Higgs–Dilaton cosmology: Are there extra relativistic species?

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A B S T R A C T

Recent analyses of cosmological data suggest the presence of an extra relativistic component beyond the Standard Model content. The Higgs–Dilaton cosmological model predicts the existence of a massless particle – the dilaton – associated with the spontaneous symmetry breaking of scale invariance and undetectable by any accelerator experiment. Its ultrarelativistic character makes it a suitable candidate for contributing to the effective number of light degrees of freedom in the Universe. In this Letter we analyze the dilaton production at the (p)reheating stage right after inflation and conclude that no extra relativistic degrees of freedom beyond those already present in the Standard Model are expected within the simplest Higgs–Dilaton scenario. The elusive dilaton remains thus essentially undetectable by any particle physics experiment or cosmological observation.

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1. Introduction

Cosmology is entering in a precision era where the interplay with particle physics is becoming more and more important. A noteworthy example is the effective number of light degrees of freedom appearing in the different extensions of the Standard Model (SM). Any extra radiation component in the Universe is usually parametrized, independently of its statistics, in terms of an effective number of neutrino species, $N_{\text{eff}} = N_{\text{eff}}^{\text{SM}} + \Delta N_{\text{eff}}$ [1], where $N_{\text{eff}}^{\text{SM}}$ stands for the number of active neutrinos in the SM.1

The strongest constraints on the effective number of neutrino species come from Big Bang Nucleosynthesis (BBN). A non-standard value of $N_{\text{eff}}$ increases the expansion rate, which results on an enhancement of the primordial helium abundance. Assuming zero lepton asymmetry, the number of effective degrees of freedom at BBN turns out to be $N_{\text{eff}} = 3.71^{+0.47}_{-0.45}$ (68% C.L.) [4]. Note that, although the existence of extra species is somehow favored, the obtained value is still compatible with the SM prediction within the 95% C.L.

Some constraints on $N_{\text{eff}}$ can be also obtained from the analysis of the Cosmic Microwave Background (CMB), although the current limits are significantly weaker than those of BBN. The combined analysis of WMAP7 results, Hubble constant measurements and baryon acoustic oscillations [5] provides a value $N_{\text{eff}} = 4.34^{+0.86}_{-0.88}$ (68% C.L.). Similar and complementary results for smaller CMB scales have been also reported by the Atacama Cosmology Telescope [6] and the South Pole Telescope [7]. It is interesting to notice the dependence of the effective number of neutrino species on the priors considered in the different Bayesian analysis existing in the literature. While in some references the SM value, $N_{\text{eff}}^{\text{SM}} = 3$, is ruled out at 95% C.L. [8,9], in others, such as [10], it is not. Besides, if the helium abundance obtained from CMB measures is taken into account, together with the most precise primordial deuterium abundance [11], the BBN result becomes perfectly consistent with the SM one at the $2\sigma$ level, $N_{\text{eff}} = 3.22 \pm 0.55$ [4]. The number of extra degrees of freedom is therefore an open question to be solved by the Planck satellite, which is expected to determine $N_{\text{eff}}$ with an accuracy of $\sim 0.3$ at $2\sigma$ [12], breaking thereby the degeneracies with nonzero neutrino masses and dynamical dark energy [13].

