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Abstract

In this work the electromagnetic radius and the polarizability of the mesons are obtained by use of the effective Lagrangians constructed on the one hand with taking into account of general principles of the relativistic quantum field theory and on the other hand with taking into account of the compound relativistic model meson representation. Also using a relativistic constituent quark model based on point form of Poincare-covariant quantum mechanics we calculated electromagnetic radius and the decay constants of the mesons with spinor quarks.
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Introduction

Relativistic few body problem has received a great attention in hadronic and nuclear physics. The most complete results here exist for the case of two particles. Description of the bound system in the relativistic quantum field theory is founded on the four-dimensional covariant Bethe-Salpeter equation [1]. However, this equation gives series of difficulties when the practical calculations are made.

There exist various reductions of the two-body Bethe-Salpeter equation. Different forms of this reduction were discussed Logunov-Tavkhelidze [2], Kadyshevsky [3], Todorov [4], Gross [5], Poluzou,Keister,Lev [6]-[7] and many others.

The relativistic two-body systems was analyzed by Weinberg [8], Frankfurt - Strikman [9], Kondratyuk-Terent’ev [10] in infinite-momentum frame. Some authors use diagrammatic approach i.e. they select leading diagrams and project them onto the three-dimensional space. Others make use effective Hamiltonians, Faddev equation.

A more general covariant perturbation theory with spurions was proposed Kadyshevsky [11]. Different forms of the quasi-potential equation can be derived using this approach [11]-[12]. The method of the quasipotential approach and the method of the contact interaction approach as consequences of the Bethe-Salpeter equation obtained broad recognition in the practical application [13].

Last years accuracy of measurement of the hadron electromagnetic characteristic improves considerably. This fact permits to analyze the existing field-theoretical and model conception of interaction of the hadrons with the electromagnetic field do better: which why we investigate some electroweak properties of the mesons.
In the paper [14] in the framework of the quasipotential approach [2], relativistic composite models for computation of the electric polarizability and the mean square radius of charge distribution and radial excitations of mesons as bound systems of valence quarks are proposed. In this work the electromagnetic radius and the polarizability of the mesons are obtained by use of the effective Lagrangians [15]-[17], constructed on the one hand with taking into account of general principles of the relativistic quantum field theory and on the other hand with taking into account of the compound relativistic model meson representation. Also using a relativistic constituent quark model based on point form of Poincare-covariant quantum mechanics we calculated electromagnetic radius and the decay constants of the mesons with spinor quarks.

**Effective Lagrangian approach**

Consider the scalar bound system which consists of the charged scalar particles. The effective Lagrangian of interaction of the electromagnetic field with the hadrons is represented as covariant expansion in terms of tensor of the electromagnetic field $F_{\mu\nu}$ and potential $A_\mu$

$$L_{eff} = \frac{1}{8M^2} \left( \partial_\mu \varphi^+ \right) \left( \partial_\nu \varphi \right) + \left( \partial_\mu \varphi \right) \left( \partial_\nu \varphi^+ \right) -$$

$$- \left( \varphi^+ \partial_\mu \varphi + \varphi \partial_\mu \varphi^+ \right) \left[ (\alpha + \beta) F^{\mu\sigma} F^\nu_{\sigma} - \frac{1}{2} \beta g_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right] +$$

$$+ e^2 A^2_{\mu} \varphi^+ \varphi + i e \left[ \varphi^+ \partial_\mu \varphi \right] A^\mu, \quad (1)$$

where $\varphi$ is the field function of scalar charge structural particle, $\alpha$ and $\beta$ is the electric and magnetic static polarizabilities. Let us consider a generating functional

$$exp(iS_{eff}) = Z(A). \quad (2)$$

For calculation $S_{eff}$ in (2) we confine ourselves by the model with two the vertex $\Gamma$. As results we have

$$iS_{eff} = \langle \Delta(A; Z_1 - Z'_1) \Gamma(A; Z'_1, Z'_2; Z) \Delta(A; Z_2 - Z'_2) \Gamma(A; Z_1, Z_2; Z) \rangle, \quad (3)$$

where $\Delta(A; Z_1 - Z_2)$ is the scalar particle propagator in electromagnetic field, $\Gamma(A; Z_1, Z_2; Z)$ is the three-point vertex function in the electromagnetic field. Using the reduction technique methods, one can get a connection of the Bethe-Salpeter function with the vertex function

$$\chi_P (Z_1, Z_2) = i \langle \exp (-iPZ) \Delta (Z_1 - Z'_1) \Gamma (Z'_1, Z'_2; Z) \Delta (Z'_2 - Z_2) \rangle, \quad (4)$$
where \( \mathcal{P} \) is the total impulse of compound system. The brackets in this equation mean the integration with respect to \( Z'_1, Z'_2 \) and \( Z \) variables. The function (4) obeys to the homogeneous Bethe-Salpeter equation [18].

