CHIRAL MULTIPLETS OF HADRON CURRENTS

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Abstract

Using group theoretical methods, we enumerate possible chiral representations in which hadron interpolating currents can be classified. We give simple examples of currents in each representation, some of which are well known. The classification enables one to find relations among current vacuum correlators in a chirally and/or axial U(1) symmetric phase of QCD in a straightforward way. Besides recovering many well-known relations among two-point correlation functions, a number of novel relations are found.

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I. INTRODUCTION

Approximate chiral symmetry is an important feature of the QCD lagrangian. Much of the low-energy behavior of QCD at zero temperature and density can be understood in terms of chiral symmetry, its spontaneous breaking and the anomalous breaking of the $U(1)_A$ subgroup. There has been considerable interest in the possibility of a chirally restored phase of QCD, as might be expected at a sufficiently high temperature, or a large number of flavors, or a high density [1]. Recently, there have also been some speculations about the possibilities that $U(1)_A$ symmetry might be restored [2–4], despite the fact that the anomaly in the singlet axial current is formally temperature-independent [5]. These issues are of obvious theoretical interest. They also may have nontrivial experimental implications since regions of the chirally restored phase might be produced in ultrarelativistic heavy-ion collisions.

An important issue in attacking this problem theoretically is finding calculable quantities which are sensitive to whether the phase breaks the symmetry. There are many well-known signatures of chiral symmetry restoration; for instance, the vanishing of the chiral quark condensates and the disappearance of Goldstone modes. A special class of tests consists of comparing different thermal two-point correlation functions of currents with hadronic quantum numbers; in the symmetric phase they are connected due to the underlying chiral (or possibly axial U(1)) symmetry. We wish to observe that it is important in practice to check many of these signatures simultaneously since practical QCD-based calculations are limited to numerical calculations on the lattice which necessarily have both statistical and systematic errors. Accordingly it is hard to tell whether the approximate vanishing of a single observable is an indication of symmetry restoration or simply an accidently small value masked by numerical noise.

Correlation functions with hadron quantum numbers are a useful window into the structure of excitations of the QCD vacuum. They have been used widely in the QCD sum rule and lattice calculations to study hadron spectrum at zero temperature. One can construct an infinite number of hadron currents of different dimensions and Lorentz symmetry by employing covariant derivatives, gluon and quark fields. However, in realistic calculations, one uses simple ones with given quantum numbers. In the chirally-symmetric phase, chiral symmetry imposes many relations among the current correlators; these are chiral Ward identities. Some of these are well known and can be derived by simple inspections.

The goal of this paper is to show that a very large class of hadron interpolating currents fall into a few chiral representations and that relations among current correlators can be systematically derived using multiplications of these representations. In particular, we will explicitly enumerate all of the chiral representations of mesonic currents that carry flavor quantum numbers of one quark and one antiquark fields and the representations of baryon currents that carry flavor quantum numbers of three quark fields. We should note that this is a very large class of interpolating currents—there are no restrictions on the number of covariant derivatives, gluon fields, and quark pairs that are coupled to chiral singlets, and indeed no restriction that the current even be local (although they must be gauge invariant). While we have focused our attention to a restricted class of currents, the techniques used in this paper can be extended straightforwardly to determine the chiral representation for arbitrary currents, such as the pion interpolating current of type $\bar{q} \tau^a q \bar{q} i \gamma_5 q$. It is also worth
noting that virtually all practical lattice gauge or QCD sum rule calculations have used currents in the class considered here.

The role of the chiral multiplet structures of the currents on two-point correlation functions in the chirally restored phase is significant in two ways. The first concerns correlation functions between two distinct currents with the same flavor, spin, and parity quantum numbers but which belong to distinct chiral representations. In a chirally-broken phase such as the $T = 0$ vacuum, such correlation functions are generically nonzero. In terms of a $T = 0$ spectral representation of the correlator, this indicates nothing more than the fact that each of these currents has a nonzero overlap between the vacuum and the same physical state. However in a chirally restored phase all such mixed correlators are identically zero. This vanishing of all these mixed correlators can be used as a signature of symmetry restoration. Some examples have already been considered in Ref. [6]. When one enumerates the chiral representations one sees that for virtually every flavor, spin and parity channel there are interpolating currents with at least two distinct sets of chiral quantum numbers, even when restricting to the class of currents considered here. This is significant when trying to interpret the nature of the chirally restored phase. For example, questions such as “does the $\rho$ meson survive the chiral transition?” becomes intrinsically ambiguous. The question cannot even be formulated without specifying the chiral quantum numbers of the current coupling to the $\rho$ channel.

