Cosmological Symmetry Breaking, Pseudo-scale invariance, Dark Energy and the Standard Model

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Abstract: The energy density of the universe today may be dominated by the vacuum energy of a slowly rolling scalar field. Making a quantum expansion around such a time dependent solution breaks fundamental symmetries of quantum field theory. We call this mechanism cosmological symmetry breaking and argue that it is different from the standard phenomenon of spontaneous symmetry breaking. We illustrate this with a toy scalar field theory, whose action displays a U(1) symmetry. We identify a symmetry, called pseudo-scale invariance, which sets the cosmological constant exactly equal to zero, both in classical and quantum theory. This symmetry is also broken cosmologically and leads to a nonzero vacuum or dark energy. The slow roll condition along with the observed value of dark energy leads to a value of the background scalar field of the order of Planck mass. We also consider a U(1) gauge symmetry model. Cosmological symmetry breaking, in this case, leads to a non zero mass for the vector field. We also show that a cosmologically broken pseudo-scale invariance can generate a wide range of masses.

1 Introduction

The current cosmological observations [1 2 3 4 5 6] suggest that the energy density of the universe gets a significant contribution from vacuum energy. For a review see [7 8 9]. This may be modelled by simply introducing a
cosmological constant \[7, 8, 10, 11, 12, 13\] or dynamically by a scalar field slowly rolling towards the true minimum of the potential \[14\]. If we assume the existence of such a field then it implies that in the current era its lowest energy state is not the true vacuum state of the theory. In order to study the spectrum of this theory one needs to make a quantum expansion around a time dependent field. This has interesting implications for the physics of such models, not explored so far in the literature. In particular since we are expanding around a time dependent field and not the ground state, the resulting physics need not display the symmetries of the original lagrangian. Hence this may provide us with another method of breaking fundamental symmetries. We point out that in general a theory displays the symmetries of the action provided the ground state of the theory is symmetric under the corresponding transformations. In the present case, however, the ground state is irrelevant since the field never reaches this state. The time dependent solution, around which we are required to make the quantum expansion, need not display the symmetries of the original action. Hence the theory may display broken symmetries over a large period in the life time of the universe.

2 Symmetry Breaking for a Scalar Field

We consider a simple model of a complex scalar field with a global U(1) symmetry

\[
\mathcal{L}[\Phi(x)] = \partial_\mu \Phi^*(x) \partial^\mu \Phi(x) - m^2 \Phi^*(x)\Phi(x) - \lambda (\Phi^*(x)\Phi(x))^2.
\]  

(1)

This Lagrangian has global U(1) symmetry, i.e., it is invariant under \(\Phi(x) \rightarrow e^{i\theta} \Phi(x)\). So far we have neglected the effect of the background metric which will also be included later. We assume that the potential is sufficiently gentle,
with mass parameter sufficiently small, such that the scalar field is slowly rolling towards its true minimum. Let $\eta(t)$ be the classical solution to the equations of motion. Here we assume that this solution is independent of space and is in general complex.

We split $\Phi(x)$ into two parts: $\Phi(x) = \eta(x) + \phi(x)$. Here $\eta(x)$ is the classical solution of the equation of motion. Let us assume the classical solution depends only on time but is in general complex, i.e, $\eta(x) = \eta(t)$.

Hence $\Phi(x) = \eta(t) + \phi(x)$ and $\Phi^*(x) = \eta^*(t) + \phi^*(x)$.

$$
\mathcal{L}[\eta, \phi] = \dot{\eta}^* \dot{\eta} + \dot{\eta}^* \dot{\phi} + \ddot{\eta} \phi + \partial_\mu \phi^* \partial^\mu \phi - m^2 (\phi^* \phi + \eta^* \eta + \phi^* + \eta^*) - \lambda \{(\phi^* \phi)^2 + (\eta^* \eta)^2 + (\phi \eta^*)^2 + 2(\eta^* \phi + \phi^* \eta)\phi^* \phi + 4\eta^* \eta \phi^* \phi + 2\eta \eta^* (\eta^* \phi + \phi^* \eta^*) \}. \quad (2)
$$

From this we identify the classical Lagrangian.

