The Draconic gearing of the Antikythera Mechanism: Assembling the Fragment D, its role and operation

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Abstract

The unplaced Fragment D of the Antikythera Mechanism with an unknown operation was a mystery since the beginning of its discovery. The gear-r1, which was detected on the Fragment radiographies by C. Karakalos, is preserved in excellent condition, but this was not enough to correlate it to the existing gear trainings of the Mechanism. The suggestion that this gear could be a part of the hypothetical planet indication gearing is still a hypothesis since no mechanical evidence has been preserved. After the analysis of AMRP tomographies of Fragment D and its mechanical characteristics revealed that it could be part of the Draconic gearing. Although the Draconic cycle was well known during the Mechanism’s era as represents the fourth Lunar cycle, it seems that it is missing from the Antikythera Mechanism. The study of Fragment D was supported by the bronze reconstruction of the Draconic gearing by the authors. The adaptation of the Draconic gearing on the Antikythera Mechanism improves its functionality and gives answers on several questions.

1. Introduction. The Antikythera Mechanism, a geared machine based on the lunar cycles

The Antikythera Mechanism a creation of an ingenious manufacturer of the Hellenistic era was a geared machine capable of performing complex astronomical calculations, mostly based on the lunar periodic cycles. After 2000 years under the Aegean/Ionian sea (Jones 2017; Voulgaris et al., 2019b), the fragments of the Antikythera Mechanism is now a permanent exhibit at the National Archaeological Museum of Athens, Greece.

At ancient time, the lunar cycles were used as the primary calendar time units (Hannah 2013). Each month of the ancient Greek calendar was based on the New Moon and Full moon phases, so the basic time unit was the synodic month (Bowen and Goldstein 1988; Hannah 2013). Even the Olympic Games started on the 8th (or 9th) Full Moon after the Winter Solstice (resulting by the two four-year periods, one of 49 months and one of 50 months)
Although one tropical year does not include an integer number of lunar synodic months, the lunar cycle prevailed instead of the tropical year, which was considered less important.

The Metonic calendar known as “enneakedekaeteris” correlates the integer number of 235 lunar synodic cycles (also 254 sidereal cycles), to the integer number of 19 tropical years. Each Metonic year of 12 or 13 synodic months deviated from the actual start of the tropical year by several days, in order to keep the lunar synodic month intact (Geminus 1880 and 2002; Theodosiou and Danezis 1995; Freeth et al., 2008; Anastasiou et al., 2016a).

The ancient astronomers chose the lunar cycles instead of the solar cycles for several obvious reasons: it is easy to calculate a time span based on the moon’s every day changing phase, it is visible both at night and at day and when the moon is on the sky it is easy to observe bright stars. In contrast, the Sun does not exhibit any phases, it is impossible to detect stars when the Sun is above the horizon and the relative position of the Sun on the sky changes by about 1/12th of the lunar angular velocity. Also a particular “solar phase” i.e. a solar eclipse is a rare astronomical event to observe from a specific place. All of the above slow solar changes lead the ancient astronomers to base their measurements and time keeping events on the easier to follow lunar synodic cycle.

By studying the Antikythera Mechanism it is obvious that the operation, design and the gear teeth selection, were mostly based on the synodic lunar month: the \( b_{in} \) axis-Lunar Disc-Input (the fastest of the rotating axes), the small Lunar phases sphere on the Lunar Disc and the cells on the two spirals (Saros and Metonic), were all precisely based on the synodic lunar month. The selection of the teeth number of most of the gear trainings was chosen in accordance to the synodic and sidereal lunar cycles, e.g. the number of the gear teeth \( e_3 \) (223 synodic months of Saros) and \( (c_2/d_1)^*d_2 \) (2 X 127 = 254 sidereal months of Metonic cycle). The Lunar Disc rotation, which is the proper driving/input for the Antikythera Mechanism gearing trains (see section 6.1), essentially defines the two lunar cycles - the sidereal and synodic month - as the basic time units for the calculations of the Mechanism. Only two out of the seven pointers of the Mechanism are related to the tropical year: the Golden Sphere-Sun pointer (Voulgaris et al., 2018b; Voulgaris et al., 2019b) i.e. one full rotation equals one tropical year, and the athletic games pointer.

All of the above lead to the conclusion that the Antikythera Mechanism was a Luni-(solar) time/calendar geared machine computer, based on the lunar synodic cycle (as the majority of the ancient Greek calendars were based on the lunar synodic cycle and not in tropical year Theodosiou and Danezis 1995).

(Vaughan 2002).
2. The Lunar motions studied in the Hellenistic era

The four well known lunar motions the synodic, sidereal, anomalistic and draconic cycles, where extensively observed and studied by the ancient astronomers, in order to find out a correlation i.e. a periodic coincidence of the start of these periodic cycles (Evans 1998; Steele 2000; Hannah 2001; Steele 2002). Ptolemy 1984, in Almagest extensively referred to the primary lunar cycles, giving a large number of calculations in tables (Pedersen 2011). The attempt of Ptolemy, Hipparchus and the previous era astronomers to (better) correlate - incorporate the four lunar cycles, it was obvious.

The ancient astronomers realized that a correlation - phase synchronization (periodicity) of two of the four lunar motions/cycles, revealed a repeatability of the solar/lunar eclipses and the synchronization of three lunar cycles, presented eclipses with highly similar geometrical characteristics and classification (Oppolzer 1889; van den Bergh 1955; Meeus et al., 1966; Neugebauer 1975; Meeus 2004). The periodicity of the lunar motions, led to the adoption of the \textit{“interrelated cycles ratio in integer numbers”}, a very useful canon, which was the key for the solar/lunar eclipses prediction. Ptolemy refers to the Saros cycle with the name \textit{“Periodic”, ... since it is the smallest single period which contains an integer number of returns of the various motions”} (Ptolemy 1984).

Certainly, the most easily observed lunar cycle was the synodic month i.e. the period between two successive New Moons or Full Moons, which was easily measured with sufficient accuracy Fig. 1A,D. For this reason the calculations of the rest three lunar cycles were based on the synodic month.

The sidereal month is the time duration between two successive transits of the Moon by the same constellation area/star Fig. 1A,C, but the start of each sidereal cycle presents a different lunar phase. As the Moon is the brighter celestial body in the sky, it far outweighs the bright stars.
A pulley is adapted on the Lunar Disc of the Antikythera Mechanism functional model (designed/constructed by the FRAMe Project) and an electrical motor rotates the Lunar Disc via a belt (images taken from video frames). The position/phases of the Moon relative to the Golden sphere-Sun and the zodiac sky are presented. A) The red dot that marks the Egyptian month of 1st MECHIR (ΜΕΧΕΙΡ) and also the 18th zodiac day of Gemini (ΔΙΔΥΜΟΙ) is the starting position of the Lunar Disc and the Golden sphere. B, C, D) During the rotation of the Lunar Disc, the Golden sphere continuously changes its position relative to the zodiac sky (the slight mismatch of the aiming, is a cause of the parallax error during the video recording). Each synodic month on the Antikythera Mechanism, started with the Full Moon phase (see section 4).

