Impact of CP phases on stop and sbottom searches

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Abstract

We study the decays of top squarks ($\tilde{t}_{1,2}$) and bottom squarks ($\tilde{b}_{1,2}$) in the Minimal Supersymmetric Standard Model (MSSM) with complex parameters $A_t$, $A_b$, $\mu$ and $M_1$. We show that including the corresponding phases substantially affects the branching ratios of $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ decays in a large domain of the MSSM parameter space. We find that the branching ratios can easily change by a factor of 2 and more when varying the phases. This could have an important impact on the search for $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ and the determination of the MSSM parameters at future colliders.
1 Introduction

The experimental studies of supersymmetric (SUSY) particles will play an important role at future colliders. Studying the properties of the 3rd generation sfermions will be particularly interesting because of the effects of the large Yukawa couplings. The lighter of their mass eigenstates may be the lightest charged SUSY particle and they could be investigated at an $e^+e^-$ linear collider. Moreover, they could also be copiously produced in the decays of heavier SUSY particles. Several phenomenological studies on SUSY particle searches have been performed in the Minimal Supersymmetric Standard Model (MSSM) with real SUSY parameters. Analyses of the decays of the 3rd generation sfermions $\tilde{t}_{1,2}, \tilde{b}_{1,2}, \tilde{\tau}_{1,2}$ and $\tilde{\nu}_\tau$ in the MSSM with real parameters have been made in Refs. [3, 4], and phenomenological studies of production and decays of the 3rd generation sfermions at future $e^+e^-$ linear colliders in Ref. [5].

In general, however, some of the SUSY parameters may be complex, in particular the higgsino mass parameter $\mu$, the gaugino mass parameters $M_{1,2,3}$ and the trilinear scalar coupling parameters $A_f$ of the sfermions $\tilde{f}$. The SU(2) gaugino mass parameter $M_2$ can be chosen real after an appropriate redefinition of the fields. The experimental upper bounds on the electric dipole moments (EDMs) of electron, neutron and the $^{199}$Hg and $^{205}$Tl atoms may pose severe restrictions on the phases of the SUSY parameters, though they are model dependent. In the constrained MSSM the phase of $\mu$ turns out to be restricted to the range $|\varphi_\mu| \lesssim 0.1$ if all SUSY masses are in the TeV range. On the other hand, in more general models, such as those with lepton-flavour violation, no restriction on the phase $\varphi_\mu$ from the electron EDM at one-loop level is obtained. Restrictions arising due to two-loop contributions to the EDMs are less severe.

Therefore, in a complete phenomenological analysis of production and decays of the SUSY particles one has to take into account that $A_f, \mu$ and $M_1$ can be complex. The most direct and unambiguous way to determine the imaginary parts of the complex SUSY parameters could be done by measuring relevant CP-violating observables. For example, for the case of sfermion decays CP-violating [9, 10] and T-violating [11] observables have been proposed. On the other hand, also the CP-conserving observables depend on the phases of the complex parameters, because in general the mass-eigenvalues and the couplings of the SUSY particles involved are functions of the underlying complex parameters. For example, the decay branching ratios of the Higgs bosons depend strongly on the complex phases of the $\tilde{t}$ and $\tilde{b}$ sectors [12, 13, 14], while those of the staus $\tilde{\tau}_{1,2}$ and $\tau$-sneutrino $\tilde{\nu}_\tau$ can be quite sensitive to the phases of the stau and gaugino-higgsino sectors. Also the Yukawa couplings of the third generation sfermions are sensitive to the SUSY phases at one-loop level [16]. Furthermore, explicit CP violation in the Higgs sector can be induced by $\tilde{t}$ and $\tilde{b}$ loops if the parameters $A_t, A_b$ and $\mu$ are complex [12, 17, 18]. It is found [12, 14, 17, 19] that these loop effects could significantly influence the phenomenology of the Higgs boson sector.

In this article we study the effects of the phases of $A_t, A_b, \mu$ and $M_1$ on the decay
branching ratios of the $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ with $\tilde{q}_1$ ($\tilde{q}_2$) being the lighter (heavier) squark. We take into account the explicit CP violation in the Higgs sector. We will show that the influence of the phases can be quite strong in a large domain of the MSSM parameter space. This could have an important impact on the search for $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ and on the determination of the MSSM parameters at future colliders.

In section 2 we discuss the SUSY CP phase dependences of masses, mixings and couplings. Section 3 contains our numerical investigation on the CP phase dependence of the branching ratios of $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ decays. In section 4 we present our conclusions.