The second class of issues concerns the equality of certain two-point correlation functions. If a current is in a nontrivial chiral multiplet, then under chiral rotations one generates new currents with distinct parity and/or flavor quantum numbers. Clearly, in a chirally restored phase these newly generated currents must yield correlation functions identical to the original ones and hence one predicts that certain correlators must be identical. Of course, many such examples have long been known. For example, for two massless flavors the correlators in the $\sigma$ (scalar-isoscalar) and $\pi$ (pseudoscalar-isovector) channels corresponding to the currents, $\bar{q}\tau^a q$ and $\bar{q}\tau^a i\gamma_5 q$, respectively are well known to be the same in the restored phase; similarly the $\rho$ (vector-isovector) and $A_2$ (pseudovector-isovector) corresponding to the currents $\bar{q}\gamma^i\tau_i q$ and $\bar{q}\gamma_5\gamma^i\tau_i q$ are also well known to be identical. However, there are many examples which are less familiar. For example, the $\rho$ and the $b_1$ (pseudovector-isovector and charge conjugation odd) corresponding to the currents $\bar{q}\tau^a \sigma^{ij} q$ and $\bar{q}\tau^a \sigma^{0i} q$, respectively, are also identical in this phase. We note that there is a very large number of these relations including a certain nucleon interpolating current whose correlator in the chirally restored phase is identical to correlators in the $\Delta (\frac{1}{2}^-)$ channel.

It is also interesting to consider correlator relations which would result if the $U(1)_A$ symmetry were to be restored. Whether the effects of the $U(1)_A$ anomaly play a role in the chirally restored phase and if they do how do, these effects die off with increasing temperature, remain interesting questions. Accordingly, it is useful to classify good signatures of the effects of a manifest $U(1)_A$ symmetric phase on correlation functions. In the course of our discussion, we will recover many relations which are widely known and have already been used frequently in the literature. However, besides organizing those into categories, we also find a number of new relations which we suggest will provide further insights for lattice or model calculations.

We divide our discussions into meson and baryon currents. For each case, we consider the possibilities of two and three massless flavors separately. Of course, in nature there are
neither two nor three flavors of massless quarks. For the analysis here to be of any use in connecting to real QCD, it is important that for quantities of interest the quark masses must be small enough to either neglect outright or to include perturbatively in some type of chiral perturbation theory. The up and down quark masses are presumably light enough for such a procedure to make sense. Moreover, the nature of the chiral symmetry of the underlying theory allows one to deduce certain relations between correlators which hold to order \( m_q^2 \) rather than \( m_q \), thereby enhancing the range of validity of the chiral expansion \( \mathcal{O} \). Treating the strange quark mass as small is clearly far more problematic. It is by no means clear that a chiral expansion in \( m_s \) will be valid for any given quantity of interest particularly in the vicinity of the phase transition. Even if it turns out that an expansion in \( m_s \) is not valid, the three massless flavor results are not without interest. One obvious use is in providing limiting cases against which to test more realistic calculations. For example, lattice calculations which hope to connect to the real world of two light and one intermediate mass flavors could be re-run comparatively easily with three light flavors. The ability of such calculations to reproduce the correlator relations for three massless flavors will be quite useful in demonstrating that the systematic and statistical errors inherent in the calculations are under control.

II. MESON CURRENTS

The simplest meson currents can be constructed from one-quark and one-hermitian-conjugated-quark fields. Although one can construct more complicated meson currents by including covariant derivatives, gluon and quark fields, a large class of meson currents belong to the chiral multiplets of one-quark and one-antiquark product representations. For \( N_f \) flavors, the relevant chiral multiplets are representations of \( SU(N_f)_L \times SU(N_f)_R \). In the following subsections, we consider the simplest chiral multiplets for two and three massless flavors.

A. Two Massless Flavors

The quark fields \((u_L, d_L)\) and \((u_R, d_R)\) belong to the basic representation \((\frac{1}{2}, 0) + (0, \frac{1}{2})\) of \( SU(2)_L \times SU(2)_R \). The convention for denoting chiral multiplets is that the first and second numbers in a bracket refer to \( SU(2)_L \) and \( SU(2)_R \) representations, respectively. Under parity transformation, the left-handed fields become right-handed and vice versa. In the case of \( SU(2) \), the hermitian-conjugation fields transform according to the same representation as the original fields.

To classify meson currents, we consider the product representation, \([ (\frac{1}{2}, 0) + (0, \frac{1}{2}) ] \times [ (\frac{1}{2}, 0) + (0, \frac{1}{2}) ] \). The simple angular momentum addition rules yield,

\[
\left[ (\frac{1}{2}, 0) + (0, \frac{1}{2}) \right] \times \left[ (\frac{1}{2}, 0) + (0, \frac{1}{2}) \right] = \left[ (\tilde{0}, 0) + (0, \tilde{0}) \right] + \left[ (\frac{1}{2}, 0) + (0, \frac{1}{2}) \right] + \left[ (\frac{1}{2}, 1) + (\frac{1}{2}, 1) \right] + \left[ (1, 0) + (0, 1) \right],
\]

where tilde on \( \tilde{0} \) has no group-theoretical role. It simply serves as a reminder that this singlet corresponds a left- or right-handed quark-antiquark pair coupled into a flavor-singlet rather
multiplets can be obtained by inserting the isospin Pauli matrices $\tau$ above currents, $\eta$ which have the quantum numbers of $\delta$.

