Link recommendations: Their impact on network structure and minorities

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ABSTRACT
Network-based people recommendation algorithms are widely employed on the Web to suggest new connections in social media or professional platforms. While such recommendations bring people together, the feedback loop between the algorithms and the changes in network structure may exacerbate social biases. These biases include rich-get-richer effects, filter bubbles, and polarization. However, social networks are diverse complex systems and recommendations may affect them differently, depending on their structural properties. In this work, we explore five people recommendation algorithms by systematically applying them over time to different synthetic networks. In particular, we measure to what extent these recommendations change the structure of bi-populated networks and show how these changes affect the minority group.

Our systematic experimentation helps to better understand when link recommendation algorithms are beneficial or harmful to minority groups in social networks. In particular, our findings suggest that, while all algorithms tend to close triangles and increase cohesion, all algorithms except Node2Vec are prone to favor and suggest nodes with high in-degree. Furthermore, we found that, especially when both classes are heterophilic, recommendation algorithms can reduce the visibility of minorities.

CCS CONCEPTS
• Information systems → Social networks; Recommender systems.

KEYWORDS
Recommendation algorithms, friendship recommendations, network science, social networks, homophily, preferential attachment.

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1 INTRODUCTION
Social networks are the infrastructure of our social and professional life. They impact, among others, our cooperation [18], our health [8], and our social perceptions [24]. The structure of modern online social networks is however not only shaped by well-studied social mechanisms (such as homophily or preferential attachment), but it is also affected by people recommender systems, complex algorithms that suggest new connections among social network users. How do these algorithms affect the structure of social networks over time? What are the consequences for different groups?
In this paper, we aim to shed light on these questions.

Problem: Previous work has shown that recommendation algorithms are prone to reinforcing popularity bias [1]. A further subtle problem is that by matching users' preferences, these algorithms often lead to the formation of filter bubbles [7], echo chambers [5], and polarization [11]. In recent years, much attention has been paid to understanding when, and to what extent, such biases are being amplified. As an example, [15] and [14] have studied the correlation between network structure and the output of ranking algorithms in social networks. While these studies highlight that homophily—the tendency to connect to similar others—and preferential attachment—the tendency to connect to those that are already well-connected—are important structural factors that impact the visibility of nodes in algorithmic rankings, they do not compare effects over time. Feedback loops, instead, have been studied in [34].
and [33], where they respectively analyze “rich-get-richer” and “glass ceiling” effects. Recently, also [16] and [9] have focused on feedback loops and long term effects of people recommender systems. The former analyzes inequalities in the exposure of minorities and the latter focuses on polarization and echo chambers. Our study integrates this body of research by providing a systematic analysis of how homophily and minority size relate to structural properties of the network and visibility of groups.

**Approach:** We systematically compare five recommendation algorithms and apply their recommendations to several synthetic networks. We focus on scale-free directed networks with adjustable homophily and minority group size [14] and quantify the global changes in network structure, as well as the changes in connectivity for the minority group over time. In particular, we assess whether certain types of links are created more often than others and whether the network becomes more cohesive or segregated. Similarly, we verify when, and at what rates, these algorithms put minorities at disadvantage by measuring the changes in their visibility, here defined as the fraction of minorities among the top most important nodes, based on their algorithmic ranking. To this end, we formulate the following research questions that will guide our analysis throughout this paper.

- **RQ1:** How do recommendation algorithms affect the structure of the network and the visibility of minorities?
- **RQ2:** To what extent is the change in visibility due to homophily?
- **RQ3:** Is the change in visibility inversely proportional to the size of the minority or proportional to the in-group links within the minority?

**Contributions:** Our contributions are the following: (1) We demonstrate that networks become more cohesive over time throughout multiple recommendations. However, the rate at which this cohesiveness gets stronger depends on the algorithm. (2) Not all algorithms suffer from the popularity bias problem, which means that certain algorithms may diversify their recommendations. (3) The visibility of the minority group gets affected differently depending on three main components: the algorithm, the initial conditions of homophily in the network, and the size of the minority group.

Moreover, our study sheds light on the weaknesses of algorithms under the initial conditions of network structure and can be used as key factors to improve recommendations, where necessary.

