STATIC BENDING OF TWO-DIRECTIONAL FUNCTIONALLY GRADED SANDWICH PLATES USING A THIRD-ORDER SHEAR DEFORMATION FINITE ELEMENT MODEL

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Abstract. In this paper, static bending of two-direction functionally graded sandwich (2D-FGSW) plates is studied by using a finite element model. The plates consist of a homogeneous core and two functionally graded skin layers with material properties being graded in both the thickness and length directions by power gradation laws. Based on a third-order shear deformation theory, a finite element model is derived and employed in the analysis. Bending characteristics, including deflections and stresses are evaluated for the plates with classical boundary conditions under various types of distributed load. The effects of material distribution and layer thickness ratio on the static bending behavior of the plates are examined and highlighted.

Keywords: 2D-FGSW plate, static bending, third-order shear deformation theory, finite element model.

Classification numbers: 2.9.4, 5.4.2, 5.4.3.

1. INTRODUCTION

Functionally graded materials (FGMs), initiated by Japanese scientists in 1984 in Sendai, are widely employed to fabricate structural components for use in severe conditions. This material is consisted of two or more component materials, usually ceramic and metal, with the volume fraction at each point depending on the coordinates of the point. This type of material has many outstanding features compared to its component materials such as high temperature resistance, good corrosion resistance etc. Many investigations on the mechanical behavior of FGM structures subjected to static and dynamic loads have been reported in the literature, contributions that are most relevant to the present work are briefly discussed herein. Zenkour and his co-workers [1-4] adopted classical and shear deformation plate theories in their static bending, free vibration, stability and thermal stability analyses of FGSW plates with or without
porosities. The results obtained in these studies show that the material distribution plays an important role on the behavior of the plates. Zaoui et al. [5] employed the two different shear deformation theories, namely the 2D Quasi-3D theories, and Navier solution to analyze the vibration of FG plates on elastic foundation. Neves et al. [6-8] used analytical methods and high-order, hyperbolic and sinusoidal quasi-3D shear deformation theories to investigate the static bending, free vibration and stability of FGSW plates. Thai and Kim [9] developed a simple quasi-3D theory with five unknowns for determining the displacements and stresses of a simply supported FGM rectangle plate. A new first-order shear deformation theory without using the shear correction factor was adopted by Thai et al. [10] to study static bending, free vibration and stability of FGSW rectangular plates. The results obtained in the work are highly accurate compared to high-order shear deformation theory and theoretical 3D strain theory. Farzam-Rad et al. [11] also considered the static bending and free vibration of FGSW plates, but using the quasi-3D shear deformation theory and the isogeometric method. Vafakhah and Neya [12] derived an exact solution for static bending problem of a simply supported rectangular FGM thick plates.

The above studies focused mainly on plates made of isotropic materials, unidirectional FGMs or unidirectional FGSW materials. Investigations on structures in general and plates in particular, made of two-dimensional FGMs are still limited. In this paper, a finite element procedure for static analysis of two-directional functionally graded sandwich (2D-FGSW) plates is presented. The formulation, namely a quadrilateral 4-node plate element with 7 degrees of freedom per node is derived from a high-order shear deformation theory and employed to compute the displacements and stresses of the plates. The effect of the material distribution and loading type on the bending behavior of the plates is examined in detail and highlighted.

2. PROBLEM FORMULATION

2.1 The mathematical model

Let us consider a rectangular 2D-FGSW plate of uniform thickness $h$, length $a$ and width $b$ in a Cartesian coordinate system $(x, y, z)$ as shown in Fig. 1. The plate with various boundary conditions is assumed under static distributed force $q(x, y)$, acting in the $z$ direction.

