I. INTRODUCTION

The superconducting phase transition temperature $T_c$ of the Bardeen-Cooper-Schrieffer (BCS) superconductors is typically of the order of $\sim 10^{-4}T_F$, where $T_F \sim 10^4\text{--}10^5$ K is the Fermi temperature [1,2]. This follows from the small density of states at the Fermi energy

$$n(0) \approx \frac{1.76k_BT}{\hbar\omega_P} \exp(-1/VN_0).$$

For example, for $T_F = 10$ K, $\Delta(0) \approx 1.5$ meV $\ll E_F \sim 1$--$10$ eV. Here $\omega_P$ is the Debye frequency, $V$ is the attractive pairing potential, and $N_0$ is the density of states at the Fermi energy $E_F$. $N_0$ has an exponential role in determining the $T_c$ which is often estimated from the semphenomenological McMillan equation [3,4]:

$$T_c \approx \frac{2\hbar\omega_P}{1.76k_B} \exp\left[\frac{-1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)}\right],$$

where $\lambda$ is the electron-phonon coupling parameter and $\mu^*$ is the screened Coulomb interaction constant. The weak-coupling BCS formula can be recovered by replacing $(\lambda - \mu^*)$ with the product $VN_0$ in the limit of small $\lambda \ll 1$. This relation was successfully applied for many intermetallic compounds with a typical carrier density $n \sim 10^{22}$ cm$^{-3}$ [5].

However, discovery of superconductivity in materials with a poor metallic normal state with $n \sim 10^{17}$--$10^{19}$ cm$^{-3}$ challenged the conventional approach. Such low $n$ superconductors include elemental bismuth [6], SrTiO$_3$ [7], and half-Heusler compound $RT$Bi ($R$=rare earth, $T$=Pd or Pt) [8–11]. The observed $T_c$’s (and often critical fields) in these materials are by orders of magnitude higher than the expected $T_c$ from the McMillan formula [12]. These low $n$ superconductors have naturally much higher values of the ratio $T_c/T_F$, pushing them closer to the Bose-Einstein condensation (BEC) regime in which the spatial range of attractive interaction in the Cooper pair, the superconducting coherence length, $\xi$, becomes comparable to the characteristic length associated with the Fermi momentum, $\xi \sim \hbar/\pi J_F$. In YPtBi the $k_F \approx 0.4$ nm$^{-1}$, whereas in SrPd$_2$Ge$_2$ with a similar $T_c$ it is $k_F \approx 10$ nm$^{-1}$. There are materials believed to be in the BCS-BEC crossover regime, notably Fe$_{1-x}$Si$_x$ [13].

Determination of the Cooper-pair density $n_s$ is required to confirm the unusual low $n$ nature of superconductivity in the material of interest. Traditionally, the normal state electronic concentration $n$ is used to estimate $n_s$ by using a simple relation $n_s = n/2$. While the relation usually holds in the normal metals, accurate measurements of $n$ in the normal state with $E_F \ll 1$ eV can be challenging due to strong temperature dependence of $n$ at low temperatures, anomalous Hall effect, and the presence of the surface states. Here we probe $n_s$ directly in the superconducting state of YPtBi by determining the theoretical critical current density $j_c$, the quantity directly proportional to $n_s$.

The half-Heusler compound YPtBi is a topological semimetal with $n \sim 10^{18}$ cm$^{-3}$ at low temperatures [9,14]. Its superconductivity is attracting considerable attention because $T_c$ is about fourfold higher than that of doped SrTiO$_3$ with a similar $n \sim 10^{18}$ cm$^{-3}$, and it was suggested that its superconductivity arises from the $j = 3/2$ Fermi surface [15]. The possible superconducting states include unprecedented spin-quintet and septet states [15,16]. The topological normal state is driven by strong spin-orbit coupling that inverts the $s$-orbital derived $\Gamma_6$ band and the $p$-orbital derived $\Gamma_8$ band [15]. The chemical potential lies about 35 meV below a quadratically
touching point of $\Gamma_8$ bands [9,15] due to naturally occurring crystal imperfection [17].

