Numerical simulation of the liquid-metal jet formation and droplet pinch-off in the cathode spot of a vacuum arc

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Abstract. A two-dimensional axisymmetric model has been developed to describe the formation of liquid-metal jet and the droplet pinch-off. These processes occur during the extrusion of the melt from the crater by the pressure of the cathode spot plasma of a vacuum arc. The jet formation has been numerically simulated for a copper cathode in the “inertial” mode of the melt splashing until the first droplet pinch-off. In this case, a jet with a longitudinal velocity gradient is formed. This gradient decreases the diameter of the jet and causes its elongation, resulting in droplet pinch-off. It has been shown that the mechanism of the droplet pinch-off is based on Rayleigh-Plateau instability. The droplet pinch-off time decreases with increasing jet velocity and increases for droplets of larger diameter. The simulation predicted the electrical explosion of the droplet-jet neck at the current density on the droplet surface \( \geq 10^7 \, \text{A cm}^{-2} \).

1. Introduction
The explosion of the liquid-metal jets formed during the formation of a crater on the cathode of vacuum arc are considered to be the basic mechanism of the initiation of cathode spot cells that are responsible for the self-sustaining of a vacuum discharge [1, 2]. According to this model, the appearance of new cathode spot cells is due to the interaction of a liquid-metal jet with a dense \( (10^{20}-10^{21} \, \text{cm}^{-3}) \) cathode spot plasma [2]. With this important part played by the liquid-metal phase in the operation of a cathode spot, a quantitative description of the hydrodynamic processes responsible for the drop formation was so far confined to estimating or very simplified calculations [1–7]. Experimentally, the emission of drops was investigated in both short pulses spark discharges [1–3] and in vacuum arcs [8–11]. By analyzing the drop deposition on probe plates, data on the number and size distribution of drops [1, 2] were obtained together with their spreading diagram [10], and also the velocity of their flight was measured [11]. The characteristics of the droplet emission for low-current sparks and for arcs are in large identical. The most probable drop diameter is 0.1–0.2 \( \mu \)m, the most probable exit angle relative to the cathode plane is 20–30 deg., and the velocity is 100–200 m s\(^{-1}\). It should be noted that the models developed by us [12, 13] have made it possible to perform a first-ever detailed quantitative analysis of the hydrodynamic processes occurring in a cathode spot by solving numerically the Navier–Stokes equations for an incompressible viscous liquid with a free surface in a two-dimensional axisymmetric approximation. The simulation results for the velocity and exit angle of a liquid-metal jet are in good agreement with experimental data on drop emission.
2. 2D axisymmetric model of the formation of liquid-metal jets in the cathode spot

2.1. Problem statement
The melt splashing process in the cathode spot at crater formation can be considered by convention to occur in three stages [14]. At the first stage, the formation of a crater, an axisymmetric liquid-metal rim is formed (see figure 1a). At the second stage, the rim disintegrates, due to the azimuthal instability developing by the Rayleigh-Plateau mechanism, into individual cylindrical jets that form a crown. The formation of quasi-one-dimensional cylindrical jets, their separation from the cathode, and disintegration into drops occur at the third stage. Obviously, the formation of jets and drops is a three-dimensional process whose end-to-end simulation is difficult to perform. However, the motion of the melt is essentially three-dimensional only at the second stage. Previously, we performed a simulation of the first stage using an axially symmetric approximation [12, 13]. The second stage was investigated by us analytically in the context of a linear instability theory [15–17]. In [14], we propose a simplified two-dimensional axisymmetric model of the third stage that describes the formation of jets on a flat surface of the melt under the influence of a ring-shaped pressure. In this paper, we propose a “spherical pipette” model that describes the formation of jets on a spherical surface of the melt (see figure 1). As in [14], we assume that the processes occurring at the second stage slightly affect the diameter of the jets, their propagation velocities, and the exit angles relative to the cathode plane. It should be noted that these parameters calculated using the first stage model are in good agreement with experimental data (see [12, 17]).

