New phases of QCD;
the tricritical point;
and RHIC as a “nutcracker”

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Abstract

It is a talk at 15-th Winter Workshop on Nuclear Dynamics, Parkcity Utah, Jan.1999. Because too many interesting things are going on now, I have tried to squeeze three different subjects into one talk. The first is a brief summary of the color super-conductivity. During the last year we learned that instanton-induced forces can not only break chiral symmetry in the QCD vacuum, but also create correlated scalar diquarks and form new phases, some similar to the Higgs phase of the Standard model. The second issue I discuss is the remnant of the so called tricritical point, which in QCD with physical masses is the endpoint of the first order transition. I will argue that exchange of sigmas (which are massless at this point even with quark masses included) create interesting event-by-event fluctuations, which can be used to locate it. Finally I describe first results in flow calculations for non-central collisions at RHIC. It was found that it is extremely sensitive to Equation of State (EOS). Furthermore, the unusual “nutcracker” picture emerges for lattice-motivated EOS, which is formation of two shells which are physically separated before the freeze-out.

1 QCD at high density

New phases of QCD at high density and the color super-conductivity issue are a part of a broader context, the studies of how the confining and chirally asymmetric QCD ground state is substituted by other phases as the temperature, the chemical potential, the number of flavors (or any combination of those) are increased. The key player in most of those effects (except confinement) happen to be instantons, see recent review [1]. In the QCD vacuum, for example,
the quark condensate is simply the density of (almost) zero modes, originating from a superposition of zero modes associated with isolated instantons and anti-instantons.

At high temperature we expect to find the quark-gluon plasma (QGP) phase in which chiral symmetry is restored. So the density of (almost) zero modes goes to zero. This can only be realized if the instanton ensemble changes from a nearly random one to a correlated system with finite clusters, e.g. instanton-anti-instanton ($\Pi\Pi$) molecules. The same is expected to happen (even at $T=0$) for sufficiently large number of flavors: in this case the expected next phase is the so called \emph{conformal} phase.

The QCD at finite baryon density lay dormant since 70’s, when basic applications of QCD like Debye screening were made. It was revived recently when it was realized that not only we expect the high density phase of QCD to be a color superconductor, as proposed in \cite{5, 6, 7} with gaps in the MeV range, but that the instanton-induced effects lead to much larger gaps on the order of 100 MeV \cite{2, 3}.

In the next year it was realized that the phase structure of QCD at finite baryon density is actually very rich. In addition to the dominant order parameter, which is a scalar-isoscalar color anti-triplet ud diquark, many other condensates form. The overall picture can be characterized by some kind of “triality”, both of three major phases under consideration, as well as of three competing attractive channels. These basic phases are: (i) the \emph{hadronic} (H) phase, with (strongly) broken chiral symmetry (ii) the \emph{color superconductor} (CSC) phase, with broken color symmetry; and (iii) the quark-gluon plasma (QGP) phase, in which there are no condensates but the instanton ensemble is non-random.

The three basic channels are the instanton-mediated attraction in (i) $\bar{q}q$ and (ii) $qq$ channels (responsible for H and CSC phases) and the (quark-mediated) attraction between $\Pi\Pi$, confining the topological charge in the QGP phase. The interrelation between these three attractive channels and phases is not straightforward: e.g. the $<\bar{q}q>$ may or may not be present in the CSC phase, and $\Pi\Pi$ molecules have non-zero presence everywhere. However, this paper is still basically about a competition between these three attractive forces in different conditions.

The overview of the situation on the phase diagram is given by Fig.1, where one can see an approximate location of color superconducting phases, as well as few schematic trajectories of excited matter, as it expands and cools in heavy ion collisions. One may see from those that unfortunately this new phase region corresponds to rather cool matter, and so it is \emph{not} crossed by them. Therefore, color super-conductivity should only exists in compact stars. This created a challenge, known as the “pulsar cooling problem”: a naked Fermi sphere is not allowed, because it generates too rapid cooling rate in contradiction to data.

The scenario depends crucially on the number of quark flavors $N_f$. We start with discussion of (i) $N_f = 2$ massless quarks, u,d; (ii) then move on to $N_f = 3$ massless quarks, and finally to (iii) real QCD with non-zero quark masses.
Figure 1: Schematic phase diagram of QCD phases as a function of temperature \( T \) and baryonic chemical potential \( \mu \), as we understand it today. The phases denoted by H and QGP are the usual hadronic phases (with nonzero \( <\bar{q}q> \) and the quark-gluon plasma (no condensates). The color super-conducting phases CSC1 and CSC2 have various \( <qq> \) condensates, the latter has broken chiral symmetry and tends asymptotically to color-flavor locking scenario. KC and QDQ are two possible phases, with Kaon condensate or quark-diquark gas. E is the endpoint of the 1-st order transition, M (from multi-fragmentation) is the endpoint of another 1-st order transition, between liquid and gas phases of nuclear matter. Two schematic trajectories corresponding to adiabatic expansion in heavy ion collisions are also indicated.

