Low lying nucleons from chirally improved fermions

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We report on our preliminary results on the low-lying excited nucleon spectra which we obtain through a variational-basis formed with three different interpolators.

There are two competing pictures for understanding baryon physics in different mass regions. For hadrons with heavy quarks linear confinement and perturbative gluon exchange corrections provide a satisfactory description. For lighter quarks towards the chiral limit we have to expect manifestations of spontaneous chiral symmetry breaking. Lattice studies can shed light on the relevant mechanisms. In particular after the advent of Ginsparg-Wilson fermions it is now also possible to run computations relatively close to the chiral limit.

However, exact chiral fermion actions (like the overlap action) are very costly. On the other hand, recently developed approximate solutions of the Ginsparg-Wilson equation allow to work at relatively small quark masses at only moderate numerical cost. Two such approximate solutions, the parametrized fixed point action and the chirally improved action, have been studied in the BGR-collaboration. Here we present results for low lying nucleon states computed with the chirally improved (CI) action. The CI operator is a parameterization of the Dirac operator with terms essentially restricted to the hypercube. It allows for simulations at pseudoscalar-mass to vector-mass ratios down to \( m_{PS}/m_{V} = 0.32 \) for a relatively large lattice spacing of \( a = 0.15 \) fm (see [3] for more details). Thus we can work with light pions on large physical volumes. The quark propagators are determined for quenched gauge configurations generated with the Lüscher-Weisz action. Here we discuss results derived from 100 configurations on \( 16^3 \times 32 \) and \( 12^3 \times 24 \) lattices at a lattice spacing of \( a = 0.15 \) fm as determined with the Sommer parameter [6]. This gives rise to spatial extensions of 2.4 fm and 1.8 fm. In this study our quark mass values cover the region \( 0.02 \leq m_q \leq 0.2 \) corresponding to values of \( m_{PS}/m_{V} \) down to 0.397. A detailed account of our setting is given in [7].

We implemented the following three interpolating operators for the nucleon

\begin{align}
\chi_1(x) &= \epsilon_{abc} \left[ u_a^T(x) C \gamma_5 d_b(x) \right] u_c(x) , \\
\chi_2(x) &= \epsilon_{abc} \left[ u_a^T(x) C d_b(x) \right] \gamma_5 u_c(x) , \\
\chi_3(x) &= i \epsilon_{abc} \left[ u_a^T(x) C \gamma_0 \gamma_5 d_b(x) \right] u_c(x) .
\end{align}

These operators were Jacobi-smeared at both source and sink. We remark that we also experimented with point sinks and wall sources but found unchanged results and no improvement of the quality of our data. We computed all cross correlations \((n, m = 1, 2, 3)\)

\begin{equation}
C_{nm}(t) = \left\langle \chi_n^*(0) \chi_m(t) \right\rangle .
\end{equation}

For a basis of infinitely many operators \((n, m = 1, 2 \ldots \infty)\) the diagonalization of the matrix \(C\)
would lead to the optimal operator combinations building the physical states. Finding these combinations is equivalent to the generalized eigenvalue problem

$$C(t) \zeta^{(k)}(t) = \lambda^{(k)}(t, t_0) C(t_0) \zeta^{(k)}(t), \quad (5)$$

with eigenvalues behaving as

$$\lambda^{(k)}(t, t_0) = e^{-(t-t_0)W_k}. \quad (6)$$

Each eigenvalue corresponds to a different energy level $W_k$ dominating its exponential decay. The optimal operators $\tilde{\chi}_i$ which have maximal overlap with the physical states are linear combinations of the original operators $\chi_i$,

$$\tilde{\chi}_i = \sum_j c^{(i)}_j \chi_j. \quad (7)$$

The coefficients $c^{(i)}_j$ are obtained from the $j$-th entry of the $i$-th eigenvector (corresponding to eigenvalue $\lambda^{(i)}$). Note that here we work with only a finite basis of operators which gives rise to corrections to the single exponential decay Eq. (6) which are discussed in more detail in [7,8].

