Electromagnetic stress on nucleon structure

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I. INTRODUCTION

Three forces influence the composition, structure, and stability of protons and neutrons (nucleons): The strong force, binding the nucleus together, the weak force, hold responsible for rare weak decays, and the electromagnetic force, typically playing a secondary role. Nucleons contain different types of particles, in particular quarks carrying charges of all three forces, and gluons propagating the strong force only.

Pictorially, the electromagnetically interacting quarks would swim in a paste of strongly interacting gluons. Even though the electrical charges of some quarks repel each other ($\sim E_{\text{int}}$), everything is held together by the dominant forces of the gluons. However, if one would expose this system for example to an external electric field $E_{\text{ext}}$, the quarks would feel this effect directly, while the gluons would feel it only indirectly, through strong interactions with those quarks.

This difference in responding to external electromagnetic forces will necessarily tend to change the spatial composition and even form of the nucleon. The amount of change in internal structure $\delta$, normalized to $|E_{\text{ext}}|/|E_{\text{int}}|$, can be expected to be in its order of magnitude quantified by the ratio between the electromagnetic coupling $\alpha_e$ and the strong coupling $\alpha_s$. Both couplings are scale dependent, in particular, $\alpha_s = \alpha_s(Q^2)$ can change significantly. Therefore, the ratio $\delta$ inherits the $Q^2$ dependence from the couplings

$$\delta(Q^2) \approx \frac{\alpha_e(Q^2)}{\alpha_s(Q^2)}.$$  

Of course, for very high energy processes ($Q^2 \to \infty$), it is well known that for $\alpha_s(Q^2) \to 0$ and thus $\delta \to \infty$. This means that small external electromagnetic fields can change the form of a nucleon completely. This effect is famously known as asymptotic freedom [1, 2]. If, to the contrary, the strong coupling totally dominates the electromagnetic coupling, all corrections to the form of the nucleon should vanish $\delta \to 0$, even for strong external electromagnetic fields. However, is this actually the case for small $Q^2$? At very low $Q^2$ one expects $\alpha_s(0) \approx 1/\pi\approx 0.137$, thus

$$\delta(0) \approx \frac{1}{\pi \cdot 137} \approx 0.002,$$

as an order of magnitude estimate. Experimental results in nuclear physics have reached remarkable precision, comparable to [2]. Thus, it should be feasible to look for corrections induced by external fields proportional to $Q^2$.

In this article, the testability of changes in the nucleon structure caused by external forces like $F_{\text{ext}} \approx qE_{\text{ext}}$ will be explored. In the following section II, deep inelastic scattering will be discussed and a novel application to the puzzling EMC effect (named after the “European Muon Collaboration”) will be presented. Finally, in section III, a summary of the results is given.

II. DEEP INELASTIC SCATTERING

A manifestation of the electromagnetic stress on the nucleon structure can be found in the context of deep inelastic scattering.

A. General discussion

The working hypothesis is that the neutron and proton structure is sensible to the force $\vec{F}$ produced by external electromagnetic fields in the rest frame. If the structure gets modified in one frame it will get modified in any other frame as well. Thus, also the parton distribution functions $f_q$ summed over the parton content $f = \sum_q f_q$ in the presence of an average external electric $F_E \sim E_q$ and magnetic $F_B \sim \nabla(\mu B)$ forces could be written as

$$f = f(x, \vec{F}_E, \vec{F}_B) \approx f(x, 0) (1 + (F_E + gF_B + s)d(x)),$$

Please note that the parton model itself is defined for a highly boosted light cone frame but the dependence on external electromagnetic fields is for convenience calculated in the rest frame of the nucleon. Lorentz contraction might lead to different modifications in different directions, when going from one frame to another. Thus one has to take the force averaged over the nucleus $\vec{F}$ instead of the local directed force $\vec{F}$. The correction due to the external fields $F_E, F_B$ is proportional to $d(x)$ which

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for small variations around a small $x_0$ can be approximated to $d(x) \approx d \cdot (x - x_0)$. The constant $s$ summarizes other short range and surface interactions. Here, $d, g, a, s, d$ are the proportionality constants in this linear expansion that have to be determined experimentally. At leading order this modification will be reflected one to one if one compares the deep inelastic $x$-dependent cross sections of a neutron (proton) with and without external electromagnetic fields and surface interactions. One finds

$$\frac{\sigma(x)F_E, F_B}{\sigma(x)_0} \sim \frac{f(x, F_E, F_B)}{f(x, 0)} \approx (1 + F_E a + (F_E + g F_B + s) d \cdot (x - x_0)),$$

where $a, g, s, d$ are proportionality constants. Thus, the $x$ derivative of this ratio is

$$\frac{d}{dx} \frac{\sigma(x)F_E, F_B}{\sigma(x)_0} \approx F_E d + F_B g d + d s.$$  

