Entanglement-enhanced phase estimation without prior phase information

G. Colangelo,1 F. Martin Ciurana,1 G. Puentes,2 M. W. Mitchell,1,3 and R. J. Sewell1

1ICFO-Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain
2Departamento de Física, Facultad de Ciencias Exactas y Naturales, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina
3ICREA – Institució Catalana de Recerca i Estudis Avançats, 08015 Barcelona, Spain

(Dated: March 30, 2018)

We study the generation of planar quantum squeezed (PQS) states by quantum non-demolition (QND) measurement of a cold ensemble of $^{87}$Rb atoms. Precise calibration of the QND measurement allows us to infer the conditional covariance matrix describing the $F_y$ and $F_z$ components of the PQS, revealing the dual squeezing characteristic of PQS. PQS states have been proposed for single-shot phase estimation without prior knowledge of the likely values of the phase. We show that for an arbitrary phase, the generated PQS gives a metrological advantage of at least 3.1 dB relative to classical states. The PQS also beats traditional squeezed states generated with the same QND resources, except for a narrow range of phase values. Using spin squeezing inequalities, we show that spin-spin entanglement is responsible for the metrological advantage.

PACS numbers: 42.50.Dv, 07.55.Ge, 03.67.Bg, 03.65.Ta

Keywords: spin squeezing, phase estimation, quantum metrology, entanglement, optical magnetometry, interferometry

Estimation of interferometric phases is at the heart of precision sensing, and is ultimately limited by quantum statistical effects [1]. Entangled states can improve sensitivity beyond the “classical limits” that restrict sensing with independent particles, and a diversity of entangled states have been demonstrated for this task, including photonic squeezed states [2] [3] and spin-squeezed states [4]. These give improved sensitivity for a narrow range of phases, but worsened sensitivity for most phases. Optical “NOON” states [5] give improved sensitivity over the whole phase range, but introduce additional phase ambiguity that increases with the size, and thus sensitivity advantage, of the NOON state. Recent proposals [6–8] suggest using planar quantum squeezed (PQS) states to obtain an entanglement-derived advantage for all phase angles, with no additional phase ambiguity. A natural application is in high-bandwidth atomic sensing [9–11], in which the precession angle may not be predictable in advance. PQS states may also be valuable for ab initio phase estimation using feedback [12] [13].

Discussion of such states under the name “intelligent spin states” [15] predates modern squeezing terminology, and analogous states have been studied with optical polarization [16] [18]. Generation of PQS states in material systems has been proposed using two-well Bose-Einstein condensates with tunable and attractive interactions [7] [8], and using quantum non-demolition (QND) measurements [19]. Here we take the latter approach, using Faraday rotation QND measurements [20] [21] applied to an ensemble of cold atomic spins with $f = 1$. As the ensemble spin precesses about the $x$ axis in an external magnetic field [22] [23], we measure the $y$ and $z$ spin components to generate measurement-induced squeezing in these two components, creating a PQS state. The resulting state has enhanced sensitivity to precession angle, i.e., to Zeeman-shift induced phase. The demonstrated PQS state beats the best possible classical state at any precession angle, and beats traditional spin-squeezed states when averaged over the possible angles. Spin-squeezing inequalities [7] [8] [25] detect spin entanglement in the PQS state, showing the sensing advantage is due to spin entanglement [26].

A spin $\mathbf{F}$ obeys the Robertson uncertainty relation

$$\Delta F_y \Delta F_z \geq \frac{1}{2} |\langle [F_y, F_z] \rangle| = \frac{1}{2} |\langle F_z \rangle|.$$ (1)

Unlike the canonical Heisenberg uncertainty relation, the rhs of Eq. (1) may vanish, e.g. for $\langle F_z \rangle = 0$, with the consequence that two spin components, e.g. $F_y$ and $F_z$, may be simultaneously squeezed, with the uncertainty absorbed by the third component, $F_x$. We refer to a state fulfilling this condition as a PQS state.

Following the approach of He et al. [7] [8], we adopt an operational definition planar squeezing. We take $\Delta^2 F_y = \Delta^2 F_z = F_\parallel / 2$ as the standard quantum limit, where $F_\parallel \equiv \sqrt{F_y^2 + F_z^2}$, so that $F_\parallel$ is the magnitude of the in-plane spin components. We define the planar variance $\Delta^2 F_\parallel \equiv \Delta^2 F_y + \Delta^2 F_z$, with standard quantum limit $\Delta^2 F_\parallel = F_\parallel$, and the planar squeezing parameter

$$\xi_\parallel \equiv \frac{\Delta^2 F_\parallel}{F_\parallel}.$$ (2)

A PQS state has $\xi_\parallel < 1$, and has individual component variances below the standard quantum limit, i.e., $\xi_y^2 < 1$, and $\xi_z^2 < 1$, where $\xi_y^4 \equiv 2 \Delta^2 F_y / F_\parallel$, so that $\xi_\parallel = (\xi_y^4 + \xi_z^4)/2$. 


