Research of combined grey model based on entropy weight for predicting anchor bolt bearing capacity

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Abstract. A combined prediction model based on entropy weight was proposed to overcome the shortcomings of single grey model for predicting anchor bolt bearing capacity. Firstly, the grey GM (1,1) model and grey Verhulst model are used to predict the bearing capacity of anchor bolt according to the P-S curve obtained from pull test. Secondly, the entropy weight method is utilized to obtain the weight indexes of the two mode. Then two models are combined by the weight index to achieve final bearing capacity prediction. The experimental results show that the combined prediction model based on entropy weight can improve the accuracy of the grey prediction model as the load increasing.

1. Introduction

Our country is short of water and reasonable exploitation of groundwater is important foundation to guarantee healthy development of national economy. In the new situation, visual and scientific construction of 3-D visualization model of underground water geologic structure has become an active demand in groundwater detection work.

In engineering project, the anchorage quality directly determines the stability of engineering structures, and the detection of the anchor bolt bearing capacity is one of the important links. Pull test method is a commonly used method to detect the bearing capacity of anchor bolts. P-S curve of anchor bolt is drawn by measuring static load and displacement data in this method. For obtaining an accurate value of ultimate bearing capacity, it is also necessary to perform a destructive pull test on some anchor bolts. Thus, it is a key research problem to predict the ultimate bearing capacity of anchor bolt by a small amount of pull-out test data to avoid affecting the engineering structure.

Professor Deng J. [1] proposed the grey theory-a new method to study the problem of little data, poor information, and uncertainty in 1982. Grey theory prediction model can be established to predict static load, displacement curve (P-S curve) and the ultimate bearing capacity of the anchor.

There are many researches on grey theory, but few studies have applied it to the prediction of anchor bolt bearing capacity. Xu M. et al.(2003) introduced a grey model prediction method for anchor bearing capacity[2]; Liu M. et al.(2006) explored grey theory to predict the pullout capacity of anchor bolts, and did engineering verification[3]; Sun X. et al.(2014) used the improved grey GM (1,1) model to predict the ultimate bearing capacity of the bolt[4]; Yang M. et al.(2018) used an improved grey model based on logarithmic transformation to predict the ultimate bearing capacity of tunnel-type anchorage, which effectively improved the prediction accuracy[5].

In this paper, we firstly analyze the mathematical model of grey GM (1,1) model and grey Verhulst model. Single grey GM (1,1) prediction model is more suitable for time series with approximative exponential changes, and grey Verhulst prediction model is mainly utilized to describe the process
with saturation. They cannot fully meet the requirements for anchor capacity prediction individually, a combination prediction model based on entropy weight method is proposed to overcome this disadvantage.

2. Combined grey model based on entropy weight
The load applied in pull-out test is generally equidistant, but the displacement of anchor bolt usually has unequal interval. Then grey GM (1,1) model to be established should be non-equidistant. The P-S curves of engineering anchor bolts often do not fully meet the exponential-like shape of theoretical curve. It may appear convex or concave. Grey Verhulst model is suitable to characterize saturation with “S” shape[6], so it’s selected for this feature of P-S curve. Entropy weight method determines the index weight of different schemes according to the size of the data provided by each variable[7]. We use the entropy weight method to combine predictions of two grey models to get more accurate results.

2.1. Non-equidistant GM(1,1) model
Assuming \( X^{(0)}(t) = \{x^{(0)}(t_1), x^{(0)}(t_2), \ldots, x^{(0)}(t_n)\} \) is original sequence. \( X^{(0)}(t_i) \) is non-equidistant sequence when time interval \( \Delta t_i = t_i - t_{i-1} \) is not constant. After an accumulation operation is performed on the original sequence, new sequence \( X^{(1)}(t_i) = \{x^{(1)}(t_1), x^{(1)}(t_2), \ldots, x^{(1)}(t_n)\} \) generate, where \( X^{(1)}(t_i) = \sum_{j=1}^{i} x^{(0)}(t_j) \Delta t_j, i = 2, 3, \ldots, n \), \( x^{(1)}(t_i) = x^{(0)}(t_i) \). A grey GM (1,1) model is established for the accumulation sequence \( X^{(1)}(t_i) \), and its whitening differential equation is:

\[
\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b
\]

The grey differential equation of GM(1,1) model is:

\[
\hat{x}^{(0)}(k) + a\hat{x}^{(1)}(k) = b \tag{2}
\]

where \( a \) and \( b \) are called development coefficient and grey action quantity respectively, \( \hat{x}^{(0)}(k) \) is the background value of \( x^{(0)}(k) \) on the interval \([k - \Delta t, k]\). In actual application, \( \hat{x}^{(0)}(k) \) is generally generated by the mean of the nearest \( x^{(0)}(k) \) neighbors, \( \hat{x}^{(0)}(k) = \frac{1}{2}[x^{(0)}(k)+x^{(0)}(k_{-1})], i = 2, 3, \ldots, n \)

