A polarised QCD condensate:

$\nu p$ elastic scattering as a probe of $U_A(1)$ dynamics

S.D. Bass

Institut für Theoretische Kernphysik, Universität Bonn,
Nussallee 14–16, D-53115 Bonn, Germany

and

Max Planck Institut für Kernphysik,
Postfach 103980, D-69029 Heidelberg, Germany

Abstract

$U_A(1)$ dynamics have the potential to induce a polarised condensate inside a nucleon. The formation of this condensate is related to the realisation of $U_A(1)$ symmetry breaking by tunneling processes such as instantons. If it is present, the polarised condensate induces a term in $g_1$ which has support only at $x = 0$. Tunneling processes then induce a net transfer of “spin” from finite $x$ to $x = 0$. The polarised condensate may be measured by comparing the flavour-singlet axial charges which are extracted from polarised deep inelastic and $\nu p$ elastic scattering experiments.
1 Introduction

Polarised deep inelastic scattering experiments at CERN [1, 2], DESY [3] and SLAC [4, 5] have revealed an apparent four standard deviations violation of OZI in the flavour-singlet axial charge $g_A^{(0)}$ which is extracted from the first moment of $g_1$ (the nucleon’s first spin dependent structure function). This result is the EMC Spin Effect. It has inspired many QCD based explanations involving the parton model [6, 7] and $U_A(1)$ dynamics [8, 9, 10].

The topological structure of the QCD vacuum [11, 12] is believed to play an important role in the physics of the $U_A(1)$ channel [12] through the axial anomaly [13, 14]. In this paper we explain how tunneling processes between vacuum states with different topological winding number may generate a polarised condensate inside a nucleon. Whether this condensate forms or not is related to the realisation of $U_A(1)$ symmetry breaking [12, 15, 16] by tunneling processes such as instantons. If a polarised condensate does form, then the vacuum inside a nucleon acquires a net axial charge relative to the vacuum outside a nucleon. The condensate induces a term in $g_1$ which has support only at Bjorken $x$ equal to zero.

Polarised deep inelastic scattering experiments measure $g_1(x, Q^2)$ between some small but finite value $x_{\text{min}}$ and an upper value $x_{\text{max}}$ which is close to one. As we decrease $x_{\text{min}} \to 0$ we measure the first moment

$$\Gamma \equiv \lim_{x_{\text{min}} \to 0} \int_{x_{\text{min}}}^{1} dx \ g_1(x, Q^2).$$

(1)

Polarised deep inelastic experiments cannot, even in principle, measure at $x = 0$ with finite $Q^2$. As noted in [17], they miss any $\delta(x)$ terms which might exist in $g_1$ at large $Q^2$. Suppose that a polarised condensate exists and that it contributes an amount $\lambda$ to the flavour-singlet axial charge $g_A^{(0)}$. The flavour-singlet axial charge which is extracted from a polarised deep inelastic experiment is $(g_A^{(0)} - \lambda)$. In contrast, elastic $Z^0$ exchange processes such as $\nu p$ elastic scattering [18] measure the full $g_A^{(0)}$ [19, 20]. One can measure a polarised condensate by comparing the flavour-singlet axial charges which are extracted from polarised deep inelastic and $\nu p$ elastic scattering experiments.

The structure of the paper is as follows. We first review (Section 2) the role of the axial anomaly in QCD based explanations of the first moment of $g_1$. We distinguish two infra-red problems: the factorisation scheme dependence of the QCD parton model [21] (a problem in perturbative QCD) and the need to ensure that the theory is invariant under topologically non-trivial gauge transformations [22, 23, 24]. The latter requirement leads us to consider the possibility that a polarised condensate may form inside a nucleon. In Section 3 we outline a dynamical mechanism for the formation of this condensate. We compare the two mechanisms of spontaneous [12] and explicit [15] breaking of $U_A(1)$ symmetry by instantons. We find that spontaneous symmetry breaking induces a polarised condensate whereas explicit symmetry breaking does not. Finally (Section 4), we discuss experiments which might be used to look for a polarised condensate in QCD.

