Inert Dark Matter and Strong Electroweak Phase Transition

Grzegorz Gil, Piotr Chankowski and Maria Krawczyk

Faculty of Physics, University of Warsaw, Hoża 69, 00-681, Warsaw, Poland

Abstract

The main virtue of the Inert Doublet Model (IDM) is that one of its spinless neutral bosons can play the role of Dark Matter. Assuming that the additional sources of CP violation are present in the form of higher dimensional operator(s) we reexamine the possibility that the model parameters for which the right number density of relic particles is predicted are compatible with the first order phase transition that could lead to electroweak baryogenesis.

We find, taking into account recent indications from the LHC and the constraints from the electroweak precision data, that for a light DM (40-60 GeV) particle $H^0$ and heavy, almost degenerate additional scalars $H^\pm$ and $A^0$ this is indeed possible but the two parameters most important for the strength of the phase transition: the common mass of $H^\pm$ and $A^0$ and the trilinear coupling of the Higgs-like particle $h^0$ to DM are strongly constrained. $H^\pm$ and $A^0$ must weight less than $\sim 440$ GeV if the inert minimum is to be the lowest one and the value of the $h^0 H^0 H^0$ coupling is limited by the XENON 100 data. We stress the important role of the zero temperature part of the potential for the strength of the phase transition.

PACS numbers:12.60.Fr, 12.15.Ji, 98.80.Cq, 95.35.+d
1 Introduction

If the scalar sector of the theory of electroweak interactions consists of only one $SU(2)$ doublet, the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry breaking down to $U(1)_{\text{EM}}$ is the only possible pattern. The first-order nature of the electroweak phase transition that had to occur during the evolution of the Universe, as it cooled down below some critical temperature $T_{\text{EW}}$, becomes less and less pronounced with the increase of the Higgs particle mass $M_h$ and turns into a continuous (second order) transition above the mass $M_h \approx 80$ GeV [1]. With the lower bound set on the mass of this particle by searches at LEP and LHC [2,3] the possibility that the baryon asymmetry of the Universe was created during the electroweak phase transition is definitely excluded within this scenario, irrespectively of the question of amount of CP violation predicted by the Standard Model to which in principle could contribute also higher order nonrenormalizable operators [4].

The possibility of electroweak baryogenesis is still open in various multiscalar extensions of the Standard Model which can exhibit more complicated symmetry breaking patterns. In such theories spontaneous breaking of CP is also possible in the scalar sector which can thus contribute significantly to the CP violation necessary in cosmology. An example worked out in detail is the electroweak baryogenesis in the Minimal Supersymmetric Standard Model (MSSM) whose Higgs sector consists of two scalar doublets. It has been shown [5] that in this model the electroweak phase transition can remain sufficiently first order even for the Higgs boson mass $\sim 100$ GeV provided the right-chiral top squark is light enough. Tight conditions under which the phase transition predicted by the MSSM is sufficiently first order are due to the very constrained form of the supersymmetric potential of the Higgs doublets and can be relaxed in the nonsupersymmetric multiscalar models whose potentials depend on many a priori free parameters.

Studies of the nature of the phase transition in the nonsupersymmetric two Higgs doublet (2HDM) extension of the Standard Model were initiated in the papers by Bochkaryev, Kuzmin and Shaposhnikov [6] and of Turok and Zadrozny [7]. Their authors, using the high temperature approximation, have found that in this class of models the rough criterion for the first-order phase transition $v(T_{\text{EW}})/T_{\text{EW}} \gtrsim 1$ can be satisfied for the mass of the scalar playing the role of the physical Higgs particle of order 100 GeV [8]. The strength of the electroweak phase transition predicted by the 2HDM was subsequently analyzed in several papers [9–13] which using the one-loop temperature dependent effective potential (with or without resummations of the so-called ring diagrams) have confirmed the original observations. Recently, the interest in the 2HDM [14–16], and in the phase transition predicted by this model in particular [17–22], has again been revived.

There are many versions of the two scalar doublet extensions of the Standard Model. A distinguished one which has recently attracted much attention is the so-called Inert Doublet Model (IDM) possessing an additional unbroken discrete $Z_2$ symmetry [14,15]. The main virtue of the IDM is that one of spin zero particles originating from its scalar sector is stable and can play the role of the Dark Matter
Phase transitions in this model were studied in [18–20, 25, 26]. Of course, the unbroken $\mathbb{Z}_2$ symmetry of the IDM (crucial for the stability of the dark matter particle) precludes any possibility of additional CP violation originating from the scalar sector. Hence, in its most attractive version, with a stable dark matter candidate, the IDM predicts no more CP violation than does the Standard Model. This means that the electroweak phase transition, even if sufficiently first order, could not yield the required baryon number asymmetry of the Universe. However if the IDM is viewed only as an effective low energy theory (effectively parametrizing the electroweak symmetry breaking) one can easily imagine that its full $\mathbb{Z}_2$-symmetric Lagrangian involves also CP violating nonrenormalizable terms [27] like

$$\Delta \mathcal{L} = \frac{1}{\Lambda^2} (c_1 \Phi_1^\dagger \Phi_1 + c_2 \Phi_2^\dagger \Phi_2) \text{tr}(W_{\mu\nu} \tilde{W}^{\mu\nu}),$$

where $\Phi_i$ ($i = 1, 2$) are the two scalar doublets, $W_{\mu\nu}$ and $\tilde{W}^{\mu\nu}$ the $SU(2)_L$ field strength and its dual, $\Lambda$ the UV cutoff and $c_i$ some coefficients. Hence, the question of amount of CP violation in the (renormalizable part of the) IDM might not be vital and the question of electroweak baryogenesis hinges essentially only on the nature of the electroweak phase transition.

