Current–oscillator correlation and Fano factor spectrum of quantum shuttle with finite bias voltage and temperature

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Received 8 November 2011, in final form 6 February 2012
Published 3 April 2012
Online at stacks.iop.org/JPhysCM/24/175301

Abstract
A general master equation is derived to describe an electromechanical single-dot transistor in the Coulomb blockade regime. In the equation, Fermi distribution functions in the two leads are taken into account, which allows one to study the system as a function of bias voltage and temperature of the leads. Furthermore, we treat the coherent interaction mechanism between electron tunneling events and the dynamics of excited vibrational modes. Stationary solutions of the equation are numerically calculated. We show that current through the oscillating island at low temperature appears to have step-like characteristics as a function of the bias voltage and the steps depend on the mean phonon number of the oscillator. At higher temperatures the current steps would disappear and this event is accompanied by the emergence of thermal noise of the charge transfer. When the system is mainly in the ground state, the zero frequency Fano factor of current manifests sub-Poissonian noise and when the system is partially driven into its excited states it exhibits super-Poissonian noise. The difference in the current noise would almost be removed for the situation in which the dissipation rate of the oscillator is much larger than the bare tunneling rates of electrons.

(Some figures may appear in colour only in the online journal)

1. Introduction
Nanomechanical oscillators have been the subject of active research during the past decade. They have potential applications in precision measurements [1] and quantum information [2]. A typical example of the structures is electromechanical tunneling called quantum shuttle which transports electrons with the help of a mechanically oscillating island [3]. Such devices with different sizes have been realized in experiments [4, 5]. Theoretically, to derive a general equation for the electromechanical system appears to be a difficult task, which in a way hinders the exploitation of some properties of the system.

When the island is very small (within a few nanometer in diameter), the mechanical oscillator can be seen as a quantum system and $I$–$V$ curves exhibit stepwise characteristics [4]. It is widely accepted in theory that the phenomenon can be interpreted based on the multi-channels which are provided by the quantized modes of the oscillating island [6–10]. In these theoretical studies the coherent coupling between electron transport and the vibrational modes, which might become important when the system works in the quantum regime, is not taken into account.

In a fully quantum mechanical description of the electromechanical process [11], the coherent coupling is included with a cost of dealing with a large matrix in the mathematical treatment. The quantum mechanical model of the coherent dynamics is further developed to investigate the shuttling mechanism [12–14]. According to the charge–position (momentum) correlation, motion of the quantum shuttle can be divided into the regimes of shuttling, tunneling and their coexistence [12]. In the shuttling regime, electron transport is highly deterministic characterized by the extraordinary sub-Poissonian Fano factor [13]. Measuring the
shot noise, transition between tunneling and shuttling can be identified [14]. As its extension to spintronics, based on the coupling between transport of spin-polarized electrons and mechanical degrees of freedom of island the shuttle instability is predicted to appear as two stationary domains such as vibronic and shuttling, depending on the applied electric and magnetic fields [15]. A good application of the quantum shuttle in spin-detection has been predicted recently [16]. The above studies are carried out in the limit of large bias [11–13, 15], or with the electron distribution functions which are independent of energy levels of the mechanical oscillator [14, 16]. Further studies about the current fluctuation, especially at finite bias voltages and large range of temperatures, is still an important issue which has not been adequately discussed in the past.

In this paper, we will develop a very general master equation for the description of the electromechanical tunneling. We consider Fermi distribution functions of the electronic leads, in which discrete energy levels of the vibrational modes are involved. We will know later that when the bias voltage is not very large the distribution functions are sensitive to the oscillator levels. In addition, both the diagonal and off diagonal terms of the coupling between oscillator dynamics and electron transfer are incorporated in the equation and both of them are shown to be very important to describe the device. Considering the position dependence of tunneling rates and all vibrational modes of the harmonic oscillator in the equation, we intend to overcome the shortcomings in a recent attempt at describing the electromechanical system where the position dependence was neglected and just two modes of the oscillator were taken into account [17]. Furthermore, we will show that our master equation can be obtained by treating the tunneling and the dissipation terms uniformly instead of with the approach of separated prescription adopted in the early derivations [11, 12, 17]. In the limit of low temperature and high bias voltage, the present master equation would be in accord with those given previously [12, 14]. Using the equation, we found some important characteristics of the electromechanical tunneling. Current steps as a function of bias voltage are correlated with the averaged phonon number of the mechanical oscillator at low temperature. The steps disappear when temperature is increased to a certain quantity and this process corresponds to the emergence of thermal noise. The voltage range of one step as a function of gate voltage is half that as a function of bias voltage. At low enough bias voltage, the system dominantly works in its ground state and the zero frequency damping of the vibrational mode.

