Supersymmetry in light of 1/fb of LHC data

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Abstract We update previous frequentist analyses of the CMSSM and NUHM1 parameter spaces to include the public results of searches for supersymmetric signals using ∼1/fb of LHC data recorded by ATLAS and CMS and ∼0.3/fb of data recorded by LHCb in addition to electroweak precision and B-physics observables. We also include the constraints imposed by the cosmological dark matter density and the XENON100 search for spin-independent dark matter scattering. The LHC data set includes ATLAS and CMS searches for jets + $\not{E}_T$ events and for the heavier MSSM Higgs bosons, and the upper limits on BR($B_t \rightarrow \mu^+\mu^-$) from LHCb and CMS. The absences of jets + $\not{E}_T$ signals in the LHC data favours heavier mass spectra than in our previous analyses of the CMSSM and NUHM1, which may be reconciled with $(g-2)_{\mu}$ if tan $\beta \sim 40$, a possibility that is, however, under pressure from heavy Higgs searches and the upper limits on BR($B_t \rightarrow \mu^+\mu^-$). As a result, the $p$-value for the CMSSM fit is reduced to $\sim 15(38)\%$, and that for the NUHM1 to $\sim 16(38)\%$, to be compared with $\sim 9(49)\%$ for the Standard Model limit of the CMSSM for the same set of observables (dropping $(g-2)_{\mu}$), ignoring the dark matter relic density. We discuss the sensitivities of the fits to the $(g-2)_{\mu}$ and BR($B_s \rightarrow \mu^+\mu^-$) constraints, contrasting fits with and without the $(g-2)_{\mu}$ constraint, and combining the theoretical and experimental errors for BR($B_s \rightarrow \mu^+\mu^-$) linearly or in quadrature. We present predictions for $m_\tilde{g}$, BR($B_s \rightarrow \mu^+\mu^-$), $M_H$ and $M_A$, and update predictions for spin-independent dark matter scattering, incorporating the uncertainty in the $\sigma$ term $\Sigma_{\pi N}$.

Finally, we present predictions based on our fits for the likely thresholds for sparticle pair production in $e^+e^-$ collisions in the CMSSM and NUHM1.

1 Introduction

In a series of papers, we and others have reported the results of global fits to pre-LHC [1–5] and LHC 2010 data [13, 14]2,3 in the frameworks of simplified variants of the minimal supersymmetric extension of the Standard Model (MSSM) [27, 28] with universal supersymmetry-breaking mass parameters at the GUT scale. We consider a class of models in which R-parity is conserved and the lightest supersymmetric particle (LSP), assumed to be the lightest neutralino $\tilde{\chi}^0_1$ [29, 30], provides the cosmological cold dark matter [31]. The specific models studied have included the constrained MSSM (CMSSM) [32–48], with parameters $m_{00}$, $m_{1/2}$ and $A_0$ denoting common scalar, fermionic and trilinear soft supersymmetry-breaking parameters at the GUT scale, and tan $\beta$ denoting the ratio of the two vacuum expectation values of the two Higgs fields. Other models

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1For a sampling of other pre-LHC analyses, see: [6–12].

2For more information and updates, please see http://cern.ch/mastercode/.

3For a sampling of other post-LHC analyses, see: [15–26].
studied include a model in which common supersymmetry-breaking contributions to the Higgs masses are allowed to be non-universal (the NUHM1) [49–51], a very constrained model in which trilinear and bilinear soft supersymmetry-breaking parameters are related (the VCMSSM) [52, 53], and minimal supergravity (mSUGRA) [52–58] in which the gravitino mass is required to be the same as the universal soft supersymmetry-breaking scalar mass before renormalization.

The impressive increase in the accumulated LHC luminosity combined with the rapid analyses of LHC data by the ATLAS [59, 60], CMS [61–63] and LHCB Collaborations [64] is putting increasing pressure on these and other supersymmetric models, in the continuing absence of any signal for supersymmetry. In this paper we update our previous frequentist fits [14] to include the analyses of ∼1/fb of LHC data made public in July and August 2011 at the EPS HEP and Lepton–Photon Conferences, termed here LHC1/fb, and also discuss the impact of the result on BR(B_s → μ⁺μ⁻) by the CDF Collaboration [65]. As in [14], we also incorporate the results of the direct search for dark matter scattering by the XENON100 Collaboration [66].

The approach we use has been documented in our previous papers [1–5, 13, 14], so we do not describe it in detail here, concentrating on relevant new aspects. We construct a global likelihood function that receives contributions from the standard portfolio of electroweak precision observables, as well as B-decay measurements such as BR(B → sγ) and BR(B → τντ). The contributions to the likelihood function from BR(B_s → μ⁺μ⁻), the XENON100 direct search for dark matter scattering and the LHC searches for supersymmetric signals using ∼1/fb of LHC data analyzed by the ATLAS and CMS Collaborations and ∼0.3/fb of data analyzed by the LHCb Collaboration. For our purposes, some of the most important constraints are provided by the ATLAS [59] and CMS [61] searches for jets + E_T events without leptons, as well as searches for the heavier MSSM Higgs bosons, H/A [60, 62]. Also important are the new upper limits on BR(B_s → μ⁺μ⁻) from the CMS [63], LHCb [64] and CDF Collaborations [65], which we incorporate in this paper as described below. In this paper we focus on the analysis of the effects in the CMSSM and the NUHM1. As discussed briefly below, the VCMSSM and mSUGRA models are further disfavoured by the LHC1/fb data.

The absences of signals in the jets + E_T searches disfavour the ranges of the model mass parameters (m_0, m_1/2) that had been favoured in our previous analyses of the CMSSM and NUHM1 [13, 14], and our current best fits have m_0 ∼ 150 to 450 GeV and m_1/2 ∼ 750 GeV. Reconciling these larger values of (m_0, m_1/2) with (g − 2)_μ favours values of tan β ∼ 40, though with a large uncertainty. The regions of parameter space with large tan β are constrained also by the new upper limits on BR(B_s → μ⁺μ⁻), as well as the LHC H/A searches. Using our standard implementation of the (g − 2)_μ constraint based on a Standard Model (SM) calculation [103], and combining the theoretical and experimental errors in BR(B → sγ) in quadrature, we find that the p-value for the CMSSM best-fit point is now ∼15%, and that for the NUHM1 is ∼16%. On the other hand, we show that if the (g − 2)_μ constraint is dropped much larger regions of the (m_0, m_1/2) and other parameter planes are allowed at the 68 and 95% CL, and these p-values increase to 38% in both models. In contrast, changing the treatment of

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4See footnote 2.
5Preliminary versions of these results were posted on http://cern.ch/mastercode/ on July 25th, 2011.
6See footnote 2.
7Information about this code is available from K.A. Olive: it contains important contributions from T. Falk, A. Ferstl, G. Ganis, A. Mustafayev, J. McDonald, K.A. Olive, P. Sandick, Y. Santoso and M. Srednicki.
8For other studies of recent data on BR(B_s → μ⁺μ⁻) in supersymmetric frameworks, see [85–87].
BR(b → sγ) by adding linearly the theoretical and experimental errors has relatively little impact on the fits and their p-values, increasing them both to 18%.

On the basis of these results, we present updated predictions for the gluino mass m_{\tilde{g}} and BR(B_s → μ^+μ^-), and the light and heavy Higgs masses M_h and M_A. We also present updated predictions for the spin-independent dark matter scattering cross section, σ_p^{SI}, stressing the importance of the uncertainty in the π-nucleon σ term Σ_{πN} [88–96].

In addition, we use our results to present likelihood functions for the thresholds for sparticle pair production in e^+e^- collisions. These results indicate that, within the CMSSM and NUHM1, the best-fit values for the sparticle thresholds lie above E_{CM} = 500 GeV. However, we emphasize that these results are derived in the context of specific models with specific universality soft supersymmetry-breaking masses at the GUT scale, and do not apply to other classes of supersymmetric models.