The Anomalous month became evident by the variable (increased-decreased) angular velocity of the moon concerning the fixed stars. Geminus 2002 (Evans and Berggren 2006), refers that at the start of the anomalous month (apogee), the minimum lunar angular velocity is 11° 06' 35"/1d and the maximum angular velocity is 15° 14' 35"/1d, measured on the middle of the anomalous month (perigee). As might be expected, this angular variation is not easily detected without the use of an astronomical instrument (astrolabe) and by recording a number of observations and measurements, as Ptolemy presents in Almagest. Geminus not only describes the mathematical process for calculating the mean Lunar angular velocity, but also refers to the measuring error as a result of the instrument’s limited resolution in practice.
The variable lunar motion is also included on the Antikythera Mechanism, which is introduced by the operation of the pin&slot design on the gears k1/k2, off-axis on board the gear e3 (Freeth et al., 2006, Wright 2006; Gourtsoyannis 2010; Voulgaris et al., 2018b). The centers of gears k1 and k2 are located in “off-axis” position and the movement transmission, from gear k2 (slot) to k1 (pin), presents a variable angular velocity, the Anomalistic lunar cycle.

*Table I:* The time measuring cycles measured in days and the corresponding duration in tropical years, as recorded by the observations of Babylonians and the ancient Greek astronomers. True and rounded values of the draconic month are presented. The 1/1000<sup>th</sup> of a synodic month is about 42.5 minutes. The ratio of draconic to synodic month is rounded to 1.08520.

| Cycle (invented for...) | Tropical years | Number of Days | Days per Year | Synodic months | Draconic months (rounded) | Ratio Draconic/Synodic month | Draconic month (days) |
|------------------------|----------------|----------------|---------------|----------------|--------------------------|-----------------------------|------------------------|
| Saros (eclipses)       | 18° 11.3<sup>o</sup> | 6585.322       | 365.2233      | 223            | 242                      | 1.0852017                   | 27.212074              |
|                        |                |                |               |                | 241.999 (true)           | 1.0851973                   | 27.212186              |
| Metonic (tropical year)| 19°            | 6940           | 365.2631      | 235            | 255                      | 1.0851063                   | 27.215686              |
|                        |                |                |               |                | 255.021 (true)           | 1.0851957                   | 27.213445              |
| Exelegmos (eclipses)   | 54° 33<sup>d</sup> | 19756         | 365.2235      | 669            | 726                      | 1.0852017                   | 27.212121              |
|                        |                |                |               |                | 725.996 (true)           | 1.0851958                   | 27.212271              |
| Callippic (tropical year)| 76°         | 27759          | 365.25        | 940            | 1020                     | 1.0851063                   | 27.214705              |
|                        |                |                |               |                | 1020.84 (true)           | 1.0851957                   | 27.192312              |
| Hipparthic (tropical year) | 345°       | 126010         | 365.246       | 4267           | 4630.5                   | 1.0851886                   | 27.213043              |
|                        |                |                |               |                | 4630.531 (true)          | 1.0851959                   | 27.212861              |
| Babylonian (eclipses)  | 441° 106.3<sup>d</sup> | 161188      | 365.2646      | 5458           | 5923                     | 1.0851960                   | 27.213911              |
|                        |                |                |               |                | 5922.999 (true)          | 1.0851958                   | 27.213916              |

The fourth, more complex lunar motion is the continuous change of the two Lunar Nodes’ position: The elliptical lunar orbit plane inclines to the Ecliptic about 5.15° and crosses the Ecliptic plane on two points, named Ascending and Descending Node. The line is connecting the two Nodes, named Line of Nodes. In order to justify the “strange” ecliptic latitude changes of the projected Lunar movements in the sky, the ancient astronomers introduced the model of the slow retrograde rotation of the Line of Nodes.

Ptolemy 1984 in Almagest calls the time span in which the Moon crosses the same Node, as “Ἀποκατάστασις κατὰ πλάτος” (*return to the same latitude*), better known as *draconic* or *draconitic month* (by the Middle Ages myth of the Dragon which “eats” the Sun during a
solar eclipse, see Kircher 1646, page 548), also known as nodal or nodical month. The duration of the draconic month is about 27.2122, a bit shorter than the sidereal month (27.3218) Barbieri 2017. The two Nodes return to the same position relative to the stars after 18.612 tropical years (larger than a Saros cycle, see Espenak and Meeus, 2008-NASA eclipse page). This implies that the Nodes change their projected position on the sky, by about 1.564°/1 synodic month, transiting each constellation in about 19.2 synodic months.

The Metonic, Callippic and Hipparchic cycles were invented in order to better correlate the synodic month to the tropical year (Oppolzer 1889; Meeus et al., 1966; Neugebauer 1975; Meeus 2004). Saros and Exeligmos offer a better correlation to integer numbers between the draconic and the synodic cycle, but not for the tropical year and for this reason were the proper cycles for the eclipse predictions.

3. Necessary Parameters for a solar/lunar eclipse - The Draconic cycle

One of the most important astronomical events in antiquity and today is a solar eclipse (https://sites.williams.edu/eclipse/2019-chile/, Voulgaris et al., 2012). In ancient Mesopotamia, Egypt and Greece, eclipses were correlated to an extended mythology regarding the Gods’ fight, the Kings, their thrones etc. The war between Medes and Lydians stopped during the eclipse of 28 May 585 BC, which Thales predicted (Panchenko 1994; Stephenson and Fatoohi 1997; Herodotus 1998). Therefore, the prediction of a solar eclipse was a considerable challenge for the astronomers of the ancient world, who used their observations and calculations to improve the accuracy of the eclipse predictions (Steele 2015).

A solar/lunar eclipse will occur only if two specific astronomical positional parameters are satisfied:

1) The Moon is at its new phase (or Full Moon) and
2) At the same time, the New Moon (or Full moon) is located at or close to the Ascending or the Descending Node, i.e. close to the beginning or the middle of the draconic month.

The resonance of these two periods in about 0 or π phase, guarantees that a solar (or lunar) eclipse will happen somewhere on the Earth. Of course, in order to predict where on Earth the eclipse will be visible, additional calculations and parameters are needed.
As the moon and the Earth have angular dimensions (about 0.5° for the moon as observed from the Earth and about 1.5° for the Earth as observed from the Moon), their corresponding shadows are extended. Hence, an eclipse can occur even if the moon is located at an angular distance from the Node. Thus, a zone on the Ecliptic can be defined with angular dimensions of about ±17° 25′ on either side of the Nodes, called the Zone of Eclipses or Ecliptic limits. The ecliptic limits vary depending on the angular dimensions of the Moon, as a result of its varying distance from the Earth (apogee /perigee) i.e. by the Anomalistic cycle Fig. 2.

![Diagram](image.png)

**Figure 2**: Harmonic graph of the draconic vs synodic cycle, assuming that the two cycles start simultaneously. On each of the resonance (or close to resonance) points (phase π or 2π) of the two graphs, a solar or lunar eclipse will occur. The units on the x-axis are synodic lunar cycles. (On the calculations for the graph presentation, the phase variation resulting from the anomalistic cycle was not included).

The exact phase of the anomalistic cycle (apogee/perigee position) during the resonance of the synodic and draconic cycles, defines the type of the eclipse event (total, annular or hybrid, also less affected by the Earth’s apogee/perigee), (Espenak and Meeus, 2008-NASA eclipse page) and the exact time in which the Moon approaches the Nodes or crosses the ecliptic limits, acting as a variable timer (see section 7.3).

