Collective Feshbach scattering of a superfluid droplet from a mesoscopic two-component Bose-Einstein condensate

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We examine the collective scattering of a superfluid droplet impinging on a mesoscopic Bose-Einstein condensate (BEC) as a target. The BEC consists of an atomic gas with two internal electronic states, each of which is trapped by a finite-depth external potential. An off-resonant optical laser field provides a localized coupling between the BEC components in the trapping region. This mesoscopic scenario matches the microscopic setup for Feshbach scattering of two particles, when a bound state of one sub-manifold is embedded in the scattering continuum of the other sub-manifold. Within the mean-field picture, we obtain resonant scattering phase shifts from a linear response theory in agreement with an exact numerical solution of the real time scattering process and simple analytical approximations thereof. We find an energy-dependent transmission coefficient that is controllable via the optical field between 0 and 100%.

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The natural way to investigate quantum objects is scattering. By selecting a convenient physical “stencil” with a large interaction cross section, one can probe the structure as well as the excitation properties of a target. It mostly happens that quantum objects are either microscopically small, like atoms or nuclei, or they are embedded inside a solid state system like electrons, electronic holes or Cooper pairs. Thus, they have an elusive character, which usually shuns direct observation.

The generic response of a compound quantum object to bombardment with projectiles is either individual, i.e., by instantaneously ejecting another single particle of the compound, or it is collective, when after some transient period the target responds as a whole. Probably, the most drastic instance of this collective behavior is nuclear fission, when a heavy nucleus breaks up into large fragments – unleashing large amounts of kinetic energy. Modeling the dynamics of the atomic nucleus as a classical liquid drop gave a very intuitive interpretation of the observed phenomenon.

Superconductivity in metals is another prominent collective effect. Within the Ginzburg-Landau theory, one associates a collective wave function with the Bose-condensed electronic Cooper-pairs, and a hydrodynamic description is again successful. The quantum mechanical nature of the fluid-like order parameter is usually discussed with the Josephson effect, but Andreev-Saint-James reflection is an equally interesting phenomenon and much more in line with collective scattering theory that will be presented in the following. This effect explains the unusual electrical transport properties of a normal metal-superconductor (N-S) junction. The occurrence of collective quantum mechanical resonances can be explained by a conversion of normal conductor electrons into hole-like excitations at the interface.

The discovery of superfluidity in bosonic and, most recently, fermionic atomic gases was an amazing achievement and is another manifestation of collective many-particle physics. Probably, the use of interatomic Feshbach scattering resonances has been the most fruitful novel concept of the past few years. Today, they are the universal tool to manipulate the binary interaction in an atomic gas in real time and they paved the way to fermionic superfluidity.

In the present Letter, we will demonstrate that the microscopic physics of binary Feshbach resonances can be also implemented at the mesoscopic level of an atomic BEC, giving rise to collective Feshbach resonances as shown in Fig. 1. In particular, we will study the scattering properties of a weak coherent perturbation on a two-component BEC confined in quasi one-dimensional square-well potentials of finite-depth. Based on a Gross-Pitaevskii (GP) mean-field picture, we derive a two-component linear response Bogoliubov theory.

![FIG. 1: Collective Feshbach resonance: complete transmission (A) or total reflection (B) of a small coherent atomic wave packet with incident momentum (A) or (B), when scattering-off a stationary, two-component BEC, trapped around \(-1 < x < 1\), with a maximal density \(n_0(x) \approx 500\), which is off-scale. The dimensionless density \(\rho_b(x, t)\) is depicted versus position \(x\) (in natural units of the trap) for three instants \(t\): initially (\(t_i\), dotted), on impact (\(t_0\), solid) and finally (\(t_f\), dashed).](image-url)
for the scattering phases of the continuum perturbations. This is compared with the numerical simulation of the nonlinear wave-packet propagation in real time. Finally, we can explain the appearance of collective Feshbach resonances quantitatively from a simple Thomas-Fermi approximation of the Bogoliubov excitations.

