Corona and XUV emission modelling of the Sun and Sun-like stars

Munehito Shoda1© and Shinsuke Takasao2©

1 National Astronomical Observatory of Japan, National Institutes of Natural Sciences, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan
e-mail: munehito.shoda@nao.ac.jp
2 Department of Earth and Space Science, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan

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ABSTRACT

The X-ray and extreme-ultraviolet (EUV) emissions from low-mass stars significantly affect the evolution of the planetary atmosphere. However, it is observationally difficult to constrain the stellar high-energy emission because of the strong interstellar extinction of EUV photons. In this study, we simulate the XUV (X-ray plus EUV) emission from Sun-like stars by extending the solar coronal heating model that self-consistently solves, with sufficiently high resolution, the surface-to-coronal energy transport, turbulent coronal heating, and coronal thermal response by conduction and radiation. The simulations are performed with a range of loop lengths and magnetic filling factors at the stellar surface. With the solar parameters, the model reproduces the observed solar XUV spectrum below the Lyman edge, thus validating its capability of predicting the XUV spectra of other Sun-like stars. The model also reproduces the observed nearly linear relation between the unsigned magnetic flux and the X-ray luminosity. From the simulation runs with various loop lengths and filling factors, we also find a scaling relation, namely log $L_{\text{EUV}} = 9.93 + 0.67 \log L_x$, where $L_{\text{EUV}}$ and $L_x$ are the luminosity in the EUV (100 Å $< \lambda \leq 912$ Å) and X-ray (5 Å $< \lambda \leq 100$ Å) range, respectively, in cgs. By assuming a power–law relation between the Rossby number and the magnetic filling factor, we reproduce the renowned relation between the Rossby number and the X-ray luminosity. We also propose an analytical description of the energy injected into the corona, which, in combination with the conventional Rosner–Tucker–Vaiana scaling law, semi-analytically explains the simulation results. This study refines the concepts of solar and stellar coronal heating and derives a theoretical relation for estimating the hidden stellar EUV luminosity from X-ray observations.

Key words. Sun: corona – stars: coronae – X-rays: stars – ultraviolet: stars

1. Introduction

The Sun has an aura of hot plasma called the corona, which has a temperature of a few million kelvin (Edlén 1943). The image of the corona has been captured by the Atmospheric Imaging Assembly (Lemen et al. 2012) of NASA’s Solar Dynamics Observatory (Pesnell et al. 2012). The corona is not unique to the Sun and has been observed to surround low-mass main-sequence stars in general (Güdel et al. 1997). Due to its high temperature, the corona is the principal source of stellar XUV (X-ray plus EUV; extreme ultraviolet) emissions.

Stellar XUV emissions drive the expansion and thermal escape of planetary atmospheres (Vidal-Madjar et al. 2003; Lecavelier des Etangs et al. 2012; Owen & Wu 2013; Ehrenreich et al. 2015; Aigrain et al. 2017). Therefore, describing the stellar XUV emission in terms of the stellar fundamental parameters (luminosity, mass, radius, etc.) is essential in understanding the evolution of a planet and its habitability. Since stellar XUV emissions decrease over time (Güdel et al. 1997; Ribas et al. 2005; Telleschi et al. 2005; Claire et al. 2012; Guinan et al. 2016), presumably in response to the stellar spin-down (Kraft 1967; Skumanich 1972; Barnes 2003; Irwin & Bouvier 2009; Matt et al. 2015) caused by magnetised stellar wind (Weber & Davis 1967; Kawaler 1988; Shoda et al. 2020), the long-term evolution of the XUV activity of the host star needs to be described as well (Tu et al. 2015; Johnstone et al. 2021).

While the observational characteristics of stellar X-ray emissions have been established, including their correlations with the rotation (Pallavicini et al. 1981; Wright et al. 2011) and magnetic field of the star (Pevtsov et al. 2003; Vidotto et al. 2014; Kochukhov et al. 2020), stellar EUV emissions are poorly characterised because they are difficult to observe. Extreme-ultraviolet photons suffer from strong absorption by the interstellar medium (Rumph et al. 1994), for which stellar EUV spectra are observable only for nearby stars in a limited range of wavelengths ($\leq 360$ Å, Ribas et al. 2005; Johnstone et al. 2021). One thus needs to indirectly estimate or reconstruct the EUV spectrum of a star from other observables. Proposed reconstruction methods include the inversion of the differential emission measure from UV and/or X-ray observations (Sanz-Forcada et al. 2011; Duvvuri et al. 2021), the empirical correlation between other observable lines and XUV emission (Linsky et al. 2014; Youngblood et al. 2017; France et al. 2018; Sreejith et al. 2020) and/or a combination of them (Diamond-Lowe et al. 2021). However, the empirical estimations of stellar EUV emission are based on observations with large uncertainty and a limited number of samples, and therefore require further validation from different perspectives.

To circumvent the intrinsic difficulty of stellar EUV observation, this study uses the solar-stellar connection to theoretically estimate the stellar XUV emission. The solar-stellar theoretical connection is a natural strategy for predicting stellar properties. For example, by extending the solar coronal theory, Shibata & Yokoyama (2002) modelled the X-ray characteristics of the stellar coronae, which was later extended by Takasao et al. (2020), who considered the size distribution of active regions. In this study, by extending the solar atmospheric model, the structure of the upper stellar atmosphere and the XUV emission are
predicted. To this end, the coronal heating problem must be explicitly solved.

To solve the coronal heating problem, the following three issues must be addressed (Klimchuk 2006). The first is energy generation and transfer. The source of the coronal thermal energy probably comes from the magneto-convection on the surface (Steiner et al. 1998). The energy generation and transfer to the corona, possibly in the form of Alfvén waves (Alfvén 1947; Osterbrock 1961; Kudoh & Shibata 1999; De Pontieu et al. 2007; McIntosh et al. 2011; Srivastava et al. 2017), needs to be solved. The second is energy dissipation in the corona. The magnetic (and kinetic) energy in the corona needs to dissipate to sustain the high-temperature corona. The field-braiiding process must be considered as a promising mechanism of magnetic-energy dissipation (Parker 1972, 1988). The third is thermal response to the heating. The density and temperature of a coronal loop are determined by the energy balance among heating, conduction, and radiation. The Rosner-Tucker-Vaiana (RTV) scaling law (Rosner et al. 1978) originates from the thermal response to coronal heating. Thus, the RTV scaling law or its generalised form (Serio et al. 1998; Zhuleku et al. 2020) should be reproduced by the model (Antolin & Shibata 2010). For these issues, a model of the coronal heating should: (1) include the photosphere (stellar surface) and chromosphere, (2) appropriately consider the field-braiiding process in the corona, and (3) implement the thermal conduction and radiative cooling.

Classically, numerical models of a corona have often focused on the thermal responses to heating events by one-dimensional (expanding) flux-tube models (Antiochos & Sturrock 1978; Peres et al. 1982; Antiochos et al. 1999). A zero-dimensional model of the coronal thermal evolution is also proposed (Klimchuk et al. 2008). The advancement of numerical techniques and increase in computational power have made models highly sophisticated. Several models deal with the realistic energy generation by explicitly solving the magneto-convection (Hansteen et al. 2015; Rempel 2017). Other models have focused more on energy dissipation with simpler numerical settings (Moriyasu et al. 2004; Rappazzo et al. 2008; van Ballegooijen et al. 2011; Dahilburg et al. 2016). These models have predicted that the signature of coronal heating could be explained by convection-driven energy injection. By extending these studies, we aimed to construct a stellar coronal model that satisfies the three requirements.

In deriving the X-ray and EUV spectra from simulations, care needs to be taken in the spatial resolution at the transition region (the temperature-jump region between the chromosphere and the corona). The numerical resolution around the transition region is found to significantly affect the coronal density (Bradshaw & Cargill 2013) and the coronal emission measure distribution (EMD). Because the coronal emission is proportional to the EMD, the XUV emission predicted by a simulation significantly depends on the numerical resolution. The required resolution is of the order of kilometres or less, which is impractical in realistic three-dimensional (3D) simulations. Previous studies have attempted to solve this ‘transition-region problem’ by introducing the artificial broadening of the transition region by tuning the magnitude of radiative cooling and thermal conduction (Johnston et al. 2017, 2021; Johnston & Bradshaw 2019; Iijima & Imada 2021). However, this treatment may yield an unrealistic EMD in the transition-region temperature and therefore is inappropriate for the calculation of the XUV emission that includes a significant contribution from the transition region.

Considering the difficulty of the transition-region problem, we perform a series of 1D, high-resolution magnetohydrodynamic (MHD) simulations for a wide range of parameters. This 1D model facilitates a coronal loop simulation with sufficiently high numerical resolution. The field-braiiding process (or turbulence), which is essentially 3D, must be appropriately modelled when solving the coronal heating problem by 1D simulation. In this study, an approximated formulation of turbulent dissipation, developed in previous studies (Dmitruk et al. 2002; Shoda et al. 2018), is employed as it is likely to reproduce the average heating rate of the coronal loop (van Ballegooijen et al. 2011). For simplicity, we focus on the Sun-like stars that exhibit solar mass, luminosity, radius, and metallicity.

The rest of the manuscript is structured as follows. In Sect. 2, the coronal model and numerical methods are detailed. Section 3 produces the numerical results, focusing on the dependences of coronal properties on the loop length and magnetic filling factor that are likely to vary with the star (Reale & Micela 1998; Reale et al. 2004; Reiners et al. 2009; See et al. 2019). The XUV spectra obtained from the simulations are also presented. In Sect. 4, analytical arguments on the energy flux injected into the corona are presented. In Sect. 5, the interpretations and limitations of the proposed model are discussed. Section 6 summarises the study. Several details are provided in the appendix, including the resolution dependence of the model (see Appendix B).

2. Model

2.1. Model overview and notation

As mentioned earlier, the aim of this study is to model the XUV (X-ray+EUV) emissions from the Sun and Sun-like stars (including young Sun) with a range of magnetic activity level. To this end, the luminosity, mass, radius, and metallicity of the stars are fixed to the solar value.

\[
L = L_\odot, \quad M = M_\odot, \quad R = R_\odot, \quad Z = Z_\odot. \tag{1}
\]

Dependence on metallicity should appear in the radiative loss function (Sect. 2.5) and spectrum calculation (Sect. 3.1). We study the dependence of coronal properties on the coronal loop length and the filling factor of magnetic fields. Because the filling factor tends to increase with the stellar rotation rate (or the Rossby number, Saar 2001; Reiners et al. 2009), the coronal dependence on the stellar rotation rate is implicitly investigated.

We model a single coronal loop rooted in the stellar surface, and self-consistently solve the energetics and dynamics inside the loop. In other words, we solve the time-dependent, 1D MHD equations for an expanding coronal loop. The differential emission measure (DEM) of a single coronal loop is directly obtained from the simulation, which is then converted to the XUV spectrum by prescribing the chemical composition and integrating the continuum and line emissions as a function of wavelength using the CHIANTI atomic database version 10.0 (Dere et al. 1997; Del Zanna et al. 2021).

Notations of the coordinates and constant parameters used in this study are listed in Table 1. \(X_2\) denotes the solar value and \(X\) the value of \(X\) measured in the photosphere.

2.2. Model of the closed flux tube

A single coronal loop is modelled by a 1D expanding flux tube rooted in the photosphere. Figure 1 illustrates a schematic of the model. As shown in Table 1, the coordinate along the axis of the flux tube is denoted by \(s\). The spatial variation along the loop is assumed to be non-zero, that is, \(\partial/\partial s \neq 0\). Similar 1D flux tube models were used in the models of solar spicules.
Table 1. Notations of the coordinates ($r$ and $s$) and constant parameters.

| Notation | Meaning |
|----------|---------|
| $r$      | Radial distance from the stellar centre |
| $s$      | Coordinate along the flux-tube axis |
| $G$      | Gravitational constant |
| $k_B$    | Boltzmann constant |
| $m_H$    | Hydrogen mass |
| $m_e$    | Electron mass |
| $h$      | Planck constant |
| $M_\odot$ | Solar mass |
| $R_\odot$ | Solar radius |
| $T_\odot$ | Solar effective temperature |
| $\Omega_\odot$ | Solar angular rotation rate |

Notes. We note that subscript $\odot$ denotes the solar fundamental parameter.

radial distance $r$

field-aligned coordinate $s$

closed flux tube

$r = R_\odot$ $s = 0$ $s = 2l_{\text{loop}}$

transition region

photosphere

Fig. 1. Schematic picture of the system. One-dimensional dynamics along the axis of the closed flux tube is simulated. The axis is indicated by the dotted line, while the flux tube surface is denoted by green lines. The flux tube is intended to be nearly vertical and super-radially expanding. The geometry of the loop is defined by the filling factor $f$ and radial distance $r$ as functions of field-aligned distance $s$.

