A New Glauber Theory based on Multiple Scattering Theory

Masanobu YAHIRO, *) Kosho MINOMO, **) Kazuyuki OGATA, ***)
and Mitsuji KAWAI †

Department of Physics, Kyushu University, Fukuoka 812-8581, Japan

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Glauber theory for nucleus-nucleus scattering at high incident energies is reformulated so as to also be applicable to the scattering at intermediate energies. We test the validity of the eikonal and adiabatic approximations used in the formulation, and discuss the relationship between the present theory and the conventional Glauber calculations using either the empirical nucleon-nucleon profile function or the modified one including the in-medium effect.

§1. Introduction

Experiments with radioactive beams of unstable nuclei have opened new frontiers in nuclear physics. New features of unstable nuclei such as a halo structure were revealed; see, for example, Ref. 1). The Glauber theory2) has widely been used as a powerful tool for studies of reactions with unstable nuclei observed at intermediate energies such as 50 – 800 MeV/nucleon.3), 4) It was reported,5), 6) however, that some modifications of the nucleon-nucleon (NN) scattering profile functions in the eikonal approximation were necessary in order to reproduce the data at energies of less than 500 MeV/nucleon.5), 6) Such phenomenological modifications obviously require theoretical foundations. In this paper, we address this problem.

The Glauber theory describes the scattering of two nuclei P and A as collisions of all nucleons in P with those in A. The theory starts with the many-body Schrödinger equation

\[ [E - K - h_P - h_A - V]\Psi = 0, \]  

(1.1)

where \( V = \sum_{i \in P, j \in A} v_{ij} \) with \( v_{ij} \) the NN interaction potential, \( E \) is the energy of the total system, \( K \) is the kinetic energy operator of relative motion between P and A, and \( h_P (h_A) \) is the internal Hamiltonian of P (A). Assuming the adiabatic approximation for the internal motion of P and A and the eikonal approximation, the theory gives the scattering amplitude at high energies and small scattering angles as

\[ f_{\beta\alpha} = \frac{ik}{2\pi} \int db \, e^{i q \cdot b} \langle \Phi_{\beta} | 1 - \prod_{i,j}(1 - \Gamma_{\text{NN}}(b_{ij})) | \Phi_{\alpha} \rangle, \]  

(1.2)

where \( \hbar k \) (\( \hbar q \)) is the initial (transferred) momentum, \( b \) is the component of the

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*) E-mail: yahiro@phys.kyushu-u.ac.jp
**) E-mail: minomo@phys.kyushu-u.ac.jp
***) E-mail: ogata@phys.kyushu-u.ac.jp
†) E-mail: kawa2scp@kyudai.jp
relative coordinate $\mathbf{R}$ between the centers of mass of $\mathbf{P}$ and $\mathbf{A}$ perpendicular to $\mathbf{k}$, $\Phi_\alpha (\Phi_\beta)$ is the antisymmetrized internal wave function of the initial (final) channel, and $\mathbf{r}_{ij} = (\mathbf{b}_{ij}, z_{ij})$ is the displacement of $i$ from $j$ with $\mathbf{b}_{ij}$ ($z_{ij}$) the component of $\mathbf{r}_{ij}$ perpendicular (parallel) to $\mathbf{k}$. Here, $\Gamma_{\text{NN}}(\mathbf{b}_{ij})$ is the profile function of the scattering of nucleon $i$ in $\mathbf{P}$ and nucleon $j$ in $\mathbf{A}$ given by

$$\Gamma_{\text{NN}}(\mathbf{b}_{ij}) = 1 - \exp\left[-i\frac{E_{ij}}{\hbar v_{\text{rel}}} \int_{-\infty}^{\infty} dz_{ij} v_{ij}(\mathbf{r}_{ij})\right], \quad (1.3)$$

where $v_{\text{rel}}$ is the relative velocity.

If the eikonal approximation is valid for NN scattering in free space, $\Gamma_{\text{NN}}(\mathbf{b}_{ij})$ in (1.3) should agree with the Fourier transform $\Gamma_{\text{em}}^\text{NN}(\mathbf{b}_{ij})$ of the NN scattering amplitude $f_{\text{NN}}$ determined from the data on NN scattering in free space,

$$\Gamma_{\text{em}}^\text{NN}(\mathbf{b}_{ij}) = -i\frac{1}{2\pi k_{ij}} \int e^{-i\mathbf{q}_{ij} \cdot \mathbf{b}_{ij}} f_{\text{NN}}(\mathbf{q}_{ij}) d\mathbf{q}_{ij}, \quad (1.4)$$

where $k_{ij}$ ($\mathbf{q}_{ij}$) is the initial (transferred) momentum of relative motion between the two nucleons $i$ and $j$ in free space and the 2-dimensional integration is over the components of $\mathbf{q}_{ij}$ perpendicular to $k_{ij}$. Instead of using $\Gamma_{\text{NN}}(\mathbf{b}_{ij})$, $\Gamma_{\text{em}}^\text{NN}(\mathbf{b}_{ij})$ has been used customarily, since first introduced in Ref. 7). However, it should be noted that the replacement of $\Gamma_{\text{NN}}(\mathbf{b}_{ij})$ by $\Gamma_{\text{em}}^\text{NN}(\mathbf{b}_{ij})$ in (1.2) is correct only if the eikonal approximation is valid for the NN scattering in free space, since (1.3) has already been used in the derivation of (1.2).

Substituting (1.4) into (1.2), one can calculate the nucleus-nucleus (PA) scattering amplitude $f_{\beta\alpha}$. In Ref. 7), $f_{\text{NN}}(\mathbf{q}_{ij})$ was given, neglecting the Coulomb effects, in the form

$$f_{\text{NN}}(\mathbf{q}_{ij}) = f_{\text{NN}}(0) e^{-\beta^2 \mathbf{q}_{ij}^2/2} \quad (1.5)$$

with

$$f_{\text{NN}}(0) = (i + \alpha) k_{ij} \sigma/4, \quad (1.6)$$

where $\sigma$ is the NN total cross section, and the constants $\alpha$ and $\beta$ were derived from the experimental data.\(^7\) This prescription works well at high energies. As already mentioned, however, (1.6) had to be modified at energies lower than 500 MeV. The parameters $\alpha$ and $\beta$ had to be adjusted away from those made to fit the PA scattering data. This cast doubt on the prescription, in particular on the validity of the eikonal approximation to NN scattering. In fact, the condition for the validity of the eikonal approximation to NN scattering is

$$|v_{ij}/E_{ij}| \ll 1, \quad k_{ij} a \gg 1, \quad (1.7)$$

where $E_{ij}$ and $k_{ij}$ are, respectively, the kinetic energy and the wave number of the relative motion of $i$ and $j$, and $a$ is the width of the region(s) in which $v_{ij}$ changes rapidly. However, since $v_{ij}$ has a strong short-range repulsive core, for example $v_{ij} \sim 2000$ MeV at $r_{ij} = 0$ in the case of the realistic NN potential AV18,\(^8\) it is obvious that the first condition of (1.7) is not satisfied.

