A Method for Stable Deployment of an Electrodynamic Tethered Satellite in Three-Dimensional Space

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Abstract. The paper presents an asymptotic stabilization strategy for the deployment of a controlled tethered satellite system in three-dimensional space, in which the tether length rate is taken as the control variable. Firstly, a rigid-rod tether model is employed to establish the nonlinear dynamic equations of in-plane and out-of-plane motions of the system. Then, by stability analysis of the linearized system at a preassigned direction to deploy, the control law and asymptotic stability condition for the deployment are obtained. The electrodynamic tethered satellite in equatorial plane is discussed. As a result, the large swing motions during deployment are stabilized asymptotically through reliance on the electrodynamic force and the tether length rate. The case studies in the paper well demonstrate the proposed stabilization control strategy.

1. Introduction

The inherent Coriolis effect of tethered satellite systems makes their deployment/retrieval unstable [1]. And, during state-keeping phase, there have always been unstable periodic motions in the electrodynamic tethered satellite system [2-4]. A large number of intensive studies have been made for the deployment/retrieval process of tethered satellite systems. For example, Barkow et al [5] investigated the deployment of a tethered satellite system in a circular orbit by using the optimal control with the free- and force-braked deployment and the Kissel’s law, respectively. Jin and Hu [6] studied the deployment and the retrieval of a tethered sub-satellite of three degrees of freedom via nonlinear optimal control. Based on a 3-D model, Mantri et al [7] identified five dimensional parameters that affect the percentage of the total length to which the tethered satellite system to be deployed, including initial separation velocity, tether tension force, orbital height, effective mass, and final desired tether length. Kumar and Patel [8] developed control laws for multi-connected satellites aligned along local horizontal configuration. Liu et al [9] designed a variable structure control for the deployment/retrieval of tethered satellite system. With electrodynamic tethered satellite systems, Wen et al [10,11] presented two nonlinear optimal control schemes for the retrieval of a tethered satellite system in inclined orbit by adjusting electrodynamic force and tether tension. Williams [12] designed a time-delayed predictive control law for the librations of electrodynamic tether. Kojima et al [13] showed that a 4-periodic motion was successfully synchronized among electrodynamic tether systems. Zhong and Zhu [14] achieved an optimal balance between a fast deorbit and a libration stability of a short electrodynamic tethered nano-satellite with the minimum control efforts. Iki et al [15] represented an electrodynamic tether deployment from a spool-type reel using thrusters, in which key parameters are estimated on the
ground-based experiments. Based on the time-delayed auto-synchronization, Iñarrea et al [16] applied two feedback control methods in suppressing the unstable periodic motions of an electrodynamic tether.

Most of the published works are devoted to control of the unstable periodic motions caused by electrodynamic forces during state-keeping phase, and have paid less attention to the use of electrodynamic forces in the deployment process of tethered satellite systems. This paper utilizes the tether length rate to stabilize the deployment of tethered satellite systems with three dimensional attitude motion. In Section 2, the study here, begins with establishing a rigid rod model of a tethered satellite system in three-dimensional space, and then in Section 3, based on Lyapunov stability theory, gives the deployment control law and stability condition. The deployment process of electrodynamic tethered satellite in equatorial plane is discussed. Finally, in Section 4, case studies are carried out numerically.

2. Mechanics model
A tethered satellite in inclined circular orbit is shown in figure 1. The system consists of a mother satellite $M$ of mass $m_M$ and a sub-satellite $S$ of mass $m_S$. The two satellites are connected by a tether of deployed length $l$ that is viewed as a massless rigid rod. An inertial geocentric frame $O-XYZ$ is established such that the $X$-axis points the direction of ascending node from the center of the Earth $O$, the $Z$-axis is aligned with the Earth’s axis, and the $Y$-axis is determined by right-hand rule. An orbital frame $o-xyz$ is on the center of mass of the system, where the $y$-axis points the direction of motion, the $z$-axis is perpendicular to the orbital plane, and the $x$-axis is given by right-hand rule. In figure 1, $\delta$ represents the angle of inclination between the orbital plane and the equatorial plane. The in-plane pitch angle $\theta$ and out-of-plane roll angle $\phi$ are selected as the generalized coordinates, respectively.