The lower half area of the Back plate of the Antikythera Mechanism was dedicated on the eclipse events information (Freeth et al., 2008; Anastasiou et al., 2016b; Freeth 2019; Iversen and Jones 2019), presented on the Saros four turns spiral, divided into 223 cells/synodic months. The ancient manufacturer calculated, designed and constructed the proper gear training for this operation.
At the same time, the manufacturer engraved the specific sequence of eclipse events on the Saros spiral,
- copying the eclipse information written in a papyrus (?),
- using the Babylonian eclipse records (?) (Carman and Evans 2014) or
- the eclipse events were directly calculated making use of his own construction, The Antikythera Mechanism(?) (see section 8).

4.1 Eclipses information/prediction mechanism on the Antikythera Mechanism
The Antikythera Mechanism could predict, the phases of the Moon, the position of the Sun on the sky (zodiac constellation) at the corresponding Egyptian and Zodiac month/date (Voulgaris et al., 2018a) and also the Metonic month. Also could predict the year in which the athletic games will occur (Freeth et al 2006 and 2008; Anastasiou et al., 2016).
Several researchers suggested/hypothetical bronze reconstructions or digital 3D representations, presenting the Antikythera Mechanism with a planet indication gearing. In these hypothetical designs, a large number of hypothetical gears (about 35) and hypothetical additional parts (about 40), is needed. Still, today no any mechanical part of these is preserved. Today, the preserved/necessary gears of the Mechanism are 35 (+1 unplaced gear/Fragment D).
Moreover, studying these suggested models, many differences and deviations by the original preserved artifact, were observed (see also Voulgaris et al., 2018b). Additionally, critical mechanical problems (especially friction, torque, weight), making difficult the hypothetical functionality, virtual reality models with planet indication gearing, arise by adapting of about 50-70 hypothetical-non existing bronze parts.
Therefore, the planet indication gearing on the Antikythera Mechanism is still hypothetical/doubtful and is not a fact.
In many axes/shafts of the Mechanism central conical holes were detected (Voulgaris et al., 2018b). Axes with conical holes also exist in places that there is no any access by someone, in order to assume that these holes were made by a careless researcher using a sharp tool (e.g. a compass), after the Mechanism retraction by the sea. These conical holes created by the ancient manufacturer using an ancient lathe (Voulgaris et al., 2019a). The existence of the central conical hole on a broken axis prevents the consideration that the lost part of the axis could have a length around 70-100mm (Voulgaris et al., 2018b). This length is necessary in order to adapt the high “tower” of the gearing for the planets motion representation.
Regarding the Fragment D/gear-r1 with 63 teeth, Freeth and Jones 2012 proposed, that it could represent the motion of planet Venus. However, this hypothesis for the r1-gear
operation is not the only one (see section 7.2 Adapting the Fragment D on the Antikythera Mechanism).

The user of the Mechanism, by turning the Lunar Disc, the proper in handling, input of the Mechanism (Voulgaris et al., 2018b, see section 6.1), he could observe when the lunar pointer aimed directly to the Golden sphere (or on the opposite position). At these positions the small half white/half blackened lunar phase sphere of the Lunar Disc, shows its black (white) hemisphere (Wright 2006; Carman and Di Cocco 2016; Voulgaris et al., 2018b). Geminus 1880 and 2002, in Chap. “VIII-About months” and “XI-Regarding the Lunar eclipses”, refers that the day of Full Moon occurred in mid-month, Διχόμηνις (Theodosiou and Danezis 1995; Jones 2017) and in Chap. VIII, IX-“About the light of the Moon”, and “X-About the solar eclipses”, the New Moon occurred at the last day of the synodic month, on (29th) or 30th day, Τριακάς. Therefore, according to Geminus, the New moon phase comes after the Full moon on each synodic month.

On the Saros spiral, a number of pairs of successive cells are preserved, presenting eclipse events on synodic months Fig. 3 (cells 25-26, 78-79 and 119-120, Anastasiou et al., 2016, Freeth 2018, Iversen and Jones 2019).

Let’s assume that each cell (synodic month) of the Saros spiral begins right after the New Moon. When the Saros pointer aims to the cell-25, a solar eclipse is occurred at the end of month 25. On the next cell-26, a lunar eclipse will occur at mid-month of the next month, i.e. 15 days after the solar eclipse. All of the eclipse events engraved in successive cells, following the pattern Η (Sun) and then Σ (Moon).

Additionally, each preserved cell with two eclipse events (Σ/Η, cells 125, 131, 137, 172, 178, and 184, Freeth et al., 2008; Anastasiou et al., 2016b, Iversen and Jones 2019), the ancient manufacturer engraved the lunar eclipse event (Σ) on the top and the solar eclipse event (Η) on the bottom, also giving a difference of 15 days Fig. 3. Therefore, the cell’s information reading is from top to bottom.

This observation leads to the conclusion that each of the Antikythera Mechanism synodic month, starting right after a New moon or a solar eclipse, as Geminus refers.

If we suppose that the synodic month starts with the New Moon, then the pattern of Η (cell-x) and then Σ (cell-x+1) cannot be functional, because the time span between the two eclipses is \(29.53+14.765=44.295^d\div 1.5 \text{ draconic months}+3.477^d = \text{Node} + [(11°/1^d) * 3.477^d] = \text{Node} + 38.247°\) (out of the ecliptic limits of 34°).
The specific pattern is also functional, if someone assumes that the synodic month starts – 1st day, with the Full Moon. But this assumption is difficult to supported, because of the complete lack of ancient references and therefore, is not considered as probable.

By each re-aiming of the Lunar Disc pointer to the Golden sphere (or in opposite position-Full Moon), there was a possibility that a solar (or lunar) eclipse would occur. By only observing the Front dial plate of the Mechanism, it was not possible to know for sure if a solar or a lunar eclipse would occur. Information about upcoming eclipses was only presented on the Saros spiral, on the Back plate of the Mechanism (Freeth et al., 2008; Anastasiou et al., 2016b; Iversen Jones 2019). It seems that the calculation of the draconic cycle, which is very critical for the eclipses prediction, there is not on the Antikythera Mechanism.

4.2 Two speeds on the Antikythera Mechanism

In the following section the “nature” of the Antikythera Mechanism as a measuring instrument, is to be presented.

By rotating the Lunar Disc-Input of the Mechanism about 389°, one synodic month is completed Fig. 1. In addition, Saros pointer transits one cell out of 223 cells (and Metonic pointer one out of 235 cells). The pointer’s angle variation is about 6.4°/cell-synodic month Fig. 3. Therefore, on the Mechanism different pointers depict the same time unit - a synodic month.

The Front plate pointers move relatively fast, making an extensive route, offering High-Resolution information of their geometrical position. In contrast, the Back plate pointers are Low-speed with Low-positional resolution.

A large number of compressed information is also engraved on the scales of the Low-Speed pointers: 235 words (months) on 235 Metonic spiral cells and a number of eclipse events on some of the 223 Saros spiral cells.

Each Saros (and Metonic) cell corresponds to 29.53° has a mean dimension of about 7mm X 4mm. Therefore, the Saros pointer changes its position about 6.4°/29.53°=0.257°/1°, Fig. 3.

