining the conditional GAN and RGAN, we propose to use the conditional RGAN to learn the generative layout distribution conditioned on the graph.

The conditional RGAN is a deep learning based generative model. Its purpose is to learn the data distribution from a collection of good layout examples $P(X|G)$. In the model architecture, there are two sub-models: the generator network $\Phi_{gen}$ and the discriminator network $\Phi_{dis}$ (see Figure 1a). To be specific, the generator is responsible for generating layouts that are as similar as possible to the good layout examples, and the discriminator estimates the goodness of layouts. Mathematically, the generator attempts to learn the generative distribution $Q(X|G)$ to approximate the distribution of good layout examples $P(X|G)$. If we regard the generator $\Phi_{gen}$ as a student and the discriminator $\Phi_{dis}$ as a teacher, the student tries to imitate the good layouts $X_r$ and aims at making the teacher believe that student-generated layouts $X_f$ are better than the good layouts $X_r$. The teacher is responsible for correctly distinguishing $X_r$ and $X_f$ such that the student will have the motivation to improve further. Therefore, their responsibility is adversarial to some extent, but they serve a common goal to help the student learn better. During the training, they are trained alternately so that they are co-evolving by trying to improve together.

More specifically, the generator will take a graph $G$ as input and generate the corresponding layout $X_f$. The discriminator predicts a goodness score $\Phi_{dis}(X_f|G)$ for any input layout $X$. The adversarial loss of conditional RGAN is the following: $t$ where $\sigma$ is the sigmoid function. In the discriminator loss, $(\Phi_{dis}(X_r|G) - \Phi_{dis}(X_f|G))$ indicates how much the good layout example $X_r$ is better than the generated layout $X_f$ in the eyes of discriminator. Thus, the discriminator will be trained to maximize the log probability that the good layout example is better than the generated layout in the belief of the discriminator. Similarly, the generator will be trained to maximize the log probability that the generated layout $X_f$ is better than good layout example $X_r$ in the belief of discriminator.

Once the conditional RGAN is trained, the $\Phi_{gen}$ encodes the learned generative layout distribution conditioned on the graph $Q(X|G)$. It has been mathematically proved that $Q(X|G)$ can well approximate the good layout distribution $P(X|G)$ such that the generated layouts $X_f$ sampled from the generative distribution $Q(X|G)$ is drawn in the similar style as the good layouts collection [19].

### 3.3 Self-Challenging GAN

Fig. 1: The high-level overview of SmartGD. The component (a) sketches the training procedure of the GAN-based deep learning model. The self-challenging mechanism is explained in component (b), which is only applied when the criterion to be optimized is given. The component (c) describes the inference procedure for drawing unseen graphs.

If the goodness of layouts can be assessed quantitatively by a criterion function $\lambda(X, G)$, all types of existing GANs including the conditional RGAN are not suitable for this scenario anymore. The reason is that the existing GANs can only make the generated examples to be as similar as possible to the good examples. However, in our cases, the globally optimal layout $X_f^*$ with respect to the criterion $\lambda$ is typically not available. Therefore, it is impossible for the existing GANs to learn the globally optimal layout distribution $P(X|G)$ with regard to the criterion $\lambda$. With this in mind, we propose a novel variant of GAN, self-challenging GAN, in which the quality of generated layout $X_f$ will not be bounded by the quality of good layout examples $X_r$ anymore.

The main difference between self-challenging GAN and the existing GANs is that the distribution of good layout examples $P(X|G)$ is dynamically changing in self-challenging GAN. In other words, $\lambda(X, G)$ is continuously improving during the training stage. In general, for every layout the student generates, we evaluate this layout by computing $\lambda(X_f, G)$ and compare it with the current good layout example $\lambda(X_r, G)$ (see Figure 1b). If the student-generated layout $X_f$ is better than the current good layout example $X_r$ given the criterion $\lambda$, the good layout collection will be updated by substituting the example with the new generated layout $X_f$. Then, the student is actually learning from the best attempt made by itself in the past and trying to outperform the layouts generated by itself. With this self-challenging mechanism, we can break through the shackle of the quality of initial good layout examples.
As long as the goodness of examples can be evaluated quantitatively, even though SmartGD is flexible and powerful in handling two different model architectures, including but not limited to GANs, which can also be applied to problems other than graph drawing.

Differentiability of criterion \( \lambda \) such that the generated layouts optimize the criterion \( \lambda \).

The generative layout distribution learned by the existing GANs can only learn a distribution which well approximates the distribution of final good layout examples. Since the existing GANs can only learn a self-challenging GAN is described in algorithm 1.

