STABILIZED QUANTUM GRAVITY:
Stochastic Interpretation and Numerical Simulation

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Abstract

Following the reasoning of Claudson and Halpern, it is shown that "fifth-time" stabilized quantum gravity is equivalent to Langevin evolution (i.e. stochastic quantization) between fixed non-singular, but otherwise arbitrary, initial and final states. The simple restriction to a fixed final state at $t_5 \rightarrow \infty$ is sufficient to stabilize the theory. This equivalence fixes the integration measure, and suggests a particular operator-ordering, for the fifth-time action of quantum gravity. Results of a numerical simulation of stabilized, latticized Einstein-Cartan theory on some small lattices are reported. In the range of cosmological constant $\lambda$ investigated, it is found that:
1) the system is always in the broken phase $<\det(e)> \neq 0$; and 2) the negative free energy is large, possibly singular, in the vicinity of $\lambda = 0$. The second finding may be relevant to the cosmological constant problem.

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1 Introduction

The "fifth-time" action is a general procedure, first proposed in ref. [1], for stabilizing theories whose Euclidean actions are unbounded from below. There are several such "bottomless action" theories which are of interest to physics; examples include D=0 matrix models, bosonic string field theory, and, especially, Einstein-Hilbert gravity. The fifth-time method has been applied by many authors [2] to D=0 matrix models. In this article I will continue the study of fifth-time stabilized quantum gravity, begun in ref. [3].

Euclidean quantum gravity is of interest for at least two reasons. First, there are the intriguing Baum-Hawking-Coleman arguments [4] for the vanishing of the cosmological constant, which are formulated in the Euclidean approach. Unfortunately, these arguments involve the semiclassical evaluation of a badly divergent path-integral, and are therefore rather suspect [5]. Secondly, Monte Carlo simulations of quantum gravity cannot avoid the Euclidean formulation. In ref. [3] it was shown how the fifth-time prescription generates a new diffeomorphism-invariant action for quantum gravity, bounded from below, which leads to the usual Einstein equations of motion in the classical limit. Moreover, the stabilized Einstein-Cartan theory appears to be reflection-positive, at least in lattice formulation, and should be a good starting point for non-perturbative investigations.

At the perturbative level, the stabilized theory flips the sign of the "wrong-sign" conformal mode at zeroth order, and is non-local at higher orders. This behavior is in agreement with two other approaches which aim at deriving the Euclidean theory from Minkowski space gravity. The first, due to Schleich [6] (see also [7]), solves the Hamiltonian constraints in the Minkowski theory prior to Wick rotation, and then reinserts redundant degrees of freedom to recover diffeomorphism invariance. The second approach, due to Biran et. al. [8] and Mazur and Mottola [9], argues that the kinetic term of the conformal factor should be Wick rotated like a potential term, when certain Jacobian factors in the integration measure are taken into account. In both cases a bounded, non-local Euclidean action is generated. These studies show that the Euclidean continuation of Einstein-Hilbert gravity is not necessarily obtained by the naive replacement of the Minkowski action by the corresponding (unbounded) Euclidean expression. The advantage of the fifth-time approach is that the 5-dimensional action is local, relatively simple, and amenable to numerical and analytical methods, whereas the non-local stabilized 4-dimensional action, obtained in any of these approaches, is not known in closed form.

A common objection to the fifth-time prescription is that it is rather ad hoc, at least as presented in the original treatment in [1]. The method generates a stabilized action which has the same classical limit and formal perturbative expansion as the
unstabilized theory, but there could be other methods to achieve those ends. In fact, for D=0 matrix models alternate stabilizations have been proposed [10, 11]. A better understanding of the physics underlying the fifth-time action would therefore be helpful in judging its merits. In the next section it will be shown that the fifth-time prescription is equivalent to stochastic quantization, with the constraint that the Langevin evolution not only begin at a fixed (arbitrary) initial state (at $t_5 \rightarrow -\infty$), but also terminate at an arbitrary non-singular final state ($t_5 \rightarrow \infty$). No claim of originality is made for this "stochastic interpretation" of the fifth-time action. The equivalence of Langevin evolution to the fifth-time action, for ordinary bounded actions, was shown by Gozzi in ref. [12]. The idea of fixing the final state of Langevin evolution for bottomless actions is due to Claudson and Halpern [13], in connection with Nicolai maps for $QCD_4$, and it was noted by Giveon et. al. [14] that Langevin evolution with fixed initial and final states is equivalent to the fifth-time stabilization prescription of ref. [1]. Section 2 contains an exposition of this reasoning, which will then be extended, in section 3, to the special case of quantum gravity. It will be shown that the stochastic interpretation fixes the integration measure, and suggests a particular choice of operator-ordering, in the fifth-time action.

In section 4 the results of a Monte Carlo simulation of stabilized, latticized, Einstein-Cartan gravity are reported. The latticization is similar to that proposed by Menotti and Pelissetto in ref. [15], in that local O(4) symmetry is exact, but diffeomorphism invariance is broken by the lattice regularization. The simulation is carried out on very tiny lattices: $2^4 \times 4$ and $3^4 \times 4$; so obviously the results must viewed with caution. Nevertheless, the outcome is interesting. It is found, for a range of positive and negative cosmological constants, that the system is always in the broken phase $<\det(e)> \neq 0$. Moreover, in the neighborhood of $\lambda = 0$: the average curvature approaches 0, the volume per lattice site diverges, and there is a peak, possibly divergent, in the negative free energy/lattice site $-F$. The peak in $-F$ near $\lambda = 0$ and infinite volume suggests that, if the cosmological constant becomes dynamical by some mechanism, then its probability distribution is peaked near $\lambda = 0$, as proposed in ref. [4]. Recently Carlini and Martellini [16] have argued that the Coleman mechanism for a vanishing cosmological term is realized in fifth-time stabilized quantum gravity; this could be related to the numerical results found here.

An alternative approach to formulating and simulating Euclidean quantum gravity is the method of summing over simplicial manifolds. In this case the action is just the Euclidean Einstein-Hilbert (Regge) action, and it is the lattice regularization (restriction to simplicial manifolds) which provides a cutoff on the number.
of manifolds with large positive or negative curvature [17]. This lattice cutoff on large curvatures makes numerical simulation possible, and interesting Monte Carlo results have recently been obtained in 4-dimensions [17-20]. It remains to be seen whether a theory whose stability is based on the lattice structure is actually independent of the lattice structure, in the sense of universality. Also, as noted above, the continuation of Minkowski space gravity to Euclidean space is not necessarily obtained by replacing the Minkowski space Einstein-Hilbert action by the Euclidean Einstein-Hilbert action. Some additional remarks are contained in section 4.