$$
\mathcal{L}_{\text{Classical}}[\eta(t)] = \dot{\eta}^* \dot{\eta} - m^2 \eta^* \eta - \lambda (\eta^* \eta)^2. \quad (3)
$$

The classical field $\eta$ satisfies the equation of motion,

$$
\ddot{\eta} + \eta(m^2 + 2\lambda \eta^* \eta) = 0. \quad (4)
$$

If we assume that the quantum fluctuations die sufficiently fast at $t = \pm \infty$, then we can write

$$
\int d^4 x \  \dot{\eta}(\dot{\phi} + \dot{\phi}^*) = - \int d^4 x \  \ddot{\eta}(\phi + \phi^*) = \int d^4 x \ \eta(m^2 + 2\lambda \eta^2)(\phi + \phi^*).
$$

Hence we get,

$$
\mathcal{L} = \mathcal{L}_{\text{Classical}}[\eta(t)] + \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda \{(\phi^* \phi)^2 + (\eta^* \phi)^2 + 2\phi^* \phi (\eta^* \phi + \phi^* \eta) + 4\eta^* \eta \phi^* \phi \}. \quad (5)
$$
Let \( \eta = \eta_0 e^{i\theta_0} \) and \( \phi(x) = (\phi_1 + i\phi_2)/\sqrt{2} \). This gives

\[
\mathcal{L} = \mathcal{L}_{\text{Classical}}[\eta(t)] + \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 + \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 - \frac{m_2^2}{2} (\phi_1^2 + \phi_2^2) - \frac{\lambda \eta_0^2}{4} [2(\phi_1^2 - \phi_2^2) \cos 2\theta_0 + 4\phi_1 \phi_2 \sin 2\theta_0 + 4(\phi_1^2 + \phi_2^2)] - \frac{\lambda}{4} [(\phi_1^2 + \phi_2^2)^2 + 4\eta_0 (\phi_1^2 + \phi_2^2)(\phi_1 \cos \theta_0 + \phi_2 \sin \theta_0)].
\] (6)

It is clear that the two modes do not have the same mass. For a complex classical solution, i.e. with \( \theta_0 \neq 0 \), the two modes are coupled. They can be decoupled by the rotation in internal space

\[
\begin{pmatrix}
    \phi_1 \\
    \phi_2
\end{pmatrix} =
\begin{pmatrix}
    \cos \beta & -\sin \beta \\
    \sin \beta & \cos \beta
\end{pmatrix}
\begin{pmatrix}
    \phi_1' \\
    \phi_2'
\end{pmatrix}.
\] (7)

We assume adiabaticity and hence we ignore the time dependence of rotation matrix in the kinetic energy term. The rotation angle \( \beta \) is found to be equal to \( \theta_0 \) and the two mass eigenvalues are found to be \( m_2^2 + 6\lambda \eta_0^2 \) and \( m_2^2 + 2\lambda \eta_0^2 \). Hence the two modes pick up different masses and the symmetry is broken. We do not find a zero mode, in contrast to the case of spontaneous symmetry breaking. We call this phenomenon Cosmological Symmetry Breaking.

3 Gravitational Background

We next consider the complex scalar field model in a background gravitational field, which would be considered classically. The action may be written as

\[
S = \int d^4 x \sqrt{-g} \left[ g^{\alpha\beta} \partial_\alpha \Phi^*(x) \partial_\beta \Phi(x) - m_2^2 \Phi^*(x) \Phi(x) - \lambda (\Phi^*(x) \Phi(x))^2 \right].
\] (8)

We assume the FRW background metric with the Hubble parameter \( H(t) \) and the expansion factor \( R(t) \). This model also displays the global U(1) symmetry \( \Phi(x) \rightarrow e^{i\theta} \Phi(x) \). We again write \( \Phi(x) = \eta(t) + \phi(x) \), where \( \eta(t) \) is
a space independent solution to the classical equations of motion. The real
and imaginary parts $\eta_1$ and $\eta_2$ satisfy

$$\frac{d^2 \eta_i}{dt^2} + 3H \frac{d\eta_i}{dt} + \frac{\partial V}{\partial \eta_i} = 0$$

