Gravitational waves and proton decay: complementary windows into GUTs

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Proton decay is a smoking gun signature of Grand Unified Theories (GUTs). Searches by Super-Kamiokande have resulted in stringent limits on the GUT symmetry breaking scale. The large-scale multipurpose neutrino experiments DUNE, Hyper-Kamiokande and JUNO will either discover proton decay or further push the symmetry breaking scale above \(10^{19}\) GeV. Another possible observational consequence of GUTs is the formation of a cosmic string network produced during the breaking of the GUT to the Standard Model gauge group. The evolution of such a string network in the expanding Universe produces a stochastic background of gravitational waves which will be tested by a number of gravitational wave detectors over a wide frequency range. We demonstrate the non-trivial complementarity between the observation of proton decay and gravitational waves produced from cosmic strings in determining SO(10) GUT breaking chains. We show that such observations could exclude SO(10) breaking via flipped SU(5) \(\times U(1)\) or standard SU(5), while breaking via a Pati-Salam intermediate symmetry, or standard SU(5) \(\times U(1)\), may be favoured if a large separation of energy scales associated with proton decay and cosmic strings is indicated.

I. INTRODUCTION

Grand Unified Theories (GUTs) combine the strong, weak and electromagnetic gauge forces of the Standard Model (SM) into a simple gauge group under which the fermions transform. In such a framework, a larger underlying gauge symmetry is subsequently broken to the SM gauge group, \(G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y\), either directly or via some symmetry breaking pattern. Following the Pati-Salam [1] and SU(5) [2] proposals, many models have been considered. Of particular interest are the SO(10) GUTs [3] which predict neutrino masses and mixing and are based on a simple gauge group.

A well known phenomenological prediction of grand unification is proton decay. At low energies, high-dimensional baryon-number-violating (BNV) operators are induced, leading to proton decay [4–10]. Super-Kamiokande has set stringent constraints on typical decay channels such as \(p \rightarrow \pi^0 e^+ + K^+ \bar{\nu}\) with the proton lifetime exceeding \(10^{34}\) years [11, 12]. There are even more exciting prospects in store during the current decade due to the rich programme of upcoming large-scale neutrino experiments, namely DUNE [13], Hyper-Kamiokande [14] and JUNO [15], which will be at the frontier of BNV searches.

Another generic consequence of GUTs is the production of topological defects when the GUT undergoes spontaneous symmetry breaking (SSB) to the SM gauge group [16]. Some of these, such as monopoles, need to be inflated away in order not to overclose the Universe. However, cosmic strings associated with the breaking of a \(U(1)\) symmetry, which can be a gauged subgroup of the GUT [17], can remain until late times and have observational consequences. The cosmic strings (cs) produced during the series of SSBs are expected to produce gravitational waves (GWs) via the scaling of the string network [17, 19]. These signals form a stochastic GW background (SGWB) today with an abundance proportional to the square of the \(U(1)\) SSB scale, \(\Lambda_{cs}\). The observation of such events provides a unique probe of physics at remarkably high scales. As such there has been recent interest in using the GWs produced from cosmic string scaling as a means of probing leptogenesis [20] and GUTs [21].

In this Letter we discuss the non-trivial complementarity between observing proton decay and GWs produced from cosmic strings in GUTs. In particular, we focus on the implications of these twin observations for determining possible SO(10) GUT breaking chains. While searches for proton decay (pd) set a lower bound on the UV-complete scale \(\Lambda_{pd}\) of BNV operators associated with proton decay, the GW observations are expected to place an upper bound on \(\Lambda_{cs}\). Moreover, in order to avoid unwanted topological defects (monopoles and domain walls) we assume an inflationary epoch, at scale \(\Lambda_{inf}\), in order to inflate away such undesirable objects. We explore the role of experimental searches for proton decay and GWs in determining these three scales: \(\Lambda_{cs}, \Lambda_{pd}\) and \(\Lambda_{inf}\).

In Section I we compare the scale of proton decay and cosmic string formation for different breaking chains of SO(10). The synergy between observation of proton decay and GWs is discussed quantitatively in all possible SO(10) breaking chains in Section II. Finally we summarise and discuss our results in Section III.

II. TERRESTRIAL AND COSMIC SIGNATURES OF GUTS

SO(10) is the minimal simple GUT which offers the possibility of cosmic string generation. The breaking of this group to the SM gauge group can occur in various ways as summarised in Fig. 1. All breaking chains are shown or can be obtained by removing the various intermediate symmetries. The following abbreviations for symmetries that may be conserved at an intermediate
scale during $SO(10)$ breaking to $G_{SM}$ are used:
\[ G_{51} = SU(5) \times U(1)_X, \]
\[ G_{51}^{\text{flip}} = SU(5)^{\text{flip}} \times U(1)^{\text{flip}}, \]
\[ G_{3211} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}, \]
\[ G_{3211}^{\text{flip}} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}, \]
\[ G_{3211}^{\text{flip}} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X, \]
\[ G_{422} = SU(4)_C \times SU(2)_L \times U(1)_Y, \]
\[ G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R. \]  

Note that $G_{3211}$ and $G_{3211}^{\text{flip}}$ are completely equivalent. All possible $SO(10)$ cases can be classified into four types denoted as (a), (b), (c) and (d) in Fig. 1. Types (a), (b) and (c) are models broken via standard $SU(5)$, flipped $SU(5) \times U(1)$ and Pati-Salam $G_{422}$ respectively. Cases with $SU(5)$ as intermediate symmetry, without a $U(1)$ symmetry, are classified as type (d). The scales of proton decay $\Lambda_{\text{pd}}$ and cosmic strings $\Lambda_{\text{cs}}$ are important testable parameters which we discuss in detail in the following.

