Effect of Stand Structure and Number of Sample Trees on Optimal Management for Scots Pine: A Model-Based Study

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Abstract: This study presents an attempt to discover the effect of sample size on the financial outcome derived by stand-level optimization with individual tree modeling. The initial stand structure was altered to reflect sparse, average, and dense Scots pine (Pinus sylvestris L.) stands. The stands had varying numbers of stems but identical weighted median diameters and stand basal areas. The hypothetical Weibull diameter distributions were solved according to the parameter recovery method. The trees were systematically sampled with respect to the tree basal area corresponding to sample sizes of 10, 20, or 40 trees. We optimized the stand management with varying numbers of sample trees and with varying stand structures and compared the optimal solutions with respect to the objective function value (maximum net present value) and underlying management schedule. The results for the pine stands in southern and central Finland indicated that the variations in the objective function value relating to sample size were minor (<2.6%) in the sparse and average stand densities but exceeded 3% in the dense stands. Generally, the stand density is not always known, and thus, we may need to generalize the average density for all cases in question. This assumption, however, resulted in overestimations with respect to the optimal rotation period and financial performance in this study. The overestimations in the net present value decreased along with the increasing sample size, from 22% to 14% in the sample sizes of 10 and 40 trees, respectively.

Keywords: sample size; individual tree models; decision variables; stand-level optimization; Maximum net present value; Scots pine

1. Introduction

The economics of timber production can be broken down into a few essential questions: how much to invest in the initial stock, when to thin, which trees to remove during thinnings, and when to perform the final cut [1]. Traditionally, forest economics literature has focused on analyzing the factors of optimum rotation (for a concise summary, see [2,3]). However, during the last few decades an increasing interest in including thinnings and silvicultural activities in stand-level optimization has emerged [1,4,5]. This has required methodological progress: moving from whole-stand models to individual tree [5–7] and stage-structured models (e.g., [8,9]). In stage-structured models, trees are grouped into size classes characterized typically by tree diameter class, and the growth is described as a transition from one class to another [8,10].

Since the late 1980s, individual tree models have been among the most popular methods of forecasting growth and yield [3,11], and they have proven to be more effective than whole-stand and
stage-structured models in capturing, for example, the consequences of silvicultural and harvesting options [1], growth conditions [12], tree-to-tree competition [3], and price effects, such as quality premium on timber logs [13]. However, there is a relevant drawback related to using individual tree models in the optimization: the number of decision variables tends to increase considerably [14,15]. For instance, if a decision variable is defined for each individual tree in a stand, the solution space becomes daunting [16], and a massive computer execution time (CPU time) is required for a single solution. This significantly limits computability [14]. In addition, it has been shown that with the increasing number of decision variables, some optimization algorithms might even converge to suboptimal solutions [11].

Because the increasing number of decision variables distinctly restricts the computability of an optimization problem with individual tree models, one needs to examine which factors are relevant with respect to the size of the stand management problem [14]. Basically, there are three main factors affecting the number of decision variables and further influencing the CPU time related to single-tree models for stand-level optimization [14]. The factors are as follows: (1) technical procedures to select harvest controls, (2) procedures to determine harvest timing, and (3) the structural information of a stand. Currently, in most applied stand-scale simulators, tree data requirements play a crucial role [17], and thus, it would be well reasoned to analyze how the structural differences between the stands might affect the stand-level optimization in individual tree models.

Sometimes, forest inventory data may be represented by a list of individually sampled trees, commonly known as tree records [14]. For each tree record, a set of characteristics is measured: tree species, diameter, age, height, and an expansion factor for the number of trees that the tree record represents in the particular stand [14]. However, comprehensive tree-level data are seldom available in practical forestry, while the most widely used forest stand simulators are based on tree-level models (cf. [18,19]). Therefore, tree size distribution models are needed for converting the stand-level information into tree-level information in order to later produce stand projections in a stand simulator with tree-level growth and mortality models. Distribution models typically produce a sample of trees from the modeled diameter range, and the number of trees affects the level of detail incorporated into the sample [20,21].

In order to reduce the size of a stand management problem and to keep the problem solvable, the number of sample trees can be reduced. In principal, the essential question, however, is the following: does the sample size have any impact on the stand-level optimum solution? Further, does the sample size have the same effect when alternative stand structures are applied? It should be emphasized that the abovementioned questions have relevancy regardless of whether a stage-structured (when trees are grouped into size classes) or an individual tree growth model is applied [22]. The process used in this study to deal with changing the number of sample trees is twofold: first, the effect of the alteration on the stand-level optimum solution is analyzed, and second, the underlying optimum management schedule is elaborated upon by a rigorous analysis of the thinnings. The latter aspect provides crucial information with respect to practical forestry, as for instance, whether to apply thinning from below or from above (see [1]). The target function to be maximized through the optimizing management regimes is the net present value (NPV), because the study scope covers only the ongoing rotation (cf. [23]).