2 RELATED WORK

The related work is organized in two parts. First, we introduce the relevant literature on the mechanisms that drive the existence of biases in network structure. Then, we focus on the creation of new ties from link recommendations. In particular, we highlight the effects of recommendation algorithms on the network structure and the visibility of minorities.

**Biases in network structure and related consequences:** The rich-get-richer or Matthew effect [28] is one of the first mechanisms of edge formation discovered by sociologists to explain cumulative advantages in real-world networks. From the network perspective, the Matthew effect operates through the preferential attachment mechanism, that is the tendency of nodes to attach preferentially to those that are already well-connected [4]. This mechanism of edge formation and other structural characteristics may impact the visibility and importance of nodes, and thus, create and enlarge inequalities. For example, [3] and [23] propose mathematical models that integrate preferential attachment and homophily (the tendency to connect to similar others [27]) to explain the emergence of the “glass ceiling” effect in social networks. Glass ceiling, as defined by the US Federal Commission, is “the unseen, yet unbreakable barrier that keeps minorities and women from rising to the upper rungs of the corporate ladder, regardless of their qualifications or achievements”. Studies on glass ceiling are expanded in [30], where the authors consider the effect of the perceived gender on the visibility of users on Twitter. In particular, they reveal how users perceived as women are hampered from attaining equal visibility. Furthermore, [23] shed light on how homophily can put minority groups at disadvantage by restricting their ability to establish links with the majority group and by limiting their access to information.

Recently, [35] observed that PageRank [31] might unfairly allocate importance scores to different classes, and proposed alternative fair versions of the algorithm. Our work is built upon this body of literature and integrates social biases in feedback loops of people recommendations. In particular, we analyze the tendencies of groups to connect to each other, how these tendencies or mechanisms of edge formation affect the recommendations, and ultimately how these recommendations affect the structure of networks and the visibility of minorities.

**Effects of recommender systems on networks:** [34] analyzes the “rich-get-richer” phenomenon through social recommendations. In particular, they study how the “Who-to-Follow” algorithm affects
the structure of the follower network on Twitter. They found that most popular users profited substantially more than average from the user suggestions. They attributed this "rich-get-richer" effect to various factors, including the mismatch between users (being recommended proportional to their degree), and the baseline growth rate of users (whose asymptotic behavior is instead sub-linear in the degree). Users’ centrality and clustering coefficient may also vary depending on the recommendation algorithm in “Social-Blue,” an internal social networking site at IBM [10]. Similar effects have been found in Tumblr and Flickr, two social media platforms, where recommendations favor popular and well-connected nodes, and at the same time limit the growth of the diameter of the network [2].

In addition to these topological effects, social recommendations may also exacerbate the under-representation of certain demographic groups in the network. For instance, [14, 15, 33] show how the visibility of minorities can be amplified or mitigated by different levels of homophily within groups when using recommendation algorithms on scale-free networks. These inequalities have been also studied over time but only recently. [16] suggests that while the homophily level of the minority affects the speed of the growth of their disparate exposure, the relative size of the minority affects the magnitude of this effect.

One of the main differences between this body of research and our work is that we vary homophily systematically. This allows us to better understand the relationship between the initial homophily of the network and the long-term effects of the recommendations. In particular, to what extent they change network structure and the visibility of minorities over time. Moreover, we study Node2Vec [20], a more recent algorithm used to generate link recommendations through node embeddings.

3 METHODS

3.1 Directed networks
We consider attributed directed networks of the following form: let $G = (V, E, C)$ be a node-attributed graph where $V = \{v_1, .., v_n\}$ is a set of $n$ nodes, $E \subseteq V \times V$ is a set of $e$ unweighted directed edges, and $C : V \rightarrow \{0, 1\}$ is a function that maps each node $v_i$ into its group (or class) membership $c_i$. For the sake of simplicity we focus on binary group membership (e.g., black/white or male/female). The function $C$, hence, divides the nodes into two groups, a minority, called $m$, and a majority, called $M$. We refer to the fraction of the minority group in the network as $f_m$.

Further definitions, peculiar to the synthetic network generation model employed, are provided in Section 3.3.

