DYNAMICAL SYSTEM OF A MOSQUITO POPULATION WITH
DISTINCT BIRTH-DEATH RATES

Z.S. BOXONOV, U.A. ROZIKOV

Abstract. We study the discrete-time dynamical systems of a model of wild mosquito population with distinct birth (denoted by \( \beta \)) and death (denoted by \( \mu \)) rates. The case \( \mu = \beta \) was considered in our previous work. In this paper we prove that for \( \beta < \mu \) the mosquito population will die and for \( \beta > \mu \) the population will survive, namely, the number of the larvae goes to infinite and the number of adults has finite limit \( \frac{\alpha}{\mu} \), where \( \alpha > 0 \) is the maximum emergence rate.

1. Introduction

In [1], [2], [3], [4], [7] and [8] (see also references therein) several kind of mathematical models of mosquito population are studied. This paper is continuation of our paper [9], where following [5] it was considered a model of the mosquito population. In the model a wild mosquito population without the presence of sterile mosquitoes is considered. For the simplified stage-structured mosquito population, due to the fact that the first three stages in a mosquitoes life cycle are aquatic, it was grouped the three aquatic stages of mosquitoes into one class and divide the mosquito population into only two classes, one of which consists of the first three stages that is called the larvae, denoted by \( x \), and one of which consists of all adults, denoted by \( y \).

The birth rate is the oviposition rate of adults denoted by \( \beta(\cdot) \); let the rate of emergence from larvae to adults be a function of the larvae with the form of \( \alpha(1 - k(x)) \), where \( \alpha > 0 \) is the maximum emergence rate, \( 0 \leq k(x) \leq 1 \), with \( k(0) = 0, k'(x) > 0 \), and \( \lim_{x \to \infty} k(x) = 1 \), is the functional response due to the intraspecific competition [6]. Let the death rate of larvae be \( d_0 + d_1 x \), and the death rate of adults be constant \( \mu \). Then in the absence of sterile mosquitoes, and in case (as in [6]) \( k(x) = \frac{x}{1+x} \), \( \beta(\cdot) = \beta \) the interactive dynamics for the wild mosquitoes are governed by the following system:

\[
\begin{align*}
\frac{dx}{dt} &= \beta y - \frac{\alpha x}{1+x} - (d_0 + d_1 x) x, \\
\frac{dy}{dt} &= \frac{\alpha x}{1+x} - \mu y
\end{align*}
\]  

Denote

\[
r_0 = \frac{\alpha \beta}{(\alpha + d_0) \mu}.
\]  

Theorem 3.1 in [5] states that:

\[
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\]

\[
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\]
- if $r_0 \leq 1$ then the trivial equilibrium $(0; 0)$ of system (1.1) is a globally asymptotically stable, and there is no positive equilibrium.
- if $r_0 > 1$ then the trivial equilibrium $(0; 0)$ is unstable, and there exists a unique positive equilibrium $(x^{(0)}, y_0)$ with

\[
x^{(0)} = \frac{\sqrt{(d_0 + d_1)^2 - 4d_1(\alpha + d_0)(1 - r_0)} - d_0 - d_1}{2d_1}, \quad y_0 = \frac{\alpha x^{(0)}}{\mu(1 + x^{(0)})},
\]

which is a globally asymptotically stable.

Define the operator $W : \mathbb{R}^2 \to \mathbb{R}^2$ by

\[
\begin{align*}
x' &= \beta y - \frac{\alpha x}{1+x} - (d_0 + d_1)x + x, \\
y' &= \frac{\alpha x}{1+x} - \mu y + y
\end{align*}
\] (1.3)

where $\alpha > 0, \beta > 0, \mu > 0, d_0 \geq 0, d_1 \geq 0$.

In this paper we study the discrete time dynamical systems generated by (1.3). In [9] for the evolution operator (1.3) the following results are obtained:
- all fixed points of the evolution operator are found. Depending on the parameters the operator may have unique, two and infinitely many fixed points;
- under some conditions on parameters type of each fixed point is determined and the limit points of the dynamical system are given.
- for the case $\beta = \mu, d_0 = d_1 = 0$ of parameters the full analysis of corresponding dynamical system is given.

