Baryogenesis in theories with large extra spatial dimensions

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Abstract

We describe a simple and predictive scenario for baryogenesis in theories with large extra dimensions which resembles Affleck-Dine baryogenesis. The Affleck-Dine field is a complex scalar field carrying a $U(1)$ charge which is dynamically broken after the end of inflation. This generates an excess of $\chi$ over $\bar{\chi}$, which then decays into Standard Model fermions to produce an excess of baryons over anti-baryons. Our model is very constrained because the Affleck-Dine field has to be sufficiently flat during inflation. It is also a source for density fluctuations which can be tested in the coming satellite and balloon experiments.

I. INTRODUCTION

Recently it has been proposed that large extra spatial dimensions can explain the apparent weakness of the electroweak scale as compared to gravity in $3 + 1$ dimensions. In such a scenario four dimensional world is assumed to be a flat hypersurface, called a brane, which is embedded in a higher dimensional space-time, which is known as the bulk. The hierarchy problem is then resolved by assuming that TeV scale can be the fundamental scale in higher dimensions $[1-3]$. This however requires the size of the extra dimensions to be large. Such a large volume can substantiate the hierarchy in energy scales. The volume suppression $V_d$, the effective four dimensional Planck mass $M_p^*$, and, the fundamental scale in $4 + d$ dimensions $M_*$ are all related to each other by a simple mathematical relation

$$M_p^2 = M_*^{2+d}V_d.$$  (1)
This automatically sets the present common size of all the extra dimensions at $b_0$. For two extra dimensions, and, $M_* = 1$ TeV, the required size is of order 0.2 mm right on the current experimental limit for the search of deviations in Newton’s gravity [4]. Recent experimental bounds suggest $M_*$ to be much larger. Naturally, such model has an important impact on collider experiments [5], and on cosmology. In this paper we address one of the most important issues in particle cosmology, the origin of baryon asymmetry in theories with large extra dimensions.

The generation of baryon asymmetry requires three well-known conditions; $C$ and $CP$ violation, $B$ or $L$ violation, and out of equilibrium decay [6]. It is quite probable that the early Universe had strong departure from thermal equilibrium due to large expansion rate in the early Universe. However, achieving out of equilibrium condition becomes more difficult when the inflationary scale and the final reheat temperature of the Universe is lowered down to the electroweak scale and below. As we shall see, this is a major obstacle for realizing baryogenesis in the context of large extra dimensions. Another major problem is a proton decay. A low fundamental scale induces fast proton decay via dimension 6 baryon number violating operator in the Standard Model (SM). With a low fundamental scale the usual coupling suppression is not sufficient.

The cosmological setup for large extra dimensions is quite different from the conventional one. Firstly, if the electroweak scale is the fundamental scale in higher dimensions then there can be no massive fields beyond the electroweak scale in four dimensions. Secondly, the size of the extra dimensions can be quite large as compared to the electroweak scale, which implies a new degree of freedom with a small mass scale related to the size of the extra dimension. This field is usually known as the radion field. Its mass can be as small as $O(\text{eV})$ for two large extra dimensions. The stabilization of such large extra dimensions is a dynamical issue because they grow from their natural scale of compactification $\sim (\text{TeV})^{-1}$ to the millimeter size in order to solve the hierarchy problem. In fact, the stabilization must take place at the very initial phase of the Universe via some trapping mechanism as discussed in Ref. [7].

Another challenge is how to realize inflation in these models such that one naturally obtains the correct amplitude for the density perturbations. There have been many proposals [8], but, the most appealing one is invoking a SM singlet scalar living in the bulk [9], and we shall point out why this is the only mechanism which works well. There is also the question of the presence of Kaluza Klein (KK) states of the graviton and any other fields residing in the bulk. At high temperatures it is quite possible to excite these KK states. Above a certain temperature known as the normalcy temperature, the Universe could be filled by the KK modes. The Big Bang Nucleosynthesis (BBN) constrains the normalcy temperature to be above $\sim 1$ MeV. In order to be safe from other cosmological bounds the final reheat temperature which is the largest temperature one can envisage during the radiation era, should be smaller than the normalcy temperature, which is constrained by cosmological considerations to be as small as 100 MeV [1,10].

There are many ways of generating baryon asymmetry in the Universe, one of which is the simplest and predictive scheme known as the Affleck-Dine mechanism [11], first discussed in the context of supersymmetry. A scalar condensate which carries non-zero baryonic, or/and
leptonic charge survives during inflation and decays into SM fermions to provide a net baryon asymmetry. The adaptation and elaboration of this particular mechanism in theories with large extra dimensions is the main goal of this paper. It has already been shown that the Affleck-Dine mechanism is the only solution for providing an adequate baryon asymmetry in this context with a very low reheat temperature, see Ref. [12].

We begin our discussion by introducing some of the salient cosmological features of the large extra dimensions. In section II, we briefly discuss the consequences of having a temperature larger than the *normalcy temperature*. This is necessary in order to judge the merit of other baryogenesis scenarios such as leptogenesis and electroweak baryogenesis. In section III we discuss inflation and its couplings to the SM fields. We shall also discuss very briefly the radion stabilization because of its importance. In section IV we consider various other proposals of baryogenesis and argue that within the context of large extra dimensions, where the inflaton is a higher dimensional field, they essentially fail to provide the observed baryon asymmetry. In section V we introduce our Affleck-Dine baryogenesis model which we embed within an inflationary scenario. In section VI we describe various facets of our model and argue why it appears to be the sole mechanism which can generate the correct baryon asymmetry. We shall also highlight various ways of constraining the model parameters. Note even though in most of the cases we specialize for the critical two extra dimensions, our applications and conclusions are valid for any number of extra spatial dimensions. Finally, in section VII we summarize our main results.

II. THERMAL HISTORY OF THE UNIVERSE

In order to appreciate thermal history of the Universe we need to estimate the temperature below which the Universe could be safely regarded as a radiation dominated one, which we call here the *normalcy temperature*, $T_c$. In this section we shall review some of the already known results and provide some new insight on how the cosmological evolution changes if the temperature of the Universe exceeds $T_c$. It is possible to excite the KK states from the plasma with a temperature $T$. The production of these states from relativistic particles depends on the cross section which is Planck mass suppressed. It is noticeable that the cross section of processes such as $\gamma + \gamma \rightarrow G$ is of order $\sigma_{\gamma + \gamma \rightarrow G} \sim (TR)^d/M_p^d$, where $R$ is the effective size of the extra dimensions. Once gravitons are produced their evolution can be traced by the Boltzmann equation

$$\frac{\partial n_G}{\partial t} + 3Hn_G = \langle \sigma v \rangle n_\gamma^2 \sim \frac{T^{d+6}}{M^{d+2} n_G} - \frac{n_{G,m}}{\tau_G}, \quad (2)$$

where $\tau_G$ is the decay lifetime of a massive KK mode, given by

$$\tau_G \approx \frac{M_p^2}{m^3}. \quad (3)$$

If we naively assume that right after the end of inflation the first term is dominating the right-hand side of Eq. (2), then by taking $a(t)T(t) = constant$, where $a$ is the scale factor of the Universe, we can in fact simplify the above equation. While doing so we may also
neglect the evolution of the individual mode and shall concentrate upon all possible KK
states excited up to a given temperature. We obtain
\[ \frac{d(n_G/n_\gamma)}{dT} = -\frac{\langle \sigma v \rangle n_\gamma}{H T}. \tag{4} \]
Once the KK states are excited they are no longer in thermal equilibrium. However, individual KK modes with a mass \( \sim T \) remain present in the Universe until they decay. We can integrate Eq. (4) while assuming that we are in a standard cosmological era such that \( H^2 \propto \rho/M_p \), to get
\[ \frac{n_G(T)}{n_\gamma} = \frac{n_\gamma(T_r)\langle \sigma v \rangle}{H(T_r)}. \tag{5} \]
The temperature \( T_r \) designates the largest temperature during radiation era, known as the reheat temperature of the Universe. We also take \( v = 1 \), henceforth. Now substituting the cross section and assuming that the relativistic particles dominate the Universe; \( n_\gamma \sim T_r^3 \), we evaluate the right-hand side of Eq. (5). The ratio thus obtained can not exceed more than one at any later times in order to maintain the successes of nucleosynthesis era and so we obtain a simple bound on \( T_r \), which is given by [1]
\[ T_r \leq T_c \sim \left( \frac{M^d+2_\ast}{M_p} \right)^{1/d}. \tag{6} \]
Note, for the preferred values \( M_\ast \sim 30 \text{ TeV} \) for \( d = 2 \), we obtain \( T_c \leq 100 \text{ MeV} \). This is an extremely strong constraint on the thermal history of the Universe. It reiterates very strongly that the radiation dominated Universe simply can not prevail beyond this temperature. Therefore, any physical phenomena such as first order phase transition, out-of-equilibrium decay of heavy particles, if at all taking place beyond \( T_c \), need to be revised. Now, it is pertinent to ask how the evolution of the Universe changes if we assume that it is possible to exceed the normalcy temperature, but let us first note that as we take larger number of extra dimensions, \( d \rightarrow \infty \), we apparently increase \( T_c \) up to the fundamental scale \( T_c \rightarrow M_\ast \) [1].

