An Analytical View for The Combination between The Soft Topological Spaces and Fuzzy Topological Spaces

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Abstract. An analytical study for the combination of the soft topological spaces and fuzzy topological spaces is presented with the reformulation of some definitions and using the new soft fuzzy points which presented in [30] to introduce new definition of separation axioms. Also base, subbase for these spaces and soft fuzzy compact space with theorems and illustrative examples

Introduction

L.Zadeh [16] in 1965 introduced the concept of fuzzy set theory since different problems in real life contains different types of uncertainty and vagueness . Chang[10] presented the definition of a fuzzy topology . Later R.Lowen [24] redefined it which is called a stratified fuzzy topology.

Studying the concept of generalized closed sets in topological spaces was introduced by Levin [6]. Chang's fuzzy topology arises interest of many researchers to introduce generalized fuzzy closed sets in it. Sostack [22] introduced the notion of smooth topology as an extension of Chang an Lowen's fuzzy topology and developed the theory of smooth topological spaces. After that several researchers studied smooth topological spaces as [4] and they assigned to the fuzzy topology in the sense of Chang- Lowen as the topology of fuzzy subsets. Atanassov[15] introduced the opinion of intuitionistic fuzzy set. By using intuitionistic fuzzy sets, Coker and his colleagues [11] introduced the notion of intuitionistic fuzzy topological space. Samanta and Mondial [13] introduced the notion of intuitionistic gradation of openness as an extension of smooth topology.

Molodtsov [9 ] Introduced soft set theory as a different method for uncertainly in real life problems.

Maji et al [14] defined new notions of soft set theory. Shabir and Naz [21] introduced soft topology by using soft sets with some basic properties. Soft fuzzy set was first introduced by Majii and his coauthor [23] in 2001 , after that many researchers improved this study and gave new results [28,29]. Tanay and Kandemir [7] defined soft fuzzy topology on a soft fuzzy set and gave an introductory theoretical base to carry further study on this topic. S.Roy and T.K.Samanta also studied soft fuzzy topological space in [27].

Several researchers Introduced a new classes of separation axioms and gave studied some of their properties . A new compactness spaces called Gem-compact space , points space under the idea of Gem-set.[18]
In this paper an analytical study is presented for the combination of the fuzzy topological spaces and soft topological spaces to produce soft fuzzy topology and redefine the axioms of separation by using the new soft fuzzy points which are produced in [3] and also base, subbase for these spaces and soft fuzzy compact space with theorems and illustrative examples.

Preliminaries
In this section we recall some definitions and notions concern our study in later sections.

Definition 2.1 [14]: Let X be universe set and I = [0,1] the function 
\[ f: x \rightarrow I \]
is called the membership of the fuzzy set \( A \) where \( A = \{(x, f_A(x)) | x \in X\} \) and \( f_A(x) = f(x), \forall x \in A \) and \( f_A(x) = 0, \text{if} x \notin A \) with \( A \subseteq X \).

Definition 2.2 [3]: Let X be a universe set and I = [0,1]. Let us denote that
1- \( M(X, I) = \{f; f : X \rightarrow I\} \) be the set of all functions \( f \) on the universe set \( X \) to the unit interval \( I = [0,1] \).
2- \( \Phi_A(X) = \{f_{|A}; \forall f \in M(X, I), \forall A \subseteq X\} \), where \( f_{|A}(x) = f_A(x) \), \( \forall x \in A \).
3- \( \Phi_{|A}(X) = \{f_{|A}; \forall f \in M(X, I), \forall A \subseteq X\} \), where \( f_{|A}(x) = 0, \forall x \notin A \).
4- \( A \subseteq B \) if \( f_A(x) \subseteq f_B(x) \), \( \forall x \in X \).

Note that :
1- \( 0 = \{(a, 0); \forall a \in A\} \) the null fuzzy set,
2- \( 1 = \{(a, 1); \forall a \in A\} \) the absolute fuzzy set.

