We investigate the azimuthal anisotropy $v_2$ of particle production in nucleus-nucleus collisions in the maximum entropy approach. This necessitates two new parameters $\delta$ and $\lambda$. The parameter $\delta$ describes the deformation of transverse configuration space and is related to the anisotropy of the overlap zone of the two nuclei. The parameter $\lambda$ defines the anisotropy of the particle distribution in momentum space. Assuming deformed flux tubes at the early stage of the collision we relate the momentum to the space asymmetry i.e. $\lambda$ to $\delta$ with the uncertainty relation. We compute the anisotropy $v_2$ as a function of centrality, transverse momentum and rapidity using gluon-hadron duality. The general features of LHC data are reproduced.
I. INTRODUCTION

The aim of this paper is to describe the azimuthal anisotropy $v_2$ of the inclusive cross section of charged particles produced in nucleus-nucleus collisions in the maximum entropy approach [1]. In this method the nonequilibrium features of heavy-ion collisions are parametrized by two parameters, the effective transverse temperature $\lambda$ and the softness parameter $w$. The first parameter $\lambda$ comes from the constraint that the total transverse energy of the produced partons is fixed. The second parameter $w$ guarantees that the sum of all partonic light cone fractions is unity for forward and backward particles separately. We assume as simplification that only gluons participate in the collision. Then the maximum entropy method yields a Bose-type distribution depending on light cone $x$ and transverse momentum as follows [1]:

$$n(x, \vec{p}_\perp) = \frac{1}{e^{\frac{|\vec{p}_\perp|}{\lambda} + xw} - 1}$$

(1)

with

$$x = \frac{\epsilon + p_z}{E + P_z}.$$  

For $pp$-collisions the transverse phase space is homogeneously distributed over the area $L_\perp^2$. Empirical values for this area give sizes $L_\perp \approx (1.2 - 1.3)$ fm for pion distributions. For nucleus-nucleus collisions we add the individual distributions of the nucleonic participants $N_{\text{part}}$. As we have shown in Ref. [2] the effective transverse temperature $\lambda$ rises with centrality because of collisional broadening of the partons. Invoking parton-hadron duality the multiplicity of produced particles in each hemisphere of the cm-system is half of the total multiplicity $N$. It is obtained by integrating the light cone distribution over the respective phase space. Note that the relativistic measure $dx$ arises from the large spatial extension in longitudinal direction of the small $x$ partons and the gluon degeneracy factor $g = 2(N_c^2 - 1)[1]$.

$$N/2 = g \frac{N_{\text{part}}}{2} L_\perp^2 \int \frac{d^2p_\perp}{(2\pi)^2} \int \frac{dx}{x} n(x, \vec{p}_\perp).$$  

(2)

We emphasize that a statistical understanding of the final state in heavy-ion collisions necessitates both a correct description of the momentum and configuration space distribution. With this in mind it is clear which changes must be implemented to describe the azimuthal anisotropy in the maximum entropy method. Clearly, we have to take into account that
phase space is deformed for noncentral collisions. This means that both configuration space and momentum space are not isotropic. The impact parameter $b$ along the $x$-axis and the momenta of the incoming particles along the $z$-axis determine the reaction plane. Orthogonal to this $x,z$-system we define the third $y$-axis. Let the angle $\varphi$ in the $x,y$-plane be the angle which the transverse momentum of the particle forms with the $x$-axis. Any asymmetry in transverse momentum space relative to the reaction plane has to be considered together with the deformed asymmetry in configuration space which is given by the overlap zone of the two nuclei. There are indications from HBT-measurements for a noticeable asymmetry in configuration space even much later in the collision cf. [3].

In the following, we concentrate on the early stage of the collision and consider the different manifestations of the fundamental QCD-degrees of freedom which are gluons and quarks. Gluons as such describe partons with high momenta at high resolution. Shortly after the collision the scattered nuclei pull many strings between the activated, color charged partons. These strings are gluonic degrees of freedom at lower resolution. They fluctuate in transverse directions as their neighbours allow them. Kinks in string configurations can be associated with harder gluons as done in the Lund model [4]. Lower energy string fluctuations form flux tubes which finally decay.