(Hollweg et al. 1982; Kudoh & Shibata 1999; Matsumoto & Shibata 2010), solar and stellar coronal heating (Moriyasu et al. 2004; Washinoe & Suzuki 2019), and solar wind acceleration (Suzuki & Inutsuka 2005; Shoda et al. 2018).

Hereinafter, the two perpendicular directions shall be denoted by $x$ and $y$. Thus, the local $xy$ plane is perpendicular to the axis of the loop. The flux-tube expansion is incorporated through the scale factors in the $x$ and $y$ directions: $h_{x,y}$. For simplicity, the loop is assumed to expand isotropically in the perpendicular directions. In terms of scale factors, the isotropic expansion is represented by

$$h_x = h_y \propto \sqrt{A(s)},$$  

where $A(s)$ is the cross section of the coronal loop. As $h_s = 1$ by definition, Eq. (2) results in

$$\nabla \cdot \mathbf{X} = \frac{1}{A(s)} \frac{\partial}{\partial s} (X_s A(s)),$$

$$\nabla \times \mathbf{X} = \frac{1}{\sqrt{A(s)}} \frac{\partial}{\partial s} \left( X_y \sqrt{A(s)} \right) \mathbf{e}_y - \frac{1}{\sqrt{A(s)}} \frac{\partial}{\partial s} \left( X_x \sqrt{A(s)} \right) \mathbf{e}_x$$

for any vector field $\mathbf{X}$, where $\mathbf{e}_{x,y}$ represent the unit vectors in the $x,y$ directions. The 1D spherical coordinate system is reproduced as a special case of $A(s) = s^2$.

The flux tube (Fig. 1) expands in the chromosphere as a response to the exponential decrease in the ambient gas pressure (Cranmer & van Ballegooijen 2005; Ishikawa et al. 2021). As a result of this expansion, the filling factor of the magnetic field (flux tube) $f$ should increase nearly exponentially with altitude. Under this assumption, we model the filling factor $f$ as

$$f = \min \left[ 1, f_s \exp \left( \frac{r - R_\odot}{H_{\text{mag}}} \right) \right], \quad H_{\text{mag}} = c_{\text{mag}} H_s,$$

where $f_s$ is the magnetic filling factor on the photosphere and

$$H_s = \frac{k_B T_\odot R_\odot^2}{GM_\odot m_H}$$

is the pressure scale height at the photosphere. By this formulation, we assume that the loop expands only in the chromosphere and exhibits a uniform cross section in the corona, which is supported by some solar observations (Klimchuk et al. 1992, but see a recent discussion by Malanushenko et al. 2021). Given that the pressure scale height is uniform from the photosphere up to the chromosphere, $c_{\text{mag}} = 2$ yields a flux-tube expansion with a constant plasma beta in altitude. In this work, we set $c_{\text{mag}} = 2.5$ that realises a slightly low-beta chromosphere. We have confirmed that the choice of $c_{\text{mag}}$ does not have a significant influence over the simulation results.

When the coronal loop extends to a region far above the surface, the flux tube also undergoes the radial expansion $\propto r^2$, where $r$ is the radial distance from the stellar centre. Considering the chromospheric and radial expansions, the cross section $A$ is expressed as

$$A \propto r^2 f.$$  

We define the inclination of the flux tube by prescribing $r$ as a function of $s$. We consider nearly vertical flux tubes, so that the vertical line of sight is nearly aligned with the axis of the flux tube (Fig. 1). It is easier to calculate the DEM along the vertical line of sight for this structure. In particular, we set

$$\frac{dr}{ds} = \tanh \left[ \frac{10 \left( l_{\text{loop}} - s \right)}{l_{\text{loop}}} \right], \quad r_{(s=0)} = R_\odot$$

where $l_{\text{loop}}$ is the half-loop length. The actual shape of the flux-tube axis is displayed in Fig. 2. Combining Eqs. (4)–(7), the cross section $A(s)$ is well defined as a function of $s$. 

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S = \frac{1}{L^2} \left( \frac{\rho}{\gamma - 1} + \frac{\rho v^2}{2} \right) \rho \int_{\nu = 0}^{\lambda} \frac{GM_e}{r^2} \frac{dr}{ds} + Q_{\text{cnd}} + Q_{\text{rad}}.

\] where \( L^{-1} = \frac{d}{ds} \ln \left( r^2 f \right) \) denotes the length scale of the flux-tube expansion. The conduction term \( Q_{\text{cnd}} \) is defined in terms of conductive flux \( q_{\text{cnd}} \) as

\[ Q_{\text{cnd}} = -\frac{1}{r^2 f} \frac{\partial}{\partial s} \left( q_{\text{cnd}} r^2 f \right). \] (13)

Because the mean free path of an electron is generally smaller than the system size, the Spitzer–Härm flux (Spitzer & Härm 1953) is applied to \( q_{\text{cnd}} \):

\[ q_{\text{cnd}} = \frac{|B_z|}{|B|} \kappa_{\text{SH}} T^{5/2} \frac{\partial T}{\partial s}, \] (14)

where \( \kappa_{\text{SH}} = 10^{-6} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-7/2} \). The radiative cooling \( Q_{\text{rad}} \) and turbulent dissipation \( D_{\nu} \) are described in Sects. 2.5 and 2.6, respectively. Because turbulence is considered not as an external force but as a dissipative process, the losses of the kinetic and magnetic energies by turbulence are locally balanced by the gain in the internal energy. Thus, the presence of the turbulence terms does not affect the conservation of the total energy. For the same reason, we do not explicitly consider the numerical dissipation of velocity and magnetic field in the energy equation. We note that the numerical dissipation is unlikely to be the dominant heating mechanism because the coronal Alfvén wave, which has a typical wavelength of \( \sim 100 \text{ Mm} \), is resolved by a sufficiently fine grid in the corona (100 km).

### 2.3. Basic equations

The MHD equations with the equation of state of partially ionised hydrogen, gravity, thermal conduction, radiative cooling, and phenomenology of turbulent heating are selected as the basic equations of the model, which are expressed in the form of conservation law (for derivation, see Appendix A)

\[
\frac{\partial}{\partial t} U + \frac{1}{r} \frac{\partial}{\partial s} \left( F \nu f \right) = S.
\] (8)

The conserved variables \( U \) and the corresponding fluxes \( F \) are given by

\[
U = \begin{pmatrix} \rho \\ \rho v_x \\ \rho v_y \\ \rho v_z \\ B_x \\ B_y \\ B_z \\ e \end{pmatrix}, \quad F = \begin{pmatrix} \rho v_x \\ \rho v_x v_y - B_y B_z / (4\pi) \\ \rho v_x v_z - B_z B_x / (4\pi) \\ v_x B_y - v_y B_x \\ (e + p_T) v_x - B_x (v_x B_x + B_y B_z / (4\pi)) \end{pmatrix},
\] (9)

where

\[
v_x = v_x e_x + v_y e_y, \quad B_x = B_x e_x + B_y e_y,
\] (10)

\[
p_T = \rho + \frac{B^2}{8\pi}, \quad e = e_{\text{int}} + \frac{1}{2} \rho v^2 + \frac{B^2}{8\pi}.
\] (11)

Here, \( e_{\text{int}} \) denotes the internal energy per unit volume and is defined in Sect. 2.4. The source term \( S \) is given by

\[
S = \left( \frac{1}{2} \rho v_x^2 \right) \left( \frac{1}{L - \rho \frac{GM_e}{r^2}} \frac{dr}{ds} \right) - \frac{1}{2} \frac{\partial}{\partial s} \left( \frac{\rho v_x^2}{r^2} \right) + \frac{\partial D_x}{\partial s} + \left( \frac{1}{2} \rho v_y v_y - B_y B_z / (4\pi) \right) + \frac{\partial D_y}{\partial s} + \left( \frac{1}{2} \rho v_z v_z - B_z B_x / (4\pi) \right) + \frac{\partial D_z}{\partial s} + \left( (v_x B_y - v_y B_x) + \sqrt{4\pi \rho D_x} \right) + \left( (v_y B_y - v_y B_y) + \sqrt{4\pi \rho D_y} \right) + \left( (v_z B_z - v_z B_z) + \sqrt{4\pi \rho D_z} \right) + Q_{\text{cnd}} + Q_{\text{rad}}.
\] (12)
2.5. Radiation

The radiative cooling rate per unit volume $Q_{\text{rad}}$ is given by

$$Q_{\text{rad}} = \xi_{\text{rad}} Q_{\text{rad}}^{\text{thick}} + (1 - \xi_{\text{rad}}) Q_{\text{rad}}^{\text{thin}},$$  \hspace{1cm} (19)

$$\xi_{\text{rad}} = 1 - \exp\left(-\frac{\rho_s}{\rho_{\text{rad}}}\right), \hspace{1cm} \rho_{\text{rad}}/\rho_s = 0.1,$$  \hspace{1cm} (20)

where $\rho_s$ is the pressure at the surface (photosphere) and $Q_{\text{rad}}^{\text{thick}}$ and $Q_{\text{rad}}^{\text{thin}}$ approximate the optically thick and thin cooling rates, respectively. The optically thick and thin functions are seamlessly connected via $\xi_{\text{rad}}$. Instead of solving the radiative transfer, we model $Q_{\text{rad}}^{\text{thick}}$ and $Q_{\text{rad}}^{\text{thin}}$ as follows.

The radiative heating and cooling are approximately balanced near the photosphere to maintain a nearly constant surface temperature. The optically thick radiative loss is approximated by an exponential cooling function that forces the local temperature to approach the reference value (Gudiksen & Nordlund 2005):

$$Q_{\text{rad}}^{\text{thick}} = \frac{1}{\tau} (e_{\text{ref}} - e_{\text{int}}), \hspace{1cm} \tau = 0.1 \left(\frac{2}{\rho_s}\right)^{1/2},$$  \hspace{1cm} (21)

where $\rho_s$ is the photospheric mass density and $e_{\text{ref}}$ is the reference internal energy density corresponding to the reference temperature $T_{\text{ref}}$. We simply assume $T_{\text{ref}} = T_\odot$, namely, the stellar atmosphere exhibits isothermal behaviour in the absence of other cooling/heating mechanisms.

The optically thin cooling is expressed in terms of the radiative loss function $\Lambda(T)$ by $n_{\text{eff}} \Lambda(T)$. In this work, we define the loss function over a wide range of temperature $(10^3$ K $\leq T \leq 10^7$ K) as follows. In the high-temperature range ($T \geq 1.5 \times 10^6$ K), the cooling rate is referred from the CHIANTI atomic database with the photospheric opacities (no first ionisation potential (FIP) effect). In the low-temperature range ($T \leq 1.0 \times 10^5$ K), the cooling function is deduced by Goodman & Judge (2012), which partially consider the non-LTE effect. In the intermediate-temperature range $(1.0 \times 10^4$ K $< T < 1.5 \times 10^5$ K), a bridging law between two loss functions are employed following the method of Iijima (2016). The chromospheric heating effect by backward coronal radiation is still missing in $\Lambda(T)$ defined above. To introduce this effect, we quench $\Lambda(T)$ in the chromospheric temperature range, which gives

$$Q_{\text{rad}}^{\text{thin}} = n_{\text{eff}} \Lambda(T), \hspace{1cm} \Lambda(T) = \Lambda(T) \exp\left(-\frac{T_{\text{cho}}}{T}\right),$$  \hspace{1cm} (22)

where $T_{\text{cho}} = 2.0 \times 10^5$ K. The effective radiative loss function $\Lambda_{\text{eff}}(T)$ is displayed in Fig. 3, along with the original radiative loss function $\Lambda(T)$ and the CHIANTI loss function defined in $T \geq 10^4$ K.