Thus, in the chirally-symmetric phase, the bilocal current correlators of each pair are the isoscalar ones with good parity correspond to the quark multiplets. The isoscalar ones with good parity correspond to the quark multiplets. The isoscalar ones with good parity correspond to the quark multiplets.

$$j_{\mu I=0}^\mu = \bar{q} \gamma^\mu q$$

where $q$ is a column vector consisting of up and down quark fields. Both currents are invariant under $SU(2)_L \times SU(2)_R$ and $U(1)_A$ transformations. Therefore, flavor symmetries impose no constraints on their correlation functions.

Next, we consider $(\frac{1}{2}, \frac{1}{2}) + (\frac{1}{2}, \frac{1}{2})$. Under the isospin subgroup, they contain both isoscalar and isovector multiplets. The isoscalar ones with good parity correspond to the quark bilinears $(\bar{u}_L u_L + \bar{d}_L d_L)$ and $(\bar{u}_R u_R + \bar{d}_R d_R)$. Two examples of the positive parity currents are,

$$j_{sL=0} = \frac{1}{2} \bar{q} q$$  \hspace{.5cm} (3)

which have the quantum numbers of $\eta(0^{++})$ and $h_1(1^{+-})$, respectively. And the corresponding examples of the negative parity currents are,

$$j_{pI=0} = \frac{1}{2} \bar{q} i \gamma_5 q$$  \hspace{.5cm} (4)

which have the quantum numbers of $\rho(0^{--})$ and $\omega(1^{--})$, respectively. Examples of isovector multiplets can be obtained by inserting the isospin Pauli matrices $\tau^a (a = 1, 2, 3)$ into the above currents,

$$j_{sI=1}^a = \frac{1}{2} \tau^a q$$

$$j_{pI=1}^a = \frac{1}{2} \tau^a q$$

where $q$ is a column vector consisting of up and down quark fields. Both currents are invariant under $SU(2)_L \times SU(2)_R$ and $U(1)_A$ transformations. Therefore, flavor symmetries impose no constraints on their correlation functions.

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$$j_{sI=1}^a = \frac{1}{2} \tau^a q$$

$$j_{pI=1}^a = \frac{1}{2} \tau^a q$$

which have the quantum numbers of $\delta(0^{++})$, $b_1(1^{+-})$, $\pi(0^{--})$, and $\rho(1^{--})$, respectively.

Under $SU(2)_L \times SU(2)_R$, the isoscalar and isovector currents transform into each other in pairs,

$$j_{sI=0} \leftrightarrow j_{pL=1}^a$$

$$j_{pI=0} \leftrightarrow j_{sL=1}^a$$

$$j_{sI=1} \leftrightarrow j_{pL=1}^a$$

$$j_{pI=1} \leftrightarrow j_{sL=1}^a$$

Thus, in the chirally-symmetric phase, the bilocal current correlators of each pair are the same,

$$\langle T j_{sL=0}(x) j_{sL=0}(0) \rangle = \langle T j_{pL=1}^a(x) j_{pL=1}^a(0) \rangle$$

$$\langle T j_{pL=0}(x) j_{pL=0}(0) \rangle = \langle T j_{sL=1}^a(x) j_{sL=1}^a(0) \rangle$$

$$\langle T j_{sL=0}(x) j_{pL=0}(0) \rangle = \langle T j_{pL=1}^a(x) j_{sL=1}^a(0) \rangle$$

$$\langle T j_{pL=0}(x) j_{sL=0}(0) \rangle = \langle T j_{sL=1}^a(x) j_{pL=1}^a(0) \rangle$$  \hspace{.5cm} (7)
Notice that there is no summation for repeated indices in the above equation. Unless stated explicitly, the same is used for other equations below. The first two relations say that the $\pi$ ($\delta$) type of correlators are the same as $\sigma$ ($\eta$) type of correlators, a result which is familiar. The second two relations say that the $\rho$ ($b_1$) type of correlators are the same as $h_1$ ($\omega$) type, which is less known.

On the other hand, under $U(1)_A$ transformations, the currents with opposite parities transform into each other. If $U(1)_A$ symmetry is restored in some phase, we then have the following relations among the correlators,

$$
\langle T_{jI=0}^\mu(x) j_{I=0}^\mu(0) \rangle = \langle T_{jI=1}^\mu(x) j_{I=1}^\mu(0) \rangle ,
\langle T_{jI=0}^k(x) j_{I=0}^k(0) \rangle = \langle T_{jI=1}^k(x) j_{I=1}^k(0) \rangle ,
\langle T_{jI=0}^{ka}(x) j_{I=1}^{ka}(0) \rangle = \langle T_{jI=1}^{ka}(x) j_{I=1}^{ka}(0) \rangle .
$$

(8)

The isovector relations are particularly interesting because they contain no disconnected contributions in the path-integral formulation. In the literature, the $\pi$ and $\delta$ types of correlators have been compared at the chiral transition region to learn about the fate of $U(1)_A$ symmetry [1,6,8,9]. The same comparison can be made of the $\rho$ and $b_1$ types of correlators.