\[
\chi_{\mathcal{P}}(Z_1, Z_2) = i(\Delta(Z_1 - Z'_1)\chi_{\mathcal{P}}(Z'_1, Z'_2)\Delta(Z'_2 - Z_2)V(Z'_1 - Z'_2)),
\]

where \( V \) is the interaction potential. In the contact interaction approximation and in impulse representation the equation (5) is:

\[
\Delta^{-1}(p_1)\Delta^{-1}(p_2)\chi_{\mathcal{P}}(p) = i\frac{g}{(2\pi)^4}\int dq\chi_{\mathcal{P}}(q),
\]

where \( p = p_1 + p_2, \mathcal{P} = \frac{1}{2}(p_1 - p_2) \). After integration over \( dp_0 \) in (6) we obtain

\[
\chi_{\mathcal{P}}(\vec{p}) = \frac{N}{E_1 + E_2} \frac{E_1 + E_2}{E_1E_2[(E_1 + E_2)^2 - E^2]},
\]

where \( E_i = \sqrt{p_i^2 + m_i^2}; \: i = 1, 2; \: E = \sqrt{\mathcal{P}^2 + M^2}, \: N \) is the normalizing factor, \( M \) is the compound system mass. Using the relativistic covariant normalization [19, 20].

\[
\frac{i}{(2\pi)^4}\int dpdp'\chi_{\mathcal{P}}(p)[\frac{\partial}{\partial E}(K - V)]\chi_{\mathcal{P}}(p') = 2E,
\]

where

\[
K(p, p'; \mathcal{P}) = \Delta^{-1}(p_1)\Delta^{-1}(p_2)\delta(p - p'),
\]

we obtain

\[
N^2 = \frac{1}{\left(\frac{2m^2}{M^2} atan\frac{M}{2\beta} - 1\right)},
\]

where \( \beta = \sqrt{m^2 - M^2} \).

**Electromagnetic radius and the polarizabilities**.

The pion \( \langle r^2 \rangle \) is calculated, using (4), is

\[
\langle r^2 \rangle = \frac{1}{8\beta^2 M^2} \frac{M^2 + \frac{M^4 + 48\beta^4}{2M^2} atan\frac{M}{2\beta} - 12\beta^2}{\frac{2m^2}{M^2} atan\frac{M}{2\beta} - 1}.
\]

It follows from (4), when the connection energy is weak (the compound system is non-relativistic)

\[
\langle r^2 \rangle \sim \frac{1}{8\beta^2};
\]
and when the connection energy is strong (the compound system is relativistic)

\[ \langle r^2 \rangle \sim \frac{0.3}{m^2}. \]  

(11)
The numerical analyze (9) gives evidence of accordance with the experimental value \( \langle r^2 \rangle \) when the mass of a \( \pi \)-meson is \( M = 140 \text{ MeV} \) and the mass of a quark is \( m = 210 \text{ MeV} \). This result is in accordance with (11). Consequently, \( \pi \)-meson is the strong-bound relativistic quark system.

In order to calculate the electrical polarizability of a \( \pi \)-meson we confined by the model of interaction of the electromagnetic field. It follows from the equations (1), (3) and (7) the electrical polarizability of a \( \pi \)-meson is defined by

\[ \alpha = \left( \frac{e^2}{4\pi} \right) \frac{16\pi N^2}{M^3} \mathcal{I}. \]  

(12)
Here

\[ \mathcal{I} = \int_0^1 dx \{ (\hat{Q}^2 + \hat{\bar{Q}}^2) \frac{\frac{1}{x^2} - \frac{1}{2} x^3 + \frac{1}{2} x^4}{[x(1-x) - \eta^2]^2} - \hat{Q} \hat{\bar{Q}} \frac{\frac{1}{2} x^2(1-x)^2}{[x(1-x) - \eta^2]^2} - \}