In this paper we consider the operator $W$ (defined by (1.3)) for the case $\beta \neq \mu, d_0 = d_1 = 0$ and our aim is to study trajectories of any initial point from the invariant (under $W$) set $\mathbb{R}_+^2$. Thus the case when $d_0 = d_1 = 0$ is not satisfied remains open.

2. Dynamics for $\beta \neq \mu, d_0 = d_1 = 0$.

We assume

\[
\beta \neq \mu, \quad d_0 = d_1 = 0
\] (2.1)

then (1.3) has the following form

\[
W_0 : \begin{cases} x' &= \beta y - \frac{\alpha x}{1+x} + x, \\
y' &= \frac{\alpha x}{1+x} - \mu y + y
\end{cases}
\] (2.2)

To interpret values of $x$ and $y$ as probabilities we assume $x \geq 0$ and $y \geq 0$. To define a dynamical system we need that $W_0$ maps $\mathbb{R}_+^2$ to itself. It is easy to see that if

\[
0 < \alpha \leq 1, \quad \beta > 0, \quad 0 < \mu \leq 1
\] (2.3)

then operator (1.3) maps $\mathbb{R}_+^2$ to itself.

A point $z \in \mathbb{R}_+^2$ is called a fixed point of $W_0$ if $W_0(z) = z$.

For fixed point of $W_0$ the following lemma holds.

**Proposition 1.** (2.2) has unique fixed point $z = (x, y) = (0, 0)$. 

Proof. We need to solve
\[
\begin{aligned}
x &= \beta y - \frac{\alpha x}{1+x} + x, \\
y &= \frac{\alpha x}{1+x} - \mu y + y
\end{aligned}
\] (2.4)
It is easy to see that \(x = 0, y = 0\).

Now we shall examine the type of the fixed point.

Definition 1. (see [4]) A fixed point \(s\) of the operator \(W_0\) is called hyperbolic if its Jacobian \(J\) at \(s\) has no eigenvalues on the unit circle.

Definition 2. (see [4]) A hyperbolic fixed point \(s\) is called:
1) attracting if all the eigenvalues of the Jacobi matrix \(J(s)\) are less than 1 in absolute value;
2) repelling if all the eigenvalues of the Jacobi matrix \(J(s)\) are greater than 1 in absolute value;
3) a saddle otherwise.

To find the type of a fixed point of the operator (2.2) we write the Jacobi matrix:
\[
J(z) = JW_0 = \begin{pmatrix} 1 - \frac{\alpha \beta}{\alpha} & \beta \\ \alpha & 1 - \mu \end{pmatrix}
\]
The eigenvalues of the Jacobi matrix are
\[
\lambda_{1,2} = \frac{1}{2} \left( 2 - \alpha - \mu \pm \sqrt{(\alpha - \mu)^2 + 4\alpha \beta} \right).
\]
For type of \(z = (x, y) = (0, 0)\) the following lemma holds.

Proposition 2. The type of the fixed point \(z\) for (2.2) are as follows:
i) if \(\beta < \mu\) then \(x^*\) is attracting;
ii) if \(\beta > \mu\) then \(x^*\) is saddle;

Proof. We need to solve \(|\lambda_{1,2}| = \left| \frac{1}{2} \left( 2 - \alpha - \mu \pm \sqrt{(\alpha - \mu)^2 + 4\alpha \beta} \right) \right| < 1\). It is easy to see that \(\beta < \mu\).

The following theorem describes the trajectory of any point \(z^{(0)} = (x^{(0)}, y^{(0)})\) in \(\mathbb{R}^2_+\).

Theorem 1. For the operator \(W\) given by (1.3), under condition (2.1) the following holds
\[
\lim_{n \to \infty} W^n(z^{(0)}) = \begin{cases} (0,0), & \text{if } \beta < \mu, \\ (+\infty, \frac{\alpha}{\mu}), & \text{if } \beta > \mu \end{cases}
\]
where \(W^n\) is \(n\)-th iteration of \(W\).

