Flow stress characterization of magnesium alloys at elevated temperatures: A review

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Abstract. The flow behaviors of magnesium alloys are too complicated to be simply formulated in a mathematical form. Most researches have based metallurgically or phenomenologically on specific functions with many constants, which could be applied only to the limited magnesium alloys under specific conditions. In this study, a review on the studies of flow stress characterization of magnesium alloys is conducted and the possibility of using the traditional piecewise C-m model and its extension to characterize the magnesium alloys is emphasized. The formulations of major flow models are given with three typical applications to magnesium alloy AZ80 and its characteristics are demonstrated through comparison of the fitted flow behaviors with their associated experiments and various flow stress models including Arrehenius model, four Ludwik family models (Johnson Cook, Modified Johnson-Cook, Hensel-Spittel, Sutton-Luo), two Voce family models (Ebrahimi et al., Razali et al.) and C-m models.

1. Introduction

Magnesium alloys are characterized by good mechanical and metallurgical features, including its high specific strength and stiffness, excellent damping performance, low density, good weldability and machinability, good heat dissipation, electromagnetic shielding capability and recycling capability [1–4]. For last two decades, very active research works on various magnesium alloys, especially of the AZ series (containing Zn and Mn elements), ZE series (containing Zn and Ce elements), and ZK series (containing Zn and Zr elements) have been thus made for their applications in automobile and aerospace industries.

Magnesium alloys are, however, difficult to be formed at room temperature because of their HCP crystal structure and low stacking fault energy and thus their applications are very limited [5,6]. Most products of magnesium alloys are still fabricated by various casting processes, even though the wrought magnesium alloy products can meet much better the requirements on mechanical and metallurgical performances from industry than the traditional methods. To the contrary, magnesium alloys have been continuously tried to be hot forged in various ways [7,8] because they can be plastically compressed at elevated temperature (>573K) at which the slip systems of the basal, prismatic and pyramidal plane can be activated [9,10].

The flow behaviors of magnesium alloys are thus quite important to find the narrow road to the final goal. In this context, their accuracy and practicability are essential for realizing process optimal design based on simulation and optimization technologies [7,8,11–13]. However, they are too complicated to
be simply formulated in a mathematical form. Compared to the steel and the aluminum alloy, the fitted flow stress error of the magnesium alloy was much greater [14].

There are three major families of flow models including the Arrhenius equation model and its variants, Ludwik model and its variants, Voce model and its variants, C-m model and the others. All the flow models have based metallurgically or phenomenologically on specific functions with the related constants, which could be applied only to the limited magnesium alloys under specific conditions.

Hyperbolic sine Arrhenius model presented by Sellars and McTegart [15] has been widely employed to characterize the flow behaviors of materials [16–37] because it has some advantages. For example, it is based on metallurgical backgrounds and it can be applied for wide range of state variables but strain. However, it lacks a sort of practicability because of difficulties in finding the material constants and excludes the strain effect. The strain effect compensation schemes have accompanied high-order polynomials (up to 14th order polynomial in case of Al6061 [38]) with so many material constants which cause flow stress instability or oscillation in terms of strain and some difficulty in calculating them.

Power-law model and its variants [10,16,18,39–43] has been employed, represented by the Fields-Backofen equation model. The material parameters including strength coefficient, strain hardening exponent and strain rate sensitivity have been formulated by the functions of state variables of strain, strain rate and temperature. It is known that the accuracy of this family is generally lower than the other families.

Various two-step models [14,18,44–47] represented by the Voce model, have been employed for modeling the complicate flow stresses with an emphasis on dynamic recrystallization. Basically, this family model divides the flow stress into two parts, i.e., the earlier part of work hardening or dynamic recovery regime and the left part of the dynamic recrystallization regime. This family require many parameters and constants to be determined and this procedure is complicated.

There are many material models developed for various magnesium alloys, which do not belong to the major three families. For example, Hadadzadeh and Wells [48] presented a mixed model of hyperbolic sine Arrhenius and exponential equations for the four divided regimes of strain rate and temperature to increase the accuracy of ZK60 magnesium alloy, as-homogenized and extruded in both the extrusion and transverse directions. During the last decade, ANN based material characterization has been applied to various magnesium alloys [49–52], showing excellent agreement between the experiments and predictions.

Traditionally, a few researchers have devoted themselves to developing closed-form material models. However, it has been concluded that the constant material parameters are not appropriate to the flow stresses partially representing dynamic recrystallization, especially in case of magnesium alloys. Therefore, more simple but flexible flow models with the related approaches have been required by the CAE application engineers. It is noteworthy that the number of material constants of a flow model is not a matter of significance in these computer ages because any type of them can be practically manageable during numerical analysis. The greater the number of the material constants, the higher its accuracy and flexibility. Unfortunately, despite remarkable dedication of researchers in this field for several decades, there has been no such material model.

Notably, hot working of magnesium alloys has been tried at relatively low strain rate because of their limited formability at the higher strain rate. The accurate description of their strain and strain rate softening phenomena at elevated temperature is thus of great importance for engineering activities of magnesium alloys. Even though most research works still focus on the accuracy of only flow stress in this field, numerical technology needs the accuracy of its slope as well as its magnitude, for example, peak strain and steady-state strain corresponding to peak and steady-state stresses, respectively, to predict microstructural phenomena with higher accuracy.

In this paper, the major material models are reviewed including Arrhenius equation model, Ludwik family model, Voce family model and power law model, i.e., C-m model with an emphasis on application to magnesium alloys.

