Enhanced optomechanical entanglement and cooling via dissipation engineering

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We propose an optomechanical dissipation engineering scheme by introducing an ancillary mechanical mode with a large decay rate to control the density of states of the optical mode. The effective linewidth of the optical mode can be reduced or broadened, manifesting the dissipation engineering. To prove the ability of our scheme in improving the performances of the optomechanical system, we studied optomechanical entanglement and phonon cooling. It is demonstrated that the optomechanical entanglement overwhelmed by thermal phonon excitations could be restored via dissipation engineering. For the phonon cooling, an order of magnitude improvement could be achieved. Our scheme can be generalized to other systems with multiple bosonic modes, which is experimentally feasible with advances in materials and nanofabrication, including optical Fabry-Perot cavities, superconducting circuits, and nanobeam photonic crystals.

I. INTRODUCTION

In recent years, optomechanical systems [1–4] have attracted considerable attention in classical and quantum information processing. Dramatic theoretical and experimental progress has been achieved in the optomechanically induced transparency [5, 6], sensing [4], frequency combs [7], and tunable optical filters [8]. For potential applications in the quantum regime [1, 9–11], ground-state cooling of the mechanical resonator [12, 13], optomechanical entanglement and squeezing [14–16], and quantum sensors [17, 18] have been studied. Most researches have focused on canonical systems with only one optical and mechanical degree of freedom. For multimode optomechanical systems [19–22], more degrees of freedom can be controlled, and new phenomena are observed and new functionalities are realized, such as optomechanically induced nonreciprocity [23], circulators [24, 25], and coherent frequency converter [26, 27]. However, the interplay between different optomechanical interactions and different mechanical or optical modes are omitted in previous studies.

On the other hand, dissipation engineering [28–31] has been developed in quantum optics. By introducing dissipative modes into the quantum system, the density of states of the system can be efficiently controlled and thus suppresses unwanted physical processes or induces desired interactions. Recently, dissipation engineering has been widely used to generate steady entanglement in superconducting qubits [32, 33], trapped ions [34, 35], superconducting resonators [36], and Rydberg atoms [37]. Therefore, it is expected to introduce this beneficial idea to the optomechanical system. In practical, most optomechanical systems naturally support the multimodes [4, 38–40], and these modes can be excited by polychromatic laser drive [41]. With the advantages of multiple degrees in multimode optomechanical systems [42], dissipation engineering [43] holds huge potential in all-optical information processing [32, 44, 45], where two or more mechanical or optical modes lead to a wealth of different possible schemes.

In this paper, we theoretically analyze a multimode optomechanical system, where Brillouin phonon mode and breath mechanical mode are coupled with a common optical mode. The Brillouin phonon mode serves as an ancillary, which is used to engineer the optical density of state and thus realize the dissipation engineering. The effect of dissipation engineering is studied in different aspects, including optomechanically induced transparency (OMIT) and optomechanical induced amplification (OMIA), entanglement between the optical mode and breath mechanical mode, and cooling of the breath mechanical mode. The enhancement of the entanglement or cooling is obtained for certain dissipation engineering strength, and the numerical results reveal the potential of our scheme. On the other hand, the extra noises introduced by the ancillary mode and strong modification of the optical mode could also suppress the performance of an optomechanical system, suggesting an optimal dissipation engineering that balances the positive and negative effects. Our scheme could be generalized to other nonlinear interactions in multiple mode systems, such as atom ensemble and Fabry-Perot (FP) cavity system [46] and superconducting circuits [47], and finds applications in quantum devices [48].

II. THE SYSTEM

We consider a multimode optomechanical system [25, 49, 50], and its schematic is shown in Fig. 1(a), where two mechanical modes \((b, m)\) are coupled with a common optical mode \(a\). The interaction Hamiltonian can be described by \((\hbar = 1)\):

\[
H_{\text{int}} = G_b b (a^\dagger + a) + G_m a (m^\dagger + m) + \text{H.c.},
\]

where \(G_{b(m)}\) is the pump laser stimulated effective coupling between the modes. Here, we focus on the modes \(a, m\), and the mode \(b\) is treated as an ancillary mode, which is encircled with a dashed line. By the interaction \(G_b\), we can modulate the mode \(a\), which further affects the effective coupling between
where the coherent couplings between the optical mode and mechanical system based on the on-chip optical microresonator, \( \text{tonic crystals} \) \[54\].

