Non leptonic $B$ decays to axial-vector mesons and factorization

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We present an analysis of two-body $B$ decays with a pseudoscalar ($P$) and an axial-vector meson ($A$) in the final state using factorization. We employ as inputs a limited number of experimental data, i.e. results for the $B \to K_1 \gamma$, and $B \to K^* \gamma$ radiative decays and the branching ratios for $B \to \pi \rho$, $\pi K^*$, $K \rho$, $K \pi$ non leptonic decays. In this way we are able to compare our predictions with recent data from the Belle and BABAR collaborations on $B \to a_1 \pi$ and make predictions on several other $B \to PA$ decay channels, which might be used as a guide for experimental researches and as tests of factorization.

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I. INTRODUCTION

Recent observations of the decays $B^0 \rightarrow a_1^\pm(1260)\pi^\mp$ from the Belle [1] and the BABAR Collaborations [2] offer the possibility of new investigations for two-body non-leptonic decay channels of the $B$ meson with an axial-vector meson in the final state. The BABAR result

$$B(B^0 \rightarrow a_1^\pm(1260)\pi^\mp) \times B(a_1^\pm(1260) \rightarrow \pi^\mp\pi^\pm\pi^\mp) = (16.6 \pm 1.9 \pm 1.5) \times 10^{-6}$$

(1)

translates into

$$B(B^0 \rightarrow a_1^\pm(1260)\pi^\mp) = (33.2 \pm 3.8 \pm 3.0) \times 10^{-6}$$

(2)

assuming [2] that $a_1$ only decays into three pions and an equal yield for $a_1^\pm(1260) \rightarrow \pi^\mp\pi^0\pi^0$ and for $a_1^\pm(1260) \rightarrow \pi^\pm\pi^0\pi^0$. On the other hand the Belle measurement gives:

$$B(B^0 \rightarrow a_1^\pm(1260)\pi^\mp) = (48.6 \pm 4.1 \pm 3.9) \times 10^{-6}$$

(3)

with an average of the two experiments

$$B(B^0 \rightarrow a_1^\pm(1260)\pi^\mp) = (40.9 \pm 7.6) \times 10^{-6}$$

(4)

In a recent paper [3], hereafter referred to as I, two of us have discussed other two-body non-leptonic decays of the $B$ meson with an axial vector meson in the final state and proposed some simple tests of factorization for them. The analysis of I was stimulated by the experimental results $B(B^+ \rightarrow K_1^+(1270)\gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}$ and $B(B^+ \rightarrow K_1^+(1400)\gamma) < 1.44 \times 10^{-5}$ (at 90% C.L.) from the Belle collaboration [4]. These numerical results are comparable with data for the analogous channels with a vector meson in the final state: $B(B^+ \rightarrow K^{*+}\gamma) = (4.18 \pm 0.31) \times 10^{-5}$ and $B(B^0 \rightarrow K^{*0}\gamma) = (4.17 \pm 0.23) \times 10^{-5}$ (averages of [5–7]). Therefore they suggest an approximate equality between the form factors for $B \rightarrow \pi$ and $B \rightarrow \pi a_1$ transitions [3]. Using this simple observation, in I we have proposed several tests of factorization for the $B$ decay channels with a strange axial-vector meson in the final state. In this paper we wish to reconsider this approach and to extend it to other decay channels with strange particles in the final state as well as final states with no strange particles. In particular we wish to compare theoretical expectations with the BABAR and Belle results (2) and (3), and to give predictions for several similar decay channels that have not been examined yet theoretically, but might be studied by the BABAR and other experimental collaborations. The study of two-body charmless $B$ decays with a light pseudo-scalar meson and an axial meson in the final states, besides providing us with information on the $B \rightarrow a_1$ and $B \rightarrow K_1$ transition form factors, could also tell us about the dynamics of these decays modes. Unlike the $B \rightarrow K\rho$ decays which is much suppressed because of the destructive interference of the $O_4$ and $O_6$ matrix elements, the decays $B \rightarrow a_1K$ could have a large branching ratio ($BR$), since the interference term becomes constructive and enhances the decay rates as in the $B \rightarrow K\pi$ decay. Therefore a large $BR$ similar to the $BR$ for $B \rightarrow K\pi$ would be a confirmation of a large $B \rightarrow a_1$ transition form factors and the penguin dominance of this decay.

Our approach is based on the idea that factorization, together with experimental data for the $BR$s of the decays $B \rightarrow K^{*+}\pi, K\pi, \rho\pi, \rho K$, can provide enough information to predict nonleptonic $B$-decays with one axial-vector meson in the final state. It is known that factorization holds only approximately and in some cases its predictions are at odds with experiment. In the last few years factorization has been proved to be a rigorous prediction of QCD in the infinite quark mass limit [8–10] and the naive factorization scheme has evolved into a more precise approach, using effective theories and an expansion in $\Lambda_{QCD}/m_b$. In this context it should be noted however that for $B$ decays into two light hadrons a proof at all orders is still
missing; moreover, for charmless $B$ decays with a strange light hadron in the final state, the tree-level $O_1, O_2$ operators are CKM suppressed compared with the $O_3, O_6$ matrix elements. This gives a numerically important contribution to the penguin-dominated decays since $O_6$ matrix elements are chirally-enhanced in naive and in QCD factorization, although power-suppressed by inverse powers of $m_b$ [10] if annihilation terms are neglected.

To our knowledge there is currently no extensive study of charmless $B$ decays with a final axial vector meson based on factorization. Therefore we feel it can be useful to collect predictions on these channels using the simple naive factorization approach, though we are conscious that these results should be interpreted with some care and used more as guidelines for experiment than as absolute predictions. The advantage over previous calculations of some related decay channels using factorization, e.g. [11–13], is the fact that we do not use predictions from theoretical models for the form factors. Therefore any discrepancy that might be found between our predictions and future data would point to a breakdown of naive factorization and suggest more refined treatments.

The plan of the paper is as follows. After a review of the approach in section II, we apply this method to get predictions for $B \to K \pi$ in section III, for $B \to a_1 K$ and $B \to b_1 K$ in section IV and for $B \to a_1 \pi, b_1 \pi$ in section V. Section VI contains our conclusions, while in Appendix A we have collected some relevant formulae used in the main text.