Recent experimental results support unconventional superconductivity in YPtBi. $T_c$ can be enhanced by physical pressure with an initial linear rate of 0.044 K/GPa [18]. The upper critical field at zero temperature is $\mu_0H_{c2}(0) = 1.5$ T [9], which is higher than the Pauli limiting field 1.4 T [9] for a weak-coupling spin-1/2 singlet superconductor. The temperature dependence of $H_{c2}(T)$ is practically linear over almost an entire superconducting temperature range; quite different from conventional parabolic behavior [9]. A muon spin rotation study determined $\lambda_L(0) = 1.6 \mu\text{m}$ [19], which is an order of magnitude greater than that of the strong type-II superconductor CeCoIn$_5$ where $\lambda(0) \approx 0.26 \mu\text{m}$ [20]. Coherence length at zero temperature is $\xi(0) = \sqrt{\Phi_0/2\pi H_{c2}(0)} \approx 15$ nm. The Ginzburg-Landau parameter is $\kappa = \lambda_L(0)/\xi(0) \approx 10^2 \gg 1/\sqrt{2}$, placing it in the strong type-II regime of superconductivity.

In the mixed state of a type-II superconductor, the small-amplitude AC magnetic penetration depth is governed by the elastic properties of the Abrikosov vortex lattice in the linear response regime. This means that the amplitude of the AC field excitation, $H_{AC}$, is not large enough to displace the vortex out of the pinning potential well, and it only perturbs the vortex position within the validity of Hooke’s law. In this case, the penetration depth is described by the Campbell penetration depth $\lambda_C$ that determines the attenuation range of the AC perturbation from the sample surface to the interior, $B_{AC}(x) \propto \mu_0H_{AC}e^{-x/\lambda_C}$ in a semi-infinite superconductor [21–27]. Here $\mu$ is magnetic permeability, and $x$ is the distance from the surface. Since $\lambda_C$ is not commonly measured due to amplitude/sensitivity limitations of the conventional AC techniques, we provide a simple derivation of $\lambda_C$ in the Appendix for completeness. The important advantage of employing $\lambda_C$ is that it gives access to the shielding current density via a relation $j_C = H_0r_p/\lambda^2_C$. Here $r_p$ is the radius of the pinning potential, and $H_0$ is the applied external DC magnetic field (see the Appendix for details). Importantly, the critical current density is estimated at the frequency of the measurement, and the rf regime gives access to almost unrelaxed values. The initial vortex relaxation is exponential, and hence the conventional techniques estimate relaxed values far from the true $j_c$ [28–30].

While analysis of the relaxed shielding current is complicated because of inclusion of the magnetic relaxation parameters, the unrelaxed critical current, $j_c = H_0r_p/\lambda^2_C$, offers direct access to the superfluid density $n_s \sim j_c$ [2,31,32]. In this work, we use a tunnel diode oscillator (TDO) technique to measure $\lambda_C(T, H)$ and determine $j_c(T, H)$ in YPtBi. The determined $j_c$ is orders of magnitude smaller than that of well-known superconductors with the typical carrier density, and its rapid suppression by the magnetic field provides valuable insight into the fascinating nature of superconductivity at the low carrier density regime exemplified by YPtBi.

II. EXPERIMENT

YPtBi single crystals were grown out of molten Bi with starting composition Y:Pt:Bi = 1:1:20 (atomic ratio) [9,15,33,34]. The starting materials Y ingot (99.5%), Pt powder (99.95%), and Bi chunk (99.999%) were put into an alumina crucible, and the crucible was sealed inside an evacuated quartz ampule. The ampule was heated slowly to 1150 °C, kept for 10 h, and then cooled down to 500 °C at a rate of 3 °C/h, where the excess of molten Bi was decanted by centrifugation.

