Single Polarisation Asymmetries for Quarkonia in Non-relativistic QCD

Sourendu Gupta
Theory Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India.

Abstract

We find that single spin asymmetries in NRQCD are non-vanishing in general. They are proportional to the imaginary parts of some non-perturbative matrix elements. With statistics of about $10^6$ identified $J/\psi$'s, or $10^5$ identified $\chi_2$, it is possible to measure these imaginary parts even if they are an order of magnitude smaller than the real parts. Such statistics are quite reasonable at polarised HERA N and other future experiments.

\footnote{E-mail: sgupta@theory.tifr.res.in}
1 Introduction

Single polarised initial states can give rise to asymmetries in two ways. If a final state polarisation is observed, then the difference between the cross sections when the initial and the final state spins are parallel and anti-parallel gives an asymmetry. These do not yield information beyond what comes from double polarised initial states. Recent interest has focussed on single spin asymmetries in which final state spins are not observed—

\[ S = \frac{\nabla \sigma}{\sigma} = \frac{\sum_h h \sigma(h)}{\sum_h \sigma(h)}, \]  

(1.1)

where \( \nabla \sigma \) is the difference of cross sections with the initial particle polarised in opposite directions. Such asymmetries do not exist in leading twist computations in perturbative QCD. They seem to arise either from next-to-leading twist-3 effects, or through non-perturbative effects like intrinsic transverse momenta of partons.

A formalism for computing the production cross-sections of heavy quarkonia, called non-relativistic QCD (NRQCD) [1], has been developed. In NRQCD the non-perturbative long-distance physics is factored from the short distance perturbative physics. This seems like an ideal place to search for single polarised asymmetries of the kind in eq. (1.1). In this paper we report a set of computations which shows that such asymmetries indeed exist in NRQCD, yield new and interesting non-perturbative information about quarkonia, and are measurable.

NRQCD is a low-energy effective theory for quarkonia. The action is written in terms of all possible operators consistent with the symmetries of QCD. All momenta in NRQCD are cut off by some scale \( \Lambda \). The coupling associated with each term is identified through a perturbative matching procedure. The applicability of NRQCD to quarkonium production depends on the proof that final state effects factorise in the hadronisation of a heavy quark pair to a quarkonium. Such a proof has been given for production at large transverse momenta [1] and successful phenomenology has been done [2]. For low-energy production of quarkonia a proof is lacking, but a reasonable phenomenology arises when cross sections are computed assuming factorisation [3, 4, 5]. Double polarisation asymmetries have been computed for quarkonia in this framework [6, 7, 8]. Numerical results for single polarisation asymmetries computed in the colour singlet model have been quoted
in the literature [9], and preliminary results in NRQCD obtained in [6].

The NRQCD factorisation formula for the inclusive production of heavy quarkonium resonances $H$ with 4-momentum $P$ can be written as

$$d\sigma = \frac{1}{\Phi} \frac{d^3 P}{(2\pi)^3 2E_P} \sum_{ij} C_{ij} \left< K_i \Pi(H) K_j^\dagger \right>, \quad (1.2)$$

$$d\nabla \sigma = \frac{1}{\Phi} \frac{d^3 P}{(2\pi)^3 2E_P} \sum_{ij} \overline{C}_{ij} \left< K_i \Pi(H) K_j^\dagger \right>. \quad (1.3)$$

where $\Phi$ is a flux factor. The coefficient functions $C_{ij}$ and $\overline{C}_{ij}$ are computable in perturbative QCD and hence have an expansion in the strong coupling $\alpha_s$ (evaluated at the NRQCD cutoff). Although each matrix element in the sum above is non-perturbative, it has a fixed scaling dimension in the quark velocity $v$. Then the NRQCD cross section is a double series in $\alpha_s$ and $v^2$. For charmonium a numerical coincidence, $\alpha_s \approx v^2$, makes this expansion a delicate one to handle. For bottomonium, the series is easier to analyse since $v^2 \ll \alpha_s$.

