Ultrawide 3D Phononic Bandgap Metastructures as Broadband Low Frequency Filter

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Graphical Abstract
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Highlights

- A novel metastructure prototype governing extremely wide three-dimensional bandgaps is established.
- By principal of mode separation, the phenomenon of bandgap generation is explained.
- The opening of bandgap is studied through analytical formulation and finite element simulations.
- By additive manufacturing 3D printed prototypes are prepared and wave attenuation over ultrawide frequency region is validated.
- The analytical model, numerical codes and vibration test on 3D printed prototypes showed excellent agreement.
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Abstract

This present study reports a novel model for the study on three-dimensional phononic metastructures endowed with ultrawide three-dimensional complete bandgaps. The phononic structure is made of a unique material without composite material composition. Based on the principal of mode separation and global and local modal masses participation, the well-engineered structural configurations give extremely wide bandgaps with the gap-to-mid gap ratio of 157.6\% and 160.1\% for two proposed prototypes that produce the widest three-dimensional bandgaps ever reported. The band structures are explained by a modal analysis and the findings are further corroborated by developing an analytical model based on monoatomic mass-spring chain and further verified with well-established FEM numerical simulations. Thanks to additive manufacturing, the prototypes are developed by using 3D printing technology and low amplitude vibration test is performed to access the real-time vibration mitigation characteristics. An excellent agreement is obtained between analytical, numerical and experimental results. The effects of material damping on transmission response is also taken into consideration that eventually merge the separated bandgaps to form a broadband vibration attenuation zone. The results reported are scale independent and the proposed strategy may pave the way for developing novel meta-devices to control the noise and vibration, and underwater acoustic waves at a wide frequency band in all directions.

Keywords: 3D bandgap; additive manufacturing; phononic metastructure
Artificial periodic structures that once begin from electromagnetic media are presently hot research topics for noise and vibration control due to their unprecedented dynamic mechanical properties that are inconceivable with respect to natural materials\textsuperscript{1,2}. The key property of interest includes formation of bandgap (BG) that is a frequency region where incident wave propagation is prohibited. Although waveguiding\textsuperscript{3}, focusing and collimation\textsuperscript{4}, negative refraction\textsuperscript{5}, topological properties\textsuperscript{6-8} and underwater acoustic applications \textsuperscript{9-11} have been explored, the multi-directional vibration control with extremely wide complete BG is also intriguing. Multiple approaches including single material\textsuperscript{12,13} and elastic impedance based multi-materials periodic structures by active\textsuperscript{14} and passive\textsuperscript{15-17} control techniques have been proposed to enlarge the BGs. Among those approaches the recently emerging 3D phononic structures with complete 3D BG\textsuperscript{18-20}, inertial amplification phenomena\textsuperscript{12}, actively controlled piezo-patches technique\textsuperscript{14,21,22}, elastic metamaterials with dissipative medium characteristics\textsuperscript{16} and multi-resonant trampoline metamaterials\textsuperscript{15} with trampoline effect\textsuperscript{23} have caught extensive attentions. The relative bandwidth $\Delta \omega/\omega_c$ or gap to mid-gap ratio is among the quantitative measures to determine the performance and robustness of BGs, i.e. a wider BG attenuates wave energy over a wide range of frequency. The width of BG is expressed by $\Delta \omega/\omega_c = 2(\omega_t - \omega_b)/(\omega_t + \omega_b)$\textsuperscript{24,25} where $\omega_t$ and $\omega_b$ are the top and bottom bounding edge frequencies, respectively. Table 1 shows a brief summary for the widest BG reported to date.

Table 1: The widest BGs reported

| Reference           | Composition type | Working direction | $\Delta \omega/\omega_c$ |
|---------------------|------------------|-------------------|--------------------------|
| Barnhart\textsuperscript{16} | Multi-material    | 1D                | 58%                      |
| Zhou\textsuperscript{14}    | Multi-material    | 1D                | 155%                     |
| Muhammad\textsuperscript{15} | Multi-material    | 2D                | 92%-29%                  |
| Acar\textsuperscript{12}     | Single-material   | 2D                | 74%                      |
| Muhammad\textsuperscript{24} | Multi material    | 2D                | 150%                     |
| Lu\textsuperscript{26}       | Multi-material    | 3D                | 100%                     |
| D’Alessandro\textsuperscript{18} | Single material  | 3D                | 132%                     |
| D’Alessandro\textsuperscript{27} | Single material  | 3D                | 134.7%                   |
| D’Alessandro\textsuperscript{20} | Single material  | 3D                | 63.8%-74.1%              |

