Exact multi-membrane solutions in $AdS_7$

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Abstract

We study the properties of an exact multi-membrane solution in seven-dimensional maximal $SO(5)$-gauged supergravity. Unlike previously known multi-centered solutions, the present one is asymptotically anti-de Sitter. We show that this multi-membrane configuration preserves only a quarter of the supersymmetries. When lifted to eleven dimensions, this solution is interpreted as a set of open membranes ending on self-dual strings on a stack of M5-branes, in the near M5 limit.

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I. INTRODUCTION

The investigation of $p$-brane solutions in supergravities has been an important aspect in the study of M-theory. Understanding the basic properties of $p$-branes has led to important ideas such as dualities, D-branes, and the AdS/CFT conjecture. By now, general $p$-brane solutions in ungauged supergravities are well understood, and much progress has been made on their classification. However much less is known about a perhaps equally important set of objects, namely $p$-branes in anti-de Sitter space, viewed as solutions of corresponding gauged supergravities. A main reason behind this lack of development is that flatness and Ricci flatness has generally been an integral part of the construction of such solutions. For branes that are asymptotically anti-de Sitter, one no longer has the simplifying condition of Ricci flatness. Along the same lines, it has been argued that static extended objects in anti-de Sitter space cannot satisfy a no-force condition, as the background curvature would otherwise provide an unbalanced cosmological force. These complications of having curved world-sheets and possible time dependence seem inherent in the study of $p$-branes in gauged supergravities.

Of course, many of the above difficulties are avoided for the case of 0-branes, or black holes. In fact both de Sitter and anti-de Sitter black holes have been known for a long time. More recently, black hole solutions to gauged supergravities have been constructed and studied in four \cite{1,2}, five \cite{3,4} and seven \cite{5,6} dimensions. However all such constructions have focused on single centered (although possibly multiply charged) black holes. Finding multi-centered solutions has generally been as elusive as finding extended object solutions for much the same reasons.\cite{1}

For the case of extended $p$-branes, an initial attempt at constructing a membrane solution to maximal gauged supergravity in seven dimensions was made in Ref. \cite{8}. While no explicit solution was given, important properties such as the nature of charges and supersymmetry were investigated. An exact magnetic string solution to $\mathcal{N} = 2$ gauged supergravity in five dimensions was also constructed and investigated in Refs. \cite{10,11}. In a later development, Lü and Pope demonstrated a braneworld reduction of gauged supergravity where the bulk theory reduces to an ungauged theory on the brane preserving half of the original supersymmetries \cite{12,13}. This Lü-Pope ansatz provided a breakthrough in the construction of $p$-branes in gauged supergravities by providing a means of lifting well-known solutions of the lower-dimensional ungauged theory to yield new solutions to the gauged supergravity theory.

Consistency of the Lü-Pope ansatz provides serious restrictions on the fields that may be lifted or reduced. For bosonic fields, in addition to the graviton, only scalars and $n$-form potentials satisfying odd-dimensional self-duality equations \cite{14,15} may be consistently reduced, yielding scalars and $n$-form field strengths, respectively, on the brane. Hence only $p$-branes charged under the appropriate fields may be constructed through this lifting technique. Furthermore, use of odd-dimensional self-dual fields suggests that the resulting

\footnote{Partial progress was made in Ref. \cite{9} which constructed multi-centered Euclidean AdS$_5$ black holes.}
$p$-branes must necessarily be dyonically charged. Nevertheless, the Lü-Pope ansatz opens up the possibility of constructing new solutions to gauged supergravities.

In this letter, we revisit the seven-dimensional system and find an exact multi-membrane solution by applying the Lü-Pope ansatz to the self-dual string in six dimensions. Examination of supersymmetry indicates that only a quarter of the original supersymmetries survive. This is in contrast to the basic membrane solution of ungauged supergravity, which preserves half of the supersymmetries.

When lifted to eleven dimensions, this multi-membrane solution corresponds to the near M5-brane horizon limit of a set of open membranes ending on self-dual strings on a stack of M5-branes [16] [17]. Since the self-dual strings of the $(2,0)$ theory preserve only eight supercharges, this explains why no supersymmetry restoration takes place in the present configuration.

