Effect of thermal fluctuations on spin degrees of freedom in spinor Bose-Einstein condensates

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We consider the effect of thermal fluctuations on rotating spinor \( F = 1 \) condensates in axially-symmetric vortex phases, when all the three hyperfine states are populated. We show that the relative phase among different components of the order parameter can fluctuate strongly due to the weakness of the interaction in the spin channel. These fluctuations can be significant even at low temperatures. Fluctuations of relative phase lead to significant fluctuations of the local transverse magnetization of the condensate. We demonstrate that these fluctuations are much more pronounced for the antiferromagnetic state than for the ferromagnetic one.

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I. INTRODUCTION

Properties of rotating spinor Bose-Einstein condensates attract a lot of attention now. First examples of these systems with hyperfine spin \( F = 1 \) were found in optically trapped \(^{23}\)Na \(^{1}\). Vortex phase diagram of spinor condensates is very rich, since the order parameter has three components in \( F = 1 \) case and five components in \( F = 2 \) case. Topological excitations in spinor condensates were studied theoretically in a large number of articles see, e.g., Refs. \( 2, 3, 4, 5, 6 \).

At the same time, an interest is now growing to temperature effects in atomic condensates. Refs. \( 7, 8, 9, 10, 11 \) study theoretically the Berezinskii-Kosterlitz-Thouless (BKT) transition associated with the proliferation of thermally-excited vortex-antivortex pairs. For instance, in Ref. \( 8 \) it was shown that in quasi two-dimensional condensates BKT transition can occur at rather low temperatures, \( T \sim 0.5 T_c \), at number of particles in the system \( N \sim 10^4 \). Recently, some signatures of possible BKT phase were also found close to the critical temperature \( T_c \) in experimental work \( 12 \), where condensates in optical lattice have been studied. Finally, experimental evidence for the BKT transition in trapped condensates was reported in Ref. \( 13 \). Refs. \( 14, 15 \) deal with the thermal fluctuations of positions of vortices in rotated scalar condensates. Note that, according to the Mermin-Wagner-Hohenberg theorem, Bose-Einstein condensation is not possible in 2D homogeneous systems. However, application of the trapping potential leads to the macroscopic occupation of the ground state of Bose gas.

The aim of the present paper is to study the effect of thermal fluctuations in rotated quasi two-dimensional spinor condensates. These systems have a specific degree of freedom, associated with the relative angle among different components of the order parameter corresponding to different hyperfine state. In other words, this angle determines coherence among components of the order parameter. Also it influences a transverse magnetization of the condensate. In this paper, we focus on thermal fluctuations of this angle. Note that experimentally, at present time, it is possible to study the condensate phase \( 16, 17, 18 \), see also Ref \( 19 \). In addition, recently, a new and nondestructive method for measuring the local magnetization of the condensate was proposed and successfully applied in Ref. \( 20 \).

We show that the relative angle among hyperfine components of the order parameter in 2D case can experience strong thermal fluctuation even at low temperatures. The reason is the weakness of the spin energy of the system as compared to interactions in density channel. Also fluctuations of this angle lead to significant relative fluctuations of the local transverse magnetization of the condensate, which are much larger in the antiferromagnetic case than in the ferromagnetic one.

This paper is organized as follows. In Section II, we give a basic formulation of the problem. In Section II, we discuss our main results for the fluctuations of angle and spin textures. We conclude in Section III.

II. BASIC FORMULATION

We consider harmonically-trapped quasi 2D Bose-Einstein condensate with spin \( F = 1 \). The trapping potential is given by

\[ U(r) = \frac{m \omega^2 \perp^2 r^2}{2}, \]

where \( \omega \perp \) is a trapping frequency, \( m \) is the mass of the atom, and \( r \) is the radial coordinate. The system is rotated with the angular velocity \( \Omega \), well below the critical rotation speed \( \omega \perp \), and the number of atoms in the cloud is \( N \). In this paper, we restrict ourselves on the range of temperatures much smaller than \( T_c \). Therefore, we can neglect a noncondensate contribution to the free energy of the cloud. The total energy of the system in this approximation coincides with the energy of the condensate. For the number of condensed particles, we use the ideal
gas result:

\[ N(T) = N \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]. \tag{2} \]

At the same time,

\[ \frac{\hbar \omega}{k_B T_c} = \sqrt{\frac{\zeta(2)}{N}}, \tag{3} \]

where \( \zeta(2) \) is a Riemann zeta function, \( \sqrt{\zeta(2)} \approx 1.28 \). Eqs. (2) and (3) remain accurate even for the case of interacting particles \([21]\). We also introduce a dimensionless temperature \( T/T_c \). Since we are considering low temperatures, \( T \sim 0.1 T_c \), temperature dependence of condensed particles number can be neglected, \( N(T) \approx N \).