In the grey modeling process, least square method is selected to estimate the parameter vector \( \hat{a} \) of the model:

\[
\hat{a} = (a, b)^T = (B^T B)^{-1} B^T Y \tag{3}
\]

Where

\[
Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -e^{a(t-1)} \\ -e^{a(t-2)} \\ \vdots \\ -e^{a(t-n)} \end{bmatrix} \tag{4}
\]

After solving the grey differential equation(formula (2)), the time-responsive result is:

\[
\hat{x}^{(1)} = \frac{b}{a} + (x^{(0)}(1) - \frac{b}{a})e^{-at}\tag{5}
\]

By discretizing equation (5), we can get Non-equidistant GM(1,1) model:

\[
\hat{x}^{(1)}(k) = \begin{cases} 
\frac{1}{\Delta t_i} \left(1 - e^{a\Delta t_i}\right) \left(x^{(0)}(t_i) - \frac{b}{a}\right)e^{-at_i}, i = 2, 3, \ldots, n 
\end{cases}\]

2.2. Grey Verhulst model
Verhulst model and GM (1,1) model have similar initial settings. The original sequence is \( x^{(0)} = \{x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)} \} \) and \( x^{(1)} = \{x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)} \} \) is the one-time accumulation (1-AGO) sequence of \( x^{(0)} \), where \( x_1^{(0)} = x_{1}^{(0)} + \sum_{k=2}^{n} x_{k}^{(0)} \), \( k = 1, 2, \ldots, n \). \( Z^{(0)} \) is the sequence generated by mean of nearest neighbors of \( X^{(0)} \), \( e^{(i)} = (z_1^{(i)}, z_2^{(i)}, \ldots, z_n^{(i)}) \), where \( z_k^{(i)} = 0.5(x_{k}^{(i)} + x_{k+1}^{(i)}) \), \( k = 2, 3, \ldots, n \). The grey Verhulst model is:

\[
X^{(0)} + aZ^{(0)} = b(e^{(i)})^2
\]  
(7)

Parameter vector \( \hat{\alpha} \) of Verhulst model satisfy \( \hat{\alpha} = (a, b)^T = (B^T B)^{-1}B^T Y \), where

\[
Y = \begin{bmatrix}
x_2^{(0)} \\
x_3^{(0)} \\
\vdots \\
x_n^{(0)}
\end{bmatrix}, \quad B = \begin{bmatrix}
-z_2^{(1)} & (z_2^{(1)})^2 \\
-z_3^{(1)} & (z_3^{(1)})^2 \\
\vdots & \vdots \\
-z_n^{(1)} & (z_n^{(1)})^2
\end{bmatrix}
\]  
(8)

The time response sequence of the grey Verhulst model is:

\[
\hat{X}^{(1)}_{k+1} = \frac{ax_0^{(0)}}{bx_1^{(0)} + (a - bx_1^{(0)})e^{ak}}
\]  
(9)

where \( x_0^{(1)} \) is taken as \( x_1^{(0)} \), then formula (9) is changed into:

\[
\hat{X}^{(1)}_{k+1} = \frac{ax_0^{(0)}}{bx_1^{(0)} + (a - bx_1^{(0)})e^{ak}}
\]  
(10)

The decremental reduction formula is:

\[
\hat{X}^{(1)}_{k+1} = \hat{X}^{(1)}_{k+1} - \hat{X}^{(1)}_k
\]

2.3. Entropy weight method

Entropy weight method is an objective method. It can exclude components that are easily affected by subjective factors when calculating weights. Assume that \( \{x_i, t=1,2,\ldots,N\} \) is a certain indicator sequence of the same object, and there are \( m \) single prediction methods in the system. The prediction value of the \( i \)-th single prediction method at time \( t \) is \( X_{it} \), \( i = 1,2,\ldots,m \), \( t = 1,2,\ldots,N \). Then the relative prediction error \( f_{it} \) of the \( i \)-th prediction method at time \( t \) is:

\[
f_{it} = \begin{cases} 
1, & \frac{|x_t - x_{it}|}{x_t} \geq 1 \\
\frac{|x_t - x_{it}|}{x_t}, & 0 \leq \frac{|x_t - x_{it}|}{x_t} \leq 1
\end{cases}
\]  
(11)

According to the definition of entropy value, we redefine the variation degree of error sequence when a single model is predicted. This algorithm is divided into five basic steps:

(1) Normalize the prediction error of each single model:

\[
U_{it} = f_{it} / \sum_{i=1}^{N} f_{it}, t = 1,2,\ldots,N
\]

then \( \sum_{i=1}^{N} U_{it} = 1, i = 1,2,\ldots,m \).

(2) Calculate the entropy \( h_i \) of the relative error of the \( i \)-th single model:

\[
h_i = -k \sum_{t=1}^{N} (U_{it} \ln U_{it}), i = 1,2,\ldots,m
\]  
(13)

where \( k \) is a constant value and \( k > 0 \). We select \( k = 1/\ln N \), then \( 0 \leq h_i \leq 1 \).

