Preliminary stability analysis of a Friedman-Lemaitre-Robertson-Walker universe

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Abstract. It is stated in many text books that the any metric appearing in general relativity should be locally Lorentzian i.e. of the type \( g_{\mu\nu} = \text{diag} (1, -1, -1, -1) \) this is usually presented as an independent axiom of the theory, which cannot be deduced from other assumptions. The meaning of this assertion is that a specific coordinate (the temporal coordinate) is given a unique significance with respect to the other spatial coordinates. It was shown that the above assertion is a consequence of requirement that the metric of empty space should be linearly stable and need not be assumed. In this work we remove the empty space assumption and investigate the consequences of spatially uniform matter on the stability of a locally Lorentzian space-time that is the Friedman-Lemaitre-Robertson-Walker space-time. It is shown that a partial stability analysis restricts the type of allowable solutions to the Friedman-Lemaitre-Robertson-Walker space-time. In particular it is shown that an open section universe is stable while an Euclidean and a closed section universes are not in accordance with observation. It will be suggested that in the presence of matter an upper limit scale to the size of a locally Lorentzian universe exists which incidentally is about the size of the observable universe.

1. Introduction

It is well known that our daily space-time is approximately of Lorentz (Minkowski) type that is, it possesses the metric \( \eta_{\mu\nu} = \text{diag} (1, -1, -1, -1) \). The above statement is taken as one of the central assumptions of the theory of special relativity and has been supported by numerous experiments. But why should it be so?

Many textbooks [3, 4] state that in the general theory of relativity any space-time is locally of the type \( \eta_{\mu\nu} = \text{diag} (1, -1, -1, -1) \), although it can not be presented so globally due to the effect of matter. This is a part of the demands dictated by the well known equivalence principle. The above principle is taken to be one of the assumptions of general relativity. Other assumption such as diffeomorphism invariance, and the requirement that theory reduce to Newtonian gravity in the proper regime lead to the Einstein equations:

\[ G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \]  

in which \( G_{\mu\nu} \) is the Einstein tensor, \( T_{\mu\nu} \) is the stress-energy tensor, \( G \) is the gravitational constant and \( c \) is the velocity of light.

\(^1\) This specific form of Einstein equations is taken from Narlikar [3].
Generic solution of Einstein equations for empty (and non-empty) spaces usually show different signatures as was shown by Eddington [5, page 25]. Among those possible solutions it was shown that for empty space-times [1, 2] only the Minkowskian solutions are stable.

Thus one need not assume that space-time is locally Minkowskian based on an empirical (unexplained) fact, rather one can derive this property from the field equations themselves based on the stability of the Minkowskian solutions. The number of assumptions needed to obtain the celebrated results of general relativity is reduced; by making the theory more compact we enhance its predictive strength. Moreover, it is shown that the existence of a temporal coordinate is a necessary consequence of the geometrical structure of the four dimensional space and not a separate ad-hoc assumption.

A different approach is due to Mukohyama & Uzan [6]. Those authors have derived a Lorentzian metric assuming a that at the microscopic level the metric is Riemannian, i.e., locally Euclidean, and that the Lorentzian structure, that we usually consider as fundamental, is in fact an effective property that emerges in some regions of a four dimensional space with a positive definite metric. In such a model, there is no dynamics nor signature flip across some hypersurface; instead, all the fields develop a Lorentzian dynamics in these regions because they propagate in an effective metric. They have shown that one can construct a decent classical field theory for scalars, vectors, and (Dirac) spinors in flat spacetime. And that gravity can be included but that the theory for the effective Lorentzian metric is not general relativity but of the covariant Galilean type. The constraints arising from stability, the equivalence principle, and the constancy of fundamental constants are detailed and a phenomenological picture of the emergence of the Lorentzian metric is also given. This construction, while restricted to classical fields, offers a new view on the notion of time. Unfortunately this approach is not consistent with Occam’s razor as one is forced to introduce several scalar fields (one of them a ”clock field”) with fine tuned coupling in order to obtain the effective sign change.