The order parameter in the \( F = 1 \) condensate has three components \( \Psi_j \) \((j = -1, 0, 1)\). The free energy of the system can be written as \([22, 23]\):

\[ F = \hbar \omega \int dS \left[ \Psi_j^\dagger \hat{h} \Psi_j + 2 \pi g_n \Psi_j^\dagger \Psi_j^\dagger (F_a)_{jk} \Psi_k \Psi_j + 2 \pi g_s \Psi_j^\dagger (F_a)_{jk} \Psi_j^\dagger \Psi_k \right. \]
\[ \left. + \left( F_a \right)_{jk} \Psi_j^\dagger \Psi_k \right], \tag{4} \]

where the integration is performed over the system area, repeated indices are summed, \( F_a \) \((a = x, y, z)\) is the angular momentum operator, which can be expressed in a matrix form through the usual Pauli matrices, \( \hat{h} \) is the one-body Hamiltonian, given by

\[ \hat{h} = -\frac{\nabla^2}{2} + \frac{\mathbf{r}^2}{2}. \tag{5} \]

Constants \( g_n \) and \( g_s \) characterize interactions in density and spin channels and are given by

\[ g_n = \frac{(a_0 + 2a_2) n_z}{3}, \tag{6} \]
\[ g_s = \frac{(a_2 - a_0) n_z}{3}, \tag{7} \]

where \( a_0 \) and \( a_2 \) are scattering lengths for atoms with total spin 0 and 2, and \( n_z \) is the concentration of atoms in longitudinal direction. In real spinor condensates, \( |g_s| \ll |g_n| \), since \( a_0 \approx a_2 \). Typically, \( |g_s/g_n| \approx 0.001 - 0.01 \), and this ratio can be tuned. In this paper, we study the case of relatively dilute condensate and take \( g_n = 10 \). We will consider different values of \( N \) but at fixed value of interaction parameter \( g_n \). This is possible, since, in the case of a single layer cloud, we can always tune the trapping frequency in the longitudinal direction keeping \( g_n \) constant. To ensure the regime of quasi-two-dimensionality, we can also tune \( \omega \). In this case, we have to change the rotation speed to keep the dimensionless rotation speed the same, and the temperature to fix dimensionless \( t \).

In real atomic condensates, \( a \) is approximately several nanometers. The most realistic value of \( N \) for this \( g_n \) is close to \( 10^3 \), and to illustrate the effect of \( N \) we will consider the following range: \( 10^2 \lesssim N \lesssim 10^4 \).

The total magnetization of the condensate is fixed:

\[ M = \int dS |\Psi_j|^2 j. \tag{8} \]

Magnetization \( M \) is normalized in terms of \( N \) and maximum of \(|M|\) is equal to 1. One has also to take into account the normalization condition for the order parameter:

\[ \int dS \Psi_j^\dagger \Psi_j^* = 1. \tag{9} \]

The spatial profiles of all the components of the order parameter in the equilibrium can be found from the condition of minimum of energy (4). It is also convenient to introduce the longitudinal \( l_z \) and transverse \( l_{tr} \) local magnetizations of the condensate:

\[ l_z = |\Psi_1|^2 - |\Psi_{-1}|^2, \tag{10} \]

\[ l_{z}^2 = l_{z}^2 + l_{y}^2 = 2 |\Psi_0|^2 |\Psi_1|^2 + 2 |\Psi_0|^2 |\Psi_{-1}|^2 + 4 (\Psi_0^\dagger \Psi_1^* \Psi_{-1}^* + c.c.). \tag{11} \]

The spin energy in this case can be represented as

\[ F_{spin} = 2 \pi g_s \hbar \omega N \int dS (l_z^2 + l_{tr}^2). \tag{12} \]

