"The Reddish Moon"--The Evaluation of Urban Air Pollution by Lunar Extinction

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Abstract. In severe polluted areas, objects tend to be redder because of the refraction of light by particulate pollutants through the atmosphere. In this study, a particle scattering model is established to simulate the scattering of pollutants based on assumptions. Then the actual data of lunar extinction is obtained by using an astronomical telescope to measure the RGB color differentiation at different altitude angles of the lunar surface in Shanghai caused by pollutants scattering. The results are combined with the particle scattering model calculation to obtain the estimated pollutant thickness in the atmosphere.

1. Introduction

1.1. Introduction to Atmospheric Particle Pollutants
Atmospheric particle pollutants are harmful solid particles in the atmosphere that cause harm to the surrounding people and the environment, produced in human or natural activities. The main chemical components of atmospheric particulate pollutants include carbon, sulfates, microelements, nitrates, etc., as well as some crustal materials such as soil and ash. The chemical composition of different regions can vary significantly. Atmospheric particulate pollutants can vary greatly in size.

Atmospheric particle pollutants are usually mixtures of larger particles called "Course particles" and smaller particles called "Fine particles." Coarse particles range in diameter from 2.5μm to 40μm, while fine particles, also known as PM2.5, range in diameter ≤2.5μm. Compared with coarse particles, fine particles are smaller and thus rich in harmful substances. It can stay in the atmosphere for a longer time and travels farther, so it has a greater impact on human health and atmospheric environmental quality. The measurement of atmospheric particulate pollutants used by most Weather Service is to isolate solid particulate pollutants and then analyze them with chemical methods, such as separating the particulate pollutants by a filter screen or a cyclone separator followed by chemically analysis, or observing pollutants under an electron microscope. Then the composition and size distribution of pollutants in a unit volume can be obtained. Or, after sampling the gas, different scattering laser intensity due to angles can be measured by laser irradiation, so as to obtain the pollutant size distribution. These methods all require sampling of gases, so they cannot produce global pollution information timely, and their subsequent chemical and physical analysis requires stringent experimental conditions. According to the US Environmental Protection Agency (EPA), chemical composition and size of atmospheric particulate pollutants have significant local and seasonal characteristics. [1]

Inside the atmosphere window, there is also some interaction between electromagnetic wave and atmosphere. These interactions generally cause light from celestial bodies to travel less in the original
1.2. Scattering Effect

Light propagates in a straight line in a vacuum, but in a non-uniform medium it changes direction due to the difference in refractive index. In practice, even in a uniform medium, dielectric particles may interact with light and change the original direction of photons. This is known as Scattering. When light from celestial bodies passes through the atmosphere, it scatters and absorbs gas molecules and particle pollutants in the atmosphere, resulting in a decrease in their flow to the earth's surface. For visible band, the scattering effect is far greater than the absorption effect despite that a few single-frequency photons correspond exactly to the absorbed energy level of the gas, so the absorption process can be ignored. For ultraviolet and infrared band, ozone absorption of ultraviolet, carbon dioxide, and water vapor become important, and thus cannot be ignored. Due to the limitations of general camera detection methods, this paper is going to discuss only the scattering effect of visible band.

The essence of scattering is the interaction between electromagnetic waves and matter. According to the wavelength of electromagnetic wave and the size of scattering particles, scattering can be divided into three theoretical categories.

Define dimensionless quantity \( \alpha = \frac{\pi D}{\lambda} \). When \( \alpha < 1 \), namely when the particle diameter is much smaller than the wavelength \( \lambda \), Rayleigh Scattering is applicable; When \( \alpha >> 1 \), namely when the particle diameter is much larger than the wavelength \( \lambda \), Geometric Scattering is applicable; When \( \alpha \) is close to 1, namely when the particle diameter is similar to the wavelength \( \lambda \), Mie Scattering is applicable.

The wavelength of visible light is about 400-800nm. The size of atmospheric particle pollutants is mainly distributed at 100-25000nm, and Rayleigh scattering or Geometric Scattering is not applicable. Therefore, Mie Scattering theory is needed in the study of atmospheric particle scattering.

