The Epoch of Reionization

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ABSTRACT

We have modelled the process of reionization of the intergalactic medium (IGM) by photoionization by galaxies, in order to learn what galaxy formation in the framework of the CDM model predicts for the epoch of reionization. We use a sophisticated semi-analytic model of galaxy formation to track the formation of these galaxies, their influence on the IGM, and the back-reaction of the state of the IGM on further galaxy formation. Our study represents a much more complete and physically consistent modelling of reionization than has been conducted in the past. In particular, compared to previous work by ourselves and others, our new calculations contain significant improvements in the modelling of the effects of reionization of the IGM on the collapse of baryons into dark matter halos (this is now computed self-consistently from the properties of model galaxies), and in the model for the cooling and condensation of gas within halos (our new model includes photoheating from a self-consistently computed ionizing background and also includes cooling due to molecular hydrogen). We find that reionization can be achieved by $z \sim 10–20$ in a $\Lambda$CDM cosmological model with $\sigma_8 \approx 0.9$. However, a cosmological model with a running spectral index is only able to achieve reionization before $z \approx 9$, and thus be consistent with an optical depth of 0.1, if very extreme assumptions are made about the physics of feedback at high redshifts. We also consider the specific galaxy formation model recently discussed by Baugh et al., which includes a top-heavy IMF in starbursts, and find that it is able to reionize the Universe by $z \approx 12$. The previous results assume that all of the ionizing photons produced by stars in galaxies are able to escape and ionize the IGM. If this is not the case, then the redshift of reionization could be substantially reduced. We find that extended periods of partial reionization and double reionizations can occur in models in which the first stars formed via cooling by $\text{H}_2$ molecules, are very massive, and in which the escape fraction of ionizing photons $\sim 10–30\%$. Such models do not fully reionize until $z \approx 6–7$, but predict an electron scattering optical depth as large as 0.15. Models with lower $\sigma_8 = 0.7 – 0.8$ as suggested by the recent Wilkinson Microwave Anisotropy Probe (WMAP) three year data have reduced redshifts of reionization, but can be consistent with the lower optical depth also suggested by the WMAP three year data.

1 INTRODUCTION

While gas in the intergalactic medium (IGM) is highly ionized in the present day Universe, it is known to have been mostly neutral in the past. Evidence for this comes from observations of the Gunn-Peterson effect in the spectra of high redshift quasars (Becker et al. 2001; Djorgovski et al. 2001; Fan et al. 2006) and from the value of the optical depth to scattering measured from the cosmic microwave background (CMB; Spergel et al. 2003). Thus, there was a period in the history of the Universe when it transitioned from being neutral to ionized—the epoch of reionization.

Measurements of the CMB polarization made during the first year of operation of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite\textsuperscript{1} have been used to estimate a value $\tau = 0.17 \pm 0.04$ for the electron scattering optical depth to reionization (Kogut et al. 2003), which, in the simple case that the IGM reionized instantaneously, implies a reionization redshift $z_{\text{reion}} = 17 \pm 5$ (Spergel et al. 2003). This reionization redshift is significantly higher than was previously thought on the basis of the observed Gunn-Peterson effect and on the basis of theoretical mod-

\textsuperscript{1} While this paper was being completed, results based upon the first three years of data from WMAP became available (Spergel et al. 2006). These show a much lower optical depth to reionization. We will comment upon these results in §5.
§2 outline the model used to follow the reionization of the Universe. In §3 we examine the reionization histories predicted by our models and assess the importance of the various physical ingredients of the models. In §4 we present results for the reionization epoch and the imprint of reionization on the cosmic microwave background. Finally, in §5 we assess the implications of our results. Several Appendices describe further details of the modelling, and present additional results.

2 MODELLING

2.1 Galaxy Formation Modelling

We employ the GALFORM semi-analytic model of galaxy formation throughout this work to predict the properties of the galaxy population as a function of redshift. Most importantly for the current work is the ability of this model to compute the star formation rates (and associated luminosities in ionizing photons) needed to predict the reionization history of the Universe. A full description of the GALFORM model is beyond the scope of this paper. We instead refer the reader to previous papers which document the model in more detail (Cole et al. 2000; Benson et al. 2002a). Here, we give a very brief description of key components of the model and provide somewhat more detail about two ingredients which are critical to the current work: the cooling of hot gas in haloes (see §2.2) and star formation (see §2.3). It should be noted that the major innovation with respect to Paper I is our improved model of gas cooling, which now includes cooling due to \( \text{H}_2 \) and which is affected by feedback from the reionization process. The implications of this new modelling are discussed in §2.2.

The basic structure of the GALFORM calculation can be summarized as follows:

- The hierarchical formation of dark matter haloes is followed using the extended Press-Schechter theory (Bond et al. 1991; Bower 1991). This allows the construction of merger trees which form the backbone of the galaxy formation calculation.
- A galaxy can form in each branch of the merger tree if the hot gas in the halo is able to cool and condense to the halo centre (see §2.2) where it forms a rotationally supported disk.
- This condensed gas is assumed to begin to form stars at a rate governed by simple, phenomenological rules (see §2.3). As a consequence of this star formation, supernovae inject energy into the galaxy causing some of the condensed gas to be re-ejected from the galaxy. This feedback loop is crucial to producing a population of galaxies which matches the observed low-redshift Universe (Benson et al. 2003a). Star formation also produces metals, enriching the gas in the galaxy. When this enriched gas is ejected from the galaxy by supernova explosions, the hot atmosphere of gas pervading the halo also becomes enriched in metals.
- When two branches of the merger tree join, the most massive galaxy from the branches becomes the central galaxy of the new branch. Any other galaxies become satellite galaxies, free to orbit within the gravitational potential of the new dark matter halo. Satellite galaxies are no longer able to gain gas from the surrounding hot atmosphere.

2 The physical plausibility of these scenarios has been questioned by Furlanetto & Loeb (2005). We present evidence, in §3.3 that such double or partial reionization may in fact occur under plausible physical conditions.
3 The running spectral index reduces the amplitude of fluctuations on small, galactic scales relative to that on large scales. As a result there are many fewer haloes present at high redshift with mass sufficient to form galaxies.
Satellite galaxies experience dynamical friction as they orbit, which eventually forces them to merge into the galaxy residing at the halo centre. These mergers are assumed to transform stellar disks into stellar spheroids if the masses of the two galaxies are comparable. Thus, in this model, mergers are the drivers of morphological evolution. Mergers of this type may also trigger bursts of star formation (see §2.3).

### 2.2 Cooling of Gas in Haloes

When a dark matter halo reaches virial equilibrium, it is assumed to have accreted into itself some mass of gas from the surrounding IGM. For high mass haloes, the mass of gas accreted is equal to $\Omega_b/(\Omega_0 - \Omega_b)$ times the dark matter mass of the halo (where $\Omega_0$ and $\Omega_b$ are the mean densities of total and baryonic mass in the Universe in units of the critical density). For lower mass haloes, the mass of gas accreted is reduced, due to the finite gas pressure in the IGM, in accordance with the filtering mass formalism of Gnedin (2000). The filtering mass, $M_F$, is discussed in detail in Appendix A3. The mass of gas accreted by a halo of mass $M_{\text{tot}}$ is then given by

$$M_{\text{gas}} = \frac{(\Omega_b/\Omega_0)M_{\text{tot}}}{1 + (2^{1/3} - 1)M_F(z)/M_{\text{tot}}}$$.

(1)

The gas is assumed to be heated by shocks as it accretes into the halo, reaching a uniform temperature equal to the halo virial temperature. The gas is then assumed to be distributed through the halo with a density profile

$$\rho_{\text{gas}}(r) \propto \frac{1}{r^2 + c^2}$$.

(2)

where $c$ is a scale radius which we set equal to $r_s/3$, where $r_s$ is the scale radius of the dark matter density profile (assumed to have the NFW form).

To compute the rate of cooling of the gas, we construct a table of net radiative cooling rates as a function of temperature, density, metallicity and redshift. (The redshift dependence arises through the inclusion of photoheating from a time-varying ionizing background which is computed self-consistently from the emission from the model galaxies.) This net cooling/heating rate includes contributions from atomic processes (e.g. recombinations, collisional excitations etc.), Compton cooling (Peebles 1968), cooling due to molecular hydrogen (see §A1) and photoheating by the ionizing background (see §2.4). Given these cooling rates and the assumed gas density profile and temperature for each halo, we then calculate, at each time step of our calculation, the cooling radius, defined as the radius in the halo at which the local cooling time equals the time since the halo formed. (A halo is defined to form anew whenever its mass is doubled by mergers.) Any gas within this cooling radius is assumed to accrete onto the galaxy forming at the centre of the halo, provided that the time for the gas to free-fall to the halo centre is less than the age of the halo. Occasionally, for example when the ionizing background is rapidly rising, the cooling radius at a given timestep may be smaller than that at the previous timestep. In such cases no gas is allowed to cool until the cooling radius once again begins to increase.

The inclusion of cooling due to H$_2$ is crucial to the present work as, at high redshifts, this is the only cooling channel available to gas in the majority of halos. Without H$_2$ cooling included, we risk seriously underestimating the epoch of reionization. The IGM pressure and the ionizing background act as sources of negative feedback on star formation (i.e. star formation leads to a rise in the IGM pressure and ionizing background, which in turn suppress later star formation). By including these effects self-consistently in our model we are therefore able to check whether this negative feedback is sufficient to significantly quench star formation. It should be noted that if we neglect H$_2$ cooling and set the filtering mass and ionizing background to zero our revised cooling model produces results identical to those of Paper I (i.e. it simply follows the cooling prescription of Cole et al. (2000) with the addition of Compton cooling). We will examine the effects of including H$_2$ cooling in our calculations in §3.1.

The effects of including the filtering mass and ionizing background in our cooling model have been explored in detail by Benson et al. (2002a). Benson et al. (2002a) find that it is the filtering mass which has the greatest effect on galaxy formation at low redshifts, resulting in a suppression of star formation after the epochs of H$\text{I}$ and He$\text{II}$ reionization. For the Cole et al. (2000) model, Benson et al. (2002a) found this suppression to be quite small. However, for a model with weak feedback from supernovae the suppression effect of the filtering mass was significant (reducing the global star formation rate by a factor of two). This occurs because, in models with weak SNe feedback, star formation can otherwise progress efficiently in low mass haloes, and therefore such haloes make a significant contribution to the global star formation rate. Once the filtering mass rises star formation is shut off in these low mass halos, thereby greatly suppressing the global star formation rate. The filtering mass also acts to reduce the number of faint galaxies which can form, as discussed in detail by Benson et al. (2002b). Note that the filtering mass begins to rise as the process of reionization begins, but does not reach its maximum value until significantly after the epoch of reionization. As such, we may expect it to have only minimal impact on the epoch of reionization, a point which we will explore in §3.2.

### 2.3 Star Formation and Supernova Feedback

Once gas has cooled inside a dark matter halo it is assumed to collapse, conserving its angular momentum, until it reaches a radius where it becomes rotationally supported against further collapse. At this point, the gas is assumed to form an exponential disk. The gas in this disk then begins to form stars at a rate $\psi$ given by

$$\psi = M_{\text{cold}}/\tau_\ast$$.

(3)

where $M_{\text{cold}}$ is the total mass of gas available in the galaxy and $\tau_\ast$ is a characteristic timescale for star formation. Following Cole et al. (2000) we adopt the parameterization

$$\tau_\ast = \epsilon^{-1} \tau_{\text{disk}}(V_{\text{disk}}/200 \text{ km/s})^{\alpha_\ast}$$.

(4)

where $\epsilon$, and $\alpha_\ast$ are dimensionless parameters, $V_{\text{disk}}$ is the circular velocity of the disk at its half-mass radius and $\tau_{\text{disk}}$ is the dynamical time of the disk at that radius.
We assume that supernova explosions in the galaxy cause gas to be ejected from the galaxy at a rate \( M_{	ext{eject}} = \beta \dot{\psi} \), where
\[
\beta = (V_{\text{disc}}/V_{\text{hot}})^{-\alpha_{\text{hot}}},
\]
where \( \alpha_{\text{hot}} \) is a dimensionless parameter and \( V_{\text{hot}} \) is a parameter with dimensions of speed.

Star formation can also occur in a burst mode as the result of a violent merger between two galaxies. When such a merger occurs, all of the gas in the two merging galaxies undergoes star formation on a very short timescale. We use the same rules as for quiescent star formation in disks, but using the dynamical properties of the newly formed spheroid instead.

