Numerical study of linearization techniques for Bouc-Wen hysteresis model considering random inputs

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Abstract. Our study focuses on the numerical solution of the stochastic vibration of elastic-plastic mechanical systems with Gaussian random excitations. It’s realized the short review about linearization techniques in stochastic dynamics mainly with application in area of elastic-plastic finite element modelling using BW (Bouc-Wen) differential mathematical model. The presented methods of statistical linearization, Hurtado-Barbat’s modification of statistical linearization, statistical quaadratization and Monte Carlo approach are applied to numerical testing.

1. Introduction
Numerical analysis of dynamic systems using linear computational tools is often just the first approximation of the actual conduct of the investigated object. The obtained results must be considered with caution, and we must decide whether they are sufficient for us, or we will require more detailed and complex study of the issue with the consequent formation of a more appropriate mechanical and mathematical model.

There are a number of independent approaches built on different principles that give acceptable results based on certain assumptions. The so-called global approaches include analysis of nonlinear dynamic systems using the theory of Markov processes, which leads to solving the familiar equation by Fokker-Planck-Kolmogorov (FPK) [1, 2]. In practice we often encounter local approaches represented by the method of statistical linearization in various modifications (Krylov, Bogoliubov, Caughey, Bolotin [1, 3], Kazakov [2], Nigam [4], Brücker and Lin [5], Elishakoff [6]), method of statistical quadratization (Spanos, Donley [7]), method of statistical cubicization (Spanos, Donley [7]), functional method by Volterra and Wiener ([4, 8]). In addition to the above-mentioned methods, which have been the most discussed ones in the recent decade, other methods have been developed in the past, such as the asymptotic method by Krylov-Bogoliubov-Mitropolsky [1, 2], suitably adapted methods of the small parameter – especially the perturbation version (Crandall [4], Nigam [4]), harmonic linearization [2] and mean values (Bogoliubov, Mitropolsky [1, 2, 8]). Thanks to computer technology also other methods have been given a green light, based on simple but time-consuming and computationally
intensive approaches. In particular, these are simulation methods in various modifications [9, 10]. Simulation approaches solve dynamic tasks directly in the time domain, which is demanding on computation time. If we add to the above also the random nature of excitation and the need to obtain a complete picture of system behavior for various excitation realizations, then we come to the Monte Carlo method, elaborated in detail for example by Rubinstein [4]. Other authors explored the ways of increasing efficiency by combining the Monte Carlo method with other methods mentioned above (for example a combination of statistical linearization and the Monte Carlo method [6, 11]).

In addition to common problems associated with nonlinear functions in the form of polynomials, or piecewise linear functions, there are also current tasks with functions describing hysteresis curves [12–14].

The above listing and analysis of the most significant methods of solving nonlinear dynamic systems with random inputs cannot of course be complete. For other, less frequently used or theoretically challenging approaches see scientific literature. In engineering practice we most often encounter methods such as equivalent statistical linearization and its modifications and equivalent statistical quadratization.

2. Shortly about linearization techniques

Equivalent statistical linearization method (ESL) has been used for a relatively long period to deal with randomly excited nonlinear systems, especially in the frequency domain. It is an approximation method, in which solving a system of nonlinear differential equations is replaced by solving an equivalent linear system suitable to obtain Fourier transform of the system. The method was first presented by Krylov and Bogoliubov (1943) [2, 4], further elaborated by Booton (1954) [8], Caughey (1959) [4], Kazakov (1965) [2, 15], Iwan, Yang (1972) [5, 16], Spanos (1978) [17], and evaluated by Roberts and Spanos in [7] (1992).

The development of ESL within nonlinear stochastic dynamics is also associated with the work by Foeter [18, 19], Malhotra and Penzien [7], Iwan and Yang [17, 20], Atalik and Utku [5], Iwan and Mason [8, 21], Brücker and Lin [5, 22], Chang and Yang [20, 23], as well as many others. Especially 1960s and 1970s were a period of enormous increase in the method applications.

We obtain the parameters of an equivalent linear system by satisfying a certain pre-selected criterion, under which we assess the conformity of the original and linearized model especially in the outputs that are characterized mainly by statistical moments. By their very nature, the presented techniques are a generalization of deterministic linearization methods by Krylov and Bogoliubov [2, 8]. The most frequently used criteria include [4, 24, 25]:

- criterion based on energy balance of an actual and equivalent model
- criterion of conformity of the corresponding mean values and response dispersions of the original and linearized nonlinear function with a random input (less suitable),
- criterion of the minimum root-mean-square deviation of an actual and approximated (linearized, or squared) function.

