Unsupervised Representation Learning in Deep Reinforcement Learning: A Review

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Abstract

This review addresses the problem of learning abstract representations of the measurement data in the context of Deep Reinforcement Learning (DRL). While the data are often ambiguous, high-dimensional, and complex to interpret, many dynamical systems can be effectively described by a low-dimensional set of state variables. Discovering these state variables from the data is a crucial aspect for improving the data efficiency, robustness and generalization of DRL methods, tackling the curse of dimensionality, and bringing interpretability and insights into black-box DRL. This review provides a comprehensive and complete overview of unsupervised representation learning in DRL by describing the main Deep Learning tools used for learning representations of the world, providing a systematic view of the method and principles, summarizing applications, benchmarks and evaluation strategies, and discussing open challenges and future directions.

Keywords  
Reinforcement Learning · Unsupervised Representation Learning · Dynamical Systems
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1 Introduction

Dynamic systems pervade our daily life. A pendulum swinging, a robot picking up objects, the flow of water in a river, the trend of economic costs, the spread of disease and the prediction of climate change are all examples of dynamic systems. Understanding how these systems evolve by discovering the governing equations, how they react to inputs and how they can be controlled are crucial aspects of science, and engineering.

Traditionally, governing equations are derived from physical principles, such as energy conservation laws and symmetries. However, many complex real-world dynamical systems exhibit strongly non-linear behaviours, making the discovery of the governing principles describing the evolution of the systems extremely challenging. Additionally, in many domains such as physical systems, robotics, economics, or health, we are interested in not only into discovering the governing principles but also into controlling the dynamical systems via additional sets of input variables, called interchangeably control variables or actions, such that the systems manifest the desired behaviours. When turning to control, understanding how the systems react to changes in the control variables is crucial for obtaining optimal control laws.

In the last decade, Artificial Intelligence (AI), Machine Learning (ML) and data-driven methods have paved the road for new approaches to studying, analyzing, understanding and controlling dynamical systems. While data, in the forms of measurements of the systems, are often abundant, physical laws or governing equations are not always available, as in finance and climate science. Even when present, they cannot fully capture the actual governing laws due to unknowns and uncertainties, e.g. turbulent flows. Traditional control techniques based on accurate models of the world are often unsuitable for such dynamical systems, making data-driven control methods, such as Reinforcement Learning (RL), even more appealing.

RL is the branch of ML studying a computation approach for learning optimal behaviours – control laws – from interacting with the world to achieve predefined goals. RL founds its success in the Markov Decision Processes (MDPs) theory, and on the Nature-inspired paradigm of an intelligent agent learning to act by interacting with an unknown environment and improving its behaviour based on the consequences of the actions taken. Consequences of the actions are quantified by a scalar signal called reward. RL faces the so-called curse of dimensionality derived by continuous state and action spaces and by the high-dimensionality of the observable variables. The curse of dimensionality is common to many dynamical systems, making algorithms computationally inefficient.

Inspired by the outstanding successes of Deep Learning (DL), RL has turned towards deep function approximators, e.g. neural networks, for representing the control strategies and tackling the curse of dimensionality. The use of deep neural networks in RL gave birth to Deep Reinforcement Learning (DRL). DRL has achieved outstanding successes in learning optimal policies directly from high-dimensional observations. However, neural networks’ drawbacks have also been inherited, namely the low sample efficiency, the consequent need for huge amounts of data, and the instabilities of learning.

The sample inefficiency of the methods is one of the significant drawbacks of DRL-based control. This aspect becomes more and more severe when controlling dynamical systems requiring real-world interaction – we cannot afford our robots to collide with obstacles multiple times before learning how to navigate around them properly. The sample inefficiency of DRL derives from disregarding any prior knowledge of the world and the sole use of the reward signal to optimize the neural network models. It is worth noticing that DRL does not rely on any labelled data as in the case of standard supervised learning. The reward signal assesses only the quality of the decision but does not directly indicate the best course of action. Therefore, DRL has recently shifted its focus toward the more appealing, unsupervised learning of data representations.

In real-world control problems, we often do not have access to the state variables unequivocally describing the evolution of the systems. However, we only have access to indirect, high-dimensional, and noisy measurements – observations in DRL terminology – e.g. noisy RGB images or, in general, any given sensory data. While data are often high-dimensional, many dynamical systems exhibit low-dimensional behaviours that can be effectively described by a limited number of latent variables that capture the systems’ essential properties for control and identification purposes. Given the observation stream, we want to learn low-dimensional state representations that preserve the relevant properties of the world and eventually use such representations to learn the optimal policies. Because the DRL methods rely on compact representations, the algorithms gain higher sample efficiency, generalization and robustness than end-to-end DRL. Figure depicts how to exploit unsupervised representation learning in DRL in the context of control of dynamical systems, e.g. a simple pendulum. Again, we do not usually have access to the labelled data, i.e. the actual state values.

---

1 The agent is the acting entity.
2 The environment is the world in which the agent lives.
3 Originally RL used look-up tables to store the value of each state or state-action pair.
Goal:
- Learning optimal control strategy for a dynamical system of unknown dynamics from high-dimensional observations

Challenges of state representation learning:
- Learning the underlying structures
- Encoding relevant features for control
- Preservation of policy properties

High-dimensional and noisy observations of the system (e.g., camera measurements)

Figure 1: State Representation Learning for Deep Reinforcement Learning.

Learning meaningful data representations for control without supervision is a significant and open challenge of DRL research, and it is often studied under the name of State Representation Learning (SRL) [12], [13], [14].

Our review paper aims to provide a comprehensive and complete review of the most relevant, influential, and newest trends of unsupervised state representation learning in DRL. Other surveys from the literature, e.g., [13], [12], [14], although they cover a considerable amount of work, they are pretty outdated, and they do not cover the latest discoveries in the field. Our review:

- describes the main DL tools used in DRL for learning meaningful representations,
- classifies and describes important trends by analyzing the most relevant contributions in the field,
- summarizes the applications, benchmarks and evaluation strategies crucial for the advance of the research, and
- provides an elaborate discussion on open challenges and future research directions.

We review over 100 publications on a wide variety of approaches for unsupervised learning in DRL. We include papers from relevant conferences in the field (e.g., NIPS, ICRL, ICML, ICRA, and IROS) and ArXiv, and we checked each contribution’s references and related work. We identified the relevant contributions using keywords such as: “State Representation Learning”, “Deep Reinforcement Learning”, and “Dimensionality Reduction”. The latest work included in the review is dated January 2022.

The review is organized as follows: Section 2 the background information related to MDPs and the notation are introduced together with the basic knowledge of unsupervised representation learning in DRL. Section 3 provides a gentle introduction to all the approaches used in the literature for learning meaningful representations. In contrast, Section 4 describes the methods from the literature. Section 5 introduces the advanced methods and the most prominent applications of SRL. Section 6 describes the evaluation metrics and the benchmarks. Section 7 uses our experience in the field to discuss the open challenges and the most exciting research directions. Section 8 concludes the review.
2 Background

In Section 2.1, we introduce the Markov Decision Processes (MDP) framework (Section 2.1.1), the Partially Observable Markov Decision Processes (POMDP), as the general framework for dealing with partial observability of the environment (Section 2.1.2), and the Block Markov Decision Process (BMDP) framework. In Section 2.2, we introduce the notation used in SRL and this review.

2.1 Background

2.1.1 Markov Decision Processes

The discrete time-steps agent-environment interaction scheme of RL can be studied by means of the MDPs. Formally, a (finite) MDP is a tuple \( \langle S, A, T, R \rangle \) where \( S \) is the set of states, \( A \) is the set of actions, \( T : S \times S \times A \rightarrow [0, 1] \); \( (s', s, a) \mapsto T(s', s, a) = p(s'|s, a) \) is the transition function and \( R : S \times A \rightarrow \mathbb{R} \); \( (s, a) \mapsto R(s, a) \) is the reward function.

The goal of the agent is to find the optimal stochastic policy \( \pi^*(s) \), i.e. the policy maximizing the expected cumulative reward in the long run. In RL, the agent does not know the transition function \( T \) and the reward function \( R \) and has to learn the optimal behaviour by interacting with the environment.

2.1.2 Partially Observable Markov Decision Processes

In many real-world problems, the state of the environment is not directly accessible by the agent, and the world is only observable via partial and often high-dimensional observations. Examples of partial observability include the Atari games \cite{atari} when played using pixel inputs, or a robot learning to navigate by relying only on sensory readings \cite{robot}. The MDP framework is insufficient in these cases as a single observation may not be sufficient to distinguish between two or more states unequivocally.

Formally a Partially Observable Markov Decision Process, or POMDP, is a tuple \( \langle S, A, O, T, \Omega, R \rangle \) where \( S \) is the set of unobservables states, \( A \) is the set of actions, \( O \) is the set of observations, \( T : S \times S \times A \rightarrow [0, 1] \); \( (s', s, a) \mapsto T(s', s, a) = p(s'|s, a) \) is the transition function, \( \Omega : O \times S \times A \rightarrow [0, 1] \); \( (o, s, a) \mapsto \Omega(o, s, a) = p(o|s, a) \) is the observation function and \( R : S \times A \rightarrow \mathbb{R} \); \( (s, a) \mapsto R(s, a) \) is the reward function.

To learn the optimal policy, the agent must first infer its state, given the history of observations.

In the most general case, the problem of learning state representation should be modeled using the POMDP framework. However, in some cases, e.g. in \cite{robot}, we can rely on a simplified version of such framework: the Block MDP (BMDP) \cite{blockmdp}.

2.1.3 Block MDP

In a BMDP \cite{blockmdp}, each observation has only one corresponding environment state and can be considered Markovian\footnote{A single observation contains sufficient information for determining the state.}.

Formally, we can identify a block MDP with a tuple \( \langle O, S, A, T, R, \Omega \rangle \) where \( O \) is the observation space here assumed Markovian, \( S \) is the set of unobservables states, \( A \) is the set of actions, \( T : S \times S \times A \rightarrow [0, 1] \); \( (s', s, a) \mapsto T(s', s, a) = p(s'|s, a) \) is the transition function, \( R : S \times A \rightarrow \mathbb{R} \); \( (s, a) \mapsto R(s, a) \) is the reward function, and \( \Omega : O \times S \times A \rightarrow [0, 1] \); \( (o, s, a) \mapsto \Omega(o, s, a) = p(o|s, a) \) is the observation function mapping each observation to a unique state.

The advantage of using the BMDP modelling framework is that we do not need any memory structure for learning the state representation, as single observations already contain all the information for inferring the states.

2.2 Notation

In the most general case, a state representation \( z_t \) is a function of all the history of data collected by the agent while interacting with the environment

\[
    z_t = \phi_{enc}(o_t, a_t, r_t, o_{t-1}, a_{t-1}, r_{t-1}, \ldots, o_0, a_0, r_0)
\]

where we refer to \( \phi_{enc} \) as the encoder, \( z_t \) is the latent state at time step \( t \), \( o_t, \ldots, o_0 \) indicate the observations from time step \( t \) to 0, \( a_t, \ldots, a_0 \) the actions taken and \( r_t, \ldots, r_0 \) the reward received.\footnote{In the deterministic case, we can write the policy as \( a = f(s) \).}
However, in practice, we rarely need the complete history, and only a subset of it is sufficient for inferring the state:

\[ z_t = \phi_{enc}(o_t, a_t, r_t, \ldots, o_{t-n}, a_{t-n}, r_{t-n}) \] (2)

For the sake of simplicity of the notation\(^6\) in this review, we consider the case of a BMDP, and we can write the latent state as:

\[ z_t = \phi_{enc}(o_t, a_t, r_t) \] (3)

In this review, we specifically focus on the methods relying on Deep Learning \(^7\) and neural networks for approximating the unknown function \(\phi_{enc}\).

3 Deep Learning for Learning State Representations

This section is composed of two parts. In the first part, we introduce the basic neural network architectures (Section 3.1) that can be used to approximate the mapping \(\phi_{enc}\) in Equation 1. Secondly, we show different unsupervised learning methods for learning the optimal mapping \(\phi_{enc}\) (Section 3.2).

3.1 Neural Network Architectures

3.1.1 Fully-Connected Neural Networks

*Fully-Connected neural networks* \(^7\) (FCNNs) \(^19\) lay the foundations of Deep Learning. FCNNs are a family of parametric models used to approximate any unknown function \(f : \mathcal{X} \rightarrow \mathcal{Y} ; x \mapsto f(x)\) by means of a parametric function \(\phi\) such that \(y = \phi(x; \theta)\). The neural network parameters \(\theta \in \mathbb{R}^\theta\) are optimized, via a training set of examples and gradient descent, to find the best approximation of \(f\). An example of a two-layer FNN is shown in Figure 2.

