A search for two body muon decay signals

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Lepton family number violation is tested by searching for $\mu^+ \rightarrow e^+ X^0$ decays among the $5.8 \times 10^8$ positive muon decay events analyzed by the TWIST collaboration. Limits are set on the production of both massless and massive $X^0$ bosons. The large angular acceptance of this experiment allows limits to be placed on anisotropic $\mu^+ \rightarrow e^+ X^0$ decays, which can arise from interactions violating both lepton flavor and parity conservation. Branching ratio limits of order $10^{-5}$ are obtained for boson masses of 10 - 80 MeV/c$^2$ and different asymmetries. For lighter bosons the asymmetry dependence is much stronger and the branching ratio limit varies up to $5.8 \times 10^{-5}$. This is the first study that explicitly evaluates the limits for anisotropic two body muon decays.

I. INTRODUCTION

The conservation of lepton family number, or flavor, in reactions involving charged leptons is a postulate of the standard model (SM). Positive muon decay ($\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$) is an excellent low energy system with which to search for charged lepton flavor violating (CLFV) interactions, such as the decay to a positron and an unknown neutral boson $\mu^+ \rightarrow e^+ X^0$, for the same reasons that it is attractive for weak interaction tests: muons can be produced in large quantities and the decay is observed with very low backgrounds.

Early studies of muon decay rejected a two body final state as the normal decay $1$, an unexpected result at the time. When the final state of the CLFV decay products can be detected, very stringent exclusive limits have been placed on the branching ratio of the decay. This is the case for the detection of $\mu^+ \rightarrow e^+ \gamma$ $2,4$ or $\mu^+ \rightarrow e^+ X^0, X^0 \rightarrow e^+ e^-$ $5$ processes. However, if the neutral $X^0$ boson or its decay products are not detected, only the shape of the positron spectrum is available to set an inclusive limit on the decay process.

Stable, non-interacting $X^0$ bosons have been associated with particles such as axions $6$ and Majorons $7,8$. The $X^0$ boson is massless when there is an associated spontaneously broken global (exact) symmetry $9$, and massive when an approximate symmetry is broken $10$. Both cases are considered in this paper.

Two body kinematics dictate that the positrons in $\mu^+ \rightarrow e^+ X^0$ decay are observable as a narrow peak at a momentum, $p_X$, determined by the mass of the $X^0$ boson:

$$p_e(m_X) = c \sqrt{\left(\frac{m^2_e - m^2_X}{2m_\mu}\right)^2 - m^2_e}$$  \hspace{1cm} (1)

where $m_\mu$ is the mass of the muon, $m_e$ is the mass of the positron, and $m_X$ is the mass of the boson generated by the LFV process.

This signal appears in addition to the three body positive muon decay spectrum:

$$\frac{\partial^2 \Gamma}{\partial x \partial \cos \theta_\mu} \propto \frac{1}{x^2} \left\{ 1 - x + \frac{2}{9} \rho (4x - 3) + \eta x_0 \left(1 - \frac{1}{x}\right) \right. $$

$$\left. + \frac{1}{3} P_\mu \cos \theta_\mu \left(1 - x + \frac{2 \delta}{3} (4x - 3) \right) \right\} + RC$$  \hspace{1cm} (2)

where

$$x = \frac{E_e}{E_{max}}, \quad x_0 = \frac{m_e}{E_{max}}, \quad E_{max} = \frac{m_\mu^2 + m_e^2}{2m_\mu}, \quad \cos \theta_\mu = \frac{\bar{p}_e \cdot \vec{p}_\mu}{| \vec{p}_e | | \vec{p}_\mu |}$$  \hspace{1cm} (3)
$P_\mu = |\vec{p}_\mu|$, and $\vec{p}_\mu$ is the polarization of the muon at the time of decay. $RC$ are the radiative corrections calculated to $O(\alpha^2)$ [11]. The muon decay parameters $\rho$, $\eta$, $\delta$, and $\xi$ are bi-linear combinations of the weak coupling constants [12] which assume values $\rho = \delta = 3/4$, $\xi = 1$, and $\eta = 0$ in the standard model.

The angular decay distribution of the positrons from the $\mu^+ \rightarrow e^+ X^0$ process has the angular dependence

$$\frac{\partial \Gamma}{\partial \cos \theta_\mu} \propto 1 - AP_\mu \cos \theta_\mu$$

We study the cases $A = 0$ (isotropic) and $A = \pm 1$ (maximally anisotropic). The definition of $A$ is motivated by the experimental geometry as described below. Asymmetric two body muon decays are predicted, for example, from Majoron production arising from a spontaneous violation of super-symmetric R-parity [13].

