Abstract
Nonlocal quantum correlation has been the main subject of quantum mechanics over the last century. Over the last few decades, various quantum technologies have been developed and applied for real matters of quantum information science. The Hong-Ou-Mandel (HOM) effect relates to the two-photon intensity correlation on a beam splitter, resulting in a quantum bunching phenomenon. In quantum information science, the HOM effect is a major test tool for the quantum features of Bell states. Here, a classically excited HOM effect is presented in a Mach-Zehnder interferometer using coherent photon pairs. In this approach, basis-product indistinguishability between paired photons on a beam splitter is achieved via fast sweeping acousto-optic modulators, whose frequency sweeping directions are opposite to each other. The resulting HOM effect is also verified via numerical calculations of the analytically driven solutions.

Introduction
Nonclassical two-photon intensity correlation is the quintessence of quantum information science [1,2]. Since 1987 [3], the Hong-Ou-Mandel (HOM) effect has been intensively studied for the anticorrelation of photon bunching [3-12]. The nonclassical feature of photon bunching of the HOM effect occurs when two indistinguishable input photons are met on a beam splitter (BS). Based on the particle nature of quantum mechanics, the HOM effect has been explained by the BS-based destructive interference between probability amplitudes of two-photon interactions without any phase information [13]. Based on this understanding, independent light sources have also been used for evidence of the HOM effect [5,9]. Although the quantum operator-based HOM analysis in conventional quantum mechanics results in destructive interference between two basis products, such an understanding should involve coherence features of each photon with a relative phase between the pair due to the rule of thumb in quantum mechanics that a photon never interferes with others [14].

Recently, wave nature-based interpretations have been performed to understand the fundamental physics of the HOM effect in an interferometric system [15,16], and established a complete understanding on such quantum feature of photon bunching in the HOM effect [17]. Moreover, nonlocal quantum correlation between space-like separated paired photons [18,19] has also been explained with the same wave nature of photons [20]. Based on these new interpretations of quantum features, observations of the HOM effect using different colored lights [9] or independent emitters [5] can also be understood coherently without violating quantum mechanics [15-17]. For the coherence interpretation of the HOM effect, an essential condition of the relative phase between the paired photons has been driven [15,17]. Here, a classically excited HOM effect is proposed and analyzed for the fundamental physics of photon bunching using coherent photon pairs in a Mach-Zehnder interferometers (MZIs). Eventually, indistinguishability between individual photons or atoms based on the particle nature in quantum mechanics should be compatible with the present wave nature of photons. The benefit of the coherence interpretation is in compatibility between quantum mechanics and classical physics that has influenced nearly all electro-optic devices used in various modern technologies.

Results
Figures 1 shows the schematics of the proposed classical model of the typical HOM measurements [3-12] that rely on entangled photon pairs generated from the spontaneous parametric down conversion (SPDC) processes [21,22]. In SPDC-entangled photon pairs, the signal and idler photons are symmetrically frequency-detuned across the line center according to phase matching conditions of the second-order (χ(2)) nonlinear optics [23]. The stability of center frequency (f₀) heavily depends on the pump laser, which is essential for the HOM effect. Each spatially separated input path in the typical HOM scheme include both signal and idler photons with the
same probability amplitudes to satisfy the definition of an entangled state, \( |\Psi\rangle = (|s\rangle_a|d\rangle_b + |d\rangle_a|s\rangle_b)/\sqrt{2} \), where \( |s\rangle \) \((|id\rangle)\) corresponds to \( |E_a\rangle \(|E_b\rangle\) in Fig. 1 [13]. On the contrary of entangled photon pairs, coherent photon pairs from an attenuated laser (L) are used for Fig. 1. For the individual output measurements in both detectors in Fig. 1(a), a pair of delay (\( \tau \)) lines (DLs) are added, where the coherent photons’ bandwidth \( \Delta \) is ~million times narrower compared with that of SPDC. The \( \tau \)-dependent coincidence measurements of the paired output photons are tested for a broad HOM dip without wavelength-dependent optical fringes [3-12,17].

Regarding the HOM effect, the proposed wave nature-based interpretation has shown that an inherent phase shift of \( \pi/2 \) between entangled photons is to be driven analytically with a broad HOM dip. This inherent \( \pi/2 \) phase shift has already been observed in an entangled ion pair [24]. This specific phase relation between entangled photon pairs does not violate quantum mechanics. In the particle nature-based quantum mechanics, however, such a phase information has never been discussed.

**Fig. 1. Schematic of the HOM effect.** (a) Classical version of HOM. L: laser, BS: nonpolarizing 50/50 beam splitter, PBS: polarizing beam splitter, AOM: acousto-optic modulator, Q: quarter-wave plate, M: mirror, DL: delay line, D: single photon detector. Inset: AOM generating spectral bandwidth. (b) Equivalent scheme of (a).