Figure 1. Experimental setup. A cloud of laser-cooled $^{87}\text{Rb}$ atoms is held in a single-beam optical dipole trap. The atoms precess in the $y-z$ plane due to an external magnetic field $B_z$. Off-resonant optical probe pulses experience Faraday rotation as they pass through the atoms by an angle $\varphi$ proportional to the collective on-axis spin component $F_z$. Rotation of the optical polarization from $S_y$ into $S'_y$ is detected by a balanced polarimeter that consists in a wave plate (WP), a polarizing beam splitter (PBS), and photodiodes PD$_2$ and PD$_3$. The input $S_y$ polarization is recorded with a reference photodetector (PD$_1$).

Entanglement is detected using the witness $\xi^2_{\text{w}} \equiv \Delta^2 F_z / \langle N\rangle$, derived in Ref. [27]: for $f = 1$ atoms, entanglement is detected if $\xi^2_{\text{w}} < 7/16$. Here $N\rangle \equiv (\eta_{\text{he}} + p(1 - \eta_{\text{he}})) N_A$ is the number of atoms remaining in the $f = 1$ state after probing, $\eta_{\text{he}}$ accounts for off-resonant scattering of atoms, and $p$ is the fraction of scattered atoms that return to $f = 1$ [27]. We also define a metrological squeezing parameter $\xi_{\text{m}}^2 \equiv F \Delta^2 F_z / F_0^2$, where $F \equiv \langle N\rangle$ is the input spin coherence, similar to the Wineland criterion [28, 29], in that it compares noise to the magnitude of the coherence $F_z$. A PQS with $\xi_{\text{m}}^2 < 1$ gives enhanced metrological sensitivity to arbitrary phase shifts.

A PQS state may be used to measure arbitrary phase angles with quantum-enhanced precision. For example, we consider an ensemble of atomic spins precessing in the $y-z$ plane in an external magnetic field $B_z$. The spin projection onto the $z$-axis is given by $F_z(t) = F_z \cos \phi - F_y \sin \phi$, where $F_y$ and $F_z$ are evaluated at $t = 0$ and the phase $\phi = \omega_z t$ is proportional to the magnetic field. The uncertainty in estimating $\phi$ of the atomic precession is

$$\Delta^2 \phi = \frac{\Delta^2 F_z(\phi)}{|d(F_z(\phi))/d\phi|^2} = \frac{\Delta^2 F_z(\phi)}{(\langle F_y \rangle \cos \phi + \langle F_z \rangle \sin \phi)^2} \tag{3}$$

where $\Delta^2 F_z(\phi) \equiv \Delta^2 F_y \sin^2 \phi + \Delta^2 F_z \cos^2 \phi + \text{cov}(F_y, F_z) \sin 2\phi$, and $\text{cov}(A, B) \equiv \frac{1}{2} \langle AB + BA \rangle - \langle A \rangle \langle B \rangle$ is the covariance. The standard quantum limit is $\Delta^2 \phi_{\text{SQL}} = 1/2 F_z$. We note that PQS states reduce the planar variance for arbitrary angles on a finite interval, except where the denominator in Eq. (3) is equal to zero. In contrast, squeezing a single spin component is only beneficial to refine the estimate of a phase over a limited range of angles, and requires prior knowledge of the phase, or adaptive procedures to determine the phase during the measurement [3].

We work with an ensemble of up to $1.75 \times 10^6$ laser-cooled $^{87}\text{Rb}$ atoms held in a single beam optical dipole trap [30–32], as illustrated in Fig. 1. The atoms are initially polarized via high efficiency ($\sim 98\%$) stroboscopic optical pumping, in the presence of a small magnetic field applied along the $x$-axis, such that $\langle F_y \rangle \approx \langle N\rangle$. $N\rangle$ is subject to Poissonian fluctuations because accumulation of independent atoms into the ensemble is a stochastic process limited by Poisson statistics $\Delta^2 N\rangle = \langle N\rangle$. We refer to this kind of state as a Poissonian coherent spin state (PCSS), with variances $\Delta^2 F_z = \Delta^2 F_\varphi = \langle N\rangle/2$ and $\Delta^2 F_y = \langle N\rangle$. Generating sub-Poissonian atom number statistics, either via strong interaction among the atoms during accumulation [33–39], or precise non-destructive measurement [40–50], remains a significant experimental challenge.