II. $N = 4, D = 7$ GAUGED SUPERGRAVITY

The bosonic sector of maximal $SO(5)$ gauged supergravity in seven dimensions contains a graviton, ten Yang-Mills fields transforming in the adjoint of $SO(5)_g$, five 3-forms in the 5 of $SO(5)_g$ and fourteen scalars which parametrize the $SO(5)_c$ indices. In addition, there are four gravitini and sixteen spin-$\frac{1}{2}$ fields transforming as the 4 and 16 of $SO(5)_c$ respectively.

The bosonic Lagrangian is given by [12,13]:

$$e^{-1} \mathcal{L} = R + \frac{1}{2} g^2 (T^2 - 2T_{ij} T^{ij}) - T r (P_\mu P^{\hat{\mu}}) - \frac{1}{2} (V_i V_j F_{ij})^2 - \frac{1}{12} (V_i^{-1} S_i^{ij})^2$$

$$+ e^{-1} g^{-1} \left[ \frac{1}{2} S_i^I \wedge d S_i^I + \frac{1}{2} \xi_{ILM} S_i^I \wedge F_{IJK} \wedge F_{LM} + \Omega_7 (A, F) \right].$$

Here we follow the notation of [12,13]. In particular, we work with a metric of signature $(- + \ldots +)$ and the 3-forms $S_i^I$ are rescaled from those given in Ref. [13]. Upper case indices $I, J = 1, \ldots, 5$ denote $SO(5)_g$ indices, while lower case $i, j, k = 1, \ldots, 5$ denote $SO(5)_c$ indices. Finally, $\Omega_7$ is a Chern-Simons form built from the gauge fields. As we will set $A_i^{[IJ]} = 0$, we will not need its explicit form.

The fourteen scalar degrees of freedom are contained in the coset elements $V^i_i$, transforming as a 5 under both $SO(5)_g$ and $SO(5)_c$. These funfbeins may then be used to build the $T$-tensor, $T_{ij} = V_i^{-1} V_j^{-1}$, with trace $T \equiv T_{ij} \delta^{ij}$. The scalar kinetic term, $P_\mu$, and composite $SO(5)_c$ connection, $Q_\mu^{i,j}$, are defined through $V_i^{-1} \mathcal{D}_\mu V_j = (Q_\mu^{i,j}) + (P_\mu^{i,j})$, where $\mathcal{D}_\mu$ is the gauge covariant derivative.

The relevant supersymmetries are given by

$$\delta \psi_\mu = \left[ \mathcal{D}_\mu + \frac{g}{20} T \gamma_\mu - \frac{1}{40} (\gamma_\mu \gamma^{\lambda} - 8 \gamma_\mu ^{\hat{\lambda}} \gamma^{\hat{\lambda}}) \Gamma^{ij} V_i V_j F_{i\hat{j}}^{j} \right] \epsilon,$$

$$\delta \lambda_i = \left[ \frac{g}{2} (T_{ij} - \frac{1}{5} \delta _{ij} T) \Gamma^j + \frac{1}{2} \gamma ^{\hat{\mu}} P_{\hat{\mu}ij} \Gamma^j + \frac{1}{16} \gamma ^{\hat{\mu} \hat{\nu}} (\Gamma^{kl} \Gamma^i - \frac{1}{5} \Gamma^{i} \Gamma^{kl}) V_K^{k} V_L^{l} F_{\mu \nu}^{KL} \right] \epsilon.$$  

Note that consistency requires both $\Gamma^i \lambda_i = 0$ and $\Gamma^i \delta \lambda_i = 0$. 

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In Refs. [12,13], it was argued that this maximal gauged $N = 4$ theory in seven dimensions may be reduced to yield ungauged $N = (2, 0)$ pure supergravity in six dimensions. We recall that the latter theory contains a $D = 6$ graviton, five 2-forms with self-dual field strength [transforming as a vector of global $SO(5)$] and four gravitini, but no scalars nor Yang-Mills fields. This suggests that an appropriate braneworld reduction is simply to take trivial scalars, $V_i^I = \delta_i^I$, and vanishing gauge fields, $A_{IJ}^I = 0$. The consistency of this truncation was demonstrated in Ref. [13], provided

\[ S_I^{[3]} \wedge S_J^{[3]} = 0, \quad *S_I^{[3]} \wedge S_J^{[3]} = 0, \quad \text{for all } I, J. \]

