In this paper we show how to control the quantum laser atoms instability using IR radiation. The control can be achieved by controlling the scattering length constant via the infrared coupling constant. This method is applied in the scheme of a continuous CW laser and involves three occupation levels in the condensed atoms. This atoms in Lambda configuration is a different picture from the conventional Lambda configuration with the corresponding rate equation involving the time derivative of the number of atoms in the state belonging to the Lamb-configuration. This method allows us a theoretical construction of any atom laser even for negative interaction constant as is the case of $^7$Li.

Since the making of the first condensed state and the later study of its coherence properties at least two other systems have been proposed for the construction of an atom laser. An atom laser is defined in analogy to optical laser and in a similar way it will operate in a dynamical steady state if some threshold requirement is satisfied. In fact, a pulsed atom laser version have been already demonstrated been constructed by Hansch and their group. Despite of this the stability control remains a difficulty to overcome, even from the onset of BEC formation, specially for those cases of negative scattering length constants as $^7$Li. The main difficulties are the dissipative mechanism given by spontaneous emission and subsequent reabsorption of photons. The first is necessary to obtain the required low temperatures but the second should be avoided because heating is introduced in the system. Several possible solutions both, dynamical and geometrical have been proposed for the reabsorption problem. All the geometrical proposals are based on the reduction of the dimensionality of the system, while dynamical ones rely almost entirely upon optical pumping mechanisms. In this paper we use a non-adiabatic approach in which we consider two IR fields with frequency $\omega_1$ and $\omega_2$ coupling the levels $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ respectively, as in Fig 1.

![FIG. 1: The three level atom in interaction with IR Radiation. $\omega_1, \omega_2$, are the coupling and probe field respectively.](image)

In this framework atoms in the ground state $|1\rangle$ are in the optically precooled reservoir before pumped to the MOT trap, where the atoms will be excited into the state $|3\rangle$ by using IR $w_1$ field. This corresponds to the thermal cloud or uncondensed phase. The state $|2\rangle$ corresponds to the trapped state that will be evaporative cooled e.g. previous to the condensed phase. The wave intensity and field detunings should be properly adjusted in order to the temporal windowing of the 1 $\leftrightarrow$ 3 transition during a time $t_d = 1/|\Delta \nu|$, with $\Delta \nu = \nu_1 - \nu_2$ and $\nu_1, \nu_2$ being the frequencies of fields 1 and 2 respectively. We are therefore quenching one transition ($1 \leftrightarrow 3$) during a time $t_d$ and allowing the other one ($2 \leftrightarrow 3$) with the appropriate value of $\Omega_{32}$.

In the method proposed the influence of reabsorption mechanism is then minimized by adjusting the dark state window at will in such a way that $\Gamma_{32} \ll \Omega_{32}$. $\Gamma_{32}$, $\Omega_{32}$ are the spontaneous decay rate from level 3, and Rabi flopping frequency associated to the probe field respectively. We point out that this is a different picture than considered in that of “Festina Lente” limit since in that method the comparison is done between the trap frequency $w$ and fluorescence rate $\gamma$.

Note that, with this mechanism we propose loading the thermal cloud into the magnetic trap directly and continuously, hence carrying out the the evaporative cooling in a steady state regime and thus a CW atom laser is obtained.

We are then left with the atomic pump from the uncondensed $|3\rangle$ level to the phase condensed state $|2\rangle$ described by the hamiltonian

$$H_i = g_2 \mathcal{V} (b_2 a_1 \phi_2 + b_2^\dagger a_2 \phi_2),$$

where $g_2$ is the IR coupling constant in the unit of volume defined as $g_2 = \mu \Omega \sqrt{\frac{2 \nu}{\gamma}}$, $b_i$, $b_i^\dagger$ are the annihilation (creation) operators for $\omega_i$ photons and $a_i$, $a_i^\dagger$ are the annihilation (creation) operators for atoms in the internal state $i$ defined in the usual matter field operator as $\psi(\vec{r}) = \sum_i \phi_i(\vec{r}) a_i$, where $\phi_i(\vec{r})$ are the spatial dependent particle wave function and $\mathcal{V}$ is the volume.
Following the procedure of Ref. [13], we calculate the IR coupling term with
\[ [\phi, H_i] = \Omega_2 V h |\phi_2|^2, \]  
where \( \Omega \rightarrow \Omega_2 = g_2 b_2 / h \) stands for the Rabi frequency, and \( V \) as the cavity volume. In the other hand, since \( \phi_2 \) is the condensed phase, we have dropped subindex 2 and substitute \( \phi_2 \rightarrow \phi \); from now on we replace \( \Omega_{32} \rightarrow \Omega_2 \rightarrow \Omega \).