### 2. Flow models employed in this study
2.1. Arrhenius equation flow model family

The Arrhenius equation flow model and its variants were utilized to characterize magnesium alloys by many researchers, which was presented by Sellars and McTegart [15] as follows:

$$\dot{\varepsilon} = A [\sinh(\alpha \sigma)]^n e^{\left(\frac{Q}{RT}\right)}$$

(1)

where $\dot{\varepsilon}$, $\sigma$, $T$ and $Q$ are strain rate, flow stress, absolute temperature, and activation energy of deformation. $A$, $\alpha$ and $n$ are material parameters and $R$ is the universal gas constant. This equation can be restated for the flow stress as follows:

$$\sigma = \frac{1}{\alpha} \sinh^{-1} \left( \frac{Z}{\alpha \dot{\varepsilon}} \right)$$

(2)

where $Z$ is the Zener-Hollomon parameter.

The original Arrhenius equation flow model cannot deal with the effect of strain on the flow stress. This problem was solved by the strain compensation scheme, formulating the material parameters as the functions of strain. Most researchers used high-order polynomials. For example, Changizian et al. [27], Dong et al. [18], Luan et al. [17], and Sutton and Luo [16] characterized as-cast AZ81, as-extruded AZ31, commercial AZ31 and ZE20 using the polynomials of order four, five, six and eight, respectively. It should be noted that increasing order of polynomial causes some oscillatory features at the higher strain, as can be found from the work of Lin et al. [53], decreasing the reliability and generality of the scheme.

All the parameters of the modified hyperbolic sine Arrhenius equation, including $\alpha$, $n$, $Q$ and $\ln(A)$, were described by the following polynomial of strain:

$$f(\varepsilon) = \sum_{i=0}^{N} C_i \varepsilon^i$$

(3)

where $N$ ranged from 4 to 14 in the literature and the constants $C_i$’s are determined by minimizing the error between the experimental and fitted flow stresses.

The hyperbolic sine Arrhenius model has been employed by many researchers, including Luan et al. [17] and Dong et al. [18] for AZ31; Arun and Chakkingal [19] for AZ31B; Cai et al. [20] for AZ41M; Wu et al. [21] for AZ61; Zhou et al. [22] Ouam et al. [23, 24] and Malik et al. [25] for AZ80; Tang et al. [26] for AZ80M, Changizian et al. [27] for AZ81, Liu and Ding [28] and Mei et al. [29] for AZ91; Wang et al. [30] for AZ91D reinforced with 10 vol. % short carbon fibers (Cf/AZ91D composites); Wu et al. [31] for ZK21; Yu et al. [32] for ZK60-T4; Malik et al. [33] for ZK61-T5; Li and Zhang [34] for Mg-9Gd-4Y-0.6Zr alloy; Zhou et al. [35] for Mg-Gd-Y-Nb-Zr alloy; Hao et al. [36] for Mg-Zn-Y-Mn alloy; Alizadeh et al. [37] for GWK940, GWK540 and GK50. Table 1 summarizes the representing research works on characterizing the magnesium alloys using the Arrhenius equation flow model.

All the hyperbolic sine Arrhenius models (except Alizadeh et al. [37]) were derived from experimental data of hot compression tests. Luan et al. [17] described the material constants ($Q$, $\ln(A)$, $n$, $\alpha$) of Arrhenius model as the 5th order polynomial of strain. The value of average absolute relative error (AARE) of their model was 5.65%. Dong et al. [18] formulated the material constants of Arrhenius model as the 5th order polynomial of strain. Furthermore, they derived the constitutive equation, considering the influence of dynamic recrystallization (DRX) and dynamic recovery (DRV). Arun and Chakkingal [19] conducted hot compression tests after ECAP, one of severe plastic deformation processes, which leads to great grain refinement due to substantial strain. They found that activation energy decreased as the number of ECAP passes increased. They calculated the material constants for each pass and showed that the peak flow stresses were in good agreement with the experimentally determined values.

Cai et al. [20] represented the material constants by the 5th order polynomial of strain. The value AARE was 3.41%. Wu et al. [21] described the material constant $\alpha$ using the hyperbolic sine function and formulated the other constants $Q$, $\ln(A)$ and $n$ as the 3rd or 4th order polynomial. They showed the flow model equation with the thus determined $\alpha$ gave accurate flow stress under the deformation
conditions used in the study. Zhou et al. [22] constructed the processing maps using the hyperbolic sine constitutive equation with the dynamic material model. The maps showed a domain of DRX which is the optimum range for hot working of the alloy. Quan et al. [23] conducted a research to understand DRX behavior of AZ80. Regression analysis was employed to determine the activation energy of DRX. The modified Avrami type equation was evaluated to characterize the evolution of DRX volume fraction as well. Quan et al. [24] used the 7th order polynomial to express the influence of strain. The value of AARE was 6.63%.

Huang et al. [25] analyzed the hot deformation behavior with the calculated material constants of the hyperbolic sine equation. They analyzed the processing maps and established the stability and instability domains from the maps. Tang et al. [26] conducted hot compression tests at the temperature ranging from 673 to 833K, which involve the solid (673-723K) or semi-solid (above 793K) region. They calculated the material constants of two constitutive equations for plastic deformation and thixotropic deformation. The accuracy of the two equations were evaluated by the values of AARE which are 2.1% and 2.6%, respectively.

