Algorithmic Assistance
with Recommendation-Dependent Preferences

Bryce McLaughlin      Jann Spiess
Stanford University

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Abstract

When we use algorithms to produce risk assessments, we typically think of these predictions as providing helpful input to human decisions, such as when risk scores are presented to judges or doctors. But when a decision-maker obtains algorithmic assistance, they may not only react to the information. The decision-maker may view the input of the algorithm as recommending a default action, making it costly for them to deviate, such as when a judge is reluctant to overrule a high-risk assessment of a defendant or a doctor fears the consequences of deviating from recommended procedures. In this article, we propose a principal–agent model of joint human–machine decision-making. Within this model, we consider the effect and design of algorithmic recommendations when they affect choices not just by shifting beliefs, but also by altering preferences. We motivate this assumption from institutional factors, such as a desire to avoid audits, as well as from well-established models in behavioral science that predict loss aversion relative to a reference point, which here is set by the algorithm. We show that recommendation-dependent preferences create inefficiencies where the decision-maker is overly responsive to the recommendation. As a potential remedy, we discuss algorithms that strategically withhold recommendations, and show how they can improve the quality of final decisions.
1 Introduction

One important application of algorithms is to turn complex data into simple predictions or recommendations that help decision-makers make better decisions, such as risk assessments presented to judges or doctors. We typically think of such algorithmic assessments as providing additional information about which choices will lead to better outcomes. Yet decision-makers may react to algorithmic input not just by shifting beliefs, but also by changing their preferences. In this article, we consider the effect and design of algorithmic advice when it also imposes a cost on the decision-maker whenever they deviate from the recommended action, such as when a judge is reluctant to overrule a jailing recommendation or a doctor fears the consequences of not testing a patient with a high predicted risk of a specific medical condition. We show that recommendation dependence creates inefficiencies where the decision-maker is overly responsive to the recommendation, and propose changes to the design of recommendation algorithms towards providing less conservative recommendations and withholding information when the algorithm is least certain of the right decision to take.

We model the interaction of a decision-maker with a recommendation algorithm in a principal–agent model of joint human–machine decision-making. The principal designs a recommendation algorithm. The agent plays the role of the human decision-maker, and makes a choice between a safe and a risky decision based on their private information along with a binary recommendation provided by the algorithm. When the state of the world is good, the risky action is best, while in the bad state, the risky action leads to high loss. The agent uses the information available to them to assess the probability that the state is bad, and chooses the risky action only if that predicted probability is low. For example, a judge who considers whether to release a defendant on bail (risky action) aims to release only those defendants with low probability of failing to appear or committing a new crime (bad state).

To this model, we add the assumption that recommendations affect decisions not only through the information they provide, but also by setting a reference point against which the agent measures their outcomes. We assume that the agent perceives an additional (personal) cost from making an error when deviating from this recommendation. Specifically, in our model, there is an additional loss when the agent takes a risky decision against the safe recommendation of the algorithm and the bad state materializes. Similarly, there may also be an additional loss from deviation when the agent opts for the safe option relative to a risky recommendation and the good state occurs.

A first motivation for such recommendation-dependent preferences stems from institutional factors, such as when deviating from recommendations triggers audits or may create backlash. For example, a judge may be reluctant to release a defendant in light of a jailing recommendation for fear of repercussions, even if they believe that the defendant represents a lower risk. Similarly, a doctor may prefer to order a test (safe decision) when the algorithm recommends so for fear of missing a bad diagnosis against algorithmic advice, and may feel more justified in taking a risky
decision when the algorithm concurs.

A second motivation is provided by established models from behavioral science that suggest expected losses impact decision-makers more than commensurate gains, relative to some reference point that we assume here is affected by the algorithmic recommendation. The combination of perceiving decision utility relative to a reference point and experiencing loss aversion relative to that reference point are two of the main features of Prospect Theory, which has been one of the most established frameworks rooted in psychology for describing systematic deviations from rational utility maximization. Here, we show that the combination of those two features predicts recommendation dependence when we assume that the reference point is obtained from the recommended action.

Having set up a model of recommendation-dependent preferences, we show that the effect of algorithmic advice generally differs from a reference-independent baseline case. Recommendation dependence increases adherence to the algorithmic recommendation. This adherence makes decisions less efficient as it reduces the amount of private information that the agent reveals through their chosen action. For example, if a judge is worried about repercussions from releasing a defendant the algorithm recommends to jail, the judge may follow the recommendation even if they have private information that suggests that the defendant is not at high risk of committing a new crime or failing to appear.

Recommendation dependence leads to inefficiencies that can be mitigated (but not completely avoided) by a better design of the recommendations. We first tackle the case where recommendations are part of the design, and the algorithm only returns binary information. In this case we show that, under regularity assumptions, if deviating from one of the recommendations becomes more costly, then this recommendation should be given less frequently. Specifically, if the agent is reluctant to overrule a safe recommendation because of additional costs from making a mistake in this case, then the algorithm should recommend the safe option less, and instead propose the risky option in some cases where a baseline algorithm would recommend the safe action.

Having shown how recommendation dependence affects the consequences and optimal design of recommendations, we discuss the benefits of allowing the algorithm to give a neutral “don’t know” recommendation when the algorithm is unsure of the best decision. With recommendation-dependent preferences, adding a third option of not providing a recommendation at all has two distinct benefits. The first benefit is that it allows the transmission of additional information through the recommendation, signaling an intermediate probability of a bad outcome occurring. The second benefit is that not providing a recommendation in some cases also reduces the cost of recommendation dependence, and allows the agent to make optimal decisions in this case. Specifically, we show in a simple example that adding such an additional “don’t know” level within our model can improve decisions relatively more in a world with recommendation-dependent preferences than in a world where the agent’s preferences are not affected by the algorithm.

So far, we have considered the case where the algorithm only returns a binary recommendation.
Yet in many applications, the algorithm may present a risk score that estimates the probability of a bad outcome, such as when a doctor receives the predicted probability that a patient suffers from a specific condition. In this case, we think of recommendations as either explicitly or implicitly derived from the risk score, as would be the case for the doctor who interprets a high risk score as a recommendation to test a patient. In this case, recommendation dependence still increases adherence to the action suggested by the risk assessment inefficiently, leading to sub-optimal outcomes. However, the recommendation may not be part of the algorithmic design in this case.

We consider an algorithm that strategically withholds risk scores in the case when recommendations are implicitly derived from algorithmic predictions, and show how it can improve outcomes. While withholding risk assessments destroys valuable information, we argue that creating instances without recommendations also reduces distortions. For example, a judge may make better decisions in borderline cases if the algorithm strategically withholds uninformative risk assessments and thereby ensures that the human decision-maker uses their private information efficiently. If properly designed, our model suggests that such strategic silence can improve overall outcomes.

We contribute to a cross-disciplinary literature that studies human–AI interaction. This includes work where the knowledge of an AI and human decision-makers (or more generally multiple knowledge sources) are combined (e.g. Lawrence et al., 2006; Palley and Soll, 2019; Steyvers et al., 2022), where humans assist an AI (e.g. Hampshire et al., 2020; Ibrahim et al., 2021), and where an algorithm optimizes advice given to human decision-makers (Bastani et al., 2021) or which instances to delegate (Raghu et al., 2019). Athen et al. (2020) discuss general trade-offs in the allocation of decision authority between human and AI. Fogliato et al. (2022b) study human overrides of algorithmic recommendations, and argue in favor of human discretion in critical applications. Hemmer et al. (2021) provides a recent review of the literature on complimentary in human–AI systems. Recent contributions to this literature emphasize that the success of human–machine collaboration is dependent on details of context, implementation, and presentation of algorithmic advice (such as Bansal et al., 2019a,b; Green and Chen, 2019; Snyder et al., 2022), including information about its uncertainty (McGrath et al., 2020; Taudien et al., 2022) and explanations of black-box classifiers (Lakkaraju and Bastani, 2020).

We also build upon a literature that brings models from psychology into economics and operations and considers behavioral aspects of the interaction between humans and machines, including algorithm aversion (Dietvorst et al., 2015, 2018), algorithm appreciation (Logg et al., 2019; Bai et al., 2021), and over-reliance on algorithms (Banker and Khetani, 2019; Buçinca et al., 2021). Closely related to our work, Fügener et al. (2021) shows how over-adherence to AI assistance may reduce the diversity of opinions and can lead to worse group decisions, and Green and Chen (2021) demonstrates in a lab experiment that providing algorithmic risk assessments may make decision-makers overly sensitive to perceived risk at the cost of other factors affecting decision quality. Boyaci et al. (2022) studies algorithms assisting humans in a model of rational inattention. Sun
et al. (2022) designs an algorithm that proactively incorporates predicted behavioral deviations in order to improve recommendations. In recent empirical work, Caro and de Tejada Cuenca (2023) studies the adherence of managers of a fashion company to price recommendations, while Albright (2023) isolates the causal effect of recommendations on bail decisions and shows that recommendations change how judges weigh risks. Beyond recommender systems, Prospect Theory (Kahneman and Tversky, 1979) is explicitly considered in the collaboration of decision-makers with robots in Kwon et al. (2020) and with an AI e.g. in Ye et al. (2022).

The remaining article is organized as follows. In Section 2, we formalize the concept of recommendation-dependent preferences within a principal–agent model of algorithm-assisted human decisions. In Section 3, we describe how this channel introduces inefficiencies that lead to lower expected loss, before discussion the better design of recommendations as a possible remedy in Section 4 and the value of strategically withholding recommendations in Section 5. In Section 6 we then consider a version of our model where the decision-maker’s information also includes the machine’s risk prediction. We also provide justifications for our specific model of recommendation dependence from institutional factors or established behavioral models in Section 7 and discuss extensions in Section 8, before concluding in Section 9.

2 A Model of Recommendation-Dependent Preferences

We model the interaction of a human decision-maker with a recommendation algorithm within a principal–agent model. The principal designs an algorithm that provides the agent with recommendations $R \in \{\text{safe, risky}\}$. The agent leverages these recommendations to take a decision $A \in \{\text{safe, risky}\}$ about an instance with outcome $Y \in \{\text{good, bad}\}$. Principal and agent both want to take the safe decision when faced with a bad outcome, but prefer the risky decision when the outcome is good. For example, the agent may be a judge who decides whether to release ($A = \text{risky}$) or jail ($A = \text{safe}$) a defendant, where the defendant may turn out to commit an offense or fail to appear ($Y = \text{bad}$) if released on bail or may appear without any new criminal activity ($Y = \text{good}$).

We assume that the agent and the algorithm both have access to the context $X$ and receive signals $H$ and $M$, respectively. The signal $X$ includes information about the instance at hand that encodes any commonly known context and information about the distribution of $Y$. In addition, the signal $H$ of the human decision-maker may include details not available to the machine, such as properties of the specific instance only visible in-person, and the machine’s signal $M$ may likewise encode information not accessible to the human decision-maker, such as information deduced from large training data. Allowing for private information of both parties creates complementarities that motivate collaborative machine–human decision-making. For example, both the judge and their algorithmic assistant may have access to the sentencing history of a defendant, while the judge learns additional information from the defendant answering questions in court and the algorithm also synthesizes systematic insights from a large database of past defendants. Jointly, the outcome
$Y$ and the signals $H$ and $M$ follow a known (prior) distribution $P$. In the judge example, the distribution $P$ represents the distribution of the probability of not appearing or committing a new crime together with the information the judge and the algorithm have about a defendant.

The game between the designer of the algorithm (principal) and the human decision-maker (agent) plays out as follows:

1. The principal chooses a recommendation algorithm $r$ that maps the machine information $(X, M)$ to a recommendation $R = r(X, M) \in \{\text{risky}, \text{safe}\}$.

2. The features $(X, H, M)$ are drawn from the distribution $P$.

3. The agent observes the features $(X, H)$ and the machine recommendation $R = r(X, M)$, then takes a decision $A \in \{\text{risky}, \text{safe}\}$.

4. The outcome $Y \in \{\text{good}, \text{bad}\}$ given the features $(X, H, M)$ is drawn and the losses of principal and agent are realized.

Both principal and agent know the joint distribution $P$ of the outcome $Y$, the context $X$, the human signal $H$, and the machine signal $M$. For example, the designer of the algorithm in the judge example chooses a mapping from information available the the machine to a recommendation, which the judge then observes together with the additional information the judge learns from the defendant in the courtroom before taking a decision whether to jail or release.

We assume that the principal aims to minimize expected loss (risk) $E[\ell(Y, A)]$ for the losses

$$\ell(Y, A) = \begin{cases} c_I, & Y = \text{good}, A = \text{safe}, \\ c_{II}, & Y = \text{bad}, A = \text{risky}, \end{cases} \quad (1)$$

where the two cases cover the two mistakes of choosing the safe option despite the outcome being good (leading to $c_I > 0$, Type-I error) or the risky decision in a bad case (leading to $c_{II} > 0$, Type-II error). For the jail decision, $c_I$ is the cost of jailing a defendant who would not engage in criminal activity, and $c_{II}$ the cost of releasing a defendant who commits a new crime or fails to re-appear. The losses are also summarized in Panel (a) of Table 1.

As a crucial deviation from standard (rational) models of human decision-making, we assume that the agent experiences a decision loss $\ell^*(Y, A, R)$ that deviates from the consequence of the action alone, and can depend on the recommendation. Starting with the principal’s loss function, we assume that the agent experiences additional loss when they make a mistake that deviates from the machine recommendation,

$$\ell^*(Y, A, R) = \ell(Y, A) + \begin{cases} \Delta_I, & Y = \text{good}, A = \text{safe}, R = \text{risky}, \\ \Delta_{II}, & Y = \text{bad}, A = \text{risky}, R = \text{safe}, \end{cases} \quad (2)$$

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with $\Delta_I, \Delta_{II} \geq 0$. Here, $\Delta_I$ describes the additional loss perceived by the human decision-maker when they play it safe against the machine’s recommendation of a risky action, and the risky action would have been optimal. Similarly, $\Delta_{II}$ quantifies the penalty of taking a risky decision in the bad state when the algorithm recommends the safe action, such as when the judge gets in trouble for releasing a defendant against the recommendation of the algorithm who then goes on to commit a crime. Table 1 summarizes the resulting losses in Panel (b), and compares them directly to the principal’s losses in Panel (a), which depend on recommendations only through final decisions.

Table 1: Losses of principal (left) and agent (right) as a function of the realized outcome $Y \in \{\text{good, bad}\}$, algorithmic recommendation $R \in \{\text{safe, risky}\}$, and decision $A \in \{\text{safe, risky}\}$.

| Decision | safe | risky |
|----------|------|-------|
| Outcome  | $c_I$ | 0     |
|          | 0    | $c_{II}$ |
(a) Welfare losses of the principal

| Recommendation | safe | risky |
|----------------|------|-------|
| Decision       | safe | risky |
| Outcome        | $c_I$ | 0     |
|                | $c_I + \Delta_I$ | 0     |
|                | 0    | $c_{II}$ |
(b) Decision losses of the agent

In this model, principal and agent preferences differ by an additional loss that the agent incurs when making mistakes and simultaneously going against the algorithm’s recommendation, but are otherwise aligned. We discuss justifications for this deviation between losses in Section 7 below, where we motivate this form of misalignment between principal (welfare) and agent (decision) losses from institutional factors and derive it from established models of reference-dependent preferences in behavioral science. We further consider cases where baseline preferences are misaligned in Section 8.

