Axial-vector transition form factors of the singly heavy baryons

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(Dated: May 2, 2022)

We investigate the axial-vector transition form factors of the lowest-lying singly heavy baryons within the framework of the chiral quark-soliton model. We consider the linear $m_s$ corrections, dealing with the strange current quark mass $m_s$ as a small perturbation. Since we have various relations between different transitions because of isospin symmetry and flavor SU(3) symmetry breaking, only two axial-vector transition form factors are independent. We present the numerical results for these form factors. The effects of the flavor SU(3) symmetry breaking turn out tiny, so we neglect them. We also compute the decay rates for several strong decays of the singly heavy baryons and compare the results with the experimental data and those from other models. While the results for the $\Sigma_c \rightarrow \Lambda_c^+ + \pi$ and $\Sigma_c^* \rightarrow \Lambda_c^+ + \pi$ decays are slightly overestimated in comparison with the corresponding experimental data, those for the $\Xi_c^* \rightarrow \Xi_c + \pi$ are in remarkable agreement with the data.

Keywords: Baryon sextet, axial-vector transition form factors, pion mean fields, the chiral quark-soliton model

I. INTRODUCTION

The structure of singly heavy baryons has been much less known than that of the light baryons, both experimentally and theoretically. Even for the charmed baryons in the ground states, we know only their masses and decay widths \cite{1}. Recently, the electromagnetic properties of the singly heavy baryons have been investigated within lattice QCD \cite{2-5}. There have also been various theoretical works on their electromagnetic structure. On the other hand, there are very few works on the axial-vector properties of the singly heavy baryons. Since the LHCb Collaborations have continuously announced a series of new experimental data on the heavy baryons \cite{6-13}, one may expect that future experiments will will reveal the axial-vector structure of the heavy baryons.

A singly heavy baryon consists of a heavy quark and two light quarks. In the limit of the infinitely heavy-quark mass ($m_Q \rightarrow \infty$), the spin of the heavy quark $S_Q$ is conserved, which brings about the conservation of the spin of the light-quark degrees of freedom: $S_L \equiv S - S_Q$ \cite{14,16}. It is called the heavy-quark spin symmetry, which allows one to take the total spin of the light quarks as a good quantum number. Thus, we can classify the singly heavy charmed baryons in the ground states according to the representation of flavor $SU(3)_f$ symmetry: $3 \otimes 3 = 3 \oplus 6$, where the baryon anti-triplet (3) has $S_L = 0$ and $S = 1/2$, whereas the baryon sextet (6) carries $S_L = 1$. Since the spin of the heavy quark is coupled to $S_L$, the baryon sextet have two degenerate representations with $S = 1/2$ and $S = 3/2$, respectively, as illustrated in Fig. 1. This degeneracy is removed by introducing the color hyperfine interaction in order $1/m_Q$.

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In the limit $m_Q \to \infty$, we regard the heavy quark inside a heavy baryon as a static color source, so the light quarks govern the structure of the singly heavy baryons. Some years ago, Yang et al. [17] proposed a pion mean-field approach to explain the masses of singly heavy baryons, following the idea proposed by Ref. [15]. Witten showed in his seminal papers [19, 20] that in the limit of the large number of colors ($N_c \to \infty$) a baryon arises as a bound state of $N_c$ valence quarks in a pion mean field with a hedgehog symmetry [21, 22] that is a minimal extension of spherical symmetry with the characteristics of the pions considered. Since the quantum fluctuation around the saddle point of the pion field is suppressed by $1/N_c$ factor, one can ignore it. In this large $N_c$ limit, the presence of $N_c$ valence quarks that constitute the lowest-lying baryons causes the vacuum polarization, which creates the pion mean field. This pion mean field makes self-consistently the $N_c$ valence quarks bound. To keep the hedgehog symmetry preserved in the case of flavor SU(3)$_f$, an SU(2) soliton is embedded into the isospin corner of SU(3)$_f$ [20].

The chiral quark-soliton model ($\chi$QSM) [23–25] was constructed such that it realizes Witten’s idea. Note that in the $\chi$QSM the right hypercharge $Y_R = N_c/3$ is constrained by the $N_c$ valence quarks, which is distinguished from the Skyrme model where the Wess-Zumino-Witten term fixes it. This indicates that the explicit valence quark degrees of freedom determine properly the baryon representations in the $\chi$QSM. This constrained right hypercharge selects allowed representations of light baryons such as the baryon octet (8), the decuplet (10), etc. The $\chi$QSM has great merit because it can extend directly to describe singly heavy baryons. In the limit of $m_Q \to \infty$, a heavy quark inside the singly heavy baryon stays dormant but can play a role only as a static color source so that a colored soliton consisting of $N_c - 1$ valence quarks emerges. The right hypercharge $Y_R = (N_c - 1)/3$ is constrained by the $N_c - 1$ valence quarks, which picks the allowed representations of singly heavy baryons such as the baryon antitriplet (3) and the baryon sextet (6) as depicted in Fig. 1 and in addition the baryon antidecapentaplet (15) [24, 27]. This extended $\chi$QSM have successfully applied in describing properties of the singly heavy baryons such as the mass splittings [17, 28, 29], isospin mass differences [30], magnetic moments [31], magnetic transitions and radiative decays [32], electromagnetic and radiative transition form factors [33–36], and gravitational form factors [37].

In the present work, we investigate the axial-vector transition form factors of the low-lying singly charmed baryons, including both the strangeness-conserving ($\Delta S = 0$) and strangeness-changing ($|\Delta S| = 1$) transitions. While there have been no theoretical works on the axial-vector transition form factors of singly heavy baryons, many theoretical groups studied their decay widths: for example, heavy hadron chiral perturbation theory (HH$\chi$PT) [38–41], a quark model (QM) [42], the light-front quark model (LFQM) [43], the relativistic three-quark model (RTQM) [44], the nonrelativistic constituent quark models (NRQM) [45, 46], the $^3P_0$ strong decay model ($^3P_0$) [47], light cone QCD sum rules (LQCDSR) [48] and lattice QCD (LQCD) [49]. Since the heavy quark is not involved in the present axial-vector transitions of the singly heavy quarks, we can concentrate on the light quark degrees of freedom to compute the axial-vector transition form factors of the singly charmed baryons. We consider the rotational $1/N_c$ corrections and explicit breaking of SU(3)$_f$ symmetry to linear order [50]. Since we have already computed the axial-vector transition form factors of the baryon decuplet [51], we will focus on how the axial-vector transition form factors of the singly charmed baryons with spin 3/2 behave differently from those of the $\Delta$ isobar.

The structure of the current work is summarized as follows: In section II, we define the axial-vector transition form factors from the baryon sextet to both the baryon antitriplet and sextet, based on the transition matrix elements of the axial-vector current. In Section III, we show how to compute the axial-vector transition form factors of the singly heavy baryons in the $\chi$QSM. In Section IV, we first compare the present results with that of the $\Delta \to p$ axial-vector transition form factor. We then discuss the effects of SU(3)$_f$ symmetry breaking. The last section is devoted to summary and conclusions of the present work.
II. AXIAL-VECTOR TRANSITION FORM FACTORS BETWEEN THE SINGLY HEAVY BARYONS

The axial-vector current of a singly heavy baryon consists of the light-quark and heavy-quark parts:

\[ A_5^a(x) = \bar{\psi}(x) \gamma_\mu \frac{\lambda^a}{2} \psi(x) + \bar{\Psi}(x) \gamma_\mu \gamma_5 \Psi(x), \]

where \( \psi(x) \) represents the light-quark field \( \psi = (u, d, s) \) in flavor space and \( \Psi(x) \) denotes the heavy-quark field generically for the charm or bottom quark. The \( \lambda^a \) denotes the well-known SU(3) Gell-Mann matrices for which the index \( \chi \) is determined by strangeness-conserving \( \Delta S = 0 \) transitions \( (\chi = 1 \pm i2) \) and for \( |\Delta S| = 1 \) ones \( (\chi = 4 \pm i5) \), respectively. Considering the Lorentz structure together with spin, parity, time reversal, and charge conjugation, we can parametrize the transition matrix elements of the axial-vector current between the baryons with spin 1/2 in terms of two different real form factors:

\[
\langle B_1' (p', J'_3)|A_5^a(0)|B_2(p, J_3)\rangle = \pi(p', J'_3) \left[ \frac{G_A^{(\chi)}(q^2)}{M_{B_1'} + M_{B_1}} \frac{q^2}{2} + \frac{G_5^{(\chi)}(q^2)}{M_{B_2'} + M_{B_2}} \frac{\gamma_5}{2} u(p, J_3), \right.
\]

where \( G_A^{(\chi)} \) and \( G_5^{(\chi)} \) are the axial-vector transition and pseudoscalar transition form factors of the corresponding baryon sextet with spin 1/2, respectively. \( u(p, J_3) \) and \( \pi(p', J'_3) \) stand for the Dirac spinors for the initial and final baryon states, respectively. \( M_{B_1} \) and \( M_{B_2} \) designate the corresponding masses, respectively. \( q_\mu \) denotes the momentum transfer and \( q^2 \) its square. The transition matrix elements between the baryon with spin 3/2 and with spin 1/2 are parametrized in terms of four real form factors \([52]:\)

\[
\langle B_1' (p', J'_3)|A_5^a(0)|B_2(p, J_3)\rangle = \pi(p', J'_3) \left[ \frac{C_A^{(\chi)}(q^2)}{M_{B_1'} + M_{B_1}} \frac{q^2}{2} + \frac{C_5^{(\chi)}(q^2)}{M_{B_2'} + M_{B_2}} \frac{\gamma_5}{2} u(p, J_3), \right.
\]

where \( g_{\alpha \beta} \) represents the metric tensor \( g_{\alpha \beta} = \text{diag}(1, -1, -1, -1) \). In the rest frame of an initial baryon, \( p^\alpha, u^\alpha(p, J_3) \) is the Rarita-Schwinger spinor that describes a baryon with spin 3/2, carrying the momentum \( p \) and \( J_3 \), which can be described by the combination of the polarization vector and the Dirac spinor, \( u^\alpha(p, J_3) = \sum_{i=1}^{3} C^{\frac{1}{2}J_3}_{i^\frac{1}{2}J_3} \epsilon_i^\dagger(p) u_s(p) \). It satisfies the Dirac equation and the auxiliary equations \( p_\alpha u^\alpha(p, J_3) = 0 \) and \( \gamma_\alpha u^\alpha(p, J_3) = 0 \). The momenta of the initial and final states \( p \) and \( p' \), and the momentum transfer are explicitly written as

\[ p = (M, 0), \quad p' = (E', -q), \quad q = (\omega, q) \]

where \( q^2 = -Q^2 \) with \( Q^2 > 0 \). Thus, the three-vector momentum and energy of the momentum transfer are expressed by

\[ |q^2| = \left( \frac{M_{B_1}^2 + M_{B_2}^2 + Q^2}{2M_{B_1}} \right)^2 - M_{B_2}^2, \quad \omega_q = \left( \frac{M_{B_1}^2 - M_{B_2}^2 + Q^2}{2M_{B_1}} \right). \]

