Spatio-Temporal Modeling of Check-ins in Location-Based Social Networks

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Abstract Social networks are getting closer to our real physical world. People share the exact location and time of their check-ins and are influenced by their friends. Modeling the spatio-temporal behavior of users in social networks is of great importance for predicting the future behavior of users, controlling the users’ movements, and finding the latent influence network. It is observed that users have periodic patterns in their movements. Also, they are influenced by the locations that their close friends recently visited. Leveraging these two observations, we propose a probabilistic model based on a doubly stochastic point process with a periodic decaying kernel for the time of check-ins and a time-varying multinomial distribution for the location of check-ins of users in the location-based social networks. We learn the model parameters by using an efficient EM algorithm, which distributes over the users. Experiments on synthetic and real data gathered from Foursquare show that the proposed inference algorithm learns the parameters efficiently and our method models the real data better than other alternatives.

Keywords Spatio-temporal · Location-based social networks · Foursquare · Check-in · Stochastic point process · Influence network · Periodic pattern · Probabilistic model · EM algorithm

1 Introduction

The advances in location-acquisition techniques and the proliferation of mobile devices have generated an enormous amount of spatial and temporal data
of users activities \[56\]. People can upload a geotagged video, photo or text to social networks like Facebook and Twitter, share their present location on Foursquare or share their travel route by using GPS trajectories to GeoLife \[51\]. A considerable amount of this spatio-temporal data is generated by the activity of users in location-based social networks (LBSN). In a typical LBSN, like Foursquare, users share the time and geolocation of their check-ins, comment about it, or unlock badges by exploring new venues.

Many techniques have been proposed for processing, managing, and mining the trajectory data in the past decade \[57\]. Several other studies try to leverage the spatial data in recommender systems \[23\]. But, a few works have attempted to model the spatio-temporal behavior of users in LBSNs \[6,5\]. Given the history of users’ check-ins, the goal is to predict the time and location of each user’s check-in by utilizing a model. This model can also be used to find the influence network between users which made up of their check-ins, detect the influential users and popular locations, predict the peak hours of a restaurant, recommend a location, and even control the movement of users.

In this paper, we propose a probabilistic spatio-temporal generative model for check-ins of users in location-based social networks. People usually have periodic patterns in their movements \[42,5,60\]. For example, a typical user may check into her office in the morning and to a nearby restaurant at noon then return home and repeat this behavior in the following days. We model the time of check-ins of each user with a novel periodic decaying doubly stochastic point process which leverages the periodicity in the movements of users and can also capture any changes in their patterns. To model the location of check-ins we use the fact that users in social media are influenced by the activities of their friends \[54,11,16\]. If many of your close friends have checked into a specific restaurant recently, then there is a high probability that you select that restaurant, next time. We model the location of check-ins by using a time-varying multinomial distribution. In summary, we propose:

- a doubly stochastic point process for modeling the time of check-ins, which captures the periodic patterns in the movement of users,
- a time-varying multinomial distribution for modeling the location of users check-ins, which incorporates the influence of their friends’ history of check-ins,
- a scalable inference algorithm based on the EM algorithm to find the model parameters, which is distributed over users,
- and created a compelling dataset of Foursquare users’ check-ins, from 12000 active users during three months in the year 2015.

2 Prior Works

Modeling information diffusion in networks has attracted a lot of attentions in recent years \[14,11,7,9\]. Given the times that users have adopted to a contagion (information, behavior, or meme), the problem is to model the time and the user of next adoption, \textit{i.e.}, predict the next event. Early methods \[14\]...
studied information diffusion using a pair-wise probability distribution for each link from node $j$ to $i$, which is the probability that node $i$ generates an event in time $t_i$ due to the event of node $j$ at time $t_j$. These methods overlook the external effects on the generation of events. In addition, they assume that each node adopts a contagion at most once, i.e., events are not recurrent. These issues were later addressed in [22,6,44,28,11,40,20,17], which use point processes for the modeling of events. In [44,15,22,28], cascades are assumed to be independent and are modeled by a special point process, called Hawkes [19]. The independence assumption is removed in [44,54], they tried to model the correlation between multiple competing or cooperating cascades. Other studies [44,20,40,15,21], use the additional information of the diffusion network such as topic of tweets or community structure to better model the influence network. Most of the previous works studied the information diffusion on microblogging networks like Twitter, whereas we try to model the time and location of users’ check-ins in the location-based networks like Foursquare. The most similar work to ours is [6], which proposes a model for the time and location of the interactions between a pair of users (in contrast to our model which considers the check-ins of each user). It uses a multivariate Hawkes process for the temporal, and GMM for the spatial model.

The prior works in location-based social networks can be categorized into three groups; location recommendation, trajectory mining and location prediction. The main approaches in location recommendation systems [23] are: content-based which uses data from a user’s profile and the features of locations [48,45,43,30]; link-based, which applies link analysis models like PageRank to identify the experienced users and interesting locations [59,44,38,29]; and collaborative filtering which infers users’ preferences from their historical behavior, like the location history [58,17,44,57,36,32]. In trajectory data mining, the source of data is usually generated by the GPS. These works include; trajectory pattern mining to find the next location of an individual [5,39,55,27], anomaly detection to detect unexpected movement patterns [25,32], and trajectory classification to differentiate between trajectories of different states, such as motions, transportation modes, and human activities [50]. A comprehensive review of these methods can be found in the recent survey [57]. We also discriminate our work from location recommendation and trajectory mining methods, because our goal is to model the check-ins of users not to recommend a location or to find the trajectory patterns of users with the position data of their routes. In location prediction, the goal is to predict the next location, given the user’s profile data and the history of check-ins [13,53,33,34]. But these methods do not consider; the relation between friends (using the influence matrix), aging effect in the history of checkins (using decaying kernel), and periodicity in users’ movement patterns.
3 Preliminaries

To model the time of occurrences of a phenomenon, which are called events, we can use point processes on the real line. The phenomena can be, an earthquake [35], a viral disease [2] or the spread of information on the network [15]. The sequence of events, as defined below, is the realization of a point process.

