Secure Image Encryption Based on Compressed Sensing and Scrambling for Internet-of-Multimedia Things

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ABSTRACT In this paper, we propose a secure image encryption system based on compressed sensing (CS) with a scrambling mechanism. For efficient encryption, we use a sparse measurement matrix, where the nonzero elements are generated by a linear feedback shift register (LFSR) based keystream generator. Then, several pairs of data scramblers, also based on LFSR, are attached behind the CS-encryption for diffusion. While guaranteeing theoretically reliable CS-decryption, numerical results indicate that the proposed cryptosystem has more reliable CS-decryption performance than other CS-based cryptosystems. Examining the histogram, entropy and correlation of the ciphertexts, experimental results demonstrate that the proposed cryptosystem has strong statistical security. Moreover, it turns out that the proposed cryptosystem has higher plaintext sensitivity than other CS-based cryptosystems, thanks to the bit-level diffusion from the scramblers.

INDEX TERMS Compressed sensing, diffusion, image encryption, plaintext sensitivity, scramblers, statistical security.

I. INTRODUCTION
Compressed sensing (CS) is a signal processing technique which allows to reconstruct a sparse signal from the incomplete measurements sampled at a rate lower than the Nyquist rate [1]. We say that a signal \(x \in \mathbb{R}^N\) is \(K\)-sparse with respect to an orthonormal basis \(\Psi \in \mathbb{R}^{N \times N}\) if \(\alpha = \Psi x\) has at most \(K\) nonzero elements, where \(K \ll N\). In CS, the sparse signal \(x\) is measured by \(y = \Phi x \in \mathbb{R}^M\), where \(\Phi \in \mathbb{R}^{M \times N}\) is a measurement matrix with \(M \ll N\). The reconstruction of the original signal \(x\) can be achieved by solving an \(l_1\)-minimization problem [2]. Thanks to the efficient data acquisition, there have been various CS applications, such as communications [3], [4], bio signal processing [5], image processing [6], RFID identification [7], etc. In particular, CS can be an attractive choice for data acquisition in a resource-limited transmitter of the Internet of Things (IoT) [8]–[11].

The CS techniques can be applied in a symmetric-key cryptosystem for information security. In CS-based cryptosystems, a plaintext \(x\) is encrypted to a ciphertext \(y = \Phi x\) using a secret matrix \(\Phi\). A sender and a legitimate recipient share a secret key to generate the elements of the secret matrix \(\Phi\). The legitimate recipient who knows \(\Phi\) can reconstruct the original plaintext through CS reconstruction algorithms. Rachlin and Baron [12] showed that the CS-based cryptosystem cannot be perfectly secure but might be computationally secure. Orsdemir et al. [13] claimed that estimating the random Gaussian secret matrix of the CS-based cryptosystem is computationally infeasible. In [14], Bianchi et al. proved that CS-based cryptosystems can resist known-plaintext attacks (KPA) by encrypting plaintexts in a one-time-sensing (OTS) manner, where the secret matrix is renewed at every encryption. Furthermore, they showed that the Gaussian-OTS (G-OTS) cryptosystem, which uses a random Gaussian secret matrix in the OTS manner, can be perfectly secure if the plaintext has constant energy. In [15], it is quantitatively shown that the Bernoulli-OTS (B-OTS) cryptosystem, which uses a random Bernoulli matrix to encrypt a plaintext in the OTS manner, can resist against KPA. Moreover, the indistinguishability [16] was discussed for the G-OTS and the B-OTS cryptosystems [17], respectively.

For image encryption, Zhou et al. proposed CS-based cryptosystems [18], [19] based on chaotic systems. In [20], a multi-class CS-encryption is proposed, which gives different decryption qualities to different class recipients. Zhang et al. [21] designed a multi-image encryption, which uses the random convolution [22] for CS-encryption. In [23], Zhu et
al. proposed a CS-based cryptosystem employing a circular shift for diffusion. Li et al. [24] proposed an efficient image encryption for IoT monitoring applications by reducing time consumption in encryption and reconstruction. Zhu et al. [25] proposed a CS-based cryptosystem, where permutation and diffusion are executed simultaneously for encryption. There are many other applications of CS for information security [26]–[29].

For processing large data size, the secret matrix of a CS-based cryptosystem may require large data storage and high computational complexity. To resolve this issue, the parallel CS-based cryptosystem was proposed in [30], where the columns of an image are encrypted in parallel with the counter mode operation. Also, a CS-based cryptosystem [31] was proposed to reduce data storage for the secret matrix by employing the semi-tensor product. In [32], Cho et al. proposed the sparse-OTS (S-OTS) cryptosystem in which a plaintext is encrypted in the OTS manner by a sparse secret matrix with only a few nonzero elements. By using the sparse matrix, the S-OTS cryptosystem can save storage space and computational cost. Therefore, it can be an attractive choice to use the S-OTS cryptosystem in a resource-limited environment, e.g., sensors of IoT.

To evaluate the security of image encryption, we can analyze the encrypted images with statistical measures, such as histogram, entropy, correlation and plaintext sensitivity [21], [24]. Although the S-OTS cryptosystem has efficiency and security, we realize in this paper that it has weak security in terms of the statistical measures. To enhance the statistical security of the S-OTS cryptosystem, a scrambler can be introduced for diffusion mechanism. Scramblers have been widely used in communications [33], [34] and information security [35]. Based on a linear feedback shift register (LFSR), a scrambler randomizes its input bitstream such that the corresponding output bitstream seems to be independently distributed at random. Moreover, a few changes of an input bitstream to a scrambler can lead to significant changes in the output by the recursive structure. Exploiting this property, a scrambler structure was proposed to give the avalanche effect for a secure key distribution to the receivers of wireless local area networks [36].

In this paper, a secure CS-based cryptosystem with strong statistical security is proposed for image encryption. The proposed cryptosystem uses a sparse matrix of the S-OTS cryptosystem and several scrambler chains for diffusion. It is noteworthy that the proposed cryptosystem can theoretically guarantee reliable CS-decryption inherited from the S-OTS cryptosystem, which will be numerically demonstrated. Each scrambler chain consists of a pair of scramblers aligned in series, where the output of the first scrambler is fed as an input to the second scrambler in reverse order. While this scrambler chain is similar to the scrambler structure in [36], we use it at a transmitter to give the avalanche effect for CS-based encryption. To prevent a malicious attacker from restoring the scrambled contents, a keystream generator of a stream cipher is also employed for simultaneous scrambling and encryption. Due to the hardware-friendly structure, the proposed cryptosystem can be implemented easily in real world applications.

