Modulation induced frequency shifts in a CPT-based atomic clock

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Abstract

We investigate systematic errors associated with a common modulation technique used for phase sensitive detection of a coherent population trapping (CPT) resonance. In particular, we show that modification of the CPT resonance lineshape due to the presence of off-resonant fields leads to frequency shifts which may limit the stability of CPT-based atomic clocks. We also demonstrate that an alternative demodulation technique greatly reduces these effects.

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Coherent Population Trapping (CPT) has recently been applied to the development of small stable clocks with observed fractional frequency stability (Allan deviation) of better than $10^{-11} \tau^{-1/2}$ for averaging times, $\tau$, around 100 seconds $^{2,3,4,5}$. CPT clocks have potential advantages relative to traditional intensity optical pumping clocks (which typically employ an optical and microwave double resonance technique) $^{2}$ including the possibility of substantial miniaturization without degradation of performance$^{6,7}$. However, various mechanisms may degrade the frequency stability of CPT clocks below that theoretically expected from the observed signal-to-noise ratio. One key mechanism for such degradation is conversion of FM laser noise to AM noise in the detected CPT transmission signal, due to the optical absorption profile when a laser with a large linewidth is used $^{8,9}$. Here we demonstrate a second important degradation mechanism: a widely-used slow phase-modulation technique leads to shifts of the clock frequency in the presence of asymmetries in the CPT resonance. We also show that a straightforward variation of this modulation technique (i.e., use of third-harmonic demodulation) can eliminate much of the systematic effect on the clock frequency.

A CPT clock employs two optical fields that are nominally resonant with electronic transitions in alkali atoms such as Rb or Cs, with the frequency difference between the optical fields being equal to the hyperfine splitting of the alkali’s electronic ground-state. Initially, these two fields optically pump the atoms into a non-interacting coherent superposition of two hyperfine states (a “dark state”) $^{1}$. The long relaxation time of the electronic ground-state leads to enhanced transmission of the optical fields in a narrow resonance around the difference frequency of the two optical fields. The center frequency of this resonance serves as the CPT clock frequency; the width of the resonance is determined by the decoherence rate of the dark state.

In the typical manifestation of a CPT clock, a current-modulated diode laser produces the two resonant optical fields (Fig. 1). Such a laser, with an optical frequency $\omega_{\text{opt}}$ and modulated at a microwave frequency $\omega_\mu$, generates an electric field,

$$
\mathcal{E} = \mathcal{E}_0 \cos \omega_{\text{opt}} t + \\
\mathcal{E}_{-1} \cos (\omega_{\text{opt}} - \omega_\mu) t + \mathcal{E}_{+1} \cos (\omega_{\text{opt}} + \omega_\mu) t + \\
\mathcal{E}_{-2} \cos (\omega_{\text{opt}} - 2\omega_\mu) t + \mathcal{E}_{+2} \cos (\omega_{\text{opt}} + 2\omega_\mu) t + \\
\ldots
$$
where $\mathcal{E}_i$ are the amplitudes of the various frequency components of the electric field. The corresponding Rabi frequencies for the atomic transitions driven by the fields are $\Omega_i = d_i \mathcal{E}_i/(2\hbar)$ where $d_i$ is the atomic transition dipole moment. If the modulation frequency is equal to half the ground-state hyperfine splitting ($\omega_{\mu} = \Delta_{\text{hfs}}/2$) and the laser carrier ($\omega_{\text{opt}}$) is tuned midway between the electronic transition frequency for the ground-state hyperfine sublevels, then the first-order ($\pm 1$) sidebands are simultaneously resonant with the electronic transitions for the two hyperfine sublevels, and a maximum optical transmission is observed.

However, additional off-resonant fields are also created by the laser current-modulation. (The carrier, $\Omega_0$, and the second-order sidebands, $\Omega_{\pm2}$, are generally the most significant off-resonant fields.) Even though these fields are far-detuned from the atomic resonances, they induce an AC Stark shift, which depends on both the optical field frequency and intensity, and causes a relative shift in the atomic levels (a “light shift” in the CPT clock frequency). This sensitivity of the CPT resonance to laser detuning and intensity can limit the stability of a CPT clock. Fortunately, with careful choice of current-modulation index, AC Stark shifts from different off-resonant fields can be arranged to cancel one another.