We probe the atomic spins via off-resonant paramagnetic Faraday-rotation. The effective atom-light interaction is given by the hamiltonian

$$H_{\text{eff}} = g S_z F_z \tag{4}$$

Here, the atoms are described by the collective spin operators $F \equiv \sum_i f^{(i)}$, with $f^{(i)}$ the spin orientation of individual atoms. The optical polarization of the probe pulses is described by the Stokes operators $S_k = \frac{1}{2}(a_L^T a_R + a_R^T a_L)\sigma_k(a_L, a_R)^T$, with Pauli matrices $\sigma_k$. The coupling constant $g$ depends on the detuning from the resonance of the probe beam, the atomic structure, the geometry of the atomic ensemble and probe beam [30–32, 44–47, 49].

Equation (4) describes a quantum non-demolition measurement of the collective atomic spin $F_z$: an input $S_z$-polarized optical pulse interacting with the atoms experiences a rotation by an angle $\varphi = g F_z$. The transformation produced by the interaction is $S_y' = S_y \cos \varphi + S_z \sin \varphi$. In our experiment we measure $S_y$ at the input by picking off a fraction of the optical pulse and...
sending it to a reference detector, and $S'_u$ using a fast home-built balanced polarimeter \[53\]. Both signals are recorded on a digital oscilloscope, from which we calculate $\hat{\phi} = \arcsin (S'_u/S'_x)$, the estimator for $\phi$. We correct for slow drifts in the polarimeter signal by subtracting a baseline from each pulse, estimated by repeating the measurement without atoms in the trap.

We probe the atoms using a train of $\tau = 0.6$ μs duration pulses of linearly polarized light, with a detuning of 700 MHz to the red of the $^{87}\text{Rb} D_2$ line, sent through the atomic cloud at 3 μs intervals. The probe pulses are $V$-polarized, with on average $n_1 = 2.74 \times 10^6$ photons. Between the probe pulses, we send $H$-polarized compensation pulses with on average $n'_{1(H)} = 1.49 \times 10^6$ photons through the atomic cloud to compensate for tensor light shifts \[21\] \[27\] \[32\]. During the measurement, an external magnetic field $B_z$ coherently rotates the atoms in the $y-z$ plane at the Larmor frequency $\omega_L$. The time taken to complete a single-pulse measurement is small compared to the Larmor precession period, i.e. $\tau \ll T_L$. Off-resonant scattering of probe photons during the measurement leads to decay of the atomic coherence at a rate $\eta = 3 \times 10^{-10}$ per photon.

The measurable signal is described by the free induction decay model \[23\]

$$\varphi(t) = g \left( F_z(t_e) \cos \phi - F'_y(t_e) \sin \phi \right) e^{-t/t_2} + \varphi_0 \quad (5)$$

where $t_e \equiv t - t_c$ and the phase $\phi = \omega_L t_e$ is proportional to the magnetic field. We record a set of measurements $\varphi(t_k)$, and detect the PQS state at time $t_c$. A typical free induction decay signal is illustrated in Fig. 2.

An independent measurement is used to calibrate $g$, while $\omega_L$, $T_2$, and $\varphi_0$ are found by fitting the measured $\varphi(t_k)$ over all the data points.

The model described in Eq. (5) allows a simultaneous estimation of $F_1 = (F_{1y}, F_{1z})$ at a time $t = t_e$ by fitting the data using the measurements from an interval $\Delta t$ prior to $t_e$ (labeled $M_1$ in Fig. 2), producing a conditional PQS at time $t_e$. We detect the PQS by comparing the first measurement outcome to a second estimate $F_2 = (F_{2y}, F_{2z})$ using the measurements from an interval $\Delta t$ after to $t_e$ (labeled $M_2$ in Fig. 2). The classical parameters $g$, $\omega_L$, $T_2$, and $\varphi_0$ are fixed beforehand. As a result, these are two linear, least-squares estimates of the vector $F$ obtained from disjoint data sets \[51\]. Statistics are gathered over 450 repetitions of the experiment, taking into account the inhomogeneous atom-light coupling \[12\] \[43\] \[55\].

The estimate of the state from the two independent measurements is subject to technical noise due to amplitude and phase fluctuations of the input state, and shot-to-shot variations of the magnetic field. In Fig. 3 a) we plot the estimate of $F_1$ at time $t_e$ for an input state with $\langle N_A \rangle = 1.75 \times 10^6$ atoms. In contrast, the conditional uncertainty of $F_2$ given $F_1$ is limited mainly by the measurement read-out noise, as shown in Figs. 3 b) and c).