2. The system model and derivation of the master equation

We consider a model in which a single-level quantum dot (QD) is connected to two leads by some elastic molecules. Mechanical vibration of the dot is described by a harmonic oscillator with an effective mass \( m \). With bias voltage, \( V \), between the two leads, electrons can be transferred from one lead into another. The sketch of the model is shown in figure 1. With respect to the energy level of the QD, the chemical potentials in the left and right leads are \( eV/2 \) and \(-eV/2\), respectively, where \( e \) is the absolute value of the electron charge. We assume the capacitance of the QD is so small that its electron occupation number is 0 or 1. Initially, the QD is empty and the harmonic oscillator is in its ground state. When an electron jumps onto the QD, the electric field caused by the bias voltage exerts a force on the charged dot and drives the mechanical oscillator into its excited states. As a consequence, the electron transfer is influenced by the vibration. The total Hamiltonian of the model is the sum of the electron tunneling Hamiltonian \( H_{\text{tun}} \), the mechanical oscillator Hamiltonian \( H_{\text{mech}} \) and the coupling between charge and the oscillator \( H_{\text{driv}} \):

\[
H = H_{\text{tun}} + H_{\text{mech}} + H_{\text{driv}}.
\]

The electron tunneling Hamiltonian is written in the form

\[
H_{\text{tun}} = \varepsilon_0 c^\dagger c + \sum_{k,\gamma=\uparrow,\downarrow} \xi_{\gamma k} d_{\gamma k}^\dagger d_{\gamma k} + \hbar \sum_{k,\gamma=\uparrow,\downarrow} (T_{\gamma k} e^{\frac{\xi_{\gamma k}}{\hbar}} a_{\gamma k}^\dagger c + T_{k\gamma}^* e^{\frac{\xi_{\gamma k}}{\hbar}} c^\dagger a_{\gamma k}),
\]

where the first term is energy of the QD with annihilation and creation operators \( c, c^\dagger \), and the second term describes noninteracting electrons in the left (\( y = \uparrow \)) and right (\( y = \downarrow \)) leads. The operator \( d_{\gamma k}^\dagger (d_{\gamma k}) \) creates (annihilates) an electron with momentum \( k \) in the lead \( y \). Spin degree of freedom is not involved here. The third term represents electron tunneling between the leads and the QD. The coupling
strength is exponentially dependent on the dot position and the coordinate is denoted in the form of dimensionless operators as \(a\) and \(a^\dagger\). For simplicity, we define \(S_L = -1\), \(S_R = 1\) and \(\alpha = \frac{h}{\hbar^2 a_0^2}\) which is a fraction of the zero point position uncertainty \(\alpha_0 = \sqrt{\frac{h}{2m_0\hbar}}\) and the tunneling length \(\lambda\). The mechanical oscillator Hamiltonian is described by

\[
H_{\text{mech}} = \hbar \omega_0 a^\dagger a + \sum_k \hbar \omega_k b_k^\dagger b_k + \sum_k \hbar g_k (b_k^\dagger a + a^\dagger b_k).
\]

(3)

where the first term is the free evolution of the mechanical oscillator with the inherent frequency \(\omega_0\). The creation and annihilation operators \(b_k^\dagger, b_k\) in the second term characterize the Bosonic thermal bath. The last term represents oscillator dissipation due to the bath. The localized charge in the QD is coupled to the oscillator in the form

\[
H_{\text{dev}} = -\hbar \Omega (a^\dagger + a)c^\dagger c,
\]