We note that polarised condensates are observed in low temperature physics. The A phase of superfluid $^3$He behaves both as an orbital ferromagnet and uniaxial...
liquid crystal with spontaneous magnetisation along the anisotropy axis $\hat{a}$, and as a spin antiferromagnet with magnetic anisotropy along a second axis $\hat{d}$ [25].

2 The axial anomaly and $g_1$

2.1 The first moment of $g_1$

When $Q^2 \to \infty$, the light-cone operator product expansion relates the first moment of the structure function $g_1$ to the scale-invariant axial charges of the target nucleon by [26, 27, 28]

$$
\int_0^1 dx \, g_1^p(x, Q^2) = \left( \frac{1}{12} g_A^{(3)} + \frac{1}{36} g_A^{(8)} \right) \left\{ 1 + \sum_{\ell \geq 1} c_{\text{NS}} \hat{g}^{2\ell}(Q) \right\} + \frac{1}{9} g_A^{(0)} \left|_{\text{inv}} \right. \right\} \left\{ 1 + \sum_{\ell \geq 1} c_{\text{S}} \hat{g}^{2\ell}(Q) \right\} + \mathcal{O}(\frac{1}{Q^2}).
$$

Here $g_A^{(3)}$, $g_A^{(8)}$ and $g_A^{(0)} \left|_{\text{inv}} \right.$ are the isotriplet, SU(3) octet and scale-invariant flavour-singlet axial charges respectively. The flavour non-singlet $c_{\text{NS}}$ and singlet $c_{\text{S}}$ coefficients are calculable in $\ell$-loop perturbation theory [27].

The first moment of $g_1$ is fully constrained by low energy weak interaction dynamics. For proton states $|p, s\rangle$ with momentum $p_\mu$ and spin $s_\mu$

$$
2ms_\mu \, g_A^{(3)} = \langle p, s \mid (\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) \mid p, s \rangle_c
$$

$$
2ms_\mu \, g_A^{(8)} = \langle p, s \mid (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s) \mid p, s \rangle_c
$$

where the subscript $c$ denotes the connected matrix element. The isotriplet axial charge $g_A^{(3)}$ is measured independently in neutron beta decays and, modulo SU_F(3) breaking [29], the flavour octet axial charge $g_A^{(8)}$ is measured independently in hyperon beta decays. The scale-invariant flavour-singlet axial charge $g_A^{(0)} \left|_{\text{inv}} \right.$ is defined by [30]

$$
2ms_\mu g_A^{(0)} \left|_{\text{inv}} \right. = \langle p, s \mid E(g) J_{\mu_5}^{GI}(z) \mid p, s \rangle_c
$$

where

$$
J_{\mu_5}^{GI} = (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)_{GI}
$$

is the gauge-invariantly renormalized singlet axial-vector operator and

$$
E(g) = \exp \int_0^g dg' \frac{\gamma(g')}{\beta(g')}
$$

is a renormalisation group factor; $\beta(g)$ and $\gamma(g)$ are the Callan-Symanzik functions associated with the gluon coupling constant $g$ and the composite operator $J_{\mu_5}^{GI}$ respectively. We are free to choose the coupling $g(\mu)$ at either a hard or a soft scale $\mu$. The singlet axial charge $g_A^{(0)} \left|_{\text{inv}} \right.$ is independent of the renormalisation scale $\mu$. It may be measured independently in an elastic neutrino proton scattering experiment [19, 20].

Polarised deep inelastic scattering experiments at CERN [1, 2], DESY [3] and SLAC [4, 5] have verified the Bjorken sum-rule [31] for the isovector part of $g_1$ to
within 15%. They have also revealed an apparent four standard deviations violation of OZI in the flavour singlet axial charge $g_A^{(0)}_{\text{inv}}$ — for recent reviews see [28, 32]. The current experimental value of

$$g_A^{(0)}_{\text{inv}} = \Delta u_{\text{inv}} + \Delta d_{\text{inv}} + \Delta s_{\text{inv}}$$

(7)

from polarised deep inelastic scattering is [33]

$$g_A^{(0)}_{\text{inv}} = 0.28 \pm 0.07.$$  

(8)

This value is extracted assuming no $\delta(x)$ term in $g_1$. It compares with $g_A^{(8)} = 0.58 \pm 0.03$ from hyperon beta-decays [29]. Deep inelastic measurements of $g_A^{(0)}_{\text{inv}}$ involve a smooth extrapolation of the $g_1$ data to $x = 0$ which is motivated either by Regge theory or by perturbative QCD. The small $x$ extrapolation of $g_1$ data is presently the largest source of experimental error on measurements of the nucleon’s axial charges from deep inelastic scattering.

