INDIRECT SIGNALS OF SUSY IN
GAUGE BOSON PAIR PRODUCTION AT LEP AND NLC *

Ralf Hempfling
Max-Planck-Institut für Physik, Werner-Heisenberg-Institut,
Föhringer Ring 6, 80805 Munich, Germany
E-mail: hempf@mppmu.mpg.de

ABSTRACT

We compute the dominant one-loop radiative corrections to the cross-section for $e^+e^- \rightarrow V_1 V_2$ ($V_{1/2} = \gamma, Z, W^\pm$) in the MSSM. We find that the genuine vertex corrections are very small. The oblique corrections are potentially large enough to be tested at LEP-II or NLC. However, the sensitivity is below the one from other high precision electro-weak measurements but can serve as a self-consistency check.

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1. Introduction

LEP-I experiments essentially rule out new physics below $m_z/2$. However, the high statistics allow even to constrain virtual effects from particles not kinematically accessible at LEP-I energy. Thus, it is natural to ask whether this is also possible at upcoming $e^+e^-$ colliders. The higher center-of-mass energy, $E_{cm}$, opens new channels even within the framework of the standard model. In addition, the clean environment of an $e^+e^-$ collider is ideal to test the triple gauge boson coupling (TGC). In light of the success of the Standard Model (SM) it is hard to imagine that the TGC will exhibit a deviation from the SU(2)×U(1) gauge structure. Nonetheless, it might be possible to detect anomalies indicating virtual effects of new physics. In table 1 we have listed upcoming collider experiments. In $e^+e^-$ colliders a pair of gauge bosons can be produced in three possible channels as depicted in fig. 1. Note, there is no s-channel (no u-channel) for $\gamma\gamma$, $\gamma Z$, $ZZ$ ($W^+W^-$) production at tree-level. In table 2 we have listed the cross-section for gauge boson pair production in an $e^+e^-$ collider obtained by integrating the differential cross-section

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow V_1V_2) = \frac{f_S \alpha^2_{em}}{4s} \sqrt{1 - \frac{4m^2_y}{s}} |A|^2 ,$$

where the symmetry factor $f_S = 1$ (1/2) for $W^+W^-$ and $\gamma Z$ ($\gamma\gamma$ and $ZZ$). We also present the expected precision to which this cross-section can be measured at LEP-II ($E_{cm} = 200$ GeV) and NLC ($E_{cm} = 500$ GeV) assuming that the statistical errors dominate. With a precision in the percent range these processes are potentially sensitive to radiative corrections (RC). The agreement (disagreement) of the theoretical and experimental values provide an important self-consistency check of the SM (hint for new physics). However, before we can appreciate the importance of this process we have to answer the question whether we can actually learn anything new from testing the TGC? One of the most important achievements at LEP-II will be...
a very precise measurement of $m_w$ to $\delta m_w = \pm 50$ MeV. This will pose the strongest constraint
on new physics via $\Delta r$ with an expected precision

$$\delta \Delta r = \pm 0.1\%(Th) \pm 1\% \frac{\delta m_w}{170 \text{ MeV}} \pm 1\% \frac{\delta m_t}{30 \text{ GeV}}.$$ 

Here the uncertainty of the top quark mass of presently $\delta m_t = 9$ GeV is expected to reach 5 GeV.

Similar constraints can be derived from measurements of the forward-backward asymmetry, neutral current processes, and the running $\alpha_{em}$. Thus, any virtual effects of new physics on $\sigma(e^+e^- \rightarrow V_1V_2)$ has to be compared to the ones on $\Delta r$ etc. In this paper we will focus on the minimal supersymmetric extension of the SM (MSSM) [8].

### 2. RC due to Squarks/Sleptons

The calculation of the RC to the gauge boson production in $e^+e^-$ collider in the SM is quite elaborate[4,5]. Thus, in extending this calculation to the MSSM we restrict ourselves to the dominant effects expected to arise from the squark/slepton sector which are enhanced by

- colour factor, $N_c = 3$
- number of generations, $N_g = 3$
- large top Yukawa coupling.

None of these enhancements exist for the selectron-chargino loops. Thus, we will rely throughout this work on the assumption that these contribution can be neglected. (For a more complete treatment see ref. 6).

