The warped extra–dimensional scenario proposed by Randall and Sundrum (RS) [1] is a particularly attractive extension of the Standard Model (SM). In addition to its original motivation, to provide a solution to the gauge hierarchy problem, this scenario turned out to represent a suitable framework to address several important phenomenological issues. For instance, the version of the model with bulk matter provides new weakly interacting candidates for dark matter in the universe [2], allows for the unification of the gauge couplings at high–energies [3] and proposes a new interpretation of SM fermion mass hierarchies based on specific localizations of the fermion wave functions along the warped extra dimension [4].

Indeed, if the fermions are placed differently along the extra dimension, hierarchical patterns among the effective four–dimensional Yukawa couplings are generated as a result of their various wave function overlapping with the Higgs boson, which remains confined on the so–called TeV–brane for its mass to be protected. A parameter denoted $c_f$ quantifies the five–dimensional mass affected to each fermion representation, $\pm c_f k$ where $1/k$ is the anti–de Sitter (AdS$_5$) curvature radius, and fixes the fermion localization with respect to the TeV–brane. As the parameter $c_f$ decreases, the zero–mode fermions become closer to the TeV–brane and acquire a larger mass. Remarkably, this geometrical mechanism of mass generation is possible for values $|c_f| \sim 1$, i.e. for fundamental mass parameters of the order of the unique scale of the theory: the reduced Planck mass scale $M_P \sim k$.

If this extra–dimensional model is to solve the gauge hierarchy problem, the masses of the first Kaluza–Klein (KK) excitations of the SM gauge bosons must be in the vicinity of the TeV scale but must be treated separately [9]. In fact, since third generation fermions interact more strongly with the KK gauge bosons, that mix with the Z–boson, one can even solve the anomaly observed in the forward–backward $b$–quark asymmetry measured at LEP [3, 10], the only (high–energy) observable that deviates significantly from the SM prediction.

In this extension of the SM group, the right–handed fermions are promoted to SU(2)$_R$ isodoublets. A new quark $b_R'$, the SU(2)$_R$ partner of the right–handed top quark $t_R$, is present and should be typically much lighter than the other SU(2)$_R$ partners and all KK excitations of the SM fermions [2], the latter ones being by construction heavier than the KK gauge bosons. Indeed, as the SU(2)$_R$ symmetry is broken by boundary conditions [7], the $b_R'$ quark has Dirichlet boundary conditions on the Planck–brane and Neumann ones on the TeV–brane [11], noted $(-+)$, so that it has no zero–mode, in contrast to the SM fermions which have $(++)$ boundary conditions. Moreover, the mass of
the first KK excitation of the $b'_R$ should be relatively low as it is controlled by the same $c_t$ parameter as the top quark $t_R$ which must be sufficiently small in order to generate a large $m_t$ value.

In this scenario, the presence of the heavy new quark, although its Higgs coupling is not proportional to the KK mass, could significantly alter the phenomenology of the Higgs particle which, otherwise, is expected to have similar properties as the SM H boson \[12\]. As a matter of fact, the $b'$ state will mix with the $b$ quark and will slightly modify the $Hbb$ Yukawa coupling which controls the decay branching ratios of the Higgs boson if its mass is in the range $M_H \lesssim 140$ GeV. A second and more drastic consequence of the presence of a relatively light $b'$ state is that it will contribute to the main production channel for a SM–like Higgs boson at the Large Hadron Collider (LHC): the gluon–gluon fusion mechanism $gg \rightarrow H$. This process proceeds through a triangular loop built up by heavy quarks, Fig. 1, and in the SM only the contribution of the top quark is significant as a result of the much larger Yukawa coupling (the $b$–quark contribution is smaller than 10% even for low Higgs masses).

![FIG. 1: Feynman diagram for the $gg \rightarrow H$ production process.](image)

We will show in this paper that the $b'$ contribution to the $Hgg$ loop amplitude can be significant for model parameters that allow to reproduce the measured $m_b$ and $m_t$ values. The Higgs production cross section in the $gg \rightarrow H$ process can be enhanced by a factor of four compared to its SM value.