Assuming the FCNN depicted in Figure 2 with \(n\) inputs, \(h\) hidden neurons and \(m\) outputs, a set weights and biases \(\theta\), i.e. the parameters, and an activation function \(\xi(\cdot)\), we can define the contribution of the \(i^{th}\)-component of the

---

\(^6\)This can be easily extended to sequences of observations by considering \(o_t = o_{t-1}, \ldots, o_{t-n}\)

\(^7\)Sometimes called multi-layer perceptrons (MLPs) or feedforward neural networks.
input \( x_i \) to the \( k^{th} \)-component of the output \( y_k \) as:

\[
\begin{align*}
  a_j &= \sum_{i=1}^{n} w_{j1}^{(1)} x_i + w_{j0}^{(1)} \\
  q_j &= \xi(a_j) \\
  a_k &= \sum_{j=1}^{h} w_{kj}^{(2)} q_j + w_{k0}^{(2)} \\
  y_k &= \xi(a_k)
\end{align*}
\]  

(4)

If the activation function \( \xi(\cdot) \) is chosen linear, the output is a linear combination of the inputs. However, in most cases, the activation function is chosen nonlinear. FCNNs with nonlinear activations are universal function approximators [20]. Common activation functions are sigmoid, \( \tanh \), rectified linear unit (ReLU), and softmax.

Independently of the structure of the network, we need a way to learn the parameters \( \theta \), i.e. weights \( w_i \) and biases \( b_i \), of the network to best approximate the unknown target function \( f \), given a set of training samples. We can do this by means of gradient descent and back-propagation by choosing a suitable loss, or cost function \( \mathcal{L}[\cdot] \).

For example, consider the SRL problem in Supervised Learning settings, we are given a training set of input data \( x^{(1)}, \ldots, x^{(l)} \), target values \( y^{(1)}, \ldots, y^{(l)} \), and the set of predicted values \( \hat{y}^{(1)}, \ldots, \hat{y}^{(l)} \), where \( \hat{y}^{(i)} = \phi(x^{(i)}; \theta) \). We can compute the prediction errors of the neural network \( \phi \) and adjust the network parameters to minimize such errors. A commonly used loss function is the mean square error (MSE) loss over the training set, as shown in Equation (5).

\[
\min_\theta \mathcal{L}(\theta) = \frac{1}{l} \sum_{i=1}^{l} || y^{(i)} - \phi(x^{(i)}; \theta) ||^2
\]

(5)

\[
\approx \mathbb{E}_{x,y \sim \mathcal{D}} || y - \phi(x; \theta) ||^2
\]

where \( l \) is the number of data in the training set, \( y^{(i)} \) is the \( i^{th} \) target output data, \( \phi(x^{(i)}; \theta) \) is the predicted output given the input \( x^{(i)} \), and we assume the input and output data, \( x^{(1)}, \ldots, x^{(l)} \) and \( \hat{y}^{(1)}, \ldots, \hat{y}^{(l)} \) respectively, to be sampled from the empirical distribution of the training set \( \mathcal{D} \). At each training operation, we compute the gradient \( \nabla_\theta \mathcal{L}(\theta) \) concerning the network parameters and then back-propagate this gradient through each layer and neuron from output to input. Computing the expectation over the whole training dataset \( \mathbb{E}_{x,y \sim \mathcal{D}} \) is usually expensive in terms of computations and hardware resources. Thus, a random subset of data, i.e. a minibatch\(^8\) is used to train the neural networks. For more details on back-propagation and optimization methods for training deep neural networks, we refer the reader to [19].

Unfortunately, in the context of SRL, we cannot rely on the true states \( s \) and directly perform regression. However, we must use auxiliary models to learn the encoder’s parameters. We will discuss this aspect in Section 3.2.2-3.2.5.

### 3.1.2 Convolutional Neural Networks

Convolutional neural networks (CNNs) [21] are neural networks specialised in efficiently handling data with grid-like structure, e.g. images (2D pixel grids) or LiDAR readings (1D data grids). CNNs inherit their name from the mathematical operator they employ in at least one of their layers, i.e. the convolution.

For example, given a sequence of 1D noisy sensor reading measuring the position of a robot over time \( x(t) \) with \( t \in (-\infty, \infty) \), we would like to obtain a better estimate of the robot position \( x(t) \) by averaging multiple measurements over time. However, past measurements’ contribution should depend on the time instant the measurements are obtained, and older measurements should be trusted less than the newer ones. Therefore, to find a smooth estimate of the robot pose, we need a weighting function \( K(a) \) dependent on the age \( a \) of the measurements we need to apply for each time instant \( t \). The function \( K \) is usually called kernel or filter. This whole operation is called convolution and is indicated by \( * \). In general, we deal with computers and sensors providing data at discrete time steps, and the smoothed position of the robot \( s(t) \) can be expressed as:

\[
s(t) = (x * K)(t) = \sum_{a=-\infty}^{t} x(a) K(t-a)
\]

(6)

\(^8\)From now on, we indicate the loss function as \( \mathcal{L}(\theta) \) to explicitly highlight the dependency of the loss on the parameters’ vector \( \theta \).

\(^9\)The size of the minibatch \( b \) is usually way smaller than the size of the training set \( l \).
We can also use the convolution operator over multi-dimensional inputs, but in particular, we are interested in using convolutions for the special case of 2D inputs, e.g. images. Given a 2D image $I$ of size $m \times n$ and a 2D kernel $K$, the convolution operation of sliding the kernel over the image can be defined as:

$$S(i, j) = (I \ast K)(i, j) = \sum_{i=1}^{m} \sum_{j=1}^{n} I(m, n)K(i - m, j - n)$$  \hspace{1cm} (7)$$

After defining the convolution operator, we want to understand how it is used in CNNs and the advantages of FCNNs. First, unlike FCNNs that use full connectivity of each input and output neuron, CNNs are an example of sparse connectivity when the kernel size is smaller than the input data size.\(^{10}\) Convolution allows reducing the number of network parameters, the physical memory required to store it and the number of computations for computing the outputs.

In a convolutional layer of a neural network, we can identify:

- Apply convolution to generate activations (see Figure 3)
- Pass the activations through non-linear activation functions (e.g. ReLU)
- Apply pooling to change (reduce) the shape of the output of the layer.\(^{11}\)

The trainable parameters of CNNs are the elements of the kernels. These parameters can be learned with the same approach used by FCNNs by defining a loss function and adjusting the network parameters according to its gradient with respect to the parameters, i.e. via back-propagation. By learning the weights of the kernels, it is possible to identify different features of the input data, for example, edges and shapes in the case of 2D images. CNNs employ parameter sharing to reduce the number of parameters with respect to FCNNs. Multiple kernels are slid with a certain stride length\(^{12}\) across the whole data grid to extract different features. Figure 3 shows an example of convolution applied to an image. It is worth highlighting that, because of parameter sharing, CNN are shift equivariant – applying a shift transformation to the image before applying the convolution operator (see (7)) is equivalent to applying the convolution and then the shift transformation. As with FCNNs, multiple convolutional layers can be stacked together. The more the convolutional layer is close to the network’s output, the higher the level of abstraction of the learned features (e.g., edges and colours to more complex shapes).

![Figure 3: example of 2D-convolution with a kernel with size 5 \times 5 and stride 5.](image)

The most widely-studied example of using CNNs is probably the problem of classifying images. Given a training set of labelled images (e.g. a data set containing images of dogs and cats and the correct labels), the neural network has to learn to properly determine if the input images belong to one of the data classes. Therefore, the network must learn to extract each class’s representative features to distinguish among them.

In the context of SRL, we often deal with image observation. Therefore CNNs are fundamental architectures for extracting the relevant features, lightening the computational burden of FCNNs, and effectively reducing the dimensionality.\(^{10}\)

\(^{10}\)For example, images can contain thousands or millions of pixels, while the kernels have way smaller dimensions. Kernels are used because we are interested in learning to extract meaningful features in the images, such as edges or geometrical shapes, which generally occupy multiple pixels.

\(^{11}\)A common type of pooling operation, called max-pooling, outputs only the maximum value of a rectangular neighbourhood. Pooling allows progressively reducing the dimensionality of the input data layer after layer.

\(^{12}\)The stride is the parameter determining how much the kernel is shifted.
3.1.3 Recurrent Neural Networks

Recurrent neural networks (RNNs) \[22\] are the class of neural networks specialized in handling sequences of data with either fixed or variable length. RNNs can be used in different contexts, for example, to map sequences to sequences, e.g. in natural language processing, or sequences to a single output, e.g. for solving POMDP in DRL and SRL.

Similar to the case of FCNNs, we can analytically define the relation between the input vector \(x\), the hidden state vector \(h\) and the predicted output \(\hat{y}\) as:

\[
a_t = W h_{t-1} + U x_t + b \\
h_t = \xi(a_t) \\
\hat{y}_t = V h_t + c
\]

where \(W, U, V\) are the weight matrices and \(b, c\) are the bias vectors.

\[8\]

Similarly to FCNNs and CNNs, RNNs are trained by defining a loss function and by applying back-propagation. A widely-used RNN architecture is the Long-Short Term Memory (LSTM) \[23\] using a sophisticated gating mechanism for preventing the vanishing of the gradient problem that affects the simple RNN architecture.

3.1.4 Neural Networks in State Representation Learning

FCNNs, CNNs, and RNNs form the building blocks for parametrizing the unknown mapping \(\phi_{enc}\) encoding observations from the world into a low-dimensional latent state space that can be used to learn the optimal behaviour 13. The neural network architecture is chosen depending on the MDP and data structure. In Figure 5, we depict the four most used neural network architectures from the literature, in the case of MDPs, BMDPs or POMDPs 14 and if grid-structured data are used or not.

3.2 State Representation Learning Methods

In Section 3.2.1 we introduce and discuss the properties of the state representations we aim at learning 15.

The section is articulated in four main blocks:

- Section 3.2.2 we describe the approaches for SRL based on Principal Components Analysis (PCA) 24 and observation reconstruction 25, 26.
- Section 3.2.3 presents the MDP models and describes their usage as auxiliary models for encoding observations into a low-dimensional latent state space,
- Section 3.2.4 discusses the so-called Robotics Priors 27, and
- Section 3.2.5 focuses on contrastive losses 28.

13In SRL, the set of input data is none other than the observation stream \(o_t, \ldots, o_0\), we do not have access to the actual output states \(s_t, \ldots, s_0\), and the network predictions are indicated by \(z\).
14In POMDPs, time matters and to reconstruct the latent state space and capture its dynamics we need to rely on sequences of observations.
15The notation is adapted to be consistent with the one introduced in Section 2.
3.2.1 Properties of the Learned State Representations

Analogously to DL, learning compact and meaningful representation is crucial for the performance of the algorithms. In DRL, the benefits are several:

- Sample efficiency. Fast learning from limited data samples is fundamental to DRL’s progress. For example, in robotics, we have limited access to real-world data while we need to learn complicated behaviours.
- Robustness. Due to the dimensionality reduction and the embedding in lower-dimensional spaces, we are less prone to overfitting and noise.
- Generalization. By encoding and disentangling the critical factor for explaining the world from the irrelevant ones, good representations are more prone to generalize to new environments with similar features.
- Interpretability. Latent representations are often composed of interpretable features and can often be visualized in 2D or 3D plots.

Given a POMPD, the goal is to encode each sequence of high-dimensional observations into a latent state space containing all the relevant information for optimally and efficiently learning to solve the task. Here, we discuss which are the desired properties for $Z$ according to [11], [12], [13]:

1. $Z$ should be smooth. If $o_{t_1} \approx o_{t_2}$, then $\phi_{enc}(o_{t_1}) \approx \phi_{enc}(o_{t_2})$, then our representation can generalize to similar features.
2. $Z$ should be a low-dimensional manifold. The latent states are concentrated in a region of much smaller dimensionality than the observation space.
3. elements in $Z$ should have simple dependencies. The transition function between latent states should be a relatively simple transformation.
4. $Z$ should be Markovian. An RL algorithm without memory should be able to solve the problem using the state representation optimally.

\[\text{Properties of the Learned State Representations}\]

These four blocks will be a recurring theme in the rest of the review.

"Why state representations?"
5. \( Z \) should be a coherent. Consecutive observations should be mapped closer in the latent space than non-consecutive ones.

