Tunneling Spectroscopy of the Underdoped High-$T_c$ Superconductors

Yun Kyu Bang$^{1,2}$
$^1$ Department of Physics, Chonnam National University, Kwangju 500-757, Korea,
$^2$ Center for Strongly Correlated Materials Research, Seoul National University, Seoul 151-742, Korea.

Han-Yong Choi$^3$
$^3$ Department of Physics and Institute for Basic Science Research, Sung Kyun Kwan University, Suwon 440-746, Korea.

The suppression of density of states around the Fermi level in the underdoped high-$T_c$ superconductors (HTSC) above $T_c$ is observed in various normal state experiments such as optical conductivity, dc-resistivity, angle resolved photoemission, NMR, tunneling spectroscopy, neutron scattering, specific heat, Raman spectroscopy etc [1], and this phenomena is termed as pseudogap (PG). Currently there is a consensus of the origin and nature of this PG is still lacking and undoubtedly it is a key issue to be resolved to make any progress toward a theory of high $T_c$ superconductivity.

About the origin of the PG state only two possibilities are logically allowed. The first possibility is that the PG is somehow related with the superconducting correlation and it develops into a real superconducting gap below $T_c$. And the second possibility is that the PG has nothing to do with a superconducting gap but with something else. Along the first line of thinking, preformed pair scenario [2], pairing fluctuations scenario [3], etc. are proposed. For the second possibility, antiferromagnetic correlation [4], charge stripes [5], etc. are considered as the origin of the PG. At present experimental evidences exist both for superconducting origin [6] and for non-superconducting origin [7-9]. Therefore it is necessary to design an experiment to identify distinct features among the proposed scenarios. Recently we proposed an experimental test using tunneling spectroscopy in the PG state, specifically, for the preformed pair scenario [10]. Namely we claim that there should be an Andreev reflection signal even above $T_c$ (PG cross-over temperature) if there exist preformed Cooper pairs without long range phase coherence. Until now there are only one positive [11] and one negative [12] experiment reported on the observation of an Andreev signal in the PG region of the underdoped HTSC compounds.

In this paper we examine the second possibility for the PG state, i.e., that the PG is irrelevant from the SC gap. We calculate the tunneling conductance ($dI/dV$) at zero temperature when the PG coexists with a SC gap. Specifically we considered two cases: (1) the PG is a simple suppression of density of states with an unknown origin; (2) the pseudogap state is due to an antiferromagnetic correlation. For both cases we calculate $dI/dV$ using the BTK theory. The results are discussed in comparison with experiments.

PACS numbers: 74.20,74.20-z,74.50

I. INTRODUCTION

The suppression of density of states around the Fermi level in the underdoped high-$T_c$ superconductors (HTSC) above $T_c$ is observed in various normal state experiments such as optical conductivity, dc-resistivity, angle resolved photoemission, NMR, tunneling spectroscopy, neutron scattering, specific heat, Raman spectroscopy etc [1], and this phenomena is termed as pseudogap (PG). Currently there is a consensus of the origin and nature of this PG is still lacking and undoubtedly it is a key issue to be resolved to make any progress toward a theory of high $T_c$ superconductivity.

About the origin of the PG state only two possibilities are logically allowed. The first possibility is that the PG is somehow related with the superconducting correlation and it develops into a real superconducting gap below $T_c$. And the second possibility is that the PG has nothing to do with a superconducting gap but with something else. Along the first line of thinking, preformed pair scenario [2], pairing fluctuations scenario [3], etc. are proposed. For the second possibility, antiferromagnetic correlation [4], charge stripes [5], etc. are considered as the origin of the PG. At present experimental evidences exist both for superconducting origin [6] and for non-superconducting origin [7-9]. Therefore it is necessary to design an experiment to identify distinct features among the proposed scenarios. Recently we proposed an experimental test using tunneling spectroscopy in the PG state, specifically, for the preformed pair scenario [10]. Namely we claim that there should be an Andreev reflection signal even above $T_c$ (PG cross-over temperature) if there exist preformed Cooper pairs without long range phase coherence. Until now there are only one positive [11] and one negative [12] experiment reported on the observation of an Andreev signal in the PG region of the underdoped HTSC compounds.

