POLYHEDRA FOR WHICH EVERY HOMOTOPY DOMINATION OVER ITSELF IS A HOMOTOPY EQUIVALENCE

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ABSTRACT. We consider a natural question: "Is it true that each homotopy domination of a polyhedron over itself is a homotopy equivalence?" and a strongly related problem of K. Borsuk (1967): "Is it true that two ANR's homotopy dominating each other have the same homotopy type?" The answer was earlier known to be positive for manifolds (Bernstein-Ganea, 1959), 1-dimensional polyhedra and polyhedra with polycyclic-by-finite fundamental groups (DK, 2005). Thus one may ask, if there exists a counterexample among 2-dimensional polyhedra with soluble fundamental groups. In this paper we show that it cannot be found in the class of 2-dimensional polyhedra with soluble fundamental groups $G$ with $cd G \leq 2$ (and soluble can be replaced here by a wider class of elementary amenable groups). We prove more general fact, that there are no counterexamples in the class of 2-dimensional polyhedra, whose fundamental groups have finite aspherical presentations and are Hopfian (or more general, weakly Hopfian). In particular, a counterexample does not exist also among 2-dimensional polyhedra, whose fundamental groups are knot groups and in the class of 2-dimensional polyhedra with one-related torsion-free, Hopfian fundamental groups. The results can be applied also, for example, to hyperbolic groups or limit groups with finite aspherical presentations.

For the same classes of polyhedra we get a positive answer to another open question: "Are the homotopy types of two quasi-homeomorphic ANR's equal?"

1. Introduction

In this paper we study two natural but still open problems: "Is it true that for every polyhedron $P$, each homotopy domination of $P$ over itself is a homotopy equivalence?" and the famous problem of K. Borsuk (1967) [B1, Ch.IX, Problem (12.7)]: "Is it true that two ANR's homotopy dominating each other have the same homotopy type?" (By a polyhedron we mean, as usual, a finite one. For convenience, we will assume without loss of generality, that each polyhedron and ANR is connected.)

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They are closely related to another open problem in geometric topology: "Are the homotopy types (or equivalently, shapes) of two quasi-homeomorphic ANR’s equal?" [B2, Problem (12.7), p. 233] that will be also considered here.

In dimension 1 the answers to all the above questions are positive. Indeed, every 1-dimensional polyhedron has the homotopy type of a finite wedge of circles $S^1$, so $K(F, 1)$, where $F$ is a free, finitely generated group. It is well-known that every finitely generated, free group is Hopfian (Nielsen 1921, Hopf 1931).

By the results of I. Bernstein and T. Ganea (1959), if $P$ is a manifold, then every homotopy domination of $P$ over itself is a homotopy equivalence [BG]. (It was generalized to the so-called Poincare complexes in [Kw]).

For polyhedra with polycyclic-by-finite fundamental groups the answers are also positive (see [K, Theorem 3, Theorem 5]). Thus, one may ask how about 2-dimensional polyhedra, in particular 2-dimensional polyhedra with soluble fundamental groups.

In this paper we prove that for polyhedra $P$ with dim $P = 2$ and soluble (or, more general, elementary amenable) fundamental groups $G$ satisfying $cdG \leq 2$, there are no counterexamples. This is a corollary to the main result that there are no counterexamples among 2-dimensional polyhedra, whose fundamental groups have finite aspherical presentations and are weakly Hopfian (see Definition 3). It should be noted that at present there is not known any example of a finitely presented group which is not weakly Hopfian.

We also consider some other classes of finitely presented groups satisfying conditions of our main theorem. As one of the corollaries, we obtain that for each 2-dimensional polyhedron whose fundamental group is a knot group, every homotopy domination over itself is a homotopy equivalence. The same we get for 2-dimensional polyhedra whose fundamental groups are one-related, torsion-free and Hopfian (note that in many cases one-related, torsion-free groups are known to be Hopfian, see, for example, [SSp]). We also obtain positive results for 2-dimensional polyhedra whose fundamental groups are hyperbolic groups or limit groups with finite aspherical presentations.

In a consequence, we also answer positively the third question for the same classes of 2-dimensional polyhedra.

