A theory of jet shapes and cross sections: from hadrons to nuclei

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For jets, with great power comes great opportunity. The unprecedented center of mass energies available at the LHC open new windows on the QGP: we demonstrate that jet shape and jet cross section measurements become feasible as a new, differential and accurate test of the underlying QCD theory. We present a first step in understanding these shapes and cross sections in heavy ion reactions. Our approach allows for detailed simulations of the experimental acceptance/cuts that help isolate jets in such high-multiplicity environment. It is demonstrated for the first time that the pattern of stimulated gluon emission can be correlated with a variable quenching of the jet rates and provide an approximately model-independent approach to determining the characteristics of the medium-induced bremsstrahlung spectrum. Surprisingly, in realistic simulations of parton propagation through the QGP we find a minimal increase in the mean jet radius even for large jet attenuation. Jet broadening is manifest in the tails of the energy distribution away from the jet axis and its quantification requires high statistics measurements that will be possible at the LHC.

I. INTRODUCTION

Fast partons propagating in a hot/dense nuclear medium are expected to lose a large fraction of their energy [1]. In fact, the stopping power of strongly-interacting matter for color-charged particles has, by far, the largest experimentally established effect: the attenuation of the cross section for final-state observables of large mass/momentum/energy. This jet quenching mechanism has been used to successfully explain the strong suppression of the hadron spectra at large transverse momentum observed in nucleus-nucleus collisions at the Relativistic Heavy Ion Collider (RHIC) [2,3]. There is mounting evidence that the quark-gluon plasma (QGP)-induced quenching can be disentangled from other nuclear effects even at the much lower Super Proton Synchrotron (SPS) center of mass energy [4]. To calculate parton energy loss in the QGP several theoretical approaches have been developed [5,6,7,8]. While tackling the same basic problem, they use different assumptions for the boundary conditions (initial/final quark or gluon virtuality), make different approximations for the parent parton and radiative gluon kinematics, and treat differently the interaction between the jet+gluon system with the medium (differentially vs on average). For discussion see [3].

At present, most measurements of hard processes are limited to single particles and particle correlations, which are only the leading fragments of a jet. There is general agreement on the physics that controls inclusive particle suppression in the QGP and the experimental methodology of determining $R_{AA}(p_T)$ (or $I_{AA}(p_{T1},p_{T2})$) [2]. Thus, for any particular combination of radiative/collisional energy loss evaluation and its phenomenological application to leading particle quenching the QGP density can be determined with $20-25\%$ “statistical” accuracy [10]. An inherent limitation of this approach is that while fits to the data can, not surprisingly, always be performed [11] they do not resolve the staggering order of magnitude “systematic” discrepancy in the extracted medium properties. Furthermore, focusing on quantities that can be constrained with little ambiguity from the measured rapidity entropy density in heavy ion collisions distracts from issues such as the approximations that go into theoretical energy loss derivations and their application to systems where more often than not these initial assumptions are violated [12]. In searching for experimental measurements which can pinpoint the framework for energy loss calculations that is applicable to heavy ion reactions, complex multi-particle correlations may not be optimal. They are very sensitive to non-perturbative effects/fragmentation and the modeling effort cannot be systematically improved due to the violation of factorization for highly exclusive observables [12]. It is, therefore, critical to find alternatives that accurately reflect the energy flow in strongly-interacting systems and have a more direct connection to the underlying quantum chromodynamics (QCD) theory.

The intra-jet energy distribution and the related cross section for jets in the case of heavy ion reaction closely match the criteria outlined above. The high rate of hard probes at the LHC and the large-acceptance calorimetry, see e.g. [13], will enable these accurate measurements. It should be noted that proof-of-principle measurements of jet cross sections have become possible at RHIC [14], but significantly better statistics will be required to quantify the QGP effects on jets. In this paper we study the magnitude of these modifications in Pb+Pb collisions at $\sqrt{s} = 5.5$ TeV at LHC. We demonstrate that a natural
generalization of leading particle suppression to jets,

\[ B_{\Delta A}^{\text{jet}}(E_T; R_{\text{max}}, \omega_{\text{min}}) = \frac{d\sigma_{\Delta A}^{\text{jet}}(E_T; R_{\text{max}}, \omega_{\text{min}})}{d\sigma^{\text{pp}}(E_T; \omega_{\text{min}})} , \]

is sensitive to the nature of the medium-induced energy loss. The steepness of the final-state differential spectra amplifies the observable effect and the jet radius \( R_{\text{max}} \) and the minimum particle/tower energy \( p_T_{\text{min}} \approx \omega_{\text{min}} \) provide, through the evolution of \( B_{\Delta A}^{\text{jet}}(E_T; R_{\text{max}}, \omega_{\text{min}}) \) at any fixed centrality, experimental access to the QGP response to quark and gluon propagation.

In the following discussion of jet shapes and jet cross sections in p+p and A+A collisions we stay as close as possible to an analytic theoretical approach. Thus, we are able to unambiguously connect the non-perturbative QCD effects, medium properties and the induced bremsstrahlung spectrum to experimental jet observables. Determination of baseline jet shapes and their generalization to finite momentum acceptance cuts builds up on the work of Seymour [15]. We refer the reader to [15, 16] for discussion of the complications in defining a jet and the related topic of jet-finding algorithms. To make the discussion simpler, we will assume that the complications of the different definitions can be subsumed into a \( R_{\text{sep}} \) parameter, as described in Appendix B. Once a jet axis and all of the jet particles / calorimeter towers “i” have been identified, the “integral jet shape” is defined as:

\[ \psi_{\text{int}}(r; R) = \frac{\sum_i (E_T)_i \Theta(r - (R_{\text{jet}})_i)}{\sum_i (E_T)_i \Theta(R - (R_{\text{jet}})_i)} \]

where \( r, R \) are Lorentz-invariant opening angles, \( R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2} \), and \( i \) represents a sum over all the particles in this jet. \( \psi_{\text{int}}(r; R) \) is the fraction of the total energy of a jet of radius \( R \) within a subcone of radius \( r \). It is automatically normalized so that \( \psi_{\text{int}}(R; R) = 1 \). To move from the integrated to the differential jet shape, we define:

\[ \psi(r; R) = \frac{d\psi_{\text{int}}(r; R)}{dr} . \]

This is the angular density of jet energy (remembering that the appropriate 3D representation would be \( \psi_{\text{int}}(r; R) = \frac{1}{2\pi r} \psi(r; R) \)). Understanding the many-body QCD theory behind jet shape calculations will naturally lead to understanding the attenuation of jets in reactions with heavy nuclei.

This article is organized as follows: in Section II we outline a calculation of the jet shape in nucleon-nucleon (N+\( N \)) collisions using the framework of perturbative QCD. We compare this calculation to existing Tevatron data and investigate the jet shapes at LHC energies. A brief discussion of final-state QGP-induced radiative energy loss in the GLV formalism is given in Section III. We prove that the cancellation of small-angle near-jet axis bremsstrahlung persists to all orders in the correlation between the multiple scattering centers and provide details of its numerical evaluation. The fully differential distribution of the energy lost by a hard parton is also shown. In Section IV we present results for the medium-modified jet shapes and cross sections as a function of the jet cone radius, \( R \), and the experimental \( p_T \) cut, \( \omega_{\text{min}} \), and discuss a simple energy sum rule. We demonstrate the connection between the characteristic properties of the QGP-induced gluon radiation and the variable suppression, at the same impact parameter \( b \), of jet rates, the modulation of the mean jet radius and the enhancement in the “tails” of the intra-jet energy flow distribution. A summary and conclusions are presented in Section V. Appendix A shows a calculation of the baseline jet cross sections at the LHC and an estimate of the accuracy with which these cross sections and jet shapes can be measured with nominal first-year integrated luminosities in p+p and A+A reactions. In Appendix B we study the influence of the different perturbative and non-perturbative contributions to the jet shape in hadronic collisions. Finally, Appendix C contains a discussion of a double differential measure of energy flow in jets and its connection to particle angular correlation measurements, currently conducted at RHIC.

II. JET SHAPES IN 'ELEMENTARY' P-P COLLISIONS

In the process of advancing perturbative QCD theory to many-nucleon systems, the final-state experimental observables should be first understood in the simpler p+p reactions.