2 Implementations of the new LHC constraints

Jets + \not{E}_T searches The CMS and ATLAS Collaborations have both announced new exclusions in the (m_0, m_{1/2}) plane of the CMSSM based on searches for events with jets + \not{E}_T unaccompanied by charged leptons, assuming tan β = 10, A_0 = 0 and μ > 0. The updated CMS σ_T analysis is based on 1.1/fb of data [61], and the updated ATLAS 0-lepton analysis is based on 1.04/fb of data [59]. It is known that 0-lepton analyses are in general relatively insensitive to the tan β and A_0 parameters of the CMSSM, as has been confirmed specifically for the CMS σ_T analysis, and they are also insensitive to the amount of Higgs non-universality in the NUHM1. Therefore, we treat these analyses as constraints in the (m_0, m_{1/2}) planes of the CMSSM and NUHM1 that are independent of the other model parameters. The ATLAS [97] and CMS Collaborations [98] have also announced new exclusions for searches for jets + \not{E}_T events with one or more charged leptons with ∼1/fb of data, but these have in general less expected sensitivity, and are more dependent on the other model parameters, so we do not include them in our analysis. A similar remark applies to the new ATLAS limits on events with b jets + \not{E}_T unaccompanied by charged leptons using 0.83/fb of data [99] and on events with b jets + \not{E}_T + 1 lepton using 1.03/fb of data [100].

The CMS and ATLAS 0-lepton searches are more powerful in complementary regions of the (m_0, m_{1/2}) plane. Along each ray in this plane, we compare the expected CMS and ATLAS sensitivities, select the search that has the stronger expected 95% CL limit, and apply the constraint imposed by that search.10 We assign ∆χ^2 = 5.99, corresponding to 1.96 effective standard deviations, along the CMS and ATLAS 95% 0-lepton exclusion contours in the (m_0, m_{1/2}) plane. In the absence of more complete experimental information, we approximate the impact of these constraints by assuming that event numbers scale along rays in this plane \propto M^{-4} where M \equiv √{m_0^2 + m_{1/2}^2}, as described in [14]. We then use these numbers to calculate the effective numbers of standard deviations and corresponding values of ∆χ^2 at each point in the plane. This procedure has been validated by comparing the likelihood it yields with results obtained independently using the generic detector simulation code DELPHES, which has also been shown to reproduce quite accurately the likelihood function evaluated by the CMS Collaboration using their data.11

Searches for heavy MSSM Higgs bosons The CMS Collaboration has announced a new constraint on the heavy MSSM Higgs bosons from a search for the neutral bosons H/A → τ^+τ^- using 1.6/fb of data [62] and ATLAS has presented a similar constraint using 1.06/fb of data [60], the results being presented as 95% CL upper limits on the production cross section times τ^+τ^- branching ratio as a function of the common mass, M_{H/A}, for masses smaller than about 500 GeV. In our analysis we use the CMS constraint, which has the greater expected sensitivity. The CMS Collaboration has also announced a constraint on the decay chain t → H^+ → τ^+ν using 1.09/fb of data [101], but this yields a constraint in a generic (M_A, tan β) plane that is much weaker than the above searches for H/A, so we do not implement it in our analysis.

We assign ∆χ^2 = 3.84, corresponding to 1.96 effective standard deviations, to model parameter sets predicting an H/A signal at the 95% CL given by the CMS constraint, for each fixed value of M_A. Other model parameter sets are assigned values of ∆χ^2 according to the numbers of effective standard deviations corresponding to the numbers of events they predict. We have verified previously [14] that, in the parameter regions of the CMSSM and NUHM1 that are relevant for this analysis, these event numbers scale approximately as (tan β)^2 for any fixed value of M_{H/A}, and use this approximation here to estimate the numbers of events expected for other values of (M_A, tan β).

9It would facilitate the modelling of LHC constraints on SUSY if each Collaboration could combine the results from its different missing-energy searches, as is already done for Higgs searches.

10It would also facilitate the modelling of LHC constraints on supersymmetry if the results from different Collaborations were combined officially, as was done at LEP, is already done for BR(B_s → μ^+μ^-) searches, and is planned for Higgs searches.

11J. Marrouche, private communication. For a description of DELPHES, written by S. Ovyn and X. Rouby, see http://www.fynu.ucl.ac.be/users/s.ovyn/Delphes/index.html.
Fig. 1 (Color online) The \((m_0, m_{1/2})\) planes in the CMSSM (left) and the NUHM1 (right). In each plane, the best-fit point after incorporation of the LHC1/fb constraints is indicated by a filled green star, and the pre-LHC fit [5] by an open star. The \(\Delta \chi^2 = 2.30\) and 5.99 contours, commonly interpreted as the boundaries of the 68 and 95% CL regions, are indicated in red and blue, respectively, the solid lines including the LHC1/fb data and the dotted lines showing the pre-LHC fits.

Constraints on \(\text{BR}(B_s \to \mu^+\mu^-)\) Three new results on \(\text{BR}(B_s \to \mu^+\mu^-)\) have been announced recently. One is an excess of candidate \(B_s \to \mu^+\mu^-\) events reported by the CDF Collaboration [65], which corresponds to \(\text{BR}(B_s \to \mu^+\mu^-) = (1.8^{+1.1}_{-0.9}) \times 10^{-8}\) or \(\text{BR}(B_s \to \mu^+\mu^-) < 4.0 \times 10^{-8}\) at the 95% CL. The other two new results are upper limits from the CMS Collaboration using 1.14/fb of data [63]: \(\text{BR}(B_s \to \mu^+\mu^-) < 1.9 \times 10^{-8}\) at the 95% CL, and from the LHCb Collaboration using 0.34/pb of data [64]: \(\text{BR}(B_s \to \mu^+\mu^-) < 1.5 \times 10^{-8}\) at the 95% CL.

These three results are reasonably compatible, though there is some tension between the CDF and CMS/LHCb results. The two latter collaborations have released an official combination of their results [102], which yields \(\text{BR}(B_s \to \mu^+\mu^-) < 1.08 \times 10^{-8}\) at the 95% CL. In our implementation of the BR constraint, we use \(\Delta \chi^2\) corresponding to the full likelihood function provided by this combination, which has a global minimum close to the SM prediction. We also comment on the changes in our results that would follow from an (unofficial) combination with the CDF result [65], which would yield a \(\Delta \chi^2\) function with a minimum at \(\text{BR}(B_s \to \mu^+\mu^-)\) at about twice the SM value.

Constraints on dark matter scattering We incorporate the upper limit on the spin-independent dark matter scattering cross section \(\sigma_p^{SI}\) provided by the XENON100 Collaboration [66] in a similar manner to [14]. In that paper we discussed extensively the uncertainty in the spin-independent scattering matrix element induced by the relatively ill-determined value of the \(\pi\)-nucleon \(\sigma\) term, \(\Sigma_{\pi N}\). In this paper we use \(\Sigma_{\pi N} = 50 \pm 14\) MeV, and neglect other uncertainties, e.g., in modelling the dark matter distribution. We also do not consider here other experiments reporting signatures that would require \(\sigma_p^{SI}\) above the XENON100 limit. Nor do we consider limits on spin-dependent dark matter scattering and astrophysical signatures of dark matter annihilations, which currently do not constrain the CMSSM and NUHM1 [14].

3 Results

The \((m_0, m_{1/2})\) planes in the CMSSM and NUHM1 Figure 1 displays contours with \(\Delta \chi^2 = 2.30\) (red) and 5.99 (blue) relative to the minimum values of \(\chi^2\) at the best-fit points (denoted by green stars) in the \((m_0, m_{1/2})\) planes for the CMSSM and NUHM1.\(^{12}\) Such contours are commonly interpreted as 68 and 95% CL contours. The solid lines are the contours after incorporation of the LHC1/fb results, and the dotted lines are the CL contours obtained from an analysis of the pre-LHC and pre-XENON100 data [5]. Consequently, the differences in the contours show the impact of the full available LHC \(\sim 1/fb\) data set. The crinkles in these contours give an indication of the sampling uncertainties in our analysis.