(4)

where \(\Omega = \frac{e V_0}{\hbar}\) and \(d\) is the effective distance between the two leads. For simplicity, we assume only bias voltage \(V\) contribute to the electric field. Other electric environments of the QD may cause bias voltage independent electric field. It also contributes to the charge--oscillator coupling, which mainly plays the role of gate voltage by shifting the QD level and changing the number of vibrational states accessible for the electron transport [7]. We are not interested in this effect and so discard this term here.

Now, to derive the master equation, we rewrite the Hamiltonian into a free \(H_0\) and an interaction part \(H_{\text{int}} + H_1\), where

\[
H_0 = \varepsilon_0 c^\dagger c + \sum_{k,y=1} \xi_d d_k^\dagger d_k + \hbar \omega_0 a^\dagger a + \sum_k \hbar \omega_k b_k^\dagger b_k.
\]

(5)

and

\[
H_1 = H - H_0 - H_{\text{dev}}.
\]

(6)

In the interaction picture, the total density matrix \(\rho_\text{int}(t)\) is \(\tilde{\rho}_\text{int}(t) = e^{iH_0/\hbar}\rho_\text{int}(t)e^{-iH_0/\hbar}\) which satisfies the Liouville–von Neumann equation

\[
\frac{\partial \tilde{\rho}_\text{int}(t)}{\partial t} = \frac{i}{\hbar} [\tilde{H}_\text{int}(t), \tilde{\rho}_\text{int}(t)] + \frac{1}{\hbar} \{\tilde{H}_1(t), \tilde{\rho}_\text{int}(t)\},
\]

(7)

where \(\tilde{H}_\text{int}(t) = e^{iH_0/\hbar}H_{\text{int}}(t)e^{-iH_0/\hbar}\) and \(\tilde{H}_1(t) = e^{iH_0/\hbar}H_1(t)e^{-iH_0/\hbar}\). Integrating equation (7) over time \(t_1\) and substituting it into the second commutator of equation (7), we obtain

\[
\frac{\partial \tilde{\rho}_\text{int}(t)}{\partial t} = \frac{i}{\hbar} \{[\tilde{H}_1(t), \tilde{H}_{\text{int}}(t_1)] + \tilde{H}_1(t_1), \tilde{\rho}_\text{int}(t_1)\} + [\tilde{H}_1(t), \rho_\text{int}(t)]
\]

(8)

It is pointed out specially that equation (8) is deduced in a way which is a bit different from the standard approach for the derivation of the master equation [23]. The first commutator in the right side of equation (7) is retained in equation (8) considering the linear response of the motion of the charged dot to the electric field. Meanwhile, the equation involves higher order of \(H_1\) to describe interaction between the opened system and the large reservoirs with a great number of degrees of freedom.

There are two kinds of environment for the system, one is the electronic reservoir as the two leads for the electrons, another is the Bosonic bath which is interacting with the mechanical oscillator. Under the Born approximation, the total density matrix can be factorized as \(\tilde{\rho}_\text{int}(t) = \tilde{\rho}(t)\rho_L\rho_B\), where \(\rho(t)\) is the density matrix of the system composed of the QD and the mechanical oscillator. Both of the leads and the Bosonic bath are assumed to be in equilibrium state all the time and described by the density matrix \(\rho_L\) and \(\rho_B\), respectively. Therefore, if initially \(\tilde{\rho}_\text{int}(0) = \tilde{\rho}(0)\rho_L\rho_B\), then at time \(t\), \(\tilde{\rho}_\text{int}(t) = \tilde{\rho}(t)\rho_L\rho_B\). This approximation is valid as long as the coupling is weak and the environment is so large that the back action from the system to the environment can be negligible. Performing trace over the leads (\(\rho_L\)) and bath (\(\rho_B\)) variables we obtain the reduced density matrix for the system \(\tilde{\rho}(t) = \text{tr}_{\rho_L}[\tilde{\rho}(t)]\). Besides, the reservoirs are assumed to have much shorter correlation time compared to the coherent evolution of the system. Hence, there is no memory about the system history available in the reservoirs. As a result, the system evolution does not have to do with its history. Then, in the Markov approximation, we replace \(\tilde{\rho}(t_1)\) by \(\tilde{\rho}(t)\), and arrive at the equation for the reduced density matrix