One consequence of our assumption is that virtual SUSY contributions only enter via the gauge boson self energy and the TGC. (In this sense, our approach is similar to ref. 7 where the supersymmetric RC to the TGC were calculated). As a result, the amplitude can be written as $A(\sigma, \epsilon_1, \epsilon_2, s, t) = \sum M_i^2(\epsilon_1, \epsilon_2) F_i^\sigma(s, t)$ with the matrix elements ($\sigma = \pm 1/2; P = 1/2 + \sigma \gamma_5$)[1,3]

\[
\begin{align*}
M_1^2(\epsilon_1, \epsilon_2) &= \bar{\nu}(p_+)[f_+^s(k_- - p_+)]\gamma^\nu Pu(p_-) \\
M_2^2(\epsilon_1, \epsilon_2) &= \frac{1}{2}\bar{\nu}(p_+)[f_+^s(k_- - k_+)](\epsilon_+^s \cdot \epsilon_-^s)Pu(p_-), \\
M_3^2(\epsilon_1, \epsilon_2) &= \bar{\nu}(p_+)[f_+^s(\epsilon_+^s \cdot k_-) - f_-^s(\epsilon_-^s \cdot k_-)]Pu(p_-), \\
M_4^2(\epsilon_1, \epsilon_2) &= \bar{\nu}(p_+)[f_+^s(\epsilon_+^s \cdot p_-) - f_-^s(\epsilon_-^s \cdot p_+)]Pu(p_-), \\
M_5^2(\epsilon_1, \epsilon_2) &= \frac{1}{2}\bar{\nu}(p_+)[f_+^s(k_- - k_+)]Pu(p_-)(\epsilon_+^s \cdot k_-)(\epsilon_-^s \cdot k_+), \\
M_6^2(\epsilon_1, \epsilon_2) &= \bar{\nu}(p_+)[f_+^s(\epsilon_+^s \cdot k_-) + f_-^s(\epsilon_-^s \cdot k_-)]Pu(p_-).
\end{align*}
\]
The formfactors for \( e^+ e^- \rightarrow W^+ W^- \) are:

\[
F^-_{1/2} = \frac{g^2}{2t} \left( 1 + 2 \frac{\delta e}{e} \right) - 4 \frac{g_{eeZ}^L \Gamma^4_{W+W^-}}{s - m_Z^2} + 4 \frac{e \Gamma^4_{W+W^-}}{s},
\]

\[
F^{+1/2}_{1} = 4 \frac{g_{eeZ}^R \Gamma^4_{W+W^-}}{s - m_Z^2} - 4 \frac{e \Gamma^4_{W+W^-}}{s},
\]

\[
F^\sigma_2 = 2 \frac{g_{eeZ}^\sigma g_{ZW+w^-}}{s - m_Z^2} \left( 1 + 2 \frac{\delta e}{e} + \frac{\delta g_{ZW+w^-}}{g_{ZW+w^-}} + \frac{A_{ZZ}(s) - A_{ZZ}(m_Z^2)}{s - m_Z^2} - \frac{Z_{ZZ}}{s} \right)
\]

\[
- \frac{e}{g_{ZW+w^-}} \frac{A_{\gamma Z}^\gamma(s)}{s} + \frac{\Gamma^2_{W+W^-}}{s} - 2 \frac{\delta e}{s} \left( 1 + 2 \frac{\delta e}{e} + \frac{A_{\gamma Z}(s)}{s} - \frac{\delta Z_{\gamma} + g_{ZW+w^-}}{s - m_Z^2} + \frac{\Gamma^4_{W+W^-}}{s} \right),
\]

\[
F^\sigma_3 = -F^\sigma_2 - \frac{\Gamma^2_{W+W^-}}{s} - \frac{\Gamma^3_{W+W^-}}{s} + 2 \frac{2 \sigma \Gamma^4_{W+W^-}}{e} - \frac{\Gamma^2_{W+W^-}}{s} - \frac{\Gamma^3_{W+W^-}}{s} + 2 \frac{\sigma \Gamma^4_{W+W^-}}{e},
\]

\[
F^\sigma_4 = -4 \sigma \frac{g_{eeZ}^\sigma \Gamma^4_{W+W^-}}{s - m_Z^2} - 4 \sigma \frac{e \Gamma^4_{W+W^-}}{s},
\]

\[
F^\sigma_5 = -2 \frac{g_{eeZ}^\sigma \Gamma^4_{WW+Z}}{s - m_Z^2} + 2 \frac{e \Gamma^4_{WW+Z}}{s} \quad P = L, R,
\]

where \( g_{eeZ} = g \sin^2 \theta_w / \cos \theta_w \), \( g_{eeZ}^{-1} = g_{eeZ} + g / 2 \cos \theta_w \), \( g_{ZW+} = g \cos \theta_w \). The formfactors for \( e^+ e^- \rightarrow V_1 V_2 \) \( (V_1/2 = \gamma, Z) \) are defined in eq. 2. Note that for squark-loops the factors \( \Gamma^4_{W+W^-} \) and all \( \Gamma^4_{V_1 V_2} \) defined in fig. 2 vanish.

