BEHAVIOR OF AN ALMOST SEMICONTINUOUS POISSON PROCESS ON A MARKOV CHAIN UPON ATTAINMENT OF A LEVEL

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We consider the almost semi-continuous processes defined on a finite Markov chain. The representation of the moment generating functions for the absolute maximum after achievement positive level and for the recovery time are obtained. Modified processes with two-step rate of negative jumps are investigated.

1 Introduction

In the present paper, we continue the investigations of almost semicontinuous processes defined on finite Markov chains originated in [1, 2]. In the first section, we consider analogs of the functionals studied in [3] (Sec. 6.3) for the scalar case. In the second section, we study overjump functionals for a modified almost semi-continuous process playing the role of an analog of a modified semicontinuous process with drift whose variations depend on the attained level (see, e.g., [4], Chap. VII and [5]; for the scalar case, see [6]).

Let \( x(t) \) be a finite irreducible Markov chain with the set of states \( E' = \{1, \ldots, m\} \) and an infinitesimal matrix \( Q \). A process \( \xi(t) \) is defined as follows: \( \xi(0) = 0 \); for \( x(t) = k, k = 1, \ldots, m \), the increments \( \xi(t) \) coincide with the increments of the process \( F_0(t) = A \left[ C (C + \alpha I)^{-1} - I \right] + \int_0^\infty (e^{tx} - 1) \Pi(dx) + Q, \) \( 0 \leq u \leq t \) where \( \Psi(\alpha) = AF_0(0) \left[ C (C + \alpha I)^{-1} - I \right] + \int_0^\infty (e^{tx} - 1) \Pi(dx) + Q, \) with cumulant

\[
\xi_k(t) = \sum_{n \leq c_k(t)} \xi_n^k - \sum_{n \leq c_k'(t)} \zeta_n^k,
\]

where \( \epsilon_k'(t) \) and \( \epsilon_k(t) \) are Poisson processes with the rates \( \lambda_k^1 \) and \( \lambda_k^2 \), respectively. \( \xi_n^k \) and \( \xi_n^k \) are independent positive random variables. Moreover, \( \xi_n^k \) have exponential distributions with parameters \( c_k \), whereas \( \zeta_n^k \) have absolutely continuous distributions with finite expectations \( m_k \). In this case, \( Z(t) = \{\xi(t), x(t)\} \) is an almost lower semicontinuous process on a Markov chain (see [1, p. 562]) with extrema

\[
\Psi(\alpha) = AF_0(0) \left[ C (C + \alpha I)^{-1} - I \right] + \int_0^\infty (e^{tx} - 1) \Pi(dx) + Q, \]

where \( A = ||\delta_{kr}(\lambda_k^1 + \lambda_k^2)||, C = ||\delta_{kr}c_k||, F_0(0) = ||\delta_{kr}\lambda_k^1/(\lambda_k^1 + \lambda_k^2)||, \Pi(dx) = AF_0(0)dF_0(x), F_0(0) = I - F_0(0) \) and \( F_0(x) = ||\delta_{kr}P \{\xi_n^k < x\}||, x < 0 \).

By

\[
\tau^+(x) = \sup_{0 \leq u \leq t} (\inf(\xi(u)), \xi^+ = \sup_{0 \leq u \leq \infty} (\inf(\xi(u)), \tau(t) = \xi(t) - \xi^+(t),
\]

we denote the extrema of the process \( \xi(t) \). The overjump functionals are specified as follows:

\[
\tau^+(x) = \inf\{t : \xi(t) > x\}, \gamma^+(x) = \xi(\tau^+(x)) - x, \gamma^-(x) = x - \xi(\tau^-(x) - 0), x \geq 0;
\]

\[
\tau^-(x) = \inf\{t : \xi(t) < x\}, x \leq 0; \tau^-(x) = 0, x > 0.
\]

Let \( \theta_s \) be an exponentially distributed random variable with parameter \( s > 0 \) independent of \( Z(t) \). The distributions of extrema and the corresponding atomic probabilities are defined as follows:

\[
P_{\pm}(s, x) = ||P \{\xi^\pm(\theta_s) < x, x(\theta_s) = r/x(0) = k\}|| = P \{\xi^\pm(\theta_s) < x, x > 0;\}
\]

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we conclude that the spectra of the matrices $R_-(s)$ and $R^-(s)$ $(\sigma(R_-(s))$ and $\sigma(R^-(s)))$ are formed by positive elements.

