Nonlinear Bethe–Heitler pair creation with attosecond laser pulses at the LHC

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Abstract

The creation of lepton pairs ($e^+e^-$ and $\mu^+\mu^-$) via multiphoton absorption in collisions of ultrarelativistic ion beams with ultrashort high-frequency laser pulses is considered. Both the free and the bound-free production channels are addressed, where in the latter case the negatively charged lepton is created in a bound atomic state. It is shown that these nonlinear QED processes are observable when a tabletop source of intense xuv or X-ray laser radiation is operated in conjunction with the LHC. We discuss the relative effectiveness of protons versus Pb ions and specify for each pair production channel the most suitable collision system.

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1. Introduction

The Large Hadron Collider (LHC) at CERN (Geneva, Switzerland) is currently starting operation [1]. It will reach ion energies up to 7 TeV/nucleon corresponding to proton beams with a Lorentz factor of $\gamma \approx 7000$ and Pb nuclei with $\gamma \approx 3300$. The main physics objectives of this new machine are the search for the Higgs boson and for physics beyond the Standard Model [2]. In this note we describe an application of the LHC for exploring a yet unobserved nonlinear QED process, namely lepton–antilepton pair creation in combined nuclear and laser fields via the multiphoton Bethe–Heitler effect. Also other QED processes probing the structure of the physical vacuum in this kind of field combination are presently under the active scrutiny of theoreticians, such as photon-fusion or Delbrück scattering [3]. The counterpropagating-beam geometry enhances the relative effectiveness of protons versus Pb ions and specifies for each pair production channel the most suitable collision system.
laser field couples nonlinearly to the leptons. Moreover, replacement of the projectile-electron by a nucleus introduces a new channel: bound-free pair production, where the electron is produced in a bound atomic state of the ion. The corresponding process is known from relativistic heavy-ion collisions, where it exhibits a pronounced nonperturbative character [14]. Similarly, in the present case the bound-free channel is beyond first order in the interaction with both the laser and the nuclear Coulomb field [see Eq. (7) below].

While the usual Bethe–Heitler process by a single high-energy photon was measured already in the late 1940s [15], its nonlinear generalization (1) has not been observed yet. The theoretical proposals have been based on the hypothetical combination of two large-scale facilities: (a) either the LHC together with a petawatt-class infrared laser of intensity $I \sim 10^{21}$ W/cm$^2$ such as the HERCULES laser at the University of Michigan (Ann Arbor, USA) [16] or the VULCAN laser at Rutherford Appleton Lab (Didcot, UK) [17]; (b) or an ordinary but still powerful ion accelerator providing $\gamma \gtrsim 50$ combined with an X-ray free-electron laser (XFEL) of frequency $\hbar \omega_0 \approx 10$ keV; corresponding XFEL beam lines are presently under development at SLAC and DESY (Hamburg, Germany) [18]. These mergings are difficult to realize since in all cases the ion accelerator and laser facility are large in size and located at different places. (Note that the HERA proton accelerator at DESY has been shut down in 2007, unfortunately.)

In the present Letter we show that a solution to this obstacle is offered by a new development in laser science. Via high-harmonic generation from laser-irradiated atomic gas jets it has been shut down in 2007, unfortunately.)

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In the present Letter we show that a solution to this obstacle is offered by a new development in laser science. Via high-harmonic generation from laser-irradiated atomic gas jets it has recently become possible in various laboratories to produce intense trains of attosecond laser pulses with frequencies in the xuv domain ($\hbar \omega_0 \sim 100$ eV) and focussed intensities of $I \sim 10^{14}$ W/cm$^2$ [19,20]. Even higher intensities might be reachable with attosecond pulses from plasma surface harmonics [21]. These ultra-short pulses hold promising prospects for time-resolved spectroscopy of the electron dynamics in atoms and molecules. Here, however, we propose an unusual application of the attosecond technology in high-energy physics. Since the corresponding experimental devices are of table-top size it is conceivable to bring them to LHC. The Doppler-shifted laser frequency in the ion frame would be sufficiently large to generate $e^+e^-$ pairs by few-photon absorption. We present corresponding production rates for proton and Pb impact, which indicate experimental feasibility of the process. We also briefly discuss $\mu^+\mu^-$ pair creation by utilizing envisaged table-top X-ray devices.

