Dynamics Analysis of Rolling Bearings for Motor of a Mobile Power Station

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Abstract. The electric motor is an important part of a mobile power station. Under the complex operating conditions, the stiffness of the rolling bearing exhibits strong time-varying characteristics and non-linear characteristics, which is the one of the main sources for the system non-linearity. Taking the angular contact bearing as the research object, the coupled dynamic model of the bearing-rotor system is established to calculate the time-varying stiffness. The bifurcation law exhibited by the system can be attributed to the excitation of the non-linear bearing force. The numerical calculation method is used to study the motion state bifurcation law and dynamic frequency response characteristics under the rated working condition of the motor. Considering bearing damping, clearance and radial load, the bifurcation and time domain diagrams of different parameters, displacements and rotation speeds in different directions are studied respectively. The results show that when the parameters change, the system will experience chaos, quasi-period and period-doubling motion state; Damping suppresses vibration very clearly, while changes in bearing clearance and radial load significantly cause the system's vibration amplitude to increase, and radial load changes accelerate system vibration. A reasonable bearing is of great significance for motor the design of a mobile power station.

Keywords: Mobile power station; Rolling bearing motor; Bearing-rotor system; non-linear dynamics

1. Introduction

With the development of modern science and technology, as the most critical component, rolling bearing plays an increasingly important role in military mobile power station. Due to the bearing clearance, non-linear Hertz contact force and other factors, many abnormal vibrations that cannot be explained by linear theory appear in the operation of the bearing supported rotor system \cite{1}. Moreover, the complex static and dynamic characteristics make the stiffness of rolling bearing show strong time-varying and non-linear characteristics under complex working conditions, which leads to the non-linear vibration of military mobile power station motor \cite{2}. Therefore, the dynamic analysis of motor system using rolling bearing-rotor system has always been a research hotspot \cite{3}. The bifurcation characteristics and the stability conditions of the system can be predicted by establishing the non-linear equations of motion of the bearing-rotor and using Hopf theory. Wang Yunlong carried out the acceleration and deceleration dynamics analysis of the rotor angular contact ball bearing model \cite{4}. The vibration response of a rolling bearing-rigid rotor was studied by numerical method in document \cite{5}, shown that the system has non-linear dynamic behaviors such as chaos, bifurcation,
quasi periodic and subharmonic waves. Hou et al. considered the load, the research on the flexible rotor ball bearing system of aero engine shown that the system not only produces 1/2 harmonic resonance, but also 1/3 or 1/4 harmonic resonance [6]. The Hopf theory provides a theoretical basis for the prediction of periodic solutions, shape and periodicity of the two-order system [7]. It is of great significance to study the influence of bearing parameters (such as speed, rolling body size, surface ripple, rolling body number, imbalance, bearing clearance, cage runout, etc.) on the system [8], and the bifurcation is a dynamic method to study the qualitative changes of the system caused by parameters. Unlike the traditional rotating machinery in military mobile power station, because of the complexity of the working environment and the difficulty of prediction, it is urgent to study the influence of supporting parts on the system.

This paper takes angular contact bearing as the research object, calculates the bearing non-linear force using Hertz theory, analyses the derivation method of bearing stiffness and damping parameters, establishes the bearing-rotor dynamic model, and studies the bifurcation rule of the system motion state and the dynamic frequency response characteristics under the rated working condition of the motor by numerical simulation method. Based on bifurcation theory, the influence of various parameters on the motion system of bearing damping coefficient, bearing clearance and radial load is studied at rated speed respectively.

2. Bearing – Rotor Modeling

The bearing-rotor model of the motor is shown in figure 1. The outer race of angular contact bearing is fixed and the inner race fits tightly with the rotor and rotates; The rotor system bears radial load \( F_r \) at the rotor center of mass. The origin of coordinates is located in the center of mass of the rotor, \( X \) is horizontal and \( Y \) is vertical.

![Figure 1. Bearing - rotor system.](image)

When the rotor of the motor spindle rotates, the spatial position of the bearing ball will change periodically with the cage. The period of change is the time when the cage rotates the angle \( \theta_j \) corresponding to each two balls. The deformation of the rotor and bearing is related to the position and load of the ball. When three balls are stressed, \( \theta_j \) is \( 2\pi/N_b \). The deformation relationship between the electric spindle rotor and the angular contact bearing rolling element is as follows:

\[
2K_r \left( \delta_j \cos \theta_j - \delta_0 \right)^{3/2} \cos \theta_j + K_r \left( \delta_j - \delta_0 \right)^{3/2} = F_r/2 + G/2
\]

\( \delta_j \) is the displacement in the \( y \) direction; \( G \) is the gravity of the rotor.

