Origin of the logarithmic correction to the Newton’s law in the presence of a homogeneous gas of wormholes

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(Dated:)

We suggest a new scenario in which the Universe starts its evolution with a fractal topological structure. This structure is described by a gas of wormholes. It is shown that the polarization of such a gas in external fields possesses a spatial dispersion, which results in a modification of the Newton’s law. The dependence on scales is determined by the distribution of distances between throat ends. The observed in galaxies logarithmic correction confirms that the distribution has fractal properties. We also discuss the possibility of restoring such a distribution from observations.

I. FRACTAL TOPOLOGICAL STRUCTURE AS INITIAL CONDITIONS

The most important problem of the modern astrophysics is the nature of dark matter (DM). While particle physics suggests too many speculative candidates, LHC does not provide even any hint for the existence of such dark particles. Moreover, the standard model in particle physics needs not their existence at all. This make us think that something wrong with our interpretation of the picture of the Universe observed.

It turns out that lattice quantum gravity, has already suggested a satisfactory principle answer to the question what DM should be. The answer lies in the very complex spacetime foam picture which exists at very small (sub-Planckian) scales and which possesses fractal properties. Indeed, the first indication that at very small scales spacetime has a universal fractal structure came from 2d-quantum gravity [1]. This result was obtained both, analytically and by means of numerical simulations and it has been developed in many papers, e.g., see [2] and references therein. Moreover, numerical simulations demonstrate that the fractal behavior retains in 4d lattice quantum gravity as well. And indeed, the spectral dimension for nonperturbative quantum gravity defined via Euclidean dynamical triangulations was calculated in [2]. It turns out that it runs from a value of $D = 3/2$ at short distance to $D = 4$ at large distance scales.

The fractal structure of the spacetime foam can be described as follows. Consider any point $x$ in space, fix a geodesic distance $R$, and consider the value of volume $\Omega_x(R)$ which gets within the ball $d(x, y) \leq R$. The homogeneity of the foam means that $<\Omega_x(R)> = \Omega(R)$, i.e., does not depend on the position of the starting point for geodesic lines. Then the scaling $\Omega(R) \propto R^{D_H}$ defines the Hausdorff dimension of space $D_H$. By the construction it is clear that in 4d gravity $D_H < 4$. Indeed, the starting point $x$ together with the omnidirectional jet of geodesic lines define the extrapolating reference system (exactly as we use in astrophysics) for which the coordinate volume scales with $D = 4$. The same behavior works for simple topology spaces. However, in the presence of wormholes for sufficiently big distances $R$ some of geodesics return and start to cover the same (already covered) physical region $\Omega(R)$. Therefore, for sufficiently complex topologies (foam) the dimension is always less than 4 (i.e., we may state that $D_H < 4$).

It is remarkable that we see on the sky exactly such a picture. Indeed, the light is too scattered upon propagating through the wormhole throats and it forms the diffused background [1]. Therefore, in the homogeneous Universe galaxies are good tracers for the actual (physical) volume which exactly demonstrate the fractal behavior $N(R) \propto R^D$ with $D \approx 2$ up to distances $R \sim 200\,\text{Mpc}$ [3]. In particular, this reflects the long standing puzzle of missing baryons. The nontrivial complex topology leads to the modification of Newton’s law [4] which is interpreted as the dark matter phenomenon and which perfectly fits the observations [5].

Thus, we see that lattice quantum gravity suggests us a new cosmological scenario. The inflationary stage in the past has enormously stretched all scales and the fractal quantum spacetime foam structure has been tempered. We may say that together with metric perturbations generated from quantum fluctuations the inflation generates topological perturbations from quantum spacetime foam as well. Such a structure represents a homogeneous and isotropic space filled with a gas of wormholes which represents the initial conditions for the standard cosmological model. In other words, such a space is homogeneous, while the topological structure is not (its mean value is homogeneous and isotropic but possesses fluctuations).

Expecting the question on the stability of such wormholes we point out to the well known fact that the non-trivial topological structure (the gas of wormholes) is consistent with the homogeneity of space. Indeed, such a structure can be realized as space of a constant negative curvature (by means of the cut and paste technique in the Lobachevsky space). In the three dimensions this however assumes that the simplest wormhole has throat sections in the form of a torus or a sphere with a single handle, since the sphere does not possess the metric of a constant negative curvature (at least one point will include a singularity). If the handle on such a sphere is sufficiently small, it will look almost as a sphere (e.g., as we know the simplest model of a horse is given by the spher-
ically symmetric horse in vacuum; In this respect all our attempts to work with spherically symmetric wormholes look like this and they may have in the first place the methodological interest. Nevertheless, upon averaging out over orientations of the torus the spherical symmetry restores and we may expect that spherical wormholes reproduce correctly some basic features. We present the simplest exact example of the constructing of a stable wormhole in the appendix.