In Fig. 1, basic properties of entangled photon pairs from SPDC nonlinear optics are classically manipulated using acousto-optic modulators (AOMs) with a double-pass scheme via a quarter-wave plate (QWP). The use of the QWP is to spatially separate the reflected photon from the incident one via opposite polarizations. Using a double-pass AOM technique, the angular deviation of the \( \delta f_j \)-dependent diffracted light is fully compensated, resulting in a jitter-free \( j^{th} \) polarization-basis pair. To mimic the random bases of the signal and idler photons in the SPDC case, the AOMs are kept for fast frequency sweeping for the given bandwidth \( \Delta \) in opposite directions across the center frequency \( f_0 \) of the undeflected original photon (see the Inset). Thus, the AOMs outputs photon pairs exactly correspond to the signal and idler photon pairs of the SPDC [21,22]. The randomly detuned photon pairs by the AOMs are superposed on the BS. Output intensity and two-photon coincidence measurements are conducted to prove the HOM effect.

Figure 1(b) shows an equivalent scheme to Fig. 1(a). In Fig. 1(b), the output photons are analyzed for the HOM effect for the full bandwidth of \( 2\Delta \), where \( 2\Delta \) is due to the double-pass AOMs. Using the BS matrix representation [25], the following relation is straightforwardly obtained in Fig. 1:

\[
\begin{bmatrix}
E_c \\
E_d
\end{bmatrix}
= i \frac{1}{\sqrt{2}}
\begin{bmatrix}
1 & i \\
i & 1
\end{bmatrix}
\begin{bmatrix}
E_a \\
E_b
\end{bmatrix}
= \frac{E_0}{\sqrt{2}} [e^{-i\eta}e^{-i\Delta_j} + ie^{i\Delta_j}],
\]

where \( E_a = E_0e^{i\eta}e^{-i\Delta_j}, \ E_b = E_0e^{i\Delta_j}, \ \Delta_j = \delta f_j \tau, \) and \( \eta \) is the inherent phase shift assigned to \( E_a \) with respect to \( E_b \). The sign of this additional phase \( \eta \) must be dependent on the sign of the detuning \( \Delta_j \). Thus, the following relations are obtained:
From the fast sweeping mode of AOMs, the detuning $\Delta_j$ is random for measurements. Due to the opposite sweeping mode between $E_a$ and $E_b$, the spectral positions of $E_a$ and $E_b$ are symmetric as shown in the Inset. Equations (2) and (3) are for the first half of the spectral distribution in the Inset.

The second half of the spectral distribution corresponds to swapping of input photons due to the sign change in $\Delta_j$, i.e., $E_a \leftrightarrow E_b$:

$$\begin{bmatrix} E'_a \\ E'_d \end{bmatrix} = \frac{E_0}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} E_a \\ E_b \end{bmatrix}$$

$$= \frac{E_0}{\sqrt{2}} \left( e^{i\eta e^{+i\Delta_j}} + ie^{-i\Delta_j} \right)$$

where $E'_a = E_0 e^{i\eta e^{+i\Delta_j}}$ and $E'_b = E_0 e^{-i\Delta_j}$. Here, the sign of the inherent phase shift $\eta$ is also changed for $E'_b$ due to the sign change in the frequency detuning. Thus, the following relations are obtained:

$$I'_a(\tau) = I_b(1 - \sin(\eta + 2\Delta_j))$$

$$I'_b(\tau) = I_a(1 - \sin(\eta + 2\Delta_j))$$

The mean values measured in each detector for full spectral distribution of the photon pairs is as follows:

$$\langle I_a(\tau) \rangle = \langle I_a(\tau) + I'_a(\tau) \rangle = I_b, \quad \text{(7)}$$

$$\langle I_d(\tau) \rangle = \langle I_d(\tau) + I'_d(\tau) \rangle = I_a. \quad \text{(8)}$$

The most important conclusion in Eqs. (7) and (8) is the detuning-independent uniform local measurements caused by the basis-randomness of entanglement as in the typical HOM effect [3]. This means that the input photon pair of $E_a$ and $E_b$ is entangled, whose quintessence is in the basis-product superposition. In other words, entanglement can be excited classically, where SPDC is one of them.