We compute the chemical enrichment of each galaxy using the instantaneous recycling approximation, with yield of heavy elements \( \dot{N} \) and recycled fraction \( R \) (as defined by Cole et al. (2000)—we use the same values for these parameters as did Cole et al. (2000) unless otherwise noted), including the exchange of metals between the galaxy and the halo gas. Once the star formation history and associated chemical enrichment history of a galaxy have been determined, we compute the SED of the galaxy at each timestep using a stellar population synthesis model based on the Padova stellar evolution tracks (Granato et al. 2000). This allows us to compute the ionizing photon luminosity for each galaxy. (as did Cole et al. (2000) unless otherwise noted), including the exchange of metals between the galaxy and the halo gas. Once the star formation history and associated chemical enrichment history of a galaxy have been determined, we compute the SED of the galaxy at each timestep using a stellar population synthesis model based on the Padova stellar evolution tracks (Granato et al. 2000). This allows us to compute the ionizing photon luminosity for each galaxy.

2.4 Evolution of the IGM and Ionizing Background

We model the process of reionization and its feedback on the IGM and galaxy formation using the methods of Benson et al. (2001; hereafter, Paper I) and Benson et al. (2002a). Briefly, we compute the volume averaged emissivity in ionizing photons at each redshift by summing the ionizing luminosity of all galaxies in our calculation. This emissivity is used to evolve a distribution of gas elements drawn from an appropriate PDF (Benson et al. 2001). We track the ionization (H and He) and thermal state of these gas elements as the Universe evolves. From the resulting thermal history of the IGM we are able to compute the filtering mass describing the effects of the IGM pressure on the collapse of baryons into haloes. This allows us to estimate the mass of baryons accreted into each dark matter halo forming at any redshift. We also have, at each redshift, the spectrum of the ionizing background, which allows us to compute the rate of photoionization heating experienced by gas in dark matter haloes. These two processes (reduced accretion due to the IGM pressure and photoheating of halo gas), which act to suppress galaxy formation, are incorporated into a second iteration of our galaxy formation calculation. This iterative process is repeated until convergence is reached (i.e. the input ionizing background and filtering mass are consistent with those resulting from the galaxy formation calculation) as in Benson et al. (2001). Schematically, the iterative procedure works as follows:

(i) Initially assume that there is no ionizing background (and associated photoheating) and that the filtering mass is always zero.
(ii) Run the GALFORM galaxy formation calculation.
(iii) Compute the evolution of the ionizing background based on the GALFORM calculation.
(iv) Compute associated photoheating rates for gas of varying densities, temperatures and metallicities.
(v) Compute the evolution of the filtering mass based on the GALFORM calculation.
(vi) Using these photoheating rates and filtering mass, go back to step (ii) and repeat.

Iteration stops once the star formation history produced in GALFORM calculations on successive iterations is sufficiently well converged.

Benson et al. (2001) and Benson et al. (2002a) considered cooling due to both atomic processes and Compton cooling due to the scattering of cosmic microwave background photons from hot electrons. Here we employ an improved cooling model which is described in detail in Appendix A. To explore the high redshifts and low mass scales of interest here we must also consider the effects of cooling due to molecular hydrogen (see §A1). We must also reconsider Compton cooling in the non-equilibrium regime (i.e. before free electrons in the gas have had sufficient time to recombine to reach their abundance in ionization equilibrium at the density of the halo gas) which occurs in some of the lower mass haloes that we consider (see §A2).

2.5 Escape Fractions

A very important parameter in our models is the fraction, \( f_{\text{esc}} \), of ionizing photons produced in a galaxy which escape into the IGM. For most of our models (and unless stated otherwise), we assume \( f_{\text{esc}} = 1 \). The only exceptions are the models H2WeakFB and H2WeakFB-VMS (to be defined in §2.7.2), for which we also present some results for lower escape fractions (for the purpose of producing models with “double reionization”—see §3.3). The assumption that \( f_{\text{esc}} = 1 \) is obviously an extreme one (although such an assumption has been used in other works studying reionization, e.g. Haiman & Holder 2003), since it ignores the absorption of ionizing photons by gas and dust in the galaxy; but it should result in the earliest possible reionization if other parameters are held fixed. We note, however, that observational evidence (and simple physical models for \( f_{\text{esc}} \)) imply a much lower escape fraction. Therefore, in Appendix B we present a simple, physically motivated calculation of \( f_{\text{esc}} \) which is in reasonable agreement with observational data, and show the resulting reionization histories.

2.6 Calculation of Filling Factor

To compute the filling factor of \( \text{H}\beta \) we follow the growth of ionized regions around each halo in the galaxy formation model using the methodology of Paper I. Specifically, we compute the total emission rate of ionizing photons from stars in each dark matter halo, \( S(t) \), and compute the number of hydrogen atoms which will be ionized by this radiation by solving
\[
\frac{dN_{\text{H}}}{dt} = S(t) - \alpha^{(2)}_{\text{H}} a^{-3} f_{\text{lump}} n_{\text{H}} N_{\text{H}},
\]
where \( N_{\text{H}} \) is the total number of hydrogen atoms ionized, \( \alpha^{(2)}_{\text{H}} \) is the case B recombination coefficient for hydrogen (for
a temperature of $10^4$K), $\alpha$ is the cosmic expansion factor, $f_{\text{clump}}$ is the clumping factor for ionized gas, and $n_H$ is the mean number density of hydrogen atoms in the Universe. We compute the clumping factor following the method described in Paper I for $f_{\text{clump}}^\text{(halo)}$, but with the cosmological parameters appropriate to the current work. In this model, the main contribution to the clumping factor comes from gas residing in haloes with virial temperatures around $10^4$K, which have deep enough potential wells to retain gas, but which are cool enough to be not completely collisionally ionized. The specific values for the clumping factors used in this work will be discussed in §2.7.1.

Note that we do not assume any particular geometry for the ionized region around each source. Ionized regions from two or more sources may overlap (particularly just before the epoch of reionization or if the sources are strongly clustered). This does not affect our results, as we are simply counting the total number of hydrogen atoms which have been ionized.

The total number of hydrogen atoms, $N_{\text{H, tot}}$, which will have been ionized in some representative volume of the Universe, $V$, is then found by simply summing over the contributions from galaxies in haloes of all masses

$$N_{\text{H, tot}} = \sum_i N_{\text{H, i}} n_i V,$$

where $N_{\text{H, i}}$ is the number of hydrogen atoms which have been ionized by galaxies in halo $i$, of which there are $n_i$ such haloes per unit volume. The sum is taken over all haloes. The mean ionized fraction in volume $V$ is found by simply dividing by the total number of hydrogen atoms in the volume, $n_\text{H} V$ where $n_\text{H}$ is the mean density of hydrogen atoms in the Universe. If we assume a uniform IGM, we can identify this mean ionized fraction with the filling factor of ionized regions,

$$F_{\text{fill}} = \frac{1}{n_\text{H} V} \sum_i N_{\text{H, i}} n_i.$$

We consider the Universe to have been fully reionized once $F_{\text{fill}} = 1$. Note that with these definitions $F_{\text{fill}}$ can exceed unity. In such cases, the Universe is fully ionized and ionizing photons will begin to build up an ionizing background rather than be absorbed by neutral hydrogen atoms.

The IGM evolution model of Benson et al. (2002a) gives an alternative means of following the reionization of the Universe. In that model, the mean ionization state of the IGM is determined by computing the ionization and recombination rates in the presence of an evolving ionizing background, allowing for a distribution of IGM densities. While these two methods for following reionization are quite different, we find that they predict very similar redshifts of reionization, with the method of Benson et al. (2002a) typically predicting $z_{\text{reion}}$ to occur roughly $\Delta z \approx 1$ earlier. Throughout this work we will show results based on the approach described in this section.

### 2.7 Cosmological and Galaxy Formation Models

We will explore three different cosmological models and several different models of galaxy formation in this work. The details of these models are outlined below.

![Figure 1. The fractional root mean square fluctuation of the density averaged in spheres, $\sigma(M)$, as a function of mass $M$, for each of our cosmological models: WMAP RSI (red line), WMAP PL (blue line) and Baugh et al. (2005; green line).](image-url)

**2.7.1 Cosmological Parameters**

For the cosmological parameters we adopt the values determined from the first-year WMAP data set (Spergel et al. 2003). In particular, we consider cosmological models with the best fit parameters for a power-law $\Lambda$CDM model with constraints from WMAP alone (Table 7 of Spergel et al. 2003) and a running spectral index $\Lambda$CDM model with constraints from WMAP, the 2dF Galaxy Redshift Survey and Lyman-$\alpha$ forest observations (Table 8 of Spergel et al. 2003). We refer to these cosmological models as WMAP PL and WMAP RSI respectively, and their parameters are given in Table 1. We also compute reionization histories in the cosmological model corresponding to the specific galaxy formation model of Baugh et al. (2005), discussed further below.

We include these three cosmologies because they have complementary strengths and weaknesses. The cosmologies based on WMAP results give good fits to the available CMB data (by definition). However, we do not have a set of galaxy formation model parameters in these cosmologies which result in galaxy populations with properties which match local observations. The Baugh et al. (2005) model, on the other hand, was not designed to match the CMB data (although its cosmological parameters are not too far from those measured by WMAP), but does reproduce the properties of galaxies in the local and high-redshift Universe.

We show in Figure 1, for all three cosmological models, the fractional fluctuation of the density measured in spheres, $\sigma(M)$, for the linear density field extrapolated to $z = 0$. The quantity $\sigma(M)$ determines the abundances of dark matter haloes at different redshifts. It is clearly seen that the WMAP RSI model has much smaller fluctuations on small scales.

The IGM clumping factor which we use depends only on the cosmological model, and is shown as a function of...
Table 1. Cosmological parameter sets used in this work. Values are taken from (Spergel et al. 2003; WMAP) and (MacTavish et al. 2006; Boomerang). \( n_s \) is the scalar spectral index at \( k = 0.05 \text{Mpc}^{-1} \), while \( \text{d}n_s/\text{d} \ln k \) is the running of the spectral index as defined by eqn. (5) of Spergel et al. (2003).

| Name               | \( \Omega_0 \) | \( \Lambda_0 \) | \( \Omega_b \) | \( H_0/\text{km s}^{-1} \text{Mpc}^{-1} \) | \( \sigma_8 \) | \( n_s \) (at \( k = 0.05 \text{Mpc}^{-1} \)) | \( \text{d}n_s/\text{d} \ln k \) |
|--------------------|----------------|----------------|---------------|---------------------------------|-------------|---------------------------------|----------------|
| WMAP PL            | 0.270          | 0.730          | 0.0463        | 72.0                            | 0.90        | 0.99                             | 0.000          |
| WMAP RSI           | 0.268          | 0.732          | 0.0444        | 71.0                            | 0.83        | 0.93                             | -0.031         |
| Baugh et al. ’05  | 0.300          | 0.700          | 0.0490        | 70.0                            | 0.93        | 1.00                             | 0.000          |
| Boomerang ’03     | 0.300          | 0.700          | 0.0469        | 69.6                            | 0.85        | 0.95                             | 0.000          |

Figure 2. The clumping factor, \( f_{\text{clump}} \), as a function of redshift for the three cosmological models considered in this work. Also shown, for reference, are the clumping factors corresponding to the cosmological parameters of Paper I (see Appendix C1) and the “Boomerang ’03” cosmology discussed in §5.

For the WMAP PL cosmology it is very similar to that found in Paper I (\( f_{\text{clump}} \approx 18 \) at \( z = 10 \), dropping to approximately 2.5 at \( z = 20 \) for example), while for the WMAP RSI cosmology it is much lower (\( f_{\text{clump}} \approx 1.5 \) at \( z = 10 \), dropping to close to 1 at \( z = 20 \)) due to the reduction of small-scale fluctuations in the latter cosmology. Inclusion of the clumping factor makes almost no difference to our results for the WMAP RSI cosmology, as the clumping factor is still small by the time reionization occurs. However, in the WMAP PL cosmology, clumping can delay reionization by \( \Delta z = 2 \)–14 (the earlier reionization occurs, the less effect clumping has). In the Baugh et al. ’05 cosmology, the clumping factor is comparable to that of the WMAP PL cosmology.

2.7.2 Galaxy Formation Parameters

To model the formation of galaxies in the Universe we employ the semi-analytic model of galaxy formation GALFORM. A full description of this model can be found in Cole et al. (2000) and Benson et al. (2002a). For our “Standard model” in the WMAP RSI and WMAP PL cosmologies, we adopt parameters which are very similar to those of Cole et al. (2000). (We ignore the cosmological parameters specified by Cole et al. (2000) and instead use those described in §2.7.1.) However, we use the improved merging calculation of Benson et al. (2002a), which results in somewhat fewer mergers on average than the simpler model used by Cole et al. (2000). Due to the change in cosmological parameters and in the merger rates, we adjust the values of a few of the parameters in the galaxy formation model, increasing the star formation efficiency \( \epsilon_* \) from 0.005 to 0.0067 and reducing the mass-to-light ratio parameter \( \Upsilon \) from 1.38 to 1.00.

