Application of computational methods for harmonic state estimation of power system networks

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ABSTRACT
In this study, a novel technique is used to estimate the power system harmonic state, as one of the biggest risks in a power system network. Nonlinear loads are widely used, which inject harmonics into a system. Such injected harmonics make networks unstable and increase power loss. The main objective of this work is to develop a new harmonic state estimator system to increase power system accuracy, stability and the wall operation state. Three computational methods are used in this study, that is, the i) proposed particle swarm optimisation-recursive least squares (PSO-RLS) algorithm, which is developed, presented and compared with the ii) discrete fourier transform (DFT) and iii) PSO algorithms. The three algorithms are tested on an IEEE 14-bus system, and simulation results show that the new PSO-RLS algorithm is more accurate than the other two algorithms (i.e. DFT and PSO algorithms), with a lower error percentage. The proposed algorithm is tested to prove its validity and effectiveness in power system networks. The capability of the PSO-RLS algorithm is apparent in the error percentage compared with that of the other two computational methods, which can be used to provide an excellent prediction rate for measurement errors in system busses.

Keywords: Computational methods, Harmonics, Hybrid algorithm, Power system, PSO, RLS, State estimation

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1. INTRODUCTION
The power system harmonics' study is of considerable interest to power system researchers owing to the importance and direct effect of power provided to consumers. Harmonics in power systems are proportional to power system quality and reliability, that is, if harmonics increase, then quality and reliability decrease. For example, nonlinear loads (e.g. power electronics circuits, adjustable speed drives, power supplies, silicon controlled rectifiers and so on) inject harmonics or can be considered as sources of harmonics in power system networks. Numerous researchers are interested in harmonics and thus find and develop algorithms that can help calculate and predict harmonics in a power system network.

A hybrid algorithm is used to estimate harmonics and amplitudes in a Kalman filter (KF) by applying a linear estimator based on least squares and solving a nonlinear equation (iterated extended KF) that describes the waveform with a phase. The least-squares and recursive least-squares (RLS) approaches used to predict amplitudes are shown in [1]. Discrete Fourier transform (DFT) is one of the most important techniques employed for waveform analysis [2-5]. However, major limitations of the periodic technical
method are explicit aliasing, leakage and picket-fence errors [6], which are the main challenges encountered by researchers in this field. Validation of a hybrid algorithm for state estimation is achieved with satisfactory performance rates [7]. Real power system load models are nonlinear and linear in phase and amplitude, respectively [8]. Nonlinear loads encounter problems with algorithm convergence speed; therefore, hybrid algorithms are used to estimate phase and amplitude values inside each load to avoid such problems [9]. In [10], FACTS devices were utilised to improve power system operation control, and PMU was used as a monitor to improve the capability of state estimation systems and full observable systems. The stability of a power system is a crucial area of research to describe the harmonics problem, and many studies employ FACT devices, such as DSTATCOM, as shown in [11, 12]. State estimation is extremely important in a power distribution system for determining the variable state in a system network, as shown in [13].

The main purpose of monitoring a system is to observe and protect a full system. However, such a system cannot monitor harmonics with a traditional meter; thus, a state estimation method is crucial for power systems. Moreover, evolutionary algorithms are also introduced, which can describe monitoring system state variables in a power network (current and voltage) [14]. Topology evaluation networks were investigated in [15], which are defined as strategic sites for meter connections, and measured values used to calculate the rest of the variable state (X series and Y shunt). In [16, 17], the topology approach was introduced and used to achieve harmonic state estimation (HSE), which is conducted by assuming that the current measured by a power quality meter is connected to a bus and can change the PQ meter location depending on the number of sections linked to the corresponding bus. However, the assumptions in this approach are not pragmatic owing to the limited number of channels for most power quality meters. Thus, amendments are made to permit the use of the topology approach to obtain optimum locations for installing power quality meters. Contingent on system availability, when a set number of channels are considered for power quality meters, a few association options become accessible at an established site. In this way, the system would characterise how the meters should be associated with a power arrangement rather than their number and establishment destination [18].