When the diameter of the particle is close to the wavelength of the scattered light \((d \sim \lambda)\), Mie scattering is applicable to the situation. The principle is to change the media to equivalent infinite spherical, and input them of boundary conditions, and to solve Maxwell's equations under the case of incident electromagnetic plane wave. See Appendix III for details. If the scattering particle is a uniform sphere with diameter D, for the light with original light intensity of \( I_0 \) and wavelength of \( \lambda \), the light intensity \( I \) at the scattering angle \( \theta \) and distance \( r \) from the sphere can be described as follows:

\[
I = \left( \frac{4}{\pi D} \right)^2 I_0 \left( \frac{I_1 + iI_2}{2} \right)
\]

In which, \( I_1, I_2 \) are functions of incident light intensity:

\[
i_1 = s_1(m, \theta, \alpha) \times s_1^*(m, \theta, \alpha)
\]

\[
i_2 = s_2(m, \theta, \alpha) \times s_2^*(m, \theta, \alpha)
\]

We can see that the scattering effect depends on the scattering angle \( \theta \), refractive index \( N \), wavelength \( \lambda \), and the diameter of the scattered particles \( D \). The light intensity of Mie scattering is asymmetric in all directions. The intensity is the strongest along the direction of incidence. The larger the particle, the greater the intensity.

1.3. Assumptions and the Scattering Model

The complex refractive index \( N \) of the scattering particles is:

\[
N = m - i k 4
\]

Where \( m \) is refractive index, and the imaginary part \( k \) is extinction coefficient, the absorption of electromagnetic waves by corresponding substances, and both are related to the wavelength of incident light. The absorption of atmospheric molecules and dust pollutant particles to visible light can be neglected. Figure 1 show some refractive index and extinction coefficient of some complex carbon compound according to the website data [2].
The polystyrene molecule \((\text{C}_8\text{H}_8)_n\) has a complex refractive index of 1.5882 for 600nm visible light, and the imaginary part is only \(6.3600 \times 10^{-7}\). The polycarbonate molecule \((\text{C}_{16}\text{H}_{24}\text{O}_3)_n\) has a complex refractive index of 1.5887 for visible light at 600nm, and the imaginary part is only \(4.80 \times 10^{-7}\). Polyetherimide molecule \((\text{C}_{37}\text{H}_{24}\text{O}_6\text{N}_2)_n\) has a complex refractive index of 1.6244 for 600nm visible light, and the imaginary part is only \(3.17 \times 10^{-5}\). Thus, the absorption part of particle pollutants can be ignored in the calculation.

Figure 1. Real and imaginary index of some complex carbon compounds

In the study, we can calculate the dimensionless quantity \(\alpha\) by size of scattered particles and wavelength of light, using Miepython Library [3]. With the refractive index, we can find the of a single particle. Define cross-section of a single particle as \(\sigma_{\text{ext}} = Q_{\text{ext}} \times 2\pi r^2\).

There are certain concentrations of air pollutants. When the distance between particles is more than three times their diameters \(D\), that is, when the particle concentration is low enough, the scattering of each particle can be considered as independent, so they do not overlap and the overall scattering effect is the sum of each particle. Relevant studies show that even in severe pollution, atmospheric particle pollutants meet this condition. Thus, the scattering effect of pollutants per unit volume is the sum of the scattering effect of all the particles in that volume.
Because of irrelevant scattering, if the number of particles per unit volume is $n$, the absorption coefficient $k_\nu = n\sigma_{\text{ext}}$. Using the material refractive index data provided by the existing data [2], the changes of the cross-section with the wavelength can be drawn. Figure 2 shows the changes of extinction coefficient $Q_{\text{ext}}$, scattering coefficient $Q_{\text{scat}}$ and absorption coefficient $Q_{\text{abs}}$ of the polycarbonate particles with the diameter of 0.2μm, 0.6μm, 1μm, and 2μm.

In this study, this method will be used to calculate the total cross section of particle pollutants per unit volume, namely the absorption coefficient $k_\nu$, by calculating the scattering cross sections of particles of various sizes per unit volume and summing them.