The variation of the rf magnetic penetration depth $\Delta\lambda_m$ was measured in a dilution refrigerator by using a TDO technique [35] (for review, see Refs. [36,37]). The sample with dimensions $(0.29 \times 0.69 \times 0.24)$ mm$^3$ positioned with the shortest direction along $H_{DC}$ was mounted on a sapphire rod and inserted into a 2-mm inner diameter copper coil that (when empty) produces rf excitation field with amplitude $H_{AC} \approx 20$ mOe and frequency of $f_0 \approx 22$ MHz. The shift of the resonant frequency (in cgs units), $\Delta f(T) = -G4\pi \chi(T)$, where $\chi(T)$ is the differential magnetic susceptibility, $G \approx f_0V_c/2V_i(1 - N)$ is a constant, $N$ is the effective demagnetization factor, $V_i$ is the sample volume, and $V_c$ is the coil volume [36]. The constant $G$ was determined from the full frequency change by physically pulling the sample out of the coil. With the characteristic sample size, $R, 4\pi R = (\lambda_m/R)\tan(hR/\lambda_m) - 1$, from which $\lambda_m$ can be obtained [36].

III. RESULTS

Figure 1(a) shows temperature variation of the rf magnetic penetration depth $\lambda_m(T)$ in a single crystal of YPtBi in various applied DC magnetic fields $H_{DC}$ from 0 to 30 kOe (bottom to top). For $H_{DC} = 0$, measured $\Delta\lambda_m(T)$ is the zero field limiting London penetration depth $\Delta\lambda_L(T)$ which exhibits a sharp superconducting phase transition at $T \approx 0.8$ K. We found $\Delta\lambda_L(T) = AT^n$ where $A = 1.98 \mu\text{m/K}^n$ and $n = 1.2$ [15]. The observed exponent $n$ is consistent with the presence of line nodes in the superconducting gap and moderate impurity scattering. The large prefactor, $A \propto \lambda_L(0)/\Delta(0)$ is compatible with a low carrier density superconductor within London theory $\lambda_L(0) = (mc^2/4\pi\hbar e^2)^{1/2}$. For comparison, $A = 4-15$ Å/K is observed in $d$-wave line-nodal high-temperature cuprate superconductors [38] and 190–370 Å/K in CeCoIn$_5$ [20,39,40].

Furthermore, YPtBi exhibits very pronounced field dependence of $\lambda_m$. It is notable even at the lowest $H_{DC} = 100$ Oe which is 0.007$H_{c2}(0)$. Here we use $H_{c2}(0) = 15$ kOe taken from Ref. [9]. By measuring $\lambda_m(H, T)$ as a function of temperature in different applied fields we constructed the magnetic field temperature $H-T$ phase diagram of YPtBi. Due to the broadness of the superconducting transition, we used three different criteria for the determination of $T_c$, as illustrated in the inset of Fig. 1(b). $T_1$ was determined at the sharp maximum of $d\lambda_m/dT$ (black squares). $T_2$ was determined at the intersection of the lines through the data in the superconducting state and the normal states (blue circles). $T_3$ was determined at the onset of $\Delta\lambda_m(T)$ deviation from the normal state behavior (red up-triangles). The phase diagram from rf magnetic penetration depth data is shown in the main panel of Fig. 1(b). For reference, we show the diagram as determined from resistivity measurements by Butch et al. [9], using zero resistivity (black solid line), crossing point of linear extrapolations (blue dashes), and onset of deviation (red dash-dot).
The lines in the main panel of (b) show for reference the $H$-$T$ phase diagram as determined from the field-dependent electrical resistivity by zero resistivity (black), crossing point of linear extrapolations (blue dashes), and onset of deviation (red dash-dot) criteria [9].

FIG. 1. (a) Temperature variation of the radio-frequency magnetic penetration depth, $\Delta \lambda_m(T)$, in various applied DC magnetic fields $H_{DC}$. (b) The $H$-$T$ phase diagram constructed from $\Delta \lambda_m(T, H)$ using characteristic temperatures, $T_1$ (maximum of $d\lambda_m/dT$, black squares), $T_2$ (crossing point of linear extrapolations, blue circles), and $T_3$ (onset of deviation, red up-triangles) as shown in the inset. The $T_1$ line is close to the $T_2$ line in the present experiment.

parallel to the field. Recently, a surface-sensitive tunneling experiment on YPtBi detected energy gap spectra at higher temperatures than $T_c \approx 0.8$ K [44]. A similar signature of the superconducting phase was reported in another half-Heusler compound LuPtBi, which was attributed to the presence of Van Hove singularity near $E_F$ [45] and surface pairing states [46]. The shape of tunneling spectra in the superconducting state is inconsistent with an isotropic $s$-wave gap in both YPtBi and LuPtBi [44,45].

