Interaction between coaxial dielectric disks enhances the Q-factor

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We study the behavior of resonant modes under variation of the distance between two coaxial dielectric disks and show an avoided crossing of resonances because of interaction between the disks. Owing to coaxial arrange of disks the consideration is separated by the azimuthal index \( m = 0, 1, 2, \ldots \). In the present paper we consider the case \( m = 0 \). For a long enough distance the resonant modes can be classified as symmetric and antisymmetric hybridizations of the resonant modes of the isolated disk. With decreasing of the distance the interaction becomes stronger that gives rise to avoided crossing of different resonances of the isolated disk. That in turn enhances the Q factor of two disks by one order in magnitude compared to the Q factor of isolated disk.

I. INTRODUCTION

Optical microcavities and various other sorts of resonators have been widely employed to tightly localize electromagnetic field in small volumes for a long durations due to high Q factors, which plays an indispensable role in lasing, sensing, filtering and many other applications in both the linear and nonlinear regimes. In the cavity the local photon density of states scales proportionally to the quality factor to volume of the cavity ratio. In general, there is a compromise between high Q factors and small mode volumes due to the fact that larger resonators are required to increase round-trip travel time for Q factor enhancement, as is the case for whispering gallery modes.

It is rather challenging for optical resonators to support resonances of simultaneous subwavelength mode volumes and high Q factors. The traditional way for increasing of the Q factor of optical cavities is a suppression of leakage of resonance mode into the radiation continua. That is achieved usually by decreasing the coupling of the resonant mode with the continua. However, microcavities and resonators based on their reflection from the boundaries demonstrate substantially low values of the Q factor by virtue of weakness of the dielectric contrast of optical materials. The conventional ways to realize high-Q resonators are the use of metals, photonic bandgap structures, or whispering-gallery-mode resonators. All of these approaches lead to reduced device efficiencies because of complex designs, inevitable metallic losses, or large cavity sizes. On the contrary, all-dielectric subwavelength nanoparticles have recently been suggested as an important pathway to enhance capabilities of traditional nanoscale resonators by exploiting the multipolar Mie resonances being limited only by radiation losses.

The decisive breakthrough came with the paper by Friedrich and Wintgen, which put forward the idea of destructive interference of two neighboring resonant modes leaking into the continuum. Based on a simple generic two-level model they formulated the condition for the bound state in the continuum (BIC) as the state with zero resonant width for crossing of eigenlevels of the cavity. This principle was later explored in open plane wave resonator where the BIC occurs in the vicinity of degeneracy of resonance frequencies.

However, these BICs exist provided that they embedded into a single continuum of propagating modes of a directional waveguide. In photonics the optical BICs embedded into the radiation continuum can be realized by two ways. The first way is realized in an optical cavity coupled with the continuum of 2d photonic crystal (PhC) waveguide that is an optical variant of microwave systems. More perspective way is to achieve the BICs in periodic PhC systems or arrays of dielectric particles in which resonant modes leak into a restricted number of diffraction continua. Although the exact BICs can exist only in infinite periodical arrays, the finite arrays demonstrate resonant modes with the very high Q factor which grows quadratically with the number of particles (quasi-BICs).

Another attractive way to achieve quasi-BICs (super cavity modes) is to use individual subwavelength high-index dielectric resonators, which exhibit also high-Q factors. Such super cavity modes originate from avoided crossing of the nearest resonant modes, specifically the TE (Mie-type) resonant mode and the TM (Fabry-Pérot) resonant mode under variation of the aspect ratio of the dielectric disk. As the result they report a significant enhancement of the Q factor. It is worthy also to notice the idea of formation of long-lived, scar like modes near avoided resonance crossings in optical deformed microcavities. The dramatic Q factor enhancement was predicted by Boriskina, for avoided crossing of highly excited whispering gallery modes in symmetrical photonic molecules of dielectric cylinders.

