Collisional effects on the formation of the second solar spectrum of the Sr II $\lambda$4078 line

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ABSTRACT

Context. One of the challenging theoretical problems in solar physics is the modeling of polarimetric observations in view of diagnostics of magnetic fields in the solar atmosphere.

Aims. This work aims to provide key elements to gain better understanding of the formation of the second solar spectrum of the Sr II $\lambda$4078 line.

Methods. The atomic states are quantified by the density matrix elements expressed in the basis of the irreducible tensorial operators. We perform accurate computation of the collisional depolarization and polarization transfer rates for all levels involved in the Sr II 4078 Å line. We solve the statistical equilibrium equations to calculate the linear polarization degree where: (a) collisions are completely ignored, (b) collisions are taken into account in the framework of a simplified atomic model, and (c) collisions are taken into account in the framework of 5-levels and 5-lines atomic model.

Results. We provide all collisional rates needed for Sr II $\lambda$4078 line modeling. Although Sr II $\lambda$4078 line is the resonance line of Sr II connecting the ground state $^2S_{1/2}$ to the excited state $^2P_{3/2}$, we show that its linear polarization is sensitive to collisions with neutral hydrogen mainly because the metastable $D$-levels are vulnerable to collisions. Thus, a 5-levels and 5-lines atomic model is needed to study this line. We determine a correction factor that one must apply to the value of the linear polarization derived from simplified two-level model.

Conclusions. In a certain range of hydrogen density, the effect of isotropic collisions between Sr II ions with hydrogen atoms is important in determining the polarization of the Sr II $\lambda$4078 line. The use of an atomic model that neglects the metastable level 4d of Sr II can induce errors of up to 25 % in the value of the scattering polarization of the solar Sr II $\lambda$4078 line.

Key words. Scattering – Collisions processes – Sun: atmosphere – Sun: magnetic fields – Line: formation - Polarization

1. Formulation of the problem

The influences of the Sun on the Earth's climate and environment are the source of new developments in astrophysics re-grouped under the name of space weather. To better understand these influences one should understand the physics of the Sun and thus should determine with good accuracy the solar magnetic field. Thanks to a vigorous technological effort, spectro-polarimetric observations with unprecedented precision and sensitivity are now available to the solar physics community (e.g. Stenflo & Keller 1997; Bianda & Stenflo 2001; Trujillo Bueno et al. 2001; Gandofer 2000, 2002, 2005; Malherbe et al. 2007; López-Ariste et al. 2009; Bianda et al. 2011; Milić & Faurobert 2012).

Diagnostics of molecular and atomic lines formed in the solar atmosphere based on the physics of scattering polarization are powerful tools to learn about unresolved magnetic fields which are very present in the solar plasma (see for instance Trujillo Bueno et al. 2004; Stenflo 2004 and the book of Landi Degl’Innocenti & Landolfi 2004). Reliable interpretation of the scattering polarization often requires solving numerically the coupled set of equations of the radiative transfer and the statistical equilibrium of a multilevel atomic model.

In the solar community, selected lines from different wavelength regimes are currently interpreted in terms of magnetic field and the scientific value of their use is quantitatively estimated. However one still must confront many serious problems facing rigorous interpretation of the observed polarization of the solar radiation. For instance, the Hanle effect of the magnetic field and isotropic collisions are mixed in the same observable (the polarization state) which makes the interpretation of the observed polarization in terms of magnetic field difficult or imprecise or controversial.

In the case of two-levels atom where the polarization of the lower level and collisions are neglected, one shows that the efficiency of the Hanle effect is controlled by the dimensionless parameter $\Gamma = \frac{2\pi \mu_B B g_J}{\lambda_{\text{eff}}}$, where $g_J$ is the Landé factor of the $J$-level under consideration and $A$ is the Einstein coefficient for the spontaneous emission of the spectral line having $J$ as upper level. $\Gamma = 1$ corresponds to $B = B_\text{c}$ which is the critical magnetic field strength where the Hanle effect is maximal. For $0.1 \, B_\text{c} < B < 10 \, B_\text{c}$, one may expect a sizable change of the scattering polarization signal with respect to the unmagnetized reference case. Numerically, one has

$$B_\text{c} \text{ (Gauss)} = \frac{1.137 \, 10^{-7} \, A \, (s^{-1})}{g_J}$$

(1)

According to L-S coupling, one finds that $g_J = \frac{3}{2}$ for the level $^2P_{3/2}$ of the Sr II ion. The Einstein coefficient is $A = 1.26 \times 10^8 \, s^{-1}$ (NIST database). Thus, for the linear scattering polarization of the Sr II $\lambda$4078 line, $B_\text{c} = 10.75$ Gauss. Therefore, the scattering polarization of the Sr II $\lambda$4078 line is of great practical interest here because the magnetic field in the chromospheric...
level is around tens of Gauss. Measurement and interpretation of the Sr \(\lambda 4078\) line gives an interesting tool to determine magnetic field in the chromosphere of the Sun where the solar magnetism is not very well known.

