Flavour and $CP$ violation in supersymmetry

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Abstract. After the shutdown of the large electron–positron collider at CERN, the search for $CP$ violation and flavour changing neutral current (FCNC) phenomena acquired a privileged role in our quest for new physics beyond the electroweak standard model (SM). Most extensions of the SM exhibit new sources of $CP$ violation and FCNC. In particular, the minimal supersymmetric extension of the SM presents several new phases and flavour structures in addition to those already present in the SM. Therefore, supersymmetry may have a good chance to manifest some departure from the SM in this particularly challenging class of rare phenomena. On the other hand, it is also apparent that $CP$ violation and FCNC generally represents a major constraint on any attempt at model building beyond the SM. In this work, we review the status of FCNC and $CP$ violation in supersymmetric extensions of the SM and discuss the possibilities of these indirect searches in the quest for supersymmetry.

1. Introduction

High-energy physics at the end of the 20th century has been strongly characterized by the impressive success of the standard model (SM) of the electroweak interactions as the correct description of fundamental interactions up to energies of $O(100$ GeV). This tremendous achievement has been possible thanks to the efforts of experimentalists and theorists all over the world and specially to the experiments at the large electron–positron collider (LEP) at CERN. Having emphasized this bright side of the last two decades, one has must also admit that the experimental and theoretical particle physics of these years has been rather unsuccessful in finding a road to follow beyond the SM. If we are all convinced that the SM cannot be the ultimate theory of ‘everything’ (if only because it does not include gravity), we are in the dark about when, where and how new physics beyond the SM should manifest itself. Searches for
new particles and interactions in leptonic and hadronic machines have failed to produce results. Theoretically, the most promising advances in recent years have led to theories (strings, branes etc) which seem rather far from experimental tests.

Although the quest for new physics finds its most convincing answer in the traditional avenue of energies sufficient to produce and observe new particles, the lesson of recent decades has clearly indicated a second, indirect way of signalling the presence of new physics. We refer to the physics related to rare processes where flavour changing neutral currents (FCNC) and/or $CP$ violation occur. These rare processes are a privileged ground for indirect signals of new physics. One can hardly believe that new physics contributions can be observable when considering phenomena that arise at the tree level in the SM. On the contrary, FCNC and/or $CP$ violating processes offer the possibility for loops containing new particles to compete with loops containing SM particles, thus allowing a fair chance for new physics to emerge against the ‘SM background’. This indirect road to new physics will become particularly relevant in the next few years. After the end of LEP, our attention in high-energy accelerator physics turns to the upgraded Tevatron at Fermilab. However, unless superpartners are light, it might be difficult to establish their existence at the Tevatron and we shall have to wait for the large hadron collider (LHC) at CERN to operate before deciding ‘directly’ whether low-energy SUSY is a reality or just an intriguing intellectual construction.

In the meantime, the FCNC and/or $CP$ violating processes come into full play in our challenging (sometimes desperate) effort to find hints of new physics. Two brand-new $B$ factories have just started operating, and there are plans for new experiments in rare kaon decays, in testing the electric dipole moments, in flavour violating leptonic processes etc. The enormous and interesting project of probing the SM in rare processes will proceed at full speed in the pre-LHC epoch. It is important to critically discuss strategies to look for new physics in such processes. Our goal here is to report on some of the most relevant aspects of the search for some signal of departure from the SM expectations in $CP$ violating phenomena. Admittedly, even if we find such signals it will be hard to establish what new physics is responsible for them. However, we indicate how, at least in the specific framework of low-energy SUSY, the combination of several observations may shed important light on the new physics, hence directing direct searches towards specific targets.

We refer to ‘FCNC and/or $CP$ violation’ in qualifying the interesting processes for indirect detection of new physics because we can consider three different classes of these: (a) phenomena that require FCNC but occur even without $CP$ violation such as $K$–$\bar{K}$ or $B$–$\bar{B}$ mixing; (b) $CP$ violating processes without FCNC, namely electric dipole moments (EDMs) of the neutron, electron etc, and finally (c) simultaneous FCNC and $CP$ violating phenomena, for instance the quantities $\varepsilon$ or $\varepsilon'/\varepsilon$ in $K$ decays into pions and the $CP$ asymmetries in $B$ decays.

In our view, among these three classes of rare processes, those related to the presence of $CP$ violation deserve special attention for at least three reasons:

After nearly four decades of intensive experimental and theoretical work $CP$ violation still appears rather mysterious and, hence, a potentially good candidate to offer surprises in future tests (for general and recent reviews on $CP$ violation see [1]). We recently witnessed two major breakthroughs in our understanding of $CP$ violation. First, from the measurements [2] of the $\varepsilon'/\varepsilon$ parameters from both sides of the Atlantic, we obtained the information that $CP$ violation occurs not only in $K$–$\bar{K}$ mixing ($\Delta S = 2$), but also in the direct $K$ decay amplitudes ($\Delta S = 1$). The second relevant piece of information is that $CP$ violation is not present exclusively in the kaon system but also in $B$ physics. Three different experiments have measured the $CP$ asymmetry in
the decays of $B$ into $J/\psi K_S$: $a_{J/\psi} = 0.59 \pm 0.14 \pm 0.05$ at BaBar [3], $a_{J/\psi} = 0.99 \pm 0.14 \pm 0.06$ at BELLE [4] and $a_{J/\psi} = 0.79^{+0.41}_{-0.44}$ at CDF [5].

Second, from the theoretical point of view, it is important to emphasize that new physics beyond the SM generically introduces new sources of $CP$ violation in addition to the usual CKM phase of the SM. Indeed, it is a common experience of model builders that if one tries to extend the SM with some low-energy new physics one must somehow control the proliferation of new $CP$ violating contributions. Significant portions of the parameter spaces of new physics models can generally be ruled out by the severe constraints imposed by $CP$ violating phenomena [7]. Even in the really minimal supersymmetric extension of the SM, that passes all the FCNC tests unscathed, one still faces severe problems in matching the experimental results concerning the constraints from EDMs.

The third reason which makes us optimistic in having new physics playing a major role in $CP$ violation concerns the matter–antimatter asymmetry in the universe. Starting from a baryon–antibaryon symmetric universe, the SM is unable to account for the observed baryon asymmetry. The presence of new $CP$ violating contributions beyond the SM looks crucial to produce an efficient mechanism for the generation of a satisfactory $\Delta B$ asymmetry [6]†.

The aim of this paper is to then review the current status of FCNC and $CP$ violation in supersymmetry and to discuss the above points with the goal of exploring the potentialities of these observables in our quest for low-energy SUSY [8]. We emphasize that low-energy SUSY does not denote a well defined model; rather, it includes a variety of SM extensions (with a variety of phenomenological implications). We characterize this huge class of models according to their main features in relation to $CP$ violation. In the following, we always stick to the minimal supersymmetric model; we introduce the minimal number of superfields that are strictly demanded to obtain a viable supersymmetrization of the SM. This means that each particle will be accompanied by a superpartner, except in the Higgs sector where we have to introduce a second Higgs doublet in addition to the usual SM Higgs doublet. A second important limitation in the class of SUSY models that we consider here concerns the imposition of $R$ parity. This discrete symmetry is usually added to the gauge and super-symmetries in order to prevent excessive baryon and lepton number violations. Even in minimal supersymmetric versions of the SM (MSSM), where the minimal number of superfields is introduced and $R$ parity is imposed, one is still left with more than 100 free parameters, almost half of them given by $CP$ violating phases [9]. Fortunately most of this huge parameter space is already phenomenologically ruled out. Indeed, FCNC and $CP$ violating processes play a major rule in drastically reducing the parameter space. Obviously it is difficult to make phenomenological predictions with so many free parameters, and so through the years many theoretical further restrictions have been envisaged for the MSSM class. The most drastic reduction on the SUSY parameter space leads to what is called the constrained MSSM (CMSSM) or minimal supergravity [8]. In the absence of phases, this model is characterized by only four parameters plus the sign of a fifth parameter.