Proof. 1) Let \(\beta < \mu\). Then there exists \(k > 1\) such that \(\beta \cdot k = \mu\). Denote \(c^{(n)} = x^{(n)} + y^{(n)}\) and \(c_0^{(n)} = k \cdot x^{(n)} + y^{(n)}\), where \(x^{(n)}, y^{(n)}\) defined by the following
\[
x^{(n)} = \beta y^{(n-1)} - \frac{\alpha x^{(n-1)}}{1 + x^{(n-1)}} + x^{(n-1)}, \quad y^{(n)} = \frac{\alpha x^{(n-1)}}{1 + x^{(n-1)}} - \mu y^{(n-1)} + y^{(n-1)}.
\] (2.5)
Both sequences \( \{c^{(n)}\} \) and \( \{c_0^{(n)}\} \) are monotone and bounded, i.e.,
\[
0 \leq \ldots \leq c^{(n)} \leq c^{(n-1)} \leq \ldots \leq c^{(0)}, \quad 0 \leq \ldots \leq c_0^{(n)} \leq c_0^{(n-1)} \leq \ldots \leq c_0^{(0)}.
\]
Thus \( \{c^{(n)}\} \) and \( \{c_0^{(n)}\} \) have limit points, denote the limits by \( c^* \), \( c_0^* \) respectively. Consequently, the following sequences have limits
\[
x^* = \lim_{n \to \infty} x^{(n)} = \frac{1}{1-k} \lim_{n \to \infty} (c^{(n)} - c_0^{(n)}) = \frac{1}{1-k} (c^* - c_0^*), \quad y^* = \lim_{n \to \infty} y^{(n)} = c^* - x^*.
\]
and by (2.5) we have
\[
x^* = \beta y^* - \frac{\alpha x^*}{1+x^*} + x^*, \quad y^* = \frac{\alpha x^*}{1+x^*} - \mu y^* + y^*,
\]
i.e., \( x^* = 0, y^* = 0 \).

2) Let \( \beta > \mu \). The proof is based on the following three lemmas.

**Lemma 1.** The sequence \( y^{(n)} \) is bounded.

**Proof.** Note that on the set \( \mathbb{R}_+ \) the function \( y(x) = \frac{x}{1+x} \) is increasing and bounded by 1. Therefore
\[
y^{(n)} = \frac{\alpha x^{(n-1)}}{1+x^{(n-1)}} + (1-\mu)y^{(n-1)} \leq \alpha + (1-\mu)y^{(n-2)} \leq \alpha + (1-\mu)(\alpha + (1-\mu)y^{(n-3)}) \leq \ldots \leq \alpha + \alpha(1-\mu) + \alpha(1-\mu)^2 + \ldots + \alpha(1-\mu)^{n-1}
\]
\[
+ (1-\mu)^n y^{(0)} = \frac{\alpha}{\mu} + (1-\mu)^n (y^{(0)} - \frac{\alpha}{\mu}).
\]
Thus if the initial point \( y^{(0)} > \frac{\alpha}{\mu} \) then for any \( m \in \mathbb{N} \) we have \( 0 \leq y^{(m)} \leq y^0 \), if \( y^{(0)} < \frac{\alpha}{\mu} \) then for any \( m \in \mathbb{N} \) we have \( 0 \leq y^{(m)} \leq \frac{\alpha}{\mu} \). \( \square \)

**Lemma 2.** For sequences \( x^{(n)} \) and \( y^{(n)} \) the following holds:

1) For any \( n \in \mathbb{N} \) and \( \beta > \mu \) the inequalities \( x^{(n)} > x^{(n+1)} \) and \( y^{(n)} > y^{(n+1)} \) cannot be satisfied at the same time.

2) If \( x^{(m-1)} < x^{(m)}, \ y^{(m-1)} < y^{(m)} \) for some \( m \in \mathbb{N} \) then \( x^{(m)} < x^{(m+1)}, \ y^{(m)} < y^{(m+1)} \).

3) If \( x^{(m-1)} > x^{(m)}, \ y^{(m-1)} < y^{(m)} \) for any \( m \in \mathbb{N} \) then for \( \beta > \mu \) the inequalities \( x^{(m)} > x^{(m+1)}, \ y^{(m)} < y^{(m+1)} \) cannot be satisfied at the same time.

4) If \( x^{(m-1)} < x^{(m)}, \ y^{(m-1)} > y^{(m)} \) for any \( m \in \mathbb{N} \) then for \( \beta > \mu \) the inequalities \( x^{(m)} < x^{(m+1)}, \ y^{(m)} > y^{(m+1)} \) cannot be satisfied at the same time.