We regard the resulting model as being a plausible model for galaxy formation in the high-redshift Universe, since it has been shown to reproduce well the properties of galaxies in the low-redshift universe, and we simply assume that the same physical processes apply to galaxy formation at high redshift as at low redshift. Our model is therefore motivated by what we currently know about the local Universe—the extrapolation to high redshifts is a simple assumption as our current understanding of the physics that governs the high redshift Universe does not allow us to devise more accurate physical prescriptions for early times. Note, however, that the Cole et al. (2000) model assumed \( \Omega_b = 0.02 \). The higher baryon densities in the cosmologies considered here will result in this model producing too many galaxies of all luminosities at \( z = 0 \). We do not attempt to correct for this deficiency for two reasons: 1) we are interested here in determining how early reionization can occur—overproduction of galaxies will push reionization to higher redshifts and so give us a conservative answer to the question of whether a particular model can reionize sufficiently early to match the WMAP results; and 2) while the Cole et al. (2000) model works well at \( z = 0 \) it does not match observational data for high-redshift galaxies and, in any case, we are here extrapolating our model to \( z > 20 \) where observational constraints on galaxy formation are practically non-existent. It is for these reasons that we consider our model to be plausible but not definitive.

While there are numerous variations of the model parameters which it would be interesting to explore, we limit ourselves here to examining a handful which might be expected to have a particularly large impact on the epoch of reionization. These various models are summarized in Table 2.

- **Standard**: For the WMAP RSI and WMAP PL cosmologies, this model is very similar to that described in Cole et al. (2000)—the differences from Cole et al. (2000) are the parameter changes described in §2.7.2 and the inclusion of Compton cooling, which is necessary for studying the high redshifts of interest here. It assumes that all stars form with
The mass of a halo with virial velocity corresponding to the feedback parameter, $V_{\text{hot}} = 200\text{km/s}$, is shown as a function of redshift for the WMAP PL cosmology (results for the WMAP RSI and Baugh et al. ’05 cosmologies are practically identical).

- **MetFree**: In this model, we compute stellar luminosities and spectra assuming that all stars are be metal free (while retaining the chemical enrichment of the interstellar and intergalactic gas, as this affects gas cooling rates), thereby enhancing the emission of ionizing photons from galaxies. In this case, we have $N_{\text{LyC}}/M_* = 2.0 \times 10^{53}$.

- **TopHeavy**: We assume an extreme IMF which has the Salpeter slope $x = 1.35$, but only covers the mass range $10 < m < 100M_\odot$, i.e. it is truncated for masses below $10M_\odot$. As massive stars produce the majority of ionizing photons, this model is expected to produce earlier reionization. For this IMF, we have $N_{\text{LyC}}/M_* = 7.1 \times 10^{53}$.

- **WeakFB**: This model has very weak feedback from supernova explosions, such that even the lowest mass galaxies which form in the model will not experience significant mass loss due to feedback. In GALFORM, the strength of feedback is controlled by two parameters $V_{\text{hot}}$ and $\alpha_{\text{hot}}$, as defined in eqn.(5). We fix $\alpha_{\text{hot}} = 2$ throughout this work, and use $V_{\text{hot}} = 200(300)\text{km/s}$ for the WMAP (Baugh et al. ’05) cosmologies except for models with weak feedback, for which we assume $V_{\text{hot}} = 1\text{km/s}$ (see Table 2). The lower value of $V_{\text{hot}} = 1\text{km/s}$ corresponds to an efficiency of supernova energy injection and of mass ejection which is lower by a factor $4 \times 10^3$ than for the standard case $V_{\text{hot}} = 200\text{km/s}$, so it is fairly extreme. This weak feedback will greatly enhance the efficiency of star formation in low-mass haloes, and hence will also increase the production of ionizing photons at high redshift.

- **H$_2$**: This model is identical to the Standard model except that it incorporates cooling due to molecular hydrogen—this should allow galaxies to form earlier, thereby shifting the epoch of reionization to higher redshifts.

- **H$_2$WeakFB**: This model combines a weak feedback with molecular hydrogen cooling.

- **H$_2$WeakFB-VMS**: This model investigates the possibility that stars forming via H$_2$ cooling (“Population III”) may be very massive stars (VMS), with $m \sim 100 - 1000M_\odot$ (Abel, Bryan & Norman 2000; Abel, Bryan & Norman 2002; Bromm, Coppi & Larson 2002). If all of the stars are very massive, then the total ionizing luminosity will be greatly enhanced relative to stars formed with a normal IMF. Therefore, in this model, we increase the UV luminosity of any stars forming in a halo with virial temperature below $10^4K$ by a factor of 20 relative to the Kennicutt IMF at all metallicities. We therefore have $N_{\text{LyC}}/M_* = 2.0 \times 10^{53}$.

Baugh et al. (2005) recently presented a GALFORM model employing a significantly different set of galaxy formation parameters from those used by Cole et al. (2000). We will use the Baugh et al. (2005) parameters to describe our “Standard model” in the Baugh et al. (2005) cosmology. The cosmological parameters of this model are listed in Table 1. The Baugh et al. (2005) model was constrained to produce a good match to observations of sub-mm galaxies and Lyman-break galaxies at high redshifts, as well as the properties of galaxies at low redshifts. In order to match the numbers of sub-mm galaxies, Baugh et al. assumed that stars which form in merger-induced bursts have a top-heavy IMF (with slope $x = 0$ covering the mass range $10^4-10^5\odot$) with a Kennicutt (1983) initial mass function (IMF), with slope $x = 0.4$ for $0.1 < m < 1M_\odot$ and $x = 1.5$ for $1 < m < 120M_\odot$. (A Salpeter IMF has $x = 1.35$.) For this IMF, the total number of ionizing photons (with $h\nu > 13.6$ eV) produced per $M_\odot$ of stars formed is $N_{\text{LyC}}/M_* = 1.0 \times 10^{53}$, for a metallicity $Z = 0.1Z_\odot$. Note that, for this model (and all others unless stated otherwise) we adopt a value of $V_{\text{hot}} = 200\text{km/s}$ for the feedback parameter. This value, chosen to reproduce the low abundance of faint galaxies in the $z = 0$ Universe, results in quite strong feedback. For the Baugh et al. (2005) cosmology, the Standard model corresponds to the galaxy formation parameters given by Baugh et al. (2005). Note that this includes a higher value for $V_{\text{hot}} = 300\text{km/s}$. Figure 3 shows the halo mass with virial velocity equal to $V_{\text{hot}} = 200\text{km/s}$ as a function of redshift. At high redshifts, the vast majority of haloes able to form will be strongly affected by feedback for $V_{\text{hot}} \geq 200\text{km/s}$.

4 Our choice of a UV enhancement factor of 20 is motivated by the results of Bromm, Kudritzki & Loeb (2001), who find an enhancement factor of 10-20. (Their enhancement is relative to a Salpeter IMF. We use the Kennicutt (1983) IMF here, but the total ionizing photon production per unit mass of star formation for this IMF differs from that of the Salpeter IMF by approximately 10% for sub-Solar metallicities.) We have taken the value at the upper end of their range to maximize the effects. We also note that theoretical considerations suggest that very massive stars should form when the metallicity of the ISM is below a critical value, and not merely when gas cooling occurs through the H$_2$ channel. If we model VMS formation in this way (i.e. we enhance the UV emissivity only for stars with a metallicity below, for example, $10^{-4}Z_\odot$ (Schneider et al. 2002; Bromm & Loeb 2004) we find little enhancement in the net UV emission. This occurs because the duration of the sub-critical metallicity phase of star formation is very short. However, our current model does not account for the different heavy element yield of these Population III stars—a low yield could dramatically extend the duration of the sub-critical metallicity phase of star formation. We defer a detailed examination of this possibility to a future work.
0.1 < m < 120M⊙), while stars which form quiescently in disks have a Kennicutt (1983) IMF (as in our Standard model). In the Baugh et al. model, the fraction of stars formed in the burst mode increases with redshift, with the burst mode dominating the total star formation density at z ≳ 3 (see Fig. 1 in Baugh et al.). In subsequent papers, Le Delliou et al. (2005a,b) have investigated the properties of Lyα-emitting galaxies at high redshift, and Nagashima et al. (2005a,b) have investigated the chemical enrichment of elliptical galaxies and intracluster gas in the same model. Given the success of the Baugh et al. model in reproducing observations of various types of star-forming galaxies at high redshift, as well as observations of chemical abundances at low redshift, it is of interest to see what it predicts for the reionization of the IGM. We have therefore used the galaxy formation and cosmological parameters of the Baugh et al. (2005) model to compute the reionization history of the Universe5.

For stars formed in bursts in the Baugh et al. model, the number of ionizing photons produced per M⊙ of stars formed N_{ion}/M∗ = 1.7 × 10^{54} for a metallicity Z = 0.1Z⊙, i.e. nearly 20 times larger than for a Kennicutt IMF. The only model variants we consider for the Baugh et al. cosmology at Standard, H2 and H2WeakFB6.

### 3 RESULTS

Below, we present results from our calculations of reionization. We begin with a summary of the key results, before concentrating on the importance of H2 cooling and photoionization feedback in setting the epoch of reionization. A comparison of the predicted epochs of reionization to the observational determinations and an exploration of the signal imprinted in the cosmic microwave background (CMB) by the patchy reionization process are given in §4.

Some further results, including comparisons to Paper I and tests of the convergence of our models are given in Appendix C. Appendix B briefly examines the effects on the reionization history of using physically motivated calculations of the escape fraction of ionizing photons.

Figure 4 shows key results from all of our calculations. Each row corresponds to a different model (as indicated along the right-hand edge of the figure) and within each panel different colours correspond to different cosmologies: WMAP RSI (red), WMAP PL (blue) and Baugh et al. ‘05 (green). The three columns respectively show the average star formation rate per unit volume, the ionizing luminosity density and the H1 filling factor (as indicated by labels at the top of the figure) all as functions of redshift. It is clear from this figure that the suppression of small mass haloes in the WMAP RSI cosmology leads to a severe reduction in the star formation rate, ionizing luminosity and H1 filling factor compared to our other cosmological models. This effect is larger than that of any of the other variations we consider in this work. As such, it may be possible to use a measured reionization epoch to place strong constraints on the cosmological model. It is also clear that reionization redshifts high enough to explain the WMAP measurement of the optical depth can be attained in the WMAP PL and Baugh et al. ‘05 cosmologies, albeit only in rather extreme models. These points will be discussed further in the remainder of this section and in §4.

#### 3.1 Does H2 Cooling Help?

We can examine whether the inclusion of H2 cooling channels in our calculations actually aids in pushing the epoch of reionization to higher redshifts. Figure 5 shows the H1 filling factor as a function of redshift for models with and without cooling due to H2. It can clearly be seen that the addition of H2 cooling channels has almost no effect on the epoch of reionization. At earlier redshifts, H2 cooling does result in an enhancement in the filling factor (e.g. by a factor of 10^4 at z = 35 in the WMAP PL cosmology). However, by the time F_{HI} is approaching unity, the ionizing emissivity is dominated by emission from haloes which are able to cool through atomic processes, and so the addition of H2 cooling channels does not help to increase z_{reion} significantly.

To understand the effects of cooling through molecular hydrogen, we can explore the cooled baryonic fraction in haloes as a function of their mass. The upper panels of Fig. 6 show the condensed baryonic fractions of haloes at redshifts 5 (left-hand panel) and 10 (right-hand panel) for models without the effects of photoionization and photo-

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5 Note that Baugh et al. (2005) included a simple model of photoionization feedback in their calculation, in which they suppressed the cooling of gas in dark matter haloes with virial velocities below 60km/s for z < 6. As we are here modelling reionization and photoionization feedback in much greater detail, we remove this simple suppression from our implementation of the Baugh et al. (2005) model.

