MULTI-OBJECTIVE LINGUISTIC-NEUTROSOPHIC MATRIX GAME AND ITS APPLICATIONS TO TOURISM MANAGEMENT

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ABSTRACT. Game theory plays an important role in numerous decision-oriented real-life problems. Nowadays, many such problems are basically characterized by various uncertainties. Uncertainties come to happen due to decision makers' collection of data, intuition, assumption, judgement, behaviour, evaluation and lastly due to the problem itself. Fuzzy concept with membership degree made an initialization towards the treatment of uncertainty, but it was not enough. Intuitionistic fuzzy concept was evolved concerning with both membership and non-membership degrees but failed to express reality more accurately. Then, neutrosophy logic was developed with a new degree in uncertainty, say, indeterminacy degree besides membership and non-membership degrees. Multi-objective optimization is an area of multiple-criteria decision making related with mathematical optimization problems involving more than one objective function to be optimized at the same time. Game theory (matrix game) problems with imprecise, vague information, like neutrosophic, can be formed with multiple objective functions. We develop and analyse a matrix game with multiple objectives, and solve the problem under a single-valued neutrosophic environment in linguistic approach. The main achievement of our study is that we here introduce a problem-oriented example to justify our designed methodologies with a successful real-life implications using linguistic neutrosophic data rather than crisp data as used in previous researches.

1. Introduction. Uncertainties and ambiguities exist everywhere in reality. Fuzzy logic (Zadeh [48]) has emerged as one of the important soft computing tools to describe the uncertainties. From Zadeh to Atanassov [2], fuzzy concept has been modified from its membership characteristics to intuitionistic fuzzy concept having non-membership characteristics. In some real-life cases, suppose that in time of choosing a candidate in voting-system, one has options to give up or to be in state of undecided besides confirm choice or no-choice. Such situations are not handled by intuitionistic characters. In these cases, neutrosophic set (NS) and neutrosophic logical concept originated and were successfully applied by Smarandache [40]. In many
situations, beyond the acceptance and the rejection degrees, the indeterminacy degree exists too. Several extensions of neutrosophic sets, such as single-valued neutrosophic set \([41]\), interval-valued neutrosophic set \([42]\), multi-valued neutrosophic set \([43]\), etc., have also been proposed. Among these, the single-valued neutrosophic set and the interval-valued neutrosophic set are highly employed for their easy applicability. Zadeh’s concept of linguistic term set \([49]\) proposed another structure towards a new approximation of reasoning to depict real-world phenomenon. Several articles \([24, 25, 26]\) have been published considering linguistic term set-based uncertainty.

Von Neumann \([28]\) invented *Game Theory* as a mathematical way in decision making problems to discover the situation related to the turns done by decision makers. A game involves a variety of players, a set of strategies plus a payoff that shows quantitatively the outcome of each play of the game in terms of the amounts that each player gains or loses. A player who chooses a pure strategy randomly selects a row or a column according to the probability process that specifies the chance with each pure strategy. These probability oriented strategies are known as mixed strategies for players. In probabilistic sense, the calculated payoffs suggest each player’s expectation to receive, and the player will actually obtain on average provided that the game is played a sufficiently high number of times. Because of the uncertainty and imprecision characteristics involved and occurred in the system, the anomaly of the judgement of players or decision makers, etc., we realize the reflection of indeterminacy and falsity characters in matrix games. Campos \([6]\) first employed fuzzy linear programming models to solve fuzzy matrix games. Li \([22]\), in his work, solved matrix games with different uncertainties using Atanassov’s intuitionistic fuzzy environment. Several monographs and articles have been published in game theory \([1, 3, 4, 5, 11, 13, 15, 17, 18, 19, 21, 33, 34, 35, 36, 37, 38, 39, 50]\) having many real-life impacts. Linguistic term set in neutrosophic environment through game theory can be viewed as a new concept. Multiple-criteria decision making problems are concerned with mathematical optimization problems involving more than one objective function, which are to be optimized simultaneously. Multi-objective optimization has been applied in many fields of science, including engineering, economics and logistics where optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. Here, we develop a two-person zero-sum matrix game model with linguistic neutrosophic numbers. Many articles and monographs \([9, 10, 27, 31]\) have been published in multi-objective and multi-criteria optimization problems. But, matrix game through multi-objective optimization under linguistic neutrosophic environment is a new one. The main contributions of this work are:

- The game-theoretical problem is solved in a neutrosophic environment.
- A new neutrosophic environment, called linguistic-neutrosophic, is assumed as the game’s environment.
- A matrix game is treated with multi-objective criteria.
- An application example of multi-objective linguistic-neutrosophic matrix game is considered in tourism management.

The rest of the paper is designed as follows. In Section 2, the basic preliminaries related to neutrosophic set, single-valued neutrosophic set, linguistic term set and their properties are briefly discussed. In Section 3, we discuss on linguistic neutrosophic set in a single-valued environment. Section 4 is based on general
multi-objective programming problem. A 2 × 2 matrix game under crisp environment is discussed in Section 5. In Section 6, a 2 × 2 matrix game in linguistic neutrosophic environment is suggested. A problem from tourism management is considered in Section 7 to validate the proposed concept and to clarify the solution of the problem. Section 8 summarizes our results and discussions. Section 9 is the conclusive part of this article which recalls the main findings, addressing scopes of future research.

2. Preliminaries. In this section, neutrosophic set, single-valued neutrosophic set, linguistic term set and their properties, and operational laws are discussed.

Definition 2.1. [40] Let \( X \) be a universe of discourse with a generic element \( x \) (\( x \in X \)). A neutrosophic set (NS) \( A \) in \( X \) is defined as: \( A = \{ (x, C_A(x)) : x \in X \} \). Here, \( C_A(x) \) is the ordered triplet \( (T_A(x), I_A(x), F_A(x)) \) of three numbers corresponding to each \( x \). Each of \( T_A, I_A, F_A: X \rightarrow (0^-, 1^+) \), with \( 0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+, \forall x \in X \). The numbers \( T_A(x), I_A(x) \) and \( F_A(x) \) are, respectively, the truth membership, indeterminacy membership and falsity membership degrees of the element \( x \) to the set \( A \).

The representation of neutrosophic set was presented philosophically, earlier. But, to express reality, standardization of non-standard subsets of \((0^-, 1^+)\) are improved to \([0, 1]\) by Wang [41] through engineering or scientific point of view.

