New Dynamical Symmetry Breaking in Electroweak Theory

Bing An Li

Department of Physics and Astronomy, University of Kentucky
Lexington, KY 40506, USA

Abstract

A new dynamical symmetry breaking of $SU(2)_L \times U(1)$ caused by the combination of the axial-vector component and the fermion mass is found in electroweak theory. The masses of the W and the Z bosons are obtained to be $m_W^2 = \frac{1}{2} g^2 m_t^2$ and $m_Z^2 = \rho m_W^2 / \cos^2 \theta_W$ with $\rho \simeq 1$. They are in excellent agreement with data. The Fermi constant is determined to be $G_F = \frac{1}{2\sqrt{2} M^2}$. Two fixed gauge fixing terms for W and Z boson fields are dynamically generated. Massive neutrinos are required.
The standard model[1] of electroweak interactions is successful in many aspects. In this model Higgs is introduced to generate the masses for the W, Z and fermions by spontaneous symmetry breaking. So far the experimental indication of the existence of the Higgs has not been found yet. On the other hand, a Higgs/Hierarchy problem[2] has been revealed. There are many different attempts[3,4] trying to solve this problem: W and Z bosons are composite; Higgs fields are bound states of fermions; supersymmetry. The top quark has been discovered in Fermi laboratory[5], whose mass has been determined to be

\[ m_t = 180 \pm 12 \text{ GeV}[6]. \] (1)

The value of \( m_t \) is at the same order of magnitude as the masses of the W and the Z bosons. As a matter of fact, before the discovery of the top quark there were attempts of finding the relationship between top quark and intermediate bosons by using various mechanism[4]. In this paper a new approach is proposed to eliminate the Higgs and keep the successes of the standard model.

This paper is organized as: 1) Lagrangian and formalism; 2) model of new symmetry breaking; 3) masses of W and Z bosons; 4) propagators of W and Z bosons; 5) Gauge fixing term; 6) theoretical values of \( m_W \) and \( m_Z \); 7) summary.
1 Lagrangian and formalism

The Lagrangian of the standard model consists of boson fields, fermions, and Higgs. The couplings between fermions and bosons have been extensively tested. Theoretical results are in excellent agreement with data. The mass terms of fermions (except for neutrinos) are well established. The Lagrangian of the boson fields is constructed by gauge principle. Therefore, in the Lagrangian of the standard model the part of the boson fields, the interactions between fermions and bosons, and the mass terms of fermions are reliable. On the other hand, the Higgs sector of the Lagrangian of the standard model has not been determined yet. In this paper we study the dynamical properties of the Lagrangian without Higgs

\[
\mathcal{L} = -\frac{1}{4} A^i_{\mu\nu} A^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{q} \{ i \gamma \cdot \partial - M \} q \\
+ \bar{q}_L \{ g^2 \frac{\tau_i}{2} \gamma \cdot A^i + g' \frac{Y}{2} \gamma \cdot B \} q_L + \bar{q}_R g' \frac{Y}{2} \gamma \cdot B q_R \\
+ \bar{l} \{ i \gamma \cdot \partial - M_f \} l + \bar{l}_L \{ g^2 \frac{\tau_i}{2} \gamma \cdot A^i - g' \frac{Y}{2} \gamma \cdot B \} l_L - \bar{l}_R g' \gamma \cdot B l_R. \tag{2}
\]

Summation over \(q_L, q_R, l_L,\) and \(l_R\) is implicated in Eq.(2).

It is necessary to point out that in eq.(2) the boson fields are still elementary fields and the couplings between the bosons and the fermions of the standard model remain unchanged, therefore, the successes of the standard model are kept. In the standard model the masses of the W and Z bosons are via spontaneous symmetry breaking mechanism generated by Higgs. In this paper we study whether \(m_W\) and \(m_Z\) can be dynamically generated from this
Lagrangian(2).

Due to the fermion mass terms the Lagrangian(2) is no longer gauge invariant. Without losing generality, we study the properties of the Lagrangian of the generation of t and b quarks. The Lagrangian of this doublet is

\[
\mathcal{L} = -\frac{1}{4} A^i_{\mu\nu} A^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{t} \{ i\gamma \cdot \partial - m_t \} t + \bar{b} \{ i\gamma \cdot \partial - m_b \} b \\
+ \bar{\psi}_L \left( \frac{g}{2} \tau_i \gamma \cdot A^i + \frac{g'}{6} \gamma \cdot B \right) \psi_L + \frac{2}{3} g' \bar{t} R \gamma \cdot B t R - \frac{1}{3} g' \bar{b} R \gamma \cdot B b R,
\]

(3)

where \( \psi_L = \begin{pmatrix} t \\ b \end{pmatrix}_L \). The quark part of the Lagrangian(3) is rewritten as

\[
\mathcal{L} = \bar{\psi} \left( i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 - m \right) \psi,
\]

(4)

where \( \psi \) is the doublet of t and b quarks, \( m = \begin{pmatrix} m_t & 0 \\ 0 & m_b \end{pmatrix} \), \( v_\mu = \tau_i v^i_\mu + \omega_\mu \), \( v^{1,2}_\mu = \frac{g}{4} A^{1,2}_\mu \), \( v^3_\mu = \frac{g}{4} A^3_\mu + \frac{g'}{4} B_\mu \), \( \omega_\mu = \frac{g'}{6} B_\mu \), \( a_\mu = \tau_i a^i_\mu \), \( a^{1,2}_\mu = -\frac{g}{4} A^{1,2}_\mu \), \( a^3_\mu = -\frac{g}{4} A^3_\mu + \frac{g'}{4} B_\mu \).

The mass term, \( \bar{\psi} m \psi \), is not invariant under the transformation

\[
\psi \rightarrow e^{i(\alpha_1 \tau_1 + \alpha_2 \tau_2)} \psi.
\]

Therefore, the charged gauge symmetry is explicitly broken by the quark masses and the charged bosons, W, are expected to gain masses. On the other hand, the quark mass term is invariant under two gauge transformations

\[
\psi \rightarrow e^{i\alpha} \psi
\]
and

\( \psi \rightarrow e^{i\alpha_3 \tau_3} \psi \).

The theory should have two massless neutral bosons. One of the two neutral bosons is photon. It is obvious that a new dynamical symmetry is needed for making the Z boson massive.