for $i = 1, 2$. Here the potential $V(\eta) = m^2 \eta^* \eta + \lambda (\eta^* \eta)^2$. The model again
displays symmetry breaking as long as $\eta(t)$ is different from zero. This is
allowed as long as the conditions for slow roll is satisfied. The Hubble pa-
rameter is determined by the entire matter content of the universe and here
we shall consider it as an independent function of $t$. We shall assume it to
be approximately constant as is the case for a vacuum dominated universe.
The slow roll condition is satisfied if the mass parameter $m << 3H$. We
may expand the potential around the classical solution and find that the
mass spectrum is same as found in the earlier section. We, therefore, find
a squared mass splitting $4\lambda \eta(t)^2$, whose scale is determined by the Hubble
parameter. Since the splitting is determined by the value of the Hubble pa-
rameter, its value in the current era is quite small. However the splitting
need not be small in the early universe. At that time it may lead to large
observable consequences.

Although the mass splitting is very small, the value of the field $\eta(t)$ can
be large. This depends on our choice of the coupling $\lambda$. By choosing $\lambda$
sufficiently small we can make $\eta(t)$ arbitrarily large and still maintain slow
roll conditions. The non-zero value of this field can lead to a wide range of
breakdown of symmetries, including Lorentz invariance. Lorentz invariance
is broken because the classical field only has time dependence. Furthermore
if we gauge the U(1) invariance then the gauge symmetry will be broken.
The mass of the gauge field in this case depends on $\eta(t)$ and can be quite
large.
4 Dark Energy

So far we have considered a slowly rolling complex scalar field and shown that it leads to breakdown of symmetries of the original lagrangian. We next consider the possibility that the complex scalar field itself leads to dark energy. We determine the range of allowed values for the background scalar field in order that it leads to vacuum energy equal to the observed dark energy density. For this purpose we consider the equation of motion in gravitational background in terms of the two real fields $\eta_1$ and $\eta_2$. For orders of magnitude estimate we may assume $\eta_1 \sim \eta_2$. The analysis is easily modified if this is not the case and does not lead to any essential difference. For slow roll, the second derivative term is negligible. We may consider two separate cases where either mass term or the quartic coupling term in the potential dominates. If the mass term dominates then the slow roll condition is satisfied if $m^2 \ll 9H^2$. If the quartic coupling term dominates then we find the condition $\lambda \ll H^2/\eta^2$.

We next require that $\rho_V = V(\eta)$. In both the cases this leads to the condition,

$$\eta >> \rho_V^{1/2}/H \approx \sqrt{\frac{3}{8\pi}} M_{\text{PL}}$$

where $M_{\text{PL}}$ is the Planck mass. Here we have used the fact that the vacuum energy density is almost equal to the critical energy density. Hence we find that in our model the value of the slow roll scalar field has to be of the order of the Planck mass or higher.

We point out that the value of the coupling constant $\lambda$ turns out to be extremely small in our model for dark energy. This by itself need not lead to a fine tuning problem since we are free to choose a value for this parameter. Indeed a parameter as small as this is expected due to the widely different scales of Planck mass and the Hubble constant. The fine tuning problem may arise if at higher orders we need to adjust this parameter to very high
accuracy. This may happen if it undergoes large quantum corrections. This is an important check of the theory which we shall address in detail in a future publication.

5 Gauge Theory

We next gauge the U(1) symmetry considered in the earlier sections. The resulting Lagrangian, in the presence of background gravity, can be written as

\[ S = \int d^4x \sqrt{-g} \left[ g^{\alpha\beta} (D_\alpha \Phi(x))^\ast D_\beta \Phi(x) - \frac{1}{4} g^{\alpha\beta} g^{\kappa\delta} F_{\alpha\kappa} F_{\beta\delta} - m^2 \Phi^\ast(x) \Phi(x) \right. \]

\[ \left. - \lambda (\Phi^\ast(x) \Phi(x))^2\right] \tag{11} \]

where \( D_\alpha = \partial_\alpha - ig A_\alpha \) is the covariant derivative and \( F_{\alpha\beta} \) the field strength tensor of the U(1) gauge field \( A_\alpha \). This Lagrangian is invariant under \( \Phi(x) \rightarrow e^{i\theta(x)} \Phi(x) \). We can parameterize \( \Phi(x) \) by:

\[ \Phi(x) = \left( \eta_0 + \rho/\sqrt{2} \right) \exp \left[ i \left( \theta_0 + \frac{\sigma}{\sqrt{2}\eta_0} \right) \right] \]

\[ = \left( \eta_0 + \frac{\rho}{\sqrt{2}} + i \frac{\sigma}{\sqrt{2}} + \ldots \right) e^{i\theta_0}. \]