At the end of training, the generator serves as the generative layout distribution. In other words, in the inference stage, only the generator is used to generate a layout sample for an unseen graph can be drawn from this distribution. In other words, in the inference stage, only the generator is needed to draw an unseen graph as shown in Figure 1c. The converged generator takes as input the adjacency matrix of an unseen graph and outputs the 2-dimensional node positions as the graph layout.

\[ L_D = -E_{X \sim P(G^*)} [\log (\sigma (D(X, G) - D(X, G)))] \]
\[ L_G = -E_{X \sim P(G^*)} \log (\sigma (D(X, G) - D(X, G))) \]

Combining self-challenging GAN with the conditional RGAN in Equation 1, the loss of self-challenging conditional RGAN is formulated as,

\[ L_{dis} = -E_{X \sim P(G^*)} \sum_{i=1}^{m} \log (\sigma (\Phi_{dis}(X_i, G) - \Phi_{dis}(X_i, G))) \]
\[ L_{gen} = -E_{X \sim P(G^*)} \sum_{i=1}^{m} \log (\sigma (\Phi_{dis}(X_i, G) - \Phi_{dis}(X_i, G))) \]

where \( P^*(X(G)) \) is the dynamic good layout distribution and \( X^*_i \) denotes the current good layout example. The training procedure of self-challenging GAN is described in algorithm 1.

At the end of training, the generator serves as the generative layout distribution which well approximates the distribution of final good layout examples \( X^*_i \). Given that \( X^*_i \) is continuously improving according to the criterion \( \lambda \), the distribution of final good layout examples \( X^*_i \) at the end of training is more likely to be closer to the globally optimal layout distribution \( P^*_D (X(G)) \), compared with the distribution of initial good layout examples. Since the existing GANs can only learn a distribution that approximates the initial good layout distribution, the generative layout distribution \( Q^*_G (X(G)) \) learned by self-challenging GAN will be closer to the globally optimal layout distribution \( P^*_D (X(G)) \) than the distribution learned by the existing GANs.

The generative layout distribution \( Q^*_G (X(G)) \) we learned takes the criterion \( \lambda \) into account such that the generated layouts optimize the criterion \( \lambda \), no matter the differentiability of criterion \( \lambda \).

It is worth mentioning that self-challenging GAN is a general GAN model which can also be applied to problems other than graph drawing. As long as the goodness of examples can be evaluated quantitatively by a criterion and the ground truth examples are not available, self-challenging GAN can better approach the optimal generative distribution with respect to the criterion. Additionally, self-challenging GAN can be combined with any flavor of GANs, including but not limited to conditional RGAN.

Algorithm 1: Self-Challenging GAN

**Input:** good layout examples \( X^{(0)} \)

1. for training epoch \( t \) do
   2. for \( k \) minibatches in the dataset \( X^{(t-1)} \) do
      3. Take \( m \) examples \( \{X^*_1, \ldots, X^*_m\} \) from the dataset
      4. Update discriminator \( \Phi_{dis} \) with gradient ascent
      5. end
   6. for \( k \) minibatches in the dataset \( X^{(t-1)} \) do
      7. Update generator \( \Phi_{gen} \) with gradient ascent
      8. Draw a sample layout from generator for \( i = 1, \ldots, m \)
      9. Update dataset \( X_i \) for \( i = 1, \ldots, m \)
      10. end
   11. end

3.4 Training and Inference

Even though SmartGD is flexible and powerful in handling two different graph drawing scenarios with conditional RGAN and self-challenging GAN respectively, the training procedure, inference procedure, and model architecture (described in subsection 3.5) for these two GANs is unified. The only difference in the training procedure between conditional RGAN and self-challenging GAN is the dynamically changing good layout collection. In this section, the training and inference procedure is explained in detail.

During the training phase (see Figure 1a), the discriminator will take one input layout at a time and output a goodness score. This input layout can be either the layouts generated by the generator or the good layout examples. For each epoch, the feedback from the discriminator, which is formulated as the adversarial loss in Equation 1 and Equation 2, is back-propagated to the generator and discriminator alternately. To be more concrete, the weight of the generator remains unchanged while the adversarial loss is back-propagated through the discriminator, and vice versa. Therefore, the generator and the discriminator are trained alternately so that they are able to co-evolve together.

After the model converged, the generator is regarded as the learned generative distribution conditioned on the graph such that the generated layout sample for an unseen graph can be drawn from this distribution. In other words, in the inference stage, only the generator is needed to draw an unseen graph as shown in Figure 1c. The converged generator takes as input the adjacency matrix of an unseen graph and outputs the 2-dimensional node positions as the graph layout.

3.5 Model Architecture

The unified model architecture is composed of two sub-models: generator and discriminator, as shown in Figure 2. The building block of these two sub-models is the GNN layer. Each GNN layer contains a graph convolutional layer [17], a dense (MLP) layer, a batch normalization layer, and an activation layer. More specifically, the graph convolutional layer is responsible for learning the latent node representation; the dense layer transforms the node representation; batch normalization [44] is adopted to accelerate the convergence by reducing the internal covariant shift; the activation layer, in particular LeakyReLU [37], introduces non-linearity in the model while alleviating the potential gradient vanishing issue.

