Tiny neutrino mass from SUSY and lepton number breaking sector

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Abstract

We suggest new setup where SUSY breaking spurion $F$-term possesses lepton number. This setup not only modifies sparticle mass spectra but also realizes several new models, where neutrino mass is naturally induced through radiative corrections. We here suggest two new models; the first one is (i): pseudo-Dirac/Schizophrenic neutrino model, and the second one is (ii): pure Majorana neutrino model. We will also show this setup can naturally apply to the supersymmetric Zee-Babu model.
1 Introduction

Recent neutrino oscillation experiments gradually reveal a structure of lepton sector\cite{0,1}. However, an origin of generation structure is still a mystery, and we do not know a mechanism of constructing generation structure in lepton sector as well as quark sector. Thus, finding a mechanism of small neutrino mass and lepton flavour structure is one of the greatest keys of revealing new physics beyond the Standard Model (SM). One interesting idea is that tiny neutrino mass is realized through radiative corrections\cite{2}. Where quantum corrections induce effective dimension five operator, and Majorana neutrino mass is obtained. The smallness of neutrino mass is natural because it is induced by quantum corrections. There are some models where tiny Majorana neutrino mass is induced through one-loop diagram\cite{3,4}, two-loop\cite{5,6,7}, three-loop\cite{8}, and so on. Anyhow, they have new extra particles, and induce new phenomenology beyond the SM, hence experimental evidence is expected in LHC.

On the other hand, supersymmetry (SUSY) is expected to be the most promising candidate of physics beyond the SM. There are rich phenomenology such as gauge coupling unification, existence of dark matter, and so on. However, experiments have not proved its existence yet, and the SUSY breaking mechanism is still a mystery. Thus, it is an important research to reveal an origin of SUSY breaking. Spurion field is sometimes introduced as an auxiliary field which has only $F$-term to represent SUSY breaking mass parameters. The origin of spurion field is considered as super-heavy particle in a underlying theory, and then it has no vacuum expectation value (VEV) of scalar component\cite{9}.

In this paper, we suggest new setup where SUSY breaking spurion $F$-term possesses lepton number. This setup not only modifies sparticle mass spectra but also realizes several new models, where neutrino mass is naturally induced by radiative corrections. We here suggest two new models; the first one is (i): pseudo-Dirac/Schizophrenic neutrino model, and the second one is (ii): pure Majorana neutrino model. We will also show this setup can naturally apply to the supersymmetric Zee-Babu model. The necessary lepton number violating couplings are induced in the soft SUSY breaking interactions, while the lepton number is kept in the superpotential.

2 Spurion fields and SUSY breaking terms

Let us consider a setup where SUSY breaking $F$-term possesses lepton number. We introduce a hidden sector, where SUSY is broken by two types of gauge singlet spurion fields. One type of the spurions has no lepton number, and the other type has a lepton number. In the following we denote the lepton number singlet spurion as $X$, and the spurion with a lepton number as $X_L$. We assume these spurion fields have non-zero $F$-terms, whose effects are transformed\cite{0}.

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to our visible sector as soft SUSY breaking terms through gravity interactions. The spurion fields are expected to have no VEV at scalar component. Of course, we can also consider other messenger of SUSY breaking, such as gauge mediation, but here we would like to show the gravity mediation, for simplicity. We consider gauge singlet spurion fields and impose \( R \)-parity conservation, hereafter.

Table 1: Matter contents of the MSSM.

| Superfields | SU(3) \(_C\) | SU(2) \(_L\) | U(1) \(_Y\) | U(1) \(_L\) |
|-------------|-------------|-------------|-------------|-------------|
| \( Q_i \)   | 3           | 2           | +\( \frac{1}{6} \) | 0           |
| \( U^c_i \)| 3           | 1           | −\( \frac{2}{3} \) | 0           |
| \( D^c_i \)| 3           | 1           | +\( \frac{1}{3} \) | 0           |
| \( L_i \)   | 1           | 2           | −\( \frac{1}{2} \) | 1           |
| \( E^c_i \)| 1           | 1           | +1           | −1          |
| \( H_1 \)   | 1           | 2           | −\( \frac{1}{2} \) | 0           |
| \( H_2 \)   | 1           | 2           | +\( \frac{1}{2} \) | 0           |

