Networked Multiple Description Estimation and Compression with Resource Scalability

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Abstract—We present a joint source-channel multiple description (JSC-MD) framework for resource-constrained network communications (e.g., sensor networks), in which one or many deprived encoders communicate a Markov source against bit errors and erasure errors to many heterogeneous decoders, some powerful and some deprived. To keep the encoder complexity at minimum, the source is coded into $K$ descriptions by a simple multiple description quantizer (MDQ) with neither entropy nor channel coding. The code diversity of MDQ and the path diversity of the network are exploited by decoders to correct transmission errors and improve coding efficiency. A key design objective is resource scalability: powerful nodes in the network can perform JSC-MD distributed estimation/decoding under the criteria of maximum a posteriori probability (MAP) or minimum mean-square error (MMSE), while primitive nodes resort to simpler MD decoding, all working with the same MDQ code. The application of JSC-MD to distributed estimation of hidden Markov models in a sensor network is demonstrated.

The proposed JSC-MD MAP estimator is an algorithm of the longest path in a weighted directed acyclic graph, while the JSC-MD MMSE decoder is an extension of the well-known forward-backward algorithm to multiple descriptions. Both algorithms simultaneously exploit the source memory, the redundancy of the fixed-rate MDQ, and the inter-description correlations. They outperform the existing hard-decision MDQ decoders by large margins (up to 8dB). For Gaussian Markov sources, the complexity of JSC-MD distributed MAP sequence estimation can be made as low as that of typical single description Viterbi-type algorithms.

The new JSC-MD framework also enjoys an operational advantage over the existing MDQ decoders. It eliminates the need for multiple side decoders to handle different combinations of the received descriptions by unifying the treatments of all these possible cases.

Keywords: Multiple descriptions, distributed sequence estimation, joint source-channel coding, hidden Markov model, forward-backward algorithm, sensor networks, complexity.

I. INTRODUCTION

We propose a joint source-channel multiple description (JSC-MD) framework for distributed communication and estimation of memory sources. The JSC-MD framework is designed to suit lossy networks populated by resource-deprived transmitters and receivers of varied capabilities. Such a scenario is common in sensor networks and wireless networks. For instance, a large number of inexpensive sensors with no or low maintenance are deployed to monitor, assess, and react to a large environment. On one hand these sensors have to conserve energy to ensure a long lifespan, and on the other hand they need to communicate with processing centers and possibly also among themselves in volatile and adverse network conditions. The energy budget and equipment level of the receivers vary greatly, ranging from powerful processing centers to deprived sensors themselves. The heterogeneity is also the norm in consumer-oriented wireless networks. A familiar and popular application is multimedia streaming with mobile devices such as handsets, personal data assistance (PDA), and notebook computers. Again battery life is a primary concern for all mobile data transmitters, while its criticality varies for receivers, depending on whether the receivers are cell phones, notebooks, base stations, etc.

Conventional source and channel coding techniques may not be good choices for networks of resource-constrained nodes, because they make coding gains proportional to computational complexity (hence energy consumption). The needs for power-aware signal compression techniques have generated renewed interests in the theory of Slepian-Wolf and Wyner-Ziv coding, which was developed more than thirty years ago [1], [2]. The key insight of these works is that statistically dependent random sources can be encoded independently without loss of rate-distortion performance, if the decoder has the knowledge or side information about such dependencies. Although originally intended for distributed source coding, the approach of Slepian-Wolf and Wyner-Ziv coding is of significance to resource-constrained compression in two aspects:

1) communication or coordination between the encoders of the different sources is not necessary to achieve optimal compression, even if the sources are statistically dependent, saving the energy to communicate between the encoders;

2) it is possible to shift heavy computation burdens of rate-distortion optimal coding of dependent sources from encoders to decoders.

Such an asymmetric codec design provides an attractive signal compression solution in situations where a large number of resource-deprived and autonomous encoders need to communicate multiple statistically dependent sources to one or more capable decoders, as is the case for some hierarchical sensor networks [3].

Recently, many researchers have been enthusiastically investigating practical Wyner-Ziv video coding schemes [4], [5], seeking for energy-conserving solutions of video streaming on mobile devices. The motive is to perform video compression...
without computationally expensive motion compensation at the encoder, departing from the prevailing MPEG practice. Instead, the decoder is responsible to exploit the interframe correlations to achieve coding efficiency.

While Wyner-Ziv coding can shift computational complexity of signal compression from encoders to decoders, it does not address another characteristic of modern communication networks: uneven distribution of resources at different nodes. As mentioned earlier, decoders can differ greatly in power supply, bandwidth, computing capability, response time, and other constraints. What can be done if a decoder has to operate under severe resource constraints as well? Despite the information theoretical promise of Wyner-Ziv coding, the rate-distortion performance of distributed compression is operationally bounded by the intrinsic complexity of the problem, or equivalently by the energy budget. It is well known that optimal rate-distortion compression in centralized form is NP-hard [6]. We have no reason to believe that approaching the Wyner-Ziv limit is computationally any easier.