For statistical security, we conduct numerical experiments to analyze the histogram, entropy, correlation and plaintext sensitivity for the proposed cryptosystem. We demonstrate that the proposed cryptosystem can achieve flat histogram, high entropy and low correlation of adjacent pixels in ciphertexts, thanks to the diffusion by the proposed scrambler structure. Especially, since the diffusion mechanism of the proposed cryptosystem is executed in ciphertext bit-level, the quantitative evaluation measures of the plaintext sensitivity are much better than those of the other CS-based cryptosystems [23]–[25], [30] that execute the diffusion mechanisms in ciphertext element-level, where each ciphertext element consists of multiple bits. In summary, the proposed CS-based cryptosystem has a hardware-friendly structure and reliable CS-decryption performance. Through numerical results, we demonstrate that the proposed cryptosystem can be statistically secure achieving flat histogram, high entropy, low correlation and high plaintext sensitivity, thanks to the proposed scrambler structure.

The rest of the paper is organized as follows. Some background knowledge for understanding this paper is given in Section II. In Section III, we briefly introduce other CS-based cryptosystems [23]–[25], [30] for comparison. The details of the proposed CS-based cryptosystem, including encryption, scrambling and decryption processes, are described in Section IV. In Section V, experimental results about the statistical security measures of the proposed CS-based cryptosystem are presented and compared to those of the other CS-based cryptosystems [23]–[25], [30]. Finally, concluding remarks will be given in Section VI.

II. BACKGROUND

A. CS-BASED CRYPTO SYSTEMS

Let \( x = \Psi^T \alpha \in \mathbb{R}^N \) be a plaintext, which is \( K \)-sparse with respect to an orthonormal basis \( \Psi \in \mathbb{R}^{N \times N} \). Then, the corresponding ciphertext of a CS-based cryptosystem is \( y = \Phi x \), where \( \Phi \in \mathbb{R}^{M \times N} \) is a secret matrix with \( M \ll N \). A legitimate recipient who knows the secret matrix can decrypt the ciphertext by CS reconstruction techniques, such as an algorithm for \( l_1 \)-minimization [37] and a greedy algorithm [38].

B. S-OTS CRYPTO SYSTEM

The secret matrix \( \Phi \) of the S-OTS cryptosystem is a sparse matrix with only a few nonzero bipolar entries, which is represented by

\[
\Phi = \frac{1}{\sqrt{Mr}}SP,
\]

where \( S \in \{-1, 0, 1\}^{M \times N} \) is a sparse matrix and \( P \in \mathbb{R}^{N \times N} \) is a column permutation matrix. The row-wise sparsity \( r \) is defined by \( r = \frac{q}{N} \), where \( q \) is the number of the nonzero elements of each row of \( S \). The nonzero elements of
S are generated by the self-shrinking generator (SSG) [39], which is an LFSR-based keystream generator. The first $qM$ bits of an SSG keystream $k = (k_1, k_2, \cdots)$ are used to generate $S$. The $i$-th row of $S$ has a column index set $\Lambda_i$ of the nonzero elements defined as

$$\Lambda_i = \{((i-1) \mod q) \cdot q + l | l = 1, \cdots, q\},$$  

where $q = \frac{N}{n}$. Then, the element in the $i$-th row and the $j$-th column of $S$ is

$$s_{i,j} = \begin{cases} b_{j \mod N} \cdot N + j, & \text{if } j \in \Lambda_i, \\ 0, & \text{otherwise}, \end{cases}$$

where $b_i = (-1)^{k_i}$ with $k_i \in \{0, 1\}$ for $i=1,2,\cdots$.

The S-OTS cryptosystem can encrypt a plaintext quickly, since the matrix-vector multiplication of the encryption process can be implemented row-wise in parallel. The CS-decryption can also be processed efficiently, exploiting the few nonzero elements of $\Phi$. In addition, the authors of [32] showed that the S-OTS cryptosystem can be computationally secure through the security analyses against ciphertext only attacks (COA) and chosen plaintext attacks (CPA). Moreover, the S-OTS cryptosystem can theoretically guarantee a stable and reliable CS-decryption for a legitimate recipient.

C. STATISTICAL SECURITY MEASURES

The histogram of an image shows the number of the occurrences of all pixel values. For secure image encryption, the histogram of an encrypted image should have a fairly uniform distribution, which means that the frequencies of the occurrences of all pixel values are almost equal. If the histogram of an encrypted image is non-uniform, it may be vulnerable to statistical attacks. To measure the randomness of an image quantitatively, we can use the entropy. An encrypted image with higher entropy means that its histogram is closer to the uniform distribution. The entropy $E$ of an 8-bit gray-scale image is defined as

$$E = -\sum_{i=0}^{l-1} p(i) \log_2 p(i),$$

where $l = 256$ is the number of the gray levels of the image, $i$ is a pixel value in $[0, 255]$, and $p(i)$ is the frequency of the occurrences of $i$. Ideally, $p(i) = \frac{1}{256}$ for all $i$, where $E$ takes the maximum of 8, and the histogram shows the ideal uniform distribution.

In correlation analysis, we measure the correlation of adjacent pixels of an encrypted image for horizontal, vertical and diagonal directions, respectively. We can quantitatively evaluate the correlation of an encrypted image using the correlation coefficient (CC) [21], which is defined as

$$CC = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}},$$

where $x_i$ and $y_i$ represent randomly selected adjacent pixel values of an encrypted image, $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$, $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$, and $n$ is the number of the selected pixel pairs.

The plaintext sensitivity shows the degree of variation in a ciphertext when we modify its plaintext slightly. Let $C_1$ be the ciphertext of a plaintext $M_1$ for a cryptosystem by a key $K$. Another plaintext $M_2$, which is different with $M_1$ by one-bit, can be encrypted to a ciphertext $C_2$ by the same key $K$. Then, we can compare $C_1$ and $C_2$ by using the unified average changing intensity (UACI) [24] and the number of pixels change rate (NPCR) [24], respectively. The UACI, which represents the average of absolute difference of pixel values for two 8-bit gray-scale images, is defined as

$$UACI(\%) = 100 \times \frac{1}{WH} \sum_{i=1}^{W} \sum_{j=1}^{H} \frac{|C_1(i,j) - C_2(i,j)|}{255},$$

and

$$NPCR(\%) = \frac{1}{WH} \sum_{i=1}^{W} \sum_{j=1}^{H} \frac{D(i,j)}{2},$$

where $D(i,j) = 1$ if $C_1(i,j) \neq C_2(i,j)$, and $D(i,j) = 0$ if $C_1(i,j) = C_2(i,j)$. The expected values of the NPCR and the UACI are theoretically calculated by treating every pixel value of two images to be uniformly distributed, which results in 99.6094% and 33.4635%, respectively [40].

