Non-Uniform Cosmological Birefringence and Active Galactic Nuclei

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Cosmological birefringence, a rotation by an angle \( \alpha \) of the polarization of photons as they propagate over cosmological distances, is constrained by the cosmic microwave background (CMB) to be \( |\alpha| \lesssim 1^\circ \) (1σ) out to redshifts \( z \approx 1100 \) for a rotation that is uniform across the sky. However, the rotation angle \( \alpha(\theta, \phi) \) may vary as a function of position \((\theta, \phi)\) on the sky. Here I discuss how a position-dependent rotation can be sought in current and future AGN data. An upper limit \( \langle \alpha^2 \rangle^{1/2} \lesssim 3.7^\circ \) to the scatter in the position-angle-polarization offsets in a sample of only \( N = 9 \) AGN already constrains the rotation spherical-harmonic coefficients to \((4\pi)^{-1/2}\alpha_{lm} \lesssim 3.7^\circ \) and constrains the power spectrum for \( \alpha \) in models where it is a stochastic field. Future constraints can be improved with more sources and by analyzing well-mapped sources with a tensor-harmonic decomposition of the polarization analogous to that used in CMB polarization and weak gravitational lensing.

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Introduction. There is a very active quest to understand dark energy \([1]\), and quintessence models \([2]\) provide a promising set of effective theories. A pseudo-Nambu-Goldstone field provides an attractive quintessence candidate, and such a field should have a coupling to the Chern-Simons term of electromagnetism \([3]\). This coupling gives rise to cosmological birefringence (CB), a frequency-independent rotation by some angle \( \alpha \) of the linear polarization of photons as they propagate over cosmological distances \([2, 4]\). In the simplest models, the rotation angle \( \alpha \) is uniform across the sky, in which case CB gives rise to parity-violating TB and EB correlations in cosmic microwave background (CMB) maps \([3]\). Null searches for such correlations now constrain the rotation over the redshift range \( 0 < z \lesssim 1100 \) to be \( \alpha = -0.25^\circ \pm 0.58^\circ \) \([6]\).

However, several recent papers have introduced quintessence models in which the rotation angle \( \alpha(\theta, \phi) \) varies as a function of position \((\theta, \phi)\) on the sky \([7]\) and similar phenomena may arise in some dark-matter models \([8]\). Refs. \([2]\) have described how to measure this rotation angle, as a function of position on the sky, with the CMB, but the algorithm has not yet been applied to data. The WMAP satellite should be sensitive (at 1σ) to spherical-harmonic coefficients of the rotation as small as \((4\pi)^{-1/2}|\alpha_{lm}| \sim 2.3^\circ\) for \( l \lesssim 400 \), and the recently launched Planck satellite should reach \((4\pi)^{-1/2}|\alpha_{lm}| \sim 0.07^\circ\) for \( l \gtrsim 800 \) \([6]\).

Radio \([4, 10, 14]\) and UV \([13, 16]\) data on active galactic nuclei (AGN) can also be used to search for CB. AGN are often elongated and polarized. While AGN may be complicated objects, symmetry considerations suggest that on average the mean polarization should be aligned or perpendicular to the position angle of the source. CB would, by rotating the polarization, give rise to a nonzero mean offset between the position angles and polarizations measured in a large number of sources \([4, 10, 11]\). Likewise, if a detailed map of the intensity and polarization of an individual source can be made \([12, 13, 17]\), then on average, the intensity gradients and polarization within that source should be aligned or perpendicular, if there is no CB. This more detailed analysis may allow competitive, or even stronger, constraints to \( \alpha \) from a smaller number of sources.

One precisely imaged radio source (3C 9) at a redshift \( z \approx 2 \) constrains \( \alpha = 2^\circ \pm 3^\circ \) out to this distance \([13, 17]\). A stronger bound, \( \alpha = -0.6^\circ \pm 1.5^\circ \), can be obtained from a larger number of well-mapped radio sources, but only at smaller redshifts \([3]\). A recent UV sample \([16]\) of eight AGN at redshifts \( z \gtrsim 2 \) constrains \( \alpha = -0.7^\circ \pm 2.0^\circ \). In this Letter, I show that AGN can be used to constrain the multipole moments \( \alpha_{lm} \) for a non-uniform rotation.