Finally, two methods are used in this study (i.e. discrete fourier transform DFT and particle swarm optimisation PSO) and compared with the proposed novel PSO-RLS method. MATLAB is employed to achieve the objective of the study, and the computational method is tested on an IEEE 14-bus standard system. The results show that the PSO-RLS method is satisfactory, fast and more accurate than the other two methods. This procedure can upgrade the number of estimations and limit estimation mistakes simultaneously. Based on the above information, the main contributions of this study are as follows:

a) Application of computational methods, such as the hybrid algorithm (PSO-RLS), for state estimation; all fundamental, inter- and subharmonics of power system signals contain various noise sources.

b) Evaluation of two important parameters of the proposed PSO-RLS algorithm, that is, the performance and best harmonic state estimator; the proposed algorithm is then compared with common DFT and PSO algorithms.

c) Evaluation of the three algorithms’ performance in estimating the harmonic parameters of standard system data precisely for finding the best and most appropriate method for harmonic estimation.

The rest of this paper is organised as follows. In Section 2, the definitions of general power system harmonics are given, and in Section 3, the state estimation formulation is provided. In Section 4, the mathematical background of the study is explained, and in Section 5, the computational PSO-RLS method is described. The results and discussion are presented in Section 6, and the conclusion is provided in Section 7.

2. POWER SYSTEM HARMONICS

In power system networks, the harmonics problem is a research hotspot for power system scholars; therefore, harmonics analysis, specifically, fourier transform, plays a substantial role in engineering studies on fourier series development. Episodic and nonsinusoidal source currents contain or consist of a series of sinusoidal harmonics; thus, the response to each harmonic can be determined by the following [19]:

\[ f(t) = A_0 + \sum_{n=1}^{\infty} (A_n \sin nt + B_n \cos nt) \]  \hspace{1cm} (1)

Where \( n = 1, 2, 3 \) is the harmonic order, \( t \) is the time variable and \( A_0, \ldots, A_n \) and \( B_1, \ldots, B_n \) are the harmonic constants. The total harmonic distortion factor is considered as the most common identification indicator for the existence of harmonics in a signal, which for voltage and current signal sources can be obtained (2) and (3):

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3. STATE ESTIMATION FORMULATION

The following mathematical equation shows the mathematical relationship between the measurements and estimated value and error [13, 22].

\[
Z_i = h(x_i) + e_i , i = 1,2,3,4,\ldots , m
\]

where \( e_i \) is the \( i \)th error in the measurement, \( h(x_i) \) is the function that relates the state variables with the measurements, \( m \) is the measurements’ numbers and \( x \) is the state variable (all bus angles and voltages, excluding the slack bus angle in this case).

4. MATHEMATICAL BACKGROUND

4.1. DFT

Fourier analysis consists of various types of signal processing, such as fourier transformation and DFT, which is a type of discrete transformation that involves a finite sequence of discrete data equivalent to the continuous fourier transform of signals known only at instants separated by sample times.

Let \( f(t) \) be a continuous signal (data source).

Let \( N \) be \( f[0], f[1], f[2], \ldots , f[k], \ldots , f[N-1] \)

\( F(jw) \) is a Fourier Transform of the original signal, which can be written as:

\[
F(jw) = \int_{-\infty}^{\infty} f(t)e^{-jwt} dt
\]

We can respect each example \( f[k] \) as a drive with zone \( f[k] \). At this point, given that the integrand exists only at the sample focuses,

\[
F(jw) = \int_{0}^{N-1} f(t)e^{-jwt} dt = f[0]e^{-j0T} + f[1]e^{-j1T} + \ldots + f[k]e^{-jKT} + \ldots + f(N-1)e^{-j(N-1)T}
\]

that is,

\[
F(jw) = \sum_{k=0}^{N-1} f[k]e^{-jkwT}
\]

On a fundamental level, we can assess this for any \( w \), yet with only information focuses, initially, only last yields are crucial [23-25].

4.2. Particle swarm optimization state estimation

PSO is an improvement procedure that models social manner and utilises a populace of particles to look inside an enquiry area using a multidimensional mode. In the hunt space inside a specified time interim,
every molecule possesses its own position, and the speed of the movement of such molecules is altered according to the sum of experiences picked up by the molecules with the experiences of the different particles in the gathering (swarm). Each molecule’s experience incorporates pertinent data from its direction in the hunt space for better storage in its best previously involved position [26]. Therefore, each molecule’s best previously involved position relates to learning retention associated with great arrangements acquired during the iterative procedure.

\[
v_{id}(t + 1) = v_{id}(t) - c_1 \times \text{rand}() \times [p_{id}(t) - x_{id}(t)] + c_2 \times \text{rand}() \times [p_{gd}(t) - x_{id}(t)]
\]

