Thermal conductivity and competing orders in d-wave superconductors

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The existence of four nodal points in d-wave superconductors provides rich and, sometimes, controllable dynamics of quasiparticle excitations at zero temperature. In particular, the expressions for electrical, thermal, and spin conductivity simplify considerably in the universal limit \( \omega \to 0, T \to 0 \) \cite{2}. It is noticeable that the role of the thermal conductivity \( \kappa \) is special: while vertex and/or Fermi-liquid corrections modify the bare, "universal", values of both electric and spin conductivities, the universal value of the thermal conductivity is not influenced by them \cite{2}. It is:

\[
\kappa_0(T) = \frac{k_B^2 v_F^2 + v_{\Delta}^2}{3 v_F v_{\Delta}} \frac{\pi^2}{T},
\]

where \( v_F \) is a Fermi velocity, \( v_{\Delta} \) is a gap velocity, and \( k_B \) is the Boltzmann constant (we use units with \( \hbar = c = 1 \)). The basis for such a remarkably simple expression is that there is a finite density of states \( N(0) \) of gapless quasiparticles down to zero energy \cite{2}.

\[
N(0) = \frac{2}{\pi^2 v_F v_{\Delta}} \frac{\Gamma_0}{\Gamma_0} \ln \frac{p_0}{\Gamma_0},
\]

where \( \Gamma_0 \equiv \Gamma(\omega \to 0) \), with \( \Gamma(\omega) \) an impurity scattering rate, and \( p_0 = \sqrt{\pi v_F v_{\Delta}}/a \) is an ultraviolet momentum cutoff (\( a \) is a lattice constant) \cite{2}. Note that expression \( 1 \) itself is valid in the so-called "dirty" limit, \( T \ll \Gamma_0 \). Therefore, although this expression does not contain \( \Gamma_0 \)

\[
\kappa_0^{(m)}(T) = \frac{k_B^2 v_F^2 + v_{\Delta}^2}{3 v_F v_{\Delta}} \frac{\Gamma_0^2}{\Gamma_0^2 + m^2}
\]

and

\[
N_m(0) = \frac{2}{\pi^2 v_F v_{\Delta}} \frac{\Gamma_0}{\Gamma_0} \ln \frac{p_0}{\sqrt{\Gamma_0^2 + m^2}}
\]

where \( m \) is a quasiparticle gap. The noticeable point is that, for all values of the gap up to \( m \approx \Gamma_0 \), the suppression of both thermal conductivity and quasiparticle density is mild: \( \kappa_0^{(m)}/\kappa_0^{(0)} \) and \( N(0)/N_m(0) \) are of order one. However, the suppression in thermal conductivity rapidly becomes strong as \( m \) crosses this threshold. The second noticeable point is that, as we will discuss below, the gap \( m \) plays a universal role and may represent different competing orders in d-wave superconductors, such as as spin density wave, charge density wave, is-pairing, etc. Although their dynamics are different, expressions \( 3 \) and \( 4 \) for \( \kappa_0^{(m)} \) and \( N_m \) are the same. This happens because, first, all those gaps \( m \) correspond to different types of "masses" in the Dirac equation describing nodal quasiparticle excitations, and, secondly, unlike electric and spin conductivities, the thermal conductivity \( \kappa_0^{(m)} \) is blind with respect to quantum numbers distinguishing those masses.

The expression \( \kappa_0^{(m)} \) corresponds to the dirty limit when \( T \ll \Gamma_0 \). In d-wave superconductors, \( \Gamma_0 \) can be as...
large as of order 1 K or even 10 K, and $k_B^{(m)}$ can be an important measurable characteristic there. Recently, two experimental groups have observed an anomalous behavior in the thermal conductivity in underdoped La$_{2-x}$Sr$_x$CuO$_4$ (Refs. [4,5] and Refs. [6,7]). One of the most interesting observations of experiment [6] is that at very low temperatures the value of the thermal conductivity in underdoped La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) is less than the absolute minimum $k_{m_{\text{min}}}/T = 2k_B^2/3$ of expression (1) for $k_B/T$, corresponding to the isotropic case with $v_F = v_\Delta$. This puzzle can be naturally explained by utilizing the modified expression (3) with a nonzero $m$ describing a competing order in the superconducting phase. We will discuss this and other results of experiments [4,5,6,7] below.

