NEWTONIAN APPROACH TO THE MATTER POWER SPECTRUM OF THE GENERALIZED CHAPLYGIN GAS

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We model the cosmic medium as the mixture of a generalized Chaplygin gas and a pressureless matter component. Within a neo-Newtonian approach we compute the matter power spectrum. The 2dFGRS data are used to discriminate between unified models of the dark sector and different models, for which there is separate dark matter, in addition to that accounted for by the generalized Chaplygin gas. Leaving the corresponding density parameters free, we find that the unified models are strongly disfavored. On the other hand, using unified model priors, the observational data are also well described, in particular for small and large values of the generalized Chaplygin gas parameter α.

Among the host of models that have been proposed for dark matter and dark energy over the last years, there are unified models of the dark sector according to which there is just one dark component that simultaneously plays the role of dark matter and dark energy. The most popular proposal along this line is the Chaplygin gas, an exotic fluid with negative pressure that scales as the inverse of the energy density [1]. This phenomenologically introduced equation of state can be given a string theory based motivation [2]. It has also been generalized in different phenomenological ways [3]. Another example for a unification scenario for the dark sector is a bulk viscous model of the cosmic substratum [4]. While the Chaplygin gas model (in its traditional and generalized forms) has been very successful in explaining the supernovae type Ia data [5], there are claims that it does not pass the tests connected with structure formation because of predicted but not observed strong oscillations of the matter power spectrum [6]. It should be mentioned, however, that oscillations in the Chaplygin gas component do not necessarily imply corresponding oscillations in the observed baryonic power spectrum [7].

The generalized Chaplygin gas is characterized by the equation of state

\[ p = -\frac{A}{\rho^\alpha}. \]  (1)

For \( A > 0 \) the pressure \( p \) is negative, hence it may induce an accelerated expansion of the universe. The corresponding sound speed is positive as long as \( \alpha > 0 \). Recently, a gauge-invariant analysis of the baryonic matter power spectrum for generalized Chaplygin gas cosmologies was shown to be compatible with the data for parameter values \( \alpha \approx 0 \) and \( \alpha \geq 3 \) [8]. This result seems to strengthen the role of Chaplygin gas type models as competitive candidates for the dark sector. The present work provides a further investigation along these lines. While we shall rediscover the mentioned results of [8], albeit in a different framework, we also extend the scope of the analysis in the following sense. The authors of [8] have shown that Chaplygin gas cosmologies are consistent with the data from structure formation because of predicted but not observed strong oscillations of the matter power spectrum [6]. It should be mentioned, however, that oscillations in the Chaplygin gas component do not necessarily imply corresponding oscillations in the observed baryonic power spectrum [7].

Our study relies on a neo-Newtonian approach which represents a major simplification of the problem. In some sense, the neo-Newtonian equations can be seen as the introduction of a first order relativistic correction to the usual Newtonian equations [13]. The neo-Newtonian equations for cosmology [12,13,14,15] modify the Newtonian equations in a way that makes the pressure dynamically relevant already for the homogeneous and isotropic background. This allows us to describe an accelerated expansion of the Universe as the consequence of a sufficiently large effective negative pressure in a Newtonian framework. While the neo-Newtonian approach reproduces the GR background dynamics exactly, differences occur at the perturbative level. However, the GR first-order perturbation dynamics

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and its neo-Newtonian counterpart coincide exactly in the case of a vanishing sound speed \[13\]. On small scales one expects the spatial pressure gradient term to be relevant and the difference to the GR dynamics should be of minor importance. Since the observational data correspond to modes that are well inside the Hubble radius, the use of a Newtonian type approach seems therefore adequate.