In order, therefore, to examine the accuracy of the eikonal approximation to NN scattering, we calculate the NN scattering amplitude at the laboratory energy $E_{\text{NN}}$ =
300 MeV with the eikonal approximation and compare the result with the exact one. Figure 1 shows the on-shell NN scattering amplitude $f_{\text{NN}}(q_{ij})$. For simplicity, we take only the central part of the realistic NN potential AV18 for the triplet-even state. The solid (dashed) and dotted (dash-dotted) lines show, respectively, the real and imaginary parts of the resulting scattering amplitude of the exact (eikonal) calculation. The eikonal amplitude deviates considerably from the exact one, even at small $q$, for both the real and imaginary parts. In particular, the deviation is serious on the imaginary part. This means that the Glauber theory cannot accurately predict the reaction cross sections of nucleon-nucleus (NA) and nucleus-nucleus (PA) scattering. This test clearly shows that the use of (1.4) in (1.2) is inaccurate.

A systematic way of making non-eikonal corrections was proposed in Ref. 9) for high-energy potential scattering. In the method, the transition matrix is expanded in a power series of $\delta = |v_0|/(k\nu_{\text{rel}})$, where $v_0$ is a typical potential strength. The method, however, has not been applied to intermediate-energy NN scattering because $\delta > 1$ in such a case.

For the NA scattering mainly at high incident energies, some methods for treating non-eikonal effects have been proposed,\textsuperscript{10,11} but they require much more difficult and/or complicated calculations than the Glauber approximation. For example, Wong and Young introduced the pseudopotential $v'_{ij}$ that reproduces the empirical NN scattering amplitude in the eikonal approximation and estimated the corrections to the Glauber amplitude when $v'_{ij}$ is used instead of $v_{ij}$.\textsuperscript{10} However, it is not easy to evaluate the corrections induced by the difference $v_{ij} - v'_{ij}$, since $|v'_{ij}| \ll |v_{ij}|$ and, therefore, $v_{ij} - v'_{ij}$ is much larger than $v'_{ij}$.

It was proposed in Ref. 5) that $f_{\text{NN}}(k', k)$ is modified from the empirical NN scattering amplitude in free space to that calculated with the Brueckner $g$-matrix.
Although this seems a reasonable proposition, it is obvious that the theoretical foundations of such a phenomenological procedure need be examined.

In this paper, we propose an accurate and practical method of treating the intermediate-energy PA scattering within the framework of the Glauber theory. For this purpose, we use the effective NN interaction $\tau$ that describes the NN collisions in PA scattering instead of the bare potential $v_{ij}$. We introduce this effective NN interaction in the Glauber theory on the basis of the multiple scattering theory (MST) of Watson\textsuperscript{12} using the formalism of Kerman, McManus, and Theler (KMT)\textsuperscript{13}. It turns out that the $\tau$ satisfies the condition \textsuperscript{(1.7)} better than $v_{ij}$ and can even take account of the modification of the NN interaction in the nuclear medium that has long been thought to be necessary for reactions at low and intermediate incident energies.\textsuperscript{5}

For the high-energy nucleon-nucleus scattering in which the Glauber approximation is good and, therefore, \textsuperscript{(1.3)} is accurate, the relationship between the Glauber theory and MST was investigated in detail.\textsuperscript{11,14} These studies, however, were mostly concerned with the cancellation in the Watson expansion between the reflection terms and the off-pole contribution of nonreflective terms.\textsuperscript{14} This is not addressed in the present paper. We present a method of going beyond the ordinary Glauber theory by calculating all the Watson series using the Glauber approximation.

This paper is organized as follows. In §2, we present a new version of the Glauber theory based on the effective NN interaction $\tau$. In §3, some important points are discussed. In §3.1 we approximate $\tau$ as a two-body operator such as the transition matrix of NN scattering in free space or the Brueckner $g$-matrix, and we present a way of localizing the two-body operator. In §3.2 we test the eikonal and adiabatic approximations used in the present formalism. Such a test is not feasible for PA scattering consisting of an infinite series of multiple NN scatterings. The test is then made for a single NN collision in PA scattering, which is an essential building block of PA scattering. In §3.3 we discuss the relationship between the present theory and the conventional Glauber calculations using either the empirical NN profile function \textsuperscript{(1.4)} or the modified one including the in-medium effects.\textsuperscript{5} Section 4 is devoted to a summary.

§2. Formulation

2.1. Multiple scattering theory for nucleus-nucleus scattering

The original KMT formalism\textsuperscript{13} is for nucleon-nucleus (NA) scattering. We first extend it to the case of nucleus-nucleus (PA) scattering between P and A. The transition matrix of PA scattering is given by $T = V(1 + G_0T)$ with

$$G_0 = \frac{\mathcal{P}}{E - K - h_P - h_A + i\epsilon}, \quad (2.1)$$

where $\mathcal{P} = \mathcal{P}_P \mathcal{P}_A$ with $\mathcal{P}_P$ ($\mathcal{P}_A$) the projection operator onto the space of antisymmetrized wave functions of P (A).
The transition matrix $T$ is given by

$$T = \sum_{i \in P, j \in A} T_{ij}, \quad (2.2)$$

where $T_{ij}$ satisfy a set of coupled equations

$$T_{ij} = \tau_{ij} + \tau_{ij} G_0 \left\{ \sum_{m \in P, n \in A} T_{mn} - T_{ij} \right\}, \quad (2.3)$$

where

$$\tau_{ij} = v_{ij} (1 + G_0 \tau_{ij}) \quad (2.4)$$

is an operator that describes the scattering of nucleons $i$ in $P$ and $j$ in $A$. The potential $v_{ij}$, in general, contains operators acting on the spins and isospins of $i$ and $j$, which we suppress for the simplicity of notation.

