Distributionally Robust Optimal Reactive Power Dispatch with Wasserstein Distance in Active Distribution Network

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Abstract—The uncertainties from renewable energy sources (RESs) will not only introduce significant influences to active power dispatch, but also bring great challenges to the analysis of optimal reactive power dispatch (ORPD). To address the influence of high penetration of RES integrated into active distribution networks, a distributionally robust chance constraint (DRCC)-based ORPD model considering discrete reactive power compensators is proposed in this paper. The proposed ORPD model combines a second-order cone programming (SOCP)-based model at the nominal operation mode and a linear power flow (LPF) model to reflect the system response under uncertainties. Then, a Wasserstein-distance-based distributionally robust optimization (WDRO) method is utilized to solve the proposed DRCC-based ORPD model. The WDRO method is data-driven due to the reason that the ambiguity set is constructed by the available historical data without any assumption on the specific probability distribution of the uncertainties. And the more data is available, the smaller the ambiguity would be. Numerical results on IEEE 30-bus and 123-bus systems and comparisons with the other three-benchmark approaches demonstrate the accuracy and effectiveness of the proposed model and method.

Index Terms—Active distribution network, chance constraint, renewable energy source, optimal reactive power dispatch (ORPD).

I. INTRODUCTION

Optimal reactive power dispatch (ORPD), also known as steady-state voltage control, is important for secure and economic operation of power systems [1]. It aims at finding the optimal control variables of the power system to minimize a certain objective function [2], [3]. Besides, ORPD satisfies a bunch of physical and operation constraints such as branch currents and bus voltage magnitudes to be within their reasonable ranges. The optimal control variables consist of both continuous variables, i.e., voltage magnitudes, and discrete variables, i.e., ratios of transformers and the number of switchable capacitors/reactors. Considering the nonlinearity of power flow equations together with the numerous continuous and discrete decision variables, ORPD problem is a rather complex optimization problem.

Traditionally, ORPD is subject to a series of nonlinear constraints that make it a mixed-integer nonlinear programming problem. Many methods have been proposed to solve the ORPD model, which can be generally divided into two categories: artificial intelligence methods (such as particle swarm optimization [4], simulated annealing [5]) and conventional methods (such as linear programming [6], gradient-based optimization [7], interior-point method [8]). In these optimization methods, the discrete variables are treated as continuous variables firstly and then rounded off to the nearest integer value, which may lead to unnecessary deviation in the objective function and more constraint violations. Nowadays, mixed-integer programming approaches have been frequently used, and they can be efficiently solved by branch-and-cut, branch-and-bound or cutting plane methods [9]. Besides, the global optimal solution can be guaranteed under the Karush-Kuhn-Tucker (KKT) conditions when the models are relaxed to be a convex problem.

Meanwhile, since the penetration of renewable energy sources (RESs) increases sharply in recent years [10], they will probably bring great challenges to the ORPD due to its stochastic nature. For example, the RES uncertainties could result in frequent voltage fluctuations due to the delay of system reaction, especially in distribution systems [11], [12]. Therefore, the integration of such volatile renewable energy into power system requires more considerations on the planning and scheduling [13]-[16]. Presently, there are several approaches to tackle the uncertainties of RES in power systems. One is stochastic programming (SP) [17]-[19], which supposes that the uncertainties follow a presumed probability distribution, and then it is feasible to be transformed into a deterministic problem. For example, a chance-constrained
programming method is proposed in [18], in which the uncertain nodal power injections and random branch outages are considered as uncertainty sources. A stochastic multi-objective ORPD problem is presented in [19], wherein a wind-integrated power system with both loads and wind power generation uncertainties is considered. The second approach is robust optimization (RO) [9], [20], [21]. For example, a two-stage robust ORPD model is proposed in [9], which can achieve a more robust solution than traditional deterministic approaches, although it has longer computation time. The RO-related methods have been demonstrated to give out rather conservative solutions, because they ignore the exact probability information of uncertainties and only search for the solution that performs best in the worst-case scenarios.

To bridge the gap between SP and RO, the third approach called distributionally robust method (DRO) has been brought up [22]-[30]. DRO assumes that the true probability distribution lies in an ambiguity set, and it minimizes the worst-case expected cost over this ambiguity set. The most popular ambiguity set is the moment-based, e.g., the set of probability distribution with the first- and second-order moments [28]-[30]. However, the information of first- and second-order moments cannot cover all the probability information of the true distribution. We do not even know the moment information of the true probability distribution. All that we have are the available historical data. Therefore, a desired ambiguity set should contain the true probability distribution. The ambiguity set will become smaller with the increasing historical data.

Currently, most existing references are using two-stage model for ORPD problem under uncertainties, and the solutions are usually complicated due to their multi-level structures [9]. Besides, non-anticipative constraints on ORPD decisions are always not considered in two-stage model formulations [31]. Moreover, the two-stage distributionally robust optimization model is much more time-consuming and difficult to be solved when more historical data are available. To address the above concerns, a new Wasserstein-distance-based distributionally robust chance-constraint (DRCC) ORPD model is proposed for active distribution networks in this paper. The main contributions are as follows:

1) In the ORPD model formulation, we propose an approximate second-order cone programming (SOCP) power flow model, which combines an exact SOCP model at the nominal operation mode and a linear power flow (LPF) model to express the system response under uncertainties. It largely inherits the accuracy of the exact SOCP model. Moreover, it is a single-level mixed-integer programming formulation rather than multi-level formulation. Therefore, it can be directly solved by popular commercial solvers like Gurobi, and no other complex algorithms for minimum-maximum structure problems are required.

2) We firstly apply Wasserstein distance to the DRCC-based ORPD model to construct the ambiguity set, so as not to presume any true probability distribution for the uncertain RESs. The Wasserstein-distance-based distributionally robust optimization (WDRO) method is a data-driven method, and larger quantity of data will lead to smaller ambiguity set and less conservative solution.