In any MSSM, at least two new ‘genuine’ SUSY $CP$ violating phases are present. They originate from the SUSY parameters $\mu$, $M$, $A$ and $B$. The first of these parameters is the dimensionful coefficient of the $H_u H_d$ term of the superpotential. The remaining three parameters are present in the sector that softly breaks the $N = 1$ global SUSY. $M$ denotes the common value of the gaugino masses, $A$ is the trilinear scalar coupling, while $B$ denotes the bilinear scalar

† Notice, that, in practice, it is not possible to have an efficient baryogenesis in the MSSM unless the SUSY phases are $O(1)$. 

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coupling. In our notation, all these three parameters are dimensionful. Two combinations of the phases of these four parameters are physical [10]. We use here the commonly adopted choice:

\[ \varphi_A = \text{arg}(A^* M), \quad \varphi_B = \text{arg}(B^* M) \]

(1)

where also \( \text{arg}(B\mu) = 0 \), i.e. \( \varphi_\mu = -\varphi_B \).

The main constraints on \( \varphi_A \) and \( \varphi_B \) come from their contribution to the EDMs of the neutron and of the electron. For instance, the effect of \( \varphi_A \) and \( \varphi_B \) on the electric and chromoelectric dipole moments of the light quarks \( (u, d, s) \) lead to a contribution to \( d_N^e \) of order [11]

\[ d_N^e \sim 2 \left( \frac{100 \text{ GeV}}{\tilde{m}} \right)^2 \sin \varphi_{A,B} \times 10^{-23} e \text{ cm}, \]

(2)

where \( \tilde{m} \) here denotes a common mass for squarks and gluinos. The present experimental bound, \( d_N^e < 1.1 \times 10^{-25} e \text{ cm} \), implies that \( \varphi_{A,B} \) should be \( <10^{-2} \), unless one pushes SUSY masses up to \( \mathcal{O} (1 \text{ TeV}) \).

In view of the previous considerations, most authors dealing with the MSSM prefer to simply put \( \varphi_A \) and \( \varphi_B \) equal to zero. Actually, one may argue in favour of this choice by considering the soft-breaking sector of the MSSM as resulting from SUSY breaking mechanisms which force \( \varphi_A \) and \( \varphi_B \) to vanish. Since \( \varphi_A \) ‘measures’ the relative phase of \( A \) and \( M \), in this case it would ‘naturally’ vanish. In some specific models, it has been shown [12] that through an analogous mechanism \( \varphi_B \) may also vanish.

If \( \varphi_A = \varphi_B = 0 \), then the novelty of SUSY in \( CP \) violating contributions merely arises from the presence of the CKM phase in loops where SUSY particles run [10,13]. The crucial point is that the usual GIM suppression, which plays a major role in evaluating \( \epsilon_K \) and \( \epsilon'/\epsilon \) in the SM, in the MSSM case (or more exactly in the CMSSM) is replaced by a super-GIM cancellation which has the same ‘power’ of suppression as the original GIM. Again, also in the CMSSM, as is the case in the SM, the smallness of \( \epsilon_K \) and \( \epsilon'/\epsilon \) is guaranteed not by the smallness of \( \delta_{\text{CKM}} \), but rather by the small CKM angles and/or small Yukawa couplings. By the same token, we do not expect any significant departure of the CMSSM from the SM predictions also concerning \( CP \) violation in \( B \) physics. As a matter of fact, given the large lower bounds on squark and gluino masses, one expects relatively tiny contributions of the SUSY loops in \( \epsilon_K \) or \( \epsilon'/\epsilon \) in comparison with the normal \( W \) loops of the SM. Several analyses in the literature tackle the above question or, to be more precise, the more general problem of the effect of light \( \tilde{t} \) and \( \chi^+ \) on FCNC processes. In this case sizeable contributions can still occur. The generic situation concerning \( CP \) violation in the MSSM case with \( \varphi_A = \varphi_B = 0 \) and exact universality in the soft-breaking sector can be summarized in the following way: the MSSM does not lead to any significant deviation from the SM expectation for \( CP \) violating phenomena such as \( d_N^e, \epsilon_K, \epsilon'/\epsilon \) and \( CP \) violation in \( B \) physics; the only exception to this statement concerns a small portion of the MSSM parameter space where a very light \( \tilde{t} (m_\tilde{t} < 100 \text{ GeV}) \) and \( \chi^+ (m_\chi \sim 90 \text{ GeV}) \) are present. In this latter particular situation, sizeable SUSY contributions to \( \epsilon_K \) are possible and, consequently, major restrictions in the \( \rho-\eta \) plane can be inferred. Obviously, \( CP \) violation in \( B \) physics becomes a crucial test for this MSSM case with very light \( \tilde{t} \) and \( \chi^+ \). Interestingly enough, such low values of SUSY masses are at the border of the detectability region at LEP II.

On the other hand, in recent years, the attitude towards the EDM problem in SUSY and the consequent suppression of the SUSY phases has significantly changed. Indeed, options have been envisaged allowing for a conveniently suppressed SUSY contribution to the EDM...
even in the presence of large (sometimes maximal) SUSY phases. Methods of suppressing the EDMs consist of cancellation of various SUSY contributions among themselves [14], nonuniversality of the soft-breaking parameters at the unification scale [15] and approximately degenerate heavy sfermions for the first two generations [16]. In the presence of one of these mechanisms, large supersymmetric phases are expected, yet EDMs should be generally close to the experimental bounds. In the next section, we focus on flavour changing CP violation in SUSY with nonvanishing SUSY phases, both without new flavour and in the presence of new flavour structures. Then in section 3 we present our conclusions and outlook.

2. Flavour changing CP violation

Despite the large sensitivity of EDMs to the presence of new phases, so far only neutral meson systems, $K^0-\bar{K}^0$ or $B^0-\bar{B}^0$, show measurable effects of CP violation. This fact is, at first sight, surprising because in the neutral mesons CP violation is associated with a change in flavour and hence is CKM suppressed, whereas EDMs are completely independent of flavour mixing. The reason for this is that, in the SM, CP violation is intimately related to flavour, to the extent that observable CP violation requires, not only a phase in the CKM mixing matrix, but also three nondegenerate families of quarks [18]. As shown in the previous section, the supersymmetrized SM contains new sources of CP, both flavour independent and flavour dependent. Although the new phases are, in principle, strongly constrained by the EDM experimental limits, we have seen that several mechanisms allow us to satisfy these constrains with large supersymmetric phases. Next, we analyse this possibility and the effects in flavour changing CP violation observables. We first concentrate on an MSSM with a flavour-blind SUSY breaking, and then we study a general MSSM in which the soft-breaking terms include all kinds of new flavour structures.