The proof of (2.3) is as follows. We define $X$ as

$$X = \sum_{i \in P, j \in A} T_{ij}. \quad (2.3)$$

Equation (2.3) is then reduced to

$$T_{ij} = (1 + \tau_{ij} G_0)^{-1} \tau_{ij} (1 + G_0 X). \quad \text{Identifying} \quad (1 + \tau_{ij} G_0)^{-1} \tau_{ij} \quad \text{with} \quad v_{ij} \quad \text{in the equation and summing} \quad T_{ij} \quad \text{over} \quad i \quad \text{and} \quad j, \quad \text{one can obtain} \quad X = V (1 + G_0 X). \quad \text{We then find that} \quad X = T.$$

Because of the antisymmetry of the nuclear wave functions, which is maintained by $\mathcal{P}$, the matrix elements of the operators $T_{ij}$ and $\tau_{ij}$ are independent of the labels $i$ and $j$, so that (2.3) can be written as

$$\theta = \tau + (Y - 1) \tau G_0 \theta, \quad (2.5)$$

where $\theta = T_{ij}$ and $Y = N_P \times N_A$, with $N_P (N_A)$ the nucleon number of $P (A)$. Multiplying (2.5) by $Y - 1$, one obtains

$$T' = U (1 + G_0 T'), \quad (2.6)$$

where

$$T' = (Y - 1) \theta, \quad U = (Y - 1) \tau. \quad (2.7)$$

Then, $T$ is obtained from $T'$ as

$$T = \frac{Y}{Y - 1} T'. \quad (2.8)$$

Equations (2.6) and (2.8) constitute the final results of the extended KMT. The antisymmetrization between the incident nucleons in $P$ and the target nucleons in $A$ has been neglected so far. It was shown \cite{15,16} however, that it can be taken care of using $\tau$ which is symmetrical with respect to the exchange of the colliding nucleons.

2.2. Glauber approximation

We proceed to the calculation of the matrix elements $T'_{\beta \alpha}$ for the transition $\alpha \rightarrow \beta$ using the Glauber approximation. We first define the wave matrix $\Omega^{(+)}_{\alpha}$ that
gives the wave function with an incident wave in channel $\alpha$ by $\hat{\Psi}^{(+)}_{\alpha} = \Omega^{(+)}_{\alpha} \Phi_{\alpha} \phi_{\alpha}$, where $\phi_{\alpha} = (2\pi)^{-3/2} \exp[i \mathbf{k} \cdot \mathbf{R}]$. The wave function $\hat{\Psi}^{(+)}_{\alpha}$ satisfies

$$(K + h_P + h_A + U - E) \hat{\Psi}^{(+)}_{\alpha} = 0.$$ \hspace{1cm} (2.9)

The matrix elements of $T'$ for the transition $\alpha \to \beta$ are then given by $T'_{\beta\alpha} = \langle \Phi_{\beta} \phi_{\beta} | U | \hat{\Psi}^{(+)}_{\alpha} \rangle$. In the present formalism, $\Phi_{\alpha}$ is an eigenstate of the internal Hamiltonian $h_P + h_A$ with the realistic NN interaction $v_{ij}$. In actual calculations, however, the $v_{ij}$ is often replaced by an effective interaction that usually has a weak repulsion. The short-range correlation neglected by the replacement may not be important at forward PA scattering with small $q$.

The potential $U$ can be written in the form

$$U = Y - \frac{1}{Y} \sum_{i \in P, j \in A} \tau_{ij}$$ \hspace{1cm} (2.10)

because of the total antisymmetry of the wave functions of $P$ and $A$. By definition (2.4), $\tau_{ij}$ is a many-body operator acting on all the nucleons of the total system. We assume, however, as commonly performed in practical applications of MST and discussed in detail in §3.1, that $\tau_{ij}$ can be well approximated by a two-body operator acting only on $i$ and $j$ and in coordinate representation by a local potential depending only on $r_{ij}$.

Equation (2.9) then has the same form as the original Schrödinger equation (1.1) for the wave function of the total system except that $V$ is replaced by $U$. The Glauber approximation can, therefore, be applied to (2.9) if the same conditions are satisfied, i.e., the adiabatic approximation to the internal motion of $P$ and $A$ is used to approximate $h_P$ and $h_A$ by their ground-state energies, and the conditions for the eikonal approximation,

$$|\frac{(Y - 1)\tau}{E}| \ll 1, \quad ka_{\tau} \gg 1,$$ \hspace{1cm} (2.11)

are satisfied, where $a_{\tau}$ is the range of the region in which $\tau$ changes rapidly. $\tau_{ij}$ has a much weaker dependence on $r_{ij}$ than $v_{ij}$, as shown in §3.1. Hence, it satisfies the conditions in (2.11) better than the $v_{ij}$ of the original Glauber theory.

We denote the center of mass of $P$ ($A$) by $R_P$ ($R_A$) and the coordinate of nucleon $i$ ($j$) in $P$ ($A$) by $\mathbf{r}_i$ ($\mathbf{r}_j$). Then, $\mathbf{R} = \mathbf{R}_P - \mathbf{R}_A$ is the relative coordinate of $P$ from $A$ by which the scattering is described, and $\mathbf{x}_i = \mathbf{r}_i - \mathbf{R}_P$ ($\mathbf{y}_j = \mathbf{r}_j - \mathbf{R}_A$) is the intrinsic coordinate of $i$ ($j$) in $P$ ($A$). With this notation, $U = U(\mathbf{R}, \xi)$ where $\xi = (\{x_i\}_P, \{y_j\}_A)$. For simplicity of notation, however, the intrinsic coordinate $\xi$ is suppressed in the following.