In fact, the Hanle effect in the solar Sr \(\lambda 4078\) line has been studied by Bianda et al. (1998) (see also Bianda 2003). They obtained a magnetic field strength in the range of 5 to 10 Gauss. They concluded that the Sr \(\nu\) line gives on average values 30% lower than the ones obtained via diagnostics based on the Ca \(\text{i}\) \(\lambda 4227\) line and they remarked that this discrepancy cannot be regarded as very significant because of the uncertainties on the collisional rate determination. They argued that in the higher layers of the solar atmosphere, where Sr \(\nu\) is formed near the solar limb, the collision rate is low. Looking to Eqs. (11), (12), and (13) of Bianda et al. (1998) giving the magnetic field strength \(B\) and the Hanle rotation angle according to the theory adopted by these authors, it seems that the obtained magnetic field is underestimated because the e and \(\gamma\) in the theory adopted by Bianda et al. (1998) are underestimated. It is possible that, if the collisions are fully taken into account in the framework of a realistic atomic model, the Sr \(\nu\) line should give magnetic field values similar to those obtained via diagnostics based on the Ca \(\text{i}\) \(\lambda 4227\) line.

2. Summary of the theory of the depolarizing collisions

Isotropic collisional interactions of the emitting Sr \(\nu\) atoms with nearby hydrogen atoms tend to reestablish thermodynamical equilibrium inside Sr \(\nu\) levels, i.e. to equalize populations of the Zeeman sublevels and to destroy their coherences, and thus partially destroy the atomic polarization. Consequently, a depolarization of the Sr \(\nu\) line arises from these isotropic collisions.

Collisions of neutral hydrogen with heavy atoms like Sr \(\nu\) and Sr \(\text{i}\) are not currently accessible to fully quantum chemistry treatments. To argue this statement, we cite an example concerning quantum calculations associated with the \(P\)-level of the Sr \(\text{i}\) atom. In fact, the depolarizing rate of \(P\)-level of the Sr \(\text{i}\) was calculated with quantum methods by Faurobert-Scholl et al. (1995) and later by Kerkeni (2002). Interestingly, Kerkeni (2002) obtained a value about a factor 2 smaller than Faurobert-Scholl et al. (1995). This disagreement was not expected since in both cases a quantum approach was used. We believe that one possible reason for this discrepancy is the significant complication of ab initio quantum calculations of the interaction potential between \(\text{H}\) and heavy atoms like Sr \(\text{i}\) leading to large error bars in the quantum calculations. Thus, for heavy atoms or ions like Sr \(\nu\) and Sr \(\text{i}\), one needs semi-classical methods to determine the depolarizing collisional rates. However, quantum chemistry rates associated with ions having small size (e.g. Na\(\text{i}\), Mg\(\text{i}\), Ca \(\text{i}\), etc.) are important in validating approximated semi-classical methods.

We used the available quantum chemistry rates associated with Ca \(\nu\) ions to validate the semi-classical method of Derouich et al. (2004). In fact, Ca \(\nu\) is smaller in size as compared to Sr \(\nu\) ions, and its quantum-chemistry study is accurate and unproblematic; for Ca \(\nu\) and neutral hydrogen collisions, the quantum interaction potentials are extensively reviewed in the literature.

Collisional depolarization and polarization transfer rates associated to levels of Sr \(\nu\) are obtained from the carefully tested semi-classical method of Derouich et al. (2004) which was developed for isotropic collisions between neutral hydrogen atoms and simple ions. In fact, the ionized alkaline earth metals Be \(\nu\), Mg \(\nu\), Ca \(\nu\), Sr \(\nu\), and Ba \(\nu\) are simple ions because they have only one valence electron above a filled subshell. Therefore, the determination of collisional rates using the approached method developed by Derouich et al. (2004) is possible with a percentage of error lower than 10% (see Sect. 6 of Derouich et al. 2004).

Derouich et al. (2004) applied their semi-classical method to obtain depolarization and polarization transfer rates for the upper level \(5p^2\ P\) of the Sr \(\lambda 4078\) line. The present work provides new depolarization and polarization transfer rates associated to the \(4d^2\ D\) which are indispensable for a rigorous analysis of the polarization of the Sr \(\lambda 4078\) line.