3) We then reformulate the original DRCC-based ORPD model to be a mixed-integer convex programming model, according to the Wasserstein-distance-based ambiguity set. Simulations are performed on IEEE standard test systems, and optimal solutions of the proposed WDRO method are compared to those of other three benchmark approaches. The proposed WDRO is able to guarantee fast computational performance as RO, which is better than moment-based distributionally robust optimization (MDRO) and SP approaches. Moreover, the proposed WDRO method is also effective when large number of historical data are available, which benefits from the unique reformulation of the proposed DRCC-based ORPD model.

The structure of the paper is as follows. Section II introduces the formulation of DRCC-based ORPD model. A WDRO method is then proposed in Section III to solve the special optimization problem. In Section IV, numerical results on IEEE 30-bus and 123-bus systems and comparisons with another three benchmark approaches are presented to demonstrate the accuracy and effectiveness of the proposed model and method. Finally, conclusions are drawn in Section V.

II. ORPD MODEL IN DISTRIBUTION NETWORKS

In this section, a novel DRCC-based ORPD model is proposed considering multiple continuous and discrete decision variables as well as the power flow constraints and uncertainties from RES. The proposed model combines an SOCP model at the nominal operation mode and an LPF model to reflect the system response under uncertainties.

A. ORPD Model Based on SOCP Relaxation

Recently, the conic relaxation technique has been deeply studied [32]-[35] to relax the nonconvex power flow equations by the use of SOCP, and it has been recognized that the conic relaxation has no gap or small gap to the original exact power flow equations in most distribution networks. An ORPD model based on SOCP has been formulated in [9]. In an RES-integrated power system, the SOCP-based ORPD model can be given as follows [9]:

\[
\min f(P^e, Q^e, v_i) \tag{1}
\]

s.t.

\[
\begin{align*}
&P^e_i + P^e_i - P^i_i = \sum_{j \in \Omega} P_{ij} - \sum_{k \in \Omega} (P_{ki} - r_{ki}l_{ki}) \quad \forall i \in B \\
&Q_i^e + Q_i^e - Q_i^i = \sum_{j \in \Omega} Q_{ij} - \sum_{k \in \Omega} (Q_{ki} - x_{ki}l_{ki}) + b_{v_i} v_i^2 \quad \forall i \notin \Omega \\
&Q_i^e + Q_i^e - Q_i^i + \frac{1}{2} \left( C_{ij}^e + s_i (2 \sigma_{ij} + 2 \sigma_{ij} + \cdots + 2 \sigma_{ij}) \right) = \sum_{j \in \Omega} Q_{ij} - \sum_{k \in \Omega} (Q_{ki} - x_{ki}l_{ki}) + b_{v_i} v_i^2 \quad \forall i \in \Omega \\
&v_i^2 = v_i^2 - 2 (r_i^e P_i^e + x_i^e Q_i^e) + (r_i^e + x_i^e) l_{ij} \quad \forall (i, j) \in ET \\
&\sum_{k \in \Omega} h_{ik} l_{ik} = v_i^2 - 2 (r_i^e P_i^e + x_i^e Q_i^e) + (r_i^e + x_i^e) l_{ij} \quad \forall (i, j) \in T
\end{align*}
\]
\[\begin{align*}
-M(1 - \beta_i) + v_i^j &\leq h_{i,j} \leq v_i^j + M(1 - \beta_i) \quad \forall (i,j) \in T \\
-M\varepsilon_{i,1} \leq h_{i,j} &\leq M\varepsilon_{i,2} \quad \forall (i,j) \in T \quad \varepsilon_{i,1}, \varepsilon_{i,2} \in \{0, 1\}
\end{align*}\]  
\[\sum_{j=1}^{n} \varepsilon_{i,j} = 1 \quad \forall (i,j) \in T \tag{4}\]

\[\begin{align*}
\nu_i^2 - M(1 - \gamma_i) &\leq \sigma_{i,k} \leq \nu_i^2 + M(1 - \gamma_i) \\
-M\gamma_{i,k} \leq \sigma_{i,k} &\leq M\gamma_{i,k} \quad \forall i \in \Omega \cap \Omega_{m}, k = 1, 2, \ldots, n_i \\
0 &\leq 2\gamma_i + \gamma_i \leq \gamma_i \leq (C_i^{\max} - C_i^{\min})/s_i \\
y_{i,0}, y_{i,1}, \ldots, y_{i,n_i} &\in \{0, 1\} \quad \forall i \in \Omega \cap \Omega
\end{align*}\]  
\[\begin{align*}
2P_{\theta} \nu_i^2 \leq I_{\theta} - v_i^2 \quad \forall (i,j) \in E \\
2Q_{\theta} \nu_i^2 \leq I_{\theta} - v_i^2 \quad \forall (i,j) \in E \tag{6}\]

\[\begin{align*}
\nu_i^2 &\leq \nu_i^2 \leq \hat{\nu}_i^2 \quad \forall i \in B \\
I_{\theta} &\leq (\hat{\nu}_i^2)^2 \quad \forall (i,j) \in E \\
P_{\theta} \leq P_{\theta} \leq \hat{P}_{\theta} \quad \forall i \in B \\
Q_{\theta} &\leq Q_{\theta} \leq \hat{Q}_{\theta} \tag{7}\]

where the subscripts \(i\) or \(j\) represent the specific bus and branch, respectively; \(P_i, Q_i, P_{ij}, Q_{ij}\) are the nominal active and reactive power for the generation from generators, injection from RES and power consumption of load, respectively; \(B_i\) or \(E_i\) are the sets of buses and branches, respectively; \(T\) is the set of branches with transformers; \(\Omega\) is the set of buses for reactive power compensators; \(\pi(i), \delta(i)\) are the sets of all parents and children of bus \(i\), respectively; \(r_{\theta}, x_{\theta}\) are the resistance and reactance of branch \((i,j)\), respectively; \(h_{i,j}\) is the shunt susceptance from bus \(i\) to the ground; \(C_i\) is the value of shunt capacitors/reactors at bus \(i\); \(s_i\) is the step size of shunt capacitors/reactors at bus \(i\); \(I_{\theta}^m\) is the current capacity limit of branch \((i,j)\); \(P_{\theta}, Q_{\theta}\) are the active and reactive power flows from bus \(i\) to \(j\), respectively; \(\tau_{\theta}\) is the tap ratio of the transformer of branch \((i,j)\); \(\nu_i\) is the nominal bus voltage magnitude; \(\nu_i\) are the upward and downward bus voltage magnitude, respectively; \(\hat{P}_{\theta}, \hat{P}_{\theta}\) are the upward and downward active power of generators, respectively; and \(\hat{Q}_{\theta}, \hat{Q}_{\theta}\) are the upward and downward reactive power of generators, respectively.