![Figure 1. Geometry and coordinates of 2D FGSW plate.](image-url)
The plate is composed of three layers, a pure ceramic core and two FGM face sheets. The material properties of the FGM sheets are assumed to vary continuously in both the thickness and longitudinal directions as

\[ P(x, z) = V_c(x, z)P_c + V_m(x, z)P_m \] (1)

where \( P_m \) and \( P_c \) are, respectively, the property of the metal and ceramic; \( V_c, V_m \) are, respectively, the volume fraction of ceramic and metal.

In the present work, \( V_c \) and \( V_m \) are assumed to vary in the x- and z-direction according to a power-law function as

\[ V_m(x, z) = \begin{cases} 
\left( \frac{z - z_1}{z_0 - z_1} \right)^{n_z} \left( 1 - \frac{x}{2a} \right)^{n_x} & \text{for } z \in [z_0, z_1] \\
0 & \text{for } z \in [z_1, z_2] \\
\left( \frac{z - z_2}{z_3 - z_2} \right)^{n_z} \left( 1 - \frac{x}{2a} \right)^{n_x} & \text{for } z \in [z_2, z_3] 
\end{cases} \] (2)

and

\[ V_c(x, z) = 1 - V_m(x, z) \] (2)

where \( n_x \) and \( n_z \) are the non-negative grading indexes, which dictate the variation of the constituent materials in the thickness and longitudinal directions, respectively.

Substituting (2) and (3) into (1) we obtain

\[ P(x, z) = \begin{cases} 
(P_m - P_c) \left( \frac{z - z_1}{z_0 - z_1} \right)^{P_z} \left( 1 - \frac{x}{2a} \right)^{P_x} + P_c & \text{for } z \in [z_0, z_1] \\
P_c & \text{for } z \in [z_1, z_2] \\
(P_m - P_c) \left( \frac{z - z_2}{z_3 - z_2} \right)^{P_z} \left( 1 - \frac{x}{2a} \right)^{P_x} + P_c & \text{for } z \in [z_2, z_3] 
\end{cases} \] (4)

2.2. Displacement field and constitutive equations

Based on the refined high-order shear deformation which proposed by Shi [13], the displacements at any point of the plate is defined as

\[ \begin{align*}
  u &= u_0 + \frac{1}{4} z \left[ 5\theta_x + \frac{\partial w_0}{\partial x} \right] - \frac{5}{3h^2} z^3 \left( \theta_x + \frac{\partial w_0}{\partial x} \right) \\
v &= v_0 + \frac{1}{4} z \left[ 5\theta_y + \frac{\partial w_0}{\partial y} \right] - \frac{5}{3h^2} z^3 \left( \theta_y + \frac{\partial w_0}{\partial y} \right) \\
w &= w_0
\end{align*} \] (5)

where \( u_0, v_0, w_0 \) are mid-plane displacements, \( \theta_x, \theta_y \) denote the rotations about the y and x axes, respectively.
Using notations \( f_1(z) = \frac{5z}{4} - \frac{5z^3}{3h^2}, f_2(z) = \frac{z}{4} - \frac{5z^3}{3h^2} \), Eq. (5) becomes
\[
\begin{align*}
\mathbf{u} &= u_0 + f_1(z) \theta_x + f_2(z) w_{0,x}, \\
\mathbf{v} &= v_0 + f_1(z) \theta_y + f_2(z) w_{0,y}.
\end{align*}
\tag{6}
\]

The strain field resulting from Eq. (5) is as follows
\[
\begin{align*}
\varepsilon_x &= u_{0,x} + f_1(z) \theta_{x,x} + f_2(z) w_{0,xx} + \varepsilon_y = v_{0,y} + f_1(z) \theta_{y,y} + f_2(z) w_{0,yy};
\gamma_{xy} &= u_{0,y} + v_{0,x} + f_1(z) (\theta_{x,y} + \theta_{y,x}) + 2f_2(z) w_{0,xy};
\gamma_{xz} &= f_1'(z) \theta_x + (1 + f_2'(z)) w_{0,x} \gamma_y = f_1'(z) \theta_y + (1 + f_2'(z)) w_{0,y}.
\end{align*}
\tag{7}
\]