**Definition 1 (Point Process)** Let \( \{t_i\}_{i \in \mathbb{N}} \) be a sequence of non-negative random variables such that \( \forall i \in \mathbb{N}, t_i < t_{i+1} \), then we call \( \{t_i\}_{i \in \mathbb{N}} \) a point process on \( \mathbb{N} \), and \( \mathcal{F}_t = \{t_i | i \in \mathbb{N}, t_i < t\} \) as its history or filtration.

There are different equivalent descriptions for the point processes such as; sequence of points \( \{t_i\} \), sequence of intervals (duration process) \( \delta t_i \), counting process \( N(t) \), or intensity process \( \lambda(t) \) [8]. In the following, we briefly explain each definition.

The counting process \( N(t) \) associated with the point process \( \{t_i\}_{i \in \mathbb{N}} \), counts the number of events occurred before time \( t \), i.e., \( N(t) = \sum_{i \in \mathbb{N}} I(t_i < t) \), where \( I(\cdot) \) is the indicator function\(^{1}\). The duration process \( \delta t_i \) associated with the point process \( \{t_i\}_{i \in \mathbb{N}} \) is defined as \( \forall i \in \mathbb{N}, \delta t_i = t_i - t_{i-1} \). Finally, the intensity process \( \lambda(t) \) is defined as the expected number of events per units of time, which in general depends on the history:

\[
\lambda(t|\mathcal{F}_t) = \lim_{dt \to 0} \frac{1}{dt} \mathbb{E}[N(t + dt) - N(t) | \mathcal{F}_t]
\]

\[
= \lim_{dt \to 0} \frac{1}{dt} \mathbb{P}[N(t + dt) - N(t) > 0 | \mathcal{F}_t]
\]

To evaluate the likelihood of a sequence of events, \( f(t_1, t_2, \ldots, t_n) \), we can use the chain rule of probability, \( f(t_1, t_2, \ldots, t_n) = \prod_i f(t_i | t_{i-1}) \). Therefore, it suffice to describe only the conditionals, which are abbreviated to \( f^*(t) \). If we use \( N(t, s) := N(s) - N(t) \) notation, then according to the definition of point processes, we can write the probability of occurring the \((n + 1)\)th event in time \( t \) as:

\[
f^*(t) dt = \mathbb{P}[N(t_n, t] = 0, N(t, t + dt] = 1 | t_{1:n}] .
\]

If we divide both sides of the above equation by \( 1 - F^*(t) \), where \( F^*(\cdot) \) is the cdf of \( f^*(\cdot) \), then in the limit as \( dt \to 0 \), we have:

\[
\frac{f^*(t) dt}{1 - F^*(t)} = \frac{\mathbb{P}[N(t_n, t] = 0, N(t, t + dt] = 1 | t_{1:n}]}{\mathbb{P}[N(t_n, t] = 0 | t_{1:n}]} = \mathbb{P}[N(t, t + dt] = 1 | t_{1:n}, N(t_n, t] = 0] = \mathbb{P}[N(t, t + dt] > 0 | \mathcal{F}_t]
\]

Therefore, according to the definition of intensity, we find the relation between conditional distribution of the time of events and the intensity function as:

\[
\lambda^*(t) = \frac{f^*(t)}{1 - F^*(t)}
\]

\(^{1}\) The indicator function \( I(x \in A) \) is 1 if \( x \in A \), otherwise it is 0.
where we use * superscript to show that a function is dependent on the history. We can also express the relation of $\lambda^*(t)$ and $f^*(t)$ in the reverse direction [1]:

$$f^*(t) = \lambda^*(t) \exp \left( -\int_{t_n}^{t} \lambda^*(s) ds \right)$$

(2)

Now, the cdf can be easily evaluated:

$$F^*(t) = 1 - \exp \left( -\int_{t_n}^{t} \lambda^*(s) ds \right).$$

(3)

A point process is usually defined by specifying its conditional distribution $f^*(t)$ or equivalently its intensity $\lambda^*(t)$. In the simplest case, the intervals $\delta t_i$ are assumed to be i.i.d., therefore the process is memoryless, and hence $\lambda^*(t) = \lambda(t)$. The Cox process [7] is a doubly stochastic point processes, and conditioned on the intensity is a Poisson process [24]. Hawkes process [19] is a special type of Cox process, where the intensity is expressed by the history as:

$$\lambda^*(t) = \mu + \int_{-\infty}^{t} \phi(t - \tau) dN(\tau) = \mu + \sum_{i=1}^{|F|} \phi(t - t_i)$$

