Analysis of the parameter extraction for on-chip transmission lines

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Abstract  Matching networks for ultra-high frequency chips are susceptible, and accurate extraction of passive devices parameters is essential. For on-chip transmission lines, the authors present a precise method of extracting transmission line propagation constant from two transmission line measurement structures with different lengths. After obtaining the propagation constant based on this method, to get the transmission line’s characteristic impedance, this paper conducts a Y-Z model analysis of the signal pad and gives mathematical expressions. This approximation method can obtain the characteristic impedance of the transmission line without solving the signal pad’s model parameters. The propagation constant and characteristic impedance derived from the methods proposed are compared with the corresponding electromagnetic simulation results, achieving fairly good agreement from 250 MHz to 110 GHz.

Keywords: transmission line, on-chip matching components, 110GHz

Classification: Integrated circuits (memory, logic, analog, RF, sensor)

1. Introduction

With the development of millimeter-wave applications such as 60GHz wireless communications [1, 2] and 77GHz automotive radars [3, 4, 5], on-chip integrated systems have attracted much more attention. As a flexible and reliable passive element, the transmission line becomes quite attractive in implementing matching functions for RF (radio frequency) integrated circuits [6, 7]. However, the on-chip transmission lines need to be precisely modeled to support the corresponding designs [8, 9]. When designing a matching network using on-chip transmission lines, the basic parameters of the transmission lines, including the propagation constant and the characteristic impedance, firstly need to be known.

A significant problem with transmission line modeling based on the chip’s measurement results is the parasitic parameters introduced by the signal pad. For modeling an on-chip transmission line, its intrinsic characteristics first must be de-embedded from the raw measurement data, usually consisting of GSG (ground-signal-ground) pads and access lines [10, 11, 12, 13, 14, 15]. The traditional open-short de-embedding method has been proven to be unsuitable for ultra-high frequencies due to its incapability for dealing with distributed nature in millimeter-wave frequency range based on assumptions of lumped equivalent circuit and ideal dummy patterns [16, 17]. On the other hand, based on cascading distributive assumption, multi-line methods, including L-2L [18, 19, 20, 21], and thru-based approaches such as thru-only [22], have been widely used for on-chip transmission line de-embedding [23, 24]. The targets of the transmission line de-embedding include the determination of complex propagation constant and characteristic impedance. It has long been concluded that the propagation constants can be accurately extracted by using the above cascading de-embedding techniques, but the relevant procedure and expressions in some reported works are somehow complicated [25, 26]. In this letter, according to the mathematical theorem of similar matrices, a straightforward method [27] is used for accurately extracting the propagation constants of on-chip transmission lines in compact and straightforward expressions. The method only needs two transmission lines of different lengths without requiring much chip area. From a mathematical point of view, this method is a method for accurately solving the propagation constant, rather than requiring some degree of approximation on the measurement structures like open-short or L-2L. This article also shows that it is impossible to directly obtain the characteristic impedance of a transmission line by merely using the two transmission lines’ measurement data. In order to directly use the measurement results to get the characteristic impedance of the transmission line, it is necessary to do approximate modeling and analysis for the signal pad.

2. Presentation of the propagation constant extraction

The proposed method for extracting the propagation constant requires only two on-wafer transmission lines of different lengths. For example, as shown in Fig. 1, the chip 1 has a longer transmission line length \(l_1\), and the chip 2 has a shorter transmission line length \(l_2\). The two chips have the same structure except that the lengths of the transmission lines are different. First of all, the ABCD matrix is always taken as 2-port parameters for the simplicity of mathematical derivation and computation. It should be noted that all S-parameters or ABCD parameters used in this article are based on the standard 50-ohm ports as the reference. In this paper, the ABCD matrix obtained from the measurement result of the chip 1 (the long measurement structure) is named \(M\), and the chip 2 (the short measurement structure)
is $T$. And $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$, $T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$. Suppose the short measurement structure is divided into two parts that are mirror-symmetrical, with ABCD matrices as $L$ and $R$, respectively. And $L = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix}$, $R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$. There is a relationship below

$$T = LR$$

(1)

Meanwhile, the long line measurement structure $M$ can be expressed as a cascading multiplication of $L$, an intrinsic part of transmission line $F$ with a length of $l_F$ ($l_F = l_2 - l_1$), and $R$ as follows

$$M = LFR$$

(2)

In this work, following [27, 28, 29], by right multiplying matrix $M$ with the inverse of the matrix $T$, we can derive the so-called hybrid structure matrix $G$ as an expression of $F$ left and right multiplied by $L$ and the inverse of $L$, respectively, below and also illustrated in Fig. 1.