The fermion bilinear operators $K_i$ are built out of heavy quark fields sandwiching colour and spin matrices and the covariant derivative $D$. The composite labels $i$ and $j$ include the colour index $\alpha$, the spin quantum number $S$, the number of derivative operators $N$, the orbital angular momentum $L$, the total angular momentum $J$ and the helicity $J_z$. The hadron projection operator

$$\Pi(H) = \sum_s |H,s\rangle \langle H,s|, \quad (1.4)$$

(where $s$ denotes hadron states with energy less than the NRQCD cutoff), is diagonal in $L, S, J$ and, when the final state spin is unobserved, connects states of opposite $J_z$. As a result, the operators $K_i$ and $K_j$ in eq. (1.3) are restricted to have equal $L, S, J$ and opposite $J_z$, for both $d\sigma$ and $d\nabla \sigma$.

The $J_z$-dependence of these matrix elements can be factored out using the Wigner-Eckart theorem—

$$\left< K_i \Pi(H) K_i^\dagger \right> = \frac{1}{2J+1} \mathcal{O}_0^H (2S+1L_j^N), \quad (1.5)$$

$$\left< K_i \Pi(H) K_j^\dagger \right> + \text{h.c.} = \frac{1}{2J+1} \mathcal{P}_0^H (2S+1L^N_j, 2S+1L^N_j). \quad (1.6)$$

The factors of $1/(2J+1)$ come from a Clebsch-Gordan coefficient and are conventionally included in the coefficient function. Each diagonal matrix
element, $O$, is the product of a reduced matrix element $\langle H||K^\dagger_i||0 \rangle$ with its complex conjugate $\langle 0||K_i||H \rangle$, and is real. However, the off-diagonal matrix elements, $P$, may have non-vanishing phase. We introduce the notation
\[ \langle K_i \Pi(H) K_j^\dagger \rangle - \text{h.c.} = \frac{1}{2J+1} T_\alpha^H (2S+1 L_j^N, 2S+1 L_j^{N'}) , \] (1.7)
where the Wigner-Eckart theorem has been used as before to factor out the $J_z$ dependence of these matrix elements. The NRQCD power counting rule for all three types of matrix elements is—
\[ d = 3 + N + N' + 2(E_d + 2M_d) , \] (1.8)
where $E_d$ and $M_d$ are the number of colour electric and magnetic transitions required to connect the hadronic state to the state $K_i|0 \rangle$.

In section 2 of this paper we present the main results for $\nabla \sigma$ after a telegraphic review of the threshold expansion technique [10], the kinematics and the appropriate Taylor series expansion of the perturbative matrix element [11]. Although the results presented in this paper are for quarkonia produced at small transverse momenta, it should be clear that similar effects should arise also at large transverse momenta. Observable consequences are discussed in the final section 3.

2 Computing the Asymmetry

We choose to construct the coefficient functions using the “threshold expansion” technique of [10]. This consists of calculating, in perturbative QCD, the matrix element $M$ connecting the initial states to final states with a heavy quark-antiquark pair ($\bar{Q}Q$), and Taylor expanding the result in the relative momentum of the pair, $q$, after performing a non-relativistic reduction of the Dirac spinors. The resulting expression is squared and matched to the NRQCD formula of eq. (1.3) by inserting a perturbative projector onto a non-relativistic $\bar{Q}Q$ state between the two spinor bilinears. The coefficient of this matrix element is the required coefficient function.

In this paper we evaluate the polarised cross sections to order $\alpha_s^2 v^9$. This requires a Taylor expansion to order, $N + N' \leq 6$, as can be seen by setting $d = 9$ and $E_d = M_d = 0$ in eq. (1.8). A simplification occurs because the
perturbative projector has only one term—

\[
\Pi(Q\bar{Q}) = |Q\bar{Q}\rangle \langle Q\bar{Q}|. \tag{2.1}
\]

In agreement with [8, 10, 11] we use the relativistic normalisation of states

\[
\langle Q(p, \xi)|\bar{Q}(q, \eta)\rangle = 4E_pE_q(2\pi)^6\delta^3(p - p')\delta^3(q - q'), \tag{2.2}
\]

with the spinor normalisations \(\xi\dagger\eta = 1\). Expanding \(E_p = E_q = \sqrt{m^2 + q^2}\) in \(q^2\) allows us to write the spinor bilinears in terms of transition operators built out of the heavy quark field.

