Photoproduction of $K^+\Lambda$ with a Regge-plus-resonance model

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Making use of the hybrid Regge-plus-resonance model, we investigate the process of kaon photoproduction off the proton target. We present a new model whose free parameters were adjusted to data in and above the resonance region and which provides an acceptable description of experimental data. The overwhelming majority of nucleon resonances selected in this analysis overlaps with those selected in our previous analyses and also with the Bayesian analysis with the Regge-plus-resonance model, which we deem to be dependable. A novel feature of our model consists in applying a different scheme for gauge-invariance restoration, which results in a need for implementing a contact current. As we further reveal, the choice of the gauge invariance restoration scheme as well as the choice of either pseudoscalar or pseudovector coupling in the strong vertex play a significant role for cross-section predictions at forward angles where data are scarce.

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I. INTRODUCTION

The main objects of exploration of kaon photo- and electroproduction from nucleons are the investigation of baryon resonance spectrum and interactions in systems of hyperons and kaons arising. It can also shed some new light on an interesting topic of so-called “missing” resonances that have been predicted by quark models \cite{1, 2}.

As the kaon production occurs in the third-resonance region, where many states possibly couple to the $KY$ channel, the number of resonances and consequently number of parameters can be relatively high. In this hadron-dynamical approach, one can either assume hadrons as effective degrees of freedom and base our calculations on effective Lagrangians. Since in these models there is no explicit connection to QCD, the number of parameters is directly related to the number of resonances introduced.

There are distinct methods of describing the elementary process of photo- and electroproduction. There are on the one hand models based on quark degrees of freedom \cite{4-6} which introduce a relatively small number of parameters and explicitly work with an inner structure of baryons. On the other hand, we can assume hadrons as effective degrees of freedom and base our calculations on effective Lagrangians. In these models there is no explicit connection to QCD, the number of parameters is directly related to the number of resonances introduced. As the kaon production occurs in the third-resonance region, where many states possibly couple to the $KY$ channel, the number of resonances and consequently number of parameters can be relatively high. In this hadron-dynamical approach, one can either assume hadrons as effective degrees of freedom and base our calculations on effective Lagrangians. Since in these models there is no explicit connection to QCD, the number of parameters is directly related to the number of resonances introduced.

A significant reduction of the number of parameters can be achieved by using Regge-plus-resonance model (RPR) constructed by the group at Ghent \cite{14-16}. This model allows us to describe the kaon-hyperon photo- and electroproduction from the threshold energy up to energies well beyond the resonance region, as it is a hybrid between the isobar model suitable for calculations in the resonance region and the Regge model \cite{17} which is applicable above the resonance region $E_{\gamma} > 3 \text{GeV}$. The Regge part of the amplitude, being a smooth function of energy, forms the background in the resonance region and dominates the predictions above the resonance region. On top of this Regge-like background, there are contributions of nucleon resonances added which then model the resonance part of the amplitude in the resonance region and vanish beyond it.

An important and often discussed issue of the Regge-type approach to photo- and electroproduction is the gauge invariance restoration method. The frequently used method is that one introduced by Guidal et al. \cite{17}. They added the proton exchange with the vector photon-proton coupling to the kaon exchange contribution to construct the residual function of the kaon-trajectory part of the amplitude. Here we utilize a prescription suggested by Haberzettl et al. \cite{15} which is based on the generalized Ward-Takahashi identities introducing a contact term. We also address the issue of the proton-kaon-Lambda coupling assuming both the the pseudoscalar and pseudovector forms that influence the gauge restoration method.

In this work, we present a new Regge-plus-resonance model for production of $K^+\Lambda$ with a special emphasis on the subject of gauge-invariance restoration. As well as in our studies of $K^+\Lambda$ production with help of isobar model \cite{11, 12}, we use a consistent formalism for description of high-spin nucleon resonances \cite{19, 20} and pay close attention to observables predictions at small kaon angles. The latter is vital for getting reliable predictions of hypernucleus production cross sections \cite{8}.

This article is organized as follows. In Sec. II the frame for Regge description of the non resonant part of the pho-
toproduction amplitude is given discussing in detail the gauge-invariance restoration method. In that part, we also introduce a novel feature of the model: the contact term. The resonant part of the amplitude is described in Sec. III. For more details on formalism of $K^+\Lambda$ photoproduction we refer to Ref. [11]. A method of adjusting free parameters of the model is described in Sec. IV. The Sec. V is devoted to comparison and discussion of model predictions with data and with results of other models. Conclusions are given in Sec. VI. More details on the Regge formalism are given in Appendices A and B.

II. REGGE MODEL

At the basis of the Regge theory [21] is the fact that, at energies where individual resonances can no longer be distinguished, the exchange of entire Regge trajectories predominates the reaction dynamics rather than the exchange of individual particles. This high-energy framework applies to the so-called “Regge limit” of extreme forward (in the case of the $t$-channel exchange) or backward (for the $u$-channel exchange) scattering angles, corresponding to small $|t|$ or $|u|$, respectively. Since the lightest hyperon, the $\Lambda$ hyperon, is significantly heavier than a kaon and, therefore, the $u$-channel poles are located much further from the backward-angle kinematical region than the $t$-channel poles are from the forward-angle region, the $u$-channel exchange Reggeization, i.e., the procedure of requiring the Regge propagator to reduce to the Feynman one at the closest crossed-channel pole, might not lead to good results [14]. What is more, the high-energy data in the backward-angle region are scarce. Therefore, we have chosen to deal with $t$-channel exchange Reggeization only.

Since in the vicinity of the $t$-channel pole the Regge amplitude is assumed to be identical with the Feynman amplitude for the exchange of the given particle, the Regge theory, in its simplest form, can be formulated by modifying the isobar model. The process of Reggeization is quite straightforward and goes as follows: one writes the amplitude for the exchange of the given particle (in the corresponding pole both Feynman and Regge propagators coincide), then interchanges the Feynman propagator with the Regge one,

$$\frac{1}{t - m_X^2} \to \mathcal{P}_\text{Regge}^X(s, \alpha_X(t)),$$

and the remnant terms in the amplitude then labels as a Feynman residuum $\beta_X$. The amplitude constructed in this way includes effectively exchanges of all particles represented by the given trajectory and reads

$$M_\text{Regge}(s, t) = \beta_X(t) \mathcal{P}_\text{Regge}^X(s, \alpha_X(t)).$$

In the case of $K^+\Lambda$ production, we Reggeize contributions of the $K^+(494)$ and $K^*(892)$ amplitudes only. For more details on Regge trajectories and propagators see Appendix [B].

The main asset of the Regge model is a reduced number of free parameters to be adjusted to experimental data. If we do not consider terms proportional to the function $A(s, t, u)$ in the transversal part of the contact current (see Eqs. (10) and (18) below), there are only three free parameters: $g_{K\Lambda N}$ and $G^{(s, t)}_{K^*}$.

A. Gauge invariance in Regge model

As in the $\gamma(k) + p(p) \rightarrow K^+(p_K) + \Lambda(p_\Lambda)$ reaction only the incoming proton and the outgoing kaon carry electric charge, the relevant contributions in view of gauge invariance stem from the $s$- and $t$-channel Born terms and a contact term. In the following we use the method of re-pairing gauge invariance broken by Reggeization of the $t$-channel exchanges which was suggested by Haberzettl et al. [18].

