Chiral Effective Lagrangian and Quark Masses

Hartmut Wittig

DESY, Theory Group, Notkestr. 85, D-22603 Hamburg, Germany

The status of lattice determinations of quark masses is reviewed (with the exception of \(m_b\)). Attempts to extract the low-energy constants in the effective chiral Lagrangian are discussed, with special emphasis on those couplings which are required to test the hypothesis of a massless up-quark. Furthermore, the issue of quenched chiral logarithms is addressed.

1. INTRODUCTION

Recently, many studies of QCD at low energies have investigated the interplay between lattice simulations and Chiral Perturbation Theory (ChPT). I shall start the review of these activities by recalling the basic features of ChPT and point out what can be learned from combining ChPT with lattice simulations.

Chiral Perturbation Theory is a systematic expansion in the 4-momentum \(p\) and the quark masses \(m_u, m_d, m_s\) about the massless limit [1]. In the low-energy regime its information content is equivalent to that of QCD, and this has been exploited in many applications, e.g. in the calculation of pion scattering amplitudes and quark mass ratios. Furthermore, ChPT provides valuable input for lattice simulations. Here, the prediction of the quark mass dependence of \(m_{\pi}^2\) is surely the most widely used piece of information. In addition, ChPT can model the volume dependence of observables, and also relates different processes. For instance, the amplitudes of \(K \rightarrow \pi\pi\) decays can be expressed in terms of the theoretically much simpler \(K \rightarrow \pi\) transition.

However, Chiral Perturbation Theory is an effective, non-renormalisable theory, parameterised in terms of a set of empirical couplings, which are usually called “low-energy constants” (LECs). At lowest order the effective chiral Lagrangian reads

\[
\mathcal{L}_{\text{eff}}^{(2)} = \frac{F_\pi^2}{2} \left\{ \frac{1}{2} \text{Tr} \left( \partial_\mu U^\dagger \partial^\mu U \right) + B_0 \text{Tr} \left( \mathcal{M} (U + U^\dagger) \right) \right\},
\]

where

\[
U = \exp \left\{ i T^a \varphi^a / (\sqrt{2} F_\pi) \right\}
\]

\(F_\pi = 93.3\,\text{MeV}\).

This shows that \(B_0\) can only be determined from \(m_\pi\) if the physical value of the quark mass \(\hat{m}\) is known already. By the same token, the quark mass \(\hat{m}\) can only be inferred if an estimate for \(B_0\) is available. However, \(B_0\) drops out in suitably chosen ratios of \(m_{\pi}^2, m_K^2, \ldots\). Thus, while ChPT enables us to compute quark mass ratios, it fails to provide an absolute normalisation of their masses.

Another reason why the complete set of LECs cannot be determined from chiral symmetry considerations alone is the fact that the effective chiral Lagrangian beyond leading order is invariant under a symmetry transformation involving the LECs and the mass matrix \(\mathcal{M}\). This is the famous “Kaplan-Manohar ambiguity” [2].

At this point it is clear that lattice simulations...
can in turn provide valuable input for ChPT. By studying the quark mass dependence of Goldstone bosons in a simulation one can determine the value of $B_0$, or, equivalently provide an absolute normalisation of quark masses. Since the Kaplan-Manohar (KM) ambiguity is not a symmetry of QCD, it is possible to resolve it by determining the values of LECs from an approach based on first principles. It must be kept in mind, though, that a successful combination of lattice QCD and ChPT requires sufficient overlap between the range of quark masses used in simulations and the region of validity of the chiral expansion. This requirement is crucial if ChPT is used to extrapolate results from simulations performed for relatively heavy quarks and the regime of the physical $u$- and $d$-quark masses.

The remainder of this review covers a summary of the current status of lattice determinations of the light quark masses, quenched chiral logaritums, and LECs at NLO.

2. QUARK MASSES

Recent years have seen a great deal of progress in lattice determinations of quark masses. Some of the dominant systematic effects are now under much better control, chiefly due to using improved lattice actions and non-perturbative renormalisation, as implemented either via the RI/MOM scheme or the Schrödinger functional (SF). Several collaborations have started to quantify quenching errors, by performing simulations with dynamical quarks. Finally, the results in the quenched approximation are also being tested in simulations employing exact chiral symmetry on the lattice. Here we will focus on determinations of $m_s$, $\hat{m}$ and the charm quark mass. The mass of the $b$-quark is discussed in R. Sommer’s contribution to this conference. Earlier reviews can be found in refs. [3,8,10].

2.1. Wilson and staggered fermions

The LEC $B_0$ is obtained straightforwardly by mapping out the quark mass dependence of pseudoscalar meson masses. In order to compute, say, $m_s + \hat{m}$ one then has to specify the lattice scale and the quantity which fixes the bare physical quark mass. For instance, using $r_0$ to set the scale \[ \frac{1}{2}(m_s + \hat{m})r_0 = Z \cdot \frac{1}{B_0 r_0} \Big|_{m_F = m_K} \times (m_K r_0)^2 \] \[ (4) \]

Here, $Z$ denotes the renormalisation factor which relates the quark mass in lattice regularisation to a reference continuum scheme (e.g. $\overline{MS}$), and $(m_K r_0)^2$ is the phenomenological estimate of the kaon mass in units of the lattice scale. In order to estimate $m_s$, one either has to extract $\hat{m}$ separately through an extrapolation to $(m_s r_0)^2$, or take the ratio $m_s/\hat{m}$ from ChPT.

