One more observational consequence of many-worlds quantum theory

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Using new cosmological doomsday argument Page predicts that the maximal lifetime of de Sitter universe should be \( t_{\text{max}} = 10^{60} \) yr which is way too small in comparison with strings predictions \((t_f > \text{googleplex})\). However, since this prediction is dependant on the total number of human observations, we show that Page arguments results instead in astounding conclusion that this number is the quantum variable and is therefore much greater then Page’s estimation. Identifying it with the number of coarse-grained histories in de Sitter universe we get the lifetime of the universe comparable with strings predictions. Moreover, it seems that this result can be considered as another one of the observational evidences of validity of the many-worlds quantum theory. Finally, we show that for the universe filled with phantom energy \( t_{\text{max}} \sim t_f \) up to very high precision.

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I. INTRODUCTION

In article [1] Don Page has presented the forcible argument that the lifetime of the de Sitter universe \( t_{\text{max}} < 10^{60} \) yr. On the other hand, the string theory prediction grants the dS universe as much time as \( t_f < \text{recurrence time} \sim 9.5 \times 10^{74} \) yr [2, 3] (the matter of whether it should be seconds, years or even millenniums is really unessential for such monstrous numbers). It is possible to lower this value to the \( t_f \sim 10^{10} \) yr even to the limit of \( t_f \sim 10^2 \) yr for models with instantons of Kachru, Pearson and Verlinde [4] and with 2 Klebanov-Strassler (see [5]) throats [6]. But, nevertheless, even with assumption that one of those models do describe our Universe, the magnitude \( t_f \) will still be way too large as compared to Page’s \( 10^{60} \) yr. Thus we are facing the following dilemma: either the dark energy is not pure positive cosmological constant at all (and stringscpe should not have any significant long-lived positive metastable minima) or the dark energy IS the cosmological constant and our ideas about stringscpe (and the strings theory in general) are absolutely false!

Actually, the nature of dark energy is one of those questions, which can be redirected to astronomers. It appears, that there exist some observational series which proved to be sufficiently difficult to explain with the assumption that the dark energy should be some scalar field (quintessence) rather then cosmological constant. (The phenomenon of this kind is, for example, the drift of unhomogeneous local volume (1 Mpc) with the regular Hubble flow inside [7]). Of course, those results can appear to be of statistically insignificant nature, but if not, then it would mean one strong graphic evidence of presence of the cosmological constant.

So, does there exist some kind of ”loophole” in the Page reasoning? Something, that would allow us to conclude that the lifetime of dS universe can be \( t_{\text{max}} \sim t_f \) with \( t_f \sim \text{googleplex} \)? As we shall show further, such loophole indeed exists. To present it we will have to re-examine the essence of the Page’s argumentation, and it will be done in the next Section. In Section III we’ll consider the case of phantom cosmology to show that it surprisingly grants us \( t_{\text{max}} = t_f \), and therefore, in such universes the Page reasoning doesn’t lead to inevitable conflict, as differs from dS models. Next Section is essentially devoted to the universes filled with vacuum energy and to the way of preventing the Page conclusion \( t_{\text{max}} \ll t_f \). Here we will show that it is possible to obtain \( t_{\text{max}} \sim t_f \) in assumption that the total number of human observations is the quantum variable. And Section V is the overall conclusion.

II. PAGE ARGUMENT

Following to Page, suppose that the process of observation is described by some localized positive operator \( A \), such that application of it to any state \( \psi \) leads to positive central tendency. This implies that every possible observation has some positive probability of occurrence in the given volume (e.g., as a vacuum fluctuation). Therefore, we can treat the observers as the standard quantum objects. With this in mind, Page has calculated the action for the brain of a human observer: \( S_{\text{br}} \sim 10^{16} J \times s \), and the probability \( p_{\text{br}} \sim e^{-S_{\text{br}}/\hbar} \sim e^{-10^{50}} \). Then, Page made an estimation for 4-volume for the brain \( V_4(\text{br}) \), taken in process of making the observation: \( V_4(\text{br}) \sim e^{331} \alpha_P^4 \).