In this case, the Lagrangian (1) simplifies to

\[ e^{-1}L = R + \frac{15}{2} g^2 - \frac{1}{12} (S_{\hat{\mu}\hat{\rho}}^{I})^2 + e^{-1} \left[ \frac{1}{2} g^{-1} S_I^{[3]} \wedge dS_I^{[3]} \right], \quad \text{(5)} \]

while the supersymmetry transformations become

\[ \delta \psi_\mu = \left[ \nabla_\mu + \frac{g}{4} \gamma_\mu + \frac{1}{60} \left( \gamma_\mu \gamma_\lambda \delta - \frac{9}{2} \delta_\mu \gamma_\lambda \delta \right) \Gamma I S_{\hat{\nu}\hat{\rho}}^{I} \right] \epsilon, \quad \text{(6)} \]

\[ \delta \lambda_I = \left[ \frac{1}{120} \gamma_\mu \delta \left( \Gamma^{IJ} - 4 \delta^{IJ} \right) S_{\hat{\mu}\hat{\nu}\hat{\rho}}^{J} \right] \epsilon. \quad \text{(7)} \]

### III. THE MULTI-MEMBRANE SOLUTION

The Lü-Pope ansatz provides a consistent means of relating gauged supergravities to ungauged counterparts in one lower dimension with half of the original supersymmetries preserved. In the context of the Randall-Sundrum braneworld, the reduction ansatz for the above seven-dimensional theory is [12,13]:

\[ ds_7^2 = e^{-2k|z|} g_{\mu\nu}(x) dx^\mu dx^n + dz^2, \]

\[ S_I^{[3]} = e^{-2k|z|} H_I^{[3]}(x). \quad \text{(8)} \]

As indicated in [19,21], preservation of supersymmetry on the brane demands that $g$ changes sign when passing through the brane, \textit{i.e.} $g = 2k \text{sgn}(z)$.

While presented above as a braneworld reduction, it is important to note that the Lü-Pope ansatz is equally valid as a consistent reduction of the bulk gauged supergravity to the ungauged $N = (2, 0)$ theory [12]. For the supergravity reduction without the brane, one simply removes the absolute values and takes $g = 2k$. The reduction ansatz may then be viewed as a reduction of AdS$_7$ along horospherical slices. This is the primary point of view we take in the present case.

Substituting this ansatz into the equations of motion resulting from (5) yields the bosonic equations of the six-dimensional $N = (2, 0)$ theory:

\[ \text{Nevertheless we maintain the absolute values in appropriate equations, since once removed they are harder to reinsert.} \]
\[ dH_3^I = 0, \quad H_3^I = *_6 H_3^I, \]
\[ R_{\mu\nu} = \frac{1}{4} H_{\mu\rho\sigma} H_{\nu}^{\rho\sigma}. \]  

(9)

While in principle the six-dimensional gravitino variation
\[ \delta \psi_\mu = [\nabla_\mu + \frac{1}{4} H_{\mu\rho\sigma} \gamma^\rho \gamma^\sigma] \epsilon, \]

(10)

may be obtained from the reduction of (3) following the procedure of Ref. [22], we will instead work directly with the seven dimensional transformations (8) and (9).

In general, six-dimensional supergravities support the construction of BPS dyonic string solutions. However only the self-dual string is of relevance in the \( N = (2, 0) \) case. By global \( SO(5) \) invariance, the self-dual string solution may be rotated so that it carries charge under a single 2-form potential. Specifying this \( SO(5) \) direction by a unit vector \( \hat{n}^I \), the multi-string solution has the form [23,24,12]
\[ ds^2_6 = \mathcal{H}^{-1}(y) \eta_{\mu\nu} dx^\mu dx^\nu + \mathcal{H}(y) dy^2 \]
\[ H_3^I = dB^I_{[2]} + *_6 dB_{[2]}^I, \quad B_{01}^I = \mathcal{H}^{-1}(y) \hat{n}^I. \]  

(11)

The \( x^\mu, \mu = 0, 1 \) are coordinates longitudinal to the string, while the \( y^i, i = 2, 3, 4, 5 \) are transverse coordinates. The function \( \mathcal{H}(y) \) is harmonic in the transverse space. For a flat transverse space, it may take on the simple multi-string form
\[ \mathcal{H}(y) = 1 + \sum_{i=1}^m \frac{q_i}{|y - \vec{y}_i|^2}, \]

(12)

where \( \{q_i\} \) and \( \{\vec{y}_i\} \) denote respectively the charges and centers of the strings.

