Experimental setup proposed to measure the quality of a spherical surface using deflectometry and a Hartmann screen

A Muñoz-Potosí 1, R Díaz-Uribe 2, M Campos-García 2, F Granados-Agustín 1
1 Coordinación de Óptica, Instituto Nacional de Astrofísica, Óptica y Electrónica, A.P. 51-216, Santa María Tonantzintla, C.P. 72000, Puebla, México
2 Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Universidad Nacional Autónoma de México, Circuito Exterior S/N, Ciudad Universitaria, A.P. 70-186, Delegación Coyoacán, C.P. 04510 México D.F., México
E-mail: amunozpotosi@inaoep.mx

Abstract. Currently there are a variety of techniques to determine the quality of optical surfaces, which must provide quantitative information of the deformation or the shape of the surface under test. This paper proposes to use the deflectometry technique using a Hartmann screen to test a spherical surface of the diameter and radius of curvature known. For this, the proposed experimental setup is presented, highlighting the importance of alignment and the measures distances, location between the screen and image capture system, measured from the center of curvature of the analyzed surface. Subsequently, by using these distances, the positions of the reflected rays and the positions of location of points on the screen, you can find the equation of the line associated with these two points and given this equation, we can deduce the value of the normal surface. With this result of normals and using the general equation of deflectometry, we will obtain the shape of the surface under test.

1. Introduction
There are several methods for evaluating and testing the performance of optical elements that provide quantitative information about the quality of surfaces under test. Among these techniques, we can cite different methods, such as interferometry [1], deflectometry [2], Hartmann [3] and Ronchi [4] tests.

The deflectometry is a technique based on reflection that suffers light rays to impinge on a surface whose topography is desired to know. It consists on impinging a known ray on the surface to be characterized, measuring the ray reflected by it, we are able to determine the normal to the surface at each point [5 6 7 8]. As the objective is to know the form of the entire surface, we have to obtain information from many points on it; for this, is necessary to scan all the surface. The fundamental equation in cartesian coordinates is given by:

$$\hat{n} \cdot ds = n_x dx + n_y dy + n_z dz = 0,$$

where $\hat{n}$ is the normal vector to the surface at the point of incidence of the light ray and $ds$ is a tangent vector determined by the sweep that will be done on the surface defined as:
\( \hat{n} = n_x \hat{i} + n_y \hat{j} + n_z \hat{k} \) \hspace{1cm} (2)

and

\( \vec{d}s = dx \hat{i} + dy \hat{j} + dz \hat{k} \). \hspace{1cm} (3)

In order to solve the equation (1) it is necessary to know the normal vector \( \hat{n} \) at each sampling point because the vector \( \vec{d}s \) is known from the path followed by the sampled rays.

2. Determination of the normal vector to the surface

To find the vector \( \hat{n} \) we will use the law of reflection in vectorial form \([9]\), where \( \vec{S}_1 \) is a vector along the direction of the incident ray, \( \vec{S}_2 \) is a vector along the reflected ray and \( \hat{n} \) is a unit vector along the normal to the reflecting surface and \( I \) is the angle between the incident ray and the normal vector of the reflector surface at the point of incidence.

\[ \vec{S}_2 = \vec{S}_1 + 2cosI|\vec{S}_1|\hat{n} \] \hspace{1cm} (4)

In this paper, we perform out the analysis for a concave spherical surface illuminated by a point source, where the incident ray, the reflected ray and the normal vector are parallel to each other, this means \( cosI = 1 \) and the equation (4) becomes:

\[ \hat{n} = \frac{\vec{S}_2 - \vec{S}_1}{2|\vec{S}_1|}, \] \hspace{1cm} (5)

Because the point source has an infinite number of rays and that the deflectometry measures the deviation that suffering one only ray, we propose to use a device to select some rays from the source, using a screen with holes that, in the literature, is known as Hartmann screen \([10,11,12]\). However, in this proposal the screen position is not in the exit pupil of the system, but can be located arbitrarily. In this case, it is located at a distance \( F \) between the surface and its center of curvature. With the device, the incident rays and its deviation are determined by using a CCD sensor located at a distance \( G \) from the center of curvature of the surface under study. The measurement of ray deflection is performed point by point until the entire surface is measured. With the largest number of holes in the Hartmann screen, the surface could be better sampled.

Because of the assumption that the normal vector is in the same direction of the incident and reflected rays, the deviation in the directions of these rays observed in the CCD sensor, will be due to the deformations of the surface of interest. The diagram of the proposed technique, where the dimensions of the spherical surface are exaggerated, is presented in Figure 4.

To perform the analysis, we define two vectors, \( \vec{P} \) and \( \vec{Q} \). Using the equation of the line, we obtained a vector \( \vec{u} \), so that is parallel to the line \( L \) that connects the points \( P, Q \) and is in the same direction of the normal vector of the surface, as shown in equation (6):

\[ \hat{n} = \frac{\vec{u}}{|\vec{u}|} = \frac{(x'^0 - x''0, y'_0 - y''0, G + F)}{\sqrt{(x'^0 - x''0)^2 + (y'_0 - y''0)^2 + (G + F)^2}}, \] \hspace{1cm} (6)

where \( (x''0, y'_0) \) are the positions of the holes in the Hartmann screen and \( (x'^0, y'_0) \) are the positions of the reflected rays measured in the CCD.

