Lattice QCD Calculations of Hadron Structure: Constituent Quarks and Chiral Symmetry

Derek B. Leinweber and Anthony W. Thomas

Special Research Center for the Subatomic Structure of Matter (CSSM) and Department of Physics and Mathematical Physics, University of Adelaide 5005

New data from parity-violating experiments on the deuteron now allow isolation of the strange-quark contribution to the nucleon magnetic moment, $G_M^s(0)$, without the uncertainty surrounding the anapole moment of the nucleon. Still, best estimates place $G_M^s(0) > 0$. It is illustrated how this experimental result challenges the very cornerstone of the constituent quark model. The chiral physics giving rise to $G_M^s(0) \sim 0$ is illustrated.

1. INTRODUCTION

Previous experimental reports [1] of the strange-quark contribution to the nucleon magnetic moment, $G_M^s(0)$, have involved the poorly understood nucleon anapole moment. However, combining the original proton data with new data from parity violation experiments on the deuteron now allow an independent isolation of $G_M^s(0)$. The preliminary analysis [2] indicates $G_M^s(0)$ is smaller than the earlier result [1] of $G_M^s(0) = +0.61 \pm 0.17 \pm 0.21 \mu_N$, but positive values for $G_M^s(0)$ are still favoured.

In this report, we elucidate the impact of this experimental result on our understanding of the concept of a constituent quark. In particular, the constituent quark approach places properties such as the magnetic moment contribution of a quark to a baryon intrinsic to the constituent quark itself and therefore not dependent upon the environment in which the quark resides.

The extraction of $G_M^s(0)$ is based on the approximation of charge symmetry in the nucleon moments; i.e. equal $u$ and $d$ current quark masses and negligible electromagnetic corrections. This approximation is expected to be valid at the 1% level [3] and is certainly sufficient for the analysis of $G_M^s(0)$.

An examination of the symmetries manifest in the QCD path integral for current matrix elements reveals various relationships among the quark sector contributions [4]. Current matrix elements of hadrons, such as the magnetic moment of the proton, are extracted from the three-point function, a time-ordered product of three operators [4]. Generally, an operator exciting the hadron of interest from the QCD vacuum is followed by the current of interest, which in turn is followed by an operator annihilating the hadron back to the QCD vacuum. In calculating the three point function, one encounters two topologically distinct ways of performing the electromagnetic current insertion. Figure 1 displays skeleton...
Figure 1. Diagrams illustrating the two topologically different insertions of the current within the framework of lattice QCD. These skeleton diagrams for the connected (left) and disconnected (right) current insertions may be dressed by an arbitrary number of gluons and quark loops.

diagrams for these two possible insertions (with Euclidean time increasing to the right). These diagrams may be dressed with an arbitrary number of gluons and quark loops. The left-hand diagram illustrates the connected insertion of the current to one of the “valence” quarks of the baryon. In the right-hand diagram the external field produces a $q\bar{q}$ pair which in turn interacts with the valence quarks of the baryon via gluons. It is important to realize that within the lattice QCD calculation of the loop diagram on the right in Fig. 1 there is no antisymmetrization (Pauli blocking) of the quark in the loop with the valence quarks. For this reason, in general only the sum of the two processes in Fig. 1 is physical.

2. CHARGE SYMMETRY EQUALITIES

Under the assumption of charge symmetry, the three-point correlation functions for octet baryons leads to the following equalities for electromagnetic current matrix elements [4]:

$$
p = e_u u_p + e_d d_p + O_N ,
\Sigma^+ = e_u u_{\Sigma^+} + e_s s_{\Sigma} + O_{\Sigma} ,
\Xi^0 = e_s s_{\Xi} + e_u u_{\Xi^0} + O_{\Xi} ,
$$

(2)

Here, $O$ denotes the contributions from the quark-loop sector – shown on the right-hand side of Fig. 1. The baryon label represents the magnetic moment. Subscripts allow for environment sensitivity implicit in the three-point function [4]. For example, the three-point function for $\Sigma^+$ is the same as for the proton, but with $d$ replaced by $s$. Hence, the $u$-quark propagators in the $\Sigma^+$ are multiplied by an $s$-quark propagator, whereas in the

2Note that the term “valence” used here differs from that commonly used in the framework of deep-inelastic scattering. Here “valence” simply describes the quark whose quark flow line runs continuously from $0 \to x_2$. These lines can flow backwards as well as forwards in time and therefore have a sea contribution associated with them [4].
proton the $u$-quark propagators are multiplied by a $d$-quark propagator. The different mass of the neighboring quark gives rise to an environment sensitivity in the $u$-quark contributions to observables, which means that the naive expectations of the constituent quark model $u_p/u_{\Sigma^+} = u_n/u_{\Xi^0} = 1$ may not be satisfied. This observation should be contrasted with the common assumption that the quark magnetic moment is an intrinsic-quark property which is independent of the quark’s environment.