Based on the Hooke’s law, the constitutive relation for the plate has the form
\[
\begin{align*}
\sigma_x &= E(x,z) \varepsilon_x + v E(x,z) \varepsilon_y; \\
\sigma_y &= E(x,z) \varepsilon_y + v E(x,z) \varepsilon_x; \\
\tau_{xy} &= \frac{E(x,z)}{2(1+v)} \gamma_{xy}; \\
\tau_{xz} &= \frac{E(x,z)}{2(1+v)} \gamma_{xz}; \\
\tau_{yz} &= \frac{E(x,z)}{2(1+v)} \gamma_{yz};
\end{align*}
\tag{8}
\]

where \( v \) is the Poisson’s ratio, assumed to be unchanged.

3. FINITE ELEMENT FORMULATION

The plate is assumed to be divided into a number of four-node rectangular plate elements, each node has seven degrees of freedom, and the vector of nodal displacements \( \mathbf{d}_e \) for a generic element is the form
\[
\mathbf{d}_e = \begin{bmatrix} d_1^T & d_2^T & d_3^T & d_4^T \end{bmatrix}^T
\tag{9}
\]

In which the \( i^{th} \) vector of nodal displacement \( (i = 1 \div 4) \) contains the following components
\[
d_i = \begin{bmatrix} u_{0,i} & v_{0,i} & w_{0,i} & w_{0,xi} & w_{0,yi} & \theta_{x,i} & \theta_{y,i} \end{bmatrix}^T
\tag{10}
\]

where \( w_{0,x} = \partial w_0 / \partial x, w_{0,y} = \partial w_0 / \partial y \).

The displacements and rotation are interpolated from the nodal displacements according to
\[
\begin{align*}
u_0 &= \mathbf{N}_u \mathbf{d}_e; \\
v_0 &= \mathbf{N}_v \mathbf{d}_e; \\
w_0 &= \mathbf{N}_w \mathbf{d}_e; \\
\theta_x &= \mathbf{N}_{\theta x} \mathbf{d}_e; \\
\theta_y &= \mathbf{N}_{\theta y} \mathbf{d}_e.
\end{align*}
\tag{11}
\]

where \( \mathbf{N}_u, \mathbf{N}_v, \mathbf{N}_w, \mathbf{N}_{\theta x}, \mathbf{N}_{\theta y} \) denote the matrices of shape functions with size of \( (1 \times 28) \)
\[
\begin{align*}
\mathbf{N}_u &= \begin{bmatrix} N_{1}^{(1)} & N_{2}^{(1)} & N_{3}^{(1)} & N_{4}^{(1)} \end{bmatrix}; \\
\mathbf{N}_v &= \begin{bmatrix} N_{1}^{(2)} & N_{2}^{(2)} & N_{3}^{(2)} & N_{4}^{(2)} \end{bmatrix}; \\
\mathbf{N}_w &= \begin{bmatrix} N_{1}^{(3)} & N_{2}^{(3)} & N_{3}^{(3)} & N_{4}^{(3)} \end{bmatrix}; \\
\mathbf{N}_{\theta x} &= \begin{bmatrix} N_{1}^{(4)} & N_{2}^{(4)} & N_{3}^{(4)} & N_{4}^{(4)} \end{bmatrix}; \\
\mathbf{N}_{\theta y} &= \begin{bmatrix} N_{1}^{(5)} & N_{2}^{(5)} & N_{3}^{(5)} & N_{4}^{(5)} \end{bmatrix}
\end{align*}
\tag{12}
\]

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The matrices \( N_i^{(j)}, (j = 1 \div 5) \) has the form
\[
N_i^{(1)} = [N_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]; \quad N_i^{(2)} = [0 \ N_i \ 0 \ 0 \ 0 \ 0 \ 0];
\]
\[
N_i^{(3)} = [0 \ 0 \ H_{3i-2} \ H_{3i-1} \ H_{3i} \ 0 \ 0]; \quad N_i^{(4)} = [0 \ 0 \ 0 \ 0 \ N_i \ 0]; \quad N_i^{(5)} = [0 \ 0 \ 0 \ 0 \ 0 \ N_i]
\]
with \( N_i (i = 1 \div 4) \) are Lagrange functions and \( H_j (j = 1 \div 12) \) are Hermite shape functions.

