The study of Anisotropic Flows at LHC with perturbative simulation

Ghi R. Shin

Department of Physics, Andong National University, Andong, South Korea

(Dated: May 7, 2014)

Abstract

We study the harmonic flows, for example, the directed, elliptic, third and fourth flow of the system of partons formed just after relativistic heavy ion collisions. We calculate the minijets produced during the primary collisions using standard parton distributions for the incoming projectile and target nucleus. We solve the Boltzmann equations of motion for the system of minijets by Monte Carlo method within only the perturbative sector. Based on the flow data calculated, we conclude the simulation can not explain the experimental results at RHIC and LHC so that the nonperturbative sector plays much more important roles even from the earliest stage of heavy ion collisions.
I. INTRODUCTION

The Quark-Gluon Plasma (QGP) is a fascinating state of matter, which is a QCD plasma consisting of quasi free quarks, antiquarks and gluons. This state of matter has been actively studied in theory and experiment. The Super Proton Synchrotron (SPS) at CERN had tried to produce the matter in the 1980s and Relativistic Heavy Ion Collider (RHIC) at BNL is continuing the search from 2000 and CERN’s Large Hadron Collider (LHC) has joined the study from 2009. It is general consensus that the QGP has been produced in laboratory.

One of reasons why so many efforts have been put on the study is the understanding of the theory of strong interaction, Quantum Chromodynamics (QCD). The QCD is well known for that it is notoriously difficult to calculate any quantitative physical observable from the theory: The static properties of the theory however can be obtained with lattice calculation and certain dynamical properties, for example, the hadron production from $e^-e^+$ scattering, can be estimated with perturbative calculations but the detailed dynamical evolution of the QCD system in general can not be addressed with neither the lattice calculation nor the perturbative one up to date. We thus have to keep in mind that the understanding of the QCD nonperturbative sector in dynamics is the primary goal of the studying a QGP. The first question for that matter is 'how is the QGP formed?'. This highly non-trivial question should be one of major problems we should answer. The next unanswered question is 'how is a hadron produced from the QGP dynamically?'.

We know that the 3+1D hydrodynamics incorporating viscous property is doing well to explain the evolution of a QGP. And the evolution of a hadronic gas is described well by hadron transport formalism, for example, uRQMD and so on. We however have to mention that the hydroformalism can not give any explanation to those two critical questions.

In order to study those unanswered problems it seems to us that it is best to use the quantum kinetic theory. Namely, we need to know how much the perturbative sector can explain the experiments and how much the non-perturbative one should contribute. This will help us to formulate a realistic model for dynamical features. Having this in mind we focus on the harmonic flows of the system formed just after a heavy ion collision as a function of time with perturbative theory.

The evolution of a heavy ion collision can be viewed in partonic point of view as follows;
A bunch of partons of a projectile nucleus (or nucleon) makes collisions with those partons of target nucleus (or nucleon) to liberate the constituent partons to quasi free particles. Those freed partons can radiate photons and partons and make collisions with other partons. During this period of evolution, the system may reach thermal and chemical equilibrium to form the QGP. The QGP will then produce (or convert into) hadrons eventually after expanding sufficient enough to break up. This hadronic gas will further evolve. We follow this viewpoint as much as we can in our study.

We calculate the primary partons which are liberated by collisions between projectile and target nucleus in Section III and briefly describe our numeric procedure which is a parton evolution code in Section IV. We present the simulation results and discussion in Section V and conclude in Section VI.

