The Gravitino-Overproduction Problem in Inflaton Decay

Masahiro Kawasaki*, Fuminobu Takahashi† and T. T. Yanagida**‡

Abstract. We show that the gravitino-overproduction problem is prevalent among inflation models in supergravity. An inflaton field generically acquires (effective) non-vanishing auxiliary field, if the Kähler potential is non-minimal. The inflaton field then decays into a pair of the gravitinos, thereby severely constraining many of the inflation models especially in the case of the gravity-mediated SUSY breaking.

Keywords: Inflation, Gravitino, Supergravity
PACS: 98.80.Cq 11.30.Pb 04.65.+e

INTRODUCTION

The gravitino is the most important prediction of unified theory of quantum mechanics and general relativity such as the superstring theory (i.e. supergravity (SUGRA) at low energies). However, the presence of the gravitino leads to serious cosmological problems depending on its mass and nature [1].

In a recent article [2], we have first pointed out that there is a new gravitino problem beside due to the thermal production of the gravitino (see also [3, 4, 5, 6, 7, 8] for the related topics). That is, an inflaton field $\phi$ has an effective nonvanishing supersymmetry(SUSY)-breaking auxiliary field $G_\phi^{(eff)}$ in most of inflation models in SUGRA, if the Kähler potential is non-minimal. This gives rise to an enhanced decay of the inflaton into a pair of gravitinos. Thus, we have stringent constraints on the auxiliary field $G_\phi^{(eff)}$ to suppress the production of gravitinos in the inflaton decay [2]. This gravitino production in inflaton decay is more effective for lower reheating temperature, while the production by particle scatterings in the thermal bath is more important for higher temperature. Therefore, the direct gravitino production discussed in this letter is complementary to the thermal gravitino production, and the former may put severe constraints on inflation models together with the latter.
INFLATON DECAY INTO A PAIR OF GRAVITINOS

The relevant interactions for the decay of an inflaton field $\phi$ into a pair of the gravitinos are [9]

$$e^{-1} \mathcal{L} = -\frac{1}{8} e^{\mu \nu \rho \sigma} (G_{\phi} \partial_{\rho} \phi + G_{z} \partial_{\rho} z - \text{h.c.}) \bar{\psi}_{\mu} \gamma_{\nu} \psi_{\sigma}$$

$$-\frac{1}{8} e^{G/2} (G_{\phi} \phi + G_{z} z + \text{h.c.}) \bar{\psi}_{\mu} [\gamma^{\mu}, \gamma^{\nu}] \psi_{\nu},$$

(1)

where $\psi_{\mu}$ is the gravitino field, and we have chosen the unitary gauge in the Einstein frame with the Planck units, $M_{P} = 1$. We have defined the total Kähler potential, $G = K + \ln |W|^2$, where $K$ and $W$ are the Kähler potential and superpotential, respectively. The SUSY breaking field $z$ is such that it sets the cosmological constant to be zero, i.e., $G^2 G_{z} \simeq 3$.

The effective coupling of the inflaton with the gravitinos is modified by the mixing between $\phi$ and $z$ [5]. According to the detailed calculation of Ref. [6], we only have to replace $G_{\phi}$ with $\mathcal{G}_{\phi}^{(\text{eff})}$ defined by [1]

$$\mathcal{G}_{\phi}^{(\text{eff})} \equiv \sqrt{3} g_{\phi z z} \frac{m_{3/2}^2}{m_{\phi}},$$

(2)

where $m_{\phi}$ is the inflaton mass. The real and imaginary components of the inflaton field have the same decay rate at the leading order [3]:

$$\Gamma_{3/2} \equiv \Gamma(\phi \to 2 \psi_{3/2}) \simeq \frac{|\mathcal{G}_{\phi}^{(\text{eff})}|^2 m_{\phi}^5}{288 \pi m_{3/2}^2 M_{P}^2}.$$

(3)

Thus the decay rate is enhanced by the gravitino mass in the denominator, which comes from the longitudinal component of the gravitino.