The OZI violation in $g_A^{(0)}_{\text{inv}}$ has a natural interpretation in terms of the axial anomaly [13, 14] in QCD. We now briefly review the theory of the axial anomaly in nucleon matrix elements of $J^{GI}_{\mu_5}$ (Section 2.2) and explain its application to the nucleon’s internal spin structure (Section 2.3).

### 2.2 The axial anomaly

The gauge invariant current $J^{GI}_{\mu_5}$ satisfies the anomalous divergence equation

$$\partial^\mu J^{GI}_{\mu_5} = 2f \partial^\mu K_\mu + \sum_{i=1}^f 2im_i q_i \gamma_5 q_i$$

(9)

where

$$K_\mu = \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[ A^\nu_a \left( \partial^\rho A^\sigma_a - \frac{1}{3} g f_{abc} A^\nu_b A^\sigma_c \right) \right]$$

(10)

is a renormalized version of the gluonic Chern-Simons current and the number of light flavours $f$ is 3.

Equ.(9) allows us to write

$$J^{GI}_{\mu_5} = J^{\text{con}}_{\mu_5} + 2f K_\mu$$

(11)

where $J^{\text{con}}_{\mu_5}$ and $K_\mu$ satisfy the divergence equations

$$\partial^\mu J^{\text{con}}_{\mu_5} = \sum_{i=1}^f 2im_i q_i \gamma_5 q_i$$

(12)

and

$$\partial^\mu K_\mu = \frac{g^2}{8\pi^2} G^\mu_\nu \tilde{G}^\nu_\mu.$$

(13)

Here $\frac{g^2}{8\pi^2} G^\mu_\nu \tilde{G}^\nu_\mu$ is the topological charge density. $J^{GI}_{\mu_5}$ is the only flavour-singlet, gauge-invariant, twist-two operator with $J^{PC} = 1^{++}$. It is multiplicatively renormalised with a finite anomalous dimension $\gamma(g)$ which starts at two loops in perturbation theory. The partially conserved current is scale invariant, viz.

$$J^{\text{con}}_{\mu_5} = J^{\text{con}}_{\mu_5} |_{\mu_0^2}.$$  

(14)
It follows that the scale dependence of $J_{\mu 5}^{GI}$ is carried entirely by $K_{\mu}$. One finds

$$K_{\mu}|_{\mu^2} = Z(\mu^2, \mu_0^2)K_{\mu}|_{\mu_0^2} + \left(Z(\mu^2, \mu_0^2) - 1\right)J_{\mu 5}^{con}$$  \hspace{1cm} (15)$$

where

$$Z(\mu^2, \mu_0^2) = \exp \int_{g(\mu_0^2)}^{g(\mu^2)} dg' \gamma(g')/\beta(g')$$  \hspace{1cm} (16)$$

is the renormalisation group factor; $Z(\mu^2, \mu_0^2) \rightarrow E(g)$ in the limit that $\mu_0 \rightarrow \infty$.

When we make a gauge transformation $U$ the gluon field transforms as

$$A_{\mu} \rightarrow UA_{\mu}U^{-1} + \frac{i}{g}(\partial_{\mu}U)U^{-1}$$  \hspace{1cm} (17)$$

and the operator $K_{\mu}$ transforms as

$$K_{\mu} \rightarrow K_{\mu} + \frac{i}{16\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial' \left(U^\dagger \partial' U A^\beta\right) + \frac{1}{96\pi^2}\epsilon_{\mu\nu\alpha\beta} \left[(U^\dagger \partial' U)(U^\dagger \partial' U)(U^\dagger \partial' U)\right].$$  \hspace{1cm} (18)$$

Gauge transformations shuffle a scale invariant operator quantity between the two operators $J_{\mu 5}^{con}$ and $K_{\mu}$ whilst keeping $J_{\mu 5}^{GI}$ invariant. Since $K_{\mu}$ satisfies the divergence equation (13) it follows that the $U$ dependent terms on the right hand side of (18) must be conserved. The second term on the right hand side of (18) is clearly conserved by itself, whence we conclude that

$$C_{\mu} = \frac{1}{96\pi^2}\epsilon_{\mu\nu\alpha\beta} \left[(U^\dagger \partial' U)(U^\dagger \partial' U)(U^\dagger \partial' U)\right]$$  \hspace{1cm} (19)$$

is also conserved.

