Local Bursts Model of CMB Temperature Fluctuations: Scattering in Primordial Hydrogen Lines

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Journal ref.: Astronomy Letters, 2015, Vol. 41, No. 10, pp. 537-548

Within the framework of a flat cosmological model a propagation of an instantaneous burst of isotropic radiation is considered from the moment of its beginning at some initial redshift $z_0$ to the moment of its registration now (at $z = 0$). Thomson scattering by free electrons and scattering in primordial hydrogen lines $H_\alpha$, $H_\beta$, $P_\alpha$ and $P_\beta$ are considered as the sources of opacity and when calculating the albedo of single scattering in the lines we take into account deactivation of the upper levels of transitions by background blackbody radiation. Profiles for these lines in a burst spectrum are calculated for different distances from the center of the burst and different values of $z_0$. In a first approximation these profiles do not depend on spectrum and intensity of a burst radiation. It is shown that lines are purely in absorption at sufficiently large distance but emission component may appear as a distance decreases and it becomes stronger while absorption component weakens with a further distance decrease. For the sum of $H_\alpha$ and $H_\beta$ lines the depth of absorption can reach $2 \cdot 10^{-4}$ while for the sum of $P_\alpha$ and $P_\beta$ lines the maximum absorption is about $7 \cdot 10^{-6}$. So that the relative magnitude of temperature fluctuations lies between $10^{-7}$ and $10^{-9}$. The calculations were fulfilled for bursts with different initial sizes. For the same $z_0$ the profiles of hydrogen lines are practically coincide for burst sizes lower than someone and for greater ones the lines weaken as the burst size grows.

PACS numbers: 98.80.-k; 98.80.Cq; 95.30.Jx

Key words: cosmology, early Universe, cosmological recombination, radiative transfer, Thomson scattering, subordinate Hydrogen lines.
Introduction

Investigations of cosmic microwave background (CMB) still continue. New results obtained by PLANCK mission (see Adam et al., 2015a) define our present day knowledge about power spectrum of primordial spatial fluctuations of matter density, about some global fundamental parameters of the Universe and about CMB polarization. Essential progress is achieved also in CMB spectroscopy. However all these important successes do not exclude further more detailed and deep investigations.

In particular medium and high resolution spectroscopy of separate (individual) objects (elements of CMB brightness map) seems to be very important. Novelity here is in turning from investigation of statistical CMB properties defined by global processes in the early Universe to searching and learning local phenomena and objects. The last ones can be somewhat rare events not affecting on the average statistical parameters of CMB. But they can carry information about physical laws. So for example one can expect new local forms of matter and fields (see, e.g. Dubrovich (2003), Grachev and Dubrovich (2011), Dubrovich and Glazyrin (2012)).

Besides of more or less probable but still hypothetical objects there exists evidently the whole class of local sources in the early Universe which can be learned individually. These are the same standard primordial CMB temperature fluctuations thoroughly learned now statistically. In fact we deal with some spatial domains of space where temperature increase or decrease takes place for some reason or other. It is very important that besides spatial apartness of these regions their temperature deviations are also nonstationary. Depending on a mechanism of a given inhomogeneity formation a typical time of its development and damping can be different. So for example if CMB temperature fluctuation forms due to acoustic waves then a typical time $\Delta t$ of its life will be of the order of a wave oscillation period i.e. for a fluctuation scale $L$ and sound speed $c_s = c/\sqrt{3}$ ($c$ is the speed of light) will be $\Delta t \approx L/c_s$ or $\Delta t/t_0 \approx \vartheta \sqrt{z_0}$ where $t_0$ is a cosmological time at the moment of fluctuation appearance and $\vartheta$ is its present-day angular size. For the angular size $\vartheta \approx 3'$ we have $\Delta t/t_0 \approx 3.5 \cdot 10^{-2}$. This time of life can be considered as a burst. On the other hand after approximately the same time the sign of the effect in this region changes and after that repeats oneself again. Then one needs to count up a summary effect. If however there is an accidental interference of several acoustic waves with different wave vectors in the same region of space then the resultant fluctuation may have appreciably larger amplitude but lower probability of a recurrence of such an event in a given spatial domain. Real observations give maximum amplitude of temperature deviation in the spots of the order of 500 $\mu$K (Adam et al., 2015b). So we have in fact local burst model of sources in the early Universe within the framework of standard scenario of matter evolution without any additional exotic hypotheses. Observed now CMB temperature map is a sum of all sources with account of their initial spatial and temporal distribution and with account of subsequent scattering of photons emitted by them. These two factors must be taken in a product of one on another. In this work we calculate a function of transition from radiation intensity of these sources to observed intensity taking into account scattering on free electrons and in hydrogen subordinate lines in a space between a source and an observer. In a first approximation this function does not depend on a source intensity.

Existing theoretical formulae correctly take into account all these effects for Thomson scattering on free electrons. However in the case of scattering in spectral lines some new additional effects take place. The most important (end evident) distinction is a different dependence on
frequency: Thomson scattering is the same for all photons in a given place and in a given time irrespective of their frequencies but scattering in lines involves only photons in a very narrow frequency band. In expanding Universe every spatial point and every moment of time define a redshift (observed frequency in fact) of photons emitted in this point and at this moment of time. Summing along a line of sight contributions from different spatial domains with a proper account of photons emission time in the case of Thomson scattering leads to averaging of fluctuations with different signs. In the case of scattering in lines CMB background distortions formed by different layers will be seen now on different frequencies. So frequency becomes the third coordinate which can be used to analyse the physics and parameters of evolution processes.

