Compact Stars in Kaluza–Klein World

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Abstract. Unification and geometrization of interactions has been extensively studied during the XX. century. In this short contribution we investigated the possible effect of an extra compactified dimension (alias hypercharge) on a flavor dependent gravitational potential, proposed by Fischbach et al.. We estimated the deviation from the 3+1 dimensional scheme and found that, although the deviation is moderate, for celestial compact object it may be higher by orders of magnitude than in terrestrial laboratory measurements.

1. Introduction
Searching for signatures of compactified extra dimension(s) is an interesting and challenging task. There are no direct observables for these topological issues in the classical physical cases, so search can start at highest energy systems, which can be created in particle accelerators or existed inside celestial objects as compact stars or black holes (Barnaföldi et al., 2006).

In this short review we would like to re-think and extend the usual description of compact objects in case of more than 3 + 1 dimensional space-time – in a Kaluza–Klein-like world with a compactified extra dimension $c$. Our aim is to highlight a possible deviation from standard gravitational potential as an influence of compactified extra dimension (or hypercharge) in a compact pulsar binary (like e.g. PSR J0737-3039).

2. Geometrization of Interactions – a Historical Review
Einstein’s general relativity (GR) was the first theory which translated an interaction of the nature i.e. gravity into a geometrical approach. The original theory was developed in 1916, assumed to have symmetric metric tensor in 3 + 1 space-time.

Later Einstein worked more on the development of the unification of gravity and electromagnetic interaction (Cartan 1923). The generalization of his model gave up the symmetric being of the metric tensor which leads to 16 free parameters in 3 + 1 dimensional space-time. In case of a non-symmetric metric tensor, gravity requires 10 independent parameter and 6 were trivially left for the Maxwell tensor. Even if this generalization looks trivial, it leads to several non-physical consequences.

Another direction was proposed by Kaluza (1921) and later on extended by Klein (1926), where they kept the assumption of the general relativity using the symmetric metric tensor, but they extended the 3 + 1 dimensional space-time including a spatial compactified extra 5th dimension. In this case the number of independent parameters become 15, separated as: 10 for gravity, 4 correspond to the vector potential, and finally setting $g_{55} = 1$ is crucial to fix the remaining part. However, this latter assumption violates the role of the ‘free choice of coordinate systems’, but Maxwell equations can be carry out in a weak-field approximation for any case.
Brans and Dicke (1961) further developed the theory introducing a time dependent 'gravitational constant', $G(t)$, giving an additional equation based on cosmological issues. In this way the whole gravitational and electro-magnetic theory can be unified and geometrized. Following this second scenario, Chodos and Detweiler (1980) did the separation of the $5^{th}$ direction for the first time in a consequent way in $4+1$ dimensional space-time. They find the 15 free 'geometrical parameters' can be separated to $10+4+1$ components, where the last 1 dimensional part must be a scalar field.

Lukács and Pacher (1985) reviewed the Brans–Dicke theory, and pointed out: the generation of extra mass by fast moving in the $5^{th}$ direction is limited due to the charge-to-mass ratio of the observed particles such as the proton. This model ruled out the electro-magnetic origin of the charge. On the other hand, applying the model for the weak charge, opened a new directions showing the possibility for the unification and geometrization of the weak and gravitational theories. In this picture the extra compactified $5^{th}$ dimension represents the hypercharge or strangeness.

3. On the Shadow of a Dimension

Obviously, a microscopical and compactified dimension has 4-dimensional consequences. However, these consequences may depend on the nature of higher dimensions, so our present study cannot give a full review. For now we use some simplifying conditions, rather accepted in general relativity but not always in particle physics. They are as follows:

1. The space-time has $(3+m_c)+1$ dimensions; and except for the last one, they are space-like, while the last is time-like.
2. The structure of general relativity is just as we have learnt in $3+1$ dimensions; especially the form of the Equivalence Principle is unchanged.
3. All causality postulates, including lightcone structure, are as they were in the $3+1$ case.
4. The $m_c$ extra space-like dimensions are microscopical, i.e. they are all compact with microscopical circumferences.
5. There is complete Killing symmetry in the $m_c$-dimensional microscopical subspace.
6. In this contribution, we focus on the simplest case, thus for now: $m_c = 1$.

