Structural damage identification based on dynamic deflection sensitivity of bridge under vehicle loading

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Abstract. Based on the concept of dynamic response sensitivity, a method of identifying bridge structural damage by using the dynamic deflection response of bridge under vehicle load is proposed. Firstly, the dynamic equation of vehicle-bridge coupling dynamic system is established. And then, the basic principle and iterative identification process of bridge damage identification based on dynamic deflection response sensitivity are derived. Finally, the effectiveness of the algorithm is verified with a numerical example of three-span continuous beam vehicle-bridge coupling system. The research results show that the proposed algorithm can overcome the influence of measurement noise and initial model error, and realize the damage location and quantification of bridge from dynamic deflections of a few measuring points.

1. Introduction
As basic civil engineering infrastructures, bridges are inevitably damaged by various factors during their long-term service, which will affect their bearing capacity. The installation of structural health monitoring systems for real-time, remote and accurate structural monitoring is a widely used safety assessment method for major bridge foundation[1]. However, due to the limitation of economic conditions, it is difficult to popularize the monitoring systems to ordinary bridges in highway and railway network. Therefore, the development of structural safety assessment and damage identification technology for medium and small span bridges has important practical engineering significance.

When a structure is damaged, its internal physical parameters will change, which leads to the change in structural characteristic parameters. Dynamic deflections of a bridge are the most real-time response to the stiffness of the bridge, and the most real response of the bridge in service under vehicle load[2]. With the rapid development of sensor technology, a variety of new bridge deflection detectors can achieve real-time dynamic deflection measurement accurately and conveniently, which makes it possible to identify bridge damage using structural dynamic deflection information[3,4]. This paper therefore will take dynamic deflections of bridge under vehicle load as the measurement response, and develop the damage identification method for bridges based on their dynamic response sensitivity. The specific work includes establishing the dynamic equation of vehicle-bridge coupling dynamic system, deriving the basic principle and iterative identification process of bridge damage identification based on dynamic deflection response sensitivity, and using the numerical calculation of three-span continuous beam vehicle-bridge coupling system to verify the effectiveness of the algorithm.

2. Basic theory

2.1. Dynamic calculation model of vehicle-bridge coupling system
Based on a urban rail transit train, a ten degree of freedom spring-damper system vehicle model is established. The model includes five parts: one carriage mass ($M_3$), two bogie masses ($M_2$), four wheelset masses ($M_1$), the primary spring damper suspension devices ($K_1$, $C_1$) connecting bogies and wheelsets, and the secondary spring damper suspension devices ($K_2$, $C_2$) connecting carriages and bogies. Assuming that the train passes through the bridge at a constant speed, the vibration equation of the vehicle-bridge coupling system can be obtained by Newton’s second law as

$$M(t) \ddot{u}(t) + C_1(t) \dot{u}(t) + K_1(t) u(t) = F_v(t)$$

(1)

where the system mass matrix $M = \text{diag}(M_3, J_3, M_2, J_2, M_2, J_2)$, $J_2$, $J_3$ are the moment of inertia of $M_2$ and $M_3$, ndof is the number of degrees of freedom of bridge structure; system damping matrix $C = [C_c, C_f]$ is composed of vehicle model damping matrix $C_c$ and bridge damping matrix $C_f$; system stiffness matrix $K = [K_c, K_f]$ is composed of vehicle model stiffness matrix $K_c$ and bridge stiffness matrix $K_f$; $F_v$ is the excitation of vehicle-bridge coupling system. The specific element composition of the above matrix is shown in reference [5]. $\ddot{u}$, $\dot{u}$ and $u$ respectively represent the acceleration, velocity and displacement response matrices of the vehicle-bridge coupling system.

If a linear elastic plane Euler-beam is used to model the bridge structure, the vibration equation of the bridge under the train wheelset load can be rewritten as

$$0 = \sum_{i=1}^{4} L_{ij}(t) \cdot P_{ij}(t)$$

(2)

$$M_i(t) \ddot{u}_i(t) + C_0(t) \dot{u}_i(t) + K_0(t) u_i(t) = F_{v_i}(t)$$

(3)

where $M_i = [M_0, 0, 0, 0, 0, 0, M + M_1 \sum_{i=1}^{4} L_{ij}(t) \cdot R_{ij}(t)]$; $C_i = [0, 0, -C_1 \cdot (L_1 + L_2), -C_1 \cdot d \cdot (L_1 - L_2), -C_1 \cdot (L_3 + L_4), -C_1 \cdot d \cdot (L_3 - L_4), C + C_1 \cdot d \cdot \sum_{i=1}^{4} L_{ij}(t) \cdot R_{ij}(t)]$; $K_i = [0, 0, -K_1 \cdot (L_1 + L_2), -K_1 \cdot d \cdot (L_1 - L_2), -K_1 \cdot (L_3 + L_4), -K_1 \cdot d \cdot (L_3 - L_4), K + K_1 \cdot d \cdot \sum_{i=1}^{4} L_{ij}(t) \cdot R_{ij}(t)]$; $M$, $C$ and $K$ represent the mass, damping and stiffness matrix of bridge structure respectively; $F_{v_i} = \sum_{j=1}^{4} \left[ \left( \frac{M_3}{4} + \frac{M_2}{2} + M_1 \right) \cdot g - C_1 \cdot \dot{y}_{ij} - K_1 \cdot y_{ij} \right] \cdot L_{ij}$.

