NON-LEPTONIC TWO-BODY WEAK DECAYS OF CHARMED
MESONS AND CP-VIOLATING ASYMMETRIES

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ABSTRACT

The non-leptonic two body decays of D mesons are studied in the framework of an improved factorization approximation. The final state interaction effects are taken into account assuming them dominated by nearby resonances. The agreement with experimental data is quite good. CP-violating asymmetries are predicted.

1. Introduction

A theoretical description of exclusive non-leptonic decays of charmed mesons based on first principles has not yet been achieved. Although the short-distance effects due to hard gluon exchange can be resummed and the effective hamiltonian has been constructed at next-to-leading order, the evaluation of its matrix elements requires non-perturbative techniques. In this respect, a classical analysis based on QCD sum rules has been presented in three papers by Blok and Shifman. However only the general trends were reproduced by their analysis, while no agreement with current experimental data was obtained. More recently, Martinelli and collaborators have proposed a procedure to study two-body non-leptonic weak decays in numerical simulation of lattice QCD. Since no numerical result have been obtained as yet, one has however to resort to models.

We here present one such model based on the factorization approximation with annihilation terms and rescattering effects due to the resonances coupled to the final states, that has been rather successful to account for the experimental data about two-body decays of charged and neutral D mesons in PP and PV final states. This feature has made it possible for us to obtain reliable predictions for the related CP-violating asymmetries.

2. Weak Decay Amplitudes

The effective weak hamiltonian for Cabibbo allowed non-leptonic decays of charmed particles is given by

\[ H_{\text{eff}}^{\Delta C=\Delta S} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \left[ C_2 \bar{s}^\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \bar{u}^\beta \gamma^\mu (1 - \gamma_5) d_\beta + C_1 \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \bar{s}^\beta \gamma^\mu (1 - \gamma_5) d_\beta \right] + \text{h.c.} \]  

(1)
For $\Delta C = -\Delta S$ processes the Hamiltonian is obtained from the previous equation with the substitution $s \leftrightarrow d$. The effective weak Hamiltonian for Cabibbo-first-forbidden (CFF) non-leptonic decays reads

$$H_{\text{eff}}^{\Delta C=\pm 1, \Delta S=0} = \frac{G_F}{\sqrt{2}} \left\{ V_{ud} V_{cd}^* \left[ C_1 Q_1^d + C_2 Q_2^d \right] + V_{us} V_{cs}^* \left[ C_1 Q_1^s + C_2 Q_2^s \right] - V_{ub} V_{cb}^* \sum_{i=3}^{6} C_i Q_i \right\} + \text{h.c.} . \quad (2)$$

In Eq. (2) the operators are defined as

$$Q_1^d = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) d_\beta \, \bar{d}^\beta \gamma_\mu (1 - \gamma_5) c_\alpha ,$$

$$Q_2^d = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \, \bar{d}^\beta \gamma_\mu (1 - \gamma_5) c_\beta ,$$

$$Q_3 = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \sum_q \bar{q}^\beta \gamma_\mu (1 - \gamma_5) q_\beta ,$$

$$Q_4 = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\beta \sum_q \bar{q}^\beta \gamma_\mu (1 - \gamma_5) q_\alpha ,$$

$$Q_5 = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\alpha \sum_q \bar{q}^\beta \gamma_\mu (1 + \gamma_5) q_\beta ,$$

$$Q_6 = \bar{u}^\alpha \gamma_\mu (1 - \gamma_5) c_\beta \sum_q \bar{q}^\beta \gamma_\mu (1 + \gamma_5) q_\alpha . \quad (3)$$

The operator $Q_1^d$ ($Q_2^d$) in Eq. (2) is obtained from $Q_1^s$ ($Q_2^s$) with the substitution $(d \rightarrow s)$. $\alpha$ and $\beta$ are colour indices (that we will omit in Eq. (3)) and in the “penguin” operators $q$ ($\bar{q}$) is to be summed over all active flavors ($u$, $d$, $s$).

For the Wilson coefficients we used the anomalous dimension matrices calculated at next-to-leading order. Assuming $A_{4}^{MS} = 300$ MeV, at the scale $\mu = 1.5$ GeV for the “scheme independent prescription” (cfr Buras et al. in Ref. [9]) we obtain $C_1 = -0.628$, $C_2 = 1.347$, $C_3 = 0.027$, $C_4 = -0.057$, $C_5 = 0.015$, $C_6 = -0.070$.

