Inflation from fermions with curvature-dependent mass

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A model of inflation realization driven by fermions with curvature-dependent mass is studied. Such a term is derived from the Covariant Canonical Gauge Theory of gravity (CCGG) incorporating Dirac fermions. We obtain an initial de Sitter phase followed by a successful exit, and moreover we acquire the subsequent thermal history, with an effective matter era, followed finally by a dark-energy epoch. This behavior is a result of the effective “weakening” of gravity at early times, due to the increased curvature-dependent fermion mass. Investigating the scenario at the perturbation level, using the correct coupling parameter, the scalar spectral index and tensor-to-scalar ratio are obtained in agreement with Planck observations. Moreover the BBN constraints are satisfied too. The efficiency of inflation from fermions with curvature-dependent mass, at both the background and perturbation level, reveals the capabilities of the scenario and makes it a good candidate for the description of nature.

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I. INTRODUCTION

The developments in cosmology have been influenced to a great extent by the idea of inflation [1–7], which provides an attractive scenario for the solution of the fundamental puzzles of the old Big Bang paradigm, such as the horizon and the flatness problems. Additionally, inflation was proved crucial in providing a framework for the generation of primordial density perturbations [8, 9]. Although the inflationary scenario is very attractive, it has been recognized that a successful implementation requires special restrictions on the underlying dynamics. The inflationary mechanism can be achieved in several different ways, considering primordial scalar fields [10, 11] or geometric corrections into the effective gravitational action [12].

On the other hand, it is known that Hamiltonian formulations of a theory may have theoretical advantages. One such framework is the covariant canonical gauge theory of gravity (CCGG) [13]. The CCGG ensures by construction that the action principle is maintained in its form by requiring that all transformations of a given system are canonical. The imposed requirement of invariance of the original action integral with respect to local transformations in curved spacetime is achieved by introducing additional degrees of freedom, namely the gauge fields [13]. In these lines, in [14–17] quadratic Riemann theories by the covariant Hamiltonian approach were formulated, which were shown to lead to inflationary models [18] based on the correspondence between the metric affine (Palatini) formalism and the metric formalism [15, 16].

The covariant Hamiltonian formulation was recently implemented to include Dirac fermions [19]. This covariant Hamiltonian incorporates additional terms with a new dependence for fermions, namely the effective mass depends linearly on the curvature through

\[ m_{\text{eff}} = m_0 + \frac{\xi}{6\kappa^2} R, \]

where \( m_0 \) is the usual fermion rest mass, \( \xi \) is the coupling to the Ricci scalar \( R \) with dimensions \([L]^3\), and \( \kappa^2 = 8\pi G_N \). For \( \xi \rightarrow 0 \), the covariant Hamiltonian approach for fermions reduces to that of standard Dirac fermions.

The curvature-dependence mass of the fermions is expected to be significant in regimes where the curvature is significant, such as close to black holes [19] or new description for Neutrino [20]. However, one can deduce that this curvature-dependent correction can be important also in the early universe, where it is known that the Ricci scalar acquires large values. Hence, it would be interesting to investigate the effect of the fermion
curvature-dependent mass in the early universe. In particular, we desire to examine whether it can drive a successful inflation. Indeed, as we will see, such a novel coupling can both drive a successful inflation at the background level, accompanied by a successful exit and the subsequent thermal history of the universe, but it can also be very efficient at the perturbation level, giving rise to a scalar spectral index and a tensor-to-scalar ratio in agreement with observations.

The plan of the work is the following: In Section II we present fermionic cosmology with curvature-dependent mass, extracting the equations of motion. In Section III we apply the scenario in the early universe, obtaining the inflation realization at the background level. In Section IV we examine the perturbation-related observables such as the spectral index and the tensor-to-scalar ratio. Finally, in Section V we summarize our results.

