INTERFERENCE PHENOMENA IN SPONTANEOUS EMISSION FROM DRIVEN MULTILEVEL ATOMS

V.O.Chaltykyan, A.D. Gazazyan, and Y.T.Pashayan

Institute for Physical Research,
Armenian National Academy of Sciences,
Ashtarak-2, 378410, Armenia
(email: alfred@ipr.sci.am)

Abstract

We study spontaneous emission from an atom under the action of laser fields. We consider two different energy level diagrams. The first one consists of two levels resonantly driven by laser radiation where either of levels may decay to a separate level. For such a system we show that the presence of the second decay channel may deteriorate the destructive interference occurring in case of one decay channel because of Autler-Townes effect. The second diagram represents two two-level resonantly driven systems with the upper levels decaying to a common level. For this diagram we show that there is no interference between the two decay channels when the laser fields are described in the Fock representation, while in case of definite-phase classical fields such interference takes place and is partially or completely destructive or constructive depending on the initial conditions and on the mutual orientation of the spontaneous emission dipole moments.
I. INTRODUCTION

The investigations of the decay of a quantum-mechanical system affected by an external electromagnetic field and the interference phenomena between the different decay channels are of fundamental as well as practical interest. All the quantum processes under the action of electromagnetic radiation can be divided into two groups: i) processes occurring also in the absence of an externally applied field and affected by this field and ii) processes that are stimulated by the electromagnetic field and do not take place when there is no radiation. In this sense, spontaneous transitions from excited states of atoms pertain to the first group, while the excitation of an atom by an external electromagnetic field with subsequent spontaneous transition to a lower state pertains to the second group.

The spontaneous emission spectrum from an isolated atomic level is known to be described by the Lorentz profile. The effect of laser radiation leads to a significant change in both the lifetime and spectral line shape of spontaneous emission. In particular, it is known that the decay law of an atomic level depends upon the state of the external quantized electromagnetic field [1]. In a multilevel system, transitions between the states make the spectrum of spontaneous emission more complicated and dependent upon the radiation intensity.

Recently much attention has been focused on the study of spontaneous emission spectrum of an atom. Of particular interest is the quantum Zeno effect, i.e., retardation (or complete stopping) of the decay of a quantum system during periodic "observations" of this system. Such an "observation" is realized, in particular, by periodic excitation with laser pulses. However, an essential modification of spontaneous emission spectrum can also be achieved by a cw laser, in particular, interference between the different decay channels in an atom can result in appearance of dark lines in the spontaneous emission spectrum if interference is destructive. The work [2] considered an atom with two excited levels coupled by external electromagnetic field and spontaneous transitions from one of these levels. Quantum interference in the spontaneous emission of a four-level atom is considered in [3] for the
case where two near-lying levels undergoing a spontaneous transition to a lower level are excited by a laser radiation from some other level. The possibility of reducing spontaneous emission due to quantum interference is studied. Spontaneous emission and the modification of the lifetime of a level is investigated in [4].

Resonance fluorescence of an atom in a high-Q resonator when the atomic transition is coupled to the "tailored" vacuum, as well as spontaneous emission from atoms in which the two transitions are coupled to different, Markovian and non-Markovian, reservoirs, are studied in [5]. It is shown that the appropriate choice of the mode density of the non-Markovian reservoir essentially affects spontaneous emission in the Markovian reservoir (free space vacuum). Ref. [6] considers the probe absorption spectrum of a Λ-system when one of the atomic levels can decay spontaneously near the edge of the photonic band gap and shows that the probe is not absorbed at this frequency under certain conditions. In Refs. [7] it is proposed to use the phase difference of two coherent fields for controlling spontaneous emission spectra.

In the present work we consider the influence of resonant laser radiation on the spontaneous emission spectra in four- and five-level atoms (Figs. 1a,b). We first obtain the wave functions of the driven atoms, as well as the wave function of spontaneous photons in frames of the Wigner-Weisskopf theory [8]. In a four-level atomic system (Fig. 1a) the two upper levels are coupled by laser field and emit spontaneous photons when decaying to different states. The work [2] considered just the system of the Fig.1a without the fourth level and showed that destructive or constructive interference may occur, depending on initial conditions, between the transitions from the components of the Autler-Townes doublet formed in laser field. In case of destructive interference a dark line appears in the fluorescence spectrum. We investigate the influence of the additional decay channel (to the fourth level) on this spectrum and show, in particular, that the dark line disappears generally, but may be observed under certain conditions.

