**Viewpoint: Toward Fractional Quantum Hall physics with cold atoms**

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**Viewpoint on**
- **Reaching Fractional Quantum Hall States with Optical Flux Lattices**  
  Nigel R. Cooper and Jean Dalibard, Phys. Rev. Lett. 110, 185301 (2013).
- **Realizing Fractional Chern Insulators in Dipolar Spin Systems**  
  N. Y. Yao, A. V. Gorshkov, C. R. Laumann, A. M. Luchli, J. Ye, and M. D. Lukin, Phys. Rev. Lett. 110, 185302 (2013).

Researchers propose new ways to recreate fractional quantum Hall physics using ultracold atoms and molecules.

In the fractional quantum Hall (FQH) effect, observed in two-dimensional electron gases in a magnetic field, the Hall resistance is quantized to non-integer multiples of $h/e^2$. The fractional values are not an accidental small deviation from the previously observed integer quantum Hall (IQH) effect, but instead point to a type of order that had not been known previously, namely “topological order”. One consequence of topological order are fractionalized excitations, e.g., carrying fractional charge, logical order are fractionalized excitations, e.g., carrying fractional values are not an accidental small deviation from the previously observed integer number of Landau levels, while the FQH states appear at fractional fillings. In both effects, magnetic fields split the electron energy levels into Landau levels, each consisting of a large number of degenerate single-particle states. The IQH states arise when there are just enough electrons to fill an integer number of Landau levels, while the FQH states appear at fractional fillings. In both effects, topological aspects of the electronic wave functions are important, as the magnetic field also modifies the eigenstates. The Hall conductivity was introduced in the context of magnetic fields, it can be calculated for any filled band in a solid, yielding the Chern number $C$. It vanishes in most cases, but a band where it is non-zero is said to be “topologically non-trivial” and a spin-polarized band with $C \neq 0$ is referred to as a “Chern band”. (Similarities and differences between a lattice supporting Landau levels and one supporting Chern bands are sketched in Fig. 1.) Accordingly, it was proposed three years ago that Chern bands may provide an alternative route to FQH-like states and substantial interest has gone in the direction of these “fractional Chern insulators” (FCI’s). Yet, despite intense research, the goal of establishing FQH states in cold gases has remained elusive.

Unsurprisingly, the goal of realizing FQH physics and topological order in ultracold atoms serving as “quantum simulators” has attracted interest and effort early on. Proposals involved rotating trapped bosonic gases, creating effective magnetic fields for lattice bosons or exploiting the internal structure of atomic states to create artificial gauge fields. Yet, despite intense research, the goal of establishing FQH states in cold gases has remained elusive.

The two papers by Cooper and Dalibard and by Yao et al. represent a new generation of such proposals. Instead of relying on Landau levels, they build on the above-mentioned generalization of IQH physics to TI’s. Even though the Hall conductivity was introduced in the context of magnetic fields, it can be calculated for any filled band in a solid, yielding the Chern number $C$. It vanishes in most cases, but a band where it is non-zero is said to be “topologically non-trivial” and a spin-polarized band with $C \neq 0$ is referred to as a “Chern band”. (Similarities and differences between a lattice supporting Landau levels and one supporting Chern bands are sketched in Fig. 1.) Accordingly, it was proposed three years ago that Chern bands may provide an alternative route to FQH-like states and substantial interest has gone in the direction of these “fractional Chern insulators” (FCI’s). Yet, despite a few materials-based proposals, an experimental realization has not been found so far.

In Ref. 2, the authors propose to use polar molecules to...
realize a bosonic FCI. In this proposal, molecules are strongly bound to their place, but have a rotational degree of freedom. A change in the rotational eigenstate can be seen as a spin flip from the “spin up” ground state, which can in turn be interpreted as a boson. Molecules at neighboring sites can exchange their rotational eigenstates, which allows the boson to hop from site to site. The crucial ingredient is now that the wave function picks up a phase during the hopping process. The phases associated with each bond are distributed in the wave function, such a way that a boson moving around one of the square plaquettes shown in Fig. 1(d) acquires a phase of $e^{\pm i\phi}$, where the fluxes $\pm \phi$ alternate between plaquettes. This is typical of Chern systems, and the two bands here have indeed Chern numbers $C = \pm 1$, similar to Fig. 1(b).

The proposal in Ref. [11] also proposes a realization of an FCI system, but is based on earlier ideas by Cooper and collaborators to formulate topological nontrivial models in momentum space. The authors propose to create a periodic spatial modulation of the coupling between lasers and bosonic atoms (such as Rb atoms). Such a modulation creates phases for the hoppings, which in turn again establish the nontrivial band topology in a similar way as a strong magnetic field would.

Since FQH states are driven by interactions between particles, these should be large compared to the kinetic energy. On the other hand, they should be smaller than the gaps between bands, because a mixture of states with $C = 1$ and $C = -1$ might cancel to topologically trivial $C = 0$. Fulfilling both criteria is easiest for nearly flat Chern bands, analogous to the high degeneracy of Landau levels. The tunability of cold gases permits this and the optimized band structure given in Ref. [2] has a “flatness ratio” (band width divided by gap to the next band) of $f = 11.5$, which has been found flat enough for FCI states in other models. The authors consider interacting bosons in this band and find a variety of phases. The FCI competes with superfluid and solid phases, but occupies a sizeable region in parameter space. In Ref. [11] the ratio is even better ($f = 46$). By interpolating between this Chern band and a Landau level, the authors provide numerical evidence for several different FCI states at different densities.

Compared to earlier proposals to find FQH states in cold quantum gases, these two do not so much propose technological advances but rather extend the potential routes to FQH-like states. Building on recent research suggesting that it may not be necessary to copy all features of Landau levels, they propose to keep only some aspects, namely the Chern number of the band of interest and its reduced dispersion, and leave out others (the Chern numbers of other bands, a constant “magnetic field”). The advantage over materials-based approaches to FCI states is the flexibility of cold gases, which makes it appear more realistic to get into the needed parameter regimes. An experimental realization of an FCI, especially a highly tunable one, would allow to study topological order in far greater depth than the original FQH setting and as a first step would establish whether the proposed generalizations from Landau levels to Chern bands hold.

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