The kinematics is very simple to leading order in \(\alpha_s\). The momenta of the initial particles are \(p_1\) and \(p_2\). We take \(p_1\) to correspond to the polarised particle and assume that it lies in the positive z-direction and that \(p_2\) is oppositely directed. The net momentum \(P = p_1 + p_2\). The 4-momenta of \(Q\) and \(\bar{Q}\) (\(p\) and \(\bar{p}\) respectively) are written as

\[
p = \frac{1}{2}P + L_jq^j \quad \text{and} \quad \bar{p} = \frac{1}{2}P - L_jq^j. \tag{2.3}
\]

Note that \(p^2 = \bar{p}^2 = m^2\), where \(m\) is the mass of the heavy quark. The space-like vector \(q\) is defined in the rest frame of the pair, and \(L^\mu_j\) boosts it to any frame. We shall use Greek indices for Lorentz tensors and Latin indices for Euclidean 3-tensors. A property of the boost matrix that we shall use many times is

\[
\epsilon_{\mu\nu\lambda\rho}p_1^\mu p_2^\nu L_1^\lambda L_2^\rho = \frac{M^2}{2} \epsilon_{ijk}\hat{z}_k, \tag{2.4}
\]

where \(\hat{z}\) is the unit vector in the z-direction. This can be derived from some of the identities listed in [10] and the kinematics given here.

The power counting rule in eq. (1.8) requires that we express the Fermion bilinear operators as spherical tensors. Recall that any vector \(a_i\) can be written as a spherical tensor of rank 1, with the components

\[
a_{\pm 1} = \mp \frac{1}{\sqrt{2}} (a_x \pm ia_y), \quad a_0 = a_z, \tag{2.5}
\]

where the subscripts \(\pm 1\), 0 are helicity indices. In NRQCD higher rank spherical tensors are constructed by coupling such rank 1 tensors successively [11]. Some useful identities are

\[
a_jb_j = a_0b_0 - (a_{+1}b_{-1} + a_{-1}b_{+1}) = -\sqrt{3}[a, b]_0^0, \quad i\epsilon_{jkl}a_jb_k\hat{z}_l = (a_{+1}b_{-1} - a_{-1}b_{+1}). \tag{2.6}
\]
We have introduced the notation $[a,b]_J^M$ to denote two spherical tensors $a$ and $b$ coupled to total rank $J$ and helicity $M$. The coefficient of the term $[a,b]^0_0$ in eq. (2.6) can be obtained from the appropriate Clebsch-Gordan coefficients.

2.1 $\bar{q}q \to \bar{Q}Q$

As a simple example of the techniques used, we write down the matrix element for the subprocess $\bar{q}q \to \bar{Q}Q$—

$$\mathcal{M} = -\frac{i g^2}{2M^2} \left[ \bar{v}(p_2, \gamma_\mu T^a u(p_1, h)) L_j^\mu \right. \\
\times \left. \left[ M \xi^\dagger \sigma^i T^a \eta - \frac{4}{M + 2m} q^i \xi^\dagger (q \cdot \sigma) T^a \eta \right] \right],$$

where $T^a$ is a colour generator, $u$ and $v$ are the light quark spinors and $\xi$ and $\eta$ are the heavy quark Pauli spinors. The equations of motion for the initial state quarks have been used to obtain the explicitly gauge invariant matrix element above. The desired Taylor series expansion is written down using the relation $M^2 = 4(m^2 + q^2)$ to expand all factors with $M$.

The squared matrix element for this process is trivial. The light quark spinor factors require the projection operator $\frac{1}{1 + h \gamma_5}/2. The polarised cross section is obtained from the part proportional to $h$. Doing the Dirac algebra and using the identity in eq. (2.4) to simplify the result, we find—

$$\nabla |\mathcal{M}|^2 = -\frac{i \alpha_s^2}{4\pi^2 M^4} e_{jkl} \hat{z}_l \mathbf{M}_j \mathbf{M}_k^\dagger,$$

where $\mathbf{M}_j$ is the heavy-quark spinor bilinear in eq. (2.8). Using eq. (2.7) and the Wigner-Eckart theorem, it is easy to see that the contribution of the diagonal operators vanishes.

