Closed universe and the first Doppler-peak of the CMB spectrum

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ABSTRACT

We study universe models which are intrinsically closed and are full of a quintessence scalar field, besides the Cold Dark Matter component. We use these to depict diverse flat Cold Dark Matter models. With the background geometry specified by the Friedman-Robertson-Walker metric, we include among them the standard Cold Dark Matter, the Cosmological Constant Cold Dark Matter and the Dark Energy or Quintessence Cold Dark Matter models. After describing these models, we determine the position of the first Doppler peak of the Cosmic Microwave Background anisotropy spectrum for $\Omega_T$ close to one; we also study the shift parameter $R$.

Key words: cosmology: theory – early Universe – cosmological parameters – cosmic microwave background.

1 INTRODUCTION

To day, we do not know precisely the exact amount of matter present in the universe, so that we ignore its geometry. Astronomical observations conclude that the matter density related to baryonic and nonbaryonic cold dark matter is much less than the critical density value (White et al. 1993). However, recent measurements of type Ia distant supernova indicate that in the universe there exists an important energy component which contributes to a large component of negative pressure, and thus it accelerates rather than decelerates the universe (Perlmutter et al. 1998; Garnavich et al. 1998).

Different interpretations have been suggested for explaining the acceleration of the universe. We distinguish here those related to the existence of a cosmological constant, and the quintessence or dark energy models. The former are characterized by a a vacuum energy density, while the latter are characterized by a scalar field $\chi$, and its potential $V(\chi)$ (Caldwell, Dave & Steinhardt 1998).

Various tests of cosmological models, including space-time geometry, galaxy peculiar velocities, structure formation, and very early universe descriptions (related to inflation (Guth1981)) support a flat universe scenario. Specifically, the redshift-distant relation for supernova of type Ia, anisotropies in the cosmic microwave background radiation (Roos & Harun 2000; Mather et al. 1994) and gravitational lensing (Mellier 1999) suggest that $\Omega_T = 1.00 \pm 0.12$ (95% cl) (De Bernardis et al. 2000).

In light of these results, one interesting question to ask is whether this flatness is due to a sort of compensation among different components that enter into the dynamical equations. In the literature we find some descriptions along these lines. For instance, a closed model with an important matter component with equation of state given by $P = -\rho/3$ has been studied (Kolb 1989). Here, the universe expands at a constant speed. Other authors, using the same equation of state, have added a nonrelativistic component in which the total matter density, $\Omega_T$, is less than one, thus describing an open universe (Kamionkowski & Toubias 1996). Also, flat decelerating universe models have been simulated (Cruz, del Campo & Herrera 1998; Cataldo & del Campo 2000; del Campo & Cruz 2000). The common fact in all of these models is that, even though the starting geometry were other than that corresponding to the critical geometry, all of these models are indistinguishable from flat models at low redshift.

In this work we want to describe a closed universe model composed of two matter components. One is the usual nonrelativistic dust matter and the other corresponds to a sort of quintessence-type matter, designated by the $Q$ scalar field, which we assume obeys an equation of state

$$P_Q = w_Q \rho_Q,$$

where, in general, the $w_Q$ parameter is a time dependent negative. We also assume that its current value is bounded from the above, $w_Q \leq -1/3$. The geometry, together with this matter component, combines in a way such that flat universe scenarios arise. Among the scenarios which we consider are the standard Cold Dark Matter or Einstein-de Sitter (sCDM), the Cosmological Constant Cold Dark Matter ($\Lambda$CDM) and the Quintessence (or dark energy) Cold Dark
Matter ($\chi$CDM) models. In the latter scenario, a scalar field $\chi$ is added to the relevant components (Wang, Caldwell, Ostriker & Steinhardt 2000). In this case, it is assumed that there exists an equation of state for the dark energy scalar field $\chi$ given by $P_\chi = w_\chi \rho_\chi$, where astronomical observations (related to type Ia supernovae measurements) set an upper limit on the present value of $w_\chi$, $w_\chi \leq -1/3$ (Garnavich et al. 1998).

From the theoretical point of view, anisotropies in the CMB are related to small perturbations, which are believed to seed the formation of large-scale structures in the universe. These anisotropies are sensitive to cosmological parameters such as the CDM, number of baryons, three-space curvature and the cosmological constant (Hu & Sugiyama 1995a). These anisotropies are enhanced by oscillations of the photon-baryon fluid before decoupling, which are driven by primordial density fluctuations that depend on the matter content (Hu, Sugiyama & Silk 1997).

Models with adiabatic and isocurvature fluctuations predict a sequence of peaks in the power spectrum which are generated by acoustic oscillations of the photon-baryon fluid at recombination. The fluctuations, as a function of the wavenumber $k$ go as $\cos(kc_s \tau_L)$ at last scattering, where $c_s$ is the sound speed and $\tau_L$ is the conformal time at recombination. In the case of primordial adiabatic fluctuations, these causes a harmonic series of temperature fluctuation peaks, where $k_m \equiv \frac{m \pi}{c_s \tau_L}$ corresponds to the $m$th peak.

In the isocurvature case, it is found that the acoustic peaks are $90^\circ$ degrees out of phase with their adiabatic counterparts (Hu & Sugiyama 1995b).