Phase sensitive detection is often used to provide a sensitive feedback signal for the local oscillator (e.g., a quartz crystal) which is typically locked to the CPT resonance to make a functioning clock. Typically, a “slow” ($< 1$ kHz) phase modulation is superimposed on the microwave source, leading to a modification of the phase of the microwave drive:

$$\omega_{\mu} t \rightarrow \omega_{\mu} t + \epsilon \sin (2\pi f_m t)$$

where $f_m$ is the modulation frequency and $\epsilon$ is the modulation index. After demodulation the (approximately) symmetric CPT transmission resonance is transformed into an (approximately) antisymmetric dispersion-like signal. Near an exactly symmetric CPT resonance, this dispersion-like signal is proportional to the frequency difference between the local oscillator and the center of the CPT resonance, and thus the local oscillator can be locked exactly to the CPT resonance by measuring the microwave frequency ($\omega_{\mu}$) corresponding to the zero-crossing of the dispersion-like signal. However, any asymmetry in the CPT transmission resonance lineshape shifts the zero-crossing of the dispersion-like signal, with a magnitude dependent upon the slow phase-modulation parameters. Instability in these modulation parameters will thus induce instability in the CPT clock microwave frequency. Here we present a detailed experimental study of such shifts, and demonstrate that the
Fig. 1. Energy level diagram for a three-level atom coupled via two near-resonant fields: the +1 and -1 sidebands from a modulated carrier laser field, with one-photon detuning Δ. Also shown are non-resonant fields (the carrier, 0, and the +2 and -2 sidebands) which can produce shifts and distortions in the CPT resonance. For the current experiments using $^{87}\text{Rb}$, the two lower levels correspond to the ground-electronic-state hyperfine levels $F=2$ and $F=1$; and the upper level corresponds to $5^2P_{1/2} F'=2$.

The main contribution is made by the far-detuned optical carrier, $\Omega_0$, (designated as field “0” in Fig. 1). Note that in principle, the slow phase-modulation and the resultant systematic effects on CPT clock performance may be eliminated. For example, the microwave signal that modulates the laser may be produced by direct feedback from a fast output photodetector.

Fig. 2 shows a schematic of our experimental set-up. We derived the two optical fields needed for the CPT clock by phase modulating the output from an external cavity diode laser tuned in the vicinity of the $D_1$ line of Rb ($5^2S_{1/2} \rightarrow 5^2P_{1/2}$, $\lambda \approx 795$ nm). An electro-optic modulator (EOM) produced the phase modulation of the optical field at half of the ground-state hyperfine frequency of $^{87}\text{Rb}$ ($\Delta_{\text{hfs}} \approx 6.8$ GHz). With the microwave power available to the EOM in these measurements (roughly 1 watt), approximately 15% of the incident laser power was transferred to all sidebands, with the remainder residing in the off-resonant carrier. Ideally, the EOM should produce equal amplitudes in the $\pm 1$ sidebands, but due to a slight misalignment of the input polarization, the +1 sideband was 1.3 dB larger than the -1 sideband, producing a 35% larger Rabi frequency ($\Omega_{+1}$) for the $|F = 1\rangle \rightarrow |F' = 2\rangle$ transition, than the Rabi frequency ($\Omega_{-1}$) $|F = 2\rangle \rightarrow |F' = 2\rangle$ transition. Following the EOM, all the optical fields were attenuated by a neutral density...
(ND) filter and an acousto-optic modulator (AOM) to approximately 10 µW total power, circularly polarized using a quarter wave ($\lambda/4$) plate, and then weakly focused to a diameter of about 0.8 mm as they passed through a Rb vapor cell.

The vapor cell was placed inside three-layers of high permeability magnetic shielding to screen out external fields, with a solenoid also inside the magnetic shields to control the magnetic field. The 2.5 cm diameter, 5 cm long vapor cell contained natural abundance Rb and 5 Torr of nitrogen buffer gas. The buffer gas slowed the Rb atomic motion through the vapor cell. The vapor cell was thermally stabilized using a blown-air oven at a temperature of 45 °C. The total Rb vapor pressure at this temperature corresponds to a $^{87}$Rb atomic density of $2 \cdot 10^{10}$ cm$^{-3}$. Under these conditions the optical depth of the vapor cell was approximately one, for a weak resonant optical field.