In this paper we examine the second possibility for the PG state, i.e., that the PG is irrelevant from the SC gap. We calculate the tunneling conductance ($dI/dV$) at zero temperature when the PG coexists with a SC gap. Specifically we considered two cases: (1) the PG is a simple suppression of density of states with an unknown origin; (2) the pseudogap state is due to an antiferromagnetic correlation. For both cases we calculate $dI/dV$ using the BTK theory. The results are discussed in comparison with experiments.

The main results are: (1) for the first case, the basic line shape of the tunneling conductance is a simple superposition of a standard BTK conductance and an assumed PG density of states. When the SC gap size is smaller than the PG size, the SC gap feature shows up as a distinguishable peak inside the PG. However if the size of SC gap is larger than the PG, the PG feature is overwhelmed by the SC gap feature; (2) for the second case, there is an interesting interplay between the AFM and SC correlations. Irrespective of the relative sizes of the PG and SC gap, the tunneling conductance shows only one gap feature at $E = \Delta_{total} = \sqrt{\Delta_{SC}^2 + \Delta_{PG}^2}$. However depending on the relative sizes of the PG and SC gap, the line shape of $dI/dV$ looks very different. When the SC gap is bigger than the PG, it looks more like a conventional NIS (Normal metal-Insulator-Superconductor) junction. But for the other case the main feature of the $dI/dV$ curve is determined by the SDW correlation and shows no diverging density of states approaching the gap energy in contrast to the NIS junction. This difference comes from the fact that the tunneling density of states of the NISDW (Normal metal-Insulator-SDW state) junction obtained by the BTK theory (even at large barrier
potential (Z) limit) is not the same as the actual density of states, which would have been obtained by the tunneling Hamiltonian method [14]. The difference of the tunneling conductance by the BTK theory and by the tunneling Hamiltonian method in the SDW state, which is in contrast to the NIS (Normal metal-Insulator-Superconductor) junction, shows that the description of tunneling process by these two methods are not the same. The important and interesting questions are then which description is more proper description for actual tunneling process and why in the case of NIS junction those two methods seem to give consistent results for a $Z >> 1$ limit. More details will be discussed in the later section.

II. PG MODEL I: SIMPLE SUPPRESSION OF DENSITY OF STATES

In this section we consider the case that the PG is a simple suppression of density of states, which has been developed above $T_c$ by an unknown origin. An assumption is that the SC gap develops below $T_c$ on top of the already developed PG density of states and there is no other correlation between the SC gap and the PG.

The tunneling conductance of NIS junction is calculated by the BTK theory as follows.

$$dI/eV/dV = 2e\nu_F A \int_{-\infty}^{\infty} dE \ N_{PG}(E) \frac{\partial f(E - V)}{\partial V} \cdot [1 + |A(E)|^2 - |R(E)|^2],$$

(1)

where

$$|A|^2 = \begin{cases} \frac{\Delta_S^2}{(E + (1 + 2Z^2)^2)\sqrt{E^2 - \Delta_S^2}} & \text{for } E > \Delta_S \\ \frac{\Delta_S^2}{E + (1 + 2Z^2)^2(\Delta_S^2 - E^2)} & \text{for } E < \Delta_S \end{cases}$$

(2)

$$|R|^2 = \begin{cases} \frac{4Z^2(\Delta_S^2 - Z^2)(Z^2 + 1)}{(E + (1 + 2Z^2)^2)(E^2 - \Delta_S^2)} & \text{for } E > \Delta_S \\ \frac{4Z^2Z^2(\Delta_S^2 - E^2)}{(E + (1 + 2Z^2)^2)(\Delta_S^2 - E^2)} & \text{for } E < \Delta_S \end{cases}$$

(3)

are the coefficients for the Andreev and normal reflections, respectively, and $Z$ is the strength of the insulating barrier ($Z = \hbar V / k_F$). The above equation is a standard formula for the tunneling conductance by the BTK theory with one modification, i.e., multiplied by the PG density of states $N_{PG}(E)$. We simulate $N_{PG}(E)$ by Dynes’ formula with the SC gap replaced by the PG ($\Delta_P$).

$$N_{PG}(E) = 2\pi N(0) R e \left[ \frac{\omega + i\Gamma}{(\omega + i\Gamma)^2 - \Delta_P^2} \right].$$

(4)

The numerical calculation has done for a tunneling into (1,0,0) direction toward the HTSC and assume that $\Delta_P$ and $\Delta_S$ has the maximum value at the same direction (1,0,0). To simulate the surface roughness we also present the angle averaged results by simply replacing $\Delta_{S,P} \Rightarrow \Delta_{S,P}^0 \cos(0)$. All the presented tunneling conductance is normalized by the normal-state resistance $R_N = (1 + Z^2)/(2N(0)\nu_F A)$ and the bias voltage is measured in unit of $\Delta_P$. Also for all the presented result we chose $\Gamma = 0.3$, which determines the shape of the PG density of states $N_{PG}(E)$ from Eq(4).