Using forest stand simulators is generally a model-based approach. A model-based approach with only one tree species was used in this preliminary study in order to minimize any disturbance factors. We believe that this kind of model-based analysis is ideal to capture whether a particular stand structure is more prone to creating noticeable differences in the financial performance with varying numbers of sample trees than by comparison with other stand structures. These differences can be assumed, because by increasing the number of trees, for example, the dynamic in a stand is better achieved (competition between trees) and the distribution of the assortment becomes wider and thus more reliable.
2. Materials and Methods

2.1. Initial Stands

This study is exclusively model-based. The initial stands represented 20-year-old Scots pine (*Pinus sylvestris* L.) stands in mesic heath sites (*Myrtillus* type (MT) by Cajander [24]) located in either southern Finland (60°48' N 23°45' E, temperature sum 1250 °C d.d. (degree days), growing season ranging from 170 to 220 days, threshold 5 °C) or central Finland (64°09' N 24°18' E, 1050 °C d.d., growing season from 140 to 180 days) [25]. The general workflow for generating the model-based input data for the simulations was as follows. We initiated the stand by first predicting the standard stand characteristics such as stand basal area per hectare (*G*), number of stems per hectare (*N*), basal area median breast height diameter (*d₇₅M*), and corresponding height (*h₇₅M*) using the family of models by Siipilehto [26]. Secondly, the breast height diameter (dbh) distribution was modeled using the parameter recovery method for the two-parameter Weibull distribution [27] for sampling tree dbh. Thirdly, the models by Siipilehto [26] also included the parameters of Näslund’s height curve, which was used for calculating the tree heights for the sampled trees according to their dbh.

Three optional stand densities were generated, namely average, dense, and sparse. The average stand density was determined by the expected values for the stand characteristics using the models by Siipilehto [26], in which the input variables were set for stand age (20 years), site type (MT), stand origin (naturally regenerated), and stand location in terms of degree days (1250 for southern Finland and 1050 for central Finland). The other options—sparse and dense stands—were modified by changing only the stem number (*N*), while *G* and *d₇₅M* were fixed to their expectation values for the average density. The arbitrary changes in *N* were large enough to cover a wide but still reasonable variation in dbh distributions (see Table A1 in the Appendix A). Note that the initial stand volumes associated with the optional densities varied only between 59.9 and 61.4 m$^3$/ha.

The structure of the dense stands was characterized by a strongly right-skewed dbh distribution (longer tail to the right), while the sparse stands were characterized by a peaked dbh distribution resulting in a relatively narrow dbh variation (see Figure A1 in the Appendix A). The cumulative Weibull distribution was $F($dbh$) = 1 – \exp\{-($dbh$/b)^c\}$, where *b* and *c* are the scale and shape parameters, respectively. We had two options for sampling the trees. The sampling could be made systematically from the predicted stem frequency–dbh distribution (each tree representing the same number of trees) or systematically from the weighted basal area–dbh distribution (each tree representing the same basal area). In this case, the trees were systematically sampled with respect to tree basal area, imitating relascope (angle gauge) samples. This means that the recovered Weibull distribution was weighted and thus resulted in Gamma distribution with parameter $k = (2/c + 1)$, as shown by Gove and Patil [28]. Systematic samples of *n* trees were taken such that each tree represented an equal percentage $p$ ($p = 1/n$) from the weighted (Gamma) distribution.

In Finland, the basal area–dbh distributions have been used traditionally instead of the stem frequency–dbh distributions. This is due to fact that relascope sampling is often used for estimating stand characteristics, such as for forest management planning purposes, as well as for the sampling of National Forest Inventory (NFI) plots, but also in order to emphasize larger and economically more valuable trees (e.g., [21,29–32]). With the same number of sampled trees, the basal area weighting gives more accurate volume estimates [29,32].

2.2. Analyses of the Varying Stand Structures and Sample Sizes

In this study the effect of the dbh distribution on the stand-level optimization was analyzed by (i) varying the number of sampled trees and (ii) by changing the initial stand structure in terms of the number of stems per hectare, resulting in different shapes of diameter distributions. The stand-level optimum represents a management schedule that maximizes the net present value of the ongoing rotation (details on technical calculation procedures below in Section 2.3). We had two different approaches to studying the differences in the stand-level optimum. First, three alternative
dbh distributions were assumed to be available due to the known stand density and the relevant prediction/recovery models, and the focus was on the sensitivity between the sample size and the optimum management within each stand structure. Secondly, the average stand density was assumed to be the only distribution type that could be predicted, and the options of dense and sparse stands represented the unpredictable realization of nature. In this case, we compared the optimum management predicted for the average density to the optimum for both extremes within the alternative sample sizes. This means that we generalized the average stand structure and the outcome of the stand-level optimum for all the stands of the same species, age, and location. The motivation for this approach was in the existing Finnish distribution models, which are still used even if they are not capable of predicting differences in stand structures within a particular basal area and median dbh (e.g., [21,31]). In addition, the traditional forest inventory field work (e.g., NFI) does not include all the information needed for accurate stand structure prediction/recovery. Indeed, the basal area and number of stems are alternative stand characteristics, and they are seldom both known. The number of stems is assessed for young stands, and the basal area is assessed for advanced stands (see [33]). Unfortunately, both are needed for determining the accurate second moment for parameter recovery.