The process of radiation transfer needs to be considered when investigating the change in the light intensity from celestial bodies passing through the atmosphere. When the radiation with an intensity of $I_\nu$ passes through the medium for a short distance $ds$, the rate of change of intensity $dI_\nu$ can be written as:

$$dI_\nu = -I_\nu k_\nu ds + j_\nu ds$$  \hspace{1cm} (5)

Where $k_\nu$ the absorption coefficient, $j_\nu$ is the emission coefficient. In reality, particle pollutants scatter the photons to other directions, resulting in absorption coefficient $k_\nu$, but they are not absorbed. For the visible light from the moon, the emission coefficient mainly comes from the thermal radiation of the atmosphere, and its intensity relative to the moonlight itself in the visible band is negligible (Otherwise the night sky would be bright).

The layer of pollutants in atmosphere is geometrically a spherical shell. Because the thickness of atmospheric pollutants is small relative to the radius of the Earth, it can be approximated as a flat plate except at very low horizon altitude angles.
The average distance from the Earth to the moon is 384401 km. Pollutant particles mainly exist in the troposphere with a thickness of 10-20 km, which is very small relative to the earth's radius (6,371 km). Thus, the angle of light received between the North Pole and the equator \( \theta \) is:

\[
\theta = \frac{6371}{384401} \times \frac{180}{\pi} \approx 0.950^\circ
\]

Thus, a light source will have an angle difference of \( 120.950^\circ \times 2 = 1.9^\circ \) between North Pole and South Pole. In the experiment, therefore, the light from moon can be perceived as parallel.

Now we consider the difference between the flat plate assumption and real situation.

Assume the center of Earth is O, the moonlight is parallel, the assumed thickness of pollutant is \( x \), the actual thickness of pollutant is \( d \), and the radius of Earth is \( r \). At some point, a light enters the atmosphere from the direction A and reaches the surface observer. Assume the angle between \( x \) and \( s \) is \( \theta \), then the distance the light will travel is

\[
s = \frac{d}{\cos \theta}
\]

(Figure 3)

Figure 3. Flat plate assumption

When \( \theta = 45^\circ \),

\[
x^2 + (x + r)^2 = (d + r)^2
\]

Radius of Earth \( r=6371\text{km} \). Even we take \( d=20\text{km} \), the positive solution of \( x \) is:

\[
x = 19.968
\]

Thus, the actual thickness of atmosphere is \( s_x = \frac{19.968}{\cos 45^\circ} = 28.23 \)

The thickness assumed in the model is \( s_d = \frac{20}{\cos 45^\circ} = 28.28 \)

They only has a \( \frac{28.28-28.23}{28.23} = 0.17\% \) difference.

When the height of the moon is lower, this difference will increase, and the model hypothesis will gradually deviate from the actual situation. However, in reality, the observation point in the city is generally not too low (at least \( > 30^\circ \)) due to surrounding buildings. Thus, the moonlight can be approximated as parallel light, and in a particular city it can be reasonably approximated as a flat plate.

For a flat plate hypothesis, without considering absorption, according to the radiation transfer equation, \( s \) is the radiation propagation direction, and the light intensity \( I_\nu \) per unit time per frequency after passing distance \( ds \) is obtained as:

\[
\frac{dI_\nu}{ds} = -I_\nu k_\nu \rho ds + j_\nu \rho ds
\]

Define optical depth \( d\tau = k_\nu \rho ds \), we can get:

\[
\frac{dI_\nu}{d\tau} = -I_\nu + \frac{j_\nu}{k_\nu}
\]

\( k_\nu \) mainly comes from absorption and scattering. The absorption of \( j_\nu \rho ds \) in the visible light band of the particle pollutants with extinction effect is very low, and the absorption of imaginary part \( j_\nu \rho ds \) have little influence on the pollutants. Therefore, the absorption of the imaginary part is not considered.
in this research, and only the scattering process is considered. Cancel the $\frac{I_v}{k_v}$ and integrate the formula, we get:

$$\ln(I_v) = -k_v + c$$  \hspace{1cm} (11)

