Simulation of a cylindrical-conical gear train on parallel axes with an adjustable side clearance

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Abstract. The geometrical synthesis of a cylindrical-conical transmission on parallel axes with external gearing is considered on the basis of the blocking contour method, which makes it possible to determine solutions for the right and left ends of the wheel. The use of the calculation for cylindrical-conical wheels is described. We used our own algorithm for constructing the blocking contour and engagement in the MathCAD program, and a 3D model of the transmission in the KISSsoft program. For a better example, the contour and engagement in the 3D model are given.

1. Introduction
Modern drive technology differs from outdated models in a variety of operating modes, in which the direction, speed and torque on the shafts change dramatically, etc. Therefore, modern drive technology is subject to technical requirements for parameters, one of which is the moment of inertia of the parts, the backlash in the mechanical transmission, etc. To reduce the inertia and increase the rigidity of mechanical transmission parts, composite materials, composite multilayer structures, etc. are used for them. But in addition to these parameters, the side clearance affects the backlash, especially in slow-moving stages. In practice, there are several ways to eliminate gaps, some of which are costly in terms of production resources, and require the skills of assemblers and adjusters. Therefore, in this article, it is worth starting to consider the design of a cylindrical-conical transmission with the continuation of research on a real model with optimal indicators of the quality of wheel engagement.

The beginning of the creation of the gear transmission of the external and internal gearing of the wheels is a geometric synthesis. Therefore, first, the main geometric parameters of the gears and the transmission itself are calculated, which ensure the efficiency and reliability of the gear, as well as the necessary values of the gearing quality indicators. The main parameters of a straight-toothed cylindrical-conical gear transmission are not only the modulus of the teeth \( m \), number of teeth \( z_1, z_2 \), but also the ranges of the displacement coefficients \( x_1, x_2 \), the angles of the cone \( \theta_1 = \theta_2 \) and the width of the wheels \( b_1 \) and \( b_2 \). A cylindrical-conical transmission on parallel axes must satisfy a number of additional conditions of geometric synthesis from the left to the right end in the cross-section of the wheels. Thus, in contrast to the flat gear, as we can observe in the spur gear, the coverage ratio, the thickness of the teeth on the top, sizes tops and bottoms of the teeth, the corners of sections in characteristic points of the teeth of wheels will be different. Therefore, this problem is related to multiparametric problems for ensuring high-quality gas-free involute engagement in all wheel cross-sections. The number of teeth of the gears is determined by the required gear ratio and the center distance according to the technical specification, especially for gears with connected wheels. The modulus, width and initial diameters of the wheels are determined based on the strength. The module is a scale factor,
and does not affect the values of dimensionless indicators of the quality of engagement, except for the parameters of machine engagement. Then the output parameters of the synthesis are only the ranges of the displacement coefficients \( x_1, x_2 \) and the corners of the cone \( \theta_1 = \theta_2 \). Hence of studying groups of solutions to the synthesis problem, building blocking contours within which the solutions to the synthesis problem will be located for all wheel cross-sections.

The coupling of the teeth of the wheels in the cylindrical-conical transmission is determined by the normal side gap, which is the gap along the line of engagement between the non-working side profiles of the two teeth that roll over each other in the working transmission. In a real cylindrical-conical transmission, the coupling of the wheel teeth is characterized by the value of the smallest guaranteed gap, since its application is caused by the use of reversible drives in the structure for platform orientation systems in different modes. The application and importance of which can be seen in figure 1 in a mechatronic electric drive developed at the Department of "Design of Mechanisms and Machine parts" [12] under the direction of prof., Doctor of Technical Sciences P G Sidorov.

