NEUTRINOLESS DOUBLE BETA DECAY AND ITS “INVERSE”

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ABSTRACT

Recent considerations by these authors pointed out the attractive features which a search for the exchange of heavy Majorana neutrinos could have for solving the mass and the lepton number puzzles for all neutrinos, in TeV-level electron-electron scattering. In the present note, we show that, contrary to subsequently published arguments, non-observation of neutrinoless double beta decay has, to date, no bearing on the promise of this important task for future linear electron colliders.

Recent developments in the planning for electron colliders in the TeV energy region have renewed interest in the possibility to investigate the reaction

\[ e^- e^- \rightarrow W^- W^-, \]  

which can proceed by means of the exchange of a Majorana neutrino. While previous work on this reaction (Refs. [1,2,3,4]) had not led to promising experimental prospects, we showed that the new generation of presently projected linear colliders can in fact deliver luminosities compatible with the production of convincing signals for reaction (1), or, failing that, important new limits on the masses and couplings of heavy Majorana neutrinos. This is predicated on the availability of highly polarized electron beams at center-of-mass energies upward of 500 GeV, where left-handed electrons will be able to interact via the exchange of heavy left-handed Majorana singlet neutrino states with masses \( \geq 1 \) TeV. For the case of
two longitudinal $W^-$ in the final state, the relevant cross-section increases $\sim s^2$ in the kinematic region $2m_W < \sqrt{s} < m_N$. The existence of two or more such singlets follows naturally from SO(10) decomposition, and could be the key ingredient for our understanding of both the observed very light masses of the three known neutrino states, and of the meaning of lepton number and its conservation or non-conservation.

Several authors [5,6] have argued that our calculations cannot serve as the basis for a possible successful observation of reaction (1): they maintain that existing limits on the related process involving neutrinoless double beta decay [hereafter $\beta\beta_{0\nu}$] already exclude it. In so doing, the authors of Ref. [5] explicitly refer to reaction (1) as “inverse neutrinoless double beta decay”.

The fallacy of their argument can, in a nutshell, be gleaned from this misnomer: in the present note, we show that only a profound misreading of the $\beta\beta_{0\nu}$ reaction mediated by heavy Majorana neutrinos can lead to the conclusions of Refs. [5,6]. Let us consider the diagrams in Fig. 1: we ask ourselves whether the lack of observation of graph 1b can serve to impose tight constraints on the observability of graph 1a.

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{fig1.pdf}
\caption{a) diagram for the quasi-elastic production of two longitudinal $W^-$ in the scattering of TeV-level left-handed electrons, mediated by massive Majorana neutrino exchange. b) Neutrinoless double beta decay \textit{via} exchange of a heavy Majorana neutrino, with the hadronic part seen at the nucleon level.}
\end{figure}

The best present evidence on the non-observation of the $\beta\beta_{0\nu}$ reaction is from the experimental limit on the process

$$^{76}Ge \rightarrow ^{76}Se \ e^-e^-,$$

with $\tau_{1/2} > 5 \times 10^{24}$ years. How do we translate this into a limit on reaction (1)?

Literally, the inverse to reaction (2) is

$$e^-e^-^{76}Se \rightarrow ^{76}Ge$$

or, somewhat less rigidly,

$$e^-e^- \rightarrow ^{76}Ge \ 76\overline{Se}.$$
Neither of these two reactions are experimentally realizable, but the argument points up a troubling question: Can the (sub-)reactions that lead to the decays $nn \rightarrow ppe^- e^-$ occur freely inside the nuclei $^{76}\text{Ge}$ and $^{76}\text{Se}$?

At this point, we have to remark that graph 1b symbolizes a process that acts over distances of order $(m_W)^{-1}$ or about $10^{-16}\text{cm}$. If, on the other hand, the exchanged neutrinos had the masses of the known light varieties, $m_\nu < 1\text{ eV}$, the range over which the interaction extends would be $> 10^{11}$ times larger. In the heavy ($\sim 1\text{ TeV}$) neutrino exchange case, the subprocess we have to study is

$$dd \rightarrow uue^- e^-,$$

$$((nn)) \quad ((pp)) \quad ((^{76}\text{Ge})) \quad ((^{76}\text{Se}))$$

where the symbols in single and double parentheses stand for the constraining configurations within which the interacting particles of the lines above have to be considered, respectively: the hadronic systems must be treated on the quark level, but are heavily constrained by nucleon-nucleon forces, and the latter are in turn constrained by the specific wave functions of their host nuclei. It is worth noting that, in this context, the study of $\beta\beta_0\nu$ via heavy $N$ exchange can be seen as a unique probing of nuclear structure on the quark correlation level, at distances $< 10^{-16}\text{cm}$ [7]. What then is our chance of observing $dd$ overlap at these distances, within the constraints of eq. (4)? Let us first determine the constraints we can easily establish:

1. The final-state electrons have to emerge in an overall $S$ state, as a spin singlet. This, in turn, imposes a spin singlet configuration on the $dd$ wave function.

