Adaptive Beamforming with Inadequate Snapshots

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Abstract. In array signal processing, the covariance matrix used to calculate the adaptive weights is often poorly estimated when the snapshot number is inadequate. The prior environmental knowledge can be used to make the estimation more accurate. In this paper, an alternative knowledge-aided adaptive beamforming approach that is robust to low sample support environment is proposed. In this algorithm the covariance matrix used to calculate the optimum weights is constructed by blending a sample covariance matrix and a priori structured covariance matrix. Numerical simulations demonstrate the proposed algorithm has the potential for substantial performance improvement.

1. Introduction
In recent decades, adaptive beamforming has been widely applied to wireless communications, radar, sonar, speech signal processing, and many other fields [1-3]. With the use of adaptive beamforming, the radiation pattern of arrays can be adaptively changed to reject interferences and can provide significant performance advantages. Capon beamformer [3] is one of the most popular criteria for beamforming, which maximize the output signal-to-interference-plus-noise ratio (SINR) adaptively. However the formulation of the Capon beamformer requires explicit knowledge of the interfering environment, namely the covariance matrix and the steering vector of the interested signal. Uncertainties in these quantities may lead performance degradation. Until recently, many works have been put forward dealing with the steering vector mismatch and improving the robustness of the adaptive beamformers. Most of the existing algorithms use diagonal loading approach to alleviate the aforementioned problems, but the loading levels are difficult to choose.

In this paper, we focus our mind on low sample support (inadequate snapshots) caused covariance matrix uncertainty problem, which may badly affects the performance of beamforming. And a novel knowledge-aided adaptive beamforming algorithm was proposed using the concept of knowledge-aided signal processing techniques. In this algorithm we make full use of the priori covariance matrix and mixing it with the estimated covariance matrix. Use the newly constructed matrix to calculate the optimum adaptive weights can bring exciting performance improvement.

2. System Model
The output of a narrowband adaptive beamformer containing $M$ sensors is given by
$$ y(k) = w^H x(k) $$
where $k$ is the time index, $x(k) = [x_1(k), \ldots, x_M(k)]^T \in \mathbb{C}^{M \times 1}$ is the $k$th output snapshot, 
$w = [w_1, \ldots, w_M]^T \in \mathbb{C}^{M \times 1}$ is the complex vector of adaptive weights, and $(\cdot)^T$ and $(\cdot)^H$ stand for the transpose and Hermitian transpose, respectively.

The observation vector is given by
$$
x(k) = x_s(k) + i(k) + n(k) = s(k)a + i(k) + n(k)
$$

(2)

where $x_s(k)$, $i(k)$ and $n(k)$ are the desired signal, interference and noise components, which are statistically independent. $s(k)$ is the signal waveform, and $a$ is the corresponding actual steering vector of the desired signal. The adaptive beamformer weights $w$ can be obtained by maximizing the signal-to-interference-plus-noise ratio (SINR).

$$
SINR = \frac{\sigma_s^2 |w^Ha|^2}{w^HR_{i\!n}w}
$$

(3)

where
$$
R_{i\!n} = E \left\{ (i(k) + n(k))(i(k) + n(k))^H \right\}
$$

(4)

is the $M \times M$ interference-plus-noise covariance matrix and $\sigma_s^2$ is the signal power.

The optimization problem of maximizing array output SINR can be equivalently written as

$$
\begin{align*}
\min_w & \quad w^HR_{i\!n}w \\
\text{s.t.} & \quad w^Ha = 1
\end{align*}
$$

(5)

which solution is the well-known Capon beamformer.

$$
w_{opt} = \frac{R_{i\!n}^{-1}a}{a^HR_{i\!n}^{-1}a}
$$

(6)

Until recently some remedies have been made to Capon algorithm, so in this paper we refer it to the standard Capon beamformer.

In practical applications, the exact interference-plus-noise covariance matrix $R_{i\!n}$ is not known. Therefore the sample covariance matrix $\hat{R}$ which is estimated using some independent identically distributed (i.i.d.) neighboring snapshots is often used instead of $R_{i\!n}$.

$$
\hat{R} = \frac{1}{N} \sum_{n=1}^{N} x(n)x^H(n)
$$

(7)

where $N$ denotes the number of i.i.d. snapshots.