The total gauge-invariant Regge current in our approach reads

$$M^\mu = M^\mu_{R,t} + M^\mu_s + M^\mu_{int}$$

$$= J^\mu_K(p_K, p_K - k)\Delta_K(p_K - k)F_{R,t}$$

$$+ J^\mu_p(p + k, p)S_p(p + k)F_s + M^\mu_{int},$$

where $J^\mu$ are electromagnetic currents, $\Delta_K$ is a kaon propagator, $S_p$ is a proton propagator, and $F_{R,t}$ and $F_s$ are hadron vertices in the $t$ and $s$ channel, respectively. The contact current is given by [18]

$$M^\mu_{int} = m_c \mathcal{F}_t(t) + G \mathcal{C}^\mu,$$

where the hadron form factor $f_s(t)$ appearing in Eq. (A1) in [18] was interchanged for the Regge residual function $\mathcal{F}_t(t) = (t - m_K^2)\mathcal{P}_\text{Regge}^K$, which corresponds with the Reggeization of the contact term. In Eq. (A2), $m_c^a$ is generally a Kroll-Ruderman-type bare contact current which results from an elementary four-point Lagrangian, $G$ is an operator describing the coupling structure of the hadron vertex, and the auxiliary current $C^\mu$ is given by Eq. (A2) in [18]. The Regge current is required to fulfill the generalized Ward-Takahashi identity

$$k_\mu M^\mu = \Delta^{-1}_K(p_K)\epsilon \Delta_K(p_K - k)F_{R,t}$$

$$- S_p^{-1}(p)S_p(p + k)F_s - e F_{R,t} + e F_s + k_\mu M^\mu_{int},$$

which warrants its gauge invariance if the contact current is constructed to have the property

$$k_\mu M^\mu_{int} = e(F_F(t) - F_s).$$

A specific form of the contact current depends on the chosen coupling in the strong vertex. Here we consider two possible forms of this coupling.

1. Pseudoscalar coupling in the strong vertex

Assuming the pseudoscalar (PS) coupling in the $K^+\Lambda p$ interaction vertex, $G_{PS} = g_{K\Lambda p}\gamma_5$, the gauge non-invariant term of the $K^+(494)$ exchange in the $t$ channel
where \( f_{\ell}(t) \) is a hadron form factor. After Reggeizing this contribution, when \( f_{\ell}(t)/(t-m_K^2) \) turns into \( \mathcal{F}_{\ell}(t)/(t-m_K^2) \), the term \( (4) \) has a form

\[
- e g_{K\Lambda p} \gamma_5 \frac{f_{\ell}(t) k \cdot \varepsilon}{k^2}, \tag{6}
\]

In the \( s \) channel, the gauge non-invariant term reads

\[
eg e g_{K\Lambda p} \gamma_5 f_s(s) \frac{k \cdot \varepsilon}{k^2}, \tag{8}
\]

where \( f_s(s) \) is a hadron form factor. These two gauge-invariance violating terms are annihilated with the presence of the contact current \( \delta \). The bare contact current \( m^\mu_c \), in the case of hadron form factors being unity, must satisfy the condition

\[
k_\mu m^\mu_c = e(F_t - F_s) = (eG_{PS} - G_{P\varepsilon}) = 0 \tag{9}
\]

where \( F_t \) and \( F_s \) is a strong vertex factor in the \( t \) and \( s \) channel, respectively. Since in the PS coupling the strong vertex factors in both channels coincide, i.e. \( F_t = F_s = G_{PS} \), the longitudinal part of the contact current is determined solely by the second term in Eq. \( (5) \). This term can be given by Eq. \( (A2) \) in \( [18] \) for \( e_m = e_b = e \) and \( e_\mu = 0 \), and assuming only the \( s \)- and \( t \)-channel contributions, i.e. \( \delta_s = \delta_t = 1 \) and \( \delta_u = 0 \).

\[
G_{PS} C^\mu \varepsilon_\mu = e g_{K\Lambda p} \gamma_5 \left\{ - (2p_K - k)^\mu \mathcal{F}_t(1 - 2p_K - k)^\mu f_s - (2p + k)^\mu f_s \frac{1}{s - m_p^2} \mathcal{F}_t \right. \\
+ \tilde{A}(s, t, u)(1 - f_s)(1 - \mathcal{F}_t) \left[ \frac{(2p_K - k)^\mu}{t - m_K^2} + \frac{(2p + k)^\mu}{s - m_p^2} \right] \varepsilon_\mu \\
= e g_{K\Lambda p} \gamma_5 \left\{ - 2 \frac{\mathcal{F}_t - 1}{t - m_K^2} f_s (\mathcal{M}_2 - \mathcal{M}_t) + (\mathcal{F}_t - 1) f_s \frac{k \cdot \varepsilon}{k^2} - 2 f_s \frac{1}{s - m_p^2} \mathcal{M}_2 - (f_s - 1) \mathcal{F}_t \frac{1}{k^2} \\
+ 2 \tilde{A}(s, t, u)(1 - f_s)(1 - \mathcal{F}_t) \left[ \frac{\mathcal{M}_2 - \mathcal{M}_t}{t - m_K^2} + \frac{\mathcal{M}_2}{s - m_p^2} \right] \varepsilon_\mu \right\}, \tag{10}
\]

where the second expression comprises the explicitly gauge invariant structures \( \mathcal{M}_i \) defined in Ref. \( [11] \). It is evident that all the gauge-violating terms proportional to \( k \cdot \varepsilon/k^2 \) in formulas \( (7), (8), \) and \( (10) \) cancel each other and that the four-divergence of the contact term \( (9) \) is

\[
k_\mu M^\mu_{int} = e g_{K\Lambda p} \gamma_5 (\mathcal{F}_t - f_s), \tag{11}
\]

which agrees with the requirement in Eq. \( (5) \) as in this case \( F_{R,t} = g_{K\Lambda p} \gamma_5 \mathcal{F}_t \) and \( F_s = g_{K\Lambda p} \gamma_5 f_s \). Contributions of the contact term to the scalar amplitudes, see Ref. \( [11] \) for more details on the general formalism, can be found in Appendix A.

The function \( A(s, t, u) \) in \( (10) \) is an arbitrary phenomenological function that is constrained only by a condition to vanish at high energies. This condition is necessary to prevent the “violation of scaling behaviour” \( [22] \). Our choice for \( A(s, t, u) \) is a “dipole” shape

\[
A(s) = A_0 \frac{\Lambda_c^4}{\Lambda_c^4 + (s - s_{thr})^2}, \tag{12}
\]

where \( s_{thr} = (m_\Lambda + m_K)^2 \) and \( A_0 \) and \( \Lambda_c \) are free parameters giving the strength of this term and cutting it off (thereby limiting the affected region), respectively.