6. \( Z \) should be sufficiently expressive and contain enough information to allow the agent to learn the optimal policy.

### 3.2.2 Observation Reconstruction

Assuming a high-dimensional observation space, one can think of projecting the observation space into a lower-dimensional space. The projection must maintain all (most of) the relevant information while discarding the irrelevant ones. This process can be done by considering the covariance matrix of the data, computing the eigenvalues and projecting the dataset in the lower-dimensional space defined by the \( n \) eigenvalues with the highest variance, i.e. the eigenvalues better explaining the data. This procedure is called Principal Component Analysis (PCA) \[24\].

One of the drawbacks of PCA is that the data are linearly projected into a low-dimensional space. However, this hinders its application to cases where no linear mapping exists between observations and latent states, and a nonlinear mapping needs to be learned.

Autoencoders (AEs) \[25\] are neural networks composed of an encoder \( \phi_{enc} \), with parameters \( \theta_{enc} \), and a decoder \( \phi_{dec} \), with parameters \( \theta_{dec} \). The encoder maps the input data \( o_t \) to a low-dimensional latent representation or abstract code \( z_t = \phi_{enc}(o_t; \theta_{enc}) \), while the decoder reconstructs the input signal \( \hat{o}_t = \phi_{dec}(z_t; \theta_{dec}) \) given the latent representation \( z_t \), see Figure 6. The low-dimensionality of the latent representations is a form of regularisation for preventing the learning of the trivial identity mapping. For more information, we refer the reader to \[11\].

The AE is trained by minimizing the so-called reconstruction loss, i.e. the Mean Squared Error loss between the original input and the reconstructed one, as shown in Equation (9).

\[
\min_{\theta_{enc}, \theta_{dec}} \mathcal{L}_{AE}(\theta_{enc}, \theta_{dec})
\]

\[
\mathcal{L}_{AE}(\theta_{enc}, \theta_{dec}) = \mathbb{E}_{o_t \sim \mathcal{D}}[ || o_t - \hat{o}_t ||^2 ]
\]

\[
= \mathbb{E}_{o_t \sim \mathcal{D}}[ || o_t - \phi_{dec}(z_t; \theta_{dec}) ||^2 ]
\]

\[
= \mathbb{E}_{o_t \sim \mathcal{D}}[ || o_t - \phi_{dec}(\phi_{enc}(o_t; \theta_{enc}); \theta_{dec}) ||^2 ]
\]

where \( o_t \) is the original input observation, \( \hat{o}_t \) is its reconstruction using the AE, and \( \mathcal{D} \) is the memory buffer containing the collected samples.

In DRL, the encoder of the AE is used to learn state representations from high-dimensional observations. Inputs can be either single observations, e.g. in BMDPs, or sequences of observations, e.g. in POMDPs. The latent state variables are then used to learn policies and value functions.

In the presence of uncertainties, the AE may not be the best option and variational autoencoders (VAEs) \[26\] can be used. Similarly to the AE, a VAE also is composed of an encoder and a decoder. However, differently from the AE, the latent space is modelled as a Normal distribution \( \mathcal{N}(\mu_t, \Sigma_t) \). Consider an observation \( o_t \), this is mapped to a Gaussian distribution \( \mathcal{N}(\mu_t, \Sigma_t) \) via the encoder \( \phi_{enc} \). The latent state \( z_t \) is sampled from \( \mathcal{N}(\mu_t, \Sigma_t) \) using the so-called reparametrization trick \[26\]. Analogously to the case of an AE, \( z_t \) is then decoded via \( \phi_{dec} \) to reconstruct the input observation \( o_t \). The VAE architecture is depicted in Figure 7. The VAE loss function, shown in Equation 10, is composed of two terms: a reconstruction loss similar to the AE, for example via maximum likelihood or MSE, see Equation 9, and a regularization term to enforce a Normal distribution of the samples \( \mathcal{N}(\mu, \Sigma) \) that is closed to a

![Figure 6: Autoencoder architecture.](image-url)
Figure 7: Variational autoencoder architecture.

normalized Normal distribution $\mathcal{N}(0, I)$. This is done using the Kullbach-Leibler divergence (KL).

$$\min_{\theta_{\text{enc}}, \theta_{\text{dec}}} \mathcal{L}_{AE}(\theta_{\text{enc}}, \theta_{\text{dec}}) + \mathcal{L}_{KL}(\theta_{\text{enc}})$$

$$\mathcal{L}_{AE}(\theta_{\text{enc}}, \theta_{\text{dec}}) = \mathbb{E}_{o_t \sim \mathcal{D}}[-\log \phi_{\text{dec}}(o_t | z_t; \theta_{\text{enc}}, \theta_{\text{dec}})]$$

$$\mathcal{L}_{KL}(\theta_{\text{enc}}) = \mathbb{E}_{o_t \sim \mathcal{D}}[\text{KL}(\mathcal{N}(\mu_t, \Sigma_t) | | \mathcal{N}(0, I); \theta_{\text{enc}})]$$

A commonly used variation of the VAE framework is the $\beta$-VAE [29]. $\beta$-VAEs are special VAEs aiming to learn representations that disentangle the underlying factors of variation. This can be done by introducing a parameter $\beta$ in the loss function in Equation (10) as follows:

$$\min_{\theta_{\text{enc}}, \theta_{\text{dec}}} \mathcal{L}_{AE}(\theta_{\text{enc}}, \theta_{\text{dec}}) + \beta \mathcal{L}_{KL}(\theta_{\text{enc}})$$

where $\beta > 1$ limits the capacity of the encoder and promotes the learning of disentangled representations [18].

The major flaw of the observation-reconstruction methods using solely the observation reconstruction framework is intrinsic in the reconstruction loss itself, see Equation (9), (10), and (11). The loss does not enforce any discrimination among features relevant for the control and features that are irrelevant [19].

3.2.3 Latent MDP models

The state representations we seek to learn in DRL must include all the relevant information for learning to make optimal decisions. Therefore, learning approaches for SRL have to be able to discriminate between task-relevant and task-irrelevant features.

Instead of relying only on observation reconstruction, we can focus on encoding the relevant features for predicting the evolution of the world in the latent state space: the MDP models. The MDP models include the forward or transition model predicting the next state given the current one and the action taken, the reward model predicting the reward given the current state and action taken, and the inverse model predicting the action between pairs of consecutive states.

In the context of state representation learning, these models act on latent state variables and describe the dynamics of the systems. Because these models utilize latent variables, we refer to them as latent MDP models. Figure 8 shows an example of a latent forward model. However, good latent MDP models are generally hard to learn, especially from high-dimensional noisy inputs.

In the following paragraphs, we explicitly focus on the latent forward model, the latent reward model, and the latent inverse model using the tuples $(o_t, a_t, o_{t+1}, r_t)$ from the memory buffer $\mathcal{D}$ collecting the samples obtained by the agent during the interaction with the environment. Then, we discuss different combinations of the MDP models that lay the foundation of two important theoretical frameworks in DRL: the MDP homomorphism and the bisimulation metric.

The forward model $T$ aims to describe the system’s forward evolution by predicting the next latent state $z_{t+1}$ from the current latent state $z_t$ and action $a_t$. Depending on the context, the forward model can be either deterministic, i.e. $z_{t+1} = T(z_t, a_t)$, or stochastic [20].

$^{18}$Notice that for $\beta = 1$ we obtain the VAE formulation of [26].

$^{19}$Images may contain many features, such as background textures, that are not relevant for learning the optimal policy, yet crucial for minimizing the reconstruction loss.

$^{20}$For the sake of conciseness, we present the MDP models in deterministic settings where a deterministic encoder maps the observations to the latent variables $z_t = \phi_{\text{enc}}(o_t; \theta_{\text{enc}})$ and the latent variables are fed to deterministic latent models to predict the latent dynamics.
In deterministic settings, see Figure 8, the latent forward model $T$ maps $z_t = \phi_{enc}(o_t; \theta_{enc})$ and $a_t$ to $z_{t+1} = T(z_t, a_t; \theta_T)$. The latent forward model can be used to compute an auxiliary loss whose gradients can be used to train the encoder $\phi_{enc}$. The loss for the deterministic formulation is shown in Equation (12).

$$
\min_{\theta_{enc}, \theta_T} \mathcal{L}_T(\theta_{enc}, \theta_T) = \mathbb{E}_{o_t, a_t \sim D} [|| \tilde{z}_{t+1} - z_{t+1} ||^2]
= \mathbb{E}_{o_t, a_t, o' \sim D} [|| \tilde{z}_{t+1} - T(z_t, a_t; \theta_T, \theta_{enc}) ||^2]
= \mathbb{E}_{o_t, a_t, o' \sim D} [|| \phi_{enc}(o_{t+1}; \theta_{enc}) - T(\phi_{enc}(o_t; \theta_{enc}), a_t; \theta_T) ||^2]
$$

where the target next latent state $\tilde{z}_{t+1} = \phi_{enc}(o_{t+1}; \theta_{enc})$ is generated by encoding the next observation $o_{t+1}$ while $z_{t+1} = T(z_t, a_t; \theta_T, \theta_{enc})$ is the predicted next latent state given the current observation $o_t$ and the action taken $a_t$.

Because the target next state $\tilde{z}_{t+1}$ is not known and generated by the same encoder, the representation may collapse to trivial embeddings. To prevent this issue other auxiliary losses such as contrastive losses can be used [30], [31]. We discuss contrastive losses in Section 3.2.5.

To encode task-relevant information into the state representation, we can make use of a latent reward model $R$ to predict the reward $\hat{r}_t = R(z_t, a_t)$ of a latent state-action pair $(z_t, a_t)$ [22] see Figure 9.

The reward model utilizes the reward samples $r_t$ from the memory buffer $D$ as target values for regression. In the deterministic case, an example of reward model loss is shown in Equation (13).

$$
\min_{\theta_{enc}, \theta_R} \mathcal{L}_R(\theta_{enc}, \theta_R) = \mathbb{E}_{o_t, a_t, r_t \sim D} [|| r_t - \hat{r}_t ||^2]
= \mathbb{E}_{o_t, a_t, r_t \sim D} [|| r_t - R(z_t, a_t; \theta_R, \theta_{enc}) ||^2]
= \mathbb{E}_{o_t, a_t, r_t \sim D} [|| r_t - R(\phi_{enc}(o_t; \theta_{enc}), a_t; \theta_R) ||^2]
$$

[21] We are studying unsupervised representation learning and we do not have access to the true environment states.

[22] The reward model may be only a function of the next latent state $z_{t+1}$. 

---
where $r_t$ is the reward sample from the memory buffer $D$, and $\hat{r}_t = R(z_t, a_t; \theta_R, \theta_{enc})$ is the reward predicted by the reward model $R$, parametrized by a neural network with parameters $\theta_R$.

However, the reward model alone may not be sufficient if the reward is sparse to generate sufficiently rich gradients and consequently rich state features. Similarly to the forward model, additional losses may be used to prevent the learning of trivial embeddings [31].

Eventually, we can learn a latent inverse model $I$, again usually modelled with a fully-connected neural network, predicting the action $a_t$ connecting two consecutive latent states $z_t$ and $z_{t+1}$. The inverse model aids the encoder in learning the features that are needed for predicting the action, i.e. the features the agent can control and change. Figure 10 depicts the latent inverse model.

![Latent inverse model](image)

Figure 10: Latent inverse model. The solid line represents the forward pass of the data through the networks, while the dash lines the gradient flow.

In DRL, the action space can be either continuous or discrete, therefore the loss function used can be either a MSE loss (see Equation 14) in deterministic settings or in discrete settings, one can use the cross-entropy loss used for solving a multi-class classification problem (see Equation 15).

$$\min_{\theta_{enc}, \theta_I} \mathcal{L}_I(\theta_{enc}, \theta_I)$$

$$\mathcal{L}_I(\theta_{enc}, \theta_I) = \mathbb{E}_{o_t, a_t, o_{t+1} \sim D}[\|a_t - \hat{a}_t\|^2]$$

$$= \mathbb{E}_{o_t, a_t, o_{t+1} \sim D}[\|a_t - I(z_t, z_{t+1}; \theta_I, \theta_{enc})\|^2]$$

$$= \mathbb{E}_{o_t, a_t, o_{t+1} \sim D}[\|a_t - I(\phi_{enc}(o_t; \theta_{enc}), \phi_{enc}(o_{t+1}; \theta_{enc}); \theta_I)\|^2]$$

(14)

$$\min_{\theta_{enc}, \theta_I} \mathcal{L}_I(\theta_{enc}, \theta_I)$$

$$\mathcal{L}_I(\theta_{enc}, \theta_I) = \mathbb{E}_{o_t, a_t, o_{t+1} \sim D}[-\sum_{i \in |A|} a_i^t \log \hat{a}_i]$$

$$= \mathbb{E}_{o_t, a_t, o_{t+1} \sim D}[-\sum_{i \in |A|} a_i^t \log I(z_t, z_{t+1}; \theta_I, \theta_{enc})]$$

(15)

where $a_t$ are the action labels, and $\hat{a}_t$ is a probability value obtained by applying a softmax activation at the output of the latent inverse model $I$.

