Consequences of Broken Time-Reversal Symmetry in Triplet Josephson Junctions

P M R Brydon¹, C Iniotakis², Dirk Manske¹ and M Sigrist²

¹ Max-Planck-Institut für Festkörperforschung, Heisenbergstr. 1, 70569 Stuttgart, Germany
² Institut für Theoretische Physik, ETH Zürich, CH-8093 Zürich, Switzerland

E-mail: p.brydon@fkf.mpg.de

Abstract. We examine a triplet Josephson junction where time-reversal symmetry is broken by the misalignment of the d-vectors of the triplet superconductors on either side of the junction. We demonstrate that a spontaneous magnetization of the tunneling barrier can be stabilized, with moment aligned along the direction mutually perpendicular to the two d-vectors. We then find that the free energy minimum lies at a phase difference intermediate between 0 and π, permitting the existence of fractional flux quanta at the junction barrier.

1. Introduction

The physics of tunneling between superconductors in Josephson junctions is of continual interest, not only as a phase-sensitive test of the order parameter symmetry in unconventional superconductors, but also in device applications [1, 2]. Although the electronic structure of the superconductors is strongly modified close to the junction interface (e.g. the formation of subgap bound states [1]), the properties of the barrier itself are regarded as independent of the bulk superconductors [3]. In this work, we describe a scenario where this fundamental assumption fails. By appropriately arranging two triplet superconductors on either side of a barrier so that time-reversal symmetry is broken, we find that the interface material can develop a spontaneous magnetic moment. This magnetic transition dramatically alters the properties of the junction, implying an exotic Josephson state distinguished by the existence of fractional flux quanta at the barrier [4].

2. Ginzburg-Landau Theory

We consider a junction between two unitary, equal-spin-pairing triplet superconductors. The d-vectors of the superconductors on the left and the right of the tunneling barrier are given by \(d_L\) and \(d_R\) respectively. We assume that time-reversal symmetry is broken at the junction by the mis-alignment of the left and right d-vectors, i.e. \(d_L \times d_R \neq 0\) (see for example Fig. (1)). We also allow a magnetic moment \(M\) at the barrier, although we assume that a magnetic state is not favourable in the absence of the superconductors and so we have a positive susceptibility of the tunneling barrier \(\chi > 0\). The free energy \(F\) of the junction can then be written as the sum of the magnetic and Josephson free energies. Keeping the lowest order terms in \(M\) of both these contributions, we obtain the total free energy

\[
F = \frac{|M|^2}{2\chi} - 2i d_L \cdot d_R \cos(\phi) + 2\gamma M \cdot (d_L \times d_R) \sin(\phi)
\] (1)
where $t$ and $\gamma$ are phenomenological constants. We find a linear term in $M$ when $d_L \times d_R \neq 0$, implying that a magnetic moment $M \perp d_L, d_R$ is stabilized by the Josephson coupling. Furthermore, since this term is also proportional to $\sin(\phi)$, the free energy minimum of the junction lies at a phase difference $\phi \neq 0, \pi$. This implies that the junction is in a so-called fractional state [4].

The description of the junction as “fractional” comes from the possible existence at the barrier of flux quanta which are not integer multiples of the flux quantum $\Phi_0 = hc/2e$. We propose the following experiment to observe this exotic behaviour: consider a tunneling barrier between two triplet superconductors with mis-aligned $d$-vectors, such that the tunneling barrier consists of a magnetic and a non-magnetic region, as shown schematically in Fig. (2). Let the phase difference across the magnetic region be $0 < \phi_m < \pi$, while the phase difference across the non-magnetic region is 0. Say we have a magnetic flux line trapped at the interface of these two materials. If we take a line integral along the contour $C$, we find that the enclosed flux $\Phi$ is given by

$$\Phi = \Phi_0 = n + \oint_C ds \cdot \nabla \phi = n + \frac{\phi_m}{2\pi}, \quad n \in \mathbb{Z}$$

Experimentally, such a fractional flux quantum could be detected by local magnetic probes like scanning SQUID microscopy. Fractional flux quanta have also been proposed to occur at grain boundaries in $d$-wave superconductors and domain walls in noncentrosymmetric superconductors [4, 5].

3. Microscopic Model

The details of the magnetic instability described above depend crucially upon the parameters $t$ and $\gamma$. It is therefore of interest to apply this theory to a specific microscopic model of a triplet superconductor junction with mis-aligned $d$-vectors on either side of the tunneling barrier.
3.1. Theoretical Description

A schematic diagram of our junction is shown in Fig. (1); it is described by the Hamiltonian

\[ H(z, z') = \int \mathcal{H}(z, z')dzdz' \]

with Hamiltonian density

\[ \mathcal{H}(z, z') = \sum_{\sigma} \psi_\sigma^\dagger(z') \delta(z' - z) \left[ \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} - \mu + U_P \delta(z) \right) \delta_{\sigma\sigma'} - \delta(z) M \hat{\sigma}_\sigma^\dagger \right] \psi_{\sigma'}(z') + \frac{1}{2} \Delta(z, z') \left\{ \sin(\eta_p) \left[ \psi_\uparrow^\dagger(z') \psi_\downarrow^\dagger(z) - \psi_\downarrow^\dagger(z') \psi_\uparrow^\dagger(z) \right] - \cos(\eta_p) \left[ \psi_\uparrow^\dagger(z') \psi_\downarrow^\dagger(z) + \psi_\downarrow^\dagger(z') \psi_\uparrow^\dagger(z) \right] + \text{H.c.} \right\} \]  

