Analysis of dynamic characteristics and seismic performance for Shengshousi Pagoda

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Abstract. To study the dynamic characteristics and seismic performance of the Shengshousi Pagoda, the present status preserved of the pagoda’s structure was investigated on site, and the numerical computation model was established based on the parameters determined by calculated value of the natural vibration frequency of the structure. The responses of acceleration, displacement, story drift and stress were analyzed under three-way seismic wave. The seismic performance of pagoda structure was evaluated according to the calculation results of displacement and stress, and the failure criterion of pagoda materials. The results indicate that the natural frequencies in the east-west are close to that in north-south. The response laws of acceleration, displacement and stress are similar under three-way seismic wave of 8-degree fortification earthquake. Under the frequent earthquakes, the structure of pagoda does not suffer from compression damage; the tensile damage area is located at the bottom floor. Under the fortification and rare earthquakes, significant compression damage area is located at lower floors of the ancient pagoda, the stress concentration occurs around the part of opening, and significant tensile damage occurs.

1. Introduction

As an outstanding example of ancient high-rise buildings, ancient masonry pagodas carry a long history of civilization and embody the superb level of architectural technology. A large number of brick towers were built during the sui and tang dynasties (581-907 AD) in China, most of which have suffered severe structural damage or collapse caused by earthquakes. The existing ancient masonry pagodas always were damaged or destroyed in some degree by wind and rain erosion, human factors, and earthquakes. If subjected to the earthquake again, they are likely damaged more seriously or even collapse. It is imperative to scientifically evaluate, effectively reinforce and protect the existing ancient masonry pagodas based on their dynamic characteristics and seismic resistance.

In order to analyze the dynamic characteristics of ancient masonry pagodas, scholars have done some research work. Li D.H. et al. [1] established a simplified cantilever model with fixed bottom and stepped constant section along the height. Wen L.h. et al. [2-4] analyzed dynamic performance and seismic capacity of Shaanxi Fanwang Pagoda by the field rocket-excited vibrating test, theoretical and numerical simulation analysis. The method of establishing dynamic analysis model of ancient building was put forward by combining experiment and theoretical analysis based on the simplified variable cross-section cantilever model with bottom fixed. Moreover, Yuan J.L. et al. [5-8] analyzed the dynamic characteristics of Suzhou Huqiu Pagoda and Zhongjiang South Pagoda by the field nondestructive environment-excited vibrating test, the classical theory analysis and computational simulation. The modeling method of dynamic characteristics was suggested. The simplified
calculation formula of basic natural vibration period of ancient masonry pagodas was obtained. The comparison between the calculated value and the field measured value showed that the calculated precision met the engineering requirement. The above method and formula can better estimate the dynamic performance of the ancient masonry pagoda. Chen T.C. et al. [9] carried out dynamic characteristics test and numerical calculation for the Jinaozhou Pagoda in Dongguan, Guangdong. CAI H.T. et al. [10-12] studied the dynamic characteristics, seismic response and seismic resistance of east and west pagodas in Quanzhou, Fujian through pulsating test and numerical simulation. the calculation formula of natural vibration period of ancient masonry pagoda was proposed. the allowable elastic storey drift index of the structure was recommended. Lu J.L. et al. [13-15] studied the dynamic characteristics, dynamic response, and failure mechanism of Xuanzang Tower of Xingjiao Temple in Xi'an, Shaanxi by field investigation, numerical simulation and shaking table test, and evaluated its seismic performance.

The above researches show that the dynamic characteristics and seismic resistance of ancient masonry pagodas should be fully understood so to reasonably evaluate and appropriately reinforce them during the protection. But the dynamic characteristics and seismic performance of the ancient pagoda vary greatly with the located region, size and shape, and protection status of the pagoda. The numerical simulation analysis is one of the effective means in the study of ancient pagodas, whose key was to establish a reasonable numerical model. Therefore, this paper investigated the preserved status of the Shengshousi masonry Pagoda of Tang Dynasty in Chang'an District, Xi'an City, Shaanxi Province. Based on similar ancient brick masonry test and theoretical analysis, a numerical computation model was established according to the principle of equivalent dynamic characteristics. Its dynamic characteristics and seismic response were analyzed, and the anti-seismic capacity of the pagoda structure was studied, which provided a reference for the protection and reinforcement of the ancient masonry pagoda structure.