![Figure 1. The simplified representation of tethered satellite system.](image)

Application of Lagrange’s equations leads to the following dynamic equations of the system [17]

$$\begin{align}
\ddot{m_l}(\phi^2 + 2(\theta + \nu)\dot{\phi}\cos\phi\sin\phi + 2\ddot{m_l}(\theta + \nu)\cos^2\phi + \frac{3\mu_E m_l^2}{R^3}\sin\theta\cos\theta\cos^2\phi = Q_\phi \\
\ddot{m_l}(\phi^2 + (\theta + \nu)^2 \sin\phi\cos\phi + 2\ddot{m_l}\phi' + \frac{3\mu_E m_l^2}{R^3}\sin\phi\cos\phi\cos^2\theta = Q_\phi
\end{align}$$

(1)

where $(\cdot)' = \frac{d}{dt}$ represents the derivative with respect to time $t$, $\nu$ the orbit true anomaly, $\mu_E$ the Earth’s gravitational constant, $R$ the radius of the running orbit, the non-dimensional parameter
\[ \dot{m} = \frac{m_b m_s}{(m_b + m_s)} \], \( Q_\theta \) and \( Q_\phi \) the generalized forces in the pitch and roll generalized coordinates. If they are produced by electrodynamic forces, one can get the following form

\[ Q_\theta = \frac{H^2 (m_b - m_s)}{2(m_b + m_s)} \cos \phi \left[ \sin \phi (B_x \cos \theta + B_y \sin \theta) - B_z \cos \phi \right] \tag{2} \]

and

\[ Q_\phi = -\frac{H^2 (m_b - m_s)}{2(m_b + m_s)} (B_x \cos \theta - B_y \sin \theta) \tag{3} \]

where \( I \) is the current in tether. It is set as positive if the current flows to the sub-satellite from the mother satellite. Based on a non-tilted dipole model, the Earth magnetic field established in the orbital frame \( o-xyz \) can be expressed as

\[
\mathbf{B} = \begin{bmatrix}
B_x \\
B_y \\
B_z
\end{bmatrix} = \begin{bmatrix}
-2(\mu_m/R^3) \sin \nu \sin \delta \\
(\mu_m/R^3) \cos \nu \sin \delta \\
(\mu_m/R^3) \cos \delta
\end{bmatrix} \tag{4}
\]

where \( \mu_m \) is the magnetic moment of the Earth’s dipole. Using the non-dimensional transformations

\[ \frac{d}{dt} = \frac{d}{dt} \cdot \frac{d \nu}{d \xi} \quad \xi = \frac{l}{l_{\text{max}}} \tag{5} \]

the original dynamic equations of the system become

\[
\begin{cases}
\ddot{\theta} + 2(\dot{\theta} + 1) \left( \frac{\ddot{l}}{\xi} - \phi \tan \phi \right) + 3 \sin \theta \cos \theta = \frac{Q_\theta}{m_c^2 l_{\text{max}}^2 \omega^2 \cos^2 \phi} \\
\ddot{\phi} + 2 \phi \left( \frac{\ddot{l}}{\xi} \right) + [(\dot{\theta} + 1)^2 + 3 \cos^2 \theta] \sin \phi \cos \phi = \frac{Q_\phi}{m_c^2 l_{\text{max}}^2 \omega^2} \tag{6}
\end{cases}
\]

in which the dot represents the derivative with respect to \( \nu \), \( \xi \) denotes non-dimensional tether length, \( l_{\text{max}} \) the maximum value of the deployed tether length, \( \omega = \sqrt{\mu_E/R^3} \) orbit angular velocity around the Earth. The simplified non-dimensional form of equation (6) is