We see that the new best-fit points with \((m_0, m_{1/2}) = (450, 780)\) GeV in the CMSSM and \((150, 730)\) GeV in the NUHM1 (denoted by solid green stars) lie well within the previous 95% CL region. On the other hand, the pre-LHC best-fit points with \((m_0, m_{1/2}) = (90, 360)\) GeV in the CMSSM and \((110, 340)\) GeV in the NUHM1 (denoted by open stars), lie far outside the regions allowed by the LHC1/fb data. Thus, we see that there is now significant tension between the LHC1/fb and pre-LHC data sets. The full

\(^{12}\) The NUHM1 analysis includes both the dedicated NUHM1 sample, which is efficient for smaller values of \((m_0, m_{1/2})\), and the basic set of CMSSM points, which provide extra NUHM1 sampling at larger values of \((m_0, m_{1/2})\).
The likelihood in the focus-point region at large $g_{\mu}^{-1}$ and the reli-
cance constraint, with large-$\beta$ in the heavy-Higgs rapid-annihilation funnel at large $\tan \beta$. The fact that much larger regions in the $(m_{0}, m_{1/2})$ are now allowed at the 95% CL, as compared to the pre-LHC fit, indicates that the tension between $(g_{\mu}^{-2})_{\nu}$, favouring relatively low SUSY scales, and the direct search limits, favouring larger SUSY scales, has significantly reduced the sen-
tivity of the fits within the CMSSM and the NUHM1 for constraining the SUSY parameters. Since the $\chi^2$ values of the best-fit points are significantly higher (see Table 1), and consequently the $\chi^2$ distribution towards higher values of $m_0$ and $m_{1/2}$ is much flatter than in the pre-LHC case, the precise location of the 68% and 95% CL contours is less precisely determined than before. The narrower range of $m_0$ allowed in the NUHM1 at the 68% CL, compared to the CMSSM, is due to the appearance of a ‘pit’ with a lower absolute value of $\chi^2$ that is attainable in this model thanks to its flexibility in reconciling the $(g_{\mu}^{-2})_{\nu}$ with the $\tilde{\chi}_1^0$ LSP and other constraints by deviating from Higgs mass univer-
sality. As in our previous analysis including 2010 LHC and XENON100 data [14], we find no distinct enhancement of the likelihood in the focus-point region at large $m_0$.

| Model                  | Minimum $\chi^2$/d.o.f. | Probability | $m_{1/2}$ (GeV) | $m_{0}$ (GeV) | $A_0$ (GeV) | $\tan \beta$ | $M_{h}$ (GeV) (no LEP) |
|------------------------|--------------------------|-------------|----------------|--------------|-------------|---------------|------------------------|
| CMSSM pre-LHC          | 21.5/20                  | 37%         | $366^{+180}_{-100}$ | $90^{+220}_{-50}$ | $-400^{+730}_{-970}$ | $15^{+15}_{-9}$ | $111.5^{+3.5}_{-1.2}$ |
| CMSSM LHC1/fb          | 28.8/22                  | 15%         | $780^{+150}_{-270}$ | $450^{+170}_{-320}$ | $-1100^{+3070}_{-3680}$ | $41^{+16}_{-32}$ | $119.1^{+3.4}_{-2.9}$ |
| Linear $\Delta$ BR($b \rightarrow s\gamma$) | 28.0/22                  | 18%         | $720^{+1170}_{-230}$ | $420^{+1270}_{-270}$ | $-1100^{+2180}_{-2750}$ | $39^{+18}_{-22}$ | $118.6^{+3.9}_{-1.9}$ |
| $(g_{\mu}^{-2})_{\nu}$ neglected | 21.3/20                  | 38%         | $2000^{+7}_{-1}$ | $1050^{+}_{-1}$ | $430^{+}_{-}$ | $22^{+}_{-}$ | $124.8^{+3.4}_{-10.5}$ |
| Both                   | 20.5/20                  | 43%         | $1880^{+}_{-7}$ | $1340^{+}_{-}$ | $1890^{+}_{-}$ | $47^{+}_{-}$ | $126.1^{+2.1}_{-6.3}$ |
| NUHM1 pre-LHC          | 20.8/18                  | 29%         | $346^{+280}_{-110}$ | $110^{+160}_{-30}$ | $520^{+730}_{-1730}$ | $13^{+2}_{-6}$ | $118.9^{+1.1}_{-11.4}$ |
| NUHM1 LHC1/fb          | 27.3/21                  | 16%         | $730^{+630}_{-170}$ | $150^{+450}_{-250}$ | $-910^{+2990}_{-1170}$ | $41^{+16}_{-24}$ | $118.8^{+1.1}_{-11}$ |
| Linear $\Delta$ BR($b \rightarrow s\gamma$) | 26.6/21                  | 18%         | $730^{+220}_{-90}$ | $150^{+280}_{-20}$ | $-910^{+2990}_{-1060}$ | $41^{+16}_{-24}$ | $118.8^{+3.1}_{-1.3}$ |
| $(g_{\mu}^{-2})_{\nu}$ neglected | 20.3/19                  | 38%         | $2020^{+}_{-3}$ | $1410^{+}_{-}$ | $2580^{+}_{-}$ | $48^{+}_{-}$ | $126.6^{+0.7}_{-1.9}$ |
| Both                   | 19.5/19                  | 43%         | $2020^{+}_{-3}$ | $1410^{+}_{-}$ | $2580^{+}_{-}$ | $48^{+}_{-}$ | $126.6^{+0.7}_{-1.9}$ |

The absolute values of $\chi^2$ at the best-fit points for the pre-LHC case and for the LHC1/fb data set using our standard imple-
mentations of the $(g_{\mu}^{-2})_{\nu}$ and BR($b \rightarrow s\gamma$) con-
straints are given in Table 1. Our updated analysis of the pre-LHC data set yields $\chi^2$/d.o.f. = 21.5/20(20.8/18) in the CMSSM and NUHM1, respectively, where the numbers of d.o.f. are those relevant at the best-fit points, corresponding to $p$-values of 37% and 29%. On the other hand, using the LHC1/fb data set, we find that the minimum values of $\chi^2$ are significantly larger than the numbers of effective degrees of freedom in the fits, which are also shown in Table 1: $\chi^2$/d.o.f. = 28.8/22(27.3/21) for the CMSSM and NUHM1, respectively.14 Correspondingly, the best fits have significantly reduced probability values, $\sim 15\%$ in the CMSSM and $\sim 16\%$ in the NUHM1.15

In the Table, the row labelled LHC1/fb includes three addi-
tional observables: the $(m_0, m_{1/2})$ constraints, BR($B_s \rightarrow \mu^+ \mu^-$), and the XENON100 constraint. In the CMSSM, be-
cause the best-fit point now lies at significantly larger $m_{1/2}$ than in the pre-LHC analysis, the constraint due to $M_{h}$ is no longer applicable. Thus we see a net increase by only

13The parameters of the pre-LHC best-fit CMSSM and NUHM1 points
given in Table 1 differ by up to 1 $\sigma$ from those given in [14]. These
differences are caused primarily by changes in the data input.

14For technical reasons, the $\Gamma_Z$ constraint was not included in our previous fits, leading to changes of one unit in the numbers of effective degrees of freedom in the fits.

15The $p$-values for the VCMSSM and mSUGRA are somewhat smaller, as was found previously [14]: $\chi^2$/d.o.f. = 31.2/23(32.5/23), respectively, corresponding to $p$-values of 12% and 9%. We do not discuss these models further, except to comment that the light-Higgs funnel region found previously in these models is now excluded by ATLAS data [59], in particular.
2 in the number of degrees of freedom. When \((g - 2)_\mu\) is dropped, \(m_{1/2}\) is pushed to still higher values and the \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) constraint also becomes ineffective, and hence the number of degrees of freedom drops by 2. In the NUHM1, there is an extra parameter and hence one degree of freedom is lost. However, in this case, \(M_H\) was ineffective even in the pre-LHC analysis, and hence there are 2 degrees of freedom less than in the CMSSM. In the LHC1/\(b\) analysis of the NUHM1, we add the three new observables as in the CMSSM, and dropping \((g - 2)_\mu\) again moves the best-fit point to large \(m_{1/2}\), and the \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) is lost as well.