Important factor is an optical thickness due to scattering. Scattering on free electrons is multiple one because the smallness of its cross-section is compensated by comparatively large photon path length and electron concentration. In hydrogen subordinate lines we have small optical thickness because scattering takes place on a small piece of path (defined by a width of line profile) and occupation numbers of excited atomic levels are comparatively small. It means that for scattering in lines we may confine ourselves to a single scattering approximation.

A various contribution of nonconservative effects of photons scattering in a resultant spectrum is a more fine and not at all evident distinction. In particular we mean deactivation of excited levels due to absorption of background photons or due to collisions with electrons and atoms. These effects are taken into account by introducing in equations of radiative transfer so-called albedo of a single scattering $\lambda$. Detailed account of nonconservative scattering effects is necessary because in our case transfer of radiation from these sources is nonequilibrium: due to small optical thicknesses in subordinate lines both occupation numbers of levels and radiation spectrum do not have a time to be thermalized.

In our case albedo of a single scattering in hydrogen lines is determined by photoionization and radiative transitions due to absorption of CMB photons. Simple estimates show that in this case $\lambda$ can be noticeably lower than unity in contradiction to scattering on free electrons with an account of double Compton effect when $\lambda = 1$ practically. As will be shown below this distinction is very important and it has significant influence on a resultant spectrum.

The present paper is a continuation of our previous work (Grachev and Dubrovich, 2011) devoted to calculation of a radiation field evolution of an instantaneous burst of isotropic radiation as a result of Thomson scattering in a homogeneous expanding and recombining Universe. Now along with Thomson scattering we take into account scattering in hydrogen subordinate lines $\text{H}_\alpha$, $\text{H}_\beta$, $\text{P}_\alpha$ and $\text{P}_\beta$ and we calculate line profiles in a burst spectrum on different distances from the burst center for different burst moments $z_0$. We do not take into account radiation polarization and we consider scattering (both Thomson one and in lines) as isotropic. Moreover we consider scattering in lines to occur with complete frequency redistribution (CFR), so that source function do not depend on frequency. We also assume that the burst radiation has no influence on electron concentration and on occupation numbers of atomic energy levels which are calculated beforehand using our code of primordial hydrogen recombination dynamics (Grachev and Dubrovich, 1991).

Rubínó-Martín et al. (2005) learned an influence of scattering in subordinate lines of primordial hydrogen on the theoretical power spectrum of CMB angular fluctuations. But we calculate profiles of spectral lines arising as a result of scattering of radiation from an external (with respect to CMB) source. Moreover as it was noticed above we take into account noncon-
servative character of radiative transfer in lines by means of albedo of a single scattering which is completely absent in previous papers.

**Main equations and relations**

We consider transfer of radiation in a homogeneous expanding Universe for initial spherically-symmetrical distribution of radiation intensity. For a flat model of the Universe corresponding nonstationary scalar transfer equation for photons occupation number $n$ is (Nagirner and Kiru-
sheva, 2005):

$$
\frac{\partial n}{\partial \eta} + \frac{\mu}{r} \frac{\partial n}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial n}{\partial \mu} - \frac{a \nu}{c} H \frac{\partial n}{\partial \nu} = -a k_e(n - s_e) - a k_{ik}(n - s_{ik}),
$$

where $\eta$ is a conformal time ($d\eta = c dt/a(\eta)$), $c$ is the speed of light, $t$ is a time, $a = a(\eta)$ is the cosmological scale factor), $r$ is a distance parameter, $H = H(\eta)$ is the Hubble factor, $n = (c^2/2h\nu^3)I$ is a radiation intensity at a frequency $\nu$ propagating at angle $\vartheta = \arccos \mu$ to a radial direction, $s_e(r, \eta, \nu)$ and $s_{ik}(r, \eta)$ are dimensionless (in average occupation numbers) source functions for Thomson scattering and for scattering in a spectral line for a transition between lower energy level $i$ and upper energy level $k$. Volume extinction coefficients corresponding to these two types of scattering are

$$
k_e(\eta) = \sigma_e n_e(\eta), \quad k_{ik}(\eta, \nu) = \bar{k}_{ik}(\eta) \varphi_{ik}(\nu), \quad \bar{k}_{ik}(\eta) = \frac{h\nu_{ik}}{4\pi} n_iB_{ik} \left(1 - \frac{n_k g_i}{n_i g_k}\right),
$$

where $\sigma_e = 6.65 \cdot 10^{-25}$ cm$^2$ is the cross-section of Thomson scattering, $n_e(\eta)$ is an electron concentration, $h\nu_{ik}$ is an energy of transition, $B_{ik}$ is the Einstein coefficient for absorption of radiation, $n_i(\eta)$ and $n_k(\eta)$ are occupation numbers of levels and $g_i$ and $g_k$ are their statistical weights, $\varphi_{ik}(\nu)$ is an absorption coefficient profile normalized as $\int_0^\infty \varphi_{ik}(\nu) d\nu = 1$. Note that in a volume coefficient of extinction in a line we consider induced radiation as a negative absorption.

