Positive spatial curvature does not falsify the landscape

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Abstract. We present a simple cosmological model where the quantum tunneling of a scalar field rearranges the energetics of the matter sector, sending a stable static ancestor vacuum with positive spatial curvature into an inflating solution with positive curvature. This serves as a proof of principle that an observation of positive spatial curvature does not falsify the hypothesis that our current observer patch originated from false vacuum tunneling in a string or field theoretic landscape. This poster submission is a summary of the work, and was presented at the 3rd annual ICPPA held in Moscow from October 2 to 5, 2017, by Prof. Rostislav Konoplich on behalf of the author.

1. Introduction: false vacuum tunneling and the landscape
Quantum tunneling and decay of a false vacuum is a means of transitioning between different low-energy vacua in a field or string theoretic landscape scenario. If the tunneling event leaves behind observable signatures, for instance positive spatial curvature which is observable in the cosmic microwave background radiation (CMB) or in the distribution of large scale structure (LSS), we may be able to test the hypothesis that our current observer patch originated in such a tunneling event. The large scale curvature of the spatial slices is quantified by the curvature parameter $\Omega_k = -\frac{3K}{8\pi G_N H^2}$, where $K$ is the inverse square of the radius of curvature, and $H$ is the Hubble parameter, all measured at the present time. A negative value for this parameter corresponds to positive curvature of the spatial slice, and vice versa. If measured, the curvature parameter will be one of the single most important parameters we can measure in cosmology, with potential implications for the existence of other vacua besides our own, and for the physics of the pre-inflationary era.

The case where false vacuum tunneling nucleates a bubble much smaller than the curvature scale was analyzed in the semiclassical approximation by [1]. The WKB amplitude for barrier tunneling can be related to the action of a solution to the equations of motion in Euclidean signature. Analytically continuing back to Lorentzian spacetime, the bubble expands after nucleation, and surfaces of constant time slicing inside the region of “true” vacuum have open or hyperbolic spatial curvature. If the inflationary epoch lasted just enough to solve the horizon and flatness problems, this curvature may be persist and be observable on the largest cosmological scales. It was argued in [4] that this may even be probabilistically favored in simple models of the landscape. This was further developed in [5] where it is argued that under fairly generic assumptions about the landscape, any spatial curvature observed today must be negative. This is often interpreted as a no-go theorem that an experimental detection of positive curvature
would falsify the landscape. It is the purpose of the present work to present a counterexample to this statement, although we do not have the tools to decide whether our model can be likely in the context of a larger landscape scenario.

It should be noted that our model is not the first to feature positive curvature from a tunneling event (see e.g. [9, 11, 10, 18], and [19]), however, what makes the model here novel is that the tunneling process is described entirely within semiclassical field theory, and the tunneling rate is tunable and dominated by the tunneling of a single scalar field. This is accomplished using the tunneling of a scalar field to rearrange the energetic contributions of the matter sector between sources with different equations of state.

2. Model: Tunneling from the simple harmonic universe

Our starting point is the simple harmonic universe studied by [2, 6, 7, 8], which was shown in [2, 3] to be classically and quantum mechanically stable at the level of linearized perturbations. The simplest solution of this type consists of positive spatial curvature, a negative cosmological constant, and a matter source with $w = p/\rho = -2/3$. Adding an additional matter source with $w = p/\rho = -1/3$ tunes the energetics of the curvature contribution and helps control certain classes of perturbations. The metric is the FRW metric $ds^2 = -dt^2 + a(t)^2 d\Sigma^2$, and the Friedmann equation for the evolution of the scale factor is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{K_{\text{eff}}}{a^2} + \frac{8\pi G}{3} \rho_{\text{eff}} = \frac{K_{\text{eff}}}{a^2} + \frac{8\pi G}{3} \left(\frac{\rho_0}{a} + \Lambda\right)$$

Here $K_{\text{eff}} > 0$ is the combination of curvature and matter with $w = -1/3$, and the term with $\rho_0$ is from the matter source with $w = -2/3$. It is simple to show that the scale factor oscillates sinusoidally in time. If the perturbations of the matter sector have positive speed of sound squared, it was shown in [2, 3] that there exists a parametric regime where the solution can be stable at the level of linearized perturbations. While there may be nonperturbative decay processes, the solution nevertheless can remain stable on exponentially long timescales.

