TOWARD A DETERMINISTIC MODEL OF PLANETARY FORMATION. IV. 
EFFECTS OF TYPE I MIGRATION

S. Ida
Tokyo Institute of Technology, Ookayama, Meguro-ku, Tokyo 152-8551, Japan; ida@geo.titech.ac.jp

AND

D. N. C. Lin
UCO/Lick Observatory, University of California, Santa Cruz, CA 95064; and Kavli Institute of Astronomy and Astrophysics, Peking University, Beijing, China; lin@ucolick.org

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ABSTRACT

In a further development of a deterministic planet formation model (Ida & Lin), we consider the effect of type I migration of protoplanetary embryos due to their tidal interaction with their nascent disks. During the early phase of protostellar disks, although embryos rapidly emerge in regions interior to the ice line, uninhibited type I migration leads to their efficient self-clearing. But embryos continue to form from residual planetesimals, repeatedly migrate inward, and provide a main channel of heavy-element accretion onto their host stars. During the advanced stages of disk evolution (a few Myr), the gas surface density declines to values comparable to or smaller than that of the minimum mass nebula model, and type I migration is no longer effective for Mars-mass embryos. Over wide ranges of initial disk surface densities and type I migration efficiencies, the surviving population of embryos interior to the ice line has a total mass of several \( M_\oplus \). With this reservoir, there is an adequate inventory of residual embryos to subsequently assemble into rocky planets similar to those around the Sun. However, the onset of efficient gas accretion requires the emergence and retention of cores more massive than a few \( M_\oplus \) prior to the severe depletion of the disk gas. The formation probability of gas giant planets and hence the predicted mass and semimajor axis distributions of extrasolar gas giants are sensitively determined by the strength of type I migration. We suggest that the distributions consistent with observations can be reproduced only if the actual type I migration timescale is at least an order of magnitude longer than that deduced from linear theories.

Subject headings: planetary systems: formation — solar system: formation

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1. INTRODUCTION

In radial velocity surveys of nearby stars, more than 200 extrasolar planets with masses \( M_p \) comparable to that of Jupiter, \( M_J \), have been discovered. Current statistics indicate that \( \eta_J > 7\% \) of all nearby F, G, and K dwarfs on various search target lists have gas giant planets with semimajor axes of \(< 5 \) AU (e.g., Marcy et al. 2005; Mayor et al. 2005). But the extrapolation of existing data suggests that a higher fraction of solar-type stars may have longer period gas giant planets (Cumming 2004).

In the previous papers of this series (Ida & Lin 2004a, 2004b, 2005, hereafter Papers I, II, and III, respectively), we carried out several sets of simulations with a numerical prescription that is based on the sequential accretion scenario. We used these results to explain the observed mass and orbital distributions of extrasolar planets and to constrain the intrinsic physics of planet formation through detailed comparisons with the observed data.

In these previous investigations, we have taken into account the effect of gas depletion by assuming that the gas surface density \( \Sigma_g \) declines everywhere exponentially over a characteristic timescale of \( \tau_{\text{dep}} \approx 1 \text{--} 10 \text{ Myr} \). With an \( \alpha \) prescription for the turbulent viscosity, this assumed global evolution of \( \Sigma_g \) can lead to declining accretion rates that are consistent with those observed (Hartmann et al. 1998). The magnitude of \( \tau_{\text{dep}} \) sets a strong constraint on the gas giant planet-building efficiency (e.g., Paper I).

We have also assumed that the dust surface density \( \Sigma_d \) in these disks is preserved from the initial value we have adopted. The dust grains’ mass inferred from millimeter observations of

\[ \text{continuum radiation from protostellar disks (e.g., Beckwith \\
\& Sargent 1996) appears to have a wide dispersion centered around} \]
\[ \text{the value of the minimum-mass solar nebula (MMSN) model} \]
\[ \text{(Hayashi 1981). We adopted a similar distribution for \( \Sigma_d \) centered} \]
\[ \text{around that of the MMSN. This assumption, although it greatly} \]
\[ \text{simplified our treatment, is less well justified. In the scaling of} \]
\[ \text{\( \Sigma_d \) with that of the MMSN, we have inherited the assumption} \]
\[ \text{that all the heavy elements were locally retained during the epoch of} \]
\[ \text{planet formation.} \]

There are two potential avenues for the depletion of heavy elements: hydrodynamic gas drag of small dust grains and type I migration of cores due to the tidal interaction with their nascent disks. Here, “cores” indicates the protoplanetary embryos that formed as a result of runaway or oligarchic growth of the accretion of planetesimals (Kokubo \\
& Ida 1998, 2002). These cores are generally not sufficiently massive to initiate runaway gas accretion. Without much additional growth before the disk gas is severely depleted, these cores would become either rocky planets or ice giants.

In the present paper, we consider the dominant processes after the formation of planetary embryos that are hydrodynamically decoupled from disk gas motions. Prior to the phase of rapid gas accretion, these embedded cores and their surrounding gas exchange angular momentum as they engage in tidal interaction (Goldreich \\
& Tremaine 1980; Lin \\
& Papaloizou 1979). Analytic studies suggest that isolated cores lose angular momentum to the disk exterior to their orbits faster than they gain it from the disk interior to their orbits. This torque imbalance leads to a “type I”
migration. From the linear analysis, the characteristic orbital decay timescale of Earth-mass cores at several AU in a MMSN model is about 1 Myr (Ward 1986; Tanaka et al. 2002). Accordingly, embryos formed well outside the ice line may migrate to the present locations of the Earth and Venus. However, the mostly refractory compositions of the terrestrial planets in the present-day solar system suggest that they probably did not migrate to their present location from regions well beyond the ice line.

Today there remain considerable uncertainties in the efficiency of this process (see § 2.2). Nevertheless, the formation of the remaining planets could have been preceded by a much larger population of cores that did migrate into the Sun. In view of this uncertainty, we carry out a parametric study on the decline of the cores’ accretion rate associated with a $\Sigma_d$ reduction due to the type I migration of cores (see below). This effect has been neglected in our previous papers.

Heavy elements are also depleted from the disks through “type II” migration. Due to runaway gas accretion onto a core, the growing gas giant planet eventually acquires sufficient mass to open a gap in the disk and lock itself into a type II migration with the viscous evolution of the disk gas (Lin & Papaloizou 1985, 1993). In the context of solar system formation, Lin (1986, 1995) has speculated that when the solar nebula was massive, several gas giants may have formed, undergone type II migration, and eventually merged with the Sun. When the disk’s value of $\Sigma_g$ decreases below that of the MMSN model, the migration has stalled. According to this scenario, Jupiter and Saturn could be the last survivors of several generations of protoplanets (also see Papaloizou & Terquem 2006), although it may not be easy to form terrestrial planets after preceding gas giants have passed through the inner disk region (Armitage 2003). A resurgence of this migration and survival scenario (Lin et al. 1996) has been stimulated by the observational discoveries of close-in gas giant planets (e.g., Marcy et al. 2005; Mayor et al. 2005). However, a majority of the known extrasolar planets have periods much longer than a few days. We showed in Papers I and II that the observed logarithmic period distribution can be used to infer comparable timescales for giant planets’ migration and disk depletion. However, on the basis of the relative abundance between planets with $a < 0.06$ AU and those that are between 0.2 and 2 AU from their host stars, we have to assume that up to 90% of the gas giants that migrated to the proximity of their host stars may have perished.

Here we focus on the repeated clearing due to type I migration. We suggest that this self-regulated clearing mechanism limits and determines the amount of heavy elements retained by the cores and residual planetesimals (also see the discussion on the metallicity homogeneity of open clusters in § 4.1). It also results in late formation of gas giants and the marginal probability of gas giants’ formation through a series of failed attempts.

The scenario we consider here is similar to the hypothesis proposed by Canup & Ward (2006) in the context of satellite formation around Jovian planets. They suggested that the total mass of the surviving satellites is self-regulated by their type I migration through their nascent circumplanetary disk, which is continuously replenished. In the context of planet formation, McNeil et al. (2005) carried out $N$-body simulations of terrestrial planets’ accretion and growth, including the effect of type I migration. They found that it is possible to retain a sufficient amount of solid materials (planetesimals and embryos) to assemble Earth-mass planets within an assumed disk lifetime of $\sim 10^6$ yr, provided that the type I migration rate is slightly slower than that predicted by linear theory. In these simulations, the initial disk mass in terrestrial planet regions is assumed to be 3–4 times that of the MMSN. On the basis of an analytic approximation for type I migration, Daisaka et al. (2006) simulated the evolution of $\Sigma_d$ under various disk conditions. Their results indicate that the depletion region expands outward, starting from the disk’s inner boundary. They also found that in disks with the same dust-to-gas ratio, the asymptotic surface density of the retained embryos decreases with the initial value of $\Sigma_d$ and $\Sigma_g$, because type I migration is faster in more massive disks.

Here we consider a much larger range of initial conditions. With a comprehensive numerical prescription, we consider the possibility of multiple generations of embryo formation. We show that in relatively massive disks, although type I migration is more effective for individual planetesimals and embryos, it does not necessarily lead to more rapid depletion of the residual population. In these disks, migration starts with smaller individual masses, but more generations of embryos form and perish before the initial supply of planet-building blocks is severely depleted. Our results indicate that the total retainable mass of heavy elements only weakly depends on the initial disk mass.

Although type I migration places a mass limit on the retainable embryos, it does not quench the formation of Earth-like terrestrial planets because they can be assembled from retainable low-mass embryos after the gas is depleted (Daisaka et al. 2006). Gas giant planets, however, must form in gas-rich disks, which appear to be depleted on the timescale of $\tau_{\text{disk}} \sim 10$ Myr. In addition, they must acquire $\sim 10 M_\oplus$ cores prior to the onset of efficient gas accretion. Although they can form rapidly outside the ice lines on massive disks, early generations of such massive cores quickly migrate into their host stars. However, Thommes & Murray (2006) showed that cores that formed after the disk gas has been severely depleted may withstand disruption by the declining type I migration. In § 3 of this paper, we present results that are consistent with those obtained by Thommes & Murray (2006). We also incorporate gas accretion onto cores and the effect of type II migration and find that under some circumstances, there is adequate residual gas to promote efficient gas accretion and the formation of gas giant planets. This “late formation” scenario is conceptually consistent with that inferred from the noble gas enrichment in Jupiter (Guillot & Hueso 2006). On the basis of this model, we are able to reproduce the observed mass-period distribution of the known gas giant planets around solar-type stars, provided that the efficiency of type I migration is at least an order of magnitude slower than that derived from linear theory (see § 3). Our conclusion is consistent with studies by Alibert et al. (2005), who also found that a necessary condition for the formation of Jupiter and Saturn is a 30 times reduction in the type I migration rate.