In the factorization approximation the matrix elements of $H_{\text{eff}}$ are written in terms of matrix elements of currents, $(V_q^a)^\mu = \bar{q}^\prime \gamma_\mu q$ and $(A_q^a)^\mu = \bar{q}^\prime \gamma_\mu \gamma_5 q$. To evaluate these matrix elements,* we adopt the usual definition of the decay constants and the form factors. The $q^2$ dependence of the involved form factors ($f_1(q^2)$, $f_0(q^2)$ and $A_0(q^2)$ for PP and PV final states) is assumed to be dominated by the nearest resonances. The values $f_1(0)$ and $f_0(0)$ are fixed by using SU(3) symmetry and the semileptonic decay rate $D^0 \rightarrow K^{-} e^+ \nu$. Since the data on $D$ meson decays show large SU(3) breaking effects, in our fit we allowed $a_{cs}(\equiv A_0^{0\rightarrow s(0)})$ to be different by $a_{cd} = a_{cu}(\equiv A_0^{0\rightarrow d(0)})$ and the values obtained by the fit are $a_{cs} = 0.59$, and $a_{cd} = 1$.†

In the W-exchange and annihilation terms, however, the large and time-like $q^2$ values needed, together with the suggested existence of resonances with masses near to the $D$-meson mass, make a prediction based on the lightest mass singularity unjustified. These terms depend on the matrix elements of current divergences between the vacuum and two-meson states. We write them, with the help of the equations of motion, in the way

*The matrix elements of axial vector current and axial density between the vacuum and $\eta(\eta')$ state are evaluated following the authors of Ref. [9].
†A direct QCD sum rule calculation of $A_0(q^2)$ shows the $q^2$ dependence compatible with the pole, but a different SU(3)-breaking effects: $a_{cs}/a_{cu} = 1.10 \pm 0.05$ at $q^2 = 0$.\[2]
indicated in the following examples:

\[ <K^-\pi^+|\partial^\mu (V_s^{\mu})|0> = i (m_s - m_d) <K^-\pi^+|\bar{s}d|0> \equiv i (m_s - m_d) \frac{M_P^2}{f_D} W_{PP}, \]

\[ <K^-\rho^+|\partial^\mu (A_s^{\mu})|0> = i(m_s + m_d) <K^-\rho^+|\bar{s}\gamma_5d|0> \equiv -(m_s + m_d) \frac{2M_P}{f_D} \epsilon^* \cdot p_K W_{PV}. \]

We use SU(3) symmetry for the matrix elements of scalar and pseudoscalar densities, and express all of them in terms of \( W_{PP}, W_{PV} \), which are, in our approach, free parameters of the fit. Their magnitude turns out to be considerably larger than what one would obtain assuming form factors dominated by the pole of the lightest scalar or pseudoscalar meson, i.e. \( K_0^*(1430) \) or \( K(497) \).

### 3. Final State Interaction Effects

As far as final state interactions (FSI) are concerned, we assume that they are dominated by resonant contributions, and we neglect the phase-shifts in exotic channels. In the mass region of pseudoscalar charmed particles there is evidence for a \( J^P = 0^- \) \( K(1830) \) (with \( \Gamma = 250 \) MeV and an observed decay to \( K\phi \)) and a \( J^P = 0^- \) \( \bar{\pi}(1770) \) with \( \Gamma = 310 \) MeV. These resonances have the right quantum numbers to construct an \( 0^- \) octect which can couple to PV final states. The hamiltonian is determined from charge conjugation and SU(3) symmetry. Analogously, the FSI for \( D \to PP \) decays should be dominated by the \( J^P = 0^+ \) octect. There is evidence for the existence of \( \bar{\bar{K}}^{*0} \) (with mass 1945\( \pm 10 \pm 20 \) MeV, width 201\( \pm 34 \pm 79 \) MeV and 52\( \pm 14\% \) branching ratio in \( K\pi \)). Unfortunately, no \( a_0 \) isovector resonance has been observed up to now in the interesting mass region. However, we assume his existence and fix the mass with an equispacing formula.

