Consistent Superalgebraic Truncations from $D=5$, $N=5$ Supergravity

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ABSTRACT

We study a novel five-dimensional, $N=5$ supergravity in the context of Lie superalgebra SU(5/2). The possible successive superalgebraic truncations from $N=5$ theory to the lower supersymmetric $N=4,3,2,$ and 1 supergravity theories are systematically analyzed as a sub-superalgebraic chain of $\text{SU}(5/2) \supset \text{SU}(4/2) \supset \text{SU}(3/2) \supset \text{SU}(2/2) \supset \text{SU}(1/2)$ by using the Kac-Dynkin weight techniques.

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I. Introduction

There have been considerable interests in superalgebras which are relevant to many supersymmetric theories.\textsuperscript{1,2} Supersymmetric extensions of Poincaré algebra in arbitrary dimensional space-time were reviewed, and their representations (reps) for the supermultiplets of all known supergravity theories were extensively searched by Strathdee.\textsuperscript{3} This work has been an extremely useful guideline for studying supersymmetric theories. Cremmer\textsuperscript{4} developed the complicated method for consistent truncations by choosing a particular rep of real symplectic metric in order to derive $N=6,4,2$ supergravities from $N=8$ in five dimensions. Recently, $M$ and $F$ theories\textsuperscript{5} have been also tackled from the point of view of the general properties of the superalgebra.\textsuperscript{6}

On the other hand, during last ten years, we have shown that superalgebras allow a more systematic analysis for finding the supermultiplets\textsuperscript{7,8} of several supergravity and superstring theories by using the Kac-Dynkin weight techniques of SU($m/n$) Lie superalgebra.\textsuperscript{9} In particular, we have shown that the massless reps of supermultiplets of maximal supergravity theories\textsuperscript{10,11} belong to only one irreducible representation (irrep) of the SU(8/1) superalgebra.\textsuperscript{12} Recently, we have shown that all possible successive superalgebraic truncations from four-dimensional maximal $N=8$ supergravity theory to lower supersymmetric ones are systematically realised as sub-superalgebra chains of SU(8/1) superalgebra\textsuperscript{13} by using projection matrices\textsuperscript{14}. Very recently, we have shown that the successive superalgebraic truncations from $D=10$, $N=2$ chiral supergravity\textsuperscript{10} to possible lower dimensional nonmaximal theories can be easily realized as sub-superalgebra chains of SU(8/1) Lie superalgebra.\textsuperscript{15}

In this paper, we show that the successive superalgebraic truncations from the novel $D=5$, $N=5$ supergravity to possible lower dimensional theories can be systematically realized as sub-superalgebra chains of SU(5/2) Lie superalgebra. In Sec. II, we briefly recapitulate the mathematical structure of the SU(5/2) superalgebra related to $D=5$, $N=5$ supergravity. Through the introduction of atypical representations of SU(5/2) superalgebra, we newly find the supermultiplets of $D=5$, $N=5$ supergravity theory. Up to now, no one mentions the existence of one irrep of SU(5/2) supermultiplets describing this theory. In Sec. III, we explicitly show that supermultiplets of possible lower supersymmetric supergravity theories can be systematically obtained from SU(5/2)
by successive superalgebraic dimensional reductions and truncations. The last section contains conclusion.

II. Kac-Dynkin Structure of SU(5/2) Superalgebra

The Kac-Dynkin diagram of the SU(5/2) Lie superalgebra is

\[ w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5 \quad w_6 \]
\[ \bigcirc \quad \bigcirc \quad \bigcirc \quad \bigcirc \quad \bigcirc \quad \bigcirc \]  

(1)

where the set \((w_1 \ w_2 \ w_3 \ w_4 \ w_5 \ w_6)\) determines the highest-highest weight vector of an irrep. Each weight component \(w_i \ (i \neq 5)\) of the highest-highest weight vector should be a nonnegative integer, while \(w_5\) could be any complex number. The first four white nodes and the last node form \(\text{SU}(5) \otimes \text{SU}(2)\) bosonic subalgebra, where \(\text{SU}(2)\) is isomorphic to \(\text{SO}(3)\) describing the massless modes of five dimensional space-time symmetry. The grey node is responsible to \(\text{U}(1)\) supersymmetric generator.