Finally, we consider $(0,1)+(0,1)$ which contain isovector multiplets only. The simplest example of the multiplets is,

$$
\tilde{j}_{vI=1}^{\mu a} = \bar{q} \gamma^\mu \frac{\tau^a}{2} q ,
\tilde{j}_{aI=1}^{\mu a} = \bar{q} \gamma^\mu \gamma^5 \frac{\tau^a}{2} q ,
$$

(9)

which have the quantum numbers of $\rho$ and $A_1$, respectively. It is worth noting that the currents written above are both conserved in the massless limit and hence do couple only to vectors (axial vectors) and not to scalars (pseudoscalars). More general realization of currents in this representation such as

$$
\tilde{j}_{vI=1}^{\mu a} = \bar{q} \gamma^\mu \frac{\tau^a}{2} F^2 q ,
\tilde{j}_{aI=1}^{\mu a} = \bar{q} \gamma^\mu \gamma^5 \frac{\tau^a}{2} F^2 q ,
$$

(10)

where $F^2 = F^{\alpha\beta} F_{\alpha\beta}$, have the quantum numbers of $(\delta, \rho)$ and $(\pi, A_1)$, respectively.

Under $U(1)_A$ transformations, the currents are invariant. Under $SU(2)_L \times SU(2)_R$ chiral transformations, they mix with each other. Thus if the vacuum is chirally-symmetric, their two-point correlation functions are equal;

$$
\langle T_{\tilde{j}_{vI=1}^{\mu a}}(x) \tilde{j}_{vI=1}^{\mu a}(0) \rangle = \langle T_{\tilde{j}_{aI=1}^{\mu a}}(x) \tilde{j}_{aI=1}^{\mu a}(0) \rangle ,
$$

(11)

which is a well-known result.

A slightly more complicated example of $(0,1)+(1,0)$ multiplet involves currents with gluon fields,
\[ \tilde{j}_{\mu 1}^{a} = \bar{q} \gamma^\nu F_{\mu \nu} \frac{\gamma^a}{2} q , \]
\[ \tilde{j}_{a 1}^{\mu} = \bar{q} \gamma^\nu \gamma_5 F_{\mu \nu} \frac{\gamma^a}{2} q . \]

The first current has the quantum number of the exotic vector meson 1\(^{-+}\) and the second has that of \(b_1(1^{+-})\). The chiral symmetry again predicts the equality of the two-point correlators in the symmetric phase.

**B. Three Massless Flavors**

For three massless flavors, the meson currents belong to the product representations of \((3,1) + (1,3)\) and \((3,1) + (1,\bar{3})\). The former correspond to the quark fields \(u_L, d_L, s_L\) and \(u_R, d_R, s_R\); the latter correspond to the conjugate quark fields \(\bar{u}_L, \bar{d}_L, \bar{s}_L\) and \(\bar{u}_R, \bar{d}_R, \bar{s}_R\). SU(3) multiplication rules give,

\[
\left[ (3,1) + (1,\bar{3}) \right] \times \left[ (3,1) + (1,3) \right] = \left[ (\bar{1},1) + (1,\bar{1}) \right] + \left[ (3,3) + (3,\bar{3}) \right] + \left[ (8,1) + (1,8) \right].
\]

The notations for SU(3) representations are such that they denote the actual dimensions.

Let us consider first the chiral-singlet \((\bar{1},1) + (1,\bar{1})\). One can easily construct currents that are generalizations of those in \((0,0) + (0,\bar{0})\) of the two-flavor case (Eq.\((2)\)),

\[
j_{\mu 1}^{\bar{c}1} = \bar{q} \gamma^\nu q , \\
j_{\mu 1}^{\bar{a}1} = \bar{q} \gamma^\nu \gamma_5 q ,
\]

where \(q\) is now a column vector consisting of up, down, and strange quark fields. Both \(j_{\mu 1}^{\bar{c}1}\) and \(j_{\mu 1}^{\bar{a}1}\) are invariant under \(SU(3)_L \times SU(3)_R\) and \(U(1)_A\) transformations, and therefore chiral symmetries do not impose any constraints on their correlators.