The strain energy of the element is given by
\[
U_e = \frac{1}{2} \int_V \left( \varepsilon_x \sigma_x + \varepsilon_y \sigma_y + \gamma_{xy} \tau_{xy} + \gamma_{xz} \tau_{xz} + \gamma_{yz} \tau_{yz} \right) dV
\]  
(14)

Substituting (11) into (7) and (8), then replace the result into Eq. (14), one gets
\[
U_e = \frac{1}{2} d_e^T K_e d_e
\]
(15)

where \( K_e \) is the element stiffness matrix.

The work done by the external force \( q(x,y) \) has the form
\[
W_e = \int_{xy} w_q(x,y) dx dy
\]  
(16)

Substituting (11) into (16), one gets
\[
W_e = d_e^T F_e
\]
(17)

where the nodal force vector \( F_e \) is of the form
\[
F_e = \int_{xy} N_w^T q(x,y) dx dy
\]
(18)

The finite element equation for the static analysis of the plate can be written in the form
\[
K D = F
\]
(19)

where \( D \) is the structural nodal displacement vector of the beam, \( K = \sum_{vel} K_e \), \( F = \sum_{vel} F_e \) respectively, are the structural stiffness matrix and the nodal force vector.

4. NUMERICAL RESULTS AND DISCUSSIONS

In this section, a plate formed from alumina \( (\text{Al}_2\text{O}_3) \) and aluminum \( (\text{Al}) \) with \( E_c = 380\text{GPa}, v_c = 0.3, E_m = 70\text{GPa}, v_m = 0.3 \) is considered. The plate is assumed simply supported and it is under a sinusoidal distributed force \( q(x,y) = q_0 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \). For the sake
of discussion, dimensionless parameters for the deflection $\bar{w}$, normal stress $\bar{\sigma}_x(z)$ and tangential stress $\bar{\sigma}_{xz}(z)$ are introduced as follows

$$\bar{w} = \frac{10E_0h}{q_0a^2} w \left( \frac{a}{2}, \frac{b}{2} \right), \quad \bar{\sigma}_x(z) = \frac{10h^2}{q_0a^2} \sigma_x \left( \frac{a}{2}, \frac{b}{2}, \frac{z}{h} \right), \quad \bar{\sigma}_{xz}(z) = \frac{h}{q_0a} \sigma_{xz} \left( 0, \frac{b}{2}, \frac{z}{h} \right)$$

(20)

where $E_0 = 1\text{GPa}$.

4.1. Comparison study

| $n_z$ | Theory        | 1-0-1 | 2-1-2 | 1-1-1 | 2-2-1 | 1-2-1 |
|-------|---------------|-------|-------|-------|-------|-------|
| 0     | Zenkour [1]   | 0.1961| 0.1961| 0.1961| 0.1961| 0.1961|
|       | Thai et al. [10] | 0.1961| 0.1961| 0.1961| 0.1961| 0.1961|
|       | Present       | **0.1957**| **0.1957**| **0.1957**| **0.1957**| **0.1957**|
| 1     | Zenkour [1]   | 0.3248| 0.3075| 0.2930| 0.2817| 0.2717|
|       | Thai et al. [10] | 0.3237| 0.3064| 0.2920| 0.2809| 0.2710|
|       | Present       | **0.3229**| **0.3056**| **0.2913**| **0.2801**| **0.2703**|
| 2     | Zenkour [1]   | 0.3751| 0.3541| 0.3344| 0.3174| 0.3037|
|       | Thai et al. [10] | 0.3737| 0.3526| 0.3330| 0.3163| 0.3027|
|       | Present       | **0.3725**| **0.3515**| **0.3321**| **0.3153**| **0.3019**|
| 5     | Zenkour [1]   | 0.4112| 0.3942| 0.3736| 0.3512| 0.3363|
|       | Thai et al. [10] | 0.4011| 0.3927| 0.3720| 0.3501| 0.3350|
|       | Present       | **0.4084**| **0.3910**| **0.3706**| **0.3485**| **0.3340**|
| 10    | Zenkour [1]   | 0.4192| 0.4066| 0.3879| 0.3640| 0.3500|
|       | Thai et al. [10] | 0.3988| 0.3894| 0.3724| 0.3492| 0.3361|
|       | Present       | **0.4170**| **0.4032**| **0.3846**| **0.3610**| **0.3474**|

