Teleparallel Energy-Momentum Distribution of Static Axially Symmetric Spacetimes

M. Sharif * and M. Jamil Amir †
Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore-54590, Pakistan.

Abstract

This paper is devoted to discuss the energy-momentum for static axially symmetric spacetimes in the framework of teleparallel theory of gravity. For this purpose, we use the teleparallel versions of Einstein, Landau-Lifshitz, Bergmann and Möller prescriptions. A comparison of the results shows that the energy density is different but the momentum turns out to be constant in each prescription. This is exactly similar to the results available in literature using the framework of General Relativity. It is mentioned here that Möller energy-momentum distribution is independent of the coupling constant λ. Finally, we calculate energy-momentum distribution for the Curzon metric, a special case of the above mentioned spacetime.

Keywords: Energy Momentum, Weyl Metrics

1 Introduction

Among all available theories of gravitation in the literature, General relativity (GR) has been accepted as a true theory of gravitation as many physical aspects of nature have been experimentally verified in this theory. However,
the localization of energy and momentum [1] in GR is still an open, unresolved and disputed problem. In GR, many attempts have been made to solve this problem but no definition has generally been accepted till now. As a pioneer, Einstein [2] used the notion of energy-momentum complex to solve this problem. Following Einstein, many scientists like Landau-Lifshitz [3], Papapetrou [4], Bergmann [5], Tolman [6], Weinberg [7] and Möller [8] have introduced their own energy-momentum complex. All these prescriptions, except Möller’s, are restricted to do calculations in Cartesian coordinates only. But this difficulty was removed in Möller’s prescription. Also, we can not define angular momentum with the help of all these prescriptions. Misner et al. [1] showed that energy can only be localized in spherical systems. But later on, Cooperstock and Sarracino [9] proved that if energy is localizable for spherical systems, then it can be localized in any system. Bondi [10] argued that a non-localizable form of energy is not allowed in GR.

After this, the idea of quasi-local energy was introduced by Penrose and other scientists [11-14]. In this method, one can use any coordinate system while finding the quasi-local masses to obtain the energy-momentum of a curved spacetime. Bergqvist [15] considered seven different definitions of quasi-local masses and showed that no two of these definitions gave the same result. Chang et al. [16] proved that every energy-momentum complex can be associated with a particular Hamiltonian boundary term. Thus the energy-momentum complexes may also be considered as quasi-local. Xulu [17-19] extended this investigation and found the same energy distribution in the case of Melvin magnetic and Bianchi type I universe.

Virbhadra and his collaborators [20-23] verified for asymptotically flat spacetimes that different energy-momentum complexes can give the same result for a given spacetime. They also found encouraging results for the case of asymptotically non-flat spacetimes by using different energy-momentum complexes. Aguirregabiria et. al. [24], by using the Einstein, Landau Lifshitz, Papapetrou, Bergmann, and Weinberg (ELLPBW) prescriptions, showed that the energy distribution within a Kerr-Schild metric is same. Recently Virbhadra [25] found that these five different prescriptions (ELLPBW) did not give the same results for the most general non-static spherically symmetric spacetime. One of the authors [26-28] found several examples which do not provide the same result for different prescriptions. The results [19,21,23,25-29] lead to know that the energy distribution in Möller’s prescription is different from Einstein’s energy for some particular spacetimes, including Schwarzschild spacetime.
Some authors [30-36] argued that this problem of energy may be settled in the context of teleparallel theory (TPT) of gravity. They showed that energy-momentum can also be localized in the framework of this theory. It has been shown that the results of the two theories agree with each other. Vargas [32] found that the total energy of the closed Friedmann-Robertson-Walker spacetime is zero by using teleparallel version of Einstein and Landau-Lifshitz complexes. This agrees with the result obtained by Rosen [37] in GR. Salti and his co-workers [33-36] considered some particular spacetimes and calculated energy-momentum densities by using different prescriptions both in GR and TPT and found the similar results. Recently, Sharif and Amir [38] evaluated the energy-momentum distribution of Lewis-Papapetrou spacetimes by using the TP version of Möller's prescription and found that the results do not agree with those available in the context of GR [39]. In this paper, we investigate energy-momentum distribution for the Weyl metrics in the context of the TPT. Further, it has been extended to the special case as a Curzon metric. We also compare our results with [40-42] in the context of GR.