A compilation of recent results for $m_s$ and the ratio $m_s/\hat{m}$ is shown in Table 1. A marked feature is that estimates for $m_s$ in the continuum limit have stabilised, in contrast to the situation seen a few years ago. The table also shows that lattice estimates for the ratio $m_s/\hat{m}$, which are all based on chiral extrapolations, are broadly consistent with the result in ChPT at NLO \[ m_s/\hat{m} = 24.4 \pm 1.4. \]

(5)

However, closer inspection reveals small but significant deviations among the results for $m_s$. Therefore, for the results to be consistent, these deviations must be due to quenching effects, which are caused by different choices for the lattice scale. We will now attempt to convert some of the results in Table 1 to a common scale, in order to check whether consistency is satisfied.

To this end we consider two lattice scales, $Q$ and $Q'$. From the definition of the quark mass in eq. (4) it follows that the strange quark mass $m_s(Q)$ in MeV, estimated using $Q$, is related to its counterpart $m_s(Q')$ via

\[ m_s(Q) \text{[MeV]} = \left( \frac{Q}{Q'} \right)_{\text{lat}} \left( \frac{Q'}{Q} \right)_{\text{exp}} \frac{m_s(Q') \text{[MeV]}}{F}. \]

(6)

Here, the subscripts “lat” and “exp” refer to lattice and experimental estimates of the scale ratio, respectively. The deviation of the factor $F$ from unity is indicative of the relative quenching effects, when either $Q$ or $Q'$ is chosen to set the scale. I have taken results for vector meson masses from refs. [1,12,17] and for $F_K$ from [10].
Table 1

Results for $m_s$ in the $\overline{\text{MS}}$-scheme at $\mu = 2$ GeV and for the ratio $m_s/\hat{m}$, obtained using Wilson (W) and staggered (KS) quarks ($K$-input). Where applicable, we include information on the implementation of $O(a)$ improvement (non-perturbative (NP), mean-field perturbative (MF), tree-level (tree)), as well as the method for non-perturbative renormalisation (RI or SF).

| Collaboration | Ref. | action | Impr. | $a$ [fm] | Scale | Ren. | $m_s$ [MeV] | $m_s/\hat{m}$ |
|--------------|------|--------|-------|---------|-------|------|-------------|---------------|
| SPQ,cdR     | [11] | W      | NP    | 0       | $m_{K^*}$ | RI   | 106(2)(8)  | 24.3(2)(6)    |
| CP-PACS     | [12] | W      | NP    | 0       | $m_{\rho}$ | 114(2)(6) | 26.5(5.1)   |
| CP-PACS     | [13] | W/lwas. MF | 0 | $m_{\rho}$ | 110(3) | 25.0(1.4) |
| APE         | [14] | W      | NP    | 0.07    | $m_{K^*}$ | RI   | 111(9)     | 23.1(3.1)    |
| QCDSF       | [15] | W      | NP    | 0       | $r_0$ | SF   | 105(4)     | 23.9(1.4)    |
| ALPHA/UKQCD | [16] | W      | NP    | 0       | $F_K$ | SF   | 97(4)      |               |
| JLQCD       | [17] | KS     |       | 0       | $m_{\rho}$ | RI   | 106(7)     | 25.1(2.4)    |
| APE         | [18] | W      | NP    | 0.07    | $m_{K^*}$ | RI   | 111(12)    | 24.7(3.4)    |
| GGRT        | [19] | W      | tree  | 0.07    | $m_{K^*}$ | RI   | 130(2)(18) | 22.8(4.5)    |

Table 2

Results for the conversion factor $F$ for various scales $Q'$ and fixed $Q = r_0^{-1}$ at zero lattice spacing. Also shown are the estimates for $m_s(Q')$ expressed through the common scale $r_0^{-1}$ in MeV ($\overline{\text{MS}}$-scheme at $\mu = 2$ GeV).

| Ref. | $m_s(Q')$ | $Q'$ | $F$ | $m_s(r_0)$ |
|------|-----------|------|-----|------------|
| [1]  | 106(7)    | $m_{\rho}$ | 0.90(1) | 95(6) |
| [2]  | 114(2)(6) | $m_{\rho}$ | 0.86(2) | 98(2)(6) |
| [3]  | 106(2)(8) | $m_{K^*}$ | 0.87(3) | 92(2)(7) |
| [4]  | 97(4)     | $F_K$     | 1.02(2) | 99(4) |

The mass of the strange quark was actually one of the first quantities computed using domain wall fermions [27]. Since then many systematic effects have been studied: discretisation errors have been estimated by comparing results at different lattice spacings, although no continuum extrapolations have been performed so far. Quenching effects have not been investigated, since the numerical effort to simulate quenched Ginsparg-Wilson fermions is already comparable to a dynamical simulation using Wilson quarks [28].