3.2 Recommendation algorithms
In this section, we define the five recommendation algorithms of interest. All algorithms are class agnostic which means that their recommendations are solely based on topology. Note as well that for each node $v_i \in V$, the recommendation algorithm suggests a ranked list of $k$ nodes that $v_i$ is not yet connected with. The ranked list is sorted in descending order in terms of relevance scores according to each algorithm. In the case of ties, where multiple nodes are equally relevant, nodes are chosen randomly.

We refer the reader to Section 3.3 for the details on the configuration of hyper-parameters for each algorithm.

**Personalized PageRank (PPR):** It is an extension of PageRank to rank nodes in a network from the perspective of a seed node [31]. In principle, random walks are performed and restarted at the origin (or seed node) multiple times to update the importance score of all nodes, see Figure 2(a). We compute the PPR vector $\pi_i$ with respect to each node $v_i \in V$ as follows:

$$\pi_i^T = (1 - \alpha)e_i^T + \alpha \pi^T W$$  \hspace{1cm} (1)

where $\alpha$ is the probability of following links, $e_i$ denotes the personalized one-hot vector, $W$ is the transition matrix inferred from $G$ and $T$ represents the transpose operator. The ranking score given to node $v_j$ is then the $j^{th}$ component of $\pi_i$.

**Who-to-follow (WTF):** This algorithm, proposed by Twitter [21], suggests users who are followed by people that are similar to the one getting the recommendation, see Figure 2(b). For each user $v_i$, the algorithm looks for its circle of trust, which is the result of an egocentric random walk (similar to personalized PageRank [22]). Then, based on this circle-of-trust $COT_i$, WTF ranks (using the SALSA algorithm [25]) users that are not yet friends with $v_i$ but are connected through the circle of trust $COT_i$.

$$WTF_i = SALSA(COT_i, \pi_{COT_i}^{out})$$  \hspace{1cm} (2)

$^{1}(e_i)_i = 1$ and $(e_i)_j = 0, \forall j \neq i$
Two-hops (2H): This algorithm follows the intuition behind friends-of-friends. In directed networks, the 2H algorithm recommends nodes \( v_j \) that are at a distance 2 from node \( v_i \), see Figure 2(c). The more such paths, the more likely the recommendation. Calling \( \Gamma_i^{out} \) the set of nodes that \( v_i \) points towards (i.e., out-links), and \( \Gamma_i^{in} \) the set of nodes pointing to \( v_i \) (i.e., in-links), we define the 2H score function as the number of possible paths of length 2 from \( v_i \) to \( v_j \):

\[
2H(v_i, v_j) := |\Gamma_i^{out} \cap \Gamma_j^{in}|
\]  

(3)

Common-followed (CF): We extend the common neighbors approach [26], which is based on the idea that two nodes \( v_i \) and \( v_j \) are more likely to connect to each other if they have multiple friends in common. In the context of directed networks, the common-followed algorithm will recommend node \( v_j \) to node \( v_i \) if they follow partially or fully the same set of nodes, see Figure 2(d). Then, the algorithm ranks all nodes \( v_j \) based on the number of common-followed nodes with \( v_i \). Let \( \Gamma_j^{com} \) be the set of nodes that \( v_j \) follows. We define the set of common-followed nodes between \( v_i \) and \( v_j \) as:

\[
CF(v_i, v_j) := |\Gamma_i^{out} \cap \Gamma_j^{com}|
\]  

(4)

Node2Vec (N2V): A popular embedding algorithm that maps nodes to a low-dimensional space of features, by maximizing the likelihood of preserving nodes’ neighborhoods [20]. It has been used for link prediction by evaluating the cosine similarity between nodes in the embedding space, see Figure 2(e). Here we use N2V to recommend to each node \( v_i \) the most similar node in the embedding space, according to cosine similarity of the embedded vectors. Calling respectively \( \mathbf{v}_i^p \) and \( \mathbf{v}_j^p \) the embedded vector projections for \( v_i \) and \( v_j \), the cosine similarity between these projections is defined as:

\[
\text{CosineSim}(\mathbf{v}_i^p, \mathbf{v}_j^p) := \frac{\mathbf{v}_i^p \cdot \mathbf{v}_j^p}{\|\mathbf{v}_i^p\| \|\mathbf{v}_j^p\|}
\]  

(5)

3.3 Experiments setup

Here we describe the networks employed in our experiments and explain how the recommendation algorithms are iteratively used to recommend new connections among nodes.