II. THE MODEL

In this section we construct the scenario of fermionic cosmology with curvature-dependent mass. We first briefly review the basics of the covariant Hamiltonian incorporation of fermions in curved spacetime, and then we present the Lagrangian of the model, extracting the equations of motion.

A. Spinors in curved spacetime

Fermionic sources in general relativity were studied in detail in [21–23], especially for cosmological applications [24–31]. The tetrad formalism was used to combine the gauge group of general relativity with a spinor matter field. The tetrad $e^a_\mu$ and the metric $g_{\mu\nu}$ tensors are related through

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}, \quad a, b = 0, 1, 2, 3,$$

(2)

with Latin indices referring to the local inertial frame endowed with the Minkowski metric $\eta_{ab}$, while Greek indices denote the (holonomic) basis of the manifold.

The spinor field Lagrangian in curved torsion-free space-time reads as

$$\mathcal{L}_f = \frac{i}{2} [\bar{\psi} \Gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \Gamma^\mu \psi] - m_{\text{eff}} \bar{\psi} \psi,$$

(3)

where $\bar{\psi} = \psi^+ \gamma^0$ is the adjoint spinor field and $m$ is the fermionic rest mass, as defined in (1). In curved space time the Dirac matrices are replaced by their generalized counterparts $\Gamma^\mu = e^a_\mu \gamma^a$, which satisfy the extended Clifford algebra $\frac{1}{2} (\Gamma^\mu \Gamma^\nu + \Gamma^\nu \Gamma^\mu) = g^{\mu\nu}$. Thus, the ordinary derivatives are replaced by their covariant versions

$$D_\mu \psi = \partial_\mu \psi - \Omega_\mu \psi,$$

(4)

Furthermore, the metric compatibility condition implies that the spin connection $\Omega_\mu$ is given by

$$\Omega_\mu = \frac{i}{4} g_{\beta\nu} \left[ \bar{\Gamma}_\alpha^\nu - \partial_\nu e^\alpha_\mu \right] \Gamma_\beta^\alpha,$$

(5)

with $\bar{\Gamma}_\sigma^\mu$ the Christoffel symbols. The original CCGG formulation assumes that the affine and spin connections are fields independent of the metric (i.e. the metric-affine or the Palatini formulation) [19], however in this work we use the metric compatibility derived from the action, that allows for the substitution (5).

The action of fermions in the gravitational background of general relativity is then

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} (\mathcal{R} - 2\Lambda) + \mathcal{L}_f \right],$$

(6)

where $\mathcal{R}$ is the Ricci scalar and $\Lambda$ the cosmological constant. Note that concerning the fermions we have assumed a minimal coupling with gravity, while their inertial mass $m_{\text{eff}}$ is given by (1). Here, rather then applying the complete CCGG action, we simplify the analysis in order to illuminate the impact of the effective spinor mass in the conventional Einstein-Hilbert theory in the metric formulation.

B. Field equations

Let us now extract the field equations of the action (6). Variation with respect to the spinor field yields the generalized Dirac equations:

$$i\Gamma^\mu D_\mu \psi - m_{\text{eff}} \psi = 0$$
$$iD_\mu \bar{\psi} \Gamma^\mu + m_{\text{eff}} \bar{\psi} = 0,$$

(7)

Moreover, variation with respect to the metric leads to the field equations

$$\frac{1}{\kappa^2} G^{\mu\nu} = T^{\mu\nu}_f + \frac{\xi}{3\kappa^2} \left[ G^{\mu\nu} \bar{\psi} \psi + \Box^{\mu\nu} (\bar{\psi} \psi) \right] + g^{\mu\nu} \Lambda \kappa^2,$$