2 Stochastic Quantization of Bottomless Actions

This section reviews a selection of ideas found in ref. [12-14], which provide a stochastic interpretation for the fifth-time action prescription of ref. [1].

What does it mean to quantize a bottomless action $S[\phi]$ in Euclidean space? Clearly, if the Boltzman factor $\exp(-S)$ is non-integrable, the path-integral formulation cannot be used directly. However, in statistical physics the Boltzman factor is just a representation of the probability distribution obtained by random Brownian motion. From this point of view it is the Brownian motion, as described by the Langevin equation, which is fundamental. Quantization based on the Langevin equation, known as ”stochastic quantization”, gives the following prescription for thermal averages of operators $Q[\phi]$:

$$<Q> = \lim_{T \to \infty} \frac{1}{Z_5} \int D\eta(x, -T < t_5 < T) \, Q[\phi(x, 0)] \exp[-\int_{-T}^{T} d^{5}x \, \eta^2/4\hbar]$$  

where

$$\partial_5 \phi = -\frac{\delta S}{\delta \phi} + \eta$$

$$\phi(x, -T) = \phi_i(x)$$  

In stochastic quantization, the role of the classical action $S$ is to supply the drift term in the Langevin equation. For a given noise configuration $\eta$, the Langevin equation is solved starting from an initial field configuration $\phi_i(x)$ at $t_5 = -T$. As $T \to \infty$, the thermal average should be independent of the initial configuration.

Applied to the problem of bottomless actions, the Langevin approach does not, at first sight, seem much of an improvement over path-integral quantization. Even if the initial configuration $\phi_i$ is a local minimum of the action, for large $t_5$ the field will almost always be driven into the bottomless region of the action, evolving towards a singular configuration as $t_5 \to \infty$. The qualifier in ”almost always” is important,
however, since there do exist noise configurations \( \eta(x, t_5) \) for which the field remains in the vicinity of a stationary point \( \delta S/\delta \phi = 0 \) throughout its Langevin evolution.

The proposal, then, is as follows: Let us retain the prescription of stochastic quantization that \( S[\phi] \), bottomless or not, supplies the drift term of Langevin evolution, but average only over noise terms which leave the field in a fixed non-singular final state \( \phi_f(x) \) at \( t_5 = T \). In other words,

\[
\langle Q \rangle = \lim_{T \to \infty} \frac{1}{Z_5} \int D\eta(x, -T < t_5 < T) \delta[\phi(x, T) - \phi_f(x)]Q[\phi(x, 0)]
\]

\[
\times e \left[ -\int_{-T}^{T} d^5x \frac{\eta^2}{4\hbar} \right]
\]

(3)

where, again,

\[
\partial_5 \phi = \frac{\delta S}{\delta \phi} + \eta
\]

\[
\phi(x, -T) = \phi_i(x)
\]

(4)

If we can show that thermal averages \( \langle Q \rangle \) are independent of the choice of both initial \( \phi_i \) and final \( \phi_f \) configurations, as \( T \to \infty \), then we have obtained a version of stochastic quantization which, for bounded actions, is equivalent to the usual version, but which is also meaningful for bottomless actions. I will refer to the standard version of eq. (1), with a fixed initial state, as “stochastic (I)” quantization, and the proposal of eq. (3), with fixed initial and final states, as “stochastic (IF)” quantization.

Following ref. [12, 13], let us change variables in the functional integral of eq. (3). Writing the Langevin equation as

\[
\eta = \partial_5 \phi + \frac{\delta S}{\delta \phi}
\]

(5)

we have

\[
Z_5 = \int D\eta \delta[\phi(x, T) - \phi_f(x)]\exp[-\int_{-T}^{T} d^5x \frac{\eta^2}{4\hbar}]
\]

= \int_{\phi_i}^{\phi_f} D\phi \det[\frac{\delta \eta}{\delta \phi}] \exp \left[ -\int_{-T}^{T} d^5x \left( (\partial_5 \phi)^2 + \frac{\delta S}{\delta \phi} \right)^2 + 2\partial_5 \phi \frac{\delta S}{\delta \phi} \right] / 4\hbar
\]

= \int_{\phi_i}^{\phi_f} D\phi \det[\frac{\delta \eta}{\delta \phi}] \exp \left[ -\int_{-T}^{T} d^5x \left( \frac{1}{4} (\partial_5 \phi)^2 + \frac{1}{4} \frac{\delta S}{\delta \phi} \right)^2 / \hbar \right]
\]

\[
\times e^{-\left( S[\phi_f] - S[\phi_i] \right) / 2\hbar}
\]

(6)

Working out the Jacobian,
\[
\det \left[ \frac{\delta \eta(x)}{\delta \phi(x')} \right] = \det \left[ \partial_5 \delta^5(x - x') + \theta(t_5 - t_5') \frac{\delta^2 S}{\delta \phi(x) \delta \phi(x')} \right] \\
= \det \left[ \partial_5 \delta^5(x - x') + \theta(t_5 - t_5') \frac{\delta^2 S}{\delta \phi(x) \delta \phi(x')} \right] \\
= \det \left[ \partial_5 \exp \left[ \text{Tr} \ln \left[ \delta^5(x - x') + \theta(t_5 - t_5') \frac{\delta^2 S}{\delta \phi(x) \delta \phi(x')} \right] \right] \right]
\]

where \( \theta(0) = \frac{1}{2} \), and the meaning of

\[
\frac{\delta^2 S}{\delta \phi(x) \delta \phi(x')} \mid_{t_5}
\]

is to carry out the D=4 dimensional functional variations, and then replace \( \phi(x) \) by \( \phi(x, t_5) \). Note that only the first term survives in the expansion of the Trace log; all other terms in the trace vanish due to the time-ordering enforced \( \theta \). Substitution of (7) and (8) into (3) gives

\[
< Q > = \lim_{T \to \infty} \frac{1}{Z_5} \int_{\phi_i}^{\phi_f} D\phi(x, \quad T < t_5 < T) \quad Q[\phi(x, 0)] \times \exp \left[ -\int_{-T}^{T} d^5 x \left[ \frac{1}{4} (\partial_5 \phi)^2 + \frac{1}{8} \frac{\delta S}{\delta \phi} \right]^2 - \frac{\hbar}{2} \frac{\delta^2 S}{\delta \phi^2} \right] / \hbar \]
\]

or, with a rescaling of \( t_5 \rightarrow t_5 / 2\hbar \)

\[
< Q > = \frac{1}{Z_5} \int D\phi \quad Q[\phi(x, 0)] e^{-S_5} \\
S_5 = \int d^5 x \left[ \frac{1}{2} (\partial_5 \phi)^2 + \frac{1}{8} \frac{\delta S}{\delta \phi} \right]^2 - \frac{1}{4} \frac{\delta^2 S}{\delta \phi^2} \]
\]