Definition 2.2. [41] A single-valued neutrosophic set (SVNS), \( \hat{A} \), in a universe of discourse \( X \), is given by \( \hat{A} = \{ (x, T_\hat{A}(x), I_\hat{A}(x), F_\hat{A}(x)) : x \in X \} \), where \( T_\hat{A}: X \rightarrow [0, 1] \), \( I_\hat{A}: X \rightarrow [0, 1] \) and \( F_\hat{A}: X \rightarrow [0, 1] \), with the condition \( 0 \leq T_\hat{A}(x) + I_\hat{A}(x) + F_\hat{A}(x) \leq 3, \forall x \in X \). The numbers \( T_\hat{A}(x), I_\hat{A}(x) \) and \( F_\hat{A}(x) \) are, respectively, the truth membership, indeterminacy membership and falsity membership degrees of the element \( x \) to the set \( \hat{A} \).

Definition 2.3. A SVNS \( \hat{A} = \{ (x, T_\hat{A}(x), I_\hat{A}(x), F_\hat{A}(x)) : x \in X \} \) is said to be neutrosophic-normal, if there exist at least three points \( a, b, c \in X \) such that \( T_\hat{A}(a) = I_\hat{A}(b) = F_\hat{A}(c) = 1 \).

Definition 2.4. A SVNS \( \hat{A} = \{ (x, T_\hat{A}(x), I_\hat{A}(x), F_\hat{A}(x)) : x \in X \} \) is called neutrosophic-convex if for all \( a, b \in X \) and \( \lambda \in [0, 1] \), the following conditions are satisfied:

2.4.1. \( T_\hat{A}(\lambda a + (1 - \lambda)b) \geq \min\{T_\hat{A}(a), T_\hat{A}(b)\} \),
2.4.2. \( I_\hat{A}(\lambda a + (1 - \lambda)b) \leq \max\{I_\hat{A}(a), I_\hat{A}(b)\} \),
2.4.3. \( F_\hat{A}(\lambda a + (1 - \lambda)b) \leq \max\{F_\hat{A}(a), F_\hat{A}(b)\} \),

i.e., the truth membership function is fuzzy convex and the indeterminacy membership function, and the falsity membership function are fuzzy concave.

Definition 2.5. Let \( \hat{A} = \{ (x, T_\hat{A}(x), I_\hat{A}(x), F_\hat{A}(x)) : x \in X \} \) be a single-valued neutrosophic set (SVNS). Then \( \hat{A} \) is said to be a single-valued neutrosophic number when

2.5.1. \( \hat{A} \) is neutrosophic-normal;
2.5.2. \( \hat{A} \) is neutrosophic-convex;
2.5.3. \( T_\hat{A} \) is upper semi-continuous, \( I_\hat{A} \) and \( F_\hat{A} \) are lower semi-continuous;
2.5.4. Support \( S(\hat{A}) \) of \( \hat{A} \) is bounded, i.e., \( S(\hat{A}) = \{T_\hat{A}(x) > 0, I_\hat{A}(x) < 1, F_\hat{A}(x) < 1, \forall x \in X \} \).
Definition 2.6. Let \( \hat{A} = \{ (x, T_\hat{A}(x), I_\hat{A}(x), F_\hat{A}(x)) : x \in X \} \) and \( \hat{B} = \{ (x, T_\hat{B}(x), I_\hat{B}(x), F_\hat{B}(x)) : x \in X \} \) be two single-valued neutrosophic sets. Then, in a neutrosophic environment, we can define order relation “\( \leq \)”, as: \( \hat{A} \leq \hat{B} \) if and only if \( T_\hat{A}(x) \leq T_\hat{B}(x) \), \( I_\hat{A}(x) \geq I_\hat{B}(x) \) and \( F_\hat{A}(x) \geq F_\hat{B}(x) \). The symbol “\( \leq \)" is a neutrosophic version of the order relation of real number system \( \mathbb{R} \).

Definition 2.7. \(^{[47]}\) Let \( x = \langle T_\hat{A}(x), I_\hat{A}(x), F_\hat{A}(x) \rangle \) and \( y = \langle T_\hat{A}(y), I_\hat{A}(y), F_\hat{A}(y) \rangle \) be two single-valued neutrosophic numbers such that \( x, y \in \hat{A} \), where \( \hat{A} \) is single-valued neutrosophic set and \( \lambda > 0 \). Then operations can be defined as follows:

2.7.1. \( x^c = \left\langle F_\hat{A}(x), 1 - I_\hat{A}(x), T_\hat{A}(x) \right\rangle \); 
2.7.2. \( x \cup y = \left\langle \max\{T_\hat{A}(x), T_\hat{A}(y)\}, \min\{I_\hat{A}(x), I_\hat{A}(y)\}, \min\{F_\hat{A}(x), F_\hat{A}(y)\} \right\rangle \); 
2.7.3. \( x \cap y = \left\langle \min\{T_\hat{A}(x), T_\hat{A}(y)\}, \max\{I_\hat{A}(x), I_\hat{A}(y)\}, \max\{F_\hat{A}(x), F_\hat{A}(y)\} \right\rangle \); 
2.7.4. \( x \oplus y = \left\langle (T_\hat{A}(x) + T_\hat{A}(y) - T_\hat{A}(x)T_\hat{A}(y)), (I_\hat{A}(x)I_\hat{A}(y)), (F_\hat{A}(x)F_\hat{A}(y)) \right\rangle \); 
2.7.5. \( x \otimes y = \left\langle (T_\hat{A}(x)T_\hat{A}(y)), (I_\hat{A}(x) + I_\hat{A}(y) - I_\hat{A}(x)I_\hat{A}(y)), (F_\hat{A}(x) + F_\hat{A}(y) - F_\hat{A}(x)F_\hat{A}(y)) \right\rangle \); 
2.7.6. \( x \oslash y = \left\langle \frac{T_\hat{A}(x) - T_\hat{A}(y)}{1 - T_\hat{A}(y)}, \frac{I_\hat{A}(x) - I_\hat{A}(y)}{1 - I_\hat{A}(y)}, \frac{F_\hat{A}(x) - F_\hat{A}(y)}{1 - F_\hat{A}(y)} \right\rangle \), provided, \( T_\hat{A}(y) \neq 1, I_\hat{A}(y) \neq 0, F_\hat{A}(y) \neq 0 \); 
2.7.7. \( x \odot y = \left\langle \frac{T_\hat{A}(x)}{T_\hat{A}(y)}, \frac{I_\hat{A}(x)}{I_\hat{A}(y)}, \frac{F_\hat{A}(x)}{F_\hat{A}(y)} \right\rangle \), provided, \( T_\hat{A}(y) \neq 0, I_\hat{A}(y) \neq 0 \), \( F_\hat{A}(y) \neq 1 \); 
2.7.8. \( \lambda x = \left\langle (1 - (1 - T_A(x))^\lambda), (I_A(x))^\lambda, (F_A(x))^\lambda \right\rangle \); 
2.7.9. \( x^\lambda = \left\langle (T_A(x))^\lambda, 1 - (1 - I_A(x))^\lambda, 1 - (1 - F_A(x))^\lambda \right\rangle \).

