Higher Spin Resolution of a Toy Big Bang

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Abstract

Diffeomorphisms preserve spacetime singularities, whereas higher spin symmetries need not. Since three dimensional de Sitter space has quotients that have big-bang/big-crunch singularities and since dS$_3$-gravity can be written as an $SL(2, \mathbb{C})$ Chern-Simons theory, we investigate $SL(3, \mathbb{C})$ Chern-Simons theory as a higher-spin context in which these singularities might get resolved. As in the case of higher spin black holes in $AdS_3$, the solutions are invariantly characterized by their holonomies. We show that the dS$_3$ quotient singularity can be de-singularized by an $SL(3, \mathbb{C})$ gauge transformation that preserves the holonomy: this is a higher spin resolution the cosmological singularity. Our work deals exclusively with the bulk theory, and is independent of the subtleties involved in defining a CFT$_2$ dual to dS$_3$ in the sense of dS/CFT.
1 Introduction

Spacetime singularities are one of the primary indications that general relativity should be modified at short distances. String theory is an arena where this question can in principle be well-posed. Various examples of singularity resolution are known in string theory, eg. [1], [2]\(^1\). But it is fair to say that a systematic understanding of singularity resolution is still lacking.

Another question where the promise of string theory has not born substantial fruit is in the understanding of time-dependent backgrounds, aka cosmologies. (See [4, 5, 7, 8] for various attempts in this direction.) This is related to the fact that typically, when it is under analytic control, string theory is tied to supersymmetry. Unfortunately, supersymmetric backgrounds are necessarily time-independent and this frustrates most attempts to make progress on time-dependence in string theory.

Together, the above two challenges imply that singularities in cosmological spacetimes are one of the hardest things to make sense of in the context of string theory\(^2\).

On a different front, after the work of Vasiliev and others [13, 14, 15], a lot of recent attention has been directed towards an understanding of interacting higher spin theories. One motivation for this interest is the belief that the tensionless (\(\alpha' \to \infty\)) limit of string theory is a higher spin theory [17], and therefore higher spin theories might be a good starting point for a tractable understanding of (some) intrinsically stringy phenomena. Besides, we know that the dynamics of spin-1 fields is that of gauge vector fields living in a fixed spacetime background and that the dynamics of spin-2 fields gives rise to metric fluctuations and a dynamical spacetime. So perhaps it is not surprising that higher spin theories turn out to be relevant in our quest for a deeper understanding of the role of spacetime in string theory, ranging from background independence and singularity resolution to non-perturbative questions and the role of boundary conditions.

Vasiliev’s intercating higher spin theories take their full glory in four and higher dimensions, and the formalism is quite complicated. But it turns out that in three dimensions, one has a poor man’s version of higher spin theories which does not require us to work with the full 2+1 dimensional Vasiliev theory\(^3\). A simple way to motivate this is to note that 2+1 dimensional gravity can be re-expressed [15] as a Chern-Simons gauge theory with the gauge group \(SL(2,\mathbb{C})\)\(^4\). It turns out that increasing the gauge group rank from 2 to \(N\) and working

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\(^1\)See [3] for reviews.

\(^2\)But see [9, 10, 11] for some progress in understanding light-like singularities in pp-waves and plane wave backgrounds using M(atrix)-theory [12].

\(^3\)The latter in the AdS context is the so-called hs[\(\lambda\)] theory, and has been conjectured to be dual to minimal model CFTs in two dimensions [15, 16].

\(^4\)This specific choice of gauge group corresponds to a positive cosmological constant \(\Lambda > 0\), which is our
with an $SL(N, \mathbb{C})$ gauge theory corresponds to working with a spin-2 gravity theory in de Sitter space, coupled to spins ranging from $s = 3, \ldots, N$. We will only be concerned with the $SL(3, \mathbb{C})$ theory in this paper, but it is evident that generalizations of our statements exist for any $N \geq 3$.

It is known that 3-D gravity has no propagating degrees of freedom and therefore all its solutions must be locally (a patch of) 2+1 dimensional de Sitter. In particular, quotients of dS$_3$ are also solutions of the theory, analogous to the case of BTZ black holes in the AdS$_3$ case. The quotients of dS$_3$ theory that we will consider are cosmological solutions that contain big bang/big crunch singularities. Our goal in this paper will be to show that these cosmological singularities can be “resolved” by a choice of gauge in the Chern-Simons formalism. This is the de Sitter analogue of the observation that higher spin black holes in AdS$_3$ have horizons and singularities that are gauge-dependent \[19, 20, 21\]. The essential physics behind this is intuitively plausible: the gauge symmetry of spin-2 fields is diffeomorphism invariance (and the implicit freedom associated to local Lorentz rotations of the frames). Diffeomorphisms are a statement about the redundancies of the spacetime description, so one might expect that the introduction of higher spin gauge symmetries can bring forth even more dramatic redundencies in the spacetime picture. What we observe is to be understood as a manifestation of this fact: the existence of the cosmological singularity in the metric depends on the choice of the higher spin gauge. It is tempting to speculate that higher spin theories require a generalization of the manifold picture of spacetime \[22, 23\].