We then consider a five-level atomic system (Fig. 1b) where the two upper levels 2 and 4 are excited by two laser fields from the lower states 1 and 3 with subsequent spontaneous
transition to the final state $f$. We will obtain that if the driving laser fields are described in the photon number representation, the system behaves like two independent two-level systems, while in definite-phase classical laser fields interference takes place between the different decay channels. We will examine the character of this interference and its results for different initial conditions and values of relevant parameters.

II. FOUR-LEVEL ATOMIC SYSTEM

Consider a four-level atom in the field of quantized laser radiation at the frequency $\omega$ close to resonance with the transition $1 \rightarrow 2$ (Fig.1a). Under the action of the laser field a coupled "atom+field" state of levels 1 and 2 is formed. Let us assume that the atom is initially in a superposition of these states and follow the evolution of the system, when spontaneous transitions $1 \rightarrow 3$ and $2 \rightarrow 4$ are possible.

The full Hamiltonian of the system reads

$$H = H_{at} + \omega c^+ c + \beta^+ c + c^+ \beta + \sum_{k, \lambda} \omega_k c^+_{k, \lambda} c_{k, \lambda} +$$

$$+ \sum_{k, \lambda} \left[ \beta^+(k, \lambda) c_{k, \lambda} + c^+_{k, \lambda} \beta(k, \lambda) \right],$$

where $H_{at}$ is the Hamiltonian of the bare atom

$$H_{at} |i\rangle = \varepsilon_i |i\rangle, \quad (i = 1, 2, 3, 4),$$

$\beta^+$ and $\beta$ are the operators of dipole transitions from state 1 to state 2 and back ($\beta = \sqrt{2\pi \omega / hV} (\mathbf{e}\mathbf{d})$, $\mathbf{e}$ being the polarization unit vector of laser field and other notation commonly used) under the action of the external field of frequency $\omega$, $\beta^+(k, \lambda)$ and $\beta(k, \lambda)$ are similar operators for spontaneous photons with the momentum $\mathbf{k}$ and polarization $\lambda$; $c^+, c, c^+_{k, \lambda}$ and $c_{k, \lambda}$ are the operators of annihilation and creation of laser field and spontaneous photons, respectively.

The solution to the Schröedinger equation with the Hamiltonian (1) can be in the interaction representation written as
\[ \Phi(t) = C_1(t) |1, n, 0\rangle + C_2(t) |2, n - 1, 0\rangle + \]
\[ + \sum_{k, \lambda} C_{3k, \lambda} |3, n, 1_k, \lambda\rangle + \sum_{k, \lambda} C_{4k, \lambda} |4, n - 1, 1_k, \lambda\rangle, \]

where \(|i, n, l\rangle\) stands for the state of the system with the atom in \(i\)-th level, \(n\) photons in the laser field, and \(l\) spontaneously emitted photons.

By substituting this expression into the Shröedinger equation we obtain a set of coupled differential equations for the expansion coefficients,

\[ i \frac{dC_1(t)}{dt} = \sqrt{n} \beta e^{-i\nu t} C_2(t) + \sum_{k, \lambda} \beta_{13}^+(k, \lambda) e^{i(\epsilon_1 - \epsilon_3 - \omega_k) t} C_{3k, \lambda}(t) \]
\[ i \frac{dC_2(t)}{dt} = \sqrt{n} \beta^* e^{i\nu t} C_1(t) + \sum_{k, \lambda} \beta_{24}^-(k, \lambda) e^{i(\epsilon_2 - \epsilon_4 - \omega_k) t} C_{4k, \lambda}(t) \]
\[ i \frac{dC_{3k, \lambda}(t)}{dt} = \beta_{31}(k, \lambda) e^{-i(\epsilon_1 - \epsilon_3 - \omega_k) t} C_1(t) \]
\[ i \frac{dC_{4k, \lambda}(t)}{dt} = \beta_{42}(k, \lambda) e^{-i(\epsilon_2 - \epsilon_4 - \omega_k) t} C_2(t), \]

where \(\nu\) is the detuning of resonance \((\nu = \epsilon_2 - \epsilon_1 - \omega)\).