It is now easy to prove that \( < Q > \) is independent of the initial and final states \( \phi_i \) and \( \phi_f \), since

\[
< Q > = \lim_{T \to \infty} \frac{\sum_{nm} \Psi_n[\phi_f] < \Psi_n | Q | \Psi_m > \Psi_m^*[\phi_i] e^{-(E_n + E_m)T / \hbar}}{\sum_{n} \Psi_n[\phi_f] \Psi_n^*[\phi_i] e^{-2E_nT / \hbar}} \\
= < \Psi_0 | Q | \Psi_0 >
\]
where the \( E_n \) are the eigenvalues, \( \Psi_{n} \) the eigenstates (\( \Psi_0 \) the ground state), of the Hamiltonian corresponding to \( S_5 \)

\[
H_5 = \int d^4x \left[ -\frac{1}{2} \delta^2 + \frac{1}{8\hbar^2} (\delta S/\delta \phi)\right]^2 - \frac{1}{4\hbar} \delta^2 S \] (12)

which is known as the Fokker-Planck Hamiltonian. Note that whether \( S \) is bottomless or not, the potential term in \( H_5 \) is bounded from below, and therefore has a well-defined ground state. Since \( <Q> \) depends only on the ground state of \( H_5 \), and not on the initial or final configurations \( \phi_i, \phi_f \), we have shown that stochastic (IF) quantization, like ordinary stochastic (I) quantization, does not depend on the the choice of the initial/final configurations.

There is one way that the reasoning above could have failed. This would be if \( \Psi_0[\phi_f] = 0 \), e.g. if \( \phi_f \) or its derivatives were infinite. Such singular configurations would be obtained in Langevin evolution for bottomless actions, as \( t_5 \to \infty \), if the final state were unconstrained. Thus, the only slight restriction we need to make on the final configuration is that it is non-singular, \( \Psi_0[\phi_f] \neq 0 \); in fact, this restriction is implicit in the choice of initial configuration as well, even for ordinary stochastic (I) quantization.

Equation (10) is the "fifth-time" stabilization prescription proposed in [1], which is now seen to be equivalent to stochastic (IF) quantization. It is easy to see from (10) that the \( \hbar \to 0 \) limit enforces the classical equations of motion \( \delta S/\delta \phi = 0 \), and it was shown in [1] that the perturbative expansion of (10) reproduces the naive perturbation expansion, to all orders in any expansion parameter, generated from Taylor expanding \( e^{-S} \) in the usual functional integral [6].

Writing the ground state \( \Psi_0 \) of \( H_5 \) in the form

\[
\Psi_0 = \exp\left[-\frac{S_{eff}}{2\hbar}\right] \] (13)

fifth-time stabilization can be regarded as replaced the bottomless action \( S \) by a bounded action \( S_{eff} \), i.e

\[
<Q> = <\Psi_0|Q|\Psi_0> = \frac{1}{Z} \int D\phi(x)Q[\phi(x)]e^{-S_{eff}/\hbar} \] (14)

which has the same classical equations of motion, and the same formal perturbative expansion, as \( S \).

**The Yang-Mills Vacuum as Stabilized Chern-Simons Theory**

\(^2\)The equality of perturbative expansions holds providing \( S \) is stable at zeroth order, which is not the case for quantum gravity, c.f. [3].
A curious example of the 5-th time approach is provided by Yang-Mills theory. The discussion in this subsection is based on the Nicolai map for $QCD_4$ found in ref. [13], but with emphasis laid on the fact that $QCD_4$ itself can be regarded as a “fourth”-time action.

It has long been known [21] that there is an exact solution of the Yang-Mills Schrodinger equation in temporal gauge

$$\int d^3x \frac{1}{2} \left[ -\frac{\delta^2}{\delta A_i^{a2}} + \frac{1}{2} F_{ij}^{a2} \right] \Psi = E \Psi$$

(15)

with energy $E = 0$; this is the Chern-Simons state

$$\Psi[A] = \exp \left[ \frac{1}{2} \int d^3x \epsilon_{ijk} A_i^a \partial_j A_k^a - \frac{2g}{3} \epsilon_{abc} A_i^a A_j^b A_k^c \right]$$

$$= \exp \{ cs[A] / 2 \}$$

(16)

But because this state is non-normalizable (the Chern-Simons action is bottomless), it must be rejected as a physical state. In fact, due to asymptotic freedom, the true Yang-Mills vacuum can be expected to look something like the abelian ground state for high frequency fluctuations

$$\Psi_{abelian}^0[A] = \exp \left[ -\frac{1}{8\pi^2} \int d^3x d^3y F_{ij}(x) F_{ij}(y) \left| x - y \right|^2 \right]$$

(17)

while for low-frequency fluctuations, it was argued in ref. [21] (see also [22]) that the true Yang-Mills vacuum has the form

$$\Psi_0[A_{low}] \approx \exp[ -\mu \int d^3x \ Tr F_{ij}^2 ]$$

(18)

This expression for the low-frequency vacuum has since been verified by Monte-Carlo simulations [23].