II. PRIMARY MINIJET PRODUCTION

Assuming that partons are produced in relativistic heavy-ion collisions by elastic scattering between the constituents of a projectile nucleus and those of a target nucleus, we can write the total number of collision events [6–10]:

\[ N_{\text{event}} = KT(b) \int dy_3 dy_4 d^2 p_T \sum_{ij, kl} [x_1 f_{i/A} (x_1, Q_0^2) x_2 f_{j/B} (x_2, Q_0^2) \frac{d\sigma_{ij\to kl}(\hat{s}, \hat{t}, \hat{u})}{dt} + x_1 f_{j/A} (x_1, Q_0^2) x_2 f_{i/B} (x_2, Q_0^2) \frac{d\sigma_{ij\to kl}(\hat{s}, \hat{u}, \hat{t})}{dt}] \frac{1}{1 + \delta_{ij}}, \]  

(2.1)

where we assume that there are no correlations between the momentum and space coordinates of a constituent parton. We explicitly neglect the transversal momentum of incoming partons and \( s \) is the CM energy squared of two mother nucleons and \( b \) is an impact parameter. \( K \) is the K-factor to include the higher-order diagrams; we will set \( K = 2 \) for the RHIC energy and 1.5 for LHC energy. A parton \( i \) of nucleus \( A \) collides with a parton \( j \) of nucleus \( B \) and produces partons \( k \) and \( l \) or a parton \( j \) of nucleus \( A \) collides with a parton \( i \) of nucleus \( B \) to produce partons \( k \) and \( l \). Each parton has rapidity \( y_1, y_2, y_3, \) and \( y_4 \), respectively. \( x_1 \) and \( x_2 \) are the Bjorken scaling variables of incoming partons. The relations between the
variables before and after the collision are given by

\[ x_1 = \frac{p_T(e^{y_1} + e^{y_2})}{\sqrt{s}}, \]  
\[ x_2 = \frac{p_T(e^{-y_1} + e^{-y_2})}{\sqrt{s}}, \]  
\[ \hat{s} = x_1 x_2 s, \]  
\[ \hat{t} = -p_T^2(1 + e^{y_2-y_1}), \]  
\[ \hat{u} = -p_T^2(1 + e^{y_1-y_2}). \]  

We write the available kinematic region for convenience \[7, 14, 15\],

\[ Q_0^2 \leq p_T^2 \leq \left( \frac{\sqrt{s}}{2 \cosh y} \right)^2, \]  
\[ -\log(\frac{\sqrt{s}}{p_T} - e^{-y}) \leq y_4 \leq \log(\frac{\sqrt{s}}{p_T} - e^{-y}), \]  
\[ |y| \leq \log(\frac{\sqrt{s}}{2Q_0} + \sqrt{\frac{s}{4Q_0^2} - 1}). \]  

\(Q_0\) is the momentum scale which we are probing the nucleus and is a minimum momentum transfer. The hat on the Mandelstam variables means that those are the variables of a parton. The processes we consider in our study are \(gg \leftrightarrow gg + q\bar{q}, gq \leftrightarrow gq, g\bar{q} \leftrightarrow g\bar{q}, q^a q^b \leftrightarrow q^c q^d, q\bar{q} \leftrightarrow q\bar{q}, q^a q^b \leftrightarrow q^c \bar{q}^d\). We do not include some basic channels, such as \(qg \rightarrow q\gamma, q\bar{q} \rightarrow \gamma\gamma\), and \(q\bar{q} \rightarrow g\gamma\), which could provide important information on the system.

Assuming the nucleons in a nucleus are treated independently except the shadow effect, we can write the parton distribution of the nucleus \(A\) as follows:

\[ f_{i/A}(x, Q^2) = f_{i/N}(x, Q^2) R_A(x, Q^2), \]  

where \(f_{i/N}(x, Q^2)\) is the parton distribution of a free nucleon and \(R_A(x, Q^2)\) is the nucleus ratio function, which is the nucleon distribution of the nucleus, \(A\). We use the CTEQ4\[11\] or the GRV98\[12\] distribution function for a free nucleon parton distribution and the EKS98 parametrization for the ratio function\[13\].