By eliminating \(C_{3,4k,\lambda}(t)\) (with the initial conditions \(C_{3,4k,\lambda}(0) = 0\)) we obtain

\[ \frac{dC_1(t)}{dt} = -i \sqrt{n} \beta e^{-i\nu t} C_2(t) - \sum_{k, \lambda} |\beta_{13}(k, \lambda)|^2 \int_0^t e^{i(\epsilon_1 - \epsilon_3 - \omega_k)(t-t')} C_1(t') dt' \]
\[ \frac{dC_2(t)}{dt} = -i \sqrt{n} \beta^* e^{i\nu t} C_1(t) - \sum_{k, \lambda} |\beta_{24}(k, \lambda)|^2 \int_0^t e^{i(\epsilon_2 - \epsilon_4 - \omega_k)(t-t')} C_2(t') dt'. \]

In further calculations we will use the Wigner-Weiskopf approximation [8]. In this case, by means of the Laplace transformation technique we obtain the following solutions to the Eqs. (5):

\[ C_1(t) = \frac{1}{\Omega} e^{-\frac{1}{2}(\Gamma_1 + \Gamma_2)t} e^{-\frac{i}{2}(\nu + \Delta_1 + \Delta_2)t} \times \]
\[ \times \left\{ [\Omega \cos \frac{\Omega}{2} t + i (\nu - \Delta_1 + \Delta_2 + \frac{\Gamma_1 - \Gamma_2}{2}) \sin \frac{\Omega}{2} t] C_1 - 2i \sqrt{n} \beta C_2 \sin \frac{\Omega}{2} t \right\} \]
\[ C_2(t) = \frac{1}{\Omega} e^{-\frac{1}{2}(\Gamma_1 + \Gamma_2)t} e^{\frac{i}{2}(\nu - \Delta_1 - \Delta_2)t} \times \]
\[ \times \left\{ [\Omega \cos \frac{\Omega}{2} t - i (\nu - \Delta_1 + \Delta_2 + \frac{\Gamma_1 - \Gamma_2}{2}) \sin \frac{\Omega}{2} t] C_2 + 2i \sqrt{n} \beta C_1 \sin \frac{\Omega}{2} t \right\}, \]

where \(C_1 = C_1(0), C_2 = C_2(0)\) and the shifts and widths of corresponding levels, and the complex Rabi frequency are defined as (the notation \(P\) stands here and below for the principal value)
\[
\Delta_1 = -P \sum_{k, \lambda} \frac{\beta_{13}(k, \lambda)^2}{\omega_k - \varepsilon_1 + \varepsilon_3}, \quad \Delta_2 = -P \sum_{k, \lambda} \frac{\beta_{24}(k, \lambda)^2}{\omega_k - \varepsilon_2 + \varepsilon_4},
\]
\[
\Gamma_1 = 2\pi \sum_{k, \lambda} |\beta_{13}(k, \lambda)|^2 \delta(\omega_k - \varepsilon_1 + \varepsilon_3), \quad \Gamma_2 = 2\pi \sum_{k, \lambda} |\beta_{24}(k, \lambda)|^2 \delta(\omega_k - \varepsilon_2 + \varepsilon_4).
\]
\[
\Omega = \sqrt{[\nu - \Delta_1 + \Delta_2 + i\frac{\Gamma_1 - \Gamma_2}{2}]^2 + 4n |\beta|^2}.
\]