The constraint (2) denotes the power balance at each bus; constraints (3)-(6) denote the Ohm’s law for each branch; constraints (4) and (5) are the constraints for discrete compensators; constraint (7) is the bounds for bus voltages, branch currents and the generator outputs. For any fixed forecasted load power \((P_i, Q_i)\) and RES \((P_n, Q_n)\), traditional OPF aims to perform an optimal dispatch of reactive power while guaranteeing the power balance and security constraints of the system, i.e., (1)-(7). With the increasing growth of RES, the power system operation is influenced more deeply by the uncertainties of RES. By using automatic generation control (AGC) and automatic voltage regulation (AVR), the adverse impact of the RES uncertainties need to be considered when arranging optimal operation modes.

**B. Response Model of Voltage-concerned System and Its Control Under Uncertainties**

Since LPF model is convenient in dealing with the uncertainties of RES, a linear approximation of the AC power flow equation can be obtained as follows [36]:

\[\begin{align*}
\begin{bmatrix}
p \\
q
\end{bmatrix} &= \begin{bmatrix}
-B' & G''
\end{bmatrix} \begin{bmatrix}
\theta \\
\nu
\end{bmatrix} \\
\begin{bmatrix}
f \\
f
\end{bmatrix} &= \frac{1}{2} G' \nu \cdot B' \theta
\end{align*}\]  
\[\begin{align*}
\begin{bmatrix}
\Delta p_s \\
\Delta q_s \\
\Delta p_L \\
\Delta q_L
\end{bmatrix} &= N^{-1} \begin{bmatrix}
\Delta p_s \\
\Delta q_s \\
\Delta p_L \\
\Delta q_L
\end{bmatrix} + (-LN^{-1}H + M) \begin{bmatrix}
0 \\
0 \\
\Delta \nu_R^2 \\
\Delta \nu_R^2
\end{bmatrix} \tag{9}
\]

\[\begin{align*}
\begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix} &= \left[\begin{array}{ccc}
-B'_{SR} & G'_{SR} & G'_{SS} \\
-B'_{SL} & G'_{SL} & G'_{SL} \\
-G'_{LS} & -B'_{LL} & -B'_{LL}
\end{array}\right] \begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix} \\
\begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix} &= \left[\begin{array}{ccc}
-B'_{SR} & G'_{SR} & G'_{SS} \\
-B'_{SL} & G'_{SL} & G'_{SL} \\
-G'_{LS} & -B'_{LL} & -B'_{LL}
\end{array}\right] \begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix}
\end{align*}\]  
\[\begin{align*}
\begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix} &= \left[\begin{array}{ccc}
-B'_{SR} & G'_{SR} & G'_{SS} \\
-B'_{SL} & G'_{SL} & G'_{SL} \\
-G'_{LS} & -B'_{LL} & -B'_{LL}
\end{array}\right] \begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix}
\end{align*}\]  
\[\begin{align*}
\begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix} &= \left[\begin{array}{ccc}
-B'_{SR} & G'_{SR} & G'_{SS} \\
-B'_{SL} & G'_{SL} & G'_{SL} \\
-G'_{LS} & -B'_{LL} & -B'_{LL}
\end{array}\right] \begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix}
\end{align*}\]  
\[\begin{align*}
\begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix} &= \left[\begin{array}{ccc}
-B'_{SR} & G'_{SR} & G'_{SS} \\
-B'_{SL} & G'_{SL} & G'_{SL} \\
-G'_{LS} & -B'_{LL} & -B'_{LL}
\end{array}\right] \begin{bmatrix}
\Delta \theta_s \\
\Delta \lambda_L \\
\Delta \nu_L^2
\end{bmatrix}
\end{align*}\]

Without the loss of generality, only wind power generation is considered as the fluctuating RES in this study, and any other types of RES can be integrated and modelled similarly. The actual active power outputs are considered as a combination of the forecasted power \(P'\) plus a random forecasting error \(\xi\). The wind farm is assumed to maintain a
fixed power factor \(\cos(\phi)\). In order to address the ORPD problem under uncertainties, the traditional ORPD model should be modified. Firstly, we use the SOCP model from Section II-A to get a nominal operation mode \(x = (v^r, P^r, Q^r, z^r, y^r)\), and \(z, y\) denote sets of discrete variables. Then, (9) and (10) can be used to calculate incremental system response under RES uncertainties. Besides, with the regulation of AGC, generators can respond affinely to the total forecasting errors of active wind power. In such circumstance, the incremental part of active and reactive nodal power injections in (9) and (10) are random variables as:

\[
\begin{align*}
\Delta \hat{p}_s &= -(1^T \hat{\xi}^r) \alpha + \hat{\xi}_s^r \\
\Delta \hat{q}_s &= \hat{\xi}_l^r \\
\Delta \hat{q}_L &= \tan \phi \hat{E}_L
\end{align*}
\]

where \(\tan \phi = \sin \phi / \cos \phi\) as the power factor of wind farms; \(\alpha\) is the AGC participation factor of generation units; and \(\hat{\xi}_s = (\hat{\xi}_s^r + \hat{\xi}_s^b + \hat{\xi}_s^L)\) denote the random forecasting errors of RES generation.

