Lensing of Stars by Spherical Gas Clouds
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Received 1998 July 13; accepted 1998 October 15; published 1998 October 29

Abstract
Cold gas clouds of ~Jovian mass and ~AU size will act as lenses for optical light. If the Galaxy contains ~10^{11} M⊙ in such clouds, background stars will be magnified at a detectable rate. The resulting light curves can resemble those due to gravitational lensing by a point mass, raising the possibility that some events attributed to gravitational lensing might instead be due to “gaseous lensing.” During a lensing event, the lens would impose narrow infrared and far-red H2 absorption lines on the stellar spectrum. Existing programs to observe gravitational microlensing, supplemented by spectroscopy, can therefore be used to either detect such events or place limits on the number of such gas clouds present in the Galaxy.

Subject headings: galaxies: halos — galaxies: ISM — Galaxy: halo — ISM: clouds — ISM: molecules

1. INTRODUCTION
It has been proposed that the Galaxy contains a population of small, self-gravitating gas clouds, in numbers sufficient to contribute an appreciable fraction of the gravitational mass of the Galaxy (Pfnenninger, Combes, & Martinet 1994; De Paolis et al. 1996; Gerhardt & Silk 1996; Combes & Pfnenninger 1997). Walker & Wardle (1998, hereafter WW98) pointed out that if such clouds existed in the Galactic halo, each would have an ionized envelope that could explain the extreme scattering events (ESEs; Fiedler et al. 1994) during which extragalactic point radio sources apparently are amplified and deamplified as a plasma “lens” moves across the line of sight. WW98 argued that the observed frequency of ESEs could be explained by a halo population of ~10^{14} clouds, with mass M ≈ 10^{-3} M⊙ and radius R ≈ 3 AU.

Gerhardt & Silk (1996) and WW98 noted that if the clouds were opaque, existing stellar monitoring programs that study gravitational “microlensing” (Paczynski 1996 and references therein) would have detected occultations of duration ~R/200 km s^{-1} ≈ 40 days. Such events are not reported. However, the clouds could be transparent at optical wavelengths: any grains present could have sedimented to form a small core, and the clouds might be of primordial composition.

In this Letter, we point out that even transparent clouds would have observable lensing effects. Stellar monitoring programs can thus test the hypothesis that cold gas clouds contribute an appreciable fraction of the Galactic mass.

2. GASEOUS LENSING
Let us consider nonrotating H2-He polytropes of radius R. The polytropic index n (T ∝ ρ^{1/n}) is assumed to be in the range 1.5 ≤ n ≤ 5; for n < 1.5, the cloud would be convectively unstable, while for n ≥ 5, the central density is infinite. Table 1 gives the half-mass radius r_h, T_r ≡ T(0), T_i ≡ T(r_i), and ρ_i ≡ ρ(0) and ρ_r ≡ ρ(r_i) relative to ρ(0) ≡ 3M/4πR^3.

The refractive index n = 1 + α(λ)p, with α(4400 Å) = 1.243 cm^3 g^{-1} and α(6700 Å) = 1.214 cm^3 g^{-1} for H2-He gas with 24% He by mass (AIP Handbook 1972, Table 6e-5). For small deflections, a light ray with impact parameter b will be deflected through an angle

\[ \phi(b) = -2ab \int_b^\infty \frac{dp}{(r^2 - b^2)^{1/2}}. \]

The additional gravitational deflection (Henriksen & Widrow 1995) is negligible compared with gaseous refraction. Let D_{Sl} and D_{Lo} be the distance from source to lens and from lens to observer, respectively. If b_0 is the distance of the lens center from the straight line from source to observer (see Fig. 1), then the apparent distance b of the image from the lens is

\[ b = b_0 + D\phi(b), \quad D = \frac{D_{Sl}D_{Lo}}{D_{Sl} + D_{Lo}}. \]

For a point source, the image magnification is given by

\[ M(b) = \frac{|b|}{b_0 - 1 - D\phi(b)}. \]

\[ \phi'(b) = -2\alpha \int_0^b dz \frac{dz}{r^2} \left( \frac{b^2 + z^2}{r^2} \right)^{1/2} \]

where r^2 = b^2 + z^2. There will be an odd number N(b_0) of solutions b_i(b_0), i = 1, ..., N. The total amplification \( A(b_0) = \sum_{i=1}^N M(b_i) \). The trajectory of the lens relative to the source is characterized by an impact parameter p and a displacement x along the trajectory; b_0 = (p^2 + x^2)^{1/2}, and the “light curve” is just A(b_0) versus x.

