A Fast Test Method of the Optical Axis Turning Angle of an Aviation Aiming POD

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Abstract. Because the aviation aiming pod has special structure, it can not install angle sensor to measure its optical axis turning angle. In order to solve this problem, a turning angle measuring method based on image technique is given in this paper. A test model based on P3P is designed and a fast calculating method is given. The application result indicates that the test range and the test precision are satisfied for the test requirements.

1. Introduction

In order to ensure that the aviation aiming POD can track and aim at the target accuracy, the precision of the optical axis turning angle of an aviation aiming POD must be measured regularly. The structure of the aviation aiming POD is shown in Figure 1. The aviation aiming POD consists of a head and other cabins. A camera platform is fixed in the head. Here, the optical axis turning angle of the aviation aiming POD is really the turning angle of the optical window axis on the camera platform. The camera platform can rotate relative to the head and the head can rotate relative to other cabins. Because the camera platform is a smooth ball and half of it covered by the head [1]. So, it is unable to install angle sensor on the camera platform. In order to resolve this problem, a turning angle measuring method based on P3P technique is given in this paper.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{ POD_structure.png}
\caption{Structure of POD}
\end{figure}

2. Structure and Working Principle of the Test System

The structure of the angle tester is shown in Figure 2. One group of control point adopts blue plastic plates and install on the end of the optical window. The other one adopts green plastic plates and install...
on the opposite end of the optical window. Each group control points include 4 circular same color control points. Three control a regular triangle and the fourth point lies in the triangular center of gravity. The direction of the optical axis of the aviation aiming POD is same as the normal vector of blue control point plane. So, if the normal vector of green control point plane is defined as $\mathbf{a}$, the vector of the optical axis of the aviation aiming POD must be proportionally. One camera is installed on the left lower of the head of the POD, and the other one is installed on the right lower.

The type of the Industrial Personal Computer is 610H+ and it is produced by Advantech Company. Its mainly frequently is 2.9G and the memory is 4G. The type of two color CCD cameras is MV-VS030FC. The resolution ratio of the camera is 640 and 480 and the frame rate is 60fps.

When the angle test system works, the image of the control points are recorded by two color cameras and it’s sent to the IPC by 1394 bus immediately. And then the turning angle of the optical axis of the aviation aiming POD is calculated by the angle test software. At last, the picture and the angle value are displayed in real time.

3. Establish Test Model of the Optical Axis Turning Angle of the Aviation Aiming POD

The position between the images control points and the corresponding real points of the control points is shown as in Figure 3.

In the Figure 3, $O$ is the focus of the camera and the quadrangle $EFGH$ is the imaging plane of the camera. Three control points $A$, $B$ and $C$ are coplanar and forms an equilateral triangle. The points $a$, $b$, and $c$ are image points of three control points $A$, $B$ and $C$ separately. The point $D$ is used to get rid of the unwanted solutions of equation group. In order to calculate easily, $D$ is set as vertical upward one centimeter to the barycenter of the equilateral triangle.
Here, suppose $\overrightarrow{A} = k_1 \overrightarrow{v}_A$, $\overrightarrow{B} = k_2 \overrightarrow{v}_B$, and $\overrightarrow{C} = k_3 \overrightarrow{v}_C$. Among them, $\overrightarrow{v}_A, \overrightarrow{v}_B, \overrightarrow{v}_C$ are unit vectors of $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$. $k_1, k_2$ and $k_3$ are radius vectors of $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ and they are variables to be solved. Let the side length of the equilateral triangle $ABC$ is $d$. Here, $d$ is 8 centimeter.

