Unparticles and CP-violation

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Abstract: We give a brief summary of the unparticle scenario proposed by Georgi. The CP-even phase
of the propagator is exploited to study the CP-asymmetry in $B^+ \rightarrow \tau^+ \nu$, which is neither experimentally searched for nor predicted by any other model. Furthermore we show that the novel CP-violation is consistent with the CPT theorem by identifying the CP-compensating mode in the unparticle sector.

1. Introduction
The possibility of a strongly coupled scale invariant sector, weakly coupled to the Standard Model (SM), was advocated by Georgi in [1, 2]. The operators of the scale invariant theory do not describe single particle excitations but entail a continuous spectrum, hence the name “unparticle”. An interesting deconstruction of this spectrum in terms of a particle tower was given in Ref. [3]. According to [1] at a very high energy scale $M_\text{UT} \gg 1$ TeV the particle world could be described by the SM and a strongly self-coupled ultraviolet (UV) sector, which interact with each other via a heavy particle of mass $M_\text{UT}$ and is described by the effective Lagrangian

$$L^\text{eff} \sim \frac{1}{M^2_{\text{UT}} + (d_{\text{SM}} - 4)} O_{\text{SM}} O_{\text{UV}} \quad \text{lower energy} \quad \lambda \frac{\Delta_\text{UT}}{M_\text{UT}^{d_{\text{UV}} - (d_{\text{SM}} - 4)}} O_{\text{SM}} O_{\text{UT}}, \quad (1)$$

and at some energy $\Lambda_\text{UT}$ the UV sector flows into a strongly coupled infrared (IR) fixed point where the UV operator undergoes dimensional transmutation $O_{\text{UV}} \rightarrow (\Lambda_\text{UT})^{d_{\text{UV}} - d_\text{d}} O_{\text{UT}}$ and the coupling indicated above is $\lambda = \alpha_\text{d}/M_\text{UT}^{d_{\text{UV}} - (d_{\text{SM}} - 4)}$, with $\alpha_\text{d}$ being a matching coefficient expected to be of order one.

From a Lagrangian of the type (1) either real [1] or virtual effects [2] can be investigated from symmetry properties and the scaling dimension $d_{\text{UT}}$ alone! The meaning of the real emission of an unparticle is at present unclear or at least model dependent. Virtual effects are described in a transparent way within the formalism of perturbative field theory by the propagator, which can be constructed from the dispersion relation

$$\Delta_\text{UT}(P^2) \equiv i \int_0^\infty d^4 x e^{i P \cdot x} \langle 0| T O_{\text{UT}}(x) O_{\text{UT}}^\dagger(0) \rangle = \int_0^\infty ds \frac{2 \text{Im}[\Delta_\text{UT}(s)]}{2\pi s - P^2 - i0} + \text{s.t.} \quad (2)$$

It is assumed that $P^2 \geq 0$ and $P_0 > 0$ and s.t. stands for possible subtraction terms due to non-convergence in the UV. The imaginary part is related to the local matrix element by the optical theorem

$$2 \text{Im}[\Delta_\text{UT}(P^2)] = \langle 0| O_{\text{UT}}(0)| P \rangle^2 P^{-2} = A_{d_\text{UT}}(P^2)^{d_\text{UT} - 2}, \quad (3)$$

whose form is dictated by the scaling dimension of $O_{\text{UT}}$. The dispersion integral is then elementary [2, 4]

$$\Delta_\text{UT}(P^2) = \frac{A_{d_\text{UT}}}{2 \sin(d_\text{d} \pi)} \left( -P^2 - i0 \right)^{2-d_\text{d}} \int_0^{P^2 > 0} d_{\text{UT}} \frac{e^{-i d_{\text{d}} \pi}}{P^2} \quad (4)$$