Definition 2.3: Let \( f: X \rightarrow I \) be function and \( I(X) \) be the collection of all fuzzy sets and \( E \) be the parameters of elements of \( X \). The soft fuzzy set denoted by \( F \), where \( F : A \rightarrow I(X) \) with respect to the universe set \( X \).

We denote that
1- \( 0 = \{(a, 0); \forall a \in A\} \) the null fuzzy set,
2- \( 1 = \{(a, 1); \forall a \in A\} \) the absolute fuzzy set.

Definition 2.4 [11]: Let \( X \) be an initial universe set and \( E \) a set of parameters of elements of \( X \). \( P(X) \) denotes the power set of \( X \) and \( \forall \subseteq E \). A pair \( (F, A) \) is called a soft set over \( X \) (simply \( F \)) parameterized family the universe \( X \).

We denote that the null soft set and absolute soft set are as follows.
1- \( \emptyset = \{(a, \emptyset); \forall a \in A\} \) the null fuzzy set.
2- \( \bar{1} = \{(x, 1); \forall x \in X\} \) the absolute fuzzy set.

Definition 2.5 [11]: Let \( X \) be an initial universe set and \( E \) a set of parameters of elements of \( X \). \( P(X) \) denotes the power set of \( X \) and \( \forall \subseteq E \). A pair \( (F, A) \) is called soft set over \( X \) (simply \( F \)) parameterized family the universe \( X \).

We denote that the null soft set and absolute soft set are as follows.
1- \( \emptyset = \{(a, \emptyset); \forall a \in A\} \) the null fuzzy set.
2- \( \bar{1} = \{(x, 1); \forall x \in X\} \) the absolute fuzzy set.

Definition 2.6 [30]: The first family of soft sets is denoted by \( SS_F(X) \) and defined as follows:
SS_1(X) = \{ F_i: A \subseteq E, \text{and } F_i: A \rightarrow P(X) \}

where X is the universe set and E is the set of parameters,

**Definition 2.7 [5]:**

Let X be a set. A fuzzy topology on X is a family \( T \) of fuzzy sets in X, which satisfies the following conditions:

1-\( \emptyset, \bar{X} \in T \),
2- If \( A \in T \), then \( A \cup B \in T \)
3- If \( A_j \in T \) for each \( j \in J \), then \( \bigvee_{j \in J} A_j \in T \)

\( T \) is called a fuzzy topology for \( X \), and the pair \( (X, T) \) is called a fuzzy topology space(in short \( fT \) fts) and \( \bar{X} \) is called fuzzy space. Every member of \( T \) is called a fuzzy open set. A fuzzy set is called fuzzy closed iff its complement is fuzzy open.

**Definition 2.8:**

A subbase for a fuzzy topological space \( (X, T) \) is a sub collection \( S \) of \( T \) such that the collection of infimum of finite subfamilies of \( S \) forms a base for \( (X, T) \).

**Definition 2.9:**

\[ \text{Let } X \text{ be an initial universal set and } A \subseteq E. \text{ Let } \mathcal{E}_0 \text{ be the collection of soft sets over } X, \text{ then } \mathcal{E}_0 \text{ is said to be a soft topology on } X \text{ if:} \]

1- \( \emptyset, X \) belong to \( \mathcal{E}_0 \).
2- The union of any number of soft sets in \( \mathcal{E}_0 \) belongs to \( \mathcal{E}_0 \).
3- The intersection of any two soft sets in \( \mathcal{E}_0 \) belongs to \( \mathcal{E}_0 \).

The triplet \( (X, \mathcal{E}_0, A) \) is called a soft topological space over \( X \). The members of \( \mathcal{E}_0 \) are called soft open sets and a soft set \( F_\alpha \) is called soft closed iff the soft complement \( X \setminus F_\alpha \) is soft open.

**Definition 2.10.**

Let \( (X, \mathcal{T}_\circ) \) be a soft topological space. A sub collection \( B \) of \( \mathcal{T}_\circ \) is called a base for a soft topological space \( (X, \mathcal{T}_\circ) \) if every member of \( \mathcal{T}_\circ \) can be expressed as union of members of \( B \).