In metal-poor disks, the formation of critical-mass cores requires a relatively large gas surface density. In these disks, type I migration is more effective in clearing the cores prior to the onset of gas accretion. Consequently, the formation of gas giant planets is suppressed. If we assume that the disks’ initial metallicity is identical to that of their host stars, this effect sharpens the dependence of the predicted formation efficiency of gas giants on the stellar metallicity, which is well established in the observational data (Fischer & Valenti 2005).

Since type I and II migrations are the essential processes that determine the properties of emerging planets, we briefly recapitulate, in § 2, their basic physical principles. We also incorporate a quantitative prescription of these processes in our existing comprehensive model for planet formation. The new model with the addition of type I migration and the decrease in planetesimal surface density due to this migration is explained in detail. In § 3 we carry out a systematic study on how cores’ migration may affect the formation of terrestrial planets and gas giant planets. We show that type I migration delays formation of gas giants and that it
leads to a mass and semimajor axis \((M_p-a)\) distribution that is consistent with that of the known gas giants around G dwarfs. We also study the dependence of the distributions on metallicity. The introduction of relatively slow type I migration results in a metallicity dependence that may be consistent with observations. Finally, we summarize our results and discuss their implications in §4.

2. PLANET FORMATION AND MIGRATION MODEL

In this paper, we simulate the \(M_p-a\) distribution of extrasolar planets, taking into account the effects of cores’ type I migration, as well as type II migration. In Papers I–III, we outlined in detail a quantitative prescription that we used to model the evolution of planetesimals, gas accretion onto protoplanetary cores, the termination of gas giant planet growth, and type II migration. In this section we briefly recapitulate the dependence of both types of migration on the background disk properties.

Here, we newly incorporate the effects of type I migration and examine the impact of type I migration on the formation of terrestrial planets and the asymptotic properties of emerging gas giant planets. In light of the theoretical uncertainties, we introduce a prescription that captures the main determining factors of type I migration and enables us to consider a wide range in the magnitude of its efficiency factor. We here consider only the case of \(M_\ast = 1\ M_\odot\), in order to focus on the effects of type I migration around solar-type stars, although we retain the dependences on stellar mass, \(M_\ast\), and stellar luminosity, \(L_\ast\).

Note that the inclusion of type I migration and the minor changes in the truncation conditions of the growth of gas giants and the disk model that are described below do not change the conclusions that have been derived in previous papers: (1) the existence of a “planet desert” (a lack of intermediate-mass planets at \(\lesssim 5\) AU) found in Paper I, (2) the metallicity dependence (the fraction of stars with giant planets increases with metallicity) found in Paper II, and (3) the stellar mass dependence (giant planets are much less abundant around lower mass stars) found in Paper III. The results with the inclusion of type I migration corresponding to conclusions 1 and 2 are shown below in Figures 5 and 7, respectively. The stellar mass dependence with type I migration is briefly commented on in §3.2.3 and will be discussed in detail in a separate paper.

2.1. Parameterized Disk Models

The main objective of this series of papers is to examine the statistical properties of emerging planets under a variety of disk environments, rather than studying the individual processes that regulate the disk structure. For computational convenience, we introduced, in Papers I–III, two multiplicative factors \((f_d\) and \(f_g\)) with which to scale \(\Sigma_g\) and \(\Sigma_d\) such that

\[
\Sigma_g = \Sigma_{d,10} f_d \left( \frac{r}{10\ \text{AU}} \right)^{-q_d},
\]

\[
\Sigma_d = \Sigma_{g,10} f_g \left( \frac{r}{10\ \text{AU}} \right)^{-q_d},
\]

where the normalization factors \(\Sigma_{d,10} = 0.32\ \text{g cm}^{-2}\) and \(\Sigma_{g,10} = 75\ \text{g cm}^{-2}\) correspond to 1.4 times the values of \(\Sigma_g\) and \(\Sigma_d\) at 10 AU of the MMSN model, and the step function \(\eta_{\text{ice}} = 1\) inside the ice line at \(a_{\text{ice eq}} (\text{[3]})\) and 4.2 for \(r > a_{\text{ice}}\) (the latter can be slightly smaller \(\sim 3.0\), Pollack et al. 1994).

We show below that the disk metallicity \([\text{Fe/H}]_d\) is an important parameter that regulates the survival of protoplanetary cores during their type I migration. The dependence on the disk metallicity is attributed to the distribution of \(f_{d,0} = f_{g,0} 10^{[\text{Fe/H}]_d}\). Solar metallicity corresponds to \([\text{Fe/H}]_d = 0\) and \(f_{d,0} = f_{g,0}\). The main advantage of these parameterized disk structure models is that they enable us to efficiently simulate and examine the dominant dependence of planet formation and dynamical evolution on the disk structure.

In the self-consistent treatment of the accretion flow, the disk temperature is determined by an equilibrium between the viscous dissipation and heat transport (Shakura & Sunyaev 1973). We neglect the detailed energy balance (Chiang & Goldreich 1997; Garaud & Lin 2007) and adopt the equilibrium temperature distribution in highly optically thin disks prescribed by Hayashi (1981), such that

\[
T = 280 \left( \frac{r}{1\ \text{AU}} \right)^{-1/2} \left( \frac{L_\ast}{L_{\odot}} \right)^{1/4} \text{K}.
\]

In this simple prescription, we set the ice line to be that determined by an equilibrium temperature (eq. [2]) in optically thin disk regions (Hayashi 1981),

\[
a_{\text{ice}} = 2.7 (L_\ast/L_{\odot})^{1/2} \text{AU},
\]

where \(L_\ast\) and \(L_{\odot}\) are the stellar and solar luminosity, respectively. The magnitude of \(a_{\text{ice}}\) may be modified by the local viscous dissipation (Lecar et al. 2006) and stellar irradiation (Chiang & Goldreich 1997; Garaud & Lin 2007). These effects do not greatly modify the disk structure during the late evolutionary stages, and they will be incorporated in subsequent papers.

2.1.1. Evolution of \(f_g\)

The surface density of the gas can be determined by a diffusion equation (Lin & Papaloizou 1985),

\[
\frac{\partial \Sigma_g}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left[ 3r^{1/2} \frac{\partial}{\partial r} (\Sigma_g \nu r^{1/2}) \right] = 0,
\]

where we have neglected the sink terms due to photoevaporation and accretion onto the cores. For computational convenience, we adopt the standard constant-\(\alpha\) prescription for viscosity (Shakura & Sunyaev 1973),

\[
\nu = \alpha c_s H,
\]

where \(c_s, H = \sqrt{2} c_s / \Omega_K\), and \(\Omega_K\) are the sound speed, the disk scale height, and the Keplerian angular frequency, respectively. Although the value of \(\alpha\) is not clear, \(\alpha \sim 10^{-3}\) may be consistent with the observational data of accretion rates of T Tauri disks (e.g., Hartmann et al. 1998) and the results of MHD simulations (e.g., Sano et al. 2004). We here use \(\alpha = 10^{-3}\) as a nominal value.

With the \(\alpha\) prescription and equation (2), we find that \(\nu \propto r\), and equation (4) is then reduced to a linear partial differential equation for which self-similarity solutions have been presented by Lynden-Bell & Pringle (1974). The numerical solution for the disk evolution equation (i.e., eq. [4] with \(\alpha = 10^{-3}\)), starting with \(\Sigma_g \propto r^{-1.5}\) for \(r < r_m\) and an exponential cutoff at \(r_m = 10\ \text{AU}\), is illustrated in Figure 1a.

After a brief initial transition, the numerical solution quickly approaches \(\Sigma_g \propto r^{-1}\) with an asymptotic exponential cutoff, which is the self-similar solution obtained by Lynden-Bell & Pringle (1974) for a linear viscosity prescription and by Lin & Bodenheimer (1982) for an \(\alpha\) model. As shown in Figure 1b, the value of \(\Sigma_g\) in the region \(r < r_m\) decreases as being uniformly
coupled to the gas. Physically, the magnitude of $\Sigma_d$ is determined by the grains’ growth rate and the planetesimals’ retention efficiency. In view of the large uncertainties in these processes, we adopt the radial gradient of the conventional MMSN model, $q_d = 3/2$. Different radial gradients between $\Sigma_d$ and $\Sigma_g$ could be produced by inward migration of dust grains due to gas drag, which tends to make the inner disk more metal-rich and the outer disk metal-poor (Stepinski & Valageas 1997; Kornet et al. 2001).

In Papers I–III, $f_d$ was assumed to be constant with time until the core mass reached the isolation mass. In the present paper, we take into account the effect of planetesimal clearing due to the cores’ accretion. Since we also include type I migration of the cores, the value of $f_d$ at a given location (i.e., at a given semimajor axis $a$) continuously decreases with time as planetesimals are accreted by cores that in turn undergo orbital decay. Note that in the present paper, $a$ is identified as $r$, since we neglect the evolution of orbital eccentricities.

The full width of the feeding zone of a core is given by (Kokubo & Ida 1998, 2002)

$$\Delta a_c = 10r_H = 10 \left( \frac{2M_c}{M_\ast} \right)^{1/3} a,$$

where $r_H$ is the two-body Hill radius. Following Papers I–III, we take into account the expansion of the feeding zones due to collisions among the isolated cores after a significant depletion of disk gas (Kominami & Ida 2002; Zhou et al. 2007), although this effect influences the results only slightly (it is important in inner regions in which cores are significantly depleted by type I migration).

An increase in the cores’ mass is uniformly subtracted from the mass in its feeding zone, keeping $q_d$ constant locally. When the cores migrate out from the feeding zone, the reduction of $f_d$ at the initial location of the cores is stopped. However, the cores can continue to accrete planetesimals along their migration path. In any given system, when a core reaches 0.03 AU (occasionally, it can attain sufficient mass to accrete gas and become a gas giant during the course of its migration), a next-generation core is launched at the premigration radius with a mass that is 100 times smaller. (Since core growth is faster during the earlier stage of disk evolution [eq. (9)], the choice of the initial mass of the next-generation core does not affect the total core accretion timescale.) Thereafter, the growth of the core and the depletion of the planetesimals in its feeding zone resumes.

This process repeats until the residual planetesimals are depleted. Along the cores’ migrating paths, they accrete residual planetesimals (until they reach $a < a_{\text{dep,mig}}$, given by eq. [27]; see below) but cannot reduce $f_d$ as fast as the cores that were formed in situ. Therefore, we neglect the evolution of $f_d$ due to planetesimal accretion by the migrating cores. The clearing of the residual planetesimal disk by migrating gas giants is also neglected because it rarely occurs and is unlikely to affect the overall distribution of the emerging terrestrial planets (see § 3.2.2). The overall evolution of $\Sigma_d$ due to core accretion and migration is described in § 3.1.