A. Proton Decay in $SO(10)$. As quarks and leptons are arranged in common multiplets in GUTs, heavy new states which mediate BNV interactions are introduced. At the low energy scale, these heavy states are integrated out and this induces higher-dimensional BNV operators which lead to proton decay.

In SUSY extensions of the SM, other renormalisable terms not directly related to the GUT scale can also trigger proton decay. SUSY BNV operators are stringently constrained by experimental bounds on the proton lifetime and an R-parity symmetry is introduced to forbid them: SM particles and their superpartners have even and odd charges respectively (see Ref. [25] for a review).

We shall first focus on the operators which have a direct connection with the GUT breaking scale. The dimension-six operators arising from gauge contributions, which respect $G_{SM}$, are given by

\[ \frac{\epsilon_\alpha}{\Lambda_1} \left[ (u_R^\alpha)^\dagger Q_u (d_R^\alpha)^\dagger L_\beta + (u_R^\alpha)^\dagger Q_u (\bar{e}_R^\alpha)^\dagger e_\beta \right], \]

where colour indices have been suppressed, $\alpha, \beta$ denote $SU(2)_L$ indices and $\Lambda_1, \Lambda_2$ are the UV-complete scales of the GUT symmetry [4,8]. $\Lambda_1$ and $\Lambda_2$ may differ from each other depending on the breaking chain. In general, the lower of these two scales will mediate the dominant proton decay channel and to simplify our discussion we define the lower scale to be $\Lambda_{pd}$. For types (a) and (d), $\Lambda_1$ and $\Lambda_2$ correspond to the $SU(5)$ and $SO(10)$ breaking scales, respectively, and thus $\Lambda_1 < \Lambda_2$, while for type (b), $\Lambda_2 < \Lambda_1$. As for type (c), $\Lambda_1 = \Lambda_2$ and there is only a single proton decay scale.

These operators induce a series of proton decay channels. The most stringently constrained is $p \to \pi^0 e^+$ as determined by Super-Kamiokande, $\tau p_{\pi e^+} > 1.6 \times 10^{34}$ years (90% C.L., 100% branching ratio assumed) [12].

The proton lifetime based on this channel is directly correlated with the proton decay scale via $\tau p_{\pi e^+} \approx 8 \times 10^{34}$ years $\times (\Lambda_1/10^{16} \text{ GeV})^4 [29]$ or $7 \times 10^{34}$ years $\times (\Lambda_2/10^{16} \text{ GeV})^4 [30]$ which places the lower limits of $\Lambda_1 > 6.7 \times 10^{15}$ GeV and $\Lambda_2 > 3.9 \times 10^{15}$ GeV, respectively. Hyper-Kamiokande offers at least an order of magnitude improvement and aims to achieve a limit of $\tau p_{\pi e^+} \gtrsim 10^{35}$ years [14] which will further push the lower bound of $\Lambda_1$ above $10^{16}$ GeV.

In SUSY GUTs, there are additional dimension-five operators constructed from two SM fermions and two superpartners which are generated via colour-triplet Higgs mediation:

\[ \frac{c_1}{M_T} (\hat{Q} \hat{Q}) (\hat{L} \hat{L}) + \frac{c_2}{M_T} (\hat{u}_R \hat{d}_R) (\hat{e}_R \hat{e}_R), \]

where $c_1$ and $c_2$ are model-dependent coefficients suppressed by Yukawa couplings and $M_T$ is the heavy colour-triplet Higgs mass which is correlated to $\Lambda_{pd}$. These operators are dressed via gluinos, charginos and neutralinos which give rise to dimension-six operators [8]. The decay width with respect to these operators is suppressed not only by $M_T^2$ but also suppressed by Yukawa...
couplings, loop factors and SUSY-breaking scale. Depending on the model, the contribution to proton decay from such SUSY GUT operators, particularly in the $K^+\bar{\nu}$ channel, can be enhanced \([31,32]\) as compared with the non-supersymmetric GUTs. This proton decay channel will be constrained by DUNE which will have the capability to identify kaon tracks using its liquid argon time projection chamber technology with a high efficiency \([33]\).

As such, DUNE will be able to place the most competitive constraint on this channel: $\tau_{K^+\bar{\nu}} \gtrsim 5.0 \times 10^{34}$ years at 90% C.L. Although JUNO has a smaller detector fiducial volume than DUNE or Hyper-Kamiokande, it can reconstruct the kaonic decay channel with high efficiency and will constrain $\tau_{K^+\bar{\nu}} \gtrsim 3.0 \times 10^{34}$ years at 90% C.L. \([15]\). The complementarity of these nucleon decay searches in the upcoming large-scale experiments will provide us with an unprecedented opportunity to probe the ultra-high energy GUT scale (see e.g., Ref. \([33]\)).

