The X-structure/mechanism approach to beneficial nonlinear design in engineering

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Abstract Nonlinearity can take an important and critical role in engineering systems, and thus cannot be simply ignored in structural design, dynamic response analysis, and parameter selection. A key issue is how to analyze and design potential nonlinearities introduced to or inherent in a system under study. This is a must-do task in many practical applications involving vibration control, energy harvesting, sensor systems, robotic technology, etc. This paper presents an up-to-date review on a cutting-edge method for nonlinearity manipulation and employment developed in recent several years, named as the X-structure/mechanism approach. The method is inspired from animal leg/limb skeletons, and can provide passive low-cost high-efficiency adjustable and beneficial nonlinear stiffness (high static & ultra-low dynamic), nonlinear damping (dependent on resonant frequency and/or relative vibration displacement), and nonlinear inertia (low static & high dynamic) individually or simultaneously. The X-structure/mechanism is a generic and basic structure/mechanism, representing a class of structures/mechanisms which can achieve beneficial geometric nonlinearity during structural deflection or mechanism motion, can be flexibly realized through commonly-used mechanical components, and have many different forms (with a basic unit taking a shape like X/K/Z/S/V, quadrilateral, diamond, polygon, etc.). Importantly, all variant structures/mechanisms may share similar geometric nonlinearities and thus exhibit similar nonlinear stiffness/damping properties in vibration. Moreover, they are generally flexible in design and easy to implement. This paper systematically reviews the research background, motivation, essential bio-inspired ideas, advantages of this novel method, the beneficial nonlinear properties in stiffness, damping, and inertia, and the potential applications, and ends with some remarks and conclusions.

Key words nonlinear stiffness, nonlinear damping, nonlinear inertia, vibration

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1 Introduction

A desired stiffness or damping property is often critical for achieving excellent performance in many engineering systems. Mismatched stiffness or damping would lead to downgrade of system performance, prolonged excessive vibration, malfunctions of machines, or even structural damage or unexpected disaster\cite{1-4}. There are several different methods for realizing a desired stiffness or damping property for a structure or system. Besides those with active controlled actuators or materials\cite{5-9}, passive methods for stiffness or damping manipulation are often more preferable in practice, including those extensively studied ones via design of meta-materials, shape-memory polymers\cite{10}, or origami structures\cite{11-12}. However, these existing passive methods for stiffness manipulation are usually difficult to be analyzed with a relatively simple and accurate mathematical model. This leads to a difficulty in the corresponding analysis and design, and consequently results in the limitation of reliability and accuracy after manufacturing. Especially for those materials with inherent nonlinearity, without an accurate analytical model, the control of nonlinear response and the selection of associated design parameters would become even more difficult, facing another risk of unexpected nonlinear dynamics to happen.

In recent years, nonlinearities have been attracting more and more attentions in system design and control for achieving superior dynamic response performance in the areas of vibration control, energy harvesting, etc. Nonlinearities can be used to improve a vibration system to shift its resonant frequency to a desired value with respect to external excitation\cite{1-3}, to reduce the resonant frequency without losing loading capacity\cite{13-17}, to lower the resonant peak without deteriorating high frequency vibration\cite{18-19}, to cover a wider frequency range for vibration suppression\cite{2-3,20-21}, or to save the active control energy without sacrificing isolation performance\cite{22-23}. Applying these nonlinear properties, the energy harvesting performance can be significantly improved\cite{24-29}, sensor systems can be enhanced to be more sensitive, reliable, and accurate\cite{30-31}, mobile robots can be more adaptable to rough grounds\cite{32}, and grasping tools or actuation systems can be much faster or more powerful\cite{33-34}. To explore beneficial nonlinear properties, special structural designs with spring or magnetic mechanisms thus have been extensively studied in recent years\cite{2-3,24,35}.

In all those cases mentioned, a basic question involved is how to design or realize desired beneficial nonlinearities for a given system subject to various practical requirements. The method should be able to provide sufficient flexibility and easiness in design and implementation. Besides high performance and low cost, in-situ adjustability is another highly relevant factor to be considered for assessing the quality of a potential solution.