We consider scattering both on electrons and in lines to be isotropic and scattering in lines to be completely incoherent as well. Therefore for radiation field with the axial symmetry the source functions in (1) are written as

$$
s_e(r, \eta, \nu) = j(r, \eta, \nu), \quad s_{ik}(r, \eta) = \lambda_{ik}(\eta) j_{ik}(r, \eta),
$$

where $\lambda_{ik}(\eta)$ is an albedo of a single scattering in a line,

$$
j(r, \eta, \nu) = (1/2) \int_{-1}^{+1} n(r, \mu, \eta, \nu) d\mu, \quad j_{ik}(r, \eta) = \int_0^\infty j(r, \eta, \nu) \varphi_{ik}(\nu) d\nu
$$

are radiation intensities averaged over directions and both over directions and over an absorption coefficient profile. When calculating a single scattering albedo we take into account spontaneous transitions and also transitions induced by blackbody CMB radiation with the exception of transitions in Lyman lines being optically very thick. As a result we have

$$
\lambda_{ik}(\eta) = R_{ki}[\sum_{i' = 2, i' \neq k}^{\infty} R_{ki'}(\eta) + R_{ke}(\eta)], \quad k > i,
$$
where
\[ R_{ki}(\eta) = \frac{g_i A_{ik}}{g_k \exp[h\nu_{ki}/kT(\eta)] - 1}, \quad k < i, \] (6)

are the coefficients of transition probabilities upwards due to absorption of the blackbody radiation with the temperature \( T(\eta) \) and

\[ R_{ki}(\eta) = \frac{A_{ki}}{1 - \exp[-h\nu_{ik}/kT(\eta)]}, \quad k > i, \] (7)

are the coefficients of transition probabilities for spontaneous and induced (due to the blackbody radiation) transitions downwards, \( R_{kc} \) are the coefficients of transition probabilities due to ionization by the blackbody radiation, \( A_{ki} \) are the Einstein coefficients for spontaneous transitions.

Further we neglect both natural and Doppler (due to thermal motion of atoms) widths of lines as compared with the width defined by cosmological space expansion. So we consider the line profile as the delta-function in a comoving frame of reference: \( \varphi_{ik}(\nu) = \delta(\nu - \nu_{ik}) \) where \( \nu_{ik} \) is the frequency of transition in the laboratory frame. Then according to eqs. (3) and (4)

\[ s_{ik}(r, \eta) = \lambda_{ik}(\eta)j(r, \eta, \nu_{ik}), \] (8)

and one can rewrite the wrighthand side of eq. (1) to obtain as a result the following basic equation:

\[ \frac{\partial n}{\partial \eta} + \mu \frac{\partial n}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial n}{\partial \mu} - \frac{a \nu}{c} H \frac{\partial n}{\partial \nu} = -a k_e(n - s) - a k_{ik} \delta(\nu - \nu_{ik})(n - \lambda_{ik}s), \] (9)

where the source function \( s = s_e = j(r, \eta, \nu) \).

Hereinafter the space expansion factor \( a \) is considered to be 1 in the present epoch (\( z = 0 \)) so that \( a(z) = 1/(1 + z) \) where \( z \) is a redshift. Therefore the distance coordinate \( r \) in eq. (9) is the distance from the center of symmetry measured in the present time (at \( z = 0 \)). For an arbitrary \( z \) the corresponding distance is \( r a(z) = r/(1 + z) \).

The problem is solved by an ordinary method widely used in the radiative transfer theory. At first one gets analytical formal (i.e. for a given source function \( s(r, \eta, \nu) \)) solution of the main eq. (9) for a given initial distribution of radiation intensity. Then this solution is substituted into the definition of the source function through the radiation intensity and one gets an integral equation for the source function which does not depend on direction. This equation is solved by any one numerical method and then the radiation field with its angular structure is obtained from the formal solution by means of simple numerical integration. We have used this method in our preceding paper. The complication consists in the presence of frequency dependence of the main functions. However since Thomson scattering is neutral (i.e. its cross-section does not depend on a frequency) and moreover we neglect frequency change for line scattering in the comoving frame then frequency changes due to cosmological space expansion only: \( \nu a(\eta) = \nu' a(\eta') \) or \( \nu' = \nu(1+z')/(1+z) \) where \( z \) is the redshift corresponding to the moment \( \eta \). Therefore one should take into account cosmological change of frequency as well when deriving the formal solution.

