Three-body properties in hot and dense nuclear matter

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We derive three-body equations valid at finite densities and temperatures. These are based on the cluster mean field approach consistently including proper self energy corrections and the Pauli blocking. As an application we investigate the binding energies of triton and determine the Mott densities and momenta relevant for a many particle description of nuclear matter in a generalized Beth-Uhlenbeck approach. The method, however is not restricted to nuclear physics problems but may also be relevant, e.g., to treat three-particle correlations in weakly doped semiconductors or strongly coupled dense plasmas.

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I. INTRODUCTION

Correlated many particle systems such as, e.g., nuclear matter or strongly coupled plasmas, have a complicated dynamical behavior. Only few areas of the density-temperature phase diagram used to characterize the state of the system in thermal equilibrium may be described in the approximation of noninteracting quasiparticles. The dynamics of the quasiparticle is determined by the mean field of the other particles and therefore some prominent features like self energy corrections and Pauli blocking (Bose enhancement) are sufficiently taken into account\textsuperscript{1}. However due to sizable residual interactions many interesting and exciting phenomena, such as clustering, formation of condensates, and phase transitions occur.

A large number of these phenomena such as, e.g., two-particle bound state formation, can already be accounted for by explicitly introducing two-body correlations into the formalism. This may be achieved in the frame work of the Green function method\textsuperscript{1} and leads to effective two-body equations that include medium effects in a consistent way.

Recently, generic three-body processes have been calculated utilizing exact few-body methods in the context of many particle systems for i) nuclear matter\textsuperscript{2} and ii) plasmas at star conditions\textsuperscript{3}. Although the relevance of three-body processes in many particle systems have already been recognized, little progress has been achieved since both fields (many-particle and few-body physics) in itself are rather elaborated, bearing their own technical problems. However, some phenomena of many particle systems require a treatment of effective few-body systems embedded in a medium, in particular three-body processes.

In the context of nuclear matter, a three-body Faddeev-type equation has been derived within the Green functions method applicable to describe three-body correlations in nuclear matter of thermal equilibrium\textsuperscript{2}. Empirical evidence, including recent experimental data on cluster formation\textsuperscript{4}, indicate that a large fraction of deuterons can be formed in heavy-ion collisions of energies below $E/A \leq 200$ MeV. The abundances of deuterons are determined by the deuteron formation via $NNN \rightarrow dN$ ($N$ nucleon, $d$ deuteron) and break-up, $dN \rightarrow NNN$, reactions. The hadronic reaction requires a proper treatment of the effective three-body problem. We have numerically solved the scattering problem to describe the deuteron break-up reaction consistently including the effects of finite densities and temperatures of the surrounding nuclear matter. Within linear response theory we have given a first estimate for the break-up time of deuterons embedded in nuclear matter\textsuperscript{3}.

Indeed, at moderate densities $n \lesssim n_0/10$ ($n_0 = 0.17$ fm$^3$ nuclear matter density) and temperatures $T \lesssim 15$ MeV nuclear matter may be considered as a mixture of light nuclei (chemical picture). The transition into that region is determined by the Mott density\textsuperscript{3} and is relevant, e.g., for the final stage of a heavy ion collision at intermediate energies, respectively on the surface of the expanding nuclear matter. The Mott effect has also been considered at the beginning of the heavy ion reaction using inverse photo disintegration\textsuperscript{6}. Here we present a calculation that determine the Mott densities for the three-body nuclear bound state.

II. BETH-UHLENBECK APPROACH TO NUCLEAR MATTER

The basis to treat correlated densities is provided by a generalization of the Beth-Uhlenbeck approach\textsuperscript{11}. The nuclear density $n = n(\mu, T)$ as a function of the chemical potential $\mu$ (for the time being we assume symmetric nuclear matter) and temperature $T$ may be written as
\[ n = n_{\text{free}} + n_{\text{corr}}, \quad n_{\text{free}} = 4 \sum f_1. \quad (1) \]

