Invited Comment

Supersymmetry: aspirations and prospects

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Abstract
The realization in the early 1980s that weak scale supersymmetry stabilizes the Higgs sector of the spectacularly successful Standard Model (SM) led several authors to explore whether low energy supersymmetry could play a role in particle physics. Among these were Richard Arnowitt, Ali Chamseddine and Pran Nath who constructed a viable locally supersymmetric grand unified theory (GUT), laying down the foundation for supergravity GUT models of particle physics. Supergravity models continue to be explored as one of the most promising extensions of the SM. After a quick overview of some of the issues and aspirations of early researchers working to bring supersymmetry into the mainstream of particle physics, we re-examine early arguments that seemed to imply that superpartners would be revealed in experiments at LEP2 or at the Tevatron. Our purpose is to assess whether the absence of any superpartners in searches at LHC8 presents a crisis for supersymmetry. Toward this end, we re-evaluate fine-tuning arguments that lead to upper bounds on (some) superpartner masses. We conclude that phenomenologically viable superpartner spectra that could arise within a high scale model tuned no worse than a few percent are perfectly possible. While no viable underlying model of particle physics that leads to such spectra has yet emerged, we show that the (supergravity-based) radiatively driven natural supersymmetry framework serves as a surrogate for a phenomenological analysis of an underlying theory with modest fine-tuning. We outline the phenomenological implications of this framework, with emphasis on those LHC and electron–positron collider signatures that might point to the underlying natural origin of gauge and Higgs boson masses. We conclude that the supergravity GUT paradigm laid down in 1982 by Arnowitt, Chamseddine and Nath, and others, remains a vibrant possibility.

Keywords: supersymmetry, naturalness, search, grand unification

1. Introduction

1.1. Historical prelude

Supersymmetry (SUSY) phenomenology has been an active area of research since the early 1980s. The direct search for the superpartners has been one of the central items on the agenda of $e^+e^-$, ep and hadron collider experiments at the energy frontier for over three decades now. In addition, there are (and have been) many experiments operating at lower energies that are also searching for quantum effects of supersymmetric particles that would modify the properties of quarks and leptons; e.g. rare decays of bottom mesons, or the magnetic moment of the muon. Finally, searches for dark matter are often interpreted in the context of supersymmetric models.

That SUSY has become a part of the mainstream of particle physics is the result of the pioneering efforts of many

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1 Indeed some ‘true believers’ of SUSY have gone out on a limb and interpreted the fact that the measured values of gauge couplings measured at LEP appear to unify in the minimal supersymmetric standard model (MSSM) but not in the standard model as evidence for the virtual corrections from sparticles. While Grand Unification is a very pretty idea, we have to keep in mind there is, as yet, no direct evidence for it.
people, including Richard Arnowitt and his collaborators\textsuperscript{2}. Space–time supersymmetry—to be distinguished from the conceptually different notion of supersymmetry on the two-dimensional worldsheet of string theory \cite{2}—was discovered as far back as 1971 when Goland and Likhtman \cite{3} introduced the supersymmetric extension of the Poincaré algebra, and independently by Volkov and Akulov \cite{4} who interpreted the (massless) neutrino as the Goldstone fermion in a model with SUSY realized non-linearly. The seminal work of Wess and Zumino \cite{5} (who were unaware of the earlier 1971 work just mentioned) presented a four-dimensional model of a quantum field theory with supersymmetry realized linearly.

SUSY, however, mostly remained a playground for quantum field theorists throughout the 1970s. Research in SUSY included, among other things, efforts to study locally supersymmetric field theories as described below. With the exception of several pioneering papers by Pierre Fayet \cite{6}, no one seemed to connect SUSY with particle physics. This situation changed dramatically when it was recognized \cite{7} that the remarkable ultra-violet properties of supersymmetric theories would protect the scalar sector of the Standard Model (SM) from enormous quantum corrections that are generically present \cite{8} when the SM is embedded in a framework with a hierarchically different mass scale, as for example, in a grand unified theory (GUT). Without supersymmetry, the corrections to the Higgs boson mass squared parameter which are typically a loop factor times $M_{2GUT}^2 \sim 10^{28}$ GeV\textsuperscript{2} and need to be cancelled to more than twenty significant figures against the corresponding Lagrangian parameter in order for the physical Higgs boson mass to be below its unitarity limit of $\sim 600–800$ GeV. While such a precise cancellation is technically always possible, the need for it is generally regarded as a flaw in the theory, and often referred to as the big hierarchy problem. The key observation was that SUSY continues to protect the SM scalar sector even if it is spontaneously broken (in this case, all SUSY breaking operators are soft \cite{9}), provided that superpartners of SM particles (at least those with significant couplings to the Higgs sector) are lighter than a few TeV.

This led many authors to construct globally supersymmetric models of particle physics with SUSY broken spontaneously \cite{10, 11} below the TeV scale. A feature of a class of these models (at least those without $U(1)$ factors in the gauge group, as in all models with grand unification) was a sum rule that implied that the sum of squared masses over all chiral fermionic degrees of freedom had to be equal to the corresponding sum over all the corresponding bosonic degrees of freedom \cite{12}. Moreover, the sum rule applied separately in each electric charge and colour sector of the theory. Assuming that there are no unknown charge 1/3 quarks, it implied that one of the charge 1/3 superpartners had to be lighter than $m_{\nu}$, which was experimentally excluded. The obvious way out—introduce new heavy charge 1/3 quarks—led to unduly complicated models, as new particles had to be included in various sectors of the theory.

A phenomenologically viable alternative that side-steps this problem is to start with a globally supersymmetric $SU(3) \times SU(2) \times U(1)_Y$ Yang–Mills gauge theory with the desired three generations of quarks and leptons, the $SU(2)$ doublet Higgs fields to spontaneously break the electroweak symmetry to electromagnetism and, of course, the gauge bosons, together with the superpartners of the SM particles. SM Yukawa interactions between the Higgs fields and matter fermions (along with interactions of the superpartners that are necessary to preserve SUSY) that are needed to give fermion masses are incorporated via a superpotential. Since the superpotential has to be a holomorphic function of the (super) fields (which in plain English means it cannot include both the field and its Hermitian conjugate), we necessarily need two independent Higgs doublet fields in order to give mass to the up- and down-type fermions. Thus, unlike the SM which has just a single physical Higgs scalar, the simplest supersymmetric model includes five spin-zero particles in the electroweak breaking sector. We note that it is possible to include renormalizable gauge-invariant interactions that violate baryon or lepton number conservation in the superpotential. Since these can be potentially dangerous, it is traditional to forbid these by imposing $R$-parity conservation. As a final step, supersymmetry breaking (SSB) is incorporated by including all soft SSB \cite{9} terms consistent with underlying Yang–Mills gauge symmetry and the assumed $R$-parity conservation. The resulting theory \cite{13} with the minimal particle content and the fewest number of new interactions necessary for a phenomenologically viable model of particle physics is called the minimal supersymmetric standard model (MSSM).

In parallel with the work that led to particle physics models with an underlying (broken) global SUSY, starting with the work of Volkov and Soroka \cite{14}, field theorists considered the possibility that SUSY, like Yang–Mills gauge symmetry, is a local symmetry. The pioneering efforts of many authors \cite{15, 16} (including Arnowitt et al \cite{17}) culminated in the work of Cremmer et al \cite{18} who, building upon their earlier work on supergravity couplings of a single chiral supermultiplet \cite{19}, successfully coupled an arbitrary number of matter and gauge fields in a locally symmetric manner\textsuperscript{3}. The resulting supersymmetric theory \cite{18, 20} included not only gauge interactions of matter particles, but also interactions of matter and gauge fields and their superpartners with gravity\textsuperscript{4}. It also included the gravitino, the spin 3/2 superpartner of the graviton, which after the spontaneous breaking of the local SUSY, acquired a mass in much the same way that the gauge bosons acquire mass when local Yang–Mills gauge symmetry is spontaneously broken.

The introduction of local SUSY had another remarkable consequence. It led to a modification \cite{18} of the mass sum rule of global SUSY mentioned earlier by the addition of a positive term proportional to the squared gravitino mass on

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\textsuperscript{2} See \cite{1} for a detailed account of this.

