Observational constraints on Tsallis holographic dark energy with Ricci horizon cutoff

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Abstract

In this research, we are interested in constraining the nonlinear interacting and noninteracting Tsallis holographic dark energy (THDE) with Ricci horizon cutoff by employing three observational datasets. To this aim, the THDE with Ricci horizon considering the noninteraction and nonlinear interaction terms will be fitted by the SNe Ia, SNe Ia+H(z), and SNe Ia+H(z)+GRB samples to investigate the Hubble ($H(z)$), dark-energy equation of state ($\omega_{DE}$), effective equation of state ($\omega_{eff}$), and deceleration ($q$) parameters. Investigating the $H(z)$ parameter illustrates that our models are in good consistency with respect to observations. Also, it can reveal the turning point for both noninteracting and nonlinear interacting THDE with Ricci cutoff in the late-time era. Next, the analysis of the $\omega_{DE}$ for our models displays that the dark energy can experience the phantom state at the current time. However, this lies in the quintessence regime in the early era and approaches the cosmological constant in the late-time epoch. Similar results will be given for the $\omega_{eff}$ parameter with the difference that the $\omega_{eff}$ will experience the quintessence region at the current redshift. In addition to the mentioned parameters, the study of the $q$ parameter indicates that the models satisfy an acceptable transition phase from the matter- to the dark energy-dominated era. After that, the classical stability ($v^2_s$) will be analyzed for our models. The $v^2_s$ shows that the noninteracting and nonlinear interacting THDE with Ricci cutoff will be stable in the past era but unstable in the present and progressive epochs. Then, we will employ the $Jerk$ ($J$) and $OM$ parameters to distinguish between our models and the $\Lambda CDM$ model. Finally, we will calculate the age of the Universe for the THDE and nonlinear interacting THDE with Ricci as the IR cutoff.

Keywords THDE · Ricci cutoff · Observational constraints · Nonlinear interaction term · Deceleration parameter · Turning point · Stability · Jerk parameter · $OM$ diagnostic · Age of the Universe

1 Introduction

Today, analysis obtained from SNe Ia (Riess et al. (1998), Perlmutter et al. (1998, 1999), Astier et al. (2006)), CMB (Spergel et al. (2003, 2007), Ade et al. (2014), Komatsu et al. (2011), Bennett et al. (2013)), and BAO (Eisenstein et al. (2005)) data confirms that the Universe is expanding with acceleration features. This characteristic of the Universe can be described by introducing a source of energy with repulsive gravitational behavior (caused by negative pressure) called dark energy (DE). Although the nature of dark energy is not clear, some candidates, such as the cosmological constant $\Lambda$, are counted as the most favorable dark energy candidate with a famous equation of state (Eos) parameter -1. However, it suffers coincidence and fine-tuning (Copeland et al. (2006), Quartin et al. (2008), Li et al. (2011), Peebles and Ratra (2003), Weinberg (1989)) problems. Therefore, many alternative dark-energy candidates have been introduced to address these difficulties.

Hence, one of the most significant dark-energy models suggested by Li (2004) is called the holographic dark-energy (HDE) model. This model is based on the Cohen et al. (1999) suggestion representing that in the effective quantum field theory, the short-distance or UV cutoff ($\Lambda$) is related to the long-distance or IR cutoff ($L$) resulting from the limit set by the black-hole information. Thus, the quantum zero-point energy ($\rho_{\Lambda}$) of a system with size $L$ should not be more than the mass of a black hole of the same size as

$$L^3\rho_{\Lambda} = M^2_{pl}L.$$
From this relation, Li (2004) introduced the following holographic dark-energy density:

\[ \rho_{HDE} = 3c^2 M_p^2 \Omega_{m} L^{-2}, \]

where \( c \) is a numerical constant and \( L \) is the size (or horizon) of the current Universe. Li (2004) examined the HDE model with particle and future-event (FE) horizons. The author found that the \( \omega_{DE} \) for \( c = 1 \) and \( c < 1 \) with the FE cutoff will be -1 and \( \omega_{DE} \) will be -0.91 and the holographic dark-energy density:

\[ \rho_{HDE} = 3c^2 M_p^2 \Omega_{m} L^{-2}. \]

Later, Huang and Gong (2004) constrained the \( \Omega_m \) and \( \omega_{HDE} \) and \( z_{ir} \) (the transition redshift) parameters using the supernova type-Ia data for fixed \( c = 1 \) and free \( c \) for the HDE with FE cutoff. The fitted parameters were measured as \( \Omega_m = 0.25, \omega_{HDE} = -0.91 \), and \( z_{ir} = 0.72 \) for the model with fixed \( c = 1 \) and \( \Omega_m = 0.46, \omega_{HDE} = -2.67, \)

\[ c = 0.21, \] and \( z_i = 0.28 \) for the model with nonfixed \( c \).

Apart from holographic dark energy, in 2007, Cai introduced the agegraphic dark energy (ADE) by applying the quantum fluctuations of spacetime and the Károlyházy relation. In this model, the author applied the age of the Universe as the infrared cutoff led to the ADE density described by (see Ref. Cai (2007))

\[ \rho_q = 3n^2 M_p^2 T^{-2}. \]

Then, the \( \omega_{DE} \) parameter for the ADE model in the Universe filled by a dark-energy component was derived. The author found that if \( n > 1 \), the Universe will be in the accelerated expansion phase. Moreover, he measured the present value of \( \omega_{DE} (\omega_{DE0}) \) for the Universe consisting of matter and ADE components with the result of \( \omega_{DE0} \) \( \leq -0.81 \) for \( n \geq 3 \).

Then, Wei and Cai (2008) using the metric fluctuations, the Károlyházy relation, and the conformal time \( (\eta) \) proposed a new agegraphic dark energy (NADE) with \( \rho_q = 3n^2 M_p^2 \eta^{-2} \). The authors investigated the \( \omega_{DE} \) and \( \Omega_{DE} \) parameters in the NADE with the noncoupling term. Then, in the continuation of their work, they calculated the \( \Omega_{DE} \) parameter with coupling ingredient between dark sectors. The noninteracting model implies \( \omega_{DE} = -\frac{3}{4} \) and \( \Omega_{DE} = \frac{3n^2}{4} \) in the matter-dominated epoch and \( \omega_{DE} = -\frac{1}{4} \) and \( \Omega_{DE} = n^2 q^2 \) in the radiation era. Hence, the \( \omega_{DE} \) in this noncoupling model cannot enter the phantom state. Despite this, the NADE with the interacting part leads the DE to pass the phantom dividing line.