In order to obtain the formal solution mentioned above let us consider propagation of radiation along a ray intersecting radial direction at an angle \( \vartheta = \arccos \mu \) on a distance \( r \).
from the center of symmetry. Let \( l \) be a coordinate measured from the nearest to the center of
symmetry point of the ray. The lefthand side of eq. (9) represents directional derivative along
the ray and so this equation assumes the form

\[
\frac{dn}{dl} = -\alpha(l, \nu(l))n(l, \nu(l)) + \beta(l, \nu(l))s(l, \nu(l)), \tag{10}
\]

where

\[
\alpha(l, \nu(l)) = a(l)k_e(l) + a(l)\tilde{\kappa}_{ik}(l)\delta(\nu(l) - \nu_{ik}), \tag{11}
\]

\[
\beta(l, \nu(l)) = a(l)k_e(l) + a(l)\tilde{\kappa}_{ik}(l)\delta(\nu(l) - \nu_{ik})\lambda_{ik}(l). \tag{12}
\]

Integrating this equation we obtain

\[
n(l, \nu(l)) = n_0(l_0, \nu(l_0))e^{-\int_{l_0}^{l} \alpha(l', \nu(l'))dl'} + \int_{l_0}^{l} \beta(l', \nu(l'))s(l', \nu(l'))e^{-\int_{l_0}^{l'} \alpha(l'', \nu(l''))dl''} dl'. \tag{13}
\]

Here \( l_0 \) is the coordinate of the point farthest from the observation point \( l \) but yet capable to
give a contribution to radiation field in the point \( l \) at the moment of time \( \eta \). Every point \( l' \) on
the ray is defined by the radial distance \( r' \) an by the angle \( \arccos \mu' \) between the ray and radial
direction. From geometry of the problem simple relations follow:

\[
l = r\mu, \quad l' = r'\mu', \quad l_0 = r_0\mu_0, \quad r\sqrt{1 - \mu^2} = r'\sqrt{1 - \mu'^2} \tag{14}
\]

and

\[
l - l_0 = \eta, \quad l - l' = \eta - \eta'. \tag{15}
\]

According to the last of these equations one can turn to integration over time in eq. (13) since
\( dl' = d\eta' \). As a result taking into account eqs. (14) and (15) the formal solution (13) is written in
the following form

\[
n(r, \mu, \eta, \nu) = n_0(r_0, \mu_0, \nu_0)e^{-\int_{0}^{\eta} \alpha(\eta', \nu')d\eta'} + \int_{0}^{\eta} \beta(\eta', \nu')s(r', \eta', \nu')e^{-\int_{0}^{\eta'} \alpha(\eta'', \nu'')d\eta''} d\eta', \tag{16}
\]

where \( n_0(r_0, \mu_0, \nu_0) \) is initial (at the moment \( \eta = 0 \)) radiation distribution over distances from
the burst center and over angles and frequencies. Here

\[
\nu_0 = \nu a(\eta)/a(0), \quad \nu' = \nu a(\eta)/a(\eta'), \quad \nu'' = \nu a(\eta)/a(\eta''), \tag{17}
\]

\[
r_0 = \sqrt{r^2 - 2r\mu\eta + \eta^2}, \quad r_0\mu_0 = r\mu - \eta, \tag{18}
\]

\[
r' = \sqrt{r^2 - 2r\mu(\eta - \eta') + (\eta - \eta')^2}, \quad r'\mu' = r\mu - \eta + \eta'. \tag{19}
\]

Instead of conformal time measured in the length units one can introduce dimensionless time

\[
u = \int_{0}^{\eta} a(\eta')k_e(\eta')d\eta' = c\sigma_e \int_{0}^{t} n_e(t')dt' = c\sigma_e \int_{z}^{z_0} \frac{n_e(z')}{(1 + z') H(z')} dz', \tag{20}
\]

which has a physical sense of optical path length (for Thomson scattering) between the moments
\( z \) and \( z_0 \). Here redshift \( z_0 \) corresponds to initial moment of time i.e. \( u = \eta = t = 0 \) at \( z = z_0 \). Conformal time \( \eta \) and dimensionless time \( u \) can be obtained by means of relation

\[
\eta = c \int_{t}^{t'} dt'/a(t') = c \int_{z}^{z'} dz'/H(z'), \tag{21}
\]
when calculating \( u \) and \( \eta \) on the same redshift grid. Here \( H(z) \) is the Hubble factor. Let us consider the integral in eq. (16):

\[
\int_{0}^{\eta} \alpha(\eta'')d\eta'' = u + \int_{0}^{\eta} \alpha(\eta'')\bar{k}_{ik}(\eta'')\delta(\nu'' - \nu_{ik})d\eta'' = \left\{ \begin{array}{ll}
u + \tau_{ik}(z_{l}), & z \leq z_{l}, \\
u_{ik}, & u, z > z_{l}, \end{array} \right. \tag{22}
\]

where

\[
\tau_{ik}(z) = \frac{hc n_{i}(z)B_{ik}}{4\pi H(z)} \left[ 1 - \frac{n_{k}(z)g_{i}}{n_{i}(z)g_{k}} \right], \tag{23}
\]

\[
z_{l} = \frac{\nu_{ik}}{\nu} - 1, \quad \frac{\nu_{ik}}{1 + z_{0}} \leq \nu \leq \nu_{ik}. \tag{24}
\]

When obtaining eq. (22) we turn to integration over \( \nu'' \) in the middle part of this equation using relation

\[
d\eta'' = -\frac{c}{H(z'')} \frac{1 + \frac{z''}{\nu}}{\nu''}d\nu'', \tag{25}
\]

which follows from equation \( \nu'' = (1 + z'')\nu \) and eq. (21). Here \( \nu \) is a radiation frequency in the present day epoch \((z = 0)\). So far as the frequency does not change in the case of Thomson scattering and moreover it does not change in fact in scattering in the lines in the adopted here approximation then the frequency \( \nu \) at \( z = 0 \) is a parameter in the problem and it can be not included into the arguments of the sought for solution.