2.6. Phenomenological model of coronal turbulence

Although the mechanism of the coronal heating is debated, it is of no doubt that magnetic field feeds heat to the corona that maintains the million-Kelvin temperature by dissipation. The formation of tangential discontinuities (electric current sheets) in response to the continuous shuffling of the foot points of coronal magnetic fields is a plausible mechanism of magnetic-field dissipation (Parker 1972, 1983; Sturrock & Uchida 1981; van Ballegooijen 1986; Galsgaard & Nordlund 1996). The ubiquitous current-sheet formation should lead to small-scale impulsive energy release, which is likely to feed a sufficient amount of energy to the corona, possibly in the form of micro- and nano-flares (Parker 1988; Shimizu 1995; Aschwanden & Parnell 2002).

The formation of current sheets can be interpreted as turbulent cascading (Rappazzo et al. 2007, 2008; Verdini et al. 2012). Because the coronal loop is threaded by a strong mean magnetic field, the MHD turbulence evolving in the coronal loop can be accurately approximated by the reduced-MHD turbulence, in which the energy cascades preferentially in the perpendicular direction (Shebalin et al. 1983; Cho & Vishniac 2000; Cho & Lazarian 2003). The energy-cascading (or heating) rate of the reduced-MHD turbulence, $Q_{\text{heat}}$, is precisely approximated by the mean-field quantities as follows (Hossain et al. 1995; Matthaeus et al. 1999; Dmitruk et al. 2002; Verdini & Velli 2007)

$$Q_{\text{heat}} \approx c_d \delta z_{\perp}^2 + z_{\perp}^2, \hspace{1cm} (23)$$

where $z_{\perp}$ denotes the (root-mean-squared (RMS)) amplitude of the perpendicular Elsässer variables and $\lambda_z$ is the correlation length of the Elsässer variable (Alfvén wave) perpendicular to the mean field. $c_d$ is a dimensionless parameter. By this approximation, we estimate the averaged heating rate using the coronal turbulence (field braiding).

The approximated heating rate in Eq. (23) is implemented by adding the source terms $D_{s,y}^p, D_{s,y}^b$ given by Shoda et al. (2018).

$$D_{s,b}^p = - \frac{c_d}{4\lambda_z} \left( |v_{s,b}^p| z_{s,b}^2 + |v_{s,b}^b| z_{s,b}^2 \right), \hspace{1cm} (24)$$

$$D_{s,b}^b = - \frac{c_d}{4\lambda_z} \left( |v_{s,b}^b| z_{s,b} - |v_{s,b}^p| z_{s,b} \right), \hspace{1cm} (25)$$

where $z_{s,b} = \nabla_s \times B_{s,b}/\sqrt{4\pi\rho}$. The role of these terms is explained below. Without the conservation part $\partial (r^2 F)/\partial s$, the perpendicular components of the equation of motion and induction equation are expressed as

$$\partial \frac{\partial}{\partial t} (pv_{s,b}) = \rho D_{s,b}^p, \hspace{1cm} \frac{\partial}{\partial t} B_{s,b} = \sqrt{4\pi\rho} D_{s,b}^b. \hspace{1cm} (26)$$
In the limit of the reduced-MHD approximation (time-independent density, $\partial \rho / \partial t = 0$), Eqs. (24)–(26) are reduced to
\[ \frac{\partial}{\partial t} \mathbf{e}_{\perp} = -c_d \rho \sum_{i=x,y} \left[ \frac{|z_i|^2 + |z^*_i|^2}{4 \lambda_{\perp}} \right], \tag{27} \]
which yields the energy conservation law of
\[ \frac{\partial}{\partial t} \varepsilon_{\perp} = -c_d \rho \sum_{i=x,y} \left[ |z_i|^2 + |z^*_i|^2 \right], \tag{28} \]
where $\varepsilon_{\perp}$ is the sum of the kinetic and magnetic energies emerging from the fluctuations of the perpendicular components:
\[ \varepsilon_{\perp} = \frac{1}{4} (z_{\perp}^2 + z_{\perp}^*)^2 = \frac{1}{2} \rho v_{\perp}^2 + \frac{B_{\perp}^2}{8\pi}. \tag{29} \]
Comparing Eqs. (23) and (28), one obtains
\[ \frac{\partial}{\partial t} \varepsilon_{\perp} \approx -Q_{\text{heat}}, \tag{30} \]
indicating that the energy dissipation by the reduced-MHD turbulence is considered appropriately.

The perpendicular correlation length is assumed to increase with the flux-tube radius, that is,
\[ \lambda_{\perp} = \lambda_{\perp, s} \sqrt{\frac{A}{A_s}} = \lambda_{\perp, s} \frac{r}{R_\odot} \sqrt{\frac{f}{f_s}}, \tag{31} \]
where the perpendicular correlation length at the photosphere is set equal to the typical width of the inter-granular lane: $\lambda_{\perp, s} = 150 \text{ km}$. However, the best possible free parameter $c_d$ is still debated. In this study, we infer $c_d = 0.1$ from the previous studies of the solar-wind turbulence (van Ballegooijen & Asgari-Targhi 2017; Chandran & Perez 2019; Verdini et al. 2019).

### 2.7 Boundary condition and simulation setting

Both boundaries of the simulation domain are located at the photosphere, and the photospheric temperature is fixed to the effective temperature:
\[ T_s = T_\odot = 5.77 \times 10^3 \text{ K}. \tag{32} \]
The photospheric magnetic field is known to form localised kilo-Gauss patches (Spruit & Zweibel 1979; Tsuneta et al. 2008). These patches are likely to be in thermal equipartition, which equates the gas and magnetic pressures. For the non-magnetised photosphere, the thermal equipartition field is given by
\[ B_{\text{eq}} = 1.34 \times 10^4 \text{ G}. \tag{33} \]
In the magnetised photosphere, as the deeper region tends to be observed (e.g. Keller et al. 2004), the ambient gas is likely to exhibit larger pressure than the non-magnetised photosphere. Thus, the equipartition magnetic field should be larger than this equipartition value. Therefore, we set the photospheric axial magnetic field equal to
\[ B_{s, \perp} = 1.5 B_{\text{eq}} = 2.01 \times 10^3 \text{ G}. \tag{34} \]

We modelled the energy injection from the photosphere by imposing the velocity and magnetic-field fluctuations. Fluctuations were imposed at both ends of the simulation domain. The vertical and horizontal velocity fluctuations were modelled separately. The upward acoustic waves were excited on the photosphere by employing the time-dependent boundary conditions on the density and axial velocity.
\[ \rho_s = \bar{\rho}_s \left( 1 + \frac{\tilde{v}_{s, \perp}}{a_s} \right), \tag{35} \]
\[ v_{s, \perp} = \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \tilde{v}_s(\omega) \sin \left( \omega t + \psi(\omega) \right), \tag{36} \]
where $\bar{\rho}_s$ is the time-averaged photospheric density, $a_s = \sqrt{k_B T_s / m_H}$ is the isothermal speed of sound on the photosphere, and $\psi(\omega)$ is a random phase function that ranges between $0$ and $2\pi$. In the numerical implementation of the integral in Eq. (36), the frequency range was evenly divided into $21$ bins and the corresponding $21$ components were summed. The time-averaged photospheric density is given by equipartition on the photosphere:
\[ \bar{\rho}_s k_B T_s / m_H = \frac{B_{s, \perp}^2}{8\pi}, \tag{37} \]
which yields
\[ \bar{\rho}_s = 4.22 \times 10^{-7} \text{ g cm}^{-3}. \tag{38} \]
Here, $\bar{\rho}_s$ is larger than the typical mass density on the solar surface because the magnetised photosphere should be deeper and denser. The (time-averaged) Alfvén speed $v_A$ on the photosphere is then expressed as
\[ \bar{v}_{A, s} = \frac{B_{s, \perp}}{\sqrt{4\pi\bar{\rho}_s}} = 8.73 \text{ km s}^{-1}. \tag{39} \]
We arbitrarily set $\tilde{v}_s(\omega) \propto \omega^{-1/2}$ with
\[ 2\pi/\omega_{\text{min}} = 300 \text{ s}, \quad 2\pi/\omega_{\text{max}} = 100 \text{ s}. \tag{40} \]
The minimum frequency corresponds to the cut-off frequency of the acoustic wave at the photosphere (e.g. Felipe et al. 2018). The magnitude of $\tilde{v}_s(\omega)$ was tuned such that the RMS amplitude of $v_{s, \perp}$ at the photosphere is $0.6 \text{ km s}^{-1}$.
\[ \sqrt{\tilde{v}_{s, \perp}^2} = 0.6 \text{ km s}^{-1}, \tag{41} \]
where the overline denotes the time average. Although the longitudinal-wave excitation on the photosphere is explicitly considered, the effect of the longitudinal-wave input is insignificant; the coronal temperature decreases by only $2\%$ when the longitudinal wave injection is terminated.

The horizontal velocity and magnetic field at the bottom boundary are expressed in terms of the Elsässer variables, which are defined as
\[ z_{s, x,y} = v_{s, x,y} \pm \frac{B_{s, x,y}}{\sqrt{4\pi\rho}}. \tag{42} \]
The free boundary condition was imposed on the downward Elsässer variables.
\[ \frac{\partial}{\partial s} z_{s, x,y} = 0. \tag{43} \]
The upward Elsässer variable was assumed to be non-monochromatic with respect to the frequency.  
\[
\begin{align*}
\Xi_{\text{El}}^{\text{up}}(\omega) = \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} d\omega \Xi_{\text{El}}^{\text{up}}(\omega) \sin(\omega t + \psi(\omega)),
\end{align*}
\]
where \(\psi(\omega)\) is a random phase function that ranges between 0 and 2\(\pi\). As with Eq. (36), the frequency range was discretized into 21 bins when numerically implementing the integral in Eq. (44). We set \(\Xi_{\text{El}}^{\text{up}}(\omega) \propto \omega^{-1/2}\) with  
\[
2\pi/\omega_{\text{min}} = 1000 \text{ s}, \quad 2\pi/\omega_{\text{max}} = 100 \text{ s}.
\]
\(\Xi_{\text{El}}^{\text{up}}(\omega) \propto \omega^{-1/2}\) corresponds to the 1/\(\omega\) energy spectrum discovered in previous solar simulations and observations (Van Kooten & Cranmer 2017). Given that the typical size of a granule is 1000 km and the typical speed of surface convection is 1 km s\(^{-1}\) (Chitta et al. 2012), the maximum wave period corresponds to the turn-over time of granular motion. Similarly, given that the typical size of an inter-granular lane is 100 km, the minimum wave period corresponds to the turn-over time of inter-granular motion. The magnitude of \(\Xi_{\text{El}}^{\text{up}}(\omega)\) was tuned such that the root-mean-squared amplitude of \(\Xi_{\text{El}}^{\text{up}}(\omega)\) at the photosphere is 1.2 km s\(^{-1}\):
\[
\frac{1}{\sqrt{2\pi}} = \left(\frac{1}{\sqrt{2\pi}}\right) = 1.2 \text{ km s}^{-1},
\]
which the overline denotes the time average. Although some solar observations have observed the suppression of convective velocity in large-filling-factor regions (e.g. Katsukawa & Tsuneta 2005), we dismiss this effect for simplicity.

By this formulation, the energy flux of the upward Alfvén wave at the footpoint of the flux tube is given by
\[
F_{\text{A,up}} = \frac{1}{4}\rho_{\text{up}}\left(\frac{z_{\text{El}}^{\text{up}} + z_{\text{El}}^{\text{down}}}{2}\right)\rho v, \approx \frac{1}{8\pi} \left(\frac{\rho z_{\text{El}}^{\text{up}} + z_{\text{El}}^{\text{down}}}{2}\right)\rho v = 2.65 \times 10^8 \text{ erg cm}^{-2} \text{ s}^{-1},
\]
which is sufficiently larger than the energy flux required to sustain the solar corona (Withbroe & Noyes 1977).