We describe the Sr \(\nu\) atomic states by the density matrix elements \(\rho_q^{\lambda}(J)\) where \(0 \leq k \leq 2J\) and \(-k \leq q \leq k\). More details about the physical meaning of the tensorial order \(k\) and the coherence \(q\) could be found for instance in Omont (1977), Sahal-Bréchet (1977) and Landi Degl’Innocenti & Landolfi (2004).

According to Eq. (1) of Derouich et al. (2004), the variation of \(\rho_q^{\lambda}(J)\) due to isotropic collisions is:

\[
\frac{d\rho_q^{\lambda}(J)}{dt}\bigg|_{\text{coll}} = - \sum_{J\neq J'} \zeta_{J \rightarrow J'} D^\lambda(J') \times \rho_q^{\lambda}(J) \times \rho_q^{\lambda}(J') + \sum_{J\neq J'} C^\lambda(J \rightarrow J') \times \rho_q^{\lambda}(J') \tag{2}
\]

where \(C^\lambda(J \rightarrow J')\) are the polarization transfer rates, \(D^\lambda(J')\) are the depolarization rates and \(\zeta(J \rightarrow J') = \sqrt{2J' + 1 \over 2J + 1} \times C^0(J \rightarrow J')\) are the fine structure transfer rates given by (Eq. 4 of Derouich et al. 2003b):

\[
\zeta(J \rightarrow J') = \sqrt{2J' + 1 \over 2J + 1} \times C^0(J \rightarrow J') \tag{3}
\]

Thus:

\[
\frac{d\rho_q^{\lambda}(J)}{dt}\bigg|_{\text{coll}} = - \sum_{J\neq J'} \sqrt{2J' + 1 \over 2J + 1} C^0(J \rightarrow J') D^\lambda(J') \times \rho_q^{\lambda}(J) \times \rho_q^{\lambda}(J') + \sum_{J\neq J'} C^\lambda(J \rightarrow J') \times \rho_q^{\lambda}(J') \tag{4}
\]

Now we denote the collisional transfer rates \(C^\lambda(J' \rightarrow J)\) by \(C^\lambda_j(J' \rightarrow J)\) if \(J' = J + J J\) (inelastic collisions) and we notice \(C^\lambda_j(J' \rightarrow J)\) by \(C^\lambda_j(J' \rightarrow J)\) if \(J' = J J J\) (super-elastic collisions). For a given collision, note that apart from the multiplicity factor \(\sqrt{2J + 1}\), the \(C^\lambda(J' \rightarrow J)\) denoted by Landi Degl’Innocenti & Landolfi (2004) are obtained from the complex configuration of a complete ion one has in the solar atmosphere, where Sr \(\nu\) is formed near the solar limb. Looking to Eqs. (11), (12), and (13) of Bianda et al. (1998) giving the magnetic field strength \(B\) and the Hanle rotation angle according to the theory adopted by these authors, it seems that the obtained magnetic field is underestimated because the e and \(\gamma\) in the theory adopted by Bianda et al. (1998) are underestimated. It is possible that, if the collisions are fully taken into account in the framework of a realistic atomic model, the Sr \(\nu\) line should give magnetic field values similar to those obtained via diagnostics based on the Ca \(\text{i}\) \(\lambda 4227\) line.

This recalls us the theoretical indications that if collisions are neglected for molecular lines, the derived magnetic field is underestimated (Berdyugina & Fluri (2004); Bomnier et al. (2006)).
We denote by $R^k(J)$ the relaxation rates of rank $k$ given by (Eq. (2) of Derouich 2008):

$$
R^k(J) = \sum_{j \neq j} \sqrt{\frac{2J_j + 1}{2J_j + 1}} C^k_{ij}(J \rightarrow J_j) \\
+ \sum_{j \neq j} \sqrt{\frac{2J_j + 1}{2J_j + 1}} C^k_{ij}(J \rightarrow J_j) + D^k(J)
$$

Then,

$$
\frac{d \rho^k(J)}{dt} \bigg|_{coll} = -R^k(J) \times \rho^k(J) + \sum_{j \neq j} C^k_{ij}(J \rightarrow J) \times \rho^k(J) \\
+ \sum_{j \neq j} C^k_{ij}(J \rightarrow J) \times \rho^k(J)
$$

which is the same as the Eq. (3) given in Derouich (2008).

Physically, the collisional relaxation rates $R^k(J)$ correspond to the lost of the atomic polarization and, in contrast, the transfer rates $C^k_{ij}(J \rightarrow J)$ and $C^k_{ij}(J \rightarrow J_j)$ correspond to the gain of atomic polarization coming from other levels.