It is impossible to detect/measure time in precision of days directly on each of the Metonic/Saros cells (7mm/29.53 =0.216mm/1°).

If the pointer aims at the middle of the cell, this does not mean that the date is precisely at the middle of the month. This calculation “inconsistency” arises from the short dimension of each minimum unit, the engagement of the gears with triangular teeth shape, the gear periodic errors, the small precession of the training axes, the (small) off-center positioning of the central gear holes, the random teeth shape mismatches, the mechanical limits of the
specific gear training etc. (Edmunds 2011). Therefore, the Saros and Metonic spirals’ minimum measurable time unit is one cell per one synodic month.

Figure 3: A) A close up of the Saros spiral with the eclipse information. The angle which the Saros pointer travels in one cell/synodic month is about 6.4°. Note also the two pairs of successive cells in which the solar eclipse (H) is referred on the first cell and the lunar eclipse (Ξ), on the next cell. Also, the cells with two eclipse events on the same synodic month are visible (the lunar eclipse on the first line and the solar on the second line). B) Close up of the micrometer dial of the tool post slide hand-wheel of a conventional lathe. C) Each line of the micrometer dial changes the tool post slide position by 0.025mm. D) The position between two lines, the movement of the tool post slide could not be defined as 0.025mm/2.

Finally, the speed of a gear train output defines the resolution of the measurement. The Low-Speed pointers manage an extensive information capacity but in a low geometrical/positional resolution. Therefore, the Saros spiral is more of an “eclipse information table”, rather than an eclipse prediction scale (as well as the Metonic spiral is a month name information table).

It seems that the Antikythera Mechanism lacks High-speed gear training/high-resolution calculations for eclipse predictions that exclusively based on the geometry of the position. The authors believe that the ancient manufacturer would not rely on a pointer of “eclipse information table” to predict of the eclipses - the most important events of the Mechanism - without taking into account geometrical/high-resolution calculations for these events.

The existence of two different speeds on the Mechanism leads the authors to consider that the Antikythera Mechanism could have had an additional gear training, with a pointer
dedicated on the eclipse prediction calculation. This pointer should be the output of High-speed gearing train, based on geometry, offering a high-resolution calculation so that the eclipse prediction results would be highly accurate and specific.

5. The Fragment D - description and analysis of the gear r1

Fragment D is an enigmatic and unplaced part of the Antikythera Mechanism. It was first noted by I. Svoronos and captured by A. Rehm in 1905/1906, and after it was misplaced, it was re-found in 1973 (Price 1974; Lazos 1994; Freeth and Jones 2012). Fragment D is strongly corroded/calcified in multiple thin layers, visible by naked eye. Its mechanical design information cannot be clearly detected Fig. 4. Fragment D is not correlated with the rest parts of the Mechanism.

By studying the AMRP Tomographies (AMRP, www.antikythera-mechanism.gr), Fragment D consists of three parts (Freeth and Jones 2012).

The first part is a partially preserved Circular plate, with a diameter of 43mm (thickness about 1mm, authors’ measurements), and a square central hole. On this Circular plate, three perpendicular pins placed symmetrically by 120° relative to the center are clearly detected Fig. 5. By observing the preserved circular perimeter shape, it arises that this plate could not be a gear. This comes from the conclusion that on this plate surface area there is no
detection of any engraved line or subdivisions that could lead someone to hypothesize the plate was a measuring scale.

Figure 5: Selected AMRP tomographies of a) the Circular plate, b) the gear r1, c) the Additional plate. A) the digital reconstruction of the Circular plate tomography, using well-preserved areas of the plate, B) the digital “cleaning” of r1 gear, from its corrosion and the deposits, in order to make its mechanical characteristics better visible. The stabilizing pin has also been placed on its correct position, C) the digital “cleaning” of the Additional plate from its corrosion and the deposits. Also, the circular hole of the plate and the circular cross section of the gear shaft (probably the end of the shaft’s edge), which is placed on the circular hole of the Additional plate, are presented. AMRP tomographies were processed by the authors. 1), 2), and 3) the corresponding original bronze material reconstructions of the three parts of Fragment D, designed/constructed by the first author.

In contact with the Circular plate, the gear r1 is clearly visible (thickness about 1.5mm) and has been preserved in great condition. Its radius slightly varies because of the contractional deformation and the random cracks (Voulgaris et al., 2019b), between 16.7mm-17.2mm. This gear also has a square central hole. C.Karakalos (Price 1974); Wright 2005; Freeth et al.,
Freeth and Jones 2012, measured 63 gear teeth (61 teeth well preserved and two teeth missing). Around the perimeter of the gear stacked deposits of salts, calcites and petrified silt are located, following the shape pattern of the teeth. However, a large percentage of these formations have been peeled out of the teeth boundaries.

On this gear, the three pins which stabilized the gear on the Circular plate are sharply detected (stabilizing pins have also been detected on the gears c1/c2 and the l1/l2). Thus, the gear-r1 and the Circular plate rotate with the same angular velocity as one body. It seems that this Circular plate is a base for r1 gear, increasing the stability of the gear.

Figure 6: The four stages for the relocation on the Additional plate’s correct position and the broken r-shaft. A) The present position of the Additional plate and the shaft. B) Digital relocation of these two parts in order to align the broken shaft to the same axial direction with the central part of the shaft (i.e. the remaining shaft on the gear). C) Relocation of the shaft and the stabilizing pin in order for the pin to be in contact up to the gear surface (white line). By the specific position of the hole on the main part of the shaft (i.e. below the gear surface), it is evident that the main part of the shaft (which is preserved inside the gear) is also displaced or shrunk because of the contractional deformation (Voulgaris et al., 2019b). D) A digital relocation of the main part of the shaft in perpendicular direction up to the stabilizing pin is presented. The relocation travel is about 0.4mm-0.5mm, (this procedure is also presented in Anastasiou 2014, without the fourth stage). E) By subtracting the two AMRP Ct’s of Fragment D of different depth (CT-1 surface of gear, CT-2, in 1mm deeper) is revealed that the pinhead of the right pin has been displaced.
Close to the gear surface and perpendicular to the gear shaft, a hole can be detected, in which the stabilizing pin of the gear was adapted. In this area, the shaft is broken. Above this hole, the AMRP tomographies reveal the stabilizing pin, the rest part of the broken axis and a strange shape piece, the Additional plate. These pieces have been displaced from their original position, as is evident by the difference in position of the corresponding formations (gear shaft, hole and pin) Fig. 5,8. The Additional plate (thickness about 1mm-1.5mm) is partially preserved in a particular shape, Fig. 5,8. Visible on the Additional plate is a circular hole, in which the broken and displaced circular axis is adapted.

The extended study of the AMRP Tomographies (original and also processed by the authors), does not reveal any proof for the existence of the three pins on the Additional plate, which could lead someone to consider that the Additional plate was fixed/stabilized on the gear r1. The three pin’s edges are clearly detected only between the Circular plate and the gear Fig. 6. Moreover, one of the three pins is totally out of the boundaries of the Additional plate Fig. 7.