(10)

\[
x_{id}(t + 1) = x_{id}(t) + v_{id}(t + 1), \quad 1 \leq i \leq n, \quad 1 \leq d \leq D
\]

(11)

Where \(\text{rand}()\) is a random number between 0 and 1; \(c1\) and \(c2\) are the positive values of acceleration constants, where \(c1\) and \(c2\) are 1.5 and 2.5, respectively; and \(w\) is the inertia weight. Implementation of the algorithm requires placing a limit on the velocity (\(V_{max}\)) in addition to the acceleration constants. The PSO can attain its best search capabilities after the \(w\) and (\(V_{max}\)) parameters are adjusted.

\[
v_{id}(t + 1) = w \times v_{id}(t) + c_1 \times \text{rand}() \times [p_{id}(t) - x_{id}(t)] + c_2 \times \text{rand}() \times [p_{gd}(t) - x_{id}(t)]
\]

(12)

\[
x_{id}(t + 1) = x_{id}(t) + v_{id}(t + 1), \quad 1 \leq i \leq n \& 1 \leq d \leq D
\]

(13)

The basic of the PSO technique is defined as shown in Figure 1.

![Figure 1. PSO algorithm flow chart](image)

**Figure 1. PSO algorithm flow chart**

5. **COMPUTATIONAL PARTICLE SWARM OPTIMIZATION-RECURSIVE LEAST SQUARE (PSO-RLS)**

The procedure for the computational PSO-RLS algorithm is as follows:

1. The standard PSO algorithm is used to achieve optimisation for the unknown parameters.
2. The output of step 1 is used to create the initial values of the RLS algorithm.
3. A comparison is conducted between the output of step 2 and the desired output.
4. The weights of the RLS responsible for minimising error in the actual output are adjusted (updated).
6. RESULTS AND DISCUSSION

In this study, three methods are used to estimate the harmonics of a power system, that is, the DFT (traditional), PSO and PSO-RLS (computational) methods, which are compared with the manual calculation of (4) and (5) based on the results. The three algorithms are tested and examined on an IEEE 14-bus standard system. As a balanced system, all buses except bus No. 7 (without sources) inject harmonics as a nonlinear loads (13 suspicious nodes). For the database, the authors take the measurements of current harmonics and voltage harmonics of branches and nodes, respectively. The proposed algorithm (PSO-RLS) for HSE is written with MATLAB as an estimator and optimiser for high-level harmonic components (amplitude and phase values of fundamentals) in a time fluctuating waveform. The proposed algorithm is validated according to the results and provides fast calculation and high estimation accuracy [27]. Table 1 presents the voltage harmonic percentage of each bus and results obtained by the manual calculation of (4) and (5), which are based on other results. Table 1 shows that the 5th harmonic value of bus 1 is the biggest value.

| Hand calculation % | 5th    | 7th    | 11th   | 13th   | 17th   |
|--------------------|--------|--------|--------|--------|--------|
| 1.07               | 0.005792 | 0.0045124 | 0.0034024 | 0.0031075 | 0.0021449 |
| 1.04               | 0.0071777 | 0.0054405 | 0.0032025 | 0.0022304 | 0.0004912 |
| 0.9703             | 0.013202 | 0.0081566 | 0.0065388 | 0.0080533 | 0.0040828 |
| 0.9854             | 0.0084957 | 0.0047932 | 0.0009719 | 0.0027266 | 0.0023329 |
| 1.0173             | 0.0087342 | 0.0060512 | 0.0032662 | 0.0023007 | 0.0009924 |
| 1.02               | 0.0037118 | 0.0022276 | 0.0009315 | 0.0005537 | 0.0000422 |
| 0.9758             | 0.0047166 | 0.0023842 | 0.0002223 | 0.0006647 | 0.0007066 |
| 0.969              | 0.0040053 | 0.0020618 | 0.0004143 | 0.0001152 | 0.0002633 |
| 0.981              | 0.0038896 | 0.0022039 | 0.0004536 | 0.0001023 | 0.0002346 |
| 0.9428             | 0.0027327 | 0.0012883 | 0.0003659 | 0.0001772 | 0.0000162 |
| 1.0641             | 0.0028041 | 0.0013013 | 0.0003593 | 0.0001704 | 0.0000197 |
| 0.9457             | 0.0035623 | 0.0016315 | 0.00011052 | 0.00030355 | 0.00036458 |
| 1.0541             | 0.0027041 | 0.0013113 | 0.0003693 | 0.00016044 | 0.0000187 |
| 0.9459             | 0.0035633 | 0.0016325 | 0.00011062 | 0.0003043 | 0.00036468 |