Assuming that the density of the nucleus is constant over the sphere of radius \(R\) with the sharp edge, we can define the nuclear thickness function,

\[ \tau(\vec{r}) = \int_V dz \rho(\vec{r}, z), \]  

where \(\vec{r}\) is the transversal vector and

\[ \tau_A(\vec{r}_A) = 2 \rho_0^A \sqrt{R_A^2 - (\vec{r} - \vec{b}/2)^2}, \]  
\[ \tau_B(\vec{r}_B) = 2 \rho_0^B \sqrt{R_B^2 - (\vec{r} + \vec{b}/2)^2}, \]
where \( \vec{b} \) is the impact parameter, the vector from the center of a target B to the center of a projectile A.

The overlap function between the target and projectile nucleus is then

\[
T_{AB}(\vec{r}; \vec{b}) = \tau_A(\vec{r} - \vec{b}/2)\tau_B(\vec{r} + \vec{b}/2),
\]

\[
= 4\rho^A_0\rho^B_0 \sqrt{R_A^2 - (\vec{r} - \vec{b}/2)^2} \sqrt{R_B^2 - (\vec{r} + \vec{b}/2)^2}, \tag{2.14}
\]

and the nuclear geometric factor \( T(b) \) at a given impact parameter \( b \) is

\[
T(b) = \int d\vec{r} T_{AB}(\vec{r}, \vec{b}). \tag{2.15}
\]

We can choose the (transversal) position according to the overlap function, Eq.2.14. To sample the position according to this probability density can be performed using Veto algorithm, namely we sample \((x, y)\) within the allowed region randomly and calculate the probability density at that position to give \( P(x, y; \vec{b}) = T_{AB}(\vec{r}; \vec{b}) \). We generate a random number \( r \) and compare to \( P(x, y; \vec{b})/P(0, 0; \vec{b}) \). If \( r \) is less than \( P(x, y; \vec{b})/P(0, 0; \vec{b}) \), we accept the \((x, y)\) but if \( r \) greater than that, we reject and sample another positions.

To obtain the longitudinal position and the collision time of a collision, we consider two nonrelativistic classical balls passing through each other and choose \( t = 0 \) as the time when two colliding nucleus make first contact at the impact parameter \( b = 0 \) in CM frame. At the given transverse collision position \((x, y)\), we consider a longitudinal tube of transversal area \( \delta A \) through the point \((x, y)\) with the half thickness \( D_A = \sqrt{R_A^2 - r_A^2} \) of nucleus A and \( D_B = \sqrt{R_B^2 - r_B^2} \) of nucleus B where \( r_A^2 = (\vec{r} - \vec{b}/2)^2 \) and \( r_B^2 = (\vec{r} + \vec{b}/2)^2 \). The tubes from both spheres, of which have different length from each other, begin to overlap one another starting at \( t_s = (R_A - D_A) + (R_B - D_B) v_A + v_B \) and completely overlap at \( t_1 = t_s + l_{min}/(v_A + v_B) \) and begins to separate at \( t_2 = t_s + l_{max}/(v_A + v_B) \) and completely separate at \( t_e = (D_A + D_B) v_A + v_B + t_s \), where \( v \) is the velocity of a sphere and \( l_{min} \) and \( l_{max} \) are the minimum and maximum of \( D_A \) and \( D_B \).

Assuming the probability density which an elastic collision can be occurred is proportional to the overlap volume, we have the probability density of collision,

\[
P(t) = (t - t_s)(v_A + v_B), \quad t_s < t < t_1, \tag{2.16}
\]

\[
= l_{min}, \quad t_1 < t < t_2, \tag{2.17}
\]

\[
= (t_2 - t)(v_A + v_B), \quad t_2 < t < t_e, \tag{2.18}
\]
so that the overall probabilities are

\begin{align*}
S_1 &= \frac{1}{2}(t_1 - t_s)l_{\text{min}}, \\
S_2 &= (t_2 - t_1)l_{\text{min}}, \\
S_3 &= \frac{1}{2}(t_e - t_2)l_{\text{min}}.
\end{align*}

(2.19)  
(2.20)  
(2.21)

We can generate a random number \( r_1 \) and choose the region depending on the overall probabilities. Once we have a region, we can generate the second random number, \( r_2 \), and the collision time is given by,

\[ t = t_s + \sqrt{S_1 r_2 v} \]  

(2.22)

for the region \( S_1 \), and

\[ t = t_1 + (t_2 - t_1)r_2, \]  

(2.23)

\[ t = t_e - \sqrt{t_e^2 + t_2^2 - 2t_2 t_e - S_3 v r_2}, \]  

(2.24)

for region \( S_2 \) and \( S_3 \) respectively.