The dynamical response of a BEC has been the subject of intensive investigations, during the last decade. So far, laser light has been mostly the method of choice to impart momentum onto a BEC, to excite collective modes and to measure the dynamic structure factor \[17\]. However, light has also been used indirectly to prepare colliding matter-wave packets that have exhibited stimulated amplification as a result of their detuning \[2\]. Today, this can be achieved experimentally with very prolate traps, which freeze out the transverse degrees of motion, effectively. The corresponding two-component GP equation reads is also known from the interal Josephson effect \[30\]

\[
\begin{align*}
\left[ i \partial_t + \frac{1}{2} \partial_x^2 - \left( V_{a}^{\text{GP}}(x) - V_{b}^{\text{GP}}(x) \right) \right] \psi_a \psi_b &= 0, \\
V_{a}^{\text{GP}}(x) &= V_a(x) + g_{aa} |\psi_a(x)|^2 + g_{ab} |\psi_b(x)|^2, \\
V_{b}^{\text{GP}}(x) &= V_b(x) + g_{ba} |\psi_a(x)|^2 + g_{bb} |\psi_b(x)|^2,
\end{align*}
\]

where \(V_a\) and \(V_b\) are external trapping potentials. For definiteness, we pick the square-well potentials as they lead to simple, analytically solvable approximations. This setup is depicted in Fig. 2. The coupling constants \(g_{aa}\), \(g_{ab} = g_{ba}\), and \(g_{bb}\) are proportional to the self- and cross-component s-wave scattering lengths. For typical experimental values of \(^{87}\text{Rb}\) see \[31\], but from the theoretical point of view the choice of parameters is uncritical \[13\]. In order to describe the scattering of a superfluid droplet from the equilibrium BEC, we will determine the linear response modes of the two-component system \[16\]. The stationary Bogoliubov ansatz with particle-like \(u = (u_a, u_b)^T\) and hole-like \(v = (v_a, v_b)^T\) excitations is

\[
\psi(x, t) = e^{-i \epsilon t} \left[ \psi^{(0)}(x) + e^{-i \epsilon t} u(\epsilon, x) + e^{i \epsilon t} v^*(\epsilon, x) \right],
\]

where \(\mu\) is the chemical potential of the spinorial ground state \(\psi^{(0)} = (\psi_a^{(0)}, \psi_b^{(0)})^T\) and \(\epsilon\) is the excitation energy. This perturbation to Eq. 11 yields the four-dimensional Bogoliubov equations

\[
\begin{align*}
\left[ \epsilon + \frac{\sigma_3}{2} \partial_x^2 - \begin{pmatrix} V^B_x & M \\ -M^* & -V^B_x \end{pmatrix} \right] \begin{pmatrix} u \\ v \end{pmatrix} &= 0, \\
M &= \begin{pmatrix} g_{aa} \psi_a^{(0)} & g_{ab} \psi_a^{(0)} \psi_b^{(0)} \\ g_{ab} \psi_a^{(0)} \psi_b^{(0)} & g_{bb} \psi_b^{(0)} \end{pmatrix}, \\
\sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\
V^B_x &= \begin{pmatrix} V_{a}^{\text{HF}}(x) & \Omega(x) + g_{ab} \psi_a^{(0)} \psi_b^{(0)} \\ \Omega^*(x) + g_{ab} \psi_a^{(0)} \psi_b^{(0)} & V_{b}^{\text{HF}}(x) \end{pmatrix}, \\
V_{a}^{\text{HF}}(x) &= V_a(x) + 2 g_{aa} |\psi_a^{(0)}|^2 + g_{bb} |\psi_b^{(0)}|^2 - \mu, \\
V_{b}^{\text{HF}}(x) &= V_b(x) + g_{ab} |\psi_a^{(0)}|^2 + 2 g_{bb} |\psi_b^{(0)}|^2 - \mu.
\end{align*}
\]