2.2. The mathematical model
To construct an axisymmetric model of the formation of liquid-metal jets, we use a local cylindrical coordinate system \( \{r', z'\} \) in which the axis is directed along the jet. The model geometry is sketched in figure 1). The initial form of the melt is a ball of radius \( r_1 \). For the jet being formed in the local coordinate system, we specify an external pressure on the ball in the following form:

\[
\text{Figure 1. Model geometry of the problem.}
\]
where $p_0$ is the external pressure amplitude, $f_p(t, t_{\text{off}})$ (equal to 1 for $t < t_{\text{off}}$ and to 0 for $t > t_{\text{off}}$) is the Heaviside function that simulates the finite time of the pressure action during the formation of the jet, and $l$ is the length of the envelope of the free surface, which is counted from $\{r_b, z'(t,r_b)\}$. Note that expression (1) specifies the constant pressure $p_0$ on the surface of the ball, with the exception of “the hole” near the $z'$ axis only for $t < t_{\text{off}}$. At $t > t_{\text{off}}$ a jet formation occurs by inertia under the action only of surface tension force.

The mathematical model of the problem is a system of Navier-Stokes equations for an incompressible viscous liquid with a free surface:

$$
\frac{\partial \tilde{u}'}{\partial t} + u' \frac{\partial \tilde{u}'}{\partial r'} + u' \frac{\partial \tilde{u}'}{\partial z'} = -\frac{1}{\rho} \frac{\partial p}{\partial r'} + \nu \left( \frac{\partial^2 \tilde{u}'}{(r')^2} + \frac{\partial}{\partial r'} \left( \frac{\partial \tilde{u}'}{\partial r'} \right) + \frac{\partial^2 \tilde{u}'}{\partial z'^2} \right),
$$

and a continuity equation:

$$
\frac{1}{r'} \frac{\partial}{\partial r'} (r' u') + \frac{\partial v'}{\partial z'} = 0.
$$

Here $\tilde{V}' = \langle u', v' \rangle$ is the hydrodynamic velocity of the melt having a temperature $T_j$, $\rho = \rho(T_j)$ is its density, and $\nu = \nu(T_j)$ is its kinematic viscosity. The impermeability condition $u' = 0$, $\partial v'/\partial r' = 0$ is set on the $z'$ axis. The hydrodynamics equations (2)–(4) are solved numerically by the projection method; therefore, the Poisson equation for pressure is solved at each time step. To do this, a Dirichlet boundary condition is used to specify the pressure on the free surface:

$$
p|_{O_{z'}} = p + \gamma C + 2\nu \rho \left[ \frac{\partial u'}{\partial r'} N_r^2 + \frac{\partial v'}{\partial z'} N_z^2 + \left( \frac{\partial v'}{\partial r'} + \frac{\partial u'}{\partial z'} \right) N_r N_z \right],
$$

where $\gamma = \gamma(T_j)$ is the surface tension coefficient, $C$ is the curvature of the free surface, and $\vec{N} = \langle N_r, N_z \rangle$ is the unit vector of the normal to the free surface. The impermeability condition $\nabla p|_{z'} = 0$ is imposed on the $z'$ axis. To calculate the motion of the liquid front, the particle level set method is used. A more detailed mathematical formulation and the methods for solving the problem are presented in [10, 11].

### 3. Simulation results

The jet formation simulation scenario is similar to the model [14]. At the first, auxiliary, stage $t < t_{\text{off}}$ (see figure 1b), the external pressure (1) acting on the ball melt surface, being at a uniform temperature $T_j$, disturbs the free surface and forms the head of a liquid-metal jet, which propagates along the $z'$ axis with a velocity $v_{zf}$. Varying the spatial parameters $r_b$ in (1), we can vary the jet diameter $d_j$. The velocity of the jet head, $v_{zf}$, can be specified by properly adjusting $p_0$ and $t_{\text{off}}$. In fact, the aim of the simulation at the first stage is to adjust the initial parameters for the jet formation.
process, $d_j$ and $\nu_{st}$, whose values are calculated in the context of a semi-phenomenological model of the formation of a crater [13] and can be evaluated from experimental data. The formation of a jet and droplet pinch-off are simulated in the second, main, stage of the scenario depending on the set parameters of the model $j_b$, $p_0$ and $t_{off}$.