## 2 SUSY CP Phase Dependences of Masses, Mixings and Couplings

In the MSSM the squark sector is specified by the mass matrix in the basis $(\tilde{q}_L, \tilde{q}_R)$ with $\tilde{q} = \tilde{t}$ or $\tilde{b}$ [20]

$$
\mathcal{M}_q^2 = \begin{pmatrix}
  m_{\tilde{q}^2} & a_{q}m_q \\
  a_qm_q & m_{\tilde{q}^R}
\end{pmatrix}
$$

(1)

with

$$
m_{\tilde{q}^L}^2 = M_Q^2 + m_Z^2 \cos 2\beta (I_3^q - e_q\sin^2\theta_W) + m_{\tilde{q}}^2,
$$

(2)

$$
m_{\tilde{q}^R}^2 = M_{\tilde{u}, \tilde{d}}^2 + m_Z^2 \cos 2\beta e_q\sin^2\theta_W + m_{\tilde{q}}^2,
$$

(3)

$$
a_qm_q = \begin{cases}
  (A_t - \mu^* \cot\beta) m_t \ (\tilde{q} = \tilde{t}) \\
  (A_b - \mu^* \tan\beta) m_b \ (\tilde{q} = \tilde{b})
\end{cases}
$$

(4)

$$
a_qm_q e^{i\varphi_q} \ (-\pi < \varphi_q \leq \pi).
$$

(5)

Here $I_3^q$ is the third component of the weak isospin and $e_q$ the electric charge of the quark $q$. $M_Q, u, d$ and $A_t, b$ are soft SUSY-breaking parameters, $\mu$ is the higgsino mass parameter, and $\tan\beta = v_2/v_1$ with $v_1$ ($v_2$) being the vacuum expectation value of the Higgs field $H_1^0$ ($H_2^0$). As the relative phase $\xi$ between $v_1$ and $v_2$ is irrelevant in our analysis, we adopt the $\xi = 0$ scheme [17]. We take $A_q$ ($q = t, b$) and $\mu$ as complex parameters: $A_q = |A_q| e^{i\varphi_{A_q}}$ and $\mu = |\mu| e^{i\varphi_{\mu}}$ with $-\pi < \varphi_{A_q, \mu} \leq \pi$. Diagonalizing the matrix (1) one gets the mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$

$$
\begin{pmatrix}
  \tilde{q}_1 \\
  \tilde{q}_2
\end{pmatrix} = R_q \begin{pmatrix}
  \tilde{q}_L \\
  \tilde{q}_R
\end{pmatrix} = \begin{pmatrix}
  e^{i\varphi_q} \cos\theta_q & \sin\theta_q \\
  -\sin\theta_q & e^{-i\varphi_q} \cos\theta_q
\end{pmatrix} \begin{pmatrix}
  \tilde{q}_L \\
  \tilde{q}_R
\end{pmatrix}
$$

(6)

with the masses $m_{\tilde{q}_1}$ and $m_{\tilde{q}_2}$ ($m_{\tilde{q}_1} < m_{\tilde{q}_2}$), and the mixing angle $\theta_q$

$$
m_{\tilde{q}_{1,2}}^2 = \frac{1}{2}(m_{\tilde{q}^L}^2 + m_{\tilde{q}^R}^2 \pm \sqrt{(m_{\tilde{q}^L}^2 - m_{\tilde{q}^R}^2)^2 + 4|m_qm_{\tilde{q}}|^2}),
$$

(7)

$$
\theta_q = \tan^{-1}(|m_qm_{\tilde{q}}|/(m_{\tilde{q}^L}^2 - m_{\tilde{q}^R}^2)) \ (-\pi/2 \leq \theta_q \leq 0).
$$

(8)
The $\tilde{q}_L - \tilde{q}_R$ mixing is large if $|m_{\tilde{q}_L}^2 - m_{\tilde{q}_R}^2| \lesssim |a_q m_q|$, which may be the case in the $\tilde{t}$ sector due to the large $m_t$ and in the $\tilde{b}$ sector for large $\tan \beta$ and $|\mu|$. From Eqs. (4), (7) and (8) we see that $m_{\tilde{h}_1,2}^2$ and $\theta_q$ depend on the phases only through a term $\cos(\varphi_A + \varphi_\mu)$. This phase dependence of the $\tilde{t}$ ($\tilde{b}$) sector is strongest if $|A_t| \simeq |\mu| \cot \beta$ ($|A_b| \simeq |\mu| \tan \beta$).