It should be noted that, on one hand, the advantages of this method certainly include its simplicity and admissible estimate of the first two statistical moments, but on the other hand, we should not forget the change in nature of the random variable in the output, that is for example Gaussian output does not correspond to Gaussian input, as contemplated by the method’s theoretical foundations. A significant shortcoming of ESL is also the possible incompatibility in the spectral response of an actual and linearized system (for example the difference in the characteristics of the function of power spectral density – PSD, etc.).

Suitability of the equivalent statistical linearization method is limited mainly to the so-called weak and symmetric nonlinearities, and it also does not sufficiently describe the response spectral properties (power spectral density), which can affect the results for example in dimensioning of structures, analysis of fatigue strength and life [23, 26], or in determining other properties associated with vibrations that are significant for correct operation [10, 27, 28]. In order to improve the results, especially the spectral ones, we can employ and apply the procedure that was first published by Spanos and Donley in [7, 17]
in the early 1990s. The authors replaced the original nonlinear system with an equivalent quadratic model that is suitable especially for polynomial types of nonlinearities, including asymmetric ones. It must be said that the equivalent system is nonlinear (quadratic) and its solution is determined by using Volterra’s method of functional series [8].

In the case of symmetric nonlinearities, the equivalent statistical quadratization method is by it very nature reduced to the equivalent statistical linearization method (ESQ). In recent years, other modifications of the method have been developed, including the method of the so-called equivalent statistical “cubization”, which appears to be a quite efficient computation procedure, even though it is theoretically challenging, especially for the cases of a “stronger” and symmetric nonlinearity. Such a computation tool works with a cubic polynomial, therefore the system response is processed as the sum of first, second and third members of Volterra series (see more details for example in [7]). Using this procedure, the investigated statistical moments are acceptable to the fourth order.

3. Implementation and comparison of chosen linearization techniques in the Bouc-Wens hysteresis system

Let’s analyse the selected methods of solving stochastic analysis of a hysteresis system. Common Gaussian distribution assumption leads to an error in estimating the response moments. This error may be too large for models that are nearly ideally elasto-plastic. We will monitor the impact of some parameters on the shape of the hysteresis curve and on its resulting shape when solving by several procedures. We will focus our attention mainly on the Monte Carlo method (MC) as a reference, equivalent statistical linearization (ESL) method, modified method of equivalent statistical linearization (ESL-HB), and equivalent statistical quadratization (ESQ) method. These methods are applied to a simple problem of a mass point oscillation, provided that the restoring force is a function of the above-mentioned Bouc-Wen endochronic type [12], that is:

$$\ddot{z} = A \cdot \dot{z} - \beta \cdot |\dot{z}|^{n-1} \cdot \dot{z} - \gamma \cdot x \cdot |x|^n,$$

where \(n, A, \beta, \gamma\) are constants affecting the hysteresis loop shape. Therefore, let’s consider the equation of motion:

$$m \cdot \ddot{x} + b \cdot \dot{x} + \alpha \cdot k \cdot x + (1 - \alpha) \cdot k \cdot z = f(t),$$

where the variable \(z\) is determined by solving the differential Eq. (1). The remaining parameters \(m, b, k\) have the usual meaning. Parameter \(\alpha\) is sometimes called the strengthening parameter and its value significantly influences the nature of the hysteresis loop and especially the stability and accuracy of solving in a linearizing manner. From the Eqs. (1) and (2) we compile a system of differential equations of the first order in the form:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ \alpha \cdot k & b & (1 - \alpha) \cdot k \\ 0 & -A + \gamma \cdot |y_3| & \beta \cdot |y_2| \cdot |y_3|^{n-1} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix},$$

where \(y_1 = x, y_2 = \dot{x}, y_3 = z\). Then we solve the nonlinear system (3) or its linear or quadratic modifications using the modified Crank-Nicolson method with controlling the weight in each time step. It should be recalled that in some cases the solving process has encountered problems with the stability of solution, which we solved by not including such realization into further calculation.