Now although the Chern-Simons state is non-normalizable, it is still an exact solution of the Yang-Mills Schrodinger equation, and an interesting question to ask is what does the stabilized theory looks like. The answer is quite remarkable: **Stabilized Chern-Simons theory is the true Yang-Mills vacuum!** This is easy to see, since the rule is to replace the unbounded distribution $\exp \{ cs[A] \}$ by $\exp[ -S_{eff} ]$, where $\Psi_0 = \exp[ -S_{eff} / 2 ]$ is the ground state of the Fokker-Planck Hamiltonian

$$H_5 = \int d^3x \left[ -\frac{1}{2} \frac{\delta^2}{\delta A_i^{a2}} + \frac{1}{8} \left( \frac{\delta cs[A]}{\delta A_i^a} \right)^2 - \frac{1}{4} \frac{\delta^2 cs[A]}{\delta A_i^{a2}} \right]$$

(19)

which in this case turns out to be the Hamiltonian of D=4 Yang-Mills theory in temporal gauge, seen in eq. [15]. As a consequence, stabilized Chern-Simons...
theory is simply the Yang-Mills vacuum in temporal gauge. Likewise, the "fifth-time" action (really a "4-th time" action, since $cs[A]$ is 3-dimensional) is just the D=4 Yang-Mills theory. From this fact, it is not hard to see that all (not just equal-time) correlators of D=4 Yang-Mills theory can be derived from stochastic (IF) quantization of the Chern-Simons action, as shown in ref. [13], and in detail in ref. [24].

3 Stabilized Gravity

To apply stochastic (IF) quantization to gravity, we must generalize the formalism somewhat to allow for field-dependent supermetrics. Denote the fields, which may represent the metric, tetrad, or spin-connection, by $g^N$, the supermetric by $G_{MN}$, and the supervielbien by $E^A_N$. The corresponding Langevin equation is

$$\partial_5 g^M = -G^{MN} \frac{\delta S}{\delta g^N} + E^M_A \eta^A$$

where

$$<\eta^A(x, t_5)\eta^B(x', t'_5)> = 2\hbar \delta_{AB} \delta^4(x-x') \delta(t_5 - t'_5)$$

$$G_{MN} = E^A_M E^A_N$$

The analogue of (3) becomes

$$Z_5 = \int Dg^N \det \frac{\delta \eta}{\delta g} \exp \left[ -\int d^5x \left( \frac{1}{4} G_{MN} \partial_5 g^M \partial_5 g^N + \frac{1}{4} G^{MN} \frac{\delta S}{\delta g^M} \frac{\delta S}{\delta g^N} \right) / \hbar \right]$$

$$\times e^{-\frac{1}{2}(S[g^N] - S[g^N]) / 2\hbar}$$

and from the Langevin equation

$$\eta^A = E^A_M (\partial_5 g^M + G^{MN} \frac{\delta S}{\delta g^N})$$

we have

$$\frac{\delta \eta^A(x)}{\delta g^L(x')} = E^A_M \left( \delta^M_L \delta_5(x-x') + \frac{\delta}{\delta g^L(x')} G^{MN}(x) \frac{\delta S}{\delta g^N(x)} |_{t_5} - \frac{\delta E^A_M(x)}{\delta g^L(x')} E^M_B \eta^B \right)$$

3This form corresponds to the $\mu = \sigma = 0$ case of the Langevin equation for gravity in ref. [25]. The most general case will not be considered here.
\[ E^A_M \partial_5 \left[ \delta^M_L \delta^5(x - x') + \int d\tau \theta(t_5 - \tau) \left\{ \frac{\delta}{\delta g^L(x')} G^{MN}(x, \tau) \frac{\delta S}{\delta g^N(x)} \right\} \right. \]
\[ \left. \quad - \frac{\delta E^M_B(x, \tau)}{\delta g^L(x')} \eta^B(x, \tau) \right\} \]

leading to the determinant

\[ \det \left[ \frac{\delta \eta^A(x)}{\delta g^L(x')} \right] \]
\[ = \det[\partial_5] \det[E] \exp \left[ \int d^5x d\tau \theta(t_5 - \tau) \left\{ \frac{\delta}{\delta g^M(x, t_5)} G^{MN}(x, \tau) \frac{\delta S}{\delta g^N(x)} \right\} \right. \]
\[ \left. \quad - \frac{\delta E^M_B(x, \tau)}{\delta g^M(x, t_5)} \eta^B(x, \tau) \right] \]

The derivatives of the supermetric and supervielbein in (25) depend on a choice of stochastic calculus. In the Ito calculus, all contractions between \( G^{MN}(x, t) \) and \( \eta(x', t) \) at equal (fifth) time \( t \) are taken to be zero, e.g.

\[ < E^N_A(x, t) \eta^A(x, t) E^M_B(x', t) \eta^B(x', t) > \]
\[ = < E^N_A(x, t) E^M_B(x', t) > < \eta^A(x, t) \eta^B(x', t) > \]

This condition can be achieved by defining

\[ E^N_A(x, t) \equiv E^N_A[g(x, t - \delta t)] \]

where \( \delta t \) is an infinitesimal fifth-time displacement, so that \( E^N_A(x, t) \) is independent of \( \eta(x, t) \). This fifth-time displacement does not affect invariance under four-dimensional, \( t_5 \)-independent, diffeomorphisms. Extending this prescription also to the supermetric in (25), i.e. \( G^{MN}(x, t) = E^M_A(x, t) E^N_A(x, t) \), results in a determinant

\[ \det \left[ \frac{\delta \eta^A}{\delta g^L} \right] = \det[\partial_5] \det[E] \exp \left[ \int d^5x \frac{\delta^2 S}{\delta g^M(x) \delta g^N(x)} \right] \]

Finally, substituting (28) into (22), we obtain the fifth-time action formulation

\[ < Q > = \frac{1}{Z_5} \int Dg^N \det[E] Q[g^N(x, 0)] e^{-S_5/\hbar} \]
\[ S_5 = \int d^5x \left( \frac{1}{4} G_{MN} \partial_5 g^M \partial_5 g^N + \frac{1}{4} G^{MN} \frac{\delta S}{\delta g^M} \frac{\delta S}{\delta g^N} - \frac{\hbar}{2} G^{MN} \frac{\delta^2 S}{\delta g^M \delta g^N} \right) \]
\[ S_{EH} = -\frac{1}{\kappa^2} \int d^4x \sqrt{g} R \]  

(30)

where \( \kappa^2 = 16\pi G \). Expanding \( g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu} \), the action to zeroth-order is

\[ S^0 = \int \frac{d^4p}{(2\pi)^4} h_{\mu\nu}(p)p^2 \left[ \frac{1}{4} P(2) - \frac{1}{2} P(0-s) \right] \mu^{\alpha\beta} h_{\alpha\beta}(-p) \]  

