Leptogenesis with Majorana neutrinos

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I review the origin of the lepton asymmetry which is converted to a baryon excess at the electroweak scale. This scenario becomes more attractive if we can relate it to other physical phenomena. For this reason I elaborate on the conditions of the early universe which lead to a sizable lepton asymmetry. Then I describe methods and models which relate the low energy parameters of neutrinos to the high energy (cosmological) CP–violation and to neutrinoless double β–decay.

1. What is known about Neutrinos?

Extensive studies with neutrinos have established many of their properties. Neutrino–induced processes are either leptonic or semileptonic. In the semileptonic processes there are also hadronic matrix elements whose properties are known to various degrees of accuracy. This was one of the main topics of this conference: to report and compare various calculations at the few percent level. I must also say that there are many calculations waiting to be compared with the data (low energy Δ–resonance, nuclear target effects, etc.). I will cover several reactions in my second talk to this conference.

Beyond their couplings, neutrinos have special properties.

i) Neutrinos of various generations mix with each other, implying that there are physical states of different masses.

ii) It is possible to construct coherent states of particles and antiparticles, known as Majorana neutrinos.

The above properties allow us to write two kinds of mass terms: Dirac $M_D \bar{\nu}_L \nu_R$ and Majorana masses $M_L \bar{\nu}_L^c \nu_L$ and $M_R \bar{\nu}_R^c \nu_R$.

The latter allow the states: $N \propto \nu_R + \bar{\nu}_R$ and $\nu_M \propto \nu_L + \bar{\nu}_L$ with which we can write the mass matrix

$$\left( \begin{array}{c} \bar{\nu}_M \ N \end{array} \right) \left( \begin{array}{cc} m_L & m_D \\ m_D & M_R \end{array} \right) \left( \begin{array}{c} \nu_M \\ \bar{\nu}_N \end{array} \right).$$

The mixing phenomena observed so far are sensitive only to mixings among generations and say nothing about the presence of Majorana mass terms. We may now ask if the Majorana nature of particles influences other phenomena, like Leptogenesis, Baryogenesis, Neutrinoless Double Beta Decay, etc.

I will describe next how the mixing and the decays of Majorana-type neutrinos produces an asymmetry between leptons and antileptons in the universe. This phenomenon provides an attractive scenario for the generation of a lepton asymmetry in the early universe, which was converted, at a later epoch, into a baryon-asymmetry.

2. Lepton Asymmetry for Heavy Majorana Neutrinos

We extend the standard model to include a right–handed neutrino in each generation $\bar{\nu}$. A typical generation is

$$\psi_L = \left( \begin{array}{c} \nu_e \\ e^- \\ (N_e)_R \end{array} \right), \ e_R, \ (N_e)_R$$

We are led to an $SU(2) \times U(1)$ invariant Lagrangian

$$\mathcal{L}_F = i \bar{\psi}_R \gamma_\mu (\partial^\mu + ig \frac{y}{2} B_\mu) \psi_R$$
produce an asymmetry
\[
\varepsilon = \frac{\Gamma(N_{R_1} \to \ell) - \Gamma(N_{R_2} \to \ell^c)}{\Gamma(N_{R_1} \to \ell) + \Gamma(N_{R_2} \to \ell^c)}
\]
with \( f(x) = \sqrt{x} \left( \frac{1}{\sqrt{x}} + 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right) \),
x = \left( \frac{M_1}{M_2} \right)^2 \text{ and } M_1 \text{ the mass of the lightest Majorana neutrino.}
The term \( \left( \frac{1}{\sqrt{x}} \right) \) comes from the mixing of the states \( \tilde{\ell} \) and the rest from the interference of vertex corrections with Born diagrams.

Figure 1. Born and vertex diagrams

and the mixing of states in the mass matrix

\[
\begin{pmatrix}
N_{R_1}^c & N_{R_2}^c
\end{pmatrix}
\]

on this basis higher order corrections produce the mass matrix

\[
\begin{pmatrix}
0 & M_1 + H_{11} & 0 & H_{12} \\
M_1 + H_{11} & 0 & \tilde{H}_{12} & 0 \\
0 & \tilde{H}_{12} & 0 & M_2 + H_{22} \\
H_{12} & 0 & M_2 + H_{22} & 0
\end{pmatrix}
\]

whose details are given in article [3]. The physical states are now superpositions of particles and antiparticles

\[
|\psi_1\rangle = C_1 [a_1 N_{R_1}^\tau + b_1 N_{R_2}^\tau + c_1 N_L + d_1 N_R]
\]

\[
|\psi_2\rangle = C_2 [a_2 N_{R_1}^\tau + b_2 N_{R_2}^\tau + c_2 N_L + d_2 N_R]
\]
a\_i, \ldots, d\_i’s are constants from the solution of the eigenvalue problem and \( C_1, C_2 \) are normalization constants. These are mixed states, analogous to \( K_L \) and \( K_S \), or \( B_H \) and \( B_L \) of the mesons. We shall call the CP–violation from the mixing of states \( \delta \), in analogy to the indirect CP–violation of the \( K \)–mesons. Similarly, we call the CP–parameter from the vertex corrections \( \varepsilon' \).