Of particular interest is the height and position of the main acoustic peak - the so called Doppler peak. This pronounced peak in the angular power spectrum occurs at multipole $l_{LS}$. The exact value of $l_{LS}$ depends on both the linear size of the acoustic horizon and the angular diameter distance from the observer to the recombination era (last scattering surface). Both of these quantities are sensitive to a number of cosmological parameters, essentially to the total density parameter $\Omega_T$, which is defined to be the ratio between the total matter density and the critical energy density. In the case in which there is no contribution from the cosmological constant, it is found that $l_{LS} \sim 200/\sqrt{\Omega_T}$ (Kamionkowski, Spergel & Sugiyama 1994; Hu & Sugiyama 1995a; Frampton, Ng & Rohm 1998; Dodelson & Knox 2000; Crooks, Dunn, Frampton & Ng 2000).

A precise measurement of $l_{LS}$ can efficiently constrain the density parameter and, specifically, the curvature of the universe. In fact, in the BOOMERAnG (Balloon Observations Of Millimeter Extragalactic Radiation and Geomagnetic) experiment the value $l_{LS} = (197 \pm 6) \left(1 - \sigma \text{ error}\right)$ has been reported (De Bernardis et al. 2000). This, in a model with $\Lambda = 0$, is consistent with an almost flat geometry, since this value leads to $\Omega_T = 1.03 \pm 0.06$ (Roos & Harun 2000) when the above expression, $l_{LS} \sim \Omega_T^{-1/2}$, is used. For instance, in the case of the $\Lambda$CDM model, we find that $\Omega_T = \Omega_M + \Omega_\Lambda$, in which $\Omega_M \equiv \frac{\rho_M}{\rho_c} = \frac{\Omega_M^0}{3H_0^2}$ is the present value of the nonrelativistic matter, the critical densities and the Hubble constant, respectively, and $\Omega_\Lambda \equiv \left(\frac{\Lambda}{8\pi G}\right)/\rho_c = \frac{\Lambda}{3H_0^2}$ Here, $\rho_M^0$, $\rho_c$ and $H_0$ are the present values of the nonrelativistic matter, the critical densities and the Hubble constant, respectively, and $G$ is the Newton constant. $\Omega_\Lambda$ and $\Omega_M$ are parameters associated with the cosmological constant $\Lambda$, and the matter density is related to the baryonic and nonbaryonic Cold Dark Matter density, respectively. From now on, all quantities with upper (or lower) zero indexes specify its current values, and we take $c = 1$ for the speed of light.

The paper is presented as follows: In section II we write the Einstein field equations. In section III we study three specific models. They are the sCDM, the $\Lambda$CDM, and the $\chi$CDM. In section IV we proceed to describe the position of the first Doppler peak for each of these models. Here, we express the first Doppler peak $l_{LS}$, in terms of the $\Omega_T$ parameter, considered to be close to one. We also determine the shift parameter $R$ for the models studied here. We conclude in section V.

## 2 THE EINSTEIN FIELD EQUATIONS

We start with the effective Einstein action given by

$$ S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R + \frac{1}{2} (\partial_a Q)^2 - V(Q) + L_M \right], $$

(2)

where $R$ is the scalar curvature; $V(Q)$ is the scalar potential associated with the scalar field $Q$; and $L_M$ is related to any other matter component.

We shall assume that the $Q$ field is homogeneous, i.e. it is a time-dependent quantity only, $Q = Q(t)$; and the spacetime is isotropic and homogeneous with its metric corresponding to the FRW metric:

$$ ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right], $$

(3)

where $a(t)$ represents the scale factor, and the $k$ parameter takes the values $k = -1, 0, 1$ corresponding to an open, flat and closed three-geometry, respectively. With these assumptions, action (2) yields the following field equations:

The time-component of the Einstein equations reads:

$$ H^2 = \frac{8\pi G}{3} \left( \rho_M + \rho_Q \right) - \frac{k}{a^2}, $$

(4)

and the evolution equation for the $Q$ scalar field becomes

$$ \ddot{Q} + 3H\dot{Q} = -\frac{\partial V(Q)}{\partial Q}. $$

(5)

Here the overdots denote derivatives with respect to $t$;

$H = \frac{\dot{a}}{a}$ defines the Hubble expansion rate; $\rho_M$ and $\rho_Q$ are the effective matter energy density and the average energy density related to $Q$, respectively. The $Q$-energy density is defined by

$$ \rho_Q = \frac{1}{2} \dot{Q}^2 + V(Q). $$

(6)

We introduce its average pressure $P_Q$ by means of

$$ P_Q = \frac{1}{2} \dot{Q}^2 - V(Q). $$

(7)

These two quantities are related by the equation of state, eq. (1). By using eqs. 6 and 7, eq. (5) becomes
which represents an energy balance for the scalar field \( Q \). Similarly, we have a relation for the nonrelativistic matter component, i.e. \( \rho_M + 3H(\rho_M + P_M) = 0 \), which we take to be characterized by the equation of state \( P_M = 0 \), corresponding to a nonrelativistic dust component, in which case this equation solves to give: \( \rho_M \propto a^{-3} \). In this way, we have a combination of two noninteracting perfect fluids: the dust matter component \((\rho_M, p_M)\) and the quintessence scalar field \((\rho_Q, p_Q)\) component.