We minimized the disturbance factors (i.e., no admixtures, no randomizing in the sampling), and thus, the stand projection results are entirely dependent on the sample size and the stand structure. Note that the number of systematically sampled trees did not result in any differences in the initial stand characteristics ($G, N, d_gM$). The stand characteristics are given in Table A1 in the Appendix A.

### 2.3. Stand-level Optimization

Having generated the alternative stands, the management was optimized in order to achieve the maximum NPV for the rest of the rotation. Currently, in optimization studies, stand growth simulators are generally used for producing stand projections (growth predictions). In the optimization framework of this study, a growth model generated the stand management alternatives, creating a solution space in which the optimization algorithm searched the maximum value for the target objective, and the optimization algorithm was controlled through a vector of control variables (an identical procedure was applied in Niinimäki et al. [4]). The stand projections here were based on the MOTTI simulator. MOTTI is a stand-level growth simulator, including specific distance–independent tree-level models for predicting such variables as growth and mortality, as well as the effects of such management on tree growth [34,35]. Generally, a 5-year period is used for the projection, but in optimization, an annual interpolation within the 5-year growth step is applied so that the management activities (such as thinnings and silvicultural actions) can be annually scheduled by the PIKAIA algorithm (see below). MOTTI is designed to simulate stand development under alternative management regimes in Finnish growth conditions [34–36]. The performance of the MOTTI simulator has been assessed in young Scots pine stands by Huuskonen [37], in mixed stands by Hynynen et al. [34], and in intensively managed Scots pine stands by Mäkinen et al. [38]. Recently, the MOTTI stand simulator has also been applied in stand-level optimization [39,40].

The objective function for maximizing the NPV ($NPV_{Max}$) was based on the discounted net revenues of the ongoing rotation and on the discounted value of the land expectation value from the end of the ongoing rotation to infinity (this is identical to the method introduced in Hyytiäinen and Tahvonen [23], (p. 444)). Because we started with existing model-based stands, regeneration was ignored, and the main focus was to maximize the NPV for the ongoing rotation with respect to the timing of the tending of a sapling stand, timing of thinnings, the thinnings’ intensity (including the thinning profile defined by four thinning points, enabling both thinning types: from below and from above), and clear-cut timing. Then, the discounted land expectation value (LEV) from the end of the
ongoing rotation was taken as a fixed term, although the discounted LEV value slightly varied due to the different optimal solutions for the ongoing rotation. The objective function was as follows:

\[
\text{Max}_{\{i_1, a_1 = 1, \ldots, i_n, a_n = 0, \ldots\}} \frac{\sum_{i=1}^{l} \left( R_{\text{pulp}}^i + R_{\text{saw}}^i \right)}{(1 + r)^{t_j}} - \frac{S_a}{(1 + r)^{t_a}} + \frac{\text{LEV}}{(1 + r)^{(n-t_0)}}
\]

where \( \text{NPV} \) = the net present value for the remaining rotation, in euros/ha; \( R_{\text{pulp}}^i \) = the cutting revenues (valued at stumpage) from pine or spruce pulpwood removal, in euros/ha; \( R_{\text{saw}}^i \) = the cutting revenues (valued at stumpage) from pine or spruce saw logs removal, in euros/ha; \( S_a \) = the tending costs of a sapling stand, in euros/ha; \( r \) = the interest rate (here, 3% or 4%); \( t_j \) = the timing of thinning or the clear-cut, years from the start of the simulation; \( t_0 = \) biological age of 20 years (\( t_l \) refers to the timing for the clear-cut); and \( t_a \) = the timing for the tending of a sapling stand, years from the start of simulation. Note that \( t_n < t_l \), and \( \text{LEV} \) = land expectation value, €/ha.

It should be emphasized that in the optimal stand management, the tending of a sapling stand did not take place in every case. In those cases, the equation was simplified to cover only the present value of the cutting revenues and the discounted LEV.

