1. INTRODUCTION

During the last years, the technology has achieved to miniaturize electronic devices to nanometer scales. In order to understand the physics of these devices, it was developed a new field of research: the mesoscopic physics [1]. There is a double motivation to study these systems: On one hand, it is the suspicion that could be possible to construct new devices that give place to a new electronic based on the principles of mesoscopic physics [2]. On the other hand, the electrons in such structures are very confined and the electron-electron interaction is very important [3]. For this reason, these devices have been excellent tools to probe correlation effects in strongly interacting systems.

It has been observed experimentally that a conductor of mesoscopic dimensions acts as a small pool where electrons are confined and where they enter one by one [4]. These conductors have received the name of Quantum Dots (QDs) [5]. A QD is an artificial device which typically consists of a small region in a semiconductor material with a size of approximately 100 nm. Due to its small size, the allowed energies for the electrons are quantized, forming a discrete spectrum of quantum states. In a typical transport experiment with QDs, it is observed a peak in the current every time one electron is added to the QD [6].

The QD is one of the central objects of this work, the other one is the mesoscopic
ring. These rings are constructed with metallic or semiconductor materials and they can be made of sizes of the order of 2500-5000 Å. If one of these rings is threaded by a constant magnetic flux, a current appears in the ring. This current has a periodic behavior as a function of the applied flux and exists in the ground state of the system [7]. Due to this property, this current has received the name of persistent current.

The system studied in this work consists of a QD inserted in a mesoscopic ring threaded by a magnetic flux [8,9]. Our aim is to present a complete description for this device and to predict the physics of an experiment with these features.

The present article is organized as follows. In section 2 we present the model and the methodology we have developed to solve it. By means of that methodology, we have found patterns of behaviour due both to electron-electron interaction and to the confinement of the QD as we explain in section 3. Finally, in section 4 we present our conclusions.

2. MODEL

To solve the ring with the QD, we separate it into two parts: one of them is a small cluster of atoms that contains the QD (see Figure 1) and the other one is a chain without interactions with a larger number of sites. Both systems are solved separately and then, they are connected to form the whole system of the QD inserted in the ring (see Figure 2). In first place, we obtain the Green’s Function (GF) for both systems separately. In second place, we find the GF of the ring with the QD (G), by solving the Dyson’s equation $\hat{G} = \hat{g} + \hat{g}\hat{T}\hat{G}$, where $\hat{g}$ is the GF of one of the systems and $\hat{T}$ is the hopping operator that connects both systems [10,11]. This operator depends on the flux, because the magnetic flux is incorporated as a phase factor in the boundary condition of the ring.

We will consider that the QD has two levels: $\alpha$ and $\beta$, which are connected to...
the rest of the system by means of hopping parameter $t'$ (see Figure 1). To model the QD, this hopping probability for the electrons to hop to the QD, will be lower than the hopping probability to hop to other sites of the system ($t$). Although the Coulomb interaction is present in the whole system, we will assume it to be restricted to the QD where, due to quantum confinement, electrons interact more strongly.

In order to reproduce some experiments, we will apply a gate voltage $V_o$ to the QD. By means of this voltage, we will be able to move the levels of the QD and study the behavior of the system as a function of $V_o$.

With this considerations in mind, the Hamiltonian of the cluster (with $N_s$ sites) can be written as sum of three terms: $\hat{H} = \hat{H}_o + \hat{H}_U + \hat{H}_T$. $\hat{H}_o$ represents essentially the kinetic energy:

$$\hat{H}_o = -t \sum_{\sigma, i=1}^{N_s} \left[ \hat{c}_{i+1,\sigma}^\dagger \hat{c}_{i,\sigma} + \hat{c}_{i,\sigma}^\dagger \hat{c}_{i+1,\sigma} \right] + V_o \sum_{\sigma, j} \hat{n}_{j,\sigma} + \sum_{\sigma, j} \epsilon_j \hat{n}_{j,\sigma}$$

where $\hat{c}_{i,\sigma}^\dagger$ ($\hat{c}_{i,\sigma}$) are the usual creation (annihilation) fermionic operators ($\sigma \equiv \uparrow, \downarrow$), $t$ is the hopping parameter between successive sites, $\epsilon_j$ are the energies of the levels of the QD (in this work we will take $\epsilon_\alpha = 0$ and $\epsilon_\beta = 2t$), and $\hat{n}_{j,\sigma}$ is the number operator.
The Coulomb interaction inside the QD is described by
\[ \hat{H}_U = U \sum_{i=\alpha,\beta} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + U \sum_{\sigma=\uparrow,\downarrow} \left[ \hat{n}_{\alpha,\sigma} \hat{n}_{\beta,-\sigma} + \hat{n}_{\alpha,\sigma} \hat{n}_{\beta,\sigma} \right] \]
where \( U \) represents the coulomb energy whenever there are two electrons inside the QD.

