Effects of an extra $Z'$ gauge boson on the top quark decay $t \to c\gamma$

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The effects of an extra $Z'$ gauge boson with family nonuniversal fermion couplings on the rare top quark decay $t \to c\gamma$ are first examined in a model independent way and then in the minimal 331 model. It is found that the respective branching fraction is at most of the order of $10^{-8}$ for $m_{Z'} = 500$ GeV and dramatically decreases for a heavier $Z'$ boson. This result is in sharp contrast with a previous evaluation of this decay in the context of topcolor assisted technicolor models, which found that $B(t \to c\gamma) \approx 10^{-4}$ for $m_{Z'} = 1$ TeV.

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Several well-motivated extensions of the standard model (SM) predict an extra neutral gauge boson $Z'$, which has both theoretical and experimental motivations. For instance, it has been conjectured that a $Z'$ boson can be helpful to describe the $Z$-pole data and atomic parity violation. Much work has been gone in studying the phenomenology of an extra $Z'$ boson $^{1}$, and experimental data have been used to obtain lower bounds on its mass and the $Z' - Z$ mixing angle $\theta$. It is not possible to obtain model-independent bounds, but current data from collider $^{2}$ and precision $^{3}$ experiments yield $m_{Z'} \gtrsim 500$ GeV and $\sin \theta \lesssim 10^{-3}$ for standard GUT models; more strong constraints on $m_{Z'}$, of the order of 1 TeV, are obtained in models with nonuniversal flavor gauge interactions such as topcolor assisted technicolor $^{4}$, noncommuting extended technicolor $^{5}$, or the ununified standard model $^{6}$. Quite recently, the CDF collaboration used the Fermilab Tevatron data to search for new massive neutral particles decaying into lepton pairs $^{7}$. As far as new gauge bosons are concerned, they considered a new SM-like $Z'$ boson ($Z'_{SM}$), the $Z'$ bosons from the $E_6$ model ($Z_X, Z_\psi, Z_\eta, Z_I$) and the one from the littlest Higgs model ($Z_H$) $^{8}$. The following lower bounds were obtained $^{7}$ for the masses of $Z'_{SM}$, $Z_X, Z_\psi, Z_\eta$ and $Z_I$: 825, 690, 675, 720 and 615 GeV, whereas $m_{Z'_{SM}}$ was found to be larger than 885, 860, 805 and 725 GeV for the following values of the mixing parameter $\cot \theta_H$: 1.0, 0.9, 0.7 and 0.5, respectively.

If the couplings of the $Z'$ to the fermions are assumed to be family universal, there is no flavor changing neutral current (FCNC) effects mediated by this particle even if there is fermion flavor mixing via the GIM mechanism. Nevertheless, it is possible that the $Z'$ couples nonuniversally to the fermions, thereby giving rise to FCNCs. For instance, some theories require that the $Z'$ couplings to the third family fermions are different than the ones to the fermions of the first two families. This is the case of the $Z'$ predicted by the 331 model $^{8}$, which is based on the $SU(3)_L \times U(1)_X$ symmetry and has attracted considerable attention recently $^{9}$. This model is particularly appealing due to its unique mechanism of anomaly cancelation: instead of the usual cancelation between each fermion family, it is necessary that all the three fermion families are summed over, which automatically requires the existence of a number of families that is multiple of 3. It has been conjectured that this may provide a hint to the solution of the family number problem. There are thus good motivations for an extra $Z'$ boson, with a mass in the range 500 GeV–1 TeV, which couples nondiagonally to the fermions.

Although the phenomenology of a $Z'$ gauge boson is interesting by itself, so is the study of FCNC effects as they would be a hint of new physics because of their large suppression in the SM. This class of effects has been considerably studied in the literature via some rare decay modes of the top quark. The interest in the top quark phenomenology stems from its large mass, which has led to some belief that new physics effects are more likely to show through processes involving this particle. Even more, the copious production of top quark pairs at the CERN large hadron collider (LHC) will allow us to examine several top quark properties and some of its rare decays. It is thus very interesting to consider the virtual effects of a $Z'$ gauge boson on rare top quark decays. Among this class of processes, the decay $t \to c\gamma$ has been analyzed both in the SM and several of its extensions $^{10, 11}$. While the SM predicts that the $t \to c\gamma$ branching fraction is of the order of $10^{-10}$ $^{10}$, other models can enhance it up to $10^{-5}$ $^{11}$.