Where $c$ is a constant determined by boundary conditions. Since when $s = 0$, $I_v = I_{v0}$, so:

$$e^c = I_{v0}$$  \hspace{1cm} (12)

Eventually, we get:

$$I_v = I_{v0}e^{-k_v s}$$  \hspace{1cm} (13)

Assuming that the properties of pollutants are exactly the same, the absorption coefficient is then a constant. Without considering atmospheric emission, using boundary conditions that when $s = 0$, $I_v = I_{v0}$ to determine the constant $c$, we get:

$$I_v = I_{v0}e^{-k_v s} = I_{v0}e^{-\tau_v}$$  \hspace{1cm} (14)

In order to calculate the total extinction coefficient of pollutants per unit volume, namely the total cross-section, it is necessary to know the size distribution of particle pollutants. The total amount of pollutants can be measured by inertial separation of a certain volume of atmospheric pollutants, which is relatively easy, and the meteorological monitoring observatory provides daily measurement results. However, the measurement of pollutant volume distribution requires electron microscopy analysis or laser scattering method, and there is no real-time measurement result at present. As an approximation, this paper uses the results of research about air pollutants in Guangzhou by Feng Zidan et al. in 2011. Researchers used electron microscope to obtain the size distribution of pollutants [4], shown in figure 4. According to the study, the main components of the pollutants are complex compounds of sulfates, nitrates, and carbon, which have a wide density distribution. For example, the nitrate density is about 1.5-1.6g/cm³, and sulfate density ranges from 1.5-4g/cm³, most of which are between 2-3g/cm³. The complex compound density range of carbon is about 1.4-2.0g/cm³. For the calculation purpose, a typical density of 2.0g/cm³ was taken as the average density. In the case of the determined chemical compositions and particle size, the change of average density will slightly affect the pollutant concentration and thus change the extinction coefficient per unit volume, and will not significantly change the following research.

![Figure 4. Diameters of equivalent spheres of particle pollutant [4]](image)

After processing the data in figure 4, we can get the following Table 1.
### Table 1. The proportion of equivalent spherical diameter of particle pollutants

| Diameter/μm | Percentage/% | Diameter/μm | Percentage/% |
|------------|--------------|-------------|--------------|
| 0.05       | 1.0          | 1.05        | 1.0          |
| 0.15       | 7.0          | 1.15        | 1.0          |
| 0.25       | 16.0         | 1.25        | 1.0          |
| 0.35       | 18.0         | 1.35        | 1.0          |
| 0.45       | 15.0         | 1.45        | 1.0          |
| 0.55       | 10.0         | 1.55        | 1.0          |
| 0.65       | 7.0          | 1.65        | 1.0          |
| 0.75       | 5.0          | 1.75        | 1.0          |
| 0.85       | 3.0          | 1.85        | 1.0          |
| 0.95       | 3.0          | 1.95        | 1.0          |

The refractive index of pollutants with different chemical compositions is also different. Major air pollutants such as sulfates, nitrates, soil, etc., have refractive indices ranging from 1.3 to 1.7, mostly from 1.4 to 1.6. As the proportion of different substances in air pollutants has not been accurately measured in real-time, it is taken as 1.5 in following discussion. By knowing concentration of pollutants and the average density, the total volume of pollutants is known. Then, distribute the total volume to different pollutants based on their size distribution, and eventually use the typical refractive index and cross-section to calculate the change of $k_p$ along with wavelength $\lambda$. My approximate calculation uses one typical pollutant density and refractive index, resulting in a deviation in the absorption coefficient of about 5%. In the following practical application, detailed data from meteorological observatory about size and concentration of particles should be used.