In the following, we focus on an experimentally optomechanical system based on the on-chip optical microresonator, where the coherent couplings between the optical mode and the Brillouin phonon mode \((\sim 10\text{GHz}) \) \[55\], breath mechanical mode \((\sim 100\text{MHz}) \) \[23\] can be stimulated simultaneously by external laser pumps. The full system can be described by the following Hamiltonian

\[ H_{\text{sys}} = H_0 + H_{\text{SBS}} + H_{\text{BM}} + H_D, \tag{2} \]

where

\[ H_0 = \omega_j a_j^{\dagger} a_j + \omega_k a_k^{\dagger} a_k + \omega_b b_j^{\dagger} b_{j-k} + \omega_m m^{\dagger} m, \tag{3} \]

\[ H_{\text{SBS}} = g_b \left( a_j^{\dagger} a_k b_{j-k} + a_k^{\dagger} a_j b_{j-k} \right), \tag{4} \]

\[ H_{\text{BM}} = g_{m,j} a_j^{\dagger} a_j (m + m^{\dagger}) + g_{m,k} a_k^{\dagger} a_k (m + m^{\dagger}), \tag{5} \]

\[ H_D = i \sqrt{\kappa_{j,ex}} a_j^{\dagger} e^{-i\omega_j t} - a_j e^{i\omega_j t} + i \sqrt{\kappa_{k,ex}} a_k^{\dagger} e^{-i\omega_k t} - a_k e^{i\omega_k t}. \tag{6} \]

Here \( H_0 \) describes four eigenmodes of the system, including two optical modes \((a_j, a_k)\) and two mechanical modes \((b_{j-k}, m)\); \( H_{\text{SBS}} \) describes the triple-resonant stimulated Brillouin scattering between the high-frequency traveling phonon mode and two optical modes, and \( H_{\text{BM}} \) is the dispersive optomechanical coupling between the breath mechanical mode and optical mode; \( H_D \) represents the external laser driving onto the optical modes. \( a_{j,k} \) is the annihilation operator for optical mode \(j,k\) with the frequency \(\omega_{j,k}\), \(b_{j,k}\) is the annihilation operator of the Brillouin mechanical mode with the frequency \(\omega_m\), \(m\) is the annihilation operator of the breath mechanical mode with the frequency \(\omega_m\), \(g_b\) is the triple-resonant coupling strength of the stimulated Brillouin scattering, \(g_{m,j}\) is the dispersive coupling rate, \(\kappa_{j,k}\) is the driving field of the optical mode \(a_{j,k}\) with a frequency \(\omega_{j,k}\), and \(\kappa_{j,k}\) is the external coupling rate. For the stimulated Brillouin scattering, we need both the energy and momentum conservation, so \(\omega_j = \omega_k + \omega_m\) is also satisfied.

For a very weak coupling rate \(g_b\), \(g_j, g_k \ll \omega_b, \omega_m\), \(\kappa_{j,k}\), where \(\kappa_{j,k} = \kappa_{j,k} + \kappa_{j,k,ex}\) is the total optical energy decay rate, the optomechanical interactions can be enhanced by the control laser \(g_{d,j,k}\). When the duration of the control pulse \(\tau_d \gg 1/\kappa_{j,k}\), the intracavity control fields can be treated classically as:

\[ \alpha_{j,k} = \frac{\sqrt{\kappa_{j,k,ex}} g_{d,j,k}}{\kappa_{j,k}/2 + i\Delta_{j,k}}, \tag{7} \]

where \(\Delta_{j,k} = \omega_{j,k} - \omega_{d,j,k}\). In the interaction picture \(H_0 = \omega_d a_j^{\dagger} a_j + \omega_d a_k^{\dagger} a_k + (\omega_d - \omega_{d,k}) b_{j-k}^{\dagger} b_{j-k} + \omega_m m^{\dagger} m + G_{b,j} b_{j-k}^{\dagger} b_{j-k} + H.c. + \left( G_{m,j} a_j^{\dagger} + G_{m,k} a_k^{\dagger} + H.c. \right) (m^{\dagger} m), \tag{8} \]

where \(G_{b,j} = \omega_b + \omega_{d,k} - \omega_{d,j}, G_{b,j,k} = g_b \alpha_{j,k}, \) and \(G_{m,j,k} = g_{m,j,k}\).