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2. Preliminaries

Definition 1. Let $S$ be a class of groups. A group $G$ is called poly-$S$ if it has a finite series $G = G_0 \triangleright G_1 \triangleright \ldots \triangleright G_l = 1$, for which each factor $G_{i-1}/G_i \in S$ (where $1 \leq i \leq l$).
DEFINITION 2. Let $\mathcal{P}, \mathcal{S}$ be some classes of groups. A group $G$ is said to be $\mathcal{P}$-by-$\mathcal{S}$ if it has a normal subgroup $A \in \mathcal{P}$ such that $G/A \in \mathcal{S}$.

DEFINITION 3. (i) A group $G$ is Hopfian if every epimorphism $f : G \to G$ is an automorphism (equivalently, $N = 1$ is the only normal subgroup for which $G/N \cong G$). (ii) A group $G$ is weakly Hopfian if $G = K \rtimes H$ and $H \cong G$ imply $K = 1$ (where $G = K \rtimes H$ means that $H$ is a retract of $G$, i.e. $G = KH$, $K \triangleleft G$, $K \cap H = 1$).

DEFINITION 4. A group $G$ is residually finite if for every $g \in G$, $g \neq 1$, there exists a homomorphism $h$ from $G$ onto a finite group $H < G$ such that $h(g) \neq 1$.

DEFINITION 5. A module $M$ is called Hopfian if every homomorphism $f : M \to M$ which is an epimorphism is an isomorphism.

REMARK 1. Let $P$ be a polyhedron such that $\pi_1(P)$ is weakly Hopfian and all the $\pi_i(X)$, for $i = 2, \ldots, \dim P$, are Hopfian modules over $\mathbb{Z}\pi_1(P)$. Then, by the Whitehead Theorem, every homotopy domination $d : P \to P$ is a homotopy equivalence.

For the following theorem, see [K, Theorem 3, Theorem 5]:

**Theorem.** Let $P$ be a polyhedron such that the group $\pi_1(P)$ is polycyclic-by-finite. Then every homotopy domination $d : P \to P$ is a homotopy equivalence. □

REMARK 2. In the proof of the above theorem in [K] we applied the fact that if $G$ is polycyclic-by-finite, then every finitely generated module over $\mathbb{Z}G$ is Hopfian (for details and necessary references, see the proofs in [K]).

Therefore it is worth to ask:

**Problem 1.** Does there exist a polyhedron $P$ with $\dim P = 2$ and soluble fundamental group and a homotopy domination of $P$ over itself that is not a homotopy equivalence?

DEFINITION 6. (i) Recall that by a cohomological dimension of a group $G$ we mean $\text{cd}G = \sup \{i \text{ such that } H^i(G, M) \neq 0, \text{ for some } \mathbb{Z}G\text{-module } M\}$. (ii) A geometrical dimension, $\text{gd}G$, of a group $G$ is a smallest dimension of a $CW$-complex $K(G, 1)$ (see [Br], [BK]).
From the results of this paper follows that for each polyhedron \( P \) with \( \dim P = 2 \) and soluble fundamental group \( G = \pi_1(P) \) satisfying \( \text{cd} G \leq 2 \), every homotopy domination of \( P \) over itself is a homotopy equivalence.

From now on \( X \leq Y \) will denote that \( X \) is homotopy dominated by \( Y \).

3. Main Theorems

**Definition 7.** A group presentation is said to be aspherical if the standard 2-dimensional CW-complex associated with it is aspherical.

**Theorem 1.** Let \( P \) be a polyhedron with \( \dim P = 2 \) such that \( \pi_1(P) \) has a finite aspherical presentation and is weakly Hopfian. Then every homotopy domination \( d : P \to P \) is a homotopy equivalence.

**Proof.** Suppose that there exists a homotopy domination \( P \geq P \) that is not homotopy equivalence. Since \( G = \pi_1(P) \) is weakly Hopfian, this domination induces an isomorphism \( \pi_1(P) \to \pi_1(P) \).