It should be mentioned that $e^+e^-$ pair production has also been observed in recent studies on relativistic laser–plasma interactions [22,23]. In these experiments, a solid target is irradiated by an intense laser pulse which creates a hot plasma and accelerates electrons to high energies. In the field of the ions, the fast electrons emit bremsstrahlung which is converted into $e^+e^-$ pairs through the ordinary (linear) Bethe–Heitler effect. The laser field plays an indirect role only in the pair production here, by serving as a particle accelerator. In contrast to that, the Bethe–Heitler processes discussed in the present Letter proceed nonlinearly via multiphoton absorption, with direct participation of the laser field in the pair creation step (1). With respect to the linear Bethe–Heitler effect by a single high-energy photon, it is interesting to note moreover that the influence of a strong low-frequency background laser field on this process has been calculated recently [24]; the laser assistance leads to a pronounced channeling of the created particles.

2. Electron–positron pair creation

From a theoretical point of view the available attosecond pulse trains (APTs) are weak in the sense that the Lorentz-invariant in-

\[ S = -ie \int d^4x \Psi_{\mu^-s_\mu}^\dagger (x) V_{\text{ion}}(r) \Psi_{\mu^+s_\nu} (x), \]

with the static nuclear potential $V_{\text{ion}}(r) = Ze/r$, the nuclear charge number $Z$, and the Volkov states $\Psi_{\mu^+s_\nu} (x)$. The leptons are characterized by their four-momenta $p_\mu$ and spin projections $s_\pm$ outside the laser field. Although the laser field is treated as a classical electromagnetic wave, photons arise from a mode expansion of the oscillatory parts in Eq. (2) which contains multiphoton processes of arbitrary order. The leading-order rate for nonlinear Bethe–Heitler pair creation by absorption of n laser photons scales like $R^{(n)} \sim \xi^{2n}$. Accordingly, the SLAC experiment found a reaction rate $\sim \xi^{10}$. Higher-order corrections stem from additional photon exchange which goes beyond the minimum photon number required from energy conservation. For example, when pair creation is energetically possible by two-photon absorption there will be small additional contributions where more than two photons have been absorbed. These corrections to the leading-order term are suppressed by an additional $\xi^2$ factor. As an alternative to the $S$-matrix approach, the laser-dressed polarization operator can be employed to calculate total pair creation rates [11].

The lowest-order term of the rate expansion in powers of $\xi^2$ gives the famous result of Bethe and Heitler for pair creation by
a single photon (see Fig. 1(a)). In the high-energy limit $\omega \gg 2m$, the rate can be written in closed-form as

$$R^{(1)} = \frac{7}{18\alpha} (\alpha Z)^2 \xi^2 \omega \left[ \ln \left( \frac{2\omega}{m} \right) - \frac{109}{42} \right].$$

(3)

referred to the rest frame of the nucleus. Here, $\alpha$ denotes the QED fine-structure constant. The rate (3) can be converted into a cross section by dividing out the photon flux $j = \omega m^2 \xi^2 / 3\pi c$. This result is independent of the photon polarization. We note that in multiphoton physics total probabilities are preferably given as reaction rates since the cross section for a multiphoton process $\sigma^{(n)} = R^{(n)} / j$ still involves powers of $\xi^2$ and thus depends on the incoming photon intensity. The next-to-leading order correction term to Eq. (3) has been obtained in [11]:

$$\Delta \sigma = -\frac{13}{90\alpha} (\alpha Z)^2 \xi \omega \left[ \ln \left( \frac{2\omega}{m} \right) - \frac{22}{13} \ln 2 - \frac{124}{195} \right].$$