### B. Gravitational Waves From Cosmic Strings

The cosmological consequence of spontaneous symmetry breaking from the GUT to the SM gauge group is the formation of topological defects. These defects generically arise from the breaking of a group, $G$, to its subgroup, $H$, such that a manifold of equivalent vacua, $M$, exists and is isomorphic to $G/H$. Monopoles form when the manifold $M$ contains non-contractible two-dimensional spheres, cosmic strings when it contains non-contractible loops and domain walls when the manifold is disconnected. Mathematically, they correspond to non-trivial homotopy groups $\pi_2(M), \pi_1(M)$ and $\pi_0(M) \neq 1$, respectively \([17]\). Different GUT breaking chains result in different combinations of topological defects affecting at various scales; these have been comprehensively categorised in \([16]\) where it was shown the vast majority of GUT breaking chains produce cosmic strings.

Cosmic strings are a source of GWs as they actively perturb the metric at all times. If cosmic strings form after inflation they exhibit a scaling behaviour where the stochastic GW spectrum is relatively flat as a function of frequency and the amplitude is proportional to the symmetry breaking scale, $\Lambda_{cs}$. This feature singles out cosmic strings as a uniquely high scale probe of the early Universe. If the inflationary period has a crossover with a stage of string formation, the standard expectation was that the string network would be diluted to an unobservable extent. However, it was recently shown that if cosmic strings form during inflation, the string network can regrow to the extent that its associated GW signal is observable \([34]\). The GWs are sourced when the cosmic strings intersect to form loops. Cusps on these strings emit strong beams of high-frequency GWs or bursts. Bursts which are unresolved over time constitute a SGWB while more recent bursts can be individually resolved.

In Fig. 2 we show sensitivities of various current and future GW experiments alongside the predicted SGWB for cosmic strings undiluted (solid curves) and diluted (dashed curves) by inflation. The $U(1)$ symmetry breaking scale $\Lambda_{cs} = 10^{10,11,\cdots,15}$ GeV corresponds to $G\mu \approx 0.7 \times 10^{-18, -16, \cdots , -8}$, respectively, where $\mu$ is the string tension and $G$ is Newton’s constant. We assume a standard cosmology and that the majority of the energy loss of the cosmic string is dominated by gravitational radiation rather than particle production \([10]\). The applied formulation of SGWB in both the undiluted and diluted cosmic strings scenario is provided in Appendix A.

Applying these standard assumptions, a large range of $\Lambda_{cs}$ can be explored using ongoing and upcoming GW detectors. LIGO O2 \([39]\) has excluded cosmic strings formation at $\Lambda_{cs} \sim 10^{15}$ GeV in the high frequency regime 10-100 Hz. While in low frequency band $10^{-9,10^{-8}}$ Hz, the null result of EPTA \([40]\) constrains the upper bound of $\Lambda_{cs}$ below $10^{14}$ GeV. Planned pulsar timing arrays such as SKA \([41]\), space-based laser interferometers such as LISA \([42]\), Taiji \([43]\), TianQin \([44]\), BBO \([45]\), DECIGO \([46]\), ground-based interferometers such as Einstein Telescope \([47]\) (ET) and Cosmic Explorer \([48]\) (CE), and atomic interferometers MAGIS \([49]\), AEDGE \([50]\), AION \([51]\) will probe $\Lambda_{cs}$ values in a wide regime $10^{10,14}$ GeV. As the spectrum of GWs produced via diluted cosmic strings decreases rapidly for $f > 10^{-6}$ Hz, this allows them to be distinguished from the undiluted cosmic strings as shown in Fig. 2.

In addition to cosmic strings, all of the $SO(10)$ breaking chains generate additional unwanted topological defects such as monopoles and domain walls. These are in

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1 New physics, which triggers an early period of matter domination, can affect the SGWB \([35,38]\).
dicated in Fig. II by the red dotted arrows. These stable defects are undesirable as they conflict with cosmological observations and inflation is a promising means to remove them. Consistent hybrid inflation models have been achieved via GUT breaking [52, 53].

The scale of inflation, \( \Lambda_{\text{inf}} \), is important as it inflates away unwanted defects and can affect the GW signal. The shape and magnitude of the inflaton potential are imprinted in the primordial density perturbations which can be characterised by the spectral index and the tensor-to-scalar ratio in cosmic microwave background (CMB) measurements, from which the upper limit on inflation is \( \Lambda_{\text{inf}} < 1.6 \times 10^{16} \) GeV (95% C.L., Planck) [54]. Future CMB measurements can improve the upper limit of the tensor-to-scalar ratio to 0.001 (95% C.L., CMB-S4) [55], corresponding to \( \Lambda_{\text{inf}} < 5.7 \times 10^{15} \) GeV.

III. SYNERGY BETWEEN PROTON DECAY AND GW MEASUREMENTS

There are several possible results of future proton decay searches. A null result will place a new lower bound on \( \Lambda_{\text{pd}} \) while the positive determination of proton decay and its nature would allow deeper insight into the GUT symmetry structure. Due to the relatively model-independent nature of the operators shown in Eq. (2), the following experimental results are of particular interest:

- Proton decay is observed in the \( \pi^0 e^+ \) channel. This possibility provides an explicit link between \( \Lambda_{\text{pd}} \) and the proton lifetime.
- Proton decay is observed in the \( K^+ \bar{\nu} \) channel. This process is dominantly associated with the SUSY version of GUTs. However, this case provides a weaker connection to \( \Lambda_{\text{pd}} \) due to the involvement of the unknown SUSY-breaking scale.