In the Glauber approximation, $T'_{\beta\alpha}$ are given by

$$T'_{\beta\alpha} = C \int d\mathbf{b} \ E^{i\mathbf{q} \cdot \mathbf{b}} |\Phi_{\beta}| U(b) |\Phi_{\alpha} \rangle,$$ \hspace{1cm} (2.12)

where $C = -i\hbar^2 k / ((2\pi)^3 \mu_{\alpha})$ with $\mu_{\alpha}$ the reduced mass in the initial channel $\alpha$, and the profile function of PA scattering is given by

$$\Gamma_U(b) = 1 - \exp[i \chi_U(b)],$$ \hspace{1cm} (2.13)
A New Glauber Theory based on Multiple Scattering Theory

\[ \chi_{U}(b) = -\frac{1}{h\nu_{rel}} \int_{-\infty}^{\infty} dz \, U(z, b), \tag{2.14} \]

where \( \nu_{rel} = \hbar k/\mu_{\alpha} \). Using (2.10), one can rewrite the phase shift function \( \chi_{U}(b) \) as

\[ \chi_{U}(b) = \frac{Y - 1}{Y} \sum_{i \in P, j \in A} \chi^{(\text{eff})}_{\text{NN}}(b_{ij}), \tag{2.15} \]

where

\[ \chi^{(\text{eff})}_{\text{NN}}(b_{ij}) = -\frac{1}{h\nu_{rel}} \int_{-\infty}^{\infty} dz_{ij} \tau(z_{ij}, b_{ij}) \tag{2.16} \]

is the phase shift function of NN scattering by effective interaction \( \tau \). The transition matrix elements are then given by

\[ T_{\beta\alpha} = \frac{Y}{Y - 1} C \int db \, e^{i q \cdot b} \langle \Phi_{\beta} | \Gamma_{U}(b) | \Phi_{\alpha} \rangle \tag{2.17} \]

\[ \Gamma_{U}(b) = 1 - \prod_{i=1}^{P} \prod_{j=1}^{A} (1 - \Gamma^{(\text{eff})}_{\text{NN}}(b_{ij})), \tag{2.18} \]

\[ \Gamma^{(\text{eff})}_{\text{NN}}(b_{ij}) = 1 - \exp \left[ \frac{Y - 1}{Y - i} \chi^{(\text{eff})}_{\text{NN}}(b_{ij}) \right]. \tag{2.19} \]

The scattering amplitude \( f_{\beta\alpha} \) and the cross section \( \sigma_{\beta\alpha} \) are given by

\[ f_{\beta\alpha} = -\frac{(2\pi)^2 \mu_{\beta}}{k^2} T_{\beta\alpha} \quad \text{and} \quad \frac{d\sigma_{\beta\alpha}}{d\Omega_{\beta}} = |f_{\beta\alpha}|^2, \tag{2.20} \]

respectively. Equations (2.17) – (2.19) with (2.16) constitute the principal result of this paper, where the conventional form of the Glauber theory with (1.2) and (1.4) is reformulated using the effective interaction \( \tau \).

Most PA collisions satisfy \( Y \gg 1 \). The factors \( (Y - 1)/Y \) and \( Y/(Y - 1) \) in \( T \) only have appreciable effects on collisions between very light nuclei. For the other collisions, the resulting scattering amplitude has the form of the original Glauber theory except that the bare NN interaction \( v \) is replaced by \( \tau \).

2.3. Elastic scattering

For elastic scattering, where \( \alpha = \beta \), one can rewrite (2.17) in the form

\[ T_{\alpha\alpha} = \frac{Y}{Y - 1} C \int db \, e^{i q \cdot b} (1 - e^{i \chi_{\text{opt}}(b)}) \tag{2.21} \]

with

\[ \chi_{\text{opt}}(b) = -i \ln \langle \Phi_{\alpha} | \exp[i \chi_{U}(b)] | \Phi_{\alpha} \rangle. \tag{2.22} \]

Since \( \chi_{\text{opt}}(b) \) is a function of \( b \), there must be a one-body potential \( U_{\text{opt}}(z, b) \) such that

\[ \chi_{\text{opt}}(b) = -\frac{1}{\hbar\nu_{rel}} \int_{-\infty}^{\infty} dz \, U_{\text{opt}}(z, b). \tag{2.23} \]
Since $U_{\text{opt}}(z, b)$ is a one-body potential that describes the elastic scattering, it is the optical potential. This definition of $U_{\text{opt}}(z, b)$ differs from the ordinary one by the overall factor $Y/(Y - 1)$ in $T$, which is, as has already been mentioned, nearly equal to 1 except for scattering between very light nuclei. The potential $U_{\text{opt}}(z, b)$ is obtained as the solution of (2.23):

$$U_{\text{opt}}(r) = \frac{\hbar v_{\text{rel}}}{\pi} \frac{1}{r} \frac{d}{dr} \int_{r}^{\infty} \frac{\chi_{\text{opt}}(b)b db}{(b^2 - r^2)^{1/2}}.$$  \hspace{1cm} (2.24)$$

One can obtain $T_{\alpha\alpha}$ by solving the Schrödinger equation using the optical potential $U_{\text{opt}}(r)$.

In the case of $|\chi_U(b)| < 1$, $\exp[i\chi_U(b)]$ in (2.22) can be expanded in powers of $\chi_U(b)$. This leads to the cumulant expansion of $\chi_{\text{opt}}(b)$. If one retains only the lowest-order term, one obtains the optical limit,

$$\chi_{\text{opt}}(0)(b) = \langle \Phi_\alpha | \chi_U(b) | \Phi_\alpha \rangle,$$

and

$$U_{\text{opt}}(0)(R) = \frac{Y - 1}{Y} \sum_{i \in P, j \in A} \tau(z_{ij}, b_{ij}) |\Phi_\alpha \rangle,$$

(2.25)

where the superscript $(0)$ stands for the optical limit. Equation (2.24) includes all orders of the cumulant expansion, although the calculation is not easy.

The optical theorem $\sigma_{\text{tot}} = 4\pi \text{Im} f_{\alpha\alpha}(k, k)/k$ for the total cross section $\sigma_{\text{tot}}$ yields from (2.20) and (2.21)

$$\sigma_{\text{tot}} = \frac{2Y}{Y - 1} \int db \text{Re} \langle \Phi_\alpha | \Gamma_U(b) | \Phi_\alpha \rangle.$$

(2.26)

The angle-integrated cross section of elastic scattering is

$$\sigma_{\text{el}} = \int d\Omega_{k'} |f_{\alpha\alpha}(k', k)|^2 \approx \left(\frac{Y}{Y - 1}\right)^2 \int db |\langle \Phi_\alpha | \Gamma_U(b) | \Phi_\alpha \rangle|^2.$$

(2.27)

When the elastic scattering is concentrated in the forward direction, $d\Omega_{k'}$ is nearly on a plane perpendicular to the direction of $k$, so that $d\Omega_{k'} \approx d^2k'/k'^2$, where $d^2k'$ is an area element on that plane. The reaction cross section $\sigma_{\text{reac}}$ is then obtained as

$$\sigma_{\text{reac}} = \sigma_{\text{tot}} - \sigma_{\text{el}} = \int db \left(1 - |Z|^2\right),$$