The scheme of this paper is as follows. In section 2, we give some basics of TPT and TP version of Einstein, Landau-Lifshitz, Bergmann and Möller prescriptions. Section 3 is devoted to the evaluation of the energy-momentum density components for static axially symmetric spacetimes and also for the Curzon metric. In the last section, we shall summarize the results.

2 TP Version of Energy-Momentum Complexes

Before giving the TP version of the energy-momentum complexes, we briefly outline the main points of the TP theory. The basic entity of the theory of teleparallel gravity (TPG) is the non-trivial tetrad [43] $h^a \mu$ whose inverse is denoted by $h_a^\nu$. They satisfy the following relations

$$h^a \mu h_a^\nu = \delta_\mu^\nu; \quad h^a \mu h_b^\mu = \delta^a_b.$$  \hspace{1cm} (1)

The theory of TPG is described by the Weitzenböck connection given as

$$\Gamma^a_{\mu \nu} = h_a^\theta \partial_\nu h^a_{\mu}$$  \hspace{1cm} (2)

which is obtained due to the condition of absolute parallelism [44]. This implies that the spacetime structure underlying a translational gauge theory
is naturally endowed with a teleparallel structure [44,45]. In this paper, the Latin alphabet \((a,b,c,... = 0,1,2,3)\) will be used to denote the tangent space indices and the Greek alphabet \((\mu,\nu,\rho,... = 0,1,2,3)\) to denote the spacetime indices. The Riemannian metric in TPT arises as a by product [44] of the tetrad field given by

\[
g_{\mu\nu} = \eta_{ab} h^a_\mu h^b_\nu, \tag{3}\]

where \(\eta_{ab}\) is the Minkowski spacetime such that \(\eta_{ab} = \text{diag}(+1,-1,-1,-1)\). In TPT, the gravitation is attributed to torsion [45] which plays the role of force here. For the Weitzenböck spacetime, the torsion is defined as [46]

\[
T^\theta_{\mu\nu} = \Gamma^\theta_{\nu\mu} - \Gamma^\theta_{\mu\nu} \tag{4}\]

which is antisymmetric in nature. Due to the requirement of absolute parallelism, the curvature of the Weitzenböck connection vanishes identically [43]. The Weitzenböck connection and the Christoffel symbols satisfy the following relation

\[
\Gamma^0_{\mu\nu} = \Gamma^\theta_{\mu\nu} - K^\theta_{\mu\nu}, \tag{5}\]

where \(\Gamma^\theta_{\mu\nu}\) are the Christoffel symbols and \(K^\theta_{\mu\nu}\) denotes the contorsion tensor and is given by

\[
K^\theta_{\mu\nu} = \frac{1}{2} \left[ T^\theta_{\mu\nu} + T^\theta_{\nu\mu} - T^\theta_{\lambda\mu} \right]. \tag{6}\]

The teleparallel version of the Einstein, Landau-Lifshitz and Bergmann energy-momentum complexes, by setting \(c = 1 = G\), are respectively given by [32]

\[
hE^\mu_\nu = \frac{1}{4\pi} \partial_\lambda (U^\mu_{\nu\lambda}),
\]

\[
hL^\mu_{\nu\rho} = \frac{1}{4\pi} \partial_\lambda (hg^{\mu\beta} U^\nu_{\beta\lambda}),
\]

\[
hB^\mu_{\nu\rho} = \frac{1}{4\pi} \partial_\lambda (g^{\mu\beta} U^\nu_{\beta\lambda}), \tag{7}\]

where \(U^\mu_{\nu\lambda}\) is the Freud’s superpotential given as

\[
U^\mu_{\nu\lambda} = hS^\mu_{\nu\lambda}. \tag{8}\]