At subkelvin temperatures relevant to the low- $T$ heat conduction experiments, we will use the continuum, low-energy, description for the nodal quasiparticles in the d-wave state. At each node, the quasiparticles are described by a two-component Nambu field. It will be convenient, following Ref. [5], to utilize four-component fields, by combining Nambu fields corresponding to the nodes within each of the two diagonal pairs. Thus we have two four-component Dirac fields. The corresponding representation for three Dirac matrices is

$$
\gamma_0 = \sigma_1 \otimes I, \quad \gamma_1 = -i\sigma_2 \otimes \sigma_3, \quad \gamma_2 = i\sigma_2 \otimes \sigma_1,
$$

(5)

where $\sigma_i$ are the Pauli matrices and while the first factor in the tensor product acts in the subspace of the nodes in a diagonal pair, the second factor acts on indices inside a Nambu field. The matrices satisfy the algebra \( \{ \gamma_\mu, \gamma_\nu \} = 2g_{\mu\nu}, \quad g_{\mu\nu} = (1, -1, -1, 1), \quad \mu, \nu = 0, 1, 2 \).

We will consider quasiparticle gaps with the matrix structure $O_i = (I, i\gamma_5, \gamma_3, \gamma_1\gamma_3\gamma_5)$. Here the matrices $\gamma_3$ and $\gamma_5$, anticommuting with matrices $\gamma_\mu$, are

$$
\gamma_3 = i\sigma_2 \otimes \sigma_2, \quad \gamma_5 = \sigma_3 \otimes I.
$$

(6)

Then, for each of the two four-component Dirac fields, the bare Matsubara Green’s function can be written as

$$
G_0(i\omega_n, \mathbf{k}) = \frac{1}{i\omega_n\gamma_0 - v_Fk_1\gamma_1 - v_\Delta k_2\gamma_2 - m_iO_i}.
$$

(7)

Therefore, different gaps $m_i$ correspond to different types of Dirac masses. As was pointed out in Refs. [9,10,11], these gaps represent different competing orders in low energy limit. In particular, the mass $m_1$, with $O_1 = I$, describes the (incommensurate) cosin-density-wave (SDW), and the mass $m_2$, with $O_2 = i\gamma_5$, describes sin SDW. The masses $m_3$ and $m_4$, with $O_3 = \gamma_3$ and $O_4 = \gamma_3\gamma_5$, correspond to the id$_{xy}$-pairing and the is-pairing, respectively. One can also consider a gap corresponding to the charge-density-wave (CDW). In that case, one should introduce a Dirac mass term mixing the four-component Dirac fields corresponding to the two different diagonal pairs of the nodes. For simplicity, we will not consider it in this letter.

The scattering on impurities can be taken into account by introducing a Matsubara self-energy $\Sigma(i\omega_n)$, so that the dressed Green’s function becomes $G(i\omega_n, \mathbf{k}) = G_0(i\omega_n - \Sigma(i\omega_n), \mathbf{k})$. As usual, retarded Green’s function is obtained by analytically continuing Green’s function $G$, $G_R(\omega, \mathbf{k}) = G(i\omega_n \to \omega + i\epsilon, \mathbf{k})$, and the impurity scattering rate is defined as $\Gamma(\omega) = -i\text{Im}\Sigma(\omega)$. At low temperatures we take $\Gamma_0 \equiv \Gamma(\omega \to 0)$. The size of $\Gamma_0$ depends on the impurity density $n_{\text{imp}}$ as well as on the scattering phase shift $\delta$. Solving the Schwinger-Dyson equation for the self-energy in the self-consistent t-matrix approximation, one can find that in the unitary limit ($\delta = \pi/2$) the equation determining $\Gamma_0$ for a nonzero $m_i$ has the form [12]

$$
\Gamma_0^2 = \pi^2 v_F v_\Delta \tilde{\Gamma} \left[ N_f \ln \frac{\nu_0}{\Gamma_0 + m_i^2} \right]^{-1},
$$

(8)

where $N_f$ is the number of four-component Dirac fields and $\tilde{\Gamma} = n_{\text{imp}}/\pi\rho_0$ with $\rho_0$ the normal state density of states. Since $v_\Delta \sim \Delta_0$, the magnitude of the superconducting gap, the scattering rate $\Gamma_0$ is proportional to $\sqrt{\Delta_0 \tilde{\Gamma}} \sim \sqrt{\Delta_0 n_{\text{imp}}}$.

