Rough horizontal plates: heat transfer and hysteresis

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Abstract. To investigate the influence of a rough-wall boundary layer on turbulent heat transport, an experiment of high-Rayleigh convection in water is carried out in a Rayleigh-Bénard cell with a rough lower plate and a smooth upper plate. A transition in the heat transport is observed when the thermal boundary layer thickness becomes comparable to or smaller than the roughness height. Besides, at larger Rayleigh numbers than the threshold value, heat transport is found to be increased up to 60\%. This enhancement cannot be explained simply by an increase in the contact area of the rough surface since the contact area is increased only by a factor of 40\%. Finally, a simple model is proposed to explain the enhanced heat transport.

1. Introduction

Despite the simplicity of the boundary conditions of a Rayleigh-Bénard (RB) cell (the hot bottom plate and the cold top plate are maintained at constant temperatures whereas the lateral sidewalls are adiabatic), it serves as the archetypal configuration for the study of natural convection and heat transfer mechanisms that occur in the atmosphere, oceans and also in many industrial processes. Indeed, RB convection displays most of the features observed in more complex systems: turbulence, viscous and thermal boundary layers, plumes, large scale circulation, a potential laminar-turbulent transition in the boundary layer beyond a critical Rayleigh number. However, to understand the high Rayleigh number behavior of RB cells, it becomes important to alter the boundary conditions and see how the plate roughness affects the heat transfer by the convecting fluid.

The Nusselt number, $N_u$, describes in dimensionless term the heat flux, $Q$, which is convected vertically upwards between the hot bottom plate and the cold top plate:

$$N_u = \frac{QH}{\lambda \Delta}$$

$N_u$ measures therefore the ratio of convective to conductive heat flux, $H$ is the height of the cell and $\lambda$ is the thermal conductivity of the fluid. The temperature difference between the hot and cold plates ($\Delta = T_h - T_c$) governs the dynamics of the convecting flow and the so-called Rayleigh number is the dimensionless parameter that controls RB cells:

$$Ra = \frac{g \alpha \Delta H^3}{\nu \kappa}$$

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α is the isobaric thermal expansion coefficient, ν the kinematic viscosity and κ the thermal diffusivity. At sufficiently large Ra for the flow to be turbulent and up to Ra ≈ 10^{12}, quantitative measurements of Nu(Ra) with smooth plates have shown that Nu can be described quite well by a power law:

\[ Nu = 0.06Ra^{1/3} \tag{3} \]

Besides, the bulk temperature \( T_b \) is nearly uniform and the mean temperature gradient differs from zero only in the thin thermal boundary layers (BLs) close to the plates. Assuming conduction is responsible for transport of heat in these BLs, their thickness is equal to:

\[ \delta(Ra) = H/2Nu \tag{4} \]

Then, according to equations 3 and 4, BL’s thickness becomes independent of the height of the cell and we can say that the cold and hot BLs behave independently of each other. Measuring the uniform bulk temperature allows to separately determine the thermal impedance of each plate: \( Nu_c = QH/\lambda \Delta c \) and \( Nu_h = QH/\lambda \Delta h \). Here \( \Delta_c = T_b - T_c \) and \( \Delta_h = T_h - T_b \) are the temperature drops across the cold and hot thermal BLs, respectively, and \( \Delta = \Delta_c + \Delta_h \).

In the case of rough bottom and top plates, Shen et al. (1996); Du & Tong (2000); Qiu et al. (2005); Roche et al. (2001a); Stringano et al. (2006) showed that the power-law behavior of Nu (Eq. 3) remains valid until the BL’s thickness is greater than the roughness height \( (\delta(Ra) > h_0) \). In other words, no effect on the heat transfer is observed when the rough elements are buried beneath the thermal BLs. But, as soon as the bulk flow feels the surface roughness \( (\delta(Ra) \approx h_0) \), a transition is observed by all previous studies and, beyond this transition, the heat transport is always enhanced compared to the smooth plate case. However, Qiu et al. (2005); Roche et al. (2001a); Stringano et al. (2006) found a change in the scaling exponent of Nu vs Ra whereas Shen et al. (1996); Du & Tong (2000) observed just a short crossover between the same regime, only the prefactor in the power law behavior of Nu is increased by the plate roughness.