II. THE TWIST EXPERIMENT

The TRIUMF Weak Interaction Symmetry Test (TWIST) has made an order-of-magnitude improvement to the precision of the muon decay parameters $\rho$, $\delta$, and $P_\mu \xi$ [11] [13] [16]. The data, consisting of $1.1 \times 10^{10}$ unbiased stopped muon events resulting in $5.8 \times 10^8$ analyzed decays, is appropriate for the search for the inclusive two-body decay. The experiment used highly polarized muons delivered by the TRIUMF M13 beam line into a parallel plane spectrometer immersed in a uniform 2 Tesla magnetic field. The spectrometer consisted of 44 drift chambers (DCs) and 12 proportional chambers (PCs) arranged symmetrically about a pure metal stopping foil. The stopping targets (75 $\mu$m Al or 30 $\mu$m Ag) also served as the central PC cathode. The design and construction of this detector has been described in detail elsewhere [17]. The spectrometer was oriented so that it had an approximate cylindrical symmetry centered on the muon beamline axis, which is then defined as the $z$-axis of the detector coordinate system. It was constructed so that the position of the detector elements, specifically the position of sense wires, is known with a total precision of parts in $10^5$. The magnetic field was mapped to a similar precision. These factors determine the absolute momentum scale for particle trajectories measured in the detector.

Figure 1 shows our measured distribution of positrons from muon decay binned by their total momentum $p_{\text{tot}} = |\vec{p}|$ and $\cos \theta = p_z / p_{\text{tot}}$. Due to the properties of pion decay at rest, positive muons at the time of pion decay have spins opposite to the muon beam direction, with the consequence that $\cos \theta = -\cos \theta_\mu$ of Eq. 2 and 4. The planar geometry of the spectrometer allows for a large angular acceptance of positrons resulting from decay in the target foil, with a relatively simple momentum calibration. The momentum resolution varies with $p_{\text{tot}}$ and $\theta$; at 52.8 MeV/c the momentum resolution is $(58 \text{ keV/c})/|\sin \theta|$ [11]. Two body decays with $m_X < 13 \text{ MeV/c}^2$, or less than 3 resolution widths from the edge of the momentum spectrum at $\cos \theta = 0.8$, are not clearly distinguishable from massless $X^0$ decays.

Almost all of the physics data collected by the TWIST collaboration during the 2006 and 2007 run periods were used for this two body decay search. These data were subject to a sequence of event selection criteria chosen to minimize the bias of comparisons between data and simulation. A total of $5.4 \times 10^8$ muon decay events were identified after the event selection cuts were applied. The event selection differs from the standard TWIST analysis [14] only through the extension of the momentum acceptance to include $p_{\text{tot}} < 53.0$ MeV/c. The kinematic fiducial region has been superimposed on the representative data spectrum shown in Fig. 1.

III. FITTING PROCEDURE

Within the context of a search for two-body decay signals the simulation of three body muon decay provides both the background for the measurement and a model for the decay signal. The response and acceptance of the detector are modeled using a detailed GEANT 3.21 simulation. A description of the simulation and its use in the TWIST experiment is given in [11] [18]. Simulated muon decay events undergo the same reconstruction as the standard data so reconstruction effects that appear in the data are reproduced in the simulation. An event will be vetoed by the analysis if additional particles are observed in association with the decay positron. Therefore, a neutral particle must persist longer than the 200 ps required for a particle to escape from the detector prior to decaying into charged daughters to be observed in this analysis.

When the lifetime of the $X^0$ particle is longer than $10^{-20}$ s the width of the two body decay resonance will be less than the momentum resolution and the signal
will be dominated by the momentum reconstruction response function. This function varies strongly with momentum and angle and is symmetric between the upstream and downstream spectra. The distribution of positrons from the two body decays was approximated from the difference of the reconstructed and true momentum, $\Delta p = p_{\text{rec}} - p_{\text{true}}$ of the TWIST high statistics muon decay simulations. The $\Delta p$ distribution from these simulations was symmetrized by adding its reflection about $\cos \theta = 0$ to remove the momentum dependent anisotropy of the source distribution. The anisotropy of the symmetric $\Delta p$ distribution can then be adjusted by scaling the distribution with the asymmetry dependence of Eq. [4]. Two-body decay distributions, $F_A(m_X)$, were generated for each of the three cases tested: the case where $A = -1$ with the same anisotropy as the three body decay spectrum, the isotropic case, $A = 0$, and the $A = +1$ case with anisotropy opposite to that of the three body decay spectrum. Effects due to reconstruction inefficiencies, detector granularity, and momentum loss are included in the decay model when it is derived from simulation in this way. The momentum response of positrons with momenta between 30 MeV/c and 45 MeV/c, which was used to define the associated isotropic two body decay distribution, is shown in Fig. 2.

