The Electron Temperature and $^{44}$Ti Decay Rate in Cassiopeia A

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Abstract. The effects of plasma elemental composition and ionization state on the effective decay rate of $^{44}$Ti are investigated. We essentially follow the methods of the first authors to treat this topic, Mochizuki et al., but use more realistic plasma models, including radiative cooling, to compute the evolution of the charge state distribution behind the reverse shock. For uniform density ejecta (i.e. no clumps or bubbles) we find a negligible change to the decay rate of $^{44}$Ti. We discuss the effects of non-uniform ejecta. We also briefly consider the effects on these calculations of collisionless electron heating associated with weak secondary shocks propagating throughout the Cas A shell as a result of forward or reverse shock encounters with density inhomogeneities, recently suggested as an explanation for the hard X-ray tail seen in BeppoSAX and RXTE/OSSE spectra.

Introduction

Radioactive nuclei in the galaxy are mainly produced in supernova explosions. Among the most important of these is $^{44}$Ti, which after $^{56}$Ni and $^{56}$Co is the main energy source for the ejecta. Its abundance is also sensitive to details of the explosion. The observation of γ rays from decay products of $^{44}$Ti (i.e. $^{44}$Sc to which $^{44}$Ti decays with a lifetime of 85.4 ± 0.9 years [1, 2] and $^{44}$Ca, to which $^{44}$Sc decays with a lifetime of a few hours) in the Cassiopeia A supernova remnant by the COMPTEL instrument on the Compton Gamma Ray Observatory [3, 4] has sparked a re-examination of some of these ideas. Cassiopeia A is the youngest known supernova remnant in the galaxy, with a likely explosion date of 1680 A.D., and so is a good target to search for emission from the decay products of $^{44}$Ti, since its lifetime is a significant fraction of the age of the remnant. The flux in the 1157 keV line of $^{44}$Ca of $4.8 \pm 0.9 \times 10^{-5}$ photons s$^{-1}$cm$^{-2}$ implies an initial mass of $^{44}$Ti of $2.6 \times 10^{-4} M_{\odot}$[3, 4]. The fluxes in the 67.9 and 78.4 keV lines of $^{44}$Sc observed by OSSE and BeppoSAX are consistent with lower values of the initial $^{44}$Ti mass synthesized in the explosion [5]. Neglecting the contribution of the continuum (a questionable assumption) the observed flux is $2.9 \pm 1.0 \times 10^{-5}$ photons s$^{-1}$ cm$^{-2}$, marginally consistent with the $^{44}$Ca flux.

The estimates of the $^{44}$Ti mass based on the $^{44}$Ca flux are at the high end of the range suggested by theory. Thielemann et al. [6] predict a $^{44}$Ti mass of $1.7 \times 10^{-3} M_{\odot}$ for a $20 M_{\odot}$ progenitor, whereas other workers [7, 8, 9] predict less than $10^{-4} M_{\odot}$. Since $^{44}$Ti is produced only in the α rich freeze out (i.e. Si burning at low density, so that reactions involving α particles have negligible rates [10]), Nagataki et al. [11] speculated that the yield of $^{44}$Ti could be increased if the explosion were axisymmetric rather than spherically symmetric, due to the existence for example of rotation or magnetic fields. The reason for this is that in the reduced symmetry, α rich freeze out occurs at higher entropy in the polar regions and hence produces more $^{44}$Ti relative to $^{56}$Ni. An axisymmetrical explosion is also quite consistent with the morphology observed today in the remnant [12].

An equally ingenious solution was that of [13] who suggested that $^{44}$Ti, if sufficiently highly ionized behind the reverse shock (see below), would have its decay rate to $^{44}$Sc reduced. This occurs because $^{44}$Ti decays mainly by capture of a K-shell electron, and if ionized to the hydrogenic or bare charge state, i.e. if the K shell electrons are removed, the decay rate is reduced. In this paper we revisit the work of [13] using some more recent ideas about the electron heating in the Cas A ejecta to investigate the effect on the $^{44}$Ti decay rate and the inferred mass of $^{44}$Ti.

The Ionization State of the Cassiopeia A Ejecta

Following a supernova explosion, a spherical shock wave moves out through the medium surrounding the progenitor (a presupernova stellar wind in the case of Cas A). As this shock sweeps up more and more mass from the sur-
FIGURE 1. Electron (solid lines) and ion (dashed lines) temperatures in pure Fe ejecta passing through the reverse shock at 50, 75, 100, 150, 200, and 250 years after the initial explosion. The ejecta shocked at 50 years undergoes radiative cooling back to temperature below $10^5$ K within about 10 years, ejecta shocked at 75 years takes 130 years to cool.