Additionally, the existence of the perpendicular stabilizing pin of the shaft, between the gear and the Additional plate prevents any contact between the surfaces of the two parts Fig. 5B, and 9C,D. Without a doubt, before the material corrosion/calcification, the distance between r1-gear and the Additional plate was at least 0.9mm-1.0mm, which is the thickness of the perpendicular stabilizing pin. Therefore, the Additional plate is not in contact with the r1-gear and seems to be utterly independent from it.

Inside the circular hole of the Additional plate, the broken part of the gear shaft is also preserved, which at this point has a circular cross section Fig. 5,9. The circular cross section design means that during the rotation of the r1 gear and its shaft/axis, the Additional plate does not rotate and, it is independent of the gear/shaft rotation.

The authors cannot find any other realistic mechanical operation for this Additional Plate, except that it was a simple oblong plate-*artistically curved* by one side (of which today only this part remains), acting as one out of two bearing points-plates for r1 shaft (Fig. 12,14): most of the Mechanism axes/shafts, are supported on two opposite points-plates. For example, the preserved shafts f, g, h, i, are supported between two parallel plates Middle and Back (this is also mandatory for the lost shafts m, n and o) and the d shaft, between the Middle plate and the Ω-shape Retention bar (Voulgaris et al., 2018b).

In Efstathiou et al., 2011; Anastasiou 2014; Basiakoulis et al., 2017; Seiradakis 2018 (slides 224-238), the Additional Plate is represented as fixed on the gear-r1 and rotating with the
same angular velocity. It is also suggested that this plate acts as a cam leading to the Equation of Time calculation. This consideration does not agree with the AMRP tomographies and the observations presented in this paper. Moreover, this suggested design has lots of additional real disadvantages, related to the proposed position of the gear $r1$, its stabilization, its operation, the way the results are displayed and the nature of these results.

It is evident that if the ancient manufacturer wanted to make the Additional plate to rotate with the same angular velocity to the $s1$ gear, he should have at least made the hole of the Additional plate square, as well as the corresponding cross section of the $r$-shaft. Moreover, the stabilizing pin between the $r1$ gear and the Additional plate prevents any immobilization of the Additional plate on the gear, even if it had square cross-section.

![Figure 7: A) A 3D representation using the 3D Slicer software (Fedorov et al., 2012), of the digitally reconstructed and “cleaned” AMRP tomographies of Fragment D, processed by the authors. B) The Fragment D is represented in transparent layers. C and D) Two side views, in order to make the pin position visible. E) Left, the transparent and Right, the 3D representation of the “artificial radiographies” of the three assembled original bronze reconstructed parts, using the 3D Slicer software. Photos and processed images by the authors.](image)

The existence and the specific position of the perpendicular stabilizing pin, leads to the conclusion that the pin was adapted in order to stabilized the gear on the specific position. It
also results that the gear shaft was extended beyond the side of the Circular plate-base (today is missing), as is also observed on most of the preserved gear shafts of the Mechanism (e.g. shafts g, h, i).

The engraved letters “ME” are detected on three different places on Fragment D. Engraved letters have also been detected on two other gears of the Mechanism, on m1 (letter H) and b1 (letter N). These letters could be the assembly numbering of the parts (?), or it could be a more secret scenario, e.g. the manufacturer could have written his name or a phrase spread out on several parts of the Mechanism (?).

6.1 The engaged gears b1-a1, the output on the a shaft
The b1 gear is the Mechanism’s larger gear (Freeth et al., 2006). In contrast to e3 gear (the second larger), which is made by one circular plate, the b1 gear consists of several assembled parts, which form four radial bars and a ring with teeth. The rest surface area of the gear does not have any bronze material. The absence of the material resulted in the irregular and relatively strong, three-dimensional deformation (shrinkage) of the b1 gear, mostly by the long-time stand still of the Mechanism on the sea bottom and its abrupt dehydration after its retraction from the sea (Voulgaris et al., 2019b).

Teeth number measurements of b1 gear (in its present condition) were made by C.Karakalos (223-226 teeth), D.S.Price (225 teeth), M.T.Wright (216-231 teeth) and Freeth et al., 2006, (223-224 teeth). An additional problem for the precise measurement of the teeth number is the difficulty for the detection of the actual teeth boundaries and tip position: many of the preserved gear teeth are filled with deposits (salts, calcites etc., as also observed on the r1 gear) that are following the shape of the gear perimeter, but at the same time they are altering or covering or erasing, the triangular shape of the teeth. The teeth are worn out or destroyed or utterly missing in some areas. Moreover, the b1 gear radius varies about ±1.1mm see the graph of Fig. 8 (also Freeth et al., 2006).

Firstly, the calculation of the total teeth number is achieved by measuring the preserved teeth and the corresponding epicenter angle (of course on their present condition and position). Secondly by according to the equation: total gear teeth = (teeth number/corresponding epicenter angle) X 360°. It is obvious that the corresponding epicenter angle is not the original-true angle, because of the gear’s 3D contractional deformation (i.e. shorter volume, shorter dimensions than the original) (Voulgaris et al., 2019b). The epicenter angle measured between the teeth tips on a part with shorter radius (i.e. shorter perimeter) results in a smaller calculated gear teeth number (see Fig. 8).
Apparently, the calculation of the mean value of the authors’ measurements was avoided, because the statistic average values of the measurements do not approach the initial/original teeth position, as most (or all) of the teeth positions are displaced by the deformation. All of these make difficult the precise detection of the actual teeth boundaries and tip position.

Naturally, before the Mechanism’s contractional deformation, the original radius of the gear was larger. The authors’ measurement of the gear teeth number is in the range of 219-225 teeth, concerning the present condition of the (deformed) gear Fig. 8. The above leads to the conclusion that the original teeth number of the (brass un-corroded/non-deformed) gear must have been towards to the upper number of this measurement.

Figure 8: A) The graph of the b1 gear radius vs angle position. Because the geometrical/mechanical centers of the two axes b_in and b_out differ by about 0.4mm (as a result of the contractional deformation), the geometrical center of b1 gear central circular hole was selected as the center for the radius and epicenter angle measurements. B) The graph of the calculated gear teeth number of b1 gear vs angle position. This graph more or less has similarities in the monotonicity to the above graph as a result of the radius variation. Authors measurements.
Gear a1 with (definite) 48 teeth (Freeth et al., 2006) is located on the right side of the Mechanism and is engaged with b1 gear Fig. 9. Gear a1 and its shaft rotate in a perpendicular direction to the rest gearing trains. The gear has a central rectangular hole, in which the poorly preserved rectangular shaft is adapted. A central conical hole results from the shaft material processing during its formation by an ancient lathe (Voulgaris et al., 2018b and 2019a), is visible by naked eye. In AMRP tomographies, a relatively thick pin with oblong cross section, perpendicular to the gear a1, is detected, Fig. 9F,G. This pin differs from the usual stabilizing pins, and it seems. It is an immobilizing pin, to fix a cylindrical bronze material (gear) to its shaft, probably before the material processing.