On this basis our analysis extends previous neo-Newtonian studies to the two-component case. One of the components is a generalized Chaplygin gas, the other one represents pressureless matter. The advantage of employing a neo-Newtonian approach is a gain in simplicity and transparency. Our neo-Newtonian approach reproduces the parameter estimations for the unified dark matter/dark energy in \[8\] also numerically. Backed up by this success of the neo-Newtonian approach we then enlarge the scope of our analysis and test the validity of the unified model itself by relaxing the unified model priors used in \[8\]. Denoting the present value of the Chaplygin gas density parameter by \(\Omega_{\text{cg}}\), we admit the total present matter density parameter \(\Omega_{\text{m0}}\) to be the sum of an additional dark matter component with density parameter \(\Omega_{\text{dm0}}\) and the baryon contribution \(\Omega_{\text{b0}}\), i.e., \(\Omega_{\text{m0}} = \Omega_{\text{dm0}} + \Omega_{\text{b0}}\). Leaving the density parameters free, we investigate whether or not the unified model with \(\Omega_{\text{m0}} \approx 0.96, \Omega_{\text{b0}} \approx 0.04\) and \(\Omega_{\text{dm0}} \approx 0\) is favored by the large-scale structure data. We mention that a similar investigation using supernova type Ia data reveals that the unification scenario is the most favored one \[5\].

In the framework of the neo-newtonian formalism, in the conservation equation one takes into account the work done by the pressure during the expansion of the universe. At the same time, the equation for the gravitational potential must be modified in order to render the equations compatible. This has been done in references \[12, 13, 14\]. The final equations are

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) + p \nabla \cdot \vec{v} = 0, \tag{2}
\]

\[
\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla p}{\rho + p} - \nabla \phi, \tag{3}
\]

\[
\nabla^2 \phi = 4\pi G (\rho + 3p). \tag{4}
\]

For the case of two non-interacting fluids with energy densities \(\rho_c\) and \(\rho_m\) and pressures \(p_c\) and \(p_m = 0\), respectively, the equations are:

\[
\frac{\partial \rho_c}{\partial t} + \nabla \cdot (\rho_c \vec{v}_c) + p_c \nabla \cdot \vec{v}_c = 0, \tag{5}
\]

\[
\frac{\partial \vec{v}_c}{\partial t} + \vec{v}_c \cdot \nabla \vec{v}_c = -\frac{\nabla p_c}{\rho_c + p_c} - \nabla \phi, \tag{6}
\]

\[
\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{v}_m) = 0, \tag{7}
\]

\[
\frac{\partial \vec{v}_m}{\partial t} + \vec{v}_m \cdot \nabla \vec{v}_m = -\nabla \phi, \quad \nabla^2 \phi = 4\pi G (\rho_m + \rho_c + 3p_c). \tag{8}
\]

The subscript \(m\) stands for pressureless matter and the subscript \(c\) for the (generalized) Chaplygin gas component. Considering now an isotropic and homogeneous universe with \(\rho = \rho(t), p = p(t)\) and \(\vec{v} = \frac{4}{3} \vec{r}\), we find

\[
\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_m + \rho_c), \tag{10}
\]

\[
\frac{\dot{a}}{a} = -\frac{4\pi G}{3} (\rho_c + \rho_m + 3p_c). \tag{11}
\]

Let us define the fractional density contrasts

\[
\delta_c = \frac{\delta \rho_c}{\rho_c} \quad \text{and} \quad \delta_m = \frac{\delta \rho_m}{\rho_m} \tag{12}
\]

for the Chaplygin gas and matter components, respectively, the first-order perturbation equations for the system \(5-11\) are

\[
\ddot{\delta}_c + \left\{2 \frac{\dot{a}}{a} - \frac{\dot{\omega}_c}{1 + \omega_c} + \frac{3}{a} \left(\frac{\dot{v}_c^2}{1 + \omega_c} - \omega_c \right) \right\} \dot{\delta}_c + \left\{3 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) - \frac{v_c^2}{1 + \omega_c} \right\} \left(\frac{\dot{\omega}_c}{1 + \omega_c} - \omega_c \right) + 3 \frac{\dot{a}}{a} \left(\frac{\dot{v}_c^2}{1 + \omega_c} - \omega_c \right) \dot{\delta}_c = 4\pi G \rho_c (1 + \omega_c), \tag{13}
\]
the fitting of the observational data for a given theoretical model with specific values of the free parameters. Hence, 

and

\[ \dot{\delta}_m + \frac{2}{a} \dot{\rho}_m - 4\pi G \rho_m \delta_m = 4\pi G \rho_m (1 + 3 \nu_c^2) \delta_c , \tag{14} \]

