DWARF GALAXIES WITH GENTLE STAR FORMATION AND THE COUNTS OF GALAXIES FROM THE HUBBLE DEEP FIELD

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ABSTRACT

In this paper the counts and colors of the faint galaxies observed in the Hubble Deep Field are fitted by means of simple luminosity evolution models that incorporate a numerous population of fading dwarfs. The observed color distribution of the very faint galaxies now allows us to put constraints on the star formation history in dwarfs. It is shown that the star-forming activity in these small systems has to proceed in a gentle way, i.e., through episodes where each one lasts much longer than a simple instantaneous burst of star formation. By allowing dwarfs to form stars in this gentle way, the number of predicted red remnants is severely reduced, in good agreement with the observations. Then, if the faint counts are to be fitted by means of dwarfs, the simple model for dwarfs forming stars in single, very short episodes is challenged, and a more complex star formation history has to be invoked. Recent observational evidence supporting this new dwarf model is also discussed.

Subject headings: galaxies: evolution — galaxies: photometry — galaxies: statistics — galaxies: stellar content

1. INTRODUCTION

The Hubble Deep Field (HDF; Williams et al. 1996) has provided the deepest view of the universe so far achieved. Metcalfe et al. (1996) have computed galaxy counts to the faintest limits \((U = 26, B = 29, R = 28, I = 28)\), which were shown to be in good agreement with ground-based data at brighter levels \((B = 28.2)\). The information on galaxy formation and evolution that can be obtained from the HDF is striking, as has been shown during the past months.

The HDF has a very small field of view \((5.3 \text{ arcmin}^2)\), and so the dominant population of galaxies is built up by very faint objects. In fact, the number of galaxies with an apparent magnitude of \(I \sim 28\) exceeds that of galaxies with an apparent magnitude of \(I \sim 23\) by almost a factor of 20. Which kind of galaxies constitutes the bulk of objects in the HDF?

The first result that becomes apparent after a first look at the HDF counts is that, even if there is some flattening in the deepest magnitude bins, down to the faintest limits the counts continue to increase with a quite steep slope. This is a difficult result to interpret in any cosmological setting, not just because of the cosmological turn-down effect of the volume (especially if \(q_0 = 0.5\), but also because of the Lyman break being redshifted into the different photometric bands (approximately \(z = 3, 4, 5, \text{ and } 6\) for \(U, B, R, \text{ and } I\), respectively), hiding any galaxies beyond those limits. Observational confirmation of the presence of the Lyman break is found in the Keck spectra of galaxies at \(z \sim 3\) (Steidel, Pettini, & Hamilton 1995). In fact, as I will show in the next sections, the level of counts is not easy to predict with pure luminosity evolution models, no matter what value of \(q_0\) is used.

The level of counts at the faint limits can be increased by including merging in the models, as was first suggested by Guiderdoni & Rocca-Volmerange (1991) and by Broadhurst, Ellis, & Glazebrook (1992) for a spatially flat cosmology. The number density of galaxies could increase as we look back toward higher redshifts, with galaxies being split up into the fragments that eventually will merge to build up the present-day population of galaxies.

However, the rate of merging has recently been claimed to be moderate (e.g., Barger et al. 1996; Dickinson 1996; Griffiths et al. 1996). Ellis et al. (1997) argue that the very little scatter in the \(U-V\) color of ellipticals in clusters at \(z \sim 0.5\) is consistent with the previous suggestion by Bower, Lucey, & Ellis (1992; from their study of galaxies in the Coma cluster) that ellipticals formed at high redshift and since then have passively evolved. Then, if ellipticals were assembled by the merging of smaller fragments, this had to occur at very early times (Kauffmann 1996).

Some authors (e.g., Koo & Kron 1992) have suggested that the bulk of faint blue galaxies in the deep counts could be intrinsically faint galaxies (dwarfs) located at low redshift. Even in an Einstein–de Sitter (EdS) model, the contribution of a (numerous) population of dwarfs to the counts will continue to increase with a Euclidean slope, becoming the dominant population at faint levels, as first noticed by Driver et al. (1994). Babul & Ferguson (1996) show that by including a population of “fading” dwarfs, the level of counts can be easily increased to the observed levels without the recourse to number density evolution (which, in any case, might exist).

In this paper I try to gain new insight into the role of dwarfs to interpret the counts of faint galaxies, in an attempt to constrain the star formation process in these low-mass systems by means of the galaxy colors in the HDF. Whereas Babul & Ferguson assume that the star formation in dwarfs takes place in single, very short \((\sim 10^7 \text{ yr})\) episodes, I will argue that in order to fit the counts to the faintest levels by means of dwarfs without overpredicting the number of red, faint remnants, the star formation should take place in a more gentle way. In fact, as discussed below, analysis of the photometry of individual stars in nearby dwarfs (Smecker-Hane et al. 1996) shows that dwarfs go through episodic bursts, with each one lasting \(\sim 1 \text{ Gyr}\).

The modeling of bright and dwarf galaxies to fit the
counts is presented in § 2. Section 3 is devoted to the comparison of the model predictions and the counts, colors, and angular sizes of galaxies in the HDF. A brief discussion of the results can be found in § 4 and the summary and conclusions in § 5.

2. MODELING GALAXY EVOLUTION TO FIT THE DEEP COUNTS

In order to get information on galaxy evolution from the deep counts of galaxies, two different approaches have been followed up to now. The classical one, pioneered by Tinsley (1972; although she was trying to understand the Hubble diagram rather than the counts), Kron (1978), Koo & Kron (1980), and Shanks (1980), takes as the starting point the population of galaxies at present (namely, the z = 0 luminosity function [LF]) and traces back the evolution of the luminosity by assuming a redshift of formation (\(z_{\text{fo}}\)) and a star formation rate. A more sophisticated approach is the one followed by, e.g., White & Frenk (1991), Kauffmann, Guiderdoni, & White (1994), Cole et al. (1994), and Baugh, Frenk, & Cole (1996), in which the starting point is the power spectrum of primordial density fluctuations predicted by the assumed theory for structure formation, followed by a recipe for the formation of the visible galaxies in which the dynamics of the gas, cooling, and feedback processes are included.