II. METHOD AND DEFINITIONS

Let us start with some relations among the various form factors of the $V - A$ currents that will be used below. We use definitions of form factors as listed in Appendix A. The main idea of I was to use ratios of $BRs$ to deduce ratios of form factors and, subsequently, to use this piece of information to predict decays of the $B$-meson into final states with an axial-vector meson. To this effect we will need below the ratios $\frac{A_{B \to V}}{F_{B \to \pi}^{V \to A}}$, $\frac{V_{B \to A}}{F_{B \to \pi}^{V \to A}}$ where $P, V$ and $A$ refer to pseudoscalar, vector and axial-vector meson. We will determine these quantities by factorization and using experimental data.

As a matter of fact factorization predicts the following results

$$\left| \frac{B(B^+ \to K^0 \rho^+)}{B(B^+ \to K^0 \pi^+)} \right| \simeq \left| \frac{4}{m_B^2} \right| \frac{W_8}{W_1} 2 \left( \frac{A_{B \to \rho}(m_K^2) + f_{\rho}(m_K^2)}{F_{B \to \pi}^{\rho}(m_K^2)} \right) \frac{q_{\rho}^3}{q_{\pi}^3}$$

$$\left| \frac{B(B^0 \to K^+ \rho^-)}{B(B^0 \to K^+ \pi^-)} \right| \simeq \left| \frac{4}{m_B^2} \right| \frac{W_6}{W_2} 2 \left( \frac{A_{B \to \rho}(m_K^2) + f_{\rho}(m_K^2)}{F_{B \to \pi}^{\rho}(m_K^2)} \right) \frac{q_{\rho}^3}{q_{\pi}^3}$$

$$\left| \frac{B(B^0 \to K^0 \rho^0)}{B(B^0 \to K^0 \pi^0)} \right| \simeq \left| \frac{4}{m_B^2} \right| \frac{W_6}{W_2} \left( \frac{A_{B \to \rho}(m_K^2)}{F_{B \to \pi}^{\rho}(m_K^2)} \right) \frac{2}{q_{\rho}^3}$$

$$\left| \frac{B(B^+ \to K^+ \rho^0)}{B(B^+ \to K^+ \pi^0)} \right| \simeq \left| \frac{4}{m_B^2} \right| \frac{W_6}{W_2} \left( \frac{A_{B \to \rho}(m_K^2)}{F_{B \to \pi}^{\rho}(m_K^2)} \right) \frac{2}{q_{\rho}^3}$$

$$\left| \frac{B(B^+ \to K^0 \pi^+)}{B(B^+ \to K^0 \pi^+)} \right| \simeq \left| \frac{4}{m_B^2} \right| \frac{W_4}{W_1} \left( \frac{f_{K^*}(m_K^2)}{f_K(m_K^2)} \right) \frac{q_{K^*}^3}{q_{K}^3}$$

$$\left| \frac{B(B^0 \to K^+ \pi^-)}{B(B^0 \to K^+ \pi^-)} \right| \simeq \left| \frac{4}{m_B^2} \right| \frac{W_5}{W_2} \left( \frac{f_{K^*}(m_K^2)}{f_K(m_K^2)} \right) \frac{q_{K^*}^3}{q_{K}^3}$$
\[
\frac{\mathcal{B}(B^0 \to K^{*0} \pi^0)}{\mathcal{B}(B^0 \to K^0 \pi^0)} \approx \frac{4}{m_B^2} \left| \frac{W_0 (f_{K^*} A_{B \to K^*}^{B \to K^*} (m_B^2)) + W_1 (f_{K'})}{W_3 (f_{K^*} A_{B \to K^*}^{B \to K^*} (m_B^2)) + W_4 (f_{K'})} \right|^2 \frac{q_{K^*}^2}{q_K^2},
\]
\[
\frac{\mathcal{B}(B^+ \to K^{*+} \pi^0)}{\mathcal{B}(B^+ \to K^{*+} \pi^0)} \approx \frac{4}{m_B^2} \left| \frac{W_0 (f_{K^*} A_{B \to K^*}^{B \to K^*} (m_B^2)) + W_1 (f_{K'})}{W_3 (f_{K^*} A_{B \to K^*}^{B \to K^*} (m_B^2)) + W_4 (f_{K'})} \right|^2 \frac{q_{K^*}^2}{q_K^2}.
\]

We have indicated the squared meson mass in the argument of the form factors to keep track of the factorization procedure, but in the numerical computations all form factors are evaluated at \(q^2 = 0\), which should be a rather good approximation; \(q_{B}, q_{\pi}, q_{K^*}\) and \(q_{K}\) are momenta in the \(B\) rest frame and \(W_i\) are combinations of Wilson coefficients and CKM matrix elements that can be found in the upper part of the Table reported in Appendix A. Using as inputs the experimental ratios for \(\mathcal{B}(B^0 \to K^{+} \rho^-) / \mathcal{B}(B^0 \to K^0 \rho^0)\) and \(\mathcal{B}(B^0 \to K^{*0} \pi^0) / \mathcal{B}(B^0 \to K^0 \pi^0)\), we have computed the entries in the first columns in Table I (the last column is obtained by the ratio of the first two data).

|                     | \(A_{B \to K}^{B \to K}(0)\) | \(F_{B \to K}(0)\) | \(A_{B \to K^*}^{B \to K}(0)\) | \(F_{B \to K^*}(0)\) | \(A_{B \to K}(0)\) | \(F_{B \to K}(0)\) |
|---------------------|-----------------------------|-----------------|-----------------------------|-----------------|----------------|----------------|
| BSW [11]            | 0.84                        | 1.15            | 0.97                        | 0.73            |
| HMEL [13]           | 0.45 ± 0.56                 | 0.92 ± 0.32     | 0.38 ± 0.46                 | 0.49 ± 0.60     |
| LCSR [14]           | 1.15                        | 1.30            | 1.38                        | 0.88            |
| CLFA [15]           | 1.12                        | 1.40            | 1.24                        | 0.80            |
| This work           | 1.63                        | 1.56            | 1.98                        | 1.02            |

We also present a comparison with other theoretical approaches. We notice that our predictions for the ratios are in general higher than other methods. The Light Cone Sum Rules (LCSR) results of Refs. [14] are however the less distant from ours.