2.1. Flavour-blind SUSY breaking and CP violation

The first step in our review of supersymmetric CP violation is the analysis of an MSSM with flavour-blind SUSY breaking. Flavour blind refers to a softly broken supersymmetric SM in which the soft-breaking terms do not introduce any new flavour structure beyond the Yukawa matrices, whose presence in the superpotential is required to reproduce correctly the fermion masses and mixing angles. Supersymmetry is broken at a large scale, that we identify with $M_{GUT}$, and from here, the parameters evolve with the standard MSSM renormalization group equations (RGE) [19, 20] down to the electroweak scale. In these conditions the most general allowed structure of the soft-breaking terms at $M_{GUT}$ is

$$
\begin{align*}
(m_Q^2)_{ij} &= m_Q^2 \delta_{ij} \\
(m_U^2)_{ij} &= m_U^2 \delta_{ij} \\
(m_L^2)_{ij} &= m_L^2 \delta_{ij} \\
(m_D^2)_{ij} &= m_D^2 \delta_{ij} \\
(m_{H1}^2)_{ij} &= m_{H1}^2 \delta_{ij} \\
(m_{H2}^2)_{ij} &= m_{H2}^2 \delta_{ij} \\
(A_U)_{ij} &= A_U e^{i \phi_A} (Y_U)_{ij} \\
(A_D)_{ij} &= A_D e^{i \phi_A} (Y_D)_{ij} \\
(A_L)_{ij} &= A_L e^{i \phi_A} (Y_L)_{ij} \\
(A_E)_{ij} &= A_E e^{i \phi_A} (Y_E)_{ij}
\end{align*}
$$

where all the allowed phases are explicitly written except possible phases in the Yukawa matrices that give rise to an observable phase in the the CKM matrix, $\delta_{CKM}$. It is important to emphasize

† In this case, two-loop contributions can become dominant and must be taken into account to constrain SUSY phases, specially in the large-tan β regime [17].
that, in this flavour-blind MSSM, \( \delta_{\text{CKM}} \) is the only physical phase in the Yukawa matrices and all other phases in \( Y_U \) and \( Y_D \) can be rephased away in the same way as in the SM. The absence of flavour structure in the scalar sector means that quarks and squarks can be rotated parallel already at the GUT scale and hence only \( \delta_{\text{CKM}} \) survives. This is not true in the presence of new flavour structures, where additional Yukawa phases cannot be rephased away from quark–squark couplings [21]. Furthermore, we also assume unification of gaugino masses at \( M_{\text{GUT}} \) and the universal gaugino mass can always be taken as real.

The soft-breaking term structure in equation (3) includes, as the simplest example, the CMSSM where all scalar masses and \( A \)-terms are universal and the number of parameters is reduced to six real parameters once we require radiative symmetry breaking, \( (m_{1/2}, m_0^2, A_0, \tan \beta, \phi_\mu, \phi_A) \) [20, 22, 23]. More general soft-breaking terms in the absence of new flavour structures can arise in GUT models [24]. For instance, in an \( SU(5) \) model, we expect common masses for the particles in the \( 5 \) and in the \( 10 \) multiplets, and, in general, different masses for the two Higgses. The new parameters in the soft-breaking sector would then be \( (m_{1/2}, m_0^2, m_{10}^2, m_{H_1}^2, m_{H_2}^2, A_5, A_{10}, \tan \beta, \phi_\mu, \phi_{A_5}, \phi_{A_{10}}) \) [25]. We take this structure as a representative example of equation (3), since it already shares all the relevant features. In any case, although the number of parameters is significantly increased with respect to the CMSSM, it can still be managed and a full RGE evolution and analysis of the low-energy spectrum is possible.

In this framework, we consider SUSY effects on flavour changing \( CP \) violation and, in particular, the \( CP \) asymmetry in the \( b \to s \gamma \) decay, \( \varepsilon_K \) and \( B^0 \ CP \) asymmetries. However, we include also two \( CP \) conserving observables that are relevant in the fit of the unitarity triangle, namely \( \Delta M_{B_d} \) and \( \Delta M_{B_s} \). All these processes receive two qualitatively different supersymmetric contributions. As shown in the previous section, supersymmetry introduces new \( CP \) violation phases that can strongly modify these observables through their effects in SUSY loops. On the other hand, even with vanishing SUSY phases, the presence of the CKM phase in loops containing SUSY particles induces new contributions that modify the SM predictions for these observables.

Concerning the first possibility, we consider the following extreme situation: we analyse the effects of both \( \phi_\mu \) and a flavour-independent \( \phi_A \) in flavour changing \( CP \) violation experiments, ignoring completely (as a first step) EDM bounds. The result looks rather surprising at first sight: in the absence of the CKM phase, a general MSSM with all possible phases in the soft-breaking terms, but no new flavour structure beyond the usual Yukawa matrices, can never give a sizeable contribution to \( \varepsilon_K, \varepsilon'/\varepsilon \) or hadronic \( B^0 \ CP \) asymmetries [26]. In other words, the effects of SUSY phases in a flavour-blind MSSM are restricted in practice to \( LR \) transitions, such as the EDM or the \( CP \) asymmetry in the \( b \to s \gamma \) decay, and the effects in observables with dominant chirality conserving contributions are negligible even with maximal SUSY phases.

Accordingly, the most interesting \( CP \) violation observable in these conditions is probably the \( CP \) asymmetry in the \( b \to s \gamma \) decay. However, we must take into account that the branching ratio itself is a very strong constraint in any SUSY model [27]. The \( CP \) asymmetry is defined as

\[
A_{CP}^{b \to s \gamma} = \frac{BR(\bar{B} \to X s \gamma) - BR(B \to X s \gamma)}{BR(\bar{B} \to X s \gamma) + BR(B \to X s \gamma)} \simeq \frac{1}{|C_{\gamma}|^2} \left( 0.012 \text{ Im}\{C_2 C_7^*\} - 0.093 \text{ Im}\{C_8 C_7^*\} + 0.001 \text{ Im}\{C_2 C_8^*\} \right),
\]

(4)
Figure 1. \( CP \) asymmetry in % versus branching ratio in the decay \( b \to s\gamma \). Black dots respect (or grey open circles violate) EDM constraints.

where the different \( C_i \) are the Wilson coefficients of the current–current, \( Q_3 = \langle \bar{s}_L\alpha\gamma_{\mu}\gamma_{\nu}\gamma_{\rho}\bar{b}_L \rangle \), magnetic, \( Q_7 = \frac{g_m}{16\pi^2} \bar{s}_L\sigma_{\mu\nu}b_{R\alpha}F_{\mu\nu} \), and chromomagnetic, \( Q_8 = \frac{g_m}{16\pi^2} \bar{s}_L\sigma_{\mu\nu}b_{R\alpha}G_{\mu\nu} \), dipole operators, evaluated at the scale \( \mu = m_b \) [28]. This asymmetry is predicted to be below 1% in the SM [28, 29]. On the other hand, we have seen that the new SUSY phases can modify the \( b \to s\gamma \) transition significantly. In fact, several studies showed that the MSSM in the presence of large \( \phi_\mu \) and \( \phi_A \) can enhance the \( CP \) asymmetry up to 15% [25], [30]–[33], which could be easily accessible at \( B \) factories. In any case, it is important to remember that this scenario is viable only if some mechanism reduces the SUSY contributions to the EDM. In the case of the CMSSM or a flavour-blind MSSM with possible EDM cancellations this analysis was repeated in [25, 32, 33] and the asymmetry can reach at most a few percent. Figure 1 shows that without EDM constraints (open grey circles) the asymmetry can be above 5% at any value of the branching ratio and can even reach 13% for low branching ratios. In figure 1 the points of the parameter space that fulfil EDM constraints are represented by black dots. The effect on the \( CP \) asymmetry can be sizeable at low values of the branching ratio [25], but for larger values of the branching ratio the asymmetry is again around 1%. In the plot, the points of the parameter space fulfilling EDM constraints are represented by black dots. In this regard, it is important to keep in mind, that the phase of the \( \mu \) term is, in any case, constrained to be \( \phi_\mu \lesssim 0.2 \) for scalar masses \( m_0 \lesssim 500 \) GeV [25]. On the other hand, the phases of the \( A \) terms are basically unconstrained by EDM in this scheme [25, 34].