Here \(S^\mu_{\nu\lambda}\) is a tensor quantity which is skew symmetric in its last two indices and is defined as

\[
S^\mu_{\nu\lambda} = m_1 T^\mu_{\nu\lambda} + \frac{m_2}{2} (T^\mu_{\rho\lambda} - T^\lambda_{\rho\mu}) + \frac{m_3}{2} (g^{\mu\lambda} T^\beta_{\rho\mu} - g^{\rho\nu} T^{\beta\lambda}_{\mu}), \tag{9}\]
where $m_1$, $m_2$ and $m_3$ are three dimensionless coupling constants of TPG [35]. It is mentioned here that $hE^0_0$, $hL^{00}$, $hB^{00}$ are the energy densities, $hE^0_i$, $hL^{0i}$, $hB^{0i}$ ($i = 1, 2, 3$) are the momentum densities and $hE^0_0$, $hL^{i0}$, $hB^{i0}$ are the current energy densities of Einstein, Landau-Lifshitz and Bergmann prescriptions respectively. Teleparallel equivalent of GR may be obtained by considering the following particular choice [44]

$$m_1 = \frac{1}{4}, \quad m_2 = \frac{1}{2}, \quad m_3 = -1. \quad (10)$$

The superpotential of the Möller tetrad theory is given by Mikhail et al. [30] as

$$U_{\mu \nu} = \sqrt{-g} \frac{2\kappa}{\kappa} P_{\chi \rho \sigma} \Phi^\rho g^{\sigma \chi} g_{\mu \tau} - \lambda g_{\tau \mu} K^{\chi \rho \sigma} - (1 - 2\lambda) g_{\tau \mu} K^{\sigma \rho \chi}, \quad (11)$$

where

$$P_{\chi \rho \sigma} = \delta_{\chi \tau} g^{\nu \beta} + \delta_{\rho \tau} g^{\nu \beta} - \delta_{\sigma \tau} g^{\nu \beta}, \quad (12)$$

while $g^{\nu \beta}$ is a tensor quantity and is defined by

$$g^{\nu \beta} = \delta_{\nu \beta} - \delta_{\nu \sigma} \delta_{\beta \rho}. \quad (13)$$

$K^{\sigma \rho \chi}$ is contortion tensor as given by Eq.(6), $g$ is the determinant of the metric tensor $g_{\mu \nu}$, $\lambda$ is the free dimensionless coupling constant of TPG, $\kappa$ is the Einstein constant and $\Phi_{\mu}$ is the basic vector field given by

$$\Phi_{\mu} = T^{\nu}_{\nu \mu}. \quad (14)$$

Now we can write the Möller energy, momentum and energy current densities as follows

$$\Xi^\mu = U_{\nu \rho}^{\mu \nu}, \quad (15)$$

where comma means ordinary differentiation. Here $\Xi^0$, $\Xi^i_0$ and $\Xi^i_0$ are the energy, momentum and energy current densities respectively in Möller’s prescription.

### 3 Static Axially Symmetric Spacetimes

The Weyl metrics are a subclass of stationary axially symmetric spacetimes. These metrics can be reduced from the Lewis-Papapetrou metric [47] (a class
of stationary axially symmetric spacetimes) by vanishing the angular velocity. In GR, Weyl exterior solutions to the Einstein field equations represent all possible static axially symmetric spacetimes [48]. They may be represented as series expansions of suitable defined relativistic multipole moments [49]. Thus each Weyl metric is characterized by a specific combination of such multipoles. It would be interesting to investigate energy-momentum distribution for this class of spacetimes. In cylindrical coordinates \((\rho, \phi, z)\), it is given by

\[
ds^2 = e^{2\psi} \, dt^2 - e^{2(\gamma - \psi)} (d\rho^2 + dz^2) - \rho^2 e^{-2\psi} \, d\phi^2,
\]

where \(\gamma\) and \(\psi\) are functions of \(\rho\) and \(z\) only. The metric functions satisfy the following constraint equations

\[
\psi_{\rho\rho} + \frac{1}{\rho} \psi_{\rho} + \psi_{zz} = 0,
\]

\[
\gamma_{\rho} = \rho (\psi_{\rho}^2 - \psi_z^2), \quad \gamma_z = 2 \rho \psi_{\rho} \psi_z.
\]

Eq.(17) implies that the function \(\psi\) satisfies Laplace equation. The general solution of this equation yields an asymptotic behavior and is given by

\[
\psi = \sum \frac{a_n}{r^{n+1}} P_n(\cos\theta),
\]

where \(r = \sqrt{\rho^2 + z^2}\), \(\cos\theta = z/r\) are Weyl spherical coordinates and \(P_n(\cos\theta)\) are Lagendre polynomials. The coefficients \(a_n\) are arbitrary constants which are called Weyl moment.

