Systematic Analysis of $B \rightarrow K\pi l^+l^-$ Decay through Angular Decomposition

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Abstract

We investigate systematically how to extract new physics contributions in $B \rightarrow K\pi l^+l^-$ decay by using the angular decomposition. The decomposition will enable us to define not only several CP averaged forward-backward (FB) asymmetries but also the direct CP asymmetry and the time-dependent mixing induced CP asymmetry for each FB asymmetry newly defined in the general 4 body angular space. The decay process involves several intermediate vector and scalar resonances as sources of strong phase difference through interference, therefore, one can expect largely enhanced CP asymmetries, if there exists any new physics with weak CP phases. The combined analysis of the FB and CP asymmetries will give us fruitful information about new physics contributions in detail.

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I. INTRODUCTION

One of the most important aims of present $B$ factories and future super-$B$ factories \[1\] would be to find out evidences of new physics beyond the standard model (SM). Indeed, to search for new physics evidence, we have investigated many penguin dominant processes, which have loop diagrams as a leading contribution. A couple of years ago, we had two such definite evidences: excitingly large discrepancies in CP asymmetries for $b \to s\bar{q}q$ decays, eg. $B \to \phi K$ \[2, 3\], and smaller but much unexpected discrepancies in $B \to K\pi$ decays, so called “$B \to K\pi$ puzzle” \[4, 5, 6, 7, 8, 9, 10\] between theoretical predictions within the SM and the experimental data.

For “$B \to K\pi$ puzzle”, the experimental data had shown three large discrepancies from the SM predictions of the branching ratios and CP asymmetries. One of them is the difference between the ratios of branching ratio for charged $B$ decays ($R_c$) and for neutral $B$ decays ($R_n$). The second one is that between the direct CP asymmetries for $B^+ \to K^+\pi^0$ and $B^0 \to K^+\pi^-$. The third one is that between the weak phase $\sin 2\phi_1$ extracted from the time-dependent CP asymmetry of $B^0 \to K^0\pi^0$ and of $B^0 \to J/\psi K_s$. Main contribution of all $B \to K\pi$ modes comes from $b \to s$ QCD penguin processes, therefore, the sub-leading electro-weak (EW) penguin type new physics contribution has been considered as the most plausible source to explain those three discrepancies \[3, 4, 5, 6, 7, 8, 9, 10\]. Recently, the $R_c - R_n$ discrepancy has been disappearing but the other two differences seem to be still remaining. It could mean that sizable parameter space for new physics is still valid in these decay modes. One of such possibilities is new physics with large CP phases in EW penguins \[5, 10\].

Investigation of the CP phase in EW penguin processes is very important to check the SM and confirm the discrepancies in $B \to K\pi$ modes at the same time. To do so, the semi-leptonic rare decays $b \to s l^+l^-$, which are pure EW penguin processes with less hadronic uncertainty than the hadronic $B$ decays, can be the best modes to search for the evidence of new CP phase in EW penguin diagram. $B \to K^*l^+l^-$ is a $b \to s$ EW transition process, so that the penguin vertex does not have large weak phase within the SM. Therefore, we have to either confirm the feature about only small CP phase coming from the CKM Matrix \[11\], or search for some evidences of new physics with large CP phases beyond the SM.

Several semi-leptonic rare decays, $B \to Ml^+l^-$ modes, have been measured \[12\] and they
will provide very useful information of new physics in EW penguin contributions \cite{13, 14, 15, 16, 17, 18, 19}. To analyze the source of new physics, we can try to extract a few hints by using radiative decays $b \to s \gamma$. However, due to the absence of large strong phase in the decay, the CP asymmetry of $b \to s \gamma$ would be very tiny. To investigate the new contributions to CP phase in $b \to s \gamma$, we seem to need new experimental technique. (For example, using photon conversion technique \cite{20} one can determine the parameters with new CP phase.)

The rare decays $b \to s l^+ l^-$ \cite{19, 21, 22} can be much more interesting process because these decays are including possibly large strong phases induced by the $(c\bar{c})$ intermediate resonance states. Furthermore, for the decays of $B \to M[\to K\pi] l^+ l^-$ ($M = K^*, K^*_0(800), ...$), if we do not constrain the invariant mass of $K$-$\pi$ system, there may exist several intermediate mesons, $M$, contributing to $B \to K\pi l^+ l^-$ decays. Therefore, through the interference we may induce large strong phases, which results possibly large CP violations if there is any new physics with weak CP phases.

We are interested in CP asymmetries and forward-backward (FB) asymmetries\cite{21} for $B \to K\pi l^+ l^-$ decays to extract information on possible new CP phases in $b \to s$ EW penguin transitions. Here the final state are including both CP odd and CP even so that it may be slightly difficult to consider the CP asymmetries. If we consider the time-dependent CP asymmetry, it cannot be even defined under this condition including both CP odd and even states. Hence we have to decompose the mode by using angular analysis. From the decomposition, one can define many observables and CP asymmetries so that one may be able to obtain fruitful information. Some of them are very sensitive to strong or EW phases. Some of them are from interference contributions between CP odd and CP even modes so that the CP asymmetry may be enhanced. Therefore, here we consider the angular analysis of 4 body decays $B \to K\pi l^+ l^-$ \cite{23, 24} and the CP asymmetries through the angular decomposition.

The important points in this work are:

- Angular decomposition of the decay rate, forward-backward asymmetries \cite{14} and CP asymmetries \cite{21} are investigated.

- Dependence of strong phases from several resonances in dilepton part and $K\pi$ part to CP asymmetries. If the intermediated states are including several meson states in addition to vector meson $K^*$, the interferences may have an important role as a
source of strong phase difference, which is one of the conditions to enhance the CP asymmetry.

- Using model-independent analysis \[25, 26\], the new physics information can be clearly classified.

This paper is organized as follows. In section 2, we show several definitions to calculate \( B \to K\pi l^+l^- \) decays and derive the branching ratio and the angular decomposition from the most general 4-fermi interaction. And we define the direct and indirect CP asymmetries of each decomposed FB asymmetry. In section 3, several figures of FB asymmetries and the CP asymmetries are plotted under some conditions. In section 4, the case with scalar resonance in addition to \( K^* \) is discussed. The interference effect may make a new source of strong phase difference to enhance the CP asymmetries. Section 5 is devoted to our summary.

II. THEORETICAL DETAILS OF \( B \to K^*[\to K\pi] l^+l^- \) DECAYS

To describe systematically the general 4 body decay, \( B(P_B) \to M[\to K(P_K) + \pi(P_\pi)] + l^+(P_+) + l^-(P_-) \), where \( P_x \) is the momentum of each particle, we define two kinetic variables, \( s \) and \( z \), and three angles, \( \theta_l, \theta_K \) and \( \phi \) \[23, 24\]. (See Fig.1.) Here \( q \) is the momentum of intermediate state \( M \), i.e. the sum of the momenta of \( K \) and \( \pi \) mesons, and \( s = q^2 = (P_K + P_\pi)^2 \). And \( z \) is defined as the square of invariant mass of dilepton, \( z = k^2 = (P_+ + P_-)^2 \). \( \theta_l \) is an angle between the momentum direction of \( l^+ \) and that of the intermediate photon (or opposite direction of the intermediate meson) at the center of mass (CM) system of \( l^+ \) and \( l^- \). \( \theta_K \) is an angle between \( K \) direction and the intermediate meson \( (M) \) direction at CM.