Equation (4) may be written as

\[
H^2 = H_0^2 \left[ \Omega_M \left( \frac{\rho_M}{\rho_M^0} \right) + \Omega_Q \left( \frac{\rho_Q}{\rho_Q^0} \right) + \Omega_k \left( \frac{a_0}{a} \right)^2 \right].
\] (9)

Here, the present curvature density parameter, \( \Omega_k \), and the quintessence (or dark energy) density parameter, \( \Omega_Q \), are defined by

\[
\Omega_k = -k \left( \frac{1}{a_0 H_0} \right)^2,
\] (10)

and

\[
\Omega_Q = \left( \frac{8 \pi G}{3 H_0^2} \right) \rho_Q^0,
\] (11)

respectively.

In the next section we study the characteristics of the different models that arise when the nonrelativistic matter component, \( \rho_M \), together with equations (4), (5) and the equation of state for the scalar field \( Q \), are considered, so that different flat universe models occur. As was mentioned in the introduction, these models are the sCDM, the ΛCDM and the \( \chi \)CDM.

In order to mimic a flat universe, we assume that \( \rho_Q \), together with the curvature term, combine in a way such that the following scenarios occur:

\[
\frac{8 \pi G}{3} \rho_Q(t) = \frac{k}{a^2(t)}
\]

where, similar to the definitions for \( \Omega_M \) and \( \Omega_Q \), we define

\[
\Omega_k = \left( \frac{8 \pi G}{3 H_0^2} \right) \rho_Q^0
\]

for the present value of the quintessence density parameter.

Some comments are in order. In the first two cases we could obtain an explicit expression (as a function of cosmological time) for the unknown energy density, \( \rho_Q \), if we know the scale factor \( a(t) \) as an explicit function of time. On the contrary, in the third case, we need not only to know the explicit expression for the scale factor, but also the explicit time dependence of the dark energy density \( \rho_Q \). Also, since in the first case the quantity located on the left hand side has to vanish and, considering that the energy density, \( \rho_Q \), cannot be negative, we are forced to consider closed geometries \((k = 1)\), in which case \( \Omega_k \) become negative. In the second and third cases we will also consider the geometry to be closed. In this case, we could take a complete range for the scale factor \( a(t) \), i.e. \( 0 \leq a(t) < \infty \).

3 THE SPECIFIC MODELS

In this section we shall impose the conditions under which a closed universe \((k = 1)\) may look like a flat universe \((k = 0)\) at low redshift. The flat models are characterized by expression (12) (or equivalently eq. (13)).

3.1 The Einstein-de Sitter or Standard Cold Dark Matter (sCDM) Model

In order to have a closed universe, but one which still has a nonrelativistic matter density whose value corresponds to that of a flat universe, we impose the first condition described by equation (12), i.e. \( \rho_Q(a) = \frac{3}{8 \pi G a^2} \) or, equivalently, from equation (13)

\[
\Omega_Q \left( \frac{\rho_Q}{\rho_Q^0} \right) = -\Omega_C \left( \frac{a_0}{a} \right)^2;
\] (14)

where \( \Omega_C \) is the density parameter for a closed universe, i.e. \( \Omega_C \equiv \Omega_{k=1} < 0 \). Note that equation (14) gives at present time \( \Omega_Q = |\Omega_C| \).

When equation (14) is substituted into equation (9), the following expression results: \( H^2 = 8 \pi G \rho_M / 3 \), which gives for a dust dominated universe: \( a(t) = a_0 (t/t_0)^{3/2} \). Notice that this expression gives \( \Omega_M = 1 \) when evaluated at present time.

Now we are in a position to obtain the intrinsic characteristics of the scalar field \( Q \). From expressions (6) and (7), together with the equation of state, eq. (1), we obtain

\[
\dot{Q}(t) = \sqrt{1 + w_Q} \rho_Q(t),
\]

which gives \( Q(t) = Q_0 (t/t_0)^{2/3} \), where \( Q_0 \) is defined by \( Q_0 = 3 \sqrt{1 + w_Q} / (a_0/a_0^0) \).

The same set of equations gives \( V_Q(t) = (1 - w_Q) \rho_Q(t)/2 = V_0 (t_0/t)^{4/3} \), where \( V_0 = \rho_c Q_0/2 \). These solutions combine in a way such that the scalar potential becomes

\[
V_Q(Q) = V_0 \left( \frac{Q_0}{Q} \right)^4,
\] (15)

which represents a typical potential for a quintessence scalar field.
In order to satisfy the field equation (5), we need to take $w_Q = -1/3$. With this value for $w_Q$, the scalar field $Q$ has properties similar to the matter described in references Kolb (1989), Cruz, del Campo & Herrera (1998), and del Campo & Cruz (2000).

### 3.2 The Lambda Cold Dark Matter Model \(\Lambda\)CDM

Following a procedure similar to that of the previous subsection, we take the second of the three constraints specified by equation (13), i.e.

\[
\Omega_Q \left( \frac{\rho_Q(t)}{\rho_C} \right) + \Omega_C \left( \frac{a_0}{a(t)} \right)^2 = \Omega_\Lambda,
\]

where the parameters $\Omega_Q$, $\Omega_C$, and $\Omega_\Lambda$ have already been defined. Equation (16) evaluated at the present epoch, gives $\Omega_Q + \Omega_C = \Omega_\Lambda$. Since $\Omega_C < 0$, we must satisfy $\Omega_Q > \Omega_\Lambda$.