D. SCRAMBLERS

An input bitstream of length $H$, denoted by $m = (m_1, m_2, \cdots, m_H)$, can be scrambled by an $L$-stage LFSR-based scrambler characterized by a polynomial $P(x) = 1 + \alpha_1 x^1 + \alpha_2 x^2 + \cdots + \alpha_L x^L$ with $\alpha_i \in \{0, 1\}$ and $\alpha_L = 1$. In Fig. 1(a), the output bitstream is $s = (s_1, s_2, \cdots, s_H)$, where

$$s_i = m_i \oplus a_1 s_{i-1} \oplus a_2 s_{i-2} \oplus \cdots \oplus a_L s_{i-L}$$

for $i = 1, 2, \cdots, H$ and $\oplus$ denotes the bit-wise XOR operation. In this paper, we assume that the initial state of the LFSR is all-zero state. We can restore the original input bitstream $m$ by using the descrambler in Fig. 1(b), where the descrambling process can be described as

$$m_i = s_i \oplus a_1 s_{i-1} \oplus a_2 s_{i-2} \oplus \cdots \oplus a_L s_{i-L}.$$  

Note that the scrambling process is recursive, so the current input bit can influence the output bits thereafter. Therefore, we expect that if a bit flip occurs in $m_i$, a scrambler can make significant changes in $s_i$’s for $i \geq t$.

E. GRAIN-128

The Grain-128 [46] is a stream cipher with a 128-bit key and a 96-bit initialization vector (IV). The stream cipher consists of a 128-stage LFSR, a 128-stage nonlinear feedback shift register (NFSR) and an output generating function, as illustrated in Fig. 2. Due to the simple structure, Grain-128 is suited for a resource-limited environment with easy implementation in hardware.
cryptosystems for comparison in this section. Note that $z$ is defined as $z = \sum_{i=1}^{a} (d_i \cdot 2^{i-1})$. Then, if $p$ and $q$ are $a$-bit integers, $r = p \oplus q$ is an $a$-bit integer, where $r_i = (r_1, r_2, \cdots, r_a)$ with $r_i = p_i \oplus q_i$ for $i = 1, 2, \cdots, a$.

A. CIRCULAR SHIFT-BASED CRYPTOSYSTEM

In the circular shift-based cryptosystem [23], a sparse representation matrix $\alpha = \Psi X \in \mathbb{R}^{n \times n}$ is randomly permuted to $\alpha'$, where $\Psi \in \mathbb{R}^{n \times n}$ is an orthonormal basis and $X \in \mathbb{R}^{n \times n}$ is an image. Then, $\alpha'$ is encrypted to $Y = \Phi \alpha' \in \mathbb{R}^{m \times m}$, where the secret matrix $\Phi \in \mathbb{R}^{m \times m}$ is generated by using the Chebyshev map [47]. Then, $Y$ is quantized to $Y_Q$ by an $a$-bit quantizer. Each row of $Y_Q$ can be concatenated into $y_Q = (y_{Q,1}, y_{Q,2}, \cdots, y_{Q,mn}) \in \mathbb{R}^{mn}$, where $y_{Q,i}$ is an $a$-bit integer for $i = 1, 2, \cdots, mn$. Each element of $y_Q$ is modified by

$$e_i = \text{circshift}(y_{Q,i}, k_i^{(1)})$$

for $i = 1, 2, \cdots, mn$, where circshift($u, v$) is a bit-wise circular shift operation to an $a$-bit integer $u$, i.e., $w = \text{circshift}(u, v)$ is an $a$-bit integer, where $w_i = u_{(i+v-1) \mod a+1}$ for $i = 1, 2, \cdots, a$. In (13), the keystream $k_i^{(1)} = (k_{i,1}^{(1)}, k_{i,2}^{(1)}, \cdots, k_{i,mn}^{(1)})$ is generated by using the 4-D hyper-chaotic system [23], where $k_{i,1}^{(1)} < a$ is an integer for $i = 1, 2, \cdots, mn$. Finally, an element-level diffusion is applied to each element $e_i$ to yield the ciphertext $c = (c_1, c_2, \cdots, c_{mn}) \in \mathbb{R}^{mn}$ by

$$c_i = \{e_i + c_{i-1}\} \mod 2^a \oplus \{(k_i^{(3)} + e_i) \mod 2^a\}$$

for $i = 1, 2, \cdots, mn$, where $c_0 = 0$, $k_i^{(3)} = (k_{i,1}^{(2)} \cdot c_{i-1} \cdot 10^i) \mod 2^a$, and $k_i^{(2)} = (k_{i,1}^{(2)}, k_{i,2}^{(2)}, \cdots, k_{i,mn}^{(2)})$ and $k_i^{(3)} = (k_{i,1}^{(3)}, k_{i,2}^{(3)}, \cdots, k_{i,mn}^{(3)})$ of (14) are generated by using the 4-D hyper-chaotic system [23], where $k_{i,1}^{(2)}$ is a real number and $k_{i,1}^{(3)}$ is an $a$-bit integer. The details of the keystream generation are described in [23].

B. KRONECKER CS-BASED CRYPTOSYSTEM

The Kronecker CS-based cryptosystem [24] encrypts an image $X \in \mathbb{R}^{n \times n}$ to $Y = \Phi \Psi X \in \mathbb{R}^{m \times n}$. In this cryptosystem, the secret matrix $\Phi$ can be constructed by

$$\Phi = A \otimes P \in \mathbb{R}^{m \times n},$$

where $\otimes$ denotes the Kronecker product [41]. In (15), $A \in \mathbb{R}^{p \times p}$ is a random chaotic matrix with each element generated from a chaotic system called the Tent map [42]. Also, $P = P_\pi \in \mathbb{R}^{p \times p}$ is a weighted permutation matrix, where $P_\pi \in \mathbb{R}^{p \times p}$ is a permutation matrix and $D \in \mathbb{R}^{p \times p}$ is a diagonal matrix with diagonal elements generated from the Logistic-Tent chaotic system [45].

After CS-encryption, $a$-bit quantization is performed to $Y$, which results in a quantized ciphertext $Y_Q$. Each row of $Y_Q$ can be concatenated into a vector $y_Q = (y_{Q,1}, y_{Q,2}, \cdots, y_{Q,mn}) \in \mathbb{R}^{mn}$, where $y_{Q,i}$ is an $a$-bit integer for $i = 1, 2, \cdots, mn$. The first diffusion is applied to each element of $y_Q$, which yields $e = (e_1, e_2, \cdots, e_{mn}) \in \mathbb{R}^{mn}$ with

$$e_i = e_{i-1} \oplus y_{Q,i} \oplus k_i^{(1)},$$

for $i = 1, 2, \cdots, mn$.
where \( i = 1, 2, \ldots, mn \) and \( c_0 = 0 \). In (16), the keystream \( k^{(1)} = (k_1^{(1)}, k_2^{(1)}, \ldots, k_{mn}^{(1)}) \) is generated by using the Logistic-Tent chaotic system [45], where \( k_i^{(1)} \) is an \( a \)-bit integer. Note that in the element-level diffusion, the \( t \)-th bit position of \( y_{Q,i} \), cannot influence the \( t \)-th bit position of \( y_{Q,j} \), where \( i \neq j \) for \( i, j = 1, 2, \ldots, mn \). Therefore, bit-level variations cannot be diffused within the whole ciphertext, which leads to weak plaintext sensitivity. The second element-level diffusion yields the final ciphertext \( c = (c_1, c_2, \ldots, c_{mn}) \in \mathbb{R}^{mn} \) by

\[
c_i = c_{i+1} \oplus e_i \oplus k_i^{(2)},
\]

(17)

where \( i = mn, mn - 1, \ldots, 1 \) and \( c_{mn+1} = 0 \). In (17), the keystream \( k^{(2)} = (k_1^{(2)}, k_2^{(2)}, \ldots, k_{mn}^{(2)}) \) is generated by using the Logistic-Tent chaotic system [45], where \( k_i^{(2)} \) is an \( a \)-bit integer. The details of the keystream generation are described in [24].