A priori it can not be ruled out that the temperature of the Universe should not exceed \( T_c \). The reheat temperature is determined by the inflaton coupling to the matter fields. The only constraint upon \( T_r \) is that it must be larger than few MeV. Therefore, there is no guarantee that the Universe can not have a radiation domination with an instantaneous temperature more than the normalcy temperature. In such a case, there is a plenty of KK states which can be excited easily beyond \( T_c \), which actually leads to enhancing the number of relativistic degrees of freedom. The number of relativistic degrees of freedom increases as \( g(T) \sim (RT)^d \), which also determines the number of degrees of freedom determining entropy. At this point one may wonder how the KK modes, which actually have Planck mass suppressed couplings, can be brought into thermal equilibrium. Indeed, they cannot if \( n_G \ll n_\gamma \). However, we are in a opposite limit when the KK gravitons have started dominating the number density of relativistic decay products of inflaton. Once the KK modes are produced they go out of equilibrium due to the fact that the self interaction among these gravitons is extremely weak. However, at the time they are being produced they introduce an extra entropy to the
already existing plasma. The entropy injection becomes important once we are above $T_c$ (the KK modes are excited with masses up to $T \gg T_c$). The distribution function for each and every KK mode has a completely different profile, for the KK mode with mass $m \sim T_c$ can be understood to be close to the relativistic particles, but the same can not be true for the heavier KK modes. As a result the effective entropy stored within the KK modes can be written as

$$s = \frac{\rho + p}{T} \sim (RT)^d T^3,$$  \hspace{1cm} (7)

neglecting factors $O(1)$. The term in the bracket corresponds to the relativistic degrees of freedom which is roughly the number of degrees of KK modes. We should mention here that while deriving Eq. (7), we have naively assumed that the final distribution function for the KK modes are peaked around the final temperature and all the KK modes below that are produced abundantly with an uniform distribution which mimics that of a relativistic species.

Now, following the fact that entropy conservation gives

$$sa^3 = T^{d+3}a^3 = constant,$$  \hspace{1cm} (8)

we obtain a simple relationship between the expansion of the Universe and the rate of change of the temperature

$$H = -\frac{d + 3 \dot{T}}{3T},$$  \hspace{1cm} (9)

which actually determines the cooling rate of the Universe provided the KK modes are fairly stable. This then leads to an approximate Hubble parameter

$$H(T) \approx \frac{T^{(d+4)/2}R^{d/2}}{M_p}.$$

However, the KK modes do decay and some of them would perhaps decay much before nucleosynthesis, but in our case this depends on the maximum temperature one could reach beyond the normalcy temperature. Note, in a standard cosmology with a radiation dominated Universe the temperature scales like inverse of the scale factor. In such a case any particle which has a Planck mass suppressed couplings with other fields can decay before nucleosynthesis provided its mass is beyond 10 TeV. Otherwise, lighter particles decay much after Nucleosynthesis and their number density is well constrained from the diffusion of gamma rays in a microwave background radiation [13]. However, if we had a KK dominated Universe, then there might be a temperature-scale factor relationship which would be governed by Eq. (9). This would also affect the life time of the massive KK modes. The simplest estimation for the masses of the KK modes which would decay before nucleosynthesis could be obtained by demanding that a KK mode with a mass just above the normalcy temperature should decay before nucleosynthesis. This determines the temperature of the Universe at the time of the decay. For instance, for $d = 2$ extra dimensions we obtain

$$T_{\text{decay}} \propto R^{-1},$$  \hspace{1cm} (11)
where $R$ is the size of the extra dimensions. In order to be consistent, the above temperature ought to be more than $\sim \mathcal{O}(\text{MeV})$. This restricts the size of the extra dimensions to be much smaller than $\sim 10^{-10}$ mm. This translates into a lower bound for the fundamental scale which is now increased to $M_\ast \gtrsim 10^8$ GeV. This result is inconsistent with our basic assumptions of having a low quantum gravity scale. This certainly may rule out the possibility of having large extra dimensions for $d = 2$. The situation improves a little if there are more than two extra dimensions. For instance, if we take $d = 6$ the above analysis gives a bound for the size $R \lesssim 2 \times 10^{-14}$ mm, and, for the fundamental scale $M_\ast \gtrsim 2 \times 10^5$ GeV. Since, the normalcy temperature also increases, which can be $\sim \mathcal{O}(1)$ TeV, baryogenesis may not be so troublesome provided we also take into account at least six extra dimensions. However, a simple hope like this seems to be a mirage once we realize that inflaton must reside in the bulk whose couplings automatically determine a reheat temperature below electro weak scale. This conclusion is actually quite robust and regardless the number of extra dimensions.

Our result asserts that we must reheat our Universe below $T_c$ given by Eq. (6), in order not to excite the KK modes with an over abundance. We have noticed that normalcy temperature is the largest temperature of the Universe below which one can safely regard the content of the Universe as a radiation dominated one.

### III. INFLATION, REHEATING AND THERMALIZATION

#### A. Inflation and density perturbations

In order to provide a relatively small reheat temperature one automatically requires very small couplings of the inflaton to the matter fields. As we already know that the dynamics of the inflaton plays a crucial role in achieving out-of-equilibrium condition for baryogenesis. Keeping this in mind we briefly review inflationary dynamics which shall also act as a preview for the AD baryogenesis.

As discussed above, the largest temperature in a radiation dominated Universe must be smaller than 100 MeV for two large extra dimensions. One way to achieve this is to assume that the inflaton is living in the higher dimensional bulk, and upon compactification it naturally admits Planck suppressed couplings to the SM fields. A simple model of inflation can be constructed using only coupled scalar fields in $4 + d$ dimensions [9] (for other attempts, see Ref. [8]). The potential can be written down as

$$V(\hat{N}, \hat{\phi}) = \lambda^2 M_\ast^d \left( N_0^2 - \frac{1}{M_\ast^d} \hat{N}^2 \right)^2 + \frac{m_\phi^2}{2} \hat{\phi}^2 + \frac{g^2}{M_\ast^d} \hat{N}^2 \hat{\phi}^2,$$

where $\hat{\phi}$ is the inflaton field, and, $\hat{N}$ is the subsidiary field which is responsible for the phase transition. The coupling constants are $g$ and $\lambda$, in general they are different, and $N_0$ determines the vacuum expectation value. Note that the higher dimensional field has a mass of dimension $1 + d/2$, which leads to non-renormalizable interaction terms. However, the suppression is given by the fundamental scale instead of the four dimensional Planck mass. Upon dimensional reduction the effective four dimensional fields, $\phi, N$ are related to their higher dimensional relatives by a simple scaling.
\[ \phi = \sqrt{V_d} \phi, \quad N = \sqrt{V_d} N. \]  
From the point of view of four dimensions the extra dimensions are assumed to be compactified on a \( d \)-dimensional Ricci flat manifold with a radii \( b(t) \), which have a minimum at \( b_0 \). The higher dimensional metric then reads
\[ ds^2 = dt^2 - a^2(t) d\vec{x}^2 - b^2(t) d\vec{y}^2, \]
where \( \vec{x} \) denotes three spatial dimensions, and \( \vec{y} \) collectively denote the extra dimensions. The scale factor of a four dimensional space-time is denoted by \( a(t) \). After dimensional reduction the effective four dimensional action reads
\[ S = \int d^4x \sqrt{-g} \left[ -\frac{M_p^2}{16\pi} R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\sigma) + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu N \partial^\mu N \right. \]
\[ \left. - \exp(-d\sigma/\sigma_0)|V(\phi, N)| \right]. \]

The radion field \( \sigma(t) \) can be written in terms of the radii of the extra dimensions
\[ \sigma(t) = \sigma_0 \ln \left[ \frac{b(t)}{b_0} \right], \quad \sigma_0 = \left[ \frac{d(d+2)M_p^2}{16\pi} \right]^{1/2}. \]
From the above equation, it is evident that \( \sigma_0 \) is proportional to the four dimensional Planck mass. For \( b(t) \sim (\text{TeV})^{-1} \), and, \( b_0 \sim 1\text{mm} \), the modulus of the radion field takes a very large initial value. The radion field has a potential which at the minimum is given by \( U(\sigma) \sim m_r^2 \sigma^2 \), where \( m_r \sim 10^{-2}\text{eV} \). In this paper we shall assume that such a potential is essential to stabilize the large extra dimensions (for discussion, see Ref. [14]).