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = MT^{-1} = LFR(LR)^{-1} = LFL^{-1}$$

(3)

Besides, as an ABCD matrix for a transmission line [30], the matrix $F$ can also be presented as

$$F = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \begin{bmatrix} \frac{\text{cosh}(\gamma F)}{\text{sinh}(\gamma F)} & \frac{Z_c \sinh(\gamma F)}{Z_c} \\ \frac{\text{sinh}(\gamma F)}{\text{cosh}(\gamma F)} & \frac{\text{cosh}(\gamma F)}{\text{sinh}(\gamma F)} \end{bmatrix}$$

(4)

Where $\gamma$ represents the complex propagation constant, $l_F = l_2 - l_1$ is the physical length of the transmission line (the length between the dotted lines in Fig. 1), and $Z_c$ stands for the characteristic impedance of the transmission line. Because the relationship between the matrix $G$ and the matrix $F$ is as (3), that means the matrix $G$ and the matrix $F$ are similar matrices, so they have the same trace which is defined as the summation of all the diagonal elements according to the mathematical theorem of similar matrices [31], giving $g_{11} + g_{22} = f_{11} + f_{22}$. As the transmission line between the dotted lines (Fig. 1) is symmetric, it is easy to know that $f_{11} = f_{22}$. From (4) and $g_{11} + g_{22} = f_{11} + f_{22}$ there is

$$f_{11} = f_{22} = \frac{\text{cosh}(\gamma F)}{2}$$

(5)

Matrix $T$ and matrix $M$ can be obtained from the short and long transmission line chips’ measurement results. And $l_1$ and $l_2$ are also known values from layout design and fabrication information. Then the complex propagation constant of the transmission line can be obtained from (6) as below

$$\gamma = \alpha + j\beta = \frac{\text{arcosh}(f_{11})}{l_F} = \frac{\text{arcosh}(\frac{g_{11} + g_{22}}{2})}{l_F}$$

(6)

The real and imaginary parts of $\gamma$ obtained from (6) are normally denoted as attenuation constant $\alpha$ and phase constant $\beta$, respectively.

From (1) and (2), there is

$$L^{-1}T = L^{-1}LR = F^{-1}L^{-1}LFR = F^{-1}L^{-1}M$$

(7)

Note that, as a good approximation, all the measurement structures in this work can be assumed to be reciprocal without losing any accuracy. As $L$ and $R$ are port swap forms of each other, there exist three unknown variables for describing them, then $l_{12} = l_{21}$, there is $L^{-1} = \begin{bmatrix} l_{22} & -l_{12} \\ l_{12} & l_{11} \end{bmatrix}$. From (4), we can also get $F^{-1}$ = \[
\begin{bmatrix}
\frac{\text{cosh}(\gamma f)}{\text{sinh}(\gamma f)} & -\frac{Z_c \sinh(\gamma f)}{Z_c} \\
\frac{\text{sinh}(\gamma f)}{\text{cosh}(\gamma f)} & \frac{\text{cosh}(\gamma f)}{\text{sinh}(\gamma f)}
\end{bmatrix}
\]

According to (7), there is

$$\begin{bmatrix} l_{22} & -l_{12} \\ l_{12} & l_{11} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

(8)

A set of homogeneous equations can be derived from (8), as follows

$$m_{11} \sinh(\gamma f) Z_c + m_{21} \sinh(\gamma f) Z_c l_{11} l_{12} = 0$$

$$m_{12} \sinh(\gamma f) Z_c + m_{22} \sinh(\gamma f) Z_c l_{11} l_{22} = 0$$

$$m_{11} \cosh(\gamma f) Z_c + (t_{11} - m_{11} \cosh(\gamma f)) Z_c l_{11} l_{12} = 0$$

$$m_{12} \cosh(\gamma f) Z_c + (t_{21} - m_{12} \cosh(\gamma f)) Z_c l_{11} l_{22} = 0$$

$$m_{11} \cosh(\gamma f) Z_c + (m_{11} \cosh(\gamma f) - t_{21}) Z_c l_{11} l_{12} = 0$$

$$m_{12} \cosh(\gamma f) Z_c + (m_{12} \cosh(\gamma f) - t_{21}) Z_c l_{11} l_{22} = 0$$

$$m_{11} \cosh(\gamma f) Z_c + (m_{11} \cosh(\gamma f) - t_{21}) Z_c l_{11} l_{12} = 0$$

$$m_{12} \cosh(\gamma f) Z_c + (m_{12} \cosh(\gamma f) - t_{21}) Z_c l_{11} l_{22} = 0$$

We can get $\gamma$ using (6), but there are still four unknowns in (9), including $Z_c$, $l_{11}$, $l_{12}$, $l_{22}$. It can be proved that the exact solution of $Z_c$ cannot be obtained [24] according to the homogeneous equations (9). This also proves that the direct use of the measurement data of two (or more) transmission line chips cannot directly solve the characteristic impedance mathematically. Therefore, to obtain the characteristic impedance of the on-chip transmission line in this paper, we need to use other methods to solve it. In the next part, we will approximate the characteristic impedance
of the transmission line by constructing a signal pad Y-Z model. And through mathematical analysis, directly find the expressions of characteristic impedance.

