Shape coexistence close to $N = 50$ in the neutron-rich isotope $^{80}\text{Ge}$ investigated by IBM-2

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Abstract

The properties of the low-lying states, especially the relevant shape coexistence in $^{80}\text{Ge}$ close to one of most neutron-rich doubly magic nuclei at $N = 50$ and $Z = 28$ have been investigated within the framework of the proton-neutron interacting model (IBM-2). Based on the fact that the relative energy of the $d$ neutron boson is different from proton bosons’, the calculated energy levels of low-lying states, $E2$ transition strengths can reproduce the experimental data very well. Particularly, the first excited state $0^+_2$ is reproduced quite nicely, which is intimately related with the shape coexistence phenomenon. And the $\rho_2^2(E0,0^+_2 \rightarrow 0^+_1)$ transition strength has been predicted.

The experimental data and theoretical results indicate that both collective spherical and $\gamma$-soft vibration structures coexist in $^{80}\text{Ge}$.

Key words: $^{80}\text{Ge}$, low-lying states, $E0$ transition strengths, shape coexistence, IBM-2.

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I. INTRODUCTION

Shape coexistence is a peculiar nuclear phenomenon when two or more states occur in the same nuclei within a very narrow energy range at low excitation energy [1]. Shape coexistence phenomenon is often found close to or at the shell closures, where deformed intruder configurations coexist with spherical shapes based on multiparticle-hole excitations across the closed shell in the nuclear chart, from light nuclei to heavy nuclei [2–4]. The presence of low-lying $0^+$ states as the first excited state in even-even nuclei is one of the signatures of shape coexistence [2, 5], which plays an important role in our understanding for the shape changes of the nuclear structure in the exotic nuclei.

In recent years, the advent of radioactive isotopes beams have been developed, which gives the access to exotic nuclei far from stability in both the neutron-deficient and neutron-rich regimes [1, 6–10]. In the neutron-rich nuclei, the empirical evidence of shape coexistence has been observed along $N = 20$, $N = 28$ and the subshell gap $N = 40$, see Refs.[2,3] for a review. A lot of theoretical works have been developed to investigate shape coexistence and shape phase transition, for example, the interacting-boson model [11, 12], the shell model [13] and the projected shell model [14, 15], the self-consistent relativistic mean-field theory [16]. Nowadays shape coexistence in nuclei close to the supposedly doubly magic nucleus $^{78}_{28}$Ni$_{50}$ is the focus of intense experimental and theoretical research (cf., for example [4, 17, 18] and references therein), because the study of shape coexistence in this region will help us to differentiate the single-particle effect from the quadrupole collective motion across $N = 50$. More recently the technique of $\beta$-delayed electron-conversion spectroscopy is applied to study $^{80}_{32}$Ge nucleus. In Ref. [18], an electric monopole $E0$ transition is observed for the first time, which points to an intruder $0^+_2$ state at $639(1)$ keV. The new state $0^+_2$ is much lower than the $2^+_1$ level in $^{80}_{32}$Ge, this characteristic implies there might exist the effect of shape coexistence near the most neutron-rich doubly magic nucleus at $N = 50$ and $Z = 28$, giving an insight into the mechanism of shape coexistence close to the neutron major shell closure at $N = 50$.

