Josephson tunnel junction controlled by quasiparticle injection

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A Josephson tunnel junction transistor based on quasiparticle injection is proposed. Its operation relies on the manipulation of the electron distribution in one of the junction electrodes. This is accomplished by injecting quasiparticle current through the junction electrode by two additional tunnel coupled superconductors. Both large supercurrent enhancement and fast quenching can be achieved with respect to equilibrium by varying quasiparticle injection for proper temperature regimes and suitable superconductor combinations. Joined with large power gain this makes the device attractive for applications where reduced noise and low power dissipation are required.

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The control of Josephson currents as for the realization of efficient transistors has gained recently a rekindled interest [1]. A novel development in mesoscopic superconductivity is indeed represented by controllable superconductor(S)-normal metal(N)-superconductor(S) metallic weak links [2], where supercurrent suppression is achieved by altering the quasiparticle distribution in the N region through current injection. So far there have been a few successful demonstrations of this operation principle [2]. On the other hand, as recently proposed [3] and experimentally demonstrated [2], a SINIS control line (where I is a tunnel barrier) is particularly suitable for tuning Josephson current, allowing both enhancement and suppression with respect to equilibrium. Operation of these devices is based on the modification of the quasiparticle distribution in the N region of the junction. In this letter, we propose an all-superconducting tunnel junction device in which transistor effect is obtained by driving the electron distribution out of equilibrium in the superconductor. This is performed by voltage biasing a SISIS line (see Fig. 1) where the inter electrode is one of the two terminals belonging to the Josephson junction.

As compared to the hybrid devices above the present one benefits from the sharp characteristics due to the presence of superconductors with unequal energy gaps. We consider different superconductors S1 and S2 with energy gaps Δ1 and Δ2 (and critical temperatures Tc1,2), respectively, and we assume Δ2 < Δ1 [4, 5]. Under voltage bias VC across the S1IS2IS1 line (see the inset of Fig. 1) the heat current from S2 to S1 is given by

\[ P = \frac{2}{\varepsilon^2 R_T} \int_{-\infty}^{\infty} d\epsilon \epsilon N_1(\epsilon) N_2(\epsilon) [f_0(\epsilon, T_{c2}) - f_0(\epsilon, T_{c1})], \]

(1)

where \( \hat{\epsilon} = \epsilon - eV_C/2 \), \( f_0(\epsilon, T_{c2}) \) is the Fermi-Dirac distribution function, \( T_{ck} \) is the electron temperature in \( S_k \), \( R_T \) is the normal-state resistance of each S1IS2 junction and \( N_k(\epsilon) = |\text{Re}(\epsilon + i\Gamma_k)/\sqrt{(\epsilon + i\Gamma_k)^2 - \Delta_k^2}| \) is the smeared BCS density of states of \( S_k \). Figure 1 shows the calculated heat current versus bias voltage \( V_C \) at constant bath temperature \( T_{bath} = T_{c1} = T_{c2} = 0.4T_{c1} \) and for different values of \( \Delta_2 \). \( P \) is symmetric in \( V_C \) and it is positive for \( V_C < 2|\Delta_1(T) + \Delta_2(T)|/e \) thus allowing heat removal from \( S_2 \), i.e., hot quasiparticle excitations are transferred to \( S_1 \); furthermore, the heat current is maximized at \( V_C = \pm 2|\Delta_1(T) - \Delta_2(T)|/e \), where the finite-temperature logarithmic singularity occurs [6] (in a real situation it will be somewhat broadened by smearing in the density of states [6, 7, 8]). From Fig. 1 it follows that a positive heat current from \( S_2 \) exists only if \( \Delta_2(T) < \Delta_1(T) \) holds. The dash-dotted line represents the heat current in the system when \( S_2 \) is in the normal state. Notably, when \( S_2 \) is in the superconducting state \( P \) can largely exceed that one in the normal state.

\[ P = \frac{2}{\varepsilon^2 R_T} \int_{-\infty}^{\infty} d\epsilon \epsilon N_1(\epsilon) N_2(\epsilon) [f_0(\epsilon, T_{c2}) - f_0(\hat{\epsilon}, T_{c1})], \]

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Then, on approaching \( V_C = \pm 2[\Delta_1(T) + \Delta_2(T)]/e \), a sharp transition brings \( \mathcal{P} \) to negative values. An additional superconducting electrode \( S_J \) is connected to \( S_2 \) through a tunnel barrier so to realize a \( S_J IS_2 \) Josephson junction. \( S_J \) is characterized by its own energy gap \( \Delta_J \) (different in general from \( \Delta_{1,2} \)) with critical temperature \( T_{cJ} \), and \( R_J \) is the normal-state resistance of the junction. As we shall prove this transistor operation relies on the quasiparticle distribution established in \( S_2 \) upon voltage biasing the control line.