In the \( N_f = 2 \) case the instanton-induced interaction is 4-fermion one. Its role in breaking chiral symmetry and making pion light and \( \eta' \) heavy is well documented, see e.g. [1]. One can Fierz transform it to diquark channels, which contain both color antisymmetric 3 and symmetric 6 terms. The scalar and the tensor are attractive:
\[
\mathcal{L}_{\text{diqu}} = \frac{g}{8N_c^2} \left\{ -\frac{1}{N_c-1} \left[ \left( \psi^T F^T C \tau_2 \lambda_A^a \lambda_B^b F \psi \right) (\bar{\psi} F^\dagger \tau_2 \lambda_A^a C F^* \bar{\psi}^T) \\
+ \left( \psi^T F^T C \tau_2 \lambda_A^a \gamma_5 F \psi \right) (\bar{\psi} F^\dagger \tau_2 \lambda_A^a \gamma_5 C F^* \bar{\psi}^T) \right] \\
+ \frac{1}{2(N_c+1)} \left( \psi^T F^T C \tau_2 \lambda_S^a \sigma_{\mu\nu} F \psi \right) (\bar{\psi} F^\dagger \tau_2 \lambda_S^a \sigma_{\mu\nu} C F^* \bar{\psi}^T) \right\}
\]

(1)

where \( \tau_2 \) is the anti-symmetric Pauli matrix, \( \lambda_{A,S} \) are the anti-symmetric (color 3) and symmetric (color 6) color generators (normalized in an unconventional way, \( tr(\lambda^a \lambda^b) = N_c \delta^{ab} \), in order to facilitate the comparison between mesons and diquarks). As discussed in ref.[2], in the case of two colors there is the so called Pauli-Gürsey symmetry which mixes quarks with anti-quarks. So diquarks (baryons of this theory) are degenerate with the corresponding mesons. It manifests itself in the Lagrangians given above: in this case the coupling constants in \( \bar{q}q \) and \( qq \) channels are the same and the scalar diquarks, like pions, have the mass vanishing in the chiral limit.

Standard BCS-type mean field treatment leads to gap equation, from which one extract all properties of the color superconductor. Let me omit details and only mention the bottom line. The chiral symmetry is restored, while color SU(3) is broken to SU(2) by the colored condensate.

**In the** \( N_f = 3 \) **case** the situation becomes more interesting. (Since the critical chemical potential \( \mu_c \approx 300 - 350 MeV \) is larger than the strange quark mass \( m_s \approx 140 MeV \), strange quarks definitely have to be included.) There are several qualitatively new features. First, since \( N_f = N_c \), there are new order parameters in which the color and flavor orientation of the condensate is locked [8]. Second, the instanton induced interaction is a four-fermion vertex, so it does not directly lead to the BCS instability, unless there is also a \( \langle \bar{q}q \rangle \) condensate as well. So we need a superconductor where chiral symmetry is still broken. This is indeed what we have found [4], after a rather tedious calculation.

**In the** \( N_f = 3 \) **case with variable strange quark mass** the algebraic difficulties increase further. There are dozens of \( q\bar{q} \) and \( \bar{q}q \) condensates present, all competing for the resources. The largest, ud one, is still in the 100 MeV range, but the smallest are just few MeV, or comparable with light quark masses. Still, those small condensates are enough to solve the “pulsar cooling problem” (while without strangeness it remained unsolved).

We have found that two cases discussed above are in fact separated by a first order transition line, as a function of density or \( m_s \). Partially this is caused by simple kinematic-al mismatch between \( p_F(u,d) \) and \( p_F(s) \) preventing their pairing, if \( m_s \) is large enough.

Finally let me mention that a transition region between nuclear matter and CSC (see Fig.1) was claimed before by such exotic phase as Kaon condensation. In [4] we propose another (also exotic) quark-diquark (QDQ in Fig.1) phase, in which nucleons dissociate into Fermi gas of constituent quarks plus Bose gas of constituent ud diquarks.
2 Event-by-event fluctuations and possible signatures of the tricritical point

We now discuss the part of the phase diagram shown in Fig.1 for densities below those for color super-conductivity. At high T and zero density it is believed to be second order if quark masses and strangeness is ignored, and a simple crossover otherwise. The discussion of the previous section (and many models e.g. the random matrix one [9]) suggest that it is likely to turn first order at some critical density. This means that there should be a tri-critical point in the phase diagram with $N_f = 2$ massless quarks, or the Ising-type endpoint E if quarks are not massless. The proposal to search for it experimentally was recently made by Stephanov, Rajagopal and myself [10]. A detailed paper about event-by-event fluctuations around this point [11] is the basis of this section.

