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An integrated interval-valued intuitionistic fuzzy technique for resumption risk assessment amid COVID-19 prevention

Ze-hui Chen a,b, Shu-ping Wan a, Jiu-ying Dong c,*

a School of Information and Management, Jiangxi University of Finance and Economics, Nanchang 330013, China
b School of Management, Jiujiang University, Jiujiang 332005, China
c School of Science, Shanghai Institute of Technology, Shanghai 201418, China

Abstract
Currently, China has achieved a remarkable achievement on the containment of COVID-19, which creates a favorable condition for the gradual resumption of normal life. However, COVID-19 infections continue to rise in many nations and some sporadic cases occur from time to time in China, which still poses some risks to the resumption. Hence, it is imperative to develop some reasonable techniques to assess the resumption risk. This paper aims to investigate an integrated interval-valued intuitionistic fuzzy (IVIF) technique to adroitly assess the resumption risk based on DEMATEL (decision making trial and evaluation laboratory), BWM (best-worst method) and SPA (set pair analysis). This integrated technique is called IVIF-DBWM-SPA, where the IVIF-DBWM (combined by the IVIF-DEMATEL and IVIF-BWM) is used to determine the global criteria weights and the IVIF-SPA is employed to generate the ranking order of the alternatives. The IVIF-DEMATEL and IVIF-BWM are used to determine the weights of dimensions and the weights of criteria under each dimension, respectively. In this IVIF-BWM, two bi-objective programming models are constructed by regarding experts’ pessimistic and optimistic attitudes, respectively. Combined experts’ intrapersonal and interpersonal uncertainties simultaneously, a bi-objective programming model is proposed to derive the dynamic weights of experts. Based on the determined weights of experts and criteria, an IVIF-SPA is developed to assess the risk levels of all alternatives. The validity of the proposed technique is demonstrated with a real case of college resumption risk assessment amid COVID-19. Some sensitivity and comparison analyses are provided to show the merits of the proposed technique.

1. Introduction
An abrupt corona-virus disease (COVID-19) invaded our planet in late 2019, bringing a range of unprecedented challenges to the world [25]. Up till now, the human activities have been severely affected by COVID-19. It has discovered that COVID-19 has two main features: long-term disruption and increasing propagation [15]. As of 31 October 2022, COVID-19 pandemic has caused more than 635.32 million global infected cases, including over 6.59 million deaths (see https://www.worldometers.info/coronavirus). Unfortunately, China was one of the serious victims affected by COVID-19. After the intrusion of COVID-19, Chinese government has taken a series of emergency measures, e.g., city lockdown, enterprise closedown and school suspension, etc., to beat back this intruder since April 2020, China has achieved a remarkable achievement on the
containment of COVID-19, which creates a favorable condition for the gradual resumption of normal life. However, since COVID-19 infections continue to rise in many nations and some sporadic cases occur from time to time in China, which surely increases the risk of COVID-19 cluster infection and poses great challenges to school resumption. Hence, it is necessary to make a concentrated effort to assess the school resumption risk.

Generally, emergency management involves four phases: strategic mitigation, tactical preparedness, operational response and long-term recovery, where the first two phases happen before a disaster and the rests happen after a disaster [10]. Currently, to abate the negative impacts of COVID-19, the problems triggered by this pandemic have been studied by numerous scholars. For example, Mishra et al. [25] extended a hesitant fuzzy method for the drug selection to treat the mild symptoms of COVID-19. Ghosh & Biswas [14] used a fuzzy inference system to develop a technique to evaluate the status of states and provinces affected by COVID-19. Govindan et al. [15] designed a practical decision support system for demand management in the healthcare supply chain. Ahmad et al. [1] proposed an uncertain decision-making model for sustainable production and waste management of COVID-19 medical equipment. Sharma et al. [30] built a meditative fuzzy logic mathematical model for the contradictory management amid COVID-19. Geetha et al. [13] put forward a fuzzy case-based technique to evaluate the admission priority of COVID-19 patients. To some extent, the above-said achievements have made some considerable contributions to lessen the negative impacts caused by COVID-19.

In fact, when COVID-19 is under control in some districts, the fourth phase of emergency management, i.e., long-term recovery, also must be valued. However, only few progresses on recovery management amid COVID-19 have been reported. For instance, Zhang et al. [46] integrated econometric and judgmental techniques to forecast the possible ways to tourism recovery in Hong Kong. Ouyang et al. [27] constructed a novel framework of collaborative early warning for COVID-19 based on blockchain and smart contracts. In order to orderly recover the social economy and daily life, Chinese government has issued a series of macro policies (e.g., guidance on resumption of work and guidance on school resumption) since April 2020. The aforementioned progresses and macro policies only provide the macro-perspective recovery guidance, they are hard to be applied to fully guide the tangible recovery activities. Obviously, school resumption is one of the important contents of recovering normal daily life. Besides, it has previously mentioned that risk assessment is essential for school resumption. Hence, it is necessary to develop some practical micro methods to assess school resumption risk.

1.1. Risk assessment framework

Generally, a holistic risk assessment framework involves the following five phases [22].

**Phase I: preliminary preparation.** In this phase, the responsible evaluator (e.g., government department) needs to form an expert group by inviting several experts from different fields. The invited experts identify the criteria (i.e., risk factors) under different dimensions for the evaluated objects (alternatives). Then, these experts give their risk evaluations on the alternatives under the selected criteria. From this perspective, the risk assessment can surely be regarded as a multi-criteria group decision making (MCGDM) problem.

**Phase II: information processing.** Due to the growing complexity of the decision environments, MCGDM is inevitably plagued by much imprecise and vague decision information. As such, numeric data is inadequate to delineate the uncertain information of real-life decision-making problems. Compared with numeric data, experts might be more inclined to use linguistic terms (e.g., low, medium, high, etc.) to express their opinions. Currently, Zadeh’s fuzzy sets theory [44] has become an extremely-used medium to handle linguistic information. However, Zadeh’s fuzzy sets only considered the membership degree, which might result in the distortion and loss of initial information. To make up this deficiency, Atanassov [2] proposed the intuitionistic fuzzy set (IFS) to express expert’s uncertain assessments by using the membership and non-membership degrees in the form of numeric values. In reality, it is difficult for experts to describe the membership and non-membership degrees with numeric values due to the lack of expertise or time, or both. Compared with IFS, interval-valued intuitionistic fuzzy set (IVIFS) [3,6] has greater freedom and flexibility to deliver expert’s uncertainty since its membership and non-membership degrees are represented by intervals. Hence, the IVIFSs are introduced to accurately express expert’s uncertainty in MCGDM. Based on this, this paper intends to encode the linguistic information into IVIFSs.

**Phase III: interdependent effect analysis.** Due to the large quantity of the selected criteria, it is hard for experts to analyze the interdependent effects between any two criteria. To this end, the interdependent effects of criteria are totally revealed by the relationship of dimensions. Currently, Choquet integral [4] and DEMATEL (decision making trial and evaluation laboratory) [11] are two efficient tools to depict the interdependent effects of dimensions. Owing to the easy enforceability of DEMATEL, this paper intends to extend DEMATEL into interval-valued intuitionistic fuzzy (IVIF) environment to analyze the effects among dimensions.

**Phase IV: risk aggregation.** In this stage, the following two issues are required to be solved.

1. Determine the weights of experts and criteria. Generally, experts’ weights can be determined based on the following two principles: (i) the lower the expert’s intrapersonal uncertainty (i.e., the expert gives lower uncertain ratings), the larger his/her weight [7]; (ii) the lower the expert’s interpersonal uncertainty (i.e., expert’s ratings are closer to the group ratings), the larger his/her weight [5]. However, Wan et al. [34] only considered expert’s interpersonal uncertainty when determining experts’ weights. Gupta et al. [16] gave the experts’ weights in advance. Deveci et al. [9] derived experts’ weights based on experts’ self-evaluated matrix. Although Li et al. [20] pondered the above two principles, the determination of experts’ weights needs several calculation steps, which unavoidably increases the
computational burden. To cover the limitations appeared in methods \cite{9,16,20,34,39}, this paper constructs a bi-objective programming model to derive experts' weights. Currently, analytic hierarchy process (AHP) \cite{31}, analytic network process (ANP) \cite{20} and best-worst method (BWM) \cite{29} are three frequently-used methods to determine criteria weights. In particular, BWM is superior to AHP and ANP since it requires fewer pairwise comparisons and receives higher reliable results \cite{29}. Given the outstanding advantages of BWM, this paper intends to use BWM to determine the weights of criteria.

(2) Assess risk. Based on the weights of criteria and experts along with experts' evaluations of alternatives, the risks of the alternatives can be synthetically assessed by using some effective methods, such as FMEA (failure mode and effects analysis) \cite{39}, fuzzy AHP \cite{31}, fuzzy TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) \cite{36}, fuzzy reasoning method \cite{28}, TODIM (Portuguese acronym for interactive multi-criteria decision making) \cite{23}, DEA (data envelopment analysis) \cite{18}, and SPA (set pair analysis) \cite{19,48}, etc. Zhao \cite{48} pioneered SPA to handle the fuzzy information by making the coordinates framework of certainty and uncertainty. SPA can manage the interaction between the certainty and uncertainty in a system through identity, discrepancy, and contrary. In retrospect, SPA has been successfully applied in various fields, e.g., career determination \cite{12}, water quality assessment \cite{24}, project assessment \cite{42}, etc.

**Phase V: risk grade.** It has analyzed in Phase II that the decision information is represented by IVIFSs. Hence, it is necessary to develop an IVIF version of SPA (denoted by IVIF-SPA) to assess the resumption risk.

1.2. Research motivations of this paper

According to the aforesaid framework of risk assessment, it is easy to find that there exist three important problems that need to be solved.

(1) Obtain experts' weights. In group decision making problems, experts' weights are crucial to decision results. Currently, some progresses \cite{9,16,20,34} developed different techniques to derive experts' weights. These techniques show a common feature that the used experts' weights are immutable, which results in the inapplicability of the MCGDM methods. Specifically, the inapplicability of using immutable experts' weights is mainly reflected in the following two aspects: (i) it is hard to reduce the impact of the abnormal evaluations on the decision results; (ii) it is difficult to complete the risk assessment of different alternatives intermittently. Hence, the first research motivation of this paper is how to develop an approach to determining mutable/dynamic experts' weights.

(2) Obtain criteria weights. Obviously, criteria weights are the basis for comprehensive evaluation of alternatives in MCGDM. At present, BWM \cite{29} has become the most popular weight determining method. It has mentioned that the decision information is represented by IVIFVs in this paper. Hence, it is necessary to extend the classical BWM into IVIF environment. Although Wang et al. \cite{40} proposed an IVIF-BWM, there exist some limitations that are described as follows: (i) the weight-determining model is extended by directly fuzzifying the numeric elements of the classical BWM into IVIF elements, which neglects the consistency of the IVIF reference comparisons (RCs); (ii) the IVIF weights are normalized by normalizing their score functions, which might cause the information loss of IVIFVs; (iii) Wang et al. \cite{40} failed to discuss the consistency ratio (CR) of the IVIF RCs. Despite the classical BWM \cite{29} proposed a CR based on the numeric RCs, the consistent transitivity of numeric RCs is quite different from that of IVIF ones. Accordingly, the CR of Ref. \cite{29} cannot be directly used to check the consistency level of the IVIF RCs. Moreover, in some complex problems, the number of criteria is usually large. In such a case, it is hard for experts to provide the RCs for numerous pairwise criteria, which limits the application of BWM. Generally, the criteria can be divided into several classes/dimensions. DEMATEL \cite{11} has the strong power to depict the interdependent effects of dimensions. Thus, the second research motivation of this paper is how to develop an integrated approach to determining the criteria weights in the case of large number of criteria.

(3) Assess resumption risk and determine risk grade. Although SPA has been successfully used to solve numerous practical decision-making problems, it is seldom employed to assess the resumption risk amid disasters. Thus, it represents a new attempt to utilize SPA to assess the resumption risk. Furthermore, since the decision information is represented by IVIFVs, it is also a deserving exploration to extend the classical SPA into IVIF environment. Consequently, the third research motivation of this paper is how to develop an IVIF version of SPA (called IVIF-SPA hereafter) to assess college resumption risk and determine the resumption risk grades of the evaluated colleges.

1.3. Contributions of this paper

In order to complete the above motivations, this paper aims to develop a fuzzy integrated method to assess college resumption risk by combining DEMATEL, BWM and SPA in IVIF environment. This integrated method is called IVIF-DBWM-SPA for short. Firstly, the IVIF-DEMATEL is proposed to analyze the effects of dimensions and determine the weights of dimensions. Then, the IVIF-BWM is developed to obtain the weights of criteria under each dimension. Subsequently, the global weights of all criteria can be calculated based on the weights of dimensions and criteria under each dimension. Lastly,
the IVIF-SPA is put forward to assess the college resumption risk. Compared with the existing achievements, some contributions of this paper are outlined as follows:

(1) In order to realize the first research motivation, a bi-objective programming model is constructed to derive the dynamic weights of experts, where the two objectives aim to minimize the intrapersonal and interpersonal uncertainties mentioned in Phase IV, respectively. The benefits of using dynamic weights are described as follows:

(i) Reduce the influence of abnormal evaluations on the assessment results. For example, the evaluations of an alternative given by expert \( e_1 \) are remarkably smaller than those given by other experts. In such a case, the evaluations given by \( e_1 \) might be regarded to be abnormal. It is logical that the weight of \( e_1 \) should be lower than those of other experts, which can efficiently reduce the influence of abnormal evaluations on the assessment results. In addition, the abnormal evaluations have no influence on the assessment results of other alternatives since experts’ weights would be updated when another alternative is assessed.

(ii) Assess alternatives flexibly. In reality, the evaluation time slots of different alternatives are usually different. In such a situation, the entire evaluation process will not end until the last alternative is evaluated. This mode is apparently no match for intermittent assessment, especially for emergency risk assessment. Fortunately, this defect can perfectly be covered by using dynamic experts’ weights since the used weights are derived only based on the evaluations of the current evaluated alternative. However, Li et al. [20] determined experts’ weights based on the evaluations of all alternatives. Thus, the proposed expert weight-determining approach has stronger flexibility in assessing alternatives than that presented in [20].

(2) This paper extends BWM into IVIF environment (called IVIF-BWM) by fully regarding the consistency of the IVIF RCs. To obtain the IVIF weights of criteria, this paper originally builds two bi-objective programming models by regarding expert’s pessimistic and optimistic attitudes, respectively. For checking the consistency level of the IVIF RCs, this paper defines a CR measure for the IVIF-BWM based on the consistent IVIFVs. The proposed IVIF-BWM can cover the limitations of Wang’s et al. IVIF-BWM [40]. Then, an integrated approach (denoted by IVIF-DBWM) is developed to determine the global weights of all criteria by incorporating IVIF-DEMATEL and IVIF-BWM, where the IVIF-DEMATEL and IVIF-BWM are respectively used to determine the weights of dimensions and the weights of criteria under each dimension. Generally, since the number of criteria under each dimension would be remarkably less than the number of all criteria, it is easier for expert to give his/her pairwise comparisons for fewer criteria. Hence, compared with ANP [20], the proposed IVIF-DBWM is easier to be implemented. This fulfils the second research motivation.

(3) In order to complete the third research motivation, this paper proposes an IVIF-SPA to evaluate the risk grades of each alternative and dimension (criterion). Based on these results, the evaluator can conveniently discover the leaks of the evaluated alternatives in the resumption preparation. Moreover, the resumption risk can be reduced by fixing the traced leaks pertinently. However, the TOPSIS-based risk assessment method [36] only calculated the overall risk levels of the evaluated objects, which makes it impossible to track the leaks of the evaluated objects.