FIG. 1. (a) The normalized tunneling conductance $R_N dI/dV$ as a function of the bias voltage in units of $\Delta_P$ with $\Delta_S = 0.5\Delta_P$ for different $Z (= 0, 0.5, 1)$. $N_{PG}(E)$ (PDOS) is also plotted for reference; (b) The same as (a) but angle-averaged; (c) The same as (a) but with $\Delta_S = \Delta_P$; (d) The same as (c) but angle-averaged.

In Fig.1(a) we show the normalized tunneling conductance $R_N dI/dV$ at zero temperature with the superconducting gap $\Delta_S = 0.5\Delta_P$ for different barrier potentials ($Z=0,0.5$, and 1). We also plot $N_{PG}(E)$ with $\Gamma = 0.3$ for comparison. The results can be trivially understood. As well known from the BTK theory, for small value of $Z$, we see the enhanced conductance below $\Delta_S$ due to the Andreev reflection and this effect quickly disappears with increasing $Z$ value. The main difference of our results from the conventional BTK calculation is that we modulate the conventional BTK tunneling conductance by multi-
plying with $N_{PG}(E)$. As a result the enhanced conductance below $\Delta_S$ and the sharp peak structure at $E = \Delta_S$ appear inside the PG density of states ($\Delta_S < \Delta_P$). In Fig.1(b) we show the same calculations but angle averaged to simulated the surface roughness in real tunneling experiment. The main features are the same as Fig.1(a) and the line shapes become more rounded off. In particular the result of $Z=1$ case in Fig.1(b) looks quite similar with the recent tunneling experiment by Krasnov et al. 9

In Fig.1(c) and Fig.1(d) we show the results of same calculations as Fig.1(a) and Fig.1(b), respectively but with $\Delta_S = \Delta_P$. When $\Delta_S$ has the same value as $\Delta_P$ the line shape of the tunneling conductance looks similar to the conventional NIS junction, in particular with a large $Z$ value. The effect of the PG is just to enhance the over-all line shape of the conductance. For smaller $Z$ value the conductance is still enhanced below $\Delta_S$ due to the Andreev reflection. However even the enhanced conductance at $E \to 0$ limit is far below than 2 (for a conventional NIS junction it approaches 2 for $E < \Delta_S$ as $Z \to 0$) because of the reduced DOS of PG origin. Again the angle averaged results in Fig.1(d) shows a more rounded off line shapes. The angle averaged PG density of states (PDOS) is shown for comparison in Fig.1(b) and (d).

In Fig2.(a-d), we plot the normalized conductance with varying $\Delta_S(=0.5, 1, and 1.5 \Delta_P)$. When $Z=1$ (Fig.2(a) and Fig.2(b)) the results are easily understood. For $\Delta_S < \Delta_P$ the distinct peak structure due to SC appears inside the PG as explained above. And when $\Delta_S > \Delta_P$ there is no more distinct peak structure and the SC feature overwhelms the PG structure. When $Z=0.1$ (Fig.2(c) and Fig.2(d)) the conductance line shapes look more peculiar, but basically it can be understood as an overlap of the Andreev enhanced conductance below $\Delta_S$ and the PG density of states $N_{PG}(E)$. G. Deutscher et al. 8 reported that there are two energy scales observed in tunneling experiments, and depending on the barrier strength ($Z$) only one of the two energy scales dominates the conductance line shape. In Fig2.(c) the line with $\Delta_S = 0.5 \Delta_P$ (solid line) shows a similar feature to the experimental data in Fig.2 of Ref. 8.

In view of the experiments of Krasnov et al. 9 and G. Deutscher et al. 8, if our calculations have any relevance with underdoped HTSC, it would be the case of $\Delta_S < \Delta_P$ (Fig.1(a)(b) and Fig.2(c)(d)) and it will be interesting if more tunneling experiments with different $Z$ parameters become available in near future.