where \( v_c^2 = \frac{\partial^2 \rho_m}{\partial \rho_c^2} \) and \( \omega_c = \frac{\rho_c}{\rho_m} \). The quantity \( k^2 \) denotes the square of the comoving wave vector. Dividing the equations (13) and (14) by \( H^2 \) and redefining the time as \( t H_0 = t \), these equations become dimensionless. In terms of the scale factor \( a \) as dynamical variable, the system (13)-(14) takes the form

\[
\begin{align*}
\delta''_c + \left\{ \frac{2}{a} + g(a) - \frac{\omega'_c(a)}{1 + \omega_c(a)} - 3 \frac{1 + \alpha}{a} \omega_c(a) \right\} \delta'_c & + \frac{3}{2 a^3 f(a)} \left[ g(a) \left( 1 + \alpha \right) \omega_c(a) + \frac{3}{a} \omega'_c(a) \right] \delta_c \\
& + \frac{3 \Omega_m a}{2 f(a)} h(a) \left[ 1 + \omega_c(a) \right] \left[ 1 - 3 \alpha v_c(a) \right] \delta_m 
\end{align*}
\]

\tag{15}

where \( k^2 \) denotes the square of the comoving wave vector and \( l_H = c H_0^{-1} \) is the present Hubble radius. The prime denotes a derivative with respect to \( a \) and the definitions

\[
\begin{align*}
f(a) & = \frac{\dot{a}^2}{H_0^2} = \left[ \frac{\Omega_m + \Omega_0 a^3 h(a)}{a} \right] + \Omega_{k0} , \tag{17} \\
g(a) & = \frac{\ddot{a}}{a^2} = - \frac{\Omega_m + \Omega_0 h(a) - 3 \dot{A} h^{-\alpha} a^3}{2a \left[ \Omega_m + \Omega_0 a^3 h(a) + \Omega_{k0} a \right]} , \tag{18} \\
h(a) & = \frac{\dot{A} + (1 - \dot{A}) a^{-3(1+\alpha)} h^{-\alpha}}{h(a)^{1+\alpha}} , \tag{19} \\
\omega_c(a) & = - \frac{\dot{A}}{h(a)^{1+\alpha}}, \tag{20}
\end{align*}
\]

with \( \tilde{\dot{A}} = \frac{\dot{A}}{\rho_{c0}^{1+\alpha}} \), \( v_{c0}^2 = \alpha \tilde{\dot{A}} \)

have been used. Recall that \( \Omega_m0 = \Omega_{dm0} + \Omega_{b0} \). For the unified model to be an adequate description one expects \( \Omega_m0 \approx \Omega_{b0} \). In case the data indicate a substantial fraction of \( \Omega_{dm0} \), the unified model will be disfavored.

The power spectrum is defined by

\[ \mathcal{P} = \delta_k^2 , \tag{22} \]

where \( \delta_k \) is the Fourier transform of dimensionless density contrast \( \delta_m \). We will constrain the free parameters using the quantity

\[ \chi^2 = \sum_i \left( \frac{P_i^O - P_i^T}{\sigma_i} \right)^2 , \tag{23} \]

where \( P_i^O \) is the observational value for the power spectrum, \( P_i^T \) is the corresponding theoretical result and \( \sigma_i \) denotes the error bar. The index \( i \) refers to a measurement corresponding to given wavenumber. The quantity \( \chi^2 \) qualifies the fitting of the observational data for a given theoretical model with specific values of the free parameters. Hence, \( \chi^2 \) is a function of the free parameters of the model. The probability distribution function is then defined as

\[ F(x_n) = F_0 e^{-\chi^2(x_n)/2} , \tag{24} \]

where the \( x_n \) denote the ensemble of free parameters and \( F_0 \) is a normalization constant. In order to obtain an estimation for a given parameter one has to integrate (marginalize) over all the other ones. For a more detailed
FIG. 1: The two-dimensional probability distribution function (PDF) for $\Omega_{dm0}$ and $\Omega_{\Lambda0}$ (left) and the corresponding one-dimensional probability distribution functions for the non-flat $\Lambda$CDM model. In the left panel: the darker the color, the smaller the probability.