A more complete analysis under these conditions was made in [25], where the flavour-blind conditions are specified at a large scale \( M_{GUT} \) and the standard MSSM RGEs [19, 35] are used to evolve the initial conditions down to the electroweak scale. Two representative examples of flavour-blind MSSM were considered, the CMSSM as the simplest model and the \( SU(5) \)-inspired model defined above. Here, we are mainly interested in the following part of the low-energy spectrum: \( \chi^+, H^+ \) and \( \tilde{t} \). Their masses are evolved to \( M_W \) and then all the relevant experimental constraints are imposed.

- Absence of charge and color breaking minima and directions unbounded from below [36].
the rare scenarios where different. This has important effects on the radiative symmetry breaking and, in fact, lower values is the fact that the Higgs masses are not tied to the other scalar masses and now may be quite SU

\[
m^{\chi^\pm}_1 < 90 \text{ GeV},
m^{\chi^0}_0 > 33 \text{ GeV and } m_{\tilde{g}} > 33 \text{ GeV}.
\]

- Neutralino as the lightest supersymmetric particle.

In this way, the complete supersymmetric spectrum at the electroweak scale is obtained in terms of six or 11 parameters in the CMSSM or \(SU(5)\)-inspired model respectively. Within an MSSM scenario, this kind of analysis was first made in the work of Bertolini et al [20] and has been updated several times since then [39]. We follow Bartl et al [25], who developed a specialized study of the spectrum relevant for FCNC and \(CP\) violation experiments. Indeed, the most interesting point of this work is the strong correlation among different SUSY masses that have a strong impact on low-energy FCNC and \(CP\) violation studies. Figure 2 shows scatter plots of the mass of the lightest chargino versus the lightest stop mass. In these plots we vary the scalar and gaugino masses at \(M_{\text{GUT}}\) as 100 GeV < \(m_i\) < 1000 GeV, the trilinear terms as 0 < |\(A_i| < m^2_{\tilde{t}_1} + m^2_{\tilde{l}_1} + m^2_{\tilde{q}_1}\) with arbitrary phases and 2 < \(\tan \beta < 50\). It is interesting to notice in this plot the very strong correlation among the chargino and stop masses. In fact, this correlation can be easily understood with the help of the one-loop RGE [19]. Neglecting for the moment the so-called D-terms and the small radiatively generated intergenerational squark mixing, we obtain for the stop masses

\[
m^{2}_{\tilde{t}_{1,2}} = \frac{1}{2} \left( M^{2}_{Q_3} + M^{2}_{U_3} + 2m^{2}_{t} + \sqrt{(M^{2}_{Q_3} - M^{2}_{U_3})^2 + 4m^{2}_{t}(A_t - \mu \cot \beta)^2} \right),
\]

in terms of the soft parameters at the electroweak scale. Thanks to the proximity of the top quark mass to its quasi-fixed point and the relative smallness of \(\mu \cot \beta\) for \(\tan \beta \geq 2.5\), we can express equation (5) as a function of the initial parameters at \(M_{\text{GUT}}\) with only a small variation of the coefficients with \(\tan \beta\). In the CMSSM case we find

\[
m^{2}_{\tilde{t}_{1,2}} = 0.43M^{2}_0 + 4.55M^{2}_{1/2} + m^{2}_{t} + 0.19 \text{ Re}(M^{2}_{1/2}A_0)\]

\[
\pm \frac{1}{2} \sqrt{2.25M^{4}_{1/2} + 1.13M^{2}_{0}M^{2}_{1/2} + 20.2m^{2}_{t}M^{2}_{1/2}}.
\]

Moreover, in the CMSSM \(|\mu| \gtrsim \sqrt{3}m_{1/2}\) is always larger than \(M_5 \approx 0.81m_{1/2}\) and, hence, the lightest chargino is predominantly gaugino. Then we can replace the initial gaugino mass in terms of the lightest chargino mass and finally obtain

\[
m^{2}_{\tilde{t}_{1,2}} = 0.43M^{2}_0 + 6.93m^{2}_{\chi_1^+} + m^{2}_{t} + 0.23 \text{ Re}(m^{2}_{\chi_1^+}A_0)\]

\[
\pm \frac{1}{2} \sqrt{5.23m^{4}_{\chi_1^+} + 1.72M^{2}_{0}m^{2}_{\chi_1^+} + 30.8m^{2}_{t}m^{2}_{\chi_1^+}}.
\]

From here, we obtain for 100 GeV < \(m_0 < 1 \text{ T eV}\) and with \(m_{\chi} = 100 \text{ GeV}\) a maximal allowed range for the lightest stop mass of 230 GeV \(\lesssim m_{\tilde{t}_1} \lesssim 660 \text{ GeV}\). As figure 2 shows, this correlation is maintained for larger chargino masses. In the case of \(SU(5)\), the main difference is the fact that the Higgs masses are not tied to the other scalar masses and now may be quite different. This has important effects on the radiative symmetry breaking and, in fact, lower values of \(\mu\) are possible such that the lightest chargino can have a predominant higgsino component. In the rare scenarios where \(|\mu| \lesssim M_2\), the stop masses are somewhat lower than for the CMSSM case. In any case, a similar correlation is still maintained. We must emphasize that, due to gluino dominance in the soft-term evolution, this kind of correlation is general in any RGE-evolved MSSM from some GUT initial conditions, assuming that gaugino masses unify as well.

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4.9

Figure 2. Chargino mass versus lightest stop mass for the parameter space described in the text in the CMSSM ($SU(5)$ case is similar). Black dots and open circles represent points satisfying or violating the $b \rightarrow s\gamma$ constraint respectively.

In summary, this implies that the ‘light stop and chargino’ scenario in any GUT-evolved MSSM must be shifted to stop masses in the range of 250 GeV and chargino masses of 100 GeV. A very similar correlation can be obtained for other squark masses, roughly

$$m_\tilde{q} \simeq 9.3 \times m_\tilde{\chi}^2 + m_0^2.$$  \hspace{1cm} (8)

In a similar way, we can discuss the charged Higgs-boson mass. The main features here are the fact that, for low $\tan \beta$, the masses of the charged Higgs boson are above 400 GeV, and that, specially in the CMSSM case, most of these light Higgs-boson masses are eliminated by the $b \rightarrow s\gamma$ constraint. For larger values of $\tan \beta$ slightly lighter masses are allowed, but it is still true that we seldom find charged Higgs masses below 300 GeV in any case. The reason for this is again the gluino dominance in the RGE. For instance, at $\tan \beta = 5$ the charged Higgs is $m_{H^+}^2 \simeq 1.23m_0^2 + 3.31m_{1/2}^2$ and at $\tan \beta = 30$, $m_{H^+}^2 \simeq 0.72m_0^2 + 1.98m_{1/2}^2$, from the one-loop RGE [25].