The question whether the IDM parameters for which the right amount of dark matter could be generated can also be compatible with the electroweak phase transition of sufficiently first order to allow for electroweak baryogenesis has been answered in the affirmative first in [25] and quite recently in [26]. In this letter we add a few new points to these analyses. Firstly, using the full one-loop effective potential (with imposed physical renormalization conditions, different than the ones used in [26]) we show that the interesting configuration of the model parameters, with the DM particle mass $\sim 40 \div 80$ GeV and the mass $M_{h^0}$ of the physical SM-like Higgs boson in the range $120 \div 130$ GeV favoured by the LHC data [3], is severely constrained by the requirement that the IDM minimum is the lowest minimum. This puts upper limit $\sim 440$ GeV on the masses of the additional heavy scalars forcing them to be within the LHC reach. Secondly, by using the full one-loop temperature dependent effective potential (in [25] only its high temperature expansion, without the zero-temperature part was used) supplemented with the resummation of the so-called ring diagrams we investigate the strength of the electroweak phase transition. We find that taking into account the zero-temperature part of the effective potential has important effect on the phase transition making it significantly stronger. However, similarly as the authors of [26], we find that the parameter space is rather limited, especially if one includes the Xenon100 data [28].

## 2 Parameters of the IDM

We consider a special case of the type I 2HDM in which only one scalar doublet ($\Phi_1$) has Yukawa couplings to fermions. Owing to this and to the following form of
its Higgs potential

\[
V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 \\
+ \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2]
\]  

(2)

the complete Lagrangian of the model possesses, in addition to the gauge SU(2)_L \times U(1)_Y symmetry, also the discrete Z_2 \times Z'_2 symmetry:

\[
Z_2 : \Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2, \\
Z'_2 : \Phi_1 \rightarrow -\Phi_1, \quad \Phi_2 \rightarrow \Phi_2, \quad f_R \rightarrow -f_R,
\]

(3)

(f_R denotes all right-chiral fermions; other SM fields are Z_2 \times Z'_2-invariant).

All parameters of V(\Phi_1, \Phi_2) are real. Depending on their values different patterns of the SU(2)_L \times U(1)_Y \times Z_2 \times Z'_2 symmetry breaking are possible - in some of them even the electromagnetic U(1)_EM symmetry can be broken [17,18,29]. The tree-level potential for the doublets VEVs v_1 and v_2 defined by

\[
\begin{align*}
\Phi_1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} G^+ \\ v_1 + h^0 + iG^0 \end{pmatrix}, \\
\Phi_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ v_2 + H^0 + iA^0 \end{pmatrix}
\end{align*}
\]

(4)

reads

\[
V_{\text{tree}} = \frac{1}{2} m_{11}^2 v_1^2 + \frac{1}{2} m_{22}^2 v_2^2 + \frac{\lambda_1}{8} v_1^4 + \frac{\lambda_2}{8} v_2^4 + \frac{\lambda_{345}}{4} v_1^2 v_2^2.
\]

(5)

In this letter we are interested in the IDM phase in which the electroweak symmetry is broken by the (real) nonzero vacuum expectation value (VEV) v_1 = v while the second doublet \Phi_2 does not develop any VEV (v_2 = 0). This requires \lambda_1 > 0 and \(m_{11}^2 < 0\); the role of the Higgs particle is then played by \(h^0\). This vacuum preserves the Z_2 symmetry and the lightest of the spinless particles \(H^0, A^0\) and \(H^\pm\), being odd under unbroken Z_2, is stable. If electrically neutral, it can therefore constitute the Dark Matter. In the IDM vacuum the (tree-level) masses of the physical spinless particles which we choose as the independent variables are given by

\[
\begin{align*}
M_{h^0}^2 &= m_{11}^2 + \frac{3}{2} \lambda_1 v_1^2 \text{tree} = -2m_{11}^2 = \lambda_1 v_1^2 \text{tree}, \\
M_{H^0}^2 &= m_{22}^2 + \frac{1}{2} \lambda_{345} v_2^2 \text{tree}, \\
M_{H^+}^2 &= m_{22}^2 + \frac{1}{2} \lambda_3 v_2^2 \text{tree}, \\
M_{A^0}^2 &= m_{22}^2 + \frac{1}{2} (\lambda_{345} - 2\lambda_5) v_2^2 \text{tree}.
\end{align*}
\]

(6)

The value of \(v_\text{tree} = 246 \text{ GeV}\) is fixed by the Fermi constant \(G_F\). For the remaining six parameters we choose the four physical masses \([6]\), and parameters \(\lambda_2\) and \(\lambda_{345} \equiv