### 3.1 Energy-Momentum Densities of Einstein, Landau-Lifshitz and Bergmann Prescriptions

Since these prescriptions can give meaningful results only in Cartesian coordinates thus we need to write tetrad in terms of Cartesian coordinates. This can be obtained by writing Eq.(16) as

\[
ds^2 = e^{2\psi} \, dt^2 - \frac{1}{\rho^2} (x^2 e^{2(\gamma - \psi}) + y^2 e^{-2\psi}) \, dx^2 - \frac{2xy}{\rho^2} (e^{2(\gamma - \psi}) - e^{-2\psi}) \, dxdy
\]

\[
- \frac{1}{\rho^2} (y^2 e^{2(\gamma - \psi}) + x^2 e^{-2\psi}) \, dy^2 - e^{2(\gamma - \psi)} dz^2,
\]

(20)
where $\rho = \sqrt{x^2 + y^2}$. The corresponding tetrad can be written as

$$h^a_{\mu} = \begin{bmatrix}
  e^\psi & 0 & 0 & 0 \\
  0 & \frac{z}{\rho} e^{\gamma - \psi} & \frac{y}{\rho} e^{\gamma - \psi} & 0 \\
  0 & -\frac{y}{\rho} e^{-\psi} & \frac{z}{\rho} e^{-\psi} & 0 \\
  0 & 0 & 0 & e^{\gamma - \psi}
\end{bmatrix} \quad (21)$$

and its inverse becomes

$$h_a^\mu = \begin{bmatrix}
  e^{-\psi} & 0 & 0 & 0 \\
  0 & \frac{z}{\rho} e^{-\gamma - \psi} & \frac{y}{\rho} e^{-\gamma - \psi} & 0 \\
  0 & -\frac{y}{\rho} e^{-\psi} & \frac{z}{\rho} e^{-\psi} & 0 \\
  0 & 0 & 0 & e^{\gamma - \psi}
\end{bmatrix}. \quad (22)$$

Here $h = \det \{h^a_{\mu}\} = \sqrt{-g} = e^{2\gamma - 2\psi}$. Using Eqs.(21) and (22) in Eq.(2), we get the following non-zero components of the Weitzenböck connection

$$\Gamma^0_{01} = \frac{x}{\rho} \psi, \quad \Gamma^0_{02} = \frac{y}{\rho} \psi, \quad \Gamma^0_{03} = \psi_z,$$

$$\Gamma^0_{11} = \frac{x}{\rho^3}(x^2 \gamma - \rho^2 \psi), \quad \Gamma^1_{12} = \frac{y}{\rho^3}(x^2 \gamma - \rho^2 \psi),$$

$$\Gamma^1_{13} = \frac{1}{\rho^2}(x^2 \gamma - \rho^2 \psi), \quad \Gamma^1_{21} = \frac{y}{\rho^3}(x^2 \gamma - \rho),$$

$$\Gamma^1_{22} = \frac{x}{\rho^3}(y^2 \gamma + \rho), \quad \Gamma^1_{23} = \Gamma^2_{13} = \frac{x y}{\rho^2} \gamma_z,$$

$$\Gamma^2_{11} = \frac{y}{\rho^3}(x^2 \gamma + \rho), \quad \Gamma^2_{12} = \frac{x}{\rho^3}(y^2 \gamma - \rho),$$

$$\Gamma^2_{22} = \frac{y}{\rho^3}(y^2 \gamma - \rho^2 \psi), \quad \Gamma^2_{23} = \frac{1}{\rho^2}(y^2 \gamma - \rho^2 \psi),$$

$$\Gamma^2_{21} = \frac{x}{\rho^3}(y^2 \gamma - \rho^2 \psi), \quad \Gamma^3_{33} = \gamma_z - \psi_z,$$

$$\Gamma^3_{31} = \frac{x}{\rho} (\gamma - \psi), \quad \Gamma^3_{32} = \frac{y}{\rho} (\gamma - \psi). \quad (23)$$

The corresponding non-vanishing components of the torsion tensor are

$$T^0_{01} = -\frac{x}{\rho} \psi = -T^0_{10}, \quad T^0_{02} = -\frac{y}{\rho} \psi = -T^0_{20},$$

$$T^0_{03} = -\psi_z = -T^0_{30}, \quad T^1_{12} = \frac{y}{\rho^2} (\rho \psi - 1) = -T^1_{21},$$