| $n_z$ | Theory        | 1-0-1 | 2-1-2 | 1-1-1 | 2-2-1 | 1-2-1 |
|-------|---------------|-------|-------|-------|-------|-------|
| 0     | Zenkour [1]   | 1.9758| 1.9758| 1.9758| 1.9758| 1.9758|
|       | Thai et al. [10] | 1.9758| 1.9758| 1.9758| 1.9758| 1.9758|
|       | Present       | **1.9889**| **1.9889**| **1.9889**| **1.9889**| **1.9889**|
| 1     | Zenkour [1]   | 1.5325| 1.4517| 1.3830| 1.2775| 1.2810|
|       | Thai et al. [10] | 1.5324| 1.4517| 1.3830| 1.2775| 1.2810|
|       | Present       | **1.5391**| **1.4579**| **1.3891**| **1.2840**| **1.2870**|
| 2     | Zenkour [1]   | 1.7709| 1.6750| 1.5824| 1.4253| 1.4358|
|       | Thai et al. [10] | 1.7709| 1.6750| 1.5824| 1.4253| 1.4358|
|       | Present       | **1.7780**| **1.6811**| **1.5883**| **1.4317**| **1.4417**|
| 5     | Zenkour [1]   | 1.9358| 1.8648| 1.7699| 1.5640| 1.5931|
|       | Thai et al. [10] | 1.9358| 1.8648| 1.7699| 1.5640| 1.5931|
|       | Present       | **1.9445**| **1.8712**| **1.7756**| **1.5704**| **1.5988**|
| 10    | Zenkour [1]   | 1.9678| 1.9217| 1.8375| 1.6165| 1.6584|
|       | Thai et al. [10] | 1.9678| 1.9217| 1.8375| 1.6165| 1.6587|
|       | Present       | **1.9788**| **1.9284**| **1.8432**| **1.6225**| **1.6642**|
In the Tables 1-3, \( \bar{w} \) and \( \bar{\sigma}_x \left( \frac{h}{2} \right), \bar{\sigma}_y \left( 0 \right) \) of a square unidirectional FGSW plate are compared with the available results of Zenkour [1] and Thai et al. [10] with the constituent materials are adopted by the references. As can be seen from the tables that the present results are in good agreements with that of the cited references.

**Table 3.** The comparison result of dimensionless shear stress \( \bar{\sigma}_x \left( 0 \right) \) of square plates \((a = 10h)\).