There are two reasons we employ the graph convolutional layer instead of the LSTM layer as DeepDrawing [52] proposed. First, it learns a hidden node representation by taking advantage of the message passing mechanism. For each convolutional layer, the representation of a node is updated according to the aggregated messages passed from its neighbors. These messages are aggregated from the node representation of the neighbors learned by the previous convolutional layer. As a result, for \( l^\text{th} \) convolutional layer, the learned node representation contains the information about nodes that are \( l \) graph theoretic distance away. By stacking multiple convolutional layers, the final node representation will not only contain the local neighborhood information but also capture the global topological structure. Therefore, this allows SmartGD to draw graphs with arbitrary topological characteristic, even
if the graph to be drawn possess a completely different topological characteristic than graphs in the training data. Another advantage of the graph convolutional layer is that it does not require the input graph to have the same number of nodes. Each convolutional layer will learn a message aggregation function to process the messages passed by the neighbors. This aggregation function, served as the kernel function, is shared within the layer across all the nodes so that the input graph can have a different number of nodes. This also endows more flexibility of the general graph drawing framework we proposed.

In the generator, multiple GNN layers are stacked together in order to wisely draw the graph according to the global graph structure. The node embedding output by the final GNN layer will be projected to 2-dimensional space by a dense layer. Similarly, the first GNN layer in the discriminator will take the 2-dimensional node embedding as the input. Then, the node representation capturing the latent characteristic in the graph layout is learned by a series of GNN layers in the discriminator. Finally, the global pooling layer aggregates all the representations of the nodes into a single graph-level layout embedding such that the dense layer can transform the layout embedding to a goodness score.

3.6 Canonicalization

As mentioned in subsection 3.4, the discriminator will alternately take the layouts generated by the generator and the good layout examples as inputs. However, the generated layout and good layout examples may have inconsistent node position distributions. For instance, the coordinate of each node in the good layout examples might be constrained within a certain numerical range, while the coordinate of the generated layouts is a real number without a certain range. Additionally, as the model is continuously evolving, the generator is also not guaranteed to produce a stable and consistent node position distribution throughout the training procedure. As a result, an inconsistent or even drastically changing input node position distribution may greatly increase the difficulty for the discriminator to learn. In order to stabilize training by avoiding out-of-distribution inputs, we introduce a canonicalization layer at the beginning of the discriminator as shown in Figure 2. The canonicalization layer stabilizes the node position distribution by throwing away all the non-essential information for determining the goodness of a layout, including center position, rotation angle, and original numerical scale of node positions. In other words, the canonicalization layer assigns each input layout a canonical representation that is geometrically similar to the original layout but consistent in node position distribution. With the canonicalized layouts, the discriminator input is metrically similar to the original layout but consistent in node position distribution, which in turn speed up convergence, and facilitate generalizability over unseen layout examples.

The canonicalization layer consists of three operations: translation, rotation, and rescaling. The translation operation translates each of the node positions in a layout by the same amount in order to make the input layout to be zero-centered. For each node $i$ in a layout $X$, $X'_i = X_i - \frac{1}{N} \sum_{j=1}^{N} X_j$, (7)

where $X'_i$ denotes the translated position for node $i$, and $N$ denotes the number of nodes in layout $X$. Then, the rotation operation rotates the entire layout by its center, such that the first principal component in the layout is aligned with the x-axis. For layout $X'$, $X'' = X' \cos(X')$, (8)

where $X''$ denotes the rotated layout positions, and $\cos(X')$ represents the covariance matrix of all the node positions in the layout. Lastly, a rescaling operation is employed to impose a canonical layout scale across different graphs. One way to achieve this is to ensure the scale of node distances in the graph space is consistent with the scale of node distances in the layout space. The discrepancy between graph space and layout space can be measured by stress energy. So we derive an optimal scaling factor by leveraging the equation of stress. For each layout $X''$, $X''' = X'' \frac{\sum_{j \neq i} ||X''_i - X''_j||/d_{ij}}{\sum_{j \neq i} ||X''_i - X''_j||^2/d_{ij}^2}$, (9)

where $X'''$ denotes the rescaled layout positions and $d_{ij}$ represents the graph theoretic distance between node $i$ and $j$.

After performing translation, rotation, and rescaling in sequence, the input layouts of the discriminator will have a canonical representation with the benefit of avoiding the out-of-distribution inputs, facilitating the convergence, and enhancing the generalizability of SmartGD over unseen graphs.

4 Evaluation

In this section, the effectiveness and efficiency of SmartGD are assessed by comparing against 12 benchmark methods quantitatively. Among the 12 benchmark methods, the competitive ones according to the quantitative evaluation are chosen to compare with SmartGD qualitatively. When no concrete aesthetic criteria can be used to specify what constitutes a good layout, we evaluate our algorithm based on its capability to draw graphs in a similar style as the good layout collection. If the goodness of layout can be evaluated by a quantitative criterion, our effectiveness of optimizing the aesthetic criterion is assessed.

