Manifestation of kaonium in the $e^+e^- \rightarrow K^+K^-$ process

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We analyze the precise data obtained by the CMD-3 experiment on the $e^+e^-$ annihilation into two charged kaons in the vicinity of the $\phi$ peak. A perfect fit is obtained only if a pole on the real axis below the reaction threshold is assumed. This can be interpreted as proof of the existence of the 2p-state of kaonium, a compound of $K^+$ and $K^-$. To determine the binding energy, more data points with the same, or better, precision would be needed. The possibility of discovering 2p kaonium in the $e^+e^- \rightarrow \pi^+\pi^-$ process is discussed. The indication of the 2p-state of $K^0$-onium on the basis of the CMD-3 $e^+e^- \rightarrow K^0\bar{K}^0$ data is inconclusive.

Kaonium is a hypothetical compound system consisting of a positively charged and a negatively charged kaon. It belongs to a wide class of onia, systems made of a particle and an antiparticle. In the lepton sector, they are the well-known positronium, true muonium and true taumonium (the latter two have yet to be observed). Quarkonia, the bound states of a quark and its antiquark are observed as truly neutral (all flavor quantum numbers vanishing) mesons. They are numerous, e.g., $\phi (ss), \eta_c$ and $J/\psi (cc)$, $\Upsilon (bb)$. Many theoretical studies, starting with the Fermi–Yang and Sakata models [1], have considered the possibilities of baryon–antibaryon bound states.

In the meson sector, pionium ($\pi^+\pi^-$) was discovered in 1993 at the 70 GeV proton synchrotron at Serpukhov [2] and intensively studied in the Dimeson Relativistic Complex (DIRAC) experiment [3] at the CERN Proton Synchrotron. Assuming pure coulombic interaction, the binding energy of pionium can be calculated from the hydrogen-atom formula

$$b_n = \frac{m_e\alpha^2}{2n^2},$$

where $m_e$ is the reduced mass in energy units (used throughout this Letter), $\alpha \approx 1/137$ is the fine-structure constant, and $n$ is the principal quantum number. Putting $n = 1$ for the ground state and $m_e = m_{\pi^+}/2$ we get $b = 1.86$ keV. The decay to two neutral pions is dominant and the measured lifetime is $3.15^{+0.29}_{-0.26} \times 10^{-15}$ s [3].

The NA48/2 Collaboration at the CERN SPS [4] studied decays $K^{\pm} \rightarrow \pi^\pm\pi^0\pi^0$ and found an anomaly in the $\pi^0\pi^0$ invariant mass distributions that can be interpreted as the production of pionia in the kaon decays and their subsequent two-$\pi^0$ decay.

The DIRAC experiment also observed and studied $\pi^-K^+$ and $\pi^+K^-$ atoms [3,5].

Up to now, the experiments concerning dimeson production were performed at proton accelerators. Electron–positron colliders, the machines that are famous for participating in the discovery of many new particles (notably quarkonia), have not yet contributed to mesonia physics. The reason is that ground-state mesonia (1s in atomic notation) are objects with $J^{PC} = 0^{++}$ quantum numbers, and as such they cannot couple to the photon. However, the DIRAC collaboration recently discovered so-called long-lived $\pi^+\pi^-$ atoms, which are the 2p atomic states with quantum numbers $J^{PC} = 1^{--}$. Therefore, they can be produced in the $e^+e^-$ collisions. The coulombic binding energy of the 2p pionium is 0.464 keV, its lifetime was determined to be $\tau_{2\pi} = 0.45^{+0.08}_{-0.30} \times 10^{-11}$ s [7]. Such a long lifetime is caused by the fact that the decay modes to the positive C-parity states $\pi^0\pi^0$ and $\gamma\gamma$ are now forbidden.

No experimental evidence of kaonium has been found yet. In the simplest way, kaonium can be considered a hydrogen-like atom (a system held together due to coulombic attraction between opposite electric charges). Equation (1) gives the binding energy of the ground state $b \approx 6.57$ keV. Unlike the hydrogen atom and leptonic onia, the constituents of pionium and kaonium also interact via strong force. Krewald, Lemmer, and Sassen [8] found “the ground state energy for the kaonium atom that is shifted above the Coulomb value by a few hundred eV". As a rule, an increase in bound state energy means a drop in binding energy. On the contrary, Zhang, Chiang, Shen, and Zou [9] found that kaonium binds more strongly ($b = 7.05$ keV) than it corresponds to Coulomb interaction. Because of this discrepancy, we will consider the coulombic binding energies.

Kaonium is not stable. Ground-state (1s) kaonium partly decays electromagnetically into two photons. In addition, the exchange of $K^*$ between kaons generates the $\pi^+\pi^-$, $\pi^0\pi^0$, and $\eta\pi^0$ decay modes. The resulting lifetime of about $2.2 \times 10^{-18}$ s is expected [10]. The corresponding decay width is about 0.3 keV, which is about four orders of magnitude smaller than the decay width of $\phi(1020)$.