From the measurement record we compute the conditional covariance matrix $\Gamma_{F_2|F_1} = \Gamma_{F_2} - \Gamma_{F_2,F_1} \Gamma_{F_1}^{-1} \Gamma_{F_1,F_2}$, which quantifies the error in the best linear prediction of $F_2$ based on $F_1$ \[24\]. $\Gamma_{F}$ indicates the covariance matrix for vector $F$, and $\Gamma_{uv}$ indicates the cross-covariance matrix for vectors $u$ and $v$. The difference between the best
linear prediction of $\mathbf{F}$ using $\mathbf{F}_1$ and the confirming estimate $\mathbf{F}_2$ is visualized using the vector $\mathbf{F} = \{F_y, F_z\} = \mathbf{F}_2 - \Gamma_{\mathbf{F}_2|\mathbf{F}_1} \Gamma_{\mathbf{F}_1}^{-1} \mathbf{F}_1$, where $\Gamma_{\mathbf{F}_2|\mathbf{F}_1} = \Gamma_{\mathbf{F}_2} - \Gamma_0$ and $\mathbf{F}_1$ is estimated at $t_c$. $\Gamma_0$ is the read-out noise, quantified by repeating the measurement without atoms in the trap. In Fig. 4, we show $\xi^2_F$ as function of the in-plane coherence $F_{||}$ of the atomic ensemble, which we vary by changing the number of atoms in the optical dipole trap. We detect a PQS for $F_{||} \geq 4 \times 10^6$ spins. With the maximum coherence $F_{||} = 1.45 \times 10^6$ spins, we observe $\xi^2_F = 0.37 \pm 0.03 < 1$, detecting a PQS with $> 20 \sigma$ significance, with $\xi^2_F = 0.32 \pm 0.03$ and $\xi^2_F = 0.42 \pm 0.04$, and $\xi^2_F = 0.32 \pm 0.02 < 7/16$, detecting entanglement among the atomic spins with $> 5 \sigma$ significance [27]. The measured conditional covariance (in units of spins$^2$) is

$$\Gamma_{\mathbf{F}_2|\mathbf{F}_1} = \begin{bmatrix} 2.32 & 0.64 \\ 0.64 & 3.00 \end{bmatrix} \pm \begin{bmatrix} 0.21 & 0.16 \\ 0.16 & 0.28 \end{bmatrix} \times 10^5. \quad (6)$$

Figure 4. Semi-log plot of the planar squeezing parameter, $\xi^2_{||}$, as function of the in-plane coherence $F_{||}$ of the atomic ensemble. We vary $F_{||}$ by changing the number of atoms loaded in the optical dipole trap. A PQS is detected for $\xi^2_{||} < 1$ (shaded region). Entanglement is detected for $\xi^2_{||} = (F_{||}/(N_A)) \xi^2_{||} < 7/16$ (dashed line). Error bars represent $\pm 1 \sigma$ statistical errors.

Empirically, we find $\Delta t = 270 \mu s$ minimizes the total variance $\text{Tr}(\Gamma_{\mathbf{F}_2|\mathbf{F}_1})$. This reflects a trade-off of photon shot noise versus scattering-induced decoherence and magnetic-field technical noise. At this point $N_L = 2.47 \times 10^8$ photons have been used in the measurement and the atomic state coherence has decayed by a factor $\eta_{bc} = 0.89$ due to off-resonant scattering, and a factor $\eta_{\text{dec}} = 0.93$ due to dephasing induced by magnetic field gradients [27]. The resulting spin coherence of the PQS is $F_{||} = \eta_{\text{dec}} \eta_{bc} N_A$ spins.

From $\Gamma_{\mathbf{F}_2|\mathbf{F}_1}$, we estimate the planar squeezing parameter $\xi^2_F = \text{Tr}(\Gamma_{\mathbf{F}_2|\mathbf{F}_1})/F_{||}$, where $\Gamma_{\mathbf{F}_2|\mathbf{F}_1} = \Gamma_{\mathbf{F}_2} - \Gamma_0$ and $F_{||}$ is estimated at $t_c$. $\Gamma_0$ is the read-out noise, quantified by repeating the measurement without atoms in the trap. In Fig. 4, we show $\xi^2_F$ as function of the in-plane coherence $F_{||}$ of the atomic ensemble, which we vary by changing the number of atoms in the optical dipole trap. We detect a PQS for $F_{||} \geq 4 \times 10^6$ spins. With the maximum coherence $F_{||} = 1.45 \times 10^6$ spins, we observe $\xi^2_F = 0.37 \pm 0.03 < 1$, detecting a PQS with $> 20 \sigma$ significance, with $\xi^2_F = 0.32 \pm 0.03$ and $\xi^2_F = 0.42 \pm 0.04$, and $\xi^2_F = 0.32 \pm 0.02 < 7/16$, detecting entanglement among the atomic spins with $> 5 \sigma$ significance [27]. The measured conditional covariance (in units of spins$^2$) is

$$\Gamma_{\mathbf{F}_2|\mathbf{F}_1} = \begin{bmatrix} 2.32 & 0.64 \\ 0.64 & 3.00 \end{bmatrix} \pm \begin{bmatrix} 0.21 & 0.16 \\ 0.16 & 0.28 \end{bmatrix} \times 10^5. \quad (6)$$