In the next section, we give a brief introduction to pure de Sitter gravity as an $SL(2, \mathbb{C})$ Chern-Simons theory, and also present $SL(3, \mathbb{C})$ gauge theory which is the (spin-3) higher spin theory that we will work with. The goal of this section is partly to fix our notation (see also the Appendices). Section 3 is devoted to the description of the quotient space, which has an interpretation as a cosmology. We describe this geometry first in a form that is analogous to that of the BTZ black hole in AdS$_3$, as well as in a Fefferman-Graham-like form that readily shows the cosmological nature of the spacetime. We also present the solution in the gauge field language and compute its holonomy. Section 4 presents the main point of the paper - we construct a class of higher spin gauge transformations that preserve the holonomy of the solution, but which can desingularize the geometry, and check that indeed the resultant solution has a smooth metric (and higher spin field) everywhere. The appendices contain some of the technical details.

There has been a lot of work recently on higher spin theories in AdS spaces and their minimal model duals \[15, 16, 19, 20, 24, 25, 26, 27, 28\]. Not much work has been done in the context of de Sitter, but see \[29\] for discussions on higher spin dS/CFT. Our work deals with main interest in the paper. For the case $\Lambda = 0$, the gauge groups is $ISO(2,1)$ and for $\Lambda < 0$ which is the case corresponding to AdS$_3$ gravity, its is $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$. 

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the bulk geometry exclusively, so the discussions and challenges in that paper regarding the CFT do not concern us. See also [30, 31, 32, 33].

2 (Higher Spin) Gravity in 2+1 Dimensional de Sitter Space

Gravity in three dimensions can be written as a Chern-Simons gauge theory. We will be interested in de Sitter gravity. Raising an lowering the local Lorentz indices using $\eta_{ab} = \text{diag}\{-1, +1, +1\}$ and using the $SL(2)$ generators $T_a$ that satisfy

$$[T_a, T_b] = \epsilon_{abc} T_c,$$  \hspace{1cm} (2.1)

the translation between the gauge field language and the gravity (i.e., vielbein and spin connection) language can be written as

$$A = \left(\omega^a_{\mu} + \frac{i}{l}\epsilon^a_{\mu}\right) T_a dx^\mu,$$  \hspace{1cm} (2.2)

$$\tilde{A} = \left(\omega^a_{\mu} - \frac{i}{l}\epsilon^a_{\mu}\right) T_a dx^\mu.$$  \hspace{1cm} (2.3)

Here $l$ is the de Sitter length scale and we have defined $\omega_a = \frac{1}{2}\epsilon_{abc}\omega^b_{\mu}dx^\mu$ in terms of the usual spin connection with two tangent space indices. This last construction works only in three dimensions, and this is the reason why the Chern-Simons formalism is natural in three dimensions. In terms of these gauge field variables, the Einstein-Hilbert action with cosmological constant takes the form

$$S = \frac{k}{4\pi y_R} \int \text{Tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) - \frac{k}{4\pi y_R} \int \text{Tr} \left( \tilde{A} \wedge d\tilde{A} + \frac{2}{3} \tilde{A} \wedge \tilde{A} \wedge \tilde{A} \right).$$  \hspace{1cm} (2.4)

where $y_R$ is defined via $\text{Tr}(T_a T_b) = \frac{y_R}{2} \eta_{ab}$. We will work with $y_R = 4$ in this paper. To make the action real, we need to choose $\tilde{A}_a T^a = A^*_a T^a$, where it is important that the $T^a$’s don’t get conjugated. Then, the flatness conditions $F = 0 = \tilde{F}$ turns into the Einstein equations when we choose the Chern-Simons level to be

$$k = \frac{il}{4G}.$$  \hspace{1cm} (2.5)

We will set $8G = 1$ in what follows. This is the connection between 3D de Sitter gravity and Chern-Simons gauge theory.

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5Note that the basis of $SL(2)$ generators and their algebra are abstract objects. But the nature of the resulting theory would depend on the field in which the coefficients are taken. We will be interested in $SL(2, \mathbb{C})$ in this paper.

6We use $\epsilon^{012} = 1$.

7This identification of the two gauge fields is one of the technical reasons why the $dS_3$ results are not merely a trivial “$l$ replaced with $il$” version of the $AdS_3$ results.
As mentioned in the introduction, working with higher spins is a complicated business in dimensions higher than 3. But in three dimensions, the Chern-Simons language allows a simple way to write down interacting higher spin theories, by increasing the rank of the Chern-Simons gauge group. We will be dealing with positive cosmological constant in this paper, and for simplicity we will restrict ourselves to $SL(3, \mathbb{C})$ Chern-Simons gauge theory. This corresponds to a spin-3 field coupled to gravity in de Sitter space. Explicitly, we introduce the extra generators $T_{ab}$ to the $T_a$ of $SL(2)$ and the full $SL(3)$ algebra takes the form

\[
[T_a, T_b] = \epsilon_{abc} T^c, \quad (2.6)
\]

\[
[T_a, T_{bc}] = \epsilon^d \eta_{a(c} T_{b)d}, \quad (2.7)
\]

\[
[T_{ab}, T_{cd}] = \sigma \left( \eta_a(c \epsilon_d) b e + \eta_b(c \epsilon_d) a e \right) T^e. \quad (2.8)
\]

The $T_{ab}$ are symmetric and traceless and therefore are five in number, adding up to a total of eight generators for $SL(3)$, as expected. Its clear from the algebra that the constant $\sigma$ can be gotten rid of by absorbing it into the $T_{ab}$ generators - it will not affect the content of our discussion, so we will choose it to be -1, in parallel with the AdS$_3$ case discussed in the literature \[21, 19\].

The above embedding of the $SL(2)$ algebra generators in $SL(3)$ is called principal embedding, and this is what we will be using in this paper.