2. Experimental set-up

The experiment is conducted in a vertical cell filled with water. Details about the convection apparatus have been described in Chillá et al. (2004); Tisserand et al. (2011) and here we mention only some key points. Two stainless steel cylinders of heights \( H = 1 \) m (the tall one) and \( H = 0.2 \) m (the small one) are used as sidewalls of the convection cell. They have the same diameter \( D = 50 \) cm and thickness 2.5 mm. As the Rayleigh number (Ra) is proportional to the cube of the height \( H \), measuring the Nusselt number in two different cylinders permits us to cover 4.5 decades of Ra (see figure 2).

The top of the tall and small cells are provided by the same copper top plate. It is plated with a thin layer of nickel to prevent oxidation and it is cooled by temperature controlled water from a refrigerator circulator. The bottom plate, made of aluminium, differs from the top one in that it contains an embedded heater and the water-contact surface is rough. Its roughness consists in a square array of square plots, \( d = 5 \) mm of side, \( h_o = 2 \) mm in height, with a period of \( 2d = 1 \) cm (figure 1). According to equations 3 and 4, the height of the thermal boundary layers is equal to \( h_o = 2 \) mm when \( Ra = 10^{11} \) for the tall cell and \( Ra = 8.10^8 \) for the small cell.

The temperatures of the two end plates are measured with type K thermocouples inserted in the plates at different points in order to check the temperature uniformity. The temperature of the bulk flow (\( T_b \)) is also measured (type K thermocouple) so that we can separately define \( \Delta_r = 2\Delta_h = 2(T_h - T_b) \) and \( \Delta_s = 2\Delta_c = 2(T_b - T_c) \). We can then assume that \( \Delta_r \) is the temperature drop across a similar RB cell but with two identical rough plates, each of which behaves like the rough plate of our nonsymmetric cell. Likewise, \( \Delta_s \) has the same behavior as the temperature drop across a symmetric cell with smooth cold and hot plates. Therefore we
can define the rough and smooth Nusselt numbers as: $N_{ur} = \frac{QH}{\lambda \Delta r}$ and $N_{us} = \frac{QH}{\lambda \Delta s}$. The main advantage of measuring Nusselt numbers in a non-symmetric cell is to test the influence of the bulk flow since the rough and smooth plates interact within the same 'large scale circulation'.

**Sidewall and non-Oberbeck-Boussinesq Corrections**

Two appropriate sets of corrections are applied to the original experimental values of the Nusselt number. The first correction concerns the sidewall effect (Roche et al., 2001b; Ahlers, 2001): lateral sidewall, made in stainless steel, is in thermal contact with the convecting fluid and carry a part of the heat flux. The analytical formula proposed by Roche et al. (2001b) is used here in order to correct this sidewall conduction effect:

$$N_{ucorrected} = \frac{N_{umeasured}}{1 + A \sqrt{2W/\Gamma \nu}}.$$  

Here $W = 4\lambda_w e/\lambda D$ is the ratio between the wall and the quiescent fluid and $A \approx 0.8$.

The second correction comes from the non-Oberbeck-Boussinesq effects (NOB). Wu & Libchaber (1991); Zhang et al. (1997) showed that the heat transfer in a RB cell is remarkably insensitive to these effects, only the temperature of the bulk is shifted from the Boussinesq case. The Oberbeck-Boussinesq (OB) approximation assumes that the fluid properties such as the viscosity, the thermal diffusivity, the heat capacity and the thermal expansion coefficient are temperature independent and thus they are constant over the cell and particularly over the two thermal boundary layers. But, to achieve high Rayleigh numbers, large temperature drops are needed and then the fluid properties do not reach the same values in the bottom BL (close to the hot plate) than in the top BL (close to the cold plate). This leads to a symmetry breaking between the two BLs: $\Delta_h \neq \Delta_c$. Thus the temperature of the bulk is shifted away from the arithmetic mean temperature $T^\text{OB}_b = (T_h + T_c)/2$ (Wu & Libchaber, 1991):

$$T^\text{NOB}_b - T^\text{OB}_b = c_2 \left( \frac{\Delta}{2} \right)^2$$

Here $c_2$ depends on the fluid properties. For water, the kinematic viscosity has the largest temperature dependence and Ahlers et al. (2006) showed that the variations of the thermal expansion coefficient have very poor influence on $c_2$. Tisserand et al. (2011) proposed an analytical formula which gives $c_2$ as a function of the kinematic viscosity:

$$c_2 = -0.06 Pr^{1/4} \frac{d \ln \nu}{dT}$$

### 3. Results

We first briefly discuss the behavior of the Nusselt number for the top smooth plate. A plot of the compensated Nusselt number $N_{us}/Ra_s^{1/3}$ is shown in figure 2 along with the experimental results of Chaumat (2002) which also used the same cell but with two smooth plates. The new...
Figure 2. The reduced Nusselt number $N_{u_s}/Ra_s^{1/3}$ as a function of the Rayleigh number (on a logarithmic scale) for the smooth plate. Blue triangles, squares and diamonds: results from the small cell for 3 different bulk temperatures: $T_b = 25 \, ^\circ \text{C}, 40 \, ^\circ \text{C}$ and $70 \, ^\circ \text{C}$, respectively. Red plusses and crosses: tall cell for $T_b = 40 \, ^\circ \text{C}$ and $70 \, ^\circ \text{C}$. Small black circles from Chaumat (2002) with two smooth plates.

Figure 3. $N_{u_r}/Ra_r^{1/3}$ vs $Ra_r$. The symbols are as in figure 2. The lines correspond to power-law behavior of $N_{u_r}$ with an exponent 1/2, that is $N_{u_r}/Ra_r^{1/3} \sim Ra_r^{1/6}$. The vertical dash lines show the transition Rayleigh numbers for which $\delta_r(Ra_r) = h_0$ using equations 3 and 4. Black stars: Reduced Nusselt number from Chaumat (2002) multiplied by a factor 1.4 which is the increase of the wet area.

Data agree well with the previous ones showing that the top smooth plate is unaffected by the presence of the bottom rough plate. Besides, measurements of $N_{u_s}$ from the small and tall cells nicely fit together, confirming the poor influence of the aspect ratio on the Nusselt number (Nikolaenko et al., 2005; Funfschilling et al., 2005).

On the contrary, we observe a significant different behavior for the bottom rough plate (figure 3) since a clear transition is observed when the thermal boundary layer is roughly equal to the height of the roughness. Before the transition, the rough Nusselt number is consistent with the smooth Nusselt number whereas beyond the transition, $N_{u_r}$ exhibits a steeper power law dependence with an exponent of about 1/2. However, it is difficult to measure precisely this exponent since the experiment spans only one and half Rayleigh decade in this regime. We do not see any saturation of the reduced Nusselt number, but it is yet an open problem at larger Rayleigh numbers.

Another instructive feature of this heat transfer enhancement is that the Nusselt number is not bounded by the increase of wet area (see figure 3). The transition cannot be explained only in terms of variation of the area of the plate or else its wet area.

Finally, it seems that before the transition, $N_{u_r}$ is slightly reduced compared to $N_{u_s}$.

4. Interpretation

The 1/2 power law of the $N_{u_r}$ vs $Ra_r$ has been already observed by Roche et al. (2001a) in a symmetric cell whose both sidewalls and plates were covered by V-shape grooves. This heat transfer enhancement is interpreted as a transition to a new regime of convection characterized by turbulent boundary layers. The effect of the plate roughness is then to alter the Rayleigh number dependence of the viscous sublayer thickness in order to cancel the logarithmic correction factor to the 1/2 power law of the $Nu$ vs $Ra$ relation predicted by Kraichnan (1962). Nusselt measurements have been carried out in a similar cell with smooth walls and plates and a transition with an increase of the heat transfer was also observed (Chavanne et al., 1997),
in contrast to the experiments presented in this paper for which no transition is detected for the smooth plate and the classical regime $Nu \sim Ra^{1/3}$ is completely unaffected. So, it may be possible that there are two distinct effects of rough elements on heat transfer. Next paragraph gives an alternative interpretation of our experimental results.

Due to large scale circulation, a part of the fluid which is close to the plate and within the notches between the plots (shaded area called sensitive area in figure 4) cannot be washed by the flow, and remains at rest, until its buoyancy destabilization. Before this buoyancy destabilization, it thus reduces the heat exchange, as it prevents the convection to go closer to the plate. On the contrary, after the buoyancy destabilization, it vigourously contributes to the heat exchange. We can see in figure 4 that the fluid at rest occupies one quarter of the whole surface (shaded area) whereas the fluid swept by the bulk flow occupies three-quarters of the surface. Then, we assume that the fluid swept by the bulk flow has the same behavior than the fluid which is close to the smooth plate so that it contributes to the Nusselt number in the same way as $Nu_s$. Thus $Nu_r = \frac{1}{4} Nu_{sens} + \frac{3}{4} Nu_s$, where $Nu_{sens}$ is the thermal impedance of the fluid at rest. This yields:

$$Nu_{sens} = 4Nu_r - 3Nu_s$$

Before the transition, the quiescent fluid, of height $h_o$, should be toped with a smooth type thermal boundary layer, of height (from equations 3 and 4):

$$\delta = H/(2 \times 0.06 \times Ra_r^{1/3})$$

Indeed, we observe that $Nu_r < Nu_s$ before the transition. The height of the whole thermal boundary layer of the sensitive area can then be expressed as:

$$\delta_{sens} = \frac{H}{2Nu_{sens}} = h_o + \delta$$

Thus:

$$\frac{2h_o}{H} Nu_{sens} = \frac{0.06 \times Ra_r^{1/3}}{1 + 0.06 \times Ra_{notch}^{1/3}}$$

Here, $Ra_{notch} = (\frac{2h_o}{H})^3 Ra_r$. 
When $Ra_{notch}$ increases, the fluid in the notch (in the sensitive area shown in figure 4) cannot stay at rest even if it is protected from the large scale circulation. Indeed, a buoyancy destabilization of the fluid in the notch is expected to happen, similar to that which one can observe in a small Rayleigh-Bénard cell of height $2h_0 = 4$ mm. This transition from conduction to convection regime appears when $Ra_{notch}$ exceeds a critical Rayleigh number, $Ra_0$, which depends only on the aspect ratio of the cell and typically varies between a few thousands and a few tens of thousands. The thermal impedance of the fluid in the notch is estimated by:

$$Nu_{notch} \approx \frac{2h_0}{H} Nu_{sens}.$$

Figure 5 shows the experimental measurements of $Nu_{notch}$ as a function of $Ra_{notch}$ and also $Nu$ vs $Ra$ in a classical RB cell from Chavanne et al. (2001). As the aspect ratios are different and in order to fit the transition Rayleigh number, we renormalize the Rayleigh numbers from Chavanne et al. (2001). Beyond the critical Rayleigh number $Ra_0$, the relation $Nu_{notch}$ vs $Ra$ is well described by the classical relation $Nu$ vs $Ra$ obtained in a cell with smooth plates (see figure 5). Before the buoyancy destabilisation, there is an agreement between the experimental measurements of $Nu_{sens}$ and our model that gives equation 8.

5. Hysteresis and relaxation

We have observed two more interesting and instructive features of turbulent convection over the rough plate: an hysteretic behaviour (figure 6) and long relaxation times (figure 7). It could mean that, maybe depending on the orientation of the large scale flow, the bifurcation of the trapped quiescent fluid (shaded area in figure 4) to a convective state can turn to subcritical. Then, on some $Ra$ range, each trapped part has a finite probability per unit time to become convective, resulting in an exponential relaxation of the global Nusselt number of the rough plate.

6. Conclusion

We have carried out measurements of the Nusselt number as a function of the Rayleigh number in a Rayleigh-Bénard cell with a smooth upper plate and a rough lower plate. Measuring the temperature in the bulk flow permits us to define two Nusselt numbers separately: one for the
smooth plate and one for the rough plate. Quite surprisingly, the Nusselt number based on the temperature of the smooth plate is perfectly consistent with previous measurements of $Nu$ in a similar cell but with two smooth plates. In addition, the smooth plate seems insensitive to the rough plate behavior since the transition of the boundary layer close to the rough plate is not detected in the results of the smooth plate. This confirms the poor influence of the large circulation flow on the behavior of both plates.

The second main result reported in this paper is that the heat transfer enhancement observed in the case of a rough plate is not simply due to the increase of the wet area. It is rather the local dynamics of the thermal boundary layer that change the heat transport over the rough plate. Despite the observed $Nu \sim Ra^{1/2}$ power law, a transition to a turbulent state is improbable since the observed transition seems to be not sensitive to the Reynolds number: $Re$ is divided by a factor of 10 between the tall and small cells. We have preferred to propose a model based on the fact that some part of the fluid close to the rough plate is not swept by the bulk flow and remains at rest until its buoyancy destabilization. The model captures most of the characteristics of the enhanced heat transfer and also the hysteretic and long relaxation time behaviors.

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