\[
\frac{\partial \tilde{\rho}(t)}{\partial t} = \frac{1}{i\hbar} [H_{\text{dev}}(t), \tilde{\rho}(t)] - \frac{1}{\hbar^2} \int_0^t \text{tr}_{\rho_L}[\tilde{H}_1(t_1), \tilde{\rho}(t_1)\rho_L\rho_B]]\, dt_1.
\]

(9)

After evaluating the commutation relation of equation (9), we transform it back to the Schrödinger picture using the unitary operator \(e^{-i(t_0\varepsilon/\hbar + h\omega_0 a^\dagger a)/\hbar}\). For convenience, we make a time displacement as \(t = t - t_1\). Denoting electron number in the left and right leads as \(w\) and \(v\), respectively, we write the density matrix in the form \(\rho_{w,v}(t)\). If an electron is annihilated (created) in the left lead it becomes \(\rho_{w-1,v}(t) (\rho_{w+1,v}(t))\), and if an electron is created (annihilated) in the right lead it would be \(\rho_{w,v-1}(t) (\rho_{w,v+1}(t))\). Additionally, the Fermi distribution function in lead \(y\) is introduced by \(\text{tr}_{\rho_B}[b_k^\dagger d_k \rho_B] = f_S(\xi_S^y)\), where \(f_S(\xi_S^y) = 1/[(\exp[\beta(\xi_S^y - \mu_\gamma)] + 1)]\) with \(\gamma = l, r\) and \(\beta = 1/k_B T\). The Bose distribution function in the thermal bath of mechanical oscillator is obtained from \(\text{tr}_{\rho_B}[b_k b_k \rho_B] = n_S(\omega_0)\), where \(n_S(\omega_0) = 1/[(\exp[\beta \omega_0] - 1)]\). Then we replace the following equation

\[
\frac{\partial \rho_{w,v}(t)}{\partial t} = \frac{1}{i\hbar} [\varepsilon_0 c^\dagger c + h\omega_0 a^\dagger a - h\Omega (a^\dagger + a)c^\dagger c, \rho_{w,v}(t)]
\]

(10)

\[
+ \int_0^t dt_1 \sum_{k=1} \sum_{y=1} T_{S_k}^2 \varepsilon^2/2 \sum_{m_1,m_2} (S_y a)^{m_1+m_2} \frac{m_1!m_2!}{m_1!m_2!} \times f_S(\xi_S^y)e^{S_y a^\dagger a + a^\dagger a} \times \rho_{w-1,v}(t)\rho_{v+1,S_2}\rho_{v+1,S_2}/(t)c^\dagger c + \rho_{w-1,v}(t)\rho_{v+1,S_2}\rho_{v+1,S_2}/(t)c^\dagger c
\]

(11)

\[
+ \int_0^t dt_1 \sum_{k=1} \sum_{y=1} T_{S_k}^2 \varepsilon^2/2 \sum_{m_1,m_2} (S_y a)^{m_1+m_2} \frac{m_1!m_2!}{m_1!m_2!} \times f_S(\xi_S^y)(a)^{m_1} e^{S_y a a^\dagger a + a^\dagger a} \frac{m_1!m_2!}{m_1!m_2!} \times \rho_{w-1,v}(t)\rho_{v+1,S_2}\rho_{v+1,S_2}/(t)c^\dagger c
\]