In conclusion, the theoretical contributions of the proposed IVIF-DBWM-SPA are mainly reflected in the following two aspects: (i) The proposed method extends the existing techniques (i.e., DEMATEL, BWM and SPA) into IVIF environment; (ii) Currently, there is no investigation to fuse the above techniques simultaneously, the proposed method completes the fusion of these techniques to assess college resumption risk.

The formation of this paper is decorated below. Section 2 introduces the concepts of interval-valued intuitionistic fuzzy preference relation (IVIFPR) and IVIFVs. Section 3 extends the classical BWM into IVIF environment. Section 4 completes the integration of IVIF-DEMATEL, IVIF-BWM and IVIF-SPA to assess college resumption risk. Section 5 conducts a real case on college resumption amid COVID-19 prevention. Section 6 terminates this paper with some remarkable conclusions and research prospects.

2. Preliminaries

This section introduces some basic concepts on IVIFPR to carry out the subsequent work of this paper. Note that \( X = \{x_1, x_2, \cdots, x_n\} \ (n \geq 3) \) is a set of objects.

**Definition 2.1** [33]. An IVFPR \( \tilde{R} \) is represented by a \( n \times n \)-order matrix \( \tilde{R} = (\tilde{r}_{ij})_{n \times n} \) with \( \tilde{r}_{ij} = [r_{ij}, \tilde{r}_{ij}] \), where \( r_{ij} \) and \( \tilde{r}_{ij} \) are non-negative real numbers, satisfying \( r_{ij} \leq \tilde{r}_{ij} \), \( r_{ij} + \tilde{r}_{ij} = r_{ji} + \tilde{r}_{ji} = 1 \) and \( r_{ij} = \tilde{r}_{ij} = 0.5 \) \((i, j = 1, 2, \cdots, n)\). \( \tilde{r}_{ij} \) denotes that \( x_i \) is between \( r_{ij} \) and \( \tilde{r}_{ij} \) times as important as \( x_j \).

**Definition 2.2** [35]. An IVFPR \( \tilde{R} = (\tilde{r}_{ij})_{n \times n} \) with \( \tilde{r}_{ij} = [r_{ij}, \tilde{r}_{ij}] \) is multiplicatively consistent, iff Eq. (1) holds.
According to Definition 2.1, it is easy to deduce that Eq. (1) can be rewritten as the following equation.

\[
\sqrt{\frac{\frac{E_k F_{jk}}{I_{ij} L_{ij}}}{(1 - r_k)(1 - r_k)}} = \sqrt{\frac{\frac{E_i F_{ij}}{I_{ij} L_{ij}}}{(1 - E_i)(1 - E_i)}} \times \sqrt{\frac{\frac{E_k F_{jk}}{I_{ij} L_{ij}}}{(1 - E_k)(1 - E_k)}} \\
(i, k, j = 1, 2, \ldots, n)
\]  

(2)

**Definition 2.3** [3]. An IVIFS \( \tilde{A} \) is defined as.

\[
\tilde{A} = \{ (x, \underline{u}_A(x), \overline{u}_A(x), [\underline{v}_A(x), \overline{v}_A(x)]) \mid x \in X \}
\]  

(3)

where \( [\underline{u}_A(x), \overline{u}_A(x)] \subset [0, 1] \) and \( [\underline{v}_A(x), \overline{v}_A(x)] \subset [0, 1] \) represent the interval-valued “membership degree” and “non-membership” degrees of \( x \) belonging to \( X \), respectively, and \( \underline{u}_A(x) + \overline{v}_A(x) \leq 1 \).

In order to facilitate the application of IVIFS, Xu [43] defined the interval-valued intuitionistic fuzzy value (IVIFV), which is formulated as \( \tilde{r} = ([\underline{u}, \overline{u}], [\underline{v}, \overline{v}]) \), where \( [\underline{u}, \overline{u}] \subset [0, 1] \), \( [\underline{v}, \overline{v}] \subset [0, 1] \) and \( \underline{u} + \overline{v} \leq 1 \).

**Definition 2.4** [43]. Let \( \tilde{r} = ([\underline{u}, \overline{u}], [\underline{v}, \overline{v}]) \) and \( \tilde{r}_k = ([\underline{u}_k, \overline{u}_k], [\underline{v}_k, \overline{v}_k]) \) \( (k = 1, 2) \) be three IVIFVs. Some operational laws of them are defined below.

1. **Addition:** \( \tilde{r}_1 + \tilde{r}_2 = ([\underline{u}_1 + \underline{u}_2 - \underline{u}_1 \underline{u}_2, \overline{u}_1 + \overline{u}_2 - \underline{u}_1 \overline{u}_2], [\underline{v}_1 \underline{v}_2, \overline{v}_1 \overline{v}_2]) \);
2. **Subtraction:** \( \tilde{r}_1 - \tilde{r}_2 = ([\underline{u}_1 - \underline{u}_2 + \underline{u}_1 \underline{u}_2, \overline{u}_1 - \overline{u}_2 + \underline{u}_1 \overline{u}_2], [\underline{v}_1 \underline{v}_2, \overline{v}_1 \overline{v}_2]) \);
3. **Multiplication:** \( \tilde{r}_1 \tilde{r}_2 = ([\underline{u}_1 + \underline{u}_2 - \underline{u}_1 \underline{u}_2, \overline{u}_1 + \overline{u}_2 - \underline{u}_1 \overline{u}_2], [\underline{v}_1 \underline{v}_2, \overline{v}_1 \overline{v}_2]) \);
4. **Division:** \( \frac{\tilde{r}_1}{\tilde{r}_2} = ([\underline{u}_1 + \underline{u}_2 - \underline{u}_1 \underline{u}_2, \overline{u}_1 + \overline{u}_2 - \underline{u}_1 \overline{u}_2], [\underline{v}_1 \underline{v}_2, \overline{v}_1 \overline{v}_2]) \);
5. **Scalar multiplication:** \( \lambda \tilde{r} = ([1 - (1 - \underline{u})^\lambda, 1 - (1 - \overline{u})], [\underline{v}^\lambda, \overline{v}]) \);
6. **Complement:** \( \tilde{r}^c = ([\underline{v}, \overline{v}], [\underline{u}, \overline{u}]) \).

An interval weight vector \( \tilde{w} = (w_i \underline{w}_i, \ldots, w_n \underline{w}_n)^T \) with \( \tilde{w} = [w_i, w_n] \) is normalized iff it holds that \( 0 \leq w_i \leq w_n \leq 1 \), \( w_i + \sum_{j=1, j \neq i}^n w_j \geq 1 \) and \( \sum_{j=1, j \neq i}^n w_j \leq 1 \) for all \( i = 1, 2, \ldots, n \) [37].

**Definition 2.5** [38]. An IVFPR \( \tilde{R} = \frac{\tilde{r}}{y} \) with \( \tilde{r} = \frac{E_i}{F_{ij}} \) is multiplicatively consistent if there exists a normalized interval priority weight vector \( \tilde{w} = (w_i \underline{w}_i, \ldots, w_n \underline{w}_n)^T \) such that.

\[
\tilde{r}_{ij} = \left[ \frac{w_i}{w_i + w_j}, \frac{w_j}{w_i + w_j} \right], \quad \text{if} \quad i = j
\]

\[
\tilde{r}_{ij} = \left[ \frac{w_i}{w_i + w_j}, \frac{w_j}{w_i + w_j} \right], \quad \text{if} \quad i \neq j
\]

(4)

**Definition 2.6** [45]. Given a judgment matrix \( \tilde{R} = \frac{\tilde{r}}{y}_{n \times n} \subset X \times X \), where \( \tilde{r}_{ij} = ([\underline{u}_j, \overline{u}_j], [\underline{v}_j, \overline{v}_j]) \) is an IVIFV, \( [\underline{u}_j, \overline{u}_j] \subset [0, 1] \) and \( [\underline{v}_j, \overline{v}_j] \subset [0, 1] \) represent the degrees to which \( x_i \) is preferred and non-preferred over \( x_j \), respectively. \( \tilde{R} \) is called an IVFPR if for all \( i, j = 1, 2, \ldots, n \), it holds that: \( \underline{u}_j + \overline{v}_j \leq 1 \), \( [\underline{u}_j, \overline{u}_j] = [\underline{v}_j, \overline{v}_j] \), \( [\underline{u}_j, \overline{u}_j] = [\underline{v}_j, \overline{v}_j] \) and \( [\underline{u}_j, \overline{u}_j] = [\underline{v}_j, \overline{v}_j] = [0, 0.5, 0.5] \).

In order to define the multiplicatively consistent IVFPR, Wan et al. [33] extracted an IVFPR \( \tilde{R} = \frac{\tilde{r}}{y}_{n \times n} \) from the IVFPR \( \tilde{R} = \frac{E_i}{F_{ij}}_{n \times n} \) by the following treatment.

\[
\tilde{r}_{ij} = \left[ \frac{E_i + \alpha_j(1 - \underline{v}_j - \underline{u}_y), \underline{u}_j + \alpha_j(1 - \underline{v}_y - \underline{u}_j)]}{\frac{1}{2}}, \quad \text{if} \quad i < j
\]

\[
\tilde{r}_{ij} = \left[ \frac{0.5, 0.5, \overline{u}_j + \alpha_j(1 - \underline{v}_j - \underline{u}_y)], \underline{u}_j + \alpha_j(1 - \underline{v}_y - \underline{u}_j)]}{\frac{1}{2}}, \quad \text{if} \quad i = j
\]

\[
\tilde{r}_{ij} = \left[ \frac{[1 - r_{ij}, 1 - \underline{r}_{ij}], \overline{u}_j + \alpha_j(1 - \underline{v}_j - \underline{u}_y)]}{\frac{1}{2}}, \quad \text{if} \quad i > j
\]

(5)
where $x_i \in [0, 1]$ represents expert's risk preference. In particular, for $i < j$, $x_{ij} \in [0, 0.5)$, $x_{ij} = 0.5$ and $x_{ij} \in (0.5, 1]$ correspond to the pessimistic, neutral and optimistic attitudes of an expert, respectively.

**Definition 2.7** [33]. An IVIFPR $\tilde{R} = (\tilde{R}_{ij})_{n \times n}$ with $\tilde{r}_{ij} = (\tilde{u}_{ij}, \tilde{v}_{ij}, \tilde{w}_{ij})$ is multiplicatively consistent if there exists a multiplicatively consistent IVFPR $R = (r_{ij})_{n \times n}$ extracted from by Eq. (5).

**Theorem 2.1** [33]. Let $\tilde{R} = (\tilde{r}_{ij})_{n \times n}$ be an IVIFPR extracted from IVIFPR $R = (r_{ij})_{n \times n}$. $\tilde{R}$ is multiplicatively consistent if $\tilde{R}$ is multiplicatively consistent following **Definition 2.5**.

Motivated by the similarity function of intuitionistic fuzzy value [47], this paper defines a new similarity function of IVIFV.

**Definition 2.8.** Let $\tilde{r} = (\tilde{u}, \tilde{v}, \tilde{w})$ be an IVIFV. The similarity function of $\tilde{r}$ is formulated as

$$L(\tilde{r}) = \frac{(1 - \tilde{v}) + (1 - \tilde{w})}{(1 - \tilde{v}) + (1 - \tilde{u}) + (1 - \tilde{v}) + (1 - \tilde{u})}$$

(6)

**Remark 1.** If $u = \tilde{u} = u$ and $v = \tilde{v} = v$, then $\tilde{r}$ will reduce to an IFV $\tilde{r} = (u, v)$. In this case, Eq. (6) can be rewritten as $L(\tilde{r}) = \frac{1}{(1 - u) + (1 - v) + (1 - u) + (1 - v)}$, which is exactly the similarity function of intuitionistic fuzzy value defined in [47]. Hence, the similarity function of IVIFV proposed by this paper can generalize that of intuitionistic fuzzy value, which justifies the validity of **Definition 2.8**.

3. Extend BWM into IVIF environment

This section extends the classical BWM into IVIF environment. For convenience, the extended BWM is called IVIF-BWM. Generally, there exist three issues that need to be solved in BWM [29]: (i) Identify the reference criteria (i.e., the best and worst criteria); (ii) Determine the optimal criteria weights; (iii) Calculate the CR of the used RCs.

3.1. Identify the best and worst criteria

Each expert identifies the best criterion $c_b$ and the worst criterion $c_w$ from a set of criteria $c = \{c_1, c_2, \ldots, c_n\}$. After identifying $c_b$ and $c_w$, each expert needs to give the RCs of $c_b$ over other criteria and other criteria over $c_w$. Due to the ambiguous of decision environment and the hesitancy of experts, it is more suitable to use IVIFVs (see Table 1) to voice experts’ RCs.

Accordingly, the RC of $c_b$ over $c_j$ can be represented by an IVIFV $\tilde{s}_{bj} = ([s_{bj}^u, s_{bj}^v], [s_{bj}^w, s_{bj}^w])$. The Best-to-Others vector (BOV) can be obtained as follows:

$$\tilde{s}_b = [\tilde{s}_{b1}, \tilde{s}_{b2}, \ldots, \tilde{s}_{bn}]$$

(7)

Similarly, the Others-to-Worst vector (OWV) can be acquired as follows:

$$\tilde{s}_w = [\tilde{s}_{w1}, \tilde{s}_{w2}, \ldots, \tilde{s}_{wn}]^T$$

(8)

where $\tilde{s}_{bw} = ([s_{bw}^u, s_{bw}^v], [s_{bw}^w, s_{bw}^w])$ denotes the IVIF RC of $c_j$ over $c_w$.

**Table 1**

| Pairwise linguistic term | Corresponding comparison IVFV |
|--------------------------|-------------------------------|
| Equal importance (EI)   | ([0.5, 0.5], [0.5, 0.5])    |
| Weak importance (WI)    | ([0.55, 0.6], [0.3, 0.35]) |
| Moderate importance (MI)| ([0.6, 0.65], [0.25, 0.3]) |
| Moderate plus importance (MPI)| ([0.65, 0.7], [0.2, 0.25]) |
| Strong importance (SI)  | ([0.7, 0.75], [0.15, 0.2]) |
| Strong plus importance (SPI)| ([0.75, 0.8], [0.1, 0.15]) |
| Very strong importance (VSI)| ([0.8, 0.85], [0.05, 0.1]) |
| Extreme importance (EXI)| ([0.85, 0.9], [0.05, 0.1]) |
| Absolute importance (AI) | ([0.9, 0.9], [0.1, 0.1]) |
Remark 2. In order to ensure that \( \tilde{s}_{bj} \) and \( \tilde{s}_{jw} \) are the upper triangular elements, the subscripts of the criteria must be renumbered such that \( c_1 \) and \( c_n \) are the best and worst criteria, respectively. Besides, since the best criterion \( c_b \) is better than other criterion \( c_i \) and other criterion \( c_j \) is better than the worst criterion \( c_w \), it is reasonable to stipulate that \( 0.5 \leq \tilde{s}^u_{bj} \leq \tilde{s}^u_j \), \( \tilde{s}^f_{bj} \leq \tilde{s}^f_j \leq 0.5 \), \( 0.5 \leq \tilde{s}^u_{jw} \leq \tilde{s}^u_j \) and \( \tilde{s}^f_{jw} \leq \tilde{s}^f_j \leq 0.5 \).

3.2. Determine the optimal weights of the criteria

This section devotes to developing an IVIF weight-determining model based on IVIF decision information.

