HEAVY FLAVOUR PRODUCTION IN $pp$ AND HEAVY ION COLLISIONS IN QCD UP TO LHC ENERGIES

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Abstract

Charm production in $pp$ collisions is considered in the framework of perturbative QCD. The values of two parameters, the charm quark mass $m_c$ and the QCD scale $\mu^2$, are determined from the comparison of the theoretical calculations with experimental data. The RHIC data on charm and beauty production are compared with the $k_T$-factorization approach predictions and with standard NLO QCD. The calculated results underestimate the STAR Collaboration data. The role of possible nuclear effects is discussed. The predictions for LHC energies are also given.

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1 Introduction

The investigation of heavy quark production in high energy collisions is an important method for studying the quark-gluon structure of hadrons and the possible nuclear effects at early stages of secondary production. The description of hard interactions in hadron collisions within the framework of QCD is possible only with the help of some phenomenological assumptions which reduce the hadron–hadron interaction to the parton–parton one via the formalism of the hadron structure functions. The cross sections of hard processes in hadron–hadron interactions can be written in a factorized form [1] as the convolutions of the squared matrix elements of the subprocess calculated within the framework of QCD with the parton distributions in the colliding hadrons.

The most popular phenomenological approach is the NLO QCD collinear approximation [2–5], where the cross sections of QCD subprocesses are calculated in the Next-to-Leading Order (NLO) of $\alpha_s$ series. The Fixed Order plus Next-to-Leading-Log (FONLL) [6] also resumes large perturbative terms proportional to $\alpha_s^n \log^k (p_T/m)$ with $k = n, n - 1$, where $p_T$ and $m$ are the heavy quark transverse momentum and the mass, respectively. In these calculations all particles involved are assumed to be on mass shell, carrying only longitudinal momenta, and the cross section is averaged over two transverse polarizations of the incident gluons. The virtualities $q^2$ of the initial partons are taken into account only through their structure functions.

The formalism which incorporates the incident parton transverse momenta is referred to as the $k_T$-factorization approach [7–10], or the theory of semihard interactions [11–19]. Here the Feynman diagrams are calculated taking into account the virtualities and all possible polarizations of the incident partons. In the small $x$ domain there is not ground to neglect the transverse momenta of gluons, $q_1T$ and $q_2T$, when compared to the quark mass and to the quark transverse momenta. In such an approach the QCD matrix elements of the partonic subprocesses are rather complicated. We have calculated them in LO. On the other hand, the multiple emission of soft gluons has been included in our calculation. In the $k_T$-factorization approach the unintegrated gluon distributions are used instead of the usual structure functions.

For the case of heavy ion collisions the linear $A$-dependence of the heavy flavour production cross sections is usually assumed, so the obtained results are usually discussed in terms of “cross section scaled to $pp$ collision”. This general assumption is experimentally supported by the result of [20], $\sigma \sim A^\alpha$ with $\alpha = 1.02 \pm 0.03 \pm 0.03$ (recent results of SELEX Collaboration [21] give significantly smaller values of $\alpha$). In perturbative QCD with the factorization approximation [1] the case of heavy ion collisions differs from $pp$ collisions only by the change of the usual nucleon structure functions or unintegrated gluon distributions by the same functions for bound nucleons. At RHIC energies the difference among parton structure functions is known from EMC and NMC experimental data [22] and their GLDAP evolution equation analysis [23]. As a result, the nuclear effects in QCD for the total cross section of charm production at RHIC are estimated to
be on the level of 5–10% [24] of the linear A-dependence values.

The experimental data of PHENIX [25, 26] Collaboration obtained at RHIC both from \( pp \) and nuclear collisions are in reasonable agreement with the NLO QCD and the \( k_T \)-factorization approach predictions. At the same time, the STAR [27] Collaboration data obtained at RHIC in nuclear collisions are in contradiction with standard QCD calculations. In the present paper we discuss the accuracy of the theoretical calculations, and the plausible reasons for the charm production enhancement on nuclear targets.