Synthetic networks: In order to systematically create networks as defined in Section 3.1, we employed the DPAH model [14]. This model allows to generate scale-free bi-populated directed networks with adjustable homophily (for each group), minority size, node activity, and edge density. DPAH is a growth model that generates networks as follows. First, \( n \) nodes are created and randomly assigned to one of two groups based on the fraction of minorities \( f_m \). Then, the following steps are repeated until the desired edge density \( d \) is fulfilled. A source node \( v_i \) is drawn from a power-law distribution, modeled through the activity parameters \( \gamma_M \) and \( \gamma_m \) for the majority and the minority group, respectively. A target node \( v_j \) is drawn with a probability that is proportional to the product of its in-degree and the pair-wise homophily between the source and the target node. Lastly, a directed edge from \( v_i \) to \( v_j \) is created. Thus, the probability of creating a link from \( v_i \) to \( v_j \) is defined as:

\[
\mathbb{P}(v_i \rightarrow v_j) = \frac{h_{ij}k_i^m}{\sum_{l=1}^{n} h_{il}k_l^m}
\]  

(6)

where \( k_i^m \) is the in-degree of \( v_j \), and \( h_{ij} \) is the homophily between \( v_i \) and \( v_j \) and it is determined by their class membership.

In this work, we systematically modify the homophily within groups and the size of the minority, leaving the variation of node activity and edge density for a further study. In particular, in order to measure the influence of algorithms (RQ1) and homophily (RQ2) in the recommendations, we generate 4 networks for each combination of homophily parameters \( h_{mm}, h_{MM} \in \{0, 0.1, \ldots, 1 \} \) (\( h_{MM} \) and \( h_{MM} \) are defined as \( 1 - h_{mm} \) and \( 1 - h_{MM} \), respectively) and fix the number of nodes \( n = 1000 \), the size of the minority \( f_m = 0.3 \), the node activity \( \gamma_M = \gamma_m = 2.5 \) and the edge density \( d = 0.03 \). We further adjust the size of the minority \( f_m \in \{0, 0.2, 0.3, 0.4\} \) to measure its influence in the visibility of minorities (RQ3).

Recommendation: Given an initial network \( G \), we apply a recommendation algorithm \( R \) to suggest to each node \( v_i \) a node \( v_j \) to connect with. Then, we create a direct link \( v_i \rightarrow v_j \) for each top-1 of these recommendations. By doing so, in what we call “one step”, we create a new out-link for each node \( v_i \). This decision is motivated by the fact that the employed acceptance policy plays only a marginal role in shaping the network [9, 16]. Then, for every addition, we remove a random out-link. This is a procedure previously employed in the literature, for example in [9]. One of the main reasons for this choice is to prevent a significant increase in the edge density of the network. The evaluation metrics considered in Section 3.4 are sensible to edge density. By removing a link every time a new one is created we ensure to keep the density constant on every step and make sure that the changes are due to the recommendations and not to an increase in the total amount of connections. The link removal procedure is also grounded on the social theory for which people exhibit a finite communication capacity and, thus, they have a limit on the number of ties that they can maintain active in time [12, 29].

We repeat the above procedure 30 times to simulate an equal amount of recommendations per node.

Hyper-parameters: For PPR, we set the probability of following links to \( \alpha = 0.85 \), as suggested by Brin and Page [6] and widely used in many applications. In N2V, we use the default values for the dimensions of the embedding space \( \text{dimensions} = 64 \), the number of visited nodes in each random walk \( \text{walk.length} = 10 \), and the number of random walks to be generated from each node in the graph \( \text{num.walks} = 200 \). For WTF, we constrain the circle of trust to include only the top-10 nodes.

Additional assumption: We assume that the recommendations of different algorithms are similarly relevant, as our goal is not to evaluate which algorithm performs better in terms of utility metrics, but rather to study their effects on the structure and their impact on the visibility of the minorities (see Section 3.4).

3.4 Evaluation metrics

We use the global clustering coefficient [17] of the network and the Gini coefficient [19] of the in-degree distribution as proxies of network structure, and the fraction of minorities among the most important nodes as visibility. We measure these metrics before and after each round of recommendations to verify whether certain
types of networks change these metrics faster or slower and by how much.

