Observing mode propagation inside a laser cavity

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\textit{New Journal of Physics} \textbf{14} (2012) 053021 (10pp)

Received 24 January 2012
Published 17 May 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/5/053021

\textbf{Abstract.} The mode inside a laser cavity may be understood as the interference of two counter-propagating waves, referred to as the forward and backward waves, respectively. We outline a simple experimental procedure, which does not require any additional components, to study the forward and backward propagating waves everywhere inside a laser cavity. We verify the previous theoretical-only prediction that the two fields may differ substantially in their amplitude profile, even for stable resonator systems, a result that has implications for how laser resonators are conceptualized and how the disparate traveling waves interact with nonlinear intra-cavity elements, for example, passive $Q$-switches and gain media.

The eigenmodes, i.e. the spatial (transverse) structure of a field that reproduces after a round trip, of an empty resonator made up of two spherical mirrors depend upon the resonator symmetry. For rectangular (circular) symmetry they are the well-known Hermite–Gaussian (Laguerre–Gaussian) modes $HG_{nm}(LG_{pl})$ \cite{1}. The particularities of these eigenmodes are: (i) only the lowest-order modes, $HG_{00}$ and $LG_{00}$, are characterized by a smooth intensity profile, and are described by the Gaussian function; (ii) the higher-order modes are made up of circular rings (spots) of light for the $LG_{pl}(HG_{nm})$ family; and (iii) the higher-order modes spread
Figure 1. Schematic representation of a simple plano-concave cavity, unfolded around M2, showing the propagating fields. Usually, the plane of calculation is taken to coincide with either M1 or M2, in which case the forward and backward propagating beams are indistinguishable experimentally. By selecting a plane at some arbitrary position inside the cavity, differences between the two beams may be observed.

Laterally concomitantly with the mode order. In general, users of laser beams are interested in achieving the highest brightness possible for a given output power, and this necessitates the forcing of the laser oscillation on the lowest transverse mode, which as a Gaussian beam will have the lowest divergence and optimal beam quality factor \( M^2 = 1 \). For that, one requires some discrimination mechanism inside the laser cavity that gives rise to optical losses increasing with mode order so that only the lowest mode order should reach the threshold oscillation condition. Usually, this transverse mode discrimination is achieved by inserting an aperture or diaphragm inside the laser resonator to increase the losses of all the undesired modes. The role of such losses is very well known and indeed was addressed in the seminal work of Fox and Li [2], and has formed the basis for how stable laser resonators are conceptualized [3, 4]. Fox and Li put forward the view that the oscillating modes are discriminated against based on differences in round trip loss, and as such, after many round trips the mode with the lowest loss dominates to become the resonant, lowest order, mode. The concept here is that of a single field propagating back and forth until it converges in the steady state to the lowest loss mode. The authors of [2] further made the observation that ‘the surface of the reflector coincides with the phase front of the wave, making it an equiphase surface’. This suggests that the mirror elements are phase-conjugating elements. These facts taken together suggest the following concept of modal propagation inside a stable laser cavity: in the steady state the mode propagates from one mirror (M1) to the other (M2) and is then returned by reverse propagation due to the phase-conjugating nature of the mirrors, as illustrated in figure 1. This accounts for the repeatability of the mode after each round trip, and suggests that the propagation of the mode in the forward direction (from M1 to M2) and that in the reverse (backward) direction (from M2 to M1) is identical. This concept has been exploited to design resonators for custom mode generation using intra-cavity phase-only elements [5–10].