We have seen that non-perturbative renormalisation is an important ingredient in order to enhance the credibility of lattice estimates for quark masses. The RI/MOM prescription has already been applied for both domain wall [29] and overlap fermions [30]. The Schrödinger functional, due to its inhomogeneous boundary conditions is somewhat harder to realise for Ginsparg-Wilson fermions [26].

2.2. Ginsparg-Wilson fermions

Lattice actions that satisfy the Ginsparg-Wilson relation [21] and thus preserve chiral symmetry at non-zero lattice spacing are now routinely used to compute many phenomenologically interesting quantities. The most widely used implementations are based on overlap [22] or domain wall fermions [23]. More recently, results for fixed-point (FP) [24] or chirally improved (CI) [25] actions, which both provide approximate solutions to the Ginsparg-Wilson relation, have also become available (see [26] for a review).

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fermions (an attempt has been made in ref. [31] in the case of domain wall fermions). The authors of [32] have proposed a general strategy to compute the renormalisation factors of quark bilinears for overlap fermions in the SF via an intermediate Wilson regularisation. This procedure avoids the direct formulation of the SF for Ginsparg-Wilson fermions, at the expense of sacrificing the ability to predict the quantity used to match to the Wilson results in the intermediate step.

Results for $m_s$ and $m_s/\hat{m}$ obtained using either domain wall or overlap fermions are compiled in Table 3. The numbers for $m_s$ are broadly consistent among each other, and also with the continuum estimates discussed earlier. However, the overall uncertainties are still somewhat larger compared with Wilson or staggered fermions. In most cases, the quoted systematic uncertainties are dominated by lattice artefacts, which have not been removed by performing continuum extrapolations. However, the overall dependence of $m_s$ on the lattice spacing appears to be fairly weak. These indications of good scaling properties of Ginsparg-Wilson fermions must be corroborated with more statistics and systematic investigations of renormalisation and finite-$a$ effects.

2.3. Dynamical simulations

The most important challenge for current lattice studies is surely the quantification of quenching effects. Perhaps the most comprehensive study so far has been published by CP-PACS some time ago [13]. They investigated $N_f = 2$ flavours of dynamical quarks, using mean-field improved Wilson fermions and the Iwasaki gauge action at three different values of the lattice spacing in the range $a \approx 0.1 - 0.2$ fm. Input values for the dynamical quark masses were chosen such that $m_{PS}/m_V = 0.58 - 0.8$. Estimates of $m_s$ based either on vector or axial vector Ward identities (VWI and AWI respectively) can be extrapolated to a common value in the continuum limit. By comparing their results to quenched data, CP-PACS find that dynamical simulations lead to a decrease in the value of $m_s^{\text{MS}}$ by about 25%. The dynamical data also appear to be less sensitive to the choice of quantity used to fix the bare strange quark mass ($m_K$ or $m_\phi$). The main findings of CP-PACS are summarised in Fig. 1, as well as in the estimates

$$m_s^{\text{MS}}(2 \text{ GeV}) = 88(4) \text{ MeV}, \quad m_s/\hat{m} \approx 26.$$  (7)

Figure 1. Scaling plot of dynamical and quenched estimates for $m_s$ by CP-PACS [13].

While CP-PACS’s results represent a major step forward, the limitations of current dynamical simulations are apparent. In particular, smaller dynamical quark masses should be simulated in order to quantify quenching effects more reliably. Moreover, simulating two flavours of dynamical quarks still does not represent the physical situation. Discretisation errors must be investigated more closely, by simulating smaller lattice spacing and comparisons with alternative fermionic discretisations. Finally, non-perturbative renormalisation should be implemented. Attempts to improve the situation in each of these areas have been reported at this conference.

The JLQCD and UKQCD Collaborations have both used non-perturbatively $O(a)$-improved Wilson fermions for $N_f = 2$ at $\beta = 5.2$. Whereas JLQCD [41] uses sea quark masses corresponding to $m_{PS}/m_V = 0.6 - 0.8$, UKQCD simulates even lighter masses, thereby running the risk of suffering from finite-volume effects. JLQCD have reported finite-size effects at the 3 – 5% level only at their lightest quark mass. They also extrapolate their results in $m_{\text{sea}}$ to the physical $u$ and $d$-quark masses. The quark masses are defined through the PCAC relation (AW1),
Table 3
Results for $m_\text{s}$ and the ratio $m_\text{s}/\hat{m}$, obtained using domain wall (DW) and overlap (Ov) quarks. Conventions are identical to Table 1.