The main idea is of course based on the existence of truly massless mode at this point, the sigma field, which is responsible for “critical opalescence” and large fluctuations. The search itself should be partially similar to “multi-fragmentation” phenomenon in low energy heavy ion collisions, which is also due to the endpoint M (see Fig.1) of another first order transition. The “smoking gun” is supposed to be a non-monotonous behavior of observable as a function of such control parameters as collision energy and centrality. One can use pions as a “thermometer” to measure this fluctuations.

(Note that in many ways it is the opposite of the DCC idea: in that case the pion was the light fluctuating field, while its coupling to heavy and wide and strongly damping sigma field is the main obstacle.)

We have studied three ways in which sigmas can show up. First and the simplest is the “thermal contact” idea: at the critical point the sigmas specific heat becomes large, and this shows up in the pion fluctuations just due to energy conservation. The second is “dynamical exchange”: pions can exchange the sigmas and this leads to long-range effects, over the whole correlation range. Both effects are in 10-20 percent range, after realistic account for correlation length is made. It is not large, but much larger than the accuracy of the measurements. The third effect is due to sigma decays into pions, which affect spectra at small $p_t$ and (even more so) the multiplicity fluctuations.

Large acceptance detectors can study the event-by-event (ebe) fluctuations quite easily. The first data by NA49 detector at CERN on distributions of $N$, the charged pion multiplicity, and $p_T$ (the mean transverse momentum of the charged pions in an event) for central PbPb collisions at 160 AGeV display beautiful Gaussians. Since any system in thermodynamic equilibrium exhibits Gaussian fluctuations, it is natural to ask how much of the observed fluctuations are thermodynamic in origin [13, 14]. We have answered this question quantitatively in this paper, considering fluctuations in pion number, mean $p_t$ and their correlation. We model the matter at freeze-out as an ideal gas of pions and resonances in thermal equilibrium, and make quantitative estimates of the
thermodynamic fluctuations in the resulting pions, many of which come from the decay of the resonances after freeze-out. The conclusion is that nearly all answers are reproduced by the resonance gas, with remaining part likely to be due to experimental corrections, due to two-particle track resolution and non-pion admixture. The good agreement between the non-critical thermodynamic fluctuations we analyze in Section 3 and NA49 data make it unlikely that central PbPb collisions at 160 AGeV freeze out near the critical point.

Estimates suggest that the critical point is located at a $\mu_f$ such that it will be found at an energy between 160 AGeV and AGS energies. This makes it a prime target for detailed study at the CERN SPS by comparing data taken at 40 AGeV, 160 AGeV, and in between. We are more confident in our ability to describe the properties of the critical point and thus how to find it than we are in our ability to predict where it is. If it is located at such a low $\mu$ that the maximum SPS energy is insufficient to reach it, it would then be in a regime accessible to study by the RHIC experiments.

3 RHIC as a nutcracker

This section is a brief account of unusual pattern of space-time evolution, found for non-central collisions at RHIC energies by D.Teaney and myself [15].

Let me begin with a pedagogic consideration of two opposite schematic models of high energy heavy ion collisions, leading to quite different conclusions about even such global thing as duration of the collision till freeze-out. This will set a stage for more elaborate considerations later, based on hydrodynamic approach.

The “model A” is just a picture of longitudinal expansion without any transverse one, except maybe very late in the process. (One may therefore call it a “late acceleration model”. ) For rapidity-independent (Bjorken) expansion the dynamics is very simple: each volume element expands linearly in proper time $\tau$. If the total entropy $S$ is conserved, its density $s \sim 1/\tau$. The initial value of entropy density at RHIC $s_i^{RHIC}$ is of course unknown, but it is believed to be several times that at SPS, say $s_i^{RHIC} = (2-4) s_i^{SPS}$. The final values should be roughly the same (At one hand, the larger the system the more should it cool down. On the other hand, at RHIC the fraction of baryons is expected to be significantly lower, and it should reduce re-scattering). Therefore, this model predicts total duration of expansion $\tau^{RHIC} = (2-4) \tau^{SPS} \sim 40 \text{fm}/c$.

The “model B” includes the transverse expansion, but in the opposite manner: the observed radial flow velocity at freeze-out $v_f \approx .4$ (the value we expect to see at RHIC) is now assumed to be there all the time. By contrast to model A, it assumes an “early acceleration”. Including simple geometric expansion in the decrease of the entropy density $s \sim 1/(\tau (r_0 + v_f \tau)^2)$ one finds then much shorter duration of the collision $\tau^{RHIC} \sim 10 \text{fm}/c$ predicted by model B.