**Definition 2.11:**

A subbase for a fuzzy topological space \( (X, T) \) is a sub collection \( S \) of \( T \) such that the collection of infimum of finite subfamilies of \( S \) forms a base for \( (X, T) \).

**Definition 2.12:**

Let \( (X, \tau_s) \) be a soft topological space. A sub collection \( S \) of \( \tau_s \) is said to be a subbase of \( \tau_s \) if the family of all finite intersections of members of \( S \) forms a base for \( (X, \tau_s) \).

**Types of fuzzy points 2.13 [3]:**

There are three types of fuzzy points as follows:

**Type I:** For some \( x \in X \) and \( 0 < \alpha \leq 1 \), we say that \( p_\alpha^x \) is classical fuzzy point on the fuzzy point of type I if

\[
p_\alpha^x(y) = \begin{cases} 
\alpha & \text{if } y = x \\
0 & \text{if } y \neq x
\end{cases}
\]

**Type II:** Let \( A \subseteq X \) and for some \( 0 < \alpha \leq 1 \), the formula \( p_\alpha^A \) is called the fuzzy point of Type II, where

\[
p_\alpha^A(y) = \begin{cases} 
p_\alpha^x & \text{if } y \in A \\
0 & \text{if } y \notin A
\end{cases}
\]

**Type III:** For any \( 0 < \alpha < 1 \), the formula \( p_\alpha \) is called the fuzzy point of type III such that

\[
p_\alpha(y) = \alpha \quad \forall \ y \in X.
\]

Noted that \( p_\alpha = \bigcup_{\alpha \in A} p_\alpha^x \).

**Types of the soft points 2.14 [30]:**

There are three types of the soft points in \( \bar{X} \) as in the following:

1- The first type of soft points in \( \bar{X} \) is denoted by \( F_e^x \) where

\[
F_e^x(a) = \begin{cases} 
\{x\} & \text{if } a = e \\
\emptyset & \text{if } a \neq e
\end{cases}
\]

2- The second type of soft points in \( \bar{X} \) is denoted by \( F_\circ \) where
\( F_e = \{(e, \{x\}) : e \in E \} \)

3- The third type of soft points in \( X \) is denoted by \( F_e \) where

\[
F_e(a) = \begin{cases} 
F(e) & \text{if } a = e \\
\emptyset & \text{if } a \neq e, \text{for all } e \in E 
\end{cases}
\]

**Definition 2.15** [30]:
Let \( F_A, G_B \in SS_3(X) \), then their Union, intersection, complement and subsets are respectively as follows:

1- \( F_A \cup G_B = \{(e, F_A(e) \cup G_B(e)) : e \in E \} \)

2- \( F_A \cap G_B = \{(e, F_A(e) \cap G_B(e)) : e \in E \} \)

3- \( F_A^c = \{(e, X - F(e)) : \forall e \in E \} \)

4- \( F_A \subseteq G_B \) iff \( F(e) \subseteq G(e), \forall e \in E \)

**Definition 2.16** [34]: Let \( U \) be an initial universe set and \( E \) be a set of parameters. Let \( P(U) \) denotes the power set of \( U \), the soft fuzzy set denoted by \( f FS \) is

\[
f_{FS} = \{(e, f(e)) : \forall e \in E, f(e) \in I^2 \}
\]

where \( f : A \rightarrow I^2 \)

\[
f_F(e) = \{(x, f_F(e)(x)) : x \in U, f : U \rightarrow I \}
\]

also \( f_{FS}(x) = f(x), \forall x \in F(e) \)

And \( f_F(x)(e) = 0, \forall x \notin F(e) \)

We denote that the null soft fuzzy set and absolute soft fuzzy set are respectively as follows:

\[
\bar{0} = \{(e, 0), \forall e \in E \} \text{ is the null soft fuzzy set.}
\]

\[
\bar{X} = \{(e, 1), \forall e \in E \} \text{ is the absolute soft fuzzy set.}
\]

**Definition 2.17** [11]: A soft fuzzy set \( f \tilde{A}_B \) is said to be a soft fuzzy subset of a soft fuzzy set \( g \tilde{C}_D \) over common universe if \( B \subseteq D \) and \( f_F(e) \subseteq g_G(e), \forall e \in B \) iff \( \forall e \in B, f(x)_{F(e)} \leq g(x)_{G(e)}, \forall x \in X \) and we denoted it by \( f \tilde{A}_B \subseteq g \tilde{C}_D \).