We focus now on the variation of $\lambda_m$ with finite $H_{DC}$ in the mixed state. The measured magnetic penetration depth satisfies the relation $\lambda_m^2 = \lambda_L^2 + \lambda_s^2$ in the approximation of a linear elastic response of a vortex lattice to a small amplitude AC perturbation $H_{AC}$ [21–23]. The TDO technique is a perfect probe for this measurement because of small frequency, $f_0 = 22$ MHz, and small amplitude of the perturbation, $H_{AC} = 20$ mOe. Since $\lambda_L(T) = \lambda_L(0) + \Delta \lambda_L(T)$ where $\lambda(0) = 1.6$ $\mu$m [19], we can readily calculate $\lambda_c(T)$ in various $H_{DC}$. However, we took a conservative approach, assuming that this approximation is valid only at temperatures below $0.5T_c$ because $\lambda(0)$ becomes comparable to the size of the sample as temperature increases towards $T_c$. The calculated $\lambda_c = \sqrt{\lambda_L^2 - \lambda_s^2}$ at various $H_{DC}$ is shown in Fig. 2(a). In all measured $H_{DC}$, $\lambda_c(T)$ shows monotonous increase with temperature as the superconductor allows more penetration of the rf field with increasing temperature.

Figure 2(b) shows the field-dependent $\lambda_c^2(H)$ at several temperatures. When critical current density does not vary much with field, we expect $\lambda_c^2(H) \sim H$ and it has been observed in most cases [41,47,48]. However, YPtBi exhibits significantly more curved $\lambda_c^2(H)$, indicating nearly logarithmic behavior at low fields. This rapid rise of $\lambda_c(H)$ at low fields is unusual but may be explained considering very large values of $\lambda_m$ leading to strong intervortex interaction due to significant overlap already in low fields.

In Fig. 3, we compare this anomalous $\lambda_c(T, H)$ in YPtBi with SrPd$_2$Ge$_2$ which has a normal carrier density and exhibits $H$-linear behavior of $\lambda_c^2(H)$. The observed power-law behavior of the Campbell penetration depth is highly unusual, and it is one of the most significant observations in this work. Typically, $\lambda_c$ varies as $H^{1/2}$, but we found that it varies as $H^{1/4}$ in YPtBi. The power-law behavior can be explained if the Labusch parameter is field dependent and varies as $H^{1/2}$, which is consistent with increasing the effective size of the pinning potential. Alternative possibilities include the anharmonic pinning potential or the breakdown of the conventional picture of Campbell penetration depth.

As noted above from the known $\lambda_c(T, H)$, one can evaluate critical current density $j_c$ via $j_c = H_{DC}\xi(0)/\lambda_c^2$ (see the Appendix for details) where $\xi(T)$ is a characteristic radius of the pinning potential, usually taken equal to the coherence length, $\xi(T) = T_c(T)$ [21,22,41,47,48]. Figure 4 shows the calculated $j_c(T)$ in YPtBi, obtained from $\lambda_c(T)$ measurements taken in minimum applied $H_{DC} = 100$ Oe. We compare $j_c(T)$ in YPtBi to the $j_c$’s determined in similar Campbell penetration depth measurements in some representative superconductors, LiFeAs [49] and SrPd$_2$Ge$_2$ [41]. The former is known as a two-band superconductor with full gaps [50], and the latter is a single-gap BCS superconductor [41]. The compared $j_c(T, H)$ in these superconductors were obtained by using the same
FIG. 2. (a) Temperature variation of the Campbell length $\lambda_C(T) = \sqrt{\lambda_2^2(T) - \lambda_1^2(T)}$ in DC magnetic fields $H_{DC}$ as indicated in the panel. (b) Isotherms of field variation of $\lambda_C^2(H)$ in YPtBi. Inset shows zoom of the low-field regime.