The condensed atoms are described by the GPE
\[ ih \dot{\phi} = \nabla^2 \phi + V_t \phi + g |\phi|^2 \phi, \]  
where \( \phi \) is the condensed wave function, \( V_t \) is the harmonic trapping potential and \( g \) the interatomic interaction constant given by \( g = 4\pi \hbar^2 a / m \). The scattering length constant is \( a \) and \( m \) the atomic mass. We adopt the generic model for an atom laser described by Kneer et al in Ref. [16] and rewrite equation (1) with terms of sink and gain (pumping and outcoupling) plus the IR coupling giving by expression 2. We then couple the resulting equation with a rate equation as proposed by Speeew et al [17],
\[ ih \dot{\phi} = \nabla^2 \phi + V_t \phi + g |\phi|^2 \phi \]
\[ + \frac{i}{2} h \Gamma_n u \phi - \frac{i}{2} h \gamma_c \phi + \Omega V h |\phi|^2 \phi. \]  
\[ n_u = R(r) - \gamma_u n_u - \Gamma n_c n_u, \]  
(4)
Here we prefer to deal with a local coupling. \( \Gamma \) is the local \((m^3 s^{-1})\) rate constant coupling the condensed field \( \phi \) with the uncondensed density \( n_u \); and is related to the global rate used in Ref. [16] \( \Gamma = \Gamma_c \)\((s^{-1})\). \( \gamma_c \) is the escape rate of condensed atoms leaving the BEC (outcoupled atoms) and \( \gamma_u \) the escape rate for the uncondensed atoms (its inverse plays as the life time of the trap). \( n_c = |\phi|^2 \) is the local density of the condensed atoms. \( R(r) \) is the non uniform, locally depleted pumping process for the uncondensed atoms coupled to the condensed phase. Because of this local depletion, the uncondensed phase \( n_u \) has a sensible space dependence and undergoes a diffusion process [18].

Besides this, the influence of \( R \) upon BEC dynamics has been shown to be critical at high pumping rates, where three-body recombination suppress the high frequency dynamics which is an important factor that limit the atom laser stability [19].

Now we have to add a real space constant diffusion term \( D_r \nabla^2 n_u \) to equation (5) and after applying an adiabatic elimination procedure [20], find a quasi stationary solution for \( n_u \) in terms of \( \phi \) and replace it into equation (4), which then becomes a closed equation for \( \phi \). All this procedure is done on the scenario for which the \( \phi \) dynamics is much slower compared to \( n_u \) dynamics.

Following the procedure of Ref. [18], we have
\[ n_u = \frac{R}{\gamma_u} \left( 1 + \frac{D_r}{\gamma_u} \nabla^2 - \frac{\Gamma}{\gamma_u} |\phi|^2 \right). \]  
(6)
As we replace this expression in Eq.(4), the operator \( \nabla^2 \) acts on its right upon the space function \( \phi \). By doing this we derive a closed equation for \( \phi \):
\[ \dot{\phi} = \left[ \frac{\Gamma R D_r}{2 \gamma_u} + \frac{i h}{2 m} \nabla^2 \phi - \left[ \frac{\Gamma^2 R}{2 \gamma_u} + i (\Omega V + \frac{g}{\hbar}) \right] |\phi|^2 \phi \right] \]
\[ + \left[ \left( \frac{\Gamma R}{2 \gamma_u} - \gamma_c \right) - i \frac{V_t}{\hbar} \right] \phi. \]  
(7)
We now write the parameterized CGL Eq.(8)
\[ \dot{\phi} = \varepsilon \phi + (1 + ic_1) \nabla^2 \phi - (1 + ic_2) |\phi|^2 \phi, \]  
(8)
where
\[ \varepsilon = \frac{1}{2} \left( \frac{\Gamma R}{\gamma_u} - 1 \right), \]  
(9)
\[ c_1 = \frac{h^2}{m R D_r \Gamma}, \]  
(10)
\[ c_2 = \frac{2 a^2}{\hbar^2} \left( \frac{\Omega V + \frac{g}{\hbar}}{\gamma_u} \right). \]  
(11)

