Application of Static Bias Game in Decision Making of International Highway Project Bidding

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Abstract. It is an urgent and practical need for international contractors to make a thorough study of the decision-making of international highway project bidding. This paper studies the application of static Bias game model in the international project bidding which provides reference for the reasonable price decision.

1. Introduction

The enterprises need to do a good job in bidding to ensure the survival and development through enhancing the bid winning rate. It has become an urgent and practical need for China's engineering enterprises to make a thorough study of the decision-making of international project bidding for international projects.

In this paper, the static Bias game model is applied to the international highway project bidding to help bidders make reasonable bidding decisions so as to improve the bid winning rate.

2. Static Bias game bid pricing model

2.1 Game theory and pricing decision

Game theory is a theory that studies the interaction between different rational actors, and each person is based on the judgment of how other people will act when they make their own actions. The game theory can be used to analyze the interaction and process of decision-making bodies more clearly. It is also applicable to the study of decision making in international highway project bidding.

In the incomplete information static game, all participants act at the same time. Although a participant can not learn the other participants' choice, he can predict the relation of other participants' decision-making and their respective types, then makes his own decisions to achieve maximum utility, by obtaining the probability distribution of other participants' previous decision results.

2.2 Expression of the static Bias game

According to the static Bias game standard formula, the decision model of international project bidding can be determined. According to the symmetry of the game, it is only need to calculate the bidder’s optimal decision: \( b = b^*(c_i) \). If the decision made by any participant \( i \) and every possible type \( t_i \in T_i \), \( S_i^*(t_i) \) of \( i \) meet \( \max_{a \in A_i} \sum_{t_{i-1}} \{u[S_i^*(t_i), a_i S_{i-1}^* t_{i-1}, a_{i-1} S_{i-2}^* t_{i-2}, \ldots, a_1 S_0^* t_0, t_i] p(t_{i-1}) | t_i \} \), then the strategy combination \( S^* = (S_i^*, \ldots, S_n^*) \) is called a Bias Nash equilibrium of G.
2.3 Bidder’s optimal decision model

The expected payoff function is:

\[ u_i = (b_i - c_i) \prod_{j \neq i} P(b_i < b_j) \]  

(1)

In the formula,

- \( i \) —the Bidders;
- \( c_i \) —cost;
- \( b_i \) —quotation;
- \( P \) —probability of bid award.

\( (b_i - c_i) \) the profit of the bidder \( i \); \( b_j \) is the pricing strategy of the bidder \( j \). In the absence of consideration of other bidders, the range of quotation shall be \([c_i, b_0]\), in which \( b_0 \) is the normal budget price with reasonable profits. Divide \( c_i \) and \( b_0 \), with \( b_0 \), the interval becomes \([c_i / b_0, 1]\), and the interval satisfies \([0, 1]\), so \( b_i = b_0 \times b \cdot c_i = b_0 \times c \). According to the above conditions and the symmetrical character, \( b_j = b^*(c_j) \) can be obtained.

Therefore, \( P(b_i < b_j) = P(b_i < b^*(c_j)) = \frac{1}{b_0} \phi(b_0) = \frac{1}{b_0} b^{*-1}(b_i) < \frac{1}{b_0} c_i = 1 - \phi(b) \)

When the bidder chooses \( b_i \), the cost is \( \phi(b_i) \), \( \phi(b) = b^* - 1(b) \) is the inverse function of \( b^*(c) \).

In the process of equation calculation, if \( \theta \) is on the uniform distribution in \([0, 1]\), then for all \( k \in [0, 1] \), \( P(\theta \geq k) = 1 - k \).

Therefore, the question faced by bidder \( i \) is,

\[ \max(u_i) = b_0(b - c) \prod_{j \neq i} P(b_i < b_j) = b_0(b - c)[1 - \phi(b)]^{n-1} \]  

(2)

The first order condition of optimization is:

\[ (1 - \phi(b))^{n-1} - (b - c)(n - 1)[1 - \phi(b)]^{n-2} \phi'(b) = 0 \]  

(3)

In the equilibrium condition \( \phi(b) = c \), and \( b^* \) is the optimal pricing decision of bidder \( i \), first order condition can be expressed as:

\[ (1 - c)^{(n-2)}[(1 - c)db - (b - c)\sigma(n - 1)dc] = 0 \]  

(4)

The full differential equation takes \( b_0 = 0 \), \( c_0 = 0 \), and uses the General integral to decompose the total differential equation:

\[ \int_0^b (1 - c)^{(n-1)} db - \int_0^c (1 - c)^{(n-2)}(c(n - 1)dv = 0, b + \frac{n-1}{n}(1 - c) - 1 = 0 \] can be obtained.