The Whitehead Theorem on Trees [Wh] states that, if \( P \) and \( Q \) are two finite 2-dimensional CW-complexes with \( \pi_1(P) \cong \pi_1(Q) \), then there exist integers \( m_P \) and \( m_Q \) such that
\[
P \lor \bigvee_{m_P} S^2 \cong Q \lor \bigvee_{m_Q} S^2.
\]

Since there exists a finite 2-dimensional CW-complex \( K = K(G, 1) \), it follows that
\[
P \lor \bigvee_{m_P} S^2 \cong K \lor \bigvee_{m_K} S^2,
\]
for some integers \( m_P \) and \( m_K \). Hence we have an isomorphism of \( ZG \)-modules
\[
\pi_2(P) \oplus (ZG)^{m_P} \cong (ZG)^{m_K}.
\]

Indeed, it is known that for any 2-dimensional polyhedron \( Q \), \( \pi_2(Q \lor S^2) \cong \pi_2(Q) \oplus \pi_2(S^2) \) as \( Z\pi_1(Q) \)-modules.

Suppose that \( d : P \to P \) is a homotopy domination but not a homotopy equivalence. Then exists a nontrivial \( ZG \)-module \( N \) such that \( \pi_2(P) \oplus N \cong \pi_2(P) \). Otherwise, \( d \) induces also an isomorphism of \( \pi_2 \), and is a homotopy equivalence, whis is a contradiction.

Therefore \( (ZG)^{m_K} \oplus N \cong (ZG)^{m_K} \). Thus \( (ZG)^{m_K} \) is isomorphic to a proper direct factor of itself. But this is impossible. Indeed, for any group \( G \), any finitely generated free \( ZG \)-module \( (ZG)^m \) (where \( m \) is an integer) cannot be isomorphic to a proper direct factor of itself — from the result of I. Kaplansky [Ka, p.122].
Remark 3. (i) Obviously, every Hopfian group is weakly Hopfian. (ii) It should be noted that there is not known at present any example of a finitely presented group that is not weakly Hopfian. (iii) On the other hand, weak hopficity was proven for some classes of finitely presented groups. For example, every nilpotent-by-nilpotent group is weakly hopfian. (iv) It is also known that any finitely generated abelian-by-nilpotent group is Hopfian (P. Hall).

So let us formulate the following:

Problem 2. Does there exist a finitely presented group which is not weakly Hopfian?

Problem 3. Does there exist a finitely presented group which has a finite aspherical presentation and is not weakly Hopfian?

Corollary 1. Let $P$ be a polyhedron with $\dim P = 2$ such that $\pi_1(P)$ has a finite aspherical presentation and is Hopfian. Then every homotopy domination $d : P \to P$ is a homotopy equivalence.

From the above we obtain:

Theorem 2. Let $P$ be a polyhedron with $\dim P = 2$ such that $\pi_1(P)$ is a knot group. Then every homotopy domination $d : P \to P$ is a homotopy equivalence.

Proof. Any knot group $G$ has a finite aspherical presentation — there exists a finite $CW$-complex $K(G, 1)$ of dimension 2 (by the result of [P], see also [Br]). It is also known that any knot group is residually finite ([Th], see, for example, [Kb]). Any finitely generated residually finite group is Hopfian [Ma]. Thus the proof is completed.

Remark 4. A special case of 2-dimensional polyhedra with fundamental groups isomorphic to the Trefoil knot group $T = \langle a, b \mid a^2 = b^3 \rangle$ was considered by the author in [K1].

Theorem 3. Let $P$ be a polyhedron with $\dim P = 2$ such that $\pi_1(P)$ is one-related, torsion-free and weakly Hopfian. Then every homotopy domination $d : P \to P$ is a homotopy equivalence.

Proof. Every one-related, torsion-free group has a finite aspherical presentation. Precisely, a $CW$-complex naturally corresponding to a given one-related presentation (created by adding a single 2-cell to a finite wedge of circles corresponding to the generators) is aspherical (compare [Ly], [Br]). So it follows from Theorem 1.
Corollary 2. Let $P$ be a polyhedron with $\dim P = 2$ such that $\pi_1(P)$ is one-related, torsion-free and Hopfian. Then every homotopy domination $d : P \to P$ is a homotopy equivalence.

Remark 5. Recall that if a finite presentation of some group has exactly one relator that is not a proper power (i.e., a power of some element of this group), then this group is torsion-free.

Remark 6. (i) In many cases one-related groups are known to be Hopfian (see, for example, [W], [W1], [ARV], [CL], [S]). (ii) A recent results of M. Sapir and I. Spakulova [SSp] show that almost surely, a one-relator group with at least 3 generators is residually finite, hence Hopfian.