3. Approximate solution of characteristic impedance

Suppose the signal pad approximately equivalent to a Y-Z model, as shown in Fig. 2, then there are

\[
\begin{array}{ccc}
1 & Z(\omega) & Y(\omega)
\end{array}
\begin{array}{ccc}
Y(\omega) & Z(\omega) & 1
\end{array} =
\begin{bmatrix}
\frac{\cosh(\gamma_1)}{Z_\text{c}} & \frac{\sinh(\gamma_1)}{Z_\text{c}} & Z_\text{c} \sinh(\gamma_1) \\
\frac{\sinh(\gamma_1)}{Z_\text{c}} & \frac{\cosh(\gamma_1)}{Z_\text{c}} & Z_\text{c} \cosh(\gamma_1)
\end{bmatrix}
\begin{bmatrix}
1 + Y(\omega)Z(\omega) & Z(\omega) & 1
\end{bmatrix} =
\begin{bmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{bmatrix}
\]

(10)

and

\[
\begin{array}{ccc}
1 & Y(\omega) & Z(\omega)
\end{array}
\begin{array}{ccc}
Z(\omega) & Y(\omega) & 1
\end{array} =
\begin{bmatrix}
\frac{\sinh(\gamma_1)}{Z_\text{c}} & \frac{\cosh(\gamma_1)}{Z_\text{c}} & Z_\text{c} \cosh(\gamma_1) \\
\frac{\cosh(\gamma_1)}{Z_\text{c}} & \frac{\sinh(\gamma_1)}{Z_\text{c}} & Z_\text{c} \sinh(\gamma_1)
\end{bmatrix}
\begin{bmatrix}
1 + Y(\omega)Z(\omega) & Z(\omega) & 1
\end{bmatrix} =
\begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix}
\]

(11)

Then the following equation (12) can be obtained from (10), and (13) can be obtained from (11).

\[
Z_\text{c} \sinh(\gamma_1) + 2 \cosh(\gamma_1)Z(\omega) + \frac{\sinh(\gamma_1)}{Z_\text{c}} Z^2(\omega) = t_{12}
\]

(12)

\[
Z_\text{c} \sinh(\gamma_2) + 2 \cosh(\gamma_2)Z(\omega) + \frac{\sinh(\gamma_2)}{Z_\text{c}} Z^2(\omega) = m_{12}
\]

(13)

We use (12) and (13) to reduce Z(\omega). Finally, a quadratic equation containing only one unknown Z_\text{c} is obtained as follows

\[
\left[4C_2^2S_2^2 - 8C_1C_2S_1^2S_2 + 4C_1^2S_1^2S_2^2\right]Z_\text{c}^2 + \\
\left[4m_{12}C_1S_1\left[C_2S_1 - C_1S_2\right] + 4t_{12}C_2S_1\left[C_1S_2 - C_2S_1\right]\right]Z_\text{c} +
\left[2m_{12}^2C_1^2S_1^2 - 4t_{12}m_{12}C_1^2S_1^2S_2 + 2t_{12}^2C_1S_1S_2^2\right] = 0
\]

(14)

In (14), for convenience of expression, C_1 = \cosh(\gamma_1), C_2 = \cosh(\gamma_2), S_1 = \sinh(\gamma_1), S_2 = \sinh(\gamma_2). Since gamma is already known by using the method of the last subsection, (14) is a quadratic equation with one unknown of Z_\text{c}. Solving the unary quadratic equation (14) can get two solutions of Z_\text{c}. So it is necessary to select artificially to choose a solution that meets the actual physical meaning.

4. Results and discussions

In this work, two microstrip transmission line test chips are designed and fabricated based on HLMC (Huail microelectronics corporation) 40 nm CMOS (complementary metal oxide semiconductor) process. Both the two structures have the same signal pads and ground pads. The thickest metal (3.3 \mu m) as the top interconnect layer serves as the signal line, and the two bottom metal layers (M1 and M2) are connected together as the ground. Fig. 3 shows the microscope photos of the two microstrip line chips.

The length of the microstrip line in the short pattern in Fig. 3(a) is 233 \mu m, and the width of the signal line is 3 \mu m. The long line pattern in Fig. 3(b) is the same as the short pattern in Fig. 3(a) except that the microstrip length is 833 \mu m. The S parameters of all the test structures are measured by using the instruments from Keysight Technologies, including network analyzer N5247A and the spread spectrum module N5250CX10. The measured S parameters are then converted to the ABCD parameters for further data processing in this paper.