\textsuperscript{3} Many years back, I had learnt from Arnowitt that Arnowitt, Chamseddine and Nath had independently obtained supergravity couplings with arbitrary number of matter multiplets that they had needed for development of locally supersymmetric particle physics models \cite{20}. Indeed, they had made crucial use of these results in their classic 1982 paper \cite{22}. See also \cite{1}.
the fermion side\footnote{We refer the interested reader to equation (10.66) of \cite{21}.}. This is exactly what was needed since now, the superpartners of the matter fermions could consistently have masses comparable to the gravitino mass which could be chosen to be at the TeV scale allowing for the construction of phenomenologically viable supergravity models of particle physics!

1.2. Supergravity grand unification

In truly pioneering work which set a new direction for supersymmetric models of particle physics, Arnowitt, Chamseddine and Nath (hereafter referred to as ACN)\footnote{The ACN model was recognized to be an effective theory obtained by integrating out Planck scale fields in the SUSY-breaking sector.} as well as a number of other groups\cite{23} worked to create realistic models of particle physics based on local SUSY\cite{24}. The common theme underlying these models is that a Planck mass field develops a SUSY breaking term $\langle F \rangle$ via its superpotential interactions, and that the superpartners of SM particles feel the effect of SUSY breaking only via their interaction with this field. The novel feature was that direct (superpotential) couplings of SM particles and their superpartners with the fields involved with the breaking of SUSY were forbidden. However, because gravity couples to energy and momentum, gravitational interactions between these are always present in locally supersymmetric models. These interactions carry the message of SUSY breaking to the SM sector, and lead to masses for SM superpartners. Dimensional analysis requires that these masses must be $m_{\text{SUSY}} \sim \langle F \rangle / M_\text{SUSY}$, since $m_{\text{SUSY}}$ has to vanish if either $\langle F \rangle \to 0$ (no SUSY breaking) or $M_\text{SUSY} \rightarrow \infty$ (no gravitational interactions), to be compared with the gravitino mass $m_{3/2} = \langle \tilde{F} \rangle / \sqrt{3} M_\text{SUSY}$. ACN developed an SU(5) supergravity GUT model whose low energy particle content was that of the MSSM augmented by a singlet Higgs field. Remarkably, they found that the gravitational interactions that are required by the underlying local SUSY trigger the spontaneous breakdown of breaking of the SU(2) $\times U(1)_Y \to U(1)_{em}$ when SUSY is broken, and generate masses for the electroweak gauge bosons at the tree level.

The ACN paper inspired the development of what became the well-known minimal supergravity (mSUGRA) model. Points worth noting are: (1) ACN assumed a minimal Kähler potential in their analysis which led to the hall-mark universality of all scalar mass parameters renormalized at an energy scale\footnote{The ACN model was recognized to be an effective theory obtained by integrating out Planck scale fields in the SUSY-breaking sector.} $Q \sim M_{\text{GUT}} - M_\text{P}$. It was pointed out that high scale non-universality can readily be incorporated by allowing non-universal Kähler potential\cite{25}. Scalar mass non-universality plays an essential role in the model discussed in section 3. (2) ACN included a SM singlet in their analysis in which SUSY breaking respects the GUT symmetry, and also that the magnitude of $\mu$ (but not its sign) is fixed to yield the observed value of $M_Z^2$. The various superpartner masses are then obtained by evolving these soft parameters from their universal values at the high scale down to the weak scale relevant for phenomenology in much the same way that the phenomenologically relevant QCD and electroweak gauge couplings are obtained from a single gauge coupling in a GUT.

1.3. Aspirations of supersymmetry

As we mentioned above, the realization that SUSY could stabilize the Higgs sector in the presence of radiative corrections led to an explosion of activity to devise strategies by which SUSY would reveal itself in various experiments. Although this may seem odd today, in the 1980s it was standard practice to motivate the examination of supersymmetric extensions of the SM, and typically a paper on SUSY phenomenology would begin by noting that:

1. supersymmetry is the largest possible space–time symmetry of the S-matrix;
2. supersymmetry provides a synthesis of bosons and fermions never previously attained;
3. local supersymmetry provides a connection to gravity;
4. assuming $R$-parity conservation (which was motivated by considerations of proton stability) supersymmetry naturally provides a candidate for particle dark matter;
5. weak scale supersymmetry solves the big hierarchy problem in that (in a softly broken supersymmetric theory) low scale physics does not exhibit quadratic sensitivity to physics at high scales; for instance, when the MSSM is embedded into a framework with a hierarchically separated mass scale such as a SUSY GUT.

While each of these items provides strong motivation for studying supersymmetric theories, it is worth stressing that it is just the last item that provides motivation for superpartners...
at the TeV scale relevant to supercolliders such as the LHC. It was indeed exciting that the measured values of the gauge couplings at LEP (at the end of the 1980s) were compatible with grand unification in the MSSM but not in the SM provided that superpartner masses were in 0.1–10 TeV range [27].

2. The mass scale of superpartners: an introspection

Since the stability of the electroweak symmetry breaking sector plays a central role in our considerations about what might lie beyond the SM, it seems worthwhile to re-assess the arguments that led us to infer the existence of new physics close to the weak scale. In a generic quantum field theory, the squared mass of a scalar boson (such as the Higgs boson of the SM) is given in terms of the corresponding Lagrangian parameter, $m^2_{\text{low}}$, by

$$m^2 = m^2_{\text{low}} + C_1 \frac{g^2}{16\pi^2} \Lambda^2 + C_2 \frac{g'^2}{16\pi^2} m^2_{\text{low}} \times \log \left( \frac{\Lambda}{m_{\text{low}}} \right) + C_3 \frac{g^2}{16\pi^2} m^2_{\text{low}}.$$  

Here, $\Lambda$ denotes the scale up to which the effective theory which is being used to evaluate the scalar mass is valid, $m_{\text{low}}$ is the highest mass scale of the low energy theory, $g$ denotes a typical coupling constant and the $C_i$ are dimensionless numbers (including spin and multiplicity factors) typically $O(1)$. The $C_3$ term could also include ‘small logarithms’ $\log (m^2_{\text{low}}/m^2_{\text{low}})$ that we have not exhibited. For instance, if the low energy theory is the SM viewed as embedded in a GUT, $m_{\text{low}}$ will be about $M_{\text{GUT}}, m_{h}, m_{t}$ or the SM vacuum expectation value $v$, while $\Lambda$ will be $M_{\text{GUT}}$, since the SM becomes invalid for energy scales higher than $M_{\text{GUT}}$ because GUT scale degrees of freedom (e.g. the GUT and coloured Higgs bosons with masses around $M_{\text{GUT}}$) are not included in the SM. In the case of the MSSM embedded in a SUSY GUT, $m_{\text{low}} \sim m_{\text{SU}}$ with $\Lambda \sim M_{\text{GUT}}$. The $C_1$ term which is quadratic in $\Lambda$ is what leads to the big hierarchy problem that destabilizes the SM if it is embedded into a theory with very heavy particles such as a GUT. Because the $C_1$ term is never present in softly broken SUSY, the big hierarchy problem is automatically solved, as long as $m_{\text{SU}} \ll M_{\text{GUT}}$.

Applying equation (2) to the MSSM embedded in a GUT, we see that the leading correction to the Higgs boson mass is given by the $C_4$ term; i.e.

$$\delta m^2 \sim \frac{g^2}{16\pi^2} m^2_{\text{SU}} \log \left( \frac{M^2_{\text{GUT}}}{m^2_{\text{low}}} \right).$$

In the early days, many authors argued that in order not to have un-natural cancellations, it is reasonable to set $\delta m^2 \lesssim m^2_{\text{SU}}$. Indeed, $\Delta_{\text{log}} = \frac{\delta m^2}{m^2_{\text{SU}}}$ was proposed as a simple measure of the degree of fine-tuning, and continued to be used by several authors [29]. Since the logarithm $\sim 30–40$ if $m_{\text{SU}}$ is near the TeV scale, this lead to the conclusion $m^2_{\text{SU}} \lesssim m^2_{\text{SU}}$, strongly suggesting that superpartners would be discovered either at LEP2 or at the Tevatron. We all know that things did not turn out this way, and it behooves us to re-examine these arguments more closely in order to assess whether we should remain optimistic about weak scale SUSY. With this in mind, we note that:

- Perhaps, $\delta m^2 < m^2_{\text{SU}}$ is too stringent a requirement; we know many examples of accidental cancellations in nature of one or two orders of magnitude.

