Compressor map prediction tool

Arjun Ravi, Lukasz Sznajder and Ian Bennett.
Shell Projects and Technology
Lange Kleiweg 40
2288GC Rijswijk, The Netherlands

Abstract. Shell Global Solutions uses an in-house developed system for remote condition monitoring of centrifugal compressors. It requires field process data collected during operation to calculate and assess the machine’s performance. Performance is assessed by comparing live results of polytropic head and efficiency versus design compressor curves provided by the Manufacturer. Typically, these design curves are given for specific suction conditions. The further these conditions on site deviate from those prescribed at design, the less accurate the health assessment of the compressor becomes. To address this specified problem, a compressor map prediction tool is proposed. The original performance curves of polytropic head against volumetric flow for varying rotational speeds are used as an input to define a range of Mach numbers within which the non-dimensional invariant performance curve of head and volume flow coefficient is generated. The new performance curves of polytropic head vs. flow for desired set of inlet conditions are then back calculated using the invariant non-dimensional curve. Within the range of Mach numbers calculated from design data, the proposed methodology can predict polytropic head curves at a new set of inlet conditions within an estimated 3% accuracy. The presented methodology does not require knowledge of detailed impeller geometry such as throat areas, blade number, blade angles, thicknesses nor other aspects of the aerodynamic design - diffusion levels, flow angles, etc. The only required mechanical design feature is the first impeller tip diameter. Described method makes centrifugal compressor surveillance activities more accurate, enabling precise problem isolation affecting machine’s performance.

1. Introduction
In the oil and gas industry, the remote performance monitoring of critical machinery is essential to reduce downtime, maintenance costs and improve production rates of entire installations. A common machine that is central to most production and processing assets is the centrifugal compressor. Continuous performance assessment plays a vital part in evaluating the health of these machines. The performance of a centrifugal compressor is usually described by how the polytropic head varies with suction volume flow over a wide range of rotational speeds. Original equipment manufacturers (OEM) typically provide the design curves of polytropic head and efficiency, which serve as a benchmark in assessing the machine’s performance - but for only a specific set of inlet conditions.

Ideally, a compressor would operate close to the OEM design conditions. In reality, the process conditions such as temperature, pressure, molecular weight and rotational speed deviate, sometimes substantially. This deviation impacts the accuracy of the performance assessment. There comes a point when the performance map provided by the manufacturer is no longer valid as a reference at new suction conditions. Accurate performance assessment of compressors requires a map that is valid for varying sets of suction conditions and rotational speeds. Unfortunately, obtaining new performance
curves for an existing compressor from the vendor can be time consuming and resource intensive. Therefore, the methodology of predicting the compressor map at changing process conditions is proposed.

There are many approaches that can be adopted to obtain a compressor map. One of these is to create a numerical model of the centrifugal compressor stage and conduct CFD simulations. An alternative approach is to use empirical correlations for predicting performance maps. A detailed information of the stage geometry is needed; at least on a one dimensional (1D) basis. The detailed geometry is often unknown as this is propriety information of the vendor. Examples of 1D methods are given by [1], [2], [5]. This paper presents a methodology that requires neither aerodynamic information (i.e. diffusion ratios, flow angles) nor the impeller geometry such as the throat areas, blade thickness and blade angles. It only requires the design performance curve from the OEM and the impeller tip diameter which is typically provided in the datasheet of the machine.

The following paper is structured as follows. Section 2 describes the methodology regarding how to correct the original performance map for a different set of suction conditions, limitations, validation and results. Conclusions are presented in the last section.