### III. DISSIPATION ENGINEERING

Firstly, we only consider the stimulated Brillouin scattering, as the mechanical mode \(b_{j-k}\) is used as an ancillary to engineer the optical mode. For the stimulated Brillouin scattering, the triple-resonant condition is required \[41, 55\]. For simplification, we assume that one driving laser is on-resonant with the optical mode, i.e. \(\Delta_j = 0\) or \(\Delta_k = 0\). By treating the laser pump on \(a_j\) as a classical field, we can write the effective Hamiltonian as \(H_{\text{SBS}} = G_{b,j} a_j^{\dagger} b_{j-k}^{\dagger} + G_{b,j} a_k b_{j-k}\). Such a stimulated Brillouin scattering is a Stokes process, which generates photon-phonon pairs into the system, thus eventually induces gain to the system and reduces the linewidth of the optical mode \(a_k\).

To observe the effect of dissipation engineering on the optical linewidth, a weak probe laser is sent into the optical mode.
\(a_k\), and the Langevin equations can be written as follows
\[
\frac{d}{dt} a_k = -\left(\frac{\kappa_k}{2} - i\delta\right) a_k - iG_{b,j}b^\dagger_{j-k} + \sqrt{\kappa_{ex}} \epsilon_{p,k},
\]
\[
\frac{d}{dt} b^\dagger_{j-k} = -\left(\frac{\gamma_b}{2} - i\delta\right) b^\dagger_{j-k} + iG_{b,j}^* a_k,
\]
where \(\delta = \omega_p - \omega_b\), \(\gamma_b\) is the Brillouin mechanical decay rate, and \(\epsilon_{p,k}\) is the probe laser with the frequency \(\omega_p\). When the system is at the steady state, which requires that the optomechanical coupling is below the lasing threshold \([56]\), we have
\(a_k(\delta) = \sqrt{\kappa_{ex}} \epsilon_{p,k} / (\kappa_{eff}/2 - i\delta)\), where the effective dissipation rate and detuning of the optical mode can be written as
\[
\kappa_{eff} = \kappa_k - \left|G_{b,j}\right|^2 \gamma_b / (\frac{\gamma_b}{2} + \delta^2),
\]
\[
\delta_{eff} = \delta + \left|G_{b,j}\right|^2 \delta / (\frac{\gamma_b}{2} + \delta^2).
\]
If these parameters satisfy \(\delta, \left|G_{b,j}\right|, \kappa_k \ll \gamma_b\), we can obtain the effective decay rate \(\kappa_{eff} \approx \kappa_k - 4 \left|G_{b,j}\right|^2 / \gamma_b\) and the effective detuning \(\delta_{eff} \approx \delta\).

If we consider the driving laser resonant with the optical mode \(a_k\), the stimulated Brillouin scattering is an anti-Stokes process, and the effective Hamiltonian can be written as
\(H_{SB} = G_{b,k} a^\dagger_{k-j} + G_{b,j}^* a b^\dagger_{j-k}\). As opposed to the Stokes process, the particle number conserves and the coupling would introduce extra loss channel to the optical mode. Similarly, then the effective decay rate is derived as \(\kappa_{eff} \approx \kappa_j + 4 \left|G_{b,k}\right|^2 / \gamma_b\), which confirms that linewidth of the optical mode \(a_j\) is broadened due to \(b\). In practical experiments, we can detect the intracavity power to show the variation of the optical mode linewidth. For the sake of illustration, we can normalize the intracavity power as
\[
P_a(\delta) = \frac{\kappa_{eff}}{2\sqrt{\kappa_{ex}} \epsilon_p} a^2,
\]
where we have omitted the subscript of some parameters for convenience.