(2.28)

where $Z = -1/(Y - 1) + Y/(Y - 1) \cdot S'_{\alpha\alpha}$ with

$$S'_{\alpha\alpha} = \langle \Phi_\alpha | \exp \left[ \frac{Y - 1}{Y} \sum_{i \in P, j \in A} i\chi_{\text{eff}}^{(\text{NN})}(b_{ij}) \right] | \Phi_\alpha \rangle.$$

(2.29)

In the case of $Y \gg 1$, which most collisions satisfy, the reaction cross section of (2.28) is reduced to that in the ordinary Glauber method with $v$ replaced by $\tau.$
§3. Discussion

3.1. Effective interaction

The effective interaction $\tau$ is the key to not only the present formalism but also to various theories of direct reactions. The effective interaction $\tau$ is a many-body operator, because Eq. (2.4) for $\tau$ includes the many-body Green’s function $G_0$, which depends on all the internal coordinates of $P$ and $A$. There is a long history of development of various approximations for $\tau$. Those widely used in many practical applications are to simply replace $\tau$ by a two-body operator. In the impulse approximation, $\tau$ is approximated by $t$, the transition matrix of NN scattering in free space.\(^\text{17}\) This has been successful for describing NA scattering at high energies of $E_{\text{NA}} > \sim 500$ MeV.\(^\text{13,18,19}\) At lower energies of $E_{\text{NA}} \lesssim 500$ MeV, the effects of the nuclear medium on $\tau$ become significant,\(^\text{13,18,19}\) and the Brueckner $g$-matrix has been used for NA and PA scattering.\(^\text{20–26}\)

The $t$ and $g$ matrices are nonlocal operators. In many of their applications to the analysis of experimental data, however, they have been given as local potentials in coordinate representation. Love and Franey\(^\text{17}\) presented the $t$ matrix elements in the form of superposition of Yukawa potentials that reproduce on-shell elements of the $t$ matrix. In many of the $g$ matrix applications, it is calculated for nuclear matter, parameterized in the form of the superposition of Yukawa or Gaussian potentials that reproduces on-shell or half off-shell elements of the $g$ matrix, and translated to finite nuclei by a local density (LD) approximation. Although the justification of the LD approximation remains as a fundamental question, the use of $g$ as $\tau$ is successful in reproducing the NA scattering over a wide energy range of $50 < \sim E_{\text{NA}} < \sim 800$ MeV.\(^\text{26}\) For PA scattering, however, there is an ambiguity in the definition of LD.\(^\text{27}\) Although further progress is required on this point, the $g$ matrix may be one of the most plausible substitutes for $\tau$.

The local potentials thus obtained are ambiguous in their $r_{ij}$ dependence particularly at small $r_{ij}$. Actually, the Yukawa-type local potential is singular at $r_{ij} = 0$, but the Gaussian-type is not. Obviously, the singularity is an artifact. Since the accuracy of the eikonal approximation is sensitive to the dependence, the localized potentials mentioned above cannot be used in the present theory.

In order to determine a local $\tau$ matrix from a nonlocal one $\tau(r_{ij}, r'_{ij})$ with no ambiguity, we introduce the trivially equivalent local (TEL) $\tau$ matrix $\tau^{\text{loc}}(r_{ij})$ as

$$v(r_{ij})\psi = \tau^{\text{loc}}(r_{ij})\phi_0$$

with $\phi_0 = (2\pi)^{-3/2}\exp(ik_{ij} \cdot r_{ij})$. In the case of $\tau = t$, $\psi$ is the solution of the Schrödinger equation of the NN system with $v$, while in the case of $\tau = g$ it is the solution of the Bethe-Goldstone equation. Since the discussion is parallel between the two cases, we consider, hereafter, the case of $\tau = t$.

The TEL $\tau$ matrix, $\tau^{\text{loc}}(r_{ij})$, is related to the nonlocal matrix as

$$\tau^{\text{loc}}(r_{ij})\phi_0(r_{ij}) = \int dr'_{ij} \tau(r_{ij}, r'_{ij})\phi_0(r'_{ij}),$$

with

$$\phi_0 = (2\pi)^{-3/2}\exp(ik_{ij} \cdot r_{ij}).$$
because the left-hand side of (3.1) is equivalent to the right-hand side of (3.2). The Fourier transform of the right-hand side of (3.2) gives the half off-shell elements of $\tau(r_{ij}, r'_{ij})$. Equation (3.2) ensures that $\tau_{\text{loc}}(r_{ij})$ gives the same half off-shell elements as the nonlocal matrix $\tau(r_{ij}, r'_{ij})$. Thus, $\tau_{\text{loc}}(r_{ij})$ is derived from the nonlocal matrix with no ambiguity.

Even if $v(r_{ij})$ is a central force, $\tau_{\text{loc}}(r_{ij})$ is not a central one, because it depends on the angle $\theta$ between $r_{ij}$ and the initial momentum $\hbar k_{ij}$ (the $z$ axis). This is not a problem in the formalism of §2, since the integration of $\tau_{\text{loc}}(r_{ij})$ over $z_{ij}$,

$$\tilde{\tau}_{\text{loc}}(b_{ij}) = \int d z_{ij} \tau_{\text{loc}}(r_{ij}), \quad (3.3)$$

is an input. The $\theta$ dependence of $\tau_{\text{loc}}(r_{ij})$ is integrated out in $\tilde{\tau}_{\text{loc}}(b_{ij})$ through the integration over $z_{ij} = r_{ij} \cos(\theta)$.

Usually, the TEL potential $\tau_{\text{loc}}$ is a function of the relative coordinate $r_{ij}$ and $\nabla r_{ij}$, and the spin and isospin of the colliding pair. When the target is a double-magic nucleus, the tensor part of $\tau_{\text{loc}}$ does not contribute to $U_{\text{opt}}^{(0)}$. For this reason, in most analyses based on the folding model, only the central and spin-orbit parts are taken into account. Actually, the tensor force is difficult to include in the Glauber theory because of its noncommutative character.