In the literature, there are different definitions of the collisional rates, and these definitions must be taken into account when writing the Eq. (9), in order to calculate the polarization signals correctly. For this reason, Eqs 4-9 are necessary for a reader who wants to exploit our paper. For example, the depolarization rates as defined in our work, as compared to those defined by Kerkeni et al. (2003), are not equivalent. Derouich et al. (2004) (Sect. 7) confronted this problem when they wanted to compare their rates with the collisional rates obtained by Kerkeni et al. (2003). In addition, there is a small difference between the definition of our collisional rates and the definition of the collisional rates given in Landi Degl’Innocenti & Landolfi (2004).

Note that the observed linear polarization of the Sr $\lambda$4078 line is the footprint of only the even orders $k$ inside the Sr $\eta$ atom, and thus only depolarization and polarization transfer rates with even $k$ are needed to study such a line. In addition, since the collisions are isotropic, all collisional rates are $q$-independent.

### 3. Calculation of the collisional rates for the modeling of the Sr $\eta$4078 $\lambda$ line

Generally speaking, studying the polarization of the Sr $\eta$4078 line implies to solve the radiative transfer equations for polarized radiation in a magnetized atmosphere. To be able to solve numerically these equations one should adopt a simplified atomic model as the one presented in Figure 1. Particularly, in the framework of the simplified model, one disregards the effects of the metastable $D$-states in the Sr $\eta$4078 line modeling (see Bianda et al. 1998).

Our intention is to complete these models by taking into account the effect of collisions on the $D$-states. To this aim, we consider a realistic multi-levels and multi-lines atomic model of Figure 2 (hereafter the full model). It contains the 5 levels: $S_{1/2}$ and $P_{3/2}$, $P_{3/2}$, $D_{3/2}$ and $D_{3/2}$ and 5 lines: $\lambda=10915$ $\AA$, $\lambda=10357$ $\AA$, $\lambda=10036$ $\AA$, $\lambda=4216$ $\AA$, and $\lambda=4078$ $\AA$.

We apply our collisional numerical code in order to calculate the collisional scattering matrix after integration of the semi-classical differential coupled equations which are derived from the time dependent Schrödinger equation. Once the scattering matrix is obtained for the $P$- and $D$-states of Sr $\eta$, we determine the transition probabilities in the tensorial irreducible basis. Afterwards, these probabilities are integrated over impact parameters and Maxwellian distribution of relative velocities to obtain the depolarization and the transfer rates. We perform calculations by varying the temperature to obtain the best analytical fit to the collisional rates. The precision on the Sr $\eta$ rates should be similar to the precision obtained on the Ca $\eta$ rates (Derouich et al. 2004), i.e. better than 10%.

Derouich et al. 2004 derived the following expressions of needed depolarization and polarization transfer rates by isotropic collisions of $P$-levels of Sr $\eta$ ions with neutral hydrogen:

$$
D^2(3/2) = 5.98 \times 10^{-9} n_H \left( \frac{T}{5000} \right)^{0.41}
$$

$$
C^1_{ij}(1/2 \rightarrow 3/2) = \frac{4.62 \times 10^{-9}}{n_H} \left( \frac{T}{5000} \right)^{0.41}
$$

In addition in the present work we provide the following new expressions of collisional rates associated to the $D$-levels:

$$
D^2(3/2) = 2.28 \times 10^{-9} n_H \left( \frac{T}{5000} \right)^{0.42}
$$

$$
D^2(5/2) = 3.08 \times 10^{-9} n_H \left( \frac{T}{5000} \right)^{0.41}
$$

$$
C^1_{ij}(3/2 \rightarrow 5/2) = \frac{2.35 \times 10^{-9}}{n_H} \left( \frac{T}{5000} \right)^{0.45}
$$

$$
C^1_{ij}(3/2 \rightarrow 5/2) = \frac{1.02 \times 10^{-9}}{n_H} \left( \frac{T}{5000} \right)^{0.35}
$$

All rates are given in $s^{-1}$. Only the depolarization rates $D^{k=2}(J)$ and inelastic collisional rates $C^k_{ij}(J \rightarrow J)$ with $k=0$ and 2 are given as a function of neutral hydrogen number density $n_H$ (in cm$^{-3}$) and kinetic temperature $T$ (in Kelvins). Note that, given the low degree of radiation anisotropy in the solar atmosphere, one can safely neglect the effect of $D^2(3/2)$ on the linear polarization of the Sr $\eta$4078 line. However, one can retrieve the values of the superelastic collisional rates $C^k_{ij}(J \rightarrow J)$ by applying the detailed balance relationship:

$$
C^k_{ij}(J \rightarrow J_j) = \frac{2 J_j + 1}{2 J_j + 1} \exp \left( \frac{E_{ij} - E_{ij}}{k_B T} \right) C^k_{ij}(J \rightarrow J_j)
$$

with $E_{ij}$ and $E_{ij}$ being the energy of the upper level $J_j$ and the lower level $J_i$ respectively, $k_B$ is the Boltzmann constant.