In Fig. 2(a), we plot the intracavity power \(P_a\) as a function of the detuning \(\delta\). The blue and red lines correspond to the case that the control laser is added into the optical mode \(a_j\) and \(a_k\), respectively, and the black line is the result without the control laser \((G_b = 0\) for comparison). The dashed and dotted lines are corresponding to the effective coupling \(G_b/(2\pi) = 2\) and \(3\) MHz, respectively. When \(\Delta_j = 0\), the blue lines show that the effective linewidth of the optical mode is reduced with the increasing of the control powers, and the intracavity power spectra agree with the theoretical result \(\kappa_{eff} = \kappa_j - 4 \left|G_{b,j}\right|^2 / \gamma_b\). For \(\Delta_k = 0\), the related results are plotted by the red lines, which show that the broadened linewidth of the optical mode as \(\kappa_{eff} = \kappa_j + 4 \left|G_{b,k}\right|^2 / \gamma_b\). Based on the analytical derivations and numerical calculation above, we conclude that the effective linewidth of the optical mode can be effectively reduced or broadened by the dissipation engineering.

![FIG. 2](Color online) Intracavity power \(P_a\) of the optical mode \(a\).

(a) Engineering the linewidth of the optical mode via only stimulated Brillouin scattering. The intracavity power spectra for different control powers \(G_{b,j}/(2\pi) = 0\) (solid line), 2 (dashed line), and 3 (dotted line) MHz, where the blue and red lines are corresponding to \(\Delta_j = 0\) and \(\Delta_k = 0\), respectively. (b) Optomechanical induced transparency and amplification are affected by the dissipation engineering, and (c) is the enlarged view of the (b) around the detuning \(\delta/(2\pi) \in [-0.01, 0.01]\) MHz. The intracavity power spectra for the red detuning \(\Delta_j(k)/2\pi = \pm 0.03\) MHz, where the blue and red arrows are corresponding to \(\Delta_j = 0\) (Stokes process) and \(\Delta_k = 0\) (anti-Stokes process), respectively. Other parameters are \(\kappa_j/2\pi = 2\) MHz, \(\kappa_j/2\pi = 2\) MHz, \(\kappa_j/2\pi = 40\) MHz, \(\kappa_\epsilon/2\pi = 10\) GHz, \(\gamma_{m}/2\pi = 10\) kHz and \(\omega_m/2\pi = 100\) MHz.

Now we consider the breath mechanical mode \(m\) for the radiation pressure optomechanical coupling, where the OMIT or OMIA can be observed due to another drive laser that induces coherent photon-phonon interaction. From Fig. 2(a), we know that the effective linewidth of the optical mode can be modulated by the stimulated Brillouin scattering, which indicates that the OMIT and OMIA can be effectively controlled by the dissipation engineering. For the radiation pressure optomechanical coupling, the drive can be either red or blue detuning \(\Delta_m(k) = \pm \omega_m\). Combining the stimulated Brillouin scattering and optomechanical couplings, we can divide possible experimental configurations into four cases: (a) anti-Stokes process and red detuning: \(\Delta_j = 0\), \(\Delta_j = \omega_m\); (b) anti-Stokes process and blue detuning: \(\Delta_k = 0\), \(\Delta_k = -\omega_m\); (c) Stokes process and red detuning: \(\Delta_j = 0\), \(\Delta_k = \omega_m\); (d) Stokes process and blue detuning: \(\Delta_j = 0\), \(\Delta_k = -\omega_m\). To illustrate different situations intuitively, we add a weak laser to probe the optical mode \(a\). Firstly, we consider the case (a) and the corresponding Hamiltonian of the system can be written as
\[
H = -\delta a^\dagger a - \delta b^\dagger b - \delta m^\dagger m + \left(G_{b,j}^* b + G_{m,a}^* m + \text{H.C.}\right) + i\sqrt{\kappa_{ex}} \epsilon_p \left(a^\dagger - a\right),
\]
where \(\delta = \omega_p - \omega_j\). We have omitted the subscript of some parameters for convenience and neglected other terms with the rotating wave approximation \(\omega_m \gg \kappa_m(k)\), \(\gamma_m\), where \(\gamma_m\) is the
breath mechanical decay rate. The steady state solution reads
\[ a(\delta) = \frac{\sqrt{\kappa_{\text{ex}}\varepsilon_p}}{\kappa/2 - i\delta + \frac{G_b^2}{\gamma_b/2 - i\delta} + \frac{G_m^2}{\gamma_m/2 - i\delta}}. \]  

For the convenience, we have assumed that \(G_b\) and \(G_m\) are real numbers. When we consider other cases (b,c,d), the steady state solution has a similar form to the above equation and we only change the couplings: \(G_b^2 \to -G_b^2\) for the Stokes process; \(G_m^2 \to -G_m^2\) for the blue detuning.