**Clustering coefficient:** This metric allows to verify whether the recommendations are making the network more cohesive by closing more triangles. The clustering coefficient of node $v_i$ is defined as:

$$c_{v_i} = \frac{2}{\text{deg}_{\text{in}}(v_i) \cdot (\text{deg}_{\text{in}}(v_i) - 1) - 2 \cdot \text{deg}_r^{\text{out}}(v_i)} \cdot T(v_i)$$

where $T(v_i)$ is the number of directed triangles through node $v_i$, $\text{deg}_{\text{in}}^{\text{out}}(v_i)$ is the sum of in-degree and out-degree of $v_i$, and $\text{deg}_r^{\text{out}}(v_i)$ is the reciprocal degree of $v_i$. The global clustering coefficient of the network is then obtained by taking the mean across all nodes: $c = 1/n \sum_{i=1}^{n} c_{v_i}$.

**Gini coefficient of the in-degree distribution:** Popularity bias is a well-known issue reinforced by certain recommendation algorithms [1]. The Gini coefficient [19] allows us to demonstrate whether this bias is exacerbated by the algorithms regardless of the initial conditions of the network structure, or whether certain types of networks are exempt from this bias. The Gini coefficient of the in-degree distribution $\pi^{\text{in}}$, sorted in ascending order, is defined as follows:

$$Gini(\pi^{\text{in}}) = \frac{\sum_{i=1}^{n} (2i - n - 1) \pi_{i}^{\text{in}}}{n \sum_{i=1}^{n} \pi_{i}^{\text{in}}}$$

The higher the Gini coefficient, the more skewed or unequal the in-degree distribution across all nodes.

**Visibility of the minority group:** First, we measure the importance of nodes by computing their PageRank [31]. Then, out of the top-10% highest-scored nodes, we measure the fraction of nodes that belong to the minority group and refer to this fraction as the visibility of the minority group $\hat{f}_m$. We use the relative visibility $\hat{f}_m = \hat{f}_m - f_m$ to verify how far the visibility of the minority is from statistical parity [13] before the recommendations. Finally, we measure the change in visibility by computing $\delta f_m$ after and before the recommendations to verify whether the minority group is gaining or losing visibility:

$$\delta f_m = \hat{f}_m(\text{after}) - \hat{f}_m(\text{before})$$

**In-group links:** We also look at the fraction of links within groups to see what type of edges are being recommended more often by the algorithms. The in-group link ratio for group $a$ is defined as:

$$I_a = \frac{e_{aa}}{e_{aa} + e_{ab}}$$
where \( a, b \in \{m, M\} \) and \( a \neq b \).

4 RESULTS

Here we address our three research questions and present the results obtained after applying the recommendation algorithms iteratively to the simulated directed networks described in Sections 3.2 and 3.3, respectively. First, we show the consequences of these recommendations on the structure of the network and on the visibility of the minority group (RQ1). Second, we explain the changes in structure and visibility as a function of homophily (RQ2). Third, we further investigate the role of the size of the minority group and in-group links in the effects of the recommendations (RQ3).

4.1 RQ1: How do recommendation algorithms affect the structure of the network and the visibility of minorities?

Changes in network structure: To address this question, we first assess the changes in network structure in terms of global clustering coefficient and Gini coefficient of the in-degree distribution. The idea is to verify whether the algorithms (while connecting people together) make the network more cohesive and whether popularity bias increases at the same rate for all algorithms. Figure 3 shows the results for both metrics (top/bottom) on different types of networks (columns) across multiple rounds of recommendations (x-axis). Note that the x-axis reflects the iteration or step of recommendation, e.g., at step=20, each algorithm has independently recommended 20 connections to each node in the network. First, we see that overall, the evolution of these metrics is consistent across types of networks (columns) and recommendation algorithms (colors). Second, all recommendation algorithms increase the clustering coefficient of the network which means that the networks are becoming more cohesive as more triangles are getting closed. However, the rate at which this clustering increases, differs across algorithms, especially for N2V which, surprisingly, is the slowest. Third, we corroborate that PPR, WTF, 2H and CF reinforce the popularity bias issue since the Gini increases over time. This means that these algorithms make popular people (in terms of high in-degree) more popular. The exception is N2V, which after several recommendations makes the in-degree distribution less skewed (i.e., the recommendations are more diverse). One possible explanation is that similarity in the embedding space is only partially sensible to popularity bias.