We can now use these data to compute the remaining \(BRs\) in Eqs.(5)-(12). The results are reported in Table II and can be considered as a consistence test for the method to be used in the subsequent Sections. In particular we note that the ratios \(\mathcal{B}(B^+ \to K^{+} \rho^-) / \mathcal{B}(B^+ \to K^{*+} \pi^0)\) and \(\mathcal{B}(B^+ \to K^{*+} \pi^0) / \mathcal{B}(B^+ \to K^{+} \pi^0)\) agree with the experimental results. It can be also noticed that \(\mathcal{B}(B^+ \to K^{*0} \pi^+) / \mathcal{B}(B^+ \to K^{*0} \pi^+)\) and \(\mathcal{B}(B^0 \to K^{*+} \pi^-) / \mathcal{B}(B^0 \to K^{*+} \pi^-)\) in the present approximation are completely independent of form factors.

We notice a discrepancy between theoretical expectations based on factorization and experimental results for the ratio \(\mathcal{B}(B^+ \to K^{*0} \rho^+) / \mathcal{B}(B^+ \to K^{*0} \pi^+)\). The data come from the experimental value \(\mathcal{B}(B^+ \to K^{0} \pi^+) = (23.1 \pm 1.0) \times 10^{-6}\) [16] and the new finding from BABAR [17]
\[
\mathcal{B}(B^+ \to K^{0} \rho^+) = (8.0^{+1.4}_{-1.3} \pm 0.5) \times 10^{-6}.
\]
TABLE II: Ratios of Branching Ratios and their comparison with experiment.

| Ratios                     | Th.     | Exp. [16],[17] | Ratios                     | Th.     | Exp. [16] |
|---------------------------|---------|---------------|---------------------------|---------|-----------|
| $B(B^0 \rightarrow K^0 \rho^-)$ | input   | 0.54 ± 0.11   | $B(B^0 \rightarrow K^+ \pi^-)$ | 0.55    | 0.69 ± 0.13 |
| $B(B^0 \rightarrow K^0 \rho^0)$ | input   | 0.44 ± 0.17   | $B(B^0 \rightarrow K^+ \pi^0)$ | input   | 0.14 ± 0.08 |
| $B(B^+ \rightarrow K^+ \rho^0)$ | 0.42    | 0.42 ± 0.10   | $B(B^+ \rightarrow K^+ \pi^0)$ | 0.71    | 0.57 ± 0.22 |
| $B(B^+ \rightarrow K^0 \rho^0)$ | 0.018   | 0.35 ± 0.07   | $B(B^+ \rightarrow K^0 \pi^0)$ | 0.46    | 0.40 ± 0.07 |

It seems to show a significant effect beyond naive factorization, e.g. one due to final state interactions, see the discussion below.

In order to apply this method to the decays with an axial-vector meson in the final state we need information on the corresponding form factors, whose definition is in the Appendix. In I we assumed that the effect of substituting $K^*$ with $K_1$ is identical in the radiative and in the non-leptonic decay, in other words that each form factor for the $B \rightarrow K_1$ transition is given by the corresponding form factor for $B \rightarrow K^*$ multiplied by the same factor $y$, once the change of parity between the two strange mesons and the kinematical factors are taken into account. For our purposes only the form factor $V_0$ (see the Appendix) for the transition $B \rightarrow$ axial-vector meson is relevant. Using the above-mentioned assumption we get

$$V_0^{B \rightarrow K_{1(1270)}(q^2)} = h \ A_0^{B \rightarrow K^*(q^2)} , \quad V_0^{B \rightarrow K_{1(1400)}(q^2)} = k \ A_0^{B \rightarrow K^*(q^2)}$$

with

$$
\begin{pmatrix}
  h \\
  k 
\end{pmatrix} = \frac{m_{K^*} - m_B + m_{K_1} - (m_B - m_{K_1})z}{m_{K_1} - m_B + m_{K^*} - (m_B - m_{K^*})z} \begin{pmatrix} y \\ y' \end{pmatrix},
$$

where $y$ and $y'$ are defined in the Appendix, while the factor $z$ is defined as

$$z = \frac{A_2^{B \rightarrow \rho}(0)}{A_1^{B \rightarrow \rho}(0)} \approx \frac{A_2^{B \rightarrow K^*}(0)}{A_1^{B \rightarrow K^*}(0)}.$$  \hspace{1cm} (16)

We take the value $z = 0.93$, intermediate between the value predicted by light cone sum rules [14] ($z = 0.9$) and that given by the BSW model [11] ($z = 0.95$). In the following we will need also of the ratio $\frac{V_0^{B \rightarrow A_1}}{A_0^{B \rightarrow \rho}}$ (with $A_1 = a_1$ or $b_1$); we can predict it from the previous result:

$$\begin{align*}
\frac{V_0^{B \rightarrow a_1}(0)}{A_0^{B \rightarrow \rho}(0)} & \approx \frac{V_0^{B \rightarrow K_{1A}}(0)}{A_0^{B \rightarrow K^*}(0)} = h \sin \theta + k \cos \theta , \\
\frac{V_0^{B \rightarrow b_1}(0)}{A_0^{B \rightarrow \rho}(0)} & \approx \frac{V_0^{B \rightarrow K_{1B}}(0)}{A_0^{B \rightarrow K^*}(0)} = h \cos \theta - k \sin \theta .
\end{align*}$$

