Entangling two Bose–Einstein condensates in a double cavity system

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Abstract

We propose a scheme to transfer the quantum state of light fields to the collective density excitations of a Bose–Einstein condensate (BEC) in a cavity. This scheme allows us to entangle two BECs in a double cavity setup by transferring the quantum entanglement of two light fields produced from a nondegenerate parametric amplifier to the collective density excitations of the two BECs. An EPR state of the collective density excitations can be created by a judicious choice of the system parameters.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Quantum entanglement has played a pivotal role in quantum information [1]. Consequently, the main aim of modern experimental quantum mechanics is to create a multiparticle entangled state. Quantum entanglement of two or more systems leads to correlations between observables of the systems that cannot be explained on the basis of local realistic theories. Quantum entanglement lies at the heart of the difference between the quantum and the classical world. Due to its vast application, quantum entanglement has been studied in different systems such as opto-mechanical systems [2] and Bose–Einstein condensates (BECs) [3–5]. Entanglement has also been observed experimentally at the NIST group [6], where the entanglement state of four ions was created successfully by using the scheme proposed by Sorensen and Mølmer [7].

In order to build a quantum information network using atomic systems, the quantum state exchange between light and matter is an essential ingredient. One prefers matter over photons as quantum memory elements since experimentally it is difficult to localize and store photons. The stimulated Raman adiabatic passage is considered to be a very convenient technique of storing light and has been used in single atom cavity quantum electrodynamics (QED) for transfer of qubits between atoms and photons and for building quantum logic gates [8]. Mapping quantum states to collective atomic spin systems [9] and quantized vibrational states of trapped atoms [10] by means of stimulated Raman adiabatic passage technology and electromagnetically induced transparency [11] has been discussed earlier.

It is always fascinating to combine two or more branches of physics. Combining the experimental and theoretical tools of cavity QED with those of ultracold gases gives rise to many new possibilities for cavity opto-mechanics [12–14]. When an ensemble of atoms is placed inside a high-finesse optical cavity, the atom–light interaction is enhanced because the atoms collectively couple to the same light mode. The motional degrees of freedom of ultracold atomic gases represent a novel source of long-lived coherence affecting light–atom interaction and other cavity mediated physics. Nonlinear quantum optics arising from the interaction of this long-lived coherent motion of ultracold atoms trapped within a high-finesse Fabry–Perot optical cavity and the cavity mode was reported recently [14]. Strong optical nonlinearities were observed even at low mean photon number of 0.05. This optical nonlinearity arising from the atom–photon coupling also gives rise to bistability in the transmitted probe light through the cavity. Experimental investigations of a combination of ultracold atoms technology and cavity QED have made significant progress over the past few years [15–17]. Theoretically, there have been many interesting works on the correlated atom–field dynamics in a cavity. The strong coupling of the condensed atoms to the cavity mode
changes the resonance frequency of the cavity [18]. The finite cavity response times lead to damping of the coupled atom–field excitations [19]. The driving optical field in the cavity can significantly enhance the localization and the cooling properties of the system [20, 21]. It has been shown theoretically that in an optical cavity the atomic backaction on the optical field introduces the atom–field entanglement which modifies the associated quantum phase transition [22]. The light field and the atoms become strongly entangled if the atoms are in a superfluid state, in which case the photon statistics typically exhibits complicated multimodal structures [23]. A coherent control over the superfluid properties of the BEC can also be achieved with the cavity and optical pump [24]. Recently, a new approach showed that atomic quantum statistics can be mapped on transmission spectra of high-$Q$ cavities, where atoms create a quantum refractive index. This was shown to be useful for studying quantum phase transitions between Mott insulator and superfluid states since various phases show qualitatively distinct optical spectra [25]. Motivated by such interesting developments in the field of cavity electrodynamics of BECs, we propose in this study a new scheme of entangling two BECs in a double cavity setup of cavity electrodynamics of BECs, which modifies the associated quantum phase transition [22].