In the present paper we consider a similar way to enhance the Q factor by variation of the distance between two identical coaxial dielectric disks. As different from consideration in papers, we consider the avoided crossing of low excited resonant modes (monopole and dipole) with variation of the distance between two dielectric disks. When the disks are separated by the enough long distance we have pairs of degenerate resonant modes such as monopole, dipole, etc. With the decrease of the distance the resonant modes interfere given rise to avoided crossing. We show that this effect is complimented by a spiral behavior of the resonant eigenvalues when the interaction between the disks is weak. With a further decrease of the distance the interaction is increasing to give rise to strong repulsion of the resonances. For this phenomenon
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II. AVOIDED CROSSING IN SYSTEM OF TWO COAXIAL DISKS

There are two limiting cases of the system of coaxial dielectric disks, the infinite periodic array of disks and the isolated disk. The former supports rich variety of resonant modes with zero resonant widths, BICs: symmetry protected BICs in the Γ-point, accidental BICs with nonzero Bloch vector, and hybrid BICs with nonzero orbital angular momentum. The symmetry protected BICs were experimentally observed in the system of ceramic disks in the THz range. The case of the isolated disk was considered in papers which have shown considerable enhancement of the $Q$ factor due to an avoided crossing of two resonant modes. In that papers the avoided crossing was achieved by a variation of aspect ratio of the disk that technologically is not simple except. In the present section we consider two identical coaxial disks as shown in Fig. 1. Each of them have the aspect ratio not obligatory tuned to the optimal $Q$-factor as in. The coaxial disks have the advantage that all resonant modes are classified by the azimuthal number $m = 0, 1, 2, \ldots$ because of the axial symmetry. Therefore one can consider subspaces with definite $m$ separately. In the present paper we follow the case $m = 0$ for which the solutions are separated by polarization with $H_z = 0$ (E modes) and $E_z = 0$ (H modes). In what follows we consider the H-modes.

In general the resonant modes and their eigenfrequencies are given by solving the time-harmonic source-free Maxwell’s equations

$$
\begin{pmatrix}
0 & iV \\
-iV & 0
\end{pmatrix}
\begin{pmatrix}
E_n \\
H_n
\end{pmatrix}
= k_n
\begin{pmatrix}
E_n \\
H_n
\end{pmatrix} \tag{1}
$$

where $E_n$ and $H_n$ are the EM field components defined in Ref as quasinormal modes which are also known as resonant states or leaky modes. It is important that they can be normalized and the orthogonality relation can be fulfilled by the use of perfectly matched layers (PMLs). With the exception very restricted number of symmetrical particles (cylinders, spheres) Eq. \ref{eq:1} can be solved only numerically.

Irrespective to the choice of dielectric particle the eigenfrequencies are complex $k_n a = \omega_n + i \gamma_n$ where $a$ is the disk radius. In what follows the light velocity is taken unit. Fig. 2 shows resonant frequencies of the single isolated disk complemented by insets with the resonant modes (only the component $E_\phi$ is shown). There are modes with nodal surfaces crossing the $z$-axis and the modes with nodal surfaces crossing the plane $z = 0$. They correspond to the Fabry-Perot resonant modes and the radial Mie modes by the terminology introduced in paper.

Fig. 3 shows the solutions of Eq. \ref{eq:1} for the case of two coaxial dielectric disks as dependent on the distance $L$ between the disks. The necessity to use PMLs restricts the distance between the disks which is to be considerably less than the distance between the PMLs in the $z$-direction. In spite of an illusive complexity in Fig. 3 the zoomed pictures reveal remarkably simple behavior of resonant frequencies in the form of a spiral convergence of avoided eigenfrequencies to the resonant frequencies of the isolated disks marked by closed circles as Fig. 3 demonstrates at zoomed plots. However when the disks approach close enough to each other the spiralling behavior is substituted by strong repulsion of resonant frequencies because of interaction enhancement.