4.1 Experimental Setup

SmartGD is implemented in Pytorch [42] and Pytorch Geometric [13]. Every model present in the following sections is trained on a single Tesla V100 GPU with memory of 32 GB. For the training configuration, stochastic gradient descent with a minibatch size of 16 graphs is adopted to train SmartGD. The optimizer we used is AdamW optimizer [30] with a decay rate of 0.99 such that the model parameters are shrunk for each optimization step. The learning rate initially is 0.001 and exponentially decays with a rate of 0.997 for each epoch. Speaking of the model architecture, the generator has 31 GNN layers and the node embedding output from each layer is 8-dimensional; the discriminator has 9 GNN layers and the node embedding output from each layer is 16-dimensional. In total, the SmartGD has about 378,000 parameters. To facilitate the model convergence, the input node embedding of the generator is initialized as a 2-dimensional node positions generated by PivotMDS (pmds) [8].

4.2 Benchmark Algorithms

To show the effectiveness of SmartGD, we compared SmartGD with 12 benchmark algorithms including force-directed layouts, energy-based layouts, heuristic-based layouts, gradient-based layouts, and deep learning based layouts. Those 12 benchmarks are widely used methods implementing various types of approaches. To be precise, spring [14], ForceAtlas2 (fa2) [25] and sfdp [23] are three force-directed layout methods aiming at reaching a balance of attractive and repulsive forces in equilibrium. Neato [15] and the method proposed by Kamada and Kawai (kk) [27] are two energy-based layouts in which the stress energy is minimized. Dot [16], circo [29] and twopi [53] are three heuristic-based layout methods that focus on drawing graphs with a certain topological characteristic. Spectral [31] layout visualizes graphs using the principal components of the graph Laplacian matrix. PivotMDS (pmds) [8] is a sampling-based layout method for efficiently approximating the classical multidimensional scaling layout. SGD2 [1] and DeepGD [51] share a common goal of optimizing the layouts according to certain aesthetic criteria, but with different approaches. SGD2 adopts stochastic gradient descent to optimize the layout but DeepGD is a GNN based deep learning model. In order to optimize some non-differentiable aesthetic criteria including the number of edge crossing, neighborhood preservation, and aspect ratio, a special accommodation is adopted in SGD2 for each of them with the purpose of making them differentiable. However, the authors of DeepGD [51] do not conduct experiments on non-differentiable criteria. Therefore, we will only compare SmartGD with DeepGD on optimizing stress.

The implementation of all the benchmarks are from three different sources including Graphviz [12], NetworkX [20] and the code repositories directly shared by the authors of the papers mentioned above. To evaluate all benchmarks for comparison, the parameter settings we employ are the default one suggested by Graphviz, NetworkX, and the authors.
4.3 Datasets

4.3.1 Graph

The graph dataset used in our experiment is Rome graphs (http://www.graphdrawing.org/data.html). It contains 11534 undirected graphs with 10 to 100 nodes. We randomly split the Rome graphs into three sets: a training set with 10000 graphs, a validation set with 534 graphs, and a test set with 1000 graphs. SmartGD was trained on the training set, validated on the validation set, and tested on the test set. All the quantitative results we presented in the following sections are evaluated on the test set.

4.3.2 Good Layout Collections

As mentioned in section 3, SmartGD learns graph drawing by imitating the good layout examples. Hence, the quality of good layout examples is essential to our model performance. If the quality of good layout collections is better, SmartGD is more likely to generate a superior layout. Therefore, for every training graph, we collect the best possible layout as the good layout examples.

In practice, the quality of layout is usually measured by some commonly agreed aesthetic criteria. Each criterion assesses one aesthetic aspect and some criteria may even contradict to each other. For this reason, it is difficult to find a layout graph which optimizes every aesthetic criterion. Therefore, in our experimental study, we collect a separate set of good layout examples for each of four aesthetic criteria respectively. These four aesthetic criteria include stress, the number of edge crossing (Xing), the acute angle formed by a pair of crossing edges (XAngle), and a weighted average of multiple criteria (Combined). The combined multi-criteria is a weighted average of 7 different criteria including stress, Xing, XAngle, the angle formed by two incident edges (IAngle), node occlusion (NodeOcc), uniform edge length (EdgeUni), and the divergence between the graph space and layout space (t-SNE) [32], with the weight of 0.2, 0.05, 0.1, 0.1, 0.2, 0.15, and 0.2, respectively.

To collect the good layout examples for each criterion, a layout of every training graph is generated by 10 existing layout methods in Table 1, which are 12 benchmarks mentioned in subsection 4.2 except DeepGD [51] and SGD2 [1]. The best layout with respect to the criterion among the layouts generated by these 10 methods is then selected as the good layout example for training purpose. If the criterion value of two layouts are tied, the stress is adopted as the tie breaker. The percentage of every layout method selected into each of the four good layout collections, i.e., generating the winning layouts for each criterion, is presented in Table 1.