There are mainly two phases of subsequent inflation in this scenario. We notice that initially \( N \) is stuck in the false vacuum determined by
\[ V \approx \left( \frac{M_p}{M_*} \right)^2 \lambda^2 N_0^4 + \frac{1}{2} m_\phi^2 \phi^2, \]
which renders \( V(\phi, N) \approx \lambda^2 M_p^2 M_*^2 \) if \( N_0 \sim M_* \), the contribution is almost constant if the vacuum dominates over the second term in Eq. (18), which is necessary for the generation of density perturbations. The first phase of inflation occurs when \( \phi \) is still rolling down the potential and has not undergone a second order phase transition. During the first phase of inflation the exponential term due to the radion field present in front of the scalar potential, see Eq. (15) is responsible for driving a power law inflation. For details we refer [7]. The exponential potential inflates not only the brane but also the bulk, which expands from \( (\text{TeV})^{-1} \) to mm size, which is equivalent to 35 e-foldings of inflation if there are only two extra dimensions. Towards the end of this phase when the radion field \( \sigma \rightarrow \sigma_0 \), the radion gets a running mass \( m_{\sigma, \text{eff}}^2 \approx \mathcal{O}(1) \mathcal{H}^2 + m_r^2 \). The Hubble parameter \( \mathcal{H}^2 \gg m_r^2 \), therefore
the Hubble correction dominates the radion mass. Once, the radion gains this mass it simply rolls down to its minimum as dictated by $U(\sigma)$. This ensures that not only the extra dimensions have already been grown to the adequate size $b_0 \sim \text{mm}$, but also have been stabilized. This takes place while the inflaton $\hat{\phi}$ is still rolling down the potential and $\hat{N}$ is still locked in the false vacuum. From the last term in Eq. (13) one finds that the exponential $\exp(\sigma/\sigma_0)$ becomes of $\mathcal{O}(1)$ when $\sigma$ is trapped in its minimum, and the effective potential is solely given by Eq. (18). Therefore the dynamics of $\sigma$ is completely frozen except for quantum fluctuations.

In a completely model independent way it can be easily argued that at least 43 e-foldings of inflation is necessary in order to produce structure formation [7]. The amount of required inflation is less than the usual 60 e-folding because of a smaller inflationary scale and reheat temperature. The scale of inflation during this second phase is simply given by

$$H_0 \approx \sqrt{\frac{8\pi}{3}} \frac{\lambda N_0^2}{M_*},$$

(19)

If $N_0 \sim M_*$, we automatically get the Hubble expansion at the end of inflation: $H_0 \approx M_*$. The adiabatic density perturbations can be generated while the inflaton $\phi$ is rolling down the potential until it reaches a critical value where a second order phase transition takes place and the effective mass square for $N$ field becomes negative. The critical point can be found from the potential Eq. (18) to be

$$\phi_c = \frac{\lambda}{g} \left( \frac{N_0 M_p}{M_*} \right).$$

(20)

Note, if $\lambda \sim g$, and, $N_0 \sim M_*$, we automatically get $\phi_c \sim M_p$.

The two slow-roll conditions for inflation can be laid down very easily

$$\epsilon = \frac{M_p^2}{16\pi} \left( \frac{V'(\phi, N)}{V(\phi, N)} \right)^2 = \frac{1}{16\pi} \left( \frac{M_p^2 \phi}{M_p N_0^2} \right)^2 \left( \frac{m_\phi}{\lambda N_0} \right)^4 \ll 1,$$

(21)

$$|\eta| = \frac{M_p^2}{8\pi} \left| \frac{V''(\phi, N)}{V(\phi, N)} \right| = \frac{1}{8\pi} \left( \frac{M_*}{N_0} \right)^2 \left( \frac{m_\phi}{\lambda N_0} \right)^2 \ll 1,$$

(22)

where prime denotes derivative with respect to $\phi$. The spectrum of the density perturbations is given by [15]

$$\delta_H^2 = \frac{32}{75} \frac{V(\phi, N)}{M_p^4} \frac{1}{\epsilon_{43}},$$

(23)

where $\epsilon_{43}$ is defined roughly 43 e-foldings before the end of second phase of inflation. From data $\delta_H \sim 1.91 \times 10^{-5}$ [18]. The interesting cosmological scale leaves the horizon when $\phi_{43} = \phi_c e^{43|\eta|} \approx \phi_c$, provided we assume $|\eta| \ll 1/43$. By proper substitution we get a simple relationship

$$\delta_H = 8.2 \lambda^2 g \frac{N_0^5}{M_*^2 m_\phi^2 M_p}.$$

(24)
In order to illustrate, let us set \( N_0 = M_* \sim 100 \text{ TeV} \) and \( m_\phi \sim 10 \text{ GeV} \); we then obtain
\[
\delta_H \sim (\lambda^2 g) \times 10^{-5},
\]
which provides the right COBE (and Boomerang) amplitude with \( \lambda \sim g \sim \mathcal{O}(1) \). Let us stress that all the above conclusions are independent of the number of extra dimensions.

Let us mention here that we actually require \( N_0 < M_* \) for two reasons. First, \( N \) provides a mass squared contribution to the Higgs potential, which has to be less than the electroweak scale in order to provide the right magnitude for the Higgs mass. The second obvious point is that effective mass for \( \phi \) and \( N \) must be greater than the Hubble rate \( H \) after the end of inflation. This is necessary to terminate inflation just after the phase transition. However, for most of our calculation \( N_0 \sim M_* \) remains a very good approximation in order to show the merit of the baryogenesis model.

The spectral index \( n = 1 + 2\eta - 6\epsilon \) is presently constrained by observations \( |n - 1| < 0.13 \). If we naively assume \( N_0 \equiv M_* \), and, \( \lambda \equiv g \), we find
\[
\lambda M_* \sim 1.27 \times 10^{15} |\eta| \text{ GeV}.
\]
This suggests that \( |\eta| \) has to be extremely small. Given that, \( \epsilon \) is also very small, see Eq. (21), we conclude that our model predicts a perfect scale invariant density perturbations. If the coupling is of order one and \( M_* \sim 100 \text{ TeV} \), the slow roll parameter
\[
|\eta| \leq 10^{-10}.
\]
If we assume that the inflaton sector is solely responsible for the adiabatic density perturbations, the small slow roll parameter constrains the amplitude of any other scalar fields we intend to introduce in our setup. This we shall elaborate when we discuss AD baryogenesis.

**B. Post inflationary dynamics of \( \phi \) and \( N \)**

Let us first discuss the classical dynamics of the fields \( \phi \) and \( N \) after the phase transition. During this era both the fields oscillate with an initial amplitude determined by
\[
N = \sqrt{2} \left( \frac{M_p}{M_*} \right) N_0, \quad \phi = \phi_c = \frac{\lambda}{g} \left( \frac{M_p}{M_*} \right) N_0.
\]
The frequencies are determined by the effective mass scales of the fields at the global minimum, given by
\[
\bar{m}_\phi = \sqrt{2} g N_0, \quad \bar{m}_N = \sqrt{2} \lambda N_0.
\]
We notice that the phase transition must terminate inflation immediately, otherwise there could be another bout of inflation which might give rise a particular signature in the spectrum of the microwave background radiation which we will discuss in a separate publication. A slow transition might provide density perturbations of order one and might produce primordial black hole formation. In this paper we do not take into account of this phase.
Therefore, in order to ensure $\bar{m}_\phi \sim \bar{m}_N > H_0$, the constraint on $N_0 < \sqrt{6/8\pi M_*}$, for $\lambda \sim g$. However, in order to illustrate our model for baryogenesis the essential physics remains for $N_0 \approx M_*$. 