6 Model H2WeakFB-VMS is not considered, as the Baugh et al. (2005) model already includes very massive stars which are assumed to form in bursts.
Table 2. Names and details of the galaxy formation models used in this work. Column 1 describes the model while column 2 gives the name by which we refer to the model throughout this work. Column 3 lists the changes in model parameters made relative to the standard model.

| Model | Name | Variation from standard model |
|-------|------|-----------------------------|
| Standard model | Standard | — |
| Metal free stellar spectra | MetFree | $Z_* = 0$ |
| Top-heavy IMF | TopHeavy | IMF Salpeter above $10M_\odot$, zero below; $R = 0.91$; $p = 0.14$ |
| Weak feedback | WeakFB | $V_{\text{hot}} = 1$ km/s |
| Compton + $H_2$ cooling and dynamical heating | $H_2$ | $H_2$ cooling on |
| Compton + $H_2$ cooling + weak feedback | $H_2$WeakFB | $H_2$ cooling on; $V_{\text{hot}} = 1$ km/s |
| Compton + $H_2$ cooling + weak feedback + VMS for $T_{\text{vir}} < 10^4$K | $H_2$WeakFB-VMS | $H_2$ cooling on; $V_{\text{hot}} = 1$ km/s; UV flux $\times 20$ for stars formed via $H_2$ cooling |

Figure 4. (cont.) The evolution of key physical parameters in our models. The three columns show mean star formation rate density, ionizing luminosity and H II filling factor as indicated at the top of each panel. Each row corresponds to a different galaxy formation model, as indicated on the right-hand side of the figure. The three lines in each panel correspond to three different cosmological models: WMAP RSI (red), WMAP PI (blue) and Baugh et al. (2005; green—not shown for models MetFree, TopHeavy, WeakFB and $H_2$WeakFB-VMS). In the H II filling factor columns, the horizontal dashed line indicates the point of complete reionization, while dashed coloured lines for the $H_2$WeakFB-VMS model show the contribution of Pop I and II stars to the total filling factor.

dissociation included. Blue circles indicate the cooled fraction, $f_{\text{cool}}$ (i.e. the mass of material which has been able to cool and condense into the galactic phase), for a model with no molecular hydrogen cooling, while red circles show results for a model with molecular hydrogen cooling. We indicate, with a vertical dashed magenta line, the halo mass for which the virial temperature corresponds to the supernova feedback parameter $V_{\text{hot}}$ in our semi-analytic model. Vertical, dashed cyan lines indicate the filtering masses for the two models. In the lower panels of Fig. 6 are curves which show the cooling measure $t_{\text{age}}/(1 + \beta)t_{\text{cool}}$, the ratio of the age of the Universe at the specified redshift $t_{\text{age}}$ to the cooling time $t_{\text{cool}}$ (calculated at 200 times the mean density of the Universe, which is approximately the mean density of haloes), and divided by $1 + \beta$, where $\beta$ (given by eqn. 5) is the ratio of the mass ejection rate due to supernova feedback to the star formation rate in model galaxies. Solid blue lines are for the model without molecular hydrogen cooling and without Compton cooling, while dashed red lines include molecular hydrogen cooling (and Compton cooling). We show curves for primordial gas and for gas of half Solar metallicity (the upper curves at large halo masses).

Our cooling measure, the quantity $t_{\text{age}}/(1 + \beta)t_{\text{cool}}$, is interesting since it roughly determines whether a halo has had time to turn most of its supply of gas into stars. Roughly speaking, a galaxy will accrete (from its surrounding hot atmosphere) all available gas in a time $t_{\text{cool}}$. Assuming haloes to have survived for a period roughly equal to $t_{\text{age}}$, the ratio of these two timescales then determines whether or not a halo has been able to accrete all available gas. However, much of this gas will be ejected once again due to feedback. This gas may be subsequently re-accreted. Consequently, a galaxy must accrete its hot halo $\sim (1 + \beta)$ times before it can turn the majority of the available gas into stars. 7

7 Assuming the cooling/reheating loop to occur $n$ times (and assuming cooling and reheating to happen instantaneously), the fraction of available gas eventually bound into stars will be $1 - (\beta/(1 + \beta))^n$. For $n = 1 + \beta$ this converges, for large $\beta$ to $1 - 1/e \approx 0.63$.

3.2 Effects of including photoionization and photodissociation feedback

We have run all models both with and without the effects of photoionization feedback (i.e. the suppression of accretion due to the IGM pressure and photoheating of gas in haloes by an ionizing background) and photodissociation feedback (i.e. the destruction of $H_2$ molecules by the dissociating background). In Fig. 7 we show the resulting star for-
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Figure 6. Measures of the ability of baryonic material to cool into the galactic phase are shown for $z = 5$ (left-hand panels) and $z = 10$ (right-hand panels) for models without the effects of photo-ionization and photo-dissociation feedback. Vertical dashed magenta lines indicate the halo mass for which the virial temperature corresponds to the feedback parameter $V_{\text{hot}}$. Vertical, dashed cyan lines indicate the filtering masses for the two models. Upper panels: The average fraction of baryonic material in haloes which has been able to cool and condense into galaxies, as a function of halo mass in the Standard model in the WMAP RSI cosmology. Blue circles indicate the cooled fraction for a model with no molecular hydrogen cooling, while red circles show results for a model with molecular hydrogen cooling.

Lower panels: Cooling measures for metallicities $Z = 0$ and $Z = 0.5Z_\odot$ (the upper curves at large halo masses): solid blue lines are for the model without molecular hydrogen cooling, while dashed red lines include molecular hydrogen cooling (and Compton cooling). The cooling measure is the ratio $t_{\text{age}}/(1 + \beta)t_{\text{cool}}$. Here $\beta$ is the ratio of rate of mass ejection due to supernova feedback to the star formation rate in model galaxies. The horizontal green lines indicate the point where the cooling measure equals 1 (i.e. the point above which cooling is efficient in the sense described in the text.

Figure 8 shows the same information as Fig. 6 but for models which now include the effects of photo-ionization and photo-dissociation feedback. By comparing this figure with Fig. 6, it can be seen from this figure that the main causes of the suppression of galaxy formation are photoheating of hot gas in haloes and photodissociation of H$_2$. The filtering mass is approximately the same in models with and without photo-

mation rates and HI filling factors as a function of redshift. Photoionization and photodissociation feedback reduce the redshift of reionization in general due to the suppression of galaxy and star formation. Fig. 7 also illustrates once more that the inclusion of H$_2$ cooling does little to increase the redshift of reionization. In fact, it appears that including H$_2$ cooling actually reduces the redshift of reionization in the curves plotted here. This is a numerical artifact, arising from the fact that the most massive haloes simulated in our calculations are of lower mass in models which include H$_2$ cooling (see §C2.2). Once this is corrected for, reionization redshifts are almost identical whether or not H$_2$ cooling is included (see §4.1).
Figure 7. Star formation histories (blue and cyan lines) and H\textsc{ii} filling factors calculated using ionized spheres (red and magenta lines). Cyan and magenta lines show results for models where photoionization and photodissociation feedback are neglected, while blue and red lines show results for models including these feedback mechanisms. Results are shown for Standard and H\textsubscript{2} models in WMAP PL, WMAP RSI and Baugh et al. '05 cosmologies as indicated for each column and row.

Note that, even when photoionization feedback is not included the filtering mass still rises due to the heating of the IGM by ionizing photons emitted by galaxies. All that is left out of the calculation is the effect of the filtering mass on subsequent galaxy formation.

It can be seen that the inclusion of photoionization and photodissociation feedback has only a small effect on the redshift of reionization. This is not surprising, as the mechanisms through which photoionization feedback works (photoheating of gas in haloes by the ionizing background and an increase in the filtering mass) become important only after reionization that the filtering mass has the chance to respond to the increase in the temperature of the gas. At \( z = 5 \) we can clearly see that the cooling times in haloes with virial temperatures below around $10^5K$ are greatly increased due to this photoheating and photodissociation. At \( z = 10 \) the effect is much weaker, due both to the lower background at that redshift and the fact that gas in haloes is denser at higher redshifts and so less affected by photoheating.
The Epoch of Reionization

Figure 8. As Fig. 6 but for models including the effects of photo-ionization and photo-dissociation feedback.

9 The “spikes” seen in the H\textsubscript{i} filling factor are an artifact of the semi-analytic model. As in Paper I, we compute the reionization process by drawing a set of halo masses from the halo mass function at some redshift \( z \) and constructing merger trees for each halo back to very high redshifts. The galaxy formation calculation then proceeds in each tree. Our merger tree algorithm is not completely accurate in the sense that, when used to produce merger trees that span large redshift ranges, it does not exactly reproduce the expected distribution of progenitor halo masses. This problem becomes more severe as the redshift range spanned increases. Consequently, we repeat our calculations for several starting redshifts \( z_i \) (ensuring that at each \( z_i \) we have the correct distribution of halo masses). The reionization histories shown are constructed by splicing together results from each calculation, using quantities between \( z_j \) and \( z_{j+1} \) from each calculation. At each \( z_j \) we find discontinuities in the predicted properties due to this splicing process.

Standard model in the WMAP RSI cosmology with and without the effects of photoionization feedback included. The filtering mass begins to rise shortly before reionization occurs, when photoheating first dominates over cooling of gas due to the expansion of the Universe, but does not achieve any significant value until after the Universe is already reionized. Similarly, the ionizing background cannot build up to any significant level (and thereby efficiently photoheat gas in haloes) until the Universe has already been ionized and thereby become transparent to ionizing radiation. Benson et al. (2002a) found that it was the filtering mass which caused the greatest suppression of galaxy formation, seemingly in contradiction with the above result. Benson et al. (2002a) were, however, considering the effects of photoionization feedback on the galaxy population at \( z = 0 \), by which time the filtering mass had had time to fully respond to the change in the IGM temperature during reionization. When considering the effects of photoionization feedback on the epoch of reionization itself, as we do here, the filtering mass
Figure 9. Indicators of the strength of photoionization feedback mechanisms as a function of redshift for the Standard model in the WMAP RSI cosmology. Results are shown for this single model and cosmology only as they are typical of all of the models we consider. The solid red (blue) line in the upper panel shows the filtering mass (scale on left-hand axis) when photoionization feedback is (is not) included. In the lower panel red (blue) circles show the ionizing background intensity (scale on left-hand axis) when photoionization feedback is (is not) included. The dashed red line in both panels indicates the H\textsubscript{ii} filling factor (scale on right-hand axis) for a model including photoionization feedback.

is much less important as it does not have time to fully respond to the heating of the IGM until well after reionization is over. The ionizing background, however, rises very rapidly as reionization occurs, allowing photoheating to have some (albeit small) effect on the redshift of reionization.

3.2.1 Effects of H\textsubscript{2} Dissociation?

In general, we find that stars formed via H\textsubscript{2} cooling contribute little to the reionization of the Universe, if they form with a normal IMF. Therefore, it is not surprising that the inclusion or otherwise of H\textsubscript{2} dissociation in our model makes no significant difference to the end results on the ionized filling factor, as shown in Fig. 10.

Figure 10. The H\textsubscript{ii} filling factor as a function of redshift for model H\textsubscript{2} with (solid line) and without (dashed line) dissociation of H\textsubscript{2} molecules included in our calculations. Results are shown for the WMAP RSI cosmology only, but are typical of all other cosmologies and models.

3.3 Enhanced Emission from Early Generations of Stars

In model “H\textsubscript{2}WeakFB-VMS” we enhance the UV emission from stellar populations which form via H\textsubscript{2} cooling (i.e. those which form in dark matter haloes with virial temperatures below 10\textsuperscript{4}K) by a factor of 20 relative to a standard IMF, to account for the possibility that these stars might be supermassive “pop III” stars. The left-hand panel of Figure 11 shows the resulting H\textsubscript{ii} filling factors vs. redshift for this model and for model H\textsubscript{2}WeakFB (which is identical apart from having no enhancement of UV emission due to supermassive stars) for the WMAP cosmologies. The results are plotted for an escape fraction chosen to give a double reionization: 25% (WMAP RSI) and 15% (WMAP PL). This results in all models producing full reionization at \( z \approx 6 \). Note however, that model H\textsubscript{2}WeakFB-VMS displays an extended period of partial reionization back to higher redshifts (from \( z \approx 8 \) for the WMAP RSI cosmology to \( z \approx 17 \) for the WMAP PL cosmology). As a consequence, model H\textsubscript{2}WeakFB-VMS has an increased electron scattering optical depth as shown in the right-hand panel of Figure 11. For example in the WMAP PL cosmology we find \( \tau = 0.15 \), compared to 0.08 for model H\textsubscript{2}WeakFB. Values of \( \tau \) are lower for the other cosmologies, but still come significantly closer to the WMAP measurement of \( \tau = 0.17 \pm 0.04 \). As such, model H\textsubscript{2}WeakFB-VMS is in much better agreement with the data obtained from the WMAP satellite (Kogut et al. 2003). Enhanced UV emission from stellar populations formed via H\textsubscript{2} cooling is therefore able to reconcile the apparent disparity between a large optical depth and evidence for reionization at \( z \approx 6 \) in the WMAP PL cosmology, by inducing a period of partial reionization.

The dotted line in Fig 11(a) shows the contribution to the H\textsubscript{ii} filling factor from stars forming via H\textsubscript{2} cooling in
the H$_2$WeakFB-VMS model for the WMAP PL cosmology. Before $z \approx 20$ these stars dominated the reionization of the Universe, as the steepness of the dark matter halo mass function results in there being very few haloes hot enough to cool via atomic processes at these redshifts. At lower redshifts the abundance of haloes with virial temperature above $10^8$K increases, while galaxy formation in lower temperature haloes becomes inefficient due to increased cooling times and the rising filtering mass$^{10}$. Therefore, haloes hotter than $10^8$K rapidly become the dominant contributors to the H$_2$ filling factor.