The axial-vector transition form factor \( G_A^{(\chi)}_{A_1B_2}(Q^2) \) can be obtained in terms of the spatial parts of the axial-vector current in the spherical tensor form

\[
G_A^{(\chi)}_{A_1B_2}(Q^2) = -2 \sqrt{\frac{M_{B_2}}{E_{B_1} + M_{B_1}}} \left[ \int d^3r j_0(|q||r|)\langle B_1' (p', S'_3)|A_5^a(0)|B_2(p, S_3)\rangle \right. \]

\[
\left. - \int d^3r j_2(|q||r|)\langle B_1' (p', S'_3)|A_5^a(0)|B_2(p, S_3)\rangle \right].
\]

Since the form factor \( C_5_{A_1B_2}(Q^2) \) is the most dominant one, we concentrate on it. Its expression is very similar to Eq. \([6]:\)

\[
C_5^{(\chi)}_{A_1B_2}(Q^2) = -\sqrt{\frac{2M_{B_1}}{E_{B_1} + M_{B_1}}} \left[ \int d^3r j_0(|q||r|)\langle B_1' (p', S'_3)|A_5^a(0)|B_2(p, S_3)\rangle \right. \]

\[
\left. - \int d^3r j_2(|q||r|)\langle B_1' (p', S'_3)|A_5^a(0)|B_2(p, S_3)\rangle \right].
\]

Note that \( G_A^{(\chi)}_{A_1B_2}(0) \) and \( C_5^{(\chi)}_{A_1B_2}(0) \) are related to the strong coupling constants \( g_{\pi BB'} \) by the Goldberger-Treiman relations.
III. A SINGLE HEAVY BARYON IN THE CHIRAL QUARK-SOLITON MODEL

The χQSM has proved great merit by showing that it can describe both the light and singly heavy baryons on the same footing. Since we want to discuss the axial-vector transition form factors of the singly heavy baryons in this work, we will first explain how a singly heavy baryon can be formulated in the pion mean-field approach. Let us define the normalization of the baryon state as

\[ \langle B(p', J^3) | B(p, J_3) \rangle = 2p_0 \delta_{J_3 J'_3} (2\pi)^3 \delta^{(3)} (p' - p). \]

In the large \( N_c \) limit, this normalization is reduced to

\[ \langle B(p', J^3) | B(p, J_3) \rangle = 2M_B \delta_{J_3 J'_3} (2\pi)^3 \delta^{(3)} (p' - p), \]

where \( M_B \) is a baryon mass. A singly heavy baryon consists of the \( N_c - 1 \) valence quarks and a static heavy quark, so the corresponding state can be written in terms of the Ioffe-type current of the \( N_c - 1 \) valence quarks and a heavy-quark field in Euclidean space as follows:

\[
|B, p\rangle = \lim_{x_4 \to -\infty} \exp(i p_4 x_4) N(p) \int d^3x \exp(i p \cdot x) (-i \Psi^\dagger_3 (x, x_4) \gamma_4 J^3_B (x, x_4)) |0\rangle,
\]

\[
\langle B, p| = \lim_{y_4 \to -\infty} \exp(-i p_4 y_4) N(p') \int d^3y \exp(-i p' \cdot y) |0\rangle J_B (y, y_4) \Psi_B (y, y_4),
\]

where \( N(p) (N(p')) \) represents the normalization factor depending on the initial (final) momentum. \( J_B (x) \) and \( J^3_B (y) \) stand for the Ioffe-type current consisting of the \( N_c - 1 \) valence quarks and a static heavy quark, so the corresponding state can be written as follows:

\[
\tilde{\Psi}_h (x) = \exp (-i m_Q v \cdot x) \tilde{\Psi}_h (x).
\]

Here \( \tilde{\Psi}_h (x) \) stands for a rescaled heavy-quark field almost on mass-shell. It carries no information on the heavy-quark mass in the leading-order approximation in the heavy-quark expansion. \( v \) is the velocity of the heavy quark with the superselection rule. We now show the normalization factor \( N^* (p') N (p) \) to be \( 2M_B \). The normalization of the baryon state can be computed as follows:

\[
\langle B(p', J^3) | B(p, J_3) \rangle = \frac{\delta_{J_3 J'_3}}{Z_{\text{eff}}} N^* (p') N (p) \lim_{x_4 \to -\infty} \lim_{y_4 \to -\infty} \exp (-i p_4 y_4 + i x_4 p_4)
\]

\[
\times \int d^3x d^3y \exp (-i p' \cdot y + i p \cdot x) \int DUD\bar{\psi} D\Phi D\bar{\Phi} \tilde{\Phi}^\dagger \tilde{\Phi}^\dagger
\]

\[
\times J_B (y) \Psi_B (y) (-i \Psi^\dagger_3 (y) \gamma_4) J^3_B (x)
\]

\[
\times \exp \left[ \int d^4z \left\{ (\tilde{\Psi}^\dagger (z))^\dagger_3 (i \partial + i M_U^2 + i M_4) \tilde{\Psi}^\dagger (z) + \tilde{\Phi}^\dagger (z) v \cdot \partial \Phi (z) \right\} \right]
\]

\[
= \frac{1}{Z_{\text{eff}}} N^* (p') N (p) \lim_{x_4 \to -\infty} \lim_{y_4 \to -\infty} \exp (-i p_4 y_4 + i x_4 p_4)
\]

\[
\times \int d^3x d^3y \exp (-i p' \cdot y + i p \cdot x) \langle B_B (y) \Psi_B (y) (-i \Psi^\dagger_3 (y) \gamma_4) J^3_B (x) \rangle_0,
\]

where \( Z_{\text{eff}} \) represents the low-energy effective QCD partition function with the quark fields integrated out

\[
Z_{\text{eff}} = \int DU U \exp (-S_{\text{eff}}).
\]
\(\langle \ldots \rangle_0\) in Eq. \([11]\) designates the vacuum expectation value of the baryon correlation function. \(S_{\text{eff}}\) is known as the effective chiral action (E\(\chi\)A) defined by
\[
S_{\text{eff}} = -N_c \text{Tr} \ln [i \partial + i M U^{\gamma_5} + i \hat{m}],
\]
which embraces the effective nonlocal interaction between the quark and pseudo-Nambu-Goldstone (pNG) fields. \(M\) is the dynamical quark mass that arises from the spontaneous breakdown of chiral symmetry. The \(U^{\gamma_5}\) stands for the chiral field that is defined by
\[
U^{\gamma_5}(z) = \frac{1 - \gamma_5}{2} U(z) + U^\dagger(z) \frac{1 + \gamma_5}{2}
\]
with
\[
U(z) = \exp \left[ i \pi^a(z) \lambda^a \right],
\]
where \(\pi^a(z)\) represents the pNG fields and \(\lambda^a\) are the flavor Gall-Mann matrices. \(\hat{m}\) displays the mass matrix of current quarks \(\hat{m} = \text{diag}(m_u, m_d, m_s)\). We regard the strange current quark mass \(m_s\) as a small perturbation.

The Green function of a light quark in the \(\chi\)QSM \([23]\) is given by
\[
G(y, x) = \left\langle y \left| \frac{1}{i \partial + i M U^{\gamma_5} + i \hat{m}} \right| x \right\rangle
= \Theta(y_4 - x_4) \sum_{E_n > 0} e^{-E_n(y_4 - x_4)} \psi_n(y) \psi_n^\dagger(x)
- \Theta(x_4 - y_4) \sum_{E_n < 0} e^{-E_n(y_4 - x_4)} \psi_n(y) \psi_n^\dagger(x),
\]
where \(\Theta(y_4 - x_4)\) is the Heaviside step function. \(\overline{m}\) denotes the average mass of the up and down current quarks: \(\overline{m} = (m_u + m_d)/2\) that constitutes an essential part in producing the correct Yukawa tail of the soliton profile function. \(E_n\) corresponds to the energy eigenvalue of the single-quark eigenstate given by
\[
H \psi_n(x) = E_n \psi_n(x),
\]
where \(H\) is the one-body Dirac Hamiltonian in the presence of the pNG boson fields, defined by
\[
H = \gamma_4 \gamma_i \partial_i + \gamma_4 M U^{\gamma_5} + \gamma_4 \overline{m} \mathbf{1}.
\]

The Green function for the heavy quark in the limit of \(m_Q \to \infty\) is given by the Heaviside step function and Dirac delta function
\[
G_h(y, x) = \left\langle y \left| \frac{1}{\partial_4} \right| x \right\rangle = \Theta(y_4 - x_4) \delta^{(3)}(y - x),
\]
which is the natural form of the heavy-quark propagator in the \(m_Q \to \infty\) limit. Using these Green functions for the light and heavy quarks and taking the limit of \(y_4 - x_4 = T \to \infty\), we arrive at expression for the baryon correlation function \(\langle J_B(y) \Psi_h(y)(-i \Psi_h^\dagger(x) \gamma_4) J_B^\dagger(x) \rangle_0\) \([36]\):
\[
\langle J_B(y) \Psi_h(y)(-i \Psi_h^\dagger(x) \gamma_4) J_B^\dagger(x) \rangle_0 \sim \exp \left[ -\{ (N_c - 1) E_{\text{val}} + E_{\text{sea}} + m_Q \} T \right] \exp[-M_B T].
\]

Since the result for the correlation function given in Eq. \([20]\) is canceled with the term \(\exp(-iy_4 \hat{p}_4' + ix_4 \hat{p}_4) = \exp[M_B T]\) in the large \(N_c\) limit, i.e., \(-iy_4 \hat{p}_4' = -ip_4 = M_B = \mathcal{O}(N_c)\). Thus, the normalization factor is reduced to the mass of a singly heavy baryon: \(N^+(p') N(p) = 2 M_B\). Combining Eq. \([21]\) with the normalization constant, we find that the classical mass of the singly heavy baryon is given by the sum of the \(N_c - 1\) soliton and the heavy-quark masses
\[
M_B = (N_c - 1) E_{\text{val}} + E_{\text{sea}} + m_Q,
\]
which was assumed in a previous work \([28]\). The classical mass given in Eq. \([21]\) comes into critical play, when we derive the axial-vector transition form factors of the singly heavy baryons, which will be mentioned in Section \(V\).
IV. AXIAL-VECTOR TRANSITION FORM FACTORS IN THE CHIRAL QUARK-SOLITON MODEL