What is deficient in the above approach (and similar approaches) is that additional theoretical structures and assumptions are needed in order to justify what appears to be a fundamental property of space-time. In previous works [1, 2, 7] it was claimed otherwise. It was shown that general relativistic equations and some ”old fashioned” linear stability analysis will lead to a unique choice of the Lorentzian metric being the only one which is linearly stable in empty space-time. Other metrics are allowed but are unstable and thus can exist in only a limited region of empty space-time. The implications of this are discussed elsewhere [8, 9, 10]. It should be mentioned that the choice of coordinates in the Fisher approach to physics [11] can be also be justified using the stability approach [7]. The nonlinear stability question of the Lorentzian metric for empty space-times was settled by D. Christodoulou and S. Klainerman [13].

This paper assumes that space-time must have four dimensions, it does not explain why this is so. For a possible explanation derived from string theory one can consult a paper by S. K. Rama [14]. The analysis discussed in this work is classical. For quantum aspects of space time difference one can consult the book by S. Hawking and R. Penrose [15]. Quantum field considerations made by A. White, S. Weinfurtner and M. Visser [16] have shown that regardless of the underlying classical theory, there are severe problems associated with any quantum field theory residing on a signature-changing background which does not respect the space-time difference (Such as the production of what is naively an infinite number of particles, with an infinite energy density). Those authors raise the question as to whether signature change transitions could be fully understood and dynamically generated within (modified) classical general relativity, or whether they require the knowledge of a theory of quantum gravity.

The plan of this paper is as follows: in the first section we outline the stability analysis of empty space-times, the next is dedicated to the modifications of those stability equations in the presence of a spatially uniform matter; this is essentially the stability analysis of a Friedman-Lemaitre-Robertson-Walker Universe. Finally we suggest an upper scale for a stable locally
Minkowskian space-time and compare this scale with the size of the observable universe.

2. Stability analysis for empty space-times

In what follows I give an outline of the proof of the linear stability of Lorentzian space-time and the linear instability of other possible flat space-times. Unfortunately the details cannot be given here for the lack of space, but can be found elsewhere [1].

Consider a flat space-time, in this case one can find a set of coordinates such that the metric will be constant everywhere. A constant metric is nothing but a 4 by 4 symmetric matrix, therefore by a suitable choice of coordinates it can be made diagonal with eigenvalues all real.

A final step would be to choose the scaling of the coordinates such that the metric will have the form:

$$\eta = \text{diag} (\pm 1, \pm 1, \pm 1).$$  \hspace{1cm} (2)

Notice that no zero eigen values are allowed by virtue of the four dimensionality of space-time.

Once the family of possible canonical flat metrics is established we make a small perturbation $h_{\mu \nu}$ of the metric and consider a new metric $g_{\mu \nu} = \eta_{\mu \nu} + h_{\mu \nu}$ in which $h_{\mu \nu} \ll 1$. Inserting $g_{\mu \nu}$ into equation (1) and keeping only first order terms in $h_{\mu \nu}$ while taking $T_{\mu \nu} = 0$ we arrive at homogenous linear equations for $h_{\mu \nu}$. Solving those equations with local but otherwise arbitrary boundary conditions we see that only in the case of a Lorentzian metric the perturbation does not exponentially grow while in any other case it does. This explains why space-time is Lorentzian and why time is unique among the coordinates of space-time [1].

