Stability-Based Parameter Selection for Data-Driven Model-Free Adaptive Controllers

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Abstracts. The algorithm of model-free adaptive control normally includes a control algorithm and a pseudo-partial derivative estimation algorithm. The selection of each parameter in the algorithm directly affects the control performance and stability of the system. Two artificial additional parameters have been introduced for the algorithm, but the corresponding theoretical basis needs to be improved. Although the range of artificial additional parameters is given in the control algorithm, the stability condition of the system cannot be guaranteed. In this paper, the theoretical basis for introducing two additional parameters is provided by modifying the cost function. Furthermore, the selection conditions of control parameters are given such that the controlled system stable and not entirely dependent on the trial-and-error. The simulation results are given to confirm the theoretical findings.

1. Introduction
For the traditional model-based control methods, the control performance excessively depends on the establishment of the mathematical model. It is well-known that accurate mathematical models are difficult to obtain for some complex objects [1-5], which will bring difficulties during the stability analysis and control design. Although accurate mathematical models can be obtained in some cases, the complexity of the controller will be increased and the difficulty of design and synthesis will be increased. Notice that the model-free adaptive control (MFAC) has attracted wide attention, which only uses the input and output data of the control system and does not contain the structural information of the system, so it can reduce the dependence on the accurate model. The MFAC algorithm was first proposed in [5], which can effectively control the discrete-time nonlinear dynamic system [6-9]. Since the system is controlled online by the MFAC only according to the data of the system I/O, no additional test signal is required, thus the stability could be achieved under some hypotheses. On the other hand, many successful applications have been reported, see [10-17], and the references therein.

Although there are many research results about the MFAC, there are still some problems that need to be discussed in the theoretical basis of the control algorithm [18-20]. In the control algorithm, two parameters are added artificially, but the reasons for these two parameters are not explained theoretically. Moreover, the relationship between the selection of the two parameters and the stability conditions of the system is an open question which deserves further study. For example, the stability of a full-form MFAC algorithm is discussed in [19], but the effect of artificially added parameters on the stability of the system is not comprehensively discussed. It is shown that adding parameters to the formula can make the formula more general and prove the stability of the formula in [7]. However, the work does not give an analysis of the existence of parameters. In [20], the influence of the artificially added parameters on the stability of the system is discussed, while only a fuzzy range is given, in
which there may be a parameter value that makes the system stable. Moreover, the influence and correlation between the parameters are not analyzed. It is pointed out in [21] that the model-free adaptive control algorithm does not imply or contain any model information of the controlled system, but the structural parameters of the controlled system should be used to verify the stability. Motivated by the above discussions, this paper obtains the effect of these two parameters on the system control by theoretical analysis, and will consider the design method of these two parameters in data-driven MFAC. Through theoretical analysis, the influence of these two parameters on system control is obtained. According to the stability condition of the system, the parameters introduced in the control algorithm are selected, which provides a theoretical reference for the design of model-free adaptive controller.

2. Problem Description
Consider a general discrete nonlinear system:

\[ y(k + 1) = f\left( {y(k), y(k - 1), \ldots ,y(k - n_y), u(k), u(k - 1), \ldots ,u(k - n_u)} \right) \quad (1) \]

where \( y(k) \) and \( u(k) \) represent the output and input of the system respectively; \( n_y \) and \( n_u \) represent the order of the system respectively. During the model-free adaptive control algorithm analysis, the following two conditions are required [21]:

- Except for a limited period of time, the partial derivative of \( f(\ldots) \) in the system (1) with respect to the \((n_y + 2)\)th variable is continuous.
- Except for a limited period of time, the system (1) meets the generalized Lipschitz condition, that is, \( |y(k + 1) - y(k)| \leq b|u(k) - u(k - 1)| \), where \( b \) is the Lipschitz parameter.

When the above conditions are met and \( |\Delta u(k)| \neq 0 \), there should be a parameter \( \Phi(k) \) called pseudo-partial derivative (PPD) such that system (1) satisfy the following dynamic linearized model. The detailed proofs can be found in the [21]:

\[ \Delta y(k + 1) = \Phi(k) \Delta u(k) \quad (2) \]

where \( \Delta y(k + 1) = y(k + 1) - y(k) \) and \( \Delta u(k) = u(k) - u(k - 1) \).