(4)

It contains the process of two-photon pair production (see Fig. 1(b)) as well as $\xi^2$-corrections to one-photon pair creation (arising, e.g., from the interference between the diagrams in Figs. 1(a) and 1(d)). The correction (4) would have an appreciable effect in stronger laser fields with $\xi \gtrsim 0.1$ where it reaches the percent level. Corresponding xuv intensities are likely to be attainable from plasma harmonics [21]. Signatures of the additional photon involved would also arise in the energy spectra of the created particles. In the opposite nonrelativistic limit close to the energetic threshold ($\omega - 2m \ll m$) the one-photon Bethe–Heitler rate reads

$$R^{(1)} = \frac{(\alpha Z)^2 \xi \omega (\omega - 2m)}{96 \pi} (\omega - m^2 / m)^3.$$

(5)

For frequencies in the range $m < \omega < 2m$ the Bethe–Heitler process can only proceed nonlinearly via two-photon absorption (see Fig. 1(b)). Close to threshold, the corresponding rate reads

$$R^{(2)} = \frac{(\alpha Z)^2 \xi \omega (\omega - m)^2}{64 \pi} (\omega - m^2 / m)^2.$$

(6)

for a linearly polarized laser wave [11]. It is interesting that the frequency scaling becomes $\sim (\omega - m)^4$ for circular laser polarization. Polarization dependence is a characteristic feature of nonlinear processes, in general.

Let us consider the collision of an intense APT ($\omega_{\text{av}} = 100$ eV, $\xi = 1.4 \times 10^{-4}$) with the LHC ion beams. For proton impact with $\gamma = 7000$ an $e^+e^-$ pair is produced by one-photon absorption since $\omega_a = 1.4$ MeV. By numerical calculations based on Eq. (2) we obtain a corresponding lab-frame rate of $R^{(1)}_{\text{lab}} \approx 5.79 \times 10^{12}$ s$^{-1}$ which is in reasonable agreement with the threshold formula (5). The latter slightly overestimates the rate by a factor of 2 since the frequency value lies above its range of applicability. Note that the rate in the lab-frame is reduced by a factor $\gamma^{-1}$ because of relativistic time dilation with respect to the ion-frame. When Pb projectiles with $\gamma = 3300$ are used instead, two photons are needed to overcome the energy barrier. The higher-order in $\xi^2$ leads to a smaller total rate of $R^{(2)}_{\text{lab}} \approx 2.53 \times 10^{11}$ s$^{-1}$. Coulomb corrections to the first-order treatment of the nuclear field in Eq. (2) could slightly modify this value. For comparison we note that a proton at the same speed leads to a rate of $R^{(2)}_{\text{ion}} \approx 3.76 \times 10^{-6}$ s$^{-1}$. These numbers agree with Eq. (6), again within a factor of 2. Our results are summarized in Table 1. The rates can be transformed into total yields by taking the laser pulse length and repetition rate as well as the projectile beam density into account. We assume that in the experiment a single LHC ion bunch containing $N_{\text{ion}} \approx 10^{11}$ particles is used. It has a transverse radius of about $\rho_{\text{ion}} \approx 16 \mu$m and circulates with a revolution frequency of $f_{\text{ion}} \approx 11$ kHz [2]. An APT of 30 fs total duration is supposed, consisting of 25 single attosecond pulses with a duration of 300 as each. The effective time when the field is present in the APT thus amounts to $\tau = 7.5$ fs. Note here that the individual attosecond bursts are separated by half a period of the driving optical laser. The train repetition rate can be synchronized with the circulating ion beam, i.e. $f_{\text{APT}} = f_{\text{ion}}$. The typical diameter of an APT is on the order of 10 μm so that we may assume perfect overlap with the ion beam. The number of pair creation events per unit of time is determined by $N_{\text{ev}} = R_{\text{lab}} N_{\text{ion}} f_{\text{ion}} \tau / 2$, with the factor of 1/2 arising from the relative beam velocity. We obtain 0.1 nonlinear Bethe–Heitler pair creation events per second via two-photon absorption from the APT colliding with the LHC Pb beam. This event rate seems to render experimental observation feasible. For comparison we note that about 2400 $e^+e^-$ pairs per second are produced through the ordinary (linear) Bethe–Heitler process when the 7 TeV proton beam is used instead.