(31)

where \( P(0-s) \) and \( P(2) \) are transverse spin-2 and spin-0 projection operators \([26]\). In this case the 10 independent fields \( g^A \) just correspond to the metric components \( g_{\mu\nu} (\mu \geq \nu) \). The supermetric \( G_{MN} \) is defined implicitly from

\[ \delta g^2 = \int d^4x G_{MN}(x) \delta g^M(x) \delta g^N(x) \]

\[ = \int d^4x G^{\mu\alpha\beta}(x) \delta g_{\mu\nu}(x) \delta g_{\alpha\beta}(x) \]

\[ G^{\mu\alpha\beta} = \frac{1}{2\sqrt{g}} g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha} + c g^{\mu\nu} g^{\alpha\beta} \]  

(32)

It is required that the arbitrary constant \( c \) in the DeWitt supermetric \( G^{\mu\alpha\beta} \) be constrained to \( c > -\frac{1}{2} \); since otherwise \( \det(G) < 0 \) and we cannot construct a supervielbein. This would break the link between stochastic (IF) quantization and the 5-th time action (in fact, \( S_5 \) would no longer be bounded from below). Applying \([24]\) to the Einstein-Hilbert action (30), one finds

\[ S_5 = \frac{1}{4} \int d^5x \left[ \frac{1}{\kappa^2} G^{\mu\alpha\beta} \partial_5 g_{\mu\nu} \partial_5 g_{\alpha\beta} + \kappa^2 G^{-1}_{\mu\alpha\beta} \frac{\delta S}{\delta g_{\mu\nu}} \frac{\delta S}{\delta g_{\alpha\beta}} \right. \\

\left. -2\hbar\kappa^2 G^{-1}_{\mu\alpha\beta} \frac{\delta^2 S}{\delta g_{\mu\nu} \delta g_{\alpha\beta}} \right] \\

= \frac{1}{4} \int d^5x \left[ \frac{1}{\kappa^2} G^{\mu\alpha\beta} \partial_5 g_{\mu\nu} \partial_5 g_{\alpha\beta} + \frac{1}{\kappa^2} g G^{-1}_{\mu\alpha\beta} (R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R) \times (R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R) - \hbar \beta \sqrt{g} R \right] 

(33)

for the corresponding 5-th time action, where \( \beta \) is a singular constant. This action was derived in \([3]\), except that the operator ordering in the last term of \( S_5 \), as well as the functional integration measure, was left undetermined. Stochastic (IF) quantization determines the integration measure to be the (D=4 dimensional) DeWitt measure \( \det(E) = \sqrt{G} = \text{const} \), while the ”retarded supervielbien” \( E_{M}^{\alpha}(x, t) \equiv E_{M}^{\alpha}[\phi(x, t - \delta t)] \) suggested by the Ito calculus determines the ordering of supermetric and functional derivatives shown above. This ordering will be particularly convenient for the Einstein-Cartan theory.

In ref. \([3]\) it was shown how to calculate the stabilized 4-dimensional action \( S_{eff} \) perturbatively, starting from \( S_5 \). At linearized level
\[ S_{\text{eff}}^0[g_{\mu\nu}] = \int \frac{d^4p}{(2\pi)^4} h_{\mu\nu}(p)p^2 \left[ \frac{1}{4} P^{(2)} + \frac{1}{2} P^{(0-s)} \right]_{\mu\nu\alpha\beta} h_{\alpha\beta}(-p) \] (34)

which, like \( S^0 \) above, is transverse; only the sign of the ”wrong-sign” conformal mode has been flipped. \( S_{\text{eff}} \) will be non-local at higher-orders, but it is guaranteed to have the same classical equations of motion as \( S_{EH} \).

The problem with (33), which is a shortcoming of the 5-th time approach in general, is that \( S_5 \) contains higher-derivative terms, in this case proportional to \( R^2 \). This means that reflection positivity for reflections across the ”ordinary” time \( x_4 \) axis cannot be guaranteed, which is problematic for continuing \( S_{\text{eff}} \) to Minkowski space.\(^4\) The presence of higher derivatives in \( S_5 \) can be traced to the 2nd derivative terms in \( S \). For this reason, it was suggested in [3] to stabilize the Einstein-Cartan theory, which, like the Chern-Simons theory discussed in the last section, contains only first order derivatives. The ”fifth”-time action corresponding to D=3 Chern-Simons theory is D=4 Yang-Mills theory, which is certainly reflection positive across any axis. As the Einstein-Cartan theory has a tensor structure similar to that of Chern-Simons theory, there is reason to expect that the stabilized version is also reflection positive.

The Einstein-Cartan theory in 4 dimensions has the action

\[ S_{EC} = -\frac{1}{4\kappa^2} \int \epsilon_{abcd} e^a \wedge e^b \wedge (d\omega^{cd} + \omega^{ef} \wedge \omega^{fd}) \] (35)

In order to write down a single Langevin equation for this system, it is necessary to have the tetrad \( e^a_\mu \) and spin connection \( \omega^{ab}_\mu \) in the same multiplet \( g^L \), and it is convenient to rescale these fields so that they have the same dimensions. The dimensional quantities in the theory are \( \kappa, \hbar \), so we rescale

\[ \bar{e} = \frac{1}{\kappa} e \]
\[ \bar{\omega} = \sqrt{\hbar} \omega \] (36)

with supermetric defined implicitly by [4]

\[ \delta g^2 = \delta \bar{e}^2 + \delta \bar{\omega}^2 \]
\[ = \int d^4x \sqrt{g} g^{\mu\nu} [\delta \bar{e}_\mu \delta \bar{e}_\nu + \delta \bar{\omega}^{ab} \delta \bar{\omega}^{ab}] \] (37)

\(^4\)Reflection positivity is not ruled out either, since \( S_5 \) certainly has this property for bounded \( S \) despite the higher derivative terms.