- It has long been emphasized that the arguments we made really apply only to those superpartners with large couplings to the Higgs sector, and so do not apply to first (or even second generation) squarks and gluinos whose masses are most stringently probed at the LHC. These superpartners couple to the Higgs sector only at two-loop so that their masses could easily be $\sim 5–10$ TeV or more because there would be an additional $16\pi^2$ in the $C_2$ term of equation (2).

- Most importantly, the argument that led us to suggest $\Delta_{\text{log}}$ as a measure of the degree of cancellation in equation (2) assumes that contributions from various superpartners are independent. It seems almost certain that we will find that various superpartner masses are correlated once we understand the mechanism of SSB so that automatic cancellations between contributions from various superpartners could well occur when we evaluate the fine-tuning in any high scale theory. Ignoring these correlations, will overestimate the ultra-violet sensitivity of any model.

These correlations are most simply incorporated into the most commonly used fine-tuning measure introduced by Ellis et al [30] and subsequently explored by Barbieri and Guidice [31]:

$$\Delta_{BG} \equiv \max_j \left| \frac{p_j \delta M_j^2}{M_j^2 p_j} \right| .$$

Here, the $p_j$’s are the independent underlying parameters of the theory. It does not matter that $M_j^2$ rather than $m_j^2$ is used to define the sensitivity measure since essentially both the quantities are proportional to the square of the Higgs field.

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7 We emphasize that $\Lambda$ in equation (2) is not a regulator associated with divergences that occur in quantum field theory but is a physical scale associated with new particles that have significant couplings to the scalar sector. In other words, the terms in equation (2) are the finite terms (after renormalization) that would result from a calculation using the high scale theory. For this same reason, although it is tempting, we refrain from choosing $\Lambda \sim M_{\text{Planck}}$. We do not know quantum gravitational dynamics, and in particular, cannot say that there are associated new particles with large couplings to the Higgs sector of the SM; see also [28].

8 The well-known $\pi^2/9$ factor in the decay rate of orthopositronium is a cancellation of one order of magnitude. Even more familiar, and perhaps more mysterious, is the fact that the angular sizes of the sun and the moon are the same to within 3%!

9 We mention that the $D$-term coupling contributions cancel.
The value of $M_Z^2$ obtained from the minimization of the one-loop-corrected Higgs boson potential of the MSSM

$$\frac{M_Z^2}{2} = \left(m_{H_u}^2 + \Sigma_d^d\right) - \left(m_{H_u}^2 + \Sigma_u^u\right) \tan^2 \beta \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} - \mu^2,$$

(4)

is the starting point for many discussions of fine-tuning. Equation (4) is obtained using the weak scale MSSM Higgs potential, with all parameters evaluated at the scale $Q = M_{\text{SUSY}}$. The $\Sigma$s in equation (4), which arise from one loop corrections to the Higgs potential, are the analogue of the $C_i$ term in (2). Explicit forms for the $\Sigma_u^u$ and $\Sigma_d^d$ are given in the appendix of [33].

Requiring that the observed value of $M_Z^2$ is obtained without large cancellations means that none of the various terms on the right-hand side of equation (4) has a magnitude much larger than $M_Z^2$. This then suggests that the electroweak fine-tuning of $M_Z^2$ can be quantified by $\Delta_{\text{EW}}$, where [32–34]

$$\Delta_{\text{EW}} \equiv \max \left| C_i \right| / \left( M_Z^2 / 2 \right).$$

(5)

Here, $C_{H_u} = m_{H_u}^2 / (\tan^2 \beta - 1)$, $C_{H_d} = -m_{H_d}^2 / (\tan^2 \beta - 1)$ and $C_\mu = -\mu^2$. Also, $C_{\Sigma_d(k)} = -\Sigma_d^d(k) \tan^2 \beta / (\tan^2 \beta - 1)$ and $C_{\Sigma_u(k)} = \Sigma_u^u(k) / (\tan^2 \beta - 1)$, where $k$ labels the various loop contributions included in equation (4). We immediately see that any upper bound on $\Delta_{\text{EW}}$ that we impose from naturalness considerations necessarily implies a corresponding limit on $\mu^2$, a connection first noted almost two decades ago [35]. We conclude that higgsino masses are necessarily bounded from above in any theory with small values of $\Delta_{\text{EW}}$. There are, however, potential loopholes in the analysis that led us to infer that higgsinos must be light that we make explicit.

- Our reasoning assumes that the superpotential parameter $\mu$ is independent of the SSB parameters. If $\mu$ were correlated to the SSB parameters (in particular with $m_{H_u}^2$), there would be automatic cancellations that would clearly preclude us from concluding that higgsinos are light. The Giudice–Masiero [36] mechanism for the origin of $\mu$ notwithstanding, we take the view that the superpotential and SSB breaking sectors likely have different physical origin, and so are unrelated.
- We assume that there is no soft SUSY breaking contribution to the higgsino mass (i.e. the $\mu^2$ that enters in equation (4) via the scalar Higgs potential is indeed the higgsino mass parameter). While it is logically possible to include a SSB higgsino mass parameter as long as there are no SM singlets with significant couplings to the higgsinos, in all high scale models with minimal low energy particle content that we are aware of, higgsino masses have a supersymmetric origin. In this connection, we mention that Nelson and Roy [37] and Martin [38] have constructed models with additional adjoint chiral superfields at the weak scale in which the parameters in the Higgs boson sector are logically independent of higgsino masses.
- It has been pointed out [39] that if the Higgs particle is a pseudo-Goldstone boson in a theory with (almost) exact global symmetry, it is possible that the Higgs boson remains light even if the higgsinos are heavy because cancellations that lead to a low Higgs mass (and concomitantly low $M_Z^2$) are a result of a symmetry. We note that the model includes several additional fields to have complete multiplets of the global symmetry, which is simply put in by hand.

Despite these caveats we find it compelling that in models with a minimal (low energy) particle content the higgsino mass enters equation (4) directly, so that a low value of $\Delta_{\text{EW}}$ implies the existence of higgsinos close in mass to $M_Z$. Since we see no strong motivation for the introduction of several extra fields at the weak scale, we will continue to regard the existence of light higgsinos as a necessary condition for natural SUSY in the rest of this paper.

Before proceeding further, we remark that $\Delta_{\text{EW}}$ as defined here entails only weak scale parameters (see also [40]) and so has no information about the log $\Lambda$ terms that cause weak scale physics to exhibit logarithmic sensitivity to high scale physics. For this reason we do not view $\Delta_{\text{EW}}$ as a fine-tuning measure in the underlying high scale theory, as already noted in [33]. Indeed precisely because $\Delta_{\text{EW}}$ does not contain information about the large logs, we expect that $\Delta_{\text{EW}} \lesssim \Delta_{\text{BG}}$. We instead regard $\Delta_{\text{EW}}^{-1}$ as the minimum fine-tuning in any theory with a given superpartner spectrum.

2.2. The utility of $\Delta_{\text{EW}}$

Although it is not a fine-tuning measure of a high scale theory, $\Delta_{\text{EW}}$ is nonetheless useful for many reasons.