**Table 1.** The combination of soft points and fuzzy points.

| Fuzzy points | Soft points | Soft fuzzy points |
|--------------|-------------|------------------|
| \( p^x \)    | \( F^x_e \)       | \( F_{p^x}^{e_F} \) |
| \( p \)      | \( F^x_e \)       | \( F_{p}^{e_F} \) |
| \( p^A \)    | \( F^x_e, x \in A \) | \( F_{p^A}^{x_F} \) |
| \( p^x \)    | \( F^x \)         | \( F_{p^x}^{x_F} \) |
| \( p \)      | \( F^x \)         | \( F_{p}^{x_F} \) |
| \( p^A \)    | \( F^x, x \in A \) | \( F_{p^A}^{x_F} \) |
| \( p^x \)    | \( F_e \)         | \( F_{p^x}^{e_F} \) |
| \( p \)      | \( F_e \)         | \( F_{p}^{e_F} \) |
| \( p^A \)    | \( F, x \in A \)   | \( F_{p^A}^{e_F} \) |

and \( F_e \cap A \neq \emptyset F_e \)
Soft Fuzzy Topological Spaces

Definition 3.1:

I. Let X be an initial universal set and E be parameters of elements of X and \( S(I^2) \) be the collection of all soft fuzzy set. A sub collection \( s_\tau \) of \( S(I^2) \) is called soft fuzzy topology of satisfy the following:

1. \( 0, 1 \in s_\tau \)
2. If \( F_A, gB \in s_\tau \), then \( F_A \cap gB \in s_\tau \)
3. For any index \( \lambda \) and \( F_A, F_{A_\lambda} \in s_\tau \), then \( \bigcup_{\lambda \in A} F_{A_\lambda} \in s_\tau \)

II. A sub collection \( s_\beta \) of \( s_\tau \) is called base for \( s_\tau \) if for any soft fuzzy operand is union of some members of \( s_\beta \)

III. A sub collection \( s_\sigma \) of \( s_\tau \) is called sub base for \( s_\tau \) if for any soft fuzzy topology \( s_\tau \) if the finite intersections of members of \( s_\sigma \) form a basis of \( s_\tau \).

An analytical study for the combination of the soft topological spaces and fuzzy topological spaces is presented with the reformulation of some definitions and using the new soft fuzzy points which presented in 3.2 to introduce new definition of separation axioms. Also base, sub base for these spaces and soft fuzzy compact space with theorems and illustrative examples.

Note: The symbols which are used in our study are agree with the lists of symbols.

Definition 3.2 [42]
A Soft fuzzy topology \( \tau \) on \((U,E)\) is a family of soft fuzzy sets over \((U,E)\) satisfying the following properties

1. \( 0, 1 \in s_\tau \)
2. If \( sF_A, sG_A \in s_\tau \), then \( sF_A \cap sG_A \in s_\tau \)
3. If \( sF_{A_\alpha} \in s_\tau \) for all \( \alpha \in \Lambda \), an index set, then \( \bigcup_{\alpha \in \Lambda} sF_{A_\alpha} \in s_\tau \).

So that

1. A sub collection \( s_\beta \) of \( s_\tau \) is called base for \( s_\tau \) if for any soft fuzzy operand is union of some members of \( s_\beta \)
2. A sub collection \( s_\sigma \) of \( s_\tau \) is called sub base for \( s_\tau \) if for any soft fuzzy topology \( s_\tau \) if the finite intersections of members of \( s_\sigma \) form a basis of \( s_\tau \).