\[
\begin{cases}
\ddot{\theta} + 2(\dot{\theta} + 1) \left( \frac{\ddot{l}}{\xi} - \phi \tan \phi \right) + 3 \sin \theta \cos \theta = H_\theta \\
\ddot{\phi} + 2 \phi \left( \frac{\ddot{l}}{\xi} \right) + [(\dot{\theta} + 1)^2 + 3 \cos^2 \theta] \sin \phi \cos \phi = H_\phi \tag{7}
\end{cases}
\]

where

\[
H_\theta = \frac{IR^3 (m_b - m_s)}{2 \mu_m m_b m_s} \left[ \sin \phi (B_x \cos \theta + B_y \sin \theta) - B_z \cos \phi \right] \tag{8}
\]

and

\[
H_\phi = -\frac{IR^3 (m_b - m_s)}{2 \mu_m m_b m_s} (B_x \cos \theta - B_y \sin \theta) \tag{9}
\]

Note that, \( H_\theta \) would vary with the current \( I \) only, say,

\[
H_\theta = -\frac{\mu_m l (m_b - m_s)}{2 \mu_m m_b m_s} \tag{10}
\]

and

\[
H_\phi = 0 \tag{11}
\]

if the system moves in an equatorial plane. Here, other parameters of the system keep unchanged. It is the motivation for utilizing the electrodynamic force to control the deployment.
3. Stabilization strategy

Clearly, the electrodynamic tethered satellite is a nonautonomous system. Therefore, Lyapunov stability theory is not suitable to the electrodynamic tethered satellite system. However, when the system moves in an equatorial plane, the expression of $H_\phi$ and $H_\theta$ can be rewritten as equation (10) and (11). Now, the electrodynamic tethered satellite system is an autonomous one. The linearization of equation (7) gives

$$
\begin{align*}
\ddot{\theta} + 2(\dot{\theta} + 1)(\frac{\dot{\theta}}{\xi} - \phi \dot{\phi}) + 3\theta &= H_\theta \\
\ddot{\phi} + 2\phi(\frac{\dot{\phi}}{\xi}) + [(\theta + 1)^2 + 3]\phi &= H_\phi
\end{align*}
$$

By setting $\varphi_1 = \theta$, $\varphi_2 = \dot{\theta}$, $\varphi_3 = \phi$, and $\varphi_4 = \dot{\phi}$, equation (12) can be recast as a set of state equations

$$
\begin{align*}
\dot{\varphi}_1 &= \varphi_2 \\
\dot{\varphi}_2 &= H_\theta - 2(\varphi_2 + 1)(\frac{\dot{\varphi}_2}{\xi} - \varphi_3 \varphi_4) - 3\varphi_1 \\
\dot{\varphi}_3 &= \varphi_4 \\
\dot{\varphi}_4 &= H_\phi - 2\varphi_4(\frac{\dot{\varphi}_4}{\xi}) - [(\varphi_2 + 1)^2 + 3]\varphi_3
\end{align*}
$$

It is easy to get the equilibrium positions

$$
\begin{align*}
\varphi_{10} &= \frac{\xi H_\theta - 2\xi}{3\xi} \\
\varphi_{20} &= 0 \\
\varphi_{30} &= \frac{-H_\phi}{4} \\
\varphi_{40} &= 0
\end{align*}
$$

It is evident that the equilibrium position of out-of-plane roll angle always equals to 0. On the other hand, for a deployment with a specified direction, say, $\varphi_{10} = \theta^*$, we have

$$
\dot{\xi} = \frac{\xi H_\theta - 3\xi \theta^*}{2}
$$

According to the tether length rate, the specified direction would be maintained over the deployment process. Next, let’s analyze the stability of the system at the specified direction. The Jacobian matrix of equation (13) is