According to the analysis of contact bearing structure, the radial load is calculated as:

\[
F_r = \sum_{j=1}^{N_b} k_j y_j \cos \left( \frac{2\pi}{N_b} (j-1) \right) = ky
\]

In equation (2), \( y \) is the radial displacement of inner ring; \( k_j \) is the stiffness of series combination; \( k \) is the overall radial stiffness of rolling bearing.

The radial stiffness is:
And the radial damping of the angular contact bearing is:

$$
C_b = \sum_{j=1}^{N_b} c_j \cos^2 \left( \frac{2\pi}{N_b} (j - 1) \right)
$$  \hspace{1cm} (4)

\(C_j\) is the damping coefficient, which is related to the angular frequency.

From the complex stiffness equation get the equation (5):

$$
k_j + i\omega c_j = \left[ \frac{1}{k_{ij} + i\omega c_{i}} + \frac{1}{k_{ij} + i\omega c_{j}} \right]^2
$$  \hspace{1cm} (5)

The bearing is only subjected to static load, and its stiffness changes with the spatial position of the bearing ball. In the bearing coordinate system, when the static load \(f_r\) acts on the \(Y\)-negative direction, once the cage turns a certain angle, the force and deformation of each ball will change, and the bearing stiffness will change accordingly. When the bearing is running at a constant speed, the bearing stiffness will fluctuate periodically according to the time when the cage rotates the corresponding angle of each two balls. When the shaft bears the dynamic load, the deformation state of each ball will change with time, and the position of all balls in the bearing will change periodically with the rotation of the cage, which will change the bearing stiffness. At high speed, the damping coefficient of the bearing is independent of the stiffness, and the damping coefficient determines the stiffness coefficient, which is the key factor affecting the dynamic characteristics of the bearing rotor.

3. Analysis of Dynamic Characteristics of Rotor-Bearing System

The motor parameters studied in this paper are shown in table 1. The front bearing is angular contact bearing (model: SKF7014 CE/P4AL1), and the main parameters are shown in table 2. The bifurcation law can be attributed to the excitation of nonlinear bearing force.

| Name                        | Value | Unit   |
|-----------------------------|-------|--------|
| stator inner diameter \(D_i\) | 84.2  | mm     |
| rotor outside diameter \(d\) | 83.2  | mm     |
| rotor inner diameter \(d_i\) | 58    | mm     |
| Length of stator and rotor \(l\) | 200  | mm     |
| Main shaft length \(L\)     | 517   | mm     |
| rotor inertia \(J/r^2\)     | 0.021 | kg·m² |
| air-gap length \(\delta\)   | 0.5   | mm     |

3.1. Bifurcation Diagram of Bearing Speed with Damping

When the motor operates at rated speed \(n = 4300\) r/min, the bearing clearance is 10\(\mu\)m, the radial load is \(21.5n\), and the bearing damping coefficient \(C\) changes between 0 ~ 400\(N/(M/s)\), and the bifurcation diagram of bearing damping change is obtained.

Figure 2 shows the law of displacement bifurcation in U and W directions with different damping. In the U direction, when the damping is small, the system moves from quasi periodic state to bifurcation state, appears a short period window, and soon enters into chaos state; when the damping is increased to 270\(n/(M/s)\), the system presents a stable period 1 motion, during which the damping is 270\(n/(M/s)\) to 385\(n/(M/s)\) appears a short period doubling motion, and 385\(n/(M/s)\) enters into chaos and period doubling alternate state.
Table 2. Bearing parameters.

| Name                  | Value | Unit  |
|-----------------------|-------|-------|
| outside diameter D    | 110   | mm    |
| inside diameter d     | 70    | mm    |
| ball screw mechanism  | 26    | /     |
| radial clearance γ    | 10    | μm    |
| Hertz contact stiffness $K_e$ | $1.332 \times 10^{10}$ | N/m$^{3/2}$ |
| Mass M                | 0.61  | kg    |

In the \( w \) direction, in the state of small damping, the motion of the system alternates from quasi periodic motion to chaotic motion, during which there is a short period of double period. When the damping is increased to 270n / (M / s), the motion of the system is similar to that in the U direction.

Figure 2. Bifurcation diagram of bearing displacement with damping changes.

Figure 3 shows the bifurcation diagrams of different damping and speed. The trend of motion state of the system is similar to that of the bearing damping and displacement. The bearing damping is between 270 ~ 385n / (M / s). The system presents stable period 1 motion, appears short period doubling motion, and enters chaos and period doubling alternate state when the damping is greater than 385n / (M / s), and the speed amplitude decreases with the increase of damping, which indicates that damping The vibration suppression is very obvious.