FIG. 2: The results for the general case with four free parameters. From left to right: the one-dimensional PDFs for $\alpha$, $\bar{A}$, $\Omega_{c0}$ and $\Omega_{dm0}$.

description of this statistical analysis see reference [5]. From now on we focus on the 2dFGRS observational data for the power spectrum [16]. We use the data that are related with the linear approximation, that is, those for which $k h^{-1} \leq 0.185 Mpc^{-1}$, where $h$ is defined by $H_0 \equiv 100 \cdot h \cdot km/s \cdot Mpc$. This definition should not be confused with the preceding definition of the function $h(a)$. To fix the initial conditions we use the BBKS transfer function [9]. This procedure is described in more detail in references [10, 11].

To “gauge” our approach, let us first consider the $\Lambda$CDM model. In the general (non-flat) case there are two parameters: $\Omega_{dm0}$ and $\Omega_{\Lambda0}$. In figure 1 we show the two-dimensional probability distribution function (PDF) as well the one-dimensional PDFs for the dark matter parameter $\Omega_{dm0}$ and for the cosmological constant parameter $\Omega_{\Lambda0}$, respectively. From the two dimensional graphic it is clear that there is a large degeneracy for the parameter $\Omega_{\Lambda0}$, while the region of allowable values for $\Omega_{dm0}$ is quite narrow. The degeneracy for the cosmological constant density is less visible in the one-dimensional PDF graphic, but it is still considerable. Incidentally, the minimum value for the $\chi^2$ parameter is 0.3822 for $\Omega_{dm0} = 0.2387$ and $\Omega_{\Lambda0} = 0.5937$, corresponding to an open universe.

The four free parameters to be constrained in our Chaplygin gas model are $\Omega_{dm0}$, $\Omega_{c0}$, $\bar{A}$ and $\alpha$. The one-dimensional PDFs for $\alpha$, $\bar{A}$, $\Omega_{dm0}$ and $\Omega_{c0}$ are displayed in figure 2. It can be seen that the preferred values are either $\alpha \ll 1$ or $\alpha \geq 2$, while the probability is higher for large values of $\Omega_{dm0}$ and small values of $\Omega_{c0}$. This show clearly that the unification scenario is disfavored.

If the unification scenario with dark matter and dark energy as a single fluid in a spatially flat universe is imposed from the beginning, the results of reference [8] are essentially confirmed: there are parameter ranges for which the data are well described by the generalized Chaplygin gas model, see figure 3. The probability distribution function for $\alpha$ is high for very small (near zero) or very large (greater than 2) values of $\alpha$. Allowing the parameter $\bar{A}$ to vary, we find that its one-dimensional PDF initially decreases with $\bar{A}$, but increases as $\bar{A} = 1$ is approached. Notice that values $\alpha > 1$ imply a superluminal sound speed and are therefore unphysical (see, however, [8]).

What is the origin of these apparently contradictory results? The first aspect to be mentioned is that the matter power spectrum data only poorly constrain the dark energy component. Even for the $\Lambda$CDM model the matter power
spectrum gives information mainly on the dark matter component, the dark energy component remaining largely imprecise. It is not by chance that the dark energy concept emerged from the supernova data. Our results for the Chaplygin gas model show that a large amount of dark matter, different from those described by the Chaplygin gas, is necessary to fit the data. However, the dispersion is quite high. For the flat case with a three-dimensional parameter space we find at $2\sigma$, that $\Omega_{dm0} = 1^{+0.09}_{-0.00}$. Another point is the use of the neo-Newtonian formalism. However, for small values of the parameter $\alpha$, the main case of interest here, the differences to the full general relativistic treatment are not expected to be substantial. Moreover, in the cases of overlap the results of the full theory are reproduced. Finally, possible statistical subtleties may influence the outcome of the investigation. But as far as we could test the statistical analysis (precision, crossing different information, etc), the results seem to be robust. If this is really the case, we must perhaps live with the fact that, while the SNe type Ia data favor a unified model of the dark sector [5], this scenario is disfavored if large scale structure data are taken into account, unless specific priors are imposed.

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