Study on the Calculation Theory of the Cable Curve for Suspension Bridge

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Abstract. In this paper, the main cable curve and calculation theory of self-anchored suspension bridge are studied based on the engineering background of the Egongyan Changjiang Bridge in Chongqing. Two kinds of calculation theories of the main cable curve based on different assumptions are discussed: The parabola method of cable gravity distributing uniformly along the span, and the catenary method of gravity distributing equally along the curve.

1. Preface
The cable curve is an important part of suspension bridge. The load on the main cable consists of two parts. One part is the load distributed along the cable curve, including the cable gravity, winding and coating weight, etc. The other part is through the concentrated force of cable clip transmission, including the weight of cable clamp and the weight of suspender cable and the weight of stiffening girders and bridge deck, etc.

2. Suspension bridge introduction
Suspension bridge refers to a bridge with a cable suspended and anchored on both sides as the main load-bearing member of the superstructure. The cable geometry is determined by the balance condition of the force and is generally close to parabola.

Suspension Bridges are mainly composed of cables, bridge towers, anchors, stiffening girders and suspenders [1]. The main load-bearing component of suspension bridge is suspension cable. It mainly bears tensile force. It is usually made of steel with high tensile strength (steel wire, steel cable, etc.). Because the suspension bridge can make full use of the strength of material and has the characteristics of saving material and light weight, the span capacity of suspension bridge in various system bridges is maximum, and the span can reach more than 1000 meters. The largest span bridge in the world today is the Akashi-Kaikyo Bridge in Japan, with a main span of 1,991 meters [2]. The main drawback of suspension Bridges is that they have less rigidity and are prone to large deformation and vibration under load.

Suspension bridge is one of the main forms of the long-span bridge. At present, most of the bridges with span of more than 1000 meters in the world are suspension bridges. If the carbon fiber material with light weight and strong strength is used as the main cable, its ultimate span can be more than 8000 meters in theory.
3. The significance of theoretical research on the cable curve of suspension bridge
The main cable is the main load bearing member in suspension bridge structure system, the main cable curve is the key control factor for structural design, calculation and guidance of construction, so it is necessary to study the calculation method of the main cable curve [3]. The structural analysis of bridge must be based on the accurate configuration, so the main cable curve directly affects the distribution of forces in the entire structure, and it determines its unstressed cable length, pickup point arrangement and the unstressed length of the suspender, all of which are indispensable in design and construction control.

4. The calculation theory of the cable curve for suspension bridge
The main cable of modern large suspension bridge is usually made of steel wire, and its relative bending rigidity is very small, which can be basically treated as a flexible cable [4]. Generally, we can make the following assumption [5]: (1) the main cable is completely flexible, it has neither pressure nor bending moment, so the influence of the flexural rigidity of its section on the cable curve can be ignored; (2) the total stress of the main cable is within the proportional limit, which conforms to hooke's law; (3) considering that the main cable will stretch, the sectional area will shrink, and the unit volume and weight will change before and after the force; (4) the geometric nonlinearity of the main cable is considered.

4.1. The parabola method
As shown in Fig. 1, we can get the following equation by taking any differential element on the cable (qx=0):

\[ H \frac{d^2y}{dx^2} - q_y = 0 \]

From:

\[ \sum y = 0 \]

According to the above assumption, we assume that the load is uniformly distributed along the span. At this time, the shape of the main cable is parabola, and the equation of the main cable curve can be obtained from Fig. 2:

\[ y = -\frac{4f_x}{L^2}(L-x) + \frac{C}{L}x \]

The parabola method assumes that the load is uniformly distributed along the span, which is an approximate method. When the span of a bridge is small, it is a practical method to determine the geometric shape and internal force of the main cable under constant load of suspension bridge.
4.2. The catenary method

The basic assumption of catenary calculation theory is that the main cable's dead weight and the local load transmitted through the sling are distributed along the cable curve. In addition to simplifying the main cable to some basic assumptions of the catenary, this calculation theory has no approximation in the derivation process. It is a more accurate calculation theory.

Fig 1. The differential element

Fig 2. The calculation icon of the parabola clue

Fig 3. The calculation icon of the catenary clue
As shown in Fig. 3, if there is no other concentrated force on the cable, only the self weight $q_1$ acts, we can deduce the catenary equation under the action of gravity:

$$y = -\frac{H}{q_1} \cosh\left(\frac{q_1 x}{H} - \alpha\right) + \alpha_1$$

Among them,

$$\alpha_1 = \frac{H}{q_1} \cosh \alpha, \quad \alpha = \text{sh}^{-1}\left(\frac{\beta C/L}{\text{sh} \beta}\right) + \beta, \quad \beta = \frac{q_1 L}{2H}$$

According to the catenary equation, the length and elastic elongation of the catenary clues can be obtained.

The main cable of suspension bridge is mainly subjected to two loads when it is completed. One is the weight of the main cable along the length of the arc, including protection and wrapping, and the other is the concentrated load transmitted by the sling, including the weight of the cable clip, the sling and the anchorage, as well as the constant load of the stiffened beam passing through the sling. Therefore, the stress diagram of the main cable of a suspension bridge can be simplified as a flexible cable with uniformly distributed load along the arc length and concentrated load acting on the sling.

As shown in Fig. 4, the cable is divided into $n$ segments by the hanger rod force $P_n$, so the cable curve equation can be obtained:

$$\alpha_i = \text{sh}^{-1}\left(\frac{\beta C/L}{\text{sh} \beta}\right) + \beta_i$$
\[ y(x) = \frac{H}{q} \left[ ch \alpha, - ch(2\beta, x, l, -\alpha) \right] \]

\[ \beta = \frac{q, l}{2H} \]

According to the condition of force balance and deformation compatibility, an iterative calculation program for the theoretical line of sectional catenary can be worked out to solve [6].

5. The Summary
Both parabola calculation theory and catenary calculation theory can be used to calculate the main cable curve of suspension bridge. The parabola method is relatively simple in calculation, but it is rough. It is practical when the bridge span is not large; the catenary method is more perfect and accurate in the process of establishing the hypothesis and derivation of the theory, and the factors to be considered are more comprehensive, so the calculation is relatively complex, but the result is more accurate. It is practical for the calculation of the cable shape of the main cable of the common suspension bridge.

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