(12)

\[
+ \int_0^t dt_1 \sum_{k=1} \sum_{y=1} T_{S_k}^2 \varepsilon^2/2 \sum_{m_1,m_2} (S_y a)^{m_1+m_2} \frac{m_1!m_2!}{m_1!m_2!} \times f_S(\xi_S^y)(a)^{m_1} e^{S_y a a^\dagger a + a^\dagger a} \frac{m_1!m_2!}{m_1!m_2!} \times \rho_{w-1,v}(t)\rho_{v+1,S_2}\rho_{v+1,S_2}/(t)c^\dagger c
\]

(13)
Considerig rapid decaying electrons in the reservoirs, the time integral regime in the equation is extended to infinity, i.e. $t \to \infty$. The integrals over time $\tau$ are performed using the formula $\int_{0}^{\infty} \exp(iv\tau) \frac{d\tau}{\tau} = \pi \delta(x)$ for any variable $x$. The imaginary part $\pm ip_{1}$ corresponds to Lamb shift in quantum optics [23]. Since we consider weak coupling between the system and its reservoirs (electronic leads and Bosonic thermal bath), the shift is very small and neglected. Subsequently, by defining the density of states in lead $N_{l}(\xi)$ and the density of states in the thermal bath as $D(\omega_{k})$, we make the conversion: $\sum_{k=1,2} \sim \int d\xi_{k} N_{l}(\xi_{k})$, $\sum_{k} \sim \int d\omega_{k} D(\omega_{k})$, and integrate over the variables of electron and phonon, respectively. Under the wide band approximation, the time integral regime in the equation is extended to infinity, i.e. $t \to \infty$. The integrals over time $\tau$ are performed using the formula $\int_{0}^{\infty} \exp(iv\tau) \frac{d\tau}{\tau} = \pi \delta(x)$ for any variable $x$.

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which works under the low bias voltage and weak dot-lead couplings. For the parameters adopted here, the probability spectrum of phonon distribution, $\rho_{00,nl} + \rho_{11,nl}$, implies that contribution from the higher levels ($n > 15$) of the vibrational modes is negligibly small. Actually, the master equation given in this paper is valid for a much wider parameter range than that used here as long as an adequate number of vibrational states are taken into account. However, the more vibrational modes considered, the longer the time spent in the numerical implementation. The large memory and long time requirements are weak points of the approach to directly solve the equation. These problems are believed to be circumvented in the iteration method in which a preconditioning is necessary to ensure the convergence [25]. The iteration reaches its end when the sum of the diagonal elements of the system density matrix is close to unity.

The current as a function of bias voltage is plotted in figure 2(a) with the solid line. Almost discontinuous transitions in the low temperature are found. It is shown that no current is available at zero bias voltage. When the bias moves away from the zero point to a small quantity, current appears and sharply increases. Then even the voltage keeps rising and the current seems to be unchanged. The chemical potential in the left lead now lies between the ground and the first excited states of the mechanical oscillator (see figure 1). When the bias voltage rises to $2h\omega_0$, the chemical potential reaches the first excited level of the oscillator, which results in the second jump of the current. More current steps emerge for further increase of the bias voltage. Each time the chemical potential reaches an additional level an extra transport channel is opened and a current step should be observed. The energy spacing of the oscillator levels is reflected by the voltage range of the current steps. The height of the steps tends to become smaller with the increase of the bias voltage and then becomes invisible. In fact the height of the steps can be controlled by tuning the parameters $\Gamma_0$, $\Gamma_\ell$, $\gamma$ and $\alpha$. The dotted line represents differential conductance corresponding to the current. The multi-vibrational modes result in many resonant conductance peaks in quantum shuttle. Every peak implies a sharp increase of the current, between two peaks the current is stationary with respect to the bias. As the left and right parts of the model are in absolutely symmetry, the current appears to be in antisymmetry for the positive and the right parts of the model are in absolutely symmetry, the current becomes smaller with the increase of the bias voltage and then it still contributes to the current due to the zero point fluctuation. We can easily prove in figure 2(a) that the first step is larger than the current of bare tunneling $\omega/(\Gamma_0 + \Gamma_\ell)$. The resonant absorption and emission peaks of the oscillator are illustrated by the dotted line in the figure. In the above analysis, we set the energy level of the QD as a constant. Now, we fix bias voltage and vary the level of the QD. In experiment, it can be realized by tuning the gate voltage. We plot the current as a function of the dot level in figure 3 for different bias voltages. As shown in the figure, the area of the transport window is equal to the bias voltage $eV$. Outside the window, the current reduces to zero and the electron transfer is prohibited. Inside the window, we obtain current with stepwise structure. The highest step corresponds to the QD level that is located in the middle of the bias. One needs to shift the gate voltage by $h\omega_0$ to obtain a new current step instead of $2h\omega_0$ in the bias voltage (see figure 2). The figure implies that, by adjusting the gate voltage, one can control the current with discrete quantities.
The current versus energy level of the QD at bias voltages $eV = 5\hbar\omega_0$ (solid line), $eV = 7\hbar\omega_0$ (dashed line) and $eV = 9\hbar\omega_0$ (dotted line). The other parameters are $\Gamma_1 = \Gamma_r = 0.005\omega_0$, $x_0/d = 0.003$, $\alpha = 0.75$, $\gamma = 0.03\omega_0$ and $\beta\hbar\omega_0 = 20$.