## 3 Consequences of Recommendation Dependence

Having set up a model of recommendation-dependent preferences, we now discuss how machine recommendations affect human choices beyond their information content. As a benchmark, we start by describing the optimal solutions a human decision-maker would take without access to algorithmic recommendations. We then describe the decision that is responsive to an algorithmic recommendation and discuss its inefficiencies, before considering how we can alleviate them in Sections 4 and 5 below. Throughout, we illustrate the main intuitions in an example.

As a baseline, we consider the choices of a human decision-maker who does not receive a recommendation (and does not exhibit any reference dependence). A decision-maker who wants to minimize expected loss compares their best prediction $P(Y=\text{bad}|H)$ of the bad outcome occurring to the critical probability threshold $p = p^*$ at which the expected loss $p c_{II}$ from the risky action
equals the expected loss \((1 - p) c_I\) from the safe action, leading to the optimal decision

\[
A = \arg \min_a E[\ell(Y, a) | X, H] = \begin{cases} 
  \text{risky}, & P(Y=\text{bad} | X, H) \leq p^* = \frac{c_I}{c_I + c_{II}}, \\
  \text{safe}, & P(Y=\text{bad} | X, H) > p^*,
\end{cases}
\]

that minimizes expected loss \(E[\ell(Y, A)]\), where we throughout resolve ties in favor of the risky action.

**Example 1** (Independent uniform signals). Consider a simple example without any context \(X\) and private signals, \(H\) and \(M\), being drawn independently from a uniform distribution on \([0, 1]\). Let the outcome \(Y\) be deterministic in terms of \(H\) and \(M\),

\[
Y = \begin{cases} 
  \text{bad}, & H + M \geq 1, \\
  \text{good}, & \text{otherwise},
\end{cases}
\]

which is presented in Panel (a) of Figure 1. When the agent decides by themselves, they need to act based solely on their observed private signal \(H\). Since \(P(Y=\text{bad} | X, H) = H\), the agent’s optimal actions can be described in terms of the threshold rule

\[
A = \begin{cases} 
  \text{risky}, & H \leq p^*, \\
  \text{safe}, & H > p^*,
\end{cases}
\]

where the threshold \(p^* = \frac{c_I}{c_I + c_{II}}\) balances Type-I and Type-II errors optimally. This rule and the resulting expected loss are illustrated in Panel (b) of Figure 1, where we assume that the cost of a risky action for a bad outcome \((c_{II})\) is higher than that of a safe decision for a good outcome \((c_I)\).

We next consider the choices of a human decision-maker who receives a machine recommendation \(R = r(X, M)\), and has potentially recommendation-dependent preferences. In this case, the decision-maker’s optimal policy applies different thresholds depending on the recommendation \(R\), since the additional cost from Type-I and Type-II errors, respectively, distorts their expected loss. The choice \(A = \arg \min_a E[\ell^*(Y, a, R) | X, H, R]\) minimizing expected decision loss \(E[\ell^*(Y, A, R)]\) given the recommendation policy \(R = r(X, M)\) is now

\[
A = \begin{cases} 
  \text{risky}, & P(Y=\text{bad} | X, H, R) \leq p^R = \frac{c_I + \Delta I}{c_I + c_{II} + \Delta I}, \\
  \text{safe}, & P(Y=\text{bad} | X, H, R) > p^R
\end{cases}
\]

where \(R = \text{risky}\) if \(R = r(X, M)\) is risky and \(R = \text{safe}\) if \(R = r(X, M)\) is safe. This choice is affected by the machine recommendation through two channels: First, the recommendation may provide additional information on the risk of a bad outcome \((R\) affects the posterior belief \(P(Y=\text{bad} | X, H, R)\)), which improves decisions. Second, the recommendation shifts the threshold the decision-maker applies when deciding in which cases to take the safe option \((R\)
(a) Distribution of human \((H)\) and machine \((M)\) signals along with resulting outcomes \((Y = \text{good}, \text{light blue}; Y = \text{bad}, \text{light red})\).

(b) Optimal threshold rule on \(H\) of the human decision-maker acting alone, where \(A = \text{risky}\) is taken on the left and \(A = \text{safe}\) on the right. Loss \(c_{II}\) is incurred in the top triangle (red) and loss \(c_I\) is incurred in the bottom triangle (blue).

Figure 1: Joint distribution of outcome, human signal, and machine signal in Example 1, along with the optimal decision of a human decision-maker acting without recommendation.

The resulting distribution of decisions and losses is depicted in Panel (a) of Figure 2. Relative to the human-only decision from Figure 1, overall losses are smaller, since this decision optimally leverages the machine recommendation \(R\) by adjusting the threshold on the human information \(H\) accordingly.

So far, we have considered a decision-maker who makes optimal decisions that minimize expected loss. We now consider a decision-maker who perceives additional reference-dependent decision loss affects \(p^R\), which distorts decisions from the perspective of the principal.

**Example 1** (Independent uniform signals, continuing from p. 8). In the example above, assume for now that the algorithm provides recommendations

\[
R = \begin{cases} 
\text{risky}, & M \leq \frac{1}{2}, \\
\text{safe}, & M > \frac{1}{2},
\end{cases}
\]

(4)

which we will later argue would be the optimal recommendation in this case if preferences were fully aligned. The agent without recommendation dependence would apply the same threshold \(p^* = \frac{c_I}{c_I + c_{II}}\) to the posterior probability \(P(Y=\text{bad}|H,R)\), irrespective of the recommendation, leading to the second-best decision

\[
A = \begin{cases} 
\text{risky}, & H \leq \frac{c_I + c_{II}/2}{c_I + c_{II}} \text{ for } R = \text{risky}, \\
\text{safe}, & H > \frac{c_I + c_{II}/2}{c_I + c_{II}} \text{ for } R = \text{risky}, \\
\text{risky}, & H \leq \frac{c_{II}/2}{c_I + c_{II}} \text{ for } R = \text{safe}, \\
\text{safe}, & H > \frac{c_{II}/2}{c_I + c_{II}} \text{ for } R = \text{safe}.
\end{cases}
\]
The machine’s recommendation (here shown as a dotted pink line) separates the space into two regions, one for each recommended action (safe on top, risky on the bottom). In each region, the human decision-maker decides according to a different threshold, choosing the risky action on the left and the safe one on the right.

When the decision-maker anticipates additional loss \( \Delta_{\text{II}} > 0 \) from mistakes when choosing the risky option against a safe recommendation, they choose a more stringent threshold when the safe action is recommended (top area). As a result, the safe action is chosen inefficiently often (blue triangle in the top half).

Figure 2: Comparison of machine-assisted decisions without recommendation dependence (left) and with recommendation dependence (right) for Example 1.

\( \Delta_{\text{II}} > 0 \) whenever they take a risky decision \( A = \text{risky} \) against a safe recommendation \( R = \text{safe} \) when that decision turns out to be a mistake, that is, when \( Y = \text{bad} \) is realized. (We set \( \Delta_I = 0 \) for simplicity.) Recommendation dependence creates a misalignment between human decisions and the preferences of the principal whenever the recommendation \( R = \text{safe} \) is given, leading to an over-adherence to that recommendation. The decision-maker is unaffected when \( R = \text{risky} \) is recommended, but observes an increased (perceived) cost of a Type-II error to \( c_{\text{II}} + \Delta_{\text{II}} \) when \( R = \text{safe} \), leading to decisions

\[
A = \begin{cases} 
\text{risky}, & H \leq \frac{c_I + c_{\text{II}}/2}{c_I + c_{\text{II}}} \\
\text{safe}, & H > \frac{c_I + c_{\text{II}}/2}{c_I + c_{\text{II}}}
\end{cases} \quad \text{for } R = \text{risky},
\]

\[
A = \begin{cases} 
\text{risky}, & H \leq \frac{c_I/2}{c_I + c_{\text{II}} + \Delta_{\text{II}}} \\
\text{safe}, & H > \frac{c_I/2}{c_I + c_{\text{II}} + \Delta_{\text{II}}}
\end{cases} \quad \text{for } R = \text{safe}.
\]

These decisions are represented in Panel (b) of Figure 2. The decision-maker now acts overly cautious in the face of a safe recommendation and overall does not choose the risky option often enough, leading to decisions that are inefficient from the perspective of the principal’s loss.

A first consequence of recommendation-dependent preferences is that a decision-maker who exhibits larger recommendation dependence (larger \( \Delta_I, \Delta_{\text{II}} \)) will follow the recommended actions relatively more. Specifically, when the risky option is recommended and \( \Delta_I > 0 \), then the decision-
maker is less likely to take the safe option, while the opposite is true when the safe option is recommended and $\Delta_{II} > 0$, relative of a reference case of no recommendation dependence ($\Delta_I = 0 = \Delta_{II}$). In the extreme case where the additional losses $\Delta_I, \Delta_{II}$ are sufficiently large, the agent avoids any errors that go against the machine recommendation altogether, and only fails to comply with the recommendation when they are sure what the optimal action is. These points are formalized in the following proposition.

**Proposition 1** (Recommendation dependence increases adherence). *Holding the recommendation policy $R = r(X, M)$ fixed, the probabilities $P(A=R|R=risky)$ and $P(A=R|R=safe)$ of adherence to the recommendation are (weakly) increasing in $\Delta_I$ and $\Delta_{II}$, respectively. Furthermore, as $\Delta_I, \Delta_{II} \to \infty$, $P(A \neq R, \ell(Y, A) > 0) \to 0$.*

A second consequence of recommendation dependence, driven directly by increased adherence, is that the principal’s loss increases. Indeed, from the perspective of the principal’s (welfare) loss, recommendation dependence creates over-adherence that is unambiguously inefficient, since it destroys valuable private information of the human decision-maker. The degree of this inefficiency generally depends on the strength of the signal available to the algorithm and the decision-maker for predicting the label of interest, with recommendation dependence having a larger effect for harder (more noisy) decisions. As one extreme, consider the case where the human decision-maker, after observing the recommendation, is sure about the label and can take the oracle action. In this case, there is no chance of an error, so the additional recommendation-dependent decision loss does not affect choices. On the other hand, if the probability of errors is large no matter the choice, then recommendation dependence may have an outsize effect on choices by making alignment with the algorithm the main driver of the decision. In the extreme case, giving a recommendation can be worse than not giving a recommendation at all. These points are formalized in the following proposition.

**Proposition 2** (Loss from over-adherence). *The principal’s expected loss $E[\ell(Y, A)]$ is (weakly) increasing in both $\Delta_I$ and $\Delta_{II}$. Furthermore, providing a recommendation can be worse than not providing a recommendation at all, that is, there are settings for which loss $E[\ell(Y, A)]$ is higher for any recommendation policy than the loss $E[\min_a E[\ell(Y, a)|X, H]]$ without any machine assistance.*

The latter result stands in contrast to the case with recommendation-independent preference ($\Delta_I = 0 = \Delta_{II}$), in which case any (correctly interpreted) recommendation (weakly) improves loss because the human decision-maker uses the additional information available through the recommendation $R = r(X, M)$ efficiently.

## 4 Implications for the Design of Recommendations

In the previous section, we argued that recommendation dependence may lead to inefficient choices because of over-adherence to the recommended action. In this section, we consider how better
design of recommendations can improve outcomes, where we assume that recommendations are an explicit choice of the algorithm designer. We first argue that optimal recommendation thresholds should be responsive to the nature and level of recommendation dependence. In Section 5, we then show how the addition of a third “don’t know” neutral recommendation level can improve outcomes in the presence of reference effects.

Throughout this section, we assume that the only information the human decision-maker receives from the machine is the binary recommendation, such as when an algorithm’s risk assessment of a defendant is summarized by a simple “jail” or “don’t jail” recommendation to the judge. In Section 6, we then consider the case where the human decision-maker receives a full risk score from the algorithm and recommendations are derived implicitly, such as when a doctor receives a probability estimate for a medical condition and associates with it an implicit recommendation to test (for a high risk score) or not to test (when the predicted probability is low).

While the results in previous sections hold irrespective of the joint distribution of outcomes, context, human signals, and machine signals, we now assume additional structure in order to solve for optimal recommendations in the principal–agent game introduced in Section 2. First, we assume that human and machine signals are unrelated to each other once we condition on the jointly known context.

Assumption 1 (Conditionally independent signals). Conditional on the context $X$, the human signal $H$ and the machine signal $M$ are independent.

We see this assumption as related to the definition of the joint signal $X$ itself; if $H$ and $M$ were related to each other conditional on $X$, this would mean that there would be additional joint information in these signals not already captured in $X$. Here, we assume instead that all common information is captured in $X$, and any additional information is independent. Next, we assume that, conditional on the context $X$, the private information of human and machine can each be summarized by a scalar-valued index.

Assumption 2 (Scalar index representation). There are measurable scalar-valued functions $f, f_H, f_M$ such that a.s. $P(Y=\text{bad}|X,H,M) = f(f_H(H;X), f_M(M;X); X)$.

This assumption means that the signals $X, f_H(H;X), f_M(M;X)$ are sufficient statistics for $Y$. This assumption allows us to express optimal strategies of the principal and the agent in terms of these simple indices only. Finally, we restrict the relationship of these two indices and the probability of a bad outcome to be monotonic, meaning that a larger value of the index corresponds to a larger probability of the bad outcomes.

Assumption 3 (Monotonicity). The function $f(\cdot;\cdot;X)$ is monotonically increasing in both arguments, given $X$.  

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This assumption allows us to relate the ordinal information in the indices to a ranking of probabilities. Together, these three assumptions imply that both optimal decision and optimal recommendations can be written as threshold rules, conditional on the context \( X \). We start with a general result on optimal decisions given the recommendation algorithm, where for simplicity we continue to resolve ties in favor of the risky decision.

**Proposition 3** (Threshold decisions). Under Assumptions 1–3, and given any recommendation policy \( R = r(X, M) \), the agent’s optimal decision is almost surely equal to

\[
A = \begin{cases} 
\text{risky}, & P(Y=\text{bad}|X, H) \leq h^R(X), \\
\text{safe}, & P(Y=\text{bad}|X, H) > h^R(X)
\end{cases}
\]

for some threshold functions \( h^\text{risky}(X) \) and \( h^\text{safe}(X) \) that vary only with the context \( X \).