| Collaboration | Ref. | action | $a$ [fm] | NP-Ren. $m_\text{s}$ [MeV] | $m_\text{s}/\hat{m}$ |
|--------------|------|--------|---------|----------------|------------------|
| RBC          | 33   | DW/DBW2 DW/W | $\sim$0.099 | RI | 133(3) 126(3) | $\sim$ 26 |
| CP-PACS      | 34   | DW     | $\geq$0.066 | 99(2)(6) | 26.3(2.3) |
| RBC          | 35   | DW     | 0.093    | RI | 105(6)(21) |
|              |      |        | 0.123    | 100(5)(20) |
| BSW          | 27   | DW     | $\geq$0.073 | 96(26) |
| DeGrand      | 36   | Ov     | 0.090    | 110(7) | 24.41(5) |
|              |      |        | 0.125    | 105(7) | 24.40(4) |
| C+H          | 37   | Ov     | 0.147    | 115(8) | 21.9(2) |
| GHR          | 38   | Ov     | 0.086    | RI | 102(6)(18) |
| HJLW         | 39   | Ov     | 0.093    | SF | 96(5)(7)   | 26.1(3.0) |
|              |      |        | 0.123    | 99(4)(7) | 25.9(3.4) |

and they expect some systematic effects to cancel, since the AWI mass is defined through a ratio of matrix elements. The results for $m_\text{s}$ and $\hat{m}$ are 20 – 30% smaller than in the quenched approximation. Scaling violations appear to be small, as there is good agreement with the previous CP-PACS results [3], but a continuum extrapolation is still lacking.

UKQCD perform extrapolations in the valence quark mass at fixed values of $m^\text{sea}$ and then monitor the results for decreasing sea quark mass, in order to search for trends which indicate the presence of dynamical quark effects. Unlike JLQCD they define the quark mass via the vector Ward identity. UKQCD’s results show no clear signs of dynamical quark effects: although there is some trend in the data as $m^\text{sea}$ is decreased, it is statistically not significant, and the same is true for the comparison with quenched data at a matched value of the lattice spacing. Their result for the ratio $m_\text{s}/\hat{m}$ is consistent with ChPT.

Hein et al. [3] have used dynamical configurations provided by the MILC Collaboration [14] for 2+1 flavours of improved staggered quarks. In this formulation the large flavour-changing interactions, which are typically encountered for conventional staggered quarks are suppressed due to the use of fat links. The lightest quark mass in [14] corresponds to $m_{1\text{PS}}/m_V \approx 0.37$, at a lattice spacing of $a \approx 0.13$ fm. Mean-field improved perturbation theory was used when matching to the $\overline{\text{MS}}$-scheme; the one-loop coefficients were found to be of the same order of magnitude as for Wilson fermions. Results for $m_\text{s} + \hat{m}$ obtained for $N_\text{f} = 0, 2$ and 2+1 dynamical flavours show that the large dependence on the quantity that sets the lattice scale is greatly reduced in the dynamical case. Low values for the light quark masses are typically preferred; for $N_\text{f} = 2 + 1$ flavours Hein et al. quote

$$(m_\text{s} + \hat{m})^{\overline{\text{MS}}}(2 \text{ GeV}) = 78 \pm 14 \text{ MeV},$$

where the scale has been taken from the $K^* - K$ mass splitting. The quoted error includes the uncertainty from neglecting the two-loop term in the perturbative matching procedure, which is estimated to be as large as 20%. The value of $(m_\text{s} + \hat{m})$ in the dynamical case is lower by 15 – 20% compared with the corresponding quenched result. For a better understanding of the systematics in the improved staggered formulation, it would, in my opinion, be extremely helpful if a quenched value for $m_\text{s}$ were available in the continuum limit.

The ALPHA Collaboration has reported results on the non-perturbative running of the quark mass in the SF scheme for the 2-flavour case [13]. This is one part of the two-step procedure, which will ultimately relate the bare current quark mass on the lattice to the renormali-
sation group invariant (RGI) quark mass $M$. In Fig. 2 the running mass in the SF scheme is plotted as a function of the renormalisation scale. One sees that the perturbative evolution follows the numerical data down to fairly small values of the scale. At its lowest value, i.e. at $\mu = 1/(2L_{\text{max}})$, ALPHA obtain their preliminary result for the matching of the running mass in the SF scheme to the RGI quark mass:

$$\frac{M}{m_{\text{SF}}} = 1.236(15), \quad \ln(\Lambda L_{\text{max}}) = -1.85(13). \quad (9)$$

The scale $L_{\text{max}}$ must still be related to some physical quantity. Further studies will include data at larger values of $L/a$, so that lattice artefacts can be eliminated completely.

![Figure 2](image.png)

Figure 2. Running quark mass in the SF scheme for $N_f = 2$.