In this paper, we restrict ourselves only to the case of axially-symmetric phases, when moduli of all the components of the order parameter are independent on the azimuthal angle and depend only on the radial coordinate \( r \). Note that equilibrium vortex phases in this situation were studied in Refs. \([2, 3]\) for the spin \( F = 1 \) condensate and in Ref. \([4]\) for \( F = 2 \) system. For the axially-symmetric phases, each component of the order parameter can be represented as

\[ \Psi_j(r, \phi) = f_j(r) \exp(-i L_j \phi - i \delta_j), \tag{13} \]

where \( \phi \) is a polar angle, \( L_j \) is a winding number, and \( \delta_j \) is a relative phase. We will denote such phases as \( (L_{-1}, L_0, L_1) \). As it was shown in Ref. \([3]\), an axial symmetry of the solution implies that winding numbers satisfy the relation: \( L_1 + L_{-1} = 2 L_0 \). In this case, according to Eqs. (11) and (12), the spin energy depends on relative angle \( \chi = 2 \delta_0 - \delta_1 - \delta_{-1} \).
It is important to note that only a spin contribution to the total energy (4) depends on phases \( \delta_j \) via the spin-mixing term. For the stationary state, which is a local minimum of Gross-Pitaevskii functional (4), the value of \( \chi \) is determined by the sign of interaction constant in a spin channel \( g_s \). For positive \( g_s \) (antiferromagnetic case), a minimum of \( F_{\text{spin}} \) is attained at \( \chi = \pi \), whereas for negative \( g_s \) (ferromagnetic case) \( \chi = 0 \).

### III. RESULTS AND DISCUSSION

According to the results of Ref. [2], for the antiferromagnetic state \( (g_s > 0) \), phases \((-1, 0, 1)\) and \((1, 1, 1)\) are energetically favorable in the region of small and moderate values of magnetization \( M \). Phase \((-1, 0, 1)\) is realized at low rotation frequencies \( \Omega \) and \((1, 1, 1)\) at higher \( \Omega \). In Ref. [2], it was shown that \((0, 1, 2)\) state is favorable in the ferromagnetic case \( (g_s < 0) \) in a region of moderate values of \( \Omega \) and \( M \). In these phases, all three hyperfine states are populated. Fluctuations of \( \chi \) have a sense only in this case, since the \( \chi \)-dependent part of the energy is equal to zero identically, if one of the components of the order parameter is zero. In this paper, we will concentrate on these three vortex states, since they are appropriate candidates for the illustration of the effect of thermal fluctuations. Note that in homogeneous spin-1 condensate atoms populate only two or one hyperfine state(s); they can populate three states only if the system is trapped and experiences rotation, which generates vortices.

An important feature of real atomic spinor Bose-Einstein condensates is a weakness of the spin interactions comparing to the interaction in density channel \((|g_s| \ll |g_n|)\). At the same time, the coherence among the different components of the order parameter (angle \( \chi \)) is fully determined by the spin interaction. Angle \( \chi \) also influences the transverse magnetization of the condensate, as seen from Eq. (11). Note that a longitudinal component of magnetization is independent on \( \chi \).

Smallness of \( g_s \) comparing to \( g_n \) leads to the fact that thermal fluctuations of relative angle \( \chi \) become significant at much lower temperatures than fluctuations of the density of particles. Therefore, at relatively low temperatures, one can assume that the moduli of all the components of the order parameter remain fixed (that can be also checked numerically), whereas \( \chi \) is fluctuating. For the case of small fluctuations of \( \chi \), one can use a harmonic approximation and represent the deviation of the energy of the system from the equilibrium, \( \delta F = F(\chi_0 + \delta \chi) - F(\chi_0) \), as a quadratic function in terms of the deviation of angle \( \chi \) from the equilibrium \( \delta \chi = \chi - \chi_0 \):

\[
\delta F = 2\pi g_s \hbar \omega_\perp N I (\cos(\chi_0 + \delta \chi) - \cos(\chi_0)) \\
\approx \pi |g_s| \hbar \omega_\perp N I (\delta \chi)^2,
\]

where \( I = \int dS \left( f_j f_{-j} f_j^2 \right) \). Under these assumptions, the average square of the deviation of \( \chi \) from the equilibrium is given by

\[
\langle (\delta \chi)^2 \rangle_T = \frac{\int d(\delta \chi) (\delta \chi)^2 \exp(-\frac{\delta F}{k_B T})}{\int d(\delta \chi) \exp(-\frac{\delta F}{k_B T})}.
\]

Integrals in Eq. (15) can be calculated analytically. After taking into account Eq. (3), we get:

\[
\langle (\delta \chi)^2 \rangle_T = \frac{t}{1.28\sqrt{N} |g_s| I}.
\]

We also introduce a quantity \( \Delta \chi = \sqrt{\langle (\delta \chi)^2 \rangle_T} \), which can be considered as an average deviation of angle \( \chi \) from the equilibrium.