Subsequently, Setare (2010) studied the NADE in the framework of f(R) gravity. First, he derived the equation-of-state parameter for the model and observed that the \( \omega_{DE} \) can pass the -1 for \( n < 0 \). Then, he reconstructed the \( f(R) \) function using the general \( f(R) \) gravity action and new agegraphic dark-energy density.

A few years later, Tavayef et al. (2018) introduced a new holographic dark energy deriving the nonadditive entropy for complex systems, including black holes (Tsallis and Cirto (2013)). They employed the Tsallis horizon entropy as

\[ S_\delta = \gamma A^\delta, \]

where \( \delta \) and \( \gamma \) are nonadditivity and unknown constant parameters, respectively. Applying the holographic principle, the relation between IR and UV cut-offs, and Eq. (1) gives \( \Lambda^4 \leq (\gamma (4\pi)^{\delta}) L^{2\delta-4} \).

Here, \( \Lambda^4 \) is the vacuum energy density that creates the acceleration of the Universe.

Hence, the new holographic dark-energy density is given by

\[ \rho_{DE} = B L^{2\delta-4}, \]

called Tsallis holographic dark energy (THDE). The authors applied the THDE with Hubble horizon assuming \( \delta = 1.4, 1.5, \) and \( 1.7 \) with \( \Omega_{DE0} = 0.70 \) and \( \Omega_{DE0} = 0.73 \) to measure the \( \omega_{DE} \), \( \Omega_{DE} \), \( q \), and \( \omega_{eff} \). The results show that at \( z \rightarrow \infty \) \( (z \rightarrow -1) \) \( \Omega_{DE} \rightarrow 0 \) \( (\Omega_{DE} \rightarrow 1) \), and \( \omega_{DE} \rightarrow 1 - \delta \) \( (\omega_{DE} \rightarrow -1) \). Also, for \( \delta = 2 \), the \( \omega_{DE} \) is -1. From studying the \( q \) parameter, \( z \rightarrow \infty \), \( q \rightarrow 0.5 \), while at \( z \rightarrow -1 \), \( q \) tends to -1. Additionally, the model has been studied in terms of \( v_2^2 \), concluding that the model is unsustainable for \( \delta > 1 \) and stable for \( \delta \leq 1 \). Moreover, the age of the current Universe was estimated for the model. For instance, \( t \approx \frac{1.5}{\pi} \) when \( \delta = 1.7, \Omega_{DE0} = 0.70, \) and \( \omega_{DE} \approx -0.87 \). This work with the Hubble cutoff satisfies the accelerating state of the Universe.

After this work, Abdollahi Zadeh et al. (2018) studied the THDE with various IR cutoffs, including the particle, Ricci, and Granda–Oliveros (GO) horizons for the noninteracting and (linear) interacting cases. Although the Hubble cutoff has been chosen for only the interacting THDE model, the authors studied the evolution of the \( \Omega_{DE}, \omega_{DE}, q, \) and \( v_2^2 \) in models with \( \Omega_{DE0} = 0.73 \) and \( H_0 = 67 \). Here, we will present some results, such as \( \omega_{DE} \) and \( v_2^2 \) features. The evolution of the \( \omega_{DE} \) states that the model with Hubble cutoff for \( b_2 = 0.03, 0.04, 0.05, \) and \( \delta = 1.4 \) and in addition, the Ricci cutoff without \( Q \) (with \( Q \)) for \( \lambda = 1 \) and \( \delta = 1 \) \( \lambda = 1 \) with \( b_2 = 0.01 \) and \( \delta = 1 \) can display passing from the quintessence to the phantom regime.

Then, the authors found the positive sign of \( v_2^2 \) at \( z \rightarrow \infty \) for the following models: First, the model with particle cutoff in the presence of a coupling term \( Q \) for different \( \delta \) values, \( b_2 = 0.1, B = 2.4, \) or various \( b_2^2 \) with \( \delta = 2.4, \) and \( B = 2.4 \). Moreover, the noninteracting and interacting models with GO cutoff taking \( \delta = 0.6, a = a = 0.8, \) and \( b = 0.5 \) and \( b_2^2 = 0 \) or \( b_2^2 = 0.01 \). Then, the THDE model with the Ricci cutoff with \( Q = 0 \) for \( \lambda = 1 \) and \( \delta = 1 \).

Later, Sadri (2019) constrained the THDE with Hubble and future event horizons using the Pantheon SN Ia, BAO,
CMB, GRB, and a local sample of H, for noninteracting and linear interacting cases (with $Q = 3Hb_0p$). The parameters that have been fitted are $H_0$, $\Omega_{DE}$, $\delta$, $b \neq 0$ or $b = 0$, $z_{tr}$, and Age (of the present Universe). Using the fitted values, the author surveyed the Information Criterion (IC), Alcock–Paczynski (AP) test, and stability situation in the THDE model. The results of IC show that selecting HDE as the reference model led to support for the THDE and ITHDE by observational data. From the AP test, the THDE and ITHDE with Hubble (future event) horizon separate from each other at $z = 1.2$ (1.5) and $z = 0.7$ (1.05). Moreover, the THDE with FE cutoff shows the lowest deviation versus the $\Lambda CDM$ and HDE rather than THDE with Hubble cutoff. Investigating stability showed that although models with Hubble cutoff in all eras are unsustainable (similar to Abdollahi Zadeh et al. (2018)), models with FE cutoff at $z \rightarrow -1$ will show sustainability.

Aly (2019) discussed the THDE model with GO cutoff in an $(n+1)$-dimensional FLRW Universe. The model is analyzed via cosmographic parameters, the statefinder pair, the $Om$ diagnostic, and the diagnostic parameter $L_n^m$ with $\alpha=0.8502$ and $\beta=0.4817$. The $Om$ plot shows the model is in a positive slope relating to the phantom era and the $L_n^m$ graph with $L_n^m \neq 0$ illustrates the model is not a $\Lambda CDM$ model. In addition, the author found the $\omega_{DE}$ and $V_s^2$ of the model for noninteracting and interacting cases. The $\omega_{DE0}$ is observed in the phantom regime in different redshifts, and $v_s^2$ has demonstrated a sustainable behavior with initial conditions $\Omega_{DE} = 0.70$, $H_0 = 73$, $\alpha = 0.8502$, and $\beta = 0.4817$.