![Figure 5. the change of $k_p$ with wavelength $\lambda$](image)

### 2. Materials and Methods

#### 2.1. Materials and Experiment Conditions

In order to compare the Model with the actual situation, I observed the lunar surface in Shanghai on the midnight of September 9th, 2020. The telescope diameter is 150mm, and the camera is cMOS camera (astronomy enthusiasts level accuracy). The actual assembly of telescope and camera is shown in Figure 6. The experiment took place on the rooftop of a six-story building in Pudong, Shanghai. Due to a small
amount of cloud in the early night, the actual observation time was from 1:30 to 5:20 am. During the observation, the exposure time of each photo was set at 3 milliseconds, a time which can produce clear lunar image and will not lead to non-linear response interval. All gain and white balance features are turned off to ensure that the count on each image has not been modified by software to reflect the brightness of the moon's surface. The time interval of photos was about 15-20 minutes, and the time of taking each photo was recorded. Combined with the latitude and longitude of the observation site, the altitude angle of the moon at the corresponding time can be calculated strictly. During observation, the PM2.5 and PM10 concentrations predicted by nearby environmental monitoring observatories were recorded in real time: the average PM2.5 concentration was 20 μg/m$^3$, and the average PM10 concentration was 35 μg/m$^3$. The total cross-section of the pollutant per cm$^3$, namely the absorption coefficient, obtained by calculation in Section 1. Figure 6 shows some of the actual photos taken.

Figure 6. Actual assembly of the telescope and the camera

2.2. Processing Method

In this study, AstroImageJ is used to calculate the counts of three bands of RGB of the lunar surface at different angels. By measuring one θ, the R, G and B counts difference corresponds to the angles can be obtained. After knowing ratios of a particular RGB set, we can fit several groups of $k_\theta * h$ and counts and estimate the absorption coefficients of the three wavebands. The most integrable one can be obtained after fitting with the least square method.

First, separate the photo into R, G, B waveband, respectively, by code from Python Library.
Software “AstroimageJ” was used to obtain the counts in photos in R, G, B wavebands. Then, manually choose surface moon area (area inside the innermost red circle), background count (area between the second and the third red circle) in the software AstroImageJ, as shown in figure 9. The software can automatically calculate the median of background count, and minus it from the lunar surface, thus generating counts of each waveband and errors.
Table 2. Counts of RGB wavebands along with altitude angle θ

| θ/°  | R Waveband/counts          | G Waveband/counts          | B Waveband/counts          |
|------|----------------------------|----------------------------|----------------------------|
| 54.57| 1142410.000(1075.314)      | 1141934.702(1075.189)      | 1142270.544(1075.270)      |
| 57.24| 1148732.983(1076.789)      | 1145632.015(1075.621)      | 1147837.405(1076.571)      |
| 61.35| 1166631.091(1090.186)      | 1148863.415(1076.040)      | 1156841.905(1064.257)      |
| 67.38| 1172257.420(1096.818)      | 1168394.985(1097.862)      | 1172264.000(1096.800)      |
| 71.21| 1176480.627(1091.202)      | 1165356.445(1033.195)      | 1176770.010(1078.445)      |
| 74.17| 1194287.000(1100.044)      | 1192248.699(1124.855)      | 1178396.825(1116.703)      |
| 76.36| 1194287.000(1121.772)      | 1192248.699(1124.855)      | 1178396.825(1116.703)      |

\[ \tau_\nu \approx k_\nu s \]

With the increase of the altitude angle, the brightness of the lunar surface increases correspondingly, which is in line with the expectation that the increase of the thickness of pollutants the light travels. Based on the data, by using the Least Square Method (detailed in Appendix C), a graph of altitude angle versus actual counts (points) and the fitting result (dashed line) is shown.

![Figure 10. Altitude angle versus actual counts (points) and the fitting result (dashed line)](image)

As optical depth \( \tau_\nu = k_\nu s \), we can obtain height of pollutants according the the result.

Table 3. Pollution heights estimated by the extinction values of R, G, B wavebands

| R Waveband | G Waveband | B Waveband |
|------------|------------|------------|
| h/m        | 3381.923   | 3391.687   | 3044.452   |

The pollutants height of the three bands is little over 3km based on the experiment data. According to relevant studies, the typical elevation of pollutants is approximately 3-5km, which is consistent with the measured value.