This is in contrast to the other fermion KK excitations which decouple from the $Hgg$ amplitude as they are expected to be much heavier \[13\]. Note, however, that within the universal extra–dimensional scenario where the Higgs boson propagates in the bulk, the KK fermion modes can be light enough to induce a sizable modification of the $gg \rightarrow H$ production rate \[14\].

To calculate the $Hgg$ amplitude, one needs first to derive the $t, b'$ couplings to the Higgs boson. The Yukawa terms take an invariant form under the custodial symmetry, as the SM SU(2)$_L$ quarks are singlets under SU(2)$_R$ whereas the $H$ field is embedded into a bidoublet. The whole $b$–quark mass matrix needs to be studied as the Yukawa couplings induce mixings with the KK excitations; the $t$–quark mass matrix has a similar structure. For simplicity, we describe only the largely dominant third family contribution and consider only the first KK excitations that we denote with the exponent $n$ in brackets $n=1, 2, \ldots$ which labels the KK–level. In the field basis $\Psi_L \equiv (b_L^{(0)}, b_L^{(1)}, b_L^{(2)}, b'_L, b'_L)^t$, $\Psi_R \equiv (b_R^{(0)}, b_R^{(1)}, b_R^{(2)}, b'_R, b'_R)^t$ where we introduce the charge conjugated fields (indicated by the superscript $c$) in order to use only left–handed SM fields, the effective four–dimensional bottom quark mass terms are of Dirac type, $\mathcal{L}_{m} = \bar{\Psi}_L \mathcal{M}_b \Psi_R + h.c.$.. After electroweak symmetry breaking, one obtains for the $5 \times 5$ mass matrix $\mathcal{M}_b$, in terms of the $c_Q$ and $c_b$ parameters associated, respectively, to the SM doublet $Q_L^c = (t_L, b_L)^t$ and singlet $b'_R$:

$$
\mathcal{M}_b = 
\begin{pmatrix}
\bar{\nu} f_{cQ}^{(0)} f_{cQ}^{(0)} & \bar{\nu} f_{cQ}^{(0)} f_{cQ}^{(1)} & \bar{\nu} f_{cQ}^{(0)} f_{cQ}^{(2)} & \bar{\nu} f_{cQ}^{(1)} f_{cQ}^{(1)} & \bar{\nu} f_{cQ}^{(1)} f_{cQ}^{(2)} \\
0 & m_{cQ}^{(1)} & m_{cQ}^{(2)} & 0 & 0 \\
0 & 0 & 0 & m_{c_b}^{(1)} & 0 \\
0 & 0 & 0 & 0 & m_{c_b}^{(2)}
\end{pmatrix}
$$

with $\bar{\nu} = -\lambda_5 v/\sqrt{2} R$, $\lambda_5$ being the five–dimensional dimensionful Yukawa coupling constant, taken equal to $k^{-1}$ as usual to avoid the introduction of an additional scale, $v \simeq 246$ GeV being the Higgs vacuum expectation value and $R$ the compactification radius. $f_{c^a}/\sqrt{\pi R}$ and $g_{c^a}/\sqrt{\pi R}$ stand for the
wave functions of the \( n \)-th KK mode of a field characterized by the \( c \) parameter and, respectively, \((++)\) and \((-+)\) boundary conditions; all wave function values are taken at the position of the TeV–brane, \( x_5 = \pi R \). Finally, \( m_c^{(n)} \) and \( m_c^{(\prime n)} \) are the \( n \)-th KK masses for, respectively, the \((++)\) and \((-+)\) fields; \( m_c^{(\prime 1)} \) decreases with the parameter \( c \) and can reach particularly small values \[\text{[2]}\]. The zeroes in the matrix eq. \([1]\) originate from the fact that the fields \( b_1^{(1)} \), \( b_L^{(1)} \) and \( b_R^{(n)} \) (with \( n = 1, 2 \)) have Dirichlet boundary conditions on the TeV–brane and, thus, do not couple to the Higgs boson. The matrix \([1]\) is diagonalized by unitary matrices \( U_{L/R} \), through the transformation \( \Psi'_{L/R} = U_{L/R} \Psi_{L/R} \):