1.1. Nomenclature

\[ \begin{align*}
D_{imp} & \quad \text{Impeller tip diameter (m)} \\
k & \quad \text{Specific heat ratio (-)} \\
\mu & \quad \text{Tip speed Mach number (-)} \\
M_W & \quad \text{Molecular weight (kg/kmol)} \\
Q & \quad \text{Volume flow rate (m}^3/\text{s)} \\
t & \quad \text{Temperature (°C)} \\
U & \quad \text{Impeller blade tip speed (m/s)} \\
R & \quad \text{Universal gas constant [J/(mol K)]} \\
Z & \quad \text{Compressibility (-)} \\
H_p & \quad \text{Polytropic Head (kJ/kg)} \\
N & \quad \text{Rotational speed (rpm)} \\
S & \quad \text{Cubic spline polynomial} \\
a, b, c, d & \quad \text{Coefficients of spline polynomial (-)} \\
\sigma & \quad \text{Second derivative of cubic spline polynomial} \\
h & \quad \text{Equidistant spacing} \\
m & \quad \text{Number of intervals in the piecewise cubic spline polynomial} \\
i & \quad \text{Variable for iteration} \\
\Phi & \quad \text{Volume flow coefficient (-)} \\
\Psi & \quad \text{Head coefficient (-)} \\
\end{align*} \]

Greek Symbols

\[ \begin{align*}
\Phi & \quad \text{Volume flow coefficient (-)} \\
\Psi & \quad \text{Head coefficient (-)} \\
\end{align*} \]

Subscripts

\[ \begin{align*}
\text{OEM} & \quad \text{Original Equipment Manufacturer} \\
\text{design} & \quad \text{design inlet conditions} \\
\text{new} & \quad \text{new inlet conditions} \\
\text{suc} & \quad \text{suction conditions} \\
p & \quad \text{polytropic} \\
\end{align*} \]

2. General methodology and verification.
The process inlet conditions such as temperature, compressibility, specific heat ratio, molecular weight and circumferential speed all determine the compressor map. One non-dimensional parameter that takes all these factors into account is the tip speed Mach number. The key principle is that if the Mach numbers are the same for two different sets of suction conditions, then their corresponding non-dimensional curve of head coefficient vs. volume flow coefficient are equivalent. This approach
follows similitude with equivalent Mach numbers, flow and head coefficient for the design and the new set of inlet conditions. An equivalent non-dimensional curve also ensures aerodynamic similarity which is important to simulate the equivalent flow of gas through the impellers. An equivalent flow coefficient for the design and the new set of suction conditions ensures the velocity triangles at the inlet of the stage to be the same. To visualize this concept a sample calculation has been done for two different set of suction conditions, for which the proximity of Mach numbers i.e. 0.517 to 0.5 was obtained. All the values are depicted in Table 1. Original polytropic head curves for these two set of inlet conditions provided by the OEM are shown in Figure 1. Corresponding non-dimensional curves obtained after conversion are displayed in Figure 2. Good agreement between these two curves validates the mentioned hypothesis. In both the graphs the red curve represents the design suction conditions and the black curve represents the new suction conditions.

**Table 1** Set of inlet conditions and Mach numbers for corresponding performance curves depicted in Figure 1 and Figure 2.

|                          | Temperature [°C] | Pressure [bara] | Molecular weight [kg/kmol] | Rotational speed [rpm] | Mach number [-] |
|--------------------------|------------------|-----------------|-----------------------------|------------------------|-----------------|
| Design inlet conditions  | 29.9             | 30              | 22.59                       | 6778                   | 0.517           |
| New inlet conditions     | 40.9             | 30              | 17.24                       | 7746                   | 0.5             |

**Figure 1** Polytropic head curves for different sets of inlet conditions.

**Figure 2** Equivalent non-dimensional curves for different sets of inlet conditions with the similar Mach number.

To present an example of the complete curve prediction methodology, data from a randomly chosen centrifugal compressor was used. The inputs are the design performance map of a variable speed compressor and its corresponding suction conditions i.e. pressure, temperature and gas
composition for which the head curves were obtained. With the use of the presented methodology the new compressor map i.e. polytropic head \((H_p)\) vs. volume flow rate \((Q)\) relationships for a new set of suction conditions will be derived.

The coordinates of \(H_p\) and \(Q\) of original compressor map provided by the OEM at design suction conditions and varied \(N\) are shown in Table 2.