The following assertion for overjump functionals is obtained from [2, p. 48]:

**Lemma 1.** For the process $Z(t)$ with cumulant $(1)$,

$$f_s(dx, dy/du) = E\left[e^{-\sigma^+(u)}, \gamma^+(u) \in dx, \gamma^+(u) \in dy, \tau^+(u) < \infty\right] =$$

$$= s^{-1}d\gamma(s, u-x)p^-(s)\Pi(dy + x)I\{x < u\} +$$

$$+ s^{-1} \int_{0<v(u-x)}^u d\gamma(s, z)R^-(s)e^{R^-(s)(u-x-z)}q^-\Pi(dx + y)dy. \quad (3)$$

$$g_s(dy/du) = E\left[e^{-\sigma^+(u)}, \gamma^+(u) \in dy, \tau^+(u) < \infty\right] = s^{-1} \int_{0}^{u} d\gamma(s, z)\times$$

$$\times \left(p^-(s)\Pi(dy + u - z) + R^-(s)\int_{u-z}^{\infty} e^{R^-(s)(u-x-z)}q^-\Pi(dx + y)dy\right). \quad (4)$$

$$g(dy/du) = \lim_{s \to 0} g_s(dy/du) =$$

$$= \int_{0}^{u} d\gamma_m(z)\left(\Pi(dy + u - z) + C \int_{u-z}^{\infty} e^{R^-(0)(u-x-z)(I - p^-(0))}\Pi(dx + y)dy\right). \quad (5)$$

where $\gamma_m(x): \int_{0}^{\infty} e^{\alpha x}d\gamma_m(x) = -\Psi^{-1}(\alpha)(C + i\alpha I)^{-1}(Cp^-(0) + i\alpha I).

Note that $d\gamma_m(x) = \lim_{s \to 0} s^{-1}d\gamma(s, x)p^-(s)$ and the matrices $p^-(0)$ and $p^-(0)$ satisfies the equations

$$(A - Q)(I - p^-(0)) = \Lambda F_0(0) + \int_{0}^{\infty} \Pi(dz)(I - p^-(0))e^{-Cp^-(0)z},$$

$$(I - p^-(0))(A - Q) = \Lambda F_0(0) + \int_{0}^{\infty} e^{-p^-(0)Cz}(I - p^-(0))\Pi(dz),$$

respectively.

## 2 Red period

In the present section, we consider functionals connected with the behavior of $\xi(t)$ upon attainment of a positive level. Denote

$$z^+(u) = \sup_{\tau^+(u) \leq t < \infty} \xi(t) - u, \quad \tau'(u) = \inf\{t > \tau^+(u), \xi(t) < u\},$$
\[ T'(u) = \begin{cases} 
\tau'(u) - \tau^+(u), & \tau^+(u) < \infty, \\
\infty, & \tau^+(u) = \infty. 
\end{cases} \]

It is worth noting that the process \( Z(t) \) can be regarded as a surplus risk process with stochastic function of premiums (the values of premiums are exponentially distributed) in a Markov environment and the functionals \( z^+(u), \tau'(u), T'(u) \) can be regarded as the total deficit after ruin, recovery time, and "red period", respectively (see [7]).

**Theorem 1.** For the process \( Z(t) \) with cumulant (1)

\[ P \{ z^+(u) < x, \tau^+(u) < \infty \} = \int_0^x g(dy/u)P \{ \xi^+ < x - y \}. \tag{6} \]

\[ sE \left[ e^{-s\tau'(u)} \right] = \int_0^u dP_+(s,x) \left( \int_\infty^{\infty} \Pi(dz)q_-(s)e^{R_-(s)(u-z)} \right) + 
\frac{C}{\int_{-\infty}^0 e^{R_-(s)y}q_-(s) \int_{u-x-y}^{\infty} \Pi(dz)q_-(s)e^{R_-(s)(u-z-y)} dz} \right), \tag{7} \]

\[ E \left[ e^{-sT'(u)} \right] = \int_0^u dM_+(x) \left( \int_0^{\infty} \Pi(dy+u-x)q_-(s)e^{-R_-(s)y} \right) + 
\frac{C}{\int_{-\infty}^0 e^{R_-(s)(u-x-z)}q_-(0) \int_{0}^{\infty} \Pi(dy+z)q_-(s)e^{-R_-(s)y} dz} \right). \tag{8} \]