Then, the incremental state variables in (9) and (10) will also become random variables. By substituting (12) into (9) and (10), the random state variables can be written as:

\[
\begin{align*}
\hat{v}_i &= (1^T \hat{\xi}) C^r \alpha + D^r \hat{\xi} + v_i^r \\
\hat{f} &= (1^T \hat{\xi}) C^f \alpha + D^f \hat{\xi} + f \\
\hat{q}_{BUS} &= (1^T \hat{\xi}) C^D \alpha + D^D \hat{\xi} + q_{BUS}
\end{align*}
\]

where \(C^r, D^r, C^f, D^f, C^D, D^D\) are the constant matrices decided by the network parameters; and \(q_{BUS}\) is the reactive power injection in the nominal operation mode, which is calculated by the SOCP-based model.

C. Formulation of DRCC-based ORPD Model

Based on the developed SOCP model and LPF model, a DRCC-based ORPD model can be formulated as (14)-(20) with constraints (2)-(7):

\[
\min_{\alpha, P, \bar{P}, \bar{F}} \sum_{i=1}^{N} f_i^0 (\bar{P}_i - (1^T \hat{\xi}) \alpha) + \sum_{j=1}^{M} f_j^0 (Q_j^r - \bar{v}_i) + c^T \bar{r} + \epsilon^T R
\]

subject to:

\[
\begin{align*}
1^T \alpha &= 1 \\
(P^r - \bar{P} &\geq P^r \geq 0) \\
(\bar{P} &\geq R \geq 0) \\
\inf_{\alpha, P, \bar{P}, \bar{F}} P \{ -\bar{r} \leq (1^T \hat{\xi}) \alpha \leq \bar{r} \} \approx 1 - \rho_1 \\
\inf_{\alpha, P, \bar{P}, \bar{F}} \{ \bar{v}_i \leq \hat{v}_i \leq \bar{v}_i \} \approx 1 - \rho_2 \quad \forall i \in L \\
\inf_{\alpha, P, \bar{P}, \bar{F}} \{ \bar{f}_j \leq \hat{f}_j \leq \bar{f}_j \} \approx 1 - \rho_3 \quad \forall j = 1, 2, ..., n_1 \\
\inf_{\alpha, P, \bar{P}, \bar{F}} \{ \bar{q}_k \leq \hat{q}_k \leq \bar{q}_k \} \approx 1 - \rho_4 \quad \forall k \in R \cup S
\end{align*}
\]

where \(P\) is a probability distribution (measure); \(\bar{r}\) is the expectation with probability distribution \(\bar{P}; \bar{F}\) are the upward and downward regulating reserve of generation units, respectively; \(\bar{v}, \bar{f}\) are the upward and downward regulating reserves

prices, respectively; \(n_1, n_2\) are the numbers of buses and lines, respectively; \(\hat{P}_N\) is the ambiguity set constructed from the historical data of forecasting error \(\hat{\xi}_i\); and \(\rho_1\) to \(\rho_4\) are the tolerable violation probability.

According to [37], the objective function for the DRCC-based ORPD model is chosen for minimizing the worst-case expectation of operation costs in this study. Only minimizing the network losses without considering generator costs may conflict with the economic operation principles [38]. The operation costs consist of the reserve costs and the production costs of both generator active and reactive power whose cost functions are obtained from MATPOWER [37]. Actually, other alternative objectives such as minimizing network losses and the operation costs of tap ratio (TR) and switchable capacitors/reactors (SCRs), can also be incorporated into the proposed model easily, as all control variables in the proposed DRCC-based ORPD model are expressed explicitly [39]. Constraints (2)-(7) ensure the power balance and security limits of state variables in the nominal operation mode. Constraint (15) denotes the basic requirement for participation factors of AGC systems; constraints (16) and (17) ensure the adequacy of the upward and downward generation reserves; chance constraint (18) is for the voltage magnitudes at \(PQ\) buses under uncertainties; chance constraint (19) is for the line flow; and chance constraint (20) is for the reactive power outputs of generators.

III. SOLVING DRCC-BASED ORPD MODEL WITH WASSERSTEIN DISTANCE

A. Ambiguity Set with Wasserstein Distance

In the real-life application, the real probability distribution \(\mathbb{P}\) for random variables \(\hat{\xi}\) is usually unknown, so we construct an empirical distribution \(\hat{\mathbb{P}}_N = \sum_{n=1}^{N} \delta_{\hat{\xi}^{(n)}} / N\) as an estimation of the true \(\mathbb{P}\), using the historical sample set \(\{\hat{\xi}^{(1)}, \hat{\xi}^{(2)}, ..., \hat{\xi}^{(N)}\}\) without any assumption of \(\mathbb{P}\). Here, \(\delta_{\hat{\xi}^{(n)}}\) is the Dirac distribution concentrating unit mass at \(\hat{\xi}^{(n)}\), and these samples are the forecasting errors of the wind power outputs. Then the Wasserstein distance can be used to measure the distance between the empirical probability \(\hat{\mathbb{P}}_N\) and the true probability \(\mathbb{P}\), given by Definition 1.

Definition 1 (Wasserstein distance) [22], [23]: for any probability distribution \(Q_1, Q_2 \in P(\mathcal{E})\), \(P(\mathcal{E})\) denotes the set of all probability distributions with support \(\mathcal{E}\); the Wasserstein distance can be defined as:

\[
W(Q_1, Q_2) = \inf_{\Pi} \left( \int_{\mathcal{E}^2} \| \xi_1 - \xi_2 \|_1 \Pi(d\xi_1, d\xi_2) \right)
\]

where \(\Pi\) is a joint distribution of \(\xi_1\) and \(\xi_2\), with marginals \(Q_1\) and \(Q_2\); \(\| . \|_1\) is a norm operator in \(\mathbb{R}^*\) used in this paper. Accordingly, we have \(W(\hat{\mathbb{P}}_N, \mathbb{P}) \leq \epsilon\) where \(\epsilon\) is some sample-dependent monotone function. In our data-driven frame, given a historical sample set with \(N\) samples, the true distribution \(\mathbb{P}\) will be included in the following ambiguity set:

\[
\hat{\mathbb{P}}_N = \left\{ \mathbb{P} \in P(\mathcal{E}) : W(\mathbb{P}, \hat{\mathbb{P}}_N) \leq \epsilon(N) \right\}
\]
As shown in (22), the performance of the DRCC-based ORPD will heavily rely on the radius $\varepsilon(N)$ of the Wasserstein ball. Several possible choices for the radius are given in [40]-[42], and a radius from [42] is selected in this study, which can be represented as:

$$\varepsilon(N)=C \sqrt{\frac{1}{N} \log \left(\frac{1}{1-\beta}\right)}$$  \hspace{1cm} (23)

where $\beta$ is the confidence level; and $C$ is the diameter of the support of the random variable that can be written as (24), where $\bar{\mu}$ is the sample mean and the minimization over $\alpha_i$ can be denoted by bisection search method.

