New inverse method of centrifugal pump blade based on free form deformation

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Abstract. In this research, a new inverse method for centrifugal pump blade based on free form deformation is proposed, the free form deformation is used to parametric the pump blade. The blade is implanted to a trivariate control volume which is equally subdivided by control lattice. The control volume can be deformed by moving the control lattice, thereupon the object is deformed. The flow in pump is solved by using a three dimensional turbulent model. The lattice deformation function is constructed according to the gradient distribution of fluid energy along the blade and its objective distribution. Deform the blade shape continually according to the flow solve, and we can get the objective blade shape. The calculation case shows that the proposed inverse method based on FFD method is rational.

1. Introduction
With the development of the flow computational technology and the modern flow measurement technology, the simulation and experiment investigation for pump inner flow has achieved rapid progress. The topics of multiphase flow, complex turbulent flow model, the transient performance and the flow structure interaction etc are the hot topics of hydraulic machinery. Many achievements have been obtained in this area. As the complexity of the inverse problem, the progress in the inverse design of hydraulic machinery impeller and its optimization are very slow [Ref.1-7], in this research, the free form deformation method is used to parametric the pump blade surface, and the new inverse method of pump blade is realized by controlling the distribution of the energy gradient of the fluid along the blade surface.

2. The Free Form Deformation Technology

2.1. The overview of FFD
In the inverse problem, alternate the flow simulation and the modelling of blade, and then we can get the optimum design. The parametric control the blade surface is one of the key problems. In this research the free form deformation is used to parametric the blade surface. In 1984 Barr has innovatively proposed the idea of solid deformation by twisting, bending and tapering; it is a very powerful and elegant design tool. On the basis Sederberg and Parry [8] proposed the free-form deformation method(FFD). The FFD method firstly constructs a parametric trivariate Bezier control volume, and then embeds the object which we wish to deform in the control volume. The control volume and the object are imagined also to be flexible. The control volume can be deformed by move
the control lattice in it, thereupon the object is deformed also. Jamshid A. Samareh[9] has introduced the FFD method for airfoil aerodynamic shape optimization, and the calculation cost is greatly reduced.

2.2. The Algorithm of the FFD

The essence of the free form deformation is form the mapping between the point on surface and the lattices, and the main algorithm of the FFD is as follows

①. First construct a parallelepiped control volume, which could embed the object to be deformed. We construct the local coordinate system $O'S'TU$ on the parallelepiped region, as shown in figure 1, then any point $X$ can be expressed as $(s,t,u)$ in the system.

\[
\overrightarrow{X} = \overrightarrow{X_0} + s\overrightarrow{S} + t\overrightarrow{T} + u\overrightarrow{U},
\]

Where $\overrightarrow{X_0}$ is the origin of the local coordinate; $0 \leq s, t, u \leq 1$ .

②. Construct the control point in the parallelepiped control volume, and these point lie on a lattice. The control can be uniformly divided into $l$, $m$, and $n$ parts by the plane parallel to $O'UT$, $O'SU$, $O'ST$ respectively. These intersections of these plan are the control lattice points $P_{i,j,k}$. $P_{i,j,k}$ can be written as

\[
P_{i,j,k} = \overrightarrow{X_0} + \frac{i}{l}\overrightarrow{S} + \frac{j}{m}\overrightarrow{T} + \frac{k}{n}\overrightarrow{U}
\]

So, the Cartesian coordinate of any point in the parallelepiped control volume can be expressed as following.

\[
X(s,t,u) = \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} P_{i,j,k}B_i(s)B_j(t)B_k(u)
\]

Where, $P_{i,j,k}$ is the control lattices coordinate, $B_i(s)$, $B_j(t)$, $B_k(u)$ are Bernstein basic function

![Figure 1. The control volume and lattices](image)

③. Embed the objective surface to be deformation into the control volume, and Cartesian coordinate of the points on the objective surface are converted to local coordinate $(s,t,u)$.

④. The control volume is deformed when the position of control points is changed, the objective surface is deformed also which is embedded in the control volume. But the local coordinate of the points on objective surface remain the same. The object surface after deformation can be expressed as following

\[
X'(s,t,u) = \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} P'_{i,j,k}B_i(s)B_j(t)B_k(u)
\]

Where, $P'_{i,j,k}$ is the control lattices coordinate after deformation, the above formula (4) is the deformation function for free surface. The deformation of surface is constructing a mapping from $\mathbb{R}^3$ to $\mathbb{R}^3$. Different deformation function will led to different deformation. So the FFD method can be
used in any type of surface and solid Modelling system, and can be act on plane, quadric surface, parametric surface etc. FFD method can intuitively control the model geometrical continuity. It can be implemented for surface local deformation. The calculation amount is not so large, and there is no need to fit or parametric the initial surface. Here, the FFD method is used for parametric design of hydraulic machinery impeller.

2.3. FFD for hydraulic machinery blade deformation control
The hydraulic machinery blade shape is complex three dimensional surface, and it is hard to parameterize. The complex blade surface is the prime determinant of the hydraulic performance. We call it functional surface, and the optimization of impeller is usually consider as shape optimization problem with flow constraint. It is the main reason that the optimization of the hydraulic machinery inverse problem is hard to implement. In this area, the author has done a lot of work. Quartic Bezier with five control point is proposed to design the hub and shroud curve. The partial differential equation is proposed to generate the blade surface. The parametric of the blade is realized, but the control variables are lack of physical significance, and we can’t control the blade surface freely. Here, the FFD is used to parameterize the blade surface, and the computational procedure is as following

1. First construct an enough large parallelepiped control volume, and embed the initial impeller blade surface into the control volume. The blade surface can be various form, regardless of it is parametric design or traditional design.
2. Construct the local coordinate system for control lattice, and the Cartesian coordinates \((x,y,z)\) and the local coordinates \((s,t,u)\) of these control lattice are achieved.
3. Calculate the local coordinates \((s,t,u)\) of these points on objective blade surface.
4. Move the control lattices, and calculate the new form of the control volume. In the inverse problem of the pump blade, the control lattices are drived by the energy gradient of the fluid along the blade, and the deformation function for lattices will be proposed in next section.
5. The blade shape will be deformed along with the control volume deformation. Calculate the new surface after deformation remaining the local coordinates \((s,t,u)\) of these points on objective blade surface the same.