5) For any \( m \in \mathbb{N} \) the inequalities \( x^{(m-1)} < x^{(m)} , x^{(m)} > x^{(m+1)} , y^{(m-1)} > y^{(m)} , y^{(m)} < y^{(m+1)} \) cannot be satisfied at the same time.

**Proof.** Adding \( x^{(n)} \) and \( y^{(n)} \) we get
\[
x^{(n)} + y^{(n)} = (\beta - \mu)y^{(n-1)} + x^{(n-1)} + y^{(n-1)} \quad (2.6)
\]
1) From (2.6) by \( \beta > \mu \) we get \( (x^{(n)} - x^{(n-1)}) + (y^{(n)} - y^{(n-1)}) > 0 \). Consequently, \( x^{(n)} > x^{(n+1)} \) and \( y^{(n)} > y^{(n+1)} \) cannot be satisfied at the same time.
2) By \(x^{(m)} - x^{(m-1)} > 0\) we have
\[
x^{(m)}\left(1 - \frac{\alpha}{1 + x^{(m)}}\right) - x^{(m-1)}\left(1 - \frac{\alpha}{1 + x^{(m-1)}}\right) > 0
\]
and
\[
\frac{x^{(m)}}{1 + x^{(m)}} > \frac{x^{(m-1)}}{1 + x^{(m-1)}}.
\]
Then
\[
x^{(m+1)} - x^{(m)} = \beta(y^{(m)} - y^{(m-1)}) + x^{(m)}\left(1 - \frac{\alpha}{1 + x^{(m)}}\right) - x^{(m-1)}\left(1 - \frac{\alpha}{1 + x^{(m-1)}}\right) > 0,
\]
\[
y^{(m+1)} - y^{(m)} = \alpha\left(\frac{x^{(m)}}{1 + x^{(m)}} - \frac{x^{(m-1)}}{1 + x^{(m-1)}}\right) + (1 - \mu)(y^{(m)} - y^{(m-1)}) > 0.
\]

3) Assume in case when \(x^{(m)} > x^{(m-1)}\), \(y^{(m)} < y^{(m-1)}\) hold for any \(m \in \mathbb{N}\) then for \(\beta > \mu\) the inequalities \(x^{(m)} > x^{(m+1)}, y^{(m)} < y^{(m+1)}\) are satisfied at the same time. Then since \(x^{(n)}\) is decreasing and bounded; \(y^{(n)}\) is increasing and bounded (see Lemma 1) there exist their limits \(x^*, y^*\) respectively. By (2.5) we obtain
\[
\begin{align*}
\beta y^* &= \frac{\alpha x^*}{1 + x^*} \\
\alpha x^* &= \mu y^*
\end{align*}
\]
i.e. \(x^* = 0, y^* = 0\). This contradiction shows that if for any \(m \in \mathbb{N}\) one has \(x^{(m-1)} > x^{(m)}, y^{(m-1)} < y^{(m)}\) then for \(\beta > \mu\) the inequalities \(x^{(m)} > x^{(m+1)}, y^{(m)} < y^{(m+1)}\) can not be satisfied at the same time (Example See Fig 2).

4) Assume if for any \(m \in \mathbb{N}\) the inequalities \(x^{(m-1)} < x^{(m)}, y^{(m-1)} > y^{(m)}\) hold then for \(\beta > \mu\) the inequalities \(x^{(m)} < x^{(m+1)}, y^{(m)} > y^{(m+1)}\) are satisfied at the same time, i.e. \(x^{(m)}\) is increasing and \(y^{(m)}\) is decreasing. Let
\[
(x^{(m+1)} - x^{(m)}) = \Delta^{(m)}, \quad (y^{(m+1)} - y^{(m)}) = \delta^{(m)},
\]

\[
(x^{(m+1)} - x^{(m)}) + (y^{(m+1)} - y^{(m)}) = \Delta^{(m)} - \delta^{(m)} < (\beta - \mu)y^{(0)}.
\]

Since \(\{\Delta^{(m)}\}\) is decreasing, \(\{\delta^{(m)}\}\) is increasing and \(\Delta^{(m)} - \delta^{(m)} > 0\) for \(\beta > \mu\) we conclude that the sequence \(\{\Delta^{(m)} - \delta^{(m)}\}\) is decreasing and bounded from below. Thus \(\Delta^{(m)} - \delta^{(m)}\) has a limit and since \(y^{(m)}\) has limit we conclude that \(x^{(m)}\) has a finite limit. By (2.5) we have
\[
\lim_{m \to \infty} x^{(m)} = 0, \quad \lim_{m \to \infty} y^{(m)} = 0.
\]

But this is a contradiction to \(\lim_{m \to \infty} x^{(m)} \neq 0\). This completes proof of part 4 (Example See Fig 3).

