Simulation of runoff and infiltration using iterative cross-coupled surface and subsurface flows

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ABSTRACT

An iterative cross-coupled surface and subsurface flows model is proposed to simulate the runoff generation and infiltration of a wide area hillside slope. Surface water on hillside slope was modeled as 2D shallow water equations and subsurface flow was modeled as 3D Richards’s equation. The infiltration capacity was estimated by Green-Ampt infiltration model. The water depth calculated by 2D shallow water equations was applied to 3D Richards’s equation as the water head boundary condition and the infiltration or exfiltration calculated by Richards’s equation was applied to the runoff simulation as source item. In this study, first, an approximation of shallow water equations that simplifies the equations of motion by considering only the main contributions was used. Then, the simplified runoff model was validated by the extensively used tilted impermeable V-catchment example with only simulating surface runoff flows. The iterative cross-coupled surface and subsurface flows model was verified by a simple 2D unsaturated-saturated model. The simulation results show that the iterative cross-coupled surface and subsurface flows model can reproduce rainfall generated runoff and infiltration. Finally, the runoff generation and infiltration of a natural mountain slope in Hokkaido were simulated, as the runoff caused several slope failures in this area during Typhoon 10 in 2016.

Keywords: surface flow, subsurface flow, infiltration, runoff simulation

1 INTRODUCTION

The Japanese archipelago is often struck by violent typhoons with extremely intense rainfalls causing risk situations in floods, debris flows and landslides (Fujisawa et al., 2010). According to the statistics of Dhual et al. (2008), from 1951 to 2005, there were 163 typhoon events hit the Japanese archipelago. The slope failures caused by typhoon-induced torrential rain continue to increase in Japan. The rainwater that infiltrates into unsaturated soil causes a decrease in suction and reduces the shear strength of the soil, thereby decreasing the stability of the slope. Many studies have pointed out that runoff water has a significant effect on the instability of slopes (e.g. Chan et al., 2018; Chiu et al., 2019), especially in the mountainous areas with catchment terrain. During the rainfall induced slope instability analysis, the impact of runoff is always neglected for the purpose of simplifying calculations, which causes the actual process of rainfall/runoff infiltration cannot be fully reflected especially under heavy rainfall conditions.

During a rainstorm or torrential rain, rainwater infiltration is a two-stage process, i.e., rainfall infiltration (rainfall derived infiltration) in the early stage of rainfall event and runoff infiltration (runoff derived infiltration, in this case, the infiltration is controlled by the surface water depth rather than the rainfall intensity) in the later stage of rainfall event. That is if rainfall intensity is smaller than the infiltration capacity of the ground surface, all rainwater will penetrate into the soil. If the rainfall intensity exceeds the soil infiltration capacity, the infiltration capacity of the ground surface is not enough to infiltrate all the rainwater into the soil. Accordingly, part of rainwater infiltrates into the ground and the rest generates runoff on the ground surface. This is a complicated process and brings challenges to its simulation.

Therefore, the objective of this study is to develop a rainfall-runoff model with the consideration of infiltration capacity of the ground surface to simulate runoff and infiltration under heavy rainfall conditions using iterative cross-coupled surface and subsurface flows. The rainfall-runoff model is governed by the coupling of the surface flow model (2D shallow water equations) and subsurface flow model (3D Richards’s equation). The Green-Ampt model is used for determining the infiltration capacity of the ground surface to divide the boundary conditions of subsurface flow analysis into flux boundary condition and water head boundary condition. Afterward, the surface flow
model is validated by the extensively used tilted impermeable V-catchment example with only simulating surface runoff flows, and the iterative cross-coupled surface and subsurface flows model is verified by a 2D unsaturated-saturated model. Finally, the runoff generation and infiltration of a natural mountain slope in Hokkaido were simulated and the results indicated that the iterative cross-coupled model of surface and subsurface flows proposed in this study can reflect the two-stage process of rainwater infiltration and it is applicable to simulate the runoff, infiltration and seepage in the wide mountain area.

