Supersymmetry Anomalies, the Witten Index and the Standard Model

John Dixon

Center for Theoretical Physics
Physics Department
Texas A & M University
College Station, Texas 77843

Abstract

The supersymmetric standard model (SSM) contains a wealth of potential supersymmetry anomalies, all of which occur in the renormalization of composite operators of the theory. The coefficients of the weak-E.M. superanomalies should be related to the Witten indices of the neutrino and photon superfields, and the coefficients of the strong superanomalies should be related to the Witten indices of the gluon and photon superfields. Assuming the coefficients are non-zero, the superanomalies break supersymmetry in observable states. However the neutral Higgs particles should remain in a supermultiplet because the Higgs supermultiplet is not coupled to any massless superfield in the SSM. Assuming that the overall Witten index is non-zero, supersymmetry is broken by superanomalies and yet the vacuum remains supersymmetric. This means that the cosmological constant is naturally zero after supersymmetry breaking, even beyond perturbation theory.

1 Introduction

All $N=1$ supersymmetric theories in four dimensions that have chiral matter have potential supersymmetry anomalies that can arise in the renormalization of certain composite operators constructed in the theories. However the

---

1Supported in part by the U.S. Dept of Energy, under grant DE-FG05-91ER40633, Email: dixon@phys.tamu.edu
action itself, at least in non-exotic theories, cannot have any supersymmetry anomalies. These statements are consequences of the structure of the BRS cohomology of supersymmetry which has been explored in references [1] [2] [3] [4] [5] [6]. This situation is more or less the opposite of what happens for the gauge and gravitational anomalies.

The basic principle obtained from the cohomology is very simple. The simplest composite operators that can develop superanomalies are the composite antichiral spinor superfields and the corresponding anomalies are products of the elementary chiral superfields in the theory, without any derivatives. The details of this structure depend on the details of the representations and the gauge structure of the theory. An examination of a number of examples in simple models indicates that the coefficients of such superanomalies are all very likely to be zero, except possibly in the case where the theory has non-Abelian gauge fields as well as chiral superfields, spontaneous breaking of the gauge symmetry (but not necessarily the supersymmetry), complex representations of the gauge group preventing bare mass terms for the chiral matter, and plenty of massless superfields after gauge symmetry breaking [7]. The standard supersymmetric model has all these properties, so one is led to look at it for superanomalies, after trying simpler possibilities without success.

The ideal would be to simply calculate the coefficients for some examples to answer this question. But this is hard. It seems to be necessary to first pick a model with a lot of structure, then find some examples of potentially superanomalous operators, then test the BRS identities in the relevant sectors, and finally remove any superanomalies that vanish by the field equations. All possible examples involve a lot of operator mixing, and a calculation needs to be guided by some theoretical ideas about how the anomalies could arise. It now seems clear that the Witten index and consequently zero mass superfields should play an essential role, as we discuss in section 6 below.

As stated above, the composite operators that are susceptible to superanomalies are the composite antichiral spinor superfields, which we will denote by $\Phi_\alpha$. The antichiral constraint is $\mathcal{D}_\alpha \Phi_\beta = 0$ where $\mathcal{D}_\alpha = \frac{\partial}{\partial \theta^\alpha} + \frac{1}{2} \sigma^\mu_{\alpha\beta} \overline{\theta}^\beta \partial_\mu$ is the superspace chiral derivative. We can define components by the expansion $\hat{\Phi}_\alpha(x, \theta, \overline{\theta}) = \phi(\overline{\theta})_\alpha + W(\overline{\theta})_{\alpha\beta} \overline{\theta}^\beta + \frac{1}{2} \overline{\theta}^2 \chi(\overline{\theta})_\alpha$ where the chirally translated
spacetime variable $y^\mu = x^\mu + \frac{1}{2} \theta^a \sigma^\mu_{\alpha\beta} \bar{\theta}^\beta$ satisfies the equation $D_\alpha y^\mu = 0$. Of course each of these components $\phi, W, \xi$ is a composite field made from the component fields of the elementary superfields in the theory.