### C. Division-Based Cryptosystem

In the division-based cryptosystem [25], an image \( X \in \mathbb{R}^{n \times n} \) is encrypted to \( Y = \Phi X \in \mathbb{R}^{n \times n} \), where \( \alpha = \Psi X \in \mathbb{R}^{n \times n} \) is a sparse representation of \( X \) with an orthonormal basis \( \Psi \in \mathbb{R}^{n \times n} \) and the secret matrix \( \Phi \in \mathbb{R}^{n \times n} \) is generated by using the Chebyshev map [47]. Then, \( Y \) is quantized to \( Y_Q \) by an \( a \)-bit quantizer. Each row of \( Y_Q \) can be concatenated into a vector \( Y_Q = (y_{Q,1}, y_{Q,2}, \ldots, y_{Q,mn}) \in \mathbb{R}^{mn} \) where \( y_{Q,i} \) is an \( a \)-bit integer for \( i = 1, 2, \ldots, mn \). Let \( y_Q^{(1)} = (y_{Q,1}, \ldots, y_{Q,w}, \ldots, y_{Q,mn}) \in \mathbb{R}^{mn} \) for \( w = 1, 2 \). Finally, elements of \( y_Q^{(1)} \) and \( y_Q^{(2)} \) are modified by an element-level diffusion to yield the ciphertext \( c = (c_1^{(1)}, c_2^{(2)}) \in \mathbb{R}^{mn} \), where \( c^{(w)} = (c_1^{(w)}, c_2^{(w)}, \ldots, c_{mn}^{(w)}) \in \mathbb{R}^{mn} \), by

\[
c_i^{(w)} = \begin{cases} \{(y_i^{(1)} + k_i^{(1)}) \mod 2^a \} & \oplus \{(y_i^{(2)} + k_i^{(2)}) \mod 2^a \} \oplus c_{i-1}^{(1)} \end{cases},
\]

(18)

for \( i = 1, 2, \ldots, \frac{mn}{2} \) and \( c_0^{(w)} = 0 \). In (18), \( \pi = (\pi_1, \pi_2, \ldots, \pi_{mn}) \) and \( \pi^t = (\pi_1^t, \pi_2^t, \ldots, \pi_{mn}^t) \) are random permutations of \( (1, 2, \ldots, \frac{mn}{2}) \), respectively, and \( k^{(w)} = (k_1^{(w)}, k_2^{(w)}, \ldots, k_{mn}^{(w)}) \) is generated by using the 6-D chaotic system [25], where \( k_i^{(w)} \) is an \( a \)-bit integer. The details of the keystream generation are described in [25].

### D. Parallel CS-Based Cryptosystem

In the parallel CS-based cryptosystem [30], the columns of an image \( X = [X_1, X_2, \ldots, X_n] \in \mathbb{R}^{n \times n} \) are encrypted in parallel. Each column \( X_i \) is encrypted to \( Y_i = \Phi_i X_i \in \mathbb{R}^{m} \) for \( i = 1, 2, \ldots, n \), where \( \Phi_i \in \mathbb{R}^{m \times n} \) is a secret matrix with each element generated from the Logistic-Tent chaotic system [45]. Then, \( Y \in \mathbb{R}^{mn \times n} = [Y_1, Y_2, \ldots, Y_n] \) is quantized to \( Y_Q \) by an \( a \)-bit quantizer. Each row of \( Y_Q \) can be concatenated into a vector \( Y_Q = (y_{Q,1}, y_{Q,2}, \ldots, y_{Q,mn}) \in \mathbb{R}^{mn} \), where \( y_{Q,i} \) is an \( a \)-bit integer for \( i = 1, 2, \ldots, mn \). The final ciphertext \( c = (c_1, c_2, \ldots, c_{mn}) \in \mathbb{R}^{mn} \) can be obtained after an element-level diffusion, which can be described as

\[
c_i = c_{i-1} \oplus y_{Q,i} \oplus k_i,
\]

(19)

where \( i = 1, 2, \ldots, mn \) and \( c_0 = 0 \). In (19), the keystream \( k = (k_1, k_2, \ldots, k_{mn}) \) is generated by using the Logistic-Tent chaotic system [45], where \( k_i \) is an \( a \)-bit integer. The details of the keystream generation are described in [30].

### IV. PROPOSED CS-BASED CRYPTOGRAPHY

In this section, we describe the details of the proposed CS-based cryptosystem. For CS-encryption, we use the secret matrix of the S-OTS cryptosystem. Then, the quantized ciphertext of the S-OTS cryptosystem is fed into the proposed scrambler structure, which enhances the statistical security of the proposed cryptosystem. Fig. 3 illustrates the overall structure of the proposed CS-based cryptosystem. In the proposed CS-based cryptosystem, it is noteworthy that the LFSR-based keystream generation by SSG and Grain-128 can be easier to be implemented in real-world applications than others, e.g., chaos-based keystream generation [43], [44].

### A. CS-Encryption

The columns of an 8-bit gray-scale \( n \times n \) image are stacked into a column vector \( x \in \mathbb{R}^{N} \), where \( N = n^2 \). We assume that each 8-bit gray-scale pixel of \( x \) takes a value in \([-128, 127]\) by shifting its original value by \(-128\). With the shared key \( K_1 \), one can generate a keystream for constructing the secret matrix \( \Phi = \frac{1}{\sqrt{Mr}} SP \in \mathbb{R}^{M \times N} \), where \( S \) is a sparse matrix with only \( q \) nonzero bipolar elements in each row, and \( P \) is a column permutation matrix. The secret matrix \( \Phi \) encrypts the plaintext \( x \) to \( y = \Phi x \in \mathbb{R}^{M} \). Then, \( y \) is quantized by an \( a \)-bit quantizer. The quantized CS-encrypted ciphertext \( y_Q \in \mathbb{R}^{M} \) is

\[
y_Q = \text{round}\left(\frac{\sqrt{2^a} - 1}{\sqrt{2^a}} \cdot (y - y_{min} \cdot 1)\right),
\]

(20)

where \( y_{max} = \frac{q}{\sqrt{Mr}} 128 \), \( y_{min} = \frac{-q}{\sqrt{Mr}} 128 \), \( 1 \) is an all-one vector of length \( M \), and \( \text{round}(\cdot) \) replaces each element of vector \( \cdot \) with the nearest integer. As each element of \( x \) is between \(-128 \) and \( 127 \), it is readily checked that the range of the elements of \( y \) is from \(-\frac{q}{\sqrt{Mr}} 128 \) to \( \frac{q}{\sqrt{Mr}} 128 \) with its resolution \( \frac{1}{\sqrt{Mr}} \), from which \( y \) has \( 2 \cdot q \cdot 128 + 1 \) possible values. Thus, the number of quantization level should be sufficiently high to represent all possible values of each element of \( y \), i.e., \( 2^a \geq 2 \cdot q \cdot 128 + 1 \). This suggests that the number of quantization bits should be \( a \geq \log_2(2 \cdot q \cdot 128 + 1) \).