In this paper I will follow the traditional approach of tracing back the evolution of the population of galaxies. Even if the semianalytical models undoubtedly give interesting insight into the problem of galaxy formation and evolution, still, the flexibility of the simpler approach and the fact that it does not rely on any specific theory for structure formation can provide useful information in our interpretation of the deep counts of galaxies.

2.1. Bright Galaxies

The population of “bright” galaxies has been split up into three main types: ellipticals (E), spirals (S), and irregulars (Irr). The \(z = 0\) LF (LF0) has been taken from Efstathiou, Ellis, & Peterson (1988) with the morphological mix by Ellis (1983). To compute the evolution of the galaxy luminosities, I used the spectrophotometric models for stellar population synthesis by Bruzual & Charlot (1993; new version of 1995). For each type of galaxy, there are three parameters that we can adjust, with the constraint that the \(z = 0\) model spectrum has to resemble the observed spectra of nearby galaxies of the type being modeled. These parameters are the redshift of galaxy formation (\(z_{\text{fo}}\)), the star formation rate (SFR), and the initial mass function (IMF). In this work I follow the suggestions given by Pozzetti, Bruzual, & Zamorani (1996) of using a Scalo IMF for E and S, because it provides of a “milder” luminosity evolution, and so the amount of high-redshift galaxies, observed to be very small in redshift surveys, is reduced. Similar results can be achieved with the Salpeter IMF, provided that extinction by dust is also included in the models (e.g., Wang 1991; Francheschini et al. 1994; Koo & Kron 1992; Campos & Shanks 1996). As shown in Table 1, the SFR for E and S is taken to decay exponentially with time, whereas for Irr I consider a Salpeter IMF with a constant SFR. For the open models \((q_0 = 0.05)\) the redshift of formation was taken to be \(z_{\text{fo}} = 4\) (age = 15 Gyr; throughout this work, \(H_0 = 50\) km s\(^{-1}\) Mpc\(^{-1}\)), whereas for the EdS model \(z_{\text{fo}} = 7\) (age = 12.7 Gyr). The dimming of the luminosity by the Lyz forest (Madau 1995) and the Lyman break is considered in the modeling of the counts and redshifts. Number density evolution is not included, even if it might exist at “moderate” rates, as was already mentioned in the introduction.

2.2. A Phenomenological Model for Dwarfs

Dwarf (intrinsically faint) galaxies do exist. We see them everywhere in large numbers. In fact, it has been shown that they may constitute up to 50% of the whole population in nearby clusters (Binggeli, Sandage, & Tamman 1985; Ferguson & Sandage 1991; see also Trentham 1996). Dwarfs display a variety of properties in terms of shapes, colors, and spectral features, while sharing in common the small sizes and low metal content. This last property strongly suggests that the star formation history in these small systems may have followed a different evolutionary path than that of normal galaxies. In this respect, an explanation was first proposed by Dekel & Silk (1986), who showed that the shallow potential well in these low-mass galaxies may not be able to retain the gas after the subsequent galactic wind following an episode of star formation. Being stripped off the gas, the galaxies can no longer form stars, and so their luminosities, as the massive stars evolve, fade away. Following this line of argument, Babul & Rees (1992) suggested that the gas may not be entirely lost but may be trapped in the outer parts of the dark halo from where it could recollapse, giving rise to new episodes of star formation. Whether the gas is completely lost will depend on the mass of the galaxy and also on the pressure of the environment, which could confine it. This scenario is in all similar to that first proposed by Davies & Phillips (1988), who suggested that the star formation in dwarfs may proceed in the form of intermittent bursts followed by long quiescent periods.

### Table 1

| Type of Galaxy | \(\Phi^*\) (Mpc\(^{-5}\)) | \(M_\text{\#}\) | \(z_{\text{fo}}\) | \(z_{\text{at}}\) |
|---------------|-----------------------|-------------|----------------|----------------|
| E.............| \(9.5 \times 10^{-4}\) | -20.9       | -0.48          | 1 (\(q_0 = 0.05\)) |
| S.............| \(1.15 \times 10^{-2}\) | -21.1       | -1.24          | 10 (\(q_0 = 0.05\)) |
| Irr................| \(5.4 \times 10^{-4}\) | -21.1       | -1.24          | Salpeter |

| SFR \(e\)-Folding Time \((\tau_\text{s})\) (Gyr) | \(z_{\text{fo}}\) |
|----------------------------------------------|----------------|
| 1 (\(q_0 = 0.05\)) | 4 (\(q_0 = 0.05\)) |
| 0.7 (\(q_0 = 0.5\)) | 7 (\(q_0 = 0.5\)) |
| 10 (\(q_0 = 0.05\)) | 4 (\(q_0 = 0.05\)) |
| 7 (\(q_0 = 0.5\)) | 7 (\(q_0 = 0.5\)) |
The importance of accounting properly for the presence of dwarfs in the modeling of the deep counts was already shown by Driver et al. (1994). A much more elaborate model was recently worked out by Babul & Ferguson (1996), who very appropriately “baptized” the dwarfs as “boojums = blue objects observed just undergoing moderate starburst.” In this work it is claimed that dwarfs may arise in large numbers in hierarchical models of structure formation at high redshifts, while the star formation is delayed until $z \sim 1$ because of the photoionization of the interstellar matter by the UV background ionization (Babul & Rees 1992).

Dwarfs are assumed to form continuously since $z = 1$ up to now (see also Babul & Ferguson 1996). Because of the “classical” approach chosen to model the deep counts, this continuous formation of dwarfs is simulated by allowing them to form in contiguous generations, each one following the previous one by 0.5 Gyr. The LF for dwarfs is assumed to have a Schechter-like form, with a slope of $\alpha = -2$. The reason for this very steep slope is based on the fact that in hierarchical clustering the distribution of small halos is a steep function of the mass. However, as discussed later, the choice of $\alpha$ will not alter any of the basic conclusions of this work.