We define a “strength” parameter

\[ S = \frac{\alpha(\rho)D}{R} = 0.36 \frac{M}{10^{-11} M⊙} \frac{\text{AU}}{10^4} \frac{D}{10 \text{ kpc}}. \]

When S is of order unity, large amplifications (and deamplifications) are possible even for b_0/R of order unity, whereas if S ≪ 1, appreciable amplification occurs only for b_0 ≪ R.
TABLE 1

| n   | ρ₀/(ρ) | ρ₀/(ρ₀) | r₀/R  | T₁⁺ | T₁⁻ | S₁ |
|-----|--------|---------|-------|-----|-----|-----|
| 1.5 | 5.9907 | 2.3228  | 0.52118 | 13.19 | 7.52 | 0.026 |
| 2   | 11.403 | 3.5602  | 0.43921 | 14.74 | 7.81 | 0.0103 |
| 2.5 | 23.407 | 5.9809  | 0.36004 | 17.14 | 8.38 | 0.0037 |
| 3   | 54.183 | 11.416  | 0.28331 | 20.93 | 9.29 | 0.0011 |
| 3.5 | 152.88 | 26.539  | 0.20879 | 27.45 | 10.76 | 0.00028 |
| 4   | 622.41 | 88.183  | 0.13650 | 40.80 | 13.32 | 3.9 × 10⁻⁵ |
| 4.5 | 6189.5 | 703.43  | 0.06671 | 81.60 | 19.31 | 2 × 10⁻⁶ |

*In kelvins, for M = 10⁻³ M⊙ and R = 10 AU.

3. SAMPLE LIGHT CURVES

For each polytropic index, we define S₁(n) (see Table 1) to be the value of S such that for b₁ → 0, there are three images for S > S₁, and one image for S < S₁. For each case with S > S₁, we define the critical impact parameter b₁(S) such that there is one image for b₁ > b₁ and three images for b₁ < b₁. For S > S₁, a light curve for which p < b₁ will have “caustics” at the points where b₁(b₁), as two additional images appear (for |x| decreasing) or merge and disappear (for |x| increasing).

Figures 2, 3, and 4 show light curves for lenses with polytropic indices n = 2, 3, and 4, respectively. The following general behavior is found. (1) For S ≤ 0.3S₁(n), the peak amplification A(0) → 2; (2) for 0.7S₁(n) ≤ S < S₁(n), A(0) ≥ 1, even though only one image is formed; (3) for S₁(n) ≤ S ≤ 5S₁(n), A(0) ≥ 1, and light curves with p < b₁ have conspicuous caustics where A → ∞ (for a point source); and (4) for S ≥ 10S₁(n), A(0) ≥ 1, but b₁ ≈ R, so the two merging images are faint except very near the caustic singularity, which is therefore of reduced importance for sources of finite angular extent.

In each figure, we also show, for comparison, an ideal “gravitational lensing” light curve fitted to the light curve for p = 0.01R. Note that for S < S₁(n) (no caustics) or S ≥ 10S₁(n) (b₁ ≈ 0.7R, caustics not important), there is considerable similarity between the gravitational lensing and gaseous lensing light curves.

4. DISCUSSION

For illustration, we consider the n = 3 polytropic model (with ρ₁ = 54(ρ)). For S ≥ 0.5S₁(n), we obtain amplifications A(p) > 1.5 for p ≤ 0.1R (see Fig. 3).

Fig. 1.—S = source, L = center of the lens, and O = observer.

Fig. 2.—The light curves, labeled p/R, are for transparent n = 2 polytropes with lensing strength S. An ideal “gravitational lensing” light curve (dashed line) fitted to the light curve for p = 0.01R is also shown.

4.1. Lensing Event Rate

Fiedler et al. (1994) report nine ESEs in 594 source yr of monitoring. Two events (0954 + 658 and 1749 + 096, with durations ∼0.35 yr) have radio light curves suggestive of lensing by the ionized atmosphere of a spherical cloud, implying a rate P_ESE ≈ 2/594 ≈ 3 × 10⁻⁴ yr⁻¹ per source, and covering fraction f_ESE ≈ P_ESE × 0.35 yr ≈ 1 × 10⁻⁴. From the 0.35 yr duration, we estimate R ≈ 200 km s⁻¹ × 0.35 yr/2 ≈ 7 AU.