Eq. (7), is based on sound physical ground, is far from being a purely conservative (GP) or purely dissipative (real Ginzburg Landau) Equation. Instead it display both characters.

The term \(-ic_0\phi \) in Eq. (7) have been eliminated by the rotation transformation \( \phi \rightarrow e^{-\text{i}c_0 t} \phi \). The model have been made dimensionless using characteristic length and time scales: \( l_0 = (\hbar / \gamma_u) \)\(^{1/2} \approx 3 \times 10^{-6} m \). Thus, dimensionless space coordinates are now \( x/l_0, y/l_0, z/l_0 \). Furthermore \( \tau = \gamma_c t \) with \( \gamma_c \approx 50 s^{-1} \), and \( |\phi|^2 \rightarrow (\frac{\Gamma^2 R}{2 \gamma_u \gamma_c}) |\phi|^2 \).

We are now ready to write down, the Benjamin-Feir criterium [21] for stability of the open system
\[ c_1(-c_2) < 1. \]  
(12)
Note that \( c_1, c_2 \) displays explicit dependence on the pumping rates and Rabi frequency allowing us a direct control on the atom laser stability throughout application of Benjamin-Feir criterium given by expression (12)

In what follows we consider two cases:

a) The \( ^{87}Rb \) (Fig 2). We have the experimental reasonable values \( \gamma_u = 1500 s^{-1}, \Gamma = 7.02 \times 10^{-16} m^3 s^{-1}, \) \( R = 2.13 \times 10^{20} m^{-3} s^{-1}, D_r = 2 \times 10^{-8} m^2 s^{-1}, g/\hbar = 4.8 \times 10^{-17} m^3 s^{-1}, V = 2.5 \times 10^{-10} m^3. \) For this case it can be seen that for any IR Rabi coupling the \( ^{87}Rb \) Atom laser, is always stable and the only possible limit is due to the upper values permitted for the \( \Omega \).

b) \( ^{7}Li \) (Fig 3). We use now the next values: \( \gamma_u = 87 s^{-1}, \Gamma = 5.4 \times 10^{-17} m^3 s^{-1}, \)
FIG. 2: The stability condition for $^{87}$Rb against IR Rabbi frequency and pumping rate R.

$R = 1.61 \times 10^{20} m^{-3}s^{-1}$, $D_r = 2 \times 10^{-8} m^2s^{-1}$, $g/\hbar = -1.61 \times 10^{-16} m^3s^{-1}$, $V = 2.5 \times 10^{-16} m^3$.

Note that in this case there is some range of Rabi frequency for non stability, running for the values $0 < \Omega < 0.35$. In the other hand there is an interesting range of values for stability as displayed in the Fig. 2 from $0.35 < \Omega < 0.6$. Discussion about the upper limit of stability of the atom laser remains. It comes to our attention a very recent work by Haine et al. [22], where strong dependence on pumping rates are displayed for stability of $^{85}$Rb atoms. However in this numerical work there is no IR interaction. In conclusion we have shown the possibility of controlling the laser atoms stability in the framework of quasi-CW optical pumping and IR coupling, even for those cases of negative scattering length as $^7$Li. As a final comment, we point out the wide spectrum of theoretical possibilities for studies of atoms laser using the CGLE which seems to be a properly equation, for describing the dynamical of an open BEC.

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FIG. 4: The stability condition for lithium against IR Rabi frequency and pumping rate $R$. 

$$F. Rabi \cdot 10^2$$ 

$$R \cdot 10^2$$ 

$$C_0 \cdot C_1$$