3. Conclusion and Evaluation

3.1. Conclusion

In this study, a particle scattering model was first established to simulate the Mie scattering of pollutants, and then the actual data was obtained by an astronomical telescope to measure the RGB color change
of the moon at different altitude angles. The measured results were combined with the model calculation to obtain an estimated pollutant thickness of about 3km in the atmosphere of the site. The results, according to relevant studies, are significant.

3.2. Weakness
The main unstable factor in the research is the weather. The experiment was done in summer, when it’s mostly cloudy and rainy, and there was a lack of consecutive sunny days. The low night sky was also whiter than night sky at the zenith, which may be due to the presence of high cirrus clouds. Cirrus clouds are common clouds in subtropical areas that extend hundreds to thousands of kilometers, and are common in subtropical areas during the summer, and almost invisible at night.

These cirrus clouds add a large number of scattering particles to the atmosphere, which will significantly increase the optical depth and estimated pollutant thickness. While there were no visible low-altitude clouds blocking the moon during the shooting, the amount of water vapor in the air might have been too high, or there may have been a few cirrus clouds. In addition, as the shooting process requires the moon to rise gradually over several hours, the cloud may also change significantly during this period, which adds to the uncertainty of weather conditions. Pollutions in China is more serious in winter and Northern areas, where further experiments can be done to compare and evaluate the effect of high-altitude clouds.

3.3. Possible Improvement and Further Application
For sure, the use of celestial objects to measure atmosphere pollutants is greatly affected by the weather and needs to combine the further composition and size experiments to produce comprehensive data to be applied, but the single point sample analysis ability is hard to replace. The physics involved in the measurement process is very simple, and the main complexity comes from the change of clouds and the uncertainty of pollutant composition. In the future, with the improvement of national monitoring capability, chemical analysis stations in many cities can effectively measure the composition and size distribution of pollutants. Combined with our extinction measurement results of celestial objects, more accurate measurement results can be obtained.

The absolute flow of a large number of stellar objects above the atmosphere has been precisely measured by astronomy, and they are distributed throughout the day, and there are no changes in the brightness caused by changes in the lunar phases. A small telescope with a diameter of 5-10cm and an astronomical camera about 1,000 yuan are enough to measure the brightness of thousands of stars with an accuracy of 1%. In sunny weather, when the chemical composition and size distribution of pollutants are relatively accurate, the pollutant height can be obtained by photographing stars in different directions with a single telescope within a few minutes, determining the distribution of pollutants over the city. Measurement by a single telescope can only obtain the accumulated pollutant height in a sight line. If there are multiple observatory stations in a city, it is possible to construct a three-dimensional distribution model of pollutants, which is difficult to obtain only from ground air sampling and analysis. In addition, the follow-up analysis of the measurement of celestial objects mainly uses software to process the photos and images, which can be completed in seconds. If measured continuously at night, the spatial distribution of pollutants can be obtained in real time, and then the source and diffusion process of pollutants can be determined.

4. Appendix

4.1. Atmospheric Extinction
The atmosphere is not completely transparent to electromagnetic waves from space. For example, the inner electron energy level of nitrogen and oxygen atoms is in the X-ray band can absorb X-ray photons; Atmospheric ozone can strongly absorb ultraviolet rays; the upper atmosphere is ionized by solar radiation and high-energy particles, forming a plasma layer and shielding the radio band above 10 meters, and so on.
Considering the composition and height of the atmosphere, the atmosphere is transparent only in the visible band, part of the infrared band (1-22micron) and the radio band (1mm-30m). These three bands are called atmospheric window in astronomy.

In the atmosphere window, there is also some interaction between electromagnetic wave and atmosphere. These interactions generally cause light from celestial bodies to travel less in the original direction as it passes through the atmosphere, known as atmospheric extinction. For visible light band, in the absence of air pollutants, the most important extinction process is Rayleigh scattering of visible light by atmospheric molecules. Because Rayleigh scattering is inversely proportional to the fourth square of wavelength, the extinction magnitude of short wavelength light is significantly greater than that of long wavelength light. This effect not only reduces the light from the celestial body, but also makes the celestial body appear redder than its true color, so it is called the celestial reddening in astronomy. Both extinction and reddening size are positively correlated with the height through the atmosphere, so the sun is redder at sunset and sunrise than it is at noon (the altitude angle is low).

