Effects of electrical properties on vibrations via electromechanical coupling in triboelectric energy harvesting

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Abstract
Vibration energy harvesting has been a popular research topic in recent years and is a promising technology in the development of the Internet of Things. Triboelectric energy harvesting, as a relatively new energy harvesting technique, is drawing attention. However, relevant studies from the perspective of structural dynamics are rare, and a study on how the electrical properties of triboelectric energy harvesters (TEHs) affect their vibration is still missing. In this paper, we perform such a study for TEHs that use the two most common working modes—namely the lateral sliding mode and the vertical contact-separation mode. In the first part of the paper, the coupled electromechanical model of a sliding mode TEH—based on a cantilever beam system involving friction—is established. The effects of the tribo-charge surface density and the load resistance on the vibration of the harvester are investigated. It is found that the effects of the tribo-charge surface density on vibrations are similar to those of mechanical damping, while the load resistance on the vibration of the harvester are investigated. It is found that the effects of the tribo-charge surface density on vibrations are similar to those of mechanical damping, while the load resistance can result in an interesting resistive shunt damping phenomenon which is distinct from the one found in piezoelectric energy harvesters. In the second part, the modelling of a vertical contact-separation mode TEH is established based on a single-degree-of-freedom vibro-impact oscillator, and the effects of the same electrical properties on vibrations are studied. The variation of the tribo-charge surface density can result in both vibration amplitude attenuation and resonance frequency shifting, while the change of the load resistance can influence vibrations only in the case of large tribo-charge surface densities. This study further unveils the electromechanical coupling mechanisms in TEHs and sheds some light on achieving desirable dynamic responses of TEHs via tuning their electrical properties.

Keywords: triboelectric energy harvesting, electromechanical coupling, non-smooth system, friction, vibro-impact

(Some figures may appear in colour only in the online journal)
1. Introduction

Vibration energy harvesters (VEH) can harvest energy from ambient vibration to output electricity. This makes them excellent candidates to provide power for remote sensor networks. Furthermore, VEH-powered or self-powered sensors are essential to the development of the Internet of Things (IoT) and smart cities, and may one day be used to power LED traffic lights and wireless sensors embedded in pavements [1].

Piezoelectric [2] and electromagnetic [3] energy harvesters have been investigated extensively, and numerous models and methods have been developed. Examples include the enhancement of energy harvesting efficiency by using frequency up-conversion [4–6], multistability [7–9], nonlinearities [10, 11], and composite materials [12–14]. A broad range of possible energy sources have been studied including roads and bridges [15, 16], vehicles [17, 18], human motion [19, 20], and fluid flow [21–24]. Actual implementations of energy harvesting devices have been made in the areas of temperature monitoring [25], structural health monitoring [26], and in some biomedical applications [27, 28].

Triboelectric energy harvesting [29], as a relatively new way to harvest vibrational energy, is receiving increasing attention because of its high conversion efficiency, simple fabrication, cost-effectiveness, and a wide variety of potential applications [30]. In addition, triboelectric energy harvesters (TEHs) offer a large degree of freedom in the selection of their constituent materials. TEHs function via a combination of triboelectric and electrostatic effects between two materials that have the opposite tribopolarities [29]. However, given the current research activity in nanotechnology, studies on TEHs are more commonly focused on the material science and fabrication aspects of the devices. On the development of novel triboelectric materials, promising dielectric materials [31] and functionalized materials with waterproofing and stretchability [32, 33] have been composited and material surface patterning techniques [34, 35] were advanced. On the applications of TEHs, various prototypes have been tested, such as wireless temperature sensors [36], respiration sensors [37], and motion sensors [38].

Some achievements in theoretical modelling have also been made. For instance, the $V - Q - x$ relationships (where $V$ is the voltage, $Q$ is the transferred charge, and $x$ is the relative displacement between two electrodes) of TEHs using the vertical contact-separation mode [39] and the lateral sliding mode [40] have been developed.