Two different types of dwarfs are tested (called $B = 0.05$ and $B = 0.5$; $B$ is the duration of the period of star formation). In both of them the star formation is assumed to proceed in the form of a single burst (i.e., constant star formation) lasting $5 \times 10^7$ and $5 \times 10^8$ yr for models $B = 0.05$ and $B = 0.5$, respectively. The masses of dwarfs are the same in both models: $4.7 \times 10^{10} M_\odot$ for an $L_\ast$ dwarf (total mass; I assume that the baryonic matter is $\sim 1\%$ of the total mass). The mass of the smallest dwarfs is $4.7 \times 10^8 M_\odot$, or 5 mag fainter than $M_\ast$. In the peak of star formation this corresponds to a magnitude in $B$ (assuming a Salpeter IMF) of $-18.5$ and $-20$ for $B = 0.5$ and $B = 0.05$, respectively.

The number density of dwarfs is a free parameter chosen to fit the $B$-band counts to the faintest limits. For each generation, $\Phi^0(g(z)) = \Phi^0[g(z = 0)](1 + z)^n$, where $g(z)$ refers to the generation formed at a redshift $z$ and $n$ is the rate of dwarf formation. Two different cases are tested: $n = 0$ and $n = 3$. In the latter, dwarfs are assumed to form more numerous as the redshift of formation is higher.

3. Dwarfs with Gentle Star Formation and the Deep Counts

3.1. Hubble Space Telescope Counts as a Function of Morphology

The unprecedented high-resolution imaging capability of the Hubble Space Telescope (HST) makes it now possible to study the shape of very distant (faint) galaxies. Deep counts as a function of morphological type have recently been published (Driver et al. 1995; Glazebrook et al. 1995; Abraham et al. 1996) down to $I = 25$. It has been shown that the counts of E galaxies increase much more slowly than the counts of irregular/peculiar systems (also called “weirdo” galaxies; hereafter W). In fact, the counts of E show small evidence for flattening at $I \sim 25$. At $I = 18$ E are more numerous than W by a factor of $\sim 3$, while at $I = 25$ W become more numerous than E by almost a factor of 2, reaching a level of counts comparable to that of S. These results have led to questioning the extent to which the Hubble system provides an adequate description of the morphology of galaxies at high redshifts.

The $I$-band counts split into the three morphological types (E, S, and W) are shown in Figure 1 together with the model predictions. As shown in the figure, the pure luminosity evolution models (PLE) provide reasonable predictions for the counts of E and S, while severely underestimating the number of W unless a (numerous) population of dwarfs is included (see Table 2 for more details on the dwarf models shown).

3.2. $B-I$ Color Distribution

The fact that PLE models may have problems predicting the level of counts at very faint magnitudes is further evidenced in Figure 2. This figure shows the distribution of $B-I$ colors for the galaxies in the Hubble Deep Field (Metcalfe et al. 1996), for galaxies selected according to their magnitudes in the $I$ band (top panels) and in the $B$ band (bottom panels). To put the HDF counts into the standard Johnson system in order to compare the HDF results with ground-based data (as in Figs. 6a–6d, where deep counts from a variety of sources are shown), it is necessary to use certain color conversions. The galaxy counts shown here were worked out by Metcalfe et al. (1996) and Metcalfe, Shanks, & Fong (1997), who used the synthetic color transforms of Holtzman et al. (1995) and the published values of the HDF zero-point conversion from STMAG to Vega system. The details of the procedure can be found in Metcalfe et al. (1997). As shown in Metcalfe et al. (1996), there is excellent agreement between space- and ground-based data down to the faintest limits (e.g., $B \sim 28.2$, the limiting magnitude of the “Herschel Deep Field”).

The distribution of $B-I$ colors, together with the data, are plotted with predictions from PLE models. It is important to notice that the models are not normalized to the number of galaxies in the HDF for each magnitude bin but just correspond to the predicted number of galaxies to be seen in the HDF field of view. The normalization is fixed by the assumed values of $\Phi^*$ (see Table 1), which already corresponds to a high normalization, $B \sim 18$. To the eye, the color distribution from PLE models seems to be a good match to the data if models had been normalized to the total number of galaxies in each magnitude bin. However, if we did so, the deep counts would be overpredicted up to very faint levels. For example, in the $q_0 = 0.05$ model, data and predictions almost agree for $24 < I < 25$, but to fit the data for fainter bins with the same model, we would have to multiply the normalization of the model by almost a factor of 3. Therefore, the result that emerges is that even if the predicted color distribution is similar to the observed one, a match to the data requires a normalization inconsistent with the counts. A remaining question is that of the very red high-$z$ Lyman-break galaxies. For the open model, because

### Table 2

| Model | $B$  | $q_0$ | $n$  | $\Phi^0[g(z = 0)]$ (Mpc$^{-3}$) | $M^*$ | $z$  |
|-------|------|-------|------|-------------------------------|------|------|
| 1     | 0.05 | 0.05  | 0    | $5 \times 10^{-3}$            | -20  | -2   |
| 2     | 0.05 | 0.5   | 0    | $5 \times 10^{-3}$            | -20  | -2   |
| 3     | 0.5  | 0.05  | 0    | $3 \times 10^{-3}$            | -18.5| -2   |
| 4     | 0.5  | 0.5   | 0    | $3 \times 10^{-3}$            | -18.5| -2   |
| 5     | 0.5  | 0.5   | 3    | $1 \times 10^{-3}$            | -18.5| -2   |
Fig. 1.—Counts of galaxies as a function of morphological type (data kindly provided by R. Abraham) in the $I$ (Kron-Cousins) band. Together with the data, pure luminosity evolution models with and without a population of dwarfs to fit the counts of “weirdo” (W) galaxies are shown. The top panels show counts of E and S galaxies. Solid and dashed lines are for PLE model predictions, respectively. For S, both a Salpeter IMF ($q_0 = 0.05$; short dashed line) and a Scalo IMF ($q_0 = 0.5$; long dashed line) were considered. In bottom panels the counts of W are plotted, together with model predictions (left panel, $q_0 = 0.05$; right panel, $q_0 = 0.5$). Solid lines are the predictions when only a population of Irr is considered, whereas dashed lines are predictions from models including a population of dwarfs with a burst length of $B = 0.5$ Gyr (dotted lines) and $B = 0.05$ Gyr (dot-dashed lines). For details on the modeling, see Tables 1 and 2 and text.

