Flow equations method of continuous unitary transformations is used to eliminate the minimal quark-gluon interaction in the light-front quantized QCD Hamiltonian. The coupled differential equations in the two lowest Fock sectors correspond to the renormalization of the light-front gluon mass and the generation of effective quark-antiquark interaction. The influence of the renormalization of the gluon effective mass on the elimination of the quark-gluon coupling and the induced quark-antiquark interaction is taken into account. Namely, the original gauge field coupling can be completely eliminated, even when the states connected by this interaction are degenerate. Furthermore, even in the case where effective interaction, obtained within perturbative schemes (bound state perturbation theory or perturbative similarity approach), is not defined, we obtain more singular behavior $1/q^4$ at small gluon momenta. This is due to asymptotic behavior of the effective gluon mass at small cutoffs. By discussing the consequences of this asymptotic behavior, it seems that our approach is superior to perturbation theory and to perturbative similarity approach.

1 Introduction

The perturbative aspects of non-abelian gauge theories were understood many years ago, and the perturbative calculations provided convincing proof that QCD is the theory of strong interactions. However nonperturbative QCD phenomena have been difficult to analyze mainly because calculational techniques are still lacking, even though the qualitative features have been more or less understood.

In particular, it is widely believed that pure Yang-Mills theory, with no dynamical quarks, possesses the features like asymptotic freedom, mass generation through the transmutation of dimensions, and confinement: linear rising potential between static (probe) quarks. Adding dynamical quarks chiral symmetry is broken spontaneously. The ultimate aim of this study is to understand these nonperturbative mechanisms in a Hamiltonian framework, solving flow equations for canonical QCD Hamiltonian in the few lowest sectors self-consistently for dynamical gluons and quarks, and their effective interactions. In this work dynamics of quarks has been excluded to disentangle the complexity of chiral symmetry breaking.

In the past few years there were several studies addressing the issue of confinement and generation of mass gap in the framework of the Schrödinger
picture 1, 2. In the relatively recent works 1 one have been using the special ansatz for a vacuum wave functional suggested by Kogan and Kovner, and integrating over all possible gauge configurations. To mention a few early works, see refs. 3. The calculational technique in this approach is still rather complicated, and allows to solve a field theory problem only in 1 + 1 dimensions 2, but treats only ground states in 3 + 1 dimensions 1.

In alternative studies of nonperturbative problems in Hamiltonian framework one considers directly the QCD Hamiltonian quantized in a special gauge, in particular the light-front gauge, $A^+ = 0$ 10. There is a belief that the light-front gauge may be the most suitable frame to study the nonperturbative QCD 10. Previous studies of confinement and bound states in the light-front frame have been done using the methods of similarity renormalization 4 and transverse lattice 5, and are based on the fact that the light-front QCD in 3 + 1 already has a confining interaction in the form of the instantaneous four-fermion interaction, $1/q^+ 2$, which is the confining interaction in 1 + 1 along the light-front direction, $x^-$. However, since instantaneous interaction does not provide confinement mechanism for quarks and gluons in 3 + 1 dimensions, the task of maintenance rotational symmetry becomes difficult to achieve when trying to extend light-front confinement to 3 + 1.

Some time ago Wilson proposed a formalism to construct a confining light-front quark-gluon Hamiltonian for light-front QCD 3+1 14. Wilson suggested that a starting point for analyzing the full QCD with confinement in light-front coordinates is the light-front infrared divergences. Based on light-front power counting, the counterterms for the light-front infrared divergences can involve the color charge densities and involve unknown nonlocal behavior in transverse direction that become a possible source for transverse confinement. However, the analysis was not complete and a scheme for practical calculation has not been developed.