But this is true only for perfect eigenmodes, i.e. the resonant modes of a cavity made up of spherical mirrors having infinite size. In fact, the selected mode is always (to some degree) distorted by diffraction upon the edge of the selecting aperture, and is therefore no longer Gaussian in shape [3, 4]. Similar arguments hold for selecting any other arbitrary high-order mode (\( \text{HG}_{nm} \) or \( \text{LG}_{pl} \)) as the single mode with the lowest loss, or for that matter...
a customized laser mode. Since a laser resonator is basically an interferometer, one has to remember that the electric field at any plane results from the interference of two traveling waves, which we refer to as the forward and backward beams, and which propagate inside the resonator in the $z > 0$ and $z < 0$ directions, respectively. That is to say, the resulting field inside a laser resonator may be conceived of as the coherent sum of these two beams, traveling in counter-propagating directions [11]. The peculiarity is that, due to diffraction upon the mode selecting aperture, the forward and backward beams can be very different in a given plane. This is in fact the design principle behind unstable resonators [3, 4], but has received very little attention in the case of stable resonator systems, and this almost always theoretical [11–16]. Unfortunately the literature almost exclusively concerns itself with the transverse shape of the mode at the output coupler plane of the laser or at least at one of the two mirrors, whereas as we explain later, these planes do not allow for an experimental observation of the differences in the two counter-propagating beams. Some attempts have been made to experimentally study the beam propagation external to the cavity in order to infer the behavior of the counter-propagating waves inside the resonator cavity [15–18], but to date there has been no experimental study on the full propagation of the forward and backward beams inside a laser cavity.

In this paper, we propose a simple experimental method that allows one to scan simultaneously, plane by plane, the forward and backward traveling waves inside any laser cavity. We execute this method on a plano-concave cavity of length 240 mm, including a gain medium of length 30 mm so that the main part of the resonator is empty, and show that we can record the intensity profile of the forward and backward beams at any plane inside the resonator. Since our method does not require any exotic or perturbing optical elements inside the cavity, we are able, for the first time, to test the theoretical predictions of the perturbations to the forward and backward beams as a result of intra-cavity diffraction. Moreover, while we validate our method by considering diffraction effects, this method is more general than this, and could be a powerful tool in the study of active resonators where the nonlinear interaction of the traveling waves with the gain medium modifies the transverse intensity pattern in comparison with the mode structure of the cold cavity.

There are several approaches to modeling the oscillating modes inside a laser cavity (see, e.g., [2–4, 10, 19] and references therein); the authors of [10] give a particularly useful summary of modeling approaches for stable and unstable resonators, given in the context of Bessel beams. To model the mode in a laser cavity as the superposition of two counter-propagating beams, we make use of the mode expansion approach [11]. In this formalism, the Laguerre–Gaussian functions make up an orthonormalized basis which is written, for the forward (subscript $f$) and backward (subscript $b$) beams, as [11]

\[ LG_{fp}(\rho, z) = L_p\left(\frac{2\rho^2}{W^2(z)}\right) \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left\{i \left[ -\frac{k\rho^2}{2R(z)} - (2p+1)\phi(z) \right]\right\}, \]  

(1a)

\[ LG_{bp}(\rho, z) = L_p\left(\frac{2\rho^2}{W^2(z)}\right) \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left\{-i \left[ -\frac{k\rho^2}{2R(z)} - (2p+1)\phi(z) \right]\right\}, \]  

(1b)

where $k = 2\pi/\lambda$ and $L_p(X)$ is the Laguerre polynomial of order $p$. Hereafter, the subscripts $f$ and $b$ denote, respectively, forward and backward quantities. The Gaussian mode of the non-apertured cavity is characterized by its beam diameter $2W(z)$ and its radius of curvature $R_c$ at position $z$. These quantities, as well as the Gouy phase shift $\phi$, are $z$ dependent and obey the
well-known propagation laws \[3\]

\[
W^2(z) = W_0^2 \left[1 + \left(z/z_0\right)^2\right],
\]

\[
R_c(z) = z \left[1 + \left(z_0/z\right)^2\right],
\]

\[
\phi(z) = \arctan(z/z_0),
\]