By using the expressions (6) in the second pair of Eqs.(4), we finally obtain

\[
C_{3k, \lambda}(t) = \frac{\beta_{31}(k, \lambda)}{2\Omega} \times
\]
\[
\left\{ \left[ (\nu - \Delta_1 + \Delta_2 + \frac{i}{2}(\Gamma_1 - \Gamma_2) + \Omega) C_1 - 2\sqrt{n} \beta C_2 \right] \times
\right.
\]
\[
e^{-i\nu_{1k} + \frac{i}{2}(\nu + \Delta_1 + \Delta_2 - \frac{i}{2}(\Gamma_1 + \Gamma_2) - \Omega)} t - 1
\]
\[
\times \nu_{1k} + \frac{i}{2} \left( \nu + \Delta_1 + \Delta_2 - \frac{i}{2}(\Gamma_1 + \Gamma_2) - \Omega \right)
\]
\[
- \left[ (\nu - \Delta_1 + \Delta_2 + \frac{i}{2}(\Gamma_1 - \Gamma_2) - \Omega) C_1 - 2\sqrt{n} \beta C_2 \right] \times
\]
\[
e^{-i\nu_{1k} + \frac{i}{2}(\nu + \Delta_1 + \Delta_2 - \frac{i}{2}(\Gamma_1 + \Gamma_2) + \Omega)} t - 1
\]
\[
\times \nu_{1k} + \frac{i}{2} \left( \nu + \Delta_1 + \Delta_2 - \frac{i}{2}(\Gamma_1 + \Gamma_2) + \Omega \right)
\}
\]
\[
C_{k, \lambda}(t) = -\frac{\beta_{24}(k, \lambda)}{2\Omega} \times
\]
\[
\left\{ \left[ (\nu - \Delta_1 + \Delta_2 + \frac{i}{2}(\Gamma_1 - \Gamma_2) - \Omega) C_2 - 2\sqrt{n} \beta^* C_1 \right] \times
\right.
\]
\[
e^{-i\nu_{2k} - \frac{i}{2}(\nu - \Delta_1 - \Delta_2 + \frac{i}{2}(\Gamma_1 + \Gamma_2) + \Omega)} t - 1
\]
\[
\times \nu_{2k} - \frac{i}{2} \left( \nu - \Delta_1 - \Delta_2 + \frac{i}{2}(\Gamma_1 + \Gamma_2) + \Omega \right)
\]
\[
- \left[ (\nu - \Delta_1 + \Delta_2 + \frac{i}{2}(\Gamma_1 - \Gamma_2) + \Omega) C_2 - 2\sqrt{n} \beta^* C_1 \right] \times
\]
\[
e^{-i\nu_{2k} - \frac{i}{2}(\nu - \Delta_1 - \Delta_2 + \frac{i}{2}(\Gamma_1 + \Gamma_2) - \Omega)} t - 1
\]
\[
\times \nu_{2k} - \frac{i}{2} \left( \nu - \Delta_1 - \Delta_2 + \frac{i}{2}(\Gamma_1 + \Gamma_2) - \Omega \right)
\}
\]

with the detunings \( \nu_{1k} = \varepsilon_1 - \varepsilon_3 - \omega_k \), \( \nu_{2k} = \varepsilon_2 - \varepsilon_4 - \omega_k \). Expressions (6) and (8) determine the wave function (3) which allows for spontaneous transitions.

It is seen from the expressions for \( C_1(t) \) and \( C_2(t) \) that these coefficients tend to zero as \( t \to \infty \): \( C_1(\infty) = C_2(\infty) = 0 \). The wave function of the system \( \Phi(t) \) will then have the form

\[
\Phi(\infty) = |ph_1\rangle |3, n\rangle + |ph_2\rangle |4, n - 1\rangle,
\]

where

\[
|ph_1\rangle = \sum_{k, \lambda} C_{3k, \lambda}(\infty) |1_{k\lambda}\rangle
\]
\[
|ph_2\rangle = \sum_{k, \lambda} C_{4k, \lambda}(\infty) |1_{k\lambda}\rangle
\]
are the wave functions of spontaneous photons emitted in the transitions 1 → 3 and 2 → 4, respectively.

From Eq. (9) it follows that there is no interference between the decay channels to levels 3 and 4 because the final atomic states are different. Now, the spontaneous emission spectrum may be obtained in terms of $|C_{3k,\lambda}(\infty)|^2 + |C_{4k,\lambda}(\infty)|^2$, which after summation over polarizations of emitted photons and integration over the angles gives $I(\omega_k) = \omega_k^2 S(\omega_k)$, where

$$S(\omega_k) \propto |d_{31}|^2 \left[ \frac{(\varepsilon_2 - \varepsilon_3 - \omega - \omega_k + \Delta_2 - \frac{i\Gamma_2}{2})C_1 - C_2\sqrt{n}\beta}{(\varepsilon_1 - \varepsilon_3 - \omega_k + \Delta_1 - \frac{i\Gamma_1}{2})(\varepsilon_3 - \varepsilon_3 - \omega - \omega_k + \Delta_2 - \frac{i\Gamma_2}{2}) - n|\beta|^2} \right]^2 +$$

$$+ |d_{42}|^2 \left[ \frac{(\varepsilon_2 - \varepsilon_4 - \omega - \omega_k + \Delta_2 - \frac{i\Gamma_2}{2})(\varepsilon_1 - \varepsilon_4 + \omega - \omega_k + \Delta_1 - \frac{i\Gamma_1}{2})C_2 + C_1\sqrt{n}\beta^*_{12}}{(\varepsilon_2 - \varepsilon_4 - \omega_k + \Delta_2 - \frac{i\Gamma_2}{2})(\varepsilon_1 - \varepsilon_4 + \omega - \omega_k + \Delta_1 - \frac{i\Gamma_1}{2}) - n|\beta|^2} \right]^2. \quad (11)$$

This is the basic result of this section; the expression (11) determines the fluorescence spectrum of the four-level atom under study.