A subtle point in the light-front field theory is that the light-front vacuum is just empty space. Therefore it seems a problem how confinement can occur in the light-front frame, and what quantity sets up a scale for a dynamical mass gap and the string tension. The infrared longitudinal cutoff properties of light-front theory suggest a fundamental role for the light-front counterterms, solving this paradox. Namely, the longitudinal infrared cutoff in light-front dynamics makes it impossible to create particles from a bare vacuum by a translationally invariant Hamiltonian and in addition the number of constituents in a given eigenstate is limited. When introducing infrared cutoff physics below the cutoff is missed, and to restore it one may introduce appropriate counterterms. Light-front counterterms to the longitudinal infrared cutoff dependence provide a nonzero amplitude of particle creation, and be-
come therefore a possible alternative source for features normally associated in
standard equal-time dynamics with a nontrivial vacuum structure, including
confinement and spontaneous symmetry breaking. Note, that small light-front
\(x\) correspond to high light-front energies. Therefore in order to remove small
\(x\), one should use renormalization group.

To be more specific we adopt the following model suggested by Susskind
and Burkardt in the context of chiral symmetry breaking in the light-front
frame\(^{11}\). In the parton model one pictures a fast moving hadron as being
some collection of constituents with relatively large momentum, such that when
one boosts the system, doubles its momentum, all these partons double their
momenta and so forth. Therefore one can formulate an effective field theory on
the axis of the light-front momentum \(x\) (or on rapidity axis, which is logarithm
of \(x\)). Partons that form a hadron are at positive, finite \(x\) and according to
Feynman and Bjorken fill the \(x\)-axis in a way which gets denser and denser as
one goes to smaller \(x\); and the vacuum is at \(x = 0\). The fundamental property of
light-front Hamiltonians, that under a rescaling of the light-front momentum,
\(x \rightarrow \lambda x\), the light-front Hamiltonian scales like \(H \rightarrow H/\lambda\), can be interpreted
as a dilatation symmetry along the \(x\)-axis, if one thinks of the \(x\)-axis as a spacial
axis. This symmetry holds on a classical level and it is broken on a quantum
level by anomali. As one approaches small \(x\), interaction between partons gets
stronger, contributing divergent matrix elements. A natural cutoff is provided
by \(\delta x = \varepsilon x\), a minimal spacing between constituents, which plays the role
of UV-regulator. As long as \(\delta x\) (or \(\varepsilon x\)) is finite, i.e. as long as the density of
partons on the \(x\)-axis is not infinite, one obtains finite matrix elements. Cutoff
\(\delta x\) breaks the dilatation symmetry along the \(x\)-axis and gauge symmetry, and
sets up an energy scale in effective light-front field theory formulated on \(x\)-
axis. In terms of effective theory a generated mass gap defines a strength
of effective interactions between quarks, in this case string tension in quark-
antiquark potential. Formation of the \(q\bar{q}\) bound state through breaking an
internal symmetry can be viewed analogously to the creation of Cooper pairs
in superconductor.

The dilatation symmetry reflects some underlying scale invariance of the
light-front field theory formulated on \(x\)-axis. Introducing the cutoff, breaks
this symmetry. Because physics should remain independent from the cutoff,
one must be looking for a fixed point of the renormalization group. Therefore
the right tool for studing such a system is the renormalization group, which is
provided by the method of flow equations for Hamiltonians\(^{6}\).

Incorporating effects from small \(x\) into an effective light-front Hamiltonian
is equivalent to integrating out high light-front energy modes in asymptotically
free domain. In terms of renormalization group, regulating small \(x\) introduces
a mass gap, which together with asymptotic freedom leads to a renormalization
group invariant scale and dimensional transmutation along $x$-axis. Mass gap
and a possible, more singular than a Coulomb, confining potential between
quark and antiquark are direct consequences of dimensional transmutation in
the effective light-front field theory, namely the light-front QCD, formulated
on the light-front $x$-axis.

In the main part of the paper, basing on the QCD Hamiltonian in the
light-front gauge, flow equations for an effective gluon mass and effective quark-
antiquark interaction are formulated in the light-front frame and solved self-
consistently within the leading iteration. By discussing solutions of these equa-
tions in concluding section, it seems that flow equation method is superior to
perturbation theory and perturbative similarity approach.

