Design of Engineering Surfaces Using Quartic Parabolas

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Abstract. The article considers the wireframe-kinematic method of hydrodynamic simulation of surfaces presented by the example of marine craft’s bottom structure. The motion trajectory for the variable shape generator is normally set by a single or a number of directing lines. The law of shape variations of the generator is represented by the graphical or analytical display, or by a set of directing lines. All of these make up the so-called determinant of the surface, which includes the geometry component, or a certain initial position of the generator, and the algorithm component, i.e. the algorithm of the wireframe sealing. The directing line may be simple, described by a single equation, or composite piecewise-smooth. In this situation, they are commonly modelled using splines, which results in piecewise-smooth surfaces.

1. Introduction

In mechanical engineering the majority of curved surfaces are referred to kinematic surfaces. This is due to the fact that when designing this type of surfaces, it is relatively easy to specify and provide their basic characteristics, and methods applied for their mechanical treatment are simple. Such surfaces can be used to solve the design and technological problems, e.g. the problems related with local shape control, calculation of mechanical characteristics, determination of cross sections and equidistance determination, etc. In certain instances, the engineering surface is designed as the wireframe, which is mainly in such industries as aircraft, shipbuilding and motor-vehicle construction, light and footwear industries. The frame of a surface is presented by the designer or is the result of scanning of the test model.

As is well-known, kinematic surfaces are formed by moving the constant or variable shape lines in accordance with a certain law [1, 7, 8]. Engineering geometry provides designers with a wide range of kinematic surfaces: spiral, cyclic, ruled surfaces with one, two and three directing lines, surfaces moved by parallel shifts and sections dependent surfaces, etc. The latter are frequently used in designing aircrafts and marine vessels.

In this paper we propose one of the wireframe-kinematic methods for hydrodynamic simulation of surfaces.

2. Research

The motion trajectory of the generator with a variable shape is generally set by one or several directing lines. The law of the generator’s shape variations may be presented graphically or analytically, as well as by the directing lines. All these factors constitute the surface determinant. The latter is related with the geometrical component, i.e. a certain initial position of the generator, and the algorithmic
parameter, i.e. the algorithm for the frame sealing. The generators may be simple, which are described by a single equation, or composite piecewise-smooth. In the given instance they are generally modelled using the splines, and the surface can be described as piecewise-smooth.

Let us take the quartic curve for the generator, which presents a segment for interpolating and integrodifferential quartic spline (ID spline) \([2, 6, 10]\)

\[
\mathbf{r}(x, u) = \mathbf{r}_1(x)\phi_1(u) + \mathbf{r}_2(x)\phi_2(u) + \left( \mathbf{r}'(x)\phi_3(u) + \mathbf{r}_2'(x)\phi_4(u) \right)h + \mathbf{I}(x)\phi_5(u),
\]

where \( h = \frac{\mathbf{r}_1' + \mathbf{r}_2'}{2} \); \( \phi_1(u) = (1 + 5u)(1 - 3u)(1 - u)^2 \), \( \phi_2(u) = (3u - 2)(6 - 5u)u^2 \);

\( \phi_3(u) = u(2 - 5u)(1 - u)^2 / 2 \), \( \phi_4(u) = (1 - u)(3 - 5u)u^2 / 2 \), \( \phi_5(u) = 30u^2(1 - u)^2 \);

\( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) are the radius-vectors of the endpoints of the arc; \( \varphi(u) \) are the basic functions, and the parameter \( u \in [0.1] \).

The vector \( \mathbf{I} \), characterizes the curve deflection (Fig. 1). It highlights the centre of the nonplanar cross-section mass bounded by an arc and a chord. The vectors \( \mathbf{r}_1' \) and \( \mathbf{r}_2' \) are tangential.

We can see that the given curve has the parameters used to control the shape: two directing parameters at the endpoints of the arc and the vector-integral. In order to set the simple surface of dependent cross sections with a quartic curve used for the generator, it is necessary to set the directing lines for the endpoints of the arc, the generators for the tangential vectors, and the directing line for the vector-integral. As an example, let us consider the initial stage of hydrodynamic simulation of the surface of a small size hard-chine vessel. (Fig.2) The surface of the hull is designed as the kinematic surface of dependent plane sections.