(13)
where \( \hat{Q} \) and \( \hat{\bar{Q}} \) are the values of quark and antiquark charges, \( \eta = \frac{m}{M} \). In the strong coupling we obtain from Eq. (13).

\[ \alpha = \frac{e^2}{4\pi m^2 M} \left[ (\hat{Q}^2 + \hat{\bar{Q}}^2) \left( \frac{1}{40} + \frac{2M^2}{315m^2} \right) - (\hat{Q} \hat{\bar{Q}}) \left( \frac{1}{120} + \frac{M^2}{140m^2} \right) \right]. \]  

(14)
The numerical estimations (14), when \( m = 210 \leq M \text{ MeV} \), have a value \( \alpha \), which agrees to an experiment [22] we shall explain how can be calculated the polarizability in the quasipotential approach [2]. Let us assume that

\[ \Gamma(A|Z_1Z_2Z) = \tilde{\Gamma}(A)\delta(Z_{10} - Z_0)\delta(Z_{20} - Z_0). \]

Then one can derive the equation

\[ iS_{eff} = \langle \tilde{\Gamma}(A)\tilde{G}^{(0)}(A)\tilde{\Gamma}(A) \rangle, \]  

(15)
where \( \tilde{G}^{(0)}(A) \) is the three-dimensional Green function in the external electromagnetic field. Let us introduce the wave function, that will obey the quasipotential equation

\[ \tilde{G}^{(0)-1}(A)\tilde{\Phi}(A) = \tilde{V}(A)\tilde{\Phi}(A), \]  

(16)
where $\tilde{V}(A)$ is the quasipotential of a system in the electromagnetic field. Then (15) takes the form

$$iS_{\text{eff}} = \langle \tilde{\Phi}(A)\tilde{G}^{(0)}(A)\tilde{\Phi}(A) \rangle. \quad (17)$$

Consider a particular case when the system interacts with the constant electric field. Restricting by the first order in field $\vec{E}$ in the left- and right-hand side (17) we get the electric radius of a system

$$\langle r^2 \rangle = \int d\vec{q}\varphi_n(\vec{q})(-\frac{i}{2}\vec{\partial}_q)^2\varphi_n(\vec{q}) + \frac{3}{16}\int d\vec{q}|\varphi_n(\vec{q})|^2\frac{\vec{q}^2}{E_q^4}; \quad (18)$$

where $\varphi_n(\vec{q})$ is the wave functions (18) of the relative movement of the quark in the rest frame system, $E_q = \sqrt{\vec{q}^2 + m^2}$, $m$ is the quark mass.

Comparing the structures on (17) being proportional to the second order in field $\vec{E}$ we obtain the expression for the electric polarizability [14]

$$\alpha = \frac{1}{|\vec{E}|^2} \left\{ \sum_\xi \int d\vec{q}\varphi_n(\vec{q})\chi_n^+(\xi)i(\vec{Q}_1 - \vec{Q}_2)(\vec{E}\vec{q}_q)\varphi_n^{(1)}(\vec{q}, \xi) - \frac{1}{M} \sum_\xi \int d\vec{q}\varphi_n(\vec{q})\chi_n^+(\xi)\Gamma^{(2)}(\vec{q})\chi_n(\xi)\varphi_n(\vec{q}) - \frac{1}{M} \sum_\xi \int d\vec{q}\varphi_n(\vec{q})\chi_n^+(\xi)\frac{1}{2}(\vec{Q}_1 - \vec{Q}_2)(\vec{E}\vec{q}_q)^2\chi_n(\xi)\varphi_n(\vec{q}) \right\}. \quad (19)$$

In the equation (18) and (19) the functions $\chi_n(\xi)$ are flavour wave function of the quarks,

$$\Gamma^{(2)}(\vec{q}) = (\vec{Q}_1^2 + \vec{Q}_2^2)(M^2 - 10E_q^2)\frac{E_q^2}{8E_q^4} +$$

$$+(\vec{Q}_1^2 + \vec{Q}_2^2)(53E_q^2 - 5M^2)\frac{(\vec{E}\vec{q}_q)^2}{16E_q^6} - 3\vec{Q}_1\vec{Q}_2\frac{2(\vec{E}\vec{q}_q)^2}{8E_q^4}. $$

