Radiation-condensation instability in a four-fluid dusty plasma

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In this work linear stability analysis of a four-fluid optically thin plasma consisting of electrons, ions, neutral atoms, and charged dust particles is performed with respect to the radiation-condensation (RC) instability. The energy budget of the plasma involves the input from heating through photo-electron emission by dust particles under external ultraviolet radiation as well as radiative losses in inelastic electron-neutral, electron-ion, neutral-neutral collisions. It is shown that negatively charged particles stimulate the RC instability in the sense that the conditions for the instability to hold are wider than similar conditions in a single-fluid description.

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I. INTRODUCTION

The radiation-condensation (RC) instability in an optically thin plasma plays an important role both in space and laboratory plasmas (cf. Refs. 1, 2, 3, 4). Dust impurities can alter dynamical and thermodynamical properties of the plasma because of their high inertia and the ability to transform thermal energy into radiation. It was shown by Ref. 5 within the framework of a two-fluid (dust and plasma) description that the presence of dust particles can change the conditions for the RC instability to grow. An essential assumption of Ref. 5 was that dust particles contribute to the radiative cooling. In many situations, however, dust provides instead the energy input to plasma due to photo-emission of electrons from the dust surface when the system is exposed to ultraviolet radiation. Such conditions can be met in interstellar plasmas (cf. Ref. 6). In this case the energy gained from non-thermal electrons escaping photo-ionized dust grains is transmitted to thermal electrons in the ambient plasma, which then in turn heat ions and neutrals in elastic collisions. Simultaneously, the electrons lose their energy and pressure radiatively, and therefore are compressed under the external pressure. In this picture dust particles and electrons are the thermodynamically active components, while neutrals and ions represent the passive components providing conditions for radiative cooling of the electrons. Therefore, at such circumstances only a full four-fluid description is adequate. For the sake of simplicity we restrict ourself in this paper to consideration of a non-magnetized plasma.

The paper is organized in the following way: in Sec. II.A we present the full set of four-fluid equations describing the evolution of the plasma under consideration, followed by the description of the energy exchange between the fluids in Sec. II.B. In Sec. III.A we present the linearized dynamical equations, and in Sec. III.B we derive the instability criterion for the condensation mode. The analysis of the instability is given in Sec. IV. We close our paper with a short summary in Sec. V.

II. DYNAMICS AND THERMODYNAMICS

A. Dynamical equations

We start from a four-fluid unmagnetized system (based on Refs. 7 and 8 for a magnetized plasma) for electrons (α = e), ions (α = i), neutral atoms (α = a) and massive charged dust particles (α = d):

\[ \partial_t n_\alpha + \nabla \cdot (n_\alpha u_\alpha) = 0, \]

\[ \partial_t (n_\alpha u_\alpha) + \nabla \cdot (n_\alpha u_\alpha u_\alpha) = -\frac{1}{m_\alpha} \nabla p_\alpha - \frac{n_\alpha \tilde{Z}_\alpha e}{m_\alpha} \nabla \Phi, \]

\[ \partial_t p_\alpha = -\nabla \cdot (p_\alpha u_\alpha) + (\gamma - 1)(-p_\alpha \nabla \cdot u_\alpha + H_\alpha), \]

\[ \Delta \Phi = -4\pi e \sum Z_\alpha n_\alpha, \]

where \( n_\alpha, \tilde{Z}_\alpha, u_\alpha \) and \( p_\alpha \) are particle number density, electrical charge number, velocity and gas pressure of the \( \alpha \)th species, respectively. As shown in Ref. 3 for a wide range of wavelengths relevant to the RC instability, separation of the dust charge can be neglected, such that the charged components are coupled through the quasi-neutrality condition

\[ n_e = \sum_{\alpha \neq e} \tilde{Z}_\alpha n_\alpha. \]