3.2.1. Irrationality of directly fuzzify BWM to IVIF fuzzy environment

If the weight-determining model (6) of BWM [29] is directly extended into IVIF environment, then an IVIF programming model similar to model (6) can be formulated as follows:

\[
\begin{align*}
\min \, & \max \left\{ \frac{w_j / w_i - \tilde{s}_{bj}}{w_j / w_i - \tilde{s}_{jw}} \right\} \\
s.t. \quad & 0 \leq w_i^u \leq w_i^u \leq 1 \quad (j = 1, 2, \ldots, n) \\
& 0 \leq w_i^v \leq w_i^v \leq 1 \quad (j = 1, 2, \ldots, n) \\
& w_i^u + w_i^v \leq 1 \quad (j = 1, 2, \ldots, n)
\end{align*}
\]

(M1)

where \( w_j = (w_j^u, w_j^v, w_j^f) \) denotes the IVIF weight of criterion \( c_j \) and 1 is an IVIF close to the real number 1.

Although it is easy to extend Rezaei’s model (6) [29] into model M1, model M1 is not suitable to derive the IVIF weights of criteria. The main reason is detailed as follows:

According to Theorem 2.1 and Definitions 2.5–2.6, it follows that if for all \( j = 1, 2, \ldots, n \), Eqs. (9) and (10) hold, then \( \tilde{s}_{bj} \) and \( \tilde{s}_{jw} \) (\( j = 1, 2, \ldots, n \)) are fully consistent.

\[
\tilde{s}_{bj} = [s_{bj}, \tilde{s}_{bj}] = [s_{bj}^u + \alpha_{bj}(1 - s_{bj}^u - s_{bj}^f), s_{bj}^u + \alpha_{bj}(1 - s_{bj}^u - s_{bj}^f)] = \begin{cases} [0.5, 0.5], & \text{if } i = j \\ \frac{w_i}{w_i + w_j}, \frac{w_j}{w_i + w_j}, & \text{if } b \neq j \end{cases}
\]

(9)

and

\[
\tilde{s}_{jw} = [s_{jw}, \tilde{s}_{jw}] = [s_{jw}^u + \alpha_{jw}(1 - s_{jw}^u - s_{jw}^f), s_{jw}^u + \alpha_{jw}(1 - s_{jw}^u - s_{jw}^f)] = \begin{cases} [0.5, 0.5], & \text{if } i = j \\ \frac{w_i}{w_i + w_j}, \frac{w_j}{w_i + w_j}, & \text{if } j \neq w \end{cases}
\]

(10)

From the objective of model M1, it is easy to know that if \( \tilde{w}_b / \tilde{w}_j = \tilde{s}_{bj} \) and \( \tilde{w}_j / \tilde{w}_w = \tilde{s}_{jw} \) (\( j = 1, 2, \ldots, n \)) hold, then the optimal objective value of model M1 is 0. In such a case, the IVIF RCs \( \tilde{s}_{bj} \) and \( \tilde{s}_{jw} \) (\( j = 1, 2, \ldots, n \)) are fully consistent. However, it has mentioned that \( \tilde{s}_{bj} \) and \( \tilde{s}_{jw} \) (\( j = 1, 2, \ldots, n \)) are fully consistent if Eqs. (9) and (10) hold. It is obvious that \( \tilde{w}_b / \tilde{w}_j = \tilde{s}_{bj} \) is nonequivalent to Eq. (9) and \( \tilde{w}_j / \tilde{w}_w = \tilde{s}_{jw} \) is nonequivalent to Eq. (10). Hence, it is illogical to infer that the IVIF RCs \( \tilde{s}_{bj} \) and \( \tilde{s}_{jw} \) (\( j = 1, 2, \ldots, n \)) are fully consistent from the conditions \( \tilde{w}_b / \tilde{w}_j = \tilde{s}_{bj} \) and \( \tilde{w}_j / \tilde{w}_w = \tilde{s}_{jw} \) (\( j = 1, 2, \ldots, n \)). As such, it is unreasonable to use model M1 to derive the optimal IVIF weights.

3.2.2. Construct the weight-determining model

To cover the above flaw, it is necessary to develop a reasonable programming model to determine the optimal IVIF weight vector. It has been analyzed that \( \tilde{s}_{bj} \) and \( \tilde{s}_{jw} \) (\( j = 1, 2, \ldots, n \)) are fully consistent if Eqs. (9) and (10) hold. However, it is difficult to attain Eqs. (9) and (10) for all \( j = 1, 2, \ldots, n \). Hence, it is expected to find the interval weights that meet Eqs. (11) and (12) as much as possible.

\[
\tilde{s}^u_{bj} + \alpha_{bj}(1 - \tilde{s}^f_{bj} - \tilde{s}^u_{bj}) = \frac{w_i}{w_i + w_j}, \quad \text{and} \quad \tilde{s}^u_{jw} + \alpha_{jw}(1 - \tilde{s}^f_{jw} - \tilde{s}^u_{jw}) = \frac{w_j}{w_j + w_w}
\]

(11)

\[
\tilde{s}^u_{bj} + \alpha_{bj}(1 - \tilde{s}^f_{bj} - \tilde{s}^u_{bj}) = \frac{w_i}{w_i + w_j}, \quad \text{and} \quad \tilde{s}^u_{jw} + \alpha_{jw}(1 - \tilde{s}^f_{jw} - \tilde{s}^u_{jw}) = \frac{w_j}{w_j + w_w}
\]

(12)

where \( \alpha_{bj}, \alpha_{jw} \in [0, 1] \) represent expert’s risk preference.
Eqs. (11) and (13) can be rewritten as Eqs. (13) and (14), respectively.

\[
\sum_{ij} x_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b \quad \text{and} \quad \sum_{ij} \bar{x}_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b
\]

(13)

\[
\sum_{ij} x_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b \quad \text{and} \quad \sum_{ij} \bar{x}_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b
\]

(14)

In real-life decision making, it is difficult to guarantee Eqs. (13) and (14) hold. One solution to this difficulty is to introduce some deviation variables into Eqs. (13) and (14). Then, Eqs. (13) and (14) can be reformulated as Eqs. (15) and (16), respectively.

\[
\sum_{ij} x_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b \quad \text{and} \quad \sum_{ij} \bar{x}_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b
\]

(15)

\[
\sum_{ij} x_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b \quad \text{and} \quad \sum_{ij} \bar{x}_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b
\]

(16)

where \( \bar{d}_b, b_j, \bar{d}_j, \) and \( \bar{d}_j (j = 1, 2, \ldots, n) \) are deviation variables.

Considering that Eqs. (15) and (16) involve parameters \( x_{ij} \) and \( \bar{x}_{ij} \) which are used to characterize expert’s risk preference and motivated by method [33], two extreme situations (i.e., the most optimistic and pessimistic situations) are simultaneously discussed to derive the IVIF weights of criteria.

(1) Optimistic model

\[
\begin{align*}
\min \ & \max_j \{ |d_{bj}|, |\bar{d}_j|, |d_{jw}|, |\bar{d}_{jw}| \} \\
\max \ & \min_j \{ x_{ij}, \bar{x}_{ij} \} \\
\text{s.t.} \ & \sum_{ij} x_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b \quad (j = 1, 2, \ldots, n) \\
& \sum_{ij} \bar{x}_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b \quad (j = 1, 2, \ldots, n) \\
& \sum_{ij} x_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b \quad (j = 1, 2, \ldots, n) \\
& \sum_{ij} \bar{x}_{ij} (1 - \bar{s}_{ij}^u - \bar{s}_{ij}^l) \bigm| \bar{w}_b + b_j \bigm| + \bar{d}_b = \bar{w}_b \quad (j = 1, 2, \ldots, n) \\
& 0 \leq w_j \leq w_j + \frac{1}{n} \sum_{i=1}^{n} w_i \geq 1, \quad w_j + \frac{1}{n} \sum_{i=1}^{n} w_i \leq 1 \quad (j = 1, 2, \ldots, n) \\
& x_{ij}, \bar{x}_{ij} \in [0, 1] \quad (j = 1, 2, \ldots, n)
\end{align*}
\]

The first objective seeks to find the interval weights that meet Eqs. (13) and (14) as much as possible. The second objective tries to maximize the minimum risk preference of expert, which reflects expert’s optimistic attitude.

Let \( \max_j \{ |d_{bj}|, |\bar{d}_j|, |d_{jw}|, |\bar{d}_{jw}| \} = \beta^p \) and \( \min_j \{ x_{ij}, \bar{x}_{ij} \} = \eta^p \). Then, it follows that

\[
\begin{align*}
\min \ & \beta^p \\
\max \ & \eta^p \\
\text{s.t.} \ & |d_{bj}| \leq \beta^p, |\bar{d}_j| \leq \beta^p, |d_{jw}| \leq \beta^p, |\bar{d}_{jw}| \leq \beta^p \quad (j = 1, 2, \ldots, n) \\
& x_{ij} \geq \eta^p, \bar{x}_{ij} \geq \eta^p \quad (j = 1, 2, \ldots, n)
\end{align*}
\]

(17)

Other constraints are the same as those of model M2

Obviously, \( |d_{bj}|, |\bar{d}_j|, |d_{jw}|, |\bar{d}_{jw}|, x_{ij} \) and \( \bar{x}_{ij} \) are all dimensionless variables between 0 and 1. Thus, \( \beta^p \) and \( \eta^p \) are also dimensionless variables between 0 and 1. That is to say, the maximum and minimum values of \( \beta^p \) (\( \eta^p \)) are 1 and 0, respectively. Denote \( \beta^p_{\min} = \eta^p_{\min} = 0 \) and \( \beta^p_{\max} = \eta^p_{\max} = 1 \). Then, \( \beta^p \) and \( \eta^p \) can be normalized as \( \beta = \frac{\beta^p - \beta^p_{\min}}{\beta^p_{\max} - \beta^p_{\min}} \) and \( \eta = \frac{\eta^p - \eta^p_{\min}}{\eta^p_{\max} - \eta^p_{\min}} \). Motivated by the multi-objective transformation approach [32], model M3 is finally converted into a single-objective programming model as follows:

\[
\min \beta \quad - \eta
\]

s.t. constraints are the same as those of model M3

where \( \beta \) and \( \eta \) have no specific meaning since \( \beta^p \) and \( \eta^p \) are dimensionless variables.

Solving model M4, the optimal objective values \( \beta^o \) and \( \eta^o \) along with the optimistic interval criteria weights \( w_j = [w_j, \bar{w}_j] \) (denoted by \( w^o_j = [\bar{w}_j^o, \bar{w}_j^o] \)) \( (j = 1, 2, \ldots, n) \) are acquired. In addition, the obtained values of \( x_{ij} \) and \( \bar{x}_{ij} \) are denoted by \( x^o_{ij} \) and
\[ \alpha_j^\mu (j = 1, 2, \ldots, n) \]. Particularly, since \( \beta^\mu \) is obtained by minimizing the maximum among the absolute deviations on both sides of Eqs. (13) and (14), \( \beta^\mu \) is expected to be used to define the CR of the IVIF-BWM.

(2) Pessimistic model.

Similarly, the pessimistic programming model can also be constructed as follows:

\[
\min \max \left\{ |d_{ij}|, |d_{ij}|, |d_{ij}|, |d_{ij}| \right\} \\
\text{s.t. constraints are the same as those of model M2}
\]

The first objective of model M5 has the same interpretation as that of model M2. The second objective seeks to minimize the maximum risk preference of expert, which reflects expert’s pessimistic attitude.

Let \( \min \{ |d_{ij}|, |d_{ij}|, |d_{ij}|, |d_{ij}| \} = \beta^\mu \) and \( \max \{ \alpha_j, \alpha_j \} = \eta^\mu \). By using the transformation of model M3 into model M4, model M5 can be converted into a single-objective programming model as follows:

\[
\min \beta^\mu + \eta^\mu \\
\text{s.t.} \left\{ \begin{array}{l}
|d_{ij}| \leq \beta^\mu, |d_{ij}| \leq \beta^\mu, |d_{ij}| \leq \beta^\mu, |d_{ij}| \leq \beta^\mu \quad (j = 1, 2, \ldots, n) \\
\alpha_j \leq \eta^\mu, \alpha_j \leq \eta^\mu \quad (j = 1, 2, \ldots, n) \\
\text{other constraints are the same as those of model M2}
\end{array} \right.
\]

where \( \beta^\mu + \eta^\mu \) has no specific meaning since \( \beta^\mu = \frac{\beta_{\min} - \beta_{\min}}{\beta_{\min} - \beta_{\min}} \) and \( \eta^\mu = \frac{\eta_{\max} - \eta_{\min}}{\eta_{\max} - \eta_{\min}} \) are dimensionless variables, \( \beta_{\min} = \eta_{\min} = 0 \) and \( \beta_{\max} = \eta_{\max} = 1 \).

Solving model M6, the optimal objective values \( \beta, \eta \) along with the pessimistic interval criteria weights \( \bar{w}_j = [\bar{w}_j, \bar{w}_j] \) (denoted by \( w_j^\mu = [w_j^\mu, w_j^\mu] \) \( (j = 1, 2, \ldots, n) \) are obtained. In addition, the obtained values of \( \alpha_j \) and \( \alpha_j \) are denoted by \( \alpha_j^\mu \) and \( \alpha_j^\mu \) \( (j = 1, 2, \ldots, n) \). Particularly, since \( \beta^\mu \) is obtained by minimizing the maximum among the absolute deviations on both sides of Eqs. (13) and (14), \( \beta^\mu \) is expected to be used to define the CR of the IVIF-BWM.

Based on the obtained \( w_j^\mu \) and \( w_j^\mu \), the IVIF weights \( \bar{w}_j = ([w_j^\mu, w_j^\mu], [w_j^\mu, w_j^\mu]) \) can be determined as \([33]\):

\[ w_j^\mu = \frac{1}{2} \min \{ w_j^\mu, w_j^\mu \}, \quad \bar{w}_j = \frac{1}{2} \max \{ w_j^\mu, w_j^\mu \}, \quad w_j^\mu = \frac{1}{2} (1 - \max \{ w_j^\mu, w_j^\mu \}) \quad \text{and} \quad w_j^\mu = \frac{1}{2} (1 - \max \{ w_j^\mu, w_j^\mu \}) \]

where \( \delta = \max \{ \max \{ w_j^\mu, w_j^\mu \} - \min \{ w_j^\mu, w_j^\mu \} + 1, 1 \} \) is used to ensure that the IVIF weights \( \bar{w}_j (j = 1, 2, \ldots, n) \) satisfy \( \bar{w}_j^u + \bar{w}_j^l \leq 1 \).

3.3. Calculate the CR

3.3.1. Determine consistency index

**Theorem 3.1** \([33]\). Let \( R = \left[ \begin{array}{c} \bar{w}_j \\ \end{array} \right]_{n \times n} \) be an IVFPR extracted from IVIFPR \( R = \left[ \begin{array}{c} \bar{w}_j \\ \end{array} \right]_{n \times n} \). \( R \) is multiplicatively consistent if \( R \) is multiplicatively consistent following Definition 2.2.

According to Theorem 3.1, it is easy to give the definition of fully consistent IVIF RCs.

**Definition 3.1.** All the IVIF RCs \( \bar{s}_{ij} \) and \( \bar{s}_{ij} \) \( (j = 1, 2, \ldots, n) \) are fully consistent if they meet the following equation:

\[
\sqrt{\frac{\bar{s}_{ij} \bar{s}_{ij}}{(1 - \bar{s}_{ij})(1 - \bar{s}_{ij})}} = \sqrt{\frac{\bar{s}_{ij} \bar{s}_{ij}}{(1 - \bar{s}_{ij})(1 - \bar{s}_{ij})}} = \sqrt{\frac{\bar{s}_{ij} \bar{s}_{ij}}{(1 - \bar{s}_{ij})(1 - \bar{s}_{ij})}} (18)
\]

where \( \bar{s} = [\bar{s}_j, \bar{s}_j] = [\bar{s}_j, \bar{s}_j] = [\bar{s}_j, \bar{s}_j] = [\bar{s}_j, \bar{s}_j] \) and \( \bar{s} = [\bar{s}_j, \bar{s}_j] \) (\( \forall i < j \)).