If we combine the \(\text{BR}(b \rightarrow s\gamma)\) errors linearly instead of quadratically, the values of \(\chi^2\) decrease by 0.8(0.7) in the CMSSM and NUHM1, respectively, and the \(p\)-values increase modestly to 18% in both cases. On the other hand, if we drop \((g - 2)_\mu\) constraint, we find \(\chi^2/\text{d.o.f.} = 21.3/20(20.3/19)\), respectively, corresponding to \(p\)-values of 38% in both cases. Thus, the qualities of our best fits are not very sensitive to the treatment of \(\text{BR}(b \rightarrow s\gamma)\), but are much more sensitive to the inclusion of \((g - 2)_\mu\), as we discuss later in more detail.

The degrees of non-universality, \(r \equiv m_H^2/m_0^2\), for the best-fit NUHM1 points in Table 1 are as follows: \(r = -57\) (pre-LHC), \(r = -54\) (LHC1/\(b\)), \(r = -54\) (linear \(\text{BR}(b \rightarrow s\gamma)\) error combination), \(r = -0.39\) (dropping \((g - 2)_\mu\) constraint), and \(r = -0.39\) (including both variants). Since \(r\) is quite poorly constrained, we do not quote its uncertainties.

We note that the best-fit values of \(M_H\) are significantly higher in the CMSSM and NUHM1 fits dropping \((g - 2)_\mu\), with their large values of \((m_0, m_{1/2})\), than in the fits that include the \((g - 2)_\mu\) constraint. An LHC measurement of \(M_H\) could provide a diagnostic discriminating between models with light and heavy spectra of third-generation squarks, and help forecast the likelihood of discovering these sparticles in future LHC runs. Within the CMSSM and NUHM1, measuring \(M_H\) could advise us whether to take \((g - 2)_\mu\) at face value or, conversely, hint towards extensions of these models.

Table 2 displays the contributions to the total \(\chi^2\) of each of the observables at the best-fit points in the CMSSM and NUHM1, revealing where this tension originates. We note, as already mentioned above, that there is an important contribution to the \(\chi^2\) function coming from the \((g - 2)_\mu\) constraint that varies significantly across the \((m_0, m_{1/2})\) planes of the CMSSM and NUHM1, as well as the LHC1/\(b\) constraints. We return later to a more complete discussion of the tension between these constraints, and also of the treatment of the \(\text{BR}(b \rightarrow s\gamma)\) constraint, which also exhibits important variations.

Table 2 also displays a similar \(\chi^2\) breakdown evaluated assuming the central values of \(m_t\), \(\Delta m_{\text{had}}^{(5)}(M_Z)\) and \(M_Z\) for a CMSSM point at very large \((m_0, m_{1/2}) = (15, 15)\) TeV, \(A_0 = 100\) GeV and \(\tan\beta = 10\), near the limit in which its predictions coincide with those of the SM for the same value of \(M_Z\). This is very similar to the global best-fit value of \(M_H\) in the SM obtained incorporating the limits from the direct Higgs searches at LEP, the Tevatron and the LHC [131]. Using the SM limit of the CMSSM within the MasterCode framework ensures that this evaluation of \(\chi^2\) in the “SM” can be compared directly with those at the best-fit points in the CMSSM and NUHM1. In the “SM” case we discard the constraints imposed by the cosmological dark matter density and XENON100, since there is no way to explain dark matter with the SM. We also discard the LHC missing-energy and \(H/A\) constraints, but all other constraints are kept. Consequently, we do list the contribution to \(\chi^2\) from \((g - 2)_\mu\). If this is included (omitted), the global \(\chi^2\) for the “SM” is 32.7 (21.5). The number of degrees of freedom for the “SM” is consequently 23 (22) and the \(p\)-value is 9% (49%). We observe that the \(p\)-value for the CMSSM is rather larger than that for the “SM” if \((g - 2)_\mu\) is included, though similar if \((g - 2)_\mu\) is not included in the “SM” and MSSM analyses.

We also note that one of the big contributors to the global \(\chi^2\) functions for all the models is \(A_b (b)\), which contributes \(\Delta \chi^2 \sim 7\) to each of the fits, suppressing all their \(p\)-values, but does not vary strongly in the CMSSM and NUHM1 parameter regions of interest. Prior to the LHC results, the CMSSM yielded a significant improvement to \(\chi^2\), with the dominant contribution coming from \((g - 2)_\mu\) (\(\Delta \chi^2 = -10.8\)). Other contributing observables were \(M_W\) (\(\Delta \chi^2 = -1.6\)) and \(A_t (SLD)\) (\(\Delta \chi^2 = -1.3\)), though \(A_b (b)\) was somewhat worse in the CMSSM (\(\Delta \chi^2 = 2.1\)). The post-LHC comparison with the “SM” is shown in Table 2. Looking at the entries for the electroweak precision observables, the only significant change is now that for \((g - 2)_\mu\) (\(\Delta \chi^2 = -6.8\)), with all other observables showing changes \(|\Delta \chi^2| < 1\). Thus, when \((g - 2)_\mu\) is dropped as a constraint, the resulting \(\chi^2\) for the best-fit points in the CMSSM and “SM” are very similar. In the case of the post-LHC NUHM1, we also see a large drop in \(\chi^2\) relative to the “SM” due to \((g - 2)_\mu\) (\(\Delta \chi^2 = -9.4\)) and again all others give \(|\Delta \chi^2| < 1\).

We also note that in both the CMSSM and NUHM1 the best fits receive significant contributions from the LHC \(\not{E_T}\) + jets searches.

To illustrate further the impact of LHC1/\(b\) experimental constraints relative to pre-LHC preferred regions, we display in Fig. 2 colour-coded contours of approximate\(^{16}\) \(p\)-values from our global fits for the CMSSM and NUHM1. Care is taken to count the effective number of degrees of freedom at each point, considering all constraints that contribute non-trivially to the \(\chi^2\) functions. Thus, for exam-

\(^{16}\)Strictly speaking, transforming a \(\chi^2\) value to a \(p\)-value, using a specified number of degrees of freedom, is valid for Gaussian-behaved constraints. Because some of the experimental limits are one-sided and modelled in a non-Gaussian manner as previously described, the \(p\)-values reported here can therefore only be considered approximate.
Table 2 List of experimental constraints used in this work, including experimental and (where applicable) theoretical errors: supersymmetric theory uncertainties in the interpretations of one-sided experimental limits are indicated by [...] Also shown are the contributions that these constraints make to the total $\chi^2$ functions at the best-fit points in the CMSSM and NUHM1, respectively, and (for comparison) in the SM limit of the CMSSM (called “SM”) including (excluding) $(g-2)_\mu$. The total values of $\chi^2$, the numbers of degrees of freedom and the $p$-values at these points are shown in the two bottom rows.