The corresponding graded Cartan matrix is

\[
\begin{bmatrix}
2 & -1 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & -1 & 2
\end{bmatrix}
\]

(2)

Let the positive and negative simple even roots of \(\text{SU}(5) \otimes \text{SU}(2)\) bosonic subalgebra be \(\alpha_i^\pm (i = 1, 2, 3, 4, 6)\), and let the positive and negative simple odd roots be \(\beta_5^{5\pm}\). Other odd roots are easily obtained by the commutation relations such as

\[
\beta_5^{i\pm} = [\alpha_i^\pm, \beta_5^{i+1\pm}], \quad \beta_6^{i\pm} = [\beta_5^{i\pm}, \alpha_6^\pm], \quad i = 1, 2, 3, 4.
\]

(3)

Then the action by an odd root \(\beta_2^{i\pm}\) alternates a bosonic (fermionic) floor with a fermionic (bosonic) one.
The fundamental rep of SU(5/2) is \( (1 0 0 0 0 0) \), and it has the substructure of \([ (5, 1, 2)_B \oplus (1, 2, 5)_F ] \) in the basis of the SU(5)\( \otimes \)SU(2)\( \otimes \)U(1) bosonic subalgebra, where the subscripts \( B \) and \( F \) stand for bosonic and fermionic degrees of freedom, respectively, as follows:

\[
\begin{align*}
(1 0 0 0 0 0) \\
\mid \text{ground} > & \quad (1 0 0 0 0 0) = (5, 1, 2)_B \\
\downarrow & \quad \beta_5^{1-} \\
\mid \text{1st} > & \quad (0 0 0 0 1 1) = (1, 2, 5)_F.
\end{align*}
\]

(4)

The U(1) supercharge generator is \( \text{Diag}(2,2,2,2,2,5,5) \) to satisfy the supertraceless condition. The complex conjugate rep of the fundamental rep is \( (0 0 0 0 0 1) = [(1, 2, -5)_F \oplus (5, 1, -2)_B] \) such as

\[
(0 0 0 0 0 1) \\
\mid \text{ground} > & \quad (0 0 0 0 0 1) = (1, 2, -5)_F \\
\downarrow & \quad \beta_5^{5-} \\
\mid \text{1st} > & \quad (0 0 0 1 -1 0) = (5, 1, -2)_B.
\]

(5)

The even and odd roots consist of the adjoint rep \( (1 0 0 0 0 1) \), which is obtained by the tensor product of the reps in Eqs.(4) and (5),

\[
(1 0 0 0 0 0) \otimes (0 0 0 0 0 1) = (1 0 0 0 0 1) \oplus (0 0 0 0 0 0),
\]

(6)

as follows

\[
\begin{align*}
(1 0 0 0 0 1) \\
\mid \text{gnd} > & \quad (1 0 0 0 0 1) = \beta_j^{1+} \\
\mid \text{1st} > & \quad (1 0 0 1 0 0) = \text{SU}(5) \\
& \quad (0 0 0 0 0 2) = \text{SU}(2) \\
& \quad (0 0 0 0 0 0) = \text{U}(1) \\
\downarrow & \quad \beta_j^{1-} \\
\mid \text{2nd} > & \quad (0 0 0 1 0 1) = \beta_j^{1-}.
\end{align*}
\]

(7)

4
In general, the irreps of SU($m/n$) are divided into two types, which are typical and atypical.\textsuperscript{1,9} All atypical reps of SU(5/2) are characterized by the fifth weight component $w_5$ of the highest-highest weight. The atypicality condition\textsuperscript{9} is given by

$$w_5 = \sum_{k=6}^{j} w_k - \sum_{k=i}^{4} w_k - 10 + i + j, \quad 1 \leq i \leq 5, \quad 5 \leq j \leq 6. \quad (8)$$

An atypical rep is obtained by terminating some odd root strings in a full weight system when $w_5$ satisfies the relation in Eq.(8) for specific $i$'s and $j$'s. Thus the atypical reps generally have not equal bosonic and fermionic degrees of freedom.

On the other hand, all the typical reps of SU(5/2) consist of eleven floors, and have equal bosonic and fermionic degrees of freedom. The lowest dimensional typical rep is $(0 \ 0 \ 0 \ w_5 \ 0) = [512_B \oplus 512_F]$ for $w_5 \neq -4, -3, -2, -1, 0, 1$. We take $w_5 = -\frac{3}{2}$ to make a real rep and normalize the U(1) supercharges by 3 such as
As you know, since the maximum dimension of supermultiplets of the consistent supergravity theories is $[128_B \oplus 128_F]$, it is impossible to accommodate the supermultiplets of five-dimensional theory in terms of the typical irrep. Thus we should search atypical cases having lower dimensions in contrast to the typical ones, and have found the atypical reps of the type $(0 \ 0 \ 0 \ 0 \ 0 \ w_6)$ for $w_6 \geq 4$ are $[8(2w_6 - 3)_B \oplus 8(2w_6 - 3)_F]$
such as