A. Theoretical considerations

1. Leading order results

In the introduction, we defined the central quantity of our study, the differential jet shape for parton “a”, \( \psi_a(r; R) \). As in [15], the starting point of the calculation is the leading order parton splitting: a suitable separation of physical time scales enables the separation of the calculation into production and jet showering. The QCD splitting functions \( P_{a \to bc}(z) \) give the distribution of the large fractional lightcone momenta (or approximately the energy fractions) of the fragments relative to the parent parton, \( z \) and \( 1 - z \) respectively. To lowest order, recalling that \( \psi_a(r; R) \) describes the energy flow \( \propto z \), we can write:

\[ \psi_a(r; R) = \frac{d\psi_{\text{int},a}(r; R)}{dr} = \sum_b \frac{\alpha_s}{2\pi} \rho \int_{z_{\text{min}}}^{1-z} d\tau z P_{a \to bc}(z). \]

In Eq. (4) \( r = (1-z)\rho \) is related to the opening angle \( \rho \) between the final-state partons.
In “elementary” p+p collisions the inclusion of soft particles ($z_{\text{min}} \approx 0$) in theoretical calculations is not a bad approximation. Even in this case, however, there are intrinsic limitations on the minimum particle calorimeter tower $p_T$ or $E_T$, related, for example, to detector acceptance. In heavy ion reactions, especially for the most interesting case of central collisions, there is an enormous background of soft particles related to the bulk QGP properties. Jet studies will likely require minimum particle energy $> 1 - 2$ GeV at RHIC and even more stringent cuts at the LHC. Furthermore, control over $z_{\text{min}}$ can provide detailed information about the properties of QGP-induced bremsstrahlung. Further kinematic constraints on the values of $z$ arise since both the resulting partons must be within an angular distance $R$ of the original jet axis, $r < R$, $rz/(1 - z) < R$. In this case they are identified with the jet. If not, they are identified as two separate jets. For a cone-based algorithm, the relative separation $R_{\text{sep}}R$ (as opposed to just the distance from the original jet axis) is an additional criterion: $\rho < R_{\text{sep}}R$. We find:

$$Z = \max \left\{ \frac{z_{\text{min}}, r}{r + R} \right\} \text{ if } r < (R_{\text{sep}} - 1)R,$$

$$Z = \max \left\{ \frac{z_{\text{min}}, r}{R_{\text{sep}}R} \right\} \text{ if } r > (R_{\text{sep}} - 1)R. \quad (5)$$

Carrying out the integration in Eq. (4) we arrive at the LO jet shape functions for quarks and gluons:

$$\psi_q(r) = \frac{C_F \alpha_s 2}{2\pi} \frac{1}{r} \left( 2 \log \frac{1 - z_{\text{min}}}{Z} - \frac{3}{2} \left[ (1 - Z)^2 - z_{\text{min}}^2 \right] \right); \quad (7)$$

$$\psi_g(r) = \frac{C_A \alpha_s 2}{2\pi} \frac{1}{r} \left( 2 \log \frac{1 - z_{\text{min}}}{Z} - \frac{11}{6} - \frac{Z}{3} + \frac{Z^2}{2} \right) \left( 1 - Z \right)^2 \right.$$

$$\left. + \left( \frac{2z_{\text{min}}^2}{3} - \frac{2}{3} z_{\text{min}}^3 + \frac{1}{4} z_{\text{min}}^4 \right) \right)$$

$$+ \left( \frac{T_R N_f \alpha_s 2}{2\pi} \left( \frac{2}{3} - \frac{2Z}{3} + Z^2 \right) \left( 1 - Z \right)^2 \right.$$ \n
$$\left. - \left( z_{\text{min}}^2 - \frac{4}{3} z_{\text{min}}^3 + \frac{4}{3} z_{\text{min}}^4 \right) \right) \right). \quad (8)$$

In the $z_{\text{min}} \to 0$ limit Eqs. (7) and (8) reduce and we recover the previously known results [15]. There is an implicit ‘plus-prescription’ in these results when calculating moments of physical quantities, as we have not considered the virtual corrections in the forward direction. Hence, the result is not applicable for $r = 0$ and does not have the correct normalization when integrated. However, the shape is reflective of final-state parton splitting and, when needed, for the leading order calculation one may apply a cutoff for small $r$ and normalize via first-bin subtraction.

In contrast to the case of $e^+ + e^-$ annihilation, hadronic scattering is accompanied by copious initial-state radiation (ISR) that can fall within the jet cone. While the contribution of the ISR is small for small values of $r/R$, it gives an essential contribution at larger angles. A simple estimate based on a dipole radiation and the kinematics of the hard parton - soft gluon coincidence within a cone [16], similar for both quark and gluon jets, yields:

$$\psi_i(r) = \frac{C_F \alpha_s 2}{2\pi} \left( \frac{1}{R^2} - \frac{1}{(1 - z_{\text{min}})^2} \right), \quad (9)$$

Again, the ‘plus-prescription’ to account for the $r = 0$ point is not explicitly shown. In Eq. (9) $C \approx C_F \approx C_A/2$.

The leading order calculation is most appropriate for the rare, hard splittings of a very high momentum jet. The use of a running coupling improves the numerical results, providing larger weight for softer events. We employ a running $\alpha_s$ evaluated at the largest $k_T$ in the problem, $\mu = r(1 - Z)E_T$ for the jet splitting and $\mu = (1 - Z)E_T$ for the initial state radiation [12].

2. Resummation - all orders and multiple emission

As $r \to 0$, in the collinear limit of parton splitting, the leading order contributions to the jet shape diverge, see Eqs. (7), (8) and (9). In fact, all orders in the perturbative expansion diverge, including powers of $\log r$ in the form $\alpha_s^n \log^{2n-1} r$. With plentiful parton showering, it becomes increasingly less likely that any particular quark and gluon will be coincident with the jet axis. Quantitatively, this is described by a Sudakov form factor. The energy density at small angles is dominated by the hard parton in the splitting. If there is a splitting that leaves the hard parton at an angle $r_1$, a subsequent splitting at $r_2 < r_1$ will not contribute to the energy density at $r_2$. Multiple independent splitting follows a Poisson distribution, hence, the probability of energy flow at an angle less than $r$ is exponentially suppressed by the integrated probability at angles greater than $r$, i.e.:

$$P(< r) = \exp(-P(> r))$$

$$= \exp \left(- \int_r^0 dr' \psi_{\text{coll}}(r') \right). \quad (10)$$

This only applies for soft emissions which do not take away (much) momentum, i.e. at leading log accuracy. Improvement can be obtained at modified leading log accuracy (MLLA), when the running of the coupling constant is included in $P_1$. We don’t take other (e.g. recoil or kinematic constraints) effects into account in evaluating the Sudakov form factor in the soft collinear approximation.

The resummed $\psi_{\text{resum}}(r) = \frac{4\pi}{\alpha_s} P(< r)$ and we carry out the integration in Eq. (10), including the running $\alpha_S(rE_T)$, to obtain modified leading logarithmic accuracy (MLLA). Note that $\alpha_S(\mu) = 1/(2\beta_0 \log \frac{r}{\Lambda_{\text{QCD}}})$, with $4\pi\beta_0 = b_0 = \frac{11}{3} C_A - \frac{4}{3} T_R N_f$. First, take the small $r$
limit in Eqs. (7), (8) and (9), keeping terms $\propto 1/r$ and $\propto 1/r \log(1/r)$. Based on $Z = \max(z_{\min}, r/R)$, in this limit we have two kinematic domains. For $r > z_{\min}R$ the results are similar to the known case of no acceptance cut-off and reduce to the known results if $z_{\min} = 0$:

$$P_q(r > z_{\min}R) = \exp \left( 2C_F \log \frac{R}{r} \right) \left( 2\beta_0 \alpha_s \log \frac{R}{r} \right)$$

$$- \left[ \frac{3}{2} C_F - C_R^2 - c_q^>(z_{\min}) \right]$$

$$\times f_2 \left( 2\beta_0 \alpha_s \log \frac{R}{r} \right),$$

(12)

$$P_g(r > z_{\min}R) = \exp \left( 2C_A \log \frac{R}{r} \right) \left( 2\beta_0 \alpha_s \log \frac{R}{r} \right)$$

$$- \left[ \frac{1}{2} b_0 - C_R^2 - c_g^>(z_{\min}) \right]$$

$$\times f_2 \left( 2\beta_0 \alpha_s \log \frac{R}{r} \right).$$

(13)

It is useful to employ the same notation as in [15] and facilitate the comparison to the case of no kinematic cuts: $f_1(x) = \log(1-x)/(2\pi\beta_0)$, and $f_2(x) = (1 – \log(1 – x)/x)/(2\pi\beta_0)$. The $z_{\min}$-dependent corrections are isolated as follows:

$$c_q^>(r > z_{\min}R; z_{\min}) = 2C_F \log(1 - z_{\min}) + \frac{3}{2} C_F z_{\min}^2,$$

(14)

$$c_g^>(r > z_{\min}R; z_{\min}) = 2C_A \log(1 - z_{\min}) + C_A \left( 2z_{\min} - \frac{2}{3} z_{\min}^3 + \frac{1}{2} z_{\min}^4 \right)$$

$$- T_R N_f \left( z_{\min} - \frac{4}{3} z_{\min}^3 + z_{\min}^4 \right).$$

(15)

When $r < z_{\min}R$ the integration in Eq. (11) has to be split in two regions: $r' \in (r, z_{\min}R)$ and $r' \in (z_{\min}R, R)$. The second integral is trivially obtained from the case that was just considered above, Eqs. (12) and (13), with the substitution $r = z_{\min}R$. When combined with the first integral, it yields:

$$P_q(r < z_{\min}R) = P_q(r > z_{\min}R; r = z_{\min}R)$$

$$\times \exp \left( - \left[ \frac{3}{2} C_F - c_q^>(z_{\min}) \right] \times f_2 \left( 2\beta_0 \alpha_s \log \frac{z_{\min}R}{r} \right) \right),$$

(16)

$$P_g(r < z_{\min}R) = P_g(r > z_{\min}R; r = z_{\min}R)$$

$$\times \exp \left( - \left[ \frac{1}{2} b_0 - c_g^>(z_{\min}) \right] \times f_2 \left( 2\beta_0 \alpha_s \log \frac{z_{\min}R}{r} \right) \right).$$

(17)

Here, we denote by $\bar{\alpha}_s = \alpha_s(z_{\min}RE_T)$, as opposed to $\alpha_s = \alpha_s(RE_T)$, and:

$$c_q^< (r < z_{\min}R; z_{\min}) = 2C_F \log \left( 1 - \frac{z_{\min}}{z_{\min}} \right) + 3C_F z_{\min},$$

(18)

$$c_g^< (r < z_{\min}R; z_{\min}) = 2C_A \log \left( 1 - \frac{z_{\min}}{z_{\min}} \right)$$

$$+ C_A \left( 4z_{\min} - \frac{2}{3} z_{\min}^2 + \frac{2}{3} z_{\min}^3 \right)$$

$$- T_R N_f \left( 2z_{\min} - 2z_{\min}^2 + \frac{4}{3} z_{\min}^3 \right).$$

(19)

Note that the Sudakov form factors evaluated here will regulate any collinear divergence present in $\psi_{\text{coll}}$.