According to (3) and (6), the \alpha and the \beta values of the intrinsic part of the microstrip line of a length of 600 \mu m (833 \mu m – 233 \mu m) are calculated and plotted for the frequency range from 250 MHz to 110 GHz as shown in Fig. 4(a) and Fig. 4(b) respectively. And the corresponding electromagnetic simulation results of the microstrip line given by a three-dimensional planar electromagnetic simulator are also shown as the comparison in Fig. 4. As seen from the figures, reasonably good agreement is obtained between the calculated and simulated results verifying the validity of the method proposed in this work. And the \gamma value obtained by

![Fig. 2](image)

Fig. 2 The signal pad is assumed to be Y(\omega) + Z(\omega).

![Fig. 3](image)

Fig. 3 Microscope photos of two microstrip line measurement structures. (a) Short measurement structure including a 233 \mu m (l_1 \times 3 \mu m) microstrip line. (b) Long measurement structure including an 833 \mu m (l_2 \times 3 \mu m) microstrip line.
Fig. 4  Comparison between our method and electromagnetic simulation. (a) \( \sigma \) of the microstrip line versus frequency from 250 MHz to 110 GHz. (b) \( \beta \) of the microstrip line versus frequency from 250 MHz to 110 GHz.

Fig. 5  Comparison between \( Z_c \) simulation and \( Z_c \) results derived from signal pad Y-Z model in this paper, thru-only in [29] and method in [32], from 250 MHz to 110 GHz.

using the similarity matrix in this paper is entirely consistent with the result obtained by the thru-only in [29]. In [29], the A and D expression in the ABCD matrix of the device under test are the same as the equation (5) in this letter. So the result of \( \gamma \) happens to be the same with this paper’s method. But unlike the work in [29], this article does not use any equivalent lumped parameter model in solving \( \gamma \) but directly uses the mathematical relationship of the matrices to solve \( \gamma \).

Fig. 5 shows the \( Z_c \) comparison between the results obtained from (14) in this paper, electromagnetic simulation, thru-only in [29], and an approximate method in [32]. When the frequency is low, the \( Z_c \) results obtained by this paper, thru-only [29], and the formula in [32] are almost coincident. The approximation method introduced in [32] is also solved under the known propagation constant, but this method approximates the test structures at both ends to be symmetrical. For this article, using the formula in [32] to solve \( Z_c \) is equivalent to treating the signal pads connected to the microstrip line as symmetric. The magnitude of \( Z_c \) decreases first and then becomes bigger with increasing frequency, which is not monotonic. The \( Z_c \) result of this paper is in good agreement with the \( Z_c \) result obtained by the thru-only solution [29], and only small differences appear at very high frequencies. However, this article directly gives the mathematical expression of the \( Z_c \) result based on the signal pad Y-Z model under the condition of known \( \gamma \). Different from [29], this article performed a Y-Z model on the signal pad itself, because the gamma was obtained first, and the transmission line part retained the transmission line expression. The thru-only method in [29] is to construct the Y-Z model of the thru structure directly.

The pad Y-Z model method used in this paper to solve \( Z_c \) requires at least two transmission line measurement chips, it can better reflect the parasitic effect and avoid the math problem of transmission line resonance. However, there are two solutions in (14). It is necessary to manually select a solution that meets the actual physical meaning of the characteristic impedance. As shown in Fig. 5, the solution 2 of the Y-Z model can be in good agreement with the other results, and the solution 1 deviates from the actual deviation too much. So the solution 1 is abandoned, and solution 2 is used as the wanted solution.

5. Conclusions

The equation set as the mathematical basis of the multi-line and thru-based de-embedding techniques have been analyzed, proving that the transmission line’s complex propagation constant can be rigorously determined. At the same time, the characteristic impedance is impossible to be strictly obtained in theory. According to the mathematical theorem of similar matrices, a simple method for precisely extracting the propagation constants of on-chip transmission lines has been used. Besides, to approximate the characteristic impedance of the transmission line simultaneously, model assumptions have been made on the signal pad. Based on the signal pad Y-Z model, using the obtained complex propagation constant, an expression for deriving the characteristic impedance is obtained without solving the pad model. Finally, the propagation constant for an on-chip microstrip line has been extracted from the measurement results of two microstrip line test structures with different lengths. As a result, the derived \( \gamma \) result using the proposed method has been found to agree reasonably well with the corresponding electromagnetic simulation data from 250 MHz to 110 GHz, which confirms our method’s validity. Combined with the signal pad’s Y-Z model assumption, the reasonable solution of the characteristic impedance solved by the pad Y-Z model agrees well with the other methods from 250 MHz to 110 GHz.
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