It is important to notice that the Eq. (10) is valid only in the case of Maxwell distribution of the velocities of the hydrogen atoms but also the dimensionless collision strength (generally denoted by $\Omega$) must be symmetrical, i.e. $\Omega(J \rightarrow J) = \Omega(J \rightarrow J)$.

The term “collision strength” was originally suggested by Seaton (1953, 1955) and is now universally used. The collision strength $\Omega$ contains the information about the collisional transition probability between two given levels $\Omega$ is ultimately related to the scattering matrix. In our collisional method, the interaction potential matrix is hermitian which implies that the scattering $S$-matrix is unitary and symmetric (Derouich et al. 2003a). Consequently $\Omega$ is symmetrical and the Eq. (10) is well satisfied.

4 Since the impact approximation is well satisfied for isotropic collisions between neutral hydrogen atoms and Sr $\eta$ atoms in the solar photosphere, the collisional depolarization rates are simply obtained by multiplying the rates for a binary collision by the hydrogen density.
By taking $E_1$ and $E_2$ from NIST database, one finds at $T=6000K$:

\[
\begin{align*}
C_5^0(P_{3/2} \rightarrow P_{1/2}) &= 0.606 \times C_1^0(P_{1/2} \rightarrow P_{3/2}) \\
C_5^0(D_{3/2} \rightarrow D_{1/2}) &= 0.713 \times C_1^0(D_{1/2} \rightarrow D_{3/2}) \\
C_5^0(D_{5/2} \rightarrow D_{3/2}) &= 0.713 \times C_1^0(D_{3/2} \rightarrow D_{5/2})
\end{align*}
\] (11)

4. Scattering polarization of the Sr $\lambda 4078$ line

In theory, the core of the Sr $\lambda 4078$ line is formed at the chromosphere but the wings are formed in deeper layers of the photosphere. Since the density of hydrogen atoms $n_H$ in the photosphere is larger than $n_H$ in the chromosphere, if the effect of collisions cannot be neglected for the core of the Sr $\lambda 4078$, then collisions cannot be neglected in the wings of that line.

We consider that the solar chromosphere is a plane-parallel layer formed by Sr ions which are excited anisotropically from below by the photospheric radiation. The components of the incident radiation field at the wavelength $\lambda=4078\,\text{Å}$ are usually denoted by $J^k_q$, where $k$ is the tensorial order and $q$ represents the coherences in the tensorial basis ($-k \leq q \leq k$); the order $k$ can be equal to 0 (with $q=0$) or 2 (with $q=0, \pm 1, \pm 2$). This radiation field with six components constitutes a generalization of the unpolarized light field where only the quantity $J^0_0$ is considered. In fact, $J^0_0$ is proportional to the intensity of the radiation. It is useful to notice that if the chromosphere is assumed to be uniform, the radiation has a cylindrical symmetry around its preferred direction implying that the coherence components with $q \neq 0$ are zero. In fact, $J^{2s+1}_q$ and $J^{2s+2}_q$ components quantify the breaking of the cylindrical symmetry around the axis of quantification.

If the incident radiation is no longer anisotropic, the components $J^{2s+2}_q$ become zero which means that no linear polarization can be created as a result of scattering processes. Regardless of the anisotropy of the incident radiation, the radiation component associated to the circular polarization usually denoted by $J^{0s}_{-1}$ is negligible. This means that no odd order $k$ can be created inside the scattering strontium ion. As a result, the Stokes $V$ of the scattered radiation at $\lambda=4078\,\text{Å}$ is zero.

We concentrate our study on the core of the Sr $\lambda 4078$ line. Neglecting the possible presence of inhomogeneities of the chromospheric regions, the radiation field is cylindrically symmetrical about the vertical direction, and its properties are fully described by the two components $J^0_0(\lambda = 4078\,\text{Å})$ and $J^0_2(\lambda = 4078\,\text{Å})$ of the radiation field tensor or, alternatively,
5. The correction factor \( f_c \)

Since we do not solve the radiative transfer equations for the Stokes parameters, our work should be considered as complementary to other investigations which focus on the radiative transfer effects but neglect multilevel and collisional effects. Our aim is to provide a correction factor on the theoretical value of the line linear polarization obtained by solving the non-LTE radiative transfer problem in the framework of two-level modeling. In this sense, we recall that Faurobert et al. (2009) used a correction factor determined by Derouich (2008) for the Ba ii \( \lambda 44554 \) line; they found that, using a correction factor, the accuracy on the magnetic field have been improved by \( \sim 50\% \).