Cond. (i) excludes inherently acausal space-times – 2 simultaneous time-like coordinates would permit time travel as an everyday profession (Lukács 2003). Conds. (ii) and (iii) are natural in general relativity contrasted to particle physical theories where the extra dimensions are 'not exactly as our familiar (iii)', e.g. that motion in that direction 'is not like other motions'. Then Cond. (iv) is for keeping the extra dimensions 'really microscopical'. The last two conditions are not really fundamental but simplifying conditions for now. Cond. (v) is for avoiding difficulties about the directly unobservable microstructures and Cond. (vi) for restricting ourselves for the simplest cases.

The general metric, satisfying Conds. (i–vi) can always be written into the form

$$ds^2 = -\Phi^{2/3}(d\Psi + A_r dx^r)^2 + \Phi^{-1/3} g_{rs} dx^r dx^s,$$

with an Einstein convention for summation in 4 dimensions, where $\Psi = x^5$, and $r, s = 0, 1, 2, 3$ and the quantities $\Phi$ and $A_r$ do not depend on $\Psi$. Then the Lagrangian of the 5 dimensional general relativity gets the form in 4 dimensions as

$$L = R^{(5)} = R^{(4)} + \Phi \frac{F_{rs}}{4} F^r_s + \frac{G^{rs}}{6} \frac{\Phi_{,r} \Phi_{,s}}{\Phi^2}.$$  

So from a purely geometrized 5-dimensional space-time you get a 4-dimensional 'projection' with a scalar coupling and a vector field as well.
The scalar field corresponds to the scalar coupling in the Brans–Dicke general relativity (Brans & Dicke, 1961). The vector field has the same structure as Maxwell field, and therefore it was the feeling of Kaluza that electromagnetism has been geometrized.

However the equations of motion give some constraint for the specific charge. The motion in a 5-dimensional vacuum must happen on a 5-dimensional geodesics on one hand and on the other, equivalent with a 4-dimensional motion in the vector and scalar fields. Now the geodesic equation formally has 5 components, but the 0th one is a consequence of the normalized nature of the velocity vector,

$$u^r u_r + u^5 u_5 = +1.$$  \hspace{1cm} (3)

Note, here we used again the 4-dimensional convention, where, according to Conds. (i) and (iii),

$$\text{sign}(g_{ik}) = \{+,-,-,-,-\}; \quad i = 0, 1, 2, 3, 5.$$  \hspace{1cm} (4)

The 5th component of the geodesic equation gives a balance equation for the extra velocity component, so approximately a conservation of something seen as a charge in 4 dimensions. The larger 5th component of the velocity, indicates the larger specific charge.

However, this 5th component is not seen, so in the normalization eq. (3) we miscalculate $u^0$. Using the full $u^i$ vector in the formalism we get that the specific charge goes to a finite value as $u^5 \rightarrow \infty$ (Lukács & Pacher, 1985). This problem is sometimes called the 'large mass problem'. Of course, it would not appear with large $u^5$ and moderate $u^0$ values; but such velocities would be space-like (Gegenberg & Kunstätter, 1985). To permit such velocities is contrary to Cond. (iii), the causality postulates in 5 dimensions.

Finally, we remain at a vector 'interaction' (geometrized in 5 dimensions but not in 4) with a coupling constant or specific charge:

$$\frac{q}{m^{(4)}} < \sqrt{\frac{16 \pi G}{3}}.$$  \hspace{1cm} (5)

Let’s take for example a proton, $p$, with 4 dimensional mass $m^{(4)} = m_p$ and a charge as $e = q$, which results: $e^2/(Gm^2_p) = 1.2 \times 10^{36}$. Based on eq. (5) this interaction cannot be the electromagnetic one, but much weaker vector fields can be the 'shadows' of the 5th dimension.

Applying quantum mechanics in 5 dimensions, the compactified extra dimension results in a quantized charge (Souriau, 1963) $q \simeq 2h \sqrt{G}/(cR_c)$ where $R_c$ is the compactified radius with value probably $10^{-13}$ cm $\leq R_c \leq 10^{-9}$ cm as we pointed out in e.g. Barnaföldi et al., (2006). Note, this is well within the spatial resolution of the Large Hadron Collider (LHC) at CERN, which is $\sim h/c/E_{beam} \sim 3 \cdot 10^{-18}$ cm, at the highest working energies.

