SCALING VIOLATION AND SHADOWING CORRECTIONS AT HERA.

A. L. Ayala Filho\textsuperscript{a,b}\textsuperscript{*}, M. B. Gay Ducati \textsuperscript{a,}\textsuperscript{**} and E. M. Levin \textsuperscript{c,d}\textsuperscript{†}

\textsuperscript{a) Instituto de Física, Univ. Federal do Rio Grande do Sul Caixa Postal 15051, 91501-970 Porto Alegre, RS, BRAZIL}
\textsuperscript{b) Instituto de Física e Matemática, Univ. Federal de Pelotas Campus Universitário, Caixa Postal 354, 96010-900, Pelotas, RS, BRAZIL}
\textsuperscript{c) HEP Department, School of Physics and Astronomy Raymond and Beverly Sackler Faculty of Exact Science Tel Aviv University, Tel Aviv 69978, ISRAEL}
\textsuperscript{d) DESY Theory, Notkestr. 85, 22607, Hamburg, GERMANY}

Abstract: We study the value of shadowing corrections (SC) in HERA kinematic region in Glauber - Mueller approach. Since the Glauber - Mueller approach was proven in perturbative QCD in the double logarithmic approximation (DLA), we develop the DLA approach for deep inelastic structure function which takes into account the SC. Our estimates show small SC for $F_2$ in HERA kinematic region while they turn out to be sizable for the gluon structure function. We compare our estimates with those for gluon distribution in leading order (LO) and next to leading order (NLO) in the DGLAP evolution equations.

\textsuperscript{*} E-mail: ayala@if.ufrgs.br
\textsuperscript{**} E-mail: gay@if.ufrgs.br
\textsuperscript{†} E-mail: leving@post.tau.ac.il
In this letter, we investigate the role of the shadowing corrections (SC) on the value of the deep inelastic structure functions and on the scaling violation mechanism in HERA kinematic region. Our estimates of the SC are done using the Glauber - Mueller formula. This formula was proven by Mueller [1] in the DLA of pQCD and was studied in details in Refs.[2][3][4]. The structure function $F_2(x, Q^2)$ reads as:

$$F_2(x, Q^2) = \frac{N_c}{6\pi^3} \sum_{q_f} \int_{Q^2/\Lambda^2}^{\infty} \frac{d^2 r}{r_1^2} \int d^2 b_\perp \{1 - e^{-\frac{2}{R}}\} ,$$

(1)

where $x$ is the Bjorken scaling variable and $Q^2$ is the photon virtuality. $N_c$ is the colour number and $Z_f$ is the charge fraction of each quark, $N_f$ is the number of flavours taken into account in the quark- nucleon scattering and $b_\perp$ is the impact parameter for the scattering of the quark - antiquark pair with transverse splitting $r_\perp$ with the nucleon target.

The opacity function $\Omega$ is, generally speaking, an arbitrary real function and it has a simple physical interpretation: $e^{-\Omega}$ is the probability that the quark-antiquark pair do not suffer an inelastic scattering. This function should be determined in QCD. For the DLA it was shown ( see Refs.[1] [5]) that the impact parameter dependence can be factorized out and $\Omega$ can be written

$$\Omega(x, Q^2, b_\perp) = \frac{4\pi^2 \alpha_s}{3Q^2} xG(x, Q^2)S(b_\perp)$$

(2)

where $S(b_\perp)$ is the profile function for a nucleon with radius $R$, and is taken as an exponential function, $S(b_\perp) = \frac{1}{\pi R^2}exp(-b_\perp^2/R^2)$, in our calculations.

It should be stressed that Eq. (1) takes into account the unitarity constraint and with $\Omega$ defined in Eq. (2) gives the DGLAP evolution equation [6] in the kinematic region where $\Omega \ll 1$.