With this enlarged gauge group, one now considers a Chern-Simons theory with the gauge field defined by

\[
A = \left( \omega^a_\mu + \frac{i}{\ell} e^a_\mu \right) T_a dx^\mu + \left( \omega^{ab}_\mu + \frac{i}{\ell} e^{ab}_\mu \right) T_{ab} dx^\mu, \quad (2.9)
\]

and its complex conjugate $\tilde{A}$, and then looks at the same action (2.4) as before. This theory is a theory of gravity coupled to a spin-3 field. By simple index counting, the obvious candidates for the metric and the spin-3 field (appropriately normalized) are \[24\]

\[
g_{\mu\nu} = \frac{1}{2} \text{Tr}(e_{(\mu} e_{\nu)}), \quad \psi_{\mu\nu} = \frac{1}{9} \text{Tr}(e_{(\mu} e_\nu e_{\alpha)}), \quad \text{with} \quad e_\mu \equiv e^a_\mu T_a + e^{ab}_\mu T_{ab}. \quad (2.10)
\]

When perturbed around the dS$_3$ background, the $e^{ab}_\mu$ satisfy the Fronsdal equations of motion \[34\], and therefore justify their identification as a spin-3 field. Just as diffeomorphisms and local Lorentz invariance of the spin-2 theory are identified with the $SL(2)$ part of the algebra, the generators $T_{ab}$ correspond to the “higher spin gauge symmetry”.

We find it is convenient to relate the above form of the algebra to the $L_m, W_n$ generators
defined by \[19, 21\] so that we can adapt their results and notations. In particular, we choose
\[
T_0 = -i L_0, \quad T_1 = i \left( \frac{2L_1 + L_{-1}}{2\sqrt{2}} \right), \quad T_2 = \frac{2L_1 - L_{-1}}{2\sqrt{2}}.
\] (2.11)
From the definition of the \(L_m\)'s in the Appendix, it is straightforward to check that the resulting \(T_a\) satisfy the \(SL(2)\) algebra\[9\].

The fact that (higher spin) de Sitter gravity has a Chern-Simons formulation means that the solutions are invariantly characterized by the holonomies of the gauge field. This is a fact that we will use in the later sections.

3 The Quotient Cosmology and its Holonomy

Solutions of 3-dimensional de Sitter gravity are locally \(dS_3\). So analogous to the BTZ black hole which can be thought of as a quotient of \(AdS_3\), the solutions of \(dS_3\) gravity can be thought of as quotients of \(dS_3\). In this section, we will discuss such a quotient \[35, 36\] which is a time-dependent background with cosmological singularities. We will compute its holonomy. In the next section, we will show that (higher spin) gauge transformations that preserve the holonomy can change the metric so drastically that the singularity is gone in the final metric.

The de Sitter quotient metric that we are interested in is written down in \[36\]. There, they call it the Kerr-d\(S_3\) geometry and write it in a form closely parallel to the BTZ black hole. But the region that contains the asymptotic region in this geometry is more appropriately thought of as a cosmology, so we will call it a quotient cosmology. The metric in BTZ like form takes the form:
\[
ds^2 = -N^2(r)dt^2 + N^{-2}(r)dr^2 + r^2 (N_\phi dt + d\phi)^2
\]
with
\[
N^2(r) = M - \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \quad N_\phi = -\frac{J}{2r^2}, \quad (3.1)
\]
One can check that the following \(SL(2, \mathbb{C})\) Chern-Simons connection can reproduce this metric:
\[
A^0 = N(r) \left( d\phi + i \frac{dt}{l} \right), \quad A^1 = \frac{i N_\phi - i dr}{N(r) \frac{l}{l}}, \quad A^2 = \left( r N_\phi + i \frac{r}{l} \right) \left( d\phi + i \frac{dt}{l} \right). \quad (3.2)
\]
\[8\]An explicit form of these \(L_m, W_n\) generators as well as the algebra they satisfy is given in an Appendix.
\[9\]The choice of \(T_a\) in terms of \(L_m\) considered in \[29\] does not reproduce the \(SL(2)\) algebra: we believe this is a typo, because the rest of the claims about the bulk theory there seem reasonable and correct.
In the context of higher spin SL(3) gravity, we use the set of SL(3) generators provided in \cite{[19]} to embed the SL(2), the specific choice we make is given in (2.11). The gauge field will then be given by $A = A^a T_a$.

Our first goal is to compute the holonomy of this connection. As done in \cite{[37]} we need to solve for $U \in \text{SL}(2, \mathbb{C})$ such that

$$A = U^{-1} dU.$$ 

These are the solutions to the flatness condition $F = 0$. Of course, the $U$’s that we get are not necessarily single valued, and that’s where the holonomy information is captured. As is worked out in the Appendix, for the quotient cosmology gauge field above, the solution is,

$$U = e^{\theta_0 T_0} e^{\theta_1 T_1},$$

for,

$$\theta_0 = \sqrt{M + i J/l}(\phi + it/l),$$

$$\cosh \theta_1 = \frac{N(r)}{\sqrt{M + i J/l}}, \quad \sinh \theta_1 = -\frac{r N_\phi(r) + ir/L}{\sqrt{M + i J/l}}.$$ (3.3) (3.4)

From $U$, one can extract the Wilson loops for loops enclosing $r = 0$ (at constant time in the $\phi$-direction),

$$W(A) = \exp(\oint A) = U^{-1}(t, r, \phi = 0)U(t, r, \phi = 2\pi)$$ (3.5)

and further, the eigenvalues of the holonomy matrix, $w$ defined by $W = \exp(w)$, by exponentiating the eigenvalues of the Wilson loop $W$.