The correction factor \( f_c \) must contain information on the difference between the value of \([p]_{\text{simplified}}\) and \([p]_{\text{full}}\). Thus it could be written as:

\[
f_c = \left( [p]_{\text{simplified}} - [p]_{\text{full}} \right) \times 100
\]  

(12)

Note that \([p]_{\text{simplified}}\) is the linear polarization inferred from considering the simplified model and \([p]_{\text{full}}\) is the linear polarization inferred from considering the full model. Obviously, \([p]_{\text{simplified}}\) and \([p]_{\text{full}}\) are calculated in the presence of collisions which means that \( f_c \) is a function of \( n_H \).

We consider that the evolution of the density-matrix elements is due to the collisional and radiative effects. To calculate the linear polarization degree \( p = Q/I \) we use the formulae of Trujillo Bueno (1999):

\[
Q/I = \frac{3}{2 \sqrt{2}} \left[ \omega_{ij}^{(2)} \sigma_0^2(u) - \omega_{ij}^{(2)} \sigma_0^2(\ell) \right].
\]

(13)

which gives the linear polarization degree at the limit of tangential observation, i.e. at zero altitude above the solar limb. The indices \( i \) (for lower) and \( a \) (for upper) denote lower level \( J_i \) and upper level \( J_a \) of the transition. \( \omega_{ij}^{(k)} \) is a numerical coefficient introduced and tabulated for various values of \( k, J, \) and \( J' \) in Landi Degl’Innocenti (1984). \( \sigma_0^2 = \rho^2_0/\rho_0^2 \), where \( \rho^2_0 \) are the solutions of the statistical equilibrium equations (SEE) including collisional and radiative rates:

\[
\left[ \frac{d \rho^2_0(J)}{dt} \right] = \left[ \frac{d \rho^2_0(J)}{dt} \right]_{\text{coll}} + \left[ \frac{d \rho^2_0(J)}{dt} \right]_{\text{rad}} = 0
\]

(14)

For atmospheric level where the hydrogen density \( n_H \sim 10^{15} \) cm\(^{-3}\) and \( T \sim 6000K \), \( D^2(P_{3/2}) \sim 6.4 \times 10^4 s^{-1} \) << \( A = 1.26 \times 10^7 s^{-1} \). Because the inverse lifetime of the upper level of the Sr ii \( \lambda 4078 \) line is greater than the value of the elastic depolarizing rate \( D^2(P_{3/2}), \) one might think that the effect of the collisions is negligible. We will demonstrate that at \( n_H \sim 10^{15} \) the effect of collisions cannot be neglected.

We solve the SEE to calculate the \( \rho^2_0 \) elements and thus we determine the linear polarization \( p = Q/I \) of the Sr ii \( \lambda 4078 \) line once by using the simplified atomic model (see Fig. 1) and once by using the full model (see Fig. 2) that accounts for the metastable D-levels. We then investigate the sensitivity of the emergent linear polarization to the above-mentioned collisions in order to point out the range of \( n_H \) where the effect of collisions is important. Since the collisional rates are proportional to the hydrogen density (the impact approximation), determining the dependence of the polarization to collisional rates amounts to the study of its dependence on \( n_H \).

Figure 5 represents the linear polarization inferred from the simplified model in the presence of collisions, \([p]_{\text{simplified}}\) divided by \([p]_{\text{max}}\) which is the zero-collisions polarization inferred from the simplified model. In addition, Fig. 5 shows the ratio \([p]_{\text{full}}/[p]_{\text{max}}\) giving the linear polarization inferred from the full model in the presence of collisions divided by the same \([p]_{\text{max}}\).

Looking to the Fig. 5, the upper horizontal line with \( \cdots \) symbols represents the polarization scattered without any sensitive collision effects. In fact, according to Fig. 5, for \( n_H < 7 \times 10^{14} \) cm\(^{-3}\) the variation of \([p]_{\text{simplified}}\) is smaller that 2%, i.e. the ratio \([p]_{\text{simplified}}/[p]_{\text{max}} \approx 1 \pm 0.08 \). For \( n_H > 7 \times 10^{14} \) cm\(^{-3}\), the ratio \([p]_{\text{simplified}}/[p]_{\text{max}}\) starts to decrease progressively which reflects the collisional destruction of the alignment of the \( 2^3P_{3/2} \) level. At \( n_H > 3 \times 10^{15} \) cm\(^{-3}\), \([p]_{\text{simplified}}/[p]_{\text{max}} < 0.1 \), thus the linear atomic polarization in the level \( 2^3P_{3/2} \) is almost completely destroyed and \([p]_{\text{simplified}} \approx 0 \).

