On digital phase detector

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Abstract. A digital phase detector for processing signals with phase modulation in a wide range of changes in the phase of the received signal is considered. A block diagram of a digital detector with minimal computational costs for the signal period is proposed. The problem of phase jumps when it changes by more than 2π is solved. An estimate is obtained for the noise immunity of a phase detector when exposed to Gaussian noise. Supposed hardware implementation of a phase detector based on field-programmable gate arrays. It can be used in devices for digital processing of the angular modulation signals, devices for controlling the phase of the reference signal and in different measurers.

1. Introduction

Phase detectors in various circuitry implementations are widely used in radio equipment in devices for modulation and demodulation of radio signals with phase modulation [1], phase locking, phase locked frequency [2]. Simple phase detectors are efficient in the range of phase change from ± π/4 to ± π [3], which is insufficient when processing signals with a large phase modulation index. In addition, they may have a non-linear transfer characteristic, which is acceptable in control devices, but unacceptable when detecting signals in receiving devices. Phase detectors on digital elements are widely used, for example, in [4].

Analog detectors are characterized by signal conversion (multiplication) errors, non-identical characteristics and non-linear signal distortions. The transition to digital signal processing makes it possible to significantly improve the characteristics of phase detectors and expand the operating range of the phase change of the received signal.
Commonly known are coherent and incoherent digital demodulators of phase-shift keyed (PSK) signals [5, 6] or quadrature amplitude modulation (QAM) signals [7]. They are based on phase detection operations.

Of particular interest are signal demodulators of multi-level phase-shift keyed (MPSK) and amplitude-phase shift keyed (APSK) signals [8], which can be implemented based on a digital phase detector.

2. Phase Modulated Signal

Phase modulated signal \( s(t) \) can be written down as

\[
s(t) = S \cos[2\pi f_0 t + \psi(t)],
\]

where \( S \) is its amplitude, \( f_0 \) is a carrying frequency and \( \psi(t) \) is a modulated phase equal to

\[
\psi(t) = bu(t)/U_{\text{max}}.
\]

In (1), \( u(t) \) is a modulating information signal, \( U_{\text{max}} \) is its maximal value and \( b \) presents a phase modulation index.

As it is shown in [9], it is enough for the analog-to-digital converter (ADC) to generate 4 samples per signal period \( T_0 = 1/f_0 \) with the sampling frequency \( f_s = 4f_0 \). This procedure is presented in figure 1, where \( i \) is the number of the current period, \( s_{1i}, s_{3i} \) are odd and \( s_{2i}, s_{4i} \) are even numbers that form sample sequences of the quadrature signal processing channels. These sequences are time-shifted by \( T_0/4 \) (that is 90 degrees by phase).

3. Digital Phase Detector

The block diagram of the digital phase detector implemented in accordance with the RF patent [10] is shown in figure 2. At the end of the \( i \)-th period, the samples \( s_{1i}, s_{3i}, s_{2i}, s_{4i} \) are saved into the 4-sample shift register of multi-digit codes (MS4), while in the subtractors \( \text{SUB}_0 \) and \( \text{SUB}_1 \) the differences of odd samples presented as \( x_{1i} = s_{1i} - s_{3i} = 2S \cos \psi_i \) and even samples presented as \( x_{2i} = s_{2i} - s_{4i} = 2S \sin \psi_i \) are generated.

![Figure 1. The time diagram of the signal sampling.](image1.png)

![Figure 2. The block diagram of the digital phase detector.](image2.png)

The values \( x_{1i} \) are accumulated in the cascading summatos from \( \text{SUM}_{01} \) to \( \text{SUM}_{0n} \) [10], and, at the output, the response
\[ y_{0i} = \sum_{j=0}^{N-1} \left[ s_i(i-j) - s_j(i-j) \right] \]  

(2)

is generated. Here \( N = 2^n \) is a number of the signal accumulation periods, \( n = \log_2 N \) is the number of the accumulation cascades and \( i \) is the number of the last received period. Likewise, at the output of \( \text{SUM}_{1n} \) one gets

\[ y_{1i} = \sum_{j=0}^{N-1} \left[ s_2(i-j) - s_4(i-j) \right] \]  

(3)

If at the last of \( N \) periods the phase \( \psi_i \) does not change significantly, then it follows from (2) and (3) that

\[ y_{0i} = 2N \cos \psi_i, \quad y_{1i} = 2N \sin \psi_i. \]  