(8)

where $G^{\mu\nu}$ is the Einstein tensor, and $\Box^{\mu\nu} = g^{\mu\nu} - \nabla_\mu \nabla_\nu$. In the above equations $T^{\mu\nu}_f$ is the kinetic part of the spinor fields, given by

$$T^{\mu\nu}_f = \frac{i}{4} \left[ \bar{\psi} \Gamma^{(\mu} D^{\nu)} \psi - D^{(\nu} \bar{\psi} \Gamma^{\mu)} \psi \right]$$
$$- g^{\mu\nu} \left\{ \frac{i}{2} [\bar{\psi} \Gamma^\Lambda D_\Lambda \psi - D_\Lambda \bar{\psi} \Gamma^\Lambda \psi] - m_{\text{eff}} \bar{\psi} \psi \right\},$$

(9)

Equation (8) can be re-written as

$$\frac{1}{\kappa^2} \left( 1 - \frac{\xi}{3} \bar{\psi} \psi \right) G^{\mu\nu} = \frac{i}{4} \left[ \bar{\psi} \Gamma^{(\mu} D^{\nu)} \psi - D^{(\nu} \bar{\psi} \Gamma^{\mu)} \psi \right]$$
$$- g^{\mu\nu} \left\{ \frac{i}{2} [\bar{\psi} \Gamma^\Lambda D_\Lambda \psi - D_\Lambda \bar{\psi} \Gamma^\Lambda \psi] - m_{\text{eff}} \bar{\psi} \psi \right\}$$
$$- \frac{\xi}{3\kappa^2} \Box^{\mu\nu} (\bar{\psi} \psi) + g^{\mu\nu} \Lambda \kappa^2.$$

(10)
Finally, note that the phase invariance of the spinor field
\[ \psi \rightarrow e^{i\theta} \psi, \quad \bar{\psi} \rightarrow e^{-i\theta} \bar{\psi}, \] (11)
through the Noether’s theorem leads to the conserved current
\[ j^\mu_\nu = 0, \] (12)
where \( j^\mu = \bar{\psi} \gamma^\mu \psi \), which proves convenient in simplifying our analysis.

III. BACKGROUND EVOLUTION

In this section we apply the above fermionic model at a cosmological framework, focusing on early-time universe, and in particular on the inflationary realization. As usual we neglect standard matter, i.e. we incorporate only the fermions with curvature-dependent mass. We consider the Friedman-Lemaitre-Robertson-Walker (FLRW) homogeneous and isotropic metric
\[ ds^2 = -dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2), \] (13)
with the scale factor \( a(t) \), and thus through (2) the tetrad components are found to be
\[ e^\mu_0 = \delta^\mu_0, \quad e^\mu_i = \frac{1}{a(t)} \delta^\mu_i. \] (14)
Moreover, the covariant version of the Dirac matrices are
\[ \Gamma^0 = \gamma^0, \quad \Gamma^i = \frac{1}{a(t)} \gamma^i, \] (15)
while the spin connection becomes
\[ \Omega_0 = \gamma^0, \quad \Omega_i = \frac{1}{2} \dot{a}(t) \gamma^i \gamma^0. \] (16)
The Dirac equation (7) then becomes
\[ \dot{\psi} + \frac{3}{2} H \psi + i m_{\text{eff}} \gamma^0 \psi = 0, \] (17)
and similarly for \( \bar{\psi} \), with
\[ m_{\text{eff}} = m_0 + \frac{\xi}{\kappa^2} (\dot{H} + 2H^2), \] (18)
and \( H = \dot{a}/a \) the Hubble function. For an isotropic and homogeneous universe the spinor field is exclusively a function of time, namely
\[ \psi = \begin{pmatrix} \psi_0(t) e^{i\phi(t)} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \] (19)
with \( \phi(t) \) the phase of the state, where we have considered the fermions to be at the minimal state. Introducing this ansatz into the generalized Dirac equation (17) yields the relation between the phase of the fermion and the effective mass, namely
\[ \frac{d}{dt} \phi(t) = -m_{\text{eff}}. \] (20)
An elegant way to extract the solution for the spinor field is by making use of the conserved current (12). For the FLRW metric the conserved current becomes
\[ \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} j^\mu) = 0 \implies \frac{1}{a^3} \partial_t (a^3 j^0) = 0, \] (21)
which then leads to a dust-like behavior for the absolute value of the spinor field, i.e.
\[ |\bar{\psi}\psi| = \psi_0(t)^2 = \frac{C}{a^3}, \] (22)
with the particle number density \( C > 0 \). The quantity \( m_0 C \) is thus the total gas energy density.