2. Pseudovector coupling in the strong vertex

In the case of pseudovector (PV) coupling, the \( K^+ \Lambda p \) vertex reads

\[
G_{PV} = - \frac{g_{K\Lambda p}}{m_\Lambda + m_p} K^\mu \gamma_\mu \gamma_5, \tag{13}
\]

where the momentum \( K \) corresponds to the kaon field coming out of the strong vertex. In the \( s \) channel, it holds \( K = p_K \), whereas in the \( t \) channel it is \( K = p - p_\Lambda \).

Gauge non-invariant terms coming from the electric part of the \( s \)-channel contribution with PV coupling read

\[
eg g'_{K\Lambda p} \gamma_5 f_s(s) [(m_\Lambda + m_p) + k] \frac{k \cdot \varepsilon}{k^2}, \tag{13}
\]

and the gauge non-invariant terms from the Reggeized \( t \)-channel \( K^+(494) \) exchange read

\[
eg g'_{K\Lambda p} \gamma_5 \mathcal{F}_{\ell}(t) (m_\Lambda + m_p) \frac{k \cdot \varepsilon}{k^2}, \tag{14}
\]

with \( g'_{K\Lambda p} = g_{K\Lambda p}/(m_\Lambda + m_p) \).

Without any \( u \) channel contribution and without the form factors, the bare contact current \( m^\mu_c \) must again satisfy the condition \( (7) \) in \( [18] \) which now reads as

\[
k_\mu m^\mu_c = e g'_{K\Lambda p} (\not{p} - \not{p}_\Lambda - \not{p}_K) \gamma_5 = e g'_{K\Lambda p} k \gamma_5. \tag{15}
\]
In contraction with the polarization vector $\varepsilon_\mu(k)$, its contribution to the amplitude can be recast as

$$\varepsilon_\mu m_\varepsilon = -eg'_{K\Lambda \gamma} k \cdot \varepsilon \cdot \frac{k \cdot k}{k^2}. \quad (16)$$

This form assures gauge invariance in the case of no hadron form factors introduced because it cancels the term proportional to $\hat{k}$ in the proton-exchange contribution, see Eq. (13) with $f_s = 1$. After Reggeization the contribution of the bare contact current is

$$\varepsilon_\mu m_\varepsilon F_\lambda(t) = -eg'_{K\Lambda \gamma} k \cdot \varepsilon \cdot \frac{k \cdot k}{k^2} F_\lambda(t), \quad (17)$$

which now cancels out with the same term, just with the opposite sign, in Eq. (18) below. Contrary to the PS-coupling case, with the PV coupling the contact current is determined by both terms in Eq. (3) where the second one has now the form

$$G_{PV}C_\mu \varepsilon_\mu = g'_K \gamma K \mu \varepsilon_\mu = g'_K \gamma K \mu \varepsilon_\mu = \gamma'_{K\Lambda \gamma} (\theta - \theta_{\Lambda}) C_\mu \varepsilon_\mu = \gamma'_{K\Lambda \gamma} (\theta_{\Lambda} + \theta_{\Lambda} - \theta_{\Lambda}) C_\mu \varepsilon_\mu = \gamma'_{K\Lambda \gamma} (\mu + m_p + k \cdot k) C_\mu \varepsilon_\mu = g'_{PV} C'_{\mu \varepsilon_\mu}$$

$$+ g'_{K\Lambda \gamma} (\theta - \theta_{\Lambda}) C_\mu \varepsilon_\mu = G_{PV} C_\mu \varepsilon_\mu + g'_{K\Lambda \gamma} \frac{k \cdot \varepsilon}{k^2}$$

$$+ (F_\lambda - 1) f_s \frac{k \cdot \varepsilon}{k^2} + 2 \frac{f_s - 1}{s - m_p^2} F_\lambda \left[ M_4 + \frac{k \cdot p}{k^2} M_6 \right] - (f_s - 1) F_\lambda \frac{k \cdot \varepsilon}{k^2}$$

$$+ 2 \hat{A}(s, t, u)(1 - f_s)(1 - F_\lambda) \left[ \frac{1}{s - m_p^2} \left( -M_4 + M_5 + \frac{k \cdot p}{k^2} \cdot M_6 \right) \right]. \quad (18)$$

It is evident again that all the gauge-violating terms proportional to $k \cdot \varepsilon / k^2$ in formulas (13), (14), (17), and (18), get mutually canceled which guarantee gauge invariance of the full Regge current. Corresponding formulas for the scalar amplitudes can be found in Appendix A.

### 3. Comparison of pseudoscalar and pseudovector couplings

The newly constructed Regge model (i.e. a background part of the photoproduction amplitude), therefore, consists of Reggeized $t$-channel contributions of $K^+(494)$ and $K^*(892)$ trajectories with no hadron form factor, $s$-channel proton exchange with a hadron form factor $f_s(s)$ and with a standard Feynman propagator, and a contact term with the Regge residual function $F_\lambda(t)$ and hadron form factor $f_s(s)$.

Before closing this section we deem important to sketch the difference the choice of either PS or PV coupling in the strong vertex makes. We do not observe any notable changes in the behaviour of either kaon trajectories or the proton exchange. What we do observe, however, is the striking transformation of the contact current contribution, see Fig. 1. With the PS coupling this contribution increases steadily, whereas with the PV coupling the contact current contributes with growing kaon angle less and less. This effect is more pronounced at the higher end of the resonance region, $W = 2.5$ GeV, where contact currents with different type of coupling produce quite contradictory shapes in the cross section. Note also the difference in magnitude of the contributions with differing coupling the origin of which is the cut-off parameter value of $\Lambda_{gfr} = 1.5$ GeV for the hadron form factor used in these calculations. Whereas this value is the usual resulting value in fits with the PS coupling, a background with the PV coupling usually needs a much smaller cut-off value around 0.5 GeV to be accordingly suppressed.

### III. REGGE-PLUS-RESONANCE MODEL

Although the Regge theory is a high-energy tool by construction, it can reproduce the order of magnitude of the forward-angle pion and kaon photoproduction [14] and kaon electroproduction [23] observables remarkably well even in the resonance region. Nevertheless, it is evident that a pure non resonant description, such as the Regge model, cannot be expected to describe the reaction at energies in the resonance region [14]. The cross section near threshold exhibits structures, such as peaks at certain energies, which might reflect the presence of individual resonances. These are incorporated into the Regge-plus-resonance (RPR) model by extending the Reggeized background with $s$-channel diagrams with exchanges of nucleon resonances. For these diagrams, standard Feynman propagators are assumed where, as in the isobar approach, the resonance finite lifetime is taken into account through the substitution

$$s - m_R^2 \rightarrow s - m_R^2 + 1 m_R \Gamma_R$$
in the propagator denominator with the $m_R$ and $\Gamma_R$ the mass and width of the propagating state, respectively. For more details on formalism for exchanges of nucleon resonances with spin up to 5/2 we refer to our work in [11].

In order to retain the RPR approach reasonable, the resonance contributions should vanish in the high-energy region. This is achieved with the help of hadron form factors which should be strong enough not to allow the resonant terms to contribute beyond the resonance region. For this purpose, one usually opts for a multipole or multidipole-Gaussian shape (see Eqs. (13) in [11]) of the hadron form factor since these two functions fall off with energy much more sharply than, e.g. the dipole form factor. Therefore, only the Regge part of the amplitude remains in the high-energy region.

