Reconstruction of DBI-essence dark energy with $f(R)$ gravity and its effect on black hole and wormhole mass accretion

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Abstract

In this paper, we have used the reconstructed DBI-essence dark energy density to modify the mass accretions of black holes and wormholes. In general, the black hole mass accretion does not depend on the metric or local Einstein geometry. That is why we have used a generalized mechanism by reconstructing the DBI-essence dark energy reconstruction with $f(R)$ gravity. We have used some particular forms of the scale factor to analyze the accretion phenomena. We have shown the effect of cosmic evolution in the proper time variation of black hole mass accretion. Finally, we have studied the validity of energy conditions and analyzed the type I - IV singularities for our reconstructed model.

Keyword: black hole mass accretion, DBI-essence dark energy, Reconstruction mechanism, $f(R)$ gravity, Cosmic evolution

1 Introduction

The black hole is one of the most important results of local solutions of the Einstein equation. The evolution of the universe follows the global solution. Although there is no direct connection between the global and local solution in geometry, the mass accretion can create a link between cosmic fluid and the black hole or wormhole like local solutions.

The nature of cosmic phases depends on the cosmic fluid with which it evolves. Various modified theories have been proposed to discuss the different cosmic phases. The Einstein Hilbert action fails to discuss the accelerated expansion of the universe, which has been experimentally proven. The introduction of the modified function of Ricci scalar in Einstein action can provide the negative pressure, which in turn can give a repulsion effect. This negativity can explain inflationary solutions as well as the late-time acceleration. From the Raychaudhuri equation, it is observed that the energy conditions must have positive values for a gravitating system. However, positive energy
conditions can never produce accelerated expansion in cosmology. Therefore, simple Einstein gravity geometry alone cannot discuss the accelerated expansion properly. Modified gravity models can resolve this problem with geometry modification. There are several types of modified gravity available in the literature. Most useful modifications are $f(R)$ gravity, Gauss-Bonnet gravity or $f(G)$ gravity, Teleparallel gravity or $f(T)$ gravity, non-Metricity of gravity or $f(Q)$ gravity, and so on. $f(Q)$ and $f(T)$ come from the modifications in the metric tensor and Christoffel connections. $f(R)$ and $f(G)$ gravity models are basically the higher-order modification of Ricci tensor in Einstein action \[1-12].

The accelerated expansion or expanding universe can also be explained with the exotic nature of matter and energy. The most highlighted exotic models are the dynamic dark energy models. There are three dark energy models, namely fluid, scalar field, and Holographic models. The primary requirement behind the use of these dynamics dark energy models is to resolve the vacuum energy problem or cosmic coincidence problem \[7,17-21].

Dirac-Born-Infeld (DBI)-essence dark energy model \[13-17\] is one of the scalar field dynamic dark energy models that are used in cosmic physics, even in string theory. There are several types of scalar field models available in the literature, viz. Quintessence model, K-essence model, Phantom field model, H-essence model, and Tachyon model. The Tachyon model is one of the most generalized scalar field models that can discuss all the cosmic phases and conditions, but it fails to bring the cosmic Brane Bulk tension. We have used this DBI-essence model of dark energy is used in our study.

All these cosmic models modify the activity of cosmic fluids, and these modifications, in turn, modify the rate of mass accretion of black holes. It was observed that the mass accretion of the black hole and wormhole strictly depends on the cosmic phases and their evolutionary scheme. Babichev et al. \[22,23\] discussed the accretion of phantom DE into a Schwarzschild black hole, using the Holographic technique, and discussed that the mass of the black hole would gradually decrease to zero near the big rip singularity. Modified gravity modifies the nature of the cosmic fluid, and that is how it can also modify the rate of mass accretion of Black holes and wormholes. The reconstruction mechanism can reproduce the evolution of energy density and pressure of the cosmic system. \[23,25,37\]

Here in this paper, we have first reconstructed the energy density and pressure of the DBI-essence Dark energy model with $f(R)$ gravity and discussed the energy conditions of the system. This modification has been used to discuss the black hole and wormhole mass accretion. We have discussed the rate of mass accretion in the presence of different scale factors and compared the rate with different cosmic phases. We have also tried to provide some physical reason behind the nature of mass accretion.

So, the paper is structured as follows.