We now show how to compute the transition matrix elements of the axial-vector current \([2]_μ\), using the functional integral. Since the heavy quark is not involved, we will only consider the light quark degrees of freedom

\[
\langle B(p', J'₃)|A_{μ}^a(0)|B(p, J₃)\rangle = \frac{1}{i} \lim_{\gamma \to \infty} \exp \left( \frac{i p_γ \gamma^a}{2} \right) \int d^3x d^3y \exp(-ip' \cdot y + ip \cdot x)
\]

\[
\int \mathcal{D}\pi^a \int \mathcal{D}\psi \int \mathcal{D}\phi J_B(y, T/2) \psi^\dagger(0)\gamma_4\gamma_μ\gamma_5 \frac{λ^a}{2} \psi(0) J_B^\dagger(x, -T/2)
\]

\[
\times \exp \left[ -\int d^4z (\psi^\dagger(z))_α^i (i\partial^i + iM U^γ_i + i\bar{m})_γ(z) \psi_γ^a(z) \right].
\]

(22)

In the large-\(N_c\) limit, we use the saddle-point approximation to get the classical soliton. However, we have to take into account the zero modes that do not change the energy of the soliton. We assume that the soliton rotates slowly so we deal with the angular velocities as a perturbative parameter. The integral over the translational zero modes in the leading order provides naturally the Fourier transform, which means that the baryon state has the proper quantization, we obtain the collective Hamiltonian as follows:

\[
H_{\text{coll}} = H_{\text{sym}} + H_{\text{sb}},
\]

(23)

where

\[
H_{\text{sym}} = M_{c1} + \frac{1}{2I_1} \sum_{i=1}^3 J_i^2 + \frac{1}{2I_2} \sum_{p=4}^7 J_p^2,
\]

\[
H_{\text{sb}} = \alpha D_{8s}^{(s)} + \beta \bar{Y} + \gamma \sqrt{3} D_{8t}^{(s)} \bar{I}_1,
\]

(24)

where \(I_1\) and \(I_2\) are the moments of inertia for the classical soliton and \(D_{8s}^{(s)}\) is the SU(3) Wigner \(D\) function. The inertial parameters \(\alpha, \beta\) and \(\gamma\), which break flavor SU(3) symmetry explicitly, are written in terms of the moments of inertia \(I_1\) and \(I_2\), and the anomalous moments of inertia \(K_1\) and \(K_2\)

\[
\alpha = \left( -\frac{\bar{Σ}_πN}{3m} + \frac{K_2}{I_2} \bar{Y} \right) m_s, \quad \beta = -\frac{K_2}{I_2} m_s, \quad \gamma = 2 \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right) m_s,
\]

(25)

where \(\bar{Σ}_πN\) is related to the pion-nucleon \(Σ\) term: \(\bar{Σ}_πN = (N_c - 1)N_c^{-1}Σ\piN\). As mentioned previously, the right hypercharge \(\bar{Y}\) is fixed by the number of valence quarks, i.e., \(Y_B = (N_c - 1)/3 = 2/3\). Diagonalizing the collective Hamiltonian, we derive the collective wavefunctions of the singly heavy baryon

\[
ψ_{B}^{(R)}(J', J₃; A) = \sum_{m₃ = ±1/2} C_{J'₆₃,J₃} \sqrt{\text{dim}(p, q)}(-1)^{-\frac{γ}{2} + J₃} D_{(Y,T,T₃),(Y',J,−J₃)}^{(R)*}(A)χ_{m₃},
\]

(26)

where \(C_{J'₆₃,J₃}\) denotes the Clebsch-Gordan coefficient for the coupling between the collective light-quark wavefunction and the heavy-quark spinor \(χ_{m₃}\), \(\text{dim}(p, q)\) represents the dimension of the \((p, q)\) representation

\[
\text{dim}(p, q) = (p + 1)(q + 1) \left( 1 + \frac{p + q}{2} \right).
\]

(27)

In the presence of the flavor SU(3) symmetry breaking term \(H_{\text{sb}}\), the collective wavefunctions of the baryon sextet should be mixed with those in higher representations. Thus, the collective wavefunctions for the baryon antitriplet and sextet are obtained respectively as

\[
|B_{3₄}\rangle = |\bar{3}_0, B\rangle + \frac{p_B}{T} |\bar{1}0, B\rangle,
\]

(28)

\[
|B_{6₁}\rangle = |6₁, B\rangle + \frac{p_B}{T} |\bar{1}5₁, B\rangle + \frac{p_B}{T} |\bar{4}_1, B\rangle
\]

(29)
with the mixing coefficients

\[
p_{T\Sigma}^B = p_{T\Sigma} \left[ -\sqrt{15}/10, -3\sqrt{5}/20 \right], \quad q_{T\Sigma}^B = q_{T\Sigma} \left[ \sqrt{5}/5 \right], \quad q_{T\Sigma}^B = q_{T\Sigma} \left[ -\sqrt{10}/10, -\sqrt{15}/10, -\sqrt{15}/10 \right],
\]

respectively, in the basis of \([\Lambda^+_c, \Xi_c]\) for the baryon anti-triplet and \([\Sigma_c(\Sigma^+_c), \Xi_c(\Xi^+_c), \Omega_c(\Omega^+_c)]\) for the baryon sextet. The parameters \(p_{T\Sigma}, q_{T\Sigma}\) and \(q_{T\Sigma}\) are given in terms of the inertia parameters \(\alpha\) and \(\gamma\)

\[
p_{T\Sigma} = \frac{3}{4\sqrt{3}} \alpha I_2, \quad q_{T\Sigma} = \frac{1}{\sqrt{2}} \left( \alpha + \frac{2}{3} \gamma \right) I_2, \quad q_{T\Sigma} = \frac{4}{5\sqrt{10}} \left( \alpha - \frac{1}{3} \gamma \right) I_2.
\]

It is straightforward to compute the transition matrix elements of the collective states, which will be expressed by the SU(3) Clebsch-Gordan coefficients. So, we arrive at the final expressions for the axial-vector transition form factors of the singly heavy baryon with spin 1/2 and 3/2 respectively

\[
G_{A, B \to B'}^{(x)}(Q^2) = \sqrt{2} \left[ \frac{\langle D^{(8)}_{a3} \rangle}{3} \left\{ A_B^{B \to B'}(Q^2) - A^{B \to B'}(Q^2) \right\} - \frac{i\langle D^{(8)}_{a3} \rangle}{6I_1} \left\{ D_0^{B \to B'}(Q^2) - D_2^{B \to B'}(Q^2) \right\} \right.
\]

\[
+ \frac{1}{3\sqrt{3}I_1} \left[ \langle D^{(8)}_{a3} \rangle \hat{j}_d + \frac{2m_s}{\sqrt{3}} K_1 \langle D^{(8)}_{a5} \rangle \right] \left\{ B_0^{B \to B'}(Q^2) - B_2^{B \to B'}(Q^2) \right\} \right.
\]

\[
+ \frac{d_{pq3}}{3I_2} \left[ \langle D^{(8)}_{a3} \rangle \hat{j}_q + \frac{2m_s}{\sqrt{3}} K_2 \langle D^{(8)}_{ap} \rangle \right] \left\{ C_0^{B \to B'}(Q^2) - C_2^{B \to B'}(Q^2) \right\} \right.
\]

\[
+ \frac{d_{pq3}}{9} \left( \langle D^{(8)}_{a3} \rangle - \langle D^{(8)}_{a5} \rangle \right) \left\{ \mathcal{H}_0^{B \to B'}(Q^2) - \mathcal{H}_2^{B \to B'}(Q^2) \right\} \right.
\]

\[
- \frac{d_{pq3}}{9} \left( \langle D^{(8)}_{a3} \rangle - \langle D^{(8)}_{a5} \rangle \right) \left\{ \mathcal{J}_0^{B \to B'}(Q^2) - \mathcal{J}_2^{B \to B'}(Q^2) \right\} \right],
\]

\[
C_{A, B \to B'}^{(x)}(Q^2) = \sqrt{3} \left[ \frac{\langle D^{(8)}_{a3} \rangle}{3} \left\{ A_B^{B \to B'}(Q^2) - A^{B \to B'}(Q^2) \right\} - \frac{i\langle D^{(8)}_{a3} \rangle}{6I_1} \left\{ D_0^{B \to B'}(Q^2) - D_2^{B \to B'}(Q^2) \right\} \right.
\]

\[
+ \frac{1}{3\sqrt{3}I_1} \left[ \langle D^{(8)}_{a3} \rangle \hat{j}_d + \frac{2m_s}{\sqrt{3}} K_1 \langle D^{(8)}_{a5} \rangle \right] \left\{ B_0^{B \to B'}(Q^2) - B_2^{B \to B'}(Q^2) \right\} \right.
\]

\[
+ \frac{d_{pq3}}{3I_2} \left[ \langle D^{(8)}_{a3} \rangle \hat{j}_q + \frac{2m_s}{\sqrt{3}} K_2 \langle D^{(8)}_{ap} \rangle \right] \left\{ C_0^{B \to B'}(Q^2) - C_2^{B \to B'}(Q^2) \right\} \right.
\]

\[
+ \frac{d_{pq3}}{9} \left( \langle D^{(8)}_{a3} \rangle - \langle D^{(8)}_{a5} \rangle \right) \left\{ \mathcal{H}_0^{B \to B'}(Q^2) - \mathcal{H}_2^{B \to B'}(Q^2) \right\} \right.
\]

\[
- \frac{d_{pq3}}{9} \left( \langle D^{(8)}_{a3} \rangle - \langle D^{(8)}_{a5} \rangle \right) \left\{ \mathcal{J}_0^{B \to B'}(Q^2) - \mathcal{J}_2^{B \to B'}(Q^2) \right\} \right],
\]

where \(\cdots\) designate the transition baryonic matrix elements of given collective operators. The explicit expressions for the quark densities \(A_{0,2}, B_{0,2}, C_{0,2}, D_{0,2}, H_{0,2}, I_{0,2}\), and \(J_{0,2}\) can be found in Appendix A. Note that the corrections from flavor SU(3) symmetry breaking are originated from two different sources: that from the effective chiral action and that from the collective wavefunctions, which we denote them respectively as \(G_{A, B \to B'}^{(x)}^{(op)}\) and \(G_{A, B \to B'}^{(x)}^{(wt)}\)

\[
G_{A, B \to B'}^{(x)}(Q^2) = \left( G_{A, B \to B'}^{(x)}(Q^2) \right)^{sym} + \left( G_{A, B \to B'}^{(x)}(Q^2) \right)^{op} + \left( G_{A, B \to B'}^{(x)}(Q^2) \right)^{wt}.
\]

We decompose \(C_{A, B \to B'}^{(x)}\) in the same manner

\[
C_{A, B \to B'}^{(x)}(Q^2) = \left( C_{A, B \to B'}^{(x)}(Q^2) \right)^{sym} + \left( C_{A, B \to B'}^{(x)}(Q^2) \right)^{op} + \left( C_{A, B \to B'}^{(x)}(Q^2) \right)^{wt}.
\]