In general, spacetime foam assumes that entrances into wormholes may be separated by time-like and space-like distances equally. However it is clear that due to rapid inflationary process the time-like separation does not survive (it remains of the order of $\ell_{pl}$) while space-like distances have enormously change $\propto e^{H\Delta t}$, where $\Delta t$ is the duration of the inflationary stage. According to [3] the volume of such a space scales as $V(R) \propto R^{d_H}$ with the Hausdorff dimension between $1 < d_H \leq 3$. At very small and very large scales (maybe even outside the Hubble radius) $d_H = 3$, while on intermediate scales the dimension changes.

The difference $d_H - 3 \neq 0$ we observe now as the dark matter phenomenon. Indeed, the simplest estimate gives for the Newton’s law $F \propto \frac{GM_\text{ph}}{R^d_H}$, where $S(R)$ is the surface of the sphere of the radius $R$ which scales as $S(R) \propto R^{d_H - 1}$. We point out that the value $d_H \approx 2$ is in agreement with observations in the range $5Kpc < R \lesssim 200 Mpc$. The value $d_H \approx 1$ gives too strong gravitational coupling and such a topological structures should decay (or even be suppressed during the inflation). Indeed, as we know metric perturbations do not grow during the radiation dominated stage. However such a behavior works only for Freedman spaces (flat space, sphere or the Lobachevsky space). In the presence of a nontrivial topological structures this, in general, is not true, e.g., see the first investigation in [8] which has reviled an essential (scale-dependent) modification of the equations for perturbations.

The negative curvature of space looks to be forbidden by the Doppler picks in $\Delta T/T$ spectrum. However the upper bounds on the curvature are model dependent (they are based on simple topology spaces). Moreover, the apparent value of the curvature is somewhat reduced by the ratio $V_{ph}(R)/V_{coord}(R)$, where $V_{ph}(R) \sim R^{d_H}$ is the actual or physical volume of space and $V_{coord}(R) \sim R^3$ is the extrapolating coordinate volume. In this estimates we should take $R = R_H$ ($R_H$ is the Hubble radius). The same ratio describes also the portion of missing baryons. In other words, these problems require the further and more careful investigation.

Thus, we see that our Universe can be rather far from the simple picture we use. It remains to be isotropic and homogeneous but may possess a rather complex local topological structure. Our phenomenological description based on the standard $\Lambda CDM$ model works well enough, but meets some small inconsistencies which permanently enforce us to add some exotic matter fields (absent in lab experiments) or consider different modifications of gravity. However it is rather clear that the polarization of matter fields on the fractal topology (bias or topological susceptibility) is rich enough and it is capable of explaining all exotic properties observed.

II. HOMOGENEOUS GAS OF WORMHOLES

Consider now the simplest model suggested by us earlier [6] to demonstrate that the topological bias allows to mimic dark matter phenomena. As it is explained in the previous section stable wormholes have throat sections in the form of torus (e.g., see Fig.3). Therefore the model based on spherical wormholes (which are not stable) has in the first place the methodological interest. Nevertheless, it contains all basic general qualitative features (topological permeability). Moreover, upon averaging over orientations of tori (wormhole throat sections) the spherical symmetry of wormholes restores and we may expect that even some quantitative features will be correct.

When the gravitational field is rather weak, the Einstein equations for perturbations reduce to the standard Newton’s law

$$\frac{1}{a^2} \Delta \phi = 4\pi G \left(\delta \rho + \frac{3}{c^2} \delta p\right),$$

here $a$ is the scale factor of the Universe, $\delta \rho$ and $\delta p$ are the mass density and pressure perturbations respectively, $G$ is the gravitational constant, and $\Delta = \nabla^2$ is the Laplace operator. Therefore, the behavior of perturbations is determined by the Green function

$$\Delta G(x, x') = 4\pi \delta(r - r').$$

In the simple flat space the Green function is well known $G_0 = -1/r$ (or for Fourier transforms $G_0 = -4\pi/k^2$). In the presence of wormholes due to polarization on throats the true Green function obeys formally to the same equation but with biased source (which is the topological bias or susceptibility)

$$\Delta G(x, x') = 4\pi \left(\delta(r - r') + b(r - r')\right).$$

In the case of a homogeneous gas of wormholes such a bias was evaluated first in [6] (see also more general consideration and details in [10]) and is given by

$$b(k) = \frac{2\pi R}{k^2} \left(\nu(k) - \nu(0)\right),$$

where $n$ is the density of wormhole throats, $R$ is the mean value of the throat radius, and $\nu(k)$ is the Fourier transform for the distribution over the distances between wormhole mouths. It is defined as $\nu(X) = \frac{1}{nR} \int F(X, R) dR$, where $F$ is the number density of wormholes in the configuration space (due to homogeneity $X = X_+ - X_-$, and $X_+$ are positions of wormhole entrances in space). This function is normalized so that
\[ \int \nu(X)d^3X = 1. \] We point out that in a more general case (non-spherical wormholes) the bias will have analogous structure with an appropriate redefinition of the dimensional parameter \(2nR \rightarrow 1/\ell^2\), though some details may change. Thus the true Green function will include some correction and look like

\[ G(k) = \frac{-4\pi}{k^2(1-b(k))} = \frac{-4\pi}{k^2} \left(1 + 2nR^2 \frac{4\pi}{k^2} (\nu(k) - \nu(0)) \right). \]

In order to get the logarithmic correction (observed in galaxies) the distribution \(\nu(k)\) should give the behavior \(b(k) \sim k^{\alpha}\) with \(\alpha \approx 1\).

