Algorithm for controlling objects implementing a semi-passive guidance technique

V I Pavlov¹, O A Belousov¹, T Y Dorokhova¹, M P Belyaev² and E V Trapeznikov³

¹ Tambov State Technical University, 106, Sovetskaya Str., Tambov, 392000, Russia
² Military Training and Scientific Center of the Air Force "Air Force Academy named after Professor N.E. Zhukovsky and Yu.A. Gagarina", Voronezh, Russia
³ Omsk State Technical University, 11, Mira av., Omsk 644050, Russia

E-mail: resbn@mail.ru, evtrapeznikov@yandex.ru

Abstract. The minimum composition of objects involved in the implementation of the developed semi-passive guidance technique is determined. The prospects of implementing the developed technique with a network-centric method of object interaction are indicated. A variant of the synthesis of trajectory control algorithms for all objects involved in the implementation of the developed technique is presented. The potential feasibility of the method is shown when using the measuring instruments available in the onboard control systems of objects. Techniques for improving the accuracy of determining the coordinates and parameters of the movement of objects have been developed and studied.

1. Introduction

Mobile controlled objects perform a wide range of tasks both in the air and water environment and on the earth’s surface [1, 2]. For the delivery of payloads to the final point of the route, self-guided unmanned mobile objects are used, among other things [3]. In addition to the existing, classified by the location of the source and receiver of radiation, active, semi-active and passive methods [4,5], a new semi-passive guidance technique for mobile objects is proposed [6]. The perspective of the proposed semi-passive method consists in a significantly longer range of information contact with the approaching object (target) in comparison with the existing homing technique [7]. In the proposed method, only the goniometric information about the target position is available for direct measurement on board the homing object. At the same time, to ensure the required accuracy of the approach to the maneuvering target, it is necessary to transmit either the closure velocity to the target or its coordinates to the homing object. Theoretically, it is possible to form an estimate of the speed of approach on board the object when it periodically performs special maneuvers. However, in practice, this technique will be ineffective while intensive maneuvering of the target. The article considers an algorithm for controlling air objects that implement a semi-passive homing technique within the network-centric approach, including measures to ensure the required accuracy of estimating the target location.
2. Problem statement

The minimum number of participants while implementing the semi-passive homing technique includes an aircraft providing emission directed to the target of a probing sound signal [8] and an unmanned aerial vehicle (UAV) that performs a direct approach to the target. As a target, an aircraft with an available natural speed of up to 700 m/s and a lateral overload of up to 10 units is considered. The target has on-board equipment that generates return emission to the probing signal. The aircraft is the source of the probing signal and the UAV together represent a multi-position air-based radar [9]. By analogy with [1], the relative position of the target G, the source aircraft of the probing signal C1 and the UAV C2 in a rectangular coordinate system combined with the aircraft C1 is shown in Figure 1. In the figure, the following designations are used: $V_1$, $V_2$, and $V_g$ are, respectively, the velocity vectors of the aircraft C1, UAV C2 and target G; $\psi_1$, $\psi_2$, and $\psi_g$ are the course angles of the aircraft C1, UAV C2 and target G; $\varepsilon_1$ and $\varepsilon_2$ are line-of-sight angles from aircraft C1 and UAV C2 in the selected coordinate system; $\varphi_1$ and $\varphi_2$ are the relative bearings of the target from the aircraft C1 and UAV C2; $\alpha_1$, $\alpha_2$, and $\alpha_g$ are the triangulation angles of a triangle C1C2G; $D_1$ and $D_2$ are the distances to the target from aircraft C1 and UAV C2, respectively; $D_b$ is the distance between C1 and C2.

Figure 1. The relative position of the source of the probing signal, the UAV and the target in the horizontal plane in the Earth coordinate system

The synthesis of the trajectory control algorithm for objects that implement a semi-passive homing technique with local optimization is performed under the following assumptions:

- there is a communication channel between C1 aircraft and C2 UAV for the mutual transmission of coordinate information, including for the transmission from the aircraft
to the UAV the calculated speed of approach to $V_{appr}$ target;

- C1 aircraft and C2 UAV can be equipped, among other things, with a satellite navigation system to accurately determine their own coordinates [10];

- UAV is guided by the proportional guidance method, in which the mismatch parameter is formed according to the rule [4]:

$$\Delta = \hat{j}_{tr} - \hat{j} = N_0 \hat{V}_{sb} \hat{\omega} - \hat{j}$$

where $\hat{j}_{tr}$ and $\hat{j}$ are, respectively, estimates of the required and current normal UAV accelerations; $N_0$ is the navigation parameter; $\hat{V}_{sb} = -\hat{D}$ is the estimate of the UAV approach speed to the target; $\hat{\omega}$ is the angular velocity of the line-of-sight;