![Figure 1](image.png)

**Figure 1.** Design of the combined mechatronic drive for the orientation of the platform to the robotic systems: 1—electric motor; 2—gearbox; 3—position sensor; 4—tachogenerator

A popular and visual method for solving the problem of geometric synthesis of gears with an involute profile of teeth in the plane is the method of blocking contours [1], [2], [3]. This method is based on the construction of a blocking contour with the domain of existence of the solution of the synthesis problem inside. By isolating specific values of quality indicators for gearing the transmission wheels, suitable variants (combinations of displacement coefficients) are cut off from unsuitable ones, in which additional synthesis conditions are met. This method is universal, and was long ago included in the current standards. Now this method is used not only in geometric synthesis, but also in the design according to the criteria of efficiency. The blocking contour has to be built in the plane of the axes for each combination of the numbers of the teeth of the gear and the wheel, not to mention the parameters
of the gear cutting tool. Therefore, the synthesis is performed to find a point, and in the case of a cylindrical-conical transmission, groups of points connected by a line with a constant angle of engagement [4, 5]. The disadvantage of the method is that we can approach the solution of a specific problem by reconstructing the blocking contour with other parameters. The task of geometric synthesis is simplified if the design specification is limited to a number of parameters, such as the center distance, the width of the gears, and the diameters of the tops and depressions of the wheels.

The use of a computer with application programs made it possible to automate the drawing of blocking contours, and to draw special blocking contours for specific synthesis conditions, which removed the restriction when changing the radial gaps. We know from the literature today a variety of software options that allow you to draw: a contour in interactive mode [6], a dynamic contour [7], [8], [9], contour for plastic wheels [10].

The system of geometric calculation of gear and wheel engagement in the standards implements the constant coefficients of standard radial clearances, but allows them to change for all engagements with the condition that all additional geometric synthesis is performed. In the absence of methods, this problem has a huge number of solutions, the volume of which may not be able to cope with a computer, especially for cylindrical transmission.

In this paper, it was also proposed to refer to the output parameters of the synthesis of radial gaps connecting through the diameters of the circles of the vertices \(d_{a1}, d_{a2}\) and depressions \(d_{f1}, d_{f2}\) wheels between spacing to machine and work engagement, which in turn is associated with the coefficients of the offset of the wheels: wheels between spacing to machine and work engagement \(a_{w12}\), which in turn relate the angle to the wheel displacement coefficients for external gearing

\[
a_{w12} = m \frac{z_1 + z_2}{2} \cos(\alpha) \cos(\alpha_{w12}).
\]

(1)

Here \(\alpha\) - angle of the initial profile of the cutting tool, \(\alpha = 20^\circ\); \(\alpha_{w12}\) – the angle of engagement, determined by the function of the involute \(\text{inv}(\alpha) = \tan(\alpha) - \alpha\) from the expression for external engagement is expressed from the formula

\[
\text{inv}(\alpha_{w12}) = \frac{x_2 + x_1}{z_2 + z_1} 2 \tan(\alpha) + \text{inv}(\alpha).
\]

(2)

In [12], it is shown that this calculation system for this synthesis problem – the geometric synthesis of internal gearing with a small difference in the number of teeth in the planetary moment multiplier - allows you to expand the domain of existence of the solution to the synthesis problem several times or create this domain if it is not available for the standard calculation system.

The solution of the synthesis problem is much more complicated in the geometric synthesis of a four-link gear drive with a parasitic wheel for mechatronic electric drives with position sensors, power supply and output, in which the parasitic wheel simultaneously participates in a group of gears. When performing the geometric synthesis of several gears, the geometric parameters of this (intermediate) wheel can be different, and at the same time the drawing for it is one single one.

In the articles [13], [14], the authors proposed to use a three-dimensional blocking contour drawn in the coordinate system \(x_1, x_2\). It very clearly and simply shows the dependence of geometric parameters, but when solving specific problems, you should use its projections on the plane of the coordinate system.

In [15], [16], it was proposed to use a calculation system for wheels with a constant tooth height in a cylindrical-conical wheel for any displacement coefficients, but with a constant sum of them.

The aim of the work is to model the engagement in two planes by two methods of a three-link cylindrical-conical gear and to analyze the fulfillment of additional conditions of geometric synthesis according to the indicators of the quality of engagement of cylindrical-conical wheels on parallel axes.