### III. PG MODEL II: SDW

In this section we consider another possibility of the PG, namely, in which the correlation, which induces the PG, coexists with a SC correlation below $T_c$ and the PG correlation interplay with the SC correlation. In this case the key difference from the case (I) is that either $\Delta_S$ or $\Delta_P$ does not show up as separate features in the tunneling conductance but the total gap $\Delta_{\text{total}} = \sqrt{\Delta_S^2 + \Delta_P^2}$ shows up as a result of the interplay of two gaps. However more detailed line shape of the tunneling conductance reveals the existence of two gaps, $\Delta_S$ and $\Delta_P$.

![FIG. 2. (a) The normalized tunneling conductance $R_n dI/dV$ as a function of the bias voltage in units of $\Delta_P$ with $Z=1$ for different $\Delta_S(=0.5, 1, 1.5 \Delta_P)$. $N_{PG}(E)(\text{PDOS})$ is also plotted for reference; (b) The same as (a) but angle-averaged; (c) The same as (a) but with $Z=0.1$; (d) The same as (c) but angle-averaged.](image)

Specifically, we assume the SDW correlation as the origin of PG. The reason for this assumption is of course that there is a strong antiferromagnetic (AFM) correlation in underdoped HTSC compounds. Particularly for the underdoped HTSC compounds there is clear experimental evidence from neutron scattering 10 for the coexistence of the AFM correlation and superconductivity below $T_c$ and also the AFM correlation length becomes much larger than the SC correlation length. This enable us to treat the short range AFM correlation by SDW from the point of view of the SC correlation.
A. tunneling with SDW only

As a prelude we first generalize the BTK theory to the NISDW (Normal metal-Insulator-SDW) junction. For simplicity we consider only a one dimension model and assume a commensurate SDW state with \( Q = k_F = \pi/a \) (\( a \) is the lattice distance) and also neglect the Fermi surface mismatch between the normal metal and the SDW state.

The wave functions of the left hand (normal metal) and the right hand (SDW) sides of the tunneling barrier is written as,

\[
\Psi_L(x) = e^{ikx} + Re^{-ikx}, \\
\Psi_R(x) = T[a(k)e^{-ikx} + b(k)e^{-(i(k-Q)x)}] \\
\]

where \( a(k)^2 = \frac{\epsilon + \xi}{2\epsilon} \) and \( b(k)^2 = \frac{\epsilon - \xi}{2\epsilon} \) are the Bogoliubov coefficients of the SDW state, \( \epsilon = \sqrt{\xi^2 + \Delta^2_{SDW}} \), and \( \xi = \hbar k^2/2m - \mu \). By matching the boundary conditions of the wave functions at \( x = 0 \), we obtained \( R \) and \( T \) as follows.

\[
R(E) = \frac{(a+b)iZ - a}{(a+b)iZ + b} \\
T(E) = \frac{1}{(a+b)iZ + b} \\
\]

For \( \epsilon > \Delta^2_{SDW} \), \( |R|^2 \) is easily calculated, and for \( \epsilon^2 < \Delta^2_{SDW} \), using \( a^2 = \frac{\epsilon - \sqrt{\Delta^2_{SDW} - \epsilon^2}}{2\epsilon} \) and \( b^2 = \frac{\epsilon + \sqrt{\Delta^2_{SDW} - \epsilon^2}}{2\epsilon} \), we can show \( |R|^2 = 1 \).