The land expectation value was based on the management schedule according to the prevailing silvicultural recommendations [41], and it was calculated by the following formula:

\[
\text{LEV} = \frac{\sum_{i=0}^{s} \left( R_i - \sum_{k=1}^{m} c_{ik} \right) \times (1 + r)^{-i}}{1 - (1 + r)^{-s}}
\]

where \( \text{LEV} \) = land expectation value, €/ha; \( R \) = harvesting revenue, value at stumpage (€/ha); \( c \) = cost for silvicultural measure \( k, k = 1, \ldots, m \) (€/ha); \( i \) = time s.t. \( s \) = time for clear-cutting, in years (based on the minimum diameter criterion).

Because the main focus of the study was to examine the effect of the dbh distribution on the stand-level optimization, we compared the maxima NPVs associated with the alternative initial stand structures and with the varying numbers of sample trees. For simplicity, we set the widely used 10-tree sample (e.g., [4,42,43]) as the reference level to which other alternatives (20 and 40 sample trees) were compared. For that purpose, we constructed a framework in which the particular pairwise comparisons were conducted in order to determine the most divergent cases with respect to the objective function value, \( \text{NPV}_{\text{Max}} \).

In searching the optimal stand management to maximize the NPV (Equation (1)), we applied PIKAIA, which is an optimization software utilizing a genetic algorithm [44,45]. In general, genetic algorithms are more effective at finding the global optimum than direct search algorithms, for example, and they have proven to be a useful method for solving various optimization problems [46]. The main advantages of genetic algorithms are their high precision, shorter calculations times, [46] and their ability to avoid local optima [47].

The parameter space in this study was concise, including 14 parameters (representing a 14-dimensional multimodal function). These 14 parameters were as follows: (1) the timing of the tending of a sapling stand, (2) the timing of the first thinning, (3) the intensity of the first thinning, (4) the timing of the subsequent thinnings, (5) the intensity of the subsequent thinnings, (6–9) the relative dbh distribution of the remaining stock after the first thinning, (10–13) the relative dbh distribution of the remaining stock after the following thinnings, and (14) the timing of the clear-cutting. Sometimes, the aforementioned parameters are referred to as decision variables (DVs) [15]. Basically, parameters 6–13 describe both the thinning intensity and thinning profile, the former showing how much is thinned in relation to the basal area (%), and the latter determining the thinning type, either from below or from above (technically, described by four thinning points). With respect to parameters 3 and 5, an additional restriction was applied. Namely, the thinning intensity was forced to fluctuate.
between the specific limits representing the silvicultural recommendations [41]. The underlying idea for this restriction was to prevent the growth models from producing biased stand projections. The risk of faulty stand projections increases, for example, when the remaining growing stock after thinning is left to 40% or less in the basal area. This is simply due to the coverage of the initial data underlying the growth models. In the Finnish data, there are only a few field observations reporting thinning intensities exceeding 60% in such basal areas. On the other hand, the silvicultural recommendations enable maximum thinning intensities between 41% and 44% (the removed basal area compared with the basal area before thinning) in southern Finland and 43% and 44% in central Finland [41].

After having used the test runs on four randomly chosen cases, we decided to keep the default values intact [44,45], that is, the population size at 100, the number of generations at 500, and the crossover rate at 0.85 (for further details on the default values of the PIKAIA algorithm, see [39]).

In this study, an average CPU time for a single stand-level optimum was 1 h and 35 min, varying between 57 min and 2 h and 37 min.

We applied a 12-year average of the stumpage prices and tending costs of a sapling stand, covering the actual years from 2000 to 2011. The original nominal stumpage prices and tending costs were deflated according to cost-of-living index in Finland [48]. We considered the underlying time series to be comprehensive to capture business cycles, so that the calculated averages would include both peak and bottom prices. The deflated stumpage price averages were 55.7 € m$^{-3}$ for pine saw logs (with a minimum top diameter of 15 cm) and 16.1 € m$^{-3}$ for pine pulpwood (with a minimum top diameter of 7 cm). The minimum length of a log for pulpwood was 3 m, and it was 3.4 m for sawn timber. The deflated tending cost of a sapling stand was 339 € ha$^{-1}$.

3. Results

**Optimized Management Schedules**

The maxima NPVs, according to the optimized management schedules, did not vary much when we looked at the results for the average and sparse stand densities. Typically, the differences in the NPVs were less than 3% (Table 1). However, when the initial stand was dense, the NPV$_{\text{Max}}$ was increased along with the sample size, with the relative differences being bigger in central Finland than in southern Finland (Table 1). Indeed, in central Finland with 4% discounting, the NPV$_{\text{Max}}$ was 1571 € ha$^{-1}$ for the 10-tree sample and 1691 € ha$^{-1}$ for the 40-tree sample, resulting in a 7.6% difference.

