Research Article

Coordinating Demand and Supply for Crowd Logistics Platforms with Network Effect

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The crowd logistics platforms connect stochastic demand with uncertain delivery supply which is provided by independent service providers. Considering direct-network effects and cross-network effects between the demand and supply side, a dynamic surge pricing model for crowd logistics platforms is built. The pricing strategy is derived to coordinate the supply with demand to equilibrium. Furthermore, the pricing strategy minimizing cumulative delivery orders is analyzed. The numerical simulation results show that the dynamic surge pricing strategies can stimulate the uncertain delivery supply for maximizing platforms’ revenue. And, direct-network effects pose a positive impact on the dynamic surge pricing strategy. In contrast, the cross-network effects have a negative impact on the pricing strategy. However, direct-network effects and cross-network effects negatively influence platforms’ revenue.

1. Introduction

Crowd logistics platforms outsource last-mile delivery services of electric commerce to a mass of independent service providers. Crowd logistics, which is derived from the crowdsourcing model in which individuals or organizations obtain services from a relatively open and often rapidly evolving network of sharing independent groups [1], has the advantages of optimizing resources’ allocation and increasing distribution services’ efficiency [2, 3]. In recent years, many crowd logistics platforms are quickly developing, such as UberEats, Dada, and Hummingbird [4]. The total amount of Dada crowd logistics services in China are up to 3.10 billion RMB in 2019. Hummingbird crowd logistics platform has three million registered independent service providers. The average daily orders amount of Hummingbird crowd logistics platform reached 4.5 million.

Crowd logistics model have drawn independent service providers with free schedule. However, the self-employed independent service providers who are not directly under the platforms’ control choose platforms by themselves and self-schedule their own work time. The delivery supply of independent service providers for platforms is uncertain. The logistics demand provided for online customer anytime is stochastic and may soar with rush hour. It is difficult for crowd logistics platforms to coordinate uncertain supply to adequately provide services for the stochastic demand. In addition, the network effects between the demand side and the supply side of crowd logistics platforms are exhibited [5]. Therefore, strategies to coordinate supply and demand under complex environments have practical implications for successful crowd logistics platforms’ management.

Surge pricing strategy wherein the price is temporarily raised above the regular price is applied to coordinate uncertain self-scheduling supply when demand exceeds supply at the rush hour by crowd logistics platforms, for instance, Dada crowd logistics platforms. Therefore, our study develops a surge pricing model of crowd logistics platforms with stochastic demand and uncertain delivery supply considering network effects based on optimal control theory. Furthermore, the paper analyzes influences of direct- and cross-network effects on surge pricing strategies for crowd logistics platforms.
Our study generates several key contributions. First, we extend the prior literature on surge pricing [6, 7] to stochastic demand and uncertain supply with network effects, which naturally existed between demand side and supply of crowd logistics platforms. Second, departing from the existing literature applying probability density function to represent stochastic demand and uncertain supply, we use optimal control theory in order to precisely describe the dynamic trajectory of crowd logistics prices. This study contributes to emerging research on the intersection of the crowd logistics, two-sided platforms, and revenue management. The research work reveals important managerial insights to guide crowd logistics platforms to choose an appropriate incentive mechanism to optimize the operations performance.

The remainder of our paper includes five sections. In Section 2, the literature of crowd logistics, dynamic pricing and network effects will be discussed. Section 3 presents the model setting and built the dynamic surge pricing model for crowd logistics platforms with network effects using optimal control theory. Section 3.1 deduces surge pricing strategy to coordinate the supply and demand to equilibrium. Section 3.2 investigates the surge pricing strategy for minimizing cumulative delay orders. Section 4 analyzes the direct-network effects and cross-network effects on the pricing strategy. Section 5 makes the numerically simulation of dynamic surge pricing models. In the end, Section 6 summarizes our research.

2. Literature Review

Crowd logistics model outsources delivery services to independent crowd service providers [8]. Existing related research includes crowds’ behavior and task analysis. Ta et al. [2] investigated crowd logistics system designs from driver disclosure perspective. Lu et al. [9] summarized influencing factors affecting crowds’ participation behavior, which are character of crowd, crowdsourcing task attention, and number of participating crowds. Rai et al. [10] identified the crucial motives of the crowd as monetary compensation and free schedule. Qi et al. [11] found out that crowdsourcing could generate economic benefits. And, it could bring operational flexibility for last-mile delivery. Chen et al. [12] studied the crowdsourcing solution for electronic commerce returned goods using taxis. Bayus [13] analyzed and studied the Dell crowdsourcing product creative community. The features of crowd participants’ efforts to create idea for new products are exploited over time. Li et al. [14] explored the crowdsourcing decision in fast fashion industry through three hierarchical submodels in terms of three key factors, which are surplus, due date, and goodwill. And, a two-sided matching model, which garment makers and online designer select each other, is proposed from a dynamic perspective. An improved Gale–Shapley algorithm is devised as well to solve the crowdsourcing matching problems.

Many researchers have studied dynamic pricing strategy on perishable products and service dynamic pricing. For example, Batur et al. [15] proposed the dynamic pricing model for Internet service based on usage and statistically changing capacity. The heuristic method is applied to solve out the optimization. Hsieh and Dye [16] established an optimal dynamic pricing for deteriorating products when inventories stimulate demand. The reference price effects are discussed to affect optimal pricing. Su et al. [17] built a fairness concern utility system for the closed-loop supply chain. The influence of competitive strength and fairness concern degree of collectors on the pricing decisions is analyzed. Cachon et al. [6] studied a kind of surge pricing strategy on crowd services’ platform with self-scheduling capacity and the incentive methods for independent providers. Lin and Zhou [18] investigated pricing policy selection for a monopoly crowd platform providing vertically differentiated services with self-scheduling capacity. They found that the dynamic pricing is the better policy to enhance the performance of the platform. Sun et al. [19] determined the dynamic pricing strategy for the crowd ride-sourcing platforms, assuming that drivers and customers maximize utility. The platform price is found to consist of a ride length-based fare and a rush hour congestion fee. Bai et al. [20] consider an on-demand crowd service platform using earning sensitive independent providers with heterogeneous reservation price to serve its time and price sensitive customers with heterogeneous valuation of the service. To coordinate supply and demand, the steady-state waiting time is included in the customer utility function to characterize the optimal price and wage rates.

