Theoretical studies of atmospheric perturbations related to inhomogeneity of field of force of gravity

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Abstract. Some physical mechanisms of perturbations of the wind field due to the inhomogeneities of gravity field have been established. The simplest analytical models of stationary linear atmospheric disturbances caused by inhomogeneities in the field of gravity are analyzed. Estimates show that the influence of gravitational force anomalies (GFAs) on the field of motion can be noticeable in some situations.

1. Introduction

In advanced models of geophysical fluid dynamics, the gravitational field is usually taken uniform and defined by the only parameter $g$. It is known, however, that the average gravitational force at the earth’s surface is superimposed upon by a broad spectrum of gravitational force anomalies (GFAs). This is due mainly to inhomogeneities of mass distribution in the earth’s crust. Variations in the gravitational force are certainly very small in magnitude compared to the average value of $g$. It is important, however, that such inhomogeneities generate a gravitational-force component tangential to the earth’s ellipsoid [1]. In mesoscale models using Cartesian coordinates (an $f$-plane or a $\beta$-plane), this means that additional volume inhomogeneous forces with a horizontal component have to be taken into account. The dynamics of the atmosphere is quite sensitive to such components. In the high-anomaly regions, the tangential (horizontal) components of the gravitational force on scales of about 100 km may be greater than 100 mGal ($10^{-3} \text{ m/s}^2$) [1]. They are therefore comparable in order of magnitude in dynamics equations to midlatitude-cyclone pressure gradient forces and to other terms, the consideration of which is beyond doubt.

At least since the 70s in the meteorological literature, it has been suggested repeatedly that gravitational-field inhomogeneities (GFIs) may significantly affect the dynamics of some atmospheric processes (see, for example, [1-3]). In particular, according to [4-6], some full-scale data indicate the possible impact of ACT on the activity of tropical cyclones. There is also some experience in numerical modeling [7-9]; in these studies, conclusions were drawn about the possibility of significant atmospheric effects of the GFIs. However, rigorous and transparent analytical models are required for a clear understanding of the physical mechanisms by which GFIs influence the dynamics of the atmosphere.

There is a theorem stating that isobars and isopycnic surfaces in a medium at rest must coincide with the equipotential surfaces [1]. If the isobars coincide with equal-potential surfaces, this means...
that the pressure-gradient force at every point and in every direction compensates the gravitational force. Therefore, a steady-state solution exists. For this reason, it is commonly believed that inhomogeneities of the gravitational field only somewhat deform or distort the hydrostatic equilibrium, but have no influence on the motion field.

However, authors' attention was paid to the existence of physical mechanisms that could disturb this static equilibrium and influence on atmospheric flows, and also lead to the generation of internal gravitational waves. These mechanisms are briefly discussed in this report.

2. Geostrophic Flow Disturbances Generated by Inhomogeneities of the Gravitational Field

First of all, the situation will change significantly in the presence of the background horizontal flows. As long as there are no gravitational-field inhomogeneities and isopycnic surfaces are strictly horizontal, such flows, moving tangentially to the isopycnic surfaces, do not break up the hydrostatic equilibrium. When the background flow encounters the gravitational-force anomaly, it, by inertia, obviously tends to retain the rectilinear motion along the horizontal axis \( x \). The deformed isopycnic surfaces related to the gravitational-field inhomogeneities are intersected by horizontal flows, so that the mass advection appears, to which the nonzero terms of the type \( u \partial \rho / \partial x \) in the transfer equations correspond (here, \( u \) is the velocity along the horizontal axis \( x \), \( \rho \) is the density of a medium). Thus, the background horizontal flows in this case should interact with the hydrostatic equilibrium of a medium – break up this state (deform isopycnic surfaces and isobars); adjust to this state (i.e., be curved); or both, in the general case. There is another factor resulting in the violation of the hydrostatic state. If the nonpermeability condition is specified at the underlying surface, the flow near this surface cannot evidently have the component normal to this surface. The equipotential surfaces may well have normal components relative to the solid underlying surface. This is one more (in addition to inertia) cause for the deviation of the flow from isosurfaces, i.e., for the disturbance of the hydrostatic balance in a nonuniform gravitational field. One cannot argue in advance that the effects of this kind are meteorologically significant, but, in any case, it makes sense to estimate them.