2. Formulation of problem
A three-link cylindrical-conical transmission with external involute engagement of cylindrical wheels with parallel axles is considered (figure 2).
The parameters of the initial contour of the cutting tool correspond to 13755-2015. Modification of the profile of the tooth head of the original contour is carried out before the finishing operation with a tooth-cutting tool, a cutter or a chisel. This modification is expressed in a change in the amount of the tool feed along the tooth. As a real object of research, according to the technical task from the enterprise (figure 2), a cylindrical-conical transmission was taken with the ability to adjust the position of the wheels along the axis to obtain a gas-free engagement with the permissible drag torque at idle. For the gear $z_1$ the final value increases from the right end of the tooth to the left end of the tooth by 1/23, and for the wheel $z_2$ from the left end of the tooth to the right is similar. As a result of this modification (correction) of the gears, the displacement coefficient of the original contour decreases. So for the gear, the displacement coefficient is in the range from $0.6087$ to $0$, and for the wheel in the range from $0$ to $-0.6087$. In addition, an additional modification is carried out on the tops of the teeth (tooth heads), which is expressed in reducing the height by cutting the heads along a conical surface with a cone angle 1°30’ with a base diameter on the left end of the teeth at the gear and the right end of the teeth at the wheel.

3. Theoretical part
In the analysis of the geometry of the transmission of the known formulas with external gear in GOST 16532-60 and internal gearing in GOST 19274-73, in the first stage, you define parameters and quality indicators with the generally accepted assumptions. That is, the deformation of the housing, shafts, rolling bearings, cylindrical-conical wheels, as well as errors in the manufacture of parts and installation of the transmission in the gearbox are not taken into account yet. Cylindrical wheels with both external teeth are traditionally cut by running-in method using a worm cutter to increase productivity, but for the universalization of the simulation of this transmission, both with external gearing and with internal gearing or a combination of them, we will use a chisel according to GOST 9323-79.
to-center distances $a_w = 80\,\text{mm}$ of external gear engagement $z_1 = 17$ and wheels $z_2 = 47$ set at the tooth module $m = 2.5\,\text{mm}$.

The diameters of the cavities of the gears cut with the help of a chisel change as can be clearly seen from figure 3, due to changes in the coefficients of displacement of the wheels and the parameters of the chisel in the machine engagement

$$d_{f1} = 2a_{w01} - d_{a0}; \quad d_{f2} = 2a_{w02} - d_{a0},$$

(3) where $a_{w01}$, $a_{w02}$ – centerline distances in machine engagements of cut gears with a chisel; $d_{a0}$ – the diameter of the circumference of the tops of the chisel teeth.

Accordingly, the diameters of the circles of the tops of the teeth of the cylindrical-conical wheels change by $3\,m$ (see Fig. 3) taking into account the formulas (1), (2) must satisfy the following dependencies,

$$d_{a1} = 2a_{w12} - d_{f2} - 2c_1^*m; \quad d_{a2} = 2a_{w12} - d_{f1} - 2c_2^*m,$$

(4)

For the synthesis of gearing in working areas $L_1L_2$ and $R_1R_2$, the engagement lines on the left $L$ and right $R$ ends of the teeth, respectively, must meet additional synthesis conditions (figure 4):

1) the coefficient of end overlap according to the condition of smoothness of engagement,
2) no interference in the working engagement of the gear tooth leg with the top of the wheel tooth,
3) no interference in the working engagement at the wheel tooth leg with the top of the gear tooth,
4) sharpening of the gear teeth to eliminate a local break at the top,
5) sharpening of the wheel teeth to avoid a local break at the top,
6) cutting the gear tooth in the machine engagement, i.e. reducing the working side profile,
7) cutting the wheel tooth in the machine engagement, i.e. reducing the working side profile,
8) no interference in the machine engagement at the leg of the chisel tooth with the top of the tooth,
9) no interference in the machine engagement at the leg of the chisel tooth with the top of the tooth,

Figure 4. Blocking contour of external engagement of wheels
The corresponding dependencies for additional conditions of geometric synthesis are given in [1], [4], [5], and the isolines of the engagement quality indicators for additional conditions are shown in figure 4 for the structure of the blocking contour. A number of isolines of quality indicators for additional conditions: 1, 5, 8 and 9, shown in figure 4, are not shown at the extreme values, in order to consider an important part of the solution area at the blocking contour. This important part has a color fill. It is worth noting that the fulfillment of additional synthesis conditions for interference of the first kind: 2, 3, 8 and 9 depends on the parameters of the chisel. Therefore, when constructing a chisel with the number of teeth from the middle of the range was chosen: 10, 15, 20, 30, 40 and 50.