The ionization state of the Cas A ejecta following passage of the reverse shock is computed using the formalism described in [14]. Assuming a total ejecta mass of $4M_\odot$, an explosion energy of $2 \times 10^{51}$ ergs and a circumstellar medium density of 3 hydrogen atoms cm$^{-3}$, the reverse shock velocity as a function of elapsed time since the explosion is taken from the analytical expressions in [15]. Ionization and recombination rates are taken from [16] (using subroutines kindly supplied by Dr P. Mazzotta). Radiative cooling is taken from [17]. Further cooling comes from the adiabatic expansion of the ejecta. A well known problem is that the shocks in supernova remnants are collisionless. Conservation of energy, momentum, and particle number results in shock jump conditions that predict shocked particle temperatures proportional to their masses, i.e. $T_{\text{ion}}/T_{\text{electron}} = m_{\text{ion}}/m_{\text{electron}} \simeq 1836 \times A_{\text{ion}}$ where $A_{\text{ion}}$ is the atomic mass. These temperatures will equilibrate by Coulomb collisions, but the possibility remains that collective effects may induce faster equilibration. For the time being we neglect this possibility.

Figure 1 shows the electron and ion temperatures in pure Fe ejecta, for plasma encountering the reverse shock at times 50, 75, 100, 150, 200, and 250 years after explosion. Ejecta shocked 100 or more years after explosion reaches electron temperatures of $2 \times 10^7$ K, where appreciable ionization of $^{44}$Ti to the H-like or bare charge states may occur. Ejecta shocked after 75 years reaches an electron temperature of $2 \times 10^7$ K briefly, but cools by radiation and adiabatic expansion to temperatures an order of magnitude lower at the present day. Ejecta shocked even earlier than this at 50 years never reaches electron temperatures above $10^7$ K, and undergoes a thermal instability, cooling catastrophically to temperatures below $10^5$ K within 15 years or so. Mochizuki et al.
FIGURE 2. Electron (solid line), Fe ion (dashed line) and O ion (dotted line) temperatures in 90% O, 10% Fe (by mass) ejecta.

[13] only included Fe charge states from Ne-like to bare in their calculation (we use the full range of 27 charge states, starting everything off in the Fe$^+$ state), and neglected to include radiation and so do not find the thermal instability. Figure 2 shows similar plots but this time of ejecta composed of 90% O and 10% Fe (by mass). Slightly lower electron temperatures are found in the plasma shocked later in the evolution of the remnant, but the reduced radiative cooling of this mixture gives higher electron temperatures in the ejecta shocked at 50-75 years. The temperatures here are very similar to those in pure O plotted in [14].

\[ 44^{\text{Ti}} \text{ DECAY RATE AND RADIOACTIVITY} \]

We now compute the ionization balance of trace amounts $^{44}\text{Ti}$ assumed to be expanding with the Fe ejecta, using the previously determined electron temperature and density profiles. $^{44}\text{Ti}$ decays by orbital electron capture, and the rate is proportional to the electron probability density at the nucleus. Thus only $s$ states contribute to the decay rate, and because the electron probability density at the nucleus varies as $n^{-3}$, the $n = 1$ shell is the dominant contribution. In the following we assume that the $n = 1$ shell is the only contribution. Thus all charge states up to He-like will decay with the neutral decay rate, H-like will decay with half this rate and the bare charge state will not decay at all. Figure 3 shows the evolution of the Ti ionization balance following the reverse shock encounter 100 years after explosion. After a further 220 years, most of the Ti is in the He-like charge state (the dotted line), but the smaller fraction that has accumulated in the H-like (solid line) and bare (dashed line) charge states is sufficient to increase the amount of Ti by a factor of 1.025 over that determined from the neutral decay rate alone. The current emission in $^{44}\text{Sc}$ and $^{44}\text{Ca}$ is enhanced by a factor 1.009. Results for various reverse shock encounter times are given in Table 1 for pure Fe ejecta, and in Table 2 for mixed 90% O, 10% Fe (by mass) ejecta. Mochizuki et al. [13] find a maximum enhancement of $^{44}\text{Ti}$ radioactivity of about 2.5 320 years after explosion for ejecta at mass coordinate $q = 0.5 - 0.6$. We find our maximum enhancement at the same mass coordinate, but only a factor of about 1.03 in pure Fe ejecta. Mochizuki et al. take similar models to us, but in seek-
FIGURE 3. Ionization balance of $^{44}$Ti in pure Fe ejecta following reverse shock at 100 years. The He-like charge state is shown as the thick dotted line, the H-like as the thick solid line and the bare charge state as the thick dashed line. The narrow solid lines represent all the lower stages of ionization. At the present day (320 years) the He-like charge state dominates the ionization balance.