2.8. Numerical method

A non-uniform grid system was used to resolve the computational domain \(0 \leq s \leq 2h_{\text{loop}}\). A uniform cell size of \(\Delta s_{\text{min}}\) was used below the critical height \(s < s_{\text{ge}}\), above which the cell size expands until it reaches the maximum \(\Delta s_{\text{max}}\). In particular, in \(0 \leq s \leq h_{\text{loop}}\), the size of the \(i\)-th cell, \(\Delta s_i\), was iteratively defined as
\[
\Delta s_i = \text{max}\left[\Delta s_{\text{min}}, \text{min}\left[\Delta s_{\text{max}}, \Delta s_{\text{min}} + \frac{2s_{\text{ge}}}{2 + e_{\text{ge}}} (s_{i-1} - s_{\text{ge}})\right]\right],
\]
\[
s_i = s_{i-1} + \frac{1}{2} (\Delta s_{i-1} + \Delta s_i).
\]
Letting \(N\) be the total number of cells, we express the cell size in the latter half of the domain \(h_{\text{loop}} \leq s \leq 2h_{\text{loop}}\) by
\[
\Delta s_i = \Delta s_{N+1-i}, \quad s_i = s_{N-i}^+ + \frac{1}{2} (\Delta s_{N-i} + \Delta s_i).
\]
The maximum cell size was fixed to \(\Delta s_{\text{max}} = 100 \text{ km}\). The relation between the minimum cell size and \(f_s\) is
\[
\Delta s_{\text{min}} = 5 \text{ km} \quad (f_s < 0.05), \quad \Delta s_{\text{min}} = 2 \text{ km} \quad (f_s \geq 0.05).
\]
Thermal conduction and the other parts are solved independently by the second-order operator-splitting procedure as follows. First, the thermal conduction is solved for a half step $\Delta t/2$.

$$\tilde{U}^n \xrightarrow{\Delta t/2} \tilde{U}^* \quad \text{(thermal conduction only)}, \quad (56)$$

where $\tilde{U}^n$ is the $n$-th step value of $\tilde{U}$. Then, the rest of the basic equations are solved for a full step $\Delta t$.

$$\tilde{U}^*_i \xrightarrow{\Delta t} \tilde{U}^{**}_i \quad \text{(without thermal conduction).} \quad (57)$$

Finally, the thermal conduction is solved again for a half step $\Delta t/2$.

$$\tilde{U}^{**}_i \xrightarrow{\Delta t/2} \tilde{U}^{***}_i \quad \text{(thermal conduction only).} \quad (58)$$

With this procedure, we avoid the severe constraints on the $\Delta t$ from thermal conduction when updating the MHD equations.

The third-order strong-stability-preserving (SSP) Runge–Kutta method is used in the time integration of the MHD equations (Shu & Osher 1988; Gottlieb et al. 2001). The super-time-stepping method (Meyer et al. 2012, 2014) is used to solve the thermal conduction, which reduce the numerical cost and time with minimum loss of accuracy.

### 3. Simulation result

#### 3.1. Fiducial (solar) case: atmosphere and spectrum

First, we discuss the simulation run with $l_{\text{loop}} = 20$ Mm and $f_e = 1.0 \times 10^{-2}$ as the fiducial case. In this case, the coronal field strength is $\approx 20$ G, which is within the range of the solar coronal magnetic field strength measured by the coronal seismology technique (Nakariakov & Ofman 2001; Verwichte et al. 2004; Jess et al. 2016), and thus the fiducial case is regarded as the solar case.

Figure 4 illustrates the time-averaged properties of a quasi-steady coronal loop. Panels show the mass density (top) and temperature (middle) along the loop axis and the differential emission measure (DEM, bottom), defined by

$$\text{DEM}(T) = n_e^2(T) \frac{dl_{\text{los}}}{dT}, \quad (59)$$

where $n_e(T)$ is the electron density with temperature $T$ and $l_{\text{los}}$ is the length along the line of sight. Practically, dividing the temperature range into bins and considering the vertical line of sight ($l_{\text{los}} = r$), the DEM and associated emission measure distribution (EMD) are numerically obtained as follows

$$\text{DEM}(T_i)\Delta T_i \equiv \text{EMD}(T_i) = n_e^2(T_i) \Delta r(T_i), \quad (60)$$

where $n_e(T_i)$ is the total number density of an electron that exhibits a temperature in $[T_i - \Delta T_i/2, T_i + \Delta T_i/2]$ and $\Delta r(T_i)$ is the total radial extension of the line of sight. The DEM is calculated in the temperature range of $10^4$ K $\leq T \leq 10^8$ K with an equal spacing in the logarithmic scale of $T$. The DEM is in the range of $T < 10^4$ K is not calculated because, in the low-temperature range, the atmosphere is not optically thin and the DEM loses its meaning. We note that the unit of EMD is cm$^{-3}$, while the ‘volume’ EMD, which has often been used in the literature (Güdel et al. 2003; Scelsi et al. 2005), represents the distribution of an emission measure over the whole coronal volume and has a different unit (cm$^{-3}$).

The high-temperature ($T > 10^6$ K) corona is successfully reproduced in the fiducial case. Since we are imposing the phenomenological, mean-field formulation of the coronal heating (field braiding/turbulence), the heating tends to be more
constant in time and more uniform in space than the actual three-dimensional case in which the heating is intermittent in time and space. Nevertheless, the time-averaged heating rate should be similar between the 1D approximation and the 3D simulation, because the previous 3D simulation of solar wind yielded a similar mean field to the 1D simulation with turbulence phenomenology (Shoda et al. 2019).

Under the assumption that the upper atmosphere ($T \geq 10^4$ K) is optically thin in the wavelength of interest, the XUV spectrum is obtained from the EMD using the open-source package ChiantiPy based on CHIANTI database ver 10.0 (Del Zanna et al. 2021). In particular, the specific intensity $I_1$ was calculated from the EMD using the ChiantiPy.core.Spectrum module with the coronal abundance given by Schmelz et al. (2012). Although the radiative loss function $\lambda(T)$ is constructed with photospheric abundance, because the loss function is nearly independent of the FIP effect in the radiation-dominated temperature range $T \lesssim 3 \times 10^5$ K, the inconsistency in abundance do not violate the simulation results. Assuming that the stellar corona is a uniformly bright sphere, the spectral flux density at the heliocentric distance $r$ is deduced by (see, for example, Sect. 1.3 of Rybicki & Lightman 1979)

$$F_\lambda = \pi I_1 \left(\frac{R_\odot}{r}\right)^2,$$

which yields the X-ray and EUV luminosities as

$$L_X = 4\pi r^2 \int_0^{100} \lambda \, d\lambda \, F_\lambda, \quad L_{\text{EUV}} = 4\pi r^2 \int_{912}^{1000} \lambda \, d\lambda \, F_\lambda.$$  

In terms of energy, X-ray photons are in the range of 0.12–2.48 keV. Caution must be exercised when calibrating the X-rays because a subtle difference in the bandpass of the instrument can result in large differences in the derived response function and X-ray luminosity (Zhuleku et al. 2020). The procedure for the translation between different instruments is available in Judge et al. (2003).

To test the capability of our model in the prediction of XUV spectrum, we compare in Fig. 5 the spectral flux density obtained from our simulation (red) and that from the observation in a solar activity minimum (blue). The observed spectrum is obtained from the coordinated observation in the Whole Heliosphere Interval (WHI, from March 20, 2008, to April 16, 2008, Woods et al. 2009; Chamberlin et al. 2009). Figure 5 shows that, below the Lyman edge ($\leq 912$ Å), the simulated spectrum is in a good agreement with the observed spectrum. Above the Lyman edge ($\geq 912$ Å), the continuum is underestimated, and the emission lines are overestimated in the simulated spectrum, possibly because the optically thin approximation is inadequate in this wavelength range. In this study, because the focus is on the spectrum below the Lyman edge, the simulation is validated with respect to spectrum prediction.

3.2. Loop-length dependence: density and temperature

The coronal loop length is a fundamental parameter that affects the coronal density and temperature. Here, we show the relation between the time-averaged coronal properties and the coronal loop length. For simplicity, we fix the magnetic filling factor to the fiducial (solar) value: $f_0 = f_s = 0.01$. Hereinafter, the time-averaged value of $X$ will be denoted by $X_{\text{ave}}$.

In the discussion on the behaviour of the coronal properties, the results must be compared with the analytical RTV scaling law (Rosner et al. 1978). We note that the half-loop length in the RTV scaling law denotes the length from the transition region to the apex of the loop, whereas $l_{\text{loop}}$ stands for the length from the stellar surface to the apex of the loop. For better comparison, instead of $l_{\text{loop}}$, we use the effective half-loop length $l_{\text{eff}}$, which denotes the coronal length, and is defined as

$$l_{\text{eff}} = \int_{r_{\text{tr}}}^{l_{\text{loop}}} ds,$$

where $r_{\text{tr}}$ ($< l_{\text{loop}}$) is where $T_{\text{ave}} = 10^5$ K.

Another factor that should be considered is the gravitational stratification. The effective half-loop length often exceeds the coronal pressure scale height. In such cases, the loop-top pressure in the original RTV scaling law is replaced by the coronal-base pressure (Serio et al. 1981). Given that the pressure is continuous across the transition region, we define the coronal-base pressure $p_{\text{base}}$ as the pressure measured at $T_{\text{ave}} = 10^5$ K. For the coronal electron density, both the loop-top value $n_e, l_{\text{loop}}$ and coronal-base value $n_e, l_{\text{base}}$ are measured. In contrast to pressure, the density is discontinuous across the transition region, and therefore the definition of the coronal-base density is not trivial. Here, we define $n_e, l_{\text{base}}$ as the value measured in the coronal base:

$$n_e, l_{\text{base}} = \frac{1}{s_{\text{cor,2}} - s_{\text{cor,1}}} \int_{s_{\text{cor,1}}}^{s_{\text{cor,2}}} n_e, l_{\text{ave}} \, ds,$$

where we set $s_{\text{cor,1}} = 10$ Mm and $s_{\text{cor,2}} = 15$ Mm. The loop-top density and temperature are given by

$$n_e, l_{\text{loop}} = n_e \left( s = l_{\text{loop}} \right), \quad T_{\text{loop}} = T \left( s = l_{\text{loop}} \right).$$

Figure 6 shows the $l_{\text{loop}}$ dependences of the time-averaged coronal properties (density, temperature and pressure). The loop-top temperature and coronal-base pressure obey a power–law relation with respect to the effective half-loop length, which is formulated as

$$T_{\text{top}} \propto l_{\text{eff}}^{-0.39},$$

$$p_{\text{base}} \propto l_{\text{eff}}^{-0.2}.$$
The (generalised) RTV scaling law predicts that the loop-top temperature obeys the following relation

\[ T_{\text{top}} \propto Q^{2/7} \ell_{\text{loop}}^{4/7}, \]  

(70)

and simulation results from Eqs. (66) and (67) indicate that \( Q = F_{\text{cor}}/\ell_{\text{loop}}^{d} \) is a decreasing function of \( \ell_{\text{loop}}^{d} \). These conclusions shall be directly validated in the following section.

The bottom panel of Fig. 6 shows the variations in loop-top and coronal-base pressure densities over \( \ell_{\text{loop}}^{d} \). Given that the RTV scaling law predicts a larger loop-top density at a higher loop-top temperature for a uniform corona, the decrease in the loop-top density is attributed to gravitational stratification. We note that the coronal-base density exhibits a weaker dependence on \( \ell_{\text{loop}}^{d} \) than the coronal-base pressure, which is contradictory if the coronal-base temperature is constant in \( \ell_{\text{loop}}^{d} \). It may be interpreted that the coronal-base temperature increases with \( \ell_{\text{loop}}^{d} \) in response to the increasing \( T_{\text{top}} \) with \( \ell_{\text{loop}}^{d} \).

### 3.3. Loop-length dependence: energy flux and XUV emission

For a better interpretation of the behaviour of coronal properties, the variation in the energy flux entering the corona \( F_{\text{cor}} \) with the effective half-loop length \( \ell_{\text{loop}}^{d} \) must be revealed.