Also, Saha and Ghose (2020) studied the correspondence between the THDE and Generalized Chaplygin Gas (GCG) in the framework of Compact Kaluza–Klein cosmology. On the ground, considering $-0.05 \leq b^2 \leq -0.03$ and $\delta = 2.2$, the $\Omega_{DE}$, $\omega_{DE(\text{eff})}$, $q$, and $v_s^2$ have been plotted. Moreover, the plane of $\omega_{DE(\text{eff})} - \Omega_{DE}$ with $b^2 > 0$ and $b^2 < 0$ (with $b^2 = \pm 0.20, \pm 0.25, \pm 0.28$) and fixed values of $\delta$ including 0.2, 0.3, 0.4, or 0.5 have been depicted. Also, the $v_s^2 - \Omega_{DE}$ plot has been graphed for different $\delta$ (for $0.3 \leq \delta \leq 0.5$), once by fixing $b^2 = -0.20$ and then $b^2 = -0.25$. Evolving the dark-energy EoS parameter versus redshift shows that for $b^2 < 0 (b^2 > 0)$, the $\omega_{DE(\text{eff})}$ has started from the phantom-like ($\omega_{DE(\text{eff})} < -1$) phase in the past to the quintessence (phantom) region at $\Omega_{DE} \rightarrow 0.80$. For the $\omega_{DE}$ with $b^2 < 0$ and $\delta = 2.2$, the dark energy behaves as the phantom at $z \rightarrow \infty$ and $z \rightarrow 0$, then approaches $-1$ at $z \rightarrow -1$. From the $v_s^2$ plots, the model is unstable for $\delta = 2.2$ and $b^2 < 0$, but it will be stable with $0.3 \leq \delta \leq 0.5$ and $b^2 > 0$. For $\delta > 0.63$, independent of the value of $b^2$, the sign of $v_s^2$ is negative.

Subsequently, Yadav (2021) regarded the THDE model in the Brans–Dicke scenario. To solve the field equations and the wave equation of the scalar field ($\phi$), the author selected the scalar field as a function of the scale factor and a constant as $\phi = a^n$. Then, solving the wave equation gives $n = \frac{5}{2}, n = \frac{3}{2}$, and $k = 0$. By choosing $\omega = -\frac{5}{2}$ and $n = -1$, the scalar field leads to consistent results. However, the relation between the scalar field and density means the conservation law for energy can not hold.

In another research work, Sharma and Srivastava (2021) worked on the THDE with Ricci horizon cutoff. The structure of this work is different from the Abdollahi Zadeh et al. (2018) project. Here, the authors took ansatz for the scale factor ($a = t^{\delta}e^\lambda$) to obtain the $H(t)$ (Hubble parameter), $\rho_{DE}$, $\rho_m$ (matter density), $P_{DE}$ (dark-energy pressure), $\Omega_{DE}$ (DE density parameter), and $v_s^2$. The plot of $\omega_{DE}$ with $\lambda = 1.5$ and $k = 0.6$ from $z \rightarrow \infty$ to $z \rightarrow 0$ indicates that the DE behaves as the quintessence (phantom) for $\delta = 1.8$ and $1.9$ ($\delta = 2.1$ and 2.2). However, they will tend to be $-1$ in the future. Also, supposing $\lambda = 1.5$ and $k = 0.6$ as fixed values, the sign of $v_s^2$ for $\delta = 1.8, 1.9$ ($\delta = 2.1, 2.2$) is $> 0$ ($< 0$) at $z \rightarrow \infty$, while it is $< 0$ ($> 0$) at $z \rightarrow 0$ and $z \rightarrow -1$. The results of $v_s^2$ for $\lambda = 1.0, 1.5$, and 2.0 as the free values show the $v_s^2 > 0$ ($v_s^2 < 0$) for $\delta = 1.8, 1.9$ ($\delta = 2.1, 2.2$) at $z \rightarrow \infty$ and $v_s^2 < 0$ ($v_s^2 > 0$) at $z \rightarrow 0$ and $z \rightarrow -1$. Then, the authors examined the THDE model with the quintessence, phantom, and k-essence fields. These fields can describe the expansion of the Universe with acceleration behavior.

Hence, motivated by these studies, we are interested in exploring the observational constraints on the Tsallis holographic dark energy regarding the Ricci scalar as IR cutoff without interaction and with nonlinear interaction components using three observational samples. For this purpose, we will fit the parameters of the models applying SNe Ia, SNe Ia+H(z), and SNe Ia+H(z)+GRB data. The confidence counters and likelihood plots (1D) for the fitted models will be graphed. Then, the models will be tested through the evolution of the Hubble plane against the observational data and searching for the turning-point feature. Additionally, we study the evolution of $\omega_{DE}$, $\omega_{DE(\text{eff})}$, and $q$ parameters. Following them, the $v_s^2$ will be studied. Next, $Jerk$ and $OM$ diagnostic parameters will be investigated. Finally, the age of the Universe will be calculated.

This manuscript is organized as follows: Sect. 2 represents the structure of the THDE model without interaction and with a nonlinear interaction term between dark matter and dark energy by considering the Ricci cutoff. In Sect. 3, the best-fitted values of the models for SNe Ia, SNe Ia+H(z), and SNe Ia+H(z)+GRB samples will be measured. In addition, we will examine the parameters’ accuracy of the models with confidence graphs and 1D likelihood plots. In Sect. 4, the Hubble graph will be studied. In Sect. 5, the dark-energy equation of state, effective equation of state, and deceleration parameters will be investigated. Section 6 is devoted to studying the classical stability, and Sect. 7 to the $Jerk$ and $OM$ diagnostic parameters. In Sect. 8, the age
of the present Universe will be estimated. Finally, the conclusion is given in the last section.

