Inflationary scenario from higher curvature warped spacetime

Narayan Banerjee
Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur Campus, Nadia, West Bengal 741246, India

Tanmoy Paul
Department of Theoretical Physics, Indian Association for the Cultivation of Science, 2A & 2B Raja S.C. Mullick Road, Kolkata - 700 032, India.

We consider a five dimensional AdS spacetime, in presence of higher curvature term like $F(R) = R + \alpha R^2$ in the bulk, in the context of Randall-Sundrum two-brane model. Our universe is identified with the TeV scale brane and emerges as a four dimensional effective theory. From the perspective of this effective theory, we examine the possibility of “inflationary scenario” by considering the on-brane metric ansatz as an FRW one. Our results reveal that the higher curvature term in the five dimensional bulk spacetime generates a potential term for the radion field. Due to the presence of radion potential, the very early universe undergoes a stage of accelerated expansion and moreover the accelerating period of the universe terminates in a finite time. We also find the spectral index of curvature perturbation ($n_s$) and tensor to scalar ratio ($r$) in the present context, which match with the observational results based on the observations of Planck 2015 [1].

I. INTRODUCTION

Over the last two decades, extra spatial dimensions [2–5] has been increasingly playing a central role in physics beyond the standard model of particle [2] and cosmology [10]. Apart from phenomenological approach, higher dimensional scenarios come naturally in string theory. Depending on geometry, the extra dimensions are compactified under various compactification schemes. Our usual four dimensional universe is considered to be a 3-brane (3 + 1 dimensional brane) embedded within the higher dimensional spacetime and emerges as a four dimensional effective theory.

Among the various extra dimensional models proposed over the last several years, Randall-Sundrum (RS) warped extra dimensional model [4] earned a special attention since it resolves the gauge hierarchy problem without introducing any intermediate scale (between Planck and TeV scale) in the theory. RS model is a five dimensional AdS spacetime with $S^1/Z_2$ orbifolding along the extra dimension while the orbifold fixed points are identified with two 3-branes. The separation between the branes is assumed to be of the order of Planck length so that the hierarchy problem can be solved. However, due to the intervening gravity, aforementioned brane configuration can not be a stable one. So, like other higher dimensional braneworld scenario, one of the crucial aspects of RS model is to stabilize the interbrane separation (known as modulus or radion). For this purpose, one needs to generate a suitable radion potential with a stable minimum. Goldberger and Wise (GW) proposed a useful mechanism [11] to construct such a radion potential by imposing a massive scalar field in the bulk with appropriate boundary conditions. Subsequently the phenomenology of radion field has also been studied extensively in [12–13]. Some variants of RS model and its modulus stabilization have been discussed in [13–19]. The Standard Big Bang model gives a predictive description of our universe from nucleosynthesis to present. But back in very early stage of evolution, the big bang model is plagued with some problems such as Horizon and Flatness problems. For a comprehensive review, we refer to [20, 21]. In order to resolve these problems, the idea of inflation was introduced by Guth [22] in which the universe had to go through a stage of accelerated expansion after the big bang. It had also been demonstrated that a massive scalar field with a suitable potential plays a crucial role in producing an accelerated expansion of the universe. This resulted in a huge amount of work on inflation based on scalar fields [20, 21, 22–31]. It is interesting to note that in the extra dimensional models, the modulus field can fulfill the requirement of the scalar field required for inflation. Thus the cosmology of higher dimensional models [22, 23] can be very different from usual cosmology of four dimensions where the inflaton field is normally invoked by handz. In our current work, we take advantage of the modulus field of extra dimensions and address the early time cosmology of our universe in the backdrop of RS two-brane model. It is well known that Einstein-Hilbert action can be generalized by adding higher order curvature terms which naturally arise from the diffeomorphism property of the action. Such terms also have their origin in String Theory due to quantum corrections. $F(R)$ [13–15], Gauss-
Bonnet (GB) \[44,46\] or more generally Lanczos-Lovelock gravity \[17,48\] are some of the candidates in higher curvature gravitational theory. Higher curvature terms become extremely relevant at the regime of large curvature. Thus for RS bulk geometry, where the curvature is of the order of Planck scale, the higher curvature terms should play a crucial role. Motivated by this idea, we consider a generalized version of RS model by replacing Einstein-Hilbert bulk gravity Lagrangian, given by the Ricci scalar \( R \) by \( F(R) \) where \( F(R) \) is an analytic function of \( R \)[43,51]. Recently it has been shown in [17], that for RS brane world modified by \( F(R) \) gravity, a potential term for the radion field is generated (in the four dimensional effective theory) even without introducing an external scalar field in the bulk and moreover the radion potential has a stable minimum for a certain range of parametric space. However, from cosmological aspect, the important questions that remain in the said higher curvature RS model [17], are:

1. Can the usual four dimensional universe undergo an accelerating expansion at early epoch, due to presence of the radion potential generated by higher curvature term?

2. If such an inflationary scenario is allowed, then what are the dependence of duration of inflation as well as number of e-foldings on higher curvature parameter? Moreover what are the values of \( n_s \) and \( r \) in the present context?

We aim to address these questions in this work and motivated by the Starobinsky model [52], the form of \( F(R) \) in the five dimensional bulk, is taken here, as \( F(R) = R + \alpha R^2 \) where \( \alpha \) is a constant.