The linear response energy matrix of Eq. 11 has the same structure as the well-known Bogoliubov equations in the case of a one-component condensate. It is sympletic and has a real-valued spectrum of pairwise positive and negative eigenvalues \[32\]. Due to the finite-depth of the trapping potentials, the spectrum supports only a finite number of bound states and has a scattering continuum above a certain excitation energy. This is now the analogous situation as required for the two-particle Feshbach scattering resonances. If quasi-bound Bogoliubov modes coincide in energy with continuum modes, we will obtain a resonance behavior. These bound, positive energy modes are depicted schematically in Fig. 2. For positive energy solutions \(\epsilon > 0\), these resonances appear in the domain \(0 < \epsilon + \mu = \hbar^2/2 = E < \Delta\). In this regime, the mode

![FIG. 2: Schematic set-up of a trapped two-component BEC with square-well potentials \(V_a(x)\), \(V_b(x)\) (solid) and a coupling laser beam \(\Omega(x)\) (long dashed) versus position \(x\) \[13\]. Scattering occurs in the two open (left/right) b-channels, while the a-channels are energetically closed with a threshold energy \(\Delta\). The chemical potential is \(\mu\) (heavy solid line), the dashed-dotted lines show the bound excitations and the numbered quasi-bound energy levels (dashed) are responsible for the collective Feshbach resonances.](image)
components \( u_a(\epsilon, x), v_a(\epsilon, x) \) and \( v_b(\epsilon, x) \) are localized on the condensate and vanish exponentially for \( |x| \to \infty \). Only the particle-like component \( u_b(\epsilon, x) \) can propagate outside
\[
\lim_{x \to \pm \infty} u_b(\epsilon, x) = \cos(kx \pm \delta_e), \text{ or } \sin(kx \pm \delta_o). \tag{3}
\]

Due to the reflection symmetry of the trapping potentials, the excitation can also be characterized by parity and we define even and odd phase shifts \( \delta_e \) and \( \delta_o \), respectively. These scattering phases have been evaluated numerically from Eqs. (1, 2, 3) and are displayed in Fig. 3. The analogy to the phenomenon of Feshbach resonances stands out very clearly. According to the Breit-Wigner parameterization of an isolated resonance \( [26] \), one finds approximately
\[
\delta_{\text{res}}(E) = \arctan \left( \frac{\Gamma / 2}{E_R - E} \right), \quad \frac{2}{\Gamma} = \left. \frac{d\delta_{\text{res}}}{dE} \right|_{E = E_R}. \tag{4}
\]

In turn, one can determine the resonance energy \( E_R \) and width \( \Gamma \) analytically from the poles of the \( S \)-matrix in the vicinity of a Feshbach resonance \( [33, 34, 35] \).

![FIG. 3: Collective Feshbach resonances in the scattering phases \( \delta_e \) (solid) and \( \delta_o \) (dashed) of a weak perturbation of a two-component BEC versus energy \( E \). Note that the \( \pi \)-jumps of the collective Feshbach resonances occur at excitation energy \( E_{\text{res}} = \mu + \epsilon_i \) of the even/odd quasi-bound Bogoliubov modes of Fig. 3. The inset shows the Lorentzian behavior of the phase derivative \( \delta_{\text{res}}(E) \) close to the resonance as in Eq. 4. For parameters see 13.](image)

From the phases shifts, one can obtain all scattering information, like reflection \( R(E) \) or transmission amplitudes \( T(E) \), by considering a causal wave \( u^{(+)}_b(\epsilon, x) \), propagating to the right \( (k > 0) \),
\[
\lim_{x \to -\infty} u^{(+)}_b(\epsilon, x) = e^{ikx} + R e^{-ikx} = e^{i\delta_e} \cos(kx - \delta_e) + i e^{i\delta_o} \sin(kx - \delta_o),
\]
\[
\lim_{x \to \infty} u^{(+)}_b(\epsilon, x) = T e^{ikx} = e^{i\delta_e} \cos(kx + \delta_e) + i e^{i\delta_o} \sin(kx + \delta_o). \tag{5}
\]

The real-valuedness of the scattering phases of Fig. 3 implies a current conservation \( |R(E)|^2 + |T(E)|^2 = 1 \), and we can write the transmission coefficient in terms of the phases
\[
|T(E)|^2 = \cos^2[\delta_e(E) - \delta_o(E)]. \tag{6}
\]

This transmission coefficient is shown in Fig. 4. In the vicinity of the resonance energies \( E \approx \mu + \epsilon_i \), it changes rapidly between 0 and 100%.