- Since it depends only on weak scale parameters, $\Delta_{\text{EW}}$ is essentially determined by the SUSY spectrum, and so is ‘measureable’, at least in principle.
- $\Delta_{\text{EW}}$ gives a bound on the fine-tuning in any theory with a given SUSY spectrum. Modulo the caveats discussed above, if $\Delta_{\text{EW}}$ turns out to be large, the underlying theory that yields this spectrum will be fine-tuned. While small
$\Delta_{\text{EW}}$ does not imply the absence of fine-tuning, it leaves open the possibility of finding an underlying theory with the same superpartner spectrum where SSB parameters are correlated so that the large logarithms automatically (nearly) cancel\textsuperscript{11}. In this underlying theory, $\Delta_{\text{BG}}$ will be numerically close to $\Delta_{\text{EW}}$, once that the correlations among the SSB parameters are incorporated in the evaluation of $\Delta_{\text{BG}}$. We emphasize that evaluation of $\Delta_{\text{BG}}$ is essential to declare that a high scale theory is free of fine-tuning, and further, that evaluation of both $\Delta_{\text{BG}}$ and $\Delta_{\text{EW}}$ are necessary to see if the underlying correlations yield the minimum fine-tuning needed for a given spectrum\textsuperscript{13}.

- Many aspects of SUSY phenomenology are determined by the superpartner spectrum. An investigation of the phenomenology of models with low $\Delta_{\text{EW}}$ is in effect an investigation of the phenomenology of (potentially) natural underlying theories. We should, however, be cautious about drawing phenomenological conclusions that are sensitive to assumed correlations (over and above those dictated by naturalness) in the spectrum.

- As we saw, low $\Delta_{\text{EW}}$ implies $\mu^2$ has to be close to $M^2_{2}$, but squarks (including $t$-squarks) and gluinos may be relatively heavy as we will see shortly.

In light of this, and despite the caveats that we mentioned above, we regard light higgsinos as the most robust feature of natural SUSY models (at least those with near-minimal low energy particle content), and focus on the phenomenology of models with small $|\mu|$ and concomitantly light higgsinos.

3. Generating spectra with low $\Delta_{\text{EW}}$

As we have already discussed, the the magnitude of $\mu$ is fixed using equation (4) which is well approximated by, $\frac{1}{2}M^2_{2} \approx -(m^2_{H_u} + \Sigma^u_u) - \mu^2$, for moderate to large $\tan \beta$. Thus, aside from radiative corrections, a weak scale value of $-m^2_{H_u}$ close to $M^2_{2}$ guarantees a correspondingly small value of $\mu^2$. Within the mSUGRA model $m^2_{H_u}$ evolves to a negative value at the weak scale (this is the celebrated mechanism of radiative electroweak symmetry breaking \textsuperscript{26}), and its magnitude is comparable to that of other weak scale SSB parameters. The resulting value of $\mu^2$ is typically much larger than $M^2_{2}$ as long as the radiative corrections contained in $\Sigma^u_u$ are of modest size, and $\Delta_{\text{EW}}$ is typically large in the mSUGRA model \textsuperscript{32}.

\textsuperscript{11} The possibility that correlations among underlying parameter reduces the fine-tuning has been noted by other authors \textsuperscript{41}.

\textsuperscript{12} See \textsuperscript{42}, section 3 for a detailed illustration of how the cancellations might occur.

\textsuperscript{13} There is an obvious technical caveat here: if a theory is really determined by a single mass scale and has no dimensionless free parameters, any reasonable fine-tuning measure should be unity. That this is not the case for $\Delta_{\text{EW}}$ is because potential correlations between weak scale parameters are ignored in its definition. This caveat is irrelevant for practical purposes in most cases (where $M^2_{2}$ depends on at least two independent parameters) and the underlying theory is defined at a very high scale.

3.1. Radiatively driven natural supersymmetry

A small weak scale value of $m^2_{H_u}$ can always be guaranteed if we relax the assumption of high scalar mass parameter universality that is the hallmark of mSUGRA, and treat the Higgs field mass parameters as independent of corresponding matter scalar masses. The non-universal Higgs mass model, which has two additional GUT scale parameters $m_{H_u}^2$ and $m_{H_d}^2$ (NUHM2 model) over and above the the mSUGRA parameter set (1), provides an appropriate setting \textsuperscript{43}. As discussed in detail in \textsuperscript{33}, the added parameter freedom in the NUHM2 model allows us to find phenomenologically viable solutions with $\Delta_{\text{EW}}$ as small as 10, corresponding to electroweak fine-tuning of no worse than 10%. We stress that from the perspective of the NUHM2 framework this necessitates a fine-tuning in that $\Delta_{\text{EW}}$ is very sensitive to the value of $m^2_{H_u}$ (GUT): see table 1 of \textsuperscript{33}. This is reflected in the large value of $\Delta_{\text{BG}}$ for the NUHM2 model points in this table, even though the corresponding $\Delta_{\text{EW}}$ is just a few percent\textsuperscript{14}. For this reason, we regard the NUHM2 model to be fine-tuned. However, as discussed at length in section 3 of \textsuperscript{42}, it still leaves open the possibility of discovering an underlying theory with essentially the same mass spectrum but where the SSB parameters $m^2_{H_u}$, $A_0$ and $m_{1/2}$ are correlated with $m_0$ for which $\Delta_{\text{BG}} \approx \Delta_{\text{EW}}$ is just a few percent. Such a theory, if it exists, would not be fine-tuned and would have essentially the same phenomenology as the NUHM2 model.

To find these low $\Delta_{\text{EW}}$ solutions, we performed scans of the NUHM2 parameter space \textsuperscript{33, 44, 45}, requiring that:

- electroweak symmetry is radiatively broken,
- LEP2 and LHC bounds on superpartner masses are respected, and
- the value of $m_0$ is consistent with the value of the Higgs boson mass measured at the LHC \textsuperscript{46} within a theoretical error that we take to be $\sim 3$ GeV.

The low $\Delta_{\text{EW}}$ solutions clearly have low values of $|\mu|$, and generally have $A_0 \sim -(1-\mu) m_{0}$, this range of $A_0$ leads to a cancellation of the $\tilde{t}$ contribution to $\Sigma^u_u$ (the $\tilde{t}$ contribution is suppressed if $m_{1/2} \sim (2.5-3) m_{0}$), and at the same time results in large intra-generational top squark mixing that raises the Higgs mass to $\sim 125$ GeV. Since the required small value of $|\mu|$ is obtained by $m^2_{H_u}$ being driven from its GUT scale choice to close to $M^2_{2}$ at the weak scale, this scenario has been referred to as RNS. We view the RNS framework as a surrogate for the underlying natural model of supersymmetry, and urge its use for phenomenological analysis of natural SUSY models.

Requiring (somewhat arbitrarily) that $\Delta_{\text{EW}} \leq 30$, the RNS spectrum is characterized by:

- The presence of four higgsino-like states $\tilde{Z}_1$, $\tilde{Z}_2$ and $\tilde{W}^0$ with masses in the 100–300 gev range, with a mass splitting $\sim 10$–30 gev between $\tilde{Z}_2$ and the lightest supersymmetric particle (LSP).

\textsuperscript{14} We refer the reader to the first row of table 1 of \textsuperscript{42}. The subsequent rows of this table show how correlations among the parameters reduce the value of $\Delta_{\text{BG}}$ until we eventually obtain $\Delta_{\text{BG}} \approx \Delta_{\text{EW}}$. 

\textsuperscript{41} We refer the reader to the first row of table 1 of \textsuperscript{42}. The subsequent rows of this table show how correlations among the parameters reduce the value of $\Delta_{\text{BG}}$ until we eventually obtain $\Delta_{\text{BG}} \approx \Delta_{\text{EW}}$. 

\textsuperscript{42} See \textsuperscript{42}, section 3 for a detailed illustration of how the cancellations might occur.

\textsuperscript{43} There is an obvious technical caveat here: if a theory is really determined by a single mass scale and has no dimensionless free parameters, any reasonable fine-tuning measure should be unity. That this is not the case for $\Delta_{\text{EW}}$ is because potential correlations between weak scale parameters are ignored in its definition. This caveat is irrelevant for practical purposes in most cases (where $M^2_{2}$ depends on at least two independent parameters) and the underlying theory is defined at a very high scale.
• $m_{\tilde{t}} \sim 1.5 - 5$ TeV, with $Z_{34}$ and $\tilde{W}_1^+$ masses fixed by (the assumed) gaugino mass unification condition.

