Quenched Kosterlitz-Thouless superfluid transitions
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Abstract
The properties of rapidly quenched superfluid phase transitions are computed for two-dimensional Kosterlitz-Thouless (KT) systems. The decay in the vortex-pair density and the recovery of the superfluid density after a quench are found by solving the Fokker-Planck equation describing the vortex dynamics, in conjunction with the KT recursion relations. The vortex density is found to decay approximately as the inverse of the time from the quench, in agreement with computer simulations and with scaling theories.

Keywords: Superfluid transition; Kosterlitz-Thouless; Vortex pairs; Quenched transition

The properties of quenched phase transitions are of interest because they may be relevant to the rapidly cooling early universe [1]. The cosmic-string phase transitions in that case have recently been linked with the vortex-loop superfluid transition in liquid helium and high-T_c superconductors [2]. In this paper we investigate a new technique for calculating the quenched superfluid transition in two dimensions, employing Kosterlitz-Thouless renormalization methods. The results are in agreement with previous studies that used either computer simulations [3] or scaling theories [4].

The stochastic dynamics of the vortex pairs of the Kosterlitz-Thouless theory are modeled by a Fokker-Planck equation [5] for the distribution function Γ(r, t), which is the density of vortex pairs of separation r:

\[ \frac{\partial \Gamma}{\partial t'} + \frac{\partial \Gamma}{\partial l} \left( \frac{\partial U}{\partial l} + \frac{\Gamma}{k_B T} \frac{\partial U}{\partial l'} \right) = 0, \]

where \( l = \ln(r/a_o) \), with \( a_o \) the vortex core radius, and \( U \) is the interaction potential between a pair. The time here is in units of the diffusion time of the smallest pairs of separation \( a_o \), \( t' = t/\tau_o \), where \( \tau_o^{-1} = 2D/a_o^2 \), with \( D \) the diffusion coefficient. From the KT theory

\[ \frac{1}{k_B T} \frac{\partial U}{\partial l} = 2\pi K, \]

where \( K = \hbar^2 \sigma_s/m^2 k_B T \) is the dimensionless areal superfluid density. \( K \) is renormalized by the presence of the vortex pairs, and the KT recursion relation for \( K \) can be written in terms of \( \Gamma \) as

\[ \frac{\partial K}{\partial l} = -4\pi^3 K^2 \Gamma e^{4t}. \]

To quench from \( T_{KT} \) to low temperature it is first necessary to compute \( K(l) \) and the vortex distribution at \( T_{KT} \). The equilibrium solution of Eq. (1) is just the Boltzmann distribution, and taking the normalized core energy of a pair to be \( \pi^2 K(0) \), iteration of the KT recursion relations gives the

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Fig. 1. Distribution function $\Gamma$ as a function of the pair separation for different times after the quench.

$t' = 0$ vortex distribution shown in Fig. 1. The iterations are taken to a maximum pair separation of $l = 8 \ (r/a_o = 3000)$.

We then (instantaneously) change the temperature to $0.1 T_{KT}$ ($K(0) \rightarrow 10 K(0)$) and compute the new renormalized values of $K(l)$ from Eq. (3), using a Runge-Kutta method. Eq. (1) is then solved for the changed $\Gamma$ after a time step $\Delta t'$ with a two-step Lax-Wendroff algorithm. The changed vortex distribution is used to recompute $K(l)$ from Eq. (3) at the increased time, and the entire sequence is continuously repeated to step out in time. In quenching to a low temperature the second term in the parentheses of the right side of Eq. (1) is the dominant term, and as a first approximation we have neglected the first term.

The results for the pair distribution function (in units $a_o^{-4}$) are shown in Fig. 1. This is a classic phase-ordering system, and as expected the smallest pairs decay away first, while the largest pairs remain "frozen" until the longest times. By integrating the distribution function the vortex density can be computed, shown in Fig. 2 (in units $a_o^{-2}$).

At long times the density decays approximately as $t'^{-1}$, in agreement with previous simulations and scaling theories [3,4], although it appears to be approaching this quite slowly (the exponent is -1.17 at $t' = 1$ and -1.12 at $t' = 250$), probably because $K$ and the superfluid fraction ($K(8)/K(0)$, dashed curve in Fig. 2) are still changing fairly rapidly with time. The $1/t'$ behavior can be traced to the nonlinear term proportional to $\Gamma^2$ that results from the substitution of Eqs. (2) and (3) into Eq. (1).

A key feature of the present results is the scaling of the vortex decay with the diffusion time $\tau_o$, something generally neglected in previous treatments [1,3,4]. Estimating for helium films $D \sim 1 \cdot 10^{-4} \text{cm}^2/\text{s}$ and $a_o \sim 1 \cdot 10^{-8} \text{cm}$ gives $\tau_o \sim 1 \cdot 10^{-12} \text{s}$, in agreement with estimates of the same quantity in three dimensions [6]. The very fast decay times may well explain the inability of millisecond-scale experiments to detect the vortices [7].

This work is supported by the U. S. National Science Foundation, DMR 97-31523.

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