3. Stability analysis for a locally Lorentzian non empty space-time

Let us now assume that space-time is locally Lorentzian and contains a spatial uniform distribution of matter; this is the case of a Friedman-Lemaitre-Robertson-Walker metric. We choose following Narlikar [3], the non-perturbed metric to be:

$$g^{(0)}_{\mu \nu} = \text{diag} \left(1, -\frac{S^2(t)}{1-k r^2}, -S^2(t) r^2, -S^2(t) r^2 \sin^2 \theta\right)$$  \hspace{1cm} (3)

for the coordinates $x^\nu = (ct, r, \theta, \phi)$. Hence for radial distances such that $r < \frac{1}{\sqrt{k}}$ (in a closed universe) and anywhere in a Euclidean or open universes, the metric is locally (up to scaling) $\eta_{\mu \nu} = \text{diag} (+1, -1, -1, -1)$ that is Lorentzian. We assume a fluid energy momentum tensor of the form [3]:

$$T_{\mu \nu} = (p + \rho c^2) u_\mu u_\nu - p g_{\mu \nu}$$  \hspace{1cm} (4)

In the above $p$ is the pressure, $\rho$ is the density and $u_\mu = dx^\mu / ds$ in which the interval $ds$ is defined as:

$$ds^2 = g_{\mu \nu} dx^\mu dx^\nu$$  \hspace{1cm} (5)

We notice that $T_{\mu \nu}$ depend on metric perturbations both directly through the term $-pg_{\mu \nu}$ and through the $u_\mu$ term. The metric $g^{(0)}_{\mu \nu}$ is a solution of the Einstein equation (1):

$$G^{(0)}_{\mu \nu} = -\frac{8\pi G}{c^4} T^{(0)}_{\mu \nu}$$  \hspace{1cm} (6)

in which $G^{(0)}_{\mu \nu}, T^{(0)}_{\mu \nu}$ are the Einstein and energy-momentum tensors calculated using the metric $g^{(0)}_{\mu \nu}$. The solution is valid provided that $p$ and $\rho$ are spatially (but not temporally) uniform. We

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2 In this paper Greek indices have the values (0, 1, 2, 3) while Latin indices are spatial and have the values (1, 2, 3).
now look at a perturbed metric \( g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} \). Inserting this metric into equation (1) and keeping only first order terms in \( g^{(1)}_{\mu\nu} \) we arrive at:

\[
G^{(0)}_{\mu\nu} + G^{(1)}_{\mu\nu} = -\frac{8\pi G}{c^4} (T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu})
\]  

(7)

in which \( G^{(1)}_{\mu\nu}, T^{(1)}_{\mu\nu} \) are the first order corrections to the Einstein and energy-momentum tensors due to the metric perturbation \( g^{(1)}_{\mu\nu} \). Combining equation (7) with equation (6) we arrive at the equation:

\[
G^{(1)}_{\mu\nu} = -\frac{8\pi G}{c^4} T^{(1)}_{\mu\nu}
\]  

(8)

Since the Einstein tensor can be calculated from the Ricci tensor \( R_{\mu\nu} \) and the curvature scalar \( R \) as follows [3]:

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R
\]  

(9)

Thus the first order correction to \( G_{\mu\nu} \) will be:

\[
G^{(1)}_{\mu\nu} = R^{(1)}_{\mu\nu} - g^{(1)}_{\mu\nu} - \frac{1}{2} g^{(0)}_{\mu\nu} R^{(1)}
\]  

(10)

However, it follows from equation (1) and equation (4) that:

\[
R = -g^{\mu\nu} G_{\mu\nu} = \frac{8\pi G}{c^4} (pc^2 - 3p)
\]  

(11)

Hence assuming that the density and pressure are not perturbed it follows that the scalar curvature is metric independent, that is:

\[
R^{(0)} = R = \frac{8\pi G}{c^4} (pc^2 - 3p), \quad R^{(1)} = 0
\]  

(12)

and thus:

\[
G^{(1)}_{\mu\nu} = R^{(1)}_{\mu\nu} - g^{(1)}_{\mu\nu} - \frac{1}{2} g^{(0)}_{\mu\nu} R^{(1)}
\]  

(13)

The Ricci tensor can be calculated in terms of the affine connection

\[
R_{\mu\nu} = \partial_{\nu} \Gamma^{\alpha}_{\mu\alpha} - \partial_{\alpha} \Gamma^{\gamma}_{\mu\gamma} + \Gamma^{\alpha}_{\mu\beta} \Gamma^{\beta}_{\alpha\nu} - \Gamma^{\beta}_{\mu\nu} \Gamma^{\alpha}_{\alpha\beta}
\]  