For abbreviation we use \( 1 \equiv \alpha_1 = \{k_1, s_1, \tau_1, \ldots\} \) denoting momenta, spin, isospin, etc. of particle 1. The one-particle Fermi function is

\[ f_1 \equiv f(\varepsilon_1) = (\exp[\beta(\varepsilon_1 - \mu)] + 1)^{-1}, \quad (2) \]

where \( \varepsilon_1 \) denotes the quasiparticle energy and \( \beta \) the inverse temperature. In first iteration the correlations may be treated on the basis of residual interactions between the quasiparticles. To do so the imaginary part of the self energy \( \Sigma(1, \omega) \) should be small and the respective spectral function may be expanded with respect to the imaginary part \( \Sigma_1(1, \omega) \) as explained in Ref. [10]. The contribution of the correlated density to the total density may then be written as [11]

\[
n_{\text{corr}}(\mu, T) = \sum_1 \int \frac{d\omega}{2\pi} \Sigma_1(1, \omega - i0^+) \\
\quad \times \left[ f(\omega) - f(\varepsilon_1) \right] \frac{d}{d\omega} \frac{\mathcal{P}}{\varepsilon_1 - \omega}, \quad (3)\]

where \( \mathcal{P} \) denotes principle part integration. To evaluate the self energy we utilize cluster decomposition taking into account two- and three-particle, in general \( n \)-particle correlations (see Fig. 1). \( \Sigma(1, \omega) \) should be small and the respective spectral function may be expanded.

\[
\Sigma(1, \omega) = \sum_{\lambda} \sum_{2} T_2(12, 12; \omega + \delta_1)G_1(2, z_1) \\
+ \sum_{\lambda, \lambda'} T_3(123, 123; \omega + \delta_1 + \delta_2)G_1(2, z_1)G_1(3, z_1') \\
+ \ldots \quad (4)
\]

where \( G_1(z) \) denotes the one-particle Green function of Matsubara frequency \( z \), and the index \( \lambda \) denotes connected terms only. The \( n \)-body \( t \)-matrices are defined in terms of the \( n \)-body Green functions \( G_n(z) \) via

\[
G_n(z) = G_n^0(z) + G_n^0(z)T_n(z)G_n^0(z). \quad (5)
\]

Using the spectral representation of the \( t \)-matrices and evaluating the Matsubara sums eventually leads to the following decomposition of the correlated density induced by the cluster decomposition Eq. (3).

\[
n_{\text{corr}} = 2n_2 + 3n_3 + \ldots, \quad n_2 = n_2^b + n_2^{sc}, \ldots \quad (6)
\]

where \( n_2(3) \) denotes the two- (three-) particle correlated densities, present as bound \( n^b \) or scattering \( n^{sc} \) states in chemical equilibrium. For the two-particle case the correlated densities have been given explicitly in Ref. [10]

\[
n_2^b = \sum_{P > P_{\text{Matt}}} 3 g(E_{\text{cont}}) \\
- \sum_{P > P_{\text{Matt}}} \int \frac{d\omega}{2\pi} \frac{d}{d\omega} g(E_{\text{cont}} + \omega) \\
\quad \times \sum_{\alpha} C_{\alpha} \cdot (\delta_{\alpha} - \sin \delta_{\alpha} \cos \delta_{\alpha}), \quad (7)
\]

where the sum over \( \alpha \) indicates partial wave decomposition with \( C_{\alpha} \) the proper Clebsch-Gordan coefficient and \( \delta_{\alpha} \) the corresponding phase shift. The energies appearing in Eq. (7) are the binding energy \( E_b \), and the continuum energy \( E_{\text{cont}} \) that includes the proper self energy shifts. The Bose function for the two-fermion system is

\[
g(\omega) = \left( e^{\beta(\omega - 2\mu)} - 1 \right)^{-1}. \quad (8)
\]

To evaluate the contribution of the three-body correlated density \( n_3 \) we presently focus on the bound state contribution. The \( t \)-matrix is then dominated by the bound state pole of energy \( E_b \). Evaluating the respective Matsubara sums for the three-body bound state contribution in Eq. (8) using Eq. (9) and the spectral representation of \( T_3 \) leads to a rather simple expression for the three-body bound state contribution \( n_3^b \) to the total density,

\[
n_3^b = \sum_{P > P_{\text{Matt}}} 4 \left( e^{\beta(E_{\text{cont}} + E_b - 3\mu)} + 1 \right)^{-1}, \quad (9)
\]

where \( E_{\text{cont}} \) the respective continuum energy.

At moderate densities not only two-particle but also three- and four-particle correlations occur. Here we consider the three-nucleon bound state within the generalized Faddeev approach discussed in the next section. One obvious feature in the above equation is the appearing of the Mott momentum \( P_{\text{Matt}} \). It is important in the framework of the cluster Hartree-Fock expansion [11] closely related to the self consistent RPA [12] and extended to finite temperature in [13] that are followed here to consistently treat few-body correlations in matter.