Because the Lagrangian(4) is not invariant under the charged gauge transformation in principle the mass term of the W boson can be added to the Lagrangian(2). This paper shows that the mass of the W boson can be correctly generated by the Lagrangian(2). Therefore, the mass term of W boson is fine-tuned to be zero. This kind of phenomena is not only for gauge symmetry broken theory, it happens to gauge invariant theory too. For example, in QED there are many other gauge invariant terms which are not forbidden by any physical principle. Anomalous magnetic moment of the lepton is a good example and it can be added to the Lagrangian. However, it is well known that the anomalous magnetic moment of the lepton is derived from the loop diagrams. There is no need to put such term into the Lagrangian by hand. This is known as the minimum coupling principle.

Using path integral to integrate out the quark fields, in Euclidean space the Lagrangian of boson fields is obtained

\[ \mathcal{L} = \ln \det D, \quad (5) \]
where

\[ D = \gamma \cdot \partial - i\gamma \cdot v - i\gamma \cdot a\gamma_5 + m. \]

The real and imaginary parts of the Lagrangian (5) are

\[ L_{Re} = \frac{1}{2} \ln \det (D^\dagger D), \quad L_{Im} = \frac{1}{2} \ln \det (\frac{D}{D^\dagger}), \]  

where

\[ D^\dagger = -\gamma \cdot \partial + i\gamma \cdot v - i\gamma \cdot a\gamma_5 + m. \]  

(7)

It is necessary to point out that \( D^\dagger D \) is a definite positive operator. In terms of Schwinger's proper time method[7] \( L_{Re} \) is expressed as

\[ L_{Re} = \frac{1}{2} \int d^Dx Tr \int_0^\infty d\tau \frac{d\tau}{\tau} e^{-\tau D^\dagger D}. \]  

(8)

Inserting a complete set of plane waves and subtracting the divergence at \( \tau = 0 \), we obtain

\[ L_{Re} = \frac{1}{2} \int d^Dx \frac{d^Dp}{(2\pi)^D} Tr \int_0^\infty \frac{d\tau}{\tau} \left\{ e^{-\tau D^\dagger D'} - e^{-\tau \Delta_0} \right\}, \]  

(9)

where

\[ D' = \gamma \cdot \partial + i\gamma \cdot p - i\gamma \cdot v - i\gamma \cdot a\gamma_5 + m, \quad D'^\dagger = -\gamma \cdot \partial - i\gamma \cdot p + i\gamma \cdot v - i\gamma \cdot a\gamma_5 + m, \]

\[ D'^\dagger D' = \Delta_0 - \Delta, \quad \Delta_0 = p^2 + m_1^2, \quad m_1^2 = \frac{1}{2}(m_t^2 + m_5^2), \quad m_2^2 = \frac{1}{2}(m_t^2 - m_5^2), \]

\[ \Delta = \partial^2 - (\gamma \cdot v - \gamma \cdot a\gamma_5)(\gamma \cdot v + \gamma \cdot a\gamma_5) - i\gamma \cdot \partial(\gamma \cdot v + \gamma \cdot a\gamma_5) \]

\[ -i(\gamma \cdot v - \gamma \cdot a\gamma_5)\gamma \cdot \partial + 2ip \cdot \partial + 2p \cdot (v + a\gamma_5) - i[\gamma \cdot v, m] \]

\[ + i\{\gamma \cdot a, m\}\gamma_5 - m_2^2\tau_3. \]  

(10)
After the integration over $\tau$, $\mathcal{L}_{Re}$ is expressed as

$$\mathcal{L}_{Re} = \frac{1}{2} \int d^Dx \frac{d^Dp}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{n (p^2 + m_i^n)^n} Tr \Delta^n. \quad (11)$$

Due to $\mathcal{L}_{IM}(-m, -\gamma_5) = -\mathcal{L}_{IM}(m, \gamma_5)$, $\mathcal{L}_{Im}$ doesn’t contribute to the masses of W and Z bosons at least at the tree level of the boson fields. Therefore, the study of $\mathcal{L}_{Im}$ is beyond the scope of this paper.

$\mathcal{L}_{Re}$ is used to investigate the symmetry breaking mechanism, the masses of the intermediate bosons, and the propagators of the boson fields. As mentioned above, the $SU(2)_L \times U(1)$ symmetry is explicitly broken by both quark and lepton masses. However, another symmetry breaking mechanism is needed.

It is necessary to emphasize on that the field theory of electroweak interactions is different from QED and QCD. In QED and QCD photon and gluons are pure vector fields. Due to parity nonconservation the intermediate bosons in the standard model have both vector and axial-vector components which are written in the forms of $v$ and $a$ in Eq.(4). In this paper it is shown that this property of the intermediate boson fields results in another $SU(2)_L \times U(1)$ symmetry breaking. From the expression of $\Delta(10)$ it is seen that in company with fermion mass the vector component $v$ of the intermediate boson field appears in a commutator $[v, m]$, while the axial-vector component $a$ appears in an anticommutator $\{a, m\}$. Due to this property the axial-vector component of boson field
causes a new symmetry breaking.

2 Model of new symmetry breaking

In order to show how the axial-vector field results in a symmetry breaking a model is studied in this section. The Lagrangian of a vector field and a fermion (QED) is

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \{ i \gamma \cdot \partial + e \gamma \cdot v \} \psi - m \bar{\psi} \psi. \]  

(12)

This Lagrangian (12) is invariant under the gauge transformation

\[ \psi \rightarrow e^{i\alpha(x)} \psi, \quad v_\mu \rightarrow v_\mu + \frac{1}{e} \partial_\mu \alpha. \]

Using the Eqs. (10, 11), the mass term of the vector field is obtained

\[ \mathcal{L}_M = \frac{1}{2} \int d^D x \int \frac{d^D p}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{(p^2 + m^2)^n} Tr(2p \cdot v - v^2)^n. \]  

(13)

Only \( n = 1, 2 \) contribute to the mass term

\[ \mathcal{L}_M = -\frac{D}{2} \int d^D x \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + m^2} v^2 + D \int d^D x \int \frac{d^D p}{(2\pi)^D} \frac{1}{(p^2 + m^2)^2} p \cdot v p \cdot v = 0. \]  

(14)

These two terms cancel each other. As expected, no mass is generated.