The authors' work [2] has been considered a two-dimensional linear stationary model of atmospheric perturbations, arising under the influence of the GFIs on the background geostrophic flow. When there are no gravitational-field inhomogeneities, a homogeneous background geostrophic flow is specified along one of the horizontal axis \( x \):

\[
U = -\frac{1}{f \rho} \frac{\partial \bar{p}}{\partial y}.
\]  

(1)

Here, \( y \) is the second horizontal coordinate (in the across-flow direction), \( f \) is the Coriolis parameter (the \( f \)-plane approximation is used), and \( \bar{p} \) is pressure. It is assumed that the background distributions of density and pressure (overlined here) depend not only on the height \( z \), but also on one of the horizontal axis \( y \). For analysis, it is convenient to use model

\[
\bar{p} = \rho_0 \exp \left( -\frac{z}{H + \frac{y}{L_\rho}} \right), \quad \bar{p} = g \rho_0 H \exp \left( -\frac{z}{H + \frac{y}{L_\rho}} \right), \quad U = \frac{gH}{fL_\rho} = \text{const}.
\]  

(2)

where axis \( z \) is directed vertically upward; the meaning of constants \( \rho_0, H, L_\rho \) is quite obvious. Such a specification of the background state reduces the problem to the system of equations with constant coefficients.

Disturbances introduced in this flow by two-dimensional inhomogeneities of the gravitational acceleration are considered in a linear approximation. The horizontal and vertical components of these additional accelerations are described, respectively, by \( g_x (x, z) \) and \( g_z (x, z) \). The total gravitational force is the vector sum of these disturbances and of the average gravitational force, which below is denoted by \( g \). If we denote the gravitational potential through \( \Phi \), then

\[
\frac{\partial \Phi}{\partial x} = -g_x, \quad \frac{\partial \Phi}{\partial z} = g - g_z, \quad \frac{\partial g_x}{\partial z} = \frac{\partial g_z}{\partial x}.
\]  

(3)
The linearized system of equations for the two-dimensional velocity, pressure, and density perturbations in an ideal incompressible medium is:

\[
\begin{align*}
\frac{\partial u'}{\partial x} - \frac{\partial p'}{\partial y} + f\nu + \rho g_z, & \quad \frac{\partial v'}{\partial x} - \frac{\partial p'}{\partial z} - f\rho u' - fU\rho', \\
\frac{\partial w}{\partial x} - \frac{\partial p'}{\partial z} - gp\rho + \rho g_z', & \quad \frac{\partial u'}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad U \frac{\partial \rho'}{\partial x} + \nu \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} = 0.
\end{align*}
\] (4)

Here, \(v, w\) are perturbations of the velocity components along the axes \(y\) and \(z\); perturbations of other quantities are indicated by a prime.

The nonpermeability condition is specified at the lower boundary. At the solid horizontal surface, it is written as

\[
w_{\mid z=0} = 0.
\] (5)

One more boundary condition is the damping of disturbances as \(z \to \infty\) (situations where the generation of vertically propagating internal waves is possible are not considered here).

The pressure perturbation is conveniently sought as

\[
\Pi(x, z) = \rho'(y, z)\Pi(x, z),
\]

where the function \(\Pi(x, z)\) satisfies the equation

\[
\frac{\partial \Pi}{\partial x} = U \frac{\partial w}{\partial z} + f\nu + g_x.
\] (6)