\(^5\)This is not the most general possible \( e - \omega \) supermetric. Only the simplest version of the stabilized theory will be considered here.
Applying this supermetric to the general formula (29), and noting that in this case (as in Chern-Simons theory) the singular term

$$G^{MN} \frac{\delta^2 S}{\delta g^M \delta g^N} = 0$$ (38)

vanishes, the stabilized Einstein-Cartan theory (including a cosmological term $\int d^4x \lambda \text{det}(e)$) is found to be

$$<Q[e, \omega] = \frac{1}{Z_5} \int D e D \omega \left[ \prod_{x,t_5} \text{det}^{10}(e) \right] Q[e(x,0), \omega(x,0)] e^{-S_5/\hbar}$$ (39)

where

$$S_5 = \frac{1}{4} \int d^5x \sqrt{g} \left[ \frac{1}{\kappa^2} g^{\mu\nu} (\partial_5 e^a_\mu \partial_5 e^a_\nu + \hbar \partial_5 \omega_{ab}^\mu \partial_5 \omega_{ab}^\nu) \ight. \\
+ 4 \left( \frac{1}{\kappa^2} R^a_\mu R^a_\nu g^{\mu\nu} - \lambda R + \kappa^2 \lambda^2 \right) \\
+ \frac{1}{\kappa^4 \hbar} T^a_{\mu\nu} T^b_{\rho\sigma} (\delta_{ab} g^{\mu\rho} + 2 e^a_\mu e^b_\rho) g^{\nu\sigma}$$

(40)

is the 5-th time action and

$$R = d\omega + \omega \wedge \omega \\
T = de + \omega \wedge e$$

are the curvature and torsion two-forms respectively. The fifth-time action $S_5$ contains no higher-derivative terms and can be shown, in the lattice version discussed in the next section, to be reflection positive across the $x_4$ axis.

The perturbative expansion of $S_5$, at $\lambda = 0$, is an expansion around the classical $R = T = 0$ solution:

$$e^a_\mu = \delta^a_\mu + \kappa b^a_\mu$$

$$\omega^a_{\mu} = \varpi^a_{\mu}(e) + \kappa^2 \sqrt{\hbar} \Omega^a_{\mu}$$

(42)

where

$$\varpi^a_{\mu} = \frac{1}{2} e^{\nu a} (\partial_\mu e^b_\nu - \partial_\nu e^b_\mu) - \frac{1}{2} e^{b a} (\partial_\mu e^a_\nu - \partial_\nu e^a_\mu) - \frac{1}{2} e^{\rho a} e^{b \rho} (\partial_\mu e_{\sigma c} - \partial_\sigma e_{\rho c}) e^c_\mu$$

(43)

is the zero-torsion spin-connection. Then the part of $S_5$ which is zeroth-order in $\hbar$ is
\[ S_5 = \frac{1}{4} \int d^5x \sqrt{g} \left[ \frac{1}{\kappa^2} g^{\mu\nu} \partial_5 e_\mu \partial_5 e_\nu + 4 \left( \frac{1}{\kappa^2} R^e_\mu R^e_\sigma g^{\mu\sigma} - \lambda R + \kappa^2 \lambda^2 \right) + (\Omega \wedge e)_\mu^a \Omega \wedge e)_\rho^b g^{\nu\sigma} (\delta_{ab} g^{\mu\rho} + 2 e_\mu^a e_\rho^b) + O(\sqrt{\hbar}) \right] \]  

(44)

where \( R = d\omega + \omega \wedge \omega \). From (44) we see that torsion propagates only at loop level.

The D=4 stabilized action \( S_{\text{eff}} \) was calculated at zeroth-order in \( \kappa \) (for \( \lambda = 0 \)) in ref. \[3\], and it was found that

\[ S_{\text{eff}}^0[e] = \int \frac{d^4p}{(2\pi)^4} h_{\mu\nu} p^2 \left[ \frac{1}{4} P^{(2)} + \frac{1}{2} P^{(0-s)} \right]_{\mu\nu\alpha\beta} h_{\alpha\beta} \]  

(45)

where \( h_{\mu\nu} = b_{\mu\nu} + b_{\nu\mu} \) is the symmetric part of the tetrad. This result is identical to the zeroth order \( S_{\text{eff}}^0 \) obtained for the stabilized Einstein-Hilbert action.

4 Numerical Simulation

There is no known lattice action for general relativity which is exactly invariant under a continuous symmetry group analogous to diffeomorphisms. The best one can do for the Einstein-Cartan theory is to preserve the invariance under the local Lorentz group (O(4) in Euclidean space), and hope that the diffeomorphism invariance of the continuum action can somehow be recovered at a fixed point. Here we follow Menotti and Pelissetto \[13\] in introducing a hypercubic lattice with link variables

\[
U_\mu(n) = \exp[aP_b \tau^b_\mu(n) + \frac{1}{2} a J_{bc} \tilde{\omega}^{bc}_\mu(n)] = \exp[aP_b e_\mu^b(n)] \exp[\frac{1}{2} a J_{bc} \tilde{\omega}^{bc}_\mu(n)] = \exp[P_b e_\mu^b(n)] \exp[\frac{1}{2} J_{bc} \omega^{bc}_\mu(n)]
\]  

(46)

where \( a \) is the lattice spacing, and from here on we set \( \hbar = 1 \). \( P_a \) and \( J_{ab} \) are the generators of the Euclidean Poincare group in the four-dimensional spinor representation

\[
J_{ab} = \frac{1}{4} [\gamma_a, \gamma_b] \quad P_a = \frac{1}{2} \gamma_a (1 + \gamma_5)
\]  

(47)

Define the plaquette variable
\[ U_{\mu\nu}(n) = U_\mu(n)U_\nu(n + \mu)U^{-1}_\nu(n + \nu)U^{-1}_\mu(n) \]  

(48)

and lattice curvature, torsion, and tetrad

\[
\begin{align*}
R_{\mu\nu}^a(n) &= -\frac{1}{2} Tr\{J_{ab}[U_{\mu\nu}(n) - U_{\nu\mu}(n)]\} \\
T_{\mu\nu}^a(n) &= -\frac{1}{4} Tr\{K_a[U_{\mu\nu}(n) - U_{\nu\mu}(n)]\} \\
e^a_\mu &= -\frac{1}{4} Tr\{K_aU_\mu(n)\gamma_5U^{-1}_\mu(n)\} 
\end{align*}
\]  

(49)

where \( K_a = -\frac{1}{2} \gamma_a(1 - \gamma_5) \). Under \( O(4) \) gauge transformations, the lattice tetrad, curvature, and torsion transform like vectors and tensors in the latin (local frame) indices.