5) Assume for any \(m \in \mathbb{N}\) one has \(x^{(m-1)} < x^{(m)}, x^{(m)} > x^{(m+1)}, y^{(m-1)} > y^{(m)}, y^{(m)} < y^{(m+1)}\). Then \(x^{(m-1)} < x^{(m+1)}, y^{(m-1)} > y^{(m+1)}\). Moreover, for each \(k \in \mathbb{N}\) for \(m = 2k\) we have \(x^{(2k-1)} < x^{(2k+1)}, y^{(2k-1)} > y^{(2k+1)}\), and for \(m = 2k + 1\) we have \(x^{(2k)} < x^{(2k+2)}, y^{(2k)} > y^{(2k+2)}\). But by Lemma 2 parts 3 and 4) these inequalities do not hold for any \(k \in \mathbb{N}\) (Example See Fig 5).
Lemma 3. There exists $n_0$ such that the sequences $x^{(n)}$ and $y^{(n)}$ are increasing for $n \geq n_0$ and $x^{(n)}$ is unbounded from above.

Proof. Monotonicity of $x^{(n)}$ and $y^{(n)}$ follow from Lemma 2. Consider

$$
\begin{align*}
(x^{(n_0+1)} - x^{(n_0)}) + (y^{(n_0+1)} - y^{(n_0)}) &= (\beta - \mu) y^{(n_0)} \\
(x^{(n_0+2)} - x^{(n_0+1)}) + (y^{(n_0+2)} - y^{(n_0+1)}) &= (\beta - \mu) y^{(n_0+1)} \\
&\vdots \\
(x^{(n-1)} - x^{(n-2)}) + (y^{(n-1)} - y^{(n-2)}) &= (\beta - \mu) y^{(n-2)} \\
(x^{(n)} - x^{(n-1)}) + (y^{(n)} - y^{(n-1)}) &= (\beta - \mu) y^{(n-1)}
\end{align*}
$$

Adding equations of (2.7) we get

$$
(x^{(n)} - x^{(n_0)}) + (y^{(n)} - y^{(n_0)}) = (\beta - \mu)(y^{(n_0)} + y^{(n_0+1)} + \ldots + y^{(n-2)} + y^{(n-1)})
$$

Figure 1. $\alpha = 0.6, \beta = 0.5, \mu = 0.48, x^{(0)} = 2, y^{(0)} = 0.1$

Figure 2. $\alpha = 0.4, \beta = 0.35, \mu = 0.3, x^{(0)} = 0.5, y^{(0)} = 2$
Let $y^{(n)}$ (see Lemma 1) is bounded by $\theta$. By (2.8) we have
\[
x^{(n)} = x^{(n_0)} + y^{(n_0)} - \theta + (\beta - \mu)(n - n_0)y^{(n_0)}.
\]
For $\beta > \mu$ from
\[
\lim_{n \to \infty} (x^{(n_0)} + y^{(n_0)} - \theta + (\beta - \mu)(n - n_0)y^{(n_0)}) = +\infty
\]
it follows that $x^{(n)}$ is not bounded from above. \hfill $\square$

Thus for $\beta > \mu$ the sequence $y^{(n)}$ has limit $y^*$ (see Lemma 1). Consequently, by (2.5) and $\lim_{n \to \infty} x^{(n)} = +\infty$ we get $y^* = \frac{\alpha}{\mu}$. Theorem is proved. \hfill $\square$

**Biological interpretation** of our result is clear: for $\beta < \mu$ the mosquito population will die and for $\beta > \mu$ the population will survive.

A point $z$ in $W_0$ is called periodic point of $W_0$ if there exists $p$ so that $W_0^p(z) = z$. The smallest positive integer $p$ satisfy $W_0^p(z) = z$ is called the prime period or least period of the point $z$.