Eliminating part of the unknowns from (4), with the use of (2), (3), and (6), we can write a system of two equations with constant coefficient

\[
\begin{align*}
\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} - \frac{1}{H} \frac{\partial w}{\partial z} + \frac{g}{HU^2} w + f \frac{\partial v}{\partial z} - \frac{1}{HU} g_x, & \quad \frac{\partial^2 v}{\partial x^2} = f \left(1 + \frac{U^2}{gH}\right) \frac{\partial w}{\partial z} - \frac{f}{UH} w + \frac{f}{gH} g_x,
\end{align*}
\] (7)

which can be reduced to one equation

\[
\frac{\partial^2}{\partial x^2} \left(\frac{1}{H} \frac{\partial w}{\partial z} + \frac{g}{HU^2} w\right) + \left(f \frac{U}{gH}\right) \frac{\partial w}{\partial z} - \frac{1}{HU} \frac{\partial^2 g_x}{\partial x^2} = f \frac{g}{gH} \frac{\partial^2 g_x}{\partial z^2} - \frac{f^2}{H} \frac{\partial g_x}{\partial z}.
\] (8)

where \(\Delta_x\) is the symbol of the two-dimensional Laplacian.

Despite the relatively cumbersome form of these equations, their analysis is in principle simple. A considerable amount of information can be obtained, for example, by considering a model with a harmonic dependence of the gravitational-field inhomogeneities on \(x\) when the problem reduces to ordinary differential equations.

We denote a characteristic spatial scale of the gravitational-field inhomogeneities and related atmospheric perturbation through \(L\) (in this section, this is assumed to be above or about 100 km). Let us introduce dimensionless variables \(X = x / L, Z = z / L\). Equation (8) becomes

\[
\frac{\partial^2}{\partial X^2} \left[\xi F^2 \left(\xi \Delta_2 w - \frac{\partial w}{\partial Z}\right) + w\right] + \xi B \frac{\partial}{\partial Z} \left[\xi (1 + F^2) \frac{\partial w}{\partial Z} - w\right] = \frac{U}{g} \left(\frac{\partial^2 g_x}{\partial X^2} - \xi B \frac{\partial g_x}{\partial Z}\right).
\] (9)

Here, notations are introduced for the three governing dimensionless parameters. The Froude number \(F = U / \sqrt{gH}\) is a ratio of the inertial and gravitational forces; \(B = f^2 L^2 / gH\) is the inverse of the Burger number, expressing the ratio between the rotation and stratification effects; and the geometric factor \(\xi = H / L\) has a meaning of the aspect ratio. The Froude number can be considered small for at least moderate background winds (at \(U = 10\) m/s, \(H = 10^4\) m, the Froude number is about 1/30). We consider sufficiently large horizontal scales of gravitational-field inhomogeneities, for example, on the order of 100 km, so that \(L \approx H\). It is easy to verify that all the above-mentioned dimensionless
coefficients are small for the typical values of the parameters in (9). If only the terms without small coefficients are retained in this equation, we obtain

\[ \frac{\partial^2 w}{\partial x^2} \approx \frac{U}{g} \frac{\partial^2 g_x}{\partial x^2} \quad \text{or} \quad w \approx \frac{U}{g} g_x. \] (10)

Approximate solution (10) does not satisfy a rigid-lid condition at the solid surface (5). This comes as no surprise because the terms with small coefficients at higher derivatives are not yet taken into account. When these terms are taken into account, the solution will contain a boundary layer such that boundary conditions are satisfied. To derive a solution in explicit analytical form, consider a simple model with an x-periodic inhomogeneity of the gravitational field:

\[ g_x = G \exp(-z/L) \cos(x/L), \quad g_z = -G \exp(-z/L) \sin(x/L). \] (11)

where \( G \) is amplitude. A solution for perturbations is also sought as a horizontal harmonic, in particular,

\[ w(x, z) = W(z) \cos(x/L). \] (12)

For the amplitude \( W \), we have

\[ \xi^2 \left[ B(1 + F^2) - F^2 \right] \frac{d^2 W}{dZ^2} + \xi(F^2 - B) \frac{dW}{dZ} - \left(1 - \xi^2 F^2\right) W = -U \frac{G}{g} \left(1 - \xi B\right) \exp(-Z). \] (13)

A general solution to the last equation is sought as the sum of a general solution of the homogeneous equation for \( W_h \) and a partial solution for the inhomogeneous \( W_n \). The latter, with the smallness of the dimensionless parameters \( F, B, \xi \), is

\[ W_n \approx \frac{U}{g} G \exp(-Z). \] (14)