Hence, for small oscillations \( \rho(x) \) and \( \sigma(x) \) are \( \phi_1'(x) \) and \( \phi_2'(x) \) respectively. We define new fields,

\[ \Phi' = \exp \left[ -i \left( \theta_0 + \frac{\sigma}{\sqrt{2}\eta_0} \right) \right] \Phi = \eta_0 + \rho/\sqrt{2}, \]

\[ B_\mu = A_\mu - \frac{1}{g} \partial_\mu \left( \theta_0 + \frac{\sigma}{\sqrt{2}\eta_0} \right). \]

If we neglect the gravitational field, the Lagrangian becomes:

\[ \mathcal{L} = (D_\mu \Phi(x))^\ast (D^\mu \Phi(x)) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - m^2 (\eta_0 + \rho/\sqrt{2})^2 - \lambda (\eta_0 + \rho/\sqrt{2})^4. \]
We now follow the same procedure as before. We split the gauge field into two parts - i) the classical gauge field, \( \beta_\mu(t) \) which depends only on time and ii) \( B_\mu(x) \), the quantum field i.e., \( B_\mu(x) = \beta_\mu(t) + B_\mu(x) \). So the classical Lagrangian becomes:

\[
L_{\text{Classical}} = \dot{\eta}_0^2 + g^2 \beta_\mu \beta^\mu \eta_0^2 - m^2 \eta_0^2 - \lambda \eta_0^4 - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}
\]

where \( f_{\mu\nu} = \partial_\mu \beta_\nu - \partial_\nu \beta_\mu \). The equations of motion are:

\[
\begin{align*}
\beta_0 &= 0, \\
\ddot{\beta}_i &= -2g^2 \beta_i \eta_0^2, \\
\ddot{\eta}_0 &= -\eta_0 (g^2 \beta^2 + m^2 + 2\lambda \eta_0^2).
\end{align*}
\]

If we assume \( \beta_\mu = 0 \), then using these equations and dropping the total derivative terms one can rewrite the Lagrangian as:

\[
L = L_{\text{Classical}} + \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + g^2 B^2 \left( \eta_0^2 + \frac{\rho^2}{2} + \sqrt{2} \rho \eta_0 \right)
\]

\[
- (m^2 + 6\lambda \rho^2) \frac{\rho^2}{2} - \frac{\lambda}{4} (\rho^4 + 4\sqrt{2} \rho^3 \eta_0) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}
\]

where \( F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \). The gauge field has acquired a mass, \( m_B = g \eta_0 \).

Hence the gauge invariance is broken in this theory. In the simplest case, discussed in the last section, this mass will be of the order of Planck mass, assuming gauge coupling of order unity and if we require that the scalar field vacuum energy gives dominant contribution to dark energy. However if we do not impose the condition that the vacuum energy associated with the field \( \Phi \) is equal to the observed vacuum energy, then the mass of the gauge field is an independent parameter which can be fixed by a suitable choice of the classical solution. In section 7 below we also provide another generalization of the lagrangian so as to generate a different mass scale.
6 Pseudo-scale Invariance

We have so far introduced a new method of breaking symmetries of a field theory. The procedure is found to naturally lead to dark energy in the form of the vacuum energy of a slowly rolling scalar field. However so far we not addressed the question of why the cosmological constant is so small. The problem is of course well known. Quantum field theory in general produces cosmological constant many orders of magnitude larger than what is observed. In the absence of any symmetry, which may demand absence of cosmological term in the action, this is a serious problem in fundamental physics. In this section we identify a symmetry which eliminates cosmological constant both at classical and quantum level.

