The Rayleigh wave scattering on a rectangular lattice of the solid roughness discontinuities

Vitalii N. Chukov
Laboratory of Acoustic Microscopy, N.M. Emanuel Institute of Biochemical Physics of the Russian Academy of Sciences, Kosygin Str. 4, Moscow 119334, Russia
E-mail: vchukov@mail.ru

Abstract. The problem of the surface acoustic Rayleigh wave scattering on a deterministic three-dimensional roughness, occupying a finite size rectangular region of an isotropic solid free surface, is solved in the Rayleigh-Born approximation of the perturbation theory in a roughness amplitude. Formula for the displacement field in the scattered Rayleigh wave at a big distance from the roughness, as compared to rough region sizes $L_{1,2}$ along the $x_{1,2}$-axes respectively, and asymptotic formulas for this displacement field in the Bragg, i.e. short-wavelength $\lambda \ll L_{1,2}$ limit, where $\lambda$ is the wavelength, are derived. The new laws of scattering are obtained. They are caused by a strong modulation of scattering by the roughness form. They exceed the fundamental physical conception, that a wave scattering in the short-wavelength limit takes place on a medium discontinuities, by the statement, that a wave strongly senses the structure of a medium in the near vicinity of discontinuities as well as the form-factor of the discontinuities lattice. This form-factor is a dependence of the discontinuity amplitude, i.e. of a difference of the left and right limit values of a roughness non-zero derivative, including one of zero order, in coordinate at a point of discontinuity, on a number of this discontinuity in a lattice. This exceeded physical conception violates the classical Laue-Bragg-Wulff laws of scattering.

1. Introduction
Phenomena of the wave scattering by the different medium inhomogeneities are known to play an important role in the nature, science and industry technologies [1]-[23]. The new topological laws of scattering are obtained in [19] - [22] for the Rayleigh wave [1] scattering on a cylindrically symmetrical solid surface roughness, occupying surface region of a finite radius. It is interesting to consider the analogical problem of scattering on the solid surface roughness, occupying solid surface region in the form of the one-dimensional along the $x_1$-axis rectangle band with length $L_1$, while this band has a finite width $L_2$ along the $x_2$-axis [23]. The scattering on this simple configuration of a surface roughness, considered from the point of view of the new topological laws of scattering, can be used as elementary basis problem of scattering for the acoustoelectronics, physical acoustics and X-rays physics [9] - [17].

2. Problem of the Rayleigh wave scattering
2.1. Statement of the problem
Let the plane surface acoustic Rayleigh wave [1], [8]-[12], [17]-[23], propagating along the $x_1$-axis of a free surface of an isotropic homogeneous solid, occupying half-space $x_3 \geq 0$ of the Cartesian coordinate system $(x_1, x_2, x_3)$, is incident on the surface rough region, having the...
form of a rectangle with the finite sizes $L_1$ and $L_2$ along the $x_1$- and $x_2$-axes respectively. That is the roughness occupies a rectangle region $-L_{1,2}/2 < x_{1,2} < L_{1,2}/2$. It is described by the next function

$$x_3 = f^{(2)}(x_1, x_2) = \delta_0 f_0(x_1, x_2) = \delta_0 f_0(x_1; -\frac{L_1}{2}, \frac{L_1}{2}) f_0(x_2; -\frac{L_2}{2}, \frac{L_2}{2}) f_1(x_1),$$

where $\delta_0$ is the roughness amplitude, having dimension of a length; the step function $f_0(x,a,b) = 1$ for $a < x < b$ and 0 otherwise; $f_1(x_1)$ is arbitrary dimensionless deterministic (not statistical) function. So the roughness itself is described by the two dimensions: $x_1$ and $x_3$, and it is homogeneous in the dimension $x_2$ inside of the band $-L_{2}/2 < x_2 < L_{2}/2$ and is equal to zero out of this band. Thus at the end points of this band $x_2 = \pm L_{2}/2$ the roughness (1) has only two discontinuities in the variable $x_2$.