![Figure 9](Image)

**Figure 9:** A) The position of the contrate gear a1 relative to the gear b1. B) Front face close-up of the contrate gear a1. The (conical) central hole on the shaft is slightly visible, C) Back face close-up of the contrate gear a1 (Credits: National Archaeological Museum, Athens, A. Voulgaris-Copyright Hellenic Ministry of Culture & Sports/Archaeological Receipts Fund Photos), D) AMRP right side view tomography of the gear a1. Some of the gear teeth are visible, and also the gear shaft with rectangular cross-section and the (conical) central hole of the shaft, E) Bronze reconstruction of the a1 contrate gear, by the first author. F) The large pin, which immobilizes the a1 gear to its shaft, and the driver/nest of the gear, are clearly visible, G) AMRP top side view tomography of the gear a1. The rectangular cross section of the large and thick immobilizing pin is visible.

Between the a1 gear and the Middle plate, a part is detected which is stabilized on the Middle plate by using an adhesive material (an alloy of and tin and lead, Voulgaris et al., 2018c). This part acts as a driver (nest) for the a1-gear in order to avoid the (probable)
precession or displacement during its rotation, causing its disengagement with the b1 gear Fig 12F.

The gear b1 is stabilized on the gear b2 by four pins and rotates with the same angular velocity (Freeth et al., 2006; Voulgaris et al., 2019b revised gearing scheme). Gear b2 belongs to the main gearing of the Mechanism and its rotation represents the tropical year on the Mechanism. The larger b1 gear does not correlate to the main gearing of the Mechanism. Regardless of its teeth number or even if it did not exist, it would not affect the main gearing sequence of the Mechanism. Therefore, this gear was adapted by the ancient manufacturer in order to introduce a new additional gearing, which started its movement by the engagement of the a1 contrate gear with b1 gear. The contrate gear transmits the movement in a perpendicular direction to the rest gearing axes direction. The presence of the a1 contrate gear leads to the conclusion that the ancient manufacturer wanted to extend the gearing in a perpendicular direction, to be continued to the right side of the Mechanism. If his purpose was to continue new gearing’s rotation in the same direction, he could simply use a 48 teeth gear instead of a contrate one.

In contrast to most of the other gearing trains, engaged in reducing/dividing ratios, the a1 gear/shaft rotation originates from a multiplying ratio: \( b1/a1 \approx 4.6 \) rotations of a shaft per one rotation of b1(b2) gear-tropical year.

The contrate gear/shaft a1 is considered the “input” of the Mechanism. For several significant mechanical and operational reasons the Input from a1 gear, presents mechanical and handling problems. The authors using, measuring and studying their Antikythera Mechanism functional models, moving the gears by the proper and ideal Input/driving, which is the Lunar Disc (Voulgaris et al. 2018b), see next section.

6.2 Why do we believe that the Input/driving of the Mechanism is from the Lunar Disc

Many researchers believe and consider that the Input/driving of the Antikythera Mechanism is from a1-gear by adapting a crank. This assumption was introduced by D.S. Price 1974. However, this common-sense assumption/traditionally accepted does not necessarily also mean that it is right. Instead of the “common-sense assumption”, we present the concept of the “measurements” and “justification”:

1) Supposing that the Mechanism’s Input is from “crank a1-gear”. On that occasion, one complete rotation of the a1 crown gear will result in 2.85 rotations of the Lunar Disc (2.85
turns=1026°), \( \{(48/225) \times (64/38) \times (48/24) \times (127/32) \times (50/50) \times (50/50) \times (32/32) \} \approx 2.85. \)

Thus, if we rotate the “crank a1” by one tooth, the Lunar Disc will change position by \( 1026° / 48 = 21.37° \) (about 71% of a zodiac dodecatemorio). This high speed rotation makes it difficult to aim the Lunar Disc pointer precisely on the Golden Sphere-Sun: E.g. if the Lunar pointer is 4° before the Golden Sphere, then the “crank a1” needs to rotate by \( (4/21.37) = 0.1871 \) tooth fraction of a1 (i.e. rotating the crank by 1.4°), in order to bring the Lunar pointer directly to the Golden Sphere (New moon, a critical position). This precision is too difficult to achieve if one takes into account the mechanical errors, backlash, friction, hand force and inertia, see Fig. 10.

Figure 10: Digitally combined two different images, which depicting the position difference of the Lunar Disc pointer, if the gear-a1 (crank), rotated by one tooth.

2) Roumeliotis 2018, which presents torque calculations on the shafts, writes: “…The next candidate is gear b1, the one already assumed to be driven by a crown. Although this is a relatively good candidate, it gives a minimum torque about 5 times smaller than the one provided by gear e6 and about one third of the torque provided by gear d2. Thus, if the friction at each shaft or axle is larger than the estimated 0.2 N mm, driving the mechanism by gear b1 may be difficult.”

3) In Voulgaris et al 2018b, was measured the kinetic energies considering the Input of the Mechanism by the crank-a1 \( (E_{a1}) \) and by the Lunar Disc \( (E_{d}) \). The results revealed that
E_{a1} = 8.28 \ E_{ld} \ i.e. \ in \ order \ to \ move \ the \ Mechanism \ gears \ from \ the \ “crank-a1” \ is \ needed \ 8.2 \ times \ more \ energy \ than \ the \ Lunar \ Disc.

The question is why the ancient manufacturer designed and constructed an Input, which is less precise, difficult/challenging to handle, and presents mechanical problems? Moreover, if the manufacturer wanted a better value of torque, he could easily decrease the teeth number of a1-gear, improving the precision of the Lunar Disc’s aiming.

On the contrary, the driving from the Lunar Disc offers easy handling, a sufficient torque, and precise aiming. In addition, is fully functional and directly related to the units of the Saros and Metonic spirals, which based on the synodic cycle (Voulgaris et al., 2018b).

For all the reasons mentioned above we do believe that the proper Input/driving of the Antikythera Mechanism is the Lunar Disc.

### 7.1 The rationale of the Draconic sequence on the Antikythera Mechanism

Based on the previous chapter arguments, the most proper, functional and easily handling Input/driving of the Antikythera Mechanism is the Lunar Disc.

1) This frees up the crown gear-a1 and its axis, which has an unknown operation.

2) Simultaneously, there is an unplaced gear, the Fragment D, which also has an unknown operation.

3) During the Mechanism’s Era four well-known Lunar cycles have been recorded, observed and used: Synodic, Sidereal, Anomalistic and Draconic (223 Synodic cycles = 239 Anomalistic = 242 Draconic= 1 Saros cycle).

4) Three - out of four - lunar cycles are inherently present in the AM: Sidereal, Synodic, and Anomalistic (see chap. 2), but not the fourth – very important – lunar motion, the Draconic cycle. As the Inscriptions of the Mechanism are partially and some areas poorly preserved, and the largest part is totally missing, it is logic that it could be not preserved inscriptive evidence for a lunar node indicator on the Mechanism.

Therefore, it makes sense for someone to look at one if the axis of gear-a1 and the unplaced gear of Fragment D, could engaged in a Draconic gearing, which can be cooperated with the rest, existing and actual gearing of the Mechanism.

### 7.2 Adapting the Fragment D on the Antikythera Mechanism

Section 4 results that there is no High Speed (High Resolution) gearing for the eclipse prediction based on geometrical (mechanical) calculations. At the same time, an unknown output of the Mechanism and an unplaced gear exists. Following is a presentation of the
thinking, the design, the gearing mathematical calculations, the use and the results of a new gearing, suggested by the authors, in order to achieve precise eclipse predictions based on geometrical calculations.