A coupled, three-level $\Lambda$ system was formed by the two first-order sidebands of the laser field, the two lower $^{87}$Rb states $F = 1$, $m_F = 0$ and $F = 2$, $m_F = 0$ and the excited state $F' = 2$, $m_{F'} = 1$ (Fig. 1). We chose these hyperfine sublevels so as to have no first-order dependence of hyperfine transition frequency on magnetic field. We applied a magnetic field of 16 mG to lift Zeeman degeneracies and remove other ground-state sublevels from two
Fig. 3. Typical dispersive-like CPT resonance measured with a slow phase-modulation frequency $f_m = 230$ Hz and index $\epsilon = 0.6$ (solid line). Also shown is a numerical calculation of the line shape expected from an ideal three-level system and just two near-resonant optical fields, for our observed Rabi frequencies and slow phase-modulation index (dotted line). [The width and center frequency of the calculated resonance were scaled to match the measured resonance.] Note the asymmetry of the measured resonance compared to the calculation for the ideal three-level/two-field system.

photon resonance, thus preventing unwanted coherences from developing.

We used phase sensitive detection to convert the approximately symmetric CPT transmission resonance of the two-photon “clock transition” into a dispersive-like resonance (Fig. 3). We applied a slow phase-modulation to the microwave source driving the EOM, and then demodulated the corresponding slow variations in the photodetector current using a lock-in amplifier. In a working CPT clock, a feedback loop locks the external oscillator to the zero-crossing of the antisymmetric, modulated line. Here, rather than closing the
loop in the feedback system, we measured the frequency of the zero-crossing (by fitting a line through the central part of the dispersive-like resonance) relative to a frequency source phase-locked to a hydrogen maser. We then varied system parameters such as the laser detuning ($\omega_{\text{opt}}$), slow phase-modulation frequency ($f_m$), or modulation index ($\epsilon$), and measured the zero-crossing as a function of these parameters.

Drifts in the zero-crossing will directly lead to changes in the clock frequency and thus degrade the frequency stability of the CPT clock. To characterize such variations in the clock frequency ($\delta$), we measured the dependence of the zero-crossing on the laser carrier detuning ($\Delta$), for various slow phase (i.e., lock-in) modulation parameters. (Note that $\Delta$ was also the detuning of the first order sidebands from the center of the Doppler broadened absorption resonance.) Fig. 4a shows the measured clock frequency shift as a function of laser detuning for various slow phase-modulation (lock-in) indices at one fixed laser modulation index (EOM power). We find that near the center of the Doppler-broadened atomic transition, the clock frequency is proportional to the laser detuning (see the linear fits in Fig. 4b), with a slope $\delta/\Delta$ that increases linearly with slow phase-modulation index (at fixed slow phase-modulation frequency) as shown on Fig. 4b. We observed similar behavior upon changing the slow phase-modulation frequency at fixed slow phase-modulation index. These shifts are significant in comparison to the desired frequency stability of a CPT clock, with fractional clock frequency sensitivity to laser detuning of order $10^{-11}$/MHz. As shown in Fig. 4c, we also observed a dependence of the clock frequency on slow phase-modulation index, with the laser fields tuned to resonance, i.e., with $\Delta = 0$. This “laser-frequency-independent” shift is of order $10^{-8}$ (fractionally) for 100% changes in the modulation index. Good CPT clock frequency stability therefore requires high stability of the slow phase-modulation frequency and amplitude, even in the absence of laser frequency variation.

