The stress state numerical study of a rectangular cross-section beam with round drillings

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Abstract. The study of the rectangular cross section beam’s stress state, weakened by round drillings has been performed. The results’ reliability is confirmed by calculation in the LIRA SAPR and ANSYS Mechanical APDL software packages. The results have been compared with the solution by the materials resistance methods.

Introduction

In previously published works [1-5], the authors investigated the perturbation of the stress state of the beams without perforation, depending on the load application method. The Saint-Venant principle applicability questions have been investigated. The distances, at which the stress state ditherings damped and it became possible to apply the materials resistance formulas, have been determined. The research has led to a generalization of the Saint-Venant principle to kinematic parameters. It turned out that the kinematic boundary conditions heterogeneity causes a perturbation of stresses, which also quickly decays along the beam’s length. This work is devoted to perforated beams, which are widely used in construction as lightweight and economically beneficial structural elements. The authors set out to check whether it is possible to use the materials resistance formulas to the perforated beams’ calculation in this work. A numerical simulation of the work of a beam with round drillings was carried out using two calculation software systems: LIRA SAPR and ANSYS Mechanical APDL. The existing approximate analytical methods for calculating perforated beams [6–10] do not answer the question of how stress peaks are distributed taking into account perforation.

To conduct a numerical study, we considered a rigidly clamped beam of a rectangular cross section 5.4 m long with perforation in the form of a circular cutout, the diameter of which is 1/3 of the beam’s height. The cross section and the perforation fragment in the beam wall are shown in Figure 1. The calculation was carried out in a two-dimensional setting, the FE size is 2x2 cm with local thickenings in the perforation region.
Study

Computation №1. In calculation № 1 vertical force was applied to the free end of the beam \( P = 100 \) kN (Figure 2). The solution of the materials resistance with constant transverse force in a continuous section beam leads to a parabolic diagram of tangential stresses that does not change along the length with an extremum at the level of the center of gravity of the section. In a beam with drillings, the tangential stresses along the beam change only because of the drillings, which makes it possible to study the perforation effect on the tangential stresses’ distribution. In this case, the integral characteristic (the area of the tangential stresses diagram) should remain constant lengthwise.

Figure 2. Design scheme of a beam with shear stress iso-poles \( \tau_zx \), MPa

Figure 3 shows a fragment of the shear stresses’ distribution obtained from calculation in the LIRA SAPR software package. Due to the fact that there is a repeating fragment with perforation in the problem under consideration, the tangential stresses’ distribution pattern also has the character of a repeating fragment.
Figure 3. Fragment of the tangential stresses’ distribution $\tau_{xz}$, MPa

For tangential stresses obtained from the materials resistance solution (Zhuravsky’s formula), their maximum value in the beam without perforation is $\tau_{xz} = 1.67$ MPa.

As it can be seen from Figure 3, the maximum tangential stress values arise directly near the drillings with local stress peaks from 2 sides. The peak values of tangential stresses occur between the sections 1-2 and 2-2 at a distance $R/2$ from the section 2-2.

We call the coefficient of increase in stress $k$ the voltage ratio obtained from a numerical experiment to the voltage determined by the formulas of the materials resistance for a continuous section. Then in this area (at a distance $R/2$ from the section 2-2) $k = 3.4$. The next peak occurs at a distance of approximately $R/3$ from the cutout (between 2-2 and 3-3) and is $\tau_{xz} = 4.1$ MPa.

Figure 4 shows the shear stresses’ plots in the sections located at a distance $R$ and $2R$ from the drilling edges (1-1, 3-3 and 4-4), as well as in the drilling center (2-2). The peculiarity of the diagrams’ presentation in the LIRA SAPR software package is that they are rotated on $90^\circ$. 
As it can be seen from Figure 4, in section 2-2 in the drilling center the values of the maximum tangential stresses (2 peaks on the lower and upper parts of the section) take the values $\tau_{xz} = 2.45$ MPa. Approximately the same value can be obtained by the methods of materials resistance, if we consider two sections of the cross section (above and below the drilling), as separate beams, perceiving half the transverse force.

The numerical solution in section 4-4 gives the value $\tau_{xz} = 2.40$ MPa, which corresponds to $k = 1.47$. On the shear stresses' plot at a distance $R$ after the drilling edge (section 3-3), the stress peak shifts toward the center of the section gravity with a maximum tangential stress 1.4 times greater than the section 2-2. The tangential stresses’ perturbation from the cross section 3-3 to 4-4 is smoothed out with a decrease in the peak value. On the distance $2R$ from the drilling (section 4-4), the tangential stresses, where the dithering influence of the stress state perturbation due to perforation somewhat decreases, the values 2.40 MPa are taken, which corresponds to $k \approx 1.44$. Before the drilling (section 1-1) at a distance $R$ it can be seen that the diagram of tangential stresses becomes gentler compared to the classical parabola obtained by the material resistance formulas.

To verify the results, this problem was solved in the ANSYS Mechanical APDL software package (Figure 5). It can be seen that the qualitative and quantitative characteristics of the results completely coincide in the section 2-2 in the drilling center.