There was a brief flurry in the 1990s of searches for a rotation with a dipole dependence on position \([11–14]\), following a claimed detection \([10]\). Here I revisit and update such measurements and generalize to higher-\( l \) moments. I search the recent UV data \([16]\), combined with the radio constraint from 3C 9 \([13]\), and find no evidence for any rotation with a dipole or quadrupole dependence on position. I constrain the \( \alpha_{lm} \) (for any \( l \)) to \((4\pi)^{-1/2}\alpha_{lm} \lesssim 3.7^\circ \) (1σ), and I place a constraint to the power spectrum for \( \alpha \) for theories that predict that \( \alpha(\theta, \phi) \) is a stochastic field. To preface, I discuss the derivation of the constraint, \( \alpha = -0.7^\circ \pm 2.0^\circ \) (to redshifts \( z \approx 2 \)), from the UV data, a result that is strengthened to \( \alpha = -0.1^\circ \pm 1.7^\circ \) if the radio data on 3C 9 is included. Finally, I discuss how the analysis of high-resolution intensity-polarization maps of individual sources can be optimized, using tensor-harmonic techniques similar to those for CMB polarization and weak gravitational lensing.

Prelude: A uniform rotation. Table \( I \) reproduces data on 8 UV sources from Ref. \([16]\) as well as radio data on 3C 9 \([13, 17]\). Listed there are the positions \((\theta_i, \phi_i)\),...
position-angle–polarization offsets \( \alpha_i \), and measurement errors \( \sigma_i \) to these offsets.

Let us first test with this data whether there is a rotation, by an angle \( \alpha \), that is uniform across the sky. We will also determine the scatter \( \sigma_p \) in the measurements of \( \alpha \). To estimate the mean offset from the data, we use the minimum-variance estimator,

\[
\hat{\alpha} = \left[ \sum_i \frac{\alpha_i}{\sigma_i^2} \right] / \left[ \sum_i \frac{1}{\sigma_i^2} \right].
\]

The error to our measurement of \( \alpha \) is then inverse root of the denominator.

The error in \( \sqrt{\sum_i \alpha_i - \hat{\alpha}} \) is obtained if an unweighted estimator,

\[
\sigma_{\alpha}^2 = \frac{\sum_i (\alpha_i - \hat{\alpha})^2}{N(N - 1)}
\]

The 9 sources in Table I result in \( \sigma_p = 2.9^\circ \), a result that will be used below. Note that the weighted estimate of the scatter in Eq. (2) is a bit smaller than the value 4.4\(^\circ\) obtained if an unweighted estimator, \( \sigma_p^2 = (N - 1)^{-1} \sum_i (\alpha_i - \hat{\alpha})^2 \), for the variance is used, an indication that the unweighted variance in this data is due primarily to measurement error, not intrinsic scatter.

### Table I: The \( \theta,\phi \) coordinates, offsets \( \alpha_i \) and measurement errors \( \sigma_i \) for the eight sources listed in Ref. [16] plus the last the radio source 3C 9 (from Ref. [13]).

| \( \theta_i \) (deg) | \( \phi_i \) (deg) | \( \alpha_i \) (deg) | \( \sigma_i \) (deg) |
|---------------------|---------------------|---------------------|---------------------|
| 1                   | 78                  | -1.0               | 3.5                 |
| 2                   | 66                  | -0.3               | 4.4                 |
| 3                   | 109                 | 1.6                | 4.5                 |
| 4                   | 90                  | -0.8               | 8.0                 |
| 5                   | 93                  | -4.0               | 8.8                 |
| 6                   | 68                  | -4.0               | 9.0                 |
| 7                   | 114                 | 4.6                | 9.7                 |
| 8                   | 103                 | 5.0                | 16                  |
| 9                   | 5                   | 5.0                | 16                  |