In previous equations we assume that the ratios satisfy $SU(3)$ flavor symmetry to a good approximation since $SU(3)$ breaking terms tend to cancel out in the ratio, see e.g. [12]. In equations (17) and (18) $\theta$ is the mixing angle between the octets $^3P_1$ and $^1P_1$ from which the states $K_{1(1270)}$ and $K_{1(1400)}$ result. To the
former octet belong $a_1$ and the unmixed strange state $K_{1A}$; to the latter $b_1$ and $K_{1B}$, see the Appendix A for further details. The mixing scheme we adopt here is analogous to that based on the conventional quark model of Ref. [18]: $\theta$ is the mixing angle between two strange P wave axial meson; therefore, differently from, e.g. $\eta-\eta'$ mixing, it should not be affected by gluonium contributions. The phenomenological analysis [3, 15, 18] gives as possible results $\theta = 32^\circ$ or $58^\circ$. In Table III we report our predictions for both values and a comparison with the result of Ref. [15].

| TABLE III: Ratio of form factors for $B$ decays to axial-vector mesons. |
|-----------------------------|-----------------------------|-----------------------------|
|                             | $V_0^{B\rightarrow K_{1A}^0}(0)$ | $V_0^{B\rightarrow K_{1B}^0}(0)$ |
| This work ($\theta = 32^\circ$) | 0.64 | 0.78 |
| This work ($\theta = 58^\circ$) | 1.02 | 0.26 |
| CLFA [15]                   | 0.45 | 1.32 |

III. $B \rightarrow K_1\pi$

These channels were already considered in I and we report them for completeness. If $q_{K_1}$ and $q_{K^*}$ are respectively the c.m. momenta of $K_1$ and $K^*$ in the reactions $B \rightarrow K_1\pi$ and $B \rightarrow K^*\pi$, one gets, using factorization:

$$\frac{B(B^+ \rightarrow K_1^{0}\pi^+)}{B(B^+ \rightarrow K^*\pi^+)} = \frac{B(B^0 \rightarrow K_1^{0}\pi^-)}{B(B^0 \rightarrow K^*\pi^-)} = \left( \frac{q_{K_1}}{q_{K^*}} \right)^3 \left( \frac{F_{1B \rightarrow \pi}(m_{K_1}^2)}{F_{1B \rightarrow \pi}(m_{K^*}^2)} \right)$$

where the subscript means that we consider only factorizable contributions. Therefore, using $f_{K_1}$ from $\tau$ decays (see the Appendix) one can predict $B(B^+ \rightarrow K_1^{0}\pi^+)$ and $B(B^0 \rightarrow K_1^{0}\pi^-)$ for both $K_1(1270)$ and $K_1(1400)$ from the knowledge of $B(B^+ \rightarrow K^*0\pi^+)$ and $B(B^0 \rightarrow K^*\pi^-)$ [19].

The reactions with a $\pi^0$ in the final state: $B^+ \rightarrow K_1^{0}\pi^0$, $B^0 \rightarrow K_1^{0}\pi^0$ involve three form factors $F_1$, $A_0$ and $V_0$ and different combinations of Wilson coefficients and CKM matrix elements. One gets ($s = h, k$, see Eq. (15), for $K_1(1270)$ and $K_1(1400)$ respectively):

$$\frac{B(B^+ \rightarrow K_1^{0}\pi^0)}{B(B^+ \rightarrow K^*\pi^0)} = \left( \frac{q_{K_1}}{q_{K^*}} \right)^3 \left[ \frac{W_0 f_{K_1}}{W_3 f_{\pi}} \frac{F_{1B \rightarrow \pi}(m_{K_1}^2)}{A_{0B \rightarrow \pi}(m_{K}^2)} \right]^2 + s$$

$$\frac{B(B^0 \rightarrow K_1^{0}\pi^0)}{B(B^0 \rightarrow K^*\pi^0)} = \left( \frac{q_{K_1}}{q_{K^*}} \right)^3 \left[ \frac{W_0 f_{K_1}}{W_3 f_{\pi}} \frac{F_{1B \rightarrow \pi}(m_{K_1}^2)}{A_{0B \rightarrow \pi}(m_{K}^2)} \right]^2 + s$$

where $W_i$ are listed in Appendix. The result of this analysis is in Table IV. For the form factors ratios, that we have considered at $q^2 = 0$, we have used the values in Table I.
TABLE IV: Theoretical branching ratios for $B$ decays into a strange axial-vector meson and a pion. Units $10^{-6}$.

| Process                    | $\mathcal{B}$ (Th.) | $\mathcal{B}$ (Exp.) [19] |
|----------------------------|----------------------|-----------------------------|
| $B^+ \rightarrow \pi^+ K^0_1(1270)$ | 5.8                  | ==                          |
| $B^+ \rightarrow \pi^0 K^+_{1}(1270)$ | 4.9                  | ==                          |
| $B^0 \rightarrow \pi^0 K^0_1(1270)$ | 0.4                  | ==                          |
| $B^0 \rightarrow \pi^- K^+_{1}(1270)$ | 7.6                  | ==                          |
| $B^+ \rightarrow \pi^+ K^0_1(1400)$ | 3.0                  | $< 260$                     |
| $B^+ \rightarrow \pi^0 K^+_{1}(1400)$ | 1.0 ($\theta = 32^0$) | ==                          |
| $B^+ \rightarrow \pi^0 K^+_{1}(1400)$ | 1.4 ($\theta = 58^0$) | ==                          |
| $B^0 \rightarrow \pi^0 K^0_1(1400)$ | 3.0 ($\theta = 32^0$) | ==                          |
| $B^0 \rightarrow \pi^- K^+_{1}(1400)$ | 1.7 ($\theta = 58^0$) | ==                          |

IV. $B \rightarrow A_1 K$

In this section we consider the decays $B \rightarrow a_1 K$, $b_1 K$. Also in this case we have some clear predictions based on factorization for the decays with a charged axial-vector meson in the final state $[3]$:

$$\frac{B(B^+ \rightarrow A_1^+ K^0)_{\text{fact.}}}{B(B^+ \rightarrow \rho^+ K^0)_{\text{fact.}}} = \left( \frac{q_{A_1}}{q_\rho} \right)^3 \left( \frac{v_{A_1 \rightarrow \rho} (m_{K^0})}{v_{A_1 \rightarrow K^0} (m_{K^0})} \right)^2 \left( \frac{f_{A_1}}{f_K} \right)^2 \left( \frac{f_{1A_1}}{f_{1K}} \right)^2 \left( \frac{f_{A_1}^{B \rightarrow K} (m_{A_1})}{f_{K}^{B \rightarrow K} (m_{A_1})} \right)^2,$$

(22)

where $\frac{v_{A_1 \rightarrow \rho} (m_{K^0})}{v_{A_1 \rightarrow K^0} (m_{K^0})}$ is given by Eq.(17) or Eq. (18) for $A_1 = a_1$ or $b_1$ respectively.