![Diagram of the kinematics of \( B \to K\pi l^+l^- \) decay.](image)

**FIG. 1:** Definition of the kinetic variables and the angles in \( B \to K\pi l^+l^- \) decay. Here \( q \) is the intermediate meson momentum and \( k \) is the intermediate photon momentum.
system of $K$ and $\pi$. And $\phi$ is an angle between the two decay planes at $B$ rest frame. Using the three angles, we can decompose the decay, $B \to K\pi l^+l^-$, completely. For simplicity, we assume the leptons and $K$ and $\pi$ mesons all massless.

As a systematic analysis, we start from the most general 4-fermi lagrangian \[25\]. It consists of 12 most general independent four-Fermi interactions,

$$
\mathcal{M}(b \to s l^+l^-) = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb}^* V_{tb} \left[ -2\bar{s}\sigma_{\mu\nu} \frac{k^\nu}{k^2} (C_7 m_b P_R \mp C'_7 m_b P_L) b l \gamma^\mu l \\
+ \bar{s} \gamma_\mu (C_9 P_L \mp C'_9 P_R) b l \gamma^\mu l \\
+ \bar{s} \gamma_\mu (C_{10} P_L \mp C'_{10} P_R) b l \gamma^\mu \gamma_5 l \\
+ \bar{s} (C_{SS} \mp C_{AS} \gamma_5) b l l \\
+ \bar{s} (C_{SA} \mp C_{AA} \gamma_5) b l \gamma_5 l \\
+ \bar{s} \sigma_{\mu\nu} b l \left( C_T \sigma^{\mu\nu} + i C_{TE} \sigma_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta} \right) l \right],
$$

where $C_X$ is the coefficient for each four-Fermi interaction. $C_7$, $C_9$ and $C_{10}$ correspond to the 3 parameters in the SM. The other coefficients will show the contributions from the interactions beyond the SM. $C'_7$ within the SM is suppressed by $m_s/m_b$ factor so that its contribution is estimated as tiny. If right-handed currents as new physics interactions exist, $C'_7, C'_9, C'_{10}$ will show non negligible contributions. $C_{SS}, C_{SA}, C_{AS}$ and $C_{AA}$ come from scalar type new physics interactions. And $C_T$ and $C_{TE}$ show the tensor type contributions. In general we can define a new CP phase as $e^{i\phi_{(l)}}$ for each Wilson coefficient $C_{(l)}$ in Eq. (1).

To calculate the process $\{ B^0, B^- \} \to M \text{ (e.g. } K^*) [\to K\pi] l^+l^-$, we are using the following parametrization \[17\] for the matrix element of the hadronic part:

$$
<K^*|\bar{s}\gamma_\mu P_{L,R} b|B> = i \epsilon_{\mu\nu\rho\sigma} \varepsilon^{\nu\rho} q^\sigma k \frac{V(z)}{m_B + m_{K^*}} + \left\{ \varepsilon^*_\mu (m_B + m_{K^*}) A_1(z) \\
- (\varepsilon^* \cdot k) (2q + k) \mu \frac{A_2(z)}{m_B + m_{K^*}} - \frac{2m_{K^*}}{z} (\varepsilon^* \cdot k) k \mu (A_3(z) - A_0(z)) \right\},
$$

$$
<K^*|\bar{s}\sigma_{\mu\nu} k^\nu P_{R,L} b|B> = -i \epsilon_{\mu\nu\rho\sigma} \varepsilon^{\nu\rho} q^\sigma T_1(z) \pm \left\{ \varepsilon^*_\mu (m_B + m_{K^*}) - (\varepsilon \cdot k) (2q + k) \mu \right\} T_2(z) \\
- (\varepsilon^* \cdot k) k \mu \left\{ \frac{z}{m_B^2 - m_{K^*}^2} (2q + k) \mu T_3(z) \right\},
$$

$$
<K^*|\bar{s} b|B> = 0,
$$

$$
<K^*|\bar{s}\gamma_5 b|B> = -\frac{2m_{K^*}}{m_B} [\varepsilon^* \cdot k A_0(z)],
$$

...
\[< K^* | \bar{s}_\mu b | B > = \epsilon_{\mu\nu\rho\sigma} [ - \varepsilon^\nu (2q + k)\sigma T_1(z) + \frac{m_B^2 - m_{K^*}^2 \varepsilon \cdot k}{z} \varepsilon^\nu k^\sigma \{ T_1(z) - T_2(z) \} \]
\[\quad - \frac{2}{z} \varepsilon^\nu \cdot k q^\lambda k^\sigma \{ T_1 - T_2 - \frac{z}{m_B^2 - m_{K^*}^2} T_3 \}], \tag{6}\]

where \(\varepsilon^\nu\) is the polarization vector of \(K^*\) meson, and \(P_{L,R}\) is the projection operator \(P_{L,R} = (1 \mp \gamma_5)/2\). For form factors \(V(z), A_i(z), T_i(z)\), we follow the definition of Ref. [17]. Considering the current conservation of the leptonic part, the terms including \(k_\mu\) will disappear. For the decay process of the intermediate vector meson, \(K^* \rightarrow K\pi\), the contribution is replaced as follows [27]:

\[< K\pi | K^* > < K^* | = g_{K\pi} (P_K - P_\pi) \frac{g^\alpha - q^\alpha q^\nu}{G} = g_{K\pi} (P_K - P_\pi) \frac{g^\alpha - q^\alpha q^\nu}{m_{K^*}^2 - q^2 - i m_{K^*} \Gamma_{K^*}} \tag{7}\]

where \(g_{K\pi}\) is the decay constant and \(\Gamma_{K^*}\) is the decay width of \(K^*\) meson.