(4)

The binary codes of the values \( y_{0i} \) and \( y_{1i} \) are poked to the normalizing unit (NU) based on the shift registers [10]. By the joint code shift, this device provides full filling of the bit-level grid with the highest modulo. Normalized codes \( y_{0i} \) and \( y_{1i} \) are transferred to the digital arctangent generator (DAG) where the value

\[ \varphi_i = \begin{cases} -\arctan(y_{1i}/y_{0i}), & \text{if } y_{0i} \geq 0, \\ -\pi - \arctan(y_{1i}/y_{0i}), & \text{if } y_{0i} < 0 \end{cases} \]  

(5)

is determined. It is equal to the current phase shift presented as \( \varphi_i = \psi_i - \psi_0 \) that takes place between the received signal (\( \psi_i \)) and the clock signal (\( \psi_0 \)), while the latter is constant and does not depend on the modulating signal. The operation (5) is implemented in hardware [10] based on ROM where the codes \( y_{0i} \) and \( y_{1i} \) generate the address of the cell where the pre-calculated code \( \varphi_i \) is stored.

For the received signal phase, one gets

\[ \psi_i = \varphi_i - \psi_0. \]

It is seen that that the response is received to the modulating signal and its accuracy is up to the constant component \( \psi_0 \). Under the variable signal \( u(t) \), this constant is easily eliminated.

The value of \( \varphi_i \) (5) changes in the interval \( 2\pi \) – from \(-3\pi/2\) to \(\pi/2\). Figure 3a exemplifies the dependence of \( \varphi(t) \) on the current normalized time \( t/T_M \) (where \( T_M \) is a period of the tonal modulating signal with 5kHz frequency) under the phase modulation index \( b = 2 \), \( f_0 = 10 \) MHz, \( N = 256 \) (\( n = 8 \)) and \( \psi_0 = 1 \) rad. Under \( b = 3 \), one can see the jumping changes of the phase (figure 3b). To eliminate them, the values of \( \varphi_i \) are transferred to the phase correction unit (PCU) shown in figure 2 where the differences \( \Delta \varphi_i = \varphi_i - \varphi_{i-1} \) are exposed at the neighbouring periods, and if \( \Delta \varphi_i > \pi/2 \), then the value \( 2\pi \) is added to \( \varphi_i \), while if \( \Delta \varphi_i < \pi/2 \), then it is subtracted. As a result, the jumps like the ones demonstrated in figure 3b are eliminated and the response takes a harmonic shape with the phase change greater than \( 2\pi \).
Figure 3. The signal phase changes over time: (a) $b = 2$; (b) $b = 3$.

Figure 4. The time diagrams a) information signal; b) results of its detection.

Figure 5. The amplitude spectrum of the received frequency-shift keyed signal.

For example, the information signal $u(t)$ introduced in (1) is presented as the sawtooth one in figure 4a, while the results of its detection under $f_0 = 10$ MHz, $N = 64$ and the modulation index $b = 20$ are shown in figure 4b. In this case, the phase modulated signal $s(t)$ is the frequency-shift keyed signal, and one can see its amplitude spectrum $G(\Delta f)$ in figure 5, where $\Delta f = f - f_0$. One can also see there that a linear phase detection is maintained within the interval of $\pm 20$ rad.

4. Noise Immunity of the Phase Detector

Mean values of the responses of the quadrature channels $y_0$ and $y_1$ are equal to (4). Under the influence of the Gaussian noise, the corresponding components of $y_0$ and $y_1$ are approximately independent and they have a normal probability distribution with the zero mean value and the dispersion equal to the sum of the sample dispersions [11]

$$\sigma^2 = 2N\sigma_n^2,$$

where $\sigma_n^2$ is the dispersion of the input noise samples.