It is necessary to solve the problem of the Rayleigh wave scattering [17]-[23] on the roughness (1) in the Laue-Bragg-Wulff [4]-[7] short-wavelength limit $\lambda \ll L_{1,2}$. The scattered bulk waves [18, 19] are not considered in the present work.

2.2. Displacement field in the scattered Rayleigh wave

The next formula for the displacement field in the scattered Rayleigh wave is obtained in the Born approximation of the perturbation theory in $f^{(2)}(x_1, x_2)$ at a big distance $x_n \gg L_{1,2}$ from the roughness by means of the Green function, calculated by the A.A. Maradudin and D.L. Mills [17] (see [18] as well)

$$\vec{u}^{(R)}(\vec{x}, t) = A\delta_0 f_0(\vec{k}_R - \vec{k}_n^{(0)}) \frac{e^{ik_R x_n - i\omega t}}{\sqrt{x_n}} \frac{e^{i\pi/4}}{2\sqrt{2\pi R_2}} \frac{\beta c_R^2}{c_t} k_R^{5/2} \times$$

$$(\cos \varphi_s - 1)(\cos \varphi_s + \gamma)(\vec{e}_1 A_1(x_3) + i\vec{e}_3 A_2(x_3)),\quad (2)$$

where $f_0(\vec{k}_n)$ is the two-dimensional Fourier transform of a function $f_0(\vec{x}_n)$ (1) in the variables $x_1, x_2$:

$$f_0(k_n) = \int f_0(x_1, x_2) e^{-i\vec{k}_n \cdot \vec{x}_n} d^2 x_n; \quad (3)$$

vectors $\vec{e}_{1,2,3}$ are the ors of the $x_{1,2,3}$- axes respectively; $\vec{e}_n = \vec{e}_1 \cos \varphi_s + \vec{e}_2 \sin \varphi_s$; $\vec{k}_n^{(0)} = (k_R, 0, 0)$, $\vec{k}_R = k_R(\cos \varphi_s, \sin \varphi_s, 0)$ are the wave vectors of the incident and scattered Rayleigh waves respectively, $k_R = \omega/c_R$; $|\vec{k}_R - \vec{k}_n^{(0)}| = k_R\sqrt{2(1 - \cos \varphi_s)} = 2k_R \sin(\varphi_s/2)$; $\varphi_s$ is the angle of scattering, i.e. angle between the vectors $\vec{k}_n^{(0)}$ and $\vec{k}_R$; $A$ is the complex amplitude of the incident Rayleigh wave $x_1$ component [17]-[19].

By the definition intensities of the scattered Rayleigh wave horizontal $I_n$ and vertical $I_3$ components have the next forms

$$I_n = I_n^{(0)}/x_n; \quad I_3 = I_3^{(0)}/x_n, \quad (4)$$

where $I_n^{(0)}, I_3^{(0)}$ are the corresponding indicatrixes of scattering:

$$I_n,3 = (A\delta_0)^2 \left| f_0(\vec{k}_R - \vec{k}_n^{(0)}) \right|^2 \frac{k_R^5}{8\pi R_2} \frac{\beta c_R^2}{c_t} \times (\cos \varphi_s - 1)^2 (\cos \varphi_s + \gamma)^2 A_{1,2}^2(k_R x_3) =$$

$$(A\delta_0)^2 I_{n,3}^{(R)}/L_1, \quad (5)$$

where $c_{l,t}$ are the velocities of the bulk longitudinal and transverse waves respectively:

$$A_1(k_R x_3) = \exp(-\alpha k_R x_3) - \gamma \exp(-\beta k_R x_3); \quad$$

$$A_2(k_R x_3) = \alpha \exp(-\alpha k_R x_3) - (\gamma/\beta) \exp(-\beta k_R x_3);$$

$$R_2 = (\alpha^2 + \beta^2 + 2\gamma^2 - 4\gamma^3)/\gamma^2; \quad \alpha = (1 - c_R^2/c_t^2)^{1/2}; \quad \beta = (1 - c_R^2/c_l^2)^{1/2}; \quad \gamma = (1 - c_R^2/2c_t^2);$$