![FIG. 4: Transmission coefficient of a weak coherent perturbation in the b-component of the trapped BEC versus energy \( E \) from the linear response Bogoliubov calculation (solid). The dashed line is the transmission coefficient obtained from propagating a Gaussian wave-packet in the nonlinear GP Eq. (1) in real time. The dotted line represents a crude Thomas-Fermi approximation. Feshbach resonances are seen clearly in all three curves superimposed on a background caused by potential scattering.](image)

In order to isolate the essential physical mechanism responsible for the collective resonance behavior, we approximate the GP Eq. (1) in the Thomas-Fermi limit. Then \( \psi_{\text{TF}}^{(0)}(x) \) is constant within the square-wells and vanishes exactly elsewhere. In an additional approximation, we disregard the matrix \( M \) in the Bogoliubov self-energy Eq. (2). In this limit, the particle and hole excitations are decoupled and \( u \) satisfies a two-component Schrödinger equation
\[
(\epsilon + \frac{\beta^2}{2} - V^B) u = 0. \tag{7}
\]

Due to the TF approximation, this equation has again a square-well character and the solution can be found analytically \( [27, 33] \). Sparing the details of the calculation of the scattering phases, we simply present the results in the dotted line in Fig. 4. A good qualitative agreement needs to be acknowledged, while there are obviously shifts in the resonance energies that are not accounted for in this simple approximation scheme.

In a final step of the analysis of the collective Feshbach resonance, we have also performed a numerical simulation of the nonlinear, time-dependent GP Eq. (1). We propagated an incident traveling Gaussian wave packet on top of the stationary
BEC solution depicted in Fig. 4

\[ \psi_b(x, t = 0) = \psi_b^{(0)}(x) + \sqrt{\frac{2\pi}{4N^2}} e^{-\frac{(x-x_0)^2}{2\sigma^2}} e^{-ik_0 x}. \]  

(8)

It can be seen in Fig. 4 that the b-component of the ground state solution \( \psi_b^{(0)} \) is well localized in the trap center and has initially no overlap with the weak Gaussian perturbation if \( \delta N \ll N \) and \( x_0 \ll -1 \). The initial momentum \( k_0 \approx 0 \) of the wave was varied to cover the energy range in Fig. 4. In order to resolve the resonance structure, one needs a small momentum spread \((\sigma = 20)\) and we find that the wave-packet transmission spectrum matches the linear response approach very well. We show two instances of the propagation of an initial wave packet. In Fig. 4A, the momentum \( k_0 \approx 2.05 \) corresponds to an energy below the resonance energy, marked as \( E_1 \) in Fig. 4, which leads to full transmission. In contrast, Fig. 4B shows the total reflection of the wave-packet if the incident momentum \( k_0 \approx 2.35 \), corresponds exactly to the resonance energy \( E_1 \).

In conclusion, we have identified collective Feshbach resonances in a trapped two component BEC. These resonances do not require a binary Feshbach resonance to modify the binary interaction, but quasi-bound Bogoliubov modes in the linear response spectrum. We have shown that this can be achieved easily with help of an optical laser beam. In contrast to the microscopic binary Feshbach resonance, this quantum mechanical phenomenon exists on a mesoscopic, possibly macroscopic scale and can be used to control the transmission of matter-waves between 0 and 100%. For conceptual simplicity, we have investigated a quasi one-dimensional geometry with square-well potentials. None of this is important for the effect and it can be implemented in various experimental configurations. However, studying this resonance in the mean-field picture can be only a first step and a proper inclusion of the thermal cloud is still lacking.

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