2 Determination of the QCD parameters

The standard QCD expression for the heavy quark production cross section in a hadron 1–hadron 2 collision has the form

\[
\sigma^{12\to QQ} = \int_{x_{a0}}^{1} dx_a \int_{x_{b0}}^{1} dx_b G_{a/1}(x_a, \mu_F^2) \cdot G_{b/2}(x_b, \mu_F^2) \cdot \hat{\sigma}^{ab\to QQ}(\hat{s}, m_Q, \mu_R^2),
\]

(1)

where \( \mu_F \) is the QCD factorization scale, \( x_{a0} = 4m_Q^2/s \), and \( x_{b0} = 4m_Q^2/(sx_a) \). Here \( G_{a/1}(x_a, \mu_F^2) \) and \( G_{b/2}(x_b, \mu_F^2) \) are the structure functions of partons \( a \) and \( b \) inside hadrons 1 and 2, respectively, and

\[
\hat{\sigma}^{ab\to QQ}(\hat{s}, m_Q, \mu_R^2) = \alpha_s^2(\mu_R^2) \cdot \sigma_{ab}^{(0)} + \alpha_s^3(\mu_R^2) \cdot \sigma_{ab}^{(1)}
\]

(2)

is the cross section of the subprocess \( ab \to QQ \), given by standard QCD as the sum of LO and NLO contributions [2, 3, 5]. These contributions depend on the parton center-of-mass energy \( \hat{s} = (p_a + p_b)^2 = x_a \cdot x_b \cdot s \), the mass of the produced heavy quark \( m_Q \) (actually only on \( \rho = 4m_Q^2/\hat{s} \)), and the QCD renormalization scale \( \mu_R^2 \).

Thus in the QCD calculation of total cross section of charm production three parameters are involved: the heavy quark mass, \( m_Q \), and the two QCD scales, \( \mu_F^2 \) and \( \mu_R^2 \). The heavy quark mass \( m_Q \) can be estimated as a half of the mass of light quarkonium. More accurately, in QCD the masses of heavy quarks should depend on the distance at which they are measured. When including this dependence one obtains [28, 29]:

\[
m_c = 1.4 \text{ GeV} , \quad m_b = 4.6 \text{ GeV} .
\]

(3)

The two QCD scales are usually assumed to be equal in numerical calculations, \( \mu^2 = \mu_R^2 = \mu_F^2 \), and from a general point of view they should be of the order of the maximal hardness of the interaction. If the produced heavy quarks are heavy enough, one can assume that

\[
\mu^2 \simeq m_Q^2,
\]

(4)

though the estimation \( \mu^2 \simeq \sqrt{m_Q^2 + p_T^2} \) used in the \( k_T \)-factorization approach seems to be also reasonable.
The next question concerns the uncertainties in the values of these parameters. Usually, the accuracy of our knowledge on heavy quark mass is assumed to be about 10% [30] (for $c$-quark), and smaller for $b$-quark. The general thinking is that the uncertainty of the value of QCD scale $\mu^2 \simeq m_Q^2$ is about a factor 2 larger. Of course, there is no reason why factor 2 should be more reasonable than, say, a factor 1.5 or a factor 3. For our purpose, the uncertainties in the values of the discussed parameters should be in any case compatible with the values of experimental data including their errorbars. As an example, the low boundary for the charm production cross section presented in [31] is in total disagreement with all available experimental data, meaning that the uncertainties assumed for the values of the parameters were too large.

We obtain the values of the QCD parameters $m_c$ and $\mu^2$ by fitting the results of the NLO QCD expression in Eqs. (1) and (2) to the fixed target experimental data in [32] and [33], the RHIC point for $pp$ collisions [34], and the UA2 Collaboration point [35]. We have used the GRV95 parton distributions [36], which are compatible with the most modern analysis (see discussion in [37]). Moreover, the difference to calculations performed with other sets of parton structure functions is small in the considered energy region. The result of our fit is shown in Fig. 1 by solid curve.