The Wilson loop is straightforward to compute, and its eigenvalues are given by $e^\lambda$ with

$$\lambda = 0, \pm \left(2\pi i \sqrt{M + i J/l}\right).$$ (3.6)

The holonomy matrix turns out to be,

$$w = \begin{pmatrix} 0 & -2\pi i \sqrt{M + i J/l} \\ 2\pi i \sqrt{M + i J/l} & 2\pi i \sqrt{M + i J/l} \end{pmatrix}.$$ (3.7)

### 3.1 Fefferman-Graham in de Sitter

The metric in the form (3.1) is best suited for $r < r_+$, where

$$r_+^2 = l^2 \left(\sqrt{M^2 + (J/l)^2} \pm M\right)/2.$$ 

In order to conduct an asymptotic symmetry analysis one needs continue the metric beyond the horizon. Now $r$ becomes a time coordinate. To facilitate comparison with the prevailing
higher spin literature \cite{19, 23} lets switch to new coordinates: $w = \phi + it/l, \quad \bar{w} = \phi - it/l$. We will also introduce a new time coordinate, $\tau$, to replace the (now time-like) $r$ coordinate:

$$
\tau = \ln \left( \frac{\sqrt{r^2 - r_+^2} + \sqrt{r^2 + r_+^2}}{2l} \right)
$$

This new coordinate system expresses the quotient cosmology in a de Sitter version of the Fefferman-Graham coordinates. From now on, since the $t$ coordinate is no longer a time coordinate, we will rename it as $z$. Therefore, really, $w$ and $\bar{w}$ are defined as

$$
w = \phi + iz/l, \quad \bar{w} = \phi - iz/l
$$

We can generalize this more. In a Fefferman-Graham gauge, the most general asymptotically dS$_3$ solution to pure gravity with a positive cosmological constant in $2 + 1$ D can be written down. This is the dS$_3$ analog of the result by Banados \cite{38} for AdS$_3$. Such a general asymptotically dS$_3$ metric can be written in terms of one complex function $L(w)$ and its complex conjugate $\bar{L}(\bar{w})$:

$$
ds^2 = \frac{l}{2} \left( L(w)dw^2 + \bar{L}(\bar{w})d\bar{w}^2 \right) + \left( \ell^2 e^{2\tau} + \frac{L(w)\bar{L}(\bar{w})}{4} e^{-2\tau} \right) dwd\bar{w} - \ell^2 d\tau^2.
$$

(3.8)

The quotient cosmology of the previous subsection corresponds to the case $L, \bar{L}$ constant.

The asymptotic symmetry analysis of \cite{36} identifies

$$
L + \bar{L} = Ml, \quad L - \bar{L} = iJ.
$$

(3.9)

The comparison of the asymptotic analysis between AdS$_3$ and dS$_3$ is straightforward \cite{29}. One can go back and forth between AdS$_3$ and dS$_3$ by using the following short cut identifications/replacements,

$$
l_{AdS} \rightarrow il_{dS}
$$

$$
L_{AdS}, \bar{L}_{AdS} \rightarrow -iL_{dS}, -i\bar{L}_{dS}
$$

$$
M_{AdS} \rightarrow -M_{dS}
$$

$$
J_{AdS} \rightarrow J_{dS}
$$

\footnote{We thank Avinash Raju for checking that this metric satisfies Einstein equation with a positive cosmological constant.}

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For example the usual asymptotic relations in AdS$_3$ are:

\[ L + \bar{L} = Ml, \quad L - \bar{L} = J \]

can be used to arrive at the dS$_3$ asymptotic relations of [36].

\[ L + \bar{L} = Ml, \quad L - \bar{L} = iJ. \]

Note however that this quick-fix match between AdS$_3$ and dS$_3$ is of limited use and does not exist in many other contexts. For example, the gauge transformations and the final form of the metric that we will discuss are entirely different from their AdS$_3$ analogues.

A convenient choice of frame fields for this general FG metric is\footnote{We are suppressing the $w/\bar{w}$ dependence of $L/\bar{L}$}

\[
\begin{align*}
e^0 &= l d\tau \\
e^1 &= -\frac{i}{2} l \left( e^\tau - \frac{L}{2l} e^{-\tau} \right) dw + c.c. \\
e^2 &= \frac{l}{2} \left( e^\tau + \frac{L}{2l} e^{-\tau} \right) dw + c.c.
\end{align*}
\]

The corresponding $SL(2,\mathbb{C})$ connection is,

\[ A = i T_0 d\tau + \left[ \left( e^\tau - \frac{L}{2l} e^{-\tau} \right) T_1 + i \left( e^\tau + \frac{L}{2l} e^{-\tau} \right) T_2 \right] dw \quad (3.10)\]

and its conjugate.

We also need the $SL(2,\mathbb{C})$ group element generating this. To this end we first rewrite the connection in the form,

\[ A^0 = d\psi_0, \quad A^1 = -\sin \psi_0 d\psi_2, \quad A^2 = \cos \psi_0 d\psi_2 \]

with,

\[ \psi_0 = i \left( \tau - \frac{1}{2} \ln \frac{L}{2l} \right), \quad \psi_2 = i \sqrt{\frac{2L}{l}} \omega. \]

Now it can be shown that one can write (see Appendix), $A = V^{-1} dV$ for

\[ V = e^{\psi_2 T_2} e^{\psi_0 T_0}. \]

which in turn gives the holonomy matrix, $w$,

\[
\begin{pmatrix}
0 \\
-2\pi i \sqrt{\frac{2L}{l}} \\
2\pi i \sqrt{\frac{2L}{l}}
\end{pmatrix}
\]
which is identical to (3.7) once we substitute, \( L = (Ml + iJ)/2 \). (Holonomies are diffeomorphism invariant).