In terms of these variables, the lattice version of the Einstein-Cartan action becomes

\[
S = \sum_n \left[ -\frac{1}{4} \epsilon^{\mu\nu\alpha\beta} \epsilon_{\alpha\beta} R_{\mu\nu}^a(n)e^c_\alpha(n)e^d_\beta(n) + \lambda det(e) \right] 
\]  

(50)

which is invariant under local Lorentz \( O(4) \) gauge transformations, but still bottomless. The lattice cosmological constant \( \lambda \) is expressed in units of the Planck length, i.e. \( \lambda = \kappa^4 \lambda_{\text{continuum}} \); note that the lattice spacing has dropped out of the action. In terms of the same variables, the latticized version of the Einstein-Cartan 5-th time action \((40)\) is

\[
S_5 = \frac{1}{4} \sum_n |det(e)| \left[ -\frac{1}{4} \epsilon^\mu_\alpha^\nu_\beta e^\mu_{c^n}(n)e^\nu_{c^n}(n)\{T^a_{\mu5}(n)T^a_{\nu5}(n) + R_{\mu5}^a(n)R_{\nu5}^a(n)\} \\
+4\epsilon\{R_{\mu\rho}^a R_{\nu\sigma}^b \epsilon^\mu_\rho e^\sigma_{c^n} e^\rho_{c^n} e^\nu_{c^n} - \lambda R_{\mu\nu}^a e^\mu_{c^n} e^\nu_{c^n} + \lambda^2\} \\
+\epsilon T^a_{\mu5} T^b_{\rho5} (\delta_{ab} e^\mu_{c^n} e^\rho_{c^n} + 2 e^\mu_{c^n} e^\rho_{c^n} e^\nu_{c^n} e^\sigma_{c^n}) \right] 
\]  

(51)

where

\[
\epsilon = \frac{a_5}{2\kappa^2}
\]  

(52)

and \( a_5 \) is the lattice spacing in \( t_5 \) direction. The lattice vectors \( n \) now label sites on a D=5 dimensional lattice, and \( e^a_\mu \) is inverse to the matrix \( e_{\alpha}^a \). Defining \( U_5(n) = 1 \), the curvature and torsion \( R_{\mu5}^a(n) \) and \( T_{\mu5}^a(n) \) are also obtained from \((49)\). The quantities in \((49)\) are dimensionless; the scalings which give the usual curvature, torsion, and tetrad in the continuum limit are

\[
R_{\mu\nu}(\text{continuum}) = \frac{1}{a^2} R_{\mu\nu}^ab
\]
\[ T^{ab(\text{continuum})} = \frac{\kappa}{a^2} T^{ab} \]

\[ \epsilon^a_{\mu(\text{continuum})} = \frac{\kappa}{a} \epsilon^a_{\mu} \]  

(53)

The action (51), together with the integration measure

\[ \int \prod_n \text{det}^{10}(e) \prod_{\mu} dU_{\mu}(n) \]  

(54)

was used for the numerical simulation. Note that the lattice spacing \( a \) in the 1 − 4 directions scales out of the action completely, and all quantities which, in the continuum, are diffeomorphism invariant, appear on the lattice in units of \( \kappa \) (c.f. [27]), e.g.

\[ \int d^4 x \sqrt{g} R = \kappa^2 \sum_n |\text{det}(e)| R^{ab}_{\mu \nu} e^a_{\mu} e^b_{\nu} \]

\[ \int d^4 x \sqrt{g} = \kappa^4 \sum_n |\text{det}(e)| \]

\[ \delta s^2_{n,n+\delta} = \kappa^2 < \epsilon^a_{\mu} \epsilon^a_{\nu} > \delta^\mu \delta^\nu \]  

(55)

The expectation values of such quantities are therefore expressed, on the lattice, in units of the Planck length \( \kappa \) rather than lattice spacing \( a \), and depend only on the parameters \( \epsilon = a_5/2\kappa^2 \), and \( \lambda \). The fact that lattice spacing \( a \) drops out of both the action and the expectation values is a remnant of diffeomorphism invariance in the continuum, and has led to the speculation that lattice quantum gravity, like string theory, has a kind of built-in minimum length \( \delta s^2 \approx \kappa^2 \) [27].

The final step in the lattice formulation above would be to symmetrize (51) with respect to all \( \pi/2 \) rotations around the axes; the resulting symmetrized action can be shown to be reflection positive [15]. In the interest of minimizing computer time however, only the unsymmetrized action (51) was used for the Monte Carlo simulation.

Computer simulation of latticized Einstein-Cartan theory is rather lengthy, even for very small lattices. The link variables are 4 × 4 matrices on a 5-dimensional lattice, and because of the plaquette-plaquette structure of the action, a single link update requires on the order of \( 10^4 \) floating-point multiplications. For this reason the simulation has only been carried out on tiny lattices of dimensions \( 2^4 \times 4 \) and \( 3^4 \times 4 \) (4 spacings in the \( t_5 \) direction), which required, despite the small lattice sizes, a total expenditure of 100 Cray YMP hours.

There is a two-parameter (\( \epsilon - \lambda \)) space of couplings for \( S_5 \). One cannot vary \( \epsilon \) at fixed \( \lambda = 0 \) because the lattice does not seem to thermalize at \( \lambda = 0 \), for reasons

\(^6\)Disappearance of the lattice spacing \( a \) in favor of \( \kappa \) is found also in the Regge lattice formulation.
discussed below. Instead I have fixed $\epsilon$ somewhat arbitrarily at $\epsilon = \frac{1}{2}$, and carried out Monte Carlo simulations for various values of $\lambda$.

Figures 1-3 plot the average curvature (Fig. 1),

$$< R > \equiv \frac{\langle \sum |\text{det}(e)\rangle R \rangle}{\langle \sum |\text{det}(e)\rangle \rangle}$$

(56)

the volume per lattice site (Fig. 2),

$$< \sqrt{g} > \equiv \frac{1}{N_{\text{sites}}} \langle \sum |\text{det}(e)\rangle \rangle$$

(57)

and the derivative of the free energy per lattice site (Fig. 3),

$$\frac{dF}{d\lambda} = \frac{1}{N_{\text{sites}}} \langle \sum |\text{det}(e)\rangle (-.5R + \lambda) \rangle$$

(58)

versus the cosmological constant $\lambda$ in a range $-5 \leq \lambda \leq 20$. Squares denote data on $2^4 \times 4$ lattices, at $\lambda = -1, .25, .5, 1, 2, 5, 10, 20$, crosses denote data on $3^4 \times 4$ lattices at $\lambda = -5, -2, -1, .5, 1, 2, 5$. The longest runs were at the smallest values of $\lambda$, e.g. 300 thermalizations and 1600 data-taking iterations at $\lambda = 0.5$ on a $3^4 \times 4$ lattice.