2 The models

The Universe with a flat homogenous and isotropic characteristic can be described by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric,
\[ ds^2 = -dt^2 + a^2(t)(dr^2 + r^2d\Omega^2), \]
in which, \( a(t) \) is the scale factor. Hence, The Universe obeys the following Friedmann equation:
\[ 3H^2 = \frac{1}{M^2_{pl}}(\rho_{DE} + \rho_m). \] (3)

Here, \( \rho_m \) and \( \rho_{DE} \) are the matter and dark-energy densities. These densities follow the conservative equation shown by
\[ \dot{\rho}_m + 3H(\rho_m + p_m) = Q, \] (4)
\[ \dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = -Q, \] (5)
where the matter is pressureless (or \( p_m = 0 \)) and dark energy is with \( P_{DE} = \omega_{DE} \rho_{DE} \). Here, \( Q \) is the interaction term between dark sectors. This term has been suggested by recent observational data (for example Wang et al. (2016), Sheykhi et al. (2012)) and can alleviate the coincidence problem (Bolotin et al. (2015), Copeland et al. (2006), Lee et al. (2006), del Campo et al. (2006), He and Wang (2008)). Therefore, many researchers have studied different types of interacting models, while in some cases, noninteracting and interacting models are compared (He and Wang (2008), Arévalo et al. (2012), Zhang and Liu (2014), Wang et al. (2014), Bolotin et al. (2015), Ebrahimim and Golchin (2016), Ebrahimim et al. (2017), Abdollahi Zadeh et al. (2018), Sadri et al. (2019), Sadri et al. (2020), Aljaf et al. (2021), Feizi Mangoudahi (2022), George (2022)).

In the previous works of the THDE with Ricci horizon cutoff, Abdollahi Zadeh et al. (2018) focused on the model with the noninteraction \( (Q = 0) \) and linear interaction \( (Q = 3h^2H(\rho_m + \rho_{DE})) \) terms. However, Sharma and Srivastava (2021) worked on the model with \( Q = 0 \). In the present research, we would like to explore the effects of the nonlinear interaction term \( Q = 3H\psi \frac{\dot{\rho}_m \rho_{DE} - \rho_m \dot{\rho}_{DE}}{\rho_m + \rho_{DE}} \) (Ebrahimim and Golchin (2016), Ebrahimim et al. (2017)) on the THDE with Ricci cutoff, although, we will focus on the model with \( Q = 0 \), as well.

The matter and DE densities with respect to \( \rho_{cr} \) (the critical energy density) will give us
\[ \Omega_m = \frac{\rho_m}{3M^2_{pl}H^2}, \quad \Omega_{DE} = \frac{\rho_{DE}}{3M^2_{pl}H^2}, \] (6)
which are the matter and dark-energy density parameters.

Now, we write the Ricci horizon cutoff (Gao et al. (2009))
\[ L = (\alpha H^2 + \beta \dot{H}) \frac{1}{\psi}, \] (7)
in which, \( \alpha = 2\beta \). Hence, from relations (2) and (7), the THDE density can be written as (Abdollahi Zadeh et al. (2018, 2020))
\[ \rho_{DE} = \lambda(2H^2 + \dot{H})^{(2-\delta)}, \] (8)
where, \( \lambda = B^{2-\delta} \). Derivative from Eqs. (3) and (6) with respect to \( X = \ln a \) and inserting Eq. (4), and \( \Omega_m = 1 - \Omega_{DE} \) for the noninteracting model we have
\[ \frac{d\Omega_{DE}}{dX} = (1 - \Omega_{DE})(2 \frac{1}{H} \frac{dH}{dX} + 3), \] (9)
and for the nonlinear model,
\[ \frac{d\Omega_{DE}}{dX} = (1 - \Omega_{DE})(2 \frac{1}{H} \frac{dH}{dX} + 3 - 3\psi \Omega_{DE}), \] (10)
where (Abdollahi Zadeh et al. (2018, 2020))
\[ \frac{dH}{dX} = H(\frac{3H^2 \Omega_{DE}}{H^2 \lambda^{1-\delta}} - 2), \] (11)

In the following section, we apply the obtained equations.

3 Constraints of the model parameters by observational data

In this section, we constrain the set of free parameters of the model \( (\theta) \) with \( \theta=(h, \Omega_{DE}, \delta, \lambda, \psi) \) for the nonlinear interacting THDE model and \( \theta=(h, \Omega_{DE}, \delta, \lambda) \) for the noninteracting THDE model by observational data.

Here, we employ the 1048 Supernovae type-Ia data (Scolnic et al. (2018)) called the Pantheon sample. Then, the Pantheon sample is combined with the 27 Hubble parameter measurements (see Table 1) as SNe Ia + H(z). Finally, the 162 GRB data (Demianski et al. (2017)) will be added to the Pantheon and H(z) data points (or SNe Ia + H(z) + GRB).

Next, to fit the models with the observational samples, we will minimize the \( \chi^2 \) function that is
\[ \chi(\theta)^2_{(SNe Ia)} = \sum_{i=1}^{1048} \left( \frac{\mu(z_i)_{obs} - \mu(z_i, \theta)_{th}}{\sigma_{\mu(z_i)_{obs}}} \right)^2, \]
\[ \chi(\theta)^2_{(H(z))} = \sum_{i=1}^{27} \left( \frac{H(z_i, \theta)_{th} - H(z_i)_{obs}}{\sigma_{H(z_i)_{obs}}} \right)^2, \]
and
\[ \chi(\theta)^2_{(GRB)} = \sum_{i=1}^{162} \left( \frac{\mu(z_i)_{obs} - \mu(z_i, \theta)_{th}}{\sigma_{\mu(z_i)_{obs}}} \right)^2, \]
where \( \mu(z)_{\text{obs}} \) and \( \mu(z)_{\text{th}} \) are the observed distance modulus and the theoretical distance modulus.

Also,

\[
\mu(z)_{\text{th}} = 5 \log_{10} d_L(z) + \mu_0, \quad \mu_0 = 42.38 - 5 \log_{10} h,
\]

\[
d_L(z) = (1 + z) \int_0^z \frac{dz}{H(z)}.
\]

The total \( \chi^2_{\text{min}}(\theta) \) for the datasets will be \( \chi^2_{\text{SNeIa}}(\theta) + \chi^2_{\text{GRB}}(\theta) \), and \( \chi^2_{\text{SNeIa}} + \chi^2_{\text{GRB}} + \chi^2_{\text{H(z)}} \).