In order to account for the apparent radiation excess one can consider several possibilities. It could be, for instance, the indication of an extra sterile neutrino [14,15], of relic gravitational waves [16], or arise from other exotic possibilities such as a decaying particle [17–19], or the interaction between dark energy and dark matter [20] or the reheating of the neutrino thermal bath [21]. In this Letter we will consider a different possibility within the minimalistic framework of Higgs–Dilaton cosmology [22–24]. This constitutes an extension of the Higgs inflation idea [25], where the Standard Model Higgs doublet $H$ is non-minimally coupled to gravity. The novel ingredient of Higgs–Dilaton cosmology is...
the invariance of the action under scale transformations. This extra symmetry leads to the absence of any dimensional parameters or scales.\footnote{In particular it forbids the appearance of a cosmological constant term in the action. In Higgs–Dilaton cosmology, the late dark energy dominated period of the Universe is recovered, at the level of the equations of motion, by replacing General Relativity with Unimodular Gravity. However, both the inflationary and reheating stages considered in this Letter take place in field space regions where the dark energy contribution is completely negligible. We will thus omit this point here. The reader is referred to Ref. [23] for details about the phenomenological consequences of Unimodular Gravity in the Higgs–Dilaton scenario.} The simplest phenomenologically viable theory of this kind requires the existence of a new scalar singlet under the SM gauge group [22,23], the dilaton $\chi$, non-minimally coupled to gravity. It corresponds to the Goldstone boson associated with the spontaneous symmetry breaking of scale invariance and it is therefore massless. This property makes it a potential candidate for contributing to the effective number of relativistic degrees of freedom at BBN and recombination. Indeed, this cosmological test seems to be the only available probe for determining the existence of the dilaton particle. The coupling between the dilaton and all the SM fields (apart from the Higgs) is forbidden by quantum numbers, which, together with the Goldstone boson nature of this particle, excludes the possibility of a direct detection in an accelerator experiment [22].

In this Letter we study the (p)reheating stage in Higgs–Dilaton cosmology, paying special attention to the dilaton production. This Letter is organized as follows. In Sections 2 and 3 we review the Higgs–Dilaton model [22,23]. In the simplest Higgs inflationary scenario [26–28], the diﬀerential structure of the non-minimal couplings to gravity, the possibility of a direct detection in an accelerator experiment [22], with the Goldstone boson nature of this particle, excludes the possibility of a direct detection in an accelerator experiment [22].

In this Letter we study the (p)reheating stage in Higgs–Dilaton cosmology, paying special attention to the dilaton production. This Letter is organized as follows. In Sections 2 and 3 we review the Higgs–Dilaton model [22,23]. In the simplest Higgs inflationary scenario [26–28], the differential structure of the non-minimal couplings to gravity, the possibility of a direct detection in an accelerator experiment [22], with the Goldstone boson nature of this particle, excludes the possibility of a direct detection in an accelerator experiment [22].

2. Higgs–Dilaton inflation

We start by reviewing the Higgs–Dilaton model [22,23]. In the unitary gauge $H^T = (0, h/\sqrt{2})$, it is described by the following Lagrangian density

$$\mathcal{L} = \frac{1}{2} \left( \xi \frac{\partial \chi}{\partial x} + \xi h^2 \right)^2 R - \frac{1}{2} \left( \partial \chi \right)^2 - U(\chi, h),$$

(1)

where we have omitted the part of the SM Lagrangian not involving the Higgs potential, $\mathcal{L}_{\text{SM}a\text{.t.}}$. The values of the non-minimal couplings to gravity can be determined from CMB observations and turn out to be highly hierarchical ($\xi \sim 10^{-3}$, $\xi h \sim 10^{-3}$–$10^{-5}$) [23,29]. The scale-invariant potential $U(\chi, h)$ is given by

$$U(\chi, h) = \frac{\lambda}{4} \left( h^2 - \frac{\alpha}{\lambda} \chi^2 \right)^2 + \beta \chi^4,$$

(2)

with $\lambda$ the self-coupling of the Higgs field. The parameters $\alpha$ and $\beta$ must be properly tuned in order to reproduce the correct hierarchy between the electroweak, Planck and cosmological constant scales. In particular, we must require $\beta \ll \alpha < 1$. The smallness of all the couplings involving the dilaton field gives rise to an approximate shift symmetry $\chi \to \chi + \text{const.}$, which, as described in Ref. [30], has important consequences for the analysis of quantum effects. For the typical energy scales involved in the (p)reheating stage we can safely set $\alpha = \beta = 0$ in all the following developments.