### III. Finite Temperature Three-Body Equations – Bound States

The formalism to derive few-body Green functions within the cluster mean field approximation at finite temperatures and densities has been given elsewhere [13]. Here we give some of the basic results and extend the formalism to include bound states. The reaction cross section based on the suitably modified AGS formalism has been given in [13] and the life time of deuteron fluctuations in nuclear matter as a first application in [13].

The hierarchy of Green function equations is truncated using cluster mean field expansion, and a calculable form is achieved by introducing ladder approximation. We consider generic elementary two-particle interactions \( V_2 \) only. The resulting equations for the decoupled one-
two- and three-body Green functions at finite temperatures (utilizing the Matsubara technique to treat finite temperatures) will be given in the following. The one-particle Green function reads

\[ G_1(z) = R_1^{(0)}(z) = (z - \varepsilon_1)^{-1}. \] (10)

In mean field approximation the single quasiparticle energy \( \varepsilon_1 \) is given by

\[ \varepsilon_1 = \frac{k^2}{2m_1} + \Sigma^{HF}(1), \]

\[ \Sigma^{HF}(1) = \sum_{2} [V_2(12, 12) - V_2(12, 21)]f_2, \] (11)

and \( f_2 = f(\varepsilon_2) \) the Fermi function given in the previous section. The equation for the two-particle Green function \( G_2(z) \) reads

\[ G_2(z) = N_2 R_2^{(0)}(z) + R_2^{(0)}(z) N_2 V_2 G_2(z), \] (12)

where the two-body resolvent \( R_2^{(0)}(z) \) is given by

\[ R_2^{(0)}(12, 1'2', z) = \frac{\delta_{11'}\delta_{22'}}{z - \varepsilon_1 - \varepsilon_2} \] (13)

and the Pauli blocking factor \( N_2 \) by

\[ N_2(12, 1'2') = \delta_{11'}\delta_{22'}(\bar{f}_1\bar{f}_2 - f_1f_2) = \delta_{11'}\delta_{22'}(1 - f_1 - f_2). \] (14)

We use the notation \( f = 1 - f \). The respective equation for the three-particle Green function relevant to describe three-body correlations in a medium is given by

\[ G_3(z) = N_3 R_3^{(0)}(z) + R_3^{(0)}(z) W_3 G_3(z), \] (15)

where the effective potential \( W_3 \) reads

\[ W_3(123, 1'2'3') = \sum_{k=1}^{3} W_3^{(k)}(123, 1'2'3'), \] (16)

\[ W_3^{(3)}(123, 1'2'3') = (1 - f_1 - f_2) V_2(12, 1'2') \delta_{33'}. \] (17)

The last Eq. is given for \( k = 3 \) and cyclic permutation is understood. Note that \( W_3 \neq W_3^{(3)} \). The Pauli factors \( N_3 \) and the resolvents \( R_3^{(0)} \) read respectively

\[ N_3(123, 1'2'3') = \delta_{11'}\delta_{22'}\delta_{33'}(\bar{f}_1\bar{f}_2\bar{f}_3 + f_1f_2f_3) \] (18)

\[ R_3^{(0)}(123, 1'2'3'; z) = \frac{\delta_{11'}\delta_{22'}\delta_{33'}}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3}. \] (19)

Note that \([N_3, R_3^{(0)}] = 0\). For convenience we may introduce the Green function of the noninteracting system, viz.

\[ G_3^{(0)}(z) = N_3 R_3^{(0)}(z). \] (20)

If we now introduce a potential \( V_3 = N_3^{-1} W_3 \) we may instead of Eq. (15) write

\[ G_3(z) = G_3^{(0)}(z) + G_3^{(0)}(z) V_3 G_3(z), \] (21)

which looks formally as the equation for the isolated case \([14]\) and allows one to use three-body techniques to arrive at a solvable form, e.g. AGS equations for the transition operator \([11,12]\). These will be used to derive numerically solvable Faddeev type equations at finite temperature. Although we assume a Fermionic system the proper symmetrization is treated separately.

Already in Eq. (14) we have introduced the channel notation that is convenient to treat systems with more than two particles \([14]\). In the three-particle system usually the index of the spectator particle is used to characterize the channel.