In Figs. 2(b) and (c), we discuss the dissipation engineering on both the OMIT and OMIA, and we plot the normalized intracavity power \(P_a\) as the function of the detuning between the probe laser and the optical cavity. When the probe drive is largely detuned from the mechanical resonance, the effect due to radiation pressure optomechanical coupling is almost not observable, where the Fig. 2(b) is almost the same as Fig. 2(a). Figure 2(c) is the magnification of Fig. 2(b) around the detuning \(\delta / (2\pi) \in [-0.01, 0.01]\) MHz. The arrows mean that the OMIT and OMIA is enhanced (the blue arrow) or suppressed (the red arrow) with the increasing of dissipation engineering \((G_b)\). Here the parameters that we choose satisfy \(\gamma_b / \kappa_{b(j)}\), \(\gamma_m\), and we only consider the detuning \(|\delta| \ll \gamma_b\). These phenomena can be described by the approximate emission power spectra
\[ P_a \approx \frac{1}{1 - 2i\delta / \kappa_{\text{eff}} \pm \frac{c}{2\delta a_{\gamma_m}}}. \]

where the cooperativity \(C = 4|G_m|^2 / (\kappa_{\text{eff}}\gamma_m)\) and \(\pm\) for the blue and red detuning. For a fixed \(G_m\), it is obvious that the OMIT and OMIA are controlled by the effective linewidth of the optical mode \(\kappa_{\text{eff}} = \kappa / 4|G_b|^2 / \gamma_b\).

From Fig. 2, it is demonstrated that the dissipation engineering could modulate the linewidth of optical mode, which can be used to control the OMIT and OMIA. Especially, it can also make the unresolved sideband in the optomechanical system, i.e. \(\omega_b \ll \kappa_{\text{eff}}\), to be resolvable by reducing the linewidth of the optical mode.

### IV. ENTANGLEMENT AND COOLING

Next, we discuss the effect of the dissipation engineering on the entanglement of radiation pressure optomechanical system and cooling of the mechanical mode \(m\). It is known that the thermal noise is bad for the entanglement [50] and cooling, so we only consider the anti-Stokes process as the dissipation engineering to cool the system. For the stimulated Brillouin scattering, we consider the driving laser is resonant with the optical mode \(a_1\). The Hamiltonian of the system can be written as
\[ H = \Delta_j a_j^\dagger a_j + \Delta_b b_j^\dagger b_j - \omega_m m^\dagger m + G_b b_j^\dagger (m^\dagger + m) + H.C. \]

We define \(R = (q_0, p_0, q_b, p_b, q_m, p_m)^T\), where \(q_0 = (\alpha^\dagger + \alpha) / \sqrt{2}\) and \(p_0 = i (\alpha^\dagger - \alpha) / \sqrt{2}\). The Langevin equations can be written as
\[ \frac{d}{dt}R = MR + R_m, \]

where
\[ M = \begin{pmatrix} -\kappa/2 & \Delta_j & 0 & G_{b,k} & 0 & 0 \\ -\Delta_j & -\kappa/2 & -G_{b,k} & 0 & -2G_{m,j} & 0 \\ 0 & G_{b,k} & -\frac{\kappa}{2} & \Delta_b & 0 & 0 \\ -G_{b,k} & 0 & -\Delta_b & -\frac{\kappa}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & \omega_m \\ -2G_{m,j} & 0 & 0 & 0 & 0 & -\omega_m - \frac{\kappa}{2} \end{pmatrix}, \]

and the thermal noise can be written as \(R_m = (\sqrt{\kappa_{a_0}^m, \kappa_{p_0}^m, \kappa_{p_0}^m, \sqrt{\kappa_{b_0}^m, \sqrt{\kappa_{b_0}^m, \sqrt{\kappa_{m}^m, \sqrt{\kappa_{m}^m}}}^m})^T\). Here we have assumed that \(G_{b,k}\) and \(G_{m,j}\) are the real numbers. The fact that the dynamics of the system is governed by a linearized Hamiltonian ensures that the evolved states are Gaussian states whose information-related properties [57] are fully represented by the \(6 \times 6\) covariance matrix with entries defined as
\[ V_{i,j} = \langle R_{i} R_j^\dagger + R_j R_i^\dagger / 2 \rangle. \]

