Landau-Zener Tunnelling in Waveguide Arrays

Ramaz Khomeriki
Department of Physics, Tbilisi State University, 3 Chavcha vadze avenue, Tbilisi 0128, Republic of Georgia

Stefano Ruffo
Dipartimento di Energetica “S. Stecco” and CSDC, Università di Firenze, and INFN, Via S. Marta, 3, 50139 Firenze, Italy

(Dated: November 8, 2018)

Landau-Zener tunnelling is discussed in connection with optical waveguide arrays. Light injected in a specific band of the Bloch spectrum in the propagation constant can be transmitted to another band, changing its physical properties. This is achieved using two waveguide arrays with different refractive indices, which amounts to consider a Schrödinger equation in a periodic potential with a step. The step causes wave “acceleration” and thus induces Landau-Zener tunnelling. The region of physical parameters where this phenomenon can occur is analytically determined and a realistic experimental setup is suggested. Its application could allow the realization of light filters.

PACS numbers: 42.82.Et; 42.25.-p; 03.65.-w

When a quantum system is subject to an external force, a non-adiabatic crossing of energy levels can occur. This phenomenon is known as Landau-Zener tunnelling and some of its recent observations are for Josephson junctions and optical effective two-level systems. On the other hand, the problem of quantum motion in a periodic potential was solved already in the 1920’s (see e.g.) and gives rise to band spectra and Bloch states. Nowadays, the observation of Landau-Zener tunnelling between Bloch waves is at the frontiers of research in Bose-Einstein condensates (BEC) in optical lattices. The external forcing, responsible for Landau-Zener tunnelling, is created by either placing the BEC in a gravitational potential or accelerating the optical lattice itself.

In this Letter we propose a new way of generating Landau-Zener tunnelling. We consider waveguide arrays, where the periodic potential of the Schrödinger equation is provided by the spatial oscillation of the refractive index in the transversal direction. Tunnelling is caused by combining two waveguide arrays with different refractive indices. As we will see, this corresponds to creating a step in a periodic potential.

For arrays of coupled waveguides, the longitudinal direction z (see Fig.1), along which the refractive index is constant, plays the role of “time” in the stationary regime. The refractive index varies only along the transversal direction x, which represents space in a 1 + 1 (space-“time”) dimensional picture. Various linear and nonlinear phenomena have been observed in waveguide arrays: discrete spatial optical solitons, diffraction management, excitation of Bloch modes, generation of multiband optical breathers and of single band-gap solitons, anomalous band-gap transmission regimes. Fast progress in discovering various nonlinear effects in waveguide arrays has been possible due to the introduction of the tight-binding approximation, which reduces the nonlinear Schrödinger equation to the discrete nonlinear Schrödinger equation.

However, such a reduction eliminates the rich band structure of the periodic medium and only a single Bloch band is left. On the contrary, we want to maintain the band structure and, hence, study transitions between the bands. Indeed, as we will see below, the coupling of two waveguides with different refractive indices introduces a step in the periodic potential and, consequently, Landau-Zener tunnelling. In the following we will study only transitions between the first two bands, denoting them upper and lower band. In particular, we propose to inject light into the left waveguide array with a given angle in order to populate the lower Bloch band, and retrieve it
We consider cases in which the acceleration constant \( \alpha \) is large. Thus, the tunnelling rate is close to one, meaning that almost all light intensity approaching the step is transferred to the upper band. This is confirmed by numerical simulations.

Outside the step, where acceleration is absent, one can write down the wave-functions and the dispersion relations in simple approximate form (see e.g. Ref. [10]).

\[
\Psi_{\pm} = \left[ \frac{\pm \sqrt{w^2 + 4 \kappa^2} - i x}{w} \right] e^{-i (\beta_{\pm} z + \kappa x)} \tag{7}
\]

where \( \kappa = K - 1 \) is the wavenumber detuning from the zone-boundary and the \( +(-) \) sign indicates the upper (lower) band. \( \beta \) is the dimensionless propagation constant. The dispersion relations (8) for the two bands are schematically shown in Fig. 3. The picture is similar to the one observed in experiments 17. Let us remark that at the zone-boundary \((\kappa \to 0)\), the amplitudes of the upper and lower band modes are \(|\Psi_+| = 2 \sin x\) and \(|\Psi_-| = 2 \cos x\), respectively. This means that the light intensity in the lower band is concentrated between the waveguides centers, while, in the upper band, intensity is concentrated on waveguides centers. This property is a clear experimental indication whether the wave is in the lower or upper band.