$\alpha\beta = \gamma^2$ is the Rayleigh wave dispersion relation, defining $c_R$ through $c_l$ and $c_t$ [8]; $I_{n,3}^{(R)}$ are the dimensionless indicatrixes of the Rayleigh wave scattering [19].
3. The new laws of the Rayleigh wave scattering in the Laue-Bragg-Wulff limit

3.1. The Rayleigh wave scattering on a rough rectangle band with the one-dimensional lattice of discontinuities

Let an arbitrary function $f_1(x_1)$ (1), describing a two-dimensional roughness along the $x_1$-axis, has $N_l + 1$ discontinuities in the next points

$$x_1 = l_1, l_2, l_3, \ldots, l_{N_l}, l_{N_l+1},$$

where $l_1 = -L_1/2$, $l_{N_l+1} = L_1/2$. $N_l + 1$ discontinuities (6) divide the rough region into $N_l$ unit cells of discontinuities lattice.

Integration by the parts gives the next expansion of the Fourier transform (3) of a function $f_1(x_1)$ in the limit $|k_1| \to \infty$

$$f_1(k_1) = \int_{-L_1/2}^{L_1/2} f_1(x_1)e^{-ik_1x_1}dx_1 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(ik_1)^{n+1}} \sum_{m=1}^{N_l+1} e^{-ik_1l_m} \left( \frac{d^n}{dx_1^n} f_1(x_1) \right)_{l_m}^{l_{m+1}}. \quad (7)$$

Expansion (7) is analog of the cylindrically symmetrical function $f_0(x_i)$ the Fourier transform $f_0(k)$ expansion (formula (28) in [19]).

The function $f_0(x_1, x_2)$ (1) has the next Fourier transform

$$f_0(k_i) = \frac{2}{k_2} f_1(k_1) \sin \left( \frac{k_2L_2}{2} \right), \quad (8)$$

where $f_1(k_1)$ is given by the formula (7), when $|k_1| > 0$.

Let us consider a periodical one-dimensional lattice of discontinuities

$$l_m^{(k)} = -\frac{L_1}{2} + ak + \frac{(m-2)}{N_l} L_1, \quad 0 \leq ak \leq \frac{L_1}{N_l},$$

$m = 2, 3, \ldots, N_l + 1; \quad k = 1, 2, \ldots, N^{(c)}$.

Discontinuities, described by inner coordinates $a_k, k = 1, 2, \ldots, N^{(c)}$, define the structure of a lattice unit cell; $m$ designates different unit cells; it is the number of boundary discontinuities, separating $m - 1$ unit cells. Summation over all the discontinuities (9) in the roughness form-factor (7), (8), using the formula for a geometrical progression sum [24] over unit cell number $m$, gives the next main term in the variable $|k_1|/(L_1/N_l) \gg 1$ for the indicatrix of scattering (4), (5) on the periodical lattice of discontinuities (9)

$$I_{n,3}^{(R)} = \beta^2 C_R^4 \frac{p_{N_l}}{c_l^2} \left( \frac{L_2/L_1}{2} \right)^2 S_{N_l}(q_1, q_2) \frac{L_1}{2^{N_l-3}} (q_1L_1/N_l)^2 A_{1,2}^{2}(k R x_3)(1 - \cos \varphi_s)^2(\gamma + \cos \varphi_s)^2 =$$

$$= \beta^2 C_R^4 \frac{p_{N_l}}{c_l^2} \left( \frac{L_1}{N_l} \right)^{2N_l-2} \frac{A_{1,2}^{2}(k R x_3)S_{N_l}(q_1, q_2)(1 - \cos \varphi_s)^2(\gamma + \cos \varphi_s)^2}{\varphi_s \neq \pi}, \quad (10)$$