The tunneling conductance of NISDW junction with different \( Z (=0,0.3, \text{and} 1) \) are shown in Fig.3(a,b,c) as dash-dot lines (\( \Delta_S = 0 \) case). The tunneling conductance line shape is qualitatively different from the actual density of states \( N_{SDW}(E) \), which would have been obtained by the tunneling Hamiltonian method. Here we clearly observe that the BTK theory and the tunneling Hamiltonian method give different results for tunneling conductance. And it has been already known that these two methods describe different physical processes for the tunneling phenomena [12]. In particular, the tunneling Hamiltonian method, although physically appealing, has never been justified in regard to the tunneling transfer matrix element \( T \) (it is always taken as a constant). Then another question may arise: why then for the BCS state does the BTK theory give a qualitatively similar result as the tunneling Hamiltonian methods at least for a large \( Z \) limit where the Andreev reflection process is suppressed, but not for the SDW state? The reason is that for the BCS state the quasiparticle is a superposition of a momentum \( k \) particle and a momentum \( -k \) hole, and all together it carries the flux of the same momentum \( k \), while for the SDW state the quasiparticle is a superposition of momenta \( k \) and \( k + Q \) particles and therefore the quasiparticle in SDW state doesn’t carry a single momentum. For the BTK theory of tunneling the main physics is the flux conservation described by Liouville’s theorem, therefore the BTK theory sensitively traces the correlation of different momentum states in the SC or SDW states, while the tunneling Hamiltonian method is completely blind of the momentum correlation. In the case of SC (BCS) state, we are in a fortunate situation as described above, therefore momentum correlation doesn’t play an important role but particle-hole correlation plays an important role in the BTK theory, which provides the main difference between the BTK theory and the tunneling Hamiltonian method through the Andreev scattering. Now for the case of SDW state, the momentum correlation is important but not the particle-hole correlation; therefore it is expected that the two methods will give different results.

An important question is then which theory should be trusted. As described above, the tunneling Hamiltonian method has never been justified, while there is no logical flow in the BTK theory based on the flux conservation. Therefore we think unless the tunneling interface is very rough (of course we need to estimate how much rough is rough) the BTK theory becomes more trustable.

B. tunneling with SDW+SC

Now we will consider the tunneling junction of a normal metal-insulator-SDW+SC (NISDWS). Again for simplicity we consider only a one dimensional model. In reality, as in HTSC, the analysis of tunneling in the two dimensional SDW state is more complicated, in particular when SDW+SC is considered. However our one dimensional model is enough to provide a qualitative understanding of the interplay of SDW and SC correlations in tunneling process.

Once the SDW state is formed, there are two branches of quasiparticles created: \( \alpha_{+,k} = a(k)c_k + b(k)c_{k+Q} \) and \( \alpha_{-,k} = b(k)c_k - a(k)c_{k+Q} \), where \( a(k) \) and \( b(k) \) are the Bogoliubov coefficients as defined above. Now we assume that the superconducting pairing occurs between \( \alpha_{+,k} \) and \( \alpha_{+,k} \), and also between \( \alpha_{-,k} \) and \( \alpha_{-,k} \), but not between \( \alpha_{+,k} \) and \( \alpha_{-,k} \), for example. Although this assumption is mainly for simplicity of the analysis, it is known that the pairing interaction between different branches are much weaker in Hubbard model [17]. Also strictly speaking the SDW state with a commensurate wave vector \( (Q = \pi/a) \) is an insulator with a fully developed gap below \( \Delta_{SDW} = \Delta_P \), in contrast to the PG state in HTSC where residual density of states still remain and the system remains a metal. Therefore it is clear that our SDW state only mimic a state with a short range AFM correlation of the underdoped HTSC and it is justified by the fact that the AFM correlation is effectively long range from the viewpoint of the SC correlation. A main drawback of this assumption is that the effect of the
residual density of states below PG in the tunneling conductance is completely missing. In summary, the purpose of this section is to study a tunneling characteristics of a SDW+SC state due to the interplay of two correlations, but not all the details of real materials.

Now for the BTK theory the wave functions of the left-hand (normal metal) and right-hand (SDW+SC) sides of the tunneling interface are written as,

$$\Psi_L(E) = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) e^{ikx} + R(E) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) e^{-ikx} + A(E) \left( \begin{array}{c} 0 \\ 1 \end{array} \right) e^{ikx},$$

$$\Psi_R(E) = C(E) \left( \begin{array}{c} u \\ v \end{array} \right) \alpha_+(-k) + D(E) \left( \begin{array}{c} v \\ u \end{array} \right) \alpha_-(k).$$ (8)

where $E^2 = \sqrt{\xi^2 + \Delta_S^2} = \sqrt{\xi^2 + \Delta_{SDW}^2 + \Delta_S^2}$ and $\xi = h^2 k^2 / 2m - \mu$. All $k \approx k_F$ approximation is taken; the error of the approximation is $O(\Delta_{total}/EF)$. Also a shorthand notation of the wave functions are

$$\alpha_+(-k) = a(\epsilon)e^{-ikx} + b(\epsilon)e^{-i(k-Q)x},$$

$$\alpha_-(k) = b(\epsilon)e^{-ikx} - a(\epsilon)e^{-i(k-Q)x}. \quad (9)$$

As usual the SC Bogoliubov coefficients are $u(E)^2 = (E + \epsilon) / 2E$ and $u(E)^2 = (E - \epsilon) / 2E$. By matching boundary conditions, which is a lengthy but straightforward calculation, we obtained the coefficients of $A(E)$ and $R(E)$ as follows.

