In the left-right symmetric models, we can take the left-right symmetry to be the charge-conjugation and then impose a global symmetry under which the left- and right-handed fermion doublets carry equal but opposite charges. Consequently, we may introduce two Higgs bi-doublets to give the desired fermion mass spectrum. The global symmetry is identified to a Peccei-Quinn symmetry. We can introduce a complex scalar singlet to break the global Peccei-Quinn symmetry at a high scale. This symmetry breaking is also responsible for generating the heavy Majorana masses of the fermion singlets which have Yukawa couplings with the lepton and Higgs doublets. In this context, we can realize the double and linear seesaw to naturally explain the small neutrino masses. Our scenario can be embedded in the SO(10) grand unification theories.

I. INTRODUCTION

In the left-right symmetric models \[1\] based on the gauge group \(SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\), the left-handed fermions are \(SU(2)_L\) doublets as they are in the \(SU(3)_c \times SU(2)_L \times U(1)_Y\) standard model (SM) while the right-handed fermions (the SM right-handed fermions plus the right-handed neutrinos) are placed in \(SU(2)_R\) doublets. Usually, we need a Higgs bi-doublet to construct the Yukawa interactions between the left- and right-handed fermion doublets for generating the Dirac masses of the SM fermions. As for the small neutrino masses, they are given by the seesaw \[2\,3\], which also accommodates the leptonogenesis \[4\] to explain the matter-antimatter asymmetry in the universe. For example, we can extend the original left-right symmetric model \[1\], where the Higgs scalars include one bi-doublet and two doublets, by introducing three fermion singlets with a Majorana mass term. In the presence of the Yukawa couplings of the fermion singlets to the lepton and Higgs doublets, we can obtain the double/inverse \[5\,6\] and linear \[7\] seesaw for generating the small neutrino masses to revive the original left-right symmetric model \[8\].

The left-right symmetry can be the parity or the charge-conjugation. In the case that the left-right symmetry is the charge-conjugation, we can impose a global symmetry under which the left- and right-handed fermion doublets carry equal but opposite charges. This symmetry breaking is also responsible for generating the heavy Majorana masses of the fermion singlets which have Yukawa couplings with the lepton and Higgs doublets. In this context, we can realize the double and linear seesaw to naturally explain the small neutrino masses. Our scenario can be embedded in the SO(10) grand unification theories (GUTs).

II. THE MODEL

The scalar fields include two Higgs bi-doublets,

\[
\phi_1(1, 2, 2^*, 0) = \begin{bmatrix} \phi_{11}^0 & \phi_{12}^0 \\ \phi_{11}^\ast & \phi_{12}^\ast \end{bmatrix},
\]

\[
\phi_2(1, 2, 2^*, 0) = \begin{bmatrix} \phi_{21}^0 & \phi_{22}^0 \\ \phi_{21}^\ast & \phi_{22}^\ast \end{bmatrix},
\]

(1)

two Higgs doublets,

\[
\chi_L(1, 2, 1, -1) = \begin{bmatrix} \chi_L^0 \\ \chi_L^\ast \end{bmatrix},
\]

\[
\chi_R(1, 1, 2, -1) = \begin{bmatrix} \chi_R^0 \\ \chi_R^\ast \end{bmatrix},
\]

(2)

and one complex singlet,

\[
\sigma(1, 1, 1, 0).
\]