![Figure 1: Total cross section of charm production in $pp$ and $\bar{p}p$ collisions at $\sqrt{s} > 25$ GeV. Experimental fixed target data are taken from [32] and [33], RHIC point from [34], and UA2 Collaboration data from [35]. The solid curve shows the result of the fit of QCD parameters $m_Q$ and $\mu^2$, and dashed curves show the edges of the uncertainties (the region between the $\chi^2$ minimal value and the $\chi^2$ minimal value plus one).](image-url)
This fit corresponds to the following parameter values:

\[ m_c = 1.48 \pm 0.03 \text{ GeV} , \quad \mu^2 = 2.10 \pm 0.21 \text{ GeV}^2 , \quad \chi^2 \]

that seem to be very natural. The value of \( \chi^2 \) is 7.3/6 ndf. The uncertainties are shown by dashed curves and they correspond to increase the value of \( \chi^2 \) by one from its minimal value.

The same analysis for the total cross section of charm production in \( \pi p \) collisions is shown in Fig. 2. Here the value of \( \chi^2 \) is 12./7 ndf and the parameter values are

\[ m_c = 1.83 \pm 0.30 \text{ GeV} , \quad \mu^2 = 2.90 \pm 0.06 \text{ GeV}^2 , \quad \chi^2 \]

One has to note that in both analysis we obtain \( \mu^2 \simeq m_c^2 \), as it is usually assumed \textit{a priori}. Also the two values of \( m_c \) almost coincide inside the error bars. On the other hand, the difference seeming to appear in the \( \mu^2 \) values can be connected to the not very good knowledge of the pion structure functions.

![Figure 2](image.png)

Figure 2: Total cross section of charm production in \( \pi^\pm p \) collisions at \( \sqrt{s} > 18 \text{ GeV} \). Experimental fixed target data are taken from [32] and [33] (they are all shown by points). The solid curve shows the result of the fit of QCD \( m_Q \) and \( \mu^2 \), and dashed curves show the edges of the uncertainties (the region between the \( \chi^2 \) minimal value and the \( \chi^2 \) minimal value plus one).

3 Charm production in different QCD approaches

The experimental data on the total cross section of charm production at high energies shown in Fig. 1 are presented again in Fig. 3, together with data on interactions with nuclei in cosmic rays [38] and RHIC nuclear data [25, 26, 27] scaled to \( pp \) interactions.
The last cross section data obtained by the STAR Collaboration, as well as cosmic rays date, are larger than the theoretical NLO QCD predictions shown by the dash-dotted curve. One can also see the different data obtained by the PHENIX and the STAR collaborations. This discrepancy is discussed in detail in references [38, 39].

Figure 3: Total cross section of charm production in $pp$ and $\bar{p}p$ collisions at $\sqrt{s} > 25$ GeV. Experimental fixed target data are taken from [32] and [33], the RHIC point [34], and the UA2 Collaboration data from [35] (all are shown by points). PHENIX data [25, 26] and STAR data [27] in nuclear collisions are shown by squares and triangles, respectively, while cosmic rays data [38] are shown by diamonds. The solid curve shows the theoretical result obtained in the $k_T$-factorization approach, the dashed curve corresponds to NLO QCD with only gluon-gluon fusion contribution, and the dash-dotted curve corresponds to the total NLO QCD result.

All other experimental points are in reasonable agreement with the NLO QCD calculations performed by taking the values of the parameters presented in Eq. (5) (dash-dotted curve), and where the GRV95 parton distributions, compatible with the most modern analysis (see discussion in [37]), were used.

