Lorentzian SRT-transformation factors as solutions of oscillation-equations -
Second announcement

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Abstract:

Shown is the derivation of Lorentz-Einstein k-factor in SRT as an amplitude-term of oscillation-differential equations of second order. This case is shown for classical Lorentz-factor as solution of an equation for undamped oscillation as well as the developed theorem as a second solution for advanced SRT of fourth order with an equation for damped oscillation-states. This advanced term allows a calculation for any velocities by real rest mass. Also accelerated coordinate -frames are discussed.

key-words: undamped oscillation; SRT; k-factor; Differential-equation of second order; Einstein-Lorentz; Amplitude-analogy; damped oscillation; developed SRT of fourth order

I. Lorentz-Einstein SRT - k-factor as an amplitude solution of undamped oscillation-equation

1. Introduction:

It is obvious to remark the similarity between the amplitude curves of an undamped oscillation and of k-factor of SRT given by Lorentz and Einstein for velocities with are smaller than light on the one hand and by Feinberg for FTL on the other [1],[2],[3],[5], if both are described and drawn together. Through this similarity there can be tried to get the Lorentzian k-factor not only from pure kinematic examinations like in [1],[2] and [6], but as an exact solution of an oscillation equation as is demanded in [4].

If the oscillation equation of second order is set in the following form there can be derived the Lorentz-k-Factor as an solution resp. an interpretation for amplitude of the oscillating system.

2. Calculation:

There is the ansatz for the following differential equation, which can be interpreted as an oscillation equation for undamped states,
\[ \ddot{\phi} + \frac{c^2}{r^2} \cdot \dot{\phi} = \bar{A}^2 \cdot e^{i \left( \frac{\nu r}{r} \right)} \]  

(1.)

where \( r \) is an unknown constant length, which value and meaning has to be discussed (maybe planck-length or the oscillation wavelengths or as rotation radius), \( \nu \) the velocity of a moving body or particle in local inertial frame of flat Minkowski-Space and \( c \) the invariance velocity by Lorentz-transformations, which occurs here in interpretation as the eigenfrequency-velocity of local spacetime.

Also is set:

\[ \phi(t) = A^2 \cdot e^{i \left( \frac{\nu r}{r} \right) t} \]  

(2a.)

as an ansatz für the solution of this equation.

Then there is derivated:

\[ \ddot{\phi} = -\frac{\nu^2}{r^2} \cdot A^2 \cdot e^{i \left( \frac{\nu r}{r} \right) t} \]  

(2b.)

If (2a.) and (2b.) are set into (1.), there follows the equation:

\[ A^2 \cdot \frac{c^2 - \nu^2}{r^2} \cdot e^{i \left( \frac{\nu r}{r} \right) t - \theta} = \bar{A}^2 \cdot e^{i \left( \frac{\nu r}{r} \right) t} \]  

(3.)

which gives the following relation:

\[ \frac{c^2 - \nu^2}{r^2} = \frac{\bar{A}^2}{A^2} \cdot e^{i \theta} \]  

(4.)

If now the terms are separated seen as a realterm \( \Re \) and an imaginary term \( \Im \), there is set:

\[ \frac{c^2 - \nu^2}{r^2} = \Re = \frac{\bar{A}^2}{A^2} \cdot \cos \theta \]  

(5a.)

and

\[ 0 = \Im = \frac{\bar{A}^2}{A^2} \cdot \sin \theta \]  

(5b.)

This term means, that \( \theta = 0^\circ \). There is no phase shifting in angle for classical SRT-term which leads to the barrier of invariance-velocity \( c \) for undamped local spacetime-states in tangent-space.

Therefore follows with theorem of Pythagoras \( \sin \theta^2 + \cos \theta^2 = 1 \) the relation of:
\[ A_{1,2,3,4} = \pm \pm i \frac{\overline{A}}{\sqrt{c^2 - v^2}} \]  

(6.)

If now is taken the positive real sign-term of the whole term (6.) which is chosen as \( A_1 \) and also is set:

\[ \overline{A}_1 = \frac{c}{r} \quad \text{or} \]

\[ \overline{A}_2 = \frac{ic}{r} \]  

(7a.) (7b.)

then follows from (7a):

\[ A_1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]  

(8a.)

and from (7b):

\[ A_1 = \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}} \]  

(8b.)

These are the classical Lorentz-Einstein-terms and the Feinberg-term for moving bodies in local inertial frames of flat space-time in classical SRT.

3. Discussion:

The similarity between Einstein-Lorentz-Feinberg k-factors and the amplitudeterm of the model of an undamped oscillation with the given bounding conditions may be coincidentally of a mere pure mathematically analogy without any physical evidence. But this derivation may throw a new light into the interpretation of local space-time-conditions. Specially the role of the supposed constant length-term \( r \) has to be discussed further. Also the phase-angle \( \theta \) must be discussed. For undamped state analogy this angle is equal to zero. Therefore can be concluded, that for phase angles with other values there can be derived a developed SRT-theory for damped states as worked out in [4.], which may unify the broken symmetry of both Einsteinian and Feinberg k-terms.