**Table 2** Polytropic head \((H_p)\) and volumetric flow \((Q)\) for design suction conditions and varying rotational speeds \((N)\) provided by the OEM. The rotational speeds are in % of the rated speed.

| \(N\)  | \(Q\) | \(H_p\) | \(Q\) | \(H_p\) | \(Q\) | \(H_p\) | \(Q\) | \(H_p\) |
|-------|------|------|------|------|------|------|------|------|
| 100%  | 20171.2 | 137.7 | 15462.1 | 110.1 | 12505.4 | 89.4 | 10559.1 | 65.4 | 8152.6 | 41.3 |
| 90%   | 22837.4 | 133.3 | 18499.0 | 110.2 | 15539.7 | 85.0 | 13594.6 | 63.2 | 10451.3 | 37.9 |
| 80%   | 25593.6 | 125.4 | 21716.0 | 103.5 | 18113.8 | 80.5 | 16535.5 | 56.4 | 12289.8 | 34.6 |
| 70%   | 27978.4 | 111.7 | 24562.2 | 92.2 | 20776.0 | 72.0 | 18923.0 | 47.4 | 14402.5 | 27.8 |
| 55%   | 31183.5 | 84.4 | 28132.2 | 59.2 | 24535.3 | 45.3 | 21764.6 | 28.1 | 17246.1 | 11.9 |

2.1. Main calculation procedure

The design compressor map conversion to new suction conditions is done with the use of following non-dimensional parameters. Head coefficient \((\Psi)\), volume flow coefficient \((\Phi)\) and Mach number \((Mu)\) are calculated using outlined set of equations (2),(3) and (4) according to [3].

Mentioned equations require the conversion of \(N\) into impeller blade tip speed \((U)\), which is done as follows:

\[
U = \pi D_{imp} \frac{N}{60} \tag{1}
\]

\[
\Psi = \frac{2H_p}{U^2} \tag{2}
\]

\[
\Phi = \frac{4Q}{\pi D_{imp}^2 U} \tag{3}
\]

\[
Mu = \frac{U}{\sqrt{k_{suc} Z_{suc} (273.15 + t_{suc}) R MW^{r(1)}}} \tag{4}
\]

The polytropic head and volume flow curves presented in Table 2 are converted to non-dimensional curves in the form of \(\Psi\) and \(\Phi\) for varied Mach numbers using the abovementioned equations. Mach number is varying as the rotational speed is changing. Obtained coordinates of converted curves, now in the form of non-dimensional curves are presented in Table 3 and visualized in Figure 3. Tip speed Mach number \(Mu_1\) corresponds to the non-dimensional curve for the reference rotational speed i.e. \(N_1\) as shown in Table 2. The same logic holds for \(Mu_2, Mu_3, Mu_4\) and \(Mu_5\). This non-dimensional map for varying Mach number is used as a baseline for the calculation of the compressor map for the new set of inlet conditions.
Table 3 The non-dimensional performance map values of head coefficient ($\Psi$) and volume flow coefficient ($\Phi$) for varying Mach number ($Mu$) at design suction conditions.

| $Mu_1$ | $Mu_2$ | $Mu_3$ | $Mu_4$ | $Mu_5$ |
|--------|--------|--------|--------|--------|
| 0.74   | 0.67   | 0.59   | 0.52   | 0.407  |



Figure 3 Invariant non-dimensional curves of head coefficient ($\Psi$) and volume flow coefficient ($\Phi$) for different Mach numbers at design inlet conditions. The legend describes the impeller tip speed Mach number ($Mu$) for each curve.

At this point the Mach numbers for new set of inlet condition and shaft speeds are calculated ($Mu_{new}$). To draw the full non-dimensional curves for new conditions i.e. new head coefficient ($\Psi_{new}$) against new volume flow coefficient ($\Phi_{new}$) the curve approximation method is used.

The mathematical method employed by the Authors was cubic spline interpolation technique, which is briefly presented in subchapter 2.2.1. It allowed obtaining coordinates of non-dimensional $\Psi_{new}$ vs. $\Phi_{new}$ curves for desired $Mu_{new}$ based on existing $\Psi$ vs. $\Phi$ curves for different $Mu$.