We study here the (p)reheating of the universe after Higgs–Dilaton inflation. As emphasized in Ref. [23], particle production is more easily analyzed in the so-called Einstein-frame, where the Higgs and dilaton fields are minimally coupled to gravity. Performing a conformal redefinition of the metric, $\bar{g}_{\mu \nu} = \Omega^2 g_{\mu \nu}$, with conformal factor $\Omega^2 = M_p^2 (\xi \chi^2 + \xi h^2)$, we obtain

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_p^2}{2} \mathcal{R} - \frac{1}{2} \tilde{K}(\chi, h) - \bar{U}(\chi, h).$$

(3)

Here $\tilde{K}(\chi, h)$ is a non-canonical kinetic term in the basis $(\phi^1, \phi^2) = (\chi, h)$

$$\tilde{K}(\chi, h) = \frac{\kappa_{ij}}{\Omega^2} g^{\mu \nu} \partial_{\mu} \phi^i \partial_{\nu} \phi^j,$$

(4)

with

$$\kappa_{ij} = \left( \delta_{ij} + \frac{3}{2} M_p^2 \partial_i \Omega^2 \partial_j \Omega^2 \right).$$

(5)

and $\bar{U}(\chi, h) = U(\chi, h)/\Omega^4$ is the Einstein-frame potential. In order to diagonalize the kinetic term we can make use of the conserved Noether’s current associated to scale invariance. It can be easily shown, via the homogeneous Friedmann and Klein–Gordon equations in the slow-roll approximation, that the field combination $(1 + 6 \xi h)^2 + (1 + 6 \xi \chi)^2$ is time-independent in the absence of any explicit symmetry breaking term [23]. This conservation suggests a field redefinition to polar variables in the $(h, \chi)$ plane

$$r = \frac{M_p}{2} \log \left[ \frac{(1 + 6 \xi h)^2 + (1 + 6 \xi \chi)^2}{M_p^2} \right],$$

$$\tan \theta = \sqrt{\frac{1 + 6 \xi h}{1 + 6 \xi \chi}},$$

(6)

(7)

In terms of the new coordinates, the kinetic term (4) becomes diagonal, although non-canonical,

$$\tilde{K} = \frac{1 + 6 \xi h}{\xi h} - \frac{1}{\sin^2 \theta + \zeta \cos^2 \theta} \left( \partial r \right)^2 + \frac{M_p^2}{\xi h} \frac{\tan^2 \theta + \mu}{\cos^2 \theta \left( \tan^2 \theta + \zeta^2 \right)} \left( \partial \theta \right)^2,$$

(8)

where we have defined

$$\mu = \frac{\xi \chi}{\xi h} \text{ and } \zeta = \frac{(1 + 6 \xi h) \xi \chi}{(1 + 6 \xi \chi) \xi h}.$$

The dilatonic field $r$ is massless, as corresponds to the Goldstone boson associated with the spontaneously broken scale symmetry. The inflationary potential depends only on the angular variable $\theta$ and it is symmetric around $\theta = 0$

$$\bar{U}(\theta) = \frac{\lambda M_p^4}{4 \xi h^2} \left( \frac{\sin^2 \theta + \zeta \cos^2 \theta}{\sin^2 \theta + \zeta \cos^2 \theta} \right)^2.$$

(9)

It can be easily seen that during the (p)reheating stage the values of the oscillating field $\theta$ are much larger than the $\mu$ parameter for a large number of oscillations, $\tan^2 \theta \gg \mu$. This allows us to neglect the $\mu$ term in Eq. (8) and perform an extra field redefinition

$$\rho = y^{-1} r, \quad |\phi| = \phi_0 - \frac{M_p}{a} \tan^{-1} \left[ \sqrt{1 - \zeta \cos \theta} \right],$$

(10)

with

$$y = \sqrt{\frac{\xi \chi}{1 + 6 \xi \chi}} \text{ and } a = \sqrt{\frac{\xi \chi (1 - \zeta)}{\zeta}}.$$  