Let us now turn to the field equations (10). Inserting the FLRW metric (13) they give rise to the Friedmann equations
\[ -H^2 \xi \psi_0^2 + 3H^2 - 2H \xi \psi_0 \dot{\psi}_0 - \Lambda - \kappa^2 m_0 \psi_0^2 = 0, \] (23)
\[ \left( 3 - \frac{2C\xi}{a^3} \right) H^2 + \left( \frac{C\xi}{3a^3} + 2 \right) \dot{H} \]
\[ + \frac{C\kappa^2}{a^5} (H - 2m_0 + 2\phi) = \Lambda = 0. \] (24)
Combining (22) with (23) we obtain the convenient form
\[ \frac{3H^2}{\kappa^2} + \frac{2\dot{\xi}H^2}{\kappa^2 \alpha^3} = \frac{\rho_0}{a^3} + \Lambda, \] (25)
with the definitions \( \rho_0 = m_0 C \) and \( \dot{\xi} = C\xi \). Finally, since \( \kappa^2 = 8\pi G_N \), from Eq. (25) we deduce that the curvature-dependent fermion mass term induces an effective Newtonian constant
\[ \frac{G_{\text{eff}}}{G_N} = \frac{3\alpha^3}{3\alpha^3 + 2\xi}. \] (26)
We proceed by investigating the inflation realization in the above construction. From now on we set \( \kappa^2 = 8\pi G_N = 1 \). For small scale factors, such that
\[ a^3 \ll \frac{\rho_0}{\Lambda} \equiv s, \quad a^3 \ll \dot{\xi}, \] (27)
the modified Friedmann equation (25) accepts the de Sitter solution \( a \sim e^{H_{\text{inf}} t} \), with
\[ H_{\text{inf}} = \sqrt{\frac{\rho_0}{2\xi}}. \] (28)
Thus, we observe that the inflation phase is driven by the effectively dust-like fermionic component. However,
after a suitable inflationary expansion the scale factor increases significantly and the approximation (27) breaks down, leading to a decrease of the Hubble function and thus to a successful inflationary exit. At later times the dust-like terms of the Friedmann equation (25) will dominate, driving the matter epoch. Finally, at late-times, the cosmological constant term will dominate in (25) and the Hubble constant asymptotically approaches the value

$$H_\infty = \sqrt{\frac{\Lambda}{3}}.$$

(29)

This thermal history of the universe is in agreement with observations. However, we should mention here that a scenario fully consistent with the observed universe evolution and complete, should include a description of the post-inflationary phase of reheating, that could produce the radiation sector, which could then dominate and thus drive the necessary radiation era before the matter one. The detailed elaboration of this intermediate evolution lies beyond the scope of the present work and will be investigated in a separate project.

We now present a convenient way to extract the above results. In particular, we introduce an “effective potential” through the relation $\dot{a}^2 + V(a) = 0$ (i.e. the “kinetic energy” $\dot{a}^2$ and the potential energy $V(a)$ add up to zero), which using the Friedmann equation (25) leads to [35]

$$V(a) = -a^2 \Lambda \left( \frac{s + a^3}{2\xi + 3a^3} \right).$$

(30)

The effective potential is presented in Fig. 1. The initial point $V(a = 0) = 0$ incorporates time scales beyond Planck time, and thus our discussion starts at point A, which corresponds to the inflationary initial scale factor $a_i$ (which is calculated in the next section). The potential contains one minimal point B and one maximal point C, with the scale factors

$$a_{B,C}^3 = \frac{1}{12} \left( 3s - 10\xi \pm \sqrt{3s - 50\xi \sqrt{3s - 2\xi}} \right),$$

(31)

where the $+$ sign corresponds to point C. One can easily see that point B is the point where inflation ends, and

the universe enters the effective dust epoch. This phase holds up to point C, after which the universe enters into the late-time, dark energy regime.