Then, the likelihood function can be calculated with

\[
L_{\text{max}}(\theta) = e^{-\frac{\chi^2_{\text{min}}(\theta)}{2}}.
\]

Our fitting process is done by an algorithm including for...do loops in Maple software. We use for...do loop for every free parameter of the models. For example, in the noninteracting THDE with Ricci cutoff with four free parameters \((h, \Omega_{DE}, \delta, \lambda)\), we will use four for...do loops. In more detail, for \( \delta \) we have: for delta from 0.12 by 0.01 to 0.14 do. Other for...do loops are applied for \( \chi^2 \) function (e.g., eight for...do loops for SNe Ia data). To find the best fit values, first, we need to choose intervals for parameters of the models to put inside the for...do loops (such as 0.1 \( \leq \delta \leq 1.3 \) for the \( \delta \) parameter). By changing the values of intervals, the value of \( \chi^2_{\text{min}} \) will increase or decrease.

Finding the best-fit parameters needs time and hard-working attempts. However, after reaching them, we have to examine the results again. For instance, if our best fit values and the minimum value of \( \chi^2 \) extracted from 0.702 \( \leq h \leq 0.703 \) (with 0.001 step), 0.71 \( \leq \Omega_{DE} \leq 0.72 \) (with 0.0001 step), 0.12 \( \leq \delta \leq 0.14 \) (with 0.01 step), and 0.0004 \( \leq \lambda \leq 0.0006 \) (with 0.0001 step), with every change in steps we will determine if our results are true or not. If the results of fitting models show that our free values will remain inside intervals and \( \chi^2_{\text{min}} \) does not increase or decrease, then we could find a suitable and reliable best-fit result.

The results of the best fit values of the Tsallis holographic dark-energy (THDE) and nonlinear interacting Tsallis holographic dark-energy (NITHDE) models have been prepared in Tables 2 and 3. We have plotted the confidence levels (68.3\% and 95.4\%) and 1D likelihood graphs in Figs. 1 and 2.

### 4. \( H(z) \) plane and turning-point feature

The \( H(z) \) plane corresponds to the comparison of the model (or theory) with observational data points during the cosmic time. Apart from this, recently, some authors have examined an exotic feature on the Hubble plot (with respect to redshift) called a turning point. Colgáin and Sheikh-Jabbari (2021) searched for the turning point of the HDE model considering the arbitrary and best-fit values with the future event horizon. The results for the \( \Omega_{DE0} = 0.7 \), \( H_0 = 100 \), and \( c < 1 \) identified that the turning point for \( c < 1 \) is inevitable. However, the measurements of the observational data, including CMB+BAO and CMB+BAO+SNe Ia, indicate that CMB+BAO data has a turning point at \( z \approx 0.04 \), while CMB+BAO+SNe Ia leads to a turning point at \( z \approx -0.1 \). Investigating the turning point in the Barrow holographic dark energy (BHDE) with Hubble and FE cutoffs has been worked on by Huang et al. (2021). The authors discussed the BHDEF model with different \( C \) and \( \Delta \). The results illustrate that increasing the \( C \) (or decreasing the \( \Delta \) parameter) causes the turning point to move to low \( z \), while it vanishes at \( C \approx 4.45 \) (or \( \Delta \approx 0.04 \)). Furthermore, the authors noted that for \( C = 3.421 \pm 1.753 \) and \( \Delta = 0.094 \pm 0.094 \) measured by Anagnostopoulos et al. (2020) for Hz+SNe Ia data, the turning point does not exist. Thus, they concluded that the BHDE models will not have a turning point. Finally, Feizi Mangoudehi (2022) observed a turning point in...
Fig. 1 2D contour levels and 1D likelihood distributions for the best-fit parameters of the noninteracting THDE with Ricci cutoff

Table 2 Best-fit parameters of the noninteracting THDE with Ricci cutoff

| Parameter       | SNe Ia                        | SNe Ia+H(z)                     | SNe Ia+H(z)+GRB                  |
|-----------------|-------------------------------|---------------------------------|----------------------------------|
| $h$             | 0.7027+0.0029+0.0047          | 0.7026+0.0029+0.0046            | 0.7036+0.0029+0.0047             |
| $\Omega_{DE}$   | 0.719+0.019+0.031             | 0.719+0.019+0.030               | 0.662+0.017+0.027                |
| $\delta$        | 0.13+0.005+0.009              | 0.13+0.005+0.009                | 0.12+0.006+0.011                 |
| $\lambda$       | 0.0005+0.00030+0.00049        | 0.0005+0.00028+0.00047          | 0.0004+0.00024+0.00041           |
| $\chi^2_{min}$  | 1065.581                      | 1073.888                        | 1289.572                        |
Fig. 2 2D contour levels and 1D likelihood distributions for the best-fit parameters of the nonlinear interacting THDE with Ricci cutoff.

Table 3  Best-fit parameters of the nonlinear interacting THDE with Ricci cutoff

| Parameter | SNe Ia | SNe Ia+H(z) | SNe Ia+H(z)+GRB |
|-----------|--------|-------------|-----------------|
| $h$       | 0.7014±0.0031±0.0049 | 0.7025±0.0029±0.0046 | 0.7052±0.0030±0.0047 |
| $\Omega_{DE}$ | 0.612±0.019±0.030 | 0.658±0.018±0.029 | 0.606±0.016±0.026 |
| $\delta$  | 0.12±0.007±0.011 | 0.14±0.006±0.010 | 0.16±0.006±0.010 |
| $\lambda$ | 0.0004±0.000024±0.000040 | 0.0005±0.000029±0.000046 | 0.0005±0.000037±0.000059 |
| $\psi$    | 0.441±0.121±0.194 | 0.218±0.115±0.199 | 0.138±0.088±0.140 |
| $\chi^2_{min}$ | 1065.180 | 1074.033 | 1290.611 |
5 Some features of the model

This section analyzes the evolution of dark-energy EoS, effective EoS, and deceleration parameters against the redshift for the fitted parameters of the models.

The dark energy and the effective equation of state parameters for the THDE with Ricci cutoff with the consideration of $Q = 0$ and $Q \neq 0$ can be given by

$$w_{DE} = -1 - \frac{1 - \Omega_{DE}}{\Omega_{DE}} - \frac{2}{3\Omega_{DE}} \left( \frac{3H^2\Omega_{DE}}{\mu} \right)^{\frac{1}{3}} - 2 \right) \tag{12}$$

and

$$w_{eff} = \frac{P_{eff}}{\rho_{eff}} = \frac{P_m + P_{DE}}{\rho_m + \rho_{DE}} = \Omega_{DE}w_{DE}. \tag{13}$$

The deceleration parameter will be

$$q = -1 - \frac{dH}{H} = \frac{1}{H^2\lambda^{\frac{1}{3}}} [H^2\lambda^{\frac{1}{3}} - (3H^2\Omega_{DE})^{\frac{1}{3}}]. \tag{14}$$

The results of $w_{DE}$, $w_{eff}$, and $q$, for both THDE models with Ricci cutoff with $Q = 0$ and $Q \neq 0$, are plotted in Figs. 4–6.