At high energies, the main contribution to the heavy flavour production cross section comes from the small $x$ region. Here there is not ground to neglect the transverse momenta of gluons, $q_1T$ and $q_2T$, with respect to the quark mass and to the quark transverse momenta, $p_T$. In particular, at high energies and high $p_T$ the main contribution to the cross sections comes from the region of $q_1T \sim p_T$, or of $q_2T \sim p_T$ (see [17] for details).

For our calculations in the framework of the $k_T$-factorization approach we have considered the heavy quark $Q = c, b$ mass [19] as $m_c = 1.4$ GeV and $m_b = 4.6$ GeV, and we
have used QCD scales $\mu_R^2 = \mu_F^2 = m_T^2$, $m_T^2 = m_Q^2 + p_T^2$. All details of the calculations in the $k_T$-factorization approach presented below can be found in [19]. The result of these calculations are shown by a solid curve in Fig. 3, where one can see that the $k_T$-factorization approach gives at not very high energies cross sections which are smaller than those obtained by NLO QCD calculations, but which, on the other hand, increase slightly faster with energy.

The cross section of beauty production in $pp$ collisions at RHIC energy $\sqrt{s} = 200$ GeV has been measured by PHENIX Collaboration [43] to be $\sigma_{b\bar{b}} = 3.2^{+1.2}_{-1.1}^{+1.4}_{-1.3}$ $\mu$b, what is in agreement with the result of our NLO QCD calculation $\sigma_{b\bar{b}} = 3.06 \mu$b, obtained at the same energy $\sqrt{s} = 200$ GeV by using GRV95 parton distributions.

The cross sections of heavy flavour production at RHIC are obtained by the extrapolation of the data measured in the central region to the whole region of rapidities. So it seems reasonable to compare the theoretical results to the experimental data before extrapolation, what will allow, as a minimum, to avoid a part of the uncertainties. For this sake, we present in Tables 1 and 2 the experimental values of $d\sigma/dy (|y| = 0)$ for charm and beauty production, together with the theoretical values obtained both in NLO QCD calculations with GRV95 parton distributions and in the $k_T$-factorization approach. Again, the STAR data for charm production appear to be several times larger than the theoretical results, whereas the PHENIX data for both charm and beauty production are in a reasonable agreement with theory.

| Collaboration/ Theory | Reaction | $d\sigma/dy (|y| = 0)$ |
|-----------------------|----------|----------------------|
| PHENIX                | Au–Au (m.b.) [40] p–p [26] | $143 \pm 13 \pm 36 \mu$b $123 \pm 12 \pm 45 \mu$b |
| STAR                  | d–Au [41] Au–Au (m.b.) [41] Au–Au (central) [41] Cu–Cu (m.b.) [42] | $301 \pm 44 \pm 67 \mu$b $267 \pm 19 \pm 49 \mu$b $283 \pm 12 \pm 39 \mu$b $260 \pm 34 \mu$b |
| NLO QCD               | p–p      | $83 \mu$b            |
| $k_T$-factorization    | p–p      | $56 \mu$b            |

Table 1. The experimental and theoretical values of $d\sigma/dy (|y| = 0)$ for charm production at the RHIC energy $\sqrt{s} = 200$ GeV, scaled to $pp$. 
Table 2. The experimental and theoretical values of \( \frac{d\sigma}{dy} (|y| = 0) \) for beauty production at the RHIC energy \( \sqrt{s} = 200 \) GeV, scaled to \( pp \).

| Collaboration/ Theory | Reaction | \( \frac{d\sigma}{dy} (|y| = 0) \) |
|-----------------------|----------|-------------------------------|
| PHENIX | \( p-p \) [43] | \( 0.92^{+0.34}_{-0.31} -0.39 \) \( \mu b \) |
| NLO QCD | \( p-p \) | \( 0.97 \) \( \mu b \) |
| \( k_T \)-factorization | \( p-p \) | \( 1.62 \) \( \mu b \) |

In Table 3 we present the predictions for heavy flavour production cross sections and midrapidity inclusive densities that we have obtained in the framework of \( k_T \)-factorization approach.