Example 3.3:
Let \( X = \{x_1, x_2, x_3\} \) and \( E = \{e_1, e_2, e_3\} \)

\[
F(e_1) = \{x_1\} \\
F(e_2) = \{x_1, x_2\} \\
F(e_3) = \{x_2, x_3\} \quad \text{where} \quad F : E \rightarrow \mathcal{P}(X)
\]

\[
F_A = \{(e_1, \{x_1\}), (e_2, \emptyset), (e_3, \{x_2, x_3\})\} \\
G_A = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\}), (e_3, \{x_2\})\} \\
H_A = \{(e_1, \{x_1\}), (e_2, \emptyset), (e_3, \{x_2\})\} \\
J_A = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\}), (e_3, \{x_2, x_3\})\}
\]

\[
\mathcal{T}_S = \{\emptyset, X, F_A, G_A, H_A, J_A\}
\]

\[
\mathcal{A} = \{(x_1, 0.1), (x_2, 0), (x_3, 0.3)\} \quad \text{with membership} \ f
\]

\[
\mathcal{B} = \{(x_1, 0.2), (x_2, 0), (x_3, 0.1)\} \quad \text{with membership} \ g
\]

\[
\mathcal{C} = \{(x_1, 0.2), (x_2, 0), (x_3, 0.3)\} \quad \text{with membership} \ h
\]

\[
\mathcal{D} = \{(x_1, 0.1), (x_2, 0), (x_3, 0.1)\} \quad \text{with membership} \ k
\]

\[
\mathcal{F} = \emptyset, \mathcal{I}, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}.
\]

Now,
for some soft fuzzy topology on $(\mathcal{F}_\mathcal{U}, \mathcal{G}_\mathcal{U})$

**Soft Fuzzy Separation axioms**

### Definition 3.4 [29]

A collection $\beta$ of some soft fuzzy sets over $(U, E)$ is called a soft fuzzy open base or simply a base for some soft fuzzy topology on $(U, E)$ if the following conditions hold:

1. $\beta \in \beta$
2. $U \beta = E$ i.e. for each $e \in E$ and $x \in U$, there exists $F_A \in \beta$ such that $\mu_{F_A} = 1$
3. If $F_A, G_B \in \beta$ then for each $e \in E$ and $x \in U$, there exists $H_C \in \beta$ such that $H_C \subseteq F_A \cap G_B$ and

   $$\mu_{F_A, G_B} = \min\{\mu_{F_A}, \mu_{G_B}\}$$
   where $C \subseteq A \cap B$.

**Note 3.5:**

1. From Definition 2.7 and Definition 2.9 we conclude that the soft fuzzy topology is extract from combination of soft topology with fuzzy topology.
2. From Definition 2.8 and Definition 2.10 we see that the base of soft fuzzy topology is output about the combination of base of soft topology with base of fuzzy topology.
3. Also from Definition 2.11 and Definition 2.12, we see that the sub base of $s$ off fuzzy topology is output about the combination of sub base of soft topology with sub base of fuzzy topology.

**Soft Fuzzy Separation axioms**

Here, we introduce various soft fuzzy separation axioms for a soft fuzzy topological spaces with definitions and theorems.

**Note 4.1:**
The properties of the separation axioms are defined on the different soft fuzzy points which are listed in table (1) above and the definition of soft fuzzy comes on view of the four different soft fuzzy points that is either on two different points of the same type or two points from two different types or valid on any two points from the four types and this means that it is more general, that is there are three definitions for \( T_0, T_1, T_2 \), regular, \( T_3 \), normal and the other kinds of separation axioms.

Theorem 4.2:

If the soft fuzzy topology \( (X, \tau_S) \) is SF\( T_0 \)-space w.r.t the soft fuzzy point (simply sfp) of the type \( F_{\alpha}^{x} \) iff the spaces that are combination to configure the fuzzy topology and soft topology be f\( T_0 \)-space w.r.t fuzzy point (simply fP) of type \( P_{\beta}^{x} \) and ST\( T_0 \)-space w.r.t soft point (simply sp) of type \( F_{\beta}^{x} \) respectively.