We assume the ions to be singly charged, so that \( \tilde{Z}_i = +1 \) and \( \tilde{Z}_e = -1 \). As the dust is usually negatively charged in the systems under consideration, we...
write \( \dot{Z}_d = -Z_d \), so that a positive value of \( Z_d \) corresponds to negatively charged dust and vice versa. The quasi-neutrality condition, thus, reads

\[
n_i - Z_d n_d - n_e = 0. \tag{6}
\]

However, the validity of this assumption is limited by the absolute value of dust charge: the characteristic time scale for a radiatively cooling plasma is of the order of \( \sim k_B T / \Lambda \sim 10^{11} n^{-1} \) s in temperature range \( T \sim 10^4 \) K. Here, \( \Lambda \sim 10^{-23} \) erg cm\(^3\) s\(^{-1}\) is the radiative cooling function at \( T \sim 10^4 \) K for the interstellar environment, and \( n \) is the total number density of all species together. Dust separation becomes important when this time is shorter than the inverse dust plasma frequency \( \omega_p^{-1} = \sqrt{m_d/4\pi Z_d e^2 n_d} \sim 10^8 n^{-1/2} \) s, for \( n_d \sim 10^{-12} n \) typical for interstellar plasma, which gives \( Z_d \ll 10^{-3} \sqrt{n} \).

For simplicity we neglect here the collisional source terms in the momentum equations. In the case when all the four fluids in the unperturbed state are at rest, \( u_o = 0 \), we consider here, this assumption can only result in an overestimate of the growth rate, but cannot change the instability criterion.

## B. Energy exchange

The source terms in the energy equations, \( \mathcal{H}_\alpha = H_\alpha - L_\alpha \), describe the generalized energy gain rate due to radiation and collisional processes. In the interstellar medium (ISM) the RC instability develops mostly in the warm neutral medium with \( T \sim 10^4 \) K, \( n \sim 0.1 \) cm\(^{-3}\) and a fractional ionization of \( x \sim 0.1 \). Small amount of trace heavy elements account for \( \sim 0.01 \) of the gas mass, and a similar amount is confined in dust grains. The sizes of grains range from \( \sim 10^\text{Å} \) to 0.4\( \mu \)m (cf. Ref. \[9\]), the charge of dust grains in the warm neutral medium is positive and varies from \( |Z_d| \sim 0.1 \) to \( |Z_d| \sim 10^2 \), e.g. Ref. \[10\] depending on size; for the sake of simplicity we will describe in what follows the dust component as an ensemble of particles of equal masses, \( m_d = 10^{-14} \)g, and equal charges, \( Z_d \), assuming the average abundance of dust particles, \( x_d = n_d/n \sim 10^{-12} \). The ion component comprises mostly of protons and singly ionized trace elements such as CII, FeII, and SiIII (ionized by the interstellar ultraviolet field), the neutrals contain hydrogen atoms, HI, and neutral oxygen, OI. Helium is neutral in the warm neutral medium, and its contribution to radiative processes is negligible at low temperatures \( T < 2 \times 10^4 \) K, therefore it can be included in the system by multiplying the mass of the neutral particles by factor of 1.4.

We have used the two sets of abundances corresponding to the standard solar abundances \( x_i = n_i/n \) for species \( i: x_{Fe} = 3.2 \times 10^{-5}, x_{Si} = 3.2 \times 10^{-5}, x_0 = 4.4 \times 10^{-4}, \) and \( x_C = 3.57 \times 10^{-4} \), and a "depleted" set of abundances with \( x_{Fe} = 6.0 \times 10^{-7}, x_{Si} = 2.0 \times 10^{-9}, x_0 = 5.0 \times 10^{-5}, \) and \( x_C = 6.0 \times 10^{-5} \), describing the mean composition of the interstellar plasma with some elements frozen on dust grains as suggested first by Ref. \[11\]. The results for the two sets are similar; later only calculations for the "depleted" set are shown.