The final results for the cross section differences, to order $v^9$, are—

$$\nabla \hat{s}^\eta_{\bar{q}q} = \nabla \hat{s}^{h \eta}_{\bar{q}q} = 0, \quad \nabla \hat{s}^{I/\psi}_{\bar{q}q} = \frac{\pi^3 \alpha_s^2}{54m^6} \delta(\hat{s} - 4m^2) \left[ \frac{2}{\sqrt{3}} \mathcal{I}^{I/\psi}(3S_0^1, 3S_1^2) \right], \quad \nabla \hat{s}^{\chi_{J}}_{\bar{q}q} = \frac{\pi^3 \alpha_s^2}{54m^6} \delta(\hat{s} - 4m^2) \left[ \frac{2}{\sqrt{3}} \mathcal{I}^{\chi_{J}}(3S_0^1, 3S_0^1) + \frac{7\sqrt{5}}{12m^2} \mathcal{I}^{\chi_{J}}(3S_0^1, 3S_1^1) \right].$$

(2.10)
where $\hat{s}$ is the parton CM energy. The expressions are exactly the same whether the initial quark or anti-quark is polarised. The matrix elements are all of order $v^9$. The corresponding unpolarised cross sections have been written down in [11]. They are of lower order in $v$.

The fact that the asymmetry is non-zero for the $^3P_0$ state, $\chi_0$, might come as a surprise. However, one should note that the matrix element involved in its production entails the radiation of at least one soft gluon. Thus, in $\bar{q}q$ annihilation a $\chi_0$ can only be produced along with some light hadrons. The total angular momentum of the final state is 1, making it possible to generate this single spin asymmetry.

2.2 $gg \to \bar{Q}Q$

The squared matrix element for the $gg$ process is technically a little more complicated. We work in a class of ghost-free gauges called planar gauges [12]. The density matrix for an initial state gluon of momentum $p$ and helicity $h$ in these gauges is given by

$$\epsilon_{\mu}(p, h)\epsilon^{*}_{\nu}(p, h) = \frac{1}{2} \left[ -g_{\mu\nu} + \frac{1}{p \cdot V} (p_{\mu} V_{\nu} + p_{\nu} V_{\mu}) + \frac{ih}{p \cdot V} \epsilon_{\mu\nu\rho\sigma} p^\rho V^\sigma \right].$$  

(2.11)

The vector $V$ defines the gauge choice. We write $V = c_1 p_1 + c_2 p_2$, with $c_1/c_2 \sim \mathcal{O}(1)$.

The sum of the matrix elements arising from the three Feynman diagrams ($s$-channel gluon exchange, $\mathcal{M}_s$, and $t$- and $u$-channel quark exchanges, $\mathcal{M}_t$ and $\mathcal{M}_u$) can be decomposed into three colour amplitudes—

$$\mathcal{M} = \frac{1}{6} g^2 \delta_{ab} S + \frac{1}{2} g^2 d_{abc} D^c + \frac{i}{2} g^2 f_{abc} F^c.$$  

(2.12)

The colour amplitudes $S$ and $D$ involve only $\mathcal{M}_t + \mathcal{M}_u$, whereas $F$ involves $\mathcal{M}_s$ as well as $\mathcal{M}_t - \mathcal{M}_u$.

In order to write down our results, we find it convenient to introduce the notation

$$\mathcal{A} = \frac{1}{M^2} \delta_{\lambda\mu\nu} P_1^\lambda P_2^\mu \epsilon_1^\nu \epsilon_2^\nu \quad \text{and} \quad S_{ij} = A_i \hat{z}_j + A_j \hat{z}_i - B_{ij} + \epsilon_1 \cdot \epsilon_2 \hat{z}_i \hat{z}_j,$$  

(2.13)
where

\[ A_i = \frac{1}{M} (\epsilon_1 \cdot L_i \epsilon_2 \cdot p_1 - \epsilon_2 \cdot L_i \epsilon_1 \cdot p_2), \]

\[ B_{ij} = \epsilon_1 \cdot L_i \epsilon_2 \cdot L_j + \epsilon_2 \cdot L_i \epsilon_1 \cdot L_j. \]  

