Observation of the vortex structure of a non-integer vortex beam

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New Journal of Physics 6 (2004) 71
Received 30 April 2004
Published 5 July 2004
Online at http://www.njp.org/
doi:10.1088/1367-2630/6/1/071

Abstract. An optical beam with an $e^{i\ell\phi}$ phase structure carries an orbital angular momentum of $\ell h$ per photon. For integer $\ell$ values, the phase fronts of such beams form perfect helices with a single screw-phase dislocation, or vortex, on the beam axis. For non-integer $\ell$ values, Berry (2004 J. Opt. A: Pure Appl. Opt. 6 259) predicts a complex-phase structure comprising many vortices at differing positions within the beam cross-section. Using a spatial light modulator we produce $e^{i\ell\phi}$ beams with varying $\ell$. We examine the phase structure of such beams after propagation through an interference-based phase-measurement technique. As predicted, we observe that for half-integer $\ell$ values, a line of alternating charge vortices is formed near the radial dislocation.

It is now well appreciated that light beams with helical phase fronts described by an azimuthal phase term of $e^{i\ell\phi}$ carry an orbital angular momentum of $\ell h$ per photon [1]. Examples of such beams include the Laguerre–Gaussian modes and high-order Bessel beams. For integer $\ell$ values, the phase fronts for a given phase, comprise $\ell$ intertwined helical surfaces giving a screw dislocation along the beam axis, and a resulting annular intensity cross-section. This screw-phase dislocation along the axis is an optical vortex of charge $\ell$. Initially, for non-integer $\ell$ values, there is a phase discontinuity, in our case, radially along the $\phi = 0$ direction, which on propagation gives rise to a line of low intensity. Recently, Berry [2] has theoretically analysed such beams showing that after propagation, they have intricate-phase structures comprising chain of alternating charge vortices along the direction of the initial radial discontinuity. In this study we experimentally produce such beams and confirm the general nature of Berry’s predictions.

One conceptually easy way to produce a helically phased beam is to pass a plane-waved beam through a spiral phase plate. A phase plate is formed from a transparent disc whose thickness is proportional to the azimuthal position, with a step at the $\phi = 0$ position; see figure 1.
Figure 1. A spiral phase plate of refractive index $n$. The thickness of the phase plate $h$ is proportional to the azimuthal position given by $\phi$.

The required step height, $s$, is related to the desired $l$ value, the refractive index, $n$, of the plate and wavelength of the light by $s = (n - 1)\lambda l$. Such a phase plate was originally used to produce helical beams at optical frequencies but the required tolerance is extremely demanding [3, 4]. By contrast, at mm-wave frequencies, the machining tolerances are relaxed and this method has become one of the choice [5, 6].

At optical frequencies, the simplest method for producing helically phased beams and indeed many other beams uses programmable spatial light modulators to modify the phase structure of the beam output from a conventional, spatially coherent, laser. The spatial light modulator is a phase-only diffractive optical component, whose design can typically be updated at video frame rates. As with conventional diffractive components, imperfections in the phase linearity and other issue mean that light is diffracted into a number of different diffraction orders. If these orders overlap, the resulting beam is degraded from that envisaged. To ensure that the beam is produced with a high accuracy, the desired phase structure is usually added to a carrier such that the first-order diffracted beam is angularly displaced from the other orders enabling it to be selected using a spatial filter. For producing helical phase fronts, the resulting design of diffractive component is the modulo $2\pi$ addition of a simple blazed grating with an azimuthal $2\pi l\phi$ phase ramp, giving the characteristic $l$-pronged fork dislocation on the beam axis (figure 2(a)) used by researchers to produce Laguerre–Gaussian modes. This design is readily adapted to any value of $l$, giving an additional radial discontinuity to the pattern (figure 2(b)) [7].

To experimentally observe these non-integer $l$ beams we used the output beam from a HeNe laser, expanded to 10 mm in diameter, incident on a programmable spatial light modulator (HoloEye). The diffracted light was collected with a 600 mm focal length lens, and a 1 mm diameter aperture positioned in the back focal plane selected the first-order diffracted beam (see figure 3). Figure 4 shows an example of the intensity cross-section 300 mm from this aperture. To enhance the image quality, the screen was rotated in the same plane to give a time-varying speckle pattern, which over the integration time of the camera results in a smooth, speckle-free, image. Note that the beam is no longer circular. Specifically, there is a radial line of low intensity with an orientation corresponding to the radial discontinuity in the design of the diffractive
Figure 2. Phase patterns represented in grey scale. (a) The addition of a carrier to the azimuthal phase such that the diffraction orders are spatially separated. (b) Examples of different patterns corresponding to different step heights.

Figure 3. The experimental configuration. The zero- and first-order diffracted light, a distance $d$ from the spatial light modulator, was imaged onto a screen for measurement.
Figure 4. Intensity cross-section of the first-order diffracted light, 300 mm from the spatial light modulator. The step height in this case is $l = 2.5$.

Determining the phase difference between two light beams can be problematic, particularly in regions of low optical intensity. In principle, three or four images taken at different phase offsets are sufficient. We can change the phase offset between the zero- and first-order beams by shifting the phase of the carrier with respect to the azimuthal phase ramp programmed to the spatial light modulator. In this study, we developed an alternative technique to determine the phase of the first-order beam which is less sensitive to noise in any one of the recorded images. For each value of $l$, 20 images were acquired with different phase offsets in intervals of $\pi/10$. At each pixel position, the 20 values of intensity were Fourier-transformed to give a complex spectrum, the argument of the lowest frequency component giving the phase difference between the beams. Plotting this angle for each pixel position gives an image corresponding to the phase difference between the zero- and first-order beams. Subtracting the carrier-phase ramp corresponding to the intersection angle of the beams gives the phase structure of the first-order beam produced by the spatial light modulator.