(4)

where $\phi(t)$ is the kernel of the Hawkes process that defines the effect of past events on the current intensity, and $\mu$ is the base intensity. For example, the exponential kernel $\phi(t) = \exp(-t)$, is used for the modeling of self-exciting events like earthquake [35]. In general, we have a multivariate process with a counting process vector $N(t) = [N_1(t), \cdots, N_n(t)]^T$ and an associated intensity vector $\lambda^*(t) = [\lambda_1^*(t), \cdots, \lambda_n^*(t)]^T$ defined as:

$$\lambda^*(t) = \mu + \int_{-\infty}^{t} \Phi(t - \tau) dN(\tau)$$

(5)

where $\Phi(t)$ is the matrix of mutual kernels, i.e., $\Phi_{ij}(t)$ models the effect of events of counting process $N_j(t)$ on $N_i(t)$, and $\mu = [\mu_1, \cdots, \mu_n]^T$ is the base intensity. Often, the point process carries other information than the time of events, which is called mark. For example, the strength of an earthquake can be considered as a mark. The mark $m$, often a subset of $\mathbb{N}$ or $\mathbb{R}$, is associated with each event through the conditional mark probability function $f^*(m|t)f^*(m|t)$:

$$\lambda^*(t, m) = \lambda^*(t) f^*(m|t)$$

(6)

The mutually-exciting property of the Hawkes process makes it a common modeling tool in a variety of applications such as seismology, neurophysiology, epidemiology, reliability, and social network analysis [8][12][15][14].
Table 1 List of symbols.

| Symbol | Description |
|--------|-------------|
| φ_l   | Identity of the l'th location |
| α_vu  | The influence of users v on u |
| β_u   | Temporal kernel parameter of user u |
| η_{uc} | Base temporal intensity of user u in category c |
| µ_{uc} | Tendency of user u to explores new locations with category c |
| w_{uc|l} | Weight of location l with category c for user u |
| m_{cl} | Overall weight of location l with category c |

4 Problem Definition

Consider a directed network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $|\mathcal{V}| = N$ users and $L$ locations in $C$ different categories. Each user can check-in to a location and influence her neighbors. We define a check-in as a 4-tuple $(t, u, c, l)$, which shows the time $t$ that user $u$ check-in to location $l$ with category $c$. We observe the sequence of all check-ins in the network $\mathcal{G}$, in the time interval $[0, T]$. The observation $\mathcal{D} = \{(t_i, u_i, c_i, l_i)\}_{i=1}^{K}$, is composed of the time $t_i \in [0, T]$, user $u_i \in \mathcal{V}$, category $c_i \in \{1, 2, \ldots, C\}$ and location $l_i \in \{\phi_1, \phi_2, \ldots, \phi_L\}$, where $\phi_i$ can be the id or geographical coordinate of the location. We use the following notation for the history of check-ins of user $u$ in location $l$ with category $c$ up to the time $t$:

$$D_{uc|l}(t) = \{(t_i, u_i, c_i, l_i) \in \mathcal{D} \mid t_i < t, u_i = u, c_i = c, l_i = \phi_l\}$$

Moreover, we use the dot notation to represent the union over the dotted variable, e.g., $D_{u.|}(t)$ represents the events of user $u$, before the time $t$, in any location with any category. Moreover, $D_{.c.|}(t)$ represents the events of all users except $u$, before the time $t$, in any location with category $c$.

Given this observations, we want to infer the latent influence network, and model the spatio-temporal behavior of users in the location-based social networks like Foursquare. In other words, we want to model the location and time of the next check-in of a user, by observing the history of the user and her friends. In this paper, we assume that users have a periodic pattern in the time of their check-ins, and are influenced by the behavior of their friends. Therefore, we model the time of check-ins by a periodic decaying point process which incorporates the periodic pattern in the users’ movements, and the location of check-ins by a time-dependent multinominal distribution which incorporates the mutually exciting effect of friends.

5 Proposed Method

5.1 Modeling the Time of Check-ins

In every working day, a user may check-in to her office in the morning then go to a restaurant at noon, and also have a weekly soccer practice program.
By observing the history of the time of check-ins of a user, if she repeats some patterns recently (within several days), for examples take a walk every afternoon, then it is more likely to repeat this pattern shortly in the following days at approximately the same time. It means, there is a periodicity in the users’ behaviors. But, there maybe also a drift or an addition of a new activity in the user’s behavior, for example, the working hour of her office may change or there may be a new weekly social gathering. Therefore, we need a periodic point process to model the time of user’s check-ins, which can also adapt to new movement patterns. This is in contrast to the self-exciting nature of the Hawkes process, which is used to model the diffusion of information [14,15].

We propose a doubly stochastic point process which is periodic, and also has a diminishing property that enables the process to change its periodic pattern and adapt to new behaviors. The proposed process, is composed of a Poisson process with the base intensity $\mu$, where each event $t_i$ of this process triggers a Poisson process with the following intensity:

$$\lambda(t) = \sum_{k=1}^{\infty} h(t - t_i - k\tau) g(k)$$

where $h(t)$ is the kernel of the process, $g(k)$ is a decreasing function to diminish the intensity in the future periods, and the hyper-parameter $\tau$ is the period. This intensity is illustrated in Fig. 1. An event in time $t = 0$, produces a process with the intensity that is schematically plotted for the Hawkes (dashed curve) and the proposed process (solid curve). The self-exciting property of the Hawkes process can be observed from its exponentially decaying kernel in Fig. 1. In the Hawkes process when an event occurs, there is a high probability to have events just after it, and this probability decreases exponentially afterward. But in the proposed process, there is a high probability to have events in the upcoming periods and this probability also decreases exponentially.