**Proof.** In view of the fact that, under the condition \( \gamma^+(u) \in dy, \tau^+(u) < \infty \), the functional is \( z^+(u) \) stochastically equivalent to \( y^+ + \xi^+ \), we find

\[ P \{ z^+(u) < x, \tau^+(u) < \infty \} = \int_0^x P \{ \gamma^+(u) < dy, \tau^+(u) < \infty \} P \{ y^+ + \xi^+ < x \}, \]

In exactly the same way as in the proof of Theorem 5.1 in [8], for the moment generating function of the time to recovery, we deduce

\[ E \left[ e^{-s\tau'(u)} \right] = \int_0^u dP_+(s,x) \left( \int_{-\infty}^{\infty} \Pi(dz)q_-(s)e^{R_-(s)(u-z)} \right) + 
\frac{C}{\int_{-\infty}^0 e^{R_-(s)(u-z-y)}q_-(0) \int_{0}^{\infty} \Pi(dy+z)q_-(s)e^{-R_-(s)y} dz} \right). \]

Combining this relation with (2), we obtain (7). By using the strict Markov property, we get

\[ E \left[ e^{-sT'(u)} \right] = \frac{r}{x(0)} = k \]

\[ = \sum_{j=1}^{\infty} \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty P \{ x(\tau^+(u)) = j, \gamma^+(u) \in dy, \tau^+(u) < \infty / x(0) = k \} \times 
\times E \left[ e^{-sT'(u)} \right] = r / \gamma^+(u) \in dy, x(\tau^+(u)) = j, \tau^+(u) < \infty, x(0) = k \]
By using (2) and (5), we establish equality (8). In deducing this equality, we have used the fact that, under the condition \( \gamma^+(u) \in dy, \tau^+(u) < \infty \), the functional \( T'(u) \) is stochastically equivalent to the time of attainment of the level \(-y\). In the matrix form, we can write

\[
E\left[ e^{-sT'(u)}, T'(u) < \infty, \tau^+(u) < \infty \right] = \int_0^\infty P \{ \gamma^+(u) \in dy, \tau^+(u) < \infty \} E\left[ e^{-s\tau^-(y)}, \tau^-(y) < \infty \right].
\]

By using (2) and (5), we establish equality (8).

\[\Box\]

### 3 Modified Process

In the present section, in addition to the results obtained for the overjump functionals, we use the relations for two-limit functionals. By

\[ \tau(u, b) = \{ t > 0 : \xi(t) \notin (u - b, u) \} \]

we denote the time of exit from the interval \((u - b, u)\). Further, we consider the events specifying the times of exit through the upper and lower boundaries of the interval:

\[ A_+(u) = \{ \omega : \xi(\tau(u, b)) \geq u \} \quad \text{and} \quad A_-(u) = \{ \omega : \xi(\tau(u, b)) \leq u - b \} , \]

and the corresponding overjumps

\[ \gamma^+_b(u) = \xi(\tau(u, b)) - u, \gamma^-_b(u) = u - \xi(\tau(u, b) - 0) \quad \text{on} \quad A_+(u); \]

\[ \gamma^-_b(u) = (u - b) - \xi(\tau(u, b)), \gamma^+_b(u) = \xi(\tau(u, b) - 0) - (u - b) \quad \text{on} \quad A_-(u). \]