In camera coordinate system $OXYZ$, the equation group (1) can be gotten with the cosine theorem.

$$
\begin{align*}
4 &= k_1^2 + k_2^2 - 2k_1k_2 \cos \theta_{AB} \\
4 &= k_2^2 + k_3^2 - 2k_2k_3 \cos \theta_{AC} \\
4 &= k_3^2 + k_1^2 - 2k_3k_1 \cos \theta_{BC}
\end{align*}
\tag{1}
$$

Among the equation group (1), $\theta_{AB}$ is the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. $\theta_{AC}$ is the angle between $\overrightarrow{OA}$ and $\overrightarrow{OC}$. $\theta_{BC}$ is the angle between $\overrightarrow{OB}$ and $\overrightarrow{OC}$. These three angles can be calculated by the coordinates of the image of the calculating control points. The parameter $d$ is side length of the triangle. In the equation group (1), the parameters $\theta_{AB}, \theta_{AC}, \theta_{BC}$ and $d$ are known; $k_1, k_2$ and $k_3$ are variables to be solved.

4. calculating test model of the optical axis turning angle of the aviation aiming POD

The calculating procedure includes 8 steps.

4.1. Picture sampling

The image point data of the MV-VS030FC camera are arranged by row column. Each image point includes three elements that are used to save R, G, B value separately. While reading the image, two images are read from two cameras at the same time, and the image data are saved in two single dimension arrays pBits1, pBits2 separately.

4.2. Picking up of the coordinates of the control points

When picking up of the coordinates of the control point, we can use the method that picks up the outline of the image of the control point firstly, and then calculates the coordinate of the image by data fitting algorithm (Xia Ruixue, 2010). But the algorithm is relatively complicated, and the real-time character is not good.

Because each group of control points are adopted same special color and whole picture background doesn’t include this color except the control point, a fast control point coordinate picking up method that gets the middle point to search in the next row is proposed in this paper. The searing processing is shown in next paragraph.

(1) Let variable $i$ equal 0, variable $j$ equal 0, and variable count equal 0. Clears empty the control point coordinate chained list BC (GC).

(2) Let variable sign equal 0 and variable $k_{max}$ equal 0. Clears empty the border chained list BB (GB).

(3) If the image point is blue (green), sets this point as the begin searching point of a new control point and Let sign equal 1. Saves the coordinate $(i, j)$ in the border chained list BB (GB) and sets this image point to black. Then, the program jumps to step (6). If the image point isn’t blue (green), lets $j$ equal $j + 1$.

(4) If $j$ is bigger than 640, Let $j$ equal 0 and $i$ equal $i + 1$.

(5) If $i$ is bigger than 480, the program jumps to step (17). Otherwise the program jumps to step (3).

(6) Let $p$ equal $i$ and $q$ equal $j$.

(7) Searches from the image point $(p, q)$ to left to count the image pinot number $m$ that these colors are blue (green) continuously and sets these image points to black. Then, the coordinate $(p, q - m)$ is saved in the border chained list BB (GB).
(8) Searches from the image point \((p, q)\) to right to count the image pinot number \(n\) that these colors are blue (green) continuously and set these image points to black. Then, the coordinate \((p, q+n)\) is saved in the border chained list \(BB (GB)\).

(9) If \(k_{\text{max}}\) is smaller than \((m+n)\), sets \(k_{\text{max}}\) equal \(m+n\).

(10) Let \(p\) equal \(p+1\) and \(q\) equal \(q+ (n-m)/2\).

(11) If \(p\) is bigger than 480 and sign equals 0, the program jumps to step (17). If \(p\) is bigger than 480 and sign equals 1, the program jumps to step (13).

(12) If image point \((p, q)\) is not blue (Green), the program jumps to step (13). Otherwise the program jumps to step (7).

(13) If \(k_{\text{max}}\) is bigger than 4, Let \(\text{count}\) equal \(\text{count}+1\). Counts the coordinate \((x, y)\) of the barycentre of the control point ellipse by the border chained list \(BB (GB)\) and save the coordinate in the control point coordinate chained list \(BC (GC)\). If \(k_{\text{max}}\) is smaller than 4, the point is believed as offending point and it is caste out.

(14) Let \(j\) equal \(y+k_{\text{max}}/2+6\).

(15) If \(j\) is bigger than 640, Let \(j\) equal 0 and \(i\) equal \(i+1\).