\[
U_L \mathcal{M}_b U_R^\dagger = \text{diag} \left( m_{b_1}, m_{b_2}, \ldots \right)
\]

where \( m_{b_i} \) corresponds to the measured value of the bottom quark mass (we take the unitary matrices such that \( m_{b_1} < m_{b_2} < \ldots \)). The \( \Psi'_{L/R} \) components are the mass eigenstates, namely \( \Psi''_{L/R} \equiv (b_{1L/R}, b_{2L/R}, \ldots)^t \), where generally \( b_{1L/R} \) is mainly composed of the bottom quark zero–mode and \( b_{2L/R} \) of the \( b_{1L/R}^{(1)} \). Then, the Higgs couplings are given by the interaction Lagrangian

\[
\mathcal{L}_{\text{int}} = \frac{H}{v} \Psi'_{L/R} \mathcal{M}_b'' \Psi'_{L/R} + \text{h.c.}
\]

where \( \mathcal{M}_b'' = U_L \mathcal{M}_b'' U_R^\dagger \) and \( \mathcal{M}_b' \) is the matrix of eq. \([1]\) but with the KK masses set to zero. Hence, the Higgs coupling to an eigenstate \( b_{L/R} \) is given by \( \mathcal{M}_{b_{ii}'} / v \) in contrast to the usual \( m_b / v \) value in the SM. Besides, the orthonormality condition for the fermion wave functions, together with the flatness of the gluon zero–mode wave function, lead to a gluon coupling with \( b_{L/R} \) states which is identical to the SM quark–gluon coupling.

In order to obtain the value for each effective quark coupling to the Higgs boson, we have diagonalized numerically the mass matrix \([1]\); to reach a good degree of convergence and an accurate determination of the quark masses and couplings, we have included states up to \( b_5 \) and \( t_5 \) in the sum over KK modes.

We are now in a position to discuss the new state contributions to the \( gg \to H \) production rate at the LHC. To lowest order, the partonic cross section reads \[\text{[15]}\]

\[
\sigma_H = \frac{G_F \alpha_s^2 M_H^2}{288\sqrt{2}\pi} \left| \sum_Q A_{1/2}^H(\tau_Q) \right|^2 \delta(\hat{s} - M_H^2)
\]

where \( \hat{s} \) is the \( gg \) invariant energy squared. The form factor \( A_{1/2}^H(\tau_Q) \) with \( \tau_Q = M_H^2 / 4m_Q^2 \) is normalized such that for \( m_Q \gg M_H \), it reaches unity while it approaches zero in the chiral limit \( m_Q \to 0 \). In fact, the approximation \( A_{1/2}^H \approx 1 \) is very good for Higgs masses below the heavy quark threshold, \( M_H \lesssim 2m_Q \) \[\text{[16]}\]. Since high–precision data suggest that the Higgs boson is relatively light, \( M_H \lesssim 200 \text{ GeV} \) \[\text{[6]}\], and the new quarks are expected to be heavier than the top quark (otherwise they would have probably been observed at the Tevatron \[\text{[6]}\]) this approximation holds and will be adopted here.

It is convenient to consider the ratio \( \mathcal{R} = \sigma_{H}^{\text{RS}} / \sigma_{H}^{\text{SM}} \) of the \( gg \to H \) production cross sections in the RS and SM models which, including the contributions of the first KK excitations reads

\[
\mathcal{R} \equiv \frac{\sigma_{H}^{\text{RS}}}{\sigma_{H}^{\text{SM}}} \approx \left| \sum_{i=1}^{5} \frac{\mathcal{M}_{bi i}'}{m_{ti}} + \sum_{j=2}^{5} \frac{\mathcal{M}_{b j j}''}{m_{b_j}} \right|^2,
\]