It is well known that the low-lying structure of Ge isotopes display the trends of coexistence of different shapes along the long isotopic chain, characterized by prolate-oblate and spherical-deformed competition. Close to the $\beta$-stability line, the shape transition of Ge isotopes is a drastic evolution from nearly spherical in $^{72}_{32}$Ge to slight prolate in $^{74}_{32}$Ge or
even triaxiality in $^{76,78}$Ge \[19, 22\] and $^{84,86,88}$Ge \[23\]. In the neutron-rich region, the $B(E2)$ behavior has a smooth decrease toward $N = 50$ \[24\]. Both the shape transition from spherical to weakly deformed and the coexistence of different types of deformation might occur in these isotopes \[25\]. A rich variety of shapes and shape coexistence in Ge isotopes provide a challenging testing ground for theoretical models. The Skyrme-Hartree-Fock (SHF) and Gogny Hartree-Fock-Bogoliubov (HFB) models imply that most Ge isotopes show the features of soft triaxial deformation \[26\]. The self-consistent total-Routhian-surface calculations show there exists the shape phase transitions from oblate deformation, through triaxial deformation, to prolate deformation in even-mass $^{64-80}$Ge isotopes \[20, 27\]. The nuclear density functional theory investigated the structural evolution from weakly triaxial deformation in $^{74}$Ge to $\gamma$ soft deformation in $^{78,80}$Ge, and finally to spherical shape in $^{82}$Ge \[28\]. The multi-quasiparticle triaxial projected shell model demonstrates that $^{76}$Ge exhibits a rigid $\gamma$ deformation in its low-lying states, which is a rare nucleus possessing this kind of nuclear structure. But its neighboring nuclei such as $^{70,72,74,78,80}$Ge isotopes show the different $\gamma$-soft features \[29\]. Moreover for $^{80}$Ge, because of subtle balance between quadrupole terms and pairing term in the interaction, each term of interaction governs two coexisting systems respectively: one for the quasiparticle type and the other for the collective triaxial type \[30\], these interactions determine the features of $^{80}$Ge.

In Ref. \[31\], the authors discussed the general properties of low-lying states of the even-even Ge isotopes through the interacting boson model (IBM-1). One does not distinct neutron pairs and proton pairs in IBM-1 \[32\]. The calculation results reproduced the available experimental data, and suggested that there exist the shape transitions from the mixture of U(5), SU(3) and O(6) symmetry to the mixture of U(5) and O(6) and finally to U(5) symmetry along the even-even isotopes of $^{64-78}$Ge \[31\]. Meanwhile, the authors of Ref. \[33\] reproduced satisfactorily the available experimental information on the energy spectrum, $E2$ transition and quadrupole moments for the even-mass $^{68-76}$Ge through the proton-neutron interacting boson model (IBM-2). In IBM-2, proton bosons and neutron bosons are independently cheated as different degree of freedom and introduces the mixing of their configurations \[32\]. Furthermore, the energy levels, $E2$ and $M1$ transition properties of even-even isotopes $^{64-68}$Ge were analyzed through the IBM model with isospin (IBM-3) \[34\].

Very recently, the shape coexistence and shape transitions in the even-even nuclei $^{66-94}$Ge were calculated by using the IBM-1 \[35\], where the authors applied a self-consistent mean-
field method on the basis of the Gogny-D1M energy density functional theory. This calculation agreed with the known experimental data of these nuclei. However, their calculated energy levels of the states $E(0^+_2)$ and $E(2^+_2)$ are a little higher than the experimental data especially for $^{80}$Ge, the reason is that the proton-neutron pairing effects could not be neglected in this case. On the other hand, the IBM-2 without introducing the configuration mixing has been used to investigate shape coexistence in some nuclei in the $A \sim 100$ mass region \cite{36,37}, and in the neutron-deficient isotopes $^{74,76}$Kr \cite{38}. The numerical calculations are in good agreement with the recent experimental values for the low-lying energy spectrum, and the key sensitive quantities such as the quadrupole shape invariants and the $B(E2)$ transition strength branch ratios. In particular, the calculation reproduces the low-lying $0^+_2$ state quite well, which is intimately related with the shape-coexistence phenomenon. However, there is short of a detailed investigation on the nuclear shape and shape coexistence in the exotic nucleus $^{80}$Ge by IBM-2. In this study, we will discuss the properties of the low-lying states of $^{80}$Ge, especially the relevant shape coexistence in the framework of IBM-2. Based on the fact that the relative energy of $d$ neutron boson is different from proton boson’s energy, we calculate the energy levels of low-lying states, and the $B(E2)$ and $\rho^2(E0)$ transition strengths. We also compare the numerical results with the recent available experimental data. Then, we will describe the shape coexistence phenomena in $^{80}$Ge with IBM-2.

The structure of this paper is listed as follows. In Sec. II we briefly describe the Hamiltonian, $E2$ and $E0$ operators used in this study, and also present the criteria adopted for determining the IBM-2 model parameters. In Sec. III we compare the numerical results and experimental data and discuss the electromagnetic transition properties. Finally in Sec. IV we give our summary and some remarks.