Usually, only \( X \) is considered in order to give soft SUSY breaking terms in the MSSM, whose fields content is shown in Table 1. The superpotential of the MSSM is given by

\[
W_{\text{MSSM}} = y_{ij}^u U^c_i Q_j \cdot H_2 + y_{ij}^d D^c_i Q_j \cdot H_1 + y_{ij}^e E^c_i L_j \cdot H_1 + \mu H_1 \cdot H_2. \tag{1}
\]

In the MSSM with spurion \( X \), the soft SUSY breaking terms are induced as

\[
\mathcal{L}^{\text{soft}}_{\text{MSSM}} = \int d^4 \theta \left\{ \frac{(c^{\Phi})^{ij}}{M^2} X^\dagger X \Phi_i^\dagger \Phi_j \right\} + \int d^2 \theta \left\{ \frac{(a_U)^{ij}}{M} X U^c_i Q_j \cdot H_2 + \frac{(a_D)^{ij}}{M} X D^c_i Q_j \cdot H_1 + \frac{(a_E)^{ij}}{M} X E^c_i L_j \cdot H_1 + b_H X H_1 \cdot H_2 + \text{h.c.} \right\} + \int d^2 \theta \left\{ \frac{(d_3)}{M} X G^\alpha G_\alpha + \frac{(d_2)}{M} X W^\alpha W_\alpha + \frac{(d_1)}{M} X B^\alpha B_\alpha + \text{h.c.} \right\}, \tag{2}
\]

where we denote the SM matter and Higgs fields as \( \Phi_i (= Q_i, L_i, D^c_i, U^c_i, E^c_i, H_1, H_2) \). When the \( F \)-term of \( X \), which is denoted as \( F_X \), has non-zero magnitude, the first line of the above equation induces the scalar masses, the second and third lines induce the tri-linear scalar couplings, and the last line gives the gaugino masses.

In addition to ordinal soft SUSY breaking terms induced by \( F_X \), we can have SUSY breaking effects from \( F \) components of \( X_L \), denoted as \( F_{X_L} \). In the hidden sector, not only the SUSY but also the lepton number is softly broken by the \( F_{X_L} \). Additional terms to \( \mathcal{L}^{\text{soft}}_{\text{MSSM}} \)
induced from the interactions between spurion field $X_L$ and the MSSM fields can be written as

$$\mathcal{L}^{\text{soft, }X_L} = \mathcal{L}^{\text{soft, }MSSM} + \int d^4\theta \left\{ \frac{(\hat{c}_N)^{ij}}{M^2} X^i_L X^j_L \Phi^i \Phi_j \right\}.$$  \hspace{1cm} (3)

Notice that $F_X$ induces all soft masses, while $F_{X_L}$ does not induce $A$-term, $B$-term, and gaugino mass terms. Thus, $F_{X_L}$ only contributes to soft “squared” masses due to the lepton number symmetry. In other words, $F_{X_L}$ can induce non-universal effects with $F_{X_L} < F_X$. Anyhow, we show that this setup can suggest new models.

3 New models

In the framework with $X_L$, new radiative induced neutrino mass models can be considered. Let us show three examples of such models. In these models, the lepton number is conserved in the superpotential, but it is softly broken in the SUSY breaking interactions.