Consider now particular examples of the above distribution. Let all distances between wormhole entrances are of the same value \(r_0\), then \(\nu(X) = \left(4\pi r_0^2\right)^{-1} \delta(|X| - r_0)\) and we find \(\nu(k) = \int \nu(X)e^{-ikX}d^3X = \frac{\sin kr_0}{kr_0}\), which gives

\[ b(k) = -2nR^2 \frac{4\pi}{k^2} \left(1 - \frac{\sin kr_0}{kr_0}\right). \]

For small \(kr_0 \ll 1\) we find

\[ b(k) = \frac{4\pi}{3} nR^2 \frac{2}{kr_0} \left(-1 + \frac{3!}{5!} (kr_0)^2 + \ldots\right). \]

The first constant simply renormalizes the gravitational constant, while next terms define corrections to the Newton’s law. At first look such a decomposition represents a rather general situation. Indeed, Consider an additional distribution over the parameter \(r_0\) with any probability density \(p(x) \left(\int_0^\infty p(x)dx = 1\right)\). Then the same decomposition works for mean values < \(\frac{\sin kr_0}{kr_0}\> = \sum \frac{(-1)^n}{(2n+1)!} < (kr_0)^{2n} > \) which means that the above expression works as well with the replacement \(r_0^{2n} \rightarrow < r_0^{2n} >\). However this is possible only for normal (Gaussian) distributions with stable (finite) momenta. In the case of fractal picture this is not true. Indeed, Consider a particular fractal distribution with infinite dispersion of the type

\[ \nu(k) = \exp(-A(ik)^\alpha - B(-ik)^\alpha). \]

In the case \(\alpha < 2\) the dispersion is divergent

\[ \sigma = \frac{d^2}{dk^2} (\ln \nu(k))|_{k=0} \propto k^{\alpha - 2} \rightarrow \infty \]

and the above decomposition does not work. However all corrections to the Newton’s law can be found from the decomposition of the characteristic function \(\nu(k)\) itself

\[ \nu(k) - 1 = k^{\alpha} \sum C_n k^{\alpha n}. \]

All the coefficients in the above decomposition reflect deviations from the standard Newton’s law and may be interpreted as the presence of dark matter. In the first place they should be fixed from observations (the observed distribution of DM) which allow us to define the actual distribution of wormholes and restore the true Green function

\[ G = \frac{-4\pi}{k^2} (C + Bk^\alpha + \ldots). \]

For observational needs as an empirical Green function (or generalized susceptibility) we may suggest the expression

\[ G_{emp} = \frac{-4\pi}{k^2 (1 + (kr_0)^{-\alpha})} \] which at small scales \((kr_0 \gg 1)\) gives the standard Newton’s law, while at large scales \(kr_0 \ll 1\) transforms to the fractal law. We also point out that the logarithmic correction (observed in galaxies) corresponds to the value \(\alpha \approx 1\) and \(r_0 \sim 5Kpc\).

### III. APPENDIX

Consider now the cut and paste technique on the Lobachevsky space which allows to construct stable wormholes. Consider the upper complex half-plane, see Fig.1, which gives the model of 2D Lobachevsky space. The metric on the half-plane has the form \(dl^2 = dx^2 + dy^2\). The absolute (infinity) is the axis \(Ox (y = 0\) or \(y \to \infty\). Geodesic lines are half circles with centers on the absolute, or perpendicular to the absolute rays. Making the cut along pies-wise geodesic lines as shown on Fig.1 (solid lines) and gluing along identical geodesics \(g_\pm\) we obtain the space of a constant negative curvature with two identical closed geodesic boundary lines \(a\) as shown on Fig.2a. Again gluing along closed geodesic circles \(a\) we get the Lobachevsky space with a handle on it, as shown on Fig.2b. It is important that such a space is the space of the constant curvature. It is also clear that repeating such a procedure we may insert an arbitrary number of handles.

Now using the axial symmetry of Lobachevsky space we add to the above 2d wormhole an angle \(0 \leq \phi < 2\pi\),
FIG. 2: a) Upon gluing along $g_{\pm}$ we get two identical closed geodesics $a$. Internal region of the circles $a$ is removed. b) Upon gluing along $a$ we get the handle in the Lobachevsky plane. Geodesics $g_{\pm}$ transform into a single continuous geodesic line.

FIG. 3: The ball is the 3D Lobachavsky space. Rotation of its section around the axis $Oz$ as shown on Fig.3, we get the simplest stable 3d wormhole. We repeat again that all these constructions represent spaces with a constant negative curvature. Their subsequent cosmological evolution is governed by the Freedman equations (i.e., the metric takes the form $ds^2 = dt^2 - a^2(t) dl^2$, where $dl^2$ corresponds to the Lobachevsky space with a set of wormholes described above).

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