A. Nucleon resonances in the $s$ channel

In order to select a set of nucleon resonances which describes the $p(\gamma, K^+\Lambda)$ data best, one has to carry out a great amount of fits assuming many combinations of $N^*$’s. Firstly, we constructed a “maximal” model including all nucleon resonances with spin up to 5/2 that might contribute in the $K^+\Lambda$ photoproduction. That resulted into model with 14 $N^*$’s. After that we systematically omitted nucleon resonances one by one and checked the $\chi^2$ values and correspondence with data. During this process, we have also been slightly manually modifying the particle’s mass and width inside the PDG limits [24] (if given). From values of particle’s mass and width shown in Table I in Ref. [11] we arrived at values only mildly different, which nonetheless play some role in reducing the $\chi^2$ value. Particularly, the masses of N3, N4, N5, N9, and P4 were shifted to values of 1510 MeV, 1670 MeV, 1750 MeV, 1675 MeV, and 1900 MeV, respectively. The widths of N3, N4, N7, P2, P3, and P5 were changed to respective values of 110 MeV, 135 MeV, 325 MeV, 400 MeV, 220 MeV, and 170 MeV. Note that the width of P2, whose value is much higher than the upper limit imposed by the PDG [24], is inspired by a previous thorough analysis at Ghent University [25]. What is more, we introduced a new resonant state in the $D_{15}$ partial wave, $N(2070)5/2^-$, which was not considered before, with mass 2070 MeV and width 200 MeV. This state was observed in recent coupled-channels analysis [26] even though with a notably smaller mass. For a notation we use here we refer our reader to the Table I in Ref. [11].

As we limit our study to the $K^+\Lambda$ channel only, we do not introduce $\Delta$ resonances since their decay to this channel is prohibited by isospin conservation.

IV. SEEKING THE BEST FIT

As the Regge and Regge-plus-resonance frameworks are effective ones with coupling parameters and cutoff values of hadron form factors not determined, our primary goal is to adjust these parameters to experimental data. The parameters in need of adjustment are the coupling constants of the Regge background, i.e. $g_{KAN}$, governing the behaviour of the $K^+(494)$ trajectory, contact current, and proton exchange in the $s$ channel, and the vector and tensor coupling constant $G_{K}^{(v,t)}$ of the $K^+(892)$ trajectory, and coupling constants of additional nucleon resonances. Each spin-1/2 resonance adds one free parameter, while higher-spin resonances add two free parameters. We also need to determine values of cutoff parameters for hadron form factors suppressing nucleon resonances (a common cutoff parameter $\Lambda_{gr}$) and also the proton exchange (cutoff parameter $\Lambda_{bgp}$). Please note that the hadron form factor for the $s$-channel Born proton exchange is a novel feature of our model. Moreover, when we assume also the transversal term in the contact current, this gives us two more parameters, i.e $A_0$ and $\Lambda_c$ in Eq. (12). In total, we need to fix around 25 free parameters depending mainly on the number of $N^*$’s we put in.

We adjusted the free parameters with help of the least-squares fitting procedure using the MINUIT code [27]. Since it is well known that MINUIT uses a nonlinear transformation for the parameters with limits, making the accuracy of the resulting parameter worse as it approaches its limiting value, we introduced limiting values only to the background cutoff parameter $\Lambda_{bgp}$ of hadron form factor and to the $g_{KAN}$ coupling parameter. The latter one was further removed and the $g_{KAN}$ coupling was therefore allowed to violate the SU(3) sym-
metry slightly more than what is normally considered, i.e. within 20% around the central values [11].

Firstly, we have been fitting on high-energy data and adjusting only the Regge-background parameters $g_{KAN}$, $G_{K*}^{(v)}$, and $G_{K*}^{(t)}$, which resulted into finding one deep minimum. We deem this hints to a strong probability for this minimum to be a global minimum. With high-energy data we mean cross-section data from CLAS 2010 collaboration [28] for $W > 2.36$ GeV (230 data), from CLAS 2005 collaboration [29] for $E_{\gamma}^{lab} > 2.56$ GeV (95 data), and 20 recent data from LEPS collaboration [30] for $E_{\gamma}^{lab} > 2.55$ GeV. The data were limited not only to high energies but also to forward kaon angles only, i.e. $\theta_{K}^{m*} < 60^\circ$, since this is the kinematical region where the $t$-channel Reggeistics takes place. Subsequently, we added data in the resonance region, namely the cross-section data from CLAS 2010 collaboration [28] for $W < 2.36$ GeV (1247 data), CLAS 2005 collaboration [29] for $E_{\gamma}^{lab} < 2.56$ GeV (1036 data), LEPS collaboration [30] for $E_{\gamma}^{lab} < 2.38$ GeV (54 data), and 91 cross-section data collected by Adelseck and Saghai in their paper [52] from various experimental facilities; hyperon-polarization data from CLAS 2010 collaboration [28] for $W < 2.23$ GeV (925 data), and a handful of beam-asymmetry data from both LEPS collaborations [30] (30 and 18 data, respectively). These data for adjusting $N^*$ parameters were naturally not restricted with respect to kaon angle $\theta_{K}^{m*}$. No weight factor was introduced to any data so they all come to the fitting process with the same importance. With this data set, we fitted the $N^*$’s coupling constants while keeping the background parameters on their values from the high-energy fit. However, soon we realised that we can achieve significantly better results when we fit all parameters simultaneously and we, therefore, merged all of these data sets into a single data file which we subsequently used for the rest of the fitting procedure.

An astute reader may have noticed that we did not include all differential-cross-section data available to us in these days. It is because there exists some ambiguities and inconsistencies between some of them, the most notable being the inconsistency between the extensive CLAS data set and the SAPHIR data. As pointed out and discussed in Ref. [33] a common fit to both data sets would be possible only after inclusion of a normalization function or factor. We do not consider this issue here (more details can be also found in Ref. [33]) and therefore restrict ourselves only to the CLAS data.

Another inconsistency apparently exists between CLAS and SLAC data which agree well in shape but the CLAS cross-section data are systematically lower than SLAC in scale by roughly a factor of two (even though a direct comparison is difficult since kinematics of both sets do not overlap much) [35]. As we know e.g. from the analysis made by Guidal et al. [17], a model resulting from fitting SLAC high-energy data and projecting down to CLAS energies consistently overpredicts the $K^+\Lambda$ cross sections (for illustration see Fig. 20 in Ref. [29]). Adjusting model parameters to the high-energy SLAC data and subsequently extrapolating them to the resonance region can, therefore, lead to a dissatisfactory description of resonance region (or it can strongly influence the interference pattern among background and resonant terms). Thus, we have decided not to use the SLAC data in our analysis.

For a much more thorough discussion of the fitting procedure, see Ref. [11].

V. DISCUSSION OF RESULTS

In this section, we present our new Regge-plus-resonance models for photoproduction of $K^+\Lambda$ and compare their predictions of the cross section, hyperon polarization, and two other double polarization observables with experimental data and results of other models.