### 2.4. The charm quark mass

The charm quark is too heavy to be described by ChPT, and too light for an efficient non-relativistic treatment. In two recent projects [46, 47] the charm quark mass was computed using the relativistic formulation. In this case, the issue of controlling lattice artefacts is even more important as in the light quark sector. The ALPHA Collaboration [47] have used $O(a)$ improved Wilson fermions in the quenched approximation at four $\beta$-values. They employ several different definitions of the RGI charm quark mass, e.g.

$$r_0 M_c = Z_M r_0 m_c [1 + (b_A - b_V) am_{q,c}] \quad (10)$$

$$r_0 M_c = Z_M Z_r r_0 m_{q,c} [1 + b_m am_{q,c}], \quad (11)$$

where $m_c$ and $m_{q,c}$ denote the bare masses defined through the axial and vector Ward identities, respectively. Non-perturbative values of renormalisation factors and improvement coefficients as reported in [48] have been used throughout, in order to guarantee a controlled extrapolation to the continuum limit. The input quantities which fix the bare charm quark mass in terms of meson masses were $m_{D_s}, m_K$ and the ratio $m_s/\bar{m}$ from ChPT. In this way chiral extrapolations can be avoided completely. By demanding that different definitions of the RGI charm quark mass extrapolate to a common continuum limit, they find $r_0 M_c = 4.19(11)$ at zero lattice spacing, which translates into

$$\frac{m_c^{\overline{\text{MS}}}(m_c)}{m_c} = 1.301(34) \text{GeV}, \quad (12)$$

where the 4-loop RG functions in the $\overline{\text{MS}}$ scheme have been used. This result agrees well with the estimate by the SPQcdR Collaboration [46]

$$\frac{m_c^{\overline{\text{MS}}}(m_c)}{m_c} = 1.26(4)(12) \text{GeV}, \quad (13)$$

which has been obtained at $a \approx 0.07 \text{fm}$.

### 2.5. Summary: Quark masses

A detailed comparison of quenched data shows that the mass of the strange quark is probably the most precisely determined quantity in the quenched approximation: for a given choice of lattice scale, the relative accuracy in the continuum limit is about 5%. If the $K$-meson is used to fix the bare $m_s$, the final result varies between 95 and 115 MeV, depending on the choice of lattice scale. There is a 20% increase in $m_s$ if the mass is fixed through a vector meson like the $\phi$.

It is now clear that current lattice estimates for the strange quark mass are completely dominated by quenching effects. First attempts to quantify the quenching error indicate a decrease in $m_s$ of about 20%, but simulations at smaller quark masses and lattice spacings are required to confirm this.

Estimates for the ratio $m_s/\bar{m}$ based on chiral extrapolations tend to agree with ChPT, both for quenched and unquenched simulations.

Finally, non-perturbative renormalisation and $O(a)$ improvement enable one to perform controlled continuum extrapolations of the charm
quark mass, thereby yielding precise quenched estimates.

3. QUENCHED CHIRAL LOGARITHMS

In this section we will be concerned with quenching artefacts in the chiral expansion. It is well known that the quark mass behaviour of hadron masses and matrix elements is modified in the quenched approximation [48, 50, 51]. The main qualitative difference is that flavour singlets do not decouple in the quenched version [49, 50, 51]. The hadron masses and matrix elements is modified estimates.

\[ L_{\text{sing}} \propto \alpha_\Phi D_\mu \Phi_0 D^\mu \Phi_0 - m_0^2 \Phi_0^2 \] (14)

The flavour-singlet propagator develops a double pole in the quenched theory, which is proportional to \( m_0^2 \). Since \( m_0 \) does not vanish in the chiral limit, this gives rise to a new type of chiral logarithm, which diverges as the octet mass vanishes. The modified chiral expansion for the pseudoscalar octet mass at NLO thus reads

\[ \frac{m_{PS}^2}{2m} = B_0 \left( 1 - \left( \delta - \frac{2}{3} \alpha_\Phi y \right) (\ln y + 1) \right) + \left( 2 \alpha_\Phi - \alpha_5 \right) \frac{2}{3} \alpha_\Phi y \] (15)

For notational convenience we have introduced

\[ \delta = \frac{m_0^2}{(4\pi F_0)^2 N_f}, \quad y = \frac{2B_0 m}{(4\pi F_0)^2} \] (16)

and rescaled the LECs \( L_i \) to \( \alpha_i = 8(4\pi)^2 L_i \). A phenomenological estimate of \( \delta \) is obtained from the relation

\[ m_0^2 = m_{PS}^2 + m_0^2 - 2m_{K}^2 \] (17)

which gives \( \delta \approx 0.18 \). There are several methods which allow the determination of \( \delta \) and \( \alpha_\Phi \). The most straightforward is to study the quark mass dependence of \( m_{PS}^2 \) and compare it with eq. (15). Another method is to compute the ratio of flavour-singlet and flavour-octet contributions to the quenched \( \eta' \) correlator. The former are the so-called “hairpin diagrams”. Finally, the Witten-Veneziano formula [52] provides a link between the flavour-singlet mass \( m_0 \) and the topological susceptibility \( \chi_1 \):

\[ m_0^2 = \frac{2N_f \chi_1}{F_\pi^2}. \] (18)

The effects of quenched chiral logarithms are expected to show up very near the chiral limit. However, in this regime quenched QCD is afflicted with the occurrence of so-called “exceptional configurations”, i.e. unphysical near-zero modes, if conventional discretisations such as Wilson fermions are used. Recently this has stimulated the use of discretisations which do not suffer from this problem, such as overlap and domain wall fermions, fixed point actions and twisted mass QCD (tmQCD) [53]. Also the “Modified Quenched Approximation” [5] in conjunction with Wilson fermions has been applied. In addition, one has to avoid finite-volume effects when going very near the chiral limit, which may fake the signature of the quenched chiral logs. It is also clear that there is a special rôle for discretisations which preserve chiral symmetry, for which the Witten-Veneziano formula is exact [55, 56]. Moreover, the comparison of lattice data with the predictions of quenched lattice QCD at non-zero lattice spacing is justified in this case.