The experimental setup could be as follows. One should inject a lower band mode with non zero but small relative wavenumber \( \kappa \). This is accomplished by choosing for the light beam a direction forming an angle \( \theta \) with respect to the \( z \) direction such that \( \tan \theta = \kappa \). Hence, the wave front will move towards the step. An analysis of the dependence of the tunnelling mechanism on the physical parameters appearing in Fig. 3 shows that the transition
FIG. 3: Schematic band-gap structure and picture of the Landau-Zener tunnelling process. \( w \) is the gap between the bands (the height of the period potential), \( d \) is the width of the upper band and \( \kappa \) is the initial detuning of the lower band mode wavenumber from zone-boundary. \( \beta \) is the dimensionless propagation constant (see formula 8). \( A \) is the height of the step. Initially, light is injected in the left array, populating the lower band mode \( \text{(a)} \). Going across the step \( \kappa \) decreases, reaching zero as the mode approaches the zone-boundary causing Landau-Zener tunnelling \( \text{(b)} \). After tunnelling, most of the light intensity is transferred to the upper band mode \( \text{(c)} \). A smaller light intensity remains in the lower band and is observed in the left array \( \text{(d)} \).

To the upper band mode verifies only if the refraction index step \( A \) fulfills the following inequalities

\[
\Delta \beta + w < A < \Delta \beta + w + d,
\]

where \( \Delta \beta = \beta_{\text{initial}} - \beta_{\text{final}} \) is the variation of the propagation constant between the initial state and zone-boundary and \( d \) is the width of the upper band.

Let us try to justify this result by commenting at the same time the results of some numerical simulations. These are performed by fixing \( w = 0.5 \), \( \kappa = 0.2 \) and varying the step size \( A \). Waveguide centers are placed every period, with the first waveguide at half a period from the left boundary. The refractive index step is placed in the middle of the array. If the refractive index step \( A \) is within the above limits \( \text{[10]} \), the lower band mode cannot overcome the step and when it reaches the zone-boundary Landau-Zener tunnelling to the upper band occurs. Light is partially transmitted to the right in the upper band and reflected to the left in the lower band. This regime is demonstrated in Fig. 4 which evidentiates, even visually, the fact that light in the left array is concentrated in-between waveguides, while it lies at waveguide centers on the right array [see also the form of wavefunctions \( \text{[11]} \)]. If the step size is higher than the upper bound of Exp. \( \text{[9]} \), the lower band mode cannot overcome the step and total reflection takes place (see the upper graph of Fig. 5). On the other hand, if

\[
A < \Delta \beta
\]

the lower band mode is able to overcome the step and to penetrate to the right side without tunnelling (see the lower graph of Fig. 5). Indeed, wave intensity is now concentrated in-between the waveguides and one can conclude that only lower band modes are present in the array.

Finally, if the step is located within the following limits

\[
\Delta \beta < A < \Delta \beta + w,
\]

the wave on the left array has no counterpart on the left with the same propagation constant, thus, no stationary penetration of the light through the step is possible, more or less like in the upper Fig. 5.

Concluding, a novel type of linear optical tunnelling effect is discovered. This is discussed in connection with waveguide arrays, which have a spatially oscillating refractive index. We explain this effect resorting to Landau-Zener model, which is commonly used for accelerated quantum-mechanical two-level systems. In our case, tunnelling between different bands takes place while a wave passes through a step in the refractive index. In waveguide arrays, Bloch bands in the propagation constant (light wavenumber) play the role of energy bands in quantum mechanics. Numerical simulations show that, if certain limits in the refractive index step are respected, a spatial separation of light in different bands can be achieved. More interesting for applications is, perhaps, the use of this mechanism to build...
and D. Mandelik for useful suggestions. This work is funded by the contract COFIN03 of the Italian MIUR Order and chaos in nonlinear extended systems. R.K. acknowledges hospitality from Dipartimento di Energetica “S. Stecco” (Florence University, Italy), the Abdus Salam International Center for Theoretical Physics (Trieste, Italy) and financial support in the frame of CNR-NATO Senior fellowship award No 217.35 S.