The nucleon matrix element of $J_{\mu 5}^{GI}$ is

$$\langle p, s | J_{5\mu}^{GI}(0) | p', s \rangle = 2m \left[s_{\mu}G_A(l^2) + l_{\mu}l.sG_P(l^2)\right]$$  \hspace{1cm} (20)$$

where $l_{\mu} = (p' - p)_{\mu}$. Since $J_{5\mu}^{GI}(0)$ does not couple to a massless Goldstone boson it follows that $G_A(l^2)$ and $G_P(l^2)$ contain no massless pole terms. The forward matrix element of $J_{5\mu}^{GI}(0)$ is well defined and

$$g_A^{(0)}|_{inv} = E(g)G_A(0).$$  \hspace{1cm} (21)$$

The matrix elements of $K_{\mu}$ need to be specified with respect to a specific gauge. In a covariant gauge we can write

$$\langle p, s | K_{\mu}(0) | p', s \rangle = 2m \left[s_{\mu}K_A(l^2) + l_{\mu}l.sK_P(l^2)\right]$$  \hspace{1cm} (22)$$

where $K_P$ contains a massless Kogut-Susskind pole [34]. This massless pole cancels with a corresponding massless pole term in $(G_P - K_P)$. In an axial gauge $n.A = 0$ the matrix elements of the gauge dependent operator $K_{\mu}$ will also contain terms proportional to the gauge fixing vector $n_{\mu}$. 

4
We may define a gauge-invariant form-factor $\chi^g(l^2)$ for the topological charge density (13) in the divergence of $K_\mu$:

$$2ml.s\chi^g(l^2) = \langle p, s | \frac{g^2}{8\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}(0) | p', s \rangle_c.$$  \hfill (23)

Working in a covariant gauge, we find

$$\chi^g(l^2) = K_A(l^2) + l^2 K_P(l^2)$$  \hfill (24)

by contracting Eq.(22) with $l^\mu$. When we make a gauge transformation any change $\delta g_t$ in $K_A(0)$ is compensated by a corresponding change in the residue of the Kogut-Susskind pole in $K_P$, viz.

$$\delta g_t[K_A(0)] + \lim_{l^2 \to 0} \delta g_t[l^2 K_P(l^2)] = 0.$$  \hfill (25)

The residue of the Kogut-Susskind pole and, hence, $K_A(0)$ is invariant under the “small” gauge transformations of perturbative QCD \footnote{The Kogut-Susskind pole corresponds to the Goldstone boson associated with spontaneously broken $U_A(1)$ symmetry. There is no Kogut-Susskind pole in perturbative QCD.}. (“Small” gauge transformations are those which are topologically deformable to the identity.) It changes only under “large” gauge transformations which change the gluonic boundary conditions at infinity or, equivalently, change the topological winding number.

The functional variation of the integral of $C_\mu[U]$ over a three dimensional sub-manifold $\mathcal{V}$ in Minkowski space can be written as an integral over the boundary $\partial\mathcal{V}$ of $\mathcal{V}$ \footnote{The Kogut-Susskind pole corresponds to the Goldstone boson associated with spontaneously broken $U_A(1)$ symmetry. There is no Kogut-Susskind pole in perturbative QCD.}. The integral is insensitive to local deformations of $U$. In contrast, the second term on the right hand side of (18) and its integral are sensitive to local deformations of both $A_\mu$ and $U$. This second term is a total divergence and its matrix elements vanish in the forward direction. The matrix elements of $C_\mu$ are sensitive to topological structure. They include the Kogut-Susskind pole. Any change in $K_A(0)$ under a “large” gauge transformation is associated with $C_\mu$.