Proof. Since, for any two different soft fuzzy points of the type \( F_{\alpha}^{x} \) are coming from the combination of \( P_{\gamma}^{x}, P_{\delta}^{x}, F_{\gamma}^{x}, F_{\delta}^{x} \) and \( P_{\gamma}^{x}, P_{\delta}^{x}, F_{\gamma}^{x}, F_{\delta}^{x} \) and \( P_{\gamma}^{x}, P_{\delta}^{x}, F_{\gamma}^{x}, F_{\delta}^{x} \) and \( P_{\gamma}^{x}, P_{\delta}^{x}, F_{\gamma}^{x}, F_{\delta}^{x} \) respectively.

Assume that \( (X, \tau_S) \) is SF\( T_0 \)-space then for any two distinct fuzzy points

1. \( P_{\alpha}^{x} \neq P_{\beta}^{x} \) and \( F_{\alpha}^{x} \neq F_{\beta}^{x} \)
2. \( P_{\alpha}^{x} \neq P_{\beta}^{x} \) and \( F_{\alpha}^{x} \neq F_{\beta}^{x} \)
3. \( P_{\alpha}^{x} \neq P_{\beta}^{x} \) and \( F_{\alpha}^{x} \neq F_{\beta}^{x} \)
4. \( P_{\alpha}^{x} \neq P_{\beta}^{x} \) and \( F_{\alpha}^{x} \neq F_{\beta}^{x} \)

But (\( \tau, sT_{P} \)) is as f\( T_0 \)-space

\( \exists \) a sf-open set \( \tilde{G} \) contains one of the two points and does not contains the other, but \( \tilde{G} \) is obtained from the combination of \( \tilde{G} \) \( f \)-open with \( G_{\alpha} \) s-open so \( \tilde{G} \) \( f \)-open contains one of the two fuzzy points in (a,b,c) and does not contains the other and this gives that (\( \tau, f \tilde{\tau} \)) is f\( T_0 \)-space and by the same method , we get (\( \tilde{\tau}, sT_{P} \)) is too ST\( T_0 \)-space

\( \Leftarrow \) suppose that (\( \tilde{\tau}, s\tilde{\tau} \)) and (\( \tau, f \tilde{\tau} \)) are s\( T_0 \)-space and f\( T_0 \)-space respectively.

Now, let \( F_{\alpha}^{x} \) \( f \)-open and \( P_{\beta}^{x} \) and \( (1),(2),(3) \) be different soft fuzzy points that can be get by the combination of the soft points with the fuzzy points, but (\( \tau, f \tilde{\tau} \)), \( \tilde{\tau}, s\tilde{\tau} \)),so \( \exists \) a \( f \)-open set \( \tilde{B} \) contains one of the two fuzzy points and does not contains the another and s-open set \( F_{\alpha}^{x} \) contains one of the two soft points and does not contains the another. By the combination between \( \tilde{B} \) with \( F_{\alpha}^{x} \) we obtain sf-open set \( s\tilde{B}_{FAS} \) which contains one of the two soft fuzzy points and does not contains the another.

Note 4.3: The theorem is valid for all four types of soft fuzzy points mentioned above.

Theorem 4.5:
The soft fuzzy topology $f_{T_s}$ is SFT$_2$-space (Hausdorff space) w.r.t the sfp of the type $F^e_{a_x}$ iff the spaces that are combination to configure the fuzzy topology and of type soft topology be f- regular space w.r.t fP of type $P^x_{a}$ and S- regular space w.r.t sp of type $F^a_{i}$, iff for two distinct soft fuzzy points there exist two soft fuzzy open sets each one contains only one soft fuzzy point and does not contain the other.