Given the conflict between energy conservation and coding performance, it is desirable to have a versatile signal coding and estimation approach whose performance can be scaled to available energy, which is the notion of resource scalability of this paper. The key design criterion is to keep the complexity of the encoders (often synonymously sensors in sensor networks) at minimum, allowing a wide range of trade-offs between the complexity and rate-distortion performance at decoders. Depending on the availability of energy, bandwidth, CPU power, and other resources, different decoders should be able to reconstruct the same coded signal(s) on best effort basis. We emphasize that a same code stream or a same set of code streams (in case of multiple descriptions) of one or more sources is generated and transmitted for an entire network. By not generating different codes of a source to different decoder specifications, encoders save the energy needed to generate multiple codes. Furthermore, this will simplify and modularize the encoder (sensor) design to reduce the manufacturing cost. Ideally, a resource-scalable code should not deny a decoder without resource constraint the possibility of approaching the Wyner-Ziv performance limit, and at the same time it should allow even the least capable decoder in the network to reconstruct the signal, barring complete transmission failure.

This paper will show how resource-scalable networked signal communication and estimation can be realized by multiple description quantization (MDQ) at encoders and joint source-channel (JSC) estimation at decoders. To keep the encoder complexity at minimum, a source is compressed by fixed rate MDQ with neither entropy nor channel coding. The code diversity of MDQ and the path diversity of the network are intended to be exploited by JSC decoding to combat transmission errors and gain coding efficiency. Various JSC estimation techniques will be introduced to provide solutions of different complexities and performances, ranging from the fast and simple hard-decision decoder to sophisticated graph theoretical decoders.

When used for MDQ decoding, the proposed JSC-MD approach has an added operational advantage over the current MDQ design. It generates an output sequence (the most probable one given the source and channel statistics) consisting entirely of the codewords of the central quantizer, rather than a mixture of codewords of the central and K side decoders. As such the JSC-MD approach offers a side benefit of unifying the treatment of the $2^K$ cases for different subsets of received descriptions. Instead of employing $2^K - 1$ decoders as required by the existing MDQ decoding process, we need only one MDQ decoder. This overcomes a great operational difficulty currently associated with the MDQ decoding process.

The presentation flow of this paper is as follows. Section II formulates the JSC-MD problem. Section III constructs a weighted directed acyclic graph to model the JSC-MD MAP estimation/decoding problem. This graph construction converts distributed MAP estimation into a problem of longest path in the graph, which is polynomially solvable. The complexity results are derived. Section IV applies the proposed JSC-MD approach to distributed MAP estimation of hidden Markov state sequences in lossy networks. This problem is motivated by sensor networks of heterogeneous nodes with resource scalability requirements. With the same MD code transmitted over the entire network, the empowered MD decoders can obtain exact MAP solution using a graph theoretical algorithm, while deprived MD decoders can obtain approximate solutions using algorithms of various complexities. Section V investigates the problem of distributed MMSE decoding of MDQ. It turns out that JSC-MD MMSE decoding can be performed by generalizing the well-known forward-backward algorithm to multiple descriptions. Simulation results are reported in Section VI. Section VII concludes.

II. PROBLEM FORMULATION

Fig. 1 schematically depicts the JSC-MD system motivated in the introduction. The input to the system is a finite Markov sequence $\mathbf{x} = x_1, x_2, \cdots, x_N$. A $K$-description MDQ first maps a source symbol (if multiple description scalar quantization (MDSQ) is used) or a block of source symbols (if multiple description vector quantization (MDVQ) is used) to a codeword of the central quantizer $q : \mathbb{R} \rightarrow \mathcal{C} = \{c_1, c_2, \cdots, c_L\}$, where $L$ is the number of codecells of the central quantizer. Let the codebooks of the $K$ side quantizers be $\mathcal{C}_k = \{c_{k,1}, c_{k,2}, \cdots, c_{k,L_k}\}$, $1 \leq k \leq K$, where $L_k \leq L$ is the number of codecells of side quantizer $k$, $L = \prod_{k=1}^{K} L_k$. The $K$-description MDQ is specified by an index assignment function $\lambda_k : \mathcal{C} \rightarrow \mathcal{C}_k$ [7]. The redundancy carried by the $K$ descriptions versus the single description can be reflected by a rate $1 - \log_2 L / \sum_{k=1}^{K} \log_2 L_k$ [8].
Due to the expediency on the part of resource-deprived MDQ encoders, a decoder is furnished with rich forms of statistical redundancy:

- the memory of the Markov source that is unexploited by suboptimal source code;
- residual source redundancy for lack of entropy coding;
- the correlation that is intentionally introduced among the $K$ descriptions of MDQ.

The remaining question or challenge is naturally how these intra- and inter-description redundancies can be fully exploited in a distributed resource-constrained environment.

Let $\mathbf{x} = x_1 x_2 \cdots x_N \in \mathbb{C}^N$ be the output sequence of $\chi^N$ produced by the central quantizer, $N = \chi$ for MDSQ, or $N = i\chi$ for MDVQ with $i$ being the VQ dimension. The $K$ descriptions of MDQ, $\lambda_k(\mathbf{x}) \in \mathbb{C}_k^N$, $1 \leq k \leq K$, are transmitted via $K$ noisy diversity channels. In this work we use a quite general model for the $K$ diversity channels. The only requirements are that these channels are memoryless, independent, and do not introduce phase errors such as insertion or deletion of code symbols or bits. In the existing literature on MDQ, only erasure errors are considered in MDQ decoding. Our diversity channel model accommodates bit errors as well. This is an important expansion because bit errors can indeed happen in a received description in reality, particularly so in wireless network communications. Denote the received code streams by $y_k = y_{k,1} y_{k,2} \cdots y_{k,N}$, with $y_{k,n}$ being the $n$th codeword of description $k$ that is observed by the decoder.