We know that for the \( K \)–Mesons \( \varepsilon_K \approx O(10^{-3}) \) and \( \varepsilon'_K \approx O(10^{-7}) \). It will be interesting to know if there is also a hierarchy among the various terms contributing to leptogenesis.
3. Properties of Heavy Majoranas

We can estimate the probability for collisions of these states. A typical interaction is shown in the diagram of Fig. 3. Taking the density of states in the early universe to be \( n = \frac{2}{\pi^2} T^3 \) and calculating the cross section at an energy \( E \) equal to the temperature \( T \), we obtain

\[
n \cdot \sigma \cdot v = \frac{|h_t|^2|h_\ell|^2}{8\pi^3} T
\]

(4)

with \( \sigma \) the cross section, \( v \) their relative velocity and \( h_t, h_\ell \) the couplings of the Higgses to quarks and leptons, respectively. At that early time the decay width of the moving leptons with mass \( M_N \) is

\[
\Gamma_N = \frac{|h_\ell|^2 M_N^2}{16\pi} \frac{1}{T}
\]

(5)

leading to

\[
\frac{n \cdot \sigma \cdot v}{\Gamma_N} \sim \left( \frac{T}{M_N} \right)^2 |h_t|^2.
\]

(6)

Thus, at early stages of the universe with \( T \gg M_N \), when the mixed states are created, they live long enough so that in one life–time they have many interactions with their surroundings. They develop into incoherent states.

When they decay, they produce more leptons than antileptons. The excess appears in each one of the decays, but does not survive on the large scale of the universe, because the inverse reaction (recombinations) washes it out. The excess survives when the recombinations cease to take place, as it happens when they decouple.

As the universe expands its temperature and the energy of the particles decrease. At some time the energy of each particle becomes smaller than half the mass of the heaviest neutrino. At that stage the heaviest neutrino decouples. These neutrinos decay, but cannot be reproduced because the decay products do not have enough energy. Over the course of time the energy of each particle becomes smaller than half the mass of the lightest Majorana neutrino. As a result the neutrinos decay but cannot be reproduced. The universe deviates from thermal equilibrium. Every heavy neutrino decays and leaves a signature of its presence in the excess of the produced leptons.

It is interesting to ask if there is a signal of this primordial CP–violation which can be observed at low energies.

The dynamical creation of the asymmetry appears in the presence of many other particles of the early universe. They are out of thermal equilibrium and develop according to the Boltzmann equations. The surviving asymmetry in the final state depends on the ratio

\[
K = \left( \frac{\langle \Gamma_{\psi_1} \rangle}{H(z)} \right)_{T=M_1}
\]

(7)

with \( H \) the Hubble constant at temperature \( T = M_1 \). Solutions of the Boltzmann equations give the development of the asymmetry as function of the inverse temperature: \( z = M_1/T \). The development is shown in figure 4 for three values of the parameter \( K \). At a temperature smaller than \( M_1 \), i.e. at an epoch which is later than the time of the \( N_1 \) decays, the asymmetry reaches constant asymptotic values \( F \), which should not be much smaller than \( F \approx 10^{-3} \).

Numerical studies have shown that a consistent picture emerges provided that

(i) the dilution factor in the out–of–equilibrium decays is \( F \sim 10^{-5} \), and

(ii) the asymmetry \( \varepsilon \) from individual decays is of order \( 10^{-4} \) to \( 10^{-5} \) for \( g_* = 100 \) degrees of freedom.
The generated lepton asymmetry survives down to the electroweak phase transition, where a fraction is converted to a baryon asymmetry. This happens in terms of topological solutions of field theories (sphalerons) and decreases the asymmetry by a factor \( \frac{28}{79} \).

To sum up, in order to obtain a large lepton and subsequently baryon asymmetry, three conditions must be satisfied.

1) The Leptogenesis must occur after Inflation, so that it is not diluted. This gives the condition \( M_{N_1} < 10^{15} \text{ GeV} \).

2) For the states to be incoherent

\[
\frac{n \cdot \sigma \cdot v}{\Gamma_N} > P
\]

with \( P \) a large number. For \( P = 10^3 \) this condition gives

\[
T > \sqrt{P}M_{N_1} \approx 30M_{N_1}
\]

(Incoherence Condition)

3) The dilution factor from thermal development should not be too large or too small. Acceptable values are \( F \sim 10^{-3} \) to \( 10^{-4} \) for \( K < 10^{-3} \) to \( 10^{-4} \).