This solution may be lifted to seven dimensions, yielding [12]
\[ ds^2_7 = e^{-2k|z|} \left[ \mathcal{H}^{-1}(y) \eta_{\mu\nu} dx^\mu dx^\nu + \mathcal{H}(y) dy^2 \right] + dz^2, \]
\[ S^I_3 = e^{-2k|z|} [dB^I_{[2]} + *_6 dB_{[2]}^I], \quad B_{01}^I = \mathcal{H}^{-1}(y) \hat{n}^I. \]  

(13)

As a braneworld configuration, this may be interpreted as a multi-string configuration on the 5-brane. Viewed from the bulk, on the other hand, this solution corresponds to multi-membranes ending on self-dual strings on the 5-brane, where coordinates on the membrane are \( x^0, x^1, \) and \( z \). This latter interpretation is obtained by removing the absolute values, thus pushing the Randall-Sundrum brane off to the boundary of AdS\( _7 \) at \( z \to -\infty \).

### IV. SUPERSYMMETRY

We examine the supersymmetry directly in seven dimensions. Starting from the spin-\( \frac{1}{2} \) variation, (7), we first compute
\[ \gamma \cdot S^I = -6 e^{k|z|} \mathcal{H}^{-3/2} \gamma^{07} \partial_i \mathcal{H} \hat{n}^I (1 + \gamma^7), \]

(14)

to obtain

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\[ \delta \lambda_I = -\frac{1}{20} e^{k|z|} \mathcal{H}^{-3/2}(\Gamma I \hat{n} \cdot \Gamma - 5 \hat{n} I) \gamma^{01} \partial_i \mathcal{H}(1 + \gamma^7) \epsilon. \]  

(15)

Note that \( \gamma^7 \) is the six-dimensional chirality operator, \( \gamma^7 = \gamma^0 \gamma^1 \cdots \gamma^5 \), which may be identified with the additional Dirac matrix in seven dimensions. As will be seen below, we find it convenient to take \( \gamma^7 = -\gamma^7 \). In any case, preservation of supersymmetry demands that the Killing spinors vanish under the projection \( P_+ \epsilon = 0 \) where

\[ P_\pm = \frac{1}{2} (1 \pm \gamma^7). \]  

(16)

This requirement on Killing spinors holds independent of the absolute value \(|z|\) in the solution, and hence is unrelated to the presence of a "kinked" Randall-Sundrum brane. In fact, this result follows simply from satisfying odd-dimensional self-duality, as was already noted in [8].

For the gravitino, we consider the three separate variations, \( \delta \psi_\mu \), \( \delta \psi_i \) and \( \delta \psi_z \). After considerable manipulation of (6), we obtain

\[ \delta \psi_\mu = [\partial_\mu + \frac{g}{4} \gamma_\mu (1 - \frac{2k \text{sgn} z}{g} \gamma^z) - \frac{1}{2} e^{k|z|} \mathcal{H}^{-3/2} \gamma^7 \partial_i \mathcal{H}(\tilde{P}^- + \frac{2}{5} \gamma^{01} \hat{n} \cdot \Gamma P_+)] \epsilon \]

\[ \delta \psi_i = [\partial_i + \frac{1}{4} \mathcal{H}^{-1} \partial_\lambda \mathcal{H} + \frac{g}{4} \gamma_i (1 - \frac{2k \text{sgn} z}{g} \gamma^z) \]

\[ - \frac{1}{2} \gamma^j \gamma^l \mathcal{H}^{-1} \partial_j \mathcal{H} \tilde{P}^- - \frac{1}{10} \hat{n} \cdot \Gamma \mathcal{H}^{-1} \partial_i \mathcal{H} (2 \delta^l_j - 3 \gamma_i^j) \gamma^{01} \tilde{P}_+ ] \epsilon \]

\[ \delta \psi_z = [\partial_z + \frac{g}{4} \gamma_z - \frac{1}{5} \gamma_z \gamma^{01} e^{k|z|} \mathcal{H}^{-3/2} \partial_i \mathcal{H} \hat{n} \cdot \Gamma P_+] \epsilon \]

(17)

where the second projection \( \tilde{P}_\pm \) is given by

\[ \tilde{P}_\pm = \frac{1}{2} (1 \pm \gamma^{01} \hat{n} \cdot \Gamma). \]  

(18)

Note that \( \tilde{P}_\pm \) commutes with \( P_\pm \) and is suggestive of an electric BPS string lying in the 0–1 directions.