We first consider actions which are scale invariant. It is clear that at the classical level scale invariance eliminates all dimensionful parameters from the action, including a cosmological constant term. However in the quantum theory, it is well known that scale invariance is anomalous and hence may generate a cosmological constant. In very interesting papers Cheng [15] and Cheng and Kao [16, 17] have argued that scale transformations can be broken into a general coordinate transformation and what is referred to as the pseudo-scale transformations. Under the pseudo-scale transformations

\[
\begin{align*}
x & \rightarrow \ x \\
\Phi & \rightarrow \Phi / \Lambda \\
g^{\mu\nu} & \rightarrow g^{\mu\nu} / \Lambda^2 \\
A_\mu & \rightarrow A_\mu.
\end{align*}
\]

The matter part of the action is invariant under this transformation. The gravitational action is not invariant but, as explained in Ref. [15, 16], it can
be easily generalized so that it is invariant. One simply replaces $\frac{1}{4\pi G} R \rightarrow \beta \Phi^* \Phi R$

$$17$$

where $G$ is the gravitational constant, $R$ the Ricci scalar and $\beta$ a dimensionless constant. Next assuming a slow role scalar field discussed in earlier sections, with its value of order the Planck mass, we find that the resulting action will have predictions identical to Einstein’s gravity, at leading order. The cosmological constant term is not invariant under pseudo-scale invariance and hence eliminated in the classical action. The theory now has exactly zero cosmological constant as long as the symmetries of the action are not broken.

The pseudo-scale invariance is, however, broken through our cosmological symmetry breaking mechanism, discussed earlier. Hence this mechanism will generate a nonzero cosmological constant, or dark energy. We can directly borrow the results obtained in section 4 with the mass parameter $m$ set to zero. The theory discussed in section 4 now displays pseudo-scale invariance, besides invariance under the $U(1)$ transformations. Both of these symmetries are broken cosmologically and the slow roll condition, along with the value of the observed dark energy density sets the scale of the classical scalar field of the order of Planck mass.

We next consider a regulated action, assuming dimensional regularization. Here we focus only on scalar fields. In this case we consider the scalar field action in $n$ dimensions,

$$S = \int d^n x \sqrt{-g} \left[ g^{\alpha \beta} \partial_\alpha \Phi^* \partial_\beta \Phi - \lambda (\sqrt{-g})^{(n-4)/n} (\Phi^* \Phi)^2 \right].$$

$18$

The action is invariant under the transformation

$$x \rightarrow x$$
\[ \Phi \rightarrow \Phi/\Lambda^{a(n)} \]
\[ g^{\mu\nu} \rightarrow g^{\mu\nu}/\Lambda^{b(n)} \]  
(19)

where \( b(n) = 4a(n)/(n - 2) \) and we may choose \( a(n) \) to be any function of \( n \). This generalizes the pseudo-scale transformations to \( n \) dimensions. The action displays exact symmetry under pseudo-scale transformations in \( n \) dimensions. However the action has general coordinate invariance only in 4 dimensions. In dimensions other than 4 the potential term violates general coordinate invariance. Here we take the point of view that the fundamental quantum theory may obey a more general transformation law rather than general coordinate invariance. We are guided primarily by data and the absence of general coordinate invariance in dimensions other than four will give modified predictions only at very high energy scale of the order of Planck mass. At this scale the theory is so far untested and we cannot rule out our action.

To summarize, we find that we can impose pseudo-scale invariance as an exact symmetry. The symmetry is not anomalous. This symmetry prohibits us to introduce a cosmological constant, both at the classical and quantum level. Hence it may provide an explanation for why the cosmological constant is so small. Alternative approaches to solve the cosmological constant problem are described in Ref. [10, 18, 19, 20, 21, 22, 23, 24, 25].

7 Generating Masses

The basic problem of generating realistic masses of the observed particles, however, still remains in our theory. Pseudo-scale invariance prohibits any mass terms in the action. Hence the standard Higgs mechanism is not applicable and all the Standard Model fields, for example, will remain massless.
The pseudo-scale invariance is of course broken cosmologically and hence one may expect that we may be able to break the standard model gauge symmetry also by a slowly rolling scalar field. However we have to do this such that the mass of the scalar field is sufficiently large and not ruled out experimentally. In the construction so far, the mass of the scalar field has been found to be very small. One possibility is that this scalar boson is eliminated from the spectrum by gauging the pseudo-scale invariance \cite{15,16}. In this case the Higgs boson will be eliminated from the spectrum. An alternate construction, which does not involve gauging the pseudo-scale invariance, is described below.