We point out that the duration of the single attosecond spikes in the APT amounts to 300 as/2γ $\sim 30$ zs in the projectile frame [27,28]. This value approaches the natural QED time scale of 1/m$^{-1}$ $\sim 1$ zs $= 10^{-21}$ s. The combination of ultrarelativistic particle beams with ultrashort laser pulses might therefore allow for pump–probe experiments on the structure of the physical vacuum when a first zepstosecond spike prepares a certain electron–positron state which is analyzed by a subsequent spike. The pump–probe technique is very successfully applied in atomic physics for time-resolved measurements of processes occurring on the fs and sub-fs scale [19].

### 3. Bound-free electron–positron pair creation

A variant of the Bethe–Heitler process (1) is represented by bound-free $e^+e^-$ pair creation [28–30]. Here the $e^+$ is created not as a free particle but in a bound atomic state of the projectile ion. The corresponding process has been observed in relativistic heavy-ion collisions, for example at CERN–SPS [14,31]. Bound-free pair creation by a high-energy photon in a nuclear field was, in principle, already calculated by Sauter in the early 1930s [32–34]. Its multiphoton generalization in laser fields can be calculated in a way similar to Eq. (2), with the electronic state given by a bound

| produced pair | projectile | $\gamma$ | $\hbar \omega_0$ [keV] | $\hbar \omega$ [MeV] | photon order | lab-frame rate [s$^{-1}$] |
|---------------|------------|---------|-----------------|-----------------|--------------|----------------|----------------|----------------|
| free $e^+e^-$ | p          | 7000    | 0.1             | 1.4             | 1            | 5.79 x 10$^4$ |
|               | Pb         | 3300    | 0.1             | 0.66            | 2            | 3.76 x 10$^4$ |
| bound-free $e^+e^-$ | p | 7000 | 0.1 | 1.4 | 1 | 6.09 x 10$^{-1}$ |
|               | Pb         | 3300    | 0.1             | 0.66            | 2            | 4.08 x 10$^{-1}$ |
| free $\mu^+\mu^-$ | p | 7000 | 12 | 168 | 2 | 8.25 x 10$^{-5}$ |
|               | Pb         | 3300    | 12              | 79              | 3            | 7.77 x 10$^{-15}$ |
|               | Pb         | 3300    | 12              | 79              | 3            | 6.59 x 10$^{-13}$ |
Coulomb–Dirac wave function. When \( n = 2 \) laser photons participate in the process, the production rate approximately scales like [29]

\[
R_{\text{bf}}^{(2)} \sim Z^2 \xi^4 \left( \frac{\omega - \omega_{\text{min}}}{m} \right)^{\kappa}.
\]  

(7)

Here, \( \omega_{\text{min}} \) denotes the threshold photon frequency determined by \( n\omega_{\text{min}} = 2m - E_\gamma \), with the atomic binding energy \( E_\gamma \). The exponent \( \kappa \) depends again on the laser polarization: \( \kappa \approx 0.5 \) for linear polarization and \( \kappa \approx 1 \) for circular polarization. The \( Z \) scaling is typical for an electron capture process [14]. Note that in Eq. (7) the interactions with both the laser and the nuclear field enter beyond first order.