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-3 & -2(\frac{\dot{\theta}}{\xi} - \varphi_3 \varphi_4) & 2(\varphi_2 + 1)\varphi_4 & 2(\varphi_2 + 1)\varphi_3 \\
0 & 0 & 0 & 1 \\
0 & -2(\varphi_2 + 1)\varphi_3 & -[(\varphi_2 + 1)^2 + 3] & -2(\frac{\dot{\phi}}{\xi})
\end{bmatrix}
$$

After Substituting equation (14) of equilibrium positions into equation (16), four eigenvalues of which are

$$
\lambda_{1,2} = -\frac{\xi}{\xi} \pm \left[\left(-\frac{\xi}{\xi}\right)^2 - 3\right]^{1/2} \\
\lambda_{3,4} = -\frac{\xi}{\xi} \pm \left[\left(-\frac{\xi}{\xi}\right)^2 - 4\right]^{1/2}
$$
Obviously, if
\[ \frac{\dot{\xi}}{\xi} < 0 \] (18)
the system becomes asymptotically stable on the basis of Lyapunov stability theory. Substitution of equation (15) into equation (18) gives
\[ \theta_e < \frac{H_0}{3} \] (19)
The same result can be obtained by substituting equation (15) into the deployment condition, i.e., \( \dot{\xi} > 0 \). Note that, if \( H_0 = 0 \) for \( I = 0 \), the magnitude of \( \theta_e \) must be less than zero. With the help of the electrodynamic force, hence, the linearized system in an equatorial plane of equation (12) can be deployed along a specified direction for \( \theta_e < 0 \) or \( \theta_e > 0 \).

In addition, the uniform deployment with an expected tether length rate \( \dot{\xi}_e \) can also be achieved through the following form
\[ I(\xi) = \frac{2\mu_t m_\eta m_\chi [2\dot{\xi} + 3\xi \dot{\theta_e}]}{\mu_\chi m_\sigma - m_\eta} \] (20)
which is obtained by substituting equation (10) into equation (15). It is easy to find that tether deployment rate of equation (15) always satisfies equation (19) even if current \( I \) change continuously.

4. Case studies
In order to verify the stabilization strategies, a set of parameters was taken as follows. The masses of mother satellite and sub-satellite were \( m_M = 500 \) kg and \( m_\eta = 20 \) kg, respectively. The system moved in an equatorial plane. The current strength was taken as \( I = -1 \) A. The initial true anomaly \( \nu_0 = 0 \). The initial non-dimensional tether length was set as \( \xi_0 = 0.05 \). The initial in-plane pitch angle and its angle velocity were \( \theta_0 = -0.02 \) rad and \( \dot{\theta}_0 = 0.05 \) rad/s, respectively. The initial out-of-plane roll angle and its angle velocity were \( \phi_0 = -0.1 \) rad and \( \dot{\phi}_0 = 0.05 \) rad/s, respectively. Based on above the initial system parameters and Runge-Kutta integration method, the following numerical simulation is carried out by the Matlab software.

According to equation (19), the expected pitch angle reads \( \theta_e < 0.16 \) rad such that \( \theta_e = 0.05 \) rad. In term of the tether length rate in equation (15), the stable deployment can be achieved within \( \nu = 18 \) rad, as shown in figure 2. From figure 2(a) and 2(b), the time histories of the pitch and roll motions in true anomaly, one can see that the amplitudes of the motions decrease gradually within \( \nu = 18 \) rad, and approach to \( \theta = 0.05 \) rad and \( \phi = 0 \) in the end. figure 2(c) shows the trajectory of the sub-satellite during deployment in a non-dimensional orbital plane frame \( o - \chi \eta \), where the \( \chi \)-axis points the opposite direction of motion, \( \eta \)-axis points the direction of the mass center of the system from the center of the Earth \( O \). The non-dimensional tether length versus the true anomaly is depicted in figure 2(d).
Next, setting the expected pitch angle $\theta_e = 0.05\text{rad}$, an uniform deployment at $\xi = 0.01$ is shown in figure 3. Based on the current change in equation (20), the duration of the deployment for $\xi = 0.95$ lasted for $\nu = 95\text{rad}$, wherein the absolute value of the current in tether decreases gradually from $-1.15\text{A}$ to $-0.35\text{A}$ with the increase of the true anomaly.