The widespread use of the Internet has resulted in a plethora of two-sided markets [21], which have network effects between same sides and cross sides [22]. Researchers intensively study network effects of two-sided platforms in recent years. Li et al. [23] examine the network effects of backers on crowdfunding projects. Hinz et al. [24] propose an influx-outflow model for estimation of network effects on two-sided business-to-consumer (B2C) platforms. Qi et al. [25] study the product line pricing with network effect. Song et al. [26] examine asymmetric cross-side network effects on different software platform sides, which are app side and user side. The governance policies of the platform on app review and platform updates has an impact on network effects and distinct value creation processes. Chao and Derdenger [27] studied mixed bundling in two-sided markets with installed base effects. And, cross-side network effects are found to have an impact on demand and prices. Chu and Manchanda [28] build a utility-based model to quantify both cross and direct-network effects on Alibaba Group’s Taobao C2C platform. The relative contributions of different factors are investigated to affect the growth of buyers and sellers on the platform.

The Internet-based crowd logistics platforms operate as two-sided markets and exhibit network effects between the stochastic customer demand side and the uncertain delivery supply side. There is existing literature on uncertain demand and supply [29], such as supply chain coordination with uncertain demand and supply applying the returns’ policy [30] and the advance purchase discount (APD) contract [31], by random variable and game theory. However, the study on crowd logistics platforms’ pricing strategy with stochastic demand and uncertain supply under network effects is rare.
In this paper, we focus on dynamic surge pricing strategy of crowd logistics platforms with stochastic demand and uncertain delivery supply considering the network effects. Moreover, optimal control theory is used to precisely depict stochastic demand and uncertain supply trajectory in this paper. The dynamic surge pricing solutions to coordinate supply and demand for crowd logistics platforms are deducted out to maximizing platforms’ revenue with Pontryagin’s maximum principle of optimal control theory. Furthermore, the impact of both direct network effects and cross-network effects on dynamic surge pricing strategies and crowd logistics platforms’ revenue is investigated.

### 3. Dynamic Surge Pricing Model

Crowd logistics two-sided platforms connect stochastic demand side with uncertain delivery supply side. The cross-network effects from cross demand side might affect delivery supply of crowd logistics platforms. And, there are direct-network effects within the same supply side with which the number of delivery service providers could attract more crowds’ participation in platforms. Using optimal control theory the dynamic surge pricing model is established to dynamically coordinate uncertain supply and stochastic demand of crowd logistics platforms, considering the direct-network effects and cross-network effects on uncertain delivery supply.

Crowd logistics platforms decide price \( p(t) \) for customers who have the demand \( D(p,t) \). Referring to prior work [32], the stochastic demand function is

\[
D(p,t) = \alpha - p(t). \tag{1}
\]

The crowd logistics demand is represented as the linear function of price. As shown in Table 1, \( \alpha \) denotes the initial crowd logistics demand value. The crowd logistics supply \( S(p,t) \) is dependent on the wage that the platform pays independent service providers and could be influenced by platforms’ same side and cross-side network effects. The wage paid independent service providers denoted by \( W(p,t) \).

The supply function of crowd logistics platforms with network effects can be assumed as

\[
S(p,t) = \varepsilon W(p,t) + \mu_1 S(p,t) + \mu_2 D(p,t). \tag{2}
\]

The crowd logistics supply are changed according to the providers’ wage. \( \varepsilon \) denotes a sensitivity coefficient. The wage function for independent service providers is represented as

\[
W(p,t) = \gamma_p(t). \tag{3}
\]

The fixed wage ratio \( \gamma \) indicates the proportion of providers’ wage to crowd logistic price [6], \( 0 < \gamma < 1 \).

The crowd logistics demand has cross influence on supply with cross-network effects intensity \( \mu_2 \), \( 0 < \mu_2 < 1 \). In this paper, independent service providers participated in crowd logistics platforms are assumed to be more sensitive to wage than cross-network effects from demand side, that is, \( \varepsilon \gamma > \mu_2 \). The intensity of direct-network effects is represented as \( \mu_1 \), \( 0 < \mu_1 < 1 \). \( S'(p,t) \) denotes the delivery supply expectation of independent service providers. Based on Katz and Shapiro’s [22] study, the rational independent service providers are supposed to form supply expectations regarding the capacity of existing delivery supply, that is, \( S'(p,t) = S(p,t) \). The crowd service providers could obtain the capacity of existing delivery supply information shared on interaction APP by the crowd logistics platforms with Internet and wireless technology. Thus, crowd logistics supply function with network effects could be transformed from equation (2) as follows:

\[
S(p,t) = \frac{1}{1 - \mu_1} (\varepsilon W(p,t) + \mu_2 D(p,t)). \tag{4}
\]

Using optimal control theory, the dynamic surge pricing model for crowd logistics platforms with network effects is built to coordinate demand and supply. Extending Herbon and Khmelnitsky [33] study, we set the cumulative delay orders when the delivery supply is not adequate to the demand as the state variable \( Y(t) \) in this paper. The initial cumulative delay orders is \( \phi_0 \). The parameter \( \phi_T \) denotes ending value of state variable. The parameter \( h \) represents the punishing cost for each delay order. The price \( p(t) \) is gradually optimized to minimize cumulative delay orders. The dynamic surge pricing model for crowd logistics platforms with network effects is proposed [34] aiming to maximize platforms’ revenue:

\[
\prod_{p(t)} \max \int_0^T [(D(p,t)p(t) - D(p,t)W(p,t) - hY(t))dt, \tag{5}
\]

which subjects to the state equation:

\[
\begin{align*}
Y(t) &= -(D(p,t) - S(p,t)), \\
Y(t_0) &= \phi_0, \tag{6}
Y(T) &= \phi_T.
\end{align*}
\]