Having the source and channel statistics and knowing the structure of MDQ, the decoder can perform JSC-MD decoding of sequences $y_k$, $1 \leq k \leq K$, to best reconstruct $\mathbf{x}$. The JSC criterion can be maximum a posteriori probability (MAP) or minimum mean-square error (MMSE). For concreteness and clarity, we formulate the JSC-MD problem for distributed MAP decoding of MDQ. As we will see in subsequent sections, the formulation for other distributed sequence estimation and decoding problems requires only minor modifications. In a departure from the current practice of designing multiple side decoders (up to $2^K - 1$ of them!), our JSC-MD system offers a single unified MDQ decoder that operates the same way regardless what subset of the $K$ descriptions are available to the decoder. For JSC decoding of single description scalar quantized Markov sequences, please refer to [9]–[13].

In JSC-MD distributed MAP decoding a decoder reconstructs, given the observed sequences $y_k$, $1 \leq k \leq K$, some of which may be empty), the input sequence $\mathbf{x}$ such that the a posteriori probability $P(\mathbf{x}|y_1, y_2, \cdots, y_K)$ is maximized. Namely, the MAP MDQ decoder emits

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in \mathbb{C}^N} P(\mathbf{x}|y_1, y_2, \cdots, y_K).$$

By Bayes’ theorem we have

$$P(\mathbf{x}|y_1, y_2, \cdots, y_K) = \frac{P(\mathbf{x})P(y_1, y_2, \cdots, y_K|\mathbf{x})}{P(y_1, y_2, \cdots, y_K)}$$

$$= P(\mathbf{x})P(y_1, y_2, \cdots, y_K|\lambda_1(\mathbf{x}), \lambda_2(\mathbf{x}), \cdots, \lambda_K(\mathbf{x}))$$

$$= P(\mathbf{x}) \prod_{k=1}^{K} P(y_k|\lambda_k(\mathbf{x}))$$

$$= \prod_{n=1}^{N} \left\{ P(x_n|x_{n-1}) \prod_{k=1}^{K} P_k(y_{kn}|\lambda_k(x_n)) \right\}.$$ 

In the above derivation, step (a) is due to the fact that $y_1$ through $y_K$ are fixed in the objective function for $\mathbf{x} \in \mathbb{C}^N$; step (b) is because of the mutual independency of the $K$ channels; and step (c) is under the assumption that $\mathbf{x}$, the output of the central quantizer, is first-order Markovian and the channels are memoryless. This assumption is a very good approximation if the original source sequence $\chi^N$ before MDQ is first-order Markovian, or a high-order Markov sequence $\chi^N$ is vector quantized into $K$ descriptions.

In (2) we also let $P(x|y_0) = P(x)$ as convention. $P_k(b'|b)$ is the probability of receiving a codeword $b = b_1 b_2 \cdots b_B$ from channel $k$ as $b' = b'_1 b'_2 \cdots b'_{B'}$. Because the channel is memoryless, we have

$$P_k(b'|b) = \prod_{i=1}^{B} P_k(b'_i|b_i).$$

Specifically, if the $K$ diversity channels can be modeled as memoryless error-and-erase channels (EEC), where each bit is either transmitted intact, or inverted, or erased (the erasure can be treated as the substitution with a new symbol ‘$\dagger$’), then $b \in \{0, 1\}^B$, $b' \in \{0, 1, \dagger\}^B$ and

$$P_k(b'_i|b_i) = \begin{cases} p_{\phi,k}, & \text{if } b'_i = \dagger; \\ (1 - p_{\phi,k})(1 - p_{c,k}), & \text{if } b'_i = b_i; \\ (1 - p_{\phi,k})p_{c,k}, & \text{otherwise} \end{cases}$$

where $p_{\phi,k}$ is the erasure probability and $p_{c,k}$ is the inversion or crossover probability for channel $k$, $1 \leq k \leq K$.

In the literature MDQ is mostly advocated as a measure against packet erasure errors in diversity networks. Such packet erasure errors can be fit by the above model $P_k(b'|b_i)$ of binary memoryless EEC, if a proper interleaver is used.

The proposed JSC-MD framework is also suitable for additive white Gaussian noise (AWGN) channels. If $b_i$ is binary phase-shift keying (BPSK) modulated and transmitted through channel $k$ that is AWGN, then

$$P_k(b'_i|b_i) = \frac{1}{\sqrt{2\pi\sigma_k}} e^{-(b'_i-b_i)^2/2\sigma_k}$$

where $\sigma_k$ is the noise power spectral density of channel $k$.

The prior distribution $P(x)$ and transition probability matrix $P(x_n|x_{n-1})$ for the first-order Markov sequence $\mathbf{x}$ can be determined from the source distribution and the particular MDQ in question.
In the case of MDSQ, if the stationary probability density function of the source is \( p_s(\chi) \) and the conditional probability density function is \( p_s(\chi|x_{n-1}) \), then
\[
P(x) = \int_{\chi:q(\chi)=x_1} p_s(\chi) d\chi
\]  
and
\[
P(x_n|x_{n-1}) = \frac{\int_{\chi:q(\chi)=x_n} p_s(\chi_1|x_2)p_s(\chi_2)d\chi_2 d\chi_1}{\int_{\chi:q(\chi)=x_{n-1}} p_s(\chi)d\chi}.
\]

If MDVQ is the source coder of the system, the transition probability matrix for \( P(x_n|x_{n-1}) \)'s can be determined numerically either from a known close-form source distribution or from a training set.