(2.14)

Here \( \epsilon_i \) is the polarisation vector for the initial gluon of momentum \( p_i \). In order to identify all terms to order \( v^9 \) we need the colour amplitude \( S \) to order \( q^5 \)—

\[ S = - \left( \frac{8im}{M} \right) A (\xi^\dagger \eta) + \frac{4}{M} S_{jm} (q^m \xi^\dagger \sigma^j \eta) - \left( \frac{32im}{M^3} \right) A \hat{z}_m \hat{z}_n (q^m q^n \xi^\dagger \eta) \]

\[ + \frac{16}{M^2} \left[ S_{jm} \hat{z}_n \hat{z}_p - \frac{M}{M + 2m} \delta_{jm} S_{np} \right] (q^m q^n q^o \xi^\dagger \sigma^j \eta) \]

\[ - \left( \frac{128im}{M^5} \right) A \hat{z}_m \hat{z}_n \hat{z}_p \hat{z}_r (q^m q^n q^o q^r \xi^\dagger \eta) \]

\[ + \frac{64}{M^6} \left[ S_{jm} \hat{z}_n \hat{z}_p - \frac{M}{M + 2m} \delta_{jm} S_{np} \right] \hat{z}_r \hat{z}_s (q^m q^n q^o q^r \xi^\dagger \sigma^j \eta). \]  

(2.15)

The amplitude \( D \) differs only through having colour octet matrix elements in place of the colour singlet ones shown above. For the colour amplitude \( F \) we need the expansion

\[ F^c = - \left( \frac{16im}{M^2} \right) A \hat{z}_m (q^m \xi^\dagger T^c \eta) + \frac{8}{M^2} S_{jm} \hat{z}_n (q^m q^n \xi^\dagger \sigma^j T^c \eta). \]  

(2.16)

In all three colour amplitudes, the terms in \( A \) are spin singlet and those in \( S \) are spin triplet.

The density matrix in eq. (2.11) yields

\[ \nabla S \cdot S^* \equiv \frac{1}{4} \sum_h h a_j b_m S_{jm} S_{j'm'} c_j d_{m'} = \left\{ [a, b]^2_{-2}[c, d]_2 - [a, b]^2_{2}[c, d]_{-2} \right\}, \]

(2.17)

where \( a, b, c \) and \( d \) are Euclidean 3-vectors, written in terms of the Euclidean components on the left and as spherical tensors on the right. Gauge invariance is obvious from the fact that the final result does not depend on the gauge choice \( V \). In addition, we find the other gauge invariant contractions, \( \nabla A \cdot A^* = 0 \) and \( \nabla A \cdot S^* \neq 0 \). Since \( A \) always comes with a spin-singlet operator and \( S \) with a spin-triplet, this last combination always gives rise to a product of two fermion bilinears of opposite parity. Such operators vanish
in NRQCD. Finally, the single spin asymmetries can be found by retaining only the terms in $S$ in the three colour amplitudes $S$, $D$ and $F$.

After retaining only those spherical tensors which contribute to order $v^9$ (they are tabulated in [11]) and dropping all the diagonal tensors, very few terms remain. It is clear, for example, that the colour amplitude $F$ could contribute through only one term. However, its contribution must then be diagonal, and as a result this colour amplitude cannot give a single spin asymmetry at this order. Similar arguments tell us that only two terms contribute from each of the $S$ and $D$ colour amplitudes. The coefficient functions can be read off from earlier computations of the unpolarized cross section [11].

The direct $J/\psi$ single polarised cross section difference is

$$\nabla \hat{\sigma}^{J/\psi}_{gg}(\hat{s}) = \varphi \frac{5}{48} \Theta^{J/\psi}_{D}(9),$$

where

$$\varphi = \frac{\pi^3 m^2}{32m^2} \delta(\hat{s} - 4m^2),$$

and $\Theta_a^{J/\psi}(d)$ denotes combinations of non-perturbative matrix elements from the colour amplitude $a (= S$, $D$ or $F$) at order $v^d$ for the cross section difference. The matrix element required here is

$$\Theta^{J/\psi}_{D}(9) = \frac{2}{\sqrt{15m^6}} T^{J/\psi}(3P_1^1, 3P_2^3).$$

The unpolarised cross section [11] starts at order $v^7$. The cross section differences for $\psi'$, $\Upsilon$ and all other $3S_1$ states are given by the same formulae; only the appropriate matrix elements have to be plugged into eq. (2.19).

