A raster polishing path-planning method based on the untethered magnetic manipulation polishing

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Abstract. In order to improve the precision of untethered magnetic manipulation polishing of material workpieces, this paper proposes a new surface path-planning method for free-form surfaces based on the refinement of the conventional polishing path method and determination of its relevant parameters. The results obtained and proposed optimization rules also make it possible to minimize the regular scratches and uneven polishing caused by the path itself in a conventional raster-shaped path and reduce the spatial error.

1. Introduction
During the polishing process, the path of the inner cavity of the workpiece and the polishing area are required to be routed and driven accordingly [1]. As an important factor in the polishing process, the polishing path directly affects the processing quality, efficiency and service life of the polishing device [2]. The spatial error caused by the polishing path superposition is an important factor affecting the polishing result [3]. Therefore, in order to achieve higher processing efficiency and accuracy, it is necessary to rationally plan the polishing path [4]. In this paper, the polishing path is theoretically analyzed for the area polishing of the free-form surface [5]. The surface-type grating-shaped polishing path is planned, the consistency of each polishing point under the path is analyzed and verified [6], and the device is proved to be able to drive the polishing tool along the path using the industrial robot technology.

2. Determination of the maximum spacing value of the path
Raster-shaped path often brings a certain regular spatial error when it is polished multiple times. Therefore, it is necessary to design and optimize the parameters based on the traditional grating-shaped path research to minimize the regular scratches and uneven polishing caused by the path itself. In the meanwhile, reduce the generation of spatial errors.

In a raster-like curved path, the maximum spacing value D of the trajectory is a very important parameter. In theory, the maximum spacing value D of the polishing path cannot exceed the diameter of the polishing tool \( d_m \), otherwise leakage will occur. Therefore, the relationship between the two is:

\[
D = \alpha \cdot d_m ,
\]

where \( \alpha \) is the polishing pitch factor and \( \alpha \in (0, 1] \).

The cylindrical polishing tool differs in polishing efficiency due to the difference in line speed at
various points of polishing during polishing. In addition, due to factors such as the shape and curvature of the surface of the workpiece being machined, as shown in figure 1, the effective polishing areas of the different forms of the surface are completely different during polishing, and the polishing forces of the polished points are also different. Efficiency is also different. Therefore, in the process of path planning, in order to reduce the unevenness of polishing due to the difference in polishing efficiency of each point and the change in the shape or curvature of the workpiece, the distance between the adjacent two polishing paths should be minimized, that is, decrease the value of $\alpha$. However, when the value of $\alpha$ is too small, the polishing time will be too long. In addition, it should be ensured that $d_m$ is an even multiple of $D$ to ensure the uniformity of the processing trajectory. Therefore, we choose $\alpha = 0.5$ and $D = 8\text{mm}$.

![Figure 1](image1.png)

**Figure 1.** Schematic diagram of the deformation of the polishing tool when processing various surfaces. (a) Convex surface, (b) Flat and (c) Concave surface.

3. **Determination of polishing offset and polishing times**

In this paper, the offset polishing method is used for multiple polishing, which is to shift the path of each polishing to a certain extent on the basis of the previous path, and each polishing starts from different directions to balance the polishing efficiency due to each point. The basic principle of the resulting polishing unevenness is shown in figure 2.

![Figure 2](image2.png)

**Figure 2.** Offset polishing track.

Assuming that the polishing offset is $t$ each time, a total of $m$ times is polished. In order to maintain uniformity at each point of the polishing area, the following formula needs to be satisfied:

$$t \cdot m = xD$$  \hspace{1cm} (2)

4. **Determination of the residence time of the polishing tool**

In this paper, the matrix solution method is used to solve the dwell time. The material removal function of untethered magnetic manipulation polishing for polishing a planar workpiece is:
\[ h = \frac{8.4 \mu_j |M||m| \cos \left( \tan^{-1} \frac{1.08r}{2|p|} \right)}{20\pi^2 |p|^2 r_n^2} \quad (3) \]

From equation (3), there is a linear relationship between the material removal function (removal rate) and the distance \( r \) between the discrete machining points and the center of the polishing tool. When the polishing tool speed \( \omega = 600 \text{rpm} \) and the distance \( |p| = 5 \text{cm} \), the relationship between the two is \( h = -8.8678r \times 10^{-7} \text{ (m/s)} \).