The multiplet \((3,3) + (3,\bar{3})\) contains \(SU(3)_V\) octets and singlets. The eighteen \(J = 0\) currents constructed from 1 and \(\gamma_5\) matrices belong to this chiral multiplet,

\[
j_{s1} = \bar{q} q / \sqrt{6} , \\
j_{p8}^a = \bar{q} i \gamma_5 t^a q , \\
j_{p1} = \bar{q} i \gamma_5 q / \sqrt{6} , \\
j_{s8}^a = \bar{q} t^a q ,
\]

where \(t^a = \lambda^a / 2\) and \(\lambda^a\) \((a = 1,\ldots,8)\) are Gell-Mann matrices. Under \(SU(3)_L \times SU(3)_R\), these currents transform into each other, and their two-point correlators equal in a chirally symmetric phase,

\[
\langle T j_{s1}(x) j_{s1}(0) \rangle = \langle T j_{s8}^a(x) j_{s8}^a(0) \rangle = \langle T j_{p1}(x) j_{p1}(0) \rangle = \langle T j_{p8}^a(x) j_{p8}^a(0) \rangle .
\]

Using the currents in this class one sees that the correlator in the pion channel is necessarily the same as that in the \(\eta'\) channel despite the existence of the anomaly. This result
has previously been obtained from arguments based on instanton contributions \[11\] and explicitly on the basis of group theory [12].

One can also construct eighteen $J = 1$ currents in the same chiral multiplet from the $\sigma^{\mu\nu}$ matrix,

$$
\begin{align*}
\mathcal{J}_{t1}^k &= \bar{q}\sigma^{0k}q/\sqrt{6}, \\
\mathcal{J}_{\bar{t}8}^{ka} &= \frac{1}{2}\epsilon^{ijk}\bar{q}\sigma^{ij}t^aq, \\
\mathcal{J}_{\bar{t}1}^k &= \frac{1}{2}\epsilon^{ijk}\bar{q}\sigma^{ij}q/\sqrt{6}, \\
\mathcal{J}_{\bar{t}8}^{ka} &= \bar{q}\sigma^{0k}t^aq, \\
\end{align*}
$$

(17)

which have the quantum numbers of $J^{PC} = 1^{\pm+}$ mesons: $\rho, \omega, \phi, K^*, b_1, h_1, K_{1B}$. Under $SU(3)_L \times SU(3)_R$, these currents transform into each other in the same way as the $J = 0$ multiplet does. In the chirally-symmetric phase, their two-point correlators have similar relations as those in Eq. (16),

$$
\langle T \mathcal{J}_{t1}^k(x)\mathcal{J}_{t1}^k(0) \rangle = \langle T \mathcal{J}_{\bar{t}8}^{ka}(x)\mathcal{J}_{\bar{t}8}^{ka}(0) \rangle = \langle T \mathcal{J}_{\bar{t}1}^k(x)\mathcal{J}_{\bar{t}1}^k(0) \rangle = \langle T \mathcal{J}_{\bar{t}8}^{ka}(x)\mathcal{J}_{\bar{t}8}^{ka}(0) \rangle.
$$

(18)

Finally, we consider the multiplet $(1, 8) + (8, 1)$ which contains $SU(3)$ flavor octets. The simplest currents in the multiplet are,

$$
\begin{align*}
\mathcal{J}_{v8}^{\mu a} &= \bar{q}\gamma^\mu t^aq, \\
\mathcal{J}_{a8}^{\mu a} &= \bar{q}\gamma^\mu\gamma_5 t^aq.
\end{align*}
$$

(19)

Under $SU(3)_L \times SU(3)_R$, they transform into each other and thus their two-point correlators equal in the chirally-symmetric phase,

$$
\langle T \mathcal{J}_{v8}^{\mu a}(x)\mathcal{J}_{v8}^{\mu a}(0) \rangle = \langle T \mathcal{J}_{a8}^{\mu a}(x)\mathcal{J}_{a8}^{\mu a}(0) \rangle.
$$

(20)

Under $U(1)_A$ transformations, the currents are separately invariant.

Since there are no two simple currents which belong to the same chiral mutiplet but transform differently under the $U(1)_A$ group, one cannot form simple two-point correlators to test the $U(1)_A$ restoration, as in the two-flavor case. One can, however, construct three-point correlators that are chiral-singlet, but transform nontrivially under $U(1)_A$. An example is presented in Ref. [12].

### III. BARYON CURRENTS

Assuming color $SU(3)$ symmetry, one can construct the simplest baryon currents out of three quark fields. However, a large class of baryon currents can be classified in the chiral multiplets derived from the product of three basic (quark) representations. In the following subsections, we again consider two and three massless flavors separately.
A. Two Massless Flavors

For two massless flavors, we consider baryon currents belonging to \([0, \frac{1}{2}) + (\frac{1}{2}, 0)]^3\). Reducing it to irreducible multiplets, we find,

\[
\begin{align*}
\left[\left(0, \frac{1}{2}\right) + \left(\frac{1}{2}, 0\right)\right]^3 &= \left[\left(\frac{3}{2}, 0\right) + \left(0, \frac{3}{2}\right)\right] + 3 \times \left[\left(1, \frac{1}{2}\right) + \left(\frac{1}{2}, 1\right)\right] \\
&+ 3 \times \left[\left(\frac{1}{2}, \frac{1}{2}\right) + \left(1, 0\right)\right] + 2 \times \left[\left(\frac{1}{2}, 0\right) + \left(0, \frac{1}{2}\right)\right].
\end{align*}
\]

Multiple appearances of the same representations are due to the permutation symmetry of three quark labellings. The tildes on \(\tilde{0}\) and \(\frac{1}{2}\) serve as a reminder that a pair of left- or right-handed quarks has been coupled to flavor-singlet.