Now we calculate $\tilde{\tau}_{\text{loc}}$ by solving the Schrödinger equation of the NN system with $v$. For simplicity, we neglect the spin and isospin of the colliding pair. As $v(r_{ij})$, we take the central part of AV18 for the triplet-even state. The results are shown in Fig. 2, which represents the $z_{ij}$ integrated TEL potential $\tilde{\tau}_{\text{loc}}$ as a function of $b_{ij}$ for the laboratory energies of $E_{\text{NN}} = 300$ and $800$ MeV. The solid (dotted) curve is the real (imaginary) part, while the dashed curve corresponds to $\tilde{v}(b_{ij})$, the $z_{ij}$ integration of $v(r_{ij})$; at $b_{ij} = 0$, $\tilde{v} = 1587$ MeV · fm and $\tilde{\tau}_{\text{loc}} = 278 - 86i$ (267 - 212i) MeV · fm for $E_{\text{NN}} = 300$ (800) MeV. As expected, $\tilde{\tau}_{\text{loc}}$ has a much weaker $b_{ij}$ dependence than $\tilde{v}$. The TEL potential $\tau_{\text{loc}}$ thus obtained is not singular at $b_{ij} = 0$. It is natural to think that the TEL potential of the $g$ matrix also maintains the same property.

### 3.2. Validity of eikonal and adiabatic approximations in derivation of $\Gamma_{\text{NN}}^{(\text{eff})}$

The effective NN profile function $\Gamma_{\text{NN}}^{(\text{eff})}(b_{ij})$ defined by (2.19), which describes a collision of the $i$-$j$ pair in the PA scattering, is the key element of the present formalism, as shown in (2.16) – (2.19). We test the validity of the eikonal and adiabatic approximations used in the derivation of $\Gamma_{\text{NN}}^{(\text{eff})}$.
Fig. 2. $z_{ij}$ integrated TEL potential as a function of $b_{ij}$ for (a) $E_{NN} = 300$ MeV and (b) $E_{NN} = 800$ MeV. The solid (dotted) curve represents the real (imaginary) part, while the dashed curve corresponds to the $z_{ij}$ integration of $v(r_{ij})$.

We assume $\gamma \gg 1$ and consider the accuracy of

$$\Gamma_{NN}^{(eff)}(b_{ij}) \approx 1 - \exp[-i\tau_{loc}/(\hbar v_{rel})], \quad (3.5)$$

taking $t_{loc}$ as $\tau_{loc}$. From (2.16) through (2.19) it is clear that

$$T_{Gl} = C \int dB_{ij} e^{i\mathbf{q} \cdot \mathbf{b}_{ij}} \Gamma_{NN}^{(eff)}(b_{ij}) \quad (3.6)$$

is the Glauber approximation of the transition matrix of a fictitious PA collision in which only nucleons $i$ in P and $j$ in A interact. The exact wave function $\psi$ describing the same process satisfies the Schrödinger equation

$$\left(K + h_P + h_A + t_{loc}(r_{ij}) - E\right)\psi = 0, \quad (3.7)$$

where $K = -(h\nabla R)^2/(2\mu_\alpha)$ with $\mu_\alpha$ the reduced mass of PA system, $r_{ij} = R + \mathbf{x}_i - \mathbf{y}_j$ and $E = E_{PA} + e_P + e_A$ with the incident energy $E_{PA}$ and the intrinsic energy $e_P$ ($e_A$) of P (A). The exact transition matrix $T_{ex}$ calculated using $\psi$ is

$$T_{ex} = t_{loc}(r_{ij}) + t_{loc}(r_{ij})G_0 T_{ex}, \quad (3.8)$$

where

$$G_0 = \frac{1}{E - K - h_P - h_A + i\epsilon}, \quad (3.9)$$

First, we test the validity of the eikonal approximation. For this purpose, we apply the adiabatic approximation to Eq. (3.7). The equation is reduced to

$$\left(K + t_{ij}^{loc}(r_{ij}) - E_{PA}\right)\psi^{AD} = 0, \quad (3.10)$$
where the adiabatic approximation $h_P + h_A$ has been replaced by the ground state energies $e_P + e_A$. We then obtain the transition matrix $T_{AD}$ under the adiabatic approximation as

$$T_{AD} = t_{loc}^{(r_{ij})} + t_{loc}^{(r_{ij})}G_0^{AD}T_{AD},$$

(3.11)

where

$$G_0^{AD} = \frac{1}{E_P A - K + i\epsilon}.$$  

(3.12)

Since $x_i$ and $y_j$ are simply parameters in (3.10), we can regard $K$ as $K = -(\hbar \nabla r_{ij})^2/(2\mu)$ and then solve (3.10) without the eikonal approximation.

Now it is possible to test the accuracy of the eikonal approximation by comparing $T_{GI}$ with $T_{AD}$. As an example, we consider $^4$He+$^{208}$Pb scattering. As mentioned in §3.1, $t_{loc}^{(r_{ij})}$ is dependent on $\theta$. In order to avoid this complication, we use the $\theta$-independent (spherical) potential

$$t_0^{loc}(r_{ij}) = -\frac{1}{\pi r_{ij}} \frac{d}{dr_{ij}} \int_{r_{ij}}^\infty \frac{\tilde{t}_{loc}(b_{ij})b_{ij}db_{ij}}{(b_{ij}^2 - r_{ij}^2)^{1/2}}.$$  

(3.13)

instead of $t_{loc}^{(r_{ij})}$. As found from the discussion in §2.3, the two potentials $t_0^{loc}(r_{ij})$ and $t_{loc}^{(r_{ij})}$ give the same transition matrix under the eikonal approximation.

Figure 3 shows the scattering amplitudes, $f_{GI}(q) = -(2\pi)^2 \mu/\hbar^2 \times T_{GI}$ and $f_{AD}(q) = -(2\pi)^2 \mu/\hbar^2 \times T_{AD}$, of a single NN collision in $^4$He+$^{208}$Pb scattering at the laboratory energy of $E_{PA} = 300$Np MeV. The solid (dashed) and dotted (dash-dotted) curves show, respectively, the real and imaginary parts of $f_{AD}(q)$ ($f_{GI}(q)$). The agreement between calculations with and without the eikonal approximation is excellent.

