Optical flat bands in 2D waveguide arrays with alternating sign of refraction index

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Abstract. We consider the coupled forward and backward waves propagating in two dimensional array of waveguide, which are featured by a positive and negative refraction indexes. The existence of the flat band under certain conditions is demonstrated.

1. Introduction

The successes of the last decade in the field of fabrication technology led to the development of optical materials with anticipated spatial characteristics, as well as with properties not found in nature. This includes materials with negative index of refraction. The advent of new materials made it possible to mimic various phenomena of physics by means of optics [1-3]. The most renowned example is transformation optics [4,5], where optical methods allow modelling the phenomenon of optical illusion, invisibility, gravitation and cosmology [3]. Works in this area include those dedicated to the photonic analogue of graphene [6], topological photon states and topological phase transitions [7], the optical Hall effect [8], and other phenomena, the list of which can be considerably extended.

Most optical analogues are based on the following idea: the electrons, localized in individual atoms, are attaining the ability of interatomic tunnelling after consolidation of atoms in the crystal lattice. Thus, they are delocalized, and their energy spectrum is considerably different from the spectrum of free electrons. In the strong-coupling approximation the electrons in the crystal lattice are characterized by Wannier functions that are localized on the lattice nodes and slightly overlap with Wannier functions of the near neighbour nodes. In the case of the electromagnetic field similar effect can be achieved by localizing field in waveguides. In this case, transfer waveguide modes serve as Wannier functions. If waveguides are consolidated in a bundle or in a one/two dimensional array, then due to the distortion of total internal reflection between adjacent waveguides, an electromagnetic wave is delocalized similar to electrons in strongly coupled atomic system. The spectrum of the electromagnetic field is changing; prohibited frequency gaps emerge in the spectrum, which give a basis to address such periodic structures as photonic crystals. However, it should be noted that in general the concept of photonic crystal is not limited to the case of strong coupling. In some cases the
electromagnetic wave propagates at an angle to the waveguides axes; this corresponds to the case of weak coupling.

Here we restrict ourselves by the electromagnetic waves propagating along the waveguide only; energy transfer between waveguides takes place only through the field tunnelling between adjacent waveguides (evanescent field). In this situation, the discrete nature of the system of ordered waveguides plays a significant role; this suggests addressing such systems as photonic lattices and introducing the concept of discrete photonics.

In this paper, we consider waveguides only as the "atoms" of the two dimensional photonic lattice, however it should be noted that there are other mechanisms of localization. An example is the localization of radiation in the micro-cavities, acting as an analogue of quantum dots [9,10].

2. 2D waveguide arrays

The study of two dimensional electron systems has demonstrated that the presence of the third node in the elementary sell as well as long range interaction in the lattice leads to emerging of a flat sheet (flat band) between conventional zones (see figure 1b). The spectrum of electronic states for two-dimensional square lattice (figure 1a) was obtained in [11,12].

An analytical expression for the energy is given by

\[
\begin{align*}
\epsilon_1(q) &= -t \sqrt{4 + 2 \cos(q_x a) + 2 \cos(q_y a)} \\
\epsilon_2(q) &= 0 \\
\epsilon_3(q) &= t \sqrt{4 + 2 \cos(q_x a) + 2 \cos(q_y a)}
\end{align*}
\]

here \( t \) is integral of hoping, \( a \) is lattice parameter (distance between nodes), \( q_x \) and \( q_y \) are projections of the wave vector on the orthogonal axes. The surface \( \epsilon_2(q) = 0 \) describes the flat band separating Dirac's points in the corners of the Brillouin zone. A similar structure for energy bands takes place in the case of Lieb lattice [13], figure 1c.