### B. Proposed Scrambler Structure

In (2), the \( i \)-th row of \( S \) has the index set \( \Lambda_i = \{(i-1) \mod \eta + l | l = 1, \ldots, q\} \) for the nonzero elements, where \( \eta = \frac{N}{M} \). Accordingly, it is clear that \( \Lambda_{i+\eta} = \Lambda_i \). Assume that \( \frac{M}{\eta} = pq \) is an integer with the compression...
ratio $\rho = \frac{M}{N}$. In $y = \frac{1}{\sqrt{M}}SPx$, if a variation occurs at
$x_k$ for $x = (x_1, x_2, \ldots, x_N)^T$, it would be dispersed to
$y_k, y_{k+\eta}, \ldots, y_{k+(\rho_1-1)\eta}$, where the index $k$ is permuted
to $k' \in \Lambda = \Lambda_{d+\eta} = \cdots = \Lambda_{d+(\rho_1-1)\eta}$ by $Px$.
Then, $y_Q = (y_{Q,1}, y_{Q,2}, \ldots, y_{Q,M})^T$ can be divided to $\rho_1$
segments, i.e., $y_Q = (y_Q, 1, y_Q, 2, \ldots, y_Q, (\rho_1)^T$, where the $w$-
th segment can be represented as

$$y_Q^w = (y_{Q, w-1+\eta+1}, y_{Q, (w-1+\eta+2), \ldots, y_{Q, M}})$$

for $w = 1, 2, \ldots, \rho_1$. Then, the variation of $x_k$ will be
embedded to each segment for the avalanche effect after scrambling.

In Fig. 3, $m^{(1)}, m^{(2)}, \ldots, m^{(\rho_1)}$, which are the corre-
sponding bitstreams of $y_Q^1, y_Q^2, \ldots, y_Q^\rho$ respectively,
are fed into the proposed scrambler structure in parallel.

In the scrambler structure, the reverse order feeding by the LIFO buffer
can cause the avalanche effect for each scrambler chain. Suppose that a bitstream
$m^i(w) = (m^i_1(w), m^i_2(w), \ldots, m^i_M(w))$ enters Scrambler 1, which
yields $s^i(w) = (s^i_1(w), s^i_2(w), \ldots, s^i_M(w))$. If another bitstream
$s_{err}^i(w) = (s_{err,1}^i(w), s_{err,2}^i(w), \ldots, s_{err,M}^i(w))$ enters Scrambler 1,
where $s_{err,w}^i = m^i_w + 1$ if $i = t$, and $s_{err,w}^i = m^i_w$ otherwise,
it yields $s_{err}^i(w)$ = $(s_{err,1}^i(w), s_{err,2}^i(w), \ldots, s_{err,M}^i(w))$. Then,
we may assume that $g \approx \frac{H}{2}$ bits of $s_{err}^i(w)$ will be different
with those of $s^i(w)$. This is because the recursion of Scrambler 1
causes almost a half of the last $(H-t)$ bits of $s^i(w)$ and $s_{err}^i(w)$
to be different. Thus, we expect that the one-bit difference
at the $t$-th position can cause the avalanche effect by Scrambler
1 only if $t \ll H$. Meanwhile, if $s^i(w)$ and $s_{err}^i(w)$ are fed in
reverse order through the LIFO buffer, there is a $g$-bit difference
between the first $(H-t)$ bits of the corresponding
encrypted LIFO outputs $m^i(w)$ and $m^i_{err}$. This results in
approximately $\frac{t}{2}$-bit difference between the last $t$ bits of $s^i(w)$
and $s_{err}^i(w)$ at the output of Scrambler 2. Finally, two output
bitstreams $s^i(w)$ and $s_{err}^i(w)$ would be almost 50% different
regardless of the one-bit difference position $t$.

When $m^i(w)$ and $m^i_{err}$ with $H = 10^4$ enter a scrambler
chain, Fig. 4 sketches the bit difference rate at the
outputs of Scrambler 1 and Scrambler 2, respectively. We
assume that every scrambler is characterized by a polynomial
$P(x) = 1 + x^{18} + x^{23}$. We denote $d_1$ as the number of
different bits between $s^i(w)$ and $s_{err}^i(w)$ for Scrambler 1 and $d_2$
as the number of different bits between $s^i(w)$ and $s_{err}^i(w)$ for
Scrambler 2. We can see that the bit difference rate $b_1 = \frac{d_1}{H}$
of Scrambler 1 decreases as $t$ increases, which means that the
avalanche effect cannot be guaranteed by a single scrambler.
On the other hand, $b_2 = \frac{d_2}{H}$ of Scrambler 2 maintains almost

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**FIGURE 3: Proposed CS-based cryptosystem**
In decryption, the received ciphertext \( \hat{c} \) can be divided to \( \hat{c}^{(1)}, \hat{c}^{(2)}, \cdots, \hat{c}^{(pq)} \), which yield the corresponding bitstreams \( \hat{s}^{(1)}_y, \hat{s}^{(2)}_y, \cdots, \hat{s}^{(pq)}_y \) respectively. Each bitstream \( \hat{s}^{(w)}_y = (\hat{s}^{(w)}_1, \hat{s}^{(w)}_2, \cdots, \hat{s}^{(w)}_{\rho q}) \) is decrypted by a Grain-128 keystream \( k \) and then fed into Descrambler 1, where \( k \) is generated by using the shared key \( K_2 \). With the LIFO buffer, \( m^{(w)}_T \) is reversely fed into Descrambler 2 as an input bitstream \( \hat{s}^{(w)}_y \) to yield \( \hat{m}^{(w)}_T \), which finally returns \( \hat{y}^{(w)}_Q \). Then, \( \hat{y}_Q = (\hat{y}^{(1)}_Q, \hat{y}^{(2)}_Q, \cdots, \hat{y}^{(pq)}_Q)T \) is scaled to the range of \( y \), i.e.,

\[
\hat{y} = \frac{\hat{y}_Q \cdot (y_{\text{max}} - y_{\text{min}})}{(2^a - 1)} + y_{\text{min}} \cdot 1.
\]

We can get the plaintext \( \hat{x} \) after CS-decryption through an l1-minimization algorithm

\[
\hat{\alpha} = \arg \min_{\alpha} \| \alpha \|_1 \text{ subject to } \hat{y} = \hat{\Psi} \hat{\Psi}^T \hat{\alpha},
\]

where \( \hat{\Psi} \in \mathbb{R}^{N \times N} \) is an orthonormal basis, such as 2D version of the Daubechies 4 (D4) wavelet basis with \( \hat{\Psi} = \Phi \hat{\Psi}^T \hat{\alpha} \).