Next, let’s assume that we are living in dS universe filled with vacuum, energy density \( \rho_\Lambda \sim 10^{-29} \text{gramme/cm}^3 \) of which greatly exceeds the total density of all other energy components in the universe.

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Then we appear to be merely prisoners in "cosmic prison" of a radius \( R = c/H \), where Hubble constant 
\( H = \sqrt{8\pi G \rho_c/3} \). After \( 10^{17} \) yr each and every star in 
the universe will be either black hole, black dwarf or neutron 
star; \( 10^{10} \) Gyr later the temperature of neutron stars 
will decrease to less than 100 K. It is mildly speaking 
unlikely that human-observers will be able to endure in 
such inhospitable universe. However, it wouldn’t really 
matter at that point, because no life (including human-
observers) will be able to exist there forever due to both 
proton decay (it’s time life \( t_{pr} \) > \( 10^{32} \) yr) and the 
exponential falloff of the density of matter (information, 
being processed in ever-expanding universes was consid-
ered in [8]. There has been shown that an infinite amount 
of information can be processed via the usage of tem-
perature gradients created by gravitational tidal energy, 
but only in assumption that the cosmological constant 
is equally zero). Therefore if \( t_f \) ∼ gooleplex then except 
for unimaginably tiny initial period from the big bang to 
\( t_{pr} \) the universe will be absolutely dead. It is definitely 
not bright future at all!

On the other hand, taking into account the unimag-
nably long lifetime of such universe we shall conclude 
that all possible events, including those with extremely 
low probability, will someday occur. One of the most 
interesting of those unlikely events would be the sponta-
naneous appearance from quantum fluctuations of "ob-
servers", surrounded by "environment" suitable to per-
mit the "observation". With this conclusion, it would 
be only natural to ask: can we in principle be one of 
those "vacuum observers"? And, more generally: un-
der what circumstances will the ordered (i.e. classical) 
observations dominate over vacuum ones? Page gives 
the following answer: if \( t < t_{max} \) = \( 10^{60} \) yr, and only 
then will the human observations be with high probabili-
ty ordered. Otherwise, almost all observations in the 
universe will have its root in vacuum fluctuations. As 
the result, in universe with \( t_{max} \) ∼ \( t_f \) ∼ gooleplex our, 
obviously ordered, observations are to be considered as 
something embarrassingly atypical. Page concludes that 
"This extreme atypicality is like an extremely low like-
lihood, counting as very strong observational evidence 
against any theory predicting such a long-lived universe 
with a quantum state that can allow localized observa-
tions", and makes the prediction that the universe just 
will not last long enough to give 4-volume \( > e^{10^{50}} \).

To show this in work, let’s consider the total 4-volume 
of universe:

\[
V_4(t) = c \int_0^t dt a^3(t).
\]  

(1)

The probability of vacuum fluctuations \( p_{vac} \) < \( p_{br} \) 
whereas the probability of ordered occurrences \( p_{ord} \) > 
\( p_{br} \). Multiplying \( V_4(t) \) by \( p_{ord} \) results in the volume of 
the part of total \( V_4(t) \) where ordered occurrences are 
dominating ones. Now let \( N \) be the number of observa-
tions, made during the past human history. The product 
\( NV_4(br) \) will mark the part of total \( V_4(t) \) where ordered 
human observations all take place. If humans are the typ-
ical observers (anthropic principle!) then one can expect that 

\[
V_4(t)p_{ord} \sim V_4(br)N.
\]  

(2)

Substituting \( a(t) = a_0 e^{Ht} \) into the (1) one get \( V_4(t) \). Follow-
ing Page we can evaluate \( N \sim e^{48} \). Substituting \( N \) and 
\( V_4(t) \) into the (2) allows us to express \( p_{ord} \). Finally, 
using the inequality \( p_{ord} > p_{br} \) one comes to conclusion 
that, under those circumstances, the timelife of the dS 
universe is indeed \( t < t_{max} = 10^{60} \) yr.