2 Flow equations in the light-front QCD

We apply flow equations to the light-front QCD Hamiltonian in order to elim-
inate the minimal quark-gluon interaction, namely to decouple matter and
gauge degrees of freedom in the leading order. In the two lowest Fock sec-
tors of the effective QCD Hamiltonian we obtain coupled differential equations
which correspond to renormalization of the light-front gluon mass (renormal-
ization of the quark mass is not considered) and generation of an effective
quark-antiquark interaction. These flow equations are solved selfconsistently
in the sense that the influence of the gluon mass renormalization on the elim-
ination of the quark-gluon coupling and the induced quark-antiquark interac-
tion is taken into account. As a result, the original gauge field coupling is
completely eliminated, even when the states connected by this interaction are
degenerate. Furthermore, in the degenerate case where effective $q\bar{q}$ interaction
obtained within perturbation theory is not defined, we obtain more singular
$1/q^4$ behavior at small gluon momenta.

2.1 Gluon gap equation

Coupled system of the light-front equations for the effective quark and gluon
masses as functions of a cut-off $\lambda$ was derived first by Glazek\(^\text{16}\). We decouple
this system of equations by neglecting the cut-off dependence of the quark
mass, i.e. $m(\lambda) = m$ with $m$ current quark mass. The light-front gluon gap
equation reads

\[
\frac{d\mu^2(\lambda)}{d\lambda} = -2T_f N_f g^2 \int_0^1 \frac{dx}{x(1-x)} \int_0^\infty \frac{d^2 k_\perp}{16\pi^3} \frac{1}{Q^2_2(\lambda)} \frac{d\mathcal{F}^2(Q^2_2(\lambda); \lambda)}{d\lambda}
\]
\[ \times \left( \frac{k^2 + m^2}{x(1-x)} - 2k^2_\perp \right) \]
\[ - 2C_ag^2 \int_0^1 \frac{dx}{x(1-x)} \int_0^\infty \frac{d^2k_\perp}{16\pi^3} \frac{1}{Q^2_1(\lambda; \lambda)} \]
\[ \times \left( k^2_\perp\left(1 + \frac{1}{x^2} + \frac{1}{(1-x)^2}\right) \right), \tag{1} \]

with

\[ Q^2_1(\lambda) = \frac{k^2 + \mu^2(\lambda)}{x(1-x)} - \mu^2(\lambda), \quad Q^2_2(\lambda) = \frac{k^2 + m^2}{x(1-x)} - \mu^2(\lambda), \tag{2} \]

where in the integral kernel gluon couples to the quark-antiquark pairs and pairs of gluons with the bare strong coupling \( g \); \( (x, k_\perp) \) is the light-front momentum in the loops. Group factors for \( SU(N_c) \) are \( T_f \delta_{ab} = \text{Tr}(T_a T_b) = \frac{1}{2} \delta_{ab} \) and the adjoint Casimir \( C_a \delta_{ab} = f^{acd} f^{bcd} = N_c \delta_{ab} \), \( N_c \) is the number of colors (i.e., \( N_c = 3 \)).

Solution of this gap equation defines an effective gluon mass at zero gluon momentum and within the leading iteration reads (for details see \( ^{21} \))

\[ \mu^2(\lambda) = \mu^2_0 + \delta \mu^2_{PT}(\lambda) + \delta \mu^2(\lambda, \lambda_0). \tag{3} \]

where \( \mu_0 \) is the ‘physical’ mass parameter, which fixes the resulting effective gluon mass at the scale \( \lambda_0 \to 0 \) as \( \mu^2_{ren}(\lambda = \lambda_0) = \mu^2_0 \) (see below); the perturbative term

\[ \delta \mu^2_{PT}(\lambda) = -\frac{g^2}{4\pi^2} \lambda^2 \left( C_a \ln \frac{\mu^2}{\mu^2_0} - \frac{11}{12} \right) + T_f N_f \frac{1}{3} \tag{4} \]