Here, \( \oslash \), \( \odot \), \( \ominus \) and \( \odot \) are used for neutrosophic addition, multiplication, subtraction and division, respectively. Also, in subtraction and division operations (given items in 2.7.6 and 2.7.7 of Definition 2.7), components of neutrosophic numbers are assumed in the interval \([0, 1]\) as classical case. But, for the general case (when dealing with neutrosophic overtet, understet and offset \(^{[47]}\)), the neutrosophic number components are in the interval \([f, g]\). Here, \( f \) is called underlimit and \( g \) is called overlimit with \( f \leq 0 < 1 \leq g \). Consequently, the components of neutrosophic numbers due to subtraction and division lie within \([f, g]\).

In some cases of the real world, variables, expressed through languages or words or sentences, are more acceptable by the society rather than numerical values. These types of variables, called linguistic variables, enhances the applicability of itself under real-life uncertainties in various fields after the proposal of the linguistic information, linguistic approach in fuzzy-set theory by Zadeh \(^{[49]}\).

Definition 2.8. \(^{[49]}\): A linguistic variable is characterized by a quintuple \((C, T(C), U, G, M)\), where \( C \) denotes the name of the variable. Here, \( T(C) \) indicates the term set of \( C \), i.e., the set of its linguistic values; \( U \) is a universe of discourse; \( G \) is the way by which the terms of \( T(C) \) are generated; and \( M \) is a semantic rule for associating each linguistic value \( X \) with its meaning; finally, \( M(X) \) is a fuzzy subset of \( U \). A linguistic variable is described logically by its semantics.

Several ways \(^{[16, 32]}\) exist to express the linguistic descriptors and the corresponding semantics. Among these, seven scales of linguistic term-based semantics
are used frequently, given as: \( S_{11} = \{ s_0 \) (nothing), \( s_1 \) (very low), \( s_2 \) (low), \( s_3 \) (medium), \( s_4 \) (high), \( s_5 \) (very high), \( s_6 \) (perfect)\}; \( S_{12} = \{ s_0 \) (very poor), \( s_1 \) (poor), \( s_2 \) (slightly poor), \( s_3 \) (fair), \( s_4 \) (slightly good), \( s_5 \) (good), \( s_6 \) (very good)\}; \( S_{13} = \{ s_1 \) (extremely poor), \( s_2 \) (very poor), \( s_3 \) (poor), \( s_4 \) (medium), \( s_5 \) (good), \( s_6 \) (very good), \( s_7 \) (extremely good)\}; \( S_{14} = \{ s_{-3} \) (none), \( s_{-2} \) (very low), \( s_{-1} \) (low), \( s_0 \) (medium), \( s_1 \) (high), \( s_2 \) (very high), \( s_3 \) (perfect)\}.

**Property 2.1.** Consider a linguistic term set \( S = \{ s_i : i = 1, 2, \ldots, t \} \). Then:

2.1.1. The set is ordered, i.e., for \( i > j \), \( s_i > s_j \);

2.1.2. A negation operator exists, i.e., \( \neg(s_i) = s_j \), for \( i + j = t + 1 \);

2.1.3. A maximizing operator exists, i.e., \( \max\{ s_i, s_j \} = s_j \), if \( s_j \geq s_i \);

2.1.4. A minimizing operator exists, i.e., \( \min\{ s_i, s_j \} = s_j \), if \( s_j \leq s_i \).

This discrete term set can be converted into continuous term set as: \( \overline{S} = \{ s_i : s_1 \leq s_i \leq s_q, i \in [1, q] \} \), where \( q \) is sufficiently large positive number. Here, \( s_i \) is called the original linguistic term if \( s_i \in S \), otherwise, the virtual linguistic term.

**Property 2.2.** Assume \( s_\gamma \) and \( s_\delta \) be two linguistic variables; \( s_\gamma, s_\delta \in \overline{S} \), \( \lambda, \kappa \in [0, 1] \). The operational laws are elucidated as [46]:

2.2.1. \( s_\gamma + s_\delta = s_{\gamma + \delta} \);

2.2.2. \( s_\gamma \odot s_\delta = s_{\gamma \delta} \);

2.2.3. \( \lambda(s_\gamma) = s_{\lambda s_\gamma} \);

2.2.4. \( (\lambda + \kappa)s_\gamma = \lambda s_\gamma + \kappa s_\gamma \);

2.2.5. \( (s_\gamma)^\lambda = s_{s_\gamma^\lambda} \).

Human judgements and perception always flow neutrosophically, and basically these environments are nurtured with linguistic characters of responses, understood in fuzziness sense. Subsequently, we demonstrate the linguistic single-valued and interval-valued neutrosophic sets.

3. **Linguistic neutrosophic set.** Based on the combination of linguistic term set and neutrosophic set, this section promotes the concept of linguistic neutrosophic set.

**Definition 3.1.** Let \( X \) be a non-empty subset of the universe and consider a linguistic term set \( S = \{ s_i : i = 1, 2, \ldots, t \} \). Then, a linguistic neutrosophic set is defined as: \( \tilde{S} = \{ \langle T_{s_i}, I_{s_i}, F_{s_i} \rangle : s_i \in X \} \). Each component of \( \tilde{S} \), i.e., \( T_{s_i}, I_{s_i}, \) and \( F_{s_i} \) are linguistic term-based semantics.