### 2.2. Core Growth and Type I Migration

We briefly recapitulate a comprehensive analysis on the growth of planetesimals and cores. Readers can find a systematic derivation of these results and the appropriate references in Paper I. During their growth through cohesive collisions, the cores’ mass
accretion rate of planetesimals) at any location $a$ and time $t$ is described by (Paper I)

$$dM_c/\text{d}t = \frac{M_c}{\tau_{\text{acc}}},$$

$$\tau_{\text{acc}} = 2.2 \times 10^5 \eta_{\text{hec}}^{-1} f_d^{-1} f_g^{-2/5} 10^{(2/5)(3/2-q_h)} \times \left(\frac{a}{1 \text{ AU}}\right)^{27/(10+(q_u-3)/2)+(2/5(q_u-3/2)} \left(\frac{M_c}{M_\odot}\right)^{-1/6}\text{yr},$$

(9)

where $f_d$ and $f_g$ change with time. In the derivation of the above expression, we have adopted the mass of the typical field planetesimals to be $m = 10^{19}$ g. The numerical factor $10^{(2/5)(3/2-q_h)}$ comes from the fact that we scale surface densities at 10 AU in equation (1).

On the basis of the results of previous numerical simulations (see references in Paper I), we assume that the runaway growth phase quickly changes to the oligarchic growth phase (Kokubo & Ida 1998). During the oligarchic growth phase, the cores’ accretion timescale is dominated by their late-stage growth ($\tau_{\text{acc}} \propto M_c^{1/3}$), so dependence on their initial mass is negligible. In equation (9), we adopt $M_c(0) = m = 10^{19}$ g. When $f_d < 10^{-3}$, we use the gas-free accretion rate rather than that in equation (9) (Paper I):

$$\tau_{\text{acc}} = 2 \times 10^7 \eta_{\text{hec}}^{-1} f_d^{-1} \left(\frac{a}{1 \text{ AU}}\right)^{3/(q_u-3/2)} \left(\frac{M_c}{M_\odot}\right)^{-1/2}\text{yr}. \quad (10)$$

In Papers I–III, we artificially terminate the cores’ accretion at $M_e = M_{\text{c,iso}}$, where the isolation mass is given by

$$M_{\text{c,iso}} \simeq 0.16 \eta_{\text{hec}}^{3/2} f_d^{3/2} \left(\frac{a}{1 \text{ AU}}\right)^{3/4-(3/2)(q_u-3/2)} \left(\frac{M_c}{M_\odot}\right)^{-1/2} M_\odot. \quad (11)$$

In the present paper, we do not need to adopt an artificial termination to the core growth, because when $M_c$ approaches the isolation mass, $M_{\text{c,iso}}$, the in situ core accretion is automatically slowed down by the reduction of $f_d$. However, in outer regions, ejection of planetesimals by large protoplanets limits their accretion at $M_{\text{c,sc}}$ (the scattering limit; Paper I). We take this effect into account by adopting an artificial termination for the cores’ accretion of planetesimals when they attain a mass of $M_{\text{c,sc}} \simeq 1.4 \times 10^4 (a/1 \text{ AU})^{-5/2}(M_c/M_\odot) M_\odot$.

Inbalance in the tidal torques from the outer and inner disks causes the type I migration of a core. Through a three-dimensional linear calculation, Tanaka et al. (2002) derived the timescale of type I migration,

$$\tau_{\text{mig}} = \frac{a}{a_\odot} = \frac{1}{C_1} \left(\frac{1}{2.2728 + 1.082 q_d} \left(\frac{c_y}{a_{\text{max}} K}\right)^2 \frac{M_c}{M_*} \frac{a_s^{2\Sigma^k}}{\Delta_{\text{He}} K} \Omega_{-1} \right. \left. \left(\frac{M_c}{M_\odot}\right) \right)^2 \text{yr},$$

(12)

where we have used equation (2) and $q_g = 1-1.5$.

In equation (12), we introduce an scaling factor $C_1$ to allow for the retardation of type I migration due to nonlinear effects. In the expression of Tanaka et al. (2002), $C_1 = 1$. Many simulations have been carried out (see, e.g., Papaloizou & Terquem 2006) with different numerical methods and resolutions, protoplanetary potentials and orbits, and disk structures. They produced a consid-

erable range for the timescale of type I migration ($\tau_{\text{mig}}$). Retardation processes for type I migration include variation in the surface density and temperature gradient (Masset et al. 2006b), intrinsic turbulence in the disk (Laughlin et al. 2004; Nelson & Papaloizou 2004), self-induced unstable flow (Koller & Li 2004; Li et al. 2005), and nonlinear radiative and hydrodynamic feedback (Masset et al. 2006a). Under some circumstances, the value of $C_1$ is reduced to $\lesssim 0.1$ (I. Dobbs-Dixon et al. 2008, in preparation).

Type I migration can occur before the core mass reaches $M_{\text{c,iso}}$. In this case, the value of $\Sigma_g$ (equivalently, $f_d$) decreases but does not completely vanish. Due to the decreased value of $\Sigma_g$, new cores accrete with reduced growth rates and isolation masses. The cycle of core growth, type I migration, and reduction of $\Sigma_g$ continues until $\Sigma_g$ decreases to the levels at which only cores small enough not to migrate are produced (Daisaka et al. 2006). We present a detailed discussion of the self-regulation mechanism in later sections.

Under some conditions, the migrating cores can capture and accumulate planetesimals along their paths (Ward & Hahn 1995; Tanaka & Ida 1999). An N-body simulation by Daisaka et al. (2006), however, showed that the trapping of planetesimals by the cores is tentative and that it does not significantly reduce their accretion rates. In our simulations, we use the accretion rate for migrating cores that is the same as the rate for nonmigrating cores (eq. [9]). For cores in systems with $q_g = 1$ and $q_d = 1.5$,

$$dM_c/\text{d}t = \frac{M_c}{\tau_{\text{acc}}},$$

$$\tau_{\text{acc}} = 3.5 \times 10^5 \eta_{\text{hec}}^{-1} f_d^{-1} f_g^{-2/5} \left(\frac{a}{1 \text{ AU}}\right)^{5/2} \left(\frac{M_c}{M_\odot}\right)^{-1/6}\text{yr},$$

(13)

$$da/\text{d}t = a/\tau_{\text{mig}},$$

$$\tau_{\text{mig}} \simeq 1.6 \times 10^5 C_1^{-1} f_d^{-1} \left(\frac{M_c}{M_\odot}\right)^{-1} \left(\frac{a}{1 \text{ AU}}\right)^{3/2} M_\odot \text{yr}. \quad (14)$$

The magnitude of $f_d$ is consistently decreased by the increase in $M_c$ (see § 3.1). Since the accretion rate is determined by the instantaneous local value of $f_d$, this limits the mass of the migrating cores in a gaseous medium. Prior to the severe depletion of the disk gas, we quench the cores’ accretion of planetesimals at $a < a_{\text{dep,mig}}$ (eq. [27]) on the assumption that there is an inadequate supply of residual planetesimals in these locations to significantly add to their masses. However, during the gas depletion, we assume that $f_d$ decays exponentially. After the disk gas is significantly depleted ($f_d \approx 10^{-5}$), equation (10) is used for $\tau_{\text{acc}}$. We also set $da/\text{d}t$ to be zero at the disk’s inner edge ($\sim 0.03$ AU).

2.3. Formation of Gas Giant Planets

Our prescriptions for the formation of gas giant planets are the same as those used in Papers I–III, although the cores’ rate of planetesimal accretion is revised from those in Papers I–III by the incorporation of type I migration and planetesimal depletion. After the formation of these gas giants, we assume that all residual planetesimals in the gap are cleared as a consequence of dynamical instabilities. These planetesimals are either accreted by the gas giants or scattered elsewhere (Zhou & Lin 2007). We neglect the emergence of second-generation cores close to the orbit of gas giants.

In principle, cores with masses much less than that of the Earth can accrete gas. However, unless the heat released during gas and
planetesimal accretion is diffused and radiated away, quasi-thermal and hydrodynamic equilibrium would be established to prevent further flow onto the cores. Around low-mass cores, the temperature and density of the envelope are low, so heat cannot be easily redistributed through their envelopes. But as the cores grow through planetesimal bombardment beyond a mass

\[ M_{\text{core, hydro}} \approx 10 \left( \frac{M_e}{10^{-5} M_\odot \text{ yr}^{-1}} \right)^{0.25} M_\odot, \]  

both the radiative and convective transport of heat become efficient enough to allow their envelopes to contract dynamically (Ikoma et al. 2000). In the above equation, we neglected the dependence on the opacity of the envelope (see Paper I). In regions in which the cores have already attained isolation, their planetesimal accretion rate \( M_e \) is much diminished (Zhou & Lin 2007), and \( M_{\text{core, hydro}} \) can be comparable to an Earth mass, \( M_\oplus \). However, gas accretion also releases energy, and its rate is still regulated by the efficiency of radiative transfer in the envelope, such that

\[ \frac{dM_p}{dt} \approx \frac{M_p}{\tau_{\text{KH}}}, \]

where \( M_p \) is the planetary mass, including the gas envelope. In Paper I, we approximated the Kelvin-Helmholtz contraction timescale \( \tau_{\text{KH}} \) of the envelope with

\[ \tau_{\text{KH}} \approx 10^{k_1} \left( \frac{M_p}{M_\oplus} \right)^{k_2} \text{ yr}. \]

In order to account for the uncertainties associated with planetesimal bombardment, dust sedimentation, and opacity in the envelope, we adopt a range of values \( k_1 = 8-10 \) and \( k_2 = 3-4 \) (see Papers I and II). Here we use \( k_2 = 3 \) and treat \( k_1 \) as a parameter.

Gas accretion onto the core is quenched when the disk is depleted either locally or globally. We assume that gas accretion is terminated if either the thermal condition or the global depletion condition is satisfied. A (partial) gap is formed when the rate of tidally induced angular momentum exchange of the planet with the disk exceeds that of the disk’s intrinsic viscous transport (Lin & Papaloizou 1985; the viscous condition),

\[ M_p \geq M_{\text{g, vis}} \approx 30 \frac{\alpha}{10^{-3}} \left( \frac{a}{1 \text{ AU}} \right)^{1/2} \left( \frac{L_p}{L_\odot} \right)^{1/4} M_\odot. \]  

Planets with \( M_p > M_{\text{g, vis}} \) have sufficient mass to induce a partial clearing of the disk near their orbit. The tidal torque on either side of the gap becomes sufficiently strong to induce the planets to adjust their positions within the gap. This feedback process leads to a transition from type I to type II migration. Therefore, we adopt the viscous condition for the onset of type II migration. In Papers I–III, we adopted \( \alpha = 10^{-4} \) as a nominal value. In order to match the simulated \( M_{\text{core}}-\alpha \) distribution to the observed data, equation (18) was arbitrarily multiplied by a scaling factor of \( A_\alpha = 10 \).