III. JOINT SOURCE-CHANNEL MULTIPLE DESCRIPTION MAP DECODING

In this section, we devise a graph theoretical algorithm for JSC-MD MAP decoding algorithm. Combining (1) and (2), we have
\[
\hat{x} = \arg\max_{x \in \mathbb{C}^N} \sum_{n=1}^{N} \left\{ \log P(x_n|x_{n-1}) + \sum_{k=1}^{K} \log P_k(y_{k,n}|\lambda_k(x_n)) \right\}.
\]

Because of the additivity of (8), we can structure the MAP estimation problem into the following subproblems:
\[
w(n,x_n) = \max_{x \in \mathbb{C}^{n-1}} \sum_{i=1}^{n} \left\{ \log P(x_i|x_{i-1}) + \sum_{k=1}^{K} \log P_k(y_{k,i}|\lambda_k(x_i)) \right\}, \quad x_n \in \mathbb{C}, \quad 1 \leq n \leq N.
\]
The subproblems \( w(\cdot, \cdot) \) can be expressed recursively as
\[
w(n,x_n) = \max_{x \in \mathbb{C}^{n-1}} \left\{ \sum_{i=1}^{n-1} \left[ \log P(x_i|x_{i-1}) + \sum_{k=1}^{K} \log P_k(y_{k,i}|\lambda_k(x_i)) \right] + \log P(x_n|x_{n-1}) + \sum_{k=1}^{K} \log P_k(y_{k,n}|\lambda_k(x_n)) \right\}
\]
\[
= \max_{c \in \mathbb{C}} \left\{ w(n-1,c) + \log P(x_n|c) + \sum_{k=1}^{K} \log P_k(y_{k,n}|\lambda_k(x_n)) \right\}.
\]

Then, the solution of the optimization problem (1) is given recursively in a backward manner by
\[
\hat{x}_N = \arg\max_{c \in \mathbb{C}} w(N,c).
\]
\[
\hat{x}_{n-1} = \arg\max_{c \in \mathbb{C}} \left\{ w(n-1,c) + \log P(\hat{x}_n|c) \right\}, \quad 2 \leq n \leq N.
\]

The recursion of \( w(n,x_n) \) allows us to reduce the MAP estimation problem to one of finding the longest path in a weighted directed acyclic graph (WDAG) [12], as shown in Fig. 2. The underlying graph \( G \) has \( LN + 1 \) vertices, which consists of \( N \) stages with \( L \) vertices in each stage. Each stage corresponds to a codeword position in \( x \). Each vertex in a stage represents a possible codeword at the position. There is also one starting node \( z_0 \), corresponding to the beginning of \( x \).

In the construction of the graph \( G \), each node is associated with a codeword \( x \in \mathbb{C} \) at a sequence position \( n, 1 \leq n \leq N \), and hence labeled by a pair \((n,z)\). From node \((n-1,b)\) to node \((n,a), a,b \in \mathbb{C} \), there is a directed edge, whose weight is
\[
\log P(a|b) + \sum_{k=1}^{K} \log P_k(y_{k,n}|\lambda_k(a)).
\]

From the starting node \( s \) to each node \((1,a)\), there is an edge whose weight is
\[
\log P(a) + \sum_{k=1}^{K} \log P_k(y_{k,1}|\lambda_k(a)).
\]

In graph \( G \), the solution of the subproblem \( w(n,a) \) is the weight of the longest path from the starting node \( s \) to node \((n,a)\), which can be calculated recursively using dynamic programming. The MAP decoding problem is then converted into finding the longest path in graph \( G \) from the starting node \( z_0 \) to nodes \((N,c)\), \( c \in \mathbb{C} \). By tracing back step by step to the starting node \( z_0 \) as given in (11), the MDQ decoder can reconstruct the input sequence \( x \) to \( \hat{x} \), the optimal result defined in (1).

Now we analyze the complexity of the proposed algorithm. The dynamic programming algorithm proceeds from the starting node \( z_0 \) to the nodes \((N,c)\), through all \( LN \) nodes in \( G \). The value of \( w(n,a) \) can be evaluated in \( O(L) \) time, according to (10). The quantities \( \log P(a|b) \) and \( \log P_k(y_{k,n}|\lambda_k(a)) \) can be precomputed and stored in lookup tables so that they will be available to the dynamic programming algorithm in \( O(1) \) time. Hence the term \( \sum_{k=1}^{K} \log P_k(y_{k,n}|\lambda_k(a)) \) in (10) can be computed in \( O(K) \) time. Therefore, the total time complexity of the dynamic programming algorithm is \( O(L^2NK) \). The reconstruction of the input sequence takes only \( O(N) \) time, given that the selections in (11) (and in (10) as well) are recorded, which results in a space complexity of \( O(LN) \).
In [12] we proposed a monotonicity-based fast algorithm for the problem of MAP estimation of Markov sequences coded by a single description quantizer, which converts the longest path problem to one of matrix search. For Gaussian Markov sequences the matrix can be shown to be totally monotone, and the search can be done in lower complexity. The same algorithm technique can be generalized to multiple descriptions and reduce the complexity of JSC-MD MAP decoding. In the appendix we prove that distributed MAP decoding of $K$-description scalar quantizer can be completed in $O(LNK)$ time for Gaussian Markov sequences. The linear dependency of the MAP MDSQ decoding algorithm in the sequence length $N$ and source codebook size $L$ makes it comparable to the complexity of typical Vertiby-type decoders for single description.