Forward, inverse, and reward models are often used and optimized jointly for better learning state representations as shown in Equation 16.

$$\min_{\theta_{enc}, \theta_T, \theta_I, \theta_R} \omega_T \mathcal{L}_T(\theta_{enc}, \theta_T) + \omega_I \mathcal{L}_I(\theta_{enc}, \theta_I) + \omega_R \mathcal{L}_R(\theta_{enc}, \theta_R)$$

(16)

where $\omega_T$, $\omega_I$, and $\omega_R$ are three scalar factors for scaling the contribution of each loss.

Similar loss functions for learning state representations for DRL are used, for example, in [32], [33], [34], and [35].

A fundamental concept in RL is the notion of MDP homomorphism [36]. The idea is to exploit the symmetries of the problem to transform a high-dimensional (in states and actions) and potentially complex MDP $\mathcal{M}$ a low-dimensional MDP $\mathcal{M}'$ that is much easier to solve. Moreover, because we learn a homomorphism between two MDPs, this

---

[31] In stochastic settings, we may want to estimate $p(a_t | z_t, z_{t+1})$. This can be done via Maximum Likelihood Estimation.
framework guarantees that the optimal policy learned for the low-dimensional MDP $\bar{M}$ can be lifted while maintaining its optimality to the original high-dimensional MDP $M$. This statement implies that once we learn the optimal policy for $\bar{M}$, this policy is the optimal policy for $M$.

Below we provide the formal definitions:

**Stochastic MDP Homomorphism:** (adapted from [36]) A stochastic MDP homomorphism $h$ from an MDP $M = \langle S, A, T, R \rangle$ to an MDP $\bar{M} = \langle \bar{Z}, \bar{A}, \bar{T}, \bar{R} \rangle$ is a tuple $(f, g_s)$, with:

- $f : S \rightarrow Z$
- $g_s : A \rightarrow \bar{A}$

such that the following identities hold:

$$\forall s, s' \in S, a \in A \quad \bar{T}(f(s')|f(s), g_s(a)) = \sum_{s'' \in [s']_f} T(s''|s, a)$$  \hspace{1cm} (17)

$$\forall s, a \in A \quad \bar{R}(f(s), g_s(a)) = R(s, a)$$  \hspace{1cm} (18)

where $[s']_f$ is the equivalence class of $s'$ under $Z$.

**Deterministic MDP Homomorphism:** (adapted from [31]): A deterministic MDP homomorphism $h$ from an MDP $M = \langle S, A, T, R \rangle$ to an MDP $\bar{M} = \langle \bar{Z}, \bar{A}, \bar{T}, \bar{R} \rangle$ is a tuple $(f, g_s)$, with:

- $f : S \rightarrow Z$
- $g_s : A \rightarrow \bar{A}$

such that the following identities hold:

$$\forall s, s' \in S, a \in A \quad T(s, a) = s' \implies \bar{T}(f(s), g_s(a)) = f(s')$$  \hspace{1cm} (19)

$$\forall s \in S, a \in A \quad \bar{R}(f(s), g_s(a)) = R(s, a)$$  \hspace{1cm} (20)

If Equation (17) and (18) (or Equation (19) and (20) in deterministic settings) are satisfied, the optimal policy $\bar{\pi}$ of the homomorphic image $\bar{M}$ can be lifted to the original MDP $M$ and we denote it as $\bar{\pi}_M$ such that for any $a \in g_s^{-1}$:

$$\bar{\pi}_M(a|s) = \pi(a|s) = \bar{\pi}(\bar{a}|f(s))/g_s^{-1}(\bar{a})$$  \hspace{1cm} (21)

where $g_s^{-1}(\bar{a})$ denotes the set of actions that have the same image $\bar{a} \in \bar{A}$.

The MDP homomorphism metric can be phrased as a loss function using latent reward and forward model to learn the state representation. The advantage of such a metric is that when minimized to zero, i.e. $L_T \rightarrow 0$ and $L_R \rightarrow 0$, we obtain a homomorphism. The policy learned using the representation is guaranteed optimal for the original MDP. Moreover, the latent state space is guaranteed to be Markovian. In Figure 11, a commutative diagram representing the MDP homomorphism is shown. However, learning forward and reward models is difficult for complex applications.

![Figure 11: Commutative diagram. Applying action $a$ in the observation space $O$ generates a transition and a reward. These transitions and rewards are equivalent to embedding the observation in $\bar{Z}$, applying the latent action $\bar{a}$ and mapping the resulting latent state back to $O$.](image)

The losses rarely converge precisely to zero, and in most cases, we end up with an approximate MDP homomorphism [37].
A closely-related metric to the MDP homomorphism that can be used for learning compact representations is the bisimulation metric \cite{40, 41}. For large MDP, we can think of partitioning the state space into sets of states with equivalent behavior, with the equivalence defined by the bisimulation relation \( E \subseteq S \times S \):

\[
  s E z \iff \forall a \ R(s, a) = R(z, a) \text{ and } \forall s' \in S/E \ p(s'|s, a) = p(s'|z, a)
\]

(22)

where \( S/E \) is the partition of \( S \) into \( E \)-equivalent subset of states. The bisimulation metric is the union of all the bisimulation relations. From \cite{22} it follows that two states are bisimilar if they have the same transition and reward and, consequently, the same value function. However, the bisimulation metric is difficult to satisfy completely and is not robust to small changes in the transition probabilities and rewards.

A lax bisimulation metric is introduced in \cite{40} to exploit symmetries of the environment by relaxing the equality of the actions in the bisimulation metric in Equation (22). We can define the lax bisimulation relation \( B \) as:

\[
  s B z \iff \forall a \exists \bar{a} \text{ such that } R(s, a) = R(z, \bar{a}) \text{ and } \forall B\text{-closed sets } X \ p(X|s, a) = p(X|z, \bar{a})
\]

(23)

Again the lax bisimulation metric is the union of the lax bisimulation relations. It is easy to see that the lax bisimulation metric is equivalent to the MDP homomorphism metric \cite{40}.

An example of a bisimulation metric in SRL is \cite{41} where a state representation is learned such that the \( l_1 \) distances correspond to bisimulation metrics. We can train the encoder \( \phi_{\text{enc}} \) by minimizing:

\[
  \min_{\theta_{\text{enc}}} \mathcal{L}_{\text{bm}}(\theta_{\text{enc}}) = \mathbb{E}_{o,a,r,o' \sim D}[||z_{t_1} - z_{t_2}||_1 - |r_{t_1} - r_{t_2}| - \gamma W_2(T(z_{t_1+1}z_{t_1}, \alpha_{t_1}), T(z_{t_2+1}z_{t_2}, \alpha_{t_2}))]
\]

(24)

where \( z_{t_1} = \phi_{\text{enc}}(o_{t_1}), z_{t_2} = \phi_{\text{enc}}(o_{t_2}) \), \( \tilde{z} \) indicates the stop of the gradients such that they do no propagate through the encoder, \( \gamma \) is a scaling factor, and \( W_2 \) is the 2-Wasserstein metric.

### 3.2.4 Priors

When the underlying MDP model is a dynamic system fully or partially governed by the laws of physics, such as in robotics, we can encode such prior physical knowledge in the form of loss functions and use them to regularize the training of the encoder \( \phi_{\text{enc}} \) such that the latent states evolve accordingly to these laws. For example, consider a mobile robot navigating based on onboard sensory readings: we know that the underlying actual state space has specific properties dictated by the laws of physics. Therefore, we can employ loss functions with shaped kernels to learn not-smooth latent spaces or the non-vicinity of two consecutive latent states connected by an action.

These loss functions, called robotics prior\cite{27}, do not rely on any auxiliary morel, e.g. forward model, and can be used to directly train the encoder, as shown in Figure \[12\].

An example of robotics prior is the so-called temporal coherence loss introduced in \cite{27} and shown in Equation (25):

\[
  \min_{\theta_{\text{enc}}} \mathcal{L}_{\text{imp}}(\theta_{\text{enc}}) = \mathbb{E}_{o,o' \sim D}||\Delta z_t||^2 \quad \mathcal{L}_{\text{imp}}(\theta_{\text{enc}}) = \mathbb{E}_{o,o' \sim D}[||z_{t+1} - z_t||^2]
\]

(25)

The temporal coherence loss enforces smooth and small changes in the learned state space between pairs of consecutive latent states. The prior knowledge introduced by this loss function is the assumption that the actual state space is smooth and continuous. This property of the state space is actual for any environment obeying, for example, physical laws, as in the case of robotics.

The temporal coherence prior alone is insufficient for learning an informative state representation. The global minimum of such loss function is reached when mapping each latent state to the zero vector. Therefore, to prevent the learning of the trivial mapping, the causality loss is introduced\cite{27} This prior, see Equation (26), utilises pairs of latent states,\cite{27}

\[\text{24}\text{Robotics priors might be seen as the precursor of the Physics Informed Neural Networks (PINNs) \cite{42}. However, while PINNs aim to solve the differential equation describing the dynamics while minimizing the regression error, the robotics priors loosely shape the latent state space according to simple physical intuitions.}\]

\[\text{25}\text{The causality loss is an example of commonly-used contrastive loss, and it is discussed in detail in Section 3.2.5.}\]
collected at different time instants $t_1$ and $t_2$, with same actions $a_{t_1}, a_{t_2}$ taken from such states, but different rewards $r_{t_1}, r_{t_2}$ received, and it is minimised when the two states predictions are pushed apart in the latent space.

$$\min_{\theta_{enc}} \mathcal{L}_{\text{cau}}(\theta_{enc})$$

$$\mathcal{L}_{\text{cau}}(\theta_{enc}) = \mathbb{E}_{o,a,r,o' \sim \mathcal{D}}[e^{-||z_{t_1} - z_{t_2}||^2} | a_{t_1} = a_{t_2}, r_{t_1} \neq r_{t_2}]$$ (26)

When the same actions $a_{t_1}, a_{t_2}$ are applied in different states, we want our state variations $\Delta z_{t_1} = z_{t_1+1} - z_{t_1}$ and $\Delta z_{t_2} = z_{t_2+1} - z_{t_2}$ to be similar in magnitude. This prior is encoded by the proportionality loss and shown in Equation (27).

$$\min_{\theta_{enc}} \mathcal{L}_{\text{pro}}(\theta_{enc})$$

$$\mathcal{L}_{\text{pro}}(\theta_{enc}) = \mathbb{E}_{o,a,o' \sim \mathcal{D}}[(||\Delta z_{t_1}|| - ||\Delta z_{t_2}||)^2 | a_{t_1} = a_{t_2}]$$ (27)

The repeatability loss, in Equation (28), enforces that state changes produced by the same action should be equal in magnitude and direction.

$$\min_{\theta_{enc}} \mathcal{L}_{\text{rep}}(\theta_{enc})$$

$$\mathcal{L}_{\text{rep}}(\theta_{enc}) = \mathbb{E}_{o,a,o' \sim \mathcal{D}}[e^{-||z_{t_1} - z_{t_2}||^2} ||\Delta z_{t_1} - \Delta z_{t_2}||^2 | a_{t_1} = a_{t_2}]$$ (28)

Eventually, to obtain an informative and coherent state representation, the encoder $\phi_{enc}$ is then usually trained by jointly minimizing the sum of multiple robotics priors losses, as shown in Equation (29).

$$\min_{\theta_{enc}} \omega_{t} \mathcal{L}_{\text{tem}}(\theta_{enc}) + \omega_{c} \mathcal{L}_{\text{cau}}(\theta_{enc}) + \omega_{p} \mathcal{L}_{\text{pro}}(\theta_{enc}) + \omega_{r} \mathcal{L}_{\text{rep}}(\theta_{enc})$$ (29)

where $\omega_{t}, \omega_{c}, \omega_{p}$, and $\omega_{r}$ are scaling factors for scaling the contribution of each loss.