For future reference, we define Fefferman-Graham gauge to be

\[ g_{\tau\tau} = -l^2, \quad g_{\tau w} = g_{\tau \bar{w}} = 0. \]

and vanishing spin-3 field. In terms of \( SL(3) \) gauge theory, one way we can realize this gauge is by turning on just the principally embedded \( SL(2) \) sector in the particular form,

\[ A_{\tau} = \pm i l T_0, \quad A_w = A_w(\tau, w) T_a, \quad A_{\bar{w}} = 0. \]

The connection (3.10) corresponding to the general Banados type solution satisfies this condition.

4 Resolution of the Cosmological Singularity

The quotient cosmology metric, where we set \( L \) and \( \bar{L} \) to constant, takes the following explicit form in Fefferman-Graham gauge (note: we have renamed \( t \) to \( z \)):

\[ ds^2 = -l^2 d\tau^2 + \left| e^{\tau} - \frac{L}{2l} e^{-\tau} \right|^2 dz^2 + \left| le^{\tau} + \frac{L}{2} e^{-\tau} \right|^2 d\phi^2 + i \frac{L - \bar{L}}{2} dz d\phi \]

It has vanishing \( g_{zz} \) at \( \tau = \frac{1}{2} \ln \frac{l}{2l} \). Equivalently, it was pointed out in [36] the metric in \( t, r, \phi \) coordinates has big bang/big crunch singularities at \( r = \pm r_+ \) with periodic spacelike \( t, \phi \). Note that this is a causal structure singularity and not a curvature singularity: the curvature is finite and constant everywhere else as it should be for a constant curvature space.

Before proceeding further, we make one observation. Consider the connection,

\[ a_0 = \left[ \left( 1 - \frac{L}{2l} \right) T_1 + i \left( 1 + \frac{L}{2l} \right) T_2 \right] dw. \] (4.1)

The quotient cosmology connection of Eq. (3.10) can be obtained by performing a single valued gauge transformation on this primitive connection \( a_0 \):

\[ A = b^{-1} a_0 b + b^{-1} db, \] (4.2)

for

\[ b(\tau) = \exp(i \tau T_0) = \exp(\tau L_0) \]

This is because \( b \) is a sole function of \( \tau \) and is therefore single valued in the \( \phi \) direction.
4.1 The class of singularity resolving spin 3 gauge transformations

The aim of this subsection is to obtain a fairly general set of holonomy preserving gauge transformations which take our quotient cosmology metric with singularities to a regular $SL(3, \mathbb{C})$ metric. The understanding is that making metric components non-vanishing eliminates all metric singularities which may or may not be true singularities.

The algorithm is as follows:

1. We first define a connection $a \in SL(3)$ which is a generalization of the $a_0$ defined in the previous subsection:

   \[ a = a_0 + Y \, dw = \left[ \left( 1 - \frac{L}{2!} \right) T_1 + i \left( 1 + \frac{L}{2!} \right) T_2 \right] \, dw + Y \, dw, \quad Y = \sum_{a=-2}^{2} C_a(\tau) W_a (4.3) \]

   Flatness of this connection i.e. $F = da + a \wedge a = 0$ demands the coefficients $C_a$’s have no $\tau$ dependence i.e. they be constants (See Appendix D). Note that the $C_a$’s are five complex numbers, so this is 10 parameters worth of freedom.

2. We will apply the gauge transformation, $U(\tau) = \exp(i\tau T_0) = \exp(\tau L_0)$ on $a$ and obtain connection, $A'$:

   \[ A' = A + X \, dw, \quad (4.4) \]

   where

   \[ X \equiv \exp(-L_0 \tau) Y \exp(L_0 \tau) = \sum_{a=-2}^{2} e^{a\tau} C_a W_a. \]

   Note that here $A$ is the connection that gives rise to the quotient cosmology metric. Our goal will be to look for gauge fields $A'$ in this class that have the same holonomy as the quotient cosmology connection $A$, but which give rise to non-singular metrics.

3. We will demand that the $g_{zz}$ and $g_{\phi\phi}$ arising from $A'$ differ by a positive quantity from those arising from $A$ (while not affecting the rest of the metric components). This is a sufficient condition for metric regularity. This requires\footnote{This is easy to demonstrate. The new vierbein derived from $A'$ is, \[ e' = e + \frac{X \, dw - X \, \bar{d} \bar{w}}{2i/l} . \] Then the new metric components are:

   \[ g'_{ww} = g_{ww} - \frac{l^2}{8} Tr(X^2), \quad g'_{\bar{w} \bar{w}} = g_{\bar{w} \bar{w}} - \frac{l^2}{8} Tr(\bar{X}^2), \quad g'_{w \bar{w}} = g_{w \bar{w}} + \frac{l^2}{8} Tr(X \bar{X}) . \]

   Noting that,

   \[ 2g'_{ww} dw d\bar{w} = 2g'_{\bar{w} \bar{w}} \left( \frac{dz^2}{l^2} + d\phi^2 \right), \]}

   \[ Tr(X^2) = Tr(\bar{X}^2) = 0, \quad Tr(X \bar{X}) > 0 \]
which are 2 (real) equations and one constraint inequality:

\[ C_0^2 - 3C_1C_{-1} + 12C_2C_{-2} = 0, \quad (4.5) \]

\[ C_0\tilde{C}_0 - 3C_1\tilde{C}_{-1} + 12C_2\tilde{C}_{-2} + c.c > 0. \quad (4.6) \]

Notice that the equations are \( \tau \) independent, even though \( X \) is \( \tau \)-dependent.