The set of nucleon resonances in the current RPR-BS model is in a good concert with the set chosen in the Ghent RPR-2011 model [25] and also with $N^*$’s in our isobar models [11, 12], see the overview in Table I. We partly corroborate the claim of Ref. [25] where authors found a decisive evidence for inclusion of $P_{11}(1880)$, $P_{13}(1900)$, and $F_{15}(2000)$ states as well as a compelling evidence for omitting $D_{13}(1700)$, $P_{13}(1710)$, and $D_{15}(1675)$ states as we include both $P_{13}(1900)$ and $F_{15}(2000)$ states but do not include the $P_{11}(1880)$ state and, on the other hand, do not include $P_{11}(1710)$ and $D_{15}(1675)$ states while including the $D_{13}(1700)$ state. From the states which were not found to be important in the Ghent analysis, we include the $D_{15}(2070)/5^-_2$ state (denoted as M4) even though its couplings are one or two orders of magnitude smaller than couplings of other spin-5/2 resonances and $F_{15}(1860)$ which plays an important role at energies $W > 2$ GeV. We include in the discussion also a model with the pseudovector coupling in the KAN vertex, coined RPR-BS(pv), although this is mainly for illustrative reasons since we were unable to get a good fit with $g_{KAN}$ coupling constant anywhere near the limits imposed on it by the SU(3) symmetry violated at around 20%. The set of $N^*$’s in this model is the same as the set in RPR-BS with only one exception which is interchanging a $D_{15}(2070)$ state for a $P_{13}(1880)$ state. In both of these models, we assume fixed decay widths of nucleon resonances. It is because when we introduce energy-dependent widths (as defined in Ref. [12]) the resulting cross-section prediction at small kaon angles and for $E_{\gamma}^{lab} > 2$ GeV plummets drastically. The resonance width increases with increasing energy and the resonance thus moves away from the physical factory description of resonance region (or it can strongly influence the interference pattern among background and resonant terms).
TABLE I. An overview of $N^*$'s included in various recent models. Our current RPR-BS models are compared with our older RPR-1 and RPR-2 models \[29\], with the Ghent RPR-2011 model and also with our recent isobar models BS1 \[11\], BS2 \[11\], and BS3 \[12\]. For the notation of resonances, we refer to Tab. I in Ref. \[11\].

|       | N3 | N4 | N5 | N6 | N7 | P1 | P2 | P3 | P4 | P5 | M1 | M4 |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|
| RPR-BS | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  |
| RPR-BS(pv) | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  |
| RPR-1 & 2 | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  |
| BS1 | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  |
| BS2 | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  |
| BS3 | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  |

and what other models produce.

When we compare the couplings of nucleon resonances in the RPR-BS model with their values in other models of ours (including the BS models), we can see that couplings of $S_{11}(1535)$ and $S_{11}(1650)$ are in the RPR-BS model approximately twice as large as in other models while retaining the same sign. The couplings of $F_{15}(2000)$ and $D_{13}(1875)$ are in the RPR-BS model an order of magnitude smaller and larger, respectively, than in BS models. The rest of the coupling parameters show only minor changes. All parameters of our new models are summarized in Tab. \[11\].

We do not include an anomalous magnetic coupling in the proton exchange proportional to $\sigma^{\mu\nu}$ because of the duality hypothesis according to which only all $s$-channel or all $t$-channel poles can be included \[14\]. A combination of both $s$- and $t$-channel contributions may lead to double counting of poles. Since in the Regge model we take into account all $t$-channel poles, the amount of additional poles in the $s$ channel should be reduced to minimum. When we, however, introduce this anomalous term into the RPR-BS model we observe a suppression of cross-section predictions in the hemisphere of forward angles and an increase at backward angles. The presence of the anomalous term in the RPR-BS(pv) model, however, hardly makes any difference.

One of the troubling ambiguities when describing the $K^+\Lambda$ photoproduction with help of effective models is an accurate selection of the hadron form factor accounting for the extended structure of hadrons. In the robust analysis of Ghent group, they selected the multi-dipole-Gaussian shape as the most suitable one. However, we reveal in our analysis that the inclusion of this kind of hadron form factor leads to a higher $\chi^2$ value. On the other hand, opting for hadron form factors which are by definition much softer, i.e. dipole or Gaussian ones, leads to unacceptable behaviour beyond the resonance region, where these form factors are not able to tame the high-spin $N^*$'s sufficiently which leads to a rapidly soaring cross-section prediction. No matter what the cutoff parameter is, dipole and Gaussian form factors work well only in the resonance region. Therefore, we turned to a multi-dipole shape of the form factor which, with a reasonably small cutoff value, works well in both worlds.

In order to illustrate the roles played by particular nucleon resonances, we include a prediction of the total cross section by the RPR-BS model in Figure \[2\]. In the upper part of this figure, where contributions of specific parts of the amplitude are shown, a noticeable feature is that the model is dominated by background (dashed line), where the contact current plays a predominant role.

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|       | RPR-BS | RPR-BS(pv) |
|-------|--------|------------|
| $g_{K\Lambda}/\sqrt{4\pi}$ | $-2.496$ | $-1.276$ |
| $G^{(s)}_{K^+\Lambda}$ | $0.085$ | $-0.0007$ |
| $G^{(s)}_{K^+\Lambda}$ | $0.186$ | $0.117$ |
| $A_2$ | $-0.971$ | $-0.050$ |
| $A_2$ | $1.430$ | $2.226$ |
| $A_3$ | $1.478$ | $0.500$ |
| $N_{res}$ | $1.901$ | $2.007$ |
| $G(N3)$ | $0.440$ | $0.104$ |
| $G(N4)$ | $-0.183$ | $-0.128$ |
| $G(N5)$ | $0.133$ | $0.070$ |
| $G(N5)$ | $-0.009$ | $-0.047$ |
| $G_1(N7)$ | $0.121$ | $0.057$ |
| $G_2(N7)$ | $0.036$ | $0.046$ |
| $G_1(N9)$ | $0.027$ | $0.005$ |
| $G_2(N9)$ | $-0.090$ | $-0.042$ |
| $G(P1)$ | $-0.038$ | $-0.290$ |
| $G_1(P2)$ | $-0.023$ | $-0.029$ |
| $G_2(P3)$ | $-0.015$ | $-0.14$ |
| $G_2(P3)$ | $0.010$ | $0.011$ |
| $G_1(P4)$ | $-0.023$ | $-0.057$ |
| $G_2(P4)$ | $0.058$ | $0.041$ |
| $G_1(P5)$ | $0.004$ | $0.015$ |
| $G_2(P5)$ | $0.035$ | $0.008$ |
| $G_1(M1)$ | $-0$ | $-0$ |
| $G_2(M1)$ | $-0$ | $-0$ |
| $G_1(M4)$ | $0.0006$ | $-0$ |
| $G_2(M4)$ | $0.0012$ | $-0$ |
| $\chi^2/n.d.f.$ | 1.42 | 1.41 |
(difference between the dashed and dotted lines), and the $N^*$ state is omitted is shown in the lower figure.

 Behaviour of the RPR-BS model when a particular $N^*$ state is active at backward angles (see e.g. the peak around $\theta_K < 30^\circ$ at $W = 2.105\text{ GeV}$). Interestingly enough, when we assume a pseudovector coupling in the $K\Lambda N$ vertex the behaviour of background changes dramatically and its contribution shows a plateau at $\theta_K < 30^\circ$ rather than a peak. This then interferes constructively with contributions of nucleon resonances and as a result we get a slightly higher prediction by the full model (RPR-BS(pv)).