Figure 3. Bifurcation diagram of bearing speed with damping.

Figure 4 shows the response time domain diagram of the bearing with the change of damping. It
can be seen that the vibration amplitude of the system is from oscillation to stability, and when the vibration amplitude reaches the maximum value.

3.2. Bifurcation Diagram of Bearing Displacement with Clearance Changes
When the motor operates at rated speed $n = 4300$ r/min, the bearing clearance changes between 0 and 3.0 $\mu$m, the damping coefficient is 50N/(M/s), and the radial load is 21.5N. The bifurcation diagram of bearing clearance change is obtained. Figure 5 shows the bifurcation law of $u$ and $w$ directions in different clearance. It can be seen that in the $u$ direction, with the increase of clearance, the motion state of the system changes from a stable period to a period. When the bearing clearance is greater than 2.35 $\mu$m, the motion of the system changes from a period to a period. When the $w$-direction gap is larger than 2.35 $\mu$m, the system moves into chaos.

Figure 5. Bifurcation diagram of bearing displacement with clearance changes.

Figure 6 shows the bifurcation diagram of different damping and speed. The bifurcation diagram of the two directions of the system is similar. When the bearing clearance is greater than 2.35 $\mu$m, the system moves into a chaotic state, while the $u$ direction bifurcates when the bearing clearance is 0.7 $\mu$M. It can be seen that the vibration amplitude of the system increases with the increase of clearance, and the increase speed of the amplitude is not affected by the change of bearing clearance. Figure 7 shows the response time domain diagram of the bearing with the change of the clearance. It can be seen that the displacement amplitudes of the two directions decrease gradually with the change of the clearance, reaching a stable state. Compared with the variable damping, the vibration amplitudes are larger, and the clearance of the bearing has a greater impact on the dynamic characteristics of the electrodynamics.
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Figure 6. Bifurcation diagram of bearing speed with clearance changes.

3.3. Bifurcation Diagram of Bearing Displacement with Radial Load Changes

When the motor operates at rated speed \( n = 4300 \text{r/min} \), the bearing clearance is \( 10 \mu \text{m} \), so that the bearing radial load \( F_r \) changes between 0 and 50N, and the damping coefficient is \( 50 \text{N/(M/s)} \), and the bifurcation diagram of bearing radial load change is obtained. Figure 8 shows different radial loads and displacement bifurcation. When the radial load is less than the rated load of 21.5n, the motion state of the system changes from quasi periodic motion to multiple periodic motion, and finally reaches periodic motion. When the radial load is greater than 21.5n, it reaches periodic motion after bifurcation.

Figure 7. Bifurcation diagram of bearing response with clearance changes.

Figure 8. Bifurcation diagram of bearing displacement with radial load changes.

Figure 9 shows the bifurcation diagram of radial load and speed of bearing. The trend of motion state of the system is consistent with that shown in figure 9. When the system is larger than the rated radial load by 21.5n through quasi period to 1 period, it will be bifurcated from the period doubling motion state to the periodic motion state. When the radial load reaches the rated load, the vibration
amplitude of bearing will increase rapidly. When the radial load increases to 40n, the vibration amplitude of bearing will increase. Speed starts to slow down. Figure 10 shows the time domain diagram of bearing response with the change of radial load. It can be seen that the time-domain change is basically consistent with the variable damping and bearing clearance. After the system reaches the stable state, the vibration amplitude is large.

![Figure 9. Bifurcation diagram of bearing speed with radial load changes.](image)

![Figure 10. Time domain graph of bearing response with radial load changes.](image)

### 4. Experimental Verification and Engineering Application

#### 4.1. Experimental Verification

The experimental data in this paper comes from MFS rotor dynamics platform of SQI company in the United States, and the experimental bearing parameters are shown in table 3. The vibration of the test-bed under constant speed is collected by the acceleration sensor.

| Table 3. ER16k parameters. |
|-----------------------------|
| Rolling element | Rolling element size | inner diameter | contact angle |
| ER16k | 8 | 7.9 | 33.5 | 0º |

On the experimental platform of bearing fixing device, it is difficult to change the parameters of the bearing. The bearing load can be changed indirectly by machining a small pit about 1.8mm wide and 0.5mm deep on the outer ring raceway.

At the speed of $n = 1800 \text{r/min}$, the vibration data collected in horizontal and vertical directions are shown in figure 11. It can be seen that when the outer ring of the rolling bearing fails, the load in different directions will also change, and the stiffness, clearance and displacement will also change accordingly.
Figure 1 shows the spectrum analysis in horizontal and vertical directions, which not only has fault vibration components, but also shows the nonlinear dynamic characteristics of the bearing.