In order to investigate the $x$ and $Q^2$ evolution of the gluon distribution in this approach, we will consider first our Born term with the gluon distribution taken in the DLA limit. Since the Born term is equivalent to the DGLAP expression for $F_2$ in the DLA limit, we will take $xG^{DLA}$ as in Ref.[7] and [8]. Thus, the gluon distribution reads

$$xG^{DLA}(x, Q^2) = G_0 I_0(y) ,$$

(3)

where the variables are $y = 2\gamma \sqrt{\ln(1/x)\ln(t/t_0)}$ and $t = \ln(Q^2/\Lambda^2)$. The QCD constants are $\gamma = \sqrt{12/\beta_0}$, $\beta_0 = 11 - \frac{2N_f}{3}$ and $\Lambda = 0.232 GeV^2$. The constant $G_0$ plays the role of the flat initial condition, since the Bessel function $I_0(y)$ goes to 1 as $y$ goes to zero. We disregard
the sub-leading corrections to the DLA gluon proportional to $\alpha_S \ln Q^2$ proposed in Ref.[7]. Integrating expression (1) over $b_\perp$ the Born term then reads

$$F_2 = F_2(x, Q_0^2) + \frac{2}{9\pi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \alpha_S(Q'^2)xG^{DLA}(x, Q'^2),$$

(4)

where we have taken $N_f = 3$. The expression (4) gives the sea component of $F_2$ generated by the gluon evolution from the initial virtuality $Q_0^2$ to $Q^2$. Going from expression (1) to (4) we have taken $Q'^2 = 1/r_\perp^2$. Thus, the lower limit $Q_0^2$ works as a cut off for the large distances effects over $F_2$. These effects are included in $F_2(x, Q_0^2)$, the value of the structure function for the virtuality $Q_0^2$. It has a nonperturbative origin and takes into account the amount of $q\bar{q}$ pairs not generated by the perturbative transition $g \rightarrow q\bar{q}$. We will parameterize the initial structure function by the expression

$$F_2(x, Q_0^2) = C_0 x^{-0.08} (1 - x)^{10},$$

(5)

where $C_0$ is a constant that adjusts the nonperturbative contribution. This expression reproduces the soft pomeron behaviour ($x^{-0.08}$ as $x \rightarrow \infty$) presented by the $\gamma^* - Nucleon$ cross section in the low $Q^2$ region[8]. Since we have used $N_f = 3$, we should add the charm component $F_c^2$. This component is generated perturbatively from the $\gamma^* - gluon$ fusion mechanism with the gluon distribution given by the DLA expression (3). This mechanism is discussed in detail in Ref.[10]. Finally, we obtain the following expression for $F_2$

$$F_2(x, Q^2) = F_2(x, Q_0^2) + F_{2, DLA}^\text{Born} + F_{2, DLA}^c.$$

(6)

To fit the expression (3) to the HERA data, we have taken the $F_2$ points which lies in the region $1 \text{GeV}^2 < Q^2 < 100 \text{GeV}^2$ and $x < 10^{-2}$, where we expect that our DLA approach to SC is valid. The H1 and ZEUS results were taken from Refs.[11] and [12], respectively.

In figure (1) we present the fit for a subset of the experimental data. The parameters used are $G_0 = 0.136$, $C_0 = 0.273$ and $Q_0^2 = 0.330 \text{GeV}^2$. The values of the parameters were chosen in such a way to minimize the $\chi^2$, which corresponds to $\chi^2/d.o.f. = 124/222$. We can see from the figure that the steep behaviour of the deep inelastic struture function is well described by the DLA evolution of the gluon distribution, regarded we have included enough nonperturbative $q\bar{q}$ pairs. With this set of parameters also the $Q^2$ scaling violation of $F_2$ can be described, as shown in figure (3). Taking a small value for the initial virtuality we can generate the DLA behavior for $Q^2 \approx 2 \text{GeV}^2$. A similar result was obtained in Ref.[8], but in a not completely DLA limit[7]. It is important to note that our aim in this letter is to describe HERA data in a completely consistent DLA limit, and not to provide an overall fit to existing high energy data.

*The authors have taken $P_{qg} = z^2 + (1-z)^2$ and the sub-leading factor $(t/t_0)^{-\delta}$ with $\delta = (11 + \frac{2N_f}{27})/\beta_0$
Since we have described the data with the Born term of the DLA expression (1), we can investigate the amount of shadowing corrections predicted for $F_2$ on Glauber - Mueller formula of Eq. (1). For that, we substitute the Born term in expression (6) by the full series, which is taken into account in expression (1). In figure (1) we present the results for $F_2$ as a function $x$, and in figure (2), as a function of $Q^2$. As we can see, the shadowing corrections are important only for very small values of $x$ and moderate values of $Q^2$. We would like to recall that in Eq. (1) we put the upper limit of integration equal to $1/Q^2_0$, or in other words we consider only the SC which are originated from sufficiently short distance, namely, $r_\perp \leq 1/Q_0 \approx 0.35 fm$. In fact, we do not take into account the SC at large distances considering that they have been included in the initial parton distribution of Eq. (4).

