Negative-Parity Baryon Spectroscopy*

Frank X. Lee and Derek B. Leinweber

aDepartment of Physics, The George Washington University, Washington, DC 20052 and Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

bDepartment of Physics and Math. Physics, University of Adelaide, Adelaide, SA 5005, Australia

Results are reported for the first calculation of the low-lying spin-1/2 odd-parity octet baryon masses using $O(a^2)$ improved gluon and fermion actions. Methods for removing even-parity ground-state contaminations from the two-point correlation functions at zero and finite momenta are outlined. We investigate the properties of two odd-parity interpolating fields based on the established interpolating fields for the nucleon ground state. Isolation of the lowest-lying odd-parity state appears to be sufficient to begin exploring odd-parity $N \to N^*$ electromagnetic transition form factors.

1. INTRODUCTION

It is well recognized that the investigation of processes involving the production of excited states of the nucleon ($N^*$) can provide relevant information on the non-perturbative regime of QCD. In the next years, the issue of $N^*$ physics will be addressed by planned experiments at Jefferson Lab, which will provide new experimental data in the resonance region with unprecedented accuracy. There is increasing demand for theoretical understanding of the $N^*$ properties from first principles calculations.

In this work, we carry out exploratory calculations for odd-parity spin-1/2 ($J^P = 1/2^-$) baryon resonances on the lattice using $O(a^2)$ improved actions. We utilize the $O(a^2)$ tree-level tadpole-improved Wilson gauge action and the $O(a^2)$ tree-level tadpole-improved D\textsubscript{34} action originally proposed by Hammer and Wu, and recently explored on the lattice.

2. INTERPOLATING FIELDS

The excitation energies of the $1/2^-$ baryons can be extracted from the standard two-point correlation function in the QCD vacuum

$$G(p, t) = \sum_x e^{-ip\cdot x} \langle 0 | \chi_{1/2}^- (x) \chi_{1/2}^- (0) | 0 \rangle$$

where $\chi_{1/2}^-$ is an odd-parity interpolating field used to excite the hadron in question. We consider odd-parity interpolating fields based on the two established even-parity interpolating fields for the nucleon ground state. Left multiplication by $\gamma_5$ provides the following odd-parity interpolating fields

$$\chi_{N_1}^- (x) = \epsilon_{abc} (u_T^a (x) C \gamma_5 d^b (x)) \gamma_5 u^c (x) ,$$

$$\chi_{N_2}^- (x) = \epsilon_{abc} (u_T^a (x) C d^b (x)) u^c (x) .$$

The interpolating fields for other members of the $1/2^-$ octet are obtained in a similar fashion. A nonrelativistic reduction of these interpolators reveals that $\chi_{1}^-$ pairs the $u$-$d$ quark field operators in parentheses into a scalar diquark while $\chi_{2}^-$ provides overlap with vector diquark pairs. Products of upper and lower components introduce derivatives of the quark field operators. Using SU(6) spin-flavor symmetry, the two nearby low-lying $N^*1/2^-$ states of the physical spectrum are described by coupling three quarks to a spin-1/2 or spin-3/2 spin-flavor wave function coupled in turn to one unit of orbital angular momentum. In a relativistic field theory one does not expect $\chi_{1}^-$ and $\chi_{2}^-$ to isolate individual states. However, it is expected that $\chi_{1}^-$ will predominantly excite the lower-lying $j = 1/2$ state associated with $\ell = 1$ and $s = 1/2$ in the quark model while $\chi_{2}^-$ will predominantly excite the higher-lying state asso-

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associated with \( s = 3/2 \). This expectation is borne out in the following.

As for the even-parity \( \Lambda \) interpolating fields \( \chi_{\Lambda C} \), we consider two full octet interpolators \( \chi_{\Lambda C1} \) and \( \chi_{\Lambda C2} \) and also consider two odd-parity interpolators containing terms common to the flavor-octet and singlet \( \Lambda \) interpolators \( \chi_{\Lambda C1} \) and \( \chi_{\Lambda C2} \). The latter two interpolating fields do not strongly bias the flavor symmetry of the odd-parity \( \Lambda \) state, but rather allow the lowest lying odd-parity state to dominate the correlator.

Despite having explicit negative-parity, these interpolating fields couple to both positive and negative parity states. Hence, the parity of the intermediate state must be projected.

### 3. Parity Projection

The ability of the interpolating fields to couple to intermediate states is described by a phenomenological parameter \( \lambda \), defined by

\[
\langle 0 | \chi_{1/2}(0) | N^\pm \rangle = \lambda_\pm \gamma_5 u(p),
\]

\[
\langle 0 | \chi_{1/2}(0) | N^- \rangle = \lambda_- u(p),
\]

where \( u(p) \) denotes a spinor. In the large Euclidean time limit

\[
G(p, t \to \infty) = e^{-E_\pm(p)t} \times \sum_s \langle 0 | \chi_{1/2}^s(0) | N_s^\pm \rangle \langle N_s^\pm | \chi_{1/2}^{-s}(0) | 0 \rangle
\]

\[
= \lambda_\pm^2 \gamma \cdot p - M_\pm e^{-E_\pm t} + \lambda_-^2 \gamma \cdot p + M_- e^{-E_- t}
\]

where \( p \) is on shell and \( E_\pm \) is the on-shell energy \((p^2 + M^2_\pm)^{1/2}\). The large Euclidean time evolution of the two-point function will ultimately be dominated by the lower-lying even-parity state. At zero momentum, even-parity states can be easily eliminated by taking the trace of \( G(p, t) \) with

\[
\Gamma_4(p) \equiv \frac{1}{4} \left( 1 + \gamma_4 \right) = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.
\]

Hence only the first two diagonal elements of the \( 4 \times 4 \) matrix \( G(p, t) \) are needed to isolate the mass of the odd-parity state. At finite momentum, the even-parity state can be eliminated by taking the trace with

\[
\Gamma_4(p) \equiv \frac{1}{4} \left( 1 + \frac{m_+}{E_+} \gamma_4 \right).
\]

Note that this matrix remains diagonal such that only four elements of \( G(p, t) \) are needed to isolate the odd-parity state of \( \chi \) at finite momenta.