One of the simplest examples of currents in multiplet \((\frac{1}{2}, \tilde{0}) + (0, \tilde{0})\), is the spin-1/2 proton interpolating field,

\[
\eta_N = \left(u^T C \gamma_\alpha u\right) \gamma_5 \gamma^\alpha d,
\]

where and hereafter color indices on the quark fields are implicit and totally antisymmetric. This current has been used in the QCD sum rule calculations [13]. The current itself couples also to a negative-parity spin-1/2 state,

\[
\langle 0 | \eta_N (0) \bar{\eta}_N (x) | N(1/2) \rangle = \lambda_5 U(p),
\]

where \(U(p)\) is a Dirac spinor. [For a recent application of this feature, see Ref. [14].] Consider the following two-point correlator:

\[
\int d^4 x e^{i p x} \langle T \eta_N (0) \bar{\eta}_N (x) \rangle = \rho_1 (p^2) p_\mu \gamma^\mu + \rho_2 (p^2).
\]

In the chirally-symmetric phase, by making chiral rotation \(U = \exp(i \pi \tau^3 / 2)\), one can show,

\[
\langle T \eta_N (0) \bar{\eta}_N (x) \rangle = -\gamma_5 \langle T \eta_N (0) \bar{\eta}_N (x) \rangle \gamma_5.
\]

Thus, we have \(\rho_2 (p^2) = 0\), i.e. the correlator contains only the chiral-even term. Since negative parity states contribute to \(\rho_2 (p^2)\) with an opposite sign compared with positive parity states, the above result implies that every intermediate state has a degenerate partner of opposite parity and their chiral-odd spectral strengths cancel. As we shall see below, this is quite a general property of two-point baryon correlators in the chirally-symmetric phase if the chiral limit can be taken uniformly.

The simplest current in the \((\frac{1}{2}, \tilde{0}) + (0, \frac{1}{2})\) multiplet is,

\[
\eta_N' = \left(u^T C \sigma_{\alpha \beta} u\right) \gamma_5 \sigma^{\alpha \beta} d,
\]

which has also been recognized in the QCD sum rule calculations [13]. Again the two-point correlator has only the chiral-even term if the vacuum has chiral symmetry.
From the group theoretical standpoint, the two multiplets discussed so far are identical. Thus, their product can produce an $SU(2)_L \times SU(2)_R$ singlet, and the correlation function,

$$\int d^4x e^{ixp} \langle T \eta_N(0) \bar{\eta}_{N'}(x) \rangle ,$$

is nonzero even if the vacuum is chirally symmetric. [Of course, in that case the $\rho_2(p^2)$ type of term does vanish.] However, since $\eta_N$ and $\eta_{N'}$ transform differently under $U(1)_A$, the chiral-even term would vanish if the $U(1)_A$ symmetry is restored. This interesting diagnosis for $U(1)_A$ restoration signature was first studied by Schafer and Shuryak [6].

The chiral multiplet $(\frac{1}{2}, 1) + (1, \frac{1}{2})$ contains both $I = \frac{1}{2}$ and $I = \frac{3}{2}$ isospin multiplets. The simplest $I = \frac{1}{2}, I_z = \frac{1}{2}$ current is [13],

$$\eta_N^\mu = (u^T C \sigma_{\alpha \beta}) \gamma_5 \sigma^{\alpha \beta} \gamma^\mu u$$

which has the quantum numbers of the nucleon($P_{\frac{1}{2}}$), as well as $S_{\frac{1}{2}}, P_{\frac{3}{2}}$ and $S_{\frac{3}{2}}$ resonances. Under $SU(2)$ chiral rotation, for instance $U = \exp(i\pi \gamma_5/4)$, the current is transformed to its $I = \frac{3}{2}$ partner,

$$\eta_\Delta^\mu = (u^T C \sigma_{\alpha \beta}) \gamma_5 \sigma^{\alpha \beta} \gamma^\mu d + (u^T C \sigma_{\alpha \beta}) \gamma_5 \sigma^{\alpha \beta} \gamma^\mu u$$

which has the quantum numbers of $\Delta(P_{\frac{3}{2}}), D_{\frac{3}{2}}, S_{\frac{3}{2}}$, and $P_{\frac{3}{2}}$ resonances. A slightly different form of $\eta_\Delta^\mu$ with three up-quark fields was first used in a QCD sum rule calculation [13]. In a chirally-symmetric phase,

$$\langle T \eta_N^\mu(x) \bar{\eta}_N^\mu(0) \rangle = \langle T \eta_\Delta^\mu(x) \bar{\eta}_\Delta^\mu(0) \rangle ,$$

and only chiral-even terms contribute. If relevant baryons survive the chiral phase transition and they couple to these currents strongly, $J = \frac{1}{2}(\frac{1}{2}^+), I = \frac{1}{2}$ resonances would be degenerate with $J = \frac{1}{2}(\frac{3}{2}^+), I = \frac{3}{2}$ resonances.