The properties of charginos $\tilde{\chi}_i^\pm$ ($i = 1, 2$; $m_{\tilde{\chi}_1^\pm} < m_{\tilde{\chi}_2^\pm}$) and neutralinos $\tilde{\chi}_j^0$ ($j = 1, \ldots, 4$; $m_{\tilde{\chi}_1^0} < \ldots < m_{\tilde{\chi}_4^0}$) are determined by the parameters $M_2, M_1, \mu$ and $\tan \beta$. We assume that the gluino mass $m_{\tilde{g}}$ is real. We write the U(1) gaugino mass $M_1$ as $M_1 = |M_1| e^{i\varphi_1}$ ($-\pi < \varphi_1 \leq \pi$). Inspired by the gaugino mass unification we take $|M_1| = (5/3) \tan^2 \theta_W M_2$ and $m_{\tilde{g}} = (\alpha_s(m_{\tilde{g}})/\alpha_2) M_2$. In the MSSM Higgs sector with explicit CP violation the mass-eigenvalues and couplings of the neutral and charged Higgs bosons, $H_1^0, H_2^0, H_3^0$ ($m_{H_1^0} < m_{H_2^0} < m_{H_3^0}$) and $H^\pm$, including Yukawa and QCD radiative corrections, are fixed by $m_{H^\pm}, \tan \beta, \mu, m_t, m_b, M_Q, M_U, M_D, A_t, A_b, |M_1|, M_2$, and $m_{\tilde{g}}$. The neutral Higgs mass eigenstates $H_1, H_2$ and $H_3$ are mixtures of CP-even and CP-odd states ($\phi_1, \phi_2$ and $a$) due to the explicit CP violation in the Higgs sector. For the radiatively corrected masses and mixings of the Higgs bosons we use the formulae of Ref. [17].

Here we list possible important decay modes of $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$:

\begin{align*}
\tilde{t}_1 & \rightarrow t\tilde{g}, t\tilde{\chi}_1^0, b\tilde{\chi}_1^+, \tilde{b}_1 W^+, \tilde{b}_1 H^+ \\
\tilde{t}_2 & \rightarrow t\tilde{g}, t\tilde{\chi}_2^0, b\tilde{\chi}_2^+, \tilde{t}_1 Z^0, \tilde{b}_1 W^+, \tilde{t}_1 H^+, \tilde{b}_1 H^+ \\
\tilde{b}_1 & \rightarrow b\tilde{g}, b\tilde{\chi}_1^0, t\tilde{\chi}_1^-, \tilde{t}_1 W^-, \tilde{t}_1 H^- \\
\tilde{b}_2 & \rightarrow b\tilde{g}, b\tilde{\chi}_2^0, t\tilde{\chi}_2^-, \tilde{t}_1 Z^0, \tilde{b}_1 W^-, \tilde{b}_1 H^+, \tilde{t}_1 H^-.
\end{align*}

The decays into a gauge or Higgs boson in (9)-(12) are possible in case the mass splitting between the squarks is sufficiently large [31, 41]. The explicit expressions of the widths of the decays (9)-(12) in case of real SUSY parameters are given in [21]. Those for complex parameters can be obtained by using the corresponding masses and couplings (mixings) from Refs.[17, 20, 21] and will be presented elsewhere [22].

The phase dependence of the widths stems from that of the involved mass-eigenvalues, mixings and couplings among the interaction-eigenfields. Here we summarize the most important features of the phase dependences.

(1) $\tilde{q}_i$ ($\tilde{q} = \tilde{t}, \tilde{b}$) sectors:

(a) The mass-eigenvalues $m_{\tilde{q}_{1,2}}$ are sensitive to the phases ($\varphi_A, \varphi_\mu$) via $\cos(\varphi_A + \varphi_\mu)$ if and only if $|a_q m_q| \sim (m_{\tilde{q}_L}^2 + m_{\tilde{q}_R}^2)/2$ and $|A_q| \sim |\mu| C_q$ (with $C_t = \cot \beta$ and $C_b = \tan \beta$).