if we neglect the relatively small \( b \)-quark contribution and use the approximation \( A_{1/2}^H \approx 1 \) discussed above. Note that the higher order QCD corrections, which are known to be rather large \[\text{[16]}\], are essentially the same for all quark species and, thus, drop in the ratio \( \mathcal{R} \).
FIG. 2: Values of the ratio \( R = \sigma_{R}^{RS} / \sigma_{SM}^{R} \) in the plane \([c_t, c_Q]\) for a fixed value \( c_b = 0.6 \). The filled regions correspond from white to darkest grey to, respectively, the intervals \( R \in [1, 1.2] \), \([1.2, 2] \), \([2, 4] \) and \( R > 4 \). The green dotted–lines are the contour levels for \( m_b = 1-4.5 \) GeV, the red solid–lines for \( m_t = 150-200 \) GeV (both from right to left), while the blue dashed –lines are associated to \( m_{b_2} = 200-1000 \) GeV (from down to up). We have set \( k_R = 10^{11} \) and \( k \) such that \( M_{KK} = 3 \) TeV and restricted to values \(|c| = O(1)|.\)

Figure 2 displays domains of the \([c_Q, c_t]\) parameter space corresponding to certain values of the ratio \( R \) for the choice \( c_b = 0.6 \); also represented are the regions in which the bottom and top quark masses \( m_b \) and \( m_t \) are close to their measured value [17]. These masses typically increase as the associated \( c \) parameters decrease, as a result of the geometrical mechanism for zero–modes mentioned previously, and the change of regime in \( m_b \) for \( c_t \lesssim -0.4 \) is due to \( b^{(0)}-b^{(1)} \) mixing. In most of the regions consistent with the correct range for both \( m_t \) and \( m_b \), the \( gg \to H \) cross section in the RS model is enhanced with respect to the SM case as a result of a constructive interference of the \( t_1 \) and \( b_2 \) loop contributions [21]. In these areas, the effective \( Ht\bar{t} \) coupling is quasi unaffected (i.e. at most at the percent level only) by mixing effects so that almost no correction to the SM amplitude is generated from the top quark exchange. The deviation of the cross section is almost entirely due to the exchange of the \( b_2 \) state in the loop as its mass is smaller than the other heavy states. In the regions with realistic \( t, b \) masses, \( m_{b_2} \) can be as low as several hundred GeV (for \( c_t \lesssim -0.3 \) leading to \( m_{c_t}^{(1)} \lesssim 1 \) TeV) while e.g. \( m_{t_2} > 3 \) TeV. For \( m_{b_2} \) values close to 200 GeV, the \( gg \to H \) production rate at the LHC is enhanced by a factor around 4 compared to the SM case.

For \( c_b \) values smaller than the one used above, \( c_b \lesssim 0.6 \), similar significant deviations to the \( gg \to H \) cross section occur. Choosing a \( c_b \) value smaller than 0.5 and in turn more far from the \( c \) values for light quarks would tend to increase the flavor changing neutral currents in the third generation, induced at the tree–level by KK gauge bosons. On the other hand, for values significantly larger than \( c_b = 0.6 \), it becomes difficult to generate a sufficiently large mass for the \( b \) quark.

Note also that the exact size of the effect could be different in models where another custodial symmetry (such as O(4) for instance) or different fermion representations (e.g. a certain freedom in the \( b_R \) embedding might exist [8, 10]) are assumed, as both can affect the quark mass matrices via the modification of Clebsch–Gordan coefficients or the introduction of mixing terms with new “custodial” partners (custodians). Nevertheless, the general aspect is that, if \( t_R \) is not a singlet under the necessary custodial symmetry, its custodial partner(s) is (are) expected to be relatively light, in order to generate a sufficiently large \( m_t \), and in turn give rise to potentially significant effects in the loop–level Higgs production. Here, we have studied the minimal custodial symmetry with the simplest fermion embedding, but clearly, some interesting extended versions of the RS model would deserve the same analysis on the Higgs production at LHC.

As in the case of top quarks, the Higgs coupling to bottom quarks is almost unchanged in this
scenario (some effects would appear only for very low $c_I$ values). Thus, no significant change is expected in the main Higgs decay branching ratios, $H \rightarrow b\bar{b}$ and $H \rightarrow WW, ZZ$ for $M_H$ respectively below and above 140 GeV. However, the gluonic decay $H \rightarrow gg$ is modified similarly to the $gg \rightarrow H$ cross section as it is generated by the same amplitude. Furthermore, the rare decays $H \rightarrow \gamma\gamma$ and $\gamma Z$ are also mediated by heavy particle loops and will be affected by the $b_2$ state but, contrary to the $H gg$ case, the dominant component of the corresponding amplitudes stems from the $W$–boson loop which is SM–like and e.g., the $H \rightarrow \gamma\gamma$ decay branching ratio is affected only at the level of a few percent at most.