We will analyze this spectrum in two special cases where i) atom is initially in the state 1 ($C_1 = 1, C_2 = 0$) and ii) the atom is initially prepared in the symmetric superposition state ($C_1 = C_2 = 1/\sqrt{2}$).

The system of Fig.1a in the absence of the level 4 has been considered in [2] where it was shown that if the atom is initially in the state 1(2), in the fluorescence spectrum a dark(bright) line appears; the phenomenon was explained as a result of destructive(constructive) interference between the splitted laser-driven levels. This result is contained as a special case in the formula (11). Indeed, if we substitute $C_1 = 1, C_2 = 0$ into (11) and assume the absence of level 4 ($d_{42} = 0, \Delta_2 = 0, \Gamma_2 = 0$), a dark state appears in the spectrum at the frequency $\omega_k = \varepsilon_2 - \varepsilon_3 - \omega$. But when the level 4 is present providing another decay channel, the dark line is absent. This case is shown in Fig.2. The figure demonstrates the modification of the fluorescence spectrum at growing value of $\Omega$. At a small value (0.5) a single peak is observed instead of a dark line. This peak corresponds to mostly the transition 1 - 3, since the level 2 is coupled to the level 1 weakly. At greater values of $\Omega$ other three peaks appear corresponding to the splitting of the levels 1 and 2 due to laser coupling (Autler-Townes effect).
Consider now the second case where the atom is initially in the superposition of the states 1 and 2. By substituting $C_1 = C_2 = 1/\sqrt{2}$ into (11) we can easily obtain that under the conditions
\begin{equation}
\varepsilon_3 - \varepsilon_4 + \omega = \nu
\end{equation}
\begin{equation}
\Delta_2 - \frac{i\Gamma_2}{2} = -(\Delta_1 + \frac{i\Gamma_1}{2}) = \sqrt{n}\beta \quad (\Gamma_1 = \Gamma_2, \quad \Delta_2 = -\Delta_1)
\end{equation}
a dark line appears at the frequency $\omega_k = \varepsilon_2 - \varepsilon_3 - \omega = \varepsilon_1 - \varepsilon_4 + \omega$ in the fluorescence spectrum. This means that if the Lamb shifts of the levels 1 and 2 are equal and opposite directed, the widths of these levels are equal, and the laser frequency is tuned to $((\varepsilon_2 - \varepsilon_1) - (\varepsilon_4 - \varepsilon_3))/2$, a complete destructive interference takes place in each transition channel to levels 3 and 4 separately due to the Autler-Townes effect in the symmetric superposition upper state.

**III. FIVE-LEVEL ATOMIC SYSTEM**

Consider now the case of a five-level atomic system in the field of two lasers at frequencies $\omega_1$ and $\omega_2$ (Fig.1b). The first field of frequency $\omega_1$ is in resonance with the transition $1 \rightarrow 2$, and the second field of frequency $\omega_2$ is in resonance with the transition $3 \rightarrow 4$. The atom initially ($t = 0$) is supposed to be in a superposition of the states 1 and 3
\begin{equation}
\Phi(0) = C_1 \vert 1 \rangle + C_3 \vert 3 \rangle.
\end{equation}

Let us investigate the evolution of such a system when spontaneous transitions from levels 2 and 4 to some intermediate level $f$ are possible. If the driving laser fields will be described in the Fock representation, the spontaneous transitions from states 2 and 4 to the level $f$ will result in the formation of a superposition of two orthogonal states, $\vert f, n_1 - 1, n_2, 1_k \rangle$ and $\vert f, n_1, n_2 - 1, 1_k \rangle$, where $n_1$ and $n_2$ are the photon numbers in the first and second laser fields, respectively. In this case we will have two different independent processes that do not interfere and the intensity of the spontaneous emission will be the sum of intensities of separate spontaneous emissions in transitions $2 \rightarrow f$ and $4 \rightarrow f$. 

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However, the situation significantly changes when the upper levels 2 and 4 are excited by classical fields with definite phases. In this case all the levels participate in the process and the interference takes place between different transition channels.