The description of rescattering effects for Cabibbo forbidden \( D^0 \) decays is complicated by the presence of a coupling with a yet unobserved \( f_0 \) and \( f'_0 \) isoscalar resonances, which should be singlet-octet mixtures. In order to reduce the number of free parameters we assume the scalar resonances behave as the tensor mesons (\( J^P = 2^+ \)), \( f_2(1270) \) and \( f'_2(1525) \). This procedure relates the coupling constant in the strong hamiltonian with the mixing angle \( \phi \) between the singlet-octect part of \( f_0 \) and \( f'_0 \).

In our model the FSI effect modifies the amplitudes, \( A_w \), in the following way:

\[ A(D \to V_h P_k) = A_w(D \to V_h P_k) + c_{hk}[\exp(i\delta_h) - 1] \sum_{h'k'} c_{h'k'} A_w(D \to V_{h'} P_{k'}) \]  

(5)

In Eq. (5) \( c_{hk} \) are the normalized (\( \sum c_{hk}^2 = 1 \)) couplings \( \bar{P}PV \) and

\[ \sin\delta_h \exp(i\delta_h) = \frac{\Gamma(\bar{P})}{2(M_{\bar{P}} - M_D) - i\Gamma(\bar{P})} \]  

(6)

where \( \bar{P} \) is the resonance appropriate to the decay channel considered (\( \bar{\pi} \) or \( \bar{K} \)).

\(^{\dagger}\)Note that for the PV final state the C symmetry forbids the coupling of the final state to a singlet part of an hypotetical \( \eta \)-resonance. Thus, we need to fit only a single phase, \( \delta \).

\(^{\ddagger}\) The \( f'_2 \) is very weakly coupled to \( \pi\pi \), and the \( f_2 \) has in turn a small coupling to \( K\bar{K} \).

\(^{\bullet}\) For further details see Ref. \(^{\ddagger}\).

\(^{\parallel}\) An analogous expression holds for PP final state.
4. Comparison with Experimental Data on Branching Ratios and Predicted CP-Violating Asymmetries

Using our model to evaluate the weak amplitudes $A_w$ and modifying them with FSI effects we are able to write the rates for all Cabibbo-allowed two-body decays and for Cabibbo-forbidden $D^+$ and $D_s^+$ as functions of eleven free parameters. Their values are fixed with a fit to the experimental data. The total $\chi^2_T = 90.0$ for 45 data ($\chi^2/dof = 2.6$): 25 are data points for Cabibbo-allowed decays ($\chi^2 = 61.8$), 12 for CFF $D^+ \to PP$ ($\chi^2 = 18.8$), 4 for CFF $D^0 \to PP$ ($\chi^2 = 1.7$), and 4 for CFF $D^0 \to PV$ ($\chi^2 = 7.7$). In all cases the agreement with the data is quite good, but the Table 1 shows that our model needs an improvement in describing the decays in PV final state.**

Table 1: Partial $\chi^2$ for each class of $D$ decays.

| Decays      | # data | $\chi^2$ |
|-------------|--------|----------|
| $D^0 \to PP$ | 8      | 8.44     |
| $D^+ \to PP$ | 5      | 9.56     |
| $D_s^+ \to PP$ | 4      | 8.79     |
| $D^0 \to PV$ | 12     | 18.35    |
| $D^+ \to PV$ | 8      | 29.55    |
| $D_s^+ \to PV$ | 8      | 15.35    |

It is well known that CP-violating effects show up in a decay process only if the decay amplitude is the sum of two different parts, whose phases are made of a weak (CKM) and a strong (final state interaction) contribution. If $A_1$ and $A_2$ denote the generic two weak amplitude contributing to the $D \to f$ amplitude, the CP-violating asymmetry in the decay rates will be:

$$a_{CP} = \frac{2 \Im(A_1 A_2^*) \sin(\delta_2 - \delta_1)}{|A_1|^2 + |A_2|^2 + 2 \Re(A_1 A_2^*) \cos(\delta_2 - \delta_1)} \tag{7}$$

where $\delta_i$ are strong phases. Now in the Cabibbo-first-forbidden $D$ decays the penguin operators in Eq. (2) provide the different phases of the weak amplitudes $A_1$ and $A_2$.