Light shifts (i.e., AC Stark shifts) associated with unequal intensities of the first-order laser sidebands are expected to scale linearly with the laser detuning:

$$\delta \propto -\Delta \frac{|\Omega_{+1}|^2 - |\Omega_{-1}|^2}{\gamma^2},$$

where $\Omega_{\pm 1}$ are the Rabi frequencies of the resonant laser sidebands and $\gamma$ is the relaxation rate of the excited state. The data in Fig. 4a exhibit this linear scaling of $\delta$ with $\Delta$. However, this simple light-shift mechanism does not account for the observed dependence of the clock frequency on the slow phase-modulation index (see Fig. 4b and 4c). The above light-shift
Fig. 4. a) Measured $^{87}\text{Rb}$ CPT clock frequency shift ($\delta$) as a function of detuning ($\Delta$) of the laser carrier frequency and resonant sidebands from $F' = 2$ resonance, for various slow phase-modulation indices ($\epsilon$). Here, a zero frequency shift ($\delta = 0$) corresponds to the free-space $^{87}\text{Rb}$ hyperfine frequency. The large offset of $\delta \approx 1360$ Hz is due to the nitrogen buffer gas pressure shift. b) Dependence of the clock frequency on laser detuning ($\delta/\Delta$), determined from the slope of each individual line on plot (a), as a function of the slow phase-modulation index. c) Measured laser-frequency-independent shift, at $\Delta = 0$, as a function of the slow phase-modulation index (see vertical line in plot (a)). All data were taken at a slow phase-modulation (lock-in) frequency $f_m = 69$ Hz. In graphs (a) and (c), measurement uncertainties were comparable to the size of the symbols shown.

expression assumes that the CPT resonance remains symmetric near the center, only shifting due to the Rabi frequency difference. However, as shown in Fig. 4, the clock frequency shift ($\delta$) depends on the modulation index ($\epsilon$) indicating that the CPT resonance is asymmetric. [A similar dependence of $\delta$ on the slow phase-modulation frequency ($f_m$) was also observed]
Additionally, these dependencies do not vanish at $\Delta = 0$, indicating that other frequency shift mechanisms are present.

One such mechanism is the light shift caused by the off-resonant carrier field. As with the Rabi frequency imbalance described above, this light shift is usually expected to cause a simple shift in the clock frequency but no distortion of the lineshape. The shift is expected to be $\simeq 2|\Omega_0|^2/\Delta_0$, where $\Omega_0$ is the carrier field Rabi frequency and $\Delta_0 = \Delta_{\text{hfs}}/2$ is the magnitude of the carrier’s detuning from both transitions in the $\Lambda$-system for $\Delta = 0$. Without a distortion of the lineshape, no dependence of the clock frequency shift ($\delta$) on the slow phase-modulation parameters should exist. Fig. 5 shows the measured clock frequency shift as a function of laser carrier field power ($\sim |\Omega_0|^2$) at two slow phase-modulation frequencies and two total powers in the first-order sidebands, all for $\Delta = 0$. In addition to the usual light shift, we observe a dependence on the slow phase-modulation frequency of approximately 20 mHz/µW/Hz. Consistent with Eq. (3) at $\Delta = 0$, we do not find a dependence of $\delta$ on the first-order sideband power, $|\Omega_{\pm 1}|$. From these measurements, we conclude that the interaction of the strong, off-resonant carrier field of the modulated laser not only shifts the clock frequency, but also modifies the transmission resonance lineshape.

To further study the dependence of the clock frequency on the carrier field, we inserted a Fabry-Perot cavity between the EOM and the vapor cell in a slightly altered experimental setup\textsuperscript{15}. This cavity had a free spectral range of 1.37 GHz (1/5 of the $^{87}$Rb hyperfine splitting) and was tuned such that it allowed the transmission of the two first-order sidebands of the laser while rejecting the carrier frequency field. A more symmetric output signal was detected (see Fig. 6), and the dependence of the clock frequency shifts on the slow phase-modulation parameters ($\epsilon$ and $f_m$) were reduced by at least an order of magnitude.

We explored alternative techniques for slow phase-modulation with the goal of reducing the dependence of the CPT clock frequency on system parameters. For example, we investigated demodulation using the third harmonic of the slow phase-modulation, a technique that is known to compensate for linear asymmetry in an underlying resonance\textsuperscript{16,17}. We found that the clock frequency for both first- and third-harmonic demodulation has a linear dependence on the slow modulation frequency (Fig. 7a). However, with third-harmonic demodulation, the slope of this dependence is a factor of five smaller, reducing the sensitivity of the clock frequency to changes in either the laser carrier frequency (e.g., Fig. 4a) or the properties of the slow phase-modulation source (Fig. 4a and b). Similarly, we found that

(Fig. 7a).
Fig. 5. Measured dependence of the CPT clock frequency on carrier field power at one-photon resonance ($\Delta = 0$), for two slow phase-modulation frequencies and two total powers in the first-order sidebands. (Uncertainties in the measured clock frequencies are approximately equal to the size of the points.) Linear fits are shown for all data points at each of the two modulation frequencies.