When the frequencies of the oscillations are different the fields lose their coherence and the motion becomes chaotic, see Ref. [17–19]. If the coupling strengths $g$ and $\lambda$ are equal we can easily obtain the field trajectories around the bottom of the potential, which follows a straight line in a phase space of $\phi - N$ [18–19]. There exists a particular solution of the classical equations of motion given by [18,19]

$$N(t) = \sqrt{2}(\phi_c - \phi(t)) \ .$$

(30)

It can be argued that in a static Universe it is possible to obtain a classical solution for either $\phi$ or $N$ near the bottom of the potential. Initial motion is an-harmonic, but the oscillations near $N_0$ can be expressed in a simple form:

$$N(t) \approx \sqrt{2} \left( \frac{M_p}{M_*} \right) N_0 \left[ 1 + \frac{1}{3} \cos(m_\phi t) \right] \ .$$

(31)

In an expanding Universe the amplitudes of the oscillations decay and

$$N(t) \approx \sqrt{2} \left( \frac{M_p}{M_*} \right) N_0 \left[ 1 + \frac{\Phi(t)}{3} \cos(m_\phi t) \right] \ ,$$

(32)

where the amplitude of the oscillations decreases as $\Phi(t) \propto 1/t$. The precise form of $\Phi(t)$ depends on the ratio $H/\bar{m}_\phi$. If the ratio is large, the decay of the amplitude is felt in a couple of oscillations, otherwise, it may take many oscillations before the expansion leads to decaying in amplitude. In our case we can make a rough estimation. We notice that $\bar{m}_\phi \sim \bar{m}_N \sim H_0$ at the time of phase transition. Therefore, the fields begin to roll down towards their respective minimum of the potential from their initial amplitude which is $\sim M_p$. The amplitude of the fields decreases very quickly until it reaches a point when $\bar{m}_\phi \sim \bar{m}_N \geq H$. This happens because the Hubble parameter $H$ is also decreasing as the fields roll down as $\sim 1/t$, where $t$ is the physical time starting from the end of inflation. In order to see this, we need to ensure

$$\frac{H(t)}{H_0} \equiv \frac{1}{tH_0} \ll 1 \ ,$$

(33)

which happens when

$$\frac{1}{H_0} \ll t \equiv \frac{2\pi N_{osc}}{\bar{m}_\phi} \ ,$$

(34)

where $N_{osc}$ is an approximate number of oscillations. For $\bar{m}_\phi \sim H_0$, we obtain $N_{osc} > 1/2\pi$. Therefore, within one oscillation the Hubble parameter decreases quite rapidly. This makes sure that the fields having masses $\bar{m}_\phi \sim \bar{m}_N \sim H_0$ can oscillate about their minimum with a common frequency for many oscillations before they completely decay. We may estimate the number of oscillations to be $N_{osc} \sim \bar{m}_\phi \tau \equiv (M_p^2/M_*^2) \sim 10^{28}$. Hence during the initial stages the dominant effect of expansion will be to render the total oscillations more harmonic around the minimum. Therefore, our assumption of the field evolution given by Eq. (32) holds well. However, in reality we need the opposite limit on masses, which would only slow down the decay of the amplitude of the oscillations. With this brief discussion on the dynamics of the fields we move on to discussing thermalization.
C. Reheating and thermalization

It is believed that the total energy density of the inflaton is transferred into radiation. The minimal requirement is to have a thermal bath with a temperature more than $O(1)$ MeV in order to preserve the successes of BBN. Recall that now we can not reheat the Universe above normalcy temperature $T_c$. The final reheat temperature depends on the decay rates $\Gamma_{\phi,N}$ of the oscillating field:

$$T_r \sim 0.1 \sqrt{(\Gamma_{\phi} + \Gamma_N) M_p}. \quad (35)$$

Therefore, we need extremely weak, non-renormalizable coupling of $\phi$ and $N$ to other SM fields. In our model this is natural because the bulk fields have Planck suppressed interactions with matter fields stuck on the brane. However, this fact also causes some problems, e.g. why the zero mode inflaton is reheating the brane and why not the bulk? The inflaton could decay into some other lighter degrees of freedom into the bulk. This point has been already addressed in Ref. [9] for the case of gravitons, and here we recapitulate the arguments.

In our previous section we found that both the fields get an effective mass term $\bar{m}_\phi \sim \bar{m}_N \propto \sqrt{2} \lambda N_0$, if we assume $\lambda = g$. Setting $N_0 \sim \mathcal{O}(\text{TeV})$, then both $\phi$ and $N$ are kinematically allowed to decay into Higgs field, $h$, with a mass $O(100\text{GeV})$. This is different from the usual Kaluza-Klein theories where the production of matter through inflaton decay occurs everywhere. The decay rate is estimated as follows:

$$\Gamma_{\phi,N \rightarrow hh} \sim \frac{f^2 M_s^4}{32\pi M_p^2 m_{\phi,N}}, \quad (36)$$

where, $f$ is the coupling constant that we take of order one. If $\bar{m}_\phi \sim \bar{m}_N \approx 0.1 M_s$, then the reheat temperature is $T_r \leq 100$ MeV, which is more than $O(1)$ MeV and below the normalcy temperature, even for the most constrained case of $d = 2$ extra dimensions. However, note that such a low reheat temperatures is a generic prediction if the inflaton field is living in the bulk.

The next point is concerning the production of KK graviton via the inflaton decay. The KK gravitons can be directly produced from the KK modes of $\phi$ and $N$, and possibly the KK modes of other scalar fields present in the bulk via $\phi_n \rightarrow \phi_l G_{n-l}$ interactions, where $n, l$ are the KK numbers, and $G$ is the KK graviton. On the other hand $\phi_n$ modes can be produced via collision processes, such as $\phi\phi \rightarrow \phi_n\phi_{-n}$, and similar reaction for $N$. The rate of exciting the KK modes and their subsequent decay rate to graviton can be estimated as:

$$\sigma_{\phi\phi \rightarrow \phi_n\phi_{-n}} \sim \lambda^2 \frac{M_s^2}{M_p^2}, \quad \Gamma_{\phi_n \rightarrow \phi_l G_{n-l}} \sim \frac{m_n m_l^2}{12\pi M_p^2}. \quad (37)$$

where $m_n^2 = \bar{m}_\phi^2 + n^2/R^2$ is the excited KK mode. Note that the $NN$ or $\phi\phi$ scattering rates for producing their KK counterparts are smaller than the direct decay of $\phi$, $N$ to the brane fields. Therefore, the zero modes of $\phi$ and $N$ still prefers the Higgs as a final decay product. However, late during the thermalization era the scattering phenomena may give rise to the
production of KK modes of inflaton. In the above equation we have only estimated a single process for graviton production. There are plenty of other accessible modes in the final channel. This enhances the decay rate

$$\Gamma_{\phi_n, \text{total}} = \sum_l \Gamma_{\phi_n \to \phi_l G_{n-l}} \sim \frac{m_n^3}{12\pi M_*^2}.$$  \hspace{1cm} (38)

The excited KK inflaton or KK partners of other scalar fields present in the spectrum are extremely short lived. The heavier KK mode decays into the lighter KK mode plus gravitons, and eventually all the KK modes of the inflaton decay into the zero mode. In case where extra bulk fields are present the reheating of the bulk can be naturally avoided, if either those modes are as heavy as the inflaton, or, the effective inflaton couplings to the bulk fields are smaller than the inflaton-brane interactions.

So far we have tacitly assumed that reheating is almost instantaneous. This might not be the case, especially when the fields are oscillating. During this period, the equation of state of the Universe in most of the cases is given by that of a matter dominated era. However, if $\phi, N$ are decaying very slowly, then they might also decay into lighter degrees of freedom, such as relativistic species directly. One might expect that this could be the most preferred channel. However, this is not the case, because again the oscillating fields have non-renormalizable couplings to these lighter fields and the decay rate follows $\Gamma_{\phi, N \to \gamma\gamma} \sim M_*^3/M_p^2$. Notice, its resemblance with that of Eq. (36), if $m_\phi \sim M_*$. This is an important lesson and all it tells us that $\phi, N$ decaying into Higgses and into lighter degrees of freedom is equally preferred. However, if there were some radiation which could thermalize, then in such a case, the Universe could in principle follow an equation of state which would be determined mainly by a mixture of relativistic species, inflaton, and non-relativistic Higgses. This would only affect the thermal history of the Universe, and not the dynamics, because inflaton energy density is dominating over all. It has been shown in the simplest situation where inflaton and radiation components are allowed, that the instantaneous temperature of a plasma might exceed the reheat temperature of the Universe. In order this to happen one must also satisfy $H \geq \Gamma_{\phi} + \Gamma_{N}$ \[13\]. We notice that this condition may be satisfied in our case at the very beginning of the oscillations, but in spite of large initial amplitudes for $\phi, N$, the oscillations are damped quickly.