While allowing efficient learning of state representation for simple robotics tasks, the robotics priors struggle in generalizing, e.g. in the presence of domain randomization, and robustness when the observations include visual distractors [43].

### 3.2.5 Contrastive Learning

Contrastive learning [28] is an unsupervised approach[26] for learning data representations by only relying on similarities and dissimilarities of data pairs. Contrastive learning has achieved impressive results in Deep Learning, and in the

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[26] Sometimes referenced as self-supervised.
most recent years, this paradigm has been used in unsupervised state representation learning more and more often. Nowadays, this is a crucial element for sample-efficient DRL [44].

The critical ingredient of contrastive learning is contrastive loss. Contrastive losses act on pairs of embeddings, and through their minimization, they push apart negative pairs, i.e. data non-belonging to the same class or non-consecutive observations in the context of DRL. Additionally, contrastive losses prevent the collapsing of the representations to trivial embeddings when no labelled data are available.

However, contrastive losses must always be combined with another loss function to effectively improve DRL algorithms’ performance.

Many contrastive losses have been proposed in the Deep Learning literature, but here we focus on the most widely used in DRL.

Contrastive Predictive Coding (CPC) was introduced in [45] for unsupervised learning of representations of future observations given the past ones. CPC has been employed in unsupervised learning of visual representation, supervised learning [46], data-efficient supervised learning [47], and DRL for sample-efficient learning of state representations [44], [48], [49].

CPC employs the so-called InfoNCE loss (Equation (30)). Optimizing InfoNCE allows the neural networks to learn (future) representations from negative representations. Given a set \(X\) of \(N\) random samples containing one positive samples from \(p(x_t|c_t)\) and \(N - 1\) negative samples from the proposal distribution \(p(x_{t+k})\), we optimize:

\[
L_{\text{InfoNCE}} = -E_X[\log \frac{f_k(x_{t+k}, c_t)}{\sum_{x_j \in X} f_k(x_j, c_t)}]
\]  

In the context of DRL, the positive example may be the actual next latent state, obtained by encoding the next observation. In contrast, the negative example may be a set of encoded randomly-sampled observations.

Another example of contrastive loss for unsupervised learning of representations is the hinge loss [50], [31], and [51].

\[
L_{\text{hinge}}(\phi_{enc}) = E_{o,o' \sim D}[\max(0, \epsilon - d(z, z'))]
\]  

where the distance \(d(.)\) between two non-consecutive embeddings \(z\) and \(z'\) and \(\epsilon\) is a scaling parameter. The minimization of \(L_{\text{hinge}}\) pushes apart non-consecutive state pairs.

Another example is the causality loss introduced by [27] again pushing apart pairs of state predictions when the same action is applied, but the reward received is different.

\[
L_{\text{cau}} = E_{o,a,r,o' \sim D}[e^{-||z_{t_1} - z_{t_2}||^2}|a_{t_1} = a_{t_2}, r_{t_1} \neq r_{t_2}]
\]  

The causality loss can be generalized by removing the conditioning on action and rewards and used to push apart non-consecutive state predictions. This loss is used in [30].

\[
L_{\text{cau}} = E_{o,o' \sim D}[e^{-||z_{t_1} - z_{t_2}||^2}]
\]  

3.3 Discussion

In this section, we briefly introduced the four major categories of SRL methods. Methods from these categories are often combined to allow the learning of better state representations.

Pure observation reconstruction-based methods tend to struggle in encoding relevant features for control as the latent code may encode features useful for reconstruction, e.g. backgrounds. However, this class of methods tends to have a more stable training procedure that hardly collapses.

Approaches relying on MDP models naturally encode environment dynamics, but they tend to be harder to train, and they often need to be coupled with contrastive losses or a decoder.

Priors allow higher sample efficiency than other methods but tend to suffer visual distractors and non-stationary environments.

Contrastive losses are always combined with other losses, e.g. forward model loss [31] or temporal difference loss [44], to improve sample efficiency and prevent representation collapsing.

In Table 1 we relate the methods introduced in the previous sections to the properties of the learned state space (see Section 3.2.1). In particular, smoothness and low-dimensionality are naturally provided by neural networks and
therefore are achieved by each method. However, obtaining simple and quasi-linear transitions between latent states or a Markovian latent state space \( Z \) is often harder in practice. The former property can be achieved by minimizing the distance between consecutive states and is related to the idea of temporal coherence. At the same time, the latter requires learning the reward and transition functions. A common trait of all methods is sufficient expressiveness, as they proved to be successful in the literature for solving various problems.

![Table 1: Relation between the SRL methods and the desired properties of the latent space \( Z \) described in Section 3.2.1.](image)

### 4 Unsupervised Representation Learning in Deep Reinforcement Learning

When defining a taxonomy diagram for DRL methods, one of the first distinctions we have to address is how the samples collected via the interaction with the environment are used. Usually, we have two options: model-free and model-based. In model-free algorithms, the samples are used to estimate value function and policy, e.g. via temporal-difference learning or policy gradient. In contrast, in model-based algorithms, the samples are used to build environment models, e.g. transition model or transition and rewards model, that are then employed to estimate value function and policy analogously to model-free DRL.

For structuring our review, see Figure 13, we utilize these two classes of DRL methods, namely model-free and model-based, to define two macro objectives of the SRL approaches:

1. Increasing the sample efficiency of model-free DRL by learning meaningful low-dimensional state representations.
2. Improving the accuracy of learned latent MDP models and consequently the quality of the policies and value functions learned in model-based DRL.

Given these two classes of methods, we review how the approaches presented in Section 3 are effectively used for improving end-to-end DRL algorithms.

#### 4.1 State Representation for Sample Efficiency in Model-free RL

This section discusses the problem of learning state representations for model-free RL. Such state representations are used to learn the value function and the policy by interacting with the environment.

##### 4.1.1 Learning State Representations with Principal Component Analysis

One of the first applications of PCA in RL was proposed by [52], where a low-dimensional state representation was learned from raw grey-scale images. Using the state representation, the RL agent was efficiently trained to solve a single level of the game Super Mario Bros®. Another example of the use of PCA for learning low-dimensional state representation was the work of [53], where the return-weighted PCA (RW-PCA) was used to learn a state representation of a continuous grid world and the simulated robot tetherball game.

In complex problems, the linear mapping learned by PCA from observations to states is insufficient for encoding all the relevant information, and commonly non-linear mappings are learned using the AE framework described in Section 3.

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27 Although we are aware of recent approaches aim at combining model-free and model-based methods and that some of the model-free SRL methods presented here could be adapted to work in model-based settings and vice versa, for the sake of simplicity, we use this explicit dichotomy in our review.

28 To explicitly maintain information relevant to the RL task, i.e. the return of the episodes.
4.1.2 Learning State Representations with Autoencoders

One of the first uses of AEs in RL was introduced in [54], where an AE is used to learn a 2D representation of a grid-world. The actual state of the environment was not available to the agent, but only grey-scale image observations corrupted by noise were. The AE not only reduced the dimensionality of the observations to a 2-dimensional state vector allowing efficient RL, but also acted as a denoiser of the images. A similar example is the work of [55], where an AE was used to learn a compact state representation from grey-scale images of a real-world inverted-pendulum in the presence of delays. The RL could quickly learn the optimal policy given the state representation concatenated to the previous action taken and the delay. The last example of AE for SRL was shown by [56] where the AE was used for learning a compressed state representation of the game VizDoom [57].

When uncertainties have to be taken into account due to the stochasticity of the underlying MDP, AEs are often replaced by the VAE framework (see Section 3).

Examples of VAEs for RL can be found in the works of [58] using a beta-VAE in combination with a Denoising AE [59] to generate target observations for the reconstruction loss, [60] using a VAE for studying the problem of continual learning in continuously changing worlds, and [61] using a beta-VAE for learning a goal-based representation from the observation of goal states.

Despite these successes, AEs and VAEs often cannot be directly used in DRL because of two major limitations:

- The reconstruction loss is often independent of the relevant features for control.
- Only the latent states are used in DRL, making the decoder an unnecessary element.

In the following sections, we discuss how to tackle these two challenges.

4.1.3 Autoencoder Regularization with Auxiliary Model

The lack of discrimination between relevant and irrelevant features for control due to the reconstruction loss translates into the lack of coherence and smoothness of the latent state space. For example, two consecutive observations that should be encoded close together in the latent space may be encoded by an AE far from each other if they present very different features. Therefore, AEs need regularization via auxiliary losses.

In this section, we first present the approaches tackling the problem of learning the state representation in an offline fashion from batches of previously collected data. In offline approaches, the SRL model is first trained, and its parameters are frozen. Only afterwards the DRL algorithm is trained given a fixed state representation.
The first example is the Deep Spatial Autoencoder introduced by [62] for learning compact state representations from RGB images in the task of learning dexterous manipulation using a robot arm. To improve the quality of the representation, next to the reconstruction loss, the authors proposed a regularization loss for keeping features from consecutive observations close to each other in the latent space. Similarly, the authors of [63] enhanced the VAE loss (see Equation (10)) with a forward model loss. Unlike other works, the forward model was defined by modelling each action with a rotational matrix with four learnable parameters.

Instead, reward and inverse models were used by [64] for improving the AE-based state representation quality and allowing transfer learning from simulation to real-world (sim2real) in a mobile robot navigation task. A VAE regularized by a linear latent forward model was employed in [33] for learning low-dimensional representations of a pendulum and a robot arm for tactile manipulation. Similarly, a non-linear forward model in combination with an AE is used by [65].

VAEs with latent forward and reward models were used by [66], [67]. The latter used the forward model and the decoder to infer an approximate metric on the learned (Riemannian) manifold. The metric was used to penalize the uncertainties in latent states and latent dynamics, and this penalization was added as an additional term in the reward functions.

The problem of imitation learning from third-person to first-person views was studied by [68]. A pair of AEs encoding third-person view observation and respective first-person view observation are regularized for disentangling view information from state space information using a permutation loss for matching third-person views to first-person views. Alternatively, one could use the DRL loss as an additional regularizer for the encoder and train the whole pipeline online as in the case of end-to-end DRL. However, this usually comes at the price of the training additional training instabilities.

Online approaches were proposed by [69] where the Double DQN (DDQN) [70] loss, reconstruction loss, reward loss, temporal coherence loss, the causality loss, and forward and inverse model losses were used. Other examples were the works of [71] where a regularized AE [72] was combined with Soft Actor-Critic (SAC) [73] and [74] where the stochastic latent variable model was used in combination with a VAE and a forward stochastic model.

In the context of multi-agent RL in partially observable settings, the authors of [75] proposed a recurrent AE combined with an Asynchronous Actor-Critic (A2C) [76] for learning the relations between the trajectory of the controlled agent and the trajectory of the modelled agent. The recurrent AE encoded sequences of observations and actions and produced a latent code that was used together with observations as input to the A2C agent.

The authors in [77] introduced a selectivity loss for specifically learning K controllable features using K independent policies. The selectivity of the features was defined as the expected difference in the representation of the next state and the current one. Selectivity can be interpreted as the reward for the control problem. The AE loss and the selectivity of the policy are jointly optimized to learn a navigation task in a grid world. Despite the success, this approach does not scale well with the size of the problem as K different policies are trained together where K can vastly grow in complex scenarios.

### 4.1.4 Learning State Representations with MDP models

In RL, we are interested in learning task-relevant and compressed representations of the observations. The latent representations are used in most cases for learning the value functions and the policies, and the observation reconstructions are rarely used in practice, except for analyzing the results and interpretability.

In this section, we review approaches that do not rely on a decoder but only on auxiliary losses using the latent MDP models.

The first important aspect is the MDP homomorphism (see Section [3]). The MDP homomorphism allows us to define a metric to train the encoder without needing a reconstruction loss and to guarantee a Markovian latent space.

In this category of approaches, we find the work of [35], where the MDP homomorphism metrics were formulated as loss functions and used as regularizers for a distributional Q-learning agent [78]. In most Atari games, the approach outperformed different baselines, such as end-to-end DRL and AE-based approaches. Another closely related example is the work of [79], which used the same metrics for continuous control of robot arms in combination with DDPG.

A closely-related concept to the MDP homomorphism is the so-called bisimulation metric. Here we find the work of [80], [81], [82], [41] combining the bisimulation metric with high-dimensional observations for learning-low dimensional representation for discrete and continuous MDPs.