We assume in this paper that the system gains the energy from photo-electrons produced by external ultraviolet radiation ionizing dust particles. Non-thermal photoelectrons emitted by dust share their energy with thermal electrons of the plasma with the rate \( \Gamma n_d \), where \( \Gamma \) is determined by the external UV radiation flux, \( G_0 \), and optical properties of the dust. In inelastic collisions with ions and neutrals the electrons lose their energy to excite internal degrees of freedom, which then decay radiatively. In the simplest case of an optically thin low-density plasma these energy losses by the electrons can, thus, be written as the sum of two processes: \( L^e_i(T_e)n_i n_e \) and \( L^e_a(T_a)n_a n_e \), where we implicitly assumed that the thermal velocity of the electrons \( v_{T,e} \gg v_{T,i}, v_{T,a} \), so that in \( L^e_i \) and \( L^e_a \) only the dependence on \( T_e \) is essential. In addition, the electrons can share their thermal energy in elastic collisions with the ions and neutrals. The coefficients are \( q^e_i \) and \( q^e_a \), respectively. In total it gives

\[
\mathcal{H}_e = \Gamma n_d - L^e_i(T_e)n_i n_e - L^e_a(T_a)n_a n_e - q^e_i(T_e)(T_e - T_i)n_i n_e - q^e_a(T_a)(T_e - T_a)n_a n_e, \tag{7}
\]

where for \( \Gamma n_d \) we have taken the value typical for the interstellar plasma illuminated by the galactic ultraviolet (UV) radiation field according to Ref. \[12\]. The cooling functions \( L^e_i \) and \( L^e_a \) are taken from Refs. \[12\] and \[13\], the coefficients for the collisional energy exchange between the electrons and ions \( q^e_i = q^e_i(T_e) \) and the electrons and neutrals \( q^e_a = q^e_a(T_e) \) are given in Refs. \[14\] and \[15\].

Energy losses of the ions in inelastic collisions are small in comparison with energy exchange rates in elastic collisions, and these losses are normally neglected. In our calculations we included, however, these losses, assuming, according to Ref. \[16\] that the cross section for direct excitation by proton impact is broadly similar to the one by electron impact if the proton has the same velocity as the electron, with the only difference that the maximum cross section by proton impacts is a factor of 10 larger than that by electrons (cf. Refs. \[17\], \[18\], \[19\]). With this assumption the derived form of the radiative cooling functions for the ions read as \( L^i_i \simeq \alpha(m_e/m_i)^{3/2} L^e_i \) for the ion-ion inelastic collisions and \( L^i_a \simeq \alpha(m_e/m_i)^{3/2} L^e_a \) for the collisions between the ions and neutrals, where we varied the factor \( \alpha \) from 1 to 10. Therefore, we adopt for the ions the generalized rate in the form

\[
\mathcal{H}_i = -L^i_i(T_i)n_i n_i - q^i_i(T_i)(T_i - T_d)n_i n_d - q^i_a(T_a, T_i)(T_i - T_a)n_i n_a, \tag{8}
\]
where \( q'^i \) depends on both \( T_i \) and \( T_n \) and is given in Refs. \([14, 15]\). \( q'^i \) describes cooling of the ions in elastic

 collisions with translationally cold dust particles – by the order of magnitude \( q'^i \sim 2k_BT(m_p/m_d)\pi\sigma_d^2v_T \), where \( m_p/m_d \) is the mass ratio of the ions (mainly protons) to dust particles, \( \sigma_d \) the geometric cross-section of a dust particle, \( v_T \) the thermal velocity of the ions, and is small compared to other terms in \([13]\): the second term on the r.h.s. describes radiative losses of the ions in collisions with neutrals; the contribution of ion-ion inelastic

 collisions in a relatively weakly ionized plasma we deal with \( (x_e \lesssim 0.1) \) is small.