| \( n_z \) | Theory | 1-0-1 | 2-1-2 | 1-1-1 | 2-2-1 | 1-2-1 |
|---------|--------|-------|-------|-------|-------|-------|
| 0       | Zenkour [1] | 0.1910 | 0.1910 | 0.1910 | 0.1910 | 0.1910 |
|         | Thai et al. [10] | 0.2387 | 0.2387 | 0.2387 | 0.2387 | 0.2387 |
|         | **Present** | **0.2251** | **0.2251** | **0.2251** | **0.2251** | **0.2251** |
| 1       | Zenkour [1] | 0.2610 | 0.2432 | 0.2326 | 0.2276 | 0.2206 |
|         | Thai et al. [10] | 0.2566 | 0.2593 | 0.2602 | 0.2582 | 0.2593 |
|         | **Present** | **0.2788** | **0.2599** | **0.2516** | **0.2511** | **0.2449** |
| 2       | Zenkour [1] | 0.2973 | 0.2675 | 0.2508 | 0.2432 | 0.2326 |
|         | Thai et al. [10] | 0.2552 | 0.2617 | 0.2650 | 0.2624 | 0.2655 |
|         | **Present** | **0.3078** | **0.2736** | **0.2603** | **0.2608** | **0.2506** |
| 5       | Zenkour [1] | 0.3454 | 0.2973 | 0.2721 | 0.2610 | 0.2460 |
|         | Thai et al. [10] | 0.2468 | 0.2576 | 0.2649 | 0.2627 | 0.2694 |
|         | **Present** | **0.3604** | **0.2930** | **0.2697** | **0.2724** | **0.2551** |
| 10      | Zenkour [1] | 0.3728 | 0.3132 | 0.2830 | 0.2700 | 0.2526 |
|         | Thai et al. [10] | 0.2419 | 0.2534 | 0.2627 | 0.2611 | 0.2698 |
|         | **Present** | **0.4036** | **0.3077** | **0.2761** | **0.2796** | **0.2572** |

### 4.2. Numerical results

**Table 4.** The dimensionless deflection \( \bar{w} \) of a square 2D-FGSW plate.

| \( n_z \) | \( n_x \) | \( a = 10h \) | \( a = 20h \) |
|---------|---------|---------------|---------------|
|         |         | (1-1-1) | (1-2-1) | (2-1-1) | (2-2-1) | (1-1-1) | (1-2-1) | (2-1-1) | (2-2-1) |
| 0       | 0       | 0.3498 | 0.2627 | 0.3488 | 0.2984 | 1.3787 | 1.0335 | 1.3687 | 1.1736 |
| 0.5     | 0.2376 | 0.1984 | 0.2346 | 0.2128 | 0.9319 | 0.7773 | 0.9155 | 0.8330 |
| 1       | 0.1894 | 0.1659 | 0.1870 | 0.1740 | 0.7400 | 0.6483 | 0.7274 | 0.6787 |
| 2       | 0.1447 | 0.1330 | 0.1433 | 0.1367 | 0.5628 | 0.5175 | 0.5552 | 0.5310 |
| 0.5     | 0.1983 | 0.1638 | 0.2059 | 0.1797 | 0.7762 | 0.6397 | 0.8053 | 0.7023 |
| 0.5     | 0.1636 | 0.1422 | 0.1673 | 0.1517 | 0.6385 | 0.5539 | 0.6519 | 0.5912 |
| 1       | 0.1434 | 0.1285 | 0.1456 | 0.1350 | 0.5585 | 0.4996 | 0.5661 | 0.5249 |
| 2       | 0.1209 | 0.1122 | 0.1220 | 0.1159 | 0.4692 | 0.4349 | 0.4726 | 0.4491 |
| 1       | 0.1564 | 0.1347 | 0.1630 | 0.1453 | 0.6100 | 0.5241 | 0.6356 | 0.5660 |
| 0.5     | 0.1373 | 0.1223 | 0.1413 | 0.1295 | 0.5344 | 0.4752 | 0.5496 | 0.5035 |
| 1       | 0.1250 | 0.1139 | 0.1277 | 0.1191 | 0.4856 | 0.4417 | 0.4958 | 0.4623 |
| 2       | 0.1101 | 0.1031 | 0.1116 | 0.1064 | 0.4264 | 0.3990 | 0.4321 | 0.4116 |
| 2       | 0.1247 | 0.1122 | 0.1294 | 0.1187 | 0.4845 | 0.4349 | 0.5029 | 0.4607 |
| 0.5     | 0.1151 | 0.1057 | 0.1184 | 0.1106 | 0.4466 | 0.4093 | 0.4594 | 0.4285 |
| 1       | 0.1084 | 0.1010 | 0.1108 | 0.1048 | 0.4198 | 0.3906 | 0.4293 | 0.4055 |
| 2       | 0.0995 | 0.0946 | 0.1011 | 0.0971 | 0.3848 | 0.3652 | 0.3907 | 0.3751 |
The Tables 4-6 list the dimensionless deflection $\overline{w}$, the dimensionless normal stress $\sigma_{n}\left(\frac{h}{2}\right)$ and $\sigma_{n}(0)$ of a square simply supported 2D-FGSW sandwich plate subjected to a sinusoidal distributed force for the various values of the grading indexes $n_x, n_z$ in two different cases, $a=10h$ and $a=20h$. In the tables and hereafter, three numbers in the brackets are used to denote the layer thickness ratio, e.g. $(2\text{-}1\text{-}1)$ means that $(h_1:h_2:h_3)=(2:1:1)$. As can be seen from the tables that the increase in the grading indexes $n_x, n_z$ leads to the decrease in dimensionless deflection $\overline{w}$, and the dimensionless normal stress $\sigma_{n}\left(\frac{h}{2}\right)$ and shear stress $\sigma_{s}(0)$.