$$C=2 \inf_{\alpha_i \geq \mu} \left( \frac{1}{2\alpha_i} \left( 1 + \ln E_x \left( e^{\|X - \bar{\mu}\|^2} \right) \right) \right)^{1/2} \leq 2 \inf_{\alpha \in \mathbb{R}} \left( \frac{1}{2\alpha} \left( 1 + \ln E_x \left( e^{\alpha |X - \bar{\mu}|} \right) \right) \right)^{1/2} \approx 2 \inf_{\alpha \in \mathbb{R}} \left( \frac{1}{2\alpha} \left( 1 + \ln \left( \frac{\mu}{\sigma} + \left( \frac{1}{\sigma^2} \right) \right) \right) \right)^{1/2} \leq 2$$  \hspace{1cm} (24)

**B. Convex Reformulation of Chance Constraints**

Considering a more general form of (17)-(20) as:

$$\inf_{x \in P(x)} \mathbb{P}(g(x, \bar{x}) \leq 0) \geq 1 - \rho$$  \hspace{1cm} (25)

where $g$ is linear with both the state variable $x$ and random variable $\bar{x}$; and $\rho$ is the tolerable violation probability.

In fact, the chance constraint is usually non-convex so that it is hard to find an equivalent formulation which is solvable. A feasible way is to find a convex conservative formulation of (25) as follows.

Firstly, find a deterministic ambiguity set that meets the robust constraint, for the random variable $\bar{x}$ as:

$$g(x, \bar{x}) \leq 0 \ \forall \bar{x} \in U$$  \hspace{1cm} (26)

For a given historical sample set $\{\bar{x}^{(1)}, \bar{x}^{(2)}, \ldots, \bar{x}^{(n)}\}$, it is easy to compute the sample mean $\bar{\mu}$ and sample covariance $\Sigma$. Then the standardised version $\tilde{\bar{\mu}} = \Sigma^{-1/2}(\bar{x} - \bar{\mu})$ of random variable $\bar{x}$ with sample set $\{\tilde{x}^{(1)}, \tilde{x}^{(2)}, \ldots, \tilde{x}^{(n)}\}$ can be obtained. Thus $\tilde{\bar{x}}$ has the sample mean as 0, sample covariance as $I$, and support as $\Theta = [-\sigma_{max}, \sigma_{max}, \ldots]$. Then, another set of $n \in \mathbb{R}^n$ needs to be found for random variable $\bar{\theta}$ that meets:

$$\sup_{Q = \mathcal{Q}_x} \mathbb{Q}(\tilde{\theta} \notin V) \leq \rho$$  \hspace{1cm} (27)

where $\mathcal{Q}$ and $\mathcal{Q}_x$ are the true distribution and empirical distribution of $\tilde{\theta}$, respectively, with ambiguity set $\mathcal{Q}_x$ constructed using (22). As a result, $U = \Sigma^{1/2}V + \tilde{\mu}$ can be utilised as a possible ambiguity set for (26). Considering the rules of sample independence and equal variance among different components of $\tilde{\theta}$, we restrict $V$ within the hypercube (28) so as to find out $V$ more efficiently.

$$V(\sigma) = \mathbb{R}^n - \sigma(1 < \tilde{\theta} < \sigma 1)$$  \hspace{1cm} (28)

Secondly, to reduce conservatism, $\sigma$ needs to be as small as possible:

$$\begin{align*}
\min_{\sigma \in [0, \sigma_{max}]} & \quad \mathbb{Q}(\tilde{\theta} \notin V(\sigma)) \\
\text{s.t.} & \quad \sup_{Q = \mathcal{Q}_x} \mathbb{Q}(\tilde{\theta} \notin V(\sigma)) \leq \rho
\end{align*}$$  \hspace{1cm} (29)

Using Lemma 1 in Appendix A, (29) is equivalent to:

$$\begin{align*}
\min_{\sigma \in [0, \sigma_{max}]} & \quad h(\sigma, \lambda) \\
\text{s.t.} & \quad h(\sigma, \lambda) \leq \rho
\end{align*}$$  \hspace{1cm} (30)

where $h(\sigma, \lambda) = -c + \sum_{i=1}^{n} \ln \left(1 - \lambda \left(1 - \|\tilde{\theta}^{(i)}\|_2\right)^{1/2}\right)$, $(x)^+ = \max(x, 0)$.

As shown in Lemma 1, (A1) is non-decreasing in $\sigma$. Therefore, (30) has a unique solution. The optimal solution can be found quickly by a nested bisection search method shown in Appendix C.

After determining the optimal $\sigma$, the hypercube $V(\sigma)$ can be expressed as the convex hull of its vertices. Then the hypercube (28) can be obtained as $V(\sigma) =$ conv $\left(\{\tilde{\theta}^{(1)}, \tilde{\theta}^{(2)}, \ldots, \tilde{\theta}^{(n)}\}\right)$, specially, $V(\sigma) =$ conv $\left(\{0, \sigma, \ldots, 0, \sigma\}\right)$ for 1-dimensional random variable, and $V(\sigma) =$ conv $\left(\{\pm 0, \pm \sigma, \ldots\}\right)$ for 2-dimensional random variable. Accordingly, the constituted ambiguity set can be expressed as:

$$U = \text{conv} \left(\{u^{(0)}, u^{(2)}, \ldots, u^{(n)}\}\right)$$  \hspace{1cm} (31)

where $u^{(i)} = \tilde{\Sigma}^{1/2} u^{(0)} + \tilde{\mu}, 1 \leq i \leq 2^n$. Then, (25) is equivalent to a deterministic expression:

$$g(x, u^{(i)}) \leq 0 \quad 1 \leq i \leq 2^n$$  \hspace{1cm} (32)

Equation (32) is a set of linear constraints with the state variable $x$. Thus, (17)-(20) can all be replaced by their corresponding deterministic linear forms using (32).