The BRS cohomology tells us that these operators are subject to superanomalies of the form $m^2 \bar{S} c_\alpha$ where $c_\alpha$ is the commuting spacetime independent supersymmetry ghost, and the composite antichiral scalar superfield $\bar{S}$ (satisfying the constraint $D_\alpha \bar{S} = 0$) consists of a product of the elementary antichiral superfields of the theory, with no derivatives or superderivatives in it. The mass parameter of the theory is $m$, and $q$ is a power determined by matching the dimensions of $\Phi_\alpha$ and $\bar{S} c_\alpha$ for each case. Evidently $\Phi_\alpha$ and $m^2 \bar{S} c_\alpha$ must have identical values for any conserved quantities such as lepton number, charge, baryon number, mass dimension, hypercharge, isospin and colour charge. Hypercharge and weak isospin are spontaneously broken down to $U(1)_{EM}$ of course in the SSM, which must be properly taken into account.

To calculate the superanomalies, one would couple these operators to a nonComposite antichiral spinor superfield source $\Phi'_\alpha$ with components $\Phi'_\alpha = \phi'_\alpha + W'_{\alpha\beta} \bar{\theta}^\beta + \bar{u}^2 \chi'_\alpha$ in the form $\text{Action}_{\Phi} = \int d^4x d^2\theta \Phi^\alpha \Phi_\alpha$. Then the anomaly would appear in the form

$$\delta \Gamma_{\Phi} = e_1 \int d^4x d^2\theta \Phi^\alpha c_\alpha m^2 \bar{S}$$

where $\Gamma$ is the one-loop 1PI generating functional and $e_1$, the coefficient of the anomaly, is calculable in one-loop perturbation theory.

Let us treat the composite superfields $\Phi_\alpha$ and $e_1 m^2 \bar{S}$ for a moment as if they were elementary fields with canonical mass dimensions. Then we could eliminate the supersymmetry anomalies by defining a new local nilpotent transformation on these elementary fields as follows:

$$\delta' \Phi_\alpha = \bar{S} c_\alpha + (c^\beta Q_\beta + \bar{c}^{\dot{\beta}} Q_{\dot{\beta}}) \Phi_\alpha$$

$$\delta' \Phi'_\alpha = (c^\beta Q_\beta + \bar{c}^{\dot{\beta}} Q_{\dot{\beta}}) \Phi'_\alpha$$

$$\delta' S = (c^\beta Q_\beta + \bar{c}^{\dot{\beta}} Q_{\dot{\beta}}) S$$

With this transformation the anomaly ceases to be an anomaly because it is now the variation of a local term:

$$\int d^4x d^2\theta \Phi^\alpha c_\alpha \bar{S} = \delta' \int d^4x d^2\theta \Phi^\alpha \Phi_\alpha$$
However now one has to deal with the new term in the ‘anomalous’ super-BRS algebra in (2). The zero ghost charge invariants of the transformations (2),(3),(4) include an explicit dependence on $\theta$. For example we have:

$$\delta' \text{Action}_{\text{anom}} = \delta' \int d^4x d^4\theta \Phi'_\alpha \bar{D}^2[\Phi^\alpha + \theta^\alpha \overline{\mathcal{S}}] = 0$$  \hspace{1cm} (6)

Presumably, this supersymmetry violating $\theta$-dependence means that the anomalies induce an effective action for the composite fields that violates supersymmetry—the term (6) is supersymmetry violating because $\delta \text{Action}_{\text{anom}} \neq 0$ where $\delta$ is the usual supersymmetry BRS operator. It may be possible to restore the supersymmetry by some non-local field redefinition, but then one is led back to the supersymmetry anomaly. Surely, this is what one should expect—the whole point of doing local BRS cohomology is to find out what cannot be eliminated by local field redefinitions and local renormalizations. An anomaly violates the supersymmetry in a way that is not removable and which should be physically significant. This leads to the conjecture that supersymmetry breaks itself in observable states through calculable supersymmetry anomalies that arise at one loop in perturbation theory.

An interesting problem arises here—to fully analyze the quadratic form of this action to see what the mass eigenstates are and to see whether they do break supersymmetry in an interesting way. This question is important even if all the coefficients of the superanomalies are zero to all orders of perturbation theory, because anomalies are also of interest in the non-perturbative regime—as is well known in instanton physics.

Sometimes we will allow $\hat{\Phi}'\alpha$ to have weak isospin or U(1) hypercharge. For example for the case of example L2 of Table 3, we get the expansion:

$$\delta \Gamma = c_1 m^4 \int d^4x c_\alpha [\chi^{\bar{i}\alpha} \mathcal{T}_i + W^{i\alpha\beta} \mathcal{T}_{i\beta} + \phi^{i\alpha} \mathcal{T}_i]$$  \hspace{1cm} (7)

Allowing these indices on $\Phi'$ would probably be wrong if we wanted to treat the source $\Phi'_\alpha$ as a fundamental field, but $\Phi'_\alpha$ in the present context is not a fundamental field.