Similarly we obtain for the neutrals

\[
\mathcal{H}_n = q' - L_n^i(T_n)n_in_a - L_n^a(T_n)n_a^2 - q'^a(T_a)n_an_d
- q'^i(T_e)(T_e - T_i)n_an_i
- q'^s(T_n)(T_n - T_e)n_an_e,
\]

where the first term in the r.h.s. describes radiative energy losses of the neutral atoms (mostly hydrogen) in collisions with the ions (mostly the ions of heavy elements, like CII, FeII), while the second term corresponds to energy losses in collisions of mostly hydrogen atoms with such heavy elements as OI. The third term in \([13]\) describes cooling of the neutrals in elastic collisions with presumably cold dust particles – loosely \( q'^i \sim q'^s \) and both contributions are small in comparison with other terms and are, thus neglected in the results shown below.

In the equilibrium state of the component the following set of equations are fulfilled

\[
\mathcal{H}_e = 0, \quad \mathcal{H}_i = 0, \quad \mathcal{H}_a = 0,
\]

where the energy gain for the ions and neutrals stems from a small difference in temperatures \( T_e > T_i, T_a \), such that \( T_e - T_i, T_e - T_a \ll T_e \) and approximate equalities \( T_e \approx T_i \approx T_a \) hold in the whole temperature range. The solution of \([10]\) is depicted as dotted lines on Figs 1-3.

### III. STABILITY ANALYSIS

#### A. Linear perturbations

We consider planar perturbations, \( \delta g \), of a quantity \( g \) in the form \( \delta g \propto \exp[i(kx - \omega t)] \), i.e. we assume the perturbations to propagate only in \( x \)-direction for simplicity. In this case the equations \([11, 12]\) read

\[
\omega v_n = -k\nu_a = 0,
\]

\[
\omega v_e = k^2\nu_e + k^2\nu_{\omega e}a_e + \frac{\tilde{Z}_e}{m_e}ke^2\Phi,
\]

\[
-\omega v_a = -i\nu_a - i\nu_{\omega a} = -i\gamma k\nu_a + \sum_{\beta} \frac{1}{T_{a,0}} \frac{\partial h_{a,\nu}}{\partial T_{a,0}} \nu_{\beta}
+ \sum_{\beta} \frac{1}{n_{a,0}} \frac{\partial n_{a,\nu}}{\partial T_{a,0}} \nu_{\beta}
\]

\[
\Phi = \frac{4\pi e}{k^2} \sum Z_{a,n} \nu_{\nu a},
\]

where \( h_{a,\nu} = (\gamma - 1)h_{a}/k_B \), \( n_{a,\nu} = \delta n_{a}/n_{a,0} \), \( \varphi_a = \delta T_a/T_{a,0} \), \( k_B \) is the Boltzmann constant. Combining \([11, 12]\), and \([13]\) one obtains

\[
i(\gamma - 1)\nu_{\omega a} = \sum_{\beta} \left( i\nu_{\omega a} + \lambda_{a,\beta} + \sum_{\kappa} \nu_{\mu,\kappa,\eta_{\beta}} \right) \nu_{\beta} = 0,
\]

where \( \alpha, \kappa \) run over the lighter species \( i, e, \) and \( a \), whereas index \( \beta \) runs over all four species, i.e. \( i, e, a, \) and \( d \). The matrix \( \eta_{\beta a} \) is given in Appendix A. This equation must be complemented by a condition that follows from inserting Eq. \([14]\) into Eq. \([12]\) for the (cold) dust:

\[
Z_{e} \frac{m_e}{m_d} \omega_{pe}^2 \nu_e - Z_{e} \frac{m_i}{m_d} \omega_{pi}^2 \nu_i + (\omega_{pd}^2 - \omega^2) \nu_d = 0,
\]

where \( \omega_{pd} \) is the plasma frequency of the \( o \)th component. Further we define the abbreviations

\[
\lambda_{a,\beta} = \frac{1}{T_{a,0}} \frac{\partial h_{a,\nu}}{\partial n_{a,0}} \nu_{\beta} = \frac{1}{T_{a,0}} \frac{\partial h_{a,\nu}}{\partial T_{a,0}} \nu_{\beta}
\]

Explicit expressions for \( \lambda_{a,\beta} \) and \( \mu_{a,\beta} \) are given in Appendix B. We concentrate in this paper on the case when the external (unperturbed) electric and magnetic fields are zero, and, thus, in linear approximation the Lorentz force can be neglected.