Similar predictions can be given also for the channels with a neutral axial-vector meson in the final state, ie the decay channels $B^+ \rightarrow a_1^0 K^+$, $B^+ \rightarrow b_1^0 K^+$, $B^0 \rightarrow a_1^0 K^0$ and $B^0 \rightarrow b_1^0 K^0$, though the corresponding formulae are more involved. In fact we have

$$\frac{B(B^+ \rightarrow A_1^0 K^+)}{B(B^+ \rightarrow \rho^0 K^+)} = \left( \frac{q_{A_1}}{q_\rho} \right)^3 \left( \frac{v_{A_1 \rightarrow \rho} (m_{K^+})}{v_{A_1 \rightarrow K^+} (m_{K^+})} \right)^2 \left( \frac{f_{A_1}}{f_K} \right)^2 \left( \frac{f_{1A_1}}{f_{1K}} \right)^2 \left( \frac{f_{A_1}^{B \rightarrow K} (m_{A_1})}{f_{K}^{B \rightarrow K} (m_{A_1})} \right)^2,$$

(24)

and

$$\frac{B(B^0 \rightarrow A_1^0 K^0)}{B(B^0 \rightarrow \rho^0 K^0)} = \left( \frac{q_{A_1}}{q_\rho} \right)^3 \left( \frac{v_{A_1 \rightarrow \rho} (m_{K^0})}{v_{A_1 \rightarrow K^0} (m_{K^0})} \right)^2 \left( \frac{f_{A_1}}{f_K} \right)^2 \left( \frac{f_{1A_1}}{f_{1K}} \right)^2 \left( \frac{f_{A_1}^{B \rightarrow K} (m_{A_1})}{f_{K}^{B \rightarrow K} (m_{A_1})} \right)^2,$$

(25)
the ratio $\frac{B(B \to K \pi)}{A_{\pi^+}}$ can be computed at $q^2 = 0$, as reported in Table I; the coefficients $W_i$ are in the Appendix A. The results obtained are reported in Table V. We have used the experimental BR for $B \to \rho^- K^+$ as given by the HFAG group [16]. For the channels $B^+ \to K^0 a_1^+$ and $B^+ \to K^0 b_1^+$ we have reported two values. The former is based on the result (13) (which supersedes an old upper limit [19]) for the BR of the channel $B^+ \to \rho^+ K^0$; the latter (given in parentheses) is based the experimental result [16] for the BR of the channel $B^+ \to \pi^+ K^0$, according to the discussion below.

TABLE V: Theoretical branching ratios for $B$ decays into one nonstrange axial-vector meson and a kaon for two different values of the mixing angle. Units $10^{-5}$. Predictions use $B \to K \rho$ decay rates for all entries except those given in parentheses, where the experimental BR for $B \to K^0 \pi^+$ is used.

| Process | $B(\theta = 32^\circ)$ | $B(\theta = 58^\circ)$ |
|---------|------------------------|------------------------|
| $B^+ \to K^+ a_1^0$ | 1.4                     | 2.8                     |
| $B^+ \to K^0 a_1^+$ | 30 (2.0)                | 75 (5.2)                |
| $B^0 \to K^0 a_1^0$ | 0.5                     | 2.5                     |
| $B^0 \to K^+ a_1^+$ | 1.6                     | 4.1                     |
| $B^+ \to K^+ b_1^0$ | 2.6                     | 0.07                    |
| $B^+ \to K^0 b_1^+$ | 4.4 (3.0)               | 5.0 (0.3)               |
| $B^0 \to K^0 b_1^0$ | 4.1                     | 0.4                     |
| $B^0 \to K^+ b_1^+$ | 2.4                     | 0.2                     |

In fact we are aware that in some cases the assumptions we make might be flawed. For example it is known that naive factorization gives a small contribution to the $B^0 \to \rho^- K^+$ channel. The experimental result $B(B^0 \to \rho^- K^+) = (9.9^{+1.6}_{-1.3}) \times 10^{-6}$ [16] is larger by one order of magnitude than theoretical predictions based on naive factorization and reasonable models for the form factors [12], which is due to large cancellations between the penguin contributions appearing in $W_i$. An enhancement with respect to naive factorization can be due to various reasons. For example one can mention $O(\alpha_s)$ corrections to the matrix elements. Moreover long-distance non-factorizable contributions, that are power suppressed, such as the charming penguin contributions [20, 21] are expected to play a role [22], as well as other power corrections terms in QCD factorization [23]. Finally including final state interactions requires both perturbative corrections at leading power, as well as power corrections. The phenomenology due these effects has been studied in detail in [9, 10]. Due to these uncertainties the results in Table V should be interpreted more as tests of the factorization model than as absolute predictions and are based on the expectations that, large as they can be, long distance effects, e.g. those described by final state interactions, cancel out in the ratios. In any case, to increase our confidence in the method, we use a different approach to get predictions for these channels, i.e we consider the ratio of $B(B \to A_1 K)$ to $B(B \to \pi K)$. In this case factorization predicts

$$\frac{B(B^+ \to K^0 A_1^+)}{B(B^+ \to K^0 \pi^+)} = \frac{B(B^0 \to K^+ A_1^-)}{B(B^0 \to K^+ \pi^-)} \approx \frac{4}{m_B^2} \left( \frac{V_{B \to A_1}^* (m_{K^+})}{F^0_B \pi (m_K)} \right)^2 q_{A_1}^2 q_\pi,$$  (26)
the parameters $W_j$ are in the Table of the Appendix A, while for $V_0^B \rightarrow A_1^0 (m_K^2)$ we use
\[
\frac{V_0^B \rightarrow A_1^0 (m_K^2)}{F_0^{B \rightarrow \pi}} \approx \frac{V_0^B \rightarrow A_1^0 (0)}{F_0^{B \rightarrow \pi}} \cdot \frac{A_0^{B \rightarrow \rho} (0)}{A_0^{B \rightarrow \pi}}.
\] (29)

We can now compute again the entries of Table V using these formulae and the experimental $BR$s for $B \rightarrow K \pi$. The interesting fact is that in general we obtain results that differ few percent at most from those found using the ratios to the decay channels $B \rightarrow K \rho$. The only exception is given by the channels $B^+ \rightarrow K^0 a_1^+$ and $B^+ \rightarrow K^0 b_1^+$; for them we present both predictions.