![Graph: Main branching ratios for the Higgs boson decay channels](image)

FIG. 3: Main branching ratios for the Higgs boson decay channels $[H \rightarrow W^+W^-; ZZ; t\bar{t}; b\bar{b}]$ as a function of the Higgs mass [in GeV], for the parameter set: $c_6 = 0.6$, $c_5 = -0.55$, $c_Q = 0.24$ which corresponds to $m_{t_1} = 180$ GeV and $m_{b_2} = 215$ GeV.

We would also like to make the following remark: if the $b_2$ quark mass is close to the lower value allowed by collider searches, $m_{b_2} \sim 200$ GeV, and the Higgs boson turns out to be heavy enough (for instance, as a result of additional corrections to electroweak observables from the light $b'$ [15]), the new decay channel $H \rightarrow b_2\bar{b}_2$ could be kinematically open. This channel could reach a branching ratio quite close to the $t\bar{t}$ one, as shown in Fig. (3). Whether this scenario is possible and to which extent the Higgs searches might be affected by these new channels deserve more detailed studies.

In addition, the deviations of the cross section for associated Higgs production with top quarks at the LHC, $pp \rightarrow Ht\bar{t}$, are also expected to be small. The other Higgs production channels, vector boson fusion and associated Higgs production with vector bosons, are of course not affected. However, an additional process, associated Higgs production with a pair of $b'$ quarks, $pp \rightarrow Hb'b'$, arises. In the regions of parameter space where the $gg \rightarrow H$ rate is enhanced by a factor $\sim 4$, i.e. for low $m_{b_2}$ values and large $Hb_2\bar{b}_2$ couplings, the cross section for this new process is similar in size as the one for $pp \rightarrow Ht\bar{t}$. More precisely, for $c_6 = 0.6$ (the dependence on the $c_6$ value being weak), the cross section ratio $\sigma(pp \rightarrow Hb_2\bar{b}_2)/\sigma(pp \rightarrow Ht\bar{t})$ is equal to the squared ratio of effective Higgs couplings to $b_2$ and $t$ which is equal to unity if $m_{b_2} = m_t = 180$ GeV (which in turn fixes the $c_Q$ and $c_I$ parameters as exhibits Fig. [2], independently of the Higgs mass.

The maximum effects of the new $b'$ state on the Higgs production and decays occur mainly for $b'$ masses around a few hundred GeV. It might turn out that, after a precise fit analysis of all the electroweak precision data in the third generation quark sector (main observables are $R_b$ and $A^{FB}$ [8, 10]) taking into account simultaneously the mixing with KK fermions and KK gauge bosons, the lowest $m_{b_2}$ values would be ruled out. At this level, we note that in the mentioned variations of the present minimal RS scenario, e.g. with an extended custodial symmetry or other fermion representations, light custodians exist in general and would have different electroweak constraints from the $b'$ here.

In conclusion, we have pointed out that in extra–dimensional models with warped geometry and a bulk custodial symmetry, as suggested by electroweak precision data, a new $b_R'$ quark is expected to be relatively light and will contribute to the Higgs–gluon–gluon amplitude. It could affect in a significant way the Higgs boson cross section in the main production channel at the LHC, $gg \rightarrow H$. The production rates can be enhanced by a factor of four at most, compared to the SM case. Even
if the $b'$ quark is heavier than 1 TeV, i.e. beyond the reach of the LHC [19], a modification of the $gg \to H$ production rate and the $H \to gg$ decay branching ratio at the level of a few 10% is possible. As the KK excitations of gauge bosons and fermions are heavy and not easy to detect at the LHC [20], the modification of the $gg \to H$ production cross section could be the only sign of warped extra–dimensions. For low $b'$ masses, the new Higgs production channel $pp \to Hb'b'$ is comparable, in rate, to the $HH$ production. The decays of the Higgs boson can also be affected by the presence of the new quark but the effects could be probed probably only in a high–precision experiment.