As is well known [5], the transverse momentum distribution of partons created in the homogeneous field of a flux tube is related to the area of the flux tube $L_{\perp}^2$ as

$$ \frac{dn}{d^2p_{\perp}} \propto e^{-\alpha p_{\perp}^2 L_{\perp}^2}. \quad (3) $$

We assume that these flux tubes repel each other because of the simple argument that the energy of two triplet-antitriplet strings is lower than the energy of a single octet-octet string. The similarity of QCD confinement in flux tubes to the behavior of vortex lines in type II-superconductors [6] motivates our ideas. Vortices in a Bose-Einstein condensate trapped in an anisotropic trap have been simulated in ref. [7]. This paper shows that the vortices fluctuate more along the extended side of the condensate than along the compressed side. In analogy, we conjecture that also the gluon flux tubes fluctuate more along the extended side than along the compressed side of the mandola formed in the nucleus-nucleus collision. Therefore, we parametrize the available configuration space for the gluons by two different extensions $L_x$ and $L_y$ in the $x$- and $y$-directions. In order to preserve our description given in eq. (2) for the angle-integrated multiplicities we have the condition:
\[ L_xL_y = L_\perp^2. \] (4)

Intuitively, this area is the average cross section of a flux tube out of which the hadrons materialize. We describe this configuration space asymmetry by the new parameter \( \delta < 0 \)

\[ \delta = \frac{L_x^2 - L_y^2}{L_x^2 + L_y^2} \] (5)

Anisotropy in momentum space necessitates a second parameter, the transverse asymmetry energy. It appears natural to use the second azimuthal moment of the total transverse energy as an additional constraint in order to define this parameter. The complete set of constraints then follows:

\[ gL_xL_y \int \frac{d^2p_\perp}{(2\pi)^2} \int \frac{dx}{x} x n(x, \vec{p}_\perp, \varphi) = 1 \] (6)

\[ gL_xL_y \int \frac{d^2p_\perp}{(2\pi)^2} \int \frac{dx}{x} |\vec{p}_\perp| n(x, \vec{p}_\perp, \varphi) = <E_\perp,pp> \] (7)

\[ gL_xL_y \int \frac{d^2p_\perp}{(2\pi)^2} \int \frac{dx}{x} |\vec{p}_\perp| \cos[2\varphi] n(x, \vec{p}_\perp, \varphi) = c_2 <E_\perp,pp> \cdot \] (8)

These three constraints are added to the entropy per participant with three Lagrange parameters. Then the sum is maximized.

\[ \delta \left( S + \frac{1}{\lambda_0} \sum |p_\perp| n(x, p_\perp, \varphi) + w \sum x n(x, p_\perp, \varphi) + \frac{1}{\lambda_2} \sum |p_\perp| \cos[2\varphi] n(x, p_\perp, \varphi) \right) \] (9)

\[ \delta n(x, p_\perp, \varphi) = 0 \]

The third constraint defines the third Lagrange parameter \( \lambda_2 \), the transverse asymmetry energy. Since all constraints are linear, the resulting maximum entropy distribution has the familiar exponential form of the light-cone Bose distribution.

\[ n(x, p_\perp, \varphi) = \frac{1}{e^{\frac{|\vec{p}_\perp|(\frac{x}{\lambda_0} + \cos[2\varphi]) + xw} - 1}} \] (10)

Higher moments on the transverse energy would generate more parameters. Fluctuating distributions of nucleons in the nuclei would also produce odd moments. These more general cases need extra Lagrange parameters which should be included in a similar way as above. Restricting ourselves to the second moment only, we get the parton distribution:
\[
\frac{dN_{AA}}{dyp_\perp dp_\perp d\phi} = g N_{part} \frac{L_x L_y}{2} \frac{1}{(2\pi)^2} \rho |p_\perp| \left( \frac{1}{\lambda_0} + \frac{\cos[2\phi]}{\lambda_2} + \frac{\cos[|\phi|]}{\lambda_0^2 \gamma_0^2} \right) - 1
\]  

(11)

If parton-hadron duality holds, this generalized maximum-entropy distribution gives the semi-inclusive cross section of hadrons including azimuthal asymmetry. We have seen in previous publications \cite{1, 2} that duality only holds approximately. Hadronic masses modify the light cone distribution. We have not yet implemented such a modification, but such an effect must be assumed to exist also for the added azimuthal asymmetry. Concentrating on the central rapidity window \( y \approx 0, \frac{w}{\sqrt{s}} \ll \frac{1}{\lambda_0} \) one can integrate the inclusive cross section with the weight function \( \cos(2\phi) \). Representing the Bose distribution by a geometric series one obtains a simple expression for the total asymmetry \( v_2 \), since \( |\lambda_2| \gg \lambda_0 \):

\[
v_2 = \frac{\int dN_{AA} |_{y=0} p_\perp \cos(2\phi) dp_\perp \phi}{\int dN_{AA} |_{y=0} dp_\perp \phi} \approx -\frac{\lambda_0}{\lambda_2}.
\]  