The Lagrangian of an axial-vector field and a fermion is

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \{ i \gamma \cdot \partial + e \gamma \cdot a_{55} \} \psi - m \bar{\psi} \psi. \]  

(15)
This Lagrangian (15) is invariant under the gauge transformation
\[ \psi \rightarrow e^{i\alpha(x)}\psi, \quad a_\mu \rightarrow a_\mu + \frac{1}{e} \partial_\mu \alpha \gamma_5. \]

Using the Eqs. (10,11), the mass term of the axial-vector field is obtained
\[ L_M = \frac{1}{2} \int d^D x \int \frac{d^D p}{(2\pi)^D} \sum_{n=1}^\infty \frac{1}{n (p^2 + m^2)^n} Tr(2p \cdot a \gamma_5 - a^2 + i\{\gamma \cdot a, m\} \gamma_5)^n. \quad (16) \]

Only \( n = 1, 2 \) contribute to the mass term
\[ L_M = -\frac{D}{2} \int d^D x \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + m^2 a^2} + D \int d^D x \int \frac{d^D p}{(2\pi)^D} \frac{1}{(p^2 + m^2)^2} \{p \cdot a, a + m^2 a^2\} \]
\[ = \frac{1}{(4\pi)^2} D \Gamma(2 - \frac{D}{2}) m^2 a_\mu a^\mu. \quad (17) \]

The axial-vector field gains mass, therefore, the symmetry is broken.

Comparing with eq. (13), in Eq. (16) there is one more term which is the anticommutator
\[ i\{\gamma \cdot a, m\} \gamma_5. \] Because of \([v_\mu, m] = 0\) there is no such term for vector field. Therefore, the theory of axial-vector field is very different from the vector field. The symmetry in the theory of axial-vector field is broken by the combination of the axial-vector field and the mass of the fermion. The mass of the axial-vector field is generated by this symmetry breaking.
3 Masses of W and Z bosons

In terms of the Lagrangian (11) the masses of the intermediate bosons are calculated. The terms related to the masses only is separated from Eq. (11)

\[
\mathcal{L}_M = \frac{1}{2} \int d^Dx \int \frac{d^Dp}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{n (p^2 + m_1^2)^n} Tr \{- (\gamma \cdot v - \gamma \cdot a\gamma_5) \\
(\gamma \cdot v + \gamma \cdot a\gamma_5) + 2p \cdot (v + a\gamma_5) + i [m, \gamma \cdot v] + i \{m, \gamma \cdot a\} \gamma_5 - m_2^2 \tau_3 \}^n.
\]  

(18)

The contributions of the fermion masses to \( m_W \) and \( m_Z \) are needed to be calculated to all orders.

Four kinds of terms of the Lagrangian (18) contribute to the masses of the bosons

\[
\mathcal{L}^1 = \frac{1}{2} \int \frac{d^Dp}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{n (p^2 + m_1^2)^n} Tr (\gamma \cdot v - \gamma \cdot a\gamma_5)(\gamma \cdot v + \gamma \cdot a\gamma_5)(-m_2^2 \tau_3)^{n-1},
\]

\[
\mathcal{L}^2 = \frac{2}{2} \int \frac{d^Dp}{(2\pi)^D} \sum_{n=2}^{\infty} \frac{1}{n} \frac{(-m_2^2)^{n-2}}{n (p^2 + m_1^2)^n} \sum_{k=0}^{n-2} (n-1-k) Tr p \cdot (v + a\gamma_5) \tau_3^k p \cdot (v + a\gamma_5) \tau_3^{n-2-k},
\]

\[
\mathcal{L}^{1+2} = \mathcal{L}^1 + \mathcal{L}^2 = -\frac{8 N_c}{(4\pi)^2} \sum_{k=1}^{\infty} \frac{1}{(2k+1)(2k-1)} (m_2^2)^{2k} (\frac{g}{4})^2 m_1^2 \sum_{i=1}^2 A_i^i A_i^{i\mu},
\]  

(19)

\[
\mathcal{L}^3 = \frac{1}{2} \int \frac{d^Dp}{(2\pi)^D} \sum_{n=2}^{\infty} \frac{1}{n} \frac{(-m_2^2)^{n-2}}{n (p^2 + m_1^2)^n} \sum_{k=0}^{n-2} (n-1-k) Tr \{\gamma \cdot v, m\} \tau_3^k \{\gamma \cdot v, m\} \tau_3^{n-2-k}
\]

\[
= 2 \frac{N_c}{(4\pi)^2} D\Gamma (2 - \frac{D}{2}) (\frac{g}{4})^2 m_1^2 \sum_{i=1}^2 A_i^i A_i^{i\mu} + 2 \frac{N_c}{(4\pi)^2} \sum_{k=2}^{\infty} \frac{1}{(2k-1)(2k-2)} (m_2^2)^{2k-2} (\frac{g}{4})^2 m_1^2 \sum_{i=1}^2 A_i^i A_i^{i\mu},
\]

(20)

\[
\mathcal{L}^4 = \frac{1}{2} \int \frac{d^Dp}{(2\pi)^D} \sum_{n=2}^{\infty} \frac{1}{n} \frac{(m_2^2)^{n-2}}{n (p^2 + m_1^2)^n} \sum_{k=0}^{n-2} (n-1-k) Tr \{\gamma \cdot a, m\} \tau_3^k \{\gamma \cdot a, m\} \tau_3^{n-2-k}
\]
\[
\left. \begin{array}{c}
N_c \left( \frac{4\pi}{2} \right) \{ \frac{1}{2} \sum_{i=1}^{2} A_i^\mu A_i^{i\mu} + m_1^2 \{ \frac{1}{4} \}^2 + \left( \frac{g'}{4} \right)^2 \} Z_\mu Z^\mu \} \\
- \frac{8 N_C}{(4\pi)^2} \sum_{k=1}^{\infty} \frac{1}{(2k-1)2k} \left( \frac{m_2^2}{m_1^2} \right)^{2k-1} \{ \left( \frac{g}{4} \right)^2 + \left( \frac{g'}{4} \right)^2 \} m_2^2 Z_\mu Z^\mu \\
+ \frac{8 N_C}{(4\pi)^2} \sum_{k=1}^{\infty} \frac{1}{(2k+1)2k} \left( \frac{m_2^2}{m_1^2} \right)^{2k} \left( \frac{g}{4} \right)^2 m_2^2 \sum_{i=1}^{2} A_i^\mu A_i^{i\mu},
\end{array} \right\}
\]

where \( m_+ = \frac{1}{2}(m_t + m_b) \) and \( m_- = \frac{1}{2}(m_t - m_b) \).

The Eqs.(19-21) show that all the four terms contribute to the mass of the W boson.