![Graphs showing total reflection and penetration through a step](image)

**FIG. 5:** Upper graph: Total reflection from the step. Step size $A = 1$ is taken higher than the upper bound of Exp. (9). Lower graph: Penetration through the step without tunnelling. Step size $A = 0.07$ is taken within the step values form the inequality (10).

* Electronic address: khomeriki@hotmail.com
† Electronic address: ruffo@avanzi.de.unifi.it

[1] L.D. Landau, Phys. Z. Sowjetunion 2, 46 (1932).
[2] G. Zener, Proc. R. Soc. London, Ser. A 137, 696 (1932).
[3] K. Mullen, E. Ben-Jacob, Y. Gefen, and Z. Schuss, Phys. Rev. Lett., 62, 2543 (1989); S. Fishman, K. Mullen, E. Ben-Jacob, Phys. Rev. A, 42, 5181 (1990).
[4] D. Bouwmeester, N. H. Dekker, F. E. v. Dorselaer, C. A. Schrama, P.M. Visser, and J. P. Woerdman, Phys. Rev. A, 51, 646 (1995).
[5] C. Kittel, *Introduction to solid state physics*, 5th edition, Wiley (1976), Chapt. 7.
[6] B.P. Anderson and M. Kasevich, Science, 282, 1686 (1998).
[7] M. Cristiani, O. Morsch, J.H. Müller, D. Ciampini and E. Arimondo, Phys. Rev. A, 65, 063612 (2002).
[8] Qian Niu, Xian-Geng Zhao, G. A. Georgakis, and M. G. Raizen, Phys. Rev. Lett., 76, 4504 (1996).
[9] B. Wu, Q. Niu, Phys. Rev. A, 61, 023402 (2000).
[10] J. Liu, L. Fu, B.-Y. Ou, Sh.-G. Chen, D.-I. Choi, B. Wu, Q. Niu, Phys. Rev. A, 66, 023404 (2002).
[11] M. Jona-Lasinio, O. Morsch, M. Cristiani, N. Malossi, J. H. Müller, E. Courtade, M. Anderlini, and E. Arimondo, Phys. Rev. Lett., 91, 230406 (2003).
[12] D.N. Christodoulides, R.I. Joseph, Opt. Lett., 13, 794, (1988); Phys. Rev. Lett., 62, 1746, (1989).
[13] H. S. Eisenberg, Y. Silberberg, R. Morandotti, A.R. Boyd and J. S. Aitchison, Phys. Rev. Lett., 81, 3383 (1998).
[14] H. S. Eisenberg, Y. Silberberg, R. Morandotti and J. S. Aitchison, Phys. Rev. Lett., 85, 1863 (2000).
[15] D. Mandelik, H. S. Eisenberg, Y. Silberberg, R. Morandotti and J. S. Aitchison, Phys. Rev. Lett., 90, 053902 (2003).
[16] D. Mandelik, H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, Phys. Rev. Lett., 90, 253902 (2003).
[17] D. Mandelik, H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, Phys. Rev. Lett., 90, 253902 (2003).
[18] R. Khomeriki, Phys. Rev. Lett., 92, 063905 (2004).
[19] A. A. Sukhorukov, Yu. S. Kivshar, O. Bang, and C. M. Soukoulis, Phys. Rev. E, 63, 016615 (2001).
[20] A. Smerzi and A. Trombettoni, Phys. Rev. A, 68, 023613 (2003).
[21] M. Öster, M. Johansson, A. Eriksson, Phys. Rev. E, 67, 056606, (2003).
[22] A.A. Sukhorukov, Yu.S. Kivshar, J. Opt. Soc. Am. B, 19, 772, (2002).