3. Power corrections - an estimate of non-perturbative effects

Inclusion of the running coupling constant under the momentum transfer integrals yields contributions from regions in which $Q \sim \Lambda_{QCD}$ or lower, i.e. there is a fundamental non-perturbative contribution to all of the integrals. An estimate of these power correction effects with finite acceptance gives the following result:

$$\psi_{PC}(r) = \frac{2C_R 2Q_0}{2\pi r E_T} \left( \alpha_0'(Q_0, k_{\min}) - \alpha_s(\mu) \right)$$

$$- 2\beta_0 \alpha_s(\mu)^2 \left( 1 + \log \frac{\mu}{Q_0} \right)$$

$$+ \frac{2C_R 2k_{\min}}{2\pi r E_T} \left( \alpha_s(\mu) \right)$$

$$+ 2\beta_0 \alpha_s(\mu)^2 \left( 1 + \log \frac{\mu}{k_{\min}} \right),$$

(20)

where $C_R = C_F, C_A$ for quarks or gluons, respectively. In Eq. (20) $k_{\min} = z_{\min}RE_T$ and $\mu$ is the renormalization scale. The term $\propto \bar{\alpha}_0'(Q_0, k_{\min})$ depends on the parametrized non-perturbative contribution, defined as:

$$\bar{\alpha}_0'(Q_0, k_{\min}) = \frac{1}{Q_0} \int_{k_{\min}}^{Q_0} dk \alpha_s(k),$$

(21)

with $Q_0$ representing the non-perturbative scale. In our numerical calculation we use

$$\bar{\alpha}_0'(2 \text{ GeV}, 0) = 0.52, \quad \bar{\alpha}_0'(3 \text{ GeV}, 0) = 0.42$$

(22)

from Ref. [13, 18] and a parametrization of the strong coupling constant at small momentum transfer given in Ref. [19]. The terms $\propto \alpha_s(\mu), \alpha_s^2(\mu)$, come from subtracting the perturbative component in the non-perturbative region [18]. Finally, the term $\propto k_{\min}$ results from the introduction of finite acceptance, $z_{\min}$. 
A similar expression is derived for initial-state radiation:

\[
\psi_{\text{PC}}(r) = \frac{2C}{2\pi} \frac{Q_0}{E_T} \left( r_0'(Q_0, k'_{\text{min}}) - \alpha_s(\mu) \right) \\
-2\beta_0 \alpha_s(\mu)^2 \left( 1 + \log \frac{\mu}{Q_0} \right) \\
+2C R_{\text{sep}} \frac{Q_0}{E_T} \alpha_s(\mu) \\
+2\beta_0 \alpha_s(\mu)^2 \left( 1 + \log \frac{\mu}{k'_{\text{min}}} \right), \tag{23}
\]

where \( k'_{\text{min}} = z_{\text{min}} E_T \), and \( C \simeq C_F \approx C_A/2 \). At lower jet energies the power corrections are, in fact, sizable even at large \( r \). This suggests that if there is deviation between the theoretical results and the experimental data it may be largely due to incomplete consideration of non-perturbative effects. We stress that the generalization of power corrections to finite acceptance implies that these should be taken into account only for \( z_{\text{min}} r E_T < Q_0 \) or \( z_{\text{min}} E_T < Q_0 \) for final-state and initial-state radiation, respectively.

4. Total contribution to the jet shape

As indicated before, the resummed jet shape at small \( r/R \) is evaluated as \( \psi_{\text{resum}}(r) = \frac{d}{dr}P(r) = \psi_{\text{coll}}(r)P(r) \). Taking all contributions to the jet shape and ensuring that there is no double counting at small \( r/R \) to \( \mathcal{O}(\alpha_s) \) we find:

\[
\psi(r) = \psi_{\text{coll}}(r) (P(r) - 1) + \psi_{\text{LO}}(r) + \psi_{\text{NLO}}(r) \\
+ \psi_{\text{PC}}(r) + \psi_{\text{PC}}(r), \tag{24}
\]

On the right-hand-side of Eq. (24) the first term comes from Sudakov resummation with subtraction of the leading \( 1/r \), \( (1/r) \log(1/r) \) contribution at small \( r/R \) to avoid double counting with the fixed order component of the differential jet shape. The second and third terms represent the leading-order contributions in the final-state and the initial-state. The last two terms represent the effect of power corrections. In a full calculation the relative quark and gluon fractions \( f_q + f_g = 1 \) are also needed:

\[
\psi(r; E_T) = f_q(E_T, \sqrt{s}) \psi_q(r; E_T) + f_g(E_T, \sqrt{s}) \psi_g(r; E_T).
\]

These fractions are calculated in Appendix A alongside the demonstration of the feasibility of jet cross section and differential jet shape measurements at \( \sqrt{s} = 5.5 \text{ TeV} \).

If the resummed part completely dominates the area under \( \psi(r) \) in Eq. (24), i.e. the power corrections and the fixed order result affect only the large \( r/R \) “tails”, the theoretically calculated differential jet shape is properly normalized. In reality, this is not the case and the first correction, \( \mathcal{O}(\alpha_s^2) \), arises from \( \psi_{\text{coll}}(r)(P(< r) - 1) = \psi_{\text{coll}}(r)(1 + C_R \alpha_s(\cdots) + \cdots - 1) \). It will be larger for gluon jets when compared to quark jets, \( C_A \) vs \( C_F \), and for lower transverse energies. The normalization can then be ensured via

\[
\psi(r) \rightarrow \psi(r) + \text{Norm} \times \psi_{\text{coll}}(r) \ln(P(< r)) , \tag{25}
\]

where Norm is determined numerically. We stress that to achieve a robust theoretical description of the differential jet shape all contributions to \( \psi(r) \) from Eq. (24) should be included. In Appendix B we elucidate their relative strength using numerical examples. We also investigate the dependence of the shape on \( R_{\text{sep}} \) and the non-perturbative scale \( Q_0 \).

B. Comparison to the Tevatron data

In Fig. 1 we show comparison of the theoretical model for the jet shape, Eq. (24), to the experimental measurements in \( p + \bar{p} \) collisions at \( \sqrt{s} = 1960 \text{ GeV} \) at Fermilab from Run II (CDF II) [20]. Our numerical results include all contributions from leading order, resummation and
power corrections with $Q_0 = 2$ GeV. The insert shows the variation of the parameter $R_{sep}$ with the transverse energy of the jet. At high jet $E_T$ our theoretical model gives very good descriptions of the large $r/R$ experimental data with $R_{sep} = 1.3 - 1.4$. For $E_T = 45 - 55$ GeV the largest meaningful value $R_{sep} = 2$ can describe the data fairly well, except at very small $r/R$ region. Extended discussion of the various contributions to the differential jet shape is given in Appendix B.

We note that for $r/R \ll 1$ and a large gluon jet fraction in conjunction with moderate $E_T \leq 50$ GeV there is still deviation between the data and the theory, e.g. the top panel of Fig. 1. This is likely related to the need for significant corrections, Eq. (26), to ensure the proper normalization of $\psi(r/R)$. Such corrections, in turn, point to NLO effects, a possible breakdown of our soft collinear jet splitting approximation for the Sudakov resummation and non-perturbative effects. Note that even Monte Carlo event generators have to be tuned to describe this data [20]. For the purpose of our manuscript the deficiencies in this specific part of phase space are not essential since, as we will see in Section IV, the experimental signatures of jet propagation in the QGP are most pronounced in the complementary $r/R \sim 1$ domain. One simply has to keep in mind that the description of $r/R < 0.25 \psi(r/R)$ in the vacuum allows for further theoretical improvement.

C. Predictions for the LHC

We employ the theoretical model that describes the CDF II data and apply the same transverse energy-dependent $R_{sep}$ parameter to obtain predictions for the LHC at $\sqrt{s} = 5.5$ TeV. The emphasis here is to produce a baseline in $p+p$ reactions for comparison to the full in-medium jet shape in $Pb+Pb$ collisions. The essential difference in going from the Tevatron to the LHC is in the production of hard jets. At the higher collision energy we observe a greater contribution from gluon jets relative to quark jets, e.g. Fig. 12 in Appendix A. Therefore, for the same $E_T$, jets at the LHC are expected to be slightly wider than at the Tevatron.