(11)

(12)
Inflation $M^2 = \frac{\lambda M_p^2}{3\xi^2}$. The same applies to the masses of gauge bosons and fermions. We obtain $(\tilde{m}^2_{\phi f})_{\text{HD}} = \tilde{m}^2_{\phi f}(1 + 6\xi_x)$, with $\tilde{m}^2_{\phi f} \simeq \frac{\alpha H^2 M^2}{4\xi^2} |\phi|$ the Einstein-frame gauge boson and fermion masses in the Higgs inflation model [26,27]. Here $g = g_2, g_2/\cos\theta_W$ and $\sqrt{2} y_f$, for $A = W, Z$ bosons and fermions $f$, respectively. We see therefore that, from the point of view of (p)reheating, all the relevant physical scales in Higgs and Higgs–Dilaton inflation coincide, up to small corrections proportional to the small parameter $\xi_x$. This allows us to apply the main results of Refs. [26,27] to the Higgs–Dilaton case. Let us summarize here those results. In Higgs inflation, the SM particles are produced through the so-called Combined Preheating mechanism [26,27]. Intermediate $W^\pm$ and $Z$ bosons are created from the oscillations of the Higgs at the bottom of the potential [16], whenever there is a violation of the adiabaticity condition. While there is no restriction on the number of created gauge bosons, the direct production of SM fermions by this mechanism is severely restricted by Fermi–Dirac statistics. The SM fermions appear as secondary products of the weak bosons created in each zero-crossing. Once produced, the gauge bosons acquire a large effective mass due to the increasing expectation value of the Higgs field and decay perturbatively into quarks and leptons within a semioscillation of the Higgs field. This decay rapidly depletes their occupation numbers and postpones the development of parametric resonance. During the first oscillations, the fraction of energy into SM particles is still very small compared with the energy in the oscillating Higgs field. A large number of oscillations ($t \sim 300 M^{-1}$) will be needed in order to transfer a significant amount of energy into the SM bosons and fermions. The decreasing of the Higgs amplitude due to the expansion of the Universe eventually reduces the decay rate and parametric resonance becomes the dominant effect. At this point, the gauge bosons start to build up their occupation numbers via bosonic stimulation and reheating occurs within a few oscillations. Soon afterwards, the Universe is filled with the remnant Higgs condensate and a non-thermal distribution of fermions and bosons, redshifting as radiation and matter respectively. From there on until thermalization, the evolution of the system is highly non-linear and non-perturbative, which makes difficult to make a clear statement about the subsequent evolution without the use of numerical lattice simulations [26,27]. However, thermal equilibrium is expected to be achieved at a reheating temperature $T_R \sim (3–15) \times 10^{13}$ GeV, much above the QCD phase transition scale, $T_{\text{QCD}} \sim 300$ MeV, due to the large SM couplings [26,27].

3. SM particle production

As shown in Fig. 1, the shape of the Higgs–Dilaton potential (15) clearly resembles that of the simplest Higgs inflationary scenario. In spite of the slight differences, both of them present an exponentially flat region for large field values and nicely agree for small ones. Indeed, the relation between them becomes explicit if we approximate Eq. (15) for small $\phi$. The potential around the minimum behaves, in a good approximation, as the standard chaotic potential

$$U(\phi) \simeq \frac{1}{2} M^2_{\text{HD}} \phi^2 + O(\phi^3),$$

where the higher order corrections can be safely neglected after a few oscillations. The curvature of the potential, $M^2_{\text{HD}} = (1 + 6\xi_x) M^2$, coincides, up to small corrections, with that of Higgs

\[\text{Fig. 1. Comparison between the Higgs–Dilaton inflationary potential (blue continuous line) obtained from Eq. (15) and the corresponding one for the Higgs inflation mode (red dotted line). In spite of the slight differences in the upper inflationary region, they nicely agree in the lower part, where the (p)reheating stage takes place. Here } U_0 = \frac{1}{2} M^2_{\text{HD}}/4\xi^2 \text{ and } \kappa = M^{-1}. \text{(For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)}\]