$$\text{floor SU(5/2) SU(5)⊗SU(2) dimension}$$

| ground > | $(0\ 0\ 0\ 0\ w_6)$ | $(0\ 0\ 0\ 0)(w_6)$ | $w_6 + 1$ |
| 1st > | $(0\ 0\ 0\ 1 - 1\ w_6 - 1)$ | $(0\ 0\ 0\ 1)(w_6 - 1)$ | $4w_6$ |
| 2nd > | $(0\ 0\ 1\ 0 - 1\ w_6 - 2)$ | $(0\ 0\ 1\ 0)(w_6 - 2)$ | $6(w_6 - 1)$ |
| 3rd > | $(0\ 1\ 0\ 0 - 1\ w_6 - 3)$ | $(0\ 1\ 0\ 0)(w_6 - 3)$ | $10(w_6 - 2)$ |
| 4th > | $(1\ 0\ 0\ 0 - 1\ w_6 - 4)$ | $(1\ 0\ 0\ 0)(w_6 - 4)$ | $5(w_6 - 3)$ |
| 5th > | $(0\ 0\ 0\ 0 - 1\ w_6 - 5)$ | $(0\ 0\ 0\ 0)(w_6 - 5)$ | $w_6 - 4$ |

Taking $w_6 = 4$, we get one graviton field in Eq.(10), since $w_6$ denotes the SU(2) ≈ SO(3) weight component. The $\beta_6^-$ string is terminated so that the fifth floor is terminated from the weight system in Eq.(10). In fact, the only atypical rep containing the desired novel $D = 5, N = 5$ supergravity multiplets is $(0\ 0\ 0\ 0\ 0\ 4) = [40_B ⊕ 40_F]$ such as

$$\text{floor SU(5/2) SU(5)⊗SU(2) field}$$

| ground > | $(0\ 0\ 0\ 0\ 4)$ | $(0\ 0\ 0\ 0)(4)$ | $e_\mu^a$ |
| 1st > | $(0\ 0\ 0\ 1 - 1\ 3)$ | $(0\ 0\ 0\ 1)(3)$ | $5\Psi_\mu$ |
| 2nd > | $(0\ 0\ 1\ 0 - 1\ 2)$ | $(0\ 0\ 1\ 0)(2)$ | $10A_\mu$ |
| 3rd > | $(0\ 1\ 0\ 0 - 1\ 1)$ | $(0\ 1\ 0\ 0)(1)$ | $10\lambda$ |
| 4th > | $(1\ 0\ 0\ 0 - 1\ 0)$ | $(1\ 0\ 0\ 0)(0)$ | $5\phi$ |

Although the reps $(0\ 0\ 0\ 0\ 0\ w_6)$ for $w_6 \geq 5$ have equal bosonic an fermionic degrees of freedom, they contain higher spin states, which are higher than spin-2 ones, leading to inconsistent theories. On the other hand, the atypical reps for $w_6 \leq 3$ are $(0\ 0\ 0\ 0\ 0\ 3) = [25_B ⊕ 24_F]$ containing a gravitino, $(0\ 0\ 0\ 0\ 0\ 2) = [13_B ⊕ 10_F]$ having a Yang-Mills field, and $(0\ 0\ 0\ 0\ 0\ 1) = [5_B ⊕ 2_F]$ including a matter field. Note that since all these
reps have larger bosonic degrees of freedom than fermionic ones, it is impossible to construct the $D=5$, $N=5$ Yang-Mills theory with any combination of these reps.

### III. Successive Possible Superalgebraic Truncations from $N=5$ to $N=4,3,2,1$

#### 3.1 $D=5$, $N=4$ Reduction

Now, let us consider the possible superalgebraic truncations from the $D=5$, $N=5$ to $D=5$, $N=4,3,2,1$ supergravities. One must carefully remove extra gravitino multiplets in a consistent manner in order to generate the existence of the $N=4,3,2,1$ theories. The massless modes of the supermultiplets are in the rep space of $SU(N) \otimes SU(2) \subset SU(N/2)$ supersymmetry. The branching rules $SU(5/2) \rightarrow SU(4/2) \rightarrow SU(3/2) \rightarrow SU(2/2) \rightarrow SU(1/2)$ are systematically attained by the successive removing of the first nodes from the Kac-Dynkin diagrams.