Reed, Mallett, and Brennan [4] have shown that the average signal-to-interference plus noise ratio (SINR) loss will be about -3dB if $N \geq 2M$, which is often called RMB rule. However, in real environment, this is no longer true if few i.i.d. snapshots are available to estimate the covariance matrix. For example $N$ is comparable with or even smaller than $M$, $\hat{R}$ usually is a poor estimation of $R_{i\!n}$. So the adaptive weights obtained from equation (6) may lead distorted mainbeam and high sidelobes. Whereas in real world, sample heterogeneity frequently occurs, which means many sample snapshots do not share the same spectral properties, the estimation of $R_{i\!n}$ may also be inaccurate. So in recent years, knowledge-aided signal processing techniques are put forwarded to deal with this effect. In these techniques, all kinds of available knowledge about the environments, the array, and the interferences are used to improve the performance.

Until recently two paradigms for knowledge-aided processing methods are presented [5-8]. One paradigm is an indirect approach where a priori knowledge is used for selecting the training data and the other one is a direct approach where a priori knowledge is used to prewhitening the samples. In the latter section, a new direct knowledge-aided adaptive beamforming algorithm will be proposed.

3. Knowledge-aided adaptive beamforming approach

In this part, we consider the eigen-decomposition of the interference-plus-noise covariance matrix first.
It can be expressed

$$R_{x,n} = E \{ x(n) * x(n)^H \} = R_x + R_n = \sum_{j=1}^{p} \lambda_j V_j V_j^H + \sum_{j=p+1}^{M} \lambda_j V_j V_j^H$$  \tag{8}$$

where $R_x$ and $R_n$ represent the covariance matrix of interference and noise respectively, $\lambda_j \geq \lambda_2 \geq \ldots \geq \lambda_1 \gg \lambda_{p+1} = \lambda_{p+2} = \ldots = \lambda_M = \sigma_n^2$ are the eigenvalues of the covariance matrix. Meanwhile, the eigenvalues of the interferences are much larger than that of noises. $V_1, V_2, \ldots, V_M$ are the corresponding eigenvectors, $P$ denotes the number of the interferences and $\sigma_n^2$ is the noise power.

In practice, in a sensor array the power of interferences is always much stronger than that of noise. Therefore, after eigen-decomposition of the covariance matrix, there exist several large eigenvalues and some small eigenvalues. According to the subspace theory, the number of large eigenvalues corresponding to the number of interferences and the number of small eigenvalues denotes the degree of freedom of the thermal noise. Consider an array which consists of 20 sensors with 2 interferences, and the interference-to-noise ratios (INR) are 30dB and 20dB respectively. Fig.1. shows the eigenspectra of the estimated covariance matrix with adequate snapshots. Obviously there existing 2 big eigenvalues and 18 small eigenvalues which correspond to the number of interferences and the noise’s degree of freedom respectively. It can also be seen from Fig.1 that the small eigenvalues have the similar magnitudes which are close to the noise level.

Theoretically if the snapshots are limited, for the aforementioned case choose the snapshots less than 40, $R$ may be a poor estimation and near singular. The eigenspectra of $R$ can be shown in Fig.2. The difference between the two cases is in the spread of the eigenvalues which correspond to the random noise. With few snapshots there is inadequate estimation of the noise and a large noise eigenvalue spread occurs. As more samples are used, the estimation improves and the eigenvalues converge on the expected value of the noise. The spread of the small eigenvalue may leads sidelobe elevation and mainbeam distortion, thus further affects the beamforming capability.

![Figure 1. Eigenspectra of covariance matrix with adequate snapshots](image1)

![Figure 2. Eigenspectra of covariance matrix with inadequate snapshots](image2)

Supposing that a priori structured covariance matrix obtained by previous experiment is available, a reconstructed covariance matrix acquired by blending the estimated covariance matrix with a priori covariance matrix according to the characteristic of eigenspectra can be used to enhance the robustness of beamforming.

Next, we will show how to build the new covariance matrix and the reason why it can be used to improve the robustness of the performance.