Changizian et al. [27] expressed the material constants as the 4th order polynomials of strain. The flow model and experiment were in good agreement with each other. Liu and Ding [28] calculated the material constants of hyperbolic Arrhenius equation and observed the microstructure evolution using optical microscope, revealing that twinning occurred initially at lower temperatures as 523 and 573K and DRX developed subsequently at twins and grain boundaries. In case of temperatures higher than 623K, DRX occurred massively at grain boundaries during deformation processes and twinning was not found at optical microscope scale.

Mei et al. [29] proposed piecewise function models considering the two-sectional flow stress behavior for both strain hardening and softening. They were based on Ebrahimi et al. model [54]. The effects of temperature and strain rate on the material constants were described by linear and non-linear functions while Ebrahimi et al. described them as constants. AAREs of the Arrhenius model, piecewise model using linear functions and piecewise model using nonlinear functions were 8.8, 3.9 and 2.5%, respectively. Wang et al. [30] used exponentials function to describe the material constants of the Arrhenius model. The model equation and experiments were in good agreement with each other. Wu et al. [31] considered the material constants as the functions of hyperbolic sine. The deformed microstructures were observed using metallographic microscope. They found the appropriate ranges of temperature (523-673K) at the strain rate (10s$^{-1}$) for ZK21 in terms of hot workability.

Yu et al. [32] formulated the material constants of Arrhenius model as the 5th order polynomial of strain, and their flow model and experiments were in good agreement with each other as the value of AARE was 2.9%. They obtained the optimum ranges of temperature (600-683K) and strain rate (0.001-0.01s$^{-1}$) from the processing maps. Malik et al. [33] calculated the material constants without considering strain-dependency. They found that strain rate range (0.001-1s$^{-1}$) at temperature 548K and strain rate range (0.001-10s$^{-1}$) at temperature 573-623K are optimum conditions. Fracture behavior showed that twinning and fracture (principal and secondary cracks) were nucleated and propagated due to the low fraction of DRX. Li and Zhang [34] calculated the stress exponents n of 3.2 and 5.1 for the low stress regime and high stress regime, respectively. They suggested that the reasonable ranges of temperature and strain rate of the employed materials are 703-723K and 0.006-0.03s$^{-3}$, respectively. Zhou et al. [35] described the material constants as the 4th order polynomial of strain and showed that the flow model used was acceptable in terms of experiments as the value of AARE was 4.8%. The processing maps showed that the instability domain involved the strain rate range of 0.3-0.5s$^{-1}$ and that the optimum working domain of the temperature and strain rate ranged from 703K to 765K and from 0.01s$^{-1}$ to 0.1s$^{-1}$, respectively.

Hao et al. [36] calculated the material constants without considering the strain dependency. They used the Avrami equation to predict the evolution of microstructure and volume fraction. The processing maps were drawn using dynamic material model, and the optimum ranges of temperature and strain rate for hot working were 673-723K and 0.001-0.01s$^{-1}$, respectively. A sample was deformed with good quality in the optimum ranges, and it verified their reasonability. Alizadeh et al. [37] investigated the
hot shear deformation behavior of the magnesium alloys. The material constants were calculated using Rieiro–Carsi’–Ruano (RCR) method without using their initial values to quantify the goodness of the fit. Processing maps of the studied alloys were in similarity. The Gd and Y contents did not have obvious effects on the recrystallization behavior but increased the high-temperature strength, that is, GWK940 (Mg–9Gd–4Y–0.4Zr) had the highest strength level in the studied conditions.

Table 1. Application of Arrhenius equation flow models to magnesium alloy.

| No. | Author’s name | Material | Range of strain rate (°/s) | Range of Temperature (K) | Parameter type | AARE (%) | Reference No. |
|-----|---------------|----------|---------------------------|-------------------------|----------------|----------|---------------|
| 1   | Luan et al.   | AZ31     | 0.01-10                   | 523-723                 | 6<sup>th</sup> order polynomial | 5.65     | [17]          |
| 2   | Dong et al.   | AZ31     | 0.01-10                   | 523-673                 | 5<sup>th</sup> order polynomial | 4.41     | [18]          |
| 3   | Arun, Chakkingal | AZ31B    | 0.01-10                   | 523-673                 | Constant      | None     | [19]          |
| 4   | Cai et al.    | AZ41M    | 0.005-1                   | 573-729                 | 5<sup>th</sup> order polynomial | 3.41     | [20]          |
| 5   | Wu et al.     | AZ61     | 0.001-1                   | 523-673                 | Polynomial    | 5.5      | [21]          |
| 6   | Zhou et al.   | AZ80     | 0.001-20                  | 693-773                 | Constant      | None     | [22]          |
| 7   | Quan et al.   | AZ80     | 0.01-10                   | 523-673                 | 7<sup>th</sup> order polynomial | 6.63     | [23,24]      |
| 8   | Huang et al.  | AZ80     | 0.0001-0.1                | 523-683                 | Constant      | None     | [25]          |
| 9   | Tang et al.   | AZ80M    | 0.001-1                   | 673-743                 | Constant      | Solid - 2.1 Semi-solid - 2.6 | [26] |
| 10  | Changizian et al. | AZ81     | 0.003-0.3                 | 523-723                 | 4<sup>th</sup> order polynomial | None     | [27]          |
| 11  | Liu, Ding     | AZ91     | 0.001-1                   | 523-723                 | Constant      | None     | [28]          |
| 12  | Mei et al.    | AZ91     | 0.001-1                   | 473-623                 | Exponential function | None     | [29]          |
| 13  | Wang et al.   | Cu/AZ91D | 0.005-0.5                 | 596-696                 | Exponential function | None     | [30]          |
| 14  | Wu et al.     | ZK21     | 0.1-50                    | 523-673                 | Constant      | None     | [31]          |
| 15  | Yu et al.     | ZK60-T4  | 0.001-1                   | 523-673                 | 5<sup>th</sup> order polynomial | 2.9      | [32]          |
| 16  | Malik et al.  | ZK61-T5  | 0.001-10                  | 298-673                 | Constant      | None     | [33]          |
| 17  | Li, Zhang     | Mg-9Gd-4Y-0.6Zr | 0.006-0.03              | 623-773                 | Constant      | None     | [34]          |
| 18  | Zhou et al.   | Mg-Gd-Y-Nb-Zr | 0.001-1       | 623-773                 | 4<sup>th</sup> order polynomial | 4.8      | [35]          |
| 19  | Hao et al.    | Mg-Zn-Y-Mn | 0.001-1          | 623-773                 | Constant      | None     | [36]          |
| 20  | Alizadeh et al. | GWK940, GWK540, GK50 | 0.0067-0.067   | 573-773                 | Constant      | None     | [37]          |