In this brief report we will consider the contribution of the $Z'$ gauge boson to the $t \to c\gamma$ decay. It has been claimed recently $^{12}$ that $B(t \to c\gamma)$ is at the $10^{-6}$ level in topcolor assisted technicolor models for $m_{Z'} = 1$ TeV, which would allow the detection of this decay channel at future particle colliders. Prompted by this assertion, we will evaluate this decay in the context of some other models. Rather than concentrating on a particular model, we will take the approach of effective theories and consider an effective interaction $Z'qq'$. We then present general expressions for the one-loop calculation. This would allow us to have an estimate of the order of magnitude of the $t \to c\gamma$ decay rate in a model-independent fashion, and will be useful to assess whether the $Z'$ virtual effects have the chance of becoming evident in future particle colliders through this decay mode.
We will consider an effective interaction of the form

\[ \mathcal{L}^{Z'}_{q,q'} = \frac{ig'}{2\epsilon_1} \sum_{i,j} \bar{q}_i \gamma_\mu \left( \xi^{ij}_V + \xi^{ij}_A \gamma^5 \right) q_j Z'^\mu, \]

(1)

where the coefficients \( \xi^{ij}_V \) and \( \xi^{ij}_A \) contain the information concerning any specific model. We refrain from discussing the mechanism of generation of the FCNCs in models in which the \( Z' \) boson has family nonuniversal couplings, but a discussion along this line can be found in Ref. \[13\]. We first present results for the decay \( t \to c \gamma \) in a model-independent fashion and then examine it in the context of the minimal 331 model. As a by-product, we reevaluate the \( t \to c \gamma \) decay in the scenario posed by topcolor assisted technicolor models and compare our result with previous ones.

\[\text{FIG. 1: Generic Feynman diagrams depicting the contribution of a Z'} \text{ gauge boson to the top quark decay t } \to c \gamma.\]

The \( Z' \) contribution to the top quark decay \( t \to c \gamma \) proceeds through the Feynman diagrams shown in Fig. 1. Electromagnetic gauge invariance restricts the amplitude of this decay to have the form

\[ \mathcal{M}(t \to c \gamma) = \frac{i e g^2 \varepsilon_\mu^\nu k_\mu}{2 m_t \pi^2} \bar{u}_c(p_2) \sigma^{\alpha \nu} (F_1 + F_2 \gamma_5) u_t(p_1), \]

(2)

where \( k_\alpha \) is the four-momentum of the photon. We will calculate the \( t \to c \gamma \) decay using the interaction 14 and verify that the resultant amplitude obeys Eq. 2, i.e. that the coefficients of the gauge noninvariant terms \( \gamma^\alpha \) and \( \gamma^\alpha \gamma^5 \) vanish. We obtain in the massless c quark limit:

\[ F_1 = \sum_{q=t,c} \left( \varepsilon_{A}^{cq} \varepsilon_{A}^{tq} A_1(m_q) + \varepsilon_{V}^{cq} \varepsilon_{V}^{tq} A_2(m_q) \right), \]

(3)

\[ F_2 = -\sum_{q=t,c} \left( \varepsilon_{V}^{cq} \varepsilon_{V}^{tq} A_1(m_q) + \varepsilon_{A}^{cq} \varepsilon_{A}^{tq} A_2(m_q) \right), \]

(4)

with \( \varepsilon_{A,V}^{ij} = g'/g \xi_{A,V}^{ij} \). We let the sum run only over the \( c \) and \( t \) quarks as the associated Feynman diagrams require only one FCNC vertex. We assume that those Feynman diagrams including two FCNC vertices are more suppressed than those containing just one of them. The \( A_i \) factors are given by

\[ A_1(m_q) = \frac{1}{m_t^2 m_{Z'}} \left( m_q^2 (m_q^2 - m_t^2) + m_{Z'}^2 ((m_q \pm m_t) (m_q \pm 2 m_t) - 2 m_{Z'}^2) \right) \delta B_0(m_q) \]

\[ - \frac{m_q^2}{m_{Z'}^2} (m_q^2 - m_t^2 + 2 m_{Z'}^2) C_0(m_q) - \frac{m_q (m_q \pm m_t)}{2 m_{Z'}^2} - 1, \]