Examination of (17) verifies the consistency of the choices \( g = 2k \text{sgn} z \) and \( \gamma^z = -\gamma^7 \). The Killing spinor equations are then solved by

\[ \epsilon = e^{-\frac{k}{2} |z|} \mathcal{H}^{-1/4} P_- \tilde{P}_+ \epsilon_0, \]  

(19)

where \( \epsilon_0 \) is a constant spinor. This demonstrates that the multi-membrane solutions preserve only a quarter of the maximum supersymmetries.

V. THE NON-EXTREMLAL MEMBRANE

While our focus has been on multi-centered BPS configurations, we note that the Lü-Pope ansatz equally well allows the lifting of a black string solution to AdS\(_7\). Blackening of the self-dual string is straightforward [25], and the resulting black membrane solution may be written as
\[ds_7^2 = e^{-2k|z|} \left[ \mathcal{H}^{-1}(-f dt^2 + dx^2) + \mathcal{H} \left( \frac{dr^2}{f} + r^2 d\Omega_3^2 \right) \right] + dz^2,\]

\[S'_{[3]} = e^{-2k|z|} [dB_{[2]} + *_6 d B_{[2]}^I], \quad B_{01}^I = \coth \mu \mathcal{H}^{-1} \hat{n}' . \]

The non-extremality function \( f \) and the harmonic function \( \mathcal{H} \) specifying the solution are given by

\[ f = 1 - \frac{k}{r^2}, \quad \mathcal{H} = 1 + \frac{k \sinh^2 \mu}{r^2}. \]

The resulting self-dual charge is given by \( q = k \sinh \mu \cosh \mu \). The black membrane has a horizon located at \( r_+ = k^{1/2} \), with topology \( R \times R^+ \times S^3 \) with the second factor corresponding to the \( z \) direction. The extremal limit may be obtained by taking \( k \to 0 \) and \( \mu \to \infty \) with \( q \) held fixed.

\section*{VI. DISCUSSION}

We again emphasize that, while motivated by the search for braneworld reductions, the above technique for constructing multi-membrane solutions in anti-de Sitter backgrounds holds equally well in the bulk. It is instructive to reexamine the form of the solution, (13), where the metric takes on a horospherical form

\[ds_7^2 = e^{-g z} \left[ \mathcal{H}^{-1} (y) \eta_{\mu \nu} dx^\mu dx^\nu + \mathcal{H} (y) d\vec{y}^2 \right] + dz^2. \]

It is evident that this solution is asymptotically anti-de Sitter far from the membranes when \( \mathcal{H} \to 1 \). In addition, however, the metric longitudinal to the membrane has the form

\[ds^2 = e^{-g z} \mathcal{H}^{-1} (-dt^2 + dx^2) + dz^2,\]

which asymptotes to AdS\(_3\). Thus, unlike their flat space counterparts, membranes in anti-de Sitter space necessarily have curved world-sheets.

It should be mentioned that the metric ansatz leading to (22) was considered in Ref. [8] without success. However in that case, the difficulty was presumably in attempting to seek a half-BPS configuration. As demonstrated above, the multi-membrane solution here necessarily preserves only a quarter of the supersymmetries, as indicated in (19). Since AdS\(_7\) is maximally symmetric and maximally supersymmetric, this quarter-BPS feature is perhaps somewhat unusual. Examining the two simultaneous broken supersymmetries,\(^3\) corresponding to projections \( P_+ \) and \( \tilde{P}_- \), we note that the former selects a definite \( \gamma^7 \) eigenvalue. Thus, for Killing spinors satisfying both projections, the latter \( \tilde{P}_\pm \) may be replaced by the equivalent

\[\tilde{P}_\pm = \frac{1}{2} (1 \pm \gamma^{012} \hat{n} \cdot \Gamma).\]

\(^3\)This presence of two complimentary supersymmetry projections was already anticipated in Ref. [8].
which has a conventional form for an electrically charged membrane.