The paper is organized as follows: Following two sections are devoted to brief reviews of RS scenario and its extension to \( F(R) \) model. Section IV is reserved for determining the solutions of effective Friedmann equations on the brane. In section V, VI, and VII, we address the consequences of the solutions that are obtained in section IV. Finally the paper ends with some concluding remarks in section VIII.

### II. BRIEF DESCRIPTION OF RS SCENARIO

RS scenario is defined on a five dimensional AdS spacetime involving one warped and compact extra spacelike dimension. Two 3-branes known as TeV/visible and Planck/hidden brane are embedded in a five dimensional spacetime. If \( \phi \) is the extra dimensional angular coordinate, then the branes are located at two fixed points \( \phi = (0, \pi) \) while the latter one is identified with our known four dimensional universe. The opposite brane tensions along with the finely tuned five dimensional cosmological constant serve as energy-momentum tensor of RS scenario. The resulting spacetime metric \( [4] \) is non-factorizable and expressed as,

\[
ds^2 = e^{-2kr_c|x|} g_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2
\]

(1)

Here, \( r_c \) is the compactification radius of the extra dimension. Due to \( S^1/Z_2 \) compactification along the extra dimension, \( \phi \) ranges from \(-\pi\) to \(+\pi\). The quantity \( k = \sqrt{\frac{12\Lambda}{\kappa^2}} \) is of the order of 5-dimensional Planck scale \( M \). Thus \( k \) relates the 5D Planck scale \( M \) to the 5D cosmological constant \( \Lambda \).

In order to solve the hierarchy problem, it is assumed in RS scenario that the branes are separated by such a distance that \( k\pi r_c \approx 36 \). Then the exponential factor present in the metric, which is often called warp factor, produces a large suppression so that a mass scale of the order of Planck scale is reduced to TeV scale on the visible brane. A scalar mass e.g. mass of Higgs boson is given as,

\[
m_H = m_0 e^{-k\pi r_c}.
\]

(2)

where \( m_H \) and \( m_0 \) are physical and bare Higgs masses respectively.

### III. RS LIKE SPACETIME IN F(R) MODEL: FOUR DIMENSIONAL EFFECTIVE ACTION

In the present paper, we consider a five dimensional AdS spacetime with two 3-brane scenario in F(\( R \)) model. The form of \( F(R) \) is taken as \( F(R) = R + \alpha R^2 \) where \( \alpha \) is a constant with square of the inverse mass dimension. Considering \( \phi \) as the extra dimensional angular coordinate, two branes are located at \( \phi = 0 \) (hidden brane) and at \( \phi = \pi \) (visible brane) respectively while the latter one is identified with the visible universe. Moreover the spacetime is \( S^1/Z_2 \) orbifolded along the coordinate \( \phi \). The action for this model is:

\[
S = \int d^4x d\phi \sqrt{-G} \left[ \frac{1}{2\kappa^2}(R + \alpha R^2) + \Lambda 
+ V_h(\phi) + V_v(\phi - \pi) \right]
\]

(3)

where \( G \) is determinant of the five dimensional metric \( (G_{MN}) \), \( \Lambda(<0) \) is the bulk cosmological constant, \( \frac{1}{\kappa^2} = M^5 \) and \( V_h, V_v \) are the brane tensions on hidden, visible brane respectively.

It is well known that a \( F(R) \) gravity model can be recast into Einstein gravity with a scalar field by means of a conformal transformation on the metric [17,53]. Thus the solutions of five dimensional Einstein equations for the action presented in eqn. (3), can be extracted from the solutions of the corresponding conformally related scalar-tensor (ST) theory and it is discussed in the following two subsections.
A. Solutions of field equations for corresponding ST theory

This higher curvature like $F(R)$ model (in eqn. 3) can be transformed into scalar-tensor theory by using the technique discussed in [12]. Performing a conformal transformation of the metric as

$$G_{MN}(x, \phi) \rightarrow \tilde{G}_{MN} = \exp\left(\frac{1}{\sqrt{3}}\kappa\Phi(x, \phi)\right)G_{MN}(x, \phi),$$

the above action (in eqn. 3) can be expressed as a scalar-tensor theory with the action given by [17]:

$$S = \int d^4x \sqrt{\tilde{G}} \left( \frac{R}{2\kappa^2} + \frac{1}{2} \tilde{G}^{MN} \partial_M \Phi \partial_N \Phi - V(\Phi) \right)$$

$$+ \Lambda + \exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right)\nu^b \delta(\phi) + \exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right)\nu^b \delta(\phi - \pi),$$

where the quantities in tilde are reserved for the ST theory. $\tilde{R}$ is the Ricci curvature formed by the transformed metric $\tilde{G}_{MN}$. $\Phi(x, \phi)$ is the scalar field which corresponds to higher curvature degrees of freedom and $V(\Phi)$ is the scalar potential which for this specific choice of $F(R)$ has the form [14],

$$V(\Phi) = \frac{1}{8\kappa^2 \alpha} \exp\left(-\frac{5}{2\sqrt{3}}\kappa \Phi\right)\left[\exp\left(\frac{3}{2\sqrt{3}}\kappa \Phi\right) - 1\right]^2$$

$$- \Lambda \left[\exp\left(-\frac{5}{2\sqrt{3}}\kappa \Phi\right) - 1\right].$$

(6)

One can check that the above potential (in eqn. 6) is stable for the parametric regime $\alpha > 0$. The stable value ($\langle \Phi \rangle$) and the mass squared ($m_\Phi^2$) of the scalar field (\Phi) are given by the following two equations

$$\exp\left(\frac{3}{2\sqrt{3}}\kappa \langle \Phi \rangle\right) = \left[\sqrt{9 - 40\kappa^2 \alpha \Lambda} - 2\right]$$