From Fig. 1, we find that $< R >$ depends linearly on $\lambda$, with $< R > \approx 1.6\lambda$, as compared to the classical value $R = 2\lambda$. The modest discrepancy in slope could have various origins: renormalization of $\lambda$, the smallness of the lattice, and the effect of the hypertoroidal lattice topology.

Figure 2 shows a clear divergence in volume/site at $\lambda = 0$. Runs at the value $\lambda = 0$ did not converge to a finite value of $|\text{det}(e)|$, instead showing a steady increase in volume with number of iterations. Apart from the divergence at $\lambda = 0$, there are two other noteworthy facts concerning $< \text{det}(e) >$: First, it is found that for all values of $\lambda \leq 10$, $< \text{det}(e) > = < |\text{det}(e)| >$, while for the largest value of $\lambda = 20$, $< \text{det}(e) >$ was only 4% less than $< |\text{det}(e)| >$. From this we conclude that in the range of $\lambda$ studied, the system is in the broken symmetry phase expected for quantum gravity, and it appears that fluctuations which change the sign of $\text{det}(e)$ are rarely generated.\footnote{Strictly speaking, transformations such as inversions $x_4 \rightarrow -x_4$, $U_4 \rightarrow U_4^{-1}$, which take $\text{det}(e) \rightarrow -\text{det}(e)$, are invariances of the symmetrized, reflection positive lattice theory. For the unsymmetrized action of eq. (51), used for the numerical simulation, invariance under such transformations is only approximate.} The second point is that the volume/site decreases smoothly away from $\lambda = 0$ for both positive and negative $\lambda$. This fact, and the fact that $< R > \propto \lambda$, show very clearly the effect of stabilization, since the cosmological term in the Einstein-Cartan/Hilbert actions is bottomless for $\lambda < 0$. In the fifth-time Einstein-Cartan, changing the sign of the cosmological constant (leaving Newton’s constant fixed) is equivalent to changing the sign of Newton’s constant (leaving the
cosmological constant fixed). In the Regge and simplicial manifold approaches, the behavior of average volume and curvature changes drastically upon changing the sign of Newton’s constant [17-20]. This is not the case in the stabilized version of the Einstein-Cartan theory, where the metric fluctuates close to zero curvature for small $\lambda$ of either sign.

In connection with simplicial manifolds, it has been noted that the lattice cut-off stabilizes the Euclidean Einstein-Hilbert action, since the entropy of simplicial manifolds with large curvatures is small [17]. If continuum quantum gravity would then be stable due to the measure, the 5th-time procedure cannot change anything, and is simply a rewriting of the original 4-dimensional theory. (This is because if $\exp(-S)$ is normalizable, then it is necessarily the ground state (squared) of $H_5$; this means that $S_{\text{eff}} = S$, where $S$ is the Einstein-Hilbert action.) On the other hand, it may be that any stability induced by the lattice cutoff in the $D=4$ theory is spurious or non-universal, in which case the 5th-time action is a much better starting point for both numerical and analytical work. Perturbative calculations in the stabilized Einstein-Hilbert theory do indicate, in fact, that $S_{\text{eff}} \neq S$, and indeed that $S_{\text{eff}}$ is non-local [3], which is in accord with the findings of ref. [3, 8, 9].

Finally, we see in Fig. 3 that the slope of the free energy is large, and possibly singular, in the neighborhood of $\lambda = 0$. Although the data is nowhere near sufficient to allow a numerical integration, it does appear that the negative free energy (and therefore $Z_5$) must have a peak, and might even be divergent, at $\lambda = 0$. This peak is intriguing, because if the cosmological constant becomes dynamical in some way, as suggested by Baum, Hawking and Coleman [4], then $Z_5(\lambda)$ has the interpretation of a probability density for the cosmological constant, i.e.

$$P(\lambda) = N \int Dg \ e^{-S_{\text{eff}}[g,\lambda]} = \int Dg \ \psi^*[g,\lambda] \psi[g,\lambda] \propto Z_5(\lambda)$$

A sharp peak in the (negative) free energy at $\lambda = 0$ is therefore an explanation of the smallness of the cosmological constant. It is too much to claim that the data shown here, obtained on tiny lattices from an action which breaks diffeomorphism invariance, is strong evidence for the vanishing of the cosmological term in quantum gravity, but at least the data does seem to support this idea. Actually the evidence for a sharp peak in $-F$ is better for $\lambda \to 0^+$ than for $\lambda \to 0^-$. Conceivably, since the volume is divergent at $\lambda = 0$, $Z_5$ itself may be discontinuous at this point, with the probability peaked at $\lambda = 0^+$. As usual, one would like to have more data, on larger lattices, in the region of interest.

I have made no attempt to search for an ultraviolet fixed point in the $\epsilon - \lambda$
parameter space; all computations at various $\lambda$ were made at the constant (and rather arbitrary) value of $\epsilon = \frac{1}{2}$. A fixed point where diffeomorphism invariance is restored (if such a point exists) would be signaled by a peak, growing sharper with lattice size, in the correlation

$$C = \frac{< (\sum \sqrt{g} R)^2 > - < \sum \sqrt{g} R >^2}{< \sum \sqrt{g} >}$$ (60)

No such peak was observed for $\epsilon = \frac{1}{2}$ on the $3^4 \times 4$ lattice; the value of $C \approx 11.5$ was almost constant, varying by less than 10%, over the full range of $\lambda$. A very interesting question is whether a peak in correlation $C$ develops as $\epsilon$ is varied; this is an issue where much additional numerical work is required.

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Figure Captions

**Fig. 1** Average curvature vs. cosmological constant, in units of the Planck length. Squares denote data on $2^4 \times 4$ lattices, crosses denote data on $3^4 \times 4$ lattices.

**Fig. 2** $< \sqrt{g} >$ (volume/site) vs. cosmological constant.

**Fig. 3** Derivative of the free energy/lattice site $dF/d\lambda$ vs. cosmological constant.