Regarding the mean two-photon intensity correlation $R_{cd}(\tau)$ between two output photons in Fig. 1, however, shows a different result from the locally measured uniform intensities in Eqs. (7) and (8) at coincidence detection:

$$\langle R_{cd}(\tau) \rangle = \langle I_a(\tau)I_d(\tau) + I'_a(\tau)I'_d(\tau) \rangle$$

$$= \langle I_b^2 \rangle \cos^2(\eta + 2\Delta_j). \quad \text{(9)}$$

To satisfy the photon bunching of the HOM effect, Eq. (9) must be zero at $\tau = 0$. Due to $\Delta_j(\tau = 0) = 0$, the inherent phase shift must be $\eta = \pm \pi/2$. This is the direct result of the wave nature-based HOM interpretation that cannot be driven by the particle nature of photons, where $\eta = \pi/2$ is automatically achieved by the BS for all coherent photon pairs in Fig. 1. So far, this inherent phase relation between paired photons has never been discussed with the particle approach in quantum mechanics, even though it has been observed [23]. For $\Delta_j(\tau = 0) \gg \pi$ or $|\tau| \gg 0$, the cosine term in Eq. (9) becomes saturated into the classical lower bound of $g^{(2)}(\tau) = 0.5$ [15]. Thus, the coherence interpretation of the HOM effect is successfully demonstrated using coherent photon pairs achieved from an attenuated laser, where oppositely scanned AOM bandwidths plays a major role for indistinguishable photon pairs on a BS.

Figure 2 shows the numerical calculations for Eqs. (7)-(9). The AOM-induced spectral bandwidth is set to $\Delta = 10^8$ Hz, and the corresponding decoherence time is $t_s = 10^{-8}$ s. Equations (1) and (4)) are numerically calculated for both first-order and second-order intensity correlations for the spectral range of $-2\Delta \leq \delta f_j \leq 2\Delta$. As shown in Fig. 2, the mean intensities of the individual output photons are uniform as expected, regardless of $\tau$ and $\eta$. For $\eta = \pi/2$, the mean coincidence detection ($R_{ab}(\tau)$) satisfies the HOM dip with $\tau$-dependent decay in both sides, as shown in the red curve in Fig. 2(a). The condition of $\eta = 0$ shows perfect coherence optics of Poisson statistics. The condition of $\eta = \pi/4$ shows a uniform intensity of perfect individuality in classical physics. Thus, the second-order intensity correlation critically depends on $\eta$ in the HOM effect. This fact has never been discussed or observed in conventional HOM effects. In both sides of the HOM dip, coherence wiggles are also demonstrated, as observed with SPDC photon pairs [6,10-12]. Thus, the quantum validity of the HOM effect in the classical scheme of Fig. 1 is successfully demonstrated with numerical
calculations. Owing to the coherence feature of the input photons, the number of coincident photons does not matter on the photon bunching of the HOM effect. In other words, cw light can also be effective via conventional correlation detection in optics, where the coincidence corresponds to many-wave interference within the given AOM’s bandwidth $\Delta$, resulting in a macroscopic HOM effect.

**Conclusion**

A classically excited HOM effect was proposed, analyzed, and numerically demonstrated for both basis randomness-induced uniform intensities in each output port and the HOM dip of second-order intensity correlation between individual detectors via a coincidence detection. For the classically excited random bases, a synchronized and opposite AOM frequency sweeping technique is taken in a double-pass scheme. The resultant HOM dip was analytically driven for a specific phase shift between two paired input photons. As a result, the AOM manipulated coherent photon pairs satisfy basis randomness in an entangled state. For the HOM effect, an essential requirement of the inherent $\pi/2$ phase difference between the randomly paired photons was analytically driven and numerically proved. In the present classical scheme, this inherent phase shift was automatically provided by a BS in an MZI. The novelty of the present paper is in the first complete model of the coherent photon-based HOM effect equivalent to the typical entangled photon-based one. Moreover, the coherence feature of paired photons can be extended into a cw scheme for a macroscopic HOM effect.

**Acknowledgments:** This work was supported by GIST via GRI 2021 and the ICT R&D program of MSIT/IITP (2021-0-01810), development of elemental technologies for ultrasecure quantum internet.

**Methods**

For the symmetric $\pm \Delta_j$ in the Inset of Fig. 1, a pair of synchronized AOMs or electro-optic modulators can be used for fast frequency sweeping (scanning) in opposite directions. To satisfy the randomness between paired photons, the AOM frequency sweeping speed must be much faster than the acquisition rate of each detector. Due to the benefit of a narrow linewidth (~MHz) in a laser at $f_0$ in frequency, even a small bandwidth $\Delta$ (~50 MHz) of the AOMs does satisfy the HOM condition of symmetric detuning, where the corresponding path-length difference is ~6 m. To fix the walk-off of diffracted lights from the sweeping AOM, a double-pass AOM scheme is applied. The double-pass AOM scheme gives an additional benefit of bandwidth doubling to $2\Delta$.