Finally, the chiral multiplet $(\frac{3}{2}, 0) + (0, \frac{3}{2})$ has isospin $\frac{3}{2}$. The simplest current in this multiplet is,

$$\eta_\Delta^{\mu \nu} = \left[ (q^T C \sigma_{\alpha \beta} q) \gamma_5 \sigma^{\alpha \beta} \sigma^{\mu \nu} q \right]_{I=3/2}$$

where the flavor indices are coupled in a totally-symmetric way. $\eta_\Delta^{\mu \nu}$ can couple to $J = \frac{1}{2}, \frac{3}{2}$ resonances. We haven’t found any previous use of this current in the literature. In a chirally-symmetric phase, the two-point correlator of the current contains chiral-even terms only, which implies that parity partners are degenerate and the chiral-odd spectral strengths cancel.

B. Three Massless Flavors

To classify baryon currents in three massless flavors, we consider decomposition of the representation $[(1, 3) + (3, 1)]^3$ of $SU(3)_L \times SU(3)_R$. The $SU(3)$ multiplication rules yield,
\[
((1, 3) + (3, 1))^3 = \left[(10, 1) + (1, 10)\right] + 3 \times \left[(6, 3) + (3, 6)\right]
+ 3 \times \left[(\bar{3}, 3) + (3, \bar{3})\right] + 2 \times \left[(8, 1) + (1, 8)\right] + \left[(\bar{1}, 1) + (1, \bar{1})\right]
\]

where \(\bar{3}\) is from an antisymmetric combination of two quark fields, and \(\bar{1}\) from an antisymmetric combination of three quark fields.

The chiral multiplet \((8, 1) + (1, 8)\) contains \(SU(3)_V\) flavor octets. Currents in this representation can be constructed as generalizations of the currents in \((\bar{1}, 0) + (0, \bar{1})\) of the two-flavor case. For instance, an extension of the \(J = \frac{1\pm}{2}\) currents in Eq. (26) is,

\[
\eta^{\alpha}_{[8]} = \left[\left(q^T C \sigma_{\alpha \beta} q\right) \gamma_5 \sigma_{\alpha \beta} q\right]^a_{[8]},
\]

where the first two quark fields are symmetrized in flavor indices to form a 6 of \(SU(3)_V\). The complete flavor-octet wave functions can be found, for instance, in Ref. [15]. In a chirally-symmetric phase, the correlators \(\langle T \eta^{\alpha}_{[8]}(x) \eta^{\alpha}_{[8]}(0) \rangle\) contain chiral-even Dirac structure only.

The multiplet \((\bar{3}, \bar{3}) + (\bar{3}, 3)\) contains both \(SU(3)_V\) octets and singlets. The baryon currents of the multiplet can be obtained from generalizations of the currents in \((\bar{1}, 0) + (0, \bar{1})\) of the two-flavor case. For instance, from Eq. (22) we can write down nine \(J = \frac{1\pm}{2}\) currents,

\[
\eta^{\alpha}_{[8]} = \left[\left(q^T C \sigma_{\alpha \beta} q\right) \gamma_5 \sigma_{\alpha \beta} q\right]^a_{[8]},
\eta^{\mu}_{[10]} = \left[\left(q^T C \sigma_{\alpha \beta} q\right) \gamma_5 \sigma_{\alpha \beta} \gamma^\mu q\right]^b_{[10]},
\]

where the first two quark fields are antisymmetrized in flavor indices to form a \(\bar{3}\) of \(SU(3)\). The implicit flavor indices in 10 are totally symmetric. \(\eta^{\mu}_{[10]}\) have been used to calculate the masses of the lowest-lying baryon decuplet in the QCD sum rule approach [16]. In a chirally-symmetric phase, we have,

\[
\langle T \eta^{\alpha}_{[8]}(x) \bar{\eta}^{\alpha}_{[8]}(0) \rangle = \langle T \eta^{\mu}_{[10]}(x) \bar{\eta}^{\mu}_{[10]}(0) \rangle,
\]

which contain only chiral-even Dirac structures. If all the currents couple to chiral resonances strongly, \(J = \frac{1\pm}{2}\) octets are degenerate with the \(J = \frac{1\pm}{2}\) singlets, which apparently is a new result [1].