TDO technique. In particular, we used the same TDO setup for YPtBi and SrPd$_2$Ge$_2$.

In YPtBi, the highest $j_c(T) \approx 40$ A/cm$^2$ is observed at $T \approx 75$ mK, and $j_c(T)$ monotonically decreases with temperature. In LiFeAs, $j_c \approx 1 \times 10^6$ A/cm$^2$ at the lowest temperature, and it monotonically decreases by two orders of magnitude upon warming. In SrPd$_2$Ge$_2$, $j_c \approx 8.3 \times 10^4$ A/cm$^2$ at the lowest temperature with $H_{DC} = 200$ Oe. However, its temperature variation is nonmonotonic and exhibits a maximum at an intermediate temperature, which was attributed to a matching effect between temperature-dependent coherence length and relevant pinning length scale [41]. At a higher $H_{DC} = 4$ kOe, $j_c(T)$ recovers a monotonic decrease with increasing temperature. Even when $T_{sc}(H)$ of SrPd$_2$Ge$_2$ was reduced to 0.86 K in $H_{DC} = 4$ kOe which is close to $T_{sc}$ of YPtBi with 100 Oe, $j_c$ is still two orders of magnitude greater, and the difference gets even bigger at base temperature. It is also instructive to calculate the depairing current density at which Cooper pairs break apart reaching critical velocity, $4\pi e^{-1}j = \phi_0/(3\sqrt{3}\lambda_2\xi) \approx 1 \times 10^7$ A/cm$^2$, which is much larger than $j_c$ due to pinning, but two orders of magnitude less than in “typical” normal carrier density superconductors [28,30].

In Table I, we compare normal state Hall constants $R_H$ reported for YPtBi [9] and SrPd$_2$Ge$_2$ [51]. In both compounds the Hall resistivity $\rho_{xy}(H)$ is field linear, which enables $R_H$ definition from the slope of the curve and sample geometry. The reported $R_H$ values are $-1.6 \times 10^{-4}$ cm$^3$/C [51] and $+2.4$ cm$^3$/C [9] for SrPd$_2$Ge$_2$ and YPtBi, respectively. In the single-band Drude model, the carrier density satisfies a simple relation, $R_H = 1/ne$ where $e$ is the electron charge. The

FIG. 3. Field-dependent Campbell penetration depth $\lambda_C^2(H)$ of YPtBi (red line with solid circles) shown in comparison with conventional metal/superconductor with similar $T_c$, SrPd$_2$Ge$_2$. The data are normalized to the values determined at 0.65$H_{c2}(0)$ for clarity.

FIG. 4. Temperature variation of the theoretical critical current density $j_c(T)$ in a selection of superconductors. We show the data in YPtBi in comparison with iron-based stoichiometric clean LiFeAs, and the low-temperature conventional superconductor SrPd$_2$Ge$_2$ measured in two different magnetic fields. $T_{sc}$ stands for the superconducting transition at a given magnetic field. For YPtBi, $T_{sc} = T_i$ (see Fig. 1).
The carrier density \( n \) is responsible for the observed theoretical current density because \( j_c \propto n v_s \), where \( v_s \) is the velocity of the supercurrent [2,31,32]. Provided \( n \approx n/2 \), \( j_c \) in YPtBi is two orders of magnitude smaller than that in SrPd\(_2\)Ge\(_2\), while the normal state \( n \) in YPtBi is smaller by four orders of magnitude. Consequently, \( v_s \) in YPtBi is two orders of magnitude greater than that in SrPd\(_2\)Ge\(_2\).