From the $w_{DE}$ plot against redshift, we can analyze the present value of $w_{DE}$ ($w_{DE0}$) and survey the quintessential, phantom, or cosmological behavior of the DE during cosmic evolution. Hence, as we see in Fig. 4 (upper panel), in the THDE with the Ricci horizon without interaction term, the $w_{DE0} = -1.094$, $w_{DE0} = -1.095$, and $w_{DE0} = -1.232$ for SNe Ia, SNe Ia+H(z), and SNe Ia+H(z)+GRB samples, respectively. However, for the nonlinear interacting THDE with the Ricci cutoff, the $w_{DE0}$ is -1.262 (for SNe Ia), -1.198 (for SNe Ia+H(z)), and -1.395 (for SNe Ia+H(z)+GRB). Also, from Fig. 4, we can observe that for the THDE and nonlinear interacting THDE with Ricci cutoff as IR, the DE behaves as quintessence in the past and phantom at the current epoch. Then, dark energy will approach the $\Lambda$ state at a late time.

Our results of $w_{DE0}$ considering the THDE and nonlinear interacting THDE with Ricci horizon as IR cutoff lies between $-1.03 \leq w_{DE0} \leq -1.55$ obtained by Planck Collaboration (2020) for the $TT + LowE$, $TT^{clean} + LowE$, $TT, TE, EE + LowE, TT, TE, EE + LowE + Lensing$, and $TT, TE, EE + LowE + Lensing + BAO$ datasets.

Following the figures, we can recognize the phase of the Universe and the current value of the $w_{eff}$ in Fig. 5. From this plot, both the THDE with noninteraction and
(nonlinear) interaction terms with Ricci cutoff can experience the quintessence era at high and current redshifts. The models will enter the phantom region and then turn to move towards -1 in the future era. Continuing to analyze Fig. 5, the $w_{DE}$ is -0.787 (-0.772) for SNe Ia, -0.787 (-0.788) for SNe Ia+H(z), and -0.816 (-0.845) for SNe Ia+H(z)+GRB in the noninteracting THDE (nonlinear interacting THDE) model.

In addition to plotting the $w_{DE}$ and $w_{\text{eff}}$ parameters versus redshift, we have depicted the $q$ parameter against redshift in Fig. 6. From this figure, today’s value of the deceleration parameter can be recognized. Moreover, when the curve of $q$ reaches $q = 0$, we can find the transition redshift from the matter-dominated to the dark energy-dominated phase (or starting point of the acceleration phase of the Universe). Hence, we have found the value of $z_{tr}$ in our models directly from the plots in Fig. 6 measuring the redshift at $q = 0$.

Hence, for the THDE with Ricci cutoff with $Q = 0$ in the upper plot of Fig. 6, we have $q_0 = -0.680$, -0.681, and -0.724 and $z_{tr} = 0.510, 0.510, and 0.446$ for the SNe Ia, SNe Ia+H(z), and SNe Ia+H(z)+GRB samples, respectively. Furthermore, the lower plot of Fig. 6 shows the evolution of $q$ versus redshift for the nonlinear interacting THDE with Ricci as IR. From this panel, the $q_0$ and $z_{tr}$ will be obtained as -0.658 and 0.552 for SNe Ia, -0.682 and 0.517 for SNe Ia+H(z), and -0.768 and 0.427 for SNe Ia+H(z)+GRB sample.

Recently, Dheepika and Mathew (2022) constrained the THDE with GO cutoff with linear interaction term $Q = 3bH\rho_m$ using SNIa, OHD, SNIa+OHD, SNIa+OHD+CMB, SNIa+OHD+BAO, and SNIa+OHD+CMB+BAO data. The
authors studied the cosmological parameters, dynamical analysis, and thermodynamics of the model. However, here we only mention the results of \(q_0\) and \(z_{tr}\) calculated from the \(q\) plot against redshift for two datasets. The authors found the \(q_0 = -0.602\) and \(z_{tr} = 0.799\) for SNIa+OHD+CMB data and \(q_0 = -0.594\) and \(z_{tr} = 0.763\) for SNIa+OHD+CMB+BAO data.

In another work, Sadri (2019) constrained the THDE with the Hubble and future event cutoffs with \(Q = 0\) and \(Q = 3bH_{\rho m}\) for BAO+CMB, BAO+CMB+SNe Ia, BAO+CMB+SNe Ia+GRB, and BAO+CMB+SNe Ia+GRB+H samples. Here, the author fitted \(z_{tr}\) and showed \(z_{tr}\) for the THDE with Hubble (future event) horizon with 0.604 \(\leq z_{tr} \leq 0.677\) (0.502 \(\leq z_{tr} \leq 0.572\)), while for the linear interacting THDE with Hubble (future event) cutoff we have 0.503 \(\leq z_{tr} \leq 0.634\) (0.541 \(\leq z_{tr} \leq 0.649\)).

### Stability

The sign of the squared sound-speed parameter can be employed to find the stability of the models. The positive value of \(v_s^2\) shows sustainability, whereas the negative sign describes the unsustainability of the models. The \(v_s^2\) is represented by

\[
v_s^2 = \frac{dP_{DE}}{d\rho_{DE}} = H \rho_{DE} \omega_{DE}' + \omega_{DE}.
\]

For the THDE with Ricci cutoff with the noninteraction term,

\[
v_s^2 = \left(\frac{\Omega_{DE}}{2(1 - \Omega_{DE})} + 3(1 - \Omega_{DE})\right) \omega_{DE}' + \omega_{DE},
\]

and,

\[
v_s^2 = \left(\frac{\Omega_{DE}}{2(1 - \Omega_{DE})} - 3\psi \Omega_{DE}(1 - \Omega_{DE}) + 3(1 - \Omega_{DE})\right) \omega_{DE}' + \omega_{DE},
\]

for the nonlinear interaction case.