It follows from the results presented in [1, p.559] that

\[ E\left[ e^{-s\tau(u, b)}, \gamma^-_b(u) \in dy, A_-(u) \right] = E\left[ e^{-s\tau(u, b)}, A_-(u) \right] C e^{-Cy}dy = B_0(s, u)Ce^{-Cy}dy, \quad (9) \]

\[ f_{\gamma^+_b,\gamma^-_b}(dx, dy/u) = E\left[ e^{-s\tau(u, b)}, \gamma^+_b(u) \in dx, \gamma^-_b(u) \in dy, A_+(u) \right] = \right| P \{ u - \xi(\theta_s) \in dx, \tau(u, b) > \theta_s, x(\theta_s) = r/x(0) = k \} \Pi(dy + x) I \{ 0 < x < b \} = d_xH_+(b, u - x) \Pi(dy + x) I \{ 0 < x < b \}. \]

Note that the representations for \( B_0(s, u) \) and \( d_xH_+(b, u, x) \) were obtained in [1] for almost upper semicontinuous processes. In this case, one can use the fact that if \( \{ \xi(t), x(t) \} \) is an almost upper semicontinuous process, then \( \{ -\xi(t), x(t) \} \) is an almost lower semicontinuous process.

We now determine the modified process \( \xi_{a,b}(t) \), \( 0 < a \leq b < \infty \). Assume that the rates of exponentially distributed negative jumps \( \xi_{a,b}(t) \) depend on the threshold levels \( a \) and \( b \) (see [6]). In the risk theory, this process has the following interpretation: As soon as the reserve of an insurance company attains a certain level, the company may decrease the value of the premium to attract additional clients. Therefore, the distribution of the values of premiums contains a parameter \( \mathcal{C} = \mathcal{C}(r) \), if the reserve of the company is equal to \( r \). Assume that \( \mathcal{C} \) takes only two values \( \mathcal{C} \) and \( \mathcal{C}^* \) equal to the initial and
lowered values of the premium, respectively, and in addition, that the transition between these values
occurs on passing through an inert zone \((a,b)\).

The increments of the process \(\xi_{a,b}(t)\) coincide with the increments of the process \(\xi(t)\) (with intensities \(C\)) between the last crossing of the level \(u-a\) from below and the next crossing of the level \(u-b\) from above. The increments of \(\xi_{a,b}(t)\) coincide with \(\xi_*(t)\) (with intensities \(C_*\)) between the last crossing of the level \(u-b\) from above and the subsequent crossing of the level \(u-a\) from below. In the notation of moment generating functions we use the symbol \(*\), which corresponds to the process \(\xi_*(t)\).

Further, if \(x(t) = k\), then

\[
d\xi_{a,b}(t) = d\xi_k(t) I\{0 \leq t \leq \tau^-(u-b)\} + d\xi_k(t) I\{\tau^-(u-b) < t \leq \tau^0_k(u)\} + d\xi_{a,b}(t - \tau^0_k(u)) I\{t > \tau^0_k(u)\},
\]

where \(\tau^0_k(u) = \inf\{t > \tau^-(u-b) : \xi_{a,b}(t) \geq u-a\}\).

By \(\bar{\tau}^+(u), \bar{\gamma}^+(u), \bar{\tau}^+(u)\) we denote the overjump functionals for the modified process \(\xi_{a,b}(t)\) (see Figure 1). We also denote

\[
f^{a,b}_s(dx, dy/u) = E\left[e^{-s\bar{\tau}^+(u)}, \bar{\gamma}^+(u) \in dx, \bar{\tau}^+(u) \in dy, \bar{\tau}^+(u) < \infty\right].
\]

Then the Gerber–Shiu function can be defined as follows (see [9])

\[
\Phi^{a,b}_s(u) = \int_0^\infty \int_0^\infty w(x, y) f^{a,b}_s(dx, dy/u),
\]

where \(w(x, y), x, y > 0\) is a nonnegative function (penalty). If the parameter \(s\) is regarded as the force of interest, then \(\Phi^{a,b}_s(u)\) can be regarded as a discounted expected penalty at the time to ruin.

Assume that the process \(e^{u - \xi_{a,b}(t)}\) describes the price of a stock whose variations have the form of random jumps. We now consider a perpetual American put option with strike price \(K\). The payoff at time \(t\) is equal to \((K - e^{u - \xi_{a,b}(t)})_+\).