2 GOVERNING EQUATIONS

The surface flow is simulated by the 2D shallow water equations and the subsurface flow is simulated by 3D Richards’s equation. The soil infiltration capacity is estimated by the Green-Ampt model.

2.1 Governing equations for surface flow

Surface flow is simulated by the 2D shallow water equations (Murillo et al., 2007):

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = R - I
\]

Equations of motion:

\[
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu)^2}{\partial x} + \frac{\partial (hv)}{\partial y} = -gh \frac{\partial h}{\partial x} - ghS_{fx} + D_x
\]

\[
\frac{\partial (hv)}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)^2}{\partial y} = -gh \frac{\partial h}{\partial y} - ghS_{fy} + D_y
\]

In which: $D_x = \frac{\partial}{\partial x} \left[ v_t \frac{\partial (uh)}{\partial x} + \frac{\partial}{\partial y} \left( v_t \frac{\partial (uh)}{\partial y} \right) \right]$

$D_y = \frac{\partial}{\partial y} \left[ v_t \frac{\partial (hv)}{\partial x} + \frac{\partial}{\partial y} \left( v_t \frac{\partial (hv)}{\partial y} \right) \right]$

where, $h$ is water depth (m), $u$ is water velocity in the $x$-direction (m/s), $v$ is water velocity in the $y$-direction (m/s), $R$ is rainfall intensity (m/s), $I$ is infiltration (m/s), $g$ is gravitational acceleration (m/s$^2$), $H$ is water surface elevation (m), $t$ is time (s), $D_x$ is advection term in the $x$-direction (m$^2$/s$^2$), $D_y$ is advection term in the $y$-direction (m$^2$/s$^2$), $v_t$ is eddy viscosity coefficient (m$^2$/s), $S_{fx}$, $S_{fy}$ is the friction slope in the $x$ and $y$ direction respectively, usually written in term of the Manning’s roughness coefficient $n_m$ (s/m$^{1/3}$):

\[
S_{fx} = \frac{n_m^2uvu^2+v^2}{h^{5/3}}, \quad S_{fy} = \frac{n_m^2uvu^2+v^2}{h^{5/3}}
\]

Di Giammarco et al. (1996) proposed an approximation of shallow water equations:

\[
S_{fx} + \frac{\partial h}{\partial x} = 0, \quad S_{fy} + \frac{\partial h}{\partial y} = 0
\]

Then, the expressions for the components of the velocity vector can be obtained as follows:

\[
u = -\frac{h^{2/3}}{n_m \sqrt{|s|}} \nabla_x (H), \quad \nu = -\frac{h^{2/3}}{n_m \sqrt{|s|}} \nabla_y (H)
\]

where, $S$ is the gradient of water surface. As the water depth gradient is much smaller than the slope gradient, it can be assumed that $S$ is equal to the slope gradient (Weill et al., 2009). The equation of continuity for surface flow is finally:

\[
\frac{\partial h}{\partial t} - \nabla \left( \frac{h^{5/3}}{n_m \sqrt{|s|}} \nabla (H) \right) = R - I
\]

2.2 Governing equation for subsurface flow

Subsurface flow is simulated by the 3D Richards’s equation (Richards, 1931):

\[
\nabla \left[ k_s k_r \nabla (H_p + \zeta) \right] + Q = [C_m + S_e S_c] \frac{\partial H_p}{\partial t}
\]

where, $C_m$ is specific moisture capacity (m$^{-1}$), $S_e$ is specific storage coefficient (m$^{-1}$), $S_c$ is the effective saturation, $H_p$ is pressure head (m), $\zeta$ is elevation head (m), $k_r$ is relative hydraulic conductivity, $k_s$ is saturated hydraulic conductivity (m/s), $Q$ is sink and/or source of water (s$^{-1}$). The nonlinear relationships with retention and permeability properties is defined according to van Genuchten (1980).