The equation (6) is an approximation for calculating the normal vector for a spherical surface; however, this equation only holds in case the spherical surface is perfect. When the surface under test has local deformations, the incident and reflected rays at that point do not follow the same path, thus the normal vector changes. The theoretical analysis of this case is still in process.
Figure 1. Scheme to determine the vector $\vec{u}$, parallel to the line L

3. Determination of vector $\vec{d}s$

To determine the vector $\vec{d}s$ is necessary taken into account the positions of the points on the Hartmann screen, which are known due to the manufacturing parameters. Subsequently the position of the rays passing over the screen after impinged on the surface being tested is determined. Figure 2 shows the geometric relationships taken into account in determining the vectors $\vec{r}_0$ and $\vec{r}_1$, considering only two points on the screen.

Figure 2. Vector position on the Hartmann screen

Once the position vectors $\vec{r}_0$ and $\vec{r}_1$ are known, the vector associated with the path is:

$$\vec{d}s = \vec{r}_0 - \vec{r}_1 = (r \sin \theta_0 - r \sin \theta_1, r \cos \theta_1 - r \cos \theta_0), \quad (7)$$
where $r$ is the radius of curvature of the spherical surface and $\theta_0$ and $\theta_1$, are the angles between the $z$ axis and the vectors $\vec{r}_0$ and $\vec{r}_1$, respectively. Considering equation [1] the coordinate $z$ of the surface is given by:

$$z = -\int \frac{n_x}{n_z} dx - \int \frac{n_y}{n_z} dy$$

(8)

where $dx$ and $dy$ are determined by the positions where the sampling is performed.

Because sampling is proposed to the surface is done continuously, the equation [8] can be approximated numerically:

$$z = -\sum_{i=1}^{N} \frac{n_x}{n_z} \Delta x_i - \sum_{i=1}^{N} \frac{n_y}{n_z} \Delta y_i,$$

(9)

where $N$ depends on the number of holes in the Hartmann screen, $n_x, n_y, n_z$ are the components of the normal vector to the surface at each point and $\Delta x, \Delta y$ are determined with the number of holes in the screen.

4. Experimental setup proposed

The experimental setup proposed to test a spherical surface of diameter 215mm and center of curvature 767.6mm, is presented in Figure 3. For this experimental setup, is necessary that the distance measured from the center of curvature of the spherical surface to the location of the Hartmann screen and the CCD sensor should be measured with major accuracy. In this case a Sony Alpha 37 camera was used.

![Figure 3](image-url)
We use a He-Ne laser in order to ensure the best alignment of the experimental setup. This laser serves as a guide to determine the optical axis and the correct centering of all the elements of the setup, including the surface under study; this, considering the laser as incident at the center of the surface, the incident and reflected rays must follow the same path. In addition, a pair of rails are placed in order to have a guide for the location of the Hartmann screen, and the CCD. For the accurate location of the Hartmann screen, we use a micro-displacement transverse to the optical axis. This lets us adjust the optical axis of the system with the mechanical axis of the screen.

Then, using a laser distance measuring tool, brand Leica model DISTOD8, were performed the measures of the distances the location of the screen and the CCD sensor. These distances are: \( F = 581.9\, \text{mm} \) for the Hartmann screen and \( G = 272.2\, \text{mm} \) for the CCD sensor, both distances measured from the curvature center of the surface but in contrary directions.

Once the measurement of both distances \( F \) and \( G \) are completed, a thresholding and binarization of the image captured by the CCD sensor [13], showed in Figure 4, is necessary in order to determine the position of the centroids for each bright circle. These centroids will provide information about the vector normal to the surface at each sampling point. Therefore, is necessary an analysis of its accuracies because, when the quality of the surface under study is determined, the accuracy of the approximation considered in order to solve the equation of deflectometry could be obtained.

![Figure 4. Centroid position for the image captured with the CCD](image)

The evaluation process of the surface shape, considering a calculation of the normals and the vector tangent to the trajectory sampling is still in process.

5. Conclusions
The deflectometry as a method of measurement is proposed.

To determine the shape of the surface, is necessary to know the normal vector of it, in this work, an experimental setup for determining the normal vector from the distances \( F \) and \( G \) is proposed. The theoretical analysis of a surface with local deformations is still in process.

A method to determine the vector \( \mathbf{d}s \), with which it can find the \( z \) coordinate of the surface under test is proposed.

An optimized experimental setup for alignment and image capture is proposed.

To calculate the normal to the surface under study and then determine the quality of spherical surface under test, it is necessary to analyze the measurement accuracy of the distances of the
experimental setup. This, later, will be useful in order to know the quality of approximation by using the equation of the deflectometry.

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