(3)
In the fermion sector, there are three neutral fermion singlets,
\[ S_R(1, 1, 1, 0), \tag{4} \]
besides three generations of quark and lepton doublets,
\[ q_L(3, 2, 1, \frac{1}{3}), q_R(3, 1, 2, \frac{1}{3}), \quad l_L(1, 2, 1, -1), \quad l_R(1, 2, 1, -1), \tag{5} \]
As the left-right symmetry is the charge-conjugation, the scalars and fermions will transform as
\[ \phi_{1,2} \leftrightarrow \phi_{1,2}^T, \quad \chi_L \leftrightarrow \chi_R, \quad \sigma \leftrightarrow \sigma, \quad q_L \leftrightarrow q_R^*, \quad l_L \leftrightarrow l_R^*, \quad S_R \leftrightarrow S_R. \tag{6} \]
We further impose a global symmetry, under which the fields carry the quantum numbers as below
\[ \begin{align*}
1 & \text{ for } q_L, q_R, l_L, l_R, S_R; \\
2 & \text{ for } \phi_1, \phi_2, \sigma^*; \\
0 & \text{ for } \chi_L, \chi_R.
\end{align*} \tag{7} \]
The full scalar potential then should be
\[ V = \mu_1^2 |\sigma|^2 + \mu_2^2 (|\chi_L|^2 + |\chi_R|^2) + \mu_3^2 \text{Tr}(\phi_1 \phi_2) + \lambda_1 |\sigma|^4 + \lambda_2 (|\chi_L|^4 + |\chi_R|^4) + \lambda_3 |\chi_L|^2 |\chi_R|^2 \\
+ \lambda_4 \text{Tr}(\phi_1^\dagger \phi_2) \text{Tr}(\phi_2^\dagger \phi_1) + \lambda_5 \phi_1^\dagger \phi_2^\dagger \phi_2 \phi_1 + \kappa_1 |\sigma|^2 (|\chi_L|^2 + |\chi_R|^2) + \rho_1 |\sigma|^2 \text{Tr}(\phi_1 \phi_2) \\
+ \kappa_2 \text{Tr}(\phi_1^\dagger \phi_2) \text{Tr}(\phi_2^\dagger \phi_1) + \rho_2 |\sigma|^2 \text{Tr}(\phi_1 \phi_2) + \tau_1 (|\chi_L|^2 + |\chi_R|^2) \text{Tr}(\phi_1 \phi_2) \\
+ \tau_2 (|\chi_L|^2 + |\chi_R|^2) \text{Tr}(\phi_1 \phi_2) + \alpha_1 |\sigma|^2 \text{Tr}(\phi_1 \phi_2) + \beta_1 |\sigma|^4 \text{Tr}(\phi_1 \phi_2) + \gamma_1 |\sigma|^4 \text{Tr}(\phi_1 \phi_2) + h.c. . \tag{8} \]
We also give the allowed Yukawa interactions,
\[ \mathcal{L}_Y = -g_L \bar{q}_L \phi_1 q_R - g_L \bar{q}_R \phi_2 q_L - h(i_L R_L S_R + i_R L_R S_R) - \frac{1}{2} g_s \sigma S_R S_R^c + h.c. . \tag{9} \]
Clearly, the lepton and quark doublets, the Higgs doublets, the Higgs bi-doublets, the fermion singlets and the scalar singlet can, respectively, belong to the 16_F, 16_R, 10_H_{1, 2}, 1_F, and 1_H representation in the SO(10) GUTs.

The symmetry breaking pattern is expected to be
\[ SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{PQ} \]
\[ \downarrow \langle \sigma \rangle \]
\[ SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \]
\[ \downarrow \langle \chi_R \rangle \]
\[ SU(3)_c \times SU(2)_L \times U(1) \]
\[ \downarrow \langle \phi_{1,2} \rangle \]
\[ SU(3)_c \times U(1)_{em} \tag{10} \]
We denote the VEVs by
\[ \langle \sigma \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle \chi_R \rangle = \left[ \begin{array}{c} v_2 \\ 0 \end{array} \right], \quad \langle \chi_L \rangle = \left[ \begin{array}{c} 0 \\ v_2 \end{array} \right], \tag{11} \]
\[ \langle \phi_{1,2} \rangle = \left[ \begin{array}{c} 0 \\ v_2 \end{array} \right], \tag{12} \]
which can be derived from the potential (6). Note that \( v_{11}, v_{12}, v_{21}, v_{22} \) and \( v_L \) should fulfill
\[ v = \sqrt{v_{11}^2 + v_{12}^2 + v_{21}^2 + v_{22}^2 + v_L^2} \approx 246 \text{ GeV}. \tag{12} \]

III. PECCEI-QUINN SYMMETRY

After the complex scalar singlet \( \sigma \) develops a VEV to spontaneously break the global symmetry, it can be described by
\[ \sigma = \frac{1}{\sqrt{2}} (f + \rho) \exp \left(i \frac{a}{f} \right). \tag{13} \]
Here \( \rho \) is the physical boson while \( a \) is the Nambu-Goldstone boson (NGB). In the presence of the \( \alpha, \beta, \gamma \)-terms in the potential (6), like the structure of the DFSZ [17] model, the NGB \( a \) can couple to the quarks,
\[ \mathcal{L} \supset - \frac{1}{2 f_\rho} (\partial_\mu a)^2 \sum_q \bar{q}_i \gamma^\mu \gamma_5 q \tag{14} \]
Therefore, through the color anomaly [18], the NGB \( a \) can pick up a tiny mass [13, 19],
\[ m_a^2 = N^2 \frac{Z}{(1 + Z)^2} \frac{f_\rho^2}{f^2} m_q^2, \tag{15} \]
where \( N = 3 \) for three families of the SM quarks while \( Z \approx m_u/m_d \). Clearly, the global symmetry is the PQ symmetry while the NGB becomes a pseudo NGB (pNGB)—the axion. We can conveniently express the axion mass as
\[ m_a = \sqrt{Z} \frac{f_\rho}{(1 + Z) f} m_q \approx 6.2 \text{ meV} \left( \frac{10^{12} \text{ GeV}}{f_\rho} \right), \tag{16} \]
where \( f_\rho \) is the axion decay constant. The PQ symmetry should be broken at a high scale to fulfill the theoretical and experimental constraints [20]. For example, the PQ symmetry breaking may happen before the inflation [21] to avoid the cosmological domain wall problem [22]. With an appropriate \( f_\rho \) the axion can act as the dark matter [23, 24].