The IVIF weights \( \bar{w}_j \) \( (j = 1, 2, \ldots, n) \) are calculated based on the interval weights derived by optimistic model M4 and pessimistic model M6. Hence, it needs to find a synthetic CR to ensure the reliability of the IVIF weights \( \bar{w}_j \) \( (j = 1, 2, \ldots, n) \). Models M4 and M6 produce two sets of risk preferences, i.e., \( \alpha^\mu = \{ \alpha_j^\mu, \alpha_j^\mu \} \) \( j = 1, 2, \ldots, n \) \( \} \) and \( \alpha^\mu = \{ \alpha_j^\mu, \alpha_j^\mu \} \) \( j = 1, 2, \ldots, n \). According to Eqs. (9) and (10), \( \alpha^\mu \) and \( \alpha^\mu \) surely yield different values of \( \bar{s}_{ij}, \bar{s}_{ij}, \bar{s}_{ij}, \bar{s}_{ij} \). Similar to the idea of constructing the optimistic and pessimistic models, this paper defines the optimistic and pessimistic consistency indices as follows:
(1) Optimistic consistency index.

When \( \alpha_{ij} = \alpha_{ij}^{o} \) and \( \alpha_{uw} = \alpha_{uw}^{o} \), the obtained \( s_{ij} \) and \( s_{uw} \) are denoted by \( s_{ij}^{o} \) and \( s_{uw}^{o} \), respectively. From Definition 3.1, it holds that all the IVIF RCs \( s_{ij}^{o} \) and \( s_{uw}^{o} \) (\( j = 1, 2, \ldots, n \)) are fully consistent if

\[
\frac{s_{ij}^{o}}{(1-s_{ij}^{o})(1-s_{bw}^{o})} = \frac{s_{bw}^{o}}{(1-s_{bw}^{o})(1-s_{ij}^{o})} = \frac{s_{uw}^{o}}{(1-s_{uw}^{o})(1-s_{bw}^{o})} (j = 1, 2, \ldots, n)
\]

(19)

where \( s_{ij}^{o} = [s_{ij}^{u}, s_{ij}^{l}] = [s_{ij}^{l} + \alpha_{ij}^{o}(1-s_{ij}^{u} - s_{ij}^{l}), s_{ij}^{u} + \alpha_{ij}^{o}(1-s_{ij}^{l} - s_{ij}^{u})] \) and \( s_{uw}^{o} = [1-s_{uw}^{u}, 1-s_{uw}^{l}] \) (\( \forall i < j \)).

In practice, it is nearly impossible for experts to provide fully consistent RCs. Thus, there might exist some \( j \in \{1, 2, \ldots, n\} \) such that the following inequality appears.

\[
\frac{s_{bw}^{o}}{(1-s_{bw}^{o})(1-s_{ij}^{o})} \neq \frac{s_{bw}^{o}}{(1-s_{bw}^{o})(1-s_{ij}^{o})} = \frac{s_{bw}^{o}}{(1-s_{bw}^{o})(1-s_{bw}^{o})} (j = 1, 2, \ldots, n)
\]

(20)

For convenience, let \( S_{bw}^{o} = \frac{s_{bw}^{o}}{(1-s_{bw}^{o})(1-s_{bw}^{o})} \), \( S_{ij}^{o} = \frac{s_{ij}^{o}}{(1-s_{ij}^{o})(1-s_{ij}^{o})} \) and \( S_{uw}^{o} = \frac{s_{uw}^{o}}{(1-s_{uw}^{o})(1-s_{uw}^{o})} \). It is clear that \( S_{bw}^{o}, S_{ij}^{o} \) and \( S_{uw}^{o} \) are all known numeric values. Obviously, the highest inequality of Eq. (20) occurs when \( S_{bw}^{o} = S_{ij}^{o} = S_{uw}^{o} \), \( \zeta^o \) is a variable that is introduced into Eq. (20) such that the highest inequality of Eq. (20) achieves. Hence, it follows that

\[
\sqrt{S_{bw}^{o} + \zeta^o} = \sqrt{S_{ij}^{o} - \zeta^o} \sqrt{S_{bw}^{o} - \zeta^o} \tag{21}
\]

As for the maximum inconsistency \( \zeta_{max}^{o} = S_{ij}^{o} = S_{uw}^{o} \), one has

\[
\sqrt{S_{bw}^{o}} = \sqrt{S_{ij}^{o} - \zeta^o} \sqrt{S_{bw}^{o} - \zeta^o} \tag{22}
\]

Eq. (22) is equivalent to a quadratic equation as follows:

\[
(\zeta^o)^2 - (1 + 2S_{bw}^{o})\zeta^o + (S_{bw}^{o})^2 - S_{bw}^{o} = 0 \tag{23}
\]

Solving Eq. (23) with different values of \( S_{bw}^{o} \), the maximum possible \( \zeta^o \) (denoted by \( \zeta_{max}^{o} \)) can be obtained. \( \zeta_{max}^{o} \) is defined as the consistency index of the IVIF RCs \( \tilde{s}_{ij} \) and \( \tilde{s}_{uw} \) (\( j = 1, 2, \ldots, n \)) for optimistic situation. For simplicity, \( \zeta_{max}^{o} \) is called the optimistic consistency index hereafter.

(2) Pessimistic consistency index.

When \( \alpha_{ij} = \alpha_{ij}^{p} \) and \( \alpha_{uw} = \alpha_{uw}^{p} \), the obtained \( s_{ij} \) and \( s_{uw} \) are denoted by \( s_{ij}^{p} \) and \( s_{uw}^{p} \), respectively. From Definition 3.1, it holds that the IVIF RCs \( s_{ij}^{p} \) and \( s_{uw}^{p} \) (\( j = 1, 2, \ldots, n \)) are fully consistent if Eq. (24) holds.

\[
\frac{s_{bw}^{p}}{(1-s_{bw}^{p})(1-s_{bw}^{p})} = \frac{s_{bw}^{p}}{(1-s_{bw}^{p})(1-s_{bw}^{p})} = \frac{s_{bw}^{p}}{(1-s_{bw}^{p})(1-s_{bw}^{p})} (j = 1, 2, \ldots, n)
\]

(24)

where \( s_{ij}^{p} = [s_{ij}^{l}, s_{ij}^{u}] = [s_{ij}^{l} + \alpha_{ij}^{p}(1-s_{ij}^{u} - s_{ij}^{l}), s_{ij}^{u} + \alpha_{ij}^{p}(1-s_{ij}^{l} - s_{ij}^{u})] \) and \( s_{uw}^{p} = [1-s_{uw}^{u}, 1-s_{uw}^{l}] \) (\( \forall i < j \)).

Let \( S_{bw}^{p} = \frac{s_{bw}^{p}}{(1-s_{bw}^{p})(1-s_{bw}^{p})} \), \( S_{ij}^{p} = \frac{s_{ij}^{p}}{(1-s_{ij}^{p})(1-s_{ij}^{p})} \) and \( S_{uw}^{p} = \frac{s_{uw}^{p}}{(1-s_{uw}^{p})(1-s_{uw}^{p})} \). Similarly, for the maximum inconsistency of RCs \( \tilde{s}_{ij} \) and \( \tilde{s}_{uw} \) (\( j = 1, 2, \ldots, n \)), one has

\[
(\zeta^{p})^2 - (1 + 2S_{bw}^{p})\zeta^{p} + (S_{bw}^{p})^2 - S_{bw}^{p} = 0 \tag{25}
\]

Solving Eq. (25) with different values of \( S_{bw}^{p} \), the maximum possible \( \zeta^{p} \) (denoted by \( \zeta_{max}^{p} \)) can be obtained. \( \zeta_{max}^{p} \) is defined as the consistency index of the IVIF RCs \( \tilde{s}_{ij} \) and \( \tilde{s}_{uw} \) (\( j = 1, 2, \ldots, n \)) for pessimistic situation. For simplicity, \( \zeta_{max}^{p} \) is called the pessimistic consistency index hereafter.

3.3.2. Define the CR for the IVIF-BWM

Since the IVIF weights \( \tilde{w}_j = ([w_{ij}^{p}, w_{ij}^{l}], [w_{uw}^{p}, w_{uw}^{l}]) \; (j = 1, 2, \ldots, n) \) are determined based on the interval weights derived by the optimistic and pessimistic models, it is reasonable to find a CR that can ensure both the optimistic and pessimistic interval RCs are acceptably consistent. Based on this, the CR is defined as follows.
Definition 3.2. The CR of the proposed IVIF-BWM is defined as:

\[ CR = \max \{ CR_t, CR_{p} \} \]

where \( CR_t = \beta^o / \gamma^o \) and \( CR_p = \beta^p / \gamma^p \).

Theorem 3.2. If \( CR = 0 \), then all the IVIF RCs \( \tilde{s}_{ij} \) and \( \tilde{s}_{ji} (j = 1, 2, \ldots, n) \) are fully consistent.

Proof. For models M4 and M6, it is obvious that if the obtained optimal objective value \( \beta^o = \beta^p = 0 \), then all the optimistic and pessimistic interval RCs are fully consistent. In terms of Eq. (25), it holds that \( CR = 0 \) iff \( \beta^o = \beta^p = 0 \). By Eqs. (11) and (12), it follows that \( \beta^o = 0 \) derived by model M4 and \( \beta^p = 0 \) derived by model M6. Therefore, the objective values \( \beta^o = \beta^p = 0 \) is equivalent to \( CR = 0 \), which means that if \( CR = 0 \), then the optimistic and pessimistic interval RCs are fully consistent. As per Definition 3.1, all the IVIF RCs are fully consistent. \( \square \).

Remark 3. If \( CR \leq 0.1 \), then both the optimistic and pessimistic interval RCs are acceptable consistency, which further indicates that the IVIF RCs \( s_{ij} \) and \( s_{ji} (j = 1, 2, \ldots, n) \) are acceptable consistency.

4. An integrated technique for resumption risk assessment

This section proposes an integrated method to assess the resumption risk by combining IVIF-DEMATEL, IVIF-BWM and IVIF-SPA.

4.1. Determine dynamic experts’ weights

Generally, experts’ weights are determined based on their intrapersonal and interpersonal uncertainties [5,7]. In this paper, experts’ weights can be divided into two types. The first type is derived by the initial direct relationship matrix of dimensions and the second type is derived by the evaluation matrix of each evaluated object (alternative). Let \( w_{ij}^{1st} = (w_{11}^{1st}, w_{12}^{1st}, \ldots, w_{1n}^{1st}) \) be the first type weight vector that is derived by the initial direct relationship judgments of dimensions given by \( t \) experts. Let \( w_{ij}^{2nd} = (w_{11}^{2nd}, w_{12}^{2nd}, \ldots, w_{1n}^{2nd}) \) be the second type weight vector that is derived by the evaluations of the alternative \( \chi_r \) given by \( t \) experts. That is, \( w_{ij}^{2nd} \) is the weight vector of experts when they evaluate alternative \( \chi_r \). In this paper, the dynamicity of experts’ weights can be interpreted as “\( w_{ij}^{1st} \) and \( w_{ij}^{2nd} \) are used to aggregate experts’ evaluations of dimensions and alternative \( \chi_r \), respectively”. Next, only the determination of \( w_{ij}^{1st} \) is presented below.

Let \( \tilde{g}^p = (g_{ij}^p)_{n \times n} \) be the initial direct relationship matrix of dimensions given by \( e_p \), where \( g_{ij}^p \) represents the direct relationship between dimensions \( D_i \) and \( D_j \). The average entropy of IVIFVs in \( \tilde{g}^p \) can be used to measure the fuzzy degree and uncertainty of \( \tilde{g}^p \) [41]. That is

\[ En(\tilde{g}^p) = \frac{1}{n \times n} \sum_{j=1}^{n} \sum_{i=1}^{n} 2 - \frac{|g_{ij}^{up} - g_{ij}^{down}|}{g_{ij}^{up} + g_{ij}^{down}} - \frac{|g_{ij}^{up} - g_{ij}^{top}|}{g_{ij}^{up} + g_{ij}^{top}} + (1 - \frac{|g_{ij}^{up} - g_{ij}^{down}|}{g_{ij}^{up} + g_{ij}^{down}}) + (1 - \frac{|g_{ij}^{up} - g_{ij}^{top}|}{g_{ij}^{up} + g_{ij}^{top}}) \]

(27)

The closeness between \( \tilde{g}^p = (g_{ij}^p)_{n \times n} \) and \( \tilde{g}^\tau = (g_{ij}^\tau)_{n \times n} \) can be measured by the following formula:

\[ CL(\tilde{g}^p, \tilde{g}^\tau) = 1 - HD(\tilde{g}^p, \tilde{g}^\tau) \]

(28)

where \( HD(\tilde{g}^p, \tilde{g}^\tau) = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} (|g_{ij}^{up} - g_{ij}^{up^\tau}| + |g_{ij}^{up} - g_{ij}^{top^\tau}| + |g_{ij}^{top} - g_{ij}^{up^\tau}| + |g_{ij}^{top} - g_{ij}^{top^\tau}|) \) is the average Hamming distance between \( \tilde{g}^p \) and \( \tilde{g}^\tau \) (\( i, j = 1, 2, \ldots, n \)).

Then, the average closeness between expert \( e_p \) and the group is calculated by the following mathematical form:

\[ CL(\tilde{g}^p) = \frac{1}{t-1} \sum_{t=1, t \neq p}^{t} CL(\tilde{g}^p, \tilde{g}^\tau) \]

(29)

Based on the principles of determining experts’ weights [5,7], a bi-objective programming model is constructed to determine the comprehensive weights of experts.
\[
\min \max \{w^1_p E(w^p) | p = 1, 2, \ldots, t\} \\
\max \min \{w^1_p CL(w^p) | p = 1, 2, \ldots, t\} \\
\text{s.t. } \sum_{p=1}^{t} w^1_p = 1; \quad w \in \Lambda
\]  

(M7)

The first objective aims to minimize the maximum among the weighted entropies of the IVIF ratings given by all experts (i.e., minimize expert’s intrapersonal uncertainty). The second objective seeks to maximize the minimum among the closeness of all experts' IVIF ratings between other experts' IVIF ratings (i.e., minimize expert’s interpersonal uncertainty). \(A\) represents the predefined preference information structure of experts’ weights.

Let \(\beta_E = \max \{w^1_p E(w^p) | p = 1, 2, \ldots, t\}\) and \(\beta_t = \min \{w^1_p CL(w^p) | p = 1, 2, \ldots, t\}\). Then, model M7 is further converted into the following single-objective programming model:

\[
\min \beta_E - \beta_t \\
\text{s.t. } \begin{cases} \\
\quad w^1_p E(w^p) \leq \beta_E & (p = 1, 2, \ldots, t) \\
\quad w^1_p CL(w^p) \geq \beta_t & (p = 1, 2, \ldots, t) \\
\quad \sum_{p=1}^{t} w^1_p = 1; \quad w \in \Lambda
\end{cases}
\]

(M8)

By solving model M8 with some mature software packages (e.g., Lingo and Matlab), the weight vector of experts \(w^{1st}(w^{2nd}, \ldots, w^{1st})\) can be determined.

Let \(Y^p_r = [y^p_{r1}, y^p_{r2}, \ldots, y^p_{rm}] \ (p = 1, 2, \ldots, t)\) be \(t\) evaluation vectors of \(x_r\) given by \(t\) experts, where \(y^p_r = (y^p_{rp}, y^p_{rp}, \ldots, y^p_{rp})\) denotes the IVIF evaluation value of element \(x_r\) under \(c_j\). Then, similar to the determination of \(w^{1st}, w^{2nd}\) can be determined based on \(Y^p_r \ (r = 1, 2, \ldots, q; p = 1, 2, \ldots, t)\).