Then the Bias equilibrium of the bidding game model is:

\[ b^*(c) = \frac{1}{n} + \frac{n-1}{n}c \]  

(5)

That is, bidder \( i \)‘s pricing decision is:

\[ b_i^*(c_j) = \left(\frac{1}{n} + \frac{(n-1)c_j}{nb_0}\right) b_0 \]  

(6)

2.4 Optimal bidding decision for bidding competitors

In the decision-making of international project bidding, the bidder must pay attention to the quotation information of himself and the participants. The analysis and prediction of competitors is based on the completeness and accuracy of their relevant information collection. In order to predict the price of competitors, the method of case guessing is usually adopted, and the quotation ratio is determined by collecting the historical information of the competitors, and then the statistical method is used to estimate the quotation ratio of competitors. According to probability theory, when the number of trials tends to infinity, the frequency value of event A can be regarded as the probability \( P(A) \) of event A. See table 1.
Table 1. Quotation ratio random number/frequency probability statistics table

| Quotation ratio(f) | times | frequency | probability |
|--------------------|-------|-----------|-------------|
| 0.75               | m1    | k1        | P1          |
| 0.8                | m2    | k2        | P2          |
| 0.85               | m3    | k3        | P3          |
| 0.9                | m4    | k4        | P4          |
| 0.95               | m5    | k5        | P5          |
| 1                  | m6    | k6        | P6          |
| 1.05               | m7    | k7        | P7          |
| 1.1                | m8    | k8        | P8          |
| 1.15               | m9    | k9        | P9          |
| 1.2                | m10   | k10       | P10         |
| sum n              | 1     | 1         | 1           |

The expected value of the bid quotation ratio is:

\[
E(f) = \sum_{i=1}^{10} P_{f_i}. \tag{7}
\]

In the case of bidders' reasonable quotations, the cost estimate of the other bidders is:

\[
c_i = \frac{b_i}{1 + L} \Rightarrow c_i = c_1 E(f)
\]

In which,

- \(b_1\) —the bidder;
- \(L\) —reasonable profit;
- \(f\) —quotation ratio.

When the bidder uses Bias equilibrium strategy, the bidding decision should be made according to the lowest cost value of each bidder.

3. Example analysis

The W19 section of a highway project, including subgrade, bridge and culvert, tunnel and other auxiliary facilities, with a construction period of 41 months. The international competitive open tender is used to determine the contractor. The lowest bidder is to be selected as the winning bidder.

A total of 4 companies participated in the tender, the four enterprises are bidders NEB1, NEB2, NEB3 and NEB4. All of them are well-known contractors in the highway industry, all of whom have participated in similar projects for many times. There is a certain degree of understanding between the bidders, and a relatively rich history of bidding information has been accumulated. Relevant historical statistics are shown in Table 2. The bidder NEB1 analyzed and estimated that the cost of the project was $253,510,000, and the normal budget price was $316,890,000.

| quotation ratio(f) | NEB1/NEB2 | NEB1/NEB3 | NEB1/NEB4 |
|--------------------|-----------|-----------|-----------|
|                    | times     | freq.     | prob.     | times     | freq.     | prob.     | times     | freq.     | prob.     |
| 0.75               | 1         | 0.03      | 0.03      | 0         | 0         | 0         | 1         | 0.03      | 0.03      |
| 0.8                | 2         | 0.07      | 0.07      | 1         | 0.04      | 0.04      | 2         | 0.06      | 0.06      |
| 0.85               | 5         | 0.17      | 0.17      | 1         | 0.04      | 0.04      | 5         | 0.14      | 0.14      |
| 0.9                | 7         | 0.24      | 0.24      | 2         | 0.07      | 0.07      | 8         | 0.23      | 0.23      |
| 0.95               | 8         | 0.28      | 0.28      | 3         | 0.11      | 0.11      | 9         | 0.26      | 0.26      |
| 1                  | 2         | 0.07      | 0.07      | 2         | 0.07      | 0.07      | 1         | 0.03      | 0.03      |
| 1.05               | 2         | 0.07      | 0.07      | 11        | 0.41      | 0.41      | 5         | 0.14      | 0.14      |
| 1.1                | 2         | 0.07      | 0.07      | 6         | 0.22      | 0.22      | 3         | 0.09      | 0.09      |
Combined with the above information, the expected price ratio of bidder NEB1 and bidder NEB2 is:

\[ E(f) = \sum_{i=1}^{10} P(f_i) E(f) \]

\[ = 0.75 \times 0.03 + 0.8 \times 0.07 + 0.85 \times 0.17 + 0.9 \times 0.24 + 0.95 \times 0.28 + 1 \times 0.07 + 1.05 \times 0.07 + 1.1 \times 0.07 + 1.15 \times 0.00 + 1.2 \times 0.00 = 0.9255 \]

Similarly, the expected quotation ratios of NEB1 and NEB3, NEB1 and NEB4 are 1.0222 and 0.9540 respectively.

Therefore, the bidder NEB1’s quotation to the bidder NEB2 is estimated as

\[ c_{NEB2} = c_{NEB1} E(f) = 253,510,000 \times 0.9255 = $234,623,505 \]

Similarly, the quotation estimates of NEB1 to NEB3 and NEB1 to NEB4 are $259,137,922 and $241,848,540 respectively.

According to Bayesian game theory, the optimal quotation can be obtained.

\[ b_i = \frac{b_{a_i}}{n} + \frac{n-1}{n} c_i \]  \hspace{1cm} (9)

In the formula, \( b_i \) - the best quotation calculated when referring to bidder \( i \).

\[ b_1 = \frac{31689}{4} + \frac{4-1}{4} 253,510,000 = $269,355,000 \]

Similarly, \( b_2 = $255,190,129 \), \( b_3 = $273,575,942 \), \( b_4 = $260,608,905 \)

According to the bidding evaluation rules, the optimal decision is \( b^* = \min (b_1, b_2, b_3, b_4) = $255,190,129 \).

4. Conclusion

The competition of international project contracting market is becoming more and more fierce, which also puts forward higher requirements for bidding quotation. International project bidding decision-making is a typical incomplete information game process. The application of static Bayesian game model has certain reference significance for improving the accuracy of bidding decision-making.

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