(b) The $\tilde{q}$-mixing angle $\theta_q$ (given by $\tan 2\theta_q = 2|a_q m_q|/(m_{\tilde{q}_L}^2 - m_{\tilde{q}_R}^2)$) is sensitive to ($\varphi_A, \varphi_\mu$) via $\cos(\varphi_A + \varphi_\mu)$ if and only if $2|a_q m_q| \gtrsim |m_{\tilde{q}_L}^2 - m_{\tilde{q}_R}^2|$ and $|A_q| \sim |\mu| C_q$. 

4
(c) The \( \tilde{q} \)-mixing phase \( \varphi_{\tilde{q}} \) in Eq.\((5)\) is sensitive to \((\varphi_{A_q}, \varphi_\mu)\), \( \varphi_{A_q} \) and \( \varphi_\mu \) if \(|A_q| \sim |\mu|C_q, |A_q| \gg |\mu|C_q \) and \(|A_q| \ll |\mu|C_q \), respectively. For large squark mixing the term \( \propto \sin 2\theta_{\tilde{q}} \cos \varphi_{\tilde{q}} \) can result in a large phase dependence of the decay widths \(\text{[22]}\) (see Eq.\((6)\)).

(II) \( \tilde{\chi}^0_i \) and \( \tilde{\chi}^\pm_j \) sectors:

(a) \( m_{\tilde{\chi}^0_i} \) (\( i = 1, \ldots, 4 \)) and the \( \tilde{\chi}^0 \)-mixing matrix are sensitive [insensitive] to the phases \((\varphi_1, \varphi_\mu)\) for small [large] \( \tan \beta \).

(b) \( m_{\tilde{\chi}^\pm_{1,2}} \) and the \( \tilde{\chi}^\pm \)-mixing matrices are sensitive [insensitive] to \( \varphi_\mu \) for small [large] \( \tan \beta \).

(III) \( H^\pm \) sector:

This sector is independent of the phases.

(IV) \( H_i^0 \) sector:

(a) \( m_{H_i^0} \) (\( i = 1, 2, 3 \)) are sensitive [insensitive] to the phase sums \( \varphi_{A_t,b} + \varphi_\mu \) for small [large] \( \tan \beta \) \(\text{[17]}\).

(b) In general the \( H_i^0 \)-mixing matrix (a real orthogonal \( 3 \times 3 \) matrix \( O_{ij} \)) is sensitive to \( \varphi_{A_t,b} + \varphi_\mu \) for any \( \tan \beta \) \(\text{[17]}\).

(V) The couplings among the interaction-eigenfields:

(a) For the decays into fermions and gauge bosons in Eqs.\((9)-(12)\), they are gauge couplings and/or Yukawa couplings \((h_{t,b})\), and are independent of the phases at tree-level.

(b) For the decays into Higgs bosons in Eqs.\((9)-(12)\), the \( \tilde{q}_L-\tilde{q}_R \) (\( \tilde{q}_L-\tilde{q}_L, \tilde{q}_R-\tilde{q}_R \)) couplings are dependent on (independent of) the phases \( \varphi_{A_t}, \varphi_{A_b} \) and \( \varphi_\mu \):

\[
\begin{align*}
C(\tilde{b}_L^i \tilde{t}_R H^-) & \sim \sin \beta \, h_t (A_t^* \cot \beta + \mu) \\
C(\tilde{t}_L^i \tilde{b}_R H^+) & \sim \cos \beta \, h_b (A_b^* \tan \beta + \mu)
\end{align*}
\]

(13)

\[
\begin{align*}
C(\tilde{t}_L^i \tilde{t}_R \phi_1) & \sim h_t \mu \\
C(\tilde{t}_L^i \tilde{t}_R \phi_2) & \sim h_t A_t^* \\
C(\tilde{b}_L^i \tilde{b}_R \phi_1) & \sim h_b A_b^* \\
C(\tilde{b}_L^i \tilde{b}_R \phi_2) & \sim h_b \mu \\
C(\tilde{b}_L^i \tilde{b}_R a) & \sim \cos \beta \, h_b (A_b^* \tan \beta + \mu)
\end{align*}
\]

(14)-(20)

with

\[
\begin{align*}
h_t & = g m_t / (\sqrt{2} m_W \sin \beta) \\
h_b & = g m_b / (\sqrt{2} m_W \cos \beta).
\end{align*}
\]

(21)-(22)
Here $\phi_i = O_{ij} H_j^0$ ($i = 1, 2$) and $a = O_{3j} H_j^0$ are the CP-even and CP-odd neutral Higgs bosons, respectively [17].