Having computed the transition matrix elements of the \(D\) functions, we find the following results respectively for
\[ G_{A_{1},B_{1}\to B'_{1}}^{(x)}(Q^{2})^{(\text{sym})}, \text{ and } G_{A_{2},B_{2}\to B'_{2}}^{(x)}(Q^{2})^{(\text{op})}, \text{ and } G_{A_{3},B_{3}\to B'_{3}}^{(x)}(Q^{2})^{(\text{vec})} \]

\[
G_{A_{1},B_{1}\to B'_{1}}^{(x)}(Q^{2})^{(\text{sym})} = -\frac{\sqrt{3}}{36} \left( \begin{array}{ccc}
\sqrt{2} & -\sqrt{2}T_{3} & 1 \\
-\sqrt{2}T_{3} & 2 & -\sqrt{2} \\
1 & -\sqrt{2} & \sqrt{2} \\
\end{array} \right) \left\{ 2\{A_{0}^{B\to B'}(Q^{2}) - A_{2}^{B\to B'}(Q^{2})\} - \frac{i\{D_{0}^{B\to B'}(Q^{2}) - D_{2}^{B\to B'}(Q^{2})\}}{I_{1}} \right\} \\
- \frac{\{C_{0}^{B\to B'}(Q^{2}) - C_{2}^{B\to B'}(Q^{2})\}}{I_{2}}, \tag{36}
\]

\[
G_{A_{1},B_{1}\to B'_{1}}^{(x)}(Q^{2})^{(\text{op})} = -\frac{\sqrt{3}m_{s}}{540} \left( \begin{array}{ccc}
\sqrt{2} & -2\sqrt{2}T_{3} & 0 \\
2\sqrt{2} & 2 & -\sqrt{2} \\
0 & -\sqrt{2} & \sqrt{2} \\
\end{array} \right) \left\{ \frac{K_{1}}{I_{1}}\{B_{0}^{B\to B'}(Q^{2}) - B_{2}^{B\to B'}(Q^{2})\} - \{I_{0}^{B\to B'}(Q^{2}) - I_{2}^{B\to B'}(Q^{2})\} \right\} \\
+ 3\sqrt{2} \left( \begin{array}{ccc}
\sqrt{2} & -2\sqrt{2}T_{3} & 0 \\
-2\sqrt{2}T_{3} & 2 & -\sqrt{2} \\
0 & -\sqrt{2} & \sqrt{2} \\
\end{array} \right) \left\{ \frac{K_{2}}{I_{2}}\{C_{0}^{B\to B'}(Q^{2}) - C_{2}^{B\to B'}(Q^{2})\} - \{J_{0}^{B\to B'}(Q^{2}) - J_{2}^{B\to B'}(Q^{2})\} \right\} \\
+ \left( \begin{array}{ccc}
7\sqrt{2} & -10\sqrt{2}T_{3} & 9 \\
-11\sqrt{2} & 0 & -12\sqrt{2} \\
9 & -12\sqrt{2} & 0 \\
\end{array} \right) \left\{ \mathcal{H}_{0}^{B\to B'}(Q^{2}) - \mathcal{H}_{2}^{B\to B'}(Q^{2}) \right\}, \tag{37}
\]

\[
G_{A_{1},B_{1}\to B'_{1}}^{(x)}(Q^{2})^{(\text{vec})} = -\frac{\sqrt{2}}{1440\pi^{15}} \left( \begin{array}{ccc}
2 & 2T_{3} & -1 \\
-5T_{3} & -1 & -3\sqrt{2} \\
1 & 3\sqrt{2} & 3 \\
\end{array} \right) \left\{ \sqrt{2} \{A_{0}^{B\to B'}(Q^{2}) - A_{2}^{B\to B'}(Q^{2})\} - \frac{i\{D_{0}^{B\to B'}(Q^{2}) - D_{2}^{B\to B'}(Q^{2})\}}{I_{1}} \right\} \\
- \frac{\{C_{0}^{B\to B'}(Q^{2}) - C_{2}^{B\to B'}(Q^{2})\}}{I_{2}}, \tag{38}
\]

in the basis of \([\Sigma_{c}^{0} \to \Lambda_{c}^{0}, \Xi_{c}^{0} \to \Xi_{c}, \Sigma_{c}^{++} \to \Xi_{c}^{+}, \Xi_{c}^{0} \to \Lambda_{c}^{0}, \Omega_{c}^{0} \to \Xi_{c}^{0}]\), and respectively for \(C_{5, B_{1}\to B'_{1}}^{(x)}(Q^{2})^{(\text{sym})}\).
$C_{5, B \rightarrow B'}^{A(x)}(Q^2)_{\text{sym}} = -\frac{1}{90} \left( \begin{array}{cc} T_3 & T_3 \\ T_3 & 1 \\ 1 & 1 \end{array} \right) \left[ 3 \left\{ 2[A_{0}^{B \rightarrow B'}(Q^2) - A_{2}^{B \rightarrow B'}(Q^2)] - \frac{i[D_{0}^{B \rightarrow B'}(Q^2) - D_{2}^{B \rightarrow B'}(Q^2)]}{I_1} \right\} \\
-3 \left[C_{0}^{B \rightarrow B'}(Q^2) - C_{2}^{B \rightarrow B'}(Q^2) \right] - 2 \left[B_{0}^{B \rightarrow B'}(Q^2) - B_{2}^{B \rightarrow B'}(Q^2) \right] \right], \tag{39} \right.$

$C_{5, B \rightarrow B'}^{A(x)}(Q^2)_{\text{op}} = -\frac{2\sqrt{2}m_{B}}{810} \left[ \sqrt{2} \left( \begin{array}{ccc} 4T_3 \\ T_3 \\ -1 \\ -3 \end{array} \right) \frac{K_1}{I_1} \left\{ B_{0}^{B \rightarrow B'}(Q^2) - B_{2}^{B \rightarrow B'}(Q^2) \right\} - \left\{ I_{0}^{B \rightarrow B'}(Q^2) - I_{2}^{B \rightarrow B'}(Q^2) \right\} \right] \\
+ \sqrt{2} \left( \begin{array}{ccc} 10T_3 \\ 14T_3 \\ -7 \\ -3 \end{array} \right) \frac{K_2}{I_2} \left\{ C_{0}^{B \rightarrow B'}(Q^2) - C_{2}^{B \rightarrow B'}(Q^2) \right\} - \left\{ J_{0}^{B \rightarrow B'}(Q^2) - J_{2}^{B \rightarrow B'}(Q^2) \right\} \right] \\
+ \left( \begin{array}{ccc} 14\sqrt{2}T_3 \\ 16\sqrt{2}T_3 \\ 19\sqrt{2} \\ 21 \end{array} \right) \left\{ H_{0}^{B \rightarrow B'}(Q^2) - H_{2}^{B \rightarrow B'}(Q^2) \right\}], \tag{40} \right.$

$C_{5, B \rightarrow B'}^{A(x)}(Q^2)_{\text{wt}} = -\frac{1}{90q_{\pi}} \left[ 2\sqrt{2} \left( \begin{array}{ccc} 8T_3 \\ 4T_3 \\ -5 \\ -3 \end{array} \right) \frac{K_1}{I_1} \left\{ A_{0}^{B \rightarrow B'}(Q^2) - A_{2}^{B \rightarrow B'}(Q^2) \right\} - \frac{i[D_{0}^{B \rightarrow B'}(Q^2) - D_{2}^{B \rightarrow B'}(Q^2)]}{I_1} \right] \\
+ \sqrt{5} \left( \begin{array}{ccc} 8T_3 \\ 5T_3 \\ -5 \\ -3 \end{array} \right) \frac{K_2}{I_2} \left\{ C_{0}^{B \rightarrow B'}(Q^2) - C_{2}^{B \rightarrow B'}(Q^2) \right\} \\
- 2 \left( \begin{array}{ccc} 6\sqrt{5}T_3 \\ 15T_3 \\ 5\sqrt{15} \\ 3\sqrt{15} \end{array} \right) \frac{K_1}{I_1} \left\{ B_{0}^{B \rightarrow B'}(Q^2) - B_{2}^{B \rightarrow B'}(Q^2) \right\} \right] \\
+ \frac{\sqrt{5}}{5400q_{\pi}} \left[ \sqrt{2} \left( \begin{array}{ccc} 0 \\ 0 \\ 2 \\ -1 \end{array} \right) \left\{ 2[A_{0}^{B \rightarrow B'}(Q^2) - A_{2}^{B \rightarrow B'}(Q^2)] - \frac{i[D_{0}^{B \rightarrow B'}(Q^2) - D_{2}^{B \rightarrow B'}(Q^2)]}{I_1} \right\} \right] \\
+ 8\sqrt{2} \left( \begin{array}{ccc} 0 \\ 0 \\ 1 \\ 0 \end{array} \right) \left\{ C_{0}^{B \rightarrow B'}(Q^2) - C_{2}^{B \rightarrow B'}(Q^2) \right\} \\
+ 40 \left( \begin{array}{ccc} 0 \\ 0 \\ 1 \\ 0 \end{array} \right) \left\{ B_{0}^{B \rightarrow B'}(Q^2) - B_{2}^{B \rightarrow B'}(Q^2) \right\} \right], \tag{41} \right.$

in the basis of $[\Sigma_{c}^{+} \rightarrow \Sigma_{c}, \Xi_{c}^{+} \rightarrow \Xi_{c}^{+}, \Omega_{c}^{++} \rightarrow \Omega_{c}^{++}, \Omega_{c}^{0} \rightarrow \Omega_{c}^{0}].$ The explicit expressions for $A_{i}^{B \rightarrow B'}, B_{i}^{B \rightarrow B'}, C_{i}^{B \rightarrow B'}, D_{i}^{B \rightarrow B'}, I_{i}^{B \rightarrow B'}, J_{i}^{B \rightarrow B'},$ and $H_{i}^{B \rightarrow B'}$ can be found in Appendix A. The matrix elements of the collective operators are given in Appendix B in detail.
Scrutinizing Eqs. [36]-[41], we find isospin relations between different axial-vector transition form factors,

\[ (\Xi_i^0 \rightarrow \Xi_i^+ = -(\Xi_i^0 \rightarrow \Xi_i^-) = \frac{1}{\sqrt{2}}(\Xi_i^0 \rightarrow \Xi_i^0) = -\frac{1}{\sqrt{2}}(\Xi_i^0 \rightarrow \Xi_i^+). \]

\[ (\Sigma_i^+ \rightarrow \Lambda_i^+ = (\Sigma_i^+ \rightarrow \Lambda_i^0) = (\Sigma_i^0 \rightarrow \Lambda_i^-) = (\Xi_i^0 \rightarrow \Lambda_i^-). \]

\[ (\Xi_i^0 \rightarrow \Xi_i^+ = \Xi_i^0 \rightarrow \Xi_i^-) = (\Xi_i^0 \rightarrow \Xi_i^0) = -\frac{1}{\sqrt{2}}(\Xi_i^- \rightarrow \Xi_i^+). \]

\[ (\Xi_i^0 \rightarrow \Xi_i^+ = (\Xi_i^0 \rightarrow \Xi_i^-) = (\Xi_i^0 \rightarrow \Xi_i^0) = -\frac{1}{\sqrt{2}}(\Xi_i^+ \rightarrow \Xi_i^-). \]

\[ (\Xi_i^0 \rightarrow \Xi_i^+ = (\Xi_i^0 \rightarrow \Xi_i^-) = (\Xi_i^0 \rightarrow \Xi_i^0) = -\frac{1}{\sqrt{2}}(\Xi_i^+ \rightarrow \Xi_i^-). \]

\[ (\Omega_i^0 \rightarrow \Xi_i^+ = (\Omega_i^0 \rightarrow \Xi_i^-) = (\Omega_i^0 \rightarrow \Xi_i^0) = -\frac{1}{\sqrt{2}}(\Xi_i^+ \rightarrow \Xi_i^-). \]