Although locally resonant multi-core structures induce wider BGs in those reported works, the width of BG largely depends upon the mass of resonator/scatterer and impedance mismatch. For a wider BG, a larger-sized resonator is required. Further, in the works as shown in Table
Recently, 3D periodic structures consisting of multi-core materials are also proposed to maximize the impedance mismatch in order to achieve wider BGs. In such an approach, multi-material based prototyping and/or adjusting the assembly phase of materials pose a significant challenge. For a single core structure, this milestone constitutes a significant advancement in terms of structure optimization to adjust the filling ratio between material regions and voids. In this regard, the present work proposes two phononic metastructure prototypes that consist of a single core structure and it is capable of inducing extremely wide BG in all three directions of the irreducible Brillouin zone. We adopt the principal of mode separation where the structural components consist of rigid masses and elastic beams/frames resembling the conventional monoatomic mass-spring system to design the periodic structures. We demonstrated the vibration mitigation capability from those structures both numerically and experimentally. For the details of theoretical framework, governing equations and design optimization, one can refer to the supplementary materials. In this study, first, an analytical model is developed to calculate the acoustic mode frequency responsible for opening the BG and then a finite element based numerical technique is implemented to obtain the band structures and frequency response spectra. By additive manufacturing technology, the 3D prototypes are printed and vibration tests are performed to validate the presence and effectiveness of ultrawide three-dimensional BGs.

Prototypes and modal comparison

The topology in Fig. 1(a) shows 3D optimized phononic metastructures that are designed in a single material in terms of $\Delta\omega/\omega_c$ for in-plane elastic wave propagation. In both prototypes, the unit cell topology is realized by an external frame assembly connected with cylindrical masses at the middle points. The optimized structures have the following internal properties: frame assembly thickness $w_h = 0.035a$, $w_k = 0.045a$ that is connected with a cylindrical mass of radius $r = 0.3a$ and height $h = 0.65r$ via a small cube of side length $l_c = 0.19a$ for Prototype 1 or an equivalent volume sphere with diameter $d$ for Prototype 2. One half of the cube/sphere is embedded inside the cylindrical mass and the other half is connected to the frame assembly. The cube/sphere is introduced for two reasons: (i) it provides strong support between the frame assembly and cylindrical masses; and (ii) it optimizes $\Delta\omega/\omega_c$ that gives a wider BG. Further, all geometric parameters are presented in terms of the lattice constant $a = 0.05\text{m}$. For numerical modelling and additive manufacturing, VeroWhite (Young
modulus $E = 1.6\text{GPa}$, mass density $\rho = 1174\text{ kg/m}^3$ and Poisson’s ratio $\nu = 0.33$ Stratasys Ltd) is used. For simplicity, the band structures are presented in the form of a general frequency $f$ and a normalized frequency $f_{nd} = f a/v$ where $v = \sqrt{E/\rho}$ is the longitudinal wave velocity. Subsequently dynamic mechanical test is performed to determine the material loss factor $\eta$ that is required for the investigation of effect of material damping on numerically obtained transmission curves, see supplementary information. We obtained the frequency response spectra by FE approach using COMSOL Multiphysics 5.4® and ANSYS 2020 R1®. For Prototype 1 the analytical model and FE results are compared in Table. 2. One can observe an excellent agreement between the two models.

![Fig. 1. Proposed prototypes for 3D phononic metastructures. (a) Schematic description for prototype 1-2 with 3D printed sample. (b) Monoatomic mass-spring chain along with simplified beam structure. (c) Vibration mode for the lower bounding edge of first BG by COMSOL Multiphysics 5.4® (left) and ANSYS 2020 R1® (right). The analytical and FEA results comparison is presented in Table. 2.](image)