Finally, the connection between the QD and other sites in the cluster is described by
\[ \hat{H}_T = -t' \sum_{\sigma=\uparrow,\downarrow} \left[ \hat{c}^\dagger_{\alpha,\sigma} \hat{c}_{\alpha,\sigma} \; \hat{c}^\dagger_{\beta,\sigma} \hat{c}_{\beta,\sigma} + \hat{c}^\dagger_{\beta,\sigma} \hat{c}_{\beta,\sigma} \; \hat{c}^\dagger_{\alpha,\sigma} \hat{c}_{\alpha,\sigma} \right] + c.c. \]
where \( t' \) is the probability for the electrons to hop to the QD.

To take the charge fluctuations inside the cluster into account, we write the GF of it, \( \hat{g} \), as a combination of the GF of \( n \) and \((n+1)\) particles with corresponding weights \((1-f)\) and \(f\), that is \( \hat{g} = \hat{g}_n (1-f) + \hat{g}_{n+1} f \). The charge inside the cluster is written as \( Q_c = (1-f)n + f(n+1) \), but it can also be expressed as \( Q_c = \int_{-\infty}^{\infty} \sum_i N_i \text{Im} G_{ii}(w) dw \). These last equations are solved self-consistently in order to obtain \( f, n \) and the GF of the system \( \hat{G} \). Once we get it, we can easily compute the density of states (DOS), the persistent current in the ring and the charge inside the QD \([10,11]\).

3. PATTERNS OF BEHAVIOUR IN TRANSPORT

We have found several patterns of behavior which are evidences that different physical phenomena are involved in the system. We have found that we can go from one regime to another either by diminishing the interaction or by increasing the hybridization between the QD and the ring. In one extreme, we have found the Coulomb Blockade regime and in the other one, we have found a quasi non interacting regime. Between these two limits, interesting intermediate regimes appear where the physics is essentially dominated by the Kondo effect. In the following, we will explain in detail every pattern.

3.a Coulomb Blockade regime

In this regime, the persistent current and the charge inside the QD as a function of the gate voltage, looks like in Figure 3.a and Figure 4.a, respectively. We can observe a peak in the current every time an electron goes into the QD and that there is a range of values of the gate voltage where we have no transport.

In order to understand this result, it is useful to take into account the local DOS at the QD. If we consider a value of gate potential such that there is one particle inside the QD in the level \( \epsilon_\alpha \), we will have two peaks in the DOS: one at \( \epsilon_\alpha \) and the other one at \( \epsilon_\alpha + U \) (indicating that to add another particle it is necessary an amount of energy \( U \)). These peaks in the DOS have a finite width \( \Delta \) and we have
found that those peaks will broaden out if the hybridization between the QD and the ring (tuned by $t'$) is increased [11].

With this considerations in mind, we can understand the graphs of the persistent current and the charge. Suppose we begin the process with a value of gate voltage in such a way that the lowest level of the QD is aligned with the Fermi level of the system. In this situation the transport is allowed, so we have current and one electron entering into the QD, as actually occurs in Figure 3.a and Figure 4.a at zero gate voltage.

Figure 3. Persistent current in the ring as a function of the gate voltage applied to the QD for the different patterns of behaviour. (a) Coulomb blockade regime. (b) Coulomb blockade-Kondo effect regime. (c) Kondo effect regime. (d) Quasi non-interacting regime. The gate voltage is normalized to $V_o/U$ to better compare among different graphs.
Figure 4. Charge inside the dot as a function of the gate voltage (normalized to $V_0/U$) for the different patterns of behaviour (a) Coulomb blockade regime. (b) Coulomb blockade-Kondo effect regime. (c) Kondo effect regime. (d) Quasi non-interacting regime.

If we lower the gate voltage, we will move the levels downwards, so there will be no states of the QD in resonance with the Fermi level in the ring. If this occurs, the transport is blocked, and we will have no current. This phenomena is known as Coulomb blockade [12]. This situation will persist during an interval of gate voltage of the order of $U$, and after that, the second level appears and the second particle goes into the QD.