#### B. The instability criterion

From system \([15]\) and Eq. \([16]\) we obtain the following dispersion, where we use the abbreviation \( N = -i\omega \) for the growth rate:

\[
\begin{vmatrix}
D_{i}N + \tilde{\Lambda}_{ii} - \eta_{ie}N + \tilde{\Lambda}_{ie} - \eta_{id}N + \tilde{\Lambda}_{id}
- \eta_{ei}N + \tilde{\Lambda}_{ei} - \eta_{ed}N + \tilde{\Lambda}_{ed}
\Lambda_{ai}
\Lambda_{ae}
\Lambda_{ao}
\Lambda_{ad}
- \frac{Z_{d}m_{e}}{m_{d}} \omega_{pd}^2
\end{vmatrix} = 0
\]

(18)

with \( \tilde{\Lambda}_{a,\beta} = \lambda_{a,\beta} + \sum_{\kappa} \mu_{a,\kappa,\eta_{\beta}} \), \( D_{a} = (\gamma - 1) - \eta_{aa} \). In the limit \( N^2 \ll \omega_{pd}^2 \) the charge separation becomes unimportant and \([18]\) converges to

\[
\begin{vmatrix}
D_{i}N + \Lambda_{ii} \Lambda_{ie} \Lambda_{ia} \Lambda_{id}
\Lambda_{ie} D_{e}N + \Lambda_{ee} \Lambda_{ea} \Lambda_{ed}
\Lambda_{ia}
\Lambda_{ae}
\Lambda_{ao}
\Lambda_{ad}
n_{e}/Z_{d}n_{d}
\end{vmatrix} = 0,
\]

(19)
where \( \Lambda_{\alpha\beta} = \lambda_{\alpha\beta} + \mu_{\alpha\beta} \). We assumed here explicitly that the heating and cooling rates of the plasma components \( e, i, a \) do not depend on the dust kinetic temperature, \( T_d \), which implies \( \mu_{ed} = \mu_{id} = \mu_{ad} = 0 \). Equation (19) is the dispersion relation for a radiating plasma, when quasi-neutrality can be assumed.

In the low-frequency limit \( |N| \ll k c e, k c i, k c a \) the coefficients \( D_\alpha \) converge to \( D_\alpha = \gamma \), and therefore the sufficient condition for the instability is that the constant term in the polynomial (19) in \( N \) is negative. Thus, the instability condition is

\[
-\frac{n_e}{Z_d n_d} \left( \lambda_{ed} \Lambda_{ei} \Lambda_{ia} + \lambda_{ed} \Lambda_{re} \Lambda_{ra} + \lambda_{ed} \Lambda_{ri} \Lambda_{aa} - \lambda_{ed} \Lambda_{ri} \Lambda_{aa} - \lambda_{ed} \Lambda_{rr} \Lambda_{aa} \right)
- \frac{n_i}{Z_d n_d} \cdot \left( \lambda_{ed} \Lambda_{ei} \Lambda_{aa} + \lambda_{ed} \Lambda_{ee} \Lambda_{ia} + \lambda_{ed} \Lambda_{ee} \Lambda_{ii} - \lambda_{ed} \Lambda_{ri} \Lambda_{aa} \right)
+ \left( \Lambda_{ee} \Lambda_{ri} \Lambda_{ia} + \Lambda_{ee} \Lambda_{ri} \Lambda_{ia} + \Lambda_{ee} \Lambda_{ri} \Lambda_{ia} \right)
- \left( \Lambda_{ee} \Lambda_{re} \Lambda_{ia} - \lambda_{ed} \Lambda_{ri} \Lambda_{aa} - \lambda_{ed} \Lambda_{ri} \Lambda_{aa} \right)
< 0, \tag{20}
\]