Figure 2 shows our numerical results for the jet shape for four different energies $E_T = 50, 100, 250, 500$ GeV and two cone radii $R = 0.7, 0.4$ in $p+p$ collisions at $\sqrt{s} = 5.5$ TeV at LHC. An interesting observation is that, when plotted against the relative opening angle $r/R$, these shapes are self-similar, i.e. approximately independent of the absolute cone radius $R$. One of the main theoretical developments in this paper is the analytic approach to studying finite detector acceptance effects or experimentally imposed low momentum cuts. In Fig. 2 this is illustrated via the selection of $z_{\min} = p_{T \min}/E_T = 0.2, 0.1, 0.04$ and 0.02 ($p_{T \min} = \omega_{\min} = 10$ GeV). Eliminating the soft partons naturally leads to a narrower branching pattern. However, for this effect to be readily observable $10 - 20\%$ of the jet energy, going into soft particles, must be missed. Thus, even with $p_{T \min} \sim$ few GeV cuts in $Pb+Pb$ collisions at the LHC aimed at reducing or eliminating the background of bulk QGP particles that accidentally fall within the jet cone, the alteration of $\psi(r/R)$ is expected to be small. We also studied the integral jet shape $\Psi_{\text{int}}(r/R)$, shown in the inserts of Fig. 2 as a tool for identifying kinematic and dynamic effects on jets [27]. Only when large differences exist between two $\psi(r/R)$ for $r/R < 0.4$ these will be reflected in the integral jet shape. If the differences are pronounced in
FIG. 3: (Color online) 3D plot of the differential jet shapes at three different jet energies \( E_T = 20 \text{ GeV} \) (top panel), \( E_T = 100 \text{ GeV} \) (middle panel), and \( E_T = 500 \text{ GeV} \) (bottom panel) with \( R = 0.7 \), \( \omega_{\text{min}} = 0 \text{ GeV} \) in p+p collisions with \( \sqrt{s} = 5500 \text{ GeV} \) at the LHC. From low jet energy to high jet energy, jet shape becomes much steeper.

The principal medium-induced contribution to a jet shape comes from the radiation pattern of the fast quark or gluon, stimulated by their propagation and interaction in the QGP. There is a simple heuristic argument which allows one to understand how interference and coherence effects in QCD amplify the difference between the energy distribution in a vacuum jet and the in-medium jet shape [21]. Any destructive effect on the integral average parton energy loss \( \Delta E_{\text{rad}} \), such as the Lundau-Pomeranchuk-Migdal effect, can be traced at a differential level to the attenuation or full cancellation of the collinear, \( k_T \ll \omega \), gluon bremsstrahlung:

\[
\Delta E_{\text{LPM suppressed}}^{\text{rad}} \Rightarrow \frac{dI^g}{d\omega}(\omega \sim E)_{\text{LPM suppressed}} \\
\Rightarrow \frac{dI^g}{d\omega d^2k_T}(k_T \ll \omega)_{\text{LPM suppressed}},
\]

and we indicate the parts of phase space where the modification of the incoherent \( dI^g/d\omega d^2k_T \) is most effective.

Indeed, detailed derivation of the coherent inelastic parton scattering regimes in QCD was given in [21]. In all cases, the origin of the LPM suppression can be traced to the cancellation of the collinear bremsstrahlung. The destructive quantum interference is most prominent for final-state radiation, where the large-angle gluon bremsstrahlung was originally discussed in Ref. [22] to first order in opacity. Even though to carry out realistic simulations to higher orders in opacity with full geometry will require computational power beyond what is currently available, we first present an analytic proof that a cone-like pattern of medium-induced emission persists to all orders in the correlations between multiple scattering centers (elementary emitters) and we focus on the case of immediate interest: light quark and gluon jets and final-state bremsstrahlung. Generalization to massive partons can easily be achieved, see e.g. [23], but it is important to note that the effect of a heavy quark mass versus the jet energy depends on the coherent scattering regime [6].
A. Radiative energy loss in the GLV formalism

In our calculation we will use the GLV formalism of expanding the medium-induced radiation in the correlations between multiple scattering centers [8]. We first recall the definitions of the Hard, Gunion-Bertsch and Cascade propagators in terms of the gluon transverse momentum \( k \) and the transverse momentum transfers from the medium \( q_i \):

\[
H = \frac{k}{k^2}, \quad C_{(i_1i_2\ldots i_m)} = \frac{(k - q_{i_1} - q_{i_2} - \cdots - q_{i_m})}{(k - q_{i_1} - q_{i_2} - \cdots - q_{i_m})^2},
\]

\[
B_i = H - C_i, \quad B_{(i_1\ldots i_m)(j_1\ldots j_n)} = C_{(i_1\ldots j_m)} - C_{(j_1\ldots j_n)}, \quad (27)
\]

The relevant inverse gluon formation times can be written as:

\[
[\tau^f_{(i_1i_2\ldots i_m)}]^{-1} = \omega(i_1i_2\ldots i_m) = [k^+|C_{(i_1i_2\ldots i_m)}|^2]^{-1}. \quad (28)
\]

For final-state radiation, the intensity spectrum reads:

\[
k^+dN^g(FS) / dk^+d^2k = C_R\alpha_s \sum_{n=1}^{\infty} \left[ \prod_{i=1}^{n} \int d\Delta z_i / \lambda_g(z_i) \right] \left[ \prod_{j=1}^{n} \int d^2q_j \left( \frac{1}{\sigma_{cl}(z_j)} \frac{d\sigma_{el}(z_j)}{d^2q_j} - \delta^2(q_j) \right) \right] \times \left[ -2C_{(1\ldots n)}, \sum_{m=1}^{n} B_{(m+1\ldots n)(m\ldots n)} \left( \cos \left( \sum_{k=2}^{m} \omega(k\ldots n)\Delta z_k \right) - \cos \left( \sum_{k=1}^{m} \omega(k\ldots n)\Delta z_k \right) \right) \right], \quad (29)
\]

where \( \sum_{i=1}^{n} \equiv 0 \) and \( B_{(n+1\ldots n)} \equiv B_n \) are understood. In the case of final-state interactions, \( z_0 \approx 0 \) is the point of the initial hard scatter and \( z_L = L \) is the extent of the medium. The path ordering of the interaction points, \( z_L > z_{j+1} > z_j > z_0 \), leads to the constraint \( \sum_{j=1}^{n} \Delta z_i \lesssim z_L \). One implementation of this condition would be \( \Delta z_i \in [0, z_L - \sum_{j=1}^{i-1} \Delta z_j] \) and it is implicit in Eq. (29).

There is an obvious limit of the GLV radiative spectrum when \( L \gg \lambda_g \gg \tau_f \), where \( \lambda_g \) is the mean free path of the gluon in a hot QGP. Here, the contributions of the \( \cos(\cdots) \) terms vanish after integration over the unobserved \( q_i \) or \( \Delta z_i \) due to rapid oscillation. It is easy to see in this limit for \( n=1 \) that,

\[
k^+dN^g / dk^+d^2k = C_R\alpha_s \left( \frac{L}{\lambda_g} \right) \int d^2k \int d^2q_1 \times \left[ \frac{1}{\sigma_{cl}(d^2q_1)} [C_1^2 - H^2 + B_1^2] \right]. \quad (30)
\]

In the very high energy limit \( E \to \infty \), leading to large \( k \) phase space, a change of variables \( k \to k - q_1 \) shows that the first two terms in Eq. (30) cancel, leading to an incoherent Bertsch-Gunion gluon emission in a hot QGP medium with \( \langle n \rangle = \frac{k}{\lambda_g} \). By direct inspection one can see that the \( n \geq 2 \) terms do not contribute. In fact, it is easy to verify that for any bremsstrahlung regime, initial-state, final-state and no hard scattering, this limit holds [9]. More generally, in this limit it can be shown that the reaction operator \( \hat{R} \to 0 \). Naturally, for finite jet energies there will be corrections when \( k^+dN^g / dk^+d^2k \) is evaluated numerically with actual kinematic bounds [9].

B. Collinear radiation in GLV formalism

While the example given above illustrates that limits can be imposed and taken in the GLV results, such limits are artificial in that the formation time of the gluon at the emission vertex spans \( \tau_f \in (0, \infty) \). The Reaction Operator approach [8], i.e. the GLV formalism, is not an approach of averages: it compares differentially \( \tau_f \) to the separation between the scattering centers. For example, even when \( k \to 0 \) the formation time can be small or large, depending on the momentum transfers for the medium. Let us investigate this case in more detail: we note that \( k^+ \approx 2\omega \) and \( k \approx r\omega \hat{n} \), where \( r \) is the angle relative to the jet axis. Here, \( \hat{n} \) is a unit vector transverse to the jet axis which defines the azimuthal angle \( \phi \) of gluon emission. Using the results of Eq. (29), the 2D \((\phi, r)\) angular distribution of gluons at \( n \)-th order in the correlated scattering expansion reads:
Here, we have already set \( r = 0 \) where possible. We can use this general notation as long as we clarify certain special cases: for \( m = n \) we have \( \cos[(q_{n+1} + q_n)^2\Delta z_{n+1}/2\omega] \equiv 1 \). For the transverse propagators and \( m \) we have \( \omega r(q_{n+1} + q_n)/(q_{n+1} + q_n)^3 \equiv \hat{n} \). It is know that the leading \( n = 1 \) contribution to final-state medium-induced radiation leads to \( \lim_{r \to 0} \frac{\omega dN^g}{d\omega d\phi dr} = 0 \) [22]. Our goal is to show that this result is general and holds to any order in the expansion. Its implications are that there is very little overlap between the techniques used to compute the “vacuum” and medium-induced contributions to the jet shape. A general proof requires demonstration of the absence of unprotected divergences for any set of momentum transfers \( \{q_i\} \), finiteness of the momentum transfer integrals as \( q_i \to \infty \) and a mechanism that kills the small-angle contribution.