(42)

the SU(3) symmetric relations

\[ (\Sigma_i^+ \rightarrow \Lambda_i^+ = -2(\Xi_i^+ \rightarrow \Xi_i^+ = -(\Xi_i^+ \rightarrow \Xi_i^+) = \sqrt{2}(\Xi_i^0 \rightarrow \Lambda_i^+) = -(\Omega_i^0 \rightarrow \Xi_i^0) \]

\[ (\Sigma_i^+ \rightarrow \Xi_i^+ = 2(\Xi_i^+ \rightarrow \Xi_i^+) = (\Xi_i^+ \rightarrow \Xi_i^+ = (\Xi_i^+ \rightarrow \Xi_i^+) = (\Xi_i^+ \rightarrow \Xi_i^+), \]

(43)

and the various sum rules as follows

\[ (\Sigma_i^+ \rightarrow \Xi_i^+ = (\Xi_i^+ \rightarrow \Xi_i^+ + (\Xi_i^+ \rightarrow \Xi_i^+) - \sqrt{2}(\Xi_i^+ \rightarrow \Xi_i^0) \]

\[ (\Sigma_i^+ \rightarrow \Xi_i^+ = \frac{1}{2}(\Xi_i^0 \rightarrow \Lambda_i^+) + \frac{1}{2}(\Xi_i^+ \rightarrow \Xi_i^+) + \frac{\sqrt{3}}{2\sqrt{2}}(\Xi_i^+ \rightarrow \Xi_i^+) \]

\[ + \frac{1}{4\sqrt{6}}(\Xi_i^+ \rightarrow \Xi_i^-) - \frac{5}{4\sqrt{6}}(\Xi_i^0 \rightarrow \Xi_i^0) \]

\[ (\Sigma_i^+ \rightarrow \Xi_i^+ = \frac{3}{2\sqrt{2}}(\Xi_i^0 \rightarrow \Lambda_i^+) + \frac{3}{2}(\Xi_i^+ \rightarrow \Xi_i^+) + \frac{3\sqrt{3}}{4}(\Xi_i^+ \rightarrow \Xi_i^0) \]

\[ + \frac{\sqrt{3}}{8}(\Xi_i^+ \rightarrow \Xi_i^-) - \frac{5\sqrt{3}}{4\sqrt{2}}(\Xi_i^0 \rightarrow \Xi_i^0) \]

\[ (\Omega_i^0 \rightarrow \Xi_i^+ = \frac{1}{2\sqrt{2}}(\Xi_i^0 \rightarrow \Lambda_i^+) + \frac{3}{2}(\Xi_i^+ \rightarrow \Xi_i^+) + \frac{3\sqrt{3}}{4}(\Xi_i^+ \rightarrow \Xi_i^0) \]

\[ + \frac{\sqrt{3}}{8}(\Xi_i^+ \rightarrow \Xi_i^-) - \frac{5\sqrt{3}}{4\sqrt{2}}(\Xi_i^0 \rightarrow \Xi_i^0). \]

(44)

These relations indicate that not all form factors are independent. As we will show soon, we will only have two different form factors, from which all other form factors can be easily obtained when flavor SU(3) symmetry is imposed.

While there are no experimental information on the axial-vector transition form factors of the singly heavy baryons, their strong decay widths are experimentally known. In particular, the Belle Collaboration has recently reported those for the \( \Sigma_c^+ \rightarrow \Lambda_c^+ + \pi^0 \) and \( \Sigma_c^+ \rightarrow \Lambda_c^+ + \pi^0 \) decays [55]. Thus, we will also compute all possible strong decay widths for the singly heavy baryons, using the following formulae

\[ \Gamma_{B_{1/2} \rightarrow B_{1/2} m_s} = \frac{1}{8\pi} \frac{|q|^3 M_f}{f_{\pi}^2 M_i} (G_{A,B \rightarrow B}(0))^2, \]

\[ \Gamma_{B_{3/2} \rightarrow B_{1/2} m_s} = \frac{1}{12\pi} \frac{|q|^3 M_f}{f_{\pi}^2 M_i} (C_{5,B \rightarrow B}(0))^2, \]

(45)

where the pion momentum \(|q|\) is given by

\[ |q| = \frac{1}{2M_i} \sqrt{(M_i^2 - (M_f^2 + m_s)^2) (M_i^2 - (M_f^2 - m_s)^2)}. \]

(46)
$M_i$, $M_f$ are the initial and final baryon masses, respectively. $m_{\pi}$ represents the mass of the pion and $f_{\pi}$ stands for the pion decay constant. The mass ratio $M_f/M_i$ in Eq. (45) arises from the recoil effect \cite{11,56}. Since the velocities of $B_i$ and $B_f$ are the same to order $O(1/m_Q)$ in effective heavy quark theory, the recoil effects can be given by the mass ratio.

V. RESULTS AND DISCUSSION

We first explain how the model parameters are fixed. In the $\chi$QSM, four different parameters need to be determined: the dynamical quark mass $M$, the cutoff mass $\Lambda$ in the regularization functions, the strange current quark mass $m_s$, and the average of the up and down current quarks $\langle\bar{m}\rangle$, as mentioned in Section III. $\langle\bar{m}\rangle$ is determined by reproducing the physical value of the pion mass, $m_{\pi} = 140$ MeV. The strange current quark mass is usually fixed by the kaon mass, $m_K = 495$ MeV. We use $m_s = 180$ MeV, which reproduces approximately the kaon mass and describes the mass spectra of the baryon octet and decuplet \cite{50,57} very well. The cutoff mass $\Lambda$ is fixed by the pion decay constant $f_{\pi} = 93$ MeV. The dynamical quark mass $M$ is regarded as a free parameter in the $\chi$QSM. Nevertheless, we use $M = 420$ MeV because one can produce various experimental data such as the radius of the proton \cite{58}, the magnetic dipole moments \cite{59}, and semileptonic decays of hyperons \cite{60,61}. Note that the values of all the parameters are the same as in the previous works \cite{28,33,34,51}.

Since we use the $1/N_c$ and $1/m_Q$ expansion as a guiding principle, we have to maintain consistency in dealing with its expansion within the theoretical framework. So, we can ignore the mass difference in the momentum transfer. It indicates that momentum transfer in Eq. (4) can be approximated to be $q^2 \approx -Q^2$. The expressions for $G_A^{(3)}$ and $C_{5}^{A(\chi)}$ contain the masses of the singly heavy baryons. Keeping the $1/N_c$ expansion in mind, we approximate $M_i$ and $M_f$ by $M_{cl} + m_c$. The baryon masses in the $\chi$QSM also include the rotational $1/N_c$ and $m_s$ corrections. If we turn off all the corrections, the singly heavy baryon mass becomes the classical $N_c - 1$ soliton mass $M_{cl}$ plus the charm quark mass. To be theoretically more consistent, hence, we will take $M_{cl} + m_c$ instead of a antitriplet and sextet baryon masses. In effect, the numerical results are improved by considering $M_{cl} + m_c$ in place of $M_i$ and $M_f$ by around 10%. Similar approximations were performed in the case of the baryon octet and decuplet \cite{51,62,63}.

![Graph](image_url)

**FIG. 2.** The axial-vector transition form factor $G_A^{(3)}_{\Sigma^+_c \rightarrow \Lambda^+_c}(Q^2)$ $(C_{5}^{A(\chi)}_{\Sigma^+_c \rightarrow \Sigma^+_c}(Q^2))$ for the transitions from the baryon sextet with spin-1/2 (3/2) to the baryon anti-triplet. In the left panel, the form factors for the $\Sigma^+_c \rightarrow \Lambda^+_c$ transition are drawn, whereas in the right panel, those for the $\Sigma^+_c \rightarrow \Sigma^+_c$ are depicted. The solid and dashed curves represent the total results with the effects of the flavor SU(3) symmetry breaking and those in the SU(3) symmetric case, respectively.

As shown in Eqs. (42) and (43), we have only two independent axial-vector transition form factors when the flavor SU(3) symmetry is considered. We will present the results for these two form factors. In left panel of Fig. 2 we draw the results for the axial-vector transition form factors for the $\Sigma^+_c \rightarrow \Lambda^+_c$ transition. All other form factors for the axial-vector transitions $B_{1/2} \rightarrow B_{1/2}'$ are related to that for the $\Sigma^+_c \rightarrow \Lambda^+_c$ transition. So, we take the $\Sigma^+_c \rightarrow \Lambda^+_c$ transition form factor as a prototype one. The dashed curve represents that in the flavor SU(3) symmetric case, while the solid one depicts that with linear $m_s$ corrections. The effects of SU(3) symmetry breaking are tiny. This
can be understood by examining Eqs. (36)–(38). The prefactors in Eqs. (37) and (38) are much smaller than that in the leading-order contribution given in Eq. (36). In addition, the first term in the bracket of Eq. (36) is the most dominant one. In the right panel of Fig. 2, we depict the results for the axial-vector transition form factors from $\Sigma_c^{++}$ belonging to the baryon sextet with spin $3/2$ to $\Sigma_c^+$ in the sextet with spin $1/2$. Note that the expression for the leading-order contribution to $C_{5,B}^A(\Sigma_c^{++} \rightarrow \Sigma_c^+)(Q^2)$ is distinguished from that for $G_{A,B}^A(\Sigma_c^+ \rightarrow \Lambda_c^+)(Q^2)$ by the last term in Eq. (39), which is proportional to $B_{5,B}^A(0)$ and $B_{5,B}^B(0)$. They provide about 9% corrections to $C_{5,B}^A(\Sigma_c^{++} \rightarrow \Sigma_c^+)(Q^2)$. The effects of the flavor SU(3) symmetry breaking turn out very small. Thus, we will neglect them in the discussion of other observables related to the axial-vector transition form factors.