These results indicate that the mass of the charged boson, W, originates in both the explicit \( SU(2)_L \times U(1) \) symmetry breaking by the fermion masses and the dynamical symmetry breaking caused by the combination of the axial-vector component and the fermion mass(\( \{a_\mu, m\} \)). Only \( L^4 \) contributes to \( m_Z \). \( L^4 \) is related to \( \{a_\mu, m\} \). Therefore, \( m_Z \) is dynamically generated by the new dynamical symmetry breaking. As expected, a U(1) symmetry remains and the neutral vector meson, the photon, is massless.

It is found that the series of the fermion masses are convergent to analytic functions. Putting all the four terms(19-21) together, the masses of W and Z are obtained in Minkowski space

\[
\mathcal{L}_M = \frac{1}{2} \frac{N_C}{(4\pi)^2} \left\{ \frac{D}{4} \Gamma(2 - \frac{D}{2}) (4\pi \frac{\mu^2}{m_1^2})^\frac{x}{2} + \frac{1}{2} \left[ 1 - ln(1 - x) - (1 + \frac{1}{x}) \sqrt{x} \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \right] m_1^2 g^2 \sum_{i=1}^{2} A_i^\mu A_i^{i\mu} \right. \\
+ \frac{1}{2} \frac{N_C}{(4\pi)^2} \left\{ \frac{D}{4} \Gamma(2 - \frac{D}{2}) (4\pi \frac{\mu^2}{m_1^2})^\frac{x}{2} - \frac{1}{2} [ln(1 - x) + \sqrt{x}ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}}] m_1^2 (g^2 + g'^2) Z_\mu Z^\mu \right\} (22)
\]
where $N_C$ is the number of colors and $x = \left(\frac{m_1^2}{m_3^2}\right)^2$. It is necessary to point out that

$$a_\mu^3 = \frac{1}{4} \sqrt{g^2 + g'^2} Z_\mu. \tag{23}$$

In the same way, other two generations of quarks, $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} c \\ s \end{pmatrix}$, and three generations of leptons $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$, $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$, and $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ contribute to the masses of W and Z bosons too. By changing the definitions of $m_1^2$ and $x$ to the quantities of other generations in Eq.(22), the contributions of the other two quark generations are found. Taking off the factor $N_C$ and changing $m_1^2$ and $x$ to corresponding quantities of leptons, the contributions of the leptons to $m_W$ and $m_Z$ are obtained. In this paper the effects of CKM matrix are not taken into account. The final expressions of the masses of W and Z bosons are the sum of the contributions of the three quark and the three lepton generations. It is learned from the processes deriving Eq.(22) that

1. Due to the U(1) symmetry the neutral vector field $\sin \theta_W A_\mu^3 + \cos \theta_W B_\mu$ (photon field) is massless;

2. The W boson gains mass from both the explicit and dynamical $SU(2)_L \times U(1)$ symmetry breaking;

3. $m_Z$ is resulted in the dynamical symmetry only. Without $\{a_\mu, m\}$ Z boson is massless.
4 Propagators of W and Z bosons

In the standard model the bosons are Yang-Mills fields and massless before the spontaneous symmetry breaking. Gauge fixing terms can be chosen and the theory is renormalizable. In the Lagrangian (2) the boson fields are still Yang-Mills fields. However after the symmetry breaking is taken into account, as done above, the W and the Z gain masses. They are massive. It is necessary to study their propagators to see whether they have right behavior for renormalization at high energy. Up to all orders of fermion masses, the kinetic terms of the intermediate boson fields are obtained from the Lagrangian (11). For the generation of t and b quarks there are nine terms

\[
\mathcal{L}^1 = -\frac{1}{2} \int \frac{d^D p}{(2\pi)^D} \sum_{n=3}^{\infty} \frac{1}{n} \frac{(-m_2^2)^{n-3}}{(p^2 + m_1^2)^n} \sum_{k=0}^{n-3-k} \sum_{k_1=0}^{k} (n - 2 - k - k_1) Tr[\gamma \cdot v, m] \partial^2 \tau_3^{k+k_1} [\gamma \cdot v, m] \tau^{n-3-k-k_1}
\]

\[
= \frac{8N_C m_2^2}{(4\pi)^2 m_1^2} \sum_{k=1}^{\infty} \frac{1}{(2k + 1)(2k - 1)} \frac{m_2^2}{m_1^2} \frac{1}{2k-2} \frac{g^2}{4} \sum_{i=1}^{2} A_i^i \partial^2 A^{i\mu},
\]

\[
\mathcal{L}^2 = 2 \int \frac{d^D p}{(2\pi)^D} \sum_{n=4}^{\infty} \frac{1}{n} \frac{(-m_2^2)^{n-4}}{(p^2 + m_1^2)^n} \sum_{k=0}^{n-4-k} \sum_{k_1=0}^{k} \sum_{k_2=0}^{k_1} (n - 3 - k - k_1 - k_2) Tr[\gamma \cdot v, m] \tau_3^{k+k_1+k_2} (p \cdot \partial)^2 [\gamma \cdot v, m] \tau^{n-4-k-k_1-k_2}
\]

\[
= -\frac{4N_C m_2^2}{(4\pi)^2 m_1^2} \sum_{k=1}^{\infty} \frac{1}{(2k + 1)(2k - 1)} \frac{m_2^2}{m_1^2} \frac{1}{2k-2} \frac{g^2}{4} \sum_{i=1}^{2} A_i^i \partial^2 A^{i\mu},
\]