CONSTRAINTS ON INFLATION MODELS

The reheating temperature $T_{R}$ is related to the total decay rate of the inflaton $\Gamma_{\text{tot}}$ by

$$\Gamma_{\text{tot}} \simeq \left( \frac{\pi^2 g_{*}}{10} \right)^{\frac{1}{2}} \frac{T_{R}^2}{M_{P}},$$

(4)

where $g_{*}$ counts the relativistic degrees of freedom and hereafter we set $g_{*} = 228.75$. In the following we assume that the reheating temperature satisfies the bounds from the thermally produced gravitinos [10]. The gravitino-to-entropy ratio is given by [3]

1. There are other contributions to $\mathcal{G}_{\phi}^{(\text{eff})}$ as shown in Ref. [6], which may the problem even worse.
2. We assume $\Gamma_{3/2} \ll \Gamma_{\text{tot}}$, since the standard cosmology would be upset otherwise.
\[ Y_{3/2} \simeq 4.5 \times 10^5 |\mathcal{G}_\Phi^{(\text{eff})}|^2 \left( \frac{m_{3/2}}{1 \text{ TeV}} \right)^{-2} \left( \frac{m_\phi}{10^{10} \text{ GeV}} \right)^4 \left( \frac{T_R}{10^6 \text{ GeV}} \right)^{-1}, \quad (5) \]

where we have neglected the gravitino production from the thermal scattering.

To be concrete let us consider unstable gravitinos with \( m_{3/2} \simeq 1 \text{ TeV} \). The gravitino abundance is then severely constrained by BBN \([10]\). We can derive the constraints on \( \mathcal{G}_\Phi^{(\text{eff})} \) as: \([2]\)

\[ |\mathcal{G}_\Phi^{(\text{eff})}| \lesssim 1 \times 10^{-11} \left( \frac{m_\phi}{10^{10} \text{ GeV}} \right)^{-2} \quad \text{for} \quad m_{3/2} \simeq 1 \text{ TeV} \quad (6) \]

for the hadronic branching ratio \( B_h \simeq 1 \), and

\[ |\mathcal{G}_\Phi^{(\text{eff})}| \lesssim 6 \times 10^{-9} \left( \frac{m_\phi}{10^{10} \text{ GeV}} \right)^{-2} \quad \text{for} \quad m_{3/2} \simeq 1 \text{ TeV} \quad (7) \]

for \( B_h \simeq 10^{-3} \).

In Fig. 1 we show the upper bounds on \( \mathcal{G}_\Phi^{(\text{eff})} \) together with predictions of new, hybrid, smooth hybrid, and chaotic inflation models for \( m_{3/2} = 1 \text{ TeV} \), where we assume that \( g_{zz} = \kappa \langle \phi \rangle \) arises from the non-minimal coupling \( K = \kappa/2|\phi|^2(zz + z^*z^*) \). Note that such couplings are expected to exist with coefficients of order unity if \( z \) is a singlet as required in the gravity-mediated SUSY breaking. The bounds are slightly relaxed for either (much) heavier or lighter gravitino mass. The smooth hybrid inflation is excluded unless \( \kappa \) is highly suppressed. Similarly, for \( \kappa \sim O(1) \), a significant fraction of the parameter space in the hybrid inflation model is excluded, while the new inflation is on the verge of. Even though the constraints on the hybrid inflation model seems to be relaxed for smaller \( m_\phi \), it is then somewhat disfavored by WMAP three year data \([11]\) since the predicted spectral index approaches to unity. The chaotic inflation model is also excluded unless \( \kappa \) is suppressed due to some symmetry (e.g. \( Z_2 \) symmetry).