In Ginzburg-Landau theory, \( j_c \) is associated with the depairing velocity \( v_s = \Delta(0)/\hbar k_F \), and since \( \Delta(0) \) values in both superconductors are of the same order, the different \( v_s \) is accounted by different \( k_F \) values in these two compounds. In SrPd\(_2\)Ge\(_2\), \( k_F \approx 10 \) nm\(^{-1} \) in free electron approximation, i.e., \( k_F = (3\pi^2n)^{1/3} \) with \( n = 2.6 \times 10^{22} \) cm\(^{-3} \) which is about two orders of magnitude greater than that in YPtBi, \( k_F = 0.37 \) nm\(^{-1} \) [15]. We note that the expression for the depairing velocity used above was originally derived in the context of the BCS theory, and it uses the concept of diminishing superconducting energy gap due to additional kinetic energy. However, this relation could be used in any type of superconductors although the magnitude of the depairing velocity cannot be determined accurately. Hence, this expression should be used with caution. We employ this expression to reconcile the orders of difference in the magnitude of the critical current density between SrPd\(_2\)Ge\(_2\) and YPtBi, and it provides a reasonable explanation about it.

There has been much effort to elucidate the unconventional superconductivity in the low carrier density superconductors including YPtBi and SrTiO\(_3\). Recently, the unexpectedly high \( T_c \) in YPtBi was explained by the electron-phonon pairing mechanism with polar optical phonon mode within the \( j = 3/2 \) Luttinger-Kohn four-band model [52]. In the similar low carrier density superconductors, the plasmonic [53] and nondiabatic [54] superconducting mechanisms were proposed in SrTiO\(_3\).

The structure of the superconducting energy gap and the symmetry of pairing interaction are prerequisites for understanding the superconducting mechanism, but low \( T_c \) in the low \( n \) superconductors makes the experimental investigation difficult. The half-Heusler compounds RTBi (\( R=\text{Y,La,Lu}; T=\text{Pt,Pd} \)) exhibit relatively high superconducting transition temperatures \( T_c \approx 1 \) K [9–11], and a nodal superconducting gap was observed in YPtBi [15]. Subsequently, various exotic pairing symmetries were proposed including nematic \( d \)-wave [55,56] and \( j=3/2 \) high-spin superconductivity [15–17,46,57,58]. In general, the high-spin superconductivity exhibits topological gap structures with the possibility of harboring the Majorana surface fluid [46,59], which makes the low carrier density superconductor RTBi a promising platform for the fault-tolerant quantum devices.

V. SUMMARY

We measured rf superconducting magnetic penetration depth in single-crystal YPtBi. The London penetration depth is consistent with the nodal superconductivity in YPtBi. In the finite DC magnetic fields, the measured Campbell penetration depth exhibits unusual subquadratic power-law behavior in the low-field range. From the variation of the Campbell penetration depth, we estimated the theoretical critical current density which is orders of magnitude smaller than that of the superconductors with a typical carrier density. Therefore, we confirmed the low carrier density nature of superconductivity in YPtBi.

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APPENDIX: ILLUSTRATION OF THE CONCEPT OF CAMPBELL LENGTH

Here we provide simple arguments behind the concept of pinning and Campbell penetration depth. This physics has been discussed multiple times in the past 80 years and is textbook material. However, we felt that it is instructive to write down the derivation with units and show step by step the flow. There is still a significant degree of confusion dealing with currents, fields, and flux in cgs and SI. There is also some confusion about the Campbell penetration depth written for a single vortex featuring single flux quanta \( \phi_0 \) vs what should be with the magnetic induction, \( B \), recognizing that this is a collective effect. This is because the critical current density is introduced via the single vortex pinning. Unfortunately, the original derivation by Campbell [21,22] is too short and too schematic to our taste and we wanted to explain everything step by step.

1. Single vortex pinning

Figure 5 shows the schematics of the vortices and pinning potential model. The vortices move along the horizontal \( x \) axis, the electrical current flows into or out of the page along the \( y \) axis, and the external magnetic field is applied along the positive vertical direction of the \( z \) axis. The displacement \( u(x) \) is the deviation of a vortex from the center of its potential well.
In equilibrium, \( u(x) = 0 \) for all vortices and their distribution is constant. Assuming a vortex along the positive \( z \) axis in a pinning potential \( U(u) \) (real units energy distance), then for a single vortex, the Lorentz force per unit length is

\[
f_L = \frac{N}{m} \dot{\phi}_0 [T \ m^2]. \tag{A1}
\]

Here, \( \phi_0 = 2.068 \times 10^{-15} \) Wb = T m². (Note, we use SI units throughout the Appendix.)