Figure 5. Estimated phase sensitivity of the PQS state as a function of the measurement phase $\phi$ (red solid line). The standard quantum limit $\Delta^2 \phi_{\text{SQL}}$ is indicated by the shaded region. For comparison, we plot the phase sensitivity of the input PCSS (blue solid line), and an ideal single-variable spin squeezed state (green solid line). We also show the metrologically significant enhancement in phase sensitivity relative to that of the PCSS, $\Delta^2 \phi/\Delta^2 \phi_{\text{PCSS}}$, for both the PQS (red dashed line) and SSS (green dot-dashed line) states.

For comparison, the estimated read-out noise is

$$\Gamma_0 = \begin{bmatrix} 1.02 & 0.14 \\ 0.14 & 1.03 \end{bmatrix} \pm \begin{bmatrix} 0.07 & 0.05 \\ 0.05 & 0.07 \end{bmatrix} \times 10^5. \quad (7)$$

For this state, the observed metrological squeezing parameter is $\xi^2_m = 0.45 \pm 0.03$, indicating that entanglement-enhanced phase sensitivity is achievable. To estimate the enhanced phase sensitivity provided by the PQS state, we evaluate Eq. 3 using the conditional covariance $\Gamma_{\mathbf{F}_2|\mathbf{F}_1}$ and the measured coherences. The PQS state achieves a maximum sensitivity $\Delta^2 \phi = 0.38 \Delta^2 \phi_{\text{SQL}}$ ($\Delta \phi = 3.6 \times 10^{-4}$ radians) at a phase $\phi = 0.68 \pi$ radians. Note that this phase is determined by the choice of measurement time $t_c$.

In Fig. 5 we plot the estimated phase sensitivity $\Delta^2 \phi$ of the observed PQS state (red solid line). For comparison purposes, we rotate the PQS so that the spin coherence is aligned along the $y$-axis, i.e. $\mathbf{F} \rightarrow R(\theta) \cdot \mathbf{F}$ and $\Gamma_{\mathbf{F}_2|\mathbf{F}_1} \rightarrow R(\theta) \cdot \Gamma_{\mathbf{F}_2|\mathbf{F}_1} \cdot R(\theta)^T$, where $\arctan \theta = \phi/F_\phi/F_z$. We compare this with the sensitivity of a PCSS with input spin coherence $\langle F_y \rangle = N_A$ (blue dashed line), and an ideal single-variable spin squeezed state (SSS) that would be produced by a single instantaneous quantum non-demolition measurement with the same precision, i.e. with $\Delta^2 F_y = \langle N_A \rangle$, $\Delta^2 F_z$ reduced by a factor $1/(1 + \eta^2 N_L N_A/2)$, and input coherence $\langle F_y \rangle = \eta_{bc} N_A$ (green dot-dashed line).

We also plot the calculated enhancement in phase sensitivity $\Delta^2 \phi$ of both the PQS and SSS states relative to the classical input PCSS. The measured PQS state
achieves \( \geq 3.1\) dB quantum-enhanced, metrologically-significant phase sensitivity with respect to the PCSS for all phases, with a maximum of 4.1 dB, enabling quantum-enhanced measurement of an arbitrary phase shift. In contrast, the SSS achieves 6.6 dB enhancement relative to the PCSS at \( \phi = 0 \), but performs worse than the PQS state outside the range \(-0.09\pi < \phi < 0.12\pi\) radians.

In contrast to the well known spin-squeezed states, planar quantum squeezed states enhance the precision of phase estimation without requiring \textit{a priori} information about the phase. Here we have shown that QND measurement can efficiently produce such states, demonstrating more than 3 dB of advantage relative to classical states over the full range of phase angles. We also detect spin-spin entanglement underlying the metrological advantage. Such states are attractive for high-bandwidth and high-sensitivity optical magnetometers [57] and other atomic sensing applications employing non-destructive spin detection [28, 58, 59].