The equation of motion corresponding to the covariance matrix can be written as follows
\[ \frac{d}{dt}V = MV + VM^T + D, \]

where \(D = \text{Diag} \left( \frac{\kappa}{2}, \frac{\kappa}{2}, \frac{\kappa}{2}, \frac{\kappa}{2}, \frac{\eta_{m(2n_m+1)}}{2}, \frac{\eta_{m(2n_m+1)}}{2} \right)\). For \(\omega_0 \gg \omega_m\), we have neglected the thermal noise of the mode \(b\).

#### A. Entanglement

The entanglement between optical mode and mechanical mode is generated by the interaction term \(G_m a_1^\dagger m^\dagger + \text{H.c.}\), so we choose the detuning \(\Delta_j = \Delta_b = -\omega_m\). The continuous variable entanglement of two modes can be calculated by the logarithmic negativity \(E_N\) [58]. This quantity is a rigorous entanglement monotone, and is zero for separable states. For the two-mode Gaussian states, it can be calculated using the expression [49]
\[ E_N = \max \{0, -\ln(2\eta)\}, \]

where
\[ \eta = \frac{1}{\sqrt{2}} \sqrt{\Sigma - \sqrt{\Sigma^2 - 4\det V}}, \]

and \(\Sigma = \det V_1 + \det V_2 - \det V_3\). The matrix \(V_1, V_2\) and \(V_3\) are \(2 \times 2\) matrices related to the covariance matrix as
\[ V = \begin{pmatrix} V_1 & V_3 \\ V_3^T & V_2 \end{pmatrix}. \]
Here we only concern the entanglement of radiation pressure optomechanical system, so the covariance matrix $V$ is about the modes $a, m$. In Fig. 3, we study the effect of dissipation engineering on the entanglement. The dynamical logarithmic negativity $E_{am}$ with dissipation engineering is shown in Fig. 3(a). The black dashed line is the result without the dissipation engineering, and the $E_{am}$ reaches a maximum at an appropriate evolution time. When the system approaches the steady-state, the entanglement disappears with $E_{am} = 0$ for the thermal noise $n_m$. The solid lines show the dissipation engineering on the entanglement, where $G_{b,k}/(2\pi) = 2$ (blue line) and 4 (red line) MHz, which mean that the steady logarithmic negativity $E_{am}$ becomes bigger with the increasing of the dissipation engineering $G_{b,k}$. However, for stronger dissipation engineering $G_{b,k}/(2\pi) = 8$ (green line) MHz, it will reduce the steady logarithmic negativity, which means that there is the optimal engineering for the logarithmic negativity.

In Fig. 3(b), we plot the optimal engineering for different couplings $G_{m,j}/(2\pi) = 0.03$ (solid line), 0.04 (dashed line), 0.05 (dotted line) MHz, and the steady logarithmic negativity $E_{am}$ has a peak. Because of the thermal noise $n_m$, there is a phase transition point for the entanglement growing out of nothing. For more thermal noise $n_m = 300$, the Figs. 3(c) and (d) show that the logarithmic negativity $E_{am}$ becomes smaller and it needs more dissipation engineering to recover the entanglement.

For dissipation engineering, we also consider the effect of the detuning $\Delta_b$ on the stationary intracavity entanglement, which is shown in Fig. 4(a). Near the resonance point $\Delta_b = -\omega_m$, we obtain the maximal $E_{am}$, which is due to the most effective coupling at the resonance point. With the increases of the coupling $G_{m,j}$, we can always find an optimal $G_{b,k}$ for the stationary intracavity entanglement, as shown in Fig. 4(b). However, for the large $G_{m,j}$, the eigenvalues of $M$ have a positive real part, which makes the system unstable. We can predicate stability conditions by the well-known Routh-Hurwitz criteria [59], which is labeled by the blank in Fig. 4(b). The maximal $E_{am}$ is obtained at the edge of the unstable region, and the stable entanglement becomes weaker for the larger $G_{m,j}$.