In the matrix solution method, a series of surface error points are obtained by discretizing the surface shape error of the inner surface of the workpiece. The total number of error points is \( M \), and the coordinates of each error point are \((x_i, y_i) (i \in [1, M])\). During the polishing process, the polishing tool is required to remove the workpiece at discrete machining points. The total machining point is \( N \), and the coordinates of each discrete machining point are \((x_j, y_j) (j \in [1, N])\). In order to facilitate control and calculation, it is common to select several evenly distributed face error points as discrete machining points.

If a point \( K(x_k, y_k) \) is within the effective polishing area \( S \) of the discrete processing point, then the material removal amount \( h_{ij} \) for each discrete machining point is:

\[ h_{ij} = h_{ij} \cdot t_j \quad (4) \]

If a point \( K(x_k, y_k) \) is within the effective polishing area \( S \) of the discrete machining point, the removal amount is \( h_{ij} \). If the point is outside the effective polishing range, the removal amount is 0:

\[ \Delta h_k = \begin{cases} h_{ij} = h_{ij} \cdot t_j, & K \in S \\ 0, & K \notin S \end{cases} \quad (5) \]

When the polishing tool walks through the entire discrete machining point along the established path and dwell time, the material removal at a point in the workpiece is:

\[ h = \sum_{j=1}^{N} \Delta h_k \quad (6) \]

If a point removal is written as a matrix, then there are:

\[ h = A \times t \quad (7) \]

\[
\begin{bmatrix}
  h_1 \\
  h_2 \\
  \vdots \\
  h_M
\end{bmatrix} =
\begin{bmatrix}
  h_{11} & h_{12} & \cdots & h_{1N} \\
  h_{21} & h_{22} & \cdots & h_{2N} \\
  \vdots & \vdots & \ddots & \vdots \\
  h_{M1} & h_{M2} & \cdots & h_{MN}
\end{bmatrix} 
\begin{bmatrix}
  t_1 \\
  t_2 \\
  \vdots \\
  t_N
\end{bmatrix}
\]

(8)

For surface machining, the polishing efficiency, effective polishing area and other parameters will change due to the shape and curvature of the surface. In the calculation process, it is necessary to consider the effective polishing field at each processed point in the effective polishing area. In the above design of the polishing path, the method of offset path is used to optimize the polishing unevenness in the polishing process. Except for the special point of special processing demand, other parts are polished with quasi-static low speed, in order to reduce the effects of regular errors due to uneven polishing of the polishing tool itself. The theoretical analysis of the basic planar polishing in the dwell time solution.
is performed, in which case the variation of the effective polishing area due to the change of the curved surface can be ignored.

![Figure 3. Single quasi-static polishing schematic.](image)

Firstly, the amount of material removed at a distance $i \left(0 \leq i < \frac{r_m}{4}\right)$ from the polishing path under a single polishing was analyzed. During the travel of the polishing tool, the value of the distance $r$ between the machining point and the center of the polishing tool is constantly changing, and the change value is $r$, as shown in figure 3. Initially, the amount of material removed at a distance $i$ from the polishing path under a single polishing was analyzed. During the travel of the polishing tool, the value of the distance $r$ between the machining point and the center of the polishing tool is constantly changing, and the change value is $r \in [i, r_m]$, as shown in figure 3.