(14)

in terms of which the first order correction is:

\[
R^{(1)}_{\mu\nu} = \partial_{\nu} \Gamma^{(1)}_{\mu\alpha} - \partial_{\alpha} \Gamma^{(1)}_{\mu\nu} + \Gamma^{(1)}_{\mu\beta} \Gamma^{(0)}_{\alpha\nu} + \Gamma^{(0)}_{\mu\beta} \Gamma^{(1)}_{\alpha\nu} - \Gamma^{(1)}_{\mu\nu} \Gamma^{(0)}_{\alpha\beta}
\]  

(15)

where \( \partial_{\nu} = \frac{\partial}{\partial x^{\nu}} \). The affine connection can be calculated as:

\[
\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left( \partial_{\mu} g_{\beta\nu} + \partial_{\nu} g_{\mu\beta} - \partial_{\beta} g_{\mu\nu} \right)
\]  

(16)

Hence to first order we have:

\[
\Gamma^{(1)}_{\mu\nu} = \frac{1}{2} g^{(0)\alpha\beta} \left( \partial_{\mu} g^{(1)}_{\beta\nu} + \partial_{\nu} g^{(1)}_{\mu\beta} - \partial_{\beta} g^{(1)}_{\mu\nu} \right) + \frac{1}{2} g^{(1)\alpha\beta} \left( \partial_{\mu} g^{(0)}_{\beta\nu} + \partial_{\nu} g^{(0)}_{\mu\beta} - \partial_{\beta} g^{(0)}_{\mu\nu} \right)
\]  

(17)

which can also be written as:

\[
\Gamma^{(1)}_{\mu\nu} = \frac{1}{2} g^{(0)\alpha\beta} \left( \partial_{\mu} g^{(1)}_{\beta\nu} + \partial_{\nu} g^{(1)}_{\mu\beta} - \partial_{\beta} g^{(1)}_{\mu\nu} \right) + g^{(1)\alpha\beta} g_{\beta\gamma}^{(0)} \Gamma^{(0)\gamma}_{\mu\nu}
\]  

(18)
It can be easily shown that up to first order:

\[ g^{(1)\mu\nu} = -g^{(0)\alpha\nu}g^{(1)\alpha\beta}g^{(0)\beta\mu} \]  

(19)

Inserting equation (19) into equation (18) we obtain:

\[ \Gamma^{(1)}_{\mu\nu} = \frac{1}{2}g^{(0)\alpha\beta}(\partial_\mu g^{(1)}_{\beta\nu} + \partial_\nu g^{(1)}_{\alpha\beta} - \partial_\beta g^{(1)}_{\mu\nu}) - g^{(1)\alpha\beta}g^{(0)\alpha\beta}\Gamma^{(0)\gamma}_{\mu\nu} \]  

(20)

We will now consider a fluid which is at rest in a comoving frame hence \( u^\mu = 0 \) for \( \mu \neq 0 \). Hence according to equation (5): \( g_{00}(u^0)^2 = 1 \) and we choose \( u^0 = \frac{1}{\sqrt{g_{00}}} \):

\[ u_\mu = g_{\mu\nu}u^\nu = g_{\mu0}u^0 = \frac{g_{\mu0}}{\sqrt{g_{00}}} \]  

(21)

By inserting \( g_{\mu\nu} = g^{(0)\mu\nu} + g^{(1)\mu\nu} \) we obtain:

\[ u^{(0)}_\mu = \frac{g^{(0)}_{\mu0}}{\sqrt{g^{(0)\mu0}}}, \quad u^{(1)}_\mu = \frac{g^{(1)}_{\mu0}g_{00} - \frac{1}{2}g^{(1)\mu0}g^{(0)}_{00}}{\sqrt{g^{(0)\mu0}}} \]  