The current of the system is correlated with energy of the mechanical oscillator as discussed in the preceding subsection. Damping of the vibrational mode influences the mean energy of the oscillator and so the electron transfer. Figure 4 illustrates such an effect on the current by changing the dissipation rate. At the bias voltages $eV = 3\hbar\omega_0$, $5\hbar\omega_0$ and $7\hbar\omega_0$, the system is driven by the electric field and excited into high levels. Current across the excited system is remarkably decreased when the dissipation rate rises from a small quantity to about $10\hbar\omega_0$ ($and 10\hbar\omega_0$). When the rate is higher than around $20\hbar\omega_0$ ($and 20\hbar\omega_0$), the currents approach a stationary value. At $eV = \hbar\omega_0$, the mechanical oscillator is mainly in the ground state, therefore, the current is almost independent of the dissipation rate (see the solid line). Approximate calculation reveals the currents for $\gamma < 0.01\omega_0$ are further intensified but have finite quantities.

![Figure 3](image3.png)

Figure 3. The current versus energy level of the QD at bias voltages $eV = 5\hbar\omega_0$ (solid line), $eV = 7\hbar\omega_0$ (dashed line) and $eV = 9\hbar\omega_0$ (dotted line). The other parameters are $\Gamma_1 = \Gamma_r = 0.005\omega_0$, $x_0/d = 0.003$, $\alpha = 0.75$, $\gamma = 0.03\omega_0$ and $\beta\hbar\omega_0 = 20$.

![Figure 4](image4.png)

Figure 4. The current versus the dissipation rate. Current across the excited system is denoted by the dotted, dashed and dot–dashed lines. Current across the system in the ground state is represented by the solid line ($\Gamma_1 = \Gamma_r = 0.005\omega_0$, $x_0/d = 0.003$, $\alpha = 0.75$, $\epsilon_0 = 0$, $\beta\hbar\omega_0 = 20$).

Next, we consider the temperature dependence of the system. As illustrated in figure 5(a), critical low temperature is required to observe the quantized current. If we increase the temperature the step-like structure tends to be smeared. Especially, the plateaus disappear at the temperature $\beta\hbar\omega_0 = 3$. In the regime of high bias in this figure, the smoothed line (the dot–dashed line) is above the other curves that are at the lower temperatures. In fact, this is not always the case. With high bare tunneling rate (e.g. $\Gamma_1 = \Gamma_r = 0.01\omega_0$), the smoothed current would be lower than the stepped ones (not shown). Such a temperature dependent effect has been previously studied both experimentally [21] and theoretically [22]. Nevertheless, it makes sense to show it as we will connect it to the behavior of zero frequency current noise in section 4. The step disappearance can also be seen in the case where the frequency-independent quality factor of the vibrational mode is very small [8].