In Fig. 1 we plot calculated dependence of \( \Delta \chi \) (measured in degrees) as a function of the number of particles in the system for different vortex phases at \( t = 0.1 \) and \( g_n = 10 \). This value of \( g_n \) is close to typical experimental ones (\( a_0 \approx 5 \) \( \text{nm} \), \( n_z \approx 2 \) \( \text{nm}^{-1} \)), see also calculations of Ref. [2]. We assume that, for \((-1, 0, 1)\) and \((1, 1, 1)\) states, \( g_s = 0.01 g_n \) and \( M = 0.1 \), whereas for \((0, 1, 2)\), \( g_s = -0.01 g_n \) and \( M = 0.5 \). Note that \( \Delta \chi \) for the particular phase is independent on \( \Omega \), since \( I \) has the same property. We see that even for quite low temperatures, \( \Delta \chi \) can be rather large and the coherence among different components of the order parameter is practically destroyed. For smaller value of \( |g_s| \), fluctuations of \( \chi \) are, of course, even stronger. To illustrate the effect of temperature, in the inset to Fig. 1, we show the dependence of \( \Delta \chi \) on \( T \) for \((0, 1, 2)\)-phase (curve 1), \((1, 1, 1)\) phase (curve 2), and \((-1, 0, 1)\) phase (curve 3) at fixed number of atoms \( N = 1000 \), \( g_s = -0.05 g_n \) for the first curve and \( g_s = 0.05 g_n \) for two others. Note that \( \Delta \chi \) is almost independent on total magnetization \( M \) of the condensate.

As we already pointed out, fluctuations of \( \chi \) lead to that of \( \delta \chi_0 \). In a harmonic approximation, one can express the average deviation of \( \delta \chi_0 \) from the equilibrium \( \langle \delta \chi_0 \rangle_T \) through the deviation of \( \chi \):

\[
\frac{\langle \delta \chi_0 \rangle_T}{|\delta \chi|} = (-1)^u \frac{1}{2} \langle (\delta \chi)^2 \rangle_T \frac{f_1 f_{-1}}{f_1 + (-1)^{u+1} f_{-1}}.
\]
where \( u = 0 \) for the antiferromagnetic case and \( u = 1 \) for the ferromagnetic one. If in the antiferromagnetic state the total magnetization is not large, \( M \lesssim 0.5 \), one can expect that \( (f_1 - f_{-1})^2 \ll f_1 f_{-1} \), and, therefore, even small fluctuations of \( \chi \) lead to strong relative fluctuations of \( |l_{tr}| \). At the same time, for ferromagnetic case, \( u = 1 \) in this equation, and relative fluctuations of \( |l_{tr}| \) are much smaller.

We have calculated \( \langle |l_{tr}| \rangle_T \) for different vortex phases and our calculations revealed that \( \langle |l_{tr}| \rangle_T / |l_{tr}| \) is almost independent on radial coordinate \( r \) for vortex phases \((-1, 0, 1) \) and \((1, 1, 1) \). This is due to the fact that \( |L_{-1}| = |L_1| \) for these states, therefore, \( f_1(r) \) is nearly proportional to \( f_{-1}(r) \), and, according to Eq. (15), \( \langle |l_{tr}| \rangle_T / |l_{tr}| \) should only slightly depend on \( r \). In Fig. 2 we present \( \langle |l_{tr}| \rangle_T / |l_{tr}| \) as a function of total magnetization of the condensate for \((-1, 0, 1) \) state at \( t = 0.1, g_s = 0.01g_n \) (antiferromagnetic case), and \( N = 1000 \). We see that relative fluctuations of transverse magnetization can be significant even at low temperature. Value of \( \langle |l_{tr}| \rangle_T / |l_{tr}| \) decreases with increasing \( M \). This result is natural, since condensate becomes more polarized with growing \( M \). An absolute value of \( \langle |l_{tr}| \rangle_T \) also remains sizable. Although value of fractional quantity \( \langle |l_{tr}| \rangle_T / |l_{tr}| \) is growing with decreasing of \( M \), the value of \( |l_{tr}| \) itself becomes smaller. Therefore, we found that the most appropriate values of \( M \) to observe fluctuations of transverse magnetization is around \( M = 0.2 \), where both \( \langle |l_{tr}| \rangle_T / |l_{tr}| \) and \( |l_{tr}| \) are high: \( \langle |l_{tr}| \rangle_T / |l_{tr}| \gtrsim 0.1 \), whereas \( l_{tr} \) is comparable to the longitudinal magnetization \( l_z \) in the fully polarized state at \( M = 1 \), where it should be easily detectable experimentally. Value of \( \langle |l_{tr}| \rangle_T / |l_{tr}| \) depends also on the vortex phase; we found that in \((1, 1, 1) \) state it is even much larger than in \((-1, 0, 1) \) state.