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31 When the loss functions approach zero.
Minimizing Lax Bisimulation and MDP homomorphism metrics guarantees a Markovian latent space homomorphic to the true MDP in terms of transition and reward function. However, two major problems arise from using these two metrics:

- Collapsing of the state representation to a trivial one in case of sparse reward functions.
- Learning of the forward model is often hard in practice.

Traditionally in DL, the collapsing of the representations is prevented using contrastive losses pushing negative embeddings apart (see Section 3.2.5). Contrastive losses are often used in approaches that do not rely on a decoder and are more prone to representation collapse.

InfoNCE loss (see Section 3.2.5) and momentum encoders were used in combination with SAC and Proximal Policy Optimization (PPO) by CURL, distributional Q-learning by [86], and PPO by for continuous and discrete control problems. In contrast, the authors in introduced PI-SAC that, similarly to CURL, used the InfoNCE bound to capture the mutual information between two random observations as an auxiliary task for continuous control from pixels. However, differently from CURL, PI-SAC captures information about the future states. Additionally to the InfoNCE, added a forward model loss and a mutual information loss between the current latent state and the next latent state to add further structure to the state representation and improve the accuracy of the forward model.

A return-based contrastive loss is used by in combination with a state and action embedding, SAC and RainbowDQN, while uses a triplet loss for learning state representations of robot behaviours from video demonstrations in combination with the PILQR algorithm.

Instead of relying only on the forward, reward, or contrastive losses, many works proposed different combinations of multiple objectives in addition to the DRL algorithm loss.

One of the earliest methods introduced by combines the A3C loss with a reward model and a pixel control loss for learning a Q-function to change the pixel values maximally. Similarly, in the context of robot navigation, uses A3C with additional objectives for inferring depth maps from RGB images and detecting loop closure.

In, the authors combine the PoPAr-t-IMPALA with a recurrent encoder, a forward model, predicting the latent states from the history of observations, and a reverse model, where the reverse model predicts the history of observations from the latent states. IMPALA is also used by in combination with a recurrent forward model for belief estimation. combines forward and inverse models for learning to poke with the Baxter robot in the real world. In contrast, uses a goal-aware cross-entropy loss for discriminating goals in the context of multi-target DRL.

In, the authors use an inverse model loss with a mutual information loss to learn a provable Markovian state abstraction without needing forward and reward models.

4.1.5 Learning State Representations with Priors

Instead of relying on the learning of the MDP models that are often hard to learn, it is possible to shape the loss functions used to train the encoder to incorporate known properties of the actual environment state (see Section 3.2.4). The so-called robotics priors were introduced in and . These loss functions were inspired and shaped accordingly to simple physical laws in the context of robotics. An example of regularization provided by the robotics priors is the proportionality loss, placing pairs of state embeddings close to each other if generated by consecutive observation and connected by an action.

In, a new set of priors was introduced. Differently from the previous set, the authors proposed priors regularizing the positions of the state embeddings and their velocities, i.e. variations between two consecutive states, to be close to each other in consecutive states better to exploit the underlying smoothness of the physical world.

In, the authors introduced a new set of robotics priors exploiting action structures in continuous action spaces, a common scenario in robotics, while in the set of original priors from were extended to multi-task learning with the addition of a task-coherence prior pushing apart embeddings from different tasks.

In, the robustness of the original priors was tested in the presence of visual distractors and domain randomization. The authors introduced the so-called reference-point prior compensating for the degrading of the performance due to
the rapid changes in the environment background. The reference-point prior uses true state samples\(^{32}\) as references for patching together the different embeddings from different trajectories\(^{33}\).

The robotics priors have been extended to deal with POMDP by \cite{15}. To do that, the authors introduced the so-call landmark prior that extends the reference prior from \cite{43} to multiple reference points. Using multiple reference points is crucial for regularizing the different trajectories in the latent space.

The robotics priors do not use any task-specific information in the reward function. One could argue that the state representation has to incorporate information about the goal. On this line of thought, the authors of \cite{103} introduced a new set of priors enforcing similarities and dissimilarities of state pairs using the rewards collected in each state. These reward-shape priors are a natural way to incorporate task-specific knowledge in the state representation.

### 4.1.6 Learning State Representation without Auxiliary Models

Improving the quality of the state representation can be done without additional objectives. An example is the work of \cite{104} where the Behavioural Cloning (BC) \cite{105} loss was used for training the encoder. Two consecutive state embeddings were concatenated to form the vector that was fed to the DRL networks. A similar approach was used by \cite{106} where consecutive state embeddings from a stack of frames were concatenated together with their latent flows, i.e. the difference of pairs of consecutive embedding, to form the augmented state vector that was used by the DRL algorithm.

The authors of \cite{102} proposed a framework for learning a context representation using the history of image stacks and force sensor readings for learning to push objects using a robot arm in combination with Qt-Opt\(^{34}\) \cite{108}. The context representation was concatenated with the encoder features and fed to the DRL algorithm.

### 4.1.7 Discussion

In Table 2, we summarize the contribution presented in this section and we highlight the methods from Section 3 that each contribution employs.

Most of the reviewed contributions rely on the (V)AE or on the CL frameworks. AE and CL are essential elements for SRL in DRL by providing the foundation for learning different MDP models. MDP models that, when used alone, are prone to representation collapsing.

Non-SRL objectives that we name in the table "other objectives" are often included when learning the representations, especially in an online fashion. These objectives are often the DRL algorithms objectives, such as minimization of the TD error. In contrast, despite being often used, the policy objective was empirically proven to be detrimental to the learning of the state representation \cite{71}.

One could think of combining many different methods to obtain the best representation, e.g. in \cite{69}. However, training the encoder using multiple loss functions needs additional care in balancing the different terms with the consequent cancellation of some of the gradients.

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\(^{32}\)The reference-point prior makes the method weakly supervised.

\(^{33}\)Because of their structure, the robotics priors can properly match latent states from the same trajectory, but struggle in enforcing coherence of embeddings from different trajectories. This problem becomes severe in the case of domain randomization.

\(^{34}\)Q-learning for continuous state and action spaces.
| Method | Contribution | PCA | AE | VAE | PW | RW | IN | MDPH | BM | PR | CL | Other Objectives |
|--------|--------------|-----|----|-----|----|----|----|------|----|----|----|------------------|
| Curran et al. [52] |                | ✓   |    |     |    |    |    |      |    |    |    |                  |
| Paris et al. [53] |                | ✓   |    |     |    |    |    |      |    |    |    |                  |
| Lange and Kuhlmann [54] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Mutter et al. [55] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Alvernas and Fugelius [56] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Kempka et al. [57] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Higgins et al. [58] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Vincent et al. [59] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Caselles-Dupré et al. [60] |             |     |    |     |    |    |    |      |    |    |    |                  |
| Nam et al. [61] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Finn et al. [62] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Caselles-Dupré et al. [63] |             |     |    |     |    |    |    |      |    |    |    |                  |
| Ratliff et al. [64] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Van Hoof et al. [65] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Kim et al. [66] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Rahinov et al. [67] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Jennehlolz et al. [68] |             |     |    |     |    |    |    |      |    |    |    |                  |
| Ma et al. [69] |                |     |    |     |    |    |    |      |    |    |    |                  |
| De Fruin et al. [70] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Vatani et al. [71] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Lee et al. [72] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Papradakakis et al. [73] |             |     |    |     |    |    |    |      |    |    |    |                  |
| Humas et al. [74] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Gamba et al. [75] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Munk et al. [76] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Castro [77] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Mihal and Platt [78] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Kemertas and Aumentado-Armstrong [79] |           |     |    |     |    |    |    |      |    |    |    |                  |
| Zhang et al. [80] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Stoeckle et al. [81] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Laskin et al. [82] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Mazou et al. [83] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Anand et al. [84] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Lee et al. [85] |                |     |    |     |    |    |    |      |    |    |    |                  |
| You et al. [86] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Liu et al. [87] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Serman et al. [88] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Radeberg et al. [89] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Matrak et al. [90] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Gne et al. [91] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Arvada et al. [92] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Kuts et al. [93] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Allen et al. [94] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Jonaschewski and Brock [95] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Jonaschewski and Brock [96] |             |     |    |     |    |    |    |      |    |    |    |                  |
| Jonaschewski et al [97] |             |     |    |     |    |    |    |      |    |    |    |                  |
| Bregoli et al. [98] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Höfer et al. [99] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Van et al. [100] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Morik et al. [101] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Bregoli et al. [102] |              |     |    |     |    |    |    |      |    |    |    |                  |
| Marckling et al. [103] |             |     |    |     |    |    |    |      |    |    |    |                  |
| Shang et al. [104] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Xu et al. [105] |                |     |    |     |    |    |    |      |    |    |    |                  |
| Koldshimov et al. [106] |             |     |    |     |    |    |    |      |    |    |    |                  |

Table 2: Which SRL methods are used by each contribution?
4.2 State Representation for Accurate Latent Models in Model-based RL

In this section, we review the most relevant model-based DRL methods for learning predictive state representations through latent forward models in Section 4.2.1 or latent forward and reward models in Section 4.2.2.

4.2.1 Learning State Representation with Latent Forward Models

Embed to Control (E2C) [109] used a VAE for learning a low-dimensional latent state space that was used as input to a linear latent forward model for obtaining a linear reduced-order model of a system from high-dimensional observations. The linear forward model forced the encoder to encode features to linearize the latent state space dynamics. VAE and the latent forward model are jointly trained together. After learning the reduced-order model, the control was performed via model-based techniques relying on the learned model, such as iterative linear quadratic regulation (iLQR) and approximate inference control (AICO).

Similarly to E2C, we find the work of [110] in which a non-linear latent forward model was combined with an AE. Model Predictive Control (MPC) is used to control the system. Another example is the work of [111] where AE with non-linear forward dynamics proposed by [110]. The authors added a latent space consistency loss enforcing the predicted latent state to be close to the embedding of the following observation to provide further regularization to the latent state space.

The State Space Models (SSMs) [112] introduced an approach for learning deterministic and stochastic state representations for state estimation in the context of model-based RL. The deterministic SSMs (dSSMs) used an AE or a VAE in combination with a deterministic forward model, while the stochastic SSMs (sSSMs) used a VAE in combination with a stochastic forward model. Differently from many works in the literature, the stochastic forward model did not directly approximate \( z_{t+1} \sim p(z_{t+1}|z_t, a_t) \), but made use of a latent variable \( \eta_{t+1} \sim p(\eta_{t+1}|z_t, a_t, o_{t+1}) \) to predict the next latent state \( z_{t+1} = T(z_t, a_t, \eta_{t+1}) \). The approach is tested on the Atari Learning Environment (ALE) [113], and the policy is then learned using the state representation via Imagination Augmented Agent (I2A) [114]. Similar approaches to sSSMs, combining VAEs and stochastic forward models, were introduced by [115], [116], [117], and [118], but with the goal of better state estimation and the control was left as future work.

All the prior approaches relied on a vector representation of the latent states. However, encoding observations as vectors is not the only option. One could think of exploiting inductive biases and encoding the observations as nodes of a graph. Contrastive Structured World Model (C-SWM) [50] encoded the latent states as an element of a graph and used a non-linear forward model approximated by a Message Passing Graph Neural Network (MPGNN) [119], [120]. Another approach embedding observation into a graph is Plan2Vec [121], where a graph was constructed by learning the distance between the nodes, i.e. the latent states and a search algorithm is used to learn the shortest path on the graph from the current state to a goal state.

4.2.2 Learning State Representation with Latent Forward and Reward Models

Continuous control, state representation learning, and latent models learning were done by PlaNet [122] and Dreamer [123] where a recurrent VAE with stochastic latent forward and reward model was learned. The policy was learned via MPC in the first case and via actor-critic DRL in the second one. Control via MPC was employed in [124] after learning a task-agnostic state representation using an AE, a linear forward model relying on the Koopman operator [4], and a reward model.

Discrete control was done in [125] via predictive autoencoding [36] and a reward prediction model for learning to play the Atari games in a model-based fashion using PPO.

Plannable MDP homomorphisms were introduced by [31], using a similar approach to [35]. However, they employ action encoding and a contrastive hinge loss (see Section 3) to prevent the state representation from collapsing due to sparse rewards. After learning the state representation, latent forward and reward models offline, they discretized the learned models and used them for planning and learning the policy without further real-world interaction via Value Iteration (VI) [5].

A combination of model-free and model-based DRL was proposed by [30]. The state representation was learned using multiple objectives: latent forward model, discount factor prediction, contrastive loss, and DQN loss. Moreover, they introduce a loss for improving the interpretability of the learned representations.