\[\text{The variable } \phi \text{ is periodic and defined in the compact interval } \phi \in [-\phi_0, \phi_0], \text{ where } \phi_0 = M_P/\alpha \tanh^{-1}[\sqrt{1 - \xi}] \text{ corresponds to the value of the field } \phi \text{ at the beginning of inflation. As happened in Higgs inflation [26,27], the absolute value in Eq. (11) is required for } \phi \text{ to maintain the symmetry of the initial field}\]

\[\text{in this figure legend, the reader is referred to the web version of this Letter.)}\]

\[\text{The constancy of the classical background component is of course coincident, up to small corrections proportional to the small parameter } \xi_x. \text{ This allows us to apply the main results of Refs. [26,27] to the (p)reheating stage. Let us summarize here those results. In Higgs inflation, the SM particles are produced through the so-called Combined Preheating mechanism [26,27]. Intermediate } W^\pm \text{ and } Z \text{ bosons are created from the oscillations of the Higgs at the bottom of the potential [16], whenever there is a violation of the adiabaticity condition. While there is no restriction on the number of created gauge bosons, the direct production of SM fermions by this mechanism is severely restricted by Fermi–Dirac statistics. The SM fermions appear as secondary products of the weak bosons created in each zero-crossing. Once produced, the gauge bosons acquire a large effective mass due to the increasing expectation value of the Higgs field and decay perturbatively into quarks and leptons within a semioscillation of the Higgs field. This decay rapidly depletes their occupation numbers and postpones the development of parametric resonance. During the first oscillations, the fraction of energy into SM particles is still very small compared with the energy in the oscillating Higgs field. A large number of oscillations (} t \sim 300 M^{-1} \text{) will be needed in order to transfer a significant amount of energy into the SM bosons and fermions. The decreasing of the Higgs amplitude due to the expansion of the Universe eventually reduces the decay rate and parametric resonance becomes the dominant effect. At this point, the gauge bosons start to build up their occupation numbers via bosonic stimulation and reheating occurs within a few oscillations. Soon afterwards, the Universe is filled with the remnant Higgs condensate and a non-thermal distribution of fermions and bosons, redshifting as radiation and matter respectively. From there on until thermalization, the evolution of the system is highly non-linear and non-perturbative, which makes difficult to make a clear statement about the subsequent evolution without the use of numerical lattice simulations [26,27]. However, thermal equilibrium is expected to be achieved at a reheating temperature } T_R \sim (3–15) \times 10^{13} \text{ GeV, much above the QCD phase transition scale, } T_{\text{QCD}} \sim 300 \text{ MeV, due to the large SM couplings [26,27].}\]

4. Dilaton production

In addition to the SM fields, the Higgs–Dilaton inflationary scenario incorporates an extra degree of freedom, the dilaton field $\rho$. The constancy of the classical background component is of course guaranteed by the scale invariance current conservation, but this reasoning does not apply to the corresponding quantum excitations. As suggested in Ref. [34], these modes can be excited in the preheating stage after inflation through the non-canonical kinetic term in the Einstein-frame Lagrangian (13), which mixes quantum excitations and background solutions. Although the perturbative dilaton production through this mixing is expected to be very small, non-perturbative effects might play an important role [34]. In this section we estimate the energy density residing in the dilaton field at the end of the preheating stage. Let us start

\footnote{It is interesting to notice at this point that this statement is only valid for values of the field such that $\tan^2 \theta \gg \mu$. For very small values of the angular variable $\theta$ we recover the standard $\lambda \phi^4$ Higgs potential. As it happens in Higgs inflation [26,27], this region turns out to be extremely small, being completely irrelevant for the study of the (p)reheating stage.}