According to the superposition theorem [23], the intensity of the proposed process can be written as follows:

$$\lambda^*(t) = \mu + \sum_{i=1}^{|\mathcal{F}_t|} \lambda_{t_i}(t) = \mu + \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} h(t - t_i - k\tau) g(k)$$

Fig. 1 An event in $t = 0$ triggers a poisson process. The solid curve shows the intensity of a periodic point process with a Gaussian kernel and period $\tau$, and the dashed curve shows a Hawkes process with an exponential decaying kernel.
To preserve the locality in time, the kernel $h(t)$ should have a peak at $t = 0$ and decay to zero in both sides when $t \to \pm \infty$. For example, the Gaussian kernel, $h(t) = \exp(-t^2/2\sigma^2)$ meets this requirements. This model has three main features:

1. **Periodic Nature.** When an event occurs in time $s$, then the intensity of events around this time in the upcoming periods, $s + k\tau$, would increase.
2. **Temporal Locality.** The intensity is high around the peak of the kernel and drops rapidly in both sides.
3. **Adaptability.** The peak of the kernel decreases by the increase of $k$, so the process can adopt its intensity to any new periodic patterns.

If we use a truncated Gaussian kernel like $h(t) = \exp(-t^2/2\sigma^2) I(-\tau/2 \leq t \leq \tau/2)$, then we can substantially reduce the complexity of the intensity function. With this kernel we can show that:

$$\lambda^*(t) = \mu + \sum_{i=1}^{\lfloor \frac{t-t_i}{\tau} \rfloor} h(t-t_i-k_i\tau) g(k_i) \quad (9)$$

where $k_i = \lfloor \frac{t-t_i}{\tau} \rfloor$ is the period number of which the event in $t_i$ affects on the current intensity. In Fig. 2 the periodic model is illustrated for a Gaussian kernel and compared to a self-exciting Hawkes model.

We propose the following point process for the time of check-ins of user $u$ in any location with category $c$:

$$\lambda_u(t, c) = \mu_{uc} + \sum_{i=1}^{\lfloor D_{\text{uc}}(t) \rfloor} \beta_u \exp \left[ -\frac{(t-t_i-k_i\tau)^2}{2\sigma^2} \right] \exp(-k_i) \quad (10)$$

The first term, $\mu_{uc}$ is the base intensity that models the external effect on user $u$ to generates check-ins with category $c$, the second term is the periodic effect of the history, $\beta_u$ is the kernel parameter, and $\tau, \sigma$ are hyper-parameters. All parameters of the model are listed in Table 1. The intuition of this model is that, if a user check-ins frequently, for example in the "restaurant" category at noon, then with high probability, she will check-ins in a restaurant at noon in the next day.
5.2 Modeling the Location of Check-ins

In this section, we propose a model for the location of users’ check-ins, given the history of check-ins. We use the fact that, users in social networks are influenced by the behavior of their neighbors. Let denote the weight of location $l$ with category $c$ for user $u$ as:

$$w_{ucl} = \sum_{i=1}^{|[\mathcal{D}_{ucl}(t)]|} \alpha_{u_i u} \exp(-(t - t_i))$$

which incorporate $\alpha_{u_i u}$, the influence of $u_i$ on $u$, and the time of check-ins with an exponentially decaying kernel. This kernel diminishes the effect of far past check-ins, so the model can adopt to any new behaviors of the users. Therefore, a location which checked in recently with many or even few but influential friends would have high weight. We also define a weight for the popularity of a location $l$ with category $c$ from the perspective of all users:

$$m_{cl} = \sum_{i=1}^{|[\mathcal{D}_{cl}(t)]|} \exp(-(t - t_i))$$

where the location that is most checked in recently, has the highest weight.

When a user decides to check-in for example, at a restaurant, she selects a location that herself or her friends have checked in frequently, recently (exploitation effect), and sometimes she check-ins to a new popular restaurant (exploration effect). Therefore, we use the following multinomial conditional distribution to define the probability that user $u$ check-ins to location $\ell$, given the time $t$ and category $c$:

$$f_u(\ell|c,t) = \frac{w_{ucl}}{\eta_{uc} + w_{ucl}} \delta_{\phi_l}(\ell) + \frac{\eta_{uc}}{\eta_{uc} + w_{ucl}} G_0(\ell)$$

The Dirac delta function $\delta_{\phi_l}(\ell)$ is 1 if $\phi_l = \ell$, otherwise it is 0, and the parameter $\eta_{uc}$ models the inclination of the user to explores the new locations. This distribution means that, with probability $w_{ucl}/(\eta_{uc} + w_{ucl})$ the current location would be a previously checked in location $\phi_l$ by the user $u$ or any of her friends (since for non visited locations the weight $w_{ucl}$ is zero), and with probability $\eta_{uc}/(\eta_{uc} + w_{ucl})$ it would be selected from all locations in the network, with a probability that is modeled by the following distribution:

$$G_0(\ell) = \sum_{i=1}^{L} \frac{m_{cl}}{m_{c}} \delta_{\phi_i}(\ell)$$

Where according to the definition of coefficient $m_{cl}$, it assigns more probability to the popular or recently frequently visited locations. The main features of the proposed location model are:
Algorithm 1: Generative model of the check-ins.

**Input**: \(N, C, L\), all parameters \(\{\mu_{uc}, \eta_{uc}, \alpha_{uv}, \beta_u\}\), history of check-ins.

**Output**: Next check-in \((t_i, u_i, c_i, l_i)\).

for \(u = 1 : N\) do
\[
\lambda(t) = \sum_c \lambda_u(t, c)
\]
\[
t_i \sim \mathcal{PP}(\lambda(t))
\]
\[
u_i \sim \text{Multi}(\frac{\lambda_u(t_i)}{\lambda(t)}, \ldots, \frac{\lambda_u(t_i)}{\lambda(t)})
\]
\[
c_i \sim \text{Multi}(\frac{\lambda_u(t_i, 1)}{\lambda_u(t_i)}, \ldots, \frac{\lambda_u(t_i, C)}{\lambda_u(t_i)})
\]
\[
l_i \sim f_{u_i}(l|c_i, t_i)
\]
return \((t_i, u_i, c_i, l_i)\)

1. **Exploitation.** The future check-ins of a user are influenced by the history of check-ins of the user and her friends.
2. **Exploration.** There is a probability that users explore and check into new unseen locations.
3. **Adaptability.** Using exponential decaying kernel for the weights, the model can adopt to new patterns in users’ behavior.