In the case of air pollution, pollutants are close to the wavelength of visible light (0.1-10μm). It is roughly estimated that the extinction value caused by the pollutants is equivalent to the Rayleigh scattering extinction value of the whole atmosphere under the typical pollutant concentration in cities.

More than 70% of pollutants is concentrated in the troposphere, at a height of about 20km.

Therefore, the ratio of gravitational acceleration at the top of the troposphere versus that at the bottom is:

\[
\frac{(6341)^2}{(6341+20)^2} = 0.99372
\]  

The difference between the two is less than 0.6%, so the difference can be ignored in the calculation and the value of surface gravity acceleration can be directly used.

One standard sea level atmospheric pressure is taken as:

\[
10^5 \text{ Pa} \approx g \ast 10^4 \text{kg} \ast \text{m}^{-2} = g \ast 1 \text{kg} \ast \text{cm}^{-2}
\]

The mass of a \( N_2 \) molecule is \( 4.63 \ast 10^{-26} \text{kg} \). If atmosphere is made of \( N_2 \) molecules, then there is \( \frac{1}{4.65312 \ast 10^{-26}} = 2.14909 \ast 10^{25} \) \( N_2 \) molecules needed on 1cm\(^2\) to generate a standard atmospheric pressure. If the atmosphere is made of 80\%\( N_2 \) and 20\%\( O_2 \) molecules, the mass of \( O_2 \) molecule is \( 5.3 \ast 10^{-26} \text{kg} \), there need \( \frac{1}{(0.8 \ast 4.65312 + 0.2 \ast 5.31486) \ast 10^{-26}} = 2.08965 \ast 10^{25} \) mixture molecules of \( N_2 \) and \( O_2 \). Their difference is much less than 1\%. In addition, the molecular size of \( N_2 \) is 65pm, the molecular size of \( O_2 \) is 60pm, the difference is also very small. Therefore, we can approximate the air is all made of \( N_2 \) molecules to do approximate calculation.

In the case of known area and refractive index, the cross-section of a single molecule can be calculated by Mie scattering python program. Since the total cross-section of the atmosphere in the optical band is very small, mutual shielding between \( N_2 \) molecules can be ignored, so the cross-section of a single molecule multiplied by number of molecules equals to the total cross-section of the molecules of pollutants. The calculation results are shown in the blue curve in Figure 11.

Use the pollutant density 15μm/m\(^3\) to calculate the extinction value. Assuming the pollutant thickness is 200m, we can draw the extinction value of total cross-section versus the extinction value of Rayleigh cross-section, shown in Figure 11.
Figure 11. Comparison of extinction value of pollutant cross-section versus extinction value of Rayleigh scattering of 200m thick atmosphere (orange: extinction value of particle pollutants at 200m; Blue: extinction value of Rayleigh scattering)

It shows that, under good air quality, the extinction value of particle pollutants in visible band is significantly greater than that caused by Rayleigh scattering in the whole atmosphere. In cities, the height of buildings is generally more than 100 meters, and the particle pollutants generated on the ground are easy to diffuse to 100 meters under low wind speed. Therefore, when considering urban air particle pollutants, especially in camera R and G bands, except in the case of extremely good air index, the background of extinction caused by atmospheric scattering can be ignored.

4.2. Shooting Settings

The telescope in the study is a hyperboloid Newton's reflecting telescope produced by RuiXing company and its primary mirror diameter is 150 mm (product page: http://www.sharpstar-optics.com/index.php/150F28HNT/239.html). As shown in figure 12, the primary mirror of the telescope (hyperboloid mirror) is located at the bottom of the lens barrel. After the parallel light from celestial bodies is gathered, the image of celestial bodies is reflected to the side of the lens barrel through the attached mirror at the front end of the lens barrel for imaging. Therefore, the camera needs to be configured on the side of the lens barrel. The telescope is fixed by a manual equatorial instrument to find the position of the moon manually during observation and fine-tune the telescope to keep the lunar surface close to the center of the field of view over time.