In this sense, the new feature of the membrane solution to gauged supergravity is the projection $P_\pm$ related to both odd-dimensional self duality and AdS Killing spinors. We recall that, for pure AdS in horospherical coordinates \[ i.e. for \mathcal{H} = 1 in (22), \] the Killing spinors take the form

\begin{align*}
\epsilon_- &= e^{-gz/4}P_-\epsilon^0, \\
\epsilon_+ &= e^{gz/4}[1 - \tfrac{g}{2}(x^\mu \gamma_\mu + y^i \gamma_i)]P_+\epsilon^0
\end{align*}

(25)

While $\epsilon_-$ is straightforward, we see that $\epsilon_+$ is sensitive to the horizon ($z \to \infty$), and furthermore has non-trivial dependence on all coordinates. Thus the latter Killing spinor cannot survive once the membrane solution is turned on. Viewing AdS$_7$ as the near-horizon limit of a stack of M5-branes, we recall that odd-dimensional self-duality for $S_I$ originates from the action of $F_4$ in eleven dimensions. The latter, and in particular the Chern-Simons term $F_4 \wedge F_4 \wedge A_3$, plays an important role in determining possible intersecting brane solutions of M-theory. Since the multi-membrane solution lifts to a configuration of open M2-branes ending on a stack of M5-branes, the projection $P_\pm$ may be identified with that of a M5-brane, while the projection $\bar{P}_\pm$ with that of a M2-brane. In the absence of a M2-brane, supersymmetry is enhanced in the near M5 limit, yielding the additional Killing spinors $\epsilon_+$ above. However, for the complete solution, the nature of the intersection prevents any enhancement of supersymmetry.

While the Lü-Pope ansatz sets the $SO(5)$ gauge fields to zero, it is curious to note that the multi-membrane solution preserves $SO(4) \subset SO(5)$. This hints that it may be possible to turn on $SO(4)$ gauge fields orthogonal to the direction given by $\hat{n}^I$ without breaking any further supersymmetries [8]. From a braneworld perspective, it is unclear what effect bulk gauge fields may have, and in particular it is expected that such fields cannot localize on the brane. Thus there may be possible braneworld implications for the localization of bulk Yang-Mills fields if any membrane configurations with further non-trivial breakings of the R-symmetry may be constructed.

Use of the Lü-Pope ansatz restricts consideration to a Poincaré patch of AdS$_7$. It remains an open issue whether the multi-membrane solution (13) may be transformed to some form of global AdS coordinates, or whether the singularity and horizon structure prevents this. Since one has the geometry of open membranes ending on self-dual strings, this issue is analogous to that of explicit construction of strings ending on the boundary of AdS, as considered in the AdS$_5$ case of the AdS/CFT duality. Curiously, in this AdS$_5$ case, a string solution was obtained in Refs. [10,11] using cylindrically symmetric coordinates. This string is magnetically charged under the $N = 2$ graviphoton and (similar to the membranes discussed here) preserves a quarter of the supersymmetries.

Finally, understanding open membranes ending on the fivebrane may be important in developing a better picture of the $N = (2,0)$ theory underlying the dynamics of the M5-brane. While much work has been focused on the world-sheet point of view, soliton solutions like the one presented here allow for a complimentary approach to such investigations. It is natural to suspect that gravity may be decoupled from the system, in which case one expects to obtain results consistent with that of open membrane theory [26,27]. In fact, this connection to open membranes was already shown to occur in the near horizon limit of the self-dual string in the M5-brane world-sheet approach [25]. It would be interesting to carry out a similar analysis for the multi-membrane solution given above.
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REFERENCES