The coulombic binding energy of the first excited state...
(n = 2) of kaonium is 1.64 keV. We will concentrate on the 2p state, which can be produced in the e+e− experiments. Its quantum numbers J^PC = 1−− forbid decays to the C = 1 states π0π0 and ηπ0. The lifetime of 2p kaonium is thus determined by the decay rate into π+π−. It is certainly longer than that of the ground state. For our purposes we can neglect the 2p kaonium decay width and consider it a stable particle.

In this Letter we analyze the existing precise data on the e+e− → K+K− process with the aim to find a pole in the amplitude corresponding to a bound state.

The reaction amplitude is a function of s, the invariant energy squared. Under very general conditions the amplitude can be continued into the complex s-plane. The resonances are represented by the poles at imaginary s. The (quasi) stable particle, or bound state, appears as a pole at real s below the reaction threshold.

The formula for the cross section of the e+e− annihilation into a K+K− pair based on the vector-meson dominance (VMD) model with two resonances is

\[ \sigma(s) = \frac{\pi \alpha^2}{3s} \left( 1 - \frac{4m_K^2}{s} \right)^{3/2} \times \frac{R_1 e^{i\delta}}{s - M_1^2 - iM_1 \Gamma_1} + \frac{R_2}{s - M_2^2 - iM_2 \Gamma_2} \right)^2, \]

where \( M_i \) and \( \Gamma_i \) determine the position and width of the \( i \)-th resonance, respectively. The residuum \( R_i \) includes the product of two constants. One characterizes the coupling of the \( i \)-th resonance to the photon (up to the elementary charge \( e \), which is taken off to form, after squaring, an \( \alpha \) in the prefactor) and the other is the coupling of the resonance to the \( K^+K^- \) pair. The phase \( \delta \) regulates the interference between the resonances.

We use the e+e− → K+K− cross section data obtained by the CMD-3 (Cryogenic Magnetic Detector) experiment at the VEPP-2000 e+e− collider in Novosibirsk. The advantage of the energy scan method used in this experiment is in getting high statistics data at precisely known energies, to which the collider is tuned up step by step. The number of data points is twenty-four, which are concentrated in a narrow region around the \( \phi(1020) \) resonance. The data are shown in Fig. 1, together with two curves depicting our fits.

We first tried to fit the data assuming just one resonance, i.e., using three free parameters. The result was disastrous, the usual \( \chi^2 \) was equal to 341.8, which together with the number of degrees of freedom \( \text{NDF}=24 \) implied a confidence level (C.L.) of zero. This fit is depicted by a dashed curve in Fig. 1. The parameters of the fit are listed in the middle column of Table I. It must be said that the authors of Ref. [11] achieved a better result. They used a more sophisticated parametrization of the resonance shape based on an energy dependent resonance width \( \Gamma(s) \) and attained \( \chi^2/\text{NDF}=25/20 \), which gave C.L.=20%.

We then tried to improve our fit by considering two resonances. To our surprise, the width of the second resonance came out close to zero and the resonance position was below the threshold, which signalized the bound state. Inspired by that, we replaced the second resonance with a bound-state pole by putting \( \Gamma_2 = 0 \) and \( M_2 = 2m_{K^+} - b \), where \( b \) is the 2p kaonium binding energy. We took \( b = 1.64 \) keV. We attained perfect agreement with the data (\( \chi^2/\text{NDF}=4.6/19 \), C.L.=100%), depicted by the full curve in Fig. 1. All parameters of the fit are listed in the rightmost column of Table I. Minimizing the \( \chi^2 \) with respect to the binding energy gives

\[ \chi^2 = 341.8 \text{ for 25/20 degrees of freedom.} \]

FIG. 1. Cross section for the e+e− annihilation into two charged kaons measured in the CMD-3 experiment and two fits to it using Eq. (2). Their parameters are given in Table I.

| one resonance | resonance & pole |
|--------------|------------------|
| \( R_1 \) (GeV²) | 0.3679(15)       | 0.3759(15)       |
| \( M_1 \) (GeV) | 1.019393(14)     | 1.019247(18)     |
| \( \Gamma_1 \) (MeV) | 4.359(32)       | 4.172(38)        |
| \( \delta \) (fixed) | 0 (fixed)       | 1.334(34)        |
| \( R_2 \) (GeV²) | 0 (fixed)        | 0.298(23)        |
| \( b \) (keV) | 1.64^a (fixed)   | 0 (fixed)        |
| \( \Gamma_2 \) |                 |                  |
| \( \chi^2 \) | 341.8            | 4.6              |
| \( \text{NDF} \) | 21               | 19               |
| Conf. level | 0                | 100%             |

\( a M_2 = 2m_{K^+} - b \)
an estimate of $b = (11 \pm 25) \text{ MeV}$ and $\chi^2 = 4.2$. Even if the number of degrees of freedom is less by 1, the confidence level remains the same (100%) as in the case of fixed $b$. Clearly, by fitting the present data we cannot say anything more definite about the 2p kaonium binding energy.