2.3 Soil infiltration capacity model

The Green-Ampt model (Green and Ampt, 1911) assumes that the soil infiltration capacity is governed by the soil properties and rainfall conditions. The soil infiltration capacity, $f_p$, can be approximated as follows (Fernández-Pato et al., 2016):

\[
f_p = k_s \left( 1 + \frac{w \Delta \theta}{F} \right)
\]

where, $f_p$ is infiltration capacity (m/s), $\Psi$ is the average suction head at the wetting front (m) and $\Delta \theta$ is the difference between the saturated volumetric water content $\theta_s$ and the initial volumetric water content $\theta_i$ (m$^3$/m$^3$). $F$ is the cumulative infiltration (m):

\[
F = \int_0^L l dt
\]

At the beginning of a rainfall event, rainfall intensity is weak and less than the infiltration capacity of the ground surface, all the rainwater will infiltrate into the soil. If the rainfall intensity exceeds the infiltration capacity of the ground surface, the infiltration, $I$, is governed by the infiltration capacity:

\[
I = \begin{cases} R, & \text{if } R < f_p \\ f_p, & \text{if } R \geq f_p \end{cases}
\]

3 COUPLED MODEL OF SURFACE FLOW AND SUBSURFACE FLOWS

In this study, an iterative cross-coupled surface and subsurface flows model with considering infiltration capacity is proposed to simulate runoff and infiltration under heavy rainfall conditions. Firstly, the Green-Ampt model is used for determining the infiltration capacity of the ground surface. Then, the infiltration capacity is used to divide the boundary conditions of subsurface flow...
analysis into flux boundary condition and water head boundary condition. In this case, if rainfall intensity \( (R) \) is smaller than the infiltration capacity \( (f_p) \), all rainwater will infiltrate into the ground and the flux boundary condition is applied to the subsurface flow analysis. If the rainfall intensity exceeds the infiltration capacity, part of rainwater infiltrates into the ground and the rest generates runoff on the ground surface. Meantime, the water depth calculated by the surface flow model is applied to the subsurface flow model as water head boundary. On the other hand, the exfiltration (positive value) and infiltration (negative value) calculated from the subsurface flow model is added to the surface flow model as a source item. Finally, the infiltrated rainwater causes the decrease of the infiltration capacity of the ground surface and the calculation of the next time step will be carried out. The illustration of the time-matching scheme in iterative cross-coupled surface and subsurface flows model is shown in Fig. 1.

4 VALIDATION OF THE ITERATIVE CROSS-COUPLED SURFACE AND SUBSURFACE FLOWS MODEL

4.1 Validation of surface flow model by the extensively used tilted impermeable V-catchment system

Di Giammarco et al. (1996) proposed a tilted impermeable V-catchment system to validate surface flow generated by rainfall event (Fig. 2). The approximation of surface flow is validated by the experimental system. The system is a V-shaped impermeable catchment with 20m depth. Two 1000 m²·800 m slopes are connected in the middle to a 1000 m²·20 m channel. Manning’s coefficient values are 0.015 s/m⁴ for the slope and 0.15 s/m⁴ for the channel. The slope of the two slopes is 0.05 along the y-direction and the slope of the channel is 0.02 along the x-direction. A rainfall rate of \( 3 \times 10^4 \) m/s is applied on the whole surface for 90 min with a total time of 180 min. Figure 3 shows the outflow rate at the exit of the channel simulated by COMSOL (COMSOL Multiphysics, 2018) compared with the measured data referred from Di Giammarco et al. (1996) and other simulation results referred from Tian and Liu (2011) and Kollet and Maxwell (2006). It suggests that the results of the approximation of surface flow simulated by COMSOL agree well with the experiment data and other simulation results meaning that the approximation of surface flow can be used to simulate surface runoff.

Fig. 1. Illustration of the time-matching scheme in iterative cross-coupled surface and subsurface flows model.

Fig. 2. Three-dimensional view of the tilted V-catchment area.

Fig. 3. Comparison of simulated results and measured data of the V-catchment case.