**Figure 5.** The tangent stress field near the drilling (left) and the normal stress diagram in the center of the drilling in the ANSYS Mechanical APDL PC (right)

**Computation №2.** In calculation № 2 a couple of forces were applied to the free end of the rod $P = 1000$ kN (Figure 6). Such a load was chosen so that the same bending moment acts along the beam’s length. The solution of the materials resistance for a continuous section beam in this case leads to a linear diagram of normal stresses that does not change along its length. The normal stresses in the cross sections in the perforated beam change only due to the drillings’ influence. Figure 7 shows a fragmented
picture of the normal stresses’ distribution obtained in the LIRA SAPR software package. Due to the fact that there is a repeating fragment with a drilling (Figure 1) in the problem under consideration, the normal stresses’ distribution also has the nature of a repeating fragment, which can be observed from the calculation results in Figure 7.

![Figure 6. Design scheme of a beam with iso-poles of normal stresses $\sigma_x$, MPa](image)

![Figure 7. Fragment of the normal stresses’ distribution $\sigma_x$, MPa](image)

We study the normal stresses’ distribution shown in Figure 8. The maximum values of normal stresses arise directly at the fibers farthest from the neutral line, as in the case of a beam without perforations. However, due to the drillings’ presence, a local stress concentration, directly affected by the cutout, arises in the area. The peak value of normal stresses arises directly along the section passing through the center of the drilling, as it can be seen from the normal stress diagrams (Figure 8). The voltage value is $\sigma_x = 43.3$ MPa, which corresponds to $k \approx 1.87$.

Figure 8 shows the normal stresses’ plots at a distance $R$ and $2R$ from perforation, as well as a plot in the section in the drilling center.

For normal stresses, we use the material resistance formulas to calculate stresses in bent elements. In a beam without perforation, normal stresses take the values $\sigma_x = 66.7$ MPa. Taking the presence of drillings into consideration, the solid section is divided into 2 rectangles, and the normal stresses at the extreme fibers take the values $\sigma_x = 69.2$ MPa. In the same section at the level of perforation boundaries, taking into account the stress concentration absence, normal stresses take the values $\sigma_x = 23.1$ MPa.

We study the normal stresses’ distribution shown in Figure 7. The maximum values of normal stresses arise directly at the fibers farthest from the neutral line, as in the case of a beam without drillings. However, due to the presence of perforation, a stress concentration occurs in the region of the local cutout influence. The peak value of normal stresses arises directly over the cross section passing through the drilling center, as it can be seen from the diagrams of normal stresses (Figure 8). The voltage value is $\sigma_x = 43.3$ MPa, which corresponds to $k \approx 1.87$. 
Figure 8 shows the normal stresses’ plots at a distance $R$ and $2R$ from perforation, as well as a plot in the section in the center of the circle.

![Normal stress diagrams](image)

**Figure 8.** Normal stress diagrams $\sigma_x$ at the sections 1-1, 2-2, 3-3 and 4-4, MPa
It is seen from the normal stresses’ diagrams that the maximum stresses on the extreme fibers do not differ much, both in cross section with the drilling and at a distance from it \( R \) and \( 2R \). The values approximately correspond to the maximum stresses in a similar beam without drillings. In general, the maximum normal stresses in the section along the center of a circle (section 2-2) are 5% less than in a section without a drilling. The greatest differences, as already mentioned, arise in the local region around the drilling, where normal stresses at the level of the drilling edge exceed those at the same level of stress in the beam without perforation \((k \approx 1.87)\).

Compared with the solution of the materials resistance for a section with drillings, where the maximum normal stresses at the extreme fibers \( \sigma_x = 69.2 \) MPa, in the section along the center of the circle (2-2), the values of the highest normal stresses are less \((\sigma_x = 64.5 \) MPa), which indicates a decrease in the overall stress level in the section with perforation and concentration of stress concentration near the boundaries of the drilling.

For additional verification, we present the results of the same problem, but solved in the ANSYS Mechanical APDL software package. As it can be seen from Figure 9, the qualitative and quantitative characteristics of the results coincide with Figure 7 and the plot in section along the center of the circle 2-2 (Figure 8).

**Figure 9.** Iso-pole of normal stresses near the drilling (left) and the diagram of normal stresses in the center of the drilling in the ANSYS Mechanical APDL PC (right)

**Summary**

1. The normal stresses’ values determined in the extreme fibers of the beam according to the formulas of the materials resistance differ little from the results of a numerical experiment.
2. The normal stresses increase near the drillings, but their peak values do not exceed the stresses in the extreme fibers, which allows using the material resistance formulas for the strength analysis.
3. The local effect of the increase in stress is estimated in the form of the coefficient values \( k \) (stress increase factor), for both tangent and normal stresses. For tangential stresses, the highest peak value \( k \approx 3.4 \) takes place near the drilling zone at a distance \( R/2 \) from the perforation center. For normal voltages, the maximum value is \( k \approx 1.87 \) directly in the center of the drilling (however, the value \( k \) on the upper and lower fibers approaches unity).
4. The presence of drillings located at the beam axis level has a greater effect on the tangential stresses’ redistribution than normal stresses due to the fact that the maximum values of tangential stresses fall on the level of the beam’s neutral axis, while normal stresses reach the highest values at the most distant from the beam fibers’ axis.
5. For the beam under consideration, despite the different nature of the stress distribution obtained by the formulas for the materials resistance and the finite element method, it is possible to carry out
the design and computational studies using the methods of materials resistance, since the maximum values of normal stresses turn out to be approximately the same for a continuous cross section beam and a beam with drillings. However, the shear stress test should be carried out taking into account the possibility of their increase by 3.4 times.

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