A branching rule of $SU(5/2) \rightarrow SU(4/2)$ for the rep in Eq.(11) is

$$\begin{align*}
(0 0 0 0 4) & \rightarrow (0 0 0 4) \oplus (0 0 0 3) \ (12)
\end{align*}$$

Then, the rep $(0 0 0 4) = [24_B \oplus 24_F]$ of $SU(4/2)$ is just the well-known graviton multiplet of $D=5$, $N=4$ supergravity$^3$ as follows

| floor   | SU(4/2)  | SU(4)⊗SU(2) | field |
|---------|----------|-------------|-------|
| ground  | (0 0 0 4) | (0 0 0)(4)  | $\epsilon^a_\mu$ |
| 1st     | (0 0 1 − 1 3) | (0 0 1)(3)  | $4\Psi_\mu$ |
| 2nd     | (0 1 0 − 1 2) | (0 1 0)(2)  | $6A_\mu$ |
| 3rd     | (1 0 0 − 1 1) | (1 0 0)(1)  | $4\lambda$ |
| 4th     | (0 0 0 − 1 0) | (0 0 0)(0)  | $\phi$    |

\[\text{(13)}\]
Note that the remaining rep \((0 0 0 3) = [16_B + 16_F]\) makes an extra gravitino multiplet, which should be removed for consistency in the \(D = 5, N = 4\) theory. On the other hand, the atypical reps for \(w_6 \leq 2\) are given by \((0 0 0 2) = [9_B \oplus 8_F]\), and \((0 0 0 1) = [4_B \oplus 2_F]\).

\[\text{3.2 } D=5, N=3 \text{ Reduction}\]

The massless modes of supermultiplets of \(D = 5, N = 3\) are described in the rep space of \(\text{SU}(3) \otimes \text{SU}(2) \subset \text{SU}(3/2)\) superalgebra. The branching rule of \(\text{SU}(4/2) \rightarrow \text{SU}(3/2)\) is

\[(0 0 0 4) \rightarrow (0 0 0 4) \oplus (0 0 0 3). \quad (14)\]

Then, the rep \((0 0 4) = [14_B + 14_F]\) of \(\text{SU}(3/2)\) is identified to the graviton multiplets of \(D = 5, N = 3\) supergravity as follows

\begin{align*}
\text{floor} & \quad \text{SU}(3/2) & \quad \text{SU}(3) \otimes \text{SU}(2) & \quad \text{field} \\
\text{ground} & \quad (0 0 0 4) & \quad (0 0)(4) & \quad e^a_\mu \\
\text{1st} & \quad (0 1 - 1 3) & \quad (0 1)(3) & \quad 3\Psi_\mu \\
\text{2nd} & \quad (1 0 - 1 2) & \quad (1 0)(2) & \quad 3A_\mu \\
\text{3rd} & \quad (0 0 - 1 1) & \quad (0 0)(1) & \quad \lambda
\end{align*}

\[(15)\]

The other rep \((0 0 3) = [10_B + 10_F]\) makes an extra gravitino multiplet, which should be removed for consistency in the \(D = 5, N = 3\) theory.

On the other hand, in contrast to the previous cases, at this level we can introduce an interesting atypical rep \((0 0 0 2) = [6_B + 6_F]\) describing a desired pure Yang-Mills multiplets such as
\begin{align}
| \text{ground} > & \quad (0 \ 0 \ 0 \ 2) \quad (0 \ 0)(2) \quad A_\mu \\
| \text{1st} > & \quad (0 \ 1 \ -1 \ 1) \quad (0 \ 1)(1) \quad 3\lambda \\
| \text{2nd} > & \quad (1 \ 0 \ -1 \ 0) \quad (1 \ 0)(0) \quad 3\phi \\
\end{align}

The rest atypical rep \((0 \ 0 \ 0 \ 1)\) is given by \([3_B \oplus 2_F]\).

### 3.3 \(D=5, \ N=2\) Reduction

The massless modes of supermultiplets of \(D = 5, \ N = 2\) are in \(\text{SU}(2) \otimes \text{SU}(2) \subset \text{SU}(2/2)\). The branching rule of \(\text{SU}(3/2) \rightarrow \text{SU}(2/2)\) is

\begin{align}
(0 \ 0 \ 0 \ 4) & \rightarrow (0 \ 0 \ 4) \oplus (0 \ 0 \ 3), & (0 \ 0 \ 0 \ 2) & \rightarrow (0 \ 0 \ 2) \oplus (0 \ 0 \ 1). \\
\end{align}