II. THEORETICAL FRAMEWORK

In IBM-2, the total bosons include proton bosons and neutron bosons, namely satisfy $N = N_\pi + N_\nu$. The boson creation operators $s_{\rho,0}^+$ and $d_{\rho,\mu}^+$ and the corresponding annihilation operators $s_{\rho,0}$ and $d_{\rho,\mu}$ constructed the generators of the group $U_\pi \otimes U_\nu$, where $\rho$ represents $\pi$ or $\nu$ and $\mu = -2, ..., 2$. The product $[N_\nu] \times [N_\pi]$ of symmetric representations of $U_\pi(6)$ and $U_\nu(6)$ constitutes the IBM-2 model space. The IBM-2 Hamiltonian used in this paper
\[ \hat{H} = \varepsilon_{d\pi} \hat{n}_{d\pi} + \varepsilon_{d\nu} \hat{n}_{d\nu} + \kappa_{\pi\nu} \hat{Q}_\pi \cdot \hat{Q}_\nu + \omega_{\pi\pi} \hat{L}_\pi \cdot \hat{L}_\pi + \hat{M}_{\pi\nu}, \]  

(1)

where \( \hat{n}_{d\rho} = d^\dagger_{\rho} \cdot \tilde{d}_\rho \) stands for \( d \)-boson number operator for neutron (\( \rho = \pi \)) and proton (\( \rho = \pi \)), respectively. \( \varepsilon_{d\rho} \) is the energy of the \( d \)-bosons relative to the \( s \) bosons. \( \hat{Q}_\rho = (s^\dagger_{\rho} \tilde{d}_\rho + d^\dagger_{\rho} s_{\rho})^{(2)} + \chi_{\rho}(d^\dagger_{\rho} \tilde{d}_\rho)^{(2)} \) denotes the quadrupole operator. \( \chi_{\rho} \) occurred in the quadrupole operator determines the type of the deformation. The third term represents the quadrupole-quadrupole interaction between proton-boson and neutron-boson with the strength parameter \( \omega_{\pi\pi} \). The fourth term of Eq. (1) denotes the dipole proton-proton interaction with strength \( \omega_{\pi\nu} \), where \( \hat{L}_\pi \) is the angular momentum operator, which can be explicitly expressed as \( \hat{L}_\pi = \sqrt{10}[^1 d_{\pi} \cdot \tilde{d}_\pi]^{(1)} \). The last term denotes the Majorana interaction, its explicit form is \( \hat{M}_{\pi\nu} = \lambda_2 (s^\dagger_{\pi} d^\dagger_{\nu} - s^\dagger_{\pi} d^\dagger_{\nu})^{(2)} \cdot (s_{\pi} d_{\nu} - s_{\pi} d_{\nu})^{(2)} + \sum_{k=1,3} \lambda_k (d^\dagger_{\rho} d^\dagger_{\nu})^{(k)} \cdot (\tilde{d}_{\rho} \tilde{d}_{\nu})^{(k)} \), where the strength of Majorana interaction are embodied by the parameters \( \lambda_k \) (\( k=1,2,3 \)).

The Hamiltonian in Eq. (1) gives rise to four dynamical symmetries \( U_{\pi\pi}(5), SU_{\pi\nu}(3), O_{\pi\nu}(6) \), and \( SU^*_{\pi\nu}(3) \), which correspond to a spherical, an axially symmetric, a \( \gamma \)-unstable, and a triaxial deformed shape respectively. For certain values of the model parameters, the Eq. (1) can reduce to contain only one kind of dynamical symmetry \( [39] \). The \( B(E2) \) transition strengths and the \( \rho^2(E0) \) values between \( 0^+ \) states could be used to search the signatures of shape coexistence. In IBM-2, the \( E2 \) transition matrix element is defined as follows

\[ B(E2, J \rightarrow J') = \frac{1}{2J + 1} |\langle J' || \hat{T}^{(E2)} || J \rangle|^2, \]  

(2)

where the \( E2 \) transition operator \( \hat{T}^{(E2)} \) is given through the quadrupole operator \( Q_\rho \) as \( \hat{T}^{(E2)} = e_\pi \hat{Q}_\pi + e_\nu \hat{Q}_\nu \). \( J \) and \( J' \) are the initial and final angular momenta, respectively. \( e_\nu (e_\pi) \) represents the effective charge of neutron (proton) bosons, one can determine the effective charges by fitting the experimental data.