The Hamiltonian function combines objective equation (5) and state equation (6) as follows:

### Table 1: Notations.

| Parameter | Explanation |
|-----------|-------------|
| \( p(t) \) | Price of crowd logistics |
| \( D(p,t) \) | Demand for crowd logistics |
| \( S(p,t) \) | Supply of crowd logistics |
| \( W(p,t) \) | Wage of service providers |
| \( Y(t) \) | Cumulative delay orders |
| \( \alpha \) | Initial demand value |
| \( h \) | Unit penalty cost on cumulative delay order |
| \( \varepsilon \) | Sensitivity coefficient of supply to wage |
| \( \mu_1 \) | Direct-network effects’ intensity, \( 0 < \mu_1 < 1 \) |
| \( \mu_2 \) | Cross-network effects’ intensity, \( 0 < \mu_2 < 1 \) |
\[ H(Y(t), p(t), \lambda(t), t) = D(p(t))p(t) - D(p, t)W(p, t) - hY(t) + \lambda(t)Y(t) \]
\[ = D(p(t))p(t)(1 - \gamma) - h\phi_o + [h(T - t) - \lambda](D(p, t) - S(p, t)). \]

(7)

It has to verify the requirement conditions for maximum optimal pricing. We calculate the second-order derivative of equation (7):

\[ \frac{\partial H^2}{\partial \rho^2} = -2(1 - \gamma) < 0. \]

(8)

Because \(0 < \gamma < 1\), the second-order derivative of Hamilton function is less than zero. Therefore, the maximum value of crowd logistics price could be inferred from the following simultaneous equation:

\[
\begin{aligned}
\bar{Y}(t) &= \frac{\partial H}{\partial \lambda}, \\
\bar{\lambda}(t) &= \frac{\partial H}{\partial Y}, \\
\frac{\partial H}{\partial \rho} &= 0.
\end{aligned}
\]

(9)

\[ \lambda^*(t) = ht + \frac{2(\phi_T - \phi_o)(1 - \mu_1)^2(1 - \gamma)}{(1 + \epsilon\gamma - \mu_1 - \mu_2)^2} - \frac{a(1 + \epsilon\gamma + \mu_1 + \mu_2)(1 - \mu_1)(1 - \gamma)}{(1 + \epsilon\gamma - \mu_1 - \mu_2)^2}, \]

(10)

\[ \lambda^*(t) = ht + \frac{2(\phi_T - \phi_o)(1 - \mu_1)^2(1 - \gamma)}{(1 + \epsilon\gamma - \mu_1 - \mu_2)^2} - \frac{a(1 + \epsilon\gamma + \mu_1 + \mu_2)(1 - \mu_1)(1 - \gamma)}{(1 + \epsilon\gamma - \mu_1 - \mu_2)^2}, \]

(11)

\[ S^*(t) = \frac{(1 + \epsilon\gamma - \mu_1 - \mu_2)(\epsilon\gamma - \mu_2)ht}{(1 - \mu_1)^2(1 - \gamma)} + \frac{(\phi_T - \phi_o)(\epsilon\gamma - \mu_2)}{(1 + \epsilon\gamma - \mu_1 - \mu_2)T} + \frac{ac\gamma}{1 + \epsilon\gamma - \mu_1 - \mu_2}, \]

(12)

\[ D^*(t) = \frac{(1 + \epsilon\gamma - \mu_1 - \mu_2)ht}{(1 - \mu_1)(1 - \gamma)} + \frac{(1 + \epsilon\gamma - \mu_1 - \mu_2)ht}{2(1 - \mu_1)(1 - \gamma)} - \frac{(\phi_T - \phi_o)(1 - \mu_1)}{(1 + \epsilon\gamma - \mu_1 - \mu_2)T} + \frac{ac\gamma}{1 + \epsilon\gamma - \mu_1 - \mu_2}. \]

(13)

Through the derivation, we could obtain the optimization values of dynamic surging pricing models for crowd logistic platforms with network effects. Equation (10) is the formula of the optimal surge pricing. Equation (11) is the Lagrange multiplier, which represents the shadow price. Equation (12) is the formula of the supply of crowd logistics platforms corresponding to the optimal surge pricing, while equation (13) is the demand:

3.1. Supply and Demand Equilibrium. In order to coordinate supply and demand for crowd logistics platforms with network effects, surge pricing is applied to stimulate crowd independent service provider participating platforms for meeting soaring demand of rush hour. Assume the supply and demand are balanced to the equilibrium state at \(T_{ba}\), namely, \(D(T_{ba}) = S(T_{ba})\). Then, the rate of delay order
accumulation is \( \mathcal{Y}(T_{ba}) = 0 \) by deducing from the state equation (6).

**Proposition 1.** Aiming to coordinate the supply and demand in the period \( [0, T_{ba}] \), the optimal surge pricing for crowd logistics platforms with network effects is

\[
p_T^* (t) = \frac{(1 + \varepsilon \gamma - \mu_1 - \mu_2)ht}{(1 - \mu_1)(1 - \gamma)} - \frac{(1 + \varepsilon \gamma - \mu_1 - \mu_2)hT_{ba}}{(1 - \mu_1)(1 - \gamma)}
\]

\[+ a \frac{(1 - \mu_1 - \mu_2)}{1 + \varepsilon \gamma - \mu_1 - \mu_2}
\]

(14)

**Proposition 2.** Aiming to coordinate the supply and demand in the period \( [0, T_{ba}] \), the optimal surge pricing for crowd logistics platforms with network effects is the strictly monotone increasing function when \( \varepsilon \gamma > \mu_2 \).

These two propositions suggest that the optimal price of crowd logistics is the strictly increasing function with respect to time \( t \) during the service period \( [0, T_{ba}] \), when supply sensitivity coefficient to price \( \varepsilon \gamma \) is greater than cross-network effects from demand side \( \mu_2 \), that is, \( \varepsilon \gamma > \mu_2 \), from proof in Appendix.