### III. PHANTOM ENERGY

Let’s see, what will happen with Page results in the 
universe filled with phantom energy. It appears, that in 
contrast to dS models, for such universes we get a re-
markable concordance: \( t_f = t_{max} \) up to very high degree 
of accuracy.

Before we start, we should mention, that the partic-
ular interest to the models with phantom fields arises 
from their prediction of so-called "Cosmic Doomsday" 
alias big rip [3]. Since for the phantom energy we have 
\( w = p/(c^2 \rho) = -1 - \epsilon \) with \( \epsilon > 0 \), the integration of the 
Einstein-Friedmann equations for the flat universe results in 

\[
a(t) = \frac{a_0}{(1 - \xi t)^{2/3 \epsilon}}, \quad \rho(t) = \rho_0 \left( \frac{a(t)}{a_0} \right)^{3 \epsilon} = \rho_0 \left( 1 - \xi t \right)^{2},
\]  

(3)

where \( \xi = \sqrt{6\pi G \rho_0} \). Choosing \( t = 0 \) as the present time, 
\( a_0 \sim 10^{28} \) cm and \( \rho_0 = 1.4 \rho_c/(2 + 3\epsilon) \) as the present 
values of the scale factor and the density (If \( \epsilon \ll 1 \) then 
\( \rho_0 \approx 0.7 \times 10^{-29} \) g/cm³), at time \( t = t_f = 1/\xi \), we 
automatically get the big rip.

Now, let’s return to our question. Equations (1) and 
(3), taken together, lead to 

\[
V_4(t) = \frac{ca_0^3 \epsilon}{\xi(2 - \epsilon)} \left( \frac{1}{(1-t/t_f)^{(2-\epsilon)/\epsilon} - 1} \right) + V_4(0),
\]

where \( V_4(0) = a_0^4 = 10^{112} \) cm⁴ = \( e^{258} \) cm⁴. Using Page 
approach we have 

\[
\frac{ca_0^3 \epsilon}{\xi(2 - \epsilon)} \left( \frac{1}{(1-t/t_f)^{(2-\epsilon)/\epsilon} - 1} \right) < V_4(br)e^{S_{br}/k} - V_4(0).
\]

The second member of the equation is 

\[
3 \times e^{48} \times 10^{12} \times e^{10^{50}} - e^{258} \sim e^{10^{50}},
\]

therefore 

\[
\left( 1 - \frac{t}{t_f} \right)^{(\epsilon-2)/\epsilon} < e^{10^{50}}.
\]  

(4)
In the case $\epsilon \ll 1$ we get

$$\frac{t_f - t}{t_f} > \exp\left(-\frac{0.5 \times 10^{50}}{t_f \sqrt{6\pi G \rho_0}}\right) = \exp\left(-\frac{1.685 \times 10^{67}}{t_f}\right).$$

Now, we have to consider 3 different cases.

a. $t_f \gg 1.685 \times 10^{67}$ s = 5.3 $\times$ 10$^{59}$ yr. In this case the power of exponent in (5) is small enough to use the expansion in Taylor’s series. It’s application results in inequality $t < t_{\text{max}} = 5.3 \times 10^{59}$ yr. This leaves us with the same problem as in dS situation: the end of the world will take place at $t = t_f$ but ordered observation will be dominating ones while $t \ll t_f$ only.

b. $t_f \ll 1.685 \times 10^{67}$ s. In this case

$$t < t_{\text{max}} = t_f \left(1 - e^{-1.685 \times 10^{67}/t_f}\right) \sim t_f.$$