7
\(T_{13}^1 = \frac{1}{\rho^2}(\rho^2 \psi_z - x^2 \gamma_z) = -T_{31}^1, \quad T_{12}^2 = \frac{x}{\rho^2}(1 - \rho \psi_\rho) = -T_{21}^2,\)

\(T_{13}^2 = T_{12}^3 = -\frac{x y}{\rho^2} \gamma_z = -T_{31}^3 = -T_{12}^2,\)

\(T_{32}^3 = \frac{y}{\rho}(\psi_\rho - \gamma_\rho) = -T_{23}^3, \quad T_{31}^3 = \frac{x}{\rho}(\psi_\rho - \gamma_\rho) = -T_{13}^3,\)

\(T_{23}^2 = \frac{1}{\rho^2}(\rho^2 \psi_z - y^2 \gamma_z) = -T_{32}^2.\) \(\tag{24}\)

The required components of the Freud’s superpotential are

\[U_{01}^0 = \frac{x}{2\rho}(2\psi_\rho - \gamma_\rho - \frac{1}{\rho}),\]

\[U_{02}^0 = \frac{y}{2\rho}(2\psi_\rho - \gamma_\rho - \frac{1}{\rho}),\]

\[U_{03}^0 = \frac{1}{2}(2\psi_z - \gamma_z).\] \(\tag{25}\)

When we use these values in Eq.(7), we obtain energy-momentum density components given in table (1)

**Table 1.** Energy-Momentum (E-M) densities in different prescriptions \((i=1,2,3)\)

| Prescription       | E. density                                           | M. density                                           |
|--------------------|------------------------------------------------------|------------------------------------------------------|
| Einstein           | \(hE_0^{01} = -\frac{1}{8\pi}(\gamma_{\rho\rho} + \gamma_{zz} + \frac{1}{\rho} \gamma_\rho)\) | \(hE_0^{0i} = 0,\)                                   |
| Landau-Lifshitz    | \(hL_0^{00} = \frac{e}{8\pi} \left[\frac{2}{\rho}(\psi_\rho - \frac{2}{\rho} \gamma_\rho - 2(\gamma_\rho - 2\psi_\rho)^2 - 2(\psi_z - 2\psi_z)^2 - (\gamma_{\rho\rho} + \gamma_{zz} + \frac{1}{\rho} \gamma_\rho)\right]\) | \(hL_0^{0i} = 0,\)                                   |
| Bergmann           | \(hB_0^{00} = \frac{e}{8\pi} \left[\frac{2}{\rho}(\gamma_\rho \psi_\rho + \gamma_z \psi_z + \frac{1}{\rho} \psi_\rho - 2(\psi_\rho + \psi_z^2)) - (\gamma_{\rho\rho} + \gamma_{zz} + \frac{1}{\rho} \gamma_\rho)\right]\) | \(hB_0^{0i} = 0,\)                                   |

We see that energy takes a well-defined and definite form in each prescription while momentum becomes constant.
3.2 Energy-Momentum Densities in Möller Prescription

By following the same procedure as given in \[50,51\], we can write tetrad of the metric (16) as

\[
h_{\mu}^{\alpha} = \begin{bmatrix}
e^{\psi} & 0 & 0 & 0 \\
0 & e^{\gamma-\psi}\cos\theta & -\rho e^{\psi}\sin\theta & 0 \\
0 & e^{\gamma-\psi}\sin\theta & \rho e^{\psi}\sin\theta & 0 \\
0 & 0 & 0 & e^{\gamma-\psi}
\end{bmatrix}
\]

with its inverse

\[
h_{\alpha}^{\mu} = \begin{bmatrix}
e^{-\psi} & 0 & 0 & 0 \\
0 & e^{-\psi}\gamma & -\frac{1}{\rho} e^{\psi}\sin\theta & 0 \\
0 & e^{-\psi}\sin\theta & \frac{1}{\rho} e^{\psi}\cos\theta & 0 \\
0 & 0 & 0 & e^{-\psi-\gamma}
\end{bmatrix}.
\]