Once we have the collision time \( t \), we can choose the longitudinal collision position \( z \) within the overlap tube by using a Monte Carlo method:

\[ z = z_c - (t - t_s)v + 2vtr, \]  

(2.25)

if \( t_s < t < t_1 \),

\[ z = z_c - (t - t_s)v + l_{\text{min}}r, \]  

(2.26)

\[ z = z_c - D_A + 9t - t_s)v + [l_A + l_B - 2(t - t_s)v]r, \]  

(2.27)

for \( t_s < t < t_1 \) and \( t_2 < t < t_e \) respectively, where \( r \) is a random number. \( z_c = \frac{(R_B - D_B) - (R_A - D_A)}{2} \) is the center of the first contact points at \( t = 0 \).

We can apply the argument to a relativistic collision. Consider a sphere of \( v \sim c \). The sphere is then contracted by the \( \gamma \)-factor so that the starting and the ending collision times for the identical spheres in the CM frame are \( t_s = (R - D)/\gamma \) and \( t_e = (R + D)/\gamma \), respectively, where \( D = \sqrt{R^2 - x^2 - y^2} \). The collision time probability is, thus, given by

\[ P(t) = \frac{\gamma^2}{2D^2}(t - t_s) \]  

(2.28)
so that the collision time and the longitudinal position can be obtained from a Monte Carlo method:

\[ t = t_s + \frac{2D}{\gamma} \sqrt{r_1}, \]
\[ z = 2(t - t_s)r_2 - (t - t_s), \]

where \((r_1, r_2)\) are random numbers between 0 and 1.

The momentum of a produced parton can be obtained from the minijet distribution, Eq. (2.1):

\[ f(p_T, y) = C \left( \frac{1}{p_T^2} \frac{dN^{jet}}{dy dp_T^2} \right), \]

where \(C\) is a normalization constant. We can choose \((p_T, y)\) for each test particle with this distribution function by using a Monte Carlo sampling method. On the other hand, the azimuthal angle \(\phi\) of the momentum can be chosen with equal weight between \((0, 2\pi)\). These give the energy-momentum of the produced parton which is on-mass shell.

The number of minijets depends on the probing momentum \(Q_0\) and K-factor. We assume that the total energy of produced minijets is about 70 - 75% of the total CM energy. This gives us \(Q_0 = 2.4\, GeV\) with \(K = 1.5\) for CTEQ4 at LHC energy \(\sqrt{s} = 2.76\, TeV\) and the number of minijets is about 9,700 partons for \(b = 0\), which are mostly gluons. At RHIC energy, \(Q_0 = 1.6\, GeV\) and \(K = 2\) to produce 3800 partons. Fig. 1 shows the rapidity distribution of the Monte-Carlo-sampled test particles. The distribution shows that the rapidity is almost flat at the central region, but falls quickly off with \(|y| > 2.5\). Figs. 2, 3 show \(p_T\) and energy distribution at LHC energy.

