Drell-Hearn-Gerasimov Sum-Rule for the Deuteron in Nuclear Effective Field Theory

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Abstract

The Drell-Hearn-Gerasimov sum rule for the deuteron is studied in nuclear effective field theory. The low-energy theorem for the spin-dependent Compton amplitude $f_1(\omega)$ is derived to the next-to-leading order in low-energy expansion. The spin-dependent photodisintegration cross section $\sigma^P - \sigma^A$ is calculated to the same order, and its contribution to the dispersive integral is evaluated.
The Drell-Hearn-Gerasimov (DHG) sum rule is a dispersive sum rule which relates the anomalous magnetic moment of a system, elementary or composite, to an integral over the spin-dependent photo-production cross section \( \sigma^P - \sigma^A \). The sum rule is derived from the low-energy theorem for the spin-dependent forward Compton amplitude \( f_1(\omega) \) and a dispersion relation. In recent years, because of rapid technological advances, it becomes possible to study this sum rule experimentally. For example, the experiments recently done at Mainz, Bonn, and Jefferson Lab were motivated by checking this sum rule for the proton and neutron.

In this paper, we examine the DHG sum rule for the deuteron (spin-1) in light of the nuclear effective field theory (EFT). The DHG sum rule for the deuteron reads,

\[
\frac{\pi^2 \alpha_{em} \kappa_D^2}{M_D^2} = \int_{\omega_{th}}^{\infty} d\omega \frac{\sigma^P(\omega) - \sigma^A(\omega)}{\omega},
\]

where \( \kappa_D = 2\mu_D M_{DC}/e \hbar - 2 \) is the deuteron’s anomalous magnetic moment in unit of \( e\hbar/(2M_{DC}) \), and \( M_D \) is the mass. Because the deuteron’s magnetic moment is \( \mu_D = 0.857 \mu_N \), where \( \mu_N = e\hbar/2M_Nc \) is the nuclear magneton and \( M_N \) the nucleon mass, \( \kappa_D = -2 \times 0.143 \). Numerically, the left-hand side is \( 0.65 \mu_B \). The \( \omega_{th} \) is the threshold photon energy for the deuteron photodisintegration; and \( \sigma^P \) and \( \sigma^A \) are the photo-production cross sections with the helicity of the photon parallel or anti-parallel to the helicity (+1) of the deuteron, respectively.

It has been realized for sometime that nuclear physics at low energy might be understood by effective field theories (EFT) which work according to the same principles as the standard model. However, constructing a workable scheme for specific systems is not necessarily straightforward. In the past few years, considerable progress has been made in two nucleon sector (see [5] for a recent review). It began with the pioneering work of Weinberg, who proposed to encode the short distance physics in the derivative expansion of local operators [4]. The problem associated with the unusually small binding energy of the deuteron was solved by Kaplan, Savage and Wise by exploiting the freedom of choosing a renormalization subtraction scheme [6], quickly followed by the pionless version [7] (see also [8, 9, 10]). Requiring reproducing the residue of the deuteron pole at next-to-leading order (NLO), a version with accelerated convergence was suggested in [11]. The use of dibaryon fields as the auxiliary fields, at first introduced in [12], was taken seriously in [13] which simplified the calculation significantly.

Using the latest formulation of EFT for the two-nucleon system, we study both the left and right hand sides of the DHG sum rule for the deuteron. The low-energy theorem is verified to NLO in low energy expansion. The spin-orbit interactions arising from non-relativistic reduction turn out to play a significant role. Then the spin-dependent photodisintegration cross section is computed to the same order. Although the leading-order result depends only on the nucleon scattering parameters, the NLO depends on two electromagnetic counter terms whose coefficients can be determined by the magnetic moment of the deuteron and the rate for \( n + p \) radiative capture. Finally, the photodisintegration contribution to the DHG integral is evaluated.