Under condition (16), the time-time component of Einstein equations becomes analogous to that for a flat universe, where the usual matter and the cosmological constant form the main matter components of the model. Thus, equation (9) reads for a nonrelativistic perfect fluid:

\[
H^2 = H_0^2 \left[ \Omega_M + \Omega_\Lambda \left( \frac{a_0}{a} \right)^3 \right].
\]

Notice that, when this expression is evaluated at present time, i.e. $t = t_0$, we obtain $\Omega_M + \Omega_\Lambda \equiv \Omega_{DM} = 1$. For numerical computations we shall take $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$. The latter choice agrees with the amount of cosmological constant, $\Omega_\Lambda < 0.7$, constrained by QSO lensing surveys (Kochanek 1996).

Using the definition of the Hubble parameter together with equation (8), we obtain

\[
P_Q = -\frac{1}{3a^2} \frac{d}{dt} \left(a^3 \rho_Q \right),
\]

for the effective pressure associated with the $Q$ field. By substituting eq. (16) into eq. (17), we obtain

\[
w_Q^{\Lambda CD M}(a) = -\frac{1}{3} \left[ \frac{\Omega_Q - \Omega_\Lambda}{\Omega_Q} \frac{a_0}{a} \right] \left( \frac{a_0}{a} \right)^2 + 3 \Omega_\Lambda \right],
\]

for the equation state parameter $w_Q$. Notice that the case $\Lambda = 0$ gives $w_Q^{\Lambda CD M}(a) = -1/3 = \text{const.}$, corresponding to the Einstein-de Sitter model described in the previous subsection. For $\Lambda \neq 0$, the parameter $w_Q^{\Lambda CD M}(a)$ is always negative, since in the limit $a \rightarrow 0$, we get $w_Q^{\Lambda CD M} \rightarrow -1/3$ and for $a \rightarrow \infty$, we find $w_Q^{\Lambda CD M} \rightarrow -1$. Thus, the parameter $w_Q^{\Lambda CD M}$ lies in the range $-1 < w_Q^{\Lambda CD M} < -1/3$.

Figure 1 shows how the equation of state parameter $w_Q^{\Lambda CD M}$ changes with time, for three different values of the scalar field density parameter, $\Omega_Q$; and the parameter $\Omega_\Lambda$ fixed at 0.6.

Another interesting characteristic of the quintessence scalar field is the form of its scalar potential, $V(Q)$. In order to determine this form, we consider the definitions (6) and (7) together with the equation of state (1), we obtain

\[
V_Q^{\Lambda CD M}(a) = V_Q^0 \left[ \frac{3 \Omega_\Lambda + 2 (\Omega_Q - \Omega_\Lambda) \left( \frac{a_0}{a} \right)^2}{\Omega_\Lambda + 2 \Omega_Q} \right],
\]

where $V_Q^0 = \frac{1}{3} \rho_C \left( \Omega_\Lambda + 2 \Omega_Q \right)$ and for $\Omega_Q = 0.75, 0.85$ and 0.95). Here, we have used $\Omega_\Lambda = 0.6$.

On the other hand, from the same equations (6) and (7) we get that, after substituting the corresponding expressions for $\rho_Q$ and $w_Q$, an explicit expression for the scalar field $Q$ as a function of the scale factor

\[
Q(a) = Q_0 \left( \frac{a}{a_0} \right)^{1/2} \times \left[ \frac{2F_1 \left( \frac{1}{2}; \frac{1}{2}; \frac{3}{2}; -\left( \frac{\Omega_Q}{\Omega_{CD M}} \right) \left( \frac{a_0}{a} \right)^3 \right) \Omega_{CD M} }{2F_1 \left( \frac{1}{2}; \frac{1}{2}; \frac{3}{2}; -\left( \frac{\Omega_Q}{\Omega_{CD M}} \right) \right) \Omega_{CD M}} \right],
\]

with $2F_1$ is the generalized hypergeometric function and $Q_0$ is defined as $Q_0 = Q(a_0) = \sqrt{\frac{\Omega_Q - \Omega_\Lambda}{\Omega_{CD M}}} - 2F_1 \left( \frac{1}{2}; \frac{1}{2}; \frac{7}{2}; -\left( \frac{\Omega_Q}{\Omega_{CD M}} \right) \right)$.

By using numerical computations, we can plot the scalar potential $V_Q$ as a function of the scalar field $Q$. Figure 2 shows the plot for three different values of the parameter density $\Omega_Q$, ($\Omega_Q = 0.75, 0.85$ and 0.95). The other parameters, $\Omega_M$ and $\Omega_\Lambda$ are fixed at values 0.35 and 0.65, respectively. Note that at sufficiently higher values of $Q$ the potential approaches a constant value given by $3V_0 \Omega_\Lambda$. This value becomes independent of the parameter $\Omega_Q$.