The Hamiltonian of the system in case of classical excitation has the form

\[
H = H_{at} + V_1(t) + V_2(t) + \sum_{\mathbf{k}, \lambda} \omega_{k} c^+_{\mathbf{k}\lambda} c_{\mathbf{k}\lambda} + \sum_{\mathbf{k}, \lambda} \left( \beta^{+}(\mathbf{k}, \lambda) c_{\mathbf{k}\lambda} + c^+_{\mathbf{k}\lambda} \beta(\mathbf{k}, \lambda) \right),
\]

where \( V_1(t) \) and \( V_2(t) \) are the energies of the interaction of the atom with the first and second lasers, respectively,

\[
V_1(t) = V_1 e^{-i\omega_1 t} + V_1^* e^{i\omega_1 t},
\]

\[
V_2(t) = V_2 e^{-i\omega_2 t} + V_2^* e^{i\omega_2 t}.
\]

The Hamiltonian of the bare atom has now the form

\[
H_{at} = \sum_{j=1}^{4} \varepsilon_j |j\rangle \langle j| + \varepsilon_f |f\rangle \langle f|.
\]

The solution to the Schrödinger equation with the Hamiltonian (14) can be written as

\[
\Phi(t) = \sum_{j=1}^{4} C_j(t) |j, 0\rangle + \sum_{\mathbf{k}, \lambda} C_{f\mathbf{k}\lambda}(t) |f, 1_{\mathbf{k}\lambda}\rangle.
\]

The expansion coefficients obey the following set of equations:

\[
\begin{align*}
    i \frac{dC_1(t)}{dt} & = V_{12}^{(1)} e^{-i\nu_1 t} C_2(t) \\
    i \frac{dC_2(t)}{dt} & = V_{12}^{(1)*} e^{i\nu_1 t} C_1(t) + \sum_{\mathbf{k}, \lambda} \beta_{f_2}^{*}(\mathbf{k}, \lambda) e^{i\nu_{k}^{(1)} t} C_{f\mathbf{k}\lambda}(t) \\
    i \frac{dC_3(t)}{dt} & = V_{34}^{(2)} e^{-i\nu_2 t} C_4(t) \\
    i \frac{dC_4(t)}{dt} & = V_{34}^{(2)*} e^{i\nu_2 t} C_3(t) + \sum_{\mathbf{k}, \lambda} \beta_{f_4}^{*}(\mathbf{k}, \lambda) e^{i\nu_{k}^{(2)} t} C_{f\mathbf{k}\lambda}(t) \\
    i \frac{dC_{f\mathbf{k}\lambda}(t)}{dt} & = \beta_{f_2}(\mathbf{k}, \lambda) e^{-i\nu_{k}^{(1)} t} C_2(t) + \beta_{f_4}(\mathbf{k}, \lambda) e^{-i\nu_{k}^{(2)} t} C_4(t),
\end{align*}
\]

where

\[
\begin{align*}
    \nu_1 & = \varepsilon_2 - \varepsilon_1 - \omega_1 \\
    \nu_2 & = \varepsilon_4 - \varepsilon_3 - \omega_2 \\
    \nu_{k}^{(1)} & = \varepsilon_2 - \varepsilon_f - \omega_{k} \\
    \nu_{k}^{(2)} & = \varepsilon_4 - \varepsilon_f - \omega_{k}
\end{align*}
\]
are the detunings of the corresponding resonances.

Now, proceeding in the same way as in the preceding section, we have

\[
\Phi(\infty) = C_1(\infty) |1, 0 \rangle + C_3(\infty) |3, 0 \rangle + |ph \rangle |f \rangle. \tag{20}
\]

The last term in (20) again determines the spontaneous emission spectrum which is proportional to \(|C^{f*}_{\text{k}, \lambda}(\infty)|^2\) and is obtained to be

\[
S(\omega_k) \propto d_{f2} \sum^4_{j=1} \frac{1}{\prod_{l \neq j} (x_j - x_l)} \left( C_1 V^{(1)*}_{12} \left[ (x_j - \varepsilon_2 + \varepsilon_4 - \nu_2)(x_j - \varepsilon_2 + \varepsilon_4 + \Delta_4 - \frac{i\Gamma_4}{2}) - |V^{(2)}_{34}|^2 \right] + \right.
\]

\[
C_3 V^{(2)*}_{34} \left[ (\tilde{x}_j + \varepsilon_2 - \varepsilon_4 - \nu_1)(\tilde{x}_j + \varepsilon_2 - \varepsilon_4 + \Delta_2 - \frac{i\Gamma_2}{2}) - |V^{(1)}_{12}|^2 \right]
\]