In Table 4 the new coordinates of head coefficient ($\Psi_{new}$) and volume flow coefficient ($\Phi_{new}$) obtained in given example are presented. The same is visualised in Figure 4.
Table 4 Volume flow coefficient ($\Phi_{\text{new}}$) and Head coefficient ($\Psi_{\text{new}}$) for new suction conditions.

| $M_u_{\text{new}}$ | $\Phi_{\text{new}}$ | $\Psi_{\text{new}}$ | $\Phi_{\text{new}}$ | $\Psi_{\text{new}}$ | $\Phi_{\text{new}}$ | $\Psi_{\text{new}}$ | $\Phi_{\text{new}}$ | $\Psi_{\text{new}}$ |
|-------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 0.65              | 0.0717              | 3.5732              | 0.0689              | 3.6353              | 0.0662              | 3.6042              | 0.0646              | 3.4536              |
| 0.62              | 0.0854              | 3.5300              | 0.0831              | 3.5410              | 0.0810              | 3.4948              | 0.0802              | 3.4024              |
| 0.56              | 0.0992              | 3.3568              | 0.0972              | 3.3219              | 0.0958              | 3.2440              | 0.0957              | 3.1760              |
| 0.50              | 0.1130              | 2.9796              | 0.1114              | 2.9206              | 0.1105              | 2.8070              | 0.1112              | 2.7046              |
| 0.44              | 0.1336              | 1.8701              | 0.1326              | 1.8525              | 0.1326              | 1.7038              | 0.1344              | 1.3848              |

**Figure 4** The interpolated non-dimensional curves of new head coefficient ($\Psi_{\text{new}}$) and volume flow coefficient ($\Phi_{\text{new}}$) for varying Mach number at new suction conditions.

In Figure 4, the black curves with different markers represent the non-dimensional curves for varying Mach numbers for the new set of inlet conditions. The red curves are the non-dimensional curves for design suction conditions from Figure 3. In the last calculation step the coordinates of non-dimensional curves consisting of $\Psi_{\text{new}}$ and $\Phi_{\text{new}}$ from Table 4 are recalculated back to obtain the actual polytropic head curves i.e. new polytropic head ($H_{p_{\text{new}}}$) vs. volumetric flow rate ($Q_{\text{new}}$) for desired set of process conditions and shaft speeds. It is done with the use of equations (5) and (6)

\[
H_{p_{\text{new}}} = \Psi_{\text{new}} \frac{U_{\text{new}}^2}{2}
\]

(5)

\[
Q_{\text{new}} = \Phi_{\text{new}} U_{\text{new}} D_{\text{imp}}^2 \frac{\pi}{4}
\]

(6)

To validate the methodology, the newly obtained compressor map was compared to the available map that was generated by the OEM at the same suction conditions during the machine’s performance test. These curves are shown in Figure 5. The accuracy achieved in polytropic head prediction with the use of described methodology reaches c.a. 2-3% of error when predicted curve is compared to the original OEM’s characteristic. The specific results regarding the accuracy of the method are depicted
in Table 5. Such level of accuracy is regarded as satisfactory.

![Comparison of OEM Vs Mapping tool. Polytropic head curves for different rotational speeds](image)

**Figure 5** Comparison of compressor map generated by compressor mapping tool and map provided by the OEM. The red markers are the polytropic head curves from the OEM. The black curves are obtained from the compressor mapping tool.

**Table 5** Validation of Compressor Mapping prediction tool with the values obtained from the OEM for the Rotational speed \(N = 9683\) rpm.

| Flow [m3/h] | Polytropic head OEM [kJ/kg] | Polytropic Mapping tool[kJ/kg] | Deviation in Polytropic head[%] |
|-------------|-----------------------------|--------------------------------|---------------------------------|
| 1           | 15991.0                     | 136.0                          | 2.0%                            |
| 2           | 19277.3                     | 133.3                          | 1.4%                            |
| 3           | 22563.7                     | 124.1                          | 2.2%                            |
| 4           | 25850.0                     | 109.1                          | 2.0%                            |
| 5           | 30779.5                     | 71.7                           | 1.8%                            |

This method is only applicable when each stage of a centrifugal compressor is treated separately. Each stage refers to the number of impellers stacked together between the suction and discharge of a compressor.
2.2. Method limitations

One should remember that the accuracy of the method depends on the range of the operating envelope of the invariant non-dimensional curve varying with Mach number. Higher inaccuracies in curve approximation are seen when curve extrapolation is used.

