The potential distribution in two-electrode gas-filled gap in weak and strong electric field

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Abstract. At theoretical analysis, the problem of distribution of the electric field intensity in a plane-parallel interval in which the current carriers are introduced, taking into account the volume charge for both the “weak” and “strong” electric fields are solved.

1. Introduction
To obtain current carriers in gas environment, sources of charged particles, such as electrons or ions, of different constructions, are often used. In some cases, it is necessary to know how the electric field intensity in the interval between the electrodes, where the current carriers are introduced, depends from performance of the source. Such information is necessary for the calculation of installations intended for the implementation of certain processes of electron-ion technologies, in the experimental study of the directed velocity of charged particles in gases, etc.

The presence of a charged particle source can be simulated by introducing of some emitter surface. Therefore, to solve the task of the distribution of the electric field, it is necessary to solve the Poisson equation for the two – electrode gap, in which one of the electrodes is an emitter, and the other is a collector of charged particles.

2. Simulation of potential distribution
Consider the interval between the electrodes of a plane-parallel construction. For such a case, the problem is solved and there is an equation that allows to determine the field strength \( E \) in an arbitrary cross section, remote by the distance \( x \) from the emitting surface. When the electric field is” weak”, that is, when the velocity of the directional motion of the current carriers is proportional to \( E \) (there is a mode of mobility), the ratio [1] is true:

\[
E(x) = \sqrt{\frac{8\pi j x}{b} + \left( \frac{j}{j_{em} - j b} \right)^2},
\]

This equation, as in [1], is written in the CGS system.

Here \( j \) is the density of the current flowing between the electrodes; \( v_m \) – is the arithmetic mean speed of charged particles; \( b \) – is their mobility; \( j_{em} \) – is the emission current density.

As you can see from the equation, to calculate you need to know \( v_m \) and \( j_{em} \). Precise information on these values is generally not available, which makes formula (1) suitable only for qualitative results.
A similar equation for \( E(x) \) is available and for the regime of “strong” electric field, when \( v_n = b' E^{1/2} \), where \( v_n \) is the directed velocity of the current carriers; \( b' \) is the coefficient depending on the type of charged particles, on the kind and pressure of the gas medium.

To obtain an equation free of the noted disadvantage for the field strength calculation, we integrate the Poisson equation and assume that the current density between the emitting surface and the collector is \( j \) and that

\[
U(x = L) = U_a, \quad U(x = 0) = 0
\]

where \( U \) – is the space potential, \( L \) – is the distance between the electrodes. As a result of integration, we obtain a condition that relates the value of the potential gradient with the electric mode and with the kind of gas medium. A similar problem, but for a vacuum two-electrode gap, was solved in [2].

For a "weak" electric field, the Poisson equation has the form:

\[
\frac{d^2U}{dx^2} = \frac{j}{\varepsilon_0b}
\]

where \( \varepsilon_0 \) – permittivity of vacuum. Having integrated this equation twice, taking into account the condition (2), we obtain:

\[
U_a = \frac{\varepsilon_0b}{3j}\left\{\left[\frac{dU}{dx}\right]_{x=0}^2 + \frac{9\varepsilon_0 j}{4L^2} j_0 \right\}^{3/2} - \left(\frac{dU}{dx}\right)_{x=0}^3 \right\},
\]

where \( j_0 = 9\varepsilon_0 bU_a^2 / 8L^3 \) – is the current density that flows between the electrodes at \( (dU/dx)_{x=0} = 0 \). Let’s take a convenient form of writing \( dU/dx = \beta U_a / L \). Denote

\[
\frac{dU}{dx}igg|_{x=0} = \beta_0 U_a / L; \quad \frac{dU}{dx}igg|_{x=L} = \beta L U_a / L
\]

In this regard the equation (4) after algebraic transformations gives a condition which allows in the result of numerical analysis to find the value of \( \beta_0 \) and, consequently, the field strength on the emitting surface. This condition has the form

\[
\left(\beta_0^2 + \frac{9 j}{4 j_0}\right)^{3/2} - \beta_0^3 = \frac{27 j}{8 j_0}
\]