A new gearing can be introduced, to represent the second necessary parameter for the eclipse prediction: a gearing representing the draconic cycle - the fourth lunar cycle, is needed (see chap. 3). The output of the draconic gearing is a draconic pointer. This gearing train must be a High Speed (High Resolution), to extract results based on geometrical calculations.

Therefore, the draconic gearing must be adapted in a position where the gears’ speed is high. Additionally, the candidate position must allow the adaptation of the additional mechanical parts.

A proper position for this new-draconic gearing can be achieved on the a-shaft-output, which starts with the engaged gears b1-a1. On the other edge of the a-shaft, the r1 gear is adapted Fig. 11. The mechanical design and the dimensions of the-a shaft allows the adaptation of r shaft/r1 gear. A hypothetical gear s1, engaged to r1-gear, is needed for the output of the draconic gearing train. The draconic pointer is adapted on the s-shaft Fig. 11,13.

One complete rotation of the draconic pointer corresponded to one draconic month. The measuring scale of the draconic pointer depicts the Ascending and Descending Node-points. Two simple pins in up/down position, anti-diametrically placed and stabilized on the External Wooden decorative casement (Voulgaris et al., 2019b), represent the two Nodes, one gold pin for the Ascending Node and one silver pin for the Descending Node. On either side of each Node-pin, there must also be an arc-shaped bronze strip, depicting the Zone of Eclipses/Ecliptic limits, about ±17° for each Node Fig. 12,15. A draconic month is completed whenever the draconic pointer returns to the same Node-pin, after one rotation.

For the gear teeth calculations, the lunar cycles of Saros period (Table I) were selected as the most proper period for the eclipse prediction.

One Saros period of 223 synodic months = 242 draconic months. Therefore, on the Antikythera Mechanism, 242 complete rotations of the draconic pointer are equal to 223 synodic rotations of the Lunar Disc (i), and also from the Metonic gearing, 235 synodic rotations = 254 sidereal rotations or 1 synodic rotation of the Lunar Disc Input = 254/235 sidereal rotations (ii).
Therefore, 242 draconic rotations = 223 * 254/235 sidereal rotations (iii). Applying this equation on the Antikythera Mechanism gearing:

\[
[223 \times (254/235)] \times (b3/e1) \times (e6/k2) \times (k1/e5) \times (e2/d2) \times (d1/c2) \times (c1/b2) \times ((b1/a1) \times (r1/s1)) = 242 \text{ rotations of draconic pointer (iv), therefore } (b1/a1) \times (r1/s1) = 13.42223271 \text{ (v).}
\]

For \(a1\) = 48 teeth (definite) and \(r1\) = 63 teeth (definite), the equation (v) becomes

\[
b1/s1 = 10.22646302 \text{ (vi).}
\]

For a gear teeth number of \(b1\) = 225 (see Chap. 6), the equation (vi) results

\[
s1 = 22.00174 \text{ teeth, rounded to 22 teeth for the gear } s1.
\]

From the above calculations, it results that for one rotation of \(b1\) gear (one metonic tropical year of 365.2631579\(^d\)), the draconic pointer rotates 13.42329545 times (draconic months), so

\[
365.2631579^d/13.42329545 = 27.2113896^d/\text{draconic month, instead of } 27.21218683^d\]

(resulting from the values of Saros cycle presented on Table I). The gearing exhibits a phase difference-error of -0.00104787\(^d\)/1 draconic pointer rotation, i.e. about -1.5 min/draconic month. Without a doubt, such an error is too small compared to the mechanical errors and manufacturing imperfections of the Mechanism gearing (Edmunds 2011).
7.3 The “scenic” operation of the draconic gearing on the Antikythera Mechanism

Introducing the draconic gearing on the Antikythera Mechanism, the operation of the high-resolution eclipse prediction based on the geometry is achieved. The specific position of the draconic pointer on the right side of the Mechanism Fig. 12, also has a special scenic operation: as the user rotates the Lunar Disc when the lunar pointer aligns with the Golden Sphere-Sun i.e. New Moon phase (or in opposite direction-Full Moon), he can also easily observe at the right side of the Mechanism, to see if the draconic pointer is located inside one of the two Ecliptic limits (arc bronze strip). If such is the case, he certainly knows that a solar (or lunar) eclipse will occur. Afterwards, he turns the Mechanism to the other side (Back plate) and by observing the cell that the Saros pointer aims, he can read the eclipse information for the time of the eclipse event and the corresponding metonic month in which the eclipse occurs. If the draconic pointer aims anywhere out of the Ecliptic limits, the corresponding Saros cell is absolutely empty (see Fig. 12, 15).

Someone may wonder why the ancient manufacturer did not place the Draconic gearing in a dial display close to the Saros dial of the Back plate:

On the Back plate there are gears with quite slow rotation (Metonic: 1 turn/3.8 years, Saros: 1 turn/4.5075 years, Athletic Games: 1turn/4 years). So it is difficult to connect a gear with fast rotation as the draconic pointer one 13.422 turns/year (about equal to Sidereal 13.368 turns/year), with a very slow gearing train. Therefore, the ancient manufacturer tried to find a place for the Draconic gearing with fast rotation.
Someone also may wonder why the ancient manufacturer did not not place the draconic gearing close/around to the Lunar Disc gearing: The Lunar Disc rotates by 13.368 turns/year, and the Draconic pointer rotates by 13.422 turns/year. For the ratio Draconic/Sidereal=1.004039, an accepted approximation of two engaged gears is 249 and 248 teeth. These gears should have a diameter of around 116mm, which needs a large space (232mmX116mm). The manufacturer could also make a combined gearing by several gears of (3X83)/(8X31) a complex combination (83 and 31 are prime numbers). Moreover, additional space is needed for the Draconic pointer and scale.

Regarding the letters “ME”, referred three times on the Fragment D parts, they could also be considered that they are the headings of the words “ΜΗΝ ΕΚΛΕΙΠΤΙΚΟΣ” or “ΜΗΝ ΕΓΛΕΙΠΤΙΚΟΣ” (Ecliptic Month), which is related to the three draconic gearing parts. The Ecliptic (ΕΚΛΕΙΠΤΙΚΗ) is referred to the Mechanism Back cover inscriptions as “ΧΡΟΝΟΙ…. ΕΓΛΕΙΠΤΙΚΟΙ” (times of eclipses) Bitsakis and Jones 2016.

7.4 The variable velocity of the Moon - pin&slot gearing motion, visible on the draconic pointer

From Voulgaris et al., 2018b, it is evident that the variable velocity achieved by the gears $k_2$-$k_1$ and transferred on the following gearing sequence is not visible to the naked eye by observing any part of the Mechanism, because the gears’ rotation is in reduced angular velocity (slower motion) than the input/pin&slot gearing. Therefore the variability in the velocity is reduced as well.