The following convention is employed to define the upper and lower boundary of Mach numbers at design inlet conditions for which the presented methodology renders accurate results.

\[ \text{Mu}_{\text{low}} \] - Mach number at design inlet conditions for lowest \( N \);

\[ \text{Mu}_{\text{high}} \] - Mach number at design inlet conditions for highest \( N \);

Extrapolation limits the accuracy of presented method. Therefore the following constraints are imposed at newly calculated \( \text{Mu}_{\text{new}} \). It is checked whether the \( \text{Mu}_{\text{new}} \) is within the range of Mach numbers calculated for the design conditions as shown below. There are three possible cases:

1. \( \text{Mu}_{\text{low}} < \text{Mu}_{\text{new}} < \text{Mu}_{\text{high}} \) – the newly obtained Mach number is between boundary values of lowest and highest Mach numbers, therefore the interpolation method will be used to calculate coordinates of new head and volume flow coefficient curve. Experience has shown that interpolation allows to obtain non-dimensional curve approximation with high accuracy;

2. \( \text{Mu}_{\text{new}} < \text{Mu}_{\text{low}} \) – the newly obtained Mach number is smaller than the lowest Mach number obtained for design inlet conditions, therefore the extrapolation method will be used to calculate coordinates of new head and volume flow coefficient curve;

3. \( \text{Mu}_{\text{new}} > \text{Mu}_{\text{high}} \) – the newly obtained Mach number is greater than the highest Mach number obtained for design inlet conditions, therefore the extrapolation method will be used to calculate coordinates of new head and volume flow coefficient curve;

In 2\text{nd} and 3\text{rd} case the following condition must be met to ensure an acceptable accuracy of the curve approximation:

- Check if \( \left( \frac{\text{Mu}_{\text{low}} - \text{Mu}_{\text{new}}}{\text{Mu}_{\text{new}}} \right) \leq 5\% \) (case 2)

- Check if \( \left( \frac{\text{Mu}_{\text{new}} - \text{Mu}_{\text{high}}}{\text{Mu}_{\text{new}}} \right) \leq 5\% \) (case 3)

The three conditions are visualised in Figure 6. The red curves are the invariant non-dimensional curves for design inlet conditions. The dotted curves are the extrapolated curves corresponding to case 2 and 3. The dashed curve is the interpolated curve corresponding to case 1.

Extrapolation or interpolation of the new non-dimensional curve of \( \Psi_{\text{new}} \) vs. \( \Phi_{\text{new}} \) for the new \( \text{Mu}_{\text{new}} \) is done depending on which of the three aforementioned conditions is met. The accuracy of the extrapolation largely deteriorates and the method is rejected for deviations in new Mach number greater than 5\% from the highest or lowest design Mach number, as it was mentioned earlier.

It must be also mentioned that this method is only applicable when each stage of a centrifugal compressor is treated separately. Each stage refers to the number of impellers stacked together between the suction and discharge of a compressor.
### 2.2.1. Curve approximation method

Mathematical method used to interpolate or extrapolate curves which was employed in the presented curve prediction tool is the Cubic Spline method. This technique provides smooth continuous approximated curves compared to piecewise linear interpolation. Cubic spline interpolation formula is continuous in the first and second derivatives and both within the intervals and interpolating nodes. It also provides an interpolating polynomial which has a smaller error compared to other interpolating polynomials such as Lagrange or Newton’s. A brief description of the original cubic spline interpolation method employed in the presented performance map prediction tool is provided hereunder.

\[
f(x) = \text{CubicSpline}(x_{\text{input}}, x, y) = S_i(x)
\]