(7)

and

$$m_\Phi^2 = \frac{1}{8\alpha} \left[\sqrt{9 - 40\kappa^2 \alpha \Lambda}\right] \left[\sqrt{9 - 40\kappa^2 \alpha \Lambda} - 2\right].$$

(8)

Furthermore, the minimum value of the potential i.e. $V(\langle \Phi \rangle)$ is non zero and serves as a cosmological constant. Thus the effective cosmological constant in scalar-tensor theory is $\Lambda_{eff} = \Lambda - V(\langle \Phi \rangle)$ where $V(\langle \Phi \rangle)$ is,

$$V(\langle \Phi \rangle) = \Lambda + \left[\sqrt{9 - 40\kappa^2 \alpha \Lambda} - 2\right]$$

$$\left[\Lambda + (1/8\kappa^2 \alpha)\left[\sqrt{9 - 40\kappa^2 \alpha \Lambda} - 3\right]^2\right].$$

(9)

This form of $V(\langle \Phi \rangle)$ with $\Lambda < 0$ clearly indicates that $\Lambda_{eff}$ is also negative or more explicitly, the corresponding scalar-tensor theory for the original $F(R)$ model has an AdS like spacetime [13]. Considering $\xi$ as the fluctuation of the scalar field over its vacuum expectation value (vev), the final form of action for the scalar-tensor theory in the bulk can be written as,

$$S = \int d^4x \sqrt{\tilde{G}} \left( \frac{\tilde{R}}{2\kappa^2} + \frac{1}{2} \tilde{G}^{MN} \partial_M \xi \partial_N \xi \right)$$

$$- \frac{1}{2} m_\xi^2 \xi^2 + \Lambda_{eff}$$

(10)

where the terms up to quadratic order in $\xi$ are retained for $\kappa \xi < 1$.

For the case of ST theory presented in eqn. (10), $\xi$ can act as a bulk scalar field with the mass given by eqn. (8). Considering a negligible backreaction of the scalar field ($\xi$) on the background spacetime, the solution of metric $\tilde{G}_{MN}$ is exactly same as RS model, i.e.,

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2,$$

(11)

where $k = \sqrt{-\Lambda_{RS}/2\kappa}$ and $r_c$ is the compactification radius of the extra dimension in ST theory. With this metric, the scalar field equation of motion in the bulk is the following,

$$- \frac{1}{r_c^2} \partial_{\phi}[\nu^{0}\nu_{0} \exp(-4kr_c|\phi|)\partial_{\phi}\xi]$$

$$+ m_\xi^2 \exp(-4kr_c|\phi|)\xi(\phi) = 0$$

(12)

where the scalar field $\xi$ is taken as function of extra dimensional coordinate only [11]. Considering non zero value of $\xi$ on branes, the above equation (12) has the general solution,

$$\xi(\phi) = e^{2kr_c|\phi|}[Ae^{\nu kr_c|\phi|} + Be^{-\nu kr_c|\phi|}]$$

(13)

with $\nu = \sqrt{4 + m_\xi^2/k}$. Moreover $A$ and $B$ are obtained from the boundary conditions, $\xi(0) = v_h$ and $\xi(\pi) = v_e$ as follows:

$$A = v_h e^{-(2+\nu)kr_c} - v_h e^{-2\nu kr_c}$$

and

$$B = v_h (1 + e^{-2\nu kr_c}) - v_h e^{-(2+\nu)kr_c}.$$
B. Solutions of field equations for original F(R) theory

Recall that the original higher curvature $F(R)$ model is presented by the action given in eqn. (4). Solutions of metric ($G_{MN}$) for this $F(R)$ model can be extracted from the solutions of corresponding scalar-tensor theory (eqn. (11) and eqn. (13)) with the help of eqn. (4). Thus the line element in $F(R)$ model turns out to be

$$ds^2 = e^{-2\Phi(x, \phi)}[e^{-2kR(x, \phi)}g_{\mu\nu}(x)dx^\mu dx^\nu - r_c^2d\Phi^2]$$

where $\Phi(x, \phi) = \phi > +\xi(\phi)$ and $\xi(\phi)$ is given by eqn. (13).

This solution of $G_{MN}$ immediately leads to the separation between hidden ($\phi = 0$) and visible ($\phi = \pi$) branes along the path of constant $x^\mu$ as follows:

$$\pi d = r_c \int_0^\pi d\phi e^{-\frac{2}{r_c^2}\Phi(\phi)}$$

where $d$ is the inter-brane separation in $F(R)$ model. A fluctuation of branes around the configuration $d$ is now considered. This fluctuation can be taken as a field (T(x)) and this new field is assumed to be the function of brane coordinates only [12]. Then the metric takes the following form,

$$ds^2 = e^{-\Phi(x, \phi)}[e^{-2kT(x, \phi)}g_{\mu\nu}(x)dx^\mu dx^\nu - T(x)^2d\Phi^2]$$

where $g_{\mu\nu}(x)$ is the induced on-brane metric and $T(x)$ is known as radion (or modulus) field. Moreover $\Phi(x, \phi)$ is obtained from eqn. (13) by replacing $r_c$ by $T(x)$.