Table 1: The composition of good layout collections. Each column corresponds to the proportion of a single good layout collection for each criterion. For instance, in the good layout collection for stress, 67.77% of layouts are generated by neato.

| Method     | Stress | Xing   | XAngle | Combined |
|------------|--------|--------|--------|----------|
| neato [15] | 67.77% | 13.64% | 16.18% | 76.71%   |
| dot [16]   | 0.00%  | 0.12%  | 0.14%  | 0.00%    |
| stfdp [23] | 0.03%  | 5.47%  | 6.85%  | 2.12%    |
| twopi [53] | 0.03%  | 0.12%  | 0.16%  | 0.03%    |
| circo [29] | 0.00%  | 0.12%  | 0.02%  | 0.00%    |
| spring [14] | 0.00% | 3.23%  | 4.62%  | 7.46%    |
| spectral [33] | 0.00% | 1.15%  | 3.54%  | 0.00%    |
| kk [27]    | 32.18% | 8.08%  | 8.07%  | 13.61%   |
| fa2 [25]   | 0.00%  | 67.83% | 60.24% | 0.08%    |
| pmds [8]   | 0.00%  | 0.35%  | 0.17%  | 0.00%    |

4.4 Quantitative Evaluation

We quantitatively evaluate the efficacy of SmartGD by different aesthetic criteria. Given one criterion function $\lambda$, we measure the relative difference in criterion $\lambda$ by comparing it against a benchmark algorithm. Similar to DeepGD [51], the symmetric percent change (SPC) of a graph $G$ ranging from $-100\%$ to $100\%$ is computed as

$$\text{SPC}_\lambda(G) = 100\% \times \frac{\lambda(X_f, G) - \lambda(X_b, G)}{\max\{\lambda(X_f, G), \lambda(X_b, G)\}},$$

where $X_f$ and $X_b$ are the layouts generated by SmartGD and a benchmark algorithm respectively. The lower the criterion $\lambda$, the better the layout. The SPC value measures the percentage of $\lambda(X_f, G)$ being better than $\lambda(X_b, G)$.

4.4.1 Optimizing Aesthetic Criteria

If the goodness of a layout can be evaluated quantitatively by a criterion function $\lambda$, SmartGD with self-challenging GAN can learn an optimal generative layout distribution such that the generated layout is optimized given the criterion. To thoroughly evaluate the effectiveness of optimizing different aesthetic criteria, in our experiments we trained 4 SmartGD models to optimize 4 different aesthetic criteria: stress, edge crossing (Xing), the angle formed by a pair of crossing edges (XAngle), and a combined criterion which is computed as the weighted average of 7 criteria (Combined). The good layout examples we used for training these 4 SmartGD models were collected as described in subsection 4.3.2.

![Fig. 3: The average test SPC of 4 SmartGD models (column) compared against the 15 benchmarks(row).](image)

Fig. 3: The average test SPC of 4 SmartGD models (column) compared against the 15 benchmarks (row). The green cell indicates that the SmartGD model (column) outperforms the benchmarks (row), whereas the red cell indicates that the benchmark outperforms the SmartGD. For example, the cell at top right corner means that the SC-SmartGD-XAngle is 43.18% better in crossing angle than neato on average.

![Fig. 4: The distribution of test SPC for 4 SmartGD models with respect to their corresponding the best performing benchmark.](image)

Fig. 4: The distribution of test SPC for 4 SmartGD models with respect to their corresponding the best performing benchmark. Given the x-axis is test SPC, the upper is the density plot and the lower is the rug plot.
SmartGD model with self-challenging GAN using the good layout collection for stress in Table 1. As shown in Figure 3, SmartGD with self-challenging GAN on optimizing stress, abbreviated as SC-SmartGD-Stress, achieves negative average stress SPCs computed against all benchmarks. It means that SC-SmartGD-Stress outperforms all benchmarks in terms of stress, among which SGD2 optimizing on stress (SGD2-Stress) is the best performing benchmark. In addition to the average stress SPC, the distribution of stress SPC for SC-SmartGD-Stress vs. SGD2-Stress is plotted in Figure 4. In the density plot, the larger area under curve to the left of zero SPC indicates that SC-SmartGD-Stress is more likely to generate layouts with better stress than SGD2-stress. In the rug plot, more marks below zero SPC also show that SC-SmartGD-Stress layouts achieved better stress than SGD2-Stress layouts for most test graphs.