Upto order $v^9$, the single polarised subprocess cross section differences for $1S_0$, $1P_1$ and $3P_{0,1}$ quarkonia vanish—

$$\nabla \hat{\sigma}^{J/\psi}_{gg}(\hat{s}) = \nabla \hat{\sigma}^{J/\psi}_{gg}(\hat{s}) = \nabla \hat{\sigma}^{J/\psi}_{gg}(\hat{s}) = \nabla \hat{\sigma}^{J/\psi}_{gg}(\hat{s}) = 0.$$  

The $3P_2$ cross section difference is

$$\nabla \hat{\sigma}^{J/\psi}_{gg}(\hat{s}) = \varphi \left( \frac{1}{18} \right) \left[ \Theta^{J/\psi}_{S}(7) + \Theta^{J/\psi}_{S}(9) \right]$$

where the combinations of non-perturbative matrix elements are

$$\Theta^{J/\psi}_{S}(7) = \frac{2}{\sqrt{15m^6}} T(3P_2^1, 3P_2^3), \quad \Theta^{J/\psi}_{S}(9) = \frac{47}{50m^8} \sqrt{\frac{3}{7}} T(3P_2^1, 3P_2^5).$$

The unpolarised cross section [11] starts at order $v^5$. 

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2.3 \( \gamma g \rightarrow \bar{Q}Q \) and \( \gamma\gamma \rightarrow \bar{Q}Q \)

The matrix elements for the two processes \( \gamma g \rightarrow \bar{Q}Q \) and \( \gamma\gamma \rightarrow \bar{Q}Q \) are closely related to the \( gg \) amplitudes. It is obvious that

\[
\mathcal{M}_{\gamma g} = geD, \quad \text{and} \quad \mathcal{M}_{\gamma\gamma} = e^2 S,
\]

(2.23)

where \( D \) and \( S \) are the colour amplitudes given in eq. (2.12), and \( e \) is the charge of the heavy quark.

The \( \gamma g \) cross sections for the production of any quarkonium state can be obtained from those for the \( gg \) process, by the prescription— replace \( \alpha_s^2 \) by \( \alpha\alpha_s \), delete the \( \Theta_D \) and \( \Theta_F \) terms, and replace the colour factor \( 5/48 \) for the terms in \( \Theta_D \) by \( 2 \). The \( \gamma\gamma \) cross sections are obtained with the prescription— replace \( \alpha_s^2 \) in \( \varphi \) by \( \alpha^2 \), delete the \( \Theta_D \) and \( \Theta_F \) terms, and replace the colour factor \( 1/18 \) for the terms in \( \Theta_S \) by \( 16 \). Only \( ^3S_1 \) quarkonia have non-vanishing single spin asymmetries in almost-elastic photo-production, and only \( \chi_2 \) can have non-zero asymmetry in photon-photon fusion.

3 Experimental Outlook

The observable asymmetries in hadro-production are obtained by polarising one of the initial hadrons. These quantities are obtained by convoluting the cross sections found in Section 2 with appropriate parton density functions. It is easy to see that

\[
S^H_{pp}(s, y) = \frac{\nabla\hat{\sigma}^H_{qq}(\hat{s})\nabla L_{qq}(s, y) + \nabla\hat{\sigma}^H_{gq}(\hat{s})\nabla L_{gq}(s, y)}{\hat{\sigma}^H_{qq}(\hat{s})L_{qq}(s, y) + \hat{\sigma}^H_{gq}(\hat{s})L_{gq}(s, y)},
\]

(3.1)

where \( \sqrt{s} \) is the centre of mass (CM) energy of the colliding protons and \( y \) is the rapidity at which the final state quarkonium, \( H \), is observed. With the convention that the positive \( z \)-direction is defined by the direction of motion of the polarised initial state hadron, the parton luminosities can be written as

\[
\nabla L_{qq} = \sum_f \Delta q_f(x_1)\bar{q}_f(x_2) + \Delta\bar{q}_f(x_1)q_f(x_2), \quad \nabla L_{gg} = \Delta g(x_1)g(x_2),
\]