Under our parametrization, the branching ratio can be expressed as

\[B(B \rightarrow K\pi ll) = \frac{\int ds dz Y B(s, z)}{\Gamma_B},\]

where

\[B(s, z) = \int d\phi d(\cos \theta_K) d(\cos \theta_l) (\Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \Gamma_7) = \int d\phi d(\cos \theta_K) d(\cos \theta_l) \Gamma_1, \tag{8}\]

\[Y = \frac{G_F^2 \alpha^2 |V_{tb}^* V_{ts}|^2 L[s, z]}{128 \times 512 \pi^8}. \tag{9}\]

\(\Gamma_B\) is \(B\)-meson total decay width, and

\[\Gamma_1 \equiv \frac{g_{K\pi}^2}{|G|^2} \left\{ (|C_{9}^{eff}(z)| - C_9'|^2 + |C_{10} - C_{10}'|^2) \right\} \]
\[\times \frac{1}{8(m_B + m_{K^*})^2} \left[ (m_B + m_{K^*})^4 (4sz B_1 + L_0^2 B_3) |A_1|^2 \right. \]
\[+ L_4 B_3 |A_2|^2 \]
\[\left. - 2L_2^2 L_0 (m_B + m_{K^*})^2 B_3 (A_1 A_2) \right] \]
\[+ 4|C_7 - C_7'|^2 \frac{m_B^2}{8sz^2} \left[ (4(m_B^2 - m_{K^*}^2)^2 sz B_1 \right. \]
\[+ \left. (L_2 - L_0(m_B^2 - m_{K^*}^2))^2 B_3 \right] |T_2|^2 \]
\[+ \frac{z^2 L_4^2}{(m_B^2 - m_{K^*}^2)^2 B_3} |T_3|^2 \]
\[
+2 \frac{zL^2}{(m_B^2 - m_{K^*}^2)}(L^2 - L_0(m_B^2 - m_{K^*}^2))B_3 (T_2T_3)
\]
\[
+4 \text{Re} \left( (C_9^{\text{eff}}(z)^* - C_9^{\text{eff}})(C_7 - C_4') \right)
\times \frac{m_b}{16(m_B + m_{K^*})z} \left[ (m_B + m_{K^*}) \left\{ 8sz(m_B^2 - m_{K^*}^2)B_1 ight. \\
-2L_0(L^2 - (m_B^2 - m_{K^*}^2)L_0)B_3 \right\} (A_1T_2) \\
-2zL^2L_0B_3 (A_1T_3) \\
+2L^2(L^2 - (m_B^2 - m_{K^*}^2)L_0)B_3 (A_2T_2) \\
+ \frac{2zL^4}{(m_B^2 - m_{K^*}^2)}B_3 (A_2T_3) \right] \\
+ \left( |C_9^{\text{eff}}(z)|^2 + |C_9'|^2 + |C_{10} + C_1'|^2 \right) \frac{1}{2(m_B + m_s)^2}L^2szB_2 |V|^2 \\
+4|C_7 + C_7'| \frac{m_B^2s}{2z}L^2B_2 |T_1|^2 \\
+4 \text{Re} \left( (C_9^{\text{eff}}(z)^* + C_9^{\text{eff}})(C_7 + C_4') \right) \frac{m_b}{2(m_B + m_{K^*})}L^2B_2 (T_1V) \\
\]
\[
+ \left( |C_{A3}|^2 + |C_{A4}|^2 \right) \frac{m_{K^*}^2}{m_B^2}2zL^2 \cos^2 \theta_K |A_0|^2
\]
\[
+ \left( |C_T|^2 + 4|C_{TE}|^2 \right) \frac{8}{z} \left[ \left\{ (m_B^2 - m_{K^*}^2 - z - L_0)^2(L_0^2S_1 + 4szS_2) + 4szL^2S_3 \right\}|T_1|^2 \\
+ \left\{ (L^2 - L_0(m_B^2 - m_{K^*}^2))^2S_1 + 4sz(m_B^2 - m_{K^*}^2)^2S_2 \right\}|T_2|^2 \\
+ \frac{z^2L^4}{(m_B^2 - m_{K^*}^2)}S_1|T_3|^2 \\
- \{ (L_0 - m_B^2 + m_{K^*}^2 + z)(L_0(L^2 - L_0(m_B^2 - m_{K^*}^2)))S_1 \\
- 4sz(m_B^2 - m_{K^*}^2)^2S_2 \}2T_1T_2 \\
- \frac{zL^2L_0}{m_B^2 - m_{K^*}^2} \left\{ (L_0 - m_B + m_{K^*}^2 + z)S_1 \right\}2T_1T_3 \\
+ \frac{zL^2}{m_B^2 - m_{K^*}^2} \left\{ (L^2 - L_0(m_B^2 - m_{K^*}^2))S_1 \right\}2T_2T_3 \right],
\]
\[
\Gamma_2 \equiv F_2(s, z) \sin^2 \theta_K \cos \theta_l
\]
\[
= \frac{g_{K\pi}^2}{|G|^2} \sin^2 \theta_K \cos \theta_l \\
\times \left\{ 2 \text{Re} \left( C_9^{\text{eff}}(z)^* C_{10} - C_9^{\text{eff}} C_1' \right) Lsz (A_1V) \\
+ 2 \text{Re} \left( (C_{10}' + C_{10}')(C_7 - C_4') \right) m_b(m_B - m_{K^*})sL (VT_2) \\
+ 2 \text{Re} \left( (C_{10}' - C_{10}')(C_7 + C_4') \right) m_b(m_B + m_{K^*})sL (A_1T_1) \right\},
\]
\[
\Gamma_3 \equiv F_3(s, z) \cos \phi \sin 2\theta_K \sin 2\theta_l
\]
\[
\Gamma_4 \equiv F_4(s, z) \sin 2\phi \sin^2 \theta_K \sin^2 \theta_l \\
= \frac{g_{K\pi}^2}{|G|^2} \sin 2\phi \sin^2 \theta_K \sin^2 \theta_l \\
x \left\{ \left( C_9^{\text{eff}}(z) - C_9^* \right)^2 + \left| C_{10} - C_{10}' \right|^2 \right\} \left( m_B + m_{K^*} \right)^2 \frac{\sqrt{s z}}{8} \left[ (m_B + m_{K^*})^2 L_0 |A_1|^2 - L^2 (A_1 A_2) \right] \\
-4 |C_7 - C_7'|^2 \frac{m_B^2 \sqrt{s z}}{8z^2} L^2 \left[ \left( m_B^2 + m_{K^*} \right)^2 \left( L^2 - L_0 (m_B^2 - m_{K^*}) \right) |T_2|^2 + 2z L^2 (T_2 T_3) \right] \\
-4 \text{Re} \left( (C_9^{\text{eff}}(z) - C_9^*)(C_7 - C_7') \right) \\
\times \frac{m_B \sqrt{s z}}{16 (m_B - m_{K^*}) z} \left[ (m_B^2 - m_{K^*}^2) (L^2 - 2(m_B^2 - m_{K^*}^2) L_0) (A_1 T_2) \right. \\
+ z L^2 (A_1 T_3) \left. + L^2 (m_B^2 - m_{K^*}^2) (A_2 T_2) \right] \right\} \\
+ 8 \left( |C_T|^2 + 4 |C_{TE}|^2 \right)^2 \frac{\sqrt{s z}}{z} \left\{ -L_0 (L_0 - m_B^2 + m_{K^*}^2 + z)^2 |T_1|^2 \\
- 4sz (m_B^2 - m_{K^*}^2) |T_2|^2 \\
+ (L^2 - 2L_0 (m_B^2 - m_{K^*}^2)) T_1 T_2 \\
+ \frac{z L^2}{m_B^2 - m_{K^*}^2} (L_0 - m_B^2 + m_{K^*}^2 + z) T_1 T_3 \\
+ z L^2 T_2 T_3 \right\} \right\}, \tag{14}
\]