$$A = \frac{uv}{F},$$

$$R = \frac{(u^2 - v^2)[Z^2(b^2 - a^2) + iZ(b^2 - a^2 + 2ab) - ab]}{F},$$

$$F = (u^2 - v^2)[Z^2(b^2 - a^2) + iZ \cdot 2ab] + b^2 a^2 + a^2 v^2. \quad (10)$$

In order to calculate $|A|^2$ and $|R|^2$ we need to divide the region of $E$ into three regions. For region (I), where $E > \Delta_{total} = \sqrt{\Delta_{PD}^2 + \Delta_S^2}$, $\epsilon = \sqrt{E^2 - \Delta_{total}^2}$ and $\xi = \sqrt{E^2 - \Delta_{total}^2}$. For region (II), where $\Delta_S < E < \Delta_{total}$, $\epsilon = \sqrt{E^2 - \Delta_S^2}$ and $\xi = i\sqrt{\Delta_{total}^2 - E^2}$, and finally for region (III), where $E < \Delta_S$, $\epsilon = i\sqrt{\Delta_S^2 - E^2}$ and $\xi = i\sqrt{\Delta_{total}^2 - E^2}$. But in the final results of $|A|$ and $|R|$, $\epsilon$ factors are all cancelled out, so that the region (II) and (III) are not distinguished. Therefore the tunneling characteristics shows distinct change only across $E = \Delta_{total}$ but no change across either $E = \Delta_S$ or $E = \Delta_P$.

The final results of the Andreev ($A$) and normal reflection ($R$) coefficients are: for region (I),

$$|A|^2 = \frac{\Delta_{PD}^2}{G},$$

$$|R|^2 = \frac{(2Z^2 \sqrt{E^2 - \Delta_P^2} - \Delta_P)^2 + 4Z^2(\sqrt{E^2 - \Delta_P^2} + \Delta_P)^2}{G},$$

$$G = [(2Z^2 + 1) \sqrt{E^2 - \Delta_P^2} + E]^2 + 4Z^2 \Delta_P^2; \quad (11)$$

and for region (II) and (III),

$$|A|^2 = \frac{\Delta_S^2}{H},$$

$$|R|^2 = \frac{(\Delta_P + 2Z^2 \sqrt{\Delta_S^2 - E^2})^2 + 4Z^2(\sqrt{\Delta_S^2 - E^2} + \Delta_P)^2}{H},$$

$$H = E^2 + [(2Z^2 + 1) \sqrt{\Delta_S^2 - E^2} + 2Z \Delta_P]^2. \quad (12)$$

FIG. 3. (a) The normalized tunneling conductance $Rn dI/dV$ for NISDWS (Normal metal-Insulator-SDW+SC) junction as a function of the bias voltage in units of $\Delta_P$ with $Z=0$ for different $\Delta_S (=0, 0.5, 1, 2 \Delta_P)$; (b) The same as (a) but with $Z=0.3$; (c) The same as (a) but with $Z=1$.