The longitudinal dc thermal conductivity is calculated by means of the Kubo formula. In the bubble approximation, following the standard procedure, it can be expressed through the quasiparticle spectral function $A(\omega, \mathbf{k})$ as follows

$$
\kappa^{(m)} = \frac{\pi N_f}{8k_B T^2} \int_{-\infty}^{\infty} \frac{d\omega \omega^2}{\cosh^2 \frac{\omega}{2k_B T}} \int \frac{d^2 k}{(2\pi)^2} \{ v_F^2 \text{tr} \gamma_1 A(\omega, \mathbf{k}) \gamma_1 A(\omega, \mathbf{k}) + v_\Delta^2 \text{tr} \gamma_2 A(\omega, \mathbf{k}) \gamma_2 A(\omega, \mathbf{k}) \}.
$$

(9)

Here the spectral function is given by the discontinuity of the fermion Green’s function

$$
A(\omega, \mathbf{k}) = -\frac{1}{2\pi i} \left[ G_R(\omega + i\epsilon, \mathbf{k}) - G_R(\omega - i\epsilon, \mathbf{k}) \right].
$$

(10)

With Green’s function at hand, we can calculate $A(\omega, \mathbf{k})$. For example, for the gap proportional to the unit Dirac matrix, it has the form $(m \equiv m_1)$ [13]

$$
A(\omega, \mathbf{k}) = \frac{\Gamma_0}{2\pi E} \left[ \frac{\gamma_0 E - v_F k_1 \gamma_1 - v_\Delta k_2 \gamma_2 + m}{(\omega - E)^2 + \Gamma_0^2} \right. + \left. \frac{\gamma_0 E + v_F k_1 \gamma_1 + v_\Delta k_2 \gamma_2 - m}{(\omega + E)^2 + \Gamma_0^2} \right],
$$

(11)

where $E(\mathbf{k}) = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2 + m^2}$ is the quasiparticle energy. Substituting the last expression in Eq. (9) and taking the limit $T \to 0$, we arrive at

$$
\frac{\kappa_0^{(m)}}{T} = \frac{2\pi N_f k_B^2}{3} \int \frac{d^2 k}{(2\pi)^2} \frac{\Gamma_0^2}{(E^2 + \Gamma_0^2)^2} = \frac{k_B^2 N_f v_F^2 + v_\Delta^2}{6v_F v_\Delta} \frac{\Gamma_0^2}{\Gamma_0^2 + m_i^2},
$$

(12)

i.e., we derived expression (3) for the thermal conductivity (in which $N_f = 2$). The result for three other gaps, $m_2$, $m_3$, and $m_4$, introduced above, is the same.
With the spectral function \(I_{11}\), the density of states (per spin)

\[
N_m(\omega) = \frac{1}{2} \int \frac{d^2k}{(2\pi)^2} \text{tr} \left[ \gamma_0 A(\omega, k) \right]
\]

is easily calculated

\[
N_m(\omega) = \frac{N_f}{2\pi^2 v_F \Delta} \left[ \Gamma_0 \ln \frac{p_0}{\sqrt{\Gamma_0^2 + (\omega - m)^2}} + \Gamma_0 \ln \frac{p_0}{\sqrt{\Gamma_0^2 + (\omega + m)^2}} + |\omega| \left( \frac{\pi}{2} + \tan^{-1} \frac{\omega^2 - m^2 - \Gamma_0^2}{2|\omega|\Gamma_0} \right) \right].
\]

It yields expression (11) for the density of states with zero energy. Therefore, in the presence of impurities, the quasiparticle band survives even for a finite \(m\). The physical reason for this is the formation of impurity bound states inside the gap (14). Overlap between these states leads to impurity band supporting the quasiparticle heat (and electric) current.

The observation of a residual linear term in \(T\) in the thermal conductivity in cuprates (YBa\(_2\)Cu\(_3\)O\(_y\)) (YBCO) [13], Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_y\) (Bi-2212) [16] as well as LSCO [4] is usually interpreted as a direct consequence of nodes in the gap. However, as it follows from Eq. (12), a subdominant order parameter, leading to a gap for nodal quasiparticles, does not exclude such a linear term in the thermal conductivity, although the latter does not have a universal form anymore.

Thus we conclude that nonperturbative dynamics, responsible for the creation of competing orders in the supercritical phase, can violate the universality in the thermal conductivity in the low temperature limit \(T \to 0\). Recent experiments indicate that the existence of such competing orders is quite possible [17]. Several theoretical models have been proposed to describe this phenomenon (for a review, see Ref. [18]). As we will now discuss, using the expression for the thermal conductivity derived above, this phenomenon can be relevant for understanding recent experiments in La\(_{2-x}\)Sr\(_x\)CuO\(_4\) [4,5,6,7].