Taking into account the relevant features of the MSSM spectrum discussed above, we can discuss the SUSY contribution in a flavour-blind scenario to the different CP violating observables. First, the $b \rightarrow s\gamma$ CP asymmetry has already been discussed in the presence of large supersymmetric phases that survive the EDM constraints through a cancellation mechanism. In this case, the asymmetry could reach a few per cent; however, with vanishing SUSY phases we again obtain an asymmetry in the range of the SM value, well below 1% [28, 29]. A similar situation is found in $\varepsilon'/\varepsilon$, where the SUSY contributions tend to lower the SM prediction [40, 41].

Second, the $\Delta F = 2$ observables, i.e. $\varepsilon_K$ and $B^0 - \bar{B}^0$ mixing, which play a fundamental role in the unitarity triangle fit, are also modified by new SUSY contributions. Taking into account the new SUSY contributions, the SM fit of the unitarity triangle is modified and one obtains different restrictions on the $\rho$ and $\eta$ parameters of the CKM matrix. Moreover this fit has to be compatible with the new direct measurements of the $B^0$ CP asymmetries [3, 4]. As explained elsewhere [25], given that the SUSY contributions tend to interfere constructively with the SM with a factorized CKM dependence, this implies that for a given SUSY contribution the values of $\eta$ required to saturate $\varepsilon_K$ are now smaller. The value of $|V_{td}V_{tb}^*|$ required to saturate $\Delta M_{B_d}$ is

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analogously decreased. Hence, it is evident that the addition of SUSY tends to lower the values of $\eta$ and increase the values of $\rho$ in the fit, therefore reducing the actual value of $\beta$. However, as shown by Buras and Buras [42], in any minimal flavour violation model at the electroweak scale, a strong correlation exists among the $\Delta F = 2$ contributions to $\varepsilon_K$ and $\Delta M_{B_d}$, and this allows only a small departure of $\sin 2\beta$ from the SM prediction. Still, different values of $\alpha$ and $\gamma$ are, in principle, allowed. Nevertheless, as shown above, the relative heaviness [25, 43] of the SUSY spectrum implies that the deviation from the SM fit in these models tends to be small for these angles. In summary, a flavour-blind MSSM cannot generate large deviations from the SM expectations in the $B^0 CP$ asymmetries [3, 4].

2.2. CP violation in the presence of new flavour structures

Flavour universality of the soft SUSY breaking is a strong assumption and is known not to be true in many supergravity and string-inspired models [44]–[47]. In these models, a nontrivial flavour structure in the squark mass matrices or trilinear terms is generically obtained at the supersymmetry breaking scale. Hence, sizeable flavour off-diagonal entries appear in the squark mass matrices, and new FCNC and $CP$ violation effects can be expected. In fact, most of these flavour off-diagonal entries are severely constrained or even ruled out by low-energy FCNC and $CP$ violation observables.

A very convenient parametrization of the SUSY effects in these rare processes is the so-called mass insertion approximation [48]. It is defined in the super CKM (SCKM) basis at the electroweak scale, where all the couplings of sfermions to neutral gauginos are flavour diagonal. In this basis, the sfermion mass matrices are not diagonal. The sfermion propagators, now flavour off-diagonal, can be expanded as a series in terms of $\delta = \Delta/\tilde{m}^2$, where $\Delta$ denotes the off-diagonal terms in the sfermion mass matrices, with $\tilde{m}^2$ an average sfermion mass [49].

As a result, FCNC and $CP$ violation constraints can be expressed as model-independent upper bounds on these mass insertions at the electroweak scale and they can readily be compared with the corresponding mass insertions calculated in a well defined SUSY model.

A complete analysis of this kind was performed (see [49, 50] for a more complete discussion) and the constraints from $\Delta S = 2$ processes were later updated [51]. In the following, we present the phenomenological constraints from [49, 51].

The main constraints on $CP$ violating mass insertions (with nonvanishing phases) come from $\varepsilon_K, \varepsilon'/\varepsilon$ and EDMs, although 'indirect' constraints from $b \to s\gamma$ and $B\bar{B}$ mixing are also relevant. These constraints are presented in tables 1–5. It is important to emphasize the strong sensitivity of $\varepsilon'/\varepsilon$ to $(\delta_{LR})_{12}$ and of $\varepsilon_K$ to $(\delta_{LL})_{12}$, which implies that it is difficult to saturate both simultaneously with a single mass insertion [52, 53].

What message should we draw from the constraints in tables 1–5? First, it is apparent that FCNC and especially $CP$ violating processes represent a significant test for SUSY extensions of the SM. Taking arbitrary sfermion mass matrices completely unrelated to the fermion mass matrices would lead to mass insertions of order unity. Consequently, the first conclusion we draw from the small numbers in tables 1–5 is that there must be some close relation between the flavour structures of the sfermion and fermion sectors. Large portions of the parameter spaces of minimal SUSY models are completely ruled out thanks to the severity of the FCNC and $CP$ constraints.

However, then an even more important question emerges after one studies tables 1–5: Given the strong constraints from FCNC and $CP$ violating processes that we have already observed, can we still hope to see SUSY signals in other rare processes? In particular, restricting this question...
Table 1. Limits from $\varepsilon_{K}$ on $\text{Im}(\delta_{A})_{12}^{d}(\delta_{B})_{12}$, with $A, B = (L, R, LR)$ including next-to-leading-order QCD corrections and lattice $B$ parameters [51], for an average squark mass $m_{q} = 500$ GeV and for different values of $x = m_{q}/m_{\tilde{q}}$.

| $x$  | $\sqrt{\text{Im}(\delta_{A})_{12}^{d2}}$ | $\sqrt{\text{Im}(\delta_{LR})_{12}^{d2}}$ | $\sqrt{\text{Re}(\delta_{A})_{12}^{d2}(\delta_{B})_{12}}$ |
|------|---------------------------------|---------------------------------|---------------------------------|
| 0.3  | $2.9 \times 10^{-3}$            | $3.4 \times 10^{-4}$            | $1.1 \times 10^{-4}$            |
| 1.0  | $6.1 \times 10^{-3}$            | $3.7 \times 10^{-4}$            | $1.3 \times 10^{-4}$            |
| 4.0  | $1.4 \times 10^{-2}$            | $5.2 \times 10^{-4}$            | $1.8 \times 10^{-4}$            |

Table 2. Limits from $\varepsilon'/\varepsilon < 2.7 \times 10^{-3}$ on $\text{Im}(\delta_{LR})_{12}$, for an average squark mass $m_{q} = 500$ GeV and different values of $x = m_{q}/m_{\tilde{q}}$.

| $x$  | $\text{Im}(\delta_{LR})_{12}$ | $\text{Re}(\delta_{LR})_{12}$ |
|------|-------------------------------|-------------------------------|
| 0.3  | $1.0 \times 10^{-1}$          | $1.1 \times 10^{-5}$          |
| 1.0  | $4.8 \times 10^{-1}$          | $2.0 \times 10^{-5}$          |
| 4.0  | $2.6 \times 10^{-1}$          | $6.3 \times 10^{-5}$          |

Table 3. Limits on $\text{Im}(\delta_{LR})_{11}$ from EDMs, for $m_{q} = 500$ GeV and $m_{l} = 100$ GeV.