| Observable | Source Th./Ex. | Constraint | $\Delta \chi^2$ (CMSSM) | $\Delta \chi^2$ (NUHM1) | $\Delta \chi^2$ (“SM”) |
|------------|----------------|------------|--------------------------|--------------------------|--------------------------|
| $m_\tau$ [GeV] | [104] | $173.2 \pm 0.90$ | $0.05$ | $0.06$ | $-$ |
| $\Delta a_\mu(M_Z)$ | [103] | $0.02749 \pm 0.00010$ | $0.009$ | $0.004$ | $-$ |
| $M_Z$ [GeV] | [105] | $91.1875 \pm 0.0021$ | $2.7 \times 10^{-5}$ | $0.26$ | $-$ |
| $I_Z$ [GeV] | [67, 68] / [105] | $2.4952 \pm 0.0023 \pm 0.001_{SUSY}$ | $0.078$ | $0.047$ | $0.14$ |
| $\sigma^0_{had}$ [nb] | [67, 68] / [105] | $41.540 \pm 0.037$ | $2.50$ | $2.57$ | $2.54$ |
| $R_l$ | [67, 68] / [105] | $20.767 \pm 0.025$ | $1.05$ | $1.08$ | $1.08$ |
| $A_{\text{fb}}(c)$ | [67, 68] / [105] | $0.01714 \pm 0.00095$ | $0.72$ | $0.69$ | $0.81$ |
| $A_{c}(P_{\tau})$ | [67, 68] / [105] | $0.1465 \pm 0.0032$ | $0.11$ | $0.13$ | $0.07$ |
| $R_b$ | [67, 68] / [105] | $0.21629 \pm 0.00066$ | $0.26$ | $0.29$ | $0.27$ |
| $R_c$ | [67, 68] / [105] | $0.1721 \pm 0.0030$ | $0.002$ | $0.002$ | $0.002$ |
| $\Delta \chi^2(\mu)$ | [67, 68] / [105] | $0.0992 \pm 0.0016$ | $7.17$ | $7.37$ | $6.63$ |
| $\Delta \chi^2(c)$ | [67, 68] / [105] | $0.0707 \pm 0.0035$ | $0.86$ | $0.88$ | $0.80$ |
| $\Delta \chi^2(b)$ | [67, 68] / [105] | $0.923 \pm 0.020$ | $0.36$ | $0.36$ | $0.35$ |
| $\Delta \chi^2(\tau)$ | [67, 68] / [105] | $0.670 \pm 0.027$ | $0.005$ | $0.005$ | $0.005$ |
| $\Delta \chi^2(SLD)$ | [67, 68] / [105] | $0.1513 \pm 0.0021$ | $3.16$ | $3.03$ | $3.51$ |
| $\sin^2\theta^{\mu}_W(Q_{\text{fb}})$ | [67, 68] / [105] | $0.2324 \pm 0.0012$ | $0.63$ | $0.64$ | $0.59$ |
| $m_W$ [GeV] | [67, 68] / [105] | $80.399 \pm 0.023 \pm 0.010_{SUSY}$ | $1.77$ | $1.39$ | $2.08$ |
| $\Delta \chi^2(\mu)$ | [118–124] / [103, 125, 126] | $(30.2 \pm 8.8 \pm 2.0_{SUSY}) \times 10^{-10}$ | $4.35$ | $1.82$ | $11.19$ (N/A) |
| $M_D$ [GeV] | [70–73] / [127, 128] | $> 114.4[\pm 1.5_{SUSY}]$ | $0.0$ | $0.0$ | $0.0$ |
| $\Delta \chi^2(SUSY)$ | [106–110] / [111] | $1.117 \pm 0.076_{\text{exp}} \pm 0.082_{3\sigma SM} \pm 0.050_{SUSY}$ | $1.83$ | $1.09$ | $0.94$ |

Example, we drop the contribution of the LHC$_{1/fb}$ missing-energy constraints where they contribute $\Delta \chi^2 < 0.1$, causing the visible changes in shading along drooping diagonal lines in both panels of Fig. 2. (This cut is applied only to the LHC$_{1/fb}$ missing-energy constraint.) Substantial non-zero $p$-values are observed to extend to high $m_0$ and $m_{1/2}$, in both pre-
The \((m_0, m_{1/2})\) planes in the CMSSM (left) and the NUHM1 (right), for the pre-LHC data set (upper) and LHC1/fb data set (lower). In each plane, different regions are colour-coded according to the \(p\)-values found in our global fits. We note that in the LHC1/fb analysis the regions with \(p > 0.05\) extend up to \(m_{1/2} \sim 2000\) GeV in each case.

Fig. 2

(upper panels) and post-LHC1/fb (lower panels), and both the CMSSM (left panels) and NUHM1 (right panels) models. As also seen earlier, the primary effect of the LHC1/fb searches for jets + \(E_T\) is most evident for \(m_{1/2}\), preferring higher values than that predicted by the pre-LHC global fits. At even higher \((m_0, m_{1/2})\), beyond the drooping diagonal line, slight increases in approximate \(p\)-values appear when comparing the pre-LHC results with the post-LHC1/fb and post XENON100 results. This is due partly to the experimental constraint on \(\text{BR}(B_s \to \mu^+\mu^-)\), which is nearing the SM prediction. Regions of the CMSSM and NUHM1 parameter spaces approaching the high-mass decoupling limit receive a non-zero contribution from \(\text{BR}(B_s \to \mu^+\mu^-)\) in the post-LHC1/fb era, resulting in \(\Delta \chi^2 < 1\), which actually improves the overall \(\chi^2\) per effective degree of freedom. Additionally, the XENON100 constraint slightly prefers high mass scales, so as to accommodate the small “excess” in events, also resulting in slightly better values of \(\chi^2\) per effective degree of freedom.

Complementing the comparison of the \(p\)-values of the “SM”, the CMSSM and the NUHM1, we now use the standard F-test to test the utility of adding one or several parameters to a model fit of data. Given a set of data comprising \(N\) observables and a model using \(m\) parameters, one may compute \(\chi^2(m)\) for \(N - m\) degrees of freedom as done above. In general, adding \(r\) parameters produces a reduced value of \(\chi^2(m + r)\), and the difference between these two \(\chi^2\) distributions is itself a \(\chi^2\) distribution for \(r\) degrees of freedom. The F-statistic is defined by

\[
F_X \equiv \frac{\chi^2(m) - \chi^2(m + r)}{\chi^2(m + r)/(N - m - r)} > 0.
\]  

The probability that introducing the \(r\) new parameters are warranted is found by integrating the F-distribution, \(p_F(f, r, N - m - r)\), from \(f = F_X\) to \(\infty\). We use the F-test to illustrate the relative preference for various models.

In our case, for the “SM” we have \(\chi^2 = 32.7(21.5)\) for 23 (22) degrees of freedom if \((g - 2)\mu\) is included in (omitted from) the fit. Using the CMSSM value of \(\chi^2 = 28.8\) for 22 degrees of freedom, we find \(F_X = 2.98\), and the probability that switching to the CMSSM is warranted is \(p_F = 90\%\). Correspondingly, using the NUHM1 value of \(\chi^2 = 27.3\) for 21 degrees of freedom, we find \(F_X = 4.15\), and the probability that switching to the NUHM1 is war-
ranted is $p_F = 97\%$. We can also compare the improvement in $\chi^2$ gained by moving from the CMSSM to the NUHM1. In this case the probability that the extra parameter needed to define the NUHM1 model is preferred over the CMSSM case is 71\%. Figure 3 uses shading to display values of $p_F$ in the $(m_0, m_{1/2})$ planes of the CMSSM and NUHM1. On the basis of these plots, we find that present data may warrant switching from the “SM” to the CMSSM or NUHM1 for values of $m_{1/2}$ out to $\sim 1500$ GeV.\(^{17}\) Beyond this range of $m_{1/2}$, the motivations for these models would be significantly reduced. We also note that the F-test indicates that there would be no advantage in switching to the CMSSM or the NUHM1 if $(g-2)_\mu$ were to be dropped.

An important part of the motivation for low-scale supersymmetry is to alleviate the fine-tuning of the Higgs mass parameter in the Standard Model. However, the problem of fine-tuning returns if the supersymmetric mass scales become large \cite{132,133}. The required amount of fine-tuning is increased significantly in our LHC\(_{1/\text{fb}}\) fits compared to our pre-LHC fits, principally because of the increases in the best-fit values of $m_0$ and $m_{1/2}$. Specifically, in the CMSSM our best pre-LHC fit required fine-tuning by a factor $\sim 100$, whereas our best LHC\(_{1/\text{fb}}\) fit requires fine-tuning by a factor $\sim 300$. The corresponding numbers for the NUHM1 are $\sim 250$ pre-LHC and $\sim 600$ with the LHC\(_{1/\text{fb}}\) data.