Our cross section parametrization with one resonance and a bound state provides a perfect fit (C.L.=100%) that is not only better than our one-resonance fit (C.L.=0), but also better than the sophisticated one-resonance fit made by the authors of the experimental paper themselves (C.L.=20%). This is strong evidence of the existence of the $K^+K^-$ bound state, kaonium, in the 2p state.

Thanks to its $\pi^+\pi^-$ decay mode, the 2p kaonium may, in principle, reveal itself as a narrow peak (or, at least, a shoulder) in the $e^+e^- \rightarrow \pi^+\pi^-$ excitation function slightly below $\sqrt{s} = 2m_{K^+}$. The contemporary experimental data do not show any anomaly that could be interpreted as a manifestation of 2p kaonium. The invariant energy region in question has been ignored by most of the experiments which have concentrated on the measurement in the $\rho/\omega$ region or at energies higher than 1 GeV. The important exception is the BaBar experiment, which covered the energies from 0.305 to 2.95 GeV by using the Initial State Radiation (ISR) method. Their data in the presumed signal region are shown in Fig. 2 together with our “conservative” fit not including kaonium. There is an insignificant excess at $\sqrt{s} = 973$ MeV, but no firm conclusion can be drawn yet. More precise and denser data, obtained preferably by the energy scan method, would be necessary to confirm or reject the existence of 2p kaonium. According to [13], we may expect new data from the CMD-3 experiment soon. Figure 4 in [15] hints at their covering energies to almost 1 GeV.

The results of the paper [9] indicate that the strong interaction between the kaons, caused by the vector mesons exchange, is attractive. One may therefore speculate about the existence of the $K^0\bar{K}^0$ bound state. This would be the first (and probably only) onium composed of neutral particles. To pursue this idea we use the high-precision data on the $e^+e^- \rightarrow K_S^0\bar{K}^0$ process coming again from the CMD-3 experiment at Budker Institute of Nuclear Physics in Novosibirsk and comprising of twenty-five data points in a narrow interval around the $\phi(1020)$ resonance.

We again compare our parametrization with the data. Our simple one-resonance fit (the dashed curve in Fig. 3) provides $\chi^2/\text{NDF}=36.7/22$, C.L.=2.5%. The sophisticated fit by experimentalists is much better ($\chi^2/\text{NDF}=15/21$, C.L.=82%). To explore the one resonance–one bound state scenario, we need to choose the binding energy $b$. Not having anything better at our disposal, we again put $b = 1.64$ keV. The result of the fit is $\chi^2/\text{NDF}=11.1/20$, C.L.=94.4%. The pertinent cross section is shown by a full curve in Fig. 3, the middle column in Table II. The existence of a 2p state of neutral kaons, the $K^0$-onium, is not excluded, but it is not required by the existing data. When looking for $K^0$-onium in the $\pi^+\pi^-$ data, the signal region should be shifted upwards by about 7.9 MeV.

To conclude: We have found strong evidence of the existence of kaonium made of charged kaons in the 2p state.
state by analyzing the data on the $e^+e^- \rightarrow K^+K^-$ process. In order to determine its binding energy, additional data are needed, which will put more severe restrictions on the fit. Data on $e^+e^- \rightarrow \pi^+\pi^-$ may provide information about the binding energy of 2p kaonium. More data on the $e^+e^-$ annihilation into two neutral kaons are needed to reach a conclusion about the mere existence of $K^0$-onium.

I thank Filip Blaschke, Martin Blaschke, and Josef Jurán for useful discussions.

This work was partly supported by the Ministry of Education, Youth and Sports of the Czech Republic Inter-Excellence project No. LTI17018.

\begin{table}
\begin{tabular}{|c|c|c|}
\hline
 & one resonance & resonance & pole \\
\hline
$R_1$ (GeV$^2$) & 0.3488(14) & 0.3490(27) & \\
$M_1$ (GeV) & 1.01921(14) & 1.019259(16) & \\
$\Gamma_1$ (MeV) & 4.144(28) & 4.151(28) & \\
$\delta$ & 0 (fixed) & 0.1\pm1.5 & \\
$R_2$ (GeV$^2$) & 0 (fixed) & -0.0179(35) & \\
b (keV) & 1.64\pm1.5 & (fixed) & \\
$\Gamma_2$ & 0 (fixed) & & \\
$\chi^2$ & 36.7 & 11.1 & \\
NDF & 22 & 20 & \\
Conf. level & 2.5\% & 94.4\% & \\
\hline
\end{tabular}

$^a M_2 = 2m_{K^0} - b$

\textbf{TABLE II. Parameters of the two fits to the CMD-3 $K^0_SK^0_L$ data \cite{16} depicted in Fig. 3.}

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