2.2. Ludwik flow model family

The Ludwik model [55] of describing strain hardening is described as follows:

\[
\sigma = \sigma_0 + K\varepsilon_p^n
\]

(4)

where \(\sigma_0\) and \(K\) are Ludwik material constants or parameters and \(n\) is Ludwik strain hardening exponent. Note that this model is the oldest among the fundamental strain hardening models including Ludwik, Ramberg and Osgood [56], Hollomon [57], Voce [58] and Swift [59] flow models.

There are several flow models which can be grouped as a family of extended Ludwik flow models in which two material constants or parameters and the exponent in the extended formulation can be expressed by the functions of strain rate and temperature or assumed at zero but the exponent. Basically, the flow stress dependences on the strain rate and temperature in this family are dealt with by formulating the material parameters or the exponent as separable functions of strain rate and temperature.

Johnson-Cook model [60] is the typical representation of the Ludwik family and employs the fewest number of material constant. It has thus been widely utilized for various engineering purposes owing to
its simplicity even though it is beneficial particularly for large strain rate problems. However, it may not give acceptable predictions for non-isothermal analysis of metal forming processes of magnesium alloys which exhibits complicated flow behaviors. Other well-known or powerful Ludwik family models include Khan-Huang-Liang flow model [61,62]. However, most traditional Ludwik family models including Johnson-Cook and Khan-Huang-Liang flow models are not appropriate in solving the practical metal forming problems related to the magnesium alloys where the complicated dynamic recrystallization occurs even though they are strong for the mechanical problems featured by their high strain rate deformation including impact, explosive forming, crash, etc.

Sutton and Luo [16] presented a remarkable extended Ludwik model starting from the following traditional strain and strain rate hardening model, called the Fields-Backofen (FB) equation model [63]:

$$\sigma = Ke^n \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^m$$

(5)

where $K$, $n$ and $m$ are strength coefficient, strain hardening exponent and strain rate sensitivity, respectively and $\dot{\varepsilon}_0$ is a reference strain rate, assumed at 1/s in this study. Assuming $dln(\sigma)/d\dot{\varepsilon}$ as a linear function of $e^{B\dot{\varepsilon}}$ where $B$ is a constant, they formulated the material parameters $K$, $n$ and $m$ as follows:

$$n = \frac{1}{ln(\dot{\varepsilon})} \left[ \frac{A}{ln(\dot{\varepsilon})} e^{B\dot{\varepsilon}} + C\dot{\varepsilon} + D \right]$$

(6)

$$m = m_0 + m_1 T$$

(7)

$$K = K_0 + K_1 T$$

(8)

where the material parameters $A$, $B$, $C$ and $D$ were formulated as

$$A, B, C, D = f(\dot{\varepsilon}, T) = C_1 + C_2 ln(\dot{\varepsilon}) + C_3 T + C_4(ln(\dot{\varepsilon}))^2 + C_5 T ln(\dot{\varepsilon}) + C_6 T^2$$

(9)

and $m_0$, $m_1$, $K_0$, $K_1$ and $C_i$’s are material constants that should be determined minimizing the error between the experimental and fitted flow stresses.

As Sutton and Luo [16] did, this kind of extended Ludwik models can be utilized for metal forming application only when the parameters are formulated by high order polynomials and/or specially constructed functions which may damage its original benefit and principle. Such mathematical formulation and the related procedure of finding the material constants cause the severe deterioration of the generality that should be considered in the stage of practical application.

Hensel-Spitell model [64] can also be classified as one of this family, which can be formulated, in case of nine material constants, as follows:

$$\sigma = A e^{(m_1 \dot{\varepsilon} + (m_2 T) + (m_3 T) + (m_4 T) + (m_5 T) + (m_6 T))} e^{(m_7 T) + (m_8 T) + (m_9 T)}$$

(10)

where all $m_i$’s are material constants.

Note that Hensel-Spitell model is basically a combination of classical power law model of strain rate and Hollomon and Swift models with their material constants considered as functions of strain and temperature. Basically, all the nine material constants are just constant and thus it describes the flow behaviors over all the coverage of state variables with nine materials constants. This advantage inevitably incurs inaccuracy in many cases, especially in expressing the strain softening behaviors owing to the dynamic recrystallization. Therefore, it is not easy to find its application to the magnesium alloys.

Note that the traditional C-m model and its extension should be grouped by this family but that we described its improved model independently in Section 2.4.

Various researches of the Ludwik model family involve Takuda et al. [40] and Cheng et al. [43] for AZ31; Tsao et al. [10] and Zhou et al. [42] for AZ61.