The observation of GWs from cosmic strings is crucially dependent on the scale of inflation. We consider two possibilities:

- If string formation occurs after inflation, namely \( \Lambda_{\text{cs}} < \Lambda_{\text{inf}} \), a SGWB is generated from undiluted strings and typical scaling behaviour is observed.
- A marginal situation occurs if string formation takes place during inflation, namely \( \Lambda_{\text{cs}} \sim \Lambda_{\text{inf}} \), and GWs from partly diluted strings may be observed [34].

There is a third possibility: inflation occurs after cosmic string formation, \( \Lambda_{\text{cs}} > \Lambda_{\text{inf}} \). In this case, cosmic strings are fully erased after inflation and no associated GW signal can be observed. This scenario offers relatively little insight into the cosmological consequences of the GUT symmetry and therefore we do not consider this possibility further. From the synergy of experimental data discussed in Section II (\( \Lambda_{\text{pd}} \geq 10^{15} \) GeV, \( \Lambda_{\text{inf}} < 10^{16} \) GeV and \( \Lambda_{\text{cs}} < 10^{14} \) GeV) certain ordering of scales are already excluded such as \( \Lambda_{\text{inf}} > \Lambda_{\text{cs}} \sim \Lambda_{\text{pd}} \) and \( \Lambda_{\text{inf}} \gtrsim \Lambda_{\text{cs}} > \Lambda_{\text{pd}} \). We first discuss the various scales for the type (a) chain and then examine the remaining breaking chains.

**Type (a)** breaking chain is shown in Fig. II(a),

\[
SO(10) \rightarrow SU(5) \times U(1)_X \rightarrow G_{\text{SM}} \times U(1)_X \rightarrow G_{\text{SM}},
\]

which is clearly characterised by \( \Lambda_{\text{pd}} > \Lambda_{\text{cs}} \). In the non-SUSY case, the main source of proton decay is provided by operators on the first line of Eq. (2) and they give rise to the \( \sigma^0 e^+ \) channel. The \( K^+ \bar{\nu} \) channel is also generated but is suppressed by nuclear matrix element and CKM mixing. The proton decay scale is given by \( \Lambda_{\text{pd}} = \mathcal{L}_3 \), where \( \mathcal{L}_3 \) is the \( SU(5) \) gauge coupling and \( M_T \) the heavy vector boson mass [4, 5].

In the SUSY case, the dimension-five operators of Eq. (3) with specified coefficients, which are the same as in the minimal SUSY \( SU(5) \) model and given in [10, 56–58], are the main contribution to the channel \( p \rightarrow K^+ \bar{\nu} \). The minimal SUSY \( SU(5) \) model exhibits a tension between the prediction of \( M_T \) from gauge unification and the lower bound constrained by Super-Kamiokande via the \( K^+ \bar{\nu} \) mode [29, 59]. For realistic models overcoming this inconsistency, see, e.g., Ref. [31, 52].

We thus obtain the same scale ordering \( \Lambda_{\text{pd}} > \Lambda_{\text{inf}} > \Lambda_{\text{cs}} \) in both non-SUSY and SUSY versions. A cosmic string network is produced at \( \Lambda_{\text{cs}} \), which is a potential source of GWs. However, the observational signal of such GWs depends on the value of \( \Lambda_{\text{inf}} \) as follows.

In addition to the cosmic strings, monopoles are produced in the first and second steps of the breaking chain as illustrated in Fig. II(a). As discussed, to ensure the GUT model is cosmologically feasible, inflation must occur to remove these defects. To achieve this, the inflationary scale \( \Lambda_{\text{inf}} \) should not be higher than the second-step breaking scale, i.e., \( \Lambda_{\text{pd}} \), which coincides with the lowest scale of unwanted defect production. Therefore, there are three possible orderings of the relevant scales. In the scenario, \( \Lambda_{\text{pd}} > \Lambda_{\text{inf}} \sim \Lambda_{\text{cs}} \), proton decay may be observed in conjunction with an undiluted GW signal. This is an ideal possibility from the experimental perspective. In the case \( \Lambda_{\text{pd}} \sim \Lambda_{\text{inf}} \sim \Lambda_{\text{cs}} \), the coupling is also generated in the undiluted GW scenario. Finally, if \( \Lambda_{\text{pd}} > \Lambda_{\text{cs}} > \Lambda_{\text{inf}} \), proton decay could be observed but no associated GW signal is detected.