(3.2)
where $x_1 = (2m/\sqrt{s})\exp(y)$ and $x_2 = (2m/\sqrt{s})\exp(-y)$ are the fractional momenta of the polarised and unpolarised initial hadrons (respectively) carried by the partons. The quantities $\Delta q_f, \Delta \bar{q}_f$ and $\Delta g$ are the usual polarised quark, anti-quark (of flavour $f$) and gluon densities, and the corresponding symbols without the $\Delta$ stand for the unpolarised densities. The unpolarised parton luminosities $L_{\bar{q}q}$ and $L_{gg}$ are given by similar expressions where the polarised parton densities are replaced by the unpolarised densities.

Numerical values of the ratios of several of these luminosities are plotted in Figure 1 as a function of $\sqrt{s}$. It is clear from the figure that $L_{gg}$ is the largest and that both the polarised and unpolarised $\bar{q}q$ luminosities are small.

The asymmetries $S_{\chi^0_{pp}}$ and $S_{\chi^1_{pp}}$ are likely to be too small to measure. The ratio of the non-perturbative matrix elements which determines the analysing power of these processes are of order $v^6$. Since $v^2 \approx 0.3$ for charmonia and about 0.1 for bottomonia, these asymmetries are unlikely to be observable. For both $S_{\chi^0_{pp}}$ and $S_{\chi^1_{pp}}$ non-vanishing contributions at order $v^9$ come only from the $\bar{q}q$ process whereas the cross sections are dominated by the $gg$ process. Thus, these asymmetries are small because of the smallness of the ratio $\nabla L_{\bar{q}q}/L_{gg}$. $S_{\chi^0_{pp}}$ is further suppressed by an analysing power which is of order $v^4$. 

Figure 1: The ratios $L_{\bar{q}q}/L_{gg}$ (full line), $\nabla L_{\bar{q}q}/L_{gg}$ (dashed line) and $\nabla L_{gg}/L_{gg}$ (dotted line) computed using the GRSV LO parton density set as a function of the CM energy $\sqrt{s}$ at zero rapidity.
The most promising observable is the single spin asymmetry for $\chi_2$—

$$S_{\chi_2}^{\chi_2}(s, y) = \left[ \frac{\sqrt{5/3} T_1^{\chi_2}(3P_1^1, 3P_2^3)}{m^2 O_1^{\chi_2}(3P_2^1) + \sqrt{5/3} P_1^{\chi_2}(3P_1^1, 3P_2^3)} \right] \frac{\Delta g(x_1)}{g(x_1)}. \tag{3.3}$$

The analysing power is of order $v^2$; the order $v^4$ correction can be easily written down using eq. (2.22), if required. Every matrix element can be written as a dimensionless number multiplying appropriate powers of $v$ and the NRQCD cutoff $\Lambda \sim m$. For the real parts of the matrix elements, there is some evidence [8] that this dimensionless number is independent of which matrix element it comes from, and depends only on the quarkonium state. We use this assumption along with the definition,

$$T_1^{\chi_2}(3P_1^1, 3P_2^3) = \tan \Phi_{\chi_2} P_1^{\chi_2}(3P_1^1, 3P_2^3), \tag{3.4}$$

of the phase of this operator to ask what is the expected sensitivity of experimental measurements of $\Phi_{\chi_2}$.

First, we rewrite the asymmetry in the form

$$S_{\chi_2}^{\chi_2}(s, y) = \frac{v^2 \sqrt{5/3} \tan \Phi_{\chi_2}}{1 + v^2 \sqrt{5/3}} \Delta g(x_1) \frac{\Delta g(x_1)}{g(x_1)}. \tag{3.5}$$