Furthermore, the rotation period of the draconic output-pointer is about equal to the Lunar Disc sidereal rotation period: from the equations (a) and (b) it results that 1 Saros = 242 draconic cycles $\approx$ 241.029 sidereal cycles i.e. 1 sidereal rotation (360°) of the Lunar Disc $\approx$ 1.004028 draconic rotations ($\approx$ 361.45°), i.e. the draconic pointer rotates a bit faster than the Lunar Disc pointer Fig. 13.
Figure 13: Left, the harmonic graph of the draconic and anomalistic cycles, assuming that they start on the same phase ( draconic in Ascending Node, anomalistic in apogee) and during the New Moon phase. The units of the x-axis (time) are in synodic cycles. Right, the white arrow represents the position of the Line of Apsides relative to the Nodes, which changes its position during the rotation of the draconic pointer (not depicted in the scheme). The draconic cycle starts with the arrow head pointing to the Ascending Node (Anomalistic month start - apogee) so that the Moon crosses the Node with the minimum velocity. After 121 Draconic cycles (half Saros-Sar), the draconic pointer crosses the Ascending Node with the maximum velocity-perigee. On the 60.5th and also on the 181.5th draconic cycle, i.e. when the draconic pointer aims to the Descending Node, the pointer crosses the two Nodes with about its mean angular velocity.

On the Saros cycle, 242 draconic months are also equal to \( \approx 239 \) anomalistic cycles i.e. 1 anomalistic month (from apogee to the next apogee) \( \approx 1.01255 \) draconic months. This means that the Line of Apsides (the line connecting the points of min and max lunar velocity) delays each draconic rotation, i.e. rotates in the opposite direction relative to the Line of Nodes Fig. 14.

On the Antikythera Mechanism, this change is visible on the draconic pointer. The draconic pointer rotation presents a variable angular velocity, and the max-min velocities (Line of Apsides) change their position relative to the Nodes. The imaginary Line of Apsides slowly rotates in opposite direction (CW) to the direction of the draconic pointer rotation (CCW). This implies that the draconic pointer returns to the same Node-pin, each time by a different velocity, as it also happens every time the Moon approaches a Node Fig. 17.

8. Epilogue

The Fragment D/gear-r1 was an unplaced part of the Antikythera Mechanism, with an unknown operation. In this work, an ideal position and function for this enigmatic part is presented, taking into account the present condition and the preserved parts of the Antikythera Mechanism. According to the authors’ opinion, the Draconic gearing existence offers the necessary precise eclipse prediction/calculation of the Mechanism. The authors tried to correlate, using the minimum hypotheses required, an unplaced gear with an
unknown gear training output of the Mechanism, without adding any too hypothetical or theoretical parts and scenarios. For this attempt, the dimensions of the preserved parts were taken into account. The simple design/construction of the draconic gearing includes the use of the three existing gears \((b1, a1, r1)\) as well as a new gear \((s1)\). No additional, hypothetical complex parts and engraved scale was needed.

This additional gearing train improves the instrument’s efficiency, which can now perform precise eclipse predictions/calculations. The adaptation of the draconic gearing/cycle on the Antikythera Mechanism presents a complete representation of the four lunar motions that were well-known and studied in the Hellenistic era. The authors strongly believe that the ancient manufacturer of the Antikythera Mechanism took into account the four integrated lunar motions when he designed his creation.

Summarizing:
1) A realistic and relevant to the Antikythera Mechanism operation for the unknown \(a1\) output was found,
2) The multiplying rotation of the \(a1\)-gear leads to a gear train output a few times shorter in rotation than the tropical gear,
3) A mechanically accepted position and role for the unplaced Fragment D/gear-\(r1\), was detected,
4) The existence of the two other parts of Fragment D was justified,
5) The mathematical calculations of the draconic gearing are highly accurate,
6) There is adequate space for the draconic training adaptation at the right side of the Mechanism, and this specific position assists the Mechanism user during the operation,
7) The teeth number of \(b1\) gear was specified,
8) The existence of the specific high-speed gear training offers the geometrical calculations needed for the eclipse events prediction/high resolution calculation of the Antikythera Mechanism, improving the accuracy of the eclipse predictions,
9) The addition of the draconic gearing on the Mechanism introduces the two mandatory lunar cycles for the eclipse prediction,
10) The invisible to the naked eye, variable angular velocity produced by the gears \(k2/k1/\) (adapted on the \(e5\) gear), becomes visible on the draconic pointer rotation,
11) Finally, by introducing the Draconic gearing on the Antikythera Mechanism, incorporating on the Antikythera Mechanism gearing of the four, well-studied in antiquity, interrelated lunar motions, is achieved, Fig. 14.
Figure 14: A comprehensive view of the four lunar cycles cooperation on the Antikythera Mechanism: the Synodic/Sidereal cycle in bronze reconstruction functional model by the authors, the Anomalistic cycle in AMRP combined tomography and a digital completion of the lost half gears e3, e4, e6 and k2, processed by the authors, and the Draconic gearing, suggested and necessary according to the authors’ opinion. The Draconic gearing b1, a1, r1, and s1 constructed by the first author, in bronze material.

Although someone should be skeptical and doubtful when introducing new hypotheses regarding the Antikythera Mechanism, the authors believe there is a large number of logical reasons justifying the existence of this new operation/gearing. According to authors’ opinion is too difficult to explain why the ancient manufacturer chose to represent on his creation the Synodic (Sidereal) and the Anomalistic lunar cycles, but not the Draconic cycle, which is critical and determinant for the eclipse prediction. Additionally, they cannot find any mechanical or malfunctioning or non-correlation or teleology reason, in order to doubt or reject this new gearing.
According to our study, which is in progress, our team has evidence to believe the ancient manufacturer engraved the eclipse events sequence on the corresponding cells of the Saros spiral, while rotating the Lunar Disc-Input of the Mechanism: By observing the relative position of the Lunar Disc pointer to the Golden sphere-Sun (and the opposite position-Full moon) and the position of the Draconic pointer relative to the Ecliptic limits, Fig. 15 (Reconstructing the Saros eclipse events using the Draconic gearing of the Antikythera Mechanism, by the authors-underwriting).

Figure 15: Top, The front dial plate of the functional model by the authors. The Lunar Disc and the Golden sphere-Sun are visible. Bottom, The Draconic scale is consisted of the draconic pointer, the Ascending (ANABIBAIZON) and the Descending (KATAKABAZON) Nodes (black dots) and the two eclipse limits (in arbitrary angle dimension, in present work), are depicted. The phase positional correlation of the Lunar Disc pointer relative to the Golden sphere-Sun and the Draconic pointer relative to the Nodes and the Ecliptic limits define an eclipse possibility event: A) The Full moon is located on the Ascending Node, a total lunar eclipse occurs. B) The New moon is located faraway from the Ascending Node, a total or partial solar eclipse occurs. C) The Full moon is located just on the ecliptic limit boundary, a partial at limit (or penumbral) lunar eclipse occurs. D) The New moon is located out of the Ecliptic limits, no solar eclipse occurs. Northern hemisphere, mid-latitude around 40° observing place is considered.

The main conclusion arising from this work is presenting the four lunar cycle cooperation on the Antikythera Mechanism’s gearing environment. Including the fourth lunar cycle, the Draconic gearing:pointer to his construction, the ancient manufacturer had the ability to predict/calculate “automatically” the unknown eclipse events. Based on the self-calculated predictions and without any previous information out of the Mechanism’s calculation, he
engraved the eclipse events on Saros corresponding cells. The ancient manufacturer “trained” his machine, in order to make predictions. This distinctive operation of the Antikythera Mechanism shall be the early first idea of the machine learning.

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