### 3.3 The Quintessence (or Dark Energy) Cold Dark Matter Model \(\chi\)CDM

In this case we consider the following constraint equation:

\[< 1.9>\]

\[\frac{3 \Omega_\Lambda + 2 (\Omega_Q - \Omega_\Lambda) \left( \frac{a_0}{a} \right)^2}{\Omega_\Lambda + 2 \Omega_Q} \]

\[\text{Figure 1. This graph shows the equation of state } w_Q^{\Lambda CD M} = P_Q/\rho_Q \text{ as a function of time (in units of } H_0 \text{, the present value of the Hubble parameter)} \text{ for three different values of the density parameter, } \Omega_Q \text{ ( } \Omega_Q = 0.75, 0.85 \text{ and } 0.95 \text{). Here, we have used } \Omega_\Lambda = 0.6.\]
tion equations for the scalar fields where, just as before, we have considered dust to be the
relevent quantities, for obtaining
where
for the quintessence scalar field components
of equations, we need to introduce the equations of state
which reduces the time-time component of Einstein equa-
tions to
where, just as before, we have considered dust to be the
regular matter, $\rho_M$.
These two latter equations together with the evolution
equations for the scalar fields $\chi$ and $Q$ form the basic
set of equations for our model. In order to solve this set
of equations, we need to introduce the equations of state
for the quintessence scalar field components $Q$ and $\chi$. We
assume that the $\chi$ field component is characterized by a
constant equation state parameter that lies in the range
$[-1 < w_\chi < -0.6]$. Instead, we consider $w_Q$ to be a variable
quantity whose actual value lies in the same range as
$w_Q$.
We can use the definition of $P_\chi$ and $\rho_\chi$ in terms of
the scalar field $\chi$, together with the equation of state that
relates these quantities, for obtaining $\chi$ field as a function
of the scale factor $a$. The result is
$$
\chi(a) = \chi_0 \left( \frac{a_0}{a} \right)^{-3w_\chi/2}
\times \left[ 2F_1 \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2} ; -\frac{\Omega_\chi}{\Omega_M} \left( \frac{a_0}{a} \right)^{-3w_\chi} \right) \right],
$$
where $\chi_0$ is given by
$$
\chi_0 = \frac{\sqrt{4\rho_C/3\Omega_Q}}{\sqrt{\Omega_\chi (1 + w_\chi)/\Omega_M w_\chi}}.
$$

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3}
\caption{Plot of the scalar potential $V_Q$ (in units of $\sqrt{3\rho_C/3H_0^2}$) as a function of the scalar field $Q$ (in units of $Q \equiv \sqrt{3\rho_C/3H_0^2}$) for three different values of the density parameter, $\Omega_Q$ ( $\Omega_Q = 0.75$, 0.85 and 0.95). We have taken $\Omega_M = 0.35$ and $\Omega_\lambda = 0.65$.}
\end{figure}

In a similar way we obtain
$$
V_\chi(a) = V_\chi^0 \left( \frac{a_0}{a} \right)^{3(1+w_\chi)}
$$
for the scalar potential $V_\chi$ as a function of the scale factor $a$, where the present value of this potential is given by
$$
V_\chi^0 = \frac{1}{2} (1 - w_\chi) \rho_C \Omega_\chi.
$$
Figure 3 shows the plot of the scalar potential $V_\chi$ as a function of the scalar field $\chi$, for different values of the state equation parameter $w_\chi$; and the parameters $\Omega_\chi$ and $\Omega_M$ have been fixed at 0.65 and 0.35, respectively. This form of potential has been described in the literature (Saini, Ray-chaudhury, Sahni & Starobinski 2000). Note that, as long as
$w_\chi \longrightarrow -1$, the potential $V_\chi \longrightarrow const. \equiv \rho_C \Omega_\chi$, i.e.,
the model becomes equivalent to the $\Lambda$CDM.

Following an approach analogous to that done in the
previous subsection, we find that $w_Q$ is given by
$$
w_Q^{\chi, CDM}(a) = - | w_Q^0 | \left[ \frac{1 + \beta}{1 - 3\beta w_\chi} \right]
\times \left[ 1 - 3\beta w_\chi \left( \frac{a_0}{a} \right)^{-3w_\chi - 1} \right],
$$
where $\beta = \Omega_\chi/\Omega_Q$ and $w_Q^0$ is the present value of $w_Q(a)$ defined by $w_Q^0 = P_Q^0/\rho_Q^0$. Figure 4 shows its dependence on the redshift $z$ defined as $z \equiv a_0/a - 1$, for three different values of $w_\chi$. As before, we have chosen $\Omega_Q = 0.85$ and $\Omega_\chi = 0.65$.

In this case the scalar field $Q$ becomes given by
$$
Q(a) = \sqrt{\Omega_\chi} \int_{a_0}^{a} \sqrt{\frac{1 + \frac{3}{2} \beta (1 + w_\chi) x^{-1 + 3w_\chi}}{x + \Omega_M x^{-1 + 3w_\chi}}} \, dx,
$$
where
$$
\sqrt{\Omega_\chi} = \left( \frac{\Omega_Q - \Omega_\chi}{\Omega_M} \right).
$$
On the other hand, the scalar potential $V_{Q}^{\chi CDM}(a)$ is given by
\begin{equation}
V_{Q}^{\chi CDM}(a) = V_{Q}^{0} \left( \frac{a_{0}}{a} \right)^{2} \times \left[ \frac{4 + 3\beta(1 - w_{x})}{4 + 3\beta(1 - w_{x})} \right]^{(1 + 3w_{x})},
\end{equation}
where $V_{Q}^{0} = \frac{c^{2}Q}{a_{0}(1 - \beta)} [1/3 + \beta(1 - w_{x})]$.  