The \( E0 \) transition matrix element \( \rho \) in the IBM-2 is defined as

\[ \rho(E0, J \rightarrow J') = \frac{Z}{e^2 R^2} \{ \beta_{0\pi} \langle J' || \hat{T}^{(E0)}_\pi || J \rangle + \beta_{0\nu} \langle J' || \hat{T}^{(E0)}_\nu || J \rangle \}, \]  

(3)

where \( R = 1.2A^{1/3} \text{fm} \), and \( \beta_{0\pi(\nu)} \) is the so-called proton (neutron) monopole boson effective charge in unit of efm\(^2\). The \( E0 \) transition operator is written as \( \hat{T}^{(E0)} = \beta_{0\pi} \hat{T}^{(E0)}_\pi + \beta_{0\nu} \hat{T}^{(E0)}_\nu = \beta_{0\pi} \hat{n}_{d\pi} + \beta_{0\nu} \hat{n}_{d\nu} \), where the \( \hat{n}_{d\rho} \) is the same in Eq. (1).
\(^{80}\text{Ge}\) is composed of \(N = 48\) neutrons and \(Z = 32\) protons, and locates at \(Z = 28, N = 50\) major shell. We take the doubly magic nucleus \(^{78}\text{Ni}\) at \(Z = 28\) and \(N = 50\) as an inert core for the description of \(^{80}\text{Ge}\). In this case, there are two proton bosons outside the \(Z = 28\) shell which are particle-like, while one neutron bosons outside the \(N = 50\) shell in \(^{80}\text{Ge}\) which is hole-like. The microscopic picture demonstrates that the valence neutrons and protons occupy different orbitals when they are added to \(^{78}\text{Ni}_{50}\) core \([4, 40]\). The four valence protons are distributed among the \(fp\) orbitals. The two hole-like valence neutrons occupy the \(g_{9/2}\) orbital \([30, 40, 41]\). In order to consist with the microscopic description and remove some of the degeneracies, we use the different energies \(\varepsilon_{d\pi} \neq \varepsilon_{d\nu}\) for \(d\) proton and neutron boson in the same way as in Refs. \([36, 42]\). In general, the parameters \(\varepsilon_{d\rho}\) and \(\kappa_{\pi\nu}\) are mainly used to reproduce the energy levels of low-lying states with positive parity. The values of \(\varepsilon_{d\rho}\) mostly contribute to the spectrum of \(U(5)\) nuclei. However \(\kappa_{\pi\nu}\) mainly characterizes the properties of deformed nuclei. The structure parameters \(\chi_{\pi}\) and \(\chi_{\nu}\) occurred in quadrupole operators are used to describe the \(B(E2)\) transition properties. Only the dipole interaction term \(\hat{L}_{\pi} \cdot \hat{L}_{\pi}\) is explicitly considered in the Hamiltonian because of only one hole-like neutron boson outside the \(N = 50\) shell in \(^{80}\text{Ge}\). \(\hat{L}_{\pi} \cdot \hat{L}_{\pi}\) plays an important role on the description of rotational energy levels \([43, 45]\). The parameter \(\omega_{\pi\pi}\) can be used to tune the order of the \(2^+_2\) state and \(4^+_1\) state. The Majoranan parameters mainly influence the mixed symmetry states, in order to reduce the number of the free parameters in Hamiltonian, for simplicity we take \(\lambda_2 = 0\) and \(\lambda_1 = \lambda_3\) in this study.

The IBM-2 parameters are determined to reproduce the the experimental data for \(^{80}\text{Ge}\):
\(\varepsilon_{d\pi} = 0.315\text{MeV}, \varepsilon_{d\nu} = 1.080\text{MeV}, \kappa = -0.150\text{ MeV}, \chi_{\pi} = -1.200, \chi_{\nu} = 0.900, \omega_{\pi\pi} = 0.063\text{ MeV}, \) and \(\lambda_1 = \lambda_3 = 0.800\text{ MeV}\) in \(^{80}\text{Ge}\). We numerically diagonalized the IBM-2 Hamiltonian by the NPBOS code \([46]\). The obtained IBM wave functions are our starting point and can be used to compute the electromagnetic properties.