| $x$  | $\text{Im}(\delta_{LR})_{11}$ | $\text{Re}(\delta_{LR})_{11}$ | $\text{Re}(\delta_{LR})_{11}$ |
|------|-------------------------------|-------------------------------|-------------------------------|
| 0.3  | $2.4 \times 10^{-6}$          | $4.9 \times 10^{-6}$          | $3.0 \times 10^{-7}$          |
| 1.0  | $3.0 \times 10^{-6}$          | $5.9 \times 10^{-6}$          | $3.7 \times 10^{-7}$          |
| 4.0  | $5.6 \times 10^{-6}$          | $1.1 \times 10^{-5}$          | $7.0 \times 10^{-7}$          |

Table 4. Limits on the $|\delta_{23}^{d}|$ from $b \rightarrow s\gamma$ decay for an average squark mass $m_{q} = 500$ GeV and different values of $x = m_{q}/m_{\tilde{q}}$.

| $x$  | $|\delta_{23}^{d}|$ | $|\delta_{LR}^{d}|_{23}$ |
|------|--------------------|--------------------------|
| 0.3  | $4.4$              | $1.3 \times 10^{-2}$     |
| 1.0  | $8.2$              | $1.6 \times 10^{-2}$     |
| 4.0  | $26$               | $3.0 \times 10^{-2}$     |

Table 5. Limits on $\text{Re}(\delta_{A})_{13}(\delta_{B})_{13}$, with $A, B = (L, R, LR)$ from $B^{0}\rightarrow\bar{B}^{0}$ mixing, for an average squark mass $m_{q} = 500$ GeV and for different values of $x = m_{q}/m_{\tilde{q}}$.

| $x$  | $\sqrt{\text{Re}(\delta_{A})_{13}^{2}}$ | $\sqrt{\text{Re}(\delta_{LR})_{13}^{2}}$ | $\sqrt{\text{Re}(\delta_{A})_{13}^{2}(\delta_{B})_{13}}$ |
|------|---------------------------------|---------------------------------|---------------------------------|
| 0.3  | $4.6 \times 10^{-2}$            | $5.6 \times 10^{-2}$            | $1.6 \times 10^{-2}$            |
| 1.0  | $9.8 \times 10^{-2}$            | $3.3 \times 10^{-2}$            | $1.8 \times 10^{-2}$            |
| 4.0  | $2.3 \times 10^{-1}$            | $3.6 \times 10^{-2}$            | $2.5 \times 10^{-2}$            |
to $CP$ violation, can we still hope to find a significant disagreement with the SM expectations when we measure $CP$ violation in various $B$ decay channels? Fortunately for us, the answer to this last question is yes. For example, it has been shown [54] that considering the $CP$ asymmetry in several $B$ decay channels, which in the SM would give just the same answer (the angle $\beta$ of the unitarity triangle), it is possible to obtain different values when SUSY effects are switched on. SUSY contributions to some of the decay amplitudes can be as high as 70% with respect to the SM contribution, whereas other decay channels are not affected at all by the SUSY presence. Hence, assuming large $CP$ violating phases in SUSY, one could find discrepancies with the SM expectations that are larger than any reasonable theoretical hadronic uncertainty in the SM computation. We refer the interested reader to [54] for a detailed discussion.

It is worth emphasizing that the above example shows that there is still room for sizeable SUSY signals in $CP$ violating processes, but this represents some kind of ‘maximal hope’ of what we can expect from SUSY. In other words, one takes the maximally allowed values of relevant $\delta$s to maximize the possible SUSY deviations from SM on $CP$ observables. A different question is what we can ‘typically’ expect in a SUSY model. As we stressed in the introduction, no ‘typical’ SUSY model exists; what we call low-energy SUSY represents a vast class of models. Yet it makes sense to try to identify some features of minimal SUSY models where no drastic departures from flavour universality are taken and to consider in this more restricted context what we can expect.

In the following, we analyse a ‘realistic’ nonuniversal MSSM, and compute the ‘reasonable’ expectations for the different mass insertions in this context. In the first place, we define our generic MSSM through a set of four general conditions.

1. **Minimal particle content.** We consider the MSSM, with no additional particles from $M_W$ to $M_{GUT}$.

2. **Arbitrary soft-breaking terms $O(m_{3/2})$.** The supersymmetry soft-breaking terms as given at the scale $M_{GUT}$ have a completely general flavour structure, but all of them are of the order of a single scale, $m_{3/2}$.

3. **Trilinear couplings originate from Yukawa couplings.** Although trilinear couplings are a completely new flavour structure they are related to the Yukawas in the usual way: $Y_{ij}^A = A_{ij} \cdot \tilde{Y}_{ij}$, with all $A_{ij} \approx O(m_{3/2})$.

4. **Gauge coupling and gaugino unification at $M_{GUT}$ and RGE evolution of the different parameters from that scale.**

In this framework, any particular MSSM is completely defined, once we specify the soft-breaking terms at $M_{GUT}$. We specify these soft-breaking terms in the basis in which all the squark mass matrices, $M_Q^2, M_U^2, M_D^2$, are diagonal. In this basis, the Yukawa matrices are $v_1 Y_d = K^{D_L \dagger} \cdot M_d \cdot K^{D_R}$ and $v_2 Y_u = K^{D_L \dagger} \cdot K^{U_R \dagger} \cdot M_u \cdot K^{D_R}$, with $M_d$ and $M_u$ diagonal quark mass matrices, $K$ the CKM mixing matrix and $K^{D_L}, K^{U_R}, K^{D_R}$ unknown, completely general, $3 \times 3$ unitary matrices.

Although our analysis is completely general within this scenario [55], we prefer to discuss a concrete example based on type-I [47] string theory (see [56] for a definition).† In this particular

† The case where large flavour-independent soft phases may give a dominant contribution to $CP$ violation has been discussed elsewhere [53].
example, gaugino masses, right-handed squarks and trilinear terms are nonuniversal. Gaugino masses are

\[ M_3 = M_1 = \sqrt{3}m_{3/2} \sin \theta e^{-i\alpha_S}, \]
\[ M_2 = \sqrt{3}m_{3/2} \cos \theta \Theta_1 e^{-i\alpha_1}, \]

whereas the \( A \)-terms are obtained as

\[ A_1 = -\sqrt{3}m_{3/2} [\sin \theta e^{-i\alpha_S} + \cos \theta (\Theta_1 e^{-i\alpha_1} - \Theta_3 e^{-i\alpha_3})] \]
\[ A_2 = -\sqrt{3}m_{3/2} [\sin \theta e^{-i\alpha_S} + \cos \theta (\Theta_1 e^{-i\alpha_1} - \Theta_2 e^{-i\alpha_2})] \]
\[ A_3 = -\sqrt{3}m_{3/2} \sin \theta e^{-i\alpha_S} = -M_3, \]

for the trilinear terms associated with the first-, second- and third-generation right-handed squarks respectively. Here \( m_{3/2} \) is the gravitino mass, \( \alpha_S \) and \( \alpha_i \) are the \( CP \) phases of the \( F \) terms of the dilaton field \( S \) and the three moduli fields \( T_i \), and \( \theta \) and \( \Theta_i \) are goldstino angles with the constraint \( \sum \Theta_i^2 = 1 \). Hence, the trilinear SUSY breaking matrix, \( (Y^A)_{ij} = (Y)_{ij}(A)_{ij} \), itself is obtained as

\[ Y^A = (Y_{ij}) \cdot \begin{pmatrix} A_{C_3} & 0 & 0 \\ 0 & A_{C_2} & 0 \\ 0 & 0 & A_{C_1} \end{pmatrix}, \]

in matrix notation [57].