From this definition, \( \langle T_{s_{0.4}}, I_{s_{0.5}}, F_{s_{0.2}} \rangle \), simply as \( \langle s_{0.4}, s_{0.5}, s_{0.2} \rangle \), is a linguistic neutrosophic number. Sometimes, it is called a linguistic single-valued neutrosophic number.

**Property 3.1.** Let \( \tilde{I}_1 = \langle T_{s_{1_1}}, I_{s_{1_1}}, F_{s_{1_1}} \rangle \), \( \tilde{I}_2 = \langle T_{s_{1_2}}, I_{s_{1_2}}, F_{s_{1_2}} \rangle \) be any three linguistic single-valued neutrosophic numbers and \( \lambda > 0 \), then the operational laws of linguistic neutrosophic numbers are defined as follows:

3.1.1. \( \tilde{I}_1 \odot \tilde{I}_2 = \langle T_{s_{1_1}} + T_{s_{1_2}}, I_{s_{1_1}} I_{s_{1_2}}, F_{s_{1_1}} F_{s_{1_2}} \rangle \);

3.1.2. \( \tilde{I}_1 \ominus \tilde{I}_2 = \langle \frac{T_{s_{1_1}} - T_{s_{1_2}}}{T_{1_1} - T_{1_2}}, \frac{I_{s_{1_1}} I_{s_{1_2}}}{I_{1_1} I_{1_2}}, \frac{F_{s_{1_1}} F_{s_{1_2}}}{F_{1_1} F_{1_2}} \rangle \), provided, \( T_{s_{1_1}} \neq T_{1_1}, I_{s_{1_1}} \neq I_{1_1}, F_{s_{1_1}} \neq F_{1_1} \);

3.1.3. \( \tilde{I}_1 \oplus \tilde{I}_2 = \langle T_{s_{1_1}} + T_{s_{1_2}}, I_{s_{1_1}} + I_{s_{1_2}} - I_{s_{1_1}} I_{s_{1_2}}, F_{s_{1_1}} + F_{s_{1_2}} - F_{s_{1_1}} F_{s_{1_2}} \rangle \);
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3.1.5. \( F_{s_{1}t_{2}} \neq F_{1} \),

3.1.5. \( \mathbf{f} = \begin{pmatrix} T_{s_{1}t_{2}}, I_{s_{1}t_{2}} - I_{s_{1}t_{2}}, F_{s_{1}t_{2}} - F_{s_{1}t_{2}} \end{pmatrix}, \) provided no, \( T_{s_{1}t_{2}} \neq T_{0}, I_{s_{1}t_{2}} \neq I_{1} \).

3.1.5. \( \lambda \mathbf{f} = (1 - (1 - T_{0})^{\lambda}, (I_{0})^{\lambda}, (F_{0})^{\lambda}); \)

3.1.6. \( \bar{\lambda} \mathbf{f} = \left( (T_{s_{0}})^{\lambda}, 1 - (1 - I_{s_{0}})^{\lambda}, 1 - (1 - F_{s_{0}})^{\lambda} \right). \)

Since calculations of linguistic neutrosophic terms depend upon \( \alpha \) of \( T_{a} \), therefore calculation of \( T_{s_{1}t_{2}} + T_{s_{1}t_{2}} - T_{s_{1}t_{1}} T_{s_{1}t_{2}} \) is nothing but \( T_{s_{1}t_{2}} + s_{1} - s_{1} t_{2} \), i.e., \( s_{1} t_{2} - s_{1} s_{2} \). Similarly, other terms are calculated during operational laws.

4. General multi-objective programming problem. A multi-objective programming problem is to find the solution vector \( x = (x_{1}, x_{2}, x_{3}, \ldots, x_{r})^{T} (\in \mathbb{X}) \) which optimizes a vector of objective functions \( f(x) \) \( (= (f_{1}(x), f_{2}(x), f_{3}(x), \ldots, f_{t}(x)) \) over the feasible region \( \mathbb{X} \) (e.g., \( \mathbb{R}^{r} \)). Considering minimization format of multi-objective optimization problem, we have:

\[
\min f(x)
\text{subject to} \quad h_{\nu}(x) = 0 \ (\nu = 1, 2, \ldots, \theta), \\
g_{j}(x) \leq 0 \ (j = 1, 2, \ldots, \kappa), \\
x_{k}^{l} \leq x_{k} \leq x_{k}^{u} \ (k = 1, 2, \ldots, r).
\]

Here, \( f_{1}(x), f_{2}(x), f_{3}(x), \ldots, f_{t}(x) \) are the individual objective functions, and \( h_{\nu}(x) \) and \( g_{j}(x) \) are equality and inequality constraints, respectively; \( x_{k}^{l} \) and \( x_{k}^{u} \) are the lower and upper bounds of \( x_{k} \), the decision variables. The concept of optimality in single objective is not directly applicable in multi-objective optimization problems. For this reason, a classification of the solutions is introduced in terms of Pareto optimality according to the following definitions, in terms of minimization.

Definition 4.1. Pareto optimality: A solution vector \( x^{*} \in \mathbb{X} \) is said to be Pareto optimal solution if there exists no \( x \in \mathbb{X} \) such that \( f_{q}(x) \leq f_{q}(x^{*}) \), for all \( q = 1, 2, 3, \ldots, t \), and \( f_{q}(x) \neq f_{q}(x^{*}) \) for at least one objective function \( f_{q} \). This solution is termed sometimes as True-Pareto optimal solution.

In general, there exist several Pareto optimal solutions to a multi-objective optimization problems. Due to multi-objectives, the selection of such objectives clearly depends upon the considered problem and the decision maker. Thus, the decision maker must select a compromise or a satisfying solution from the Pareto optimal solution set according to the preference orders.

5. Matrix game in crisp environment. A two-person zero-sum matrix game is exhibited as \( A = (a_{ij}) \ (i = 1, 2, \ldots, p; \ j = 1, 2, \ldots, q) \) with entries being real numbers; this matrix is called the payoff matrix. The row player, considered here as player I (PI), chooses to play row \( i \) and player II (PII), the column player, chooses to play column \( j \). The payoff to PI is \( a_{ij} \) and that of PII is \( -a_{ij} \). Consequently, the strategies which are beneficial for the player’s individual payoffs are picked out by the players.