reproduces the result of the light-front perturbation theory \(^{12} \). Renormalizing the effective Hamiltonian through the second order in coupling \( O(g^2) \), the perturbative mass correction is absorbed by the mass counterterm, \( m^2_{CT}(\Lambda_{UV}) = -\delta \mu^2_{PT}(\Lambda_{UV}) \) with \( \Lambda_{UV} \to \infty \), and the renormalized effective gluon mass reads \( \mu^2_{ren}(\lambda) = \mu^2(\lambda) + m^2_{CT}(\lambda) \) for \( \lambda = \Lambda_{UV} \to \infty \). Though the perturbative renormalization is applied at large cut-off scales, \( \Lambda_{UV} \), we assume that the leading cutoff dependence is absorbed by the mass counterterm for all \( \lambda \). Therefore the resulting effective mass, renormalized in the second order, is given

\[ \mu^2_{ren}(\lambda) = \mu^2_0 + \delta \mu^2(\lambda, \lambda_0) = \mu^2_0 + \sigma(\mu_0, u) \ln \frac{\lambda^2}{\mu^2_0} \tag{5} \]

\[ \sigma(\mu_0, u) = -\frac{g^2}{4\pi^2} \mu^2_0 \left( C_a \left( \frac{u^2}{\mu^2_0} + \ln \frac{u^2}{\mu^2_0} - \frac{5}{12} \right) + T_f N_f \left( \frac{1}{3} + \frac{m^2}{\mu^2_0} \right) \right), \]
where scale $u$ has been introduced to regulate small light-front $x$ divergences, $x \sim 0$ and $x \sim 1$, which correspond to high light-front energies. In asymptotic free theories, such as QCD, the regulating scale can be related to the gauge invariant scale, using Callan-Semanzchik type equation. Namely scale $u$ can be expressed through $\Lambda_{QCD}$, solving the third order flow equations for the strong coupling constant $\alpha_s(\lambda)$. Some calculations have been recently done in this direction for the asymptotic free toy model and for the three-gluon vertex in QCD. However, here we do not perform these calculations and assume that the value $u$ is given by the hadron scale, $u \sim \Lambda_{hadron}$.

The resulting effective Hamiltonian is defined at the scale $\lambda \to 0$, therefore an effective gluon mass equals the 'physical' mass, $\mu^2_{ren}(\lambda = \lambda_0 = 0) = \mu_0^2$. In particular, when the 'physical' mass is set to zero, $\mu_0 = 0$, the effective QCD Hamiltonian has zero mass gauge fields, therefore our unitary transformation does not violate gauge invariance (though at the intermediate stages for finite $\lambda$ gauge invariance is broken by the cutoffs). From Eq. (5) one has in the limit $\mu_0 \to 0$

$$\sigma' = \lim_{\mu_0 \to 0} \sigma(\mu_0, u) = u^2 g^2 C_a \pi^2,$$

(6)

and, as shown below, $\sigma'$ plays the role of the string tension between quark and antiquark. In Eq. (5) the squared of the light-front cutoff, $u^2$, defines the rate how fast the effective gluon mass approaches asymptotically $\lambda \to \lambda_0 \sim 0$ (from above $\lambda \geq \lambda_0$) the 'physical' value $\mu_0$. Asymptotic behavior of the effective gluon mass near the renormalization point $\lambda \to \lambda_0$ Eq. (5) is important to take into account when solving flow equations for effective quark interactions at vanishing gluon momenta. In the next section this dependence, Eq. (5), is used to find an effective quark potential, generated by flow equations.

2.2 Effective quark-antiquark interaction

Eliminating the quark-gluon coupling generates an effective interaction in the quark-antiquark sector. In the light-front frame an effective quark-antiquark interaction is given

$$V_{q\bar{q}} = -4\pi\alpha_s C_f \langle \gamma^\mu \gamma^\nu \rangle \lim_{(\mu_0, \lambda_0) \to 0} B_{\mu\nu},$$