If the plasma envelope extends to ∼2R, the neutral cloud covering factor is f ≈ 2f_ESE ≈ 5 × 10⁻⁴, and the rate per source

Fig. 3.—Same as Fig. 2, but for n = 3. When S > S₁(n), light curves with p < b₁(S) contain caustics where A → ∞ (see text); the A(s) light curves shown here remain finite because of finite sampling.
of \( p < 0.1R \) optical lensing events is

\[
\dot{P}_{\text{OL}} \approx (0.1R/2^{1/2}R)\dot{P}_{\text{ES}} \approx 2 \times 10^{-4} \text{ yr}^{-1}. \tag{6}
\]

The optical light curve would have a characteristic timescale \(~0.2R/(200 \text{ km s}^{-1})\) \( \approx 10 \) days, which is short compared with the observed lensing events toward the LMC (Alcock et al. 1997) but within the detectable range: with an effective exposure of \( \sim 2 \times 10^6 \) source yr, the absence of \( \sim 10 \) day events exceeding the \( A = 1.75 \) MACHO threshold implies an upper limit of \( \sim 1.5 \times 10^{-6} \) yr \(^{-1} \), which is 100 times smaller than \( \dot{P}_{\text{OL}} \) from equation (6)! We can therefore exclude the possibility that the typical cloud can produce amplifications \( A \geq 2 \): gas clouds associated with ESEs must have \( S \leq 0.3S_r \) for \( D \approx 10 \) kpc. Thus,

\[
M \leq 8.5 \ M_\odot (R/10 \text{ AU})^2(10 \text{ kpc}/D)S_r. \tag{7}
\]

4.2. Other Limitations

The total mass in clouds should not exceed \( \sim 10^{11} \ M_\odot \):

\[
M \leq 1 \times 10^{-3} \ M_\odot (5 \times 10^{-4}/f) \times (R/10 \text{ AU})^2(10 \text{ kpc}/D_{\text{LS}})^2. \tag{8}
\]

\( \dot{P}_{\text{OL}} \) and \( f \approx 5 \times 10^{-4} \) are estimated from only two ESEs and hence are quite uncertain; \( D_{\text{LS}} \) is also uncertain, so that the limits (7) and (8) are uncertain by perhaps a factor of \( \sim 10 \).

A third requirement is that the gravitational binding energy of gas at the surface exceed the thermal energy for plausible surface temperature \( T \approx 10 \) K:

\[
M \geq 6 \times 10^{-4} \ M_\odot (R/10 \text{ AU}). \tag{9}
\]

Conditions (7)–(9) are plotted in Figure 5. We see that the parameters favored by WW98—\( M = 10^{-3} \ M_\odot \) and \( R = \)

![Figure 4](image1.png)

**Fig. 4.**—Same as Fig. 2, but for \( n = 4 \)

![Figure 5](image2.png)

**Fig. 5.**—Allowed values of \( M \) and \( R \), for gas clouds at \( D \approx 10 \) kpc. Below the line \( T_c = 10 \) K, the cloud surface would be unbound for \( T = 10 \) K. Above the line \( S = 0.5S_r \) the cloud would be capable of amplification \( A \geq 2 \), and hence this region is ruled out by searches for optical lensing toward the LMC. Above the line of \( M_{\text{tot}} = 10^{11} \ M_\odot \), clouds with a covering fraction \( f = 5 \times 10^{-4} \) would contribute more than \( 10^{11} \ M_\odot \).

3 AU—are ruled out by equation (8). However, an allowed region does remain, including \( M \approx 10^{-3} \ M_\odot \) and \( R \approx 10 \) AU (black dot in Fig. 5).

4.3. Demagnification

For \( p > b_{\text{in}} \) (no caustics), light curves for spherical gaseous lenses bear considerable similarity to light curves for gravitational lensing. As a consequence, it would not be trivial to distinguish gravitational lensing from gaseous lensing purely on the basis of optical light curves, particularly since departures from the ideal gravitational lensing of point sources by point masses are anticipated because of blending with other stars, lensing by stars with companions, and sources of finite angular extent. The most conspicuous difference in the light curves is the fact that gaseous lensing always has a range of \( b_{\text{in}} \) for which \( A < 1 \). For lenses with \( S \ll 1 \), the demagnification is very slight: for example, for \( n = 3 \) and \( S < S_r, A > 0.88 \). Very accurate photometry would be required to detect such demagnification.