It is a partial case of approximate solution (10). To satisfy the boundary condition on the surface, it is necessary to add the corresponding solution of the homogeneous equation. Its characteristic equation is written as

\[ \xi^2 \left[ B(1 + F^2) - F^2 \right] \sigma^2 + \xi(F^2 - B) \sigma - \left(1 - \xi^2 F^2\right) = 0. \] (15)

Different situations are generally possible depending on the ratio between dimensionless parameters \( F, B, \xi \). The problem we consider has common features with the well-known problem on disturbances of a stratified flow over a sinusoidal terrain. In the latter, there is a region where internal waves are generated within the range of scales \( L \) between \( L_N \equiv U / N \) (where the buoyancy frequency \( N = \sqrt{(g/\rho)(\partial \rho / \partial z)} \) in the given model is \( \sqrt{g/\rho \gamma} \)) and \( L_f \equiv U / f \). On both sides of this region are areas in which the waves are not generated, and disturbances in varying degrees are enclosed at the lower boundary. In the present paper, according to the scale of heterogeneity considered, the emphasis is on the region

\[ L \geq L_f = U / f \quad \text{or} \quad B \geq F^2, \] (16)

in which the internal waves are not generated (a more correct condition for the absence of waves is cumbersome; it is somewhat less strict than (16)). In this region, the discriminant of (15) is negative (i.e., the radicand in the formula for \( \sigma \) is positive). Following the boundary conditions, from the two values of \( \sigma \) we take the negative one, so that the solution is
\[ w \approx U \frac{G}{g} \left[ \exp \left( -\frac{z}{L} \right) - \exp \left( -|\sigma| \frac{z}{L} \right) \right] \cos \frac{x}{L}, \]
\[ u' \approx U \frac{G}{g} \left[ \exp \left( -\frac{z}{L} \right) - |\sigma| \exp \left( -\frac{|\sigma| z}{L} \right) \right] \sin \frac{x}{L}, \]
\[ v \approx fL \frac{G}{g} \left[ \exp \left( -\frac{z}{L} \right) - \left(|\sigma| + \xi^{-1}\right) \exp \left( -\frac{|\sigma| z}{L} \right) \right] \cos \frac{x}{L}, \]
\[
\sigma = \frac{B - F^2 - \left(\frac{B - F^2}{2}\right) + 4(1 - \xi^2 F^2)\left(\frac{B}{1 + F^2} - F^2\right)^{1/2}}{2\xi^2 B^2} < 0.
\]

In this case, therefore, the solutions of the type of (10) and (14) are supplemented with a boundary layer of thickness \( L/|\sigma| \), thus providing the fulfillment of boundary condition (5). Given the smallness of the dimensionless parameters, (18) is significantly simplified:
\[
\sigma \approx -\frac{1}{\xi^2 B^2} \left(\frac{B}{1 + F^2} - F^2\right)^{1/2}.
\]

Note that the aforementioned boundary layer is relatively thin \(|\sigma| > 1\). With increasing \(|\sigma|\), there is an increase in \( \frac{\partial w}{\partial z} \) and, hence, in the absolute values of the horizontal value of velocity, divergence, and vorticity (the latter is seen, in particular, from the second equation of (7)). Because \(|\sigma| > \xi^{-1}\) according to (19), both horizontal components of the velocity perturbation in (17) have nearly the same dependence on the height.

Let, for example, \( B = 2F^2 \) (i.e., condition (16) is satisfied with a double margin). Then \( \sigma \approx -1/\xi F = -(L/U)\sqrt{g/H} \), and the thickness of the boundary layer that now arises is \( h_{bl} \approx L/|\sigma| \approx U\sqrt{H/g} \). If \( L = 150 \) km, \( U = 10 \) m/s, and \( H = 10^{4} \) m, then \(|\sigma| \approx 500\), \( h_{bl} \approx 300 \) m. If the amplitude of the gravitational-field inhomogeneity is taken to be \( G = 10^{-3} \) m/s², then, according to (17), in the boundary layer \( u' \approx -|\sigma| U G/g \approx LG/g \approx 1 \) m/s and \( v \approx fL G/g \approx fL^2 G/U \approx 1 \) m/s (the latter holds at \( f = 10^{-4} \) m/s). In the second equation of (4), there is an approximate boundary-layer balance \( U \partial v / \partial x \approx -f u' \).