Also we have calculated \( \langle |l_{tr}| \rangle_T \) for the ferromagnetic \((0, 1, 2) \) phase. As can be expected, in this case, relative fluctuations of \( |l_{tr}| \) are much weaker. Physically, this is because \( |l_{tr}| \) is proportional to the ferromagnetic order parameter \( \delta \), which is responsible for the ferromagnetic ordering. Therefore, one can expect that in the ferromagnetic phase this order parameter is more robust with respect to thermal fluctuations, than in the antiferromagnetic one. In addition, average deviation of \( |l_{tr}| \) from the equilibrium is negative and its modulus is growing with increase of \( M \), in contrast to the antiferromagnetic system.

Thermal fluctuations should also be important in the case of \( F = 2 \) condensate, where there are two interaction constants in spin channel and two characteristic angles. Therefore, one can expect more complicated behavior, as compared to \( F = 1 \) condensate. For instance, in homogeneous \( F = 2 \) system, a cyclic state can have a lowest energy; in this case atoms populate three hyperfine states, and the spin energy depends on the coherence among them. Fluctuation problem for this system was analyzed in Ref. 24. A new method to create such entangled states in spin-1 condensate was recently applied experimentally in Ref. 19, where a microwave energy was injected to the system. As a result, particles redistribute from spin \(-1 \) state to spin 0 and 1 states, and all three magnetic sublevels become populated. The spin-mixing dynamics in \( F = 1 \) condensate was studied theoretically in Ref. 25.

Note that in Eq. (14) we have assumed that fluctuating \( \chi \) is spatially independent that is not true in general case. However, spatial gradients of \( \chi \) give some additional contribution to the kinetic energy of the system, which is much larger than the spin energy. Therefore, gradients of \( \chi \) result in rather large increase of total energy, and we can neglect them for the trapped system, at least for our range of parameters. In other words, healing length for \( \chi \) far exceeds the Thomas-Fermi radius of the system, and, therefore, although \( \chi \) is fluctuating inside the cloud, it remains nearly constant 24, except of the surface layer, where the density of particles is low.

Thermal fluctuations of \( \chi \) should be also noticeable in three-dimensional condensates at low and moderate temperatures. In general, the dependences of the number of condensed particles on the reduced temperature and critical temperature on the total number of atoms for 3D case are similar to that in 2D system, which are described by Eqs. (2) and (3). The main difference is the powers of \( t \) and \( N \) in the right hand sides of Eqs. (2) and (3) that are 3 and \(-1/3 \) (\( \hbar \omega_\perp/kT_c \sim N^{-1/3} \)), respectively. However, in this case one has to take accurately into account the possibility of long wave length fluctuations of \( \chi \) in longitudinal direction and formation of kinks 24.