IV. DISTRIBUTED MULTIPLE-DESCRIPTION ESTIMATION OF HIDDEN MARKOV SEQUENCES

In this section we apply the proposed JSC-MD MAP estimation technique to solve the problem of hidden Markov sequence estimation in a resource-constrained network. For single description hidden Markov sequence estimation an extensively-studied problem with many applications [14]. As a case study, consider a sensor network in an inaccessible area to monitor the local weather system for years with no or little maintenance. Our objective is to remotely estimate the sequence of weather patterns: sunny, rainy, cloudy and so on. To this end the sensors collect real-valued data vector: temperature, pressure, moisture, wind speed, etc., and communicate them to processing nodes of various means in the network. Some are well-equipped and easily-maintained processing centers, while others need to run autonomously on limited power supply and react to certain weather conditions on their own rather than being instructed by the central control.

A. Problem Formulation

Our task is to estimate the state sequence of a hidden Markov model (HMM), which is, in our example, the time sequence of weather patterns that are not directly observable by processing nodes in the sensor network. Let the state space of the HMM be $S = \{s_1, s_2, \ldots, s_M\}$, being sunny, rain, and so on. For state transition from $s_i$ to $s_j$ (weather change) of Markov state transition probability $P_{S}(s_j/s_i)$, the HMM output to be observed by the sensors is a real-valued random vector $x \in \mathbb{R}^d$ (temperature, pressure, moisture, wind speed, etc.) with probability $P_{O}(x|s_i, s_j)$. The observations need to be communicated to data processing centers at a low bit rate against channel noise and losses. To maximize their operational lifetime the sensors have to do without sophisticated source coding and forgo channel coding altogether. A viable solution under such stringent conditions is to produce and transmit $K \geq 2$ descriptions of $x$ in fixed length code without entropy coding. There are many ways for inexpensive and deprived encoders (sensors) to code $x$ into multiple descriptions in collaboration. One possibility is the use of multiple description lattice vector quantizer (MDLVQ) [7], [15], [16].

Among known multiple description vector codes, MDLVQ is arguably the most resource-conserving with a very simple implementation. A $K$-description MDLVQ uses a fine lattice in $\mathbb{R}^d$ as its central quantizer codebook $C$ and an accompanying coarse lattice $C_s$ in $\mathbb{R}^d$ as its side quantizer codebooks $C_k$, $1 \leq k \leq K$. Therefore we have $C_1 = C_2 = \cdots = C_K = C_s$, and typically $C_s \subset C$. An MDLVQ index assignment is depicted in Fig. 3 for $K = 2$. Each fine lattice point in $C$ is labeled by a unique ordered pair of coarse lattice points in $C_s$.

Upon observing an HMM output sequence $\chi^N$, the central quantizer first quantizes $\chi^N$ to a sequence of the nearest fine lattice points $x = q(\chi^N)$. Then the MDLVQ encoder generates $K$ description sequences of $x$: $\lambda_k(x)$ and transmits them through $K$ diversity channels (or diversity paths in the network).

A decoder can reconstruct $\lambda_k(x_n)$ to $x_n$ with the inverse labeling function $\lambda^{-1}$, if all $K$ descriptions are received. In the event that only a subset $\Psi$ of the $K$ descriptions are received, the decoder reconstructs $x_n$ to the average of the received coarse descriptions:

$$\hat{x}_n = \frac{1}{|\Psi|} \sum_{k \in \Psi} \lambda_k(x_n)$$  \hspace{1cm} (12)

where $|\cdot|$ is the cardinality of a set. This is the simplest MDLVQ decoder possible, which is also asymptotically optimal for $K = 2$ [17].

B. Distributed MAP Sequence Estimation

Let $y_k = y_k, 1 \leq y_k, 2 \cdots y_k, N$ be the received sequence from channel $k$, $1 \leq k \leq K$. Our task is to estimate the hidden state sequence $z = z_1, z_2, \cdots, z_N \in S^N$ of weather patterns, given the $K$ noisy time sequences of atmosphere attributes produced by the HMM: $y_1$, $y_2$, ..., $y_K$. With the resource-scalability in mind, we take an approach of MAP estimation:

$$\hat{z}, \hat{x} = \arg\max_{z \in S^N, \hat{x} \in C^N} P(x, z|y_1, y_2, \cdots, y_K).$$  \hspace{1cm} (13)
Analogously to (12) we use Bayes’ theorem and the independence of the $K$ memoryless channels to obtain

$$P(x, z|y_1, y_2, \ldots, y_K) \propto P(z|y_1, y_2, \ldots, y_K) P(x|z) P(y_1, y_2, \ldots, y_K|x, z)$$

$$= \prod_{n=1}^{N} \left\{ P_S(z_n|z_{n-1}) P_O(x_n|z_n, z_{n-1}) P(y_1, y_2, \ldots, y_K|x) \right\}$$

From the starting node $z_0$ to each node $(1, a)$, there is an edge whose weight is

$$\log P_S(b|a) + \xi(b, a).$$

In graph $G$, the solution of the subproblem $w(n, s)$ is the weight of the longest path from the starting node $z_0$ to node $(n, s)$, which can be calculated recursively using dynamic programming. The distributed MAP estimation problem is then converted into finding the longest path in graph $G$ from the starting node $z_0$ to nodes $(N, s), s \in S$. Tracing back step by step to the starting node $z_0$ generates the optimally estimated HMM state sequence $\hat{z}$.