Here \( \Lambda_{\alpha\beta}' = \lambda_{\alpha\beta} - \mu_{\alpha\beta} \). This condition is an extension of the standard (one-fluid) condition for the condensation mode to grow as given by Ref. 1.

\[
\frac{\partial \mathcal{H}}{\partial n} < \frac{T_0}{n_0} \frac{\partial \mathcal{H}}{\partial T} \tag{21}
\]

IV. RESULTS

In a one-fluid description under the conditions considered here: \( H \propto n_4 \propto n \) and \( L = \Lambda(T)n^2 \) with fixed (temperature independent) ionization states, the instability criterion reads as \( d \ln \Lambda/d \ln T < 1 \) and restricts a density independent temperature range where the plasma is unstable: for the diffuse warm neutral phase of the ISM the instability can occur in the temperature range \( 100 \lesssim T \lesssim 7000 \) K. The situation changes dramatically, when a possible separation in the dynamics of the four plasma components is taken explicitly into account. For the sake of simplicity we consider first constant dust heating rate \( \Gamma = \text{const} \) in the whole range of temperature and density. This suggests that the dust charge and correspondingly the fraction of the UV radiation transformed into gas heating are kept constant in the whole range of plasma parameters. It should be stressed though that at realistic conditions in the interstellar plasma dust charge, and as a result the heating rate, depend on temperature and density, cf. Ref. 1: \( Z_d = Z_d(T, n_e), \Gamma = \Gamma(T, n_e) \).

However, we believe it worthwhile to restrict ourselves with a simplified scheme in order to understand better how dust particles affect the dynamics of the RC instability in a four-fluid description. In the models shown below we assume also a constant fractional ionization, \( x_e = \text{const} \). This assumption is justified by the fact that the instability growth time is expected to be of the order of the radiative cooling time, which is always shorter than the relaxation time for recombination (see Ref. 21); for conditions, e.g., in the warm phase of the interstellar medium with \( T \sim 10^4 \) K the radiative cooling time is \( \sim 10^{11} n^{-1} \) s\(^{-1}\), while the recombination time is \( \sim 10^{12} n^{-1} \) s\(^{-1}\).

Figure 1 shows the domains of instability in the temperature-density plane for several values of the dust charge. Panel a) corresponds to the standard one-fluid case for a constant fractional ionization \( x_e = n_e/n = 0.1 \): the whole plane is separated onto two regions independent of the density, where the plasma is stable (white) or unstable (gray), corresponding to the criterion \( d \ln \Lambda/d \ln T < 1 \). Panel b) shows the instability domain in the four-fluid description with neutral dust particles \( Z_d = 0 \): the domain is clearly density-independent and slightly wider than on the panel for the one-fluid case - this corresponds to the condition \( d \ln \Lambda/d \ln T < 2 \), which holds either when radiative losses are unbalanced by heating, or when heating is independent of the density, i.e. \( H = \text{const} \). As we neglect in this study collisional friction between the components, the dust component is decoupled from the plasma when \( Z_d = 0 \), which is equivalent to the latter case, i.e. \( \delta H = 0 \).

The panels c), d), and e) correspond to positively
charged dust grains, \( Z_d < 0 \) with increasing absolute values (given in the Figure caption) of the dust charge. It is clearly seen that positively charged dust particles destabilize the system, widening the instability domain to higher temperatures where formally in one-fluid approximation \( d \text{ln} \Lambda / d \text{ln} T > 1 \).