Similarly one can transform the second (integral) term in the right side of eq. (16). As a result the formal solution (16) becomes:

\[
n(r, \mu, u) = n^{0}(r, \mu, u)\text{ for } u < u_{l} \text{ and }
\]

\[
n(r, \mu, u) = n_{0}(r_{0}, \mu_{0})e^{-u - \tau_{i}}, + \int_{u_{l}}^{u} s(r', u')e^{u' - u}du' +
\]

\[
e^{-\tau_{i}} \int_{0}^{u} s_{0}(r', u')e^{u' - u}du' + \lambda_{i}s(r_{i}', u_{i})e^{u_{i} - u}(1 - e^{-\tau_{i}}), \quad u > u_{l}, \tag{26}
\]

where \( \tau_{i} = \tau_{ik}(z_{l}), \eta_{i} = \lambda_{ik}(z_{l}), r_{i}' = r'(z_{i} - \eta_{i}) \) and \( n^{0}(r, \mu, u) \) and \( s^{0}(r, u) \) are solutions in the absence of scatterings in spectral lines (i.e. for \( z_{l} = 0 \)). By its physical sense \( z_{l} \) is a resonance redshift at which scattering of photons with the frequency \( \nu \) takes place and \( \tau_{i} \) is the Sobolev optical thickness of the medium for this redshift. The mentioned above quantities appear in the theory of primordial hydrogen recombination lines formation (see e.g. Dubrovich and Grachev, 2004).

By definition the source function \( s(r, u) \) is an averaged over angles intensity \( j(r, u) \). Inserting eq. (26) into (21) gives \( s(r, u) = s^{0}(r, u) \) for \( u < u_{l} \) while for \( u > u_{l} \) it leads to the following integral equation for \( s(r, u) \):

\[
s(r, u) = s_{0}(r, u)e^{-u - \tau_{i}} + \frac{1}{2r} \int_{u_{l}}^{u} e^{u' - u} \frac{du'}{\eta - \eta'} \int_{r_{l}' - \eta_{l}' + \eta}^{r_{l}' + \eta_{l}'} s(r', u')r'dr' +
\]

\[
+ \frac{e^{-\tau_{i}}}{2r} \int_{0}^{u_{l}} e^{u' - u} \frac{du'}{\eta - \eta'} \int_{r_{l}' - \eta_{l}' + \eta}^{r_{l}' + \eta_{l}'} s^{0}(r', u')r'dr' +
\]

\[
+ \frac{\lambda_{i}}{2r}(1 - e^{-\tau_{i}}) \int_{r_{l}' - \eta_{l}' + \eta}^{r_{l}' + \eta_{l}'} s^{0}(r', u_{l})r'dr', \tag{27}
\]

where

\[
s_{0}(r, u) = \frac{1}{2r\eta} \int_{r_{l}' - \eta}^{r_{l}' + \eta} n_{0}(r_{0}, \mu_{0})r_{0}dr_{0}, \tag{28}
\]
and according to eq. (18)

$$\mu = (r^2 - r_0^2 + \eta^2)/2r\eta, \quad \mu_0 = (r^2 - r_0^2 - \eta^2)/2r_0\eta.$$  \hspace{1cm} (29)

When deriving the main eq. (27) we turn from integration over \( \mu \) to integration over \( r' \) in the integral terms and to integration over \( r_0 \) in the free term. In the first case we use eq. (19) which yields

$$d\mu = -r'dr'/r(\eta - \eta'),$$  \hspace{1cm} (30)

and in the second case we use the first of eqs. (29) which gives

$$d\mu = -\tau_0 dr_0/r\eta.$$  \hspace{1cm} (31)

It’s worse noting that for \( \tau_1 = 0 \) the main eqs. (26) and (27) turn to the scalar equations obtained by us earlier (Grachev and Dubrovich, 2011) for the case of purely Thomson scattering. As concerned to the quantity of \( \tau_1 \) it does not exceed \( 4 \cdot 10^{-4} \) according to our calculations of primordial hydrogen recombination dynamics. So it is expedient to expand solution in a power series of \( \tau_1 \) and to find the first correction to the known solution for \( \tau_1 = 0 \). So we seek solution in the form \( s(r, u) = s^0(r, u) \) for \( u < u_1 \) and

$$s(r, u) = s^0(r, u) + \tau_1 s^1(r, u), \quad u > u_1.$$  \hspace{1cm} (32)