\[
\mathcal{L}^3 = \frac{1}{2} \int \frac{d^D p}{(2\pi)^D} \sum_{n=3}^{\infty} \frac{1}{n} \frac{(-m_2^2)^{n-3}}{(p^2 + m_1^2)^n} \sum_{k=0}^{n-3-k} \sum_{k_1=0}^{k} (n - 2 - k - k_1) Tr[\gamma \cdot a, m] \partial^2 \tau_3^{k+k_1} [\gamma \cdot a, m] \tau^{n-3-k-k_1}
\]
\[
L^4 = -2 \int \frac{d^D p}{(2\pi)^D} \sum_{n=4}^\infty \frac{1}{n} \frac{(-m_2^2)^{n-4}}{(n^2 + m_1^2)^n} \sum_{k=0}^{n-4-k} \sum_{k_1=0}^{n-4-k-k_1} (n - 3 - k - k_1 - k_2)
\]
\[
Tr\{\gamma \cdot a, m\} \tau^{k+k_1+k_2} (p \cdot \partial)^2 \{\gamma \cdot a, m\} \tau^{n-4-k-k_1-k_2}
\]
\[
= -\frac{4N_c}{(4\pi)^2} \sum_{k=2}^\infty \frac{1}{(2k-1)(2k-2)} \left(\frac{m_2^2}{m_1^2}\right)^{2k-2} \left(\frac{g_4}{4}\right)^2 A_\mu \partial^2 A^{i\mu} - \frac{4N_c}{3(4\pi)^2} a_\mu^3 \partial^2 a^{3\mu},
\]
\[
L^{1+2+3+4} = \frac{4N_c}{(4\pi)^2} \sum_{k=2}^\infty \frac{1}{(2k-1)(2k-2)} \left(\frac{m_2^2}{m_1^2}\right)^{2k-2} \left(\frac{g_4}{4}\right)^2 A_\mu \partial^2 A^{i\mu} + \frac{4N_c}{3(4\pi)^2} a_\mu^3 \partial^2 a^{3\mu},
\]
\[
L^5 = -\frac{1}{2} \int \frac{d^D p}{(2\pi)^D} \sum_{n=2}^\infty \frac{1}{n} \frac{(-m_2^2)^{n-2}}{(n^2 + m_1^2)^n} \sum_{k=0}^{n-2-k} (n - 1 - k) Tr\{\gamma \cdot v - \gamma \cdot a a_5\} \partial^2 \tau^k \tau^{n-2-k}
\]
\[
= -\frac{N_c}{(4\pi)^2} \frac{D}{4} \Gamma(2 - \frac{D}{4}) Tr(v_\mu \partial^2 v^{\mu} + a_\mu \partial^2 a^{\mu})
\]
\[
-\frac{4N_c}{(4\pi)^2} \sum_{k=2}^\infty \frac{1}{(2k-1)(2k-2)} \left(\frac{m_2^2}{m_1^2}\right)^{2k-2} \left(\frac{g_4}{4}\right)^2 A_\mu \partial^2 A^{i\mu}
\]
\[
-\frac{2N_c}{(4\pi)^2} \sum_{k=2}^\infty \frac{1}{2k-2} \left(\frac{m_2^2}{m_1^2}\right)^{2k-2} \left\{v_\mu^3 \partial^2 v^{3\mu} + \omega_\mu \partial^2 \omega^{\mu} + a_\mu^3 \partial^2 a^{3\mu}\right\}
\]
\[
+\frac{4N_c}{(4\pi)^2} \sum_{k=1}^\infty \frac{1}{2k-1} \left(\frac{m_2^2}{m_1^2}\right)^{2k-1} \omega_\mu \partial^2 v^{3\mu},
\]
\[
L^6 = 2 \int \frac{d^D p}{(2\pi)^D} \sum_{n=3}^\infty \frac{1}{n} \frac{(-m_2^2)^{n-3}}{(n^2 + m_1^2)^n} \sum_{k=0}^{n-3-k} \sum_{k_1=0}^{n-3-k-k_1} (n - 2 - k - k_1)
Tr p \cdot (v + a a_5) \tau^{k+k_1} \partial^2 p \cdot (v + a a_5) \tau^{n-3-k-k_1}
\]
\[
= \frac{2}{3} \frac{N_c}{(4\pi)^2} \frac{D}{4} \Gamma(2 - \frac{D}{4}) Tr(v_\mu \partial^2 v^{\mu} + a_\mu \partial^2 a^{\mu})
\]
\[
+\frac{8N_c}{(4\pi)^2} \sum_{k=2}^\infty \frac{1}{(2k+1)(2k-1)(2k-2)} \left(\frac{m_2^2}{m_1^2}\right)^{2k-2} \left(\frac{g_4}{4}\right)^2 A_\mu \partial^2 A^{i\mu}
\]
\[
+\frac{4N_c}{3(4\pi)^2} \sum_{k=2}^\infty \frac{1}{2k-2} \left(\frac{m_2^2}{m_1^2}\right)^{2k-2} \left\{v_\mu^3 \partial^2 v^{3\mu} + \omega_\mu \partial^2 \omega^{\mu} + a_\mu^3 \partial^2 a^{3\mu}\right\}
\]
\[ - \frac{8N_C}{3(4\pi)^2} \sum_{k=1}^{\infty} \frac{1}{2k-3} \left( \frac{m_2^2}{m_1^2} \right)^{2k-3} \omega_\mu \partial^2 v^3\mu, \quad (29) \]

\[ \mathcal{L}^7 = -8 \int \frac{d^D p}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{-m_2^2}{n(p^2 + m_1^2)} \right)^n \sum_{k,k_1,k_2} (n - 3 - k - k_1 - k_2) \]

\[ T r p \cdot (v + a_\gamma_5) \mathcal{T}^{k+k_1+k_2} (p \cdot \partial)^2 p \cdot (v + a_\gamma_5) \mathcal{T}^{n-4-k-k_1-k_2} \]

\[ = - \frac{1}{3(4\pi)^2} \frac{D}{4} \Gamma(2 - \frac{D}{4}) T r (v_\mu \partial^2 v^\mu - 2 \partial_\mu v^\mu \partial_\nu v^\nu + a_\mu \partial^2 a^\mu - 2 \partial_\mu a^\mu \partial_\nu a^\nu) \]

\[ - \frac{4N_C}{(4\pi)^2} \sum_{k=3}^{\infty} \frac{1}{(2k - 1)(2k - 3)(2k - 4)(m_2^2)} \left( \frac{m_1^2}{m_1^2} \right)^{2k-2} \left( \frac{g_1}{4} \right)^2 (A_\mu^i A_i^\mu - 2 \partial_\mu A_i^\mu \partial_\nu A_i^{\nu i}) \]

\[ - \frac{2N_C}{3(4\pi)^2} \sum_{k=3}^{\infty} \frac{1}{2k - 4} \left( \frac{m_2^2}{m_1^2} \right)^{2k-4} \{ v_\mu^3 \partial^2 v^3\mu - 2 \partial_\mu v^3\mu \partial_\nu v^3\nu + \omega_\mu \partial^2 \omega^\mu \]

\[ - 2 \partial_\mu a^\mu \partial_\nu a^\nu \}

\[ + \frac{4N_C}{3(4\pi)^2} \sum_{k=2}^{\infty} \frac{1}{2k - 3} \left( \frac{m_2^2}{m_1^2} \right)^{2k-3} \omega_\mu \partial^2 v^3\mu - 2 \partial_\mu a^\mu \partial_\nu a^\nu \}, \quad (30) \]