This results says that the human decision after receiving a recommendation has a similar structure to unassisted decisions: the agent compares the best prediction of the bad outcome occurring using their information \((X, H)\), and takes the risky decision only if that probability is low. However, the probability threshold to decide between risky and safe actions now depends on the recommendation \( R \) (as well as the context \( X \), which may be required to interpret the recommendation). This is in contrast to the unassisted case, for which the threshold is simply \( p^* = \frac{c_I}{c_I + c_{II}} \).

While the above representation holds for any recommendation policy, we now specifically consider recommendations that can similarly be written as a threshold rule of the best machine prediction \( P(Y=\text{bad}|X, M) \), that is,

\[
R = \begin{cases} 
\text{risky}, & P(Y=\text{bad}|X, M) \leq m(X), \\
\text{safe}, & P(Y=\text{bad}|X, M) > m(X).
\end{cases}
\]

(5)

The class of these recommendations includes recommending the decision that the algorithm would take, in which case the threshold would simply be \( p^* = \frac{c_I}{c_I + c_{II}} \). As a consequence, we can describe recommendation algorithms and resulting decisions in terms of the thresholds they imply on \( P(Y=\text{bad}|X, H) \) and \( P(Y=\text{bad}|X, M) \), respectively. Before describing some general properties of optimal human and machine thresholds, we return to our simple example of independent uniform signals, which fulfills the above assumptions.

**Example 1** (Independent uniform signals, continuing from p. 8). We now consider how thresholds should be optimally set in the example, which follows Assumptions 1–3 with \( f_H(H) = H, f_M(M) = \)

\[1\]While such threshold rules are optimal for decisions, they are not generally optimal for recommendations, and we may theoretically be able to do better by allowing for more complex mapping between machine information and recommendation. However, we think that simple threshold rules are realistic restrictions in many cases and may be better understood by a human decision-maker than more complex rules. We therefore focus on optimal thresholds. Solving for optimal recommendation rules more generally (under realistic transparency restrictions) could be a promising direction for future research.
M, and \( f(h,m) = 1(h + m \geq 1) \) (without a context \( X \)). In this example, \( P(Y=\text{bad}|H) = H \) and \( P(Y=\text{bad}|M) = M \). Following (5), for thresholds \( m \in [0,1] \) we now consider algorithms of the form

\[
R = \begin{cases} 
\text{risky}, & M \leq m, \\
\text{safe}, & M > m,
\end{cases}
\tag{6}
\]

to which an optimal agent response for \( \Delta_I = 0 \leq \Delta_{II} \) is

\[
A = \begin{cases} 
\text{risky}, & H \leq \frac{c_I + (1-m)c_{II}}{c_I + c_{II}} \text{ for } R = \text{risky}, \\
\text{safe}, & H > \frac{c_I + (1-m)c_{II}}{c_I + c_{II}} \text{ for } R = \text{safe}.
\end{cases}
\]

We first note that these thresholds are generally above (risky recommendation) and below (safe recommendation) the threshold \( \frac{c_I}{c_I + c_{II}} \) that the agent would choose without obtaining a recommendation. Also, the threshold for the safe recommendation depends on the degree of recommendation dependence, \( \Delta_{II} \), in which it is monotonically decreasing, as well as the recommendation threshold \( m \), in which it is monotonically increasing. This means that the agent will avoid the risky action when receiving a safe recommendation, especially when the safe recommendation is given less often.

We now turn to different choices of the principal’s threshold. When there is no recommendation dependence, \( \Delta_{II} = 0 \), the optimal choice of threshold is \( m^* = \frac{1}{2} \), which is already different from the optimal threshold \( \frac{c_I}{c_I + c_{II}} \) of an algorithm that took the decision directly, rather than merely providing a recommendation. Hence, the optimal algorithmic recommendation is not the same as the optimal algorithmic decision. The optimal threshold \( m^*_{\Delta_I,\Delta_{II}} \) that minimizes the expected loss of the principal generally depends on the degree of recommendation dependence. In particular, with a positive degree of recommendation dependence, \( \Delta_{II} > 0 \), decisions in the region with safe recommendation are inefficient from the perspective of the principal, and the safe action is chosen too often overall. The principal therefore optimally shifts the threshold towards giving the safe recommendation less (Panel (a) of Figure 3), thereby reducing the probability of ending up in the region with inefficient decisions. As a response, the agent slightly adjusts their threshold towards taking the risky action less often in both recommendation regimes (Panel (b) of Figure 3), but overall efficiency still increases from the perspective of the principal.

The example demonstrates the following implications of recommendation dependence: First, recommendation dependence increases adherence to the recommendation, which reduces the information revealed by the decision-maker when a distortionary recommendation (here, the safe recommendation) is given. Second, in response to this distortion, the optimal recommendation that takes recommendation dependence into account differs from the optimal recommendation without recommendation dependence, and suggests the relatively more distortionary recommendation less often. Finally, these effects get stronger the stronger the recommendation dependence. We now show that the insights from the example generalize to other cases for which the above assumptions
(a) The machine shifts its threshold in order to reduce the probability of the region with misaligned decision losses.

(b) In response the decision-maker becomes more likely to take the risky decision in both recommendation regions.

Figure 3: Optimal decision thresholds are adjusted in response to recommendation dependence. The dotted pink lines show the thresholds in Figure 2, while the arrows depict the optimal change in machine threshold (left) and resulting adjustment of conditional decision-maker choices (right).

Proposition 4 (Optimal agent thresholds). Assume that Assumptions 1–3 hold, and that the principal’s threshold policy $m(X)$ is optimal. Then, for any $\Delta_I, \Delta_{II} \geq 0$, we can choose thresholds in Proposition 3 such that

$$h_{\text{risky}}(X) \geq p^* \geq h_{\text{safe}}(X)$$

where we note that $p^* = \frac{c_I}{c_I + c_{II}}$ is the threshold the agent could choose if they chose an action directly, without a recommendation.

Next, we consider how the optimal policy of the agent changes as the degree of recommendation dependence changes. As in the example, we find that an increasing level of recommendation dependence leads to thresholds that make the recommended action more likely to be taken. Also, decreasing the threshold of the algorithm means that the agent thresholds both increase to compensate for a lower implied probability of the bad outcome occurring.

Proposition 5 (Change in optimal agent thresholds). Assume that Assumptions 1–3 hold. Then we can choose thresholds in Proposition 3 across values of $\Delta_I, \Delta_{II} \geq 0$ such that:

1. Assuming the principal follows threshold policy as in (5) with some fixed threshold $m(X)$ that only depends on $X$, then $h_{\text{risky}}(X)$ can be chosen such that it (weakly) increases in $\Delta_I$ and
We now turn to changes in the optimal algorithmic recommendation itself. A natural starting
point for giving recommendations is to have the algorithm recommend the optimal action it would
take if it were to make the decision itself. However, as the example above shows, this optimal
decision would not generally correspond to an optimal recommendation. Furthermore, the optimal
recommendation itself depends on the degree of recommendation dependence.

**Proposition 6** (Optimal algorithmic decision vs optimal algorithmic recommendation). The opti-
minal threshold \( m^*_{\Delta_I, \Delta_{II}}(X) \) for (5) is not generally the same as \( p^* = \frac{c_I}{c_I + c_{II}} \), which is the threshold in
(5) that leads to a loss-minimizing decision of the algorithm if the algorithm were to be implemented
directly. Furthermore, the optimal threshold \( m^*_{\Delta_I, \Delta_{II}}(X) \) generally depends on \( \Delta_I, \Delta_{II} \).

The result that optimal decisions are not the same as optimal recommendations relates to
Andrews and Shapiro (2021), which shows that optimal statistics given to decision-makers with
private information or varying priors are different from optimal statistical decisions. The result also
relates to recent research by Grand-Clément and Pauphilet (2022) showing that optimal advice is
not the same as best decisions in a Markov decision process.

As the main result of this section, we now consider how the optimal threshold of the algorithm
itself depends on the level of recommendation dependence. In order to simplify the derivation of
some of these comparative statics, we make the additional assumption that human and machine in-
formation are continuously distributed with full support, that the function \( f(\cdot, \cdot; X) \) is continuously
differentiable and strictly positively increasing, and that the optimal threshold in the reference-
indepedent case is unique with well-behaved expected loss around the optimum.

**Assumption 4** (Continuously distributed signals and differentiable outcome probabilities). Con-
ditional on the context \( X \), \( f_H(H; X) \) and \( f_M(M; X) \) are a.s. continuously distributed on \( \mathbb{R} \) (that is,
their measures are absolutely continuous with respect to Lebesgue measure) with positive density,
and \( f(\cdot, \cdot; X) \) is continuously differentiable and strictly monotonically increasing, given \( X \).
Furthermore, almost surely we have that the optimal threshold \( m^*(X) = \arg \min_m E[\ell(Y, A)|X] \) for the
reference-independent case \( \Delta_I = 0 = \Delta_{II} \) is unique with \( \frac{\partial^2}{\partial^2 m} E[\ell(Y, A)|X]\bigg|_{m=m^*(X)} > 0. \)

As suggested by the example, we would generally expect that the optimal threshold \( m^*(X) \)
decreases in \( \Delta_I \) and increases in \( \Delta_{II} \), that is, the recommendation to which the agent adheres
too much should be given less. While there are pathological cases in which the comparative statics
can move in the opposite direction, that statement holds under regularity assumptions in a
neighborhood around the benchmark \( \Delta_I = 0 = \Delta_{II} \) without recommendation dependence.
Proposition 7 (Threshold monotonicity). Assume that Assumptions 1–4 hold. Then the optimal threshold $m^*_{\Delta_I, \Delta_{II}}(X)$ is almost surely continuously differentiable for small $\Delta_I, \Delta_{II} \geq 0$, with $rac{\partial}{\partial \Delta_I} m^*_{\Delta_I, \Delta_{II}}(X) < 0$ and $rac{\partial}{\partial \Delta_{II}} m^*_{\Delta_I, \Delta_{II}}(X) > 0$.

In particular, increasing the decision loss when the bad outcome materializes leads to a recommendation that is more likely to recommend the risky decision. The reason is that increased recommendation dependence in the case of a safe recommendation (higher $\Delta_{II}$) means that the decision-maker does not make the risky decision enough. As an optimal response, the algorithm recommends the safe action less, thereby shifting away from the inefficient decision region.

5 The Value of Strategic Non-Recommendations

Above, we have shown that recommendation dependence introduces inefficiencies that make the value of the recommendation ambiguous and affect its optimal design. When recommendations distort choices, one solution is to strategically withhold recommendations in cases where the decision-maker knows better which decisions to take. Shashikumar et al. (2021) propose training a recommendation algorithm to return an “I don’t know” response and apply the idea in the context of sepsis prediction. Within our formal model, we capture this approach by considering recommendations of the type

$$R = r(X, M) \in \{\text{risky, neutral, safe}\}$$ \hspace{1cm} (7)

Such a recommendation structure relaxes the restriction that the provided information is binary to allow for three levels, so we would expect it to improve outcomes even in a model without recommendation dependence. However, with recommendation dependence, there can be an additional gain: if there is no additional cost from mistakes in the neutral case, then allowing for this third level also reduces the cost from recommendation dependence. As a consequence, providing strategic non-recommendations can have a strictly higher benefit in our model relative to a rational baseline, as we illustrate in an application to Example 1.

Example 1 (Independent uniform signals, continuing from p. 8). In the example with uniform independent signals $H$ and $M$, consider algorithmic recommendations

$$R = \begin{cases} \text{risky,} & M \leq m^\downarrow, \\ \text{neutral,} & m^\downarrow < M \leq m^\uparrow, \\ \text{safe,} & M > m^\uparrow. \end{cases}$$

with thresholds $0 \leq m^\downarrow \leq m^\uparrow \leq 1$. Without considering recommendation dependence, adding a neutral option improves decisions by increasing the amount of information about the machine signal $M$ preserved in the recommendation $R$. In the baseline case without recommendation dependence, the
Machine would optimally provide recommendations based on thresholds \( m^\dagger = 1/3, m^\rceil = 2/3 \), equally dividing the signal space in order to maximize the amount of information in the recommendation. Recommendation dependence changes optimal recommendations by reducing the frequency of situations in which the safe recommendation is given, as this recommendation distorts decisions. Thus, both thresholds will increase, as we visualize in Figure 4. The resulting reduction in expected loss is larger than in the case without recommendation dependence.

![Figure 4: Incorporating a neutral recommendation provides additional information to the human decision-maker, while also limiting the region in which recommendation dependence distorts choices.](image)

Having discussed the effect of an additional recommendation option in the example, we now show that adding a third option can achieve combined human–machine decisions that are at least as good (from the principal’s perspective) as implementing either human or machine decisions alone, without having to impose any substantial assumptions.

**Proposition 8** (Human–machine complementarity). Assume that recommendations take the form from (7), where the neutral recommendation does not imply any additional decision loss. Then there are recommendation policies \( R = \tau(X, M) \) such that the expected loss (weakly) improves over both machine-only and human-only decisions, that is,

\[
E[\ell(Y, A)] \leq \min \left( E[\min_a E[\ell(Y, a)|X, M]], E[\min_a E[\ell(Y, a)|X, H]] \right).
\]

We finish this discussion by considering the design of recommendations when a third option is
available. We again invoke our assumptions from Section 4 and consider machine recommendations

\[ R = r(X, M) = \begin{cases} 
  \text{risky}, & P(Y=\text{bad}|X, M) \leq m^\downarrow(X), \\
  \text{neutral}, & m^\downarrow(X) < P(Y=\text{bad}|X, M) \leq m^\uparrow(X), \\
  \text{safe}, & P(Y=\text{bad}|X, M) > m^\uparrow(X)
\end{cases} \]  

(8)

based on simple thresholds on the machine prediction. We note that the complementarity result from Proposition 8 still applies if we restrict recommendation to take this form. As in the case of simple binary recommendations, optimal thresholds are still monotonic in the strength of recommendation dependence in a neighborhood around the benchmark case without recommendation dependence, under the same assumptions.

**Proposition 9** (Threshold monotonicity with non-recommendation). Assume that Assumptions 1–4 hold.\(^2\) Then the optimal thresholds \(m^\downarrow(X, \Delta_I, \Delta_{II}), m^\uparrow(X, \Delta_I, \Delta_{II})\) are almost surely continuously differentiable for small \(\Delta_I, \Delta_{II} \geq 0\), with \(\frac{\partial}{\partial \Delta_I} m^\downarrow(X, \Delta_I, \Delta_{II}) < 0\) and \(\frac{\partial}{\partial \Delta_{II}} m^\uparrow(X, \Delta_I, \Delta_{II}) > 0\).