In order to make the actual output of the discrete nonlinear system tends to the required output and limits the input range, the following cost functions are established:

\[ J(u(k)) = (y^d(k + 1) - y(k + 1))^2 + \lambda(u(k) - u(k - 1))^2 \quad (3) \]
\[ J(\hat{\Phi}(k)) = (\Delta y(k) - \hat{\Phi}(k)\Delta u(k - 1))^2 + \mu\Delta^2(k) \quad (4) \]

where, \( \hat{\Phi}(k) \) is the estimation of \( \Phi(k) \). By substituting the equation (2) into the equation (3) and then making the partial derivatives of equations (3) and (4) equal to 0 respectively, one has:

\[ u(k) = u(k - 1) + \rho\hat{\Phi}(k)\frac{y^d(k + 1) - y(k)}{\lambda + \hat{\Phi}^2(k)} \quad (5) \]

and

\[ \hat{\Phi}(k) = \hat{\Phi}(k - 1) + \frac{\eta\Delta u(k - 1)(\Delta y(k) - \hat{\Phi}(k - 1)\Delta u(k - 1))}{\mu + \Delta^2(k - 1)} \quad (6) \]

Parameters \( \rho \) and \( \eta \) are added to generalize the formula (5) and (6). The range of parameters in equations (5) and (6) are given as follows [21]:

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0 < \rho \leq 1, 0 < \eta \leq 2; \lambda, \mu > 0 \hspace{1cm} (7)

In order to make the estimation algorithm of pseudo-partial derivative parameters have stronger ability to track time-varying parameters, the pseudo-partial derivative updating algorithm needs to be introduced [21], that is, \( \hat{\Phi}(k) = \hat{\Phi}(1) \), when \( |\hat{\Phi}(k)| \leq \varepsilon \), \( |\Delta u(k - 1)| \leq \varepsilon \) and \( \text{sign}(\hat{\Phi}(k)) \neq \text{sign}(\hat{\Phi}(1)) \), where \( \hat{\Phi}(1) \) is the initial value of \( \Phi(k) \) and \( \varepsilon \) is a sufficiently small positive number.

From the analyses above, the control input \( u(k) \) and pseudo-partial derivative \( \hat{\Phi}(k) \) are obtained as shown in equations (5) and (6).

**Remark 1.** Some reasons affecting the design of model-free adaptive controllers include:

1) As \( \eta \) and \( \rho \) are not included in the cost functions (3) and (4), equations (5) and (6) cannot be directly derived from (3) and (4). Though it was stated in [21] that the parameters \( \rho \) and \( \eta \) are added to generalize the formula (5) and (6), a theoretical explanation is needed to clarify the use of the two parameters.

2) The selection of the parameters given in (7) is not fully based on the system stability conditions. Thus, it may not guarantee the system stability and a repeated trial-and-error process might be needed for the selection of the parameters.

3) The proof of the system stability needs to know the maximum value of \( \Phi(k) \), but the selection of the parameters given in (7) is not related to the maximum value of \( \Phi(k) \).

3. Modification of Cost Functions and Derivation Of \( \rho \) and \( \eta \)

To derive control algorithm (5) with \( \rho \), a new cost function is defined as follows:

\[
J(u(k)) = \rho^2 \left( y^d(k + 1) - y(k) - \frac{\hat{\Phi}(k)}{\rho} \Delta u(k) \right)^2 + \lambda (u(k) - u(k - 1))^2 \hspace{1cm} (8)
\]

where \( y^d(k) \) is a reference input signal. Letting \( \partial J(u(k))/\partial u(k) = 0 \), we immediately have equation (5).

Comparing equation (8) with equation (3), the role of \( \rho \) in the model-free control algorithm can be obtained as:

- If the difference between \( \Phi(k) \) and \( \hat{\Phi}(k) \) is too large, it will lead to poor control performance. Thus, \( \hat{\Phi}(k) \) divided by an adjustment coefficient \( \rho \) may be a better estimation of \( \Phi(k) \).

- As the weighted coefficient \( \rho \) is added in cost function (8), this makes the cost function and the control algorithm (5) more general, and better control performance may be obtained.

To derive the estimation algorithm of the pseudo-partial derivative (6) with \( \eta \), a new cost function is defined as follows:

\[
J(\hat{\Phi}(k)) = \eta^2 \left( \Delta y(k) - \frac{1}{\eta} \left( \hat{\Phi}(k) + (\eta - 1)\hat{\Phi}(k - 1) \right) \Delta u(k) \right)^2 + \mu \hat{\Phi}^2(k) \hspace{1cm} (9)
\]

Setting \( \partial J(\hat{\Phi}(k))/\partial \hat{\Phi}(k) = 0 \), we immediately have equation (6).