Figure 5 shows the scalar potential $V_{Q}^{\chi CDM}$ (in units of $\rho_c/2$) as a function of the scalar field $Q$ (in units of $1/\sqrt{4\pi G}$) for different values of the equation state parameter $w_{x}$ ($w_{x} = -0.7, -0.8, -0.9$). Here, we have taken $\Omega_Q = 0.85$, $\Omega_M = 0.35$ and $\Omega_X = 0.65$.

Note that this scalar potential decreases when $Q$ increases. This potential asymptotically tends to vanishing for $a \to \infty$. This implies that, asymptotically, the effective equation of state becomes $P_{Q} = \rho_{Q}$ (for $\dot{Q} \neq 0$), corresponding to a stiff fluid.

### 4 THE FIRST DOPPLER PEAK OF THE CMB SPECTRUM

In this section, we are going to describe the position of the first Doppler peak ($l_{LS}$) for the different models studied in the previous section.

The scales, which are important in determining the shape of the CMB anisotropy spectrum are the sound horizon $d_s$ at the time of recombination, and the angular diameter distance $d_A$ to the last scattering surface. The former defines the physical scales for the Doppler peak structure that depends on the physical matter density ($\Omega_M$), but not on the value of the cosmological constant ($\Omega_{\Lambda}$) or spatial curvature ($\Omega_c$), since these are dynamically negligible at the time of recombination (Efstathiou & Bond 1998). The latter depends practically on all of the parameters and is given by (for a closed universe):

\begin{equation}
l_{LS} \propto \frac{d_A}{d_S}.\end{equation}

where the constant of proportionality depends on both the shape of the primordial power spectrum and the Doppler peak number (Hu & White 1996). Since we are going to keep the $\Omega_M$ parameter fixed, we shall take $l_{LS} \approx d_A$, up to a factor that depends on $\Omega_M$ and $z_{LS}$ only.

For the $\Lambda$CDM model we find that
\begin{equation}
l_{LS}^{\Lambda CDM} \approx \frac{1}{H_0 (1 + z_{LS})} \frac{1}{\sqrt{\Omega_M}} \times \sin \left[ 2 \sqrt{\frac{\Omega_Q}{\Omega_M}} \left( 1 - \frac{1}{1 + z_{LS}} \right) \right].
\end{equation}

Note that, for $\Omega_Q \ll \Omega_M$ (which means that the curvature term is very small), we find that $l_{LS}^{\Lambda CDM} \sim 1/\sqrt{\Omega_M} = 1/\sqrt{\Omega_T}$ which agrees with the result obtained by Frampton et al (Frampton, Ng & Rohm 1998) for $\Omega_{\Lambda} = 0$.

In the $\Lambda$CDM model, we find that
\begin{equation}
l_{LS}^{\Lambda CDM} \approx \frac{1}{H_0 (1 + z_{LS})} \frac{1}{\sqrt{\Omega_M - \Omega_{\Lambda}}} \times \sin \left[ \sqrt{\Omega_Q - \Omega_{\Lambda}} \int_0^1 \frac{dx}{\sqrt{\Omega_M x + \Omega_{\Lambda} x^2}} \right].
\end{equation}
where we have set the lower limit on the integral equal to zero, since $z_{LS} \gg 1$, and therefore we may take $x_{LS} \equiv 1/(1+z_{LS}) \approx 0$.

From the equation $\Omega_Q + \Omega_C = \Omega_0$ together with the expression $\Omega_M + \Omega_\Lambda = \Omega_T = 1$, we may write $\Omega_C = 1 - \Omega_M - \Omega_Q$. Now, following Weinberg (Weinberg 2000), we will keep fixed the $\Omega_M$ parameter with $\Omega_T$ close to one and $\Omega_Q$ close to $\Omega_\Lambda$. Thus, we find that

$$I_{LS}^{\Lambda CDM} \sim \Omega_T^{-\eta},$$

where

$$\eta = \left( \frac{\partial \ln I_{LS}^{\Lambda CDM}}{\partial \Omega_C} \right)_{\Omega_M=1-\Omega_M} = \frac{1}{6} I_1^2 - \frac{1}{2} I_2^2,$$

with

$$I_1 \equiv \int_0^1 \frac{dx}{[(1-\Omega_M)x^4 + \Omega_M x]^{1/2}},$$

and

$$I_2 \equiv \int_0^1 \frac{x^4 dx}{[(1-\Omega_M)x^4 + \Omega_M x]^{3/2}}.$$

These expressions yield $\eta = 2.45$ for $\Omega_M = 0.2$ and $\eta = 11/18$ for $\Omega_M = 1.0$. The latter value should be compared with that corresponding to the $\Omega_0 = 0$ case, in which $\eta = 1/2$.