Finally, the decrypted plaintext \( \hat{x} \) is mapped to the range \([0, 255]\) by adding 128 to each element.

V. EXPERIMENTAL RESULTS

In this section, we demonstrate the statistical security of the proposed CS-based cryptosystem through numerical experiments. The statistical security measures are compared to those of the circular shift-based [23], the Kronecker CS-based [24], the division-based [25], and the parallel CS-based [30] cryptosystems, which are referred to as circular CS, Kronecker CS, division CS, and parallel CS, respectively. To demonstrate the statistical security enhancement by the proposed scrambler structure, we encrypt plaintexts only with the S-OTS cryptosystem [32], which is called S-OTS only. In contrast, the proposed cryptosystem, called S-OTS scrambled, includes the proposed scrambler structure, where Scrambler 1 and Scrambler 2 are characterized by a polynomial \( P(x) = 1 + x^{18} + x^{23} \), respectively. We use ‘Lena’, ‘Barbara’, ‘Boat’, ‘Plane’, and ‘Peppers’ of \( 256 \times 256 \) test images, which are shown in Fig. 5. In S-OTS encryption, the SSG operates with a 128-stage LFSR, the number of the nonzero elements of each row of the secret matrix is \( q = 8 \), the compression ratio is \( \rho = 0.625 \), and the quantization bit size is \( a = 12 \). For the Kronecker CS, we select the parameter \( \rho = 8 \) for the secret matrix. With the same quantization bit size.
TABLE 1: PSNR (dB) of decrypted images in noiseless condition

| Scheme                  | Image          | ‘Lena’ | ‘Barbara’ | ‘Boat’ | ‘Plane’ | ‘Peppers’ |
|-------------------------|----------------|--------|-----------|--------|--------|----------|
| S-OTS scrambled         |                |        |           |        |        |          |
| circular CS [23]        |                | 34.7884| 32.0400   | 31.9992| 33.1949| 35.0885  |
| Kronecker CS [24]       |                | 32.9776| 28.7020   | 28.5125| 29.0521| 31.4574  |
| division CS [25]        |                | 30.0425| 27.2451   | 26.2842| 27.5739| 29.1625  |
| parallel CS [30]        |                | 31.8795| 28.2499   | 28.0019| 28.6990| 29.7942  |

![Figures of histograms](image_url)

FIGURE 6: The histograms of original and encrypted images (S-OTS only and S-OTS scrambled)

size and compression ratio, the other experimental settings of circular, Kronecker, division, and parallel CS, such as initial values and control parameters for chaotic systems and initial conditions, are selected randomly to meet the constraints presented in [23]–[25], [30], respectively. For all CS-based cryptosystems, each $a$-bit ciphertext $c$ is mapped to $\hat{e} = \lfloor \frac{x}{2^a} \rfloor c$ in $[0, 255]$ for statistical analysis.

A. CS-DECRYPTION PERFORMANCE

In [32], reliable and stable CS-decryption of the S-OTS cryptosystem have been theoretically guaranteed. Moreover, the phase transition shown in [32] demonstrated that the CS-decryption performance of the S-OTS cryptosystem is similar to that of the B-OTS cryptosystem with random Bernoulli secret matrices. Clearly, the S-OTS scrambled inherits the theoretically guaranteed reliable performance of CS-decryption from the S-OTS cryptosystem.

The peak signal-to-reconstruction noise ratio (PSNR), defined by $\text{PSNR} = 10 \log_{10} \left( \frac{N \times 255^2}{\|x - \hat{x}\|^2} \right)$, can be used to evaluate the CS-decryption performance, where $x$ and $\hat{x}$ are original and decrypted plaintexts, respectively. For CS-decryption, we employ SPGL1 [48] for the basis pursuit (BP) with D4 wavelet basis. The decryption results of the proposed cryptosystem in noiseless condition are shown in Fig. 5, which demonstrates that the decrypted images are visually acceptable. Table 1 shows that the proposed cryptosystem numerically guarantees higher PSNR for decrypted images than circular, Kronecker, division, and parallel CS. Therefore, we conclude that the proposed cryptosystem can achieve more reliable CS-decryption performance than the other CS-based cryptosystems, while guaranteeing theoretically reliable and stable CS-decryption performance.

B. HISTOGRAM

Fig. 6 shows the histograms of original and encrypted images of S-OTS only and S-OTS scrambled, respectively. A secure cryptosystem should conceal the histograms of original images and have ciphertexts with uniform histograms. We can see that the encrypted images of the S-OTS only show non-uniform histograms, which can be vulnerable to statistical attacks. The encrypted images of the S-OTS scrambled show uniform histograms, which means that the proposed scrambler structure can successfully flatten the histograms of the ciphertexts of the S-OTS only. We also observed that the encrypted images of ‘Lena’, ‘Barbara’, ‘Boat’, ‘Plane’, and...
‘Peppers’ have uniform histograms for circular, Kronecker, division, and parallel CS. Therefore, our proposed CS-based cryptosystem is statistically secure with the uniform histograms of ciphertexts.

### C. ENTROPY

In Table 2, the entropies of encrypted images of each CS-based cryptosystem are shown. We observe that the S-OTS scrambled can increase entropies of the encrypted images of the S-OTS only by means of scramblers, which demonstrates the statistical security enhancement by the proposed scrambler structure. Similar to circular, Kronecker, division, and parallel CS, the S-OTS scrambled has entropies close to the ideal value 8. Therefore, the S-OTS scrambled is sufficiently secure in terms of entropy.

### D. CORRELATION

In correlation analysis, we randomly select 2,000 adjacent pixel pairs from an encrypted image and calculate the correlation coefficients by using (5) in horizontal, vertical and diagonal directions, respectively. Table 3 shows that the encrypted images of the S-OTS scrambled present low correlation coefficients, similar to the other CS-based cryptosystems, which demonstrates the statistical security of the proposed cryptosystem. We observed similar results of low correlation coefficients for the other test images of ‘Boat’, ‘Plane’ and ‘Peppers’ for all the CS-based cryptosystems, respectively.