(22)

For a diagonal metric \( g^{(0)\gamma} \):

\[ u^{(0)}_a = 0, \quad u^{(0)}_0 = \sqrt{g^{(0)\mu0}} \quad \text{and} \quad u^{(1)}_a = \frac{g^{(1)}_{a0}}{\sqrt{g^{(0)\mu0}}}, \quad u^{(1)}_0 = \frac{1}{2}g^{(1)}_{00}\sqrt{g^{(0)\mu0}}. \] Thus zeroth and first order contributions to the energy-momentum tensor are thus:

\[ T^{(0)}_{\mu\nu} = (p + \rho c^2)u^{(0)}_\mu u^{(0)}_\nu - p g^{(0)\mu\nu} \]  

(23)

\[ T^{(1)}_{\mu\nu} = (p + \rho c^2)(u^{(0)}_\mu u^{(1)}_\nu + u^{(1)}_\mu u^{(0)}_\nu) - p g^{(1)\mu\nu} \]  

(24)

From this one can easily calculate:

\[ T^{(1)}_{ab} = -p g^{(1)\mu\nu} \]  

(25)

Hence combining the above equations with equation (13) and equation (8) we have

\[ R^{(1)}_{ab} - g^{(1)\mu\nu} \frac{4\pi G}{c^4}(\rho c^2 - p) = 0, \quad R^{(1)}_{\mu0} + g^{(1)\mu0} \frac{4\pi G}{c^4}(\rho c^2 + 3p) = 0 \]  

(26)

We will now assume a perturbation of the type \( h \equiv g^{(1)}_{11}(t, r) \) and will look only at the (1, 1) component of equation (26). This will take the form:

\[ \frac{1}{2}\partial_0^2 h + \frac{(1 - kr^2)}{rS(t)^2}\partial_1 h + \frac{(S'(t)^2 - 2c^2 k)}{c^2 S(t)^2} h - \frac{4\pi G}{c^2} \rho h = 0 \]  

(27)

in case pressure is negligible with respect to \( \rho c^2 \) as is the case in the present epoch. This can also be written as:

\[ \partial_1^2 h + \frac{2c^2 (1 - kr^2)}{rS(t)^2}\partial_1 h + 2 \left( H^2(t) - 2c^2 \frac{k}{S(t)^2} - 4\pi G\rho \right) h = 0 \]  

(28)

\(^3\) We remind the reader that Latin indices are spatial.
in which \( H(t) = \frac{S'(t)}{S(t)} \) is the Hubble function. Assuming that the spatial derivative of \( h \) are negligible we arrive at the following equation.

\[
\partial_t^2 h + 2 \left( H^2(t) - 2c^2 \frac{k}{S(t)^2} - 4\pi G\rho \right) h = 0 \tag{29}
\]

The solution of which will be stable if:

\[
H^2(t) - 2c^2 \frac{k}{S(t)^2} - 4\pi G\rho > 0 \tag{30}
\]

At the present epoch this takes the form:

\[
\frac{H_0^2}{4\pi G} - c^2 \frac{k}{2\pi GS_0^2} - \rho_0 > 0 \tag{31}
\]

Since the critical density is \( \rho_c = \frac{3H_0^2}{8\pi G} \), we can write:

\[
\frac{2}{3}\rho_c - c^2 \frac{k}{2\pi GS_0^2} - \rho_0 > 0 \tag{32}
\]

This is indicates that an Euclidean section universe with \( k = 0 \) is unstable since it requires \( \rho_0 = \rho_c \), the same can be said about a universe with a closed section in which \( k = 1 \). This universe requires \( \rho_0 > \rho_c \), this clearly cannot be satisfied at the same time with equation (32). The open section universe with \( k = -1 \) is stable to this type of perturbation provided that its density satisfies:

\[
\frac{2}{3}\rho_c + c^2 \frac{k}{2\pi GS_0^2} > \rho_0 \tag{33}
\]

Since \( \rho_c = 1.83 \cdot 10^{-26} \text{Kg/M}^3 \), while \( \rho_0 \) is estimated to be \( 4.5 \cdot 10^{-28} \text{Kg/M}^3 \), the stability condition for this type of perturbation is satisfied. Thus the current stability analysis is consistent with observations of an expanding universe.