Figures 2(a), 2(b) and 2(c) illustrate the comparison between the three computational methods, that is, the DFT, PSO and proposed PSO-RLS methods, to estimate the voltage harmonic state of power system networks. Bus 1 has the largest voltage harmonic value, followed by buses 3, 4, 7, 5, 2, 14, 12, 8, 9, 6, 11, 13 and 10. In terms of percentage values, the 5th harmonic is between (0.006042 and 0.0029827), the 7th harmonic is between (0.0047625 and 0.0015383), the 11th harmonic is between (0.0036524 and 0.00061595), the 13th harmonic is between (0.0033575 and 0.00042728) and the 17th harmonic is between (0.00233949 and 0.0002662) for the traditional DFT method.

The results of the computational PSO method show that the order of the buses with the highest voltage harmonic percentage value is 3, 4, 1, 7, 5, 2, 14, 12, 8, 9, 6, 11, 13 and 10. In terms of voltage harmonic percentage values, the 5th harmonic is between (0.014102 and 0.0036327), the 7th harmonic is between (0.0090566 and 0.0021883), the 11th harmonic is between (0.0074388 and 0.00126595), the 13th harmonic is between (0.0089535 and 0.00107728) and the 17th harmonic is between (0.0049828 and 0.0009162).

The PSO-RLS method is the third computational method used in this study, and the results demonstrate that the order of the buses with the highest voltage harmonic percentage value is 3, 4, 1, 7, 5, 2, 14, 12, 8, 9, 6, 11, 13 and 10. For the voltage harmonic percentage values, the 5th harmonic is between (0.013442 and 0.002973), the 7th harmonic is between (0.008397 and 0.001528), the 11th harmonic is between (0.006779 and 0.000606), the 13th harmonic is between (0.008293 and 0.000417) and the 17th harmonic is between (0.004323 and 0.000256) for each bus. Figure 2 exhibits the error percentage of each harmonic order, and the accuracy of the proposed PSO-RLS computational method is better than that of the other two methods, with a lower voltage error percentage. Overall voltage harmonic error as shown in Figure 3.

Table 2 shows that the harmonic current percentage of each branch in the IEEE 14-bus system depends on (5). The 5th harmonic is between (0.0066786 and 0.000596), the 7th harmonic is between (0.003016 and 0.000615), the 11th harmonic is between (0.003292 and 0.003416), the 13th harmonic is between (0.002384 and 0.002629) and the 17th harmonic is between (0.001263 and 0.000234).

Table 3 shows the voltage bus angle of all 14 IEEE buses with the three methods, and Table 4 presents the voltage bus angle error of each bus and method. The results indicate that the PSO-RLS method has fewer errors than the other two methods, with a percentage value of 0.17632%.
Figure 2. Voltage harmonics of each of the 14 IEEE buses: (a) DFT method; (b) PSO method; (c) PSO-RLS method

Figure 3. Overall voltage harmonic error

Table 2. Harmonic current of each branch in the IEEE 14-bus system [5]

| branch | 5th%  | 7th%  | 11th% | 13th% | 17th% |
|--------|-------|-------|-------|-------|-------|
| I1-2   | 0.0066786 | 0.003016 | 0.003292 | 0.002384 | 0.001263 |
| I1-5   | 0.008224  | 0.00323 | 0.002707 | 0.003162 | 0.000128 |
| I2-3   | 0.031552  | 0.01618 | 0.009676 | 0.01247 | 0.018209 |
| I2-4   | 0.0034164 | 0.00692 | 0.000547 | 0.002465 | 0.004287 |
| I2-5   | 0.011856  | 0.005496 | 0.002637 | 0.002666 | 0.000273 |
| I3-4   | 0.029031  | 0.018965 | 0.025964 | 0.005899 | 0.001182 |
| I4-5   | 0.0075226 | 0.005565 | 0.008078 | 0.03581 | 0.003679 |
| I4-6   | 0.009596  | 0.00615 | 0.003416 | 0.002629 | 0.000234 |
| I6-9   | 0.011354  | 0.006608 | 0.002233 | 0.002452 | 0.000742 |
| I6-10  | 0.01178   | 0.006824 | 0.002161 | 0.002411 | 0.000725 |
| I6-11  | 0.0040914 | 0.001784 | 0.003763 | 0.002447 | 0.000147 |
| I7-8   | 0.0056124 | 0.002481 | 0.00448 | 0.003116 | 0.000192 |
| I7-12  | 0.0014406 | 0.000851 | 0.004799 | 0.003801 | 0.000933 |
| I8-9   | 0.0016693 | 0.00091 | 0.004849 | 0.003851 | 0.000956 |
| I8-10  | 0.011354  | 0.006608 | 0.002233 | 0.002452 | 0.000742 |
| I9-12  | 0.000596  | 0.000615 | 0.003416 | 0.002629 | 0.000234 |
| I10-11 | 0.0014406 | 0.000851 | 0.004799 | 0.003801 | 0.000933 |
| I11-12 | 0.011856  | 0.005496 | 0.002637 | 0.002666 | 0.000273 |
| I12-13 | 0.01178   | 0.006824 | 0.002161 | 0.002411 | 0.000725 |
| I13-14 | 0.0178    | 0.0064 | 0.00211 | 0.00211 | 0.00025 |
Figures 4(a), 4(b) and 4(c) present the current harmonic percentage of each bus in the IEEE 14-bus system for the three methods, namely, the a) DFT, b) PSO and c) PSO-RLS methods. In the PSO-RLS method, the inputs are chosen as the current, and the outputs are chosen as the magnitude of the 3rd, 5th, 7th and 11th harmonics.