The details of the predicted phase structure critically depends on a number of factors including the size of the illuminating beam and the propagation distance from the phase plate. For our particular experiment, we used a 10 mm Gaussian beam and observed the phase structure 300 mm behind the phase plate. In all cases, however, there are a number of key features. For $l$ values greater than 1.5, multiple vortices of the same sign are present near the beam axis, but are only coincident for integer $l$ values. For half-integer $l$ values, the radial dislocation in the phase plate gives a radial line of low intensity which is associated with a string of single change vortices of alternating sign. It is this last feature of Berry’s prediction which is most striking and which we seek primarily to observe.

To verify our experimental observations, we compare our results with a numerical model of the same experimental configuration. Our model is based on a Fourier decomposition of beam into its plane-wave components, equivalent to the analytical expressions of Berry [2]. Propagation of this beam is modelled by multiplication of each component by the appropriate phase factor and an inverse Fourier transform gives the new distribution [8, 9]. To aid interpretation of both the modelled and observed data, we adopted a full-colour representation of the beam’s phase structure. Relative phase values of $0, \pi/8, \pi/4, 3\pi/8, \ldots$ are coloured as red, orange, yellow, etc.
A single change vortex is recognized in the figures as a meeting of all eight colours at a single point, the handedness of the vortex corresponding the handedness of the colours. Multiple-charge vortices appear as the convergence of 16, 24, ... phase lines.

Figures 5–7 show the observed and modelled phase distributions for various \( l \) values from \( l = 1 \) to \( l = 3.5 \). The corresponding intensity distributions are also displayed. Note specifically that vortices of charge greater than one are only observed for exact integer values of \( l \) (e.g. see \( l = 3 \) in figure 5). As has been widely recognized and observed these multiple-charge vortices are unstable and slight astigmatism or other experimental defects leads to splitting into several single-charge vortices. For non-integer \( l \) values greater than 1.5, the number of same-sign vortices near the beam axis is equal to the value of \( l \) rounded to the nearest integer (see e.g. \( l = 2.7 \)). Most strikingly, however, is for exact half-integer values of \( l \), we confirm the presence of a string of single-charge vortices of alternating sign extending near the line of the initial radial discontinuity (see e.g. \( l = 3.5 \)). Although we do not observe the ringing in the intensity that is predicted, this feature is observed in the phase.

These features can be observed in the two movies; one for the modelled results and one for the experimental results (figure 8). These both show the evolution of the phase and intensity as the step height is increased from \( l = 0 \) to \( l = 4 \) in steps of \( l = 0.025 \). Note the chain of alternating sign vortices and the splitting of the vortices at the centre of the beam at half-integer step heights.
Figure 6. Modelled (top) and experimental (bottom) results corresponding to non-integer \( l \) step heights. The red and green circular arrows indicate the first two alternating sign vortices along the line of the initial radial discontinuity.

Figure 7. Detail of modelled (left) and experimental (right) patterns. The line of alternating sign vortices along the dislocation for a step height corresponding to \( l = 3.5 \) can be seen in both patterns.

The same phase measurements used to identify the nature of the vortex structure can be combined with the intensity measurements to give the transverse momentum distribution the light beam, i.e. the local inclination of the phase front multiplied by the corresponding intensity. These transverse components can then be multiplied by the radius vector to give the \( z \)-component of the orbital angular momentum density of the beam [1]:

\[
j_z = (r \times p)_z = \left( r \times i\omega \frac{\varepsilon_0 c_0}{2} (u^* \nabla u - u \nabla u^*) \right)_z.
\]

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Figure 8. Screen shots from the movies for modelled results (left) and experimental results (right). Both movies show the evolution of the phase and intensity as the step height is increased from $l = 0$ to $l = 4$.

Figure 9. The relationship between the step height of the phase plate and the orbital angular momentum content of the beam.

Integrating this over the beam cross-section gives the total orbital angular momentum. It is usual to divide this by the energy in the beam and express the result as the angular momentum per photon [10, 11].

Clearly, this analysis depends upon both the direction and position of the axis about which the calculation is performed. As has been previously identified [12], providing the direction of the calculation axis (i.e. the $z$-direction) is the one for which the net transverse momentum of the beam is zero, the calculated value of the orbital angular momentum is invariant to transverse displacement of the axis, i.e. in this situation, the orbital angular momentum is intrinsic.

We find that, when the angular momentum is calculated about the $z$-axis, it is only proportional to the step height for integer and half-integer values. The predicted value of the angular momentum per photon is given by $l - \sin \frac{2\pi l}{2\pi}$ [13]. This agrees with our experimentally observed values, derived from the phase measurements and calculated using the above equation. This deviation away from a linear dependence of angular momentum on step height...
height arises due to the fact that for non-integer values of step height, the transverse momentum as defined with respect to the fixed $z$-axis is no longer zero. This non-zero transverse momentum results in an extrinsic contribution to the total orbital angular momentum and the observed deviation (see figure 9).

Although, as predicted, the vortex structure of these beams is complicated, one should not confuse this with the nature of the beams’ orbital angular momentum. As these vortices occur at regions of zero intensity they carry no linear or angular momentum in themselves. Rather the momentum is associated with the bright areas of the beams that surround these vortices. Irrespective of the evolving vortex structure, in free space, the orbital angular momentum integrated over the whole beam is invariant under propagation. As has been noted \cite{11, 14} even the inversion of the vortex sign after focussing by a cylindrical lens does not change the orbital angular momentum of the beam.

Acknowledgments

We thank Professor M Berry and Dr M Dennis for helpful discussions.

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