For the $\chi CD M$ model we find that

$$I_{LS}^{\chi CDM} \approx \frac{1}{H_0(1+z_{LS})} \frac{1}{\sqrt{\Omega_Q - \Omega_x}} \times \sin \left[ \sqrt{\Omega_Q - \Omega_x} \int_0^1 \frac{dx}{\sqrt{\Omega_M x + \Omega_\chi + \Omega_M x x^{-3w_\chi}}} \right],$$

and

$$\tilde{I}_1 \equiv \int_0^1 \frac{dx}{[(1-\Omega_M)x^{1-3w_\chi} + \Omega_M x]^{1/2}},$$

and

$$\tilde{I}_2 \equiv \int_0^1 \frac{x^{1-3w_\chi} dx}{[(1-\Omega_M)x^{1-3w_\chi} + \Omega_M x]^{3/2}}.$$

The values of the exponent $\eta$ appearing in equation (32) are tabulated in Table 1 for the different models treated here.

One important parameter that describes the dependence of the first Doppler peak position on the different parameters that characterize any model is the shift parameter $R$. This parameter is related to the geometry of the universe and, for closed models, it may be defined as (Efstathiou & Bond 1998; Melchiorri & Griffiths 2000; Melchiorri 2002)

$$R = \sqrt{\frac{\Omega_M}{\Omega_Q - \Omega_x}} \sin \left[ \frac{\Omega_Q}{\Omega_Q - \Omega_x} y_{LS} \right],$$

where $y_{LS}$ is defined by equation (29).

For the sCDM model, this parameter is given by

$$R_{sCDM} = \sqrt{\frac{\Omega_M}{\Omega_Q}} \sin \left[ 2 \sqrt{\frac{\Omega_Q}{\Omega_M} \left( 1 - \frac{1}{1+z_{LS}} \right)} \right],$$

which, at first glance, seems to be a quantity that depends on $\Omega_Q$ and $\Omega_M$ parameters. But we know that, in this model, the parameter $\Omega_M$ gets the value one. Here, we have used the equality $|\Omega_C| = \Omega_Q$. For the $\Lambda CDM$ model we find that the parameter $R$ is given by

$$R_{\Lambda CDM} = \sqrt{\frac{\Omega_M}{\Omega_Q - \Omega_x}} \sin \left[ \frac{\Omega_Q}{\Omega_Q - \Omega_x} \right] \times \int_0^1 \frac{dx}{\sqrt{\Omega_M x + \Omega_\chi + \Omega_M x x^{-3w_\chi}}}.$$

Here, we have used the relation $|\Omega_C| = \Omega_Q - \Omega_\Lambda$ (with $\Omega_Q > \Omega_\Lambda$) and we have considered $x_{LS} \approx 0$.

In Figure 6 we have plotted a set of lines $R = \text{const.}$ for different values of the parameter $\Omega_Q$. The values of $R$ and $\Omega_Q$ are given next to each line. We have also included in this plot the line that joins the points $(1.0; 0.0)$ and $(0.0; 1.0)$ (dashed line).

In Figure 7 we have plotted two sets of $R_{sCDM} = \text{const.}$ contours for three different values of $w_\chi$ ($w_\chi = -0.3, -0.5, -0.9$). In these curves we have kept fixed $\Omega_Q$.
Table 1. This table shows the exponent parameters \( \eta \) of Eq. (32) for the \( \Lambda \)CDM and the \( \chi \)CDM models, where we have used different values of the parameter \( \Omega_M \). For the latter model we have taken the values \(-0.7, -0.8\) and \(-0.9\) for the parameter \( w_\chi \).

| \( \Omega_M \) | \( \eta^{\Lambda \text{CDM}} \) | \( \eta^{\chi \text{CDM}} \) |
|------------|-----------------|-----------------|
| 0.2        | 2.422           | 2.152           |
| 0.3        | 1.727           | 1.571           |
| 0.4        | 1.350           | 1.248           |
| 1.0        | 0.595           | 0.571           |

We should note that the \( s \)CDM and \( \Lambda \)CDM models are special cases of the \( \chi \)CDM model. They are obtained from equation (41) by taking \( \Omega_\chi = 0 \) and \( \Omega_\chi = \Omega_\Lambda \) with \( w_\chi = -1 \), respectively. At this point, we should add that the \( \chi \)CDM model, and its corresponding characteristics will certainly supply information on which of these models (or another one) is more appropriate for describing the universe we live in.

5 CONCLUSIONS

In this paper we have described closed universe models in which, apart from the usual Cold Dark Matter component, we have included a quintessence scalar field \( Q \). We have fine tuned the quintessence component together with the curvature term for getting a flat model in which three different models were described. These models were the standard Cold Dark Matter (\( s \)CDM model, characterized by \( \Omega_T \equiv 1 \)), the Cosmological constant Cold Dark Matter (\( \Lambda \)CDM model, characterized by \( \Omega_T = \Omega_M + \Omega_\Lambda \equiv 1 \)) and the Quintessence (or Dark Energy) Cold Dark Matter (\( \chi \)CDM model, characterized by \( \Omega_T = \Omega_M + \Omega_\chi \equiv 1 \)). In all of them we have described the properties of the scalar field \( Q \). The characterization of the scalar field \( Q \) in the different models comes from the determination of the scalar potential \( V(Q) \). In all of these models, this potential decreases when the scalar field \( Q \) increases. This property seems to be common to all of the dark energy potentials. In the \( \Lambda \)CDM model, this potential approaches a constant given by \( 3V_Q \Omega_\Lambda \) at a large scale factor. In the other two models, the potential \( V(Q) \) goes to zero, asymptotically. Since all of these models are indistinguishable from flat models at enough low redshift (say \( z \sim 1 \)), we expect that with an appropriate fine tuning, it will be possible to consolidate the supernova measurements with closed universe models.