(7)

which includes dynamical interactions generated by flow equations and the instantaneous term present in the original light-front gauge Hamiltonian. The gluon renormalization mass parameter ('physical' gluon mass) $\mu_0$ and the renormalization point $\lambda_0$ are set to zero at the end of calculations. Here the
current-current term in the exchange channel reads
\[
\langle \gamma^\mu \gamma^\nu \rangle = \frac{\langle \bar{u}(-k_\perp, (1-x)) \gamma^\mu u(k_\perp, x) \rangle \langle \bar{v}(k'_\perp, x') \gamma^\nu v(-k'_\perp, (1-x')) \rangle}{\sqrt{x x'(1-x)(1-x')}} \tag{8}
\]
where helicities of quarks are suppressed for simplicity; and the interaction
kernel is given in full analogy with an effective electron-positron interaction
in the light-front frame\textsuperscript{20}, except for keeping the cutoff dependence in the
four-momentum transfers, as
\[
B_{\mu\nu} = g_{\mu\nu} (I_1 + I_2) + \eta_\mu \eta_\nu \frac{\delta Q^2}{q^+} (I_1 - I_2) \tag{9}
\]
where \(g_{\mu\nu}\) is the light-front metric tensor, and the light-front unity vector \(\eta_\mu\)
is defined as \(\eta \cdot k = k^+\). The cutoff dependence of four-momentum transfers
along quark and antiquark lines is accumulated in the factor
\[
I_1 = \int_0^\infty d\lambda \frac{1}{Q^2_1(\lambda)} \frac{d f(Q^2_1(\lambda); \lambda)}{d\lambda} f(Q^2_2(\lambda); \lambda), \tag{10}
\]
\(I_2\) is obtained by the interchange of indices 1 and 2; \(f(z)\) is a similarity function;
and the light-front four-momentum transfers are defined
\[
Q^2_1(\lambda) = Q^2_1 + \mu^2_{\text{ren}}(\lambda) \\
Q^2_2(\lambda) = Q^2_2 + \mu^2_{\text{ren}}(\lambda), \tag{11}
\]
with
\[
Q^2_1 = \frac{(x' k_\perp - x k'_\perp)^2 + m^2 (x-x')^2}{x x'} \\
Q^2_2 = Q^2_{2|x\rightarrow(1-x);x'\rightarrow(1-x')} \tag{12}
\]
where \(\mu_{\text{ren}}\) is given in Eq. (5); and the average momenta are \(Q^2 = (Q^2_1 + Q^2_2)/2\)\textsuperscript{20} and
\(\delta Q^2 = (Q^2_1 - Q^2_2)/2\). Calculating an effective kernel with explicit similarity
functions, one has (for details see\textsuperscript{21})
\[
\lim_{(\mu_0, \lambda_0) \rightarrow 0} B_{\mu\nu} = g_{\mu\nu} \left( \frac{1}{Q^2} + \frac{\sigma'}{Q^2} \right) + O \left( \frac{\delta Q^2}{Q^2} \right)^n \tag{13}
\]
where \(n = 2\) for smooth and \(n = 1\) for sharp cut-off functions, and the first term
does not depend on the cut-off function. The four-momenta can be represented
in the 'mixed' light-front \((x, k_\perp)\) and instant \(\bar{k} = (k_z, k_\perp)\) frames as
\[
Q^2 = \bar{q}^2 - (2x - 1)(2x' - 1)(M_1 - M_2)^2/4 \\
\delta Q^2 = -(x - x')(M^2_1 - M^2_2)/2 \tag{14}
\]
where \( \vec{q} = \vec{k} - \vec{k}' = (q_z, q_\perp) \) is the gluon three-momentum transfer, and \( M_1^2 = 4(\vec{k}^2 + m^2) \) and \( M_2^2 = 4(\vec{k}'^2 + m^2) \) are the total energies of the initial and final states. Therefore in the limit of vanishing gluon momentum \( \vec{q} \to 0 \), which defines mainly the bound state spectrum because then the effective \( q\bar{q} \) interaction is singular, the four-momenta are \( Q^2 \to \vec{q}^2 \) and \( \delta Q^2 \to 0 \), and the effective interaction is given

\[
V_{q\bar{q}} = -\langle \gamma^\mu \gamma_\mu \rangle \left( C_f \alpha_s \frac{4\pi}{\vec{q}^2} + \sigma \frac{8\pi}{\vec{q}^4} \right),
\]

where a new constant \( \sigma \) is introduced instead of \( \sigma' \), given by Eq. (6), as \( \sigma = \sigma' \alpha_s C_f / 2 \). One recovers the standard Coulomb and linear rising confining potentials, \(-C_f \alpha_s/\rho + \sigma \cdot \rho\), in configuration space. It is remarkable, that, though calculations are done in the light-front frame, the result for the leading quark-antiquark effective interaction, Eq. (15), is rotationally invariant. Confining term in Eq. (15) with singular behavior like \( 1/\vec{q}^4 \), arises from elimination of the quark-gluon coupling at small gluon momenta, that is governed by the asymptotic behavior of the effective gluon mass.