The temperature of the plasma may reach its maximum when $a/a_0 \sim 1.48$, where $a$ denotes the scale factor of the Universe and the subscript 0 denotes the era when inflation comes to an end. The maximum temperature is given by \[20\]

$$\frac{T_{\text{max}}}{T_r} = 0.77 \left( \frac{9}{5\pi^3 g_*} \right)^{1/8} \left( \frac{H_0 M_*}{T_r^2} \right)^{1/4},$$  \hspace{1cm} (39)

where $g_*$ denotes the effective number of relativistic degrees of freedom. For the purpose of illustration if we fix $H_0 \sim M_*$ and $T_r \sim 100$ MeV we found $T_{\text{max}} \sim 10^5$ GeV. This temperature is much higher than the actual reheat temperature, and it seems that this is a generic prediction of inflationary scenarios in extra dimensions. However, this warrants a preferred production of relativistic species from the decay of inflaton. This is unfortunately certainly not the case with the present situation. In order to proceed with our present
discussion, we note that after reaching the maximum temperature, the temperature of the intermediate plasma decreases as

\[ T \sim 1.3 \left( \frac{g_*(T_{\text{max}})}{g_*(T)} \right)^{1/4} T_{\text{max}} a^{-3/8}, \tag{40} \]

The important thing to notice here is that the thermal history of the Universe is again different: the temperature does not drop like \( T \propto a^{-1} \) so that the entropy of the plasma \( s \propto a^{15/8} \). This will eventually dilute any KK graviton being produced during this era, and, as long as \( n_G/n_\gamma < 1 \), we would not expect any further alteration of the results mentioned in Eqs. (39) and (40).

**IV. BARYOGENESIS**

The constraints on inflationary parameters which we have been discussing so far must be bear in mind when discussing baryogenesis. We certainly need a concrete mechanism from particle physics in order to address baryogenesis. The problem here is that we should introduce some baryon number violating processes. However, in theories where the fundamental scale is low naturally introduces dangerous higher order operators. For instance, dimension 6 operators such as \( QQQL/\Lambda^2 \), where \( Q \)'s and \( L \) correspond quark and lepton SM doublets, which can mediate proton decay, unless \( \Lambda \geq 10^{15} \) GeV. This operator violates baryon and lepton number conservation by \( \Delta B = \Delta L = 1 \). There is also a possibility of having right handed singlets \( uudd/\Lambda^2 \), for which \( \Lambda \geq 10^{12} \) GeV. There are other processes which violate baryon number, such as neutron anti-neutron oscillations with a dimension 7 operator \( uudd/\Lambda^2 \), which implies \( \Lambda \geq 10^5 \) GeV. An alternative is the dimension 9 operator \( QQHQHQH/\Lambda^5 \) with \( \Lambda \geq 10^4 \). (The experimental bound on proton life time is \( \tau_p \geq 10^{33} \) years [21], and for neutron anti-neutron oscillations \( \tau_{\bar{n}n} > 1.2 \times 10^8 \) seconds [22,23].)

We assume that the above mentioned operators can be avoided in some way or other. It is very difficult to come up with a model where baryon violating operators are not constrained at a TeV scale. Thus, one has to ensure that such operators are not being reintroduced by the mechanism of baryogenesis. Especially, in our case we ought to be careful with an operator such as \( \chi QQQL \). In case \( \chi \) develops a vacuum expectation value \( \sim M_* \), fast proton decay is inevitable.

Moreover, in the context of extra dimensions, leptogenesis is not a viable mechanism as we shall argue now. In leptogenesis a net lepton number is produced in the decay process of a heavy fermionic singlet such as a right handed neutrino, which is then processed into baryon number by anomalous \( B + L \) violating sphaleron interactions [24]. However, the electro weak sphaleron transitions are active only up to 100 GeV [25]. In our case viability of leptogenesis has a simple catch. A singlet right handed neutrino can naturally couple to the SM lepton doublet and the Higgs field through \( yLHN \). This leads to a potentially large Dirac mass term unless the Yukawa coupling \( y \sim 10^{-12} \). Moreover, now the see-saw mechanism fails to work, since, the largest Majorana mass we may expect can never be larger than the fundamental scale. Therefore, given a very small neutrino mass \( \sim y^2 \langle H \rangle^2/M \sim y^2 \cdot \mathcal{O}(1) \) GeV, we still have to fine tune \( y^2 \lesssim 10^{-10} \), in order to obtain the right order of magnitude.
for the neutrino mass. In any case the decay rate of the right handed neutrino is very suppressed and it is similar to Eq. (36). This means that when the right handed neutrino decays into the SM fields, the background temperature is of order of the reheat temperature $\sim \mathcal{O}(1 − 10)\ MeV$. At this temperature the sphaleron rate is exponentially suppressed, which is actually a setback for leptogenesis. A reheat temperature of at least $\mathcal{O}(1 − 100)\ GeV$ is required for making this scenario viable [26], which could in principle be attained if the number of extra spatial dimensions is increased to six, at least from the point of view of the normalcy temperature. However, as we observed before, in the class of models we considering this is not the case. Indeed, as mentioned above already, unless we increase the fundamental scale, the largest reheating temperature we can get from the inflaton decay is just barely about $100\ MeV$, regardless the number of extra dimensions. This makes leptogenesis even more difficult.

A different possibility that sphalerons can reprocess a pre-existing charge asymmetry into baryon asymmetry [27] reflected in an excess of $e_L$ over anti-$e_R$ created during inflaton oscillations. This mechanism requires $(B + L)$ violating processes to be out of equilibrium before $e_R$ comes into chemical equilibrium, such that the created baryon asymmetry could be preserved. Again, this has to happen at or above $100\ GeV$. One could then assume that inflaton decays preferably into relativistic species such that the plasma thermalizes to a temperature above $100\ GeV$ [28,29]. However, this scenario cannot be implemented in the context of large extra dimensions because in this case the oscillating inflaton field injects more entropy to the thermal bath as discussed earlier. This leads to an immediate dilution of whatever baryon asymmetry has been created prior to the reheating era. The dilution factor is given by:

$$\gamma^{-1} = \left( \frac{s(T_r)}{s(T_{EW})} \right) = \left( \frac{g_s(T_r)}{g_s(T_{EW})} \right) \left( \frac{T_r}{T_{EW}} \right)^3 \left( \frac{a(T_r)}{a(T_{EW})} \right)^3,$$

where $s$ is the entropy and $T_{EW} \sim 100\ GeV$. For a lower reheat temperature such as $T_r \sim 1\ MeV$, the above expression gives $\gamma^{-1} \gtrsim 10^{25}$ (assuming $T \propto a^{-3/8}$ and $g_s(T_c) \approx g_s(T_r)$). Therefore, one concludes that the initial $n_b/s$ has to be extremely large $\gtrsim 10^{15}$, this is certainly an extraordinary requirement on any natural model of baryogenesis and practically impossible to achieve. Even if we increase reheat temperature $T_r \sim 100\ MeV$, we would still require $n_b/s$ of order one.

We have learned two important lessons. First, the large entropy production during the last stages of reheating can in principle wash away any baryon asymmetry produced before electroweak scale. Second, it is extremely unlikely that leptogenesis would work. The only simple choice left is to produce baryon asymmetry directly. The sole mechanism which can be successful in these circumstances seems to be the Affleck-Dine (AD) baryogenesis [11], which we are going to discuss now.

V. AFFLECK-DINE BARYOGENESIS

For the details of the AD baryogenesis we refer the readers to Ref. [30]. In our case the relevant questions are following:
(1) Can we have a condensate?

Unlike in the usual AD mechanism, which is based on Minimal Supersymmetric Standard Model, we do not have flat directions automatically in-built in our model, or, protected by supersymmetry. Even if we invoke some flat directions, it cannot be associated with a condensate carrying $B$ or $L$. The condensate can not be protected alone due to lack of any symmetry argument. One can not form a SM condensate in the bulk because bulk gravity is color and flavor blind. This suggests that we necessarily have to assume some fundamental scalar field.

(2) What charge should it carry?

The AD field, which we denote here by $\chi$ has to be a gauge singlet carrying some global charge under $U(1)_\chi$. This global charge has to be broken dynamically in order to provide a small asymmetry in the current density. This shall be reflected by generating an excess of $\chi$ over $\bar{\chi}$. The charge associated with this AD field must be such that baryon number is violated maximally. All that we need is to ensure that the SM quarks maintain the small asymmetry between baryons and anti baryons. Notice that $\chi$ field as such does not create the baryon asymmetry. The asymmetry is produced due to the difference in number density of $\chi$ over $\bar{\chi}$. This is the most important aspect of our model. This small asymmetry is then transferred via the decay of $\chi$ and $\bar{\chi}$ into the SM quarks and leptons. This constrains the decay channel for $\chi$ and $\bar{\chi}$ which we discuss next.

(3) What kind of interactions should it have?