- \( x_{\text{input}} \) is the new Mach number for new set of inlet conditions;
- \( x \) is the dataset comprising known values of \( \Phi \) obtained at design inlet conditions;
- \( y \) is the dataset comprising known values of \( \Psi \) obtained at design inlet conditions;

Spline interpolation is divided into small sub-interval and each of these sub-intervals is interpolated by using a third degree polynomial. According to [4], for a set of \( m+1 \) data points \( (x_i, y_i) \) and where no two \( x_i \) are the same on a given interval, a cubic spline \( (S_i) \) can be defined on each sub-interval \( (x_i, x_{i+1}) \) by

\[
S_i(x) = a_i (x_{\text{input}} - x_i)^3 + b_i (x_{\text{input}} - x_i)^2 + c_i (x_{\text{input}} - x_i) + d_i
\]

For \( m \) piecewise polynomial, four times \( m \) boundary conditions are needed to determine the coefficients \( a_i, b_i, c_i, \) and \( d_i \). The boundary conditions are described in the equations outlined below:

\[
S_i(x_i) = y_i \quad \text{for} \quad i = 0 : m - 1
\]

\[
S_{m-1} = y_m
\]

\[
S_i(x_{i+1}) = S_{i+1}(x_{i+1}) \quad \text{for} \quad i = 0 : m - 2
\]
\[ S_i(x_{i+1}) = S_{(i+1)}(x_{i+1}) \quad \text{for } i = 0: m - 2 \] (12)

\[ S_i(x_{i+1}) = S_{(i+1)}(x_{i+1}) \quad \text{for } i = 0: m - 2 \] (13)

\[ S_0(x_0) = S_{(m-1)}(x_m) = 0 \] (14)

A new variable \( \sigma \) for the second derivative of the polynomial is introduced. Another variable \( h \) is defined for equidistant spacing.

\[ \sigma_i = S'(x_i) \] (15)

\[ h = x_{i+1} - x_i \]

By using the equations (8), (9), (10), (11), (12), (13), (14) and (15) the coefficients \( a_i, b_i, c_i, d_i \) are obtained.

\[ d_i = \frac{y_i - b_i}{2}; a_i = \frac{\sigma_{i+1} - \sigma_i}{6h}; c_i = \frac{y_{i+1} - y_i}{h} - h \frac{2\sigma_i + \sigma_{i+1}}{6} \] (16)

3. Conclusion and Limitations

In this paper, a generic methodology is proposed, which enables correction of the original polytropic head curves for actual operating conditions based on the equivalence of non-dimensional parameters. The general procedure and validation of the method has been described. With minimal inputs of process conditions, first tip impeller diameter and a design performance map from the manufacturer, a compressor performance map for a new set of inlet conditions can be predicted within an accuracy of 2-3%.

The effective implementation of this method in remote condition monitoring system enables actual compressor performance assessment with the use of corrected polytropic head curves, which more accurately reflect the theoretical behaviour of a healthy machine at given conditions. Additionally, it can aid the performance review of the machine for new operating conditions and advise the engineers on site if the compressor can operate at the new conditions. Another key feature of this method is the accurate locations of the operating points on the compressor maps, which is essential for surge and choke protection. The presented feature can eventually facilitate isolation of undesirable compressor behaviour at an early stage and, therefore, help identify potential failure modes, thus reducing downtime and maintenance costs of the compressor and surrounding equipment. The method has been validated for 20 different centrifugal compressors, thus proving its readiness.

References

[1] Aungier R H 2000 Centrifugal Compressor - A Strategy for Aerodynamic Design and Analysis (New York, USA: ASME Press).
[2] Cumpsty N A 1989 Compressor Aerodynamics (Harlow, Essex, UK: Longman Scientific &Technical).
[3] Lüdtke K H 2004 Process Centrifugal Compressors (Berlin: Springer).
[4] Plato R. 2003 Concise Numerical Mathematics vol 57 (Providence R.I: American Mathematical Society).
[5] Swain E 2005 Improving a one-dimensional centrifugal compressor performance prediction method Proc of the Institution of Mechanical Engineers Part A: Journal of Power and Energy 219(8) pp.653-659.