(5)

where the \( + (-) \) sign stands for \( A_1 (A_2) \), and

\[ \delta B_0(m_q) = 1 + \frac{\xi}{m_t^2} \arccosh \left( \frac{m_q^2 - m_t^2 + m_{Z'}^2}{2 m_q m_{Z'}} \right) - \frac{1}{2} \left( \frac{m_q^2 - m_{Z'}^2}{m_t^2} \frac{m_q (m_q \pm m_t)}{m_{Z'}^2} \right) \log \left( \frac{m_q^2}{m_{Z'}^2} \right), \]

(6)
\[ C_0(m_q) = \frac{1}{m_q^2} \left[ \text{Li}_2 \left( 1 - \frac{m_{Z'}^2}{m_q^2} \right) - \text{Li}_2 \left( -\frac{2m_q^2}{\eta + \xi} \right) - \text{Li}_2 \left( -\frac{2m_q^2}{\eta - \xi} \right) \right], \]

with \( \xi^2 = (m_q^2 + m_t^2 - m_{Z'}^2)^2 - 4m_q^2m_t^2 \) and \( \eta = m_q^2 + m_t^2 - m_{Z'}^2 \). We have verified that ultraviolet divergences cancel out. From \( \text{Eq.}\), the decay width for \( t \rightarrow c\gamma \) follows easily:

\[ \Gamma(t \rightarrow c\gamma) = \frac{\alpha^3 m_t \sum_i |F_i|^2}{2^{10/2} \pi^4 s_W^2 e_W^4} \approx 2.35 \times 10^{-7} \text{GeV} \times \sum_i |F_i|^2. \]  

Instead of evaluating the \( t \rightarrow c\gamma \) branching ratio, it is worth first evaluating the single contribution of an internal quark to the \( A_i \) coefficients. In this way we can get a rough estimate of the order of magnitude of the branching ratio \( B(t \rightarrow c\gamma) \approx \Gamma(t \rightarrow c\gamma)/\Gamma(t \rightarrow bW) \). In Fig. 2 we show the \( A_i \) coefficients as functions of \( m_{Z'} \), for \( m_q = m_t \) and \( m_q = m_c \). We can see that both \( A_1 \) and \( A_2 \) are of the order of \( 10^{-1} \). For \( m_{Z'} \approx 500 \text{ GeV} \) and decrease dramatically for a heavier \( Z' \) boson. Even more, since \( A_2 \) changes sign for \( m_q = m_c \), there is the possibility of strong cancelations when summing over the internal c and t quarks. We can thus expect that the square \( F_i \) form factors are of the order of \( 10^{-2} \) times the square of some products of the \( \epsilon_{V,A} \) coefficients. In view of Eq. \( 8 \) we can conclude that even if the \( \epsilon_{V,A} \) coefficients are of the order of the unity and there is no strong cancelations, \( B(t \rightarrow c\gamma) \) could be of the order of \( 10^{-8} \) at most for \( m_{Z'} \approx 500 \text{ GeV} \).

![Graph](image_url)