By solving equations (1) and (2) together with the Newmark method, the response of vehicle-bridge coupling system can be calculated, and then the dynamic deflections of bridge under vehicle loading can be from the vertical displacement of bridge nodes.

2.2. Bridge damage identification theories based on dynamic deflection response sensitivity

When a part of the bridge structure is damaged, the dynamic deflection of the bridge will change when the vehicle is running. Suppose that there are $n$ elements in the finite element model of bridge structure, and the characteristics (such as element stiffness, etc.) of $n$ ($n \leq N$) elements may change. At this time, the element stiffness or damage factor of $n$ elements can be taken as independent variable, and the dynamic deflection response $u$ of a bridge node is taken as dependent variable. Then every data of $u$ is a function of these $n$ damage factors, namely,

$$u_j = f_j(\alpha_1, \alpha_2, \ldots, \alpha_n) \quad 1 \leq j \leq n$$

(4)
where $\alpha_i$ is the damage factor of the $i$-th unit; $u_j$ is the data of the $j$-th sampling point of $u$.

The damage factors of each element can be obtained by combining all equations. However, since equation (4) is a nonlinear equation, in order to solve it conveniently, it can be expanded as the Taylor series. After the higher-order term is ignored in the Taylor series, the equation is simplified as

$$u_j = u_{j,0} + \sum_{i=1}^{n} \frac{\partial u_j}{\partial \alpha_i} (\alpha_j - \alpha_{j,0})$$

(5)

where $u_{j,0} = f(\alpha_{i,0}, \alpha_{2,0}, \ldots, \alpha_{n,0})$. Write (5) as an increment

$$\Delta u_j = \sum_{i=1}^{n} \frac{\partial u_j}{\partial \alpha_i} \Delta \alpha_i + \sum_{j=2}^{n} \frac{\partial u_j}{\partial \alpha_j} \Delta \alpha_j + \cdots + \frac{\partial u_j}{\partial \alpha_n} \Delta \alpha_n$$

(6)

where $\Delta u_j = u_j - u_{j,0}$, $\Delta \alpha_i = \alpha_j - \alpha_{i,0}$.

Equation (6) can be abbreviated as

$$\Delta u = S \Delta \alpha$$

(7)

where $\Delta u$ is the change of deflection response, $S$ is the sensitivity matrix and $\Delta \alpha$ is the change of damage factor.

Using this method, the nonlinear equation (4) is transformed into the linear equation (7). When the change of deflection response $\Delta u$ and sensitivity matrix $S$ are known, the change of damage factor $\Delta \alpha$ can be obtained by solving the equation.

The sensitivity matrix $S$ can be obtained by deriving the partial derivative of equation (3) to the damage factor. After the derivation, equation (8) is gotten.

$$M_e(t) \frac{\partial u_e(t)}{\partial \alpha_j} + C_e(t) \frac{\partial u_e(t)}{\partial \alpha_j} + K_e(t) \frac{\partial u_e(t)}{\partial \alpha_j} = -K_e(t) u_e(t)$$

(8)

where $\frac{\partial u_e(t)}{\partial \alpha_j}$, $\frac{\partial u_e(t)}{\partial \alpha_j}$, $\frac{\partial u_e(t)}{\partial \alpha_j}$ respectively represent the sensitivity of the acceleration, velocity and displacement of the system node to the $j$-th structural damage parameter, and the right end term of the equation is equivalent to the equivalent load of equation (8). Therefore, the sensitivity of structural dynamic response to stiffness damage parameters can be obtained by the Newmark method. Among them, the matrix $\frac{\partial u_e(t)}{\partial \alpha_j}$ corresponding to the vertical degree of freedom of the bridge node is the dynamic deflection sensitivity.

The specific steps of using the sensitivity method to identify bridge damage are as follows.

Step 1: Measure the time history of dynamic deflection $u$.

Step 2: Assume that the damage factor vector of each element is $\tilde{\alpha}^k = \tilde{\alpha}_1^k, \tilde{\alpha}_2^k, \ldots, \tilde{\alpha}_n^k$ (the initial value can be assumed as 0 vector) in the $k$-th iteration, so as to modify the bridge model, and calculate $u^k$ and $S^k$ in the $k$-th iteration.