Some interesting predictions can be read from Table V. For $\theta = 32^\circ$, for all the decay channels, with the exception of $a_1^0 K^0$, we predict $BR$s of the order of $10^{-5}$. This holds also for the channel $K^0 a_1^+$, as the prediction based on $K^0 \pi^+$ is more reliable. For $\theta = 58^\circ$ we have $BR$s of similar sizes only for $B \rightarrow a_1 K$.

Summing up one can say that nonleptonic $B$ decays with a kaon and a light non-strange axial-vector meson in the final state represent interesting decay channels with generally large $BR$s.

V. $B \rightarrow A_1 \pi$

In this section we consider the decays $B \rightarrow a_1 \pi$, $b_1 \pi$. To start with, we consider the channel with at least one neutral particle in final state. We get the following results for the ratios $\frac{B(B \rightarrow A_1 \pi)}{B(B \rightarrow \rho \pi)}$:

\[
\frac{B(B^+ \rightarrow A_1^0 \pi^+)}{B(B^+ \rightarrow \rho^0 \pi^+)} = (q_{A_1} / q_{\rho})^3 \left( \frac{V_0^B \rightarrow A_1^0 (m_K^2)}{A_0^{B \rightarrow \rho} (m_\pi^2)} \right) \left( \frac{A_0^{B \rightarrow \rho} (m_\pi^2)}{w_5 + w_0 \frac{f_R}{f_\pi} A_0^{B \rightarrow \pi} (m_\pi^2)} \right)^2,
\] (30)

\[
\frac{B(B^+ \rightarrow A_1^+ \pi^0)}{B(B^+ \rightarrow \rho^+ \pi^0)} = (q_{A_1} / q_{\rho})^3 \left( \frac{V_0^B \rightarrow A_1^+ (m_K^2)}{A_0^{B \rightarrow \rho} (m_\pi^2)} \right) \left( \frac{A_0^{B \rightarrow \rho} (m_\pi^2)}{w_5 + w_0 \frac{f_R}{f_\pi} A_0^{B \rightarrow \pi} (m_\pi^2)} \right)^2,
\] (31)

\[
\frac{B(B^0 \rightarrow A_1^0 \pi^0)}{B(B^0 \rightarrow \rho^0 \pi^0)} = (q_{A_1} / q_{\rho})^3 \left( \frac{V_0^B \rightarrow A_1^0 (m_K^2)}{A_0^{B \rightarrow \rho} (m_\pi^2)} \right) \left( \frac{A_0^{B \rightarrow \rho} (m_\pi^2)}{w_5 + w_0 \frac{f_R}{f_\pi} A_0^{B \rightarrow \pi} (m_\pi^2)} \right)^2,
\] (32)
TABLE VI: Theoretical branching ratios for $B$ decays into one nonstrange axial-vector meson and a pion for two different values of the mixing angle. Units are $10^{-6}$.

| Process                  | $\mathcal{B}(\theta = 32^\circ)$ | $\mathcal{B}(\theta = 58^\circ)$ | Exp.     |
|--------------------------|----------------------------------|----------------------------------|----------|
| $B^+ \to \pi^+ a_1^0$   | 3.9                              | 8.8                              | < 900 [19]|
| $B^+ \to \pi^0 a_1^+$   | 4.8                              | 10.6                             | < 1700 [19]|
| $B^0 \to \pi^0 a_0^0$   | 1.1                              | 1.7                              | < 1100 [19]|
| $B^0 \to \pi^+ a_1^-$   | 4.7                              | 11.8                             | 40.9 ± 7.6 [1, 2]|
| $B^0 \to \pi^- a_1^+$   | 11.1                             | 12.3                             |          |
| $B^+ \to \pi^+ b_1^0$   | 4.5                              | 0.4                              | ==       |
| $B^+ \to \pi^0 b_1^+$   | 4.8                              | 0.5                              | ==       |
| $B^0 \to \pi^0 b_1^0$   | 0.5                              | 0.01                             | ==       |
| $B^0 \to \pi^+ b_1^-$   | 6.9                              | 0.7                              | ==       |
| $B^0 \to \pi^- b_1^+$   | ≈ 0                              | ≈ 0                              | ==       |

We use the ratios in Table I and the experimental BRs for $B \to \rho \pi$ as given by the HFAG group [16]. The results are reported in Table VI.

Let us now consider the channels having only charged mesons in the final state. In order to use the same method employed in the previous sections we would need the BRs from the decays $B^0 \to \rho^+ \pi^-$ and $B^0 \to \rho^- \pi^+$. Only their sum: $\mathcal{B}(B^0 \to \rho^+ \pi^-) = \mathcal{B}(B^0 \to \rho^+ \pi^-) + \mathcal{B}(B^0 \to \rho^- \pi^+)$ is at the moment known $\mathcal{B}(B^0 \to \rho^+ \pi^+) = (24.0 ± 2.5) \times 10^{-6}$ [16], therefore we consider the following ratios ($A_1 = a_1, b_1$):

$$
\frac{\mathcal{B}(B^0 \to \pi^+ A_1^0)}{\mathcal{B}(B^0 \to \pi^+ \rho^+)_{\text{fact.}}} = \left( \frac{q_{A_1}}{q_\rho} \right)^3 \frac{w_1 V_6^{B \to A_1^0}(m_\pi^2)^2}{1 + \left( \frac{w_3 f_0 F_1^{B \to \pi}(m_\rho^2)^2}{w_2 f_\rho A_0^{B \to \rho}(m_\rho^2)^2} \right)^2},
$$