As was said before, none of the models are able to provide a reasonable fit when using the appropriate normalization. Interestingly, as we approach fainter limits, the models more severely underpredict the number of red faint galaxies (i.e., $B - I \sim 1$–2) observed.

The same plot is shown in Figures 3 and 4 (for $q_0 = 0.05$ and 0.5, respectively), although now the models include the dwarf population ($n = 0$ case). As expected, the level of counts increases and the color distribution is much better reproduced. However, it becomes clear that the star formation rate assumed for the dwarfs is a “key” issue in reproducing the data, with the $B = 0.5$ model giving a much better fit to the color distribution than the $B = 0.05$ one.

As shown in Table 2, the number density of dwarfs in the $B = 0.05$ model is larger than that in the $B = 0.5$ one by almost a factor of 2. The reason is found in the imposed restriction that the model has to be able to (approximately) reach the observed level of counts in the $B$ band. In the $B = 0.05$ model, because the galaxies form the bulk of stars during a shorter period, fading away afterward, the number of dwarfs required to fit the $B$-band counts is larger. This is simply due to the fact that the probability of observing a $B = 0.05$ dwarf while exhibiting blue colors (i.e., in the “boojum” phase) is smaller than for a $B = 0.5$ one. As a result, the $B = 0.05$ model predicts a large population of red remnants, which is not seen in the color distribution of the HDF galaxies. The disagreement is even worse if $q_0 = 0.5$.

The fit to the color distribution provided by the $B = 0.5$ model is quite reasonable. It is only for the $I = 26$–27 magnitude bin where model and data show disagreement, with the model predicting a larger number of red ($B - I \sim 2$–3) galaxies than are observed. However, for this magnitude bin the incompleteness is large (see the box in the figures), and
the galaxies without measured color (i.e., those detected in the I-band image but not in the B-band one) are expected to be red, i.e., too faint ($B > 29-30$) to be detected.

It is interesting to note that the shorter the star formation period in dwarfs (notice that Babul & Ferguson use $\sim 10^7$ yr), the larger the number of (unobserved) remnants, and vice versa. Therefore, if we want to fit the counts to the faintest levels by means of dwarfs, the star formation has to proceed in a more "gentle" way.

In order to test the effect of the rate at which dwarfs are being formed, I show in Figure 5 the same plot with predictions from $B = 0.5$ models with two different rates of dwarf formation: $n = 0$ (formation of dwarfs is constant with time) and $n = 3$ (formation of dwarfs decreases with time). The differences between the two are not as large as between the $B = 0.05$ and $B = 0.5$ models. Still, it can be seen that in the $n = 3$ case the number of red remnants in the $I = 26-27$ bin is larger, as is expected. Nevertheless, the data do not show a clear distinction between the two models.

3.3. Deep Counts and Redshift Distributions

The deep counts in the $K$ band (data taken from Djorgovski et al. 1995; Gardner et al. 1996a, 1996b; Soifer et al. 1994; McLeod et al. 1995; Glazebrook et al. 1995), $I$ band (Metcalf, Shanks, & Fong 1997; Metcalf et al. 1996; Driver et al. 1994, 1995; Glazebrook et al. 1995; Smail et al. 1996; Lilly, Cowie, & Gardner 1991; Tyson 1988; Hall & Mackay 1984; Koo 1986), $B$ band (Metcalf et al. 1991, 1995, 1996; Lilly et al. 1991; Tyson 1988; Couch & Newell 1984; Infante, Pritchet, & Quintana 1986; Jones et al. 1991; Koo 1986; Kron 1987; Maddox et al. 1990), and $U$ band (Jones et al. 1991; Metcalf et al. 1996; Guhathakurta, Tyson, & Majewski 1990; Koo 1986) are shown in Figures 6a–6d, together with various model predictions for $B = 0.5$ and 0.05 and $q_0 = 0.05$ and 0.5. (Because the counts are plotted on a logarithmic scale, they have been normalized by subtracting the corresponding "best" fit at brighter levels in order to expand the scale.) It must be pointed out here that the HST F300W filter is not very close to the standard $U$-band, which complicates any comparison between HDF and ground-based data (see Metcalf et al. 1997). Because the main source of data at the very faint limits is the HDF, the comparison between models and data in the $U$ passband must be taken with caution not only because of the uncertainties in the color conversion but because it can affect the number of Lyman dropouts, as the F300W wavelength is shorter than the standard $U$-band one.

![Figure 2](image-url)

Fig. 2.—$B-I$ color distribution of faint galaxies in the HDF selected in the $I$ band (top panels) and $B$ band (Johnson; bottom panels) for different magnitude bins. The boxes in each panel correspond to the incompleteness. Also shown are predictions from PLE models. The models are not normalized to the number of galaxies in each magnitude bin but correspond to the number of galaxies expected to be seen in the HDF field of view for each model.
In the $K$ band, all five models give good predictions, although it would seem that the EdS case fits the data slightly better. In particular, the shoulder (clear change of slope) seen at $K \sim 10$ is very nicely reproduced. Notice that there is very little difference between the $B = 0.5$ and $B = 0.05$ models, the reason being that the contribution of the dwarfs to the $K$-band counts is noticeable only at very faint levels. In the other bands, all five models are marginally consistent with the data, although the $B = 0.5$ case gives more consistent fits. For example, the $B = 0.05, q_0 = 0.5$ model clearly overpredicts the $I$-band counts while it underpredicts the $U$-band counts. (Notice that the number of Lyman dropouts could be underpredicted because, as was pointed out above, the F300W wavelength is shorter than the standard $U$-band one used in the models. This would affect the modeling in the sense of “overpredicting” the counts.) Notice also that the $I$-band counts are better fit by EdS models than by open ones. Faintward of $I \sim 26$, open models overpredict the counts, especially when $B = 0.05$.