The structure of the forward Compton scattering amplitude for a general spin target is,

\[
f = f_0 \epsilon^* \cdot \bar{\epsilon} + f_1 i \epsilon^* \times \bar{\epsilon} \cdot \vec{S} + f_2 (\hat{k} \otimes \hat{k})^{(2)} \cdot (\vec{S} \otimes \vec{S})^{(2)} \epsilon^* \cdot \bar{\epsilon} + \ldots
\]

where \( \bar{\epsilon}(\epsilon') \) is the initial (final) photon polarization, \( \vec{S} \) is the angular momentum operator of the target, and \( \otimes \) indicates a tensor coupling. The vector amplitude \( f_1 \) is related to those
with the target magnetic quantum number $m_S$,
\[
 f_1 = -\frac{3}{S(S+1)} \frac{1}{2S+1} \sum_{m_S} m_S f^{(m_S)} .
\]
(3)

The amplitude has a low-energy expansion \[2\],
\[
 f_1 = -\frac{\alpha_{em} \kappa^2}{4S^2 M^2} \omega + 2\gamma \omega^3 + ... ,
\]
(4)

where the first term corresponds to the famous low-energy theorem with the anomalous magnetic moment $\kappa$ defined as $\mu - 2S$ \[2\], where $\mu$ is the magnetic moment in unit of $\hbar/2Me$. The next term defines the forward spin-polarizability $\gamma$ which has been studied in EFT in \[14\].

In a pionless effective field theory for the deuteron \[10, 12, 13\], the nucleon field $N$ and the $^3S_1$-channel dibaryon field $t_j$ are introduced. The leading-order effective lagrangian is
\[
 L = N^\dagger \left( iD_0 + \frac{D^2}{2M_N} \right) N - t_j^\dagger \left[ iD_0 + \frac{D^2}{4M_N} - \Delta \right] t_j - y \left[ t_j^\dagger N^T P_j N + \text{h.c.} \right] ,
\]
(5)

where $P_i = \tau_2 \sigma_2 \sigma_i / \sqrt{8}$ is the $^3S_1$ two-nucleon projection operator and $y$ is a coupling constant between the dibaryon and two-nucleon in the same channel. The covariant derivative is $D = \partial + ieQ A$ with $Q = (1 + \tau^3)/2$ as the charge operator and $A$ the photon vector potential. The NN scattering amplitude is reproduced by the following choice of parameters
\[
y^2 = \frac{8\pi}{M_N^2 r^{(3S_1)}}, \quad \Delta = \frac{2}{M_N r^{(3S_1)}} \left( \frac{1}{a^{(3S_1)}} - \mu \right) ,
\]
(6)

where $a^{(3S_1)}$ is the scattering length, $r^{(3S_1)}$ is the effective range, and $\mu$ is the renormalization scale. Similarly, one can introduce the dibaryon field to describe the scattering in the $^1S_0$ channel as well.

Let us compute the spin-dependent forward Compton amplitude $f_1(\omega)$ on the deuteron. The Feynman diagrams to NLO are shown in Fig. 1, where the crossing diagrams are omitted. The shaded circles represent the photon magnetic coupling with the nucleon,
\[
 L_{em}^{LO} = \frac{e}{2M_N} N^\dagger \left( \mu^{(0)} + \mu^{(1)} \tau_3 \right) \sigma \cdot B N
\]
(7)

where $\mu^{(0)} = (\mu_p + \mu_n)/2$ and $\mu^{(1)} = (\mu_p - \mu_n)/2$ are the isoscalar and isovector nucleon magnetic moments in unit of nuclear magneton, $B$ is an external magnetic field. The contribution from the pure magnetic photon coupling are shown in diagrams (a) and (b). A straightforward calculation yields
\[
f_1(\omega)|^{(a)+(b)} = -\frac{e^2}{4\pi M_N^2} (\mu^{(0)})^2 \omega + ... ,
\]
(8)

Although it appears as a $N^2$LO contribution in EFT power counting, it is actually a leading-order one, proportional to $q^2/\omega$, before setting the photon momentum $q = \omega$. 

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FIG. 1: Feynman diagrams for spin-dependent forward Compton scattering on the deuteron. The thick initial and final state arrows represent the deuteron. The thick arrows in the middle of the diagrams denote the dibaryon states in the $^3S_1$ and $^1S_0$ channels. The shaded circles denote the magnetic moment interactions, and the open circles the electric current interactions. The solid squares are from electromagnetic counter terms ($L_1$ and $L_2$) at NLO. The seagull vertices and the solid circles are from relativistic spin-orbit interactions.