### 3. Numerical Result

Our sfermion mass spectrum is characterized by a universal mass parameter \( m_0 \) (here, we omit the possibility of mass-splitting due to renormalization group evolution), a \( L/R \) mixing parameter \( A_0 \) and the ratio of Higgs VEVs \( \tan \beta \). In fig. 3 we see that the differential cross-sections with \( \gamma \)'s in the final state diverges for \( |\cos \theta| = 1 \). In this case, we choose the range

Fig. 2. The Lorentz invariant decompositions of the gauge boson self-energies and the TGC. The ellips in the first diagram stands for all contributions proportional to \( k^\alpha_i, k^3_i, (k_1 + k_2)^\alpha, \ldots \).
of integration as $|\cos \theta| < 0.95$. Furthermore, we find that the dominant RC come indeed from the third generation squarks due to large mass splitting within SU(2) multiplet. For the superpartners of the light fermions this splitting is generated by the $SU(2)$ term, $M^2_{u_L} - M^2_{d_L} = g \langle D^3 \rangle = m^2_w \cos 2\beta$.

The counter terms in eq. 2 depend on the renormalization scheme, i.e. the parameterization of the Born term. So far we have used $g^2 = 4\pi \alpha_{\text{em}} / \sin^2 \theta_w$. It is easy to understand that the RC to $e^+e^- \rightarrow \gamma\gamma$ vanish in this scheme while those to $e^+e^- \rightarrow W^+W^-$ can become quite large (see fig. 4b). However, if we change the renormalized coupling constant to $g^2 \rightarrow \sqrt{2}m^2_w G_\mu$ then we have to replace $\delta g / g \rightarrow \delta g / g + \Delta r$ and $\Delta \sigma / \sigma \rightarrow \Delta \sigma / \sigma - 2\Delta r$. In this scheme the RC to $e^+e^- \rightarrow W^+W^-$ are very tiny (fig. 4b). For $\sigma(e^+e^- \rightarrow ZZ)$ and $\sigma(e^+e^- \rightarrow \gamma Z)$ the situation is similar. In the SM it is convenient to use $G_\mu$, $\alpha_{\text{em}}$, $m_z$ and possibly $m_w$ to parameterize the tree-level term because these observables are known to such a high precision. However, in the MSSM the largest uncertainty arise from our ignorance of the SUSY parameters. Thus, it is convenient to parameterize in such a way that RC cancel, even if that means to use observables with larger errors. E.g.: it is easy to see that no sfermionic RC exist to the relation

$$\frac{d\sigma}{d\cos \theta}(e^+e^- \rightarrow \gamma Z) = \alpha_{\text{em}} \Gamma(Z \rightarrow e^+e^-) \times \ldots,$$

where the ellipsis stands for some kinematical factors. Thus, with an error of 0.3% in the leptonic width of the $Z$ boson an expected precision of at best 0.4% for $\sigma(e^+e^- \rightarrow \gamma Z)$ at NLC this process is not suited to yield new information on sfermions. A similar relation holds between $\sigma(e^+e^- \rightarrow ZZ)$, $\Gamma(Z \rightarrow e^+e^-)$, and $\Gamma(Z \rightarrow \text{hadrons})$. A deviation of the TGC from the SM prediction (assuming $\Delta r$ agrees with the SM prediction) will indicate new physics other than the MSSM.

4. Summary

We have investigated virtual effects of sfermions on the cross-sections of gauge boson pair productions. We find that

- the genuine vertex corrections of sfermions to $ZW^+W^-$ and $\gamma W^+W^-$ are very small;
The sfermionic RC to the total cross-sections vs. $m_0$ and $A_0$. The born term is parameterized by $\alpha_{em}$ and $G_\mu$. 

- the genuine vertex corrections of sfermions to $V_1V_2V_3$ vanish if $V_i = \gamma, Z$;
- all oblique corrections can be absorbed by a suitable parametrization of the Born term;
- the experimental precision on $\sigma(e^+e^- \rightarrow V_1V_2)$ is lower than the precision of the corresponding EW observables $\Gamma(Z \rightarrow \text{hadrons})$ or $\Gamma(Z \rightarrow \text{leptons})$ etc;
- sfermions cannot explain any anomalous TGC that may be observed at LEP-II unless there is also evidence of new physics in other observables (e.g. $\Delta R$);
- if a deviation in e.g. $\Delta r$ is found then $\sigma(e^+e^- \rightarrow V_1V_2)$ provides an additional cross-check;

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