1. We first look at the large \( q_i \) limit. The transverse propagator contribution itself in Eq. (31) behaves as \( \sim 1/q_i^2 \). Furthermore, irrespective of the small \( q_i \), behavior of the momentum transfer distribution from the medium, for large momentum transfers the collision cross section is suppressed by the Rutherford \( \sim 1/q_i^4 \) behavior, ensuring the finiteness of the integrals.

2. Next, we examine the potential singularity as \( |q_1 + \cdots + q_n| \to 0 \). The difference in the LPM interference terms in this limit goes as \( \mathcal{O}((q_1 + \cdots + q_n)^2) \) and for the most problematic transverse propagator term \( (m=1) \) even as \( \mathcal{O}((q_1 + \cdots + q_n)^3) \). In summary, not only is there no divergence in this case, but the integrand in Eq. (31) vanishes.

3. We now collect the interference phases associated with problematic propagators as \( |q_k + \cdots + q_n| \to 0 \), \( 1 < k \leq n \). Expanding for a small net transverse momentum-sums we find that the singularity is canceled:

\[
\begin{align*}
&\left[ \sin \sum_{j=2}^{k-1} \frac{(q_1 + \cdots + q_n)^2}{2\omega} \Delta z_j \right] - \sin \sum_{j=1}^{k-1} \frac{(q_1 + \cdots + q_n)^2}{2\omega} \Delta z_j \right] \frac{(q_k + \cdots + q_n)^2}{2\omega} \Delta z_k .
\end{align*}
\]

Actually, the lack of singularities persists also away from the small \( r \) limit.

4. With the integrand well behaved and all integrals finite we see that the phase space factor \( r \) in the numerator is sufficient to ensure vanishing medium-induced bremsstrahlung contribution at the center of the jet. It is assisted by partial cancellation from angular integrals of the type \( \int f(q_i \cdot q_j) \, d\phi_{ij} \). It is only for the special case of \( \hat{n} \cdot (q_1 + \cdots + q_n) \) where the antisymmetric integrand under \( q_i \to -q_i \) for all \( i \) fully ensures the vanishing zero-angle radiative contribution.

This completes our proof that at any order in opacity

\[
\lim_{r \to 0} \frac{\omega dN^g}{d\omega d\phi dr} = 0 .
\]

Numerical simulations, using Monte-Carlo techniques, confirm independently that \( d\sigma/d\omega d^2k \) vanishes as \( k \to 0 \).

C. Numerical methods and QGP properties

Results relevant to the LHC phenomenology are calculated using full numerical evaluation of the medium-induced contribution to the observed jet shapes and the modification of the in-medium jet cross sections. Jet production, being rare in that \( \sigma(E_T > E_{T \text{min}}) T_{AA}(b) \ll 1 \), follows binary collision scaling \( \sim d^2N_{\text{bin}}/d^2x_L \). In contrast, the medium is distributed according to the number of participants density \( \sim d^2N_{\text{part}}/d^2x_L \). Soft particles that carry practically all of the energy deposited in the fire ball of a heavy ion collision cannot deviate a lot from such scaling. We take into account longitudinally Bjorken expansion since transverse expansion leads to noticeable corrections only in the extreme \( \beta_T \to 1 \) limit [24]. In our approach all relevant finite time and finite kinematics integrals, such as the ones over the separation between the scattering centers \( \Delta z_i = z_i - z_{i-1} \), the bremsstrahlung gluon phase space \( \Lambda_{QCD} < \omega < E_{\text{jet}} \), \( \Lambda_{QCD} < k_\perp < 2\omega \) [42], and the transverse momentum transfers \( 0 < q_\perp < \sqrt{s}/4 = m_D E_{\text{jet}}/2 \), are done numerically [4]. In our simulation we generated in-plane jets, \( \varphi_{\text{jet}} - \varphi_{\text{reaction plane}} = 0 \). This is of little importance in central Pb+Pb collisions (b=3 fm), where the medium effects on jet propagation are most pronounced, but in semi-central (b=8 fm) and peripheral (b=13 fm) reactions this will lead to smaller than average energy loss.
The evolving intrinsic momentum and length scales in the QGP expected to be created at the LHC are determined as follows: we first estimate the QGP formation time $\tau_0 = 1/(g_T) = 0.23$ fm, where $\langle p_T \rangle \approx 850$ MeV is obtained from extrapolations to LHC energies made by using Monte Carlo event generator results, fit to the CDF collaboration data from $\sqrt{s} = 1.8$ GeV $p + \bar{p}$ collisions [22]. Here we account for the observed $\sim 25\%$ increase in the mean transverse momenta in going from $N + N$ to $A + A$ collisions at RHIC. Gluons dominate the soft parton multiplicities at the LHC and their time- and position-dependent density can be related to charged hadron rapidity density in the Bjorken expansion model [20]:

$$\rho = \frac{1}{\tau} \frac{d^2(dN^g/dy)}{d^2x_\perp} \approx \frac{1}{\tau} \frac{1}{2} \left( \frac{dN^\text{ch}/dy}{d^2x_\perp} \right).$$  (34)

Here, $dN^\text{ch}/dy = \kappa N_{\text{part.}}/2$ with $\kappa \approx 9$ for $\sqrt{s} = 5.5$ TeV.

Table I summarizes characteristics of Pb+Pb collisions at the LHC and initial QGP properties. An inelastic cross section $\sigma_{\text{in}} = 65$ mb has been used in an optical Glauber model where necessary. Assuming local thermal equilibrium one finds:

$$T(\tau, x_\perp) = \frac{3}{\tau^2} \rho(\tau, x_\perp)/16\zeta(3), \tau > \tau_0.$$  (35)

The Debye screening scale is given by $m_D = gT$, recalling that we work in the approximation of a gluon-dominated plasma and $N_f = 0$. The relevant gluon mean free path is easily evaluated: $\lambda_\perp = 1/\sigma_{gg} \rho$ with $\sigma_{gg} = (9/2)\pi a_s^2/m_T^2$. Note that in Table I the quoted initial mean temperatures, Debye screening scales and gluon mean free paths are obtained as averages with the binary collisions weight $T_{AA}(x_\perp;b)$. In our evaluation we use $g_s = 2.5$, $(a_s = 0.5)$ to describe the scattering of the jet with the medium but the QGP-induced bremsstrahlung is calculated with a running $a_s(k_T)$ for emission vertex, similar to the MLLA approach for the vacuum jet shapes.

Evaluation of the medium-induced energy loss and its contribution to jet shapes is numerically expensive. Exact results have been obtained only for $E_{\text{jet}} = 20$, 100 and 500 GeV. We interpolate for other values of interest.

| Class   | Central | Mid-central | Peripheral |
|---------|---------|-------------|-------------|
| $b$ [fm] | 3       | 8           | 13          |
| $N_{\text{part}}$ | 361     | 165         | 18          |
| $dN^g/dy$ | 2800    | 1278        | 137         |
| $\langle T(\tau_0) \rangle$ [MeV] | 751     | 693         | 426         |
| $\langle m_D(\tau_0) \rangle$ [GeV] | 1.89    | 1.73        | 1.07        |
| $\langle \lambda_\perp(\tau_0) \rangle$ [fm] | 0.25    | 0.27        | 0.46        |

TABLE I: Summary of the relevant energy loss parameters and initial QGP properties for central, semi-central and peripheral collisions at $\sqrt{s} = 5.5$ TeV collisions at the LHC.

D. Energy loss distribution

A consistent energy loss theory provides complete information for the differential distribution of the lost $\Delta E_{\text{rad}}$, i.e. the bremsstrahlung spectrum in Eq. (29). The main point that we make here is that this distribution is completely determined by the properties of the QGP and the mechanisms of energetic quark and gluon stopping in hot and dense matter. Therefore, selecting different jet radii $R$ and $p_T$ of the particles will significantly alter both the jet shape and the amount of energy lost by the hard parton which can be recovered in the experimental measurement. In contrast, we have seen in Fig. 2 that the jet shapes scale approximately as a function of $R/R$, i.e. they are independent of the selection of cone opening angle $R$. The jet cross section weakly depends on $R$, unless $R \to 0$. Finally, $z_{\text{min}} = 0.1 - 0.2$ is necessary to noticeably alter the jet shape, implying that $\sim 20\%$ of the parent parton energy has to be missed via $p_T$ cuts to observe significant effects on $\psi_{\text{vac}}(r)$.