**Figure 2.** Jet formation and droplet pinch-off in the inertial melt splashing mode at various pressures: a) $p_0 = 2.5 \times 10^8$ Pa, b) $p_0 = 3.14 \times 10^8$ Pa, c) $p_0 = 4.39 \times 10^8$ Pa. Calculated parameters: $r_{jb} = 0.5 \, \mu$m, $r_i = 1.5r_{jb}$, $t_{off} = 1$ns, $T_j = 5000$ K.

Calculations were carried out for a copper cathode in the melt temperature range of 2000–5000 K. In this paper, we confine ourselves to considering the jet dynamics only until the first droplet pinch-off for the jets with $d_j \sim 0.1–0.5 \, \mu$m and $\nu_{st} \sim 100–300$ m s$^{-1}$. As the main calculated parameters of the jet dynamics, we will use $t_n$, $t_d$, $<v_d>$ and $r_d$, where $t_n$ – the time of the beginning of the formation of the droplet-jet neck, $t_d$ – the droplet pinch-off time, $<v_d>$ – droplet average velocity along the $z'$ axis, $r_d$ – droplet radius. The space-time characteristics of the jet formation process calculated at different values of the problem parameters $j_b$, $p_0$ and $t_{off}$ are presented in table 1 and figures 2–6. The effect of jet velocity on its dynamics is shown in figures 2a–c. Note that all calculations were carried out in the “inertial” mode of the melt splashing $t_{off} \ll t_d$. In this case, a jet with a longitudinal velocity gradient is formed. During jet formation, the jet head contracts, due to surface tension, into a spherical drop. This process is accompanied by the excitation of a capillary wave that propagates toward the base of the jet and harmonically modulates its diameter. Firstly this forms a neck between the jet and the head droplet. If there is a longitudinal velocity gradient, the length $\lambda$ of the capillary wave increases.
| N  | \(r_j\)   | \(p_0\times10^8\) (Pa) | \(t_{\text{off}}\) (ns) | \(t_0\) (ns) | \(t_d\) (ns) | \(<v_d>\) (m/s) | \(r_d\) (µm) | B²×10⁵ | \(j_{\text{exp}}^0\times10^6\) (A·cm⁻²) |
|----|----------|-----------------|-----------------|-----------|---------|-----------------|---------|--------|-----------------|
| 1  | 0.25     | 2.01            | 1               | 2.52      | 7.81    | 98.3            | 0.085   | 7.37   | 7.46            |
| 2  | 0.25     | 2.50            | 1               | 2.18      | 7.10    | 148             | 0.081   | 5.66   | 8.51            |
| 3  | 0.25     | 3.76            | 1               | 1.79      | 6.26    | 253             | 0.078   | 5.04   | 9.02            |
| 4  | 0.5      | 3.14            | 1               | 4.89      | 16.3    | 110             | 0.14    | 22.2   | 4.30            |
| 5  | 0.5      | 3.76            | 1               | 4.32      | 14.2    | 150             | 0.13    | 15.7   | 5.11            |
| 6  | 0.5      | 4.39            | 1               | 3.90      | 12.5    | 188             | 0.12    | 11.3   | 6.03            |
| 7  | 1.0      | 5.02            | 1               | 8.56      | 31.3    | 112             | 0.22    | 24.0   | 4.13            |
| 8  | 1.0      | 5.64            | 1               | 7.53      | 27.4    | 137             | 0.20    | 16.5   | 4.98            |
| 9  | 1.0      | 6.27            | 1               | 6.70      | 24.3    | 163             | 0.19    | 12.9   | 5.63            |

When \(\lambda\) becomes larger than the jet perimeter, so that the Rayleigh-Plateau instability starts developing, the droplet-jet neck radius is rapidly decreases and there is a pinch-off of the droplet. As can be seen from figure 2, as the jet velocity increases, the droplet pinch-off time \(t_d\) and the droplet radius \(r_d\) decrease.