The two body decay distribution is added to the spectrum expansion used by the standard TWIST analysis [11] to obtain the decay positron spectrum $S_F$:

$$S_F = S_M + \frac{\partial S_M}{\partial \rho} \Delta \rho + \frac{\partial S_M}{\partial p_\mu \xi} \Delta P_\mu \xi +$$

$$\frac{\partial S_M}{\partial (P_\mu \xi \delta)} \Delta (P_\mu \xi \delta) + F_A(m_X)B_A(m_X)$$

(5)

where $S_M = \partial^2 \Gamma(\mu^+ \to e^+ \nu_e \bar{\nu}_\mu) / \partial \rho \partial \cos \theta$ is the decay spectrum from simulation as defined by Eq. [2] and $\Delta \rho$, $\Delta P_\mu \xi$, and $\Delta (P_\mu \xi \delta)$ are the fitted differences in the muon decay parameters from those used to simulate $S_M$. $F_A(m_X)$ is normalized to have the same integrated area as the 3-body decay spectrum by construction so $B(m_X) = \Gamma(\mu^+ \to e^+ X^0) / \Gamma(\mu^+ \to e^+ \nu_e \bar{\nu}_\mu)$ is the branching ratio of a two body decay which produces a boson with a mass $m_X$. The associated signal occurs at a momentum, $p_\mu(m_X)$, defined in Eq. [1].

To maximize the sensitivity to a narrow peak the data and simulation are binned more finely than is optimum for the determination of the muon decay parameters. The branching ratio and decay parameters are obtained from a $\chi^2$ fit of the data to Eq. [5]. The fit for the signal amplitude and the muon decay parameters $\rho$, $P_\mu \xi$, and $P_\mu \xi$ is performed for values of $p_\mu(m_X)$ at 0.05 MeV/c intervals between 17.03 MeV/c and 52.83 MeV/c. This choice of interval size was made to limit running time of the algorithm. The value of $\eta$ was fixed to $-0.0036$ [19] in line with the TWIST muon decay parameter analysis [11]. The decay parameters obtained from these fits are consistent with those obtained when the two body decay signal is omitted from the fit at the level of the measured statistical uncertainty, or a part in $10^5$.

IV. VALIDATION OF FITTING METHOD

The fitting procedure was assessed for potential biases by applying the algorithm to the comparison of statistically independent simulations of three body muon decay. A histogram of the measured branching ratios, $B$, divided by their statistical uncertainties is shown in Fig. 3. A Gaussian fit to this distribution produces a mean, $\mu = 0.01 \pm 0.03$, and a standard deviation, $\sigma = 0.98 \pm 0.03$, with a $\chi^2$ of 43 for 54 degrees of freedom, showing no evidence of bias.

A mono-energetic positron peak at 30 MeV/c, with a known amplitude was added to a simulation of three body muon decay to determine if the results of the fit are self consistent. The difference between the altered simulation and the standard simulation integrated over angles is shown in Fig. 4 with the best fit two body decay signal overlaid. The branching ratio of the added signal is $8.1 \times 10^{-4}$; the measured branching ratio is $(7.3 \pm 0.1) \times 10^{-4}$ for this signal. The 10% area difference is the result of the deviation of the most probable momentum of the test signal from the nearest $p_\mu(m_X)$ grid point which is 0.024 MeV from the introduced peak or half the grid width. Further tests of signals at other

![FIG. 2. (color online) The distribution of the momentum loss $\Delta p$ between the momentum of a simulated positron track at the time of decay and its momentum reconstructed from the positron track as a function of $\cos \theta$. This distribution, generated with 30 MeV/c < $p$ < 45 MeV/c is used to model a two body decay signal within that range after applying an offset in momentum.](image-url)
momenta confirm that this is the maximum deviation between the result of the fit and the signal amplitude. The bias created by the choice of the fitting grid motivates a 10% increase in the measured upper limit of the measured branching ratio which is reported in the results that follow.