Plugging back the solutions presented in eqn. (15) into original five dimensional F(R) action (in eqn. (9)) and integrating over $\phi$ yields the four dimensional effective action as follows [17]

$$S_{eff} = \int d^4x\sqrt{-g} \left[ M_{(4)}^2 R_{(4)} + \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - V(\Psi) \right]$$

where $M_{(4)}^2 = \frac{M_{(5)}^2}{k} \left[ \sqrt{9-40k^2\alpha\Lambda} - 2 \right]^{-5/6}$ is the four dimensional Planck scale, $R_{(4)}$ is the Ricci scalar formed by $g_{\mu\nu}(x)$. Moreover, $\Psi(x) = \sqrt{\frac{24M_{(5)}^2}{k^2}} \left[ 1 + \frac{20}{\sqrt{3}} \alpha k^2 \kappa v_h \right] e^{-kR(x)} = e^{-kR(x)}$ (with $f = \sqrt{\frac{24M_{(5)}^2}{k^2}} \left[ 1 + \frac{20}{\sqrt{3}} \alpha k^2 \kappa v_h \right]$), is the canonical radion field and $V(\Psi)$ is the radion potential with the following form [17]

$$V(\Psi) = \frac{20}{\sqrt{3}M_{(5)}} \psi^4 v_h - \left( v_h - \frac{\kappa v_h^3}{2\sqrt{3}} \right) (\psi/f)^2$$

where the terms proportional to $\epsilon$ $(= \frac{m^2}{r_c^2})$ are neglected [11, 12]. It may be observed that $V(\Psi)$ goes to zero as $\alpha$ tends to zero. This is expected because for $\alpha \to 0$, the action contains only the Einstein part which does not produce any potential term for the radion field [12].

Thus for five dimensional warped geometric model, the radion potential is generated from the higher order curvature term $\alpha R^2$. In figure (1), we plot $V(\Psi)$ against $\Psi$.

The potential in eqn. (17) has a vev at

$$< \Psi > = f \left( \frac{v_h}{v_h} \right)^{1/2} \left[ 1 - \frac{\kappa v_h^4}{2\sqrt{3} v_h} \right]^{-1/2}$$

as long as $\alpha > 0$. Correspondingly the squared mass of radion field is as follows,

$$m^2_{rad}(F(R)) = \frac{20}{\sqrt{3}M_{(5)}} \epsilon^2 e^{-2kR} \frac{v_h^2}{2\sqrt{3} v_h}$$

$$\left[ 1 + \frac{40}{\sqrt{3}} \alpha k^2 \kappa v_h \right] \left( \frac{v_h}{v_h} - 1 \right)^2.$$  

Due to the presence of $V(\Psi)$, radion field has a certain dynamics governed by effective field equations. In the next few sections, we examine whether the dynamics of radion field can trigger an inflationary scenario for the four dimensional universe or not.

IV. SOLUTIONS OF EFFECTIVE FRIEDMANN EQUATIONS

Considering the on-brane metric ansatz as flat FRW one i.e.

$$ds^2_{(4)} = g_{\mu\nu}(x)dx^\mu dx^\nu$$

$$= dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]$$

where $a(t)$ is the scale factor of the visible universe. The effective field equations (obtained from the effective action presented in eqn. (13)) take the following form,

$$H^2 = \frac{1}{3} \left[ V(\Psi) + \frac{1}{2}(\dot{\Psi})^2 \right]$$
where an overdot denotes the derivative \( \frac{d}{dt} \), \( H = \frac{\dot{a}}{a} \) is known as Hubble parameter and the form of \( V(\Psi) \) is given in eqn.\((17)\). To derive the above equations, we assume that the radion field (\( \Psi(t) \)) is homogeneous in space.

In order to solve the effective Friedmann equations, the potential energy of radion field is taken as very much greater than the kinetic energy (known as slow roll approximation) i.e.

\[
V(\Psi) \gg \frac{1}{2}(\dot{\Psi})^2.
\]

With this approximation, eqn.\((20)\) and eqn.\((21)\) are simplified to,

\[
H^2 = \frac{1}{3}V(\Psi)
\]

and

\[
3H\dot{\Psi} + V'(\Psi) = 0
\]

respectively. Substituting \( H(t) \) from eqn.\((22)\) to eqn.\((23)\) and using the explicit form of \( V(\Psi) \), we get the equation of motion for radion field as,

\[
\frac{d\Psi}{dt} = -8\nu_v\sqrt{\frac{5}{3\alpha k \sqrt{M_6}}}\Psi(B\Psi^\epsilon - 1)
\]

where \( B = \frac{1}{v_h\sqrt{3\alpha k \sqrt{M_6}}} \). Eqn.\((24)\) immediately leads to the dynamics of radion field as,

\[
\Psi(t) = \Psi_0 \left[ B\Psi_0^\epsilon - (B\Psi_0^\epsilon - 1)\exp \left( -8\nu_v\sqrt{\frac{5}{3\alpha k \sqrt{M_6}}}t \right) \right]^{-1/\epsilon}
\]

where \( \Psi_0 \) is the value of radion field (\( \Psi(t) \)) at \( t = t_0 \).