Such optical lattices can be realized by means of waveguides as nodes of the lattice. System of equations describing coupled waves in two dimensional arrays of waveguides shown in figure 2 has the following form

\[
\begin{align*}
i(\partial_t + \sigma_3 \partial_x) A_{n,m} + \alpha_1 (B_{n,m} + B_{n-1,m}) + \alpha_2 (C_{n,m} + C_{n,m-1}) &= 0, \\
i(\partial_t + \sigma_3 \partial_x) B_{n,m} + \alpha_1 (A_{n,m} + A_{n+1,m}) &= 0, \\
i(\partial_t + \sigma_3 \partial_x) C_{n,m} + \alpha_2 (A_{n,m} + A_{n,m+1}) &= 0.
\end{align*}
\]
Here $A_{n,m}$, $B_{n,m}$, and $C_{n,m}$ are dimensionless slowly varying amplitudes of coupled waves, sub-indices are numbering elementary cells. Phase matching condition is assumed to be satisfied, $\sigma_j = \pm 1$ stand for the sign of the index of refraction or projection of the wave vector on the axis of the $j$-th waveguide, $\alpha_1$ and $\alpha_2$ are the coupling constants.

Since system of equation (2) is linear, the dispersion relation can be found in a standard way. General expression for dispersion relation in the case of an infinite lattice can be obtained by solving the following cubic equation:

$$
(\omega - \sigma_1 k)(\omega - \sigma_2 k)(\omega - \sigma_3 k) - \gamma_1(\omega - \sigma_1 k) - \gamma_2(\omega - \sigma_2 k) = 0,
$$

(3)

\[
\gamma_1 = 4\alpha_1^2 \cos^2(k_x l/2), \quad \gamma_2 = 4\alpha_2^2 \cos^2(k_y l/2).
\]

Note that in the limit of an infinite lattice all mode indices turning into continuous variable – two dimensional vector with components $k_x l$, and $k_y l$, here $l$ is a distance between neighboring waveguides.

An analytical solution of equation (3) can be found if one of the waveguides (the first one) has the positive and other two the negative signs of refraction index: $\sigma_1 = 1$, $\sigma_{2,3} = -1$. Corresponding surfaces of the constant frequency (dispersion surfaces) are defined by the following expressions

$$
\omega_{k_{1,2}}(k_x, k_y) = \pm \sqrt{k^2 + 4\alpha_1^2 \cos^2(k_x l/2) + 4\alpha_2^2 \cos^2(k_y l/2)},
$$

(4)

$$
\omega_3(k_x, k_y) = -k.
$$

(5)

The surface determined by equation (5) at fixed value of $k$ describes a flat band.

For a waveguides with $\sigma_1 = -1$ and $\sigma_{2,3} = +1$, equation (4) remains unchanged, and the surface of the flat band is determined by the equation $\omega_3(k_x, k_y, k) = k$. In the case, when for all waveguides $\sigma_{1,2,3} = +1$, equation (3) has the solution:

$$
\omega_{k_{1,2}}(k_x, k_y) = k \pm 2\sqrt{\alpha_1^2 \cos^2(k_x l/2) + \alpha_2^2 \cos^2(k_y l/2)},
$$

(6)

$$
\omega_3(k_x, k_y) = k.
$$

(7)

Therefore there is always a flat sheet in the considered lattice of waveguides.
3. Conclusion
In this article, we have considered several phenomena of discrete photonic for two-dimensional arrays of waveguides. An analysis of the optical properties of discrete media structured with dielectric micro cavities or dielectric spheroids [9, 10] remained outside of this work.

Note that the above we considered quasi-harmonic electromagnetic waves with carrier frequency $\omega_0$ and with corresponding wave number $k_{x0}$. For this reason, all $\omega$ and $k$ are small with compare to $\omega_0$ and $k_{x0}$ respectively. Hence, expression (2) describes an electromagnetic wave with a narrow spectrum, localized in a small neighborhood of $(\omega_0, k_{x0})$. The existence of the flat band $\omega_3(0, k_x, k_y) = 0$ means that the part of the radiation carried by this mode is not involved in the diffraction or exhibits very small leakage to a neighboring waveguides. In the 2D case, if the frequency belongs to the flat band, then the electromagnetic wave is not experiencing the discrete diffraction.

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