In order to quantitatively evaluate this interaction we start consideration with an isolated disk for which the matrix of derivatives in Eq. \ref{eq:1} becomes diagonal with the complex eigenfrequencies $k_0$ in the eigenbasis presented in Fig. 2. It is reasonable to consider that for enough separation of two disks the matrix is still diagonal with pairs of degenerate $k_n$ shown in Fig. 4 by blue closed circles. Rigorously speaking for the large distance between the disks $L \gg a/\gamma_n$ the interaction via the resonant modes can grow exponentially. In view of that we restrict the distance $L < a/\gamma_n$. As the distance between the disks is reduced the interaction between the disks via the resonant modes splits the degenerate resonant modes $k_n$ giving rise to the avoided crossing. Assume also that a value of splitting

![Fig. 1. Two coaxial dielectric disks separated by distance $L$ referred between the centers of disks.](image1)

![Fig. 2. The resonant eigenfrequencies (close circles) and corresponding resonant modes (the component $E_\phi$ of dielectric disk with the height $h = a$ and permittivity $\varepsilon = 40$.](image2)
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\[
H_{\text{eff}}^{(n)} = H_{\text{eff}}^{(0)} + V = \begin{pmatrix}
 k_n a & 0 \\
 0 & k_n a \\
\end{pmatrix} + \begin{pmatrix}
 u_n & v_n \\
 v_n & u_n \\
\end{pmatrix},
\]

where \( v_n \) is responsible for interaction between the disks via the resonant modes while \( u_n \) is the result of the backscattering by the first disk. Therefore one can expect that \( \arg(v_n) = \arg(u_n) = 20a/L/a \). Fig. 3 shows the behavior of the absolute value and phase both of the matrix elements. The matrix elements \( v_n \) and \( u_n \) can be easily found from numerically calculated resonances shown in Fig. 4

\[
k_{n,\alpha} = k_n a + u_n \pm v_n,
\]

as \( v_n = \frac{k_n^{(a)} - k_n^{(b)}}{2}, u_n = \frac{k_n^{(a)} + k_n^{(b)}}{2} - k_n \). From Fig. 4 one can evaluate that the interaction term in (2)

\[
v_n \sim \frac{\alpha k_n L}{L^2}, \quad u_n \sim \frac{\gamma k_n L}{L^2}.
\]

The distance behavior (3) is observed with good accuracy for all resonances shown in Fig. 3 however, for only spiral convergence of the resonances. Numerically calculated behavior of the matrix elements \( v_n \) and \( u_n \) for \( n = 2 \) is shown in Fig. 4. In spiralling around the resonances of the isolated disk the hybridized resonant eigenmode is given by symmetric and antisymmetric combinations of the resonant modes of the isolated disk

\[
\psi_{n,\alpha}(\vec{r}) = \psi_n(\vec{r}_z - \frac{1}{2}L\vec{z}) \pm \psi_n(\vec{r}_z + \frac{1}{2}L\vec{z})
\]

where \( \vec{r}_z, \vec{r}_\phi, \vec{z} \) is unit vector along the z-axis and \( \psi_n(\vec{r}_z) \) are the corresponding resonant mode of the isolated disk shown in the insets in Fig. 2.

At first the resonant frequencies slowly spiral away from the limiting point given by \( k_n \). Respectively the Q factor in Fig. 3(c) demonstrates oscillating behavior exceeding the Q factor of the isolated disk a few times. With approaching of disks spiral behavior of the pair of resonances \( k_{n,\alpha}^{(a)} \) is replaced by strong repulsion as shown Fig. 3(a). Fig. 3(d) shows a remarkable feature caused by avoided crossing of resonances with different \( n \). To be specific there is avoided crossing of symmetric resonances \( k_n^{(2)} \) and \( k_n^{(5)} \) according to enumeration in Fig. 2. Because of the same symmetry of resonances relative to \( z \rightarrow -z \) these resonances undergo typical avoided crossing with a considerable decrease of the imaginary part of the resonant frequency and correspondingly enhancement of the Q factor by one order in magnitude. Respectively the two mode approximation (2) breaks down.