\footnote{In this paper we ignore possible existence of U(1)_{EM} breaking vacua.}
\[\lambda_3 + \lambda_4 + \lambda_5.\] We also take \(\lambda_5\) real and negative\(^2\) so that \(H^0\) is lighter than \(A^0\). With \(\lambda_1 > 0, m^2_{11} < 0\) and positive masses squared of all spinless particles the IDM minimum \(v_1^2 = v_{\text{tree}}^2 = -2m^2_{11}/\lambda_1 \neq 0, v_2 = 0\) is at least a local minimum\(^3\) of the potential (5). The absolute stability (boundedness of the potential from below) along the directions preserving the \(U(1)_{\text{EM}}\) symmetry requires in addition \(\lambda_2 > 0\) and \(|\lambda_{345}| < \sqrt{\lambda_1 \lambda_2}\) while boundedness of the potential along the electromagnetic symmetry breaking directions requires also that \(|\lambda_3| < \sqrt{\lambda_1 \lambda_2}\). However if the IDM is treated as an effective theory valid only up to some high cut-off scale \(\Lambda\), boundedness of the potential from below for arbitrarily large field values is not a physical requirement: it is sufficient to require that the electroweak symmetry breaking vacuum be the deepest minimum\(^4\) in the domain in which \(|v_{1,2}| < \Lambda\).

In the following we fix the SM-like Higgs boson \(h^0\) mass \(M_{h^0}\) to 125 GeV, consistently with the recent indications from the LHC\(^\text{[3]}\). There are then three possible ranges of the \(H^0\) mass \(M_{H^0}\) for which the right relic density of these particles can be generated during the evolution of the Universe\(^\text{[19,23,24]}\): i) \(M_{H^0} \gtrsim 40 \div 80\) GeV, and iii) \(M_{H^0} \lesssim 8\) GeV. We will not consider the first possibility: with very heavy \(A^0\) and \(H^\pm\) \((M_A \sim M_{H^\pm} \gtrsim M_{H^0} \gtrsim 1\) TeV) the temperature properties of the potential should be identical to the ones of the Standard Model potential (decoupling!), that is the electroweak phase transition would be of second order. With the DM particle mass \(M_{H^0}\) smaller than \(M_W\) (scenarios ii) and iii)), too light \(A^0\) and \(H^\pm\) would not produce in the high temperature expansion (13) a cubic term of a magnitude necessary for a sufficiently strong electroweak phase transition. As in\(^\text{[25]}\) we concentrate therefore on heavy \(A^0\) and \(H^\pm\). The electroweak precision data require then the masses of these particles to be degenerate - otherwise the contribution of the extended scalar sector to the Peskin-Takeuchi \(T\) and \(S\) parameters would be too large. We thus take \(M_{H^0} \ll v_{\text{tree}}\) and \(M_{A^0} \approx M_{H^\pm} > M_{h^0}\)\(^\text{[19,24]}\). This mass configuration allow also to satisfy the existing collider limits\(^\text{[31,32]}\).

Because

\[
\lambda_5 = (M^2_{H^0} - M^2_{A^0})/v^2_{\text{tree}}, \quad \lambda_3 = 2(M^2_{H^\pm} - M^2_{H^0})/v^2_{\text{tree}} + \lambda_{345}, \tag{7}
\]

\(A^0\) and \(H^\pm\) significantly heavier than \(H^0\) imply large values of the couplings \(\lambda_3\) and \(\lambda_5\) so that unitarity of the tree-level scattering amplitudes in the scalar sector imposes an upper bound of \(\sim 700\) GeV on the \(A^0\) and \(H^\pm\) masses\(^\text{[33]}\). Alternatively, as done in\(^\text{[25]}\), the coupling \(\lambda_3\) (and consequently also \(\lambda_5\) if \(A^0\) and \(H^\pm\) are to be degenerate) can be constrained by imposing a (to a large extent arbitrary) bound on the growth of \(\lambda_1\) with the renormalization scale. However, as we show in the next section, the requirement that the minimum \(v_1 = v_{\text{tree}}, v_2 = 0\) be the absolute

\(^2\)Any phase factor of \(\lambda_5\) can be removed by a suitable redefinition of the fields. For real \(\lambda_5\) the \(H^0\) particle corresponds to the real part of the lower component of \(\Phi_2\); the redefinition needed to make \(\lambda_5 < 0\) out of \(\lambda_5 > 0\) amounts to multiplying \(\Phi_2\) by \(i\), that is, to interchanging \(H^0\) and \(A^0\)\(^\text{[18]}\).

\(^3\)Owing to the \(Z_2\) symmetry the derivative with respect to \(v_2\) automatically vanishes at this point.

\(^4\)Barring the possible metastability of the present phase of the Universe\(^\text{[30]}\).
minimum (in the domain $|v_{1,2}| \lesssim \Lambda$) leads to the bound $M_{A0}, M_{H^\pm} \lesssim 440$ GeV. This result, obtained after taking into account quantum corrections to the zero-temperature effective potential, weakly depend on the values of $\lambda_2$ and $\lambda_{345}$.