The measurements of the thermal conductivity in LSCO at low temperature [4,5,6,7] showed the following characteristic features:

a) At subkelvin temperatures, the value of \(\kappa/T\) decreases with \(x\) [4,8]. At temperature as low as 40 mK, the value of \(\kappa/T\) in some underdoped samples is either less than the absolute minimum \(\kappa_{\text{min}}/T = 2k_B/3\) of expression (11) (for \(x = 0.06\)) or quite close to it (for \(x = 0.07\) and \(x = 0.09\)) [6]. On the other hand, this anomalous behavior in the thermal conductivity disappears in overdoped samples \((x = 0.17\) and \(x = 0.20\)) [6].

b) The evolution of \(\kappa/T\) across optimum doping is smooth [4,5].

c) The thermal conductivity is sensitive to magnetic field. While in overdoped samples it increases with magnetic field, in underdoped samples the thermal conductivity decreases with increasing magnetic field [17]. The authors of Refs. [3,4] describe this as a field-induced thermal metal-to-insulator transition.

d) Although remaining smooth, the evolution of \(\kappa/T\) across optimum doping becomes visibly faster with increasing magnetic field [5].

The results of item a) can be easily understood if one assumes that there exists a competing order, described by the Dirac mass \(m\), in the superconducting phase of underdoped LSCO. Then an appropriate value of \(m\) in expression (9) will provide the necessary suppression of the thermal conductivity. The fact that such an anomalous behavior in \(\kappa/T\) disappears with increasing \(x\), in overdoped samples, can be understood if one assumes that the dynamical gap ("mass") \(m\) decreases with increasing \(x\). As to this assumption, it is well known in quantum field theory that, indeed, an increase of the fermion density often suppresses a dynamical Dirac mass. The reasons for that are simple. With increasing the fermion density, the screening effects become stronger and the quasiparticle interactions become weaker. In addition, at a sufficiently large quasiparticle density, the energy gain from creating a gap \(m\) in the quasiparticle spectrum will be surpassed by the energy loss of pushing up the energy of all states in the band above the gap. In the case of the model with Dirac fermions describing highly oriented pyrolytic graphite (HOPG) [19,20], this fact was explicitly shown in Ref. [20]. Although the present system is quite different from HOPG, that example supports plausibility of this assumption.

It is tempting to speculate that the dynamical gap \(m\) disappears close to optimum doping \((x_0 = 0.16\) in LSCO). A smooth evolution of \(\kappa/T\) across optimum doping then suggests that it could be a continuous phase transition with the scaling law of the form \(m \sim (x_c - x)^\nu\) in the scaling region with \(0 < (x_c - x)/x_c \ll 1\), where the critical value \(x_c \simeq x_0\). The critical index \(\nu = 1/2\) would correspond to the mean-field phase transition. In that case, there would be a kink in expression (5) at the critical point \(x = x_c\). Indeed, since the thermal conductivity (6) depends on \(m^2\), and there is a linear in \(m^2\) term as \(m^2 \to 0\), its derivative with respect to \(x\) will have a finite discontinuity at \(x = x_c\) for \(\nu = 1/2\). In the case of a non-mean-field continuum phase transition, with \(\nu > 1/2\), the evolution of \(\kappa/T\) across \(x_c \simeq x_0\) would be smoother.

This picture, with appropriate modifications, can survive in the presence of a magnetic field. In particular, the fact that in overdoped samples \(\kappa\) increases with magnetic
field as \( \sqrt{H} \) [17], implies that the dynamics in a magnetic field in overdoped samples is apparently conventional. Indeed, the \( \sqrt{H} \) behavior is well described by semiclassical models [21]. This seems to suggest that there is no gap \( m \) (competing order) in overdoped samples.

The situation is different in underdoped samples. The magnetic field enhances the suppression in \( \kappa \) observed in the same samples at zero field (item c) above). Moreover, the evolution \( \kappa/T \) across optimum doping becomes visibly faster with increasing \( H \) (item d)). This suggests that magnetic field plays here the role of a catalyst, enhancing the gap \( m \). For sufficiently large values of \( m \), the suppression in \( \kappa \) will be so large that a sample effectively becomes a thermal insulator as was observed in experiments [5,7].