**Corollary 1.** Aiming to coordinate the supply and demand in the period \( [0, T_{ba}] \), the supply for crowd logistics platforms with network effects increases according to the optimal price when \( \varepsilon \gamma > \mu_2 \).

The first-order derivative of supply to price is greater than zero from proof in Appendix. Based on Proposition 1, we could conclude that the supply for crowd logistics platforms with network effects increases according to the optimal price when the crowd logistics supply is more sensitive to the price than cross-network effects from demand side, that is, \( \varepsilon \gamma > \mu_2 \), aiming to coordinate the supply and demand in the period \( [0, T_{ba}] \).

3.2. Minimizing Cumulative Delay Orders. In order to minimize cumulative delay orders of crowd logistics platforms with network effects to maximize the expected revenue of platforms, surge pricing is further applied to stimulate more crowd independent service providers participating platforms for soaring demand of rush hour. The crowd logistics platforms could coordinate supply and demand equilibrium with surge pricing strategy at \( T_{ba} \). Nevertheless, the delay orders are accumulated in the period \( [0, T_{ba}] \), namely, \( Y(T_{ba}) > 0 \). Assuming cumulative delay orders is minimized at \( T_{ma} \), due to increasing independent service providers participation by surge price strategy of crowd logistics platforms with network effects, that is, \( Y(T_{ma}) = 0 \).

**Proposition 3.** Aiming to coordinate the supply and demand to minimize cumulative delay orders in the period \( [0, T_{ma}] \), the optimal surge pricing for crowd logistics platforms with network effects is

\[
p_T^* (t) = \frac{(1 + \varepsilon \gamma - \mu_1 - \mu_2)ht}{(1 - \mu_1)(1 - \gamma)} - \frac{(1 + \varepsilon \gamma - \mu_1 - \mu_2)hT_{ma}}{2(1 - \mu_1)(1 - \gamma)}
\]

\[+ a \frac{(1 - \mu_1 - \mu_2)}{1 + \varepsilon \gamma - \mu_1 - \mu_2}
\]

(15)

**Corollary 2.** Aiming to coordinate the supply and demand to minimize cumulative delay orders in the period \( [0, T_{ma}] \), the supply for crowd logistics platforms with network effects increases according to the optimal price when \( \varepsilon \gamma > \mu_2 \), until the cumulative delay orders has been minimized to zero at the \( T_{ma} \) moment.

The crowd independent service providers’ supply increases according to crowd logistics price if the crowd logistics supply is more sensitive to the price than cross-network effects from demand side, that is, \( \varepsilon \gamma > \mu_2 \), from proof in Appendix. Based on Proposition 4, we could conclude that the supply for crowd logistics platforms with network effects increases according to the optimal surge pricing strategy when cross-network effects is greater than the price. More precisely, the cumulative delay orders is minimized at \( T_{ma} \), due to increasing independent service providers participation by surge price strategy of crowd logistics platforms with network effects, that is, \( Y(T_{ma}) = 0 \).
effects increases according to the optimal price within the period \([0, T_{ma}]\) when \(\epsilon Y > \mu_2\), until the cumulative delay orders has been minimized to zero at the \(T_{ma}\) moment. Therefore, the supply for crowd logistics platforms with network effects to minimize cumulative delay orders is similarly increasing with surge pricing as the supply and demand equilibrium scenario, which is in accordance with the reality of crowd logistics platforms.

Within the period \([0, T_{ma}]\), crowd logistics platforms coordinate the supply and demand by dynamic surge pricing strategy to minimize cumulative delay orders through two phases. The crowd logistics platforms coordinate the supply and demand by dynamic surge pricing strategy to reach the equilibrium moment \(T^*\) in the first phase and then continuously minimize cumulative delay orders at \(T_{ma}\) in the second phase. The equilibrium moment \(T^*\) could be derived out as equation (28) in the proof in Appendix.

In the period \([0, T^*]\), surge pricing strategy for crowd logistics platforms with network effects is applied to coordinate the supply and demand to equilibrium moment \(T^*\) in the first phase. Then, surge pricing strategy for crowd logistics platforms with network effects is applied in the period \([T^*, T_{ma}]\) in order to minimize cumulative delay orders at \(T_{ma}\) in the second phase, namely, \(Y(T_{ma}) = 0\). Because the punishing cost of cumulative delay orders is minimized to zero, namely, \(hY(T_{ma}) = 0\), the crowd logistics platforms’ revenue is maximized. However, the crowd logistics platforms’ revenue is hard to be described from equation (4) and will be investigated in the numerical simulation section.

Therefore, the optimal surge pricing strategies of crowd logistics platforms with network effects in both scenarios, which are equilibrium and minimizing cumulative delay orders, could stimulate the growth of delivery supply to coordinate with demand to the equilibrium state and optimize expected revenue.

4. The Network Effects’ Impact Analysis

Crowd logistics platforms link the customer demand side and supply side provided by independent service providers. There are the same-side direct-network effects within crowd logistics supply side. And, the other demand side has cross-network effects on crowd logistics supply side as well. In this paper, both the same-side direct-network effects and cross-side cross-network effects on crowd logistics supply side will be investigated for their influence on optimal surge pricing strategies and crowd logistics platforms’ revenue.

4.1. Direct-Network Effects. The influence of direct-network effects on both optimal surge pricing strategies for crowd logistics platforms to equilibrium and delay orders’ minimization is analyzed in this section.

**Proposition 5.** The growth rate of optimal surge pricing for crowd logistics platforms increase with the intensity of direct-network effects \(\mu_1\) within delivery supply side when \(\epsilon Y > \mu_2\).

**Corollary 3.** The supply for crowd logistics platforms at the equilibrium moment is increasing with respect to the intensity of direct-network effects.