2. Overview of Shengshousi Pagoda
Shengshousi Pagoda shown in figure 1 is located in the valley at the northern foot of Qinling Mountains in Chang'an District of Xi'an City. It was built in the Tang Dynasty (776-779 A.D.) (Also said in the Sui Dynasty(601-604)). The Pagoda was established as the Sixth Batch of national cultural relic’s protection units in China in 2006. The pagoda is a 7-story square attic brick tower with a height of 29.15m and a base length of 7.2m. Vaulted doors are arranged in the north-south of odd floors and the east-west of even floors. The cornices between stories were stacked by bricks. There are wooden-like elements such as columns, architrave, bucket arches etc. on the wall of the pagoda. The spires of the pagoda were made of black bricks flat. The size of the black brick is 325 mm x 175 mm x 65 mm, which was built with lime mortar and clay mortar. The dimensions of each part of the pagoda structure were shown in Table 1.
According to field investigation, there were accumulated soils with height of 0.2-1.5m in the northwest side of the pagoda. Drainage of the pagoda base surface was not smooth. Rainwater was easily poured into the tower room from the door hole. The plant growth on the pagoda wall caused partial cornice brick carving, brick rafters to damage, a layer of wall plastering to fall off, mortar to run off. In order to avoid the structural damage caused by local damage of the Shengshousi Pagoda, the cultural relic’s management department strengthened and repaired the pagoda in 2016.

### Table 1. Structure size of Shengshousi Pagoda

| Layer | Floor height(m) | Side length(m) | Vaulted hole height(m) | Vaulted hole width (m) |
|-------|-----------------|----------------|------------------------|------------------------|
| 1     | 7.345           | 7.2            | 1.967                  | 1.53                   |
| 2     | 3.73            | 6.86           | 1.20                   | 1.11                   |
| 3     | 3.46            | 6.33           | 1.13                   | 1.01                   |
| 4     | 2.87            | 5.83           | 1.00                   | 0.9                    |
| 5     | 2.85            | 5.24           | 0.9                    | 0.71                   |
| 6     | 2.52            | 4.57           | 0.75                   | 0.6                    |
| 7     | 3.3             | 3.92           | 0.65                   | 0.49                   |

### 3. Numerical Computation Model

The structure of Shengshousi pagoda was made of bricks distributed evenly and reasonably which was equivalent to a continuum. The Pagoda structure was analyzed by the whole modeling method. The density of wall masonry is 1900 kg/m³, the elastic modulus is 1078MPa, Poisson's ratio is 0.15, the axial tensile strength is 0.089MPa, the axial compressive strength is 1.97MPa and the shear strength is 0.04MPa on account of field investigation, masonry strength test and literature [16].

A three-dimensional numerical computation model without the tower spire of the Shengshousi Pagoda was established by using the six-node space tetrahedral element C3D10 of the finite element software Abaqus as shown in Figure 2. The model size is real structural size. The bottom of the pagoda model is rigidly connected with the foundation.
4. Dynamic Characteristics
Dynamic characteristics of ancient masonry pagodas was calculated by empirical formulas and numerical simulation in this paper.

4.1. Estimation by Empirical Formula
Considering the influence of main factors such as structural height, height-width ratio and mechanical parameters of structural materials on the natural frequencies of the pagodas, the formula for calculating the horizontal natural frequencies of ancient masonry pagodas was selected as following [17].

\[ f_j = \frac{\alpha_j b_0}{2\pi H^2} \psi \]

In which, \( f_j \) is the \( j \) th natural frequency of the structure (Hz); \( \alpha_j \) is the comprehensive deformation coefficient of the \( j \) th natural frequency of the structure and is selected in the given table [17] according to \( H/b_n \) and \( b_0/b_n \); \( b_0 \) is the width of the structure bottom, i.e. the distance between two sides (m); \( H \) is the height of the structure, i.e. the height from the top of the pagoda foundation to the bottom of the spire (m); \( b_n \) is the weighted average value of the width of each layer to the floor height in the range of the calculated height of the structure; \( \psi \) is the mass stiffness parameter of the structure (\( m/s^2 \)), and \( \psi = 5.4H + 615 \) for ancient masonry pagodas.