V. SYSTEMATIC UNCERTAINTIES FOR ENDPOINT FITS

In the standard TWIST analysis, a momentum calibration is performed by matching the endpoint of the data spectrum to that of the simulated spectrum assuming that any differences are linear with respect to $sec \theta$. The motivation for this procedure is to remove small differences in the momentum reconstruction of the positrons passing through the detector in data versus the simulation, consistent with differences of the energy loss on the order of 10 keV/c. The differences between data and simulation at the momentum edge of the spectrum at $E_{max}$ can be characterized by the momentum offset at $|cos \theta| = 3/4$, $\Delta p_{\pm 3/4} = p_{data} - p_{sim}$ where $p_{data}$, $p_{sim}$ are the momenta of the spectrum endpoint reconstructed from data and simulation.

While the full momentum calibration as determined at the endpoint is applied for the fits for massive two body signals, the calibration procedure is not valid when a fit with a two body decay is conducted near the endpoint of the muon decay spectrum as the net effect of the two body peak and the momentum calibration is similar. This is demonstrated in Fig. 5 where the impact of various two body decay signals of various anisotropies on the endpoint muon decay spectrum are shown. Fig. 5(c) explicitly compares a two body decay signal to the effect of a change in the measured energy calibration. In the analysis for light or massless boson production the calibration and its uncertainty is obtained, with less precision, from known differences and uncertainties in the simulation inputs.

The corrections and uncertainties affecting the endpoint are summarized in Table I for signals corresponding to massless $X^0$ production. The offset and uncertainty in the spectrum endpoint at $cos \theta = 3/4$ indicates a magnitude for the associated effect. These are translated to branching ratio uncertainties using sensitivity of two body signals to variations in the momentum calibration. Correlations between the endpoint calibration parameters are included to reflect upstream/downstream relationships for each contribution to the systematic uncertainties.

The uncertainties in the stopping power of the detector materials and the thickness of the muon stopping target produce a leading contribution to $B(m_X)$, as shown in Table II. The momentum loss in the stopping target alters the momentum offsets $\Delta p_{-3/4}$ and $\Delta p_{+3/4}$ by the same amount, with a 100% positive correlation between these parameters. The measured difference in the muon stopping target thickness from the value used in the simulation is $1.4 \pm 0.6 \mu m$ for the silver target and $0.6 \pm 0.5 \mu m$ for the aluminum target. The measurement was a destructive process conducted well after the simulation was programmed and run. Averaging this effect over all data sets yields a contribution of $-0.6 \pm 0.4$ keV/c to the momentum offsets. Further uncertainties in the energy loss are associated with the simulation of the target material which uses values taken from the Berger-Seltzer report [20]. In this case there is a 2% uncertainty in the

![FIG. 3. Branching ratio normalized by the measured uncertainty of the rare decay search applied to two uncorrelated simulations. A fit of the distribution to a Gaussian function is overlaid with a mean, $\mu = 0.01 \pm 0.03$, and a width, $\sigma = 0.98 \pm 0.03$.](image)

![FIG. 4. (color online) The normalized counts of a simulation altered to include an artificial decay signal with the background subtracted (○) accumulated in momentum bins 10 keV/c wide. The peak fit is overlaid (○).](image)
calculated ionization energy loss and a 3% uncertainty in the radiative energy losses. In the detector stack, events with large energy loss components are suppressed by the track fitting procedure, which dissociates the trajectory into multiple instances rather than changing the effective momentum of the fitted helix. As a result, only the ionization energy loss uncertainties are included for those materials.

The differences between the simulated and the true stopping position of the muon introduces anti-correlated contributions to \( \Delta p_{-3/4} \) and \( \Delta p_{+3/4} \). These were estimated to be 1.6 \( \mu \)m in Ag and 3.8 \( \mu \)m in Al. Averaging over all data sets, this produces a change in the offsets of \( \Delta p_{-3/4} = -(+0.9 \pm 1.0) \) keV/c.

The space time relationship (STR) within the drift cell [21], magnetic field, and detector dimension uncertainties all affect the momentum offset and the angular dependence of the endpoint. These systematic uncertainties are independent upstream and downstream. A difference between data and simulation of \( 1.4 \times 10^{-3} \) T in the average magnetic field at the position where it is monitored is predicted from a study of the field mapping systematics [11]. This alters the positron energy scale so that there is a change in the momentum offset of \(-2.8 \pm 1.5\) keV/c at \( \cos \theta = -3/4 \). A fractional uncertainty of \( 5 \times 10^{-5} \) in the detector length scale and thus the position of the wire planes was calculated from the uncertainties of the detector components. The uncertainty due to the STRs was estimated from their difference when the STR for each wire plane is separately determined from the data and when a plane-averaged STR is determined from the simulation. There is negligible evidence of corrections due to the STRs or a mis-calibration of the detector length scale.