6 Implicit Recommendations and Strategic Silence

So far we have considered the explicit design of recommendations, where the only information the decision-maker receives from the algorithm is a discrete recommendation that explicitly suggests a course of action. Yet in many applications, the human decision-maker may get access to a full risk score provided by the algorithm. In this section, we therefore extend our model to assume that the information available to the decision-maker consists of the context \(X\), their private signal \(H\), and a continuous machine prediction \(c_M \in [0, 1]\) of the bad state occurring, such as the prediction \(\widehat{M} = P(Y=\text{bad}|X, M)\). For example, a judge may receive an algorithmic prediction of a defendant committing a crime or failing to appear, and a doctor may obtain a risk score that expresses the probability that a patient has some medical condition.

In this framework where the algorithm provides a continuous probability score, we then consider the consequences of recommendation-dependent preferences when recommendations are associated with the machine risk assessment \(\widehat{M}\). Such a recommendation may be explicit, such as when a judge obtains a probability score along with an explicit recommendation based on a probability threshold. Alternatively, the recommendation could be implicit, for example when a doctor interprets a high-risk assessment as a recommendation to test. The former case could be captured by our model of explicit recommendations by assuming that the machine assessment becomes part of the context \(X\) available to the decision-maker. But in that case, our above results suggest that it is optimal from the perspective of the principal not to add any explicit recommendations, as they only distort

\(^2\)Here, we interpret the assumption on the second derivative of the expected loss function in Assumption 4 to mean that the Hessian matrix at the unique optimal thresholds \(m^\downarrow(X), m^\uparrow(X)\) without recommendation dependence is positive definite.
decisions. Here, we instead focus on the latter case, where recommendation dependence is relative to the recommendation implicit to the machine’s risk score.

We consider a specific form of additional decision loss related to implicit recommendations that generalizes the setup from Section 2. Specifically, we assume that the decision-maker anticipates decision loss

\[
\ell^*(Y, A, c_M) = \ell(Y, A) + \begin{cases}
\delta_I(M), & Y=\text{good}, A=\text{safe} \\
\delta_{II}(M), & Y=\text{bad}, A=\text{risky}
\end{cases}
\]

when given the probability assessment \( c_M \in [0, 1] \). Here, \( \delta_I(M) \) and \( \delta_{II}(M) \) represent additional (perceived) losses that come from reference effects through the risk assessment \( M \) when the decision-maker makes an error. We assume that these additional losses are larger the less likely the chosen action is according to the risk score (and are zero if the risk score implies that the chosen action is optimal):

**Assumption 5.** The additional loss functions \( \delta_I, \delta_{II} : [0, 1] \rightarrow [0, \infty) \) fulfil \( \delta_I(1) = 0 = \delta_{II}(0) \) with \( \delta_I \) monotonically decreasing and \( \delta_{II} \) monotonically increasing.

For example, we could recover losses similar to Section 2 if we assume that \( \delta_I, \delta_{II} \) express recommendation dependence relative to the implied machine decision \( A = \text{risky} \) for \( M < p^* = \frac{c_I}{c_I + c_{II}} \) and \( A = \text{safe} \) for \( M > p^* \), in which case

\[
\delta_I(M) = \Delta_I \mathbb{1}(M < p^*), \quad \delta_{II}(M) = \Delta_{II} \mathbb{1}(M > p^*). \tag{9}
\]

In contrast to previous sections, the setup above also allows the magnitude of the predicted probability to matter for reference effects. For example, if we choose

\[
\delta_I(M) = \Delta_I (1 - M), \quad \delta_{II}(M) = \Delta_{II} M \tag{10}
\]

then the additional cost is proportional to the predicted probability of the corresponding adverse outcome: if the probability assessment suggests a high probability of the bad action occurring, then the cost of taking the risky action and encountering a bad outcome is higher than if the prediction suggests a low probability of the bad action. As in Section 3, recommendation dependence implies inefficient decisions since the decision-maker follows the (implicit) recommendations too much. Specifically, if \( \Delta_{II} \) is large and the machine prediction suggests a substantial probability of the bad outcome occurring, then the decision-maker will choose the safe action too often.

The case of recommendation-dependent preferences with binary recommendations \( R \in \{ \text{risky, safe} \} \) can be seen as a special case with \( \widehat{M} = 1 \) corresponding to \( R = \text{safe} \) and \( \widehat{M} = 0 \) corresponding to \( R = \text{risky} \). Similarly, we could identify the third (neutral) option with the probability prediction \( \widehat{M} = p^* \) that signals indifference between risky and safe option, and does not induce any recom-
mendation dependence in either of the above specifications. Before considering similar remedies to recommendation dependence in the general model with continuous machine predictions, we first illustrate potential inefficiencies in an example.

**Example 2** (Independent signal with symmetric losses). As in Example 1, we consider private signals $H$ and $M$ that are drawn independently from a uniform distribution on $[0, 1]$. But unlike in Example 1, we now assume $Y$ is stochastic conditional on $H$ and $M$,

$$P(Y=\text{bad} | M, H) = \frac{M + H}{2}.$$  

The probability of the bad outcome occurring is illustrated in Panel (a) of Figure 5.

Assuming symmetric error costs $c_I = 1 = c_{II}$, the optimal decision given both signals $M$ and $H$ is $A = \text{risky}$ if $H + M \leq 1$ and $A = \text{safe}$ otherwise. This optimal decision is illustrated in Panel (b) of Figure 5. In this example, the machine prediction of the bad outcomes is $\hat{M} = P(Y=\text{bad} | M) = \frac{1 + 2M}{4}$. Since the machine signal $M$ can be recovered from the machine prediction $\hat{M}$, a human decision-maker without recommendation dependence takes the optimal decision. With recommendation-dependent preferences as in (9), the human decision-maker instead chooses

$$A = \begin{cases} \text{risky}, & H \leq 1 - M - \frac{\Delta_{II}}{2 + \Delta_{II}} 1(M > 1/2), \\ \text{safe}, & H > 1 - M - \frac{\Delta_{II}}{2 + \Delta_{II}} 1(M > 1/2), \end{cases}$$

which is illustrated in Panel (c) of Figure 5. From the perspective of the principal, this choice creates inefficiencies where the risky decision is taken too little, especially for high values of $\Delta_{II}$.

Despite the decision-maker now having access to a continuous algorithmic risk assessment, recommendation-dependent preferences still lead to inefficient choices because of over-adherence to the recommendation implicit to the probability assessment and can lead to outcomes that are worse than a decision-maker deciding by themselves without any risk score or recommendation. The results from Section 3 still apply.

When recommendations are directly tied to machine predictions, we may not be able to change recommendations explicitly as we discussed in Section 4. Instead, we consider in this section the merits of withholding the machine risk prediction $P(Y=\text{bad} | X, M)$ itself in order to reduce distortions through recommendation dependence in return for a loss of information. Specifically, we assume that the machine assessment is now given by

$$\hat{M} = \begin{cases} P(Y=\text{bad} | X, M), & P(Y=\text{bad} | X, M) \notin [p^I(X), p^I(X)], \\ \text{withheld}, & P(Y=\text{bad} | X, M) \in [p^I(X), p^I(X)]. \end{cases}$$  \hspace{1cm} (11)$$

That is, the algorithm withholds a score when it is intermediate (and thus may have limited helpful
(a) The square represents the uniform distribution over signals $H$ and $M$, with the lines illustrating the probability $P(Y=\text{bad}|H,M)$ at different levels.

(b) The optimal decision from knowing both signals $H$ and $M$ (as well as the decision taken by a machine-assisted human decision-maker without recommendation dependence) is to take the risky action in the lower left quadrant where the outcome is more likely to be good than bad, and the safe action otherwise.

(c) Recommendation-dependence with $\Delta_{II} > 0$ leads to excess safe action, which in turn produces excess loss from Type-II errors (dark blue region) from the perspective of the principal.

(d) Strategically withholding predictions around $M = 1/2$ reduces the region in which recommendation dependence distorts decisions, and improves expected loss for the principal.

Figure 5: Distribution of outcome (top left), optimal decision (top right), recommendation-dependent decision (bottom left), and recommendation-dependent decision with withheld machine prediction (bottom right) in Example 2.
information about the optimal action).\(^3\) In order to fit such recommendations within our setup of recommendation dependence, we assume that the decision-maker interprets the withheld recommendation as a risk assessment \(\hat{M} = P(Y=\text{bad}|X, M) \in [p^I(X), p^O(X)]\).\(^4\) The risk assessment \(\hat{M}\) thus represents a coarsening of the full prediction \(P(Y=\text{bad}|X, M)\) that loses information about variations in risk scores between \(p^I(X)\) and \(p^O(X)\). Despite losing information, withholding information strategically in this way can improve outcomes in the presence of recommendation dependence, as we first demonstrate in the example before stating general results.

**Example 2** (continuing from p. 21). In the example, consider the risk score

\[
\hat{M} = \begin{cases}
    P(Y=\text{bad}|M), & M \notin [\frac{1}{2}-\epsilon, \frac{1}{2}+\epsilon], \\
    \text{withheld}, & M \in [\frac{1}{2}-\epsilon, \frac{1}{2}+\epsilon],
\end{cases}
\]

where \(P(Y=\text{bad}|M) = \frac{1+2M}{4}\) and we assume that the agent interprets the withheld risk score as \(\hat{M} = P(Y=\text{bad}|M \in [\frac{1}{2}-\epsilon, \frac{1}{2}+\epsilon]) = \frac{1}{2}\). As a consequence, there is no recommendation dependence when the score is withheld, and the decision-maker takes actions

\[
A = \begin{cases}
    \text{risky}, & H \leq \frac{1 - M - \frac{\Delta I}{2\Delta II}}{1/2} \mathbb{1}(M > 1/2) \quad M \notin [\frac{1}{2}-\epsilon, \frac{1}{2}+\epsilon] \\
    \text{safe}, & H > \frac{1 - M - \frac{\Delta I}{2\Delta II}}{1/2} \mathbb{1}(M > 1/2) \quad M \notin [\frac{1}{2}-\epsilon, \frac{1}{2}+\epsilon],
\end{cases}
\]

Withholding information around \(M = \frac{1}{2}\) eliminates recommendation dependence for \(M \in [\frac{1}{2}-\epsilon, \frac{1}{2}]\) (although decisions are still not first best), while also leading to inefficient decisions for \(M \in (\frac{1}{2}, \frac{1}{2}+\epsilon]\). For small \(\epsilon\), the gain from reducing recommendation dependence outweighs the cost from withholding information.

Having discussed the idea that withholding the score strategically can improve outcomes, note that an analog of Proposition 8 holds for the case of continuous risk scores. Specifically, we now provide conditions under which we can find a scoring rule of the form (11) that always (weakly) improves over machine-only and human-only decisions. In order to formulate our result, we call a risk value \(\hat{m} \in [0, 1]\) recommendation-neutral if it does not imply any recommendation dependence, that is, if \(\delta_I(\hat{m}) = \delta_{II}(\hat{m}) = \frac{\delta_I(\hat{m})}{c_I} = \frac{\delta_{II}(\hat{m})}{c_{II}}\). For example, \(\hat{m} = p^I = \frac{c_I}{c_I+c_{II}}\) is recommendation-neutral for the specifications (9) and (10).

---

\(^3\)As previously, such simple threshold rules are not necessarily optimal. However, we believe that more complex policies may not be understood by human decision-makers, and that such threshold rules represent a natural starting point.

\(^4\)We could alternatively assume that there is no recommendation dependence when the risk prediction is withheld, but this assumption may be unrealistic when a withheld risk score signals a particularly high or low risk score.
Proposition 10 (Human–machine complementarity from destroying information). Assume that risk scores take the form (11) and that Assumption 5 holds. If \( p^* = \frac{c_I}{c_I + c_{II}} \) is recommendation-neutral, then there is a risk score of the form (11) (with \( p^I(X) \leq p^* \leq p^V(X) \)) that (weakly) improves over the best machine-only decision,

\[
E[\ell(Y, A)] \leq E[\min_a E[\ell(Y, a)|X, M]].
\]

If \( P(Y|X) \) is recommendation-neutral almost surely, then there is a risk score of the form (11) that (weakly) improves over the best human-only decision,

\[
E[\ell(Y, A)] \leq E[\min_a E[\ell(Y, a)|X, H]].
\]

This approach adapts the idea of Bayesian persuasion (Kamenica and Gentzkow, 2011) to our context: by changing the structure of the information and coarsening the signal strategically, the designer of the algorithm can improve outcomes through increasing the alignment between their goal and the misaligned choices of the decision-maker. However, unlike the baseline Bayesian persuasion case, the signal structure affects the preferences themselves through the implied recommendations.

We note that unlike the setting from Section 4, where adding a neutral option added information, this modification of the risk assessment strictly decreases the information given by the machine. In the rational baseline of no recommendation dependence (\( \Delta_I = 0 = \Delta_{II} \)) this modification would strictly worsen outcomes. Yet in the recommendation-dependent case, there is room for net improvements through (strategic) silence about the risk score.

7 Foundations of Recommendation Dependence

In the previous sections, we have explored the consequences of recommendation dependence on chosen actions and optimal design of the algorithm. Here, we discuss sources for our model of recommendation-dependent choices, focusing on institutional factors along with established models from behavioral economics. We then mention available empirical evidence that may relate to reference effects from algorithmic recommendations.

7.1 Motivation from institutional factors

In many critical applications, negative outcomes can trigger additional scrutiny and formal audits. When recommendations are part of a decision process, ex-post suboptimal decisions that lead to undesirable outcomes may be seen as particularly problematic when they go against underlying recommendations. Doctors who are found to have caused medical harm with a procedure need to show their actions do not “deviate from accepted norms of practice in the medical community” to avoid a malpractice lawsuit (Bal, 2009). Deviations that lead to bad health outcomes are likely to
draw additional scrutiny in this regard compared with physicians following an algorithmic standard. The specific outcomes of individual trials are not the basis for judge performance evaluations, but the reasoning of the judge’s written opinions are (IAALS, 2022), which would likely address any deviation from recommended practice. Although evaluators are not supposed to consider the outcome when evaluating the judge’s opinion, the evaluators have more information about the outcome than the judge did when they made their opinion. This may taint their perception of the logic used regardless of the conscious intent to do so. Hiring managers will be able to explain the hiring of an underperformed employee more easily if all indications of candidate quality are positive rather than if an algorithm or pre-employment evaluation recommends not hiring the individual.

Institutional constraints may in many cases also predict an asymmetry in the penalty associated with taking ex-post suboptimal decisions that deviate from recommendations. Deviations that lead to safe decisions against risky advice may not be seen as equally problematic as risky decisions against safe advice, even if they are ex-post suboptimal. Doctors who order an extra test that was unnecessary may face some penalty from an insurer but are unlikely to face a large outcry or malpractice lawsuit. A judge who jails a defendant who turns out to be of low risk of committing a new crime or failing to appear may not face scrutiny because the behavior outside jail is never observed. When a good applicant is not hired, there may be limited repercussions for the manager since performance is not visible.