Referring again to the Fig. 5, the upper horizontal line with \( \cdots \cdots \) symbols corresponds to the scattering polarization obtained using the full model in a range of \( n_H \) where the collisions are negligible, i.e. for \( n_H < 1 \times 10^{14} \) cm\(^{-3}\) where \([p]_{\text{full}}/[p]_{\text{max}} \approx \text{constant} \approx 0.93 \). It is important to notice that \([p]_{\text{full}}/[p]_{\text{max}} \approx 1 \) even in the range where collisions are negligible, this is because the simplified model overestimates the polarization degree leading to the fact that \([p]_{\text{full}}/[p]_{\text{max}} \approx 1\) Usually. For \( n_H > 3 \times 10^{14} \) cm\(^{-3}\), the ratio \([p]_{\text{full}}/[p]_{\text{max}}\) starts to decrease progressively which reflects the collisional destruction of the alignment of the \( 2^3D_{5/2} \) and \( 2^3D_{3/2} \) levels. At \( n_H \approx 3 \times 10^{14} \) cm\(^{-3}\), the alignment of the level \( 2^3P_{3/2} \) is not affected yet by collisions but the polarization of Sr ii \( \lambda 4078 \) line decreases via the destruction of the alignment of the D-levels. For \( n_H = 4 \times 10^{15} \) cm\(^{-3}\), the
\[ |p|_{\text{full}} / |p|_{\text{max}} = 0.74 \] meaning that the collisions and multilevel effects result in a decrease of the polarization degree by more than 25%; note that at the same \( n_H = 4 \times 10^{15} \text{ cm}^{-3} \) one has \( |p|_{\text{simplified}} / |p|_{\text{max}} = 0.9 \).

In order to take into account the effect of elastic collisions and of alignment transfer due to multi-level coupling with the metastable \( D \) and of alignment transfer due to multi-level coupling with the metastable \( D \) levels, one must determine the correction factor \( f_c = (|p|_{\text{simplified}} - |p|_{\text{full}}) \times 100 \) for each \( n_H \). For instance at \( n_H = 4 \times 10^{15} \text{ cm}^{-3} \), \( f_c = (0.9 - 0.74) \times 100 = 16 \% \). Figure 6 gives the variation of \( f_c \) as a function of \( n_H \). For \( \approx 4 \times 10^{14} < n_H < 5 \times 10^{17} \), the determination of \( f_c \) is crucial for a correct determination of the linear polarization.

6. Conclusion

We use our collisional approach and our numerical code to calculate new collisional rates associated with the D-states of Sr \( \text{ii} \), and using another numerical code, we calculate the polarization of the Sr \( \text{ii} \) line by including our collisional rates in the statistical equilibrium of a Sr \( \text{ii} \) model atom. Our aim was to pay particular attention to a subtle collisional depolarizing effect of the Sr \( \text{ii} \) \( \lambda 4078 \) line due to the metastable \( D \)-levels, this should help to provide key elements to analyze quantitatively the scattering polarization of the Sr \( \text{ii} \) \( \lambda 4078 \) line. In fact, we find that the Sr \( \text{ii} \) \( \lambda 4078 \) line could be collisionally depolarized even when its upper level \( 2\, P_{3/2} \) is not affected by collisions. A given model can correctly reproduce the observed scattering polarization only if one takes properly into account the collisional perturbation of the atomic levels by the interaction with nearby hydrogen atoms. It is important to notice that our results have to be considered as information complementary to the models taking radiative transfer into account but without taking properly into account the effect of collisions in realistic multilevel schemes.

As the scattering polarization degree is usually rather small (e.g. polarization of the order of 1% in Fraunhofer lines in the photosphere and chromosphere of the Sun), careful and rigorous modelling of that polarization is of fundamental importance for learning especially about a weak and even unresolved magnetic field by its Hanle effect.