4. Finally, we want to ensure that both \( A' \) and \( A \) have the same holonomy, so that they are related by a single valued gauge transformation. Computing the \( U \) matrix and explicitly evaluating the eigenvalues of the holonomy matrix like we did before is tiresome now because there are five more generators coming from the spin 3 charges.

Instead we will follow [21] and fix the holonomy matrix in terms of its characteristic polynomials. The idea is that any \( SL(3, \mathbb{C}) \) matrix \( M \) satisfies the equation \( M^3 = \Theta_0 I + \Theta_1 M \) where

\[ \Theta_0 = \det(M), \quad \text{and} \quad \Theta_1 = \frac{1}{2} \text{Tr}(M^2) \quad (4.7) \]

So we will equate the characteristic coefficients of the holonomy matrix for \( A' = A + Xdw \) with that of \( A \). The holonomy matrix is easily computed by integrating the gauge field around the \( \phi \)-circle at fixed \( z \). Demanding that the \( \Theta_0 \) and \( \Theta_1 \) are the same for \( A \) and \( A' \) holonomies gives

\[ \Theta_0 : C_0^3 - \frac{9}{4} \frac{L}{l} C_0 + \frac{27}{2} \left( C_1^2 C_{-2} + C_2^2 C_2 \right) - \frac{9}{2} C_0 \left( C_1 C_{-1} + 8C_2 C_{-2} \right) + 27C_{-2} + \frac{27}{16} \frac{L}{l} C_2 = 0. \quad (4.8) \]

\[ \Theta_1 : C_0^2 - 3C_{-1}C_1 + 12C_{-2}C_2 = 0. \quad (4.9) \]

The first equation is obtained straight from\(^{13}\)

\[ \text{Det} \left( \int d\phi(A + Xdw)|_{z=\text{const.}} \right) - \text{Det} \left( \int d\phi(A)|_{z=\text{const.}} \right) = 0 \]

A similar equation for the Tr gives rise to the second equation.

Together these give rise to 2 new real constraints (instead of 4, because the \( \Theta_1 \) constraint is the same as the already found \( Tr(X^2) = 0 \) constraint).

\(^{13}\)The specific form of the \( \Theta_0 \) equation here depends on the specific choice of \( T_a \) that we made (2.11). There are other choices of \( T_a \) in terms of \( L_a \) which result in the same \( \text{Tr}(T_a T_b) \) and \( SL(2) \) algebra (and therefore the metric), but which can change the determinant that we are computing here. Note that the \( \Theta_1 \) equation will not change because the traces are protected.
5. We can simplify the condition (4.8) a bit by inserting (4.5),
\[ C_0 \left( C_1 C_{-1} + 32 C_2 C_{-2} + \frac{3L}{2l} \right) - 3 \left( C_1^2 C_{-2} + C_{-1}^2 C_2 + 6 C_{-2} + \frac{3L}{8l} C_2 \right) = 0 \] (4.10)

6. So we are left with \( 10 - 2 - 2 = 6 \) parameter family of singularity eliminating spin-3 transformations. The general form of the resultant regular metric is,
\[ ds^2 = -l^2 d\tau^2 + \left( \left| e^\tau - \frac{L}{2l} e^{-\tau} \right|^2 + \alpha \right) dz^2 + \left( \left| le^\tau + \frac{L}{2} e^{-\tau} \right|^2 + \alpha t^2 \right) d\phi^2 + i \frac{L - L}{2} dz d\phi \]
with
\[ \alpha = \frac{2}{3} \left( C_0 \bar{C}_0 - 3 C_1 \bar{C}_{-1} + 12 C_2 \bar{C}_{-2} + c.c \right). \]

7. A convenient choice is to set, \( C_{\pm 2} = 0 \). Then the equations are:
\[ C_0^2 - 3C_1 C_{-1} = 0 \]
\[ C_0^2 - \frac{9L}{4l} - \frac{9}{2} C_1 C_{-1} = 0 \]
which solves to give,
\[ C_0 = i \sqrt{\frac{L}{2l}}, \quad C_1 C_{-1} = -\frac{3L}{2l} \]
and we also need to make \( g_{zz}, g_{\phi\phi} \) positive definite, i.e. satisfy
\[ \frac{9L}{2l} - 3C_1 \bar{C}_{-1} + c.c > 0 \]

8. As an example, for the case when asymptotic charge \( L > 0 \), we can further choose, \( C_{\pm 1} \in \mathbb{R} \) and then we have automatically satisfied the metric positivity constraint,
\[ |C_0|^2 - 3C_1 \bar{C}_{-1} + c.c = 18 \frac{L}{l} > 0 \]
and the connection,
\[ a = \left[ \left( 1 - \frac{L}{2l} \right) T_1 + i \left( 1 + \frac{L}{2l} \right) T_2 \right] dw + \left( C_1 W_1 + C_{-1} W_{-1} + i 3 \sqrt{\frac{L}{2l}} W_0 \right) dw\] (4.11)
and the metric,
\[ ds^2 = -l^2 d\tau^2 + \left( \left| e^\tau - \frac{L}{2l} e^{-\tau} \right|^2 + \frac{6L}{l} \right) dz^2 + \left( \left| e^\tau + \frac{L}{2l} e^{-\tau} \right|^2 + \frac{6L}{l} \right) \right) l^2 d\phi^2. \] (4.12)