Experimentally, a clear strategy will be to use the leverage arms provided by $R (= R_{\text{max}}$ in the evaluation of the $\Delta E_{\text{rad}}$) and $p_T$ of the particles in the evaluation of the $\Delta E_{\text{rad}}$ to determine the distribution of the lost energy. This is illustrated schematically in Fig. 4. Theoretically, the first quantity to be calculated is:

$$\frac{\Delta E^{\text{min}}}{E} (R_{\text{max}}, \omega_{\text{min}}) = \frac{1}{E} \int_{\omega_{\text{min}}}^{E} d\omega \int_0^{R_{\text{max}}} dr \frac{dI^g}{d\omega dr}(\omega, r).$$  (36)

We present in Fig. 5 this fractional energy loss for a quark jet and a gluon jet of energy $E_{\text{jet}} = 20$ GeV inside a jet cone of radius $R_{\text{max}}$ and with acceptance cut $\omega_{\text{min}}$. Increasing $R_{\text{max}}$ or decreasing $\omega_{\text{min}}$ we will recover more of
the parent parton energy, lost via gluon bremsstrahlung. We note that in Fig. 5 the mean energy loss was calculated as an average over the probability distribution \( P(\epsilon; E) \) [27], reflective of multi-gluon fluctuations:

\[
\langle \epsilon \rangle = \frac{\langle \Delta E \rangle}{E} = \int_0^1 \, d\epsilon \, \epsilon P(\epsilon; E) .
\]  

(37)

For large fractional energy losses, such as in the illustrative example of a 20 GeV jet in central \( Pb + Pb \) collisions at the LHC, \( \Delta E_g/\Delta E_g \) is much smaller than the asymptotic ratio \( C_A/C_F = 9/4 \) due to the kinematic constraint \( \Delta E < E \) [27].

The separate dependence of \( \Delta E^{\text{in}}(R^{\text{max}}, \omega^{\text{min}})/E \) on the cone radius and the momentum acceptance cut is more clearly illustrated in Fig. 6. We show central, mid-central and peripheral collisions, impact parameters \( b = 3, 8, 13 \) fm, respectively, in \( Pb + Pb \) reactions at LHC at nominal \( \sqrt{s} \). We notice that, not surprisingly, the ratio \( \Delta E^{\text{in}}(R^{\text{max}}, \omega^{\text{min}})/E \) goes down at larger impact parameters because the energy loss of the jet decreases in peripheral collisions. More importantly, at each impact parameter there is a variation of the amount of the bremsstrahlung energy, recovered in the cone. This is precisely the variation that will map on the \( R_A^{\text{jet}}(R^{\text{max}}, \omega^{\text{min}}) \) observable. For example, in the limit of a very small opening angle and/or large momentum cut to eliminate the QGP-induced radiation the suppression should approximate that of leading hadrons (up to differences arising from the possibly softer particle spectra due to fragmentation):

\[
R_A^{\text{jet}}(R^{\text{max}}, \omega^{\text{min}} = 0) = R_A^{\text{leading parton}} \approx R_A^{b^\pm} .
\]  

(38)

One can see that the typical choices, \( R = 0.4 \) and \( \omega^{\text{min}} = 2 \) GeV are a good starting point to explore the variable quenching of jets.

IV. TOMOGRAPHY OF JETS IN HEAVY ION COLLISIONS

The purpose of this Section is to relate the theory of jet propagation in the QGP to experimentally measurable quantities.

A. Experimental observables

An essential ingredient that controls the relative contribution of \( \psi_{\text{vac}}(r/R) \) and \( \psi_{\text{med}}(r/R) = (1/\Delta E_{\text{rad}}) dP^{\psi}/dr \) to the observed differential jet shape in heavy ion reactions and also determines the attenuation of the jet cross sections is:

\[
f \equiv f \left( \frac{R}{R = R^{\infty}}, \omega^{\text{min}} = 0 \right) = \frac{\Delta E_{\text{rad}} \{ (0, R); (\omega^{\text{min}}, E) \}}{\Delta E_{\text{rad}} \{ (0, R^{\infty}); (0, E) \} ,}
\]  

(39)

the fraction of the lost energy that falls within the jet cone, \( r < R \), and carried by gluons of \( \omega > \omega^{\text{min}} \) relative to the total parton energy loss without the above kinematic constraints. If this fraction is known together with the probability distribution \( P(\epsilon) \) for the parton energy loss...
the medium-modified jet cross section per binary N + N scattering can be calculated as follows:

\[
\frac{\sigma^{AA}(R, \omega_{\text{min}})}{d^2 E_T dy} = \int_{r=0}^{1} \frac{de}{\sigma_{q,g}(R, \omega_{\text{min}})} \int_{\epsilon=0}^{1} \frac{1}{(1 - (1 - f_{q,g}) \cdot \epsilon)^3} \times \sigma_{q,g}^{NN}(R, \omega_{\text{min}}) \times \frac{1}{d^2 E_T dy},
\]

where \( E'_T = E_T/(1 - (1 - f_{q,g}) \cdot \epsilon) \). The \((1 - f_{q,g}) \cdot \epsilon\) factor accounts for the total "missed" energy in a jet cone measurement, which necessitates \( E'_T > E_T \), and the Jacobian \( J = |d^2 E'_T / d^2 E_T| \) is properly accounted for. In this paper possible fluctuations of \( f_{q,g} \) independent of \( \epsilon \) are not considered. Simple analytic limits illustrate the physics represented by Eq. \( 38 \): if there is no energy loss, \( P(\epsilon) = \delta(\epsilon) \), \( J = 1 \) and the cross section is unaltered. In the opposite limit, \( P(\epsilon) = \delta(\epsilon - 1) \), the quenching of jets, if any, is completely determined by the fraction of the lost energy \( f_{q,g} \) that is recovered in the experimental acceptance. When \( f_{q,g} = 1 \) once again there will be no attenuation of jets and when \( f_{q,g} \to 0 \) our result approximates the inclusive particle \( R_{AA}(p_T) \) \([22] \), see also Eq. \( 38 \).

Next, we obtain the full jet shape, including the contributions from the vacuum and the medium-induced bremsstrahlung:

\[
\psi_{\text{tot}}.(r/R) = \frac{1}{\text{Norm}} \int_{\epsilon=0}^{1} \frac{de}{(1 - (1 - f_{q,g}) \cdot \epsilon)^3} \times \sigma_{q,g}^{NN}(R, \omega_{\text{min}}) \times \frac{1}{d^2 E_T dy}
\times \left[ (1 - \epsilon) \psi_{\text{vac}}.(r/R) + f_{q,g} \cdot \epsilon \psi_{\text{med}}.(r/R) \right].
\]

We recall that, by definition, the area under any differential jet shape, \( \psi_{\text{tot}}.(r/R) \), \( \psi_{\text{vac}}.(r/R) \) and \( \psi_{\text{med}}.(r/R) \), is normalized to unity. Integrating over \( r \) in Eq. \( 38 \), it is easy to see that the correct "Norm" is the quenched cross section, Eq. \( 36 \). The interested reader can independently carry out the analysis of the simple limiting cases and gain insight into the dominant contribution to the full jet shape. Proper treatment of isospin is implicit in Eqs. \( 36 \) and \( 37 \).

**B. Energy sum rule**

Sum rules provide useful integral representation of conservation laws, originating from symmetries in QCD. One such example is momentum conservation in independent fragmentation:

\[
\sum_{h} \int_{0}^{1} z D_{h/q,g}(z, Q^2) dz = 1,
\]

where \( z = p_{T,h}/p_{T,q,g} \) is the momentum fraction of parent partons carried by fragmentation hadrons. The same sum rule will hold in the presence of a medium since the total momentum of the partons must be conserved irrespective of whether they are propagating in vacuum or in the QGP with/without medium-induced bremsstrahlung \([22, 27] \). For jets, taking a monochromatic pulse in the vacuum,

\[
\frac{1}{\sigma} \int d^2 E_T \frac{1}{\sigma^{AA}(R, \omega_{\text{min}} \to 0)} \frac{1}{d^2 E_T} = \delta^2(E_T - E_0),
\]

we can easily verify that in the presence of a QGP

\[
\int d^2 E_T \frac{1}{\sigma} \sigma^{AA}(R \rightarrow \infty, \omega_{\text{min}} \rightarrow 0) \frac{1}{d^2 E_T} = E_T = E_0,
\]

in case of perfect experimental acceptance. More generally, only a fraction, \( 1 - (1 - f)(\epsilon) \), of \( E_0 \) is recovered.