Eqn.\((25)\) clearly indicates that \( \Psi(t) \) decreases with time. Comparison of eqn.\((18)\) and eqn.\((25)\) clearly reveals that the radion field reaches at its vev asymptotically (within the slow roll approximation) at large time \( t \gg t_0 \) i.e.

\[
\Psi(t \gg t_0) = f\left(v_h\right)^{1/\epsilon}[1 - \frac{\kappa v_h}{2\sqrt{3}}(v_h - 1)]^{-1/\epsilon}
\]

where \( \Psi_0 \) is given in eqn.\((18)\). Putting the solution of \( \Psi(t) \) into eqn.\((22)\) one gets, on integration, the evolution of scale factor as,

\[
a(t) = C\exp \left[ 2\nu_v\sqrt{\frac{5}{3\alpha k \sqrt{M_6}}} (g_1(t) - g_2(t)) \right]
\]

where \( C \) is an integration constant and \( g_1(t) \) has the following form,

\[
g_1(t) = -\frac{B\Psi_0^\epsilon}{(B\Psi_0^\epsilon - 1)}\frac{1}{16\nu_v\sqrt{\frac{5}{3\alpha k \sqrt{M_6}}}\Psi_0^\epsilon *}
\]

\[
2F1 \left( 1, 1, 2 + \frac{2}{\epsilon}, B\Psi_0^\epsilon - 1 \right) \exp \left( 8\nu_v\frac{\sqrt{5\alpha k \sqrt{M_6}}}{3\sqrt{3}}(t - t_0) \right)
\]

\[
\exp \left( -8\nu_v\sqrt{\frac{5}{3\alpha k \sqrt{M_6}}}t \right) \left( B\Psi_0^\epsilon - (B\Psi_0^\epsilon - 1) \right)\]^{-2/\epsilon}
\]

where \( 2F1 \) symbolizes the hypergeometric function. Similarly the form of \( g_2(t) \) is given by,

\[
g_2(t) = -\frac{\Psi_0^\epsilon}{(B\Psi_0^\epsilon - 1)}\frac{1}{16\nu_v\sqrt{\frac{5}{3\alpha k \sqrt{M_6}}}\Psi_0^\epsilon *}
\]

\[
2F1 \left( 1, 1, 1 + \frac{2}{\epsilon}, B\Psi_0^\epsilon - 1 \right) \exp \left( 8\nu_v\frac{\sqrt{5\alpha k \sqrt{M_6}}}{3\sqrt{3}}(t - t_0) \right)
\]

\[
\exp \left( -8\nu_v\sqrt{\frac{5}{3\alpha k \sqrt{M_6}}}t \right) \left( B\Psi_0^\epsilon - (B\Psi_0^\epsilon - 1) \right)^{-1/2}\epsilon
\]

It may be noticed from eqn.\((25)\) and eqn.\((28)\) that for \( \alpha \rightarrow 0 \), the solution of radion field and Hubble parameter become \( \Psi(t) = \Psi_0 \) and \( H(t) = 0 \) respectively. It is expected because in the absence of higher curvature term, \( V(\Psi) \) goes to zero and thus the radion field has no dynamics which in turn vanishes the evolution of scale factor of the universe.

V. BEGINNING OF INFLATION

After obtaining the solution of \( a(t) \) (in eqn.\((28)\)), we can now examine whether this form of scale factor corresponds to an accelerating era of the early universe (i.e. \( t \gg t_0 \)) or not. In order to check this, we expand \( a(t) \) in the form of Taylor series (about \( t = t_0 \)) and retain the terms only up to first order in \( t - t_0 \).

\[
a(t \gg t_0) = a_0
\]

\[
\exp \left[ 2(B\Psi_0^\epsilon - 1)\Psi_0^\epsilon \nu_v\sqrt{\frac{5}{3\sqrt{3}}M_6}(t - t_0) \right]
\]

where \( a_0 \) is the value of the scale factor at \( t = t_0 \) and related to the integration constant \( C \) as,

\[
a_0 = C\exp \left[ -\Psi_0^\epsilon \frac{1}{8} \right].
\]
It is evident from eqn. (31) that \( a(t) \) corresponds to an exponential expansion at early age of the universe where \( t_0 \) specifies the onset of inflation. Moreover the Hubble parameter (\( H = \frac{\dot{a}}{a} \)) depends on the higher curvature parameter \( \alpha \) and for \( \alpha \to 0, a(t) = a_0 \). Thus the accelerating period of the early universe is triggered entirely due to the presence of higher curvature term in the five dimensional bulk spacetime.

VI. END OF INFLATION

In the previous section, we show that the very early universe expands with an acceleration and this accelerating stage is termed as the inflationary epoch. In this section, we check whether the acceleration of the scale factor has an end in a finite time or not. In the case of inflation, \( \ddot{a} > 0 \). By relating the definition of inflation to the Hubble parameter, one readily obtains,

\[
\ddot{a} = \frac{\dot{a}^2}{a} = H^2 > 0.
\]

We now estimate the time interval which is consistent with this condition. Recall the slow roll equation (eqn. (24)) as,

\[
H^2 = \frac{1}{3} V(\Psi) = \frac{20}{3\sqrt{3} M^6} v_0^2 \Psi^4 (B \Psi^4 - 1)^2.
\]

Differentiating both sides of this equation with respect to \( t \), we get the time derivative of the Hubble parameter as follows,

\[
\dot{H} = -\frac{160}{3\sqrt{3} M^6} \dot{\Psi}^2 (B \Psi^4 - 1)^2.
\]

where we use the expression of \( \dot{\Psi} \) from eqn. (24). Plugging back the expressions of \( H^2 \) and \( \dot{H} \) into eqn. (24) one gets the following condition on radion field,

\[
\Psi > 2\sqrt{2} = \Psi_f = \Psi(t_f)
\]

where \( t_f \) is the time when the radion field acquires the value \( 2\sqrt{2} \) (in Planckian unit). Eqn. (34) clearly indicates that the inflationary era of the universe continues as long as the radion field remains greater than \( \Psi_f = (2\sqrt{2}) \). Correspondingly the duration of inflation (i.e. \( t_f - t_0 \)) can be calculated from the solution of \( \Psi(t) \) as follows,

\[
\Psi_f = \frac{\Psi'_0}{B \Psi'_0 - (B \Psi'_0 - 1) \exp \left(-8e v_0 \frac{5\sqrt{3}}{3\sqrt{4} M^6} (t_f - t_0)\right)}
\]