The electrical current density \( j \) along the positive \( y \) direction would push the vortices to the positive \( x \) direction with the magnitude, \( f_L = j \phi_0 \). It is usually assumed that the pinning potential is given by

\[
U(u) = \frac{1}{2} \alpha u^2 \left[ \frac{J}{m} \right], \tag{A2}
\]

where \( \alpha \) is the so-called Labusch parameter:

\[
\alpha = \frac{d^2U(u)}{du^2} \left[ \frac{1}{m^3} \right]. \tag{A3}
\]

The pinning force due to \( U(u) \) is defined by

\[
f_p = -\frac{dU(u)}{du} = -\alpha u. \tag{A4}
\]

In the presence of the electrical current, the two forces, \( f_L \) and \( f_p \), will act on a vortex in the opposite directions, i.e., \( f_L + f_p = 0 \). In this case, the critical current density, \( j_c \), is reached when the magnitudes of two forces become equal at a distance \( u = r_p \) that is called the “radius of the pinning potential.” In equilibrium, \( j_c \phi_0 = \alpha r_p \), and therefore \( j_c \) can be expressed as

\[
j_c = \frac{\alpha r_p}{\phi_0} \left[ \frac{A}{m^2} \right]. \tag{A5}
\]

The major contribution to pinning comes from the gain of free energy in the normal core volume of a vortex and, therefore, \( r_p \) is usually assumed to be equal to the superconducting coherence length \( \xi \). Of course, the pinning theory is much more complex, and the readers are referred to the excellent review articles, Refs. [23,28].

2. The Campbell penetration depth

The Campbell length \( \lambda_C \) is defined for a large number of vortices since this is a wavelike perturbation in the vortex lattice treated as an elastic medium [23,28]. In other words, \( \lambda_C \) determines how far a small-amplitude AC perturbation on the superconductor edge propagates into the vortex lattice. [21]. Let us assume a uniform distribution of vortices [e.g., after field cooling (FC)] and hence a uniform magnetic induction \( B_0 \). We apply a small AC field on the sample edge, i.e., \( B = B_0 + B_{AC} \), where \( B_{AC} \ll B_0 \). In equilibrium, the vortices are equally spaced by the distance \( d_0 \) found from the condition that each vortex carries a single flux quantum, \( \phi_0 \):

\[
d_0 = \frac{\sqrt{\phi_0}}{B_0} = \frac{45.473}{\sqrt{B_0 [T]}} \text{ [nm]}. \tag{A6}
\]

Here we assumed a square vortex lattice instead of triangular for simplicity, which does not alter the results.

Consider a one-dimensional problem (semi-infinite superconductor positioned at \( x \geq 0 \)) with a magnetic field applied along the positive \( y \) axis and electric current flowing along the positive \( x \) axis. At a distance, \( x \), from the edge, a row of vortices uniformly spaced along the \( y \) axis is displaced by \( u(x) \) from their equilibrium positions. The next row of vortices is displaced by the distance \( d_0 + u(x + d_0) - u(x) \) counted from the first row of vortices (see Fig. 5). Therefore, the distance between the vortices, \( d(x) \), satisfies

\[
d(x) = d_0 \left[ 1 + \frac{u(x + d_0) - u(x)}{d_0} \right] = d_0 \left( 1 + \frac{du}{dx} \right) \tag{A7}
\]

because \( d_0 \) is the smallest physical distance in the problem. Therefore, the magnetic induction \( B \) at the location \( x \) is given by

\[
B(x) = \frac{\phi_0}{d d_0} \frac{\phi_0}{d_0^2 (1 + \frac{du}{dx})} = B_0 \left( 1 - \frac{du}{dx} \right), \tag{A8}
\]

where we assume that \( \frac{du}{dx} \ll 1 \), which can be easily checked with the final solution for \( u(x) \). Note that if all vortices were displaced uniformly, \( u = \text{const} \), then \( B(x) \) remains unchanged.