\[
+ \left. d_{f4} \sum^4_{j=1} \frac{1}{\prod_{l \neq j} (\tilde{x}_j - \tilde{x}_l)} \left( C_3 \sqrt{\frac{\Gamma_2 \Gamma_4}{2}} V^{(1)*}_{12} (q + i)(\tilde{x}_j - \nu_2) + \right. \right.
\]

\[
\left. \frac{C_3 \sqrt{\frac{\Gamma_2 \Gamma_4}{2}} V^{(1)*}_{12} (q + i)\tilde{x}_j - \nu_1) \right) \frac{1}{\omega_k - \varepsilon_2 + \varepsilon_f - x_j} \right)
\]

where \(C_1(0) = C_1, C_3(0) = C_3, (C_2(0) = C_4(0) = C^{f*}_{\text{k}, \lambda}(0) = 0), \Delta_2, \Gamma_2 \) and \(\Delta_4, \Gamma_4\) are the shifts and widths of levels 2 and 4 defined as in preceding section, \(q\) is the Fano parameter \([9]\)

\[
q = \frac{2}{\sqrt{\Gamma_2 \Gamma_4}} P \sum_{k, \lambda} \frac{\beta_{f2}(k, \lambda) \beta_{f4}(k, \lambda)}{\omega_k + \varepsilon_f - \varepsilon_2} \approx \frac{2}{\sqrt{\Gamma_2 \Gamma_4}} P \sum_{k, \lambda} \frac{\beta_{f2}(k, \lambda) \beta_{f4}(k, \lambda)}{\omega_k + \varepsilon_f - \varepsilon_4} \tag{22}
\]

(the levels 2 and 4 are supposed to be close having nearly equal Fano parameters, and the \(\beta\)'s are taken to be real). The quantities \(x_j\) and \(\tilde{x}_j\) \((j = 1, 2, 3, 4)\) are the roots of the fourth order polynomials \(f(x)\) and \(\tilde{f}(x)\), respectively,
\[ f(x) = \left[ (x - \nu_1)(x + \Delta_2 - \frac{\Gamma_2}{2}) - \left| V_{12}^{(1)} \right|^2 \right] \times \]
\[ \times \left[ (x - \varepsilon_2 + \varepsilon_4 - \nu_2)(x - \varepsilon_2 + \varepsilon_4 + \Delta_4 - \frac{\Gamma_4}{2}) - \left| V_{34}^{(2)} \right|^2 \right] - \]
\[ - \frac{\Gamma_2 \Gamma_4}{4} (q + i)^2 (x - \nu_1)(x - \varepsilon_2 + \varepsilon_4 - \nu_2) \]
\[ \bar{f}(x) = \left[ (x - \nu_2)(x + \Delta_1 - \frac{\Gamma_1}{2}) - \left| V_{34}^{(2)} \right|^2 \right] \times \]
\[ \times \left[ (x + \varepsilon_2 - \varepsilon_4 - \nu_1)(x + \varepsilon_2 - \varepsilon_4 + \Delta_2 - \frac{\Gamma_2}{2}) - \left| V_{12}^{(1)} \right|^2 \right] - \]
\[ - \frac{\Gamma_2 \Gamma_4}{4} (q + i)^2 (x - \nu_2)(x + \varepsilon_2 - \varepsilon_4 - \nu_1). \]

It can be shown that all the poles determined by these roots are in the upper half-plane and satisfy the inequality
\[ 0 < \text{Im}(x_j, \bar{x}_j) < \frac{\Gamma_2 + \Gamma_4}{2}. \] (24)

This is the main result of this section; the expression (21) determines the fluorescence spectrum of the five-level atom under study.

We will analyze this spectrum in two special cases where i) atom is initially in the state 1 \((C_1 = 1, C_3 = 0)\) and ii) the atom is initially prepared in the symmetric or antisymmetric superposition state \((C_1 = C_2 = \pm 1/\sqrt{2})\).

In case where the atom is initially in the state 1, by substituting \(C_1 = 1, C_3 = 0 \) into (21) we obtain that the destructive interference may take place if \(d_{f2}\) and \(d_{f4}\) are parallel or antiparallel, but this destructive interference is not complete and there is no dark line in the spectrum of spontaneous transition.