Inserting this expression into the formal solution (26) and retaining the terms not above the first power on \( \tau_1 \) leads to

$$\frac{n - n^0}{n^0} = -\tau_1 \left\{ 1 - \frac{\lambda_1}{r^0} \left[ e^{u_1} s^0(r'_1, u_1) + \int_{u_1}^{u} \tilde{s}(r', u') e^{u'-u} du' \right] \right\},$$  \hspace{1cm} (33)

where \( \tilde{s} = s^0 + s^1 \). Substitution of eq. (32) into eq. (27) gives for \( \tilde{s} \) the following equation

$$\tilde{s}(r, u) = \tilde{s}_0(r, u) + \frac{1}{2r} \int_{u_1}^{u} e^{u'-u} \frac{du'}{\eta - \eta'} \int_{|r - u + \eta|}^{r + \eta - \eta'} \tilde{s}(r', u') r'dr', \quad u > u_1,$$  \hspace{1cm} (34)

where the free term

$$\tilde{s}_0(r, u) = \frac{1}{2r} \frac{e^{u-u}}{\eta - \eta'} \int_{|r - u + \eta|}^{r + \eta - \eta'} s^0(r', u_1) r'dr'$$  \hspace{1cm} (35)

is expressed through the source function \( s^0(r, u) \) which is the solution of the problem without scattering in spectral lines. Hence at first the function \( s^0(r, u) \) should be calculated by the same method as in our previous work (Grachev and Dubrovich, 2011). Then eq. (34) is solved and further the dimensionless profile (33) is calculated. In general case all calculations must be fulfilled for each frequency separately. However in the case of Thomson scattering radiation spectrum changes due to cosmological redshift only. So if a frequency dependence of initial (for \( u = 0 (z = z_0) \)) intensity \( n_0(r, \mu, \nu) \) is detached from its spatial and angular dependences i.e. if \( n_0(r_0, \mu_0, \nu_0) = f(\nu_0) n_0(r_0, \mu_0) \) then using eq. (17) for \( \nu_0 \) the searched radiation intensity and source function can be written as

$$n^0(r, \mu, u, \nu) = f(\nu(1 + z_0)/(1 + z)) n_0^0(r, \mu, u),$$  \hspace{1cm} (36)

$$s^0(r, u, \nu) = f(\nu(1 + z_0)/(1 + z)) s_0^0(r, u),$$  \hspace{1cm} (37)
where along the photon path \( \nu/(1 + z) = \nu(0) \) is the photon frequency in the present epoch (at \( z = 0 \)) so that the right sides (and hence the left sides) of equations depend on the contemporaneous frequency. Substitution of eqs. \( (36) \) and \( (37) \) into eq. \( (9) \) (without an account of scattering in lines) instead of \( n \) and \( s \) leads to disappearance of the term derivative with respect to a frequency and after division of the both sides of the equation by the constant multiplier \( f(\nu(0)(1 + z_0)) \) we obtain an equation for intensity \( n^0(r, \mu, u) \) independent on frequency. We solved such an equation earlier (Grachev and Dubrovich, 2011) by reducing it to an integral equation for a source function \( s^0(r, u) \). Further we used obtained solutions to solve eq. \( (34) \). It turned out that the function \( \tilde{s}(r, u) \) is very close to \( s^0(r, u) \) and eq. \( (34) \) can be written as

\[
\frac{n - n^0}{n^0} = -\tau_l \left\{ 1 - \frac{\lambda_l}{n^0} \left[ e^{u_l - u} s^0(r'_l, u_l) + \int_{u_l}^u s^0(r', u') e^{u - u'} du' \right] \right\}. \tag{38}
\]

Here the right side does not depend on an initial radiation spectrum because it enter in \( n^0 \) and \( s^0 \) as a constant multiplier \( f(\nu(0)(1 + z_0)) \). By designation of the right side of eq. \( (38) \) as \( \varphi_\nu(r, \mu, u) \) where \( \nu \equiv \nu(0) \) is a contemporaneous frequency we obtain that according to eq. \( (38) \) radiation intensity (in terms of the mean photons occupation numbers) is defined as follows

\[
n(r, \mu, u_0, \nu) = f(\nu(1 + z_0)) n^0(r, \mu, u_0)[1 + \varphi_\nu(r, \mu, u_0)], \tag{39}
\]

where \( \nu \) is a frequency, \( \arccos \mu \) is an angular distance from the burst center, \( r \) is a distance from the burst center in the present day epoch (for \( z = 0(u = u_0) \)). Here both dimensionless profile \( \varphi_\nu \) and \( n^0 \) do not depend on a spectrum \( f(n_0) \) of initial radiation.

As concerns \( n^0 \) we assume that at the initial time moment \( t = 0 (\eta = 0) \) corresponding to some redshift \( z_0 \) it does not depend on angular variable \( \mu \) and it has spherically-symmetrical distribution:

\[
n^0(r, \mu, 0) = n_0(r), \tag{40}
\]

where \( n_0(r) \) is defined as

\[
n_0(r) = \pi^{-3/2} r_*^{-3} \exp[-(r/r_*)^2] \to \delta(r)/4\pi r^2 \quad r_* \to 0, \tag{41}
\]

where \( r_* \) is a parameter which defines characteristic size of the burst.

### Method of solution and main results

Using the codes devised in our previous work (Grachev and Dubrovich, 2011) we build summary profiles of H\( \alpha \) and H\( \beta \) lines and (separately) of P\( \alpha \) and P\( \beta \) lines. The calculations were carried out for two initial time moments \( z_0: 1600 \) and 1200 and for a few angular distances from the direction to the burst center.