## 2 Supersymmetric Standard Model

The chiral superfields of the supersymmetric standard model (SSM) are summarized in Table [3]. But we will not add any explicit supersymmetry breaking
terms, the hope being that such terms are not actually needed because sup
persymmetry breaks itself in physical states in a way that does not show up in the fundamental action. It is worth noting that if explicit ('soft' or other
wise) supersymmetry breaking terms are put into the action, they are very likely to create interesting problems because of the superanomalies discussed here.

The electromagnetic charge is $Q = I_3 + 2Y$. The components of a chiral superfield are denoted $\hat{J}_i = J_i + \theta^\alpha j^{\alpha}_i + \bar{\theta}^2 \tilde{J}_i$. Here $\hat{J}_i$ is the chiral superfield, $J_i$ is the scalar, $j_\alpha$ the spinor and $\tilde{J}_i$ the auxiliary component. Weak isospin is $I$, weak hypercharge is $Y$, baryon number is $B$ and lepton number is $L$.

In Table 1, $c = 1, 2, 3$ is an index labelling the 3 of color SU(3), $i = 1, 2$ is an isospin $\frac{1}{2}$ index, $\alpha = 1, 2$ is a two component Weyl spinor index, $\hat{Q}^{i\alpha} = \hat{U}^{\alpha}_L$ and $\hat{Q}^{2\alpha} = \hat{D}^{\alpha}_L$ are the left-handed chiral superfields that contain the up and down quarks, and $\hat{Q}^{ia}$, which transforms as a $\mathbf{3}$ of SU(3), is the complex conjugate of $\hat{Q}^{ia}$. The Yukawa interactions are:

$$
\int d^4x d^2\theta \{ g_1 \hat{J}_i \hat{L}_i \hat{E}_R + g_2 \hat{J}_i \hat{K}_i \hat{M} + g_3 m^2 \hat{M} + g_4 \hat{Q}^{ia} \hat{K}_i \hat{U}^{\alpha}_R + g_5 \hat{Q}^{ia} \hat{J}_i \hat{D}^{\alpha}_R \} \quad (8)
$$

Our notation for the (gluon) vector superfields, in the Wess-Zumino gauge, is $\hat{G}^a = [\theta^\alpha \sigma^{\mu \nu}_\alpha \tilde{G}_\mu + \theta^2 \tilde{G}_\alpha + \tilde{g}^\bar{\alpha} \bar{G}_\alpha + \theta \theta^2 G]_a$, and the gauge-covariant chiral spinor superfield is $\hat{G}^{a\bar{b}} = [\mathcal{D}^2 (e^{-\hat{G}} \mathcal{D}_a e^{\hat{G}})]^a_b$, which has the expansion $\hat{G}^{a\bar{b}} = [g_a + \sigma^{\mu \nu}_a \bar{G}_\mu \bar{G}_\nu + \bar{G} \theta_a + \sigma^{\mu \nu}_a \mathcal{D}_\mu \hat{g}^\bar{b}]_b$. The $SU(2) \; \hat{W}^i_j$ and $U(1) \; \hat{\Psi}^i$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Field & $SU(3)$ & $\mathcal{I}$ & $\mathcal{Y}$ & $B$ & $L$ & Field & $SU(3)$ & $\mathcal{I}$ & $\mathcal{Y}$ & $B$ & $L$ \\
\hline
$\hat{J}_i$ & 0 & $\frac{1}{2}$ & $-1$ & 0 & 0 & $\hat{J}_i$ & 0 & $\frac{1}{2}$ & 1 & 0 & 0 \\
$\hat{K}_i$ & 0 & $\frac{1}{2}$ & +1 & 0 & 0 & $\hat{K}_i$ & 0 & $\frac{1}{2}$ & -1 & 0 & 0 \\
$\hat{M}$ & 0 & 0 & 0 & 0 & 0 & $\hat{M}$ & 0 & 0 & 0 & 0 & 0 \\
$\hat{L}_i$ & 0 & $\frac{1}{2}$ & -1 & 0 & 1 & $\hat{L}_i$ & 0 & $\frac{1}{2}$ & 1 & 0 & -1 \\
$\hat{E}_R$ & 0 & $\frac{1}{2}$ & +2 & 0 & -1 & $\hat{E}_R$ & 0 & $\frac{1}{2}$ & -2 & 0 & 1 \\
$\hat{Q}^{i\alpha}$ & 3 & $\frac{1}{2}$ & $\frac{1}{3}$ & $\frac{1}{3}$ & 0 & $\hat{Q}^{i\alpha}$ & 3 & $\frac{1}{2}$ & -$\frac{1}{3}$ & -$\frac{1}{3}$ & 0 \\
$\hat{U}^{\alpha}_R$ & 3 & 0 & $\frac{-4}{3}$ & $\frac{-1}{3}$ & 0 & $\hat{U}^{\alpha}_R$ & 3 & 0 & $\frac{4}{3}$ & $\frac{1}{3}$ & 0 \\
$\hat{D}^{\alpha}_R$ & 3 & 0 & $\frac{-2}{3}$ & $\frac{-1}{3}$ & 0 & $\hat{D}^{\alpha}_R$ & 3 & 0 & $\frac{-2}{3}$ & $\frac{1}{3}$ & 0 \\
\hline
\end{tabular}
\caption{Chiral superfields of the SSM}
\end{table}
gauge fields are defined with a similar notation. Usually the $e^V$, $e^G$ and $e^W$ factors will be omitted with the understanding that they are to be supplied as necessary to make the gauge invariance work properly.