where $p_{N_l} = k_R(L_1/N_l),

$$S_{N_l}(q_1, q_2) = S_{N_l}^{(f)}(q_1, q_2);$$

$$S_{N_l}^{(f)}(q_1, q_2) = S_{c}(q_1, N^{(c)}) \sin \left( \frac{q_2L_2}{2} \right) \sin \left( \frac{q_1L_1}{2N_l} \right);$$

$$\lim_{\frac{q_1L_1}{2N_l} \to -\pi n} S_{N_l}^{(f)}(q_1, q_2) = N_l S_{c}(q_1, N^{(c)}) \sin \left( \frac{q_2L_2}{2} \right) \sin \left( \frac{q_1L_1}{2N_l} \right), \quad n = 0, 1, 2, \ldots;$$

$$\frac{q_1L_1}{2N_l} \to -\pi n$$
\[ S_c(k_1, N^{(c)}) = \left( \sum_{k=1}^{N^{(c)}} F_k^{(c)}(n_d) \cos(k_1 a_k) \right)^2 + \left( \sum_{k=1}^{N^{(c)}} F_k^{(c)}(n_d) \sin(k_1 a_k) \right)^2 \]

is a contribution to scattering, defined by a lattice unit cell form-factor;

\[ F_k^{(c)}(n_d) = L_1^{n_d} \left( \frac{d^{n_d}_1 f_1(x_1)}{dx_1} \right) \bigg|_{l_m^{(k)}=0}^{l_m^{(k)}}, \ m = 2, \ k = 1, 2, \ldots, N^{(c)} \] (11)

are dimensionless amplitudes of the \( n_d \)-order derivatives of the function \( f_1(x_1) \) discontinuities. They are the same for each unit cell. Natural number \( n_d \) is the smallest order of a non-zero derivative of the roughness \( f_1(x_1) \) (7) at least at any one point of discontinuity. It is the same for all the discontinuities of the periodical lattice in the consideration, even if the amplitude (11), corresponding to the concrete another discontinuity of this order \( n_d \) is equal to zero. Vector \( \vec{q}(k_R, \varphi_s) \) is the wave vector, transmitted from the incident to the scattered Rayleigh wave in a scattering:

\[ \vec{q}(k_R, \varphi_s) = \vec{k}_R - \vec{k}_{n=0}^{(0)} = (q_1(k_R, \varphi_s), q_2(k_R, \varphi_s)) \]; \( q_1(k_R, \varphi_s) = - k_R (1 - \cos \varphi_s) = -2 k_R \sin^2(\varphi_s/2); \)

\[ q_2(k_R, \varphi_s) = 2 k_R \cos(\varphi_s/2) \sin(\varphi_s/2) = k_R \sin \varphi_s; \]

\[ q_1(k_R, \varphi_s)/q_2(k_R, \varphi_s) = - \tan(\varphi_s/2). \] (12)

On the analogy of [19] an applicability condition of the formula (10) has the next form

\[ |q_1/L_1/N_l | \gg 1. \] (13)

So the forward scattering \( \varphi_s = 0 \), when the transmitted wave vector \( \vec{q}(k_R, \varphi_s) = 0 \), is not considered in the present work.

Formula (10) gives the new laws of the Rayleigh wave diffuse scattering \( |q_1|/L_1/N_l \gg 1 \) for a rectangle periodical lattice of a solid surface roughness discontinuities, containing \( N_l \) unit cells along the \( x_1 \)-axis and bounded only by the two ends \( x_2 = \pm L_2/2 \) along the \( x_2 \)-axis. It is analogous of the diffuse scattering laws for a cylindrically symmetrical roughness [19]. It is the diffuse scattering law for lattice of a roughness discontinuities or for a single continuous roughness, occupying rectangle region of a surface when \( N_l = 1 \).