2. To achieve an overall antisymmetric wave function for the $l = 0$ $dd$ system, the product of space and $\text{SU}_3^c$ wave functions has to be symmetrical. This leaves only the 6 representation of $\text{SU}_3^c$, imposing a suppression factor of $2/3$.

3. To evaluate the strong Hamiltonian density involved in the $dd \rightarrow uue^- e^-$ subgraphs of Fig. 2a, we have to keep the constraints imposed by the surrounding nuclear and nucleon environment in mind, as schematically shown in Fig. 2b. This leads to two further suppression factors to be determined: one is due to the color Coulomb repulsion of the $d$ quarks, the other to the collective pull which the saturated nucleon configurations of two neutrons exert on each quark that may be drawn into an interaction with a quark from another nucleon.

4. Finally, the resulting Hamiltonian density operator will have to include the leptonic weak current operator, integrated over the appropriate interaction
volume, and then sandwiched between the mother and daughter nuclei’s wave functions.

Fig. 2. a) Two quark-level sub-diagrams for the neutrinoless double beta decay process of Fig. 1b, prior to the identification of the exchanged neutrino in terms of a massive Majorana particle. b) Schematic picture of the agents in neutrinoless double beta decay: 6 quarks in two neutrons inside the nuclear environment. c) Schematic arrangement of quarks in the process of a): the color sextet interaction repels the two central $d$ quarks; the color singlet interaction pulls the central $d$ quarks in opposite directions, toward the CM of the remaining $ud$ pairs. See text.

Each of these effects will lead to a suppression factor. In an attempt to write down the Hamiltonian density of the highly local quark-quark Hamiltonian density in the overall expression

$$H = G_F^2 \left[ \bar{e}_\alpha \bar{u}^{ca} d_\gamma^c (x_2) \bar{e}_\beta \bar{u}^{b\beta} d^{b\gamma} (x_1) \right]_{x_2 \rightarrow x_1}$$

we can write, using the customary lepton-hadron factorized expression,

$$H \propto U_{eN}^2 / m_N \left[ \bar{e}_\alpha \bar{e}_\beta \right] \left[ \bar{u}^{ca} d^{\gamma c} \bar{u}^{b\beta} d^{b\gamma} \right]$$

$U_{eN}$ is the mixing angle for electron/heavy neutrino $N$ with mass $M$, the $b, c$ are color indices. The high degree of locality that governs the interaction involving both sub-diagrams of Fig. 2a permits us to rewrite the hadronic Hamiltonian density in eq. (6) such as to pair like-flavor quarks:

$$H_q(x) = \left[ \bar{u}_\alpha^b \bar{u}^{ca}(x) \right] \left[ d^{\gamma c} d^{b\gamma}_\gamma (x) \right] .$$
This density operator can then conveniently be sandwiched between the nuclear state vectors for mother \((A, Z)\) and daughter \((A, Z + 2)\) nucleus, for a hadronic matrix element

\[
\langle A, Z + 2; p_2 | H_q(x) | A, Z; p_1 \rangle = V^{-1} e^{iqx} \rho_{21} ; \quad q = p_2 - p_1 .
\] (8)

\(V\) is a normalizing volume, \(\rho\) is a density matrix that contains the Fermi and Gamow-Teller structure of the interaction when expressed in the current–current form (VV and AA on the quark level). It can be saturated with a complete set of the possible intermediate states with the requisite energy and momentum \([8]\). We will narrow our interest down to two-nucleon correlations; they will dominate the small-distance behavior in the case of \(N_M\) exchange. Recall that \(H_q\) in eq. (8) is a four-quark operator: the \(H_q\) operator does not “see” the Ge, As, Se nuclei; rather, its interaction involves the 76 nucleons in terms of their 228 quarks.