Let the symmetry plane of a vessel coincide with the plane \( xOy \). The axis of distances is parallel to the axis \( x \), respectively. Then the dependent sections of the surface, or frames (which are also called lines), are parallel to the plane \( yOz \). The surface of the hull (we consider only the left side) consists of two imperfectly joined parts: the side and the bottom. At the first stage, we set three directing factors: the side line, the bow and keel lines. The side line and the bow constitute the surface of the board side. When planning, the given surface is not washed by sea water and does not affect the quality characteristics of the vessel. Therefore, the surface is designed as the cylindroid, i.e. the surface with curved directing lines, the linear generator, and the parallelism plane \( yOz \). The formula of the surface is given by

\[
\mathbf{r}(x, v) = (\mathbf{r}_s(x) - \mathbf{r}_{bo}(x))u + \mathbf{r}_b(x, v).
\]

The bottom is constituted by the bow and keel. This part of the hull surface determines all the hydrodynamic parameters (Fig. 3).
Fig.2. The bow-to-stern view of the surface
s – board side, b – bottom, sl - side line, bw – bow, \( \alpha_d \) – deadrise angle, \( \alpha_r \) – rebound angle

The directing lines are piecewise smooth. Each directing line is defined by twelve points. The point basis is interpolated by the spline. The cubic spline is commonly applied for the given purpose. As a result, we obtain two piecewise-smooth curves with the second order smoothness \([2, 3]\). At developing the surface, the generator (1) is shifted along these directing lines. To ensure the smoothness of the surface, it is necessary to provide the continuity and smoothness of the functions of cross-section parameters within the nodes of the directing lines.

For the cross-section surface we select the quartic parabola represented by the parameters (1)
\[
\bar{r}(x, u) = \bar{r}^x(x)\phi_1(u) + \bar{r}^{bw}(x)\phi_2(u) + \left( \bar{r}^x(x)\phi_3(u) + \bar{r}^{bw}(x)\phi_4(u) \right) h + \bar{I}(x)\phi_5(u),
\]
where
\[ h = \frac{|\bar{r}^{bw} + \bar{r}^x|}{2} ; \]
\( \bar{r}^x \) and \( \bar{r}^{bw} \) are the corresponding points on the keel and bow, having identical abscissas, i.e. they are on the single frame.

The indicated generator is characterized for five parameters: \( \bar{r}^x \) and \( \bar{r}^{bw} \) are the endpoints of the arc found on the directing lines; \( \bar{r}^x \) and \( \bar{r}^{bw} \) are the tangent vectors, which determine the deadrise angle \( \alpha_d \) and the rebound angle \( \alpha_r \), respectively. The vector-integral \( \bar{I} \) is the parameter, which ensures the control of the hull shape without altering the deadrise and the rebound. The given vector determines the area bound by the arc of the cross-section.
In order to develop the surface with the necessary shape, each frame must be presented as a straight line. To present the quartic curve in the form of the straight line, we need to set the parameters of the frame into initial positions along the entire length of the directing lines.

At the point \( r^k_i \) we set the tangent vector \( r^k_i \) to the upcoming section, whereas at the point \( r^{bw}_i \) the tangent vector \( r^{bw}_i \), i.e. the point of vector intersections, we shift to the middle of the chord

\[
\frac{r^k_i - r^{bw}_i}{2}, \quad \frac{r^k_i - r^{bw}_i}{2}.
\]

Now we reduce the integral parameter to its original position. For this let us assign

\[
I_i = \frac{r^k_i + r^{bw}_i}{2}, \quad i = 0, \ldots, 13.
\]

We can now modify the wireframe surface, altering each frame separately and providing each \( I_i \) with an additional vector \( \Delta I_i \) and to the vectors \( r^k_i \) and \( r^{bw}_i \) we add the vectors \( \Delta r^k_i \) and \( \Delta r^{bw}_i \), respectively (Fig. 4).

If we interpolate the values \( I_i \) by the parameter \( x \), then we will not receive the ruled surface, since only the reference sections will have the straightforward form, and hence we need to interpolate the vector functions \( \Delta I_i \), \( \Delta r^k_i \) and \( \Delta r^{bw}_i \). For this purpose, let us use the same polynomial as in the case of the longitudinal lines.

The results are presented graphically in the form of scalar functions, or the components of the vector function (Fig. 5).
Finally, the equation for the bottom surface takes the form

\[ f(x, u) = f^x(u) + f_{bw}(x)f_2(u) + \left( f^x(x) + \Delta f^x(x)f_2(u) + f_{bw} + \Delta f_{bw}(x)f_3(u) \right)h + f(x) + \Delta f(x)f_5(u) \] (3)

where \( \bar{f}(x) = \frac{f^x + f_{bw}}{2} \), \( f_{2} = f_{bw} - f_{ee} \), \( f_{bw} = f_{ee} - f_{bw} \).