| Reaction  | Quantity | \( \sqrt{s} = 5.5 \) TeV | \( \sqrt{s} = 7 \) TeV | \( \sqrt{s} = 14 \) TeV |
|-----------|----------|----------------|----------------|----------------|
| \( pp \to c\bar{c} \) | \( \sigma \) | 9 mb | 11.3 mb | 21 mb |
|          | \( \frac{d\sigma}{dy} (|y| = 0) \) | 1.1 mb | 1.35 mb | 2.3 mb |
| \( pp \to b\bar{b} \) | \( \sigma \) | 0.44 mb | 0.57 mb | 1.2 mb |
|          | \( \frac{d\sigma}{dy} (|y| = 0) \) | 0.063 mb | 0.079 mb | 1.5 mb |

Table 3. The predictions for charm and beauty production cross sections \( \sigma \), and for the inclusive densities \( \frac{d\sigma}{dy} (|y| = 0) \), obtained in the \( k_T \)-factorization approach for \( pp \) collisions at LHC energies.

4 Transverse momentum distributions

The STAR Collaboration also presents the \( p_T \)-distributions of \( D \)-mesons produced in \( d+Au \) collisions and scaled to \( pp \) collisions at \( \sqrt{s} = 200 \) GeV. These data, taken from [44], are presented in Fig. 4 together with the corresponding theoretical calculations.

The NLO QCD result shown by the dashed curve is in an evident disagreement (significantly below) with the experimental data on the same level as in Fig. 3. The upper curve of FONLL calculations for charmed quark production taken from [44] (dash-dotted curve) also underestimates the data. The result of the \( k_T \)-factorization approach (solid curve) presents a reasonable slope in \( p_T \)-dependence, but also it underestimates the data. In this case, the level of disagreement at high \( p_T \) is, however, smaller than in the case of the total cross sections, what should be connected with the fact that a rather large contribution to the total cross section comes from the low-\( p_T \) region \( (p_T \leq m_c) \), where both experimental and theoretical uncertainties are rather large.
Figure 4: The STAR Collaboration data for $p_T$-distributions of $D$-mesons produced in $d+Au$ collisions and scaled to $pp$ collisions at $\sqrt{s} = 200$ GeV, together with the corresponding calculations in the $k_T$-factorization approach (solid curve), NLO QCD (dashed curve), and FONLL [44] (dash-dotted curve).

It is necessary to note that in Fig. 4 we compare the experimental points for $D$-meson production to theoretical curves for charmed quark distributions. In contradiction to what is claimed in [44], on top of fragmentation processes where the momentum of the $D$-meson is smaller than the momentum of the $c$-quark, also recombination processes where the momentum of $D$-meson is larger than the momentum of $c$-quark are active. The existence of recombination processes in charm production seems to be evident from the experimental data on the asymmetry in yields of the so-called favoured and unfavoured $D$-mesons (see discussions in [45–48]). As a matter of fact, though the produced heavy and light quarks have very different transverse momenta, the difference in the components of their velocities can be not so large. It is very possible that the fragmentation and recombination processes in charm quark hadronization balance each other in the processes with not very high $p_T$, and as a hint of this, the calculated Feynman-$x$ distributions of produced charm quarks in $\pi p$ collisions are in good agreement with the experimental distributions of produced $D$-mesons (see Fig. 5 in [30]).

The total cross section of charm production was not measured at Tevatron-collider energies. However, data on $p_T$-distributions of $D$-mesons exist at these energies [49]. They are presented in Fig. 5, where one can see that they are in good agreement with NLO QCD calculations that use GRV95 parton distributions for charm quarks (dashed curve).
Figure 5: The cross sections for $D$-meson production in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV [49] with $|y_1| \leq 1$, together with calculations in the $k_T$-factorization approach (solid curve) and in NLO QCD (dashed curve).