It is easy to verify that \( \nabla \rho' / \partial y \) in the last equation (4) is negligible, whence
\[
\frac{\partial \rho'}{\partial x} \approx -\frac{w}{U} \frac{\partial \rho}{\partial z}; \quad \rho' \approx \rho \frac{L}{H} \frac{G}{g} \left[ \exp \left( -\frac{z}{L} \right) - \exp \left( -\frac{|\sigma| z}{L} \right) \right] \sin \frac{x}{L}.
\]

Outside the boundary layer, the isopycnals are close to the equipotential surfaces: they differ little from the case with no background flows. Near the lower boundary, however, \( \rho' \rightarrow 0 \); hence, the isopycnals do not generally coincide with the equipotential surfaces. This means deviations from the hydrostatic balance and may give rise to noticeable perturbations of horizontal velocity in the boundary layer considered here.

3. Generation of internal gravitational waves under the action of inhomogeneities in the field of gravity on the atmospheric flow

Above we have been considered the case of inhomogeneities of sufficiently large horizontal scales, when trapped perturbations are generated. At smaller scales (but larger than \( U / N \)), GFIs result in the generation of internal gravitational waves (IGW). This case was considered in the work of the authors [10]. As it has been shown, the situation in some respects is analogous to the generation of IGW in the
flow of surface irregularities in the relief of the underlying surface. In the coordinate system associated with the background flow, the solution for the vertical velocity has the form

\[
\begin{aligned}
\left\{ \exp(-kz)\cos[k(x + Ut)] - \exp\left(\frac{z}{2H}\right)\cos\left[\frac{z}{\lambda} + k(x + Ut)\right]\right\}
\end{aligned}
\]

Here \( k = L^{-1}, \quad \tilde{k} = k + 1/2H, \quad \lambda = \left[\left(1/N \right)^2 - (2H)^2 - k^{-2}\right]^{-1/2} \approx U/N \). The first of the terms in the curly brackets directly describes flowing of equipotential surfaces. These terms are not wave-like (they do not contain waves propagating along the vertical axis) and slowly damp with altitude on the same scales \( k^{-1} \) as the anomaly of gravity. The second term describes the internal gravitational waves, the phase velocity of which is directed downward, and the group one is directed upward. The amplitude of these waves in the velocity field increases with height as \( \exp(z/2H) \). The vertical component of the wave vector is \( \lambda^{-1} \), frequency is \( \omega = k \lambda U \), vertical phase velocity is \( k \lambda U \). For the vertical flux of wave energy, we obtain an approximate expression that does not depend on the height:

\[
F_z \approx \frac{1}{2} \rho_0 N U^2 G^2 / g^2 k \quad (22)
\]

This result is clearly interpreted: taking into account the effect of GFAs (curved equipotential surfaces) leads to an effect analogous to the effect of irregularities in the relief with the same amplitude and horizontal scales. If we take the amplitude GFAs \( G = 10^{-3} \text{ m/s}^2, \quad k = 2 \times 10^{-5} \text{ m}^{-1} \) (which corresponds to a half-wave length of about 150 km), \( U = 20 \text{ m/s}, \quad N = 10^{-2} \text{ s}^{-1}, \quad \rho_0 = 1 \text{ kg/m}^3 \), then we obtain an energy flux of about \( 10^{-3} \text{ W/m}^2 \). If \( G = 2 \times 10^{-3} \text{ m/s}^2, \quad k = 4 \times 10^{-5} \text{ m}^{-1}, \quad U = 30 \text{ m/s}, \quad N = 10^{-2} \text{ s}^{-1}, \quad \rho_0 = 1 \text{ kg/m}^3 \), then the energy flux is of the order of \( 0.5 \times 10^{-2} \text{ W/m}^2 \). Such flows, in some situations, may seem to be significant, although they can not often compete with the most efficient mechanisms of IGW generation in the atmosphere (slopes of equipotential surfaces are usually quite flat compared with the relief of the underlying surface). For illustration we mention that, according to [11], the average energy flux coming from the lower atmosphere to the upper one, due to wave disturbances and tidal oscillations, is of the order of \( 2 \times 10^{-4} \text{ W/m}^2 \).