**C. Reformulation of Objective Function**

After the chance constraints being reformulated to a solvable form, there is still an obstacle $\sup_{x \in P(x)}$ in the objective function of the DRCC-based ORPD model. The objective function (14) inside the $\sup_{x \in P(x)}$ can be rewritten as:

$$l(x, \tilde{\omega}) = c_1 \tilde{\omega}^2 + c_1 \tilde{\omega} + c_0 \tilde{\omega} - 1^{T} \tilde{x}$$  \hspace{1cm} (33)

Note that the objective function would be a convex quadratic function of $x$ and (33) is also a convex quadratic function of $\tilde{\omega}$. Given a sample set $\{\tilde{\omega}, \tilde{\omega}_1, \ldots, \tilde{\omega}_n\}$ and its support $[\omega, \tilde{\omega}]$, using Lemma 2 in the Appendix B and let $\lambda = \max \{l'(x, \tilde{\omega}), -l'(x, \omega)\}$ with $l'(x, \tilde{\omega}) = 2c_2 \tilde{\omega} + c_1$, then:

$$\begin{align*}
l(x, \omega) + \lambda (\omega - \omega) & \leq l(x, \omega) \quad \forall \omega \in [\omega, \tilde{\omega}] \\
l(x, \tilde{\omega}) - \lambda (\tilde{\omega} - \omega) & \leq l(x, \tilde{\omega}) \quad \forall \omega \in [\omega, \tilde{\omega}]
\end{align*}$$  \hspace{1cm} (34)

A close upper approximate of the worst-case evaluation of the cost in (14) can be rewritten as:

$$\begin{align*}
\sup_{x \in P(x)} \left\{l(x, \tilde{\omega})\right\} \leq \inf_{i \in [1, \hat{N}]} \left(\frac{1}{N} \sum_{i=1}^{n} l_i(x, \tilde{\omega})\right) \\
\text{s.t.} & \quad l'(x, \tilde{\omega}) \leq \lambda, -l'(x, \omega) \leq \lambda
\end{align*}$$  \hspace{1cm} (35)

Equation (35) can be used as the worst-case cost with good computation performance, because the number of con-
strains and decision variables of (35) remain unchanged when using a larger historical data set in our WDRO method.

IV. NUMERICAL TESTS

A. Description of Two Test Systems

In the case study, the proposed DRCC-based ORPD model and the WDRO method are tested on both IEEE 30-bus system and IEEE 123-bus distribution system [43]. YALMIP is used as the modelling tool and Gurobi 7.5.2 as the solver, running on a 12-core 2.4 GHz Workstation.

The IEEE 30-bus system consists of 6 generators and 41 branches. The total load of the system is 189.2 MW. Five wind farms are connected to buses 3, 7, 17, 20, 24, respectively, and the capacity of each wind farm is set to 30 MW. The forecasted values of wind power output are assumed to be 50% of their capacities. Two transformers are installed at branches 11 and 16, respectively, whose ranges of TR are both [0.95, 1.05] with the step size of 0.01. And two SCRs are installed at buses 7 and 26, respectively, whose capacities are both [−0.12, 0.12] p.u. with the step size of 0.01 p.u.. The base is 100 MVA.

For the IEEE 123-bus distribution system, the total load is 3490 kW. Ten wind farms are connected to buses 5, 16, 29, 33, 46, 59, 64, 71, 75, 79, respectively, with 240 kW wind power capacity for each wind farm. The forecasted values of wind power output are also assumed to be 50% of their capacities. There are one transformer installed at the substation whose range of tap ratio is [0.95, 1.05] with the step size of 0.01. And there are four SCRs installed at buses 12, 35, 54 and 108, whose capacities are [−0.006, 0.006] p.u. with the step size of 0.002 p.u.. The base is 1000 kVA. Besides, numerical tests on other three-benchmark approaches are performed to compare with the proposed model.

1) RO: it requires (17)-(20) to be satisfied for all possible scenarios of the random variable.

2) SP: it presumes that the random variable follows Gaussian distribution with a pre-given mean and covariance, and (17)-(20) are formulated as SOCP constraints by using inequality \( \mathbb{P}\left\{ \left| \xi - \mu \right| \geq \Phi^{-1}\left(1 - \rho/2\right)\sigma \right\} \leq \rho \), where \( \Phi \) is the cumulative distribution function of standard Gaussian random variable.

3) MDRO: the ambiguity set of MDRO is a set of probability distribution with a pre-given mean and variance, then (17)-(20) are formulated as SOCP constraints by using inequality \( \mathbb{P}\left\{ \left| \xi - \mu \right| \geq \sqrt{1/\rho} \sigma \right\} \leq \rho \).

In this paper, the tolerable violation probability in (17)-(20) are set to \( \rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.05 \), and the confidence level in (23) is set to \( \beta = 0.9 \). According to [44], Laplace distribution is used to generate realistic historical data whose sizes are ranging from 10\(^3\) to 10\(^6\), with typical standard forecasting error variance [45]. Then, we use a set of 10\(^6\) samples to estimate the simulated costs of the true distribution, so as to test the practical and out-of-sample performance of the corresponding method.

B. IEEE 30-bus System

1) The accuracy of the proposed model: the proposed DRCC-based ORPD model combines an SOCP model in the nominal operation mode and an LPF model to reflect the system response under uncertainties. Therefore, the accuracy of the proposed model should lie between the SOCP-based model and the LPF-based model with deterministic power flow. The comparisons of operation costs in different models are shown in Fig. 1, when the total wind power output forecasting error varies deterministically from −30 MW to 30 MW. The results demonstrate that the operation costs of the proposed model lie exactly between the SOCP model and the LPF model. And when the total forecasting error is −30 MW, the percentage difference between the proposed model and the SOCP model reaches a maximum value of −0.43%. Besides, when the total forecasting error is zero, the proposed model coincides with the SOCP model.