By comparing equation (9) with the original cost function (4), the role of the added parameter \( \eta \) in equation (6) can be listed as follows:

- The added parameter \( \eta \) makes equation (9) and the estimation algorithm of the pseudo-partial derivative (6) more general, and better estimation performance of \( \Phi(k) \) may be achieved.
• In the cost function (9), $\Phi(k-1)$ is replaced by $\frac{1}{\eta} \hat{\Phi}(k) + \frac{(\eta-1)}{\eta} \hat{\Phi}(k-1)$. Thus, we have:

$$\hat{\Phi}(k) = (1-\eta)\hat{\Phi}(k-1) + \eta\Phi(k-1)$$  \hspace{1cm} (10)

From equation (10), the iterative speed can be adjusted by changing $\eta$.

4. Selection of $\eta$ and $\rho$ Based on System Stability

First, we discuss the selection of $\eta$. As the cost function (9) implies the iterative process (10), the iterative process must be stable, leading to that $0 < \eta < 2$ must hold. However, the range of parameter $\eta$ was given as $0 < \eta \leq 2$ in (7), which was based on the stability proof [21] and later selected in [22]. Obviously, if $\eta = 2$, the iterative process (10) may not be stable, resulting in that the estimation algorithm of the pseudo-partial derivative (6) may not be stable.

Then, we discuss the selection of $\rho$. Defining the tracking error $e(k) = y^d(k) - y(k)$, and according to equation (2), one has:

$$e(k+1) = y^d(k+1) - y(k+1) = y^d(k+1) - y(k) - \Phi(k)\Delta u(k)$$  \hspace{1cm} (11)

Substituting equation (5) into equation (11), we have:

$$e(k+1) = \left[1 - \frac{\rho\Phi(k)\hat{\Phi}(k)}{\lambda + \hat{\Phi}^2(k)}\right]e(k) + \left[1 - \frac{\rho\Phi(k)\hat{\Phi}(k)}{\lambda + \hat{\Phi}^2(k)}\right](y^d(k+1) - y^d(k))$$  \hspace{1cm} (12)

Equation (12) can be changed as:

$$e(k+1) = \left[1 - \frac{\rho\Phi(k)\hat{\Phi}(k)}{\lambda + \hat{\Phi}^2(k)}\right]e(k) + \left[1 - \frac{\rho\Phi(k)\hat{\Phi}(k)}{\lambda + \hat{\Phi}^2(k)}\right](y^d(k+1) - y^d(k))$$  \hspace{1cm} (13)

As $y^d(k)$ is bounded, $\left|y^d(k+1) - y^d(k)\right|$ must be bounded. To make the error dynamic (13) stable, $\left|1 - \rho\Phi(k)\Phi(k)(\lambda + \hat{\Phi}^2(k))^{-1}\right| < 1$ must hold [21] and satisfy $0 < \rho\Phi(k)\Phi(k)(\lambda + \hat{\Phi}^2(k))^{-1} < 2$. Because $\lambda + \hat{\Phi}^2(k) \geq 2\Phi(k)\sqrt{\lambda}$, one has:

$$0 < \frac{\rho\Phi(k)\Phi(k)}{2\Phi(k)\sqrt{\lambda}} < \frac{\rho\Phi(k)\Phi(k)}{2\Phi(k)\sqrt{\lambda}} < 2$$  \hspace{1cm} (14)

where $|\Phi(k)| \leq b$ with $b$ being a positive constant. From (14), we have:

$$0 < \rho < \frac{4\sqrt{\lambda}}{b}$$  \hspace{1cm} (15)

The inequality (15) shows that the closed-loop control system must be stable when parameter $\rho$ satisfies the condition (15). However, in [21], the range of $\rho$ and $\lambda$ that make the error stable is $0 < \rho \leq 1$, $\lambda > 0$, The connection between $\rho$ and other parameters and whether the stability condition (15) is met are not take into account. It is clear from the equation (15) that the range of $\rho$ cannot simply be defined as a certain number between 0 and 1.

From the stability condition (15), we can see that the error dynamics (13) may not be stable when giving $0 < \rho \leq 1$ because $4\sqrt{\lambda}/b$ may also be less than 1 and the given $\rho$ may be larger than $4\sqrt{\lambda}/b$.