Table 2 summarizes the representing research works on characterizing the magnesium alloys using the Ludwik model family.

Takuda et al. [40] expressed the flow behavior using the FB equation and made the formula in simple form with the parameters $(K, n$ and $m$) formulated by functions of temperature. Cheng et al. [43] used a
modified FB equation to describe the softening behavior much better. It showed better results in the softening stage (the strain of 0.15-0.45, temperature of 473-573K and strain rate of 0.0001-0.1s\(^{-1}\)) compared to the traditional FB equation. Tsao et al. [10] also applied the modified FB equation to AZ61. The calculated flow curves were in good agreement with experimental flow curves in both the strain-hardening and softening stages. Zhou et al. [42] used Zener-Hollomon parameter with FB equation and added a softening term to apply the influence of the softening to their flow model. The fitted flow stresses and experiments of them were in good agreement as the maximum difference between the calculated and experimental peak stresses was less than 8%.

Table 2. Application of Ludwik family model to magnesium alloys.

| No. | Author’s name | Material | Experiment type | Range of strain rate (/s) | Range of Temperature (K) | Reference No. |
|-----|---------------|----------|-----------------|--------------------------|--------------------------|--------------|
| 1   | Takuda et al. | AZ31     | Tension         | 0.01-1                   | 423-573                  | [40]         |
| 2   | Cheng et al.  | AZ31     | Tension         | 0.0001-0.1              | 423-573                  | [43]         |
| 3   | Tsao et al.   | AZ61     | Compression     | 0.01-10                 | 523-673                  | [10]         |
| 4   | Zhou et al.   | AZ61     | Compression     | 0.001-1                 | 523-673                  | [42]         |

2.3. Voce model family

Voce [58] represented a flow stress as a function of strain, strain rate and temperature with an emphasis on microstructural evolution as follows:

\[
\sigma = \sigma_p - (\sigma_p - \sigma_o) e^{(be)} - [\sigma_p - \sigma_S] X_{dy} \tag{11}
\]

where \(X_{dy}\) is the volume fraction of dynamic recrystallization defined as

\[
X_{dy} = \begin{cases} 
0 & ; \varepsilon < \varepsilon_p \\
1 - \exp \left[-2.996 \left(\frac{\varepsilon - \varepsilon_p}{\varepsilon_s - \varepsilon_p}\right)^2\right] & ; \varepsilon \geq \varepsilon_p 
\end{cases} \tag{12}
\]

where \(\sigma_o, \sigma_p, \sigma_S, \varepsilon_p\) and \(\varepsilon_s\) are the initial yield stress, peak stress, steady-state stress, peak strain and steady-state strain, respectively, which are formulated by Zener-Hollomon parameter \(Z\) with many material parameters or constants as follows:

\[
\sigma_o = c_o \cdot d_o \cdot c_1^Z \cdot c_2
\]

\[
\sigma_p = d_o \cdot \sinh^{-1}(Z \cdot d_1 / d_2)
\]

\[
\sigma_S = e_o \cdot \sinh^{-1}(Z \cdot e_1 / e_2)
\]

\[
\varepsilon_p = f_o d_o \cdot f_1 Z f_2
\]

\[
\varepsilon_s = g_o d_o \cdot g_1 Z g_2
\]

where \(b, c_o, d_o, d_1, d_2, c_1, c_2, e_o, e_1, e_2, f_0, f_1, f_2, g_0, g_1\) and \(g_2\) are material constants to be obtained for the specific material.

This model has many advantages. For example, the strain hardening effect which should be dominant in the smaller strain than peak strain decays very quickly as the strain increases, which can be compatible with the actual situations occurring in hot forming of commercial metals. It is also friendly with the typical strain softening owing to the dynamic recrystallization. Unfortunately, there are some difficulties in identifying the material constants or parameters and thus its applications are quite few, despite the model being based on metallurgical theory.

Considering this difficulty, certain practical approaches based on phenomenological models with fewer material constants have been developed. Ebrahimi et al. model [14] can be classified as the Voce model family, even though their backgrounds are quite different from each other. The Ebrahimi et al.
model stands on the fully phenomenological model side while the Voce model on the neighborhood of the physical model side. However, they are the same in the viewpoint of describing the dynamic recrystallization. Ebrahimi et al. model can be stated as follows:

\[ \sigma = \sigma_p \left[ \left( \frac{\varepsilon}{\varepsilon_p} \right) \exp \left( 1 - \frac{\varepsilon}{\varepsilon_p} \right) \right]^{C_h} ; \varepsilon \leq \varepsilon_p \quad [65] \]  \tag{18}

\[ \sigma = \sigma_s + (\sigma_p - \sigma_s) \exp \left[ C_s \left( \varepsilon - \frac{\varepsilon_p}{2} - \frac{\varepsilon^2}{2\varepsilon_p} \right) \right] ; \varepsilon \geq \varepsilon_p \quad [54] \]  \tag{19}

where \( C_h \) and \( C_s \) are material parameters or constants. Ebrahimi et al. derived a general systematic scheme using experimental plots showing relationships between variables and/or parameters, not only to calculate \( C_h \) and \( C_s \) but also to formulate \( \varepsilon_p, \sigma_p \) and \( \sigma_s \) as functions of strain rate and temperature. The scheme assumes that the experimental plots are linear. The fitted linear functions were used to interrelate variables and/or parameters to identify flow stresses at particular strains, strain rates and temperatures.