Focusing now on the nucleon, we note that for magnetic properties, $O_N$ contains sea-quark-loop contributions from primarily $u$, $d$, and $s$ quarks. In the SU(3)-flavor limit ($m_u = m_d = m_s$) the charges add to zero and hence the sum vanishes. However, the heavier strange quark mass allows for a result which is non-zero. By definition

$$O_N = \frac{2}{3} \ell G_M^u - \frac{1}{3} \ell G_M^d - \frac{1}{3} \ell G_M^s,$$

and the leading superscript, $\ell$, reminds the reader that the contributions are loop contributions. Note that, in deriving Eq. (3), we have set $\ell G_M^u = \ell G_M^d$, corresponding to $m_u = m_d$ [4]. In the constituent quark model $\ell R_d^s = m_d/m_s \simeq 0.65$.

With no more than a little accounting, the strange-quark loop contributions to the nucleon magnetic moment, $G_M^s$ may be isolated from (2) and (4) in the following two phenomenologically useful forms,

$$G_M^s = \left( \frac{\ell R_d^s}{1 - \ell R_d^s} \right) \left[ 2p + n - \frac{u_p}{u_{\Sigma^+}} (\Sigma^+ - \Sigma^-) \right],$$

and

$$G_M^s = \left( \frac{\ell R_d^s}{1 - \ell R_d^s} \right) \left[ p + 2n - \frac{u_n}{u_{\Xi^0}} (\Xi^0 - \Xi^-) \right].$$

As we explained above, under the assumption that quark magnetic moments are not environment dependent, these ratios (i.e. $u_p/u_{\Sigma^+}$ and $u_n/u_{\Xi^0}$) are taken to be one in many quark models. Incorporating the experimentally measured baryon moments leads to:

$$G_M^s = \left( \frac{\ell R_d^s}{1 - \ell R_d^s} \right) \left[ 3.673 - \frac{u_p}{u_{\Sigma^+}} (3.618) \right],$$

and

$$G_M^s = \left( \frac{\ell R_d^s}{1 - \ell R_d^s} \right) \left[ -1.033 - \frac{u_n}{u_{\Xi^0}} (-0.599) \right],$$

where all moments are expressed in nuclear magnetons ($\mu_N$). (Note that the measured magnetic moments are all known sufficiently accurately [12] that the experimental errors play no role in our subsequent analysis.) We stress that these expressions for $G_M^s$ are exact consequences of QCD, under the assumption of charge symmetry. Equation (8) provides a particularly favorable case for the determination of $G_M^s$ with minimal dependence on the valence-quark ratio.
Figure 2. The consistency relation between $u_p/u_{\Sigma^+}$ and $u_n/u_{\Xi^0}$ which must be satisfied within QCD. The part of the straight line which is dashed corresponds to $G_M^s(0) < 0$, while the solid part of the line has $G_M^s(0) > 0$. The standard quark model assumption of intrinsic quark moments independent of their environment is indicated by the filled square at (1, 1). The diamonds on the constraint line mark 0.1 $\mu_N$ changes in $G_M^s(0)$ when $\ell R_d^s = 0.55$ [13]. The lattice QCD prediction (after an appropriate chiral extrapolation, discussed in following sections) is illustrated by the filled circle with standard errors indicated.

If one considers the quark model suggestions of $u_n/u_{\Xi^0} = 1$ and $\ell R_d^s = 0.65$ in (8), one finds $G_M^s = -0.81 \mu_N$, a significant departure from the experimental indication of positive values.

Equating (7) and (8) provides a linear relationship between $u_p/u_{\Sigma^+}$ and $u_n/u_{\Xi^0}$ which must be satisfied within QCD. Figure 2 displays this relationship by the dashed and solid line, the latter corresponding to values for which $G_M^s(0) > 0$ when $\ell R_d^s$ is in the anticipated range $0 < \ell R_d^s < 1$. Since the line does not pass through the point (1, 1) corresponding to the simple quark model assumption of universality, the experimentally measured baryon moments are signaling that there must be an environment effect exceeding 12% in both ratios or approaching 20% or more in at least one of the ratios. Moreover, a positive value for $G_M^s(0)$ requires an environment sensitivity exceeding 70% in the $u_n/u_{\Xi^0}$ ratio. Hence the experimental indication that $G_M^s(0) > 0$ challenges the intrinsic magnetic moment concept which is fundamental to the constituent quark model.
3. CHIRAL CORRECTIONS