Non-uniform rotation. Now suppose we wish to test if there is a single \( lm \) spherical-harmonic variation in \( \alpha(\theta, \phi) \): i.e., that \( \alpha(\theta, \phi) = \alpha_{lm}(Y_{lm}(\theta, \phi)) \), for some given \( l \) and \( m \). Then each data point would provide an estimator, \( \hat{\alpha}_{lm} = (\alpha_i - \hat{\alpha}) / Y_{lm}(\theta_i, \phi_i) \), with variance \( \langle |\hat{\alpha}_{lm}|^2 \rangle = \sigma_{\hat{\alpha}}^2 / |Y_{lm}(\theta_i, \phi_i)|^2 \). I include the \( \hat{\alpha} \) term in the estimator to avoid confusing a higher moment (e.g., a dipole) with uniform rotation in case of limited or irregular sky coverage. It should become irrelevant in an ideal experiment, with \( N \rightarrow \infty \) and a population of sources spread uniformly throughout the sky. Note that \( \hat{\alpha}_{lm} \) is complex, and the variances to the real and imaginary parts are each \( \langle |\hat{\alpha}_{lm}|^2 \rangle / 2 \).

The minimum-variance estimator \( \hat{\alpha}_{lm} \) obtained from all \( N \) data points is obtained by adding all the \( N \) individual \( \hat{\alpha}_{lm} \) estimators with inverse-variance weighting; i.e.,

\[
\hat{\alpha}_{lm} = \left( \sum_i \frac{\alpha_i - \hat{\alpha}}{\sigma_i^2} Y_{lm}(\theta_i, \phi_i) \right) / \left( \sum_i \frac{|Y_{lm}(\theta_i, \phi_i)|^2}{\sigma_i^2} \right).
\]

with variance given by the inverse of the denominator in this expression. The results of such an analysis of the 9 sources in Table I are presented in Table II. There is no evidence for any nonzero \( \alpha_{lm} \) for \( l \leq 2 \).

| \( l \) | \( \text{measured} (4\pi)^{-1/2} \hat{\alpha}_{lm} \) (deg) | \( (4\pi)^{-1/2} \langle |\alpha_{lm}|^2 \rangle^{1/2} \) (deg) |
|-------|-----------------------------|--------------------------------|
| 1     | -2.9                        | 3.4                            |
| 1 1   | -0.7 - 0.3i                  | 1.4                            |
| 1 0   | 0.2                         | 2.0                            |
| 1 2   | 1.1 + 0.2i                   | 2.3                            |
| 2 2   | 0.2 - 0.5i                   | 1.3                            |

### Table II: The measured \( \alpha_{lm} \) obtained from the data in Table II.

The values of the individual \( \hat{\alpha}_{lm} \)'s, for a given \( l \), depend on the choice of coordinate system. To test for a non-uniform CB in a rotationally-invariant way, one must evaluate the rotational invariants \( C_l = \sum_{m=-l}^{l} |\hat{\alpha}_{lm}|^2 / (2l + 1) \). Doing so, no evidence of a non-uniform CB is found for the dipole (\( l = 1 \)) and quadrupole (\( l = 2 \)). Rough upper limits to the dipole and quadrupole amplitudes can be obtained from the noise: \( \sqrt{C_1/(4\pi)} \lesssim 2.3^\circ \) and \( \sqrt{C_2/(4\pi)} \lesssim 1.9^\circ \).