We consider a transport regime where strong inelastic electron-electron interaction forces the system to retain a local thermal (\( \text{quasi} \))equilibrium, so that the quasiparticle distribution in \( S_2 \) is described by a Fermi function at temperature \( T_{c2} \) differing in general from \( T_{\text{bath}} \). In order to determine the actual \( T_{c2} \) upon biasing with \( V_C \) we need to include those scattering mechanisms that transfer energy in \( S_2 \). At the typical operation temperatures the predominant contribution comes from electron-phonon scattering that transfers energy between electrons and phonons. This heat flux is given by \( \mathcal{P}_{\text{e-ph}} = \mathcal{P}_{\text{e-ph}}(T_{c2} - T_{\text{bath}}) \), where \( \mathcal{P} \) is a material-dependent parameter and \( V \) is the volume of \( S_2 \). The temperature \( T_{c2} \) is then determined by solving the energy-balance equation \( \mathcal{P}(V_C, T_{\text{bath}}, T_{c2}) + \mathcal{P}_{\text{e-ph}} = 0 \).

The supercurrent \( (I_J) \) flowing through the \( S_J IS_2 \) junction can be calculated from [12]:

\[
I_J = -\frac{\sin \phi}{2eR_J} \int_{-\infty}^{\infty} d\varepsilon \{ f_2(\varepsilon) \Re F_2(\varepsilon) \Im F_J(\varepsilon) + f_J(\varepsilon) \Re F_J(\varepsilon) \Im F_2(\varepsilon) \},
\]

where \( \phi \) is the phase difference between the superconductors, \( f_{2,J}(\varepsilon) = \tanh[\varepsilon/2k_B T_{c2,\text{bath}}] \) and \( F_{2,J}(\varepsilon) = \Delta_{2,J}/(\varepsilon + i\Delta_{2,J})^2 - \Delta_{2,J}^2 \). In the aforementioned expressions we set \( \Delta_2 = \Delta_2(T_{c2}) \) and \( \Delta_J = \Delta_J(T_{\text{bath}}) \). Equation (2) shows that, for fixed \( T_{\text{bath}} \) and phase difference, the Josephson current is controlled by \( T_{c2} \). The solution of the balance equation for \( T_{c2} \) combined with Eq. (2) yields the dimensionless transistor output characteristic shown in Fig. 2(a) [12], where \( I_J \) is plotted versus \( V_C \) at different bath temperatures, for \( T_{c2} = 0.3 T_{c1} \) and \( T_{cJ} = T_{c1} \). For \( T_{\text{bath}} < T_{c2} \), \( I_J \) first increases monotonically up to \( eV_C = 2[\Delta_1(T_{\text{bath}}) - \Delta_2(T_{c2})] \), where the cooling power is maximized; then it starts to slightly decrease after which it is rapidly quenched at \( eV_C = 2[\Delta_1(T_{\text{bath}}) + \Delta_2(T_{c2})] \). Notably, even at bath temperatures exceeding \( T_{c2} \) (i.e., for \( T_{\text{bath}} > T_{c2} \) where \( I_J \) is zero at equilibrium), a finite supercurrent is obtained at a voltage for which \( S_2 \) is brought into the superconducting state, after which \( I_J \) is recovered up to a large extent. The influence of different \( S_J \) on the supercurrent is displayed in Fig. 2(b) that shows \( I_J \) versus \( V_C \) at \( T_{\text{bath}} = 0.8 T_{c2} \) for different \( T_{cJ}/T_{c1} \) ratios. As a consequence \( I_J \) is enhanced upon increasing \( \Delta_J \), being nearly doubled for \( T_{cJ}/T_{c1} = 10 \).

Figure 3(a) displays the transistor power dissipation \( P = V_C I_C \), where \( I_C \) is the control current in the

**FIG. 2:** (a) Supercurrent \( I_J \) vs control voltage \( V_C \) calculated at different bath temperatures \( T_{\text{bath}} \), for \( T_{c2} = 0.3 T_{c1} \) (corresponding roughly to the Ti/Al combination) and \( T_{cJ} = T_{c1} \). Note the sharp \( I_J \) suppression at \( eV_C = 2[\Delta_1(T_{\text{bath}}) + \Delta_2(T_{c2})] \). (b) Supercurrent vs \( V_C \) for several \( T_{cJ}/T_{c1} \) ratios at \( T_{\text{bath}} = 0.8 T_{c2} \) and for \( T_{c2} = 0.3 T_{c1} \).