**Type (b)** is associated with flipped \( SU(5) \times U(1) \) and proton decay proceeds dominantly via the pion channel. As with the scenario of type (a), the string network formation occurs in the final breaking step. This case is characterised by \( \Lambda_{\text{pd}} \sim \Lambda_{\text{cs}} \), so that the scale of proton decay and GW production is approximately the same. Given the current limits on proton decay and GWs (which implies \( \Lambda_{\text{pd}} \geq 10^{15} \) GeV and \( \Lambda_{\text{cs}} \leq 10^{14} \) GeV for the undiluted cosmic string scenario) it may seem like

\[ 2 \text{The latter ordering is not predicted by any breaking chain of } SO(10) \text{ but in enlarged symmetry groups. For example, in } E_6, \text{ this ordering can arise [10].} \]
\( \Lambda_{\text{pd}} \sim \Lambda_{\text{cs}} \) is already excluded. However, as before, the observability of GWs depends on the scale of inflation \( \Lambda_{\text{inf}} \) as we now discuss.

If the scale of inflation is high, then indeed the scale ordering \( \Lambda_{\text{inf}} > \Lambda_{\text{pd}} \sim \Lambda_{\text{cs}} \) can already be excluded. However, if it is lower, then \( \Lambda_{\text{inf}} \sim \Lambda_{\text{pd}} \sim \Lambda_{\text{cs}} \) remains viable as the SGWB produced from dilute strings is suppressed relative to the undiluted case. Given the sensitivities, this ordering can be tested in the next generation experiments. In the future, experimental limits on proton decay and inflation may constrain \( \Lambda_{\text{pd}} > \Lambda_{\text{inf}} \). Then \( \Lambda_{\text{pd}} \sim \Lambda_{\text{cs}} \) would imply \( \Lambda_{\text{cs}} > \Lambda_{\text{inf}} \) leading to the falsifiable prediction that GWs from this model are unobservable.

**Type (c)** represents a complicated class of cases which all have the common feature that proton decay is associated with the first step of symmetry breaking \( SO(10) \rightarrow G_{422} \), the Pati-Salam gauge group. In this chain, cosmic strings are generated by the last step of breaking. Hence, \( \Lambda_{\text{pd}} > \Lambda_{\text{cs}} \) here, as in type (a). As before, the observability of GWs depends on the scale of inflation \( \Lambda_{\text{inf}} \), to which we turn. The breaking of \( G_{422} \) results in the production of unwanted defects at each stage of SSB prior to the final breaking that produces the string network. Therefore, \( \Lambda_{\text{inf}} \) must occur below the breaking of \( G_{422} \), \( \Lambda_{\text{inf}} < \Lambda_{\text{cs}} \). Apart from that, the scale ordering of this class of models can be determined in a similar way to type (a), (see previous discussion).

In order to distinguish between type (a) and (c) further specification of the model and particle representations is required. From this, predictions of nucleon decay branching ratios could be used as a means of differentiation between the breaking chains (see Ref. [32] for an overview). Furthermore, \( \Lambda_{\text{pd}} \) in type (c) chains can be significantly higher than \( 10^{16} \) GeV if there are threshold corrections from intermediate symmetries at low scale, e.g., \( 10^{10-12} \) GeV \([60, 61]\). Such a low scale symmetry breaking may be linked to the origin of neutrino masses and leptogenesis. An observation of low scale GWs may favour some specific breaking chains of this type. In summary, SGWB can be a useful means of disentangling the scale ordering of type (a) and (c) but not in distinguishing these two cases from each other, which will depend on detailed information about proton decay channels.

**Type (d)** has the same \( SU(5) \) intermediate symmetry as type (a) and therefore the similar predictions for proton decay as in type (a) but with \( \Lambda_{\text{cs}} > \Lambda_{\text{pd}} \). However, the inflation scale must be lower than the proton decay scale \( \Lambda_{\text{pd}} > \Lambda_{\text{inf}} \), since monopoles generated in the final step of symmetry breaking must be inflated away. Unfortunately this also inflates away the cosmic strings. Hence, any associated GW detection via cosmic strings (diluted or undiluted) would exclude this class of breaking chains under our assumption the GW signal is associated to the \( SO(10) \) breaking.

Our analysis is summarised in Fig. 3. In the right panel we tabulate how observing proton decay via the pion channel in conjunction with GWs can be used to exclude or favour certain breaking chains and also provide information on the scale ordering. The consequence of null observations are not given in the table of Fig. 3. In the event proton decay is not observed in the next generation of neutrino experiments, the limit on the UV-complete scale \( \Lambda_{\text{pd}} \) will be pushed even higher. On the other hand, future non-observation of cosmic string-induced GWs would suggest an inflationary era occurred after cosmic string formation. In addition, improved CMB measurements will allows more stringent upper bound for \( \Lambda_{\text{inf}} \) to be placed which will in turn be an upper bound for \( \Lambda_{\text{cs}} \) if cosmic strings are to be observed. This is schematically shown in the left panel of Fig. 3 where coloured and hatched regions indicate current and future experimental limits to probe these scales. For example, future experiments may constrain \( \Lambda_{\text{pd}} > \Lambda_{\text{inf}} \).

**IV. SUMMARY AND CONCLUSION**

We have proposed a strategy to use both proton decay and gravitational waves (GWs) as a means of identifying possible breaking chains of Grand Unified The-
ories (GUTs). We focus on $SO(10)$ GUT models and categorise them according to their symmetry breaking patterns as shown in Fig. 1(a)-(d), corresponding to $SU(5) \times U(1)$, Pati-Salam, flipped $SU(5) \times U(1)$ and $SU(5)$, respectively.