(5)

### B. EMC effect

In the most simple version of an atomic nucleus model, one would expect that the deep inelastic scattering (DIS) cross section is given by the cross section of a neutron (proton) with and without external electromagnetic fields and surface interactions. One finds

$$\sigma^{A,Q}(x) = A \sigma^{1,1}(x).$$  

(6)

Thus, the DIS data for heavy nuclei should be predictable from the DIS data obtained from lighter nuclei and vice versa. In particular, the ratio

$$\Delta_{EMC}^{A,Q} = \frac{\sigma^{A,Q}(x)}{A \cdot \sigma^{1,1}(x)}$$

was expected to be constant (one) and not a function of Bjorken $x$. The EMC effect, is the famous observation that $\Delta_{EMC}^{A,Q} = \Delta_{EMC}^{A,Q}(x)$. This observation has triggered numerous experimental tests and theoretical explanations (for a review see, for example [5]). In particular, it was noted that the size of the EMC effect depends on the charge $Q$ of the atomic nuclei [6]. Below, this effect will be studied in the light of our hypothesis of electromagnetic stress acting on nucleons, a perspective which we have not found in the literature.

Let’s return to the most simple version of an atomic nucleus model but allowing for contributions of electromagnetic stress. Larger atomic nuclei will have larger charge $Q$ and thus accumulate larger average electric fields $\bar{E}(Q)$ of the surrounding protons. Thus, one has to correct the relation [6] by

$$\sigma^{A,Q}(x) = A \sigma_F(Q)(x).$$

(8)

In this straightforward model, one can compare the normalized DIS $x$-dependent cross section of large nuclei with the DIS cross section of small nuclei

$$\Delta_F^{A,Q} = \frac{\sigma^{A,Q}(x)}{A \cdot \sigma^{1,1}(x)}$$

(9)

$$\approx (1 + (\bar{F}_E(Q) + g \bar{F}_B + s) d \cdot (x - x_0)).$$

where the expansion point $x_0$ was chosen to be the point for which this ratio crosses the value of one with a negative slope. For a given isotope this linear dependence reads $\sim a' + b' \cdot x$. It can be obtained by fitting the experimental data for [7], as shown in figure [8] for the case of an aluminium isotope.

![FIG. 1: $x$-dependence of the ratio](image)

This fit was repeated for the elements ($He, Be, C, Al, Ca, Fe, Ag, Au$), using data from [6] and the digitalization tool [7]. One finds for example that $x_0 = 0.27 \pm 0.02$.

Even more interesting information can be obtained from the $x$-slope, because it will allow relating to $\bar{F}_E = \bar{F}_E(Q)$. Deriving [9], with respect to $x$, gives

$$\frac{d}{dx} \Delta_F^{A,Q} \approx (\bar{F}_E(Q) + g \bar{F}_B + s)d.$$  

(10)

The magnetic field within the nucleus will be produced by the surrounding spin 1/2 protons and neutrons. However, those are oriented randomly [8] and thus one can expect the $Q$ dependence of $\bar{F}_B$ in [10] to be subdominant in comparison with $\bar{F}_E$. The same holds for the short-range contributions $s$. Let’s now estimate the average electric force as function of nucleon charge $\bar{F}_E \sim q \bar{E}(Q)$ by assuming a constant average charge density $\rho$. The total charge of such a spherical nucleus is

$$Q = \frac{4}{3} \pi R^3 \rho,$$

(11)

which can be solved for the nucleon radius. The radial electric field which is produced by the surrounding protons of a nucleon within the same nucleus is obtained

\[ E(r) = \frac{Q}{4\pi\varepsilon_0 r^2} \]
from Gauss law

$$E_r(r) = \frac{1}{\varepsilon_0} r \rho. \quad (12)$$

The average value of this field is

$$\bar{E} = \frac{1}{\varepsilon_0 4} \rho \left( \frac{3Q}{4 \pi x} \right)^{1/3} \sim Q^{1/3}. \quad (13)$$

Note that the field (13), produced by the neighboring protons, is the most dominant one. All other contributions, like the field produced by the eventually surrounding electrons or any other external field of the experimental apparatus can be neglected.