Using Eqs.(26) and (27) in Eq.(2), we get the following non-vanishing components of the Weitzenböck connection

\[
\Gamma_{01}^0 = \psi_{\rho}, \quad \Gamma_{03}^0 = -\Gamma_{23}^2 = \psi_z, \quad \Gamma_{21}^2 = \frac{1}{\rho} - \psi_{\rho},
\]

\[
\Gamma_{11}^1 = \Gamma_{31}^3 = \gamma_{\rho} - \psi_{\rho}, \quad \Gamma_{12}^2 = \frac{1}{\rho} e^\gamma, \quad \Gamma_{12}^2 = -\rho e^{-\gamma},
\]

\[
\Gamma_{13}^1 = \Gamma_{33}^3 = \gamma_z - \psi_z
\]

The corresponding components of the torsion tensor and the basic vector field will become

\[
T_{01}^0 = -\psi_{\rho}, \quad T_{03}^0 = \Gamma_{23}^2 = -\psi_z, \quad T_{12}^2 = \frac{1}{\rho} (1 - e^\gamma) - \psi_{\rho},
\]

\[
T_{13}^1 = T_{01}^0 = \psi_z - \gamma_z, \quad T_{31}^3 = \psi_{\rho} - \gamma_{\rho}
\]

and

\[
\Phi^1 = e^{2(\psi-\gamma)} \{ \gamma_{\rho} - \psi_{\rho} + \frac{1}{\rho} (1 - e^\gamma) \}, \quad \Phi^3 = e^{2(\psi-\gamma)} (\gamma_z - \psi_z)
\]

respectively. The required non-vanishing components of the superpotential in Möller tetrad theory are

\[
U_{0}^{01} = \frac{1}{\kappa} (2 \rho \psi_{\rho} - \rho \gamma_{\rho} + e^\gamma - 1),
\]

\[
U_{0}^{03} = \frac{\rho}{\kappa} (2 \psi_z - \gamma_z).
\]
Substituting these results in Eq.(15) and $c, G = 1$, it yields energy and momentum densities in Möller’s prescription

$$\Xi_0^0 = \frac{1}{8\pi}(2\rho\psi^2 + \gamma\rho^e \gamma),$$
$$\Xi_i^0 = 0, \quad \Xi_i^i = 0. \tag{32}$$

This shows that momentum becomes constant and the energy density turns out as a definite and well-defined quantity. If we take $\gamma \rho^e \gamma = -2\rho\psi^2$ then energy also becomes constant which coincides with the energy in GR [40-42]. It is worth mentioning here that the results are independent of the coupling constant $\lambda$, that is, these results are valid for any teleparallel theory.

The Curzon metric [52] is a special case of static axially symmetric space-times and can be obtained by substituting

$$\gamma(\rho, z) = -\frac{m^2 \rho^2}{2(\rho^2 + z^2)^2}, \quad \psi(\rho, z) = -\frac{m}{\sqrt{\rho^2 + z^2}} \tag{33}$$

in Eq.(16). The energy and momentum density components turn out in a simple form as given in the table (2).

| Prescription    | E. density                                                                 | M. density                                                                 |
|-----------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Einstein        | $hE_0^0 = \frac{m^2 \rho^2}{4\pi \rho^2} \left[ 2 - \frac{m^2 \rho^2 - 3\rho^2}{r^4} - 4\rho^2 - \frac{3m^2 \rho^2 - 3m^2}{r^4} \right]$ | $hE_i^0 = 0$                                                               |
| Landau-Lifshitz | $hL_0^0 = \frac{m^2 \rho^2}{4\pi \rho^2} \left[ 2 - \frac{m^2 \rho^2 - 3\rho^2}{r^4} - 4\rho^2 - \frac{3m^2 \rho^2 - 3m^2}{r^4} \right]$ | $hL_0^i = 0$                                                               |
| Bergmann        | $hB_0^0 = \frac{m^2 \rho^2}{4\pi \rho^2} \left[ 2 - \frac{m^2 \rho^2 - 3\rho^2}{r^4} - 4\rho^2 - \frac{3m^2 \rho^2 - 3m^2}{r^4} \right]$ | $hB_0^i = 0$                                                               |
| Möller          | $\Xi_0^0 = \frac{m^2 \rho^2}{4\pi \rho^2} \left[ e^\gamma(\rho^2 - z^2) + 2z^2 \right]$ | $h\Xi_0^i = 0$                                                             |

Here $\gamma(\rho, z)$ and $\psi(\rho, z)$ are given by Eq.(32), and $r = \sqrt{\rho^2 + z^2}$. This table gives the energy-momentum distribution for the Curzon metric in four different prescriptions.