As was shown in \(^{20}\) with \( \sigma = 0 \), solving QED effective interaction for positronium spectrum numerically, rotational symmetry holds with high accuracy for smooth cut-off functions and even for a sharp cutoff if the collinear singular part is subtracted. Based on our analyses here, we anticipate to see in numerical calculations, that also in the QCD effective interaction rotationally nonsymmetric part contributes negligible and meson spectrum manifests rotational invariance.

3 Conclusions and outlook

An effective QCD Hamiltonian in the light-front gauge has been obtained, solving flow equations for the two lowest Fock sectors selfconsistently. It has been shown that it is possible to eliminate the minimal quark-gluon interaction by using continuous unitary transformation. This elimination causes the renormalization of the coupling functions of the Hamiltonian described by the flow equations. In the two lowest Fock sectors this change of the couplings corresponds to the renormalization of the one-particle energies and to the generation of effective interactions between quarks, in particular quark-antiquark interaction. In oder to set up these differential equations the generated new interactions, with more than three intermediate states, have to be neglected. Truncating in number of particles participating in intermediate states is not equivalent to perturbation theory in coupling constant, but rather is close to Tamm-Dancoff approach.
Our approach has the following advantages: (i) The original gauge field coupling can be completely eliminated, even when the states connected by this interaction are degenerate. The continuous transformation is chosen in such a way that the transformed Hamiltonian does not contain any interactions between one (anti)quark and the creation or annihilation of one gluon. These interactions, connecting the states with energy differences less than a cutoff scale $|E_p - E_q| \leq \lambda$, are still present in similarity approach due to neglecting the renormalization of single particle energies. They may cause mixing between the low and high Fock sectors, and it is problematic to include them perturbatively. Ignoring these low-energy interactions may break the gauge invariance, and in the light-front frame the rotational symmetry. (ii) Effective quark-antiquark interaction is rotationally symmetric at small gluon momenta $q$, while all collinear singular terms $\sim 1/q^+$ and $\sim 1/q^2 + 2$ cancel; and contains in addition to a perturbative term $1/q^2$, which can be obtained in the second order perturbation theory, also a more singular behavior of $1/q^4$ type. Our result for the induced $q\bar{q}$-interaction differs also from the result of similarity scheme, where the collinear singular part of the uncanceled instantaneous interaction $\sim 1/q^+ + 2$, produces a logarithmic potential, which is not rotationally symmetric. The origin of these differences lies in the fact that in our approach all couplings depend on $l$. In order to obtain properties (i) and (ii) the influence of the renormalization of the one-particle energies, in particular the gluon effective energy, on the elimination of the quark-gluon coupling has to be taken into account. By doing so the renormalization of the light-front gluon mass $\mu(\lambda)$ at zero gluon momentum in the asymptotic regime of small cutoff scales is described by an integro-differential equation. This equation can be solved, assuming the renormalization condition that the renormalized gluon mass is given at some small cutoff scale $\lambda_0$ by the ‘physical’ value $\mu_0$, $\mu_{\text{ren}}(\lambda = \lambda_0) = \mu_0$. Renormalization is understood in perturbative sense, i.e. the renormalized effective gluon mass is obtained by absorbing the leading cutoff dependence into the second order mass counterterm. As a result the asymptotic behavior of the renormalized gluon mass $\mu_{\text{ren}}(\lambda)$ for small cutoffs $\lambda$, approaching the renormalization point from above $\lambda \geq \lambda_0$, has been obtained. As a consequence of the properties of $\mu_{\text{ren}}(\lambda)$ the quark-gluon coupling is always eliminated even in the case of degeneracies, namely for vanishing gluon momenta. A similar argumentation was used by Kehrein, Mielke and Neu for the spin-boson model, where the authors argued that the coupling to the bosonic bath always is eliminated because of the renormalization of the tunneling frequency. Also, in a complete analogous to our problem of interacting electrons in BCS-theory, Lenz and Wegner showed that the elimination of electron-phonon coupling for all states is a direct consequence of the renormalization of phonon frequency.
Furthermore it is shown that due to the asymptotic behavior of the gluon energy, elimination of quark-gluon coupling at small gluon momenta gives rise to the enhancement of 'zero modes' in the effective quark-antiquark interaction, i.e. in the infrared region $q \sim 0$ a more singular potential $1/q^4$, than in the perturbative case, is induced. By discussing the consequences of this asymptotic behavior it becomes clear that the approach of flow equations is superior to perturbative calculations, and probably also to (perturbative) similarity scheme which works in terms of bare unrenormalized fields. It can be seen that the flow of the coefficients (couplings) changes the generator $\eta$ of the unitary transformation. Even if the flow of couplings is obtained within the perturbative frame, the unitary transformation based on this generator $exp(\int d\eta(l))$, includes infinite many orders in perturbation theory, corresponding to (leading log) resummation of diagrams. It is worth mentioning, that Lenz and Wegner found in the system of interacting electrons that carrying out the $l$-ordering of $\eta$, the induced electron-electron interactions differ from the Fröhlich’s, where the unitary transformation based on the second order bound state perturbation theory is used. Moreover, this interaction is attractive in all momentum space, providing binding electrons in Cooper pairs. Kehrein and Mielke obtained similar modifications due to $l$-dependent generator by eliminating the hybridization term in the single impurity Anderson model by continuous unitary transformations. The authors showed that their approach generates a spin-spin interaction which differs from the one obtained by the famous Schrieffer-Wolff transformation. Their induced interaction has the right high-energy cutoff, as compared to the Schrieffer-Wolff’s result. Summarizing, within flow equations approach it is possible to make statements which can not be obtained by perturbation theory.