Razali et al. [14] developed a systematically formulating and identifying scheme of the material parameters formulated by piecewise bilinear functions (called thus PLF model), which can be applied regardless of their dependence on state variables. They applied their scheme successfully to commercial steel 20MoCrS4, aluminum alloy AHS-2 and magnesium alloy AZ80 and concluded that the fitting error in case of the AZ80 is the greatest among the three test materials because of its high complicated strain softening behaviors.

Fereshteh-Saniee et al. [46] conducted flow behavior characterization of various magnesium alloys including AZ31, AZ80 and AZ81 using the Ebrahimi et al. model approach [54] in which the material constants were graphically determined. Graphical approach is inherently poor in terms of practicability and computerization. It hinders the material constants or parameters from being formulated as functions of state variables including strain, strain rate and temperature, which is essential to describe the complicated flow behaviors with higher accuracy. To cope with this matter, Razali et al. [14] suggested a systematic scheme of calculating the material parameters. They were formulated as the functions of state variables with material constants that were determined by optimization techniques. Razali et al. applied the scheme for the Ebrahimi et al. model to the same magnesium alloy AZ80 studied by Fereshteh-Saniee et al. They compared their scheme with original Ebrahimi et al. graphical approach to emphasize the advantage of the former. However, their approach also needs some routine work to calculate the best material constants which may be hardly programmed for general usage.

This model family have the problem of formulating the peak strain and stress as functions of state variables as can be found from the work of Zeng et al. [47] for GW83 alloy, AZ31B and ZK61, which causes complexity and difficulty and deteriorates the practicability of these flow models.

This flow behavior family also include Qin et al. [44] for ZK60. Compared with magnesium alloys, this family have many applications to the steels.

Table 3 summarizes the representing research works on characterizing the magnesium alloys using the Voce model family.

Fareshteh-Saniee et al. [46] used an improved Ebrahimi’s model [54] to magnesium alloys and showed a good agreement between the calculated flow stresses and both tensile and compressive experiments. Their maximum errors in both tension and compression were approximately 11%. Activation energies of AZ80 and AZ81 in tension, however, were found to be larger than those in compression.

Zeng et al. [47] proposed a new model from functional relationship between normalized stress and normalized strain with respect to their peak values, based on isothermal compression stress-strain curves of GW83 alloy. They applied the model to AZ31B and ZK61 alloys, showing a good agreement between fittings and experiments.

Razali et al. [14] proposed a modified Ebrahimi et al. model, called Razali et al. model and compared it with Ebrahimi et al. model on AZ80 alloy. They represented the material parameters as a piecewise
bilinear function of strain rate and temperature and applied an optimization scheme to find the material constants. They found that the model could describe the flow stress curves of various metals at different temperatures and strain rates with quite good accuracy.

Table 3. Application of Voce flow model to magnesium alloys

| No. | Author’s name | Material | Experiment type | Range of strain rate (/s) | Range of Temperature (K) | Reference No. |
|-----|---------------|----------|-----------------|--------------------------|--------------------------|---------------|
| 1   | Fereshteh-Sanee et al. | AZ31 | Compression | 0.00025-0.025 | 473-513 | [46] |
|     |                | AZ80 | Compression | 0.001-0.1 | 493-573 |                      |
|     |                |      | Tension | 0.0005-0.005 | 548-623 |                      |
| 2   | Zeng et al.   | GW83 | Compression | 0.001-1 | 623-723 | [47] |
| 3   | Razali et al. | AZ80 | Compression | 0.001-0.1 | 495-575 | [14] |

2.4. Piecewise C-m family models

The most common and general flow model for the material at elevated temperature is the following C-m model [45], i.e., the power-law model:

$$\sigma = C_0 \varepsilon^{m_0}$$  \hspace{1cm} (20)

where $C_0$ is the strength parameter and $m_0$ the strain rate sensitivity. Equation (20) is a traditional description. However, its good characteristics are not known sufficiently.

$C_0$ and $m_0$ values can be considered as constants, especially in case of isothermal analysis. For the non-isothermal analysis, traditionally $C_0$ and $m_0$ have been modeled as piecewise bilinear functions of strain and temperature. When $C_0$ and $m_0$ are described in the piecewise way as shown in Figure 1, their values at a point $A^*$ inside a patch can be bilinearly interpolated by the following function with the known nodal values of $C_0$ and $m_0$ defined at the grid points:

$$\phi = \frac{(T_{j+1}-T)(\varepsilon_{i+1}-\varepsilon)}{(T_{j+1}-T)(\varepsilon_{i+1}-\varepsilon)} \phi_1 + \frac{(T_{j}-T)(\varepsilon_{i+1}-\varepsilon)}{(T_{j}-T)(\varepsilon_{i+1}-\varepsilon)} \phi_2 + \frac{(T_{j+1}-T)(\varepsilon_{i}-\varepsilon)}{(T_{j+1}-T)(\varepsilon_{i}-\varepsilon)} \phi_3 + \frac{(T_{j}-T)(\varepsilon_{i}-\varepsilon)}{(T_{j}-T)(\varepsilon_{i}-\varepsilon)} \phi_4$$  \hspace{1cm} (21)

Figure 1. Piecewise bilinear expression of flow stress using the sample points of (strain, temperature).
where $\phi$ represents $C_0$ or $m_0$ and $\phi_i (i = 1,2,3,4)$ is its nodal value. The nodal values of $C_0$ and $m_0$ at each grid point where its temperature and strain are fixed are calculated by an optimization technique. $C_{0i}$ and $m_{0i}$ represent the flow behaviors at the fixed temperature $T_i$ and fixed strain $\epsilon_i$ and they are thus calculated by the optimally fitting scheme in the stress-strain rate domain. They are easily calculated and the procedure can be automated with ease.