• $m_{\tilde{g}} = 1 - 2$ TeV, $m_{\tilde{t}_1}, m_{\tilde{b}_2} \sim 2 - 4$ TeV; this is in contrast to other studies that suggest that naturalness requires that top squarks should all be in the few hundred GeV range, and so likely be accessible at the LHC [29]. The difference arises because we allow for the possibility that SSB parameters may be correlated.

• First and second generation sfermions in the tens of TeV range. This is not required to get low $\Delta_{EW}$, but compatible [47] with it. Sfermion masses in this range ameliorate the SUSY flavour and CP problems [48], and also raise the proton lifetime [49].

We stress that attaining a small value of $\Delta_{EW}$ (in a manner consistent with phenomenological constraints) is not trivial in high scale models. Within the mSUGRA framework $\Delta_{EW} \gg 200$ [32] because we need to be in the so-called hyperbolic branch/focus point region (HB/FP) [35, 50] in order to generate the required small, negative weak scale value of $m_{\tilde{H}}^2$. This, however, requires multi-TeV values of $m_0$ and results in relatively large contributions to $\Sigma_u^u$ from loops involving top-squarks. Indeed Baer et al [51] have emphasized that of the many high scale SUSY models that have been considered in the literature, only the NUHM2 model allows for spectra with $\Delta_{EW} \lesssim 30$ that we have adopted as a necessary criterion for naturalness.

3.1.1. Fine-tuning. Before closing this section, we stress that our interpretation of $\Delta_{EW}$ differs sharply from that in [52] where it is argued that $\Delta_{EW}$, correctly used, is the appropriate measure of fine-tuning. These authors use the RNS framework to illustrate their argument. They note that in a gravity-mediated SUSY breaking framework, the various soft-SUSY-breaking parameters (but not $\mu$) are all proportional to the gravitino mass with proportionality constants $\sqrt{a_i}$, leading them to rewrite the expression for $M_Z^2$ in terms of high scale parameters (used for evaluating $\Delta_{BG}$) as

$$M_Z^2 = a \frac{m^2_{3/2}}{2} - 2.18 \mu^2_{\text{GUT}}.$$  (6)

Here $a$ is the coefficient of $m_{3/2}^2$ that results after combining the various contributions from the SSB terms to $M_Z^2$, and 2.18 arises from the (small) renormalization of the $\mu$-parameter. They then note that for low $\Delta_{BG}$, $\mu^2_{\text{GUT}}$ and $a m_{3/2}^2$ must both be comparable to $M_Z^2$, and argue that in this framework $\Delta_{BG}$ automatically approaches $\Delta_{EW}$ because all SSB parameters are written in terms of the single parameter $m_{3/2}$. We would agree with this reasoning if we had a theory that fixed the various coefficients $a_i$ so that there was an automatic cancellation that resulted in $a m_{3/2}^2 \sim M_Z^2$. In the gravity-mediated scenario envisioned in [52], however, the coefficients $a_i$ are fixed in terms of the parameters of the hidden sector superpotential, Kähler potential and gauge kinetic functions. In the absence of an underlying theory of hidden sector physics, we regard equations (31)–(35) of [52] simply as a re-parametrization of the MSSM SSB parameters, which cannot alter any conclusions as to whether or not the theory is fine-tuned! Stated differently, we would agree with the authors of [52] that the fine-tuning measure reduces to $\Delta_{EW}$ in the RNS framework if we had a theory that fixed the $a_i$ to automatically give a small value of $a m_{3/2}^2$ in equation (6). Such a theory would then fix the SSB parameters so that the large logarithms automatically cancel in the way that we have already mentioned above\textsuperscript{15}[42]. Needless to say, we do not have such a theory. We stress though that this difference of interpretation of the meaning of $\Delta_{EW}$ is unimportant for many practical purposes, in particular for providing motivation for the examination of the phenomenology of SUSY models with low $\Delta_{EW}$. It also does not detract the importance of $\Delta_{EW}$.

4. Phenomenology

We have argued that 100–300 GeV charged and neutral higgsinos, with a mass gap of 10–30 GeV with the LSP, characterize natural SUSY scenarios with $\Delta_{EW} \lesssim 30$. In this section, we present an overview of various SUSY signals in such scenarios, with emphasis on signatures suggestive of light higgsinos in the spectrum.

4.1. LHC

In natural SUSY the light higgsinos are likely to be the most copiously produced superpartners at the LHC [44]. This is illustrated in figure 1 where we show various -ino production cross sections versus $m_{1/2}$ for the RNS model-line with

$$m_0 = 5 \text{ TeV}, \quad A_0 = -1.6 m_0, \quad \tan \beta = 15, \quad \mu = 150 \text{ GeV}, \quad \text{and} \quad m_A = 1 \text{ TeV},$$  (7)

at LHC14. The cross sections for the production of higgsino-like charginos and neutralinos ($\tilde{W}_1, \tilde{Z}_{1,2}$) whose masses are $\sim |\mu| = 150$ GeV across most of the plot remain flat, while cross sections for the gaugino-like states ($\tilde{W}_2, \tilde{Z}_{3,4}$) fall off

\textsuperscript{15} Indeed it is not necessary for all the SSB parameters to be correlated ($\alpha_{\text{SSB}} = 1$) in order to obtain $\Delta_{BG} \approx \Delta_{EW}$, since many of these have only a weak effect on $M_Z^2$ in equation (4).
because their masses increase with $m_{/Z}$. Cross sections for associated gaugino-higgsino pair production are dynamically suppressed.

Despite the sizeable production rate, the small energy release in their decays makes signals from higgsino pair production difficult to detect over SM backgrounds. We are thus led to investigate other strategies for discovery of SUSY.

4.1.1. Gauginos. Gluino pair production leads to the well-known cascade decay signatures in the widely explored multi-jet + multi-lepton channels. That lighter charginos and neutralinos are higgsino-like rather than gaugino-like affects the relative rates for the various topologies with specific lepton multiplicity, relative to expectations in mSUGRA. However, the gluino mass reach which is mostly determined by the gluino production cross section (for very heavy squarks, the gluino pair production rate is essentially determined by $m_{/g}$) and is not significantly altered. An examination of the gluino reach within the RNS framework shows that experiments at LHC14 should be sensitive to $m_{/g}$ values up to 1700 GeV (1900 GeV), assuming an integrated luminosity of 300 (1000) fb$^{-1}$. It may also be possible to extract the neutralino mass gap, $m_{/Z} - m_{/2}$, from the endpoint of the mass distribution of opposite sign/same flavour dileptons from the leptonic decays of $Z_{2}$ produced in gluino decay cascades, if the mass $Z_{2} - Z_{1}$ mass gap is large enough [44]. We note, however, that experiments at the LHC can discover gluinos only over part of the range allowed by naturalness considerations.

4.1.2. Same sign dibosons. In a typical scenario based on naturalness considerations we expect that $|\mu| \ll M_{/Z}$ so that $\tilde{W}_{l}$ and $Z_{2}$ are higgsino-like and only 10–30 GeV heavier than $Z_{1}$. $Z_{1}$ is dominantly a bino, and $\tilde{W}_{l}$ and $Z_{4}$ are winos. For heavy squarks, electroweak production of the bino-like $Z_{2}$ is dynamically suppressed since gauge invariance precludes a coupling of the bino to the $W$ and $Z$ bosons. However, winos have large ‘weak isovector’ couplings to the vector bosons so that wino production cross sections can be substantial. Indeed we see from figure 1 that for high values of $m_{/Z}$ the kinematically disfavoured $\tilde{W}_{s}^{\pm}Z_{2}^{\mp}$ and $\tilde{W}_{l}Z_{4}$ processes are the dominant sparticle production mechanisms with large visible energy release and high $E_{T}^{miss}$. Winos pair production leads to a novel signature involving same-sign dibosons produced via the process, $pp \rightarrow \tilde{W}_{l}^{+}\tilde{W}_{l}^{-}(\rightarrow W^{\pm}Z_{2}^{\mp}) + \tilde{Z}_{4}(\rightarrow W^{\pm}W_{1}^{\mp})$. The decay products of $\tilde{W}_{l}$ and $Z_{2}$ tend to be soft, so that the signal of interest is a pair of same sign, high $p_T$ leptons from the decays of the $W$-bosons, with limited jet activity in the event. This latter feature serves to distinguish the signal from wino pair production from same sign dilepton events that might arise at the LHC from gluino pair production [53]. We note also that $pp \rightarrow \tilde{W}_{l}\tilde{W}_{l}^{*}$ production (where one chargino decays to $W$ and the other to a $Z$) also makes a non-negligible contribution to the $E_{T}^{miss}$ channel when the third lepton fails to be detected. The same sign dilepton signal with limited jet activity is a hallmark of all low-$\mu$ models, as long as wino pair production occurs at substantial rates.