The \(\omega_{DE}'\) is

\[
\omega_{DE}' = -\frac{4(3H^2\Omega_{DE})^{1/3}}{3(2 - \delta)H\Omega_{DE}^2} \frac{dH}{dX} - \frac{2(3H^2\Omega_{DE})^{1/3}}{3\lambda} \frac{d\Omega_{DE}}{dX} + 2(3H^2\Omega_{DE})^{1/3} \frac{d\Omega_{DE}}{dX} + \frac{4(3H^2\Omega_{DE})^{1/3}}{3\lambda} \frac{dH}{dX} - \frac{Hd\Omega_{DE}}{dX} - \frac{3\Omega_{DE}^2}{2\Omega_{DE}^2}.
\]

Now, we would like to use the evolution of the square sound of speed to identify the stability (\(v_s^2 > 0\)) or instability (\(v_s^2 < 0\)) of our models during the evolution of the Universe (see Fig. 7). As is manifest, both the THDE models with a noninteraction and nonlinear interaction terms for the SNe Ia, SNe Ia+H(z), and SNe Ia+H(z)+GRB data display the stable models at the early time and unstable models in the current and future times.

Now, what we wish to do here is to compare the THDE with the Ricci cutoff between this manuscript and the Abdollahi Zadeh et al. (2018) project in terms of \(v_s^2\) (see Table 4). From this table, we observe that the noninteracting THDE with Ricci horizon in both works shows stability in the past and instability in the current and future eras. Although the linear interacting THDE with Ricci as L does not provide a stable model, the THDE with Ricci horizon regarding the nonlinear interaction term displays a sustainable
Table 4: Comparison of $v_s^2$ for the THDE model with Ricci as IR cutoff. Here, ITHDE and NITHDE are the linear and nonlinear interacting THDE with Ricci horizon, respectively.

| Model                        | Past | Present | Future | Ref.                        |
|------------------------------|------|---------|--------|-----------------------------|
| THDE (Ricci cutoff)          | $> 0$| $< 0$   | $< 0$  | This project                |
| NITHDE (Ricci cutoff)        | $> 0$| $< 0$   | $< 0$  | This project                |
| THDE (Ricci cutoff)          | $> 0$| $< 0$   | $< 0$  | Abdollahi Zadeh et al. (2018) |
| ITHDE (Ricci cutoff)         | $< 0$| $< 0$   | $< 0$  | Abdollahi Zadeh et al. (2018) |

7 Jerk and $OM$ diagnostic tool

What we investigate in this section is a survey of the evolution of the Jerk parameter and the $OM$ diagnostic to discriminate between the $LCDM$ and the noninteracting and nonlinear interacting THDE with Ricci horizon as IR cutoff.

7.1 The Jerk parameter

The *Jerk* parameter ($J$) is mainly a way to compare the models with the $LCDM$ model characterized by $J = 1$ in the flat Universe. Also, the *Jerk* parameter with a positive value can describe the acceleration phase of the Universe. The $J$ is expressed by

$$ J = \frac{\ddot{H}}{H^3} - 3q - 2. $$

Hence,

$$ J = \frac{dH}{H} \left( -4 + \frac{2(3H^2\Omega_{DE})^{\frac{1}{3}}}{H^2(2-\delta)\lambda^{\frac{1}{3}}} \right) $$

$$ + \frac{(3H^2\Omega_{DE})^{\frac{1}{3}}}{H^2(2-\delta)\Omega_{DE}\lambda^{\frac{1}{3}}} \frac{d\Omega_{DE}}{dX} - 3q - 2. $$

The Jerk parameter of the THDE without interaction and with nonlinear interaction term with Ricci as IR L (see relation 16) have been graphed in Fig. 8. As is clear, our models deviate from the $LCDM$ model from the past to the present. However, the models will approach the $LCDM$ model in the future. In addition, from Fig. 8, the present value of $J$ ($J_0$) is 1.574 (SNe Ia), 1.575 (SNe Ia+H(z)), and 1.916 (SNe Ia+H(z)+GRB) for the THDE with $Q = 0$, and 1.581 (SNe Ia), 1.652 (SNe Ia+H(z)), and 2.214 (SNe Ia+H(z)+GRB).
Table 5 Comparison of \( v_2 \) for the THDE model with different IR cutoffs. Here, ITHDE and NITHDE are the linear and nonlinear interacting THDE models, respectively.

| Model                        | Past  | Present | Future | Ref.                        |
|------------------------------|-------|---------|--------|-----------------------------|
| THDE (Ricci cutoff)          | > 0   | < 0     | < 0    | This project                |
| NITHDE (Ricci cutoff)        | > 0   | < 0     | < 0    | This project                |
| THDE (Ricci cutoff)          | > 0   | < 0     | < 0    | Abdollahi Zadeh et al. (2018) |
| ITHDE (Ricci cutoff)         | < 0   | < 0     | < 0    | Abdollahi Zadeh et al. (2018) |
| THDE (GO cutoff)             | > 0   | < 0     | < 0    | Abdollahi Zadeh et al. (2018) |
| ITHDE (GO cutoff)            | > 0   | < 0     | < 0    | Abdollahi Zadeh et al. (2018) |
| THDE (particle cutoff)       | < 0   | < 0     | < 0    | Abdollahi Zadeh et al. (2018) |
| ITHDE (particle cutoff)      | < 0   | < 0     | < 0    | Abdollahi Zadeh et al. (2018) |
| THDE (Hubble cutoff)         | < 0   | < 0     | < 0    | Sadri (2019)                |
| ITHDE (Hubble cutoff)        | < 0   | < 0     | < 0    | Sadri (2019)                |
| THDE (future event cutoff)   | < 0   | < 0     | > 0    | Sadri (2019)                |
| ITHDE (future event cutoff)  | < 0   | < 0     | > 0    | Sadri (2019)                |

Fig. 8 Evolution of \( \text{Jerk} \) parameter versus redshift for the noninteracting THDE with Ricci cutoff (the upper plot) and nonlinear interacting THDE with Ricci cutoff (the lower plot).

for the THDE with nonlinear interaction between dark sectors. It is interesting to note that the highest deviation with respect to the \( \Lambda \text{CDM} \) model at the present epoch relates to the nonlinear interacting THDE with Ricci scale as \( L \) with \( J_0 = 2.214 \) for SNe Ia+H(z)+GRB data, whereas the lowest deviation corresponds to the \( J_0 = 1.574 \) with SNe Ia sample for the THDE with Ricci horizon (with \( Q = 0 \)).