For analyzing the correlation, we also examine the correlation distribution, which is a scatter diagram plotting randomly selected 2,000 adjacent pixel pairs of an encrypted image. Fig. 7 displays the correlation distributions of original and encrypted images of ‘Lena’. We also observed that the other test images show similar correlation distributions for original and encrypted images of ‘Lena’. We also observed that the adjacent pixel pairs are highly correlated. The encrypted

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**TABLE 2: Entropies of encrypted images**

| Scheme           | Image    | ‘Lena’ | ‘Barbara’ | ‘Boat’ | ‘Plane’ | ‘Peppers’ |
|------------------|----------|--------|-----------|--------|---------|-----------|
| S-OTS only [32]  |          | 6.1137 | 6.2853    | 6.0534 | 6.6391  | 6.2931    |
| S-OTS scrambled  |          | 7.0956 | 7.79956   | 7.9958 | 7.99954 | 7.99954   |
| circular CS [23] |          | 7.9961 | 7.9956    | 7.9956 | 7.9956  | 7.9954    |
| Kronecker CS [24]|          | 7.9955 | 7.9946    | 7.9956 | 7.9956  | 7.9954    |
| division CS [25] |          | 7.9950 | 7.9958    | 7.9956 | 7.9951  | 7.9953    |
| parallel CS [30] |          | 7.9954 | 7.9950    | 7.9951 | 7.9954  | 7.9954    |

**FIGURE 7:** Correlation distributions of original and encrypted ‘Lena’ in the horizontal (first row), vertical (second row) and diagonal (third row) directions.

---

**TABLE 3: Correlation coefficients ($\times 10^{-4}$) of encrypted images.**

| Scheme           | Image    | ‘Lena’ | ‘Barbara’ |
|------------------|----------|--------|-----------|
| S-OTS only [32]  |          | (91, 307, 251) | (−130, −279, 215) |
| S-OTS scrambled  |          | (27, 485, 185)  | (−78, −156, 218)  |
| circular CS [23] |          | (271.108, −350)  | (273.96, −416)  |
| Kronecker CS [24]|          | (111, −41, −172)  | (−259, −134, 4)  |
| division CS [25] |          | (−135, 253.3)  | (140, 278, −123)  |
| parallel CS [30] |          | (438, 73, −66)  | (−56, −429, −382)  |
image of the S-OTS only has a concentrated distribution. On the other hand, the uniform distribution of the S-OTS scrambled suggests that adjacent pixel pairs of the encrypted image are uncorrelated to each other. Therefore, we conclude that the proposed scrambler structure can successfully reduce the correlation between adjacent pixel pairs.

### E. PLAINTEXT SENSITIVITY

To analyze the plaintext sensitivity, we calculate the NPCR and the UACI for test images. For secure encryption, the NPCR and the UACI should be close to the theoretically expected values, 99.6094% and 33.4635%, respectively.

In numerical experiments, we randomly select a pixel position and flip the least significant bit of the pixel for test images. To evaluate the plaintext sensitivity, we examine the average NPCR and UACI over 100 trials of each cryptosystem. Table 4 shows that circular, Kronecker, division and parallel CS with ciphertext element-level diffusion mechanisms show lower average NPCR and UACI than the theoretically expected values, respectively, while the S-OTS scrambled has average NPCR and UACI close to the theoretically expected values, respectively. This demonstrates that our proposed CS-based cryptosystem has high plaintext sensitivity. Moreover, the variances of NPCR and UACI for ‘Lena’ are 0.0027 and 0.0272 for the S-OTS scrambled, 271.3460 and 30.7554 for the circular CS, 1120 and 24.3601 for the Kronecker CS, 262.2163 and 1.0785 for the division CS, 1179 and 22.3574 for the parallel CS, respectively. We observed similar results for the other test images of ‘Barbara’, ‘Boat’, ‘Plane’ and ‘Peppers’. Since circular, Kronecker, division, and parallel CS have large variances in NPCR and UACI, their diffusion performance for the plaintext sensitivity is not stable. Meanwhile, extremely small variances of the S-OTS scrambled indicate that our diffusion mechanism can stably guarantee high plaintext sensitivity. As the S-OTS only rarely achieves high plaintext sensitivity, the stable diffusion performance of the S-OTS scrambled demonstrates security enhancement in terms of plaintext sensitivity by the proposed scrambler structure.

Fig. 8 shows the differential images of the CS-based cryptosystems for test image ‘Lena’, respectively. In this experiment, the original ‘Lena’ is encrypted to the ciphertext $C_1$, while the other ciphertext $C_2$ is obtained from a modified ‘Lena’ with one pixel modification. Then, bit-wise XOR operation between each pixel pair of $C_1$ and $C_2$ yields the differential image. The noise pattern of Fig. 8(a) demonstrates security enhancement in terms of plaintext sensitivity by the proposed scrambler structure.

### F. KEY SPACE

It is important to have a large key space to resist a brute-force attack for a secure cryptosystem. In the proposed cryptosystem, we use a 128-bit key for the SSG to construct the secret matrix $\Phi$ and a 128-bit key for Grain-128 for diffusion. This means that the key space of the proposed cryptosystem is $2^{128} \times 2^{128} = 2^{256}$, which implies that the proposed cryptosystem can be secure against a brute-force attack with

| Scheme                  | Measure NPCR ($\times 10^{-4}$) | Image ‘Lena’ | ‘Barbara’ | ‘Boat’ | ‘Plane’ | ‘Peppers’ |
|-------------------------|---------------------------------|--------------|-----------|--------|---------|-----------|
| S-OTS only [32]         |                                 | 19           | 17        | 17     | 16      | 17        |
| S-OTS scrambled         |                                 | 0.0727       | 0.0670    | 0.0679 | 0.0641  | 0.0670    |
| circular CS [23]        | NPCR                            | 99.6299      | 99.6224   | 99.6109 | 99.6121 | 99.6114   |
|                         | UACI                            | 33.4677      | 33.4666   | 33.4660 | 33.4718  | 33.4821   |
| Kronecker CS [24]       | NPCR                            | 62.6405      | 50.0851   | 46.9720 | 66.5797  | 63.9393   |
| division CS [25]        | UACI                            | 21.0327      | 16.8183   | 15.8107 | 20.3693  | 21.4801   |
| parallel CS [30]        |                                 | 35.9286      | 29.2818   | 33.2155 | 28.4033  | 36.2758   |