\[ \mathcal{L}^8 = 2 \int \frac{d^D p}{(2\pi)^D} \sum_{k=3}^{\infty} \frac{1}{n} \left( \frac{-m_2^2}{n(p^2 + m_1^2)} \right)^n \sum_{k=0}^{n-3} \sum_{k_1=0}^{n-k-3} (n - 2 - k - k_1) \]

\[ T r p \cdot (v + a_\gamma_5) \mathcal{T}^{k+k_1+k_2} p \cdot \partial_\gamma \cdot \partial (\gamma \cdot v + \gamma \cdot a_\gamma_5) \mathcal{T}^{n-3-k-k_1}, \]

\[ = \frac{2N_C}{3(4\pi)^2} \frac{D}{4} \Gamma(2 - \frac{D}{4}) T r (v^\mu \partial_\mu v^\nu + a^\mu \partial_\mu a^\nu) \]

\[ + \frac{8N_C}{(4\pi)^2} \sum_{k=2}^{\infty} \frac{1}{(2k - 1)(2k - 3)(2k - 2)(m_2^2)} \left( \frac{m_1^2}{m_1^2} \right)^{2k-2} \left( \frac{g_1}{4} \right)^2 (A_\mu^i \partial_\nu A_i^{\nu i}) \]

\[ + \frac{4N_C}{3(4\pi)^2} \sum_{k=2}^{\infty} \frac{1}{2k - 2} \left( \frac{m_2^2}{m_1^2} \right)^{2k-2} \{ v^3\mu \partial_\mu v^3\nu + \omega^\mu \partial_\mu \omega^\nu + a^3\mu \partial_\mu a^3\nu \}

\[ - \frac{8N_C}{3(4\pi)^2} \sum_{k=2}^{\infty} \frac{1}{2k - 3} \left( \frac{m_2^2}{m_1^2} \right)^{2k-3} \omega_\mu \partial_\mu v^3\nu, \quad (31) \]

\[ \mathcal{L}^9 = 2 \int \frac{d^D p}{(2\pi)^D} \sum_{n=3}^{\infty} \frac{1}{n} \left( \frac{-m_2^2}{n(p^2 + m_1^2)} \right)^n \sum_{k=0}^{n-3} \sum_{k_1=0}^{n-k-3} (n - 2 - k - k_1) \]

\[ T r (\gamma \cdot v - \gamma \cdot a_\gamma_5) \gamma \cdot \partial p \cdot \partial_\tau_3^{k+k_1+k_2} p \cdot (\gamma \cdot v + \gamma \cdot a_\gamma_5) \mathcal{T}^{n-3-k-k_1}, \]
Taking other two generations of quarks and three generations of leptons into account in
Eqs.(24-32) and adding them together, in Minkowski space the kinetic terms are obtained

\[ \mathcal{L}_K = -\frac{1}{4} \sum_{i=1,2} (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)^2 \{1 + \frac{1}{(4\pi)^2} g^2 \sum_{q,l} N[\frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)\frac{\mu^2}{m_1^2}]\}
\]

\[ -\frac{1}{4} (\partial_\mu A^3_\nu - \partial_\nu A^3_\mu)^2 \{1 + \frac{1}{(4\pi)^2} g^2 \sum_{q,l} N[\frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)\frac{\mu^2}{m_1^2}]\}
\]

\[ -\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \{1 + \frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)\frac{\mu^2}{m_1^2}\}
\]

\[ + \frac{1}{(4\pi)^2} g^2 \sum_{l} [\frac{D}{12} \Gamma(2 - \frac{D}{2})(4\pi)\frac{\mu^2}{m_1^2}]\}
\]

\[ -\frac{1}{12} g g' \frac{1}{12} \{\partial_\mu A^3_\nu - \partial_\nu A^3_\mu\} \{\partial_\mu B_\nu - \partial_\nu B_\mu\} \{N_G - \frac{2}{3} N_C \sum_{q} f_3 + 2 \sum_{l} f_3\}
\]

\[ + \frac{1}{(4\pi)^2} N_G \frac{g^2}{12} \sum_{i=1,2} (\partial_\mu A^i_\mu)^2 + \frac{1}{(4\pi)^2} N_G \frac{1}{12} (g^2 + g'^2)(\partial_\mu Z^\mu)^2, \]  

(33)

where \( \sum_q \) and \( \sum_l \) stand for summations of generations of quarks and leptons respectively,

\( N = N_C \) for q and \( N = 1 \) for l, \( N_G = 3N_C + 3 \), \( x \) depends on fermion generation and is defined in Eq.(22),

\[ f_1 = \frac{4}{9} - \frac{1}{6x} - \frac{1}{6} \ln(1 - x) + \frac{1}{4\sqrt{x}} (\frac{1}{3x} - 1) \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}}, \]

\[ f_2 = -\ln(1 - x), \quad f_3 = \frac{1}{2} \frac{1}{\sqrt{x}} \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}}. \]  

(34)

Following results are obtained from Eq.(33)

16
1. The boson fields and the coupling constants $g$ and $g'$ have to be redefined by multiplicative renormalization

$$
A_{\mu}^{1,2} \rightarrow Z_{1}^{1/2}A_{\mu}^{1,2},
A_{\mu}^{3} \rightarrow Z_{3}^{1}A_{\mu}^{3},
B_{\mu} \rightarrow Z_{B}^{1}B_{\mu},
$$

$$
g_{1} = Z_{1}^{-1/2}g, g_{3} = Z_{3}^{-1/2}g, g_{B} = Z_{B}^{-1/2}g', \tag{35}
$$

where

$$
Z_{1} = 1 + \frac{1}{(4\pi)^{2}}g^{2}\sum_{q,l}N[D_{12}\Gamma(2 - D/2)(4\pi)^{2}(\frac{m_{1}^{2}}{2})^{2}(\frac{1}{6} + f_{1})]
$$

$$
Z_{3} = 1 + \frac{1}{(4\pi)^{2}}g^{2}\sum_{q,l}N[D_{12}\Gamma(2 - D/2)(4\pi)^{2}(\frac{m_{1}^{2}}{2})^{2}(\frac{1}{6} + f_{2})]
$$

$$
Z_{B} = 1 + \frac{N_{C}}{(4\pi)^{2}}g^{2}\sum_{q}[D_{12}\Gamma(2 - D/2)(4\pi)^{2}(\frac{m_{1}^{2}}{2})^{2}(\frac{1}{6} + f_{2} - \frac{1}{18}f_{3})]
$$

$$
+ \frac{1}{(4\pi)^{2}}g^{2}\sum_{l}D_{4}\Gamma(2 - D/2)(4\pi)^{2}(\frac{m_{1}^{2}}{2})^{2}(\frac{1}{6} + f_{2} + \frac{1}{6}f_{3}). \tag{36}
$$

The divergent terms in $Z_{1}$ and $Z_{3}$ are the same.