II. Developed Lorentz-Einstein SRT – k-factor of fourth order as an amplitude solution of damped oscillation-equation

4. Result:
In analogy to I. there is calculated the differential equation of second order for damping oscillation processes:

The result of calculating this equation is given in short form (without further algorithm processes):

It is easy shown, that the equation for these processes must have the form:

\[ \ddot{\varphi} + \beta \cdot \dot{\varphi} + \gamma \cdot \varphi = A^2 \cdot e^{\frac{ivr}{c^2}} \tag{9.} \]

with the boundary conditions for the solution-function and the variables:

\[ \varphi(t) = A^2 \cdot e^{\frac{ivr}{c^2} \cdot t} \tag{10.a} \]

and

\[ \beta = \frac{a}{r} \cdot \gamma = \frac{c^2}{r^2} = A^2 \tag{10.b} \]

So there is finally the form of oscillation-equation for advanced SRT of fourth order as:

\[ \ddot{\varphi} + \frac{a}{r} \cdot \dot{\varphi} + \frac{c^2}{r^2} \cdot \varphi = \frac{c^2}{r^2} \cdot e^{\frac{ivr}{c^2}} \tag{11.} \]

and there is the term of amplitude given for this equation as solution:

\[ A = \pm \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)^2 + \frac{4 \cdot a^2 \cdot v^2}{c^4}}} \tag{12.} \]

which is the right form of developed K-factor for advanced SRT in fourth order if the positive real sign is chosen because in the case \( a \equiv 0 \) this leads to classical k-term of Lorentz-Einstein-SRT. The factor \( a \) is interpreted as the damping velocity of the relativistic system which can be coupled with an equivalent circle-frequency \( \omega \).

The result for the phase angle forms to:

\[ \theta_v |v| = \arctan \left( \frac{2 \cdot a \cdot v}{c^2 - v^2} \right) . \tag{13.} \]

Also here is seen: for \( a \equiv 0 \) there is the phaseangle \( \theta_v |v| = 0 \) and no phase shifting in classical SRT-term which agrees in line with I but there is a special phase angle for the bothside limit:

\[ \lim_{v \to c} \theta_v |v| = \frac{\pi}{2} \quad \text{with} \quad v < c \quad \text{and} \quad v > c . \tag{14.} \]

So the classical Lorentz-Einstein-factor can be interpreted as a solution of an oscillation equation with phase-angle \( \theta \) equivalent to zero. For the advanced system there are phase angles \( \theta \)
different from zero, so there are given phase-shifted states in the fundamental function of the system.

IIIA. Lorentz-factor for undamped but accelerated process

5. Calculation:

Since accelerated coordinate-frames are as dynamical processes not pure kinematic SRT-description but originally an ART-problem, they are here discussed nevertheless because the question of the force which causes the acceleration is not considered here in this paper. Nevertheless also frames without constant velocity \( \nu \) but with \( \nu = \nu(t) \), velocity as a function of time can be described in the connection of analogy amplitudes by oscillation equations. The consideration here deals with linear velocity, so the second derivative is always zero. Therefore all terms with \( \ddot{\nu} = 0 \) are left out of the calculation. If there will be a discussion of spacecraft-equation with no constant acceleration, these terms must be taken into account, but then the dependence of velocity in time would be no longer linear.

If there is \( \nu \) as a function of \( t \), which means \( \nu \)-non-constant, then there is with the same ansatz for undamped but accelerated movement:

\[
\ddot{\phi} + \frac{c^2}{r^2} \cdot \phi = \overline{A}^2 \cdot e^{\frac{i}{r} \cdot \nu(t)}
\]  
(15.)

This leads with the boundary conditions of:

\[
\dot{\nu} = b = \text{const. and } \ddot{\nu} \equiv 0
\]  
(16.)

to the result of:

\[
\dot{\phi} = \overline{A} \cdot e^{\frac{i}{r} \cdot \nu(t)} \cdot \left( i \cdot \frac{2}{r} \cdot \dot{\nu} - \left( \frac{\nu}{r} + \frac{\dot{\nu}}{r} \cdot t \right)^2 \right)
\]  
(17.)

This relation leads to the final amplitude-term of accelerated coordinate frames with:

\[
A = \frac{1}{\sqrt{\left( 1 - \left( \frac{\nu + b \cdot t}{c} \right)^2 \right)^2 + \frac{4b^2 \cdot r^2}{c^4}}}
\]  
(18.)

if \( \overline{A} = \frac{c}{r} \) is chosen like before.