IV. DOUBLE AND LINEAR SEESAW

After the symmetry breaking (10), the fermions will obtain their mass terms by their Yukawa couplings.
Specifically, the quarks and charged leptons can have the usual $3 \times 3$ Dirac mass matrices, i.e.

$$\mathcal{L} \supset -\bar{m}_u u_L u_R - \bar{m}_d d_L d_R - \bar{m}_e e_L e_R + \text{H.c.}, \quad (17)$$

with

$$\bar{m}_u = \frac{1}{\sqrt{2}} g_\nu y_{11} v_{11} + \frac{1}{\sqrt{2}} g_{22} v_{21},$$
$$\bar{m}_d = \frac{1}{\sqrt{2}} g_\nu y_{12} v_{12} + \frac{1}{\sqrt{2}} g_{22} v_{22},$$
$$\bar{m}_e = \frac{1}{\sqrt{2}} g_\nu y_{12} v_{12} + \frac{1}{\sqrt{2}} g_{22} v_{22}. \quad (18)$$

As for the neutral fermions including the left- and right-handed neutrinos and the fermion singlets, they will form a symmetric $9 \times 9$ mass matrix as below,

$$\mathcal{L} \supset -\bar{\nu}_L \nu_R - \frac{1}{\sqrt{2}} h v_L \bar{e}_L S_R - \frac{1}{\sqrt{2}} h v_R \bar{e}_R S_R$$
$$- \frac{1}{2} M_S \bar{S}_R S_R + \text{H.c.},$$
$$\quad = \frac{1}{2} \left[ \begin{array}{c} \bar{\nu}_L \\ \bar{e}_R \\ S_R \end{array} \right] \left[ \begin{array}{ccc} 0 & \bar{m}_\nu & h v_L \\ \bar{m}_\nu^T & 0 & h v_R \\ h^*_v & h^*_v & M_S \end{array} \right] \left[ \begin{array}{c} \nu_L \\ \nu_R \\ S_R \end{array} \right] + \text{H.c.}, \quad (19)$$

with

$$\bar{m}_\nu = \frac{1}{\sqrt{2}} y_1 y_{11} + \frac{1}{\sqrt{2}} y_2^* y_{21}, \quad M_S = \frac{1}{\sqrt{2}} g f. \quad (20)$$

For $\frac{1}{\sqrt{2}} h v_R$ and/or $M_S$ much bigger than $\bar{m}_\nu$ and $\frac{1}{\sqrt{2}} h v_L$, we can make use of the seesaw formula \([2]\) to derive the neutrino masses,

$$\mathcal{L} \supset -\frac{1}{2} m_\nu \nu_L \nu_R^T + \text{H.c.}, \quad (21)$$

where the mass matrix $m_\nu$ contains two parts,

$$m_\nu = \bar{m}_\nu \frac{1}{\sqrt{2}} h v_R M_S \frac{1}{\sqrt{2}} h v_R \bar{m}_\nu^T \bar{m}_\nu - (\bar{m}_\nu + \bar{m}_\nu^T \bar{v}_L \nu_R). \quad (22)$$

The first term is the double seesaw $[3, 4]$ for $M_S \gg \frac{1}{\sqrt{2}} h v_R$ or the inverse seesaw $[6]$ for $M_S \ll \frac{1}{\sqrt{2}} h v_R$. Clearly, the double seesaw should be our choice because of the large PQ symmetry breaking scale. As for the second term, it is the linear seesaw $[8]$. We can understand the double and linear seesaw by Fig. 1. In the double and linear seesaw context, the leptogenesis $[4]$ can be realized by the decays of the right-handed neutrinos $[8]$.

V. Summary

In this paper we embedded the PQ symmetry in a left-right symmetric theory and then in an $SO(10)$ GUT. In our model, the left- and right-handed fermion doublets can naturally carry equal but opposite PQ charges because the left-right symmetry is the charge-conjugation. This implies we need two Higgs bi-doublets with the PQ charge so that the fermions can obtain a desired mass spectrum. The PQ symmetry breaking is driven by a complex scalar singlet at a high scale. The invisible axion can pick up a tiny mass through the color anomaly. In our model, the PQ symmetry breaking is also responsible for generating the heavy Majorana masses of the fermion singlets to realize the double and linear seesaw. Therefore, the neutrino mass-generation is naturally related to the PQ symmetry breaking $[22]$.

Acknowledgement: I thank Manfred Lindner for helpful discussions. This work is supported by the Alexander von Humboldt Foundation.

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FIG. 1: Double seesaw (top) and linear seesaw (bottom).

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