The previously adopted value of \( \alpha \) is smaller than that inferred from the models of protostellar disk evolution (Hartmann et al. 1998). In this paper, we use \( \alpha = 10^{-3} \) without imposing the scaling factor. As a result, the condition for the onset of type II migration is the same as that in Papers I–III.

A clear gap is formed and gas accretion is terminated when the planet’s Hill radius becomes larger than the disk scale height (Lin & Papaloizou 1985; the thermal condition), which is given by (Paper I)

\[ M_p > M_{\text{g, th}} \approx 0.95 \times 10^3 \frac{a}{1 \text{ AU}}^{3/4} \left( \frac{L_p}{L_\odot} \right)^{3/8} \left( \frac{M_p}{M_\odot} \right)^{-1/2} M_\odot. \]  

While the thermal condition \( r_H > 1.5H \) was used in Papers I–III, we used the condition \( r_H > 2H \) here in order to make it clear that the asymptotic mass of gas giants is determined by the global depletion of disk gas rather than by the local thermal condition in the cases with inclusion of type I migration in which gas giants are formed in relatively late stages. (It can also reflect the effect of gas flowing into the gap mentioned below.)

Some numerical simulations indicate that a residual amount of gas may continue to flow into the gap after both the viscous and thermal conditions are satisfied (D’Angelo et al. 2003; Kley & Dirksen 2006; Tanigawa & Ikoma 2007). However, recent numerical simulations (Dobbs-Dixon et al. 2007) also show that the azimuthal accretion flow from the corotation regions onto the planet is effectively quenched despite a diminishing flux of gas into the gap. Since the thermal condition usually requires a larger range of \( M_p \) than the viscous condition \( (M_{\text{g, th}} > M_{\text{g, vis}}) \), we adopt it as the criterion for the termination of gas accretion in the determination of the asymptotic mass of gas giant planets. However, we take into account a residual amount of gas that may leak through the gap and provide an effective avenue for angular transfer between the inner and outer regions of the disk via the gas giant’s corotation resonance (§ 2.4).

Gas accretion may also be limited by the diminishing amount of residual gas in the entire disk, even for planets with \( M_p < M_{\text{g, th}} \). In Papers I–III, we assumed that the maximum available mass was determined simply by \( M_{\text{g, no iso}} \sim \pi a^2 \Sigma_p \). However, because the global limit plays a more important role when type I migration is incorporated, here we use the more appropriate condition \( M_{\text{g, no iso}} \sim \int_0^{\infty} 2\pi a^2 \Sigma_p \, da \) (neglecting disk gas inflow from far outer regions). For \( q_\beta = 1 \),

\[ M_{\text{g, no iso}} \approx 3.5 \times 10^2 \frac{a}{1 \text{ AU}} \exp \left( \frac{r_H}{r_{\text{dep}}} \right) \left( \frac{a}{1 \text{ AU}} \right)^{1/2} M_\odot. \]  

When \( M_{\text{g, no iso}} \) diminishes below \( M_p \) or when \( M_p \) exceeds \( M_{\text{g, th}} \), gas accretion is terminated.

### 2.4. Type II Migration

In our previous analysis in Papers I–III, we adopted a simple analytic prescription for type II migration: (i) before the disk gas mass decays to the value comparable to the planet mass, the planet migrates with (unperturbed) disk accretion, and (ii) when the disk gas mass is comparable to the planet mass, a fraction \( (C_2 \simeq 0.1) \) of the total (viscous plus advective) angular momentum flux transported by the disk gas (which is assumed to be independent of the disk radius) is utilized by the planet in its orbital evolution.

If the planet’s tidal torque can severely clear a gap in the vicinity of its orbit, the value of \( \Sigma_p \) in the inner disk region would decrease faster than that in the outer region. The full torque asymmetry leads to \( C_2 \simeq 1 \) in case (ii). However, when the truncation condition is marginally satisfied, the disk interior to the planet’s orbit has a total mass \( \gtrsim M_p \), such that it can effectively replenish the angular momentum lost by the planet to the outer disk region. There may also be a leakage of gas through the gap region (D’Angelo et al. 2003), which would suppress the degree of
torque asymmetry. The protoplanet’s corotation resonance may drive an effective angular momentum transfer across the two disk regions separated by the protoplanet. Protoplanets are formed in disk regions where the midplane is inactive to magnetorotational instabilities (e.g., Sano et al. 2000). It is possible that the modest accretion rate onto the host stars flows through this region via an active layer that is exposed to external ionizing photons and cosmic-ray particles (Gammie 1996). Different values of the effective α may contribute to the mass flow through the disk and the planet–disk interaction, especially if there is some leakage across the gap region. All of these possible scenarios can lead to $C_2 \ll 1$. The uninterrupted replenishment of gas into the corotation region may also maintain a finite vortensity gradient and an unsaturated corotation torque (Masset et al. 2006a), which may reduce the efficiency of type II migration from disk gas accretion even in case (i) (Crida & Morbidelli 2007). However, the results of another set of two-dimensional numerical hydrodynamic simulations (D’Angelo et al. 2006) essentially reproduces the one-dimensional simulation (Lin & Papaloizou 1986) in which the contribution from the corotation resonance is neglected.

In order to take into account these uncertainties, we reduce the migration rate by a factor of $C_2$ for case (ii), as in Papers I–III:

$$
\tau_{\text{mig2, ii}} = \frac{1}{C_2} \left( \frac{1}{2} \frac{M_p \Omega_K(a) a^2}{3 \pi \Sigma_g(r_m)^{1/2} \mu^2 \Omega_K(r_m)} \right)
\simeq 5 \times 10^5 \frac{f_g}{r} \left( \frac{C_2 \alpha}{10^{-4}} \right)^{-1} \left( \frac{M_p}{M_\odot} \right)^{-1/2} \text{yr. (21)}
$$

When $\tau_{\text{mig2, ii}}$ is shorter than the migration timescale for case (i), we use the latter timescale,

$$
\tau_{\text{mig2, i}} = \frac{|a|}{\text{AU}} = \left( \frac{3}{2} \right) \frac{a}{\mu \text{AU}}
\simeq 0.7 \times 10^5 \left( \frac{\alpha}{10^{-3}} \right)^{-1} \left( \frac{1}{\text{AU}} \right)^{-1/2} \text{yr. (22)}
$$

Due to more accurate estimations, the numerical factors in $\tau_{\text{mig2, i}}$ and $\tau_{\text{mig2, ii}}$ slightly differ from those in Papers I–III for the same values of $C_2$ and $\alpha$. However, there are still uncertainties in the formula for $\tau_{\text{mig2, ii}}$. Furthermore, it is not easy to theoretically evaluate the value of $C_2$, as well as that of $\alpha$. Hence, we vary the magnitude of $C_2$ and compare the simulated results with the observed data to obtain a calibration. For $\alpha \sim 10^{-3}$, in the limit that $C_2 \sim 1$, most of the gas giants are removed from the regions beyond 1 AU, which is inconsistent with observed data of extrasolar planets. As is shown in Paper II, in order to reproduce the $M_p$-a distribution of observed extrasolar planets, the disk depletion timescale $\tau_{\text{depl}}$ must be $\tau_{\text{mig2}}$ at a few AU. In the present paper, we mostly adopt $C_2 \alpha = 10^{-4}$. For $\alpha \sim 10^{-3}$, this corresponds to $C_2 \sim 1/10$. As a result, the value of $\tau_{\text{mig2, ii}}$ that we adopt here is almost identical to that used in Papers I–III.

3. EFFECTS OF TYPE I MIGRATION

In our previous simulations in Papers I–III, the effect of type II migration was included, but that of type I migration was neglected. In this section, we investigate the effects of type I migration on the planets’ $M_p$-a distribution.

![Graph](image)

**Figure 2.** Evolution of $\Sigma_d$ due to dynamical sculpting by type I migration in the case of $C_1 = 1$ and $f_{\text{dip}} = 3$. Here we assume a constant value of $f_{\text{dip}}$. (a) The $\Sigma_d$ distributions at $t = 10^5$, $10^6$, and $10^7$ yr, shown by dashed, dotted, and solid lines, respectively. The initial distribution is also shown by a dotted line. (b) Evolution of the $f_d$ distributions. (The styles of the lines are the same as in panel a.) (c) Number of generations of protoplanetary cores formed at each value of $a$. (d) Planet distributions at $t = 10^7$ yr. The dashed line shows the core isolation mass and the scattering limit.

3.1. Surface Density Evolution Due to Type I Migration

In order to analyze the asymptotic $M_p$-a distribution from the Monte Carlo simulations, we first present the results on the reduction of $\Sigma_d$ due to type I migration. The relevant timescale here is a function of $C_1$, $f_{\text{dip}}$, and $f_{\text{dip}}$ (eq. [12]). The magnitude of the cores’ values of $M_p$ is a function of $f_{\text{dip}}$, $f_{\text{dip}}$, and $t$.

Since type I and II migrations involve the tidal interaction of embedded cores with the disk gas, the dynamical clearing of the residual planetesimals is suppressed after the gas is severely depleted (except for outer regions in which ejection by massive embryos can be efficient). Since $t \sim \tau_{\text{depl}}$ is a critical stage for the buildup and retention of the cores and the onset of gas accretion onto the cores, we are particularly concerned with the residual $\Sigma_d$ distribution at this stage. The asymptotic $\Sigma_d$-a distribution is determined by $C_1$, $f_{\text{dip}}$, and $\tau_{\text{depl}}$. In this subsection, we show the simulation results with $f_{\text{dip}} = f_{\text{dip}}$, $q_d = 1.5$, and $q_{\text{dip}} = 1.0$ at $t = 0$. The results with other reasonable values of $f_{\text{dip}}$, $f_{\text{dip}}$, and $q_d$ are qualitatively similar. With these initial conditions, we can compute the emergence of cores during the epoch of gas depletion.

Figure 2a shows $\Sigma_d$ at $t = 10^5$, $10^6$, and $10^7$ yr (dashed, dotted, and solid lines, respectively) with $C_1 = 1$ and $f_{\text{dip}} = 3$. These results correspond to surviving protoplanets at $t \sim \tau_{\text{depl}}$. For $\tau_{\text{depl}} = 10^5$, $10^6$, and $10^7$ yr, although the depletion of $f_{\text{dip}}$ on timescales of $\tau_{\text{depl}}$ is not taken into account here (in the Monte Carlo simulations in §3.2, the exponential decay, eq. [7], is considered). We generated 1000 semimajor axes with a log-uniform distribution in the
range 0.05–50 AU and simulated the growth and orbital migration of cores there. In these simulations, gas accretion onto cores is neglected so that we can clearly see the effects of type I migration. The dynamical interactions among the cores are also neglected. With N-body simulations, Komimami et al. (2005) showed that the dynamical interactions with other cores and planetesimals do not change the cores’ type I migration speed. The results presented in this figure confirm the findings of the previous generations of cores limits the growth of the migrating cores and the gravitational interactions between them. In order to take into account the effect of dynamical clearing in the Monte Carlo simulations (presented in the next subsection), we terminate the growth of cores after their semimajor axes have decreased below $a_{\text{dep,mig}}$, which is given by equation (27).