According to formula (1), the first three orders of natural frequencies of Shengshousi Pagoda are calculated as follows: \( f_1 = 1.54 \) Hz, \( f_2 = 6.24 \) Hz, \( f_3 = 12.28 \) Hz.

4.2. Numerical calculation
The numerical model was modally analysed to calculate the dynamic characteristics of the ancient pagoda, as shown in Figure 3 and Figure 4. The results show the stiffness in the north-south and east-west main axes of the pagoda are close, and the each order natural frequencies in the direction of the two main axes are close. The difference between the numerical value of natural frequencies of structures and the calculated value of empirical formulas is less than 10%, which indicates that the numerical model reflected the dynamic characteristics of real ancient pagoda structures.
5. Seismic Response Analysis

5.1. Seismic wave selection and calculation parameters
The Shenghousi Pagoda is located in the 8-degree earthquake fortified area without site foundation data. The site was preliminarily determined to be a type III referring to the site conditions. The three-way Tianjin wave was selected as the ground motion input, in which, the north-south direction was X, the east-west direction was Y, and the vertical direction was Z. The ratio of acceleration peak values in the X,Y and Z directions, i.e. $a_x : a_y : a_z$ is 1:0.85:0.65. The record of the first 18 seconds of the earthquake wave was intercepted as shown in Figure 5.
Amplitude of seismic wave was modulated according to the corresponding fortification intensity requirements. Acceleration peaks of small, medium and large earthquakes are $70 \text{cm/s}^2$, $220 \text{cm/s}^2$ and $400 \text{cm/s}^2$ respectively. The Rayleigh damping coefficient $\alpha$ is 0.479 and $\beta$ is 0.0013 according to the dynamic principle [18].

The acceleration, displacement, and stress response of the Shengshousi pagoda structure were calculated by input three-way seismic wave selected.

5.2. Acceleration response

The acceleration response curves of the 1st, 4th and 7th floor were shown in Figure 6. The variation law of acceleration of each floor with time is shown to be basically the same. The variation laws of peak acceleration of each floor were shown in figure 7. The maximum acceleration of the top floor along horizontal direction were shown as $4.02 \text{cm/s}^2$, $12.59 \text{cm/s}^2$ and $22.76 \text{cm/s}^2$ respectively under small, medium and large earthquakes. As shown in Figure 8, the change rule of acceleration response with the height of the structure is basically the same under small, medium and large earthquakes, and the response value increases with the height of the structure. The top acceleration amplification effect is not obvious in small earthquakes, but most significant and the dynamic magnification factor is approximately 6 in large earthquakes.

5.3. Displacement response

The calculation results show that the time of the peak displacement response of pagoda structure is the same, and the wave shape of response curve is similar, and the maximum horizontal displacements are
0.03m, 0.094m and 0.17m respectively under small, medium and large earthquakes. The change of the story drift with the height of the pagoda under three loading conditions was shown in Figure 9. The story drift increases uniformly with the pagoda height which indicates no significant sudden change of story drift in every floor. The story drift of the top of the 6th story is the largest. The story drift increases with the seismic intensity. The maximum story drifts of the structural are 1/609, 1/192, 1/106 in small, medium and large earthquakes, respectively.

5.4. Stress response
The peak stress response was only analyzed as follows because the stress of the pagoda structure changes dynamically under the excitation of seismic waves under different loading conditions. The first principal stress, the third principal stress and the shear stress at the peak time of the pagoda stress response were extracted and shown in Figures 10-12.