(16) If \(i\) is bigger than 480, the program jumps to step (17). Otherwise the program jumps to step (2).

(17) If the camera is left camera, saves the control point coordinate chained list \(BC (GC)\) to the chained list \(LBC (LGC)\). Otherwise, saves the control point coordinate chained list \(BC (GC)\) to the chained list \(RBC (RGC)\).

4.3. Finding out the coordinates of the control points that are used to calculated
The control point coordinate list \(LBC\) of left camera is searched and the number \(Lb\) of blue control points is counted. Then, the control point coordinate list \(LGC\) of left camera is searched and the number \(Lg\) of green control points is counted. The control point coordinate list \(RBC\) of right camera is searched and the number \(Rb\) of blue control points is counted. Then, the control point coordinate list \(RGC\) of right camera is searched and the number \(Rg\) of green control points is counted. If the value that equals four is found out from \(Lb, Lg, Rb\) and \(Rg\), the corresponding control points is used to the calculating control points. If there is more than one value that equals four, the blue control points will be given first rank to act as the calculating control points.

After four calculating control points are found out, three calculating points and one control point that is the barycenter of the equilateral triangle composed by three calculation control points and is used to get rid of the unwanted solutions of equation group are found out from four calculating control points. The searching method is shown as follow. Firstly, the point that \(y\) coordinate is smallest is searched and is signed as \(A\). then, the remainder three control points are sorted by \(x\) coordinate from small to large. The point that its \(x\) coordinate is largest is signed as \(B\) and the point that its \(x\) coordinate is smallest is signed as \(C\). The remainder point is signed as \(D\). So, the points \(A, B\) and \(C\) are acted as the calculating control points, and the point \(D\) is acted as the barycenter of the equilateral triangle.

4.4. Calculating the coordinates of control point in the camera coordinate system
Because the explicit solutions of the equation group (1) can’t be gotten, the numerical iterative method is often used to solve this type equation group. The equation group (1) can be solved not only by the method that solve the equation group by calculating the Jacobian matrix \(J\) presented by Hu Xiaoping\[2\], but also other methods by Liu Chongliang, etc.\[3\][4]\[5\][6][7] Because these methods are relatively complicated, a simple calculating method is presented in this paper.

Firstly, the equation group (2) is constructed according to the equation group (1).

\[
\begin{align*}
\xi(x, y, z) &= k_x^2 + k_y^2 - 2k_xk_y \cos \theta_{xy} - d^2 \\
\eta(x, y, z) &= k_x^2 + k_z^2 - 2k_xk_z \cos \theta_{xz} - d^2 \\
\zeta(x, y, z) &= k_y^2 + k_z^2 - 2k_yk_z \cos \theta_{yz} - d^2
\end{align*}
\]
According to the literature we can get

\[
\begin{bmatrix}
\Delta k_x \\
\Delta k_y \\
\Delta k_z 
\end{bmatrix} = -J^{-1} \begin{bmatrix}
f'(k_x, k_y, k_z) \\
f'(k_x, k_y, k_z) \\
f'(k_x, k_y, k_z)
\end{bmatrix} 
\] (3)

Here, \( J^{-1} \) is the inverse matrix of the Jacobian matrix. Because

\[
J = \begin{bmatrix}
2k_x - 2k_y \cos \theta_x & 2k_y - 2k_z \cos \theta_z & 0 \\
2k_y - 2k_x \cos \theta_y & 2k_z - 2k_x \cos \theta_x & 0 \\
0 & 2k_z - 2k_y \cos \theta_y & 2k_x - 2k_y \cos \theta_y 
\end{bmatrix} 
\] (4)

If the equation (4) is substituted in the equation group (3) and settles the equation, the equation group (5) is gotten.