Besides to this comparison with perturbative schemes, due to complete elimination of the quark-gluon coupling, flow equation for an effective quark-antiquark interaction can be integrated for all cutoffs down to $\lambda = 0$. In similarity approach one removes couplings perturbatively until the finite cutoff, below which perturbation theory breaks down. The choice of this cutoff depends on the problem considered, that might be ambiguous. In QCD treatment this cutoff introduces the nonzero scale in the theory, which breaks gauge and rotational invariance. In our approach, the regulator of small light-front $x$ sets up a scale in the effective theory, in particular for the string tension in the effective quark-antiquark potential. Besides of the nonzero scale, the resulting renormalized gluon mass vanishes asymptotically, maintaining gauge invariance. As a consequence, the effective quark-antiquark interaction is rotationally symmetric. However, in this work the small light-front $x$ cutoff scale $u$ enters as an input parameter, and is fitted to the string tension from lat-
tice calculations. To improve this, one should be looking for the fixed point of renormalization group and possible relate the cutoff $u$ with renormalization group invariant scale $\Lambda_{QCD}$. In this way one should include higher Fock sectors in the intermediate states. By doing so it is desirable to establish the independence on the regularization scheme, used to regulate small light-front momenta $x$.

The ultimate goal of the study is to solve the coupled chain of flow equations in different sectors selfconsistently. As has been shown, even an approximate solution of the gluon gap equation together with the flow equation for the effective interaction between probe quarks provides some information beyond the perturbation theory. The next step is to introduce dynamical quark degrees of freedom, formulating quark gap equation, and study the influence of the renormalization of the light-front quark mass on the effective interaction between quarks.

It seems, that it is possible to isolate in the light-front frame the degrees of freedom which are responsible for the long-range properties of QCD, and obtain some insight into the nonperturbative QCD phenomena. This suggests that probably the light-front formulation may be the most suitable frame to solve the system of flow equations selfconsistently on computer.

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