Negative dust particles, shown in panel f), strongly stabilize the system even in the limit of small \( Z_d \) – dust charges in the range \( Z_d = +[0.01 – 100] \) give identically stability in the whole domain of the temperature-density plane shown in Fig. 1. Stabilization of the thermal instability by negatively charged dust can be understood as follows: due to the quasineutrality condition negative dust particles repel the electrons out of the compressed plasma, and thus the radiative cooling rate provided basically by the electrons decreases. Only when dust charge is high in absolute value, \( Z_d = 10^3 \), a narrow region at temperatures \( 100 \text{ K} < T < 10^4 \text{ K} \), where the instability condition is fulfilled, does appear in the high density range as seen in Figure 2.

This instability at high densities in turn seems to stem from a tighter collisional coupling between the electrons and ions. Figure 3 shows the dependence of the instability domains in the temperature-density plane for an artificially enhanced rate of the elastic energy exchange between electrons and ions; the left and right panels show the instability domain for positively \((Z_d = -10^3)\) and negatively \((Z_d = 10^3)\) charged grains, respectively. The collisional frequency is multiplied by a factor of 0.01 \((a,b)\), 10 \((c,d)\), 20 \((e,f)\), 50 \((g,h)\), 100 \((i,j)\).

The instability criterion \((20)\) the derivatives of the elastic collisional rates over temperatures, proportional to the density square, do contribute; at high densities this contribution becomes dominant.

For a constant dust charge \( Z_d \) variations of the UV radiation flux, \( G_0 \), result in a simple shift of the instability domain and the equilibrium temperature curve along the \( n \)-axis – this is a consequence of the fact that at such an assumption the heating rate linearly depends on density \( n \), while the cooling rate is proportional to the square of density \( n^2 \): the higher is \( G_0 \) the larger is the density needed for radiation losses to balance the heating at given temperature. This behavior may change to some extent when the dependence of dust charge on \( G_0 \) is accounted.

V. SUMMARY

The radiation-condensation (thermal) instability in a dusty plasma heated through photo-ionization of dust particles is described in a four-fluid approximation. It is shown that in this approximation with admitted separate motions of the species, the instability criterion changes dramatically compared to the standard one-fluid case. In particular, positively charged dust particles are found to destabilize the system, while negatively charged ones strongly stabilize perturbations: only in the limit \( Z_d > 1 \) negative dust leaves a relatively narrow domain of insta-

FIG. 2: The instability criterion on the temperature-density plane for highly charged negative dust grains: \( Z_d = 10^3 \).

FIG. 3: The instability criterion in the temperature-density plane for enhanced rate of elastic energy exchange between electrons and ions; the left and right panels show the instability domain for positively \((Z_d = -10^3)\) and negatively \((Z_d = 10^3)\) charged grains, respectively. The collisional frequency is multiplied by a factor of 0.01 \((a,b)\), 10 \((c,d)\), 20 \((e,f)\), 50 \((g,h)\), 100 \((i,j)\).
bility in the high density end, where elastic energy exchange between the electrons and ions dominates.

In the interstellar medium the dust charge in low-density regions (diffuse neutral HI phase) is positive due to photo-ionization by stellar ultraviolet light, cf. Ref. [10]. In the light of our results this means that low-density intercloud gas is highly unstable against the formation of condensations in the whole temperature range. A significant fraction of dust is also positive in denser regions, such as diffuse HI clouds and even in molecular clouds (see Ref. [10]). Therefore, contrary to a common understanding that dense phases of the interstellar gas are stable against the formation of condensations, they turn out to be unstable when separated motions of the species are accounted. One may speak thus about possible fragmentation of interstellar clouds through the RC instability on smaller structures.