3.1. Implementation of equivalent statistical linearization

This is one of the most popular and frequently used techniques. A nonlinear differential system (3) is substituted by a statistically equivalent linear one in the form:
where the linearization parameters \( s_c, c_c, k_c \) are numerically calculated as follows:

\[
E \left( \begin{bmatrix} y_1^2 & y_1 \cdot y_2 & y_1 \cdot y_3 \\ y_2 \cdot y_1 & y_2^2 & y_2 \cdot y_3 \\ y_3 \cdot y_1 & y_3 \cdot y_2 & y_3^2 \end{bmatrix} \right) \cdot \begin{bmatrix} s_c \\ c_c \\ k_c \end{bmatrix} = E \left( \begin{bmatrix} y_3 \cdot y_1 \\ y_3^2 \cdot y_2 \\ y_3 \cdot y_3 \end{bmatrix} \right).
\]

(5)

Analytical calculation of coefficients \( s_c, c_c, k_c \) can be found in the relevant literature [4] and will be as follows:

\[
s_c = E \left( \frac{\partial z}{\partial c} \right), \quad c_c = E \left( \frac{\partial z}{\partial c} \right), \quad k_c = E \left( \frac{\partial z}{\partial c} \right),
\]

(6)

and after performing derivations we obtain:

\[
s_c = 0, \quad c_c = \frac{E}{2 \cdot \sigma_y} \cdot F_1 - \frac{E}{2 \cdot \sigma_y} \cdot F_2, \quad k_c = \frac{E}{2 \cdot \sigma_y} \cdot F_3 - \frac{E}{2 \cdot \sigma_y} \cdot F_4.
\]

(7)

The coefficients \( F_1 \) to \( F_4 \) are calculated as follows:

\[
F_1 = \frac{\sigma_y^2}{\pi} \cdot \Gamma \left( \frac{n+2}{2} \right) \cdot 2^{n^2} \cdot I_s, \quad F_2 = \frac{\sigma_y^3}{\pi} \cdot \Gamma \left( \frac{n+1}{2} \right) \cdot 2^{n^2},
\]

\[
F_3 = \frac{n \cdot \sigma_c \cdot \sigma_z^{n-1}}{\pi} \cdot \Gamma \left( \frac{n+2}{2} \right) \cdot 2^{n^2} \cdot (2 \cdot (1 - p^2) \cdot \frac{n+1}{2}) + \rho \cdot \cdot I_s, \quad I_s = \frac{\pi}{2} \int \sin^n(\varphi) \cdot d\varphi
\]

(8)

\[
F_4 = \frac{n \cdot \rho \cdot \sigma_c \cdot \sigma_z^{n-1}}{\pi} \cdot \Gamma \left( \frac{n+2}{2} \right) \cdot 2^{n^2},
\]

where \( I_s = 2 \cdot \int \sin^2(\varphi) d\varphi \) and \( l = \left( \frac{\sqrt{4 \cdot \rho^2 + \rho^2 \cdot \cdot \cdot}}{\rho} \right)^{-1} \).

3.2. Implementation of Hurtado-Barbat’s modified equivalent statistical linearization

This technique was designed by Hurtado and Barbat in [11] as more stable and accurate especially for small values of the parameter \( \alpha \). The linearization strategy is non-Gaussian in nature and linearization coefficients are determined as follows:

\[
\begin{bmatrix} s_v \\ c_v \\ k_v \end{bmatrix} = (1 - 2 \cdot p) \cdot \begin{bmatrix} s_c \\ c_c \\ k_c \end{bmatrix} + 2 \cdot p \cdot \begin{bmatrix} s_d \\ c_d \\ k_d \end{bmatrix} \cdot \begin{bmatrix} y_1^2 & y_1 \cdot y_2 & y_1 \cdot y_3 \\ y_2 \cdot y_1 & y_2^2 & y_2 \cdot y_3 \\ y_3 \cdot y_1 & y_3 \cdot y_2 & y_3^2 \end{bmatrix}^{-1} \]

(9)
where

\[ s_d = E\left( y_1 \cdot y_2 \right) \cdot \left( A - \gamma \cdot z^a \right), \quad c_d = E\left( y_2^2 \right) \cdot \left( A - \gamma \cdot z^a \right), \quad k_d = -E\left( y_2^2 \right) \cdot \beta \cdot z^{n+1} \cdot \frac{2}{\sqrt{\pi}}, \]  

(10)

and \( z^a \) is the maximum value of the variable \( y_3 \) and the weighting parameter is determined from the equation:

\[ p = r \cdot \Phi(z^a). \]  

(11)

\( \Phi(z^a) \) is the value of the distribution function for \( z = z^a \) and \( r \) is the weighting parameter of the hysteresis loop shape [14, 29].