### III. RESULTS AND DISCUSSION

The calculated results of the low-lying energy levels compared with the corresponding available experimental data are shown in Fig. 1. The experimental values are taken from Refs. \([18, 30]\). Each panel includes two different parts: the yrast band up to the \(6^+\) state and the nonyrast, low-spin, positive-parity levels. Fig. 1 shows that the calculated energy
FIG. 1: The energy scheme for low-lying states of $^{80}$Ge with positive-parity, the left panel shows the experimental data and the right panel denotes the calculated results from IBM-2. The experimental energy levels are taken from Ref. 18, 30.

levels from IBM-2 for the low-lying states agree with experimental data very well. The experimental energy levels of the yrast states are reproduced precisely by the theoretical calculations. At the same time, the calculated ordering of the nonyrast states consists with the experimental data, although the theoretical prediction of the $2^+_2$ state is lower than the experimental value. In particular, the calculated result of the first excited $0^+_2$ state is almost equal to the experimental data of 639(1) keV, which is lower than the $2^+_1$ state.

The energy ratio $R_{4/2} = E(4^+_1)/E(2^+_1)$ of $2^+_1$ state and $4^+_1$ is a well-known observable to measure the extent of quadrupole deformation. $R_{4/2}$ reaches the limit of 2.00 for the U$_{\pi\nu}(5)$ dynamical symmetry (the spherical vibration), 2.50 for the O$_{\pi\nu}(6)$ dynamical symmetry (the $\gamma$-unstable rotor), and the maximum 3.33 for the SU$_{\pi\nu}(3)$ dynamical symmetry (the axially rotor) [39]. The experimental result of $R_{4/2}$ is 2.64 for $^{80}$Ge, and the calculated value is 2.68. Both the experimental and theoretical value of $R_{4/2}$ predict that $^{80}$Ge has a mostly typical $\gamma$-soft triaxial feature. At the same time, from Fig. 1 one can see that both of the experimental and calculated energy levels of the $2^+_2$ state lie below the corresponding $4^+_1$ state, and they form a pair of $2^+_2$ and $4^+_1$, which also indicates $^{80}$Ge exhibits a characteristic of O$_{\pi\nu}(6)$ symmetry because the fact of the second $2^+$ state below the $4^+_1$ state is the manifestation of a $\gamma$-soft spectrum. However, the appearance of the $4^+_1$ state at an energy of nearly 2.5 times that of $2^+_1$ level alone does not uniquely determine the O$_{\pi\nu}(6)$ structure [47]. In the
The $\pi\nu(6)$ limit of IBM-2, the $2^+_2$ state and $4^+_1$ state belong to the $\tau = 2$ multiplet, but the $6^+_1$, $0^+_2$, $3^+_1$, $4^+_2$ states belong to $\tau = 3$ multiplet. As a consequence, the $0^+_2$ state ($\tau = 3$ multiplet) locates at much higher energy level and can decay to the second $2^+$ state with $\tau = 2$ rather than to the $2^+_1$ state. But the $0^+_2$ state of $^{80}$Ge actually lies at lower energy level than the $4^+_1$, $2^+_2$ states, even below the $2^+_1$ level both in experiment and theory. From the above discussion it is clear that the $^{80}$Ge is not a typical $\gamma$-soft nucleus, at least deviates from the pure $\pi\nu(6)$ limit, although the yrast states have approximately the $\gamma$-soft rotor picture. More importantly, it is an important evidence of shape coexistence if a deformation state occurs near the almost spherical ground state or much lower than the first-excited $2^+$ state $[5]$. Therefore, both the experimental and theoretical energy levels imply that shape coexistence occurs in $^{80}$Ge.