Only a few studies on the structural dynamics of TEHs have been conducted. In a study of a vibro-impact TEH, it has been found that the harvesting frequency bandwidth can become wider because of impact [41]. Another study, based on a similar TEH configuration but involving mechanical bistability via a buckled beam, showed promise as a shock sensor [42]. Since the contact-separation mode TEHs are based on vibro-impact (non-smooth) systems, the study of the vibro-impact dynamics is essential to analyse and optimize the harvesters [43]. In addition to using piece-wise stiffness to model the impact and restitution [41], velocity-dependent coefficient of restitution can be experimentally identified and used in simulation [44]. Further, the ordinary differential equation modelling the electrical portion of a TEH can be stiff. Since the related mechanical system is non-smooth, this can raise issues in numerically solving the combined systems. Thus, a numerical scheme based on the TR-BDF2 (a composite of the trapezoidal rule and the second order backward differentiation formula) method is proposed to address them [44]. In the context of electrostatic energy harvesting, a model considering the free-flight and impact motions of an electrostatic generator was established and validated through experiments [45]. An accurate analytical model of an electret-based electrostatic converter was developed and experimentally verified in [46], and the impact of the parasitic capacitance on the output power was emphasized.

Few reports on the effects of electrical properties on the vibration of mechanical energy harvesters can be found. It has been shown that the resistive shunt damping can affect the vibrational characteristics (resonance frequency and amplitude) of piezoelectric energy harvesters owing to electromechanical coupling [2, 47]. On the other hand, harvesters can be tuned to increase the output power using a generalized electrical load because of the electromechanical coupling effect [48], and similar work has been done for an electromagnetic energy harvester [49] as well. However, a comprehensive study of the effects of electrical properties on the mechanical behaviour of TEHs is still missing. In [44], the effect of the load resistance on the vibration of a contact freestanding triboelectric-layer mode TEH was experimentally investigated but no obvious influence was observed. However, this study was based on a model of a TEH with a small tribo-charge surface density (around $6 \mu \text{C m}^{-2}$) and a relatively large physical volume. For TEHs having larger tribo-charge surface densities and/or smaller volumes, the resultant effects might be more significant. In addition, the environment, such as the temperature, humidity and adsorption, can affect the triboelectrification and the tribo-charge surface density [50]. Different combinations of triboelectric materials also result in different tribo-charge surface densities. Hence, it is important to find out how the variation of the tribo-charge surface density can affect the mechanical behaviours of TEHs in the two most common working modes—the vertical contact-separation mode and the lateral sliding mode. Another electrical property that is worth investigating is the load resistance, since it can be used to electrically tune the harvesters. Therefore, in this paper, the effects of both the tribo-charge surface density and the load resistance on the vibration of TEHs of two most common working modes are investigated.

The outline of the rest of this paper is as follows: section 2 first presents the modelling of a lateral sliding mode TEH and then shows the effects of tribo-charge surface density and load resistance on its vibration. In section 3, the model of a vertical contact-separation mode TEH is first established, and the effects of the same electrical properties on its vibration are studied. The last section draws the main conclusions of this study.
2. Effects of electrical properties on vibrations of a lateral sliding mode TEH

2.1. Modelling of the mechanical system of the TEH

The configuration of a typical sliding mode TEH is shown in figure 1. The system consists of a beam with one end clamped onto a base and the other end attached to a slider. The base is under sinusoidal excitation. The slider is attached with an electrode (top electrode) and a dielectric layer (e.g. PTFE) on its bottom surface. The slider moves laterally and is in frictional sliding on the patches without vertically separating from them. The patches consist of two types: an electrically active patch (EAP) and an electrically inert patch (EIP) (as shown in figure 1(b)). Triboelectrification only happens between the contact of the slider and the EAP. The dielectric layer on the cantilever beam, and only the first mode is excited, between the slider and the EAP and between the slider and the EIP, respectively. The parameter \( w_{\text{rel}} \) is the lateral displacement of the slider relative to the centre of the EAP.

Considering that the cantilever beam is a uniform Euler-Bernoulli beam and only the first mode is excited, \( w_{\text{rel}} \) can be written as

\[
w_{\text{rel}}(x,t) = \phi(x) q(t)
\]

where \( \phi(x) \) is the mode shape function and \( q(t) \) is the temporal modal coordinate.