FIGURE 8: Differential images of ciphertexts $C_1$ and $C_2$ with size $160 \times 256$. The ciphertext $C_1$ is obtained by original ‘Lena’, while the other ciphertext $C_2$ is obtained from ‘Lena’ with one pixel modification. (a) S-OTS scrambled (b) circular CS (c) Kronecker CS (d) division CS (e) parallel CS
TABLE 5: NPCR (%) and UACI (%) for key sensitivity in CS-based cryptosystems

| Scheme          | Measure | 'Lena' | 'Barbara' | 'Boat' | 'Plane' | 'Peppers' |
|-----------------|---------|--------|-----------|--------|---------|-----------|
| S-OTS scrambled | NPCR    | 99.6102| 99.6049   | 99.6014| 99.6091| 99.6124   |
|                 | UACI    | 33.4638| 33.4567   | 33.4595| 33.4675| 33.4640   |
| circular CS [23]| NPCR    | 99.6094| 99.6104   | 99.6070| 99.6113| 99.6062   |
|                 | UACI    | 33.4651| 33.4652   | 33.4691| 33.4593| 33.4639   |
| Kronecker CS [24]| NPCR   | 99.6120| 99.6038   | 99.6104| 99.6078| 99.6071   |
|                 | UACI    | 33.4562| 33.4641   | 33.4689| 33.4554| 33.4578   |
| division CS [25]| NPCR    | 99.5990| 99.5829   | 99.6111| 99.5861| 99.6052   |
|                 | UACI    | 33.5579| 33.5472   | 33.5888| 33.3956| 33.5644   |
| parallel CS [30]| NPCR    | 99.6089| 99.6126   | 99.6068| 99.6209| 99.6071   |
|                 | UACI    | 33.4634| 33.4750   | 33.4741| 33.4370| 33.5171   |

TABLE 6: PSNR (dB) of decrypted images of CS-based cryptosystems with a wrong key

| Scheme          | Image  | 'Lena' | 'Barbara' | 'Boat' | 'Plane' | 'Peppers' |
|-----------------|--------|--------|-----------|--------|---------|-----------|
| S-OTS scrambled | PSNR   | 9.2766 | 8.8251    | 9.4506 | 7.3328  | 8.6472    |
| circular CS [23]|        | 5.9674 | 6.0264    | 5.8058 | 3.6655  | 6.0358    |
| Kronecker CS [24]|       | 6.1381 | 6.1648    | 6.0751 | 6.2047  | 4.6905    |
| division CS [25]|        | 7.0838 | 7.1050    | 7.0187 | 5.1825  | 7.1631    |
| parallel CS [30]|        | 6.5898 | 6.4962    | 6.3569 | 5.5337  | 6.3787    |
| benchmark       |        | 9.2749 | 8.9388    | 9.3759 | 8.0782  | 8.9232    |

(a) Decrypted ‘Lena’ with a correct key
(b) Decrypted ‘Lena’ with a one-bit wrong key

FIGURE 9: Decrypted ‘Lena’ images from the S-OTS scrambled with a one-bit wrong key

a sufficiently large key space.

G. KEY SENSITIVITY

For a secure cryptosystem, if we modify the key slightly, the ciphertext should be changed significantly. In numerical experiments for key sensitivity, we change either K1 or K2 by one-bit randomly for the S-OTS scrambled, while one of the chaotic parameters is changed by 10−15 randomly for circular, Kronecker, division, and parallel CS, respectively. Table 5 shows the average NPCR and UACI over 100 trials of the ciphertexts of all the test images for the CS-based cryptosystems, when we use a wrong key. For a cryptosystem with high key sensitivity, it is reasonable to assume that a decrypted image by a wrong key has the uniform distribution of the pixel values to provide no useful information about the original image. Under this assumption, Table 6 also presents the average PSNR of the decrypted images of uniformly distributed pixel values as a benchmark. Compared to the other CS-based cryptosystems, the PSNR of the decrypted images of the S-OTS scrambled is relatively high, but close to the benchmark PSNR, where the decrypted images with a one-bit wrong key are not recognizable, as shown in Fig. 9(b).

H. COMPUTATIONAL COMPLEXITY

For each CS-based cryptosystem, Table 7 shows the numbers of addition, multiplication, and XOR operations for encryption, respectively. Since the CS-decryption performance of the S-OTS cryptosystem is known to be irrelevant to q [32], we can select q ≪ n to reduce the computational complexity for the S-OTS scrambled, while maintaining reliable CS-decryption performance. Given identical compression ratio ρ and quantization bit size a for all the CS-based cryptosystems, the computational complexity including all operations of the S-OTS scrambled is O(n^2). For the Kronecker CS, the minimum computational complexity is O(n^2) for ρ = 2, since ρ ≥ 2 is an integer from the secret matrix structure of the Kronecker CS. This implies that the S-OTS scrambled can have similar computational complexity to the Kronecker CS. Also, the computational complexity of the S-OTS scrambled is lower than the computational complexity O(n^4) of circular, division, and parallel CS, respectively. Note that the multiplication operations with bipolar entries of the secret...
matrix of the $S$-OTS scrambled can be implemented as simple operations, i.e., sign change for the numbers multiplied with the negative entries. Meanwhile, the other CS-based cryptosystems require more complex multiplication with floating point numbers. Therefore, we conclude that the proposed cryptosystem has a benefit in terms of computational complexity with a proper selection of $q$.

VI. CONCLUSION

In this paper, we proposed a secure CS-based cryptosystem for image encryption with hardware-friendly structure, which can be easily implemented in real world applications. In addition to a sparse secret matrix for efficient CS-encryption, a scrambling mechanism has been proposed for enhancing the statistical security of the CS-encryption. We numerically confirmed that the proposed CS-based cryptosystem has more reliable CS-decryption performance than the other CS-based cryptosystems. For analyzing the statistical security, we investigated the histogram, entropy, correlation and plaintext sensitivity of the proposed cryptosystem, which demonstrates that the proposed cryptosystem is sufficiently secure in terms of the statistical measures. In particular, we confirmed that our bit-level diffusion by the proposed scrambler structure has superior performance in plaintext sensitivity than the element-level diffusion mechanisms of the other CS-based cryptosystems. In conclusion, the proposed cryptosystem can be statistically secure thanks to the proposed scrambler structure.

Despite the strong statistical security, the proposed CS-based cryptosystem may have some potential drawbacks. First, the bit-level diffusion by the proposed scrambler structure takes longer time than the element-level diffusion. Also, the proposed scrambler structure requires memory space for the LIFO buffers as well as the scramblers. For a future work, we will further study to reduce the time complexity and the memory requirement for the proposed cryptosystem. We found that the CS-based encryption of [49] employs the hash of a plaintext for keystream generation. Our future work may exploit the hash-based structure to improve our proposed CS-based cryptosystem further.

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| Scheme       | Operation           | Addition   | Multiplication | XOR       |
|--------------|---------------------|------------|----------------|-----------|
| S-OTS scrambled | $p^q n (q-1)$ | $p^q q$   | $p^q n^2$ | $5p^q n^2$ |
| Circular CS  | $p^q (n-1) + 2p^q$ | $p^q + 2p^q$ | $p^q n^2$ | $2p^q n^2$ |
| Kronecker CS | $p^q (n-1)$ | $p^q$       | $p^q n^2$ | $2p^q n^2$ |
| Division CS  | $p^q (n-1)$ | $p^q$       | $p^q n^2$ | $2p^q n^2$ |
| Parallel CS  | $p^q (n-1)$ | $p^q$       | $p^q n^2$ | $2p^q n^2$ |
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