### III. PARTON EVOLUTION SIMULATION

The evolution of a system of partons can be best described by the quantum transport equations based on the field theory. But it is too complicated to solve even numerically. The semi-classical Boltzmann equations of motion thus are used to capture the main physics. To solve the partonic Boltzmann equations, we use partonic Monte Carlo simulation (PCC) which implements the main features of perturbative QCD, which
FIG. 1: Rapidity distribution of the test partons sampled by a Monte Carlo method

FIG. 2: $p_T$ distribution of the test partons sampled by a Monte Carlo method

FIG. 3: Energy distribution of the test partons sampled by a Monte Carlo method

includes the gluon radiation ($gg \rightarrow ggg$) channel in the secondary collision. The algorithm of the simulation is simple and straightforward: A parton, which is a minijet and was produced from the primary collisions between projectile and target nucleus (or nucleon), is following the straight classical trajectory depending on the initial momentum and position until it hits other parton. In order to decide whether the two particles makes a collision or not, we calculate the impact parameter or the closest distance, $r_{\text{min}}$, between two particles and compare the distance with the radius of cross section, $r_c = \sqrt{\sigma/\pi}$. If $r_{\text{min}} < r_c$, those two
have scattering. This decision making is rather deterministic even though it should be probabilistic in quantum nature. We however note that although the decision is deterministic, the outcomes, for example, scattering channels and energy-momentum of outgoing particles, are stochastic and Markobian. Namely we choose the scattering channel out of many possible ones based on the probabilistic weight with no history. Once the channel is chosen, we sample by Monte Carlo the outgoing momentum according to the differential cross section. In this way the parton system evolves up to the time set by outside.

In this study, the small angle scatterings between test partons, \( \sin \theta \geq \frac{p_{\min}}{E} \) where \( p_{\min} \) is the minimum momentum transfer, is set at \( p_{\min} = 0.3 \text{GeV}/c \). We put the QCD coupling constant to be \( \alpha_s = 0.3 \) throughout the simulation. The realistic value of K-factor is 1 to 2 to include the higher-order diagrams, but we will set \( K = 2 \) or \( K = 20 \). We know these values of basic parameter for the simulation are beyond the limit of perturbative calculation in some case. The idea of this setting is to get the maximal outcomes from the perturbative sector.

The processes we consider in this study are \( gg \leftrightarrow gg + q\bar{q}, \) \( gq \leftrightarrow gq, \) \( g\bar{q} \leftrightarrow g\bar{q}, \) \( q^a q^b \leftrightarrow q^c \bar{q}^d, \) \( q\bar{q} \leftrightarrow q\bar{q}, \) \( \bar{q}^a \bar{q}^b \leftrightarrow \bar{q}^c \bar{q}^d, \) and \( gg \rightarrow ggg \). The cross sections for the processes up to the leading order (LO) can be found in Ref. \cite{16}; the total cross section of \( gg \rightarrow gg \) with the momentum cutoff, for example, is about \( 10/\text{GeV}^2 \), which is about 4 \( \text{mb} \).

IV. SIMULATION RESULTS AND DISCUSSION

The primary partons, which are produced directly from the colliding nuclei, have azimuthal symmetry in the momentum space since the collision cross section between partons is independent of azimuthal angle. The partons will have numerous collisions among themselves after they are born and may develop anisotropy in the momentum distribution if there is spatial anisotropy. This azimuthal anisotropy, which is called the harmonic flow collectively, provides very important information of the system. This azimuthal anisotropy can be extracted systematically by expanding the number density as a function of azimuthal angle, \( \phi \), with respect to the reaction plane \cite{17},

\[
E \frac{d^3N}{dp^2} = \frac{1}{\pi} \frac{d^2N}{dp_T^2 dy} [v_0 + 2v_1 \cos\phi + 2v_2 \cos2\phi + ...],
\]

(4.1)
where $v_1$ is the directed flow and $v_2$ the elliptic flow. When a parton’s momentum is known, the directed and elliptic flow can be calculated by using the equation

$$v_1 = <\cos \phi> = <\frac{p_x}{\sqrt{p_x^2 + p_y^2}} >$$

$$v_2 = <\cos 2\phi> = <\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2}>,$$

where the bracket denotes the average over the partons, and the impact parameter vector and collision axis define the reaction plane. The directed flow tells us the sideward motion of particles in heavy ion collisions and it carries information developed in the earliest stage of collisions. It is argued that the directed flow could reveal a signature of a possible phase transition from normal nuclear matter to a QGP\[18\]. The elliptic flow is a fundamental observable and is known as one of probes of QGP formation. It reflects how the initial spatial anisotropy of the nuclear overlap region of primary nuclei collision is translated into the asymmetric momentum distribution of final particles.