As is discussed in [8], there are three ‘Higgs’ fields $J$, $K$ and $M$, so that the gauge symmetry naturally breaks from $SU(2) \times U(1)$ down to $U(1)_{E.M}$. This set of Higgs fields also gives both the up and down quark superfields a mass, which is not possible in the SSM with just one weak isospin doublet Higgs multiplet. The electrically neutral superfields $\hat{J}$ and $\hat{K}$ develop equal VEVs

$$m_h = \langle J^1 \rangle = \langle K^2 \rangle$$

where $h = \frac{M}{\sqrt{g_2}}$. This gauge breaking leaves the vacuum energy zero of course, because supersymmetry is not spontaneously broken in this theory—all auxiliary fields have zero VEV.

## 3 Equations of Motion

One more point must be made before we turn to the superanomalous operators. An ‘anomaly’ is not an anomaly if it vanishes when the equations of motion are used. More accurately, one needs to know the BRS cohomology of the full BRS operator which includes the variations of the antifields, which include the equations of motion of the fields. The antifields are introduced into the action with terms like $\int d^4x d^2\theta \hat{\Phi}^{i\alpha} c_\alpha \hat{\Phi}^{i\alpha} \hat{L}_i$, where $\hat{L}_i$ is the anti-super-field that couples to the left-handed lepton superfield $\hat{\bar{L}}_i = (\hat{\bar{N}}_L, \hat{\bar{E}}_L)$. In Table 2 we show these equations of motion, leaving off the supersymmetry and gauge parts of the variation. Now it is easy to see that these variations do remove parts of the cohomology space that are proportional to the equations of motion. For example we have

$$A' = \int d^4x d^2\theta \Phi^{i\alpha} c_\alpha \hat{\bar{L}}_i \hat{\Phi}^{i\alpha} = \delta\{ \int d^4x d^2\theta \Phi^{i\alpha} c_\alpha \hat{\bar{L}}_i - \int d^4x d^2\theta \Phi^{i\alpha} c_\alpha \hat{\bar{L}}_i \}$$

Without the use of the equation of motion, $A'$ would appear to be a supersymmetry anomaly of the theory. It is clear that one must calculate any real supersymmetry anomalies by removing all such spurious anomalies [7]. Of course we must translate the Higgs fields by their vacuum expectation values when using these equations of motion: $\hat{J}^i \rightarrow m h \delta_1^i + \hat{J}^i$; $\hat{K}^i \rightarrow m h \delta_1^i + \hat{K}^i$. The equations of motion of the antichiral source $\Phi'_\alpha$ should not be used, because it is not a fundamental field and it does not propagate.
Variation = Equation of Motion

\[ \delta \hat{J}_i = \mathcal{D}^2 \hat{J}_i + g_1 L_i E_R + g_2 K_i M - g_5 Q^c_i D_{Re} \]

\[ \delta \hat{L}_i = \mathcal{D}^2 \hat{L}_i - g_1 \hat{J}_i E_R \]

\[ \delta \hat{E}_R = \mathcal{D}^2 \hat{E}_R + g_1 \hat{J}_i \hat{L}_i \]