for appropriate $d_{\text{UT}}$ to be discussed below. The normalization factor $A_{d_\text{UT}}$, which is arbitrary up to the requirement $\Delta_\text{UT}(P^2)^{d_{\text{UT}} - 1}/P^2$, has been chosen to be $A_{d_\text{UT}} = 16\pi^{5/2} \Gamma(d_{\text{d}} + 1)/2 \Gamma(d_{\text{d}} - 1) \Gamma(2d_{\text{d}}))^{-1} [1]$. It is the analytic continuation of the phase space volume of $d_{\text{UT}}$ massless particles based on the observation that the matrix element $\langle 0| O_{\text{UT}}(0)| P \rangle$ behaves as such. This led to the statement that an unparticle looks like a non-integral number $d_{\text{UT}}$ of massless particles [1]. The propagator (4) exposes power like scaling, unlike the logarithmic scaling of the trivial UV fixed point of QCD, and a CP-even phase factor $e^{-i d_{\text{d}} \pi}$, whose consequences have been investigated in many papers and constitutes the central ingredient to the analysis presented here. The identification $d_{\text{UT}} = 1 + \gamma$ of the scaling dimension and the anomalous dimension follows from the limit to the free propagator. The lower bound of values for the scaling dimension is $d_{\text{UT}} \geq 1 + j_L + j_R$, where $j_L(R)$ is the Lorentz spin, for which the four dimensional conformal group admits unitary representations [5]. This bound assures the IR convergence of the dispersion integral (2). The integral diverges in the UV for $d_{\text{UT}} \geq 2$, but on the other hand the theory is described in the UV by the non-scale invariant theory of operators $O_{\text{UV}}$, which alters the dispersion integral.
in the UV. In principle there is no upper boundary but nevertheless in the literature most often the values 
1 < dU < 2 are assumed without much loss for the phenomenological analyses. Scale invariance is expected to
be broken at lower energies, first by the emergence of the weak scale, coupling to the Higgs VEV for instance
[6], and second in concrete realizations discrete parameters, such as the number of colours, might only allow for
a near critical behaviour only.

The discussion up to now has been mostly formal based on symmetries. This raises the question of whether
there are indeed such theories in four dimension that flow into a non-trivial IR fixed point. In Ref. [1] the
(perturbative) Banks-Zaks [7] fixed-point was given as an illustrative example. Very recently it was shown that
half of the supersymmetric gauge theories and around a quarter of the the non-supersymmetric gauge theories
do indeed flow into a scale invariant phase [8]. Furthermore, it was pointed out that in an appropriate limit
the so-called higher dimensional (HEIDI) models [9] assume the unparticle spectral relation (3). The role of
the non-trivial anomalous dimension is mimicked through, possibly fractional, flat extra dimensions accessible
to SM singlet fields.

Unparticle like behaviour of the propagator (4) is observed elsewhere. For instance the resummation of
logarithms due to the emission and absorption of the massless photon in QED leads to an electron propagator
\( S(p) \sim (p^2 - m^2 + i0)\)\(^{-1-\gamma} \) [10]; the analogous case was considered in QCD in Ref. [11]. Or in the scale
invariant solvable two dimensional Thirring model the propagator behaves as \( S(x) \sim \frac{1}{(-x^2 + i0)^{1+\gamma}} \) [12].

2. CP-violation in \( B^+ \rightarrow \tau^+\nu \)

It is well-known that direct CP-asymmetry occurs if there are two amplitudes with different weak (CP-odd)
and strong (CP-even) phases, e.g. [13]. In the SM the decay \( B^+ \rightarrow \tau^+\nu \) is mediated by the diagram Fig. 1(left)
and receives negligible radiative corrections and has therefore no sizable CP asymmetry. Moreover the CP-
asymmetry is not even searched for in experiment [13]!

Figure 1: (left) SM diagram for \( B \rightarrow \tau\nu \) (right) unparticle diagram with CP odd phase \( e^{i\Delta_\nu \pi} \).