ACKNOWLEDGEMENTS

We thank Q. Y. He, M. Reid, P. Drummond, G. Vitagliano, G. Tóth, E. Distanza, V.G. Lucivero, L. Blanchet, N. Behbood and M. Napolitano for helpful discussions. Work supported by MINECO/FEDER, MINECO projects MAQRO (Ref. FIS2015-68039-P), XPLICA (FIS2014-62181-EXP) and Severo Ochoa grant SEV-2015-0522, Catalan 2014-SGR-1295, by the European Union Project QUIC (grant agreement 641122), European Research Council project AQUMET (grant agreement 280169) and ERIDIAN project (grant agreement 713682), by Fundación Privada CELLEX. GP gratefully acknowledges funding from the Agencia Nacional de Promoción Científica y Tecnológica (ANPCyT), PICT2014-0710, and UBACYT PDE 2017.

1 giorgio.colangelo@icfo.es

2 morgan.mitchell@icfo.es

3 robert.sewell@icfo.eu

[1] H.M. Wiseman and G.J. Milburn, Quantum Measurement and Control (Cambridge University Press, 2010).

[2] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, “Observation of squeezed states generated by four-wave mixing in an optical cavity,” Phys. Rev. Lett. 55, 2409–2412 (1985).

[3] Ling-An Wu, H. J. Kimble, J. L. Hall, and Huifa Wu, “Generation of squeezed states by parametric down conversion,” Phys. Rev. Lett. 57, 2520–2523 (1986).

[4] V. Meyer, M. A. Rowe, D. Kielpinski, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, “Experimental demonstration of entanglement-enhanced rotation angle estimation using trapped ions,” Phys. Rev. Lett. 86, 5870–5873 (2001).

[5] M. W. Mitchell, J. S. Lundeen, and A. M. Steinberg, “Super-resolving phase measurements with a multiphoton entangled state,” Nature 429, 161–164 (2004).

[6] Géza Tóth, Christian Knapp, Otfried Gühne, and Hans J. Briegel, “Spin squeezing and entanglement,” Phys. Rev. A 79, 042334 (2009).

[7] Q. Y. He, Shi-Guo Peng, P. D. Drummond, and M. D. Reid, “Planar quantum squeezing and atom interferometry,” Phys. Rev. A 84, 022107 (2011).

[8] Q Y He, T G Vaughan, P D Drummond, and M D Reid, “Entanglement, number fluctuations and optimized interferometric phase measurement,” New J. Phys. 14, 093012 (2012).

[9] V. Shah, G. Vasilakis, and M. V. Romalis, “High bandwidth atomic magnetometry with continuous quantum nondemolition measurements,” Phys. Rev. Lett. 104, 013601 (2010).

[10] G. Vasilakis, V. Shah, and M. V. Romalis, “Stroboscopic backaction evasion in a dense alkali-metal vapor,” Phys. Rev. Lett. 106, 143601 (2011).

[11] R. J. Sewell, M. Koschorreck, M. Napolitano, B. Dubost, N. Behbood, and M. W. Mitchell, “Magnetic sensitivity beyond the projection noise limit by spin squeezing,” Phys. Rev. Lett. 109, 253605 (2012).

[12] G. Y. Xiang, B. L. Higgins, D. W. Berry, H. M. Wiseman, and G. J. Pryde, “Entanglement-enhanced measurement of a completely unknown optical phase,” Nat. Photon. 5, 43–47 (2011).

[13] Hidehiro Yonezawa, Daisuke Nakane, Trevor A. Wheatley, Kohjiro Iwasawa, Shuntaro Takeda, Hajime Arao, Kentaro Ohki, Koji Tsumura, Dominic W. Berry, Timothy C. Ralph, Howard M. Wiseman, Elanor H. Huntington, and Akira Furusawa, “Quantum-enhanced optical-phase tracking,” Science 337, 1514–1517 (2012).

[14] Adriano A. Berni, Tobias Gehring, Bo M. Nielsen, Vitus Händchen, Matteo G. A. Paris, and Ulrik L. Andersen, “Ab initio quantum-enhanced optical phase estimation using real-time feedback control,” Nat. Photon. 9, 577 (2015).

[15] C Aragone, G Guerri, S Salamo, and J L Tani, “Intelligent spin states,” J. Phys. A 7, L149 (1974).

[16] Natalia Korolkova, Gerd Leuchs, Rodney Loudon, Timothy C. Ralph, and Christine Silberhorn, “Polarization squeezing and continuous-variable polarization entanglement,” Phys. Rev. A 65, 052306 (2002).

[17] Roman Schnabel, Warwick P. Bowen, Nicolas Treeps, Timothy C. Ralph, Hans-A. Bachor, and Ping Koy Lam, “Stokes-operator-squeezed continuous-variable polarization states,” Phys. Rev. A 67, 012316 (2003).

[18] A. Predojević, Z. Žaia, J. M. Cáballero, and M. W. Mitchell, “Rubidium resonant squeezed light from a diode-pumped optical-parametric oscillator,” Phys. Rev. A 78, 063820 (2008).