We next construct a toy model such that, besides generating dark energy, it also breaks another $U(1)$ symmetry with a sufficiently large mass of the scalar field. This is a toy model which can be generalized to construct an acceptable Standard Model of particle physics. We consider a model with two complex scalar fields $\Phi$ and $\Psi$. Here $\Phi$ will be considered as a slowly rolling field which gives rise to dark energy. We construct an action such that it is invariant under the transformation $\Phi \to e^{i\theta}\Phi$ and $\Psi \to e^{i\xi}\Psi$ as well as the pseudo-scale transformations. Here we restrict ourselves to four dimensions.

\[
S = \int d^4x \sqrt{-g} \left[ g^{\alpha\beta} \partial_\alpha \Phi^* (x) \partial_\beta \Phi(x) + g^{\alpha\beta} \partial_\alpha \Psi^* (x) \partial_\beta \Psi(x) - \lambda (\Phi^* (x) \Phi(x))^2 - \lambda_1 (\Psi^* \Psi - \lambda_2 \Phi^* \Phi)^2 \right].
\]

(20)

Here $\lambda_1$ is taken to be of order unity. Its precise value will be fixed by the mass of the $\Psi$ particle. The coupling $\lambda_2 << 1$ and will be fixed by the magnitude of the classical solution of the field $\Psi$, which is eventually determined by the scale of symmetry breaking of the $U(1)$ group of transformation $\Psi \to e^{i\xi}\Psi$.

We again expand these two fields as $\Phi = \eta(t) + \phi$ and $\Psi = \zeta(t) + \psi$,
where $\eta(t)$ and $\zeta(t)$ are the time dependent classical fields and $\phi$ and $\psi$ are the quantum fluctuations. The classical fields satisfy,

$$\frac{d^2 \eta_i}{dt^2} + 3H \frac{d \eta_i}{dt} + \lambda(\eta_1^2 + \eta_2^2)\eta_i + \lambda_1 \lambda_2^2 [\zeta_1^2 + \zeta_2^2 - \lambda_2^2 (\eta_1^2 + \eta_2^2)] \eta_i = 0, \quad (21)$$

$$\frac{d^2 \zeta_i}{dt^2} + 3H \frac{d \zeta_i}{dt} + \lambda_1 [\zeta_1^2 + \zeta_2^2 - \lambda_2^2 (\eta_1^2 + \eta_2^2)] \zeta_i = 0. \quad (22)$$

We consider a slow roll solution such that all the second derivative terms are negligible. We set $\zeta_i = (1 + \delta) \lambda_2 \eta_i$, where $\delta << 1$. We can determine $\delta$ perturbatively by solving the differential equations. As we have seen earlier, slow roll condition requires that $\lambda << H^2/\eta^2$. Here we have an additional term proportional to $\lambda_1$ in the equation of motion for $\eta_i$, eq. [21]. Substituting $\zeta_i = (1 + \delta) \lambda_2 \eta_i$ in eq. [22] we get an estimate of $d\eta_i/dt$. Substituting this in eq. [21] we find

$$\delta \approx \frac{\lambda}{2\lambda_1 \lambda_2^2}. \quad (23)$$

We want to choose $\lambda_2$ such that $\zeta_i$ is of order of the Weinberg Salam symmetry breaking scale. It is clear that in this case $\delta << 1$, which is required for the self consistency of the perturbative solution.

We can now determine the contribution of the term proportional to $\lambda_1$ to the equation for $\eta_1$. We find that it gives a contribution of order $\lambda_2^2 \lambda (\eta_1^2 + \eta_2^2)\eta_i$, which is much smaller compared to leading order term $\lambda(\eta_1^2 + \eta_2^2)\eta_i$. Hence the term proportional to $\lambda_1$ can be treated perturbatively. We also find the $\lambda_1$ term gives a correction of order $(\lambda/\lambda_1)\lambda_2 \phi^4$ to the vacuum energy. Since $\lambda/\lambda_1 << 1$, this correction is negligible. We, therefore, find that the new term in the lagrangian, proportional to $\lambda_1$, can be ignored at the leading order in the equation of motion for $\eta_1$ and also gives negligible correction to the vacuum energy density. Furthermore the complete solution in its presence can be determined by treating this term perturbatively.
Finally we estimate the mass of the $\Psi$ particle. For this we expand the potential in terms of the fields $\phi$ and $\psi$ and collect terms which are second order in these fields. We find