The equation of motion including damping can be derived as [44]

\[
\ddot{q} + 2\zeta\dot{q} + \omega^2 q = \alpha\ddot{\phi} + F_s\phi(L_b)
\]

where the over-dot denotes a derivative with respect to time \( t \); \( \zeta \) is the damping ratio; \( \ddot{q} \) is the base excitation in the form of acceleration and \( \ddot{\phi} = \dot{A}\sin(2\pi ft) \) where \( A \) is the acceleration amplitude and \( f \) is the excitation frequency in Hz; \( \omega \) is the undamped natural frequency, and \( F_s \) is the kinetic friction force. The expressions for \( \omega \) and \( \alpha \) are given as

\[
\omega = \beta^2 \sqrt{\frac{EI}{\rho b L_b^4}}
\]

\[
\alpha = -\rho b A_b \int_0^{L_b} \phi dx - m_s \dot{\phi}(L_b)
\]

where \( \beta \) can be determined from the associated eigensystem, \( E \) is the Young’s modulus of the beam, \( I \) the second moment of area of the beam’s cross section, \( \rho \) the density of the beam, \( A_b \) the area of the beam’s cross section, \( L_b \) the length of the cantilever beam, and \( m_s \) the mass of the slider.

The kinetic friction force exerted on the slider during sliding is given by

\[
F_s = -(\mu_k A_1 + \mu_k A_2) \frac{N}{A} \operatorname{sgn}(\dot{q}\phi(L_b))
\]

where \( \mu_k \) and \( \mu_k \) are the kinetic coefficients of friction between the slider and the EAP and between the slider and the EIP, respectively; \( N \) is the normal force and \( N = m g + F_e \), where \( g \) is the gravitational acceleration and \( F_e \) is the electrostatic force whose expression will be given in equation (16); \( A \) is the total contact area, \( A = \pi r^2 \), where \( r \) is the radius of the slider and the EAP, and \( \operatorname{sgn}(\cdot) \) is the signum function. The expressions of \( A_1 \) and \( A_2 \) are given as

\[
A_1 = \begin{cases} 2r^2 \cos^{-1} \left( \frac{|q\phi(L_b)|}{2r} \right) - \frac{1}{2} |q\phi(L_b)| \sqrt{4r^2 - |q\phi(L_b)|^2}, & |q\phi(L_b)| < 2r \\ 0, & |q\phi(L_b)| \geq 2r \end{cases}
\]
and the corresponding governing equation can be written as

\[ A_2 = \begin{cases} A - A_1, & |q\phi(L_b)| < 2r \\ A, & |q\phi(L_b)| \geq 2r \end{cases} \] (7)

Stick-slip may happen during oscillation, and the conditions for stick are

\[ q\phi(L_b) = 0 \] (8)

\[ |f_s| \leq f_{\text{max}} \] (9)

where equation (8) indicates that the relative sliding velocity is zero while equation (9) denotes that the static friction force, i.e., \( f_s \), should not be bigger than the maximal static friction capacity, i.e., \( f_{\text{max}} \), and \( f_s \) and \( f_{\text{max}} \) are expressed as

\[
\begin{align*}
f_s &= -[E\xi[q''']_{x=L_b} -(m_s + m_e)g] \\
f_{\text{max}} &= \begin{cases} (\mu s_1 A_1 + \mu s_2 A_2) \xi, & |q\phi(L_b)| < 2r \\
\mu s_2 N, & |q\phi(L_b)| \geq 2r \end{cases}
\end{align*}
\] (10) (11)

where a prime in equation (10) denotes a derivative with respect to \( x \); \( m_e \) is the equivalent mass of the cantilever beam given by \( m_e = \frac{12}{13} \mu s_1 A_1 L_b \) [51], and \( \mu s_1 \) and \( \mu s_2 \) are the static coefficients of friction between the slider and the EAP and between the slider and the EIP, respectively.