Corresponding to this condition the dependence \( \beta_0 \) from the relationship \( jj_0 \) presented in table 1. The integration of equation (3) allows us to relate the potential gradient in an arbitrary section with the field strength on the emitting surface and with the current density

\[
\frac{dU}{dx} = \left(\frac{\beta_0^2 + \frac{9 j}{4 j_0}}{L}ight)^{1/2} U_a / L
\]

From here we find that for the surface of the collector of charged particles the following relation is true

\[
\left(\frac{dU}{dx}\right)_{x=L} = \beta L U_a / L; \quad \beta_L = \left(\beta_0^2 + \frac{9 j}{4 j_0}\right)^{1/2}
\]

Table 1. Dependence \( \beta_0 \) from the relationship \( jj_0 \) for a weak electric field.

| \( jj_0 \) | 0  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( \beta_0 \) | 1.0 | 0.94 | 0.88 | 0.82 | 0.75 | 0.68 | 0.60 | 0.52 | 0.42 | 0.29 | 0 |

Consider the case of a “strong” electric field. In this case, the Poisson equation is written as follows:

\[
\frac{d^2U}{dx^2} \left(\frac{dU}{dx}\right)_{x=0}^{1/2} = \frac{j}{\varepsilon_0b'}
\]
Integrating of this equation gives the following condition for determining $\beta_0$:

$$\left(\frac{5\sqrt{3}}{3\sqrt{3} j_0} + \beta_0^{3/2}\right)^{5/3} - \beta_0^3 = \frac{25\sqrt{3}}{9\sqrt{3}} j_0$$

The numerical data of the dependence $\beta_0$ ($j/j_0$) for the case of a strong field are presented in table 2.

| $j/j_0$ | 0   | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\beta_0$ | 1.0 | 0.93| 0.86| 0.80| 0.78| 0.70| 0.61| 0.52| 0.43| 0.32| 0.20| 0   |

The electric field intensity in an arbitrary section for the case of a strong field is determined by the expression obtained as a result of the first integration of the equation (6)

$$\frac{du}{dx} = \left[\frac{3}{3} \frac{2}{3} \frac{x}{L} + \beta_0 \frac{2}{3} \frac{u_0}{L}\right].$$

The results show that the potential distribution in the considered case can be represented as a function of the ratio $j/j_0$, and the field strength can be calculated without data on the thermal velocity of the current carriers and the emission capacity of the source of charged particles.

As a result of the work performed, it is shown that the ratio $j/j_0$ not only for vacuum but also for gas environment is a convenient measure for practical use, which allows to estimate the effect of the volume charge of the current carriers on the potential distribution between the electrodes. The proposed calculation ratios make it possible to optimize the geometry and electrical mode of sources operating at high gas pressure.

The external zone of corona discharge was used as an emitter in experimental studies of the emission capacity of clusters of similarly charged particles. The studies were carried out using a three-electrode device. For the formation of the corona discharge, many-row many-needle and grid electrodes were used. A flat collector of the investigated particles was used as the third electrode. The geometry of the many-needle electrode provided sufficient homogeneity of the beam on the plane of its cross-section. The extraction of charged particles by the collector field was carried out through a grid electrode.

The value of the interelectrode gap grid-flat electrode $L_2$ is determined based on the geometry of the grid cells. In practice, the distance $L_2$ must be selected within 3-5 times the step of the grid cells. In this case, the unevenness of the current density distribution in the cross section of the charge flow in the area of the flat electrode will be approximately half of the unevenness of the current density distribution over the corona discharge section on the surface of the grid from the side facing to the corona electrode.