The wave functions $\varphi_n^{(1)}(\vec{q}, \xi)$ obey the quasipotential equation in the first order in the external field. The interaction of the dipole moment with the electric field yields the first term (19) ($\alpha_{\text{dip}}$). All the other terms (19) are determined by the relativistic interaction of the quarks with the
external field \((\alpha_{rel})\). The quasipotential equation \((16)\) for the system of scalar quarks in the first order in the field \(\vec{E}\) is the form
\[
[\vec{\nabla}^2 - \vec{V}(r) - m^2 + \frac{M_n^2}{2} \left( \frac{\vec{Q}_2 - \vec{Q}_1}{2e} \right) (e \vec{E} r)] \varphi_n^{(1)}(\vec{q}, \xi) = 0.
\]

We shall consider the model with Coulomb and oscillator quasipotential \((C = const)\)
\[
V_{coul} = -g r - C, \\
V_{osc} = -g r^2 - C.
\]

The wave function for the quasipotential \((20)\) have the form \((n = 0)\)
\[
\varphi_{0}^{coul}(r) = Aexp(-\beta_{coul}^{0}r), \quad \varphi_{0}^{osc}(r) = Aexp(-\frac{1}{6} \beta_{osc}^{0}r^2).
\]

The parameters of wave functions \(\varphi_{0}(r)\) are determined by the condition of the energy quantum
\[
\beta_{n}^{coul} = \frac{g^2}{4(n+1)^2}, \quad \beta_{n}^{osc} = -\sqrt{g(n + \frac{3}{2})},
\]
where
\[
\beta_{n}^{coul} = m^2 - C - \frac{M_n^2}{4}, \quad \beta_{n}^{osc} = \frac{M_n^2}{4} - m^2 + C.
\]

The parameters \(g\) and \(C\) are in agreement with the masses \(\pi'(1300)\) and \(K'(1400)\). For \(\pi\)-mesons \(m = 330 MeV\), for \(K\)-mesons \(m = 420 MeV\).

Under these parameters the averaged squared radiuses of \(\pi\)- and \(K\)-mesons \((18)\) have the same order in Coulomb and oscillator models:
\[
\sqrt{\langle r_{\pi}^2 \rangle^{coul}} = 0.25 f_m, \quad \sqrt{\langle r_{\pi}^2 \rangle^{osc}} = 0.21 f_m,
\]
\[
\sqrt{\langle r_{K}^2 \rangle^{coul}} = 0.26 f_m, \quad \sqrt{\langle r_{K}^2 \rangle^{osc}} = 0.23 f_m.
\]

We obtain the same order for polarizability in Coulomb and oscillator models (the numerical values are given in units of \(10^{-4}f_m^3\)).
\[
\alpha_{\pi coul}^{0} = 1.83, \quad \alpha_{\pi coul}^{\pm} = 3.63, \quad \alpha_{K coul}^{0} = 0.24, \quad \alpha_{K coul}^{\pm} = 0.93;
\]
\[
\alpha_{\pi osc}^{0} = 1.68, \quad \alpha_{\pi osc}^{\pm} = 4.02, \quad \alpha_{K osc}^{0} = 0.27, \quad \alpha_{K osc}^{\pm} = 1.05.
\]

Due to the factor \(m^2\) in \((13)\), the relativistic correction to the polarizability \((\alpha_{rel})\) make the essential contribution compared with an ordinary polarizability \((\alpha_{dip})\).
RQM formalism for $q\bar{q}$ bound states

The formulation of relativistic quantum mechanics (RQM) differs from nonrelativistic quantum mechanics by the replacement of invariance under Galilean transformations with invariance Poincare transformations. The dynamics of many-particle system in the RQM is specified by expressing the ten generators of the Poincare group $\hat{M}_{\mu\nu}$ and $\hat{P}_\mu$ in terms of dynamical variables. In the constructing generators for interacting systems it is customary to start with the generators of the corresponding noninteracting system (we shall write this operators without "a hat") and then add interactions in the a way that is consistent with Poincare algebra. In the relativistic case it is necessary to add an interaction $V$ to more than one generator in order to satisfy the commutation relations of the Poincare algebra. Dirac [23] observed, that there is no unique way of separating the generators into dynamical subset (the generators including interaction $V$) and kinematical subset. Kinematical subset must be associated with some subgroup Poincare group, usually called stability group [24] or kinematic subgroup [23].