Obtaining a period of extended partial reionization, or a double reionization, requires an interplay of several key factors. Firstly, the cosmological model must produce enough haloes with virial temperatures below $10^5$K to permit a significant H$_2$ filling factor to be obtained from the early generation of stars (assumed to be supermassive). The enhancement factor (taken to be 20 in this work) is also important: if it is too high then reionization may happen once only, if it is too low the early generation of stars will not be able to produce a significant H$_2$ filling factor. The decline in the contribution from early generation stars as time progresses must also be sufficiently rapid (otherwise the filling factor will continue to rise, leading to a single reionization event)—requiring either that the filtering mass grows rapidly or that cooling times for haloes cooler than $10^5$K increase rapidly shortly after the first reionization. Despite this requirement that many factors combine to produce a double reionization, we find, for quite reasonable values of the UV enhancement factor, that double reionization is a natural outcome of our H$_2$WeakFB-VMS model in all cosmologies considered. An extensive survey of parameter space would be required to determine how common such double reionizations are, but this is beyond the scope of this work.

While our modelling of this enhanced UV emission from early stars is very simplified (e.g. we assume that supernova feedback is the same for Very Massive Stars and for normal stellar populations), and based on uncertain assumptions (e.g. that all haloes which cool by H$_2$ produce Very Massive Stars), it is still relatively sophisticated compared to previous studies of double reionization (Cen 2003; Wyithe & Loeb 2003) to the extent that ours is the only model which follows the full hierarchical merging history of each forming galaxy and, furthermore, computes the fraction of baryons able to cool and the fraction able to turn into stars using physically motivated prescriptions rather than simply making these fractions parameters of the model.

4 OBSERVABLE QUANTITIES

4.1 The Epoch of Reionization

Table 3 lists the redshifts of reionization in all of the models considered (all results shown include the effects of photoionization and photodissociation feedback), along with the corresponding electron-scattering optical depths. Observations of the cosmic microwave background suggest a large value for $z_{\text{reion}}$. Kogut et al. (2003) used WMAP measurements of the temperature-polarization (TE) cross-power spectrum to estimate the optical depth $\tau$. They found a best-fit value $\tau = 0.17$, with a 95% confidence range $0.09 \leq \tau \leq 0.28$. For the WMAP cosmology, this corresponds to a best-fit value of the reionization redshift $z_{\text{reion}} = 17$, and a 95% confidence range $11 < \tau < 24$, if one assumes instantaneous reionization, i.e. if the ionized fraction increased from 0 to 1 at $z = z_{\text{reion}}$. Spergel et al. (2003) found very similar results from their multi-parameter fit to the whole WMAP dataset combined with data on galaxy clustering.

Since our models predict a gradual rather than an instantaneous reionization, we compare our models with the WMAP data using the value of $\tau$ rather than the value of $z_{\text{reion}}$. From Table 3 we see that all of our models based on the WMAP PL cosmology and assuming $f_{\text{esc}} = 1$ are consistent with the measured $\tau$ (at 95% confidence), apart from the H$_2$WeakFB-VMS model, which predicts $\tau = 0.30$, marginally too high. These models predict $z_{\text{reion}}$ in the range 11–22. However, when $f_{\text{esc}}$ is reduced to 15%, the H$_2$WeakFB-VMS model in this cosmology predicts $\tau = 0.15$ (and $z_{\text{reion}} = 17$), in good agreement with the WMAP measurement. In contrast, all but one of our models based on the WMAP RSI cosmology, and assuming $f_{\text{esc}} = 1$, are excluded (at 95% confidence) by the WMAP measurement of $\tau$, due to the predicted optical depths being too low, the one exception being the H$_2$WeakFB-VMS model, which (with $\tau = 0.10$) is marginally compatible. The models in the WMAP RSI cosmology predict $z_{\text{reion}}$ in the range 6–11. Thus, while the WMAP RSI cosmology cannot be completely excluded based on the WMAP measurement of $\tau$, it does appear very unlikely. If the escape fraction $f_{\text{esc}}$ is in fact significantly less than unity, then the predicted reionization epochs will be shifted to considerably lower redshifts in all of the models. For the Baugh et al. ’05 cosmology, we find that the Standard and H$_2$ models are marginally consistent with the WMAP constraint, while models with weak feedback are able to match that constraint within current errors.

Observations of the spectra of the highest redshift quasars at $z \sim 6$ show evidence of Gunn-Peterson absorption by neutral hydrogen in the IGM (Becker et al. 2001; Djorgovski et al. 2001; White et al. 2003; Fan et al. 2003; Fan et al. 2006), which has been interpreted as evidence that the Universe reionized at redshifts only slightly above 6, since simple theoretical models predict that the transition from the IGM being almost completely neutral to almost completely ionized should have happened over a rather narrow redshift range. A low $z_{\text{reion}} \sim 7$ is in apparent contradiction with the results from WMAP. This interpretation of the observed quasar spectra has however been questioned by some authors (e.g. Songaila 2004). Even if current measurements of the Gunn-Peterson optical depth $\tau_{\text{GP}}$ at $z \sim 6$ are correct, they can be explained by very small neutral hydrogen fractions in the IGM, e.g. White et al. (2003) measure $\tau_{\text{GP}} \sim 5 - 20$ for two quasars at $z \approx 6.3$, which can be produced with neutral fractions of only $f_{\text{HI}} \sim 1.5 - 6 \times 10^{-5}$. Therefore, in principle the IGM might have ceased to be mainly neutral at much higher redshifts, and still produce detectable Gunn-Peterson absorption at $z \sim 6$. Note that the only way in which we are able to produce a “double-reionization” is by assuming that stars forming via H$_2$ cool-

$^{10}$ The filtering mass in the H$_2$WeakFB-VMS model in the WMAP PL cosmology rises above the mass corresponding to a $10^4$K halo at $z = 15$. 

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Cosmology comes closest, achieving $\tau \approx 0.03$ by $z = 6$ which is sufficiently high to explain the observed Gunn-Peterson optical depth $\tau_{GP}$. The neutral fraction falls very rapidly during reionization as can be seen from the fifth and sixth columns of Table 3, which list the redshifts at which neutral fractions of $10^{-2}$ and $10^{-5}$ are reached. Neutral fractions comparable to those required by the observational data could be obtained if $f_{esc}$ were reduced, at the expense of a slightly lower reionization redshift.

We find that the Baugh et al. (2005) model achieves reionization at $z \approx 12$ for an escape fraction of 100%, while adding H$_2$ cooling to this model raises the reionization redshift to $z \approx 12.5$. The corresponding values of the electron scattering optical depth are $\tau = 0.108$ and 0.118 respectively. This model is therefore consistent with the WMAP measurement of the optical depth, given the rather large errors on that datum. However, the neutral fraction in this model is too low at $z \approx 6$ to explain observations of the Gunn-Peterson trough (the optical depth at $z = 6$ for the Baugh et al. (2005) model lies in the range 0.01 to 0.1 for the various models considered here). While this could be rectified by lowering the escape fraction in order to reduce the reionization redshift to be closer to $z \approx 6$, this would also reduce the optical depth.

### 4.2 CMB Anisotropies

We have computed the temperature and polarization power spectra predicted by our model reionization histories. Figure 13 shows results for the WMAP PL and WMAP RSI cosmologies. To calculate CMB temperature spectra, we employed the Boltzmann code of Sugiyama (1995), which

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11 That is, $f_{HI} = \int n_{HI} dV/\int n_{HI} dV$. This is equivalent to a mass weighted average of the local neutral fraction $x_{HI}$.

12 The Baugh et al. '05 cosmology was not designed to accurately match available CMB data and so is not considered here.
Table 3. The epochs of reionization for all models considered in this work. Columns 1 through 3 specify input parameters of the models: column 1 specifies the cosmological model while column 2 specifies the galaxy formation model used and column 3 specifies the value of $f_{\text{esc}}$ used. The remaining columns list output quantities. Column 4 lists the redshift of reionization—defined as the redshift at which the filling factor in the ionized sphere reionization calculation reaches unity. The first value gives $z_{\text{reion}}$ as found directly from our calculations, while the second value accounts for the finite upper mass limit of the haloes which we simulate. The true $z_{\text{reion}}$ should lie between these two values. The next column gives the electron scattering optical depth (assuming that helium is singly ionized at the same time as hydrogen is ionized, and that helium is doubly ionized below $z = 3$). The two values again refer to the calculation without and with the correction for the finite upper mass limit of simulated haloes. The next two columns list the redshifts at which the neutral hydrogen fraction in the IGM drops to $10^{-2}$ and $10^{-5}$ respectively, as determined using the IGM evolution model of Benson et al. (2002a). The final column lists the redshift at which the mean Gunn-Peterson optical depth reaches a value of 3.

| Cosmology | Model         | $f_{\text{esc}}$ | $z_{\text{reion}}$ | $\tau$ | $z_{\text{HI}=10^{-2}}$ | $z_{\text{HI}=10^{-5}}$ | $z_{\text{GP}=3}$ |
|-----------|---------------|------------------|--------------------|--------|-------------------------|-------------------------|------------------|
| WMAP RSI  | Standard      | 1.00             | 6.9–6.9            | 0.061–0.061 | 7.36                     | 7.25                     | 7.10             |
| WMAP RSI  | MetFree       | 1.00             | 7.3–7.4            | 0.064–0.068 | 7.60                     | 7.50                     | 7.38             |
| WMAP RSI  | TopHeavy      | 1.00             | 8.7–8.7            | 0.079–0.080 | 9.02                     | 8.94                     | 8.79             |
| WMAP RSI  | WeakFB        | 1.00             | 8.1–8.1            | 0.073–0.079 | 8.50                     | 8.45                     | 8.33             |
| WMAP RSI  | H$_2$         | 1.00             | 6.1–6.6            | 0.055–0.063 | 6.91                     | 6.69                     | 6.51             |
| WMAP RSI  | H$_2$WeakFB   | 1.00             | 7.8–8.1            | 0.074–0.075 | 8.44                     | 8.38                     | 8.22             |
| WMAP RSI  | H$_2$WeakFB   | 0.25             | 4.4–4.4            | 0.048–0.055 | 7.60                     | 7.34                     | 7.17             |
| WMAP RSI  | H$_2$WeakFBVMS | 1.00           | 10.6–10.6         | 0.102–0.102 | 8.46                     | 8.39                     | 8.21             |
| WMAP RSI  | H$_2$WeakFBVMS | 0.25           | 4.4–4.4            | 0.076–0.080 | 7.55                     | 7.36                     | 7.13             |
| WMAP PL   | Standard      | 1.00             | 10.8–10.8          | 0.141–0.141 | 11.38                    | 10.74                    | 10.54            |
| WMAP PL   | MetFree       | 1.00             | 11.4–11.4          | 0.157–0.157 | 11.92                    | 11.20                    | 10.93            |
| WMAP PL   | TopHeavy      | 1.00             | 15.3–15.3          | 0.213–0.213 | 14.54                    | 14.46                    | 14.33            |
| WMAP PL   | WeakFB        | 1.00             | 14.1–14.1          | 0.191–0.191 | 13.73                    | 12.77                    | 12.42            |
| WMAP PL   | H$_2$         | 1.00             | 9.2–10.8           | 0.131–0.141 | 7.87                     | 7.79                     | 7.73             |
| WMAP PL   | H$_2$WeakFB   | 1.00             | 14.1–14.1          | 0.206–0.206 | 13.87                    | 12.56                    | 12.31            |
| WMAP PL   | H$_2$WeakFB   | 0.15             | 6.1–8.3            | 0.095–0.107 | 7.99                     | 7.85                     | 7.84             |
| WMAP PL   | H$_2$WeakFBVMS | 1.00           | 22.2–22.2         | 0.299–0.299 | 13.77                    | 12.71                    | 12.40            |
| WMAP PL   | H$_2$WeakFBVMS | 0.15           | 6.9–8.8            | 0.183–0.196 | 7.98                     | 7.85                     | 7.84             |
| Baugh et al. (2005) | Standard   | 1.00             | 11.9–11.9          | 0.108–0.108 | 7.97                     | 7.44                     | 7.39             |
| Baugh et al. (2005) | H$_2$         | 1.00             | 12.5–12.5         | 0.118–0.118 | 7.55                     | 7.23                     | 6.87             |
| Baugh et al. (2005) | H$_2$WeakFB | 1.00             | 14.4–14.4         | 0.144–0.144 | 8.62                     | 8.19                     | 7.76             |

Figure 12. The mean mass-weighted H$_i$ fraction (left-hand panel) and Gunn-Peterson optical depth (right-hand panel) as functions of redshift for various models in our three cosmologies (see key in figure for colour and line type coding). The model results plotted here all assume $f_{\text{esc}} = 1$. The dashed black line in the right-hand panel indicates the fit to the mean optical depth reported by Fan et al. (2006).
has been used to calibrate the accuracy of publicly available codes, e.g., CMBFast (Seljak et al. 2003). The models shown in the figure have $\sigma_8 = 0.9$ for the WMAP PL cosmology and $\sigma_8 = 0.83$ for the WMAP RSI cosmology. Furthermore, all models are shown for $f_{\text{esc}} = 1$, except for the model labelled “grad:$f_{\text{esc}} = 0.15$” for which we plot the $H_2$WeakFB – VMS model with $f_{\text{esc}} = 0.15$ (which exhibits a double reionization). For comparison, we also show the CMB anisotropies predicted for a toy model of instant reionization, in which the IGM instantaneously becomes fully reionized at $z_{\text{reion}} = 17$, chosen to give the optical depth $\tau = 0.17$ that best fits the WMAP data.