4. Current fluctuation

In this section, let us focus our attention on the Fano factor, $F(\omega) = S(\omega)/2eI$, which is given by ratio of the actual noise spectrum $S(\omega)$ and the Poissonian noise $2eI$. Concerning the probability of $\nu$ electrons collected in the
Figure 6. Fano factor spectrum at the dissipation rate $\gamma = 0.001\hbar\omega_0$. (a) $F(0) < 1$, (b) $F(0) > 1$, (c) $F(0) > 1$ and (d) $F(0) > 1$. The other parameters are $\Gamma_r = 0.005\hbar\omega_0$, $x_0/d = 0.003$, $\omega_0 = 0.75$, $e_0 = 0$ and $\beta\hbar\omega_0 = 20$.

right lead, we employ the McDonald formula [26],

$$S(\omega) = 2e^2 \omega \int_0^\infty dt \sin(\omega t) \frac{\hbar}{2\pi} \sum_{v=0}^{\infty} v^2 P_v(t),$$

to calculate shot noise of the system. Figure 6 shows the Fano factor spectrum for the dissipation rate which satisfies $\gamma < \Gamma_l, \Gamma_r$. At the bias voltage $eV = \hbar\omega_0$, mainly the channel of the ground state is opened and the probability of electrons passing through the system is concentrated in this channel. Hence, the zero frequency Fano factor appears to be sub-Poissonian as illustrated in figure 6(a). It is similar to the bare tunneling process. The difference is that noise peaks appear at frequencies $\pm\omega_0$. This indicates the contribution from an additional channel with small probability. Figure 6(b) reveals that, at voltage $eV = 2\hbar\omega_0$, the Fano factor appears to be super-Poissonian at the frequencies which are integrals of $\omega_0$. At that voltage, excited states of the mechanical oscillator begin to effectively contribute to the electron flow, which destroys the zero frequency sub-Poissonian statistics. This effect can be observed more clearly at higher voltages (see figures 6(c) and (d)). Since at the higher voltages, more channels are opened for the transport. The super-Poissonian noise at the zero frequency implies that the electron transfer through the ground state channel is interrupted by the tunneling through the channels of the excited levels. Due to the Coulomb blockade effect, there will be a competition between these channels with different tunneling probabilities and correlation occurs among these transport processes of the multi-channels. Electron transport in the present situation is not so deterministic as that in the so-called shuttling regime where zero frequency sub-Poissonian noise is predicted [13]. It may be due to the extremely large bias voltage applied in their model, consequently multi-channels are not involved and electrons are forced to transfer mainly in one direction without being reflected back to the original lead. In the area of off resonant frequencies, we can see noise suppression which is consistent with the result achieved from the model of incoherent dynamics [27].

In figure 7, the Fano factor is shown for the situation $\gamma \gg \Gamma_l, \Gamma_r$. Due to the fast damping effect, the contribution from excited states of the mechanical resonator is very small. An electron transports with the dominant probability through the channel provided by the ground state of the system. It causes suppression of the noise. In particular, the zero frequency Fano factor exhibits sub-Poissonian noise approaching 0.5 even in the case where the bias voltage is increased to a finite quantity. In fact, the result is also held in the limit of large bias voltage [13].

The above interpretation of the physical picture of the super-Poissonian statistics is consistent with the previous results. In a movable QD array [25], the different current channels are formed due to different resonant quantum states connecting the neighboring dots in the co-tunneling regime. The switching between those channels gives rise to super-Poissonian noise in the regime of small damping rate. In the semiclassical case, electron transport through bistable coexistent channels of shutting and tunneling causes a super-Poissonian noise spectrum both at zero [13] and finite
Figure 7. Fano factor spectrum at the dissipation rate $\gamma = 0.3\hbar\omega_0$. (a) $F(0) < 1$, (b) $F(0) < 1$, (c) $F(0) < 1$, and (d) $F(0) < 1$. The other parameters are the same as in figure 6.

frequencies [28]. The bistable of the quantum shuttle is further illustrated with full counting statistics [29].