\[ \delta \hat{K}_i = \mathcal{D}^2 \hat{K}_i - g_2 \hat{J}_i \hat{M} - g_4 \hat{Q}^c \hat{U}_{Re} \]

\[ \delta \hat{M}_i = \mathcal{D}^2 \hat{M} + g_2 \hat{J}_i \hat{K}_i + g_3 m^2 \]

\[ \delta \hat{Q}^{ic} = \mathcal{D}^2 \hat{Q}^{ic} + g_4 \hat{K}_i \hat{U}_{Re} + g_5 \hat{J}_i \hat{D}_{Re} \]

\[ \delta \hat{U}_{Re} = \mathcal{D}^2 \hat{U}_{Re} + g_4 \hat{Q}^{ic} \hat{K}_i \]

\[ \delta \hat{D}_{Re} = \mathcal{D}^2 \hat{D}_{Re} + g_5 \hat{Q}^{ic} \hat{J}_i \]

Table 2: Equations of Motion of Chiral Superfields

4 Composite Superoperators and their Anomalies

A selection of candidate superanomalous operators and their anomalies for the standard model can be found in Table 3. The labels L,M,B,V,H stand for leptons, mesons, baryons, vector bosons and Higgs respectively. One of these particles can be found in each set. Sometimes it will be in the operator and sometimes in the anomaly.

A comparison of Tables 2 and 3 reveals that each of the above anomalies does represent a class that does not vanish by the equations of motion. It is also clear that to find the coefficient of the anomaly, one needs to evaluate a large set of diagrams and compare coefficients to see if they can all be eliminated by the equations in Table 2 or not. So it looks much harder to calculate these coefficients than it is for the gauge anomalies, where mixing is not much of a problem at all. For example, one can add the term \( \int d^4x \epsilon^{\mu\nu\lambda\sigma} V_\mu A_\nu \partial_\lambda V_\sigma \) to the action for the VVA triangle anomaly to conserve one type of current at the expense of another, but this is not a major mixing problem like we get for the superanomalies.

The leptons occur in the anomaly in operators L1,L2 and L3. The most important thing to notice is that the electron superfields \( E_L, E_R \) and the neutrino superfield \( N_L \) appear linearly in these expressions (see equation (4) above). This implies that the elementary lepton fields are being mixed with...
the composite field $\Phi_\alpha$ in a non-supersymmetric way, which will result in some mass splitting of both multiplets (see equation (6) above), assuming that the coefficients of the anomalies are not zero.

In the quark model the $\Pi^-$ would naturally be produced by such operators as:

$$\Pi^- = \bar{\tau}_{La} \hat{\theta}_R^{a}$$

(10)

The corresponding anomaly would be M1. The expansion of $\hat{\theta}_{ic} \hat{\theta}_R^{ic}$ superfield gives the pion operator above (plus other isospins and terms for supersymmetry) as its $F$ term.

Let us now write down some typical interpolating operators that could be used to create a proton:

$$P_{La} = u^a_{L\beta} \hat{d}^b_{\beta} L \; u^c_{La} \epsilon_{abc}$$

(11)

$$P_{Ma} = \bar{u}^a_{R\beta} \hat{d}^b_{\beta} R \; u^c_{La} \epsilon_{abc}$$

(12)

$$P_{Ra} = \bar{u}^a_{R\beta} \hat{d}^b_{\beta} R \; \epsilon_{abc}$$

(13)

$$P_{Ma} = u^a_{L\beta} \hat{d}^b_{\beta} R \; u^c_{La} \epsilon_{abc}$$

(14)