This is a non-trivial issue because we do not have a condensate made up of SM quarks and leptons. We need to assume that $\chi$ interactions with SM fields conserve $U(1)_\chi$ symmetry. Therefore, the quarks and leptons must carry a non zero global $\chi$ charge. However, we do not want the $\chi$-$\bar{\chi}$ asymmetry to be transferred to non-baryon number violating interactions such as interactions involving the Higgses. Therefore, the Higgs field should not carry a global $\chi$ charge, forbidding $\chi$ decay into Higgses.

Regarding the decay channels, coupling of $\chi$ to SM dimension 3 operators such as $Q\bar{Q}$, which has $\Delta B = 0$, cannot provide baryon asymmetry. Similarly, for higher dimensional operators such as $Qhq$, where $Q$ ($q$) is the right (left) handed SM quark and $h$ is the Higgs doublet. A dimension 5 operator cannot be constructed at a quark level because of the color symmetry. The lowest order turns out to be the dimension 6 operator $QQQl$, for which $\Delta B = \Delta L \neq 0$, which can certainly transmute any asymmetry in $\chi$ to the quark sector. Thus, the global charge carried by $\chi$ has to be chosen such that $QQQl$ carries the opposite one. Thus, forbidding the presence of this operator alone on the theory and making the coupling $\chi QQQl$ the lowest possible order for $\chi$. Also note this operator can mediates proton decay too, unless one ensures that $\chi$ does not develop a vacuum expectation value, which is a severe constraint on model building, but not a difficult one to realize. In the same spirit one may check those operators which induce $n - \bar{n}$ oscillations. Again, an effective $\Delta B = 2$ operator $UDDUDD$, or, such as $(QQQh)^2$ cannot be induced by, say, integrating out $\chi$. To avoid the decay of $\chi$ into Higgs fields we take $h$ to be chargeless under $U(1)_\chi$.

(4) Can AD field be either $\phi$, or, $N$?

Given the constraints, neither $N$ nor the inflaton field $\phi$ can act as an AD field. The auxiliary field $N$ develops a vacuum expectation value after the end of inflation, which would
immediately induce proton decay. Regarding $\phi$, the bad news is that, since the asymmetry in AD field depends on the initial configuration, a large amplitude oscillations in $\phi$ can induce an undesirably large $\phi - \bar{\phi}$ asymmetry. Hence we invoke a separate AD field, a fundamental scalar field $\chi$ with a global charge $U(1)_\chi$.

We remind the readers that the inflaton energy density $\rho_I$ should govern the evolution of the Universe. Eventually the decay products of the inflaton should be responsible for reheating the Universe. The inflaton decays before $\chi$ decays via baryon violating interaction and generates a baryon asymmetry given by

$$\frac{n_b}{s} \approx \frac{n_b}{n_\chi} \frac{T_r}{m_\chi} \rho_I.$$  \hspace{1cm} (42)

The final entropy released by the inflaton decay is given by $s \approx \rho_I/T_r$. The ratio $n_b/n_\chi$ depends on the total phase accumulated by the AD field during its helical motion in the background of an oscillating inflaton field. In our calculation we shall always approximate the total phase $\sim O(1)$.

In theories with extra dimensions there are two choices for the AD field. It could either be a brane field or a bulk field like $\phi$ and $N$. If we assume that the AD field is a brane field, then it cannot have an effective mass higher than the fundamental scale, and the effective initial field amplitude $|\chi| \leq M_*$. These constraints suggest that the energy stored in $\chi$ can at most be $\rho_\chi \approx m_\chi^2 M_p^2$, where $m_\chi \leq M_*$. The energy density stored in $\phi$ and $N$ is quite large because these fields can have initial amplitudes close to Planck scales so that $\rho_I \approx M_\phi^2 M_p^2$. Thus we would find

$$\frac{n_b}{s} \sim \frac{T_r}{M_p} \frac{m_\chi}{M_*} \approx \frac{10^{-34}}{M_*} \ll 10^{-10}.$$  \hspace{1cm} (43)

for $T_r \sim O(1-10) \text{ MeV}$, we conclude that $\chi$ cannot be a brane field.

Let us therefore promote AD field to the bulk whence the energy density stored in the AD field is $\rho_\chi \sim m_\chi^2 M_p^2$ since now $\chi$ can have large initial amplitude $|\chi| \geq M_*$. This implies that the maximum baryon to entropy ratio is given by

$$\frac{n_b}{s} \approx \left( \frac{T_r}{M_*} \right) \frac{m_\chi}{M_*} \sim 10^{-10} \left( \frac{m_\chi}{1\text{GeV}} \right),$$  \hspace{1cm} (44)

where we assumed $T_r \sim 10 \text{ MeV}$ and $M_* \sim 10 \text{ TeV}$ for concreteness. Thus we can achieve an adequate baryon to entropy ratio from $\chi$ decaying into SM quarks and leptons. Of course, the numeric value of the ratio depends on the initial amplitude for $\chi$ and its mass. Note, if we set $m_\chi \sim M_*$, we obtain the initial amplitude $\chi \sim M_{GUT} \sim 10^{16} \text{ GeV}$.

Now we can also estimate the decay rate for $\chi$ field and $\bar{\chi}$ fields. The interaction is given by

$$\kappa \frac{\chi}{M_p^2 M_p} \chi Q Q l.$$  \hspace{1cm} (45)

We sum over all possible channels, such as various color and family index combinations which can be of the order of thousand. On the other hand we strictly assume that inflaton is decaying into Higgses. Hence we estimate the decay rate as
In order that the decay products of $\chi$ thermalizes before Nucleosynthesis we have to assume $m_\chi \approx M_*$. This does not leave much freedom for the masses which makes the model more predictive but also demands some level of fine tuning if AD baryogenesis is to be successful, as we shall discuss below.

VI. THE MODEL

Let us now describe our model. We assume that the inflaton sector is responsible for breaking $U(1)_\chi$ charge dynamically. The AD potential can be written as

$$V_{AD}(\phi, N, \chi_1, \chi_2) = \kappa_1^2 \left( \frac{M_*}{M_p} \right)^2 N^2 (\chi_1^2 + \chi_2^2) + \frac{\kappa_2^2}{4} \left( \frac{M_*}{M_p} \right)^2 (\chi_1^2 + \chi_2^2)^2$$

$$+ \kappa_3^2 \left( \frac{M_*}{M_p} \right)^2 \phi N (\chi_1^2 - \chi_2^2),$$

(47)

where $\kappa_1, \kappa_2, \kappa_3$ are constants, and $\chi_1$ and $\chi_2$ are the real and imaginary components of the complex field $\chi$. Note, all the terms are Planck mass suppressed, because Eq. (47) depicts an effective four dimensional potential derived from higher dimensional Lagrangian by integrating out the extra spatial dimensions. From Eq. (47), it is evident that since during inflation the auxiliary field is trapped in the false vacuum $N = 0$, this renders $\chi_1$ and $\chi_2$ massless and the AD potential becomes almost flat. We also notice that the last term is also not present during inflation. Therefore, the only contributions comes from the self coupling which obviously allows the AD field to evolve. The details of the field evolution shall be discussed in the coming subsections. Note that we do not include a term of the type $(M_* / M_p)^2 \phi^2 \chi^2$ as it would ruin both inflation and baryogenesis because of the large-amplitude oscillation of $\chi$; this is a fine tuning we have to perform on our model for which we have no dynamical explanation.

A. Constraining the initial amplitude for the AD field

Dynamical evolution of multi scalar fields induces density perturbations of two kinds; adiabatic and isocurvature. Since, in our case $\chi_1$ and $\chi_2$ are massless during inflation they contribute to the adiabatic fluctuations. In order to obtain a scale invariant spectrum, the fluctuations should be mainly generated by $\phi$ and $N$. However, this imposes an upper bound on the allowed amplitude for the AD field at the time when the modes relevant for large scale structure formation are leaving the horizon during inflation, which happens in our case during the last 43 e-foldings of inflation.

In the regime when the main inflationary potential is due to the inflaton sector, the slow-roll parameter $|\eta|$ can be written as [31]

$$|\eta| \approx \frac{M_p^2}{8\pi} \frac{V''(\phi, N)}{V(\phi, N)} - \frac{M_p^2}{8\pi} \frac{V'_{AD} V''_{AD}}{V(\phi, N)V'(\phi, N)},$$

(48)
where prime in $V_{AD}$ denotes derivative with respect to AD field $\chi$. According to Eq. (27), the second term in the above equation must be smaller than $10^{-10}$. This leads to

$$|\chi(0)|^5 \leq 32\pi \times 10^{-10} \times \frac{\lambda^2}{\kappa_2^2} \left( \frac{N_0}{M_*} \right)^5 M_p^5.$$  \hfill (49)

If $\lambda, \kappa_2 \sim \mathcal{O}(1)$, and $N_0 \sim M_*$, we obtain an initial amplitude for $|\chi(0)| \leq 10^{-2} M_p$. This is a constraint which we should bear in mind while estimating the total baryon asymmetry. Note, for a reasonably fast phase transition $N_0 < M_*$, the amplitude for $\chi(0)$ becomes even smaller than the above limit.