In sections 2 and 3, we have discussed the basics of the modified gravity and scalar field theory. Section 4 contains the coupling calculations that help us to discuss the reconstruction formalism. Sections 5 and 6 have the basics of thermodynamic energy conditions and condensed body mass accretion. In section 7, we have discussed the analysis for different scale factors and the comparison of their results. In section 8, we have finally tried to resolve the finite-time future singularities w.r.t. the scale factors, energy densities, and pressures. We have provided the details of the results in the discussion section 9 and the overall outcome of the work in section 10.
2 Overview of $f(R)$ gravity

The study of $f(R)$ gravity should start from its action as follows [53]

$$S = \frac{1}{16\pi} \int f(R)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x.$$  \hspace{1cm} (1)

Here $f(R)$ is a function of the Ricci scalar curvature $R$, $g$ is the determinant of 4D metric $g_{\mu\nu}$ and $L_m$ is the matter Lagrangian. The line element for the isotropic, homogeneous, and curvature free Friedmann-Robertson-Walker (FRW) model of the universe is as follows

$$ds^2 = dt^2 - a(t)(dx^2 + dy^2 + dz^2),$$  \hspace{1cm} (2)

where $a(t)$ is the scale factor of the universe. So for flat FRW model, $R = 6(2H^2 + \dot{H})$, where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. The field equation corresponding to the above action is given by

$$f(R)R_{\nu\mu} - \frac{1}{2}f(R)g_{\nu\mu} + (g_{\nu\mu}\Box - \nabla_\mu \nabla_\nu)f(R) = 8\pi T_{\nu\mu},$$  \hspace{1cm} (3)

where $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$ is the energy-momentum tensor. Here $\rho$ and $p$ are respectively the energy density and pressure of the fluid and $u^\mu$ is the fluid four vector.

In the context of our work we are using the isotropic and homogeneous FRW model and its evolutionary equation to reconstruct the DBI-essence dark energy model.

3 Overview of DBI-essence model

The DBI-essence model starts from the following Lagrangian. [16]

$$L = T(\phi)\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} + V(\phi) - T(\phi).$$  \hspace{1cm} (4)

Here $T(\phi)$ is a Bulk Brane tension and $V(\phi)$ is the potential.

From the scalar field Lagrangian, the pressure and energy density for DBI-essence model can be given as follows [16]

$$\rho_\phi = (\gamma - 1)T(\phi) + V(\phi)$$  \hspace{1cm} (5)

and

$$p_\phi = (1 - \frac{1}{\gamma})T(\phi) - V(\phi).$$  \hspace{1cm} (6)

The conservation equation for any scalar field always produce the Klein-Gordon equation which gives us the dynamical nature of scalar fields. In the case of the DBI-essence model, the modified scalar field Klein-Gordon equation can be written as follows [16]

$$\ddot{\phi} - \frac{3T'(\phi)}{2T(\phi)}\dot{\phi}^2 + T'(\phi) + \frac{3}{\gamma^2} \frac{\dot{a}}{a} \dot{\phi} + \frac{1}{\gamma^3}(V'(\phi) - T'(\phi)) = 0.$$  \hspace{1cm} (7)

We assume $V(\phi) = T(\phi) = n\dot{\phi}^2$ as used in [33], where $\gamma$ is given by $\gamma = \sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}}$.

Using this assumption in equations (5) and (6) we get

$$\frac{1}{2}\dot{\phi}^2 = \frac{1}{2}\sqrt{\frac{n-1}{n}}[\rho_\phi + p_\phi].$$  \hspace{1cm} (8)

Rewriting the scalar field potential and tension in terms of scalar field density and pressure, we obtain

$$V(\phi) = T(\phi) = \sqrt{n(n-1)[\rho_\phi + p_\phi]}.$$  \hspace{1cm} (9)


4 Coupling between $f(R)$ gravity and DBI-essence DE and its reconstruction

We proceed with the reconstruction of the DBI-essence model by introducing coupling between the DBI-essence model and $f(R)$ model. In our study, modified gravity is the reconstructing system, and the DBI-essence scalar field is reconstructed. The coupled action can be written as follows

$$S = \int d^4x \sqrt{-g} [f(R) + T(\phi) \sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} + V(\phi) - T(\phi) + L_m].$$

(10)