![Directed Flow](image)

**FIG. 4**: Directed flow for RHIC and LHC at $b = 7$ fm.

Figure 4 shows the directed flow as a function of transverse momentum for the parton system. The data have been obtained by averaging over 100 runs. We include only partons with $|y| < 2$. The impact parameter is $b = 7$ fm which corresponds to 20 - 30 % in centrality\[19\]. There are big error bars for small and large $p_T$ since the number of partons in the region is small. The simulation shows no directed flow while significant amount of $v_1$ have been reported in experiments \[20\]. Here the RHIC energy means $\sqrt{s} = 200$ GeV per
pair of nucleons and LHC energy $\sqrt{s} = 2.76$ TeV per pair.

![Elliptic Flow](image1)

**FIG. 5**: Elliptic flow for RHIC and LHC energy at $b = 7$ fm.

Figure 5 shows the elliptic flow as a function of transverse momentum for the parton system. The simulation data show both energy have no elliptic flows while ALICE\cite{21} reported the elliptic flow as big as RHIC\cite{22–25}. This is particularly interesting; even though we use much bigger perturbative cross sections than the reasonable ones by setting $K = 20$, we have null elliptic flows over the transverse momentum. This clearly shows the failure of naive perturbative calculation.

![Fourth Flow](image2)

**FIG. 6**: Fourth flow for RHIC and LHC at $b = 7$ fm.
Figure 6 shows the fourth harmonic flow as a function of transverse momentum for the parton system formed just after heavy ion collisions.

To understand the flow data of simulations further, we calculate the number of collisions as a function of time. Figure 7 shows the cumulative number of collisions per parton. The figure tells us that the increasing rate at early stage is much higher at LHC than at RHIC and perturbative collisions occur within 2 fm/c in both cases. This is interesting because this tells us that all happen within 1-2 fm/c and then the system is free streaming no matter what the density is. We also calculated the number of collision at LHC with $K=2$ and found that the number is almost same as that of $K=20$. We can understand this as follows; The available phase space, especially the momentum space, at LHC is so large that the number of collision per parton do not increase as expected as the cross section does.

**V. CONCLUSIONS**

We study relativistic heavy ion collisions with Monte Carlo simulation; Using CTEQ4 and GRV98 we obtain the parton phase space distribution of nuclei (Au and Pb). When two colliding nuclei overlap, the constituent partons make collisions and are freed if the momentum transfer is greater than $Q_0$. Those partons formed just after primary collision evolve further. We stop the evolution at $t = 10 fm/c$ and analyze the results, especially the harmonic flows. All the simulation data is compelling us that the naive perturbative calcu-
lation cannot explain the results of RHIC and LHC. We further calculated the simulations using GRV98 distribution for a proton and found no difference from what we concluded here.

There are two places to improve the perturbative sector, which are not implemented yet in our study: The first one is parton radiation. The partons of a system are off-mass shell and are surely subject of parton shower, namely $1 \rightarrow 2$ branching. These will introduce many new partons into the system and could increase the number of collisions and flow effects. The other important component which is missing in our simulations is a color electromagnetic force\cite{26}. We expect this color force will not change the harmonic flows since the force does not depend on the azimuthal angle of partons but could excel the parton’s thermal equilibration. These two issues are under investigation by us.

*Acknowledgements:* This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0022228)

[1] B. Müller, The Physics of the Quark-Gluon Plasma, Lecture Notes in Physics, Vol. 225 (Springer-Verlag, 1985); L. McLerran, Rev. Mod. Phys. 58, 1021 (1986); Quark Gluon Plasma, edited by R. C. Hwa (World Scientific, Singapore, 1991).