A further test of the models can be done by means of the redshift distribution of galaxies, $n(z)$, selected in different photometric bands. In fact, the very early PLE models (e.g., Metcalfe et al. 1991) that were successful in fitting the deep counts faced some problems with the absence of a high-redshift tail in the observed $n(z)$. These first discrepancies were solved by including further ingredients in the modeling, like the presence of dust (e.g., Wang 1991; Franchescini et al. 1994; Campos & Shanks 1996) or variations in the IMF (Pozzetti et al. 1996). Also, the introduction of merging (Guiderdoni & Rocca-Volmerange 1991; Broadhurst et al. 1992) helped to solve the problem.

Here I will use the sample of Colless et al. (1990, 1993) for galaxies with $B = 21-22.5$, the recent survey by Cowie, Songalia, & Hu (1997) for galaxies with $B = 22.5-24$ and $K = 18-19$, and the Canada-France Redshift Survey (CFRS; Crampton et al. 1995) for galaxies in the magnitude range $I = 17-22$. The $n(z)$ for the four samples are shown in Figure 7, together with model predictions. As can be seen, the $n = 0$ models face some problems, as they slightly overpredict the number of low-$z$ galaxies in Colless et al. (which is an almost “complete” sample). The number of very low-$z$ galaxies is severely reduced when $n > 0$ (i.e., if the formation of dwarfs decreases with time). It is in any case interesting to point out that it is usually claimed (e.g., Glazebrook et al. 1995) that the unidentified galaxies in deep redshift surveys are likely to be at high $z$, with the reason given being that the redshifts could not be measured because the main spectral features (like the frequently strong [O ii] $\lambda 3727$ emission line) are redshifted outside the optical spectral window.

Fig. 3.—Same as Fig. 3, but models now incorporate a population of dwarf galaxies with the star formation lasting 0.05 and 0.5 Gyr ($q_0 = 0.05$).
However, it may happen that some “still blue and luminous” dwarfs show a featureless spectrum, and so their redshifts are difficult to measure. To illustrate this, I show in Figure 8 the evolution with time of the [O II] $\lambda3727$ equivalent width and $B-I$ color after a single burst of star formation. As soon as the massive stars evolve (which happens very fast), the equivalent width drops to zero, as there are no UV photons capable of ionizing the interstellar medium, while the colors remain blue for quite a longer period of time because of the bulk of intermediate-mass stars still in the main sequence. Therefore, some of the unidentified objects might be dwarfs at low $z$, still blue and luminous, but in which the star formation just ceased.

In order to give the reader a better view on the behavior of the models, predictions are plotted in Figure 9 for the redshift distribution of galaxies at fainter magnitude bins (i.e., $24 < B < 25$, $25 < B < 26$, $26 < B < 27$, and $27 < B < 28$). As expected for dwarf-rich models, at the very faint limits most galaxies are located at low redshifts (i.e., $z < 1$). There are, however, differences from model to model. For example, it can be seen that the $B = 0.05$ models predict a higher percentage of galaxies at $z < 0.2$ than the $B = 0.5$ ones, especially in the faintest bin. Also, the distribution is shifted toward higher redshifts when $n = 3$ instead of $n = 0$.

As a further test of the models, in Figure 10 the $z = 0$ LF used is plotted, together with the LF measured by Loveday et al. (1992) and Marzke, Huchra, & Geller (1994) (arbitrarily normalized, because of the inconsistency between the two LFs). The LF0 shown here corresponds to the $B = 0.5$ model. We see that the steep slope measured by Marzke et al. (1994) is, within the errors, consistent with the model. Notice that the slope of the LF for each generation of dwarfs is very steep, but the superposition of all dwarfs from the different generations plus giants at $z = 0$ gives a much flatter slope, except at magnitudes fainter than $\sim -15$.

3.4. Angular Size Distribution

One of the most “puzzling” results that came from the HDF concerns the sizes of the galaxies. The HDF is apparently filled with too many “tiny” galaxies. Merging may help to solve the problem, by assuming that galaxies at high redshifts are split into the (smaller) fragments that will eventually merge to build up the present-day population (see Im et al. 1995; Roche et al. 1997). Another possible explanation is found in the dwarf-rich models, where a large contribution to the faint counts comes from intrinsically small (dwarf) galaxies. In order to check the viability of the models analyzed here, I next show predictions for the
angular size distribution of the galaxies in the HDF. (The isophotal angular sizes of the HDF galaxies shown in the figures were measured by N. Metcalfe, who kindly provided me with the data previous to publication.)

To compute the angular size distribution, it is first antecessor to find relationships between physical size and the absolute $z = 0$ $B$-band luminosity of the galaxies. As in Im et al. (1995), for ellipticals the scaling laws derived by Binggeli, Sandage, & Tarenghi (1984) were used: log $(r_{hl}/\text{kpc}) = -0.3(M_B + 18.75)$ if $M_B < -20$ and log $(r_{hl}/\text{kpc}) = -0.1(M_B + 15.7)$ if $M_B > -20$, where $r_{hl}$ is the model-independent half-light radius. The profiles are assumed to fit a de Vaucouleurs law, and the Kormendy relation is used to relate the effective radius with the effective surface brightness ($\mu_{\text{eff}}$): $\mu_{\text{eff}} = -2.5 \log (r_{\text{eff}}) + 20.2$ (the zero point of the relation was taken from the work by Jørgensen, Franx, & Kjærgaard 1995 for ellipticals in the Coma cluster). Notice that here I assume the half-light radius to be the same as the de Vaucouleurs effective radius, which will be true only as long as the de Vaucouleurs law is a good fit to the light profiles (see Binggeli et al. 1985). The angular sizes of the HDF shown in the figures are isophotal sizes, defined by the isophote $\mu_B = 28.5$ mag per square arcsecond. Assuming a de Vaucouleurs profile, the physical $r_{28.5}$ size corresponding to the $\mu_B = 28.5$ isophote is computed by taking into account the surface brightness dimming: $\mu_{28.5}^* = 28.5 - 10 \log (1 + z)$. $\mu_{28.5}^*$ is the surface brightness of the ($z = 0$) isophote that, at a redshift $z$, would be observed as 28.5 mag per square arcsecond (see, e.g., Sandage 1961). Then, $r_{28.5} = r_{\text{eff}}[(\mu_{28.5}^* - \mu_{\text{eff}})/8.325 + 1]^4$. Notice that the evolution of the luminosity (and the $K$ correction) is included in the modeling, but no size evolution has been assumed (this means a uniform fade of the luminosity is considered). Regarding the modeling of ellipticals, consider the work by Im, Griffiths, & Ratnatunga (1997), who tested luminosity evolution and merging models by means of the distribution of angular sizes and colors in the HST Medium Deep Survey. The modeling of the ellipticals follows the same approach as in Im et al. (1995), which, as quoted before, is also followed in the present work. Im et al. (1997) showed that for $20 < I < 21$ and $21 < I < 22$, simple luminosity evolution models (with $q_0 = 0.5$ and $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$) nicely reproduced the half-light angular size distribution, whereas merging models predict too many small galaxies compared with the data.