For the width of initial intensity distribution as a function of \( r \) (see eq. \( (41) \)) we use two values \( r_* = 1.5 \) and 50 Mpc in a distance scale at \( z = 0 \). But in a distance scale corresponding to a burst moment (at \( z = z_0 \)), the width of initial distribution for \( z_0 \gg 1 \) will be significantly lower: \( a(z_0) r_* = r_*/(1 + z_0) \).

As to another parameters in the problem they enter in particular in the Hubble factor

\[
H(z) = H_0 \sqrt{\Omega_\Lambda + (1 - \Omega_0)(1 + z)^2 + \Omega_M(1 + z)^3 + \Omega_{rel}(1 + z)^4}, \tag{42}
\]

9
where \( H_0 = 2.4306 \cdot 10^{-18} h_0 \text{ s}^{-1} \), \( h_0 \) is the Hubble constant in units 75 km/(s·Mpc); \( \Omega_M, \Omega_\Lambda \) and \( \Omega_{\text{rel}} \) are the density ratios of the matter, dark energy and relativistic particles (radiation, massless neutrino) to the critical density \( \rho_c = 3H_0^2/(8\pi G) \) in the present epoch; \( \Omega = \Omega_M + \Omega_\Lambda + \Omega_{\text{rel}}, \Omega_{\text{rel}} = \rho_{\text{R}}^0/(1 + f_n)/\rho_c, \rho_{\text{R}}^0 = a_R T_0^4/c^2 \) is the density of radiation mass now (\( T_0 \) is an average CMB temperature), \( f_n \) is the part of relativistic (massless) neutrino (usually \( f_n = 0.68 \)). For the flat model of the Universe \( \Omega = 1 \) and then \( \Omega_M = 1 - \Omega_\Lambda - \Omega_{\text{rel}} \).

Furthermore an electron concentration enters into equations. Usually it is measured in the units of the total number density \( n_H \) of hydrogen atoms and ions: \( n_e(z) = x_e(z)n_H(z) \) where \( x_e(z) \) is so-called recombination history of the Universe and

\[
n_H(z) = n_H^0 (1 + z)^3, \quad n_H^0 = 0.63144 \cdot 10^{-5} X \Omega_B h_0^2 \text{ cm}^{-3},
\]

where \( \Omega_B \) is the ratio of the baryons density to the crytical density now, \( X \) is the hydrogen mass abundance. Recombination history is calculated separately and it is an entry file. We calculate it using the code recfast.for (Seager et al., 1999). As the base values we use: \( \Omega = 1, \Omega_\Lambda = 0.7, \Omega_B = 0.04, T_0 = 2.728 \text{ K}, \) hydrogen abundance \( X = 0.76, \Omega_{\text{rel}} = 0.85 \cdot 10^{-4} \), Hubble constant \( H_0 = 70 \text{ km/(s·Mpc)}. \)

As to the optical depths \( \tau_{ik} \) in subordinate lines they were calculated by means of our codes (Dubrovich and Grachev, 2004) for the learning of primordial hydrogen recombination and occupation numbers of levels behavior. Results of these calculations were also used to obtain time dependencies of single scattering albedos \( \lambda_{ik}(z) \).

On the fig. 1 are the graphs of the optical depths in H\( \alpha \), H\( \beta \), P\( \alpha \) and P\( \beta \) lines and on the fig. 2 are the graphs of albedos of single scattering in these lines. On the fig. 3 are the space profiles of the average intensity of scattered radiation in the present epoch \( (z = 0) \).

The results of the profiles calculations for H\( \alpha \) and H\( \beta \) lines are presented on figs. 4 – 8. The typical feature of the profiles is the presence of absorption jumps caused by the source (burst) appearance at a given redshift \( z_0 \) so that jumps arise on the frequencies \( \nu = \nu_{ik}/(1 + z_0) \) where \( \nu_{ik} \) is the frequency of transition \( i \to k \). Another feature consists in the presence of emission components which appear in the profiles for distances on the back side of spatial distribution of average radiation intensity (see fig. 3) where photons “lagged behind” due to scatterings are disposed. Photons scattered in line sideways return into a given direction due to scatterings on free electrons with some delay. Just they form emission components of the line profiles and there is a minimum of “lagged behind” photons in the direction of the burst center. So when moving off this direction emission components enhance and absorption components weaken (see fig. 8). When passing from the back front of an average intensity distribution to the front one (see crosses on the curves on fig. 3) emission components weaken and absorption ones enhance and in the end the line becomes purely absorption one (see figs. 4 – 7). This is connected with the decrease of contribution of photons “lagged behind” due to scatterings.

From comparison of the profiles for the bursts with great (50 Mpc) and small (1.5 Mpc) size it follows that in the first case absorption is less deep than in the second one. Further an appearance of narrow emission component on the fig. 7 for the burst at \( z_0 = 1200 \) with characteristic size 1.5 Mpc is connected with a comparatively small optical thickness \( (\approx 3) \) of the Universe due to Thomson scattering between the moments \( z = 1200 \) and \( z = 0 \). So we see photons arrived from the burst directly without scatterings. On the fig. 3 they form a narrow peak with a width \( \approx 1.5 \text{ Mpc} \) at the distance equal to conformal time of the burst center \( \eta = 13635 \text{ Mpc} \). On the fig. 7, where the profiles for different distances from the burst
center are displayed, the maximum height of emission peak is achieved namely on the distance close to mentioned above conformal time. On the smaller distances overwhelming contribution give scattered photons which have significantly wider density redistribution in space (see fig. 3). So the height of emission peaks turns out to be smaller and their width greater.