Recently, Joun et al. [66] proposed an extended $C_0$-$m_0$ model which is able to describe complicated flow stresses. In the extended $C_0$-$m_0$ model, the simple model of Equation (20) was reformulated as follows:

$$\sigma = (C_0 + C_i \dot{\epsilon}) \dot{\epsilon}^{(m_0 + m_i \dot{\epsilon})}$$  \hspace{1cm} (22)

where $C_i$ and $m_i$ represent all flow properties defined at the fixed sample strains and temperatures, similar to $C_0$ and $m_0$ in Equation (20). They can be treated either as just constants or as piecewise bilinear functions of strain and temperature. They are calculated by minimizing the error between experimental and fitted flow stress functions. Some of them can be considered as fixed during curve fitting. With these extended functions of $C_i$ and $m_i$, all of the aforementioned problems of the $C_0$-$m_0$ model were solved.

Joun et al. [66] applied the extended piecewise bilinear $C_0$-$m_0$ model to ZE60 and ZK61, as summarized in Table 4. They verified that the new approach was superior to the other flow models in terms of accuracy and practicability.

### Table 4. Application of piecewise bilinear $C_0$-$m_0$ flow model to magnesium alloys.

| No. | Author’s name | Material | Range of strain rate (/s) | Range of Temperature (K) | AARE (%) | Reference No. |
|-----|---------------|----------|--------------------------|----------------------------|-----------|---------------|
| 1   | Joun et al.   | ZE20     | 0.001-1                  | 523-723                    | None      | [66]          |
|     |               | ZK61     | 0.001-10                 | 473-673                    | None      |               |

3. **Flow behaviors of AZ80 magnesium alloys**

Fereshteh-Saniee et al. [46] conducted experimental works to obtain the flow stress information of AZ80 (chemical composition in wt%: Al 7.83; Zn 0.46; Mn 0.25; Si 0.03; Ni 0.001; Mg bal.) using T-shape tests and compression tests with cylindrical specimens which have a diameter of 6 mm and a length of 9 mm. The tests were conducted at sample temperatures of 493K, 523K and 573K and sample strain rates of 0.001/s, 0.01/s and 0.1/s without any lubricant.

Fereshteh-Saniee et al. applied the flow model suggested by Ebrahimi et al. to characterize their experiments based on its graphical method. Razali et al. [14] also conducted flow stress characterization of the same experiments using Ebrahimi et al. model with the scheme of PLF description of material parameters and an optimization technique.

We characterized additionally these flow stress curves using not only Sutton-Luo, Arrhenius, and Hensel-Spittel models but also the four cases of the $C$-$m$ model. The optimized material constants of all the flow models were listed in Tables 5-6. We selected the 0.5/s - strain rate case for the sake of comparison because it is enough to represent the three sample strain rates. We compared them with the experimental flow curves in Figure 2.

### Table 5. Material constants for $C$-$m$ models used to evaluate AZ80.

|          | 493K | 523K | 573K |
|----------|------|------|------|
| $C_{0-m_0}$ |      |      |      |
| $C_{0}$  | 0.2  | 198  | -    |
| $C_1$    | 0.4  | 210  | -    |
| $m_0$    | 0.6  | 196  | -    |
| $m_1$    | 0.8  | 182  | -    |
| $C_{0-m_0}$ |      |      |      |
| $C_{0}$  | 0.2  | 198  | -    |
| $C_1$    | 0.4  | 210  | -    |
| $m_0$    | 0.6  | 196  | -    |
| $m_1$    | 0.8  | 182  | -    |

$C_{0-m_0}$ model $C_{0}$ and $m_0$ model.
Table 6. Material constants for other models used to evaluate AZ80

| Models          | Material constants |
|-----------------|--------------------|
|                 | $c_i$ | $\alpha$ | $n$ | $Q$ | $A$ |
| Arrhenius       | $c_0$  | 0.0244  | 2.3805 | 187.72 | 0.9631 |
|                 | $c_1$  | 0.3851  | 162.79 | 47.673 | -3.1569 |
| Sutton-Luo      | $c_2$  | 2.5091  | 1038.5 | -319.52 | 21.869 |
|                 | $c_3$  | -7.2321 | 2951.8 | 936.07  | -65.322 |
|                 | $c_4$  | 10.2224 | -4120.8 | -1332.7 | 94.001 |
|                 | $c_5$  | -6.9411 | 2769.2 | 907.47  | -64.379 |
| Ebrahim         | $c_6$  | 1.8101  | -716.26 | -236.82 | 16.854 |
| Hensel-Spitell  | $m$    | 0.005   | 1.69  | 14.773 | -61.1903 | 0.1328 | -0.8891 |
|                 | $K$    | 2.044   | -625  | 2.2671 | -4.3025  | 0.2305 | -0.2047 |
|                 | $c_i$  | -0.0879 | 0.0659 | 0.0007  | 0.0045  |
|                 | $c_4$  | -0.0109 | 0.0848 | 0.0088  | 0.0012  |
|                 | $c_5$  | 0.0011  | 0.0069 | -0.0004 | 0.0003  |
|                 | $c_6$  | 0.0002  | -1.710^5 | -2.810^6 | -4.810^6 |
| B_p             | $m_1$  | -0.007 | 0.015 | -0.025 | -0.134 | - |
| B_k             | $m_2$  | -0.115 | 0.08  | 0.025 | 0.134 | - |
| 2×10^2          | $n_s$  | 9.49   | $b$   | 0.22   | $k$   | 0.8 |
| 3×10^4          | $d_0$  | 16560  | $d_1$  | 4×10^5 | $C_h$  | 0.22 |
| 8×10^6          | $R$    | 170500 | $R_1$  | 8.314  | $C_g$  | 4.25 |
| 0.01/s          | $\dot{\varepsilon}$ | 493 | 0.32 | 7.20 | 0.248 | 164.09 | 129.41 |
| PLF             | $\dot{\varepsilon}$ | 523 | 0.39 | 7.40 | 0.198 | 152.75 | 120.81 |
| 0.1/s           | $\dot{\varepsilon}$ | 573 | 0.51 | 7.73 | 0.247 | 83.96 | 64.27 |
|                 | $\dot{\varepsilon}$ | 493 | 0.25 | 4.68 | 0.255 | 180.12 | 146.58 |
|                 | $\dot{\varepsilon}$ | 523 | 0.32 | 4.88 | 0.245 | 170.51 | 136.01 |
|                 | $\dot{\varepsilon}$ | 573 | 0.45 | 5.21 | 0.299 | 113.41 | 93.961 |