The next frequency law follows from the (10) for the lateral diffuse scattering \( \varphi_s \neq \pi \) \( (x_3 = 0) \)

\[ I_{i,3}^{(R)} \sim P_{N_l}^{1-2n_d} S_{N_l}(q_1, q_2), \quad \varphi_s \neq \pi, \] (14)

and for the backward diffuse scattering \( \varphi_s = \pi \) \( (x_3 = 0) \), when \( q_2 = 0 \); this new law has the next form due to frontal incidence of the Rayleigh wave on the discontinuities lattice

\[ I_{i,3}^{(R)} \sim P_{N_l}^{3-2n_d} S_{N_l}(q_1, q_2), \quad \varphi_s = \pi. \] (15)

The frequency laws of the diffuse scattering, obtained in [19], and (14), (15) differ from each other because of different geometry of the Rayleigh wave scattering.

3.2. The Rayleigh wave scattering on the lattice of discontinuities, having a single unit cell

Let us consider the next special simple form of the discontinuities lattice (9)

\[ N^{(c)} = 2, \quad a_1 = 0, \quad a_2 = L_1/N_l, \quad F_1^{(c)} = F_2^{(c)} = F_0^{(c)}, \] (16)

According to a lattice form-factor (7), (8) and this special lattice model (9), (16) all the inner discontinuities of the lattice \( -L_1/2 + (m-1) L_1/N_l, m = 2, 3, \ldots, N_l \) are summarized twice in the sum (10) over discontinuities coordinates \( l_m \) (7), (9), but the end discontinuities \( x_1 = \pm L_1/2 \) - only once. It corresponds to the double inner discontinuities amplitude \( 2 F_0^{(c)}(n_d) \), but the discontinuities amplitude at the end points \( \pm L_1/2 \) is equal to \( F_0^{(c)}(n_d) \). Unit cell form-factor (10) of the lattice (16) gives the next expression

\[ S_c(k_1, N^{(c)}) = 2 F_0^{(c)}(n_d) \cos \left( \frac{k_1 L_1}{2 N_l} \right). \] (17)

Indicatrix of scattering (10) - (13) for the discontinuities lattice (16) gives the next resonance reciprocal lattice in the space of the transmitted wave vectors \( \vec{q} = (q_1, q_2) \).
\[
\hat{q}(n_1, n_2) = -\frac{2\pi n_1 a_1^{(N)}(n_1)}{(a_1^{(N)})^2} \pm \frac{2\pi(n_2 + 1/2) a_2^{(1)}}{(a_2^{(1)})^2}, \tag{18}
\]

\[
n_1 = 1, 2, \ldots; \ n_2 = -1/2, 0, 1, 2, \ldots,
\]

where

\[
\vec{a}_1^{(N)} = \frac{L_1}{N_l} \vec{e}_1; \ \vec{a}_2^{(1)} = L_2 \vec{e}_2 \tag{19}
\]

are the basis vectors of the discontinuities lattice (16); \(\vec{e}_1, \vec{e}_2\) are the orts of the \(x_1\)- and \(x_2\)-axes respectively (2).

It follows from (18), that the next necessary condition

\[
\hat{q} \vec{a}_1^{(N_i)} = q_1(L_1/N_l) = -2\pi n_1, \ n_1 = 1, 2, \ldots \tag{20}
\]

of the Laue-Bragg-Wulff resonances (18) is caused by the one-dimensional lattice of discontinuities, arranged along the \(x_1\)-axis, contrary to the phenomenon of the Laue-Bragg-Wulff condition and resonances mismatch for a cylindrically symmetrical lattice of discontinuities (formula (94) in the [19] and text below it).

Existence of the roughness (9), (16) two ends, arranged along the \(x_2\)-axis, adds the second necessary condition

\[
\hat{q} \vec{a}_2^{(1)} = q_2 L_2 = \pm 2\pi(n_2 + 1/2), \ n_2 = -1/2, 0, 1, 2, \ldots \tag{21}
\]

of the classical Laue-Bragg-Wulff resonances in the \((q_1, q_2)\) space. This second condition (21), that is a finiteness of roughness along the \(x_2\)-axis, modifies the positions of the global classical Laue-Bragg-Wulff resonances, caused by the condition (20), and thus violates the classical Laue-Bragg-Wulff law (20) and in particular the Laue conditions [4 - 7] of scattering.