The atomic backaction on the field is represented by the parameter $U_0 = \frac{\omega_a}{\omega_p}$, which is defined as the optical lattice barrier height per photon. Here, we introduce $V_{cl}(\mathbf{r})$ as the external classical potential. We will always take $U_0 > 0$. Along the $x$-axis, the cavity field forms an optical lattice potential of period $\lambda/2$ and depth ($hU_0|\hat{a}|\hat{a}$) + $V_{cl}$. We now write the Hamiltonian in a second quantized form including the two-body interaction term:

$$H = \int d^3x \Psi^\dagger(\mathbf{r})H_0\Psi(\mathbf{r})$$

$$+ \frac{1}{2} \int d^3x \Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\Psi(\mathbf{r})$$

(3)

where $\Psi(\mathbf{r})$ is the quantum field operator for the condensate atoms. Here, $a_s$ is the usual two-body $s$-wave scattering length. We can now derive the corresponding opto-mechanical-Bose–Hubbard (OMBH) Hamiltonian by writing $\Psi(\mathbf{r}) = \sum_j \hat{b}_j w(\mathbf{r} - \mathbf{r}_j)$, where $w(\mathbf{r} - \mathbf{r}_j)$ is the Wannier function and $\mathbf{r}_j$ is the position of the $j$th atom.
In the above model, the tunnelling terms scale of atomic tunnelling.

time scale of light measurements is much faster than the time by varying the optical lattice depth. Under such circumstances,

\[ c = -j \frac{\hbar a_i \hbar^2}{m} \int d^3 x \int d^3 y \int d^3 z \int d^3 \rho w(\tilde{\rho}) \]

where \[ U = \frac{4 \pi a_i \hbar^2}{m} \int d^3 x \int d^3 y \int d^3 z \int d^3 \rho w(\tilde{\rho}) \]

\[ E_0 = \frac{\hbar}{2} \int d^3 x \int d^3 y \int d^3 z \int d^3 \rho w(\tilde{\rho}) \]

\[ J_0 = \frac{\hbar}{2} \int d^3 x \int d^3 y \int d^3 z \int d^3 \rho w(\tilde{\rho}) \]

In the above model, the tunnelling terms \[ E = \frac{\hbar a_i \hbar^2}{m} \int d^3 x \int d^3 y \int d^3 z \int d^3 \rho w(\tilde{\rho}) \]

\[ \{ -\frac{\hbar^2 V^2}{2m} \} w(\tilde{\rho}) \]

and \[ J = \frac{\hbar a_i \hbar^2}{m} \int d^3 x \int d^3 y \int d^3 z \int d^3 \rho w(\tilde{\rho}) \]

\[ \{ -\frac{\hbar^2 V^2}{2m} \} w(\tilde{\rho}) \]

are neglected as they can be made small in experiments by varying the optical lattice depth. Under such circumstances, the matter-wave dynamics is not important for light scattering. Such a situation can be realized in experiments because the time scale of light measurements is much faster than the time scale of atomic tunnelling.

The dynamics of the system can be described by the following quantum Langevin equations:

\[ \dot{c} = -i\Delta c - iU_0 c J_0 \sum_i b_i \bar{b}_i + \sqrt{2\gamma} c^{in} - \gamma c \]

(6)

\[ \dot{b}_i = -i\frac{E_0 b_i}{\hbar} - i \left[ \frac{V_0}{\hbar} + U_0 c \right] J_0 b_i - \frac{U}{\hbar} b_i \bar{b}_i - \Gamma b_i . \]

(7)

Here, \( \gamma \) is the cavity decay rate and \( \Gamma \) is the condensate decay rate. The noise operators for the input field obey the following correlation functions: \( \langle c^{in}(t) c^{in}(t') \rangle = n_p \delta(t - t') \), \( \langle c^{in}(t) c^{out}(t') \rangle = (n_p + 1) \delta(t - t') \). The backaction heating by the cavity light field fluctuation is the primary source of heating of the atomic sample which leads to loss of condensate atoms. The atom heating rate is then directly related to the spectral density of photon fluctuations in the cavity. To measure the backaction heating, atoms are loaded into the cavity. The cavity is then driven with a detuned probe light. Light transmitted through the cavity is then directed to a single photon counting device [14]. \( \Gamma \) actually also depends on the chemical potential, Rabi frequency and atomic detuning [26]. Recent experiments coupling a BEC to an optical resonator [13] suggest that the Bogoliubov modes interacting significantly with the cavity field are those with momentum \( \pm k_{cav} \) (\( k_{cav} \) is the cavity mode momentum) while the condensate can be considered to be initially at zero temperature. Consequently, we will assume that only one BEC mode with frequency given by the Bogoliubov dispersion relation (the Bogoliubov frequency depends on \( |k_{cav}| \) and the two-body atom–atom interaction \( U \)) interacts with the cavity mode.