To analyze the complexity of the proposed algorithm, we notice that the dynamic programming algorithm proceeds through all $MN$ nodes in $G$. The value of $w(n, s)$ can be evaluated in $O(M)$ time, according to (18). The quantities $\log P_S(b|a)$ and $\log P_O(x|z_n, s)$ can be precomputed and stored in lookup tables so that they will be available in the dynamic programming process in $O(1)$ time. The term $\xi(b, a)$ can be computed in $O(KL)$ time. Therefore, the total time complexity of this algorithm is $O(M^2NKL)$. The space complexity is $O(MN)$.

### C. Resource Scalability

If a network node is not bounded by energy and computing resources, it can use the relatively expensive MAP algorithm that taps all available redundancies to obtain the best estimate of HMM state sequence, knowing the statistics of HMM and underlying noisy diversity channels. This JSC-MD framework can be used as an asymmetric codec in the Wyner-Ziv spirit, which stripes the encoders to the bone while empowering the decoders. More importantly, it also offers a resource-scalability. If a node in the sensor network needs to estimate $z$ but is severely limited in resources, it can still do so using the same MDLVQ code, albeit probably at a lesser estimation accuracy. The simplest hence most resource-conserving approximate solution is to first perform a hard-decision MDLVQ decoding of received descriptions $y_{k,n}$’s to $\hat{x}_n$ using (12), and then estimate $z_n$ to be

$$\hat{z}_n = \max_{z \in S} P_S(z) P_O(\hat{x}_n|z).$$ (19)
by $P_S(z_n)$ in (14), another possible JSC-MD estimation emerges:

$$\hat{z}_n = \arg\max_{z \in S} P_S(z) P_O \left( \max_{x \in \mathcal{C}} \prod_{k=1}^{K} P_k(y_{k,n} | \lambda_k(x)) \right). \quad (20)$$

This leads to an $O(N(KL + M))$ HMM state sequence estimation algorithm. The algorithm is slightly more expensive than the one based on (19) but offers better performance because the MDLVQ decoding is done with the knowledge of channel statistics.

D. Application in Distributed Speech Recognition

Given the success of HMM in speech recognition [14], we envision the potential use of the JSC-MD estimation technique for remote speech recognition. For instance, the cell phones transmit quantized speech signals via diversity channels to processing centers and the recognized texts are sent back or forward to other destinations. This will offer mobile users speech recognition functionality without requiring heavy computing power on handsets and fast draining batteries. Also, the network speech recognizer can prompt a user to repeat in case of difficulties, the user’s revocalization can be used as extra descriptions to improve the JSC-MD estimation performance.

V. RESOURCE-SCALABLE JSC-MD MMSE DECODING

The JSC-MD distributed MAP estimation problem discussed above is to track the discrete states of a hidden Markov model. Likewise, the cost function (1) for distributed MAP decoding of MDQ requires the output symbols to be discrete codewords of the central quantizer. This may be desirable or even necessary, if the quantizer codewords communicated correspond to discrete states of semantic meanings, such as in some recognition and classification applications. But in network communication of a continuous signal $\chi = \chi_1, \chi_2, \cdots, \chi_N$, the JSC-MD output can be real valued. In this case a JSC-MD distributed MMSE decoding scheme of resource scalability is preferred, which is the topic of this section.

The goal of the JSC-MD MMSE decoding is to reconstruct $\chi_n$ as

$$E(\chi_n | y_1, y_2, \cdots, y_K) = \sum_{l=1}^{L} P(x_n = l | y_1, y_2, \cdots, y_K) \int_{\chi \in \mathcal{C}} \chi \, \rho(\chi) d\chi,$$ \quad (21)

where $V_l$ is cell $l$ of the central quantizer. Hence we need to estimate the a posteriori probability $P(x_n | y_1, y_2, \cdots, y_K)$. Equivalently, we estimate

$$P(x_n = l, y_1, y_2, \cdots, y_K), \quad l \in \mathcal{C}. \quad (22)$$

We can solve the above estimation problem for the JSC-MD distributed MMSE decoding by extending the well-known BCJR (forward-backward) algorithm [18] to multiple observation sequences. For notational convenience let $y_{k,n}^{a-b}$, $a < b$, be the consecutive subsequence $y_{k,a}, y_{k,a+1}, \cdots, y_{k,b}$ of an observation sequence $y_k$.

Define

$$\alpha_n(l) = P(x_n = l, y_1^{1-n}, y_2^{1-n}, \cdots, y_K^{1-n})$$

$$\beta_n(l) = P(y_1^{n+1-N}, y_2^{n+1-N}, \cdots, y_K^{n+1-N} | x_n = l)$$

$$\gamma_n(l', l) = P(x_n = l, y_1, y_2, y_3, \cdots, y_K, x_{n-1} = l').$$

Then we have

$$P(x_n = l, y_1, y_2, \cdots, y_K) = P(x_n = l, y_1^{1-n}, y_2^{1-n}, \cdots, y_K^{1-n})$$

$$\cdot P(y_1^{n+1-N}, y_2^{n+1-N}, \cdots, y_K^{n+1-N} | x_n = l)$$

$$= \alpha_n(l) \cdot \beta_n(l).$$

The last step is due to the fact that $y_k^{1-n}$ and $y_k^{n+1-N}$ are independent given $x_n$, and that $y_k$ and $y_j$ are independent for $k \neq j$. The terms $\alpha_n(l)$ and $\beta_n(l)$ can be recursively computed by