For \( b \equiv 0 \) there is the limit of the transition to classical Einstein-Lorentz-Feinberg formulas for inertial frame transport of
\[ A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad A = \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}} \quad \text{(8a./8b.)} \]

6. Discussion:

Although this term (18.) is the result for an undamped ansatz of the original differential equation, it looked like a damping term because of the similarity between this formula and the formula which is a solution of the whole differential equation with existence of derivation of first order term. The cause for this result is the existence of a classical accelerating term which makes velocity as a variable but linear function of time. Maybe this solutions are formal similar both. So although they results from different equations-ansätze, there may be set a formal identity of the adding velocity factor \( a \) for damped seen processes with \( b \cdot t \) seen here for the assumption of no damping. Therefore can another term be described for the whole differential equation which uses acceleration and damping term. This description follows now:

IIIb. Lorentz-factor for damped and accelerated process

7. Calculation:

If there is the ansatz:

\[ \ddot{\phi} + \frac{a}{r} \cdot \dot{\phi} + \frac{c^2}{r^2} \cdot \phi = \bar{A} \cdot e^{i \left( \frac{v}{r} \cdot t \right)} \quad \text{(19.)} \]

with the derivations:

\[ \ddot{\phi} |_{v(t)} = \left( A^2 \cdot i \cdot 2 \cdot \frac{v}{r} - A^2 \cdot \left( \frac{\dot{v}}{r} \cdot t + \frac{v}{r} \right)^2 \right) \cdot e^{i \left( \frac{v}{r} - \theta \right)} \quad \text{(20.)} \]

then there yields the main result to:

\[ A = \frac{\bar{A}}{\sqrt{c^2 - \frac{|b \cdot t + v|^2}{2} + \frac{|2 \cdot b \cdot r + b \cdot a \cdot t + v \cdot a|^2}{2}}} \quad \text{(21.)} \]

where acceleration \( v = b \) is set.

With the in the meantime well-known boundary condition of \( \bar{A} \equiv \frac{c}{r} \) there is the special end-result for damped and accelerated amplitude-term of:
\[
A = \frac{1}{\sqrt{\left(1 - \frac{b \cdot t + v}{c^2}\right)^2 + \left(\frac{2 \cdot b \cdot r + b \cdot a \cdot t + v \cdot a}{c^2}\right)^2}} \quad (22.)
\]

**8. Remark:**

Since there is first of all made the assumption that \( v = v(t) \) is linear as a function, with constant acceleration \( \dot{v} = b \) as first derivative of \( v \) against time \( t \), so the second derivative of \( v \) is zero. Terms including this second derivative are also left out in this summary description here above. If there is a rocket equation necessary with non constant, but also time-dependend acceleration, then this term has to be included.

For \( a = b = 0 \) this advanced term goes also into the limit of classical Lorentz-Einstein-Feinberg-terms of

\[
A = \frac{1}{\sqrt{\left(1 - \frac{\nu^2}{c^2}\right)}} \quad \text{and} \quad A = \frac{1}{\sqrt{\left(\frac{\nu^2}{c^2} - 1\right)}} \quad . \quad (8a./8b.)
\]

**9. Discussion:**

May be these terms (18.), (22.) are fully not the right cases of description of accelerated frames in approximation of SRT because they are derived from an oscillation-equation and not by the discussion of kinetic processes but in first case the original lorentz-term was also derived by this equations and all limits of further descriptions go over into these classical case. So there also may be a sort of confidence in the further descriptions of accelerated frames. They can be considered as a limit of ART-case into SRT for weak curvature and small acceleration.

**10. Conclusions and summary:**

If differential-equations of second order for undamped or damped oscillating systems are set with special bounding conditions like proposed in [4.] and solved, then the Lorentz-Einstein-k-Factor from Special Relativity Theory (SRT) can be derived from this equation for undamped oscillation as a solution. Also the advanced Lorentz-factor for SRT of fourth order can be derived from the equation for damped oscillation states. If the model is advanced to accelerated frames, lorentsimsial-term can also be formulated.

**11. References:**
[1.] Einstein, A., über die spezielle und allgemeine Relativitätstheorie, Vieweg Braunschweig, WTB Nr. 59, 21.Auflage, (1979)

[2.] Einstein, A., Grundzüge der Relativitätstheorie, Vieweg Braunschweig, WTB Nr. 58, 5.Auflage, (1979)

[3.] Feinberg, G., Possibility of Faster-Than-Light Particles, Phys. Rev. 159, 1089, (1967)

[4.] Döring, H., SRT as a Fourth-order-theory with Analogy-model of Damped Resonance. Preprints 2021, 2021040433 (doi: 10.20944/preprints202104.0433.v1).

[5.] Einstein, A., Zur Elektrodynamik bewegter Körper, Annalen der Physik, 322, 10, 1905; https://doi.org/10.1002/andp.19053221004

[6.] Lorentz, H.A., Elektromagnetische Erscheinungen in einem System, das sich mit beliebiger, die des Lichtes nicht erreichender Geschwindigkeit bewegt. In: Das Relativitätsprinzip - Eine Sammlung von Abhandlungen, Seiten 6–25. Wissenschaftliche Buchgesellschaft, Darmstadt, 6. Auflage, 1958.