$$V = \lambda(\Phi^*\Phi)^2 + \lambda_1(\Psi^*\Psi - \lambda_2^2\Phi^*\Phi)^2$$

$$= \lambda \left[ \eta_1^2\phi_1^2 + \eta_2^2\phi_2^2 + 2\eta_1\eta_2\phi_1\phi_2 + \frac{1}{2}(\eta_1^2 + \eta_2^2)(\phi_1^2 + \phi_2^2) \right]$$

$$+ \lambda_1\lambda_2^2(\eta_1\phi_1 + \eta_2\phi_2)^2 - \delta\lambda_1\lambda_2^2(\eta_1^2 + \eta_2^2)(\phi_1^2 + \phi_2^2)$$

$$+ \lambda_1[(\zeta_1\psi_1 + \zeta_2\psi_2)^2 + \delta\lambda_2^3(\eta_1^2 + \eta_2^2)(\psi_1^2 + \psi_2^2)$$

$$- 2\lambda_2^4(\zeta_1\psi_1 + \zeta_2\psi_2)(\eta_1\phi_1 + \eta_2\phi_2)] + \ldots$$

(24)

where we have only displayed the quadratic terms in the fluctuations. We now need to diagonalize the mass matrix. The form of these terms suggests that we define the fields

$$\phi_+ = \phi_1 \cos \theta_0 + \phi_2 \sin \theta_0$$

$$\phi_- = -\phi_1 \sin \theta_0 + \phi_2 \cos \theta_0$$

$$\psi_+ = \psi_1 \cos \theta_0 + \psi_2 \sin \theta_0$$

$$\psi_- = -\psi_1 \sin \theta_0 + \psi_2 \cos \theta_0$$

(25)

where $\cos \theta_0 = \eta_1/\sqrt{\eta_1^2 + \eta_2^2}$ and $\sin \theta_0 = \eta_2/\sqrt{\eta_1^2 + \eta_2^2}$. In terms of the rotated fields the potential can be written as

$$V = \left\{ \frac{3}{2}\lambda + (1 - \delta)\lambda_1\lambda_2^4 \right\}\phi_+^2 + \left( \frac{\lambda}{2} - \delta\lambda_1\lambda_2^2 \right)\phi_-^2$$

$$+ \lambda_1(1 + 3\delta)\lambda_2^2\psi_+^2 + \delta\lambda_1\lambda_2^2\psi_-^2$$

$$- 2\lambda_1\lambda_2^3(1 + \delta)\phi_+\psi_+ \left( \eta_1^2 + \eta_2^2 \right).$$

(26)

We find that the states $\phi_+$ and $\psi_+$ mix with one another. The mass matrix can be diagonalized and we find the four states

$$\psi_+ = \psi_+ \cos \theta_1 - \phi_+ \sin \theta_1$$

$$\phi_+ = \psi_+ \sin \theta_1 + \phi_+ \cos \theta_1$$

(27)
ψ_ and φ_ with mass squared eigenvalues, \(2\lambda_1 \lambda_2^2 (\eta_1^2 + \eta_2^2), 3\lambda(\eta_1^2 + \eta_2^2), \lambda(\eta_1^2 + \eta_2^2)\) and \(\lambda(\eta_1^2 + \eta_2^2)\) respectively. The mixing angle \(\theta_1 \ll 1\) is approximately equal to \(\lambda_2\). We find one particle with relatively large mass of order \(2\lambda_1 \lambda_2^2 (\eta_1^2 + \eta_2^2)\). By adjusting \(\lambda_2\) we can choose this to be of order of 100 GeV and hence can model the Higgs particle. The remaining three particles have very small masses. If we gauge one of the U(1) symmetry then one of these particles will be eliminated from the spectrum and will instead give rise to a massive gauge boson. The theory then predicts two very light weakly coupled particles.