According to items (I)-(V) we expect that the widths (and hence the branching ratios) of the decays (9)-(12) are sensitive to the phases (\(\varphi_{At}, \varphi_{Ab}, \varphi_\mu, \varphi_1\)) in a large region of the MSSM parameter space.

3 Numerical Results

Now we turn to the numerical analysis of the $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ decay branching ratios. We calculate the widths of all possibly important two-body decay modes of Eqs.(9)-(12). Three-body decays are negligible in the parameter space under study. In order to improve the convergence of the perturbative expansion [11, 23] we calculate the tree-level widths by using the corresponding tree-level couplings defined in terms of “effective” MSSM running quark masses $m_{t,b}^{\text{run}}$ (i.e. those defined in terms of the effective running Yukawa couplings $h_{ij}^{\text{run}} \propto m_{t,b}^{\text{run}}$). For the kinematics, e.g., for the phase space factor we use the on-shell masses obtained by using the on-shell (pole) quark masses $M_{t,b}$. We take $M_t = 175$ GeV, $M_b = 5$ GeV, $m_t^{\text{run}} = 150$ GeV, $m_b^{\text{run}} = 3$ GeV, $m_Z = 91.2$ GeV, $\sin^2 \theta_W = 0.23$, $m_W = m_Z \cos \theta_W$, $\alpha(m_Z) = 1/129$, and $\alpha_s(m_Z) = 0.12$ (with $\alpha_s(Q) = 12 \pi/(33 - 2n_f) \ln(Q^2/A_{\text{run}}^2)$), $n_f$ being the number of quark flavors). In order not to vary too many parameters we fix $|A_t| = |A_b| \equiv |A|$ and $M_2 = 300$ GeV, i.e. $m_{\tilde{g}} = 820$ GeV. In the following we will assume that $m_{\tilde{g}} > m_{\tilde{t}_{L-R}}$ so that the decays $\tilde{t}_{1,2} \rightarrow t\tilde{g}$ and $\tilde{b}_{1,2} \rightarrow b\tilde{g}$ are kinematically forbidden. In our numerical study we take $\tan \beta, m_{\tilde{i}_1}, m_{\tilde{i}_2}, m_{\tilde{b}_1}, |A|, |\mu|, \varphi_{At}, \varphi_{Ab}, \varphi_\mu, \varphi_1$ and $m_{H^+}$ as input parameters, where $m_{\tilde{i}_{1,2}}$ and $m_{\tilde{b}_1}$ are the on-shell squark masses. Note that for a given set of the input parameters we obtain two solutions for $(M_{\tilde{Q}}, M_{\tilde{t}})$ corresponding to the two cases $m_{\tilde{i}_L} \geq m_{\tilde{i}_R}$ and $m_{\tilde{i}_L} < m_{\tilde{i}_R}$ from Eqs. (11-14) and (7) with $m_t$ replaced by $M_t$. In the plots we impose the following conditions in order to respect experimental and theoretical constraints:

(i) $m_{\tilde{g}} > 103$ GeV, $m_{\tilde{t}_{L-R}} > 50$ GeV, $m_{\tilde{t}_{L-R}} > 100$ GeV, $m_{\tilde{t}_{L-R}} > m_{\tilde{t}_{L-R}}$,

(ii) $|A_t|^2 < 3 (M_{\tilde{Q}}^2 + M_{\tilde{U}}^2 + M_{\tilde{D}}^2)$, and $|A_b|^2 < 3 (M_{\tilde{Q}}^2 + M_{\tilde{D}}^2 + M_{\tilde{U}}^2)$, where $m_1^2 = (m_{H^+}^2 + m_Z^2 \sin^2 \theta_W) \sin^2 \beta - \frac{1}{2} m_Z^2$ and $m_2^2 = (m_{H^+}^2 + m_Z^2 \sin^2 \theta_W) \cos^2 \beta - \frac{1}{2} m_Z^2$,

(iii) $\Delta \rho (\tilde{t} - \tilde{b}) < 0.0012$ [24] using the formula of [25],

(iv) $2.0 \cdot 10^{-4} < B(b \rightarrow s\gamma) < 4.5 \cdot 10^{-4}$ [26] assuming the Kobayashi-Maskawa mixing also for the squark sector.

Condition (i) is imposed to satisfy the experimental mass bounds from LEP [24]. (ii) is the approximate necessary condition for the tree-level vacuum stability [28]. (iii) constrains
$\mu$ and $\tan\beta$ (in the squark sector). For the calculation of the $b \to s\gamma$ width we use the formula of [29] including the $O(\alpha_s)$ corrections as given in [30].