5.3 Summary of Generative Model

The proposed generative model is summarized in Algorithm 1. Using the superposition theorem, first the time \(t\) of check-in is sampled from the proposed periodic point process \(\lambda(t) = \sum_{u, c} \lambda_u(t, c)\), then the user \(u\) which generated this event is selected in proportion to its intensity \(\lambda_u(t)\). The category \(c\) of the check-in is also selected in proportion to \(\lambda_u(t, c)\). Finally, the location \(l\) is sampled from the proposed location model.

5.4 Inference

We propose a Bayesian inference algorithm based on the EM algorithm to find the model parameters. To find the maximum likelihood solution, for each check-in \((t_i, u_i, c_i, l_i)\), we define a latent variable \(z_i\) as the user that caused \(u_i\) to check into location \(l_i\), given the time \(t_i\) and category \(c_i\). We use 1-of-\(N\) coding to represent \(z_i\)’s. For notational convenient, let:

\[
\gamma^v_{ucf} = \frac{w^v_{ucf}}{\eta_{uc} + w_{uc}} \mathbb{I}(v > 0) + \frac{m_{cf} \eta_{uc}}{m_{cf} (\eta_{uc} + w_{uc})} \mathbb{I}(v = 0)
\]

\[
w^v_{ucf} = \sum_{i=1}^{\lvert D_{cfc}(t)\rvert} \alpha_{uv} e^{-(t-t_i)} = \alpha_{uv} \sum_{i=1}^{\lvert D_{cfc}(t)\rvert} e^{-(t-t_i)}
\]
where \( \gamma_{uv} \) is the contribution or influence of user \( v \) in the check-in of user \( u \) at location \( l \) with category \( c \). Now, we define:

\[
\begin{align*}
f_{u}(l, z_{1}, c_{i}) &= \prod_{v=0}^{N} (\gamma_{uv}^{v} c_{i} l_{v})^{z_{uv}} \tag{17}
\end{align*}
\]

where \( z_{uv} \) is the \( v \)’th element of \( z_{i} \), or the index of the user that caused \( i \)’th check-ins. But, \( v = 0 \) is not the index of a user, it represents the exploration effect. It can be verified that marginalizing out the \( z_{i} \), \( \sum_{z_{i}} f_{u}(l, z_{1}, c_{i}) \), results in the probability distribution \( \text{[13]} \). Now, to evaluate the complete likelihood \( p(D, Z|\theta) \) of the data \( D \) and hidden variables \( Z = \{z_{i}\}_{i=1}^{K} \), given the parameters \( \theta = \{\mu_{uc}, \eta_{uc}, \alpha_{uv}, \beta_{uv}\} \), \( u, v = 1 \ldots N \) and \( c = 1 \ldots C \), we use the following proposition.

**Proposition 1** \( \text{[54]} \) Let \( N_{u} \), \( u = 1, 2, \ldots, N \) be a multivariate marked point process with the associated intensity \( \lambda_{u}(t) \), and the mark probability \( f_{u}(m|t) \). Let \( D = \{(t_{i}, u_{i}, m_{i})\}_{i=1}^{K} \) be a realization of the process over \([0, T]\). Then the likelihood of \( D \) on model \( N_{u} \) with parameters \( \theta \) can be expressed as follows.

\[
p(D|\theta) = \exp \left( - \int_{0}^{T} \sum_{u=1}^{N} \lambda_{u}(\tau) \, d\tau \right) \prod_{i=1}^{[D]} \lambda_{u_{i}}(t_{i}) f_{u_{i}}(m_{i}|t_{i})
\]

If we consider \( (c_{i}, l_{i}, z_{i}) \) as the mark \( m_{i} \) of the process, according to this proposition the complete likelihood is,

\[
p(D, Z|\theta) = \exp \left( - \int_{0}^{T} \sum_{u=1}^{N} \lambda_{u}(\tau) \, d\tau \right) \prod_{i=1}^{[D]} \lambda_{u_{i}}(t_{i}) f_{u_{i}}(c_{i}, l_{i}, z_{i}|t_{i}) \tag{18}
\]

where using Bayes’ rule and equation \( \text{[17]} \) it can be evaluated as follows.

\[
p(D, Z|\theta) = \exp \left( - \int_{0}^{T} \sum_{u=1}^{N} \lambda_{u}(\tau) \, d\tau \right) \prod_{i=1}^{[D]} \lambda_{u_{i}}(t_{i}) f_{u_{i}}(c_{i}|t_{i}) f_{u_{i}}(l_{i}, z_{i}|t_{i}, c_{i})
\]

\[
= \exp \left( - \sum_{u=1}^{N} \sum_{c=1}^{C} \int_{0}^{T} \lambda_{u}(\tau, c) \, d\tau \right) \prod_{i=1}^{[D]} \lambda_{u_{i}}(t_{i}, c_{i}) f_{u_{i}}(l_{i}, z_{i}|t_{i}, c_{i})
\]

\[
= \exp \left( - \sum_{u=1}^{N} \sum_{c=1}^{C} \int_{0}^{T} \lambda_{u}(\tau, c) \, d\tau \right) \prod_{i=1}^{[D]} \lambda_{u_{i}}(t_{i}, c_{i}) \prod_{v=0}^{N} (\gamma_{uv}^{v} c_{i} l_{v})^{z_{uv}}
\]