3.1 Loop induced Majorana neutrino mass model I

The first model contains the right-handed neutrinos in addition to the MSSM particles. Imposing the $U(1)_L$ symmetry, the Majorana mass term of the right-handed neutrinos in the superpotential is forbidden. Then the superpotential is given by

$$W_I = W_{\text{MSSM}} + y_{ij} N^c_i L_j \cdot H_2.$$  \hspace{1cm} (4)

As mentioned above, the lepton number is kept as a good quantum number in the superpotential. In our framework, the lepton number violation occurs only in the soft SUSY breaking terms induced by $F_{X_L}$. Considering here the spurion possessing the lepton number 2, which is denoted as $X_{L_2}$, we can write the soft SUSY breaking Lagrangian as

$$\mathcal{L}_{\text{Model-1}} = \mathcal{L}^{\text{soft, }X_L} + \int d^4\theta \left\{ \frac{(\hat{c}_N)^{ij}}{M^2} X^i_L X^j_L + \frac{(\hat{c}_N)^{ij}}{M^2} X^i_{L_2} X^j_{L_2} \right\} N^c_i N^c_j \right\}$$

$$+ \int d^2\theta \left\{ \frac{a_N}{M} X N^c_i L_j \cdot H_2 + \frac{b_2 N^c_i N^c_j} + h.c. \right\}.$$  \hspace{1cm} (5)
The $A$-term can be parametrized as $(a_N)_{ij} = (\bar{a}_N)_{ij} y'_{ij}$. Then, the right-handed sneutrino mass matrix is given by

$$\tilde{m}^2_{\tilde{\nu}_R} \simeq \begin{pmatrix} \frac{c_N}{M} |F_X|^2 + \frac{\hat{b}_N}{M} |F_{X_L_2}|^2 & \hat{b}_N F_{X_{L_2}}^* \\ \hat{b}_N F_{X_{L_2}} & \frac{c_N}{M} |F_X|^2 + \frac{\hat{b}_N}{M} |F_{X_L_2}|^2 \end{pmatrix}$$

in the basis of $(\tilde{\nu}^R, \tilde{\nu}_R)$. Lepton number is violated by non-vanishing value of $\hat{b}_N F_{X_{L_2}}$.

In this model, the Majorana mass terms for the neutrinos do not appear at the tree level. It is naively expected that the light neutrinos are Dirac type. However the right-handed sneutrino mass term breaks lepton number and dimension five lepton number violating operator is induced through one-loop diagram shown in the Fig. 1 as

$$\mathcal{L}_{\text{LNV}} = (C_L)_{ij} (\ell_i \cdot \Phi_2) (\ell_j \cdot \Phi_2),$$

where $(C_L)_{ij}$ is determined by sneutrino and SUSY breaking mass parameters. The right-handed Majorana mass terms might be induced at the higher loop level, but they are suppressed much stronger. Taking account of the $n_N$ generation ($n_N \leq 3$) of the right-handed neutrinos, the $(3 + n_N) \times (3 + n_N)$ neutrino mass matrix is induced after the electroweak symmetry breaking as

$$m_\nu \simeq \begin{pmatrix} m_{LL} & m_D \\ m_D & 0 \end{pmatrix},$$

where $m_{LL}$ and $m_D$ are $3 \times 3$ matrix and $n_N \times 3$ matrix, respectively. $m_D$ is the Dirac mass matrix as

$$(m_D)_{ij} = (y_\nu)_{ij} \langle \Phi_2 \rangle. $$

On the other hand, $m_{LL}$ is induced from dimension five operator in Eq. (7), which is given by

$$(m_{LL})_{ij} \simeq \frac{g^2}{(4\pi)^2} \frac{\langle \Phi_0 \rangle^2}{m_{\chi_0}^6} \frac{\Gamma_N^{(a)}}{\Gamma_N} \frac{\Gamma_N^{(a)}}{\Gamma_N} (A_\nu)_{ki} (A_\nu)_{nj} (\hat{b}_N)_{kn} F_{X_{L_2}}$$