TABLE 1. Enhancements in $^{44}$Ti density and $^{44}$Sc and $^{44}$Ca emission due to suppression of the $^{44}$Ti decay rate by plasma ionization. $^{44}$Ti assumed embedded in pure Fe ejecta, for various reverse shock times after explosion.

| time (years) | mass coordinate | $^{44}$Ti density | $^{44}$Sc, $^{44}$Ca emission |
|--------------|-----------------|-------------------|-----------------------------|
| 50           | 0.72            | 1.0               | 1.0                         |
| 75           | 0.63            | 1.0               | 1.0                         |
| 100          | 0.55            | 1.025             | 1.009                       |
| 150          | 0.43            | 1.001             | 1.0                         |
| 200          | 0.34            | 1.0               | 1.0                         |
| 250          | 0.28            | 1.0               | 1.0                         |

TABLE 2. Same as Table 1, but giving $^{44}$Ti enhancements etc in mixed 90% O, 10% Fe (by mass) ejecta.

| time (years) | mass coordinate | $^{44}$Ti density | $^{44}$Sc, $^{44}$Ca emission |
|--------------|-----------------|-------------------|-----------------------------|
| 50           | 0.72            | 1.0               | 1.0                         |
| 75           | 0.63            | 1.11              | 1.04                        |
| 100          | 0.55            | 1.09              | 0.995                       |
| 150          | 0.43            | 1.01              | 1.002                       |
| 200          | 0.34            | 1.0               | 1.0                         |
| 250          | 0.28            | 1.0               | 1.0                         |

ing to match a present day blast wave velocity of $\sim 2000$ km s$^{-1}$ inferred in [18], adopted a circumstellar medium density much higher than ours. We took a lower circumstellar medium density to match the blast wave velocity of $\sim 5000$ km s$^{-1}$ found by [19, 20]. The effect of this change is that the reverse shock in our model accelerates more slowly, generally producing lower temperatures in the shocked ejecta, with correspondingly lower degrees of ionization for the $^{44}$Ti. Mochizuki et al. also assumed the Fe ejecta to expand in overdense (by a factor $\sim 10$) clumps, which reduces the reverse shock velocity and temperature somewhat, but increases the electron density and hence the ionization time, defined as the product of the electron density and the elapsed time since shock passage, assuming an electron temperature constant with time during this period.
FURTHER ELECTRON HEATING?

The fundamental reason for not obtaining the same suppression of the $^{44}$Ti decay rate as Mochizuki et al. [13] is that the electron temperatures in our probably more realistic model for Cas A do not reach high enough temperatures. Here we will briefly discuss two possibilities so far neglected.

First, [13] assumed the Fe to exist in clumps a factor of 10 more dense than the surrounding ejecta. In connection with Type Ia supernova, [21] argue that the Fe in such clumps is unlikely to have a radiogenic origin, and is probably $^{54}$Fe. The reason is that the likely mechanisms to form clumps are hydrodynamic instabilities following Ni-Co-Fe bubble formation as a result of the extra pressure resulting from energy input mainly due to the Ni radioactivity in the first 10 days following explosion. Evidence for such bubbles is found in SN 1987A [22], and some modeling has already been performed [23]. The $^{44}$Ti is likely to reside in these bubbles, being formed in the same regions as $^{56}$Ni. If these ejecta bubbles persist into the remnant phase, then the reverse shock will accelerate into them due to their lower density. Higher temperatures in the shocked plasma will result, with correspondingly higher degrees of ionization and possibly an effect on the $^{44}$Ti decay rate.