\footnote{Any dilaton production previous to this stage is completely diluted by the inflationary expansion.}
by considering the linearized\textsuperscript{5} equations of motion for dilaton perturbations $\delta \rho_k$ in Fourier space
\begin{equation}
\delta \ddot{\rho}_k + (3H + 2\dot{b}) \dot{\delta \rho}_k + \frac{k^2}{a^2} \delta \rho_k = 0,
\end{equation}
where we have ignored metric perturbations and taken into account the constancy of the background field $\rho$ during the Higgs oscillations at the end of inflation. The function $b = b(\phi)$ plays the role of an additional oscillatory damping term for the dilaton perturbations and depends on the absolute value of the inflaton field $\phi$, cf. Eq. (14). This dependence makes it cumbersome the direct application of the techniques presented in Ref. [34] for the study of particle creation in models with non-canonical kinetic terms. The field redefinitions used there would imply delta functions coming from the derivatives of the absolute value, which substantially complicates the analytical and numerical treatment of the problem. On the other hand, although the rephrasing of Eq. (17) as a Hill’s equation is extremely useful for the understanding of the particle creation mechanisms, it is not necessary for a precise computation in an expanding background, as the one needed to estimate $N_{\text{eff}}$. For this reason, we will adopt an alternative approach, dealing only with non-singular evolution equations in an expanding Universe. Let us rewrite Eq. (17) as
\begin{equation}
\frac{1}{a^2 e^{2b}} \frac{d}{dt} \left( a^2 e^{2b} \frac{d}{dt} \delta \rho_k \right) + \frac{k^2}{a^2} \delta \rho_k = 0,
\end{equation}
which, after a redefinition of time, $dt = a^{-3} e^{-2b} dt$, can be recast in the form of a time-dependent harmonic oscillator
\begin{equation}
\delta \rho'_k + \omega_k^2(\tau) \delta \rho_k = 0,
\end{equation}
with frequency $\omega_k^2(\tau) = k^2 a^4 e^{4b}$. Here the prime denotes derivative with respect to the new time $\tau$. Choosing an initial vacuum state with zero particle content,\textsuperscript{6} the number of created dilatons is given by
\begin{equation}
\rho_\chi = \frac{1}{2} \frac{1}{\pi k^2} \int_0^\infty dk k^2 \omega_k \alpha_k,
\end{equation}
and its associated energy density.
\begin{equation}
\rho_\chi = \frac{1}{2 \pi^2} \frac{1}{\pi k^2} \int_0^\infty dk k^2 \omega_k \alpha_k,
\end{equation}
can be computed numerically by solving Eq. (19), together with the background evolution equations. The resulting energy density must be compared with the energy density in SM particles at the end of the preheating stage, $\rho_0 \equiv \rho_0^\text{SM}$. This quantity can be easily related to the effective number of light degrees of freedom $N_{\text{eff}}$. In order to do that, let us note that, once produced, the dilaton particles are completely decoupled from the SM particles, being its energy density only diluted by the expansion of the Universe. $\rho_0^\text{SM} = \rho_0 a_0^2$. Here the subscripts ‘0’ and ‘$\tau$’ stand for the end of the preheating stage and the BBN epoch respectively. On the other hand, the total entropy of SM particles after thermalization remains constant, $s_{\text{SM}} a_0^3 = s_{\text{SM}} T_0^3$. Taking into account the relation between the entropy density and the number of relativistic degrees of freedom $g$, the previous expression can be rewritten as $g_0 T_0^3 a_0^3 = g_f T_f^3$. By combining this expression with the evolution equation for the dilaton energy density described above and dividing the result by the energy density stored in a single neutrino species, $\rho_\nu = \frac{3}{40} g_f T_f^3$, we get
\begin{equation}
\Delta N_{\text{eff}} \equiv \frac{(\rho_\chi / \rho_0)}{g_f / g_0} = \frac{g_0}{g_f} \left( \frac{g_f}{g_0} \right)^{4/3} C \simeq 2.85 C.
\end{equation}
In the last equality we have made use of the number of SM degrees of freedom at the end of inflation ($g_0 = 106.75$) and at BBN ($g_f = 10.75$). Therefore, we see that, in order to have a contribution to the effective number of relativistic degrees of freedom within the reach of the Planck satellite, roughly a 10% of the energy density at the end of inflation must be converted into dilatons. Nevertheless, the transferred fraction turns out to be significantly smaller. Evaluating the numerical solution of Eq. (19) at the time at which the energy density in SM particles roughly equals the initial energy density of the inflaton field, $\tau (\tau_0 \sim 300 M_{\text{Pl}}^2)$ [27], we get $C \sim 10^{-7}$. The precise value of the $C$ parameter weakly depends on the ratio $\xi_n / \sqrt{\zeta}$, which determines the total energy density available at the end of inflation [23], and it is quite insensitive to the particular value of the small non-minimal coupling $\xi_n$.