From [12], one knows the characteristics of the random vector having either Gaussian Cartesian coordinates $y_0$ and $y_1$ that characterize the mean values $a$ and $b$ and the dispersion $\sigma^2$ or the polar coordinates $\rho$ (modulo) and $\psi$ (phase). Then, according to (4), for the responses of the quadrature channels of the phase detector $y_0$ and $y_1$, one gets:
\[ a = 2NS \cos \varphi , \quad b = 2NS \cos \varphi , \quad \rho = \sqrt{a^2 + b^2} = 2NS. \]  
\[ \varphi = \cos^2 NSa, \quad \varphi = \cos^2 NSb, \quad \varphi^2 = \frac{\rho^2}{2} + \frac{\rho^2}{2}. \]  

Under the higher values when \( \rho/\sigma > 10 \), the probability density of the noise component \( \xi \) of the phase takes the Gaussian one with the mean value \( \langle \xi \rangle = \psi \) and the dispersion \( \sigma_\xi^2 = (\sigma/\rho)^2 \) [10]:

\[ w(x) = \rho \exp \left[ -\rho^2 (x - \psi)^2 / 2\sigma^2 \right] / 2\sigma^2. \]

From (6) and (7), one gets

\[ \sigma_\xi^2 = \frac{2\sigma_n^2}{2NS^2}. \]  

The noise immunity of the signal reception is usually determined in terms of the ratio between the signal power and the noise power at the output of the detector, that, in this very case, is equal to

\[ h^2 = \rho^2 / 4N\sigma_n^2 = NS^2 / \sigma_n^2. \]

Thus, from (8), (9), it follows that

\[ \sigma_\xi^2 = \frac{1}{2h^2}. \]

In real conditions, as a rule, \( h^2 > 10 \) and then, from (10), one sees that the mean square deviation of the noise component of the phase \( \sigma_\xi \) is less than 0.22 rad. Under the index (amplitude) \( b \) of the phase modulation equal to 10 one gets the ratio \( b/\sigma_\xi \) greater than 44, which means that the noise immunity of the phase modulated signal reception is quite high.

The operation of the introduced phase detector has been tested by means of statistic simulation. For example, in figure 6a and figure 6b, the time diagrams of the phase detector response are presented for the signal shown in figure 4b which is received in the presence of the Gaussian noise with independent samples. In the first case, \( h = 12 \) dB, and in the second – \( h = 6 \) dB. As it can be seen, reliable signal detection is provided even if a signal-to-noise ratio is low enough.

![Figure 6](image-url)

**Figure 6.** The time diagrams of the phase detector response in the case when the received signal is distorted by the Gaussian noise with independent samples: (a) \( h = 12 \) dB; (b) \( h = 6 \) dB.

5. **Demodulation of the Discrete Signals**

Under coherent demodulation in the digital communication systems, the binary phase-shift keyed (BPSK) and \( M \)-ary phase-shift keyed (MPSK) signals are widely used. To eliminate “backward operation” of the demodulator, a \( M \)-ary differential phase-shift keying (MDPSK) is used in, for
example, TETRA trunking radio communication network (π/4 DQPSK and π/8 D8PSK) and in broad-band networks like WiFi (DBPSK and DQPSK) [13].

For their demodulation, the considered phase detector can be used, if only the duration \( T_I \) of the information signal (either binary or multi-level one) is

\[
T_I = NT_0.
\]

While carrying out the coherent demodulation of the MPSK signals one should maintain the phase locking of the clock pulse generator of the detector. The specified generator generates the value of the received phase at the end of the information symbol. During the processing of the differential phase-shift keyed signals the phase difference of the received and the preceding symbols is calculated:

\[
\varphi_i = \psi_i - \psi_{i-N}.
\]

In this case, the random unknown initial phase \( \psi_0 \) is subtracted and does not influence the detection result.

In figure 7, one can see the time change of the phase detector response \( \psi_i \) to the 8PSK signal (the number of positions is \( M = 8 \)) under the absence of interferences and the equiprobable symbol choice. Under integer \( i/N \), the decisions are made on the received signal phase or on the phase difference of the neighboring symbols.

![Figure 7. The time diagram of the phase detector response to the 8PSK signal.](image)

6. Conclusion
The digital phase detector is introduced providing extracting the analog or digital information signal that modulates the phase and is characterized by a high phase modulation index and low necessary computational costs and hardware requirements. It is demonstrated that such detector maintains high linearity in a wide phase change range. Detector noise immunity is evaluated and the corresponding statistic simulation is carried out verifying the calculation results. It is proven that an acceptable signal reception remains possible under the noise-signal ratio up to 6-10 dB. In this context, demodulation of the discrete multi-level phase-shift keyed signals is considered. It is concluded that the use of the differential phase-shift keying eliminates the negative influence of an indefinite initial signal phase.

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