For the time being it is not clear where in Cas A this Fe ejecta exists. We expect it to be synthesized in the innermost regions of the progenitor, and so naively assuming spherical symmetry we should expect the Fe to pass through the reverse shock relatively late in the evolution of the remnant. However in the SE quadrant of the remnant, (the region most extensively studied in Chandra data to date) the Fe rich knots are remarkable for being exterior to those rich in Si and O [24, 25, 26]. These Fe rich knots also have high ionization times, implying high electron density and/or early encounter with the reverse shock. The high density also possibly suggests that this Fe is not associated with $^{44}$Ti. However the Fe rich region gives mass ratios in the following ranges; Fe/O = 0.2 - 0.4, Si/O = 0.03 -0.06 [27]. Fe/O is consistent with or slightly higher than with solar system values, while Si/O is lower. Earlier X-ray observations, essentially spatially unresolved, of large regions of the remnant find Si/O, S/O, Ar/O broadly consistent with solar system values, while Fe/O is significantly lower [28, 29].

The second possibility is that the electrons are heated following shock passage by processes that are faster than the ion-electron Coulomb equilibration. Most discussion of this in the literature has focused on plasma wave excited by instabilities at the forward shock [see e.g. 30, 31, 32]. At the reverse shock most electrons are bound to heavy ions, and so cannot participate in collisionless heating [33]. However it is important to realize that most of the ejecta emission comes from plasma that passed through the reverse shock in the first 75 years or so after explosion. From Figures 1 and 2 it is apparent that the temperature of these ejecta at the present day is rather lower than the observed value of $\sim 4 \times 10^7$ K, spectacularly so for the ejecta shocked at around 50 years. This is due to the combined effects of radiation losses and adiabatic expansion. Additionally, this plasma has reached ion-electron temperature equilibrium, and so the inclusion of extra collisionless processes at the reverse shock itself, however unlikely, would not produce any change to this conclusion.

Recently it has been suggested that this ejecta is re-heated by weak secondary shocks produced as the forward or reverse shocks run into density inhomogeneities producing reflected and transmitted shocks [34, 14]. The precise mechanism is a modified two stream instability arising as upstream ions are reflected from the shock front, move back through the preshock plasma and excite lower-hybrid waves as they go. These waves are electrostatic ion oscillations with wavevectors at angle nearly perpendicular to the magnetic field direction. Thus the wave phase velocity perpendicular to the magnetic field can resonate with the ions, and the wave phase velocity parallel to the field can resonate with the electrons, allowing fast energy transfer between the two [35, 36]. Such a mechanism is particularly appealing in Cas A. Arguing from the radio synchrotron luminosity, which is maximized when the cosmic ray electrons and magnetic field have similar energy densities, leads to the inference of magnetic field strengths of order 1 mG; significantly higher than usually assumed for supernova remnants. The electrons are accelerated into a characteristic distribution function along the magnetic field. Calculations of the bremsstrahlung spectrum emitted by such an accelerated electron distribution produced a remarkably good match to the BeppoSAX MECS and PDS spectra [see 34, for fuller description]. In fact the computed continuum spectrum for certain parameters (basically the Alfvén speed in the ejecta) can be consistent with the seemingly extreme assumption made in [5] of neglecting the continuum under the $^{44}$Sc emission to derive a flux of $^{44}$Sc decay photons consistent with the observation of the $^{44}$Ca flux. This accelerated electron distribution equilibrates with the ambient plasma electrons by Coulomb collisions. The extra heating thus produced would increase the ionization of the plasma above that modeled in this paper, and could plausibly produce more $^{44}$Ti in H-like or bare charge states, hence further suppressing its decay. However in order to avoid heating the ejecta of Cas A to temperatures significantly higher than those observed, such electron acceleration must have commenced relatively recently (i.e. within 50-100 years of the present day), if it is also responsible for the hard X-ray emission.
CONCLUSIONS

Cassiopeia A is an exciting physics laboratory for a wide variety of phenomena; particle heating and acceleration, magnetic field amplification, hydrodynamic instabilities and of course stellar nucleosynthesis. Initial ideas to study the element abundances produced in a core-collapse supernova can often end up with arguments based on observed element abundances to infer some other characteristic of the supernova or its remnant. Examples of this are provided in this paper by the plasma processes that may effect the decay rate of $^{44}\text{Ti}$. We find significantly less change to the decay rate produced by the plasma ionization than previous work [13], but our model for the ejecta, while arguably more realistic, is still probably a long way from reality itself.

X-ray astronomers have only just begun to get to grips with the wealth of new data on Cassiopeia A and other supernova remnants, and we can expect many exciting new results as this effort matures. This, coupled with hard X-ray and $\gamma$-ray observations with the forthcoming INTEGRAL mission, might finally allow to unambiguously infer exactly how much $^{44}\text{Ti}$ was produced in the explosion of Cassiopeia A.

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