where \(z_0 = \pi W_0^2/\lambda\) is the Rayleigh range and \(W_0\) is the beam waist radius expressed as \(W_0^2 = (\lambda L/\pi) \sqrt{g/(1-g)}\) for a plano-concave cavity (see the experimental discussion later in this paper), with the waist plane at \(z = 0\), taken to be that of the plane mirror (M1) with the concave mirror (M2) at \(z = L\). The forward and backward fields are assumed to be linearly polarized and are expressed as linear combinations of the basis functions

\[
U_f(\rho, z) = \exp\left[i (kz - \omega t)\right] \sum_p a_{fp} LG_{fp}(\rho, z),
\]

\[\text{(2a)}\]

\[
U_b(\rho, z) = \exp \left\{i[k(2L - z) - \omega t]\right\} \sum_p a_{bp} LG_{bp}(\rho, z).
\]

\[\text{(2b)}\]

The objective of the calculation is to find the coefficients \(a_{fp}\) and \(a_{bp}\), which are \((\rho, z)\) independent, and which change after one round trip (for each direction) into new coefficients \(\Gamma a_{fp}\) and \(\Gamma a_{bp}\), where \(\Gamma\) is the eigenvalue corresponding to the eigenvector of a round trip matrix, with eigenvalues associated with the round trip loss \[11\]. For the cavity illustrated in figure 1 and conceptualized in figure 2, the method of finding these eigenvalues and eigenvectors is known \[11\]. Theoretically, the basis functions in equations \((1a)\) and \((1b)\) may be infinitely expanded and correspond to the eigenmodes of the unperturbed cavity where the difference in the basis functions is purely dependent on their phase expressions. To accurately describe the resonant field we need not exceed 80–100 terms in the series expansion presented in equations \((2a)\) and \((2b)\), thus truncating the series. This results in the expansion coefficients \(a_{fp}\) and \(a_{bp}\) being truncated without loss of accuracy as the magnitude of the coefficients decreases with the expansion. In addition, for a basis size corresponding to the beam waist size of the physical cavity, minimum truncation errors are incurred \[20\]. This formalism is consistent with the resonance condition requirement that the resonant field of the cavity does not change during a round trip, except for a complex constant; in this scenario it is applied to both the forward and backward traveling fields, which are related to one another through the boundary conditions at the mirrors, e.g. for the apertured mirror M2 we have

\[
U_b(\rho, L) = \begin{cases} 
    r_{M2} U_f(\rho, L), & \rho < d/2, \\
    0, & \rho > d/2,
\end{cases}
\]

\[\text{(3)}\]

where \(r_{M2}\) is the reflectivity function of mirror M2 and \(d\) is the aperture diameter. The position of the aperture inside the cavity is not restricted to a mirror surface and this technique is valid for an aperture positioned elsewhere between the mirrors. The two traveling waves in this case are truncated and yield higher losses for a particular transverse mode discrimination \[21\]. It is evident that inspecting the field at the mirror surfaces will not reveal a significant difference in the two propagating waves. It should also be noted that calculation of the mode at one of the mirrors is the ubiquitous test in most experimental and theoretical studies. The final field at any plane inside the cavity is therefore made up of the coherent sum of two counter-propagating fields, the forward and the backward waves, and is given by

\[
U(\rho, z) = U_b(\rho, z) + U_f(\rho, z).
\]

\[\text{(4)}\]
Figure 2. The experimental setup, which comprises an L-shaped cavity, with a 99% reflecting mirror at 45°, which allows for the forward beam (solid red line) and the backward beam (blue dashed line) to be directed in different directions. The L-shaped cavity has a cavity length $L = 240 \text{ mm} (90 + 84 + 66)$ where the Nd : YAG crystal (pink rectangle) is end-pumped (green arrow) with a 75 W multi-mode fiber-coupled diode with the aperture located on mirror M2. Appropriate relay imaging using lenses $f_1$ and $f_2$ allows one to observe the propagation inside the cavity without any magnification.