Similarly one might expect isocurvature perturbations arising from the AD field. Notice that the angular direction of AD field is effectively massless, but gains a dynamical mass $\geq H$ just after inflation. This is unlike the case of a supersymmetric AD baryogenesis where the field remains massless even after the end of inflation provided there is no supergravity correction to the $U(1)$ violating term. Nevertheless, in our case we would expect isocurvature fluctuations during inflation because there are more than one dynamical scalar fields present during inflationary phase. It is well known that in the case of isocurvature fluctuations it is easier to constrain the spectral index \cite{32}, rather than the amplitude of perturbations. This restricts the initial amplitude for the AD field very much in a similar vein as in Eq. (49). However, since we are assuming that the main contribution to the density perturbations is coming from the adiabatic sector, rather than the isocurvature fluctuations, we would not expect any further improvement on the limit we have already obtained upon the initial amplitude for $\chi_1$ and $\chi_2$ from Eq. (49).

**B. Dynamics of AD field**

The dynamics of $\chi$ is complicated. Even though $\chi$ has no effective mass during inflation, it has an effective quartic self coupling which determines its dynamics (see Eq. (17)). The equations of motion in terms of the component fields $\chi_1, \chi_2$ are given by

$$\ddot{\chi}_1 + 3H\dot{\chi}_1 = -\kappa_2^2 \left( \frac{M_*}{M_p} \right)^2 (\chi_1^2 + \chi_2^2)\chi_1,$$  \hfill (50)

$$\ddot{\chi}_2 + 3H\dot{\chi}_2 = -\kappa_2^2 \left( \frac{M_*}{M_p} \right)^2 (\chi_1^2 + \chi_2^2)\chi_2.$$  \hfill (51)

We remind that $\chi_1$ and $\chi_2$ are the components of a a bulk field, therefore, they would have naturally taken initial values close to the fundamental scale in higher dimensions but close to the Planck scale in four dimensions. From the point of view of four dimensions, the fields simply roll down from the Planck scale because the self coupling induces a curvature terms for $\chi_1$ and $\chi_2$ fields. We note that the suppression in the couplings is very small $\sim (M_*/M_p)^2$ if $\kappa_2 \sim \mathcal{O}(1)$. The fields are very weakly self coupled. However, in the process of rolling down the potential their effective running mass becomes of $\sim H$, given by Eq. (19). When this happens their dynamics is effectively frozen at a particular amplitude which observes the constraint derived independently in the earlier section, see Eq. (19).
As we have discussed, we may assume that $\chi_1$ and $\chi_2$ collectively follow a similar trajectory. The dynamics of $|\chi|$ freezes when $3H|\dot{\chi}| \approx -\kappa_2^2(M_*/M_p)^2|\chi|^3$. This immediately gives us a simple solution for the largest values of the fields:

$$\chi_1(0) \sim \chi_2(0) \leq \frac{1}{\kappa_2} \left( \frac{M_p}{M_*} \right)^2 \sqrt{\frac{3H}{2\Delta t}} \approx \frac{\lambda}{\kappa_2} \sqrt{\frac{4\pi}{N_e}} M_p,$$

(52)

where $N_e = H\Delta t$ is the total number of e-foldings, which could be larger than $\sim 130$. The total number also takes into account of inflation during the radion stabilization, but for the structure formation it is the last 43 e-foldings of inflation which matters. For a simple estimate, if $\lambda \sim \kappa_2$, we obtain $|\chi(0)| \leq 10^{18}$ GeV, in well accordance with Eq. (19). When inflation comes to an end all the fields begin oscillations; for $\phi$ and $N$, the initial amplitudes are given by Eq. (28), while for $\chi_1$ and $\chi_2$ the initial amplitudes are determined by Eq. (19).

The post-inflationary dynamics of $\chi_1$ and $\chi_2$ depends on $\phi$ and $N$, which are both oscillating. As a result $\chi_1$ and $\chi_2$ become massive fields (see Eq. (17)). This leads to the equations of motion

$$\ddot{\chi}_1 + 3H \dot{\chi}_1 = -2\kappa_1^2 \left( \frac{M_*/M_p}{2} \right)^2 N^2 \chi_1 - \kappa_2^2 \left( \frac{M_*/M_p}{2} \right)^2 (\chi_1^2 + \chi_2^2) \chi_1 - 2\kappa_3^2 \left( \frac{M_*/M_p}{2} \right)^2 \phi N \chi_1,$$

(53)

$$\ddot{\chi}_2 + 3H \dot{\chi}_2 = -2\kappa_1^2 \left( \frac{M_*/M_p}{2} \right)^2 N^2 \chi_2 - \kappa_2^2 \left( \frac{M_*/M_p}{2} \right)^2 (\chi_1^2 + \chi_2^2) \chi_2 + 2\kappa_3^2 \left( \frac{M_*/M_p}{2} \right)^2 \phi N \chi_2.$$

(54)

There is an effective mass term for $\chi$ fields which is again field dependent. The last term in the above equations comes with an opposite sign and this is responsible for giving rise to a small asymmetry in $\chi$ over $\bar{\chi}$. As a simplest approximation, we neglect the back reaction of $\chi$ fields on the background fields, which is actually true as long as $N, \phi$ are responsible for generating the final entropy of the Universe. This assumption simplifies the situation and allows us to estimate $\chi$ asymmetry created by the motion of $\chi_1$ and $\chi_2$ analytically.

We have earlier pointed out that $\phi$ and $N$ oscillate several times before they decay at the reheating temperature. While the initial oscillations are quite large $\sim M_p$, the energy density stored in the oscillations is quite small due to the presence of very small couplings of order $(M_*/M_p)^2 \sim 10^{-28}$. Likewise, $\chi_1$ and $\chi_2$ also oscillate with a large amplitude and they also go through several oscillation periods before they decay at a rate given by Eq. (16). By substituting Eq. (28) in Eq. (17), we may estimate the effective masses

$$m_{\chi_1} \approx \sqrt{2}\kappa_1 N_0, \quad m_{\chi_2} \approx \sqrt{2}\kappa_1 N_0.$$

(55)

The coupling constants $\kappa_1, \kappa_2$ can be tuned to obtain $m_{\chi_1} \sim m_{\chi_2} \approx M_*$ in order to match the decay rate of $\Gamma_\chi \sim \Gamma_\phi \sim \Gamma_N$. This finally ensures that the AD field decays along with $\phi$ and $N$ fields, and this also prevents the decay of $\phi$ and $N$ into $\chi$ fields. If we compare Eqs. (29), and (53), we notice that all the masses are of same order with a small variation due to different couplings. If we set $\kappa_1 \geq \kappa_2 \sim \lambda \sim g$, we can ensure that $\chi_1$ and $\chi_2$ go through more oscillation periods than the background fields $\phi$ and $N$. Here we tacitly assume that the oscillations in $\chi$ do not have any dynamical back reaction on $\phi$ and $N$ oscillations. This is true if the field couplings are weak and indeed, in our case the couplings
such as $\kappa_1^2(M_* M_p)^2$, $\kappa_3^2(M_* M_p)^2$ are extremely small even if we choose $\kappa_1 \sim \kappa_3 \approx \mathcal{O}(1)$. Such a weak coupling also ensures that the field trajectories do not give rise to a chaotic behavior. In order to avoid any chaotic behavior it is necessary that all the fields oscillate with a similar frequency around the bottom of their respective potential. Hence we choose $\kappa_1 \equiv \lambda \equiv g$ while $\kappa_2$ is constrained by the expressions Eq. (49), and (52), and $\kappa_3$ shall be constrained by the baryon to entropy ratio. However, in order to have a helical motion for $\chi_1$ and $\chi_2$, we ought to have $\kappa_1 > \kappa_3$, and $\kappa_2 > \kappa_3$. We still have to estimate the classical behavior of $\chi_1$ and $\chi_2$, which do not develop any vacuum expectation value. Their classical evolution can be estimated by

$$\chi_1(t) \sim \chi_1(0) A(t) \cos(m_\chi t), \quad \chi_2(t) \sim \chi_2(0) A(t) \cos(m_\chi t), \quad (56)$$

where the amplitudes $\chi_1(0)$ and $\chi_2(0)$ are constrained by Eqs. (49) and (52). Both the amplitudes decrease in time as $A(t) \propto 1/t$. The frequency of the oscillations are determined by Eq. (55). In what follows we shall always assume that the relative phase between the fields $\chi_1$ and $\chi_2$ is of order $\mathcal{O}(1)$. We now have all the tools to address the generation of $\chi \sim \bar{\chi}$ charge asymmetry from the classical evolution of all the fields. This shall provide us with a final baryon to entropy ratio in this particular model.