[19] Graciana Puentes, Giorgio Colangelo, Robert J Sewell, and Morgan W Mitchell, “Planar squeezing by quantum non-demolition measurement in cold atomic ensembles,” New J. Phys. 15, 103031 (2013).

[20] M W Mitchell, M Koschorreck, M Kubasik, M Napolitano, and R J Sewell, “Certified quantum non-demolition measurement of material systems,” New J. Phys. 14, 085021 (2012).

[21] R. J. Sewell, M. Napolitano, N. Behbood, G. Colangelo,
and M. W. Mitchell, “Certified quantum non-demolition measurement of a macroscopic material system,” Nat. Photon. 7, 517–520 (2013).

[22] N. Behbood, G. Colangelo, F. Martin Ciurana, M. Napolitano, R. J. Sewell, and M. W. Mitchell, “Feedback cooling of an atomic spin ensemble,” Phys. Rev. Lett. 111, 103601 (2013).

[23] N. Behbood, F. Martin Ciurana, G. Colangelo, M. Napolitano, M. W. Mitchell, and R. J. Sewell, “Real-time vector field tracking with a cold-atom magnetometer,” Appl. Phys. Lett. 102, 173504 (2013).

[24] N. Behbood, F. Martin Ciurana, G. Colangelo, M. Napolitano, Géza Tóth, R. J. Sewell, and M. W. Mitchell, “Generation of macroscopic singlet states in a cold atomic ensemble,” Phys. Rev. Lett. 113, 093601 (2014).

[25] Giuseppe Vitagliano, Philipp Hyllus, Inigo L. Egusquiza, and Géza Tóth, “Spin squeezing inequalities for arbitrary spin,” Phys. Rev. Lett. 107, 240502 (2011).

[26] Federica A. Bedin, Joanna A. Zielinska, Vito G. Lucivero, Yannick A. de Icaza Astiz, and Morgan W. Mitchell, “Macrosopic quantum state analyzed particle by particle,” Phys. Rev. Lett. 114, 120402 (2015).

[27] Giorgio Colangelo, Robert J Sewell, Naimeeh Behbood, Ferran Martin Ciurana, Gil Triginer, and Morgan W Mitchell, “Quantum atom–light interfaces in the gaussian description for spin-1 systems,” New J. Phys. 15, 093007 (2013).

[28] D. J. Wineland, J. J. Bollinger, W. M. Itano, F. L. Moore, and D. J. Heinzen, “Spin squeezing and reduced quantum noise in spectroscopy,” Phys. Rev. A 46, R6797–R6800 (1992).

[29] D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, “Squeezed atomic states and projection noise in spectroscopy,” Phys. Rev. A 50, 67–88 (1994).

[30] M. Kubasik, M. Koschorreck, M. Napolitano, S. R. de Echaniz, H. Crepaz, J. Eschner, E. S. Polzik, and M. W. Mitchell, “Polarization-based light-atom quantum interface with an all-optical trap,” Phys. Rev. A 79, 043815 (2009).

[31] M. Koschorreck, M. Napolitano, B. Dubost, and M. W. Mitchell, “Sub-projection-noise sensitivity in broadband atomic magnetometry,” Phys. Rev. Lett. 104, 093602 (2010).

[32] M. Koschorreck, M. Napolitano, B. Dubost, and M. W. Mitchell, “Quantum nondemolition measurement of large-spin entanglement by dynamical decoupling,” Phys. Rev. Lett. 105, 093602 (2010).

[33] Nicolas Schlosser, Georges Reynold, Igor Protsenko, and Philippe Grangier, “Sub-poisonious loading of single atoms in a microscopic dipole trap,” Nature 411, 1024–1027 (2001).

[34] Y. R. P. Sortais, A. Fuhrmanek, R. Bourgain, and A. Browaeys, “Sub-poisonious atom-number fluctuations using light-assisted collisions,” Phys. Rev. A 85, 053403 (2012).

[35] C.-S. Chuu, F. Schreck, T. P. Meyrath, J. L. Hanssen, G. N. Price, and M. G. Raizen, “Direct observation of sub-poisonious atom statistics in a degenerate Bose gas,” Phys. Rev. Lett. 95, 260403 (2005).

[36] Amir Ihå, Hagar Veksler, Oren Lahav, Alex Blumkin, Coral Moreno, Carmit Gordon, and Jef Steinhauer, “Direct observation of a sub-poisonious number distribution of atoms in an optical lattice,” Phys. Rev. Lett. 104, 113001 (2010).

[37] Christian Sanner, Edward J. S., Aviv Keshet, Ralf Gommers, Yong-il Shin, Wujie Huang, and Wolfgang Ketterle, “Suppression of density fluctuations in a quantum degenerate fermi gas,” Phys. Rev. Lett. 105, 040402 (2010).