Figure 2d shows that the mass of the remaining population of cores is $\sim 0.1 M_\odot$. For these retained cores, $\tau_{\text{mig}} > 3 \tau_{\text{acc}}$ ($\tau_{\text{acc}} = M_\odot / M_*$), where the factor of 3 reflects the actual timescale to reach $M_*$, because $\tau_{\text{acc}} \propto M^{1/3}$. From equations (13) and (14), the maximum mass of the remaining cores ($M_{\text{c,max}}$) is given by

$$M_{\text{c,max}} \approx 0.21 C_1^{-3/4} \left( \frac{f_{\text{g,0}}}{3} \right)^{3/10} \left( \frac{\eta_{\text{acc}} f_{\text{g,0}}}{f_{\text{g,0}}} \right)^{3/4} \left( \frac{a}{1 \text{ AU}} \right)^{-9/8} \left( \frac{M_\odot}{M_\odot} \right)^{5/4} M_\odot. \quad (23)$$

The above expression with $f_{\text{g,0}} = f_{\text{g,0}}$ reproduces the result at $a_{\text{dep,mig}} \sim 1$ AU in Figure 2d, as well as the weak dependence of $M_{\text{c,max}}$ on $f_{\text{g,0}}$ and $C_1$ (Figs. 3d and 4d). In order to further examine the $f_{\text{g,0}}$ and $C_1$ dependences, we run a set of models with $f_{\text{g,0}} = 30 (C_1 = 1)$ and $C_1 = 0.1 (f_{\text{g,0}} = 3)$ and show the results in Figures 3 and 4, respectively. In the former case, the disks are marginally self-gravitating and contain a significant fraction of the central stars’ mass. The emergence of relatively massive cores is more sensitively determined by $C_1$, the inefficiency of type I migration. A large amount of solid material in the disk does not efficiently promote the formation of massive cores, because the associated dense gas increases the type I migration speed, as has already been pointed out by Daisaka et al. (2006).

In the inner regions at $a \lesssim 0.3$ AU, the cores’ accretion proceeds on very short timescales, and they reach their isolation masses near their birthplaces. Thereafter, most of these cores migrate into their host stars within $10^5$ yr. Consequently, the local value of $\Sigma_d$ is essentially depleted by the formation and migration of the very first generation of cores (Fig. 2c). This domain is determined by the condition $M_{\text{c,max}} \gtrsim M_{\text{c,iso}}$. From equations (11) and (23), this condition implies

$$a \lesssim a_{\text{iso,mig}} \approx 0.45 C_1^{-2/5} \left( \frac{f_{\text{g,0}}}{3} \right)^{-2/3} \left( \frac{\eta_{\text{acc}} f_{\text{g,0}}}{f_{\text{g,0}}} \right)^{-2/5} \left( \frac{M_\odot}{M_\odot} \right)^{14/15} \text{AU}. \quad (24)$$

This expression approximately reproduces the result in Figure 2, and it accounts for the dependence on $C_1$ and $f_{\text{g,0}}$ (see Figs. 3c and 4c). The location of $a_{\text{iso,mig}}$ does not vary with the slope of the surface density distribution, because both competing processes are determined by local properties of the disk.

In the limit of efficient type I migration (with $C_1 = 1$), most of the cores undergo orbital decay before they attain their isolation masses in the intermediate regions (for Fig. 2, 0.3 AU $\lesssim a \lesssim 10$ AU). Consequently, a significant fraction of $\Sigma_d$ remains to
promote the formation of cores in subsequent generations. The results of our simulations (Fig. 2c) show that many generations of cores may emerge at the same disk location. This repeated formation and self-destruction process is even more efficient in massive disks where \( f_d,0 \gg 1 \) (see Fig. 3c).

In the inner region, the surface density is significantly depleted by the \( N_{\text{gene},1} \) generations of core formation and disruption, where

\[
N_{\text{gene},1} \simeq \frac{M_{\text{c, max}}}{M_{\text{c, max}}} \simeq 4.13^{3/4} \left( \frac{f_{g,0}}{3} \right)^{6/5} \times \left( \frac{\eta_{\text{he, fd},0}}{f_{g,0}} \right)^{3/4} \frac{a}{1 \text{ AU}}^{15/8} \left( \frac{M_{\text{c}}}{M_{\odot}} \right)^{-7/4}.
\]

(25)

As the planetesimal building blocks become depleted, the cores formed at later epochs have masses smaller than \( M_{\text{c, max}} \). Therefore, the above expression for \( N_{\text{gene},1} \) slightly underestimates the number of populations of cores that may emerge.

At large disk radii, the number of generations is limited by the disk depletion time. In this region,

\[
N_{\text{gene},2} = \frac{t}{\tau_{\text{c, mig}}(M_{\text{c, max}})} \simeq 3.95^{1/4} \left( \frac{f_{g,0}}{3} \right)^{11/10} \left( \frac{\eta_{\text{he, fd},0}}{f_{g,0}} \right)^{3/4} \times \left( \frac{a}{1 \text{ AU}} \right)^{-17/8} \frac{t}{10^6 \text{ yr}} \left( \frac{M_{\text{c}}}{M_{\odot}} \right)^{-1/4}.
\]

(26)

This estimate completely reproduces the results in Figures 2c, 3c, and 4c. The actual number of the generation is given by \( \min(N_{\text{gene},1}, N_{\text{gene},2}) \). Significant depletion of the original inventory of heavy elements occurs in the limit that \( N_{\text{gene},1} \lesssim N_{\text{gene},2} \); that is, within the location

\[
a \lesssim a_{\text{dep, mig}} \simeq C_1^{-1/8} \left( \frac{f_{g,0}}{3} \right)^{1/4} \left( \frac{t}{10^6 \text{ yr}} \right)^{1/4} \left( \frac{M_{\text{c}}}{M_{\odot}} \right)^{3/8} \text{ AU}.
\]

(27)

This boundary of the disruption zone is in excellent agreement with the critical width within which \( \Sigma_d \) (equivalently, \( f_d \)) has been reduced from its initial values by an order of magnitude. Note that the dependences of \( a_{\text{dep, mig}} \) on \( C_1 \) and \( f_{g,0} \) are very weak. As long as \( N_{\text{gene},2} > 1 \), the disruption zone is confined to \( a \lesssim (t/10^6 \text{ yr})^{1/4} \) AU, independent of disk surface density and migration speed \( C_1 \) (in the limit of small values of \( C_1, N_{\text{gene},2} < 1 \) and \( N_{\text{gene},1} < 1 \), so that no depletion occurs).

These results indicate that, during the early epoch of disk evolution, cores form and migrate repeatedly to clear out the residual planetesimals. This self-regulated process provides an avenue for the host stars to acquire most of the heavy elements retained by the planetesimals. In large, well-mixed molecular clouds where star clusters form, this self-regulated mechanism would lead to stellar metallicity homogeneity (§ 4.1). The inner disk region contains cores that have started their type I migration but have not yet reached the disk's inner edge. The average value of \( \Sigma_d \) at these locations, including the contribution of these migrating cores, is 2 orders of magnitude smaller than its initial value.

Eventually the disk gas is so severely depleted that relatively massive cores no longer undergo a significant amount of type I migration. At any given disk radius, the condition for retaining 90% of the initial solid surface density is \( N_{\text{gene},2} \simeq 0.1N_{\text{gene},1} \). We find, from equations (25), (26), and (27), that this condition is satisfied in regions with

\[
a \gtrsim a_{\text{surv, mig}} = 10^{1/4} a_{\text{dep, mig}} \simeq 2a_{\text{dep, mig}}.
\]

(28)

Type I migration leads to a transition in the \( \Sigma_d \) distribution at this orbital radius.

Inside the ice line, type I migration limits the mass of individual surviving cores. But these cores can coalesce through giant impacts during and after the severe depletion of the disk gas (Kominami & Ida 2002; Ida & Lin 2004a). Provided that the total mass of the residual planetesimals and cores is \( \sim O(1) M_{\odot} \) at \( 1 \text{ AU} \lesssim a \lesssim 1 \text{ few AU} \), Earth-mass terrestrial planets may form near 1 AU. Previous simulations (Chambers & Wetherill 1998; Agnor et al. 1999; Kominami & Ida 2002; Raymond et al. 2004) show that the most massive terrestrial planets tend to form in inner regions of the computational domain, where the isolated cores are initially placed. In these simulations, strong gravitational scattering processes can inject planetesimals close to the host stars to form smaller planets with relatively close-in orbits.

In general, type I migration leads to clearing of planetesimals close to their host stars and sets the inner edge of the cores’ population at \( \sim 1 \text{ AU} \). The lack of planets inside Mercury’s orbit in our solar system might also be attributed to this result (Daisaka et al. 2006). In addition, the self-regulated clearing process also leads to \( f_d \sim O(1) \) near 1 AU for a wide variety of initial conditions \( (f_{g,0}) \). This residual distribution of heavy elements at \( \tau_{\text{dep}} \) ensures the formation of Mars- to Earth-sized terrestrial planets in habitable zones (see § 3.2). Even in the limit that the disk’s initial \( \Sigma_d \) distribution is much larger than that of the MMSN model, the reduction of \( \Sigma_d \) at \( \lesssim 1 \text{ few AU} \), the planetesimals in the ice line. In contrast to the results in Papers I–III, the inclusion of type I migration suppresses the rapid and prolific formation of hot Jupiters, which in turn facilitates the formation and retention probability of habitable terrestrial planets.