The magnetic coupling is generated from a relativistic interaction, which, after non-relativistic reduction, also produces a “spin-orbit” interaction

$$L_{em}^{N^2LO,SO} = N t_i \left[ (2\mu_p - \frac{1}{2}) + (2\mu_n - \frac{1}{2}) \tau_3 \right] \frac{e}{8M_N^2} \sigma \cdot (D \times E - E \times D) N ,$$

where $E$ is an external electric field. The above lagrangian contains a seagull interaction for the proton as shown in diagram (c), which contributes to $f_1(\omega)$ at the same order as (a) and (b) do. Moreover, there is a derivative coupling which is shown as solid circles in diagrams in Fig. 1. This coupling, when combined with a current interaction from the gauged part of the proton’s kinetic energy, generates a contribution to $f_1(\omega)$ as shown in diagrams (d) and (e). The combined result from the spin-orbit interaction acting on the proton is

$$f_1(\omega)|^{(c)+(d)} = \frac{e^2}{16\pi M_N^2} (2\mu_p - 1) \omega + ... .$$

When the spin-orbit term acts on the neutron (e), the result is proportional to the magnetic moment of the neutron,

$$f_1(\omega)|^{(e)} = \frac{e^2}{16\pi M_N^2} 2\mu_n \omega + ... .$$

Summing over the above contributions, one has in EFT

$$f_1(\omega)|^{LO} = -\frac{e^2}{16\pi M_N^2} (\mu_n + \mu_p - 1)^2 \omega + ... .$$
At this order, the magnetic moment and mass of the deuteron are the sum of those of the neutron’s and proton’s, $\mu_d = \mu_n + \mu_p$ nuclear magneton, $M_D = M_p + M_n \approx 2M_N$, respectively. The above result is clearly the same as the low-energy theorem.

There is a new electromagnetic counter-term at NLO,

$$\mathcal{L}_{\text{em},2}^{\text{NLO}} = -i \frac{e}{M_N} \left( \mu^{(0)} - \frac{L_2}{r^{(3S_1)}} \right) \epsilon^{ijk} t_i^t B_j t_k ,$$

where we introduce a $\mu^{(0)}$ term in the definition, chosen to cancel the wave function renormalization contribution, which is present in the leading order result in the di-baryon formulation but has been omitted so far. The above contributes to the magnetic moment of the deuteron is

$$\mu_d = 2\mu^{(0)} + \frac{2\gamma L_2}{1 - \gamma r^{(3S_1)}} ,$$

in unit of nuclear magneton, where $\gamma = \sqrt{M_NB} = 45.703$ MeV with $B = 2.225$ MeV the deuteron binding energy. Fitting to the experimental value, one finds, $L_2 = -0.03$ fm.

Using a solid square to denote the above interaction, its contribution to the NLO Compton amplitude is shown by the three Feynman diagrams (f-h) in Fig. 1. In addition, there is an associated term from relativistic correction,

$$\mathcal{L}_{\text{em},2}^{\text{NLO,SO}} = \frac{e}{2M_N} \left( \mu^{(0)} - \frac{L_2}{r^{(3S_1)}} \right) - \frac{1}{4} \epsilon^{ijk} t_i^t (D \times E - E \times D)_j t_k ,$$

which generates a sea-gull contribution shown in diagram (i). Summing over the above contributions, we find the spin-dependent Compton amplitude to NLO,

$$f_1(\omega) |^{\text{LO+NLO}} = -\frac{e^2}{4\pi(2M_N)^2} (\mu_d - 1)^2 \omega + ... ,$$

where $\mu_d$ is the NLO result shown in eq. (14). The result is again consistent with the low-energy theorem. Additional relativistic corrections will systematically converts $2M_N$ into a deuteron mass.

We now turn to the right-hand side of the DHG sum rule in eq. (1)—the photon-energy integration over the entire spin-dependent production cross section from the threshold to infinity. Experimentally, there have been preliminary data from Mainz on meson production and more data will be analyzed soon [15]. But there is no direct data on the cross-section asymmetry $\sigma^P - \sigma^A$ in the region of deuteron photodisintegration. The HIGS facility at Duke University is poised to make this measurement in the near future [16].

There are theoretical estimates on the cross section asymmetry from nuclear and hadronic models. A most complete and up-to-date study was made by Arenhövel [17, 18], who classifies the cross section into three types:

- photo-disintegration $\gamma + d \rightarrow n + p$,
- single-pion production including coherent pion production $\gamma + d \rightarrow d + \pi^0$, and incoherent production $\gamma + d \rightarrow N + N + \pi$,
- two and more pion and other meson production.
FIG. 2: Cross section asymmetry for the deuteron photodisintegration, shown in two different photon-energy scales. The solid line is the full EFT result to NLO, the dashed line does not contain the relativistic spin-orbit effects. The squares show the potential model result of Arenhovel [17].