The solid curve corresponds to the $k_T$-factorization approach and it slightly overestimates the data, but the agreement should become better in the future when the contribution of charmed antibaryons will be added to the experimental data. The results of FONLL calculations [44] with fragmentation functions for $D$-mesons production presented in [49], slightly underestimate the yields of $D$-mesons. Thus, all QCD approaches (see also [50]) are in reasonable agreement with the experimental data at $\sqrt{s} = 1.96$ TeV. Some disagreement of these data with [51] can be expected by the QCD parameter values used there, where the charm production cross section increases with energy very fast. This should probably result in a rather good agreement of [51] with the STAR data presented in Fig. 4, but also in the overestimation of the Tevatron-collider data presented in Fig. 5.

Finally, the theoretical predictions for $d\sigma/dp_T$ distributions of charm and beauty quarks produced in the rapidity window $|y_1| \leq 1$ at three LHC energies, $\sqrt{s} = 5.5$ TeV, 7 TeV, and 14 TeV, calculated in the $k_T$-factorization approach are presented in Fig. 6.

5 Nuclear effects in charm production at RHIC

First, it can be said that the EMC-effect, i.e. the nuclear deformation of parton distributions [23], should increase the total cross sections for charm production calculated in
Figure 6: The predicted cross sections of charm (solid curves) and beauty (dashed curves) production in $pp$ collisions at LHC energies, from bottom to top, $\sqrt{s} = 5.5$ TeV, 7 TeV, and 14 TeV, in $|y_1| \leq 1$, obtained in the $k_T$-factorization approach.

the NLO QCD linear approximation at RHIC energy by 5–10% [24] with respect to the linear A-dependence.

The data of PHENIX and STAR Collaborations scaled to binary interactions are presented together in [39]. There one can see the absence of nuclear effects in the PHENIX data from $pp$ to central Au-Au collisions, and in STAR data from d-Au to central Au-Au collisions, in both cases at the level of 20% accuracy. Concerning the PHENIX data presented in [38], they do not present any visible dependence of the charm production cross section on the number of collision participant nucleons, $N_{\text{part}}$, in Au-Au collisions at different centralities.

One factor $\sim 2–3$ discrepancy [39] exists between the total cross sections of charm production obtained by PHENIX and STAR Collaborations. It seems evident that the explanation of this discrepancy could be completely connected to some experimental problem. However, meanwhile the experimental collaborations will not clarify this point, two scenarios can be scripted: either the PHENIX data are right and the NLO QCD calculation reasonably describes all experimental data except for two cosmic ray points presented in Fig. 3, and then nuclear effects in total cross sections of charm production are small, or, on the contrary, the STAR data are correct and the nuclear effects increase about 4–5 times the total cross sections of charm production. In the case the $N_{\text{part}}$ dependence of these nuclear effects would saturate very fast, even the cosmic ray data could be included in the theoretical description.
In this second scenario, so large nuclear effects could be connected to large non-perturbative contributions (which could be larger than perturbative contributions) to high density states, e.g. string fusion [52], percolation [53], strong colour field [54], or colour glass condensate effects [55, 56] in the interactions with nuclei.

In these non-perturbative approaches, the strong colour field inside the cluster formed by the overlapping strings produces, above some scale $\eta_c$ [53] given by the critical percolation string density, heavy $QQ$ pairs via the Schwinger mechanism in the same way as a single string produces light $q\bar{q}$ pairs. Thus, in the colour glass condensate approach [55, 56] the significant scale is the saturation momentum $Q_s$, which grows with energy and nuclear size. When $Q_s > m_T(\bar{c}c)$, the classical colour field is strong enough and it produces heavy pairs $Q\bar{Q}$. In summary, the production pattern for heavy quarks in these non-perturbative approaches becomes similar to that of the light quarks and so an overall enhancement of heavy quark production cross section is expected. However, to get numerical estimation of these nuclear effects a set of additional assumptions is required.