Step 3: The sensitivity equation $S^k \Delta \alpha^k = \Delta u^k = u^k - u^k$ is constructed and the increment of damage factor $\Delta \alpha^k$ is resolved.

Step 4: Modify the damage factor of each element with $\tilde{\alpha}^{k+1} = \tilde{\alpha}^k + \Delta \alpha^k$.

Step 5: If the convergence condition $|\Delta \alpha^k / \tilde{\alpha}^k| < \text{Tolerance}_1$ or $|\Delta u^k| < \text{Tolerance}_2$ is satisfied, the iteration ends. Otherwise, repeat (2) - (4) until the convergence condition is satisfied.
3. Numerical example verification

A three span 30m + 30m + 30m continuous box girder bridge in a subway line is taken as the research object to verify the proposed algorithm. Its finite element model consists of 90 plane Euler beam elements of equal length, and the node numbers are set as 1-91 from left to right of the bridge. The detailed values of the material and section size parameters include elastic modulus $E$ of $3.45 \times 10^{10}$ N/m², moment of inertia $I$ of 2.07m⁴, Poisson's ratio $\gamma$ of 0.2, density $\rho$ of 2500kg/m³, and cross-sectional area $A$ of 5.06m². The damages of the bridge are set as the stiffness of element 2 near the side span support is reduced by 10%, the stiffness of element 45 near the middle span is reduced by 15%, the stiffness of element 58 near the middle support is reduced by 10%, and the stiffness of element 75 near the middle span is reduced by 5%. At the same time, a group of Gauss random numbers are added to the stiffness coefficient of bridge element to simulate the model error of bridge structure. The basic parameters of the vehicle model include: the masses of vehicle body, bogie and wheelset are $3.9 \times 10^4$kg, 4360kg and 1770kg respectively; the stiffness coefficient and damping coefficient of primary suspension are $2.976 \times 10^6$N/m and 15000kNꞏs/m respectively; the stiffness coefficient and damping coefficient of secondary suspension are $1.06 \times 10^6$N/m and $3.0 \times 10^4$Nꞏs/m respectively. The distance between front and rear wheelsets of a single train is 18.1m, the distance between front and rear bogies is 15.6m, and the distance between two wheelsets under the same bogie is 2.5m. The running speed of the train is 80km/h, the sampling frequency of dynamic response is 200Hz, and the sampling time is 4.8645s. According to Table 1, the dynamic deflection of the bridge with different measuring points is taken as the measured responses to identify the damage of the bridge. At the same time, Gaussian white noise with 5% standard deviation is added to the deflection time histories to simulate the influence of measurement noise on dynamic responses.

The last column of Table 1 shows the 2-norm error between the identified value and the real value of the damage stiffness parameters with and without measurement noise. It can be seen from the identification error that the accurate identification of the stiffness parameters of the whole bridge unit can be realized only from the deflection of one measuring point under the condition without noise. The appearance of test noise brings errors into the identification value of the stiffness parameters, but the identification error decreases with the increase of the number of deflection measuring points.

| Condition number | Number of measuring points | Deflection measuring point position (node number) | Noise level (%) | Recognition error (%) |
|------------------|-----------------------------|-----------------------------------------------|----------------|----------------------|
| 1                | 1                           | Midspan of the middle span (46)                | 0              | 0                    |
| 2                | 1                           | Midspan of the middle span (46)                | 5              | 1.02                 |
| 3                | 3                           | Midspan of each span (16, 46, 65)              | 5              | 0.28                 |
Figure 1. Identification results of bridge damage and model error with 5% test noise

Figure 1 shows the comparison between the damage and model error of bridge structure identified in case 3 and its real value. It can be seen from the identification results in Figure 1 that although the appearance of measurement noise reduces the identification accuracy of stiffness parameter changes of some elements, the damage location and damage degree can still effectively be identified. These results indicate that the proposed method based on bridge dynamic deflection sensitivity can realize the identification of multi damage under vehicle load. By using multi-point dynamic deflection to participate in identification, the influence of noise on identification effect can be effectively reduced, and the occurrence of misjudgment of damage can be reduced.

4. Conclusion
In this paper, based on the sensitivity of dynamic deflection response of bridge under vehicle load, a series of studies are carried out to identify the damage of bridge structure, and the effectiveness of the algorithm is verified by the numerical example of a three span continuous beam vehicle-bridge coupling system. The research results show that the damage and model error of the whole bridge can be accurately realized by using only the dynamic deflection of a single measuring point under the condition without noise. Noise has a certain influence on the recognition effect, but the influence of noise can be weakened by increasing the number of measuring points appropriately. The proposed algorithm can overcome the influence of measurement noise and initial model error, and realize the damage location and quantification of the bridge from the dynamic deflection of a few measuring points. The proposed method can provide technical support for the fast and convenient state assessment of small and medium span bridges.

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