Figure 12. The inner structure of telescopes

The camera used is a full-frame color astronomy camera produced by Suzhou Photoelectric Production company (product page: http://zwoasi.com/product-detail/asi6200mc-pro-%e5%85%a8%e7%94%bb%e5%b9%85-%e5%a4%a9%e6%96%87%e7%9b%b8%e6%9c%ba). The camera's imaging chip is a Sony's full-frame chip, with each pixel measuring 3.76 microns in size, a readout noise of 1.2e and a saturation count of 51,000.
In the actual observation, the count of single pixel is hundreds to thousands, which can be considered to be in a good linear response interval. The response curve of camera chip provided by the manufacturer is shown in Figure 13, and the equivalent peak wavelength of RGB tri-colors is 460nm, 540nm and 620nm. As the absorption coefficient calculated by the scattering model is relatively smooth in the optical band, the calculation ignores the variation of the response curve within the bandwidth and takes the value of the absorption coefficient curve at the equivalent peak value, which is a good approximation as the average of the whole band.

At work, the camera is connected to the home computer via usb3.0 port and is driven by software provided by the company. The shortest exposure time of the camera is 32 microseconds, which is achieved through the electronic system of the camera in a relatively short time to generate an electronic count of incoming photons. The actual exposure time of a single photo is 3 milliseconds, much longer than the minimum exposure time, which can ensure a good stability of the actual exposure time of each image.

![Figure 13. The response curve of camera chip [5]](image)

4.3. Least Square Method

Assuming there is a set of experimental data \((x_i, y_i)\), and we know in advance that it satisfies some functional relationship, but the function contains a set of parameters, and we need to determine the best estimate of the parameters through a limited number of experimental data points. If \(P\) is used to represent the parameters to be determined, then the problem can be transformed to find a set of \(P\) to minimize the value of the function \(S\):

\[
S(P) = \sum_{i=1}^{n} [y_i - f(x, P)]^2
\]

This process is called least square fitting.

For example, use the above principle for point (1,1) and point (2,2) fitting. Let the equation of this line be \(y = ax + b\), so that:

\[
S(P) = (ax_1 + b - y_1)^2 + (ax_2 + b - y_2)^2
\]

Is the smallest. We integrate \(a\), \(b\):

\[
\frac{\partial S}{\partial a} = 2(ax_1 + b - y_1) + 2(ax_2 + b - y_2) = 0
\]

\[
\frac{\partial S}{\partial b} = 2(ax_1 + b - y_1) + 2(ax_2 + b - y_2) = 0
\]

Then get:

\[
\begin{cases}
10a + 6b = 10 \\
6a + 4b = 6
\end{cases}
\]

Solve:

\[
\begin{cases}
a = 1 \\
b = 0
\end{cases}
\]

That is, the \(P\) that minimizes \(S\) in this example is \((1, 1)\), and the equation is \(f(x) = x\).
Under the specific measurement condition, the count of lunar surface photos at different altitude angles is measured. Then, $y_i$ is the count of lunar surface brightness $C_i$, $x_i$ is the altitude angle $\theta_i$.

According to the radiation transfer equation, assuming the original count of lunar surface brightness is $C_0$ (when the camera is placed beyond atmosphere), the thickness of atmosphere is $h$, the average extinction coefficient at some waveband is $k$, $C_i$ and $\theta_i$ should satisfy the function relation:

$$\frac{C_i}{C_0} = e^{-kh \sin(\theta_i)}$$  \hspace{1cm} (23)

In this function relation, the unknown parameters are $C_0$ and $kh$, and the measurement is $(\theta, C_i)$.

In practice, the best estimate of $C_0$ and $kh$ can be obtained by using the least square fitting with some input of $C_i$ and $kh$. It is assumed that extinction is mainly caused by atmospheric particulate pollutants. Under certain chemical composition and particle size distribution, it can be directly calculated from pollutant concentration, so that pollution-free characteristic height $h$ can be estimated by measuring the brightness (counts) of celestial bodies at different altitude angles.

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