There are three forms of the dynamics in the relativistic quantum mechanics, called "instant", "point", "light-front" forms [23]. The description in the instant form implies that the operators of three-momentum and angular momentum do not depend on interactions i.e. $\hat{\vec{P}} = \vec{P}$ and $\hat{\vec{J}} = \vec{J}$ ($\vec{J} = (\hat{M}^{23}, \hat{M}^{31}, \hat{M}^{12})$) and interactions may be presents in operator $\hat{P}^0$ and generators of the Lorentz boosts $\hat{N} = (\hat{M}^{01}, \hat{M}^{02}, \hat{M}^{03})$. The description in the point form implies that the operators $\hat{M}_{\mu\nu}$ are the same as for noninteracting particles, i.e. $\hat{M}_{\mu\nu} = M_{\mu\nu}$, and thus interaction terms can be present only in the four-momentum operators $\hat{P}$. In the front form with the marked $z$ axis we introduce the + and - components of the four-vectors as $p^+ = (p^0 + p^z)/\sqrt{2}$, $p^- = (p^0 - p^z)/\sqrt{2}$. We require that in the front form the operators $\hat{P}^+, \hat{P}^0, \hat{M}^{12}, \hat{M}^{+-}, \hat{M}^{+j}$ ($j = 1, 2$) are the same as the corresponding free operators, and interaction terms may be present in the operators $\hat{M}^{--}$ and $\hat{P}^-$. It is significant that four-momentum of the two-particle bound system $P$ is not conserved i.e. $P \neq p_1 + p_2$, where $p_1$ and $p_2$ are four-momenta of the particles of noninteracting system.

The three momenta $\vec{p}_1$, $\vec{p}_2$ of the particles (in our case, the quarks) with the masses $m_1$ and $m_2$ of relativistic system can be transformed to the total $\vec{P}$ and relative momenta $\vec{k}$ to facilitate the separation of the center
mass motion:
\[ \vec{P}_{12} = \vec{p}_1 + \vec{p}_2, \]
\[ \vec{k} = \vec{p}_1 + \frac{\vec{P}_{12}}{M_0} \left( \frac{\left( \vec{P}_{12} \cdot \vec{p}_1 \right)}{\left( \vec{P}_{12} \cdot \vec{p}_1 \right)} + \omega_{m_1} (\vec{p}_1) \right), \]
\[ (21) \]
where
\[ M_0 = \omega_{m_1} (\vec{k}) + \omega_{m_2} (\vec{k}), \omega_{m_1} (\vec{p}_1) = \sqrt{\vec{p}_1^2 + m_1^2}. \]
\[ (22) \]
The eigenvalue problem for the mass of $q\bar{q}$ system can be expressed in the two equivalent forms [26]:
\[ \hat{M} \mid \Psi > \equiv (\hat{M}_0 + \hat{V}) \mid \Psi > = M \mid \Psi >, \]
\[ (k^2 + \hat{W}) \mid \Psi > = \eta \mid \Psi >, \]
\[ (23) \]
where the mass of bound $q\bar{q}$ system $M$ and $\eta$ have relationship:
\[ M^2 = 2\eta + m_1^2 + m_2^2 + 2\sqrt{\eta(\eta + m_1^2 + m_2^2)} + m_1^2 m_2^2 \]
and $k \equiv |\vec{k}|$.

The solution any of the above eigenvalue problems will lead to eigenfunctions of the form
\[ o \left< \vec{V}_{12} \mu, [Jk] \parallel Jk \right> \mid \vec{V} \mu, [JM] > = \]
\[ = \left( \frac{M}{M_0} \right)^{3/2} \delta_{jj'} \delta_{\mu\mu'} \delta(\vec{V} - \vec{V}_{12}) \Psi^{J\mu} (k l s; M) \]
\[ (24) \]
with the velocities of bound system $\vec{V} = \vec{P}/M$ and noninteracting system $\vec{V}_{12} = \vec{P}_{12}/M_0$. The function $\Psi^{J\mu} (k l s; M)$ satisfies in the point form a following equation [26]:
\[ \sum_{l' s'} \int_0^\infty < k l s \parallel W^J \parallel k' l' s'> \Psi^{J} (k' l' s'; M) k'^2 dk' + \]
\[ + k^2 \Psi^{J} (k l s; M) = \eta \Psi^{J} (k l s; M) \]
\[ (25) \]
with reduced matrix element of operator $\hat{W}$.