**IV. CONCLUSIONS**

In this paper, we have studied the effect of thermal fluctuations on the coherence among different components of the order parameter in quasi 2D rotating \( F = 1 \) Bose-Einstein condensate, when all three hyperfine states are populated. Different axially-symmetric vortex phases were considered. We have shown that the deviation of the relative phase \( \chi = 2\delta_0 - \delta_1 - \delta_{-1} \) from the equilibrium can be very significant even at low temperatures, much smaller than \( T_c \). Fluctuations of relative angle induce sizable fluctuations of the spin texture, namely, local transverse magnetization of the condensate. We have shown that these fluctuations are much more pronounced in antiferromagnetic case than in the ferromagnetic one. The recently proposed in Ref. 24 direct and nondestructive method for the imaging of spinor BEC spatial magnetization (or some of its modification) can be applied for the experimental study of the thermal fluctuations of spin textures, since it enables multi-shot imaging and one can directly observe the dynamics of a single sample.
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[1] J. Stenger, D. M. Stamper-Kurn, H. J. Miesner, A. P. Chikkatur, and W. Ketterle, Nature 396, 345 (1999).
[2] T. Mizushima, K. Machida, and T. Kita, Phys. Rev. Lett. 89, 030401 (2002).
[3] T. Isoshima and K. Machida, Phys. Rev. A 66, 023602 (2002).
[4] J. W. Reijnders, F. J. van Lankvelt, K. Schoutens, and N. Read, Phys. Rev. Lett. 89, 120401 (2002).
[5] M. H. Wheeler, K. M. Mertes, J. D. Erwin, and D. S. Hall, Phys. Rev. Lett. 93, 170402 (2004).
[6] W. V. Pogosov, R. Kawate, T. Mizushima, and K. Machida, Phys. Rev. A 72, 063605 (2005).
[7] A. Trombettoni, A. Smerzi, and P. Sodano, New J. Phys. 7, 57 (2005).
[8] T. P. Simula, M. D Lee, and D. A. W. Hutchinson, Phys. Rev. Lett. 89, 030401 (2002).
[9] M. Holzmann, G. Baym, J. P. Blaizot, and F. Laloe, cond-mat/0508131.
[10] T. P. Simula and P. B. Blakie, Phys. Rev. Lett. 96, 020404 (2006).
[11] D. Schumayer and D. A. W. Hutchinson, cond-mat/0601500.
[12] S. Stock, Z. Hadzibabic, B. Battelier, M. Cheneau, and J. Dalibard, Phys. Rev. Lett. 95, 190403 (2005).
[13] Z. Hadzibabic, P. Kruger, M. Cheneau, B. Battelier, and J. Dalibard, to be published in Nature; cond-mat/0605291.
[14] Y. Castin, Z. Hadzibabic, S. Stock, J. Dalibard, and S. Stringari, cond-mat/0511330.
[15] W. V. Pogosov and K. Machida, cond-mat/0601604.
[16] Y. J. Wang, D. Z. Anderson, V. M. Bright, E. A. Cornell, Q. Dio, T. Kishimoto, M. Prentiss, R. A. Saravanav, S. R. Segal, and S. Wu, Phys. Rev. Lett. 94, 090405 (2005).
[17] M. H. Wheeler, K. M. Mertes, J. D. Erwin, and D. S. Hall, Phys. Rev. Lett. 93, 170402 (2004).
[18] M. Saba, T. A. Pasquini, C. Sanner, Y. Shin, W. Ketterle, and D. E. Pritchard, Science, 307, 1945 (2005).
[19] M. S. Chang, Q. Qin, W. Zhang, L. You, and M. S. Chapman, Nature Physics 1, 111 (2005).
[20] J. M. Higbie, L. E. Sadler, S. Inouye, A. P. Chikkatur, S. R. Leslie, K. L. Moore, V. Savalli, and D. M. Stamper-Kurn, Phys. Rev. Lett. 95, 050401 (2005).
[21] C. Gies, B. P. van Zyl, S. A. Morgan, and D. A. W. Hutchinson, Phys. Rev. A 69, 023616 (2004).
[22] T. Ohmi and K. Machida, J. Phys. Soc. Jpn. 67, 1822 (1998).
[23] T. L. Ho, Phys. Rev. Lett. 81, 742 (1998).
[24] W. V. Pogosov and K. Machida, cond-mat/0604505.
[25] H. Pu, C. K. Law, S. Raghavan, J. H. Eberly, and N. P. Bigelow, Phys. Rev. A 60, 1463 (1999).

V. FIGURE CAPTIONS

Fig. 1. Dependences of $\Delta\chi$ (in degrees) on the number of particles in the system for different vortex phases at fixed value of interaction constant $g_n = 10$ (see in the text) and $t = 0.1$. In the (-1, 0, 1) phase, $g_s = 0.01g_n$, $M = 0.1$; in the (1, 1, 1) state, $g_s = 0.01g_n$, $g_n = 0.05g_n$; and in the (0, 1, 2) state, $g_s = -0.01g_n$, $M = 0.5$. Inset shows $\Delta\chi$ as a function of temperature for (0, 1, 2) phase (curve 1), (1, 1, 1) phase (curve 2), and (-1, 0, 1) phase (curve 3) at the same values of $M$, $g_n$, and $t$. Number of atoms is $N = 1000$, interaction constants are $g_s = -0.05g_n$ for the first curve and $g_s = 0.05g_n$ for two others.

Fig. 2. Dependences of $\langle |\delta l_{tr}| \rangle_T / |l_{tr}|$ on the total magnetization for (-1, 0, 1) phase at $N = 1000$; $g_n = 10$, $t = 0.1$, $g_s = 0.01g_n$. 