We now write each canonical operator of the system as a sum of its steady-state mean value and a small fluctuation with the zero mean value, i.e. \( c \rightarrow c_i c + c_i' \) and \( b_i \rightarrow (\sqrt{N/M} + \bar{b}) \) (this is known as the Bogoliubov approximation in the literature [27]), and linearize equations (6) and (7) to obtain the following Heisenberg–Langevin equations for the fluctuation operators:

\[ \dot{c} = -i\Delta c - iU_0 c J_0 \sqrt{N} c_i (b + \bar{b}) + \sqrt{2\gamma} c^{in} - \gamma c \]

(8)

\[ \dot{b} = -i[\nu + 2U_0 \bar{b}] - iU_0 c b_i - ig_c (c_i c_i' + c_i' c_i) - \Gamma b . \]

(9)

Here, \( U_0 \equiv \frac{U_0}{M}, g_c = U_0 J_0 \sqrt{N} |e_i|, \nu = U_0 J_0 |c_i|^2 + \frac{V_0}{\hbar} + \frac{U_0}{\hbar}, \) and \( \Delta = \Delta_c + U_0 N J_0 \) is the renormalized detuning. Also \( N \) is the total number of condensate atoms in \( M \) sites. The above-mentioned Bogoliubov approximation is made in order to separate the condensate part, which corresponds to the macroscopic occupation of a single quantum state, from the remaining part of the Bose field operator. This split is essentially equivalent to the separation of the zero-momentum mode in the usual textbook discussion of the Bose–Einstein condensation in a homogenous system. Note that the Bogoliubov approximation is valid only if the number of condensate atoms in each lattice site is large such that \( M/N \rightarrow 0 \). The condensate single-particle operators can be thought of as approximately commuting. All operator dependence is contained in the fluctuation operator. The Bogoliubov approximation is a somewhat drastic approximation which has the direct consequence that the physical state described by such a decomposition does not satisfy the same symmetries as the original Hamiltonian. This leads to the non-conservation of the total number of particles. The consequence of violation of particle number conservation is evident, since in this approximation one assumes that the addition or removal of a particle in the condensate does not affect the state of the system for large atom numbers. This approximation is equivalent to the statement that the ensemble average of the Bose field operator is well defined and nonzero and the fluctuations satisfy \( \langle b \rangle = 0 \). We drop the site index \( j \) from the atomic operators since we have ignored tunnelling. We now make the transformations \( c = \tilde{c} e^{i\theta}, c_i = \tilde{c}_i e^{i\theta} \) and \( b = \tilde{b} e^{i\theta} \). Neglecting the fast rotating terms, we obtain the following equations:

\[ \dot{\tilde{c}} = -i\Delta \tilde{c} - ig \tilde{c} \tilde{b} e^{i\theta} + \sqrt{2\gamma} \tilde{c}^{in} - \gamma \tilde{c} \]

(10)

\[ \dot{\tilde{b}} = -i[\nu + 2U_{eff}] \tilde{b} - ig_c \tilde{c} \tilde{b} e^{i\theta} - \Gamma \tilde{b} . \]

(11)

If we assume that the decay rate \( \gamma \) of the cavity field is very large, we can adiabatically eliminate the dynamics of the cavity mode. Consequently, we have the steady-state value of \( \tilde{c} \):

\[ \tilde{c} = \frac{1}{\gamma + i\Delta} (-iU_0 J_0 \sqrt{N} \tilde{c} \tilde{b} e^{-i\theta} + \sqrt{2\gamma} \tilde{c}^{in}) . \]

(12)

On substituting (12) into (11) and assuming \( \theta = -\pi/2 \) for simplicity, we obtain

\[ \dot{\tilde{b}} = -i[\nu + 2U_{eff}] \tilde{b} - \beta \tilde{b} - \sqrt{2\gamma} \tilde{b} e^{i\theta} - \Gamma \tilde{b} . \]

(13)

Here, \( \beta = g_c^2 |c_i|^2 \). If \( \omega_{in} = \nu + 2U_{eff} \gg \Gamma \), the statistics of the input light field can be transferred to the collective density excitations of the BEC.
frequency degenerate but polarization is nondegenerate. The light fields are generated by a NOPA. The output fields are in two independent cavities which are physically separated.