APPENDIX A: $\eta_{\alpha\beta}$

The matrix $\eta_{\alpha\beta}$ is determined from the momentum equations for the components and connects the dimensionless temperature and density

$$ \vartheta_{\alpha} = \sum_{\beta} \eta_{\alpha\beta} \nu_{\beta} \quad \text{(A1)} $$

The elements of the matrix are

$$
\begin{pmatrix}
  \eta_{ii} & \eta_{ie} & \eta_{ia} & \eta_{id} \\
  \eta_{ei} & \eta_{ee} & \eta_{ea} & \eta_{ed} \\
  \eta_{ai} & \eta_{ae} & \eta_{aa} & \eta_{ad} \\
  \eta_{di} & \eta_{de} & \eta_{da} & \eta_{dd}
\end{pmatrix} = 
\begin{pmatrix}
  \frac{\omega^2 - \omega_p^2}{k^2 c_s^2} - 1 & \frac{m_e}{m_i} \frac{\omega^2 - \omega_p^2}{k^2 c_s^2} & 0 & \frac{m_d}{m_i} \frac{\omega^2 - \omega_p^2}{k^2 c_s^2} \\
  \frac{m_e}{m_i} \frac{\omega^2 - \omega_p^2}{k^2 c_s^2} & \frac{\omega^2 - \omega_p^2}{k^2 c_s^2} - 1 & 0 & -\frac{m_d}{m_i} \frac{\omega^2 - \omega_p^2}{k^2 c_s^2} \\
  0 & 0 & \frac{\omega^2 - \omega_p^2}{k^2 c_s^2} - 1 & 0 \\
  0 & 0 & 0 & \frac{\omega^2 - \omega_p^2}{k^2 c_s^2}
\end{pmatrix}
\quad \text{(A2)}
$$

APPENDIX B: $\lambda_{\alpha\beta}$ AND $\mu_{\alpha\beta}$

For simplicity we assume further that $\Gamma$ does not depend on $n_a$ and $T_e$. In addition, we assume that in equilibrium $|T_e - T_i|, |T_e - T_a|, |T_i - T_a| \ll T_e$. The collision coefficients with the dust, $q_d^i$ and $q_d^a$ give here, were neglected in the results shown. The coefficients are:

$$
\begin{align*}
\lambda_{ii} & = - \frac{2 L^i_n a_i + L^i_d n_d}{T_i} + q_d^i n_d \\
\lambda_{ic} & = 0 \\
\lambda_{ia} & = - \frac{L^i_d n_d}{T_i} \\
\lambda_{id} & = - q_d^i n_d \\
\lambda_{ei} & = - \frac{L_e^i n_i + L_e^a n_a}{T_e} \\
\lambda_{ee} & = 0 \\
\lambda_{ea} & = - \frac{L_e^a n_a}{T_e} \\
\lambda_{ed} & = - \frac{\Gamma n_d}{T_e n_e}
\end{align*}
\quad \text{(B2)}
$$

$$
\begin{align*}
\lambda_{ai} & = - \frac{L^a_n a_i}{T_a} \\
\lambda_{ae} & = 0 \\
\lambda_{aa} & = - \frac{L^a_n n_i + 2 L^a_d n_d}{T_a} - q_d^a n_d \\
\lambda_{ad} & = - q_d^a n_d
\end{align*}
\quad \text{(B3)}
$$

$$
\begin{align*}
\mu_{ii} & = - \frac{d L^i_n a_i}{d T_i} - \frac{d L^i_d n_d}{d T_i} - q_d^i n_d \\
\mu_{ic} & = q_d^i n_e \\
\mu_{ia} & = q_d^i n_a \\
\mu_{id} & = 0 \\
\mu_{ei} & = q_d^e n_i \\
\mu_{ee} & = - \frac{d L^e_n a_i}{d T_e} - \frac{d L^e_d n_d}{d T_e} - q_d^e n_d \\
\mu_{ea} & = q_d^e n_a \\
\mu_{ed} & = 0 \\
\mu_{ai} & = q_d^a n_i \\
\mu_{ac} & = q_d^a n_e \\
\mu_{aa} & = - \frac{d L^a_n n_i}{d T_a} - \frac{d L^a_d n_d}{d T_a} - q_d^a n_d \\
\mu_{ad} & = 0
\end{align*}
\quad \text{(B4)}
$$

We accounted here that direct injection of energy from dust particles to the ions and neutrals is zero.

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