The above uncertainties assume a linear dependence of the momentum calibration with respect to \( \cos \theta \). However, the \( \chi^2 \) determined from the fits of the upstream momentum calibration exceed the number of degrees of freedom by a factor of 2, suggesting that the model used to determine the energy calibration is not an ideal model of the angular behavior at the endpoint. In absence of a motivated correction to the model, an additional systematic uncertainty in the values of the momentum offsets of 1.6 keV/c was assessed to account for this effect.

The contributions of each of these systematics to the value of the endpoint offset is shown in Table I. The values of \( \Delta p_{+3/4} =-2.5 \pm 6.1\) keV/c and \( \Delta p_{-3/4} =-4.3 \pm 6.1\) keV/c are consistent at the 1.5\( \sigma \) level with the offset obtained from fitting the endpoints of the data to the simulation [11] that were used in the decay parameter analysis.

The sensitivity of the measured branching ratio to each of the momentum calibration parameters was determined by adding 50 keV/c separately to each parameter and fitting for the \( \mu^+ \rightarrow e^+ X^0 \) branching ratio for all accessible \( m_X \). These combined sensitivities are used to transform momentum calibration parameter uncertainties into an uncertainty in the branching ratio. For example, when \( A=1 \), this procedure shows that the momentum calibration contributes approximately 20 parts per million (ppm) to the branching ratio for every keV/c offset in the momentum at the endpoint so that the systematic uncertainties dominate the total uncertainty. For momenta less than 52 MeV/c this sensitivity decreases to less than 0.2 ppm per keV/c offset meaning the statistical errors dominate for signals from massive \( X^0 \). The same pattern is followed for the other signal asymmetries.

The momentum resolution difference between data and simulation has an upper limit of 3 keV/c based on the comparisons of fits to the endpoint spectra using an error function convolved with a linear approximation of the muon decay spectrum [11]. Momentum resolution differences produce structure in the endpoint region that will alter two body fits at the endpoint. To evaluate the resolution sensitivity, the simulation was smeared on an event-by-event basis by an exaggerated amount and a signal search was conducted on the altered spectrum. The resulting branching ratios were added to the other uncertainties in quadrature to produce the total uncertainties.
TABLE I. Biases and uncertainties introduced to the momentum edge of the positron spectrum by various systematic effects. The endpoint offset is given as the change in the momentum edge at the center of the angular fiducial, \( \cos \theta = -3/4 \). The uncertainties in the offsets corresponding to each of these systematic effects produce the uncertainties in the two body decay branching ratios shown in the right three columns.

\[
\begin{array}{cccc}
\text{Detector Property} & \text{Offset} & \text{Uncertainty in B (in ppm)} \\
& (\text{in keV/c}) & A = -1 & A = 0 & A = +1 \\
\hline
\text{Stopping Distribution} & -0.9\pm1.0 & 17.3 & 0.4 & 1.8 \\
\text{Target Thickness} & -0.6\pm0.4 & 7.6 & 2.5 & 0.8 \\
\text{Field Map Correction} & -2.8\pm1.5 & 6.0 & 3.8 & 0.6 \\
\text{Energy Loss in Target} & 0.0\pm4.7 & 89.8 & 32.2 & 11.3 \\
\text{Detector Length} & 0.0\pm4.3 & 12.9 & 9.3 & 0.9 \\
\text{STRs} & 0.0\pm3.1 & 49.1 & 10.8 & 4.3 \\
\text{Calibration Model} & 0.0\pm1.6 & 21.8 & 8.1 & 2.3 \\
\text{Resolution} & 0.0\pm3.0 & 21.4 & 7.6 & 3.1 \\
\hline
\text{Total} & -4.3\pm6.1 & 107.3 & 36.4 & 12.6 \\
\end{array}
\]

at each trial momentum. The resolution uncertainties obtained at momenta less than 52 MeV/c are consistent with statistical noise as expected. The contribution for massless decays is given in Table [1].