**C. Numerical results**

Combining our full theoretical model for the jet shape and the jet cross section in heavy ion collisions with realistic numerical simulations of parton propagation in the QGP, see Section \( \text{III.C} \), we first evaluate the nuclear modification factor \( R_{AA}^{jet}(R_{\text{max}}, \omega_{\text{min}}) \) in Pb+Pb collisions with center of mass energy \( \sqrt{s} = 5.5 \text{ TeV} \) at the LHC. Fig. \( 7 \) illustrates the attenuation of the measured jet rate as a function of the jet transverse energy, \( E_T \), at impact parameters \( b = 3 \) fm (circle), \( b = 8 \) fm (square) and \( b = 13 \) fm (diamond) in Pb+Pb collisions with \( \sqrt{s} = 5.5 \text{ TeV} \).
unity. For $R_{\text{jet}}^{\text{max}} \leq 0.2$ the magnitude of jet quenching approaches the suppression for leading hadrons. A good starting point is a cone radius selection $R_{\text{jet}}^{\text{max}} = 0.4 - 0.7$ if the experimental statistics allows for positive identification of 30% to a factor of 2 in the measured cross section. In the bottom panel of Fig. 8 we present the sensitivity of jet attenuation to the minimum particle momentum/calorimeter tower energy deposition cut $\omega_{\text{min}}$. For a finite $R_{\text{jet}}^{\text{max}} = 0.7$ even if $\omega_{\text{min}} = 0$ GeV $R_{AA}^{\text{jet}}(R_{\text{jet}}^{\text{max}}, \omega_{\text{min}})$ does not reach unity, see our discussion above. The largest variation in the quenching strength is observed between $\omega_{\text{min}} = 2$ GeV and $\omega_{\text{min}} = 5 - 10$ GeV, and reflects the typical energy of the stimulated gluon emissions. We emphasize that, in the GLV approach [6], partons lose energy through $\sim$ few GeV bremsstrahlung gluons [27]. For $\omega_{\text{min}} > 10$ GeV, $R_{AA}^{\text{jet}}(R_{\text{jet}}^{\text{max}}, \omega_{\text{min}})$ approaches again the characteristic leading particle suppression. In summary, for the same centrality, $E_T$ and $\sqrt{s}$ the continuous variation of quenching values may help differentiate between competing models of parton energy loss [28], thereby eliminating the order of magnitude uncertainty in the extraction of the QGP density.

Detailed investigation of $R_{AA}^{\text{jet}}(R_{\text{jet}}^{\text{max}}, \omega_{\text{min}})$ can also indicate whether “elastic” $2 \rightarrow 2$ processes, such as collisional energy loss [29], or “inelastic” $2 \rightarrow 2 + n$ processes, such as bremsstrahlung [2] and hadron dissociation in the QGP [30], dominate the inclusive particle and particle correlation quenching. If the energy loss per interaction in the first scenario $\Delta E_{\text{coll}} / E \lesssim 5\%$, the recoil parton form the medium will accelerate almost transversely relative to the jet axis and will not be part of the jet for any reasonable selection of $R_{\text{jet}}^{\text{max}}$. Therefore, for collisional energy loss, in contrast to the well-defined evolution of the jet suppression with cone radius and the acceptance cut seen in Fig. 8 the cross section attenuation will be large and constant and will approximate the quenching of leading hadrons. Note that $R_{AA}^{\text{jet}} < 1$ has also been observed in Monte-Carlo simulations of jet quenching [31].

We now turn to the numerical results for the jet shape in Pb+Pb collisions at $\sqrt{s} = 5.5$ TeV at the LHC. In Fig. 8 we first explore the difference between the vacuum and the medium-induced only ($E_T$ given for the parent parton) $\psi(r/R)$ as a function of the impact parameter, jet energy, and the cone radius. We note that in central heavy ion reactions for lower $E_T$ and, in particular, for $R \geq 0.7$ the two differential shapes can be quite similar. The differences become more pronounced for smaller jet radii where the experimental acceptance will subtend the part of phase space with the most effective cancellation of the collinear medium-induced radiation [22]. It is interesting to observe that in going to more peripheral collisions $\psi_{\text{med}}(r/R)$ becomes slightly wider. The underlying reason is that the LPM destructive interference between the radiation induced by the large $Q^2$ scattering and the radiation induced by the subsequent interactions in the QGP determines the angular distribution in the bremsstrahlung spectrum. Thus, a small medium size facilitates the resulting cancellation for glu-
Jets are inherently narrower, see Figs. 2 and 3.

The pattern of energy flow for in-medium jets is shown in Fig. 10 together with \( \psi_{\text{vac}}(r/R) \) and \( \psi_{\text{med}}(r/R) \) for comparison. We used \( E_T = 20 \) GeV to 200 GeV and \( R = 0.4 \) to 0.7 to cover a wide range of measurements that will become accessible during the first year of heavy ion running at the LHC. One observes that there is no significant distinction between the jet shape in the vacuum and the total in-medium \( \psi(r/R) \). The underlying reason for this surprising result is that although medium-induced gluon radiation produces a broader \( \psi_{\text{med}}(r/R) \), this effect is offset by the fact that the jets lose a finite amount of their energy, see Figs. 5 and 6. Furthermore, when part of the lost energy is missed due to finite experimental acceptance, the required higher initial virtuality jets are inherently narrower, see Figs. 2 and 3.

In Table II we show the mean relative jet radii \( \langle r/R \rangle \) in the vacuum and in the QGP medium created at the LHC for two different cone selections \( R = 0.4 \) and \( R = 0.7 \) and four transverse energies \( E_T = 20, 50, 100, 200 \) GeV, respectively, in central Pb+Pb collisions at the LHC.

Lastly, we point out where the anticipated jet broadening effects will be observed in the differential shape by studying the ratio \( \psi_{\text{tot}}(r/R)/\psi_{\text{vac}}(r/R) \) in Fig. 11. We have used the same transverse energies and cone radii as in Fig. 10. We recall that the small \( r/R < 0.25 \) region of the intra-jet energy flow in p+p collisions in our calculation has uncertainties associated with the normalization of the jet shape. In the moderate and large \( r/R > 0.25 \) region our theoretical model gives excellent descriptions of the Fermilab Run II (CDF II) data, as shown in Fig. 11.

The QGP effects are manifest in the “tails” of the energy flow distribution and for a cone radius \( R = 0.4 \) the ratio could reaches \( \sim 1.75 \) when \( r/R \rightarrow 1 \). However, for
the total jet shape in medium to jet shape in vacuum at \( r/R > 0.5 \), high statistics measurements will be needed. This precision can hopefully be achieved with the large acceptance experiments at the LHC.

### V. CONCLUSIONS

The unprecedentedly high center of mass energies at the LHC will usher in a new era of precision many-body QCD. The theory and phenomenology of jets in nuclear collisions are expected to evolve as the new frontier in the perturbative studies of parton propagation in the QGP \[21\]. In this paper we discussed three important aspects of such studies: a generalization of the analytic approach for calculating differential jet shapes \[15\] that can accommodate experimental acceptance cuts needed to isolate jets in the high multiplicity environment of heavy ion collisions; the theory of the intra-jet energy flow redistribution through large-angle medium-induced gluon bremsstrahlung; and a comprehensive new set of experimental observables that can help identify and characterize the mechanisms of parton interaction in nuclear matter.

In elementary nucleon-nucleon collisions we compared our theoretical model to the CDF II Tevatron data on jet shapes \[20\] and investigated the baseline \( \psi_{\text{vac.}}(r/R) \) at the higher \( \sqrt{s} = 5.5 \text{ TeV} \) at the LHC. We found that in the absence of a hot and dense QGP matter these shapes are self-similar and approximately independent of the cone radius \( R \). Elimination of low momentum particles of up to \( \sim \)few GeV is not likely to significantly alter the pattern of intra-jet energy flow for \( E_T > 50 \text{ GeV} \) jets.

In nucleus-nucleus reactions we demonstrated that the characteristic large-angle QGP-stimulated gluon emission \[22\] persists to all orders in the correlation between the elementary bremsstrahlung sources. We showed that this intensity spectrum can be fully characterized by the amount of the lost energy that falls inside the jet cone \( (r < R_{\text{max}}, \omega > \omega_{\text{min}}) \) and derived the medium modifica-

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**TABLE II**: Summary of mean relative jet radii \( ⟨r/R⟩ \) in the vacuum, with complete energy loss, and in the QGP medium. Shown are results for cone radii \( R = 0.4 \) and \( R = 0.7 \) and transverse energies \( E_T = 20, 50, 100, 200 \text{ GeV} \) at \( \sqrt{s} = 5.5 \text{ TeV} \) central Pb+Pb collisions at the LHC.

| \( R = 0.4 \) | Vacuum | Complete E-loss | Realistic case |
|--------------|--------|----------------|---------------|
| \( ⟨r/R⟩, E_T = 20\text{GeV} \) | 0.41   | 0.55           | 0.45          |
| \( ⟨r/R⟩, E_T = 50\text{GeV} \) | 0.35   | 0.48           | 0.38          |
| \( ⟨r/R⟩, E_T = 100\text{GeV} \) | 0.28   | 0.44           | 0.32          |
| \( ⟨r/R⟩, E_T = 200\text{GeV} \) | 0.25   | 0.40           | 0.28          |

| \( R = 0.7 \) | Vacuum | Complete E-loss | Realistic case |
|--------------|--------|----------------|---------------|
| \( ⟨r/R⟩, E_T = 20\text{GeV} \) | 0.41   | 0.44           | 0.42          |
| \( ⟨r/R⟩, E_T = 50\text{GeV} \) | 0.33   | 0.39           | 0.37          |
| \( ⟨r/R⟩, E_T = 100\text{GeV} \) | 0.27   | 0.34           | 0.29          |
| \( ⟨r/R⟩, E_T = 200\text{GeV} \) | 0.24   | 0.31           | 0.26          |

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**FIG. 11**: (Color online) The ratios of total jet shape in heavy-ion collisions to the jet shape in the vacuum for jet energies \( E_T = 20, 50, 100, 200 \text{ GeV} \) at \( \sqrt{s} = 5.5 \text{ TeV} \) in Pb+Pb at the LHC. Experimental acceptance cuts needed to isolate jets in the high multiplicity environment of heavy ion collisions; the theory of the intra-jet energy flow redistribution through large-angle medium-induced gluon bremsstrahlung; and a comprehensive new set of experimental observables that can help identify and characterize the mechanisms of parton interaction in nuclear matter.
tion of the jet shapes and jet cross sections in the QGP, subject to an intuitive energy sum rule.