2. There is a crossing term between $A_{\mu}^{3}$ and $B_{\mu}$, which is written as

$$
g_{3}g_{B}(\partial_{\mu}A_{\nu}^{3} - \partial_{\nu}A_{\mu}^{3})(\partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}) = e^{2}(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu})^{2} - e^{2}(\partial_{\mu}Z_{\nu} - \partial_{\nu}Z_{\mu})^{2}
$$

$$
+ e(g_{3}\cos\theta_{W} - g_{B}\sin\theta_{W})(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu})(\partial_{\mu}Z_{\nu} - \partial_{\nu}Z_{\mu}), \tag{37}
$$

where $\sin\theta_{W} = \frac{g_{3}}{\sqrt{g_{3}^{2} + g_{B}^{2}}}$, $\cos\theta_{W} = \frac{g_{B}}{\sqrt{g_{3}^{2} + g_{B}^{2}}}$, and $e = \frac{g_{3}g_{B}}{\sqrt{g_{3}^{2} + g_{B}^{2}}}$. Therefore, the photon and the $Z$ fields are needed to be renormalized again

$$
(1 + \frac{\alpha}{4\pi}f_{4})^{1/2}A_{\mu} \rightarrow A_{\mu}, \ (1 - \frac{\alpha}{4\pi}f_{4})^{1/2}Z_{\mu} \rightarrow Z_{\mu}. \tag{38}
$$
where

\[ f_4 = \frac{1}{3} N_G - \frac{2}{3} \sum_q f_3 + \frac{2}{3} \sum_l f_3. \]

After these renormalizations (35,39), \( \mathcal{L}_K(33) \) is rewritten as

\[
\mathcal{L}_K = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4} \sum_{i=1,2} (\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)^2 - \frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2
\]

\[
- \frac{1}{4} \frac{\alpha}{4\pi} \left( \frac{g_3}{g_B} - \frac{g_B}{g_3} \right) (1 - \frac{\alpha^2}{(4\pi)^2} f^2_4) - f_4 (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu)
\]

\[
- \frac{N_G}{(4\pi)^2} \frac{g^2}{12} (\partial^\mu A^i_\mu)^2 - \frac{N_G}{(4\pi)^2} (1 - \frac{\alpha}{4\pi} f_4)^{-1} \frac{1}{12} (g^2_3 + g^2_B)(\partial_\mu Z^\mu)^2. \tag{39}
\]

3. The interaction between photon and Z boson is predicted in Eq.(39).

4. For very small neutrino masses it is derived from Eq.(34)

\[ f_3 = -\ln\left( \frac{m_\nu}{m_l} \right). \tag{40} \]

Therefore, if neutrino is massless, \( f_4 \) is logarithmic divergent. This divergence is in contradiction with that the physical coupling between photon and z boson must be finite. Therefore, this theory requires massive neutrinos.

5 **Gauge fixing term**

Before the spontaneous symmetry breaking the standard model is gauge invariant. Gauge fixing terms can be chosen artificially. In the Lagrangian(2) the strengths of the boson fields
have the structure of the Yang-Mills fields. However, W and Z are massive. Eq.(39) shows that fixed gauge fixing terms of W- and Z- fields are dynamically generated

$$\xi_W = \frac{N_G g_1^2}{(4\pi)^2} \frac{1}{3}, \quad \xi_Z = \frac{N_G}{(4\pi)^2} (1 - \frac{\alpha}{4\pi}f_4)^{-1}\frac{1}{3}(g_3^2 + g_B^2).$$

(41)

The propagator of W field is derived from Eq.(39)

$$\frac{i}{q^2 - m_W^2} \{ -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \} - \frac{i}{\xi_W q^2 - m_W^2} \frac{q_{\mu}q_{\nu}}{q^2}.$$

(42)

Changing the index W to Z in Eq.(42), the propagator of Z boson field is obtained. Obviously, due to the gauge fixing terms the propagators of W an Z bosons do not affect the renormalizability of the theory(2). It is necessary to point out that the W and the Z fields are massive and no longer gauge fields. The "gauge fixing" terms of W and Z bosons(41) are derived from this theory and they are not obtained by choosing gauge.

6 Theoretical values of $m_W$ and $m_Z$

Now we can study the values of the masses of W and Z bosons. After the renormalizations(35,38) there are still divergences in the mass formulas of $m_W$ and $m_Z$(22). The boson fields are already renormalized and the kinetic terms of the boson fields are already in the
standard form. Therefore, the divergences in the formulas of \( m_W \) and \( m_Z \) cannot be absorbed by the boson fields. On the other hand, the divergences in Eq.(22) are fermion mass dependent, while the coupling constants should be the same for all fermion generations. It is difficult that these divergences are absorbed by the coupling constants. In Eq.(22) the fermion masses are bare physical quantities. It is reasonable to redefine the fermion masses by multiplicative renormalization

\[
Z_m m_1^2 = m_{1,P}^2,
\]

\[
Z_m = \frac{N}{(4\pi)^2} \left\{ N_G \frac{D}{4} \Gamma(2 - \frac{D}{2}) (4\pi)^\frac{D}{2} \left( \frac{\mu^2}{m_1^2} \right)^\frac{D}{2} + \frac{1}{2} \left[ 1 - \ln(1-x) - (1 + \frac{1}{x}) \sqrt{x} \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \right] \right\},
\]

for each generation of fermions. The index "P" is omitted in the rest of the paper. Now the mass of W boson is obtained from Eq.(22)

\[
m_W^2 = \frac{1}{2} g^2 \left( m_t^2 + m_b^2 + m_c^2 + m_s^2 + m_u^2 + m_d^2 + m_c^2 + m_u^2 + m_c^2 + m_d^2 + m_c^2 + m_d^2 \right)
\]

(44)

Obviously, the top quark mass dominates the \( m_W \)

\[
m_W = \frac{g}{\sqrt{2}} m_t.
\]

(45)

Using the values \( g = 0.642 \) and \( m_t = 180 \pm 12 \text{GeV} \)[6], it is found

\[
m_W = 81.71(1 \pm 0.067) \text{GeV},
\]

(46)
which is in excellent agreement with data $80.33 \pm 0.15\text{GeV}[6]$. The Fermi coupling constant is derived from eq.(45)

$$G_F = \frac{1}{2\sqrt{2}m_t} = 0.96 \times 10^{-5}m_N^2,$$

where $m_t = 180\text{GeV}$ is taken.