Figure 1: Loop diagrams relevant to the left-handed Majorana neutrino mass.
where \((A_{\nu})_{ij} = \frac{(\hat{a}_N)^i_j y_{\nu}^j}{M} F_X\), and \(\hat{m}\) is the diagonal element of right-handed sneutrino mass matrix in Eq. (6). Notice that lepton number violating squared mass component, \((\hat{b}_N)_{ij} F_{X_{L_2}}\), plays a crucial role of generating Majorana masses of light neutrinos, and Majorana nature disappear as \((\hat{b}_N F_{X_{L_2}}) \to 0\) (and also \(\hat{a}_N \to 0\) as vanishing \(A\)-term). The coupling \(\Gamma_N^{(a)}\) is defined as

\[
\Gamma_N^{(a)} = \sqrt{2} \left( \frac{1}{2} (U_{a2}^*) - \frac{1}{2} \tan \theta_W (U_{a1}) \right),
\]

where the unitary matrix \(U_N\) is a neutralino mixing matrix as

\[
\tilde{\chi}_a = (U_N)_{ai} \bar{\psi}_i^0, \quad \bar{\psi}_i = (\bar{B}_i^0, \bar{W}_i^0, \bar{H}_i^0, \bar{H}_i^0).
\]

We then obtain radiative Majorana neutrino mass through the effective dimension five operator.

Choosing the structure of the \(y_{\nu}, \hat{a}_N,\) and \((\hat{b}_N F_{X_{L_2}})\), we can obtain Dirac-, pseudo-Dirac-, or Schizophrenic-\cite{9,10} types of light neutrinos. For example, the Dirac-type neutrino is obtained when \(\hat{a}_N = 0\) or \((\hat{b}_N F_{X_{L_2}}) = 0\). As for pseudo-Dirac-type neutrinos, there are constraints for neutrino absolute mass, \(\hat{a}_N,\) and \((\hat{b}_N F_{X_{L_2}})\), since neutrino oscillation experiments suggest no large mixing to the sterile neutrinos. The induced Majorana neutrino \(m_{LL}\) should be suppressed much smaller than the mass splitting of the active neutrinos, \(\Delta m_{\odot}, \Delta m_A\), which implies

\[
\frac{m_{\nu}^2 m_{\chi^0}^2 \hat{a}_N^2 \hat{b}_N F_X^2 F_{X_{L_2}}}{M^2 \hat{m}_6^6} \ll \mathcal{O}(10^{-19} \text{GeV}^2).
\]

For example, \(m_{\chi^0} \simeq \hat{m} = 500 \text{ GeV}, m_{\nu} = 0.05 \text{ eV},\) and \((\hat{b}_N F_{X_{L_2}}) = 100 \text{ GeV}^2\) are taken, the bound on the \(A\)-term is read as \((a_N F_X/M) \ll \mathcal{O}(10^2 \text{GeV})\).

Next example is Schizophrenic-type neutrinos, which is realized with the specific structure of \(y_{\nu}, \hat{a}_N,\) and \((\hat{b}_N F_{X_{L_2}})\). For simplicity, let us consider one right-handed neutrino generation case \((n_N = 1)\). With the following parameters,

\[
y_{\nu} = (2.7 \times 10^{-14} \quad 2.7 \times 10^{-14} \quad 2.7 \times 10^{-14}),
\]

\[
(a_N F_X/M) = (0 \quad 68 \quad -68) \text{ [GeV]},
\]

\(m_{\chi^0} = M = 500 \text{ GeV},\) and \((\hat{b}_N F_{X_{L_2}}) = 100 \text{ GeV}^2,\) the \(4 \times 4\) neutrino mass matrix becomes

\[
m_{\nu} = \begin{pmatrix}
0 & 0 & 0 & 0.005 \\
0 & 0.025 & -0.025 & 0.005 \\
0 & -0.025 & 0.025 & 0.005 \\
0.005 & 0.005 & 0.005 & 0
\end{pmatrix} \text{ [eV]}
\]

which has the same structure as Eq. (4) with \(m_1 = 0\) in Ref. [10]. The mass matrix gives the tri-bimaximal neutrino mixing with massless lightest neutrino. With this parameter set, the second lightest neutrino is a Dirac type and the heaviest one is Majorana type.
Notice that in the case of extended gauge symmetry with $U(1)_{B-L}$ as in Ref. [9], Majorana mass $m_{RR}$ is also induced through one-loop diagram. Where right-handed sneutrino and $B-L$ gaugino are propagating in internal lines, and which can make a TeV-scale seesaw work.