6 Conclusion

We show that the $k_T$-factorization approach predictions are in reasonable agreement with the NLO QCD calculations for the total cross section of charm and beauty production at RHIC energies.

We obtain a reasonable description of the PHENIX Collaboration data including the points obtained from nuclear collisions, and our results are clearly in contradiction with the STAR Collaboration data.

It seems that the main disagreement of our calculations with the STAR Collaboration data comes from the low-$p_T$ region ($p_T \leq 1$ GeV/c), where both experimental and theoretical uncertainties are rather large.

The predicted $p_T$-distribution of the produced charm at high $p_T$ in the $k_T$-factorization approach is higher than the NLO QCD and FONLL predictions. The $k_T$-factorization approach only slightly underestimates the experimental data of the STAR Collaboration.

Since the NLO QCD calculation basically agrees with the PHENIX data, one would be in principle prompted to think that the PHENIX data are right. Either way, the question of the existence of a factor $\sim 2$–3 discrepancy between the total cross section of charm production obtained by PHENIX and by STAR Collaborations has to be experimentally unraveled in order to understand the basic mechanisms working in charm production.

The predictions for LHC energies are also presented.

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References

[1] J.C. Collins, D.E. Soper, and G. Sterman, Nucl. Phys. B308 (1988) 833.
[2] P. Nason, S. Dawson, and R.K. Ellis, Nucl. Phys. B303 (1988) 607.
[3] G. Altarelli, M. Diemoz, G. Martinelli, and P. Nason, Nucl. Phys. B308 (1988) 724.
[4] P. Nason, S. Dawson, and R.K. Ellis, Nucl. Phys. B327 (1989) 49.
[5] W. Beenakker, H. Kuijf, W.L. Van Neerven, and J. Smith, Phys. Rev. D40 (1989) 54.
[6] M. Cacciari, M. Greco, and P. Nason, JHEP 9805 (1998) 007 and hep-ph/9803400.
[7] S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B242 (1990) 97; Nucl. Phys. B366 (1991) 135.
[8] J.C. Collins and R.K. Ellis, Nucl. Phys. B360 (1991) 3.
[9] G. Marchesini and B.R. Webber, Nucl. Phys. B310 (1988) 461; Nucl. Phys. B386 (1992) 215.
[10] P. Hägler et al., Phys. Rev. D62 (2000) 71502.
[11] L.V. Gribov, E.M. Levin, and M.G. Ryskin, Phys. Rept. 100 (1983) 1.
[12] E.M. Levin and M.G. Ryskin, Phys. Rept. 189 (1990) 267.
[13] E.M. Levin, M.G. Ryskin, Yu.M. Shabelski, and A.G. Shuvaev, Sov. J. Nucl. Phys. 53 (1991) 657; 54 (1991) 867.
[14] V.A. Saleev and N.P. Zotov, Mod. Phys. Lett. A9 (1994) 151; A11 (1996) 25.
[15] S.P. Baranov and M. Smizanska, Phys. Rev. D62 (2000) 014012.
[16] Yu.M. Shabelski, Surveys of High Energy Phys. 11 (1997) 169; 14 (2000) 357.
[17] M.G. Ryskin, Yu.M. Shabelski, and A.G. Shuvaev, Phys. Atom. Nucl. 64 (2001) 120; 64 (2001) 1995.
[18] N.P. Zotov, A.V. Lipatov, and V.A. Saleev, Phys. Atom. Nucl. 66 (2003) 755.
[19] Yu.M. Shabelski and A.G. Shuvaev, Phys. Atom. Nucl. 69 (2006) 314.
[20] M.J. Leitch et al., Phys. Rev. Lett. 72 (1994) 2542.
[21] A. Blanco-Covarrubias et al., SELEX Collaboration, arXiv:0902.0355[hep-ex].
[22] M. Arneodo, Phys. Rept. 240 (1994) 301.

[23] K.J. Eskola, Nucl. Phys. B400 (1993) 240.