Although the differences in the NPV$_{\text{Max}}$ between the varying numbers of sample trees and alternative stand densities were mainly reasonable, the underlying management schedules were quite divergent. The extreme stand structures were more prone to variation in the timing of the first thinning, as well as in the rotation period between the sample sizes. If we look at the differences greater than or equal to 2 years between the 10-tree sample and the bigger samples, we found only one such case for the first thinning and two cases for the rotation period (out of 8 cases) within the average stand structure (Table 2). The corresponding differences were found two times for the first thinnings and six times in the rotation period for the sparse stands, while the respective numbers were four and five times for the dense stands. Furthermore, these ≥2-year differences were more abundant, with a 3% discount rate (total of 12 cases) as opposed to a 4% discount rate (total of 8 cases). If we also consider the 1-year differences, the difference between the 3% and 4% discount rates is more obvious, namely 22 cases versus 13 cases, respectively. Note that the timing of the first thinning and the optimal rotation period could vary by more than 10 years, as was the case in the dense stands in central Finland (Table 2). Furthermore, the aggregate removal (m$^3$ ha$^{-1}$) associated with the thinnings with the 10-tree sample was considerably higher than the corresponding number for the 40-tree sample (Table 2).
Table 1. The maximum net present value, NPV$_{\text{Max}}$ (€/ha) associated with the optimal solutions. The comparisons are made in relation to (i) the NPV$_{\text{Max}}$ of the 10-tree sample and (ii) in relation to that of the average stand density. The location (Loc) was in southern Finland (SF) or central Finland (CF) having a temperature sum of 1250 and 1050°C degree days, d.d., respectively. The discount rate (Dr) is expressed in %.

| Stand Density | Loc | Dr, % | NPV$_{\text{Max}}$ of the Optimal Stand Management, €/ha | NPV$_{\text{Max}}$ in Relation to That of n = 10, % | NPV$_{\text{Max}}$ in Relation to That of Average Density, % |
|---------------|-----|-------|----------------------------------------------------------|-----------------------------------------------|----------------------------------------------------------|
|               |     | n: 10 | 20 | 40 | 20 | 40 | 10 | 20 | 40 |
| Sparse        | SF  | 3     | 6281 | 6310 | 6336 | 0.5 | 0.9 | −2.4 | −1.2 | −0.3 |
|               |     | 4     | 4522 | 4593 | 4611 | 1.6 | 2.0 | −2.6 | 0.0  | 1.0  |
|               | CF  | 3     | 3170 | 3225 | 3219 | 1.7 | 1.5 | 3.3  | 5.9  | 6.7  |
|               |     | 4     | 2083 | 2067 | 2065 | −0.8 | −0.9 | 3.7  | 3.8  | 5.5  |
| Average       | SF  | 3     | 6435 | 6389 | 6353 | −0.7 | −1.3 |       |       |       |
|               |     | 4     | 4644 | 4591 | 4567 | −1.1 | −1.7 |       |       |       |
|               | CF  | 3     | 3069 | 3046 | 3018 | −0.7 | −1.7 |       |       |       |
|               |     | 4     | 2009 | 1992 | 1957 | −0.8 | −2.6 |       |       |       |
| Dense         | SF  | 3     | 5827 | 5878 | 6006 | 0.9  | 3.1  | −9.4 | −8.0 | −5.5 |
|               |     | 4     | 4123 | 4157 | 4277 | 0.8  | 3.7  | −11.2 | −9.5 | −6.3 |
|               | CF  | 3     | 2513 | 2626 | 2616 | 4.5  | 4.1  | −18.1 | −13.8 | −13.3 |
|               |     | 4     | 1571 | 1623 | 1691 | 3.3  | 7.6  | −21.8 | −18.5 | −13.6 |

Regarding the other approach of the study, we assumed that the extreme stand structures (dense and sparse) were unable to be predicted due to missing input data (e.g., the number of stems is not measured in the forest inventory field work) or the application of such dbh distribution models (e.g., by Kilkki and Päivinen [30]). According to Kangas and Maltamo [32], Siipilehto et al. [49], and Siipilehto and Mehtätalo [27], together with traditional G and dgM, additional knowledge of N is required to obtain an accurate dbh distribution. Thus, we generalized the optimization results obtained for the average stand density to represent the extreme stand densities—sparse and dense. Further, if we denote the results of the average stand density as 100% with respect to the optimal rotation period and the NPV$_{\text{Max}}$, there were considerable differences in both the optimal rotation period and the NPV$_{\text{Max}}$ between the average and sparse stand densities, as well as between the average and dense stand densities (Table 2). For instance, the optimal rotation period in southern Finland was overestimated by as much as 22.9% (70 versus 54 years) when the average stand density represented the sparse stand, with the discount rate being 4% and the number of sample trees being 10 (Table 2). On the other hand, in central Finland, with 20 sample trees, the average stand density representing the dense stand underestimated the optimal rotation period by 21.3% (80 versus 97 years, Table 2). Further, the maxima NPVs were overestimated in central Finland if the average stand density represented the dense stand, with the range of overestimations being between 13.3% and 21.8%, depending on the number of sampled trees and the discount rate (Table 2). It is worth noting that differences decreased with the increase in the sample size (Table 2).
Table 2. Thinning characteristics and rotation periods associated with the optimal solutions. The location (Loc) was in southern Finland (SF) or central Finland (CF). The discount rate (Dr) was expressed in %.