Higher-\( l \) moments. Since we have in the current analysis only 8 data points, it is not really possible to measure any \( \alpha_{lm} \)'s with \( l \gtrsim 2 \). However, if there were a nonzero \( \alpha_{lm} \) for some high \( l \), it would give rise to a scatter in the measured \( \alpha \)'s with variance, \( \langle \sigma^2 \rangle = (4\pi)^{-1} \int d\theta d\phi |\alpha(\theta, \phi)|^2 = \alpha_{lm}^2 / (4\pi) \). If the sources are randomly distributed on the sky at points with angular separations \( \Delta \theta \gg \pi/l \), then this variance \( \langle \sigma^2 \rangle \) cannot be larger than the variance in the data. The variance measured from the data in Table II is roughly \( (2.9^\circ)^2 \), but there is a sample error to this variance, of roughly
\[I \text{ nates in the image plane. One first Fourier transforms,}
\[\delta \sim (3.7^\circ)^2. \text{ The upper limit to any } C_l \text{ are similar:}
\[\alpha \text{ is small in the sources where it is measured, suggest that}
\text{the existing measurements, which show that the offset } \alpha
\text{ should be no preference for a given handedness when averaging over many sources. More importantly,}
\text{the existing measurements, which show that the offset } \alpha
\text{ is small in the sources where it is measured, suggest that}
\text{IB and EB correlations will be small.}
\]
\[\text{If CB rotates the polarization by an angle } \alpha, \text{ then part of the E mode is rotated into a B mode,}
\text{so we should have } P_l^{EB} = 0, \text{ if parity is preserved.}
\text{While any given source may in principle have some hand-
\text{edness, and thus possibly nonzero IB or EB correlations,}
\text{there should be no preference for a given handedness when averaging over many sources. More im-
\text{portantly, the existing measurements, which show that the offset } \alpha
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\text{ is small in the sources where it is measured, suggest that}
\text{IB and EB correlations will be small.}
\]
\[\text{A similar analysis can be done, alternatively and equi-
\text{valently, using temperature-polarization two-point corre-
\text{lalion functions } P_l^{XX}. \text{ This involves taking all pairs}
\text{of points in the map, and then measuring correlations-
\text{between the intensity } I \text{ and Stokes parameters}
\text{Q_r and } U_r, \text{ measured in a coordinate system that is}
\text{aligned with the line connecting the two points. Again,}
\text{symmetry considerations suggest that, in the absence of}
\text{CB, } \langle I(\theta_1)U_r(\theta_2) \rangle = 0. \text{ If the polariza-
\text{tions are rotated by an angle } \alpha, \text{ then these parity-odd correlations are induced, with magnitudes}
\langle I(\theta_1)Q_r(\theta_2) \rangle = 2\alpha \langle I(\theta_1)Q_r(\theta_2) \rangle \text{ and } \langle Q_r(\theta_1)U_r(\theta_2) \rangle = 2\alpha \langle Q_r(\theta_1)Q_r(\theta_2) \rangle. \text{ The decision as to whether to use power spectra or correlation functions will depend on the}
\text{noise properties of the map.}
\]
\[\text{Discussion. Here I have discussed measurements of a}
\text{CB rotation of the linear polarization that varies as a}
\text{function of position on the sky and derived rough upper}
\text{limits to rotation-angle multipole moments and power}
\text{spectra. I discussed how the analysis of future high-
\text{resolution intensity-polarization images of high-redshift}
\text{sources can be optimized with techniques analogous to}
\text{those in CMB-polarization studies.}
\]
\[\text{The analysis presented here is meant primarily to be}
\text{illustrative. The existing data are far from optimized for this}
\text{particular measurement. First of all, I used only 9 sources at redshifts } z > 2, \text{ and the statistical weight is}
\text{dominated by only half of those. Moreover, they are not uniformly spread on the sky (which is why the errors to}
\]
the different $m$ moments for a given $l$ vary so widely), and this could give rise to pitfalls. Had my analysis found evidence for a signal, this may have been cause for concern. But given that the results are null, the derived upper limits are probably sound.

Although a comparable sensitivity to a position-dependent rotation can in principle be obtained from existing CMB data, the analysis is difficult and has not yet been done. The simple exercise I have performed here is thus the strongest existing constraint to a position-dependent rotation, at least for a rotation that occurs at redshifts $0 < z \lesssim 2$. A model that predicts rotation at $3 < z \lesssim 1100$ could still produce a signal in the CMB without violating the constraint I have derived. Likewise, a slightly stronger constraint can probably be obtained from the radio-galaxy data in Ref. [13], although for lower-redshift sources, and thus over a smaller baseline. Whether that constraint would be competitive with the one I have derived would, again, depend on the redshift dependence of $\alpha$ in any given model. Of course, one has a particular model that makes a specific prediction for the redshift dependence of $\alpha(z)$, then the data from sources at all redshifts can be combined to provide optimal estimators for the parameters of the model.

Improvements to the illustrative analysis I have done here should be straightforward. The error to the $\alpha_{\text{lin}}$ should scale simply as $N^{-1/2}$ with the number $N$ of sources, assuming similar image qualities to those obtained so far. Thus, for example, if the sample I used of $N \sim 4$ well-measured offsets can be improved to $N \sim 400$, the sensitivity will be competitive with that expected from Planck. However, progress can be accelerated, beyond $N^{-1/2}$, if more precise offset measurements can be obtained for at least some of these individual sources, either from better images, an improved analysis, or both. The goal of identifying the new physics responsible for cosmic acceleration will hopefully motivate such empirical investigations.

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