Ferromagnetic phase transition in charged spin-1 Bose gases

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Abstract. Within mean-field theory, we investigate the ferromagnetic phase transition of charged spin-1 Bose gases with ferromagnetic interactions. It is shown that the internal field due to spontaneous magnetization can not prevent the occurrence of spontaneous magnetization. There exists a phase transition from paramagnetic phase to ferromagnetic phase, and the critical value of reduced ferromagnetic coupling $I_c$ increases with increasing temperature.

1. Introduction
Recently a series of experiments have stimulated research interest in magnetic properties of quantum Bose gases. The realization of spinor Bose-Einstein condensation (BEC) in optical traps [1] offers the possibility of studying condensates with spin degree of freedom. Furthermore, an ultracold plasmas can be created by photoionization of laser-cooled neutral atoms [2]. The temperatures of electrons and ions are as low as 100 mK and 10 $\mu$K, respectively. The ions can be regarded as charged bosons [3, 4, 5] if their spin is an integer. Moreover, the discovery of several ferromagnetic (FM) superconductors in experiments [6] recasts interest of coexistence of spin-triplet superconductivity and ferromagnetism. The spin-triplet pairs behave somewhat like charged spin-1 bosons.

It is known that the orbital motion due to charge degree of freedom in magnetic field induces the diamagnetism. While the paramagnetism is produced by spin degree of freedom of particles. The diamagnetism of charged spinless Bose gases has been discussed based on different groups [7, 8]. Furthermore, spin degree of freedom has been introduced to study the magnetic properties of charged spin-1 Bose gases in external magnetic field, and the competition between paramagnetism and diamagnetism has been discussed with different Lande-factors [9].

The purpose of this paper is to discuss the FM phase transition of charged spin-1 Bose gases with FM interactions. It will help to understand exotic magnetic properties observed in FM superconductors. In our earlier work [10], it is indicated that FM phase transition occurs in chargeless spinor Bose gases with FM couplings, in spite of the magnitude of the FM coupling. Once charge degree of freedom is involved, accordingly the magnetic behavior will become more complex since diamagnetism, paramagnetism and ferromagnetism compete with each other in such case.
2. The model
We consider a charged Bose gas of \( N \) particles, with FM interactions is described by the following Hamiltonian

\[
H - \mu N = D_L \sum_{j,k_x,\sigma} \left( \epsilon^I_{jk_x} + \epsilon^{ze}_{\sigma} + \epsilon^m_{\sigma} - \mu \right) n_{jk_x,\sigma},
\]

where \( \mu \) is the chemical potential and the Landau levels of bosons with charge \( q \) and mass \( m^* \) in the effective magnetic field \( B \) is

\[
\epsilon^I_{jk_x} = \left( \frac{1}{2} + j \right) \hbar \omega + \frac{\hbar^2 k_x^2}{2m^*},
\]

where \( j = 0, 1, 2, \ldots \) labels different Landau levels and \( \omega = qB/(m^*c) \) is the gyromagnetic frequency. The magnetic field is assumed to be along the \( z \) direction. \( D_L \) is the degeneracy \([9]\) of the energy level and the Zeeman energy levels split in the magnetic field is \( \epsilon^{ze}_{\sigma} = -g_\sigma \hbar \omega \), where \( g \) is the Lande-factor and \( \sigma \) denotes the spin-\( z \) index of Zeeman state \( |F = 1, m_F = \sigma\rangle \) \((\sigma = +1, 0, -1)\). The contribution to the effective Hamiltonian from the FM coupling is

\[
\epsilon^m_{\sigma} = -2I_\sigma (m + \sigma n_\sigma),
\]

where \( I \) denotes FM coupling and spin polarization \( m = n_1 - n_{-1} \). The particle density and the magnetization density can be expressed as \( n_{T \neq 0} = -\partial \Omega/T_\mu/V \) and \( M_{T \neq 0} = -\partial \Omega/T_\mu/B/V \), respectively, where \( \Omega \) is the grand thermodynamic potential. The relation among \( B \), external magnetic field \( H \) and \( M \) is formally expressed as

\[
B = H + 4\pi M,
\]