4.2 Determine the weights of dimensions with IVIF-DEMATEL

Step 1. Calculate the group direct relationship matrix of dimensions.

Based on the obtained value of \(w^{1st}\), the individual direct relationship matrices \(g^p \ (p = 1, 2, \ldots, t)\) are aggregated into a group direct relationship IVIF matrix \(g^p = (\tilde{g}^p)_{n \times n}\), where

\[
\tilde{g}^p = (\prod_{p=1}^{t} (1 - g^p_{iy} w^1_p), \prod_{p=1}^{t} (1 - g^p_{iy} w^1_p), \prod_{p=1}^{t} (1 - g^p_{iy} w^1_p), \prod_{p=1}^{t} (1 - g^p_{iy} w^1_p), \prod_{p=1}^{t} (1 - g^p_{iy} w^1_p))
\]

Step 2. Transform \(g\) into a normalized numeric matrix \(F\).

The similarity function of IVIFV, i.e., Eq. (6), is used to transform \(g\) into a numeric matrix \(g = (g^p)_{n \times n}\), where

\[
g^p = L(\tilde{g}^p)
\]

(30)

Then, \(g\) can be transformed into a normalized matrix \(F\) by using Eq. (31) as follows:

\[
F = (f^p)_{n \times n} = (g^p / g_{max})_{n \times n}
\]

(31)

where \(g_{max} = \max(\max_i \sum_{j=1}^{n} g^p_{ij}, \max_j \sum_{i=1}^{n} g^p_{ij})\).

Step 3. Construct the total direct-relationship matrix of dimensions.

The total direct-relationship matrix of dimensions \(\Phi = (\phi^p)_{n \times n}\), which represents the direct and indirect relationships between the pairwise dimensions, can be derived by Eq. (32) as follows:

\[
\Phi = (\phi^p)_{n \times n} = F(1 - F)^{-1}
\]

(32)

where \(I\) is \(n\)-order identity matrix.

Step 4. Calculate the importance degree (i.e., weight) of each dimension.

The row vector \(RT\) and the column vector \(CT\) can be derived from \(\Phi\) as follows:

\[
RT = (rt_p)_{n \times 1} = \left(\sum_{j=1}^{n} \phi^p_{ij}\right)_{n \times 1} \quad \text{and} \quad CT = (ct_p)_{1 \times n} = \left(\sum_{i=1}^{n} \phi^p_{ij}\right)_{1 \times n}
\]

(33)
Particularly, \( r_{ij} \), is the sum of the values that lie in the \( i \)-th row, indicating the total influence degree of dimension \( D_i \) on other dimensions, i.e., the influencing degree of \( D_i \); \( c_{ij} \) is the sum of the values that lie in the \( j \)-th column, meaning the total influence degree of other dimensions on dimension \( D_i \), i.e., the influenced degree of \( D_i \).

When \( i = j \), \( r_{ij} + c_{ij} \) is the sum of influencing and influenced degrees of \( D_i \), representing the importance degrees of \( D_i \) (\( i = 1, 2, \ldots, n \)). Hence, the weights of \( D_i \) (\( i = 1, 2, \ldots, n \)) can surely calculated by Eq. (34) as follows:

\[
w_{D_i} = \frac{r_{ij} + c_{ij}}{\sum_{i=1}^{n} (r_{ij} + c_{ij})} (i = 1, 2, \ldots, n) \tag{34}
\]

4.3. Determine the weights of criteria under each dimension with IVIF-BWM

Since the number of criteria is large, it is hard for experts to provide their pairwise comparisons for so many criteria. In reality, it is easy for experts to provide their pairwise comparisons of the criteria under a dimension. Hence, the criteria weights can be determined by the following steps.

Step 1. Obtain the BOV and OWV of each set of criteria.

Experts give their RCs for the set of criteria \( c_{D_k} = \{c_{k1}, c_{k2}, \ldots, c_{kn} \} \) under dimension \( D_k \). The BOV of \( c_{D_k} \) given by expert \( e_p \) is obtained as follows:

\[
S_{b} = \left[ s_{b1}^{p}, s_{b2}^{p}, \ldots, s_{bn}^{p} \right]
\]

The OWV of \( c_{D_k} \) given by expert \( e_p \) is acquired as follows:

\[
S_{w} = \left[ s_{w1}^{p}, s_{w2}^{p}, \ldots, s_{wn}^{p} \right]^T
\]

Step 2. Use the proposed IVIF-BWM to determine criterion weights with respect to each expert.

By using the IVIF-BWM presented in subsections 3.2 and 3.3, the IVIF weight vector for each set of criteria \( c_{D_k} \) with respect to expert \( e_p \) can be determined and denoted by \( \tilde{w}_{D_k}^{p} = (\tilde{w}_{k1}^{p}, \tilde{w}_{k2}^{p}, \ldots, \tilde{w}_{kn}^{p}) \) \((k = 1, 2, \ldots, n; p = 1, 2, \ldots, t)\).

4.4. Calculate the global weights of all criteria

In order to further acquire the global weights of all criteria, it is required to incorporate \( w_{D_k} \) into \( \tilde{w}_{D_k}^{p} \). That is,

\[
\tilde{w} = \left( \tilde{w}_{1}, \tilde{w}_{2}, \ldots, \tilde{w}_{n} \right) = \left( \tilde{w}_{11}, \tilde{w}_{12}, \ldots, \tilde{w}_{1n} \right) \left( \tilde{w}_{21}, \tilde{w}_{22}, \ldots, \tilde{w}_{2n} \right) \ldots \left( \tilde{w}_{n1}, \tilde{w}_{n2}, \ldots, \tilde{w}_{nn} \right) (p = 1, 2, \ldots, t)
\]

where \( \tilde{w}_{k} = (\tilde{w}_{k1}, \tilde{w}_{k2}, \ldots, \tilde{w}_{kn}) \) is the global IVIF weight of criterion \( c_k \) with respect to \( e_p \).

The criterion weight vectors \( \tilde{w}^p (p = 1, 2, \ldots, t) \) are aggregated into the global IVIF criteria weight vector \( \tilde{w} = (\tilde{w}_{1}, \tilde{w}_{2}, \ldots, \tilde{w}_{n}) \), where

\[
\tilde{w}_{k} = (\tilde{w}_{1k}, \tilde{w}_{2k}, \ldots, \tilde{w}_{nk}) = [1 - \frac{1}{t} \prod_{p=1}^{t} (1 - \tilde{w}_{kp})^{w_{Dk}}], 1 - \frac{1}{t} \prod_{p=1}^{t} (1 - \tilde{w}_{kp})^{w_{Dk}}] \prod_{p=1}^{t} \tilde{w}_{kp}^{w_{Dk}}(k = 1, \ldots, m)
\]

Furthermore, the global IVIF criteria weight vector can be defuzzified into a global numeric criteria weight vector \( \omega = (\omega_1, \omega_2, \ldots, \omega_m) \), in which

\[
\omega_k = L(\tilde{w}_k) = \frac{1}{n} \sum_{k=1}^{n} L(\tilde{w}_k)(k = 1, \ldots, m)
\]

where \( L(\tilde{w}_k) \) is the similarity function of \( \tilde{w}_k \).

4.5. Assess risk and rank alternatives with IVIF-SPA

Step 1. Normalize the individual decision matrix.

Generally, the criteria can be divided into two classes: negative criterion and positive criterion. To facilitate calculation, this paper converts the individual evaluation value \( y_{ij}^p \) into a normalized one \( h_{ij}^p (j = 1, \ldots, n) \), where

\[
h_{ij}^p = (h_{ij}^{p}, h_{ij}^{n}, h_{ij}^{r}, h_{ij}^{s}) = \begin{cases} (\min y_{ij}, y_{ij}, \min y_{ij}, y_{ij}), & \text{if } c_j \text{ is positive} \\ (\max y_{ij}, y_{ij}, \max y_{ij}, y_{ij}), & \text{if } c_j \text{ is negative} \end{cases}
\]

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Step 2. Defuzzify the IVIF individual decision matrices into numeric ones.

Eq. (6) is used to defuzzify the IVIF individual decision matrices \( h = (h_i)_{q \times n} \) into numeric ones \( h^p = (h^p_i)_{q \times n} \) (\( p = 1, 2, \ldots, t \)).

Step 3. Calculate the five-unit connection degree of each criterion \( c_i \) with respect to \( \chi_i \).

This paper extracts five risk grades (see Table 2) from the linguistic terms and their corresponding IVIFVs presented in [17]. The set \( z = \{ z_1, z_2, z_3, z_4, z_5 \} \) is called risk grade set. The numeric evaluations of alternative \( \chi_i \) under all criteria form a set \( h^p = \{ h^p_1, h^p_2, \ldots, h^p_m \} \) which can be regarded as the sample set. Then, sets \( h^p \) and \( z \) constitute a set pair \( H^p = \langle h^p, z \rangle \). Taking criterion \( c_i \) as an example, according to the SPA theory [24,48], the five-unit connection degree between sets \( h^p \) and \( z \) is formulated as follows:

\[
\mu^p_i(c_i) = a^p_i + b^p_{ij} i_1 + b^p_{ij} i_2 + b^p_{ij} i_3 + \rho^p_{ij} j = \begin{cases} 1 + 0i_1 + 0i_2 + 0i_3 + 0j, & \text{if } h^p_i \geq z_1 \\ \frac{h^p_i - z_2}{z_1 - z_2} + \frac{h^p_i - z_3}{z_2 - z_3} i_1 + 0i_2 + 0i_3 + 0j, & \text{if } z_2 \leq h^p_i < z_1 \\ 0 + \frac{h^p_i - z_1}{z_2 - z_1} i_1 + \frac{h^p_i - z_2}{z_1 - z_2} i_2 + 0i_3 + 0j, & \text{if } z_3 \leq h^p_i < z_2 \\ 0 + 0i_1 + \frac{h^p_i - z_3}{z_2 - z_3} i_1 + \frac{h^p_i - z_4}{z_3 - z_4} i_2 + 0i_3 + 0j, & \text{if } z_4 \leq h^p_i < z_3 \\ 0 + 0i_1 + 0i_2 + \frac{h^p_i - z_4}{z_3 - z_4} i_1 + \frac{h^p_i - z_5}{z_4 - z_5} i_2 + \frac{h^p_i - z_5}{z_3 - z_5} j, & \text{if } z_5 \leq h^p_i < z_4 \\ 0 + 0i_1 + 0i_2 + 0i_3 + 1j, & \text{if } h^p_i \geq z_5 \end{cases} \tag{41}
\]

where \( i_1, i_2, i_3 \) and \( j \) denote the risk factors of mild discrepancy, moderate discrepancy, severe discrepancy and contradictory, respectively. Generally, \( i_1 \in (0, 1), i_2 \in [-1, 0), j = -1 \) and \( i_3 \) is a real number close to 0. \( a^p_i, b^p_{ij}, b^p_{ij}, b^p_{ij} \) and \( \rho^p_{ij} \) denote identity degree, mild discrepancy degree, moderate discrepancy degree, severe discrepancy degree and contradictory degree, respectively. In addition, it holds that \( a^p_i + b^p_{ij} + b^p_{ij} + b^p_{ij} + \rho^p_{ij} = 1 \).

Step 4. Assess the risk grade of alternative.

By using Eq. (42), we can calculate the weighted five-unit connection degree of criterion \( c_i \) with respect to \( \chi_i \) as follows:

\[
\mu_i = \sum_{j=1}^{m} \alpha_j \mu^p_i(c_i) = \sum_{j=1}^{m} \alpha_j a^p_i + \sum_{j=1}^{m} \alpha_j b^p_{ij} i_1 + \sum_{j=1}^{m} \alpha_j b^p_{ij} i_2 + \sum_{j=1}^{m} \alpha_j b^p_{ij} i_3 + \sum_{j=1}^{m} \alpha_j \rho^p_{ij} j \tag{43}
\]

For simplicity, let \( a_1^i = \sum_{j=1}^{m} \alpha_j a^p_i, a_2^i = \sum_{j=1}^{m} \alpha_j b^p_{ij}, a_3^i = \sum_{j=1}^{m} \alpha_j b^p_{ij}, a_4^i = \sum_{j=1}^{m} \alpha_j b^p_{ij}, a_5^i = \sum_{j=1}^{m} \alpha_j \rho^p_{ij} \).

Lastly, the risk grade of \( \chi_i \) can be determined according to the following rule [21]:

\[
\begin{cases} a_1^i + a_2^i + \cdots + a_5^i \geq \sigma \\
 a_1^i + a_2^i + \cdots + a_5^i < \sigma \end{cases} \tag{44}
\]

where \( \sigma \in [0.5, 0.7] \) is a parameter determined by the evaluator. In this paper, the value of \( \sigma \) is taken as 0.55. The index of \( a_5^i \) corresponds to the risk grade of \( \chi_i \). For example, if \( \kappa = 3 \) meets Eq. (44), then the risk grade of \( \chi_i \) is determined as grade III.

Step 5. Rank the alternatives.

Similarly, the overall five-unit connection degrees of other alternatives can be calculated. When \( i_1, i_2 \) and \( i_3 \) are assigned with the specific values, the values of \( \mu_r \) (\( r = 1, 2, \ldots, q \)) can be calculated. Then, the alternatives \( \chi_r \) (\( r = 1, 2, \ldots, q \)) can be ranked according to the values of \( \mu_r \). The larger the value of \( \mu_r \), the higher the priority of \( \chi_r \).

Based on the above steps, the framework of the proposed method is depicted in Fig. 1. Obviously, Fig. 1 can visually reveal the interaction of the various approaches in the proposed IVIF-DBWM-SPA. For example, the interaction between the IVIF-SPA and IVIF-BWM is reflected in the fact that the IVIF-SPA needs to use the criteria weights determined by the IVIF-BWM (see the bold arrow numbered 4 in Fig. 1).

| Table 2 |
|-----------------------------------------------|
| **Risk grades and their corresponding IVIFVs.** |
| **Risk grade** | **IVIFV (\( z_i \))** | **\( z_i = I(z_i) \)** |
|----------------|---------------------|---------------------|
| Extremely low (I) | \( \tilde{z}_1 = (0.9 \ 0.9, \ 0.1 \ 0.1) \) | 0.9 |
| Low (II) | \( \tilde{z}_2 = (0.6 \ 0.7, \ 0.1 \ 0.2) \) | 0.723 |
| Medium (III) | \( \tilde{z}_3 = (0.5 \ 0.5, \ 0.5 \ 0.5) \) | 0.5 |
| High (IV) | \( \tilde{z}_4 = (0.25 \ 0.35, \ 0.5 \ 0.6) \) | 0.391 |
| Extremely high (V) | \( \tilde{z}_5 = (0.1 \ 0.1, \ 0.9 \ 0.9) \) | 0.1 |
5. Application to the risk assessment of college resumption

After COVID-19 is basically controlled, government department is faced with the problem of restoring normal life order amid COVID-19 prevention. Particularly, school resumption is a major event that affects thousands of families. In April 2020, China made some giant strategic achievements on the prevention and control of COVID-19, which provides some favorable conditions for the resumption of normal life order. The overall strategy of epidemic prevention and control in China has shifted from acute and unconventional prevention and control to the normal prevention and control stage. Coupled with the risk of overseas input, China is still facing a great pressure on preventing the coronavirus from re-entering the country to cause a new epidemic, which means that there is also a certain risk of epidemic aggregation in the process of school resumption. In addition, since college students come from many different places in China, the resumption risk of colleges is more severe than that of primary and secondary schools. In order to conduct the mission of resumption, this section uses the proposed IVIF-DBWM-SPA to evaluate the college resumption risk amid COVID-19 prevention. The validity and superiority of the IVIF-DBWM-SPA are demonstrated with some sensitivity and comparative analyses.