**FIG. 2:** The coefficients \( A_i \) as functions of the \( Z' \) mass for \( m_q = m_t \) and \( m_q = m_c \).

Let us now analyze the scenario posed by the minimal 331 model \( \text{Ref.}\ [8] \). The details of this model are discussed for instance in Ref. \( \text{Ref.}\ [14] \), here we will only present its most relevant features to contextualize our results. In the \( SU(3)_L \times U(1)_X \) model the gauge interactions are non flavor-universal since fermion generations are represented differently under the \( SU_L(3) \) group. The leptons do not carry quantum number \( X \) and all the three generations are accommodated as antitriplets of \( SU_L(3) \). To cancel the \( SU_L(3) \) anomaly, the same number of fermion triplets and antitriplets is necessary, thereby requiring two quark generations to be accommodated as triplets and the other one as antitriplet. This is accomplished by introducing three new exotic quarks, which are denoted by \( D, S, \) and \( T \). The electric charge of \( D \) and \( S \) is \( -4/3e \), whereas that of \( T \) is \( 5/3e \). As far as the Higgs sector is concerned, it is comprised by three \( SU(3)_L \) triplets (\( \phi_Y, \phi_1 \) and \( \phi_2 \)) and one sextet (\( \eta \)). \( \phi_Y \) is required to break \( SU(3)_L \times U_X(1) \) into \( SU_L(2) \times U_Y(1) \), whereas the next stage of spontaneous symmetry breaking (SSB) occurs at the Fermi scale and is accomplished by the remaining triplets \( \phi_1 \) and \( \phi_2 \). The scalar sextet \( \eta \) is necessary to provide realistic masses for the leptons \( \text{Ref.}\ [12] \). In the gauge sector, the 331 model predicts new particles: one pair of singly charged gauge bosons \( Y^\pm \), one pair of doubly charged gauge bosons \( Y^{\pm \pm} \), and an extra neutral boson \( Z' \). These new gauge bosons and the exotic quarks get their masses at the first stage of SSB. The charged gauge bosons appear in an \( SU_L(2) \) doublet with hypercharge +3 and carry two units of lepton number, so they were dubbed bileptons. As for the neutral fields, the symmetry breaking \( SU_L(2) \times U_Y(1) \rightarrow U_Q(1) \) yields the photon and two massive neutral gauge bosons \( N \) and \( N' \), which are linked to the mass eigenstates \( \tilde{Z} \) and \( \tilde{Z}' \) via an unitary rotation. Since current constraints indicate that \( \sin \theta < 10^{-3} \), it is reasonable to assume that the \( Z - Z' \) mixing is negligible, in which case \( Z \) (\( Z' \)) almost coincides with \( N \) (\( N' \)). In this scenario the flavor diagonal neutral currents mediated by the photon and the \( Z' \) boson can be written as

\[ \mathcal{L}^{FD} = ig_f \bar{f} \gamma_\mu f A^\mu + \frac{ig}{2c_W} \bar{f} \gamma_\mu (g'_{V(Z')} - g'_{A(Z')} \gamma_5) f Z'^\mu. \]  

\( \text{FD} \)
All the coefficients \(g_{VZ}^f\) and \(g_{AZ}^f\) are listed in Ref. \[14\], but we show the ones necessary for our calculation in Table 4. As far as the FCNCs are concerned, they are essentially mediated by the \(Z'\) boson because the couplings of the quarks to the \(Z\) boson are proportional to \(\sin \theta\). This class of effects arises from the left-handed sector only and is a result of the different \(X\) quantum number assignments existing between the fermion families. Assuming that there is no \(Z - Z'\) mixing, the FCNC Lagrangian for the up sector can be written as

\[
\mathcal{L}^{FC} = \frac{ig}{2\hat{c}_W} Z'^\mu V_{3q}^* \bar{u}_i \gamma_\mu P_L U_j,
\]

with \(\delta_L = \frac{2}{\sqrt{3}} \frac{c_\theta^q}{1+4\eta^q}\). \(V_{ab}\) is the unitary matrix linking gauge states to mass eigenstates, and \(U_a = u, c, t\). In the right-handed sector there is no FCNC as these fermions transform identically. The \(V_{ab}\) matrix elements have to be constrained via some FCNC processes \[13\].

**TABLE I: Diagonal couplings of the \(Z'\) boson to up quarks.**

| q   | \(\epsilon_V\) | \(\epsilon_A\) |
|-----|-----------------|-----------------|
| \(t\) | \(\frac{1+4\eta_t}{2\sqrt{3}\hat{c}_W}\) | \(\frac{1+4\eta_t}{2\sqrt{3}\hat{c}_W}\) |
| \(u, c\) | \(\frac{-1+4\eta_u}{2\sqrt{3}\hat{c}_W}\) | \(\frac{-1+4\eta_c}{2\sqrt{3}\hat{c}_W}\) |

Once the main features of the model were described, we proceed to calculate the corresponding \(t \to c\gamma\) branching ratio, which is shown in Fig. 3 as a function of \(m_{Z'}\). From that Figure one can conclude that \(B(t \to c\gamma) < 10^{-8}\) for \(m_{Z'} > 500\) GeV. The inequality stems from the fact that the mixing matrix elements \(V_{3c}\) and \(V_{3t}\) are smaller than the unity and so is the coefficient \(|V_{3c}V_{3t}|^2\).