Definition 8 (Hyperbolic Groups). Recall that a finitely generated group is called hyperbolic if its Calley graph with respect to some finite generatic set is hyperbolic [Gr].

Remark 7. (i) Almost every finitely presented group is hyperbolic (see M. Gromov [Gr1]). (ii) Examples of non-hyperbolic groups are $\mathbb{Z} \times \mathbb{Z}$ and just considered here knot groups. (iii) All the hyperbolic groups without torsion have finite Eilenberg-Mac Lane CW-complexes (see, for example, [Kt]).

Theorem 4. Let $P$ be a polyhedron with $\dim P = 2$ such that $\pi_1(P)$ is a hyperbolic group and has a finite aspherical presentation. Then every homotopy domination $d : P \to P$ is a homotopy equivalence.

Proof. By the result of Z. Sela [Se], every hyperbolic group is Hopfian. Hence it follows from Corollary 1.

In the sequel, we will use properties of Baumslag-Solitar groups $BS(m, n)$ and consider soluble groups (and more general, elementary amenable groups).

Definition 9 (Baumslag-Solitar Groups). For each pair of integers $0 < m \leq |n|$, $BS(m, n) = \langle a, b \mid ab^m a^{-1} = b^n \rangle$ (compare [BS]).

Remark 8. It is known that (i) $BS(m, n)$ soluble (but non-nilpotent) if and only if $m = 1$. These groups are, in particular, metabelian, hence Hopfian. (ii) in general, $BS(m, n)$ is Hopfian if and only if $m = 1$ or $m$ and $n$ have the same prime divisors.

Definition 10 (Elementary Amenable Groups). Elementary amenable groups is the smallest class of groups that contains all abelian and all finite groups, and is closed under extensions and directed unions (see [KLL]).
Theorem 5. Let $P$ be a polyhedron with $\dim P = 2$ such that $G = \pi_1(P)$ is elementary amenable and $cdG = 2$. Then every homotopy domination $d : P \to P$ is a homotopy equivalence.

Proof. Let $G$ be a finitely generated elementary amenable group and $cdG = 2$. Then, $G$ has a presentation of the form $G = \langle a, b \mid aba^{-1} = b^m \rangle$, for some $m \in \mathbb{Z} - \{0\}$ [KLL, Theorem 3], i.e. is a Baumslag-Solitar group $B(1,m)$. Then, there exists a finite $CW$-complex $K(G,1)$ of dimension 2 (as in the proof of Theorem 2). Moreover, every Baumslag-Solitar group $B(1,m)$ is metabelian, hence (in the case of finitely generated groups) Hopfian (compare Remark 8). So, we apply Corollary 1, and the proof is complete. \hfill \Box

As a corollary, we obtain the following:

Theorem 6. Let $P$ be a polyhedron with $\dim P = 2$ such that $G = \pi_1(P)$ is soluble and $cdG = 2$. Then every homotopy domination $d : P \to P$ is a homotopy equivalence.

Proof. This is a corollary to Theorem 5. Any soluble group is poly-abelian, hence elementary amenable. \hfill \Box

Our results will be completed by the following.

Definition 11 (Limit Groups). A finitely generated group $G$ is a limit group if, for any subset $S \subset G$, there exists a homomorphism $f : G \to F$ (where $F$ is a free group of finite rank) so that the restriction of $f$ to $S$ is injective.

Remark 9. (i) Examples of limit groups include finite-rank free abelian groups. (ii) Limit groups are non-soluble except of free abelian groups. (iii) Note that all the limit groups have finite Eilenberg-Mac Lane $CW$-complexes.

The following useful lemma can be drawn, for example, from [AB]:

Lemma 1. Any limit group is Hopfian.

Proof. Given a limit group $G$, any sequence of epimorphisms $G = G_0 \to G_1 \to \cdots$ eventually consists of isomorphisms (see [AB, the proof of Lemma A.1, p. 269]). Therefore, there is no an epimorphism of $G$ onto $G$ which is not an isomorphism (take $G_i = G$, for all $i$, with the same given epimorphism between $G_i$ and $G_{i+1}$). Hence $G$ is Hopfian. \hfill \Box

Theorem 7. Let $P$ be a polyhedron with $\dim P = 2$ such that $\pi_1(P)$ is a limit group and has a finite aspherical presentation. Then every homotopy domination $d : P \to P$ is a homotopy equivalence.