and

$$
\frac{\mathcal{B}(B^0 \to \pi^- A_1^+)}{\mathcal{B}(B^0 \to \pi^+ \rho^+)_{\text{fact.}}} = \left( \frac{q_{A_1}}{q_\rho} \right)^3 \frac{w_4 f_A F_1^{B \to \pi}(m_\rho^2)^2}{1 + \left( \frac{w_3 f_0 F_1^{B \to \pi}(m_\rho^2)^2}{w_2 f_\rho A_0^{B \to \rho}(m_\rho^2)^2} \right)^2},
$$

where the parameters $w_k$ are defined in the Appendix A. In this way we can complete the inputs of Table VI. There is an independent analysis, given by Höcker et al. [24], which extracts from $B^0 \to \rho^+ \pi^+$ the values of the single channels with the result $\mathcal{B}(B^0 \to \rho^+ \pi^-) = (15.3^{+3.7}_{-3.3}) \times 10^{-6}$ and $\mathcal{B}(B^0 \to \rho^- \pi^+) = (14.5^{+3.8}_{-3.4}) \times 10^{-6}$. Using these values one would estimate the BRs of $B^0 \to A_1^0 \pi^-$ and $B^0 \to A_1^- \pi^+$ with results $\sim 20\%$ greater than those in Table VI, i.e. within our estimated theoretical error. A greater confidence can be obtained using a slightly different approach. One might note that the ratio $\frac{\mathcal{B}(B^0 \to A_1^- \pi^+)}{\mathcal{B}(B^0 \to \rho^- \pi^+)}$ is independent of the Wilson coefficient and CKM matrix elements. Using as an experimental input $\mathcal{B}(B^0 \to \rho^- \pi^+)$
π⁻ π⁺ = (4.5 ± 0.4) × 10⁻⁶ [16], one gets values for \( B(B^0 \rightarrow A^- \pi^+) \) in agreement with Table VI within \( \sim 10\% \). We note that the prediction for \( B^0 \rightarrow \pi^+ a_1^+ \) is somewhat smaller than the result from the Belle[1] and BABAR Collaborations [2], although the value 58° for the mixing angle offers a better agreement. This is an indication that this value of the angle is to be preferred. If subsequent analyses would lead to prefer the solution \( \theta = 32° \), this would mean either a failure of some of our assumptions or that there are non-resonant effects, not included in the theoretical analysis, and implicitly taken into account in the data. This might happen because, for non-resonant diagrams, some particles in the final state might fall in the same kinematical range as the \( a_1 \) state, with an effect similar to what discussed for \( B \rightarrow 3\pi \) in Ref. [25].

VI. CONCLUSIONS

In conclusion we have presented predictions for the nonleptonic \( B \)–meson decay channel with one axial-vector meson in the final state. We have used uniquely experimental data, e.g. the decay rates for \( B \rightarrow K^+ \pi, K \pi, \rho \pi, \rho K \) and, as a theoretical input, the assumption of naive factorization. Our results may provide a useful benchmark for the future searches of the decay channels \( B \rightarrow K_1 \pi, B \rightarrow a_1 K, B \rightarrow b_1 K, B \rightarrow a_1 \pi, B \rightarrow b_1 \pi \) that might be investigated by the experimental collaborations.

APPENDIX A

In this appendix we list the values of the Wilson coefficients and the CKM matrix elements we have used in the main text.

Wilson coefficients (using the results of [26] for \( \Lambda_{MS}^{(5)} = 225 \) MeV in the HV scheme) and current quark masses:

\[
\{a_1, a_2\} = \{1.029, 0.140\}, \quad \{a_3, \cdots a_{10}\} = \{33, -246, -10, -300, 2, 4.8, -93, -12\} \times 10^{-4},
\]

\[
\{m_u, m_d, m_s, m_b\} = \{4, 8, 150, 4600\} \text{MeV}.
\]

CKM matrix elements:

\[
V_{ud} = 0.97, \quad V_{us} = 0.22, \quad V_{ub} = 0.0018 - 0.0032i, \quad V_{td} = 0.0074 - 0.0031i, \quad V_{ts} = -0.04 - 0.00072i, \quad V_{tb} \simeq 1.
\]

In the text we use some combinations of Wilson coefficients and CKM matrix elements as reported in Table VII. One may note the correct treatment of isospin invariance [27] in these results.

We have used the following definitions for the form factors. If \( |V\rangle \) is a vector meson state (\( \rho, K^* \)) and \( |A\rangle \) an axial-vector meson state (i.e. one of the states \( K_{1A}, K_{1B}, a_1, b_1 \)) we use

\[
\langle V(\epsilon, p')|V^\mu - A^\mu|B(p)\rangle = -i(m_B + m_V)\epsilon^{\mu\nu}A_1(q^2) + i(\epsilon^* \cdot q)(p + p')^{\mu}A_2(q^2)
\]

\[
+ i(\epsilon^* \cdot q)\frac{2m_V}{q^2}q^\mu [A_3(q^2) - A_0(q^2)] + \frac{2V(q^2)}{m_B + m_V}q^{\mu\nu\alpha\beta}\epsilon^*_{\nu p}\epsilon_{p' \beta}
\]

\[\langle A(\epsilon, p')|V^\mu - A^\mu|B(p)\rangle = i(m_B + m_A)\epsilon^{\mu\nu}V_1(q^2) - i(\epsilon^* \cdot q)(p + p')^{\mu}V_2(q^2)
\]

\[
- i(\epsilon^* \cdot q)\frac{2m_A}{q^2}q^\mu [V_3(q^2) - V_0(q^2)] + \frac{2A(q^2)}{m_B + m_A}q^{\mu\nu\alpha\beta}\epsilon^*_{\nu p}\epsilon_{p' \beta}.
\]
In these equations
\[ A_3(q^2) = \frac{m_V - m_B}{2m_V} A_2(q^2) + \frac{m_V + m_B}{2m_V} A_1(q^2), \quad V_3(q^2) = \frac{m_A - m_B}{2m_A} V_2(q^2) + \frac{m_A + m_B}{2m_A} V_1(q^2) \] (5)
with \( V_3(0) = V_0(0) \) and \( A_3(0) = A_0(0) \).