To derive the second line, we used the superposition theorem, and the fact that the probability of category is \( f_{u_{i}}(c_{i}|t_{i}) = \lambda_{u_{i}}(t_{i}, c_{i})/\lambda_{u_{i}}(t_{i}) \), according to our generative model. Given the joint distribution of the observed and latent variables \( p(D, Z|\theta) \), we use EM algorithm to maximize the likelihood function \( p(D|\theta) \) with respect to \( \theta \). In the E-step we evaluate \( p(Z|D, \theta) \). Using Bayes’ rule we can write the posterior distribution of the latent variables as,

\[
p(Z|D, \theta) \propto \prod_{i=1}^{[D]} \prod_{v=0}^{N} (\gamma_{uv}^{v} c_{i} l_{v})^{z_{uv}} \tag{19}
\]
which factorizes over \( i \), so that \( z_i \)'s are independent with multinomial distribution and we can write the expected of \( z_{iv} \) under this distribution as follows.

\[
E[z_{iv}] = \frac{\sum_{iv} z_{iv} (\tilde{\gamma}_{\alpha_{iv}}^{u})^{z_{iv}}}{\sum_{iv} \prod_{v=0}^{N} (\tilde{\gamma}_{\alpha_{iv}}^{v})^{z_{iv}}} = \frac{\tilde{\gamma}_{\alpha_{iv}}^{v}}{\tilde{\gamma}_{\alpha_{iv}}^{v}}
\]  

(20)

In the M-step we maximize \( E[Z \ln p(D, Z|\theta)] \) the expected complete log-likelihood, which can be decomposed to the sum of expected log-likelihoods of users \( E[Z_u \ln p(D_u, Z_u|\theta_u)] \).

\[
E[Z \ln p(D, Z|\theta)] = - \sum_{u=1}^{N} \sum_{c=1}^{C} \int_{0}^{T} \lambda_u(\tau, c) \, d\tau + \sum_{i=1}^{|D_u|} \log \lambda_u(t_i, c_i) + \sum_{i=1}^{N} \sum_{v=0}^{N} E[z_{iv}] \log \tilde{\gamma}_{\alpha_{iv}}^{v}
\]

\[
= \sum_{u} \left( - \int_{0}^{T} \sum_{c=1}^{C} \lambda_u(\tau, c) \, d\tau + \sum_{i=1}^{|D_u|} \log \lambda_u(t_i, c_i) + \sum_{i=1}^{N} \sum_{v=0}^{N} E[z_{iv}] \log \tilde{\gamma}_{\alpha_{iv}}^{v} \right)
\]

\[
= \sum_{u} E[Z_u \ln p(D_u, Z_u|\theta_u)]
\]  

(21)

Where \( Z_u = \{ z_i \in Z \mid u_i = u \} \) and \( \theta_u = \{ \mu_{uc}, \eta_{uc}, \alpha_{uv}, \beta_u \}, \quad v = 1 \ldots N, \quad c = 1 \ldots C \). Accordingly, the M-step can be decomposed to multiple maximizations over users, which can be done in parallel. The two steps of the EM algorithm can be summarized as follows.

**E-Step:** \( E[z_{iv}] = \frac{\tilde{\gamma}_{\alpha_{iv}}^{v}}{\sum_{v=0}^{N} \tilde{\gamma}_{\alpha_{iv}}^{v}} \)

**M-Step:** \( \theta_u^{*} = \arg \max_{\theta_u \geq 0} - \int_{0}^{T} \sum_{c=1}^{C} \lambda_u(\tau, c) \, d\tau + \sum_{i=1}^{N} \sum_{v=0}^{N} E[z_{iv}] \log \tilde{\gamma}_{\alpha_{iv}}^{v} \)

In the following proposition, we prove that the maximization in M-step is concave, so it has a unique and optimal solution.

**Proposition 2** The expected log-likelihood of a user, \( E[Z_u \ln p(D_u, Z_u|\theta_u)] \) as a function of \( \{ \mu_{uc}, \eta_{uc}, \alpha_{uv}, \beta_u \} \) is concave, where \( \alpha_{uv} = \exp(\bar{\alpha}_{uv}) \) and \( \eta_{uc} = \exp(\bar{\eta}_{uc}) \).

**Proof.** According to equation (21) the log-likelihood of user \( u \) is:

\[
E[Z_u \ln p(D_u, Z_u|\theta_u)] = - \int_{0}^{T} \sum_{c=1}^{C} \lambda_u(\tau, c) \, d\tau + \sum_{i=1}^{N} \sum_{v=0}^{N} E[z_{iv}] \log \tilde{\gamma}_{\alpha_{iv}}^{v}
\]

The first term is a linear function of \( \{ \mu_{uc}, \beta_u \} \), so it is both convex and concave. The second term is the log of a linear function which is concave, according to composition rules [3]. The third term is composed of \( \log \tilde{\gamma}_{\alpha_{iv}}^{v} \), which for \( v > 0 \),

\[
\log \tilde{\gamma}_{\alpha_{iv}}^{v} = \bar{\alpha}_{vu} - \log \left( e^{\tilde{\eta}_{uc}} + \sum_{j=1}^{|D_{uc}|} c^{\tilde{\alpha}_{uv}} e^{-(t-t_j)} \right) + \text{const}
\]
and for $v = 0$,

$$
\log \gamma_{uc_i} = \tilde{\eta}_{uc_i} - \log \left( e^{\tilde{\eta}_{uc_i}} + \sum_{j=1}^{|[D_{uc_i}(t)]|} e^{\tilde{\alpha}_{uv} e^{-|t-t_j|}} \right) + \text{const}.
$$

In both cases $\log \gamma_{uc_i}$ is concave according to Lemma 1 of [54] which state that logarithm of sum of linear exponentials is convex. So, the overall expression is concave. Actually, we use $\tilde{\eta}_{uc}, \tilde{\alpha}_{uv}$ instead of $\eta_{uc}, \alpha_{uv}$ in the implementations, and solve the resulting concave optimization.