Based on the comparison between the three methods and the manual calculation, the steady-state percentage error is computed and tabulated in Figure 5. The simulation results demonstrate that the three methods, namely, the a) DFT, b) PSO and c) PSO-RLS methods, perform well and can estimate harmonic state percentages with the steady-state error, that is, 4.11%, 3.75% and 3.4%, respectively. Figure 5 illustrates that the proposed computational PSO-RLS method is more accurate than the other two methods.

![DFT method](image1.png)  ![PSO method](image2.png)  ![PSO-RLS method](image3.png)

Figure 4. Current harmonics of each of the 14 IEEE buses; a) DFT method; b) PSO method; c) PSO-RLS method

![Overall current harmonic error](image4.png)

Figure 5. Overall current harmonic error
7. CONCLUSION

In this study, a new algorithm for HSE in power systems is proposed, which is a hybrid between the PSO and RLS algorithms, namely, the PSO-RLS algorithm. The new method is compared with the two methods (i.e. the DFT and PSO methods), and the results show that the PSO-RLS algorithm has satisfactory accuracy and computational time as a state estimator for amplitudes and phases. The evaluation of the performance of the proposed algorithm (i.e. the PSO-RLS algorithm) compared with that of the DFT and PSO algorithms is reported. A comparison among the three algorithms (i.e. the DFT, PSO and PSO-RLS algorithms) shows that the proposed PSO-RLS algorithm is the best method. Moreover, based on the simulation results and analysis, the new proposed PSO-RLS work proves that it can be used as an estimator for harmonics in a power system.

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### Table 3. Voltage bus angle of 14 IEEE buses

| Bus No. | Calculation | DFT | PSO | PSO-RLS |
|---------|-------------|-----|-----|---------|
| 1       | 0           | 0   | 0   | 0       |
| 2       | -0.0785     | -0.0861 | -0.20188 | -0.0852 |
| 3       | -0.21407    | -0.223 | -0.29067 | -0.22277 |
| 4       | -0.1699     | -0.195 | -0.20745 | -0.1789 |
| 5       | -0.22231    | -0.2321 | -0.23794 | -0.23091 |
| 6       | -0.14413    | -0.1543 | -0.18416 | -0.1528 |
| 7       | -0.24301    | -0.2461 | -0.27163 | -0.2515 |
| 8       | -0.22331    | -0.2321 | -0.25706 | -0.23092 |
| 9       | -0.24993    | -0.2653 | -0.2485 | -0.26853 |
| 10      | -0.25363    | -0.26223 | -0.23705 | -0.25223 |
| 11      | -0.25037    | -0.2797 | -0.20774 | -0.2598 |
| 12      | -0.25785    | -0.2745 | -0.21501 | -0.2564 |
| 13      | -0.2585     | -0.291 | -0.21958 | -0.2771 |
| 14      | -0.27158    | -0.2848 | -0.23374 | -0.38018 |

### Table 4. Voltage bus angle error of 14 IEEE buses

|               | DFT    | PSO    | PSO-RLS |
|---------------|--------|--------|---------|
| Overall Error % | 0.21115 | 0.19014 | 0.17632 |

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