Urban Traffic Dynamics: A Scale-Free Network Perspective

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This letter proposes a new model for characterizing traffic dynamics in scale-free networks. With a replotted road map of cities with roads mapped to vertices and intersections to edges, and introducing the road capacity $L$ and its handling ability at intersections $C$, the model can be applied to urban traffic systems. Simulations give the overall capacity of the traffic system which is quantified by a phase transition from free flow to congestion. Moreover, we report the fundamental diagram of flow against density, in which hysteresis is found, indicating that the system is bistable in a certain range of vehicle density. In addition, the fundamental diagram is significantly different from single-lane traffic model and 2-D BML model with four states: free flow, saturated flow, bistable, and jammed.

Traffic and transportation is nowadays one of the most important ingredients of modern society. We rely greatly on networks such as communication, transportation, and power systems. Ensuring free traffic flow on these networks is therefore of great research interest [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Recently, more and more empirical evidence indicates that these networked systems are of small-world and scale-free structural features [12, 13, 14, 15]. And so the traffic flow on scale-free networks is being widely investigated [16, 17, 18, 19, 20]. In the present work, we propose a new model for the traffic dynamics of such networks. The potential application of this research will be urban road traffic networks. Previously work on urban traffic normally maps roads to edges and intersections to vertices. In 1992, Biham, Middleton and Levine (BML) [8] proposed an ideal 2-dimensional model for studying urban traffic. They used a $N \times N$ grid to represent the urban transportation network. Initially, cars are placed at the sites. Each car is either East-facing or North-facing. At odd time steps, each North-facing car moves one unit North if there is a vacant site before it. At even time steps, East-facing cars move East in the same way. The rules of BML model can be considered as controlling urban traffic by traffic lights. The BML model reproduced the transition from free-flow phase to jammed phase. Since then, many studies have been carried out on the base of the BML model. For example, Nagatani [9] investigated the effect of car accident induced jam on the phase transition of urban traffic; Chung et al. [10] investigated the effect of improperly placed traffic lights; and recently, Angel et al. [11] discussed the jammed phase of the BML model.

The models mentioned above always adopt a $N \times N$ grid (or close to that) to represent the urban traffic system. However, real urban traffic system is obviously much more complicated. Perhaps, the most natural way is to map each intersection to a vertex and each segment of road to an edge between vertices. However, obviously, this kind of simulation is computation-consuming and the interaction of any two neighboring segments of a straight road is in most cases wiped off.

The unique feature of our model is that we look at the urban traffic networks in a different point of view. We will create traffic flow networks with roads mapped to vertices and intersections to edges between vertices, as was inspired by the information networks suggested in [12] for an information view of a city. In this way (See FIG. 1), the degree $k$ of vertex is the number of intersections along the street and a major road with many minor roads connected to it can be seen as a major vertex. Empirical observations demonstrate that the remapped urban networks exhibit scale free structural properties.

With this new paradigm, we can look at urban traffic networks from a novel perspective. In network language, a trajectory of a vehicle can then be mapped in an urban traffic network map from a road (vertex) to another road (vertex) through a one directional channel (edge) of an intersection. In this work we will take this perspective to the extreme, and assume that the travel time/cost of just driving along a given road can be zero [12]. Our

FIG. 1: Illustration of the road network mapping. Each straight road in (a) is mapped to a vertex in (b). And intersections are mapped to edges between the vertices.

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The model is partially inspired by the work of information packet flow on the Internet [16,17,18,19,20]. The phase transition from free flow to jammed state, and the plot of flow against density that all have been empirically observed can be well reproduced by this model.

Though the previous work found that the replotted road networks of cities have scale-free characteristics [12], there is no well-accepted model for these networks up to now. Without losing generality, our simulation starts from generating the urban transportation network according to the most general Barabási-Albert scale-free network model [14]. In this model, starting from $m_0$ fully connected vertices, one vertex with $m$ edges is attached at each time step in such a way that the probability $\Pi_i$ of being connected to the existing vertex $i$ is proportional to the degree $k_i$ of the vertex, i.e. $\Pi_i = \frac{k_i}{\sum_j k_j}$, where $j$ runs over all existing vertices. The capacity of each vertex (road) is controlled by two parameters: (1) its maximum cars number $L$, which is proportional to its degree $k$ (a long road ordinarily has more intersections and can hold more cars): $L = \alpha \times k$; (2) the maximum number of cars handled per time step, which reflects the capability of intersections: $C = \beta \times L$. Motivated by the Internet information flow models [16,17,18,19,20], the system evolves in parallel according to the following rules:

1. Add Cars - Cars are added with a given rate $R$ (cars per time step) at randomly selected vertices and each car is given a random destination.