To summarise, in a covariant gauge the forward matrix elements of $K_\mu$ change under “large” gauge transformations but not under “small” gauge transformations. This change is associated with $C_\mu$, which acts as a topological current for gauge transformations \footnote{The Kogut-Susskind pole corresponds to the Goldstone boson associated with spontaneously broken $U_A(1)$ symmetry. There is no Kogut-Susskind pole in perturbative QCD.}.

One can find axial gauges where the forward matrix element of a given component of $K_\mu$ is invariant under residual gauge transformations. In the light-cone gauge $A_+ = 0$ the non-abelian three-gluon part of $K_+$ and, hence, $C_+$ vanishes. The forward matrix elements of $K_+$ are invariant under all residual gauge degrees of freedom in the light-cone gauge.

\subsection*{2.3 The anomaly and the first moment of $g_1$}

We now discuss the application of this theory to polarised deep inelastic scattering. The structure function $g_1$ appears in the anti-symmetric, spin dependent part of the hadronic tensor $W^{\mu\nu}$ for $ep$ scattering, viz.

$$\frac{1}{2m}W_A^{\mu\nu} = i\epsilon^{\mu\nu\rho\sigma} q_\rho s_\sigma \left( \frac{1}{p.q} g_1(x, Q^2) + [p.qs_\sigma - s.qp_\sigma] \frac{1}{m^2 p.q} g_2(x, Q^2) \right).$$  \hfill (26)
Let $q_\mu$ denote the momentum of the exchanged photon. The kinematics of polarised deep inelastic scattering are $Q^2 = -q^2$ and $p.q \to \infty$ with $x = \frac{q^2}{2p.q}$ held fixed. In light-cone coordinates $q_- \to \infty$ with $q_+ = -xp_+$ finite. In this Bjorken limit the leading term in $W_{\mu\nu}^A$ is obtained by taking $\sigma = +$. (Other terms are suppressed by powers of $\frac{1}{q_-}$.) Understanding the first moment of $g_{1}$ in terms of the matrix elements of anomalous currents ($J_{5\mu}^\text{con}$ and $K_{\mu}$) is a problem in understanding the forward matrix element of $K_+$. Here we are fortunate in that the parton model is formulated in the light-cone gauge where the forward matrix elements of $K_+$ are invariant. Furthermore, in this gauge, $K_+$ measures the gluonic “spin” content of the polarised target [36, 37]. We find [6, 7]

$$G_A^{(A_+ = 0)}(0) = \sum_q \Delta q_{\text{con}} - \int \frac{\alpha_s}{2\pi}\Delta g$$

(27)

where

$$\langle p, s|J_{5\mu}^\text{con}(0)|p, s\rangle_c = 2ms_+\Delta q_{\text{con}}$$

(28)

and

$$\langle p, s|K_{\mu}(0)|p, s\rangle_c = 2ms_+\left(-\frac{\alpha_s}{2\pi}\Delta g\right).$$

(29)

(Here $\alpha_s = \frac{g^2}{4\pi}$.) To obtain the parton model description of $g_A^{(0)}$ the factor $-\frac{\alpha_s}{2\pi}$ is then disassociated with the operator $K_+$ and reassigned with the first moment of the coefficient function of the polarised gluon distribution, viz. [6, 7]

$$\int_0^1 \Delta C_{A_+ = 0}^{(g)} = -\frac{\alpha_s}{2\pi}.$$  

(30)

The gluonic term in Eq. (27) offers a possible source for the OZI violation in $g_A^{(0)}_{\text{inv}}$. In the parton model, the axial anomaly provides a local measurement of the gluon polarisation $\Delta g$. If we calculate the first moment of the box graph for photon-gluon fusion then we find the sum of two contributions [21]:

$$\int_0^1 \,dx g_1^{\gamma\gamma^*g} = \frac{\alpha_s}{2\pi}\left[1 - \frac{2m_q^2}{p^2}\frac{1}{\sqrt{1 - 4m_q^2/p^2}}\ln\left(\frac{1 - \sqrt{1 - 4m_q^2/p^2}}{1 + \sqrt{1 - 4m_q^2/p^2}}\right)\right].$$

(31)