The extraction of the same sign dilepton signal from wino production requires a detailed analysis to separate the signal from SM backgrounds: see section 5 of [44]. The most important cuts necessary for suppressing backgrounds are a hard cut on $E_{T}^{miss}$, together with a cut on

$$m_{T}^{min} = \min \left[m_{T}\left(\ell, E_{T}^{miss}\right), m_{T}\left(\ell_{2}, E_{T}^{miss}\right)\right].$$

The $5\sigma$ reach of the LHC for the NUHM2 model line (7), chosen to yield low $\Delta_{EW}$, is illustrated in figure 2 as a function of the gaugino mass parameter $m_{/Z}$. We show results for relatively soft cuts (dashed line) and hard cuts (solid lines) on $E_{T}^{miss}$ and $m_{T}^{min}$. We see that with 300 fb$^{-1}$ (1000 fb$^{-1}$) of integrated luminosity, experiments at the LHC will probe $m_{/Z}$ values up to 840 GeV (1 TeV), well in excess of what can be probed via cascade decays of gluinos.

4.1.3. Hard trileptons. It is natural to examine the SUSY reach via the trilepton channel from wino pair production, i.e. from the reaction $pp \rightarrow \tilde{W}_{l}\tilde{Z}_{2} + X \rightarrow W + Z + E_{T}^{miss} + X$, long considered to be the golden mode for SUSY searches [54]. Here the $E_{T}^{miss}$ arises from the $\tilde{W}_{l}/\tilde{Z}_{2}$ (whose visible decay products are very soft) daughters of the winos. A detailed analysis [44] shows that the LHC14 reach extends to $m_{/Z} = 500$ (630) GeV for an integrated luminosity of 300 (1000) fb$^{-1}$. This is considerably lower than the reach via the SSdB channel.

4.1.4. Four lepton signals. Light higgsino models also offer the opportunity for detecting SUSY via $ZZ + E_{T}^{miss}$ events.
background distribution extends over a much broader range.

It was found that there should indeed be an enhancement of the dimuon mass spectrum may well reveal the signal if\(^{17}\) \(m_{1/2} < 400 - 500\) GeV, for \(\mu = 150\) GeV. For larger values of \(m_{1/2}\) the mass gap becomes so small that the resulting spectral distortion is confined to just one or two low mass bins. In our view, the soft-trilepton signal is unlikely to be a discovery channel, though it could serve to strikingly confirm a SUSY signal in the SSdB or multilepton channels. Perhaps more importantly, an \(m_{\text{miss}}\) spectrum distortion in \(e\mu\mu + E_T^{\text{miss}}\) events would point to a small value of \(|\mu|\), if model parameters happen to lie in a fortuitous mass range.

4.1.6. Mono-jet and mono-photon signals. Many groups have suggested that experiments at the LHC may be able to identify the pair production of LSPs via high \(E_T\) mono-jet or mono-photon plus \(E_T^{\text{miss}}\) events, where the jet or the photon arises from QCD or QED radiation. Many of these studies have been performed using non-renormalizable contact operators for LSP production \([56–58]\). This grossly overestimates the rates for mono-jet/mono-photon production at high \(E_T\), especially in models such as RNS where s-channel \(Z\) exchange dominates LSP pair production \([59]\). A careful study of this signal for the case of light higgsinos, incorporating the correct matrix elements for all relevant higgsino pair production processes within the RNS framework, shows that it will be very difficult to extract the signal unless SM backgrounds can be controlled at the better than the percent level \([60]\). The problem is that the jet/photon \(E_T\) distribution as well as the \(E_T^{\text{miss}}\) distribution has essentially the same shape for the signal and the background.

In \([61]\) it was suggested that it may be possible to enhance the mono-jet signal relative to background by requiring additional soft leptons in events triggered by a hard mono-jet. Reference \([62]\) examined the mono-jet signal requiring, in addition, two opposite-sign leptons in each event, and showed that the SUSY signal could indeed be observable at the LHC. A subsequent detailed study of monojet events with opposite-sign, same-flavour dileptons with low invariant mass showed that experiments at LHC14 would be able to detect a 5\(\sigma\) signal from higgsino pair production for \(|\mu| < 170\) (200) GeV, assuming an integrated luminosity of 300 (1000) fb\(^{-1}\) \([63]\). We conclude that while LHC experiments will be sensitive to the most promising part of the parameter of natural SUSY models, they would not be able to probe the entire RNS region with \(\Delta_{\text{EW}} \lesssim 30\).

4.1.7. Same sign charginos from vector boson fusion. The ATLAS and the CMS experiments have reported a measurement of the cross section for same-sign \(W\) pair production via the ‘vector boson fusion’ process \(qq \to q'q'W^\pm W^\mp\) at the LHC \([64]\). This process leads to events with two high rapidity jets in opposite hemispheres, together with a pair of (leptonically decaying) same sign \(W\)-bosons. The observed rate is compatible with SM expectations. Motivated by this observation together with the fact that chargino masses are expected to be close to \(M_Z\) in natural SUSY models, we were led to examine same-sign

\(^{17}\) This range of \(m_{1/2}\) is nearly excluded by lower limits on the gluino mass \([55]\), assuming that gaugino mass parameters unify.
chargino production from vector boson fusion. We examined the cross section along the RNS model line (7) for which the lighter chargino mass remains close to 150 GeV. To our surprise, we found that the cross section for $pp \rightarrow \tilde{W}_1^\pm \tilde{W}_1^0 j j$ evaluated for the RNS model line (7) falls rapidly from $\sim 0.1$ fb for $m_{1/2} \sim 200$ GeV (already excluded by LHC gluino searches [55]) to $< 0.01$ fb for $m_{1/2} > 800$ GeV [67]. We stress that the sharp fall-off of the cross section with $m_{1/2}$ cannot be for kinematic reasons since the charged higgsino mass $m_{\tilde{W}_1^0}$ remains close to 150 GeV.

To better understand the rapid fall-off of the production rate for same sign chargino pairs, we show the cross section for the underlying sub-process $W^+W^- \rightarrow \tilde{W}_1^0 \tilde{W}_1^0$ for the same model line in (7) in figure 3, taking $\sqrt{s} = 1$ TeV. We see from the figure that this cross section continues to drop off with increasing $m_{1/2}$ even though the lighter chargino mass is essentially unchanged over the entire plot.

To explain the suppression, we first note that for $m_{1/2} \gg |\mu|$ (or more generally, $M_1, M_2 \gg |\mu|$), there are two Majorana neutralinos and a chargino, all with a mass $\sim |\mu|$, while the gauginos are essentially decoupled. In the limit $M_{1,2} \rightarrow \infty$, the two degenerate neutralinos can be combined into a single Dirac neutralino, $\tilde{Z}_D$, with couplings to the $W\tilde{W}_1$ system given by

$$\mathcal{L} = -|\mu|(\overline{\tilde{W}_1^0}\tilde{W}_1^0 + \overline{\tilde{Z}_D}\tilde{Z}_D) + \frac{g}{\sqrt{2}}(-i)^{\alpha+1}\overline{\tilde{W}_1}\gamma^\mu\tilde{Z}_D W^\alpha + \text{h.c.}. \quad (8)$$

We see that the Lagrangian in equation (8) conserves ino-number, defined to be $+1$ for the Dirac particles $\tilde{W}_1$ and $\tilde{Z}_D$, $-1$ for the corresponding anti-particles, and 0 for sfermions.