Simplifying the above expression, we obtain

\[
t_f - t_0 = \frac{1}{8e v_0 \frac{5\sqrt{3}}{3\sqrt{4} M^6} \ln \left[\frac{\Psi'_0 (B \Psi'_0 - 1)}{\Psi'_0 (B \Psi'_0 - 1)}\right]}.
\]

So the inflation comes to an end in a finite time. In order to estimate the duration of inflation explicitly, one needs the initial value of the radion field (i.e. \( \Psi_0 \)) which can be determined from the expression of number of e-foldings, discussed in the next section.

VII. NUMBER OF E-FOLDINGS AND SLOW ROLL PARAMETERS

Total number of e-foldings (\( N_0 \)) of the inflationary era is defined as,

\[
N_0 = \int_{t_0}^{t_f} H(t)dt.
\]

Using the slow roll equation, the above expression is simplified to the form,

\[
N_0 = \int_{\Psi_0}^{\Psi_f} \sqrt{\frac{V(\Psi)}{3}} \frac{1}{\dot{\Psi}} d\Psi
\]

Putting the explicit form of \( V(\Psi) \) (eqn. (17)) and the time derivative of \( \Psi(t) \) (eqn. (21)) into the right hand side of eqn. (37) and integrating over \( \Psi \), one obtains the final result of number of e-foldings, given by

\[
N_0 = \frac{1}{16} (\Psi_f^2 - \Psi_0^2).
\]

It may be mentioned that the total number of e-foldings is independent of mass of the radion field (or inflaton field).

We may define,

\[
N(\Psi) = N_0 - \int_{\Psi_0}^{\Psi} \sqrt{\frac{V(\Psi)}{3}} \frac{1}{\dot{\Psi}} d\Psi,
\]

the number of e-foldings remaining until the end of inflation when the inflaton field crosses the value \( \Psi(t) \). Simplifying the above expression, one obtains:

\[
N(\Psi) = \frac{1}{16} (\Psi^2 - \Psi_f^2).
\]

In order to test the broad inflationary paradigm as well as particular models against precision observations [1], it is crucial to calculate the slow roll parameters (\( \epsilon_V \) and \( \eta_V \)), which are defined as follows:

\[
\epsilon_V = \frac{1}{2} \left(\frac{V''(\Psi)}{V(\Psi)}\right)^2
\]

and

\[
\eta_V = \frac{V''(\Psi)}{V(\Psi)}
\]

The slow roll condition demands that the parameters \( \epsilon_V \) and \( \eta_V \) should be less than unity as long as the inflationary era continues. By using the form of inflaton potential
(V(Ψ), in eqn.17), the above expressions can be simplified and turn out to be,

\[ \epsilon_V = \frac{8}{16N(Ψ) + Ψ_f^2} \] (39)

and

\[ \eta_V = \frac{6}{16N(Ψ) + Ψ_f^2} \] (40)

Using these expressions of slow roll parameters, one determines the spectral index of curvature perturbation \( n_s \) and tensor to scalar ratio \( r \) in terms of \( N_* \) (= \( \frac{1}{16}(Ψ_f^2 - Ψ_f^2) \)), the number of e-foldings remaining until the end of inflation when the cosmological scales exit the horizon and \( Ψ_\ast \) is the corresponding value of the inflaton field [25–27]:

\[ n_s = \frac{16N_* - 28}{16N_* + 8} \] (41)

and

\[ r = \frac{128}{16N_* + 8} \] (42)

To derive eqn.41 and eqn.42, we use the value of \( Ψ_f = 2√2 \) that has been obtained earlier (see eqn.34). From observational results (Planck 2015) \[ 1 \] \( n_s \) and \( r \) are constrained to be \( n_s = 0.968 \pm 0.006 \) and \( r < 0.12 \) respectively. Using eqn.41 and eqn.42, it can be easily shown that in order to make agreement between the theoretical and observational results, \( N_* \) should be equal to 60. Putting this value of \( N_* \) into eqn.41 and eqn.42, we obtain the following results of \( n_s \) and \( r \):

\[ n_s = 0.963 \]
\[ r = 0.11 \]

Moreover, at the pivot scale \( (N(Ψ) = N_\ast) \), \( \epsilon_V \) and \( η_V \) acquire the values as 0.009 and 0.007 respectively. In table 1, we now summarize our results.

| Parameters (\( n_s \) and \( r \)) | Theoretical results from the present model (for \( N_* = 60 \)) | Observational results from Planck 2015 |
|-----------------------------------|-------------------------------------------------|-----------------------------------|
| \( n_s \)                       | 0.963                                           | 0.968 ± 0.006                     |
| \( r \)                         | 0.11                                            | < 0.12                           |

TABLE I. Theoretical and observational results of \( n_s \) and \( r \)