On the other hand, according to the equation (13), $1 - \rho\Phi(k)\Phi(k)(\lambda + \hat{\Phi}^2(k))^{-1}$ shows that the
parameter $\lambda$ as a weight, play the role of adjusting the control input. Generally speaking, we hope that the control input cannot change dramatically, thus, the value of $\lambda$ can be large to constrain the change of the control input. However, if $\rho$ is too small, such as between 0 and 1, it may cause $1 - \rho \hat{\Phi}(k) \Phi(k) (\lambda + \hat{\Phi}(k))^{-1}$ to be close to 1, so that the system settling time is too long, which may reduce the control performance of the system. At this point, the actual value of $\rho$ should be calculated based on the equation (15), rather than simply limiting the range of $\rho$ to between 0 and 1.

The stability condition (15) also shows that the maximum value $b$ of $\Phi(k)$ must be known so that the selection of $\lambda$ and $\rho$ can make the error dynamics (13) stable. Otherwise, a repeated trial-and-error process might not be avoided for the selection of the parameters.

5. Simulations Experiments

The following two models are selected for simulation to verify the above theoretical investigation.

5.1. Example 1

This example shows that the closed-loop system may be unstable even if the value of parameter $\rho$ is selected to be less than 1. Thus, the value of $\rho$ should be selected according to the stability condition (15). The system is described as follows:

$$y(k+1) = \frac{y(k)u(k)}{1+y^3(k)} + u^3(k)$$

The reference input that needs to be tracked is as $y(r)(k) = 0.5 \times (-1)^{\text{round}(k/100)}$

Parameter selection for MFAC: $u(1) = u(2) = 0$, $y(1) = y(2) = y(3) = 0$, $\Phi(1) = \Phi(2) = 2$, $\eta = 0.1$, $\mu = 0.5$, $\lambda = 0.2$. It is estimated here that the value of $b$ is 1.5. $\rho \approx 0.39$ can be calculated by equation (15). $\rho = 0.4$ and 0.2 are selected respectively for simulation. The simulation results are shown in Fig. 1 to Fig. 4. From the simulation results, we can see that the closed-loop system is unstable even if $\rho$ is selected to be between 0 and 1. This demonstrates that the selection of $\rho$ must satisfy the stability condition (15). Simply letting $\rho$ be between 0 and 1 may make the closed-loop system unstable.

![Figure 1. Tracking performance when $\rho = 0.2$](image1.png)

![Figure 2. Tracking errors when $\rho = 0.2$](image2.png)
Figure 3. Tracking performance when $\rho = 0.4$

Figure 4. Tracking errors when $\rho = 0.4$

5.2. Example 2

This example shows that the value of $\rho$ calculated by the equation (15) is greater than 1. However, if the value of $\rho$ is still selected to be between 0 and 1, the control performance of the system will be reduced. The system is described as follows:

$$y(k+1) = \frac{y(k)u(k)}{1 + y^2(k)} + u(k)$$

The reference input signal is as $y_d(t) = \sin \frac{\pi k}{250}$, $k \in [0, 1000]$.

Parameter selection for MFAC: $x(1) = x(2) = x(3) = 0$, $u(1) = u(2) = 0$, $y(1) = y(2) = y(3) = 0.5$, $\Phi(1) = \Phi(2) = 1$, $\eta = 1$, $\mu = 0.5$, $\lambda = 1$. The estimated value of $b$ is 1.5. $\rho < 2.66$ can be calculated according to the stability condition (15) so that $\rho = 2.5$ and 0.8 are selected for simulations. The results are shown in Fig. 5 to Fig. 8. From the simulation results, we can see that when the parameter values are selected as $\rho = 2.5$, the tracking performance of the system is better, and the tracking error is less than that when the parameter values are selected as $\rho = 0.8$. In summary, the controller parameters $\lambda$ and $\rho$ cannot be simply set to $0 < \rho \leq 1$, $\lambda > 0$, and the range of values for $\lambda$ and $\rho$ should be determined according to the stability condition (15).

Figure 5. Tracking performance when $\rho = 0.8$

Figure 6. The tracking errors when $\rho = 0.8$
Figure 7. Tracking performance when \( \rho = 2.5 \)

Figure 8. Tracking errors when \( \rho = 2.5 \)

6. Conclusion
By establishing new cost function, the general form of pseudo-partial derivative estimation and control algorithm is established in this paper. The significance of the two added controller parameters are discussed and the stability conditions of parameter selection in MFAC algorithm are also investigated. Through theoretical analysis, it is pointed out that if the parameters of the controller are not selected according to the stability conditions of the system, the control performance of the system may be reduced, and even the system may be unstable. Two simulation examples verify the results of the theoretical analyses, which provides a reference for the selection of the parameters in MFAC.

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