**Theorem 2.** For $p \geq 2$ the operator (2.2) does not have any $p$-periodic point in the set $\mathbb{R}^2_+$.  

**Proof.** This is a corollary of Theorem 1. Here we give an alternative proof. Let us first describe periodic points with $p = 2$ on $\mathbb{R}^2_+$, in this case the equation $W_0(W_0(z)) = z$. That is
\[
\begin{aligned}
x &= \beta\left(\frac{\alpha x}{1 + x} - \mu y + y\right) + (\beta y - \frac{\alpha x}{1 + x} + x)(1 - \frac{\alpha y - \alpha x}{1 + \beta y - \frac{\alpha x}{1 + x}}), \\
y &= \alpha(\beta y - \frac{\alpha x}{1 + x}) + (1 - \mu)(\frac{\alpha x}{1 + x} - \mu y + y).
\end{aligned}
\]
Simple calculations show that the last equation is equivalent to the following
\[
\frac{\alpha x}{1 + x} = (\mu - 2)y, \quad \frac{\alpha x}{1 + x} \geq 0, \quad (\mu - 2)y \leq 0.
\]
Solutions to (2.10) are $x = 0, y = 0$. Thus the operator (2.2) does not have any two periodic point in the set $\mathbb{R}^2_+$. 

Now we show that $W_0$ does not have any periodic point (except fixed). Rewrite operator (2.2) in normalized form:

$$
U : \begin{cases}
    x' = \frac{(1+x)(x+\beta y) - ax}{(1+x)(x+(\beta - \mu + 1)y)}, \\
y' = \frac{ax + (1+x)(1-\mu)y}{(1+x)(x+(\beta - \mu + 1)y)}.
\end{cases}
$$

(2.11)

It is easy to see that the operator (2.11) satisfies the conditions (2.3), hence $U : S \rightarrow S$, where

$$S = \{(x, y) \in \mathbb{R}^2_+ : x + y = 1\}.$$ 

Using $x + y = 1$, from (2.11) we get

$$T : x' = \frac{(1-\beta)x^2 + (1-\alpha)x + \beta}{(\mu - \beta)x^2 + x + \beta - \mu + 1}.$$  

(2.12)

Note that (2.12) maps $S^* = [0, 1]$ to itself, i.e., $T : S^* \rightarrow S^*$.

Let us first describe periodic points of $U$ with $p = 2$ on $S$, in this case the equation $U(U(z)) = z$ can be reduced to description of 2-periodic points of the function $T$ defined in (2.12), i.e., to solution of the equation

$$T(T(x)) = x.$$  

(2.13)

Note that the fixed points of $T$ are solutions to (2.13), to find other solution we consider the equation

$$\frac{T(T(x)) - x}{T(x) - x} = 0,$$

simple calculations show that the last equation is equivalent to the following

$$Ax^2 + Bx + C = 0.$$  

(2.14)

where

$$A = (1-\beta)(\beta - 2) + (\beta - \mu + 1)(\beta - \mu),$$

$$B = (\beta - 2)(\beta - \mu - \alpha + 2) - \beta(\beta - \mu),$$

$$C = (\beta - \mu + 1)(\alpha + \mu - \beta - 2) + \beta(\beta - 1).$$

By (2.3) we have $A+B+C < 0, B < 0, C < 0$. Therefore since $x \geq 0$ the LHS of (2.14) is $< 0$. Consequently, the equation (2.14) does not have solution in $S^*$. Thus function (2.12) does not have any 2-periodic point in $S^*$. Since $T$ is continuous on $S^*$ by Sharkovskii’s theorem ([4], [10]) we have that $T^p(x) = x$ does not have solution (except fixed) for all $p \geq 2$. Hence it follows that the operator (2.2) has no periodic points (except fixed) in the set $\mathbb{R}^2_+$. \hfill $\square$

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Z. S. BOXONOV, INSTITUTE OF MATHEMATICS, 81, Mirzo Ulug’bek str., 100170, Tashkent, Uzbekistan.
E-mail address: z.b.x.k@mail.ru

U. A. ROZIKOV, INSTITUTE OF MATHEMATICS, 81, Mirzo Ulug’bek str., 100170, Tashkent, Uzbekistan.
E-mail address: rozikovu@yandex.ru