Changes in the visibility: Now, we explore to what extent each recommendation algorithm changes the visibility of the minority group after several recommendations. We show the results in Figure 4. Each violin refers to one algorithm and the distribution of the violin represents the variation across a multiplicity of networks with different initial homophily values, fixed number of nodes and minority size (see Section 3.3 for details). PPR, WTF and 2H show similar patterns: they have median close to zero but denser tails in the negative direction. This indicates that these algorithms mostly keep the visibility of the minority unchanged, but, in certain cases, they decrease this visibility. CF shows the opposite behavior. First, it keeps the visibility unchanged for a few cases, but most of the time it drastically changes this visibility in either direction. Among all, N2V reveals more symmetric and smaller effects. Summarizing, Figure 4 suggests that four out of five algorithms are more prone to keep the visibility of the minority unchanged. Nevertheless, in certain regimes (explored next in RQ2) this visibility can be increased or reduced depending on the levels of homophily.

4.2 RQ2: To what extent is the change in visibility due to homophily?

To understand how the initial levels of homophily in the network affect the recommendations, we compare the visibility of the minority before and after the recommendations for each algorithm, see Figure 5. We control for the number of nodes and the fraction of minorities by keeping them fixed, and vary homophily values (see Section 3.3 for more details). PPR, WTF, 2H and CF reinforce the popularity bias issue since the Gini increases over time. This means that these algorithms make popular people (in terms of high in-degree) more popular. The exception is N2V which, surprisingly, is the slowest. Third, we corroborate that PPR, WTF, 2H and CF reinforce the popularity bias issue since the Gini increases over time. This means that these algorithms make popular people (in terms of high in-degree) more popular. The exception is N2V, which after several recommendations makes the in-degree distribution less skewed (i.e., the recommendations are more diverse). One possible explanation is that similarity in the embedding space is only partially sensible to popularity bias.

Visibility before the recommendations: Figure 5(a) shows the relative visibility of the minority before the recommendations. White regions (neutral visibility) represent statistical parity [13], in which the fraction of the minority in the top-10% is equal to the fraction of minority populating the whole network. Orange regions (positive visibility) represent higher amount of minority
Figure 5: Visibility of the minority group as a function of homophily. Heatmaps show the visibility of the minority group before and after the recommendations for different algorithms and different combinations of homophily within the majority (y-axis) and the minority (x-axis) groups. The visibility of the minority group is measured by the fraction of minorities in the top-10% of nodes ranked by their PageRank. In (a), colors show the relative visibility of the minority group w.r.t., the fraction of minorities in the network $f_m = 0.3$ before the recommendations. Positive visibility means that the minority is over-represented (orange), and negative visibility means that the minority is under-represented or the majority is over-represented (blue). Zero visibility refers to those cases where the top rank does not include any node from the minority group. In (b-f), colors represent the variation in the visibility due to different recommendation algorithms. For PPR, WTF and 2H one can see that the minority loses more visibility than the majority (especially in the heterophilic regime), while CF and N2V show more symmetric effects on the visibility of the minority and majority. Notice that the homophily values shown in the x- and y-axis of all plots represent the initial levels of homophily in the network before the recommendations.

At first glance, we see that the visibility gets affected differently depending on the algorithm and the initial values of homophily. We further notice that there are slightly more blue than orange regions in almost all plots (i.e., the majority increases its visibility more often than the minority across all regimes).

Among all the algorithms CF produces the strongest changes, while N2V is the most balanced. PPR, WTF and 2H, on the other hand, show a similar behavior. They penalize minorities especially in the heterophilic regimes for both classes, i.e., $h_m < 0.5$, bottom-left corners of Figures 5(b) to 5(d).

Furthermore, when only one group is homophilic, PPR, WTF and 2H do not change the initial over-representation of the homophilic group, see top-left and bottom-right corners in Figures 5(b) to 5(d).