### 4 Summary and Discussion

The problem of localization of energy has been re-considered in the framework of TPG by many scientists. The authors [30-36] showed that energy-momentum can also be localized in this theory. It has been shown that
results of the two theories can agree with each other. Möller found that a
tetrad description of a gravitational field equation allows a more satisfactory
treatment of the energy-momentum complex than does GR. Vargas [32] found
that the total energy of the closed Friedmann-Robertson-Walker spacetime is
zero by using teleparallel version of Einstein and Landau-Lifshitz complexes
which agreed with the results of GR [37]. Recently, Sharif and Jamil [38] eval-
uated the energy-momentum distribution of Lewis-Papapetrou spacetime by
using Möller’s prescription and found that the results of TPG and GR [39]
are not consistent.

In this paper, we have explored the energy-momentum distribution for
static axially symmetric spacetimes by using the TP version of Einstein,
Landau-Lifshitz, Bergmann and Möller’s prescriptions. We see from the table
1 that the energy density turns out to be different but momentum becomes
constant in each prescription. Further, the expressions for energy density do
not coincide with those given in the framework of GR [40-42] but momentum
is the same. Finally, we have considered the Curzon metric, a special case
of the Weyl metrics. This also leads to different expressions for the energy
density but same for the momentum. It is interesting to note from table 2
that for the Curzon metric energy also becomes constant in the limiting case
when \( r \to \infty \) and hence coincides with GR. While the energy density in each
case will diverge at \( r = 0 \), that is, along \( \phi - \)axis.

In recent papers [26-28,39-42], Sharif and his collaborators used different
prescriptions to determine the energy-momentum distribution for various
spacetimes in GR. These results do not coincide for any of the prescriptions.
Here we have used the TP version of different energy-momentum complexes
and found that the energy density is different for the four prescriptions but
the momentum becomes constant. It is mentioned here that these results
turn out to be the same under the limiting case of the Curzon metric which
is a special solution of the Weyl metrics.

We would like to mention here that the results of energy-momentum dis-
tribution for the Weyl metrics are not surprising. This justifies that different
energy-momentum complexes, which are pseudo-tensors, are not covariant
objects. This is in accordance with the equivalence principle [1] which im-
plies that the gravitational field cannot be detected at a point. This supports
the well-defined proposal developed by Cooperstock [9] and verified by many
authors [26-28,39-42].
Acknowledgment

We would like to acknowledge Higher Education Commission Islamabad, Pakistan for its financial support through the Indigenous PhD 5000 Fellowship Program Batch-I.

References

[1] Misner, C.W., Thorne, K.S. and Wheeler, J.A.: *Gravitation* (Freeman, New York, 1973).

[2] Trautman, A.: *Gravitation: An introduction to Current Research*, ed. Witten, L. (Wiley, New York, 1962).

[3] Landau, L.D. and Lifshitz, E.M.: *The Classical Theory of Fields* (Addison-Wesley Press, New York, 1962).

[4] Papapetrou, A.: *Proc. R. Irish Acad.* A52(1948)11.

[5] Bergman, P.G. and Thompson, R.: Phys. Rev. 89(1958)400.

[6] Tolman, R.C.: *Relativity Thermodynamics and Cosmology* (Oxford University Press, Oxford, 1934).

[7] Weinberg, S.: *Gravitation and Cosmology* (Wiley, New York, 1972).

[8] Möller, C.: Ann. Phys. (N.Y.) 4(1958)347.

[9] Cooperstock, F.I. and Sarracino, R.S.: J. Phys. A: Math. Gen. 11(1978)877.

[10] Bondi, H.: *Proc. R. Soc. London* A427(1990)249.

[11] Penrose, R.: *Proc. Roy. Soc. London* A388(1982)457.

[12] Penrose, R.: GR10 Conference eds. Bertotti, B., de Felice, F. and Pascolini, A. Padova 1(1983)607.