The energetic dependence of the cross section is shown in the Fig. 4 for four angles in the forward hemisphere. The data reveal a two-peak structure and so do the models as all of them are in accordance with experimental data but the behaviour of background changes dramatically and its contribution shows a plateau at $\theta_K < 30^\circ$ rather than a peak. This then interferes constructively with contributions of nucleon resonances and as a result we get a slightly higher prediction by the full model (RPR-BS(pv)).

The model behaviour at forward-angles seems to be strongly influenced by the choice of gauge-invariance restoration method. In the GLV procedure for gauge-invariance restoration, which is implemented in our older RPR-1 and RPR-2 models, the crucial contribution at forward angles stems from the proton exchange, which is governed by $g_{K\Lambda N}$ coupling constant. Generally, the smaller is the coupling constant the higher is the cross section at forward angles. In the RPR-1 model, this coupling parameter acquires a value of $-1.45$ whereas in the RPR-2 model the $g_{K\Lambda N}$ is much lower, specifically $-3.00$. The proton exchange contribution in the RPR-2 model is thus much stronger at very small kaon angles. An interested reader may find a much more thorough analysis of this topic in Ref. [37].

The energetic dependence of the cross section is shown in the Fig. 4 for four angles in the forward hemisphere. The data reveal a two-peak structure and so do the models as all of them are in accordance with experimental data except for the sharp peak right above the threshold at $\cos \theta_K^m = 0.8$ in the CLAS 2010 data set which is RPR-2011B model [10], and with the Ghent RPR-2011 model [25]. For obtaining results of the RPR-2011 model we made use of the online interface in Ref. [38]. The resulting angular dependence of the cross section is compared with experimental data in Fig. 3 for six various energies in and above the resonance region. The models differ particularly in regions of kaon angles $\theta_K < 90^\circ$, which allows for its remarkable behaviour at backward angles (most striking for $W < 2\text{ GeV}$). Our new models, on the other hand, are a bit more moderate in their forward-angle predictions which are approximately $1\mu\text{b/sr}$ below the RPR-2011 model at zero kaon angle and subsequently decrease more gradually. The RPR-BS model even produces a plateau-like behaviour above the resonance region ($W > 2.5\text{ GeV}$). In this model, the background terms, with a significant support from the contact current, contribute most significantly at kaon angles approximately from $30^\circ$ to $90^\circ$ and thus create the structure at around $\theta_K^m = 30^\circ$. However, their contribution for $\theta_K < 30^\circ$ is negligible and therefore the strength in this region comes from $N^*$’s contributions; generally, the higher the spin of the resonance the higher its contribution in forward regions but $N^*(5/2)$ are also active at backward angles (see e.g. the peak around $\theta_K^m = 140^\circ$ at $W = 2.105\text{ GeV}$). Interestingly enough, when we assume a pseudovector coupling in the $K\Lambda N$ vertex the behaviour of background changes dramatically and its contribution shows a plateau at $\theta_K^m < 30^\circ$ rather than a peak. This then interferes constructively with contributions of nucleon resonances and as a result we get a slightly higher prediction by the full model (RPR-BS(pv)).

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FIG. 3. Angular dependence of the cross section calculated with RPR-BS (solid line), RPR-BS with pseudovector coupling in the $K\Lambda N$ vertex (dashed line), RPR-1 [36] (dotted line), RPR-2 [36] (dashed double-dotted line), and RPR-2011 [25] (dash-dotted line) are shown for six values of the center-of-mass energy. The data stem from the CLAS 2005 [29] and CLAS 2010 [28] collaborations.

apparently excluded also by the older CLAS 2005 measurement. Most probably, the $S_{11}(1650)$ state, together with its constructive interference with other terms, plays the decisive role in creating or not creating that sharp peak in model predictions. In the RPR-2 model, coupling parameters of this state are twice as large as in other RPR models of ours and an order of magnitude higher in comparison with our isobar models. No wonder then that this peak is formed in the RPR-2 model and not in the other models. The description of the cross section by the RPR-BS model above the threshold is shaped predominantly by an interplay of $S_{11}(1535)$ and $S_{11}(1650)$ while the dip around $W \approx 1.9$ GeV is modelled by $D_{13}(1875)$ which interferes destructively with other terms. Above 2 GeV, the main (destructive) contributions come from $F_{15}(1680)$ and $F_{15}(1860)$ and they decrease with kaon angle. Even though these spin-5/2 resonances resonate higher than where their masses are, they eventually get suppressed so that the high-energy region is completely modelled by Regge-like background. This aptly illustrates the sufficiency of the multi-dipole hadron form factor for taming even the high-spin $N^{*}$'s contributions. Note also that the LEPS 2018 data in Fig. 4 are slightly above both CLAS data sets at higher energies, which is mainly because both LEPS data sets shown in the upper left-hand side and right-hand side figures are binned for $\cos \theta_{c.m.}^{K} = 0.85$ and 0.65, respectively, i.e. slightly above the binning of the CLAS data and we know that the differential cross section decreases with kaon angle so data at higher $\theta_{c.m.}^{K}$ must lie slightly below data at smaller $\theta_{c.m.}^{K}$.

In the very forward-angle region with $\theta_{c.m.}^{K} < 20^\circ$, the experimental data are highly insufficient which leaves us with predictions of our models, see Fig. 5. We can, however, get a helpful hint on how the model prediction should behave from one photoproduction datum by Bleckmann et al. [40], two electroproduction data [41, 42] with a very small virtual-photon mass, and from our knowledge from hypernuclei calculations that the c.m. cross section at $\theta_{c.m.}^{K} = 2^\circ$ and around $W = 2.2$ GeV should be at least 0.4 µb/sr [3]. See also the discussion on the elementary reaction in Garibaldi et al. [9]. Complying with the latter condition in particular helped us selecting the final models from the plenty of fits. While the RPR-2011 model predicts a predominantly structure-
less cross section, our models reveal some resonant features, see Fig. 5. The first peak in our new models is most probably the result of an interplay of many nucleon states, where the $S_{11}(1535)$ and $S_{11}(1650)$ are the most prominent ones. The second peak is then created by spin-$5/2$ states, which is in accord with what we stated above about the magnitude of high spin resonances at forward-angle predictions. This second peak is, however, missing in the RPR-BS(pv) model prediction where the background terms at forward angles and higher energies dominate and the $N^*$'s thus cannot create any palpable shapes in the prediction. The older RPR models of ours give predictions that do not differ much in shape but they differ in magnitude. We have already discussed the role the proton exchange plays at very forward angles but let us point out that another difference between RPR-1 and RPR-2 models is a cut-off parameter value for the hadron form factor of nucleon resonances (please note that there is no hadron form factor introduced for background terms in the GLV gauge-invariance restoration scheme). In the RPR-1 model the hadron form factor with a cutoff value 2 GeV suppresses nucleon resonances efficiently, which results in recession above $E^{\text{lab}}_\gamma = 2$ GeV. On the other hand, the RPR-2 model prediction diminishes slowly as the cutoff value for its hadron form factor is 3 GeV. Both RPR-1 and RPR-2 models exploit the multidipole-Gaussian shape of hadron form factor.