If the correlated pair, e.g. (12) and the spectator particle, e.g. 3, are uncorrelated in the channel \(1\) we may define a channel Green function \( G_3^{(3)}(z) \). We generalize to the channel \((\gamma)\). The channel Green function is then defined by

\[ G_3^{(\gamma)}(z) = \frac{1}{-i\beta} \sum_{\lambda} iG_2(\omega_\lambda) G_1(z - \omega_\lambda). \] (22)

The summation is done over the Bosonic Matsubara frequencies \( \omega_\lambda, \lambda \) even, \( \omega_\lambda = \pi \lambda/(-i\beta) + 2\mu \). The equation for the channel Green function is derived in the same way as for the total three-particle Green function given in Eqs. (15) and (21). The result is

\[ G_3^{(\gamma)}(z) = G_3^{(0)}(z) + G_3^{(0)}(z) V_3^{(\gamma)} G_3^{(\gamma)}(z), \] (23)

(no summation of \(\gamma\)). Introducing the notation \( V_3^{(\gamma)} = V_3 - V_3^{(\gamma)} \) we arrive at the following equation for \( G_3^{(\gamma)}(z) \) expressed through the channel Green functions \( G_3^{(\gamma)}(z) \), i.e.

\[ G_3(z) = G_3^{(\gamma)}(z) + G_3^{(\gamma)}(z) V_3^{(\gamma)} G_3(z). \] (24)

Now we have set the necessary equations, i.e. Eqs. (21), (23) and (24) to derive a proper integral equation for scattering (see Ref. [2,3]) and bound states at finite temperatures and densities.

To arrive at a homogeneous three-body equation for the bound state \( |\Psi_4\rangle \) in medium we insert Eq. (21) into the Lippmann-Schwinger equation

\[ |\Psi_4\rangle = \lim_{\epsilon \to 0} i\epsilon G_3(E_\epsilon + i\epsilon) |\Psi_4\rangle. \] (25)

After performing the limit we find

\[ |\Psi_4\rangle = G_3^{(0)}(E_\epsilon)V_3 |\Psi_4\rangle. \] (26)

The Faddeev components are given by

\[ |\Psi^{(\alpha)}\rangle = G_3^{(0)} V_3^{(\alpha)} |\Psi_4\rangle \] (27)
and

$$|\Psi_t\rangle = \sum_{\alpha} |\Psi^{(\alpha)}\rangle.$$  \hfill (28)

To arrive at a conveniently solvable version of the three-body bound state problem in medium we introduce form factors. To this end Eq. (23) is inserted into Eq. (25). The limit $\epsilon \to 0$ results in

$$|\Psi_t\rangle = G_{3}^{(\alpha)}(E_{t}) V_{3}^{(\alpha)} |\Psi_{t}\rangle.$$  \hfill (29)

Introducing the usual form factors

$$|F^{(\alpha)}\rangle = \bar{V}_{3}^{(\alpha)} |\Psi_{t}\rangle$$  \hfill (30)

eventually leads to an integral equation

$$|F^{(\alpha)}\rangle = \sum_{\beta} (1 - \delta_{\alpha\beta}) T_{3}^{(\beta)} G_{3}^{(0)} |F^{(\beta)}\rangle$$  \hfill (31)

and finally to the bound state given in terms of the form factors

$$|\Psi_{t}\rangle = \sum_{\beta} G_{3}^{(0)} T_{3}^{(\beta)} G_{3}^{(0)} |F^{(\beta)}\rangle.$$  \hfill (32)

The transition channel operator $T_{3}^{(\beta)}$ is defined via

$$G_{3}^{(\beta)} = G_{3}^{(0)} + G_{3}^{(1)} T_{3}^{(\beta)} G_{3}^{(0)},$$  \hfill (33)

inserting this equation into Eq. (23) leads to an equation for the channel $t$ matrix

$$T_{3}^{(\beta)} = V_{3}^{(\beta)} + G_{3}^{(0)} V_{3}^{(\beta)} T_{3}^{(\beta)},$$  \hfill (34)

and to $V_{3}^{(\beta)} G_{3}^{(\beta)} = T_{3}^{(\beta)} G_{3}^{(0)}$. Writing the Pauli factors explicitly in the integral equation (31) results in

$$|F^{(\alpha)}\rangle = \sum_{\beta} (1 - \delta_{\alpha\beta}) T_{3}^{(\beta)} N_{3} R_{3}^{(0)} |F^{(\beta)}\rangle.$$  \hfill (35)