To depart from previous studies that have only considered the fields at the mirrors or external to the cavity, we construct our laser cavity as shown in figure 2. The essential feature of the cavity is the L-shaped design with a ‘leaky’ 99% reflecting 45° mirror. The forward beam is directed along a path at 90° to that of the backward beam, so that the two traveling waves may be studied independently. To measure the propagation of each beam inside the cavity, the planes of the two mirrors are relay imaged to conjugate planes outside the cavity (recall that relay imaging preserves both the amplitude and phase of the field). For example, the starting plane of the forward beam, mirror M1, is relay imaged to plane M1* outside the cavity without any magnification, so that the propagation of the forward beam from M1 to M2 may be measured external to the cavity from planes M1* to M2*, and similarly for the backward propagating beam. This simple technique allows us to measure both the forward and backward propagating beams everywhere inside the cavity, simultaneously if so desired. The cavity, comprising a gain medium of Nd : YAG (4 mm $\times$ 30 mm rod), was end-pumped with a Jenoptik (JOLD-75-CPXF-2P W) 75 W multi-mode fiber-coupled diode. The geometrical parameter $g = 1 - L/R$ of the resonator was chosen to be 0.4, which corresponds to the output coupler having a curvature of...
With the aperture set for an unperturbed Gaussian beam, the measured propagation of the forward and backward beams inside the cavity is in good agreement with the analytical theory for such a propagation. $R = 400\ \text{mm}$ at a cavity length of $L = 240\ \text{mm}$. The Gaussian beam width on the concave mirror (M2) without internal aperture was calculated to be $407\ \mu\text{m}$. The light that was transmitted through the $45^\circ$ high reflector was used to relay image, by afocal telescopes, the plane of the output coupler (M2) and the plane of the flat mirror (M1) to respective positions outside the cavity. The telescope used to relay image the plane on mirror M1 to the plane $M1^*$ consisted of two lenses, $f_2 = 200\ \text{mm}$, where the first lens was positioned $200\ \text{mm}$ ($90 + 110$) from M1 and the second lens was positioned $400\ \text{mm}$ ($65 + 335$) from the first lens. The plane $M1^*$ was thereafter located at $200\ \text{mm}$ from the second lens. Similarly, the telescope used to relay image the plane on mirror M2 to the plane $M2^*$ consisted of two lenses, $f_1 = 250\ \text{mm}$, where the first lens was positioned $250\ \text{mm}$ ($66 + 84 + 29 + 71$) from M2 and the second lens was positioned $500\ \text{mm}$ ($145 + 355$) from the first lens. The plane $M2^*$ was thereafter located at $250\ \text{mm}$ from the second lens. We were thus able to scan the intensity profiles of the forward and backward components external to the cavity for the propagation from the plane to the concave mirror and vice versa by using a CCD camera (Spiricon LB-USB L130).

Before proceeding, it is important to verify that there were no perturbing influences inside the cavity from the gain medium (e.g. gain effects or thermal effects), and to confirm that the imaging system was correctly configured. The aperture at M2 was adjusted for selecting a pure Gaussian mode, i.e. with a very small clipping, so that the measurement of the forward and backward beam widths could be compared to the analytically calculated values for this cavity using the well-known Gaussian beam propagation expressions; the experiment was found to be in excellent agreement with theory, as shown in figure 3.

The result confirms that when the perturbations due to diffraction are small, the forward and backward beams are nearly identical at a given plane—in this example they are Gaussian at all planes and follow the Gaussian beam propagation equations. With the efficacy and accuracy

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**Figure 3.** With the aperture set for an unperturbed Gaussian beam, the measured propagation of the forward and backward beams inside the cavity is in good agreement with the analytical theory for such a propagation.
of the experimental setup verified, the aperture was reduced to a radius $d = 400 \, \mu m$. This value of the aperture was judiciously chosen for the cavity to be in a regime where the forward and backward propagating beams are predicted not to be the same everywhere. The measurement of the full propagation of these beams inside the cavity is shown in figure 4. The distance scale represents the propagation distance inside the cavity from M1($z = 0$) to M2($z = 240$ mm). We are not aware of any such measurement in the literature, with most researchers considering only the output field from the cavity. The measurements conclusively show that indeed the forward and backward beams are not the same, and therefore that the concept of a single field reversing on itself is not in general true, even for stable resonators. At the plane $z = 144$ mm, the forward beam is Gaussian-like, while the backward beam has a central ‘dip’ in intensity. The propagation of the two beams clearly differs significantly.