[1] L.J. Romans, “Supersymmetric, cold and lukewarm black holes in cosmological Einstein-Maxwell theory, Nucl. Phys. B 383 (1992) 395 [hep-th/9203018].
[2] M.M. Caldarelli and D. Klemm, Supersymmetry of Anti-de Sitter Black Holes, Nucl. Phys. B 545 (1999) 434 [hep-th/9808097].
[3] D. Klemm, BPS Black Holes in Gauged $N = 4, D = 4$ Supergravity, Nucl. Phys. B 545 (1999) 461 [hep-th/9810090].
[4] W.A. Sabra, Anti-De Sitter BPS Black Holes in $N = 2$ Gauged Supergravity, Phys. Lett. B 458 (1999) 36 [hep-th/9903143].
[5] K. Behrndt, A.H. Chamseddine and W.A. Sabra, BPS black holes in $N = 2$ five-dimensional AdS supergravity, Phys. Lett. B 442 (1998) 97 [hep-th/9807187].
[6] K. Behrndt, M. Cvetič and W.A. Sabra, Non-extreme black holes of five-dimensional $N = 2$ AdS supergravity, Nucl. Phys. B 553 (1999) 317 [hep-th/9910227].
[7] M. Cvetič and S.S. Gubser, Phases of R-charged Black Holes, Spinning Branes and Strongly Coupled Gauge Theories, JHEP 9904 (1999) 024 [hep-th/9902195].
[8] J.T. Liu and R. Minasian, Black holes and membranes in $AdS_7$, Phys. Lett. B 457 (1999) 39 [hep-th/9903280].
[9] J.T. Liu and W.A. Sabra, Multi-centered black holes in gauged $D = 5$ supergravity, Phys. Lett. B 498 (2001) 123 [hep-th/0101023].
[10] A.H. Chamseddine and W.A. Sabra, Magnetic strings in five dimensional gauged supergravity, Phys. Lett. B 477 (2000) 329 [hep-th/9911193].
[11] D. Klemm and W.A. Sabra, Supersymmetry of black strings in $D = 5$ gauged supergravities, Phys. Rev. D 62 (2000) 024003 [hep-th/0001131].
[12] H. Lü and C.N. Pope, Branes on the Brane, Nucl. Phys. B 598 (2001) 492 [hep-th/0009183].
[13] M. Cvetič, H. Lü and C.N. Pope, Brane-world Kaluza-Klein Reductions and Branes on the Brane, [hep-th/0009183].
[14] P.K. Townsend, K. Pilch and P. van Nieuwenhuizen, Selfduality In Odd Dimensions, Phys. Lett. B 136 (1984) 38 [Addendum–ibid. 137 (1984) 443].
[15] M. Pernici, K. Pilch and P. van Nieuwenhuizen, Gauged Maximally Extended Supergravity in Seven Dimensions, Phys. Lett. B 143 (1984) 103.
[16] A. Strominger, Open p-branes, Phys. Lett. B 383 (1996) 44 [hep-th/9512053].
[17] P.K. Townsend, D-branes from M-branes, Phys. Lett. B 373 (1996) 68 [hep-th/9512062].
[18] M. Pernici, K. Pilch, P. van Nieuwenhuizen and N.P. Warner, Noncompact Gaugings and Critical Points of Maximal Supergravity in Seven-dimensions, Nucl. Phys. B 249 (1985) 381.
[19] N. Alonso-Alberca, P. Meessen and T. Ortin, Supersymmetric brane-worlds, Phys. Lett. B 482, 400 (2000) [hep-th/0003248].
[20] A. Falkowski, Z. Lalak and S. Pokorski, Supersymmetrizing branes with bulk in five-dimensional supergravity, Phys. Lett. B 491, 172 (2000) [hep-th/0004093].
[21] E. Bergshoeff, R. Kallosh and A. van Proeyen, Singularity in singular spaces, [hep-th/0009212]. Nucl. Phys. B 605 (2001) 234 [hep-th/0009212].
[22] M.J. Duff, J.T. Liu and W.A. Sabra, Localization of supergravity on the brane, Nucl. Phys. B 605 (2001) 234 [hep-th/0009212].
[23] M.J. Duff and J.X. Lu, *Black and super p-branes in diverse dimensions*, Nucl. Phys. B 416 (1994) 301 [hep-th/9306052].

[24] M.J. Duff, S. Ferrara, R.R. Khuri and J. Rahmfeld, *Supersymmetry and dual string solitons*, Phys. Lett. B 356 (1995) 479 [hep-th/9506057].

[25] M.J. Duff, H. Lü and C.N. Pope, *The black branes of M-theory*, Phys. Lett. B 382 (1996) 73 [hep-th/9604052].

[26] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, *OM theory in diverse dimensions*, JHEP 0008 (2000) 008 [hep-th/0006062].

[27] E. Bergshoeff, D.S. Berman, J.P. van der Schaar and P. Sundell, *Critical fields on the M5-brane and noncommutative open strings*, Phys. Lett. B 492 (2000) 193 [hep-th/0006112].

[28] D.S. Berman and P. Sundell, *AdS3 OM theory and the self-dual string or membranes ending on the five-brane*, hep-th/0105288.