Assuming the matrix game with the set of pure strategies \( S_{1}, S_{2} \) and mixed strategies \( Y, Z \) for PI and PII, respectively, which are defined as:

\[
S_{1} = \{ \alpha_{1}, \alpha_{2}, \ldots, \alpha_{p} \}, \quad S_{2} = \{ \beta_{1}, \beta_{2}, \ldots, \beta_{q} \},
\]
respectively, and the game is denoted by $G := (Y, Z, A)$. Therefore, the payoff matrix for PI & PII can be described as: $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1q} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2q} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & a_{p3} & \ldots & a_{pq} \end{bmatrix}$.

Definition 5.1. [29, 30] Considering a choice of mixed strategies $Y$ for PI and $Z$ for PII, chosen independently, then the expected payoff to PI of the game is given as:

$$E(Y, Z) = \sum_{i=1}^{p} \sum_{j=1}^{q} a_{ij} \text{Prob}(\text{PI uses } i \text{ and PII uses } j)$$

$$= \sum_{i=1}^{p} \sum_{j=1}^{q} a_{ij} \text{Prob}(\text{PI uses } i) \text{Prob}(\text{PII uses } j)$$

$$= \sum_{i=1}^{p} \sum_{j=1}^{q} y_i a_{ij} z_j = Y^T A Z.$$

6. Mathematical Model: 2×2 matrix game in linguistic neutrosophic environment. In this Section, we describe two-person matrix game in linguistic neutrosophic environment, and the game is represented as $\tilde{G} := (Y, Z; A_1)$ shortly, $\tilde{G}_{(A_1)}$, where:

1. the set for strategies of PI is $Y$, a non-empty set,
2. the set for strategies of PII is $Z$, another non-empty set, and
3. $A_1$ is linguistic neutrosophic set over $Y \times Z$, defined as: $A_1 = \{(y, z), (s_{T_{A_1}}(y, z), s_{I_{A_1}}(y, z), s_{F_{A_1}}(y, z)) \} : (y, z) \in Y \times Z\}$.

Here, for the sake of simplicity, we consider a 2×2 matrix game, i.e., when PI uses $y_1$ and PII uses $z_2$, the payoff for PI is described as: $(s_{T_{A_1}}(y_1, z_2), s_{I_{A_1}}(y_1, z_2), s_{F_{A_1}}(y_1, z_2))$. We derive the matrix of the game problem in 2×2 format as:

$$A = \begin{bmatrix} a(T_{s_{11}, I_{s_{11}}, F_{s_{11}}}) & a(T_{s_{12}, I_{s_{12}}, F_{s_{12}}}) \\ a(T_{s_{21}, I_{s_{21}}, F_{s_{21}}}) & a(T_{s_{22}, I_{s_{22}}, F_{s_{22}}}) \end{bmatrix};$$

its $p \times q$ format is:

$$A = \begin{bmatrix} a(T_{s_{11}, I_{s_{11}}, F_{s_{11}}}) & a(T_{s_{12}, I_{s_{12}}, F_{s_{12}}}) & a(T_{s_{13}, I_{s_{13}}, F_{s_{13}}}) & \ldots & a(T_{s_{1q}, I_{s_{1q}}, F_{s_{1q}}}) \\ a(T_{s_{21}, I_{s_{21}}, F_{s_{21}}}) & a(T_{s_{22}, I_{s_{22}}, F_{s_{22}}}) & a(T_{s_{23}, I_{s_{23}}, F_{s_{23}}}) & \ldots & a(T_{s_{2q}, I_{s_{2q}}, F_{s_{2q}}}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a(T_{s_{p1}, I_{s_{p1}}, F_{s_{p1}}}) & a(T_{s_{p2}, I_{s_{p2}}, F_{s_{p2}}}) & a(T_{s_{p3}, I_{s_{p3}}, F_{s_{p3}}}) & \ldots & a(T_{s_{pq}, I_{s_{pq}}, F_{s_{pq}}}) \end{bmatrix}.$$
Definition 6.1. Let the linguistic neutrosophic game be \( \tilde{G} \equiv (Y, Z; A_1) \), where \( A_1 = \{(y, z), (T_{s}, I_{s}, F_{s}) : (y, z) \in Y \times Z \} \). If \( \max_{y_i} \{T_{s}I_{s}F_{s}(y, z)\} = \min_{y_j} \{T_{s}I_{s}F_{s}(y, z)\} \), then \((y_i, z_j)\) is called a linguistic neutrosophic saddle point of the matrix game.

Definition 6.2. Let PI choose any mixed strategy \( y \in Y \) and let PII select any mixed strategy \( z \in Z \), then the expected payoffs for PI are \( E(A_1) = y^T A_1 z \) and the expected payoffs for PII are the corresponding negation of \( E(A_1) \). It is generally assumed that PI is considered as a maximizing player, and PII is defined as minimizing player. Generally, PII’s interest consists in finding a mixed strategy \( z \in Z \) so as to minimize \( E(y, z) \), denoted by

\[
v(y) = \min_{z \in Z} \{y^T A_1 z\}.
\]

Hence, PI should choose a mixed strategy \( y \in Y \) that maximizes the minimum expected gain, i.e.,

\[
v^* = \max_{y \in Y} \min_{z \in Z} \{y^T A_1 z\}.
\]

Such a value \( v^* \) is called as PI’s gain-floor. Similarly, PII’s loss-ceiling is the value \( w^* \), given by

\[
w^* = \min_{z \in Z} \max_{y \in Y} \{y^T A_1 z\}.
\]

According to maximin and minimax principles for PI and PII, the Nash equilibrium strategy for the matrix game in a linguistic neutrosophic environment is defined in the following way.

Definition 6.3. A pair \((y^*, z^*) \in Y \times Z\) is a Nash equilibrium strategy for matrix game \( A_1 \) in a linguistic neutrosophic environment if

1. \( y^* A_1 z^* \leq y^* A_1 z^* \) for \( y \in Y \) in a linguistic neutrosophic environment;
2. \( y^* A_1 z^* \leq y^* A_1 z^* \) for \( z \in Z \) in a linguistic neutrosophic environment.

Based on the ranking method of neutrosophic sets as in Definition 2.6 with the linguistic version, the subsequent theorem is obtained.