It is interesting to trace the behavior of resonances and Q factors for the aspect ratio \( a/h \approx 0.7 \) and \( \varepsilon = 40 \) for which the isolated disk shows the maximal Q factor. The results are presented in Fig. 5. One can see that with decrease of the distance between the disks we have the same spiralling behavior of the hybridized resonances around the resonances of the isolated disks which is terminated by strong repulsion of the symmetric and antisymmetric resonances for \( L \to h \).
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![Graphs and diagrams showing the dependence of matrix elements on the distance between disks.](image)

**FIG. 4.** Dependence of the matrix elements $v_n$ and $u_n$ on the stance between disks $L$. (a) and (b) corresponds to Fig. 3(d) and (c) and (d) does to Fig. 5(b).

However, we have no pronounced effect of the avoiding crossing of hybridized resonances with different $n$ and respectively have no enhancement of the $Q$ factor by one order as it was achieved for the aspect ratio $a/h = 1$ (see Fig. 4(e)).

Till now we considered the permittivity $\varepsilon = 40$ and $a = 1cm$ (ceramic disks) that enters the resonant frequencies into the THz range. Finally, we consider $\varepsilon = 12$ (silica disks) and $a = h = 1\mu$m with the resonant frequencies in the optical range.

Results of computations are presented in Fig. 5 which show that there is no qualitative difference between the ceramic disks with $\varepsilon = 40$ and silica disks with $\varepsilon = 12$. Similar to Fig. 3 and Fig. 5 we observe spiral behavior of the resonant frequencies around the resonances of the isolated disk for the enough distance between the disks. But what is more remarkable also we observe the avoided crossing of the resonances with different $n$ as shown in Fig. 5(d) with corresponding strong enhancement of the $Q$ factor by one order in magnitude (Fig. 5(e)).

### III. CONCLUSIONS

The recept to enhance the $Q$ factor by means of the avoided crossing of resonances is well known. Friedrich and Wintgen were the first who investigated the quantitative influence of the interference of resonances on their positions and widths. Moreover, in the framework of two-level effective Hamiltonian they found out that one of the widths can turn to zero to identify the BIC. A single isolated dielectric particle of finite dimensions can not trap light because of the infinite number of radiation continua or diffraction channels. However, for sufficiently large refractive index the particle shows distinctive Mie resonances with the Q-factors which can be substantially enhanced owing to the avoided crossing of the resonances under variation of aspect ratio of the disk.

Technologically it might be challengeable to vary the size of the disk in the optical range. In the present paper we propose to vary the distance between two coaxial disks that is preferable from the experimental viewpoint. Continuous variation of the distance gives rise to an avoided crossing of the Mie resonances due to interaction between the disks through radiating resonant modes.

For the enough distance we can consider that two disks have degenerate resonances $k_n$. However with drawing closer of the disks the disks begin weakly interact with each other via the leaking resonant modes that lifts a degeneracy of the resonances according to Eq. and respective symmetric and antisymmetric hybridizations of resonant modes $k^{(s)}_n$ and $k^{(a)}_n$ accord
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FIG. 6. (a) Behavior of resonant eigenfrequencies under variation of the distance between the disks $L$ with $\varepsilon = 12$ (silica in optical range) and aspect ratio $a/h = 1$. (b) and (d) zoomed areas highlighted in (a) with symmetric (solid lines) and antisymmetric (dash lines) hybridization \( k_s \) of resonant modes of the isolated disk. (c) and (e) show behavior of the Q factor vs the distance for corresponding insets at the left. Closed circles mark the eigenfrequencies of isolated disks and respectively the Q factors while crosses mark the limiting case $L = 1$ when two disks stick together.

As the result we observe ”soft” avoided crossing of the resonances around the points $k_0$. The further decrease of the distance between the disks enhances the interaction and respectively gives rise to strong repulsion of $k_s(n)$ and $k_a(n)$. However what is the most remarkable there are events of the avoided crossing of the resonances with different $n$. Respectively one can observe strong enhancement of the $Q$ factor around one order in magnitude.

Although in the present paper we considered only the dielectric disks, it is clear that the phenomenon of the avoided crossing and respective enhancement of the $Q$ factor would occur with particles of arbitrary shape when the distance between them is varied. The case of two coaxial disks simplifies computations because the solutions with different angular momentum $m$ are independent. In the present paper we have presented only the case $m = 0$ because of a possibility to consider separately E and H polarizations.

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