For the WMAP PL cosmology, it is clear that the $H_2$WeakFB – VMS model with $f_{\text{esc}} = 1.0$ is ruled out from the TT-spectrum. The reason for the failure of this model is the combination of the large optical depth (which suppresses CMB temperature anisotropies on small scales) and $\sigma_8 = 0.9$. One could try to make this model agree with the TT data by increasing $\sigma_8$ by $\sim 20\%$. However, this increase leads to galaxy formation at higher redshifts and hence a larger optical depth. Thus, while increasing $\sigma_8$ helps to reconcile this model with the high-$\ell$ part of the TT spectrum, it produces a very large polarization signal at small $\ell$, which is inconsistent with that measured by WMAP.

The $H_2$ and Standard models in the WMAP PL cosmology also do not agree well with the WMAP TT-spectrum, due to their optical depths being too low.

It is very interesting that the gradual reionization model (the $H_2$WeakFB – VMS model with $f_{\text{esc}} = 0.15$; light blue line in Fig. 13) shows a unique behaviour in the EE-spectrum compared with the $H_2$WeakFB and instant reionization models, while the optical depths of these three models are similar. In particular, this model shows a feature (a bump) at $\ell \approx 20$. The location of the $\ell \approx 8$ peak and the width of this peak are slightly different from other models of comparable optical depth (i.e., the width is slightly greater and the peak location is shifted to the right). These differences are due to the fact that the reionization process occurs over a longer period in this model. On the other hand, there are only small differences in TT and TE spectra. We expect the EE-spectrum to be the best probe of the details of reionization such as the duration and the epoch (Holder et al. 2003).

None of the models in the WMAP RSI cosmology fits the WMAP CMB data, since we employ the $\sigma_8$ normalization here (i.e. $\sigma_8 = 0.83$ in all of the RSI models plotted), and the optical depths of all of the models are very small.

The calculations here do not include the effects of patchiness in the reionization process. Patchy reionization can produce secondary anisotropies in the CMB on small angular scales, $\ell \gtrsim 3000$ (Paper I). These anisotropies must be studied in detail using numerical simulations which provide the spatial distribution and peculiar motions of the ionized gas. We defer this work to a future paper.

5 DISCUSSION

We have described the most detailed and physically realistic semi-analytic calculation of the reionization of the Universe that has been carried out to date. These calculations rely on a physical model of galaxy formation which includes cooling through Compton and molecular hydrogen channels, a physically motivated model of star formation and feedback, and the feedback exerted on galaxy formation by the process of reionization itself.

We find that, in the WMAP PL cosmological model, the Universe can be reionized by $z \approx 14$–15 either if early generations of stars are preferentially very massive or if feedback in the high-redshift Universe is much weaker than it is today. A direct measurement of the reionization redshift is not yet possible. Instead, we can compute, from our reionization histories, optical depths for electron scattering which can be compared directly with the measurement of this quantity from WMAP. Reionization at these redshifts produces optical depths in the range $\tau = 0.19$–0.21. This is consistent with the WMAP result of $\tau = 0.17 \pm 0.04$. In the WMAP RSI cosmology however, reionization cannot occur before $z \approx 9$, due to the much smaller number of massive haloes which are able to form at high redshifts in this model. As such, the WMAP RSI cosmological model is barely consistent with the WMAP result for $\tau$, achieving at most $\tau \approx 0.1$. This suggests one of three possibilities:

(i) the WMAP estimate of $\tau$ is too high (either due to systematic effects or a simple statistical fluctuation);

(ii) the RSI cosmology is incorrect (perhaps due to problems in determining the normalization of the Lyman-α forest power spectrum; see, for example, Viel, Weller & Haehnelt 2004)

(iii) our understanding of galaxy formation, or the CDM hierarchical model itself, are incorrect at high redshifts.

Option (iii) is almost certainly true to some degree—there are clearly large gaps remaining in our understanding of galaxy formation. We include within option (iii) the possibility that some neglected galaxy formation physics, for example a contribution to the ionizing emissivity from super-massive black holes forming in high redshift galaxies (Ricotti & Ostriker 2004), affects our results. However, in the WMAP RSI cosmology, the basic problem is that there are simply too few haloes existing at high redshifts to allow enough galaxies to form and so cause reionization. Only if $100\%$ of the baryons which have the potential to cool in high redshift haloes are allowed to turn instantly into stars can a sufficiently large $\tau$ be attained in the WMAP RSI cosmology (Haiman & Holder 2003; Somerville, Bullock & Livio 2003). Within our models, even with rather extreme assumptions about the strength of feedback, the IMF of early generations of stars, and the escape fraction of ionizing photons, we are unable to achieve reionization prior to $z \approx 9$ for the WMAP RSI cosmology. Thus, the WMAP RSI cosmology seems hard to reconcile with our understanding of galaxy formation in the CDM hierarchical model.

The recent three year data release from the WMAP satellite confirms (i) above. Spergel et al. (2006) report $\tau = 0.093 \pm 0.029$. This optical depth is well within the reach of our models with $f_{\text{esc}} = 1$, although physically motivated escape fractions (see Appendix B) would result in models still struggling to reach such large optical depths.

Using the model of Baugh et al. (2005), which was designed to match the properties of low and high-redshift galaxies, we find that reionization can occur by $z \approx 12$ for $f_{\text{esc}} = 1$, producing an optical depth of $\tau = 0.11$ which is marginally consistent with the WMAP data. This optical
Figure 13. CMB anisotropy power spectra for the WMAP PL (left-hand panels) and WMAP RSI (right-hand panels) cosmologies. The power spectra of temperature fluctuations (TT) are shown in the upper panels, while the middle and lower panels show polarization power spectra (EE in the middle panels and TE in the lower panels). The circles with error bars show observational data from the WMAP satellite (Hinshaw et al. 2003; Kogut et al. 2003). The dashed black lines (labelled instant) show results for a simple toy model in which reionization is instantaneous, and occurs at a redshift selected to best fit the WMAP data. The coloured lines show the predictions for various models from this work, assuming $f_{esc} = 100\%$ unless indicated otherwise. The cyan lines (labelled grad) show results for the $H_2$WeakFB-VMS model with an escape fraction of 15\%, which exhibits gradual reionization.
depth can be increased to \( \tau = 0.12 \) by including H\(_2\) cooling (which does not affect the ability of this model to match local galaxy data), and can be increased further if the strength of feedback is reduced (which would affect the ability of the model to match local galaxy data).

Recently, the Boomerang experiment has provided an improved measurement of the cosmological parameters (MacTavish et al. 2005). We have computed reionization models using these revised cosmological parameters (specifically, we use the “CMBall+B03+LSS” parameter set from MacTavish et al. (2005), as listed in Table 1). We find that at high redshifts (\( \gtrsim 20 \)) the Boomerang cosmology produces a lower H\(_\text{I}\) filling factor than the WMAP PL cosmology (for all of our different galaxy formation model parameter sets). However, this difference becomes less towards lower redshifts, and we find that the epoch of reionization in the Boomerang cosmology is usually very close to that for the corresponding model in the WMAP PL cosmology. The one exception is for the H\(_2\)WeakFB-VMS model, in which reionization occurs significantly later in the Boomerang cosmology (since reionization happens early in the H\(_2\)WeakFB-VMS model, the differences between the Boomerang and WMAP cosmologies are more pronounced). Nonetheless, double reionization can still occur in our model, even in the Boomerang cosmology. We conclude, therefore, that the conclusions we present for the WMAP cosmology can also be applied to the revised cosmological parameter set suggested by the Boomerang data.

Throughout this work we have used the values of \( \sigma_8 \) which best fit the WMAP data (\( \sigma_8 = 0.90 \) and 0.83 for the PL and RSI models respectively, Spergel et al. 2003). However, the value of \( \sigma_8 \) remains somewhat uncertain, with plausible values lying between about 0.7 to 1.1. In particular, Spergel et al. (2006) report \( \sigma_8 = 0.76 \pm 0.05 \). We have re-run our WMAP PL models using these two extreme values of \( \sigma_8 \). Results for the ionized filling factor are shown in Fig. 14. Not surprisingly, lowering \( \sigma_8 \) reduces the redshift of reionization, while increasing \( \sigma_8 \) increases it. The effect is greater for models in which reionization happens at higher redshifts, since here those haloes which are able to form galaxies and emit ionizing photons correspond to very rare fluctuations in the initial Gaussian density field. As such, their abundance is very sensitive to the normalization of that density field. Typically, models which reionize at \( z \approx 10 \) have their epoch of reionization reduced/increased by \( \Delta z \approx 1 \) for a reduction/increase of 0.2 in \( \sigma_8 \), while models with reionization at \( z \approx 20 \) experience a \( \Delta z \) of 4–5 for the same change in \( \sigma_8 \) (these values of \( \Delta z \) do depend also on the parameters of the galaxy formation calculation). Thus, \( \sigma_8 \) can have a large effect on predictions of the epoch of reionization.

These results show that, even for the low \( \sigma_8 \) reported by Spergel et al. (2006) sufficiently early reionization can be achieved (i.e. to match the optical depth also reported by Spergel et al. 2006) providing \( f_{\text{esc}} = 100\% \). Most of the results we present here are based on the assumption that the fraction of ionizing photons which escape from galaxies into the IGM is \( f_{\text{esc}} = 1 \). In the local Universe, and at \( z \approx 3 \), observational estimates give much smaller values. If \( f_{\text{esc}} \) were low at higher redshifts also, then our estimates of \( \tau_{\text{reion}} \) would be substantially too high. A value of \( f_{\text{esc}} = 0.1 \) would reduce the reionization redshift in even our best models to \( z \lesssim 10 \). In Appendix B we present reionization histories computed using a simple physically-motivated model for the escape fraction. Clearly, a better theoretical understanding of \( f_{\text{esc}} \) is crucial if we are to use measures of the reionization epoch to put interesting constraints on the process of galaxy formation.

We have found that extended periods of partial reionization and double reionization can occur in models in which early generations of stars formed via H\(_2\) cooling are very massive (with enhanced ionizing luminosities relative to a standard IMF). Such models are able to delay full reionization until \( z \approx 6–7 \), while achieving an electron scattering optical depth large enough to be consistent with the results from WMAP. While a more extensive study of such models would be required to determine whether or not they can match all of the current observational constraints, they appear to be a promising route to reconciling the apparently discrepant estimates of the reionization redshift from the CMB polarization measured by WMAP and from the Gunn-Peterson absorption detected in the spectra of \( z \sim 6 \) quasars. It should be noted, however, that these models rely on the assumption of very weak feedback from supernovae. Within CDM cosmogonies, it has long been argued that feedback from supernovae must be very effective in order to reduce the number of low-luminosity galaxies at low redshifts which are otherwise produced (see Benson et al. 2003a and references therein). Thus, if double reionization (or an extended period of partial reionization) is required to meet observational constraints, then it appears that the strength of supernova feedback must vary with redshift, or else some other feedback mechanism must act at low redshifts to reduce the number of faint galaxies formed. This underlies the urgent need for the inclusion of physical recipes for feedback into
semi-analytic models of galaxy formation, rather than the phenomenological feedback rules currently employed.

In conclusion, the WMAP RSI cosmology is very difficult to reconcile with our current understanding of galaxy formation (although a wide parameter space of models remains to be searched) if the optical depth were as large as suggested by the first year WMAP data. The WMAP PL model, in contrast, is able to achieve a sufficiently high optical depth with reasonable assumptions about the nature of galaxy formation at high redshifts. The three year WMAP data imply an optical depth significantly lower than that found in the first year data (i.e. $\tau = 0.093 \pm 0.029$; Spergel et al. 2006). Reionization can then be achieved sufficiently early in both the WMAP PL and WMAP RSI cosmologies, providing $f_{\text{esc}}$ is close to 1 and $\sigma_8$ is not too low. We will present a more detailed analysis of the implications of the 3-year WMAP data in a follow-up paper.