4. Possible Experimental Evidences

As it was shown earlier by Barnaföldi et al. (2006) and Kan & Shiraishi (2002) in a Kaluza–Klein like world one kind of 5 dimensional particles generate series of particles in the 4 dimensional world, following the Kaluza–Klein mass-ladder. However, it is still a question which quantum number corresponds to the mass-steps of this ladder. One can assume strangeness, $S$ is related to the radius of the compactified extra dimension with size $\sim 10^{-13}$ cm. In this case a 5 dimensional compact baryon star would mimic a hyperon star in 4 dimensions, but with a somewhat different mass distribution, $M(R)$ (see further details in Barnaföldi et al., (2006)).

In this contribution we focus on another possible signal, following the popular idea of Fischbach et al. from the mid '80s. They re-analyzed the Eötvös experiment about the equivalence of inertial and gravitational masses, and found a small, but systematic deviation from chemical composition of the test bodies (Eötvös et al., 1922). Fischbach and collaborators find the best linear fit using hypercharge, $H = B + S$ as the source of deviation. Therefore, they
believed to found the so called '5th force', which may even be responsible for the CP violation of the weak interaction (Fischbach et al. 1986).

Here, we do not want to argue either for or against the Fischbach’s scheme however, note, their fitted coupling is \( q^2/e^2 \approx 10^{-38} - 10^{-41} \), which is in agreement with eq. (5). Thus the results of Eötvös’s experiment may originate from multi-dimensional general relativity.

5. Summary – Compact Stars in Kaluza–Klein World

To evaluate the above predicted effect of an extra dimension, let’s use an astrophysical scenario: taking a compact binary star with typical physical parameters \( R \approx 10 \text{ km}, M \approx 2 \times M_\odot, B \approx 2.6 \times 10^{57} \), and baryon density, \( \rho_B \approx 0.15 \text{ fm}^{-3} \).

Following Fischbach’s idea the presence of the 5th force in 3 + 1 dimensional space-time appears as an extra term additional to the gravitational potential,

\[
V(r) = -G_\infty \frac{m_1 m_2}{r} \left(1 + \frac{\alpha \cdot e^{-r/\lambda}}{r} \right),
\]

where \( G_\infty \) is the gravitational constant at distance \( r \to \infty \) and \( m_1, m_2 \) are the masses. Parameters, based on Fischbach’s re-analysis are: \( \alpha = -(7.2 \pm 3.3) \times 10^{-3} \) and \( \lambda = 200 \pm 50 \text{ m} \).

However, the 3-dimensional effective potential is somewhat misleading not including the equivalence principle, well above 200 m distance the '5th force’ cannot be felt, but eq. (6) implies that gravitating mass also contains corrections. Note, our present scheme is completely general relativistic, so the gravitational mass must be equal to the inertial one.

Obviously the maximal deviation from the '4-dimensional equivalence principle’ would appear as using strangeness content test particles. For example if one could have \( \Lambda^0 \) or \( K^0 \)-matter content test bodies, for the gravitational experiment, the difference would appear in the 2nd or 3rd digits not only at the 9th as proposed by Fischbach et al., 1986.

Test bodies would feel their strangeness and in astrophysical situations hyperonic masses can occur. Especially, the 'first compact pulsar binary', PSR J0737-3039 was discovered in 2004 by Lyne et al., with \( 8 \cdot 10^5 \text{ km} \) orbit, is the best known candidate for experimental test of the extra-dimensional influence on gravitating mass in the 2nd or 3rd digit.

Acknowledgments

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\(^1\) Note, mesons with zero baryon number, \( B = 0 \) and with non-zero strangeness (or hypercharge) \( S \neq 0 \), would have higher possibility to move to the 5th direction. Thus deviation effect is expected to be stronger compared to ordinary baryonic matter. On the other hand \( \Lambda^0 \) content compact star is more physically than \( K^0 \) content one.