## 3 One-loop effective potential at T=0

The one-loop effective potential $V_{\text{eff}}(v_1, v_2)$ is given in the Landau gauge by the standard formula

$$V_{\text{eff}}^{(1L)} = V_{\text{tree}} + \frac{1}{64\pi^2} \sum_{\text{fields}} C_s \left\{ \mathcal{M}^4_s \left( \ln \frac{\mathcal{M}_s^2}{4\pi\mu^2} - \frac{3}{2} + \frac{2}{d-2} - \gamma_E \right) \right\} + \text{CT}, \quad (8)$$

where $\mathcal{M}^2_s(v_1, v_2)$ are field dependent masses squared (eigenvalues of the appropriate matrices), $C_s = (-1)^{2s}(2s+1)g_s$ accounts for the number of states and $V_{\text{tree}}$ is given by (5). The sum over fields does not include ghost but does include the would-be Goldstone bosons. We specify the counterterms $\text{CT}$ by imposing the following renormalization conditions.

Firstly we require that the first derivative of $V_{\text{eff}}$ with respect to $v_1$ vanishes at $v_1 = v_{1,\text{tree}} = -2m^2_{11}/\lambda_1$. This is equivalent to fixing the Lagrangian counterterm linear in the $h^0$ field

$$\delta L_{\text{lin}} = -[\delta m^2_{11} + m^2_{11}\delta Z_1 + \frac{1}{2}(\delta \lambda_1 + 2\delta Z_1)(v_{1,\text{tree}})^2] v_{1,\text{tree}} h^0$$

$$= -[\delta m^2_{11} - m^2_{11}\delta Z_1 - \frac{m^2_{11}}{\lambda_1}(2 v_{1,\text{tree}})^2] v_{1,\text{tree}} h^0,$$ \quad (9)

($\delta Z_i, i = 1, 2$ are the renormalization constants of the two doublets $\Phi_i$) so that it cancels the one-loop 1PI tadpole $-i\mathcal{T}_{h}^0$ of the field $h^0$. This automatically ensures that the would-be Goldstone boson propagators have, in the Landau gauge, poles at $p^2 = 0$. Next we require that the $h^0$ field propagator has the pole for the tree-level mass-squared with the residue equal $i$. Together these conditions determine the combinations ($\Sigma_h(p^2)$ is the $h^0$ field self-energy):

$$\delta m^2_{11} + m^2_{11}\delta Z_1 = -\frac{1}{2} \left( \frac{3}{v_{1,\text{tree}}^2} - \Sigma_h(M^2_h) + M^2_h\Sigma'_h(M^2_h) \right),$$

$$\delta \lambda_1 + 2\lambda_1\delta Z_1 = -\frac{\lambda_1}{2m^2_{11}} \left( \frac{3}{v_{1,\text{tree}}^2} - \Sigma_h(M^2_h) + M^2_h\Sigma'_h(M^2_h) \right), \quad (10)$$

which renormalize the divergent parts of $V_{\text{eff}}^{(1L)}$ proportional respectively to $v^2_1$ and $v^4_1$. As long as the effective potential is probed only along the $v_1$ direction these two combinations are all what is needed; in particular, switching off the $\mathbb{Z}_2$-odd fields one recovers the effective potential of the SM with all its well known features (including the Linde-Weinberg [35] lower bound on the $h^0$ Higgs boson mass).

\footnote{Since the formula (8) is obtained in the dimensional reduction rather than in the dimension regularization, the tadpole $-i\mathcal{T}_{h}^0$ and the self-energies $\Sigma_h$ and $\Sigma'_h$ must also be computed using the dimensional reduction.}
Figure 1: Behaviour of the zero temperature effective potential for $\lambda_{345} = 0.2$, $\lambda_2 = 0.2$, $M_{h^0} = 125$, $M_{A^0} = 65$ and $M_{H^\pm} = M_{A^0} = 300$ GeV (solid line), 400 GeV (long-dashed) 450 GeV (short-dashed) and 500 GeV (dotted).

For the other counterterms needed to renormalize the effective potential there does not seem to exist equally obvious physical conditions. Any choice of these counterterms corresponds to some particular definition of the renormalized couplings $\lambda_2$ and $\lambda_{345}$. The parameter $\lambda_{345}$ determines the coupling of the SM-like Higgs particle to the DM $H^0$ particles and the remaining counterterms could in principle be chosen so that $\lambda_{345}$ is directly related to the physical $h^0 \rightarrow H^0 H^0$ decay amplitude. On the other hand $\lambda_2$ cannot be directly measured in the foreseeable future so its precise definition at the loop-level is not important. Here for simplicity we choose to subtract the divergences of $V_{\text{eff}}^{(1L)}$ proportional to $v^2_1$ and $v^2_2 v^2_1$ using the $\overline{MS}$ scheme. This fixes the combinations $\delta \lambda_2 + 2 \lambda_2 \delta Z_2$ and $\delta \lambda_{345} + \lambda_{345} (\delta Z_1 + \delta Z_2)$. Once the latter counterterm is fixed the last necessary combination $\delta m^2_{22} + m^2_{22} \delta Z_2$ is determined by renormalizing the $H^0$ propagator on-shell. The counterterms $\delta \lambda_3$ and $\delta \lambda_5$ can be then used to enforce that the tree-level masses $M_{A^0}$ and $M_{H^\pm}$ remain unchanged by one-loop corrections (they do not need to be determined explicitly).