3.2 Loop induced Majorana neutrino mass model II

The second model also contains right-handed neutrinos, however, we assign the $U(1)_L$-charge of $N^c_i$ as quite different from conventional models as shown in Table 3. We introduce three kinds of spurion fields $X, X_{L_2}$, and $X_{L_6}$, which have lepton number 0, 2, and 6, respectively. Note that Yukawa interaction of $N^c_i L_i H_2$ is forbidden by $U(1)_L$ symmetry. The soft SUSY

| Table 3: Additional matter in radiative induced neutrino mass model II. |
|-----------------|-----------------|
| **Superfields** | **SU(3)$_C$** | **SU(2)$_L$** | **U(1)$_Y$** | **U(1)$_L$** |
| $N^c_i$         | 1               | 1             | 0             | -3            |

breaking terms replaced to the right-handed neutrino are generated from

$$
\mathcal{L}_S = \mathcal{L}_{\text{MSSM}}^{\text{soft}} + \int d^4\theta \left\{ \left( \frac{(c_N)^{ij}}{M^2} X^\dagger X + \frac{(-\tilde{c}_N)^{ij}}{M^2} X_{L_2}^\dagger X_{L_2} \right) N^c_i \right\} - 3| \right\}

$$

(16)

We assume that the right-handed supersymmetric Majorana mass term of $M_{ij} \bar{\nu}_{R_i} \bar{\nu}_{R_j}$ is induced by a mechanism associated with $U(1)_{B-L}$ breaking. Notice that Dirac neutrino mass is forbidden, while Majorana neutrino mass is allowed. Majorana masses of $m_{LL}$ are induced through the one-loop diagram as Fig.1. After the electroweak symmetry breaking, the $(3 + n_N) \times (3 + n_N)$ neutrino mass matrix is induced as

$$
m_\nu \simeq \begin{pmatrix} m_{LL} & 0 \\ 0 & M \end{pmatrix},
$$

(17)

where the formula for $m_{LL}$ is given in Eq. (10). Let us take a suitable parameter set for $n_N = 2$ case, for example,

$$
(a_N F_X/M) = \begin{pmatrix} 0 & 68 \\ -23 & -68 \end{pmatrix} [\text{GeV}], \quad (b_N F_{X_{L_2}}) = 100 \text{ GeV}^2,
$$

(18)

with $m_{\tilde{\chi}_0^0} = M = 500 \text{ GeV}$ and $(\tilde{b}_N F_{X_{L_2}}) = 100 \text{ GeV}^2$, we can obtain neutrino masses and mixing angles as

$$
m_1 = 0.0 \text{ eV}, \quad m_2 = 0.0088 \text{ eV}, \quad m_3 = 0.05 \text{ eV},
$$

(19)
\[
\sin^2 2\theta_A = 1.0, \quad \tan^2 \theta_\odot = 0.5, \quad \sin^2 \theta_{13} = 0.0, \quad (20)
\]

which can explain the neutrino oscillation data with tri-bimaximal mixing.

We should comment that, if there exist \(U(1)_{B-L}\) gauge symmetry, Dirac mass is also induced through one-loop diagram. Where, by \(b_N\) coupling, sneutrino mass is modified, but its effect is negligible comparing to SUSY Majorana mass.