4.2 Validation of the iterative cross-coupled surface and subsurface flows model

The iterative cross-coupled surface and subsurface flows model is verified by the experimental system presented by Abdul and Gillham (1984). The
experimental system is composed of a 140 cm×120 cm×8 cm Plexiglas box filled with medium-fine sand as shown in Fig. 4. The surface slope is equal to 12° and the free water is drained off at the toe of the slope located at a height of 76 cm. The initial water table is located at the toe of the slope. The soil porosity is 0.34 and saturated hydraulic conductivity is 3.5×10⁻⁵ m/s. The van Genuchten parameters n=5.5 and a=2.3 m⁻¹. Manning’s coefficient value is 0.185 s/m³/². A rainfall rate of 1.2×10⁻¹ m/s is applied on the whole surface domain for 20 min with a total time of 25 min. Figure 5 shows the fluxes along the land surface (normalized by the rainfall flux imposed at the land surface and entering fluxes are negative by convention) calculated by COMSOL compared with the simulated results of Cast3M referred from Weill et al. (2009). The results imply that the model implemented in COMSOL is able to describe the three surface regimes (infiltration, runoff and exfiltration) along the land surface: in a small area at the top of the slope, all rainwater infiltrates into the soil (normalized flux equals -1); in the upper half of the slope, part of the rainwater infiltrates, and another rainwater flows in the form of runoff on the land surface (normalized flux between -1 and 0); at the lower half of the slope, groundwater exfiltrates to the land surface and flows out with the runoff from the upper part (normalized flux positive).

5 WIDE AREA RUNOFF AND INFILTRATION ANALYSIS

5.1 Outline of disasters

During August 29th-31st, 2016, Typhoon 10 (Lionrock) landed on Hokkaido, Japan, closely following three other typhoons. Near to Nissho Pass along the National Highway Route 274 in Hokkaido, Japan, Typhoon 10 triggered several intense landslides, embankment collapses and debris flows as shown in Fig. 6(a). According to the rainfall records obtained from the Automated Meteorological Data Acquisition System (AMeDAS) shown in Fig. 6(b), the observed rainfall peaked at 55 mm at 01:00 on August 31st, 2016 and the maximum observed cumulative rainfall as shown by the red line in Fig. 6(b) was 488 mm which far exceeded the average rainfall in the area.

5.2 Simulation of surface and subsurface flows for a natural mountain area

A three-dimensional model of the natural mountain area surrounded by the blue dashed box in Fig. 6(a) for surface and subsurface flows analysis is built as shown in Fig. 7. The model is composed of three parts: bedrock, topsoil and embankment. Soil properties are listed in Table 1. The initial groundwater level is set to -5.5 m based on the historical measured average value in the same period of

![Fig. 4. Abdul and Gillham system.](image)

![Fig. 5. Comparison of simulated results of the normalized flux along the land surface.](image)

![Fig. 6. (a) Locations of slope failure induced by Typhoon 10 along National Highway Route 274; (b) Rainfall recorded during Typhoon 10 at Nissho Pass.](image)
previous years. Manning’s coefficient value is 0.3 s/m$^{1/3}$ for the slope which is the recommended value of Japan Institute of Country-ology and Engineering (JICE) for mountain grassland. The simulation time is a total of 70 hours from 20:00 on August 28th, 2016 to 18:00 on August 31st, 2016.

Fig. 7. Three-dimensional numerical model of the natural mountain area at Nissho Pass.

Table 1. Soil properties used for the simulation of the natural mountain area.

| Parameters                  | Embankment | Topsoil | Bedrock  |
|-----------------------------|------------|---------|----------|
| Saturated hydraulic conductivity, $k_s$ (m/s) | 1.12×10^{-5} | 1.4×10^{-6} | 3.47×10^{-9} |
| Saturated water content, $\theta_s$ (m$^3$/m$^3$) | 0.36 | 0.63 | 0.48 |
| Residual water content, $\theta_r$ (m$^3$/m$^3$) | 0.035 | 0.190 | 0.008 |
| van Genuchten parameter, $\alpha$ (1/m) | 0.538 | 0.810 | 0.012 |
| van Genuchten parameter, $m$ | 0.468 | 0.437 | 0.246 |