Therefore, we are calculating only perturbative shadowing. In the kinematic region of present data, the corrections lie within the experimental error.

We plot also in figures (3) and (4) the SC for $F_2$ predicted by the Glauber approach taking into account the leading order (LO) and next to the leading order (NLO) gluon. In both cases, we have used the modified Mueller formula discussed in Ref.

In this formula, the Born term is taken in leading $\alpha_S\ln Q^2$ approximation (LLA($Q^2$)), while the correction term is taken in DLA. For practical purpose, we use the structure function $F_2$, solution of DGLAP equations, as the Born term in expression (6). We have taken only the GRV distribution since those distributions evolve from small virtualities and can be compared with our DLA approach. We see from the figures that the LO gluon predicts much more SC to $F_2$. It means that the scaling violation suffers a stronger modification for the LO gluon when compared to the simple DLA gluon and to the NLO gluon.

The Glauber - Mueller approach cannot be considered as a full description of the SC, because it was assumed that only quark - antiquark pair embodies a multi rescatterings with the target. As was shown in Refs. [3] [4] [13] the gluon rescatterings turn out to be more essential. To demonstrate this fact we calculate here the Born term of Eq. (1) but given by expression:

$$F_2 = F_2(x, Q^2_0) + \frac{2}{9\pi} \int_{Q^2_0}^{Q^2} \frac{dQ^2}{Q^2} \alpha_S(Q^2) x G^{GM}(x, Q^2)$$

(7)

where $xG^{GM}$ is the gluon structure function calculated in the Glauber - Mueller approach, namely

$$xG^{GM}(x, Q^2) = \frac{2}{\pi^2} \int_{Q^2_0}^{Q^2} \frac{dr^2_\perp}{r^4_\perp} \int_{x}^{1} \frac{dx'}{x'} \int_{0}^{\infty} db^2_\perp \left\{ 1 - e^{-\Omega_G(x', r^2_\perp, b_\perp)} \right\}$$

(8)

where the opacity $\Omega_G = \frac{4}{3}\Omega$. Expression (8) is the Mueller formula which was discussed in detail in Ref. [4]. When Eq. (8) is included in expression (7), the Born term reproduces...
Eq. (4), since the Born term is the DGLAP equation in the DLA limit. The other terms take into account the shadowing corrections to the gluon distribution. The results are shown in figure (5). Comparing figures (2) and (5), one can see that the SC due to gluon rescattering is bigger than the corrections due to quark rescattering.

In order to complete our discussion, we plot in figure (6) the DLA gluon distribution given by expression (3) and the corrected gluon distribution given by the modified Mueller formula (8). The LO and NLO gluon distribution given by the parameterization GRV95 are plotted also. As we can see, the DLA distribution predicts an amount of gluons closer to the NLO DGLAP evolution. It is not a coincidence, since the NLO GRV distribution has a flat behaviour for $Q^2 = 0.4 \text{GeV}^2$ while the LO gluon distribution has already a steep behaviour in the small $x$ region for this low value of $Q^2$.

Fig. (6) shows the main conclusion of this letter: the SC turns out to be big (about 40% - 50%) in the gluon structure function but their manifestation in $F_2$ is rather small as we have discussed (see Fig. (5)). Comparing also figures (2), (4) and (5) we can see that the SC for $F_2$ have a strong dependence on the amount of gluons taken into account in the QCD evolution. This conclusion calls for new measurements in the high energy kinematic region more sensitive to the value of the gluon structure function than the measurements of $F_2$.

ALAF2 acknowledges CAPES and MBGD acknowledges CNPq for partial financing. EML thanks E. Gotsman and U. Maor for everyday discussions on the subject.

References

[1] A.H. Mueller: *Nucl. Phys.* B335 (1990) 115.

[2] A.L.Ayala, M.B.Gay Ducati and E.M. Levin: *Phys. Lett.* B388 (1996) 188.