| Stand Density | Loc | Dr % | First Thinning: Timing (Years)/Removal Volume (m³/ha) | Intermediate Thinnings: Number of Thinnings/Removal Volume | Clear-cutting, Years/Pulpwood/Logs (m³/ha) |
|---------------|-----|------|-----------------------------------------------------|---------------------------------------------------------|------------------------------------------|
|               | n: 10 | 20 | 40 | 10 | 20 | 40 | 10 | 20 | 40 |
| Sparse        | SF   | 3  | 29/49.4 | 29/49.8 | 29/47.1 | 3/215.4 | 3/216.1 | 3/211.9 | 64/65/182 | 64/66/182 | 64/65/182 |
|               | 4    | 29/49.7 | 29/50.9 | 29/47.5 | 3/215.8 | 3/215.0 | 3/210.9 | 61/62/179 | 63/65/181 | 63/64/180 |
|               | CF   | 3  | 40/45.0 | 38/46.4 | 38/45.9 | 3/180.2 | 2/115.6 | 2/115.8 | 75/47/162 | 78/51/180 | 78/51/179 |
|               | 4    | 38/45.0 | 38/46.5 | 38/46.5 | 2/118.5 | 2/115.5 | 2/115.6 | 74/46/166 | 77/51/180 | 77/50/178 |
| Average       | SF   | 3  | 30/57.3 | 30/57.3 | 30/44.6 | 4/315.0 | 3/209.5 | 3/210.2 | 71/58/167 | 70/94/235 | 69/67/220 |
|               | 4    | 30/53.1 | 30/57.4 | 30/57.6 | 4/298.0 | 4/312.0 | 4/309.4 | 70/62/169 | 71/58/168 | 71/58/167 |
|               | CF   | 3  | 40/45.0 | 40/45.9 | 42/45.0 | 3/180.2 | 3/183.6 | 2/116.6 | 83/75/131 | 80/67/117 | 80/73/162 |
|               | 4    | 40/45.2 | 40/47.3 | 40/52.3 | 3/181.0 | 3/187.0 | 3/190.7 | 80/75/117 | 80/69/115 | 80/75/108 |
| Dense         | SF   | 3  | 32/42.4 | 31/47.3 | 32/54.9 | 3/216.8 | 4/284.0 | 4/298.4 | 70/74/197 | 71/74/165 | 72/74/166 |
|               | 4    | 31/42.3 | 31/55.5 | 32/54.3 | 3/213.4 | 3/208.0 | 4/299.1 | 70/81/201 | 70/114/203 | 72/74/166 |
|               | CF   | 3  | 53/68.2 | 50/65.5 | 42/45.3 | 2/139.3 | 3/214.6 | 2/112.2 | 84/112/106 | 97/78/137 | 80/118/121 |
|               | 4    | 53/65.9 | 50/65.4 | 42/48.6 | 2/136.6 | 2/132.0 | 2/112.0 | 81/105/96 | 84/120/112 | 80/129/116 |
4. Discussion

A significant advantage of using individual tree growth models in stand-level optimization is the ability to include the structural information of a stand into the analysis [1,14]. However, there is a drawback involved with applying individual tree models to stand-level optimization: as the number of tree records increases, the solution space (problem size) expands, which gradually becomes daunting, demanding an immense CPU time [14]. Thus, there is a trade-off between the number of tree records and the ability to keep the optimization problem solvable. The objective here was to evaluate the effect of tree data descriptions (sample size and dbh distribution) on the stand-level optimization. The trees were generated by existing models, indicating a pure model-based approach.

There have been earlier attempts to evaluate the effect of uncertainty related to the sample size when estimating [20] or applying dbh distribution [21], in addition to growth projection errors, on the forest value (e.g., [50]). In a study by Kilkki et al. [21], 10 trees seemed to be a critical number when the sample size and sampling method from the Weibull basal area–dbh distribution were analyzed against the RMSE in the generated stem volume. Shiver [20] estimated the Weibull dbh–frequency distributions and found that 50-tree random samples were adequate to reproduce the distributions into the dbh classes with less than 10% error in any class. In our study, we wanted to emphasize that no matter the sample size, the initial stands were free of errors in the stand basal area, number of stems, and median diameter due to the recovery method and systematic sampling [27]. Additionally, the systematic sample was taken from the basal area distribution (imitating relascope sample), as recommended by Kilkki et al. [21], and also due to the Finnish tradition of predicting the basal area–dbh distributions instead of the stem frequency distributions [21,30–32]. Yet, an interesting question arose: is the sample size adequate to describe the competition conditional to tree size for tree-level growth and mortality models in the simulations? Nevertheless, a sample size of 10 trees has been used quite frequently in stand-level optimization applying either empirical or process-based individual tree growth models (e.g., [3,4,51]).