The figure shows that the axis track of the bearing is irregular, which is different from the regular circle or ellipse without fault. It can be judged from the experiment that when the outer ring of the rolling bearing is in fault, the load in different directions of the bearing movement is changed, which can be equivalent to running under the condition of different radial clearance and bearing stiffness. The chaos state of the bearing movement can be seen from the multi frequency spectrum and the irregular axis track.

Although the experimental results verify the above results to a certain extent, the research object is limited to small and medium-sized motors, and the dynamic characteristics of high-power motor rolling bearing need further study.

4.2. Engineering Application
The research results of this paper provide theoretical basis and fault diagnosis theoretical support for motor design and selection of military mobile power station. During the operation of bearing, its parameters change with time. In order to ensure the reliable operation of the motor, through the research method of this paper, the above bearing parameters should be controlled in a certain range, as shown in table 4

| Parameter            | Range     | Unit     |
|----------------------|-----------|----------|
| Damping coefficient  | 270–385   | N/(m/s)  |
| Bearing clearance    | 0–10      | μm       |
| Radial load          | 21.5–46.9 | kN       |

Table 4. The selection range of bearing parameters.
5. Conclusion

Bifurcation is the study of the qualitative and quantitative behavior of a dynamical system with parameters. The GB (geometric boundary) coupling dynamic model of the motor spindle rotor-bearing system supported by the angular contact bearing is established. The bifurcation law and the dynamic frequency response characteristics of the system under the rated working condition of the motor are studied by numerical simulation. When the motor is running at rated speed, the influence of various parameters on the motion system is studied when the damping coefficient of bearing, bearing clearance and radial load change, and the experimental verification is carried out. The research results are as follows:

(1) With the increase of damping coefficient, the motion state of the system will change from chaotic motion through period doubling to bifurcation, and reach stable periodic motion. When the damping coefficient reaches a certain value, it will enter into the alternate state of chaos and period doubling, and the speed increase will decrease with the increase of damping coefficient.

(2) With the increase of the clearance, the motion state of the system changes from a stable period to a period. When the bearing clearance reaches a certain value, the motion state of the system alternates between a period and a period or enters into a chaotic state. The vibration amplitude of the system increases with the increase of the bearing clearance, and the increase speed of the amplitude is not affected by the change of the bearing clearance.

(3) When the motor is at rated speed, constant damping and constant bearing clearance, when the radial load reaches rated load, the system motion state changes from quasi periodic motion to multiple periodic motion, and finally reaches periodic motion. When the radial load reaches a certain value, the system motion amplitude reaches periodic motion through bifurcation, and the system motion amplitude increases rapidly. The growth rate slows down.

(4) Through the study of the above parameters, it can be seen that the damping can restrain the vibration obviously, while the change of bearing clearance and radial load can obviously increase the vibration amplitude of the system, and the change of radial load can accelerate the vibration of the system.

References

[1] Cao H, Niu L, Xi S, et al. 2018 Mechanical model development of rolling bearing-rotor systems: A review Mechanical Systems & Signal Processing 102: 37-58.
[2] Zhang Z, Chen Y and Cao Q 2015 Bifurcations and hysteresis of varying compliance vibrations in the primary parametric resonance for a ball bearing Journal of Sound & Vibration 350: 171-184.
[3] Bai Y, Yang B, Li H 2018 Analysis of nonlinear vibration in permanent magnet synchronous motors under unbalanced magnetic pull Applied Sciences 8(1): 113.
[4] Wang Y L, Wang W Zh, Qing T, et al. 2018 Angular contact ball bearing rotor dynamic analysis of acceleration and deceleration Chinese Journal of Mechanical Engineering 54(9): 9-16.
[5] Cao H, Li Y, Niu L, et al. 2015 A general method for the dynamic modeling of ball bearing-rotor systems Journal of Manufacture Science and Engineering Transactions of the ASME 137(2).
[6] Hou L, Chen Y, Cao Q, et al. 2015 Turning maneuver caused response in an aircraft rotor-ball bearing system Nonlinear Dynamics 79(1): 229-240.
[7] Miraskari M, Hemmati F, Gadala M S 2017 Nonlinear dynamics of flexible rotors supported on journal bearings - Part []: Numerical bearing model Journal of Tribology 140(2).
[8] Cao H, Niu L, Xi S, et al. 2018 Mechanical model development of rolling bearing-rotor systems: A review Mechanical Systems & Signal Processing 102: 37-58.