Minimizing Edge Crossing and Crossing Angle In addition to stress, we also evaluated the effectiveness of SmartGD on optimizing non-differentiable criteria including edge crossing and crossing angle. The non-differentiable criteria cannot be directly optimized by gradient based methods such as SGD2 [1] and DeepGD [51]. Hence, SGD2 approximately optimizes edge crossing and crossing angle by reformulating them into a differentiable function, whereas DeepGD does not explore the optimization of non-differentiable criteria. However, even though GAN is also a gradient-based method, SmartGD can directly optimize non-differentiable criteria without any special accommodation. To optimize edge crossing (Xing) and crossing angle (XAngle), we train two SmartGD models with self-challenging GAN respectively: SC-SmartGD-Xing and SC-SmartGD-XAngle. From Figure 3, we can see that SC-SmartGD-Xing and SC-SmartGD-XAngle can generate layouts with significantly better edge crossing and crossing angle than all benchmarks according to the average Xing SPC and the average XAngle SPC. Specifically, compared with the best performing benchmark, SGD2-Xing, SC-SmartGD-Xing is 25.88% better than SGD2-Xing on edge crossing. Compared with the best performing benchmark, fa2, SC-SmartGD-XAngle is 34.78% better than fa2 on crossing angle. The distribution of the test SPC for SC-SmartGD-Xing vs. SGD2-Xing and SC-SmartGD-XAngle vs. fa2 is plotted in Figure 4. The distribution plot clearly shows that we can achieve better quality for most of the test graphs than the best benchmark according to the larger area under curve to the left of zero SPC in the density plot and more marks below zero in the rug plot.

Combined Criterion Some research works have shown that optimizing multiple aesthetic criteria is more likely to generate a visually pleasing graph layout [24]. Therefore, to show the generalizability and flexibility of SmartGD, we also conducted experiments on training SmartGD to optimize a combination of 7 different aesthetic criteria. Empirically, the combined criterion is computed as the weighted average of stress(0.2), Xing (0.05), XAngle (0.1), IAngle (0.1), NodeOcc (0.1), EdgeUni (0.15), t-SNE (0.2). We train a model, SC-SmartGD-Combined, to optimize this combined criterion with self-challenging GAN by using the good layout collection for the combined criterion in Table 1. From Figure 5, we can see that SmartGD with self-challenging GAN on optimizing the combined criterion obtained superior layouts compared with all the 12 benchmarks from 7 different aesthetic aspects. Compared with the best performing benchmark, SGD2-Stress, the distribution of test SPC in Figure 6 indicates that SC-SmartGD-Combined can consistently produce layouts with better quality from 7 different aesthetic perspectives, especially the crossing angle, edge uniformity, and edge crossing.

4.4.2 Learning Similar Drawing Style As described in subsection 3.2, if there is a lack of concrete aesthetic criteria to specify what constitutes a good layout, SmartGD attempts to visualize graphs in a similar drawing style as the good layout collection. In other words, SmartGD attempts to draw graphs according to the implicit but unknown layout preference which inherently exists in the good layout collection. However, it is difficult to quantitatively evaluate how well the generated layouts align with the implicit layout preference. To better show our capability of learning implicit layout preference from examples, we employ a good layout collection whose inherent layout preference is quantifiable but unknown to SmartGD. To be specific, we trained a SmartGD with conditional RGAN using the layout collection for edge crossing in Table 1, abbreviated as SmartGD-Xing. Since the self-challenging mechanism in Figure 1c was not applied, SmartGD did not know that minimizing edge crossing was the target layout preference. In other words, the numbers of edge crossings in both the generated layouts and the good layout examples were never explicitly evaluated during training, but SmartGD was expected to learn the essence of the layout examples even without being instructed what criterion to optimize. Therefore, to assess how well the generated layout align with the implicit preference in good layout collections, we measure the number of edge crossings in the generated layouts.

The quantitative evaluation in Figure 3 shows that SmartGD-Xing outperformed all benchmarks on edge crossing with a negative Xing SPC. Among all benchmarks, SGD2 optimizing on edge crossing (SGD2-Xing) was the best performing model, but SmartGD-Xing was still 11.85% better than SGD2-Xing on edge crossing. In addition to the average test Xing SPC, the distribution of Xing SPC for SmartGD-Xing vs. SGD2-Xing for each test graph is shown in Figure 4. As we can see from the density plot, the larger area under curve to the left of zero SPC indicates that the SmartGD-Xing layouts are more likely to be better than SGD2-Xing layouts regarding edge crossing. In conclusion, even though SmartGD-Xing does not explicitly know the target layout preference (i.e. minimizing edge crossing), it can still generate layouts which perfectly aligns with the inherent preference in the good layout collection. It further demonstrates our capability of drawing the graph...
in a similar style as the good layout collection if a concrete aesthetic criteria to specify what constitutes a good layout is absent.

### 4.5 Qualitative Evaluation

In addition to the quantitative evaluation, we qualitatively compare 5 SmartGD models against the benchmarks on 12 graphs, which includes 10 real-world large graphs [10] with hundreds to thousands of nodes and 2 Rome graphs in the test set. However, due to the page limit, only the competitive and representative benchmarks (i.e. SGD2-Stress, SGD2-Xing, fa2, neato) on quantitative evaluation are selected to be evaluated qualitatively. To be specific, SGD2-Stress is the best performing benchmark on stress and the combined criterion; SGD2-Xing is the best performing benchmark on edge crossing; fa2, as a traditional force-directed layouts, achieves the best performance among all benchmarks on crossing angle; neato, as a classical layout methods minimizing stress, achieves competitive performance on stress and the combined criterion.