\[
\Gamma_5 \equiv F_5(s, z) \sin \phi \sin 2\theta_K \sin^2 \theta_l \\
= \frac{g_{K\pi}^2}{|G|^2} \sin \phi \sin 2\theta_K \sin^2 \theta_l \\
x \left\{ \text{Im} \left( (C_9^{\text{eff}}(z) - C_9^*)(C_7 + C_7') \right) m_b L (m_B + m_{K^*}) (A_1 T_1) \right\} \\
+ \text{Im} \left( (C_9^{\text{eff}}(z) + C_9^*)(C_7 - C_7') \right) m_b L (m_B - m_{K^*}) (VT_2) \\
- \text{Im} (C_7^* C_7^\dagger) \frac{8m_b^2 L (m_B^2 - m_{K^*}^2)}{z} (T_1 T_2) \right\} \right\}, \tag{15}
\]

\[
\Gamma_6 \equiv F_6(s, z) \cos \phi \sin 2\theta_K \sin \theta_l \\
= \frac{g_{K\pi}^2}{|G|^2} \sin \phi \sin 2\theta_K \sin \theta_l \\
x \left\{ \text{Im} \left( (C_9^{\text{eff}}(z) + C_9^*)(C_7 + C_7') \right) \right\} \frac{m_b \sqrt{s z L}}{4(m_B + m_{K^*}) z} \left[ \frac{L_0 (m_B^2 - m_{K^*}^2)}{(A_1 T_1)} - L^2 (A_2 T_1) \right] \right\} \\
+ \text{Im} \left( (C_9^{\text{eff}}(z) + C_9^*)(C_7 - C_7') \right) \frac{m_b \sqrt{s z L}}{4(m_B - m_{K^*}) z} \left[ \left( L^2 - (m_B^2 - m_{K^*}^2) L_0 \right) (VT_2) \right. \\
- \frac{z L^2}{(m_B^2 - m_{K^*}^2)} (VT_3) \right] \right\} \right\} \\
- \text{Im} (C_7^* C_7^\dagger) \frac{m_b^2 \sqrt{s z L}}{(m_B^2 - m_{K^*}^2) z^2} \left[ \left( m_B^2 - m_{K^*}^2 \right) \left( L^2 - (m_B^2 - m_{K^*}^2) L_0 \right) (T_1 T_2) + z L^2 (T_1 T_3) \right\} \right\}, \tag{16}
\]

\[
\Gamma_6 \equiv F_6(s, z) \cos \phi \sin 2\theta_K \sin \theta_l \tag{17}
\]
due to the canceling angular dependence with an over-all factor of 

\[ \frac{\sqrt{s}L}{2(m_B + m_{K*})^2} \left[ (m_B + m_{K*})^2 L_0 \ (A_1 V) - L^2 \ (A_2 V) \right] \]

\[ - \text{Re} \left( (C_{10}^* + C_{10}')(C_7 - C_7') \right) \frac{m_b L \sqrt{s} \pi}{2(m_B + m_{K*})} \left[ (L^2 - (m_B^2 - m_{K*}^2) L_0) \ (VT_2) \right. \]

\[ + \frac{L^3 m_b}{(m_B + m_{K*})(m_B^2 - m_{K*}^2)} \ (VT_3) \right] \]

\[ + \text{Re} \left( (C_{10}^* - C_{10}')(C_7 + C_7') \right) \frac{m_b L \sqrt{s} \pi}{2(m_B + m_{K*})} \left[ (m_B + m_{K*})^2 L_0 \ (A_1 T_1) - L^2 \ (A_2 T_1) \right] \}

\[ \Gamma_7 \equiv F_7(s, z) \sin \phi \sin 2\theta_K \sin \theta_l \]

\[ = \frac{g_{K\pi}^2}{|G|^2} \sin \phi \sin 2\theta_K \sin \theta_l \]

\[ \times \left\{ \text{Im} \left( (C_{10}^* - C_{10}')(C_7 - C_7') \right) \frac{m_b L^2 \sqrt{s} \pi}{2(m_B - m_{K*})} \left[ (m_B^2 - m_{K*}^2) \ (A_1 T_2) + z \ (A_1 T_3) \right. \right. \]

\[ - (m_B - m_{K*})^2 \ (A_2 T_2) \right]\]

\[ + 8 \text{Re} \left( (C_{10}^* C_T - 2C_{10} C_{TE}) \frac{m_{K*} \sqrt{s} \pi L^2}{m_B} T_1 A_0 \right) \}

where

\[ L \equiv \sqrt{(s - z)^2 - 2m_B^2(s + z) + m_B^4}, \]

\[ L_0 \equiv \sqrt{L^2 + 4sz} = m_B^2 - s - z. \]

Angular functions in \( \Gamma_1 \) are

\[ B_1 = \sin^2 \theta_K - \cos^2 \phi \sin^2 \theta_K \sin^2 \theta_l, \]

\[ B_2 = \sin^2 \theta_K - \sin^2 \phi \sin^2 \theta_K \sin^2 \theta_l, \]

\[ B_3 = \cos^2 \theta_K \sin^2 \theta_l, \]

\[ S_1 = \cos^2 \theta_K \cos^2 \theta_l, \]

\[ S_2 = \sin^2 \theta_K \sin^2 \theta_l \cos^2 \phi, \]

\[ S_3 = \sin^2 \theta_K \sin^2 \theta_l \sin^2 \phi. \]