Figure 6 shows the final general view of the entire surface of the vessel. It may be possible to smoothly join the side and the bottom, and thus make them into a single smooth surface.

Let us examine the surface of the hull of the round-bilge vessel (Fig.7).
As is seen from figure 7, the cross-section of the surface may have a double curvature and an inflection point. The given line may be presented as the cubic curve. However, let us make it composite for the accuracy and convenience purposes (Fig. 8).

If we have two component parts, then as in the previous case, we will have three directing lines. In this case, the inner directing line can be called considered as bow only as conditional assumption. As a component of the vessel design, it is missing.

It is recommended the points rc across all the sections be selected on the assumed points of the inflection, if there are any.

The line consists of two arcs of a quartic parabola (1). Let us write the equations to these arcs with respect to Fig.8:

\[
\overline{r}_i^b(u) = \overline{r}_i^e(x)\phi_1(u) + \overline{r}_i^{bw}(x)\phi_2(u) + \left(\overline{r}_i^{ew}(x)\phi_3(u) + \overline{r}_i^{bw}(x)\phi_4(u)\right)h_k + \overline{I}_i^b(x)\phi_5(u) \tag{4}
\]

\[
\overline{r}_i^s(u) = \overline{r}_i^{bw}(x)\phi_1(u) + \overline{r}_i^{sl}(x)\phi_2(u) + \left(\overline{r}_i^{bw}(x)\phi_3(u) + \overline{r}_i^{sl}(x)\phi_4(u)\right)h_s + \overline{I}_i^s(x)\phi_5(u) \tag{5}
\]

where, \(\overline{r}_i^e\) is the upper arc; \(\overline{r}_i^b\) is the lower arc; \(\overline{r}_i^{ew}, \overline{r}_i^{bw}, \text{ and } \overline{r}_i^{sl}\) are the nodal points of the line; \(\overline{r}_i^{ew}, \overline{r}_i^{bw}\) and \(\overline{r}_i^{sl}\) are the tangents to the line at the nodal points; \(\phi_j(u), j=1...5\) are the basis functions (1), \(u \in [0,1]\) is the local parameter. The vectors \(\overline{I}_i^b\) and \(\overline{I}_i^s\) determine the deflection of the upper and lower arcs of the i-th section.

The design of the surfaces is to be conducted in line with the following rules:

The lines of the keel and the side are marked by the points. The points are interpolated by cubic splines [3, 4, 5, 9].

The cross-sections are pre-formed (Fig.7). A mid-point \(\overline{r}_i^{bw}\) is set over each section.

The mid-points of sections are also interpolated by the cubic splines. A system of equations solved at constructing the spline, ensure smoothness for the curve being not lower than the second order [3]. However, we must be careful in order to make the bow line smooth without any oscillations or undesired points of inflection. Taking into account these requirements, the points shift their positions. It is also important that the first derived equations in the points \(\overline{r}_i^e\) were equal left and right at each section.

The smooth vector functions \(\overline{r}_i^{ew}(x), \overline{r}_i^{bw}(x), \overline{r}_i^{sl}(x)\), \(\overline{I}_i^b(x)\) and \(\overline{I}_i^s(x)\) are composed and presented in the drawing by their scalar components (Fig.9).

To achieve the required characteristics for the engineering surface, its shape is changed by manipulating the functions \(\overline{I}_i^b(x)\) and \(\overline{I}_i^s(x)\).
The vectors $\overline{T}^b$ and $\overline{T}^s$ can be modified unrestricted, while maintaining the smoothness functions $\overline{T}^b(x)$ and $\overline{T}^s(x)$. Any intermediate cross-section will retain the first order smoothness which is sufficient for hydrodynamic properties of the surface. Thus, we can design surfaces with cross sections composed of a larger number of arcs.

3. Conclusion
The above considered mathematical parameters allow for the design of complex engineering surfaces on the basis of dependent sections, and at the same time efficiently alter the shape of the surface locally. These alterations do not require any recalculation for the entire surface. The algorithms based on the given design method are simple and easy to program. Using this method, we can simulate the aero- and hydrodynamic properties of surfaces, complex surfaces of buildings and structures under the interactive mode, while adding appropriate tools to the existing computer-aided design systems.

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