We also simulated several models with \( q_{d,1} = 2 \) that correspond to a steeply declining initial surface density distribution. Such a steep gradient of \( \Sigma_d \) could be produced by the inward migration of dust due to the hydrodynamic drag on the dust particles by the disk gas (Stepinski & Valageas 1997; Kornet et al. 2001). In this model, a significant amount of solid mass is contained in the inner disk regions, where cores quickly form and undergo type I migration. Nearly all the initial mass of the solid components in disks is accreted by the host stars through the self-regulation by type I migration. The results of this model are consistent with the discovery of metallicity homogeneity among the stars in young open clusters (see § 4.1)

The above discussions indicate that the formation of Mars- to Earth-sized habitable planets depends only weakly on the type I migration speed. However, it is critical for the formation of cores of gas giant planets, because \( M_{\text{c, max}} \propto C_1^{-3/4} \) (eq. [23]). For giant planets to actually form, sufficiently massive cores must be able to accrete gas on timescales that are at least shorter than a few folding times of \( \tau_{\text{dep}} \). For \( \tau_{\text{dep}} \sim 10^6 \text{ to } 10^7 \text{ yr} \), we deduce, from equation (16) with \( k_1 = 9 \) and \( k_2 = 3 \), that gas giant formation is possible only for \( M_{\text{c}} \gtrsim 1 \text{ few } M_{\odot} \). However, type I migration suppresses the emergence of such massive cores in disk regions with relatively large values of \( \Sigma_d \). The results with \( C_1 = 1 \) in Figures 2d and 3d show that the asymptotic core masses are generally much smaller than those needed to launch efficient gas accretion, even though there is little decline in the magnitude of \( \Sigma_d \).

In the case of \( C_1 = 0.1 \) (Fig. 4d), the cores formed at a few AU originally have \( \sim M_{\odot} \) and that the planetesimals along their
migration paths are not captured into their mean motion resonances, these cores can grow up to a few $M_\odot$ through accretion during migration. The core mass is marginal for rapid gas accretion. As shown in equation (23), in metal-rich disk regions (where $f_{d,0}/f_{g,0} > 1$), more massive cores can be retained with the same amount of solid materials. In this expression, the effects of gas depletion have not been taken into account. However, this process is included in the numerical simulations in § 3.2, and it also enhances the masses of retainable cores, especially those that emerge during the advanced stages of gas depletion. These results suggest that gas giants can be formed for $C_1 \lesssim 0.1$. (Alibert et al. [2005] derived a similar condition for the formation of Jupiter and Saturn.) In the next subsection, we will show that values of $C_1 \lesssim 0.1$ reproduce the mass and semimajor axis distribution of extrasolar gas giants that is comparable to those observed.

3.2. Mass–Semimajor Axis Distributions

3.2.1. Dependence on Type I Migration Speed

We now consider the signature of type I migration on the $M_p$–a distribution for a population of emerging planets. In the Monte Carlo simulations, we first generate a set of 1000 disks with various values of $f_{d,0}$ (the initial value of $f_d$) and $\tau_{\text{dep}}$. We adopt the same prescriptions for the distributions of $f_{d,0}$ and $f_{g,0}$ as those in Paper II. For the gaseous component, we assume that $f_{g,0}$ has a lognormal distribution that is centered on the value of $f_{g,0} = 1$ with a dispersion of 1 ($\log f_{g,0} = 1.0$) and an upper cutoff at $f_{g,0} = 30$, independent of the stellar metallicity. For the heavy elements, we choose $f_{d,0} = 10^{[\text{Fe/H}]_d} f_{g,0}$, where $[\text{Fe/H}]_d$ is the metallicity of the disk. We assume that these disks have the same metallicities as their host stars.

Following our previous papers, we also assume that $\tau_{\text{dep}}$ has log-uniform distributions in the range $10^6$–$10^7$ yr. For each disk, 15 values of $a$ for the protoplanetary seeds are selected from a log-uniform distribution in the range $0.05$–$50$ AU, assuming that the averaged orbital separation between planets is 0.2 in a logarithmic scale (the averaged ratio of semimajor axes of adjacent planets is $\approx 1.6$). This procedure is the same as that adopted in Paper II. Constant spacing in the logarithm corresponds to the spacing between the cores being proportional to $a$, which is the simplest choice and a natural outcome of dynamical isolation at the end of the oligarchic growth. The logarithmic constant spacing for planets and cores with similar masses also maximizes dynamical stability. In the present paper, we neglect dynamical interaction between planets (this issue will be addressed in future papers), and the growth of individual planets is integrated independently. Although the choice of averaged orbital separation is arbitrary, it would not change the overall results with regard to the effects of type I migration.

In all the simulations presented here, the values $\alpha = 10^{-3}$ and $M_i = 1 M_\odot$ are assumed. Since the ongoing radial velocity surveys are focusing on relatively metal-rich stars, we present the results with $[\text{Fe/H}] = 0.1$ in most cases. The dependence on $[\text{Fe/H}]$ is presented in § 3.2.3. In order to directly compare with observations, we determine the fraction of stars with currently detectable planets as $\eta$. In the determination of $\eta$, we assume that the detection limit is set by the magnitude of the radial velocity ($v_r > 10$ m s$^{-1}$) and the orbital periods ($T_K < 4$ yr). According to the following uncertainty, we exclude close-in planets with $a < 0.05$ AU in the evaluation of $\eta$.

We artificially terminate type I and II migration near the disk inner edge at a 2 day period ($\approx 0.03$ AU for $M_i = 1 M_\odot$) in a similar way to that used in Papers I–III. We have not specified a survival criterion for the close-in planets because we do not have adequate knowledge about the planets' migration and their interaction with their host stars near the inner edge of their nascent disks. Hence, we record all the planets that have migrated to the vicinity of their host stars. In reality, a large fraction of the giant planets that have migrated to small disk radii may either be consumed (e.g., Sandquist et al. 1998) or tidally disrupted (e.g., Trilling et al. 1998; Gu et al. 2003) by their host stars. Cores that have migrated to the inner edge of the disk may also coagulate and form super-Earths (e.g., Terquem & Papaloizou 2007). We also neglect such core coagulation near the inner edge.

In a set of fiducial models, we adopt $M_i = 1 M_\odot$, $[\text{Fe/H}] = 0.1$, and $(k_1, k_2) = (9, 3)$ for the gas giants' growth rate (eq. [17]). The analytic deductions in the previous subsection suggest that relatively massive cores can be retained to form gas giants, provided that $C_1 \lesssim 0.1$. Figure 5 shows the predicted $M_p$–a distributions for $C_1 = 0$ (Fig. 5b), $C_1 = 0.01$ (Fig. 5c), $C_1 = 0.03$ (Fig. 5d), $C_1 = 0.1$ (Fig. 5e), and $C_1 = 0.3$ (Fig. 5f). In order to directly compare the theoretical predictions with the observed data, we plot in (Fig. 5a) the values of $M_p$ that are a factor of 1.27 times the values of $M_p$ sin $i$ determined from radial velocity measurements. This correction factor corresponds to the average value of $1/\sin i = 4/\pi$ for a sample of planetary systems with randomly oriented orbital planes. To compare with the theoretical results with $M_i = 1 M_\odot$, we plot only the data of planets around stars with $M_i = 0.8$–1.2 $M_\odot$ that have been observed by radial velocity surveys.

All results show the “planet desert,” which is a lack of intermediate-mass ($M_p \sim 10$–100 $M_\oplus$) planets at $a \lesssim$ a few AU. However, the formation probability of gas giants dramatically changes with $C_1$. The fraction of stars with gas giants, $\eta_g$, changes from 22.9% for the model with $C_1 = 0$ to 0.2% for $C_1 = 0.3$ (see Table 1). In the observed data, $\eta_g \approx 5\%$–8% around $[\text{Fe/H}] \approx 0.1$.

In contrast, the distributions of the retained gas giants are similar to each other except for those with relatively large values of $C_1$. The theoretical predictions are also consistent with the observed distribution in Fig. 5a. We carry out a Kolmogorov-Smirnov (K-S) test for statistical similarity between the simulated models and the observed data for the parameter domain of $0.1 \text{ AU} < a < 2.5 \text{ AU}$ and $M_p > 100 M_\oplus$. This range corresponds to a maximum rectangular region in which planets are detectable by radial surveys with a precision of $v_r > 10$ m s$^{-1}$ and a duration of $< 4$ yr. Since we have not imposed any criterion for determining the survival probability of short-period planets, the predicted population of planets with $a < 0.1$ AU is excluded in the quantitative statistical significance test. Except for the model with $C_1 = 0.3$ (Fig. 5f), these models are statistically similar to the observed data within a significance level of $Q_{\text{KS}} \gtrsim 0.3$ for both the semimajor axis and mass cumulative distribution functions. In particular, the model with $C_1 = 0.03$ (Fig. 5d) shows an excellent agreement with $Q_{\text{KS}} = 0.86$ for the mass function.

In models with $C_1 = 0.3$, only the low-mass cores can survive type I migration. The envelope contraction timescales for these low-mass cores are generally much longer (eq. [17]) than the gas depletion timescales. Consequently, the value of $\eta_g$ is very small (<1%) for the simulated model with $C_1 = 0.3$. Since type II migration occurs after planets have acquired a mass that is adequate to open up gaps, the close-in planets with $\gtrsim 100 M_\oplus$ are rare for models with $C_1 = 0.3$, in contrast to the models for which type I migration is neglected or sufficiently reduced (Figs. 5b–5e).

If we assume that the survival fraction of close-in planets is independent of their value of $M_i$, the magnitude of $C_1$ can be calibrated from the observed mass distribution of close-in planets. In

\footnote{See http://exoplanet.eu/}
Figure 6a, we plot the mass function for all the planets that are halted artificially at $a = 0.03$ AU. In this panel, we neglect any further evolution, including both disruption and collisions. We also consider an alternative limit, which is that after the gas depletion, the multiple generations of cores that migrated to the proximity of any given star are able to merge into a single terrestrial planet with a mass $M_{\text{mer}}$ (Fig. 6b). These results suggest the potential findings of many hot Earths, including Neptune-mass planets, with either transit or radial velocity surveys.