2. Navigate Cars - If a car’s destination is found in its next-nearest neighborhood, its direction will be set to the destination vertex. Otherwise, its direction will be set to a neighboring vertex $h$ with probability: $P_h = \frac{k_h}{\sum_j k_j}$. Here the sum runs over the neighboring vertices, and $\phi$ is an adjustable parameter. It is assumed that the cars are unaware of the entire network topology and only know the neighboring vertices’ degree $k_h$.

3. Cars Motion - At each step, only at most $C$ cars can leave a vertex (road) for other vertices and FIFO (first-in-first-out) queuing discipline is applied at each vertex. When the queue at a selected vertex is full, the vertex won’t accept any more vehicles and the vehicle will wait for the next opportunity. Once a car arrives at its destination, it will be removed from the system.

We first simulate the traffic on a network of $N = 100$ vertices (roads) with $m_0 = m = 2$, $\alpha = 5$ and $\beta = 0.2$. This relatively small system can be seen as simulating the backbone of a city’s urban traffic network. The selection of $\beta$ is based on the single-road traffic flow theory which shows that the maximum flux on a highway is about 20% of its maximum capacity [14,15,16,17]. For simplicity, we do not consider the phenomenon that the flux decreases when the density is above 20%. The combination of $\alpha$ and $\beta$ can be also interpreted as: each intersection can handle one car turning for one road at each step.

To characterize the system’s overall capacity, we first investigate the car number $N_c$ increment rate $\omega$ in the system: $\omega(R) = \lim_{t \to \infty} \frac{\langle N_c(t+\Delta t) - N_c(t) \rangle}{\Delta t}$. Here $\langle N_c(t + \Delta t) - N_c(t) \rangle$ takes average over time windows of width $\Delta t$. Fig.2(a) shows the variation of $N_c$ with different $R$ for $\phi = 0.1$. One can see that there is a critical $R_c = 13$ at which $N_c$ runs quickly towards the system’s maximum car number and $\omega(R)$ increases suddenly from zero. $\omega(R) = 0$ corresponds to the cases of free flow state, which is attributed to the balance between the number of added and removed cars at the same time. However, if $R$ exceeds the critical value $R_c$, cars will in succession accumulate in the system and then congestion emerges and diffuses to everywhere. Ultimately almost no cars can arrive at their destinations.

Evidently, $R_c$ is the onset of phase transition from free flow to jammed state. Hence, the system’s overall capacity can be measured by the critical value of $R_c$ under which the system can maintain its normal and efficient functioning. Fig.2(b) depicts the variation of $R_c$ versus $\phi$. The maximum overall capacity occurs at $\phi = 0.1$ (slightly greater than 0.0) with $R^{\text{max}}_c = 13$.

Here we give a heuristic analysis for determining the optimal value of $\phi$ with the maximum capacity. If we neglect the queue length $L$ of each vertex, for $\phi = 0$, cars will move in the system nearly like random walk. There is a well-known result from graph theory that if a particle performs a random walk, in the limit of long times, the time the particle spends at a given vertex is proportional to the degree $k$ of the vertex [21]. One can easily find out that, the number of cars observed averagely at a vertex is proportional to its degree. Thus in the case of $\phi = 0$, this rule produces an average effect that no congestion occurs earlier on some vertices with particular degree than on others. Accordingly, $\phi = 0$ results in the maximum system capacity. However, in our model, each vertex has a limited queue length $L = \alpha \times k$ and $R$ cars are generated randomly among all vertices at each time step, so small degree vertices are slightly more easily congested.
Therefore, for our traffic model, a $\phi$ slightly larger than zero can enhance the system’s capacity maximally.

Then we simulate the cars’ travel time spent in the urban transportation system. It is also an important factor for measuring the system’s efficiency. In Fig.3(a), we show the average travel time $\langle T \rangle$ versus $\phi$ under traffic load $R = 1$ and 2. The data are truncated because the system jams when $\phi$ is either too large or too small. The right panel shows the variation of $\langle T \rangle$ versus $R$ when $\phi$ is fixed. The data are also truncated when the system jams.

FIG. 3: (color online). Average travel time $\langle T \rangle$ versus $\phi$ for $R = 1$ and 2. The data are truncated because the system jams when $\phi$ is either too large or too small. The right panel shows the variation of $\langle T \rangle$ versus $R$ when $\phi$ is fixed. The data are also truncated when the system jams.

Finally, we try to reproduce the fundamental diagram (flux-density relation) of urban traffic system. It is one of the most important criteria that evaluates the transit capacity for a traffic system. Our model reproduces the phase transition and hysteresis in fundamental diagram.