Uncertainties in the analysis In assessing the compatibility of the CMSSM and the NUHM1 with the experimental data it is important also to examine carefully the most important systematic uncertainties in the constraints that drive the fit in this global likelihood analysis. We saw in the Table 2 that the most important contributions to the global $\chi^2$ functions at the best-fit points in these models originate from the $(g-2)_\mu$ and LHC\(_{1/\text{fb}}\) constraints. Exploring this further, Fig. 4 displays the CMSSM and NUHM1 $(m_0, m_{1/2})$ planes again, exhibiting the contribution to $\chi^2$ from the $(g-2)_\mu$ constraint, evaluated for the model parameter sets that minimize the total $\chi^2$ at each point in the plane. Their shapes are different in the CMSSM and NUHM1, reflecting the existence of the previously mentioned ‘pit’ in the latter model where $\chi^2$ may be reduced by some judicious choice of the degree of Higgs non-universality. Prima facie, there is tension between the $(g-2)_\mu$ constraint, which prefers small values of $(m_0, m_{1/2})$, and the LHC\(_{1/\text{fb}}\) constraints, which prefer larger values of $(m_0, m_{1/2})$. This tension is alleviated for larger values of $\tan \beta$, which is why post-2010-LHC \cite{14} and post-LHC\(_{1/\text{fb}}\) fits have favoured larger values of $\tan \beta$ than pre-LHC fits \cite{5}, albeit with large uncertainties.

Figure 5 again displays the $(m_0, m_{1/2})$ planes in the CMSSM (left) and the NUHM1 (right), this time showing as solid lines the 68% and 95% CL contours obtained by dropping the $(g-2)_\mu$ constraint, the contours obtained applying the $(g-2)_\mu$ constraint as in Fig. 1 being shown here for comparison as dotted lines. We see that, in the absence of the $(g-2)_\mu$ constraint, the outer parts of the 68% CL contours are expanded outwards, close to the 95% CL contours that are themselves close to the boundary set by the $\Omega_\chi h^2$ constraint.\(^{18}\) Within the overall range allowed by this constraint, the most important constraint is that provided by the LHC data. We note that the global likelihood functions in

\(^{17}\)We note, however, that these results may be too favourable to the CMSSM or NUHM1, since they do not include the impacts of the many lower-sensitivity constraints from CMS and ATLAS. This problem could be avoided if the Collaborations publish official combinations of the sensitivities of their searches.

\(^{18}\)This constraint is not sacrosanct, but could be relaxed by postulating some amount of R-violation, or some other source of dark matter, or by modifying the expansion history of the Universe, e.g., by altering the expansion rate during freeze-out, or by postulating some subsequent injection of entropy.
Fig. 4 The \((m_0, m_{1/2})\) planes in the CMSSM (left) and the NUHM1 (right), with shading displaying the contribution to the global \(\chi^2\) function from \((g - 2)_\mu\) as calculated using low-energy \(e^+e^-\) data to evaluate the SM contribution. These contributions are evaluated for the parameter sets that minimize \(\chi^2\) at each point in the planes.

Fig. 5 The \((m_0, m_{1/2})\) planes in the CMSSM (left) and the NUHM1 (right). Here we show as solid lines the 68\% and 95\% CL contours obtained by dropping the \((g - 2)_\mu\) constraint. The contours obtained applying the \((g - 2)_\mu\) constraint as in Fig. 1 are shown here for comparison as dotted lines.

The two uncertainties are of similar size, and the issue of how they are combined is more severe than for other observables. In our default implementation of \(\text{BR}(b \rightarrow s\gamma)\) we add the quoted errors in quadrature. However, it might be argued that the theoretical error should be regarded as an overall range of uncertainty that cannot be treated as an effective Gaussian error to be added in quadrature to the experimental error. Therefore, we consider as an alternative implementation of \(\text{BR}(b \rightarrow s\gamma)\) the possibility of adding linearly the theoretical and experimental errors. As seen in Fig. 6, the region of the CMSSM \((m_0, m_{1/2})\) plane that is favoured at the 68\% CL contracts significantly if the errors in \(\text{BR}(b \rightarrow s\gamma)\) are added linearly, whereas there is little effect in the NUHM1.

This effect arises because the treatment of the \(\text{BR}(b \rightarrow s\gamma)\) errors does not change the global \(\chi^2\) function at large \((m_0, m_{1/2})\), where its value approaches that of the “SM”, since the experimental measurement of \(\text{BR}(b \rightarrow s\gamma)\) is in

The CMSSM and NUHM1 are very flat, and that the best-fit points found dropping the \((g - 2)_\mu\) constraints are correspondingly quite uncertain. However, it is clear that the amounts of fine-tuning at \((g - 2)_\mu\)-less best-fit points are much higher than if \((g - 2)_\mu\) is included.

We also comment on the treatment of \(\text{BR}(b \rightarrow s\gamma)\), where different points of view have been taken concerning the combination of the theoretical and experimental errors.\(^{19}\)

\(^{19}\)As in our previous analyses, we do not take into account constraints from exclusive \(b \rightarrow s\gamma\) transitions. In particular, we do not impose any constraint on the SUSY parameter space from the isospin asymmetry in \(B \rightarrow K^+\gamma\), as included for instance in the \texttt{SuperIso} package [76–78]. A conservative treatment of non-factorizable contributions to this observable suggests a SM error exceeding \(\pm 0.05\), i.e., a relative error exceeding 100\% (see, e.g., [134]), and even larger errors are associated to the contributions of non-SM operators (see, e.g., [135]). These uncertainties obscure possible SUSY contributions within the ranges currently of interest in the CMSSM and the NUHM1 models.
good agreement with the SM prediction. On the other hand, adding the errors linearly relaxes the BR($b\to s\gamma$) constraint at smaller $(m_0, m_{1/2})$ in the CMSSM, reducing the tension with other observables and hence also the minimum of $\chi^2$, as seen in Table 1. The net result is to enhance the rate of increase of $\chi^2$ for CMSSM parameters departing from the best fit, implying that the 68% CL is reached more quickly. On the other hand, this effect is absent in the NUHM1 because the freedom to adjust the degree of Higgs non-universality already mitigates the tension of BR($b\to s\gamma$) with the other observables, leading to the 'pit' in the global $\chi^2$ function mentioned above.

The (tan $\beta$, $m_{1/2}$) planes in the CMSSM and NUHM1

Figure 7 displays the (tan $\beta$, $m_{1/2}$) planes in the CMSSM and NUHM1, exhibiting clearly the movement of the best-fit points and 68% and 95% CL contours to larger tan $\beta$ that is driven by the tension between ($g - 2$)$_\mu$ and the LHC push to larger $m_{1/2}$. Comparing the dotted and solid contours, we see that the LHC $\not{E}_T$ constraints force the new best-fit points into what were previously the ‘tails’ of the 95% CL regions at large $m_{1/2}$ and hence tan $\beta$. However, it is clear that the range of tan $\beta$ allowed at the 68% CL is still very broad, extending from <20 to >50 in both the CMSSM and the NUHM1. On the other hand, any future substantial increase in the LHC lower limit on $m_{1/2}$ would push tan $\beta$ in both models into a narrower range ~50, where it encounters pressure from BR($B_s \to \mu^+\mu^-$) as discussed below.

The (MA, tan $\beta$) planes in the CMSSM and NUHM1

We now turn to the (MA, tan $\beta$) planes of the CMSSM and NUHM1, shown in Fig. 8, which are affected directly
by the new CMS constraints on the heavy MSSM Higgs bosons \(H/A, H^\pm\) \([60, 62, 101]\), and by the CMS \([63]\) and LHCb \([64]\) constraints on \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\). As already discussed, we include the heavy Higgs constraints via the most sensitive CMS search for \(H/A \rightarrow \tau^+\tau^-\) \([62]\). In evaluating the constraint on the \((M_A, \tan \beta)\) planes of the CMSSM and NUHM1 imposed by the new measurements of \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\), we use the official combination of the recent upper limits on \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) from the CMS and LHCb Collaborations \([102]\), which yields \(\text{BR}(B_s \rightarrow \mu^+\mu^-) < 1.08 \times 10^{-8}\) at the 95% CL. Our implementation actually includes the full likelihood function arising from this combination. The measurement of \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) by the CDF Collaboration \([65]\) is in some tension with the CMS/LHCb combination, but only at the \(\Delta \chi^2 \approx 1\) level. However, since there is so far no official combination of the CDF result with those of CMS and LHCb, we limit ourselves to discussing later its compatibility with the predictions for \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) of our global fits. We see in Fig. 8 that the general effect of the LHC1/\(b\) data is to push the preferred range of \(M_A\) to larger values, as well as pushing \(\tan \beta\) towards larger values.