\( S_1 IS_2 IS_1 \) line, calculated for \( T_{c2} = 0.3 T_{c1} \) and \( T_{cJ} = T_{c1} \) at different bath temperatures. The plot shows that at the lowest temperatures \( P \) obtains values of the order of some fW in the regime of supercurrent enhancement while of some hundred of fW around the \( I_J \) quenching. This is because of low control currents through the structure. As far as noise is concerned, the total input noise per unit bandwidth \( \langle \delta I^2_C \rangle \) [13] in the control line can be expressed as

\[
\langle \delta I^2_C \rangle = \langle \delta I^2_C \rangle - 2S_{1C} \langle \delta \mathcal{P} \delta I_C \rangle + S_{2C}^2 \langle \delta \mathcal{P}^2 \rangle,
\]

where

\[
\langle \delta I^2_C \rangle = \frac{1}{R_T} \int_{-\infty}^{\infty} d\varepsilon N_1(\varepsilon) N_2(\varepsilon) W(\varepsilon, \varepsilon),
\]

\[
\langle \delta \mathcal{P}^2 \rangle = \frac{1}{e^2 R_T} \int_{-\infty}^{\infty} d\varepsilon \varepsilon^2 N_1(\varepsilon) N_2(\varepsilon) W(\varepsilon, \varepsilon),
\]

\[
\langle \delta \mathcal{P} \delta I_C \rangle = -\frac{1}{eR_T} \int_{-\infty}^{\infty} d\varepsilon \varepsilon N_1(\varepsilon) N_2(\varepsilon) W(\varepsilon, \varepsilon),
\]
FIG. 3: (a) Dissipated power $P$ and (b) total input noise $\langle \delta I^2_{\text{tot}} \rangle$ in the $S_1S_2|S_3$ line against $V_C$. The transistor current gain $G_T(V_C)$ is shown in (c) and (d) in two different ranges of $V_C$. All these calculations are performed for $T_{c2} = 0.3 T_{c1}$, $T_{c3} = T_{c1}$ and at three different bath temperatures.

The total dissipated power is $P = C V^2/(\Delta^2 + V^2)$ and the total input noise $\langle \delta I^2_{\text{tot}} \rangle$ is $\langle \delta I^2_{\text{tot}} \rangle = \langle \delta I^2_{\text{line}} \rangle$. The figure shows that $\langle \delta I^2_{\text{line}} \rangle$ is plotted in Fig. 3(b) for the same parameters as in Fig. 3(a), and that shows that input noise as low as some $10^{-30}$ A$^2$ Hz$^{-1}$ can be achieved in the enhancement regime while of some $10^{-29}$ A$^2$ Hz$^{-1}$ at the quenching voltage. Thin lines are the uncorrelated noise power, i.e., the noise obtained by adding the contributions of Eqs. (4) and (5) only. Notably, the impact of mutual correlations (Eq. (6)) is easily recognized leading to significant noise reduction (\sim 50\%) in the range of supercurrent enhancement.

We shall further comment on the available gain. Input ($V_{\text{in}} = V_C \sim \Delta_1$) and output ($V_{\text{out}} = I_J R_J \sim \Delta_2$) voltages allow a voltage gain $G_V = V_{\text{out}}/V_{\text{in}} \sim \Delta_2/\Delta_1$ so that with realistic parameters $G_V$ is not much smaller than 1. The difference current gain, defined as $G_T = dI_J/dI_C = (dI_C/dV_C)/(dI_C/dV_C)$, is plotted in Fig. 3(c,d) in two different bias ranges for some values of $T_{\text{bath}}$. The figure shows that $G_T$ obtains large values with some $10^2$ in the regime of supercurrent enhancement and several $10^3$ below the quenching. The corresponding input impedance ranges from hundreds of kΩ to tens of MΩ, respectively. In order to exploit the power gain ($G_P$) the Josephson junction needs to be operated in the dissipative regime; in such a situation an estimate for the achievable power gain $G_P$ yields $G_P \sim 10^2$...$10^3$ depending on the operating $V_C$ and bias current $I_J$ across the junction (see Fig. 1). One should note that such a large power gain, not achievable, e.g., using a SINIS controlled SNS transistor in the same transport regime, is an additional advantage of the present scheme.

We conclude with some further benefits of our proposal. Due to the presence of the superconducting inter-electrode, highly transmissive tunnel junctions are not necessary unlike in SINIS devices. The device is also less sensitive to thermal fluctuations as compared to SNS junctions. Furthermore, it is easier to fabricate taking advantage of the well established metal-based tunnel junction technology. A promising choice for transistor and switch implementations could be a combination of Al and Ti.

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