5.1. Risk assessment process

Phase I: preliminary preparation

With the improvement of COVID-19 prevention, it is imperative for colleges to reopen their campuses in an orderly manner. Since college resumption will bring the latent infection risk of COVID-19, it is extremely necessary to assess and identify the latent infection risk amid college resumption. The main information about the studied case is described as follows:

(i) Alternatives. The alternatives in this case refer to the colleges to be evaluated. Suppose that there are five colleges (denoted by \( v_1, v_2, v_3, v_4 \) and \( v_5 \)) that apply to the local government department for resumption, then the local government department needs to assess the resumption risks of these colleges.

Fig. 1. Framework of the proposed method.
(ii) Decision-makers (experts). In China, the education department of the local government (denoted by $e_d$) is directly responsible for college resumption. In order to complete these assessments, $e_d$ recruits five acknowledged experts $e_p$ ($p = 1, 2, 3, 4, 5$) from different professional fields (see Table A.1).

(iii) Criteria. According to Technical scheme for COVID-19 prevention and control in colleges and universities (see https://www.gov.cn) and the criteria collected by Chen et al. [8], the invited experts extract eighteen criteria which can be classified into three dimensions to assess the college resumption risk, shown in Table 3.

Phase II: information processing

Each expert carefully analyzed the importance of the dimensions (i.e., $D_1$, $D_2$ and $D_3$) and conducted the field survey on the resumption preparations of the five colleges. Then, based on their professional knowledge, experts provide three types of decision information about this case, described as follows:

(i) The direct influence matrix between the dimensions. Experts give their judgments (see Table A.2) on the influence between any two dimensions with the linguistic terms shown in Table 1.

(ii) The RCs of criteria. Experts give their RCs (see Table A.4) of the criteria under each dimension with the linguistic terms shown in Table 1.

(iii) The evaluations of the evaluated colleges (i.e., the alternatives). After completing the field survey on the evaluated colleges, experts give their evaluations (see Table A.5) of the evaluated colleges under all criteria with the linguistic terms shown in Table A.3.

It should be noted that the first two types of linguistic information can be encoded into IVIFVs according to Table 1 and the last type of linguistic information can be encoded into IVIFVs according to Table A.3.

Phase III: interdependent effect analysis

(1) Determine the first type of experts’ weights. Solving model M8 with the IVIF judgments encoded from Table A.2, the

### Table 3

| Dimension               | Criterion                                                                 | Explanation                                                                 |
|-------------------------|---------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| Management system ($D_1$) | Epidemic prevention program ($c_{11}$)                                    | The more perfect the epidemic prevention program, the lower the risk of school resumption. |
|                         | Emergency response plan ($c_{12}$)                                       | The more perfect the emergency response plan, the lower the risk of school resumption. |
|                         | Health management system ($c_{13}$)                                      | The more perfect the health management system, the lower the risk of school resumption. |
|                         | Hygiene inspection system ($c_{14}$)                                     | The more perfect the hygiene inspection system, the lower the risk of school resumption. |
|                         | Investigation-registration of absentees due to illness ($c_{15}$)         | The more detailed the investigation-registration system, the lower the risk of school resumption. |
|                         | Dining system in canteen ($c_{16}$)                                      | The more reasonable the dining system, the lower the risk of school resumption. |
| Personnel & materials ($D_2$) | Stock of epidemic protection supplies ($c_{21}$)                   | The more materials (e.g., disinfectants, masks) have been prepared, the lower the risk of school resumption. |
|                         | Medical staff matching ($c_{22}$)                                        | The number of medical staff must be matched according to the total number of students. |
|                         | The geographical distribution of school-returnees ($c_{23}$)              | This data can be obtained by the daily health registration system. The wider geographical distribution of school-returnees, the higher the risk of school resumption. |
|                         | School-returnees’ health ($c_{24}$)                                      | This data can be obtained by the daily health registration system. The healthier the school-returnees, the lower the risk of school resumption. |
|                         | Population density ($c_{25}$)                                            | This data can be calculated by the ratio of the total number of students and teachers over the available public area. |
|                         | Isolation place setting ($c_{26}$)                                       | The more isolation place meets the epidemic prevention standards, the lower the risk of school resumption. |
| Epidemic prevention measures ($D_3$) | Epidemic prevention publicity ($c_{31}$)                                | The richer the epidemic prevention channels (e.g., WeChat official account, micro-blog, official website, radio, brochure, etc.), the lower the risk of school resumption. |
|                         | Psychological counseling service ($c_{32}$)                              | Psychological counseling service can intervene and repair the psychological trauma of teachers and students, which is beneficial for them to devote themselves to teaching and learning. |
|                         | Epidemic prevention skills training ($c_{33}$)                            | Carry out epidemic prevention system and personal protection training for teachers and students. The better the training effect, the lower the risk of school resumption. |
|                         | School resumption simulation ($c_{34}$)                                   | Simulation can improve the college’s ability of risk prevention and emergency response, and minimize the risk of school resumption. |
|                         | Campus gate control ($c_{35}$)                                           | Before entering the campus, visitors need to register their personal information and take their temperatures. Strict campus gate control can effectively prevent the epidemic source from entering the campus. |
|                         | Teaching modes ($c_{36}$)                                                | Reasonable teaching modes include online & offline teaching, small class & peak shifting teaching, outdoor teaching. The more reasonable the teaching modes, the lower the risk of school resumption. |
first type of experts’ weight vector can be derived as \( w_{1st} = (0.182, 0.238, 0.203, 0.201, 0.176) \).

(2) According to Tables A.2 and Eq. (32), the total direct-relationship matrix of dimensions can be obtained as follows:

\[
\Phi = \begin{pmatrix}
2.318 & 3.003 & 2.77 \\
2.501 & 2.556 & 2.597 \\
2.622 & 3.116 & 2.481
\end{pmatrix}
\]

Then, it is easy to yield that \( r_1 + c_1 = 15.53, r_2 + c_2 = 16.33 \) and \( r_3 + c_3 = 16.07 \). Hence, according to Eq. (34), the weights of dimensions can be determined as \( w_{c_1} = 0.324, w_{c_2} = 0.341 \) and \( w_{c_3} = 0.335 \).

Phase IV: risk aggregation

(1) Determine the second type of experts’ weights. Solving model M8 with the IVIF judgments encoded from Table A.5, the second type of experts’ weight vector can be derived. For example, the experts’ weight vector associated with alternative \( \chi_1 \) is determined as \( w_{1st} = (0.204, 0.227, 0.159, 0.221, 0.188) \). Similarly, experts’ weight vectors associated with other four alternatives can also be acquired.

(2) Determine the global weights of all criteria. The value of \( \tilde{\lambda}_i^p \) can be calculated as follows:

\[
\tilde{\lambda}_i^p = \left( \tilde{\lambda}_{i1}^p, \tilde{\lambda}_{i2}^p, \cdots, \tilde{\lambda}_{ik_i}^p \right)
\]

According to Section 4.4, the global numeric weights of all criteria can be acquired and shown in the second column of Table 4.

(3) Assess the resumption risk. According to Table A.4, Eqs. (42) and (43), it is easy to obtain the weighted five-unit connection degree for all criteria and the overall five-unit connection degree for \( \chi_1 \).

Similarly, the overall five-unit connection degrees for \( \chi_2, \chi_3, \chi_4 \) and \( \chi_5 \) can be calculated as follows:

\[
\mu_{\chi_2} = 0.222 + 0.368i_1 + 0.238i_2 + 0.139i_3 + 0.034j + \mu_{\chi_1} = 0.182 + 0.417i_1 + 0.218i_2 + 0.131i_3 + 0.052j
\]

\[
\mu_{\chi_3} = 0.193 + 0.441i_1 + 0.246i_2 + 0.12i_3 + 0j \quad \text{and} \quad \mu_{\chi_5} = 0.232 + 0.42i_1 + 0.234i_2 + 0.104i_3 + 0.01j
\]

### Table 4

| Criterion | weight \( w_{1st} \) | Five-unit connection degree derived by \( w_{1st} \) | Risk grade | Five-unit connection degree derived by \( w_{1st} \) | Risk grade |
|-----------|-----------------|-----------------|----------|-----------------|----------|
| \( c_{11} \) | 0.0795 | 0.913 + 0.087i_1 + 0i_2 + 0i_3 + 0j | I | 0.9 + 0.1i_1 + 0i_2 + 0i_3 + 0j | I |
| \( c_{12} \) | 0.0779 | 0 + 0i_1 + 0i_2 + 0.632i_3 + 0.368j | IV | 0 + 0i_1 + 0i_2 + 0.604i_3 + 0.396j | IV |
| \( c_{13} \) | 0.059 | 0 + 0.558i_1 + 0.442i_2 + 0i_3 + 0j | II | 0 + 0.531i_1 + 0.469i_2 + 0i_3 + 0j | II |
| \( c_{14} \) | 0.0428 | 0 + 0.3i_1 + 0.431i_2 + 0.569i_3 + 0j | IV | 0 + 0.3i_1 + 0.412i_2 + 0.588i_3 + 0j | IV |
| \( c_{15} \) | 0.0341 | 0 + 0.656i_1 + 0.344i_2 + 0i_3 + 0j | II | 0 + 0.627i_1 + 0.373i_2 + 0i_3 + 0j | II |
| \( c_{16} \) | 0.0306 | 0 + 0.1i_1 + 0.111i_2 + 0.658i_3 + 0.231j | IV | 0 + 0.1i_1 + 0.148i_2 + 0.632i_3 + 0.22j | IV |
| \( c_{21} \) | 0.0445 | 0.453 + 0.547i_1 + 0i_2 + 0i_3 + 0j | IV | 0.424 + 0.576i_1 + 0i_2 + 0i_3 + 0j | IV |
| \( c_{22} \) | 0.0289 | 0.12 + 0.29i_1 + 0.41i_2 + 0.183i_3 + 0j | III | 0.106 + 0.307i_1 + 0.407i_2 + 0.18i_3 + 0j | III |
| \( c_{23} \) | 0.0952 | 0 + 0.47i_1 + 0.472i_2 + 0.058i_3 + 0j | II | 0 + 0.415i_1 + 0.518i_2 + 0.066i_3 + 0j | II |
| \( c_{24} \) | 0.0781 | 0.232 + 0.199i_1 + 0.452i_2 + 0.116i_3 + 0j | III | 0.202 + 0.173i_1 + 0.5i_2 + 0.125i_3 + 0j | III |
| \( c_{25} \) | 0.0618 | 0.101 + 0.178i_1 + 0.651i_2 + 0.069i_3 + 0j | III | 0.117 + 0.221i_1 + 0.598i_2 + 0.064i_3 + 0j | III |
| \( c_{26} \) | 0.0322 | 0 + 0i_1 + 0.002i_2 + 0.593i_3 + 0.059j | IV | 0 + 0i_1 + 0.002i_2 + 0.519i_3 + 0.079j | IV |
| \( c_{27} \) | 0.026 | 0.102 + 0.596i_1 + 0.239i_2 + 0.062i_3 + 0j | V | 0.118 + 0.626i_1 + 0.205i_2 + 0.051i_3 + 0j | V |
| \( c_{32} \) | 0.074 | 0.085 + 0.703i_1 + 0i_2 + 0.282i_3 + 0.56j | V | 0.114 + 0.098i_1 + 0i_2 + 0.253i_3 + 0.535j | V |
| \( c_{33} \) | 0.039 | 0.489 + 0.511i_1 + 0i_2 + 0i_3 + 0j | IV | 0.524 + 0.476i_1 + 0i_2 + 0i_3 + 0j | IV |
| \( c_{34} \) | 0.08 | 0 + 0.266i_1 + 0.596i_2 + 0.137i_3 + 0j | III | 0 + 0.335i_1 + 0.541i_2 + 0.123i_3 + 0j | III |
| \( c_{35} \) | 0.08 | 0 + 0.16 + 0.636i_1 + 0.142i_2 + 0.062i_3 + 0j | II | 0.213 + 0.619i_1 + 0.116i_2 + 0.051i_3 + 0j | II |
| \( c_{36} \) | 0.036 | 0.811 + 0.189i_1 + 0.142i_2 + 0.062i_3 + 0j | I | \( \mu_{c_{36}} = 0.809 + 0.191i_1 + 0i_2 + 0i_3 + 0j \) | I |
| Overall | 0.191 + 0.286i_1 + 0.307i_2 + 0.166i_3 + 0.05j | III | 0.195 + 0.287i_1 + 0.305i_2 + 0.164i_3 + 0.049j | III |
Phase V: Risk grade

By using Eq. (40), the overall risk grade of \( v_1 \) and the criteria risk grades of \( v_1 \) can be calculated, shown in Table 4.

Let \( i_1 = 0.8, i_2 = 0.05 \) and \( i_3 = -0.8 \). Then, it yields that \( \mu_{X_1} = 0.252, \mu_{X_2} = 0.382, \mu_{X_3} = 0.37, \mu_{X_4} = 0.462 \) and \( \mu_{X_5} = 0.487 \). Hence, the ranking order of \( X_r (r = 1, 2, 3, 4, 5) \) is \( X_5 > X_4 > X_3 > X_1 \). In addition, according to Eq. (40), it is easy to determine the overall risk grades of \( X_r (r = 1, 2, 3, 4, 5) \) are III (medium), II (low), II (low), II (low) and II (low), respectively. Hence, the proposed method can rank the evaluated colleges and determine their risk grades, simultaneously.

From Table 4, it is easy to conclude that the proposed method not only can measure the overall risk grade of the evaluated college \( v_1 \), but also can identify the high or extremely high risk indices for \( v_1 \) that need to be improved, i.e., \( c_{12}, c_{14}, c_{16}, c_{26} \) and \( c_{32} \). Hence, the proposed method is helpful for the evaluated colleges to dig out the resumption weaknesses amid COVID-19 prevention.

5.2. Sensitivity analysis

From Eq. (43), it is easy to know that the final ranking result depends on the values of \( i_1, i_2 \) and \( i_3 \), where \( i_1, i_2 \) and \( i_3 \) represent the risk factors of mild discrepancy, moderate discrepancy and severe discrepancy, respectively. The setting of \( i_1, i_2 \) and \( i_3 \) hinges on expert’s personal intention. An optimistic expert would like to set the values of \( i_1, i_2 \) and \( i_3 \) as large as possible, while a pessimistic expert is just the opposite. To clear the influence of expert’s personal intention (i.e., the values of \( i_1, i_2 \) and \( i_3 \)) on the ranking result, this paper varies the value of \( i_1 \) (\( i_2 \) and \( i_3 \)) from 0.05 to 1 (-0.05 to 0.05 and /0.05 to /0.05). The final ranking results for three different sets of parameter combinations \((i_2, i_3), (i_1, i_3)\) and \((i_1, i_2)\) are graphically shown in Fig. 2.

(i) By observing Fig. 2(c), it is easy to see that the ranking results of \( X_r (r = 1, 2, 3, 4, 5) \) remain unchanged, which indicates that the parameter combination \((i_1, i_2)\) has no influence on the ranking result.

(ii) By observing Fig. 2(a) and (b), it is not difficult to discover that only the priorities of \( X_3 \) and \( X_1 \) are swapped, which shows that the parameter combinations \((i_2, i_3)\) and \((i_1, i_3)\) only have little influence on the ranking result.

Therefore, the ranking results are relatively stable for different sets of parameter combinations, which shows that the expert’s personal intention only has little influence on the evaluated results. As such, the proposed method is robust and reliable.