![Comparison of Q^2 dependence](image)

**FIG. 3.** Comparison of $Q^2$ dependence of $C_{5,B}^A(\Sigma_c^{++} \rightarrow \Sigma_c^+)(Q^2)$ to $C_{5,B}^A(\Delta^+ \rightarrow p)(Q^2)$. The solid curve represents the result for $C_{5,B}^A(\Sigma_c^{++} \rightarrow \Sigma_c^+)(Q^2)$, whereas the dashed one depicts $C_{5,B}^A(\Delta^+ \rightarrow p)(Q^2)$.

It is of great interest to compare the $Q^2$ dependence of the result for $C_{5,B}^A(\Sigma_c^{++} \rightarrow \Sigma_c^+)(Q^2)$ to that for $C_{5,B}^A(\Delta^+ \rightarrow p)(Q^2)$, since both form factors describe the axial-vector transitions from the spin-$3/2$ baryon to the spin-$1/2$ baryon. To compare more closely, we normalize them by the corresponding values of the form factors at $Q^2 = 0$. As shown in Fig. 3, the axial-vector form factor for the $\Sigma_c^{++} \rightarrow \Sigma_c^+$ transition starts to fall off more fast than that for the $\Delta^+ \rightarrow p$ transition. It indicates that the mean square radius for the $\Sigma_c^{++} \rightarrow \Sigma_c^+$ transition is larger than that for the $\Delta^+ \rightarrow p$ one. We want to emphasize that the pion mean field for the singly heavy baryons is different from that for the light baryons. This makes main difference between the $\Sigma_c^{++} \rightarrow \Sigma_c^+$ and $\Delta^+ \rightarrow p$ transition form factors, as exhibited in Fig. 3.

**TABLE I.** Numerical results for the axial-vector transition constants $G_{\Delta_c^+ \rightarrow \Lambda_c^+}(0)(C_{5,\Sigma_c^{++} \rightarrow \Sigma_c^+}(0))$, and the corresponding axial masses and mean square radii.

| $G_{\Delta_c^+ \rightarrow \Lambda_c^+}(0)(C_{5,\Sigma_c^{++} \rightarrow \Sigma_c^+}(0))$ | $\Sigma_c^+ \rightarrow \Lambda_c^+$ | $\Sigma_c^{++} \rightarrow \Sigma_c^+$ |
|----------------|----------------|----------------|
| $M_A$ [GeV] | -0.955 | -0.443 |
| $(r_A^2)$ [fm$^2$] | 0.759 | 0.728 |

In Table I, we list the values of the $G_{\Delta_c^+ \rightarrow \Lambda_c^+}(0)$ and $C_{5,\Sigma_c^{++} \rightarrow \Sigma_c^+}(0)$ at $Q^2 = 0$. These values will be used for determining the decay widths for the strong decays of the singly heavy baryons. The axial-vector transition form factor $G_{\Delta_c^+ \rightarrow \Lambda_c^+}(Q^2)$ presented in Fig. 2 can be parametrized by the dipole-type parametrization

$$G_A = \frac{G_A(0)}{1 + Q^2/M_A^2},$$

where $M_A$ is called the axial mass. We can parametrize $C_{5,\Sigma_c^{++} \rightarrow \Sigma_c^+}(Q^2)$ in the same manner. The numerical results for $M_A$ are given in the third row in Table I which indicates that the $Q^2$ dependence of the $\Sigma_c^+ \rightarrow \Lambda_c^+$ and $\Sigma_c^{++} \rightarrow \Sigma_c^+$ form factors are similar. The results for the mean square radii are listed in the last row of Table I.
TABLE II. Numerical results for the strong decay widths in comparison with the experimental data.

| Decay modes | \( \Gamma \) [MeV] | Exp. [1] | FOCUS Coll. [64] | CLEO II [65] | Belle [55, 66, 67] |
|-------------|-------------------|---------|------------------|-------------|--------------------|
| \( \Sigma_c^{++} \to \Lambda_c^+ + \pi^+ \) | 2.80 | 1.89\(^{+0.09}_{-0.18} \) | 2.05\(^{+0.38}_{-0.18} \) | 2.3 \( \pm 0.2 \) | 1.84 \( \pm 0.04\) | |
| \( \Sigma_c^{0} \to \Lambda_c^+ + \pi^0 \) | 3.39 | < 4.6 | - | - | 2.3 \( \pm 0.3 \) | |
| \( \Sigma_c^{0} \to \Lambda_c^+ + \pi^- \) | 2.76 | 1.83\(^{+0.11}_{-0.19} \) | 1.55\(^{+0.41}_{-0.37} \) | 2.5 \( \pm 0.2 \) | 1.76 \( \pm 0.04\) | |
| \( \Sigma_c^{++} \to \Lambda_c^+ + \pi^+ \) | 21.0 | 14.76\(^{+0.39}_{-0.40} \) | - | - | 14.77 \( \pm 0.25\) | |
| \( \Sigma_c^{0} \to \Lambda_c^+ + \pi^0 \) | 22.1 | < 17 | - | - | 17.2\(^{+3.1}_{-0.7} \) | |
| \( \Sigma_c^{0} \to \Lambda_c^+ + \pi^- \) | 21.0 | 15.3\(^{+0.4}_{-0.5} \) | - | - | 15.41 \( \pm 0.32 \) | |
| \( \Xi_c^+ \to \Xi_c + \pi \) | 2.12 | 2.14 \( \pm 0.19 \) | - | - | 2.6 \( \pm 0.2 \) | |
| \( \Xi_c^0 \to \Xi_c + \pi \) | 2.30 | 2.35 \( \pm 0.22 \) | - | - | - | |

In Table II we compare the current results for the strong decay widths of the singly heavy baryons, of which the experimental data are available. Thus, we consider the strong decays of \( \Sigma_c \), \( \Sigma_c^* \), and \( \Xi_c^* \). The third column of Table II are the experimental data taken from the PDG [1]. In the second, third, and fourth columns of Table II we list the experimental data taken from the FOCUS Collaboration [64], the CLEO Collaboration [65], and the Belle Collaboration [55, 66, 67], respectively. The values of the decay widths for the strong decay of \( \Sigma_c \) and \( \Sigma_c^* \) are overestimated, compared with the experimental data. On the other hand, those of the \( \Xi_c^* \) decays are in good agreement with the data. In Tables III and IV we compare the current results with those from other works. We find that the results from the present work are consistent with those from Refs. [14, 15].

TABLE III. Numerical results for the strong decay widths in comparison with those from various works.

| Decay modes | \( \Gamma \) [MeV] | Yan et al. [38] | Huang et al. [39] | Rosner [42] | Pirjol et al. [40] | Tawfik et al. [12] | Ivanov et al. [44] |
|-------------|-------------------|-----------------|------------------|-------------|------------------|-------------------|------------------|
| \( \Sigma_c^{++} \to \Lambda_c^+ + \pi^+ \) | 2.80 | 2.5 | 3.2 | 2.5 | - | - | 2.85 \( \pm 0.19 \) |
| \( \Sigma_c^{0} \to \Lambda_c^+ + \pi^0 \) | 3.39 | 2.5 | 3.2 | - | 1.70 | - | 3.63 \( \pm 0.27 \) |
| \( \Sigma_c^{0} \to \Lambda_c^+ + \pi^- \) | 2.76 | 2.45, 4.35 | 2.4 | 1.32 \( \pm 0.04 \) | - | - | 2.65 \( \pm 0.19 \) |
| \( \Sigma_c^{++} \to \Lambda_c^+ + \pi^+ \) | 21.0 | 25 | 20 | - | 12.84 | - | 21.99 \( \pm 0.87 \) |
| \( \Sigma_c^{0} \to \Lambda_c^+ + \pi^0 \) | 22.1 | 25 | 20 | - | - | - | - |
| \( \Sigma_c^{0} \to \Lambda_c^+ + \pi^- \) | 21.0 | 25 | 20 | - | 12.40 | - | 21.21 \( \pm 0.81 \) |
| \( \Xi_c^+ \to \Xi_c + \pi \) | 2.12 | - | 2.3 \( \pm 0.1 \) | - | 1.12 | - | 1.78 \( \pm 0.33 \) |
| \( \Xi_c^0 \to \Xi_c + \pi \) | 2.30 | - | 2.3 \( \pm 0.1 \) | - | - | - | 2.11 \( \pm 0.29 \) |

VI. SUMMARY AND CONCLUSION

We investigated the axial-vector transition form factors of the baryon sextet within the framework of the chiral quark-soliton model. Assuming that the heavy-quark mass is infinitely heavy, the \( N_c - 1 \) light valence quarks govern the quark dynamics inside a singly heavy baryon. In contrast, the singly heavy quark is merely a static color source, making singly heavy baryons the color singlet. The presence of the \( N_c - 1 \) valence quarks creates the pion mean field, so that they are also influenced by it self-consistently. The \( N_c - 1 \) valence quarks also constrain the right hypercharge \( Y_R = (N_c - 1)/3 = 2/3 \), allowing the flavor SU(3) representations such as the baryon antitriplet, baryon sextet, and higher representations. Based on this framework, we studied the axial-vector transitions from the baryon sextet with spin 1/2 and 3/2 to the baryon sextet with spin 1/2 and antitriplet, considering the rotational 1/\( N_c \) and linear \( m_s \) corrections. We presented the results for the \( \Sigma_c^+ \to \Lambda_c^+ \) and \( \Sigma_c^{++} \to \Sigma_c^{++} \) form factors. These are the first results for the axial-vector transition form factors of the singly heavy baryons. For all other decay channels are related either by isospin symmetry or by flavor SU(3) symmetry. We found that the effects of the flavor SU(3) symmetry breaking were tiny. Thus, we neglected them to compute other observables. We also obtained the corresponding axial masses employing the dipole-type parametrizations for the form factors. We derived the axial-transition mean square radii. Using the values of the form factors at \( Q^2 = 0 \), we got the decay rates for the strong decays of \( \Sigma_c \), \( \Sigma_c^* \), and \( \Xi_c^* \). The decay rates of \( \Sigma_c \) and \( \Sigma_c^* \) decays are overestimated in comparison with the data but those of the \( \Xi_c^* \) decays are in good agreement with the data.
TABLE IV. Numerical results for the strong decay widths in comparison with those from various works.