After integrating out whole angular space, we get the values of \( \Gamma_2, ..., \Gamma_7 \) becoming zero, due to the canceling angular dependence with an over-all factor of \( F_{2,7}(s, z) \). However, partial angular integration asymmetries becoming non-zero values, like FB asymmetries, can give us possibly very important information on new physics contributions. For each
Because modes, investigating CP asymmetry of the decay is not so simple, even though it is very also considered similarly. The CP eigen-mode for each \( \Gamma_i \) states with CP eigen-mode. The mixing induced time-dependent CP asymmetry can be asymmetries with previously defined FB asymmetries, we can clearly separate the final important to extract new physics information beyond the SM. However, by combining CP CP odd, and \( \Gamma_3 \) conjugate \( B \) meson decays. Usual definition of FB asymmetry of leptons with the narrow
resonance (via e.g. $K^*$) assumption is
\[
A^{FB_2}(M_{K^*}, z) = \frac{8\pi F_2(M_{K^*}, z)}{3B(M_{K^*}, z)},
\]
(38)
If no new CP phases are present, $A^{FB_i}(s, z) = FB_i \Gamma_i/B(s, z)$. We can also define several CP asymmetries,
\[
A_{CP}(s, z) = \frac{\bar{\Gamma}_1 - \Gamma_1}{\Gamma_1 + \Gamma_1},
\]
\[
A^{FB_i}_{CP}(s, z) \equiv \frac{FB_i[\eta_{CP}\bar{\Gamma}_i - \Gamma_i]}{B(s, z) + B(s, z)},
\]
(39)
where $A^{FB_i}_{CP}$ is the CP asymmetry for each $FB_i$ asymmetry. (The CP asymmetry for $FB_2$ was also defined in [21].) Similarly, the time dependent CP asymmetries of $B^0 \rightarrow K^0\pi^0l^+l^-$ are defined after combined with the FB asymmetries,
\[
S^{FB_i}_{CP}(s, z) = \frac{2\eta_{CP}Im[e^{-2i\phi_1}]}{FB_i[\eta_{CP}\bar{\Gamma}_i + \Gamma_i]} \left[ \frac{C^*_x C_y \rightarrow C^*_x \bar{C}_y \Gamma_{total}}{FB_i[\eta_{CP}\bar{\Gamma}_i + \Gamma_i]} \right],
\]
(40)
where $C^*_x C_y \rightarrow C^*_x \bar{C}_y$ means the Wilson coefficients of all $C^*_x C_y$ in $\Gamma_{total}$ is replaced to the charge conjugated Wilson coefficients like $C^*_x \bar{C}_y$. If there is no new CP phase in the Wilson coefficients except the CKM phase, $S^{FB_i}_{CP}$ becomes exactly $\eta_{CP} \sin 2\phi_1$ after the cancellation of $FB_i$ in Eq. (39). However, if there exists any new CP phase beyond the SM, the values would change appropriately. Therefore, investigating the time-dependent CP asymmetries will be very important to find hints of new physics.

III. FORWARD-BACKWARD ASYMMETRIES OF LEPTONS

In Fig. 2, we show those newly defined CP averaged FB asymmetries as functions of the invariant mass square ($z$) of dilepton, where the red (solid) line, the green (dashed) line, the blue (dash-dotted) line and the purple (dotted) line show the SM case, the SM case with $-C_7$, the case with $C_7^\prime = |C_7|$, and the case with $-C_7^\prime$, respectively. In Fig. 2, we have not assumed any new CP phase and the magnitude of the parameters are the SM predictions, except for $C_7^\prime$. We note that in the SM $C_7$ and $C_{10}$ are almost real, so that the origin of CP violation is from the imaginary part of $C_9^{eff}$, whose contributions come from intermediate $\bar{c}c$ bound states. Due to the absence of strong phase in $C_9^{eff}(z)$ at low dilepton invariant mass region within the SM, some asymmetries show very strong sensitivity to such
FIG. 2: FB asymmetries, which defined in Eqs. (31)- (36), are plotted, where the solid (red) line shows the SM case, and the dashed (green) line show the case with $-C_7$, the dash-dotted (blue) is the pure $C'_7 = |C_7|$ case and the dotted (purple) line is $-C'_7$ case. Here, we did not assume any new CP phase and the parameters are SM predictions except for $C_7$ and $C'_7$. 
FIG. 3: The imaginary part of $C^\text{eff}_9$ and the direct CP asymmetry, $A_{CP}$, for varying the CP phase of $C_9$.

extra CP phase, if exists, which can be an undeniable evidence of new physics with new CP phase. Note that $FB_4$ $\Gamma_4$ and $FB_5$ $\Gamma_5$ can easily extract the imaginary part of $C^\text{eff}_9$, whose contributions within the SM come only from intermediate $\bar{c}c$ bound stats in high $z$ region. Hence, those observables are very sensitive to new phase in low $z$ region. $FB_7$ $\Gamma_7$ does not include $C_9$ and is proportional to $Im[C^*_7 C_{10}]$, so that it can be sensitive to new CP phase in $C_7$ and $C_{10}$. In the usual case, $C_7$ and $C_{10}$ are almost real (except for overall factor) so that FB asymmetry for $\Gamma_7$, $FB_7$ $\Gamma_7$, should be zero. $A^{FB_2}$ is the usual FB asymmetry for leptons. Therefore, proving the zero point of the asymmetry, $A^{FB_2}(z) = 0$, in low $z$ region can show the evidence of new physics contribution. For $A^{FB_0}$, it is very similar to $A^{FB_2}$ but with the slightly different behavior.

In Fig. 3, we show the imaginary part of $C^\text{eff}_9$ and the direct CP asymmetry $A_{CP}$ as a function of $z$. One can find that within the SM the direct CP asymmetry $A_{CP}$ in Fig. 3 is quite small because it is directly proportional to $C^\text{eff}_9 C_7$ terms, which is small and also suppressed by $1/z$. In general a CP asymmetry for modes with both CP odd and CP even is canceling each other, becoming small. On the other hand, the CP asymmetry for $FB_i$ may not be so because they are enhanced by the angular decomposition.

CP asymmetry for each FB asymmetry, $A_{CP}^{FB_i}$, is plotted in Fig 4 as a function of the dilepton’s invariant mass. Here we have introduced new CP phase in $C_9$ and $C_{10}$. The lines for $A_{CP}^{FB_2}$, $A_{CP}^{FB_6}$, $A_{CP}^{FB_7}$ are showing the case that $C_{10}$ has a pure imaginary CP phase.
FIG. 4: For $A_{CP}^{FB2}, A_{CP}^{FB6}, A_{CP}^{FB7}$, we show the case that $C_{10}$ has a pure imaginary CP phase. For $A_{CP}^{FB3}, A_{CP}^{FB4}, A_{CP}^{FB5}$, we introduced a pure imaginary CP phase in $C_9$. 
For $A_{CP}^{FB_5}$, $A_{CP}^{FB_4}$ and $A_{CP}^{FB_3}$, we introduced a pure imaginary CP phase in $C_9$. Please note a condition to have CP asymmetry is the existence of strong phase differences among several contributions. In the figures of $A_{CP}^{FB_2}$ and $A_{CP}^{FB_6}$, one can find the dependence of imaginary part of $C_9^{eff}$, where large CP asymmetries do not appear in low $z$ region. On the other hand, the figures of $A_{CP}^{FB_5}$ and $A_{CP}^{FB_7}$ show large CP asymmetries in low $z$ region. This is a very interesting feature. It is because $\Gamma_7$ is proportional to $Im[C_{10}^* C_7]$ so that the CP asymmetry has to be $Re[C_{10}^* C_7] \sin[\phi_{10}]$, where $\phi_{10}$ is newly introduced CP phase of $C_{10}$ as $C_{10} e^{i\phi_{10}}$. Hence from these observables in low $z$ region, we can extract a few hints about new CP phase.