| Label | Operator | Anomaly | Dim $L$ $B$ $Y$ $I$ |
|-------|----------|---------|-------------------|
| L1    | $D^2[\hat{j}^i D_\alpha \hat{E}_\alpha]$ | $m^2 \hat{T}^i_{\alpha}$ | $\frac{7}{2}$ $-1$ $0$ $1$ $\frac{1}{2}$ |
| L2    | $D^2[\hat{j}^i \hat{W}^i_{\alpha} \hat{E}_\alpha]$ | $m^2 \hat{T}^i_{\alpha}$ | $\frac{7}{2}$ $-1$ $0$ $1$ $\frac{1}{2}$ |
| L3    | $D^2[\hat{j}^i \hat{W}^i_{\alpha} L_j]$ | $m^2 \hat{E}_R \epsilon_{\alpha}$ | $1$ $0$ $-2$ $0$ |
| M1    | $D^2[\hat{U}_{Ra} D_\alpha \hat{Q}_c]$ | $m^2 \hat{U}_{ic} \hat{D}_R \epsilon_{\alpha}$ | $0$ $0$ $-1$ $\frac{1}{2}$ |
| B1    | $D^2[\hat{Q}^a_{\alpha} \hat{Q}^b_{\beta} \hat{D}_a \hat{Q}^c_{\epsilon} \epsilon_{abc}]$ | $\hat{U}_R^{a} \hat{D}_R^{b} \hat{D}_R^{c} \epsilon_{abc} \epsilon_{\alpha}$ | $\frac{7}{2}$ $0$ $1$ $1$ $\frac{1}{2}$ |
| B2    | $D^2[\hat{Q}^a_{\alpha} \hat{Q}^b_{\beta} \hat{K}^{ij} \hat{D}_a \hat{Q}^c_{\epsilon} \epsilon_{abc}]$ | $m^2 \hat{U}_R^{a} \hat{D}_R^{b} \hat{D}_R^{c} \epsilon_{abc} \epsilon_{\alpha}$ | $\frac{7}{2}$ $0$ $1$ $2$ $0$ |
| B3    | $D^2[\hat{Q}^a_{\alpha} \hat{D}_R^{a} \hat{D}_R^{b} \hat{D}_R^{c} \epsilon_{abc}]$ | $m^2 \hat{Q}^a_{\alpha} \hat{Q}^b_{\beta} \hat{Q}^c_{\epsilon} \epsilon_{abc} \epsilon_{\alpha}$ | $\frac{7}{2}$ $0$ $1$ $1$ $\frac{1}{2}$ |
| B4    | $D^2[\hat{D}_R^{a} \hat{D}_R^{b} \hat{D}_R^{c} \epsilon_{abc}]$ | $m^2 \hat{Q}^a_{\alpha} \hat{Q}^b_{\beta} \hat{Q}^c_{\epsilon} \epsilon_{abc} \epsilon_{\alpha}$ | $\frac{7}{2}$ $0$ $1$ $2$ $0$ |
| V1    | $D^2[\hat{W}^i_{\alpha} \hat{J}_j]$ | $m^2 \hat{T}^i_{Lj} \hat{E}_R \epsilon_{\alpha}$ | $0$ $0$ $-1$ $\frac{1}{2}$ |
| H1    | $D^2[\hat{W}^i_{\alpha} \hat{K}_j]$ | $m^2 \hat{T}^i_{j} \epsilon_{\alpha}$ | $\frac{7}{2}$ $0$ $0$ $1$ $\frac{1}{2}$ |

Table 3: Some candidates for superanomalies in the SSM
The supersymmetric generalization of the familiar composite operators (11) are to be found in B1 and B2. For (13) we have B4. The third $Q'$ in the anomalies must involve new flavours so that the expression does not vanish. $C_R'$ is the right-handed charm superfield, also needed so the anomaly does not vanish. Evidently there is a lot of structure here that involves the flavour symmetry.

Now let us consider the weakly interacting vector boson supermultiplets. Here the relevant observable states occur both in the operator (V1) and in the anomaly (H1). Recall that in the SSM, the Higgs chiral superfields combine to form part of the supermultiplet that contains the vector bosons. Evidently, there is a lot of analysis to be done in this sector too. Also, the cohomology of the vector superfields is not yet known completely, so there could be even more anomalies than the ones that are discussed here. But it does not appear necessary to find more anomalies to have a good chance of breaking supersymmetry in all the observed particles.

It is worthwhile to note that if there are superanomalies in the lowest dimension operator with given quantum numbers, this will spread to all the higher dimensional operators with those quantum numbers. The reason is that higher dimensional operators will necessarily mix with the lower dimension operators with the same quantum numbers, with coefficients which involve the appropriate power of mass. So supersymmetry breaking could not be rescued by using a different interpolating operator.

Finally we note that the true neutral massive Higgs supermultiplet $\hat{H} = \frac{1}{\sqrt{2}}[\hat{J}^1 - \hat{K}_1]$ could also appear in an anomaly in a term like H1. However the coefficient here may be zero, for reasons discussed in section (3) below.

5 Higher Spins

One possible expression for the spin $\frac{3}{2}$ operator corresponding to the $\Delta^{++}$ member of the baryon decuplet is

$$\Delta_{\alpha\beta\gamma}^{++} = u^a_L u^b_L u^c_L \epsilon_{abc}$$

Is there any chance of splitting the mass of this particle from those of its superpartners in analogy with the above?