Here $-p^2$ is the virtuality of the gluon and $m_q$ is the mass of the quark which is liberated into the final state. (The photon-gluon fusion is calculated in the Bjorken limit $Q^2 \gg -p^2, m_q^2$.) The unity term is the anomaly. It comes from the region of phase space where the hard photon scatters on a quark or antiquark carrying transverse momentum squared $k_T^2 \sim Q^2$ [5]. The second term comes from the kinematic region $k_T^2 \sim -p^2, m_q^2$ — it is the gluon matrix element of $m_q [\bar{q}^{\gamma\gamma^*}q]$. If we make a cut-off $\lambda$ on the quark transverse momentum where $Q^2 \gg \lambda^2 \gg -p^2, m_q^2$, then we recover just the anomaly term as a local measurement of the gluon’s spin [4]. If we make a cut-off on, say, the quark virtuality $-k^2$ which mixes transverse and longitudinal momentum components, then we obtain half of the anomaly in the corresponding gluon coefficient [21] when the cut-off $\lambda$ is chosen such that $Q^2 \gg \lambda^2 \gg -p^2, m_q^2$.

\[\text{The transverse momentum is defined as orthogonal to the two-dimensional plane spanned by} \]

the momenta of the incident photon and the target gluon.
The factorisation scheme dependence of the decomposition of $G_A(0)$ into quark and gluonic contributions has been studied at length in Refs.\cite{21, 37, 38}. The transverse momentum cut-off approach provides the most natural link to the anomaly.

If we were to work only in the light-cone gauge we might think that we have a complete parton model description of the first moment of $g_1$. However, one is free to work in any gauge including a covariant gauge where the forward matrix elements of $K_+$ are not invariant under “large” gauge transformations. It remains an open question whether the net non-perturbative quantity which is shuffled between $K_A(0)$ and $(G_A - K_A)(0)$ under “large” gauge transformations is finite or not. If it is finite and, therefore, physical, then, when we choose $A_+ = 0$, it must be frozen into some combination of $\Delta q_{\text{con}}$ and $\Delta g$ in Eqs.(27-29).

The topological winding number depends on the gluonic boundary conditions at infinity. It is insensitive to local deformations of the gluon field $A_\mu(z)$ or of the gauge transformation $U(z)$. When we take the Fourier transform to momentum space the topological structure induces a light-cone zero-mode which can contribute to $g_1$ only at $x = 0$. Hence, we are led to consider the possibility that there may be a term in $g_1$ which is proportional to $\delta(x)$.

It is worthwhile to stop and contrast the two different infra-red problems that one has to worry about when understanding the first moment of $g_1$. The factorisation scheme dependence of the decomposition of $G_A(0)$ into a quark term and a gluonic term is a problem in perturbative QCD, which is derived from the QCD Lagrangian by demanding invariance under “small” gauge transformations. The problem of the (non-)invariance of the forward matrix elements of $K_+$ is about the topological structure of the QCD vacuum — that is, invariance under “large” gauge transformations.

3 The polarised condensate and $U_A(1)$ symmetry

We now explain how tunneling processes may induce a polarised condensate inside a nucleon. The formation of this polarised condensate is related to the realisation of $U_A(1)$ symmetry breaking \cite{12, 15, 16} by instantons.

3.1 Realisations of $U_A(1)$ symmetry

Let us choose $A_0 = 0$ gauge and define two operator charges:

$$X(t) = \int d^3z J_{05}^{GI}(z) \tag{32}$$

and

$$Q_5(t) = \int d^3z J_{05}^{\text{con}}(z) \tag{33}$$

corresponding to the gauge-invariant and partially conserved axial-vector currents respectively.

When topological effects are taken into account, the QCD vacuum $|\theta\rangle$ is a coherent superposition

$$|\theta\rangle = \sum_m e^{im\theta} |m\rangle \tag{34}$$
of the eigenstates $|m\rangle$ of $\int d\sigma K^\mu \neq 0$ \cite{11, 29} (for a recent review, see \cite{40}). Here $\sigma^\mu$ is a large surface which is defined \cite{39} such that its boundary is spacelike with respect to the positions $z_k$ of any operators or fields in the physical problem under discussion. For integer values of the topological winding number $m$, the states $|m\rangle$ contain $mf$ quark-antiquark pairs with non-zero $Q_5$ chirality $\sum_1 \chi_l = -2fm$ where $f$ is the number of light-quark flavours. Relative to the $|m = 0\rangle$ state, the $|m = +1\rangle$ state carries topological winding number $+1$ and $f$ quark-antiquark pairs with $Q_5$ chirality equal to $-2f$.