Also the profiles for the case of pure scattering ($\lambda_{23} = \lambda_{24} = 1$) were calculated and they were compared with the profiles for real dependence of a single scattering albedo on redshift $z$ (see fig. 2). As it should be expected in the last case the jumps of absorption appear on the frequencies corresponding to the moments of the bursts appearance and as a whole the profiles go lower than in the case of pure scattering. The difference is especially great for the burst of large size.

The summary profiles of $P_\alpha$ and $P_\beta$ lines were also calculated. $P_\alpha$ and $P_\beta$ lines turn out to be 10–30 times weaker than $H_\alpha$ and $H_\beta$ lines because of the lower optical depths (see fig. 1) and moreover they are much more weaker depend on parameters because of the lower albedo of single scattering (see fig. 2). In the limit of $\lambda \to 0$ the line profile is purely absorption one and it is defined only by an optical depth profile (as it is seen from eq. (38)) and by the burst moment $z_0$ which defines position of the jump in the profile at the low frequency edge for $\nu = \nu_{ik}/(1 + z_0)$.

It should be stressed once more that the profiles displayed on figs. 4 – 7 are calculated with the proper account of the real dependence of single scattering albedos $\lambda_{ik}$ in lines on redshift $z$.

**Conclusions**

The local bursts model of primordial plasma and radiation temperature fluctuations in the early Universe is considered. These fluctuations can be represented as fastly variable sources with the initial blackbody spectrum of radiation with the temperature which slightly differs from the average CMB temperature. More generally, one may consider as a source any real picked out object: primordial accreting black hole for example. In this work we calculate transitional function from radiation intensity of these sources to an observed intensity taking into account photons scatterings on free electrons and in hydrogen subordinate lines. In the first approximation this function makes sense of an optical depth in regard to scattering in lines. However due to multiplicity of scattering on free electrons a part of photons scattered in atomic lines returns on the line of sight in another space point which leads to an appearance of emission components in line profiles.

In the capacity of the model we consider scattering of continuous radiation of the source (instantaneous burst of radiation at a given redshift $z_0$) on electrons and in $H_\alpha$, $H_\beta$, $P_\alpha$ and $P_\beta$ lines of primordial hydrogen at the recombination epoch. It is shown that thus arising lines in a source spectrum are generally absorption ones with the depths from $3 \cdot 10^{-5}$ to $2 \cdot 10^{-4}$ (for $H_\alpha$ line) depending on characteristic initial size of the source and on the distance from its center. For $P_\alpha$ line the depth of absorption is by 10 – 30 times smaller. Real observations give maximum amplitude of the order of 500 $\mu$K for temperature deviation in the spots. So relative magnitude of temperature fluctuations lies within the limits of $10^{-7} – 10^{-9}$. The profiles may contain emission components with lower intensities (with respect to source continuum) as compared with absorption components. It’s worse noting also that the most deep absorption is at the low frequency edge of the profile.

This work was supported in part by St. Petersburg State University grant No. 6.38.18.2014.
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Figure 1: The profiles of optical depths in $\text{H}_\alpha$, $\text{H}_\beta$, $\text{P}_\alpha$ and $\text{P}_\beta$ lines.
Figure 2: The profiles of albedos of a single scattering in $\text{H}_\alpha$, $\text{H}_\beta$, $\text{P}_\alpha$ and $\text{P}_\beta$ lines.
Figure 3: The profiles of average intensity of scattered radiation for the bursts at $z_0 = 1200$ and 1600 with characteristic sizes 1.5 and 50 Mpc (at $z = 0$). The crosses mark the points where the profiles of subordinate lines were calculated.
Figure 4: Summary profiles of $H_\alpha$ and $H_\beta$ lines for the burst at $z_0 = 1200$ with the characteristic size 50 Mpc (at $z = 0$) in the direction of the burst center for different distances $r$ (in Mpc) from the center.
Figure 5: The same as on fig. 4 but for $z_0 = 1600$. 

\[(n-n^0)/n, 10^{-4}\]
Figure 6: Summary profiles of H\(_\alpha\) and H\(_\beta\) lines for the burst at \(z_0 = 1600\) with the characteristic size 1.5 Mpc (at \(z = 0\)) in the direction of the burst center for different distances \(r\) (in Mpc) from the center.
Figure 7: The same as in the preceding figure but for $z_0 = 1200$. 

\[ \frac{(n-n^0)}{n^0}, 10^{-4} \]
Figure 8: Summary profiles of $H_\alpha$ and $H_\beta$ lines on different angular distances $\theta$ from the burst center for the burst at $z_0 = 1600$ with the characteristic size $r_\ast = 1.5$ Mpc. The distance from the center of the burst is $r = 13630$ Mpc.