(12)

II. GEOMETRY OF THE COLLISION AND MOMENTUM ASYMMETRY

On first sight, cf. eqs. (11) and (12), the configuration space asymmetry \( L_x \neq L_y \) cancels out in the ratio for \( v_2 \). We should not leave out, however, quantum mechanics, which generates a momentum anisotropy from the shape asymmetry \( \delta \) of the deformed flux tubes. The transverse extensions of the flux tubes depend on the geometrical arrangement of the participants which pull the strings making up the flux tubes. If we further assume that the deformations of the flux tubes and the overlap zone are isomorphic (see Fig. 1), we can calculate \( \delta \) as a function of impact parameter \( b \):

\[
\delta(b) = \frac{L_x^2 - L_y^2}{L_x^2 + L_y^2} = \frac{\int n_{part}(\rho \cos(\phi), \rho \sin(\phi), \vec{b}) \cos(2\phi) \rho d\rho d\phi}{\int n_{part}(\rho, \vec{b}) \rho d\rho d\phi}.
\]  

(13)

\[
(14)
\]

The numerator gives the expectation value of the asymmetry \( \cos(2\phi) \) for a given overlap density \( n_{part} \) at the impact parameter \( b \). The denominator gives the total integral of the overlap density at the same impact parameter, i.e. the total number of participants

\[
N_{part}(\vec{b}) = \int n_{part}(\rho \cos(\phi), \rho \sin(\phi), \vec{b}) \rho d\rho d\phi.
\]  

(15)
FIG. 1. Schematic picture of the overlap zone for a heavy-ion collision and flux tubes with equal deformation.

This average is defined consistently with eq. (11) where interaction area scales with $N_{\text{part}}$. The average $\cos(2\varphi)$ gives the deformed extensions $L_x, L_y$ of the flux tube, whose total area $L_x L_y = L^2_\perp$ is constrained from the pp-data

$$L_x = L_\perp \left( \frac{1 + \delta}{1 - \delta} \right)^{1/4}$$  \hspace{1cm} (16)

$$L_y = L_\perp \left( \frac{1 - \delta}{1 + \delta} \right)^{1/4}. \hspace{1cm} (17)$$

With $L_x < L_y$ the asymmetry parameter $\delta < 0$. Note the parameter $\delta$ differs from the parameter $\varepsilon$ given in the literature e.g. [8] which represents the average of $< x^2 - y^2 > / < x^2 + y^2 >$.

As we do not yet have precise theoretical tools to handle the hadronisation of the gluons resulting from the deformed flux tubes, we stay with the gluons in the flux tubes and estimate their momentum asymmetry from the uncertainty principle

$$\frac{< p_x^2 - p_y^2 >}{< p_x^2 + p_y^2 >} = \frac{1}{L_x^2} - \frac{1}{L_y^2} \hspace{1cm} (18)$$

$$= -\delta(b). \hspace{1cm} (19)$$

A similar idea has been recently implemented in a cascase model [9].

We can calculate the same asymmetry in leading order $\frac{\lambda_0}{\lambda_2}$ from the deformed light-cone plasma distribution of gluons given in eq. [10]:

6
FIG. 2. The theoretical azimuthal asymmetry $v_2$ is shown as a function of centrality in percent. The data points represent measurements of $v_2\{2\}$ (blue points) and $v_2\{4\}$ (red points) for charged particles from ALICE [10] at $\sqrt{s} = 2.76$ TeV.

\[
\frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} = -\frac{2\lambda_0}{\lambda_2}.
\] (20)

Combining the two equations we are able to relate the momentum space asymmetry $\lambda_2$ to the configuration space anisotropy $\delta$. This is the main result of this section

\[
\lambda_2 = \frac{2\lambda_0}{\delta}.
\] (21)

Finally, we can compute the momentum integrated $v_2$ from the anisotropic light-cone plasma distribution with the parameter $\lambda_2$ (eq. (13)). The resulting $v_2$ is determined from the geometrical function $\delta(b)$ only

\[
v_2(b) \approx -\frac{\delta(b)}{2}.
\] (22)

In Fig. 2 we compare the calculated azimuthal asymmetry $v_2$ for massless gluons from eq. (20) with the data for $v_2\{2\}$ (blue points) and $v_2\{4\}$ (red points) of charged particles as
a function of the centrality. Pb-Pb collisions at LHC were at $\sqrt{s} = 2.76$ TeV. The general behavior of $v_2$ is reproduced, but the measured asymmetry saturates for large centralities at very peripheral collisions, where the overlap becomes very small and our simple calculation fails. We also underestimate $v_2$ for low centralities where fluctuations in the positions of the participants are important. In addition the effect of hadronic masses has to be considered in further work. We also calculated $v_2$ for Au-Au collisions at RHIC $\sqrt{s} = 0.2$ TeV. A very small difference results, which comes from the slightly different participant distributions at RHIC ($\sigma_{NN}^{in} = 42mb$) and LHC ($\sigma_{NN}^{in} = 64mb$).