### 7.2 Derivation from reference-dependent preferences with loss aversion

Above, we have considered institutional justifications for considering recommendation dependence. In this section, we instead consider a psychological motivation from behavioral science that similarly yields that recommendations do not only affect decisions through the provision of information, but also by affecting decision utility. Specifically, we derive the specific decision loss $\ell^*$ in (2) from Prospect Theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), which has been one of the most established frameworks for describing systematic deviations from rational utility theory in behavioral economics and its applications (see e.g. Barberis, 2013, for an overview and assessment). Like Kleinberg et al. (2022), we therefore assume that there is a gap between welfare-relevant utility and the decision-maker’s perceived utility when making the decision.

We consider two central tenets of Prospect Theory to the decision-makers choice between safe and risky actions. First, we assume that choices are evaluated relative to a reference point, which we here assume is induced by the action $R$ recommended by the algorithm. This means that the decision-maker evaluates losses relative to the reference loss $\ell(Y, R)$ that they would achieve if they followed the recommendation. The second aspect of Prospect Theory we adopt to our setting is that the decision-maker puts more emphasis on losses relative to the reference point than on gains. Specifically, we assume that loss aversion takes the form of a factor $\lambda > 1$ by which losses are multiplied. This means that decision loss from outcome $Y$ relative to the reference point $\ell(Y, R)$
from taking action $A$ given recommendation $R$ is given by

$$\ell^{PT}(Y, A, R) = \lambda[\ell(Y, A) - \ell(Y, R)]_+ - \left[\ell(Y, A) - \ell(Y, R)\right]_-,$$

where by $[\cdot]_+$ and $[\cdot]_-$ we denote the (absolute value of the) positive and negative parts, respectively. For the specific loss function from (1), the Prospect-Theory loss takes the form of a recommendation-dependent decision loss from (2).

**Proposition 11** (Derivation from Prospect Theory). *Decision-maker choices according to $\ell^{PT}$ are equivalent to choices according to $\ell^*$ with $\Delta_I = (\lambda - 1)c_I, \Delta_{II} = (\lambda - 1)c_{II}$.***

We note that this justification implies additional structure relative to the ad-hoc construction of recommendation-dependent losses above. Specifically, additional costs are larger in the case where the baseline cost of an error is larger. In the canonical case where taking the risky decision in the bad case has higher cost ($c_{II} > c_I$), this model of behavioral decision-making justifies a focus on the case with large $\Delta_{II}$. The derivation extends to the setup in Section 6 where a continuous risk assessment $\widehat{M}$ is given, assuming that recommendation dependence is now relative to the implied recommendation

$$R = \begin{cases} \text{risky,} & \widehat{M} < p^* = \frac{c_I}{c_I + c_{II}}; \\ \text{safe,} & \widehat{M} > p^*. \end{cases}$$

**corresponding to the optimal action of the machine, as in (9).**

### 7.3 Related empirical findings

Many empirical studies of algorithm-assisted human decision-making (see Lai et al. (2021) for a literature review) have made observations that we believe can be related to recommendation dependence. Green and Chen (2019) and Fogliato et al. (2022a) provide evidence for anchoring effects, where adherence to algorithmic recommendations is larger when these are revealed initially rather than after eliciting provisional human judgements. Banker and Khetani (2019) documents cases of over-dependence on algorithmic recommendations across multiple experiments in which algorithmic advice pushes human decision-makers towards making inferior, dominated choices. Fügener et al. (2021) hypothesizes and demonstrates empirically that excess coordination due to algorithmic advice may destroy unique knowledge and reduce performance in an aggregated ‘wisdom of the crowds’ scenario. Albright (2023) isolates the causal effect of recommendations on bail decisions and shows that recommendations affect judges’ risk preferences.

Models of recommendation dependence may also relate to variation in leniency, as our model predicts that decision-makers with less experience or information about specific cases exhibit more recommendation dependence. Similar patterns have been observed in social services (Cheng and Chouldechova, 2022) and healthcare (Kiani et al., 2020). Caro and de Tejada Cuenca (2023)
studies status-quo bias in managers’ price setting, and analyzes how salience affects adherence to algorithmic recommendations. Studies of pretrial safety assessment document systematic ways in which judges adhere – and do not adhere – to recommendations. Imai et al. (2020) finds that release decisions were generally more lenient for women when risk assessment algorithms were used and much stricter for men who the algorithm perceived as risky. Stevenson and Doleac (2019) finds that judges were less willing to accept harsh recommendations for the young and lenient recommendations for older defendants. The authors hypothesize this is due to a long-standing norm in sentencing of treating age as a mitigating factor, pointing to an additional channel that may affect reference points.

8 Discussion of Model Assumptions and Potential Extensions

We close our investigation by briefly mentioning relevant extensions to the baseline models.

8.1 Alignment of baseline preferences

We have assumed throughout that decision-maker and algorithm agree on their costs $c_I$ and $c_{II}$ of making mistakes, and only differ with respect to recommendation-dependent losses of the decision-maker. If the baseline costs $c_I, c_{II}$ are already misaligned, recommendation dependence may improve decisions by increasing adherence to the preferred action of the algorithm designer, even if it comes at the cost of reducing revealed information.

We have also assumed that the additional loss associated with deviating from recommendations only affects the perceived loss of the decision-maker, and not directly the loss of the designer of the algorithm. As an alternative extension to our model, we could also assume that the designer aims to minimize (part of) this additional loss. This modification would change optimal thresholds. In the case where the designer of the algorithm fully incorporates the decision-makers perceived loss, choices are now perfectly aligned, but the additional cost associated with deviations from recommendations means that the loss of the designer is directly affected by costs from recommendations that the decision-maker does not follow through on.

8.2 Simple cost from deviation

In our main model, we assume that there are additional costs $\Delta_I, \Delta_{II}$ affecting the decision loss that only come from (expected) Type-I and Type-II errors (when deviating from the recommendation). Here, we instead consider the case where any deviation from the recommendation is perceived as costly by the decision-maker, no matter whether it leads to errors or not. Assuming that there is a cost (in addition to expected loss $\mathbb{E}[\ell(Y, A)]$ of $d^{\text{risky}}$ of deviating from the risky recommendation $R = \text{risky}$ and $d^{\text{safe}}$ of deviating from the safe recommendation $R = \text{safe}$, the resulting optimal
decision is the same as (3) with thresholds

\[ p^r = \begin{cases} \min \left( \frac{c_I + p^{\text{risky}}}{c_I + c_{II}}, 1 \right), & r = \text{risky}, \\ \max \left( \frac{c_I - p^{\text{safe}}}{c_I + c_{II}}, 0 \right), & r = \text{safe}. \end{cases} \]

Our main results and comparative statics therefore still apply since the costs shift the recommendation-specific thresholds similarly to \( p^r \) from (3). However, relative to our baseline model, there are now cases where the decision-maker may go with the recommendation even when they know with certainty that it leads to an error. While this prediction may appear less realistic for modeling application areas and behavioral effects like those discussed in Section 7, it captures cases where a designer imposes a deviation cost on the decision-maker irrespective of the actual outcome.

### 8.3 Mis-interpretation by the decision-maker

Throughout, we have assumed that the decision-maker is able to interpret recommendations correctly. However, in practice, the decision-maker may have a hypothesis about the recommendation that may not be fully accurate. As an extension, three approaches may be particularly relevant. The first is that the decision-maker assumes that the recommendation is an optimal machine decision. The second approach considers a decision-maker who is naive about their own reference dependence, so they assume that the recommendation is optimal for the case where they do not exhibit recommendation dependence. The third approach would be one where we assume that the decision-maker can only understand a simple representation or explanation of the recommendation.

### 9 Conclusion

When we provide a decision-maker with a recommendation, they may not only react to its information content, but also see it as a default action that affects their preferences. In this article, we illustrate in a simple example and with general results how recommendation-dependent preferences create inefficiencies and affect the design of optimal recommendations. Our model suggests practically implementable modifications that reduce distortions by strategically altering or even withholding recommendations for instances where they may otherwise hurt more than they help.

With our work, we hope to provide an example of the integration of more realistic models of human behavior into the design of algorithms, and hope that it can contribute to improving human–AI interaction in critical applications.

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Proofs

Proof of Proposition 1. We have that

\[ P(A = R | R = \text{risky}) = P(A = \text{risky} | R = \text{risky}) \]
\[ = P\left( P(Y = \text{bad} | X, H, R = \text{risky}) \leq p^{\text{risky}} | R = \text{risky} \right) \]
\[ = P\left( P(Y = \text{bad} | X, H, R = \text{risky}) \leq \frac{c_I + \Delta_I}{c_I + c_{II} + \Delta_I} | R = \text{risky} \right) \]

where \( P(Y = \text{bad} | X, H, R = \text{risky}) \) is unaffected by \( \Delta_I \) and \( \frac{c_I + \Delta_I}{c_I + c_{II} + \Delta_I} \) is monotonically increasing in \( \Delta_I \), which means that \( P(A = R | R = \text{risky}) \) can not decrease as \( \Delta_I \) increases. The result for \( P(A = R | R = \text{safe}) \) follows similarly.

For the second result, assuming that \( c_I, c_{II} > 0 \), we have that

\[ P(A \neq R, \ell(Y, A) > 0) = P(A = \text{risky}, R = \text{safe}, Y = \text{bad}) + P(A = \text{safe}, R = \text{risky}, Y = \text{good}) \]
\[ = P\left( P(Y = \text{bad} | X, H, R = \text{safe}) \leq \frac{c_I}{c_I + c_{II} + \Delta_{II}}, R = \text{safe}, Y = \text{bad} \right) \]
\[ + P\left( P(Y = \text{bad} | X, H, R = \text{risky}) > \frac{c_I + \Delta_I}{c_I + c_{II} + \Delta_I}, R = \text{risky}, Y = \text{good} \right) \]
\[ = P\left( P(Y = \text{bad} | X, H, R = \text{safe}) \leq \frac{c_I}{c_I + c_{II} + \Delta_{II}}, Y = \text{bad} | R = \text{safe} \right) P(R = \text{safe}) \]
\[ + P\left( P(Y = \text{good} | X, H, R = \text{risky}) \leq \frac{c_{II}}{c_I + c_{II} + \Delta_I}, Y = \text{good} | R = \text{risky} \right) P(R = \text{risky}) \]
\[ = E\left[ 1 \left( P(Y = \text{bad} | X, H, R = \text{safe}) \leq \frac{c_I}{c_I + c_{II} + \Delta_{II}} \right) P(Y = \text{bad} | X, H, R = \text{safe}) | R = \text{safe} \right] P(R = \text{safe}) \]
\[ + E\left[ 1 \left( P(Y = \text{good} | X, H, R = \text{risky}) \leq \frac{c_{II}}{c_I + c_{II} + \Delta_I} \right) P(Y = \text{good} | X, H, R = \text{risky}) | R = \text{risky} \right] P(R = \text{risky}) \]
\[ \leq \frac{c_I}{c_I + c_{II} + \Delta_{II}} P(R = \text{safe}) + \frac{c_{II}}{c_I + c_{II} + \Delta_I} P(R = \text{risky}) \rightarrow 0 \]
as \( \Delta_I, \Delta_{II} \rightarrow \infty \).

Proof of Proposition 2. Relative to the optimal agent decision

\[ A^* = \begin{cases} 
\text{risky}, & P(Y = \text{bad} | X, H, R) \leq p^*, \\
\text{safe}, & P(Y = \text{bad} | X, H, R) > p^*,
\end{cases} \]

that minimizes expected loss for the principal (and is not affected by \( \Delta_I, \Delta_{II} \)), the principal expe-
riences additional expected loss

\[ E[\ell(Y, A)] - E[\ell(Y, A^*)] \]

\[ = E \left[ \mathbf{1} \left( \frac{c_I}{c_I + c_{II} + \Delta_{II}} < P(Y=\text{bad}|X, H, R=\text{safe}) \leq \frac{c_I}{c_I + c_{II}} \right) \right] P(R=\text{safe}) \]

\[ + E \left[ \mathbf{1} \left( \frac{c_I}{c_I + c_{II}} < P(Y=\text{bad}|X, H, R=\text{risky}) \leq \frac{c_I + \Delta_{I}}{c_I + c_{II} + \Delta_{I}} \right) \right] \frac{(c_{II} P(Y=\text{bad}|X, H, R=\text{risky}) - c_I P(Y=\text{good}|X, H, R=\text{risky}))}{\geq 0} P(R=\text{risky}) P(R=\text{risky}) \]

(12)

where the indicator functions are picking up more cases as \( \Delta_{I}, \Delta_{II} \) increase, thus increasing the additional expected loss.

For the case where not providing a recommendation can be better, consider the case where \( \Delta_{I}, \Delta_{II} \) are very high and the private machine information is substantially less helpful than the private information available to the agent. In this case, with a recommendation, the agent follows the recommendation closely to take a decision that is only weakly correlated with the optimal decision. Without a recommendation, on the other hand, the agent takes a decision that better tracks the optimal decision.

For the following proofs that rely on Assumptions 1–3, we note that we can consider all statements to be a.s. conditional on \( X \) (and omitting \( X \) in our notation), since principal and agent have access to \( X \) and all policies are allowed to depend on its realization. Furthermore, writing \( \tilde{H} = f_H(H; X) \) and \( \tilde{M} = f_M(M; X) \), we obtain the representation

\[ P(Y=\text{bad}|H, M) = P(Y=\text{bad}|\tilde{H}, \tilde{M}), \quad P(Y=\text{bad}|\tilde{H}=\tilde{h}, \tilde{M}=\tilde{m}) = f(\tilde{h}, \tilde{m}) \]

for \( \tilde{h} \) and \( \tilde{m} \) in the support of \( \tilde{H} \) and \( \tilde{M} \), respectively, with \( f \) monotonically increasing in both (scalar) arguments and \( \tilde{H} \) independent of \( \tilde{M} \) and \( R \). This notation improves the readability of the following proofs, and we maintain it throughout.