Changes in the visibility after the recommendations: Figures 5(b) to 5(f) show the change in visibility after 30 recommendations per node. A positive change (orange) indicates that the visibility of the minority increased after the recommendations (relative to the initial visibility they had before the recommendations). Actual values in each cell denote the magnitude of this change. Conversely, a negative change (blue) indicates that the majority increased its visibility at the cost of reducing the visibility of the minority. No changes (white) indicate that the visibility did not vary across time.

nodes at the top of the rank compared to the statistical parity condition. Blue regions (negative visibility), instead, represent under-representation of minorities in top ranks. We see that the minority is over-represented mostly when the majority is heterophilic $h_{MM} < 0.5$ or when the minorities are more homophilic than the majority $h_{mm} > h_{MM}$.
Figure 6: Changes in the visibility of minorities as a function of the minority size. The y-axis shows the change in visibility for the minority group after the recommendations. A positive (negative) change indicates that minorities appeared more (less) often in the top-10% compared to their initial representation before the recommendations. If this change is around zero, the visibility of minorities remained constant or invariant. The x-axis shows the size of the minority group as a fraction of all nodes in the network. In general, we see that larger minorities get penalized less than smaller ones when the majority is heterophilic (a,b). When the majority is homophilic, however, the changes in visibility not only depend on the fraction of the minority but also on its homophily level. For instance, when the minority is heterophilic (c), its visibility remains mostly constant for all algorithms except CF, and when the minority is homophilic (d), its visibility drops for larger-size minorities, unless N2V and CF are used.

4.3 RQ3: Is the change in visibility inversely proportional to the size of the minority or proportional to the in-group links within the minority?

Size of the minority: To answer RQ1 and RQ2, we kept the size of the minority fixed ($f_{m} = 0.3$) to study the effects of homophily on the visibility of minorities after the recommendations. However, it is unclear whether the changes in visibility are inversely proportional to the size of the minority (e.g., larger changes for smaller minorities), or whether these are steady-state changes that appear regardless of the size of the minority. In Figure 6, we show how the change in visibility (y-axis) is affected by multiple factors including the size of the minority. First, we see a concordance among algorithms when the majority is heterophilic, Figures 6(a) and 6(b). In these cases, the larger the minority, the smaller the change in the visibility of the minority, except for CF which drastically reduces this visibility when the minority is more homophilic than the majority, Figure 6(b). When only the majority is homophilic, Figure 6(c), i.e., most out-links point to nodes in the majority group, the size of minorities has almost no effect on their final visibility in algorithmic rankings unless CF is used as recommendation algorithm. When both groups are homophilic, Figure 6(d), however, only CF and N2V increase the visibility of larger minorities more than the visibility of smaller minorities.

In-group links: As we have seen previously, the visibility of the minority can be affected by different factors, including the initial homophily of the network. Since homophily depends on the mixing of types of edges (see [14] for a detailed derivation of homophily in DPAH networks), we further investigate the evolution of in-group links over time, see Figure 7. Here, we found two main patterns. First, results from PPR, WTF and 2H are consistent in each type of network (columns). These algorithms mostly increase the number of in-group minority links, see Figures 7(a), 7(b) and 7(e). Surprisingly, this advantage does not guarantee an increase in visibility for the minority group. On the contrary, they lose visibility, see Figures 6(a) and 6(b) for $f_{m} = 0.3$. Second, results from CF and N2V are also consistent in each type of network. We see in Figures 7(a) and 7(b) that these two algorithms increase the in-group majority links when the majority is initially heterophilic, and reduce them when the majority is initially homophilic, see Figures 7(c), 7(d) and 7(e).

Now, we analyze in details different possible homophily configurations.
When one class is homophilic and the other class is heterophilic, the links coming from both classes are mostly directed to nodes in the homophilic class. Let us consider PPR, WTF and 2H where the values of homophily are $h_{MM} = 0.2$ for the majority and $h_{mm} = 0.8$ for the minority and vice-versa, see Figures 7(b) and 7(c), respectively. In these situations, these recommendation algorithms will keep increasing the in-group proportions of the homophilic group since the recommended links mostly point to nodes in this group.
These correspond to situations in the white regions at the top-left and bottom-right of Figures 5(b) to 5(d). Hence, this shows that the absence of variation in the fraction of minority is due to the fact that PPR, WTF and 2H do not modify connections between classes in these cases. This does not hold for CF and N2V. In fact, under the same homophily conditions, these methods make the in-group links for both classes more similar, decreasing structural differences between classes, see Figures 7(b) and 7(c).