Directly measuring \( F_{\text{cor}} \) from the simulation data is, however, not a trivial task. To obtain \( F_{\text{cor}} \), one needs to measure the energy flux at the base of the corona, which moves in time and is broadened after time averaging. Because a significant amount of energy is reflected back at the transition region, a slight difference in the position of the coronal base yields a significant error or uncertainty in \( F_{\text{cor}} \). Therefore, instead of directly measuring \( F_{\text{cor}} \), we measure the backward conductive flux \( F_{\text{cnd}} \), which should be balanced with \( F_{\text{cor}} \) by energy conservation.

The top and bottom panels in Fig. 7 show the loop-length \( (\ell_{\text{loop}}^{p} \text{ and } \ell_{\text{loop}}^{c}) \) dependence of the coronal conductive flux. The panels display the simulation runs with a fixed magnetic filling factor of \( f_{c} = 1.00 \times 10^{-2} \). The red circles and blue diamonds represent the conductive flux measured at \( T_{\text{base}} = 1.0 \times 10^{5} \) K and averaged over 10 Mm \( \leq s \leq 15 \) Mm, respectively. The black solid lines represent the power-law fittings to the blue diamonds. The coronal conductive flux increases with the loop length. However, the trend deviates from a simple power law. When the loop length is sufficiently small \( (\ell_{\text{loop}}^{d} \leq 10^{1.5} \text{ Mm}) \) or large \( (\ell_{\text{loop}}^{d} \geq 10^{2.5} \text{ Mm}) \), the conductive flux weakly depends on the loop length. The energy flux increases around \( \ell_{\text{loop}}^{d} \sim 10^{2.0} \) Mm, with the minimum and maximum values differing by a factor of four.

An approximate power-law fit to the blue diamonds yields the following scaling law

\[ F_{\text{cnd}} \propto \ell_{\text{loop}}^{d}^{0.48}, \]  

(72)

which, in combination with the RTV scaling law, predicts

\[ T_{\text{top}}^{\text{RTV}} \propto \left( F_{\text{cor}}^{\ell_{\text{loop}}^{d}} \right)^{2/7} \ell_{\text{loop}}^{d}^{0.42}. \]  

(73)

The simulated dependence of \( T_{\text{top}} \) on \( \ell_{\text{loop}}^{d} \), Eq. (66), is reproduced by the semi-analytical arguments. Thus, the results shall

---

**Fig. 6.** Coronal-property dependences on the effective loop length. **Top:** relation between the effective half loop length \( \ell_{\text{loop}}^{d} \) (see Eq. (63) for definition) and the time-averaged loop-top temperature \( T_{\text{top}} \) (red circles) and coronal-base pressure \( p_{\text{base}} \) (blue diamonds). **Bottom:** relation between the effective half loop length \( \ell_{\text{loop}}^{d} \) and the time-averaged loop-top electron density \( n_{e,\text{top}} \) (red circles) and coronal-base electron density \( n_{e,\text{base}} \) (blue diamonds). In both panels, lines represent the power-law fittings to the symbols.

\[ p_{\text{base}} \propto \ell_{\text{loop}}^{d}^{0.25}. \]  

(67)
be explained semi-analytically once the theoretical behaviour of $F_{\text{cor}}$ (or equivalently $F_{\text{cnd}}$) has been derived. In Sect. 4, we propose a simple model to produce $F_{\text{cor}}$.

The enhanced energy injection to the corona produces enhanced XUV emissions. Figure 8 depicts the loop-length dependence of the predicted XUV luminosity. For each coronal loop calculation, the XUV spectrum is calculated as in Fig. 5 and converted to XUV luminosity through Eq. (62). Both $L_X$ and $L_{\text{EUV}}$ exhibit increasing trends as those found in the coronal energy flux $F_{\text{cnd}}$. In particular, $L_X$ exhibits this trend significantly. The inferred power-law relations are

$$
L_{\text{EUV}} \propto l_{\text{loop}}^{0.29}, \\
L_X \propto l_{\text{loop}}^{0.65},
$$

The dependence of $L_X$ on $l_{\text{loop}}^{\text{eff}}$ is further explained semi-analytically. The X-ray luminosity should be proportional to the emission measure of the corona, that is,

$$
L_X \propto n_{\text{cor}}^{2} l_{\text{loop}}^{\text{eff}},
$$

where $n_{\text{cor}}$ is the typical number density of the coronal loop that lies between $n_{\text{base}}$ and $n_{\text{top}}$. Assuming that $n_{\text{cor}}$ follows the prediction of the RTV scaling law,

$$
r_{\text{cor}} \propto T_{\text{top}}^{1/2} l_{\text{top}}^{1/4} F_{\text{cor}}^{-3/7},
$$

the X-ray luminosity is connected to the coronal energy flux $F_{\text{cor}}$ and loop length $l_{\text{loop}}^{\text{eff}}$ by

$$
L_X \propto F_{\text{cor}}^{-3/7} l_{\text{loop}}^{1/7} n_{\text{cor}}^{0.69} l_{\text{loop}}^{0.69},
$$

where the simulated scaling law Eq. (72) is used in the second proportional relation. The actual scaling relation Eq. (75) is in good agreement with the semi-analytical prediction Eq. (78).
validating the RTV scaling law in describing the stellar coronal properties.

We note that EUV luminosity $L_{\text{EUV}}$ is weakly dependent on the loop length compared with $L_X$ because a portion of the EUV photons originates from the upper chromosphere and transition region, which are barely affected by the variation in the coronal loop length. Within the investigated loop-length range, the ratio $L_{\text{EUV}}/L_X$ decreases from ~10 to ~3 as the effective loop length increases.

### 3.4. Filling-factor dependence: density and temperature

The magnetic filling factor of the photosphere appears to have scaled with the Rossby number, or equivalently the magnetic activity level (Saar 1996, 2001; Reiners et al. 2009). Hence, the dependence of the coronal properties on the filling factor is worth investigating as a proxy of the dependence of the stellar corona on the activity level. The simulation results with various magnetic filling factors shall be discussed below, with the half-loop length fixed to $l_{\text{loop}} = 20 \, \text{Mm}$.

Figure 9 shows the filling-factor dependence of the time-averaged loop-top temperature ($T_{\text{top}}$, top panel) and loop-top electron density ($n_{e,\text{top}}$, bottom panel). The half loop length is fixed to $l_{\text{loop}} = 20 \, \text{Mm}$. Red circles show the simulation results and the solid lines show the power-law fittings to them.

$$n_{e,\text{top}} \propto f_0^{0.49}$$

$$T_{\text{top}} \propto f_0^{0.29}$$

Figure 10. Relation between filling factor and coronal conductive flux. Top: magnetic filling factor on the photosphere $f_*$ versus forward conductive flux measured in the corona. The half loop length is fixed to $l_{\text{loop}} = 20 \, \text{Mm}$. The red circles and blue diamonds represent the flux measured at $T_{\text{ave}} = 10^6 \, \text{K}$ and averaged in $[10 \, \text{Mm}, 15 \, \text{Mm}]$. The black solid line is the power-law fitting to the blue diamonds. Bottom: same as the top panel but the vertical axis denotes $F_{\text{cnd}}^{\text{ave}}$.

$$F_{\text{cnd,loop}} = f_0^{0.95}$$

$$F_{\text{cnd}} \propto f_0^{0.29}$$

Although not explicitly demonstrated, the effective half-loop length also weakly depends on $f_*$ as

$$l_{\text{loop}}^{\text{eff}} \propto f_0^{0.04}.$$

Figure 9. Magnetic filling factor on the photosphere $f_*$ versus time-averaged loop-top temperature ($T_{\text{top}}$, top panel) and loop-top electron density ($n_{e,\text{top}}$, bottom panel). The half loop length is fixed to $l_{\text{loop}} = 20 \, \text{Mm}$. Red circles show the simulation results and the solid lines show the power-law fittings to them.
We note that the RTV scaling law semi-analytically predicts a relation of
\[ T_{\text{top}}^\text{RTV} \propto \sqrt{\eta_{\text{e,loop}} f_{\text{eff}}^\text{loop}} \propto f_s^{0.27}, \]  
(82)
which is close to the simulation result Eq. (79). Alternatively, in terms of the heating rate per unit volume \( Q \), the RTV scaling law coupled with Eqs. (79) and (81) yields
\[ Q \propto f_s^{0.935}, \]  
(83)
while the RTV scaling law Eqs. (80) and (81) yield
\[ Q \propto f_s^{0.868}. \]  
(84)
The two trends are mostly consistent with each other.

The RTV scaling law predicts that the loop-top temperature \( T_{\text{top}} \) depends on the coronal energy flux \( F_{\text{cor}} \) and effective half-loop length \( l_{\text{eff,loop}} \) as Eq. (69). A comparison between Eqs. (69) and (79) reveals the mostly proportional relationship between \( F_{\text{cor}} \) and \( f_s^* \):
\[ F_{\text{cor}} l_{\text{eff,loop}} \propto f_s^*, \]  
(85)
which shall be further investigated in the following section.

### 3.5. Filling-factor dependence: energy flux and XUV emission

The energetics of the corona with various magnetic filling factors shall be discussed here. The top panel of Fig. 10 shows the relation between the filling factor \( f_s \) and the coronal backward conductive flux measured at \( T_{\text{ave}} = 10^6 \) K (red circles) and averaged over 10 Mm \( \lesssim s \lesssim 15 \) Mm (blue diamonds). The two conductive fluxes exhibit different behaviours, with the red circles appearing to saturate within the large \( f_s \) limit. This difference may arise from the different physical meanings of \( T_{\text{ave}} = 10^6 \) K, where the values of the red circles are measured.

\[ T_{\text{ave}} = 10^6 \text{ K}, \]

located near the loop top for the small \( f_s \) cases and near the transition region for the large ones. Therefore, the blue diamonds may be interpreted as the coronal energy flux.

Given the dependence of the loop-top temperature and RTV scaling law on the filling factor, the relation among the coronal energy flux \( F_{\text{cor}} \) and effective half-loop length \( l_{\text{eff,loop}} \) is defined by Eq. (69). This prediction is confirmed in the bottom panel of Fig. 10, which shows the following scaling relation:
\[ F_{\text{cor}} l_{\text{eff,loop}} \propto f_s^{0.95}. \]  
(86)
This consistency indicates that the RTV scaling law must be applicable to stellar coronae with varying activity levels. The simulation results validate the previous studies on stellar coronal energetics based on the RTV scaling law (Shibata & Yokoyama 2002; Takasao et al. 2020).

Figure 11 shows the spectra of XUV emission for different magnetic filling factors. For all wavelengths below the Lyman edge (912 Å), the XUV emissions increase as the magnetic filling factor increases. This is a natural consequence of the increased coronal density shown in Fig. 9. In particular, as seen in the central panels of Fig. 11, the enhancement of the energy flux from \( f_s = 0.01 \) to \( f_s = 1 \) is centred on \( \sim 10 \) with a variance of factor \( \sim 5 \) in the EUV range (100 Å \( < \lambda \lesssim 912 \) Å), whereas a significantly larger enhancement occurs in the X-ray range (5 Å \( < \lambda \lesssim 100 \) Å). This is because of the sensitivity of the X-ray emissivity to the temperature. Also shown in the right panels of Fig. 11 are the spectra of the total photon number emitted from the star per unit time (photon number luminosity) given by \( 4\pi^2 L_f \Phi_{\text{mag}}/hc \). For all \( f_s \), the spectrum of the photon number luminosity is flatter than the energy spectrum, implying that the photon population with respect to wavelength is approximately uniform in the XUV range.