In Fig. 5, as an example, $A_{FB_2}^{CP}$, $A_{CP}^{FB_2}$ and $S_{CP}^{FB_2}$ are plotted as functions of $z$, where the

$$\theta_9 = 0, \pi/8, \pi/4, \pi/3.$$
new CP phase of $C_9$ is taken as $0, \pi/8, \pi/4$ and $\pi/3$. If there is no new CP phase introduced, $S_{CP}^{FR_t}$ becomes exactly $\sin 2\phi_1$ (red line in the figure). However, if there exists new CP phase, the changes are drastic with new CP phase. For the case with scalar and/or tensor type new interactions, the effects will appear in the branching ratio, $\Gamma_3$ and $\Gamma_7$. E.g., if $C_7$ and $C_{10}$ are real values, $A^{FR_t}$ appears as nonzero value only with the scalar and/or tensor type new interactions.

IV. CASE WITH SCALAR RESONANCE IN ADDITION TO $K^*$

In previous section, we have examined the case with a single narrow $K^*$ resonance limit. Only with a single narrow resonance, we do not have the required large strong phase difference for direct CP violations. However, there exist also scalar resonances in the decay mode of $B \to K\pi l^+l^-$, e.g., $K^*_0(800)$, $K^*_0(1410)$. In this section, we consider effects of the interference from the scalar resonances with the existing vector $K^*_0(892)$ state. We assume Wigner type resonance formula for simplicity to express the effects, even though it is known that this formula cannot describe the effects precisely.

The matrix element is

$$<M> = <K, \pi|\{K^* <K^*| + S > <S|}M|B>, \quad (41)$$

where $S$ expresses a scalar resonance state. If the mass of $S$ is very close to $K^*(892)$ mass, the cross term between the two resonance states will make large strong phase difference. To calculate the decay rate, we use the following parametrization for the hadronic matrix elements,

$$<S|\bar{s}\gamma_\mu b|B> = 0, \quad (42)$$

$$<S|\bar{s}\gamma_\mu \gamma_5 b|B> = \left(2q + k\right)_\mu - \frac{m_B^2 - m_0^2}{z} k_\mu \right) F_1(z) + \frac{m_B^2 - m_0^2}{z} k_\mu F_0(z), \quad (43)$$

$$<S|\bar{s}i\sigma_{\mu\nu} k^\nu b|B> = 0, \quad (44)$$

$$<S|\bar{s}i\sigma_{\mu\nu} k^\nu \gamma_5 b|B> = -\frac{1}{m_B + m_0} [(2q + k)_\mu q^2 - q_\mu (m_B^2 - m_0^2)] F_T(z), \quad (45)$$

$$<S|\bar{s}b|B> = 0, \quad (46)$$

$$<S|\bar{s}\gamma_5 b|B> = -\frac{m_B^2 - m_0^2}{m_B} F_0(z), \quad (47)$$

$$<S|\bar{s}\sigma_{\mu\nu} k^\nu b|B> = 0, \quad (48)$$
where \( F_x \) are the form factors. Here we are using the definitions of Ref. [17]. We also assume that the scalar resonance states as follow:

\[
<K \pi|S><S| = m_0 g_0 \frac{1}{G_0} = -m_0 g_0 \frac{1}{m_0^2 - s - im_0 \Gamma_0},
\]

(49)

where \( m_0 \) is the mass and \( \Gamma_0 \) is the decay width of the scalar resonance. Here we assumed the mass and width of the scalar particle, e.g., \( K^*_0(800) \) as,

\[
m_0 = 0.658 \text{ GeV}, \quad \Gamma_0 = 0.557 \text{ GeV},
\]

and also assumed that \( K^*(800) \) decays only to \( K \pi \).

Using the parameterizations, the differential decay rate from scalar resonance is

\[
\Gamma_1^s = \frac{g_0^2}{|G_0|^2} \times \frac{m_0^2L^2}{2} \sin^2 \theta_l \left\{ \left( |C_9^{eff}(z)| - C_9' \right)^2 + |C_{10} - C_{10}'|^2 \right\} (F_1^2)
\]

\[
+ 4 Re \left( (C_9^{eff}(z) - C_9')(C_7 - C_7') \right) \frac{m_b}{m_B + m_0} (F_1F_T)
\]

\[
+ 4 |C_7 - C_7'|^2 \frac{m_b^2}{(m_B + m_0)^2} |F_T|^2 \right\}
\]

\[
+ \left( |C_{AS}|^2 + |C_{AA}|^2 \right) \frac{2z m_0^2 (m_B^2 - m_0^2)}{m_B^2} |F_0|^2].
\]

(50)

And the cross terms with vector \( K^* \) resonance contribution are

\[
\Gamma_2^s \equiv F_2^s \cos \theta_K \sin^2 \theta_l + F_2^{s'} \cos \theta_K
\]

\[
= \frac{g_{K^*} g_0}{|G|^2 |G_0|^2} \cos \theta_K \sin^2 \theta_l
\]

\[
\times \left\{ \left( |C_9^{eff}(z)| - C_9' \right)^2 + |C_{10} - C_{10}'|^2 \right\} \ Re[GG_0^*] \left[ \frac{m_0(m_B + m_{K^*})LL_0}{2} (A_1F_1)
\]

\[
- \frac{m_0 m_3}{2(m_B + m_{K^*})} (A_2F_1) \right]\]

\[
- 4 |C_7 - C_7'|^2 \ Re[GG_0^*] \left[ \frac{m_0 m_2 L}{2(m_B + m_0) z} (L^2 - (m_B^2 - m_{K^*}) L_0) (T_2F_T)
\]

\[
- \frac{m_0 m_3}{2(m_B^2 - m_{K^*}^2)(m_B + m_0)} (T_3F_T) \right]
\]

\[
- 4 Re \left( (C_9^{eff}(z) - C_9')(C_7 - C_7')GG_0^* \right) \left[ \frac{m_0 m_B L_0(m_B + m_{K^*})}{4(m_B + m_0)} (A_1F_T)
\]

\[
- \frac{m_0 m_3 m_B L}{4(m_B + m_{K^*})(m_B + m_0)} (A_2F_T) \right].
\]