### 2.2. Modelling of the electrical system of the TEH

The equivalent electric circuit for the sliding mode TEH connected with an external resistive load is given in figure 2 and the corresponding governing equation can be written as [29, 40]

\[ V = -\frac{1}{C_e}Q + V_{\text{oc}} \] (12)

where \( V \) is the voltage across the load resistance \( R \), \( Q \) is the amount of transferred charges between two electrodes, \( C_e \) is the equivalent capacitance, and \( V_{\text{oc}} \) is the open-circuit voltage, and this equation or model has been widely used in the study of the sliding mode TEHs and has been validated through experiments by Khorsand et al [52] and Zhang et al [53], which gives some confidence to this study. The expressions for each of these quantities are [29, 40]

\[ V = RI = R\frac{dQ}{dt} \] (13)

\[ C_e = \frac{\varepsilon_0 \varepsilon_e A_1}{l_0} \] (14)

\[ V_{\text{oc}} = \frac{\sigma A_2 f_d}{\varepsilon_0 \varepsilon_e A_1} \] (15)

where \( \varepsilon_0 \) is the vacuum permittivity; \( \varepsilon_e, l_0 \) and \( \sigma \) are the relative permittivity, the thickness and the tribo-charge surface density of the dielectric layer attached to the slider.

The electrostatic force between the slider and the EAP can be expressed as

\[ F_e = \frac{Q^2}{2\varepsilon_0 \varepsilon_e A_1} \] (16)

During sticking, the amount of transferred charges between electrodes remains unchanged and no current is induced in the circuit, so

\[ Q(q = q_s) = \text{const.} \] (17)

\[ \frac{dQ}{dt} = 0 \] (18)

where \( q_s \) is the temporal modal coordinate and \( t_s \) is the time instant at which the slider is sticking.

### 2.3. Effects of charge density on vibrations

Since the mechanical system of the harvester is non-smooth and the related electrical system is likely to be stiff, solving the combined system can present numerical difficulties. A numerical scheme, which is based on the TR-BDF2 (trapezoidal rule backward differentiation formula of order two) method, has been proposed to integrate such systems [44] and will be used in numerical simulations in this paper too. The main parameters used in simulation are given in table 1.

### Table 1. The values of the main parameters used in simulation.

| Parameter | Value |
|-----------|-------|
| \( m_e \) | 2 g |
| \( L_b \times A_b \) | \( 150 \times (10 \times 0.5) \text{ mm}^2 \) |
| \( \rho_b \) | 7800 kg · m⁻³ |
| \( E \) | 210 GPa |
| \( \zeta \) | 0.0035 |
| \( r \) | 20 mm |
| \( \mu s_1 \) | 0.35 |
| \( \mu s_2 \) | 0.25 |
| \( \mu s_2 \) | 0.30 |
| \( \mu s_2 \) | 0.20 |
| \( t_0 \) | 0.05 mm |
| \( \varepsilon_s \) | 2.0 |
The frequency response of the cantilever tip with different tribo-charge surface densities and with $R = 1\, \text{M}\Omega$ and $A = 0.25\, \text{g}$ are shown in figure 3(a), and the corresponding frequency response of the RMS voltage are given in figure 3(b). It can be seen that the increase of the tribo-charge surface density reduces the vibration amplitude and shifts the resonance peak to slightly lower frequencies. This is because a larger tribo-charge surface density corresponds to a stronger electric field and a larger electrostatic attractive force, which has a quite similar effect to that of structural damping. Since tribo-charge surface density is a key factor affecting electrical output, by comparing figures 3(a) and (b), it can be seen that large tribo-charge surface densities can result in high electrical output despite the correspondingly low vibration amplitudes.

2.4. Effects of load resistance on vibrations

For a relatively small tribo-charge surface density of $\sigma = 5\, \mu\text{C m}^{-2}$, the frequency response of the cantilever tip under different load resistances are obtained and given in figure 4(a), and a local enlargement of the peaks is shown in figure 4(b). Overall, the effect of load resistance on the vibration is not obvious. Only around the resonance peak do the vibration amplitudes decrease with increasing resistance, and the resonance frequency first shifts to the left slightly and then seems to undergo a very tiny shift to the right. Nevertheless, it can be concluded that the resistive shunt damping effect is not noticeable at small tribo-charge surface densities.