[24] N. Armesto, C. Pajares, C.A. Salgado, and Yu.M. Shabelski, Phys. Lett. B366 (1996) 276.

[25] K. Adcox et al., PHENIX Collaboration, Phys. Rev. Lett. 88 (2002) 192303 and nucl-ex/0202002.

[26] K. Adare et al., PHENIX Collaboration, Phys. Rev. Lett. 97 (2006) 252002 and hep-ex/0609010.

[27] J. Bielcik, STAR Collaboration, nucl-ex/0606010.

[28] S. Narison, Phys. Lett. B341 (1994) 73.

[29] P. Ball, M. Beneke, and V.M. Braun, Phys. Rev. D52 (1992) 3929.

[30] S. Frixione, M.L. Mangano, P. Nason and G. Ridolfi, hep-ph/9702287.

[31] R. Vogt, Eur. Phys. J. ST155 (2008) 213; arXiv:0709.2531 [hep-ph].

[32] C. Lourenco and H.K. Wöhri, Phys. Rept. 433 (2006) 127.

[33] M. C. Abreu et al., NA38 and NA50 Collaborations, Eur. Phys. J. 69 (2000) 443.

[34] Y. Morino et al., PHENIX Collaboration, J. Phys. G35 (2008) 104116 and arXiv:0805.3871[hep-ex].

[35] O. Botner et al., UA2 Collaboration, Phys. Lett. B236 (1990) 488.

[36] M. Gluck, E. Reya, and A. Vogt, Z. Phys. C67 (1995) 433.

[37] M. Gluck, E. Reya and A. Vogt, Eur. Phys. J. C5 (1998) 461.

[38] Zhangbu Xu, J. Phys. G32 (2006) S309 and nucl-ex/0607015.

[39] A.A.P. Suaide, J. Phys. G34 (2007) S369 and nucl-ex/0702035.

[40] S.S. Adler et al., PHENIX Collaboration, Phys. Rev. Lett. 94 (2005) 082301 and nucl-ex/0409028.

[41] Y. Zhang, STAR Collaboration, J. Phys. G32 (2006) S523 and nucl-ex/0607011.

[42] S. Baumgart, STAR Collaboration, arXiv:0709.4223[hep-ex].

[43] A. Adare et al., PHENIX Collaboration, arXiv:0903.4851[hep-ex].
[44] M. Cacciari, P. Nason, and R. Vogt, Phys. Rev. Lett. **95** (2005) 122001 and hep-ph/0502203.

[45] A.K. Likhoded and S.R. Slabospitsky, Phys. Atom. Nucl. **60** (1997) 981; **62** (1999) 693; hep-ph/0002202.

[46] T. Tashiro et al., hep-ph/0002202.

[47] J. Dias de Deus and F. Duraes, Eur. Phys. J. **C13** (2000) 647.

[48] Yu.M. Shabelski, hep-ph/0011032.

[49] D. Acosta et al., Phys. Rev. Lett. **91** (2003) 241804 and hep-ph/0307080.

[50] R. Vogt, hep-ph/9702287.

[51] I.M. Dremin and V.I. Yakovlev, Astropart. Phys. **26** (2006) 1.

[52] H.D. Möhring, J. Ranft, C. Merino, and C. Pajares, Phys. Rev. **D47** (1993) 4142.

[53] N. Armesto, M.A. Braun, E.G. Ferreiro, and C. Pajares, Phys. Rev. Lett. **77** (1996) 3736.

[54] V. Topor Pop, J. Barrette, and M. Gyulassy, Phys. Rev. Lett. 102:232302, 2009, and arXiv:0902.4028[hep-ph].

[55] F. Gelis and R. Venugopalan, Phys. Rev. **D69** (2004) 014019; H. Fujii, F. Gelis, and R. Venugopalan, hep-ph/0702174.

[56] D. Kharzeev and K. Tuchin, Nucl. Phys. **A735** (2004) 248.