\[
\begin{align*}
E(k_x, k_y, k_z) + (2k_x - 2k_y \cos \theta_x) \Delta k_x + (2k_y - 2k_z \cos \theta_z) \Delta k_y + 0 &= 0 \\
E(k_x, k_y, k_z) + (2k_y - 2k_x \cos \theta_y) \Delta k_y + (2k_z - 2k_x \cos \theta_x) \Delta k_z + 0 &= 0 \\
E(k_x, k_y, k_z) + (2k_z - 2k_y \cos \theta_y) \Delta k_z + (2k_x - 2k_y \cos \theta_y) \Delta k_x &= 0
\end{align*}
\] (5)

Because the equation group (5) is a linear equation group with three variables, its explicit solution of \( \Delta k_A, \Delta k_B \) and \( \Delta k_C \) can be gotten directly. After \( \Delta k_A, \Delta k_B \) and \( \Delta k_C \) are calculated, the values of \( k_A, k_B \) and \( k_C \) can be calculated by Newton iteration method. \( X^K \) is the iteration vector \([k_A, k_B, k_C]^T\) of the step \( k \) and \( \Delta K \) is the vector \([\Delta k_A, \Delta k_B, \Delta k_C]^T\). So the iteration value of the step \( k+1 \) is shown in equation (6).

\[
X^{k+1} = X^k + \Delta K
\] (6)

The equation (6) is used to execute the iterative computation. If \(|f(K^{k+1})| \) is smaller than \( \varepsilon \) or the iteration is \( N \), the iterative computation will stop and outputs the calculating result \([k_A, k_B, k_C]^T\). Really application indicates that the iterative condition will be approached after calculates form 5 to 7 times.

4.5. Picking up the right solution of the equation

Because the equation (1) is a quadratic group with three variables, it may have four real solutions according to the physical meanings of the solutions. But only one solution is the right solution. So, the fourth control point \( D \) is adapted to gotten rid of unwanted solutions [8].

Four points \( D \) of four solutions are calculated one by one. The calculating process is shown as follow. Firstly, Coordinates \( A, B \) and \( C \) that form a group of control point are calculated by a solution. Then, the normal vector and the barycentre coordinate of the triangle \( ABC \) are calculated. The straight linear equation that it passes the barycentre of the triangle and its direction vector is same as the normal vector of the triangle is computed. Two points that are on this straight line and there distance relative to the barycentre of the triangle are one centimeter are calculated. The unwanted point is gotten rid of according to direction requirement. If there isn’t error, the point \( D \) of right solution must be on the straight line \( Od \). But the error can’t be avoided, the solution that the distance between its point \( D \) and the straight line \( Od \) is shortest is considered to the right solution.
4.6. Calculating the normal vector of the control point panel in the camera coordinate system

After the right solution \( k_A, k_B, \) and \( k_C \) are picked up, the coordinates of three control points \( A(x_A, y_A, z_A) \), \( B(x_B, y_B, z_B) \) and \( C(x_C, y_C, z_C) \) can be calculated. Let the normal vector of the control panel that is in the camera coordinate system is \( \vec{n} \). So, three coordinate values of \( \vec{n} \) can be calculated by formula (7).

\[
\begin{align*}
x &= (y_C - y_A) \times (z_B - z_A) - (y_B - y_A) \times (z_C - z_A) \\
y &= (z_C - z_A) \times (x_B - x_A) - (z_B - z_A) \times (x_C - x_A) \\
z &= (x_C - x_A) \times (y_B - y_A) - (x_B - x_A) \times (y_C - y_A)
\end{align*}
\]

(7)

4.7. Calculating the normal vector of the control point panel in the aviation aiming POD coordinate system

Because the azimuth angle and pitch angle is relative the aviation aiming POD, the normal vector must be transformed from the camera coordinate system to the aviation aiming POD coordinate system. Let the angle between the left camera coordinate system and the POD coordinate system is \( \alpha_1, \beta_1, \) and \( \gamma_1 \), and the angle between the right camera coordinate system and the POD coordinate system is \( \alpha_2, \beta_2, \) and \( \gamma_2 \). Because these angles are known, the transition matrixes \( K_1 \) and \( K_2 \) can be calculated by these transition angles. Let the normal vectors of the control point panel in left and right camera coordinate system are \( \vec{n}_{LR} \) separately and the normal vector of the control point panel in the aviation aiming POD coordinate system is \( \vec{n}' \). \( \vec{n}' \) can be computed by the formula (8).