Third-harmonic demodulation reduces the laser-frequency-independent shift by at least an order of magnitude to fractional shifts of less than $10^{-9}$ for 100% changes in modulation index.

One must also consider the change in measurement sensitivity to the clock transition when changing the slow phase-modulation technique. We define the measurement sensitivity as the ratio of the fitted central slope of the dispersive-like resonance divided by the RMS fluctuations in the fit residuals. Optimal measurement sensitivity occurs for the slow phase-modulation index and frequency that provide maximum signal at the photodetector in the slow phase-modulation sidebands demodulated by the lock-in amplifier. This optimal sensitivity is achieved when the width of the “comb” of slow phase-modulation sidebands transmitted through the atomic medium is comparable to the CPT resonance width. At the optimal slow phase-modulation index for first harmonic demodulation, we found that the measurement sensitivity when using third harmonic demodulation is substantially reduced;
Fig. 6. CPT resonances with (solid line) and without (dotted line) the carrier field present. Both data sets were taken with $f_m = 98$ Hz and $\epsilon \approx 0.6$ and with a vapor cell containing isotopically enriched $^{87}$Rb and 22 Torr of Ne buffer gas.\textsuperscript{15}

However, we recovered the original sensitivity by increasing the slow phase-modulation index (Fig. 7b).

The measurement sensitivity’s dependence on modulation index is consistent with a simple Lorentzian model. We calculated the dispersive curves obtained by applying slow modulation at frequency $f_m$ and index $\epsilon$ to a Lorentzian lineshape and determined the slope of the curve near the zero-crossing. The lines in Fig. 7b show this slope as a function of modulation index with only the ratio of the Lorentzian resonance linewidth to the modulation frequency and the overall amplitude adjusted to fit the data. Note, however, that this model does not include a realistic CPT lineshape or the effects of off-resonant fields and thus does not reproduce the data in Fig. 7a.

In conclusion, we have quantitatively studied systematic effects on CPT clock frequency due to asymmetries in the two-photon CPT resonance induced by strong off-resonant laser fields. These asymmetries can produce significant shifts in the CPT clock frequency when
Fig. 7. (a) Clock frequency dependence on laser frequency ($\delta/\Delta$) as a function of slow phase-modulation frequency ($f_m$) for both first and third harmonic demodulation. The modulation index $\epsilon = 1.87$ for both data sets. (b) Dependence of the clock transition measurement sensitivity (defined in text) to the slow phase-modulation index ($\epsilon$) for $f_m = 153$ Hz. A much larger slow phase-modulation index is required for optimal sensitivity in third harmonic modulation. The lines are the results of a fit to the sensitivity expected for an ideal Lorentzian lineshape.
slow phase-modulation is used to determine the center of the CPT resonance. While lowering the frequency of the slow phase-modulation decreases the sensitivity of the clock to such systematic frequency shifts, technical noise reduces the signal-to-noise ratio (and hence the clock’s short-term frequency stability) at very slow phase-modulation frequencies.

To achieve good fractional frequency stability $\sim 10^{-11}$ in a CPT clock, these systematic frequency shifts impose demanding requirements on the stability of the elements that control the slow phase-modulation ($f_m$ and $\epsilon$) and the laser carrier frequency ($\omega_{\text{opt}}$). Fortunately, there are promising alternative modulation techniques that can mitigate the effects of CPT lineshape asymmetry in the presence of non-resonant laser fields. A careful choice of the fast (microwave) modulation index allows the AC Stark shifts from different off-resonant laser fields to cancel each other$^2$. As demonstrated here, third-harmonic demodulation of the slow phase-modulation greatly reduces the sensitivity to asymmetry in the CPT resonance.

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