Integrating over all impact parameters one obtains average values $\lambda_2$ for RHIC and LHC

$$\lambda_2(RHIC) = -4.49\text{ GeV}$$
$$\lambda_2(LHC) = -5.84\text{ GeV}.$$  \hfill (23)

The large transverse asymmetry energy $\lambda_2$ compared with the transverse effective temperature $\lambda_0 \approx 0.38$ GeV at LHC generates a small anisotropy. One should consider that the presented model is based on the early stage of the collision, where the flux tube geometry plays an important role. As noted in ref. [8] any sort of interaction among the primordial QCD degrees of freedom will cause non-vanishing radial and anisotropic flows even before the system has thermalized and viscous hydrodynamics becomes applicable. Of course strong interactions of the finally produced hadrons also may modify the angular anisotropy. Recently another model of the initial state interaction based on AdS/CFT [11] has come to a different conclusion about the deformation of the flux tubes. It proposes a widening of the flux tubes along the impact parameter axis. Since AdS/CFT has special problems to describe QCD in the region between $T_c$ and $3T_c$ [12, 13] we think its predictive power for such complicated processes is limited.

The momentum dependent $v_2(p_{\perp})$ is computed in our model from the azimuthal integrals of the anisotropic light-cone plasma distribution. It gives sums of modified Bessel-functions $I_1$ and $I_0$ of the first kind.
\[ v_2(p_{\perp}) = \frac{\int \frac{dN_{AA}}{dy p_{\perp} dp_{\perp}} |_{y=0} \cos(2\varphi) d\varphi}{\int \frac{dN_{AA}}{dyp_{\perp} dp_{\perp}} |_{y=0} d\varphi} \]  

(24) 

\[ = \frac{\sum_{n=1}^{\infty} (-1)^n \exp(-np_{\perp}/\lambda_0) I_1(n p_{\perp}/\lambda_2)}{\sum_{n=1}^{\infty} \exp(-np_{\perp}/\lambda_0) I_0(n p_{\perp}/\lambda_2)}. \]  

(25) 

For LHC at \( \sqrt{s} = 2.76 \) TeV we have the parameters given in Table I.

| Centrality | \( \lambda_0 \) (GeV) | \( \lambda_2 \) (GeV) | \( w \) |
|------------|-----------------|-----------------|-----|
| 10-20\%   | 0.396           | -8.65           | 7.2 |
| 20-30\%   | 0.383           | -5.53           | 6.73|
| 30-40\%   | 0.376           | -4.26           | 6.49|

TABLE I. Parameters of the anisotropic light-cone distribution for different centralities in Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV.

For the explanation of the rise of the effective temperature \( \lambda_0 \) for more central collisions we refer to ref. [2]. The transverse size is constant \( L_{\perp} = 0.766 \) fm. These parameters are fitted at each centrality bin to the constraints \( dN/dy \) at \( y = 0 \), \( < p_{\perp} > \) and the \( x \)-sum rule. The effective transverse temperatures and the transverse size for massless gluons differ from the corresponding parameters for massive pions. In Fig. 3 we show the resulting momentum dependent asymmetries for LHC at \( \sqrt{s} = 2.76 \) TeV in these three different centrality bins. The increase of \( v_2 \) is reproduced, as before for peripheral collisions \( v_2 \) is overestimated. With masses of the hadrons included in the constraints of the maximum entropy distribution the anisotropy will be more suppressed for small momenta.

Since \( |\lambda_2| > \lambda_0 \), we can expand the Bessel functions for small transverse momentum \( p_{\perp} \rightarrow 0 \) and obtain a formula which is good for \( 0.1 \) GeV < \( p_{\perp} \) < \( 1 \) GeV. One sees that negative \( \lambda_2 \) gives positive values for \( v_2(p_{\perp}) \). The intercept is determined by the geometrical quantity \( \delta \) and the slope by the inverse of the transverse asymmetry energy. Therefore, peripheral collisions have a larger intercept and a steeper slope

\[ v_2(p_{\perp}) = -\frac{1}{\lambda_2} \left( \frac{\lambda_0}{2} + \frac{p_{\perp}}{4} + \frac{p_{\perp}^2}{24\lambda_0} \right). \]  

(26)
FIG. 3. The azimuthal asymmetry \( v_2 \) is shown as a function of transverse momentum, in three different centrality bins (30% − 40%, 20% − 30%, 10% − 20%) from top to bottom for LHC at \( \sqrt{s} = 2.76 \) TeV. The data points are \( v_2 \{4\} \) measurements from ALICE [10].