We close this section by examining the behavior of the effective Newton’s constant $G_{\text{eff}}$ from (26), as well as the effective (curvature-dependent) fermion mass $m_{\text{eff}}$ from (18), which using the Friedmann equation (25) becomes

$$m_{\text{eff}} = \frac{3\xi}{2} \left[ 12a^6 + a^3(14\xi + 3s) + 8\xi s \right] + \frac{m_0 s}{f_0} a^3,$$

(32)

In Fig. 2 we depict their normalized evolution, for various values of $\xi$. As we observe, at early times the curvature-dependent fermion mass is small, which makes the effective Newton’s constant small too, and this effective “weakening” of gravity is the reason for the inflation realization. In particular, the effective Newtonian constant starts from 0 and results at $G_N$ (we mention that we can choose $\xi$ in order to obtain a small post-inflationary variation of $G_{\text{eff}}$, in order to be in agreement with the Big Bang Nucleosynthesis (BBN) constraints [36, 37]), while $m_{\text{eff}}$ starts from $\Lambda \left( 3s + \frac{\text{max}}{f_0} \right)$ and results to $2\Lambda \xi$, in agreement with the fact that fermionic matter is cold. Nevertheless, apart from the above BBN bound on $G_{\text{eff}}$ itself, we should check the full BBN constraints on the scenario, i.e. focusing on the ratio of the standard radiation to the above dark component. Although a complete analysis lies beyond the scope of the present work, we can explicitly check the compatibility of fermionic inflation within the standard BBN using the average bound on the possible variation of the BBN speed-up factor [42, 43]. First of all, since the model at hand includes only the fermionic sector, as we mentioned above if we want to additionally describe the radiation epoch (and hence the BBN phase) we need to include a reheating mechanism that could produce the radiation sector. For the purpose of the BBN analysis we assume that such a mechanism took place and thus at post inflationary universe the Friedmann equation (25) extends to

$$\frac{3H^2}{\kappa^2} + \frac{\dot{\rho}_{\text{eff}}}{\kappa^2a^3} = \frac{\rho_{f0}}{a^3} + \Lambda + \frac{\rho_{f0}}{a^4},$$

(33)

with $\rho_{f0}$ a parameter. Now, the BBN speed-up factor is defined as the ratio of the difference of the expansion rate predicted in a given model versus that of $\Lambda\text{CDM}$ with standard radiation at the BBN epoch, i.e. at $z_{\text{BBN}} \sim 10^9$ or at $a_{\text{BBN}} = \frac{1}{\sqrt{1+z_{\text{BBN}}}} \sim 10^{-9}$ (setting the current value $a_0 = 1$), namely $\Delta H^2/H_{\Lambda\text{CDM}}^2$ [42, 43]. Thus, in our case, using (33) (which for $\xi = 0$ coincides with $\Lambda\text{CDM}$) we obtain

$$\frac{\Delta H^2}{H_{\Lambda\text{CDM}}^2} = \frac{2\xi}{2\xi + a_{\text{BBN}}^3} < 10\%.$$
inflation (indeed in all the above figures $\tilde{\xi}$ has been chosen accordingly).

FIG. 2: The evolution of the normalized effective Newton’s constant (upper graph), and of the normalized curvature-dependent fermion mass (lower graph), for various values of $\tilde{\xi}$, for $\rho_f = 0.3$ and $\Lambda = 0.7$, in units where $\kappa^2 = 1$.