(3)

where \( \psi_\sigma^\dagger(z) \) (\( \psi_\sigma(z) \)) is the fermionic creation (annihilation) operator for a spin-\( \sigma \) particle with co-ordinate \( z \). The quasiparticles have effective mass \( m \) and chemical potential \( \mu \) in the superconductors. The superconducting gap is antisymmetric with respect to particle interchange, i.e. \( \Delta(z, z') = -\Delta(z', z) \). We assume \( p_y \) pairing orbitals in both superconductors and equal gaps displaying BCS temperature-dependence. A phase difference \( \phi \) is assumed between the two condensates. In the interests of simplicity, we neglect the spatial variation of the gaps within the superconducting regions. The \( \mathbf{d} \)-vectors of each superconductor lie within the \( x,z \) plane, parameterized by the angle \( \eta_{\sigma=L,R} \). On the right hand side, the \( \mathbf{d} \)-vector is inclined at an angle \( \eta_R = \eta \) to the \( z \)-axis, whereas the \( \mathbf{d} \)-vector on the left makes the angle \( \eta_L = \pi - \eta \) to the \( z \)-axis. The orientations of the \( \mathbf{d} \)-vectors discussed here could be realized at a break discontinuity in a crystal, see the inset of Fig. (1) [6]. As the two \( \mathbf{d} \)-vectors have no \( y \)-component, any induced magnetic moment will lie along the \( y \)-axis. At the barrier, we therefore allow potential scattering by \( U_P \) and magnetic scattering by the moment \( \mathbf{M} = M \hat{\mathbf{y}} \); \( \hat{\sigma}_\sigma^\dagger \) is the \( \sigma \) Pauli matrix. In what follows, we set \( \hbar = 1 \) and quote values of \( M \) and \( U_P \) in units of \( m/k_F \) where \( k_F \) is the Fermi momentum.

We diagonalize the Hamiltonian Eq. (3) in order to obtain the Andreev bound state energies. Working within the quantization basis where the \( y \)-component of spin is a good quantum number \( \sigma \), we obtain the spin-dependent Andreev bound states

\[ E_\sigma = \frac{|\Delta(\varphi, \vartheta)|}{\sqrt{2}} \sqrt{1 + T_\sigma^2(\vartheta)} \left[ \cos(\varphi - 2\sigma\eta) - 1 \right] \]  

(4)

where \( \varphi \) and \( \vartheta \) are respectively the azimuthal and polar angles describing the position on the spherical Fermi surface, \( T_\sigma(\vartheta) = \sqrt{\cos^2(\vartheta) + (M - \sigma U_P)^2 + \cos^2(\vartheta)} \) is the transparency of the barrier in the spin-\( \sigma \) channel, and \( \Delta(\varphi, \vartheta) = \Delta(T) \sin(\varphi) \sin(\vartheta) \) is the \( p_y \) gap with \( \Delta(T = 0) = \Delta_0 \). We then proceed to calculate the total free energy \( F \) of the junction using the full expression for the electronic free energy

\[ F = \frac{|\mathbf{M}|^2}{2\chi} - \frac{1}{\beta\pi} \int d\Omega \cos(\vartheta) \sum_\sigma \log(2 \cosh(\beta E_\sigma/2)) \]  

(5)

where the integration is performed over half the Fermi sphere. The free energy is then minimized with respect to \( M \) and \( \phi \) to obtain the stable values of the magnetization and phase difference.

3.2. Results

We find that the free energy displays a global minimum at \( M \neq 0 \), \( 0 < \phi < \pi \) over a wide region in the parameter space defined by \( \eta, U_P, \chi \) and \( T \). The maximum temperature \( T_M \) at which the junction displays a magnetic instability is always less than \( T_L \). In Figs. (3)(a) and (b) we respectively show typical results for the induced magnetization and stable phase difference as a
The induced magnetic moment as a fraction of $U_p$ (a) and the stable phase difference (b) as a function of temperature for different values of $\chi$. We take $\eta = 0.2\pi$ and $U_p = 0.7$.

function of $T$, obtained by minimization of Eq. (5). As expected, both $T_M$ and the maximum magnetization at $T = 0$ increase with increasing $\chi$. For given $\eta$, $U_P$ and $T$, there is a critical value of $\chi > 0$ below which a magnetic instability is forbidden. Although both the magnetization and the stable phase difference display monotonic dependence on $T$, this is specific to the junction geometry adopted.

4. Conclusions
Using general arguments, we have demonstrated that an insulating tunneling barrier between two unitary, equal-spin-pairing triplet superconductors can display a magnetic instability if the $d$-vectors of the two triplet superconductors are mis-aligned. We have shown that this stabilizes the junction in an exotic fractional state. We proposed an experimental setup according to Fig. 2, for which these characteristic fractional flux quanta could be detected, e.g. by scanning SQUID microscopy. We have also examined the appearance of a magnetic instability of the tunneling barrier for a specific microscopic model of a Josephson junction. Within this model we find that a spontaneous magnetization is stable over a wide region of parameter space.

Acknowledgments
PMRB gratefully acknowledges the support and hospitality of A. Simon. A part of this work was financially supported by the Swiss Nationalfonds and the NCCR MaNEP, as well as the Center for Theoretical Studies of ETH Zurich.

References
[1] Kashiwaya S and Tanaka Y 2000 Rep. Prog. Phys. 63 1641-1724.
[2] Tsuei C C and Kirtley J R 2000 Rev. Mod. Phys. 72 969-1016.
[3] Zaitzev A V 1984 Sov. Phys. JETP 59 1015-1024.
[4] Sigrist M, Bailey D B and Laughlin R B 1995 Phys. Rev. Lett. 74 3249-3252.
[5] Iniotakis C, Fujimoto S and Sigrist M 2008 J. Phys. Soc. Japan 77 083701.
[6] Asano Y 2006 Phys. Rev. B 74 220501(R).