References

Berdygina, S. V.; Fluri, D. M., 2004, 417, 775
Bianda, M.; Stenflo, J. O.; Solanki, S. K., 1989, A&A, 337, 565
Bianda, M., & Stenflo, J. O., 2001, in Advanced Solar Polarimetry - Theory, Observation, and Instrumentation, ed. M. Sigwarth (San Francisco: ASP), ASP Conf. Ser., 236, 117
Bianda, M., 2003, PhD thesis, ETH Zurich (Göttingen: Cuvillier)
Bianda, M., Ramelli, R.; Anusha, L. S.; Stenflo, J. O.; Nagendra, K. N.; Holzreuter, R.; Sampoorna, M.; Frisch, H.; Smitha, H. N. 2011, A&A, 530, L13
Bonmier, V; Landi Degl’Innocenti, E.; Feautrier, N.; Molodij, G., 2006, 458, 625
Cox, A. N. 2000, Allen’s Astrophysical Quantities, 4th ed. (New York: Springer Verlag and AIP Press)
Derouich M., Sahal-Bréchot S., Barklem P.S., & O’Mara B. J., 2003a, A&A, 404, 763
Derouich M., Sahal-Bréchot S., & Barklem P.S., 2003b, A&A, 409, 369
Derouich M., Sahal-Bréchot S., & Barklem P.S., 2004, A&A, 426, 707
Derouich M., Sahal-Bréchot S., & Barklem P.S., 2005, A&A, 434, 779
Derouich M., 2008, A&A, 481, 845
Faurobert, M.; Derouich, M.; Bonmier, V.; Arnaud, J., 2009, A&A, 493, 201
Faurobert-Scholl, M.; Feautrier, N., Machefer, F.; Petrovay, K., & Spießfeld, A., 1995, A&A, 298, 289
Gandorfer A., 2000, The Second Solar Spectrum: A high spectral resolution polarimetric survey of scattering polarization at the solar limb in graphical representation, vol. 1: 4625 Å to 6995 Å (Hochschulverlag AG an der ETH Zurich)
Gandorfer, A., 2002, The Second Solar Spectrum: A high spectral resolution polarimetric survey of scattering polarization at the solar limb in graphical representation, vol. 2: 3910 Å to 4630 Å (Hochschulverlag AG an der ETH Zurich)
Gandorfer, A., 2005, The Second Solar Spectrum: A high spectral resolution polarimetric survey of scattering polarization at the solar limb in graphical representation, vol. 3: 3160 Å to 3915 Å (Hochschulverlag AG an der ETH Zurich)
Kerkens, B. 2002, A&A, 390, 783
Landi Degl’Innocenti, E., 1984, Solar Physics 91, 1
Landi Degl’Innocenti E., & Landolfi, M. 2004, Polarization in Spectral Lines (Dordrecht: Kluwer)
Deb & Derouich: Collisional effects

| line wavelengths (Å) | 10915 | 10327 | 10036 | 4216 | 4078 |
|----------------------|-------|-------|-------|------|------|
| $\bar{n}$            | $5.528 \times 10^{-3}$ | $4.6875 \times 10^{-3}$ | $4.3115 \times 10^{-3}$ | $1.757 \times 10^{-3}$ | $1.450 \times 10^{-3}$ |
| $w$                  | 0.0541 | 0.0636 | 0.06660 | 0.1754 | 0.1816 |
| Einstein Coefficient $A$ (s$^{-1}$) | $9.5 \times 10^{6}$ | $8.7 \times 10^{6}$ | $1.1 \times 10^{8}$ | $1.41 \times 10^{8}$ | $1.26 \times 10^{8}$ |

Table 1. $w$ and $\bar{n}$ are the anisotropy factor and the number of photons per mode for the Sr $\Pi$ lines.

López-Ariste, A.; Asensio Ramos, A.; Manso Sainz, R.; Derouich, M.; Gelly, B., 2009, A&A, 501, 729
Malherbe, J.-M.; Moity, J.; Arnaud, J.; Roudier, Th., 2007, A&A, 462, 753
Manso Sainz R., & Landi Degl’Innocenti E., 2002, A&A, 394, 1093
Milić I. & Faurobert M., 2012, A&A, 547, 7
NIST web page: [http://www.nist.gov/pml/data/asd.cfm](http://www.nist.gov/pml/data/asd.cfm)
Omont, A. 1977, Prog. Quant. Electron., 5, 69
Sahal-Bréchot S., 1977, ApJ., 213, 887
Seaton M J. Proc. Roy. Soc. A 1953, 218, 400
Seaton M J. Proc. Roy. Soc. A 1955, 231, 37
Stenflo, J.O. & Keller, C.U., 1997, A&A, 321, 927
Stenflo, J. O., 2004, Nature, 430, 304
Trujillo Bueno, J. 1999, in Solar Polarization, KN Nagendra and JO Stenflo. (Eds.), Kluwer Academic Publishers, 73
Trujillo Bueno, J.; Collados, M.; Paletou, F.; Molodij, G., 2001, in Advanced Solar Polarimetry - Theory, Observation, and Instrumentation, ed. M. Sigwarth (San Francisco: ASP), ASP Conf. Ser., 236, 141
Trujillo Bueno, J. 2001, in Advanced Solar Polarimetry - Theory, Observation, and Instrumentation, ed. M. Sigwarth (San Francisco: ASP), ASP Conf. Ser., 236, 161
Trujillo Bueno, J., Shchukina, N., & Asensio Ramos, A. 2004, Nature, 430, 326