Note that $G_{3}^{(0)} = N_{3} R_{3}^{(0)} = R_{3}^{(0)} N_{3}$. We may now introduce $T_{3}^{(\beta)} = N_{3}^{1/2} T_{3}^{(\beta)} N_{3}^{1/2}$ and $|F^{(\alpha)}\rangle = N_{3}^{1/2} |F^{(\beta)}\rangle$. The resulting equation is

$$|F^{(\alpha)}\rangle = \sum_{\beta} (1 - \delta_{\alpha\beta}) T_{3}^{(\beta)} R_{3}^{(0)} |F^{(\beta)}\rangle.$$  \hfill (36)

The equation for the transition channel operator $T_{3}^{(\beta)}$ is then

$$(V_{3}^{(\alpha)} = N_{3}^{1/2} V_{3}^{(\alpha)} N_{3}^{1/2})$$

$$T_{3}^{(\beta)} = V_{3}^{(\alpha)} T_{3}^{(\beta)} R_{3}^{(0)} |T_{3}^{(\beta)}\rangle.$$  \hfill (37)

Inserting all definitions the explicit form of the effective potential arising in this equation reads

$$V_{3}^{(\alpha)} (123, 1'2'3') = N_{2}^{1/2} (12) (1 - f_{1} - f_{2}) (1 - f_{1} + g(\epsilon_{1} + \epsilon_{2})))^{-1/2}$$

$$\times V_{2} (12, 1'2') \delta_{33'} (1 - f_{3} + g(\epsilon_{1'} + \epsilon_{2'}))^{1/2} N_{2}^{1/2} (1'2')$$

$$\simeq (1 - f_{1} - f_{2})^{1/2} V_{2} (12, 1'2') (1 - f_{1'} - f_{2'})^{1/2}.$$  \hfill (38)

where we have used $(f_{1} f_{2} + t_{1} t_{2} f_{3}) = (1 - f_{1} - f_{2}) (1 - f_{3} + g(\epsilon_{1} + \epsilon_{2}))$ valid for all permutations of $ijk = 123$. Note that $1 - f_{1} - f_{2} > 0$ which is the case for low densities and the last equality in Eq. (38) holds for $f^{2} \ll f$. Utilizing this approximation the corresponding scattering solution using a separable ansatz for the strong nucleon-nucleon potential has been given in [3]. In Ref. [3] we have calculated the break up cross section $Nd \rightarrow NNN$ and found considerable dependence of the deuteron fluctuation time on the proper treatment of the medium dependence [3]. Here we solve Eq. (30) for the Yamaguchi [17] and a high rank separable version (Paris (EST)) of the Paris potential [18]. For a detailed overview on the procedure we refer to [19]. To compared to the perturbation theory result we use the respective wave function of the isolated triton and evaluate the medium dependent part, i.e., the Pauli blocking $-(f_{1} + f_{2}) V_{2}$ and self energy corrections $\Sigma_{HF}^{(1)}$ of the effective Hamiltonian in the standard fashion.

IV. KINEMATICS

Unlike the isolated three-body problem Galilei invariance (for the three-body system embedded in a medium) is not satisfied, since the Fermi functions depend explicitly on the relative momentum of the three-nucleon system with respect to the medium.

As a consequence one has to solve the three-body problem at a finite center of mass momentum $P_{cm} = k_{1} + k_{2} + k_{3}$ in a medium that may be considered at rest $P_{med} = 0$ for simplicity, see Fig. 2. However, technically it is more convenient to let the three-nucleon system rest $P_{cm} = 0$ and the surrounding medium move with $P_{med} = -(k_{1} + k_{2} + k_{3})$. This procedure results in the least change of the three-body algebra and is possible since the dependence on $P_{cm} - P_{med}$ is only through the Pauli blocking factors thus parametric.

For simplicity we use angle averaged Fermi function $<\langle N_{2} \rangle>$ and $<\langle N_{3} \rangle>$, i.e.

$$<\langle N_{3} \rangle> = \frac{1}{(4\pi)^{2}} \int d\cos \theta_{q} d\cos \theta_{p} d\phi_{q} d\phi_{p} N_{3}(p, q, P_{cm})$$  \hfill (39)

where the angles are taken with respect to $P_{cm}$ and $p, q$ are the standard Jacobi coordinates [4].