It is evident from table 1, that the present model of five dimensional higher curvature gravity predicts the correct values for \( n_s \) and \( r \) as per the observations of Planck 2015. However, the required value of \( N_* \) (= 60) can be achieved if \( Ψ_\ast \) is adjusted to the value as \( Ψ_\ast \approx 31 \) (in Planckian unit). Also demanding the total number of e-foldings of inflationary era to be equal to 70 (i.e. \( N_0 = 70 \)), we obtain the initial value of inflaton field, \( Ψ_0 = 33.5 \) (in Planckian unit). With this value of \( Ψ_0 \), duration of inflation \( (t_f - t_0) \) comes as \( \sim 10^{-32} \) sec (or \( 10^{-8} (GeV)^{-1} \), see eqn.35) if the higher curvature parameter \( α \) and \( κv_v \) are taken as

\[ α \sim 10^{-22} (GeV)^{-2} \] (43)

and

\[ κv_v = 0.01 \]

respectively. Furthermore, the effective gravitational constant \( (M(4)) \) is \( \sim 10^{19} \) GeV for the estimated value of \( α \) presented in eqn.43. Once we find the initial \( (Ψ_0) \) and final \( (Ψ_f) \) values of the inflaton field, we now give the plots (figure 2 and figure 3) between the slow roll parameters and \( Ψ \) (by using eqn.39 and eqn.40).

**FIG. 2.** \( \epsilon_V \) vs \( Ψ \)

**FIG. 3.** \( η_V \) vs \( Ψ \)

Figure 2 and figure 3 clearly demonstrate that as long as inflation continues, both the slow roll parameters \( \epsilon_V \) and \( η_V \) remain less than unity. This behaviour of \( \epsilon_V \)
and $\eta_V$ are expected from the slow roll approximation. Furthermore, the value of $\epsilon_V$ and $\eta_V$ increase with the evolution of universe during the inflationary epoch and at the end of inflation, $\epsilon_V$ becomes one i.e. $\epsilon_V(\Psi_f) = 1$.

VIII. COMPARISON OF SOLUTIONS WITH AND WITHOUT SLOW ROLL APPROXIMATION

In this section, we solve the radion field and Hubble parameter numerically from the complete form of effective Friedmann equations (eqn. (20) and eqn. (21), without slow roll approximations). These numerical solutions are then compared with the solutions (in eqn. (25) and eqn. (28)) obtained by solving the slow roll equations. Eqn. (20) and eqn. (21) lead to the equation of $\Psi(t)$ as follows:

$$\ddot{\Psi} + \frac{\sqrt{3}}{\psi} \left[ V(\Psi) + \frac{1}{2} (\dot{\Psi})^2 \right] \dot{\Psi} + V'(\Psi) = 0$$

Using the form of $V(\Psi)$, above differential equation is solved numerically for $\Psi(t)$. The comparison between this numerical solution and the solution obtained in eqn. (25) is presented in figure (4).

![Fig. 4. $\Psi(t)$ vs. $t$ with/without slow roll approximation](image)

Figure (4) demonstrates that the plotted result of $\Psi(t)$ based on solving the slow roll equations and the plotted result of $\Psi(t)$ based on solving the full Friedmann equations (in presence of $\dot{\Psi}^2$ and $\ddot{\Psi}$) are almost same during the inflation. But after the inflation the acceleration term of inflaton (the term containing $\ddot{\Psi}$) starts to contribute and as a result the two solutions (with and without slow roll conditions) differ from each other. Moreover in the slow roll approximation, $\Psi(t)$ does not exhibit oscillatory phase at the end of inflation, but it tends to its minimum value asymptotically. Such an oscillatory character of $\Psi(t)$ occurs when the term $\ddot{\Psi}$ is taken into account in the equation of motion.

The numerical solution of Hubble parameter and correspondingly the deceleration parameter ($q(t) = -(\dot{H} + H^2)$) are also obtained from eqn. (20). The variation of $q(t)$ versus $t$ (with/without slow roll approximation) is shown in figure (5).

![Fig. 5. $q(t)$ vs. $t$ with/without slow roll approximation](image)

Figure (5) clearly depicts that the duration of inflation predicted from the numerical solution of complete Friedmann equations is longer than that predicted from the solutions of slow roll equations.

IX. SUMMARY AND CONCLUDING REMARKS

In this work, we consider a five dimensional compactified warped AdS model with two 3-branes embedded within the spacetime. Due to large curvature ($\sim$ Planck scale) in the bulk, the spacetime is considered to be governed by higher curvature like $F(R) = R + \alpha R^2$. Our visible universe is identified with the TeV scale brane, which emerges out of the four dimensional effective theory. On projecting the bulk gravity on the brane, the extra degrees of freedom of $R(5)$ appears as a scalar field on the brane and is known as radion field. The potential term ($V(\Psi)$) of radion field is proportional to the higher curvature parameter $\alpha$ and goes to zero as $\alpha \rightarrow 0$. Thus it is clear that the radion potential is generated entirely due to the presence of higher curvature term in the five dimensional bulk spacetime. The form of $V(\Psi)$ (in eqn. (17)) indicates that the radion potential is stable as long as $\alpha$ is considered to be positive and the minimum of $V(\Psi)$ is zero.