Proof. It follows from Corollary 1 and Lemma 1. \hfill \Box
4. On different homotopy types of ANR's dominating each other

In [B1, Ch.IX, (12.7)] K. Borsuk stated the following question:

**Problem 4.** Is it true that two ANR's, P and Q homotopy dominating each other have the same homotopy type?

By the previous results of this paper (Theorems 1-7, Corollaries 1-2), we obtain:

**Corollary 3.** Let $P, Q \in \text{ANR}$, $\dim P = 2$, $\pi_1(P)$ has a finite aspherical presentation and is weakly Hopfian. Then $P \geq Q$ and $Q \geq P$, imply that $P \simeq Q$.

**Corollary 4.** Let $P, Q \in \text{ANR}$, $\dim P = 2$, $\pi_1(P)$ has a finite presentation with one relation, is torsion-free and weakly Hopfian. Then $P \geq Q$ and $Q \geq P$, imply that $P \simeq Q$.

**Corollary 5.** Let $P, Q \in \text{ANR}$, $\dim P = 2$, and $G = \pi_1(P)$ satisfies one of the following conditions:

(i) $G$ is soluble and $\text{cd} G \leq 2$,
(ii) $G$ is elementary amenable and $\text{cd} G \leq 2$,
(iii) $G$ is a limit group and has a finite aspherical presentation,
(iv) $G$ is a hyperbolic group and has a finite aspherical presentation,
(v) $G$ is a knot group.

Then $P \geq Q$ and $Q \geq P$, imply that $P \simeq Q$.

One may ask the following questions (compare strongly related Problem 2):

**Problem 5.** Does there exist a finitely presented one-related group $G$ and an $r$-homomorphism $r : G \to H$, where $H \simeq G$, such that $r$ is not an isomorphism?

**Problem 6.** Do there exist finitely presented one-related, non-isomorphic groups $G$ and $H$ and $r$-homomorphisms $h : G \to H$, and $h : H \to G'$, where $G' \simeq G$?

5. On two quasi-homeomorphic ANR's of different homotopy types

The main problem we consider here is also related to the other question published by K. Borsuk in [B2]. Let us recall two definitions (see [MS] and [B2], respectively).

**Definition 12 (S. Mardešić, J. Segal).** Let $X$ and $Y$ be compacta. $X$ is said to be $Y$-like, if for every $\varepsilon > 0$ there exists a continuous map $f : X \to Y$ such that, for all $y \in Y$, $\text{diam}(f^{-1}(y)) < \varepsilon$.

**Definition 13 (K. Kuratowski, S. Ulam).** Two compacta $X$ and $Y$ are quasi-homeomorphic if $X$ is $Y$-like and $Y$ is $X$-like.
Borsuk found two quasi-homeomorphic compacta of the different shapes and asked [B2, Problem (12.7), p. 231-233]:

**Problem 7.** Is it true that the shapes of two quasi-homeomorphic ANRs are equal?

It is well-known that on ANR’s shape and homotopy theory coincide, thus in this problem shapes can be replaced by homotopy types.

From Corollary 5 we can drawn:

**Corollary 6.** Let $P, Q \in \text{ANR}$, $\dim P = 2$ and $G = \pi_1(P)$ satisfies one of the following conditions:

(i) $G$ is soluble and $\text{cd}G \leq 2$,
(ii) $G$ is elementary amenable and $\text{cd}G \leq 2$,
(iii) $G$ is a limit group and has a finite aspherical presentation,
(iv) $G$ is a hyperbolic group and has a finite aspherical presentation,
(v) $G$ is a knot group.

If $P$ and $Q$ are quasi-homeomorphic, then $P \simeq Q$.

**Proof.** It is known that, if $P$ and $Q$ are two ANRs, and $Q$ is $P$-like, then $Q \leq P$ (compare [B2, (12.6) p. 233]). Thus, if $P$ and $Q$ are two quasi-homeomorphic ANRs, then $P \leq Q$ and $Q \leq P$. We apply Corollary 5 and the proof is complete. $\square$

6. **Final Remarks**

(1) Another classes of groups satisfying the assumptions of the main theorem one can find among subgroups of the Coxeter groups, small cancelations groups, Fuchsian groups, and among others.

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