When an LHC proton collides with an xuv pulse, bound-free pair creation proceeds by single photon absorption (Sauter limit) with a rate of \( R_{\text{bf}}^{(1)} \approx 6.09 \times 10^{-3} \text{ s}^{-1} \), as shown in Table 1. For Pb impact the absorption of two photons is required so that an additional \( \xi^2 \approx 10^{-8} \) factor appears in the total rate. The reduction by this factor, however, is overcompensated by the strong \( Z \) dependence in Eq. (7). As a result, the two-photon rate from Pb is larger than the one-photon rate from protons and amounts to \( R_{\text{bf}}^{(1)} \approx 4.08 \times 10^{-1} \text{ s}^{-1} \). In both cases the electron is assumed to be captured to the 1s1/2 ground state which gives the major contribution to the process due to its broad momentum distribution. The contribution from higher atomic levels slightly enhances the rate by \( \approx 15\% \) [29,33]. The rate for nonlinear bound-free pair creation by Pb impact is larger by an order of magnitude than the corresponding creation rate of free pairs. This is due to the stronger \( Z \) scaling in combination with the weaker frequency dependence of the bound-free channel [cf. Eqs. (6) and (7)]. The total number of events, taking the spatio-temporal beam parameters into account, is \( 1.6 \text{ s}^{-1} \) which should be accessible to experiment. It is interesting to note that antihydrogen atoms could be produced in collisions of APTs with antiprotons, with the rate \( R_{\text{bf}}^{(1)} \) given above. The corresponding process in relativistic antiproton–ion collisions was realized a decade ago at the CERN-LEAR facility [35].

4. Muon pair creation

With X-ray rather than xuv frequencies also nonlinear \( \mu^+ \mu^- \) pair creation could be realized at LHC [36]. Corresponding table-top sources of intense APTs with frequencies up to the keV domain are anticipated by surface harmonics in the relativistic regime of laser–plasma interaction [21]. Moreover, apart from the large-scale XFEL facilities presently under development, there are also efforts to build table-top XFEL devices [37]. According to optimistic estimates, these might reach comparable operation parameters \( (\omega_{\gamma} \sim 10 \text{ keV}, I \sim 10^{16} \text{ W/cm}^2) \) due to very high quality of the laser-generated driving electron beam with a peak current of \( \sim 100 \text{ kA} \) (see Table 1 in [37]). \( \mu^+ \mu^- \) production requires higher laser frequencies because of the large muon mass \( m_\mu \approx 207\text{m} \) and the rate scaling \( \sim \xi_{\mu}^4 \), where \( \xi_\mu = \xi m_\mu/m_\gamma \) and \( n \sim m_\mu/\omega \). In contrast to the e\(^+\)e\(^-\) pair production, the finite size of the projectile nucleus has to be taken into account here since the typical momentum transfer \( q \sim m_\mu \) is of the order of the inverse nuclear radius. As usual, this introduces the nuclear form factor \( F = F(q^2) \) into the reaction rate by virtue of [38]

\[
R_{\mu} = R_0 [Z^2 \xi^2 + Z (1 - \xi^2)].
\]  

(8)