Microscopic dynamics responsible for creating competing orders can be quite sophisticated [18]. This issue is outside the scope of this letter. Here we will only comment on the role of a magnetic field as a catalyst in generating the gap \( m \). In non-superconducting systems, it is well known that a magnetic field is indeed a strong catalyst in generating gaps (masses) for Dirac fermions [22]. In particular, that a magnetic field is indeed a strong catalyst in generating (or nearly mean-field) one, and b) the gap increases as \( \omega \rightarrow \omega - \nu_s(r)k \), [23] \( \nu_s(r) \) is the superfluid velocity at a position \( r \) which depends on the form of vortices distribution. In this case, the local thermal conductivity \( \kappa(r) \) has to be averaged over the unit cell of the vortex lattice [20],

\[
\kappa(H,T) = \frac{1}{A} \int d^2r \kappa(r) = \int d\epsilon \mathcal{P}(\epsilon) \kappa(\epsilon,T),
\]

where

\[
\mathcal{P}(\epsilon) = \frac{1}{A} \int d^2r \delta(\epsilon - \nu_s(r)k)
\]

is the vortex distribution, and \( A = \pi R^2 \) is the area of the vortex unit cell. We use the Gaussian distribution function \( \mathcal{P}(\epsilon) = (1/\sqrt{\pi E_F}) \exp[-\epsilon^2/E_F^2] \) which is believed to be the most suitable distribution in the presence of high disorder [30]. Thus we need to calculate

\[
\kappa(H,T) = \frac{\pi N_f}{8k_B T^2} \int_{-\infty}^{\infty} \frac{d\omega^2}{\cosh^2 \frac{\omega}{2k_BT}} \int_{-\infty}^{\infty} d\epsilon \mathcal{P}(\epsilon) \int \frac{d^2k}{(2\pi)^2} \times \left\{ v_F^2 \text{tr} \left[ \gamma_1 A(\omega - e, k) \gamma_1 A(\omega - e, k) \right] + v_F^2 \text{tr} \left[ \gamma_2 A(\omega - e, k) \gamma_2 A(\omega - e, k) \right] \right\}
\]

(compare with Eq. (9)).

Taking the limit \( T \rightarrow 0 \) in the last equation, we arrive at the following expression:

\[
\frac{\kappa(H,0)}{\kappa_0} = \frac{1}{2} \int_{-\infty}^{\infty} d\epsilon \mathcal{P}(\epsilon) \left[ \frac{1}{2} + \frac{\epsilon^2 - m^2 + I_0^2}{2|\epsilon| I_0} \right] \left( \frac{\pi}{2} - \tan^{-1} \frac{\epsilon^2 + m^2 - \epsilon^2}{2|\epsilon| I_0} \right),
\]

where \( \kappa_0 \) is the step function, \( E_F = hv_F/2R = (\hbar v_F/2)\sqrt{E_H/\hbar c} \) is a characteristic energy scale in the presence of a magnetic field in the vortex state (2R is the average distance between vortices), and \( m_0 \) and \( b \) are free parameters. Taking \( v_F = 2.5 \times 10^5 \text{cm/s} \) for LSCO cuprates [17,27], we find \( E_H = 38 K \cdot \sqrt{H(T)} \) where the field \( H(T) \) is taken in Teslas. The constant \( b \) is of order 1 (for numerical calculations we take \( b = 2.2 \)). As to the parameter \( m_0 \) that determines the gap for \( H = 0 \), it can be found from the ratio \( \kappa/\kappa_0 = 2/3 \) (i.e., \( \kappa/T \simeq 12 \mu\text{WK}^{-2}\text{cm}^{-1} \)) for \( x = 0.06, H = 0 \) and \( T \rightarrow 0 \), as reported in Ref. [6]. Then, taking \( \kappa/\kappa_0 = \kappa_H^0/\kappa_0 \), with \( \kappa_H^0 \) from equation (4), we get

\[
m_0 = a I_0^2
\]

where the constant \( a \simeq 0.9 \).
effect that is valid even for gapped quasiparticles \cite{31} when the vortex scattering is neglected.

Fig. 2 shows the dependence of $\kappa$ on the doping for two different values of the magnetic field. One can see the suppression of $\kappa$ in the underdoped regime as a result of the presence of the magnetic-field-induced gap. Note that both curves grow fast near the critical doping $x_c = 0.16$ where the gap disappears. It is also noticeable that this growth is much faster for the $H = 13 \ T$ curve than that for the $H = 1 \ T$ curve. These facts agree with the experimental data \cite{5} discussed in item d) above.

Although the present analysis is based on the particular ansatz \cite{14} for $m(H, x)$, one can expect that the main characteristics in the behavior of the thermal conductivity will retain qualitatively the same for a wide class of gaps $m(H, x)$ sharing the features that they are generated below a critical doping and increase with a magnetic field.

In conclusion, we derived the expression for the thermal conductivity in d-wave superconductors in the presence of competing orders. The derived expression \cite{33} for $\kappa_0^{(m)}/T$ is simple and transparent. We also analyzed the dependence of the thermal conductivity on a magnetic field and a doping in the vortex state. Our results strongly suggest that the presence of competing orders can be crucial for understanding recent experiments in LSCO \cite{14, 15}.

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