In Fig. 6 there are results for hyperon polarization $P$ for several kaon angles. Our models are in a good agreement with data except for $\cos \theta_k = 0.9$ where they show much larger value of $P$ for $W > 2.2$ GeV and where the actual shape of hyperon polarization near the threshold is hard to guess thanks to considerable inconsistencies in data. The dominant contributions to $P$ in the forward-angle hemisphere come from background and higher-spin nucleon resonances whereas the contact term contributes at central angles mainly and the presence of spin-1/2 nucleon resonances is noticeable only at the threshold area. The role of nucleon states then lies especially in interfering among themselves and other terms and thus creating the subtle shapes as both spin-3/2 and spin-5/2 nucleon resonances on their own produce shapes which are far from what can be seen in the result of the complete model. The RPR-2011 model, unlike our fits, is able to capture the hyperon-polarization data also at very forward angles and at $\cos \theta_k = -0.5$ in the transition between resonant and high-energy regions produces a structure according to experimental data.

During the fitting procedure, we did not fit to experimental data on double-polarization observables $C_x$, $C_z$, $O_x$, and $O_z$ and hence the Figures 7 and 8 show mere predictions of the models. For $C_x$ we have a reliable prediction by the RPR-BS model for all angles shown except for the threshold region and the region around $W = 1.9$ GeV where the model on the one hand captures the shape of data but on the other hand its predictions do not lie within the data errorbars. The RPR-BS(pv) model with pseudovector coupling in the $K\Lambda N$ vertex works similarly and the Ghent RPR-2011 model shows slightly less structures than what we see in the data. However, it is the RPR-2011 model which captures the $C_z$ data best while the RPR-BS at forward angles fails to reproduce even the shape of data. The agreement with $O_x$ and $O_z$ data is much worse since in many cases the models predict structures with opposite sign in comparison with data or their predictions lack any structure, opposing to data. Generally, we observe more structures in the model predictions at higher energies and for $E^{\text{lab}} = 1.222$ GeV the RPR-BS model can at least capture the shape of the data.

In Figures 9, 10, and 11 we show an overall descrip-
FIG. 8. Predictions of the double-polarization observables $O_x$ and $O_z$ for several kaon c.m. angles. The data originate from the GRAAL [46] experiment and notation of the curves is the same as in the Fig. 3.

tion of the cross section for the $p(\gamma, K^+)\Lambda$ production process by the RPR-BS model, its set of nucleon resonances and by the mere background terms, respectively, for all angles and for energies from the threshold up to 4 GeV, i.e. well beyond the resonance region. We can see that at higher energies, there is some strength only at very forward angles, $\theta_{K^{cm}} < 30^\circ$, which is apparently caused by the background terms. The nucleon states do not contribute anywhere above the resonance region, i.e. for $W > 2.5$ GeV, as requested, and the high-energy region is therefore described by the Regge background only. This shows beyond any doubt that even the multipole hadron form factor can suppress contributions of nucleon resonances sufficiently so that they vanish at the edge of the resonance region. The complicated shape produced in the resonance region seems to be a result of a rather intricate interference among many $N^*$ and background terms. In the forward angles, there appears to be a constructive interference producing the second peak around approximately 2 GeV whereas we surely observe a destructive interference at backward angles leading to a suppressed cross section.

VI. CONCLUSION

We have constructed a new version of the Regge-plus-resonance model for $p(\gamma, K^+)\Lambda$ utilizing a new method to maintain gauge invariance that is based on generalised Ward-Takahashi indentities. In this method a contact term is included together with the proton exchange in the $s$ channel. Another novel feature of the model is the presence of the hadronic form factor in the proton exchange that constitutes an important contribution to the non resonant part of the photoproduction amplitude.
The nucleon resonances with higher spins are treated in the frame of consistent formalism that we used also in our recent isobar models. In the analysis we considered both pseudoscalar and pseudovector forms of couplings in the strong $KAN$ vertex. Parameters of the model were adjusted to ample data in the resonance region and to available CLAS and LEPS data above this region with $\chi^2 = 1.42$. A set of nucleon resonances contributing significantly to the process was carefully selected and some resonance parameters (mass and width) were gently modified. The chosen set of $N^*$’s agrees quite well with that selected in the Ghent analysis.

Satisfactory description of the cross sections and polarizations was achieved. However, the model predictions still diverge for very small kaon angles. In the new RPR model the background part is dominated by the contact term, which mimics the higher-order effects, pointing out to importance of the final-state interactions.

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**Appendix A: Contact current contributions**

In the case of pseudoscalar coupling in the strong vertex, the contact term contribution to scalar amplitudes defined in Ref. [11] reads

$$A_2 = -2eg_{K\Lambda p} \left[ \frac{\mathcal{F}_t - 1}{t - m_K^2} f_s + \frac{f_s - 1}{s - m_p^2} \frac{1}{t - m_K^2} \right],$$

(A1a)

$$A_3 = 2eg_{K\Lambda p} \left[ \frac{\mathcal{F}_t - 1}{t - m_K^2} f_s - 2\hat{A}(s, t, u)(1 - f_s)(1 - \mathcal{F}_t) \frac{1}{t - m_K^2} \right].$$

(A1b)

Electric part of the Born $s$-channel contribution with the pseudovector coupling can be recast to the compact form

$$\mathcal{M}_{s-channel}^{PV} = \frac{e}{s - m_p^2} f_s \frac{\alpha}{2} \cdot \frac{1}{k} \cdot \frac{1}{k} \gamma^\mu \varepsilon^\mu u(p)$$

$$= \bar{u}(p)\gamma_5 \gamma^\mu f_s \left[ (m_\Lambda + m_p) (\mathcal{M}_1 + 2\mathcal{M}_2) \right]$$

$$- (1 + 2 k \cdot p / k^2) \mathcal{M}_6$$

$$+ (s - m_p^2)(m_\Lambda + m_p + k) \frac{k \cdot \varepsilon}{k^2} u(p).$$

(A3)

Scalar amplitudes resulting from the electric part of the

Born $s$-channel then are

$$A_1 = f_s \frac{e}{s - m_p^2} \frac{\alpha}{2} A_2, \quad A_6 = -f_s \frac{e}{s - m_p^2} \left[ 1 + \frac{2 k \cdot p}{k^2} \right].$$

(A4)

Magnetic part of the Born $s$-channel contribution in the pseudovector coupling

$$\mathcal{M}_{s-channel}^{PV} = \bar{u}(p)\gamma_5 \gamma^\mu f_s \left[ (m_\Lambda + m_p) (\mathcal{M}_1 + 2\mathcal{M}_2) \right]$$

$$\times \frac{i}{2m_p} \sigma_{\mu \nu} k^\nu \varepsilon^\mu u(p).$$

(A5)
can be recast into the compact form

\[ M^{(PV)}_{Bs-mg} = \bar{u}(p_{\Lambda}) \gamma_5 \frac{eg'_K}{s-m_p^2} f_s \frac{K_p}{2m_p} [2(k \cdot p) + k^2]M_1 \\
+ 2(m_{\Lambda} + m_p)M_4 - (m_{\Lambda} + m_p)M_6]u(p), \]