### 7.2 The \( OM \) Diagnostic

Here, we benefit from the \( OM \) as a geometrical diagnostic to show that the noninteracting THDE and nonlinear interacting THDE models will behave as the cosmological constant, quintessence, or phantom model in different redshifts. For this, if the evolution of the \( OM \) does not show any slope, the models are the \( \Lambda \text{CDM} \). However, deviation from constant \( OM \), namely positive slope or negative slope corresponds to the phantom or quintessence models. The relation of the \( OM \) diagnostic is given by (Sahni et al. (2008))

\[
OM = \left( \frac{H(z)}{H_0} \right)^2 - 1 - \frac{1}{(z + 1)^3} - 1.
\]

where \( H_0 \) is the present value of the Hubble parameter and in our model using the best-fit values of \( h \) in Tables 2 and 3, it will be equal to 100 \times \( h \).

The results of the \( OM \) diagnostic can be observed in Fig. 9. The solid “GreenYellow” line in this figure belongs to the \( \Lambda \text{CDM} \) model for the SNe Ia, SNe Ia+H, and the SNe Ia+H+GRB data (see Table 6).

In the upper panel related to the SNe Ia data for \( OM \) plot, the THDE with noninteraction term (written as the THDE in the panel) behaves as the quintessence at \( 0.540 < z \leq 4.5 \), and then it behaves as the phantom at \( -1 \leq z < 0.540 \). This panel also shows the quintessential behavior of the nonlinear interacting THDE with Ricci cutoff (written as the NITHDE...
in the panel) at $0.523 < z < 4.5$ and phantom behavior at $-1 < z < 0.523$.

However, the middle plot belongs to the evolution of the $O\M$ parameter with respect to the $z$ for our models (THDE and NITHDE models) with the SNe Ia+H sample. From this plot, the THDE with Ricci horizon experiences the quintessence phase in $0.577 < z < 4.5$ and the phantom phase at $-1 < z < 0.577$. However, the nonlinear interacting THDE with Ricci as IR L shows the quintessence region for the model at $0.572 < z < 4.5$ and phantom regime at $-1 < z < 0.572$.

Finally, the lower graph of Fig. 9 describes the $O\M$-z plot in our models for the SNe Ia+H+GRB dataset. From this plot, the THDE with Ricci cutoff ($Q = 0$) lies in the quintessence phase at $0.591 < z < 4.5$ and phantom state at $-1 < z < 0.591$. However, the nonlinear interacting THDE with Ricci scale as IR will behave as the quintessence at $0.582 < z < 4.5$ and phantom at $-1 < z < 0.582$.

### 8 The age of the Universe

Now, we estimate the Universe age for the noninteracting and (nonlinear) interacting THDE with Ricci horizon utilizing the best-fit values in Sect. 3. To measure the age of the Universe we use the following relation (Tavayef et al. (2018), Huang et al. (2019))

$$t = \int \frac{dH}{dH} \approx \frac{1}{2\H}(z=0) \int \frac{dH}{H^2} \approx \frac{1}{2 - \left(\frac{3H_0^2\O_{DE}}{H^2}\right)^{\frac{1}{\beta}}},$$

where $H_0 = \sqrt{\frac{\O_M}{\Lambda}}$. The age of the Universe $t$ is given by Eq. (17).

The results of the age of the Universe for the noninteracting THDE with Ricci as IR are obtained as $t \approx \frac{3.1}{H_0}$, $\frac{3.1}{H_0}$, and $\frac{3.6}{H_0}$ for the SNe Ia, SNe Ia + H(z), and SNe Ia + H(z) + GRB, respectively. Also, we have $t \approx \frac{2.9}{H_0}$, $\frac{3.1}{H_0}$, and $\frac{4.3}{H_0}$ for the SNe Ia, SNe Ia + H(z), and SNe Ia + H(z) + GRB samples for the nonlinear interacting THDE with Ricci cutoff. These results are comparable with the work done by Huang et al. (2019) for the THDE with Hubble cutoff with $t \approx \frac{2.7}{H_0}$ calculated using assumed values $\delta = 2.1$, $\alpha = 0.5$, $\beta = 0.002$, and $t \approx \frac{2.3}{H_0}$ extracted from the fixed $\delta = 2.1$, $\alpha = 0.7$, $\beta = 0.1$.

### 9 Conclusions

In this paper, we have studied the observational limits on the THDE and nonlinear THDE with the Ricci cutoff as IR L using the SNe Ia, SNe Ia+H(z), and SNe Ia+H(z)+GRB datasets. We have graphed the $1\sigma$ and $2\sigma$ confidence levels and one-dimensional likelihood distribution.

At the continuation of our work, we have depicted the $H(z) - z$ plot in which the theoretical function of $H(z)$ has been compared with 27 measurements of the Hubble parameter $H(z)$. This graph identifies the suitable agreement between our models and the observational data. Apart from this, the $H(z)$ graph has demonstrated the turning-point feature for our models in the future epoch.

After the $H(z)$ plot, we analyzed the $\omega_{DE}$, $\omega_{eff}$, and $q$ parameters against redshift for the models. Investigating the evolution of $\omega_{DE}$ versus $z$ elucidates a quintessence-like behavior of dark energy in the THDE and (nonlinear) interacting THDE with the Ricci horizon at an early epoch and phantom-like behavior in the current era. After that, dark energy moves toward the $\Lambda$ barrier state in the future era. The analysis of the $\omega_{eff}$ shows that this parameter passes the phantom divide line in the future and then tends to -1 at $z \rightarrow -1$.

The classical stability trajectories with respect to redshift indicate that the noninteracting and nonlinear interacting THDE with Ricci cutoff behaved stably in the early time and unstably in the present and future eras.

In the next step of our work, to distinguish the THDE and nonlinear interacting THDE models with the $\Lambda CDM$ model, we used the $J$ and $O\M$ parameters. Both parameters denote that our models deviate from the $\Lambda CDM$ model during the Universe evolution. The $J$ parameter indicates that the noninteracting and nonlinear interacting THDE models (with Ricci scale as L) will move toward the $\Lambda CDM$ model in the late-time era. However, the $O\M$ parameter shows more details than the $J$ parameter. From the $O\M$ diagnostic...
tool, we have found that our models had a quintessence nature in the past, while it behaves as the phantom in the future era.

In the final step, we calculated the age of the Universe for our models. The results show that our models lead to a cosmic age older than the age measured by observations. However, our models will not show the age problem.

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Declarations

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