The direct-network effects of delivery supply side has positive impact on optimal price of crowd logistics if the independent service providers are more sensitive to wage than cross-network effects from the demand side (\(\epsilon Y > \mu_2\)). And, the growth speed of surge crowd logistics price increases with direct-network effects \(\mu_1\) increment from proof in Appendix. The crowd logistics supply at equilibrium moment \(T^*\) is increasing with respect to the direct-network effects \(\mu_1\) within delivery supply side in both the supply and demand equilibrium and minimizing cumulative delay orders’ scenarios. Thus, the intensity of direct-network effects \(\mu_1\) has the positive impact on delivery supply of crowd logistics platforms at equilibrium moment \(T^*\) and \(T_{ba}\) of the supply and demand. However, the impact of direct-network effects \(\mu_1\) on supply growth, crowd logistics platforms’ revenue, and independent service providers’ income are hard to be described, and they will be investigated in the numerical simulation section.

In summary, the direct-network effects have positive impact on the growth of optimal surge crowd logistics price. The intensity of direct-network effects has positive impact on the crowd logistics supply under the equilibrium state as well. The higher intensity of direct-network effects within supply side will have the positive influence on delivery supply enhancement of crowd logistics platforms.

4.2. Cross-Network Effects. The influence of cross-network effects between the demand side and the supply side on both optimal surge pricing strategies for crowd logistics platforms to equilibrium and delay orders minimization scenarios are analyzed.

**Proposition 6.** The growth rate of optimal surge pricing for crowd logistics platforms decreases with the intensity of cross-network effects \(\mu_2\) on the delivery supply side from the demand side.

**Corollary 4.** The supply for crowd logistics platforms at the equilibrium moment is increasing with respect to the intensity of cross-network effects \(\mu_2\).

The growth rate of optimal crowd logistics service price \(F_{ma}(t)\) is a strictly monotone decreasing function to cross-network effects \(\mu_2\) from proof in Appendix. So, the speed of optimal crowd logistics price growth is decreasing with cross-network effects \(\mu_2\) increment. And, the cross-network effects on the delivery supply side from the demand side have the negative influence on optimal crowd logistics price. The crowd logistics supply at equilibrium moment \(T^*\) and \(T_{ba}\) is increasing with respect to the cross-network effects \(\mu_2\) on the supply side from the demand side in both the supply and demand equilibrium and minimizing cumulative delay orders’ scenarios from proof in Appendix. Hence, the intensity of cross-network effects \(\mu_2\) has the positive impact on delivery...
supply of crowd logistics platforms at equilibrium moment $T^*$ and $T_{ba}$. However, the influence of cross-network effects $\mu_2$ on supply growth, crowd logistics platforms' revenue, and independent service providers' income is hard to be described and will be investigated in Section 5.

In summary, cross-network effects of the demand side on the supply side have negative impact on the growth of optimal crowd logistics price while the direct-network effects have positive impact. Both cross-network effects and direct-network effects' intensities have positive impact on the crowd logistics supply at the equilibrium moment. Therefore, the crowd logistics supply will be influenced by the demand and other delivery providers. The higher network effects' intensity from both the same supply side and the other cross demand side could enhance the crowd logistic supply. The direct-network effects within the supply side could make the optimal logistics price growing faster. Nevertheless, the cross-network effects of the demand side on the supply side make the increase of optimal logistics price slower.

5. Numerical Study

The optimal surge pricing model for crowd logistics platforms with the stochastic demand and uncertain delivery supply under network effects will be tested for its validity in this section. We will analyze the dynamic trajectories for optimal crowd logistics price to coordinate supply with demand in equilibrium and cumulative delay orders minimization scenarios. And, the influence of the direct- and cross-network effects is examined on crowd logistics pricing and platforms’ revenue. We investigated on crowd logistics platforms’ practice. Accordingly, the simulation parameters are set as $a = 50$, $h = 0.2$, $\mu_1 = 0.3$, $\mu_2 = 0.4$, $\gamma = 0.3$, $\epsilon = 2$, $\varphi_0 = 10$, $T_{ba} = 25$, and $T_{ma} = 45$.

The optimal surge pricing strategy for crowd logistics platforms under network effects is applied to coordinate the supply and demand to equilibrium scenario at $T_{ba} = 25$. The dynamic surge price trajectory to encourage crowd independent service providers’ participation for crowd logistics supply to satisfy soaring demand in the period [0, 25] is shown in Figure 1(a). Proposition 2 is proved by numerical results shown in Figure 1(a) in which crowd logistic price curve is linear increasing with time $t$. Correspondingly, crowd independent service providers' supply is increasing with time $t$ shown in Figure 2(a). So, the surge pricing strategy could stimulate more crowd independent service providers' participation to increase the supply for crowd logistics platforms. Figure 2(a) illustrates that growth of crowd independent service providers' supply for crowd logistics is coordinated to reach equilibrium with decreasing logistics demand by surge pricing at the point $T_{ba} = 25$. These results verify Corollary 1.

The optimal surge pricing strategy is further applied by crowd logistics platforms with network effects to coordinate the supply and demand to minimize cumulative delay orders’ scenario at $T_{ma} = 45$. The simulation results are illustrated in Figures 1(b) and 2(b), respectively. The dynamic surge pricing trajectory to encourage crowd independent service providers' participation for crowd logistics supply satisfying soaring demand and minimizing cumulative delay orders in the period [0, 45] is shown in Figure 1(b). Proposition 4 is proved by numerical results shown in Figure 1(b) in which crowd logistics price curve is linearly increasing with time $t$. Figure 2(b) illustrates that the crowd independent service providers' supply increases with $t$ as well, while crowd logistics demand reduces when crowd logistics price increases with $t$. So, the surge pricing strategy could stimulate participation of more crowd independent service providers to increase the supply for crowd logistic platforms in minimizing cumulative delay orders’ scenario. These results verify Corollary 2.