4. Model development
The problem of synthesis for the right and on the right side, located on the line of constant spacing of 80 mm and marked by extreme points inside blocking the path to the right and on the right side, close to the borders of the outline left by condition 6, right, and bottom condition 4 condition 2. This should solve the problem of optimization of the synthesis strategy additive compensation contradictions criteria [18], [19]. In the first stages of optimization, the center distance can be increased by 1...3 mm. It is more preferable to reduce the gear ratio by reducing the number of wheel teeth by 1...3 teeth, because in many drives, the gear ratio can be deviated to 3...5 %, and it is not necessary to change the position of the bearing seats in the device body (Fig. 5). The first optimization option is marked with a dotted line with displacement coefficients for the right end of the gear 0.71 and for the wheel – 0.29. As you can see, this solution is less dependent on the tool when the coefficient of end overlap under condition 1 is not less than 1.45.

![Figure 5. Transmission diagram in the gearbox](image1)

![Figure 6. Basic 3D model of a gearbox without a housing with a cylindrical-conical transmission](image2)

The geometry-optimized gearing is used to simulate a solid-state model of a cylindrical-conical transmission in the KISSsys program of KISSsoft AG under a license agreement with Tula State University. In the program, along with a pair of cylindrical-conical wheels, a group of shafts, bearings and their connections is considered. The main window of the program shows the model tree, its diagram, and a 3D view in figure 5 with the display of the housing and the oil level. The simulation model of involute gearing cylinder-conical wheels that enable an analysis of the coupling teeth deformations of
main parts of speed reducer, precision manufacturing, and engineering this re-transfer, adapting them to real-world conditions.

The study is carried out in two stages: on the 1st-analysis of the coupling of the teeth without taking into account the rigidity of the main parts; on the 2nd – taking into account the rigidity and accuracy of their paired coupling.

The creation of a tree of the gearbox model with this gear engagement is carried out according to the drawing of the enterprise using the well-known technique [17, 18]. At the same time, setting the source data and getting it is a quick and intuitive process. To automate the creation of such a model, you can use ready-made templates. The process of transferring data from KISSsys model to calculate the transmission, axles, joints and bearings can be made in two ways: in a direct input to specify the main parameters of the calculation of the elements of the gearbox and optional parameters through or ready designed templates, and data storage in a single file, calculations, heat balance, modal analysis, etc. for the visual correction of the display elements of the diagram in the gearbox (figure 5).

Calculations performed by individual modules of the KISSsoft program can be saved for further optimization in a file that can be replaced with another file with the appropriate extension or optimized using files with ready-made calculation templates according to the standards. Standards for the calculation of drive elements, their manufacture and quality regulation are very diverse, among which we can distinguish the standards used in modeling: GOST21354–87, ISO 6336-2019, etc.

Upon completion of the calculations, you can upload the calculation protocol in text format with the ability to edit the template and add dependency graphs to it in two-dimensional and three-dimensional format. At the same time, it is possible to visualize the process of cutting gears and wheels from the workpiece with the output of a technological drawing, including the ability to load any profile of one tooth of the tool in the form of a drawing file with the extension dxf. When visualizing, you can change not only the color, line thickness and viewing angle, but also the speed of the wheel engagement animation, which you can imagine without stopping the animation, and export the model to a graphic file with the extension of programs: AutoCAD, Solidworks, Siemens NX, etc.

Three-dimensional models of the gear and wheel are built with the adjustment of the number of tool cuts. This changes the number and length of the splines depending on the curvature of the profile, especially on the transition curve. A coupling is established between the side profiles of the gear and the wheel, taking into account possible interference by the 2nd and 3rd additional synthesis conditions. In KISSsys, a constant speed for the gear and a constant drag torque for the wheel are set via the couplings on the input and output shafts. As a result of this simulation in this formulation, we obtain a number of characteristics, such as the contact spot, taking into account the deformation wheel, shaft.