4. Atmospheric disturbances caused by vertical heat transfer in a inhomogeneous field of gravity

Above, we considered a model of an ideal medium, without taking heat exchange into account. During these studies, another mechanism is found for disturbing hydrostatic equilibrium and the appearance of atmospheric flows in an inhomogeneous field of gravity.

The geometry of the problem is schematically illustrated in figure 1. Horizontal and vertical coordinate axes are denoted as \( x \) and \( z \), respectively (for simplicity, we are limited to a two-dimensional problem). Dashed lines denote equipotential surfaces with which, according to the aforementioned theorem, coincide isobars, isopycnics, and, consequently, isotherms in an idealized medium. In an area of gravity force anomaly these surfaces are deformed (in case of negative GFA, they are curved down). Certain temperature inhomogeneity should exist in this area (in the atmosphere, the so-called potential temperature is more convenient to use as the corresponding variable). Accounting for heat exchange with an ambient environment (in the atmosphere, turbulent exchange with the lower boundary, i.e., the underlying surface, is foremost substantial), this inhomogeneity should relax to some extent (area influenced by heat exchange with lower boundary, the temperature of which is assumed to be constant, is shaded in the figure). In turn, this should be reflected in the pressure field (changes in temperature lead to changes in the weight of the medium column). As such, taking into account the heat exchange leads to changes in spatial distribution of the pressure field and, thus, to the disruption of the aforementioned balance of the gravity and pressure gradient balance. The tangential (horizontal) component of gravity field is now (accounting for heat exchange) not fully compensated, and this means the generation of flows. This is a principal difference...
from the case of a homogeneous gravity force, where taking into account the vertical heat exchange changes only the vertical distributions of temperature, density, and pressure fields and may not lead to the origin of uncompensated horizontal forces.

If we consider the inhomogeneities of the gravity force field with horizontal scales on the order of 100-1000 km, then these scales are much larger than the characteristic atmosphere thickness. For this reason, heat diffusion in the vertical direction is most substantial. If the temperature of the horizontal lower boundary (underlying surface) is assumed to be fixed and constant in the simplest model, this boundary, which takes into account the heat exchange, will apparently influence on the temperature field in the lower layer of the medium. Now, isotherms and isobars near boundary do not coincide with equipotential surfaces, and the balance of the horizontal forces is distorted in favor of variations in the gravity force.

This should lead to the formation of flows.

The linearized hydrothermodynamic set of equations for a stationary problem in the Boussinesq approximation with allowance for planetary rotation has the form [12-13]:

\[
\begin{align*}
0 &= -\frac{1}{\rho} \frac{\partial \rho'}{\partial x} + f v + \nu \Delta_2 u + g z, \\
0 &= -fu + \nu \Delta_2 v, \\
0 &= -\frac{1}{\rho} \frac{\partial \rho'}{\partial z} + \nu \Delta_2 w - g \frac{\rho'}{\rho} + g z, \\
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0, \\
\gamma w &= \kappa \Delta_2 \theta, \\
\rho' &= -\rho \alpha \theta
\end{align*}
\]

Here \( \theta \) are the disturbances of potential temperature; \( \nu \) is the kinematic viscosity coefficient; \( \kappa \) is the temperature conductivity coefficient (both assumed constant); \( \gamma > 0 \) is the background vertical gradient of the potential temperature (stable background density stratification is assumed); \( \alpha \) is the temperature expansion coefficient; and \( \Delta_2 \) is symbol of two-dimensional Laplacian.