This perturbation of \( B(x) \) corresponds to the current density from the Maxwell equation, \( \mu_0 j = \nabla \times B \). Assuming \( B = B(x) \xi \),

\[
\mu_0 j_y = -\frac{\partial B(x)}{\partial x} = B_0 \frac{d^2 u}{dx^2}. \tag{A9}
\]

The Lorentz force on vortices per unit volume is

\[
F_L = JB_0 = B_0^2 \frac{d^2 u}{\mu_0 \frac{dx^2}{}}. \tag{A10}
\]

which must be balanced by the pinning force. From the previous (single vortex) section, each vortex experiences pinning force per unit length, \( f_p = -\alpha u \), and there are approximately \( N = B_0/\phi_0 \) vortices per unit area. The total pinning force per unit volume, \( F_p \), can be written in the form

\[
F_p = NF_p = -\frac{\alpha B_0}{\phi_0} u. \tag{A11}
\]
The two forces, $F_L$ and $F_p$, balance each other in the steady state, and the characteristic penetration depth is determined from the following relation:

$$F_L + F_p = \frac{B_0^2}{\mu_0} \frac{d^2u}{dx^2} - \alpha B_0 \frac{u}{\phi_0} = 0$$  \hspace{1cm} (A12)

or

$$\lambda_C^2 \frac{d^2u}{dx^2} = u.$$  \hspace{1cm} (A13)

Here we introduced the Campbell length:

$$\lambda_C^2 = \frac{\phi_0 B_0}{\mu_0 \alpha} \left[ \frac{T^2 m^2}{\frac{H}{x}} \right] = m^2.$$  \hspace{1cm} (A14)

Note that the radius of the pinning potential, $r_p$, does not explicitly enter here. This is true only for parabolic pinning potential within the validity of Hooke’s law for vortex displacement. The nonparabolic potentials have also been considered and lead to a variety of interesting effects [24, 26, 27, 60].

The Labusch constant $\alpha$ can be evaluated from the measured $\lambda_C$ by using (A14). The solution of Eq. (A13) for $u$ is

$$u(x) = u_0 e^{-x/\lambda_C}.$$  \hspace{1cm} (A15)

Therefore the magnetic induction can be found by using (A8) as follows:

$$B(x) = B_0 \left( 1 - \frac{du}{dx} \right) = B_0 \left( 1 + \frac{u_0}{\lambda_C} e^{-x/\lambda_C} \right).$$  \hspace{1cm} (A16)

At the boundary, $x = 0$, and $B = B_0 + B_{AC}$ where

$$B_{AC} = B_0 \frac{u_0}{\lambda_C}, \quad u_0 = \lambda_C B_{AC} B_0.$$  

The displacement $u(x)$ in terms of $B_{AC}$ and $B_0$ can be written in the form

$$u(x) = \lambda_C \frac{B_{AC}}{B_0} e^{-x/\lambda_C},$$  \hspace{1cm} (A17)

and we can express $B(x)$ as

$$B(x) = B_0 \left( 1 + \frac{\lambda_C B_{AC}}{B_0} \frac{u_0}{\lambda_C} e^{-x/\lambda_C} \right) = B_0 + B_{AC} e^{-x/\lambda_C},$$  \hspace{1cm} (A18)

which is expected from the boundary conditions.

Finally, we derive a practical expression for $J_c$ in terms of $\lambda_C$ that can be experimentally determined. Using (A18),

$$\lambda_C^2 = \frac{\phi_0 B_0}{\mu_0 \alpha} = \frac{\phi_0 H}{\alpha},$$  \hspace{1cm} \(\text{a)}\)

$$\frac{\phi_0}{\alpha} = \lambda_C^2 \frac{B_0}{B_0}.$$  \hspace{1cm} (A19)

Thus, $J_c$ is related to $\lambda_C$ as follows:

$$J_c = \frac{\alpha r_p}{\phi_0} = \frac{B_0 r_p}{\mu_0 \alpha \lambda_C^2} = \frac{H_0 r_p}{\lambda_C^2}.$$  \hspace{1cm} (A19)

We use the relation (A19) to calculate the critical current density from the measured Campbell penetration depth in the main text.
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