![FIG. 3: Individual contributions of the \(c\) and \(t\) quarks to \(B(t \to c\gamma)\) as a function of \(m_{Z'}\) in \(Z'\) models with left-handed couplings to up quarks. The solid line is for the total contribution in the minimal 331 model.](image)

The \(Z'\) contribution to \(t \to c\gamma\) was previously calculated in the context of topcolor-assisted technicolor (TC2) models \[12\]. Those authors claim that, in that class of models, \(B(t \to c\gamma)\) ranges between \(1.3 \times 10^{-6}\) and \(1.7 \times 10^{-7}\) for \(1\) TeV \(\leq m_{Z'} \leq 2\) TeV. Since our previous findings seems to be in contradiction with that result, it is convenient to reevaluate \(B(t \to c\gamma)\) in the scenario discussed in \[12\]. The flavor diagonal and flavor changing couplings of the \(Z'\) boson relevant for our calculation are

\[
\mathcal{L}^{FD}_{Z'} = \frac{1}{6} g_1 \cot \theta' (\bar{t}_L \gamma_\mu t_L + 4 \bar{t}_R \gamma_\mu t_R) Z'^\mu,
\]

\[
\mathcal{L}^{FC}_{Z'} = -\frac{1}{6} i g_1 K_{tc} (\bar{t}_L \gamma_\mu c_L + 4 \bar{t}_R \gamma_\mu c_R) Z'^\mu,
\]

where \(g_1\) is the hypercharge coupling constant, \(\tan \theta' = g_1/\sqrt{3\hat{c}_W}\), and \(K_{tc}\) is a flavor mixing factor. It is required that \(\tan \theta' \ll 1\) to have top quark condensation rather than \(b\bar{b}\) condensation \[12\].
It is straightforward to cast Eqs. (11) and (12) in the form of Eq. (11). After identifying the coefficients that enter into Eqs. (3) and (4), numerical evaluation yields $B(t \to c\gamma) \approx 10^{-11}$ for $m_{Z'} = 500$ GeV and the same set of parameters used in (12), i.e. $K_{tc} = 0.8$ and $k_l = 1$. For a heavier $Z'$ boson $B(t \to c\gamma)$ is negligibly small. This result is several orders of magnitude below than the one found in Ref. [12]. Unfortunately those authors did not report the analytical result used for their calculation and we are unable to compare it with ours.

Finally, we would like to consider the scenario in which the $Z'$ boson has only left-handed couplings to up quarks. This means that we will take $\epsilon_V = \epsilon_A = \epsilon$ for both diagonal and nondiagonal $Z'$ couplings. In Fig. 3, we have plotted the single contributions of those loops carrying the same internal quark to the $t \to c\gamma$ branching ratio. These results show that $B(t \to c\gamma)$ is unlikely to be above the $10^{-5}$ level unless the $\epsilon$ coefficient is much larger than unity.

In closing, we would like to remark that the $Z'$ contribution to the $t \to c\gamma$ decay is highly suppressed and it is unlikely that this class of effects is at the reach of future particle colliders. Our result is consistent with previous evaluations of this decay in other models [10], which found, after considering the current bounds, that the main contributions arise from scalar particles rather than gauge bosons. As for the contribution to the decays $t \to cg$ and $t \to cZ$, explicit evaluation of the former shows that it can be of the order of $10^{-6}$ at most for $m_{Z'} = 500$ GeV and coupling constants of the order $O(1)$ [we only need to make the replacement $\alpha \to 4/3\alpha_s$ in Eq. (8) to obtain $\Gamma(t \to cg)$], while the latter is expected to be more suppressed due to kinematics.

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