(A6)

from which one can extract the scalar amplitudes

\[ A_1 = \frac{eg'_K}{s-m_p^2} f_s \frac{K_p}{2m_p} [2(k \cdot p) + k^2], \quad (A7a) \]
\[ A_4 = \frac{eg}{s-m_p^2} f_s \frac{K_p}{m_p} = -2A_6. \quad (A7b) \]

Born \( t \)-channel contribution with the pseudovector coupling in the strong vertex can be recast into the compact form

\[ M^{(PV)}_{Bt} = \bar{u}(p_{\Lambda})(-eg'_K p_{\Lambda}) \gamma_5 \times \frac{(2p_K - k_p)\varepsilon}{t-m_K^2} u(p) \]
\[ = \bar{u}(p_{\Lambda}) \gamma_5 eg_K p_{\Lambda} \frac{F_t}{t-m_K^2} \left[ 2(M_2 - M_3) \right. \]
\[ - \left. (t-m_K^2) \frac{k_2 \cdot \varepsilon}{k_2^2} \right] u(p). \]

(A8)

Scalar amplitudes of this contribution then read

\[ A_2 = 2F_t \frac{eg_K p_{\Lambda}}{t-m_K^2} = -A_3. \quad (A9) \]

**Appendix B: Regge Trajectories and Propagators**

At the energies of a few GeV and higher, where no individual resonances can be distinguished, the dynamics of the process is governed by the exchange of \( t \)-channel Regge trajectories. This choice is motivated by the shape of the \( K^+ \Lambda \) photoproduction cross section which is peaked on small \(|t|\), i.e., on small kaon angles \( \theta_{Km} \). This behaviour indicates a dominant role played by \( t \)-channel Regge exchanges.

The Regge trajectories, which are often called after a lightest member (so-called first materialization) of the particular trajectory, connect spin and mass squared of the exchanged particle. When the spins of a set of resonant states are plotted against their mass squared in a Chew-Frautschi plot, see Figure 12, it is observed that all Regge trajectories can be reasonably well parameterized by means of a linear function

\[ \alpha_X(t) = \alpha_{X,0} + \alpha'_X(t - m_X^2), \quad (B1) \]

with \( m_X \) and \( \alpha_{X,0} \) the mass and spin of the trajectory lightest member \( X \), respectively. What is more, \( \alpha'_X \), which is the slope of the trajectory, happens to be close to an universal constant for all trajectories and acquires the value of 0.8 GeV\(^2\). Trajectory equations for \( K^+(494) \) and \( K^+(892) \) read

\[ \alpha_{K^+(494)}(t) = 0.70 (t - m_{K^+}^2), \quad (B2a) \]
\[ \alpha_{K^+(892)}(t) = 1 + 0.85 (t - m_{K^+}^2), \quad (B2b) \]

respectively. Note that \( t = m_X^2 \) can never be reached in the physical region of the process as \( t \) is negative in this region.

An efficient way to model trajectory exchanges involves embedding the Regge formalism into a tree-level effective-field model. The amplitude for the \( t \)-channel exchange of a linear kaon trajectory \( \alpha(t) \) can be obtained from the standard Feynman amplitude by replacing the usual pole-like Feynman propagator of a single particle with a Regge one of the form

\[ \mathcal{P}_\text{Regge}^{\zeta=\pm 1}(s,t) = \left( \frac{s}{8\alpha} \right)^{\alpha(t)} \frac{\pi\alpha'}{\sin(\pi\alpha(t))} \left[ 1 + \zeta e^{-i\pi\alpha(t)} \frac{1}{\Gamma(\alpha(t) + 1)} \right] \]

(B3)

while keeping the vertex structure given by the Feynman diagrams which correspond to the first materialization of the trajectory.

While deriving the Regge propagator, one has to differentiate between two signature parts of the trajectories, \( \zeta = \pm 1 \), in order to obey the convergence criteria: \( \zeta = +1 \) corresponds with the even and \( \zeta = -1 \) with the odd partial waves. Thus, a summation over this factor is to be done in the propagator. Unfortunately, the theory does not allow to determine the relative sign between the even and odd parts of the trajectory. We, therefore, end up either with a so-called constant phase, identical to 1, or

![FIG. 12. Chew-Frautschi plot for the two lightest kaon trajectories assumed in our analysis. The squares and dots represent trajectories with parity +1 and −1, respectively. Both trajectories are linear to a very good approximation.](image-url)
a rotating phase which gives rise to a complex factor of \( \exp(-i\pi\alpha(t)) \). As was revealed in Ref. [25], both trajectories with rotating phases are clearly favoured by data.

In our treatment of \( K^+\Lambda \) photoproduction, we identify the \( K^+(494) \) and \( K^*(892) \) trajectories as the dominant contributions to the high-energy amplitude. The corresponding propagators for the \( K^+(494) \) and \( K^*(892) \) trajectories have the following form [14]

\[
P_{\text{Regge}}^{K(494)}(s,t) = \frac{(s/s_0)^{\alpha_K(t)}}{\sin(\pi\alpha_K(t))} \frac{\pi\alpha'_K}{\Gamma(1 + \alpha_K(t))} \times \left\{ \frac{1}{e^{-i\pi\alpha_K(t)}} \right\} \tag{B4a}
\]

\[
P_{\text{Regge}}^{K*(892)}(s,t) = \frac{(s/s_0)^{\alpha_{K^*}(t)-1}}{\sin(\pi\alpha_{K^*}(t))} \frac{\pi\alpha'_{K^*}}{\Gamma(\alpha_{K^*}(t))} \times \left\{ \frac{1}{e^{-i\pi(\alpha_{K^*}(t)-1)}} \right\} \tag{B4b}
\]

As can be seen from the definition of the Regge propagators, there are poles at non negative integer values of \( \alpha_X(t) \), which correspond to the zeroes of the sine function which are not compensated by the poles of the \( \Gamma \) function. Here comes the interpretation of the Regge propagator effectively incorporating the exchange of all members of the \( \alpha_X(t) \) trajectory. In the physical region of the process under study (with \( t < 0 \)), these poles cannot be reached.

The separation of the Regge amplitude into two different signatures is a theoretical request to ensure convergence, experimentally both trajectories shown in (B2) coincide with one another. The residue for the lowest materialisation is, therefore, assumed to be used for the combined trajectory of both odd and even parity. This assumption is then called degeneracy. Whether a trajectory should be treated as degenerate or non degenerate, depends less on the trajectory equations themselves than on the process studied. It is the structure of the observed cross section that gives a hint whether the degeneracy is a valid supposition for a given channel or not. Non degenerate trajectories lead to peaks in the differential cross section while a smooth differential cross section indicates degenerate trajectories [21]. Since no obvious structure is present in the \( \rho(\gamma, K^+)\Lambda \) cross-section data for \( E_{\text{lab}} \geq 4 \text{ GeV} \), both the \( K^+(494) \) and \( K^*(892) \) trajectories are supposed to be degenerate [14].

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