6 Experiments

In this section, we evaluate the performance of the proposed method by using both synthetic and real data. Synthetic data is generated from our model and real data is crawled from Foursquare. To better measure the performance, we evaluate the spatial and temporal models separately in both synthetic and real experiments.

6.1 Synthetic Data

We use synthetic data to measure the ability of the inference algorithm to learn the model parameters, given the data that is generated from the model with a known set of randomly selected parameters.

6.1.1 Dataset Preparation

We generated a random network with $N = 50$ nodes and the edge sparsity of 0.2. We set the number of categories to $C = 4$ and consider eight locations in each category, in total $L = 32$ locations. The temporal and spatial parameters are randomly drawn from the uniform distributions $\mu_{uc}, \eta_{uc} \sim U(0, 0.05)$, $\alpha_{uv} \sim U(0, 0.5)$ and $\beta_u \sim U(0, 0.1)$. The period and standard deviation in temporal model are $\tau = 12$ and $\sigma = 0.5$, respectively. We generated 10000 check-ins from our model, using the Ogata method [54], and used the first 80% for the training and the remaining 20% for the test.

6.1.2 Evaluation Criteria

To evaluate the performance of inference algorithm in parameter learning, we use the following measures.

- **Parameter Estimation Error.** It measures the distance between the inferred $\hat{\theta}$ and the real parameters $\theta$, which includes; mean squared error (MSE), mean absolute error (MAE), and mean relative error (MRE):

$$
\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (\theta_i - \hat{\theta}_i)^2, \quad \text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |\theta_i - \hat{\theta}_i|, \quad \text{MRE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{\theta_i - \hat{\theta}_i}{\theta_i} \right|
$$
– **Likelihood Measure.** To use this measure, we learn the model with different percentage of the available train data and then evaluate the average log-likelihood (AvgPredLogLik) of the model on test data. For a properly learned model, we expect that this measure increases with the size of train data.

– **Network Structure Prediction.** To measure the strength of models in network structure inference, we introduce AUC measure. We assume that the network has a complete structure, then train the model to infer \( \alpha_{ij} \)'s. Using a threshold on \( \alpha_{ij} \)'s, we extract the network adjacency matrix and evaluate the ROC curve. The AUC measure is the area under the ROC curve.

### 6.1.3 Parameter Learning

We learn the model with different percentage of the available training data, and evaluate the average predicted log-likelihood on a separate test data. The EM algorithm is implemented in parallel for all users. All source codes for our experiments and datasets are available in the git repository.\footnote{https://github.com/azarezade/stp} We have plotted the evaluation measures with respect to the number of events per user in Fig. 3. The likelihood increases with the increase of size of the train data in both spatial and temporal models. It means that the inference algorithm works properly. In Fig. 4 the parameter estimation error for the temporal model is plotted vs the number of events in train data per user. We expect MSE, MAE and MRE, decrease by the number of train events. The mean square error, MSE, reduces to about \( 10^{-4} \). But, this measure is not enough alone, since if we select small values for the model parameters, then we may have small MSE even with unsatisfactory models. For this reason, the mean relative error, MRE is a better measure. It reaches to about 20% with full train data. The same performance is also achieved in the spatial model. In the first row of Fig. 5 we have plotted the logarithm of the parameter estimation error measures for 10 iterations of EM algorithm. In the second row, the measures are plotted for 40 iterations of EM, where we used all train data. Again, the measures decreases with the size of train data and EM iterations, for example the mean
relative error reaches to about 25%. Finally, to evaluate the performance of our method in the network structure prediction, we have plotted the AUC curve for different percentage of the train data. The curve is increasing, as we expected, and it achieves about 66% performance with all train data.

6.2 Real Data

We used real data to evaluate the ability of the proposed model to predict the future check-ins, in comparison with other methods.

6.2.1 Dataset Preparation

We used both Twitter and Foursquare API’s to crawl the check-ins data of the users in Foursquare because the Foursquare API doesn’t provide check-ins data. Specifically, we crawled the tweets of the users that have Swarm application. This app is connected to Twitter and Foursquare account of the users. When a user check-ins, this app tweets the URL of that location in
the Foursquare website. Therefore, we have access to the location details (via Foursquare API) and the time of check-ins (via Twitter API).

Using the Twitter search API we find active users with high check-ins rate in Foursquare. By querying the API with “I am at”, the default template of Swarm app for check-ins, we select the top 12000 users, and crawl their tweets in 10 weeks during the year 2015. We pruned the data by selecting 1000 active users that were in the same country, to better see the effect of neighbors on the users’ check-ins. In the 10 weeks that we crawled the data, users from Brazil have the highest number of check-ins. The average degree of the network is 6.4. The total number of check-ins is about 60000, i.e., average 60 check-ins per user. The number of unique locations is about 10000 in 10 categories.