If \( P, P' \) are pseudoscalar mesons, we have used
\[ \langle P'(p')|V_\mu|P(p)\rangle = F_1(q^2) \left( p_\mu + p'_\mu - \frac{m_\mu^2 - m_{P'}^2}{q^2} q_\mu \right) + F_0(q^2) \frac{m_\mu^2 - m_{P'}^2}{q^2} q_\mu. \] (6)

We do not make assumptions on the \( q^2 \) behavior of the \( F_1^{B \rightarrow \pi} \) form factor as we only need its value at \( q^2 = 0 \).

Finally we have used the following definitions for the leptonic decay constants
\[ \langle 0|A_\mu|P(p)\rangle = i f_P p_\mu, \quad \langle V(\varepsilon, p)|V_\mu|0\rangle = f_V m_V \varepsilon_\mu^*, \quad \langle A(\varepsilon, p)|A_\mu|0\rangle = f_A m_A \varepsilon_\mu^*, \] (7)
with the following numerical values \( (f_{\pi^+}, f_K, f_{\rho^+}, f_{K^*}) = (132, 161, 210, 210) \) MeV, and, from \( \tau \) decays, \( (f_{K_1(1270)}, f_{K_1(1400)}) = (171, 126) \) MeV [3].

For the determination of the analogous constants for the \( a_1 \) and \( b_1 \) nonstrange axial-vector mesons one has to take into account that the strange axial-vector mesons \( K_1(1270) \) and \( K_1(1400) \) are the result of the mixing of \(^3P_1\) and \(^1P_1\) states. Denoting by \( K_{1A} \) and \( K_{1B} \) the \(^3P_1\) and \(^1P_1\) states of \( K_1 \) one has
\[ K_1(1270) = K_{1A} \sin \theta + K_{1B} \cos \theta, \quad K_1(1400) = K_{1A} \cos \theta - K_{1B} \sin \theta. \] (8)

\( K_{1B} \) belongs to the same nonet as the states \( b_1(1235), h_1(1170) \) and \( h_1(1380); K_{1A}, a_1(1260), f_1(1285) \) and \( f_1(1400) \) are also in one nonet. The mixing angle \( \theta \) and the masses of the \( K_1 \) states have been determined

| Coefficient | Formula |
|-------------|---------|
| \( W_1 \) | \( V_{ib}^*V_{is} \left( a_4 - \frac{1}{2}a_{10} + \frac{2m_\pi^2}{m_{b1} - m_{a1}} (a_6 - \frac{1}{2}a_8) \right) \) |
| \( W_2 \) | \( V_{ic}^*V_{ia} \left( a_4 + a_{10} + \frac{2m_\pi^2}{m_{b1} - m_{a2}} (a_6 + a_8) \right) \) |
| \( W_3 \) | \( V_{ib}^*V_{is} \left( a_4 + a_{10} + \frac{2m_\pi^2}{m_{b1} - m_{a2}} (a_6 + a_8) \right) \) |
| \( W_4 \) | \( V_{ib}^*V_{is} \left( a_4 + \frac{1}{2}a_{10} \right) \) |
| \( W_5 \) | \( V_{ib}^*V_{ia} - V_{ib}^*V_{is} \left( a_4 + a_{10} \right) \) |
| \( W_6 \) | \( V_{ib}^*V_{ia} - V_{ib}^*V_{is} \left( a_4 + a_{10} - \frac{2m_\pi^2}{m_{b1} - m_{a2}} (a_6 + a_8) \right) \) |
| \( W_7 \) | \( V_{ib}^*V_{is} \left( a_4 - \frac{1}{2}a_{10} \right) \) |
| \( W_8 \) | \( V_{ib}^*V_{is} \left( a_4 - \frac{1}{2}a_{10} - \frac{2m_\pi^2}{m_{b1} - m_{a2}} (a_6 - \frac{1}{2}a_8) \right) \) |
in [3] (but see also [18], [15]) up to a twofold ambiguity

\[ \text{Sol.}[a]: \ \theta = 32^\circ, \ (m_{K_{1B}}, m_{K_{1A}}) = (1310, 1367) \text{ MeV}, \]

\[ \text{Sol.}[b]: \ \theta = 58^\circ, \ (m_{K_{1B}}, m_{K_{1A}}) = (1367, 1310) \text{ MeV}. \]  

(9)

Using this result and SU(3) symmetry one gets [3]:

\[ \text{Sol.}[a] (\theta = 32^\circ) : (f_{b1}, f_{a1}) = (74, 215) \text{ MeV}, \]

\[ \text{Sol.}[b] (\theta = 58^\circ) : (f_{b1}, f_{a1}) = (-28, 223) \text{ MeV}. \]  

(10)

The two values for \( f_{b1} \) given above refer to the strange partner in the octet and come from SU(3) violations [18], the non strange partners having zero coupling.

We have also used the matrix element describing radiative transitions:

\[ \langle K_1(p', \epsilon)|\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)q'\gamma_b|B(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}p'^{\rho}p^{\sigma}q_2T_1(q^2) + [\epsilon^{*\mu}(m_B^2 - m_{K_1}^2) - (\epsilon^{*} \cdot q)(p + p')_\mu]T_2(q^2) + [(\epsilon^{*} \cdot q)q_\mu - q_2^2(m_B^2 - m_{K_1}^2)(p + p')_\mu]T_3(q^2), \]  

(11)

with \( T_1(0) = T_2(0) \) (\( T_3 \) does not contribute to the radiative decay). For \( B \to K^* \) an analogous formula can be written. From experiment one has [3],

\[ y = \frac{T_{B \to K_1(1270)}^{1}(0)}{T_{B \to K^*}^{1}(0)} \approx 1.06, \quad y' = \frac{T_{B \to K_1(1400)}^{1}(0)}{T_{B \to K^*}^{1}(0)} \approx \begin{cases} 0.14 & (\theta = 32^\circ) \\ 0.35 & (\theta = 58^\circ) \end{cases}. \]  

(12)

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