The magnetic-flux dependences of the X-ray luminosity \( L_X \) and EUV luminosity \( L_{\text{EUV}} \) are also investigated. Figure 12 reveals the variation in \( L_X \) (blue diamonds) and \( L_{\text{EUV}} \) (red circles) with the unsigned magnetic flux \( \Phi_{\text{mag}} \) defined by
\[ \Phi_{\text{mag}} = 4\pi R_\odot^2 B_{\text{sun}} f_s^*. \]  
(87)
Because the photospheric magnetic field is fixed, the filling factor \( f_s \) is the only variable in the definition of \( \Phi_{\text{mag}} \). We find that both \( L_X \) and \( L_{\text{EUV}} \) follow power laws with respect to \( \Phi_{\text{mag}} \) and are expressed as follows
\[ L_{\text{EUV}} \propto \Phi_{\text{mag}}^{0.81}, \]  
(88)
\[ L_X \propto \Phi_{\text{mag}}^{1.19}, \]  
(89)
which are represented by the solid and dashed lines in Fig. 12, respectively. A scaling law is observationally discovered between the unsigned magnetic flux and the X-ray luminosity (Pevtsov et al. 2003)
\[ L_{\text{obs}} \propto \Phi_{\text{mag}}^{1.15}, \]  
(90)
which is shown by the dotted line in Fig. 12. Although the magnitudes of the X-ray luminosity from the simulation are smaller than the observed values, possibly because the proposed model reproduces the coronal properties in the activity minimum (a quiet, quasi-steady corona without flaring activities), the power-law index from the simulation (1.19) is nearly identical to that from the observation (1.15). The reproduction of the observational \( L_{\text{obs}} \)–\( L_X \) relationship provides another support to our model. The proposed \( \Phi_{\text{mag}} \)–\( L_{\text{EUV}} \) relationship, Eq. (88), can be tested by the Sun-as-a-star observations in the EUV range (e.g. Toriumi et al. 2020).

The filling-factor dependence of the EUV photon number luminosity \( \Phi_{\text{EUV}} \) is also investigated. \( \Phi_{\text{EUV}} \) is defined as the total number of EUV photons emitted from the star, that is,
\[ \Phi_{\text{EUV}} = 4\pi^2 \int_{100 \text{ Å}}^{912 \text{ Å}} \frac{dA_f}{hc}, \]  
(91)
which acts as a proxy of the photo-ionised hydrogen number per unit time. Figure 13 shows the EUV photon number luminosity with varying magnetic filling factors. As with \( L_{\text{EUV}} \), \( \Phi_{\text{EUV}} \) also follows a power law with respect to \( f_s \), which is given by
\[ \Phi_{\text{EUV}} \propto f_s^{0.78} \propto \Phi_{\text{mag}}^{0.78}. \]  
(92)
A comparison of Eqs. (88) and (92), reveals that \( L_{\text{EUV}} \) and \( \Phi_{\text{EUV}} \) follow marginally different power laws with respect to \( f_s \) (or equivalently \( \Phi_{\text{mag}} \)).
3.6. \( L_X - L_{\text{EUV}} \) and \( L_X - \Phi_{\text{EUV}} \) photon relations

One of the objectives of this study is to propose a method to estimate the stellar EUV luminosity \( L_{\text{EUV}} \) from other observable quantities. Because the X-ray luminosity \( L_X \) is measurable and should be correlated with \( L_{\text{EUV}} \) (see e.g. Sanz-Forcada et al. 2011), the relationship between \( L_X \) and \( L_{\text{EUV}} \) over a wide range of filling factors and loop lengths warrants investigation.

The top panel of Fig. 14 highlights the correlation between X-ray luminosity \( L_X \) and EUV luminosity \( L_{\text{EUV}} \). The black circles correspond to the simulation runs listed in Table 2. Although the simulation runs are spread over wide ranges in the filling factor and loop length, a single power-law fit adequately applies to the \( L_X - L_{\text{EUV}} \) relation, which is given by

\[
\log L_{\text{EUV}} = 9.93 + 0.67 \log L_X, \tag{93}
\]

where \( L_{\text{EUV}} \) and \( L_X \) are measured in the unit of erg s\(^{-1}\). The red points with the vertical error bars indicate the observational estimation by Sanz-Forcada et al. (2011). The pure theoretical estimation of \( L_{\text{EUV}} \) in this study conforms with the pure observational estimation of the same by Sanz-Forcada et al. (2011). Although the power-law index of \( L_X - L_{\text{EUV}} \) relation is 0.86, according to Sanz-Forcada et al. (2011), the coefficient 0.67 in Eq. (93) is, in fact, close to the observational value (0.681) derived from the Extreme Ultraviolet Explorer (EUV) observations (Johnstone et al. 2021). The fact that the independent methods produces a similar \( L_X - L_{\text{EUV}} \) relation validates the proposed model of stellar coronae and their XUV emissions.

The correlation between the X-ray luminosity \( L_X \) and the EUV photon-number luminosity \( \Phi_{\text{EUV}} \) is also highlighted in the bottom panel of Fig. 14. They, too, follow a power-law

---

**Table 2. Simulation runs conducted in this study.**

| Magnetic filling factor (photosphere) | Half-loop length [10\(^3\) km] |
|--------------------------------------|----------------------------------|
| \( f_e = 1 \)                       | \( l_{\text{loop}} = [20, 30, 40] \) |
| \( f_e = 0.5 \)                      | \( l_{\text{loop}} = [20, 30, 40] \) |
| \( f_e = 0.333 \)                    | \( l_{\text{loop}} = [20, 30, 40, 60] \) |
| \( f_e = 0.2 \)                      | \( l_{\text{loop}} = [20, 30, 40] \) |
| \( f_e = 0.1 \)                      | \( l_{\text{loop}} = [20, 30, 40, 60, 80] \) |
| \( f_e = 0.05 \)                     | \( l_{\text{loop}} = [20, 30, 40] \) |
| \( f_e = 0.0333 \)                   | \( l_{\text{loop}} = [20, 30, 40, 60, 80, 120, 160, 240] \) |
| \( f_e = 0.01 = f_0 \)               | \( l_{\text{loop}} = [20, 30, 40, 60, 80, 120, 160, 240, 320, 480, 640] \) |
| \( f_e = 0.005 \)                    | \( l_{\text{loop}} = [20] \) |

**Notes.** The magnetic filling factor on the solar photosphere is set to \( f_0 = 0.01 \) (Cranmer 2017).
4. Analytical arguments on coronal energy flux

Several scaling relations with respect to the loop length and magnetic filling factor are provided in Sect. 3. In this section, we analytically describe the coronal energy injection process governing the scaling relations. Because energy is mainly transported as transverse fluctuations of the velocity and magnetic field (Alfvén waves), the efficiency of Alfvén-wave transmission into the corona is key to the analysis. This section is devoted to reviewing the linear theory of Alfvén-wave propagation in a stellar atmosphere and to provide analytical arguments for the origin of the scaling laws discovered through the simulation.

Fig. 12. Unsigned magnetic flux $\Phi_{\text{mag}}$ versus X-ray luminosity $L_X$ (blue diamonds) and EUV luminosity $L_{\text{EUV}}$ (red circles). Results for the fixed half loop length $h_{\text{loop}} = 20$ Mm are shown. Solid and dashed lines are the power-law fittings to the simulation results. Also shown, by the dotted line, is the observational relation by Pevtsov et al. (2003).

Fig. 13. Magnetic filling factor on the photosphere $f_*$ versus photon number luminosity in the EUV range. Each simulation run corresponds to each red circle. The black solid line shows the power-law fitting to the simulation results.

Fig. 14. Inferred relations between X-ray luminosity and EUV emission. Top: correlation between the X-ray luminosity $L_X$ and EUV luminosity $L_{\text{EUV}}$. X-ray and EUV are defined in terms of wavelength $\lambda$ as 5 Å $< \lambda \leq 100$ Å and 100 Å $< \lambda \leq 912$ Å, respectively. Each black circle corresponds to the simulation run listed in Table 2, and the black solid line shows the power-law fitting to them. Also shown, by the red points, with vertical error bars, are the empirical estimation by Sanz-Forcada et al. (2011). Bottom: same as the top panel but for the X-ray luminosity $L_X$ and photon number luminosity $\Phi_{\text{photon}}$.

### 4.1. Linear theory of Alfvén-wave propagation and reflection

A significant amount of Alfvén waves excited at the photosphere is reflected before reaching the solar corona (Cranmer & van Ballegooijen 2005; Verdini & Velli 2007). The relationship between the Alfvén-wave transmission efficiency and the loop properties in the linear regime remains unclear and shall be discussed in this section.

Figure 15 illustrates the schematic of the concerned system. The convection–magnetic field interaction excites the Alfvén waves propagating upwards along the expanding flux tube. In our simulation setting, the Alfvén speed is maintained at a nearly constant value during the tube expansion (Eq. (4)). Because the Alfvén-wave reflection is triggered by a gradient in the Alfvén speed (Stein 1971; An et al. 1989; Vasquez 1990), the Alfvén waves reflected in the flux-tube expansion region are feeble. Therefore, the bulk of the reflection occurs in a layer between the flux-tube merging height (where the flux-tube expansion ends)
and the coronal base, which is in between the two vertical dashed lines in Fig. 15.

The Alfvén waves travelling through the reflection layer is considered in the following analysis. For simplicity, the thickness of the layer is disregarded. This approximation is validated by the fact that the reflection-layer thickness (∼10^0 Mm) is notably smaller than the typical wavelength of Alfvén waves therein (∼10^1–2 Mm). Then, the problem is simplified as a linear-wave transmission through a discontinuity, as described in the lower part of Fig. 15. Hereinafter, we denote the variables in the region on the left/right of the discontinuity as region I/II, respectively.

The equation describing the Alfvén-wave propagation is

\[ \frac{\partial^2}{\partial t^2} - \frac{v_{\text{A,I}/\text{II}}^2}{\partial s^2} \delta n_{\text{I}/\text{II}} = 0, \]  

the solution of which is given by

\[ \delta n_{\text{I}/\text{II}} = a_{\text{I}/\text{II}} e^{i(k_{\text{I}/\text{II}}s + \omega t)} + b_{\text{I}/\text{II}} e^{-i(k_{\text{I}/\text{II}}s + \omega t)}. \]  

We note that \( a_{\text{I}/\text{II}} \) (\( b_{\text{I}/\text{II}} \)) denotes the amplitudes of the upward (downward) propagating Alfvén wave in region I/II, respectively. Given the frequency \( \omega \), the dispersion relation yields the wavenumber \( k_{\text{I}/\text{II}} \) in each region.

\[ k_{\text{I}/\text{II}} = \omega / v_{\text{A,I}/\text{II}}. \]  

The corresponding magnetic-field fluctuation is deduced from the linearised induction equation, that is,

\[ \frac{\partial}{\partial t} \delta B_{\text{I}/\text{II}} = -B_{\text{I}/\text{II}} \frac{\partial}{\partial s} \delta n_{\text{I}/\text{II}}. \]  

Because the fluctuations of the velocity and magnetic field are required to be continuous across the boundary, the amplitudes must satisfy the following relations.

\[ a_1 + b_1 = a_{\text{II}} + b_{\text{II}}, \]  

\[ - k_1 a_1 + k_1 b_1 = -k_{\text{II}} a_{\text{II}} + k_{\text{II}} b_{\text{II}}. \]  

Solving these two equations yields

\[ a_1 = \frac{k_1 + k_{\text{II}}}{2k_1} a_{\text{II}} + \frac{k_1 - k_{\text{II}}}{2k_1} b_{\text{II}}. \]  

The energy transmission rate \( T_{\text{AW}} \) is defined as the ratio of the forward-propagating Alfvén-wave energy in regions I and II. Namely,

\[ T_{\text{AW}} = \frac{\rho_{\text{II}} v_{\text{A,II}} |a_{\text{II}}|^2}{\rho_{\text{I}} v_{\text{A,I}} |a_{\text{I}}|^2}. \]  

We obtain a general form of the energy transmission rate within the limit of \( \rho_{\text{II}} / \rho_{\text{I}} \ll 1 \) by combining Eqs. (101) and (102):

\[ T_{\text{AW}} = \frac{4 \sqrt{v_{\text{A,I}} / v_{\text{A,II}}}}{\rho_{\text{II}} / \rho_{\text{I}}} \left( |a_{\text{II}}|^2 + |a_{\text{II}} b_{\text{II}}^* + a_{\text{II}}^* b_{\text{II}} + |b_{\text{II}}|^2 \right)^{1/2}. \]  

where \( X^* \) denotes the complex conjugate of \( X \). It is interesting to see that the energy transmission rate depends on the values of \( a_{\text{II}} \) and \( b_{\text{II}} \), namely, the wave population in region II (corona).

The transmission rate in the two limiting cases are investigated below. When \( b_{\text{II}} = 0 \) (no Alfvén waves propagating towards the chromosphere), the energy transmission rate is given by (Hollweg 1984)

\[ T_{\text{AW}} = \frac{4 \sqrt{v_{\text{A,I}} / v_{\text{A,II}}}}{\rho_{\text{II}} / \rho_{\text{I}}}, \]  

where \( c_{\text{ref}} \) depends on the population ratio of the forward and backward Alfvén waves in region II.