The path $s$ that the polishing tool walks through when polishing the machining point is:

$$s_i = 2\sqrt{r_m^2 - i^2}$$  \hspace{1cm} (9)

The dwell time of the polishing tool at this processing point is:

$$t_i = \frac{s_i}{v} = \frac{2\sqrt{r_m^2 - i^2}}{v}$$  \hspace{1cm} (10)

The material removal of the processing point under a single polishing is:

$$h_i = \frac{\int_0^\pi 8.4 \mu_0 |M| \mu_0 (i + r_m) \sqrt{r_m^2 - i^2} \cos \left(\tan^{-1} \frac{1.08 r_m}{5 |p|}\right)}{20 \pi i^2 |p| r_m^2}$$  \hspace{1cm} (11)

According to the polishing path offset $t = \frac{D}{4} = \frac{r_m}{4} = 2$mm selected in the previous section, the processing point has been polished 8 times after multiple polishing, and the distance between the polishing point and the polishing path is $i + \frac{3}{4} r_m, i + \frac{1}{2} r_m, i + \frac{1}{4} r_m, i, -i + \frac{1}{2} r_m, -i + \frac{1}{4} r_m, -i + \frac{3}{4} r_m, -i + r_m$ respectively. Then the total material removal amount $h$ of the processing point is:

$$h = \sum h_i$$  \hspace{1cm} (12)

therefore, after determining the total amount of material removed, the dwell time $t$ can be obtained, thereby obtaining the quasi-static motion velocity $v$ of the polishing tool.

At the polishing tool speed $\omega = 600 \text{rpm}$, the distance $|p| = 5$cm, and the polishing tool motion speed $v = 0.01 \text{m/s}$, substitute the constant value in the formula (12), and the total removal amount $h$ when $0 \leq i < 2$cm can be obtained by the Matlab software programming. As the trend of $i$ changes, as shown
in figure 4, the x-axis is in meters and the y-axis is in microns. It can be seen from the figure that the total removal amount $h$ has a small tendency to change with $i$, and the value of each point removal amount is relatively balanced, both being about $5\mu m$. If you want to further optimize the uniformity of the total removal, you can continue to reduce the value of the path offset, but the required polishing time will also increase exponentially.

![Figure 4. Relationship between total removal amount $h$ and distance $i$.](image)

5. **Planar polished grating path verification test**

During the polishing process of the free-form surface, the posture of the polishing tool needs to maintain a dynamic change due to the change of the curved surface. In the theoretical calculations in Chapter 2, it has been verified that the polishing tool has a tendency to keep coaxial with the external permanent magnets, which makes it possible to drive the polishing tool by external permanent magnets. In this section, the polishing tool is raster-driven on and in the placed plates to verify that the polishing tool can move along the grating path in three poses: horizontal, oblique, and vertical.

The planar path driven test setup is shown in figure 5. In the plane polishing, it is not necessary to adjust the azimuth of the external permanent magnet by the industrial robot, only the movement of the industrial robot on the x-axis, the y-axis and the z-axis is controlled; the speed of the external permanent magnet is $0.01m/s$, and the distance between the polishing tool and the external permanent magnet is $5cm$. The test results are shown in figure 6. The results show that the polishing tool can be moved in the form of a grating-shaped trajectory by external permanent magnets in three postures, and the polishing tool maintains a stable state during low-speed uniform speed operation.

![Figure 5. Plane grating path verification test device. (a) 0° plane, (b) 45° plane and (c) 90° plane.](image)
6. Conclusion

In this paper, based on the research of the traditional polishing trajectory method, a planning method of grating-like curved path is proposed for the free-form surface based on the grating path. The maximum spacing value $D=8$ mm is determined under the premise of ensuring no over-polishing and leakage-polishing phenomenon and the polishing path is uniform; the offset path polishing method is proposed to perform multiple polishing on the workpiece to reduce the surface error caused by multiple polishing unevenness. And determine the offset $t=2$ mm and the number of polishing $m=4$. According to the workpiece material removal function, the relationship between the total material removal amount $h$ and the residence time $t$ is determined, and the removal amount $h = 5 \mu m$ is calculated when the movement speed of the polishing tool during quasi-static operation is $v = 0.01 m/s$.

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