One can easily check that at the erstwhile singularity, \( \tau = \frac{1}{2} \ln \frac{L}{2l} \) the scalar curvature is currently finite, \( R = \frac{176}{117} \frac{1}{l^2} \). We have checked that the curvature scalars are finite for all finite values of \( \tau \).
We make one observation. The specific gauge field configuration that we have chosen \((4.11)\) is not continuously connected in the \(C_\alpha\) parameter space with the original connection \(A\) corresponding to the quotient cosmology. This is because we have imposed the condition \(C_1 C_{-1} = \text{const}\), so \(C_1\) and \(C_{-1}\) cannot both be tuned to zero at the same time. Another way to say this is to note that in our choice of parameters, the resolution parameter \(\alpha\) is determined in terms of \(L\), with no dependence on \(C_\alpha\). It is not clear to us if this is a general feature of gauge transformations that allow resolution of the singular cosmology or a feature of the specific ansatzes that we chose.

In any event, the conclusion is that we have resolved the singularity in the quotient cosmology metric. We accomplished this by (effectively) doing a higher spin gauge transformation that preserved the holonomy and the flatness. Note however that the asymptotic geometry is no longer of the conventional asymptotically dS\(_3\) form. A similar price had to be paid for resolving the black holes singularity in \([21]\) where one ended up losing the conventional asymptotically AdS\(_3\) form. This is to be expected: a spin-3 field is non-normalizable and corresponds to deforming the boundary theory. This is also reflected in the fact that the higher spin field components can vanish in the interior or reach the future or past boundary in our resolved solution. Similar features were again seen in \([21]\). To complete the presentation of our resolved version of the solution, we present these spin-3 fields in an appendix.

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A \(SL(2, \mathbb{C})\) gauge field in terms of (complex) Euler angles

The \(SL(2)\) generators we use are
\[ T_0 = -i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad T_1 = \frac{i}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \quad T_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \]

and the corresponding subgroups are,

\[ U_0 = e^{\theta_0 T_0} = I + \sin \theta_0 T_0 + 2 \sin^2 \frac{\theta_0}{2} T_0^2, \]
\[ U_1 = e^{\theta_1 T_1} = I + \sinh \theta_1 T_1 + 2 \sinh^2 \frac{\theta_1}{2} T_1^2, \]
\[ U_2 = e^{\theta_2 T_2} = I + \sinh \theta_2 T_2 + 2 \sinh^2 \frac{\theta_2}{2} T_2^2. \]

For \( U = U_0 U_2 U_1 \), one has,

\[ U^{-1} dU = (U_2 U_1)^{-1} (U_0^{-1} dU_0) (U_2 U_1) + U_1^{-1} (U_2^{-1} dU_2) U_1 + U_1^{-1} dU_1. \]

Using,

\[ U_1^{-1} dU_1 = d\theta_1 T_1, \]

\[ U_1^{-1} (U_2^{-1} dU_2) U_1 = (\cosh \theta_1 T_2 - \sinh \theta_1 T_0) d\theta_2 \]

and,

\[ (U_2 U_1)^{-1} (U_0^{-1} dU_0) (U_2 U_1) = (\cosh \theta_1 \cosh \theta_2 T_0 + \sinh \theta_2 T_1 - \sinh \theta_1 \cosh \theta_2 T_2) d\theta_0 \]

we finally have the expression for a pure gauge field \( A = U^{-1} dU \)

\[ A^0 = \cosh \theta_1 \cosh \theta_2 d\theta_0 - \sinh \theta_1 d\theta_2 \]
\[ A^1 = d\theta_1 + \sinh \theta_2 d\theta_0 \]
\[ A^2 = - \sinh \theta_1 \cosh \theta_2 d\theta_0 + \cosh \theta_1 d\theta_2 \]

(A.1)

**B Wilson Loop in the Fefferman-Graham coordinates**

Proceeding identically as in the last section for \( V = e^{\psi_2 T_2} e^{\psi_0 T_0} \), we get,

\[ V^{-1} dV = d\psi_0 T_0 - \sin \psi_0 d\psi_2 T_1 + \cos \psi_0 d\psi_2 T_2. \]

with,

\[ \psi_0 = i \left( \tau - \frac{1}{2} \ln \frac{L}{2l} \right), \quad \psi_2 = i \sqrt{\frac{2L}{l}} \left( \phi + \frac{iz}{l} \right), \]
The corresponding Wilson loop in the $\phi$- direction,

$$W = V^{-1}(z, \tau, 0)V(z, \tau, 2\pi)$$

is,

$$
\begin{pmatrix}
\cosh^2 \frac{\Delta \psi_2}{2} & \frac{e^{i\psi_0} \sinh \Delta \psi_2}{\sqrt{2}} & \frac{e^{2i\psi_0} \sinh^2 \frac{\Delta \psi_2}{2}}{\sqrt{2}} \\
\frac{e^{-i\psi_0} \sinh \Delta \psi_2}{\sqrt{2}} & \cosh \Delta \psi_2 & \frac{e^{i\psi_0} \sinh \Delta \psi_2}{\sqrt{2}} \\
e^{-2i\psi_0} \sinh^2 \frac{\Delta \psi_2}{2} & \frac{e^{-i\psi_0} \sinh \Delta \psi_2}{\sqrt{2}} & \cosh^2 \frac{\Delta \psi_2}{2}
\end{pmatrix}
$$

where,

$$\Delta \psi_2 = 2\pi i \sqrt{\frac{2L}{l}}.$$

The Eigenvalues of this matrix are,

$$(1, e^{-\Delta \psi_2}, e^{\Delta \psi_2}).$$

C  The SL(3) basis

Consistent with [19, 21], we have employed the following set of generators which we use to furnish a basis for $sl(3)$:

$$L_1 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad L_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad L_{-1} = \begin{pmatrix} 0 & -2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix},$$

$$W_1 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \quad W_0 = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -4/3 & 0 \\ 0 & 0 & 2/3 \end{pmatrix}, \quad W_{-1} = \begin{pmatrix} 0 & -2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$W_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}, \quad W_{-2} = \begin{pmatrix} 0 & 0 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Note that we have set $\sigma = -1$ in [19]. The algebra they satisfy is

$$[L_m, L_n] = (m - n)L_{m+n} \quad (C.1)$$

$$[L_m, W_p] = (2m - p)W_{m+p} \quad (C.2)$$

$$[W_p, W_q] = \frac{\sigma}{3} (p - q)(2p^2 + 2q^2 - pq - 8)L_{p+q}. \quad (C.3)$$

Using these we define the $T_a$ as in (2.11) and the SL(3) algebra takes the following form:

$$[T_a, T_b] = \epsilon_{abc} T_c;$$

$$[T_0, W_a] = i a W_a,$$

and the algebra between the $W$'s is of course the same as (C.3).
D Flatness of the Primitive $SL(3, \mathbb{C})$ connection

In this appendix we will show that the primitive connection $a$ of the type (4.2) or (4.3) that we use to construct our solutions are flat. By directly plugging in the matrix-valued one-form $a = (a_i^\mu L_a + a_m^\mu W_m) dx^\mu$, the field strength

$$F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu + [a_\mu, a_\nu]$$

(D.1)

can be explicitly computed using the $SL(3)$ algebra relations in [21] (or our Appendix C). The $\bar{w}\bar{w}$ and the $\rho\bar{w}$ components of the equation of motion are manifestly satisfied (due to the fact that every component is at best a function of $\rho$ and in our gauge $a_{\bar{w}}, a_\rho = 0$). The $\rho w$ component equation is,

$$\partial_\rho a_w = 0$$

so the components are forced to be $\rho$-independent.

$$\implies C_a = \text{constant}$$

So this is the condition for a connection like $a$ to be flat. Then it follows, that $A = b^{-1}ab + b^{-1}db$, being a gauge transform of $a$ will also flat. A similar statement also holds for $A'$.

Even though we don’t look at more general gauge fields, it is worthwhile mentioning that $C_a$ can be arbitrary functions of $w$, and the connection will still be flat.

E The Spin-3 Field on the Resolved Geometry

For the connection in Eq.(4.4) obtained by performing a spin-3 transformation on a singular pure $SL(2, \mathbb{C})$ geometry, one turns on the spin-3 field (following conventions of [21]),

$$\phi_{\mu\nu\rho} = \frac{1}{3!} \text{Tr} (e'_\mu e'_\nu e'_\rho)$$

where

$$e' = e + \frac{Xdw - \bar{X} d\bar{w}}{2i/l} = e^a T_a - i \frac{l}{2} \sum_{a=-2}^2 e^{a\tau} C_a W_a dw + i \frac{l}{2} \sum_{a=-2}^2 e^{a\tau} \bar{C}_a W_a d\bar{w}.$$ 

Explicitly

$$e'_w = -\frac{i}{2} l \left( e^\tau - \frac{L}{2l} e^{-\tau} \right) T_1 + \frac{l}{2} \left( e^\tau + \frac{L}{2l} e^{-\tau} \right) T_2 - \frac{i}{2} l \sum_{a=-2}^2 e^{a\tau} C_a W_a$$
\[ e'_\bar{\omega} = \frac{il}{2} \left( \tau - \bar{L} e^{-\tau} \right) T_1 + \frac{l}{2} \left( \tau + \bar{L} e^{-\tau} \right) T_2 + \frac{il}{2} \sum_{a=-2}^{2} e^{a\tau} \bar{C}_a W_a \]

\[ e'_\tau = e_\tau = l T_0, \]

which then determine the components of the spin-3 field. For the case where we have chosen the parameters \( C_a \) as in (4.12) the result is

\[ \phi_{\tau\tau\tau} = \phi_{\tau\tau\bar{\omega}} = \phi_{\bar{\omega}\bar{\omega}\bar{\omega}} = 0, \]

\[ \phi_{\tau\tau\bar{\omega}} = \phi_{\bar{\omega}\bar{\omega}\omega} = -\sqrt{\frac{2l^3}{6}} \sqrt{\frac{L}{l}}, \]

\[ \phi_{\tau\bar{\omega}\bar{\omega}} = \frac{l^2 (4C_{-1} l - C_1 L)}{12\sqrt{2}}, \phi_{\bar{\omega}\bar{\omega}\omega} = -\sqrt{\frac{LL}{l}} \left( 4e^{2\tau} l^2 + 20Ll + e^{-2\tau} L^2 \right) \]

\[ \phi_{\bar{\omega}\omega\tau} = \frac{l^2 (C_1 + 2e^{-2\tau} C_{-1}) (L - 2e^{2\tau} l)}{24\sqrt{2}}. \]

Here \( C_1 \) and \( C_{-1} \) could be any pair of real numbers satisfying,

\[ C_1 C_{-1} = \frac{3L}{2l}. \]

An observation worthy of remark here is that some components of the spin-3 fields vanish and some reach all the way to the (future or past) boundary. This means that the higher spin gauge transformations that we used have come at a price: a similar phenomenon was observed in the context of higher spin black holes in AdS3.

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