The above results clearly highlight the competing effects of type I migration and gas accretion. In these models, we approximate the gas accretion process with $(k_1; k_2) = (9, 3)$. The early models of proto–gas giant planet formation (Pollack et al. 1996) yield slower growth rates, and they are better fitted with $(k_1; k_2) = (10, 3.5)$. However, recent revisions (Ikoma et al. 2000; Hubickyj et al. 2005) of these models indicate that the proto–gas giants’ growth rates can be significantly enhanced by the opacity reduction associated with grain growth or boundary conditions at different regions of the disk (Ikoma et al. 2001). The Kelvin–Helmholtz contraction timescale may also be reduced by turbulent heat transport in the outer envelope of the protoplanets. In view of these uncertainties, we also simulated models with $k_1 = 8$ and 10, with

![Figure 5](image_url)  
**FIG. 5.**—Planetary mass and semimajor axis distribution. Units of the mass ($M_p$) and semimajor axis ($a$) are Earth masses ($M_E = M_0$) and AU. (a) Observational data of extrasolar planets (based on data from http://exoplanet.eu/) around stars with $M_* = 0.8–1.2$ $M_\odot$ detected by radial velocity surveys. The determined value of $M_p \sin i$ is multiplied by $1/(\sin i) = 4\pi \simeq 1.27$, where we assume a random orientation of the planetary orbital planes. Subsequent panels show (b) the distribution obtained from Monte Carlo simulations without taking into account the effect of type I migration, and distributions that include type I migration, with (c) $C_1 = 0.01$, (d) $C_1 = 0.03$, (e) $C_1 = 0.1$, and (f) $C_1 = 0.3$. The dashed lines express the observational limit with a radial velocity measure precision of $v_r = 10$ m s$^{-1}$. In these models, $M_* = 1 M_\odot$, the magnitude of the metallicity is $[\text{Fe/H}] = 0.1$, and the contraction timescale parameters in eq. (17) are assumed to be $(k_1; k_2) = (9, 3)$. [See the electronic edition of the Journal for a color version of this figure.]

| TABLE 1 |
|-----------------------------------|
| Envelope Contraction Time | $C_1 = 0$ | $C_1 = 0.01$ | $C_1 = 0.03$ | $C_1 = 0.1$ | $C_1 = 0.3$ |
|-----------------------------------|
| $k_1 = 8$ | 23.7 | 19.1 | 15.9 | 6.7 | 2.0 |
| $k_1 = 9$ | 22.9 | 17.2 | 12.1 | 4.3 | 0.2 |
| $k_1 = 10$ | 21.3 | 12.0 | 7.9 | 2.3 | 0.0 |

**NOTES.**—The quantity $q_0$ is given in units of percent. Other parameters are fixed: the metallicity $[\text{Fe/H}] = 0.1$, and the stellar mass $M_* = 1 M_\odot$. 

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are coagulated into one planet in each system. Includes the maximum effect of coagulation after the gas is depleted. All the planets are dynamically gas accretion (which is represented by $k$) and collisions is neglected. Filled circles, open circles, squares, and triangles represent the models to suppress the emergence of gas giant planets, Earth-mass planets with $M < 1 M_\oplus$, and very fast type II migration, some of the embryos along the path of the migrating gas giants may also be scattered to large radial distances and form later generation terrestrial planets (Raymond et al. 2006).

The survival of terrestrial planets depends on their postformation encounter probability with migrating giant planets. In the absence of any type I migration, this probability is modest. But the inclusion of a small amount of type I migration significantly reduces the fraction of stars with massive close-in gas giants, because the retention of the progenitor cores becomes possible only at the late stages of disk evolution when the magnitude of $\Sigma_g$ is reduced. With a limited supply of the residual disk gas, the growth of gas giants and their type II migration are suppressed.

We find that repeated migration of gas giants is less common in models with $C_1 \geq 0.01$ than in those with $C_1 = 0$. The low type II migration probability reduces the need for efficient disruption of largely accumulated close-in planets (see Paper II). It also ensures that most of the terrestrial planets formed in the habitable zones are not removed by the migrating gas giants. Note that type I migration also inhibits in situ formation of gas giants near 1 AU (see § 3.1). Thus, a small amount of type I migration facilitates the formation and retention of terrestrial planets in habitable zones in extrasolar planetary systems, rather than inhibiting them.

3.2.3. Metallicity Dependence

We also study the dependence of other parameters on metallicity ([Fe/H]). As fiducial models, we set $C_1 = 0.03$ and $k_1 = 9$ to compare the metallicity dependence. The most massive cores that can form and be retained prior to gas depletion have masses of $M_{c, \text{max}}$ given by equation (23). Smaller values of $f_{\text{d},0}$, larger values of $f_{\text{d},0}$, or smaller values of $C_1$ increase the value of $M_{c, \text{max}}$ and would enhance the formation of gas giants. The models in Figure 5 have already illustrated the dependence on $C_1$.

In Paper II, we showed that relatively large values of [Fe/H] (or equivalently, $f_{\text{d},0}$) enhance the growth rates and increase the isolation masses for the cores for disks with the same value of $f_{\text{d},0}$. Fast emergence of massive cores can lead to the rapid onset of gas accretion. On the basis of that model, we found that $\eta_\text{II}$ increases with [Fe/H], and the simulated dependence of $\eta_\text{II}$ on [Fe/H] is qualitatively consistent with observed data (Fischer & Valenti 2005).

Here we study the effect of type I migration on the $\eta_\text{II}$-[Fe/H] correlation. In Figure 5, [Fe/H] = 0.1 is assumed. We carried out similar simulations with various values of [Fe/H], and the simulated $\eta_\text{II}$-[Fe/H] relation is plotted in Figure 7. A comparison between the results here and those in Paper II shows that type I migration enhances the $\eta_\text{II}$-[Fe/H] correlation. In metal-poor disks, $f_{\text{d},0}$/$f_{\text{d},0} < 1$, and $\eta_\text{II}$ is significantly reduced by type I migration. But in more metal-rich disks, relatively massive cores can be retained before the disk gas is severely depleted. The resultant steep dependence is in better agreement with the observed data (Fig. 7, open circles) than in the dependence without type I migration (Fig. 7, filled circles with dashed line), although a different choice of the assumed averaged orbital separation may change $\eta_\text{II}$ slightly.

Formed planetary systems are affected by the stellar mass, $M_*$, as well as by the metallicity. The dependence on $M_*$ was studied in Paper III and in Burkert & Ida (2007), using a simple model without the effects of type I migration. In the former paper, we predicted that gas giants are much more rare around M-type dwarfs than around FGK dwarfs, whereas super-Earths are abundant around M-type dwarfs, which is consistent with radial velocity surveys and microlensing surveys (Beaulieu et al. 2006). Adding type I migration to the simple prescription in Paper III, we found that the above conclusions do not change. Around M-type dwarfs,

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{fig6.pdf}
\caption{Predicted mass functions, which are halted artificially at $a = 0.03$ AU. (a) Mass function for all the planets, where further evolution due to both disruption and collisions is neglected. Filled circles, open circles, squares, and triangles represent the results with $C_1 = 0$, 0.001, 0.01, and 0.1, respectively. [Other parameters are $M_*=1 M_\odot$, [Fe/H] = 0.1, and $(k_1, k_2) = (9, 3).$] (b) Mass function that includes the maximum effect of coagulation after the gas is depleted. All the planets are coagulated into one planet in each system.}
\end{figure}
Fig. 7.—Metallicity dependence of the fraction of stars with planets that are detectable by radial velocity surveys with measurement precisions of \( v_r > 10 \text{ m s}^{-1} \) and coverage periods of <4 yr. Open circles with error bars represent observed data (Fischer & Valenti 2005). Dashed, solid, and dotted lines with filled circles indicate the theoretically predicted dependences with \( C_1 = 0, 0.03, \) and 0.3, respectively. Other parameters are \((k_1, k_2) = (9, 3)\) and \(M_\star = 1 M_\odot\). [See the electronic edition of the Journal for a color version of this figure.]

super-Earths at 1–3 AU, which are inferred to be abundant by microlensing surveys, survive type I migration. In a separate paper, we will address the details of the \( M_\star \) dependence of planetary systems, taking into account the \( M_\star \) dependences of many physical quantities.

4. SUMMARY AND DISCUSSION

In our previous Monte Carlo simulations of planet formation processes (Papers I, II, and III), we neglected the effects of type I migration. In the present paper, we have investigated its effects on the formation of terrestrial planets and cores of gas giants. We found that type I migration provides a self-clearing mechanism for planetesimals in the terrestrial planet region. Although the planetesimal disk at \( a \leq 1 \text{ AU} \) is significantly cleared, the total mass of residual planetary embryos at regions within a few AU is comparable to the Earth’s mass, almost independent of the disk and migration parameters. Earth-like planets can be assembled in habitable zones after the depletion of the disk gas.

But this self-regulated clearing process does prevent giant planets from forming at \( \sim 1 \text{ AU} \) even in very massive disks. In general, the clearing of cores leads to the late formation of gas giants. When the surface density of the disk gas is reduced below that of the MMSN, type I migration would no longer be able to remove cores that are sufficiently massive to initiate the onset of rapid gas accretion. This late-formation tendency also reduces the fraction of gas giants that undergo extensive type II migration. Since migrating gas giants capture and clear cores along their migration paths, type I migration also facilitates the retention of Earth-mass planets in the habitable zones.

In the limit that type I migration operates with an efficiency comparable to that deduced from the traditional linear torque analysis (i.e., with \( C_1 = 1 \)), all cores would be cleared prior to the gas depletion, such that gas giant formation would be effectively suppressed. However, the catastrophic peril of type I migration would be limited by at least a 10-fold reduction in the type I migration speed \((C_1 \leq 0.1)\). In this limit, a substantial fraction of the cores may survive, and gas giants would form and be retained with an efficiency \((\eta_I)\) comparable to that observed.

The above discussions indicate that the formation probability of gas giants is delicately balanced by various competing processes. Since these processes have comparable efficiency, the fraction of solar-type stars with gas giants appears to be the “threshold” quantity. Small variations in the strength of one or more of these effects can strongly modify the detection probability of extrasolar planets. For example, \(\eta_I\) is observed to be a rapidly increasing function of its host star’s metallicity (Fischer & Valenti 2005). Although we were able to reproduce this observed trend in Paper II, the simulations presented here indicate that this effect is enhanced by a small amount of type I migration because it is more effective in metal-poor disks.

The \( M_\star-\alpha \) distribution in Figure 5 also shows its sensitive dependence on various other model parameters. The most noticeable dependence is the relative frequency between gas and ice giant planets at several AU from their host stars. These distributions also predict a modest population of close-in cores, a fraction of which may survive and be observable as hot Earths.

4.1. Metallicity Homogeneity of Open Clusters

In the modern paradigm of star formation (Shu et al. 1987), most of the stellar content is processed through protostellar disks. Gas diffusion in these disks is regulated by the process of turbulent transport of angular momentum, whereas the flow of heavy elements is determined by the orbital evolution of their main carriers; i.e., through the gas drag of grains and the tidal interactions of planetesimals and cores with the gas. In general, the accretion rates of these two components are not expected to match each other. In fact, the formation of gas giant planets around \( \sim 10\% \) of solar-type stars and the common existence of debris disks requires the retention of heavy elements with masses at least comparable to that of the MMSN. Yet, stars in young stellar clusters are chemically homogeneous (Wilden et al. 2002; Quillen 2002; Shen et al. 2005). The observationally determined upper limit in the metallicity dispersion \((<0.03–0.04 \text{ dex})\) among the stars in the Pleiades and IC 4665 open clusters implies a total residual heavy-element mass (including the planets) of less than twice that in solar system planets.

Clues to the resolution of this paradox can be found in the protostellar disks. Recent models of the observed millimeter continuum spectral energy distributions suggest that some fraction of classical T Tauri disks have dust masses \( \geq 10 \text{ times the total heavy-element mass in the solar system} \) (Hartmann et al. 2006). Their host stars would acquire a significant metallicity dispersion if a large fraction of the heavy elements in these disks was retained as terrestrial planets and cores, while most of their gas components were accreted by the stars. Hence, an efficient process to clear heavy elements is required.