[38] S. Whitlock, C. F. Ockeloen, and R. J. C. Sreekun, “Sub-poisonious atom-number fluctuations by three-body loss in mesoscopic ensembles,” Phys. Rev. Lett. 104, 120402 (2010).

[39] C. S. Hofmann, G. Günter, H. Schempp, M. Robert-de Saint-Vincent, M. Gärtnner, J. Evers, S. Whitlock, and M. Weidemüller, “Sub-poisonious statistics of rydberg-interacting dark-state polaritons,” Phys. Rev. Lett. 110, 203601 (2013).

[40] John Kenton Stockton, “Suppression of density fluctuations in a degenerate bose gas,” Phys. Rev. Lett. 117, 260403 (2015).

[41] T. Takano, M. Fuyama, R. Namiki, and Y. Takahashi, “Spine squeezing of a cold atomic ensemble with the nuclear spin of one-half,” Phys. Rev. Lett. 102, 033601 (2009).

[42] J. Appel, P. J. Windpassinger, D. Oblak, U. B. Hoff, N. Kjærgaard, and E. S. Polzik, “Mesoscopic atomic entanglement for precision measurements beyond the standard quantum limit,” Proc. Nat. Acad. Sci. 106, 10960–10965 (2009).

[43] Monika H. Schleier-Smith, Ian D. Leroux, and Vladan Vuletić, “States of an ensemble of two-level atoms with reduced quantum uncertainty,” Phys. Rev. Lett. 104, 073604 (2010).

[44] D. B. Hume, I. Stroescu, M. Joos, W. Muessel, H. Strobel, and M. K. Oberthaler, “Accurate atom counting in mesoscopic ensembles,” Phys. Rev. Lett. 111, 253001 (2013).

[45] J.-B. Béguin, E. M. Bookjans, S. L. Christensen, H. L. Sørensen, J. H. Müller, E. S. Polzik, and J. Appel, “Generation and detection of a sub-poisonious atom number distribution in a one-dimensional optical lattice,” Phys. Rev. Lett. 113, 263603 (2014).

[46] J. G. Bolmet, K. C. Cox, M. A. Norcia, J. M. Weiner, Z. Chen, and J. K. Thompson, “Reduced spin measurement back-action for a phase sensitivity ten times beyond the standard quantum limit,” Nat. Photon. 8, 731–736 (2014).

[47] M. Gajdacz, A. J. Hilliard, M. A. Kristensen, P. L. Pedersen, C. Klempt, J. J. Arlt, and J. F. Sherson, “Preparation of ultracold atom clouds at the shot noise level,” Phys. Rev. Lett. 117, 073604 (2016).

[48] Onur Hosten, Nils J. Engelsen, Rajiv Krishnakumar, and Mark A. Kasevich, “Measurement noise 100 times lower than the quantum-projection limit using entangled atoms,” Nature 529, 505–508 (2016).

[49] Hao Zhang, Robert McConnell, Senka Ćuk, Qian Lin, John Kenton Stockton, Continuous quantum measurement of cold alkali-atom spins., Ph.D. thesis, California Institute of Technology (2007).
and measurement of atomic spins in polarization spectroscopy,” Opt. Commun. 283, 681 – 694 (2010).

[52] A Kuzmich, L Mandel, and N P Bigelow, “Generation of spin squeezing via continuous quantum nondemolition measurement,” Phys. Rev. Lett. 85, 1594–1597 (2000).

[53] F. Martin Ciurana, G. Colangelo, Robert J. Sewell, and Morgan W. Mitchell, “Real-time shot-noise-limited differential photodetection for atomic quantum control,” Opt. Lett. 41, 2946–2949 (2016).

[54] Further details of the fitting procedure are given in Ref. [55].

[55] G. Colangelo, F. Martin Ciurana, L. Bianchet, R. J. Sewell, and M. W. Mitchell, “Simultaneous tracking of spin angle and amplitude beyond classical limits,” in press (2017).

[56] M. Kendall and A. Stuart, The advanced theory of statistics, Vol.2 (London, Griffin, 1979).

[57] IK Kominis, TW Kornack, JC Allred, and MV Romalis, “A subfemtotesla multichannel atomic magnetometer,” Nature 422, 596–599 (2003).

[58] Jérôme Lodewyck, Philip G. Westergaard, and Pierre Lemonde, “Nondestructive measurement of the transition probability in a sr optical lattice clock,” Phys. Rev. A 79, 061401 (2009).

[59] D. Sheng, S. Li, N. Dural, and M. V. Romalis, “Subfemtotesla scalar atomic magnetometry using multipass cells,” Phys. Rev. Lett. 110, 160802 (2013).