**VI. RESULTS**

The \( B(m_X) \) confidence intervals for \( m_X > 13 \text{ MeV/c}^2 \) are shown in Fig. [a] for the three signal asymmetries. These intervals were defined using the Feldman-Cousins (FC) approach [22] and include both statistical and systematic uncertainties. As expected from the number of \( m_X \) grid points on which the search is conducted, some of these lower limits are non-zero. The significance (p-value) of these B values is assessed by calculating the probability that a peak with the same or greater B/\( \sigma \) will occur at any of the \( m_X \) grid points due to a random fluctuation.

The FC confidence bands for a massless \( X^0 \) signal for the three asymmetry values are shown in Fig. [b]. The vertical black, dotted line indicates the endpoint offset value recorded in Table I. The 90% upper limits for signals at the endpoint shown in Table I were obtained from this estimated calibration of the endpoint.

To study this, the probability distribution function (PDF) for observing a \( \mu^+ \rightarrow e^+X^0 \) signal B/\( \sigma \) given the null hypothesis was obtained by running the two body decay search on 1000 sets of randomized spectra and collecting the most significant signal from each search. The randomized spectra were generated by applying Poisson noise to the data and simulation. The signal amplitudes measured from the randomized spectra less the observed signal amplitude produces a PDF consistent with the null hypothesis. The resulting PDF has an appearance similar to the distribution shown in Fig. [a] and it is used to evaluate the p-values for the most significant peaks. It is consistent with a simpler approach to obtaining these p-values which assumes normally distributed uncertainties.

The branching ratios for massive decays, summarized in Table I, are averaged over all momenta between 17 MeV/c and 52.8 MeV/c, while the p-value is given for the most significant signal observed in the data. The limits quoted at the endpoint are based on the single fit of a \( \mu^+ \rightarrow e^+X^0 \) signal at \( p_X = 52.83 \text{ MeV/c} \), using the momentum calibration calculated from the systematic bias. All of the observed branching ratios are consistent with statistical fluctuations.

The isotropic results can be compared directly to those of Jodidio et al. [24] and Bryman and Clifford [23]. Jo-
TABLE II. The 90% upper limits for $\mu^+ \rightarrow e^+ X^0$ processes which produce positron signals with positive, negative, and no anisotropy. The average of the upper limits of $e^+$ signals produced in the presence of massive $X^0$ particles is shown for all three cases as well as similar limits associated with massless $X^0$ particles. The momentum, 90% upper confidence limits, and p-value of the most significant massive signal is also given. The results of Bryman and Clifford are directly comparable to the case of $\mu^+ \rightarrow e^+ X^0$ decays producing massive bosons with no anisotropy ($A = 0$), while the results of Jodidio are comparable to the production of massless $X^0$ bosons, also assuming $A = 0$.

| Decay Signal | Upper Limit (in ppm) | p-value |
|--------------|----------------------|---------|
| $A = 0$      |                      |         |
| Average      | 9                    | 0.66    |
| Endpoint     | 26                   | 0.81    |
| $A = -1$     |                      |         |
| Average      | 10                   | 0.60    |
| Endpoint     | 26                   | 0.80    |
| $A = +1$     |                      |         |
| Average      | 6                    | 0.59    |
| Endpoint     | 6                    | 0.90    |

Previous Results

|                         | Upper Limit (in ppm) | p-value |
|-------------------------|----------------------|---------|
| Bryman and Clifford [23]| 300                  |         |
| Jodidio et al. [21]     | 2.6                  |         |

The review of results presented by Bryman and Clifford is partially derived from an uncorrelated data set collected by the same apparatus as Jodidio et al. in addition to data collected from a $\pi \rightarrow e\nu$ measurement. It is directly comparable to the average branching ratio for isotropic massive decays.

VII. CONCLUSIONS

No significant evidence for $\mu^+ \rightarrow e^+ X^0$ decays has been found in this search. The limits on these decays for $13 \text{ MeV}/c^2 < m_{X^0} < 80 \text{ MeV}/c^2$, where the $X^0$ decay is not observed, have been improved by a factor of 32 over previously published limits. The dependence of these limits on the decay anisotropy has been studied for the first time.

Due to the systematics associated with the detailed understanding of the decay positron spectrum endpoint, our limits on $\mu^+ \rightarrow e^+ X^0$ processes with $m_{X^0} < 80 \text{ MeV}/c^2$ are much less restrictive. For this range we have reported the first inclusive limit on decays having the same anisotropy as ordinary muon decay, while for other anisotropies the Jodidio et al. measurement is more sensitive.

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