Here we come to remarkable difference between phantom and dS cosmologies. While in the last case we have $t_{\text{max}} = 10^{60}$ yr $\ll t_f$ > googleplex, where $t_{\text{max}}$ follows from Page’s reasoning and $t_f$ is the string theory prediction, in the former case the situation can be much more agreeable: in fact, the validity of the $b$ condition ensures that $t_{\text{max}} \sim t_f$. As we can see from Tab. 1, $t_{\text{max}} \to t_f$ very fast when $t_f$ decreases. If $t_f = 5.3 \times 10^{50}$ yr then $t_{\text{max}} = t_f(1 - e^{-10^{50}})$ and $t_f = 22$ Gyr stands for $t_{\text{max}} = t_f(1 - e^{-10^{50}})$, thus actually erasing the very difference between $t_{\text{max}}$ and $t_f$.

c. $t_f \sim 1.685 \times 10^{67}$ s. This case implies

$$t_{\text{max}} = \left(1 - \frac{1}{e}\right) t_f \sim 0.63 t_f.$$

Therefore, in such Universe only about half of all observers can assuredly consider themselves classical and having the naturally ordered observations, which is sufficiently better then what we had in dS universe, yet still being far from perfect.

Summarizing all of the above, we can conclude that the one and essentially the only convenient case is $b$. After all, for $t_f < 10^{59}$ yr it gives us $t_f \sim t_{\text{max}}$ for granted!

### IV. THE NUMBER OF COARSE-GRAINED HISTORIES

For the time being, the physical meaning of phantom fields is as yet unclear. For this reason let’s return back to the realistic case of dS universe and seeming inconsistency between $t_{\text{max}}$ and $t_f$ ($t_{\text{max}} \ll t_f$), that has been found in it. The core of Page’s argumentation is the equation (2). But let’s inspect carefully the quantities, forming it. It is clear that, by complete analogy with $p_{br}$, quantity $p_{ord}$ should be calculated by quantum laws. Indeed, in the framework of Page approach one need make a comparison $p_{br}$ with $p_{ord}$. It is as well to remember that $p_{br}$ is the quantum quantity, therefore, generally speaking, the same must be true for the $p_{ord}$. As a matter of fact, $p_{ord} = e^{-S_{\text{nrm}}/\hbar}$. Therefore, l.h.s of equation (2) is dependant on $h$. But the equivalence will hold only if the same will be true for the r.h.s. If the value $V_4(\text{br})$ is purely classical, then $N$ is the only remaining candidate for the dependancy on $h$.

At a first glance this conclusion seems absolutely grotesque, but it appears to be right in touch with Page resonings. As a matter of fact, in his article Page deals with quantum (or semi-classical) observers. The number of quantum observers $N$ is the quantum quantity and hence, must be calculated by the quantum laws. From this point of view, it is no wonder that $N$ will depend on $h$.

But if this is correct, then one can’t use Page estimate ($N \sim e^{48}$) anymore. Unfortunately, we can’t calculate $N$ explicitly, but we can evaluate it upon usage of very simple quantum-based reasoning. It is already clear that ”new” $N$ should be much greater then $e^{48}$. As we shall see, this number can exceed even googleplex, thus totally refuting Page argument.

One can roughly evaluate the number $N$ as the number of coarse-grained histories: $N = N_b = N_b^{N_c}$ where $N_c$ is the number of spacetime cells and $N_b$ is the number of relevant bins in field space. In the article [10] Garriga and Vilenkin have made this for the spacetime volume with the size $R = c t_0$ where $t_0 = 10^{10}$ yr. As a result they got $N \sim e^{10^{64}}$. Substituting this value into the (2) one get $t_{\text{max}} = 10^{261}$ yr. This number is by many orders greater than Page’s $10^{60}$ yr but is still too small in comparison with $e^{10^{311}}$. However, the number $N$ easily allows for additional increase up to to the point, where $t_{\text{max}}$ will be comparable with strings predictions.