Consider the case where the atom is initially in a symmetric or antisymmetric superposition state, i.e., \(C_1 = C_3 = \pm 1/\sqrt{2}\). Substitution of these values into the expression (21) demonstrates that when the dipole moments \(d_{f2}\) and \(d_{f4}\) are perpendicular to each other, the interference term disappears and the intensity of the spontaneous emission is equal to the sum of the intensities of the separate transitions. When the dipole moments are antiparallel, the levels 2 and 4 are close \((\varepsilon_2 \approx \varepsilon_4)\), the matrix elements \(V_{12}\) and \(V_{34}\) are approximately equal \((V_{12} \approx V_{34})\), and \(d_{f2} \approx d_{f4}\), \(\Delta_2 \approx \Delta_4\), \(\Gamma_2 \approx \Gamma_4\), \(\nu_1 \approx \nu_2\), complete destructive interference takes place throughout the spectrum and the intensity of spontaneous emission
becomes negligible. When the dipole moments are parallel or antiparallel, but the conditions above are not met, constructive or destructive interference takes place at certain frequencies depending on the sign of the interference term. Figs.3a and 4a show this peculiarity (for the parallel case) for the values of the Fano parameter equal to 1 and 5, respectively. For the perpendicular dipole moments Figs. 3b and 4b demonstrate the absence the absence of the dark line for the same values of the Fano parameter.

IV. CONCLUSION

We have investigated spontaneous emission of atoms under the action of laser radiation. We showed that the external electromagnetic field affects essentially the spectrum of spontaneous emission by stimulating interference between different decay channels.

In a four-level system (Fig.1a) without resonant laser radiation no interference occurs between decay channels to levels 3 and 4. However, when the laser radiation is switched on and there is no fourth level, the Autler-Townes effect is known to cause interference and appearance of a dark line in the fluorescence spectrum [2] depending on the initial conditions.

We show that if an additional level, 4, is present providing another decay channels, this interference may partially or completely be destroyed depending on the values of parameters. We also show that if the atom is prepared initially in a superposition of upper states, destructive interference occurs, under specific conditions, in both decay channels separately, and a dark line appears at a certain frequency.

We then show that in a five-level system where the upper states of two two-level systems driven by two lasers can decay to the same, fifth, level, interference between the decay channels takes place only in classical driving fields (no interference if the fields are quantized and described in the Fock representation) and the result of this interference depends strongly on the mutual orientation of the dipole moment matrix elements of the spontaneous transitions, as well as, again, on the initial conditions.

We, specifically, show that when the dipole moments of spontaneous transitions are
perpendicular to each other, the interference term disappears and the intensity of the spontaneous emission is equal to the sum of the intensities of the separate transitions. But when these dipole moments are parallel or antiparallel, the destructive or constructive interference takes place depending on the sign of the interference term. In case where the atom is initially in the state 1, the destructive interference is not complete and dark line is absent in the spectrum of spontaneous emission. When the atom is initially in a superposition state, a complete destructive interference takes place under certain conditions and the intensity of spontaneous emission becomes very small throughout the spectrum, while if these conditions are not met, constructive or destructive interference takes place at certain frequencies depending on the sign of the interference term.

So, the presence of additional levels enabling alternative decay channels may modify significantly the spectrum of fluorescence from atomic media.

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FIGURE CAPTIONS

Fig.1. Energy-level diagrams of the four-level (a) and five-level (b) atomic systems.

Fig. 2. The spontaneous emission spectra $S(\omega_k)$ for $\varepsilon_2-\varepsilon_3-\omega+\Delta_2 = 2, \varepsilon_1-\varepsilon_3+\Delta_1 = 2.5, \varepsilon_2-\varepsilon_4+\Delta_2 = 3.5, \varepsilon_1-\varepsilon_4+\omega +\Delta_1 = 4, \Gamma_1 = \Gamma_2 = \Gamma = 1$ and (a) $\Omega = 0.5$, (b) $\Omega = 2.5$, (c) $\Omega = 3.0$, (d) $\Omega = 4.0$. All parameters are in units of $\Gamma$.

Fig.3. The spontaneous emission spectra $S(\omega_k)$ for the value of Fano parameter $q = 1$ and following values of other parameters: $V_{12} = 1$, $\varepsilon_2 = \varepsilon_4 = 8$, $\varepsilon_f = 1$, $\Delta_2 = \Delta_4 = 0$, $\Gamma_2 = \Gamma_4 = 1$, $V_{34} = 1.5$. All parameters are in units of $\Gamma$. In case (a) the transition dipole
moments are parallel, while in case (b) they are perpendicular.

Fig. 4. The same as in Fig. 3, but with $q = 5$. 
\[ S(\omega_k) \]

(a)

\[ S(\omega_k) \]

(b)

\[ \omega_k / \Gamma \]