[3] A.L.Ayala, M.B.Gay Ducati and E.M. Levin: *Nucl. Phys.* B493 (1997) 305.

[4] A.L.Ayala, M.B.Gay Ducati and E.M. Levin: *Parton Densities in a nucleon.* hep-ph/9706448.

[5] E.M. Levin and M.G. Ryskin: *Sov. J. Nucl. Phys.* 45 (1987) 150; B. Blättel, G. Baym, L.L Frankfurt, M. Strikman: *Phys. Rev. Lett.* 70 (1993) 896.

[6] V.N. Gribov and L.N. Lipatov: *Sov. J. Nucl. Phys.* 15 (1972) 438; L.N. Lipatov: *Yad. Fiz.* 20 (1974) 181; G. Altarelli and G. Parisi: *Nucl. Phys.* B 126 (1977) 298; Yu.L. Dokshitzer: *Sov. Phys. JETP* 46 (1977) 641.
[7] R.D. Ball and S. Forte, *Phys. Lett.* B335 (1994) 77.

[8] Z. Huang, H. Jung Lu and I. Sarcevic: hep-ph/9705250 (1997).

[9] A. Donnachie and P.V. Landshoff: *Phys. Lett.* B244 (1984) 322, *Z. Phys.* C61 (1994) 139.

[10] M. Glück, E. Reya and A. Vogt: *Z. Phys* C67 (1995) 433.

[11] H1 collaboration, S. Aid et al.: *Nucl. Phys.* B470 (1996) 3; C. Adloff et al.: *Nucl. Phys.* B497 (1997) 3.

[12] ZEUS collaboration, M. Derrick et al.: *Z. Phys.* C 69 (1996) 607; M. Derrick et al.: *Z. Phys.* C 69 (1996) 399.

[13] L. V. Gribov, E. M. Levin and M. G. Ryskin: *Phys.Rep.* 100 (1983) 1.
Figure 1: The structure function $F_2$ evolution from the scaling violation mechanism in DLA approximation as a function of $x$. The solid line represents the Born term and the dashed line includes the shadowing corrections (SC).
Figure 2: $F_2$ evolution from the scaling violation mechanism in DLA as a function of $Q^2$ (for the scaling violation figures, the value of $i$ goes from 0 for $x = 2.0 \times 10^{-3}$ to 7 for $x = 8.0 \times 10^{-5}$).
Figure 3: The structure function $F_2$ evolution in LO and NLO as a function of $x$. The Born term (DGLAP evolution) for $F_2$ NLO and LO numerically coincide for $Q^2 > 10 \text{GeV}^2$.

Figure 4: $F_2$ evolution from the scaling violation mechanism in LO and NLO as a function of $Q^2$. 
Figure 5: $F_2$ evolution from the scaling violation mechanism in DLA with the gluon distribution $xG^{GM}$ given by the Glauber - Mueller approach.

Figure 6: The gluon distributions in DLA with and without SC - calculated from the Glauber - Mueller approach, compared with LO and NLO gluon distribution from GRV95 set.
\[ F_2 + i^0.5 \]

\[ \log_{10}(Q^2/\text{GeV}^2) \]
$F_2^{i+1}$

$Q^2 = 1.5 \text{ GeV}^2$

$Q^2 = 2.0 \text{ GeV}^2$

$Q^2 = 3.5 \text{ GeV}^2$

$Q^2 = 8.5 \text{ GeV}^2$

$Q^2 = 15 \text{ GeV}^2$

$Q^2 = 20 \text{ GeV}^2$

$Q^2 = 35 \text{ GeV}^2$

$Q^2 = 60 \text{ GeV}^2$

$Q^2 = 90 \text{ GeV}^2$

$F_2$ DGLAP NLO

$F_2$ SC NLO

$F_2$ DGLAP LO

$F_2$ SC LO
The figure shows a plot of $F_2 + i^* 1$ versus $x$ for various values of $Q^2$ and $i$. The graph includes data points for $F_2$ H1 and fitted curves for $F_2$ DLA and $F_2$ SC DLA. The regions covered by $Q^2$ are marked with different colors and line styles. The $x$-axis is logarithmic, spanning from $10^{-4}$ to $10^0$. The $y$-axis shows values ranging from 0 to 7.0.
$Q^2 = 3.5 \text{ GeV}^2$