Coupling a scalar unparticle, in an ad-hoc fashion, to the flavour sector similar to the charged Higgs sector
\[ L^{\text{eff}} = \frac{\lambda_{U}^{D}(\gamma_{D})D}{\Lambda_{U}^{D}}(U_{P}(\gamma_{D})D) U + \frac{\lambda_{U}^{P}(\gamma_{P})}{\Lambda_{U}^{P}}(U_{P}^{D})D U + \text{h.c.}, \]
gives rise to the tree-level contribution shown in Fig. 1(right). The CP-even phase of the propagator and a weak
phase \( \lambda^{\text{ub}(\tau\nu)} \) different from either the CKM or the PMNS phase then opens the door to novel CP-violation,
\[ \mathcal{A}_{\text{CP}}(\tau\nu) = \frac{2\Delta_{\tau\nu} \sin(\phi) \sin(d_{U}\pi)}{1 + 2\Delta_{\tau\nu} \cos(\phi) \cos(d_{U}\pi) + \Delta_{\tau\nu}^{2}}, \]
where \( \phi = \delta_{\text{ub}} - \delta_{\text{ud}} \) is the non-CKM(PMNS) weak phase and the ratio of unparticle to SM amplitude is
\[ \Delta_{\tau\nu} = \rho_{\tau\nu} \frac{\lambda_{U}^{D}(\gamma_{D})D}{\Lambda_{U}^{D}} m_{B}^{2} \sin(d_{U}\pi) m_{B} \left( \frac{m_{B}^{2}}{\Lambda_{U}^{D}} \right)^{d_{U}^{-1}} \frac{1}{m_{B}^{2}} \rho_{\tau\nu} = \frac{\lambda_{U}^{\text{ub}(\tau\nu)}}{|V_{ub}U_{\tau\nu}|}. \]
The first factor is the ratio of flavour couplings, the second is a normalization factor of the order of one, the
third is kinematical, the fourth measures the relvance of the operator and the fifth factor is an enhancement
factor \( \sqrt{\frac{G_{F} m_{B}^{2}}{\Lambda_{U}^{D}}} \sim 5 \cdot 10^{3} \) which is peculiar to the tree-level weak unparticle sector [12] and allows for large
effects unlike in other sectors [6]. Two simplifying assumptions were made, first \( \lambda \equiv \lambda_{S} = \lambda_{P} \) and second it
was assumed that the ratio of \( \lambda^{\nu_{e}}/U_{\nu_{e}} \) to be independent of \( l' \) which otherwise leads to more complicated
formulae [12] because the neutrino flavour is not observed in experiment. The main results are [12]

1. The measured branching ratio \( B(B \rightarrow \tau\nu) \) [13] allows for maximal CP-violation (for suitable parameters)
2. Up to \( \rho_{\tau\nu} \sim 10^{-3} \) parameters \( (d_{U}, \phi) \) exist for which \( \mathcal{A}_{\text{CP}} \sim 80\% \), due to the enhancement factor (7)
3. Consistent with CPT?

It is well known that the CPT symmetry enforces the equality of the total rate of particle and antiparticle. In fact, the equality already holds for the sum of partial rates of particles rescattering into each other, e.g. [14].

\[ \sum_{i \in I} \Delta \Gamma(B \to f_i) = 0, \quad \langle f_i | S \hat{S} | f_j \rangle \neq 0 \quad i, j \in I, \]

where \( \Delta \Gamma(B \to f) \equiv \Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f}) \) is the width difference. It is not clear which decay channel compensates for the novel CP asymmetry \( A_{CP}(\tau \nu) \sim \Delta \Gamma(B^+ \to \tau^+ \nu) \) in the unparticle world.

In the SM there is no appropriate final state since \( \tau^+ \nu \) is essentially a class on its own. We are led to look in the unparticle sector for a suitable candidate. A hint can be gained from counting the weak coupling constants. The processes \( B^+ \to U^+ \) with an interference of the two amplitudes depicted in Fig. 2 has the same counting in the coupling constants. One amplitude corresponds to a tree decay and the other incorporates a virtual correction due to a fermion loop of the \( \tau \) and the \( \nu \). The process \( B^+ \to U^+ \) is kinematically allowed since the unparticle has a continuous mass spectrum. It does not proceed at resonance, but rather behaves like a multiparticle final state and is a realisation of Georgi’s observation that the unparticle field in a final state behaves like a non-integral number \( d_U \) of massless particles. We refer to Ref. [12] for further details where the exact verification of \( \Delta \Gamma(B^+ \to \tau^+ \nu) + \Delta \Gamma(B^+ \to U^+)_{\tau \nu - \text{loop}} = 0 \) is demonstrated explicitly.

Conclusions

The unparticle scenario gives rise to spectacular phenomena. For example it permits CP-violation in leptonic decays unprecedented so far in other models. We have shown that the CP-violation is consistent with the CPT theorem. Yet a concrete realization of the unparticle scenario remains to be worked out, where questions such as the nature of the real unparticle, the breaking of scale invariance and the fate of unparticles at low energies can be studied in a concrete and quantitative way.

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