(51)
\[-4Re \left((C_9^{eff}(z) - C_9')(C_7 - C_7')G^*G_0\right) \left[\frac{m_0m_bL}{4z}(L^2 - (m_B^2 - m_{K*}^2)L_0) (T_2F_1) + \right.\]
\[\left.\frac{m_0m_bL^3}{4(m_B^2 - m_{K*}^2)} (T_3F_1) \right] \} \}
\[\cos \theta_K \{ (|C_{AS}|^2 + |C_{AA}|^2) Re[GG_0^*] \frac{4zLm_0m_{K*}(m_B^2 - m_0^2)}{m_B^2} F_0A_0 \}, \] (52)

\[
\Gamma_3^s \equiv F_3^s \cos \phi \sin \theta_K \sin \theta_l \]
\[= \frac{g_{K*}g_0}{|G|^2|G_0|^2} \cos \phi \sin \theta_K \sin \theta_l \times \left\{ -2 \left(Re(C_9^{eff}(z)C_{10} - C_9'C_{10})Re(GG_0^*) \right) \frac{m_0\sqrt{s\xi}L^2}{(m_B + m_0)} V F_1 \right.\]
\[\left. -4Re \((C_{10} + C_{10}')(C_7 - C_7')GG_0^*) \left[\frac{m_0\sqrt{s\xi}L^2}{2(m_B + m_{K*})(m_B + m_0)} (V F_T) \right] \right\}, \] (54)

\[
\Gamma_4^s \equiv F_4^s \sin \phi \sin \theta_K \sin \theta_l \]
\[= \frac{g_{K*}g_0}{|G|^2|G_0|^2} \sin \phi \sin \theta_K \sin \theta_l \times \left\{ -2 \left(Re(C_9^{eff}(z)C_{10} - C_9'C_{10})Im(GG_0^*) \right) m_0L\sqrt{s\xi}(m_B + m_{K*}) (A_1F_1) \right.\]
\[\left. -4Im \((C_{10} - C_{10}')(C_7 - C_7')GG_0^*) \left[\frac{m_0\sqrt{s\xi}L^2}{2(m_B + m_0)} (A_1F_T) \right] \right\}, \] (55)

\[
\Gamma_5^s \equiv F_5^s \sin \phi \sin \theta_K \sin 2\theta_l \]
\[= \frac{g_{K*}g_0}{|G|^2|G_0|^2} \sin \phi \sin \theta_K \sin 2\theta_l \times \left\{ - \left(|C_9^{eff}(z)|^2 + |C_{10}|^2 - |C_9'|^2 - |C_{10}'|^2 \right) Im[GG_0^*] \left[\frac{m_0L^2\sqrt{s\xi}}{2(m_B + m_{K*})} (F_1 V) \right] \right.\]
\[\left. -4Im \((C_7 + C_7')(C_7 - C_7')GG_0^*) \frac{m_0m_bL^2\sqrt{s\xi}}{2z(m_B + m_0)} (T_1F_T) \right.\]
\[\left. -4Im \((C_9^{eff}(z) + C_9')(C_7 - C_7')GG_0^*) \frac{m_0m_bL^2\sqrt{s\xi}}{4(m_b + m_{K*})(m_B + m_0)} (V F_T) \right.\]
\[\left. +4Im \((C_9^{eff}(z) - C_9')(C_7 + C_7')G^*G_0) \frac{m_0m_bL^2\sqrt{s\xi}}{4z} (T_1F_1) \right\}, \] (56)

\[
\Gamma_6^s \equiv F_6^s \cos \phi \sin \theta_K \sin 2\theta_l \]

\[= \frac{g_{K*}g_0}{|G|^2|G_0|^2} \cos \phi \sin \theta_K \sin 2\theta_l \times \left\{ \right\}, \] (57)
\begin{align}
&= \frac{g_{K\pi g_0}}{|G|^2|G_0|^2} \cos \phi \sin \theta_K \sin 2\theta_l \\
&\times \left\{ \left( |C_9^{\pi\pi}|^2 + |C_{10} - C_{10}'|^2 \right) Re[GG_0^*] \left[ \frac{m_0L\sqrt{s\bar{s}(m_B + m_K^*)}}{2} \right] (A_1 F_1) \\
&+ 4 |(C_7 - C_7')|^2 Re[GG_0^*] \frac{m_0m_8L\sqrt{s\bar{s}(m_B^2 - m_K^2)}}{2z(m_B + m_0)} (T_2 F_T) \\
&- 4 Re((C_9^{\pi\pi} - C_9')^*(C_7 - C_7')GG_0^*) \frac{m_0m_8L\sqrt{s\bar{s}(m_B + m_K^*)}}{4(m_B + m_0)} (A_1 F_T) \\
&+ 4 Re((C_9^{\pi\pi} - C_9')^*(C_7 - C_7')G^*G_0) \frac{m_0m_8L\sqrt{s\bar{s}(m_B^2 - m_K^2)}}{4z} (T_2 F_1) \right\}, \quad (58)
\end{align}

\begin{align}
\Gamma_7^s &\equiv F_7^s \cos^2 \theta_K \cos \theta_l \\
&= \frac{g_{K\pi g_0}L_0}{|G|^2|G_0|^2} \cos^2 \theta_K \cos \theta_l \\
&\times \left\{ 8 Im((C_{AA}^*C_T - 2C_{AS}^*C_{TE})GG_0^*) \frac{m_0(m_B^2 - m_K^2)}{m_B} (L_0 - m_B^2 + m_K^2 + z)T_1 F_0 \right\}, \quad (59)
\end{align}

\begin{align}
\Gamma_8^s &\equiv F_8^s \cos \phi \sin 2\theta_K \sin \theta_l \\
&= \frac{g_{K\pi g_0}\sqrt{s\bar{s}}}{|G|^2|G_0|^2} \cos \phi \sin 2\theta_K \sin \theta_l \\
&\times \left\{ 8 Im((C_{AA}^*C_T - 2C_{AS}^*C_{TE})GG_0^*) \frac{m_0(m_B^2 - m_K^2)}{m_B} (L_0 - m_B^2 + m_K^2 + z)T_1 F_0 \right\}. \quad (60)
\end{align}

Note that \( \Gamma_7^s \) and \( \Gamma_8^s \) have the same angular distributions as \( \Gamma_2 \) and \( \Gamma_6 \), respectively, and therefore, their contributions can be extracted by \( FB_2 \) and \( FB_6 \) integration operators. All the other \( \Gamma_i^s \)'s are independent of previously defined \( FB_i \) of Eqs. (31)-(36). To extract these contributions, we need new definitions of FB asymmetries from new integration operators, \( FB_i^s \). Namely, these contributions appear only in the case with the scalar resonance effects.

If any one of the following type \( FB_i^s \) asymmetries appears, it can be a strong evidence for the scalar resonance contributions. The new operators \( FB_i^s \) are defined as follow:

\begin{align}
FB_2^s \Gamma_{\text{total}} &= \int_0^{2\pi} d\phi \int_0^K \sin \theta_l d\theta_l \left( \int_0^{\pi} - \int_0^{\pi} \right) \sin \theta_K d\theta_K \frac{8\pi F_2^s}{3} + 4\pi F'_2, \quad (61)
\end{align}

\begin{align}
FB_3^s \Gamma_{\text{total}} &= \left( \int_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \right) \phi \int_0^K \sin \theta_K d\theta_K \int_0^{\pi} \sin \theta_l d\theta_l \Gamma_3^s = \pi^2 F_3^s, \quad (62)
\end{align}

\begin{align}
FB_4^s \Gamma_{\text{total}} &= \left( \int_0^{\pi} - \int_0^{\pi} \right) \phi \int_0^K \sin \theta_K d\theta_K \sin \theta_l d\theta_l \Gamma_4^s = \pi^2 F_4^s, \quad (63)
\end{align}