4.4. Spectroscopy

The cold molecular gas will be nearly transparent at infrared and optical frequencies but could be detected through \( \text{H}_2 \) absorption lines. The characteristic \( \text{H}_2 \) column density is

\[
N(\text{H}_2) \approx \frac{M R^2}{2.64 m_{\text{H}}} \approx 2 \times 10^{25} \times \frac{M}{10 \ M_\odot} \left( \frac{10 \text{ AU}}{R} \right)^2 \text{ cm}^{-2}. \tag{10}
\]

The \( \text{H}_2 \) will be mainly in \((v, J) = (0, 0)\) and \((0, 1)\). Quadrupole transitions out of these levels are listed in Table 2, with line-center absorption cross sections \( \sigma_0 \) computed by assuming only thermal broadening at \( T = 20 \) K. For the \( N(\text{H}_2) \) in equation

(10), the 0–1 transitions would have optical depths $\tau_v \approx 10^2$, but the increased stellar brightness near $\sim 8300 \, \text{Å}$, plus sensitive CCD detectors, may make the 0–3 transitions the best to use. Detection of these absorption features during a lensing event would both confirm the gaseous nature of the lens and determine its radial velocity, expected to be $\sim 200 \, \text{km s}^{-1}$ if the lens is in the halo.

### 4.5. Chromaticity

The optical dispersion results in a slightly larger value of $S$ in the blue. However, noting the similarity in light curves for $S = 0.01$ and $S = 0.001$ for $n = 3$, it is clear that changing $S$ by 0.8% will be expected to produce only a slight increase in $A(0)$ (“blueing”). However, caustics, if present, will occur at different times for different colors, which could be detectable with multicolor photometry with good time resolution.

The increased amplification at shorter wavelengths is counteracted by Rayleigh scattering, which will redden light passing through the cloud (WW98). The Rayleigh scattering cross section is $\sim 8.4 \times 10^{-27} (\mu m^2) \lambda^2$, resulting in reddening of a magnified star by $E(B - V) \approx 1.3 \times 10^{-27} N(H_2) \, \text{cm}^{-2}$, giving $E(B - V) \approx 0.03$ for $N(H_2)$ from equation (10).

The gaseous lens would be opaque for $\lambda \approx 1150 \, \text{Å}$ because of the damping wings of the Lyman band transitions of $H_2$, which overlap to form an opaque continuum shortward of $\sim 1110 \, \text{Å}$ for $N(H_2) \approx 10^{23} \, \text{cm}^{-2}$ (Draine & Bertoldi 1996). Since most target stars are faint in the vacuum ultraviolet, it seems unlikely that this absorption could be observed.

### 5. Conclusions

It is not obvious how cold, self-gravitating gas clouds, with the properties suggested by WW98, might have formed, or whether they would be stable for $\sim 10^{10} \, \text{yr}$. If they do exist, WW98 show that they could help solve two long-standing problems: (1) their ionized envelopes could account for some of the ESEs, and (2) they could contain the “missing” baryons in the Galaxy. It is notable that these same clouds could ameliorate a third problem: the fact that microlensing searches detect a larger number of amplification events toward the LMC than expected for lensing by stars and stellar remnants. Some of these events could be due to gaseous lensing. Indeed, lensing by the hypothesized clouds would be so frequent that existing gravitational microlensing searches can already place limits on the cloud parameters (see Fig. 5), but clouds with $M \approx 10^{-3} \, M_\odot$ and $R \approx 10 \, \text{AU}$ are still allowed. With a predicted lensing rate $P_{\odot} \approx 2 \times 10^{-4} \, \text{yr}^{-1}$, the typical lens must be weak, with $S \lesssim 0.3S_*$. Occasional gaseous lensing events with $S \approx 0.5S_*$ could perhaps account for some of the events attributed to gravitational microlensing.

For noncaustic lensing, the dispersion would produce slightly larger amplification in the blue, but this is counteracted by Rayleigh scattering. A number of quadrupole lines of $H_2$ could be detectable in absorption during the lensing event; this would be an unambiguous signature of “gaseous lensing.”

Existing programs to observe gravitational lensing, supplemented by spectroscopy during lensing events, can therefore be used to either detect gaseous lensing events or place limits on the number of $\sim 10^{-3} \, M_\odot$ $H_2$ clouds in the Galaxy.

I am grateful to B. Paczyński, J. Wambsganss, and an anonymous referee for helpful comments and to R. H. Lupton for availability of the SM package. This work was supported in part by NSF grant AST-9619429.

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