The Field equations corresponding to the above action are as follows

$$3H^2 = \frac{1}{f'(R)} [\rho_m + \rho_R + \rho_\phi]$$

(11)

and

$$3H^2 + 2\dot{H} = \frac{1}{f'(R)} [-p_m - p_R - p_\phi],$$

(12)

where $\rho_R$ and $p_R$ represent the energy density and pressure in context of $f(R)$ gravity. Their expressions are as follows

$$p_R = \frac{1}{2} [f(R) - Rf'(R)] + [2H\dot{R} + \ddot{R}]f''(R) + \dddot{R}f'''(R)$$

(13)

and

$$\rho_R = \frac{1}{2} [-f(R) + Rf'(R)] - 3H\dot{R}f''(R).$$

(14)

Here, prime denotes derivative w.r.t. $R$. The energy density due to dark matter is given by

$$\rho_m = \rho_{m0}a^{-3},$$

(15)

where $\rho_{m0}$ is present value of the energy density. Using equations (10)-(15), the expression of the reconstructed scalar field density and pressure for the coupled system are as follows:

$$\rho_\phi = 3H^2f'(R) - \rho_m - \frac{1}{2} [-f(R) + Rf'(R)] + 3H\dot{R}f''(R)$$

(16)

and

$$p_\phi = -(3H^2 + 2\dot{H})f'(R) - p_m - \frac{1}{2} [f(R) - Rf'(R)] - [2H\dot{R} + \ddot{R}]f''(R) - \dddot{R}f'''(R).$$

(17)

5 Overview of different types of singularities and thermodynamic energy conditions

The types of finite time singularities are given as follows [42,56,57]:

- For any finite time, $t = t_s$ if we have $a \rightarrow \infty; \rho \rightarrow \infty; p \rightarrow \infty$, it is known as Type-I or Big Rip singularity.
For any finite time, $t = t_s$, if we have $a \to a_s; \rho \to \rho_s; p \to \infty$, it is Type-II or Sudden singularity.

For any finite time, $t = t_s$ if we have $a \to a_s; \rho \to \infty; p \to \infty$, it is called Type III singularity. It is found in EOS of type $p = -\rho - A\rho^\alpha$.

At any finite time, $t = t_s$, if we have $a \to a_s; \rho \to \rho_s; p \to \infty$ and also the Hubble parameter and its first derivative remain finite, but its higher derivatives diverge, it is Type IV singularity. This kind of singularity is found in $p = -\rho - f(\rho)$.

The thermodynamic energy conditions can be derived from the Raychaudhuri equation. For a congruence of time-like and null-like geodesics, the Raychaudhuri equations are given in the following forms that can be also found in the following expression in 21,56,57

$$\frac{d\theta}{d\tau} = -\frac{1}{3} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} + w_{\mu\nu} w^{\mu\nu} - R_{\mu\nu} u^\mu u^\nu$$

and

$$\frac{d\theta}{d\lambda} = -\frac{1}{3} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} + w_{\mu\nu} n^\mu n^\nu - R_{\mu\nu} n^\mu n^\nu,$$

where $\theta$ is the expansion factor, $n^\mu n^\nu$ is the null vector, and $\sigma_{\mu\nu} \sigma^{\mu\nu}$ and $w_{\mu\nu} w^{\mu\nu}$ are the shear and rotation associated with the vector field $w^\mu w^\nu$. For attractive gravity we may write followings

$$R_{\mu\nu} u^\mu u^\nu \geq 0 \text{ and } R_{\mu\nu} n^\mu n^\nu \geq 0.$$

For our matter-fluid distribution, the energy conditions are as follows:

i. Null energy condition (NEC) = $\rho + p \geq 0$.

ii. Weak energy condition (WEC) = $\rho \geq 0$ and $\rho + p \geq 0$.

iii. Strong energy condition (SEC) = $\rho + 3p \geq 0$ and $\rho + p \geq 0$.

iv. Dominant energy condition (DEC) = $\rho \geq 0$ and $-\rho \leq p \leq \rho$.