As already mentioned the experimental upper limits on the EDMs of electron, neutron, 199Hg and 205Tl strongly constrain the SUSY CP phases [2]. One interesting possibility for evading these constraints is to invoke large masses (much above the TeV scale) for the first two generations of the sfermions [31], keeping the third generation sfermions relatively light ($\lesssim 1$ TeV). In such a scenario ($\varphi_1$, $\varphi_\mu$) and the CP phases of the third generation ($\varphi_{A_t}$, $\varphi_{A_b}$, $\varphi_{A_\mu}$) are practically unconstrained [31]. We adopt this scenario. Furthermore, we have checked that the electron and neutron EDM constraints at two-loop level [8] are fulfilled in the numerical examples studied in this article.

In Fig.1 we plot the contours of the branching ratios of the $\tilde{t}_1$ decays $B(\tilde{t}_1 \to t\tilde{\chi}_1^0)$ and $B(\tilde{t}_1 \to b\tilde{\chi}_1^+)$ in the $|A| - |\mu|$ plane for $\varphi_{A_t} = 0$ and $\pi/2$ with tan $\beta = 8$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (400,700,200)$ GeV, $\varphi_{A_b} = \varphi_1 = 0$, $\varphi_\mu = \pi$, and $m_{H^+} = 600$ GeV in the case $m_{\tilde{t}_{1L}} \geq m_{\tilde{t}_{1R}}$. In the case $m_{\tilde{t}_{1L}} < m_{\tilde{t}_{1R}}$ we have obtained similar results. As expected, these branching ratios are sensitive to the phase $\varphi_{A_t}$ in a sizable region of the $|A| - |\mu|$ plane. The difference between $\varphi_{A_t} = 0$ (Figs. 1a and c) and $\varphi_{A_t} = \pi/2$ (Figs. 1b and d) can be explained by item (I). Especially the strong $\varphi_{A_t}$ dependence of the decay width $\Gamma(\tilde{t}_1 \to b\tilde{\chi}_1^+)$ is a result of item (I)(c) [32]. Moreover, the $t$-mixing angle $\theta_t$ is sensitive to $\cos(\varphi_{A_t} + \varphi_\mu)$ for large $|A_t| = |A|$ with $|A_t| \sim |\mu|/8$, which is a consequence of item (I)(b).

In Fig.2 we plot the contours of the $\tilde{t}_1$ decay branching ratios $B(\tilde{t}_1 \to t\tilde{\chi}_1^0)$ and $B(\tilde{t}_1 \to b\tilde{\chi}_1^+)$ in the $\varphi_{A_t} - \varphi_\mu$ plane for tan $\beta = 8$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (400,700,200)$ GeV, $|A| = 800$ GeV, $|\mu| = 500$ GeV, $\varphi_{A_b} = \varphi_1 = 0$, and $m_{H^+} = 600$ GeV in the case $m_{\tilde{t}_{1L}} \geq m_{\tilde{t}_{1R}}$. In the case $m_{\tilde{t}_{1L}} < m_{\tilde{t}_{1R}}$ we have obtained similar results. One sees that these branching ratios depend quite strongly on the CP phases $\varphi_{A_t}$ and $\varphi_\mu$, as follows from item (I).

In Fig.3 we show the $\varphi_\mu$ dependence of the $\tilde{t}_1$ decay branching ratios $B(\tilde{t}_1 \to t\tilde{\chi}_1^0)$ and $B(\tilde{t}_1 \to b\tilde{\chi}_1^+)$ for $\varphi_1 = 0$ and $\pi/2$ with tan $\beta = 5$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (400,700,200)$ GeV, $|A| = 800$ GeV, $|\mu| = 500$ GeV, $\varphi_{A_t} = \varphi_{A_b} = 0$, and $m_{H^+} = 600$ GeV in the case $m_{\tilde{t}_{1L}} \geq m_{\tilde{t}_{1R}}$. For $m_{\tilde{t}_{1L}} < m_{\tilde{t}_{1R}}$ we have obtained similar results. As can be seen in Fig.3 the branching ratios are very [somewhat] sensitive to $\varphi_\mu$ [\varphi_1] for small tan $\beta$=5. For this choice of parameters the masses and mixings of $\tilde{t}$, $\tilde{b}$, $\tilde{\chi}^0$ and $\tilde{\chi}^\pm$ are sensitive to $\varphi_\mu$, while only the masses and mixings of $\tilde{\chi}^0$ are sensitive to $\varphi_1$, as can be seen from items (I) and (II). In general, according to items (I), (II) and (V), the $\tilde{t}_1$ decay branching ratios tend to be sensitive [insensitive] to $\varphi_{A_t}$, $\varphi_{A_b}$, $\varphi_\mu$, and $\varphi_1$ for large tan $\beta$ ($\gtrsim 15$) in case the width of the $b\tilde{t}_1 H^+$ mode in Eq. (9) (which can be sensitive to $(\varphi_{A_t}, \varphi_{A_b})$ as seen from Eqs. (6) [13] [41]) is relatively small. We have confirmed this.