For each pattern of breaking, we compare the scale of proton decay, $\Lambda_{pd}$, with the cosmic string formation scale, $\Lambda_{cs}$. These scales can have important testable consequences as they are related to the proton lifetime and the generation of GWs via cosmic strings. Observations of proton decay and GWs can be used to constrain the scales $\Lambda_{pd}$ and $\Lambda_{cs}$, respectively. The determination of these scales, in particular their ordering, provides useful information in assessing the viability of a given class of breaking chains within $SO(10)$ GUTs.

Our results are summarised in Fig. 3. In particular, such observations could exclude $SO(10)$ breaking via flipped $SU(5) \times U(1)$ or standard $SU(5)$, while breaking via a Pati-Salam intermediate symmetry, or standard $SU(5) \times U(1)$, may be favoured if a large separation of energy scales associated with proton decay and cosmic strings is indicated.

In conclusion, we are entering an exciting era where new observations of GWs from the heavens and proton decay experiments from under the Earth can provide complementary windows to reveal the details of the unification of matter and forces at the highest energies.

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A. NUMERICAL METHOD TO CALCULATE GWs FROM COSMIC STRINGS

We list numerical methods to calculate the SGWB released from cosmic strings. Those from the general undiluted cosmic strings and from inflation-diluted ones will be considered separately.

1. GWs via undiluted cosmic strings. We follow [30] to estimate the emission of stochastic gravitational wave background (SGWB) from cosmic string scaling. Considering ideal Nambu-Goto strings, the dominant radiation emission is in the form of GWs. The large loops give the dominant contribution to the GW signal and therefore, we focus on them. The initial large loops have typical length $l_i = \alpha t_i$ with $\alpha \sim 0.1$ and $t_i$ the initial time of string formation. The length of loops decreases as they release energy to the cosmological background,

$$l(t) = \alpha L(t_i) - \Gamma G\mu (t - t_i).$$  \hfill (A1)

Frequencies of GW released from the loops are given by $2k/l_i$ where $k = 1, 2, \cdots$.

We denote the $\Lambda_{cs}$ as the scale of symmetry breaking leading to GW. The tension of the string (energy per unit length) $\mu$ is typical taken to be $\Lambda_{cs}^2$. After strings form, loops are found to emit energy in the form of gravitational radiation at a constant rate

$$\frac{dE}{dt} = -\Gamma G\mu^2,$$  \hfill (A2)

where numerically $\Gamma$ is found to be $\Gamma \approx 50$ [62, 63].

Assuming the fraction of the energy transfer in the form of large loops is $F_\alpha$, the relic GW density parameter is given by

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\log f},$$  \hfill (A3)

This can be written as a summation of mode $k$

$$\Omega_{GW}(f) = \sum_k \Omega_{GW}^{(k)}(f),$$  \hfill (A4)

with

\begin{align*}
\Omega_{GW}^{(k)}(f) &= \frac{1}{\rho_c} \frac{2k}{\int \alpha} (\alpha + \Gamma G\mu)^2 \\ &\times \int_{t_F}^{t_0} dt \frac{C_{eff}(\frac{t_i^{(k)}}{k})}{t_i^{(k)^4}} \left(\frac{a^2(t)}{a^2(t_0)} \theta(t_i^{(k)} - t_F)\right),
\end{align*}

\hfill (A5)

where $\rho_c$ is the critical energy density of the Universe given by

$$\rho_c = \frac{1}{3.6} \sqrt{\frac{c}{k} - 4/3},$$

$$t_i^{(k)} = \frac{1}{\alpha + \Gamma G\mu} \left(\frac{2k}{\int a(t)} a(t_0) + \Gamma G\mu \right),$$

\hfill (A6)

$C_{eff}$ is numerically obtained as $C_{eff} = 5.7, 0.5$ [64, 65] for radiation and matter domination, respectively, and $t_F$ is the time of string network formation.

2. GWs of inflation-diluted cosmic strings. Our assumptions follow those outlined in [41] where it is assumed during inflation the Hubble expansion rate is constant with $H = H_1 = \sqrt{V_1/3M_{Pl}^2}$ with $V_1 = \Lambda_{inf}^4$ and $M_{Pl}$ the reduced Planck mass.

Cusps on string loops lead to bursts of GWs, which can potentially be resolved as individual events [67–70]. Assuming the correlation length of strings as $L$, together with the speed of string $v$, satisfy

\begin{align*}
\frac{dL}{dt} &= (1 + v^2)HL + \frac{\tilde{c} v}{2}, \\
\frac{dv}{dt} &= (1 - v^2) \left(\frac{k(\tilde{v})}{L} - 2HV\right),
\end{align*}

\hfill (A7)

where $\tilde{c} \approx 0.23$ parametrises the loop chopping efficiency [74] and

$$k(\tilde{v}) = \frac{2\sqrt{2}}{\pi} (1 - \tilde{v}^2)(1 + 2\sqrt{2}\tilde{v}^3) \left(1 - \frac{8\tilde{v}^6}{1 + 8\tilde{v}^6}\right).$$  \hfill (A8)

Kinks also leads to bursts of GWs but subdominant [64, 71], which will not be considered here.
During inflation, the solution is simplified to
\[ L(t) = L_F e^{H_I (t - t_F)}, \]
\[ \bar{v}(t) = \frac{2\sqrt{2}}{\pi} \frac{1}{H_I L_I}, \]  
(A10)

where \( L_F = L(t_F) \) is the initial condition, and \( t_F \) is the network formation time (assuming it happens after the beginning of inflation). Inflation results in the string out of horizon \( HL \gg 1 \). After inflation ends, \( HL \) reduces and eventually evolves back to the horizon. We denote \( z \) as the redshift when strings returns to the horizon, \( H(\bar{z}) L(\bar{z}) = 1 \).