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35 This is similar to what the temporal coherence prior, in Equation 25, does.
36 AE predicting next frame given the current one and the action.
Eventually, we find the work of [126] learning a state representation via the DQN loss, the forward, reward, and terminal condition loss. However, instead of relying on vectorized representations, they used a set-based one through a set-based encoder [127]. Moreover, the proposed the so-called consciousness bottleneck using a multi-layer self-attention to enforce sparsity of the dependencies the forward, reward, and terminal condition model could relate. The set representation and the attention mechanism allowed outstanding generalization properties of the agent when places in randomly generated and unseen mazes.

4.2.3 Discussion

Figure 13 shows an overview of the methods presented in this section. We reviewed how the SRL are used in model-free and model-based methods by identifying how the approaches presented in Section 3 are combined and exploited to improve the quality of the learned representations.

In Table 5 we summarize the most prominent SRL methods in the context of model-based DRL.

The contributions utilizing only the forward model often rely on MPC to derive the optimal controller. At the same time, when the reward model is also learned, DRL algorithms can be used directly by sampling from the latent models and by limiting the real-world interaction.

In model-based DRL, AE and VAE tend to be preferred to CL, especially when training in an online fashion encoder, models and policy. Contribution relying on a decoder often outperforms CL-based ones. An empirical comparison was provided in [122].

Stochastic models are more often employed than deterministic ones. However, the quantification of the uncertainties is understudied and either the latent model variances are kept constant with all the uncertainties quantified by the encoder, e.g. [122], [123], or when the variances of the latent models are learned no analysis on how these uncertainties can be used in control is done, e.g. [115], [116], [117], and [118].

| Method | Contribution | PCA | AE | VAE | RW | IN | MDQ | BM | PR | CL | Other Objectives |
|--------|--------------|-----|----|-----|----|----|-----|-----|----|----|------------------|
| Watter et al. [109] | | | | | | | | | | | |
| Wahlström et al. [110] | | | | | | | | | | | |
| Assael et al. [111] | | | | | | | | | | | |
| Deilin et al. [112] | | | | | | | | | | | |
| Karl et al. [113] | | | | | | | | | | | |
| Krizhevsky et al. [114] | | | | | | | | | | | |
| Pelleg et al. [115] | | | | | | | | | | | |
| Kipf et al. [50] | | | | | | | | | | | |
| Yang et al. [121] | | | | | | | | | | | |
| Hafner et al. [122] | | | | | | | | | | | |
| Hafner et al. [123] | | | | | | | | | | | |
| van der Heijden et al. [124] | | | | | | | | | | | |
| Kaiser et al. [125] | | | | | | | | | | | |
| van der Pol et al. [31] | | | | | | | | | | | |
| François-Lavet et al. [30] | | | | | | | | | | | |
| Zhao et al. [126] | | | | | | | | | | | |

Table 3: Which SRL methods is used by each contribution?

5 Advanced Methods and Applications

In this section, we provide a different dimension of the SRL methods we discussed in Section 4 see Figure 13. In particular, in Section 5.1 we focus on advanced derivations of the SRL methods that we name advanced methods for solving the SRL problem and also more general problems of DRL. In contrast, in Section 5.2 we focus on the major application domains of the SRL methods (see Figure 14).

The terminal-condition model was trained to predict the end of the episodes.

A set can be viewed as a particular case of a graph with no edges [128]. We will discuss more about Geometric Deep Learning in Section 7.
5.1 Advanced Methods

We enhance the review by studying two crucial challenges for scaling DRL to complex applications:

- Enhancing exploration skills of the DRL agents by using the latent representations to generate intrinsic rewards.
- Transforming large action spaces in lower-dimensional latent action spaces for easier policy learning.

5.1.1 State Representation for Efficient Exploration in Complex Environments

The exploration-exploitation dilemma is one of the most studied problems in RL. Efficient exploration while exploiting rewards is especially problematic in large and complex environments where random exploration or heuristics cannot guarantee sufficient coverage of underlying state space. This aspect is even more severe when the state of the environment is not available to the agent, and only high-dimensional observations are fed to the agent. Learning state representations reveal to be a crucial aspect even in the context of better exploring the world.

Curiosity-driven exploration [130], [131] founds its success in the concept of rewarding the agent for exploring unexplored areas of the world. Unexplored areas are areas without or with limited samples, therefore hard to predict using, for example, forward models. By rewarding the agent with a scalar proportional to the prediction error, we invite the agent to visit uncertain states more often and enhance its exploration skills.

Despite the plethora of work on this topic [132], in this section, we focus on a curiosity-driven exploration through learning state representations and latent MDP models to highlight further the versatility of SRL and its benefits in many faces of DRL. Encoding observations to a low dimensional space by maintaining the relevant information for exploring is again crucial here.

One of the first works combining learned state representations and latent MDP models to generate curiosity rewards is [133], where an encoder trained by an inverse model and a forward model is used to reduce the dimensionality of the observations. The inverse model is used to focus the encoder on features that are controllable by the agent and, therefore, more interesting for exploration. The latent forward model and the embedding of the successive observations are used to compute the prediction error that is summed to the reward to promote exploration. However, this method presented several limitations that were highlighted in [134]. The major one arises when the environment dynamic is stochastic. The curiosity, as it is, could not distinguish uncertainties due to the lack of data from uncertainties due to the stochasticity of the environment dynamics. Therefore the agent was subjected to learning policies that exploit the stochasticity of the environment without any exploratory meaning.
Improvements to curiosity-driven exploration to limit the problem of the stochasticity of the environment were proposed by [135] where an ensemble of latent forward models is used to make N independent prediction-error terms, and the curiosity bonus is computed by looking at the variance of the error terms as a mean of exploiting disagreement of the models.

Exploration can also be promoted if the agent learns to avoid terminal states that the actions cannot reverse. With this idea in mind, the authors in [136] developed an approach for intelligent exploration that penalized visiting irreversible states.

Novelty cannot be defined only by prediction error but also in terms of how distant a state is from other visited states. This idea is exploited by [137] with the method named episodic curiosity. First, observations were embedded in the latent state space. Then each embedding is compared with the novel embeddings stored in the so-called reachability buffer to generate a score depending on how far that embedding is from the others in the buffer. If the score is above a certain threshold, i.e. the observation is deemed novel, the observation is added to the reachability buffer. The score is used as a curiosity bonus to reward the agent for visiting states far from the set of known ones.

A similar concept is used by [138], where the curiosity bonus was defined by looking at the latent states’ K-Nearest Neighbours (KNN) to reward visiting states with high KNN scores. States with high KNN scores belong to areas with more exploratory potential. Closely related, we found the work of [139], where an inverse model aided the learning of the latent states used for computing the KNN score to focus on controllable features, the work of [140] where a contrastive loss was used instead of the inverse model, and the work of [141] that, differently from the other approaches, simultaneously learned a state representation that was directly used by the policy and to generate the exploration bonus. Randomly initialized AEs can also generate exploration bonuses to visit different features of the latent state space [142].

Eventually, we find the methods keeping track of the visiting count of each state and rewarding exploration strategies achieving uniform coverage [143, 144, 145]. However, these methods struggle in large discrete and continuous state spaces where complicated and expensive density models have to be learned and updated during the exploration process.

5.1.2 Learning Action Representations

Reducing the dimensionality of the observations by encoding and reducing their dimensionality is a powerful and crucial tool for DRL. In this section, we argue that the same mechanism of encoding can be used for the agent’s actions. While many works have only focused on learning state representations, only a few have focused on learning action representations. This section reviews the methods relying on action (and state) representation for directly learning policies and value functions.

One of the first examples is the work of [146] in the context of Natural Language Processing (NLP). The goal was to train a DQN agent that, by looking at a sentence, i.e. the state, could predict the sentence that would follow correctly. State and actions are recurrently and independently encoded, and their embeddings, i.e. the state and the action representations, were fed to the Q-network to estimate the Q-value. Learning in the state-action representation space proved to be the critical ingredient for outperforming other approaches in the literature.

Large and discrete action spaces are traditionally a challenge for DRL algorithms. However, one could think of embedding the action space in continuous latent spaces to exploit similarities among actions and allow generalization. Despite different effects, actions with similar effects on the states are mapped together in the latent action space, making learning the value function and the policy more accessible. In this context, we find the work of [147] that, given an action embedding, computed the K closest actions via KNN and consequently evaluated the Q-value of those actions only. This approach efficiently saved evaluations of the Q-value, making the DRL algorithm efficient for large action spaces. Another way to learn action embeddings was proposed by [148]. They introduced a variant of an inverse model with a bottleneck. Two consecutive states were fed to the inverse model to predict the latent action between two states. Then the latent action was fed to a decoder network mapping the latent action to the original action space. The policy was trained to map states to latent actions.

State representation and action representations were combined by [149] using similar principles to [148] with the difference of using a forward model instead of an inverse one. Again embedding states and actions proved to be a powerful way to achieve sample efficiency in DRL, especially in the case of large discrete action spaces where embedding the actions adds structures and exploits the similarities.

A similar architecture to [149] was proposed by [150], but, similarly to [146], the action representation was learned from a sequence of actions and not from a single one. Moreover, state and action representations are enforced to be Gaussian distributions using a KL-divergence loss. Because sequences of actions were mapped to a latent action vector, decoding this vector back to a sequence of actions was not trivial and required the complete training of the action encoder and the freezing of its weights. In the same line of thought, the authors of [51] proposed a framework for jointly
learning state and action representations from high-dimensional observations by relying on the MDP homomorphism metrics for guaranteeing optimality and equivalence of the latent policy mapping latent state to latent actions and the overall policy from observations to actions.

5.1.3 Discussion

In Table 5.1, we summarize how the contributions presented utilize the SRL methods. In particular, while additional objectives are very often used in this context, especially in online exploration, we notice that forward, and inverse models are used to extract the set of epistemically uncertain yet controllable features.

Similarly to model-based DRL, the exploration via forward models requires precise quantification and discrimination of aleatoric and epistemic uncertainties. Deepening the connection between uncertainty quantification, SRL and learning latent dynamical models is an important aspect to address for further progress.

Combining the learning of state and action representations from high-dimensional observation has been proposed only in [150] and [51]. These approaches may open the door to more sample-efficient DRL algorithms combined with temporal abstractions of action and states.

| Method Contribution | PCA | AE | VAE | PW | RW | IN | MDPH | BM | PR | CL | Other Objectives |
|---------------------|-----|----|-----|----|----|----|-------|----|----|----|------------------|
| Pathak et al. [133] |     | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Badia et al. [134]  |     | ✔  |     | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Pathak et al. [135] | ✔   | ✔  |     | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Grinsztajn et al. [136] |     | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Arjovsky et al. [137] |     | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Tao et al. [138]    | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Badia et al. [139]  |     | ✔  |     | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Liu and Abbeel [140] | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Tarini et al. [141] |     | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Barbi et al. [142]  | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Martin et al. [143] | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Tang et al. [144]   | ✔   | ✔  |     | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Machado et al. [145] | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| He et al. [146]     | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Dulac-Arnold et al. [147] | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Chang et al. [148]  |     | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Pilor et al. [149]  | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Whitney et al. [150] | ✔   | ✔  | ✔   | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |
| Botteghi et al. [51] |     | ✔  |     | ✔  | ✔  | ✔  |       | ✔  | ✔  | ✔  |                  |

Table 4: Which SRL methods are used by each contribution?

5.2 Applications

After an in-depth presentation of the basic approaches, methods and advanced methods, we focus on the most relevant applications of SRL.

State estimation is one of the most well-studied problems in dynamical systems evolving over time. Predicting such systems’ natural and controlled evolution is crucial when we talk about physical systems, such as robotics and fluid dynamics, games, e.g. chess or Atari, and other fields such as health and finance.

5.2.1 Physical Systems

Physical systems often present a crucial challenge: real-world interaction. The interaction with the real world, and the consequent need for learning from limited data, is possibly the most significant obstacle DRL-based control systems have to overcome. In the context of physical systems, robotics is one of the most researched application domains due to the possibility of studying how to deal with the real world and tackle the problem of learning from limited samples.
Simulators are often used as a proxy for the real world. However, the simulation-to-reality gap\footnote{Simulators rely on models of the world (e.g. perception, and actuation) and even minor discrepancies can hinder the transferability of the DRL algorithm.} often hinders the direct transfer of the learned behaviours to the real world \cite{151}. SRL plays an important role here. The enforced dimensionality reduction makes the low-dimensional state representations and the policies learned given such representations more robust to noise, visual distractors, and discrepancies between the simulator and the real world compared to end-2-end DRL\footnote{Namely DRL algorithms learning directly from high-dimensional measurements.}. Moreover, due to the higher sample efficiency of the DRL algorithm, it is possible to fine-tune the performance with real-world samples in some cases.