Although, due to gas drag, dust grains can undergo inward migration and be accreted onto their host stars, we here assume that the formation of planetesimals is a more efficient self-clearing process. In active protostellar disks in which the values of \( \Sigma_g \) and \( \Sigma_d \) are much larger than those of the MMSN, planetesimals quickly grow into cores that undergo rapid type I migration. Through a series of numerical simulations, we show that most cores formed interior to \( a_{\text{dep,mig}} \sim 1 \text{ AU} \) are lost before the disk gas is severely depleted. On larger spatial and temporal scales, this process is also effective in transporting most of the original heavy elements to their host stars. Provided that their progenitor molecular clouds are thoroughly mixed, this self-regulated clearing process would ensure that stars in young clusters acquire a nearly uniform metallicity.
4.2. Diversity of Giant Planets around Solar-Type Stars

Among the stars with known gas giants, a large fraction of them show signs of additional planets. This special multiplicity function is associated with the formation of gaps around the first-born gas giants. Beyond the outer edge of the gap, positive pressure gradient leads the gas to attain a local super-Keplerian velocity. Dust particles and planetesimals accumulate in this region and grow into sufficiently massive cores to initiate the formation of additional planets (Bryden et al. 2000). It is also possible that planet formation is a threshold phenomenon; i.e., the conditions needed to form multiple-planet systems are marginally more stringent than those for the formation of single gas giants.

The simulations presented here consider the formation probability of individual planets. Although the impact of type I migration on the residual disk and the formation of multiple generations of cores have been taken into account, we have neglected the impact of gas giant formation on the residual disks. Nevertheless, this algorithm can be used to qualitatively describe a threshold scenario for the formation of multiple planets.

The total formation timescale of gas giants is \( \tau_{\text{form}} \sim \tau_{\text{c, acc}} + \tau_{\text{KH}} \). In comparison with both type II migration and the disk depletion timescales, we find that, provided that \( \tau_{\text{c, acc}} < \tau_{\text{mig1}} \),

1. Solar-system–like giant planets would form if \( \tau_{\text{form}} \sim \tau_{\text{dep}} \), because gas giants would not have enough time to undergo extensive type II migration.

2. Eccentric giants would form if \( \tau_{\text{form}} < \tau_{\text{dep}} \) and if orbital instability does not occur on a timescale of \( \tau_{\text{dep}} \), the multiple planets could be locked into mean motion resonances during migration. According to the above conditions, the regions in which close-in, eccentric, and solar-system–like giant planets are likely to form are schematically plotted in the \( f_{\text{dep}}-a_0 \) plane (Ida & Lin 2004a, Paper I). Light gray, dark gray, and medium gray regions correspond to the regions for close-in, eccentric, and solar-system–like giants, respectively. A more quantitative set of simulations will be presented in a subsequent paper.

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REFERENCES

Agnor, C. B., Canup, R. M., & Levison, H. F. 1999, Icarus, 142, 219
Alibert, Y., Mousis, O., Mordasini, C., & Benz, W. 2005, ApJ, 626, L57
Armitage, P. J. 2003, ApJ, 582, L47
Beaulieu, J.-P., et al. 2006, Nature, 439, 437
Bryden, G., & Bodenheimer, P. 1997, Icarus, 128, 139
Bryden, G., Rozyczka, M., Lin, D. N. C., & Bodenheimer, P. 2000, ApJ, 540, 1091
Canup, R. M., & Ward, W. R. 2006, Nature, 441, 834
Crida, A., & Morbidelli, A. 2007, MNRAS, 377, 1324
Cumming, A. 2004, MNRAS, 354, 1165
Daisaka, K. J., Tanaka, H., & Ida, S. 2006, Icarus, 185, 492
D’Angelo, G., Kley, W., & Henning, T. 2003, ApJ, 586, 540
D’Angelo, G., Lubow, S. H., & Batu, M. R. 2006, ApJ, 652, 1698
Dobbs-Dixon, I., Li, S.-L., & Lin, D. N. C. 2007, ApJ, 660, 791
Fischer, D. A., & Valenti, J. A. 2005, ApJ, 622, 1102
Gu, P., & Lin, D. N. C. 2007, ApJ, 654, 606
Goldreich, P., & Tremaine, S. 1980, ApJ, 241, 425
Garaud, P., & Lin, D. N. C. 2007, ApJ, 654, 606
Goldreich, P., & Tremaine, S. 1980, ApJ, 241, 425
Gu, P., Lin, D. N. C., & Bodenheimer, P. H. 2003, ApJ, 588, 509
Guillot, T., & Hueso, R. 2006, MNRAS, 367, L47
Hartmann, L., Calvet, N., Guiblin, E., & D’Alessio, P. 1998, ApJ, 495, 385
Hartmann, L., D’Alessio, P., Calvet, N., & Muzerolle, J. 2006, ApJ, 648, 484
Hubickyj, O., Bodenheimer, P., & Lissauer, J. J. 2005, Icarus, 179, 415
Kokubo, E., & Ida, S. 1998, Icarus, 131, 171
Kokubo, E., & Ida, S. 2002, ApJ, 581, 666
Kollmer, J., & Li, H. 2004, in AIP Conf. Proc. 713, The Search For Other Worlds, ed. S. S. Holt & D. Deming (New York: AIP), 63
Kominami, J., & Ida, S. 2002, Icarus, 157, 43
Kornet, K., Stepinski, T. F., & Rozyczka, M. 2001, A&A, 378, 180
Laughlin, G., Steinacker, A., & Adams, F. C. 2004, ApJ, 608, 489
Lecar, M., Podolak, M., Sasseleov, D., & Chiang, E. 2006, ApJ, 640, 1115
Lin, D. N. C. 2005, ApJ, 624, 1003
Lin, D. N. C. 1986, in The Solar System, ed. M. G. Kivelson (New Jersey: Prentice Hall), 28
———. 1995, in Molecular Clouds and Star Formation, ed. C. Yuan & J.-H. You (Singapore: World Scientific), 261
———. 2000b, ApJ, 616, 567 (Paper II)
———. 2004a, ApJ, 604, 388 (Paper I)
———. 2004b, ApJ, 616, 567 (Paper II)
———. 2005, ApJ, 626, 1045 (Paper IV)
Lin, D. N. C., & Bodenheimer, P. 1982, ApJ, 262, 768
Lin, D. N. C., Bodenheimer, P., & Richardson, D. 1996, Nature, 380, 606
Lin, D. N. C., & Ida, S. 1997, ApJ, 479, 781
Lin, D. N. C., & Papaloizou, J. C. B. 1979, MNRAS, 188, 191
———. 1985, in Protostars and Planets II, ed. D. C. Black & M. S. Mathew (Tucson: Univ. Arizona Press), 981

We are thankful for detailed helpful comments by an anonymous referee. This work is supported by NASA (NAGS5-11779, NNG04G-191G, and NNG06-GH45G), JPL (1270927), the NSF (AST-0507424 and PHY99-0794), and the JSPS.
Lin, D. N. C., & Papaloizou, J. C. B. 1986, ApJ, 309, 846
———. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 749
Lynden-Bell, D., & Pringle, J. E. 1974, MNRAS, 168, 603
Marcy, G., Butler, R. P., Fischer, D., Vogt, S., Wright, J. T., Tinney, C. G., & Jones, H. R. A. 2005, Prog. Theor. Phys. Suppl., 158, 24
Masset, F. S., D'Angelo, G., & Kley, W. 2006a, ApJ, 652, 730
Masset, F. S., Morbidelli, A., Crida, A., & Ferreira, J. 2006b, ApJ, 642, 478
Mayor, M., Pont, F., & Vidal-Madjar, A. 2005, Prog. Theor. Phys. Suppl., 158, 43
McNeil, D., Duncan, M., & Levison, H. F. 2005, AJ, 130, 2884
Nelson, R. P., & Papaloizou, J. C. B. 2004, MNRAS, 350, 849
Papaloizou, J. C. B., & Terquem, C. 2006, Rep. Prog. Phys., 69, 119
Pollack, J. B., Hollenbach, D., Beckwith, S., Simonelli, D. P., Roush, T., & Fong, W. 1994, ApJ, 421, 615
Pollack, J. B., Hubickyj, O., Bodenheimer, P., Lissauer, J. J., Podolak, M., & Greenzweig, Y. 1996, Icarus, 124, 62
Quillen, A. C. 2002, AJ, 124, 400
Rasio, F. A., & Ford, E. B. 1996, Science, 274, 954
Raymond, S. N., Mandell, A. M., & Sigurdsson, S. 2006, Science, 313, 1413
Raymond, S. N., Quinn, T., & Lunine, J. I. 2004, Icarus, 168, 1
Sandquist, E., Taam, R. E., Lin, D. N. C., & Burkert, A. 1998, ApJ, 506, L65
Sano, T., Inutsuka, S., Turner, N. J., & Stone, J. M. 2004, Prog. Theor. Phys. Suppl., 155, 409
Sano, T., Miyama, S. M., Umebayashi, T., & Nakano, T. 2000, ApJ, 543, 486
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Shen, Z.-X., Jones, B., Lin, D. N. C., Liu, X.-W., & Li, S.-L. 2005, ApJ, 635, 608
Shu, F. H., Adams, F. C., & Lizano, S. 1987, ARA&A, 25, 23
Stepinski, T. F., & Valageas, P. 1997, A&A, 319, 1007
Tanaka, H., & Ida, S. 1999, Icarus, 139, 350
Tanaka, H., Takeuchi, T., & Ward, W. 2002, ApJ, 565, 1257
Tanigawa, T., & Ikoma, M. 2007, ApJ, 667, 557
Terquem, C., & Papaloizou, J. C. B. 2007, ApJ, 654, 1110
Thommes, E., & Murray, N. 2006, ApJ, 644, 1214
Trilling, D. E., Benz, W., Guillot, T., Lunine, J. I., Hubbard, W. B., & Burrows, A. 1998, ApJ, 500, 428
Ward, W. 1986, Icarus, 67, 164
Ward, W., & Hahn, J. 1995, ApJ, 440, L25
Weidenschilling, S. J., & Marzari, F. 1996, Nature, 384, 619
Wilden, B. S., Jones, B. F., Lin, D. N. C., & Soderblom, D. R. 2002, AJ, 124, 2799
Zhou, J.-L., Aarseth, S. J., Lin, D. N. C., & Nagasawa, M. 2005, ApJ, 631, L85
Zhou, J.-L., & Lin, D. N. C. 2007, ApJ, 666, 447
Zhou, J.-L., Lin, D. N. C., & Sun, Y. S. 2007, ApJ, 666, 423