It is shown in figure 10 that the maximum first principal stresses of the pagoda structure are located at the northeast corner of the first floor under all loading conditions. The maximum tensile stresses are 0.58MPa, 2.92MPa and 5.69MPa in small earthquakes, medium earthquakes and large earthquakes respectively. It is shown in figure 11 that the maximum third principal stresses of the pagoda structure are located at the northeast corner of the first floor under all loading conditions. The maximum compressive stresses are 1.88MPa, 4.44MPa and 7.53MPa in small earthquakes, medium earthquakes and large earthquakes respectively. It is seen from figure 12 that the shear stresses are distributed uniformly along the structure, and the maximum shear stresses are located at the top of the first floor portal. Their values are 0.52MPa, 1.47MPa and 3.21MPa for small, medium and large earthquakes.
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6. Seismic Performance Assessment

6.1. Evaluation Criteria

The horizontal deformation of ancient masonry pagoda under earthquake includes shear deformation and bending deformation. Therefore, the story drift was used to judge the whole damage state of the structure. The ultimate story drift in reference [10] was chosen as the criterion for judging the damage of ancient pagodas, as shown in Table 2.
Table 2. Displacement failure criteria

| Story drift | State of structure    | Damage state of structure |
|-------------|-----------------------|---------------------------|
| ≤ 1/565     | Elastic stage         | Unbroken                  |
| 1/565-1/360 | Initial yield stage   | slight damage             |
| 1/360-1/260 | Initial stage of stiffness degradation | Moderate damage |
| >1/260      | Exceeding the limit of story drift | Serious damage or collapse |

Because the tensile strength of masonry material of the ancient pagoda is far lower than its compressive strength, the structural failure of the pagoda is the shear failure as a result of the main tensile stress exceeding the ultimate tensile stress of the material. Therefore, the main tensile stress was taken as the index to judge the element failure, and the failure criteria were given in Table 3.

Table 3. Principal tensile stress failure criteria

| Main Tensile Stress of Masonry | Stage of Structures | Structural damage |
|--------------------------------|---------------------|-------------------|
| ≤0.089MPa                      | Elastic stage       | Unbroken          |
| >0.089MPa                      | Plastic stage       | Cracking          |

6.2. Seismic Performance Assessment
The seismic performance of Shengshousi ancient masonry pagoda was evaluated in base of comprehensive judgement of the ultimate displacement and ultimate bearing capacity. The ultimate displacement is used to judge the weak layer, and the maximum principal tensile stress is used to judge the damage site.

The results of seismic displacement response of Shengshousi pagoda show that under 8-degree frequent earthquakes the maximum story drift is 1/609 and occurs in the 6th floor of the pagoda. The structure is judged to be basically intact and in the elastic stage at this time from Table 2. While the main tensile stress occurs in the cornices and portals of the south elevation of the pagoda which is prone to cracking if the main tensile stress reaches the axial tensile strength. Under 8-degree fortification earthquake the main tensile stress occurs in the 1st to 4th floor of the pagoda and is greater than the axial tensile strength of the material, and the maximum principal tensile stress is distributed at the bottom of each layer, especially at the northeast and southwest corners, and the structure is in the failure stage.

7. Conclusion
The numerical model and calculation parameters were determined according to the principle that the natural frequencies of the structure are consistent with the calculation results of the dynamic characteristics of the Shengshousi Pagoda. Seismic waves under 8-degree of small, medium and large earthquakes were input to calculate the seismic acceleration, and displacement, and stress of the ancient pagoda. The main conclusions are as follows:

1) The numerical computational model of the pagoda structure was determined according to the principle of the consistent dynamic response that the numerical calculation results of the first three natural frequencies of the structure are consistent with the theoretical calculation results. The numerical model reveals the real structure of Shengshousi Pagoda.

2) There is no compressive damage and the tensile damage zone is located on the bottom floor in Shengshousi pagoda under small earthquakes. Under the medium and large earthquakes, there is a significant compressive damage zone around the entrance of the middle and lower floors of the ancient pagoda, and the all structural floors are significantly damaged by tension.
3) Under the 8-degree earthquake, the seismic response of each floor increases rapidly with the height of the storey, and the tensile failure origins in the bottom floor. The shear stress around the opening and the middle part of the pagoda structure is large, which causes the cracking around the opening.

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