Using Eqs.(22,35,38), the mass formula of the Z boson is written as

$$m_Z^2 = \rho m_W^2(1 + \frac{g_B^2}{g_1^2}), \quad (47)$$

where

$$\rho = (1 - \frac{\alpha}{4\pi}f_4)^{-1}\sum_{q,l} N\{\frac{D}{4}\Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2}(\frac{\mu^2}{m_1^2})^\frac{D}{2} - \frac{1}{2}[\ln(1 - x) + \sqrt{x}\ln\frac{1 + \sqrt{x}}{1 - \sqrt{x}}]\}$$

$$/ \sum_{q,l} N\{\frac{D}{4}\Gamma(2 - \frac{D}{2})(4\pi)^\frac{D}{2}(\frac{\mu^2}{m_1^2})^\frac{D}{2} + \frac{1}{2}[1 - \ln(1 - x) - (1 + \frac{1}{x})\sqrt{x}\ln\frac{1 + \sqrt{x}}{1 - \sqrt{x}}]\} \frac{g_3^2 + g_B^2}{g_1^2 + g_2^2} \quad (48)$$

Comparing with the infinities in Eqs.(35,36,48), the finite terms can be ignored in Eqs.(35,36,48).

We have

$$g_1 = g_3 \equiv g_A, \quad m_Z^2 = \rho m_W^2/cos^2\theta_W, \quad cos\theta_W = g_A/\sqrt{g_A^2 + g_B^2}, \quad \rho = (1 - \frac{\alpha}{4\pi}f_4)^{-1}, \quad (49)$$

where $g_A$ and $g_B$ are $g$ and $g'$ of the GWS model respectively. The finiteness of the $\rho$ factor requires a finite $f_4$. Once again massive neutrinos are required. Due to the smallness of the factor $\frac{\alpha}{4\pi}$ in the reasonable ranges of the quark masses and the upper limits of neutrino masses we expect

$$\rho \simeq 1. \quad (50)$$
Therefore,

\[ m_Z = m_W / \cos \theta_W \] (51)

is a good approximation. Eq.(51) is the prediction of the standard model.

Introduction of a cut-off leads to

\[ \frac{D}{4} \Gamma(2 - \frac{D}{2})(4\pi)\hat{\chi}^2 \left( \frac{\mu^2}{m^2_1} \right)^2 \to \ln(1 + \frac{\Lambda^2}{m^2_1}) - 1 + \frac{1}{1 + \frac{\Lambda^2}{m^2_1}}. \] (52)

Taking \( \Lambda \to \infty \), Eq.(49) is obtained. On the other hand, the cut-off might be estimated by the value of the \( \rho \) factor. The cut-off can be considered as the energy scale of unified electroweak theory.

### 7 Summary

In the Lagrangian(2) the boson fields are elementary fields. The Lagrangian of boson fields, the couplings between the fermions and the bosons, and the fermion mass terms are the same as in the standard model. The boson fields are different from photon and gluons, they have both vector and axial-vector components. It shows in this paper that axial-vector field is very different from vector field. A new dynamical symmetry breaking caused by the combination of the axial-vector component and fermion mass is found. In this theory there are explicit symmetry breaking by fermion masses and dynamical symmetry breaking.
The charged boson, $W$, gains mass from both the symmetry breaking, while the mass of the neutral boson, $Z$, originates in the dynamical symmetry breaking only. Due to the $U(1)$ symmetry and the vector nature of photon the photon remains massless. Upon the scheme of multiplicative renormalization the values of $m_W$ and $m_Z$ are determined. They are in excellent agreement with data. After $W$ and $Z$ gain masses the theory is no longer gauge invariant. Two gauge fixing terms for $W$ and $Z$ are dynamically generated respectively. The propagators of $W$- and $Z$-fields have no problem for renormalization. On the other hand, the finiteness of $m_Z$, the $\gamma - Z$ coupling, and the fixed gauging fixing terms require massive neutrinos.

In this paper[8] the path integral is used to derive the Lagrangian of boson fields without any additional assumption. The results are rigorous. The Lagrangian(2) is the one of the standard model without Higgs. Therefore, the successes of the standard model are kept. $m_W$, $m_Z$, and gauge fixing terms are dynamically generated. This theory is equivalent to the standard model without Higgs.

This research was partially supported by DOE Grant No. DE-91ER75661.

References
[1] S.L.Glashow, Nucl.Phys. B22,579(12961); S.Weinberg, Phys.Rev. Lett. 19,1264(1967); A.Salam, Proc. of the 8th Nobel Symposium, p.367, ed. by N.Svartholm, Almqvist and Wiksell, Stockholm 1968.

[2] see review article by P.Langacker, Advanced Series on Direction in High Energu Physics-Vol.14, p.13, edited by P.Langacker, World Scientific.

[3] see review articles: T.Appelquist, in Mexican school of particles and Fields, 1990,p.1; H.P.Nilles in Testing the standard model, p.633 and Phy.Rep.C110,1(1984); H.E.Haber and G.Kane, Phys.Rep.C117,75(1985).

[4] Y.Nambu, Enrico Fermi Institute preprint 88-39(1988), 88-62(1988) and 89-08(1989); V.A.Miransky, M.Tanabashi and K.Yamawaki, Mod. Phys. Lett. A4(1989) 1043; Phys.Lett. B221,177(1989); W.J.Marciano, Phys.Rev.Lett. 62, 2793(1989); W.A.Bardeen, C.T.Hill and M.Linder, Phys.Rev., D41, 1647(1990); R.D.Peccei and X.Zhang, Nucl.Phys. B337,269(1990); D.E.Kahana and S.H.Kahana, Phys.Rev. D43,2361(1991).

[5] F.Abe et al., Phys.Rev.Lett. 74,2626(1995); S.Abachi et al., Phys.Rev.Lett. 74,2632(1995).

[6] Particle Data Group, Phys.Rev.D54 1996.
[7] J. Schwinger, Phys. Rev. 93, 613 (1954).

[8] Some of the results of this paper have been briefly reported in B. A. Li, hep-ph/9709332.