In addition, universal soft scalar masses for quark doublets and the Higgs fields are obtained,

\[ m_{L_i}^2 = m_{3/2}^2 [1 - \frac{3}{2} \cos^2 \theta (1 - \Theta_i^2)]. \]

Finally, the soft scalar masses for quark singlets are nonuniversal,

\[ m_{R_i}^2 = m_{3/2}^2 (1 - 3 \cos^2 \theta T_i), \]

with \( T_i = (\Theta_3^2, \Theta_2^2, \Theta_1^2) \).

To complete the definition of the model, we also need to specify the Yukawa textures. The only available experimental information is the CKM mixing matrix and the quark masses. We choose our Yukawa texture following two simple assumptions: (a) the CKM mixing matrix originates from the down-type Yukawa couplings (as done in [58]) and (b) our Yukawa matrices are Hermitian [59]. With these two assumptions we obtain \( K_{DL} = K \) and \( K_{UL} = 1 \). However, it is important to emphasize that given that now \( K_{DL} \) and \( K_{UL} \) measure the flavour misalignment between quarks and squarks, and that we already use the rephasing invariance of the quarks to make \( \delta_{\text{CKM}} \) real, we can expect new observable (unremovable) phases in the quark–squark mixings, and in particular in the first two-generation sector. That is,

\[ K_{DL} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda e^{i\alpha} & Ap \lambda^3 e^{i\beta} \\ -\lambda e^{-i\alpha} & 1 - \lambda^2/2 & A \lambda^2 e^{-i\gamma} \\ A\lambda^3 (e^{-(\alpha+\gamma)} - p' e^{-i\beta}) & A \lambda^2 e^{i\gamma} & 1 \end{pmatrix} \]

(14)

to \( O(\lambda^4) \); \( A \) and \( p' = |p + i\eta| \) are the usual parameters in the Wolfenstein parametrization, both \( O(1) \). We must emphasize here that the observable phase in the CKM mixing matrix corresponds to the combination \( \delta_{\text{CKM}} = \beta - \alpha - \gamma \); hence it is transparent that we can have a vanishing \( \delta_{\text{CKM}} \) while being left with large observable phases in the SUSY sector [21]. Hence, the Yukawa matrices are \( v_1 Y_d = K_{DL} \cdot M_d \cdot K_{DL} \) and \( v_2 Y_u = M_u \). It is important to remember that this is the simplest structure consistent with all phenomenological constraints.
Now, the next step is to use the MSSM RGEs [19, 20] to evolve these matrices down to the electroweak scale. The main RGE effects from $M_{GUT}$ to $M_W$ are those associated with the gluino mass and the large third-generation Yukawa couplings. Regarding squark mass matrices, it is well known that diagonal elements receive important RGE contributions proportional to gluino mass that dilute the mass eigenstate nondegeneracy, $m_{q_D}^2 (M_W) \simeq c_A^1 \cdot m_\tilde{g}^2 + m_{q_D}^2 \Delta [19, 20, 23]$, with $c_1^L \simeq (6.5, 6.5)$, $c_2^L \simeq (5.5, 4.6)$, $c_1^R \simeq (6.1, 6.1)$ and $c_2^R \simeq (6.1, 4.3)$ for $(\tan \beta = 2.5, 40)$ [25]. In the SCKM basis, the off-diagonal elements in the sfermion mass matrices are given by $(K^A \cdot M^2_{D_A} \cdot K^{A\dagger})_{ij \neq j}$, up to smaller RGE corrections. Similarly, gaugino effects in the trilinear RGE are always proportional to the Yukawa matrices, not to the trilinear matrices themselves, and so they are always diagonal to extremely good approximation in the SCKM basis. Once more, the off-diagonal elements will be approximately given by $(K^{Q_L} \cdot Y^A \cdot K^{Q_R \dagger})_{ij \neq j}$.

The $LR$ and $RR$ mass insertions are defined as $(\delta^Q_{LR})_{ij} = (M^2_{Q,LR})_{ij} / m_q^2$ and $(\delta^Q_{RR})_{ij} = (M^2_{Q,RR})_{ij} / m_{\tilde{q}}^2$ respectively. Hence, in our example defined in equations (9)–(14), we have $LR$ and $RR$ off-diagonal mass insertions, which can be estimated as

\begin{equation}
(\delta^q_{LR})_{ij} = \frac{1}{m_q^2} m_i [K^D_{i1} K^D_{j2} \cdot (A_2 - A_1) + K^D_{i3} K^D_{j3} \cdot (A_3 - A_1)] \tag{15}
\end{equation}

and

\begin{equation}
(\delta^q_{R})_{ij} = \frac{1}{m_q^2} [K^D_{i2} K^D_{j2} \cdot (m_R^2 - m_{R_1}) + K^D_{i3} K^D_{j3} \cdot (m_R^2 - m_{R_1})]. \tag{16}
\end{equation}

Equation (15) reveals an important feature of the $LR$ mass insertions. Because of the trilinear term structure in generic models of soft breaking, the $LR$ sfermion matrices are always suppressed by $m_{\tilde{q}}/m_q$, with $m_{\tilde{q}}$, the mass of one of the quarks involved in the coupling and $m_q$ the average squark mass [57]. In any case, this suppression is necessary to avoid charge and colour breaking and directions unbounded from below [36]. We can easily estimate the different mass insertions with these formulae. First we must take into account that, owing to the gluino dominance in the squark eigenstates at $M_W$, $m_{\tilde{q}}^2 (M_W) \approx 6 m_{\tilde{q}}^2 (M_{GUT})$. In the kaon system, we can neglect $m_{\tilde{d}}$; replacing the values of masses and mixings in equations (9)–(14) we obtain

\begin{equation}
(\delta^q_{LR})_{12} \simeq \frac{m_s}{m_q} \frac{(A_2 - A_1)}{m_{\tilde{q}}} K^D_{12} K^D_{22} \cdot \\
\simeq 2.8 \times 10^{-5} \cdot (\Theta_2 \cdot e^{-i\alpha_2} - \Theta_3 \cdot e^{-i\alpha_3}) \cdot \left( \frac{100 \text{ GeV}}{m_{3/2}} \right) \tag{17}
\end{equation}

where we have used $\theta \approx 0.7$ as in [56]. Comparing this value with the bounds in table 2, we see that it could indeed give a very sizeable contribution to $\epsilon^\prime / \epsilon$ [56, 58, 60]. The phases $\alpha_2$ and $\alpha_1$ are actually unconstrained by EDM experiments as emphasized in [56]†. This important result means that even if the relative quark–squark flavour misalignment is absent and the only flavour mixing is provided by the usual CKM matrix, i.e. $K^{D_L} = K^{CKM}$, the presence of nonuniversal flavour-diagonal trilinear terms is enough to generate large FCNC effects in the kaon system.

Similarly, in the neutral $B$ system, $(\delta^q_{LR})_{13}$ contributes to the $\bar{B}_d - B_d$ mixing parameter, $\Delta M_{B_d}$. However, in our minimal scenario, $K^{D_L} \approx K$, we obtain

\begin{equation}
(\delta^q_{LR})_{13} \simeq \frac{m_b}{m_q} \frac{(A_3 - A_1)}{m_{\tilde{q}}} K^D_{13} K^D_{33} \cdot \\
\end{equation}

† Again, in the large-$\tan \beta$ regime, two-loop contributions [17] must be taken into account to constrain these phases.