**C. Baryon to entropy ratio**

The $CP$ violation in our model is given by the last term in Eq. (47). For a charged scalar field this is equivalent to $C$ violation. The $B$ violation arise via the decay of $\chi$, because the decay products have $\Delta B \neq 0$, and we have a non-trivial helical oscillations in $\chi$ which accumulates net $CP$ phase which is transformed into asymmetric $\chi$. The net $\chi$ asymmetry; $n_\chi$ can be calculated by evaluating the Boltzmann equation:

$$n_\chi = \frac{i}{2} \left( \bar{\chi}^* \chi - \chi^* \bar{\chi} \right). \quad (57)$$

With the help of Eqs. (53) and (54), we can rewrite the above expression as

$$\dot{n}_\chi + 3 H n_\chi = 4 \kappa_3^2 \left( \frac{M_*}{M_p} \right)^2 \langle N(t) \phi(t) \rangle \chi_1(t) \chi_2(t). \quad (58)$$

The right hand side of the above equation is the source term which generates a net $\chi$ asymmetry through a non-trivial motion of $\chi_1$ and $\chi_2$ fields. We integrate Eq. (58) from $t_0$ which corresponds to the end of inflation up to a finite time interval.

$$n_\chi a^3 = 4 \kappa_3^2 \left( \frac{M_*}{M_p} \right)^2 \int_{t_0}^t \langle N(t') \phi(t') \rangle a^3(t') \chi_1(t') \chi_2(t') dt'. \quad (59)$$

The upper limit of integration signifies the end of reheating. We assume that the oscillations continue until the fields decay completely. Before we perform the integration, we notice that the integrand decreases in time. This can be seen as follows; first of all notice that the approximate number of oscillations are quite large before the fields decay $\sim (M_p/M_*)^2$. This
allows us to average $\phi$ and $N$ oscillations; from Eqs. (31) and (32) we get the time dependence $\langle \phi N \rangle \sim \Phi^2(t) \sim 1/t^2$. Similarly, from Eq. (56), $\langle \chi_1 \chi_2 \rangle \sim 1/t^2$ provides another suppression. While taking care of the expansion, where the scale factor behaves like $a(t) \sim t^{2/3}$, the overall behavior of the integrand follows $\sim 1/t^2$. This suggests that the maximum contribution to $\chi$ asymmetry comes only at the initial times when $t_0 \sim 1/H_0$, where $H_0$ is determined by Eq. (19). The right hand side of the above equation turns out to be

$$n_\chi \approx \frac{2}{27} \kappa_2^2 M^2 \frac{\langle \gamma^2 \rangle}{H_0}. \quad (60)$$

We have assumed that the total $CP$ phase, which is given by two factors: an initial phase determined arbitrarily during inflation and the final dynamical phase which is accumulated during the oscillations, is of the order $\sim O(1)$. We have also assumed $N_0 \sim M_*$. The final ratio of $\chi$ number density produced and the entropy is given by

$$\frac{n_\chi}{s} \approx \frac{2 \kappa_2^2}{27 \lambda^2} \left( \frac{\langle \gamma^2 \rangle}{M_*} \right)^2 \left( \frac{T_r}{H_0} \right) \leq \frac{2 \sqrt{6} \pi}{27} \left( \frac{\kappa_3}{\kappa_2} \right)^2 \left( \frac{1}{\lambda N_{e}} \right) \left( \frac{T_r}{M_*} \right), \quad (61)$$

where we have used the fact that $s \propto a^3$ in our case. This is the final expression for $\chi$ asymmetry produced during the helical oscillations of $\chi$ and is to be compared with the observed baryon asymmetry, whose range is $4(3) \times 10^{-10} \leq \eta_B \equiv n_B/n_\gamma \leq 7(10) \times 10^{-10}$ \cite{13}. The upper bound on the baryon to entropy ratio given by Eq. (61) depends on the amplitude of $\chi_1$ and $\chi_2$, which were frozen during inflation, it also depends on the number of e-foldings, which could be $\sim 100$. In order to evaluate Eq. (61), we may take an example: $T_r \sim 100$ MeV, and $M_* \sim 100$ TeV. This gives an asymmetry of order $\sim 10^{-10}$, if we choose the couplings of order one. The final asymmetry in $\chi$ is transferred to the SM quarks via baryon number violating interaction mentioned in Eq. (45). The asymmetry is injected into thermal bath along with the inflaton decay products. It is essential that the thermalization takes place after AD field has decayed in order not to wash away the total baryon asymmetry.

Finally, we mention that our model is quite generic. The robustness of the model is that it can work for arbitrary number of extra spatial dimensions. The model predicts baryogenesis just above nucleosynthesis scale. The model does not rely on first order phase transition of electroweak baryogenesis, nor does it depend on sphaleron transition rate. Our approach is similar to Ref. \cite{33}, where this model has been embedded in a supersymmetric theory in four dimensions. Like in the case of AD baryogenesis \cite{11}, our setup can be tested by the forthcoming accurate measurements of the spectral index of the microwave background; our model predicts a very flat spectrum.

VII. CONCLUSION

In summary, we have presented a natural mechanism of baryogenesis in the context of low quantum gravity scale with large compact extra dimensions. Our mechanism is generic and it is independent of the fundamental scale and the number of compact extra dimensions. This mechanism does not rely on any extra assumption other than invoking a fundamental scalar field that lives in the $4 + d$ dimensional space time. The baryogenesis scheme can work
at any temperature lower than the electro weak scale. This is because of the presence of non-renormalizable couplings which automatically reheats the Universe with a temperature close to BBN. This feature is independent of the number of extra dimensions. Our mechanism does not depend on the rate of sphaleron transition and also does not rely on leptogenesis.

The three requirements for baryogenesis is fulfilled in our case as follows. The $C$ and $CP$ violation is a dynamical process in our case. The AD field has a charge associated with a global $U(1)_{\chi}$ symmetry. This symmetry is broken once the AD field gains a dynamical mass from its coupling to the inflaton sector. This happens once inflation comes to an end and when all the fields start oscillating around their respective minima. The dynamics of the AD field is solely responsible for $CP$ violation and the initial $CP$ phase is determined during inflation which we have assumed to be of order one. During post inflationary era the real and the imaginary parts of the AD field has different equations of motion and as a result the motion of AD field is helical in the field space. During this helical motion the AD field accumulates a net asymmetry in $\chi$ over $\bar{\chi}$. This asymmetry is then transferred to the SM sector via a baryon violating interaction which does not allow fast proton decay.

The decay of inflaton into Higgses and their subsequent decay generates a net entropy in the Universe. The thermalization of the Universe is quite late and the final reheat temperature is as low as 100 MeV for a fundamental scale as small as 100 TeV. The AD field decays into quarks and leptons directly imparting the total baryon asymmetry it has generated during the helical motion, obviously, the net $CP$ phase generated during this helical motion is understood to be also of order one. We note that the baryon to entropy ratio is determined by the ratio of the reheat temperature and the fundamental scale. The ratio also depends on the number of e-foldings. This tells us that the evolution of the massless AD field during the inflationary stage also counts in producing a net baryon asymmetry, through fixing the initial amplitude of the AD field. The most important aspect of our model is that the model parameters are very constrained from the density perturbations during inflation. The initial amplitude of the AD field is also constrained in order not to depart from the scale invariance of density perturbations measured by the satellite and balloon experiments. The constraint which we apply on the amplitude of the AD field comes purely from adiabatic perturbations. However, presence of many fields automatically introduces a possibility of isocurvature fluctuations which we shall work out in future. In this sense, our model can be falsifiable. As a final note we mention that our dynamical mechanism of producing baryon asymmetry can be used even in four dimensions and the mechanism is capable of generating baryon asymmetry at a scale much lower than the electro weak scale.

**VIII. ACKNOWLEDGEMENTS**

The authors are thankful to Mar Bastero-Gil, Zurab Berezhiani, Pierre Binetruy, Sacha Davidon, Alexandre Dolgov, and Katrin Heitmann for helpful discussions. R.A. is supported by “Sonder-forchschungsberich 375 für Astro-Teilchenphysik” der Deutschen Forschungs gemeinschaft, K.E. partly by the Academy of Finland under the contract 101-35224, and A.M. acknowledges the support of The Early Universe network HPRN-CT-2000-00152. A.M. acknowledges the hospitality of the Helsinki Institute of Physics where part of the work has been carried out.
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