Evidently, this operator occurs in the following spin $\frac{3}{2}$ superfield:

$$\Phi^{(ijk)}_{(\alpha\beta\gamma)} = D^i_{(\alpha} \hat{Q}^{ia} D^j_{\beta} \hat{Q}^{jb} D^k_{\gamma} \hat{Q}^{kc} \epsilon_{abc}$$

9
where the round brackets in \((ijk)\) and \((\alpha\beta\gamma)\) denote symmetrization.

Is there any anomaly available for this case? It appears quite likely. In \cite{[6]}, it was shown that there are polynomials in the BRS cohomology space for spinor superfields of all spins. The anomaly for spin \(\frac{3}{2}\) would have spin 1. Reference \cite{[6]} needs to be extended to the case where gauge fields are included to ensure that this works out properly.

The \(\rho^+\) meson corresponds to operators such as:

\[
\rho^+_{\alpha\beta} = u^c_{a\alpha} \bar{d}_{Lc\beta} \tag{17}
\]

Particles such as these will presumably appear in the spin 1 anomalies of spin \(\frac{3}{2}\) superfields and perhaps also in M1 of Table \cite{[4]}.

Therefore it seems quite possible that there are superanomalies available for all the myriad of hadronic resonances that have been observed. The question of course is whether nature makes use of them, and that is most easily determined for the low spin cases discussed above.

## 6 The Witten Index and Superanomalies

The lepton supermultiplets, the \(0^-\) meson supermultiplets and the Higgs supermultiplets all occur as possible superanomaly terms for various composite antichiral spinor superfields, chosen to have the right quantum numbers. On the other hand, the \(1^+\) baryons are naturally written as composite antichiral spinor superfields that can develop superanomalies with the appropriate quantum numbers. The vector boson supermultiplets also can get mixed with other operators through superanomalies.

Are there any guidelines that can aid one to find a way through all the complexity of the necessary calculations? If there are superanomalies, where do they come from? Fortunately, there is a very plausible and simple conjecture that naturally arises here, although there is still much mystery connected with the details.

The well-known gauge and gravitational anomalies of quantum field theory are associated with index theorems for the Dirac operator \cite{[9]}. The anomaly measures the appropriate index, which is equal to the number of left handed fermion zero modes minus the number of right handed fermion zero modes in the relevant background gauge or gravitational field.
If the superanomalies discussed above do exist with non-zero coefficients, it would be natural to expect that they also measure an index. The obvious choice for such an index is the Witten index \cite{10}, which measures the number of bosonic zero modes minus the number of fermionic zero modes in these supersymmetric theories. For present purposes, the Witten index would have to be defined for each relevant zero mass supermultiplet separately, since different superanomalies would evidently measure different Witten indices.

Thus the Witten indices of the neutrino and photon superfields must be associated with the weak-EM type superanomalies L1, L2, L3, V1 and H1 of Table 3. The hadronic superanomalies M1 and B1-B4 would be associated with the Witten indices of the gluon and photon superfields.

It is well known and also clear from the Feynman diagrams that the supersymmetry BRS identities work by cancellation of the fermionic and bosonic modes. From the results of \cite{10}, we know that these can be unmatched only for zero mass fields at zero momentum and in fact frequently are unmatched in these very circumstances. In the context of looking for non-perturbative spontaneous breaking of supersymmetry, this was a disappointing result, because a theory with a non-zero Witten index cannot break supersymmetry spontaneously \cite{10}, even beyond perturbation theory– it necessarily must possess some zero-energy state. However, these very theories that cannot break supersymmetry spontaneously are the theories which are most likely to possess superanomalies, precisely because they have non-zero Witten indices.

This leads to a puzzle. In reference \cite{11} it is demonstrated that ‘the invariance of the vacuum is the invariance of the world’. This means that if the vacuum is supersymmetric, and it must be if the Witten index is non-zero, then all the states must be in supermultiplets, even non-perturbatively. If this theorem applies to the present situation, it means that superanomalies can not possibly break the supersymmetry in a theory with non-zero Witten index, even if one goes beyond perturbation theory.