One of the major challenges at present in connecting lattice calculations of hadronic properties with the physical world is that computational limitations restrict the accessible quark masses to values an order of magnitude larger than the physical values. At such large masses one is far from the region where chiral perturbation theory is applicable. Yet one knows that for current quark masses near zero there is important non-analytic structure (as a function of the quark mass) which must be treated correctly if we are to compare with physical hadron properties. Our present analysis of the strangeness magnetic form factor has been made possible by a recent breakthrough in the treatment of these chiral corrections for the nucleon magnetic moments [14, 16]. In particular, a study of the dependence of the nucleon magnetic moments on the input current quark mass, within a chiral quark model which was fitted to existing lattice data, suggested a model independent method for extrapolating baryon magnetic moments which satisfied the chiral constraints imposed by QCD. We briefly summarize the main results of that analysis:

- a series expansion of \( \mu_{p(n)} \) in powers of \( m_\pi \) is not a valid approximation for \( m_\pi \) larger than the physical mass,

- on the other hand, the behavior of the model, after adjustments to fit the lattice data at large \( m_\pi \) is well determined by the simple Padé approximant:

\[
\mu_{p(n)} = \frac{\mu_0}{1 - \frac{\chi}{\mu_0} m_\pi + c m_\pi^2},
\]

as illustrated in Fig. 3.

- Eq.(9) not only builds in the usual magnetic moment of a Dirac particle at moderately large \( m_\pi^2 \) but has the correct leading non-analytic (LNA) behavior of chiral perturbation theory

\[
\mu = \mu_0 + \chi m_\pi,
\]

with \( \chi \) a model independent constant.

- fixing \( \chi \) at the value given by chiral perturbation theory and adjusting \( \mu_0 \) and \( c \) to fit the lattice data yielded values of \( \mu_p \) and \( \mu_n \) of 2.85 \( \pm \) 0.22 \( \mu_N \) and -1.96 \( \pm \) 0.16 \( \mu_N \), respectively, at the physical pion mass. These are in remarkably good agreement with the experimental values – certainly much closer than the usual linear extrapolations in \( m_q \).

Clearly it is vital to extend the lattice calculations of the baryon magnetic moments to lower values of \( m_\pi \) than the 600 MeV used in the study just outlined. It is also vital to include dynamical quarks. Nevertheless, the apparent success of the extrapolation procedure gives us strong encouragement to investigate the same approach for resolving the strange quark contribution to the proton magnetic moment.
Figure 3. Extrapolation of lattice QCD magnetic moments (● Ref. [3] and ■ Ref. [13]) for the proton (upper) and neutron (lower) to the chiral limit. The solid curves illustrate a two parameter fit of Eq. (9) to the simulation data in which the one-loop corrected chiral coefficient of $m_\pi$ is taken from $\chi$PT. The dashed curves illustrate the leading non-analytic contribution of Eq. (10). The experimentally measured moments are indicated by asterisks.

Our focus here is to extrapolate existing lattice QCD estimates of valance quark contributions to baryon moments ($u_p$, $u_n$, $u_{\Sigma^+}$, $u_{\Xi^0}$) to the physical mass regime. The leading non-analytic pieces of these various contributions to the baryon magnetic moments are proportional to

$$\beta \frac{m_N}{8\pi f_\pi^2} m_\pi \equiv \chi m_\pi,$$

where the pion decay constant $f_\pi = 93$ MeV. Isolating a particular quark flavour contribution only requires setting the electric charge of all other quark flavours to zero. Separation of the contributions of the LNA terms into valance and sea-quark sectors has been resolved in Ref. [17]. The coefficients $\beta$ and $\chi$ for various quark sectors and baryons are summarized in Table I. In removing the $u$ sea-quark contribution from the $u$-quark sector, one eliminates the contributions of the $\pi$ sea-quark to the LNA term. Since the $z$-component of angular momentum in the pion-baryon system is positive, the $\pi$ contribution to the moment is negative. Hence its removal suppresses the non-analyticity in the neutron moment and enhances the non-analyticity in $\Xi^0$, $p$, and $\Sigma^+$ where the pion cloud acts to enhance the $u$-quark sector contribution to these baryon moments.

For the calculation of $G_M^s(0)$, the LNA chiral behavior of $u_n$ and $u_{\Xi^0}$ is crucial, as the corresponding coefficients are of opposite sign. As a result, the ratio of these $u$-
Table 1

Coefficients $\beta$ and $\chi$ of (9) and (11) for the leading nonanalytic (LNA) contribution of the $u$-quark(s) to the magnetic moment of various baryons. We express the LNA coefficients $\chi$ in terms of the usual SU(6) constants $F$ and $D$, as well as giving numerical values for $\chi$ and $\beta$ (which are equivalent through Eq.(11)). The $u$-quark charge has been normalized to 1.