Although the non-perturbative creation of dilatons due to the background field $\phi$ turns out to be extremely small, one should consider the possibility of producing them as secondary products of the Higgs particles created at the preheating and thermalization stages. The Lagrangian (13) leads to a number of perturbative processes, such as the decay of Higgs particles into dilatons ($\phi \to \rho \rho$) or Higgs–Higgs scatterings ($\phi \phi \to \rho \rho$). The corresponding decay rate and cross-section are given respectively by
\begin{equation}
\Gamma \simeq \frac{M_{\text{HD}}^3}{192 \pi M_P^4} \quad \text{and} \quad \sigma \simeq \frac{E^2}{576 \pi M_P^4}.
\end{equation}
where $M_{\text{HD}}$ is the effective Higgs mass in the region $M_P / \sqrt{\lambda} < \phi \ll M_P$ (cf. Eq. (16)) and $E$ is the Higgs energy in the center-of-mass frame. Assuming this energy to be of the order of the temperature of the thermalized SM plasma, $T_0$, it can be easily seen that the contribution of these processes to the $C$ parameter is of order
\begin{equation}
C \propto \left( \frac{T_0}{M_P} \right)^3,
\end{equation}
and therefore much smaller than the non-perturbative contribution found above.

5. Conclusions

We have considered particle production in a scale-invariant extension of Higgs inflation known as Higgs–Dilaton inflation. This model predicts the existence of an extra massless particle – the dilaton – which might contribute to the effective number of light degrees of freedom. After recasting the problem in the appropriate set of variables, all the particle masses and energy scales in the model turn out to coincide, up to small corrections, with those of Higgs inflation. Gauge bosons and fermions are therefore produced through the so-called Combined Preheating mechanism. On the other hand, the production of dilatons could take place only through the non-canonical kinetic term for the dilaton field, fed by the absolute value of the Higgs field. We have shown that the number of non-perturbatively created particles can be easily evaluated thanks to a particular time redefinition. The dilaton energy density computed this way turns out to be extremely small, which is translated into an effective number of relativistic degrees

\textsuperscript{5} The coupled Higgs–Dilaton equations further simplify since the Higgs fluctuations are not significantly amplified [27], and thus can be treated as decoupled equations.

\textsuperscript{6} This corresponds to the initial vacuum conditions $\delta \rho_k(0) = 1 / \sqrt{2 \pi a_0}$ and $\delta \rho'_k(0) = - \log \delta \rho_k$. 
of freedom very close to the SM one. This result is not modified by any of the subsequent perturbative processes involving the Higgs particles produced at the bottom of the potential. We thus conclude that, in spite of the potential value of BBN and CMB for testing the existence of the elusive dilaton particle, its subdominant production at the (p)reheating stage after inflation makes it completely undetectable by any particle physics experiment or cosmological observation. The only remnant of the dilaton field is a dynamical Dark Energy stage with an equation of state close, but slightly different, to that of a cosmological constant, which leads to a power-like expansion of the Universe in the far future [22].

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