**Proof of Proposition 3.** The optimal action is almost surely given by (3) (where ties are broken in favor of the risky decision), so our goal is to show that there are functions \( h^{\text{risky}}, h^{\text{safe}} \) such that for all \( \tilde{h} \) in the support of \( \tilde{H} \) and all \( \tilde{r} \in \{\text{risky}, \text{safe}\} \),

\[ P(Y=\text{bad}|\tilde{H}=\tilde{h}, R=\tilde{r}) \leq p^{\tilde{r}} \quad \iff \quad P(Y=\text{bad}|\tilde{H}=\tilde{h}) \leq h^{\tilde{r}}. \]
Here, we invoke the notation from above this proof, and assume that $P(R = \text{risky}), P(R = \text{safe}) > 0$, since the case where $P(R = \text{risky}) = 0$ or $P(R = \text{safe}) = 0$ is trivial. Note first that almost surely

$$P(Y = \text{bad} | \tilde{H} = \tilde{h}, R = \tilde{r}) = E[f(\tilde{H}, \tilde{M}) | \tilde{H} = \tilde{h}, R = \tilde{r}] = E[f(\tilde{h}, \tilde{M}) | R = \tilde{r}],$$

and the right-hand side is monotonically increasing in $\tilde{h}$ by independence and monotonicity of $f$, and the same holds for

$$P(Y = \text{bad} | \tilde{H} = \tilde{h}) = E[f(\tilde{H}, \tilde{M}) | \tilde{H} = \tilde{h}] = E[f(\tilde{h}, \tilde{M})]$$

$$= E[f(\tilde{h}, \tilde{M}) | R = \text{risky}] P(R = \text{risky}) + E[f(\tilde{h}, \tilde{M}) | R = \text{safe}] P(R = \text{safe})$$

where we have used independence of $\tilde{H}$ and $R$ (a.s. conditional on $X$, which is implicit here). As a consequence, for all $\tilde{h}_1, \tilde{h}_2$ in the support of $\tilde{H}$, all $\tilde{r} \in \{\text{risky, safe}\}$, and all $\varepsilon > 0$,

$$E[f(\tilde{h}_1, \tilde{M}) | R = \tilde{r}] + \varepsilon \leq E[f(\tilde{h}_2, \tilde{M}) | R = \tilde{r}]$$

$$\implies E[f(\tilde{h}_1, \tilde{M})] + \varepsilon P(R = \tilde{r}) \leq E[f(\tilde{h}_2, \tilde{M})]$$

since from the left it also follows that $\tilde{h}_1 < \tilde{h}_2$ and thus $P(Y = \text{bad} | \tilde{H} = \tilde{h}_1, R = \tilde{r}') \leq P(Y = \text{bad} | \tilde{H} = \tilde{h}_2, R = \tilde{r}')$ for the other $\tilde{r}' \neq \tilde{r}$ (while the opposite implication does not generally hold). Let now

$$h^\tilde{r} = \sup_{\tilde{h} \in \text{the support of } \tilde{H}} E[f(\tilde{h}, \tilde{M}) | R = \tilde{r}] \leq p^\tilde{r} E[f(\tilde{h}, \tilde{M})]$$

(13)

where we define the supremum over the empty set as 0. We have that

$$P(Y = \text{bad} | \tilde{H} = \tilde{h}, R = \tilde{r}) \leq p^\tilde{r} \implies P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h^\tilde{r}.$$

by the definition of $h^\tilde{r}$ and

$$P(Y = \text{bad} | \tilde{H} = \tilde{h}, R = \tilde{r}) > p^\tilde{r}$$

$$\implies \exists \varepsilon > 0 : E[f(\tilde{h}, \tilde{M}) | R = \tilde{r}] \geq E[f(\tilde{h}', \tilde{M})] + \varepsilon$$

$$\forall \tilde{h}' \text{ in the support of } \tilde{H} \text{ with } E[f(\tilde{h}', \tilde{M}) | R = \tilde{r}] \leq p^\tilde{r}$$

$$\implies \exists \varepsilon > 0 : E[f(\tilde{h}, \tilde{M})] \geq E[f(\tilde{h}', \tilde{M})] + \varepsilon$$

$$\forall \tilde{h}' \text{ in the support of } \tilde{H} \text{ with } E[f(\tilde{h}', \tilde{M}) | R = \tilde{r}] \leq p^\tilde{r}$$

$$\implies P(Y = \text{bad} | \tilde{H} = \tilde{h}) > h^\tilde{r}.$$
Hence

\[ P(Y = \text{bad} | \tilde{H} = \hat{h}, R = \tilde{r}) \leq p^\hat{r} \quad \iff \quad P(Y = \text{bad} | \tilde{H} = \hat{h}) \leq h^\hat{r}. \]

\[ \square \]

Proof of Proposition 4. We employ the simplified notation above the proof of Proposition 3 (and condition on \( X \) throughout). Consider decision thresholds

\[
\begin{align*}
    h^\text{safe}_0(p, m) &= \sup \hat{h} \text{ in the support of } \tilde{H}; E[f(\hat{h}, \hat{M})] | P(Y = \text{bad} | M) \leq p \leq P(Y = \text{bad} | \tilde{M})], \\
    h^\text{risky}_0(p, m) &= \sup \hat{h} \text{ in the support of } \tilde{H}; E[f(\hat{h}, \hat{M})] | P(Y = \text{bad} | M) > m \leq p \leq P(Y = \text{bad} | \tilde{M})] \end{align*}
\]

(14)

defined by (13) in the proof of Proposition 3 for the threshold recommendations from (5), where the supremum over the empty set is again 1. In addition, define the analogous threshold

\[
h^*_0(p) = \sup \hat{h} \text{ in the support of } \tilde{H}; E[f(\hat{h}, \hat{M})] \leq P(Y = \text{bad} | \tilde{M})] \]

(15)

for decisions that do not use machine input. By the proof Proposition 3, we obtain thresholds with

\[
P(Y = \text{bad} | \tilde{H} = \hat{h}, P(Y = \text{bad} | \tilde{M}) \leq m) \leq p^\text{risky}_0 \quad \iff \quad P(Y = \text{bad} | \tilde{H} = \hat{h}) \leq h^\text{risky}_0(p^\text{risky}, m),
\]

\[
P(Y = \text{bad} | \tilde{H} = \hat{h}, P(Y = \text{bad} | \tilde{M}) > m) \leq p^\text{safe}_0 \quad \iff \quad P(Y = \text{bad} | \tilde{H} = \hat{h}) \leq h^\text{safe}_0(p^\text{safe}, m)
\]

\[
P(Y = \text{bad} | \tilde{H} = \hat{h}) \leq p^* \quad \iff \quad P(Y = \text{bad} | \tilde{H} = \hat{h}) \leq h^*_0(p^*).
\]

By construction, the \( h^\text{safe}_0(p, m), h^\text{risky}_0(p, m), h^*_0(p) \) are monotonically increasing in \( p \). By monotonicity of \( f \) and independence of \( \tilde{H} \) and \( \tilde{M} \), we also have that

\[
P(Y = \text{bad} | \tilde{H} = \hat{h}, P(Y = \text{bad} | \tilde{M}) > m) = E[f(\hat{h}, \hat{M})] | E[f(\tilde{H}, \tilde{M}) | \tilde{M}] > m],
\]

\[
P(Y = \text{bad} | \tilde{H} = \hat{h}, P(Y = \text{bad} | \tilde{M}) \leq m) = E[f(\hat{h}, \hat{M})] | E[f(\tilde{H}, \tilde{M}) | \tilde{M}] \leq m]
\]

are monotonically increasing in \( m \), and \( h^\text{safe}_0(p, m), h^\text{risky}_0(p, m) \) thus monotonically decreasing in \( m \). Finally, \( h^\text{safe}_0(p, 1) = h^*_0(p) \) and \( h^\text{risky}_0(p, 0) \geq h^*_0(p) \). As a consequence, using \( p^\text{risky} \geq p^* \geq p^\text{safe} \),

\[
h^\text{risky}_0(p^\text{risky}, m) \geq h^\text{risky}_0(p^*, m) \geq h^\text{risky}_0(p, 0) \geq h^*_0(p^*) = h^\text{safe}_0(p^*, 1) \geq h^\text{safe}_0(p^*, m) \geq h^\text{safe}_0(p^\text{safe}, m).
\]

As a last step, we now need to transform thresholds \( h^\text{risky}_0(p^\text{risky}, m) \geq h^*_0(p^*) \geq h^\text{safe}_0(p^\text{safe}, m) \) into thresholds \( h^\text{risky}(p^\text{risky}, m) \geq h^*(p^*) \geq h^\text{safe}(p^\text{safe}, m) \) with \( h^*(p^*) = p^* \) (where we note that \( h^*_0(p^*) \leq p^* \), but the inequality can be strict). To this end for \( h \in [0, 1] \) let

\[
h^{-1}(h) = \begin{cases} p^*, & h \leq p^* < p \text{ for all } p \text{ with } h^*_0(p) > h, \\ h, & \text{otherwise.} \end{cases}
\]

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for which \( h^{-1}(h_0^*(p^*)) = p^* \) since \( h_0^*(p^*) \leq p^* \) and whenever \( h_0^*(p) > h_0^*(p^*) \) we must have that \( p > p^* \) by monotonicity of \( h_0^* \).

First, \( h^{-1} \) is monotonically increasing on \([0, 1]\). Indeed, this is straightforward for any \( h_1, h_2 \) that either both fulfill or both do not fulfill the condition in the first line. For the remaining case, assume that \( h_1 \leq p^* < p \) for all \( p \) with \( h_0^*(p) > h_1 \) (which implies \( h^{-1}(h_1) = p^* \)), while \( h_2 > p^* \) or there is some \( p_2 \leq p \) with \( h_0^*(p_2) > h_2 \) (both of which imply \( h^{-1}(h_2) = h_2 \)). If \( h_2 > p^* \) then \( h_2 > p^* \geq h_1 \) and \( h^{-1}(h_2) > p^* = h^{-1}(h_1) \), so monotonicity holds. If \( h_2 \leq p^* \) and such a \( p_2 \) exists then we must have \( h_0^*(p_2) \leq h_1 \); with \( h_0^*(p_2) > h_2 \) this implies \( h_1 > h_2 \) and \( h^{-1}(h_1) = p^* \geq h_2 = h^{-1}(h_2) \), so monotonicity holds again.

Second,

\[
P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h \iff P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h^{-1}(h),
\]

where \( \iff \) follows from \( h^{-1}(h) \geq h \). For \( \iff \) we note that \( h_0^*(h^{-1}(h)) \leq h \), which holds for \( h^{-1}(h) = h \) by \( h_0^*(p) \leq p \) and otherwise since \( h_0^*(p^*) \leq h \) for all \( p \leq p^* \) such that \( h_0^*(p) > h \) implies that \( p > p^* \), since in this case for all \( p \leq p^* \) it follows that \( h_0^*(p) \leq h \). Hence, from \( P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h^{-1}(h) \) we obtain \( P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h_0^*(h^{-1}(h)) \) by the properties of \( h_0^* \) and thus \( P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h \) from \( h_0^*(h^{-1}(h)) \leq h \).

As a consequence of these properties of \( h^{-1} \) along with those of \( h_0^\text{risky}, h^*, h^\text{safe} \), we can define

\[
h^\text{risky}(p^\text{risky}, m) = h^{-1}(h_0^\text{risky}(p^\text{risky}, m)), \quad h^\text{safe}(p^\text{safe}, m) = h^{-1}(h_0^\text{safe}(p^\text{safe}, m))
\]

for which

\[
P(Y = \text{bad} | \tilde{H} = \tilde{h}, P(Y = \text{bad} | \tilde{M} \leq m) \leq p^\text{risky}
\]

\( \iff \) \( P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h_0^\text{risky}(p^\text{risky}, m) \iff P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h^\text{risky}(p^\text{risky}, m), \)

\[
P(Y = \text{bad} | \tilde{H} = \tilde{h}, P(Y = \text{bad} | \tilde{M} > m) \leq p^\text{safe}
\]

\( \iff \) \( P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h_0^\text{safe}(p^\text{safe}, m) \iff P(Y = \text{bad} | \tilde{H} = \tilde{h}) \leq h^\text{safe}(p^\text{safe}, m)
\]

and \( h^\text{risky}(p^\text{risky}, m) \geq h^*(p^*) \geq h^\text{safe}(p^\text{safe}, m) \) by monotonicity.

**Proof of Proposition 5.** For a fixed threshold \( m \), the thresholds constructed in the proof of Proposition 4 are monotonically increasing in \( p^\text{risky} \) and \( p^\text{safe} \), respectively. Since \( p^\text{risky} = \frac{c_I + \Delta_I}{c_I + c_{II} + \Delta_{II}} \) is monotonically increasing in \( \Delta_I \) and \( p^\text{safe} = \frac{c_I}{c_I + c_{II} + \Delta_{II}} \) is monotonically decreasing in \( \Delta_{II} \), these thresholds have the desired properties.

Similarly, for fixed thresholds \( p^\text{risky} \) and \( p^\text{safe} \), the thresholds constructed in the proof of Proposition 4 are similarly monotonically decreasing in \( m \), since monotonicity holds for \( h^\text{safe}(p, m), h^\text{risky}(p, m) \) by construction.
Proof of Proposition 6. An instance is provided by Example 1.

\[ \text{Proof of Proposition 7.} \text{ Using the simplified notation from above the proof of Proposition 3, note that we can express (by monotonicity of } E[f(\tilde{h}, \tilde{M})], E[f(\tilde{H}, \tilde{m})]) \text{ the threshold-based policies by the agent and the principal as} \]

\[ R = \begin{cases} \text{risky,} & \tilde{M} \leq \tilde{m}, \\ \text{safe,} & \tilde{M} > \tilde{m}, \end{cases} \quad A = \begin{cases} \text{risky,} & \tilde{H} \leq \tilde{h}_R, \\ \text{safe,} & \tilde{H} > \tilde{h}_R. \end{cases} \]

Given thresholds \( \tilde{m}, \tilde{h}_{\text{risky}}, \tilde{h}_{\text{safe}}, \) expected losses of principal and agent are

\[ L(\tilde{m}, \tilde{h}_{\text{risky}}, \tilde{h}_{\text{safe}}) = E[\ell(Y, A)] = E[\mathbb{1}(\tilde{M} \leq \tilde{m}, \tilde{H} \leq \tilde{h}_{\text{risky}}) f(\tilde{H}, \tilde{M})] c_I + E[\mathbb{1}(\tilde{M} \leq \tilde{m}, \tilde{H} > \tilde{h}_{\text{risky}})(1 - f(\tilde{H}, \tilde{M}))] c_I + E[\mathbb{1}(\tilde{M} > \tilde{m}, \tilde{H} \leq \tilde{h}_{\text{safe}}) f(\tilde{H}, \tilde{M})] c_I + E[\mathbb{1}(\tilde{M} > \tilde{m}, \tilde{H} > \tilde{h}_{\text{safe}})(1 - f(\tilde{H}, \tilde{M}))] c_I, \]

\[ L^*(\tilde{m}, \tilde{h}_{\text{risky}}, \tilde{h}_{\text{safe}}) = E[\ell^*(Y, A, R)] = L(\tilde{m}, \tilde{h}_{\text{risky}}, \tilde{h}_{\text{safe}}) + E[\mathbb{1}(\tilde{M} \leq \tilde{m}, \tilde{H} > \tilde{h}_{\text{risky}})(1 - f(\tilde{H}, \tilde{M}))] \Delta_I + E[\mathbb{1}(\tilde{M} > \tilde{m}, \tilde{H} \leq \tilde{h}_{\text{safe}}) f(\tilde{H}, \tilde{M})] \Delta_{II}. \]