5.3. Comparative analysis

5.3.1. Comparison with the existing methods

In order to demonstrate the availability and advantages of the proposed method, the ranking result derived by the proposed method is compared with those derived by TOPSIS [16], VIKOR (VIsekriterijumska optimizacija i Kom-promisno Resenje) [26], CODAS (Combinative Distance based Assessment) [9] and TODIM (TOmada de Decisão Iterativa Multicritério)
The ranking results derived by five different methods are listed in Table 5, where the number in brackets after $\chi_r$ is the synthetic evaluation value of $\chi_r$.

From Table 5, it is easy to know that the ranking orders of $\chi_r$ $(r = 1, 2, 3, 4, 5)$ derived by these five methods are all the same, which shows the availability of the proposed method on ranking the alternatives. Moreover, compared with methods [9,16,20,26], some real advantages of the proposed method are summarized as follows:

1. Advantage in picking out the best alternative. According to Table 5, it is not difficult to calculate the dominance of the best alternative (i.e., the difference between the synthetic evaluation values of the ranking first and second alternatives), shown in Table 5. It is easy to observe that the dominance of the best alternative derived by the proposed method is evidently larger than those derived by other methods [9,16,20,26]. In practice, the greater the difference between the synthetic evaluation values of the ranking first and second alternatives, the more satisfactory and convincing of the ranking first alternative. Thus, the proposed method has the real advantage in picking out the satisfactory and convincing alternative.

2. Advantage in identifying the risk grade. Although methods [9,16,20,26] can be used to rank the alternatives, they are powerless in identifying the risk grades of the alternatives. However, the proposed method can rank the alternatives and identify their risk grades, simultaneously. It is obvious that different risk grades usually correspond to different moves. For example, if the grade level is greater than III, the evaluated college must delay its resumption. Hence, the proposed method is more suitable for the risk assessment of college resumption.

3. Advantage in determining experts’ weights. In the proposed method, experts’ weights are determined based on the uncertainty (entropy) of experts’ IVIF ratings and the consensus (closeness) between each expert’s ratings and group ratings, which can effectively avoid the subjective casualness of experts’ weights. However, experts’ weights in method [9] are derived based on experts’ self-evaluated matrix and experts’ weights in method [16] are pre-given. Obviously, it is hard to eliminate the subjectivity of the experts’ weights used in methods [9,16]. Hence, the proposed method has the real advantage in avoiding some undesirable results that caused by the subjective casualness of experts’ weights.

4. Advantage in determining the criteria weights. Li et al. [20] developed an effective IVIF-DANP to obtain the criteria weights. However, the IVIF-DANP becomes powerless when the number of criteria is large enough since it is nearly impossible for experts to give their pairwise comparisons for so many criteria. To avoid this obstruction, this paper develops the IVIF-DEMATEL and IVIF-BWM to respectively determine the weights of dimensions and the weights of criteria under each dimension. Compared with the IVIF-DANP [20], the proposed criteria weights determining approach has the real advantage in handling the complex decision-making problems with large number of criteria.

### 5.3.2. Comparison with other approaches of obtaining experts’ weights

(1) Comparison with the approach based on all experts’ evaluations of all alternatives.

In the evaluation stage, this paper uses $q$ weight vectors $w_{2,1}^q$ $(r = 1, 2, \cdots, q)$ to determine the risk grades of alternatives and rank the alternatives. The weight vector $w_{2,1}^q$ is derived from all expert’s evaluations on the alternative $\chi_r$ (see the evaluation matrix listed in columns 2 to 5 of Table A.5). In fact, experts’ weights can also be derived from all expert’s evaluations on all alternatives (see Table A.5). Let $\tilde{y}_q^p = (\tilde{y}_{q,1}^p, \cdots, \tilde{y}_{q,m}^p)$ be $t$ evaluation matrices given by $t$ experts, where

$$
\begin{align*}
\chi_1 & \begin{bmatrix} 
\tilde{y}_{1,1}^p & \tilde{y}_{1,2}^p & \cdots & \tilde{y}_{1,m}^p \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{y}_{q,1}^p & \tilde{y}_{q,2}^p & \cdots & \tilde{y}_{q,m}^p 
\end{bmatrix} 
\end{align*}
$$

Based on these evaluation matrices and Section 4.1, the experts’ weight vector, denoted by $w_{2,1}^q = (w_{2,1}^q, \cdots, w_{2,1}^q)$, can be determined. In this case, only one weight vector $w_{2,1}^q$ is obtained. The relation among $w_{2,1}^q$ and $w_{2,1}^q$ $(r = 1, 2, \cdots, q)$ is visually depicted in Fig. 3.

| Method       | Ranking order of alternatives | Dominance |
|--------------|-------------------------------|-----------|
| CODAS [9]    | $\chi_5(0.659) > \chi_4(0.646) > \chi_2(0.552) > \chi_1(0.518) > \chi_3(0.263)$ | 0.013     |
| TOPSIS [16]  | $\chi_5(0.688) > \chi_4(0.675) > \chi_2(0.637) > \chi_1(0.618) > \chi_3(0.565)$ | 0.013     |
| TODIM [20]   | $\chi_5(1) > \chi_4(0.994) > \chi_2(0.638) > \chi_3(0.428) > \chi_1(0)$ | 0.006     |
| VIKOR [26]   | $\chi_5(0) > \chi_4(0.021) > \chi_2(0.053) > \chi_3(0.063) > \chi_1(0.083)$ | 0.021     |
| The proposed method | $\chi_5(0.487) > \chi_4(0.462) > \chi_2(0.382) > \chi_3(0.37) > \chi_1(0.252)$ | 0.025     |
Next, the advantages of using different sets of experts' weights, i.e., $w^{2nd}_r (r = 1, 2, \ldots, 5)$, are outlined as follows:

(i) Advantage in obtaining more stable ranking orders. When $w^{2nd}_r (r = 1, 2, \ldots, 5)$ are all replaced by $w^{2nd}$, similar to the sensitivity analysis in section 5.2, the influence of $i_1, i_2$ and $i_3$ on the connection degrees (ranking orders) of $X_r (r = 1, 2, 3, 4, 5)$ can be graphically delineated in Fig. 4. From Fig. 4, it is easy to see that the ranking order of $X_r (r = 1, 2, 3, 4, 5)$ is influenced by the parameter combinations $(i_2, i_3), (i_1, i_3)$ and $(i_1, i_2)$ when using $w^{2nd}$. However, the ranking order of $X_r (r = 1, 2, 3, 4, 5)$ is only influenced by the parameter combinations $(i_2, i_3)$ and $(i_1, i_2)$ when using $w^{2nd}_r (r = 1, 2, \ldots, 5)$. Hence, the ranking order of $X_r (r = 1, 2, 3, 4, 5)$ derived by $w^{2nd}_r (r = 1, 2, \ldots, 5)$ is more stable than that derived by $w^{2nd}$.

(ii) Advantage in reducing the influence of abnormal evaluations on the assessment results. It can be seen from Table A.5 that the evaluations of $X_3$ given by $e_1$ are remarkably different from those given by other experts. In such a case, the evaluations of $X_3$ given by $e_1$ might be regarded to be abnormal. Normally, it is expected to reduce the effect of abnormal evaluations on the assessment results. Fortunately, the approach of determining $w^{2nd}$ (denoted by approach I) has the ability to assign the expert who gives abnormal evaluations with smaller weight. That is to say, the effect of abnor-

![Fig. 3. Relation among different sets of experts' weights.](image)

![Fig. 4. Influence of $i_1$, $i_2$ and $i_3$ on the connection degrees (one set of experts' weights).](image)
normal evaluations on the assessment results can be reduced through the smaller expert weight. However, it is hard for the approach of determining $w^{\text{ind}}$ (denoted by approach II) to identify expert’s abnormal evaluations for each alternative. For example, the weight of $e_1$ equals 0.145 in $w^{\text{ind}}$, while it equals 0.167 in $w^{\text{ind}}$. Since 0.145 is smaller than 0.167, the former has higher power in reducing the effect of abnormal evaluations on the assessment results. The assessment results associated with $X_3$ derived by $w^{\text{ind}}$ and $w^{\text{ind}}$ are shown in Fig. 5.

It is easy to observe from Fig. 5 that the connection degree, identity degree $(\alpha)$, mild discrepancy degree $(b_1)$, moderate discrepancy degree $(b_2)$ derived by $w^{\text{ind}}$ are larger than those derived by $w^{\text{ind}}$, while the severe discrepancy degree $(b_3)$ and contradictory degree $(\rho)$ derived by $w^{\text{ind}}$ are both smaller than those derived by $w^{\text{ind}}$. This indicates that the former has lower risk than the latter, implying that the former can decrease the impact of abnormal evaluations on the assessment results by assigning the expert who gives abnormal evaluations with a small weight.

(iii) Advantage in flexibly assessing alternatives. In reality, the risk assessment process might be intermittent since the application time slots of the alternatives (colleges) are usually different. For example, colleges $X_1$ and $X_2$ applied for reopening their campuses on 10 May and 20 May, respectively. Generally, the education department (evaluator) needs to evaluate the applicants 15 days in advance. That is, colleges $X_1$ and $X_2$ might be evaluated by the evaluator on 15 April and 5 May, respectively. In such a situation, it is impractical for the evaluator to assess the resumption risk of college $X_1$ by using $w^{\text{ind}}$. It is obvious that $w^{\text{ind}}$ is determined only based on all experts’ evaluations for the current college $X_1$ (see the evaluations of $X_1$ in Table A.5). Hence, it highlights the flexibility of using $w^{\text{ind}}$ to assess $X_1$. In addition, the evaluation result of $X_1$ can be acquired on 15 April. However, if we use $w^{\text{ind}}$ (which is determined based on the evaluations of $X_1$ and $X_2$) to evaluate colleges $X_1$ and $X_2$, the evaluation results of them can be obtain on 5 May. In this situation, the resumption risk assessment of $X_1$ is seriously delayed, which is not only conducive to the normal resumption of college $X_1$, but also violates the principle of “staggered resumption of school” advocated by the education department.

(2) Comparison with Li’s et al. weight-determining approach [20].

By using Li’s et al. approach [20], the values of $w^{\text{ind}}$ and $w^{\text{ind}}$ ($r = 1, 2, \cdots, q$) can be determined and listed in Table 6. It is easy to see from Table 6 and Fig. 6 that the difference between the largest and the smallest weights derived by model MB is remarkably larger than that derived by approach [20], which means that the proposed expert weight determining approach has higher power in distinguishing experts’ discrepancies than approach [20].

5.3.3. Comparison with Wang’s et al. IVIF-BWM [40]

In this subsection, we use Wang’s et al. IVIF-BWM [40] (denoted by BWM-I) and the proposed IVIF-BWM (denoted by BWM-II) to determine the weights of criteria $c_{11}$, $c_{12}$, $c_{13}$, $c_{14}$, $c_{15}$ and $c_{16}$ with respect to five experts. Limited by space, only the weights of $c_{11}$, $c_{12}$, $c_{13}$, $c_{14}$, $c_{15}$ and $c_{16}$ with respect to expert $e_5$ are listed in Table 7. The relation between the results derived by BWM-I and BWM-II is portrayed in Fig. 7.

From Table 7 and Fig. 7, the advantages of BWM-II can be collected as follows:

(i) Advantage in obtaining reasonable ranking order of criteria. The weights of $c_{11}$, $c_{12}$, $c_{13}$, $c_{14}$, $c_{15}$ and $c_{16}$ derived by BWM-I and BWM-II are $\lambda_{13} > \lambda_{11} > \lambda_{14} > \lambda_{12} > 0.15 = 0.16$ and $\lambda_{13} > \lambda_{11} > \lambda_{14} > \lambda_{12} > 0.15 > 0.16$, respectively. It is easy to observe from Table A.5 that the preference relations of $c_{11}$, $c_{12}$, $c_{13}$, $c_{14}$, $c_{15}$ and $c_{16}$ in BOV and OWV are both
(ii) The IVIF variables are transformed into intervals by regarding expert’s risk attitudes, then the IVIF weights can be normalized by using the normalization approach [37]. This flexible transformation can effectively reduce the loss and distortion of IVIF information. However, BWM-I directly normalized the score functions of IVIF weights, which might result in serious loss and distortion of IVIF information.

(i) The weight-determining model in BWM-II considers the consistency of the IVIF RCs, while that in BWM-I (which was constructed by directly fuzzifying model (6) of Ref. [29]) ignored this core issue.

(ii) Advantage in distinguishing the importance of criteria. It is easy to see from Fig. 7 that the differences among the criteria derived by BWM-II are remarkably larger than those derived by BWM-I. For example, the difference between the criterion derived by BWM-I obviously does not tally with the preference information presented in BOV and OWV (see Table A.5).

Table 6
Experts’ weights with different approaches.

| Expert weight vector | Derived by model M8 | Derived by Li’s et al. [20] approach |
|----------------------|---------------------|-------------------------------------|
| $w_{11}^{M8}$        | (0.167, 0.208, 0.212, 0.196, 0.217) | (0.19, 0.204, 0.202, 0.199, 0.205) |
| $w_{11}^{M2}$        | (0.204, 0.227, 0.159, 0.221, 0.188) | (0.197, 0.204, 0.193, 0.204, 0.202) |
| $w_{12}^{M2}$        | (0.195, 0.193, 0.192, 0.216, 0.204) | (0.193, 0.207, 0.201, 0.201, 0.199) |
| $w_{12}^{M2}$        | (0.145, 0.226, 0.221, 0.214, 0.195) | (0.194, 0.207, 0.199, 0.192, 0.208) |
| $w_{13}^{M2}$        | (0.186, 0.213, 0.199, 0.198, 0.205) | (0.192, 0.191, 0.207, 0.2, 0.209) |
| $w_{14}^{M2}$        | (0.177, 0.192, 0.227, 0.167, 0.237) | (0.177, 0.192, 0.227, 0.167, 0.237) |

C13 > C11 > C14 > C12 > C15 > C16. Hence, it is reasonable that the weights of C11, C12, C13, C14, C15 and C16 meet $\hat{\lambda}_{13} > \hat{\lambda}_{11} > \hat{\lambda}_{14} > \hat{\lambda}_{12} > \hat{\lambda}_{15} > \hat{\lambda}_{16}$, which exactly coincides with the one derived by BWM-II. However, the relation $\hat{\lambda}_{13} > \hat{\lambda}_{11} > \hat{\lambda}_{14} > \hat{\lambda}_{12} > \hat{\lambda}_{15} = \hat{\lambda}_{16}$ derived by BWM-I obviously does not tally with the preference information presented in BOV and OWV (see Table A.5).

Fig. 6. Difference between the largest and the smallest weights for different approaches.

Table 7
Criteria weights with different approaches.

| Criterion | BWM-I | BWM-II |
|-----------|-------|--------|
|           | IVIF weight | Numeric normalized weight | IVIF weight | Numeric normalized weight |
| c11       | (0.368, 0.418, 0.051, 0.101) | 0.181 | (0.134, 0.163, 0.714, 0.745) | 0.183 |
| c12       | (0.234, 0.270, 0.151, 0.201) | 0.157 | (0.07, 0.098, 0.794, 0.818) | 0.133 |
| c13       | (0.503, 0.507, 0.101, 0.101) | 0.193 | (0.362, 0.444, 0.474, 0.596) | 0.341 |
| c14       | (0.301, 0.349, 0.101, 0.151) | 0.169 | (0.1, 0.125, 0.755, 0.784) | 0.157 |
| c15       | (0.179, 0.268, 0.196, 0.251) | 0.15 | (0.043, 0.047, 0.848, 0.848) | 0.104 |
| c16       | (0.268, 0.268, 0.257, 0.279) | 0.15 | (0.029, 0.034, 0.883, 0.884) | 0.082 |

BWM-II can receive more reasonable results should be credited to the following two aspects:

(i) The weight-determining model in BWM-II considers the consistency of the IVIF RCs, while that in BWM-I (which was constructed by directly fuzzifying model (6) of Ref. [29]) ignored this core issue.