For the scalar case, the optimal strategy is as follows

\[
\tau_\beta = \inf\{t > 0 : e^{u - \xi_{a,b}(t)} < e^\beta\},
\]

Figure 1: Modified Risk Process
Corollary 1. For the scalar modified process $\xi_{a,b}(t)$, assume that the market is risk neutral. Then the price of an option is defined as the expected discounted payoff

$$
E \left[ e^{-s\tau_{\beta}}(K - e^{a-x_\beta}(\tau_{\beta}))^+ \right]
$$

or, in view of the fact that $\tau_{\beta} = \tilde{\tau}^+(u - \beta)$, as follows

$$
E \left[ e^{-s\tilde{\tau}^+(u-\beta)}(K - e^{\beta-\gamma^+(u-\beta)})^+ \right].
$$

Therefore, $\Phi_{a,b}^\tau(u - \beta)$ with $w(x, y) = (K - e^{\beta-y})^+$ can also be regarded as the price of perpetual American put option [10, p.12].

**Theorem 2.** For the modified process $\{\xi_{a,b}(t), x(t)\}$

1) if $0 < u \leq b$, then

$$
\Gamma_{a,b}(dx, dy/u) = \Gamma_{a,b}^+(dx, dy/u) + B_b(s, a, u) \int_0^\infty C e^{-Cz} \Gamma_{a,b}(dx, dy/z + b) dz; \quad (10)
$$

2) if $b < u$, then

$$
\Gamma_{a,b}(dx, dy/u) = \Gamma_{a,b}^+(dx - a, dy + a/u - a) I\{x > a\}+
\int_0^a g_a^+(dz/u - a) \left\{ \Gamma_{a,b}^+(dx, dy/a - z) + B_b(s, a - z) \int_0^\infty C e^{-Cz} \Gamma_{a,b}(dx, dy/v + b) dv \right\}, \quad (11)
$$

where

$$
\left( 1 - \int_0^\infty C e^{-Cz} \int_0^a g_a^+(dv/b - a + z) B_b(s, a - v) dz \right) \int_0^\infty C e^{-Cz} \Gamma_{a,b}(dx, dy/z + b) dz =
\int_0^\infty C e^{-Cz} \left( \Gamma_{a,b}^+(dx - a, dy + a/b - a + z) I\{x > a\} + \int_0^a g_a^+(dv/b - a + z) \Gamma_{a,b}^+(dx, dy/a - v) \right) dz. \quad (12)
$$

**Proof.** Relation (10) is an analog of the result obtained in [6]. By using the formula of total probability and the strict Markov property, for $u > b$, we find

$$
E \left[ e^{-s\tilde{\tau}^+(u)}, \tilde{\gamma}^+(u) \in dx, \tilde{\gamma}^+(u) \in dy, \tilde{\tau}^+(u) < \infty \right] =
E \left[ e^{-s\tilde{\tau}^+(u-a)}, \tilde{\gamma}^+(u-a) \in dx, \tilde{\gamma}^+(u-a) \in dy, \tilde{\tau}^+(u-a) < \infty \right] +
\int_0^a E \left[ e^{-s\tilde{\tau}^+(u-a)}, \tilde{\gamma}^+(u-a) \in dz, \tilde{\tau}^+(u-a) < \infty \right] \times
E \left[ e^{-s\tilde{\tau}^+(a-z)}, \tilde{\gamma}^+(a-z) \in dx, \tilde{\gamma}^+(a-z) \in dy, \tilde{\tau}^+(a-z) < \infty \right]. \quad (13)
$$

This yields relation (11). Relation (12) is obtained from (11) as a result of the integral transform. \qed

In the scalar case ($m = 1$) the matrix relations become somewhat simpler. If we set $w(x, y) = 1$, then $\Phi_{a,b}^\tau(u)$ is the ruin probability for the modified process.