3. Entangling two BECs

Here, we now utilize the results obtained in the previous section to show that how we can transfer the entanglement of a pair of quantum correlated light fields into a pair of BECs in two independent cavities which are physically separated. The scheme is shown in figure 2. The quantum correlated light fields are generated by a NOPA. The output fields are frequency degenerate but polarization is nondegenerate. The coupling between the two intracavity field modes $c_1$ and $c_2$ can be represented by $i\hbar\chi (c_1 c_2 - c_1^\dagger c_2^\dagger)$. Here, $\chi$ is the coupling strength that is proportional to the nonlinear susceptibility of the intracavity medium and the intensity of the coherent pump field. We take $k_c$ as the damping rate of the cavity modes. The equations of motion for the mode operators can be written as

$$c_1 = -k_c c_1 - \chi c_1^\dagger + \sqrt{2k_c} c_1^{\text{in}}, \tag{14}$$

$$c_2 = -k_c c_2 - \chi c_2^\dagger + \sqrt{2k_c} c_2^{\text{in}}, \tag{15}$$

where $c_1^{\text{in}}$ and $c_2^{\text{in}}$ are the vacuum input fields of the two cavity modes of the NOPA. The output fields from the NOPA follows from the boundary conditions (given in the appendix):

$$c_1^{\text{out}} = \sqrt{2k_c} c_1 - c_1^{\text{in}}, \tag{16}$$

$$c_2^{\text{out}} = \sqrt{2k_c} c_2 - c_2^{\text{in}}. \tag{17}$$

The quadrature amplitudes $X_i^{\text{out}} = c_i^{\text{out}} + c_i^{\text{out}\dagger}$ and $Y_i^{\text{out}} = i(c_i^{\text{out}} - c_i^{\text{out}\dagger})$, $i = 1, 2$, in Fourier space are found as

$$X_i^{\text{out}}(\omega) + Y_i^{\text{out}}(\omega) = \frac{k_c - \chi + i\omega}{k_c + \chi - i\omega} \left[ X_i^{\text{in}}(\omega) + X_i^{\text{in}}(\omega) \right] \tag{18}$$

$$Y_i^{\text{out}}(\omega) - Y_i^{\text{out}}(\omega) = \frac{k_c - \chi + i\omega}{k_c + \chi - i\omega} \left[ Y_i^{\text{in}}(\omega) - Y_i^{\text{in}}(\omega) \right]. \tag{19}$$

The highly correlated light fields from the NOPA are incident on the two space-separated cavities with two identical BECs. We can now write the BEC mode operators of the two condensates as

$$\hat{b}_1 = -i(v + 2U_{\text{eff}}) \hat{b}_1 - \frac{\beta}{\gamma + i\Delta} \hat{b}_1 - \frac{\sqrt{2\gamma\beta}}{\gamma + i\Delta} \hat{c}_1^{\text{in}} - \Gamma \hat{b}_1, \tag{20}$$

Here, we assume that the coupling between the NOPA and the cavities is unidirectional. If system parameters are chosen such that $c_1^{\text{out}}$ and $c_2^{\text{out}}$ can be regarded as quantum white noise operators, then the variances of the positions $(\Delta X^2 = \langle \delta^2 (\Delta X_{\text{in}}) \rangle)$ and momenta $(\Delta Y^2 = \langle \delta^2 (\Delta P_{\text{in}}) \rangle)$ of the two BECs are given by

$$\Delta X^2 = \Delta Y^2 = \frac{\beta \gamma}{C} \left( \frac{A^2}{B^2} + 1 \right) \times \left\{ \frac{4(A^2 + B)\chi k_c - (A^2 + B)^2 (k_c + \chi) + (k_c - \chi)^2 (k_c + \chi)}{(A^2 + B)(k_c + \chi)(k_c + \chi)^2 - (A^2 + B)^2} \right\}. \tag{21}$$

Here, $A = v + 2U_{\text{eff}} - \Delta \beta / (\gamma^2 + \Delta^2)$, $B = \Gamma + \beta \gamma / (\gamma^2 + \Delta^2)$ and $C = \gamma^2 + \Delta^2$. The two identical BECs are entangled if $\Delta X^2 + \Delta Y^2 < 1$. For certain set of parameters, the variances $\Delta X^2 = \Delta Y^2 \to 0$, i.e., an EPR state in the positions and momenta of the two BECs. A plot of the EPR variance $\Delta X^2 + \Delta Y^2$ as a function of $\omega_{\text{in}} / \gamma$ for two values of the detuning $\Delta / \gamma = 0.3$ (thin line) and $\Delta / \gamma = 0.6$ (thick line). The other parameters are $\Gamma / \gamma = 0.0001$, $\beta / \gamma = 3$, $k_c / \gamma = 1$ and $\chi / \gamma = 1$. Quantum entanglement is said to exist when $\Delta X^2 + \Delta Y^2 < 1$. The above expressions show that significant entanglement is achieved when the numbers of atoms are in the strong coupling regime since the atom–field coupling is about $g_0 = 2\pi \times 10.9$ MHz [13] ($2\pi \times 14.4$ MHz [14]) and is greater than the decay rate of the intracavity field.
\( \kappa = 2\pi \times 1.3 \text{ MHz} \) [13]. Typically, the atom–pump detuning is \( 2\pi \times 32 \text{ GHz} \). The rate \( \Gamma \) at which atoms are coupled out of the BEC is about \( 2\pi \times 7.5 \times 10^3 \text{ Hz} \) [13]. The kinetic energy and potential energy contribution \( \nu \) is about \( 35 \text{ kHz} \) [13].