Theorem 6.4. Let the matrix game be \( A_1 \). For \((y^*, z^*) \in Y \times Z\) to be a Pareto Nash equilibrium strategy for the matrix game \( A_1 \) in the linguistic neutrosophic environment if and only if the following conditions are fulfilled:

1. \( \exists y \in Y \), such that \( y^T \mu z^* \leq y^T \mu z^* \) and \( y^T \nu z^* \geq y^T \nu z^* \) and \( y^T \tau z^* \geq y^T \tau z^* \),
2. \( \exists z \in Z \), such that \( y^T \mu z \leq y^T \mu z^* \) and \( y^T \nu z \geq y^T \nu z^* \) and \( y^T \tau z \geq y^T \tau z^* \).

Proof. The proof of this theorem can be seen as an extended version of [23] in the neutrosophic environment.

Theorem 6.5. The strategy \( y^* \) is a Pareto Nash equilibrium strategy and \((\mu^*, \nu^*, \tau^*)\) is the player I’s gain floor for the linguistic neutrosophic matrix game \( A_1 \) if and only if \((y^*, \mu^*, \nu^*, \tau^*)\) is a Pareto-optimal solution (or efficient solution) of the following multi-objective programming problem:

- maximize \( \mu \),
- minimize \( \nu \),
- minimize \( \tau \),
Theorem 6.6. The strategy $z^*$ is a Pareto Nash equilibrium strategy and $(\sigma^*, \rho^*, \eta^*)$ is the PII’s loss ceiling for the linguistic neutrosophic matrix game $A_1$ if and only if $(z^*, \sigma^*, \rho^*, \eta^*)$ is an efficient solution of the following multi-objective programming problem as follows:
minimize $\sigma$,  
maximize $\rho$,  
maximize $\eta$,  

subject to 

\[ \sum_{j=1}^{q} \sigma_{ij} z_j \leq \sigma \quad (i = 1, 2, \ldots, p), \]
\[ \sum_{j=1}^{q} \rho_{ij} z_j \geq \rho \quad (i = 1, 2, \ldots, p), \]
\[ \sum_{j=1}^{q} \eta_{ij} z_j \geq \eta \quad (i = 1, 2, \ldots, p), \]
\[ z_1 + z_2 = 1, \]
\[ 0 \leq \sigma + \rho + \eta \leq 3, \]
\[ z_1, z_2 \geq 0, \quad \sigma, \rho, \eta \geq 0. \]

(15)

**Proof.** The proof of the theorem is analogous to the one of Theorem 6.5. \qed

7. **An Application.** We consider a real-life example from hotel recommendation system in tourism management. While planning of a trip is booked, it is customary to choose a good place to stay. For this reason booking of hotel via on-line can be an overwhelming task with thousands of hotels to choose from. We decide to booking hotels after reading the statements of previous users of the recommending hotels. After reading the linguistic statements, we have to put some semantic values corresponding to each criteria based on which the hotels are considered. Management bodies of hotels also pay close notice on user-recommended reviews. Suppose there are two hotels in a hill-station aiming to get travellers and tourists as their customers. If one get booked, other not, i.e, if one hotel increases its market-share, the market share of the other decreases. This formatively decides the game structure. These two hotels not only care for getting customers, but also maintain many other elements (cf. [44]), like tourists’ travel demand, tourists’ preference for tourism products, hotel environment, tourism banking service, etc. Among these we only take three objectives in this study: tourists’ travel demand, preference for tourism products and tourism banking. Due to lack of information or imprecise information, the hotel managers are unable to get exact output figures towards payoffs for recommendation. The recommendation list shows the outcomes in linguistic values also. Sometimes optimistic recommendations arise, sometimes pessimistic recommendations are seen and sometimes recommendations have indeterminacy. So, neutrosophic characteristics are seen promptly. Both of these two hotels use two strategies to increase their market shares, namely,

- $\zeta_1$: reduction of rent.
- $\zeta_2$: available car facilities.

The above problem may be regarded as a matrix game. Namely, the hotels $H_1$ and $H_2$ are regarded as the PI and PII, respectively. They may adopt strategies $\zeta_1$ and $\zeta_2$. According to the opinions of the user-recommendation, outcomes are assessed and expressed using linguistic terms as follows. Thus, the game problem in linguistic neutrosophic environment can be put in matrix form, as a matrix game $\tilde{G}_1$: 

[The proof is analogous to Theorem 6.5.]

[The application example is described in detail.]
\[ \tilde{G}_1 = \frac{\zeta_1}{\zeta_2} \begin{pmatrix} \text{Good} & \text{Slightly Good} \\ \text{Fair} & \text{Very Good} \end{pmatrix}. \]

Therefore, \( \tilde{G}_1 \) can be expressed as:

\[
\begin{align*}
\tilde{G}_1 &= \frac{\zeta_1}{\zeta_2} \begin{pmatrix} (T_{s_0,75}, T_{s_0,25}, T_{s_0,45}) & (T_{s_0,70}, T_{s_0,30}, T_{s_0,25}) \\ (T_{s_0,50}, T_{s_0,45}, T_{s_0,37}) & (T_{s_0,90}, T_{s_0,20}, T_{s_0,70}) \end{pmatrix} \\
&= \frac{\zeta_1}{\zeta_2} \begin{pmatrix} (s_{0.75}, s_{0.25}, s_{0.45}) & (s_{0.70}, s_{0.30}, s_{0.25}) \\ (s_{0.50}, s_{0.45}, s_{0.37}) & (s_{0.90}, s_{0.20}, s_{0.70}) \end{pmatrix}.
\end{align*}
\]

Human intuition and description depend upon mind mapping languages, and these languages are mainly based on linguistic semantics. Linguistic terms are put as:

\[ S = \{ s_0 \text{ (very poor), } s_1 \text{ (poor), } s_2 \text{ (slightly poor), } s_3 \text{ (fair), } s_4 \text{ (slightly good), } s_5 \text{ (good), } s_6 \text{ (very good)} \}, \]

in numerical scales, when we consider seven scales of semantics. We convert the numerical scales of semantics within the interval \([0,1]\) into numerical data. The corresponding relations between linguistic terms and neutrosophic sets of the game problem are given in Table 1.