$$\alpha_n(l) = \sum_{l'}^{L-1} \gamma_n(l', l) \cdot \alpha_{n-1}(l').$$

$$\beta_n(l) = \sum_{l'=0}^{L-1} P(x_n+1 = l', y_1^{n+1-N}, y_2^{n+1-N}, \cdots, y_K^{n+1-N} | x_n = l)$$

$$= \sum_{l'=0}^{L-1} \left\{ \left[ P(x_n+1 = l', y_1^{n+1-N}, y_2^{n+1-N}, \cdots, y_K^{n+1-N} | x_n = l) \right. \right.$$

$$\left. \cdot P(y_1^{n+2-N}, y_2^{n+2-N}, \cdots, y_K^{n+2-N} | x_{n+1} = l') \right\}$$

$$= \sum_{l'=0}^{L-1} \gamma_{n+1}(l, l') \cdot \beta_{n+1}(l').$$

By definition the term $\gamma_n(l, l')$ can be computed by

$$\gamma_n(l', l) = P(x_n = l, y_1, y_2, \cdots, y_K, x_{n-1} = l')$$

$$= P(x_n = l | x_{n-1} = l') \cdot P(y_1, y_2, \cdots, y_K | x_n = l)$$

$$= P(x_n = l | x_{n-1} = l') \cdot \Pi_{k=1}^{K} P_k(y_{k,n} | \lambda_k(l)).$$

If the input sequence $x$ is i.i.d. the above is reduced to

$$P(x_n = l, y_1, y_2, \cdots, y_K)$$

$$= P(x_n = l, y_1, y_2, \cdots, y_K)$$

$$= P(x_n = l) \cdot P(y_1, y_2, \cdots, y_K | x_n = l)$$

$$= P(x_n = l) \cdot \Pi_{k=1}^{K} P_k(y_{k,n} | \lambda_k(l)), \quad 1 \leq n \leq N.$$
This is also the scheme for hard-decision MDQ MMSE decoding in midst of inversion and erasure errors.

Now we analyze the complexity of the proposed JSC-MD MMSE algorithm. For each \( n, 1 \leq n \leq N \), we need to calculate the value of \( \alpha_n(l), \beta_n(l) \) and \( \gamma_n(l', l) \). As explained in the complexity analysis of JSC-MD MAP algorithm, the term \( \sum_{k=1}^{K} \log P_k(y_{k,n} | a_k(n)) \) in (27) can be computed in \( O(K) \) time. Thus, the matrix \( \gamma_n(l', l) \), \((l', l) \in \mathbb{C}^2\), can be computed in \( O(L^2K) \) time. The value of \( \alpha_n(l) \) and \( \beta_n(l) \) can be computed in \( O(L) \) time. Therefore, the total complexity is \( O(L^2KN) \), which has the same order with the complexity of JSC-MD MAP algorithm as derived in Section III.

If sequence \( x \) is i.i.d. the complexity of JSC-MD MMSE decoding is reduced to \( O(LKN) \) as exhibited by (28). For memoryless sources, MMSE sequence estimation is degenerated to MMSE symbol-by-symbol decoding. Even if \( x \) is not memoryless, in consideration of resource scalability, (28) can still be used as a less demanding alternative for network nodes not having sufficient resources to perform full-fledged JSC-MD MMSE decoding. The approximation is good if the source memory is weak. To the extreme, the weakest network nodes of severe source constraints can always resort to a hard-decision MD decoding (e.g., using the MD decoder (12)), which takes only \( O(KN) \) time to decode a multiple-description coded sequence \( x \) of length \( N \). The important point is that all three decoders of complexities ranging from \( O(L^2KN) \) to \( O(KN) \) operate on the same MD code streams distributed in the network. The reader can continue to the next section for further discussions on the issue of resource scalability.

VI. SIMULATION RESULTS

The proposed resource-aware JSC-MD distributed MAP and MMSE decoding algorithms are implemented and evaluated via simulations. The simulation inputs are first-order, zero-mean, unit-variance Gaussian Markov sequences of different correlation coefficient \( \rho \). A fixed-rate two-description scalar quantizer (2DSQ) proposed in [15] is used as the encoder in our simulations. The 2DSQ is uniform and is specified by the index assignment matrix shown in Fig. 4. The central quantizer has \( L = 21 \) codecells and the two side quantizers each has \( L_1 = L_2 = 8 \) codecells. For each description \( k, k = 1, 2 \), the codeword index \( a_k(x) \) is transmitted in fixed length code of three bits.

The channels are simulated to be error-and-erasure channels with identical erasure probability \( p_\varnothing \) and inversion probability \( p_c \). We report and discuss below the simulation results for different combinations of \( p_c, p_\varnothing \) and \( \rho \).

First, we evaluate the performance of the JSC-MD distributed MAP decoder. The performance measure is symbol error rate (SER), which is the probability that a symbol of the input Markov sequence is incorrectly decoded. Since the input source is Gaussian Markov, the \( O(LKN) \) MAP algorithm of Section III can be used by the resource-rich network nodes to obtain the optimal estimation. However, resource-deprived network nodes can also decode whatever received description(s) of the same 2DSQ code, using a simple energy-conserving \( O(KN) \) hard-decision MDQ decoder. The simulation results are plotted in Fig. 5. Over all values of \( \rho, p_c \) and \( p_\varnothing \), the JSC-MD MAP decoder outperforms the hard-decision MDQ decoder. As expected, the performance gap between the two decoders increases as the amount of memory in the Markov source (\( \rho \)) increases. This is because the hard-decision MDQ decoder cannot benefit from the residual source redundancy left by the suboptimal primitive 2DSQ encoder.