V. RESULTS

Since the Green functions have been evaluated in an independent particle basis the one-, two-, and three-particle Green functions are decoupled in hierarchy, as given in Eqs. (14), (23), (13). To solve the in medium problem up to three-particle clusters the one-, two- and three-particle problems are consistently solved. This
leads to the single particle self energy shift Eq. (11), the two-body input including the proper Pauli blocking Eq. (67), and eventually to the three-body bound state Eqs. (12) and (33) (or scattering state).

For technical reasons we have approximated the nucleon self energy calculated via Eq. (11) by use of effective masses for the nucleon, which is reasonable for the small nuclear densities considered here.

For comparison we have calculated the energy shift of the three-body bound state using perturbation theory and the solutions of the respective isolated system.

The resulting binding energies per nucleon in case where triton and medium are both at rest, i.e. $P_{cm} = P_{med} = 0$, as a function of the nuclear uncorrelated density $n$ are shown in Fig. 3 for two different temperatures $T = 10, 20$ MeV. The rank one Yamaguchi potential has been used for the solid lines, also used earlier in the context of in medium break up reactions ($Nd \to NNN$) [2]. The dashed line originating at the deuteron binding energy $E_d/2 \simeq -1.11$ MeV reflects the respective continuum threshold (since the deuteron also changes the binding energy the continuum threshold changes). The intersection between the triton binding energy and the continuum defines the Mott transition density. The Mott transition of the triton leads for both temperatures considered directly to a three-body break-up.

For $T = 20$ MeV the long dashed line shows the result of the Paris (EST) potential. The difference between the dashed line and the respective result of the simple Yamaguchi potential is mostly due to the difference in binding energy. The shape of the curves, also for the case $T = 10$ and $30$ MeV (not shown) are very similar. The dashed dotted line is the result of a perturbative calculation of all medium effects evaluated by using the triton wave functions for the isolated case. The dominant effect comes from Pauli blocking. The self energy correction including the continuum shift (due to effective masses) is smaller.

The momentum dependence of the binding energy per nucleon for $T = 10$ MeV is shown in Fig. 4. As expected and known from the deuteron case the influence of the surrounding matter on the binding energy is decreasing with increasing momentum. Therefore the Mott transition moves to higher densities for higher momenta. As a consequence in a moving system (like in a heavy ion collision) deuterons, tritons and presumably other light clusters may be formed at higher densities than expected from the simple considerations at rest. The momentum where the triton binding energy crosses the continuum is referred to as Mott momentum $P_{Mott}$.

Finally, Fig. 5 shows the dependence of the Mott momentum on the density for the triton and the deuteron. The momenta and the densities related to the area above the respective curves allow bound states to be formed (i.e. $E_{t,d} < 0$). At higher densities (respectively at higher momenta) the temperature dependence is less pronounced than for the lower densities.

VI. SUMMARY AND CONCLUSIONS

We have given proper equations to treat three-body correlations in matter and solved them for the nuclear bound state. The framework is provided by the cluster mean field (cluster Hartree-Fock) approximation closely related to the self consistent RPA approach, known for $T = 0$, and generalized to $T \neq 0$ recently [3].

We find substantial changes of the triton properties (being a much simpler problem the deuteron has been discussed much earlier [3]) and give the respective Mott densities and momenta.

The application of rigorous few-body methods in the context of a many-particle description of nuclear matter provides a fruitful method not only useful for nuclear physics. Potential areas for a new application may be, e.g., weekly doped semiconductors to include the effects of bound $eeh$ charged exciton states (trions) or strongly coupled dense plasmas where ionisation rates may rely on few-body reactions if the charge of the ions become large.

The method is capable to be applied to the $\alpha$ particle as well, which is the strongest bound nucleus and therefore very important for nuclear matter. Recently, indication for $\alpha$-particle quartetting (condensation) has been found in a rather simple approach that need to be confirmed [7].