[2] B. Schenke, S. Jeon, C. Gale, Phys. Rev. Lett. 106, 042301 (2011).

[3] H.-Th. Elze and U. Heinz, Phys. Rep. 183 (1989) 81 and references therein.

[4] K. Geiger, Phys. Rev. D 54, 949 (1996) 949; Phys. Rev. D 56, 2665 (1997).

[5] K. Geiger and B. Müller, Nucl. Phys. B 369, 600 (1992); K. Geiger, Phys. Rev. D 46, 4965, and 4986 (1992); K. Geiger and J. I. Kapusta, Phys. Rev. D 47, 4905 (1993).

[6] E. Eichten, I. Henchiliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. 56, 579(1984).

[7] N. Hammon, H. Stocker, and W. Greiner, Phys. Rev. C61 (2000), 014901.

[8] G. C. Nayak, A. Dumitru, L. McLerran, and W. Greiner, Nucl. Phys. A687 457 (2001).

[9] J. Eskola, Nucl. Phys. A702:249-258 (2002)

[10] F. Cooper, E. Mottola, and G. Nayak, Phys. Lett. B555, 181-188 (2003).

[11] H. L. Lai, J. Huston, S. Kuhlmann, J. Morfin, F. Olness, J. F. Owens, J. Pumplin, W. K.
Tung, Eur. Phys. J. C 12 (2000) 375; H. L. Lai, J. Huston, S. Kuhlmann, F. Olness, J. F. Owens, D. Soper, W. K. Tung, H. Weerts, Phys. Rev. D55, 1280 (1997).

[12] M. Glueck, E. Reya, and A. Vogt, Eur. Phys. J.C5:461-470 (1998); J. Eskola, V.J. Kolhinen, and C.A. Salgado, Eur. Phys. J. C9 61 (1999); K. J. Eskola, V. J. Kolhinen, and P. V. Ruuskanen, Nucl. Phys. B535 351 (1998).

[13] J. Eskola, V.J. Kolhinen, and C.A. Salgado, Eur. Phys. J. C9, 61 (1999); \texttt{hep-ph/9807297} K. J. Eskola, V. J. Kolhinen, and P. V. Ruuskanen, Nucl. Phys. B535, 351 (1998).

[14] G. R. Shin and B. Mueller, J. Phys. G 28, 2643 (2002).

[15] G. R. Shin and B. Mueller, J. Phys. G 29, 2485 (2003).

[16] M. E. Peskin and D. V. Schroeder, \textit{An Introduction to Quantum Field Theory}, (Addison Wesley, New York, 1995).

[17] J.-Y. Ollitrault, Phys. Rev. D 46, 229 (1992).

[18] J. Brachmann, S. Soff, A. Dumitru, H. Stocker, J. A. Mahruhn, W. Greiner, L. V. Bravina, and D. H. Rischke, Phys. Rev. C 61, 024909 (2000); L. P. Csernai and D. Rohrich, Phys. Lett. B, 458, 454 (1999); H. Stocker, Nucl. Phys. A 750, 121 (2005).

[19] W. Broniowski and W. Florkowski. \texttt{arXiv:nucl-th/0110020v1}

[20] E. Retinskaya, M. Luzum, and Jean-Yves Ollitrault. Phys. Rev. Lett. 108, 252302 (2012).

[21] K. Aamodt et al, Phys. Rev. Lett. 105, 252302 (2010).

[22] BRAHMS Collaboration, Nucl. Phys. A757, 1 (2005).

[23] PHOBOS Collaboration, Nucl. Phys. A757, 28 (2005).

[24] STAR Collaboration, Nucl. Phys. A757, 102 (2005).

[25] PHENIX Collaboration, Nucl. Phys. A757, 184 (2005).

[26] S. Mrowczynski, Phys. Lett. B393, 26 (1996); J.Phys.Conf.Ser. 27 (2005) 204-216.