The typical behaviour of the zero-temperature effective potential $V_{\text{eff}}^{(1L)}$ along the direction $v_2 = 0$ for different values of the heavy $\mathbb{Z}_2$-odd particles masses is shown in fig. 1. It is clear that for too heavy $H^\pm$ and $A^0$ the electroweak symmetry breaking minimum of $V_{\text{eff}}^{(1L)}$ becomes metastable because while still remaining a local minimum, it becomes higher than the minimum at $v_1 = 0$. With two doublets it is however also possible that the full one-loop $T = 0$ potential develops other deeper minima and this is indeed what happens: as the masses of the $H^\pm$ and $A^0$ particles grow (remaining almost degenerate) for fixed value of $\lambda_{345}$ a new minimum of $V_{\text{eff}}^{(1L)}$ appears along the direction $v_2 \neq 0$, $v_1 = 0$ and, above some critical value

\footnote{Some constraints on $\lambda_2$ follow from the DM relic density [19].}
of $M_{A^0} \approx M_{H^\pm}$, it becomes deeper than the minimum at $v_1 = v_{\text{tree}}$, $v_2 = 0$ whose existence - at least as a local one - is enforced by our renormalization condition. As illustrated in fig. 2 in the plane $(\lambda_{345}, M_{A^0})$ one can distinguish three domains: in the first one - denoted “Inert” - the minimum $v_1 = v_{\text{tree}}$, $v_2 = 0$ is the global minimum and the only other (local) minimum can be the one at $v_1 = v_2 = 0$ (cf. the long-dashed line in fig. 1). In the domain denoted “Inert+inert-like” the minimum at $v_1 = 0$, $v_2 \neq 0$ exists but the one at $v_1 = v_{\text{tree}}$, $v_2 = 0$ is still the deepest minimum. Finally, in the domain denoted “Inert-like” the minimum at $v_1 = 0$, $v_2 \neq 0$ becomes the deepest one and the inert phase could only exist as metastable one (and probably would not be reached in the course of the thermal evolution of the Universe). The upper (almost horizontal) line in fig. 2 delimits the region in which the minimum at $v_1 = v_2 = 0$ is deeper than the one at $v_1 = v_{\text{tree}}$, $v_2 = 0$ (i.e. it corresponds to the metastability of the inert phase illustrated in fig. 1). The central vertical band marked in fig. 2 shows the range of the coupling $\lambda_{345}$ still allowed by the negative results of the XENON 100 experiment [28]. (One should however remember that the coupling $\lambda_{345}$ defined in our renormalization scheme can differ by from the effective $h^0 H^0 H^0$ coupling tested in this experiment.)

Figure 2 shows that imposing the condition that the Inert phase of the model be an absolutely stable after including the one-loop quantum corrections constrains the (degenerate) masses of $A^0$ and $H^\pm$ to be smaller than $\sim 440$ GeV (smaller than $\sim 380$ GeV if the XENON 100 limit on $\lambda_{345}$ can be trusted). Thus, the upper limit imposed on these masses in [25] by appealing to a rather at hoc criterion can be replaced by a more physical one. Note, that the regions marked in fig. 2 do not change considerably for $M_{h^0}$ in the range $120 \div 130$ GeV and are very weakly sensitive to the DM mass $M_{H^0} \lesssim 80$ GeV and to the value of $\lambda_2$ (between 0 and 1).
4 Temperature-dependent effective potential

In this section we investigate the strength of the electroweak phase transition predicted by the IDM. Our analysis goes beyond that of [25] which was limited by the use of the high temperature expansion, not well justified for the realistic values of the particle masses. Concentrating only on the relevant variables allows us to display the most characteristic details of the electroweak phase transition in the scenario whose more broad aspects were analyzed in [26].

The one-loop temperature dependent effective potential is given by [34]

$$V_{T}^{(1L)}(v_1, v_2) = V_{\text{eff}}^{(1L)}(v_1, v_2) + \Delta^{(1L)}V_{T\neq 0}(v_1, v_2).$$

(11)

$V_{\text{eff}}^{(1L)}$ has been specified in the preceding section (eq. 8) and

$$\Delta^{(1L)}V_{T\neq 0}(v_1, v_2) = \frac{T^4}{2\pi^2} \sum_{\text{fields}} C_s \int_0^\infty dx \, x^2 \ln \left[ 1 - (-1)^2 \exp \left( -\sqrt{x^2 + M_s^2/T^2} \right) \right].$$

(12)

For $T^2 \gg M_s^2$ the contribution of $M_s^2$ to (12) can be expanded:

$$(\Delta^{(1L)}V_{T\neq 0})_B = |C_s| \left\{ \frac{-\pi^2}{90} T^4 + \frac{1}{24} T^2 M_s^2 - \frac{T}{12\pi} |M_s| \right\} - \frac{M_s^4}{64\pi^2} \ln \left( \frac{M_s^2}{T^2} - C_B \right)$$

$$(\Delta^{(1L)}V_{T\neq 0})_F = |C_s| \left\{ \frac{-7\pi^2}{720} T^4 + \frac{1}{48} T^2 M_s^2 + \frac{M_s^4}{64\pi^2} \ln \left( \frac{M_s^2}{T^2} - C_F \right) \right\}$$

(13)

$$(C_B = 5.40762, \quad C_F = 2.63503).$$

In the opposite limit $T^2 \ll M_s^2$ one has

$$(\Delta^{(1L)}V_{T\neq 0})_s = -|C_s| T^4 \left( \frac{M_s}{2\pi T} \right)^{3/2} \left( 1 + \frac{15}{8} \frac{T}{|M_s|} + \ldots \right) \exp \left( -\frac{|M_s|}{T} \right),$$