**Table 1. Relations between linguistic terms and neutrosophic sets.**

| Linguistic terms | Semantic neutrosophic sets | Neutrosophic terms |
|------------------|-----------------------------|-------------------|
| Good             | \( (s_{0.75}, s_{0.25}, s_{0.45}) \) | \( (0.75, 0.25, 0.45) \) |
| Slightly Good    | \( (s_{0.70}, s_{0.30}, s_{0.25}) \) | \( (0.70, 0.30, 0.25) \) |
| Very Good        | \( (s_{0.90}, s_{0.20}, s_{0.70}) \) | \( (0.90, 0.20, 0.70) \) |
| Very Poor        | \( (s_{0.05}, s_{0.58}, s_{0.85}) \) | \( (0.05, 0.58, 0.85) \) |
| Slightly Poor    | \( (s_{0.30}, s_{0.45}, s_{0.35}) \) | \( (0.30, 0.45, 0.35) \) |
| Fair             | \( (s_{0.50}, s_{0.45}, s_{0.37}) \) | \( (0.50, 0.45, 0.37) \) |

Using the data from Table 1, we express the matrix game, now as:

\[
\tilde{G}_1 = \frac{\zeta_1}{\zeta_2} \begin{pmatrix} (0.75, 0.25, 0.45) & (0.70, 0.30, 0.25) \\ (0.50, 0.45, 0.37) & (0.90, 0.20, 0.70) \end{pmatrix}.
\]

Thus the multi-objective programming problem for PI is constructed as Model 1 (from Theorem 6.5), given by:

**Model 1**

maximize \( \mu \),

minimize \( \nu \),

minimize \( \tau \),

subject to \( 0.75y_1 + 0.50y_2 \geq \mu \),

\( 0.70y_1 + 0.90y_2 \geq \mu \),

\( 0.25y_1 + 0.45y_2 \leq \nu \),
Similarly, the multi-objective programming problem for PII is constructed as Model 2 (from Theorem 6.6), given by:

**Model 2**

\[ \begin{align*}
\text{minimize} & \quad \sigma, \\
\text{maximize} & \quad \rho, \\
\text{maximize} & \quad \eta, \\
\text{subject to} & \quad 0.75z_1 + 0.70z_2 \leq \sigma, \\
& \quad 0.50z_1 + 0.90z_2 \leq \sigma, \\
& \quad 0.25z_1 + 0.30z_2 \geq \rho, \\
& \quad 0.45z_1 + 0.20z_2 \geq \rho, \\
& \quad 0.45z_1 + 0.25z_2 \geq \eta, \\
& \quad 0.37z_1 + 0.70z_2 \geq \eta, \\
& \quad z_1 + z_2 = 1, \\
& \quad 0 \leq \sigma + \rho + \eta \leq 3, \\
& \quad z_1, z_2 \geq 0, \sigma, \rho, \eta \geq 0.
\end{align*} \]

Now we convert the multi-objective criteria of Models 1 and 2 into a single-objective programming problem using weighted models obtained in Model 3 and Model 4 (respectively, from Model 1 and Model 2).

**Model 3**

\[ \begin{align*}
\text{maximize} & \quad w_1 \mu - w_2 \nu - w_3 \tau, \\
\text{subject to} & \quad 0.75y_1 + 0.50y_2 \geq \mu, \\
& \quad 0.70y_1 + 0.90y_2 \geq \mu, \\
& \quad 0.25y_1 + 0.45y_2 \leq \nu, \\
& \quad 0.30y_1 + 0.20y_2 \leq \nu, \\
& \quad 0.45y_1 + 0.37y_2 \leq \tau, \\
& \quad 0.25y_1 + 0.70y_2 \leq \tau, \\
& \quad y_1 + y_2 = 1, \\
& \quad 0 \leq \mu + \nu + \tau \leq 3, \\
& \quad w_1 + w_2 + w_3 = 1, \\
& \quad y_1, y_2 \geq 0, \mu, \nu, \tau \geq 0, w_1, w_2, w_3 \geq 0.
\end{align*} \]

and **Model 4**

\[ \begin{align*}
\text{minimize} & \quad \omega_1 \sigma - \omega_2 \rho - \omega_3 \eta, \\
\text{subject to} & \quad 0.75z_1 + 0.70z_2 \leq \sigma,
\end{align*} \]

\[ \]
0.50z_1 + 0.90z_2 \leq \sigma, \\
0.25z_1 + 0.30z_2 \geq \rho, \\
0.45z_1 + 0.20z_2 \geq \rho, \\
0.45z_1 + 0.25z_2 \geq \rho, \\
0.37z_1 + 0.70z_2 \geq \eta, \\
z_1 + z_2 = 1, \\
0 \leq \sigma + \rho + \eta \leq 3, \\
\omega_1 + \omega_2 + \omega_3 = 1, \\
z_1, z_2 \geq 0, \sigma, \rho, \eta \geq 0, \omega_1, \omega_2, \omega_3 \geq 0.

Now, for PI, y_1, y_2, \mu, \nu \text{ and } \tau \text{ are considered as decision variables. Table 2 shows the solution results of Eq. (18). From Table 2 we get the Pareto Nash equilibrium strategy for PII is given by (0.30, 0.20, 0.50) (0.8888889, 0.1111111) (0.7222222, 0.2888889, 0.4411111), and Pareto-optimal gain-floor is expressed in (0.47, 0.30, 0.40) (0.8888889, 0.1111111) (0.7222222, 0.2888889, 0.4411111).

Similarly, we can construct the solution sets for PII, where we assume \( z_1, z_2, \sigma, \rho \text{ and } \eta \) as decision variables. The efficient solution set can be easily obtained from Table 3. Table 3 shows the solution results from Eq. (19). Here, \( (z_1^*, z_2^*) = (0.4444444, 0.5555556) \) represents Pareto Nash equilibrium strategy for PII, and loss-ceiling for PII is given by \( (\sigma^*, \rho^*, \eta^*) = (0.7222222, 0.2888889, 0.4411111) \).

Here, we use the simulation tool LINGO 14.0 to solve the problems of Eqs. (18) and (19). Maximizing player with the set of strategies and objective values are shown in Figure 1 and for minimizing player, the set of strategies with corresponding set of objective values are illustrated in Figure 2.