When performing calculations based on contact analysis, including the finite element method, the stress-strain state of the teeth of cylindrical-conical wheels is considered as a three-dimensional three-dimensional problem. The contact between the teeth of the gear and the wheel, which is subject to investigation, is adjusted with certain characteristics, such as stiffness, coefficient of friction, etc. During the first calculation, load distribution coefficients are determined, which can be adjusted with the help of design experience and data from previous tests you can gradually monitor the adequacy of the mathematical model embedded in the software code of the KISSsoft AG complex. This is partly due to the variety of drive solutions and the limitations of the algorithms outlined in the standards. Along with the standards, there are technical regulations that have not yet received an official status, but expand the designer's capabilities for predicting the behavior of the drive during operation. For example, these include technical regulations for the calculation of the drive temperature, oil, and efficiency.

The stress-strain state of gears and shafts in the moment of engagement with the use of this model we analyzed the influence of load and stiffness on the nature of the beginning and end of contact of the teeth that causes changes in the size and shape of the work area of the line of action and $L_1L_2$ and $R_1R_2$ (figure 3). To solve this problem in statics with all the stiffness in their contact irrationally. Therefore, it is possible to perform a dynamic analysis indicating the number of natural frequencies with the output of visualization of the deformations of the parts: house, shaft and gear with a configurable display scale.
In the simulation, it is necessary to obtain a gas-free cylindrical-conical transmission (figure 6), and therefore it is necessary to control the moment of resistance at idle. Due to the location of the normal engagement force of the wheels outside the plane perpendicular to the axis of rotation of the wheels, radial thrust bearings with a sufficiently high rigidity, namely tapered rolling bearings, are installed in the supports. This will reduce the impact of the bearings on the transmission operation, as well as the connections, which were taken as spline connections between the shaft, wheel or clutch.

The development of the model will consist, in particular, in taking into account the flexibility of the drive housing, the stiffness matrix of which can be downloaded after preparation in the program provided free of charge to KISSsys for a 3D model based on drawings provided by the company. The manufacturing error of the gears can be compared with the calculated model and take into account the deviations in the KISSsys 2021 program. This will help to make an important contribution to the analysis of the nature of the mating teeth of spur-bevel gears with preloaded tapered bearings support the input shaft to eliminate the gap by sets of rings are installed between the bearing and cover. Preload in the tapered bearings of the output shaft supports is carried out by tightening the nut on the output shaft, provided that these bearings are installed in the spacer, as can be clearly seen in figure 6.

The result of checking the quality of the drive as a whole can be considered a change in its temperature in different modes or a combination of them, as it happens in real operating conditions in different climates, etc. Thus, the calculation of the efficiency of the entire drive with this transmission at the maximum permissible load was made from the condition of the strength of the teeth for contact stresses and bending stresses according to the technical regulation ISO TR 14179-1. As can be seen from figure 7 calculation results revealed acceptable losses of mechanical energy with the ability to track the distribution of thermal energy in the main of the elements drives based on the drive part of the machine.

5. Results and discussions
To check the operability of an approximate model of a cylindrical-conical transmission as part of a mechanical drive and to test the proposed method of geometric synthesis with design in a well-known software package, a set of calculations of variants of this transmission was performed. Figure 4 shows for comparison the best transmission option, marked with a dashed line inside the locking contour, for which the strength of the gears increases by 5-15 %, the probability of interference of the teeth and the dependence on the parameters of the gear tool decreases.

6. Conclusions
Thus, the implemented approach to modeling the gas-free engagement of cylindrical-conical wheels with parallel axles allows the design to form design schemes with the actual operating conditions of this transmission in a mechatronic drive, and significantly reduce the costs of obtaining optimal design solutions in a short time according to the technical specification in parallel with production and testing.

This is the first stage of modeling, and therefore it is necessary not only to obtain the first "raw" design solutions, but also to prepare a list of questions on operational and technological data, based on the capabilities of the software package and the method of geometric synthesis. The proposed method of geometric synthesis of this gear, based on a new calculation system, allows increasing the number of output synthesis parameters from two to four [11, 12]. Thus, it is proposed to further consider the model of a cylindrical-conical transmission with a variable radial clearance along the wheel axis, describing the tops of the teeth not only one conical, but also locally on a cylindrical surface or a combination of them in the absence of a CNC machine. As it is expected in the future, at the second stage, to obtain a better engagement in any end section during the research, by performing a group of transmission calculations for strength, reliability, efficiency, etc.

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