Having obtained a quite good description of the rates we may give reliable prediction on CP-violating asymmetries for $D^+$ and $D^0$ Cabibbo-first-forbidden decays. In Table 2 we report CP-violating asymmetries in $10^{-3}$ unit; the central values are obtained choosing $\rho = 0.2, \eta = 0.3$ and $V_{cb} = 0.040$. The errors result from the variation of $\rho$ and $\eta$ in the one-sigma region obtained in Ref. 11.

As we can see in Table 2 large asymmetries ($\approx 10^{-3}$) are predicted in our model; at this end large final state phase shifts and penguin contributions played the fundamental role.

**This is the starting point for the authors of Ref. 10.
Table 2: CP-violating decay asymmetries for some $D^+$ and $D^0$ Cabibbo forbidden decays.

| decay channel | $10^3 \times a_{CP}$ | decay channel | $10^3 \times a_{CP}$ |
|---------------|----------------------|---------------|----------------------|
| $D^+ \to \rho^0 \pi^+$ | $-1.17 \pm 0.68$ | $D^+ \to K^0\pi^+$ | $-0.51 \pm 0.30$ |
| $D^+ \to \rho^+\pi^0$ | $+1.28 \pm 0.74$ | $D^0 \to \pi^0\eta$ | $-1.43 \pm 0.83$ |
| $D^0 \to K^+\rho^0$ | $-0.67 \pm 0.39$ | $D^0 \to \pi^0\eta'$ | $+0.98 \pm 0.57$ |
| $D^0 \to \overline{K}^0\rho^0$ | $-0.67 \pm 0.39$ | $D^0 \to \eta\eta'$ | $-0.50 \pm 0.29$ |
| $D^0 \to K^{*+}\pi^-$ | $+0.038 \pm 0.022$ | $D^0 \to \eta\eta'$ | $-0.28 \pm 0.16$ |
| $D^0 \to K^{*-}\rho^+$ | $+0.16 \pm 0.09$ | $D^0 \to \pi^0\pi^0$ | $+0.54 \pm 0.31$ |
| $D^0 \to \rho^+\pi^-$ | $+0.37 \pm 0.22$ | $D^0 \to \pi^+\pi^+$ | $-0.02 \pm 0.01$ |
| $D^0 \to \rho^-\pi^+$ | $-0.36 \pm 0.21$ | $D^0 \to K^+K^+$ | $-0.13 \pm 0.08$ |
| $D^0 \to D^0\eta$ | $+0.28 \pm 0.16$ |

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6. Bibliography

1. G. Altarelli, G. Curci, G. Martinelli and S. Petrarca, *Nucl. Phys.* **B187** (1981) 461; A.J. Buras, M. Jamin, M.E. Lautenbacher and P.E. Weisz, *Nucl.Phys.* **B370** (1992) 69 and *Nucl.Phys.* **B375** (1992) 501 (addendum); M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Nucl. Phys.* **B415** (1994) 403.
2. B. Blok and M. Shifman, *Sov.Jour.Nucl.Phys.* **45** (1987) pp. 135, 301, 522.
3. M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, Preprint CERN-TH/96-31, ROME prep. 96/1131 and hep-ph/9604240.
4. F. Buccella, M. Lusignoli, G. Miele, A. Pugliese and P. Santorelli, *Phys. Rev.* **D51** (1995) 3478.
5. D.I. D’yakonov and M.I. Eides, *Sov.Phys.JETP* **54** (1981) 232; G. Veneziano, *Nucl. Phys.* **B159** (1979) 213; I. Halperin, *Phys. Rev.* **D50** (1994) 4602.
6. M. Wirbel, B. Stech and M. Bauer, *Zeits.f.Phys.* **C29** (1985) 637.
7. P. Colangelo, F. De Fazio and P. Santorelli, *Phys. Rev.* **D51** (1995) 2237.
8. F. Buccella, M. Forte, G. Miele and G. Ricciardi, *Zeits.f.Phys.* **C48** (1990) 47.
9. Rev. of Part. Prop., Particle Data Group, *Phys. Rev.* **D50** (1994) part I.
10. F. Buccella, M. Lusignoli and A. Pugliese, *Roma1-1130/96, Napoli-DSF-T-2/96*, hep-ph/9601343, to appear in *Phys. Lett.* **B**.
11. M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Roma preprint* n. 1024-1994.