This is known as the out–of–equilibrium condition and leads to \( M_{N_1} > 10^7 \text{ GeV} \).

All three conditions can be satisfied in the early universe with a large range for the masses still being possible.

The lepton asymmetry created by the above method can be converted into a Baryon asymmetry at the electroweak scale. This phenomenon takes place through topological solutions of non–abelian gauge theories. The new solutions are called sphalerons and create a baryon excess

\[
\frac{n_B}{S} = \frac{8N_G + 4N_H}{22N_G + 13N_H} \left( \frac{n_L}{S} \right)_{\text{initial}}
\]

with \( n_L \) and \( n_B \) the excess of leptons and baryons, respectively. They are normalized to the entropy

\[
S = m_B \frac{1}{3} \frac{n_L}{S} = \frac{1}{3} \frac{\delta + \varepsilon'}{g} F
\]

With the numerical values, mentioned in the previous section, we obtain a Baryon asymmetry consistent with the one observed in the universe.

4. Possible Observables

Leptogenesis will become more interesting and important when it will be related to other physical phenomena. Two interesting questions arise in this respect:

1) Are the CP asymmetries observed in low energy laboratory experiments related to the CP–violation in the early universe?

2) Are there other macroscopic remnants of the cosmological CP–violation, besides the matter asymmetry, which we can observe?

These are important questions whose consequences are beginning to emerge. In the past year, it was recognized that for specific models there are relations between the high and low en-
nergy phenomena. There are already models where a connection has been established.

The Majorana mass matrix is symmetric and can be diagonalized by a unitary matrix \( U_R \)

\[
U_R M_R U_R^+ = \text{diag.} \ (M_1, M_2, M_3).
\] (11)

On this basis the current states \( N'_R \) are related to the physical states \( N_R \) by \( N_R = U_R N'_R \). The transformation also changes the Dirac mass matrix

\[
m_D = \tilde{m}_D U_R^+
\]

where \( \tilde{m}_D \) is the original Dirac mass matrix appearing in the Lagrangian. This demonstrates how the right–handed mixing matrix enters the Dirac mass matrix and consequently the lepton asymmetry.

The low energy phenomena, on the other hand, are determined by the matrix

\[
m_\nu = -m_D \ \text{diag.} \ (M^{-1}_1, M^{-1}_2, M^{-1}_3) \ m_D^T.
\] (12)

In fact, \( m_\nu \) is determined by the masses, mixing angles and phases occurring in low energy phenomena. In specific models for \( \tilde{m}_D \) and \( M_R \), it is possible to invert Eq. (12), to obtain \( m_R \) and consequently the lepton asymmetry \( \varepsilon \) in Eq. (3). This has been done in several articles [10]–[18] and has been discussed in a more general framework of \( SU(2) \times U(1) \) [19]. Many of these models consider \( SU(2) \times U(1) \) theories with the see–saw mechanism.

An alternative approach considers the left–right symmetric models based on \( SU(2)_L \times SU(2)_R \). The left– and the right–handed mass matrices are now related

\[
m_L = f v_L \quad \text{and} \quad m_R = f v_R
\] (13)

with \( f \) the entries of the mass matrices and \( v_L, v_R \) the vacuum expectation values.

In case that the low energy phenomena are determined by \( m_L \), the lepton asymmetry is predicted [20]. Fig. 5 shows the results of an analysis [20] where \( m_L \) is determined by the observed neutrino mass differences and mixing angles. In particular, the analysis adopted the hierarchical mass scheme and calculated the asymmetry for the three solar solutions. We note that the large–mixing–angle and the vacuum–oscillation solutions produce acceptable values for the baryon asymmetry over an extended region of \( \sin^2 \theta_3 \).

In the same model, it is possible to calculate the lepton mass parameter

\[
\langle m_{ee} \rangle = \sum_i U_{ei}^2 m_i
\]

entering the neutrinoless double beta decay. In special cases large values of

\[
\langle m_{ee} \rangle \sim 10^{-3} \ \text{to} \ 10^{-2}\text{eV}
\]

are allowed which are close to the bound established in the Heidelberg–Moscow experiment [21]

\[
\langle m_{ee} \rangle < 0.35 \ \text{eV}.
\]

5. Summary

Neutrino physics remains an active field of research with the development of new topics for investigation. An open issue is the origin of the unique properties of neutrinos. We would like to know the nature of the neutrinos: are they Dirac or Majorana particles? In case they are Majorana particles, there is the attractive possibility that they generated a lepton–asymmetry in the early universe, which was later converted to a baryon asymmetry.

Several articles have been published during the past year, relating the masses and mixings ob-
served in the oscillation experiments to the high energy phenomena that took place in the early universe. This is a welcome development, because several apparently remote phenomena seem to be related to each other.

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