Table 1. Reach of LHC14 for SUSY in terms of gluino mass, $m_{\tilde{g}}$ (TeV) for various values of integrated luminosity values along an RNS model line introduced in (7).

| Int. lum. (fb$^{-1}$) | $\overline{g}\overline{g}$ | SSdB | $WZ \rightarrow 3\ell$ | $4\ell$ |
|----------------------|----------------|------|----------------|------|
| 10                   | 1.4            | —    | —              | —    |
| 100                  | 1.6            | 1.6  | —              | $\sim 1.2$ |
| 300                  | 1.7            | 2.1  | 1.4            | $\geq 1.4$ |
| 1000                 | 1.9            | 2.4  | 1.6            | $\geq 1.6$ |

and all SM particles. Ino-number conservation then requires the cross section for the process $W^+W^- \rightarrow \tilde{W}_1^0 \tilde{W}_1^0$ must vanish in the limit$^{20} M_{1,2} \rightarrow \infty$. The couplings of the higgsinos to the fermion–sfermion system violate ino-number conservation, but the corresponding amplitudes are very small because we have taken the squark masses around 5 TeV. We thus understand why the cross section in figure 3 is strongly suppressed for $m_{1/2} \gg |\mu|$, and are forced to conclude that same-sign chargino production is not a viable avenue for searching for the light higgsinos of natural SUSY [67].

4.1.8. Recap of the LHC14 reach in the RNS framework.

Table 1 summarizes the projected reach of LHC14 in terms of the gluino mass within the RNS framework that we advocate for phenomenological analyses of natural SUSY. We see that for an integrated luminosity in excess of $\sim 100$ fb$^{-1}$ the greatest reach is attained via the SSdB channel, if we assume gaugino mass unification. More importantly, the SSdB channel provides a novel signature for a SUSY signal in any natural model of supersymmetry without a proliferation of (beyond MSSM) particles at the weak scale. In this case, there may be striking confirmatory signals in the $4\ell$ and soft-trilepton channels in addition to the much-discussed clean trilepton signal from wino pair production.

4.2. Electron–positron colliders

Since light higgsinos are $SU(2)$ doublets, they necessarily have sizeable electroweak couplings, and so must be copiously produced at $e^+e^-$ colliders, unless their production is kinematically suppressed. This can be seen from figure 4.

$^{18}$ Superpartner production by vector boson fusion has been suggested by several authors going back nearly a decade [65], and has received recent attention in [66, 61]. Since $\tilde{W}_1^0$, $\tilde{W}_1^\pm$, and $Z, Z'$ production in association with high rapidity jets also occurs by conventional $q\bar{q}$ initiated processes, we have confined our attention to same-sign chargino pair production which (for heavy squarks) dominantly occurs via vector boson fusion: contributions from $s$-channel processes $W^\pm W^\pm \rightarrow \tilde{W}_1^0 \tilde{W}_1^0$, $W^\pm W^\pm \rightarrow \tilde{W}_1^0 \tilde{W}_1^0$ are suppressed, and also do not lead to hemispherically separated jets. Our results for same-sign chargino production are in agreement with [61] but at variance with those in [65]. We also differ by a factor of about 20 with the topmost curve in figure 2 of the second paper of [66]. We have contacted these groups and they have since confirmed that they agree with our calculation. We thank B. Dutta, T. Ghosh, A. Gurrola, T. Komon and especially T. Plenh and S. Wu for extensive communications and discussion about this discrepancy.

$^{19}$ Since we take squarks to be heavy, $2 \rightarrow 4$ amplitudes proportional to $s^2W^2$ are suppressed.
where we illustrate the variation of various particle production cross sections at an electron–positron collider with the centre-of-mass energy $\sqrt{s}$, for the NUHM2 point with $m_{\tilde{g}} = 7025$ GeV, $m_{\tilde{Q}} = 568.3$ GeV, $A_0 = -11424$ GeV, $\tan \beta = 10$, $\mu = 115$ GeV and $m_A = 1$ TeV. Indeed we see that the cross sections for higgsino pair production processes are comparable to the cross section for muon pair production if higgsino production is not kinematically suppressed. Moreover, the higgsino pair production rate, for higgsinos with masses comparable to $m_h$ exceeds that for $Z\gamma$ production, so that these facilities may well be higgsino factories in addition to being Higgs boson factories. Electron–positron linear colliders that are being envisioned for construction are thus the obvious facility for definitive searches for natural SUSY. The real question is whether, in light of the small visible energy release in higgsino decays, it is possible to extract the higgsino signal above SM backgrounds. These dominantly come from two-photon-initiated processes because those $2 \to 2$ SM reactions can be efficiently suppressed by a cut on the visible energy in the event.

The higgsino signal was examined in [68] where the authors studied two cases. For Case A (which is just the NUHM2 point shown in figure 4), $m_{\tilde{g}} = 117.3$ GeV, $m_{\tilde{Z}} = 124$ GeV and $m_{\tilde{\chi}} = 102.7$ GeV, with $\Delta_{\text{EW}} = 14$, and a neutralino mass gap of 21 GeV. Case B was chosen so that $m_{\tilde{g}} \approx m_{\tilde{Z}} = 158$ GeV, and a mass gap with the neutralino of just $\sim 10$ GeV. This case has $\Delta_{\text{EW}} = 28.5$, close to what we consider the maximum for natural models, and a neutralino mass gap that is nearly as small as it can be, consistent with $\Delta_{\text{EW}} \leq 3\%$. The small mass gap severely limits the visible energy, and in this sense Case B represents the maximally challenging situation within the RNS framework.

The most promising signals come from $e^+e^- \to \tilde{W}_1^-(\to t\tilde{Z}_1)\tilde{W}_1^+(\to q\bar{q}\tilde{Z}_1)$ which leads to $n_1 = 1$, $n_2 = 1$ or $2$ plus $E_T^{\text{miss}}$ events, and from $e^+e^- \to \tilde{Z}_1\tilde{Z}_2^-(\to t\tilde{Z}_1)$ (with 90% electron beam polarization to reduce $WW$ background) processes. SM backgrounds can be nearly eliminated using judicious cuts on the visible energy (signal events are very soft), $E_T^{\text{miss}}$ and transverse plane opening angles between leptons and/or jets. The signal is observable at the $5\sigma$ level assuming $\sqrt{s} = 250$ GeV (this is the energy for the initial phase of the linear collider that is being envisioned for construction in Japan) for Case A, and $\sqrt{s} = 340$ GeV for Case B, with an integrated luminosity of just a few fb$^{-1}$. We refer the reader to [68] for details. Based on this study, we infer that an electron–positron collider will be able to detect higgsino-pair production nearly all the way to the kinematic limit provided that the neutralino mass gap is not smaller than $\sim 10$ GeV, and further, that an electron–positron collider with $\sqrt{s} = 600$ GeV will probe the entire parameter space with $\Delta_{\text{EW}} \leq 30$ in the RNS framework.

Aside from discovery, the clean environment of electron–positron collisions also enables precise mass measurements. For example, even in the maximally difficult Case B considered in [68], a fit to the invariant mass distribution of dileptons in $\tilde{Z}_1\tilde{Z}_2$ events allows the determination of the neutralino mass gap, $m_{\tilde{Z}} - m_{\tilde{\chi}} = 9.7 \pm 0.2$ GeV. A subsequent fit to the distribution of the total energy of the two leptons then allows the extraction of individual neutralino masses: $m_{\tilde{Z}} = 158.5 \pm 0.4$ GeV and $m_{\tilde{\chi}} = 148.8 \pm 0.5$ GeV. These mass determinations, together with cross section measurements using polarized beams, point to the production of higgsinos as the underlying new physics, and suggest a link to a natural origin of gauge and Higgs boson masses [68].