\begin{align}
FB_5^s \Gamma_{\text{total}} &= \left( \int_0^{\pi} - \int_0^{2\pi} \right) \phi \left( \int_0^{\pi} - \int_0^{\pi} \right) \sin \theta_l d\theta_l \int_0^{\pi} \sin \theta_K d\theta_K \Gamma_5^s = \frac{8\pi F_5^s}{3}, \quad (64)
\end{align}

\begin{align}
FB_6^s \Gamma_{\text{total}} &= \left( \int_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \right) \phi \left( \int_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \right) \sin \theta_l d\theta_l \int_0^{\pi} \sin \theta_K d\theta_K \Gamma_6^s = \frac{8\pi F_6^s}{3}. \quad (65)
\end{align}
Also note that $FB^s_2 \Gamma^s_2$ is the FB asymmetry for $K$ meson (or $\pi$). Similarly $FB^s_3 \Gamma^s_3$ and $FB^s_4 \Gamma^s_4$ are the asymmetries for the angle $\phi$ between two decay planes. $FB^s_5 \Gamma^s_5$ and $FB^s_6 \Gamma^s_6$ are defined as the double FB asymmetries. The CP averaged FB asymmetries are defined as

$$A^{FB^s_i}(s, z) = \frac{FB^s_i[\eta_{CP}\bar{\Gamma}^s_i + \Gamma^s_i]}{B(s, z) + B(s, z)}, \quad (66)$$

where $\eta_{CP} = +1$ for CP even case and $-1$ for CP odd. We can also define several CP asymmetries,

$$A^{FB^s_{CP}}(s, z) = \frac{FB^s_i[\eta_{CP}\bar{\Gamma}^s_i - \Gamma^s_i]}{B(s, z) + B(s, z)}. \quad (67)$$

In Fig. 6, we plot the asymmetries defined in Eqs. (61)-(65). Here we have not assumed any new CP phases. The red (solid) curve is the case of the SM with the scalar resonance. The FB asymmetry of $K(\pi)$ meson, $A^{FB^s_2}$, can be relatively large but the other asymmetries cannot be so large without any new physics CP phases. $A^{FB^s_5}$ and $A^{FB^s_6}$ are actually tiny because they are extracted only by the double asymmetries. In Fig. 7, we show the dependence of new CP phase for $FB^s_2$, $FB^s_3$ and $FB^s_4$, $A^{FB^s_i}$ and $A^{FB^s_{CP}}$ as functions of $z$, where the new phases of $C_9$ (for $FB^s_2$) and $C_{10}$ (for $FB^s_{(3, 4)}$) are taken as $0, \pi/8, \pi/4$ and $\pi/2$. The direct CP asymmetries at low $z$ region seem to be enhanced by strong phase differences induced by interferences with the scalar resonance. Unfortunately, $A^{FB^s_{CP}}$ will be quite small because it is proportional only to $C^{'eff}_{9, 7}$ term. However, $A^{FB^s_5}$ and $A^{FB^s_6}$ are very interesting at low $z$ region because we see the enhancement effects through the interference. And the contributions from the nonstandard interactions $C^i_l$ can be enhanced in newly defined CP asymmetries.

Nonzero values for newly defined FB asymmetries will indicate strong evidences for the existence of the scalar resonance in addition to vector $K^*$ meson. Indeed, future super-$B$ factories and LHC-b experiment can measure and count the events for some regions of phase space after separating several bins. Surely we can detect such contributions from these measurements. If we find these contributions quite large, the interferences may have an important role as a source of strong phase difference, which is one of the conditions to enhance CP asymmetries.
FIG. 6: The asymmetries defined in Eqs. (61)-(65), where the solid (red) line shows the SM case with scalar resonance, the dashed (green) shows the $-C_7$ case, the dash-dotted (blue) line is the pure $C'_7 = |C_7|$ case and the dotted (purple) line is for $-C'_7$ case. Here we did not assume any new CP phase.
FIG. 7: $A^{FB_s^*}$ and $A^{FB_i^*}_{CP}$ are plotted as functions of $z$, where new phases of $C_9 (A^{FB_2^*})$ and $C_{10} (A^{FB_3^*}$ and $A^{FB_4^*})$ are taken as 0, $\pi/8$, $\pi/4$ and $\pi/2$. 
V. SUMMARY

Based on the most general 4-fermi interaction, which includes all types of possible interactions with new CP phases, we have investigated the general 4-body decay process, \( B \to K\pi l^+ l^- \), through the angular decomposition method. As is well known, this 4-body decay process can be described in general by using 3 angles, so that we can extract many useful information from the angular decomposition analysis. Similar to \( B \to K^* l^+ l^- \), we can probe the region of zero point for the leptonic FB asymmetry, \( A_{FB} = 0 \), as well. However, here in this general 4-body analysis we can obtain much more information to extract the sources of new physics. We can define several CP averaged FB asymmetries, direct CP asymmetries as well as time dependent CP asymmetries as functions of the 3 angles in terms of the general 4-fermi interaction parameters. We found that some of them are very sensitive to strong or EW phases, and some of them are from interference contributions between CP odd and CP even modes so that the CP asymmetry can be enhanced.

Note that for the decays of \( B \to M [\to K\pi] l^+ l^- \), if we do not constrain the invariant mass of \( K^-\pi \) system, there exist several intermediate mesons contributing to \( B \to K\pi l^+ l^- \) decays. Therefore, through the interference we may induce large strong phases, which result possibly large CP violations if there exist any new physics CP phases beyond the CKM phase. We considered the case with the scalar resonance decay \( B \to K_0^*(800) [\to K\pi] l^+ l^- \) in addition to the vector resonance decay \( B \to K^*(892) [\to K\pi] l^+ l^- \). Again we can define new type of several FB asymmetries and direct CP asymmetries resulted from the interference of vector and scalar intermediate mesons. We investigated the interference effects as a source of strong phase difference, the same as imaginary part of \( C_{eff} \) within the SM, to obtain a few hints of new physics effects. By considering these asymmetries systematically, we can obtain several hints for new CP phases in EW penguin decays, and find that the angular decomposition analysis for the general 4-body decay process can be very useful tool to understand new physics, which may be hiding in EW penguin. If the interference effect is fortunately quite large, we can use it as an enhancement of CP asymmetries to find new CP phases very clearly.

Future super-\( B \) factories \[1\] and LHC-b may be able to find out unknown resonance states and investigate the dependence of new physics in detail. At very low region of dilepton invariant mass \[29\] by using photon conversion technique \[20\], the new contribution from
right-hand current and the CP phase of $C^{(r)}_{7}$ type interaction may be measured. Then, we have to consider more carefully on measuring the angular distributions of $B \rightarrow K\pi l^+l^-$ and the related CP asymmetries to find out information of not only new CP phases of $C_9$ and $C_{10}$ type interactions but also the most general 4-fermi interaction type new physics. Hence, we expect our analysis will be very useful to find new physics hiding beyond $C_9$ and $C_{10}$ with new CP violating phases.

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