The growth of crowd independent service providers' supply for crowd logistics platforms with network effects which are stimulated by dynamic surge pricing strategy is coordinated to minimize cumulative delay orders through two phases as illustrated in Figure 2(b). The growth of crowd independent service providers' supply for crowd logistic platforms with network effects is coordinated to reach equilibrium with decreasing logistics demand by surge pricing in the first phase at the equilibrium time point $T^* = 23$. Furthermore, the growth of crowd independent service providers' supply for crowd logistics platforms is continuously coordinated to minimize cumulative delay orders until time point $T_{ma} = 45$ in which $Y(T_{ma}) = 0$. The delay orders of crowd logistics is accumulated in the first phase due to insufficient supply provided by crowd independent service providers for platforms. At the end of first phase $T^* = 23$, the crowd independent service providers' supply has been controlled to reach equilibrium with soaring demand, in which the rate of delay order accumulation is $\lambda(T^*) = 0$. Nevertheless, the cumulative delay orders is greater than zero, that is, $Y(T^*) > 0$. Hence, the surge pricing strategy for crowd logistics platforms in the second phase tries to increase the supply of crowd independent service providers to finish remaining delay orders accumulated in the first phase. At the end of the second phase $T_{ma} = 45$, remaining delay orders accumulated in the first phase have been finished, that is, $Y(T_{ma}) = 0$, by increasing crowd independent service providers supply encouraged by surge pricing strategy for crowd logistics platforms.

The revenue of crowd logistics platforms with network effects are found to grow according to increasing crowd logistics price in the numerical simulation results illustrated in Figure 3. The growth of crowd independent service providers' supply for crowd logistic platforms with network effects which are stimulated by dynamic surge pricing strategy minimizes cumulative delay orders, which reduce the punishing cost of delay orders. So, the optimal surge pricing strategy for crowd logistic platforms with network effects could increase the platforms’ revenue. Figures 3(a) and 3(b) also reveal that crowd logistics platforms’ revenue is reduced with intensity of direct-network effects and cross-network effects.

The numerical simulation of the direct-network effects $\mu_1$ and the cross-network effects $\mu_2$ on the surge pricing of logistics services, delivery supply growth, equilibrium moment value, crowd logistics platforms’ revenue, and independent service providers’ income are shown in Figures 4–7,
respectively. Figure 4(a) illustrates that optimal logistics service price value declines as the direct-network effects $\mu_1$ increase. However, it is found that the growth of optimal crowd logistics price increases with respect to direct-network effects $\mu_1$ increment in Figure 4(b) which verified Proposition 5. Figure 6 shows that the delivery supply growth is increasing as the direct-network effects $\mu_1$ increase. And, the delivery supply at equilibrium moment is increasing with the direct-network effects $\mu_1$ increase, which verified Corollary 3. Figure 7(a) reveals that the crowd logistics platforms’ revenue and the independent service providers’ income are decreasing as the direct-network effects $\mu_1$ increase. Therefore, the intensity of direct-network effects has positive impact on logistics service price growth and delivery supply. Thus, higher expectation of delivery supply will urge more potential service providers to join the platform to enhance delivery supply satisfying the huge demand. However, the intensity of direct-network effects within the same supply side has negative impact on the price. Thus, the income of independent service providers is decreasing with the direct-network effects’ increment due to declining logistics service price. Correspondingly, declining logistics service price makes the revenue of the platform reduce with respect to the direct-network effects’ increment.

The growth of optimal crowd logistics price decreases to the cross-network effects $\mu_2$ increment, as shown in Figure 5(b), which verified the Proposition 6. Figure 5(a) shows that optimal crowd logistics price declines as the cross-network effects $\mu_2$ increases. And, the slope of dynamic optimal price trajectory shown in Figure 5(a) is becoming smaller when the cross-network effects $\mu_2$ increase. Figure 6 indicates that the growth of delivery supply is decreasing as the cross-network effects $\mu_2$ increase because of the declining growth rate of price. However, the delivery supply at the equilibrium moment increases with the cross-network effects $\mu_2$ increment that verified Corollary 4. Figure 7(b) reveals that crowd logistics platforms’ revenue is decreasing as the cross-
network effects $\mu_2$ increases. And, the income of independent service providers is decreasing with the cross-network effects $\mu_2$ as well. Therefore, the intensity of cross-network effects from the demand side on the supply side has negative impact on both optimal logistics services’ price and logistics services’ price growth. However, the intensity of cross-network effects has positive impact on delivery supply. Thus, the supply for crowd logistics platforms could be positively influenced by demand. Nevertheless, the income of independent
Figure 5: The cross-network effects’ crowd logistics price: (a) price and (b) price growth.

Figure 6: The network effects on crowd logistics supply: (a) supply growth and (b) supply.

Figure 7: The network effects on platforms’ revenue and crowd independent providers’ income: (a) direct-network effects and (b) cross-network effects.
service providers is decreasing with the cross-network effects due to declining optimal logistics services’ price and slow growth of logistics services’ price. Correspondingly, declining logistics services price makes the revenue of the platform decline with respect to the cross-network effects’ increment.

6. Conclusions

This study focuses on surge pricing strategy for crowd logistics platforms with stochastic demand and uncertain supply under network effects. The optimal pricing strategies for crowd logistics platforms with network effects are investigated to coordinate the supply and demand under two scenarios, which are equilibrium and minimizing cumulative delay orders by applying numerical simulation. Considering the same-side direct-network effects and cross-side cross-network effects, we established an optimal surge pricing model for crowd logistics platforms applying optimal control theory.

Our theoretical findings suggest that surge pricing strategy for crowd logistics platforms with network effects could quickly coordinate the supply with demand to the equilibrium state. Moreover, the surge pricing strategy for crowd logistics platforms with network effects could coordinate supply and demand to minimize cumulative delay orders. And, the surge pricing strategy for crowd logistics platforms could increase crowd logistics platforms’ revenue. Furthermore, we find that both direct- and cross-network effects have influence on optimal pricing, crowd independent service providers supply, crowd logistics platforms’ revenue, and independent providers’ income. The direct-network effects within the delivery supply side have positive impacts on the surge pricing and crowd independent service providers’ supply for crowd logistics platforms. There are negative impacts on the surge pricing from cross-network effects between the customer demand side and delivery supply side. The cross-network effects have positive impact on the crowd logistics supply. However, the cross-network effects have negative impacts on the growth of crowd logistics supply. In addition, we find that both direct- and cross-network effects have negative impact on crowd logistics platforms’ revenue and the independent providers’ income. Our research findings have important managerial insights to guide crowd logistics platforms to coordinate supply and demand and improve operations performance.