Results under a large tribo-charge surface density of $\sigma = 200\, \mu\text{C m}^{-2}$ are presented in figure 5, in which the resistive shunt damping phenomenon can be observed. With the increase of the load resistance, the peak is first shifted to the left and its amplitude decreases slightly. Upon a further increase in resistance, the peak is then shifted to the right with an additional decrease in amplitude. This is different from the resistive shunt damping effects in piezoelectric energy harvesters [2, 47] where the resonant frequencies shift monotonically to the right with increasing resistance.
The resonance amplitudes in piezoelectric energy harvesters also behave slightly differently, with the largest amplitudes occurring in the short-circuit \( R = 0 \) and open-circuit \( R = \infty \) cases, and with somewhat lower amplitudes occurring at intermediate values of resistance.

To get a clearer view of the variation of the peaks, the resonance frequencies, \( f_r \), and the corresponding amplitudes of the tip displacement, \( w_r \), are plotted against the load resistance in figure 6. The resistive shunt damping results in a looping curve in the 3D space shown in figure 6(a). With the increase of the resistance, the frequency resonance first experiences a drop and then can eventually increase beyond the zero-resistance value, as shown in figure 6(b). Under increasing resistance, the resonant amplitude first undergoes an increase and then a gradual drop, and it eventually levels out, as shown in figure 6(c).

### 3. Effects of electrical properties on vibrations of a vertical contact-separation mode TEH

#### 3.1. Modelling of the mechanical system of the TEH

The configuration of a typical vertical contact-separation mode TEH is shown in figure 7, where \( m \) is the mass, \( k \) is the stiffness, \( d_0 \) is the initial separation gap between the top electrode and the bottom dielectric, \( c \) is the damping and can be given as \( c = 2 \zeta \sqrt{km} \), where \( \zeta \) is the viscous damping ratio. The whole system is under sinusoidal base excitation.

The equation of motion of the mechanical system away from any two consecutive impacts can be given by

\[
m \ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = -F_e = 0
\]

where \( y = A \sin(\omega t) \) and \( \omega = 2\pi f \). \( F_e \) is the electrostatic attractive force. The separation distance between the top electrode and the bottom dielectric is expressed as

\[
d = d_0 + (x - y).
\]

To model the impact, Newton’s law of restitution can be used. Ouanes et al [54] employed it in the modelling of a new concept impact-based TEH and showed a good correlation between their theoretical and experimental models. At impact, Newton’s law of restitution gives

\[
(\dot{x}_+ - \dot{y}) = -\mu (\dot{x}_- - \dot{y})
\]

where \( \dot{x}_- \) is the velocity just before impact, \( \dot{x}_+ \) is the velocity just after impact, and \( \mu \) is the coefficient of restitution. The coefficient of restitution can be velocity-dependent, and an experimentally identified relationship between the just-before-impact velocity and the coefficient of restitution [44] is employed in the present simulation.

Sticking may happen between the mass (or the top electrode) and the base (or the bottom dielectric). During sticking, the mass has the same velocity and acceleration as the base, the separation distance \( d \) equals zero, and a contact force exists between them. This contact force is given by

\[
N = m\ddot{y} - kd_0 - F_e
\]

where it is noted that charge transfer ceases during sticking but the electrostatic force is not zero.

#### 3.2. Modelling of the electrical system of the TEH

The equivalent electric circuit for the vertical contact-separation mode TEH connected with an external resistive load and considering the parasitic capacitance is similar to figure 2 but with a parasitic capacitor connected in parallel with the resistive load, and according to [39, 46], the differential equation modelling the electrical system is

\[
\dot{Q} = \frac{1}{1 + \frac{C_p}{C_e}} \left[ \frac{V_{oc}}{R} - Q \left( \frac{1}{RC_e} - \frac{C_p}{C_e^2} \right) \right]
\]

where \( Q \) is the charge transferred between electrodes, \( R \) is the load resistance in the circuit, \( C_p \) is the parasitic capacitance, which mainly comes from the structural interconnection and the external signal cables [55] and acts in parallel with the harvester and the load resistance and tends to have the largest effects on the model when the load resistance is large [46]. Hinchet et al [56] also considered the parasitic capacitance in their study of a vertical contact-separation mode TEH and measured its exact value in experiments, and they also successfully extracted other model parameters of their proposed TEH to facilitate more realistic modelling and practical applications.