We propose a model where the energetics of the matter sector depend on the vacuum expectation value of a scalar field $\phi$, and the effective potential $V_{\text{eff}}$ has multiple degenerate minima in $\phi$ with different equations of state. In particular, as $\phi$ tunnels from its initial to its final value, we have $\rho_0(\phi_i) > 0$, $\Lambda(\phi_i) < 0$, and $\rho_0(\phi_f) = 0$, $\Lambda(\phi_f) > 0$. The tunneling potential is constructed so that the tunneling is energetically possible only when the scale factor is at or near the minimum of the bounce. This must be true for the Euclidean as well as the Lorentzian solution, and so it is also necessary to make the tunneling transfer energy to a radiation sector as well, so that the tunneling will be energetically unfavorable for the classically forbidden region $a < a_{\text{min}}$ as well as for the classically allowed region $a > a_{\text{min}}$. The full potential for the matter sector is therefore given by

$$V_{\text{eff}}(\phi, a) = \frac{\rho_0(\phi)}{a} + \Lambda(\phi) + \frac{\rho_r(\phi)}{a^4},$$

where the energy of the various matter sources depends on the vacuum expectation value of the scalar field so that $V_{\text{eff}}(\phi_i, a) \leq V_{\text{eff}}(\phi_f, a)$, with equality only for $a = a_{\text{min}}$, and $\rho_0(\phi_i) > 0$, $\Lambda(\phi_i) < 0$, $\rho_r(\phi_i) = 0$; $\rho_0(\phi_f) = 0$, $\Lambda(\phi_f) > 0$, $\rho_r(\phi_f) > 0$. The scalar field tunnels over the entire volume of the ancestor at once, and is effectively a 0+1-dimensional process. As it tunnels through the barrier from the ancestor to the daughter vacuum, the universe transitions from the simple harmonic universe solution to an inflating solution with positive curvature. There is also a radiation bath present in the daughter vacuum, but this redshifts away within a few e-folds.

Since the energy is conserved, as long as the evolution of the scalar field in the Euclidean solution proceeds much faster than the evolution of the scale factor, gravitation effectively
decouples and the scalar field evolves in a fixed background, with the transition taking place over the entire spherical spatial slice. On longer timescales, the Euclidean solution for the evolution of the scale factor can be analyzed with and without the presence of the scalar bounce, and it is found that the change in the Euclidean action is dominated by the scalar evolution. The scalar bounce evolution therefore decouples from gravity and proceeds without gravitational backreaction in this limit. Furthermore, there exists a parametric limit where the scalar field decay dominates the total decay rate over any purely global quantum gravitational processes (such as tunneling of the scale factor, as discussed in [3, 12, 17] which may be present.

Although gravity decouples from the homogeneous bounce solution, it is necessary to include backreaction effects when considering the second variation of the Euclidean action around the bounce solution. It was shown in [13], working separately with the eigenmodes of this operator which are odd/even in Euclidean time, that there cannot be more than one nonpositive eigenmode for the second variation if the Euclidean solution is to maximize the tunneling probability. This can be shown to be equivalent to finding the number of odd and even bound states of the potential \( V_{eff}(\phi, \tau) \) in flat space, which is replaced by a more complicated potential when gravitational backreaction is taken into account. For the bounce of a scalar field in flat space, it can be shown that there is a single even eigenmode with negative eigenvalue, and a single odd mode with zero eigenvalue. Here things are different because of the background evolution of the metric, and we find a cluster of odd negative eigenmodes closely spaced in energy, and a cluster of even negative eigenmodes closely spaced in energy. Within each cluster, however, these solutions for the wavefunction differ only in a small singular region of the effective potential, where the wavefunction undergoes rapid oscillations that may arise from quantum gravitational effects. This class of eigenvalues also arose in models studied in [14], and are believed to arise from quantum gravitational effects when the tunneling instanton includes most or all of the volume of the Universe. Although the interpretation of these eigenmodes is not fully understood, they do not seem to invalidate the interpretation of the Euclidean bounce solution as a tunneling process. The bounce action is not strictly speaking a decay rate when there are multiple negative eigenvalues present, but rather should be interpreted as a tunneling between degenerate minima.

3. Conclusion and experimental constraints

To conclude, we have presented a proof-of-principle model where an inflationary vacuum with positive spatial curvature is the endpoint of a scalar field tunneling event. The tunneling can be analyzed in the semiclassical regime and there exists a parametric limit where it proceeds self-consistently and dominates the tunneling rate. Whether such a process can be generic in the context of a larger landscape of possibilities will have to await further studies explicitly constructing the simple harmonic universe in the context of string or field theory, and it would be interesting to pursue this further. For the moment, however, we wish to make the point that positive curvature does not falsify the possibility that our present vacuum originated from tunneling in a landscape, and it is worth including positive values for the curvature in experimental searches. Together with the tensor to scalar ratio of primordial perturbations, if observed it is perhaps as significant as a single number can be.

The curvature parameter \( \Omega_k \) must have an absolute value of at least \( 10^{-5} \) in order to be distinguishable from gauge-dependent primordial fluctuations of the same type that generated the seeds of structure visible in the CMB. Current observational limits from CMB and LSS surveys constrain the curvature parameter at the 1 per cent level, and upcoming experiments from will push this by at least a further order of magnitude and may even close the observationally interesting window entirely [15, 16].
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