In Fig.3(a-c) we plot the numerical calculations of the normalized tunneling conductance as a function of bias voltage (in unit of $\Delta_P$) with varying size of $\Delta_S (=0,0.5,1,2)$ for different barrier potentials ($Z=0, 0.3,$ and 1). The main features of the tunneling conductance are very different from a pure SC case. When $\Delta_S$ is much bigger than $\Delta_P$ (say, $\Delta_S/\Delta_P = 2$ in Fig.3) the line shape is similar to the pure SC case. But even in this case the position of a gap in the conductance is determined by $\Delta_t$ and the presence of SC gap $\Delta_S$ only show up through an Andreev scattering coefficient, which enhances the conductance below the total gap. On the other hand when $\Delta_S \leq \Delta_P$, the line shape looks closer to the pure SDW case, which is qualitatively different from a real density of states as explained in the previous section. Again the presence of SC gap $\Delta_S$ show up only through an Andreev scattering and enhances the conductance below the total gap.
the PG is a simple suppression of density of states of a unknown origin and when the system goes to SC state there is no direct interplay between the PG correlation and the SC correlation. In this case the characteristics of tunneling conductance is a simple superposition of a standard tunneling conductance with SC gap $\Delta_S$ and the PG density of states $N_{PG}(E)$. Despite the simplicity of the model, the results seem to explain some of the recent tunneling experiments by Krasnov et al. \cite{9} and G. Deutscher \cite{8}, which indicate non-superconducting origin of the PG. In the model II, we assumed that the PG is caused by an AFM correlation and it is simulated by SDW state. Below $T_c$ the SDW correlation and SC correlation show an interesting interplay. As a result, the tunneling gap is given by $\Delta_{total} = \sqrt{\Delta_S^2 + \Delta_P^2}$ and individual gaps, $\Delta_S$ and $\Delta_P$, do not show up explicitly. In particular when $\Delta_P, SDW > \Delta_S$, the line shape of the tunneling conductance looks qualitatively different from a conventional NIS junction. In view of available tunneling experiments in underdoped HTSC, the relevance of the model II is possible only when $\Delta_P, SDW < \Delta_S$ below $T_c$, which is quite unlikely at present. To clarify the issue of the PG more experiments on tunneling are essential and our study should serve as a useful benchmark.

We would like to thank H.J. Lee for invaluable discussion of his data and G. Deutscher for sending us their preprint prior to publication, respectively. This work was supported by the Korean Science and Engineering Foundation (KOSEF) through the Center for Strongly Correlated Materials Research (CSCMR) (2000)(YB) and through the Grant No. 1999-2-114-005-5 (YB and HYC).

\begin{thebibliography}{10}
\bibitem{1} For a review, see, for example, T. Timusk and B. Statt, Rep. Prog. Phys. 62, 61 (1999); M. Randeria, cond-mat/9710223.
\bibitem{2} V. J. Emery and S. A. Kivelson, Nature 374, 434 (1995).
\bibitem{3} J. R. Engelbrecht, A. Nazarenko, M. Randeria, and E. Dagotto, Phys. Rev. B 57, 13406 (1998); Q. Chen, I. Kosztin, B. Janko, and K. Levin, Phys. Rev. Lett. 81, 4708 (1998).
\bibitem{4} D. Pines, Physica C 282-287, 273 (1997); A. V. Chubukov, D. Pines, and B. P. Stojkovic, J. Phys. Condens. Matter 8, 10017 (1996); A. V. Chubukov and J. Schmalian, Phys. Rev. B 57, R11085 (1998).
\bibitem{5} C. Castellani et al., cond-mat/0001231; R.S. Gonnelli et al., cond-mat/0003100.
\bibitem{6} Ch. Renner et al., Phys. Rev. Lett. 80, 149 (1998); N. Miyakawa, et al., Phys. Rev. Lett. 83, 1018 (1999).
\bibitem{7} J.L.Tallon, cond-mat/9911422.
\bibitem{8} G. Deutscher, Nature 397 410 (1999).
\bibitem{9} V.M. Krasnov, A.Yurgens, D. Winkler, P. Delsing and T.Claeson, cond-mat/0002172.
\end{thebibliography}
[10] H.Y. Choi, Y. Bang, and D. K. Campbell, cond-mat/9902125 (to appear in Phys. Rev. B).
[11] H.J. Lee et al., (private communication).
[12] G. Deutscher et al., preprint.
[13] G.E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B 25, 4525 (1982).
[14] A.M. Gabovich and A.I. Voitenko, Phys. Rev. B 52, 7437 (1995).
[15] Ch. Niedermayer et al., Phys. Rev. Lett. 80, 3843 (1998); K. Yamada et al., Phys. Rev. B 57, 6165 (1998); Y.S. Lee et al., cond-mat/9902157.
[16] J. Bardeen, Phys. Rev. Lett. 6, 57 (1961); M.H. Cohen, L.M. Falicov, and J.C. Phillips, Phys. Rev. Lett. 8, 316 (1962).
[17] J.R. Schrieffer, X.G. Wen and S.C. Zhang, Phys. Rev. B 39, 11663, (1989).