**Corollary 1.** For the scalar modified process $\xi_{a,b}(t)$

1) if $0 < u \leq b$, then

$$
\Phi_{a,b}^\tau(u) = 1 - B_b(u) \left( 1 - \int_0^a g_a^+(dz/b - a + \theta_c^a) B_b(a - z) \right)^{-1} P_a^+(b - a + \theta_c^a); \quad (14)
$$
2) if $b < u$ then

$$
\Phi_{0,b}^a(u) = P_\times^i(u - a) - \int_0^a g_* (dz/u - a) B_b(a - z) \times \\
\left(1 - \int_0^a g_* (dz/b - a + \theta') B_b(a - z) \right)^{-1} P_\times^i (b - a + \theta'),
$$

where

$$
P_\times^i (b - a + \theta') = \int_0^\infty c e^{-c x} P \{ \xi^t < b - a + x \} dx,
$$

$$
g_* (dz/b - a + \theta') = \int_0^\infty c e^{-c x} P \{ r_i^t (b - a + x) \in dz \}, \text{ and } r_i^t (b - a + x) < \infty \} dx.
$$

**Example.** Assume that, for the scalar risk process, the premiums $\xi^t_i$ have exponential distributions with parameter $\tilde{c}$, whereas the claims obey the Erlang distribution (2):

$$
P \{ \xi^t < x \} = \delta^2 x e^{-dx}, x > 0.
$$

It is necessary to determine the corresponding ruin probability for $u \leq b$.

According to Example 5.2 [3], for $E \xi(1) < 0$ and $E \xi_s (1) < 0$ we find

$$
P \{ \xi^t < u \} = P_\times^i (u) = 1 - a_i e^{-r_i u} - a_i e^{-r_i u},
$$

$$
m_i^t(0+) = \frac{1}{\lambda} dM^*_i(x) = \frac{1}{c_i |E_{x^*}(1)|} dP_\times^i (x), R^-(0) = p^- (0) = 0,
$$

$$
B_b(u) = (1 - a_1 e^{-r_1 u} - a_2 e^{-r_2 u}) (1 - b_1 e^{-r_1 b} - b_2 e^{-r_2 b})^{-1},
$$

where the quantities $a_i, a_i^t$, and $b_i$ are independent of $u$ and $b$; $r_i$ and $r_i^t$ are positive roots of the Lundberg equation for the processes $\xi(t)$ and $\xi_s (t)$, respectively. By using (5) and (14), we conclude (for $u \leq b$):

$$
P \{ \bar{r}^+(u) < \infty \} = 1 - \frac{(1 - a_1 e^{-r_1 u} - a_2 e^{-r_2 u}) (1 - f_1^* e^{-r_1 (b-a)} - f_2^* e^{-r_2 (b-a)})}{P(u, a, b)},
$$

$$
P(u, a, b) = 1 - f_2 e^{-r_1 b} - f_3 e^{-r_2 b} + (g_{11} + g_{12} (b - a)) e^{-\delta (b-a)} +
$$

$$
+ (g_{21} + g_{22} (b - a)) e^{-\delta (b-a) - r_1 u} + (g_{31} + g_{32} (b - a)) e^{-\delta (b-a) - r_2 u},
$$

where $f_i, f_i^t$, and $g_i, j$ are independent of $u, a$ and $b$.

Assume that $c = 1, c_* = 4, \delta = 20, \lambda_1 = 2, \text{ and } \lambda_2 = 2$. Then $E \xi(1) = -19/10$ and $E \xi_s (1) = -2/5$.

The corresponding Lundberg roots are $r_1 = 8, r_2 = 95/3$ and $r_1^* = 20/3, r_2^* = 32$.

In addition, the distribution of the absolute maximum of the process $\xi_s (t)$ and the probability of exit of the process $\xi(t)$ from the interval $(u - b, b)$ through the lower boundary are given by the formulas

$$
P \{ \xi^t < u \} = 1 - \frac{32}{57} e^{-20 x/3} - \frac{9}{95} e^{-32 x};
$$

$$
B_b(u) = P \{ \xi(u, b) \leq u - b \} = \frac{1 + \frac{49}{420} e^{-95u/3} - \frac{171}{355} e^{-8u}}{1 + \frac{49}{285} e^{-95b/3} - \frac{171}{355} e^{-8b}}.
$$

For $a = b$, by using (16), we arrive at the following expression for the ruin probability

$$
P \{ \bar{r}^+(u) < \infty \} = 1 - \frac{1 + \frac{49}{420} e^{-95u/3} - \frac{171}{355} e^{-8u}}{1 + \frac{49}{852} e^{-95b/3} + \frac{9}{852} e^{-8b}}.
$$
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