| Decay modes       | Γ [MeV] | Albertus et al. [45] | Chen et al. [47] | Azizi et al. [48] | Cheng et al. [41] | Nagahiro et al. [46] | Can et al. [49] |
|-------------------|---------|----------------------|-----------------|-------------------|-------------------|----------------------|-----------------|
| Σ⁺⁺→Λ+c+π⁺       | 2.80    | 2.41 ± 0.07 ± 0.02   | 1.24            | 2.16 ± 0.85       | -                 | 4.27 − 4.33          | 1.65 ± 0.28 ± 0.30 |
| Σ⁺⁺→Λ+c+π⁰       | 3.39    | 2.79 ± 0.08 ± 0.02   | 1.40            | 2.16 ± 0.85       | 2.3 ± 0.1         | -                    | 1.65 ± 0.28 ± 0.30 |
| Σ⁺⁺→Λ+c+π⁻       | 2.76    | 2.37 ± 0.07 ± 0.02   | 1.24            | 2.16 ± 0.85       | 1.65 ± 0.28 ± 0.30 |
| Σ⁺⁺⁺→Λ+c+π⁺      | 21.0    | 17.52 ± 0.74 ± 0.12 | 11.9            | -                 | 14.5 ± 0.5        | 30.3 − 31.6         | -               |
| Σ⁺⁺⁺→Λ+c+π⁰      | 22.1    | 17.31 ± 0.73 ± 0.12 | 12.1            | -                 | 15.2 ± 0.6        | -                    | -               |
| Σ⁺⁺⁺→Λ+c+π⁻      | 21.0    | 16.90 ± 0.71 ± 0.12 | 11.9            | -                 | 14.7 ± 0.6        | -                    | -               |
| Ξ⁺⁺⁺→Ξ+c+π       | 2.12    | 1.84 ± 0.06 ± 0.01  | 0.64            | -                 | 2.4 ± 0.1         | -                    | -               |
| Ξ⁺⁺⁺→Ξ+c+π      | 2.30    | 2.07 ± 0.07 ± 0.01  | 0.54            | -                 | 2.5 ± 0.1         | -                    | -               |

ACKNOWLEDGMENTS

The authors are grateful to Gh.-S. Yang and J.-Y. Kim for fruitful discussions. We also want to express our gratitude to Y.-S. Jun for his contribution to the initial stage of the current work. The present work was supported by an Inha University Research Grant in 2022.
In this Appendix, the expressions for the axial-vector transition form factors in Eqs. (32), (33) will be given explicitly

\[ \begin{align*}
    A_0^{B \rightarrow B'}(Q^2) &= \frac{\sqrt{M_B}}{\sqrt{E_B' + M_B'}} \int d^3r j_0(|q||\mathbf{r}|) \left[ (N_c - 1) \phi_{\text{val}}^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_{\text{val}}(r) 
    + N_c \sum_n \phi_n^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_n(r) R_1(E_n) \right], \\
    B_0^{B \rightarrow B'}(Q^2) &= \frac{\sqrt{M_B}}{\sqrt{E_B' + M_B'}} \int d^3r j_0(|q||\mathbf{r}|) \left[ (N_c - 1) \sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_n(r) \cdot \langle n | \mathbf{\tau} | \text{val} \rangle 
    - \frac{1}{2} N_c \sum_{n,m} \phi_n^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_m(r) \cdot \langle m | \mathbf{\tau} | n \rangle R_5(E_n, E_m) \right], \\
    C_0^{B \rightarrow B'}(Q^2) &= \frac{\sqrt{M_B}}{\sqrt{E_B' + M_B'}} \int d^3r j_0(|q||\mathbf{r}|) \left[ (N_c - 1) \sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_n(r) \cdot \langle n | \mathbf{\tau} | \text{val} \rangle 
    - N_c \sum_{n,m_0} \phi_n^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_{m_0}(r) \langle m_0 | \mathbf{\tau} | n \rangle R_5(E_n, E_{m_0}) \right], \\
    D_0^{B \rightarrow B'}(Q^2) &= \frac{\sqrt{M_B}}{\sqrt{E_B' + M_B'}} \int d^3r j_0(|q||\mathbf{r}|) \left[ (N_c - 1) \sum_{n \neq \text{val}} \frac{\text{sgn}(E_n)}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(r) (\mathbf{\sigma} \times \mathbf{\tau}) \phi_n(r) \cdot \langle n | \mathbf{\tau} | \text{val} \rangle 
    + \frac{1}{2} N_c \sum_{n,m} \phi_n^\dagger(r) \mathbf{\sigma} \times \mathbf{\tau} \phi_m(r) \cdot \langle m | \mathbf{\tau} | n \rangle R_4(E_n, E_m) \right], \\
    H_0^{B \rightarrow B'}(Q^2) &= \frac{\sqrt{M_B}}{\sqrt{E_B' + M_B'}} \int d^3r j_0(|q||\mathbf{r}|) \left[ (N_c - 1) \sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_n(r) \cdot \langle n | \gamma^0 | \text{val} \rangle 
    + \frac{1}{2} N_c \sum_{n,m} \phi_n^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_m(r) \langle m | \gamma^0 | n \rangle R_2(E_n, E_m) \right], \\
    I_0^{B \rightarrow B'}(Q^2) &= \frac{\sqrt{M_B}}{\sqrt{E_B' + M_B'}} \int d^3r j_0(|q||\mathbf{r}|) \left[ (N_c - 1) \sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(r) \mathbf{\sigma} \phi_n(r) \cdot \langle n | \gamma^0 | \mathbf{\tau} \rangle \mathbf{\tau} | \text{val} \rangle 
    + \frac{1}{2} N_c \sum_{n,m} \phi_n^\dagger(r) \mathbf{\sigma} \phi_m(r) \cdot \langle m | \gamma^0 | \mathbf{\tau} | n \rangle R_2(E_n, E_m) \right], \\
    J_0^{B \rightarrow B'}(Q^2) &= \frac{\sqrt{M_B}}{\sqrt{E_B' + M_B'}} \int d^3r j_0(|q||\mathbf{r}|) \left[ (N_c - 1) \sum_{n \neq \text{val}} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_{n_0}(r) \langle n_0 | \gamma^0 | \text{val} \rangle 
    + N_c \sum_{n,m_0} \phi_n^\dagger(r) \mathbf{\sigma} \cdot \mathbf{\tau} \phi_{m_0}(r) \langle m_0 | \gamma^0 | n \rangle R_2(E_n, E_{m_0}) \right], \\
    A_2^{B \rightarrow B'}(Q^2) &= \frac{\sqrt{M_B}}{\sqrt{E_B' + M_B'}} \int d^3r j_0(|q||\mathbf{r}|) \left[ (N_c - 1) \phi_{\text{val}}^\dagger(r) \left\{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \right\}_1 \cdot \mathbf{\tau} \phi_{\text{val}}(r) 
    + N_c \sum_n \phi_n^\dagger(r) \left\{ \sqrt{2\pi} Y_2 \otimes \sigma_1 \right\}_1 \cdot \mathbf{\tau} \phi_n(r) R_1(E_n) \right].
\end{align*} \]
where the regularization functions are defined by

\[
\mathcal{R}_1(E_n) = \frac{-E_n}{2\sqrt{\pi}} \int_0^\infty \frac{du}{\sqrt{u}} e^{-uE_n^2},
\]

\[
\mathcal{R}_2(E_n, E_m) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{du}{\sqrt{u}} \frac{E_m e^{-uE_n^2} - E_n e^{-uE_m^2}}{E_n - E_m},
\]

\[
\mathcal{R}_4(E_n, E_m) = \frac{1}{2\pi} \int_0^\infty d\phi(u) \int_0^1 \frac{d\alpha}{\sqrt{\alpha(1-\alpha)}} e^{-\alpha uE_n^2 - (1-\alpha)uE_m^2} (1-\alpha)E_n - \alpha E_m,
\]

\[
\mathcal{R}_5(E_n, E_m) = \frac{\text{sgn}(E_n) - \text{sgn}(E_m)}{2(E_n - E_m)}.
\]

Here, \(|\text{val}\rangle\) and \(|n\rangle\) represent the state of the valence and sea quarks with the corresponding eigenenergies \(E_{\text{val}}\) and \(E_n\) of the one-body Dirac Hamiltonian \(h(U)\), respectively.
Appendix B: Matrix elements of the SU(3) Wigner $D$ function

In the following we list the results for the matrix elements of the relevant collective operators for the axial-vector transition form factors of the singly heavy baryons in Table IV to XII.

### TABLE V. The matrix elements of the single and double Wigner $D$ function operators when $a = 3$.

| $B \to B'$ | $\Sigma_c^+ \to \Lambda_c^+$ | $\Xi_c' \to \Xi_c$ | $\Sigma_c'^+ \to \Lambda_c^+$ | $\Xi_c' \to \Xi_c$ | $\Sigma_c^+ \to \Sigma_c$ | $\Xi_c^+ \to \Xi_c'$ | $\Omega_c^{*0} \to \Omega_c^0$ |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $\langle B' \mid D_{33}^{(8)} \mid B \rangle$ | $-\frac{1}{2\sqrt{6}}$ | $\frac{1}{2\sqrt{6}}T_3$ | $\frac{1}{2\sqrt{6}}T_3$ | $-\frac{1}{2\sqrt{6}}T_3$ | $-\frac{1}{2\sqrt{6}}T_3$ | $0$ |
| $\langle B' \mid D_{33}^J \hat{J}_3 \mid B \rangle$ | $0$ | $0$ | $0$ | $0$ | $\frac{1}{2\sqrt{6}}T_3$ | $0$ |
| $\langle B' \mid d_{abc} D_{33}^{(8)} \hat{J}_c \mid B \rangle$ | $-\frac{1}{4\sqrt{6}}$ | $-\frac{1}{4\sqrt{6}}T_3$ | $-\frac{1}{4\sqrt{6}}T_3$ | $\frac{1}{4\sqrt{6}}T_3$ | $\frac{1}{4\sqrt{6}}T_3$ | $\frac{1}{10\sqrt{2}}T_3$ | $0$ |
| $\langle B' \mid D_{33}^{(8)} D_{33}^{(8)} \mid B \rangle$ | $-\frac{1}{20\sqrt{6}}$ | $\frac{1}{5\sqrt{6}}T_3$ | $\frac{1}{20\sqrt{6}}T_3$ | $-\frac{1}{5\sqrt{6}}T_3$ | $-\frac{1}{5\sqrt{6}}T_3$ | $-\frac{1}{5\sqrt{6}}T_3$ | $0$ |
| $\langle B' \mid D_{33}^{(8)} D_{33}^J \hat{J}_3 \mid B \rangle$ | $-\frac{7}{20\sqrt{6}}$ | $0$ | $\frac{7}{20\sqrt{6}}$ | $0$ | $0$ | $-\frac{7}{20\sqrt{6}}T_3$ | $0$ |
| $\langle B' \mid d_{abc} D_{33}^{(8)} D_{33}^{(8)} \mid B \rangle$ | $-\frac{1}{10\sqrt{2}}$ | $\frac{1}{10\sqrt{2}}T_3$ | $-\frac{1}{10\sqrt{2}}T_3$ | $-\frac{1}{10\sqrt{2}}T_3$ | $-\frac{1}{10\sqrt{2}}T_3$ | $0$ |