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\begin{equation}
\simeq 2.5 \times 10^{-5} \cdot (\Theta_1 e^{-i\alpha_1} - \Theta_3 e^{-i\alpha_3}) \cdot \left( \frac{100 \text{ GeV}}{m_{3/2}} \right),
\end{equation}

clearly too small to generate sizeable $\bar{b} \rightarrow \bar{d}$ transitions, as the bounds in Table 5 show. Notice that larger effects are still possible in a more ‘exotic’ scenario with a large mixing in $K_{13}^{DL}$. For instance, with a maximal value, $|K_{13}^{DL} K_{33}^{DL^*}| = 1/2$, we would obtain $(\delta_{LR})_{13} \simeq 2 \times 10^{-5} \cdot (100 \text{ GeV}/m_{3/2})$. Even in this limiting situation, this result is roughly one order of magnitude too small to saturate $\Delta M_B$, though it could still be observed through the $CP$ asymmetries. Hence in the $B$ system we reach a very different result: it is not enough to have nonuniversal trilinear terms; large flavour misalignment among quarks and squarks is also required.

A similar analysis can be made with the chirality conserving mass insertions. From equation (16), in the kaon system, we obtain
\begin{equation}
(\delta_{R})_{12} \simeq \frac{\cos^2 \theta (\Theta_1^2 - \Theta_2^2)}{6 \sin^2 \theta} K_{12}^{DL} K_{22}^{DL^*} + \frac{\cos^2 \theta (\Theta_3^2 - \Theta_2^2)}{6 \sin^2 \theta} K_{13}^{DL} K_{23}^{DL^*} \\
\simeq \frac{\cos^2 \theta (\Theta_3^2 - \Theta_2^2)}{6 \sin^2 \theta} \bar{A} e^{i\alpha}.
\end{equation}

This value has to be compared with the mass insertion bounds required to saturate $\varepsilon_K$ [48], which in this case are $(\delta^d_{R})_{12}^{\text{bound}} \leq 0.0032$. Using $\theta \simeq 0.7$, we obtain
\begin{equation}
(\delta_{R})_{12} \simeq 0.035 (\Theta_3^2 - \Theta_2^2) \sin \alpha \lesssim 0.0032.
\end{equation}

Hence, it is clear that we can easily saturate $\varepsilon_K$ without any special fine-tuning. Indeed, this constraint, which is one of the main sources of the so-called supersymmetric flavour problem, in this generic MSSM amounts to the requirement that $(\Theta_3^2 - \Theta_2^2) \sin \alpha \lesssim 0.1$ with all the different factors in this expression $\Theta_3^2, \Theta_2^2, \sin \alpha \lesssim 1$ [21].

Now we turn to the $CP$ asymmetries in the $B$ system. Once more, with equation (15) we have
\begin{equation}
(\delta_{R})_{13} \simeq \frac{\cos^2 \theta (\Theta_2^2 - \Theta_1^2)}{6 \sin^2 \theta} K_{12}^{DL} K_{32}^{DL^*} + \frac{\cos^2 \theta (\Theta_3^2 - \Theta_1^2)}{6 \sin^2 \theta} K_{13}^{DL} K_{33}^{DL^*} \\
\simeq A \lambda^3 \frac{\cos^2 \theta}{6 \sin^2 \theta} \left[ - (\Theta_2^2 - \Theta_1^2) e^{i(\alpha + \gamma)} + (\Theta_3^2 - \Theta_1^2) (e^{-i(\alpha + \gamma)} - \rho e^{-i\beta}) \right] \lesssim 10^{-3},
\end{equation}

to be compared with the mass insertion bound $(\delta^d_{R})_{12}^{\text{bound}} \leq 0.098$ required to not over-saturate the $B^0$ mass difference.

We conclude that large effects are expected in the kaon system in the presence of nonuniversal squark masses even with a ‘natural’ CKM-like mixing for both chirality changing and chirality conserving transitions. The $B$ system is much less sensitive to supersymmetric contributions, so observable effects are expected only with approximately maximal $\bar{b} \rightarrow \bar{d}$ mixings.

Recently, the arrival of the first measurements of $B^0 CP$ asymmetries from the $B$ factories has caused a great excitement in the high-energy physics community.

\begin{equation}
a_{J/\psi} = \begin{cases} 
0.59 \pm 0.14 \pm 0.05 & \text{(Babar [3])} \\
0.99 \pm 0.14 \pm 0.06 & \text{(Belle [4])} \\
0.79^{+0.41}_{-0.44} & \text{(CDF [5])}.
\end{cases}
\end{equation}
The errors are still too large to draw any firm conclusion. Still, these measurements leave room for an asymmetry sizeably different from the SM expectations corresponding to $0.59 < a_{J/\psi}^{SM} = \sin(2\beta) \leq 0.82$. This possible discrepancy, if confirmed, would be a first sign of the presence of new physics in CP violation experiments. Several papers have discussed the possible implications of a nonstandard CP asymmetry [55, 61] and pointed out two possibilities. A small asymmetry can be due to a large new physics contribution in the $B$ system and/or to a new contribution in the $K$ system modifying the usual determination of the unitarity triangle. Taking into account the results above, in a nonuniversal MSSM it is realistic to reproduce the CP violation in the kaon system through SUSY effects, while being left with a small $a_{J/\psi}$ in the $B$ system. Indeed the role of the CKM phase could be confined to the SM fit of the charmless semileptonic $B$ decays and $B^0_d-\bar{B}^0_d$ mixing, while predominantly attributing to SUSY the $K$ CP violation ($\varepsilon_K$ and $\varepsilon'/\varepsilon$). In this case the CKM phase can be quite small, leading to a lower $a_{J/\psi}$ CP asymmetry [21].

3. Conclusions and outlook

The main points of our discussion can be summarized as follows. (i) There exist strong theoretical and ‘observational’ reasons to go beyond the SM. (ii) The gauge hierarchy and coupling unification problems favour the presence of low-energy SUSY (either in its minimal version, CMSSM, or more naturally, in some less constrained realization). (iii) Flavour and CP problems constrain low-energy SUSY, but, at the same time, provide new tools to search for SUSY indirectly. (iv) In general, we expect new CP violating phases in the SUSY sector. However, these new phases are not going to produce sizeable effects as long as the SUSY model we consider does not exhibit a new flavour structure in addition to the SM Yukawa matrices. (v) In the presence of a new flavour structure in SUSY, large contributions to CP violating observables are indeed possible.

In summary, in a flavour-blind SUSY there exist quite a few special places where we can hope to ‘see’ SUSY in action: the EDMs, the $A_{CP}^{b\to s\gamma}$ and, as emerged recently, the anomalous magnetic moment of the muon. On the other hand, in the more general (and, in our view, also more likely) case where, indeed, SUSY breaking is not insensitive to the flavour mechanism, there exist a rich variety of FCNC and CP violation potentialities for SUSY to show up. As we have seen, $K$ and $B$ physics offer appealing possibilities: $\varepsilon_K$, $\varepsilon'/\varepsilon$, CP violating rare kaon decays, CP asymmetries in $B$ decays, rare $B$ decays, .... In fact, we think that the relevance of SUSY searches in rare processes is not confined to the usually quoted possibility that indirect searches can arrive ‘first’, before direct searches (Tevatron and LHC), in signalling the presence of SUSY. Even after the possible direct production and observation of SUSY particles, the importance of FCNC and CP violation in testing SUSY remains of utmost relevance. They are and will be complementary to the Tevatron and LHC establishing low-energy supersymmetry as the response to the electroweak breaking puzzle.

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