With regard to the parameters of the model, we have found this regime when correlations are much more greater than the width of the levels ($\Delta$) due to hybridization.
3.b Kondo effect - Coulomb blockade regime

By diminishing the value of the interaction $U$ - maintaining it greater than $\Delta$-we find a pattern where Kondo effect and Coulomb blockade coexist [10].

In this regime, we can note in Figure 3.b an excess of current between the Coulomb Blockade peaks that indicate the entrance of particles inside the QD. This behaviour can be easily understood if we note that in the DOS, a peak at the Fermi level appears, whenever we have an odd number of electrons inside the QD [10]. This peak is evidence of the Kondo effect which appears in metals with magnetic impurities and it occurs because the spin of the magnetic impurity is coupled anti-ferromagnetically with the spin of the conduction electrons. It shows up as a resonance in the Fermi level, which is usually called Kondo resonance (KR) [13]. On the other side, when there is an even number of electrons, the resonance at the Fermi level disappears.

When we have an odd number of electrons, there is a net spin inside the QD, so it behaves as a magnetic impurity whose spin can be coupled anti-ferromagnetically with the spins of the electrons in the ring. In this situation, the QD behaves as a magnetic impurity and we have Kondo effect. The Kondo resonance at the Fermi level gives a new channel for the transport and it is responsible of the excess of current between the CB peaks. On the other side, when we have an even number of electrons, we have no net spin inside the QD, so it is no possible to have Kondo effect. The fact that a QD could exhibit Kondo effect has been predicted theoretically [14] and it has been observed in recent experiments [15].

3.c Kondo effect regime

If we diminish $U$ or increase $t'$ again, we get the pure Kondo effect regime. A typical graph for the persistent current in this regime can be seen in Figure 3.c.

It is interesting to point out that if we increase $t'$ or diminish $U$ in such a way that the parameter $J = \frac{4t'^2}{U}$ remains constant, we observe the same behaviour for the persistent current in the ring. This feature can be observed in Figure 5. The parameter $J$ is essentially the coupling between the spin of the QD and the electrons in the ring. In other words, we have found that the parameter $J$ governs the whole physics in this regime, and it occurs because in the present regime we have the pure Kondo effect.

In a recent work [10], we have found that the regime where the Kondo effect coexists with the Coulomb blockade, the KR is very narrow and as a consequence, the persistent current presents an scaling with the length of the ring ($L$) as $\frac{1}{\sqrt{L}}$. In the present regime the KR widens, so the current presents the usual $\frac{1}{L}$ scaling of a perfect ring. However, as the Kondo temperature can be roughly estimated as the width of the KR [13], we can conclude that this temperature will be greater than that of the previous regimes.

In Figure 4.c we can observe that the charge begins to lose its discreteness, because charge enters into the QD in a almost continuous way but it is still entering one by one.
Figure 5. Persistent current as a function of the gate voltage in the pure Kondo effect regime. The ring is constructed connecting a 6-sites cluster with a chain of 2000 sites. The dash line is the current for $U = 2t$ and $t' = 0.16t$ and the circles correspond to $U = 0.5t$ and $t' = 0.08t$. In both cases the value of the coupling parameter is the same, that is: $J = 0.0512t$.

Regarding the parameters of the model, this regime appears when the coupling $J$ is lower than the width of the peaks in the local DOS [11]. That is, $J = \frac{4t'^2}{U} < \Delta$.

3.d Quasi non-interacting regime

For lower values of $U$, we find the quasi non-interacting regime with a typical curve for the persistent current as in Figure 3.d.

If $U \ll \Delta$, the peaks in the DOS merge and we get one doubly degenerated peak. In this regime, the KR is absent because the charge inside the QD changes in a continuous way from 0 to 2 particles (as can be seen in Figure 4.d). So, we can say that the system behaves essentially as if it were non-interacting.

4. CONCLUSIONS

In the present work we have studied the transport properties of QD inserted in
a mesoscopic ring which is threaded by a magnetic flux.

We have proposed a model that takes into account the conditions that are usual in experiments with QDs [8,9]. In order to solve the model, we develop a methodology to find the Green’s functions of the system by means of proper approximations.

This system presents persistent currents as a function of a gate voltage applied to the QD. We have studied the influence of both the interaction between electrons and the hybridization between the ring and the QD. We have found several regimes that describe different physical phenomena involved in the system. These regimes range from the phenomenon of Coulomb Blockade (in the high correlation limit) to a quasi non-interacting regime. Between these two limits we have found an intermediate regime where the Kondo effect shows up. Similar results has been recently reported in other systems [16].

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