According to Freeman’s law (Freeman 1970), the central surface brightness of the disk component in spiral galaxies is nearly constant [$\mu_0(B) \sim 21.3$]. Even if not properly understood yet, results have accumulated indicating the
validity of this empirical law (Boroson 1981; Kent 1985; Bosma & Freeman 1993). The fact that the central surface brightness is constant (though with a scatter of ~0.5–1 mag., see, e.g., de Jong 1995; Marquez & Moles 1997) implies that the scale length and the luminosity of disk components are correlated: \( \log (r_0 / \text{kpc}) = -0.2M_B - 3.45 \). As pointed out by Im et al. (1995), in modeling the spirals complications arise because the galaxies are not pure disks but also show a bulge component. To overcome this problem, Roche et al. (1997) model the half-light radii of spirals, \( r_{hl} \), by considering that \( r_{hl} = 1.4r_0 \) for the early types, instead of the "pure disk" relationship \( r_{hl} = 1.68r_0 \), which is used for the late types. In the present work, because a single spiral type is considered, all spirals have been modeled as pure disks. Therefore, we will have to keep in mind that the sizes will be slightly overestimated. As for the ellipticals, the surface brightness dimming is taken into account in computing the isophotal angular sizes, and no size evolution is considered. I will get back to this point at the end of the section.

The above empirical relationships allow us, in a first simple approach, to compute predictions for the angular size distribution of "bright" galaxies. For dwarfs, the situation is much more uncertain. As pointed out by Impey, Bothun, & Malin (1988), studies of the structural parameters of dwarf galaxies are "obscured" by all kinds of selection effects due to the intrinsic faintness of the objects. Nevertheless, it is already well known that the light profiles of dwarf ellipticals (dEs) obey a simple exponential law (Binggeli et al. 1984; Ichikawa, Wakamatsu, & Okamura 1986; Caldwell & Bothun 1987; Impey et al. 1988). However, for the dwarf irregulars (dIrrs) the situation is less clear, as some of them show an excess of light over the exponential fit in the central parts (Bothun et al. 1986). With respect to the blue compact dwarfs (BCDs), at least some of them seem to have surface brightness profiles that can be fitted by exponentials (Bothun et al. 1986; but see also Kunth, Maurogordato, & Vigroux 1988). This situation has led to the suggestion that (some of) the BCDs might be the truly progenitors of the dEs, but it is less clear that dIrrs could evolve into dEs. In this respect, Bothun et al. propose that there might be a continuous spectrum of dwarfs, and even if they all could eventually be stripped off the gas, the gas depletion process as well as the star formation efficiency might be quite complex, probably related to the gravitational potential well in which the gas is embedded.

Compared to the picture that emerges from the observations, the modeling of dwarfs in the present work is obvi-
The $n(z)$ redshift distribution of galaxies selected in different magnitude ranges (data taken from the literature. See text for references). The incompleteness rate (i.e., galaxies for which the redshift could not be measured) are $D_{12}\%$ for $B \geq 21 - 22.5$, $D_{15}\%$ for $I \geq 17 - 22$, and $D_{20}\%$ for $K \geq 18 - 19$ and $B \geq 22.5 - 24$. Lines are predictions from $n = 0$ dwarf models as in Fig. 6. Notice that $B = 0.5$ models predict fewer very low-$z$ dwarfs than $B = 0.05$ models. The $n = 3$ model ($B = 0.5, q_0 = 0.5$; triangles) provides the best fits to all the data. The percentage of galaxies predicted by the models to be located beyond the highest redshifts shown in the figure are $D_{4}\%$ and $D_{1}\%$ for $B \geq 21 - 22.5$, $D_{10}\%$ and $D_{2}\%$ for $I \geq 17 - 22$, $D_{6}\%$ and $D_{0.4}\%$ for $K \geq 18 - 19$, and $D_{17}\%$ and $D_{24}\%$ for $B \geq 22.5 - 24$. The three flat models, on the one hand, and the two open models, on the other, give very similar percentages for the predicted high-redshift population.