To simulate a conservative system (constant density), we count the number of arriving cars at each time step and add the same number of cars to randomly selected vertices of the system at the beginning of next step. The flux is calculated as the number of successful car turnings from vertex to vertex through edges per step, as is similar to the Internet information flow. Here we ignore the movement of cars on a given road. In fact, the flux of car turnings at intersections can, to some extent, reflect the flux on roads. In Fig.4 the fundamental diagrams for $\phi = 0.1, 0.0, 1.0, -0.5$ are shown. The curves of each diagram show four flow states: free flow, saturate flow, bistable and jammed. For simplicity, we focus on the $\phi = 0.1$ chart in the following description. As we can see, when the density is low (less than $\approx 0.1$), all cars move freely and the flux increases linearly with car density. It is the free-flow state that all vertices (roads) are operated below its maximum handling ability $C$. Then the flux’s increment slows down and the flux gradually comes to saturation ($0.10 \sim 0.45$). In this region, the flux is restricted mainly by handling ability $C$ of vertices. One can see that when $\phi$ is close to zero, the saturated flux ($\approx 360$) is much higher than other values.

At higher density, the model reproduces an important character of traffic flow - “hysteresis”. It can be seen that two branches of the fundamental diagram coexist between 0.45 and 0.76. The upper branch is calculated by adding cars to the system, while the lower branch is calculated by removing cars from a jammed state and allowing the system to relax after the intervention. In this way a hysteresis loop can be traced (arrows in Fig.4). The hysteresis loop indicates that the system is bistable in a certain range of vehicle density. And as we know so far, it is the first time that our model reproduces the hysteresis phenomenon in scale-free network traffic and in urban network traffic.

To test the finite-size effect of our model, we simulate some bigger systems with much more vertices/roads. The simulation shows similar phase transition and hys-
Moreover, an important conclusion can be drawn by comparing the \( \phi = 0.1 \) chart with the \( \phi = 0.0 \) chart in Fig. 4 that the \( \phi = 0.1 \) chart has a much broader bistable region than the \( \phi = 0.0 \) one. This means, when the system retreats from a heavy load jammed state, it is more difficult to reach a high efficiency state if \( \phi \) is greater than zero that cars are more likely to move to main roads. In other words, though it is wise to take full advantage of the main roads when the entire traffic is light, it won’t be so happy to do so at rush hours.

In conclusion, a new traffic model for scale-free networks is proposed. In the new perspective of mapping roads to vertices and intersections to edges, and incorporating road/intersection capability limits, the model can be applied to urban traffic system. In a systemic view of overall efficiency, the model reproduces several significant characteristics of network traffic, such as phase transition, travel time, and fundamental diagram. A special phenomenon - the “hysteresis” - can also be reproduced. Although the microscopic dynamics of urban traffic are not well captured, such as the movement of cars along streets, the interaction with traffic lights, and the differences between long and short streets, our model is still a simple and good representation for urban traffic, since much empirical evidence is well reproduced by the model. Further effort is deserved to consider more detailed elements for better mimicking real traffic systems. Moreover, by choosing other values of the parameters, the model may be applied to other networked systems, such as communication and power systems.

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[1] K. Nagel, M. Schreckenberg, J. Phys. I France 2, 2221 (1992).
[2] D. Helbing, B.A. Huberman, Nature (London) 396, 738(1998).
[3] D. Helbing, Rev. Mod. Phys. 73, 1067(2001).
[4] B.S. Kerner and H. Rehborn, Phys. Rev. Lett. 79, 4030(1997).
[5] B.S. Kerner, S.L. Klenov, J. Phys. A 35, L31(2002).
[6] B.S. Kerner, The Physics of Traffic. (Springer, Berlin, New York, 2004).
[7] X. Li, Q. Wu, R. Jiang, Phys. Rev. E 64, 066128(2001).
[8] O. Biham, A.A. Middleton, D. Levine, Phys. Rev. A 46(10), R6124 (1992).
[9] T. Nagatani, J. Phys. A 26, 781(1993).
[10] K.H. Chung, P.M. Hui, G.Q. Gu, Phys. Rev. E 51(1), 772(1995).
[11] O. Angel, A.E. Holroyd, J.B. Martin, Elec. Comm. In Prob. 10, 167(2005).
[12] M. Rosvall, A. Trusina, P. Minnhagen and K. Sneppen, Phys. Rev. Lett. 94, 028701(2005).
[13] R. Albert, H. Jeong, and A.-L. Barabási, Nature (London) 401, 130(1999).
[14] R. Albert, A.-L. Barabási, Rev. Mod. Phys. 74, 47(2002).
[15] M. E. J. Newman, Phys. Rev. E 64, 016132 (2001).
[16] R.V. Sole, S. Valverde, Physica A 289, 595(2001).
[17] A. Arenas, A. Díaz-Guilera, and R. Guimerá, Phys. Rev. Lett. 86, 3196(2001).
[18] B. Tadić, S. Thurner, G.J. Rodgers, Phys. Rev. E 69, 036102(2004).
[19] L. Zhao, Y.C. Lai, K. Park, N. Ye, Phys. Rev. E 71, 026125(2005).
[20] W.X. Wang, B.H. Wang, C.Y. Yin, Y.B. Xie, T. Zhou, Phys. Rev. E 73, 026111(2006).
[21] B. Bollobás, Modern Graph Theory (Springer-Verlag, New York, 1998).