In Fig.4 we show the $\varphi_{A_t}$ dependence of the $\tilde{t}_2$ decay branching ratios for $\varphi_\mu = 0$ and $\pi/2$ with tan $\beta=8$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (400,700,200)$ GeV, $|A| = 800$ GeV, $|\mu| = 500$ GeV, $\varphi_{A_b} = \varphi_1 = 0$, and $m_{H^+} = 600$ GeV in the case $m_{\tilde{t}_{1L}} \geq m_{\tilde{t}_{1R}}$. The case $m_{\tilde{t}_{1L}} < m_{\tilde{t}_{1R}}$ leads to similar results. We see that the $\tilde{t}_2$ decay branching ratios are very sensitive to $\varphi_{A_t}$ and depend significantly on $\varphi_\mu$. This can be expected from items (I), (II), (IV) and (V)(b).
From item (I)(c) [(IV)(b) and (V)(b)] we expect that the widths of the decays $\tilde{t}_2 \rightarrow b\tilde{\chi}^{+}_{1,2}$ [$\tilde{t}_1 H^0_k$ and $\tilde{b}_{1,2} H^+$] in Eq. (10) can be sensitive to $\varphi_{A_t}$ [$\varphi_{A_{t,b}}$ and $\varphi_\mu$] for large $\tan \beta (\gtrsim 15)$ if they are kinematically allowed. We have confirmed this.

For $\tilde{b}_{1,2}$ decays we have obtained results similar to those for the $\tilde{t}_{1,2}$ decays. Here we show just a few typical results for them. In Fig. 5 we show the $\varphi_{Ab}$ dependence of the $\tilde{b}_{1,2}$ decay branching ratios for $\tan \beta = 30$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (200,700,400)$ GeV, $(|A_t|, |\mu|) = (800,700)$ GeV, $\varphi_{A_t} = \varphi_\mu = \pi$, $\varphi_1 = 0$, and $m_{H^+} = 180$ GeV in the case $m_{\tilde{t}_L} \geq m_{\tilde{t}_R}$. For $m_{\tilde{t}_L} < m_{\tilde{t}_R}$ our results are similar. We find that the $\tilde{b}_{1,2}$ decay branching ratios are very sensitive to $\varphi_{Ab}$ as expected from items (IV)(b) and (V)(b). The main reason is that the decay widths for $\tilde{b}_{1,2} \rightarrow \tilde{t}_1 H^-$ strongly depend on $\varphi_{Ab}$ (and $\varphi_\mu$) for large $\tan \beta$ (see Eqs. (13,14)). This explains also the tendency of the $\varphi_{Ab}$ dependence of the branching ratios for the other decays shown in Fig. 5. For small $\tan \beta \sim 8$ we expect that the $\tilde{b}_{1,2}$ decay branching ratios can be somewhat sensitive to $\varphi_{A_{t,b}}$ and $\varphi_\mu$ (see items (I) and (V)(b)). Similarly, we expect that the decay widths of $\tilde{t}_{1,2} H^-$ can be fairly sensitive to $\varphi_{A_{t,b}}$ and $\varphi_\mu$. We have confirmed this.

4 Conclusions

We have calculated the branching ratios of the two-body decays of $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ and studied their CP phase dependence within the MSSM with complex parameters $A_t$, $A_b$, $\mu$ and $M_1$. We have shown that the effect of the SUSY CP phases on the branching ratios can be quite strong in a large domain of the MSSM parameter space. The $\varphi_{Ab}$ dependence of the $\tilde{b}_{1,2}$ decay branching ratios is mainly due to the $\varphi_{Ab}$ dependence of their couplings to the Higgs bosons. In the case of the $\tilde{t}_{1,2}$ decays the branching ratios depend on $\varphi_{A_t}$ also via the $t$-mixing phase $\varphi_t \approx \varphi_{A_t}$ for $|A_t| \gg |\mu|/\tan \beta$, in addition to the $\varphi_{A_t}$ dependence of the mixing angle $\theta_{\tilde{t}}$ and their couplings to the Higgs bosons. Some of the branching ratios can change by a factor of 2 or more when varying the phases. This CP phase dependence of the branching ratios could have an important impact on the search for $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ and on the determination of the MSSM parameters at future colliders.