Continuing with the simple approach in the modeling of dwarfs, here I will assume a single exponential fit for all dwarfs with constant central surface brightness at the peak of the star formation process [$\mu_{0}(\text{peak}) \sim 21$], together with a uniform fade in luminosity. As for the spirals, an exponential profile with a constant central surface brightness (which, of course, will fade at the same ratio than the luminosity) implies a correlation between total luminosity at the peak of star formation and scale length. For the $B = 0.5$ model, where $M^*_{\text{peak}} = -18.5$ (see Table 2), an $M^*$ dwarf would have a disk scale length of $\sim 3.5$ kpc, whereas for the smallest dwarfs ($M^*_{\text{peak}} = -13.5$) $r_0 \sim 0.4$ kpc. The existence of a correlation between luminosity and scale length for dwarf ellipticals has been shown by Binggeli et al. (1985), among others. For BCDs, Campos, Moles, & Masegosa (1990) have shown the existence of a well-defined correlation between luminosity and isophotal radius of the form $\log L_{25}(B) \propto 1.88 \log \left( \frac{R_{25}}{r_0} \right)$.$^{1}$ The slope of the correlation, which is very close to 2, implies a nearly constant surface brightness inside the 25 mag per square arcsecond isophote, as it is in fact observed ($\mu_{25} \sim 23.5$), although with a large ($\sim 1.5$ mag) scatter. The scatter is interpreted in this paper as reflecting the fade in luminosity already at the BCD

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No. 2, 1997

DWARF GALAXIES WITH GENTLE STAR FORMATION

615
stage. Assuming an exponential surface brightness profile for dwarfs, and taking \( \mu_{23} \sim 23.5 \) for BCDs that could be considered as dwarfs observed approximately very close to the peak of the star formation process, it is derived that \( \mu_{0(\text{peak})} \sim 21 \).

In Figure 11 predictions are shown for the \( B = 0.5 \) model for three different cases: \( q_0 = 0.5 \) (with \( n = 0 \) and \( n = 3 \)), and \( q_0 = 0.05 \) (with \( n = 0 \)). Solid lines are predictions for the total angular size distribution, whereas dotted lines correspond to the dwarfs. The histograms are normalized to the total number of galaxies in each of the magnitude bins. As can be seen, for the faintest bin (\( 27 < B < 28 \)) the models, in spite of their simplicity, fit the data remarkably well (especially if \( q_0 = 0.5 \)). For the brighter bins, in particular for the \( 25 < B < 26 \) one, the models predict too large sizes compared with the data. It should be noticed that the contribution to the “unobserved large” galaxies comes from the “bright” population, as dwarf sizes fall well into the observed range.

In order to get some insight into the sources of the discrepancy between data and models (even if this goes beyond the goals of this paper, which are mainly to address the role of dwarfs in the faint counts), let us go back to the modeling of the spirals. As was said before, spirals are treated as single
“pure” disks with no size evolution. Already, the first assumption, i.e., neglecting the bulge, causes the sizes to be overestimated. Also, it has been largely suggested in most models about the formation and evolution of disk components (see e.g. Lacey & Fall 1985; Wang & Silk 1994; Cayon, Silk, & Charlot 1996) that disks are very likely formed from the inside out, as the gas is slowly falling from the halo to develop the disk. Just to check the effect of the “disk growth” in the predictions, let us model it in a very crude way as \( \log (r_0/\text{kpc}, z) = -0.2M_B + 3.45(1 + n \times z) \) with \( n = 0.03 \). (This means that for \( z = 1 \) and \( z = 2 \) galaxies, the disk lengths will be a factor of \( \sim 1.2 \) and \( \sim 1.5 \) smaller than for \( z = 0 \) galaxies). Adding this (totally) ad hoc disk evolution, the predictions (Fig. 11, triangles) now fit the data also for the brighter bins, and the discrepancies are smoothed away.

4. DISCUSSION

The phenomenological model for dwarfs shown here is somewhat ad hoc, due to the lack of knowledge about how dwarfs are formed and evolve. Nevertheless, it is based on reasonable assumptions such as the formation at low \( z \) when the UV-background ionization decreases (Babul & Rees 1992) or formation of stars during a “finite” period,
with the luminosity of the galaxy fading away afterward (if stars were continuously formed, the metallicity would not remain at the low levels observed among dwarfs).

How long is the star formation period? If counts and colors in the HDF are to be fitted by means of the dwarf population, the star formation period has to be quite long, at least a few times $10^8$ yr. As was shown in the last section, using shorter periods requires larger numbers of dwarfs to fit the deep counts, which results in a large number of red, faint remnants not seen in the HDF. [It should be said here that, for the $B = 0.05$ model, a time-consuming trial and error test—i.e., variations of $a, n, M_\star(\text{peak})$—showed that there is no choice of a set of parameters that is able to provide good fits to the data.]

In the “standard” model for dwarfs, it is suggested that the star formation takes place in the form of single, very short bursts. As was first shown by Dekel & Silk (1986), the shallow potential well of the galaxy is not likely to be able to retain the gas after the explosions of few supernovae. This vision has been challenged by the analysis of the photometry of individual stars in nearby dwarfs, e.g., in the Carina dwarf spheroidal galaxy by Smecker-Hane et al. (1996). In this very low-mass galaxy, the star formation history has been very complex: several bursts each lasting $\sim 1$ Gyr followed by quiescent periods. In order to explain this complex history together with the low metal content in the Carina galaxy, the authors suggest that the “gentle” star formation during the “active” phases generated winds that expelled the metal-enriched gas. But in order to keep the star formation without stripping the galaxy off, these winds cannot be strongly coupled with the general interstellar medium, and so denser gas clouds are not expelled from the galaxy. Therefore, there must exist a sort of self-regulation mechanism able to keep a low-rate (gentle) star formation process for long periods without increasing the metal content. The gas might be eventually lost, but the gas depletion could be a very slow process, perhaps due to the presence of a massive dark halo.

A case of anomalous chemical enrichment in local dwarfs was found in GR 8. This low-metallicity galaxy ($Z \sim 1/18 Z_\odot$) shows a high helium content, a fact interpreted as the result of a selective metal loss during a gentle process of star formation (Moles, Aparicio, & Masegosa 1990). In the same line, Masegosa, Moles, & Campos (1994) analyzed a sample of 121 HI galaxies and found no trend between the helium abundance and the metallicity of the systems. Again, this result was interpreted as reflecting the inability of the galaxy to retain the supernova ejecta, although the star formation phase in those systems is still an ongoing process.

In the best model ($B = 0.5$) for dwarfs used here, each galaxy undergoes a single period of star formation lasting 0.5 Gyr and fades away afterward. If the period of activity was longer and/or galaxies underwent multiple bursts, the number of dwarfs requires to fit the deep counts would be smaller, possibly in better agreement with the observed $n(z)$ distributions.