Lattice data for \( m_{PS}^2 \) show very low sensitivity on \( \alpha_\Phi \) in general. The RBC Collaboration finds [58] that in the studied mass range the combination \( \alpha_\Phi m_{PS}^2 \ln m_{PS}^2 \) (or \( \alpha_\Phi y \ln y \) in eq. (14)) stays practically constant, so that there is hardly any sensitivity to the value of \( \alpha_\Phi \). CP-PACS report [1] that fits in which \( \alpha_\Phi \) is treated as a free parameter are either unstable or yield results that are consistent with zero. Thus, in most studies it is assumed that \( \alpha_\Phi = 0 \).

Table [5] shows a compilation of recent determinations of \( \delta \). Overall one observes large variations in the current estimates for \( \delta \), with results from simulations employing conventional fermionic discretisations being somewhat lower that those using Ginsparg-Wilson fermions (exceptions are the
Table 4
Recent determinations of $\delta$. The second and third columns show the minimum (bare) quark mass in MeV and the smallest pion mass in units of the spatial lattice size, respectively.

| Collaboration | Ref. | $m_q$ [MeV] | $m_{\pi}^{\text{min}} L$ | $a$ [fm] | action | $\delta$ |
|--------------|------|-------------|------------------------|---------|--------|--------|
| CP-PACS      | 12   | 20          | 6                      | 0.05 − 0.1 | W      | 0.10(2) |
| QCDSF        | 57   | 22          | 4.7                    | 0.05 − 0.1 | SW     | 0.14(2) |
| FNAL         | 54   | 14          | 3                      | 0.17     | SW     | 0.065(13) |
| MILC         | 14   | 15          | 3.4                    | 0.13     | KS     | 0.061(3) |
| Kentucky     | 59   | 19          | 3.2                    | 0.157    | Ov     | 0.2 − 0.4 |
| RBC          | 68   | 28          | 3.8                    | 0.104    | DW     | 0.05(2) |
| DeGrand/Heller | 56  | 30          | 0.13                   |          | Ov     | 0.093(28) |
| Chiu+Hsieh   | 60   | 80          | 2.5                    | 0.147    | Ov     | 0.203(14) |
| Bern         | 61   | 38          | 2.6                    | 0.13     | FP     | 0.23(7) − 0.30(18) |
| Kentucky     | 62   | 14          | 2.9                    | 0.20     | Ov     | 0.26(3) |
| BGR          | 63   | 2.7         | 0.16                   |          | FP     | 0.17(2) |
|              |      | 2.9         | 0.15                   |          | CI     | 0.18(3) |

low values quoted in refs. [58,59]. Despite the fact that the sensitivity to quenched chiral logs is still quite low, there is mounting evidence for a non-zero value for $\delta$. In fact, more recent simulations, which employ lattice chiral symmetry, produce results that are compatible with the phenomenological estimate based on eq. (17). However, a remaining matter of concern is the possibility that finite-volume effects may distort the mass dependence towards the chiral regime. This must be addressed in simulations on larger volumes. So far no convincing signal for $\alpha_8$ has been observed, mainly because the sensitivity of lattice data is even worse than for $\delta$.

4. IS THE UP-QUARK MASSLESS?

Let us now return to the Kaplan-Manohar ambiguity. The fact that the effective chiral Lagrangian beyond leading order is invariant under simultaneous transformations of the mass matrix and a subset of LECs implies an uncertainty in the size of the NLO correction to the mass ratio $m_u/m_d$. This correction, $\Delta_M$, is given by

$$\Delta_M = \frac{(m_K^2 - m_{\pi}^2)}{(4\pi F_{\pi})^2} (2\alpha_8 - \alpha_5) + \text{chiral logs}.$$ (19)

While $\alpha_5$ can be estimated from the ratio $F_K/F_{\pi}$, there exists no experimental information which would allow an unambiguous determination of $\alpha_8$ or indeed the linear combination $2\alpha_8 - \alpha_5$. The value of $\alpha_8$ and thus $\Delta_M$ must then be fixed by invoking (plausible) assumptions beyond chiral symmetry considerations. Proceeding in this way Leutwyler [20] obtained

$$0 < \Delta_M \leq 0.13.$$ (20)

This result excludes the possibility that $m_u/m_d = 0$, which is only possible for a large negative value for $\Delta_M$. Therefore, a massless up-quark, which presents a simple and elegant solution to the strong CP problem is an unlikely scenario. However, an estimate of $\Delta_M$ based on first principles is still lacking. Given the importance of the strong CP problem, it is desirable to tackle this question by means of lattice determinations of $2\alpha_8 - \alpha_5$.