The estimate shows that the first process contributes about $-383 \, \mu b$ to the DHG integral, the second $299 \, \mu b$, and the third $70 \, \mu b$. A large cancellation occurs between meson production and photodisintegration, as is dictated by the sum rule. A simplified way of understanding the cancellation is to imagine a complete scale separation between the deuteron structure physics and the nucleon physics. Photoproduction, independently on the proton and neutron in the deuteron, yields a contribution about $438 \, \mu b$. This must be largely cancelled by the photodisintegration contribution. In reality, of course, the deuteron structure physics can strongly affect the outcome of meson production in individual channels, even at very high energy. For instance, a significant effect is coherence pion production, which contributes about $99 \, \mu b$ to the integral according to Ref. [18]. Moreover, charged single-pion production off the proton and neutron is strongly modified by the final-state interactions. It is likely, however, that the complete integral is less sensitive to the deuteron structure and final-state interaction effects.

The nuclear EFT allows calculation of the deuteron photodisintegration cross section at low energy. Using the optical theorem, one can obtain the cross section through the imaginary part of the forward Compton scattering amplitude, for which Feynman diagrams are again those shown in Fig. 1. At NLO, there is an additional electromagnetic counter-term that couples the $^3S_1$ and $^1S_0$ channels,

$$L_{\text{em},1}^{\text{NLO}} = \frac{L_1}{M_N \sqrt{\gamma r(1S_0) r(3S_1)}} t^j_s B_j + \text{h.c.}$$

(17)

where $s_a$ is the dibaryon field with quantum number of isovector $^1S_0$. The coupling constant $L_1$ has been determined by the rate of $n + p \rightarrow d + \gamma$. The measured cross section $\sigma = 334.2 \pm 0.5 \, \text{mb}$ with an incident neutron speed of 2200 m/s fixes $L_1 = -4.42 \, \text{fm}$.

To NLO, the cross section difference for the photo-disintegration is

$$\sigma^p - \sigma^A \bigg|_{\tilde{q}d \rightarrow np} = -\frac{e^2 p \gamma}{2 M_N^2 (p^2 + \gamma^2)} \left(1 - \gamma r(3S_1)\right) \times \left\{ \frac{2 \mu^{(1)}(\gamma - 1/a(1S_0) + \frac{1}{2} r(1S_0) p^2)}{1 - \gamma r(1S_0) + \frac{1}{2} r(1S_0) p^2} + \frac{(p^2 + \gamma^2)}{1 - \gamma r(1S_0) + \frac{1}{2} r(1S_0) p^2} \right\} ,$$

(18)
where \( p = \sqrt{M_N \omega - \gamma^2} \) and \( \omega \) is the photon energy. The first term in the braces comes from the production of the \( ^1S_0 \) proton-neutron scattering state, and the second term from the \( ^3S_1 \) scattering state. In the latter case, the contribution from the magnetic coupling alone vanishes because of the orthogonality of the scattering and bound state wave functions. The third term comes from the interference between the current interaction and the spin-orbit term, involving multiple nucleon-nucleon partial waves in the final state.

In Fig. 2, we have shown, using solid lines, the cross section difference as a function of photon energy in two different scales. Near the threshold, the dominant contribution comes from the \( ^1S_0 \) final state. It is negative because only an anti-parallel photon-deuteron configuration has a non-zero cross section. The \( L_2 \) contribution is small throughout the region. When the photon energy is 10 MeV and higher, the spin-orbit interaction becomes significant. It fact, it changes the cross section asymmetry from positive to negative, as is made clear by the difference between the solid and dashed lines. The potential model calculation in Ref. [18] is shown by the squares, for the sake of clarity. The agreement between the EFT and model calculation is excellent below \( \omega = 10 \) MeV. The former becomes less trustworthy at higher photon energy.

Finally, we consider the contribution of photodisintegration to the DHG integral in EFT. In principle, one should cut-off the integral at some photon-energy, beyond which the nuclear EFT is no longer applicable. Indeed the model calculation shows a strong effect from the \( \Delta \) resonance [18], which is clearly beyond the nuclear EFT. However, because the integral is manifestly convergent, we extend the integration all the way to infinity as our crude estimate. This yields a contribution \(-385 \mu b\) to the DHG integral.

To summarize, we verify that the nuclear EFT reproduces the low-energy theorem for the spin-dependent deuteron Compton amplitude. In the same framework, we calculate the spin-dependent photo-disintegration cross section, which has been compared to a potential model calculation and can be tested by future experimental data.

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