On the lower boundary (underlying surface), it is assumed that the conditions of no permeability and no-slip, as well as a fixed temperature (no temperature disturbances) are met:

\[
\begin{align*}
u = w = 0 \quad \text{on} \quad z = 0.
\end{align*}
\]

Far from the surface, it is assumed that the static regime exists, which take place in absence of heat exchange, i.e., without taking into account the influence of the underlying surface (horizontal heat exchange in the considered geometry is negligible). The latter means that isobars, isopycnics, and

\[\begin{align*}
\begin{array}{c}
\text{Figure 1. Geometry of the problem} \\
\text{(explained in the text).} \\
\text{Lower part of area of gravity field} \\
\text{anomaly, which is influenced} \\
\text{by thermal influence of lower} \\
\text{boundary, is shaded.} \\
\text{Solid lines schematically show flows} \\
\text{that originate due to} \\
\text{the disruption of the hydrostatic} \\
\text{balance associated with} \\
\text{heat exchange.}
\end{array}
\end{align*}\]
isotherms far from lower boundary coincide with equipotential surfaces, and velocity disturbances damp. We denote the deviations of the gravity force potential and vertical deviations of equipotential surfaces associated with gravity field inhomogeneities as $\Phi$ and $\eta$, respectively. On definition

$$\int g \, dx / g = \Phi - \eta,$$

where lower integration boundary is a reference point at which the mentioned disturbances are absent. Correspondingly, the upper boundary condition for the temperature disturbance is written as

$$\theta \to -\gamma \eta = -\gamma \int g \, dx / g \quad \text{on} \quad z \to \infty. \tag{25}$$

It is convenient, as above, to analyze a model with a sinusoidal dependence of gravity field inhomogeneities on a horizontal coordinate $x$. In this case, the problem reduces to solving the system in ordinary derivatives and is analyzed quite simply. The solution depends essentially on the values of the dimensionless parameters $R = N^2 / k^3 \nu$, $Ta = f^2 / \nu^2 k^4$ (the analogs of the Rayleigh and Taylor numbers, respectively). Their values in the problems under consideration are usually very large. But in the narrow equatorial zone the value of $Ta$ is small. This case, when the rotation is insignificant, has been investigated in the article [12].

Solutions for the two components of velocity (motion in the “transverse” direction $y$ without taking Coriolis accelerations into account does not arise) have the form

$$u \approx -\frac{1}{2} \frac{\kappa}{\nu} \sqrt{\frac{\nu}{\kappa}} \left[ \exp \left( \frac{z}{h} \right) - \frac{2}{\sqrt{3}} \exp \left( -\frac{z}{2h} \right) \cos \left( \frac{\sqrt{3}z}{2h} - \frac{\pi}{6} \right) \right] \cos kx,$$

$$w \approx \frac{1}{2} \frac{hk\Xi}{\nu} \left[ \exp \left( \frac{z}{h} \right) - \frac{2}{\sqrt{3}} \exp \left( -\frac{1}{2} \frac{z}{h} \right) \cos \left( \frac{\sqrt{3}z}{2h} + \frac{\pi}{6} \right) \right] \sin kx,$$

where we introduced the scales of length $h = (\kappa \nu / N^2 k^2)^{1/6} = k^{-1} R^{-1/6}$ (vertical scale of flows originating near the underlying surface) and velocity

$$\Xi = \frac{N G}{k g} \sim N \eta. \tag{26}$$

Velocity disturbances oscillate with height, damping on vertical scales on the order of $h$. If $\kappa = \nu = 1 \, \text{m}^2 / \text{s}$, $k = 0.5 \times 10^{-3} \, \text{m}^{-1}$ (which corresponds to the length of horizontal half-wave of about 600 km) then $h = 300 \, \text{m}$. In the area of negative GFA (for example, near vertical line $x = -\pi / 2$) horizontal spreading out (positive horizontal divergence) and descending motion prevail near surface. This is understandable, since the balance of horizontal forces is disrupted in the specified area in favor of the gravity field, which defines the direction of horizontal motion.