6 Overview of black Hole and Wormhole Mass accretion

We start with the study of mass accretion of black hole which is derived in [33]. The mass accretion of black holes can be written as

$$\dot{M} = -4\pi r^2 T_0^1. \quad (18)$$

Here $T_{\mu\nu}$ is the energy momentum tensor for the fluid given by $T_{\mu\nu} = (\rho_\phi + \rho_m + p_\phi + p_m) u_\mu u_\nu - g_{\mu\nu}(\rho_\phi + p_m)$. Therefore, the above equation becomes as follows

$$\dot{M} = -4\pi AM^2(\rho_\phi + \rho_m + p_\phi + p_m), \quad (19)$$

where $A$ is a positive constant (details can be found in [22]). Similarly, the mass accretion of wormholes can be written as follows [33]

$$\dot{M} = 4\pi r^2 T_0^1. \quad (20)$$
We can again re-write this equation using the functional form of energy momentum tensor $T^1_0$ for wormhole, as follows
\[ \dot{M} = 4\pi BM^2 (\rho_\phi + \rho_m + p_\phi + p_m), \] (21)
where $B$ is a positive constant which can be found in [42]. We will discuss the mass accretion with the reconstructed scalar field energy density and pressure for different scale factors ($A$ and $B$ are the constants to differentiate the black holes and Wormholes mass accretion). To discuss the accretion phenomena, we consider pressureless dark matter or cold dark matter i.e., $p_m = 0$ in the subsequent sections.

7 Different scale factors and mass accretion analysis

We will proceed with the discussion of the application of our reconstructed system by considering four different types of scale factor. We study the simplest form of $f(R)$ modified gravity [6] as
\[ f(R) = \lambda R^2, \] (22)
where $\lambda$ is constant and taken as $\lambda \in (I^-)$ for first two scale factor and $\lambda \in (I^+)$ for last two scale factors.

7.1 Scale factor $a(t) = a_0(a_1 + nt)^m$

According to the results of section 4 with power-law form of scale factor $a(t) = a_0(a_1 + nt)^m$ [32], where $a_0, a_1, n, m$ are positive constants, we can represent $\rho(\phi)$ and $p(\phi)$. Here $a_1$ removes the initial singularity and $m > 1$ for accelerated universe. We plot $\rho(\phi)$ and $p(\phi)$ in Figs. 1 and 2. This $\rho(\phi)$ and $p(\phi)$ can be used in equations (14) and (15) and the variations of kinetic energy and potential energy can be observed in figures 3 and 4.

![Figure 1: Plot of $\rho_\phi$ vs $t$ with $m = 2$, $n = 1/2$, $a_1 = a_0 = 1$ and $\lambda = -5$](image1.png)

![Figure 2: Plot of $p_\phi$ vs $t$ with $m = 2$, $n = 1/2$, $a_1 = a_0 = 1$ and $\lambda = -5$](image2.png)
7.1.1 Energy conditions with reconstructed DBI-essence model and effective system

We will discuss the thermodynamic energy conditions w.r.t. the scalar field energy density and pressure that are obtained for the first assumed scale factor. The graphical representations for energy conditions are shown in Figs. 5 to 7.

7.1.2 Mass accretion formalism

The basics of mass accretion has been discussed in section 6. The mass accretion of black hole and wormhole is shown graphically in in figs. 8 and 9.
7.2 Scale factor $a(t) = g + a_0(a_1 + nt)^m$, $g = \text{constant} > 0$

The second scale factor is of the form $a(t) = g + a_0(a_1 + nt)^m$, where $m$ is even power. The importance of this constant $g$ is that it resolves any past-time singularity. We can obtain the expression $\rho(\phi)$ and $p(\phi)$ by the method discussed in section 4. The variation of $\rho(\phi)$ and $p(\phi)$ can be observed graphically in figures 10 and 11.

The time variation of scalar field kinetic energy and potential energy can be observed in Figs. 12 and 13.

7.2.1 Energy conditions with reconstructed DBI-essence model and effective system

We study the thermodynamics energy conditions w.r.t. the scalar field energy density and pressure shown graphically in Fig 10 and 11 in the previous subsection. The graphical representations for energy conditions have been shown below in Figs. 14 to 16.
Figure 14: Plot of \( \rho_\phi + 3p_\phi \) with \( t \) for \( m = 2, n = 1/2, a_1 = a_0 = 1, f = 5 \) and \( \lambda = -5 \)

Figure 15: Plot of \( \rho_\phi + p_\phi \) with \( t \) for \( m = 2, n = 1/2, a_1 = a_0 = 1, f = 5 \) and \( \lambda = -5 \)

Figure 16: Plot of \( |\rho_\phi| - |p_\phi| \) with \( t \) for \( m = 2, n = 1/2, a_1 = a_0 = 1, f = 5 \) and \( \lambda = -5 \)

7.2.2 Mass accretion formalism

The graphical representation of mass accretion of black hole and wormhole is given in Figs. 17 and 18.