There are two schools of thought \cite{12, 16} about how instantons break $U_A(1)$ symmetry. Both of these schools start from 't Hooft’s observation \cite{13} that the flavour determinant

$$\langle \text{det} \left[ q_L^i q_R^j(z) \right] \rangle_{\text{inst.}} \neq 0 \quad (35)$$

in the presence of a vacuum tunneling process between states with different topological winding number. (We denote the tunneling process by the subscript “inst.”. It is not specified at this stage whether “inst.” denotes an instanton or an anti-instanton.)

(a) **Explicit $U_A(1)$ symmetry breaking**

In this scenario \cite{12, 16} the $U_A(1)$ symmetry is associated with the current $J_{\mu 5}^{GI}$ and the topological charge density is treated like a mass term in the divergence of $J_{\mu 5}^{GI}$. The quark chiralities which appear in the flavour determinant (35) are associated with $X(t)$ so that the net axial charge $g_A^{(0)}$ is not conserved ($\Delta X \neq 0$) and the net $Q_5$ chirality is conserved ($\Delta Q_5 = 0$) in quark instanton scattering processes.

In QCD with $f$ light flavoured quarks the (anti-)instanton “vertex” involves a total of $2f$ light quarks and antiquarks. Consider a flavour-singlet combination of $f$ right-handed ($Q_5 = +1$) quarks incident on an anti-instanton. The final state for this process consists of a flavour-singlet combination of $f$ left-handed ($Q_5 = -1$) quarks; $+2f$ units of $Q_5$ chirality are taken away by an effective “schizon” which carries zero energy and zero momentum \cite{16}. The “schizon” is introduced to ensure $Q_5$ conservation. The non-conservation of $g_A^{(0)}$ is ensured by a term coupled to $K^\mu$ with equal magnitude and opposite sign to the “schizon” term which also carries zero energy and zero momentum. This gluonic term describes the change in the topological winding number which is induced by the tunneling process. The anti-instanton changes the net $U_A(1)$ chirality by an amount $(\Delta X = -2f)$.

This picture is the basis of ’t Hooft’s effective instanton interaction \cite{15}.

(b) **Spontaneous $U_A(1)$ symmetry breaking**

In this scenario \cite{12, 33} the $U_A(1)$ symmetry is associated with the partially-conserved axial-vector current $J_{\mu 5}^{\text{con}}$. Here, the quark chiralities which appear in the flavour determinant (35) are identified with $Q_5$. With this identification, the net axial charge $g_A^{(0)}$ is conserved ($\Delta X = 0$) and the net $Q_5$ chirality is not conserved ($\Delta Q_5 \neq 0$) in quark instanton scattering processes. This result is the opposite to what happens in the explicit symmetry breaking scenario.
When \( f \) right-handed quarks scatter on an instanton the final state involves \( f \) left-handed quarks. There is no "schizon" and the instanton induces a change in the net \( Q_5 \) chirality \( \Delta Q_5 = -2f \). The conservation of \( g_A^{(0)} \) is ensured by the gluonic term coupled to \( K_\mu \) which measures the change in the topological winding number and which carries zero energy and zero momentum. The charge \( Q_5 \) is time independent for massless quarks (where \( J_{\mu 5}^{\text{con}} \) is conserved).

Since \( \Delta Q_5 \neq 0 \) in quark instanton scattering processes we find that the \( U_A(1) \) symmetry is spontaneously broken by instantons. The Goldstone boson is manifest \(^{[12]} \) as the massless Kogut-Susskind pole which couples to \( J_{\mu 5}^{\text{con}} \) and \( K_\mu \) but not to \( J_{\mu 5}^{\text{GI}} \) — see Eq.(22).

In the rest of this paper we explain why these two possible realisations of \( U_A(1) \) symmetry have a different signature in \( \nu p \) elastic scattering.