Alternatively, one could think of training the algorithms directly in the real world. While model-free approaches are predominant with simulators, model-based approaches become more appealing when learning directly in the real world. Therefore, learning accurate and reduced-order latent models from measurements is crucial. Again, sample efficiency and robustness are the two essential concepts that must be considered.

The literature proposed a wide variety of robotics applications from mobile robot navigation, dexterous manipulation with robot arms, autonomous driving, and continuous control\footnote{Namely control via continuous variables, e.g. pendulum, legged robots, and cart-pole.}.

In the context of continuous control with end-2-end DRL, especially in recent years, fluid dynamics has gathered more attention \cite{152}. While traditionally, fluid dynamics has focused only on model-order reduction and state estimation, SRL principles have yet to be exploited in this field, but they can be crucial for systems without known physical models.

\subsection{Games}

Games such as chess, Go, Atari and Montezuma’s Revenge are other examples of successful application of SRL and DRL. While sample efficiency might not appear critical due to no real-world interaction, solving games with long-term dependencies, multiple levels of difficulty, requiring complex exploration policies, against or collaborating with multiple players requires unobtainable amounts of data. Therefore, learning good and general state representations and latent models can allow the development of new approaches and new, more advanced solutions, e.g. efficient exploration or credit assignment.

Unlike physical systems, where a continuous action space is often preferred for the smoothness of the control laws, games present a discrete action space from simple north-south-west-east movements on a grid to more complex actions such as grabbing objects opening doors and moving different chess pieces.

\subsection{Other fields}

DRL has become increasingly popular in many applications in different fields in recent years. However, SRL methods remain understudied. Here, we show how the problem-specific challenges of each field can enormously benefit from SRL methods.

The first example is economics. Decision-making problems such as the stock market, portfolio management and allocation of capital, online services and recommendations systems can all be tackled with DRL \cite{153}. Improving information compression, quantifying uncertainties in the measured variables over wide windows of time, and providing interpretable results are some problems the SRL methods could potentially solve.

Secondly, the health domain \cite{154} has recently shifted its focus towards dynamic treatment regimes with long-term effects for chronic disease, automated medical diagnoses, and health resource scheduling and allocation, e.g. time slots, nursing resources, and diagnostic devices. Many challenges arise in this field, such as sample efficiency due to the limited data, state-action spaces engineering due to multiple high-dimensional measurements over long time windows and potentially coming from different devices, interpretability of the decisions and the learned latent variables, development of patient-specific models, dealing with uncertainties, exploration of new treatment regimes, and the integration of prior knowledge\footnote{Priors.}. These challenges perfectly fit the scope of SRL methods, and SRL can open new frontiers in health research.

DRL has invaded other domains such as the Internet of Things (IoT) and communication networks \cite{155} for solving power control and power management, reducing latency and traffic load problems, and cloud computing, smart grid, and resource allocation. With many users involved, the key challenges include dealing with multi-agent systems and encoding partial information over time.
Natural Language Process (NLP) presents problems that DRL has started to solve, such as language understanding, text generation, machine translation and conversational systems [156]. The proper encoding and representation of sentences and sounds and the generalization to new speakers are crucial for decision making. SRL may be again an essential element for further progress.

Eventually, we find the field of Explainable AI (ExAI) [157], [158]. ExAI aims to open the black box and, in DRL, understand the agent’s behaviour. SRL can be seen as a way to simplify the problem of decision-making by separating feature extraction and policy learning by identifying the low-dimensional set of important variables for control. These SRL goals align with ExAI, and further research should be conducted to strengthen this connection.

6 Evaluation and Benchmark

6.1 Evaluation Metrics

Evaluating the learned representations is crucial for assessing and comparing new approaches, especially in DRL, where most of the results are empirical. We can identify three major ways of evaluating state representations:

- Reward over training and testing.
- Structural metrics.
- Qualitative metrics.

6.1.1 Reward Over Training

The reward over training and testing is the most used of the metrics. The quality of the different learned representations is indirectly compared using the training speed of the learned policy and its robustness when tested. The policy can be learned jointly with the representation or after the representation is entirely learned.

The performance of the agent is ultimately the critical metric in DRL. However, it is usually computational expensive to train multiple agents [165], the performance is heavily dependent on the choice of the DRL algorithm and its hyperparameters, making it difficult to disentangle the benefits of the learned representation from the overall increment of the cumulative rewards.

6.1.2 Structural Metrics

An alternative and appealing way of assessing the learned representations’ quality is using the so-called structural metrics. Structural metrics are all the metrics evaluating the properties of learned representations independently of the performance of the DRL agent. An example is a relation between the learned states and the actual states of the environment [44].

Common structural metrics are:

- Ground Truth Correlation (GTC) [160] measuring the correlation between the learned states and the true environment states.
- K-Nearest Neighbour (K-NN) [160] measuring the distance between K neighbours of latent states and K neighbours of true states.
- Linear Reconstruction Error (LRE) [28] measuring the disentangle of the state features by linearly mapping them to their respective true state value.
- Validation Decoder [15] measuring if the latent states can be used to reconstruct the observations without disambiguities [45].
- Ground Truth Error (GTE) [161] measuring the error to the ground truth with Euclidean or geodesic distance.
- Normalization Independent Embedding Quality Assessment (NIEQA) [162] measures the quality of the latent manifold in terms of preservation of the true underlying structure.

43 Due to the intrinsic stochasticity of RL, experiments with multiple random seeds (5 to 10 nowadays) have to be performed. A complete discussion on the topic is provided in [159].

44 The state of the environment is still considered non-observable over training, but in most cases, e.g. games and physics simulators, it is available for evaluation.

45 The gradients of the validation decoder are not propagated through the encoder as we aim at validating a fixed state representation.
• Feature predictiveness measuring how many of the features learned by the encoder are used by the forward model for predicting the future states.

• Hits at rank K measuring if the predicted latent state belongs to the K-NN of the true encoded observation.

6.1.3 Qualitative Metrics

Eventually, qualitative evaluations are often provided by the authors of the papers, especially when the representation is attributable to a lower-dimensional domain, e.g. robot control, where the features can be directly related to physical quantities, such as positions and velocities.

Visualization of latent representations is crucial for interpretability. Because in most cases, it is preferred to keep the dimension of the latent representation higher than the actual state space, usually PCA or t-distributed Stochastic Neighbour Embedding (t-SNE) are used to project, linearly and non-linearly, respectively, the latent state variables into a visualizable space, i.e. 2D or 3D, that can be plotted. However, complex problems cannot often be reduced to plottable dimensions. Therefore, their interpretability remains an open problem in the literature.

6.2 Benchmarks

SRL algorithms are usually evaluated using the same environments built for DRL. Their domains span from the Atari games to continuous control tasks such as pendulum and cheetah. Below, we list the most used platforms:

• Atari Learning Environment (ALE) providing an RL interface to the Atari games with the possibility of playing them from pixels. The Atari games offer the possibility of testing discrete state and action spaces algorithms and present several challenges to the algorithms, such as partial observability, long-term exploration, and reward sparsity.

• DeepMind Control Suite including a vast range of continuous control tasks such as pendulum, cheetah, and humanoid. Continuous control tasks require the algorithms to deal with continuous state and action spaces, multiple degrees of freedom, and partial observability when played from pixels. The physics is simulated by Mujoco.

• DeepMind Lab introducing a first-person 3D environment for testing the navigation, exploration, and generalization skills of the agents.

• SRL-Toolbox offering a complete package with many of the early SRL approaches from the literature, evaluation functions, and a set of testing environments, smaller if compared to the other platforms.

7 Discussion

To conclude the review, in Figure we provide a graphical overview of the SRL methods in model-free and model-based DRL and their role in advanced methods and applications.

Eventually, we discuss two final aspects: open challenges and future directions.

7.1 Open Challenges and Future Directions

DRL and SRL are two quickly-evolving fields, and here we summarize the most exciting future directions. From our review of the literature, we have identified several aspects that are currently understudied in the field:

• the incorporation of inductive biased and prior knowledge in the neural network architectures using the tools provided by Geometric Deep Learning,

• the proper quantification of uncertainties in the latent variables, observations, and MDP dynamics,

• the learning of general representations using the Meta-Learning paradigm, and

• the evaluation and comparison of the approaches and the interpretability and explainability of the latent representations.

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46 This helps the training of the encoder and the disentangling of the features.
7.1 Incorporation of Inductive Biases

In the last decade, DL has solved many high-dimensional problems considered beyond reach by developing increasingly complex architectures with a continuously-growing number of parameters and requiring more data. When scaling to larger problems, simply increasing the model complexity and the size of the training set do not seem viable options. However, many complex problems present low-dimensional structures, symmetries, and geometry deriving from the physical world. Incorporating this knowledge into the neural network architectures and the learned representations is essential for tackling the curse of dimensionality, reducing the number of parameters needed by each model, and improving generalization to unseen data \cite{128}. For example, when learning a latent forward model for a mobile robot, we would like the model to learn invariant features to background changes or noise in the measurements.

The SRL methods described in this review have prepared the ground for incorporating geometry, group theory and symmetries in DRL methods. Exploiting symmetries, structures, and prior knowledge are three critical ingredients for sample efficient SRL and DRL that are yet under-investigated but have shown outstanding potential for future research \cite{168}, \cite{169}.

7.1.2 Uncertainty Quantification in Unsupervised Representation Learning

Uncertainties may arise from different sources, e.g. the measurement noise or the chaotic dynamics of the system at hand. Quantifying and disentangling these data uncertainties is crucial to the generalization of state representations and latent models. While this problem is primarily studied in supervised learning \cite{170}, it is even more complex in unsupervised representation learning in dynamical systems. The goal of understanding and discovering the low-dimensional set of variables describing the evolution of the systems cannot be achieved without proper uncertainty quantification.

In particular, we highlight Deep Kernel Learning \cite{171}, \cite{172} as a computationally-efficient approach, scalable to high-dimensional data, for combining the expressive power of neural networks with the uncertainty quantification abilities of Gaussian processes and kernel methods \cite{173}.

7.1.3 Meta-Learning General Representations

DRL agents have faced an uncountable amount of different tasks, and this trend is only destined to grow in the future. Learning to adjust the behavior to new problems from limited data is still an open question in DRL research. While the Meta-Learning has been studied in DRL for learning general-purpose policies \cite{174}, \cite{175}, and \cite{176}, this paradigm has yet to be exploited for learning general-purpose state and action representations and MDP models. In contrast, with the expensive Meta-DRL from high-dimensional observations, Meta-Learning of representations may provide
new ways to tackle the problem of generalization and transfer learning of the learned policy to new tasks and new environments in an efficient sample manner.

7.1.4 Evaluation and Interpretability of the Latent Representations

In Section 6, we discussed different metrics for assessing the quality of the representations, e.g. through the reward over training, structural and qualitative metrics.

However, here we argue that none of these metrics provides the ultimate answer to which method is the best. While the reward over training is the most popular metric, this is subjected to the intrinsic stochasticity of the DRL process, and the results need to be evaluated over multiple runs. The need for multiple runs makes the process extremely computational and resource-intensive for complex environments and high-dimensional observations. On the other side, structural metrics provide a good way of comparing approaches without high-computational demands. However, high metric scores do not necessarily translate into optimal policy results and the highest reward over training. The same holds for qualitative metrics. High human-interpretability of the representations does not necessarily mean optimal behaviors of the agents.

Therefore, we believe that further research must be done to understand what the optimal state representation is, how to learn it, and how to interpret and explain representations and agents’ behaviors.

8 Conclusion

This paper reviewed the most important and newest research trends on unsupervised SRL in DRL. From the literature, we identified four major classes of SRL methods: observation reconstruction, learning of latent MDP models, using priors, and contrastive learning. Moreover, we study how these methods are combined to develop more complex approaches. Eventually, we extended the review to include the advanced methods and applications of SRL.

Unsupervised representation learning has proven to be a crucial element of the best-performing DRL algorithms for improving sample efficiency, robustness, generalization, and interpretability. Despite the progress in the field, many issues remain open for tackling complex real-world problems.

The incorporation of prior knowledge and geometry, the exploitation and learning of symmetries, the quantification of the uncertainties, the generalization to different tasks, and the evaluation and interpretability of the representations are important problems to address in the future.

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