The optimal surge pricing strategies for crowd logistics platforms with network effects established in our work are proved to enhance crowd logistics platforms performance. However, the crowd logistics platforms fiercely compete in practical operations. Future research could be conducted on multiple platforms’ competing pricing. Additionally, wage ratios that are also typical incentive mechanisms for independent service providers are differently adopted by practical crowd logistics platforms. The dynamic wage mechanism for independent service providers could be taken into consideration in further research on crowd logistics management.

Appendix

Proof of Proposition 1. (the supply and the demand balance at the equilibrium moment $T_{ba}$). At the equilibrium moment, the crowd logistics price could be deduced out by simultaneously solving equations (1) and (3):

$$p(T_{ba}) = \frac{a(1 - \mu_1 - \mu_2)}{1 + \epsilon y - \mu_1 - \mu_2}. \tag{A.1}$$

The optimal crowd logistic pricing to coordinate the supply and demand could be derived out from equation (9) as well:

$$p^*(T_{ba}) = \frac{(1 + \epsilon y - \mu_1 - \mu_2) h T_{ba}}{2(1 - \mu_1)(1 - \gamma)} + \frac{(\phi_T - \phi_0)(1 - \mu_1)}{(1 + \epsilon y - \mu_1 - \mu_2) T_{ba}} + \frac{a(1 - \mu_1 - \mu_2)}{1 + \epsilon y - \mu_1 - \mu_2}. \tag{A.2}$$

Simultaneously, solving the crowd logistics service optimal price equations (A.1) and (A.2), we can obtain

$$\phi_T = \phi_0 - \frac{(1 + \epsilon y - \mu_1 - \mu_2)^2 h T_{ba}^2}{2(1 - \mu_1)^2(1 - \gamma)}. \tag{A.3}$$

Substituting $T = T_{ba}$ and $\phi_T$ represented by equation (A.3) into equations (9) and (11), we could derive the optimization values of dynamic surging pricing models for crowd logistic platforms with network effects at equilibrium moment $T_{ba}$. Equation (A.4) is the formula of the optimal surge pricing at the equilibrium moment $T_{ba}$. Equation (A.5) is the Lagrange multiplier, which represents the shadow price at the moment $T_{ba}$. Equation (A.6) is the formula of the crowd logistics supply corresponding to the optimal surge pricing, while equation (A.7) is the demand at the equilibrium moment $T_{ba}$.

$$p^*_{T_{ba}}(t) = \frac{(1 + \epsilon y - \mu_1 - \mu_2) h t}{(1 - \mu_1)(1 - \gamma)} - \frac{(1 + \epsilon y - \mu_1 - \mu_2) h T_{ba}}{(1 - \mu_1)(1 - \gamma)} + \frac{a(1 - \mu_1 - \mu_2)}{1 + \epsilon y - \mu_1 - \mu_2}. \tag{A.4}$$

$$\lambda^*_{T_{ba}}(t) = h t - \frac{a(-1 + \epsilon y + \mu_1 + \mu_2)(1 - \mu_1)(1 - \gamma)}{(1 + \epsilon y - \mu_1 - \mu_2)^2} - h T_{ba}. \tag{A.5}$$
\begin{align}
S^*_{T_{bs}}(t) &= \frac{(1 + ey - \mu_1 - \mu_2)(ey - \mu_2)ht - (1 + ey - \mu_1 - \mu_2)(ey - \mu_2)hT_{bs}}{(1 - \mu_1)^2(1 - \gamma)} + \frac{ae\gamma}{1 + ey - \mu_1 - \mu_2}, \\
D^*_{T_{bs}}(t) &= \frac{(1 + ey - \mu_1 - \mu_2)ht}{(1 - \mu_1)(1 - \gamma)} + \frac{(1 + ey - \mu_1 - \mu_2)hT_{bs}}{(1 - \mu_1)(1 - \gamma)} + \frac{ae\gamma}{1 + ey - \mu_1 - \mu_2}.
\end{align}

\textbf{Proof of Proposition 2.} In order to prove Proposition 2, we calculate the first-order derivative for optimal surge price of crowd logistics platforms with network effects in the period $[0, T_{bs}]$ from equation (A.4) as follows:

\[ P^*_{T_{bs}}(t) = \frac{h(1 + ey - \mu_1 - \mu_2)}{(1 - \gamma)(1 - \mu_1)} \]

On account of $0 < \mu_1 < 1$, so $1 - \mu_1 > 0$. And, $0 < \gamma < 1$, so $1 - \gamma > 0$. Therefore, when $ey > \mu_2$, $\frac{dP^*_{T_{bs}}(t)}{dp} > 0$. \hfill \Box

\textbf{Proof of Corollary 1.} In order to prove Corollary 1, we calculate the first-order derivative for the supply of crowd logistics platforms with network effects to price in the period $[0, T_{bs}]$ by equation (3):

\[ \frac{dS^*_{T_{bs}}(t)}{dp} = \frac{(ey - \mu_2)}{(1 - \mu_1)}. \]