- on board aircraft and C1 UAV C2, their course ($\psi_1, \psi_2$), onboard bearings ($\varphi_1$ and $\varphi_2$) and their derivatives ($\dot{\Psi}_1, \dot{\Psi}_2, \dot{\varphi}_1, \dot{\varphi}_2$), positioning data -coordinates $x_1, z_1$ and $x_2, z_2$ and the speed of their change are measured $\dot{x}_1, \dot{z}_1, \dot{x}_2, \dot{z}_2$ [11];

- on board an UAV, the angular velocity of the target line of sight $\omega$ is measured by the radar semi-passive homing head protractor, while the current accelerations are measured by accelerometers oriented along the control planes;

- all objects are at the same height, while the algorithms for controlling C1 aircraft and C2 UAV in the vertical and horizontal planes do not affect each other.

It is necessary to synthesize an algorithm for trajectory control of C1 aircraft, which provides the necessary accuracy in determining the location of the target both for tracking it and for forming an estimate of the speed of approach to it.

3. A variant of the synthesis of the trajectory control algorithm

The motion of C1 aircraft is described by a system of equations [4]

$$\dot{x}_y = F_y x_y + B_y u + \xi_y$$

$$x_{tr} = F_{tr} x_{tr} + \xi_{tr}$$

where $x_y, x_{tr}$ are, in general case, $n = dimensional$ vectors of the controlled and required coordinates of the source C1; $F_y, F_{tr}$ are the dynamic matrices that take into account the internal connections of processes (2) and (3); $B_y$ is the efficiency matrix of the $r$-dimensional ($r < n$) vector of the control signal $u$; $\xi_y, \xi_{tr}$ are random components with zero mathematical expectation. Aircraft C1 performs the measurement

$$z = H x + \xi_u$$

where $H$ is the modulation matrix of the relationship between the state vector and the measurements; $\xi_m$ is the random component of the measurements with zero mathematical expectation for the of the control formation

$$u = K^{-1} B_y^T Q [\hat{x}_{tr} - \hat{x}_y]$$

optimal by minimum local quality functional

$$I = M \left\{ [x_{tr} - x_y]^T Q [x_{tr} - x_y] + \int u^T u dt \right\}$$

where $Q$ and $K$ are the penalty matrices for the accuracy of the operation and the magnitude of the control signals. To use this synthesis device, it is necessary to specify the state models (2),
(3), measurements (4), and the quality functional (6). Based on the conclusions of the separation theorem (statistical equivalence), the synthesis of control and filtering signals is carried out independently (separately) using deterministic state models. Since the theory of observations control [12,13] involves the use of curvilinear trajectories, and the goal can maneuver, the model should be taken into account side bearing of the target with respect to C1, the dependence of the angular velocity of the line of sight from the distance and speed and maneuver of the goal G and the plane C1. In general, these requirements are satisfied by the kinematic relations [4] which, when applied to the horizontal plane, have the following forms:

\[ \dot{\phi}_1 = \omega_1 + \frac{j_1}{D_1}, \phi_1(0) = \phi_0 \]  
\[ \dot{\omega}_1 = -\frac{2\dot{D}_1}{D_1}\omega_1 - \frac{j_1}{D_1}, \omega_1(0) = \omega_0 \]

where \( \phi_1 \) and \( \omega_1 \) are the onboard bearing and the angular velocity of the line-of-sight in the horizontal plane; \( D_1, \dot{D}_1 \) is the range from C1 to the target and the closure velocity to it; \( j_1 \) is the transverse acceleration of C1, which acts as a control signal. In [14], based on the analysis of the geometric relations between the objects in Figure 1, an expression is obtained for the standard deviation of the error in determining the location of the target

\[ \sigma_g = \frac{0.0175}{\sin \alpha_g} \sqrt{\sigma_{\phi_1}^2 + \sigma_{\phi_2}^2 D_1^2 + D_2^2} \]

where \( \sigma_{\phi_1} \) and \( \sigma_{\phi_2} \) are the standard deviations of the angular errors of the direction finders of C1 aircraft and C2 UAV. It follows from (9) that in order to ensure high accuracy, it is necessary to control the C1 aircraft so that \( a_g = 90^\circ \) and, respectively,

\[ \alpha_1 = 90^\circ - \alpha_2 \]

In mathematical terms, the synthesis problem is formulated as follows. For the system (7, 8), it is necessary to generate a control signal \( u = [j_1] \), which is optimal for the minimum of the local quality functional

\[ I = M \left\{ \begin{bmatrix} \phi_{tr1} - \phi_1 \\ \omega_{tr1} - \omega_1 \end{bmatrix}^T \begin{bmatrix} q_{\phi_1} & 0 \\ 0 & q_{\omega_1} \end{bmatrix} \begin{bmatrix} \phi_{tr1} - \phi_1 \\ \omega_{tr1} - \omega_1 \end{bmatrix} + \int_0^t j_1^2 k_j dt \right\} \]

provided that the restrictions (10) are met. In the quality functional (11), \( \phi_{tr1} \) and \( \omega_{tr1} \) are the required values of the onboard bearing and the angular velocity of the line of sight; \( q_{\phi_1}, q_{\omega_1} \) are the penalty coefficients for control errors according to \( \phi_1, \omega_1 \); \( k_j \) is the penalty coefficient for the value of the control signal [14].