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[1] for a textbook treatment see, e.g., A.L. Fetter, J.D. Walecka, Quantum Theory of Many-Paricle Systems, (Mc Graw Hill, New York, 1971).
[2] M. Beyer, G. Röpke, and A. Sedrakian, Phys. Lett. B376, 7 (1996);
[3] M. Beyer and G. Röpke, Phys. Rev. C56 2636 (1997), M. Beyer Few Body Systems Supplement 10 (1998) in print
[4] V.V. Belayev, Nucl. Phys. A613, 132 (1997), Nucl. Phys. A 635, 257 (1998)
[5] G. Kunde, PhD. thesis, GSI Darmstadt (1994), unpublished.
[6] D.O. Handzy et al., Phys. Rev. Lett. 75, 2916 (1995); F. Zhu, W.G. Lynch, D.R. Bowman, R.T. de Souza, C.K. Gelbke, Y.D. Kim, L. Phair, M.B. Tsang, C. Williams, and H.M. Xu, Phys. Rev. C 52, 784 (1995).
[7] P. Danielewicz and G.F. Bertsch, Nucl. Phys. A 533, 712 (1991).
[8] G. Röpke, L. Münchow, and H. Schulz, Nucl. Phys. A379, 536 (1982); G. Röpke, M. Schmidt, L. Münchow, and H. Schulz, Nucl. Phys. A 399, 587 (1983);
[9] P. Bożek, P. Danielewicz, K. Gudima, and M. Płozańczak, Phys. Lett. B 421, 31 (1998).
[10] M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. (NY) 202, 57 (1990);
[11] M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. (Leipzig) 3, 145 (1994); G. Röpke, T. Seifert, H. Stolz, and R. Zimmermann, Phys. Stat. Sol. (b) 100, 215 (1980).
[12] P. Schuck, S. Ethofer, Nucl. Phys. A212, 269 (1973); J. Dukelsky, P. Schuck, Nucl. Phys. A512, 466 (1990); P. Schuck, Z. Phys. 241, 395 (1971); J. Dukelsky, P. Schuck, Mod. Phys. Lett. A26, 2429 (1991); P. Krüger, P. Schuck, Europhysics Lett. 72, 395 (1994); J. Dukelsky, P. Schuck, Phys. Lett. B 387, 233 (1996).
[13] J. Dukelsky, G. Röpke, and P. Schuck, Nucl. Phys. A628, 17 (1998).
[14] W. Glöckle, The Quantum Mechanical Few-Body Problem, (Berlin, New York, Springer, 1983); I.R. Afnan and A.W. Thomas in: Modern Three Hadron Physics ed. A.W. Thomas, p.1, (Berlin, Springer, 1977); E.W. Schmidt and H. Ziegelmann, The Quantum Mechanical Three-Body Problem, (Oxford, Pergamon Press, 1974); W. Sandhas, Acta Physica Austriaca, Suppl. IX, 57 (1972); V.B. Belyaev, Lectures on the Theory of Few-Body Systems (Spinger, Berlin 1990).
[15] E.O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B 2, 167 (1967).
[16] W. Sandhas, Acta Physica Austriaca, Suppl. IX, 57 (1972).
[17] Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).
[18] J. Haidenbauer, private communication.
[19] L. Canton and W. Schadow, Phys. Rev. C 56, 1231 (1997); W. Schadow, W. Sandhas, J. Haidenbauer, and A. Nogga, nucl-th/9810073.
[20] G. Röpke, A. Schnell, P. Schuck, P. Nozieres, Phys. Rev. Lett.

![FIG. 1. Cluster decomposition of the self energy Σ in terms of n-body t-matrices. The sum is over irreducible contributions only.](image)

![FIG. 2. Kinematical variables of the three-body system embedded in a medium.](image)

![FIG. 3. Triton binding energy per nucleon (solid lines, Yamaguchi potential) as a function of the uncorrelated nuclear density at a given temperature T; long-dashed lines: corresponding perturbation result, dashed-dotted lines: Paris (EST) potential. Dashed lines show the Nd continuum threshold for T = 10 MeV (left) and T = 20 MeV (right).](image)

![FIG. 4. Triton binding energy as a function of nuclear density at T = 10 MeV, and total momentum relative to the medium. From left to right: P = 0, 1, 2, 3, 4, 5 fm⁻¹.](image)
FIG. 5. Mott momentum per nucleon $P_{\text{Mott}}/A$ for triton - solid ($T = 10$ MeV) and dashed line ($T = 20$ MeV) and deuteron - dashed dotted ($T = 10$ MeV) and short dashed line ($T = 20$ MeV) as a function of nuclear density. Below the respective lines tritons (deuterons) do not exist as bound states.