6.2.2 Evaluation Criteria

In real data there is no ground truth, therefore we can’t use the parameter estimation error measures. Instead, we use the likelihood and the following two measures.

– Check-ins Time Accuracy. These measures are used to evaluate the performance of the model to predict the time of future check-ins. The first measure is $l_1$ distance. To evaluate it, we divide the 2-week interval of the test data into 12-hour bins and count the number of check-ins in each bin for the real data and the predicted check-ins by the model. For each user-category, we would have a 28-dimensional vector. The measure is the

![Fig. 6](image-url)  
Fig. 6 Network structure prediction accuracy for different sizes of train data.

![Fig. 7](image-url)  
Fig. 7 Average predicted log-likelihood of test data in spatial and temporal models, versus the size of train data.
average of $\ell_1$ distance between the predicted vector and its corresponding real vector for all users and categories. The second measure, # users, counts the number of users that better modeled the time of checkins (have lower $\ell_1$ distance) compared to the alternative methods.

- **Check-ins Location Accuracy.** To measure the performance of the model to generate locations, we use precision at $k$. Given the time of check-ins from real data, for each time, we evaluate the probability of each location. Precision at $k$ is the percent of check-ins that their real location are among the top $k$ predicted locations.

- **Temporal Inference Time.** The optimization time to infer the temporal model parameters.

### 6.2.3 Baselines

In the temporal model, the baselines are:

- **Hawkes.** The time of check-ins of user $u$ is modeled by a simple Hawkes process, $\lambda_u(t) = \mu_u + \alpha_u \sum_{i=1}^{\left| D_u \right|} \exp(-(t - t_i))$, which depends on its own history of check-ins.

- **MultiHawkes.** The intensity of check-ins of user $u$ is modeled by a multivariate Hawkes process $\lambda_u(t) = \mu_u + \sum_{i=1}^{\left| D_u \right|} \alpha_{u_i} \exp(-(t - t_i))$, which depends on herself and her friends’ history.

In the spatial model the baselines are:

- **MostPopular.** In this method, the weight of locations is considered as $w_{ucl} = \sum_{i=1}^{\left| D_{ucl} \right|} \alpha_{u_i}$, instead of the weight (11) in our model. So in this model, the most checked in location, disregarding the time of check-in, have the highest probability.

- **PeriodicLoc.** This model assumes periodicity in the location of check-ins. So, the weight is modeled by $w_{ucl} = \sum_{i=1}^{\left| D_{ucl} \right|} \alpha_{u_i} \exp(-|t - w - t_i|)$.

### 6.2.4 Parameter Learning

We use the first eight weeks of the real data as the train data, and the remaining two weeks as the test data. The hyper-parameters of temporal model are tuned to $\tau = 24$ and $\sigma = 1$. The model is trained on different sizes of the train data and the average log-likelihood of the test data is plotted in Fig. 7. The proposed method compared to the other alternatives, has better modeled the time and location of check-ins. In the right diagram of Fig. 7, the likelihood

| Method    | $\ell_1$ distance ($\times 10^{-4}$) | # users |
|-----------|-----------------------------------|---------|
| Hawkes    | $2.02 \pm 1.32 \times 10^{-4}$    | 44      |
| MultiHawkes| $1.91 \pm 1.10 \times 10^{-4}$    | 311     |
| Our       | $1.89 \pm 1.02 \times 10^{-4}$    | 404     |
of MultiHawkes is lower than other methods. It shows that the time of users’ check-ins is more affected by their own check-ins instead of their friends. Also, the superior performance of our model, shows that the periodic kernel compared to the exponential kernel, successfully modeled the time of check-ins. In the right digram of Fig. 7, the weak performance of PeriodicLoc shows that the location of users’ check-ins is not periodic in contrast to the time of check-ins.

To evaluated the prediction accuracy of check-ins time, we use two measures; $\ell_1$ distance between real and predicted time vector, and the # users that better modeled the time of checkins, which are reported in the first and second row of Table 2 respectively. It can be seen that our model has the lowest mean and variance of $\ell_1$ distance, i.e.,. In the left diagram of Fig. 8, the accuracy at top $k$ for $k = 1, 3, 10$ is plotted. For higher values of $k$, the accuracy in location is higher, since the measure becomes simpler. Compared to other methods, the proposed method has higher accuracy for all values of $k$. In the right diagram of Fig. 8, the inference time of temporal model parameters is plotted against the size of train data. The proposed method and Hawkes both performed well with respect to MultiHawkes which is multivariate and has $O(N^2)$ parameters. In spatial models, the inference time is nearly the same for all models, since all of them are multi-user.

7 Conclusion

In this paper, to model the periodic behavior in the time of users’ check-ins, we proposed a point process with a periodic decaying kernel. In addition, we modeled the mutually-exciting effect of friends in the selection of the location of next check-ins by a multinomial distribution, where the frequently visited locations have a higher probability. The experiments show that most of users have periodicity in the time of their check-ins. Moreover, the spatial model, even with a large number of locations (about 10000), has 20% accuracy at top-10 prediction. But, our model lacks in some aspects: all users as in Table 2 are not periodic so the temporal model can’t accurately predict their time of check-in; also the location model does not consider the relation between adjacent locations, because a user that checked-in to a cinema, with high
probability will check into a nearby restaurant. The home location of users is another important aspect in defining the probability of check-ins location, which can be incorporated into our model by modifying the weight of locations in equation (11), which is being considered as a future work. In addition, we can investigate the utilization of a non-parametric spatial model in our future work.

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