|                     | Analytical Model | COMSOL Multiphysics® | ANSYS 2020 R1® |
|---------------------|------------------|-----------------------|----------------|
| Prototype 1         | 1279.2 Hz (0.05478) | 1247.2 Hz (0.05341)   | 1240 Hz (0.05312) |
The modal analysis by both FE codes shows a dominant mixed compressional-bending resonant mode that initiates BG where the cylindrical masses and cubes/spheres work as rigid masses while box-like frame assembly exploits the flexural stiffness of frame assembly when subjected to incident elastic waves. For better understanding, a monoatomic mass-spring chain is introduced as shown in Fig. 1(b). For details, see supplementary information. For a general monoatomic mass-spring chain, the acoustic mode frequency \( \omega \) responsible for opening the BG is \( \omega = 2\omega_0 \) and \( \omega_0 = \sqrt{k/m} \) is natural frequency of the system\(^1\). For present monoatomic chain the acoustic mode frequency is \( f_{nd} = 2/2\pi \sqrt{k/m} (a/v) \) that is listed in Table. 2. The parameter \( m \) incorporates the mass of cylinders and cube for Prototype 1 or sphere for Prototype 2 while \( k \) represents the longitudinal stiffness of supporting frame structure with \( k = k\gamma k_{ss} \). Here \( \kappa \) and \( \gamma \) are associated with the stiffness of two sets of beams and the summation of two orthogonal beam stiffnesses, respectively, that are connected with cylindrical masses via cubes/spheres. For the present symmetric frame assembly \( \kappa = 0.5 \) and \( \gamma = 2^{31} \). Furthermore, as shown in Fig. 1(b), for a single beam with the effective length \( L_{el} = L/3 - w_b/2 \) the stiffness is \( k_{ss} = 24EI/L_{el}^3 \) where \( I = \frac{1}{12}w_b w_h^3 \) is second moment of area and \( L_{el} \) is beam effective length. The results comparison for Prototype 1 is presented in Table 2 and there exists an error of 2.5-3% between numerical and analytical solutions for the opening edge of the first BG. For Prototype 2 the BG opening edge obtained from COMSOL Multiphysics® and ANSYS 2020 R1® are 929.24 Hz (0.03981) and 939.86 Hz (0.04026), respectively. The vibration modes obtained from COMSOL and ANSYS is shown in Fig. 1(c) to double check the accuracy. An excellent agreement can be observed in term of deformation mechanism and displacement field distribution. Further details can be found in the supplementary materials.

Results and discussion

For Prototype 1, the numerical band structure and BGs determined by COMSOL Multiphysics5.4® is shown in Fig. 2(a-b). The first BG is the widest with the relative bandwidth 160.2%. The vibration modes corresponding to the bounding BG edges are shown in Fig. 2(c). The vibration modes associated to BG opening and closing edges are designated with red and green stars, respectively. The single material 3D structure proposed here possesses the widest 3D BG with the capability of attenuating mechanical vibration and noise in all directions as compared to the reported works listed in Table. 1.
Fig. 2. Prototype 1 - numerical dispersion spectra: (a) complete band structure with normalized frequency; (b) the widest first BG with $\Delta \omega/\omega_c$ of 160.2%; (c) vibration modes corresponding to the lower and upper bounding edges of BGs.
Similarly, the band structure with BGs and vibration modes corresponding to the bounding BG edges for Prototype 2 are shown in Fig. 3 (a-c). It is noticed that replacing cube masses with spheres of an equivalent volume results in two wide BGs with $\Delta \omega_1/\omega_c = 157.6\%$ and $55\%$, respectively. Interestingly both BGs are very close and they are separated by some narrow passbands. The material damping/viscoelasticity effects will weaken this passband that eventually results in a broadband BG covering an extremely wide frequency range.
For both Prototype 1 and Prototype 2, the nature of the widest first BG is due to structural optimization. Significant contribution is made by the rigid masses and flexural stiffness of the frame assembly as indicated by the vibration modes. Further, the mechanical features of these modes (i.e. deformation mechanism of modal stiffness and modal masses) are identical at different points within a particular band in the irreducible Brillouin zone. However, with a change in the wave propagation direction, the deformation phase varies but the deformation mechanism remains identical. In addition, for both prototypes the modal analysis result also indicates that significant contribution is made by the complete unit cell for the lower bounding edge of the first BG. In other words, the rigid motion of lumped cylindrical-cube/sphere masses and flexural deformation of elastic frame assembly play an important role in the opening of the first BG. Since the complete unit cell structures are in vibration, thus we call it a global resonant mode. Also, due to involvement of complete unit cell structure, the effective mass increases and eigenmode is situated at relatively lower frequency region. The analytical model also validates this finding as shown in Fig. 1(b).