8. Results and discussions. Solving the multi-objective optimization problem, we display the Pareto-optimal solutions of two-person \( 2 \times 2 \) matrix game in linguistic neutrosophic term-set based expressions. Here, hotel \( H_1 \)'s Pareto Nash equilibrium strategy is: (0.8888889, 0.1111111), and Pareto-optimal gain-floor is expressed in linguistic neutrosophic semantics as: \( (T_{s_0,0.7222222}, I_{s_0,0.2888889}, F_{s_0,0.4411111}) \). This value in language can be explained within slightly good and good. One can also express this by saying almost good or more than slightly good or near about good or near about slightly good. So, linguistic recommendation towards booking of hotel \( H_1 \) via online...
Table 3. Tabular values of strategies and loss-ceilings for minimizing player.

| \((\omega_1, \omega_2, \omega_3)\) | \((z_1, z_2)\) | \((\sigma, \rho, \eta)\) | objective value |
|-----------------------------|-----------------|-----------------|-----------------|
| (0.40, 0.30, 0.30)         | (0.8490566, 0.1509434) | (0.7424528, 0.2575472, 0.4198113) | 0.0937736       |
| (0.40, 0.50, 0.10)         | (0.4444444, 0.5555556) | (0.7222222, 0.2777778, 0.3388889) | 0.1161111       |
| (0.50, 0.30, 0.20)         | (0.4444444, 0.5555556) | (0.7222222, 0.2777778, 0.3388889) | 0.2100000       |
| (0.30, 0.30, 0.40)         | (0.8490566, 0.1509434) | (0.7424528, 0.2575472, 0.4198113) | 0.0937736       |
| (0.30, 0.10, 0.60)         | (0.8490566, 0.1509434) | (0.7424528, 0.2575472, 0.4198113) | 0.0937736       |
| (0.70, 0.20, 0.10)         | (0.4444444, 0.5555556) | (0.7222222, 0.2777778, 0.3388889) | 0.2100000       |
| (0.65, 0.25, 0.10)         | (0.4444444, 0.5555556) | (0.7222222, 0.2777778, 0.3388889) | 0.3661111       |
| (1.00, 0.00, 0.00)         | (0.4444444, 0.5555556) | (0.7222222, 0.0000000, 0.0000000) | 0.7222222       |
| (0.00, 1.00, 0.00)         | (0.3333333, 0.6666667) | (0.7666667, 0.2833333, 0.0000000) | -0.2833333      |
| (0.00, 0.00, 1.00)         | (0.8490566, 0.1509434) | (0.7424528, 0.0000000, 0.4198113) | 0.4161111       |
| (0.25, 0.25, 0.50)         | (0.8490566, 0.1509434) | (0.7424528, 0.2575472, 0.4198113) | 0.2018868       |
| (0.50, 0.25, 0.25)         | (0.8490566, 0.1509434) | (0.7424528, 0.2575472, 0.4198113) | 0.3661111       |
| (0.37, 0.33, 0.30)         | (0.8490566, 0.1509434) | (0.7424528, 0.2575472, 0.4198113) | 0.0637736       |
| (0.47, 0.23, 0.30)         | (0.8490566, 0.1509434) | (0.7424528, 0.2575472, 0.4198113) | 0.1637736       |
| (0.30, 0.20, 0.50)         | (0.8490566, 0.1509434) | (0.7424528, 0.2575472, 0.4198113) | -0.0386792      |

Figure 1. Maximizing player with strategies and objective values.

booking system pays customers’ attraction by the terms good or near about good, etc. Similarly, hotel \(H_2\)’s Pareto Nash equilibrium strategy is: \((0.4444444, 0.5555556)\), and Pareto-optimal loss-ceiling is expressed in linguistic neutrosophic semantics as: \((T_{0.7222222}, I_{0.0000000}, F_{0.0000000})\). These values in language can be determined as slightly good, only. Thus, recommendation plays an important role through languages. Here, we observe that different weights to decision variables sets \((\mu, \nu, \tau)\) and \((\sigma, \rho, \eta)\) correspond to different objective values and different strategy sets. So, it is obvious that weights play here the significant roles towards optimal values. Since weights are considered at the end eventually by the decision maker, this decision maker (here, the hotel) plays the most significant role in game-theory supported decision making.

9. **Conclusion and outlook.** In this work, we have proposed linguistic neutrosophic environment. We have chosen here linguistic semantics under neutrosophy logics. To solve our designed multi-objective game phenomena, we have considered a weighting method. In this study, we have considered an indeterminacy characteristic with membership and falsity characteristics assuming neutrosophic sets.
and logic. Moreover, we have testified our proposed multi-objective game model in tourism-management oriented recommendation systems, and have achieved some attractive results. In Table 4, we have compared our way of thinking about the hotel recommendation problem via game phenomena in multi-objective programming with some selected other literatures. We have noticed that others have solved the problems basically in crisp data sets, whereas, we have addressed linguistic and neutrosophic data sets which are more closely related to real-life problems’ expressions. If we assume a \( p \times 2 \) or \( 2 \times q \) or \( p \times q \) matrix game, the calculation pro-

Table 4. Selected studies on hotel recommendation through optimization processes.

| References     | Topic                                      | Used methods                                      | Study space          |
|---------------|--------------------------------------------|--------------------------------------------------|----------------------|
| Carrasco et al. [7] | Hotel-service quality evaluation         | Linguistic MCDM method                            | Linguistic web data  |
| Chan and Wong [8] | Hotel selection                             | Statistical methods                               | Real-life survey data|
| Etaati and Sundaram [12] | Adaptive tourism recommendation system     | ATRS & MCDM models with double loop learning model | Survey data          |
| Godinho et al. [14] | Hotel-location through competitor’s reactions | Game-theoretic gravitational method               | Game and location problem related data |
| Lee et al. [20]     | FIT guests’ perception of hotel location    | Cronbach’s alpha-test, ANOVA                     | Location problem oriented data |
| Our paper          | Hotel recommendation                        | Game-theoretic multi-objective programming       | Linguistic-neutrosophic data |

cedures become time-consuming. This can be seen as a limitation in generalization of our study. However, some theoretical frameworks may be extended to various circumstances and other real-life problems with different degrees of preconditions. In future, research can be conducted to game theory under different uncertainties with multi-objective programming in different fields, like medical diagnosis, business management optimization, aerospace engineering, space design management, manufacturing industry management, weaponry, laboratory research management, waste water management, optimization in renewable energy sources, supply chain
management, etc. Game theory with neutrosophic characteristics can be solved using different methodologies from neuroscience, robotics, humanitarian operations, humanitarian logistics, artificial intelligence, etc.

**Conflict of interest.** The authors would like to announce that there is no conflict of interest.

**Acknowledgments.** The authors would like to thank to the Editors-in-Chief, the Associate Editors and the anonymous Reviewers for very helpful suggestions and comments, which helped to improve the paper.

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