The optimal agent thresholds \( \tilde{h}_{\text{risky}}^*(\tilde{m}), \tilde{h}_{\text{safe}}^*(\tilde{m}) \) minimize \( L^*(\tilde{m}, \tilde{h}_{\text{risky}}, \tilde{h}_{\text{safe}}) \) given \( \tilde{m} \), which yields the first-order conditions

\[ E[f(\tilde{h}_{\text{risky}}, \tilde{M}) | \tilde{M} \leq \tilde{m}] = \frac{c_I + \Delta_I}{c_I + c_{II} + \Delta_{II}}, \quad E[f(\tilde{h}_{\text{safe}}, \tilde{M}) | \tilde{M} > \tilde{m}] = \frac{c_I}{c_I + c_{II} + \Delta_{II}}. \]

with unique solutions \( \tilde{h}_{\text{risky}}^*(\tilde{m}) > \tilde{h}_{\text{safe}}^*(\tilde{m}) \) by monotonicity of \( f \) and our regularity assumptions, which by the implicit function theorem are continuously differentiable in \( \tilde{m} \) with

\[ \frac{\partial}{\partial \tilde{m}} \tilde{h}_{\text{risky}}^*(\tilde{m}) = \mu_M(\tilde{m}) \frac{\frac{c_I + \Delta_I}{c_I + c_{II} + \Delta_{II}} - f(\tilde{h}_{\text{risky}}^*(\tilde{m}), \tilde{m})}{E[\partial f / \partial h(h_{\text{risky}}^*(\tilde{m}), \tilde{M}) \mathbb{1}(\tilde{M} \leq \tilde{m})]} < 0, \]

\[ \frac{\partial}{\partial \Delta_I} \tilde{h}_{\text{risky}}^*(\tilde{m}) = \frac{E[(1 - f(\tilde{h}_{\text{risky}}^*(\tilde{m}), \tilde{M})) \mathbb{1}(\tilde{M} \leq \tilde{m})]}{(c_I + c_{II} + \Delta_{II}) E[\partial f / \partial h(h_{\text{risky}}^*(\tilde{m}), \tilde{M}) \mathbb{1}(\tilde{M} \leq \tilde{m})]} > 0, \]

\[ \frac{\partial}{\partial \tilde{m}} \tilde{h}_{\text{safe}}^*(\tilde{m}) = \mu_M(\tilde{m}) \frac{f(\tilde{h}_{\text{safe}}^*(\tilde{m}), \tilde{m}) - \frac{c_I}{c_I + c_{II} + \Delta_{II}}}{E[\partial f / \partial h(h_{\text{safe}}^*(\tilde{m}), \tilde{M}) \mathbb{1}(\tilde{M} \leq \tilde{m})]} < 0, \]

\[ \frac{\partial}{\partial \Delta_{II}} \tilde{h}_{\text{safe}}^*(\tilde{m}) = \frac{-E[f(\tilde{h}_{\text{safe}}^*(\tilde{m}), \tilde{M}) \mathbb{1}(\tilde{M} > \tilde{m})]}{(c_I + c_{II} + \Delta_{II}) E[\partial f / \partial h(h_{\text{safe}}^*(\tilde{m}), \tilde{M}) \mathbb{1}(\tilde{M} > \tilde{m})]} < 0. \]

The optimal principal threshold \( \tilde{m}_{\Delta_I, \Delta_{II}}^* \) then minimizes \( L(\tilde{m}, \tilde{h}_{\text{risky}}^*(\tilde{m}), \tilde{h}_{\text{safe}}^*(\tilde{m})) \).

Writing \( \frac{d}{dm} \) for the (total) derivative of \( L(\tilde{m}, \tilde{h}_{\text{risky}}^*(\tilde{m}), \tilde{h}_{\text{safe}}^*(\tilde{m})) \) with respect to \( \tilde{m} \) and \( \mu_M, \mu_H \)
for the density functions of $\tilde{M}, \tilde{H}$, respectively, we have that

$$\frac{dL}{dm} = \frac{\partial L}{\partial m} + \frac{\partial h_{\text{risky}}}{\partial m} \frac{\partial L}{\partial h_{\text{risky}}} + \frac{\partial h_{\text{safe}}}{\partial m} \frac{\partial L}{\partial h_{\text{safe}}}$$

$$= \frac{\partial L}{\partial m} + \frac{\partial h_{\text{risky}}}{\partial m} \frac{\partial L_s}{\partial h_{\text{risky}}} + (\Delta I \frac{\partial L}{\partial m} + \frac{\partial h_{\text{risky}}}{\partial m} \frac{\partial L}{\partial h_{\text{risky}}}) E[1(\tilde{M} \leq \bar{m}, \tilde{H} > h_{\text{risky}}(\bar{m}))(1 - f(\tilde{H}, \tilde{M}))]
+ \frac{\partial h_{\text{safe}}}{\partial m} \frac{\partial L_s}{\partial h_{\text{safe}}} + \Delta II \frac{\partial h_{\text{safe}}}{\partial m} \frac{\partial L}{\partial h_{\text{safe}}} E[1(\tilde{M} > \bar{m}, \tilde{H} \leq h_{\text{safe}}(\bar{m}))(1 - f(\tilde{H}, \tilde{M}))]
= \mu_M(\bar{m}) E[1(\tilde{h}_{\text{safe}}(\bar{m}) < \tilde{H} \leq h_{\text{risky}}(\bar{m}))(c_I + c_{II})f(\tilde{H}, \bar{m}) - c_I]
- \Delta I \frac{\partial h_{\text{risky}}}{\partial m} \mu_H(h_{\text{risky}}(\bar{m})) E[1(\tilde{M} \leq \bar{m})(1 - f(\tilde{h}_{\text{risky}}(\bar{m}), \tilde{M}))]
+ \Delta II \frac{\partial h_{\text{safe}}}{\partial m} \mu_H(h_{\text{safe}}(\bar{m})) E[1(\tilde{M} > \bar{m})f(\tilde{h}_{\text{safe}}(\bar{m}), \tilde{M})] = F_{\Delta I, \Delta II}(\bar{m}),$$

where $F_{\Delta I, \Delta II}(\bar{m})$ is continuously differentiable in $\bar{m}, \Delta I, \Delta II$ with

$$\frac{\partial}{\partial \Delta I} F_{0,0}(\bar{m}) = \boxed{\frac{\partial}{\partial \Delta I} F_{0,0}(\bar{m})}$$

$$= \mu_M(\bar{m}) \mu_H(h_{\text{risky}}(\bar{m}))(c_I + c_{II} f(h_{\text{risky}}(\bar{m}), \bar{m}) - c_I)$$

$$- \mu_H(h_{\text{risky}}(\bar{m})) E[1(\tilde{M} \leq \bar{m})(1 - f(h_{\text{risky}}(\bar{m}), \tilde{M}))] > 0,$$

$$\frac{\partial}{\partial \Delta II} F_{0,0}(\bar{m}) = \boxed{\frac{\partial}{\partial \Delta II} F_{0,0}(\bar{m})}$$

$$= -\mu_M(\bar{m}) \mu_H(h_{\text{safe}}(\bar{m}))(c_I + c_{II} f(h_{\text{safe}}(\bar{m}), \bar{m}) - c_I)$$

$$+ \mu_H(h_{\text{safe}}(\bar{m})) E[1(\tilde{M} > \bar{m})f(h_{\text{safe}}(\bar{m}), \tilde{M})] < 0,$$

The optimal threshold $m_{\Delta I, \Delta II}^\star$ fulfills the first-order condition $F_{\Delta I, \Delta II}(m_{\Delta I, \Delta II}^\star) = 0$. Furthermore, by assumption, the solution at $\Delta I = 0 = \Delta II$ is unique with $\frac{\partial}{\partial m} F_{0,0}(m_{0,0}^\star) > 0$. By the implicit function theorem, there is a neighborhood of $\Delta I = 0 = \Delta II$ in which $m_{\Delta I, \Delta II}^\star$ is continuously differentiable in $\Delta I, \Delta II$ with derivatives

$$\frac{\partial}{\partial \Delta I} m_{\Delta I, \Delta II}^\star = -\frac{\partial}{\partial m} F_{\Delta I, \Delta II}(m_{\Delta I, \Delta II}^\star), \quad \frac{\partial}{\partial \Delta II} m_{\Delta I, \Delta II}^\star = -\frac{\partial}{\partial m} F_{\Delta I, \Delta II}(m_{\Delta I, \Delta II}^\star).$$

By continuity of the derivatives, the first one is negative and the second one is positive in a sufficiently small neighborhood of $\Delta I = 0 = \Delta II$. 

**Proof of Proposition 8.** For the comparison to machine decisions, consider recommending the op-
timal machine decision,

\[
R = \begin{cases} 
    \text{risky}, & P(Y=\text{bad}|X, M) \leq p^* = \frac{c_I}{c_I + c_{II}}, \\
    \text{safe}, & P(Y=\text{bad}|X, M) > p^*.
\end{cases}
\]

For the action \( A \) chosen by the agent to be different from the recommendation, we must have that

\[
P(Y=\text{bad}|X, H, R=\text{risky}) \geq \frac{c_I + \Delta_I}{c_I + c_{II} + \Delta_I} \geq p^* \quad \text{(safe} = A \neq R = \text{risky}),
\]

\[
P(Y=\text{bad}|X, H, R=\text{safe}) \leq \frac{c_I}{c_I + c_{II} + \Delta_{II}} \leq p^* \quad \text{(risky} = A \neq R = \text{safe}),
\]

and both cases can only improve over implementing \( R \) directly. Specifically, it follows that

\[
P(Y=\text{bad}|\text{safe}=A \neq R=\text{risky}) = E[P(Y=\text{bad}|X, H, R=\text{risky})|\text{safe}=A \neq R=\text{risky}] \geq p^*,
\]

\[
P(Y=\text{bad}|\text{risky}=A \neq R=\text{safe}) = E[P(Y=\text{bad}|X, H, R=\text{safe})|\text{risky}=A \neq R=\text{safe}] \leq p^*
\]

and thus

\[
E[\ell(Y, A)] = E[\ell(Y, A)1(A=R)] + E[\ell(Y, A)1(\text{safe}=A \neq R=\text{risky})] + E[\ell(Y, A)1(\text{risky}=A \neq R=\text{safe})]
\]

\[
\leq E[\ell(Y, A)1(A=R)] + c_I(1 - p^*) E[1(\text{safe}=A \neq R=\text{risky})] + c_{II}p^* E[1(\text{risky}=A \neq R=\text{safe})]
\]

\[
= E[\ell(Y, A)1(A=R)] + c_{II}p^* E[1(\text{safe}=A \neq R=\text{risky})] + c_I(1 - p^*) E[1(\text{risky}=A \neq R=\text{safe})]
\]

\[
\leq E[\ell(Y, R)1(A=R)] + E[\ell(Y, R)1(\text{safe}=A \neq R=\text{risky})] + E[\ell(Y, R)1(\text{risky}=A \neq R=\text{safe})]
\]

\[
= E[\ell(Y, R)] = E[\min_a E[\ell(Y, a)|X, M]].
\]

For the comparison to human decisions, consider the recommendation \( R \equiv \text{neutral} \), which will lead to the same decision as if the human is acting by themselves. Hence, \( E[\ell(Y, A)] \leq E[\min_a E[\ell(Y, a)|X, H]] \) for this recommendation.

Putting both parts together, actions given an optimal recommendation policy do (weakly) better than each of these two policies, and thus must fulfill the inequality. \( \square \)

**Proof of Proposition 9.** Using the simplified notation from above the proof of Proposition 3 and following the proof of Proposition 7, note that we can express the threshold-based policies by the agent and the principal as

\[
R = \begin{cases} 
    \text{risky}, & \tilde{M} \leq \tilde{m}^\dagger, \\
    \text{neutral} & \tilde{m}^\dagger < \tilde{M} \leq \tilde{m}^\uparrow, \\
    \text{safe}, & \tilde{M} > \tilde{m}^\uparrow,
\end{cases}
\]

\[
A = \begin{cases} 
    \text{risky}, & \tilde{H} \leq \tilde{h}^R, \\
    \text{safe}, & \tilde{H} > \tilde{h}^R.
\end{cases}
\]
Given thresholds $\bar{m}^\downarrow, \bar{m}^\uparrow, \bar{h}_\text{risky}, \bar{h}_\text{neutral}, \bar{h}_\text{safe}$, expected losses of principal and agent are

\[
L(\bar{m}^\downarrow, \bar{m}^\uparrow, \bar{h}_\text{risky}, \bar{h}_\text{neutral}, \bar{h}_\text{safe}) = E[\ell(Y, A)]
\]

\[
= E[\mathbb{1}(\bar{M} \leq \bar{m}^\downarrow, \bar{H} \leq \bar{h}_\text{risky})f(\bar{H}, \bar{M})]c_{II} + E[\mathbb{1}(\bar{M} > \bar{m}^\downarrow, \bar{H} > \bar{h}_\text{risky})f(\bar{H}, \bar{M})](1 - f(\bar{H}, \bar{M}))c_I
\]

\[
+ E[\mathbb{1}(\bar{M} < \bar{m}^\downarrow, \bar{H} \leq \bar{h}_\text{neutral})f(\bar{H}, \bar{M})]c_{II} + E[\mathbb{1}(\bar{M} > \bar{m}^\downarrow, \bar{H} > \bar{h}_\text{neutral})f(\bar{H}, \bar{M})](1 - f(\bar{H}, \bar{M}))c_I
\]

\[
+ E[\mathbb{1}(\bar{M}^\uparrow < \bar{m}^\downarrow, \bar{H} \leq \bar{h}_\text{safe})f(\bar{H}, \bar{M})]c_{II} + E[\mathbb{1}(\bar{M}^\uparrow > \bar{m}^\downarrow, \bar{H} > \bar{h}_\text{safe})f(\bar{H}, \bar{M})](1 - f(\bar{H}, \bar{M}))c_I
\]

\[
L^*(\bar{m}^\downarrow, \bar{m}^\uparrow, \bar{h}_\text{risky}, \bar{h}_\text{neutral}, \bar{h}_\text{safe}) = E[\ell^*(Y, A, R)] = L(\bar{m}^\downarrow, \bar{m}^\uparrow, \bar{h}_\text{risky}, \bar{h}_\text{neutral}, \bar{h}_\text{safe})
\]

\[
+ E[\mathbb{1}(\bar{M} \leq \bar{m}^\downarrow, \bar{H} > \bar{h}_\text{risky})(1 - f(\bar{H}, \bar{M}))] \Delta_I + E[\mathbb{1}(\bar{M}^\uparrow > \bar{m}^\downarrow, \bar{H} \leq \bar{h}_\text{safe})f(\bar{H}, \bar{M})] \Delta_{II}.
\]