The impacts of the \(H/A\) and \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) constraints are less important in the CMSSM than in the NUHM1, so we discuss the latter in more detail. Figure 9 shows four versions of the \((M_A, \tan \beta)\) plane in the NUHM1, with all the LHC1/\(b\) constraints applied (upper left, equivalent to the right panel of Fig. 8), dropping the \(H/A\) constraint but keeping \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) (upper right), dropping \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) but keeping the \(H/A\) constraint (lower left), and dropping both constraints (lower right). Comparing the two upper panels, we see that the \(H/A\) constraint is relevant for \(M_A \lesssim 450\) GeV, and that applying it impacts the low-\(M_A\) sides of the 68% and 95% CL contours, whereas the best-fit point is unaffected. Comparing the upper and lower panels, we see that the \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) constraint is relevant for low \(M_A\) values. In particular, the 68% CL contours extend to slightly lower \(\tan \beta\) values and the best-fit points (green stars) move to significantly lower \(\tan \beta\) when \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) is included as seen in Fig. 9. Both the \(H/A\) and \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) constraints have the potential for more significant impacts in the future.

**Predictions for \(m_{\tilde{g}}\)** In Fig. 10 we show the one-dimensional \(\chi^2\) functions predicted by global fits for \(m_{\tilde{g}}\) in the CMSSM (left) and the NUHM1 (right). The solid lines are based on our global fits including the LHC1/\(b\) constraints, whereas the dotted lines are based on our previous global fits based on the pre-LHC constraints \([5]\).\(^{20}\) We see that the best-fit estimates of \(m_{\tilde{g}}\) have increased substantially to \(\sim 1600\) GeV as a result of the LHC1/\(b\) data, but we also see that there is considerable uncertainty in this estimate, with \(m_{\tilde{g}} > 2500\) GeV subject to a penalty \(\Delta \chi^2 \sim 2\) only.

**Predictions for \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\)** In Fig. 11 we show the one-dimensional \(\chi^2\) functions predicted by our global fits for \(\text{BR}(B_s \rightarrow \mu^+\mu^-)\) in the CMSSM (left) and the NUHM1 (right). The solid lines are based on the official combination of the CMS and LHCb constraints on this decay \([102]\),

\(^{20}\)The ‘stalactites’ at \(m_{\tilde{g}} \sim 400\) GeV in the pre-LHC fits \([5]\) were due to the light-Higgs funnel that has now been excluded by the LHC1/\(b\) data. Likewise, the ‘stalactite’ in the CMSSM LHC1/\(b\) curve also originates in the light-Higgs funnel region, and comes from points with large \(m_0 > 3\) TeV—which is why they are not seen in Fig. 1—and large \(A_0\). These points might be excluded by the ATLAS 1/\(b\) 0-lepton search, whose published \((m_0, m_{\tilde{g}})\) exclusion for \(\tan \beta = 10\) and \(A_0 = 0\) extends only to \(m_0 = 3\) TeV.
The $(M_A, \tan \beta)$ planes in the NUHM1 including both the $H/A$ [62] and $BR(B_s \to \mu^+ \mu^-)$ [102] constraints (upper left), dropping the $H/A$ constraint but keeping the $BR(B_s \to \mu^+ \mu^-)$ constraint (upper right), dropping $BR(B_s \to \mu^+ \mu^-)$ but keeping $H/A$ (lower left), and dropping both constraints (lower right).

Fig. 10 The one-dimensional $\chi^2$ functions for $m_\tilde{g}$ in the CMSSM (left) and the NUHM1 (right). The solid lines are for fits including the LHC1/1b data, and the dotted lines are for fits based on the pre-LHC data [14].

whereas the dashed lines show results using an unofficial combination of these constraints with the CDF measurement [65], and the dotted lines represent pre-LHC predictions [5]. We see that the best-fit estimates of $BR(B_s \to \mu^+ \mu^-)$ are somewhat above the SM value, as a result of the push towards larger $\tan \beta$ required to accommodate the LHC data while reconciling them with $(g - 2)_\mu$. In both the CMSSM and the NUHM1, the estimates of $BR(B_s \to$
The one-dimensional $\chi^2$ functions for $BR(B_s \rightarrow \mu^+\mu^-)$ in the CMSSM (left) and the NUHM1 (right). The solid lines are for fits including the official combination of the results from the CMS and LHCb Collaborations [102], the dashed lines are for fits using an unofficial combination of these results with the CDF result [65], and the dotted lines represent pre-LHC predictions [5].

$\mu^+\mu^-$ are quite compatible with an unofficial combined fit to CDF, CMS and LHCb data, where the main effect is a reduction of $\chi^2$ in a somewhat broader range of $BR(B_s \rightarrow \mu^+\mu^-)$.

**Predictions for $M_h$** In Fig. 12 we show the one-dimensional $\chi^2$ functions predicted by our global fits for $M_h$ in the CMSSM (left) and the NUHM1 (right). In this figure we do not include the direct limits from LEP [127, 128] or the Tevatron, so as to illustrate whether there is a conflict between these limits and the predictions of supersymmetric models. For each model we display the new likelihood functions corresponding to the LHC $1\text{ fb}$ data set, indicating the theoretical uncertainty in the calculation of $M_h$ of $\sim 1.5$ GeV by red bands. We also show, as dashed lines without red bands, our previous predictions based on the pre-LHC results (also discarding the LEP constraint). We see that the LHC data improve the consistency of the model predictions with the LEP exclusion, removing whatever tension existed previously. We cannot resist pointing out that the best-fit value for $M_h$ found recently in a SM fit including LEP, Tevatron and LHC exclusions as well as precision electroweak data $\sim 120$ GeV [131], and that this is also the value of the SM Higgs mass that is most compatible with the ongoing LHC searches [136, 137].

**Predictions for $M_A$** In Fig. 13 we show the one-dimensional $\chi^2$ functions predicted by our global fits for $M_A$ in the

Theoretical

Inaccessible

Inaccessible

The lighter yellow shading in the right panel reflects the fact that this mass range is not completely excluded in the NUHM1 due to a possible suppression of the $ZZh$ coupling. The beige shading in both panels indicates values of $M_h$ inaccessible in the supersymmetric models studied with GUT-scale unification.
CMSSM (left) and the NUHM1 (right). We see that the best-fit values of $M_A$ have increased in both models, by $\sim 350$ GeV and $\sim 250$ GeV, respectively.

**Dark matter scattering cross sections** In Fig. 14 we show the 68% and 95% CL contours in the $(m_{\tilde{\chi}_1^0}, \sigma_p^{SI})$ planes for the CMSSM (left) and the NUHM1 (right). The solid lines are based on our global fits including the LHC1/fb constraints, whereas the dotted lines correspond to our previous fits using the pre-LHC constraints. In both cases, we assume $\Sigma_{\pi,N} = 50 \pm 14$ MeV [88–96], and we include with the LHC1/fb data the XENON100 constraint on $\sigma_p^{SI}$ [66]. We see that the LHC1/fb data tend to push $m_{\tilde{\chi}_1^0}$ to larger values $^{22}$, and that these are correlated with lower values of $\sigma_p^{SI}$, though with best-fit values still $\sim 10^{-45}$ cm$^2$. We do not present here predictions for spin-dependent scattering or signatures of astrophysical dark matter annihilations, which are further removed from the prospective experimental sensitivities in the near future.