### TABLE VI. The transition matrix elements of the single Wigner $D$ function operators coming from the $\Sigma^-$-plet component of the baryon wavefunctions when $a = 3$.

| $B \to B'$ | $\Sigma_c^+ \to \Lambda_c^+$ | $\Xi_c' \to \Xi_c$ | $\Sigma_c'^+ \to \Lambda_c^+$ | $\Xi_c' \to \Xi_c$ | $\Sigma_c^+ \to \Sigma_c$ | $\Xi_c^+ \to \Xi_c'$ | $\Omega_c^{*0} \to \Omega_c^0$ |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $\langle B'_{\Sigma^-} \mid D_{33}^{(8)} \mid B \rangle$ | $-\frac{1}{6\sqrt{6}}$ | $\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $0$ |
| $\langle B'_{\Sigma^-} \mid D_{33}^J \hat{J}_3 \mid B \rangle$ | $0$ | $0$ | $0$ | $0$ | $-\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $0$ |
| $\langle B'_{\Sigma^-} \mid d_{abc} D_{33}^{(8)} \hat{J}_c \mid B \rangle$ | $-\frac{1}{6\sqrt{6}}$ | $\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $0$ |
| $\langle B'_{\Sigma^-} \mid D_{33}^{(8)} D_{33}^{(8)} \mid B \rangle$ | $-\frac{1}{6\sqrt{6}}$ | $\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $0$ |
| $\langle B'_{\Sigma^-} \mid d_{abc} D_{33}^{(8)} D_{33}^{(8)} \mid B \rangle$ | $-\frac{1}{6\sqrt{6}}$ | $\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $-\frac{1}{6\sqrt{6}}T_3$ | $0$ |

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TABLE VII. The transition matrix elements of the single Wigner $D$ function operators coming from the $\Xi$-plet component of the baryon wavefunctions when $a = 3$.

| $B \rightarrow B'$ | $\Sigma_c^+ \rightarrow \Sigma_c^+$ | $\Xi_c^+ \rightarrow \Xi_c^+$ | $\Omega_c^0 \rightarrow \Omega_c^0$ |
|---------------------|---------------------------------|--------------------------|------------------------------|
| $\langle B_1^\prime | D^{(8)}_{33} | B \rangle$ | $-\frac{1}{9\sqrt{2}} T_3$ | $\frac{1}{2\sqrt{3}} T_3$ | 0 |
| $\langle B_1^\prime | D^{(8)}_{33} J_3 | B \rangle$ | $\frac{1}{\sqrt{6}} T_3$ | $\frac{2}{45} T_3$ | 0 |
| $\langle B_1^\prime | d_{ab} D^{(8)}_{3a} J_b | B \rangle$ | $\frac{1}{4\sqrt{2}} T_3$ | $\frac{2}{45} T_3$ | 0 |
| $\langle B_1^\prime | D^{(8)}_{3} | B_2 \rangle$ | $-\frac{1}{9\sqrt{2}} T_3$ | $-\frac{1}{2\sqrt{3}} T_3$ | 0 |
| $\langle B_1^\prime | D^{(8)}_{3} J_3 | B_2 \rangle$ | $-\frac{1}{\sqrt{6}} T_3$ | $\frac{2}{45} T_3$ | 0 |
| $\langle B_1^\prime | d_{ab} D^{(8)}_{3a} J_b | B_2 \rangle$ | $-\frac{1}{4\sqrt{2}} T_3$ | $-\frac{2}{45\sqrt{3}} T_3$ | 0 |

TABLE VIII. The matrix elements of the single and double Wigner $D$ function operators when $a = 4 + i5$.

| $B \rightarrow B'$ | $\Sigma_c^{++} \rightarrow \Xi_c^+$ | $\Sigma_c^{++} \rightarrow \Xi_c^+$ |
|---------------------|---------------------------------|--------------------------|
| $\langle B_2^\prime | D^{(8)}_{33} | B_6 \rangle$ | $\frac{1}{\sqrt{6}}$ | $-\frac{1}{\sqrt{6}}$ |
| $\langle B_2^\prime | D^{(8)}_{33} J_3 | B_6 \rangle$ | 0 | 0 |
| $\langle B_2^\prime | d_{bc} D^{(8)}_{3b} J_c | B_6 \rangle$ | $-\frac{1}{4\sqrt{2}}$ | $\frac{1}{4\sqrt{2}}$ |
| $\langle B_2^\prime | D^{(8)}_{3} D^{(8)}_{3} | B_6 \rangle$ | $-\frac{1}{20\sqrt{2}}$ | $\frac{1}{20\sqrt{2}}$ |
| $\langle B_2^\prime | d_{bc} D^{(8)}_{3b} D^{(8)}_{3} | B_{10} \rangle$ | $-\frac{1}{20\sqrt{2}}$ | $\frac{1}{20\sqrt{2}}$ |

TABLE IX. The transition matrix elements of the single Wigner $D$ function operators coming from the $\Xi$-plet component of the baryon wavefunctions when $a = 4 + i5$.

| $B \rightarrow B'$ | $\Sigma_c^{++} \rightarrow \Xi_c^+$ | $\Sigma_c^{++} \rightarrow \Xi_c^+$ |
|---------------------|---------------------------------|--------------------------|
| $\langle B_3^\prime | D^{(8)}_{33} | B \rangle$ | $\frac{1}{18\sqrt{10}}$ | $-\frac{1}{18\sqrt{10}}$ |
| $\langle B_3^\prime | D^{(8)}_{33} J_3 | B \rangle$ | 0 | 0 |
| $\langle B_3^\prime | d_{ab} D^{(8)}_{3a} J_b | B \rangle$ | $\frac{1}{12\sqrt{10}}$ | $-\frac{1}{12\sqrt{10}}$ |
| $\langle B_3^\prime | D^{(8)}_{3} | B_3 \rangle$ | $\frac{1}{2\sqrt{15}}$ | $\frac{1}{\sqrt{30}}$ |
| $\langle B_3^\prime | D^{(8)}_{3} J_3 | B_3 \rangle$ | 0 | 0 |
| $\langle B_3^\prime | d_{ab} D^{(8)}_{3a} J_b | B_3 \rangle$ | $-\frac{1}{4\sqrt{15}}$ | $\frac{1}{2\sqrt{30}}$ |
TABLE XII. The transition matrix elements of the single Wigner D function operators when $a = 4 - i 5$.

| $B \to B'$ | $\Xi_c^0 \to \Lambda_c^+$ | $\Omega_c^0 \to \Xi_c^+$ | $\Xi_c^+ \to \Lambda_c^+$ | $\Omega_c^+ \to \Xi_c^+$ | $\Xi_c^{++} \to \Sigma_c^{++}$ | $\Omega_c^{+0} \to \Xi_c^{++}$ |
|-------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\langle B' | D_{\Xi^0 - 3}^{(8)} | B \rangle$ | $- \frac{1}{3 \sqrt{7}}$ | $\frac{1}{2 \sqrt{7}}$ | $\frac{1}{2 \sqrt{7}}$ | $- \frac{1}{3 \sqrt{7}}$ | $- \frac{1}{3 \sqrt{7}}$ |
| $\langle B' | D_{\Xi^0 - 8}^{(8)} J_3 | B \rangle$ | $0$ | $0$ | $0$ | $1 \frac{8 \sqrt{3}}{5}$ | $1 \frac{5 \sqrt{6}}{6}$ |
| $\langle B' | d_{a_{bc}} D_{\Xi^0 - b}^{(8)} \hat{J}_3 | B \rangle$ | $\frac{1}{8 \sqrt{3}}$ | $- \frac{1}{4 \sqrt{6}}$ | $- \frac{1}{4 \sqrt{6}}$ | $0$ | $1 \frac{10 \sqrt{2}}{2}$ | $1 \frac{10 \sqrt{2}}{2}$ |
| $\langle B' | D_{\Xi^0 - 8}^{(8)} | B \rangle$ | $\frac{\sqrt{3}}{40}$ | $0$ | $- \frac{\sqrt{5}}{40}$ | $0$ | $1 \frac{90 \sqrt{2}}{3}$ | $1 \frac{30 \sqrt{2}}{3}$ |
| $\langle B' | D_{\Xi^0 - 3}^{(8)} | B \rangle$ | $\frac{1}{40 \sqrt{3}}$ | $- \frac{1}{10 \sqrt{6}}$ | $\frac{1}{20 \sqrt{6}}$ | $0$ | $1 \frac{90 \sqrt{2}}{3}$ | $1 \frac{30 \sqrt{2}}{3}$ |
| $\langle B' | d_{a_{bc}} D_{\Xi^0 - a}^{(8)} D_{\Xi^0 - b}^{(8)} | B \rangle$ | $\frac{1}{30}$ | $- \frac{1}{20 \sqrt{2}}$ | $\frac{1}{20 \sqrt{2}}$ | $\frac{1}{30}$ | $0$ | $0$ |

TABLE XII. The transition matrix elements of the single Wigner D function operators when $a = 4 - i 5$.

| $B \to B'$ | $\Xi_c^0 \to \Lambda_c^+$ | $\Omega_c^0 \to \Xi_c^+$ | $\Xi_c^+ \to \Lambda_c^+$ | $\Omega_c^+ \to \Xi_c^+$ | $\Xi_c^{++} \to \Sigma_c^{++}$ | $\Omega_c^{+0} \to \Xi_c^{++}$ |
|-------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\langle B' | D_{\Xi^0 - 3}^{(8)} | B \rangle$ | $\frac{\sqrt{3}}{40}$ | $0$ | $- \frac{\sqrt{5}}{40}$ | $0$ | $1 \frac{90 \sqrt{2}}{3}$ | $1 \frac{30 \sqrt{2}}{3}$ |
| $\langle B' | D_{\Xi^0 - 8}^{(8)} J_3 | B \rangle$ | $- \frac{2 \sqrt{2}}{15 \sqrt{3}}$ | $- \frac{1}{15}$ | $\frac{2 \sqrt{2}}{15 \sqrt{3}}$ | $\frac{1}{15 \sqrt{3}}$ | $- \frac{1}{30 \sqrt{3}}$ | $- \frac{1}{15 \sqrt{3}}$ |
| $\langle B' | d_{a_{bc}} D_{\Xi^0 - a}^{(8)} J_3 | B \rangle$ | $\frac{2}{45}$ | $\frac{1}{15 \sqrt{3}}$ | $\frac{2}{45}$ | $- \frac{1}{15 \sqrt{3}}$ | $- \frac{1}{30 \sqrt{3}}$ | $- \frac{1}{15 \sqrt{3}}$ |