We project the operators in the correlation matrix (4) to definite parity. This allows us to disentangle states with positive and negative parity. We use the eigenvalues of the full correlation matrix to identify the plateaus in the effective mass. In this region we then use fully correlated two-parameter fits for the largest and second largest eigenvalues to determine the energy of the ground state and first excited state. We also experimented with Bayesian fitting techniques but do not discuss these results here.

In our data we clearly identify the nucleon, the two lowest negative parity states N(1535) and N(1650) and an excited state of positive parity with a mass well above the masses of the two excited negative parity states and thus too high to be identified with the Roper state at 1440 MeV. We denote this state as $N'_+$. For negative parity and for the excited state in the positive parity sector the quality of the fits decreases towards smaller quark masses and we cannot maintain the standards of our fitting procedure. We do not give results in these cases.

Our results for masses the are given in Fig. 1. Like previous studies [9,10,11,12] we clearly observe a splitting between the nucleon and the negative parity states. At large quark masses this is attributed to the orbital excitation of the quark motion in the confining color-electric field. The splitting slowly increases towards the chiral limit. This implies that near the chiral limit there is another mechanism contributing in addition to confinement. This is quite consistent with the chiral constituent quark model [2] where a considerable part of the splitting is related to the flavor-spin residual interaction between valence quarks. Also the splitting between the two negative parity states is clearly visible.

At smaller quark masses (below $a m_q = 0.05$) plateaus in the effective mass plots for both negative parity states and for $N'_+$ completely disappear and hence we cannot trace these states closer to the chiral limit. At small Euclidean time separation there is a hint from our data (for smeared source and point sink) that there may be one more positive parity excited state. However, our analysis did not provide consistent results for this weak signal.

We performed our analysis for two different lattice sizes $16^3 \times 32$ (full curves in Fig. 1) and $12^3 \times 24$ (dashed curves). Fig. 1 shows that the two data sets fall on top of each other and we do...
Figure 2. The mixing coefficients $c_i$ of the optimal operators plotted as a function of pseudoscalar mass. By definition, the basis of the spectral decomposition has $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1$. Data from $16^3 \times 32$ is plotted with filled symbols, for $12^3 \times 24$ we use crosses.

not suffer from finite size effects.

Interesting physical insight can be obtained from the mixing coefficients $c_j^{(i)}$ of Eq. (1). In Fig. 2 we show their dependence on the pseudoscalar mass and again compare $16^3 \times 32$ (filled symbols) and $12^3 \times 24$ (crosses) to illustrate the absence of finite volume effects. Operator $\chi_1$ clearly dominates the nucleon at all quark masses, whereas in QCD sum rule analyses it is assumed that the equally weighted superposition of $\chi_1$ and $\chi_2$ is the optimal combination coupling to the nucleon.

The nucleon belongs to a $56$-plet which would require that under the permutation of two quarks a diquark subsystem has positive parity since the color wave function is completely antisymmetric. In the large $N_c$ limit the nucleon should not contain pseudoscalar or vector diquark subsystems and one expects the interpolator content from $\chi_2$ and $\chi_3$ to be minimal as is indeed seen in our data.

Fig. 2 shows that at large quark masses the $N(1535)$ is dominated by $\chi_1$ and $N(1650)$ by $\chi_2$. As the chiral limit is approached $N(1535)$ couples optimally to $(\chi_1 - \chi_2)$ while $N(1650)$ couples to $(\chi_1 + \chi_2)$ and the $\chi_3$ contribution is suppressed in both cases. $N(1535)$ and $N(1650)$ belong to the negative parity $L = 1$ $70$-plet of $SU(6)$ and both contain scalar and pseudoscalar diquark components in their wave functions. So the mixing in the vicinity of the chiral limit is very different from the mixing in the heavy quark region. Within the quark model this mixing is attributed to the tensor force and our results hint at its relation to spontaneous chiral symmetry breaking.

In [12] the subtraction of a parametrization of possible quenched artifacts ($\eta' N$), giving rise to negative values of the correlation function for central $t$-values, was crucial for the identification of a Roper signal. We also observe such negative values but postpone a detailed analysis to future studies with better statistics.

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