Now, we will consider regimes where both classes are heterogeneous, \( h_{\text{mm}} = h_{\text{MM}} = 0.2 \), see Figure 7(a). Here, CF and N2V are the only algorithms in which the initial conditions of in-group links are flipped. Note that the proportion of links within the majority group gets larger than the proportion of links within the minority after multiple rounds of recommendations. Consequently, the majority increases its visibility even further by pushing minorities to lower ranks, see Figures 5(e) and 5(f). Interestingly, the visibility of minorities decreases even if the flip does not occur in these heterogeneous settings for PPR, WTF and 2H, see bottom-left of Figures 5(b) to 5(d).

On the other extreme of homophily, when both groups are homophilic, \( h_{\text{mm}} = h_{\text{MM}} = 0.8 \), we found two main patterns, see Figure 7(d). First, PPR, WTF and 2H tend to strengthen the connections towards the majority group by either recommending majority-to-majority or minority-to-majority links. This in turn penalizes the minorities at the top of the rank, see \( h_{\text{mm}} = h_{\text{MM}} = 0.8 \) in Figures 5(b) to 5(d). In contrast, CF and N2V slowly increase the number of connections within the minority group. For N2V, one possible explanation is that the homophily levels are high enough so that the two classes (especially the minority class), are represented in the embeddings as, at least partially, separated clusters.

Lastly, the possibility to systematically vary the initial levels of homophily for both classes allows us to identify tipping points. For instance, in a homophilic regime, where both groups have the same level of initial homophily, \( h_{\text{MM}} = h_{\text{mm}} = 0.8 \), we found that PPR, WTF and 2H increase the number of links within the majority group after multiple recommendations, see Figure 7(d). However, the same algorithms may also increase the number of links within the minorities, and thus their visibility, if the minority group is initially more homophilic than the majority, \( h_{\text{MM}} = 0.7 \) and \( h_{\text{mm}} = 0.9 \), see Figure 7(e). CF and N2V, on the other hand, do not show this tipping effect when both groups of nodes are initially homophilic. In either case, these two algorithms keep increasing the proportion of in-group links which induces segregation.

5 LIMITATIONS AND FUTURE WORK

We have limited our study to five recommendation algorithms, and in future work we aim to include more algorithms into this investigation, especially recent versions of popular algorithms that have been developed with the goal to increase fairness.

Furthermore, we focused on scale-free directed networks with homophily which represent a plausible configuration of online social networks. As next steps, we would like to include in our analysis different network simulation models that include other factors in the network generation process, such as multiple node-attributes, heterogeneous group mixing, the presence of communities, and triadic closure. We also acknowledge the fact that our analysis is theoretical and has not been validated with real data. We plan to extend our study by considering empirical networks.

Importantly, link recommendation algorithms and datasets are generally proprietary. This is why simulation-based approaches are often necessary for this kind of investigations. In addition, the simulation approach enables us to examine different scenarios which might not occur in one instance of the data [32].

6 CONCLUSIONS

In this work, we systematically studied five link recommendation algorithms and quantified their feedback loop effects on bi-populated scale-free directed networks with homophily. In particular, we assessed two types of changes in these networks due to multiple link recommendations. First, we measured the changes in network structure in terms of clustering and in-degree distribution. Second, we measured the changes in the visibility of minorities at the top-10% of the rank with respect to their PageRank (importance) scores, highlighting the effects of homophily, minority size, and in-group links.

Our results show that four out of the five algorithms reduced on average the visibility of minorities more often than to the majority counterpart. In particular, PPR, WTF and 2H when both groups are initially heterogeneous, and CF when the minority is initially more homophilic than the majority.

We also found that while all algorithms tend to close triangles and increase the clustering coefficient, all algorithms except N2V are prone to favor and suggest nodes with high in-degree. This is known as popularity bias, rich-get-richer effect or cumulative advantage, a well-known mechanism that contributes to inequality.

Link recommendations based on N2V rely on the proximity of nodes in the embedding space, which does not necessarily imply closeness to nodes with high in-degree. Consequently, N2V is a promising alternative to other link recommendation algorithms since it mitigates cumulative advantage.

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