Indeed, remaining in framework of quantum theory we should consider all possible observers, including those who are living in much older universes where vacuum energy already exceeds the total density of all the other energy components in the universe. In such universes

$$V_4(t) \sim \frac{c_0^3}{\sqrt{24\pi G \rho_\Lambda}} e^{\sqrt{2 \pi G \rho_\Lambda}} = e^{0.5 \times 10^{-17} t}.$$  

For example, if $t = 10^{17}$ yr (the era of black holes) one have $V_4 = e^{0.2 \times 10^8}$ and if $t = 10^{32}$ yr (the low bound
of proton lifetime) then \( V_4 = \frac{e^{0.2 \times 10^{23}}}{L^4} \). The number of spacetime cells of size \( L \) will be \( N_c \sim V_4(t)/L^4 \sim e^{10^8} \) in first case and \( N_c \sim e^{10^{23}} \) in the second one. But in all cases the values of \( N \) are given by "supergoogleplex" numbers:

\[
N = e^{10^{23}}, \quad N = e^{10^8}.
\]

Substituting them in (2) we finally get \( t_{max} = e^{10^8} \) yr or

\[
t_{max} = e^{10^{23}} \text{ yr}.
\]

The interesting fact here is that both these numbers lie in remarkable agreement with the results of [6]. In particular, for the case of 3 D3-branes with some parameters there have been obtained theoretical value \( t_f \sim e^{10^{19}} \) (lifetime on the NS5-brane). Decays due to decompactification are much faster: \( t_f \sim e^{10^{17}} \). Those are results of the models with the single KS throat. Consideration of 2 KS throats (such models are more interesting since they result in positive cosmological constant whereas the models with single KS throat result in \( \Lambda < 0 \) in case of KPV instantons leads to such value as \( t_f \sim e^{10^6} \) - very good agreement with the previously obtained \( t_{max} = e^{10^{18}} \).

In the case of general position one can conclude that

\[
t_{max} = 10^{17} e^{10^{-17} t} \text{ yr},
\]

where \( t \) is the maximal possible lifetime of "human-observers". Thus, if \( t_{max} = \text{googleplex} \) one get \( t = 10^{117} \) yr while \( t_{max} = e^{10^{23}} \) yr implies \( t = 10^{140} \) yr. Of course, it can be difficult to imagine that \( 10^{117} \) yr later the universe will be filled by "human-observers". Besides, it can be argued whether such "observers" fits into the set being reviewed or not. But the answer is very simple: whenever the probability of finding ourselves in such universe has the nonzero value, we have to take it into account.

Finally, we should answer the following question: are those "auxiliary" observers real, or not, i.e. can we ascribe all of them to some really existing Universes, or are they nothing more then "vacuum probabilities"? The answer is: yes, they have to be real; otherwise, we are facing the situation, where the \( 10^{10^{10}} \) quantum objects are required to explain the existence of \( e^{18} \) (real) objects. Here is the same Page's paradox, only in other form and aggravated by much worsened numbers!

V. CONCLUSIONS

As we have seen, the assumption that \( N \) is the total number of quantum observers results in such lifetime of universe which is comparable with strings predictions. This creates the very strong grounds for serious consideration of such strange possibility. After all, the quantum nature of \( N \) seems to be absolutely inevitable in quantum cosmology.

Of course, such state of affairs is something highly unusual in "everyday" quantum mechanics. It has already become a common fact, that in laboratory research with neutron interferometer the neutron passing through a beam splitter will split into "two neutrons". But in lab we don’t expect that the same will be true for us. Observers are classical objects "ad definition".

However, in quantum cosmology this situation changes drastically. Since we are nothing but the part of the universe we have no choice but to consider ourselves as quantum objects. Page has shown in [11] that quantum cosmology can give observational consequences of many-worlds quantum theory. We think that our results can be considered as one more observational evidence of validity of many-worlds quantum theory.

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