The qualitative comparison in Table 8 shows that SmartGD models optimizing different aesthetics can indeed visualize the graphs with various sizes in a visually pleasing and informative way, by satisfying certain aesthetic aspect. It is interesting to observe that SmartGD-Xing and SC-SmartGD-Xing tends to bundle edges together to avoid edge crossing. Besides, for visualizing large graphs with SGD2-Xing, we made our best effort to obtain reasonably good layouts within 100,000 iterations. However, we do not observe any visible improvement after tens of hours of computation. In particular, it takes more than 5 hours for SGD2-Xing to visualize the graph named ex4 in Table 8 with no meaningful outcome in the end. We suspect that directly minimizing edge crossings on large graphs might be a potential weakness of SGD2 [1], since the loss landscape can be particularly rough for a highly intertwined layout in which an extremely small perturbation in node positions may lead to drastically changing edge crossing numbers.

### 4.6 Discussion

In addition to the performance evaluation of SmartGD, there are four additional issues that we want to discuss. First, to evaluate the robustness and stability of SmartGD, 10-fold cross validation was performed on SC-SmartGD-XAngle over 10 random train-test splits of Rome graphs. The arithmetic mean of the average XAngle SPC over SGD2 was -35.404%, after averaging over 10 folds. Comparing to the XAngle SPC of SC-SmartGD-XAngle vs. fa2 on a single fold in Figure 3, 34.78%, we can see that the performance of SmartGD is very robust to the potential variation in the training data.

Secondly, SC-SmartGD-Xing and SmartGD-Xing share the same good layout collection for training but with different GANS. Comparing the performance of SC-SmartGD-Xing and SmartGD-Xing in Figure 3, we can clearly see that SC-SmartGD-Xing achieves better performance on edge crossing than SmartGD-Xing. It proves that the self-challenging GAN indeed can break through the shackles of the quality of initial good layout examples and thus further improve the generated layouts on edge crossing with significant success, if the quantitative description of layout preference is known.

Additionally, we also explore the effect of SmartGD with self-challenging GAN but without using any good layout examples. To be more clear, at the first epoch of the training procedure, the initial good examples are generated by the generator itself instead of the layout examples collected in Table 1. In this case, SmartGD solely learns from the layouts generated by itself and utilizes the quantitative criteria as guidance to select good layout examples, without the help of layout examples generated by others. The quantitative result shows that SC-SmartGD-Xing without using any good layout collection in Table 1 achieves an average Xing SPC of -3.48% compared with fa2. From this result, we can infer that the powerfulness of self-challenging GAN endows even more flexibility of SmartGD because self-challenging SmartGD can perform reasonably well even without being provided with initial good layout examples.

Lastly, the essence of self-challenging GAN is continuously improving the good layout examples during the training, which is accomplished by replacing the good layout examples with the better layout generated by itself. For SC-SmartGD-Xing which uses a good layout collection for Xing in Table 1 during training, the replacement pattern is shown in Figure 7. As we can see, at the early stage during training, many initial good layout examples were replaced by the generated layouts. After about 1000 epochs, SmartGD kept replacing the layout generated by itself instead of the initial good layouts. Cumulatively, until the SC-SmartGD-Xing converged, there were more than 9000 training graphs whose initial good layout examples were replaced by the layouts generated by SC-SmartGD-Xing.

#### 4.7 Layout Computation Time

Table 2: Average layout computation time per graph with 10-100 nodes.

| Method    | Time  | Method    | Time  |
|-----------|-------|-----------|-------|
| neat [15] | 0.34s | pmds [8]  | 0.02s |
| dot [16]  | 0.29s | SGD2-Stress [1] | 13.19s |
| sfdf [23] | 0.28s | SGD2-Xing [1] | 142.06s |
| twopi [53] | 0.26s | SGD2-XAngle [1] | 16.75s |
| circo [29] | 0.35s | DeepGD on CPU [51] | 0.27s |
| spring [14] | 0.01s | DeepGD on GPU [51] | 0.05s |
| spectral [31] | 0.01s | SmartGD on CPU | 0.19s |
| kk [27]   | 0.04s | SmartGD on GPU | 0.03s |
| fa2 [25]  | 0.37s |           |       |

To assess the efficiency of SmartGD, the layout computation time is evaluated for all 12 benchmarks and SmartGD. Specifically, the computation time we report in Table 2 is calculated as the average time over 1000 test graphs. Note that the computation time for all graph drawing methods is usually proportional to the graph size. Therefore, the computation time we evaluate can be an approximation of the average drawing time per graph with 10-100 nodes. Given that SmartGD and DeepGD is a deep learning model, it can take advantage of parallelism on GPU so that their computation time on GPU is also evaluated.