The chiral multiplet \((6, 3) + (3, 6)\) contains both \(SU(3)_V\) octets and decuplets. Again, the currents in the multiplet can be obtained as generalizations of those in \((\bar{1}, 0) + (0, \bar{1})\) of the two-flavor case (Eqs. (28),(29)). For instance,

\[
\eta^{\mu\alpha}_{[8]} = \left[\left(q^T C \sigma_{\alpha \beta} q\right) \gamma_5 \sigma_{\alpha \beta} \gamma^\mu q\right]^a_{[8]},
\eta^{\mu\alpha}_{[10]} = \left[\left(q^T C \sigma_{\alpha \beta} q\right) \gamma_5 \sigma_{\alpha \beta} \gamma^\mu q\right]^b_{[10]}
\]

The implicit flavor indices in 10 are totally symmetric. \(\eta^{\mu\alpha}_{[10]}\) have been used to calculate the masses of the lowest-lying baryon decuplet in the QCD sum rule approach [16]. In a chirally-symmetric phase, we have,

\[
\langle T \eta^{\mu\alpha}_{[8]}(x) \bar{\eta}^{\mu\alpha}_{[8]}(0) \rangle = \langle T \eta^{\mu\alpha}_{[10]}(x) \bar{\eta}^{\mu\alpha}_{[10]}(0) \rangle\]
which contain chiral-even structures only. If these currents are dominated by the lowest resonances, then \( J = \frac{1}{2} \pm (\frac{3}{2} \pm) \) octets are degenerate with \( J = \frac{1}{2} \pm (\frac{3}{2} \pm) \) decuplets.

Finally, the simplest currents in multiplet \((10, 1) + (1, 10)\) are the three-flavor generalization of those in \((\frac{3}{2}, 0) + (0, \frac{3}{2})\) in the two-flavor case,

\[
\eta^{\mu \nu b}_{[10]} = \left[ (q^T C \sigma_{\alpha \beta} q) \gamma_5 \sigma^{\alpha \beta} \sigma^{\mu \nu} q \right] b_{[10]},
\]

where three implicit flavor indices are symmetric. An example of the chiral-singlet current in \((\tilde{1}, 1) + (1, \tilde{1})\) is

\[
\eta^{\mu}_{[1]} = \left[ \left( q^T C \gamma_5 q \right) D^{\mu} q - \left( q^T C q \right) \gamma_5 D^{\mu} q \right]_{[1]},
\]

which would vanish if without the covariant derivative. In the chirally-symmetric vacuum, the two-point correlators of the above currents contain only chiral-even terms.

Since there are no two simple currents in the same chiral \( SU(3) \) representation having different \( U(1)_A \) properties, a test of \( U(1)_A \) would involve at least four baryon currents. Correlators with three baryon currents vanish due to the baryon number \( U(1) \) symmetry. This generalizes the result in Ref. \[12\].

**IV. COMMENTS**

In this paper, we have systematically enumerated the simplest chiral multiplets which have meson and baryon quantum numbers. The reason for doing this is the prospect that QCD may have a chiral-restored phase. If so, the chiral symmetry will be reflected explicitly in the correlators of hadronic currents, as we mentioned earlier in the Introduction.

Of course, many of the results we have stated are well known. We repeat them here so that the reader can clearly see a group-theoretical organization. However, we have also found a number of new results which we summarize here:

- A test of unbroken \( U(1)_A \) symmetry in the two massless flavor case is that the correlators of \( \rho \) and \( b_1 \) currents in parity-conjugating \((\frac{1}{2}, \frac{1}{2}) + (\frac{1}{2}, \frac{1}{2})\) multiplets are equal.
- A test of unbroken \( SU(2) \) chiral symmetry is the equality of \( \rho \) and \( h_1 \) types of correlators and \( b_1 \) and \( f_1 \) types of correlators in \((\frac{1}{2}, \frac{1}{2}) + (\frac{1}{2}, \frac{1}{2})\). This result has a three-flavor generalization.
- We found several new interpolating currents for baryons: the currents for \( I = \frac{3}{2} \) baryons in \((\frac{3}{2}, 0) + (0, \frac{3}{2})\) multiplet and its three-flavor generalization, and the current for singlet \( \Lambda \) in \((\tilde{1}, 1) + (1, \tilde{1})\).
- A test of unbroken \( SU(2) \) chiral symmetry in baryon sector is the equality of the \( I = \frac{1}{2} \) (nucleon) and \( I = \frac{3}{2} / 2 \) (\( \Delta \)) two-point current correlators in \((1, \frac{1}{2}) + (1, \frac{1}{2})\). (1, 1/2) + (1/2, 1). Besides a generalization of this result to the three flavor case, we found that the singlet \( \Lambda \) and the baryon-octet correlators in \((3, 3) + (3, 3)\) are equal.

Finally, the chiral multiplets are a useful way to organize baryon interpolating currents, which are closely related to independent Bethe-Salpeter amplitudes. Thus, we expect the present work to be useful also for the study of hadron structure.
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