IV. PERTURBATIONS

In this section we investigate the perturbations of the above background evolution, and in particular we focus on the inflationary observables such as the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$. As usual, we introduce the Hubble slow-roll parameters $\epsilon = -\dot{H}/H^2$, $\eta = -\ddot{H}/(2H\dot{H})$ [38, 39], which in our case using the Friedmann equation (25) read

$$\epsilon(a) = \frac{3s}{2(a^3 + s)} - \frac{3\xi}{3a^3 + 2\xi}, \quad (35)$$

$$\eta(a) = \frac{3}{2} - \frac{6\xi}{3a^3 + 2\xi}. \quad (36)$$

Inflation ends when $\epsilon(a_f) = 1$, and as mentioned above using the “effective” potential one can see that $a_f = a_B$, with $a_B$ the scale factor value at the minimal point $B$ of Fig. 1. Thus, if the initial scale factor is $a_i$ then the number of $e$-folding is

$$N = \log \left( \frac{a_f}{a_i} \right). \quad (37)$$

Using the slow-roll parameters, one can calculate the values of the scalar spectral index and the tensor-to-scalar ratio respectively as [32, 33]

$$r = 16\epsilon = \frac{24s}{a_i^2 + s} - \frac{48\xi}{3a_i^3 + 2\xi}, \quad (38)$$

$$n_s = 1 - 6\epsilon + 2\eta = 4 - \frac{9s}{a_i^2 + s} + \frac{6\xi}{3a_i^3 + 2\xi}. \quad (39)$$

As we can see, the first slow-roll condition $0 < \epsilon \ll 1$ leads to

$$0 < a_i^3 \lesssim \frac{4\xi}{3}, \quad \frac{2\xi}{3} < s, \quad 0 \ll s, \quad (40)$$

while the second slow-roll condition $0 < \eta \ll 1$ yields

$$\frac{2\xi}{3} \leq a_i^3 \ll \frac{10\xi}{3}. \quad (41)$$

Hence, combining them we obtain the requirement

$$\frac{2}{3} \tilde{\xi} \lesssim a_i^3 < \frac{4}{3} \tilde{\xi}, \quad \frac{2}{3} \tilde{\xi} \lesssim s. \quad (42)$$

Finally, from (38),(39) we can obtain

$$r = \frac{8(4 - n_s)}{3} - \frac{32\xi}{3a_i^3 + 2\xi}. \quad (43)$$

As one can see, by suitably choosing the values of $\tilde{\xi}$ and $s$ we obtain $r$ and $n_s$ well inside the Planck observed values [34]. For instance, taking according to (42) that $a_i^3 \approx \frac{2}{3} \tilde{\xi}$ we obtain that $3r \approx 8(1 - n_s)$, which gives $n_s = 0.97$ and $r = 0.08$. This is one of the main results of the present work and reveals the capabilities of the scenario at hand, which can give a solution in agreement with observations, both at background and perturbation levels.

V. CONCLUSIONS

In this work we constructed a model of inflation realization, driven by fermions with curvature-dependent mass. We started from the Covariant Canonical Gauge Theory of gravity (CCGG) formulation, which imposes the requirement of invariance with respect to local transformations in curved spacetime through the insertions of gauge fields [13]. Incorporation of Dirac fermions in such a framework implies that the fermions acquire a curvature-dependent mass. The effect of such a correction is expected to be significant in regimes where the Ricci curvature is large, such is the case in the early
universe. In particular, the dark spinor dust is indeed repulsive due to the $m_{\text{eff}}$ term, as long as we are close to a singularity, either close to black holes or close to the Big Bang.

We first investigated the scenario at the background level. As we showed, we obtained the inflation realization, with an initial de Sitter phase followed by a successful exit. Furthermore, we acquired the subsequent thermal history, with an effective matter era, followed finally by a dark-energy epoch. This behavior is qualitatively expected, since as we showed the curvature-dependent fermion mass induces a smaller effective Newton’s constant at early times, and this effective “weakening” of gravity is the reason for the inflation realization.