In contrast, for the closing bounding edge of the first widest BG as shown in Fig. 2(c) and Fig. 3(c), the rigid masses remain almost unaffected while the vibration mode is characterized by local deformation of the frame-like beam assembly. Here, we call it a local resonant mode where vibration is observed only in frame assembly. A comparison between these two opening and closing BG edges indicate the modal mass participation in the closing BG edge is much smaller than the opening counterpart mode while the flexural stiffness of the frame assembly has reminiscent effect. Thus, due to insufficient modal mass participation and flexural deformation of elastic frame assembly, the upper bounding BG edge is located at a significantly higher frequency compared to the BG opening mode. In summary, the structural optimization results in significant difference between the opening (global resonant mode) and closing (local resonant mode) of BG edges that gives ultrawide BG. In the present study, we intend to optimize the difference between global and local resonant modes to obtain an extremely wide BG. Thus, modal masses participation and flexural stiffness of supporting frame assembly in the proposed 3-D prototypes play an important role for the opening and closing of BG. A similar conclusion can be derived from the vibration modes for higher BGs where the
contribution of rigid masses and flexural stiffness of frame-assembly can be seen. Except the lower bounding edge of the first BG, all other BGs have an almost negligible effect from the modal masses hence they are positioned in a higher frequency region. For preliminary designs and optimizations, one can refer to the supplementary materials.

Frequency response spectrum

The band structure presented above is obtained from COMSOL Multiphysics® where the Floquet-Bloch periodicity condition is applied on all edges of cylindrical mass that made the structure infinitely periodic in the x-y-z directions. Some reported studies\textsuperscript{14,15,20,27,29} indicate one possible way to visualize the vibration mitigation capability from the proposed metastructures is to build a finite array of unit cells and to perform a frequency response study. In this regard, a 3x3x1 supercell structure is constructed. As shown in Fig. 4(a), a harmonic excitation force is applied at the left-edge and the response in the form of displacement is record at the right-edge. The input and output displacement fields are recorded with the help of probes and the transmission ratio $T = 20\log_{10}\left(\frac{u_{\text{out}}}{u_{\text{in}}}\right)$ is calculated.

![finite supercell with input (blue) and output (red) probes](image1)

![Finite supercell and response spectrum](image2)

**Fig. 4.** (a) Finite supercell with input (blue) and output (red) probes; (b-c) response spectrum for Prototype 1 and Prototype 2 obtained from COMSOL Multiphysics (red dashed line) and ANSYS workbench (black solid line).
The transmission obtained by FE numerical simulation presented in Fig. 4(b, c) correspond to the \( \Gamma - X \) direction of the irreducible Brillouin zone. Since BGs are uniformly distributed in all directions of the Brillouin zone, thus wave propagation in any direction leads to an identical response spectrum. Inside BG, one can physically visualize the vibration attenuation capability from the proposed prototypes. Further, as shown in Fig. 5, different probe positions on Prototype 2 are selected to investigate the influence of BG width and attenuation depth. The BG width remains persistent for all the cases while the attenuation depth varies. For probe 1 the attenuation is around -50dB while probe 2-3 being equidistance have identical attenuation depth of around -150dB. Similarly, probe 4-5 have attenuation depth of approximately -250dB. In conclusion, an increase in the number of unit cell robustly attenuate the propagating incident waves.

Fig. 5. Prototype 2: displacement fields recorded at various probe locations numbered from 1 to 5. The BG width is independent of probe location while the attenuation depth increases if the point probe is away from the excitation source.