Theorem 4.6:

The soft fuzzy topology $f_{T_s}$ is SF Regular space w.r.t the sfp of the type $F^e_{a_x}$ iff the spaces that are combination to configure the fuzzy topology and of type soft topology be f- regular space w.r.t fP of type $P^x_{a}$ and S- regular space w.r.t sp of type $F^a_{i}$.

Note 4.7: The soft fuzzy topology $(X, \tau)$ is called SFT$_3$ if it is regular space and $T_1$.

Theorem 4.8:

Soft fuzzy topological space is soft fuzzy compact iff it is soft compact and fuzzy compact.

Proof: $\Leftarrow$ For any cover of soft fuzzy open sets to the space $\tilde{X}$ we get that $\tilde{X}$ is cover by soft open sets and cover by fuzzy open sets, But the spaces are soft compact and fuzzy compact so there exists finite sub cover by soft set and fuzzy set, and we may put $m \in N$ and is the number of the soft open sets $n \in N$ is the number of the fuzzy open sets, so the combination $(m,n)$ is also finite so that $(m,n)$ are sub covers of soft fuzzy open sets for the space. Therefore the space is soft fuzzy compact.

$\Rightarrow$ Suppose that the space is soft fuzzy compact .To show that it is soft compact and fuzzy compact, for every soft open cover for the soft space $X$ and and every fuzzy open cover to fuzzy space an by combination of soft space and fuzzy space we get soft fuzzy space covered by soft fuzzy open sets, so there exists , finite sub cover and again we used the combination to have finite soft cover and finite fuzzy cover respectively.

Therefore the soft space is soft compact and fuzzy space is also fuzzy compact.

Proposition 4.9:

Let $F_{a_x}$ be a soft fuzzy closed set in soft fuzzy compact space $(X, f_{T_s})$. Then $F_{a_x}$ is also soft fuzzy compact.

Proof. Let $\mathcal{L}$ be a collection of soft fuzzy open cover to $F_{a_x}$, s the cover $\mathcal{L}$ with complement of $F_{a_x}$ its cover to $\tilde{X}$, by soft fuzzy compact of $\tilde{X}$, there is finite subcover to $\tilde{X}$ which also cover to $F_{a_x}$.

Definition 4.10:

A soft fuzzy set $F^e_{a_x}(\varepsilon) = \{ p^x_{a} \quad \text{if} \quad \varepsilon = e \quad \text{for} \quad 0 < \alpha \leq 1 \}
\begin{align*}
0, \quad \text{if} \quad \varepsilon \neq e
\end{align*}

1- A soft fuzzy set $F^e_{a_x} = \{(e, p^x_{a}), \forall e \in E, 0 < \alpha \leq 1 \}$ is a soft fuzzy set of type ii

2- A soft fuzzy set $F^e_{a_x} = \{(e, p^x_{a}), \forall e \in E, 0 < \alpha \leq 1 \}$ is a soft fuzzy set of type ii

3- A soft fuzzy set $F^e_{a_x}$ is a soft fuzzy point of type ii

such that $F^e_{a_x} = \begin{align*}
F^e_{a_x} \quad \text{if} \quad x \in F(e) \\
0 \quad \text{if} \quad x \notin F(e)
\end{align*}

4- A soft fuzzy set $F^e_{a_x}$ is a soft fuzzy point of type iv where

$F^e_{a_x} = \begin{align*}
p^e_{a} \quad \text{if} \quad \varepsilon = e \\
0 \quad \text{if} \quad \varepsilon \neq e
\end{align*}

5- A soft fuzzy set $F^e_{a_x}$ is a soft fuzzy point of type v such that $F(e) \cap A = \emptyset$.

As shown in table (1) below.

Conclusion

Soft fuzzy sets which represent the combination between fuzzy sets and soft sets are applied in wide area of sciences. In this paper, we introduce definitions of soft fuzzy topology and soft fuzzy separation.
axioms with respect to soft fuzzy points which are introduced in [4] with some definitions and theorems. Also, base, subbase and soft fuzzy compact space with illustrative examples.

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