**Sparticle thresholds in $e^+e^-$ annihilation** In view of the interest in building an $e^+e^-$ collider as the next major project at the energy frontier, we now analyze the implications of the LHC1/fb and XENON100 data for expectations for sparticle production in $e^+e^-$ annihilation within the CMSSM and NUHM1. In this respect it has to be kept in mind that the LHC searches are mainly sensitive to the production of coloured particles, whereas lepton colliders will

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$^{21}$We recall the sensitivity of predictions for $\sigma_p^{SI}$ to the uncertainty in $\Sigma_{\pi,N}$ [14].

$^{22}$The slivers of points at $m_{\tilde{\chi}_1^0} \sim 60$ GeV originate in the light-Higgs funnel region with large $m_0 > 3$ TeV mentioned earlier, which might be excluded by the ATLAS 1/fb 0-lepton search.
have a high sensitivity in particular for the production of colour-neutral states, such as sleptons, charginos and neutralinos, as well as yielding high-precision measurements that will provide indirect sensitivity to quantum effects of new states. Anything inferred from the coloured sector concerning the uncoloured sector depends on the underlying model assumptions, and in particular on assumptions about the possible universality of soft supersymmetry breaking at the GUT scale. Non-universal models, e.g., low-energy supersymmetric models, or models with different GUT assumptions, could present very different possibilities.

Figure 15 compares the likelihood functions for various thresholds in the CMSSM (upper panel) and the NUHM1 (lower panel), based on the global fits made using the LHC1 fb and XENON100 constraints. The lowest thresholds are those for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_1$, $\tilde{\tau}_1\tilde{\tau}_1$, $\tilde{e}_R\tilde{e}_R$ and $\tilde{\mu}_R\tilde{\mu}_R$ (the latter is not shown, it is similar to that for $\tilde{e}_R\tilde{e}_R$). We see that, within the CMSSM and NUHM1, it now seems that these thresholds may well lie above 500 GeV, though in the CMSSM significant fractions of their likelihood functions still lie below 500 GeV. The thresholds for $\tilde{\chi}^0_1\tilde{\chi}^0_2$ and $\tilde{e}_R\tilde{e}_L + \tilde{e}_L\tilde{e}_R$ are expected to be somewhat higher, possibly a bit below 1 TeV. The preferred value for the threshold for $\tilde{\chi}^+\tilde{\chi}^-_1$ lies at about 1700 GeV in both the CMSSM and NUHM1 scenarios, that for the $HA$ threshold lies above 1 TeV, and that for first- and second-generation squark–antisquark pair production lies beyond 2.5 TeV in both models. It should be kept in mind that these high thresholds are linked with the reduced $p$-value of the model. Further increases in the excluded regions would yield even higher thresholds, but would also make the CMSSM or NUHM1 seem even less likely.

4 Summary and conclusions

There is some disappointment in the air that the LHC has found no signs of supersymmetry in its first $\sim 1/fb$ of data. However, it should be kept in mind that the searches performed at the LHC so far have essentially only been able to set limits on the production of the gluino and the squarks of the first two generations, and the resulting limits depend sensitively on the mass assumed for the lightest supersymmetric particle [138, 139]. On the other hand, the sensitivities

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15.png}
\caption{The $\chi^2$ likelihood functions for various pair-production thresholds in $e^+e^-$, as estimated in the CMSSM (upper panel) and the NUHM1 (lower panel) after incorporating the XENON100 [66] and LHC1 fb constraints. The likelihood function for the $\tilde{\mu}_R\tilde{\mu}_R$ threshold (not shown) is very similar to that for $\tilde{e}_R\tilde{e}_R$
\end{figure}
of direct searches for stops and sbottoms and colour-neutral superpartners are very limited up to now. This situation will improve, as several times more data can be expected by the end of 2012, there is the prospect subsequently of an increase in the energy by a factor up to two, and the LHC is expected eventually to accumulate orders of magnitude more data.

The initial optimistic prospects for SUSY searches at the LHC were largely driven by two indications that the supersymmetric mass scale might not be very high: $(g-2)_\mu$ and the need for dark matter that should not be overdense. Neither of these indications has weakened recently. Indeed, the $(g-2)_\mu$ hint has even strengthened with the convergence of the previously discrepant SM calculations using low-energy $e^+e^-$ and $\tau$ decay data [103, 140]. However, as we have discussed in this paper, significant tension is now emerging between the $(g-2)_\mu$ constraint and LHC data within the specific context of the CMSSM and NUHM1. A priori, in a general SUSY model there is not necessarily a tension between a heavy gluino and heavy squarks of the first two generations on the one hand, as favoured by the LHC limits, and light colour-neutral states on the other hand, as favoured by $(g-2)_\mu$.

The tension within the CMSSM and NUHM1 can be reduced to some extent by adopting a larger value of $\tan\beta$, but this may eventually lead to subsidiary tension with the LHC $H/A$ constraints and the tightening experimental visor on $\text{BR}(B_s \to \mu^+\mu^-)$. In any case, it will be important to subject the $(g-2)_\mu$ constraint to closer scrutiny, and the upcoming Fermilab and J-PARC experiments on $(g-2)_\mu$ [141] are most welcome and timely in this regard. In parallel, refinements of the experimental inputs for the prediction of $(g-2)_\mu$ from both low-energy $e^+e^-$ and $\tau$ decay data would also be welcome. It will be also necessary to subject the theoretical calculations within the SM and the corresponding estimates of the remaining theoretical uncertainties to further scrutiny.

The dark matter upper limit on the sparticle mass scale remains unchanged, and is responsible for the disfavoured region above $m_{1/2} \sim 2500$ GeV visible in our figures for the CMSSM and the NUHM1. On the other hand, the dark matter constraint on $m_0$ is not so strong, as also seen in the figures, extending well beyond the range displayed. Considering the impact of direct jets + $\not{E}_T$ searches only, the regions of the CMSSM and NUHM1 ($m_0, m_{1/2}$) planes in Fig. 2 with $p$-values significantly non-zero extend beyond the likely reach even of the full-energy LHC in its high-luminosity incarnation. A fortiori, the same is true for the regions of these planes allowed at the current $95\%$ CL ($\Delta \chi^2 = 5.99$ relative to the global minima, bounded by the blue contours in Fig. 1). This is even more true of the full regions of the CMSSM and NUHM1 ($m_0, m_{1/2}$) planes that are allowed by the dark matter constraint.

In light of this discussion, under what circumstances could one conclude that the CMSSM or NUHM1 is excluded? Currently, our best fits in both these models have $p$-values above $10\%$, comparable to that of SM fits to precision electroweak data from LEP and SLD, and the F-test shows that both the CMSSM and NUHM1 are warranted extensions of the SM, in the sense that introducing their parameters provides an improvement in $\chi^2$ that is valuable in both cases. Moreover, it seems unlikely that the LHC will soon be able to explore all the region of the $(m_0, m_{1/2})$ planes in Fig. 2 where the models’ $p$-values exceed $5\%$, nor does the LHC seem likely soon to push $F_{\mu}$ (see Fig. 3) to the uninteresting level of $60\%$ or less. This is not surprising, as in the high-mass limit the superpartners decouple and one is left essentially with the SM with a light Higgs.

One way for the LHC to invalidate the models studied here would be to discover an SM-like Higgs boson weighing substantially more than the range $\sim 120$ GeV predicted in Fig 12. A value of $M_h \sim 125$ GeV or more would be in some tension with $(g-2)_\mu$, and perhaps hint towards models beyond the CMSSM and NUHM1, whereas a value of $\sim 130$ GeV or more would cast severe doubt on most simple GUT-based models. As already mentioned, range $M_h \sim 120$ to 130 GeV is precisely that currently favoured independently by precision electroweak data and by LEP, Tevatron and LHC searches. If a Higgs-like signal were to be discovered in the lower part of this range, supersymmetry might not be far away, whereas if $M_h$ is in the upper part of this range, indicating that at least the third-generation squarks could be heavy, one might for some time be in the frustrating situation of acquiring ever more circumstantial hints for supersymmetry, but with no direct evidence. On the other hand, if the LHC discovers that $M_h > 130$ GeV, the time might come to take another look at non-minimal supersymmetric models.

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