$B(E2)$ transition probability and its branching ratios can also give important information on the nuclear structure. Unfortunately, only absolute $B(E2)$ transition strengths of $2^+_1 \rightarrow 0^+_1$, and $2^+_2 \rightarrow 0^+_1$ in $^{80}$Ge have been observed so far. However, one can further explore shape coexistence in $^{80}$Ge based on the other key sensitive quantities $[47, 48]$. To calculate the $E2$ transition strengths, the effective charges of proton and neutron bosons were determined to reproduce the experimental data of $B(E2, 2^+_1 \rightarrow 0^+_1)$ and $B(E2, 2^+_2 \rightarrow 0^+_1)$. By fitting the experimental data of the $B(E2, 2^+_1 \rightarrow 0^+_1) = 200(26) \ e^2\text{fm}^4$, we obtain the $e_\nu = 13.9$, and $e_\pi = 6 \ e\text{fm}^2$ for $^{80}$Ge. The effective charge of neutron boson is much larger than proton boson’s probably due to the effect of a valid proton midshell around $Z = 34$. For the protons, the state space beyond the $Z = 20$ shell closure and up to $Z = 32 - 34$ is indeed made of the full $pf$ shell $[4]$, which might lead to a very valid proton subshell closure at $Z = 32$ and 34. The other reason is that the parameters $e_\nu$ and $e_\pi$ incorporate a (length)$^2$ factor, simultaneously, the neutrons are occupying higher shells than protons in $^{80}$Ge $[49]$.

TABLE I: The experimental and calculated $B(E2)$ values (in $e^2\text{fm}^4$) and $\rho^2(E0)$ values in $^{80}$Ge are listed. We take the experimental data from Refs. $[18, 30]$.

|               | $B(E2, 2^+_1 \rightarrow 0^+_1)$ | $B(E2, 2^+_2 \rightarrow 0^+_1)$ | $\rho^2(E0, 0^+_2 \rightarrow 0^+_1)$ |
|---------------|----------------------------------|----------------------------------|-------------------------------------|
| Exp.          | 200(26)                          | 23(7)                            |                                     |
| Cal.          | 200.0                            | 21.3                             | 0.001                               |

The calculated $B(E2)$ transition strengths comparing with the recent experimental values
are listed in Table I. The theoretical calculations are in consistence with the experimental data quite nicely. The calculated transition strength of $B(E2, 2^+_2 \rightarrow 0^+_1)$ is in agreement with the experimental value within the experimental uncertainty. In the IBM, the key sensitive quantities $R_1 = B(E2, 2^+_2 \rightarrow 2^+_1)/B(E2, 2^+_2 \rightarrow 0^+_1)$ and $R_2 = B(E2, 2^+_2 \rightarrow 0^+_1)/B(E2, 2^+_2 \rightarrow 2^+_1)$ are usually considered as one of the most crucial available structure indicators [47] to distinguish the dynamical symmetry limits. The U(5) symmetry is realized when $R_1 = 1$ and $R_2 = 0$, while it is the O(6) symmetry when $R_1 = 0.79$ and $R_2 = 0.07$ [50]. The calculation result of $B(E2, 2^+_2 \rightarrow 2^+_1)$ is 187.13 $e^2$fm$^4$. The calculated $R_1$ and $R_2$ are 0.94 and 0.11 respectively, which are much closer to O(6) symmetry. Obviously, the predict ratios of $R_1$ and $R_2$ are consistent with the character of the yrast states, but do not match with the feature of the nonyrast states. Thus, the above result has confirmed the existence of shape coexistence in $^{80}$Ge.

One can obtain valuable information from the electric monopole transition strengths $\rho^2(E0)$ on the excited $0^+$ states of different features coexisting in the same nucleus [51–53]. In order to further understand the properites of shape coexistence in $^{80}$Ge, we calculate $\rho^2(E0, 0^+_2 \rightarrow 0^+_1)$ transition strength. Since the experimental data about $E0$ transition is still scarce in $^{80}$Ge, we choose the parameters $\beta_{0\nu}$ and $\beta_{0\pi}$ as the values derived in ref. [54] from a detailed analysis of $E0$ transition in O(6)-like nuclei, namely, $\beta_{0\nu} = 0$ and $\beta_{0\pi} = 0.20$ $e$fm$^2$. The calculated transition strength is also listed in Table I. Because the $E0$ operator is proportional to $\hat{n}_d$, no $E0$ transitions occur in the U(5) dynamical limit [55]. Within the O(6) limit, the selection rules are $\Delta \sigma = 0, \pm 2$, $\Delta \tau = 0$, so the $0^+_2 \rightarrow 0^+_1$ transition is forbidden [56]. The present calculation value of $\rho^2(E0)$ is comparable with those observed in $^{72}$Ge, $^{102}$Pd and $^{120}$Xe [51, 56], which implies that different nuclear shapes coexist in $^{80}$Ge.