Figures 2 and 3 show the distribution of dimensionless normal stress $\sigma_{n}\left(\frac{h}{2}\right)$ and tangential stress $\sigma_{s}(0)$ of the square 2D-FGSW plate for various values of the index $n_x$. As seen from the figure, the effect of the index $n_x$ on the normal stress and tangential stress is different. When increasing the index $n_x$, the maximum normal stress decreases but the maximum tangential stress increases, regardless of the layer thickness ratio. In Figure 4, the effect of the layer thickness ratio on the distribution of dimensionless normal stress $\sigma_{n}\left(\frac{h}{2}\right)$ and the dimensionless tangential stress $\sigma_{s}(0)$ of square 2D-FG sandwich plate with $n_z=1, n_x=1$ and is illustrated. The layer thickness ratio, as seen from the figure significantly influences on the distribution of the stresses, especially when the layers are not symmetrical with respect to the mid-plane.

Table 5. The dimensionless normal stress $\sigma_{n}\left(\frac{h}{2}\right)$ of a square 2D-FGSW plate.

| $n_z$ | $n_x$ | $a=10h$ | | $a=20h$ |
|------|------|--------|---|--------|
|      |      | (1-1-1) | (1-2-1) | (2-1-1) | (2-2-1) |
|      |      | (1-1-1) | (1-2-1) | (2-1-1) | (2-2-1) |
| 0    | 0    | 1.6980  | 1.2729  | 1.4690  | 1.2650  | 1.6931  | 1.2685  | 1.4629  | 1.2603  |
| 0.5  | 1    | 1.7507  | 1.4722  | 1.5751  | 1.4295  | 1.7451  | 1.4735  | 1.5598  | 1.4242  |
| 1    | 1.7811 | 1.5917  | 1.6372  | 1.5337  | 1.7739  | 1.5866  | 1.6285  | 1.5270  |
| 2    | 1.8316 | 1.7235  | 1.7390  | 1.6732  | 1.8218  | 1.7156  | 1.7284  | 1.6638  |
| 0.5  | 2    | 0.9555  | 0.7873  | 0.9021  | 0.7933  | 0.9515  | 0.7836  | 0.8977  | 0.7894  |
| 0.5  | 1    | 1.2279  | 1.0711  | 1.1652  | 1.0658  | 1.2234  | 1.0666  | 1.1600  | 1.0611  |
| 1    | 1.3958 | 1.2611  | 1.3344  | 1.2492  | 1.3902  | 1.2557  | 1.3280  | 1.2434  |
| 2    | 1.5987 | 1.5032  | 1.5486  | 1.4876  | 1.5907  | 1.4956  | 1.5399  | 1.4794  |
| 1    | 0    | 0.7506  | 0.6447  | 0.7257  | 0.6521  | 0.7470  | 0.6413  | 0.7220  | 0.6486  |
| 0.5  | 1    | 1.0346  | 0.9231  | 1.0017  | 0.9259  | 1.0300  | 0.9185  | 0.9969  | 0.9213  |
| 1    | 1.2285 | 1.1248  | 1.1934  | 1.1238  | 1.2229  | 1.1192  | 1.1875  | 1.1181  |
| 2    | 1.4790 | 1.3980  | 1.4472  | 1.3932  | 1.4712  | 1.3903  | 1.4390  | 1.3853  |
| 2    | 0    | 0.5959  | 0.5346  | 0.5861  | 0.5411  | 0.5926  | 0.5315  | 0.5827  | 0.5379  |
| 0.5  | 2    | 0.8688  | 0.7976  | 0.8546  | 0.8034  | 0.8642  | 0.7931  | 0.8499  | 0.7988  |
| 1    | 1.0723 | 1.0013  | 1.0561  | 1.0056  | 1.0666  | 0.9956  | 1.0502  | 0.9998  |
| 2    | 1.3549 | 1.2942  | 1.3387  | 1.2961  | 1.3471  | 1.2863  | 1.3308  | 1.2882  |
Table 6. The dimensionless shear stress $\bar{\sigma}_{xz}(0)$ of square 2D-FGSW plate.