In addition, the proposed model has much higher accuracy than the LPF model in minimizing the operation costs with different total forecasting errors, which are depicted clearly in Fig. 1. Besides, the comparisons in Table I and Fig. 2 further presents the accuracy of the proposed model in voltage magnitudes and reactive power outputs, with the total forecasting error of −30 MW. Therefore, the proposed model has a considerably high accuracy as the SOCP model and a mathematical tractability as the LPF model, which makes it more attractive for ORPD under uncertainties.

2) Effectiveness of reactive power-related chance constraints: the optimization results of the proposed model with
different chance constraints are shown in Table I. The results show that compared with the complete proposed model to ensure a safe operation under uncertainties.

3) Comparison of discrete reactive power variables at different penetration levels of wind power: the optimized results of the discrete reactive power variables at different penetration levels of wind power are shown in Table III and Table IV. Under low wind power penetration condition (wind power output is $P_w = 5$ MW, with a maximum load of 189.2 MW), the reactive power compensators, i.e., SCR1, SCR2, TR1, stay nearly unchanged as the forecasting error variance increases from 0.03 p.u. to 0.12 p.u., because such low wind power penetration would not induce much voltage fluctuations in the system. However, the situation varies with higher penetration of wind power. For higher wind power penetration ($P_w = 50$ MW), the transformer taps change significantly with the a small step size of 0.01 p.u. The operation statuses of SCRs also vary significantly with small step size,

el, the active power of generators in the model without (18) does not have much difference. But the reactive power related variables, i.e., the discrete control devices TR1, TR2, SCR1 and the continuous reactive power output of generators varies a lot, which will make the voltages at generator buses closer to their upper or lower limits. For example, the voltage magnitudes of G3 and G6 increase to 1.043 p.u. and 1.041 p.u., respectively, which are under potential risk of over-voltage; and the voltage magnitude of G4 decreases to 0.967 p.u., which may violate the under-excitation limit. For another comparison, if the reactive power chance constraints (20) is excluded from the complete proposed ORPD model, the active power does not have much difference. However, the active power of generators, e.g., G1, G2, G3, G6 decreases, which might lead to the decrease of voltage magnitudes at the corresponding generator buses. The consequence of this change makes the voltage magnitude of G3 under a risk of violating the under-excitation limit. The results show that (18) and (20) are significant in the DRCC-based ORPD model to ensure a safe operation under uncertainties.

| Model                | Generator No. | $P_w$ (p.u.) | $Q_w$ (p.u.) | Voltage (p.u.) | SCR1 capacity (p.u.) | SCR2 capacity (p.u.) | TR1 (p.u.) | TR2 (p.u.) |
|----------------------|---------------|--------------|--------------|----------------|----------------------|----------------------|------------|------------|
| Complete proposed model | G1            | 0.308        | 0.186        | 0.999          | 0.09                 | 0.09                 | 1.02       | 0.96       |
|                      | G2            | 0.434        | 0.219        | 0.989          |                      |                      |            |            |
|                      | G3            | 0.079        | 0.155        | 0.961          |                      |                      |            |            |
|                      | G4            | 0.186        | 0.055        | 0.977          |                      |                      |            |            |
|                      | G5            | 0.073        | 0.110        | 0.997          |                      |                      |            |            |
|                      | G6            | 0.087        | 0.195        | 1.040          |                      |                      |            |            |
| Without voltage chance constraint | G1            | 0.307        | 0.183        | 0.996          |                      |                      |            |            |
|                      | G2            | 0.433        | 0.215        | 0.986          |                      |                      |            |            |
|                      | G3            | 0.078        | 0.148        | 1.043          |                      |                      |            |            |
|                      | G4            | 0.185        | 0.055        | 0.967          |                      |                      |            |            |
|                      | G5            | 0.074        | 0.110        | 0.989          |                      |                      |            |            |
|                      | G6            | 0.090        | 0.200        | 1.041          |                      |                      |            |            |
| Without reactive power chance constraint | G1            | 0.308        | 0.177        | 0.997          |                      |                      |            |            |
|                      | G2            | 0.434        | 0.208        | 0.988          |                      |                      |            |            |
|                      | G3            | 0.079        | 0.151        | 0.951          |                      |                      |            |            |
|                      | G4            | 0.186        | 0.065        | 0.977          |                      |                      |            |            |
|                      | G5            | 0.076        | 0.136        | 1.001          |                      |                      |            |            |
|                      | G6            | 0.085        | 0.170        | 1.037          |                      |                      |            |            |
whereas SCRs with bigger step size of 0.02 p.u. only change when the forecasting error variance increases to a big enough value. In summary, no matter with high or low penetration of wind power, the reactive power compensators respond accurately under uncertainties, which verifies the effectiveness and accuracy of the proposed model.

| Method | Objective cost | Simulated cost |
|--------|---------------|----------------|
| RO     | 6.8754        | 6.8683         |
| Proposed method (10^3) | 5.8452 | 5.8134 |
| Proposed method (10^5) | 5.4835 | 5.4748 |
| Proposed method (10^6) | 5.3238 | 5.3207 |
| Proposed method (10^7) | 5.2511 | 5.2498 |
| MDRO   | 5.7035        | 5.6964         |
| SP     | 5.1704        | 5.1634         |

C. IEEE 123-bus Distribution System

The performances and advantages of the WDRO method on the proposed DRCC-based ORPD model are further compared with other three-benchmark approaches on the IEEE 123-bus distribution system.

1) Method conservatism and reliability comparison: the comparisons of objective operation costs on IEEE 123-bus distribution system are shown in Table V. Among the simulated costs of different approaches, we can sort the method conservatism as: RO > WDRO (10^3) > MDRO > WDRO (10^4, 10^5, 10^6) > SP. In addition, it can be seen clearly from Fig. 3 that the simulated cost of RO is the highest and that of SP is the lowest, due to the fact that RO ignores most of the probabilistic information while SP assumes the precise knowledge about the presumed true probability distribution. In other words, RO outputs the most conservative solutions and SP provides the most optimistic solutions. Since MDRO assumes partial knowledge, namely the first- and second-order moments of the probability distribution, it certainly reduces conservatism to some extent compared with RO. However, MDRO is still too conservative.