Fig. 3. On-shell scattering amplitudes, $f_{GI}(q)$ and $f_{AD}(q)$, of a single NN collision in $^4$He+$^{208}$Pb scattering $t_0^{loc}(r_{ij})$ at the laboratory energy of $E_{PA} = 300$Np MeV.
The denominator of $G_0^{\text{AD}}$ in the momentum representation is

$$\frac{\hbar^2(k^2 - k'^2)}{2\mu_\alpha} = \hbar v_{\text{rel}}\left(q_z - \frac{q^2\hbar}{2\mu_\alpha v_{\text{rel}}}\right), \quad (3.14)$$

where $v_{\text{rel}} = \hbar k/\mu_\alpha$, $E_{\text{PA}} = \hbar^2 k^2/2\mu_\alpha$, $q = k - k'$ and $q_z$ is the $z$ component of $q$. In the eikonal approximation, the $q^2$ term in the denominator is neglected. This is realized in the large-$\mu_\alpha$ limit with $v_{\text{rel}}$ fixed. In general, PA scattering has a larger $\mu_\alpha$ than NA scattering, and $\mu_\alpha$ becomes minimum for NN scattering. Thus, the eikonal approximation is generally better for PA scattering than for NA scattering but becomes worse for NN scattering, when the scatterings of different systems with common $v_{\text{rel}}$ are compared with each other.

In order to confirm the validity of the adiabatic approximation, we compare the solution $T_{\text{ex}}$ of Eq. (3.13) with $T^{\text{AD}}$ of Eq. (3.11) for nucleus-proton scattering in which the nucleus is described by a core+nucleon $(c+n)$ two-body system. We consider the two reactions $^{11}\text{Be}+p$ and $^{40}\text{Ca}+p$ at an incident energy of 300 MeV per nucleon. We use a Woods-Saxon interaction between $c$ ($^{10}\text{Be}$ or $^{39}\text{Ca}$) and $n$

$$V_{cn}(r) = V_0 \left(1 + \exp \frac{r - R_0}{a_0}\right)^{-1}, \quad (3.15)$$

where $r$ is the displacement of $n$ from $c$. The potential parameters and the resulting neutron separation energy $S_n$ are shown in Table I. We solve the three-body scattering problem by the method of continuum-discretized coupled channels (CDCC)\textsuperscript{(28),29)} with and without adiabatic approximation to the $c-n$ internal Hamiltonian, i.e., $h_P$ in Eq. (3.7). For simplicity, we neglect the internal degrees of freedom of the target proton; we then have $h_A = 0$ in the present case. The maximum $r$ is 60 fm, and $s$-, $p$-, $d$- and $f$-waves for the $c-n$ relative wave function are included. The momentum bin is truncated at 1.5 fm$^{-1}$ for each partial wave and divided into 30 (15) discretized states for the $s$-wave ($p$-, $d$- and $f$-waves). The coupled-channel potentials are calculated using $t_0^{\text{oc}}(r_{ij})$ of Eq. (3.13). Note that the interaction between $c$ and the target proton is not included since we solve the Schrödinger equations (3.7) and (3.10). The relative wave functions $\psi$ and $\psi^{\text{AD}}$ are integrated up to 20 fm and connected to the standard asymptotic form.

In Fig. 4 we show the total breakup cross sections for $^{11}\text{Be}+p$ (left panel) and $^{40}\text{Ca}+p$ (right panel) at 300 MeV per nucleon as a function of the center-of-mass scattering angle of the $c-n$ two-body system. In each panel the solid and dashed lines indicate the results with and without adiabatic approximation, respectively. One sees clearly that the adiabatic approximation works very well for not only $^{11}\text{Be}$ with small

|         | $V_0$ (MeV) | $R_0$ (fm) | $a_0$ (fm) | $S_n$ (MeV) |
|---------|-------------|------------|------------|-------------|
| $^{10}\text{Be}-n$ | 51.60 | 2.996 | 0.52 | 0.503 |
| $^{39}\text{Ca}-n$ | 51.68 | 4.343 | 0.67 | 15.64 |
Fig. 4. Total breakup cross sections for $^{11}$Be+$p$ (left panel) and $^{40}$Ca+$p$ (right panel) at 300 MeV per nucleon. The horizontal axis is the scattering angle of the center of mass of the fragments $c$ and $n$. The solid and dashed lines show the results of CDCC calculation with and without adiabatic approximation, respectively.

S_n but also $^{40}$Ca with quite large $S_n$. Thus, in the energy region considered here, namely, a few hundred MeV per nucleon, the adiabatic approximation and hence Eq. (3.10) is shown to be valid.

3.3. Relationship between present theory and conventional Glauber calculations

In this subsection, we clarify the relationship between the present theory and the conventional Glauber calculations using the empirical NN profile function (1.4) or using the modified profile functions including the in-medium effects. For simplicity, we consider the case of $Y \gg 1$, and the superscript loc of $\tau^{\text{loc}}$ is suppressed.

In the present theory, the transition matrix of the PA scattering in coordinate representation is

$$T = C \left[ 1 - \prod_{i,j} \exp(-i\tilde{\tau}_{ij}/\hbar v_{\text{rel}}) \right].$$

(3.16)

The corresponding matrix in the Glauber theory is

$$T_{\text{GI}} = C \left[ 1 - \prod_{i,j} \left( 1 - i\tilde{\tau}_{ij}^{\text{em}}/\hbar v_{\text{rel}} \right) \right],$$

(3.17)

where $\tilde{\tau}_{ij}^{\text{em}} = -i\hbar v_{\text{rel}} \Gamma_{\text{NN}}^{\text{em}}$ using $\Gamma_{\text{NN}}^{\text{em}}$ of (1.4). Since $\tilde{\tau}_{ij}^{\text{em}}$ is adjusted to fit the data on NN scattering, it is essentially identical to $\tilde{\tau}_{ij}$. One sees, therefore, that the conventional Glauber calculation with (1.4) is justified provided that (a) $\tau \approx t$ is a good approximation and (b) $\tilde{\tau}_{ij}/\hbar v_{\text{rel}}$ is small enough to warrant $\exp(-i\tilde{\tau}_{ij}/\hbar v_{\text{rel}}) \approx 1 - i\tilde{\tau}_{ij}/\hbar v_{\text{rel}}$. At very high energies, where the replacement of $\tau$ by $t$ is valid and the amount of multiple scatterings by $t$ is negligible, that is, in the limit that the
impulse approximation is good, the present theory agrees using the conventional Glauber theory with the empirical profile function $\Gamma_{\text{em}}^{\text{NN}}$ of (1.4). Conversely, if either of conditions (a) and (b) is not satisfied, the conventional Glauber procedure is not justified by the present approach.