The energy of the cavity mode decreases due to the photon loss through the cavity mirrors, which lead to a reduced atom–field coupling and subsequent loss of entanglement. Photon loss can be minimized by using high-\( Q \) cavities. In order to experimentally realize our proposed scheme, the coherent dynamics should dominate over the losses, i.e. the characteristic time scales of coherent dynamics are significantly faster than those associated with losses.

Entanglement dynamics with two thermal atoms in two physically separated optical cavities have been studied earlier [28, 29]. There are couple of distinct advantages of using a BEC instead of thermal atoms. Since the \( N \) two-level atoms are identically coupled to the single mode photon field, this gives the BEC–photon field coupling a collective enhancement of a factor \( \sqrt{N} \) [30, 31]. In fact, such strong atom–photon couplings are very useful in performing quantum information processing before decoherence sets in. The potential applications include long-lived quantum memory [32] and quantum networks for the light–matter interface [33]. In addition, the BEC with reduced Doppler broadening leads to much longer coherence times than those of the thermal atoms [34]. Long coherence times provide a robust way to generate entangled light [35]. These are advantages relative to having a similar scheme with a thermal cloud of atoms where all the atoms do not interact with the light equally and the interaction Hamiltonian can only be an approximation obtained by integrating the light–atom coupling strength along the light propagation. Thus, for the BEC, on one hand, the common coupling of all atoms to the same mode introduces cavity-mediated long-range atom–atom interactions, and on the other hand, atomic backaction on the field introduces the strong atom–field entanglement. A BEC is a macroscopic quantum object where entanglement arises quite naturally due to the two-body interaction. The two-body interaction \( U_{\text{eff}} \) is a new parameter to control the entanglement compared to the case of thermal atoms. Bogoliubov theory of Bose condensation reveals that in the ground state of condensate, two particles with opposite momentum are maximally entangled [36] in momentum variables as in a EPR state. This unique feature makes a BEC a good source of entanglement in motional degrees of freedom. One major limitation is spontaneous emission and backaction heating which destroys the BEC and eventually the entanglement between the two BECs. In order to reduce the spontaneous emission, the atom–cavity detuning should be kept large. The stored EPR state in the proposed scheme would persist for a duration set by the time scale \( 1/\Gamma \) for the BEC decoherence.

### 4. Conclusions

In conclusion, we have proposed a novel scheme to transfer the quantum entanglement between two light fields emerging from a nondegenerate parametric amplifier (NOPA) to the collective density excitations of two Bose–Einstein condensates (BECs) which are physically separated in two independent optical cavities. An EPR state of the collective density excitations can be created by a judicious choice of the parameters. This study could be of use in studying entanglement in macroscopic quantum objects and for high precision metrology.

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### Appendix

We consider a single cavity mode interacting with an external field in a one sided cavity, i.e. a cavity with significant loss through only one mirror. Following [37, 38], the quantum Langevin equation for the cavity mode is

\[
\frac{dc}{dt} = -\frac{i}{\hbar} [c, H_{\text{sys}}] - \gamma c + \sqrt{2\gamma} c^\text{in},
\]

where, \( H_{\text{sys}} \) is the free Hamiltonian for the intracavity field mode, \( \gamma \) is the cavity damping constant, and \( c^\text{in} \) is the vacuum input field. The operators \( c \) and \( c^\text{in} \) satisfy \([a, a^\dagger] = 1\) and \([c^\dagger(t), c^\text{in}(t)] = \delta(t - t')\). Time reversal of equation (23) must be equivalent to a change of sign in the systematic part, and replacement of the incoming field by the outgoing one \( c^\text{out} \), to give

\[
\frac{dc}{d(-t)} = \frac{i}{\hbar} [c, H_{\text{sys}}] - \gamma c + \sqrt{2\gamma} c^\text{out}.
\]

A relation between the external fields and the intracavity field is obtained by subtracting equation (A.2) from equation (A.1):

\[
c^\text{out} + c^\text{in} = \sqrt{2\gamma} c.
\]

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