The effect of the melt temperature \(T_j\) on the jet formation dynamics is presented in figure 3. In the considered temperature range, it changes slightly. The decrease \(\rho(T_j)\) and \(\gamma(T_j)\) with increasing temperature leads to a decrease in \(t_d\) and \(r_d\). But this increases \(<v_d>\) and the length of the jet before droplet pinch-off.

The formation of jets with significantly different diameters (but with about the same droplet velocity \(<v_d>\)) is presented in figure 4. As can be seen from figure 4, as the jet diameter increases, the droplet pinch-off time \(t_d\) increase significantly. According to the obtained results, it takes \(\sim 25\) ns to the drop pinch-off with a diameter of \(\sim 0.5\) µm and the velocity \(<v_d>\sim 150\) m/s⁻¹.

The time dependence of the radii of the droplet-jet neck \(r_n\) and droplet \(r_d\) for various parameters of the task is presented in figure 5. Their relationship is shown in figure 6. The numbers of the curves correspond to the numbers of lines in table 1. As can be seen from figure 5, the neck formation process has two stages. In the first stage \(r_n/r_d > 0.3\), the neck radius \(r_n\) decreases linearly with time. In the second nonlinear stage \(r_n/r_d < 0.3\), the neck radius decreases much faster. According to the simulation results, the neck formation time \(\Delta t_n = t_d - t_a\) increases linearly with the droplet radius \(r_d\) and decreases with increasing the drop velocity \(<v_d>\).

![Figure 3](image_url)  
*Figure 3. Jet formation and droplet pinch-off at the same initial jet velocities \(v_{zf}\), but different melt temperatures: solid curve – \(T_j = 2000\) K, \(p_0 = 3.67 \times 10^8\) Pa; dotted curve – \(T_j = 5000\) K, \(p_0 = 5.33 \times 10^8\) Pa. Calculated parameters: \(r_j = 0.5\) µm, \(r_i = 1.5r_j\), \(t_{\text{off}} = 1\) ns.*
Figure 4. Jet formation and droplet pinch-off at $T_j = 5000$ K and the different $j_b$: a) – line 2, b) – line 5, c) – line 9 in table 1.

Figure 5. The time dependence of the radii of the droplet-jet neck $r_n$ and droplet $r_d$ for various parameters of the task. The numbers of the curves correspond to the numbers of lines in table 1.

Figure 6. The time dependence of the radii of the droplet-jet neck $r_n$ and droplet $r_d$ for various parameters of the task. The numbers of the curves correspond to the numbers of lines in table 1.
The calculated dependences of $\beta_j$ on time. The numbers of the curves correspond to the numbers of lines in table 1.

The simulation of the process of the droplet pinch-off also made it possible to calculate the time dependence of the coefficient of enhancement of current density in the droplet-jet neck $\beta_j = S_d/S_n$, where $S_d$ is the total area of the droplet surface and $S_n$ is the cross-sectional area of the droplet-jet neck [2, 7]. The calculated dependences of $\beta_j$ on time are presented in figure 7. As can be seen from figure 7, the main growth of $\beta_j > 100$ occurs in the nonlinear stage of the droplet-jet neck formation.

Let us estimate the current density $j^\text{exp}_{S_j}$ at which we can expect the development of temperature runaway in the neck due to Joule heating, leading to its electrical explosion. In the studies of the electrical explosion of conductors, specific action integral $\bar{h} = \int_0^{t_{\text{exp}}} j^2 dt$ (with $t_{\text{exp}}$ being the explosion delay time) acts as an explosion criterion. According to the data given in [18], as the cross-sectional area of conductor is changed by a factor of 2000 and the current density by a factor of 10, the quantity $\bar{h}$ changes by no more than 10%. Therefore, we can assume that the quantity $\bar{h}$ characterizes the physical properties of the metal and for copper $\bar{h} = 4.1 \times 10^9 \text{A}^2\text{s}\cdot\text{cm}^{-4}$. With reference to the problem under consideration, for the current density in the neck we may write $j_n = j_{S_j} \beta_j$, where $j_{S_j}$ is the current density on the droplet surface. Then for $j^\text{exp}_{S_j}$ we have

$$j^\text{exp}_{S_j} = \sqrt{\bar{h}/B_2}$$

where $B_2 = \int_{S_j} \beta_j^2 dt$ (6)