Here, partially quenched simulations of lattice QCD play an important rôle: for unequal sea and valence quark masses the effective chiral Lagrangian is parameterised in terms of the same LECs as the physical theory. Thus, as long as the correct number of dynamical quark flavours (i.e. $N_f = 3$) is used, simulations based on unphysical mass combinations provide phenomenological information [60], provided that the regime of validity of ChPT and the range of simulated masses overlap.

A general method which allows for the extraction of LECs with good statistical accuracy has been introduced by ALPHA [67]. Since the ex-
pressions of ChPT are valid for arbitrary quark mass, one can introduce a reference value \( m_{\text{ref}} \) and define the ratios

\[
R_F(x) = \frac{F_{PS}(m)}{F_{PS}(m_{\text{ref}})}
\]

\[
R_M(x) = \frac{\left( \frac{2m}{m_{PS}(m)} \right)}{\left( \frac{2m_{\text{ref}}}{m_{PS}(m_{\text{ref}})} \right)}
\]

where \( x = m/m_{\text{ref}} \). The ratios \( R_F, R_M \) are universal functions of the dimensionless mass parameter \( x \). All renormalisation factors drop out, so that these quantities can be extrapolated to the continuum limit straightforwardly. Due to the correlations between the numerator and denominator one can expect good statistical precision in numerical evaluations. The LECs at NLO can then be determined by comparing numerical data for \( R_F, R_M \) with the corresponding expressions in quenched \([49,50,51]\) or partially quenched \([68,69]\) ChPT.

\[\text{Figure 3. Results for } R_F \text{ and } R_M \text{ in the quenched approximation [67].}\]

\[\text{ALPHA have tested their method in the quenched approximation for } m_{\text{ref}} \approx m_s, \text{ and } x \text{ in the range } 0.75 \leq x \leq 1.4 \text{ [67]. Their results are shown in Fig. 3. The ratio } R_F, \text{ which is predicted by quenched ChPT to rise linearly with the quark mass, is modelled very well by the data, resulting in stable estimates for } \alpha_5. \text{ Numerical data for the ratio } R_M, \text{ however, show an almost constant behaviour, while quenched ChPT predicts the presence of linear terms, as well as chiral logs. Thus, if the range of simulated masses lies indeed within the regime where ChPT at NLO is valid, then the constant behaviour of the data must be the result of a strong cancellation between the various contributions at NLO.}\]

Bardeen et al. \([54]\) have performed global fits to pseudoscalar correlation functions to extract masses and various LECs. Using the pole shifting prescription of the “Modified Quenched Approximation” (MQA) at \( \beta = 5.7 \) and mean-field improved Wilson fermions, they report an estimate for \( \alpha_5 \) which is about 3 times larger than ALPHA’s. A large fraction of this discrepancy can be attributed to the fact that renormalisation factors for the axial current have not been taken into account \([70]\). In fact, if the data for the decay constant published in \([54]\) are subjected to ALPHA’s ratio method, then the estimates for \( \alpha_5 \) are consistent among the two studies. Thus, the source for the original discrepancy is a combination of renormalisation effects and lattice artefacts, with the ratio method being a lot more stable against these systematic effects.

The ratio method has also been applied in two recent simulations of partially quenched QCD. UKQCD \([71]\) have simulated two flavours of \( O(a) \) improved dynamical Wilson fermions at \( a = 0.1 \) fm and a fixed value of the sea quark mass of \( m_{\text{sea}} \approx 0.7m_s \), which corresponds to \( m_{PS}/m_V \approx 0.58 \text{ [72].} \) The mass dependence of the ratios \( R_F \) and \( R_M \) was subsequently studied as a function of the valence quark mass at fixed \( m_{\text{sea}} \). The overall behaviour of \( R_F \) and \( R_M \) was found to be similar to what is observed in the quenched case. However, the strict linearity of \( R_F \) for \( N_f = 0 \) is modified by a slight curvature, which may signal the expected presence of chiral logs. UKQCD’s results for the LECs are listed in Table 5.

The Ohio group \([73]\) have reported results for LECs from simulations using \( N_f = 3 \) flavours of dynamical staggered quarks. The sea quarks are somewhat lighter than in UKQCD’s study, while the lattice spacings are larger (\( a \approx 0.15 \) and 0.28 fm). The dominant systematic effects are large flavour-symmetry violations, which are estimated by applying hypercubic blocking to the gauge configurations before the computation of
Table 5
Results for the LECs at NLO for several values of \(N_f\). The last two lines display the results for the “standard” phenomenological estimates, as well as the numbers required for the up-quark to be massless.

| Collab. | Ref. | \(N_f\) | \(\alpha_5\) | \(\alpha_8\) | \(2\alpha_8 - \alpha_5\) | comments |
|---------|------|---------|-------------|-------------|-----------------|----------|
| ALPHA   | 27   | 0       | 0.99(6)(20) | 0.50(4)(20) | 0.35(5)(15)     | \(\delta = 0.12, \alpha_\Psi = 0\) |
|         | “3”  |         |             |             |                 |          |
| UKQCD   | 7    | 2       | 1.22(13)(25) | 0.79(6)(21) | 0.36(10)(22)    |          |
|         | “3”  |         |             |             |                 |          |
| OSU     | 72   | 3       | 0.5(6)      | 0.76(4)     |                 | “standard” |
|         | "1"  |         |             |             |                 | m_\Psi = 0 |
|         | "2"  |         |             |             |                 |          |

observables. The value of \(\alpha_5\) differs significantly when it is evaluated on blocked and unblocked configurations. In the future this issue will be addressed by comparing Monte Carlo data to ChPT for which the effects of flavour-symmetry breaking are included [74].