As a result of comparing (7) with (2), and (11) with (6), we obtain:

\[ x_T = \begin{bmatrix} \phi_{tr1} \\ \omega_{tr1} \end{bmatrix}; x_y = \begin{bmatrix} \phi_1 \\ \omega_1 \end{bmatrix}; u = j_1 \]

\[ B_y = \begin{bmatrix} 1 \\ -\frac{\dot{D}_1}{D_1} \end{bmatrix}; Q = \begin{bmatrix} q_\phi & 0 \\ 0 & q_\omega \end{bmatrix}; K = k_j \]

When using (12) and (13) in (5), we can obtain:

\[ j_{tr1} = \begin{bmatrix} q_\phi \\ D_1 k_j \end{bmatrix} \Delta \phi_1 - \begin{bmatrix} q_\omega \\ D_1 k_j \end{bmatrix} \Delta \omega_1 \]
where $\Delta\phi_1 = \hat{\phi}_{tr1} - \hat{\phi}_1$, $\Delta\phi_1 = \hat{\phi}_{tr1} - \hat{\phi}_1$. Then the algorithm for trajectory control of the C1 aircraft has the following form:

$$\Delta_1 = \hat{j}_{tr1} - \hat{j}_1 = \frac{q_\phi}{\hat{D}_1 k_j} \Delta\phi_1 - \frac{q_\omega}{\hat{D}_1 k_j} \Delta\omega_1 - \hat{j}_1$$

(15)

in which $\hat{j}_1$ is the estimate of its own transverse acceleration; $\hat{D}_1$, $\hat{D}_1$ are the estimates of the range to the target and the speed of its change.

4. Investigation of the trajectory control algorithm

From the geometry of the relative position of objects (Figure 1) and the expression (10), it follows that

$$\varphi_{tr1} = 90^\circ - \varphi_b - \psi_1 - \alpha_1 = \alpha_2 - \varphi_b - \psi_1 = 90^\circ - \varphi_2 + \psi_2 - \psi_1$$

(16)

Besides, when performing a maneuver by the C1 aircraft in the area of increasing the accuracy of determining the location of the target, the angular velocity of the target line-of-sight depends on changes in the onboard bearing and heading angle

$$\omega_{tr1} = \hat{\epsilon}_1 = \dot{\phi}_{tr1} + \dot{\psi}_1$$

(17)

**Figure 2.** Dependence of the root-mean-square deviation of the target location error $\sigma_g$ on the angle $\alpha_g$ of the triangulation triangle C1C2G

The analysis of expressions (15) – (17) allows us to draw the following conclusion. Signal $\Delta_1$ of the trajectory control of the source of the probing signal, which is C1 aircraft, depends on both the angle errors and the angular velocity of the target line of sight. At the same time, in the origin of the trajectory, the signal directs C1 aircraft away from the target, which leads to a
Figure 3. Dependence of the standard deviation of the target location error $\sigma_g$ on the distance between the C1 aircraft and the target

faster fulfillment of the condition $\alpha_g = 90^\circ$, providing an increase in the accuracy of determining the target location. These conclusions are confirmed by the simulation results shown in Figures 2 and 3. In addition, the analysis of the graphs in Figure 3 shows that the accuracy of determining the location of the target increases as a UAV approaches the target.

Conclusion

The synthesized algorithm can be used for trajectory control of objects that implement a semi-passive homing technique. When controlling the aircraft—the source of the probing signal in accordance with the algorithm (15), the aircraft maneuvers away from the current direction to the target in order to increase the accuracy of assessing its position. It is important to form an estimate of the closure velocity to the target and transfer it to the homing UAV. UAV homing is carried out by the method of proportional guidance in accordance with the algorithm (1). All the parameter estimates necessary for the implementation of algorithms (1) and (15) are formed in the onboard control systems of the corresponding objects. It should also be noted that the accuracy requirements for determining the homing parameters of a UAV with a semi-passive homing head and the trajectory of the source of the provoking sounding signal when pointing at non-intensively maneuvering targets can be reduced compared to the considered option, and the estimates of the target range and the closure velocity are "averaged" and "taken into account" in the navigation parameter $N_0$.

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