The $B = 0.5$ model should be taken as a simple “statistical” approach to model dwarfs. This means that, on average, star formation in dwarfs may proceed in the form of intermittent periods of star-forming activity, with each one lasting longer than a single, instantaneous burst of star formation. The implication is that dwarfs should have a sort of self-regulation mechanism capable of inhibiting an effective gas loss while keeping the metal content low. It is very plausible that this mechanism is related not only to the mass of the galaxy (or the dark halo) but also to the environment. The latest observational evidence is found in the dependence of the early dwarf-to-giant ratio on the richness of the environment (Ferguson & Sandage 1991). Our local group of galaxies is completely dwarf dominated, with an LF extending over more than $\sim 13$ mag (van der Berg 1992). Some of the local dwarfs are red spheroidals depleted of gas, while some others (e.g., Carina, GR 8) show evidence that the star-forming activity has been proceeding over periods of several Gyr. Therefore, a much more realistic model for dwarfs should account for the different evolutionary paths that dwarfs, depending on the dark halo mass, the environment, or both, have followed. Unfortunately, little is yet known about the star formation history in these faint, small (slippery) systems.

In any case, the purpose of the present work is to show that the inclusion of dwarfs is a “key” issue in the proper interpretation of the deep counts of galaxies. The success of the very simple models in fitting both the counts in all different photometric bands and the colors of the faint galaxies in the HDF, while giving reasonable predictions for the redshift distributions, supports this conclusion. Moreover, it seems clear that the very faint end of the counts is dominated by these intrinsically faint objects, i.e., reflects the faint end of the local LF. The inclusion of dwarfs also provides of a simple explanation of the large amount of “weirdo” objects observed in the deep HDF images. Nevertheless, it is important to point out that some authors (e.g., Ellis et al. 1997) have suggested that some of these peculiar systems may be subsystems that will eventually merge to form ellipticals, although, as already mentioned, this merging process very likely occurred at high redshift ($z > 3$).

Further success of the models shown here comes from the fit to the angular size distribution of the galaxies in the HDF. Although the modeling of dwarfs is very simplistic, mainly due to the lack of enough observational evidence, it was still shown that the predicted angular sizes of the dwarf population fall well into the observed range. Discrepancies between data and models were found to be due to the “bright” galaxies, and in fact these are smoothed out when some disk size evolution is introduced.

The model of dwarfs proposed here to fit the counts, colors, redshifts, and angular size distribution of faint galaxies relies on two parameters: the normalization (i.e., number density) of dwarfs and the star formation history. Whereas the first one is difficult to constrain because of the intrinsic faintness of these galaxies, the star formation history can be tested using local dwarfs, both by following the approach by Smecker-Hane et al. of resolving individual stars but also by means of detailed studies of the chemical abundances.

The note of “pessimism” is attributable to the fact that adding “gentle” dwarfs to the models makes it possible to fit counts, colors, and redshifts both in the open and in the EdS case. Because the faint end of the counts is now dominated by low-$z$ dwarfs, for which the normalization is, as remarked above, very uncertain (not to say totally unknown), $q_0$ will not be easily constrained by comparing the level of faint galaxies with the availability of large volumes at high $z$. A way out is to measure the amount of high-$z$ (e.g., $z > 3$) galaxies at faint levels, following, for example, the $U$-dropout procedure developed by Steidel et al. (1995). The ratio of low-$z$ to high-$z$ giants is more likely
to provide a closer constraint on the value of \( q_0 \). For the \( I \) band, where counts are comparatively deeper than in the other bands, the open models overpredict the counts at the faint levels, while the EdS models provide a much better fit. However, the uncertainties in the modeling of dwarfs do not allow us to extract any strong conclusion from this.

Finally, it is worth noticing that testing the dwarf model—i.e., proving by means of redshifts that the bulk of faint galaxies in the deep counts is actually located at low redshifts—not only has important implications for our understanding of the galaxy formation and evolution processes, but it provides of a further test of the standard cosmological framework. If bright counts were dominated by giants while the main contribution to the faint counts was provided by dwarfs at lower redshifts, this would imply that the counts of giants flatten, i.e., level off as the volume elements at high redshifts are increasing smaller than in a Euclidean (\( d^3 \)) geometry.

5. SUMMARY AND CONCLUSIONS

1. The counts and colors of galaxies in the Hubble Deep Field can be easily fitted by simple luminosity evolution models that incorporate a numerous population of dwarf galaxies. Because of the present lack of knowledge about the space density of dwarfs, reasonable fits can equally be provided in both open and flat cosmologies. To constrain \( q_0 \), the high-z to low-z bright galaxy ratio is then needed, which requires knowledge of the redshifts of the very faint population.

2. The incorporation of dwarfs in the models provides of a simple explanation of the large number of “weirdo” galaxies seen in the HDF images. While \( E \) and \( S \) counts can be fitted by means of simple pure luminosity evolution models, the high level of \( W \) counts requires of an extrapolation of dwarfs. Previous claims that the results from the counts as a function of morphology either challenge the morphological classification system and/or support the idea that strong merging is present at low to moderate redshifts are no longer sustained.

3. In order to fit the counts to the very faint levels by means of dwarfs, it is necessary to consider that the star formation history in dwarfs is more complex than was previously thought. Models in which the star formation takes place in single, very short episodes predict large amounts of red remnants not seen in the HDF deep counts. By allowing the star formation to take place over longer periods, the number of remnants is severely reduced, and the color distribution of very faint galaxies in the HDF can be nicely fitted. Then, a kind of self-regulation mechanism capable of keeping the metal content in dwarfs low while the star formation lasts for longer periods has to exist. Observational evidence for the complexity of the evolutionary path followed by dwarfs has been found (e.g., Smecker-Hane et al. 1996) in local dwarfs.

4. As a further test of the simple dwarf-rich models shown here, the isophotal size distribution of the HDF galaxies was compared with predictions from the models. It was found that model dwarf sizes are comparable to the observed sizes of the HDF galaxies, although to fit the distribution of the whole population it was necessary to include some disk size evolution for the spirals.

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