Now, let us briefly discuss the influence of temperature on the zero frequency current fluctuation. As illustrated in figure 5(b), a large Fano factor is predicted in the case $\beta\hbar\omega_0 \sim 1$. The curves at different bias voltages have a common feature that they do not obviously depend on temperature until $\beta\hbar\omega_0$ is decreased to about 3. When $\beta\hbar\omega_0 \lessapprox 3$, the noise is supposed to be dominated by thermal noise. We connect the Fano factor with the I–V curves at different temperatures in figure 5(a). It is revealed that noise increase with temperature is accompanied by disappearance of the current steps near $\beta\hbar\omega_0 = 3$. It seems to imply that the thermal noise that emerges due to the finite temperature removes the quantum mechanical characteristics of the current.

5. Discussion and conclusions

First we discuss the effect that comes from the coherent coupling between the charge transport and dynamics of the mechanical resonator. Now we calculate the system current using an incoherent model in which only diagonal terms of the density matrix, such as $\rho_{00,nn}$, $\rho_{11,nn}$, in the equation (11) and the current formula are involved. As illustrated in figure 8, for the lower bias $eV \lesssim 4\hbar\omega_0$, the contribution from the off diagonal part is negligibly small. However, in the case $eV > 4\hbar\omega_0$, the current calculated from the coherent model including both diagonal and off diagonal terms of the density matrix is lower than that achieved from the incoherent model. The suppression of current in the coherent model may be due to the destructive interference between different transport channels. The incoherent model applied here is not absolutely the same as those considered previously because of the different derivation methods [6–10]. Whereas, here we just...
intend to make clear the importance of the coherent coupling in the system.

The relation between resonator state and charge transport is an interesting issue. Measuring the character of charge transport in the system one expects to find out information about the mechanical resonator. In section 3, we show that the current is sensitive to the mean phonon number of the resonator with varying bias voltage. Then, we point out that the Fano factor spectrum of the charge transfer is dependent on the motion of the mechanical oscillator. Recently, the current noise together with the phonon statistics has been considered [30]. As illustrated in the reference, a uniform relation of statistical characteristics between the localized phonon and electron current does not exist. The relation is determined by the system parameters. As an example, in the parameter area where the charge–oscillator coupling is weak and two tunneling barriers are very asymmetric, one can expect a super-Poissonian current Fano factor associated with sub-Poissonian phononic population. The sub-Poissonian statistics of phonon distribution is also predicted in a system where a resonator is coupled to a superconducting single-electron transistor [31]. They found the system behaves as a micromaser and can generate a number-squeezed state of the resonator. Since the phonon number distribution would be narrowed in the squeezed state [23], the phonon noise is reduced to be sub-Poissonian.

Finally, we conclude that a master equation is developed to describe coherent dynamics of the electromechanical quantum transport as a function of the bias voltage and temperature. In the Born–Markovian approximation, the general master equation can be explicitly derived including position dependence of tunneling rates and Fermi distribution functions of the electron reservoirs. The mechanical motion of the charged QD is modeled by the quantum harmonic oscillator and coherently coupled to the electron transfer. The equation of motion is numerically solved in the Fock state Hilbert space of the electron and phonon. It is found that the steps of current are related to the steps of mean energy in the mechanical oscillator. The dissipation rate of the resonator significantly affects the current intensity, especially when it is comparable with the bare tunneling rate. The gate voltage can be applied to control the current with discontinuous quantities. In the extremely low bias voltage, only the channel of the ground state is opened. In this case, we observe sub-Poissonian noise of electron transfer. When the bias is high enough to drive the oscillator partially into its excited states the system current manifests super-Poissonian noise. It reveals, in the Coulomb blockade regime, that positive correlation is generated between the multi-channel electron transports. If the dissipation rate of the vibrational modes is much faster than bare tunneling rates, the channels of excited states become unimportant and the shot noise would be suppressed remarkably. At higher temperature, smoothing of the current steps is observed. This event is accompanied by the increase of current shot noise due to temperature. Our present work enriches the researches on phonon assisted electron tunneling and the nano-electromechanical oscillator.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under grant Nos 10874002 and 91021017.

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