In the case of JSC-MD distributed MMSE decoding, we evaluate three decoders of different complexities (hence different resource requirements): the exact \( O(L^2KN) \) algorithm derived in Section VII, the simplified \( O(LKN) \) algorithm given in (28), and the conventional \( O(KN) \) hard-decision MDQ decoder. The performance measure for MMSE decoding is naturally the signal-to-noise ratio (SNR). The simulation results are plotted in Fig. 6 with the correlation coefficient being 0, 0.5 and 0.9 respectively. The trade-offs between the complexity and performance of a decoder can be clearly seen in these figures. Given \( \rho, p_c, p_\varnothing \), the SNR decreases as the decoder complexity increases. The JSC-MD MMSE decoder achieves the highest SNR, because it utilizes both inter- and intra-description correlations. The performance of the algorithm given in (28) is in the middle, which is \( O(L) \) faster than the full-fledged JSC-MD MMSE decoder but \( O(L) \) slower than the hard-decision MDQ decoder. This decoder reduces complexity or energy requirement by making use of...
still have an advantage over the hard-decision MDQ decoders. Distributed decoder can make a better use of inter-description correlation in the event of packet loss. As the erasure error probability gap between different algorithms increases as the erasure redundancy increases. When \( \rho = 0 \), the first two algorithms become the same.

Under both MAP and MMSE criteria, the performance gap between different algorithms increases as the erasure error probability \( p_c \) increases, indicating that the JSC-MD distributed decoder can make a better use of inter-description correlation in the event of packet loss. As the erasure error probability increases in the network, the proposed JSC-MD decoder enjoys up to 8 dB gain over the hard-decision MD decoders.

Finally, we point out that even when source memory is weak (see the curves for \( \rho = 0 \)), the JSC-MD distributed decoders still have an advantage over the hard-decision MDQ decoders that cannot handle the bit errors within a received description effectively.

VII. CONCLUSIONS

We propose a joint source-channel multiple description approach to resource-scalable network communications. The encoder complexity is kept to the minimum by fixed rate multiple description quantization. The resulting MD code streams are distributed in the network and can be reconstructed to different qualities depending on the resource levels of receiver nodes. Algorithms for distributed MAP and MMSE sequence estimation are developed, and they exploit intra- and inter-description redundancies jointly to correct both bit errors and erasure errors. The new algorithms outperform the existing hard-decision MDQ decoders by large margins (up to 8dB). If the source is Gaussian Markov, the complexity of the JSC-MD distributed MAP estimation algorithm is \( O(LNK) \), which is the same as the classic Viterbi algorithm for single description.

Operationally, the new MDQ decoding technique unifies the treatments of different subsets of descriptions available at a decoder, overcoming the difficulty of having a large number of side decoders that hinders the design of a good hard-decision MDQ decoder.

APPENDIX

COMPLEXITY REDUCTION OF JSC-MD PROBLEM

The complexity of the JSC-MD MAP decoding problem in Section III can be reduced because it has a strong monotonicity property, if the source is Gaussian Markovian and is coded by multiple description scalar quantizer (MDSQ). To show this we need to convert the recursion formula in Section III into a matrix search form [12]. We rewrite (10) as

\[
w(n, a) = \max_{b \in \mathbb{C}} \left\{ w(n-1, b) + \log P(a|b) + \sum_{k=1}^{K} \log P_k(y_{n,k}|\lambda_k(a)) \right\}.
\]  

Fig. 6. SNR performances of different MDQ decoders (\( \rho = 0 \)).

Fig. 7. SNR performances of different MDQ decoders (\( \rho = 0.5 \)).

Fig. 8. SNR performances of different MDQ decoders (\( \rho = 0.9 \)).
Then for each $1 \leq n \leq N$, we define an $L \times L$ matrix $A_n$ such that

$$A_n(a, b) = w(n - 1, b) + \log P(a|b) + \sum_{k=1}^{K} \log P_k(y_{k,n}|\lambda_k(a)).$$  

(30)

Now one can see that the computation task for JSC-MD MAP decoding is to find the row maxima of matrix $A_n$.

A two-dimensional matrix $A = A(a, b)$ is said to be totally monotone with respect to row maxima if the following relation holds:

$$A(a, b) \leq A(a', b) \Rightarrow A(a', b') \leq A(a', b'), \quad a < a', b < b'.$$

(31)

A sufficient condition for (31) is

$$A(a, b') + A(a', b) \leq A(a, b) + A(a', b'), \quad a < a', b < b'$$

(32)

which is also known as the Monge condition. If an $n \times n$ matrix $A$ is totally monotone, then the row maxima of $A$ can be found in $O(n)$ time [19].

To apply the linear-time matrix search algorithm to the joint source-channel MDSQ decoding problem, we only need to show that matrix $A_n$ satisfies the total monotonicity. Substituting $A_n$ in (30) for $A$ in (32), we have

$$\log P(a'|b') + \log P(a'|b) \leq \log P(a|b) + \log P(a'|b'), \quad a < a', b < b'$$

(33)

which is a sufficient condition for $A_n$ to have the total monotonicity and therefore, for the fast algorithm to be applicable. This condition, which depends only on the source statistics not the channels, is exactly the same as the one derived in [12]. It was shown by [12] that (33) holds if the source is Gaussian Markovian, which includes a large family of signals studied in practice and theory.

Finally, we conclude that the time complexity of MAP decoding of MDSQ can be reduced to $O(LNK)$ for Gaussian Markovian sequences.

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