4. The scale of the observable universe

Although the current analysis is not comprehensive, we would like to point out a general feature which will negatively effect the stability of any spatial perturbation, that is the existence of a critical wave number in equation (26) [20]:

\[
k_{\text{crit}} = \frac{4}{c} \sqrt{2\pi G\rho} \tag{34}
\]

In terms of wavelengths we see that an upper scale for stable perturbations is:

\[
\lambda < \lambda_{\text{max}} = \frac{2\pi}{k_{\text{crit}}} = \frac{c}{2} \frac{\sqrt{\pi}}{2\sqrt{2G\rho}} \tag{35}
\]

The effect of an upper stability wavelength is well known in astrophysical systems [21, 22, 23]. For example Jeans [21] has studied the stability of a self gravitating uniform medium and has determined that if the system is smaller that a certain size instability will not set in to change the system configuration. But if it is bigger than a certain size (Jeans length) the system will become unstable and move to different configuration thus structure will spontaneously appear in uniform medium such structures are galaxies and stars.
The same happens in a cosmological scale. If the universe is smaller than a certain scale the instability will not set in but if it is bigger than this scale the instability will set in and push the universe to a different stable equilibrium which may not be uniform and may effect the metric locally in certain space-time events.

Inserting the density of the universe as estimated by [17]: $\rho \simeq 4.5 \cdot 10^{-28} \frac{Kg}{m^3}$ which leads to a $\lambda_{\text{max}} \simeq 1.08 \cdot 10^{27} \text{Meter}$. This is slightly larger than the radius of the observable universe: $[18] \ 14 \cdot 10^9 \text{Parsec} \simeq 4.32 \cdot 10^{26} \text{Meter}$. Hence in "surprising" coincidence the diameter of the observable universe is about the size of the largest scale stable perturbation. At this time the size of the universe is not known [19] but it is suspected [9, 10] that above the stability scale the metric of space-time and hence physics may be quite different.

5. Conclusion
Mathematically speaking one of the main differences between time and space is encapsulated in the flat metric of our space-time which is locally of the Lorentzian type $\eta_{\mu\nu} = \text{diag} (1,-1,-1,-1)$. But this is an empirical fact or a mathematical postulate thus unexplained. One can imagine also other flat metrics such as the Euclidian metric: $\text{diag} (1,1,1,1)$. In Euclidian metrics there is no restriction on the speed of any moving body as the speed of light restricts the speed of propagation only in the presence of a Lorentzian metric. Why is our space-time Lorentzian and not Euclidean? The answer is that only the Lorentzian metric is stable [1] for an (almost) empty space. But space-time is not empty and thus the notion of time always progressing forward with the increase of entropy is probably just a consequence of the scales of reality that we are exposed to. In huge cosmological scale the Friedman-Lemaitre-Robertson-Walker universe may lose its Lorentzian character [9, 10]. The horizon problem related to the homogeneity of cosmic microwave background can be solved if one takes into account the superluminal motion of particles for $r > r_c$ and the same particles moving into the $r < r_c$ Lorentzian domain. What is shown here is that a locally Lorentzian type universe although stable for all scales in an empty universe has a size limitation for a non empty universe and hence the suggested solution to the horizon problem is not only plausible but in fact strongly suggested by the analysis presented here.

It should be noted that the stability analysis given here is rather restricted as we only allowed $h_{11}$ perturbations. A full stability analysis is expected to involve a set of partial differential equations for the ten free linear metric components which probably can only be solved numerically. However, we can also learn something about the structure of the equations and there solution by looking at specific perturbations as was done here.

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