The results of this experiment can be explained by the predicting theory succinctly presented earlier but available in more detail in [12, 13]. We are able to explain the experimental results by considering the mode at any plane in the cavity to be the superposition of the forward and backward propagating fields, as given by equation (4), with the theoretical prediction illustrated in figures 5(a) and (b). There is clearly good agreement with the experimental data, shown in figure 5(c). It is also clear that the Fox–Li approach can account for the differences, as indicative of figure 5(b). This is achieved by considering the usual unfolded resonator (figure 1), but calculating the field at the observation plane for both the $n$th and $n+1$ one-pass trips in the steady state. This is not usually necessary because the observation plane is often the plane of the mirrors, in which case the two passes would coincide at the same plane. However, with a plane of reference inside the cavity, the forward beam can be monitored in the $n$th pass, and the backward beam in the $n+1$ pass. The fact that the two planes reveal differing fields can only be explained by a difference in phase between the forward beam at the mirror and the mirror surface; that is, it is in general not true that the mirrors form equiphase surfaces. On inspection one finds that this was not a conclusion of the study in [2], but rather a remark from the particular
Figure 5. Theoretical prediction of the forward and backward beans for: (a) the Laguerre–Gaussian expansion method and (b) the Fox–Li method. In (c) we show the experimental cross-section from the intensity plot of figure 4 ($z = 144 \text{ mm}$) and (d) the phase of the mode at mirror M2.

example they used to illustrate the method. It may be that this remark has unduly influenced our conceptualization of modes in optical resonators. An extraction of the phase of the mode at mirror M2 shows a clear departure from an equiphase surface, as shown in figure 5(d), for the case of a strongly truncated field. This effect is probably linked to the well-known converging effect associated with the diaphragm clipping which is able to focus laser light [22]. When the aperture is large, we note that the phase of the mode matches the mirror surface, and hence the concept of a single mode propagating back and forth is valid in this regime.

In conclusion, we have revisited the problem of mode propagation inside laser resonators, and have demonstrated experimentally that the mode may be considered as a superposition of a forward and a backward traveling wave. Moreover, we confirmed the theory that these two waves may differ substantially due to intra-cavity diffraction and that as a consequence the mode should not be thought of as a single field propagating back on itself. For demonstrating these effects we have achieved an experimental setup allowing one to scan, plane by plane,
inside the laser cavity without the need for additional perturbing optical components. In addition to outlining a new experimental technique, we remark that such diffraction effects, which render substantial differences in the forward and backward beams, should be accounted for, particularly when placing optical elements inside the cavity whose response is dependent on the intensity distribution, e.g. saturable absorbers, nonlinear optical elements, passive $Q$-switches and gain media. We show for the first time experimental confirmation of previous theoretical predictions that the forward and backward waves can differ inside the cavity. Our results highlight the physics of laser cavities in general, and outline an approach to studying them both experimentally and theoretically. Of particular importance is that these techniques are very general, and may be applied to virtually any laser cavity or interferometer. It is clear from these results that in some cases the response of such elements may be compromised if both propagating waves are not considered. It is now possible to study such effects experimentally with this approach. In a more general context involving optical devices based on interference, the experimental setup we have proposed allows one to scan from the outside the waves interfering in any internal plane.

Acknowledgments

The authors acknowledge that this work is based upon research supported by the National Research Foundation (NRF) of South Africa. The authors also acknowledge support from the Conseil Régional Basse Normandie and the Partenariat Hubert Curien (France) under the join grant PROTEA no. 25886XC.

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