Figure 2. Stabilization strategy with constant current.
5. Conclusions
In three-dimensional space, the asymptotic stabilization of deployment of a tethered satellite system along a preassigned direction can be achieved via tether length rates. In equatorial plane, the tether can be deployed along the desired pitch angle through reliance on electrodynamic forces in conjunction with tether length rates. In this situation, the deployment control with a constant tether length rate can also be performed by electrodynamic forces. Next, the proposed method would be extended to elliptical orbits in which tethered satellite systems can be also asymptotically stable deployed.

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References
[1] Barkow B, Steindl A and Troger H 2005 A targeting strategy for the deployment of a tethered satellite system *IMA Journal of Applied Mathematics* 70 626-644
[2] Peláez J and Andrés Y N 2005 Dynamic stability of electrodynamic tethers in inclined elliptical orbits *Journal of Guidance, Control, and Dynamics* 28 611-622
[3] Zanutto D, Curreli D and Lorenzini E C 2011 Stability of electrodynamic tethers in a three-body system, *Journal of Guidance, Control, and Dynamics* 34 1441-1456
[4] Sánchez-Arriaga G, Bombardelli C and Chen X 2015 Impact of nonideal effects on bare electrodynamic tether performance *Journal of Propulsion and Power* 31 951-955
[5] Barkow B, Steindl A, Troger H and Wiedermann G 2003 Various methods of controlling the deployment of a tethered satellite *Journal of Vibration and Control* 9 187-208
[6] Jin D P and Hu H Y 2006 Optimal control of a tethered subsatellite of three degrees of freedom *Nonlinear Dynamics* 46 161-178
[7] Mantri P, Mazzoleni A P and Padgett D A 2007 Parametric study of deployment of tethered satellite systems *Journal of Spacecraft and Rockets* 44 412-424
[8] Kumar K D and Patel T R 2009 Dynamics and control of multi-connected satellites aligned along local horizontal *Acta Mechanica* 204 175-191
[9] Liu Y Y, Zhou J and Chen H L 2012 Variable structure control for tethered satellite fast deployment and retrieval *Future Control and Automation* 172 157-164
[10] Wen H, Jin D P and Hu H Y 2008 Retrieval control of an electro-dynamic tethered satellite in an inclined orbit *Chinese Journal of Theoretical and Applied Mechanics* 40 375-380
[11] Wen H, Jin D P and Hu H Y 2008 Infinite-horizon control for retrieving a tethered subsatellite via an elastic tether *Journal of Guidance, Control, and Dynamics* **31** 899-906

[12] Williams P 2009 Libration control of electrodynamic tethers using predictive control with time-delayed feedback *Journal of Guidance, Control, and Dynamics* **32** 1254-1268

[13] Kojima H, Iwashima H and Trivailo P M 2011 Libration synchronization control of clustered electrodynamic tether system using Kuramoto model *Journal of Guidance, Control, and Dynamics* **34** 706-718

[14] Zhong R and Zhu Z H 2013 Long term dynamics and optimal control of nano-satellite deorbit using a short electrodynamic tether *Advances in Space Research* **52** 1530-1544

[15] Iki K, Kawamoto S and Morino Y 2014 Experiments and numerical simulations of an electrodynamic tether deployment from a spool-type reel using thrusters *Acta Astronautica* **94** 318-327

[16] Iñarrea M, Lanchares V, Pascual A I and Salas J P 2014 Attitude stabilization of electrodynamic tethers in elliptic orbits by time-delay feedback control *Acta Astronautica* **96** 280-295

[17] Williams P 2005 Optimal orbital transfer with electrodynamic tether *Journal of Guidance, Control, and Dynamics* **28** 369-372