To demonstrate the connection between the QGP properties, the mechanisms of parton interaction and energy loss in hot and dense matter, and a new class of jet-related experimental observables, we carried out realistic simulations of quark and gluon production and propagation in the medium created in relativistic heavy ion collisions at the LHC. We introduced a natural generalization of the leading particle suppression to jets and showed that it is a more differential and powerful tool that can be used to assess in approximately model-independent way the characteristic properties of the induced gluon intensify spectrum. Consequently, in the future progress can be made toward identifying the set of approximations that most adequately reflect the dynamics of hard probes in the QGP. We also discussed how the evolution of the jet cross sections, should include cold nuclear matter effects, such as nuclear shadowing, the consideration of jet cross sections, and the evolution of jet cross sections in heavy ion collisions can easily be generalized to light ion collisions.

APPENDIX A: JET CROSS SECTIONS

In this paper we focus exclusively on large momentum transfer processes, \( Q^2 \gg \Lambda^2_{\text{QCD}} \), that can be systematically calculated in the framework of a reliable theory, the perturbative QCD factorization approach. Factorization not only separates the short- and long-distance QCD dynamics but implies universality of the parton distribution functions (PDFs) and fragmentation functions (FFs) and infrared safety of the hard scattering cross sections. For hadronic collisions, one of the most inclusive processes is jet production. To lowest order (LO) the invariant differential cross section reads:

\[
\frac{d\sigma_{h_b h_b}}{dy_c d^2 p_T} = K \sum_{abcd} \int \frac{d\sigma_{b_d}^{\text{max}}}{x_a x_b} \phi_{a/h_a}(x_a, \mu_T) \phi_{b/h_b}(x_b, \mu_T) \times \frac{\alpha^2_s(\mu_r)}{s^2} |M_{ab \rightarrow cd}|^2.
\]

(A1)

Here, \( s = (p_T^a + p_T^b)^2 \) is the squared center of mass energy of the hadronic collision and \( x_a = p_T^a / p_T^c, \; x_b = p_T^b / p_T^c \) are the lightcone momentum fractions of the incoming partons. In this formulation, for massless initial-state quarks and gluons,

\[
y_c^{\text{max(min)}} = (+)(-1) \ln \left( \frac{\sqrt{s} - m_T^\text{max(c)}}{m_T^c} e^{(+)(-)} \right),
\]

(A2)

where \( m_T^2 = m_T^2 + p_T^2 \). In Eq. (A1), \( \phi_{i/h_i}(x_i, \mu_{fi}) \) is the distribution function of parton \( i \) in the hadron \( h_i \) and \( \mu_r \) and \( \mu_{fi} \) are the renormalization and factorization scales, respectively. In this work calculations are done strictly in the collinear factorization approach and we use the CTEQ6.1 LO PDFs and the MOE of China under Project No. 1RT0624.

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\[\text{\textbf{APPENDIX A: JET CROSS SECTIONS}}\]
of narrow jets $R_{AA}^{jet} \approx R_{AA}^{jet+} = 0.25 - 0.5$ \cite{27, 29}. Taking this into account, but neglecting for the moment the complications associated with jet reconstruction in the high particle multiplicity environment of heavy ion collisions \cite{14}, from Fig. \ref{fig:fig12} we find that excellent \(< 10\%\) statistical precision can be achieved for inclusive measurements of jets of $p_T$ as high as 160 GeV in Pb+Pb reactions and 1.3 TeV in p+p reactions. Jet shape measurements require higher statistics since the shape functions are precipitously falling as of $r/R \rightarrow 1$. We expect that very good, \(< 30\%\) at large $r/R \sim 1$, jet shape measurements will be possible to $p_T$ as high as 100 GeV and 900 GeV in Pb+Pb and p+p collisions, respectively. These will give an indication as to whether medium-induced broadening is present in the tails of the intra-jet energy distribution. The estimates presented in this appendix are conservative, $\Delta y = 1$.

APPENDIX B: CONTRIBUTIONS TO THE VACUUM JET SHAPE

It is important that a jet finding algorithm be infrared and collinear safe. In full Monte-Carlo simulations of high-energy hadronic events the algorithm can be “tested” and matched to the experimental measurement techniques. In analytic calculations an approximate way to mimic the effect of jet splitting/merger is to introduce an adjustable parameter, $R_{sep}$, for cone type algorithms. If two partons are within an angle $\theta < \frac{\pi}{2}$ of each other, they should be merged into one jet \cite{38}. This approach may not be optimal \cite{15} since it does not generalize intuitively for NLO jet shape calculations. When comparing theoretical results to experimental data one finds that $R_{sep}$ is a function of the event kinematics, i.e. it is jet momentum (energy) dependent. Nevertheless, at lowest order this is a useful phenomenological approach to obtain the best possible description of the baseline differential jet shapes in nucleon-nucleon collisions, needed for the study of QGP-induced effects. Also, it known that for jet cross sections at NLO the results from other jet finding algorithms, such as the fully infrared and collinear safe $k_T$ algorithm with a jet-size parameter $D$, coincide within a few % with the results from the cone algorithm with appropriate matching/choice of $R$ and $R_{sep}$ \cite{16}.
In this Appendix we achieve via first bin subtraction and the energy flow distribution. At low transverse energy, $E_T = 45 - 55$ GeV, a leading-order (LO) calculation with initial-state radiation already gives a good description of the experimental data. However, below $\approx 75$ GeV it is impossible to fit the data using only leading order, even with the maximum $R_{sep} = 2$. Other contributions should be considered in an improved theoretical description of jet shapes. These include the effect of the running coupling constant in the momentum transfer integrals (MLLA) and power corrections (PC) $\propto Q_0/E_T$. Sudakov resummation (RS) ensures the finiteness of $\psi(r,R)$ in the region $r \to 0$. The explicit formulas for these two contributions are given in Section II A. It can be seen from Fig. 14 that all perturbative and non-perturbative effects should be taken into account if reliable description of the experimental results is to be achieved.

**APPENDIX C: DOUBLE DIFFERENTIAL MEDIUM-INDUCED JET SHAPE**

In this paper we have investigated extensively the differential jet shapes $\psi(r,R) = \frac{d^2I_{\text{med}}(r/R)}{drdz}$ in vacuum and in heavy-ion collisions at the LHC. This quantity integrates over the energy distribution of the partonic jet fragments. Thus, the information about the angular distribution of soft vs hard shower partons is lost. At the LHC it might be possible, via particle tracking or jet reanalysis, to recover this information on an event-by-event basis and construct $\frac{d^2\Psi_{\text{med}}(r,R)}{drdz}$, where:

$$\frac{d^2\Psi_{\text{med}}(r,R)}{drdz} = \frac{1}{\Delta E_{\text{jet}}(R,0)} \frac{d\sigma(\omega = zE_{\text{jet}}, r)}{d\omega dr}.$$  \hspace{1cm} (C1)

In Eq. (C1) the QGP-induced double differential shape is normalized such that it integrates to unity with $R^{\text{max}} = R$, $\omega^{\text{min}} = 0$ and $z = \omega/E_{\text{jet}}$. To illustrate the additional insight that can be gained through such studies we show numerical results for a simplified case where a quark jet of $E_{\text{jet}} = 100$ GeV propagating through a QGP of length $L = 6$ fm. While the medium is not expanding and characterized by $\lambda_g = 1.5$ fm, $m_D = 0.7$ GeV and $\alpha_s = 0.3$, the $\langle \Delta E/E \rangle$ approximates the full numerical result for central Pb+Pb collisions at the LHC.

Fig. 13 shows a 3D plot of $r_{\text{med}}(r/R)$ for momentum fraction $z = k^+ / E^+ \sim \omega_{\text{gluon}} / E_T = 0.01, 0.03, 0.1, 0.3$ and $R = 1$, $\omega^{\text{min}} = 0$ GeV. We can observe that at $z = 0.01 (\omega = 1$ GeV) the double differential medium-induced jet shape is dominated by gluon radiation at large opening angle $r/R$. At $z = 0.03 (\omega = 3$ GeV) the peak lies in the intermediate $r/R$ region with significantly suppressed gluon radiation at small open angle $r/R$. Increasing $\omega$ further narrows the medium-induced intensity profile. It is tempting to associate the characteristic...
shapes for small values of $z$ in Fig. 15 with measurements of enhanced away-side large-angle particle-triggered correlations [39, 40, 41]. However, the current RHIC data presents challenges in separating the jet from the background or even distinguishing between events with 1 or 2 minijets (recall that $N_{coll} \sim 1000$ in central Au+Au). Analysis on an event-by-event basis can help reveal unambiguously the QGP medium response to jets. When future experimental measurements of jet in heavy ion collisions are perfected at the LHC full numerical simulations of the double differential shape, including the vacuum and medium-induced components, will soon follow.

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