The rate of bursts per volume per length \( d^2R/dV dl \) observed today can be transferred to the rate of per redshift per waveform as
\[ \frac{d^2R}{dz dh} (h, z, f) = \frac{2^{3(q-1)} \pi G \mu N_q}{2 - q} \frac{r(z)}{(1 + z)^3 H(z)} \frac{n(l, z)}{h^2 f^2}, \]  
(A11)

where \( h \) is the waveform, \( r(z) = \int_z^\infty dz'/H(z') \) is the proper distance to the source and \( q = 4/3 \) for cusps. During the inflationary era, \( r(z) \) is simplified to \( r(z) = r_R + (z - z_R)/H_I \), where \( r_R = r(2) \) represents the reheating period in the end of inflation.

\[ n(l, t) \] is the differential number density of long loops per unit length given by
\[ n(l, t) = \frac{F_\alpha}{\sqrt{2}} \frac{(z(t) - 1)^3/(z(t_i) + 1)^3}{\alpha dL/dl|_{t=t_I} + \frac{1}{2G\mu} \alpha L^4(t_i)}, \]  
(A12)

where \( l \) is the length of string given in Eq. [A1]. \( l \) is correlated with the waveform of loops \( h \). Given the redshift \( z \), the frequency \( f \) and \( h, l \) is determined to be
\[ l(h, z, f) = \left( \frac{f^q(1 + z)^{q-1} h r(z)}{G\mu} \right)^{1/(2-q)} \]  
(A13)

for cusps.

From this correlation, one can determine \( t_i \) for given \( t, h \) and \( f \). In order to ensure a solution for \( t_i, l(h, z, f) < \alpha L(z) \) is satisfied, it is equivalent to to setting an upper bound value of \( h \). Furthermore, the above formulas are valid only for small angle radiation, i.e., \( \theta_m = 1/(f(l(1 + z))^{1/3} < 1 \), which provides a lower bound value of \( h \). In summary, \( h \) is restricted in the interval \( (h_{\text{min}}, h_{\text{max}}) \) with
\[ h_{\text{min}} = \frac{1}{(1 + z)}} f^2 r(z), \]  
(A14)
\[ h_{\text{max}} = \frac{\alpha L(z)^{2-q} G\mu}{(1 + z)^{q-1} f^q r(z)}. \]  
(A15)

Bursts contribute to the SGWB as
\[ \Omega_{\text{GW}}(f) = \frac{1}{\rho_c 2\pi^2} f^3 \int_{z_*}^\infty dz \int_{h_{\text{min}}}^{h_{\text{max}}} dh h^2 \frac{d^2R}{dz dh}(h, z, f), \]  
(A16)

where \( z_* \) enforces the rate condition and solved via
\[ f = \int_0^{z_*} dz \int_{h_{\text{min}}}^{h_{\text{max}}} dh \frac{d^2R}{dz dh}(h, z, f). \]  
(A17)