The parameters \( C_e \) and \( V_{oc} \) are the equivalent capacitance and the open-circuit voltage, given by [29, 39]

\[
C_e = \frac{\sigma d}{\varepsilon_0 \varepsilon_r + d}
\]

\[
V_{oc} = \frac{\sigma d}{\varepsilon_0} \frac{S}{l_d}
\]

where \( S \) is the circular area of each electrode, \( \varepsilon_0 \) and \( \varepsilon_r \) are the vacuum permittivity and the relative permittivity of the bottom dielectric (PTFE), \( l_d \) is the thickness of the dielectric layer, and \( \sigma \) is the tribo-charge surface density.
Figure 6. The relationships among load resistance, resonance frequency and amplitude: (a) a 3D plot, (b) load resistance against resonance frequency, and (c) load resistance against resonance amplitude; \( \sigma = 200 \, \mu \text{C m}^{-2} \), \( A = 0.25 \text{g} \).

Thus, the electrostatic force is given as

\[
F_e = -\frac{dU_e}{dx_t} = -\frac{\varepsilon_0 SQ^2}{2[\varepsilon_0 S + C_p (t_d/\xi_t + d)]^2} \tag{28}
\]

where \( x_t \) is the displacement of the mass relative to the frame and \( x_t = x - y \).

3.3. Effects of charge density on vibrations

The main parameters used in the simulation are given in Table 2. The thickness of the dielectric layer or the PTFE layer is identical to that of the sliding mode TEH presented in section 2.

The frequency responses of the amplitude of the separation distance under different tribo-charge surface densities at low and high load resistances of 1 M\( \Omega \) and 100 M\( \Omega \), and at an excitation amplitude of \( \hat{A} = 0.30 \text{g} \) where \( \hat{A} = A \omega^2 \), are shown.
As tribo-charge surface density is increased, it can be seen that the vibration amplitude is reduced and the resonance band is shifted towards the left. Further, the resonance band becomes wider, especially in the high load resistance case as shown in figure 8(b). This is because a larger tribo-charge surface density induces a larger electrostatic attractive force which, in turn, confines the oscillation of the mass to a relatively smaller range close to the bottom dielectric layer. Since the electrostatic attractive force can cause stiffness softening, i.e. the total displacement-related force is $f_d = k(x - y) - F_e$, so that the equivalent ‘spring’ force is $\frac{\partial f_d}{\partial x} = k - \frac{C_p \varepsilon_0 S}{\varepsilon_0 S + C_p (\varepsilon_r + \varepsilon_0)} < k$, it results in a lower stiffness and thus shifts the resonance band towards the left.

Additionally, at larger tribo-charge surface densities and higher resistances, the mass (i.e. the top electrode) can get into contact with the bottom dielectric layer at smaller excitation frequencies. This is due to the increase of the electrostatic attractive force. The corresponding time histories of $d$ at $f = 13.05$ Hz (blue dash-dot line) and $f = 13.10$ Hz (red solid line) are shown in figure 9(a), while those at $f = 17.60$ Hz (red solid line) and $f = 17.65$ Hz (blue dash-dot line) are shown in figure 9(b). It can be seen that the mass suddenly begins impacting with the bottom dielectric after just a small increase of excitation frequency from $f = 17.60$ Hz to $f = 17.65$ Hz, though there is a relatively long free-flight state between two

![Figure 8. Frequency responses of the separation distance under different tribo-charge surface densities at (a) $R = 1\text{M}\Omega$ and (b) $R = 100\text{M}\Omega$ and $\Lambda = 0.30\text{g.}$](image)

![Figure 9. Time histories of the separation distance $d$ at (a) $f = 13.05$ Hz (blue dash-dot line) and $f = 13.10$ Hz (red solid line) and (b) $f = 17.60$ Hz (red solid line) and $f = 17.65$ Hz (blue dash-dot line); $\sigma = 130$ $\mu$C m$^{-2}$, $R = 100\text{M}\Omega$ and $\Lambda = 0.30\text{g.}$](image)

| Table 2. The values of the main parameters used in simulation. |
|-----------------|-------------------|
| Parameter       | Value             |
| $m$             | 20 g              |
| $\zeta$         | 0.005             |
| $k$             | 240 N m$^{-1}$    |
| $S$             | 4 cm$^2$          |
| $d_0$           | 3 mm              |
| $C_p$           | 1 pF              |

in figure 8.
Figure 10. Bifurcation diagram of the amplitude of the displacement of the mass when $\sigma = 130 \mu C m^{-2}$, $R = 100 M\Omega$ and $\dot{A} = 0.30 g$.