\[
\begin{align*}
\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} &= \begin{bmatrix} x_L \\ y_L \\ z_L \end{bmatrix} K'_1 \\
\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} &= \begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} K'_2
\end{align*}
\]

(8)

4.8. Calculating the turning angles of the optical axis of the aviation aiming POD

After \( \vec{n}' \) has been calculated, the azimuth angle and pitch angle that the optical axis of the aviation aiming POD is relative the aviation aiming POD coordinate system are calculated by position relation between the vector \( \vec{n}' \) and the aviation aiming POD.

5. Measure Result Analysis

The measure software based on VC ++6.0 by the method that is provided in last festival is designed and its interface is shown in Figure 4.

Figure 4. interface of the measure system.

The focus of the camera is 0.5 centimeters, and its size of image point is \( 5.6 \mu m \times 5.6 \mu m \). The turning angle by Z-Y-X method from the aviation aiming POD coordinate system to the left camera coordinate system is \( (0^\circ, 30^\circ, 120^\circ) \). So, the transformation matrix \( K_i \) from the aviation aiming POD coordinate system to the left camera coordinate system can be computed by formula (10) [9].
\[ \mathbf{K}_i = \begin{bmatrix} 0.866 & 0 & -0.499 \\ 0.433 & -0.499 & 0.750 \\ -0.249 & -0.866 & -0.433 \end{bmatrix} \] (10)

The azimuth angle of the aviation aiming POD axis is preset -77° and the pitch of the aviation aiming POD axis is preset -18°. Then, we use the angle test software measure the turning angle.

The coordinates of the image points of the control points that are picked up from the image of the left camera are

\[ \begin{aligned} &A(-37*5.6*0.0001,27*5.6*0.0001,-0.5), \\
&B(-82*5.6*0.0001,-58*5.6*0.0001,-0.5), \\
&C(11*5.6*0.0001,-58*5.6*0.0001,-0.5), \\
&D(-40*5.6*0.0001,-31*5.6*0.0001,-0.5). \end{aligned} \]

The unit is centimeter. The origination parameter of iterative computation \(K_0\) is preset \([60, 80, 60]\), and finishing condition of the iterative computation is \(|f(K^{k+1})|<0.001|\). After the iterative computation is executed 6 steps, the finishing condition of the iterative computation is satisfied. Then, The values of \(k_A, k_B\) and \(k_C\) are shown in table 1. So, the normal vector \(\mathbf{n}\) that the control point panel is relative to the left camera coordinate system is computed, and its value is \([43.898, -11.102, -31.963]\). After coordinate transformation computing, the normal vector \(\mathbf{n}'\) is computed and its value is \([53.998, -18.420, 12.481]\).

At last, the value of the calculated azimuth angle is -76.98° and the value of the calculated pitch angle value is -18.38°. So, the error of azimuth angle is +0.02°, and the error of pitch angle is -0.38°. It is fit for the precision requirement of angle test.

| times | 1   | 2    | 3    | 4    | 5    | 6    |
|-------|-----|------|------|------|------|------|
| \(k_A\) | 128.387 | 102.990 | 88.4086 | 85.4103 | 85.3577 | 85.3577 |
| \(k_B\) | 137.174 | 106.278 | 90.1302 | 87.1335 | 87.0811 | 87.0811 |
| \(k_C\) | 126.387 | 98.9959 | 83.2911 | 80.2616 | 80.2093 | 80.2093 |

6. Conclusions

Application indicates that the test range and precision are satisfied for the test requirement, and the test speed is fast. The method given in this paper offers a new thinking to solve the problem of the turning angle and turning angular speed of a class of equipment that its structure is unsuited to fix angle test equipment directly.

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