As we can see from Table 2, spring, pmds, spectral, kk, SmartGD on GPU and DeepGD on GPU are the first-tier algorithms regarding efficiency because they are significantly faster than others. SGD2 with different criteria is less efficient than others, even though SGD2 tends to be the best performing benchmark on stress, edge crossing, and the combined criterion. Among all first-tier efficient algorithms, SmartGD achieves the best quantitative and qualitative performance according to different aesthetic criteria (see Figure 3, Figure 5 and Table 8).

### 5 Conclusion

In this paper, we propose SmartGD, a Generative Adversarial Network (GAN) based graph drawing framework that can learn to visualize arbitrary graphs. Specifically, if there is a lack of concrete aesthetic criteria to specify what constitutes a good layout, SmartGD can learn to draw graphs in a similar style by imitating the good layout examples. On the other hand, if the goodness of layout can be assessed by quantitative criteria, we propose a novel variant of GAN, self-challenging GAN, to generate an optimal layout with respect to the desired criteria.
| Graph       | Benchmark Methods | SmartGD | SC-SmartGD-Stress | SC-SmartGD-Xing | SC-SmartGD-XAngle | SC-SmartGD-Combined |
|------------|------------------|--------|------------------|----------------|------------------|------------------|
| rome-7554  | N = 38           | t = 0.31s | t = 0.015s       | t = 290.38s    | t = 0.154s       | t = 0.144s       |
| N = 119    |                  |        |                  |                |                  |                  |
| rome-1746  | N = 77           | t = 0.25s | t = 0.010s       | t = 7.92s      | t = 0.162s       | t = 0.164s       |
| N = 270    |                  |        |                  |                |                  |                  |
| plat32   | N = 3074         | t = 0.65s | t = 0.12s        | t = 327.10s    | t = 0.153s       | t = 0.162s       |
| N = 1746   |                  |        |                  |                |                  |                  |
| bor_131   | N = 434          | t = 0.30s | t = 0.14s        | t = 97.00s     | t = 1.15s        | t = 1.23s        |
| N = 2172   |                  |        |                  |                |                  |                  |
| dnc_583   | N = 301          | t = 0.83s | t = 0.19s        | t = 101.10s    | t = 1.15s        | t = 1.23s        |
| N = 3205   |                  |        |                  |                |                  |                  |
| can_634   | N = 614          | t = 0.94s | t = 0.27s        | t = 102.80s    | t = 1.37s        | t = 1.73s        |
| M = 3931   |                  |        |                  |                |                  |                  |
| bfwa782   | N = 782          | t = 2.12s | t = 0.36s        | t = 103.32s    | t = 4.99s        | t = 4.99s        |
| M = 4176   |                  |        |                  |                |                  |                  |
| can_838   | N = 838          | t = 1.16s | t = 0.41s        | t = 121.36s    | t = 7.40s        | t = 7.40s        |
| M = 5424   |                  |        |                  |                |                  |                  |
| radf1     | N = 1044         | t = 2.55s | t = 0.68s        | t = 315.92s    | t = 25.82s       | t = 25.82s       |
| M = 12944  |                  |        |                  |                |                  |                  |
| plantmarop | N = 1600        | t = 4.06s | t = 1.28s        | t = 391.05s    | t = 45.20s       | t = 45.20s       |
| M = 12741  |                  |        |                  |                |                  |                  |
| exi       | N = 1500         | t = 4.01s | t = 1.30s        | t = 629.20s    | t = 18.02s       | t = 26.90s       |
| M = 16500  |                  |        |                  |                |                  |                  |
| sntl700h  | N = 1700         | t = 8.08s | t = 1.44s        | t = 641.68s    | t = 58.74s       | t = 57.81s       |
| M = 16226  |                  |        |                  |                |                  |                  |
|.........    |                  |        |                  |                |                  |                  |

Fig. 8: The qualitative evaluation of 5 SmartGD models by comparing with 4 competitive and representative benchmarks. The name of the graphs with the number of nodes $N$ and the number of edges $M$ is presented in the row header. For each layout, the computation time $t$ on the CPU is computed and reported in seconds.

Thanks to the flexibility of self-challenging GAN, it can be adopted to optimize any quantitative criterion regardless of the differentiability, and without the need of special accommodation. If multiple criteria are considered to be important, SmartGD with self-challenging GAN is also applicable.

We conduct extensive experiments to evaluate the effectiveness of SmartGD quantitatively and qualitatively against 12 widely used layout methods. The quantitative evaluation on Rome graphs demonstrates that SmartGD can consistently generate superior layouts compared against all the benchmarks according to the criterion to be optimized. The qualitative evaluation on Rome graphs and real-world graphs with hundreds to thousands of nodes shows that the layouts generated by SmartGD are visually pleasing and informative. Lastly, the time efficiency of SmartGD is competitive among all methods.
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