To summarise, the transmission rate of Alfvén wave is given by

\[ T_{\text{AW}} = c_{\text{ref}} \sqrt{\frac{\rho_{\text{II}}}{\rho_{\text{I}}}}, \quad 1 \leq c_{\text{ref}} \leq 4, \]  

where the value of \( c_{\text{ref}} \) depends on the density ratio of the forward and backward Alfvén waves.

In the numerical model implemented in this study, the density in the region I should be the mass density at the flux-tube merging height \( s = h_{\text{merge}} \). In the flux-tube model employed in this model, \( h_{\text{merge}} \) is expressed as

\[ f_c \exp \left( \frac{h_{\text{merge}}}{H_{\text{mag}}} \right) = 1, \quad H_{\text{mag}} = 2.5 H_c. \]
where \( H_s \) is the density scale height at the photosphere (see Eq. (5) for definition). Assuming that the density scale height is uniform in \( 0 \leq s \leq h_{merge} \), we can write the mass density at the merging height \( \rho_{merge} \) as

\[
\rho_{merge} \approx \rho_s \exp \left( -\frac{h_{merge}}{H_s} \right) = \rho_s f_s^{2/5}.
\]  

(108)

Then, using Eq. (102), we obtain the Alfvén-wave transmission rate for our model as

\[
T_{AW} \approx c_{ref} \sqrt{\frac{\rho_{cor}}{\rho_{merge}}} = c_{ref} \sqrt{\frac{\rho_{cor}}{\rho_s} f_s^{-1/5}},
\]  

(109)

where \( \rho_{cor} \) denotes the mass density at the coronal base. We note that a large \( f_s \) could lead to a small transmission rate, when \( \rho_{cor} \) does not increase faster than \( f_s^{2/5} \).

### 4.2. Loop-length dependence of the coronal energy flux

In Sect. 3.3, it is shown that the coronal energy flux increases with \( l_{loop} \) but saturates at approximately four times the value of the short-loop limit (see Fig. 7). This behaviour is naturally explained by the theory of Alfvén-wave transmission.

The bottom panel of Fig. 6 shows that the mass density of the coronal base remains mostly constant with respect to the loop length. Therefore, the population ratio \( z_\perp \) is uniform in this case.

The enhancement of the coronal energy flux at \( l_{loop} \) can be explained in terms of the timescales of Alfvén-wave propagation and dissipation. The backward waves in the coronal loop significantly dissipate during the propagation through the chromosphere, which results in a smaller amplitude at the footpoint of the flux tube, which is fixed for all the simulation runs.

\[
\tau_{diff} = \int_{z_{ref}}^{z_{trim}} \frac{d s}{v_A}, \quad \tau_{diss} = \frac{1}{2} \frac{\tau_{loop}}{\tau_{ref}} \int_{z_{ref}}^{z_{trim}} \frac{d s}{c d z_{rms}^2}.
\]  

(110)

When \( \tau_A/\tau_{diss} < 1 \), the coronal Alfvén waves propagating from one coronal base to the other hardly experience any turbulent dissipation. Therefore, the ratio \( \left| z_\perp^2 / z_\parallel^2 \right| \) should tend to unity in this case. When \( \tau_A/\tau_{diss} > 1 \), the coronal Alfvén waves significantly dissipate during the propagation through the corona. Thus, the ratio \( \left| z_\perp^2 / z_\parallel^2 \right| \) should be far from unity in this case.

This argument is substantiated by Fig. 16, in which \( \tau_A \) (red circles) and \( \tau_{diss} \) (blue diamonds) are plotted against the half-loop length \( l_{loop} \). The Alfvén crossing time is shorter than the turbulent dissipation timescale \( \tau_A \) in the coronal loop. Given that a larger \( f_s \) yields a smaller \( \tau_A/\tau_{diss} \) (as well as a smaller \( c_{ref} \)), the predicted dependence of \( c_{ref} \) on \( f_s \) is

\[
c_{ref} \propto f_s^{1/7}.\]

(114)

Eqs. (113) and (114) yield

\[
F_{cor}^{l_{loop}} \propto c_{ref} F_{loop}^{7/10} f_s^{28/25} \propto c_{ref} f_s^{1.15},
\]  

(115)

where we use \( h_{loop} = f_s h_s \). As addressed in the previous section, the value of \( c_{ref} \) depends on the balance between the Alfvén-crossing timescale \( \tau_A \) and the turbulent dissipation timescale \( \tau_{diss} \) in the coronal loop. Given that a larger \( f_s \) yields a smaller \( \tau_A/\tau_{diss} \), the predicted dependence of \( c_{ref} \) on \( f_s \) is

\[
c_{ref} \propto f_s^{\alpha_{ref}},
\]  

(116)

where \( \alpha_{ref} > 0 \). Given the limited range of \( c_{ref} \) (1 \( \leq \alpha_{ref} \leq 4 \)), the power index \( \alpha_{ref} \) is small. Then, the semi-analytical scaling relation

\[
F_{cor}^{l_{loop}} \propto f_s^{1.15-\alpha_{ref}},
\]  

(117)

conforms with the obtained scaling relation Eq. (112).
The phenomenological turbulent dissipation implemented in this study yields the time-averaged heating rate, whereas the actual coronal heating is highly intermittent in space and time, as indicated by previous simulation studies (Einaudi et al. 1996; Cassak et al. 2008; Dahlburg et al. 2016; Kanella & Gudiksen 2018) and observations (Cirtain et al. 2013; Testa et al. 2014; Ishikawa et al. 2017; Antolin et al. 2021). In the presence of localised heating, the DEM extends to the high-temperature side, possibly forming the bi-modal structure observed in active stellar coronae (Güdel et al. 1997). The accuracy of the DEM obtained in this 1D simulation must be tested in the future.

The coronal loop is assumed to be isolated from the ambient plasma. However, previous studies have stated that the interface between the coronal flux tube and its surrounding is a preferential region for heating by phase mixing and resonant absorption (Jonsson 1978; Heyvaerts & Priest 1983; Hollweg & Yang 1988; Sakurai et al. 1991; Hood et al. 2002; Pagano & De Moortel 2019; Van Damme et al. 2020). In fact, resonant absorption is observed to occur in the solar corona (Antolin et al. 2015; Okamoto et al. 2015). Although there is no scientific consensus on the ability of phase mixing and/or resonant absorption to feed sufficient thermal energy to the corona, the relation between these processes and XUV emissions must be investigated.

Energy injection by the emergence of magnetic field from the stellar interior (magnetic flux emergence, Takasao et al. 2013) was dismissed. Recent observations of the solar corona indicate that the coronal loops are preferentially heated near the bottom (Berghmans et al. 2021), possibly by its interaction (magnetic reconnection) with adjacent small-scale loops (Chen et al. 2021). Because the (small-scale) coronal loops are formed by the magnetic flux emergence, these observations imply that a fraction of the coronal energy must be supplied by the emergence. Indeed, a recent work on the solar wind indicates the considerable role of magnetic flux emergence in the injection of energy to the open-field regions (Wang 2020).

Throughout this study, the energy spectrum is reconstructed from EMD (DEM) under the optically thin approximation. Although this approximation has been widely used (Sanz-Forcada et al. 2011; Duvvuri et al. 2021), it must be tested in future under a realistic treatment of radiative transfer. For example, the Lyman continuum appears to be optically thick, with an extended source region in the upper chromosphere (Avrett & Loeser 2008). Other studies have claimed that a simple optically thin treatment of the coronal lines may produce incorrect results (Schrijver et al. 1994). To solve this issue, the frequency-dependent radiative transfer equation must be solved in the future.

In converting the single-loop properties retrieved from the simulation results to a global quantity such as $L_X$, we assume that the stellar corona has a uniform brightness. The brightness of the actual stellar coronae, however, may be highly structured, as seen in the solar corona. Because emissions from a single active region depends on its size, the size distribution of active regions over the entire stellar surface affects the measured distribution of the total coronal emissions from the star (Takasao et al. 2020). The effect of coronal structuring must be considered in the future by, for instance, the global modelling of stellar coronae (van der Holst et al. 2014; Alvarado-Gómez et al. 2016; Oran et al. 2017; Airapetian et al. 2021).

These issues need to be considered in future model improvements.

5.3. Extension to other types of stars

In this study, we focused on the main-sequence Sun-like stars that exhibit solar mass, solar radius, solar luminosity, and solar
metallicity. Therefore, the application of Eqs. (93) or (94) to other classes of stars would require additional considerations.

First, M-type stars are of particular interest in the context of exoplanet because they occupy a large fraction of the local stellar population (Bochanski et al. 2010; Winters et al. 2019) and often host terrestrial planets within the habitable zones (Bonfils et al. 2013; Dressing & Charbonneau 2015). The applicability of the proposed theoretical predictions to low-mass stars (especially, M dwarfs) must be investigated in the future. Given the lower metallicity of old M-type stars, the dependence of XUV emissions on metallicity must also be studied (Washinoue & Suzuki 2011).

This work made use of matplotlib, a Python library for Transport, and Heating Dynamics in Laboratory and Solar/ Astrophysical Plasmas for cross-disciplinary study (grant Nos. 01321802 and 01311904) on Turbulence, and Society for the Promotion of Science (JSPS) Fellows and by the NINS program

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6. Conclusion

In this study, we theoretically investigate the coronal properties of the Sun and Sun-like stars. The simulation domain extends from the solar surface to the corona, with energy transport from the surface and dissipation in the corona being solved in a self-consistent manner. The thermal structuring of the stellar atmosphere is also reproduced by implementing thermal conduction and radiative cooling. The behaviours of the coronal properties with respect to the loop length and magnetic filling factor are explained by a combination of the behaviour of the coronal energy flux and the RTV scaling law. With the parameter survey, we reproduce the nearly linear relation between the EUV properties (luminosity and photon number luminosity) and the X-ray luminosity, which will be useful in constraining the stellar EUV parameters from the observable quantity $L_X$.

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Antolin, P., Pagano, P., Testa, P., Petrakia, A., & Reale, F. 2021, Nat. Astron., 5, 54.
Appendix A: Derivation of basic equations

In this Appendix, the basic equations are derived from the conventional MHD equations (e.g., Priest 2014) given by

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (A.1)
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{1}{4\pi \rho} (\nabla \times \mathbf{B}) \times \mathbf{B} + \mathbf{g}, \quad (A.2)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \quad (A.3)
\]

\[
\frac{\partial e}{\partial t} + \nabla \cdot \left[ (e + p) \mathbf{v} - \frac{1}{4\pi} (\mathbf{v} \times \mathbf{B}) \times \mathbf{B} \right] = \rho \mathbf{v} \cdot \mathbf{g} + Q_{\text{cond}} + Q_{\text{rad}}, \quad (A.4)
\]

From Eq. (3) and (6), for any vector field \( \mathbf{X} \), its divergence and rotation are given by

\[
\nabla \cdot \mathbf{X} = \frac{1}{r^2 \sqrt{f}} \frac{\partial}{\partial s} \left( X_r r^2 f \right), \quad (A.5)
\]

\[
\nabla \times \mathbf{X} = -\frac{1}{r \sqrt{f}} \frac{\partial}{\partial s} \left( X_r r \sqrt{f} \right) e_s + \frac{1}{r \sqrt{f}} \frac{\partial}{\partial s} \left( X_r r \sqrt{f} \right) e_y, \quad (A.6)
\]

Under the assumption that gravity works only in the field-aligned direction, the gravitational force is simplified as

\[
g = (g \cdot e_s) e_s = g_s e_s, \quad (A.7)
\]

where \( e_s \) is the unit vector in the \( s \) direction. Combining Eqs. (A.1), (A.4), and (A.5), the mass and energy conservation laws are written as