[13] Brown, J.D. and York Jr., J.W.: Phys. Rev. D47(1993)1407.

[14] Hayward, S.A.: Phys. Rev. D497(1994)831.

[15] Bergqvist, G.: Class. Quantum Gravit. 9(1992)1753.
[16] Chang, C.C., Nester, J.M. and Chen, C.: Phys. Rev. Lett. 83(1999)1897.
[17] Xulu, S.S.: Int. J. Mod. Phys. A15(2000)2979.
[18] Xulu, S.S.: Mod. Phys. Lett. A15(2000)1151.
[19] Xulu, S.S.: Astrophys. Space Sci. 283(2003)23.
[20] Virbhadra, K.S.: Phys. Rev. D42(1990)2919.
[21] Virbhadra, K.S. and Parikh, J.C.: Phys. Lett. B317(1993)312.
[22] Virbhadra, K.S. and Parikh, J.C.: Phys. Lett. B331(1994)302.
[23] Rosen, N. and Virbhadra, K.S.: Gen. Relativ. Gravit. 25(1993)429.
[24] Aguirregabiria, J.M., Chamorro, A. and Virbhadra, K.S.: Gen. Relativ. Gravit. 28(1996)1393.
[25] Virbhadra, K.S.: Phys. Rev. D60(1999)104041.
[26] Sharif, M.: Int. J. Mod. Phys. A17(2002)1175.
[27] Sharif, M.: Int. J. Mod. Phys. A18(2003)4361.
[28] Sharif, M.: Int. J. Mod. Phys. D13(2004)1019.
[29] Radinschi, I.: Mod. Phys. Lett. A16(2001)673.
[30] Mikhail, F.I., Wanas, M.I., Hindawi, A. and Lashin, E.I.: Int. J. Theo. Phys. 32(1993)1627.
[31] Nashed, G.G.L.: Phys. Rev. D66(2002)060415.
[32] Vargas, T.: Gen. Relativ. Gravit. 30(2004)1255.
[33] Salti, M., Havare, A.: Int. J. of Mod. Phys. A20(2005)2169.
[34] Aydogdu, O. and Salti, M.: Astrophys. Space Sci. 229(2005)227.
[35] Aydogdu, O., Salti, M. and Korunur, M.: Acta Phys. Slov. 55(2005)537.
[36] Salti, M.: Astrophys. Space Sci. 229(2005)159.
[37] Rosen, N.: Gen. Relativ. Gravit. 26(1994)323.
[38] Sharif, M. and Amir, M.J.: Mod. Phys. Lett. A22 (2007)425.

[39] Sharif, M. and Azam, M.: Energy-Momentum Distribution of the Wyell-Lewis-Papapetrou and the Levi-Civita Metrics Int. J. Mod. Phys. A (to appear 2007).

[40] Sharif, M. and Fatima, T.: Nouvo Cim. B120(2005)533.

[41] Sharif, M. and Fatima, T.: Int. J. Mod. Phys. A20(2005)4309.

[42] Sharif, M. and Fatima, T.: Astrophys. Space Sci. 302(2006)217.

[43] Aldrovendi, R. and Pereira, J.G.: An Introduction to Gravitation Theory (preprint).

[44] Hayashi, K. and Shirafuji, T.: Phys. Rev. D19(1979)3524.

[45] Hehl, F.W., McCrea, J.D., Mielke, E.W. and Neeman, Y.: Phys. Rep. 258 (1995)1.

[46] Aldrovandi and Pereira, J.G.: An Introduction to Geometrical Physics (World Scientific, 1995).

[47] Kramer, D., Stephani, H., Herlt, E. and MacCallum, M.: Exact Solutions of Einstein’s Field Equations (Cambridge University Press, 1985).

[48] Synge, J.L.: Relativity: The General theory (North-Holland Pub. Co. Amsterdam, 1960).

[49] Hernandez-Pastoral, J. L. and Martin, J.: Gen. Relativ. Gravit. 26(1994)877.

[50] Pereira, J.G., Vargas, T. and Zhang, C.M.: Class. Quantum Grav. 18(2001)833.

[51] Sharif, M. and Amir, M.J.: Gen. Relativ. Gravit. 38(2006)1735.

[52] Curzon, H.E.J.: Proc. Math. Soc. London 23(1924)477.