The numerical transmission response is first compared by FEA simulations without considering material damping on the response spectrum. The dynamic mechanical analysis
(DMA) test is performed on VeroWhite specimen to determine the material loss factor $\eta$\textsuperscript{15} and this parameter is incorporated in the FEA codes to investigate the effect of material losses on the BGs. Further details about DMA testing is given in the supplementary materials. Thanks to additive manufacturing, by using 3D printer (Strata sys Ltd) both prototypes are constructed and a low amplitude vibration test is conducted to investigate the real time vibration attenuation characteristics. In Fig. 6(a), the experiment setup and details are presented\textsuperscript{25} while Fig. 6(b-c) compare the numerical and experiment transmission spectra for both prototypes. An excellent agreement between the results is obtained. One can observe identical attenuation bandwidths however due to accelerometers limited precision compare to both FEA codes, there is a discrepancy between numerical and experiment attenuation depth. Other possible reasons for minor discrepancy between numerical and experimental results are (1) manufacturing issues \textsuperscript{32,33} and anisotropic feature of the samples (2) back-reflection of the incident waves from the sample boundaries that may have obscure the results. Further, it is observed the presence of $\eta$ on numerical transmission curve merges the first widest BG with other higher frequency BGs that results in a broadband vibration attenuation zone. This zone is, in fact, much wider than the multi-resonant structures reported previously\textsuperscript{15,16}.
Fig. 6. (a) Experiment setup; (b- c) Prototype 1 and Prototype 2 experiment and numerical result. For numerical transmission curve, the effect of material loss factor is $\eta = 0.06$. Excellent agreement between numerical and experiment result is obtained.

**Experiment setup**

The experiment setup is illustrated in Fig. 6(a). A vibration excitor (Brüel & Kjær type 4809 vibration excitor) is used as an actuator to transmit sine waves of varying frequencies. The sine waves are generated by NI PCI 6251 and are amplified with a Brüel & Kjær type 2706 power amplifier. A sine-sweep vibration testing approach is adopted where sine waves are swept from 100Hz-20,000Hz. By drilling small holes at both ends of the sample, the vibration excitor nob and two accelerometers (Kistler 8774 A50 with sensitivity 100 mV/g) are mounted.
The input and output acceleration data are acquired by data acquisition module NI USB 9162 and 24 bit NI 9234. The result obtained is postprocessed by a computer system that is connected with data acquisition module and built-in LabView program. The response spectrum is calculated by $T(\text{dB}) = 20 \log_{10}(a_{\text{out}}/a_{\text{in}})$ where $a_{\text{out}}$, $a_{\text{in}}$ are the output and input acceleration quantities obtained from the output and input accelerometers, respectively.

Method

A monoatomic mass-spring chain is developed to calculate the acoustic mode frequency and compare it with numerically obtained BG opening frequency. Numerical simulation is conducted by two different FEA codes, COMSOL Multiphysics 5.4® and ANSYS R1 2020® to double check the accuracy. The band structures are obtained from COMSOL Multiphysics due to the flexibility in applying Floquet-Bloch periodicity condition. Subsequently a frequency response study is performed by both FEA codes to visualize the vibration attenuation capability for three-dimensional BG frequencies. Both FEA codes yield very agreeable response spectra. The 3D prototypes are developed by using 3D printing technology (OBJECT60 Strata Sys Ltd) and vibration test is conducted to validate the numerical findings. Throughout the study, an excellent agreement between theoretical, numerical and experimental results is reported.

Conclusion

This study proposes two optimized phononic metastructures prototypes that govern extremely wide bandgap for vibration and noise filtration and attenuation. The study is conducted by a finite element approach and the numerical solutions are validated by experiments. Initially, an analytical model based on monoatomic mass-spring chain is established to study the bandgap opening and later numerical simulation is performed to obtain the band structure and global and local vibration modes. Both results are in excellent agreement. The band structure of proposed prototypes witnesses the presence of an extremely wide bandgap that is distributed over a wide frequency range. A global mode participation by rigid masses and flexural stiffness of beams results in the opening of bandgap. The bandgap is closed by local mode participation caused by flexural stiffness of the frame assembly while rigid cylindrical masses have no significant contribution. The significant differences between these opening and closing eigenmodes caused by proper engineering design result in ultrawide bandgaps. Further, a supercell structure is created and by conducting a frequency response
study, transmission curves are obtained that confirm the vibration attenuation inside the bandgaps. By additive manufacturing, 3D prototypes are printed and vibration tests are performed to further corroborate our findings. Both numerical simulation and experimental results showed excellent agreement. In the light of this study, the proposed designs can have potential applications in vibration absorption facilities to attenuate noises and vibrations at a wide frequency spectrum. The usage of a single material and simple structural designs make the fabrication and manufacturing works easy. This study may contribute in the design of novel metadevices for elastic and acoustic waves manipulation, and underwater acoustic applications where ultrawide bandgap and wave attenuation is desirable.

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**Supplementary materials**

The supplementary material is available at (URL).

**Declaration of Competing Interest**

The authors declare no competing financial interest or personal relationship that could appear to influence the work reported in this paper.

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