Table 7. Maximum and average errors of different models used to evaluate AZ80

| Model          | Temperature (K) | Max. error (%) | Avg. error (%) |
|----------------|-----------------|----------------|----------------|
| Sutton-Luo     | 493             | 3.72           | 1.73           |
|                 | 523             | 3.85           | 2.15           |
|                 | 573             | 6.82           | 2.14           |
| Total          | 6.82            | 2.01           |
| Arrhenius      | 493             | 3.57           | 1.04           |
|                 | 523             | 3.95           | 3.18           |
|                | 493    | 573     | Total  |
|----------------|--------|---------|--------|
| **Hensel-Spittel** | 17.76  | 20.92   | 4.41   |
|                | 16.40  | 15.92   | 5.52   |
|                | 9.71   | 9.36    | 2.57   |
| **Ebrahimi**   | 28.37  | 15.92   | 4.41   |
|                | 9.21   | 9.52    | 5.52   |
|                | 6.40   | 9.36    | 2.57   |
| **PLF**        | 5.36   | 7.84    | 4.41   |
|                | 9.21   | 15.92   | 5.52   |
|                | 15.55  |         |        |
| **C₀-m₀**      | 3.25   | 4.03    | 4.03   |
|                | 1.67   | 2.13    | 1.80   |
| **C₀₁-m₀**     | 3.27   | 4.05    | 4.05   |
|                | 1.01   | 1.84    | 1.14   |
| **C₀-m₀₁**     | 3.33   | 4.19    | 4.19   |
|                | 1.40   | 1.68    | 1.51   |
| **C₀₁-m₀₁**    | 0.06   | 0.13    | 0.13   |
|                | 0.04   | 0.06    | 0.05   |

Figure 2 shows that the C₀₁-m₀, C₀-m₀₁ and C₀₁-m₀₁ models and the PLF model are excellent in terms of average error summarized in Table 7 and that the C₀-m₀ model, Sutton-Luo model and Arrhenius model are acceptable. To the contrary, the errors of Hensel-Spittel and graphical Ebrahimi et al. models were quite great.

Note that all the flow models but the C-m model family employ the closed-form functions which are hard to be locally adjustable. To the contrary, the C-m model family can be locally improved by changing the material constants affecting only the region. The procedure of calculating them can also be generalized [45] and its automation can be easily accomplished. However, there is a drawback that they cannot be extrapolated for the flow stress over the state variables out of their coverages. Thus, a coupling of the C-m model with the other closed-form function models may be the solution of practical characterization of the magnesium alloys, which are thermomechanically complicated.
4. Conclusion
In this paper, we reviewed the flow models of describing complicated flow stresses of magnesium alloys, which exhibit the strain, strain rate and temperature hardening and softening as well as dynamic recrystallization. We sorted all the research works on flow characterization of metallic materials into Ludwik model family (involving Hollomon and Swift), Voce model family, Hyperbolic sine Arrhenius model family, C-m model family and the miscellaneous models. It was shown that the Arrhenius was overwhelmed in number while the extended piecewise bilinear C-m model is the best promising in terms of CAE application.

We compared the experimental flow curves with the fitted flow curves of the AZ80 obtained by the Johnson-Cook, Hensel-Spittel, Sutton-Luo flow models among the Ludwik model family, graphical
Ebrahimi et al. and PLF Ebrahimi et al. models among the Voce model family, hyperbolic sine Arrhenius flow model with the 6th order polynomial description, and the several cases of the extended C-m model.

Hyperbolic sine Arrhenius flow model and Sutton-Luo flow model with high-order polynomials or linear combination of purposely chosen functions, respectively, for the material parameters fitted quite accurate flow curves.

To the contrary, graphical Ebrahimi et al. model exhibited some weakness at the large strain when the flow stress increased after the steady-state point. This weak point could be covered by the PLF description of its material parameters together with the optimization technique. The Ebrahimi et al. model with the PLF scheme produced quite acceptable flow stress curves. However, it cannot stay out of troublesome procedure of finding the material constants as well as the effect of assumed type of functions for the material parameters.

It was found out that the traditional piecewise C-m model, that is, the constant C and m case at the test grid point of sample strain and temperature exhibited quite great error because it cannot deal with the effect of strain rate itself on the strain rate sensitivity. However, the traditional C-m model showed some possibility to fit the complicated flow behaviors for both mechanical and metallurgical predictions in metal forming.

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