However, one of the explicit assumptions in the theorem of reference \cite{11} is that there are no zero mass particles, and of course these are precisely the origin of the superanomalies. So theorem \cite{11} does not apply to our situation! It appears to be possible to have a supersymmetric vacuum without having the states in supermultiplets precisely in the case when the total Witten index is non-zero. There is a bonus involved in having the vacuum supersymmetric of course–the cosmological constant is exactly zero even after supersymmetry
breaking, and this statement survives even beyond perturbation theory!

But the comparison with the gauge anomalies raises another issue. The zero modes are related to the gauge and gravitational anomalies in situations where the topology of the gauge fields is non-trivial, e.g. in the field of an instanton. So perhaps we should be looking at topologically non-trivial field theories for supersymmetry anomalies—particularly in theories where the topology of the Higgs sector chiral superfields is non-trivial, as in the ‘t Hooft-Polyakov magnetic monopole solution for example. In fact, there is evidence that supersymmetry has problems with zero modes in this very situation \[12\].

All of the above discussion applies only to the rigidly supersymmetric theory of course. Obviously there is no unitarity problem created by the superanomalies in this case, because it is only a rigid symmetry that is violated.

What happens when we couple the theory to a supergravity theory derived from a superstring? On the positive side, this will generate a zero cosmological constant, so long as supersymmetry is not spontaneously or explicitly broken. Does the above mechanism for supersymmetry breaking survive with a zero cosmological constant? In fact it does appear to have a chance of doing so, because we do not want to couple operators like those in Table 3 to the supergravity theory anyway—it would be wrong to couple, say, both a fundamental proton operator and the quarks to gravity in the same theory. The basic idea would be that the scale of gravity is so huge compared to the scale of the leptons and hadrons that we should be dealing with the rigid theory when considering the observable particles anyway. It may therefore be comforting, rather than disappointing, that it is very difficult to couple antichiral spinor superfields $\Phi_\alpha$ to supergravity theory. So the supergravity would still be unitary and supersymmetric—this appears to imply that the gravitino should be massless.

It is remarkable that for nearly all of the examples in this standard model, there are in fact zero mass fields which do contribute to the Feynman diagrams which can take one from the operator to the superanomaly. This is the main reason why it seems best to look for these anomalies in the context of the standard model. Simpler examples that look as promising are hard to find and not so interesting. But note that the Higgs particle does not couple to any massless field in the rigidly supersymmetric theory. So the Higgs supermultiplet may not be involved in any superanomaly. Assuming
that the scheme works as outlined above, this has the surprising consequence that the Higgs particles should remain in a supermultiplet.

Acknowledgments

It is a pleasure to recall many useful remarks made to me in the course of this work by R. Arnowitt, M. Duff, S. Ferrara, R. Khuri, U. Lindstrom, R. Minasian, H. Pois, S. Polyakov, C. Pope, J. Rahmfeld, P. Ramond, M. Rocek, E. Sezgin, K. Stelle, C. Thorn, and E. Witten.

References

[1] Piguet, O.; Sibold, K., and Schweda, M: Nucl. Phys. B174 (1980) 183, Preprint UGVA-DPT 1992/5-763.

[2] Brandt, F., Dragon, N., Kreuzer, M.: Phys. Lett. B231, 263-270 (1989), Nucl. Phys. B332, 224-249 (1990), Nucl. Phys. B332, 250-260 (1990), Nucl. Phys. B340, 187-224 (1990).

[3] Brandt, F.: Nucl. Phys. B392, 928 (1993), Preprints NIKHEF-H-93-22 and NIKHEF-H 93-12.

[4] Dixon, J. A.: Commun. Math. Phys. 140, 169-201 (1991); Class. Quant. Grav. 7, 1511-1521 (1990); Phys. Rev. Lett. 67, 797-800 (1991).

[5] Dixon, J. A., Minasian, R.: Preprint CPT-TAMU-13/93 (hep-th/9304035).

[6] Dixon, J. A., Minasian, R., Rahmfeld, J.: Preprint CTP-TAMU-20/93 (hep-th/9308013).

[7] Dixon, J. A.: Preprint CPT-TAMU-46/93 (hep-th/9309254).

[8] Haber, H. and Kane, G.: Phys. Rep. 117 (1985) 76.

[9] Alvarez-Gaume, L. and Ginsparg, P.: Ann. Phys. 161 (1985) 423.

[10] Witten, E.: Nucl. Phys. B202 (1982) 253.

[11] Coleman, S.: J. Math. Phys. 7 (1966) 787.

[12] Casher, A. and Shamir, Y.: Phys. Lett. B274 (1992) 381.