![Fig. 3. Comparisons of simulated costs among different methods.](image)

All the simulated costs of the three conventional approaches are nearly irrelevant with the quantity of data, except the proposed WDRO method. Since the proposed WDRO fully relies on the available historical data, the WDRO will provide a conservative solution as RO when it is short of data. And it can get closer to the result of the SP approach when more historical data is available. Although SP obtains the lowest simulated costs, it fails to guarantee the reliability level of the security constraints, which can be seen in Fig. 4.

This is because the true probability distribution would always be different from the presumed Gaussian distribution used in the SP approach. While all the other tested approaches, including RO, MDRO and the proposed WDRO, are able to ensure higher reliability level (>95%) due to the robust nature.

2) Data-driven characteristic of the WDRO method: as analyzed above, all the simulated costs of the other three approaches are nearly irrelevant with the quantity of the data samples, except the proposed WDRO method. In fact, the WDRO method would safely reduce the reliability level to a slight extent when more data is available. The objective oper
Fig. 4. Comparisons of reliability among different methods.

Fig. 5. Percentage differences between objective function and simulated costs with increasing data samples.

3) Comparison of computation performance: the comparison of the computation performance for all the methods is shown in Table VI, and the wind capacity is 10×240 kW. The calculation of the whole WDRO method consists of two parts. One part is the construction of $U$ in (26) with available historical data, which can be completed before optimization. It can be concluded from the IEEE 123-bus distribution system case that the computation time of uncertainty set does not increase with the order of magnitude of the available dataset. This characteristic guarantees that the proposed WDRO method stays effective when more data is available.

The other part of the calculation is the optimization process. The computation time of WDRO is almost the same as RO approach, and much less than those of SP and MDRO methods. The proposed WDRO method reformulates the chance constraints into a bunch of linear constraints, which is similar to RO method. However, MDRO and SP methods both reformulate the chance constraints into SOCP constraints that induce larger computation burden. Most importantly, the computation time of WDRO is not sensitive to the quantity of historical data, namely that the solver time would not increase when more historical data are used, which further demonstrates better computation performance of the proposed WDRO method.

V. CONCLUSION

This paper proposes a DRCC-based ORPD model under uncertainties, whose ambiguity set is constructed by Wasserstein distance. Different from the conventional ORPD model, the proposed model is a combination of an exact SOCP model in nominal operation mode and an LPF model to reflect the system response under uncertainties.

1) Numerical case studies on IEEE 30-bus system demonstrate that the proposed model largely inherits both the accuracy of exact SOCP model and the tractability of LPF model. The model is also able to deal with discrete control variables such as transformer tap ratios and switchable capacitors/reactors, which ensures a reliable and efficient ORPD under uncertainties from volatile RES.

2) A WDRO method is proposed only based on historical data without any assumption on the specific probability distribution of the uncertainties. When more historical data are available, the solution will become less conservative to guarantee the required reliability level of security constraints.

3) Numerical studies on IEEE 123-bus distribution system further verifies the DRCC-based ORPD model and the WDRO method, by comparing the conservatism, reliability, data-driven characteristic and computation performance to those of the other three approaches. Compared with RO and SP approaches, the proposed WDRO method can robustly give a less conservative solution. Compared to MDRO, WDRO can extract more information of the true probability distribution by directly using the historical dataset. Besides, the proposed WDRO is able to guarantee fast computation performance as RO, which is better than MDRO and SP approaches, and stays effective when large number of histori-
cal data are available.

APPENDIX A

The underlying formulation is developed in [26].

Lemma 1:

$$\sup_{\sigma \in \mathbb{R}^N} \mathcal{Q} \left( \hat{\sigma} \notin V(\sigma) \right) = \inf_{\lambda \geq 0} \left\{ \lambda \epsilon + \frac{1}{N} \sum_{k=1}^{N} \left[ 1 - \lambda \left( \sigma - \frac{1}{N} \sum_{i=1}^{N} s_i \right) \right] \right\} \quad (A1)$$

APPENDIX B

The underlying formulation is developed in [24].

Lemma 2: given a random variable $\tilde{\sigma} \in \mathbb{R}^N$ with closed and convex support $\Xi$, the Wasserstein ball $\mathcal{B}_\rho (\tilde{\mathcal{P}})$ is constructed from sample set $\left\{ \tilde{\sigma}_1, \tilde{\sigma}_2, \ldots, \tilde{\sigma}_N \right\}$. If the loss function $l(x, \tilde{\sigma})$ is upper semi-continuous, the worst-case expectation is as:

$$\sup_{\sigma \in \mathbb{R}^N} \mathbb{E} \left( l(x, \tilde{\sigma}) \right) = \inf_{\tilde{\sigma} \in \mathcal{B}_\rho (\tilde{\mathcal{P}})} \mathbb{E} \left( l(x, \tilde{\sigma}) \right) \quad (B1)$$

APPENDIX C

The method is summarised in Algorithm 1 in which the function bisearch $f(\cdot) \cdot a, b)$ returns the minimum of $f(\cdot)$ in the interval $[a, b]$ by performing a bisection search. Note that $h(\alpha, \lambda)$ is convex in $\lambda$ for a fixed $\alpha$, so the bisection search in Step 4 of Algorithm 1 is well-defined. Since Algorithm 1 only involves function evaluations, it efficiently solves the problem (30).

Algorithm 1 Nested bisection search
1: Initialize $\sigma = 0, \hat{\sigma} = \sigma_{\text{max}}$;
2: while $(\hat{\sigma} - \sigma > 10^{-4})$ do
3: $\sigma = (\hat{\sigma} + \sigma) / 2$;
4: $\gamma = \text{bisearch} (h(\cdot, \cdot), 0, 100)$;
5: if $\gamma > \rho$ then
6: $\sigma = \gamma$;
7: else
8: $\hat{\sigma} = \sigma$;
9: end if
10: end while
11: Output $\sigma = \hat{\sigma}$.

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