Since we have the explicit form of the light-cone plasma distribution depending on rapidity, we can also calculate the momentum integrated flow parameter as a function of the rapidity \( y \) with the parameters of Table I used for the calculation of the momentum dependence and \( K = 0.35 \) given in ref. [2]

\[
v_2(y) = -\frac{1}{\lambda_2} \left( \frac{1}{\lambda_0} + \frac{w y}{K \sqrt{s}} \right).
\]  

(27)

The anisotropy \( v_2(y) \) depends on rapidity in the above specific combination including the effective transverse temperature \( \lambda_0 \), the softness parameter \( w \) and the effective cm-energy \( K \sqrt{s} \). In Fig. 4 we show the resulting rapidity dependent asymmetries. For each centrality there is one calculated \( \lambda_2 \) in the light-cone plasma distribution to describe the dependences of the anisotropy parameter \( v_2 \) on momentum and rapidity. This is a definite advantage of the light-cone plasma distribution compared with other statistical distributions. The centrality dependent data points in Fig. 1 are somewhat larger than the preliminary data in
FIG. 4. The impact parameter integrated azimuthal asymmetries $v_2$ are shown as a function of rapidity using the deformed light-cone plasma distribution for centrality bins (30% − 40%, 20% − 30%, 10% − 20%) from top to bottom. The preliminary data points are from a talk by A. Hansen [http://qm2012.bnl.gov].

Fig. 3 at $y=0$. The tendency of theory to overshoot experiment for peripheral collisions is consistent with the comparisons shown before. The theoretical $y$-dependence does not seem strong enough which may point to less deformed flux tubes at the sources of the strings.

III. DISCUSSION

We have presented a model of the anisotropy parameter $v_2$ based on the non-equilibrium maximum entropy distribution. We consider deformed gluon flux tubes to cause the observed momentum anisotropy. Assuming the deformation isomorphic to the nucleus-nucleus overlap zone, we calculated the flux tube deformation parameter $\delta$ from the participant density. With the uncertainty relation we connected the momentum asymmetry of gluons to their spatial asymmetry $\delta$. Identifying this asymmetry with the momentum asymmetry resulting from the maximum entropy distribution we determined the new third parameter
in the maximum entropy distribution, the transverse asymmetry energy $\lambda_2$. The scenario underlying our picture sees the anisotropy $v_2$ developing in the very early stage of the collision where a non-equilibrium gluon light-cone plasma describes the situation well. The origin of anisotropy really lies in the quantum properties of hadronization from flux tubes. Quantum mechanically the flux tube size gives the right size of transverse momenta. In the EPOS-code, even the inclusive cross sections at LHC are derived from flux tubes created in the nucleus-nucleus collision cf. [14]. In our model the total cross section $L^2_{\perp}$ of the flux tube is constrained from the integrated multiplicity. Consequently, the anisotropy $v_2$ becomes strongly correlated with the anisotropy of the average flux tube,

$$\delta = \langle \cos(2\varphi) \rangle .$$

We found $v_2 \approx -0.5 \delta$. The similarity of the configuration space asymmetry to the negative momentum asymmetry has been known for a long time and guided intuition towards a hydrodynamical evolution. The anisotropy $v_2$ and the asymmetry parameter of the energy density $\varepsilon$ are only weakly correlated, however, as $v_2 \approx -0.2 \varepsilon$, where

$$\varepsilon(b) = \langle \rho^2 \cos(2\varphi) \rangle / \langle \rho^2 \rangle .$$

Our study focusses on the very early stage of the collision after which a hydrodynamical flow and finally a blast wave may occur. The presented paper restricts itself to the asymmetry of the gluons, therefore it presents only a crude picture of the full collisions dynamics. Further work is necessary to investigate additional consequences following from our picture. On a purely phenomenological level it is also worthwhile to determine the transverse asymmetry energy parameter $\lambda_2$ in Eq. (8) directly from the experimental data. The more facets we learn about high energy $pp$ and nucleus-nucleus collisions the more the very early stages of the collision seem to become important reflecting the light-like trajectories of the partons and their dynamics.
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