(14)

for bosons and fermions alike. In our numerical investigations we perform the resumation of the higher-order diagrams. For the contribution of the scalar sector to the temperature-dependent effective potential this is achieved by interpreting the $T^2$ dependent terms in the expansion (13) as the corrections to the Lagrangian mass parameters $m_{ii}^2 \quad i = 1, 2: \quad m_{ii}^2 \rightarrow m_{ii}^2(T) \equiv m_{ii}^2 + c_i T^2$ where

$$c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{16} + \frac{g_t^2 + g_b^2}{4},$$

$$c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{16},$$

(15)

($g$, $g'$ and $g_t$, $g_b$ are the gauge and Yukawa couplings, respectively) and using $m_{ii}^2(T)$ obtained in this way to calculate the field dependent masses squared $M_s^2$ which are reinserted back into the formula (12). For the contribution of the gauge boson sector we follow the prescription of [36]. Unlike the authors of [22], in the zero-temperature part $V_{\text{eff}}^{(1L)}$ of the potential we use the temperature-independent masses.
Figure 3: The strength of the electroweak phase transition for $M_{A^0} = 125$ GeV, $M_{H^0} = 65$ GeV, $\lambda_2 = 0.2$ as a function of the coupling $\lambda_{345}$ for different values of $M_{A^0} = M_{H^\pm}$. Left panel: with the zero-temperature potential $V_{\text{eff}}^{(1L)}$ included. Right panel: without $V_{\text{eff}}^{(1L)}$. The shaded vertical band corresponds to the region allowed by the Xenon 100 data.

Squared $M_{h^0}$; inserting temperature dependent masses into $V_{\text{eff}}^{(1L)}$ would amount to generating inadmissible UV divergences depending on temperature $T$.

We have probed the potential $V_{T}^{(1L)}(v_1, v_2)$ as a function of two variables $v_1$ and $v_2$ looking for minima appearing away from the origin $v_1 = v_2 = 0$ as the temperature $T$ is lowered. The critical temperature $T_{\text{EW}}$ is defined as the one for which the value of $V_{T}^{(1L)}$ at the new minimum is equal to its value for $v_1 = v_2 = 0$. The measure of the strength of the phase transition is given by the ratio $v(T_{\text{EW}})/T_{\text{EW}}$, where $v = \sqrt{v_1^2 + v_2^2}$ determines the $W$ boson mass.

Even if the free parameters of the model correspond to the domain “Inert” in fig. 2 thermal evolution of the system can be quite complicated [18,19]: it can first go to a minimum other than $v_1 \neq 0, v_2 = 0$ and jump to it only after further cooling. The electroweak phase transition consists then of two consecutive transitions of different strengths. We have found that in the considered scenario this indeed can happen: for $\lambda_{345}$ larger than some critical value (which depends on the $A^0$ and $H^\pm$ masses) there appears first the minimum at $v_1 = 0, v_2 \neq 0$. This is seen in fig. 3 where we show $v(T_{\text{EW}})/T_{\text{EW}}$ as a function of $\lambda_{345}$ for several values of the $A^0$ and $H^\pm$ masses.

The left panel of this figure shows result obtained using the temperature-dependent potential with the Coleman-Weinberg term while the right one - without this term. To the left of the discontinuities (or cusps) of the curves the electroweak phase transition occurs in one step: $(0,0) \rightarrow (v_1,0)$. The discontinuities (cusps) mark the critical values of $\lambda_{345}$ for which the system goes first to the minimum $(0,v_2)$. However, for the allowed by Xenon 100 values of $\lambda_{345}$ (vertical bands) the phase transition occurs in one step and for $M_{A^0} \approx M_{H^\pm}$ in a rather narrow window between $\sim 275$ GeV and $\sim 380$ GeV (the upper limit of the “Inert” domain allowed by the XENON 100 experiment in fig. 2) it is sufficiently strong to allow for electroweak baryogenesis. The corresponding values of the temperatures $T_{\text{EW}}$ are shown in fig. 4.
Figure 4: $T_{EW}$ as a function of $\lambda_{345}$ for the same parameters as in fig. 3.

The turnover points correspond to the changes in the character of the symmetry breaking minimum discussed above. For the considered masses of the scalars the temperatures of the EW transition lay between 100 and 150 GeV.

Comparison of the left and right panels of fig. 3 illustrates the important impact of the zero-temperature potential $V^{(1L)}_{\text{eff}}$ on the strength of the phase transition. It is clear that neglecting $V^{(1L)}_{\text{eff}}$ underestimates the value of $v(T_{EW})/T_{EW}$. The second effect of $V^{(1L)}_{\text{eff}}$ is the welcome shift of the maximal value of this ratio to the left, closer to the band of $\lambda_{345}$ values allowed by the XENON 100 results.

The values of $v(T_{EW})/T_{EW}$ shown in fig. 3 depend rather weakly on the coupling $\lambda_2$ and the masses $M_{h^0}$ and $M_{H^0}$ in the considered ranges. For instance changing $M_{h^0}$ from 120 GeV to 130 GeV decreases $v(T_{EW})/T_{EW}$ by a factor $\sim 0.85$. The ratio $v(T_{EW})/T_{EW}$ also mildly increases with decreasing mass of the DM particle.