### 3.3 Application to SUSY Zee-Babu model

We can also apply our setup to known models, where the lepton number violation is required for generating neutrino masses. A famous example is Zee-Babu model\[^5\], and trial of SUSY extension of the model has been first suggested in Ref. \[^7\]. This model has extra two pairs of the \(SU(2)\)-singlet fields (and their hypercharges are ±1 and ±2, respectively) in addition to the MSSM as Table 4. In this model, the superpotential is written as

| Superfields | \(SU(3)_C\) | \(SU(2)_L\) | \(U(1)_Y\) | \(U(1)_I\) |
|-------------|-------------|-------------|-------------|-------------|
| \(\Omega_{\pm}\) | 1 | 2 | ±1 | \(\mp 2\) |
| \(K_{\pm}\) | 1 | 2 | ±2 | \(\mp 2\) |

\[
W = W_{\text{MSSM}} + f_{ij}L_i \cdot L_j \Omega + g_{ij}E_i^c E_j^c K - \mu_{\Omega} \Omega_+ \Omega - + \mu_K K_+ K_-. \quad (21)
\]

Let us introduce spurion fields, \(X\) and \(X_{L_2}\), which have lepton number 0 and 2, respectively. Then, interactions between these spurion fields and the scalar components of the visible sector fields are given by

\[
\mathcal{L}_S = \mathcal{L}_{\text{MSSM}} + \int d^4\theta \left\{ \left( \frac{c_{\Psi}}{M^2} X^\dagger X + \frac{\tilde{c}_{\Psi}}{M^2} X^\dagger_{L_2} X_{L_2} \right) \Psi \Psi \right\} \\
+ \int d^2\theta \left\{ b_{\Theta} X \Omega_+ \Omega_- + b_K X K_+ K_- + \frac{(a_{LL})^{ij}}{M} X L_i \cdot L_j \Omega + \frac{(a_{EE})^{ij}}{M} X E_i^c E_j^c K \\
+ \frac{\tilde{a}_{M}}{M} X_{L_2} K_- \Omega_+ \Omega_- + \text{h.c.} \right\}, \quad (22)
\]

where \(\Psi\) represents \(\Psi = \Omega_+, \Omega_-, K_+, K_-\). Notice that the lepton number is softly broken by two units when the SUSY is broken by the \(F\)-term of the spurion, \(X_{L_2}\).

The neutrino mass in the model is induced through two-loop diagram in Fig. 2 which is the same as non-SUSY Zee-Babu model\[^5\]. The neutrino mass in the model is evaluated as\[^5\], \[^7\],

\[
(m_{\nu})_{ij} = \left( \frac{1}{16\pi^2} \right)^2 \frac{16\mu_B f_{ik} m_{\ell_k} g_{ikm_{\ell_k} f_{j\ell}}}{m_{\kappa}^2} I \left( \frac{m_{\kappa}^2}{m_{\omega}^2} \right), \quad (23)
\]
where $m_\kappa$ and $m_\omega$ are the masses of the scalar components of $K_-$ and $\Omega_+$, $I(x)$ is the loop function given by

$$\begin{equation}
I(r) = -\int_0^1 dx \int_0^{1-x} dy \frac{1}{x + (r-1)x + y^2} \ln \frac{y(1-y)}{x + ry},
\end{equation}$$

and the lepton number violating coupling $\mu_B$ is

$$\mu_B = \frac{\tilde{a}_- \langle F_{XL2} \rangle}{M}.$$  

In this setup, the lepton number is a good quantum number with softly breaking parameter. Therefore, our suggesting setup can cure the model and we can obtain accurately modified SUSY Zee-Babu model. Since the lepton number symmetry is broken softly, lepton number violating processes can be strongly suppressed comparing to lepton number hardly breaking model\cite{7}. For example, a lepton number violating cross section of $\mu \rightarrow e \nu \nu$ is tiny since it is induced through two-loop diagram in our softly breaking setup, while this process appears in one-loop diagram in hardly breaking setup.

4 Conclusion

We have suggested new setup where SUSY breaking spurion $F$-term possesses lepton number. This setup not only modifies sparticle mass spectra but also realizes several new models, where neutrino mass is naturally induced by radiative corrections. We have suggested two new models; the first one is (i): pseudo-Dirac/Schizophrenic neutrino model, and the second one is (ii): pure Majorana neutrino model. We have also shown this setup can naturally apply to the supersymmetric Zee-Babu model. The necessary lepton number violating couplings are induced in the soft SUSY breaking sector, while the lepton number is kept in the superpotential.
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