In the heuristic approach in Ref. 5), $\tilde{t}_{ij}^{\text{em}}$ is replaced by the $z$ integration of $g_{ij}$, $\tilde{g}_{ij}$, in the conventional Glauber theory. The corresponding transition matrix of the PA scattering is

$$T^{(g)} = C \left[ 1 - \prod_{i,j} (1 - i\tilde{g}_{ij}/\hbar v_{\text{rel}}) \right]. \quad (3.18)$$

One can find from (3.16) and (3.18) that even if $\tilde{t}_{ij} \approx \tilde{g}_{ij}$, $T^{(g)}$ agrees with $T$ in the lowest (first) order of $\tilde{g}/\hbar v_{\text{rel}}$, but not in higher orders. Thus, the heuristic approach in Ref. 5) gives a better description of PA scattering than the Glauber theory, but it is still not perfect, since it contains no higher order terms of $\tilde{g}/\hbar v_{\text{rel}}$.

For the calculation of optical potential, it is common to take the optical limit in which only the first order of $\tilde{g}/\hbar v_{\text{rel}}$ is taken into account. In this limit, the present theory with $g$ as $\tau$ reduces to the folding model with $g$, which has been successfully used in reproducing NA scattering data.

\section*{§4. Summary}

We present in this paper a new version of the Glauber theory for nucleus-nucleus collisions based on the multiple scattering theory of Watson (MST), making use of the KMT formalism. The input of the theory is the effective nucleon-nucleon (NN) interaction $\tau$ of MST, which has a weaker short-range repulsion than the bare NN nuclear force potential, which makes the eikonal approximation much more reliable. We tested the validity of the eikonal and adiabatic approximations to NN collision using $\tau$ in nucleus-nucleus scattering at the laboratory energy of $E_{\text{PA}} = 300N_{\text{P}}$ MeV and showed that these approximations are good at the intermediate energies.

At very high energies where the replacement of $\tau$ by $t$ is valid and multiple scattering by $t$ is negligible, that is, in the limit that the impulse approximation is good, the present theory gives the same $T$ matrix of PA scattering as the conventional Glauber theory using the empirical profile function $\Gamma_{\text{em}}^{\text{NN}}$ of (1.4).

When the $g$ matrix is used as $\tau$, the present theory can take into account the nuclear-medium effects in the analyses of NA and PA scattering. Therefore, the present theory is also applicable in the intermediate-energy region. The present theory also provides a theoretical foundation to heuristic Glauber calculations in which the profile function is modified to reproduce the in-medium NN cross section calculated from the $g$ matrix, although the heuristic calculations include no multiple scattering by $g$. For the calculation of optical potential, the present formalism reduces in the optical limit to the folding model using $g$ matrix, which has been successfully used in reproducing nucleon-nucleus scattering data.

Thus, the present theory unifies standard methods such as the conventional Glauber method using the empirical NN profile function (1.4), heuristic Glauber calculations with the modified profile functions including the in-medium NN effects,
and the folding potential method using the $g$ matrix.\textsuperscript{20)--26) In our forthcoming paper, we propose a way of deriving $\tau$ with no local density approximation, and compare the result with those obtained using the $t$ and $g$ matrices.

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**References**

1. I. Tanihata et al., Phys. Rev. Lett. 55 (1985), 2676; Phys. Lett. B 287 (1992), 307.
2. R. J. Glauber, Lectures in Theoretical Physics (Interscience, New York, 1959), Vol. 1, p. 315.
3. J. S. Al-Khalili and J. A. Tostevin, Phys. Rev. Lett. 76 (1996), 3903.
4. Y. Ogawa, T. Kido, K. Yabana and Y. Suzuki, Prog. Theor. Phys. Suppl. No. 142 (2001), 157, and references cited therein.
5. C. Xiangzhou, F. Jun, S. Wenqing, M. Yuguang, W. Jiansong and Y. Wei, Phys. Rev. C 58 (1998), 572.
6. W. Horichi, Y. Suzuki, B. Abu-Ibrahim and A. Kohama, Phys. Rev. C 75 (2007), 044607.
7. R. J. Glauber and G. Matthiae, Nucl. Phys. B 21 (1970), 135.
8. R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. C 51 (1995), 38.
9. S. J. Wallace, Ann. of Phys. 78 (1973), 190; Phys. Rev. D 8 (1973), 1934, and references cited therein.
10. C. W. Wong and S. K. Young, Phys. Rev. C 12 (1975), 1301.
11. K. M. Watson, Phys. Rev. 89 (1953), 575.
12. A. K. Kerman, H. McManus and R. M. Thaler, Ann. of Phys. 8 (1959), 135, and references cited therein.
13. N. M. Queen, Nucl. Phys. 55 (1964), 177.
14. D. R. Harrington, Phys. Rev. 184 (1969), 1745.
15. G. Takeda and K. M. Watson, Phys. Rev. 97 (1955), 1336.
16. A. Picklesimer and R. M. Thaler, Phys. Rev. C 23 (1981), 42.
17. W. G. Love and M. A. Franey, Phys. Rev. C 24 (1981), 1073.
18. M. A. Franey and W. G. Love, Phys. Rev. C 31 (1985), 488.
19. L. Ray, Phys. Rev. C 41 (1990), 2816.
20. H. F. Arellano, F. A. Brieva and W. G. Love, Phys. Rev. C 52 (1995), 301.
21. G. Bertsch, Direct Nuclear Reactions (Oxford University Press, New York, 1983), and references cited therein.
22. G. R. Satchler, Direct Nuclear Reactions (Oxford University Press, New York, 1983), 399.
23. J.-P. Jeukenne, A. Lejeune and C. Mahaux, Phys. Rev. C 16 (1977), 80; Phys. Rep. 25 (1976), 83.
24. N. Yamaguchi, S. Nagata and T. Matsuda, Prog. Theor. Phys. 70 (1983), 459.
25. L. Rikus, K. Nakano and H. V. von Geramb, Nucl. Phys. A 414 (1984), 413.
L. Rikus and H. V. von Geramb, Nucl. Phys. A 426 (1984), 496.
26) K. Amos, P. J. Dortmans, H. V. von Geramb, S. Karataglidis and J. Raynal, Adv. Nucl. Phys. 25 (2000), 275.
27) T. Furumoto and Y. Sakuragi, Phys. Rev. C 74 (2006), 034606.
28) M. Kamimura, M. Yahiro, Y. Iseri, Y. Sakuragi, H. Kameyama and M. Kawai, Prog. Theor. Phys. Suppl. No. 89 (1986).
N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. H. Rawitscher and M. Yahiro, Phys. Rep. 154 (1987), 125.
29) N. Austern et al., Phys. Rev. Lett. 63 (1989), 2649; Phys. Rev. C 53 (1996), 314.