\begin{align}
P^*_{T_{mas}}(t) &= \frac{(1 + ey - \mu_1 - \mu_2)ht - (1 + ey - \mu_1 - \mu_2)hT_{mas}}{(1 - \mu_1)(1 - \gamma)} - \frac{\phi_0(1 - \mu_1)}{(1 + ey - \mu_1 - \mu_2)} + \frac{a(1 - \mu_1 - \mu_2)}{1 + ey - \mu_1 - \mu_2}, \\
\lambda^*_{T_{mas}}(t) &= ht - \frac{2\phi_0(1 - \mu_1)^2(1 - \gamma)}{(1 + ey - \mu_1 - \mu_2)^2T_{mas}} - \frac{a(-1 + ey + \mu_1 + \mu_2)(1 - \mu_1)(1 - \gamma)}{(1 + ey - \mu_1 - \mu_2)^2}, \\
S^*_{T_{mas}}(t) &= \frac{(1 + ey - \mu_1 - \mu_2)(ey - \mu_2)ht}{(1 - \mu_1)^2(1 - \gamma)} - \frac{(1 + ey - \mu_1 - \mu_2)(ey - \mu_2)hT_{mas}}{2(1 - \mu_1)^2(1 - \gamma)} - \frac{\phi_0(ey - \mu_2)}{(1 + ey - \mu_1 - \mu_2)T_{mas}} + \frac{ae\gamma}{1 + ey - \mu_1 - \mu_2}, \\
D^*_{T_{mas}}(t) &= \frac{(1 + ey - \mu_1 - \mu_2)ht}{(1 - \mu_1)(1 - \gamma)} + \frac{(1 + ey - \mu_1 - \mu_2)hT_{mas}}{2(1 - \mu_1)(1 - \gamma)} + \frac{\phi_0(1 - \mu_1)}{(1 + ey - \mu_1 - \mu_2)T_{mas}} + \frac{ae\gamma}{1 + ey - \mu_1 - \mu_2}.
\end{align}

\textbf{Proof of Proposition 4.} In order to prove Proposition 4, we calculate the first-order derivative for optimal surge price of crowd logistics platforms with network effects in the period $[0, T_{mas}]$ from equation (A.10) as follows:

\[ P^*_{T_{mas}}(t) = \frac{h(1 + ey - \mu_1 - \mu_2)}{(1 - \gamma)(1 - \mu_1)} \]

On account of $0 < \mu_1 < 1$, so $1 - \mu_1 > 0$. And, $0 < \gamma < 1$, so $1 - \gamma > 0$. Therefore, when $ey > \mu_2$, $\frac{dP^*_{T_{mas}}(t)}{dp} > 0$. \hfill \Box

\textbf{Proof of Corollary 2.} In order to prove Corollary 2, we calculate the first order derivative for the supply of crowd logistics platforms with network effects to price in the period $[0, T_{bs}]$ by equation (4) as the supply and demand equilibrium scenario:

\[ S^*_{T_{bs}}(t) = S^*_{T_{bs}}(t) = S(p,t) = \frac{1}{1 - \mu_1} \left[ \mu_2 a + (ey - \mu_2) p(t) \right], \]

\[ \frac{dS^*_{T_{bs}}(t)}{dp} = \frac{dS^*_{T_{bs}}(t)}{dp} = \frac{(ey - \mu_2)}{(1 - \mu_1)}. \]

On account of $0 < \mu_1 < 1$, so $1 - \mu_1 > 0$. When $ey > \mu_2$, $\frac{\partial S^*_{T_{bs}}(t)}{\partial p} > 0$.

We can deduce out the crowd logistics price at equilibrium moment $T^*$ according to equations (1) and (2):
Simultaneously, solving the price equations (A.10) and (A.16), time point $T^*$ could be derived out:

$$T^* = \frac{\phi_0(1 - \mu_1)^2(1 - \gamma)}{kT_{ma}(1 + \epsilon \gamma - \mu_1 - \mu_2)^2} + \frac{T_{ma}}{2} \tag{A.17}$$

Proof of Proposition 5. In order to prove Proposition 5, we calculate the first-order derivative for optimal surge price of crowd logistics platforms with network effects by equations (A.3) and (A.10):

$$\frac{dP_{ma}^*(t)}{dt} = \frac{dP_{ba}^*(t)}{dt} = \frac{(1 + \beta \gamma - \mu_1 - \mu_2)h}{(1 - \mu_1)(1 - \gamma)} \tag{A.18}$$

Furthermore, we take the first-order derivative of pricing growth rate shown in equation (A.18) to direct-network effects $\mu_1$:

$$\frac{d\bar{P}^*(t)}{d\mu_1} = \frac{(\epsilon \gamma - \mu_2)h}{(1 - \mu_1)(1 - \gamma)} \tag{A.19}$$

Because $0 < \gamma < 1$, $0 < \mu_1 < 1$, so $(1 - \gamma) > 0$ and $(1 - \mu_1) > 0$. When $\epsilon \gamma > \mu_2$, $d\bar{P}_{ma}^*(t)/d\mu_1 > 0$.

Proof of Corollary 3. According to equations (A.15) and (A.16), the delivery supply function at $T^*$ could be deducted out:

$$S_{ma}^*(T^*) = \frac{ac_{ma}}{1 + \epsilon \gamma - \mu_1 - \mu_2} = S_{ba}^*(T_{ba}). \tag{A.20}$$

In order to prove Corollary 3, we calculate the first-order derivative for the supply of crowd logistics platforms to direct-network effects $\mu_1$ by equation (A.20) in both the supply and demand equilibrium and minimizing cumulative delay orders scenarios:

$$\frac{dS_{ma}^*(T^*)}{d\mu_1} = \frac{dS_{ba}^*(T_{ba})}{d\mu_1} = \frac{ac_{ma}}{(1 + \epsilon \gamma - \mu_1 - \mu_2)^2} > 0. \tag{A.21}$$

Proof of Proposition 6. In order to prove Proposition 6, we take the first-order derivative of pricing growth rate shown in equation (A.18) to cross-network effects $\mu_2$:

$$\frac{d\bar{P}^*(t)}{d\mu_2} = \frac{h}{(1 - \mu_1)(1 - \gamma)} < 0. \tag{A.22}$$

Because $0 < \gamma < 1$ and $0 < \mu_1 < 1$, so $d\bar{P}_{ma}^*(t)/d\mu_2 > 0$.

Proof of Corollary 4. In order to prove Corollary 4, we calculate the first-order derivative for crowd logistics supply at equilibrium moment $T^*$ and $T_{ba}$ with respect to the cross-network effects $\mu_2$ from equation (A.20):
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