The eigen-decomposition of the estimated covariance $\hat{R}$ is written below:

$$\hat{R} = \sum_{i=1}^{P} \gamma_i U_i U_i^H + \sum_{j=p+1}^{M} \gamma_j U_j U_j^H$$  \tag{9}$$

where $\gamma_i$ and $U_i$ are eigenvalue and eigenvector, respectively. $P$ denotes the number of interferences which equals the number of large eigenvalues.
The prior covariance matrix in ideal environment with known array structure and interferences can be expressed

\[ R_{\text{prior}} = \sum_{j=1}^{p} \alpha_j C_j C_j^H + \sum_{j=p+1}^{M} \alpha_j C_j C_j^H \]  

(10)

where \( \alpha_j \) and \( C_j \) denote the eigenvalue and eigenvector respectively. \( p \) also indicate the number of interferences.

Build a new matrix \( R_{\text{const}} \) using the noise subspace of the prior covariance matrix \( R_{\text{prior}} \) and the interference subspace of the estimated covariance \( R \).

\[ R_{\text{const}} = \sum_{j=1}^{p} \gamma_j U_j U_j^H + \sum_{j=p+1}^{M} \alpha_j C_j C_j^H = QGQ^H \]

(11)

\[ Q = [U_1, \ldots, U_p, C_{p+1}, \ldots, C_M] \]

\[ G = \begin{bmatrix} \gamma_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & \gamma_p & 0 & 0 \\ 0 & 0 & 0 & \alpha_{p+1} & 0 \\ 0 & 0 & 0 & 0 & \ddots \\ 0 & 0 & 0 & 0 & \alpha_M \end{bmatrix} \]

(12)

Using \( R_{\text{const}} \) instead of \( R_{\text{ana}} \) in equation (6) to acquire the adaptive weights may generate stable beam patterns. That is because the reconstructed matrix \( R_{\text{const}} \) overcomes the spread of the small eigenvalues.

In equation (11), \( p \) should be a known quantity. However, in practical applications, we may not exactly know the number of interferences. Fortunately, we can obtain it by analyzing the characteristic of the eigenspectra of the estimated covariance matrix and the number of the large eigenvalue corresponds to the number of interferences. So, to use this algorithm one must first observe the eigenspectra of the estimated covariance matrix \( R \) and distinguish the large and small eigenvalues.

Generally in real systems, interferences are much stronger than thermal noise, so the large eigenvalues are easy to separate from their magnitude. The new approach is easy to be implemented and the simulations in the next section validate the superiority over the Capon beamformer.

4. Simulation results

In our simulations, a uniform linear array with \( M = 20 \) omnidirectional sensors which has half-wavelength inter-element spacing is considered. Through all simulations, two interfering sources are assumed with plane wave fronts and the directions of arrival are 50° and 70°, respectively. The additive noise is modeled as a complex Gaussian zero-mean spatially and temporally white process. The INR in a single sensor is equal to 20dB. The desired signal is assumed to be from the direction \( \theta_s = 0^\circ \). For each scenario, 200 Monte-Carlo runs are used to obtain each simulated point. The proposed knowledge-aided algorithm is compared to Capon beamformer.

The output SINR versus the number of i.i.d. samples is illustrated in Fig.3. In this simulation the signal-to-noise ratio SNR=2dB. It is clear from the picture that the proposed knowledge-based algorithm has better performance compared to the Capon beamformer. Especially, when the i.i.d. samples are limited, the knowledge-based method still has stable capability while the Capon beamformer behaves invalid. As the number of i.i.d. samples becomes larger the performances of two algorithms tend to be the same.

Fig.4 shows the output SINR versus SNR for the fixed training sample size \( N=200 \). It can be seen from the figure that the knowledge-based method still has advantages over Capon beamformer when the training samples are plenteous.
5. Conclusions
To overcome the covariance matrix uncertainty in low sample support environment a novel knowledge-aided beamforming process is proposed. Unlike the existing diagonal loading based methods in which the loading level should be known, in this new approach we made full use of the prior information of the environment. The covariance matrix used to calculate the optimum weights is constructed by blending a sample covariance matrix and a priori structured covariance matrix. The approach is easy to be implemented and the experiment results demonstrate that in low sample support environment it has superior performance than the Capon beamformer.

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