Implement the above calculation, and then we can realize the deformation of blade surface. According to the calculation of the blade surface deformation, the parametric control of the blade surface is realized, the blade surface can be freely deformed by moving the control points, the curve and surface is smooth and continuous. There is no need to parametric the initial surface to be deformed.

3. The inverse method of centrifugal impeller based on FFD
The inverse methods of centrifugal impeller include the pure inverse method and the iteration method based on the direct problem and inverse problem. In the former, the flow field is solved with complex turbulent model, and the blade shape is modified according to the flow simulation. The inverse problem and the direct problem are performed independently, and the blade shape is hard to control. In the latter, the blade shape is modelled by the boundary condition \((w \cdot n = 0)\). The inverse problem and the direct problem are coupled and are solved simultaneously, and the flow model is simplified with many assumptions. In this method the circulation distribution or the loading distribution are assigned. Both inverse methods have advantages and disadvantages. In this research the proposed inverse method based on FFD method can bring the two methods together in highly effective ways.

M Zangeneh[1,10,11] proposed the circulation distribution with parabolic segment and linear segment distribution. As the fluid machinery is used to transform the energy between the fluid energy and the shaft mechanical energy, the fluid energy distribution or the loading distribution along the blade are the key control variables. We consider the fluid energy gradient \(\frac{\partial H}{\partial m}\) along the blade as the design variable, which is agree with the theory that modelling the free surface based on physics model proposed in reference [12]. In this inverse method the FFD is used to parametric the blade surface, and then the fluid energy gradient along the blade is assigned. Solve the flow in pump and adjust the blade shape according the flow solution, then we can get the objective design. The difficult is how to modify
the blade shape according to the flow solution. Here, the deformation function for the lattices is proposed as following.

\[ \Delta P_{i,j}(r,\theta) = k \left( \frac{\partial H_i(r)}{\partial m} \right)_o - \left( \frac{\partial H_i(r)}{\partial m} \right)_j \cdot f(\theta) \] (5)

Where \( \Delta P_{i,j} \) is the displacement of the control lattices

\( k \) is the underrelaxation coefficient

\( \frac{\partial H_i(r)}{\partial m} \) is the gradient of fluid energy along the meridional direction.

\( f(\theta) \) is the circular angle contribution coefficient

The subscript “o” denote the objective

According to the inverse method above, we can get the objective blade shape easily, and the calculation cost is greatly reduced.

4. Calculation case

The low specific speed centrifugal pump in the optimization research [13] done by the author is consider as the inverse design object. The flow rate is 12.5 \( m^3/h \), the head is 30.7 \( m \), and the rotational speed is 2900 \( r/m \). As the specific speed is very low, the blade is design as the cylindrical blade. The blade shape after one optimization cycle in reference [13] is consider the initial blade shape as show in figure 2, and the energy gradient distribution along the optimized blade is consider as the design object in this inverse problem. The blade shape is renewed according to the flow solution and the objective flow field. The pump inner flow field is simulated by using FLUENT 13.0. structured grid were generated for the press and suction side of the impeller blade, and the RNG \( k-\varepsilon \) turbulent model and SIMPLEC algorithm are used for the flow simulation.

![Figure 2. The initial blade and the control lattices.](image)

![Figure 3. The objective and the initial energy gradient distribution along the blade.](image)

Construct the control volume, and then construct the local coordinate system for control lattices in cylindrical coordinate system as shown in figure 2. According to the objective and the initial energy gradient distribution along the blade, we can calculation the deformation using lattices deformation function. The position of the control lattices are renewed according to following formula.

\[ P_{i,j} = P_{i,j} + \Delta P_{i,j} \] (6)

So, we can get the deformed control volume. Remain the local coordinates(\( s,t,u \)), we can renew the blade shape using the formula (4). The deformed volume and the deformed blade shape are shown in figure 4. Construct the three dimensional model of the new blade and recalculate the flow field, and
then we can get the energy gradient distribution. We can get the expect blade shape by adjust the
under relaxation coefficient $k$ in the lattices deformation function. The blade shape obtained by the
new inverse method and the objective blade shape is shown in figure 6. As we can see that the blade
shape are almost the same. The energy gradient distribution of the blade design by FFD method and
the objective energy gradient distribution are shown in figure 5. We can see that the energy gradient
distribution by proposed inverse method agree well with the objective distribution.

![Figure 4](image1)

**Figure 4.** The initial blade shape and the blade shape design by FFD method

![Figure 5](image2)

**Figure 5.** The energy gradient distribution of the blade design by FFD method and the objective energy gradient distribution

![Figure 6](image3)

**Figure 6.** The blade shape design by FFD method and the objective blade shape

5. **Conclusion and future work**

In this research, the inverse method of centrifugal impeller based on free form deformation method is
proposed. The lattices deformation function is proposed, and we can deform the control volume and
renew the blade shape according to the fluid energy gradient distribution. According to the proposed inverse method, we can obtain the blade shape with the given energy distribution easily. But, what kind of distribution of the fluid energy gradient will result in high performance. That will be discussed in my future work.

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