Inserting (13) into (10) one obtains a prediction of the charge dependence of the slope data

$$\frac{d}{dx} A \cdot \sigma^A(x) \approx g' + Q^{1/3} d' \quad (14)$$

The constant $g'$ is expected to originate for example from the largely $Q$ independent average magnetic field $F_B$, or other mean field and boundary effects.

The EMC-type ratio has been measured and fitted like in figure 1 for eight different atomic nuclei ($\text{He, Be, C, Al, Ca, Fe, Ag, Au}$), which allows to extract the observed data for the predicted charge dependence (14). As shown in figure 2, one gets a good match between (14) and the data obtained from the slope fitting like the one plotted in figure 1. Given the simplicity of the underlying idea and model, the good agreement between the model (14) and the data in figure 2 is remarkable. This is the main result of the paper.

The calculations and estimates can be improved in numerous ways: One can use more sophisticated and realistic models for heavy nuclei [9]. One can try to predict the numerical values of the proportionality constants in (4) with explicit nucleon models [10–14], which would have to be generalized to cases including external electromagnetic fields. For example one should promote the propagators to those including external electro-magnetic fields [15]. This would allow to explore whether it is just a coincidence that the order of magnitude estimate (1), which at a typical $Q^2$ for small $x$ gives $\delta(Q^2 = 5\text{GeV}^2) \approx 0.024$, is only by a factor of three away from the proportionality $d' = 0.07$ in figure 2. One should also try including further observables.

However, all those improvements go beyond the scope of this paper.

III. SUMMARY

This short article explored the impact of electromagnetic stress on the nucleon structure. As a possible manifestation of this effect we studied the dependence of parton distribution functions on external electromagnetic forces. Simple and straightforward assumptions led to a remarkably good description of the charge dependence of the EMC effect.

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[1] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973). doi:10.1103/PhysRevLett.30.1343

[2] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973). doi:10.1103/PhysRevLett.30.1346

[3] A. Deur, doi:10.1142/9789812774132 0007 hep-ph/0509188

[4] J. J. Aubert *et al.* [European Muon Collaboration], Phys. Lett. **123B**, 275 (1983). doi:10.1016/0370-2693(83)90437-9

[5] D. F. Geesaman, K. Saito and A. W. Thomas, Ann. Rev. Nucl. Part. Sci. **45**, 337 (1995). doi:10.1146/annurev.ns.45.120195.002005

[6] J. Gomez *et al.*, Phys. Rev. D **49**, 4348 (1994). doi:10.1103/PhysRevD.49.4348 (see figure 14)

[7] Ankit Rohatgi. https://automeris.io/WebPlotDigitizer.

[8] B. Q. Ma, I. Schmidt and J. Soffer, Phys. Lett. B **441**, 461 (1998) doi:10.1016/S0370-2693(98)01158-7 [hep-ph/9710247].

[9] M. Baranger, Phys. Rev. **120**, no. 3, 957 (1960). doi:10.1103/PhysRev.120.957

[10] R. L. Jaffe and F. E. Low, Phys. Rev. D **19**, 2105 (1979). doi:10.1103/PhysRevD.19.2105

[11] G. E. Brown and M. Rho, Phys. Lett. **82B**, 177 (1979). doi:10.1016/0370-2693(79)90729-9

[12] S. J. Brodsky, T. Huang and G. P. Lepage, Springer Tracts Mod. Phys. **100**, 81 (1982).

[13] S. J. Brodsky, D. S. Hwang, B. Q. Ma and I. Schmidt, Nucl. Phys. B **593**, 311 (2001) doi:10.1016/S0550-3213(00)00626-X [hep-th/0003082].

[14] T. Gutsche, V. E. Lyubovitskij and I. Schmidt, Eur. Phys. J. C **77**, no. 2, 86 (2017) doi:10.1140/epjc/s10052-017-4648-5 [arXiv:1610.03526 [hep-ph]].

[15] A. Di Piazza, C. Müller, K. Z. Hatsagortsyan and C. H. Keitel, Rev. Mod. Phys. **84**, 1177 (2012) doi:10.1103/RevModPhys.84.1177 arXiv:1111.3886 [hep-ph].