For computational convenience, some dimensionless parameters are introduced below. \( k_T = 2\pi h^2 n^2/m^* \), \( t = T/T^* \), \( M = m^* c M/(n\hbar q) \), \( \varpi = \hbar \omega/(k_T^*) \), \( \tilde{I} = I n/(k_T^*) \), \( \bar{\mu} = \mu/(k_T^*) \), \( \bar{m} = m/n \), \( \bar{n}_\sigma = n_\sigma/n \) and \( h = \hbar H/(m^* c k_T^*) \), and then \( x = \varpi/t \). Based on the self-consistent calculations \([9, 11]\), dimensionless \( \bar{M} \) can be obtained \([11]\) as,

\[
\bar{M}_{T \neq 0} = \frac{3}{2} \sum_\sigma \left\{ \Sigma'_{1\sigma} [-D, 0] + x (\varpi^\sigma D - \frac{1}{2}) \Sigma'_{1\sigma} [2 - D, 0] \right. \\
- \left. \left( \Sigma'_{2\sigma} [2 - D, 1] \right) \right\},
\]

with

\[
\Sigma'_{\alpha\sigma}[\alpha, \delta] = \sum_{l=1}^{\infty} \rho l^2 e^{-l\alpha} (1 - e^{-l\alpha})^\delta,
\]

where \( \rho = g_\sigma \varpi + 2I_\sigma (\bar{m} + \sigma \bar{n}_\sigma) = (\frac{1}{2} - \pi)\varpi \), with \( \varpi = h + 4\pi \alpha \bar{M} \), where \( \alpha = q^2 n^{1/3}/(2\pi m^* c^2) \). In our calculations, the characteristic parameter \( \alpha \approx 10^{-10} \). This value is estimated for a system with the charge and mass of \( ^4\text{He} \), and the particle density being set as \( (1/nm)^{-3} \).

3. Results and discussions
The curves of dimensionless magnetization density \( M \) and \( m = m_1 - m_{-1} \) as a function of reduced FM coupling \( \bar{I} \) with different Lande-factors \( g \) are plotted in Figure 1(a) and Figure 1(b), respectively. \( \bar{I}_c \) is used to denote the critical value of reduced FM coupling of paramagnetic (PM) phase to FM phase transition. The value of \( M \) presents a turning point from zero to nonzero
with increasing $\bar{T}$. It is shown that the spontaneous magnetization exists in this system. The curves of $\bar{m}$ versus $\bar{T}$ are almost superposed for different Lande-factors. This suggests that $\bar{m}$ is independent with the Lande-factor $g$, so $T_c$ at a certain temperature possesses the same value for any Lande-factor. When $\bar{T}$ is fixed, the value of $\bar{M}$ is larger for larger $g$. The increase of $\bar{M}$ with $g$ is attributed to the paramagnetic effect [9]. There exists a competition among diamagnetism, paramagnetism and ferromagnetism in this system. The result of the competition shows that the internal field from spontaneous magnetization can not destroy the occurrence of spontaneous magnetization.

In order to manifest the relation between the critical value of reduced FM coupling and the reduced temperature. We plot $T_c$ dependence of $\bar{T}$ at $h = 0.00001$ in Figure 2. The region below $T_c$ is PM phase and the region above it is FM phase. $T_c$ increases with increasing temperature. At high temperature, the FM transition occurs at a relatively large $T_c$. That is to say, spontaneous magnetization become more and more difficult for high temperature. In this situation, the Bose statistics reduces to Boltzmann statistics.

4. Summary
In summary, the FM phase transition of charged spin-1 Bose gases with FM interactions is studied via mean-field theory. We show that the occurrence of spontaneous magnetization can not be destroyed by the internal field originating from spontaneous magnetization. The critical value of reduced FM coupling $T_c$ of PM phase to FM phase transition increases with increasing temperature.

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