5 Conclusions

We have reconsidered the electroweak phase transition predicted by the inert doublet model with a stable DM candidate. For the DM particle below the electroweak scale the ratio $v(T_{EW})/T_{EW}$ which determines the strength of the phase transition depends mainly on the coupling $\lambda_{345}$ and the masses of the additional spinless particles $A^0$ and $H^\pm$. For $v(T_{EW})/T_{EW} \gtrsim 1$ these states must be sufficiently heavy. They are then forced to be highly degenerate (to satisfy the constraints from the electroweak precision data) and, as we have found, their (common) mass is strongly bounded from above by the requirement that the inert vacuum is reached at the end of the thermal evolution (i.e. that this minimum is the deepest one). If the XENON 100 constraint on the effective $h^0 H^0 H^0$ coupling of the DM to the SM-like Higgs boson can be applied directly to $\lambda_{345}$, one concludes that the portion of the IDM parameter space in which it can predict the right relic density of DM particles density and a sufficiently strong electroweak phase transition is rather limited. In particular, the

\[7\] The contribution of $V^{(1L)}_{\text{eff}}$ was also taken into account in [26] but was omitted in [25].
masses of the $H^\pm$ and $A^0$ states are constrained to a narrow window $275 - 380$ GeV. We also stress that it is the zero temperature part of the potential which allows to reconcile the requirement $v(T_{EW})/T_{EW} \gtrsim 1$ with the constraints following from stability and the XENON 100 results.

Finally, if the necessary additional source of CP violation is due to the operator (1), the mechanism producing the excess of baryons should be due to the so-called local baryogenesis for which the generated value $n_B/s$ of the baryon number to entropy ratio can reliably be estimated only in the quasi-static regime (thick, slowly moving walls of the bubbles of the new phase) \cite{27,38}; in the opposite regime of fast change the value $n_B/s$ is rather hard to estimate even if the scale $\Lambda$ in (1) is known \cite{4,27,37}. It is also interesting to note that the operator (1) would contribute to the nowadays very important decay $h^0 \rightarrow \gamma \gamma$:

$$\Gamma(h^0 \rightarrow \gamma \gamma) = \frac{\alpha_{EM}^3 G_F M_{h^0}^3}{128 \pi^3} \left\{ |A_{SM} + A_{H^+}|^2 + \left| 4 c_W \frac{v_{tree}^2}{\Lambda^2} \right|^2 \right\},$$ \hspace{1cm} (16)

where $A_{SM} \approx -6.5$ for $M_{h^0} = 125$ GeV, $A_{H^+}$ is the $H^+$ contribution \cite{39} and where as in \cite{38} we have written the coefficient $c_1$ of the operator (1) as $c_W g^2/8\pi^2$. With $c_W \sim 0.1 \div 1$ \cite{38} and $\Lambda$ in the TeV range this contribution is small but potentially distinguishable (though not at the LHC) as the two photons originating from the interaction (1) are polarized differently compared to the photons originating from the ordinary loop-induced coupling.

**Acknowledgments.** We thank D. Sokołowska and M. Carrington for valuable discussions. The work of G.G. and M.K. was partly supported by the Polish Ministry of Science and Higher Education Grant N202 230337.
References

[1] K. Kajantie et al. Phys. Rev. Lett. 77 (1996) 2887 [hep-ph/9605288]; Nucl. Phys., B493 (1997) 413 [hep-lat/9612006]; M. Gurtler, E.-M. Ilgenfritz and A. Schiller, Phys. Rev. D56 (1997) 3888 K. Jansen Nucl. Phys. Proc. Suppl., 47 (1996) 196.

[2] LEP Collaborations, LEP Electroweak Working Group, SLD Electroweak Group and SLD Heavy Flavor Group, Report CERN-EP/2003-091, LEP-EWWG/2003-02, [hep-ex/0312023]

[3] G. Aad et al. (ATLAS Collaboration), Phys. Lett. B710 (2012) 49 [arXiv:2012.1408 [hep-ex]], S. Chatrchyan et al. (CMS Collaboration), Phys. Lett. B710 (2012) 26 [arXiv:2012.1488 [hep-ex]].

[4] M. Trodden, Rev. Mod. Phys. 71 (1999) 1463 [hep-ph/9803479].

[5] M.S. Carena, M. Quiros and C. E. M. Wagner, Phys. Lett. B380 (1996) 81 [hep-ph/9603420], M. Carena, G. Nardini, M. Quiros and C. E. M. Wagner, Nucl. Phys. B812 (2009) 243 [arXiv:0809.3760 [hep-ph]].

[6] A.I. Bochkarev, S.V. Kuzmin and M.E. Shaposhnikov, Phys. Lett. B244 (1990) 275.

[7] N. Turok and J. Zadrozny, Nucl. Phys. B358 (1991) 471, Nucl. Phys. B369 (1992) 729.

[8] M.E. Shaposhnikov, JETP Lett. 44 (1986) 465 [Pisma Zh. Eksp. Teor. Fiz. 44 (1986) 364]; Nucl. Phys. B287, 757 (1987).

[9] D. Land and E.D. Carlson, Phys. Lett. B292 (1992) 107 [arXiv:hep-ph/9208227].