3.3. Implementation of equivalent statistical quadratization

This is a theoretically and computationally more demanding technique. The nonlinear differential system (3) is substituted by a statistically equivalent quadratic one in the form:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & m & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}
+ \begin{bmatrix}
0 & -1 & 0 \\
\alpha \cdot k & b \cdot (1 - \alpha) \cdot k & 0 \\
-s_{q1} & -c_{q1} & -k_{q1}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}
= \begin{bmatrix}
\gamma y_1^2 + y_1 \cdot y_2 + y_1 \cdot y_3 \\
y_2^2 + y_2 \cdot y_1 + y_2 \cdot y_3 \\
y_3^2 + y_3 \cdot y_1 + y_3 \cdot y_2
\end{bmatrix}
- \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}
\]

(12)

The vector of quadratization coefficients \( q \) is calculated as follows:

\[ E\left( y_q \cdot y_q^T\right) \cdot q = E\left( y_q \cdot y_q \right), \]

(13)

where

\[ q = \begin{bmatrix}
s_{q1} & c_{q1} & k_{q1} \\
s_{q2} & c_{q2} & k_{q2}
\end{bmatrix} \]

and

\[ y_q = \begin{bmatrix}
y_1, & y_2, & y_3, & y_1^2 + y_1 \cdot y_2 + y_1 \cdot y_3, & y_2^2 + y_2 \cdot y_1 + y_2 \cdot y_3, & y_3^2 + y_3 \cdot y_1 + y_3 \cdot y_2
\end{bmatrix}. \]

In this case, it is not possible to exactly express the quadratization parameters. Their calculation must be carried out only numerically, resulting in significant problems mainly in the calculations of the time domain, but especially in the frequency domain.

3.4. Monte Carlo method

We will realize a series of simulation calculations (in the order of hundreds), from which we obtain the results by averaging [2, 3, 10, 30]. It is a time consuming but relatively reliable approach. Its core is repetitive numerical solving of the Eq. (3) and subsequent evaluation of all analyses.
3.4.1. Numerical tests and comparison of results
We compared individual methods for the known parameters of the system (3) with the system parameters as follows: \( m = 1 \) kg, \( b = 5 \) N s m\(^{-1}\), \( k = 8000 \) N m\(^{-1}\). The hysteresis loop parameters were \( A = 1 \), \( \alpha = 0.15 \), \( \beta = 10 \), \( \gamma = 10 \) and \( n = [1; 1.5; 2; 2.5] \). In the time period of three seconds we randomly generated 5000 values of the excitation function \( f(t) \) and the calculation was repeated 100 times. Individual results were obtained by averaging throughout all numerically stable realizations. Calculation was repeated with various values of the coefficient \( n \), by which we also changed the hysteresis loop shape.

We observed the first two statistical moments, the course of the output power spectral density and the hysteresis loop shape. Analysis was performed in the time domain so as to obtain the observed results in the same numerical conditions.

The generated excitation force \( f(t) \) was of a random nature with an exponential autocorrelation function in the form [3]:

\[
R(\tau) = \sigma^2_f e^{-|\tau|},
\]

and power spectral density [3]:

\[
S_f(\omega) = \frac{\sigma^2_f a}{\pi \omega^2 + a^2},
\]

where the standard deviation of the excitation force was \( \sigma_f = 1000 \) N, and the parameter \( a = 150 \). Generating the random excitation function was carried out using the classic Monte Carlo approach.

If \( S_f(\omega) \) is power spectral density of the random function \( f(t) \), then its course in discrete equidistant points is obtained as follows [3, 29]:

\[
f(t) = \sum_{i=1}^{m} \sqrt{2 \cdot d\omega_i \cdot S(\omega_i) \cdot \cos(\omega_i \cdot t - \varphi_i)},
\]

where \( \omega_i \) is the discrete value of circular frequency with a corresponding respective value \( S(\omega_i) \), \( d\omega_i \) is the \( i \)th frequency increment, \( \varphi_i \) is the random phase angle with uniform distribution on the interval \((0, 2\pi)\) and \( m \) is the total number of considered harmonic oscillations (waves). Chosen results of analyses are processed in tables 1 to 4 and in a graphic manner in figures 1 to 6 where we present the hysteresis loop shapes for various \( n \).