In this section we have considered a toy model which illustrates that we can generate any mass scale by cosmological symmetry breaking in a theory with pseudoscale invariance. The precise gauge group used \(U(1) \times U(1)\) is not essential for this purpose and the construction can be easily generalized to the standard model. It is of course important to check that quantum corrections do not lead to acute fine tuning problems. We postpone this necessary check to future research. We point out that an alternative to the construction in this section is to simply gauge the pseudo-scale invariance \[15, 16, 26, 27, 28, 29, 30, 31, 32, 33\]. This eliminates the Higgs boson from the particle spectrum and instead predicts a new vector boson with mass of the order of Planck mass.

8 Conclusions

We have shown that a slowly rolling solution to scalar field theories, leads to breakdown of symmetries of the action. We call this phenomenon cosmological symmetry breaking and show that it is intrinsically different from spontaneous symmetry breaking. We argue that if we impose pseudo-scale
invariance on the action then it sets the cosmological constant to zero both in the classical and the quantum theory. The pseudo-scale invariance is also broken cosmologically, leading to a slowly varying cosmological constant. We further show that cosmologically broken pseudo-scale invariance can lead to a wide range of particle masses and it appears possible to impose this symmetry on the full action of fundamental particle physics.

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**References**

[1] A. G. Riess et al., Astron. J. **116**, 1009 (1998).

[2] P. M. Garnavich et. al., ApJ **509**, 74 (1998).

[3] S. Perlmutter et al., ApJ **517**, 565 (1999).

[4] J. L. Tonry et al., ApJ **594**, 1 (2003).

[5] B. J. Barris et al., ApJ **602**, 571 (2004).

[6] A. G. Riess et al, ApJ **607**, 665 (2004).

[7] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).

[8] T. Padmanabhan, Phys. Rep. **380**, 235 (2003).

[9] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. **D 15**, 1753 (2006).

[10] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[11] S. M. Caroll, W. H. Press and E. L. Turner, Ann. Rev. Astron. Astrophys. **30**, 499 (1992).
[12] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 2000.

[13] J. R. Ellis, Phil. Trans. Roy. Soc. Lond. A 361, 2607 (2003).

[14] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998).

[15] H. Cheng, Phys. Rev. Lett. 61, 2182 (1988).

[16] H. Cheng and W. F. Kao, MIT preprint Print-88-0907 (1988).

[17] W. F. Kao, Phys. Lett. A 154, 1 (1991).

[18] A. Aurilia, H. Nicolai and P.K. Townsend, Nucl. Phys. B 176, 509 (1980).

[19] J. J. Van Der Bij, H. Van Dam and Y. J. Ng, Physica A 116, 307 (1982).

[20] M. Henneaux and C. Teitelboim, Phys. Lett. B 143, 415 (1984).

[21] J. D. Brown and C. Teitelboim, Nucl. Phys. B 297, 787 (1988).

[22] W. Buchmuller and N. Dragon, Phys. Lett. B 223, 313 (1989).

[23] M. Henneaux and C. Teitelboim, Phys. Lett. B 222, 195 (1989).

[24] A. Daughton, J. Louko and R. D. Sorkin, Talk given at 5th Canadian Conference on General Relativity and Relativistic Astrophysics (5CCGRRRA), Waterloo, Canada, 13-15 May 1993, published in Canadian Gen. Rel. 0181, (1993).

[25] D. E. Kaplan and R. Sundrum, JHEP 0607, 042 (2006).

[26] D. Hochberg and G. Plunien, Phys. Rev. D 43, 3358 (1991).
[27] W.R. Wood and G. Papini, Phys. Rev. D 5, 3617 (1992).

[28] J. T. Wheeler, J. Math. Phys. 39, 299 (1998).

[29] A. Feoli, W.R. Wood and G. Papini, J. Math. Phys. 39, 3322 (1998).

[30] M. Pawlowski and Turk. J. Phys. 23, 895 (1999).

[31] H. Nishino and S. Rajpoot, hep-th/0403039.

[32] D. A. Demir, Phys. Lett. B 584, 133 (2004).

[33] R. Foot, A. Kobakhidze and R. R. Volkas, hep-ph/0704.1165.