ESTIMATE ON DECELERATION PARAMETER IN A UNIVERSE WITH VARIABLE FINE STRUCTURE “CONSTANT”.

Marcelo S.Berman(1)
and Luis A.Trevisan(2)

(1) Tecpar-Instituto de Tecnologia do Paraná
Grupo de Projetos Especiais.
R.Prof Algacir M. Mader 3775-CEP 81350-010
Curitiba-PR-Brasil
Email: marsambe@tecpar.br

(2) Universidade Estadual de Ponta Grossa,
Demat, CEP 84010-330, Ponta Grossa, Pr,
Brasil email: latrevis@uepg.br

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Abstract

We determine a cosmological model that includes acceleration of the Universe and time-decreasing fine structure “constant”, as found recently. We found that the present-day deceleration parameter should be \( q \simeq -1.1 \times 10^{-5} \). The variation in \( \alpha \) is attributed to a slowly increasing speed of light.

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The latest developments of physics, when we enter this century, are the discovery that the Universe is accelerating, and that the fine structure "constant " is varying with the age of the Universe.

The first conclusion, based on Supernovae [1] observations may mean that the value of the cosmological constant is non-null; however, what we know for sure is that the deceleration parameter,

\[ q = -\frac{\ddot{R} R}{R^2} \] (1)

is negative. In (1), \( R = R(t) \) stands for the scale-factor, and overdots are time-derivatives, while the metric is Robertson-Walker’s.

The second finding is that the “constant ”

\[ \alpha \equiv \frac{e^2}{\hbar c}, \]

(2)
is varying with time.

Let us call \( F = \Delta \alpha/\alpha \Delta t \) (experimental). We can make a rough estimate for \( F \) by employing the values found by Webb et al for the deviation from the average:

\[ \frac{\Delta \alpha}{\alpha} \approx -0.72 \times 10^{-5} \] (3)

spanning 23% to 87% of the age of the Universe [2]. A rough estimate for \( \Delta t \) is given by \( \Delta t = (0.87 - 0.23)t \) where \( t \) stands for the age of the Universe. Any competent reader will be able to modify our results when a better estimate shall be available in the literature for \( F \).

As in cosmology, the charge of the electron, or Planck’s constant, do not play a large-scale rôle, we fix our attention on a time-varying speed of light, \( c = c(t) \).
Webb et al.\cite{2} pointed out that a common property of unified theories, applied to Cosmology, is that they predict space and time-dependence of the coupling constants. Let us suppose that Newton’s gravitational “constant” $G$ is also time varying; the natural framework we need is JBD (Jordan-Brans-Dicke) theory with varying speed, as Barrow has been studying \cite{3}.

It is well accepted that, for the present Universe a power-law of time can represent the scale-factor. We are thus led to consider constant deceleration parameter models, as studied initially by Berman \cite{4}, and Berman and Gomide\cite{5}. On defining $m = 1 + q = \text{constant}$, we find,

$$R = R(t) = (mDt)^{\frac{1}{m}}$$  \hspace{1cm} (4)

where $D = \text{constant}$.

Einstein-de Sitter model, with $k = 0$ ($k$ is the tricurvature), is of this kind, with $m = 3/2$. As our time scale for the present Universe duration so far is about $10^{10}$ years, and there is not evidence for a significant time variation of $q$, for the present Universe, we find that a constant deceleration parameter model is reasonable for the present Universe. In reference \cite{7} there is a comment on one single evidence for a changing $q$ for the Universe in its present phase, but obviously this is not a conclusive evidence up to now. We shall show that the solution with variable $\alpha$ differs from Einstein-de Sitter’s model.

Barrow’s equation read:

$$H^2 = \frac{8\pi\rho}{\phi} - \frac{\dot{\phi}}{\phi}H + \frac{\omega\dot{\phi}^2}{6\phi^2} - \frac{kc^2(t)}{R^2}$$  \hspace{1cm} (5)

$$\frac{3\ddot{R}}{R} = \frac{-8\pi}{(3 + 2\omega)\phi} \left[ (2 + \omega)\rho + \frac{3(1 + \omega)p}{c^2(t)} \right] - \frac{\ddot{\phi}}{\phi} - \frac{\omega\dot{\phi}^2}{\phi^2}$$  \hspace{1cm} (6)

$$\ddot{\phi} + 3H\dot{\phi} = \frac{8\pi}{3 + 2\omega} \left( \rho - \frac{3p}{c^2(t)} \right)$$  \hspace{1cm} (7)

Unlike Barrow, we shall not impose a definite equation of state; the values for energy density $\rho$ and cosmic-pressure $p$ will arise naturally in our solution.

From the theory of constant deceleration parameter we have:

$$H = (mt)^{-1}$$  \hspace{1cm} (8)
We find the following solution:

\[ R = (mDt)^{\frac{1}{m}} \]  
\[ \phi = At^n \]  
\[ \rho = ABt^{n-2} \]  
\[ c = Ct^{\frac{1}{m}-1} \]  
\[ p = Ft^{\gamma-4+2/m} \]

where \(A, B, \gamma, F, C\) are constants, related by means of equations (4), (5) and (6).

From (11) and (2), we find,

\[ \dot{\alpha} \alpha = Hq \]  
\[ \frac{\Delta \alpha}{\alpha \Delta t} = \frac{\Delta \alpha}{\alpha \left[0.64 \frac{H^{-1}}{m}\right]} \simeq -0.72 \frac{10^{-5} H m}{0.64} \]  
\[ \frac{\Delta \alpha}{\alpha \Delta t} \simeq 1.1 \times 10^{-5} H m. \]

Let us now compare with the experimental values:

For the age of the Universe, we make use of relation (7). We see that \(q \simeq -1.1 \times 10^{-5}\) and \(m \simeq 0.99999\).

The speed of light is, then increasing slowly with time. The Universe has a minimal, though finite, acceleration, as can be checked from (8), with the values of \(q\) and \(m\).

It has been a happy coincidence, that when the \(\alpha\) varying experiments were published, we were able to apply the recent Barrow-Jordan-Brans-Dicke equations, in order to derive a value for \(q\), which was lacking in the literature. Barrow and Magueijo [6] explained how a changing \(\alpha\) could yield the Supernova results with a different argumentation than ours. For up to date
references in relation to variable $G$ cosmologies, see ref [8] [9]. We refer to these papers for further information.

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