### Table 1. Mean value and standard deviation of parameters \( x \) and \( z \) for \( n = 1 \).

|       | MC   | SL   | SLHB  | SQ   |
|-------|------|------|-------|------|
| \( m_y \) [m] | 0.0115 | 0.0067 | 0.0064 | -0.0041 |
| \( m_z \) [m] | -0.0010 | -0.0002 | -0.0002 | 0.0014 |
| \( S_y \) [m] | 0.2015 | 0.2983 | 0.2812 | 0.2467 |
| \( S_z \) [m] | 0.0378 | 0.0170 | 0.0185 | 0.0453 |

### Table 2. Mean value and standard deviation of parameters \( x \) and \( z \) for \( n = 1.5 \).

|       | MC   | SL   | SLHB  | SQ   |
|-------|------|------|-------|------|
| \( m_y \) [m] | 0.0093 | 0.0078 | 0.0037 | 0.0377 |
| \( m_z \) [m] | -0.0009 | -0.0008 | -0.0001 | -0.0060 |
| \( S_y \) [m] | 0.1346 | 0.1146 | 0.1098 | 0.1228 |
| \( S_z \) [m] | 0.0706 | 0.0636 | 0.0595 | 0.0726 |
Table 3. Mean value and standard deviation of parameters $x$ and $z$ for $n = 2$.

|       | MC  | SL  | SLHB | SQ  |
|-------|-----|-----|------|-----|
| $m_x$ [m] | 0.0646 | 0.0102 | 0.0019 | 0.0334 |
| $m_z$ [m] | -0.0115 | -0.0016 | -0.0001 | -0.0059 |
| $S_x$ [m] | 0.1229 | 0.1203 | 0.1219 | 0.1163 |
| $S_z$ [m] | 0.0997 | 0.0681 | 0.0861 | 0.0989 |

Table 4. Mean value and standard deviation of parameters $x$ and $z$ for $n = 2.5$.

|       | MC  | ESL | ESLHB | ESQ |
|-------|-----|-----|------|-----|
| $m_x$ [m] | 0.0253 | 0.0102 | 0.0008 | 0.0735 |
| $m_z$ [m] | -0.0046 | -0.0013 | 0.0004 | -0.0130 |
| $S_x$ [m] | 0.1398 | 0.1249 | 0.1379 | 0.1330 |
| $S_z$ [m] | 0.1273 | 0.0879 | 0.0987 | 0.1237 |

From the above tables, we can conclude good suitability of the above-mentioned methods particularly in terms of the second statistical moments – dispersion, or standard deviation. Figures 1–6 allow us to observe and compare the hysteresis loop shapes obtained from a series of simulation calculations using the Monte Carlo principle (dashed curve) with the statistically equivalent models SL, SLHB and SQ. We can conclude from the graphic output that the above linearization as well as quadratization approaches are suitable for larger values of the parameter "$n$" ($> 2$).

Figure 1. Hysteresis loop $z$-$x$ for $n = 1$ (—— ESL, ------ MC).
Figure 2. Hysteresis loop $z-x$ for $n = 1$ (— SLHB, —— MC).

Figure 3. Hysteresis loop $z-x$ for $n = 1$ (— ESQ, —— MC).
Figure 4. Hysteresis loop $z\rightarrow x$ for $n = 1.5$ (--- ESL, ------ MC).

Figure 5. Hysteresis loop $z\rightarrow x$ for $n = 1.5$ (---- SLHB, ----- MC).
4. Conclusion
Our attention focused on a differential system with nonlinear function of restoring force describing a hysteretic phenomenon. Assuming random excitation, the most frequently applied method in the past used to be statistical linearization. However, Hurtado and Barbat have recently designed a modified calculation of the linearization coefficients, which led to some improvements in the results. Further improvements were caused by the statistical quadratization method. This method, however, is computationally demanding especially when solving in the frequency domain. The results documented in the above figures show that a statistically equivalent model does not necessarily describe the course of the hysteresis loop with sufficient precision even at a solid conformity of the first statistical moments, or of other spectral properties (PSD, etc.). The primary aim of the example was to study linearization methods of solving hysteretic systems that have been used or designed in recent years.

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