From Figs. 3 and 4, we conclude that the noise-overnorled steady entanglement between the optical mode and mechanical mode can be restored by the dissipation engineering, which is robust to the thermal noise of the mechanical mode. It is due to the anti-Stokes process from the stimulated Brillouin scattering to cool the optomechanical system. For the strong dissipation engineering, the entanglement abates for the strong nonlinearity.

**B. Cooling**

The dissipation engineering can be also used to enhance the cooling of the mechanical mode $m$. From the covariance matrix, the final steady thermal occupation of the mode $m$ is calculated by

$$\langle m^\dagger m \rangle = n_f = \frac{V_{a,5} + V_{b,6} - 1}{2}. \quad (25)$$

The cooling is from the interaction term $G_{m,j}a_j^\dagger m + H.c. [4]$, so we choose the detuning $\Delta_j = \Delta_b = \omega_m$. In Fig. 5, we discuss the cooling effect of dissipation engineering. We plot the thermal occupation $n_f$ as a function of $G_{m,j}$ in Fig. 5(a). The black dashed line is the result without the dissipation engineering,
For the weak coupling, the dissipation engineering still has a bad effect on the cooling, which is observed by the black dashed line in Fig. 5(b). The blue and red lines show that there is a minimal value of $n_f$ by the dissipation engineering for the strong coupling. Taking the coupling $G_{m,j}$ and the dissipation engineering $G_{b,k}$ into consideration, we plot the Fig. 5(c). It is obviously shown that it can improve the cooling effect by an order of magnitude compared to the case without dissipation engineering.

For different $n_m$ and $\omega_m$, we can always obtain the optimal cooling by tuning the coupling $G_{m,j}$ and the dissipation engineering coupling strength $G_{b,k}$, as shown in Fig. 5(c). In Fig. 6, we discuss the optimal cooling as a function of $n_m$ for $\omega_m/(2\pi) = 50$ (red line), 100 (blue line) and 200 (black line) MHz. From Fig. 6(a), it is observed that the thermal occupation $n_f$ is linear to the thermal noise $n_m$ and the cooling effect is better for the higher frequency $\omega_m$, which is due to the reasonable condition for the rotating wave approximation. Compared with no dissipation engineering $G_{b,k} = 0$, the ratio $\eta = n_f(G_{b,k}=0)/n_f$ is plotted in Fig. 6(b), and we observe that the cooling effect is more obvious for the more thermal noise and the ratio value eventually stabilizes. For the frequency $\omega_m$, the ratio has a similar phenomenon with the thermal occupation $n_f$. In conclusion, we can improve the cooling effect via dissipation engineering by about an order of magnitude for the low-frequency mechanical modes.

V. CONCLUSION

We theoretically studied a multimode optomechanical system and proposed the optomechanical dissipation engineering by an ancilla mechanical mode. It is demonstrated that the dissipation engineering could enhance the system performances on the optomechanical induced transparency and amplification, optomechanical entanglement generation, and the
cooling of the mechanical mode. It is shown that the optomechanical induced transparency and amplification can be enhanced or suppressed due to the controlling of the optical linewidth through coupling to ancilla mechanical mode. In the weak regime of the optomechanical coupling, the thermal noise from the mechanical mode destroys the entanglement between the optical mode and mechanical mode. However, we can recover the entanglement by the anti-Stokes process. In the strong coupling regime, we also find that the optomechanical cooling can be greatly enhanced by dissipation engineering. For the entanglement and cooling, the destruction is from the thermal noise, and the anti-Stokes process provides a cooling effect, which is beneficial to the optomechanical system. However, the strong dissipation engineering ultimately has a bad effect on the multimode optomechanical system because of strong nonlinearity, therefore we can always find the optimal dissipation engineering to realize the best entanglement and cooling. The proposed optomechanical dissipation engineering opens up novel prospects for the phonon-based quantum information processing and macroscopic quantum phenomena.

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