The optimal agent thresholds $\bar{h}_\Delta^I(\bar{m}^\downarrow), \bar{h}_\Delta^I(\bar{m}^\downarrow, \bar{m}^\uparrow), \bar{h}_\Delta^I(\bar{m}^\uparrow)$ now are determined uniquely by the first-order conditions

\[
E[f(\bar{h}_\text{risky}, \bar{M})|\bar{M} \leq \bar{m}^\downarrow] = \frac{c_I + \Delta_I}{c_I + c_{II} + \Delta_I},
\]

\[
E[f(\bar{h}_\text{neutral}, \bar{M})|\bar{M} < \bar{m}^\downarrow] = \frac{c_I}{c_I + c_{II}},
\]

\[
E[f(\bar{h}_\text{safe}, \bar{M})|\bar{M} > \bar{m}^\downarrow] = \frac{c_I}{c_I + c_{II} + \Delta_{II}}
\]

with unique solutions $\bar{h}_\Delta^I(\bar{m}^\downarrow) > \bar{h}_\Delta^I(\bar{m}^\downarrow, \bar{m}^\uparrow) > \bar{h}_\Delta^I(\bar{m}^\downarrow, \bar{m}^\uparrow)$ by monotonicity of $f$ and our regularity assumptions, which by the implicit function theorem are continuously differentiable in $\bar{m}$ as in the proof of Proposition 7 with

\[
\frac{\partial}{\partial \bar{m}^\downarrow} \bar{h}_\Delta^I(\bar{m}^\downarrow) < 0,
\]

\[
\frac{\partial}{\partial \bar{m}^\downarrow} \bar{h}_\Delta^I(\bar{m}^\downarrow, \bar{m}^\uparrow) < 0,
\]

\[
\frac{\partial}{\partial \bar{m}^\downarrow} \bar{h}_\Delta^I(\bar{m}^\downarrow, \bar{m}^\uparrow) < 0,
\]

\[
\frac{\partial}{\partial \bar{m}^\downarrow} \bar{h}_\Delta^I(\bar{m}^\downarrow, \bar{m}^\uparrow) < 0,
\]

The optimal principal thresholds $\bar{m}^\uparrow_\Delta^I, \bar{m}^\downarrow_\Delta^I, \bar{m}^\downarrow_\Delta^I, \bar{m}^\uparrow_\Delta^I$ then minimize

\[
L(\bar{m}^\downarrow, \bar{m}^\uparrow, \bar{h}_\text{risky}, \bar{h}_\text{neutral}, \bar{h}_\text{safe}) = E[\ell(Y, A)]
\]
Using the same notation as in the proof of Proposition 7, we have that

\[
\frac{dL}{dm^\downarrow} = \frac{\partial L}{\partial m^\downarrow} + \frac{\partial h_{\text{risky}}}{\partial m^\downarrow} \frac{\partial L}{\partial h_{\text{risky}}}
\]
\[
= \frac{\partial L}{\partial m^\downarrow} + \Delta_I \frac{\partial h_{\text{risky}}}{\partial m^\downarrow} \frac{\partial}{\partial h_{\text{risky}}} E[1(\bar{M} \leq \bar{m}^\downarrow, \bar{H} > \bar{h}_{\Delta_I} (\bar{m}^\downarrow))(1-f(\bar{H}, \bar{M}))]
\]
\[
= \mu_M(\bar{m}^\downarrow) E[1(\bar{h}_{\text{neutral}}(\bar{m}^\downarrow, \bar{m}^\uparrow) < \bar{H} \leq \bar{h}_{\Delta_I} (\bar{m}^\downarrow))(c_I + c_{II} f(\bar{H}, \bar{m}^\downarrow) - c_I)]
\]
\[
- \Delta_I \frac{\partial h_{\text{risky}}}{\partial m^\downarrow} \mu_H(\bar{h}_{\Delta_I} (\bar{m}^\downarrow)) E[1(\bar{M} \leq \bar{m}^\downarrow)(1-f(\bar{h}_{\Delta_I} (\bar{m}^\downarrow), \bar{M}))] = F_{\Delta_I}^\downarrow (\bar{m}^\downarrow, \bar{m}^\uparrow),
\]

\[
\frac{dL}{dm^\uparrow} = \frac{\partial L}{\partial m^\uparrow} + \frac{\partial h_{\text{safe}}}{\partial m^\uparrow} \frac{\partial L}{\partial h_{\text{safe}}}
\]
\[
= \frac{\partial L}{\partial m^\uparrow} + \Delta_{II} \frac{\partial h_{\text{safe}}}{\partial m^\uparrow} \frac{\partial}{\partial h_{\text{safe}}} E[1(\bar{M} > \bar{m}^\uparrow, \bar{H} \leq \bar{h}_{\Delta_{II}} (\bar{m}^\uparrow))(1-f(\bar{H}, \bar{M}))]
\]
\[
= \mu_M(\bar{m}^\uparrow) E[1(\bar{h}_{\Delta_{II}} (\bar{m}^\uparrow)) < \bar{H} \leq \bar{h}_{\text{neutral}}(\bar{m}^\uparrow, \bar{m}^\uparrow))(c_I + c_{II} f(\bar{H}, \bar{m}^\uparrow) - c_I]
\]
\[
+ \Delta_{II} \frac{\partial h_{\text{safe}}}{\partial m^\uparrow} \mu_H(\bar{h}_{\Delta_{II}} (\bar{m}^\uparrow)) E[1(\bar{M} > \bar{m}^\uparrow) f(\bar{h}_{\Delta_{II}} (\bar{m}^\uparrow), \bar{M})] = F_{\Delta_{II}}^\uparrow (\bar{m}^\uparrow, \bar{m}^\uparrow),
\]

where \((F_{\Delta_I}^\downarrow (\bar{m}^\downarrow, \bar{m}^\uparrow), F_{\Delta_{II}}^\uparrow (\bar{m}^\uparrow, \bar{m}^\uparrow))\) is continuously differentiable in \(\bar{m}^\downarrow, \bar{m}^\uparrow, \Delta_I, \Delta_{II}\) with

\[
\frac{\partial}{\partial \Delta_I} F_{\Delta_I}^\downarrow (\bar{m}^\downarrow, \bar{m}^\uparrow) = \mu_M(\bar{m}^\downarrow) \mu_H(\bar{h}_{\text{risky}}(\bar{m}^\downarrow)) ((c_I + c_{II} f(\bar{h}_{\Delta_I} (\bar{m}^\downarrow), \bar{m}^\downarrow) - c_I) \frac{\partial h_{\text{risky}}}{\partial \Delta_I}
\]
\[
- \frac{\partial h_{\text{risky}}}{\partial m^\downarrow} \mu_H(\bar{h}_{\Delta_I} (\bar{m}^\downarrow)) E[1(\bar{M} \leq \bar{m}^\downarrow)(1-f(\bar{h}_{\Delta_I} (\bar{m}^\downarrow), \bar{M}))] > 0,
\]

\[
\frac{\partial}{\partial \Delta_{II}} F_{\Delta_{II}}^\uparrow (\bar{m}^\downarrow, \bar{m}^\uparrow) = - \mu_M(\bar{m}^\uparrow) \mu_H(\bar{h}_{\text{safe}}(\bar{m}^\uparrow)) ((c_I + c_{II} f(\bar{h}_{\Delta_{II}} (\bar{m}^\uparrow), \bar{m}^\uparrow) - c_I) \frac{\partial h_{\text{safe}}}{\partial \Delta_{II}}
\]
\[
+ \frac{\partial h_{\text{safe}}}{\partial m^\uparrow} \mu_H(\bar{h}_{\Delta_{II}} (\bar{m}^\uparrow)) E[1(\bar{M} > \bar{m}^\uparrow) f(\bar{h}_{\Delta_{II}} (\bar{m}^\uparrow), \bar{M})] < 0,
\]

As in the proof of Proposition 7, the result follows from the implicit function theorem, where we note that

\[
\left( \begin{array}{c}
\frac{\partial m^\downarrow_0}{\partial \Delta_I} \\
\frac{\partial m^\uparrow}{\partial \Delta_I}
\end{array} \right) = - \left( \begin{array}{cc}
\frac{\partial m^\downarrow_0}{\partial m^\downarrow} F^\downarrow_{\Delta_I} (\bar{m}^\downarrow_0, \bar{m}^\uparrow_0) & \frac{\partial m^\uparrow}{\partial \Delta_{II}} F^\downarrow_{\Delta_{II}} (\bar{m}^\downarrow_0, \bar{m}^\uparrow_0) \\
\frac{\partial m^\downarrow}{\partial m^\downarrow} F^\downarrow_{\Delta_I} (\bar{m}^\downarrow_0, \bar{m}^\uparrow_0) & \frac{\partial m^\uparrow}{\partial \Delta_{II}} F^\downarrow_{\Delta_{II}} (\bar{m}^\downarrow_0, \bar{m}^\uparrow_0)
\end{array} \right)^{-1} \left( \begin{array}{c}
\frac{\partial \Delta_I}{\partial m^\downarrow} F^\downarrow_{\Delta_I} (\bar{m}^\downarrow_0, \bar{m}^\uparrow_0) \\
0
\end{array} \right)
\]

\[
= \left( \frac{\partial^2 E[(Y, A)]}{\partial (\bar{m}^\downarrow, \bar{m}^\uparrow) \partial (\bar{m}^\downarrow, \bar{m}^\downarrow)} \right) \left( \begin{array}{c}
\bar{m}^\downarrow_0 - \bar{m}^\uparrow_0 > 0
\end{array} \right)
\]

\[
\Delta_I = 0 = \Delta_{II}
\]

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which implies \( \frac{\partial}{\partial \Delta I} m_{0,0}^{1*} < 0 \), \( \frac{\partial}{\partial \Delta II} m_{0,0}^{1*} > 0 \), and extends to a neighborhood of \( \Delta I = 0 = \Delta II \). \( \square \)

Proof of Proposition 10. For the comparison to the machine decision, note that the optimal machine-only decision (assuming ties are broken in favor of the risky decision) is given by

\[
\arg \min_a E[\ell(Y, a)|X, M] \geq A^* = \begin{cases} \text{risky}, & \widehat{M} \leq p^* = \frac{c_I}{c_{II} + c_{III}}, \\ \text{safe}, & \widehat{M} > p^*. \end{cases}
\]

Similarly to the proof of Proposition 8, for the action \( A \) chosen by the agent to be different from \( A^* \), we must have that

\[
P(Y=\text{bad}|X, H, A^*=\text{risky}) = P(Y=\text{bad}|X, H, \widehat{M} \leq p^*) \\
\geq \frac{c_I + \delta_I(\widehat{M})}{c_I + c_{II} + \delta_I(\widehat{M}) + \delta_{III}(\widehat{M})} \geq \frac{c_I + \delta_I(p^*)}{c_I + c_{II} + \delta_I(p^*) + \delta_{III}(p^*)} = p^* \quad (\text{safe} = A \neq A^* = \text{risky}),
\]

\[
P(Y=\text{bad}|X, H, A^*=\text{risky}) = P(Y=\text{bad}|X, H, \widehat{M} > p^*) \\
\leq \frac{c_I + \delta_I(\widehat{M})}{c_I + c_{II} + \delta_I(\widehat{M}) + \delta_{III}(\widehat{M})} \leq \frac{c_I + \delta_I(p^*)}{c_I + c_{II} + \delta_I(p^*) + \delta_{III}(p^*)} = p^* \quad (\text{risky} = A \neq A^* = \text{safe}),
\]

where we have used monotonicity from Assumption 5 and that \( p^* \) is recommendation-neutral. Hence,

\[
P(Y=\text{bad}|\text{safe}=A \neq A^* = \text{risky}) = E[P(Y=\text{bad}|X, H, A^*=\text{risky})|\text{safe}=A \neq A^* = \text{risky}] \geq p^*,
\]

\[
P(Y=\text{bad}|\text{risky}=A \neq A^* = \text{safe}) = E[P(Y=\text{bad}|X, H, A^*=\text{safe})|\text{risky}=A \neq A^* = \text{safe}] \leq p^*
\]

and thus

\[
E[\ell(Y, A)] = E[\ell(Y, A) \mathbb{1}(A=A^*)] + E[\ell(Y, A) \mathbb{1}(\text{safe}=A \neq A^* = \text{risky})] + E[\ell(Y, A) \mathbb{1}(\text{risky}=A \neq A^* = \text{safe})] \\
\leq E[\ell(Y, A) \mathbb{1}(A=A^*)] + c_I(1 - p^*) E[\mathbb{1}(\text{safe}=A \neq A^* = \text{risky})] + c_{III} p^* E[\mathbb{1}(\text{risky}=A \neq A^* = \text{safe})] \\
= E[\ell(Y, A) \mathbb{1}(A=A^*)] + c_{III} p^* E[\mathbb{1}(\text{safe}=A \neq A^* = \text{risky})] + c_I(1 - p^*) E[\mathbb{1}(\text{risky}=A \neq A^* = \text{safe})] \\
\leq E[\ell(Y, A^*) \mathbb{1}(A=A^*)] + E[\ell(Y, A^*) \mathbb{1}(\text{safe}=A \neq A^* = \text{risky})] + E[\ell(Y, A^*) \mathbb{1}(\text{risky}=A \neq A^* = \text{safe})] \\
= E[\ell(Y, A^*)] = E[\min_a E[\ell(Y, a)|X, M]].
\]

For the comparison to human decisions, choosing \( p^i \equiv 1, p^i \equiv 0 \) means that the score is always withheld and interpreted as \( \widehat{M} = P(Y=\text{bad}|X) \). Since \( P(Y=\text{bad}|X) \) is recommendation-neutral and does not contain any new information, it does not affect the final action, so it leads to the same decision as the human-only decision. Hence, \( E[\ell(Y, A)] \leq E[\min_a E[\ell(Y, a)|X, H]] \) for this recommendation. \( \square \)

Proof of Proposition 11. Writing out this reference-dependent loss with loss aversion for the specific
loss functions, we find that

\[
\ell^{PT}(Y, A, R) = \lambda[\ell(Y, A) - \ell(Y, R)]_+ - [\ell(Y, A) - \ell(Y, R)]_-
\]

\[
= \ell(Y, A) - \ell(Y, R) + (\lambda - 1)[\ell(Y, A) - \ell(Y, R)]_+
\]

\[
= \ell(Y, A) - \ell(Y, R) + \begin{cases} 
(\lambda - 1)c_I, & Y = \text{good}, A = \text{safe}, R = \text{risky}, \\
(\lambda - 1)c_{II}, & Y = \text{bad}, A = \text{risky}, R = \text{safe}.
\end{cases}
\]

Since \(\ell(Y, R)\) is not affected by the decision-maker’s choice, their preferences are as if they are minimizing expected loss with loss function

\[
\ell^{*}(Y, A, R) = \ell(Y, A) + \begin{cases} 
(\lambda - 1)c_I, & Y = \text{good}, A = \text{safe}, R = \text{risky}, \\
(\lambda - 1)c_{II}, & Y = \text{bad}, A = \text{risky}, R = \text{safe},
\end{cases}
\]

as in (2) with \(\Delta_I = (\lambda - 1)c_I, \Delta_{II} = (\lambda - 1)c_{II}\.\)