The term \( Z^2 \) accounts for the elastic channel where the nucleus remains in its ground state and the Z protons act coherently; the term \( Z \) describes the inelastic channel involving nuclear excitation and incoherent proton action. \( R_0 \) denotes the reaction rate for a pointlike proton; for the case of two-photon muon creation close to threshold, \( R_0 \) is given by Eq. (6) with the replacements \( m \rightarrow m_\mu \) and \( \xi \rightarrow \xi_\mu \). For simplicity we employ in our calculations a spherically symmetric Gaussian charge distribution, whose free parameter is adjusted to reproduce the measured rms charge radius. We assume an X-ray beam of frequency \( \omega_{\gamma} = 12 \text{ keV} \) and intensity parameter \( \xi_\mu = 6.8 \times 10^{-5} \) [18]. This corresponds to an intensity of order \( 10^{22} \text{ W/cm}^2 \) which can be achieved by improved X-ray focussing techniques. When colliding with the LHC proton beam, muon pair creation is possible via two-photon absorption and occurs with a rate of \( R_{\mu}^{(2)} \approx 8.25 \times 10^{-5} \text{ s}^{-1} \) (see Table 1). This result has already been reported in [36]; a similar rate can be expected for \( \pi^+\pi^- \) creation. We note that because of pronounced recoil effects, the \( \gamma \)-factor which would be required for two-photon \( \mu^+\mu^- \) production by a projectile electron is much larger: \( \gamma \sim 10^6 \), corresponding to a yet unavailable electron energy in the TeV range. The LHC Pb beam produces muons with much lower efficiency since three photons are needed to bridge the energy gap (see Fig. 1(c)), which suppresses the process by a factor \( \xi_{\mu}^2 \sim 10^{-8} \). Besides, the production mainly proceeds through the inelastic channel because of the large nuclear size of Pb, so that its charge enhances the rate by a factor \( Z \) only. As a consequence, Coulomb corrections are of minor importance here [14]. The resulting rate is \( R_{\mu}^{(3)} \approx 6.59 \times 10^{-13} \text{ s}^{-1} \) which seems hardly observable. Experimental studies of \( \mu^+\mu^- \) production should therefore rely on the LHC proton beam. The total event rates are reduced by the fact that the beams overlap only partially since a sub-\( \mu \)m waist size of the X-ray pulse is required to obtain sufficiently high intensity. As a background process, also e\(^+\)e\(^-\) pairs are produced by single-photon absorption in the nuclear field. The rate is given by Eq. (3) and amounts to \( \sim Z^2 \times 10^{11} \text{ s}^{-1} \) in the lab frame. We emphasize that this rather strong background process does not deplete the X-ray beam: assuming a pulse duration of 100 fs and a focus radius of 100 nm, the latter contains about \( 10^{15} \) photons, whereas only \( \sim Z^2 \times 10^{-2} \) e\(^+\)e\(^-\) pairs are generated per ion. Finally, we point out that e\(^+\)e\(^-\) pair production might also become feasible within an all-optical setup where a relativistic proton beam is generated by laser wakefield acceleration instead of the LHC. In the so-called laser-piston regime, proton energies corresponding to \( \gamma \approx 50 \) have been predicted [39]. According to the simulations, each proton bunch contains \( \sim 10^{10} \) particles and has a transverse radius of about 5 \( \mu \)m. When combined with an ultrashort X-ray pulse produced from plasma harmonics \( (\omega_{\gamma} \approx 5–10 \text{ keV}) \), the energy threshold for e\(^+\)e\(^-\) pair creation would be overcome by two-photon absorption leading to a total rate of \( R_{\text{lab}}^{(2)} \approx 5 \text{ s}^{-1} \).

5. Conclusion

In conclusion, lepton pair creation via the nonlinear Bethe–Heitler process in relativistic laser–ion collisions has been considered. A feasible setup for the first observation of this process could be obtained by combining two cutting-edge technologies which are usually considered unrelated, namely an attosecond xuv photon source with CERN-LHC. In contrast to earlier proposals this combination is, in principle, readily available since such laser pulses are produced by table-top devices. In conjunction with the LHC Pb beam, event rates approaching \( N_{\text{ev}} \sim 1 \text{ s}^{-1} \) could be achieved for the creation of free or bound-free e\(^+\)e\(^-\) pairs via absorption of two real photons. By envisaged compact X-ray devices and the LHC proton beam also \( \mu^+\mu^- \) pairs could be produced in this manner, though at lower rates [40]. Nonlinear laser polarization effects could be tested and contributions from next-to-leading order terms revealed. The zeptosecond duration of the pulses in the ion frame holds prospects for time-resolved pump-probe studies on the QED vacuum.
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