Next we note that the measured asymmetry,

$$S = \frac{N_+ - N_-}{N_+ + N_-}, \tag{3.6}$$

($N_+$ is the number of events observed when the initial hadron is polarised in the positive direction, and $N_-$ when it is negative) is smallest and most susceptible to error when $N_+ \approx N_- = N/2$; and the expected error in $S$ is less than or equal to $1/\sqrt{N}$ (the equality is reached when the errors in $N_+$ and $N_-$ are perfectly correlated). By inverting eq. (3.5) we can readily see that, near $S = 0$, the sensitivity of the experiment is

$$|\tan \Phi_{\chi_2}| \geq \frac{1 + v^2 \sqrt{5/3}}{v^2 \sqrt{N 5/3}} \frac{\Delta g(x_1)}{\Delta g(x_1)}. \tag{3.7}$$

If the scaling rule in eq. (1.8) is to determine the importance of various operators, then we should expect that the phase is independent of the operator and depends only on the hadron.
Recent experiments at $\sqrt{s} = 38.8$ GeV have observed a $J/\psi$ cross section of about 400 nb in proton nucleon collisions \[14\]. About 15\% of these come from decays of $\chi_2$. At the future polarised-HERA N the CM energy is likely to be about 39.3 GeV with an integrated luminosity of 80 pb$^{-1}$ per year \[1]\.

Even if only the forward moving $J/\psi$'s are recorded, this should yield about a million observed $J/\psi$ in the dimuon channel per year of run, and hence about $10^5$ identified $\chi_2$, assuming a reconstruction efficiency of about 67\% for the $\chi$'s. The appropriate ratio of parton densities\[3\] is around 0.4. If the degree of polarisation of the target is about 0.8, polarised-HERA N should be able to measure

$$|\tan \Phi_{\chi_2}| \geq 0.03 \quad \text{(single polarised – HERA N).} \quad (3.8)$$

Thus, single spin asymmetries in $\chi_2$ should be measurable at polarised-HERA N even if the imaginary part of the matrix element is more than an order of magnitude smaller than the real part. Current Fermilab experiments have statistics of about $10^5$ observed $J/\psi$, and hence about 10000 $\chi_2$ (assuming a reconstruction efficiency of 67\%). With similar target polarisation their sensitivity is only a factor 3 less.

For direct production of $^3S_1$ quarkonia the single spin asymmetry is given by

$$S^\psi_\psi(s, y) \approx \left[ \frac{\Theta_{\psi'}(9)}{\Theta_{\psi'}(7) + \Theta_{\psi'}(9) + (9/5)\Theta_{\psi'}(9)} \right] \frac{\Delta g(x_1)}{g(x_1)}. \quad (3.9)$$

where the unpolarised matrix elements are given in \[11\]. With the same assumptions as before, we can rewrite this as

$$S_{pp}(s, y) \approx \left[ \frac{0.65v^2 \tan \Phi_{\psi'}}{1 + 1.1v^2} \right] \frac{\Delta g(x_1)}{g(x_1)}. \quad (3.10)$$

The analysis is very similar to that for $\chi_2$. However, the results are much weaker, since the $\psi'$ cross sections are two orders of magnitude smaller. In this case the polarised-HERA N and Fermilab experiments can only measure an imaginary part which is no smaller than a third of the real part.

The asymmetry for $J/\psi$ is the hardest to predict, due to feed down from radiative decays of $\chi$ and $\psi'$. The problem can be simplified by working with

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\[3\] By our convention, $x_1$ is the momentum fraction of the parton from the polarised target, and $x_2$ from the unpolarised beam.
a data set consisting of directly produced $J/\psi$. Then the analysis follows from the analysis of $\psi'$ above. A million $J/\psi$'s can easily be obtained at polarised-HERA N. The sensitivity is similar to that for $\chi_2$. At Fermilab fixed target experiments statistics are poorer by a factor of 10, and the sensitivity is smaller by a factor of 3.

We conclude with a summary of our results. NRQCD predicts non-vanishing single polarised asymmetries for $^3S_1$ and $^3P_2$ quarkonia. The asymmetries are proportional to imaginary parts of some off-diagonal non-perturbative matrix elements. Since these are unknown, it is not possible to predict the values of these asymmetries. However, at polarised-HERA N and with polarised targets at Fermilab it is possible to measure the values of these imaginary parts even if they are an order of magnitude smaller than the real parts. Such experiments are extremely exciting, since they will probe an as yet unknown sector of the theory of quarkonia.
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