In a study by Wikström [14], mixed stands had the most unstable optimization (i.e., the relative NPV changed the most among the changes in the number of decision variables). On the other hand, almost pure and relatively sparse spruce stands achieved the most stable optimization results. Hyytiäinen et al. [1] analyzed the effects of timber quality on the stand-level optimization for stands with varying initial juvenile densities using empirical data. We decided to minimize all disturbance factors, and thus, we excluded species admixtures, empirical data, and the juvenile state of the stand. Instead, we compared the stand-level optimal solutions for Scots pine only, with alternative sample sizes describing similar initial total volumes distributed differently in terms of dbh distribution (recovered Weibull) for young thinning stands (dominant height, \( H_{\text{dom}} \geq 7 \) m) close to first thinning state (as in [51]). This kind of model-based two-dimensional analysis is ideal to capture whether a particular stand structure is more prone to creating noticeable differences in the financial performance with varying numbers of sample trees rather than other stand structures.

In general, when the dbh distribution represented the sparse or average stand structures, the differences in the NPV\(_{\text{Max}}\) in relation to the sample sizes were less than 2%. However, when simulating dense stands, differences greater than 100 euros/ha were found five times out of eight cases. These differences typically corresponded to a 3%–4% higher NPV\(_{\text{Max}}\) than that of the 10-tree sample, while the maximum difference was as high as 7.6%. A small sample size seemed to underestimate the NPV\(_{\text{Max}}\) in the dense stands. This finding was in line with the study by Wikström [14], in which data aggregation into dbh classes lowered the NPV\(_{\text{Max}}\); the wider the class the smaller the NPV\(_{\text{Max}}\). Note that the stand structure in Wikström [14] resembled our dense stands most closely, but they were Norway spruce-dominated. Within the average stand structure, however, the NPV\(_{\text{Max}}\) decreased slightly but systematically along with the increasing sample size, even if the management schedules were identical.

The differences between the stand structures turned out to be quite large. The optimal rotation periods and maxima net present values fluctuated by as much as 20% between the different stand
densities (see also [1]). Thus, it seems that the assumption of average density might lead to considerable errors in stand-level optimization, at least in our case of Scots pine stands. In this context, increasing the sample size clearly diminished the difference in the NPV_{Max} (Table 1). In dense stands, these differences in the NPV_{Max} diminished by 4–8 percentage points (e.g., from 21.8% with a 10-tree sample to 13.6% with a 40-tree sample). As we can see, the differences were bigger with a higher discount rate (Table 1). It is noteworthy that traditional forest inventories or available distribution models do not include enough information on the initial stand density. These problems do not concern only Finland. Major or minor incompatibility between known and predicted stand basal areas and stem numbers is present whenever prediction models are used instead of parameter recovery for dbh distribution [52]. This means that either the number of stems or the basal area is correct for predicting distribution but not both of them (e.g., [28,53–56]).

The underlying optimal management schedules fluctuated quite considerably, especially with the extreme stand structures (Table 2). The extreme stand structures seemed to be more prone to variation in the timing of the first thinning, as well as the variation in the optimal rotation period between the sample sizes. Furthermore, these differences were more abundant with a lower discount rate.

One interpretation for the robustness of the NPV_{Max} (i.e., objective function) might be the optimization algorithm, PIKAIA, itself; it is capable of efficiently seeking the global optimum with alternative sample sizes, indicating that the number of sample trees or the stand structure might not be the bottleneck, as long as a minimum of 10 trees is used. This argument, however, has to be tested with alternative optimization algorithms to confirm its veracity (cf. [4,15,57,58]). Another explanation could be the fact that we started from existing 20-year-old stands, and the major silvicultural costs (i.e., stand establishment costs) were, at that point, already sunk. Thus, the optimization algorithm may have had merely “limited power” to seek the global optimum, because forthcoming thinnings and clear-cutting do not involve cost variables but only revenues. This kind of simplistic variable space could indicate that the objective function value does not change abundantly (e.g., [59]). Finally, it is worth mentioning that we also tried smaller 5-tree samples during test simulations, but then our optimization problem did not converge.

In this study, we demonstrated the effect of both the dbh distribution and sample size on the stand-level optimal solution when the NPV was maximized. According to our results, more stable optimizations were achieved with bigger sample sizes. Apparently, this was due to the fact that the bigger samples enabled a more detailed description of tree competition, as well as more flexible thinning profiles in the simulations and more accurate assortment volumes. Further, we do not know if even higher sample sizes, say of 50 or 60 trees, would have performed better than our maximum of 40 trees. Further, optional sampling from the dbh-frequency distribution instead of the basal area-dbh distribution could also have some impact on the stand dynamics and, thus, on the optimization results due to the fact that the smaller trees would have been better represented.