[10] A.B. Lahanas, V.C. Spanos and V. Zarikas, Phys. Lett. B472 (2000) 119 [hep-ph/9812535].

[11] J.M. Cline and P.A. Lemieux, Phys. Rev. D55 (1997), 3873 [arXiv:hep-ph/9609240].

[12] S. Kanemura, Y. Okada and E. Senaha, Phys. Lett. B606 (2005), 361 [arXiv:hep-ph/0411354].

[13] L. Fromme, S.J. Huber and M. Seniuch, JHEP 0611 (2006), 038 [arXiv:hep-ph/0605242].

[14] E. Ma, Phys. Rev. D73 077301 (2006) [hep-ph/0601225].
[15] R. Barbieri, L.J. Hall and V.S. Rychkov *Phys. Rev.* **D74**:015007 (2006) [hep-ph/0603188].

[16] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, *Phys. Rept.* **516**, 1 (2012) [arXiv:1106.0034 [hep-ph]].

[17] I.F. Ginzburg, I.P. Ivanov, K.A. Kanishev, *Phys. Rev.* **D81**:085031 (2010) [arXiv:0911.2383 [hep-ph]].

[18] I.F. Ginzburg, K.A. Kanishev, M. Krawczyk, D. Sokolowska, *Phys. Rev.* **D82**, 123533 (2010); PoS **QFTHEP2010** (2010) 067.

[19] D. Sokolowska, [arXiv:1104.3326 [hep-ph]], *Acta Phys. Polon.* **B42**, 2237 (2011) [arXiv:1112.2953 [hep-ph]].

[20] G. Gil, Master Thesis, Faculty of Physics, University of Warsaw, Badanie przejść fazowych w Inert Doublet Model, August 2011

[21] A. Kozhushko and V. Skalozub, *Ukr. J. Phys.* **56** (2011) 431 [arXiv:1106.0790 [hep-ph]].

[22] J.M. Cline, K. Kainulainen and M. Trott, *JHEP* **1111** (2011) 089 [arXiv:1107.3559 [hep-ph]].

[23] L. Lopez Honorez, E. Nezri, J. F. Oliver and M. H. G. Tytgat, *JCAP* **0702**, 028 (2007) [hep-ph/0612275]; L. Lopez Honorez and C. E. Yaguna, *JHEP* **1009**, 046 (2010) [arXiv:1003.3125 [hep-ph]]. L. Lopez Honorez and C. E. Yaguna, *JCAP* **1101**, 002 (2011) [arXiv:1011.1411 [hep-ph]].

[24] E. M. Dolle and S. Su, *Phys. Rev.* **D80**, 055012 (2009) [arXiv:0906.1609 [hep-ph]].

[25] T.A. Chowdhury, M. Nemevsek, G. Senjanovic and Y. Zhang, *JCAP* **1202** (2012) 029 [arXiv:1110.5334 [hep-ph]].

[26] D. Borah and J.M. Cline, [arXiv:1204.4722 [hep-ph]].

[27] M. Dine, P. Huet, R.L. Singleton and L. Susskind, *Phys. Lett.* **B257** (1991) 351, *Nucl. Phys.* **B375** (1992) 625.

[28] E. Aprile *et al.* (XENON100 Collaboration), *Phys. Rev. Lett.* **105** (2010) 131302 [arXiv:1005.0380 [astro-ph.CO]].

[29] A. Barroso, P. M. Ferreira and R. Santos, *Phys. Lett.* **B632**, 684 (2006) [hep-ph/0507224].

[30] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto and A. Strumia, *Phys. Lett.* **B709**, 222 (2012) [arXiv:1112.3022 [hep-ph]]; G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, [arXiv:1205.6497 [hep-ph]].
[31] E. Lundstrom, M. Gustafsson and J. Edsjo, Phys. Rev. D79, 035013 (2009) [arXiv:0810.3924 [hep-ph]]; M. Gustafsson, PoS CHARGED 2010, 030 (2010) [arXiv:1106.1719 [hep-ph]].

[32] J. Abdallah et al. [DELPHI Collaboration], Eur. Phys. J. C34, 399 (2004) [hep-ex/0404012].

[33] B. Gorczyca and M. Krawczyk, Acta Phys. Polon. B42 (2011) 2229 [arXiv:1112.4356 [hep-ph]]; arXiv:1112.5086 [hep-ph].

[34] L. Dolan i R. Jackiw, Phys. Rev. D9 (1974), 3320; S. Weinberg, Phys. Rev. D9 (1974), 3357.

[35] A.D. Linde, Pis’ma w Zh. Eksp. Teor. Fiz. 23 (1976) 73; S. Weinberg, Phys. Rev. Lett. 36 (1976) 294.

[36] M.E. Carrington, Phys. Rev. D45 (1992), 2933.

[37] A. Lue, K. Rajagopal and M. Trodden, Phys. Rev. D56 (1997) 1250 [hep-ph/9612282].

[38] X. Zhang and B.L. Young, Phys. Rev. D49 (1994) 563 [hep-ph/9309269].

[39] P. Posch, Phys. Lett. B696 (2011) 447 [arXiv:1001.1759 [hep-ph]]; A. Arhrib, R. Benbrik and N. Gaur, Phys. Rev. D85 (2012) 095021 [arXiv:1201.2644 [hep-ph]].