Figure 17: Plot of black hole mass accretion \( M(t) \) vs \( t \) with \( t \) for \( m = 2, n = 1/2, a_1 = a_0 = 1, f = 5 \) and \( \lambda = -5 \)

Figure 18: Plot of wormhole mass accretion \( M(t) \) vs \( t \) with \( t \) for \( m = 2, n = 1/2, a_1 = a_0 = 1, f = 5 \) and \( \lambda = -5 \)

7.3 Scale factor \( a(t) = a_0 \exp \alpha t^2 \)

The third assumed scale factor is of the form \( a(t) = \exp \alpha t^2 \) as discussed in [32], we can derive \( \rho(\phi) \) and \( p(\phi) \). The time variation of \( \rho(\phi) \) and \( p(\phi) \) can be observed in Figs. 19 and 20.

The time variation of kinetic energy and potential energy for this scale factor is shown in Figs. 21 and 22.
7.3.1 Energy conditions with reconstructed DBI-essence model and effective system

The energy conditions corresponding the the above discussed density and pressure are graphically shown in Figs. 23 to 25.

7.3.2 Mass accretion formalism

The mass accretion for the third type of scale factor is given in Figs. 26 and 27.
Figure 26: Plot of black hole mass $M(t)$ vs $t$ for $\alpha = 5, m = 2, n = 1/2, a_1 = a_0 = 1$ and $\lambda = 5$

Figure 27: Plot of wormhole mass $M(t)$ vs $t$ for $\alpha = 5, m = 2, n = 1/2, a_1 = a_0 = 1$ and $\lambda = 5$

7.4 Scale factor $a(t) = a_0(\exp \alpha t^2 + \exp \alpha^2 t^4)$

The fourth assumed scale factor is $a(t) = a_0(\exp \alpha t^2 + \exp \alpha^2 t^4)$ as discussed in [32], we can derive $\rho(\phi)$ and $p(\phi)$. The time variation of $\rho(\phi)$ and $p(\phi)$ is shown in Figs. 28 and 29.

Figure 28: Plot of $\rho(\phi)$ with $t$ for $\alpha = 5, m = 2, n = 1/2, a_1 = a_0 = 1$ and $\lambda = 5$

Figure 29: Plot of $p(\phi)$ with $t$ for $\alpha = 5, m = 2, n = 1/2, a_1 = a_0 = 1$ and $\lambda = 5$

The time variation of kinetic energy and potential energy is shown in Figs. 30 and 31.

Figure 30: Plot of $\frac{1}{2}(\dot{\phi})^2$ with $t$ for $\alpha = 5, m = 2, n = 1/2, a_1 = a_0 = 1$ and $\lambda = 5$

Figure 31: Plot of $V(\phi)$ with $t$ for $\alpha = 5, m = 2, n = 1/2, a_1 = a_0 = 1$ and $\lambda = 5$
7.4.1 Energy conditions with reconstructed DBI-essence model and effective system

The graphical representations of the energy conditions corresponding to the above assumed scale factor are Figs. 32 to 34.

Figure 32: Plot of $\rho_\phi + 3p_\phi$ with $t$ for $\alpha = 5$, $m = 2$, $n = 1/2$, $a_1 = a_0 = 1$ and $\lambda = 5$

Figure 33: Plot of $\rho_\phi + p_\phi$ with $t$ for $\alpha = 5$, $m = 2$, $n = 1/2$, $a_1 = a_0 = 1$ and $\lambda = 5$

Figure 34: Plot of $|\rho_\phi| - |p_\phi|$ with $t$ for $\alpha = 5$, $m = 2$, $n = 1/2$, $a_1 = a_0 = 1$ and $\lambda = 5$

7.4.2 Mass accretion formalism

The variation of mass accretion of black hole and wormhole are shown in Figs. 35 and 36.