From the compilation of results in Table 5 one concludes that the \(N_f\)-dependence of LECs is quite weak. If one uses the expressions of ChPT for \(N_f = 3\), even though actually \(N_f = 0\) or 2 was used to generate the data, one still observes consistency within errors (cases labelled \(N_f = "3"\) in Table 5). A comparison with the “standard” values for the LECs estimated in the continuum, as well as with those values that would be required to support the notion of a massless up-quark, shows that the latter scenario is strongly disfavoured by lattice calculations: the quark mass behaviour of \(R_M\) would have to be radically different to accommodate a large negative value of \(\alpha_8\). In short, calculations based on first principles support the analysis of ref. [20].

The most important issue at this stage is whether or not the quarks used in the simulations are light enough to justify the comparison with ChPT. This has been addressed in several contributions to the panel discussion on chiral extrapolations at this conference [3]. Although lattice estimates for some of the LECs make sense phenomenologically (e.g. UKQCD’s result for \(\alpha_5\) in [71] is consistent with the experimental value of \(F_K/F_\pi\) [7]), there is evidence that the dependence of \(m_{PS}^2\) and \(F_{PS}\) on the sea quark mass is not in agreement with ChPT if the sea quark mass corresponds to \(m_{PS} = 550 - 1000\) MeV [74]. This affects attempts to perform extrapolations in \(m_{sea}\) to the physical point defined by \(m_{PS}/m_V \approx 0.17\). A similar study [77] concluded that sea quark masses of current simulations and ChPT at NLO overlap only marginally.

In addition to many numerical results, there are also new analytic developments. Aubin et al. [74] have analysed the structure of chiral logarithms under staggered flavour-symmetry breaking for \(N_f = 2+1\) flavours. The basic idea is to generalise the Lagrangian of Lee and Sharpe [78], which describes a single staggered field up to \(O(a^2)\), to the 3-flavour case. Normally, each staggered flavour describes four internal fermions (called “tastes” in [74]). In order to obtain one taste per flavour one then takes the fourth root of the determinant. For the actual calculation of chiral logs in the 2+1 flavour case, this implies that one starts with \(4+4\) “tastes” and then cancels the unwanted loop contributions by multiplying them with simple weight factors. The resulting expressions for pseudoscalar meson masses fit the data by MILC [44] very well, unlike the corresponding formulae in “ordinary” partially quenched ChPT [68,69].

In a recent paper [79] Rupak and Shoresh have incorporated the explicit chiral symmetry breaking of Wilson fermions directly into the effective Lagrangian. The starting point is Symanzik’s effective action. Including all operators up to dimension 5, they find that chiral symmetry is broken not only by the quark density \(\bar{\psi}\psi\), but also
by the Pauli term $\bar{\psi} \gamma^\mu F_{\mu\nu} \psi$, which is multiplied by the coefficient $c_{sw}$. Rupak and Shoresh then consider simultaneous chiral expansions in the parameters

$$\epsilon \sim \frac{2B_0 m}{\Lambda^2}, \quad \delta \sim \frac{2W_0 a c_{sw}}{\Lambda^2}.$$  \hspace{1cm} (23)

The Pauli term generates additional operators in the effective chiral Lagrangian, starting at leading order. Consequently, there are a number of additional LECs. Up to NLO these are $W_0, W_4, \ldots, W_8$, where the subscripts are chosen in analogy with the operators appearing in the conventional Lagrangian. The resulting expressions for pseudoscalar masses and decay constants in the partially quenched case may be used to extract the LECs at small but non-zero lattice spacing.

5. SUMMARY

Lattice determinations of quark masses have entered a mature stage, where the dominant systematic error is quenching. The quantification of dynamical quark effects will be the main subject of investigation in the coming years. The comparison of lattice data with effective low-energy theories such as ChPT has turned into a major activity. Here, the goal is not just to verify ChPT but to exploit its predictions in order to make contact with the regime of very light quarks, which is difficult to access directly in simulations. So far it is not entirely clear whether the sea quark masses used in simulations are small enough to justify extrapolations to the chiral regime based on ChPT at NLO. Simulations of partially quenched QCD show that the scenario of a massless up-quark is not reflected at all in the valence quark mass behaviour of pseudoscalar quantities. Further simulations with smaller quark masses should be performed to corroborate these findings and to settle the issue of the mass range in which ChPT at NLO can be expected to be valid.

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