**Figure 1.** The classical Laue-Bragg-Wulff resonances of the Rayleigh wave scattering on the rectangular lattice of discontinuities (1)-(10), (16). \(N_l = 10, n_d = 0, \varphi_s = \pi/2, P_0^{(c)} = 1, L_2/L_1 = 21/20, m_l = 1/21 (22), \rho_{p_{N_l}}^{(RC)} = 10\pi; I_3^{(RC)} = I_3^{(R)}.\) The Poisson’s ratio \(\sigma = 0.25\) everywhere.

The nodes of the reciprocal lattice (18) and conditions (20), (21), corresponding to them, give the next relation between the angle of the lateral scattering \(\varphi_s^{(max)} \neq \pi\), for which the indicatrix of scattering (10) passes through the classical Laue-Bragg-Wulff resonances, defined by the integer \(n_1\) and \(n_2\) (18), and the parameters of the rectangular discontinuities lattice

\[
\tan\left(\frac{\varphi_s^{(max)}}{2}\right) = \pm \frac{n_1}{(n_2 + 1/2) (L_2/L_1)} \equiv \pm 2m_l \frac{L_2}{L_1/N_l}, \ \varphi_s^{(max)} \neq \pi, \tag{22}
\]

\[
\ m_l = n_1/(2n_2 + 1); \ n_1 = 1, 2, \ldots; \ n_2 = 0, 1, 2, \ldots; \ N_l = 1, 2, 3, \ldots.
\]

This condition (22) is a necessary condition of the classical Laue-Bragg-Wulff resonances (18) for the lateral scattering, when \(\varphi_s \neq \pi\). Analogical relation for the backward scattering \(\varphi_s^{(max)} = \pi\) is given by the formula (20), when \(q_2 \equiv 0\).

Figs. 1, 2 illustrate the laws of the Rayleigh wave scattering on the rectangular discontinuities lattice (6)-(10), (16) of a solid surface roughness (1). The condition (21) is satisfied as well as
(20) only for odd values of the $n_1 = 1, 3, 5, \ldots$ and gives the next values of the $n_2 = 10, 31, 52, \ldots$ respectively. Even values of the $n_1 = 2, 4, 6, \ldots$ give $n_2 = 41/2, 83/2, 125/2, \ldots$ according to (18)-(21). These values of the $n_2$ correspond to the integer value of the $(n_2 + 1/2)$ in the conditions (18), (21) and exactly give zeroes of the indicatrix of scattering (10). For the indicatrix of scattering maxima, presented in the Fig.2, the classical Laue-Bragg-Wulff resonances, corresponding to the nodes of the reciprocal lattice (18), have the next numbers $n_1=1, 3, 5, \ldots$, while $n_1=2, 4, 6, \ldots$ correspond to the exact zeroes of the indicatrix of scattering. These zeroes (Figs. 1, 2) are caused by existence of the rough band ends $x_2 = \pm L_2/2$, arranged along the $x_2$-axis. These end discontinuities of a roughness have the amplitudes analogous to (11), which are equal in absolute value to $|\delta_0 f_1(x_1)|$ but have opposite sign. These phenomena (18) are violation of the classical Laue-Bragg-Wulff law of scattering [4]-[7], [13, 14] due to opposite sign of the amplitudes of discontinuities at the points $x_2 = \pm L_2/2$.

4. Conclusion

The new phenomena of a strong modulation of the surface acoustic Rayleigh wave scattering by the form of a deterministic (non-statistical) roughness profile, occupying the region of a solid surface in the form of the rectangle band, having the finite sizes $L_{1,2}$ along the $x_{1,2}$-axes respectively, are obtained and investigated theoretically in the Laue-Bragg-Wulff limit $\lambda \ll L_{1,2}$. The new laws of scattering are obtained. It is obtained first from the first principles of the dynamical theory of elasticity [8] that amplitude form-factor (11) of the roughness discontinuities lattice has dominant influence on the laws of the wave scattering in the Laue-Bragg-Wulff limit, up to violation of the classical Laue-Bragg-Wulff laws [4]-[7] of the wave scattering.

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