The rainfall-runoff analysis is performed based on the proposed coupled iterative cross-coupled surface and subsurface flows model. The simulation results of runoff are compared with the results of two-dimensional (2D) impermeable plane flow simulation using Nays2D Flood Solver software (Nays2D Flood Solver, 2015). Figure 8 displays the distribution of water depth and the vector of flow velocity calculated by COMSOL and iRIC. The legends show the simulated water depth and the length of the black arrow indicates the magnitude of the flow velocity. It is recognized that both COMSOL and iRIC can describe the behavior of the surface runoff. Since both Location 1 and Location 2 are located at the exit of the valley where the water from the watershed on both sides is gathered, the water depth is much higher than other parts of the highway. It means that the runoff from upstream allows more water to infiltrate into the embankment causing the possibility of slope failure at the exit of the valley to be much greater than other locations along the highway.

A representative point (Point A in Fig. 7, located in the valley upstream of Location 1) is selected for discussing the relationship of the infiltration rate (the velocity of rainwater infiltrates into the soil, which is related to the rainfall intensity or water depth) on the hillside slope and the rainfall intensity. The results shown in Fig. 9(a) suggest that at the beginning of the rainfall event, all rainwater infiltrated into the ground and the infiltrated rainwater caused the decrease of infiltration capacity of the ground surface. Surface water depth located at the center of the road at Location 1 and Location 2 is shown in Fig. 9(b). It can be identified that
the runoff simulated by iRIC was generated when the rainfall event happened since iRIC does not consider infiltration, while the runoff simulated by COMSOL was generated from 22 hours after the rainfall event happened. Nearly at the same time, rainfall intensity exceeded the infiltration capacity of the ground surface in Fig. 9(a), meaning that runoff is generated when the rainfall intensity is larger than the infiltration capacity. Due to the non-consideration of infiltration, the water depth calculated by iRIC was larger than that calculated by COMSOL at both Location 1 and Location 2. From Fig. 9(a), it can be seen that in the early stage of rainfall event, as the ground surface was dry, suction was large at surface resulting in the large infiltration capacity of the ground surface. While the rainfall intensity was weak causing the infiltration rate was less than the infiltration capacity, all rainwater penetrated into the soil. With the continuation of rainfall, the infiltrated rainwater decreased the infiltration capacity of the ground surface. After the rainfall intensity exceeded the soil infiltration capacity, runoff was generated and the infiltration rate was controlled by the surface water depth. Accordingly, the iterative cross-coupled model of surface and subsurface flows proposed in this study can reflect the two-stage process of rainwater infiltration, i.e., rainfall infiltration in the early stage of rainfall event and runoff infiltration in the later stage of rainfall event as shown in Fig. 9(a) and it is also applicable to simulate the runoff, infiltration and seepage in the wide mountain area. This study provides a viable approach for simulating the runoff generation and infiltration of a natural mountain slope.

6 CONCLUSIONS

In this study, an iterative cross-coupled surface and subsurface flows model with considering infiltration capacity is proposed to simulate the runoff generation and infiltration of a wide area hillside slope. After the validation of the surface flow model and the iterative cross-coupled surface and subsurface flows model by two extensively used experimental system, the proposed iterative cross-coupled surface and subsurface flows model was used to simulate the runoff generation and infiltration of a natural mountain slope in Hokkaido. The findings from this study can be outlined as follows:

1. The iterative cross-coupled model of surface and subsurface flows proposed in this study can reflect the two-stage process of rainwater infiltration, i.e., rainfall infiltration in the early stage of rainfall event and runoff infiltration in the later stage of rainfall event, and it is applicable to simulate the runoff, infiltration and seepage in wide mountain area.

2. Runoff is the main cause of the embankment slope failures at the exit of the valley and its effects cannot be neglected. The runoff from upstream allows more water to infiltrate into the embankment and causes the possibility of slope failure at the exit of the valley to be much greater than other locations along the highway.

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