We proceeded to the investigation of the scenario at the perturbation level, focusing on observables such as the scalar spectral index and the tensor-to-scalar ratio. We extracted analytical expressions for their values, and we showed that by suitably choosing the coupling parameter, we can obtain values in agreement with Planck observations. The efficiency of inflation from fermions with curvature-dependent mass at both the background and perturbation level reveals the capabilities of the scenario and makes it a good candidate for the description of nature.

One could try to investigate extensions of the above basic scenario. In particular, in this work we treated the spinor fields classically, assuming them to lie at the minimal state, namely being cold. Nevertheless, incorporation of high-temperature effects, as for example in the case of warm inflation [40, 41], could lead to the appearance of an additional friction term, and this could further improve the values of the spectral index and the tensor-to-scalar ratio. Another possible generalization is to consider a quadratic curvature dependence, alongside the linear one, in the fermion mass, or even more general functions of the form $m_{\text{eff}} = f(m, R)$.

We close this work by mentioning that in scalar field models of inflation the transition into the radiation and matter fields is realized by introducing additional fields, such as the curavton field. The exact way with which the inflationary fermions decay into other particles is an important question that should be investigated in a future project.

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[1] A. A. Starobinsky, JETP Lett. 30 (1979) 682 [Pisma Zh. Eksp. Teor. Fiz. 30 (1979) 719].
[2] D. Kazanas, Astrophys. J. 241, L59 (1980).
[3] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980) [Adv. Ser. Astrophys. Cosmol. 3, 130 (1987)].
[4] A. H. Guth, Phys. Rev. D 23 (1981) 347 [Adv. Ser. Astrophys. Cosmol. 3 (1987) 139].
[5] A. D. Linde, Phys. Lett. 108B, 389 (1982) [Adv. Ser. Astrophys. Cosmol. 3, 149 (1987)].
[6] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982) [Adv. Ser. Astrophys. Cosmol. 3, 158 (1987)].
[7] S. K. Blau, E. I. Guendelman and A. H. Guth, Phys. Rev. D 35 (1987) 1747.
[8] V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. 33, 549 (1981)].
[9] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49 (1982) 1110.
[10] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro and M. Abney, Rev. Mod. Phys. 69, 373 (1997) [astro-ph/9508078].
[11] B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. 78, 537 (2006) [astro-ph/0507632].
[12] S. Nojiri and S. D. Odintsov, Phys. Rev. D 68, 123512 (2003) [hep-th/0307288].
[13] J. Struckmeier, J. Muench, D. Vasak, J. Kirsch, M. Hanauske and H. Stoecker, Phys. Rev. D 95, no. 12, 124048 (2017) [arXiv:1704.07246 [gr-qc]].
[14] D. Benisty, E. I. Guendelman, D. Vasak, J. Struckmeier and H. Stoecker, Phys. Rev. D 98, no. 10, 106021 (2018) [arXiv:1809.10447 [gr-qc]].
[15] D. Benisty, E. I. Guendelman and J. Struckmeier, arXiv:1808.01978 [hep-th].
[16] D. Benisty and E. I. Guendelman, Phys. Rev. D 98, no. 4, 044023 (2018) [arXiv:1805.09667 [gr-qc]].
[17] D. Vasak, J. Kirsch, D. Kehm and J. Struckmeier, J. Phys. Conf. Ser. 1194, no. 1, 012108 (2019) [arXiv:1812.00578 [gr-qc]].
[18] R. Myrzakulov, S. Odintsov and L. Sebastiani, Phys. Rev. D 91, no. 8, 083529 (2015) [arXiv:1412.1073 [gr-qc]].
[19] J. Struckmeier, D. Vasak, A. Redelbach, P. Liebrich and H. Stoecker, arXiv:1812.09669 [gr-qc].
[20] R. Onofrio, neutrinophilic two-Higgs-doublet models, Phys. Rev. D 86 (2012) 087501 doi:10.1103/PhysRevD.86.087501 [arXiv:1310.5165 [hep-ph]].
[21] J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and G. Volkov, Gen. Rel. Grav. 32, 1777 (2000) [gr-qc/9911055].
[22] M. O. Ribas, F. P. Devecchi and G. M. Kremer, Phys. Rev. D 72, 123502 (2005) [gr-qc/0511099].
[23] L. L. Samojeden, F. P. Devecchi and G. M. Kremer, Phys.
Rev. D 81, 027301 (2010) [arXiv:1001.2285 [gr-qc]].