The state vector $\left| \vec{V}_{12} \mu, [Jk] \parallel (l s) >$ is the eigenstate of operators $\vec{V}_{12}$, $\hat{J}^2$ (angular momentum), $\hat{J}_3$ and also $\vec{L}^2$, $\vec{S}^2$, where $\vec{L}$ and $\vec{S}$ are relative
orbital momentum and total spin momentum accordingly. This vector of the noninteracting $q\bar{q}$ system transforms irreducibly under Poincare transformations. The vector $\left| \hat{\vec{V}} \mu, [J M] \right\rangle$ is eigenstate of the interacting system, that also transforms irreducibly.

In this approach the meson state is defined by a state of on-shell quark and antiquark with the wave function $\Psi^{J\mu} (k l s; M)$

$$\left| \hat{P}_\mu [JM] \right\rangle = \sum_{ls} \sum_{\lambda_1 \lambda_2} \int d^3 \vec{k} \sqrt{\omega_{m_1} (\vec{p}_1) \omega_{m_2} (\vec{p}_2)} \omega_{M_0} (\vec{P}_{12})$$

$$\Psi^{J\mu} (k l s; M) \sum_{m\lambda \nu_1\nu_2} \langle s_1 \nu_1, s_2 \nu_2 | s \lambda \rangle \langle l m, s \lambda | J \mu \rangle$$

$$Y_{lm} (\theta, \phi) D^{1/2}_{\lambda_1\nu_1} (\vec{n} (p_1, P_{12})) D^{1/2}_{\lambda_2\nu_2} (\vec{n} (p_2, P_{12}))$$

$$\langle p_1 \lambda_1 | p_2 \lambda_2 \rangle$$

(26)

where $\langle s_1 \nu_1, s_2 \nu_2 | s \lambda \rangle, \langle l m, s \lambda | J \mu \rangle$ are Clebsh-Gordan coefficients of $SU(2)$-group, $Y_{lm} (\theta, \phi)$ - spherical harmonic with spherical angle of $\vec{k}$. Also, in Eq.(26)

$$D^{1/2} (\vec{n}) = \frac{1 - i (\vec{n} \vec{\sigma})}{\sqrt{1 + \vec{n}^2}}$$

is $D$-function of Wigner rotation, which determined by vector-parameter

$$\vec{n} (p_1, p_2) = \frac{\vec{u}_1 \times \vec{u}_2}{1 - (\vec{u}_1 \cdot \vec{u}_2)}$$

with $\vec{n} = \vec{p} / (\omega_m (\vec{p}) + m)$.

**Basic Requirements of Current Operator**

Current operators $\hat{J}^\mu (x)$ of bound systems is required for a calculation of the decay constants, the charge form factors and the other properties of relativistic particles. Since $\hat{J}^\mu (x)$ is a four-vector operators, it has same transformation properties as the four-momentum $\hat{P}_\mu$ under Poincare transformation. It implies that the commutation relations between $\hat{J}^\mu (x)$ and Poincare generators $\hat{M}^{\rho\sigma}, \hat{P}_\mu$ is identical to the commutation relations between the generators of the Poincare group and four momentum:

$$[\hat{M}^{\rho\sigma}, \hat{J}^\mu (x)] = i \left( g^{\rho\sigma} \hat{J}^\mu (x) - g^{\mu\sigma} \hat{J}^\sigma (x) \right) - i \left( x^\rho \frac{\partial \hat{J}^\mu (x)}{\partial x_\sigma} - x^\sigma \frac{\partial \hat{J}^\mu (x)}{\partial x_\rho} \right)$$

(27)
\[ \left[ \hat{P}_\mu, \hat{J}_\gamma(x) \right] = -i \frac{\partial \hat{J}_\mu(x)}{\partial x^\gamma} \]  

(28)

The translational invariance of the current operators implies that

\[ \hat{J}_\mu(x) = \exp \left( i \hat{P} x \right) \hat{J}_\mu(0) \exp \left( -i \hat{P} x \right) \]  

(29)