In principle, the value of the optical depth to reionization is one of the few strong constraints on galaxy formation at high redshifts. The uncertainty mentioned in option (i) has already been reduced by the WMAP three year data. Thus, the ability to reionize the Universe sufficiently early will now become one of the key tests that any model of galaxy formation must pass before it can be considered to be viable.

ACKNOWLEDGEMENTS

AJB acknowledges support from a Royal Society University Research Fellowship. NS is supported by a Grant-in-Aid for Scientific Research from JSPS (No. 17540276). This work was also supported by the PPARC rolling grant for extragalactic astronomy and cosmology at Durham. We wish to thank Carlton Baugh, Shaun Cole and Carlos Frenk for permitting us to use the GALFORM model of galaxy formation in this work. We also acknowledge helpful suggestions from an anonymous referee.

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APPENDIX A: REVISED COOLING MODEL

In this Appendix, we discuss how we incorporate cooling by molecular hydrogen and Compton cooling into our model and how we apply the effects of the filtering mass.

A1 Cooling by Molecular Hydrogen

We incorporate cooling due to molecular hydrogen in our model following the algorithms of Yoshida et al. (2003a). Below, we give a brief account of our calculation of the $H_2$ contribution to the cooling rate. The reader is referred to Yoshida et al. (2003a) and references therein for a full description.

We begin by estimating the fraction, $f_{H_2}$, of molecular hydrogen which would be present if there were no background of $H_2$-dissociating radiation from stars. As shown by Tegmark et al. (1997), at the redshfits of interest here, $H_2$ is formed mostly via the formation of $H^-$ through the reaction $H^+ + e^- \rightarrow H^-$, with rate coefficient

$$k_{H^-} = 1.83 \times 10^{-18} (T/K)^{0.88} \text{ cm}^3\text{s}^{-1}$$

(Hutchins 1976). The rate of formation of $H^-$ depends on the abundance of free electrons, which depends on recombinations $H^+ + e^- \rightarrow H + \gamma$ with rate coefficient

$$k_1 = 1.88 \times 10^{-10} (T/K)^{-0.04} \text{ cm}^3\text{s}^{-1}$$

(Hutchins 1976). We ignore photoionization and collisional ionization of hydrogen in this part of the calculation, since cooling by $H_2$ is only important in low-temperature haloes at high redshifts. Tegmark et al. (1997) solved the rate equations for electron recombination and $H_2$ formation to obtain a characteristic $H_2$ abundance $f_{H_2,c}$ for gas with hydrogen number density $n_H$ and temperature $T$ at redshift $z$:

$$f_{H_2,c} = 3.5 \times 10^{-4} T_4^{1.52} \left[1 + (7.4 \times 10^8/n_{H,1}) \times (1 + z)^{2.13} \exp(-3173/[1 + z]) \right]^{-1},$$

where $n_{H,1}$ is the hydrogen number density in units of cm$^{-3}$ and $T_4$ is temperature in units of 1000K. (The above result depends on the values of various rate coefficients, and is consistent with the rate coefficients given in eqns. A1, A2 and A8.) The factor in square brackets corresponds to photodissociation of $H^-$ by CMB photons, but is important only for $z \gtrsim 100$. For our own estimate of the $H_2$ abundance in the absence of a photodissociating background from stars, we then use

$$f_{H_2,0} = \max \left[f_{H_2,prim}, f_{H_2,c}\right],$$

where $f_{H_2,prim}$ is the primordial $H_2$ fraction, which we take to be $3.5 \times 10^{-7}$ (Anninos & Norman 1996) (although our results are highly insensitive to this choice of value).

Next, we account for the effects of a background of $H_2$-dissociating radiation from stars. If we assume equilibrium between $H_2$ production via $H^-$ and $H_2$ destruction by photodissociation, we obtain an $H_2$ abundance

$$f_{H_2,eq} = f_{H_2,0} n_H / k_{diss},$$

We calculate the electron fraction, $x_e$, appearing in eqn. (A5) from

$$x_e = \frac{x_{e,0}}{1 + x_{e,0} t_{1/2} / \tau_{rec}},$$

based on eqn. (6) from (Tegmark et al. 1997), where

$$\tau_{rec} = 1/(k_1 n_H).$$

This estimate of $x_e$ assumes that recombinations have been proceeding in gas at its present density and temperature for a time $t_{1/2}$ equal to half the age of the Universe at redshift $z$, starting from the primordial post-recombination electron fraction $x_{e,0}$. We calculate $x_{e,0}$ using RECAST (Seager, Sasselov & Scott 2000). Eqn. (A6) accounts crudely for the age of the halo, and therefore the time available for recombinations. The electron fraction is never allowed to fall below the value corresponding to collisional ionization equilibrium at the specified temperature. Note that, post-reionization, the electron fraction should in fact tend towards the equilibrium value for photoionization equilibrium (as opposed to collisional ionization equilibrium). Our current model does not account for this fact, but this is unimportant here as the determination of electron fractions in the post-reionization Universe does not affect our determination of the epoch of reionization.

For the photodissociation rate in eqn. (A5), we assume

$$k_{diss} = 1.1 \times 10^{-13} J_{21} F_{\text{shield}} \text{s}^{-1},$$

(Machacek, Bryan & Abel 2001), where $J_{21}$ is average flux in units of $10^{-21}\text{ergs/cm}^2\text{s}/\text{Hz}$ in the Lyman-Werner bands (with photon energy around 12.87 eV), and is computed from the radiation background tracked by our model of IGM evolution (Benson et al. 2002a). We account for the effects of self-shielding using the algorithm of Yoshida et al. (2003a). Specifically, we estimate the shielding factor

$$F_{\text{shield}} = \min \left[1, \left(\frac{N_{H_2}}{10^{14}\text{cm}^{-2}}\right)^{-3/4}\right],$$

which depends on the $H_2$ column density $N_{H_2}$. In order to avoid the computational cost and complexity of solving iteratively for the radial abundance profile of $H_2$ within each
halo, we estimate this shielding factor using the abundance $f_{\text{H}_2,0}$ computed in the absence of a photodissociating background, using

$$N_{\text{H}_2} = C f_{\text{H}_2,0} \tau_{\text{H},\text{vir}} R_{\text{vir}}$$

with $C = 0.2$, where $R_{\text{vir}}$ is the virial radius of the dark matter halo and $\tau_{\text{H},\text{vir}}$ is the mean hydrogen density within $R_{\text{vir}}$. (For a more detailed justification of this approach, see Yoshida et al. 2003a.)

We then calculate the final $\text{H}_2$ abundance used in our cooling calculation from

$$f_{\text{H}_2} = \min \left[ f_{\text{H}_2,0}, f_{\text{H}_2,\text{eq}} \right] \exp \left( -\frac{T}{51.920 \text{K}} \right).$$

We have introduced an exponential cut-off in the molecular hydrogen fraction at temperatures corresponding to the dissociation energy of $\text{H}_2$, to account for collisional dissociation of $\text{H}_2$. (Tegmark et al. 1997) ignored this because they were interested only in the relatively low temperature regime. The exact form and position of the cut-off are unimportant, as the cooling function is dominated by atomic processes at the temperatures for which collisional dissociation is important.)

Finally, we compute the cooling timescale due to molecular hydrogen using

$$\tau_{\text{H}_2} = 48,200 \text{ yr} \left( 1 + \frac{10 T_{3}^{7/2}}{60 + T_{3}} \right)^{-1} \times \exp(\frac{512 \text{K}}{T})(f_{\text{H}_2} \tau_{\text{H},1})^{-1}.$$  

(A12)

The net radiative cooling timescale for gas, $\tau_{\text{cool}}$ is then given by

$$\tau_{\text{cool}}^{-1} = \tau_{\text{H}_2}^{-1} + \tau_{\text{other}}^{-1},$$

(A13)

where $\tau_{\text{other}}$ is the cooling timescale due to all other cooling mechanisms (e.g. atomic processes, Compton cooling).

Yoshida et al. (2003a) recommend the inclusion of a “dynamical heating” term which accounts for the accretion of new material onto a halo. This accretion is assumed to heat the gas already in the halo (as the kinetic energy of the infalling material is converted into thermal energy), reducing the rate at which it can cool. Yoshida et al. (2003a) find that the inclusion of this term in their analytic model of gas cooling in haloes produces good agreement with the results of their gas-dynamical calculations. We have experimented with including such a term in our calculations. We note, however, that our semi-analytic model already effectively includes such dynamical heating. Whenever a halo’s mass doubles we consider the halo to have reformed (Cole et al. 2000). The temperature of the gas in the halo is then recomputed, and the cooling clock is reset to zero. It is simple to show that this results in an effective heating rate for the gas which is comparable to that supplied by the dynamical heating term of Yoshida et al. (2003a). For example, following the approach of Yoshida et al. (2003a) and taking the virial temperature of a halo to be given by

$$T = \frac{\gamma M^{2/3}}{3} (1 + z),$$

(A14)

where $\gamma$ is a normalization constant, we find by differentiation that

$$\dot{T} = 2 \frac{2}{3} \gamma \left( 1 + z \right) \left( M + M_{1/3} \frac{z}{1 + z} \right).$$

(A15)

(Note that Yoshida et al. (2003a) include only the first of the two terms on the right-hand side of this equation. However, we expect them to be of similar magnitude.) In the semi-analytic calculation, the virial temperature is also given by eqn. (A14) and is updated after each halo reformation event. Consequently, when averaged over a halo lifetime, the amount of dynamical heating in the semi-analytic model must approximately equal that proposed by Yoshida et al. (2003a).

Not surprisingly therefore, we find that the explicit inclusion or otherwise of a dynamical heating term (of the form proposed by Yoshida et al. 2003a) makes no significant difference to the results of our calculations.

While in primordial plasma $\text{H}_2$ must form in the gas phase via the $\text{H}^-$ channel described above, in enriched gas there is the possibility of $\text{H}_2$ formation on the surfaces of dust grains. While we do not include this formation mechanism in our cooling model we have checked that it makes no significant difference to the evolution of the filling factor. To do this, we used the methodology of Cazaux & Spaans (2004) to compute the ratio of $\text{H}_2$ formation rates on dust grains and in the gas phase. We then scaled the abundance of $\text{H}_2$ in our cooling model appropriately and computed a selection of our models using this revised cooling model. We found no significant change in the evolution of the $\text{H}_2$ filling factors.

### A2 Compton Cooling

The Compton cooling timescale does not explicitly depend on the gas density, but does depend on the electron abundance, and therefore on the ionization state of the gas. The electron abundance, in turn, does depend on the gas density, due to the density dependence of the electron recombination timescale.

Prior to halo formation, the electron abundance in the IGM gas is computed from our model of the ionization state of the IGM. Prior to IGM reionization, the electron abundance is low, corresponding to that left over from recombination. Post-reionization, the electron fraction becomes large as the Universe is almost entirely ionized. During and after halo formation, the electron fraction will move towards the appropriate equilibrium value as described in §A1. The Compton cooling timescale is then computed using the recombinde electron abundance from eqn.(A6), or the equilibrium electron abundance, whichever is the larger.

### A3 Filtering Mass

The filtering mass is similar to the Jeans mass, but is the relevant quantity for describing the effect of pressure on the gravitational collapse of gas in an expanding Universe with a time-varying temperature. It is defined by

$$M_p = \left( 4\pi/3 \right) \rho_0 (2\pi a/kp)^{3},$$

(A16)

where $\rho_0$ is the mean density of the Universe, $a$ is the expansion factor and...
\[
\frac{1}{k^2(t)} = \frac{1}{D(t)} \int t dt' a^2(t') \frac{\bar{D}(t') + 2H(t') \bar{D}(t')}{k^2(t')} \int_0^t \frac{dt''}{a^2(t'')}, \tag{A17}
\]
where \(D(t)\) is the linear theory growth factor, \(H(t)\) is the Hubble variable and \(k(t)\) is the Jeans wavenumber, defined in the usual way. The collapse of gas into halos with masses below \(M_\odot\) is greatly suppressed relative to the case of zero pressure.

**APPENDIX B: ESCAPE FRACTIONS**

Here, we apply simple, physical models to predict the escape fraction for each galaxy in our model. We follow Paper I and use the model of Dove & Shull (1994) (and the development thereof carried out in Paper I) to compute the amount of ionizing radiation absorbed by the gas in each galaxy. The resulting escape fractions are shown in Table B1 and in Fig. B1.

The mean escape fraction for H\textsc{i} ionizing photons for galaxies due to absorption by neutral gas are found to be a few percent. These numbers agree reasonably well with the limited observational determinations of escape fractions, e.g. Leitherer et al. (1995) find that less than 3% of ionizing photons escape from low redshift starburst galaxies, and similar results have been obtained by Tumlinson et al. (1999), while Steidel, Pettini & Adelberger (2001) estimate escape fractions \(\sim 10\%\) for Lyman-break galaxies at \(z = 3.4\). More recently, Bergvall et al. (2006) found an escape fraction of 4–10% for a local starbursting dwarf galaxy.