Finally, additional economic research using alternative optimization algorithms, different tree species (especially shade-tolerant Norway spruce), distance-dependent growth models, and perhaps process-based growth models are needed to verify and generalize our preliminary results. Before this additional research is done, extra caution is warranted when interpreting the results obtained by individual tree models and the PIKAIA optimization algorithm. As a conclusion of our results, one could quite safely apply various numbers of sample trees ranging from 10 to 40 into the stand-level optimization, if the stand structure is sparse or average. However, in dense stands, a sample size of 10 trees seemed to be too small. Thus, a word of a caution must be said: varying results in dense stands is worrisome because increasing interest has been directed to simulating extensive forest management (i.e., keeping high stand density) for carbon storage, for example (e.g., [60,61]). Most of all, generalizing the typical stand structure and its optimal solution for all stands of the same species and site (e.g., general management guidelines) may cause considerable economic losses, especially in initially dense stands.
The next logical step after this study would be to compare the optimal solutions associated with the alternative numbers of sample trees (here, 10, 20, or 40) against actual, measured tree records. This would reveal the underlying bias related to tree data descriptions of individual tree models within the stand-level optimization context. Such an analysis, however, requires a distinctively more powerful computability than that applied here, as well as a totally new study design involving large-scale field measurements. Also, including random variation in tree height is an interesting issue, because it will result in a reasonable marginal distribution of tree heights (Siipilehto et al. [62]) and simultaneously more realistic timber assortment and quality distribution. This, in turn, would further increase the needed sample size. Also, stand establishment costs and tending costs of sapling stands should be included in the analyses, as well as the effect of a sudden shift of unit price increase when a stem exceeds the limit for saw log dimensions, and its value is multiplied. This sudden price difference might have an effect on the existing weighting in the sampling trees. Furthermore, one can argue that this shift becomes smoother with increasing number of description trees (see [63]).

5. Conclusions

This study demonstrated that stand structure (sparse, average, or dense) plays a relevant role when optimizing stand management in simulation studies. However, an underlying stand structure related to the initial stands (to be simulated and further optimized) has seldom been described and/or reported in articles tackling stand-level optimization. Thus, such a shortcoming might lead to possible misinterpretations between existing articles.

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Appendix A

Table A1. Stand characteristics, parameters of the predicted Näslund’s height curve ($b_0$, $b_1$), and recovered parameters of a two-parameter Weibull distribution ($b$, $c$). The stands represented Myrtillus type (MT) fresh sites in southern (temperature sum 1250 degree days (d.d.)) and central (1050 d.d.) Finland. (For Näslund’s height curve and Weibull distribution, see e.g., [26,27]).

| d.d. | $G$, m$^2$ha$^{-1}$ | $N$, ha$^{-1}$ | $d_{GM}$, cm | $h_{GM}$, m | $H_{dom}$, m | $b_0$ | $b_1$ | $b$ | $c$ |
|------|--------------------|----------------|--------------|-------------|-------------|--------|--------|-----|-----|
| 1250 | 14.4               | 3500           | 11.3         | 7.9         | 8.7         | 0.951  | 0.310  | 6.250| 1.320|
| 1250 | 14.4               | 2276           | 11.3         | 7.9         | 8.7         | 0.979  | 0.307  | 8.176| 2.300|
| 1250 | 14.4               | 1600           | 11.3         | 7.9         | 8.6         | 1.017  | 0.302  | 11.310| 6.460|
| 1050 | 9.17               | 3500           | 10.0         | 6.6         | 7.0         | 1.093  | 0.325  | 4.486| 1.119|
| 1050 | 9.17               | 2150           | 10.0         | 6.6         | 7.0         | 1.129  | 0.321  | 7.130| 1.753|
| 1050 | 9.17               | 1200           | 10.0         | 6.6         | 6.9         | 1.174  | 0.316  | 10.158| 16.396|
**20-year old Scots pine stands on MT site**

![Graph showing diameter distributions](image)

**Figure A1.** An example of the diameter distributions in the 20-year-old Scots pine stands on fresh mesic heath site (*Myrtillus* type, MT). The distributions represented dense (N 3500 ha$^{-1}$), average (N 2276 ha$^{-1}$), and sparse (N 1600 ha$^{-1}$) stands, where the expected $d_{GM}$ was 11.3 cm and G 14.4 m$^2$ ha$^{-1}$. Tree heights of 10- (♦) and 40- (○) tree samples taken from the average density distribution are indicated with the secondary axis (on the right-hand side).

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