Figure 11. Frequency responses of the amplitude of the separation distance under different load resistances at (a) $\sigma = 1 \mu C m^{-2}$, (b) $\sigma = 100 \mu C m^{-2}$, and (c) a local view of (b); $\dot{A} = 0.30 g$. 
consecutive impact events. At $f = 17.60\text{Hz}$, the impacts are more frequent and intense, but disappear altogether when the frequency is increased to $17.65\text{Hz}$.

Returning attention to figure 8, there exist some ripples on the left part of the resonance band. These ripples indicate non-periodic vibrations at the corresponding frequencies. This is because in generating the frequency response, the maximal vibration amplitude of the steady-state response over a hundred excitation periods is taken at each excitation frequency. Thus, the response curve fluctuates at frequencies where non-periodic vibrations are present. To give a clearer view, a bifurcation diagram of the system at $\sigma = 130 \mu\text{C m}^{-2}$ and $R = 100\text{M}\Omega$ is shown in figure 10. It can be seen that the system is in periodic motion before the mass experiences vibro-impact motion. With the increase of the excitation frequency, impact occurs and the system response becomes non-periodic, and grazing bifurcation [58] might happen in this process. However, the system will eventually return to stable periodic motion with the increase of the excitation frequency.

3.4. Effects of load resistance on vibrations

For different load resistances, the corresponding frequency response of the separation distance are shown in figures 11(a) and (b) for tribo-charge surface densities of $\sigma = 1 \mu\text{C m}^{-2}$ and $\sigma = 100 \mu\text{C m}^{-2}$. In figure 11(a), the variation of the load resistance does not have an obvious effect on the frequency responses when the tribo-charge surface density is small. However, the influence can be observed at a larger tribo-charge surface density, as shown in figures 11(b) and (c). The resonance band becomes slightly wider with the increase of the load resistance.

4. Conclusions

The effects of electrical properties on the vibration of TEHs (triboelectric energy harvesters) in the lateral sliding mode and the vertical contact-separation mode are investigated and shown in two separate parts. In the first part, the mechanical and electrical models of a lateral sliding mode TEH, which is based on a cantilever beam system involving friction, are established, and the effects of the tribo-charge surface density and the load resistance on its vibrations are studied separately. In the second part, the modelling of a vertical contact-separation mode TEH, which is based on a single-degree-of-freedom vibro-impact oscillator, is first established, and then the effects of the same electrical properties on vibrations are investigated. This brief study intends to fill an existing research gap by analysing the effects of electrical properties on the vibration of TEHs. The main conclusions are as follows:

For the lateral sliding mode TEH, the increase of the tribo-charge surface density has an effect that is similar to mechanical damping, i.e. increasing tribo-charge surface density attenuates the vibration amplitudes and shifts the resonance peaks towards slightly lower frequencies. At large tribo-charge surface densities, the increase of the load resistance results in an interesting resistive shunt damping phenomenon involving both vibration amplitude attenuation and resonance frequency shifting, which is quite different from what appears in piezoelectric energy harvesters.

For the vertical contact-separation mode TEH, the increase of the tribo-charge surface density leads to both obvious vibration amplitude attenuation and resonance frequency shifting at both low and high load resistances, and it also results in a wider resonance band and complex oscillations. While the increase of the load resistance has only a minor influence at large tribo-charge surface densities, it tends to widen the resonance band slightly.

These studies and findings further reveal the electromechanical coupling mechanisms in TEHs and shed some light on tuning the electrical properties of the devices.

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Conflict of interest

We declare no conflict of interest.

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