This relation makes it possible to reduce problem of seeking \( \hat{J}_\mu(x) \) to the problem of seeking \( \hat{J}_\mu(0) \) (see, for example [27], [3]). The requirements of Lorentz invariance (27) reduces to

\[ \left[ \hat{M}^{\mu\sigma}, \hat{J}_\mu(0) \right] = i \left( g^{\mu\sigma} \hat{J}_\rho(0) - g^{\mu\rho} \hat{J}_\sigma(0) \right) \]  

(30)

If the theory is invariant under the space reflection and time reversal, and \( \hat{U}_P, \hat{U}_T \) are the corresponding representation operators, then the current operator should be satisfy the following conditions:

\[ \hat{U}_P (\hat{J}^0(x^0, \vec{x}), \vec{J}(x^0, \vec{x})) \hat{U}_P^{-1} = (\hat{J}^0(x^0, -\vec{x}), -\vec{J}(x^0, -\vec{x})) \]

\[ \hat{U}_T (\hat{J}^0(x^0, \vec{x}), \vec{J}(x^0, \vec{x})) \hat{U}_T^{-1} = (\hat{J}^0(-x^0, \vec{x}), -\vec{J}(-x^0, \vec{x})) \]  

(31)

In addition to these equations continuity equation \( \partial \hat{J}_\mu(x)/\partial x^\mu = 0 \) can be used. As follows from (28) this requirements can be written in the form

\[ \left[ \hat{P}_\mu, \hat{J}_\nu(0) \right] g^{\mu\nu} = 0. \]  

(32)

Finally, the operator \( \hat{J}_\mu(x) \) should be also satisfy the cluster separability [26, 6, 7, 28] for many-particle system.

For calculations many authors assumed that the mathematical expressions of current operators for bound system and for the noninteracting system are equal. This condition (so called, impulse approximation)

\[ \hat{J}_\mu(0) = J_\mu(0) \]  

(33)

reasonable only in point form of the Poincare relativistic mechanics ([29]). This result is evident from Eq. (30).

**Decay constants and charge radius of pseudoscalar mesons in RQM formalism**

10
The decay constant $f_p$ for $\pi^\pm$ is defined by

$$\langle 0 \left| \hat{J}^\mu (0) \right| \vec{P}, M \rangle = i (1/2\pi)^3 \frac{1}{\sqrt{2\omega_M (\vec{P})}} P^\mu f_p, \quad (34)$$

where $\hat{J}^\mu (0)$ is the operator axial-vector part of the charged weak current after a Cabibbo-Kobayashi-Maskawa mixing matrix element $V_{qq'}$ has been removed. The state vector is normalized by $\langle \vec{P}, M \left| \vec{P}', M' \rangle = \delta (\vec{P} - \vec{P}') \rangle$.

The current operator of the meson can be defined by the axial-vector current of the free quarks fields:

$$\hat{J}^\mu (0) = \hat{d} (0) \gamma^\mu \gamma^5 \hat{u} (0).$$

Using Eq.(26) and Eq.(34) we found, that

$$f_p = \frac{\sqrt{2m N_c}}{\pi \sqrt{M}} \int_0^\infty \frac{dk \, k^2}{\omega_m (\vec{k})} \Psi (k, M), \quad (35)$$

where $N_c$-number of colors and the wave function for pseudoscalar meson have normalization condition

$$\int_0^\infty \, dk \, k^2 N_c |\Psi (k, M)|^2 = 1$$

For pion we assumed, that the masses of $u$ and $d$-quarks are equal i.e. $m_u = m_d \equiv m$. In the nonrelativistic quark model, when $k^2 \ll m^2$, meson decay constants are given

$$f_p \sim \frac{1}{\sqrt{M}} \Psi (r = 0) \quad (36)$$

with $\Psi (r = 0) \sim \int d^3 k \, \Psi (k)$.

Amplitude for decay $\pi \rightarrow \gamma + \gamma$ can be parameterized as

$$M_{\pi^0 \rightarrow \gamma \gamma} (k_1, k_2) = (-1) \frac{4\alpha}{(2\pi)^{3/2}} g_{\pi\gamma\gamma} \epsilon_{\mu\nu\rho\sigma} k_1^\mu k_2^\nu \xi^\rho (k_1) \xi^\sigma (k_2), \quad (37)$$

where $k_1, k_2$ are momenta of photons having polarization vectors $\xi^\rho (k_1)$ and $\xi^\sigma (k_2)$. The calculations of the matrix element $M_{\pi^0 \rightarrow \gamma \gamma} (k_1, k_2)$ for real photons in the lowest order implies that