[24] R. Myrzakulov, arXiv:1011.4337 [astro-ph.CO].

[25] L. P. Chimento, F. P. Devecchi, M. I. Forte and G. M. Kremer, Class. Quant. Grav. 25, 085007 (2008) [arXiv:0707.4455 [gr-qc]].

[26] M. O. Ribas, F. P. Devecchi and G. M. Kremer, Mod. Phys. Lett. A 31, no. 06, 1650039 (2016) [arXiv:1602.06874 [gr-qc]].

[27] M. O. Ribas, F. P. Devecchi and G. M. Kremer, EPL 81, no. 1, 19001 (2008) [arXiv:0710.5155 [gr-qc]].

[28] M. Wali Hossain, R. Myrzakulov, M. Sami and E. N. Saridakis, Int. J. Mod. Phys. D 24, no. 05, 1530014 (2015) [arXiv:1410.6100 [gr-qc]].

[29] S. Basilakos, N. E. Mavromatos and J. Sol, Universe 2, no. 3, 14 (2016) [arXiv:1505.04434 [gr-qc]].

[30] C. Q. Geng, C. C. Lee, M. Sami, E. N. Saridakis and A. A. Starobinsky, JCAP 1706, no. 06, 011 (2017) [arXiv:1705.01329 [gr-qc]].

[31] C. F. Paganini, arXiv:1812.09056 [gr-qc].

[32] S. Nojiri, S. D. Odintsov and E. N. Saridakis, arXiv:1904.03455 [gr-qc].

[33] I. Dalianis, A. Kehagias and G. Tringas, JCAP 1901, 037 (2019) [arXiv:1805.09483 [astro-ph.CO]].

[34] Y. Akrami et al. [Planck Collaboration], arXiv:1807.06211 [astro-ph.CO].

[35] D. Benisty, E.I. Guendelman, E. N. Saridakis, In preparation.

[36] S. Nesseris, G. Pantazis and L. Perivolaropoulos, Phys. Rev. D 96, no. 2, 023542 (2017) [arXiv:1703.10538 [astro-ph.CO]].

[37] L. Kazantzidis and L. Perivolaropoulos, and implications for modified gravity theories,” Phys. Rev. D 97, no. 10, 103503 (2018) [arXiv:1803.01337 [astro-ph.CO]].

[38] J. Martin, C. Ringeval and V. Vennin, Phys. Dark Univ. 5-6 (2014) 75 doi:10.1016/j.dark.2014.01.003 [arXiv:1303.3787 [astro-ph.CO]].

[39] K. Bamba, S. D. Odintsov and E. N. Saridakis, Mod. Phys. Lett. A 32, no. 21, 1750114 (2017) doi:10.1142/S0217732317501140 [arXiv:1605.02461 [gr-qc]].

[40] A. Berera, Phys. Rev. Lett. 75, 3218 (1995) [astro-ph/9509049].

[41] V. Kamali, S. Basilakos, A. Mehrabi, M. Motaharfar and E. Massaeli, Int. J. Mod. Phys. D 27, no. 05, 1850056 (2018) [arXiv:1703.01409 [gr-qc]].

[42] J. P. Uzan, Living Rev. Rel. 14, 2 (2011) [arXiv:1009.5514 [astro-ph.CO]].

[43] J. Sol, A. Gmez-Valent and J. de Cruz Prez, Astrophys. J. 836 (2017) no.1, 43 [arXiv:1602.02103 [astro-ph.CO]].