**CONCLUSION**

In this paper we have shown that an inflation model generically leads to the gravitino overproduction, which can jeopardize the successful standard cosmology. We have explicitly calculated the gravitino abundance for several inflation models. The new inflation is on the verge of being excluded, while the (smooth) hybrid inflation model is excluded if \( \kappa = O(1) \). To put it differently, the coefficient of the non-minimal coupling in the Kähler potential, \( \kappa \), must be suppressed especially in (smooth) the hybrid inflation model. Therefore those inflation models required to have \( \kappa \ll 1 \) involve severe fine-tunings on the non-renormalizable interactions with the SUSY breaking field, which makes either the inflation models or the SUSY breaking models containing the singlet \( z \) (with \( G_z = O(1) \)) strongly disfavored. One of the most attractive ways to get around this new gravitino problem is to postulate a symmetry of the inflaton, which is preserved at the vacuum, to forbid the mixing with the SUSY breaking field. Among the
Upper bound on the effective auxiliary field of the inflaton $G_{\Phi}^{(\text{eff})}$ as a function of the inflaton mass $m_\phi$, for $m_{3/2} = 1 \text{ TeV}$. $T_R$ is set to be the largest allowed value, and the bound becomes severer for lower $T_R$. The typical values of $G_{\Phi}^{(\text{eff})}$ and $m_\phi$ for the single(multi)-field new, hybrid, smooth hybrid, and chaotic inflation models with $\kappa = 1$ are also shown. The chaotic inflation can avoid this bound by assuming $\mathbb{Z}_2$ symmetry. The solid and dashed lines are for the hadronic branching ratio $B_h = 1$ and $10^{-3}$, respectively.

known models, such a chaotic inflation model can avoid the potential gravitino overproduction problem by assuming $\mathbb{Z}_2$ symmetry. Another is to assign some symmetry on the SUSY breaking field $z$ as in the gauge-mediated [12] and anomaly-mediated [13] SUSY breaking models.

**ACKNOWLEDGMENTS**

F.T. is grateful to Motoi Endo and Koichi Hamaguchi for a fruitful discussion, and thanks Q. Shafi for useful communication on the hybrid inflation model.

**REFERENCES**

1. S. Weinberg, Phys. Rev. Lett. 48, 1303 (1982).
2. M. Kawasaki, F. Takahashi and T. T. Yanagida, Phys. Lett. B 638, 8 (2006); Phys. Rev. D 74, 043519 (2006).
3. M. Endo, K. Hamaguchi and F. Takahashi, Phys. Rev. Lett. 96, 211301 (2006); S. Nakamura and M. Yamaguchi, Phys. Lett. B 638, 389 (2006).
4. T. Asaka, S. Nakamura and M. Yamaguchi, Phys. Rev. D 74, 023520 (2006).
5. M. Dine, R. Kitano, A. Morisse and Y. Shirman, Phys. Rev. D 73, 123518 (2006).
6. M. Endo, K. Hamaguchi and F. Takahashi, Phys. Rev. D 74, 023531 (2006).
7. M. Endo and F. Takahashi, Phys. Rev. D 74, 063502 (2006).
8. M. Endo, M. Kawasaki, F. Takahashi and T. T. Yanagida, Phys. Lett. B 642, 518 (2006); M. Endo, F. Takahashi and T. T. Yanagida, arXiv:hep-ph/0611055.
9. J. Wess and J. Bagger, Supersymmetry and Supergravity, (Princeton University Press, 1992).
10. M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B 625, 7 (2005); Phys. Rev. D 71, 083502 (2005).
11. D. N. Spergel et al., arXiv:astro-ph/0603449.
12. M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D 51 (1995) 1362; M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D 53 (1996) 2658; For a review, see, for example, G. F. Giudice and R. Rattazzi, Phys. Rep. 322 (1999) 419, and references therein.
13. L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999);
   G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP 9812, 027 (1998);
   J. A. Bagger, T. Moroi and E. Poppitz, JHEP 0004, 009 (2000).