| Baryon | $\beta, \chi$ | $u$-flavor sector | $u$-sea-quark loop | $u$-valence sector |
|--------|---------------|--------------------|--------------------|--------------------|
| $n$    | $\beta$       | $(F+D)^2$          | $(9F^2-6FD+5D^2)/3$ | $2(-3F^2+6FD-D^2)/3$ |
|        | $\beta$       | 1.020              | 0.612              | 0.408              |
|        | $\chi$        | 4.41               | 2.65               | 1.76               |
| $\Xi^0$ | $\beta$       | $-(F-D)^2$         | $(F-D)^2$          | $-2(F-D)^2$        |
|        | $\beta$       | -0.0441            | 0.0441             | -0.0882            |
|        | $\chi$        | -0.191             | 0.191              | -0.381             |
| $p$    | $\beta$       | $-(F+D)^2$         | $(9F^2-6FD+5D^2)/3$ | $-4(3F^2+2D^2)/3$ |
|        | $\beta$       | -1.020             | 0.612              | -1.632             |
|        | $\chi$        | -4.41              | 2.65               | -7.06              |
| $\Sigma^+$ | $\beta$ | $-2(3F^2+D^2)/3$ | $2(3F^2+D^2)/3$ | $-4(3F^2+D^2)/3$ |
|        | $\beta$       | -0.568             | 0.568              | -1.136             |
|        | $\chi$        | -2.46              | 2.46               | -4.91              |

Quark contributions, extrapolated with the correct chiral behavior, will be completely different from the linearly extrapolated ratio. That the signs of these chiral coefficients are opposite may be traced back to the charge of the predominant pion cloud associated with these baryons. The chiral behavior in the neutron case is dominated by the transition $n\rightarrow p\pi^-\rightarrow n$, where there is a contribution from a $\bar{u}$ loop. On the other hand, for the $\Xi^0$ the LNA $u$-quark contribution is dominated by the process $\Xi^0\rightarrow \Xi^-\pi^+\rightarrow \Xi^0$. In this case the virtual $\pi^+$ involves a valence $u$-quark – and hence the sign change.

In order to extrapolate the lattice data for valence quarks to the physical pion mass we use the same approach which worked so well for the neutron and proton moments in Ref. [14, 16]. That is, we fit the lattice data for the valence quark of flavor $q$ in baryon $B$ with the form:

$$\mu(m_\pi) = \mu^0 \frac{1}{1 - \frac{\chi}{\mu^0} m_\pi + c m_\pi^2}.$$  (12)

Here $\chi$ are the modified LNA chiral coefficients for the valence quarks given in Table 1 (last column) and $\mu^0$ and $c$ are fitting parameters.

Figures 4 and 5 illustrate the extrapolations of the lattice data. The results of these fits shown at $m_\pi = 140$ MeV are the results of the extrapolation procedure, including fitting errors. Naive linear extrapolations are also shown to help emphasize the dominant role of chiral symmetry in the extrapolation process.

Clearly the effect of the chiral corrections on the extrapolated values of the key magnetic moment ratio $u_n/u_{\Xi^0}$ at $m_\pi = 140$ MeV is dramatic. Instead of the value $0.72 \pm 0.46$ obtained by linear extrapolation one now finds $1.51 \pm 0.37$. It is vital that this ratio is much more consistent with the range of values found necessary to yield a positive value of $G_M^s$. 
Figure 4. Extrapolation of the valence $u$-quark magnetic moment contributions to the neutron (solid symbols) and $\Xi^0$ (open symbols). Naive linear extrapolations (dashed lines) are contrasted with the chiral extrapolations incorporating the leading nonanalytic behavior. The lattice data is taken from Ref. [6].

as illustrated in Fig. 2 – values which seemed quite unreasonable in the constituent quark model. By comparison, the ratio $u_p/u_{\Sigma^+}$ at $m_\pi = 140$ MeV is quite stable, changing from $1.14 \pm 0.08$ in the case of linear extrapolation to $1.14 \pm 0.11$ when the correct chiral extrapolation is used. The lattice point illustrated in Fig. 2 is consistent with the constraint obtained from charge symmetry. This suggests that any charge symmetry breaking in the experimentally measured moments is small.

The effect of the chiral extrapolation on the ratio $u_n/u_{\Xi^0}$ is the most dramatic, changing it from $0.72 \pm 0.46$ to $1.51 \pm 0.37$. Thus Eq. (8), which seemed certain to guarantee $G_M^s < 0$, no longer does. Indeed, using the recent estimate [13] of $\ell_R^s = 0.55$, we obtain $G_M^s = -0.16 \pm 0.18\mu_N$. While this is still negative it does permit a small positive value within the error.

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Figure 5. Extrapolation of the valence $u$-quark magnetic moment contributions to the proton (solid symbols) and $\Sigma^+$ (open symbols). Naive linear extrapolations (dashed lines) are contrasted with the chiral extrapolations based on (12). The lattice data is taken from Ref. [6].

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