**Reference**

1. Bouchard, F., Sit, A., Zhang, Y., Fickler, R., Miatto, F. M., Yao, Y., Sciarrino, F. & Karimi, E. Two-photon interference: the Hong-Ou-Mandel effect. Rep. Prog. Phys. **84**, 012402 (2021).
2. Nielsen, Michael A.; Chuang, Isaac L. Quantum Computation and Quantum Information. (Cambridge University Press, NY, 2000).
3. Hong, C. K., Ou, Z. Y. & Mandel, L. Measurement of subpicosecond time intervals between two photons by interface. Phys. Rev. Lett. **59**, 2044-2046 (1987).
4. Kaltenbaek, R., Blauensteiner, B., Zukowski, M., Aspelmeyer, M. & Zeilinger, A. Experimental
interference of independent photons. *Phys. Rev. Lett.* **96**, 240502 (2006).

5. Lettow, R. *et al.* Quantum interference of tunably indistinguishable photons from remote organic molecules. *Phys. Rev. Lett.* **104**, 123605 (2010).

6. Lopez-Mago, D. & Novotny, L. Coherence measurements with the two-photon Michelson interferometer. *Phys. Rev. A* **86**, 023820 (2012).

7. Lang, C., Eichler, C., Steffen, L., Fink, J. M., Woolley, M. J., Blais, A. & Wallraff, A. Correlations, indistinguishability and entanglement in Hong-Ou-Mandel experiments at microwave frequencies. *Nature Phys.* **9**, 345-248 (2013).

8. Kobayashi, T. *et al.* Frequency-domain Hong-Ou-Mandel interference. *Nature Photon.* **10**, 441-444 (2016).

9. Deng, Y.-H. *et al.* Quantum interference between light sources separated by 150 million kilometers. *Phys. Rev. Lett.* **123**, 080401 (2019).

10. Edamatsu, K., Shimizu, R. & Itoh, T. Measurement of the photonic de Broglie wavelength of entangled photon pairs generated by spontaneous parametric down-conversion. *Phys. Rev. Lett.* **89**, 213601 (2002).

11. Thomas, R. J., Cheung, J. Y., Chunnilall, C. J. & Dunn, M. H. Measurement of photon indistinguishability to a quantifiable uncertainty using a Hong-Ou-Mandel interferometer. *Appl. Phys.** **49**, 2173-2182 (2010).

12. Poulios, K., Fry, D., Politi, A., Ismail, N., Worhoff, K., O’Brien, J. L. & Thompson, M. G. Two-photon quantum interference in integrated multi-mode interference devices. *Opt. Exp.* **21**, 23401-23409 (2013).

13. Gerry, C. & Knight, P. *Introductory to Quantum Optics.* (Cambridge Univ. Press, Cambridge, 2005).

14. Dirac, P. A. M. *The principles of Quantum mechanics* (4th ed., Oxford university press, London, 1958), Ch. 1, p. 9.

15. Ham, B. S. The origin of anticorrelation for photon bunching on a beam splitter. *Sci. Rep.* **10**, 7309 (2020).

16. Ham, B. S. Coherently controlled quantum features in a coupled interferometric scheme. *Sci. Rep.* **11**, 11188 (2021).

17. Ham, B. S. Coherence interpretation of Hong-Ou-Mandel effects. arXiv:2203.13983 (2022).

18. J. D. Franson, Bell inequality for position and time. *Phys. Rev. Lett.* **62**, 2205-2208 (1989).

19. Kwiat, P. G., Steinberg, A. M. & Chiao, R. Y. High-visibility interference in a Bell-inequality experiment for energy and time. *Phys. Rev. A* **47**, R2472–R2475 (1993).

20. Ham, B. S. The origin of Franson-type nonlocal correlation. *AVS Quantum Sci.* **4**, 021401 (2022).

21. Cruz-Ramirez, H., Ramirez-Alarcon, R., Corona, M., Garay-Palmett, K. & U’Ren, A. B. Spontaneous parametric processes in modern optics. *Opt. Photon. News* **22**, 36-41 (2011), and reference therein.

22. Zhang, C., Huang, Y.-F., Liu, B.-H., Li, C.-F. & Guo, G.-C. Spontaneous parametric down-conversion sources for multiphoton experiments. *Adv. Quantum Tech.* **4**, 2000132 (2021).

23. Boyd, R. W. *Nonlinear Optics* (Academic Press, San Diego, 2003), ch. 2.

24. Solano, E., Matos Filho, R. L. & Zagury, N. Deterministic Bell states and measurement of motional state of two trapped ions. *Phys. Rev. A* **59**, R2539–R2543 (1999).

25. Degiorgio, V. Phase shift between the transmitted and the reflected optical fields of a semireflecting lossless mirror is $\pi/2$. *Am. J. Phys.* **48**, 81–82 (1980).