It should be noted that these escape fraction calculations are extremely simplified—they model the interstellar medium (ISM) as a smooth distribution of gas (a clumpy distribution could enhance the escape fraction as noted by Wood & Loeb 2000) and ignore dynamical effects (such as expanding superbubbles as considered by Dove, Shull & Ferrara 2000) which significantly lower the escaping fraction. More detailed calculations, employing hydrodynamical simulations, suggest that high redshift dwarf galaxies might have much higher escape fractions (Fujita et al. 2003). A better theoretical understanding of escape fractions is a crucial missing component of accurate calculations of reionization.

We estimate the effects of dust extinction on the ionizing radiation escape fractions by following the methods of Paper I. Briefly, we extrapolate the Milky Way extinction curve to the wavelengths corresponding to the ionization threshold of each species, \(H_\text{\tiny i}, \text{He}\text{\tiny i}, \text{He}\text{\tiny ii}\), and compute an extinction averaged over galaxy inclinations using the calculations of Ferrara et al. (1999). The results of including dust extinction on escape fractions are shown in Table B1 and in Fig. B1. It is seen that dust can cause severe (frequently an order of magnitude) attenuation in the escaping fraction, at least in this highly simplified calculation of a smooth ISM. Clearly, a more detailed analysis of the effects of dust on the escaping fraction is urgently needed if we are to gain a better understanding of how the IGM was reionized.

The effects on the ionized filling factor of using different assumptions for the escape fraction are shown in Fig. B2, for different models and different redshifts. It is interesting to note that, in the WMAP cosmologies, only the TopHeavy model in the WMAP PL cosmology is able to achieve reionization by \(z = 6\), and then only if the effects of dust on the escape fraction are ignored. In the Baugh et al. '05 the majority of ionizing photons are produced during bursts of star formation. As such, dust (assumed to be located in galaxy disks in our simple model) has little effect on the escape fraction. It should be noted that our model for the escape fraction is likely to be very unreliable in cases where most of the ionizing photons are produced in bursts rather than in quiescent disks.

**APPENDIX C: TESTS OF THE MODEL**

In this Appendix we present results of various tests of our model. We begin by comparing to the results of Paper I

| \(z\) | Cosmology | Model | No dust \(f_{\text{esc}}\) | With dust \(f_{\text{esc}}\) |
|---|---|---|---|---|
| 11 | WMAP RSI | Standard | 0.06 | 0.019 |
| 5 | WMAP RSI | Standard | 0.03 | 0.005 |
| 11 | WMAP RSI | MetFree | 0.05 | 0.012 |
| 5 | WMAP RSI | MetFree | 0.03 | 0.005 |
| 11 | WMAP RSI | TopHeavy | 0.05 | 0.006 |
| 5 | WMAP RSI | TopHeavy | 0.02 | 0.002 |
| 11 | WMAP RSI | WeakFB | 0.02 | 0.002 |
| 5 | WMAP RSI | WeakFB | 0.02 | 0.002 |
| 11 | WMAP RSI | H\text{\tiny II} | 0.06 | 0.016 |
| 5 | WMAP RSI | H\text{\tiny II} | 0.05 | 0.011 |
| 11 | WMAP RSI | H\text{\tiny II}WeakFB | 0.06 | 0.007 |
| 5 | WMAP RSI | H\text{\tiny II}WeakFB | 0.02 | 0.002 |
| 11 | WMAP RSI | H\text{\tiny II}WeakFB-VMS | 0.04 | 0.001 |
| 5 | WMAP RSI | H\text{\tiny II}WeakFB-VMS | 0.03 | 0.004 |
| 11 | WMAP PL | Standard | 0.03 | 0.005 |
| 5 | WMAP PL | Standard | 0.03 | 0.004 |
| 11 | WMAP PL | MetFree | 0.03 | 0.004 |
| 5 | WMAP PL | MetFree | 0.04 | 0.008 |
| 11 | WMAP PL | TopHeavy | 0.03 | 0.001 |
| 5 | WMAP PL | TopHeavy | 0.02 | 0.001 |
| 11 | WMAP PL | WeakFB | 0.02 | 0.001 |
| 5 | WMAP PL | WeakFB | 0.03 | 0.001 |
| 11 | WMAP PL | H\text{\tiny II} | 0.05 | 0.014 |
| 5 | WMAP PL | H\text{\tiny II} | 0.07 | 0.022 |
| 11 | WMAP PL | H\text{\tiny II}WeakFB | 0.02 | 0.001 |
| 5 | WMAP PL | H\text{\tiny II}WeakFB | 0.03 | 0.002 |
| 11 | WMAP PL | H\text{\tiny II}WeakFB-VMS | 0.02 | 0.001 |
| 5 | WMAP PL | H\text{\tiny II}WeakFB-VMS | 0.03 | 0.002 |
| 11 | Baugh et al. 05 | Standard | 0.05 | 0.036 |
| 5 | Baugh et al. 05 | Standard | 0.08 | 0.076 |
| 11 | Baugh et al. 05 | H\text{\tiny II} | 0.08 | 0.066 |
| 5 | Baugh et al. 05 | H\text{\tiny II} | 0.13 | 0.130 |
| 11 | Baugh et al. 05 | H\text{\tiny II}WeakFB | 0.06 | 0.062 |
| 5 | Baugh et al. 05 | H\text{\tiny II}WeakFB | 0.12 | 0.117 |
| 11 | Baugh et al. 05 | H\text{\tiny II}WeakFB-VMS | 0.07 | 0.069 |
| 5 | Baugh et al. 05 | H\text{\tiny II}WeakFB-VMS | 0.12 | 0.120 |
after which we check that our methodology leads to model calculations that are suitably converged.

C1 Comparison with Paper I Results

We begin by comparing our current results with those of Paper I. Our galaxy formation model, galform, has undergone significant development over that used in Paper I. As such, we expect some differences in the results obtained. To assess the degree of change in our results we have performed the same calculations as in Paper I. Specifically, we adopted the same model parameters as in that paper and computed the H\text{\textsc{i}} filling factor as a function of redshift. The models of Paper I use cosmological parameters (\(\Omega_0, \Lambda_0\)) = 0.3, 0.7 and \(\sigma_8 = 0.9\), while using similar galaxy formation prescriptions and parameters as used in this work. The Paper I models are similar to our Standard model, with major differences being a weaker feedback, \(V_{\text{hot}} = 150\,\text{km/s}\) as opposed to 200\,km/s as used here, and a simpler model for galaxy merging than used here (Paper I uses the model of Cole et al. (2000) whereas the current work uses that of Benson et al. (2004) which gives somewhat longer merger timescales). Furthermore, in Paper I a fraction of all star formation was assumed to result in the formation of brown dwarfs, such that the effective ionizing luminosity per unit mass of stars formed was reduced by a factor of 0.65.

Calculations were performed for both the low and high baryon fraction models used in Paper I (\(\Omega_b = 0.02\) and 0.04 respectively).

As can be seen in Fig. C1, differences do exist between the calculations of this work and those of Paper I. The revised modelling tends to produce slightly earlier reionization (by \(\Delta z \approx 1\)). These changes are due to several improvements and corrections made to the semi-analytic model and our calculation of ionized sphere growth made since Paper I.

C2 Tests of Model Convergence

We have examined two possible sources of numerical error in our computational method. We outline both below.

C2.1 Convergence of Ionizing Emissivity

As noted in §2 we use an iterative procedure to produce a self-consistent model of galaxy formation and IGM evolution. To be self-consistent, the ionizing emissivity as a function of redshift produced by the galaxy formation model should be identical to that used to evolve the IGM in the previous iteration. We have checked, for all models used in this work, that such agreement has in fact been met to within the accuracy of the model calculation (which is limited by the number of merger trees we are able to simulate).

C2.2 Upper Limit to Halo Masses

With finite computing resources, we can only simulate haloes with masses up to some maximum value, if we always resolve the formation histories of haloes down to the same minimum progenitor mass. Our approach is to ensure that the resolution of the merger trees used in the semi-analytic model is sufficient to resolve all haloes in which gas could cool (i.e. haloes with virial velocities above 13\,km/s for models with no \(H_2\) cooling or 1.3\,km/s for models with \(H_2\) cooling). In our standard runs, the most massive haloes we consider are a factor \(1.3 \times 10^5\) more massive than the lowest mass haloes. The contribution to the ionizing emissivity from galaxies
Figure B2. The H\textsc{ii} filling factor for all cosmologies and models for a fixed escape fraction of $f_{\text{esc}} = 1$ (blue lines) and $f_{\text{esc}}$ computed using the model described in Appendix
B both for absorption by neutral gas only (red lines), and for absorption by gas and dust (green lines).
in more massive haloes is therefore lost from our calculation. Figure C2 shows the median H\textsc{i} ionizing luminosity from haloes as a function of mass weighted by the halo mass function, \(dn/d\ln M_{\text{halo}}\). This therefore gives an indication of the relative contribution to the total ionizing luminosity from haloes of different masses. At high redshifts, the contribution to the total luminosity declines with increasing halo mass (as the halo mass function is steeply declining), so we may expect the contribution from higher mass haloes to be negligible. At lower redshifts the contribution is an increasing function of halo mass. However, at the highest masses shown the relation is quite flat and is expected to turn over at still higher masses once the exponential break in the mass function is reached.

We can estimate the effects of the non-inclusion of higher mass haloes as follows. We find that, for all models, the dependence of the mean ionizing luminosity per halo on halo mass can be well represented by a power-law.\(^{14}\) We can use this relation, combined with the known mass function of dark matter haloes to determine the filling factor we would have obtained had we been able to model haloes of arbitrarily large mass. We have checked the accuracy of this correction by repeating some of our calculations including haloes up to a factor of 10 more massive than in our standard calculations.\(^{15}\) We find that our correction does indeed overestimate the filling factor (as described above), but is typically accurate to better than 10%. The correction is smaller for higher redshifts (due to the ever more rapidly declining halo mass function found at those redshifts), and is therefore less important prior to reionization (which is the period of interest in this work).

The relation between halo mass and ionizing luminosity

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\(^{14}\) For even larger masses this relation flattens (see, for example, Figure 8 of Benson et al. (2000) which shows that, for high mass haloes, galaxy formation becomes gradually less efficient as halo mass increases), but this will merely make our estimate of the effects of ignoring high mass haloes a conservative one.

\(^{15}\) While this is possible, the computational time involved makes it impractical to do this for all of our models.
correction reaches a factor of 2–3 for models including H$_2$ cooling. In models which ignore H$_2$ lines) and for a model in which the effect of missing high mass haloes (Standard model in the WMAP RSI cosmology; blue lines) is least affected by the lack of high mass haloes and from the H$_2$ model in the WMAP RSI cosmology (green lines) which is least affected by the lack of high mass haloes and from the H$_2$ model in the WMAP RSI cosmology (green lines) in which the effect of missing the highest mass haloes is more typical. The effects of including H$_2$ cooling can be clearly seen when quoting results for the epoch of reionization we list the value computed directly from our calculations along with the value obtained after the above correction has been applied. The true epoch of reionization should then lie between these two values.

**APPENDIX D: CONDENSED FRACTIONS**

Figures D1 and D2 show condensed fraction (i.e. the mass of material condensed into the galaxy phase divided by $\Omega_0 M_{\text{halo}}/\Omega_b$) as a function of halo mass $M_{\text{halo}}$ for redshifts $z = 5$ and 10 respectively. Columns indicate the cosmology (see labels in upper row) while rows indicate the model (see labels at right-hand edge of each row). In each panel, the vertical dashed magenta line indicates the virial temperature corresponding to the feedback parameter $V_{\text{hot}}$, while the vertical dashed cyan line indicates the filtering mass. Red points indicate the condensed fraction (note that the vertical scale is logarithmic). The effects of weak feedback are also apparent, resulting in a small increase in $f_{\text{cond}}$ in low mass halos.
Figure D1. The condensed fraction, $f_{\text{cond}}$, defined as the mass of material in a halo which has condensed into the galaxy phase, divided by $\Omega_b M_{\text{halo}}/\Omega_0$ is shown by red circles. Results are shown for $z = 5$ and for all three cosmological models (one per column as indicated by the labels in the upper row of panels), and for all models computed (one model per row as indicated at the right-hand edge of each row). The upper x-axis indicates the virial temperature corresponding to halo mass. Vertical dashed magenta lines indicate the virial temperature corresponding to the feedback parameter $V_{\text{hot}}$ in each model, while vertical cyan lines indicate the filtering mass in each model.
Figure D2. As Fig. D1 but for $z = 10$. 