As an applicability of the different models described above, we have determined the position of the first Doppler peak together with the shift parameter \( R \). For \( |\Omega_C| \approx 0 \), we have found that the first Doppler peak is quite sensitive to the mass density values (\( \Omega_M \)).

For a fixed \( \Omega_M \) at 0.4 and \( \Omega_T \approx 1 \), the first Doppler peak behaves as \( \Omega_\chi^\eta \), with the exponent \( \eta \) given by 0.500 (for the \( s \)CDM model), 1.350 (for the \( \Lambda \)CDM model), 1.248 (for the \( \chi \)CDM model with \( w_\chi = -0.7 \)) and 1.322 (for the \( \chi \)CDM model with \( w_\chi = -0.9 \)). We may compare these values with that specified by Weinberg, which results to be 1.244. This value agrees with that obtained from the \( \chi \)CDM model, in which \( w_\chi = -0.7 \). Some values of the \( \eta \) exponent have been tabulated in table 1. In this table we have included the value corresponding to the \( s \)CDM model, where \( \eta = 1/2 \) is found. Precise measurements of the location of the first Doppler peak (together with the other peaks and their properties) can supply information on the parameters \( \Omega_\Lambda \) (or \( \Omega_\chi \)) and \( \Omega_T \), from which we could obtain an appropriate value for the parameter \( \Omega_Q \).

Secondly, we have determined the shift parameter \( R \). In the \( \Lambda \)CDM model, this parameter is highly sensitive to the value of \( \Omega_T \). Something similar occurs in the \( \chi \)CDM model. We have plotted curves \( R = \text{const.} \) in the graph \( \Omega_\chi \) v/s \( \Omega_M \) for different values of the \( w_\chi \) parameter. For \( \Omega_\chi \approx 1 \) the \( R = \text{const.} \) curves became separated. But, for \( \Omega_\chi \approx 0 \), they began to come together. It is interesting to consider the value \( R = 1 \), since this case allows us to determine a precise value.
for the $\Omega_Q$ parameter. For the sCDM model we obtain that $\Omega_Q = 0$; meanwhile, for the other two cases, $\Omega_Q = \Omega_\Lambda$ and $\Omega_Q = \Omega_\chi$ for the $\Lambda$CDM and $\chi$CDM models, respectively. We may conclude that, as far as we are concerned with the observed acceleration detected in the universe and the location of the first Doppler peak, we will be able to utilize a closed model to describe the universe we live in.

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REFERENCES

Caldwell R., Dave R. and Steinhardt P., 1998, Phys. Rev. Lett. 80, 82.
Crooks J. L., Dunn J. O., Frampton P. H. and Ng Y. J., astroph/0005406.
Cruz N., del Campo S. and Herrera R., 1998, Phys. Rev. D 58, 123504.
Cataldo M. and del Campo S., 2000, Phys. Rev. D 62, 025455.
del Campo S. and Cruz N., 2000, MNRAS, 317, 825.
de Bernardis P. et al., 2000, Nature, 404, 955.
Dodelson S. and Knox L., 2000, Phys. Rev. Lett., 84,3523.
Efstathiou G. and Bond J. R., astro-ph/9807103.
Frampton P., Ng Y. J. and Rohm R., 1998, Mod. Phys. Lett. A13, 2541.
Garnavich P. M. et al, 1998, ApJ, 509, 74.
Guth A., 1981, Phys. Rev. D, 23, 347.
Hu W. and Sugiyama N., 1995a, ApJ, 444, 489.
Hu W. and Sugiyama N., 1995b, Phy. Rev. D, 51, 2599.
Hu W., Sugiyama N. and Silk J., 1997, Nature, 386, 37.
Hu W. and White M., 1996, ApJ, 471, 30.
Kamionkowski M., Spergel D. N. and Sugiyama N., 1994, ApJ, 426,L57.
Kamionkowski M. and Toubas N., 1996, Phys. Rev. Lett., 77, 587.
Kochanek C. S., 1996, ApJ, 466, 638.
Kolb E. W., 1989, ApJ, 344, 543.
Mather J. et al., 1994, ApJ, 420, L439.
Melchiorri A. and Griffiths L. M., astro-ph/0011147.
Melchiorri A., astro-ph/0201237.
Mellier Y., 1999, ARA&A, 37, 127.
Perlmuter S. et al., 1998, Nature 391, 51.
Roos M. and Harun-or-Rashid S., astro-ph/0005541.
Saini T. D., Raychaudhury S., Sahni V. and Starobinski A.,2000, Phys. Rev. Lett., 85, 1162.
Wang L., Caldwell R. R., Ostriker J. P. and Steinhardt P. J., 2000, ApJ, 530, 17.
Weinberg S., 2000, Phys. Rev. D, 62, 127302.
White S. et al., 1993, Nature, 366, 429.