Then, the reps \((0 \ 0 \ 4) = [8_B + 8_F], (0 \ 0 \ 2) = [4_B + 4_F]\), and \((0 \ 0 \ 1) = [2_B + 2_F]\) of \(\text{SU}(2/2)\) is graviton multiplets, Yang-Mills multiplets, and matter multiplets of \(D = 5, \ N = 2\) supergravity, respectively, as follows;

\begin{align}
| \text{ground} > & \quad (0 \ 0 \ 4) \quad (0)(4) \quad e_\mu^a \\
| \text{1st} > & \quad (1 \ -1 \ 3) \quad (1)(3) \quad 2\Psi_\mu \\
| \text{2nd} > & \quad (0 \ -1 \ 2) \quad (0)(2) \quad A_\mu \\
\end{align}

\begin{align}
| \text{ground} > & \quad (0 \ 0 \ 2) \quad (0)(2) \quad A_\mu \\
| \text{1st} > & \quad (1 \ -1 \ 1) \quad (1)(1) \quad 2\lambda \\
| \text{2nd} > & \quad (0 \ -1 \ 0) \quad (0)(0) \quad \phi \\
\end{align}
Note that the rep $(0\ 0\ 3) = [6_B + 6_F]$ denotes an extra gravitino multiplet, which should be removed for consistency in the $D = 5,\ N = 2$ theory.

### 3.4 $D=5,\ N=1$ Reduction

The massless modes of supermultiplets of $D = 5,\ N = 1$ are in $U(1) \otimes SU(2) \subset SU(1/2)$ superalgebra. The branching rule of $SU(2/2) \rightarrow SU(1/2)$ is

$$(0\ 0\ 4) \rightarrow (0\ 4) \oplus (0\ 3),\ (0\ 0\ 2) \rightarrow (0\ 2) \oplus (0\ 1),\ (0\ 0\ 1) \rightarrow (0\ 1) \oplus (0\ 0).$$

Then, the rep $(0\ 4) = [5_B \oplus 4_F]$ of $SU(1/2)$ may play a role of the graviton multiplet of $D = 5,\ N = 1$ supergravity due to the existence of graviton as follows

| floor | SU(1/2) | SU(2) | field |
|-------|---------|-------|-------|
| ground > | (0\ 4) | (4) | $e_{\mu}^{a}$ |
| 1st > | (−1\ 3) | (3) | $\Psi_{\mu}$ |

However, since this rep has asymmetry between bosons and fermions, we should couple this graviton multiplet to the following asymmetric matter one $(0\ 1) = [1_B \oplus 2_F]$:

| floor | SU(1/2) | SU(2) | field |
|-------|---------|-------|-------|
| ground > | (0\ 1) | (1) | $\lambda$ |
| 1st > | (−1\ 0) | (0) | $\phi$ |
Then, the coupled reps \((0 \ 4) \oplus (0 \ 1) = [6_B \oplus 6_F]\) have same bosonic and fermionic degrees of freedom which make consistent graviton multiplets. Note that the rest rep \((0 \ 3) = [3_B + 4_F]\) makes an extra gravitino multiplet, which should be removed for consistency in the \(D = 5, \ N = 1\) theory. On the other hand, the contents of the atypical rep \((0 \ 2) = [3_B \oplus 2_F]\) are given by

\[
\begin{array}{ccc}
\text{floor} & \text{SU(1/2)} & \text{SU(2)} & \text{field} \\
| \text{ground} > & (0 \ 2) & (2) & A_\mu \\
| 1\text{st} > & (-1 \ 1) & (1) & \lambda
\end{array}
\]

Note that in contrast to the \(D=5, \ N=5\) case, we can construct the desired \(D=5, \ N=1\) Yang-Mills theory by using the coupled reps \((0 \ 2) \oplus (0 \ 1) = [4_B \oplus 4_F]\) because \((0 \ 1)\) rep in Eq.(23) has larger fermionic degrees of freedom than bosonic ones.

**IV. Conclusion**

In conclusion, we have newly studied a novel supermultiplets of \(D=5, \ N=5\) supergravity in the context of SU(5/2) superalgebra. We have obtained possible regular maximal branching patterns in terms of Kac-Dynkin weight techniques. Then, we have shown that the possible superalgebraic truncations from the \(D=5, \ N=5\) supergravity theory to the \(D=5, \ N=4,3,2,1\) theories can be systematically realized as sub-superalgebra chains of the SU(5/2) superalgebra. As results, we have explicitly identified the supermultiplets of the possible relevant lower supersymmetric theories, which have been classified in terms of super-Poincaré algebra by Strathdee, with irreps of SU\((N/2)\) superalgebra by using the systematic superalgebraic truncation method. Finally, since several authors\(^{16,17}\) have recently considered \(D = 5\) supergravity theories\(^{18}\) through the compactification of \(M\) theory, we hope through further investigations that our superalgebraic branching method will provide a deeper understanding of the structure of the supersymmetric systems including the \(M\) and \(F\) theories.
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