Which model is closer to reality depends on the real acceleration history,
which is in turn determined by the interplay of the collision geometry, the energy
and the EOS. Qualitatively speaking, the main message of the previous section
is that while at AGS/SPS energy domain the collective flow appears late, like in
model A, at RHIC/LHC it is expected to be generated early, as in the model B.
The reason for that is specific behavior of the QCD EOS, which is soft in the
“mixed phase” region of energy density, very soft in QGP at $T \approx T_c$, and then
rapidly becoming hard at $T \approx 2 - 3T_c$.

One consequence is that duration of the collision grows with energy in
the AGS domain, but expected to decrease from SPS to RHIC. The maximal
is expected to be when the initial conditions hit the “softest point” of the EOS,
roughly at beam energy 30-40 GeV*A. Somewhat counter-intuitive, around the
same energy one expects also a maximum value of the radial flow: longer accel-
eration time seem to win over softness! Especially interesting is the dynamics
of the “elliptic” flow in the SPS-low RHIC energy domain. It is quite possible
that its energy dependence would be sufficient to see the onset of QGP plasma.

Of course, the magnitude of the flow depends not only on hydro-EOS but
also on kinetics of the freeze-out itself. We have already mentioned two factors
which enter into consideration here: the absolute size of the system (hydro itself
is scale-invariant!) and the baryon/meson ratio. Only careful systematic study
of various systems at various energies will clarify the actual role of all these
effects.

The magnitude of collective flow and its acceleration history can be un-
derstood as follows. We expect to have rapid change of pressure to energy density
ratio in QGP around the phase transition. Higher density QGP has $p/\epsilon \approx 1/3$,
but at the transition there is the minimum of this ratio (the so called “softest
point” [8]) where $p/\epsilon$ is small (0.1-0.05). So at AGS/SPS energies the ex-
ansion is slow and QGP just “burns inward”. At some point, the outward
expansion of the QGP and the inward burning may cancel each other, leading
to near-stationary “burning log” picture [7]. At higher collision energies, the
burning discontinuity is blown out and the situation returns to much simpler
hydro picture typical for simple EOS $p = \epsilon/3$.

We have recently found that the lattice-inspired EOS leads to very unusual
picture of the expansion, with quite characteristic inhomogeneous matter dis-
tribution (to be referred below as a *nutshells*). Stiff QGP at the center pushes
against soft matter in the transition region: as a result some piling of matter
occurs, in a shell-like structure. Furthermore, for non-central collisions the ge-
ometry drives expansion more to the direction of impact parameter (called x
axis) rather than y, starting rather early. As a result, the two half-shells separate
by freeze-out, and so (at least) two separate fireballs are actually produced.
(We called this scenario a *nutcracker*.) Nothing like this happens for simple
EOS, which always lead to matter distribution with a maximum at the center.

There is not much place here to display this interesting phenomenon. In
Fig.4 we show a typical mater distribution. The time 11 fm/c is around (or
slightly before) the freeze-out for most matter (it is not changed much anyway,
the longitudinal expansion simply dilute it more). One can clearly see two shells in x direction and holes in y ones.

How can such phenomenon be seen experimentally?
(i) We have calculated several harmonics (in angle $\phi$) of flow, $v_n$. We found quite observable deviations from a directed+elliptic (n=1 and 2 only) distributions seen before up to n=6.
(ii) The distribution of pions can be sufficiently accurate to see it, but with nucleons and, even better, heavier particles like deuteron-s we find much stronger signals for “nutcracker” scenario in production/flow patterns.
(iii) Another dramatic changes are found if one calculates correlators used by two-particle interferometry (HBT). Strong flow plus inhomogeneous distribution make visible HBT radii to be significantly smaller to what one might naively expect: we see only smaller “patches” of the picture in any given direction.

(But by the way, it significantly reduces conditions for momentum resolution of the detectors.). But for the same reason taking these patches all together, into a unified picture, is becoming more complicated.

Finally, let me emphasize it once again: the expected “nutcracker” pattern is supposed to be seen in typical non-central events. Because most of the RHIC detectors are able to detect the impact parameter plane in most events, there is no doubt that absolutely any phenomenon, from particle single-body distribution to $J/\psi$, $\Upsilon$ suppression or “jet-quenching” would be found strongly $\phi$ dependent, if it takes place. We will see exciting results on that, right from the first day of RHIC operation.
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Figure 2: Typical matter distribution for AuAu collision at $b=8\text{ fm}$, at time $t=11\text{ fm/c}$, in the transverse plane $x-y$, resulting from hydro calculation with lattice-inspired EOS. Transverse expansion is assumed to be rapidity-independent, while the longitudinal expansion is Bjorken-like. The final multiplicity assumed is $dN_{ch}/dy = 850$. Lines show levels with fixed energy density, with step $??\text{ GeV/fm}^3$. The dotted contours are for $T_f = 120\text{ MeV}$ (the outer one), and $T=140\text{ MeV}$ (two inner ones).