From the perspective of four dimensional effective theory, we examine the possibility of “inflationary scenario” by taking the on-brane metric ansatz as a spatially flat FRW one. In the presence of radion potential, $V(\Psi)$,
we determine the solutions (in eqn. (25) and eqn. (28)) of effective Friedmann equations by considering the potential energy of radion field as very much greater than the kinetic energy (also known as slow roll approximation). The solution of scale factor corresponds to an accelerating expansion of the early universe and the rate of expansion depends on the parameter $\alpha$. It may be mentioned that the radion field as well as the scale factor become constant as $\alpha$ goes to zero. Thus it can be argued that due to the presence of higher curvature term, the radion field has a certain dynamics which in turn triggers an exponential expansion of the universe at an early epoch. The expression of duration of inflation $(t_f - t_0)$ is also obtained in eqn. (35) which reveals that the accelerating phase of the universe terminates within a finite time.

We determine the slow roll parameters ($\epsilon_V$ and $\eta_V$) and it is found that both $\epsilon_V$ and $\eta_V$ remain less than unity as long as the inflation continues. The expressions of slow roll parameters yield the spectral index of unity as long as the inflation continues. The expressions and it is found that both $\epsilon_V$ and $\eta_V$ remain less than unity as long as the inflation continues. The expressions and it is found that both $\epsilon_V$ and $\eta_V$ remain less than unity as long as the inflation continues.

Finally we find the solution for the radion field and Hubble parameter numerically from the complete form of Friedmann equations (without the slow roll approximations). During the inflation, these numerical solutions are almost same with the solutions of slow roll equations, as demonstrated in figure (4) and figure (5). Another important point to note is that in the slow roll approximation, $\Psi(t)$ does not exhibit oscillatory phase at the end of inflation, but it tends to its minimum value asymptotically, while such an oscillatory behaviour of inflaton is indeed there if the slow roll approximation is relaxed, i.e., the acceleration term of $\Psi(t)$ in the equation of motion is not dropped, as depicted in figure (4).

[1] Planck Collaboration (P.A.R. Ade (Cardiff U.) et al.); Planck 2015 results. XX. Constraints on inflation; Astron.Astrophys. 594 A20 (2016). arXiv:1502.02114.
[2] N. Arkani-Hamed, S. Dimopoulos, G. Dvali; Phys. Lett. B 429 263 (1998); N. Arkani-Hamed, S. Dimopoulos, G. Dvali; Phys. Rev. D 59 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali; Phys. Lett. B 436 257 (1998).
[3] P. Horava and E. Witten; Nucl. Phys. B475, 94 (1996); B460, 506 (1996).
[4] L. Randall and R. Sundrum; Phys. Rev. Lett. 83, 3370 (1999).
[5] N. Kaloper; Phys. Rev. D60, 123506 1999; T. Nihei; Phys. Rev. D 59, 064003 (2000).
[6] A. G. Cohen and D. B. Kaplan; Phys. Lett. B470, 52 (1999).
[7] C. P. Burgess, L. E. Ibanez, and F. Quevedo; ibid. 447, 257 (1999).
[8] A. Chodos and E. Poppitz; ibid. 471, 119 (1999); T. Gherghetta and M. Shaposhnikov; Phys. Rev. Lett. 85, 240 (2000).
[9] G. F. Giudice, R. Rattazzi and J. D. Wells; Nucl. Phys. B 544, 3 (1999).
[10] R. Marteens and K. Koyama; Living Rev. Rel. 13, 5 (2010).
[11] W. D. Goldberger and M. B. Wise; Phys.Rev.Lett.83, 4922 (1999).
[12] W. D. Goldberger and M. B. Wise; Phys.Lett.B 475 275-279 (2000).
[13] C. Csaki, M. L. Graesser, and G. D. Kribs; Phys. Rev. D 63, 065002 (2001).
[14] J. Lesgourgues, L. Sorbo; Phys. Rev. D69 084010 (2004).
[15] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch; Phys. Rev.D 62, 046008 (2000).
[16] S. Das, D. Maity, and S. SenGupta; J. High Energy Phys. 05 042 (2008).
[17] A. Das, H. Mukherjee, T. Paul and S. SenGupta; arXiv:1701.01571 [hep-th].
[18] T. Paul and S. SenGupta; Phys.Rev. D93 no.8, 085035 (2016).
[19] A. Das, T. Paul and S. SenGupta; arXiv:1609.07787 [hep-ph].
[20] Donald Perkins; Particle Astrophysics, Oxford University Press, 1st edition, 2005.
[21] Gary Scott Watson; astro-ph/0005003, 2000.
[22] A. H. Guth; Phys.Rev. D23 347-356 (1981).
[23] A. Linde; Particle Physics and Inflationary Cosmology. (Electronic version of the book from 1990), hep-th/0503203, 2005.
[24] W. H. Kinney; astro-ph/0310148 2004.
[25] D. Langlois; hep-th/0405053 2004.
[26] S. Habib, A. Heinen, K. Heitmann, G. Jungman; Phys. Rev. D 71, 043518 (2005).
[27] A. Riotto; hep-th/0210162, (2002).
[28] N. Banerjee, S. Sen; Phys.Rev. D57, 4614 (1998).
[29] J. D. Barrow and P. Saich; Class. Quantum Grav. 10, 279 (1993).
[30] J. D. Barrow and P. Mimoso; Phys. Rev. D 50, 3746 (1994).
[31] J. P. Mimoso and D. Wands; Phys. Rev. D 51, 477 (1995).
[32] C. Csaki, M. Graesser, L. Randall, J. Terning; Phys.Rev. D62 045015 (2000).
[33] P. Binetruy, C. Deffayet, and D. Langlois; Nucl. Phys. B565, 269 (2000).
[34] C. Csaki, M. Graesser, C. Kolda, and J. Terning; Phys.