### 4.3. Precision measurements

Generally speaking, precision measurements of SM particle properties offer an independent avenue (from direct production of new particles) for discovery of new physics. This is not, however, the case for the RNS scenario where our assumption that GUT scale matter fermion masses are very large essentially precludes any observable effect. (We emphasize that this choice was not required by naturalness considerations, but made to alleviate issues with flavour physics.) For instance, the RNS contributions to the rate for the inclusive $b \to s\gamma$ decay are very suppressed. This is in keeping with the fact that the SM prediction [69] for the branching ratio $B(b \to s\gamma)$ is compatible, within errors, with its measured value [70]. Likewise, it is not possible to attribute the reported deviation of the measured value of the muon anomalous magnetic moment [71] from its expectation in the SM [72] to SUSY contributions within the RNS framework. New physics beyond the RNS framework will be needed to account for this discrepancy, if the SM computation of $(g_{\mu}-2)$ holds up to scrutiny. Finally, we note that though SUSY contributions to the rare decay rate for the exclusive decay $B_s \to \mu^+\mu^-$ do not couple with the super-partner mass scale, these are strongly suppressed for large values of $m_{\tilde{A}}$. Recall that for moderate to large values of $\tan \beta$, $m_{\tilde{A}} \approx m_{\tilde{W}} - m_{\tilde{\chi}}$ aside from radiative corrections. Thus, for large values of $m_{\tilde{W}}$, $m_{\tilde{\chi}}$ can easily be in the multi–TeV range without jeopardizing electroweak fine-tuning because the contribution of the $m_{\tilde{W}}$ term in equation (4) is suppressed by the $(\tan^2 \beta - 1)$ factor. This is fortunate because the measured value [73] for the branching fraction for this process is also in good agreement with the SM prediction [74] so that any new physics contribution is strongly constrained.

### 4.4. Dark matter

Since the LSP is likely higgsino-like in all simple models with natural supersymmetry, it will annihilate rapidly to gauge bosons (via its large coupling to the Z boson, and also via t-channel higgsino exchange processes) in the early universe. Thus, in natural supersymmetry the measured cold dark matter density cannot arise solely from thermally produced higgsinos in standard Big Bang cosmology. Dark matter is thus likely to be multi-component. It is important to note that because naturalness considerations also impose an upper bound on $m_{\tilde{g}}$ and corresponding limits on electroweak gaugino masses (via gaugino mass unification), the thermal higgsino relic density cannot be arbitrarily small. Indeed, within the RNS framework, $\Omega_{\tilde{g}}h^2$ must be between $\sim 0.004$–0.03, as shown by Baer et al [75]. This has implications for DM detection experiments. Specifically, ton-size
direct detection experiments such as Xe-1 Ton that are sensitive to a spin-independent nucleon-LSP cross section at the $10^{-47}$ pb level will be able to detect a signal over the entire range of RNS parameters with $\Delta_{EW} \lesssim 30$. Thus, the outcome of these experiments has an important impact on naturalness.

4.5. Non-universal gaugino masses

The RNS framework assumes gaugino mass universality. It is, however, possible that the bino and wino mass parameters are independent of the gluino mass and fortuitously small. This does not have any impact on $\Delta_{EW}$ but will affect both collider as well as dark matter phenomenology [78]. In particular, if bins and/or winos are accessible at the LHC (with $|\mu|$ also small for naturalness reasons), signals from $Z_{3,4}$ as well as $W_2$ production at the LHC would occur at observable rates and be relatively straightforward to detect because the mass difference between these states and the higgsinos is typically large. Multilepton $+ E_{T}^{miss}$ events, $WZ + E_{T}^{miss}$ events [79] and $Wt + E_{T}^{miss}$ events [80] would be typical in such scenarios. Experiments at the LHC are already searching for these signals [81]. The dark matter could be a well-tempered neutralino which saturates the dark matter relic density if the bino is light, but would necessarily have to have other components (the axion and its associated SUSY partners, or hidden sector particles are obvious candidates) if instead $M_2$ is small.

5. Concluding remarks

Weak scale supersymmetry stabilizes the electroweak scale and, in our view, offers the best solution to the big hierarchy problem. The non-observation of superpartners at LHC8 have led some authors [82] to express reservations about this far-reaching idea. As far as we can ascertain, these are largely based on the early notions of fine-tuning that ignore the possibility that the underlying soft-SUSY-breaking parameters of the underlying theory might be correlated. We acknowledge that a credible high scale model of SUSY breaking that predicts appropriate correlations among the SSB parameters and so automatically has a modest degree of fine-tuning has not yet emerged, obituaries of supersymmetry when the LHC has run at just 60% of its design energy and collected $\lesssim$10% of the anticipated integrated luminosity seem to be premature.

SUSY GUT ideas pioneered by Arnowitt, Chameddine and Nath [22] and others in the 1980s remain as promising as ever. Moreover, the original aspirations of early workers on weak scale supersymmetry outlined in 1.3 remain unchanged, if we accept that:

- ‘accidental cancellations’ at the few percent level are ubiquitous and may not require explanation, and
- dark matter may be multi-component.

Viable natural spectra with light higgsinos exist without a need for weak scale new particles beyond the MSSM. We have argued that light higgsinos are necessary at least for the most economic realizations of the ideas of SUSY naturalness (see, however, the qualifying discussion in section 2.1), and may yield novel signals for supersymmetry at the LHC. We have analysed these within the RNS framework which has a low value of $\Delta_{EW}$ by construction, and so leads to a SUSY spectrum that could have its origin in an underlying natural theory. Since many phenomenological results are sensitive to just the particle spectrum, these can be abstracted using the RNS framework which we view as a proxy (for phenomenological purposes) for the underlying natural SUSY theory.

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21 We remind the reader that there are the usual caveats to this conclusion. For instance, if physics in the sector that makes up the remainder of the dark matter entails late decays that produce SM particles, the neutralino relic density today could be further diluted, reducing the signal; see e.g. [76]. On the other hand, late decays of associated saxion, axino or even string-moduli fields to the neutralino could enhance the neutralino relic density from its thermal value. The important lesson is that while the thermal relic density is interesting to examine, it would be imprudent to categorically exclude a new physics scenario based on relic density considerations alone, because the predicted relic density can be altered by the unknown (and, perhaps, unknowable) history of the Early Universe [77].

22 Given that visible matter which comprises a small mass fraction of the total matter content already consists of several components, this is hardly a stretch.
RNS phenomenology is discussed in section 4 and summarized in figure 5, where we show contours of $\Delta_{\text{EW}}$ in the $m_{1/2} - \mu$ plane of the NUHM2 model with large $m_0$ and $A_0 = -1.6m_0$. Above and to the right of the $\Delta_{\text{EW}} = 30$ contour, we regard the spectrum to be fine-tuned: in this region the fine-tuning must be worse than $\Delta_{\text{EW}} \sim 3\%$. The light-shaded (green) region is where the thermal higgsino relic density is smaller than its measured value, with the balance being made up by something else. It is worth stressing that despite the fact that thermal relic higgsinos of RNS comprise only a fraction of the dark matter, ton scale direct detection experiments will be able to detect the higgsino signal. The dashed line shows the LHC14 reach via the canonical search for gluinos, while the dotted–dashed line shows the projected reach via searches in the novel SSdB channel discussed in section 4.1. The region with $\mu < 170$ (200) GeV may be probed via searches for hard mono-jet events with low mass, same-flavour, opposite sign dileptons as discussed in section 4.1.6. We see, however, that LHC searches will, by themselves, not be able to cover the entire parameter space with $\Delta_{\text{EW}} < 30$. The remainder of this parameter space should be accessible, via a search for higgsinos at an $e^+e^-$ collider operating at $\sqrt{s} = 600$ GeV. Such a facility will be a decisive probe of light higgsinos associated with a natural origin of Higgs and gauge boson masses.

In summary, the fact that low scale physics is only logarithmically (and not quadratically) sensitive to the scale of ultraviolet physics remains a very attractive feature of softly broken SUSY models that provides an elegant resolution of the big hierarchy problem. That it is possible to find phenomenologically viable models with low electroweak fine-tuning leads us to speculate that our understanding of UV physics is incomplete, and that there might be high scale models with the required parameter correlations that will lead to comparably low values of the true fine-tuning parameter $\Delta_{BG}$. The supergravity GUT paradigm remains very attractive despite the absence of sparticle signals at LHC8. We remain hopeful that experiments at the new run of the LHC will unearth new physics and perhaps realize the vision laid out by ACN and other colleagues during the 1980s.

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