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Figure Captions

Figure 1: Contours of the $\tilde{t}_1$ decay branching ratios $B(\tilde{t}_1 \to t\tilde{\chi}_1^0)$ (a,b) and $B(\tilde{t}_1 \to b\tilde{\chi}_1^+)$ (c,d) in the $|A| - |\mu|$ plane for $\varphi_{\tilde{t}_1} = 0$ (a,c) and $\pi/2$ (b,d) with $\tan\beta = 8$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (400,700,200)$ GeV, $\varphi_{\tilde{A}_t} = \varphi_1 = 0$, $\varphi_{\tilde{\mu}} = \pi$, and $m_{H^+} = 600$ GeV in the case $m_{\tilde{t}_L} \geq m_{\tilde{t}_R}$. The blank areas are excluded by the conditions (i) to (iv) given in the text.

Figure 2: Contours of the $\tilde{t}_1$ decay branching ratios $B(\tilde{t}_1 \to t\tilde{\chi}_1^0)$ (a) and $B(\tilde{t}_1 \to b\tilde{\chi}_1^+)$ (b) in the $\varphi_{\tilde{A}_t} - \varphi_{\tilde{\mu}}$ plane for $\tan\beta = 8$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (400,700,200)$ GeV, $|A| = 800$ GeV, $|\mu| = 500$ GeV, $\varphi_{\tilde{A}_b} = \varphi_1 = 0$, and $m_{H^+} = 600$ GeV in the case $m_{\tilde{t}_L} \geq m_{\tilde{t}_R}$.

Figure 3: $\varphi_{\tilde{\mu}}$ dependence of the $\tilde{t}_1$ decay branching ratios $B(\tilde{t}_1 \to t\tilde{\chi}_1^0)$ and $B(\tilde{t}_1 \to b\tilde{\chi}_1^+)$ for $\varphi_1 = 0$ (solid lines) and $\pi/2$ (dashed lines) with $\tan\beta = 5$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (400,700,200)$ GeV, $|A| = 800$ GeV, $|\mu| = 500$ GeV, $\varphi_{\tilde{A}_t} = \varphi_{\tilde{A}_b} = 0$, and $m_{H^+} = 600$ GeV in the case $m_{\tilde{t}_L} \geq m_{\tilde{t}_R}$.

Figure 4: $\varphi_{\tilde{A}_t}$ dependence of the $\tilde{t}_2$ decay branching ratios for $\varphi_{\tilde{\mu}} = 0$ (a) and $\pi/2$ (b) with $\tan\beta = 8$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (400,700,200)$ GeV, $|A| = 800$ GeV, $|\mu| = 500$ GeV, $\varphi_{\tilde{A}_b} = \varphi_1 = 0$, and $m_{H^+} = 600$ GeV in the case $m_{\tilde{t}_L} \geq m_{\tilde{t}_R}$. Note that the $\tilde{t}_1 H^0_{2,3}$ and $\tilde{b}_1 H^+$ modes are kinematically forbidden here.

Figure 5: $\varphi_{\tilde{A}_b}$ dependence of the $\tilde{b}_1$ decay branching ratios for $\varphi_{\tilde{\mu}} = 0$ (a) and $\pi/2$ (b) with $\tan\beta = 30$, $(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}) = (200,700,400)$ GeV, $(|A|, |\mu|) = (800,700)$ GeV, $(|A|, |\mu|) = (600,500)$ GeV, $\varphi_{\tilde{A}_t} = \varphi_{\tilde{\mu}} = \varphi_1 = 0$, and $m_{H^+} = 180$ GeV in the case $m_{\tilde{t}_L} \geq m_{\tilde{t}_R}$. Only interesting modes are shown in Fig.b where we have $m_{\tilde{b}_2} \sim 570$ GeV and $(m_{H^0_1}, m_{H^0_2}, m_{H^0_3}) \sim (114,156,158)$ GeV.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