On the other hand, the choice of the parameters to reproduce the properties of the low-lying states might give us a clue to understand shape coexistence in nuclei. Recalling the best fit parameters in the present calculation, we found that the $\varepsilon_{d\rho}$ is much larger than $\kappa_{\pi\nu}$, which reflects that $^{80}$Ge mainly exhibits the character of spherical vibration or U(5) dynamical symmetry. At the same time, the structure parameter of the quadrupole operator $\chi_{\pi} = -1.200$, and $\chi_{\nu} = 0.900$ were adopted in this paper. The sum $\chi_{\pi} + \chi_{\nu} = -0.3$ indicates that $^{80}$Ge nucleus is close to the O(6) dynamical symmetry or $\gamma$-soft in IBM. As mentioned above, combing the information from the best fit parameters and the properties of the low-lying states, the physical picture from the IBM point of view is clear: both collective
spherical and $\gamma$-soft vibration structures coexist in $^{80}$Ge. Microscopically, the recent shell model calculations in the $pf gd$ model space suggest that the tensor force play an important role on setting up a shape coexistence environment and the tensor effect changes dynamically with orbital occupation and spin $^{[57]}$. For $^{80}$Ge, many neutrons occupying $g_{9/2}$ orbital reduce the proton $f_{7/2} - f_{5/2}$ gap, much more particle-hole excitations occur over the gap, which lead to much stronger shell evolution $^{[58]}$. The other studies have clearly shown that the $\nu s_{1/2}$ shell drops in energy and becomes almost degenerate with the lower-lying $\nu d_{5/2}$ shell at $Z = 32$ $^{[18]}$. Therefore, neutron pair excitations across $N = 50$ are likely to include both orbitals which result in significant configuration mixing. The deformation and change of shell structure driven by the combination of the tensor force and changes of major configurations can occur and can enhance shape coexistence in $^{80}$Ge.

IV. CONCLUSION

In summary, we discussed the properties of the low-lying states, especially the relevant shape coexistence in $^{80}$Ge near one of most neutron-rich doubly magic nuclei at $N = 50$ and $Z = 28$. Based on the different relative energy for $d$ proton boson neutron boson, i.e., $\varepsilon_{d\pi} \neq \varepsilon_{d\nu}$, the low-lying positive parity states consist with experimental data very well in IBM-2. More importantly, the calculated energy level of the first excited $0^+_2$ state, which associates with the shape coexistence phenomenon, is almost equal to the experimental data at 659 keV, which is lower than the $2^+_1$ state. Both the experimental and theoretical energy spectrum indicated the shape coexistence exists in $^{80}$Ge structure, although the value of the characteristic ratio of $R_{4/2}$ suggests that $^{80}$Ge is a mostly typical $\gamma$-soft triaxial feature.

The calculated $B(E2)$ transition strengths agreed with the experimental data within the experimental uncertainty. The key sensitive quantities do not match with the feature of the nonyrast states, which demonstrates the different property of $^{80}$Ge compared with its energy spectrum structure. Therefore, the above result has just confirmed the existence of shape coexistence in $^{80}$Ge. Furthermore, the $\rho^2(E0, 0^+_2 \rightarrow 0^+_1)$ transition strength has been calculated. The theoretical result of $\rho^2(E0, 0^+_2 \rightarrow 0^+_1)$ transition also indicates the different nuclear shapes exist in the same time in $^{80}$Ge.

The best fit values of $\varepsilon_{d\rho}$ is much larger than $\kappa_{\pi \nu}$, which implies that $^{80}$Ge has the property of U(5) dynamical symmetry. While the sum $\chi_\pi + \chi_\nu = -0.3$ indicates that $^{80}$Ge is
close to the $\gamma$-soft or O(6) dynamical symmetry in IBM. Combing the results of the best fit parameters in present calculations and the properties of the low-lying states, we found that both collective spherical and $\gamma$-soft vibration structures coexist in $^{80}$Ge from the IBM point of view. However, the experimental information on $E2$ and $E0$ transition from $0^+_2$ state to other states in $^{80}$Ge is still scarce. As a result, our theoretical analysis for the associated $0^+_2$ level might be incomplete. More theoretical calculations and experimental investigations on these aspects are needed.

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