(ii) The IVIF variables are transformed into intervals by regarding expert’s risk attitudes, then the IVIF weights can be normalized by using the normalization approach [37]. This flexible transformation can effectively reduce the loss and distortion of IVIF information. However, BWM-I directly normalized the score functions of IVIF weights, which might result in serious loss and distortion of IVIF information.
### Table A1
Brief information about experts.

| Experts | Education level | Professional position | Experience time |
|---------|-----------------|-----------------------|-----------------|
| $e_1$   | Ph. D. degree in management science | Professor of emergency management | 12 years |
| $e_2$   | M. S. degree in public safety management | National first class safety assessor | 8 years |
| $e_3$   | M. S. degree in epidemiology | Professor of epidemiology | 13 years |
| $e_4$   | M. S. degree in public safety management | Associate professor of safety management | 12 years |
| $e_5$   | Ph. D. degree in respiratory diseases | Professor of respiratory diseases | 10 years |

### Table A2
Influence between dimensions.

| Influence degree | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|------------------|-------|-------|-------|-------|-------|
| $D_1$            | AL    | ML    | MH    | AL    | VL    | H     | AL    | VL    | H     | AL    | ML    | H     | AL    | H     |
| $D_2$            | H     | AL    | ML    | MH    | AL    | ML    | M     | AL    | L     | H     | AL    | VH    | M     | AL    |
| $D_3$            | L     | VH    | AL    | VL    | EH    | AL    | MH    | H     | AL    | ML    | H     | AL    | MH    | VH    | AL    |

### Table A3
Linguistic evaluations and their corresponding IVIFVs.

| Linguistic term | IVIFV         |
|-----------------|--------------|
| Absolutely low  | ([0,0], [1,1]) |
| Very low        | ([0.1, 0.1], [0.9, 0.9]) |
| Low             | ([0.15, 0.2], [0.6, 0.75]) |
| Medium low      | ([0.25, 0.35], [0.5, 0.6]) |
| Medium high     | ([0.35, 0.45], [0.4, 0.55]) |
| High            | ([0.45, 0.6], [0.15, 0.25]) |
| Very high       | ([0.6, 0.75], [0.1, 0.21]) |
| Extremely high  | ([0.75, 0.85], [0.05, 0.15]) |
| Absolutely high | ([0.9, 0.9], [0.1, 0.1]) |

### Table A4
RCs of criteria under each dimension given by each expert.

| Criterion | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|-----------|-------|-------|-------|-------|-------|
|           | BOV   | OWV   | BOV   | OWV   | BOV   | OWV   | BOV   | OWV   | BOV   | OWV   | BOV   | OWV   | BOV   | OWV   |
| $c_{11}$  | EI    | VSI   | WI    | VSI   | EI    | VSI   | WI    | EXI   | MPI   | VSI   |       |       |       |       |
| $c_{12}$  | WI    | SPI   | EI    | AL    | WI    | SPI   | EI    | AL    | SPI   | SI    |       |       |       |       |
| $c_{13}$  | MI    | MI    | MI    | SPI   | SPI   | MPI   | SI    | EI    | Al    |       |       |       |       |       |
| $c_{14}$  | SI    | MPI   | VSI   | MI    | MPI   | WI    | SI    | SPI   | SI    | SPI   |       |       |       |       |
| $c_{15}$  | MPI   | WI    | AL    | EI    | SI    | SPI   | VSI   | MPI   | EXI   | MPI   |       |       |       |       |
| $c_{16}$  | VSI   | EI    | SI    | SI    | VSI   | EI    | Al    | EI    | Al    |       |       |       |       |       |
| $c_{17}$  | MI    | SPI   | SI    | MPI   | SPI   | MPI   | SI    | SPI   | SI    |       |       |       |       |       |
| $c_{18}$  | AI    | EI    | AI    | EI    | AI    | EI    | AI    | EI    | EI    |       |       |       |       |       |
| $c_{19}$  | AI    | MI    | VSI   | MI    | SPI   | SI    | SPI   | EI    | Al    | MPI   | VSI   |       |       |       |
| $c_{20}$  | SI    | SI    | MI    | SI    | MI    | VSI   | MI    | VSI   | EI    | Al    |       |       |       |       |
| $c_{21}$  | VSI   | MI    | SPI   | WI    | EXI   | WI    | EXI   | WI    | Al    | EI    |       |       |       |       |
| $c_{22}$  | SI    | SI    | MI    | EXI   | VSI   | WI    | VSI   | MPI   | SPI   | SI    |       |       |       |       |
| $c_{23}$  | MI    | EXI   | SPI   | SI    | SPI   | MPI   | WI    | EXI   | VSI   | MPI   |       |       |       |       |
| $c_{24}$  | EXI   | MI    | VSI   | MPI   | MI    | VSI   | SI    | SI    | WI    | EXI   |       |       |       |       |
| $c_{25}$  | MPI   | SPI   | SI    | SI    | SPI   | SI    | SPI   | SI    | SPI   |       |       |       |       |       |
| $c_{26}$  | AI    | EI    | AI    | EI    | Al    | EI    | AI    | EI    |       |       |       |       |       |       |
| $c_{27}$  | MI    | VSI   | MI    | VSI   | MI    | VSI   | MI    | VSI   | EI    | Al    |       |       |       |       |
| $c_{28}$  | AI    | EI    | Al    | EI    | Al    | EI    | Al    | EI    |       |       |       |       |       |       |
6. Conclusions

Based on the obtained decision-making results, sensitivity and comparative analyses results, this section summarizes some quantitative results as follows.

(1) The proposed IVIF-DBWM-SPA not only has the ability to assess the comprehensive risk grades of the evaluated colleges, but also to identify the existing deficiencies during the college resumption preparation. For example, it is easy to know from Section 5.1 that there exist five high risk indices (i.e., $c_{12}$, $c_{14}$, $c_{16}$, $c_{26}$ and $c_{32}$) in the resumption preparation of college $\chi_1$. These indices can be regarded as the risk warning, which offers a guidance for college $\chi_1$ to make the targeted improvements.

| $Z_1$ | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{15}$ | $c_{16}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ | $c_{25}$ | $c_{26}$ | $c_{31}$ | $c_{32}$ | $c_{33}$ | $c_{34}$ | $c_{35}$ | $c_{36}$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $e_1$ | AH      | ML      | MH      | ML      | MH      | VL      | VH      | ML      | MH      | VH      | M       | L       | ML      | EL      | VH      | M       | ML      | AH      |
| $e_2$ | EH      | ML      | MH      | ML      | VL      | ML      | VH      | ML      | ML      | VL      | MH      | ML      | EH      | ML      | VH      | ML      | ML      | EH      |
| $e_3$ | AH      | ML      | MH      | L       | ML      | VH      | ML      | ML      | M       | ML      | MH      | VL      | VH      | EH      | ML      | EH      | AH      |
| $e_4$ | VH      | L       | M       | ML      | VH      | ML      | MH      | L       | ML      | VH      | ML      | VH      | ML      | VH      | ML      | VH      | VH      |
| $e_5$ | VH      | L       | M       | ML      | VH      | ML      | MH      | L       | ML      | VH      | ML      | VH      | ML      | VH      | ML      | VH      | VH      |

| $Z_2$ | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{15}$ | $c_{16}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ | $c_{25}$ | $c_{26}$ | $c_{31}$ | $c_{32}$ | $c_{33}$ | $c_{34}$ | $c_{35}$ | $c_{36}$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $e_1$ | EH      | VL      | ML      | ML      | M       | L       | VH      | ML      | H       | VH      | M       | L       | ML      | EL      | VH      | H       | ML      | VH      |
| $e_2$ | VH      | MH      | VH      | H       | MH      | H       | VH      | MH      | H       | VH      | ML      | H       | MH      | VL      | H       | H       | H       | VH      |
| $e_3$ | AH      | MH      | MH      | VH      | M       | VL      | VH      | ML      | M       | VL      | VH      | ML      | H       | VH      | ML      | H       | EH      | EH      |
| $e_4$ | AH      | VH      | ML      | M       | H       | VH      | L       | VH      | L       | VH      | ML      | M       | L       | VL      | VH      | ML      | H       | EH      |
| $e_5$ | VH      | VH      | M       | ML      | VH      | M       | VH      | L       | VH      | ML      | VH      | ML      | VH      | ML      | VH      | ML      | VH      | VH      |

| $Z_3$ | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{15}$ | $c_{16}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ | $c_{25}$ | $c_{26}$ | $c_{31}$ | $c_{32}$ | $c_{33}$ | $c_{34}$ | $c_{35}$ | $c_{36}$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $e_1$ | ML      | VL      | ML      | VL      | VL      | MH      | L       | ML      | VL      | L       | MH      | L       | M       | L       | VL      | L       | MH      | VH      |
| $e_2$ | MH      | H       | VH      | H       | MH      | H       | VH      | MH      | VH      | EH      | ML      | H       | MH      | VH      | ML      | H       | ML      | VH      |
| $e_3$ | VH      | H       | MH      | H       | VH      | MH      | VH      | EH      | ML      | H       | MH      | VH      | ML      | H       | ML      | H       | ML      | VH      |
| $e_4$ | ML      | VH      | MH      | ML      | ML      | VH      | MH      | L       | ML      | L       | H       | ML      | VH      | ML      | VH      | ML      | VH      | ML      |
| $e_5$ | ML      | VH      | M       | H       | VH      | VH      | H       | VH      | VH      | ML      | VH      | VH      | EH      | VH      | EH      | VH      | EH      | VH      |

| $Z_4$ | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{15}$ | $c_{16}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ | $c_{25}$ | $c_{26}$ | $c_{31}$ | $c_{32}$ | $c_{33}$ | $c_{34}$ | $c_{35}$ | $c_{36}$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $e_1$ | VH      | H       | ML      | MH      | H       | MH      | L       | ML      | MH      | L       | M       | L       | VL      | L       | VL      | L       | VH      | EH      |
| $e_2$ | MH      | H       | VH      | H       | MH      | H       | VH      | MH      | VH      | EH      | ML      | H       | MH      | VH      | ML      | H       | ML      | VH      |
| $e_3$ | ML      | VH      | VH      | H       | ML      | VH      | M       | VH      | MH      | MH      | H       | VH      | VH      | VH      | VH      | VH      | VH      | VH      |
| $e_4$ | ML      | VH      | MH      | ML      | VH      | MH      | L       | ML      | L       | H       | ML      | VH      | ML      | VH      | ML      | VH      | ML      | VH      |
| $e_5$ | VH      | VH      | M       | H       | VH      | VH      | H       | VH      | VH      | VH      | VH      | VH      | EH      | VH      | EH      | VH      | VH      | EH      |

| $Z_5$ | $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{14}$ | $c_{15}$ | $c_{16}$ | $c_{21}$ | $c_{22}$ | $c_{23}$ | $c_{24}$ | $c_{25}$ | $c_{26}$ | $c_{31}$ | $c_{32}$ | $c_{33}$ | $c_{34}$ | $c_{35}$ | $c_{36}$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $e_1$ | EH      | ML      | MH      | ML      | VL      | VH      | ML      | H       | VH      | ML      | L       | MH      | VL      | H       | ML      | EH      |
| $e_2$ | EH      | ML      | MH      | ML      | VH      | ML      | VH      | L       | ML      | L       | M       | ML      | VH      | ML      | VH      | ML      | ML      | MH      |
| $e_3$ | VH      | H       | MH      | VH      | ML      | VH      | M       | VH      | MH      | VH      | M       | VH      | MH      | VH      | VH      | VH      | VH      | VH      |
| $e_4$ | VH      | VH      | M       | H       | VH      | VH      | L       | VH      | VH      | VH      | H       | VH      | H       | H       | MH      | VH      | VH      | VH      |
| $e_5$ | VH      | VH      | H       | ML      | VH      | ML      | ML      | VH      | ML      | VH      | ML      | VH      | EH      | H       | EH      | EH      | EH      | VH      |

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Fig. 7. Normalized weights of criteria.
(2) Since the invited experts are the risk evaluators, the accuracy and reliability of decision-making results will be surely affected by their evaluations. Therefore, how to avoid the impact of abnormal evaluations on decision-making results is an important issue that needs to be solved during the decision-making. From Section 5.3.2, it is easy to learn that the weight of \( e_1 \) equals 0.145 in \( w^{2nd} \), while it equals 0.167 in \( w^{3rd} \). This result shows that the dynamic experts' weights used in this paper can effectively reduce the impact of abnormal evaluations on decision-making results. Therefore, we suggest that the decision-makers can also adopt dynamic experts' weights when conducting the group decision-makings.

(3) Fuzzy information usually appears in practical management decision-making problems. It is an important prerequisite to reasonably process the fuzzy decision-making information to obtain the reliable results. In order to reduce the loss or distortion of fuzzy information, this paper firstly transforms IVIFVs into intervals. Then, based on the mature interval theory, the classical BWM is extended to obtain the criteria weight. From Section 5.3.3, it is easy to see that the difference between the weights of the best and worst criteria derived by the proposed IVIF-BWM is 0.259, while that derived by Wang's et al. IVIF-BWM [40] is only 0.043. Thus, the proposed IVIF-BWM has more ability in distinguishing the importance of criteria, which is helpful for decision-makers to identify the significant and insignificant criteria when solving some real-world decision-making problems.

(4) The validity and superiorities of the proposed IVIF-DBWM-SPA is illustrated by a college resumption case. In future, the IVIF-DBWM-SPA is anticipated to be applied to solve other real-life decision-making problems, e.g., career determination [12], water quality assessment [24] and project assessment [42]. When the IVIF-DBWM-SPA is applied to other decision-making problems, the following two key problems need to be solved: (i) how to build a new criteria system for other decision-making problems; (ii) how to handle the fuzzy decision information offered by experts.

Despite the proposed IVIF-DBWM-SPA has strong feasibility and rationality, it still has the following disadvantages that need to be improved in the further.

(1) In order to propose the IVIF-BWM, this paper converts the IVIF information into interval information, which can reduce the information loss of IVIFVs to some certain extent. However, the conversion of IVIFVs into intervals still exists information loss. Therefore, how to construct an IVIF weight-determining model is one of the directions worth studying in the future. In addition, the extended IVIF-BWM is proposed based on the multiplicatively consistent IVIF preference relations. In fact, there also exist additively consistent IVIF preference relations. Therefore, it is expected to develop a novel IVIF-BWM based on additively consistent IVIF preference relations.

(2) Since the decision information used in this paper is described by IVIFVs, the numeric weight information might be insufficient to reflect the uncertainty of the decision environment. However, experts' weights derived by the proposed approach are still numeric numbers, which may affect the reliability of decision results. Thus, how to develop a novel weight-determining approach to obtaining fuzzy experts' weights is also a deserving research field.

(3) Since the proposed IVIF-DBWM-SPA is combined by several mature methods, the calculation workload is heavier than other methods [9,16,20,26]. If the proposed IVIF-DBWM-SPA is applied to solve more complex decision-making problems, it is suggested to design a running program to improve the solution efficiency.

CRediT authorship contribution statement

**Ze-hui Chen:** Conceptualization, Investigation, Software, Writing – original draft, Writing – review & editing. **Shu-ping Wan:** Supervision, Data curation, Writing – review & editing, Validation, Funding acquisition. **Jiu-ying Dong:** Resources, Methodology, Formal analysis, Project administration, Validation, Writing – review & editing.

Data availability

Data will be made available on request.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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