The calculated values of $B_2$ and $j^\text{exp}_{S_j}$ based on the simulation results are presented in table 1. As can be seen from last column of the table 1, in all cases the current density $j^\text{exp}_{S_j}$ does not exceed $10^7 \text{A}\cdot\text{cm}^{-2}$.

4. Conclusion

In the context of a 2D axisymmetric statement of the problem, a hydrodynamic model of the formation of liquid-metal jet in the cathode spot has been developed to describe the droplet pinch-off. The jet formation has been simulated for a copper cathode in the “inertial” mode of the melt splashing until the first droplet pinch-off. It has been shown that for the jets with diameter $\sim 0.1–0.5 \mu\text{m}$ and velocity $\sim 100–300 \text{m}\cdot\text{s}^{-1}$, the droplet-jet pinch-off time is $\sim 7–30 \text{ns}$. The droplet-jet pinch-off time decreases with increasing jet velocity and increases for droplets of larger diameter. The simulation predicted the electrical explosion of the droplet-jet neck on the droplet surface $\geq 10^7 \text{A}\cdot\text{cm}^{-2}$. 

Figure 7. The calculated dependences of $\beta_j$ on time. The numbers of the curves correspond to the numbers of lines in table 1.
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References

[1] Mesyats G A and Proskurovsky D I 1989 Pulsed Electrical Discharge in Vacuum (Berlin: Springer) pp 110–114
[2] Mesyats G A 2000 Cathode Phenomena in a Vacuum Discharge: The Breakdown, the Spark and the Arc (Nauka, Moscow) pp 167–170
[3] Hantzsche E, Juttner B and Puchkarev V F 1976 J. Phys. D: Appl. Phys. 9 1771
[4] Hantzsche E 1977 Beiträge aus der Plasmaphysik 17 65
[5] McClure G W 1974 J. Appl. Phys. 45 2078
[6] Mesyats G A 1994 JETP Lett. 60 593
[7] Litvinov E A, Mesyats G A, Parfenov A G and Fedosov A I 1985 Zh. Tekh. Fiz. 55 2270
[8] Juttner B 1979 Beitr. Plasma Phys. 19 25
[9] Daalder J E 1978 Cathode erosion of metal vapor arcs in vacuum (PhD dissertation Tech. Univ. Eindhoven)
[10] Daalder J E 1976 J. Phys. D: Appl. Phys. 9 2379
[11] Utsumi T and English J H 1975 J. Appl. Phys. 46 126
[12] Mesyats G A and Uimanov I V 2015 IEEE Trans. on Plas. Sci. 43 2241
[13] Mesyats G A and Uimanov I V 2017 IEEE Trans. on Plas. Sci. 45 2087
[14] Mesyats G A and Uimanov I V 2018 Proc. of the 28th Inter. Symp. on Discharges and Electrical Insulation in Vacuum vol 2 (New York: IEEE) pp 397–400
[15] Gashkov M A, Zubarev N M, Zubareva O V, Mesyats G A and Uimanov I V 2016 J. Exper. and Theor. Phys. 122 776
[16] Gashkov M A, Zubarev N M, Mesyats G A and Uimanov I V 2016 Tech. Phys. Lett. 42 852
[17] Gashkov M A, Zubarev N M, Mesyats G A and Uimanov I V 2018 Proc. of the 28th Inter. Symp. on Discharges and Electrical Insulation in Vacuum vol 1 (New York: IEEE) pp 365–368
[18] Chace W G and Moore H K 1959 Exploding Wires 1 (New York: Plenum Press)