We therefore have to try and evaluate the quark-quark suppression factors in the density matrix \(\rho_{21}\) of eq. (8). They are a function of the relative distance \(r_{12}\) of any two interacting quarks, at \(r_{12}\) values in the region of the “hard-core”, \(r_{12} < 0.3\)fm. First, there is the 2/3 factor due to the spin singlet requirement (see above). Second, the repulsive color sextet interaction can be reasonably estimated by a WKB method for an evaluation of the color Coulomb barrier. A straightforward relativistic treatment leads to a barrier penetration/inhibition factor

\[
F_B = e^{-\frac{\pi\alpha_s}{3}},
\] (9)

irrespective of the nuclear environment.

Third, as Fig. 2b indicates, there are similar inhibition factors to be expected from the interaction of the remaining two quarks in the two interacting neutrons, two each that are not directly overlapping. We illustrate this schematically in Fig. 2c: each of the two central \(d\) quarks is being “pulled on” by a \(u\) and a \(d\) quark from its “own” neutron, trying to keep the straying companion in the color singlet configuration. Although the relevant Clebsch Gordan coefficient may make these inhibition factors somewhat stronger, we approximate them by a joint ansatz of

\[
F_{nn} \approx F_B^2 = e^{-2/3\pi\alpha_s}.
\] (10)

This factor of order 1/9 is very conservatively estimated, given that the color force increases considerably above the \(r^{-2}\) level known for the small-distance behavior valid for the initial \(dd\) interaction at \(r_{12}\) values of \(> 1/3\)fm, for this somewhat longer-range color singlet restoration force. We feel justified in regarding this suppression as amply supported by such evidence as the loose binding of the deuteron and the non-existence of bound \(nn\) and \(pp\) states. Note that this repulsive vector interaction can also be modeled in terms of omega meson exchange between the
two neutrons, leading to an inhibition factor stronger than the one resulting from eq. (10).

Lastly, let us recall that the entire process thus inhibited by a factor bounded from above by $2/3 \times (1/3)^3 = 2/81$, or about 0.025, but likely to be considerably smaller, has to happen inside the “mother-daughter” nuclear system. For the $^{76}\text{Ge}$ $\beta\beta_0\nu$ decay with the best present limits, this means that the above suppression modifies the nuclear wave function overlap symbolized by the projection operator

$$\hat{P} \equiv |^{76}\text{Se}^{-2u}\rangle \langle ^{76}\text{Ge}^{-2d}|,$$

which effectively sums over all intermediate states in the density matrix $\rho_{21}$ that contain two $d$ quarks less than $^{76}\text{Ge}$, two $u$ quarks less than $^{76}\text{Se}$.

Finally, we compare our conservative estimate of the overall inhibition factor with the recent literature: it reduces the exclusion zone for observable heavy Majorana neutrino masses from the estimate made by Pantis et al. from $(1/m_N)^{-1} \gtrsim 6.7 \times 10^3$ TeV to

$$\frac{(m_N)_L}{|U_{eN}|^2} = 0.025 \times 6.7 \times 10^3 \text{ TeV}. \tag{12}$$

With the mixing parameter $|U_{eN}|^2 = (2 - 40) \times 10^{-4}$ Ref. [11], this puts

$$(m_N)_L > \begin{cases} 0.67 \text{ TeV} & \text{for the upper limit on } |U_{eN}|^2. \\ 0.033 \text{ TeV} & \text{for the lower limit on } |U_{eN}|^2. \end{cases} \tag{13}$$

Similarly, it moves the exclusion zone advocated in Ref. [8] for the possible observation of process (1) in the face of existing evidence from neutrinoless double beta decay searches, as drawn in the $U_{eN}^2$ vs. $m_N$ plane, well out of danger’s way.

We conclude that established limits on the observation of neutrinoless double beta decay do not in any way preclude the observability of process (1), and thereby the possible discovery of TeV-level Majorana masses in electron-electron scattering. The next generation of electron linear colliders thus has a highly attractive chance of unraveling the major mystery that shrouds our understanding of the observed lepton spectrum and forces an illogical treatment of the lepton sector on the Standard Model.

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[8] In the case of 0+ ground state transitions between $^{76}\text{Ge}$ and $^{76}\text{Se}$, this means all bound states of $^{76}\text{As}$ plus many hadronic combinations with the same quantum numbers.

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[12] Note that Fig. 5 of Ref. [5] has to be corrected for the presence of a high degree of polarization in both incident electron beams. With $P_e = 90\%$, this raises the sensitivity by a factor 3.24 at the same time that the lower limits to the exclusion zone are decreased by the factor 0.025.