[1] J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973).
[2] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972).
[3] H. Fritzsch and P. Minkowski, Annals Phys. 93, 193 (1975).
[4] S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979).
[5] F. Wilczek and A. Zee, Phys. Rev. Lett. 43, 1571 (1979).
[6] S. Weinberg, Phys. Rev. D22, 1694 (1980).
[7] S. Weinberg, Phys. Rev. D 26, 287 (1982).
[8] N. Sakai and T. Yanagida, Nucl. Phys. B 197, 533 (1982).
[9] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Lett. B 112, 133 (1982).
[10] J. R. Ellis, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B202, 43 (1982).
[11] K. Abe et al. (Super-Kamiokande), Phys. Rev. D 90, 072005 (2014) arXiv:1408.1195 [hep-ex].
[12] K. Abe et al. (Super-Kamiokande), Phys. Rev. D 95, 012004 (2017) arXiv:1610.05597 [hep-ex].
[13] R. Acciarri et al. (DUNE), (2015), arXiv:1512.06148 [physics.ins-det].
[14] K. Abe et al. (Hyper-Kamiokande), (2018), arXiv:1805.04163 [physics.ins-det].
[15] F. An et al. (JUNO), J. Phys. G43, 030401 (2016) arXiv:1507.05613 [physics.ins-det].
[16] R. Jeannerot, J. Rocher, and M. Sakellariadou, Phys. Rev. D68, 103514 (2003) arXiv:hep-ph/0308134 [hep-ph].
[17] A. Vilenkin, Phys. Rept. 121, 263 (1985).
[18] R. R. Caldwell and B. Allen, Phys. Rev. D45, 3447 (1992).
[19] M. B. Hindmarsh and T. W. B. Kibble, Rept. Prog. Phys. 58, 477 (1995) arXiv:hep-ph/9411342 [hep-ph].
[20] J. A. Dror, T. Hirama, K. Kohri, H. Murayama, and G. White, Phys. Rev. Lett. 124, 041804 (2020) arXiv:1908.03227 [hep-ph].
[21] W. Buchmuller, V. Domcke, H. Murayama, and K. Schmitz, (2019), arXiv:1912.03695 [hep-ph].
[22] S. J. King, S. F. King, and S. Moretti, Phys. Rev. D 97, 115027 (2018) arXiv:1712.01279 [hep-ph].
[23] S. M. Barr, Phys. Lett. 112B, 219 (1982).
[24] J. Derendinger, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. B 139, 170 (1984).
[25] A. De Rujula, H. Georgi, and S. Glashow, Phys. Rev. Lett. 45, 413 (1980).
[26] I. Antoniadis, J. R. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. B 231, 65 (1989).
[27] J. C. Pati and A. Salam, Phys. Rev. D10, 275 (1974) [Erratum: Phys. Rev.D11,7063(1975)].
[28] H. Dreiner, Perspectives on Supersymmetry II, 565 (2010).
[29] H. Murayama and A. Pierce, Phys. Rev. D65, 055009 (2002), arXiv:hep-ph/0108104 [hep-ph].
[30] J. Ellis, M. A. Garcia, N. Nagata, D. V. Nanopoulos, and K. A. Olive, (2020), arXiv:2003.03285 [hep-ph].
[31] S. Raby, Rept. Prog. Phys. 67, 755 (2004), arXiv:hep-ph/0401155 [hep-ph].
[32] P. Nath and P. Fileviez Perez, Phys. Rept. 441, 755 (2004), arXiv:hep-ph/0401155 [hep-ph].
[33] J. Heeck and V. Takhistov, Phys. Rev. D101, 015005 (2020), arXiv:1910.07647 [hep-ph].
[34] Y. Cui, M. Lewicki, D. E. Morrissey, and J. D. Wells, Phys. Rev. D97, 123505 (2018), arXiv:1711.03104 [hep-ph].
[35] Y. Cui, M. Lewicki, D. E. Morrissey, and J. D. Wells, JHEP 01, 081 (2019), arXiv:1803.02886 [hep-ph].
[36] Y. Cui, M. Lewicki, D. E. Morrissey, and J. D. Wells, (2019), arXiv:1912.03245 [hep-ph].
[37] B. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. D100, 061101 (2019), arXiv:1903.02886 [gr-qc].
[38] L. Lentati et al., Mon. Not. Roy. Astron. Soc. 453, 2576 (2015), arXiv:1504.03692 [astro-ph.CO].
[39] G. Janssen et al., PoS AASKA14, 037 (2015), arXiv:1501.00127 [astro-ph.IM].
[40] P. Amaro-Seoane et al. (LISA), (2017), arXiv:1709.03845 [astro-ph.CO].
[41] W.-H. Ruan, Z.-K. Guo, R.-G. Cai, and Y.-Z. Zhang, (2018), arXiv:1807.09495 [gr-qc].
[42] J. Luo et al. (TianQin), Class. Quant. Grav. 33, 035010 (2016), arXiv:1512.02076 [astro-ph.IM].
[43] N. Seto, S. Kawamura, and T. Nakamura, Phys. Rev. Lett. 87, 221103 (2001), arXiv:astro-ph/0108011 [astro-ph].
[44] B. Sathyaprakash et al., Class. Quant. Grav. 29, 124013 (2012) [Erratum: Class. Quant. Grav.30,079501(2013)], arXiv:1206.0331 [gr-qc].
[45] B. P. Abbott et al. (LIGO Scientific), Class. Quant. Grav. 34, 044001 (2017), arXiv:1607.08697 [astro-ph.IM].
[46] P. W. Graham, J. M. Hogan, M. A. Kasevich, S. Rajendran, and R. W. Romani (MAGIS), (2017), arXiv:1711.02225 [astro-ph.IM].
[47] Y. A. El-Neaj et al. (AEDGE), EPJ Quant. Technol. 7, 6 (2020), arXiv:1908.00802 [gr-qc].
[48] L. Badurina et al., JCAP 05, 011 (2020) arXiv:1911.11755 [astro-ph.CO].
[49] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, Phys. Rev. D 49, 6410 (1994), arXiv:astro-ph/9401011.
[50] G. Dvali, Q. Shafi, and R. K. Schaefer, Phys. Rev. Lett. 73, 1886 (1994), arxiv:hep-th/9406319.
[51] Y. Akrami et al. (Planck), (2018), arXiv:1807.06211 [astro-ph.CO].
[52] K. Abazajian et al., (2019), arXiv:1907.04473 [astro-ph.IM].
[53] P. Nath, A. H. Chamseddine, and R. L. Arnowitt, Phys. Rev. D 32, 2348 (1985).
[54] P. Nath and R. L. Arnowitt, Phys. Rev. D38, 1479 (1988).
[55] J. Hisano, H. Murayama, and T. Yanagida, Nucl. Phys. B402, 46 (1993) arXiv:hep-ph/9207279 [hep-ph].