Figure 35: Plot of black hole mass $M(t)$ vs $t$ with $t$ for $\alpha = 5$, $m = 2$, $n = 1/2$, $a_1 = a_0 = 1$ and $\lambda = 5$

Figure 36: Plot of wormhole mass $M(t)$ vs $t$ with $t$ for $\alpha = 5$, $m = 2$, $n = 1/2$, $a_1 = a_0 = 1$ and $\lambda = 5$

8 Removal of singularities

In section 5, we have discussed the various types of singularities that can be present in any model. In this paper, we have studied four types of scale factors. It is observed that the scalar field density and pressures of all the four types of scale factors are continuous and finite and hence there is no singularity at an initial time as well as at a future time. It is also observed that for the first scale factor in the time range $t > 0$ and for all other scale factors in the time range $t \in (-\infty, \infty)$ we did not find any kind of singularities. Thus from fluid and geometry parameters, we can conclude that our model is free of any singularity.
9 Physical Analysis and Discussions

In this work, we have discussed four types of scale factors, along with their pictorial representation of reconstructed scalar field energy densities, pressures, kinetic energies, and potential energies. Later, we discussed energy conditions and mass accretion with respect to the reconstructed scalar field of black holes and wormholes, respectively. The detailed discussions of those results have been given in the proceeding paragraphs.

The energy densities have similar nature for the third and fourth type scale factors. Both of them have increasing nature after the point of cosmic bounce. The first type scale factor provides decreasing type energy density, whereas the second type bouncing scale factor gives some variation in nature with time. For the second type scale factor, the energy density firstly decreases with time, then after cosmic bounce, it increases with time and then again decreases. The turning points provide some phase transition whose details discussion is beyond the scope of that work.

The pictorial representations of the pressures provide exactly opposite nature of the energy densities of them. This does not happen for the second type scale factor. For the second type, the energy density and pressure have similar nature. Except for the second kind scale factor, the pressures of all other scale factors have negative values, and hence we may conclude except for the second type scale factor, all others can represent just the accelerated expansion of the universe. The pressure representation of the second scale factor can conclude that we can have a deceleration phase also somewhere after the bounce and late-time acceleration.

The nature of scalar field kinetic energy and potential energy changes with changing the scale factors, but they are surprisingly evolving in similar ways when we analyze them with fixed type scale factor. The potential of the first type scale factor provides the idea of slow roll variation. The potential for second type scale factor can provide negative results, which also proves the existence of an attractive gravity-dominated universe for some time after the cosmic bounce.

The mass accretion rate decreases for both the inflationary and bouncing type scale factors, i.e., for third and fourth type scale factors. But for first and second type scale factors, the mass accretion rate of the black hole is increasing, and for the wormhole, it is decreasing in nature.

In the analysis of energy conditions, we can find that the strong energy conditions are mostly violated. The other energy conditions are also violated except for the first and second type scale factors.

Thus, we have found a comparison between the results we found for four types of scale factors with reconstructed Dirac-Born-Infeld scalar field model with modified $f(R) = \lambda R^2$ gravity.

10 Concluding remarks

In this paper, we have discussed the reconstructed scalar field theory in terms of the DBI-essence model. The reconstruction has been done by introducing coupling between the DBI-essence scalar field dynamic dark energy model and modified gravity, particularly $f(R)$ gravity. We have observed late-time decreasing nature of reconstructed energy density for the first two power-law scale factors whereas an increasing nature for the last two exponential scale factors. The pressures for the first two scale factors have increasing nature and for the last two have to decrease nature.

In the analysis of mass accretion, we have found different natures for Black holes and Wormholes mass accretion for different scale factors. The variation of mass accretion for the inflationary cosmology remains similar in nature, irrespective of the nature of the scale factor.

The last two scale factors have been taken to discuss the bouncing universe and inflationary
phase at the same time. The second scale factor is our assumption on power-law cosmology to resolve the initial time singularity at \( t \to 0 \) as well as \( t \to 0^- \) or \( t < 0 \). The first scale factor also resolves the initial singularity but only at \( t \to 0 \).

Overall, we have discussed the reconstruction mechanism and mass accretion with the discussion of energy conditions in some cosmic phases at primordial times of the universe.

**Limitations of this work**

The first two scale factors have been assumed, and the last two have been taken from works of Bamba et al. [32]. We have investigated the results using reconstructed scalar field theory but didn’t use any theoretical analysis on mass accretion.

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