$$g_{\pi\gamma\gamma} = \frac{m N_c}{\pi M^{3/2}} \int_0^\infty \frac{dy \, y}{\sqrt{y^2 + 1}} \ln \left| \frac{\sqrt{y^2 + 1} + y}{\sqrt{y^2 + 1} - y} \right| \Psi (y m, M) \quad (38)$$
The equation of motion of the bound $\bar{q}q$ states \cite{29} in the RQM is relativistic equation with appropriate effective potential $W$ (or $V$). However, it is hard problem to obtain wave function $\Psi(k,M)$ as solution of this equation. Therefore, we use simple model wave function depending on length scale parameter $\beta$:

$$\Psi(k,M) \equiv \Psi(k,\beta) = 2/\sqrt{N_c\beta^{3/2}\pi^{1/4}} \exp(-k^2/2\beta^2).$$ (39)

And second parameter of this model is the constituent quark mass $m$. They are fixed by fitting the relevant experimental data. We use the following values for decay constant $f_p = 130.7 \pm 0.1 \pm 0.36 \text{ MeV}$ \cite{30} and for coupling constant $g_{\pi\gamma\gamma} = 0.0839 \pm 0.0013 \text{ MeV}^{-1}$, which determined from radiative widths $\Gamma(\pi \rightarrow \gamma + \gamma) = 7.25 \pm 0.13 \text{ eV}$ \cite{31}. From these values we obtain the allowed region for the mass $m$ of quark and the parameter $\beta$ in wave function \cite{33}:

$$m = 155 \pm 5 \text{ MeV}, \quad \beta = 150 \pm 5 \text{ MeV}. \quad (40)$$

Our optimal value of light quark mass $m = 155 \text{ MeV}$ is not in agreement with result of $m = 250 \text{ MeV}$ (see, \cite{32}, \cite{33}), but is roughly in agreement with result of $m = 170 \text{ MeV}$ \cite{34}.

The most general form of the hadronic matrix element of the vector current must be represented in terms of two form factors

$$\langle P, M' | \bar{q}'(0)\gamma^\mu q(0) | P, M \rangle = \frac{1}{2 \ast (2\pi)^3 \sqrt{\omega_M(P) \omega_{M'}(P')}} (F_+ (t) (P\mu' + P_\mu) + F_- (t) (P_\mu' - P_\mu)),$$

where $t = -(P' - P)^2$, $q(x)$—operator of the quark field. In the case of a pion we shall consider only $F_+ (t)$. On rearrangement, as a result we obtain following expression for the form factor:

$$F_+ (t) = \frac{1}{2} \int_0^\infty dk k^2 \Psi(k) \int_0^1 dx [\Psi(k_1) + \Psi(k_2) -$$

$$- \frac{k}{\omega_m(k^2)} x (\Psi(k_1) - \Psi(k_2))] +$$

$$+ \frac{1}{2} \int_0^\infty dz \frac{1}{z^2} \left[ \frac{1}{2} \int_0^\infty dk' k'^2 \Psi(k') \int_0^1 dx' [\Psi(k_1') + \Psi(k_2') -$$

$$- \frac{k'}{\omega_m'(k'^2)} x' (\Psi(k_1') - \Psi(k_2'))] \right].$$
where $k_i^2 = k^2 + \Delta_i$, $\Delta_i = 4\gamma^4 |V|^2 \left( \omega_m^2 (\vec{k}^2) + k^2 x^2 \right) + (-1)^i |V|^2 (1 + |V|^2)$, $\gamma = 1 / \left( 1 - |V|^2 \right)$, $|V|^2 = t/(t + 4M^2)$.

Using the definition mean square radius: $\langle r^2 \rangle = -6 \, dF_+ (t) / dt$ at $t = 0$ we obtain the final relativistic pion charge radius for the wave function $\Psi (k)$ in simple form:

$$\langle r^2 \rangle = \frac{3}{M^2} \left( \frac{m^2}{2\beta^2} + \frac{5}{4} \right).$$  \hfill (41)

Unfortunately for authors, numerical calculation of the mean square radius is not in agreement with experimental result $\langle r^2 \rangle = 0.44 \, fm^2$ for pion. But we thinks, that more appropriate form of the wave function $\Psi (k)$ furnishes the desired result.

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