| $n_z$ | $n_x$ | $a = 10h$ | $a = 20h$ |
|-------|-------|-----------|-----------|
|       |       | (1-1-1)   | (1-1-1)  |
|       |       | (1-2-1)   | (1-2-1)  |
|       |       | (2-1-1)   | (2-1-1)  |
|       |       | (2-2-1)   | (2-2-1)  |
| 0     | 0     | 0.3337    | 0.1489    |
|       | 0.5   | 0.3473    | 0.2096    |
|       | 1     | 0.3548    | 0.2405    |
|       | 2     | 0.3644    | 0.2747    |
| 0.5   | 0     | 0.2940    | 0.1940    |
|       | 0.5   | 0.2984    | 0.2127    |
|       | 1     | 0.3016    | 0.2250    |
|       | 2     | 0.3065    | 0.2410    |
| 1     | 0     | 0.2745    | 0.1970    |
|       | 0.5   | 0.2768    | 0.2072    |
|       | 1     | 0.2787    | 0.2146    |
|       | 2     | 0.2820    | 0.2249    |
| 2     | 0     | 0.2580    | 0.1970    |
|       | 0.5   | 0.2590    | 0.2019    |
|       | 1     | 0.2600    | 0.2058    |
|       | 2     | 0.2619    | 0.2118    |

Figure 2. The distribution of dimensionless stress of a square 2D-FGSW (1-2-1) plate with $a=20h$, $n_z = 1$.

Figure 3. The distribution of dimensionless stress of a square 2D-FGSW (2-2-1) plate with $a=20h$, $n_z = 1$. 

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5. CONCLUSIONS

In this paper, a high-order shear deformation theory which allows to evaluate the shear stresses in the thickness has been adopted to derive a finite element plate element for static bending of 2D-FGSW plate. The plate is considered to consist of three layers, a homogeneous ceramic core and two FGM skin layers with material properties vary in both thickness and length directions by power-law function. The plate element with 4-node and seven degrees of freedom per node is derived using Lagrange functions and cubic Hermite polynomials to interpolate the displacement field. Using the derived element, bending characteristics, including the deflection and stresses of the simply supported plate under sinusoidal distributed load are computed. The obtained numerical results show that the effect of material distribution and the layer thickness ratio have a significant influence on the static response of the plate. The results obtained in the present paper can be used as references when analyzing 2D-FGSW sandwich plates according to different plate theories.

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