Implicit in \( G \) are two assumptions. First the spectral representation of the odd-parity resonance has been taken to be a pole. For the quark masses considered in this simulation, this is in fact correct. For example, the \( \pi N \) decay channel of the low-lying \( N^* 1/2^- \) is closed by energy conservation. The issue of odd-parity projection centers on the elimination of the ground-state pole. As such, the procedure outlined here continues to be valid as the resonance decay channel opens upon decreasing the quark masses.

In addition, it is assumed there is only one positive parity state lying below the lowest-lying \( N^* 1/2^- \) state. However, in the physical nucleon spectrum there is both the ground state and the \( N^* 1/2^+ (1440) \) Roper resonance. Since the mass of the Roper and the odd-parity states of interest are quite close, the prefactors of the exponentials in \( G \) will govern the relative contributions of these states over a wide range of Euclidean time. For small momenta, \( \lambda \) indicates the Roper will contaminate the projected \( N^* 1/2^- \) state at the level of a few percent.

### 4. Results

We analyze 100 configurations on a \( 10^3 \times 16 \) lattice at \( \beta = 7.0 \) \((a \approx 0.24 \text{ fm})\). Additional details of the simulation may be found in Ref. [3].

Figure 1 displays effective masses for the \( N^* 1/2^- \) and \( N^* 3/2^- \) states compared to nucleon and rho masses. A reasonable plateau in the effective mass is seen for time slices 7 through 10 for all five quark masses considered.

For the first qualitative look at the lattice data, the \( 1/2^- \) masses are linearly extrapolated to the chiral limit. Mass ratios of the \( 1/2^- \) states to the nucleon mass are summarized in Table 1 at three quark masses and the chiral limit. Clear separations of the odd-parity states from the ground...
Table 1
Mass ratios of the $1/2^-$ baryons to the nucleon. The uncertainties are statistical. The three pion to rho mass ratios correspond to quark masses of approximately 210, 180, 90 MeV. The assignment of the state labels is our conjecture.

| Interpolator | Ratio | $\pi/\rho = 0.82$ | $\pi/\rho = 0.78$ | $\pi/\rho = 0.73$ | Chiral | Expt. |
|--------------|-------|-----------------|-----------------|-----------------|--------|-------|
| $\chi_{N1}$  | $N/N(1535)$ | 0.89(4) | 0.85(5) | 0.84(6) | 0.71(10) | 0.61 |
| $\chi_{N2}$  | $N/N(1650)$ | 0.77(5) | 0.71(6) | 0.62(9) | 0.42(11) | 0.57 |
| $\chi_{A^c1}$ | $N/\Lambda(1670)$ | 0.89(5) | 0.85(5) | 0.81(6) | 0.65(8) | 0.56 |
| $\chi_{A^c2}$ | $N/\Lambda(1800)$ | 0.77(5) | 0.71(6) | 0.64(9) | 0.45(9) | 0.52 |
| $\chi_{A^c3}$ | $N/\Lambda(1405)$ | 0.87(5) | 0.83(6) | 0.80(7) | 0.66(10) | 0.67 |
| $\chi_{\Sigma^c1}$ | $N/\Sigma(1620)$ | 0.91(5) | 0.85(5) | 0.81(6) | 0.62(7) | 0.58 |
| $\chi_{\Sigma^c2}$ | $N/\Sigma(1750)$ | 0.76(6) | 0.71(6) | 0.67(8) | 0.51(9) | 0.54 |
| $\chi_{\Xi^c1}$ | $N/\Xi(1620)$ | 0.90(5) | 0.85(5) | 0.81(6) | 0.63(5) | 0.58 |
| $\chi_{\Xi^c2}$ | $N/\Xi(1690)$ | 0.77(6) | 0.71(6) | 0.66(7) | 0.48(8) | 0.56 |

Figure 1. Effective hadron masses. The five quark masses, heaviest (top) to lightest (bottom), are offset for clarity. The source is at $t = 3$.

state nucleon are displayed. Furthermore, it appears $\chi_{1}^-$ couples predominantly to the ground state in the $1/2^-$ spectrum, while $\chi_{2}^-$ mostly to the next state as anticipated. This is further supported by the fact that the off-diagonal correlation function $(\chi_{1}^- \overline{\chi}_{2}^- + \chi_{2}^- \overline{\chi}_{1}^-)/2$ is smaller by more than a factor of two compared to the diagonal correlation functions $\chi_{1}^- \overline{\chi}_{1}^-$ or $\chi_{2}^- \overline{\chi}_{2}^-$. We have also been investigating the odd-parity spin-$3/2$ nucleon resonance $\chi$. The computational demand increases dramatically relative to the proton. We are currently exploring ways to reduce this factor. Preliminary analysis based on 25 configurations indicates a clear splitting from the ground state nucleon, an encouraging result.

This exploratory study shows promise in extracting $N^*$ structure from lattice QCD. To improve the correlation function data for $N^*$ states, we plan to fine tune the source/sink smearing functions, use anisotropic lattices with a fine lattice spacing in the time-dimension, and substantially increase statistics. After the spectrum of the odd-parity states is understood, we plan to explore quantities of greater phenomenological interest, such as the $N \rightarrow N^*$ electromagnetic transition form factors.

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