(3) Determine the variation coefficient \( s_i \) of the relative error sequence of the \( i \)-th method:

\[
s_i = 1 - h_i
\]  
(14)
(4) Then the weighting coefficient $w_i$ of each single prediction model is:

$$w_i = \frac{1}{m-1} \left(1 - \frac{S_i}{\sum S_i} \right)$$

(15)

(5) The final prediction of the combined grey model is:

$$X = \sum_{i=1}^{m} w_i x_{it}, t = 1, 2, ..., N$$

(16)

3. Prediction analysis of anchor bolt bearing capacity

The theoretical P-S curve of anchor bolt is generally divided into three stages: elastic stage, elastic-plastic stage and failure stage. The curve of elastic stage is approximately a sloped straight line. The elastic-plastic stage is a similar exponential curve. The failure stage is almost a straight line parallel to the S axis. In actual engineering, the P-S curve may be different from the theoretical curve, sometimes there are one or several concave points or convex starting points. The bearing capacity of three bolts with different characteristics is predicted here. Their load and displacement data are from Wang X. et al. experiments (as shown in Table 1)[8]. The ultimate bearing capability of the bolts are separately 700kN, 700kN and 750kN when they are failure.

| No. | Anchor bolt A | Anchor bolt B | Anchor bolt C |
|-----|---------------|---------------|---------------|
|     | Load /kN | Displacement S/mm | Load /kN | Displacement S/mm | Load /kN | Displacement S/mm |
| 1   | 80        | 1.09          | 350     | 4.86          | 55      | 1.6             |
| 2   | 240       | 3.94          | 420     | 6.82          | 165     | 5.5             |
| 3   | 400       | 6.02          | 490     | 8.85          | 275     | 10.2            |
| 4   | 480       | 8.14          | 560     | 20.55         | 330     | 13.5            |
| 5   | 560       | 11.01         | 595     | 23.82         | 385     | 15.5            |
| 6   | 640       | 18.41         | 630     | 29.24         | 440     | 18.5            |
| 7   | 680       | 33.97         | 665     | 40.63         | 533     | 21.0            |
| 8   |           |               |         |               | 583     | 37.3            |
| 9   | 700       | failure       | 700     | failure       | 750     | failure         |

Grey GM (1,1) model, grey Verhulst model and Combined grey model based on entropy weight are utilized to predict the bearing capacity of the three anchor bolt samples, as shown in Table 2. The P-S curves excluding the ultimate bearing capacity are shown in Figure 1, 2 and 3.

| No. | Anchor bolt A | Anchor bolt B | Anchor bolt C |
|-----|---------------|---------------|---------------|
|     | Prediction load /kN | Prediction load /kN | Prediction load /kN |
| 80  | 80            | 279.1         | 160.5         | 226.7 |
| 240 | 420.8         | 422.4         | 421.6         | 165   |
| 400 | 476.6         | 490.4         | 483.8         | 275   |
| 480 | 616.1         | 549.6         | 581.2         | 330   |
| 560 | 628.8         | 597.9         | 612.6         | 385   |
| 640 | 440           | 459.9         | 375.8         | 394.0 |
The P-S curve of anchor bolt A is close to the theoretical curve. The prediction curves of three models are all in good agreement with the actual curve (Figure 1). The predicted results of grey Verhulst model are all less than actual values, and grey GM(1,1) model are more than actual value when load exceeds 600kN. The combined grey model is closest to the actual data when load is beyond 600kN, especially for the ultimate bearing capacity with an error of only 5.6kN.

The P-S curve of anchor bolt B has several concave points according to theoretical curve (Figure 2). For those points, the predicted results of grey GM(1,1) are higher than actual values, and Verhulst model has the most accurate predictions. But for the ultimate bearing capacity, the predictions of combined grey model have the minimum error.

When comparing with theoretical curve, the P-S curve of anchor bolt C has convex point (Figure 3). When the load is lower than 533kN, the actual P-S curve is approximately linear, and the grey GM(1,1) model has the most accurate prediction. As the load increases, the P-S curve tends to saturation, the prediction error of grey GM(1,1) model is also the largest. The predicted results of grey Verhulst model are all less than actual values. The prediction error of the combined grey model is in the middle when load is less than 533kN and highest when load is more than 533kN.

From the P-S curve prediction results of three different bolts, the combined grey model based on entropy weight has the most accurate prediction results under high loads, especially for the prediction of ultimate bearing capacities.

4. Conclusions
Aiming at the anchor bolt bearing capacity, we propose a combined grey model based on entropy weight to improve the prediction accuracy. We analyzed three kinds of P-S curve of anchor bolts with grey GM(1,1) model, grey Verhulst model and combined grey model. The results show that three models all have acceptable predictions for P-S curve similar to theoretical shape, grey Verhulst model is best for P-S curve with concave points in elastic and elastic-plastic stage, grey GM(1,1) model has minimum prediction error for P-S curve with convex points in in elastic and early elastic-plastic stage, but our combined grey model gets highest prediction accuracy when the load approaches the failure stage for all situations.

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