To calculate the value of the current density $j_1$, flowing in the interval of the corona electrode-grid, you must use the following relations:

$$j_1 = \frac{L_2}{a}, \quad \alpha = \alpha_0 \sqrt{\frac{E_{02}}{E_{01}}},$$

where $\alpha$ – is the current extraction coefficient, $\alpha_0$ – is the optical transparency of the grid, $E_{01}$ – is the electric field intensity on the grid surface facing to the corona electrode, $E_{02}$ – is the electric field intensity on the grid surface facing to the flat electrode.

The voltage value on the electrodes of the corona electrode – grid and the distance between the electrodes are selected based on a given uneven distribution of charges on the surface of the flat electrode. The size of the step between the needles $l$ is directly dependent on the distance from the corona electrode-grid $L_1$. Typically, the ratio of $l/L_1$ is in the range 0.3-0.5. At the same time, the
unevenness of the current density distribution over the corona discharge section in the grid area does not exceed 7%.

From the existing family of volt-ampere characteristics \( j_1 = f(U_1) \), constructed for many-needle electrodes with needles of known tip radius and for known values of \( U/L_1 \), by the calculated current density \( j_1 \), we determine the value of the corresponding voltage on the electrodes \( U_1 \).

The obtained values of \( U_1 \) and \( L_1 \) allow us to calculate the electric field intensity on the grid surface facing the corona electrode:

\[
E_{L1} = \beta_{L1} \frac{U_1}{L_1},
\]

where \( \beta_{L1} \) – dimensionless coefficient.

The value of the electric field intensity \( E_{02} \) is associated with the voltage between the electrodes grid-flat electrode \( U_2 \) and the distance between these electrodes \( L_2 \) by following ratio:

\[
E_{02} = \beta_{02} \frac{U_2}{L_2},
\]

where \( \beta_{02} \) – dimensionless coefficient.

The numerical values of \( \beta_{L1} \) and \( \beta_{02} \) are determined from the graphs shown in figure 1., where \( j \) – is the density of the current flowing between the electrodes, \( j_0 \) – is the value of the current density that would flow between the electrodes at zero field intensity on the emitting surface.

![Figure 1](image-url)  

**Figure 1.** To determination of the electric field intensity.

In equation (10), the coefficient is a function of the ratio \( j/j_0 \). The value \( j_2 \) is known as the value found from the condition (8). In order to find the value of \( U_2 \), we use the equation for \( j_0 \) (10):

\[
j_0 = \frac{\varepsilon_0 b j_2}{\varepsilon_2},
\]

where \( \varepsilon_0 \) – dielectric constant of vacuum, \( b \) – coefficient of mobility of the carriers of current.

Having made simple transformations in equation (11), we obtain:

\[
U_2 = L_2 \frac{\theta_{L1} j_2}{\varepsilon_0 b \sqrt{j_2}}.
\]

Substituting the expression (12) in (10), we obtain:

\[
E_{02} = \xi \frac{\theta_{L1} \sqrt{j_2}}{\varepsilon_0 b}.
\]
where

$$\xi(j_2/j_{02}) = \beta_{02} \sqrt{\frac{j_{02}}{j_2}}. \quad (14)$$

The graph of the function $\xi(j_2/j_{02})$ is shown in figure 2.

Thus, by calculating using the graph shown in figure 2, find the ratio $j_2/j_{02}$.

$$\xi(j_2/j_{02}) = \frac{E_{02}}{\frac{8j_2}{9e_0 b}} \quad (15)$$

Given that the value of $j_2$ is already defined, we find $j_{02}$, which allows using the expression (11) to calculate $U_2$.

![Figure 2. To the calculation of the sources of charged particles.](image)

3. Conclusion

Thus, the presented method of calculation of three-electrode sources of charged particles allows to calculate their electrical modes of operation and geometric parameters. Quite good (within 10%) coincidence of experimental and calculated data allows us to conclude about the suitability of the proposed method and the correctness of the calculated ratios in its justification.

References

[1] Granovsky V L 1971 Electrical current in gas. The steady current. (Moscow: Science)

[2] Ivey H F 1979 Phys. Rev. 4 21