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r^2 \sqrt{f}} \frac{\partial}{\partial s} \left( \rho v_r r^2 f \right) = 0, \quad (A.8)
\]

\[
\frac{\partial e}{\partial t} + \frac{1}{r^2 \sqrt{f}} \frac{\partial}{\partial s} \left[ \left( e + p + \frac{B_r^2 + B_\perp^2}{8\pi} \right) v_r - \frac{B_s}{4\pi} \left( v_s B_x + v_y B_y \right) \right] = \rho v_s g_s + Q_{\text{cond}} + Q_{\text{rad}}, \quad (A.9)
\]

where we use

\[
\left( e + p \right) \mathbf{v} - \frac{1}{4\pi} (\mathbf{v} \times \mathbf{B}) \times \mathbf{B} = \left( e + p + \frac{B_r^2 + B_\perp^2}{8\pi} \right) v_r - \frac{B_s}{4\pi} \left( v_s B_x + v_y B_y \right). \quad (A.10)
\]

The advection term and Lorentz force in Eq. (A.2) are written as

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \frac{1}{4\pi \rho} (\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{1}{4\pi \rho} \left( \frac{B_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) \right) e_s
\]

\[
+ \frac{B_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) e_y, \quad (A.11)
\]

\[
(\nabla \times \mathbf{B}) \times \mathbf{B} = \left[ -\frac{B_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) - \frac{B_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) \right] e_s
\]

\[
+ \frac{1}{4\pi \rho} \left[ \frac{B_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) + \frac{B_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) \right] e_y. \quad (A.12)
\]

The \( s \)-component of Eq. (A.2) is given by

\[
\frac{\partial}{\partial t} \left( v_s^2 + v_r \frac{\partial}{\partial s} v_s \right) - \frac{1}{2} \left( \frac{v_s^2 + v_r^2}{\sqrt{f}} \right) \frac{d}{ds} \ln \left( r^2 f \right) + \frac{1}{\rho} \frac{\partial}{\partial s} p
\]

\[
+ \frac{1}{4\pi \rho} \left[ \frac{B_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) + \frac{B_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) \right] = g_s. \quad (A.13)
\]

which is, after some algebra, written in terms of conservation law as

\[
\frac{\partial}{\partial t} \left( \rho v_s + \frac{1}{r^2 \sqrt{f}} \frac{\partial}{\partial s} \left( \rho v_r r^2 f \right) \right) = \rho \left( \frac{1}{2} v_s^2 + v_r \frac{\partial}{\partial s} v_s \right) + \rho \frac{\partial}{\partial s} p + g_s. \quad (A.14)
\]

The transverse components \( (v_r, v_\perp = v_x e_x + v_y e_y) \) of Eq. (A.2) are

\[
\frac{\partial}{\partial t} v_r + \frac{v_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} v_r \right) = -\frac{B_s}{4\pi \rho r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} B_s \right) = 0, \quad (A.15)
\]

which is written in terms of conservation law as

\[
\frac{\partial}{\partial t} (\rho v_r) + \frac{1}{r^2 \sqrt{f}} \frac{\partial}{\partial s} \left( \rho v_r r^2 f \right) = -\frac{1}{2} \left[ \rho v_r v_r - \frac{B_s B_s}{4\pi} \right] \frac{d}{ds} \ln \left( r^2 f \right) + \rho \frac{\partial}{\partial s} p. \quad (A.16)
\]

The actual basic equation is obtained by adding a phenomenological turbulence term \( \rho \mathbf{D}_\perp \) on the right-hand side, namely,

\[
\frac{\partial}{\partial t} \left( \rho v_\perp + \frac{v_s}{r \sqrt{f}} \frac{\partial}{\partial s} \left( r \sqrt{f} v_\perp \right) \right) = -\frac{1}{2} \left[ \rho v_\perp v_\perp - \frac{B_s B_s}{4\pi} \right] \frac{d}{ds} \ln \left( r^2 f \right) + \rho \mathbf{D}_\perp. \quad (A.17)
\]

The rotation of the Lorentz force in Eq. (A.3) is given by

\[
\nabla \times (\mathbf{v} \times \mathbf{B}) = \frac{1}{r \sqrt{f}} \frac{\partial}{\partial s} \left[ (v_r B_x - v_s B_y) r \sqrt{f} \right] e_s
\]

\[
- \frac{1}{r \sqrt{f}} \frac{\partial}{\partial s} \left[ (v_s B_x - v_r B_y) r \sqrt{f} \right] e_y, \quad (A.18)
\]

Then, the transverse components \( (\mathbf{B}_\perp = B_r e_x + B_\perp e_y) \) of the induction equation are

\[
\frac{\partial}{\partial t} \mathbf{B}_\perp + \frac{1}{r \sqrt{f}} \frac{\partial}{\partial s} \left[ (v_r B_x - v_s B_y) r \sqrt{f} \right] = \frac{1}{2} \left( v_s B_x - v_r B_y \right) \frac{d}{ds} \ln \left( r^2 f \right). \quad (A.19)
\]

which is rewritten in terms of conservation law as

\[
\frac{\partial}{\partial t} (B_r e_x + B_\perp e_y) + \frac{1}{r \sqrt{f}} \frac{\partial}{\partial s} \left[ (v_r B_x - v_s B_y) r \sqrt{f} \right] = \frac{1}{2} \left( v_s B_x - v_r B_y \right) \frac{d}{ds} \ln \left( r^2 f \right). \quad (A.20)
\]
A crucial difficulty in the simulation of coronal heating is that the transition region needs to be resolved with extremely fine grids, otherwise the coronal density is significantly underestimated (Bradshaw & Cargill 2013). This complex problem should be referred to as the ‘transition-region problem’. Here, we investigate the relation between the coronal density and grid size in the transition region and the numerical scheme.

The numerical settings of the test calculations are listed in Table B.1. Two types of the approximated Riemann solver are used in this test: Harten-van Leer-Lax (HLL) and HLL-Disscontinuity (HLLD) schemes. In exchange for its simplicity, the HLL scheme has the disadvantage that the contact discontinuities (entropy modes) suffer from significant numerical diffusion. Meanwhile, by explicitly considering the substructures inside the Riemann fan, the contact (and rotational) discontinuities are better resolved in the HLLD scheme. Because of the transition region is a type of contact discontinuity, its numerical diffusion in the HLL and HLLD schemes should behave differently even with the same grid spacing. Accordingly, the transition-region problem should also change. Therefore, dependence on the choice of the Riemann solver, as well as the spatial resolution, is also investigated.

Figure B.1 shows the time evolution of the loop-top electron density ($n_{e,\text{top}}$, top panel) and loop-top temperature ($T_{\text{top}}$, bottom panel) for fixed filling factor $f_s = 0.05$ and half loop length $l_{\text{top}} = 20 \text{ Mm}$. Three lines show the results with $\Delta s_{\text{min}} = 5 \text{ km}$ and HLL solver (red), $\Delta s_{\text{min}} = 5 \text{ km}$ and HLLD solver (green), and $\Delta s_{\text{min}} = 2 \text{ km}$ and HLLD solver (blue), respectively.

### Appendix B: Transition-region problem and dependence on numerical methods

The basic equation is obtained after adding the phenomenological turbulence term:

$$\frac{\partial}{\partial t} B_\perp + \frac{1}{r^2 f} \frac{\partial}{\partial s} \left[ (v_s B_\perp - v_\perp B_s) r^2 f \right]$$

$$= \frac{1}{2} (v_s B_\perp - v_\perp B_s) \frac{d}{ds} \ln (r^2 f) + \sqrt{\pi \rho D_s^*}. \quad (A.21)$$

Eqs. (A.8), (A.9), (A.14), (A.17) and (A.21) are the basic equations used in this study.

#### Table B.1. List of the simulation runs executed to study the transition-region problem

| Riemann solver | $\Delta s_{\text{min}}$ | filling factor $f_s$ |
|----------------|-------------------------|---------------------|
| HLL            | 5 km                    | [1, 0.33, 0.1, 0.05] |
| HLLD           | 5 km                    | [1, 0.33, 0.1, 0.05] |
| HLLD (high-reso)| 2 km                    | [1, 0.33, 0.1, 0.05] |

Bradshaw & Cargill (2013). Although Bradshaw & Cargill (2013) state that the transition region must be resolved within 1 km to ensure the numerical convergence of coronal density, the test in this study proves the numerical convergence for $\Delta s_{\text{min}} = 5 \text{ km}$, when $f_s = 0.05$. This gap can be attributed to the different numerical settings. Bradshaw & Cargill (2013) consider the thermal response to strong impulsive heating, whereas this study considers quasi-steady uniform coronal heating.

Figure B.2 is the same as Figure B.1 but for $f_s = 1$. The difference in the loop-top density is more significant than $f_s = 0.05$. The HLL and HLLD schemes produce largely different results despite the same grid size at the transition region. The reason is that, in the HLL method, the numerical diffusivity of the entropy mode is roughly proportional to the speed of the fastest wave. At the transition region, the fastest wave speed is the fast magneto-acoustic velocity that increases with the field strength. Thus, with HLL, $f_s = 1$ yields a higher wave speed and larger...
Fig. B.3. Comparison of the time-averaged loop-top temperature $T_{\text{top}}$ and the loop-top temperature predicted by the RTV scaling law $T_{\text{RTV}}$.

value. We note that the deviation of the results of the simulation from those of the RTV prediction is not necessarily evidence of inaccuracy, although, as a general trend, a higher resolution of the transition region should lead to a value closer to the RTV prediction. Deviations from the RTV value in a low-resolution run is prominent at higher coronal temperatures. However, it remains to be seen if the large discrepancy from the RTV value at high coronal temperatures is due to the large numerical diffusion at the transition region or the increase in the required resolution. The coronal-temperature dependence of the transition-region problem should be investigated in the future.

Finally, we note that there are physical mechanisms that can produce the broadening of the transition region, including ambipolar diffusion (Fontenla et al. 1990; Lanzafame 1995). The transition-region problem should also be revisited considering the additional diffusive processes in future.

Appendix C: XUV spectrum from different atmospheric layers

The XUV photons of a specific wavelength (band) do not originate from the plasma with a specific temperature. Indeed, narrow-band filters in the Atmospheric Imaging Assembly of the Solar Dynamics Observatory have broad response functions in terms of temperature (Boerner et al. 2012). Although there is no direct correlation between temperature and wavelength, it is still useful to understand the energy spectrum emanating from plasma with a specific temperature.

Figure C.1 shows the correlation between the emission measure distribution in a given temperature bin (top panel) and the corresponding spectral energy flux normalised at 1 au (bottom panel) for the fiducial case ($f_\alpha = 0.01$, $h_{\text{loop}} = 20 \text{ Mm}$). For better visualisation, the spectral energy fluxes averaged over 10 Å are shown in the bottom panel. The black dotted line in the top panel represents the total emission measure distribution, while the black dotted line represents the corresponding spectral energy flux in the bottom panel. We consider three temperature
As a general trend, the low-energy photons tend to originate from the low-temperature plasma and vice versa. We note that a non-negligible fraction of EUV photons originates from the chromosphere and the transition region, and therefore including them in the model is essential in predicting the XUV spectrum.

![Graph showing emission measure distribution and corresponding spectral energy flux at 1 au.](image)

**Fig. C.1.** Emission measure in the limited range of temperature and corresponding spectral energy flux at 1 au. For better visualisation, the spectral energy fluxes in the bottom panel are averaged over 10 Å. Orange, green and blue lines correspond to the emission measures and spectral energy fluxes of $T \approx 10^4$ K (chromosphere), $T \approx 10^5$ K (transition region) and $T \approx 10^6$ K (corona), respectively. Black dotted lines show the full emission measure distribution and corresponding full spectral energy flux.

Bins centred on $T = 10^4$ K, $T = 10^5$ K, and $T = 10^6$ K corresponding to the upper chromosphere, transition region, and corona, respectively. The corresponding spectral energy fluxes are highlighted with the same colours in the bottom panel, which showcase the following three properties. First, emissions from the upper chromosphere ($T \approx 10^4$ K) dominate the Lyman continuum but negligible in $\leq 600$ Å. Second, emissions from the transition region ($T \approx 10^5$ K) are nearly uniform in the middle of the EUV wavelength ($200 \text{ Å} \leq \lambda \leq 800 \text{ Å}$) but negligible in the X-ray range. Third, emissions from the corona ($T \approx 10^6$ K) are the dominant sources of stellar X-rays. The contribution to the EUV emission is also non-negligible, especially in the shorter wavelength.