Acknowledgments: This work is supported by the French ANR for the project PHYS@COL&COS. G.M. thanks E. Kou and A. M. Teixeira for useful discussions.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370. See also, M. Gogberashvili, Int. J. Mod. Phys. D11 (2002) 1635.
[2] K. Agashe and G. Servant, Phys. Rev. Lett. 93 (2004) 231805; JCAP 0502 (2005) 002; G. Bélanger, A. Pukhov and G. Servant, arXiv:0706.0526 [hep-ph].
[3] A. Pomarol, Phys. Rev. Lett. 85 (2000) 4004; L. Randall and M. D. Schwartz, JHEP 0111 (2001) 003; Phys. Rev. Lett. 88 (2002) 081801.
[4] T. Gherghetta and A. Pomarol, Nucl. Phys. B586 (2000) 141; S. J. Huber and Q. Shafi, Phys. Lett. B498 (2001) 256; B512 (2001) 365; S. Chang et al., Phys. Rev. D73 (2006) 033002; G. Moreau and J. I. Silva-Marcos, JHEP 0601 (2006) 048; 0603 (2006) 090.
[5] C. Csaki et al., Phys. Rev. D66 (2002) 064021; G. Burdman, Phys. Rev. D66 (2002) 076003; J. Hewett, F. Petriello and T. Rizzo, JHEP 0209 (2002) 030.
[6] Particle Data Group, J.-W. Yao et al., J. Phys. G33 (2006) 1.
[7] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP 0308 (2003) 050.
[8] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, Phys. Lett. B641 (2006) 62.
[9] M. Carena et al., Nucl. Phys. B759 (2006) 202.
[10] A. Djouadi, G. Moreau and F. Richard, Nucl. Phys. B773 (2007) 43.
[11] See e.g. C. Csáki et al., Phys. Rev. D70 (2004) 015012.
[12] For reviews, see A. Djouadi, arXiv:hep-ph/0503172 and arXiv:hep-ph/0503173 to appear in Phys. Rept.
[13] B. Lillie, JHEP 0602 (2006) 019.
[14] F. J. Petriello, JHEP 0205 (2002) 003. For the process of gluon fusion into a pair of Higgs bosons, see: H. de Sandes and R. Rosenfeld, arXiv:0706.2665 [hep-ph]. Note also, that a significant reduction of the $gg \to H$ rate can occur in some supersymmetric higher–dimensional models; see G. Cacciapaglia, M. Cirelli and G. Cristadurol, Phys. Lett. B531 (2002) 105.
[15] H. M. Georgi, S. L. Glashow, M. E. Machacek and D. V. Nanopoulos, Phys. Rev. Lett. 40 (1978) 692.
[16] See e.g., M. Spira et al., Nucl. Phys. B453 (1995) 17.
[17] We do not attempt to reproduce the precise experimental mass values as we neglect flavor mixing effects in the mass matrices and possible small deviations of the Yukawa couplings from their universal values; these approximations should not modify our discussion, though.
[18] We thank F. Richard for a discussion on this point.
[19] C. Dennis et al., arXiv:hep-ph/0701158. See also for other exotic heavy quarks, J. A. Aguilar-Saavedra, Phys. Lett. B625 (2005); G. Azuelos et al., Eur. Phys. J. C39S2 (2005) 13; M. Carena et al., arXiv:hep-ph/0610156.
[20] The production of KK gauge bosons at the LHC has been discussed recently; see: K. Agashe et al., arXiv:hep-ph/0612015; B. Lillie et al., arXiv:hep-ph/0701166; F. Ledroit et al., JHEP 0709 (2007) 071; B. Lillie et al., arXiv:0706.3960 [hep-ph]; A. Djouadi et al., arXiv:0706.4191 [hep-ph]; K. Agashe et al., arXiv:0709.0007 [hep-ph].
[21] Note that these interferences are always constructive as the possibly negative sign of the effective Yukawa coupling is systematically compensated by the same–sign mass term needed to flip the chirality for each fermion in the loop.