But outside the narrow equatorial zone, the values of $Ta$ are large. For example, when $N = 10^{-2} \, \text{s}^{-1}$, $\kappa = \nu = 1 \, \text{m}^2 / \text{s}$, and $k = 2 \times 10^{-5} \, \text{m}^{-1}$ (which corresponds to the length of horizontal half-wave of about 150 km) $R \sim 10^{15}$, $Ta \sim 10^{11}$. In work [13], a very general case is analyzed $1 \ll R^{2/3} \ll Ta \ll R$. The approximate solution has the form

$$u \approx \nu k \left( \frac{\nu}{\kappa} \right)^{1/2} \frac{G}{f} \left( N / f \right) \frac{N^3}{R^2} \left[ -\exp \left( -\frac{z}{h_B} \right) + \left( \frac{4T a^3}{R^2} \right) \exp \left( -\frac{z}{h_E} \right) \sin \left( \frac{z}{h_E} \right) \right] \cos kx,$$

$$v \approx \left( \frac{\kappa}{\nu} \right)^{1/2} \Xi \left[ (1 - \delta) \exp( -kz) - \exp \left( -\frac{z}{h_B} \right) + \delta \exp \left( -\frac{z}{h_E} \right) \cos \left( \frac{z}{h_E} \right) \right] \cos kx,$$

$$w \approx \nu k \frac{G}{f} \left( \frac{N}{f} \right)^2 \left[ \exp \left( -\frac{z}{h_B} \right) - 2^{1/2} \exp \left( -\frac{z}{h_E} \right) \cos \left( \frac{z}{h_E} - \frac{\pi}{4} \right) \right] \sin kx,$$
\[ \theta \approx -\gamma G \left( \exp(-kz) - \exp\left(-\frac{z}{h_B}\right) \right) \sin kx. \]

Here we introduce a dimensionless parameter \( \delta = (2R)^{1/2} / Ta^{3/4} = \frac{\nu k}{(k \kappa^2 / 2)^{3/2}} \ll 1 \) and length scales

\[ h_B = \left( k \sigma_1 \right)^{-1} = \frac{1}{k} \left( \frac{Ta}{R} \right)^{1/2} = \frac{\left( \frac{\kappa}{\nu} \right)^{1/2} f}{kN}, \quad h_E = \frac{1}{k} \left( \frac{Ta}{4} \right)^{-1/4} = \left( \frac{2\nu}{f} \right)^{1/2}. \]

A characteristic feature of this problem, taking into account the effects of planetary rotation, is the occurrence of vortex perturbations under the influence of the GFIs. Convergent flows (see the figure) in the field of Coriolis forces lead to the appearance of positive vorticity. Therefore, the tangential velocity is positive up to levels of order \( k^{-1} \). Near the underlying surface, in the boundary layer, divergent flows, in principle, should generate a negative vorticity. But since this is a thin layer in which the flows are effectively suppressed by viscosity, in the approximation considered, these weak motions are lost. Therefore, in the region of the negative gravity anomaly, the mechanism in question mainly leads to the generation of a cyclonic vorticity. We note that this is in qualitative agreement with the analysis of field data presented in [4-6]. In the papers mentioned, a positive correlation of the activity of tropical cyclones with regions of reduced values of the vertical component of the gravity anomaly was noted. If the exchange coefficients \( \nu, \kappa \) are of the same order, then the velocity \( v \) of the resulting vortex motion is of the order of \( \eta \Xi N \). For \( N = 10^{-2} \) \( \text{s}^{-1} \) and the deviation of the geoid \( \eta = 30 \) m, \( v \sim 0.3 \) m/s.

5. Conclusion
The analysis performed above and the publications cited show that, contrary to popular belief, the inhomogeneities of the field of gravity can lead to perturbations of the wind field. Some physical mechanisms of these disturbances have been established. The simplest analytical models of stationary linear atmospheric disturbances caused by inhomogeneities in the field of gravity are analyzed. Estimates show that the influence of the GFAs on the field of motion can be noticeable in some situations.

This material does not completely exhaust the possibilities of analytical studies. For example, only two-dimensional models are considered so far. Three-dimensional problems in the simplest cases, apparently, can also be investigated by analytical methods. Probably, one can go beyond the considered models of an incompressible atmosphere, or Boussinesq approximation. In addition to internal-gravity waves, it may also be worthwhile to consider the generation of inertia-gravitational waves, i.e. consider a similar problem for large horizontal scales of inhomogeneities, when Coriolis forces are important. It is also of interest to consider the influence of slow vertical movements (which, as it is shown above, arise under the influence of the GFAs) on the atmospheric processes.

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