### 3.2 Formation of the polarised condensate

In both the explicit and spontaneous symmetry breaking scenarios we may consider multiple scattering of the incident quark first from an instanton and then from an anti-instanton. Let this process recur a large number of times. When we time-average over a large number of such interactions, then the time averaged expectation value of the chirality \( Q_5 \) carried by the incident quark is reduced from the naive value \(+1\) that it would take in the absence of vacuum tunneling processes. Indeed, in one flavour QCD the time averaged value of \( Q_5 \) tends to zero at large times \(^{[9, 23]} \).

In the spontaneous \( U_A(1) \) symmetry breaking scenario \(^{[12]} \) any instanton induced suppression of the flavour-singlet axial charge which is measured in polarised deep inelastic scattering is compensated by a net transfer of axial charge or "spin" from partons carrying finite momentum fraction \( x \) to a polarised \( U_A(1) \) condensate at \( x = 0 \). The polarised condensate induces a flavour-singlet \( \delta(x) \) term in \( g_1 \) which is not present in the explicit \( U_A(1) \) symmetry breaking scenario. The net polarised condensate is gauge invariant. In the \( A_0 = 0 \) gauge the condensate polarisation is "gluonic" and is measured by \( \int d^3 z K_0 \). In the light-cone gauge the polarisation of the condensate may be re-distributed between the "quark" and "gluonic" terms measured by \( J_{\mu 5}^{\text{con}} \) and \( K_\mu \) respectively.

### 4 How to measure a polarised condensate

Polarised deep inelastic scattering experiments measure \( g_1 \) between a small but finite value of \( x \) and a value of \( x \) which is close to one. They cannot, even in principle, measure directly at \( x = 0 \). However, there are rigorous sum-rules for the first moment of \( g_1 \) for both proton and photon targets. These sum-rules include any contribution from the end-point \( x = 0 \).

In the case of a polarised real photon target one finds \(^{[11]} \)

\[
\int_0^1 dx g_1^\gamma(x, Q^2) = 0. \tag{36}
\]

\(^{5}\text{cf. an anti-instanton in the explicit } U_A(1) \text{ symmetry breaking scenario.} \)
This sum-rule follows from electromagnetic gauge invariance together with the absence of any massless pole contributions to the photon matrix elements of the axial vector currents. It has been generalised to a virtual photon target in [12]. If a polarised condensate exists in, for example, the hadronic (vector-meson dominance) part of the photon wavefunction, then we would expect to see a violation of this sum-rule in a polarised deep inelastic experiment.

For a proton target, the scale-invariant flavour singlet axial charge can be measured independently in an elastic neutrino proton scattering experiment [18]. Rigorous QCD renormalisation group arguments tell us that the neutral current axial charge which is measured in elastic $\nu p$ scattering is [19]

$$g_A^{(Z)} = \frac{1}{2} g_A^{(3)} + \frac{1}{6} g_A^{(8)} - \frac{1}{6} (1 + C) g_A^{(0)} \mid_{\text{inv}} + O(\frac{1}{m_h}).$$

(37)

Here $C$ denotes the leading order heavy-quark contributions to $g_A^{(Z)}$ and $m_h$ is the heavy-quark mass. Numerically, $C$ is a $\simeq 6 - 10\%$ correction [19, 20] — within the present experimental error on $g_A^{(0)} \mid_{\text{inv}}$. The flavour-singlet axial charge in Eq.(37) includes any contribution from a polarised condensate [6].

A polarised $U_A(1)$ condensate contributes to the value of $g_A^{(0)} \mid_{\text{inv}}$ which is extracted from $\nu p$ elastic scattering but not to the flavour-singlet axial charge which is extracted from a polarised deep inelastic experiment. A precise measurement of $\nu p$ elastic scattering would make an important contribution to our understanding of the internal spin structure of the nucleon and help to resolve the issue of spontaneous versus explicit breaking of $U_A(1)$ symmetry by instantons.

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6 Heavy-quark condensates contribute only at $O(1/m_h)$ in the heavy-quark mass $m_h$ [13]. It follows that the coefficient of any heavy-quark $\delta(x)$ term in $g_1$ decouples as $O(1/m_h)$. It does not affect our derivation of the relation between polarised deep inelastic scattering and $\nu p$ elastic scattering.
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