To this aim, a bio-inspired structure/mechanism design method has been consequently proposed and systematically investigated in recent years, referred to as the X-structure/mechanism approach. The method is inspired from animal leg/limb skeletons, and can provide a flexible solution to most engineering applications demanding high-quality nonlinear stiffness and damping properties with a convenient and reliable structure or linkage mechanism design. Studies have revealed that the X-structure/mechanism can provide tunable and beneficial nonlinear stiffness, damping, and inertial characteristics and meet most requirements from engineering practices. The resulting nonlinear characteristics can be employed for different applications in vibration control, energy harvesting, sensor systems, robotic technology, etc. This paper presents an updated summary of this topic and those recently published results, and some further development perspectives would also be discussed accordingly. The main results and advances of the bio-inspired X-structure/mechanism approach are introduced systematically in this paper. The rest of this paper is organized as follows. The general idea, motivation, and methodology are discussed firstly, followed by a discussion on the design flexibility and parameter selection freedom. Thereafter, the main nonlinear properties of the X-structure/mechanism are summarized, and then followed by a brief introduction of potential applications. A conclusion is drawn at the end.
2 Idea and methodology

To seek effective methods for the efficient employment of beneficial nonlinearity extensively demanded in engineering systems for achieving advantageous performance (in vibration control, energy harvesting, etc.) with the minimum implementation cost and complexity, the X-structure/mechanism is thus proposed and developed with the following bio-inspired idea. All natural creatures are equipped with superior features over environmental challenges (see Fig. 1). Legs or limbs of animals can easily suppress vibration and shock impact, while the osteoporosis bone structures are obviously weak in impact protection due to loosened interlayer connection. Woodpeckers do not suffer from brain damage after daily-repeated 500-600-time drumming a tree with a surprising speed of about 20 beats per second and a deceleration up to 1200 g[36–37]. A reason is the special cranial bone structure, which has two outside layers. The interlayer supporting structure does not support the outside layer in a normal direction, but takes an inclination link like a limb-joint system. Considering human body, it can act as a very good shock absorber to prevent from brain injury caused by vibration and impact, and provide excellent vibration isolation. Noticeably, human body tends to swing arms during walking or jumping, which is a special behavior since the arms play no obvious role in bipedal gait. A study on whole-body vertical angular momentum and ground reaction moment when human walking indicates that arm swinging does improve the walking stability[38–39].

Fig. 1  (a) Scanning electron microscope (SEM) image of woodpecker’s cranial bone; (b) SEM image of hoopoe’s cranial bone; (c) avian leg system; (d), (e) human arms, legs, and bone structures; (f) billfish/swordfish and its head structure. The inside supporting structure of the cranial bone in (a) or (b) or the limb bone in (e) does not support the outside layer in a normal direction, but takes an inclination link like a limb-joint system in (c) or (d). The limb structures and inside bone structures of animals all take a “Z” shape or polygon shape with spring-like muscles or tissues for supporting or joining. The billfish head bone takes a polygonal structure, forming an excellent mounting system for the spear. These structures can successfully absorb or suppress the shock or vibration impact, but have never been systematically explored or employed for vibration isolation/suppression. The osteoporosis bone structures obviously lose impact protection due to weak or loosening inside connection (in the bottom of (e)) (color online)
The observations above motivate a series of innovative studies which well answer the fundamental questions as mentioned before, present new understanding of bio-inspired nonlinear dynamics and their application potential, and consequently lead to the X-structure/mechanism approach established in the past several years. To start this series of cutting-edge theory and methods for the design of nonlinear dynamics, a key issue is to seek for a generic structure/mechanism from those observed biological bone structures, animal legs or limbs, which can have supervisor nonlinear features at least in equivalent stiffness and be easy and flexible to implement in practice. Motivated by this, the X-shaped structure or linkage mechanism (initially referred to as scissor-like structure or limb-like structure) is therefore proposed\[15,40–41\]. It is simple and very easy to implement, has no strict material restriction, can have different variants, and take different forms but which all share similar nonlinear geometric relationship between the structural suppression displacement and the external exerted force. The nonlinear features provided by the X-structure/mechanism are demonstrated to be superior in terms of potential stiffness, damping, and inertia properties. These results will all be introduced in the following sections.

3 Flexibility in structure/mechanism design and implementation

In this section, the flexibility of the X-structure/mechanism approach in design and implementation is demonstrated.

3.1 Basic symmetric X-structure

The basic symmetric X-structure/mechanism can be designed in the form as shown in Fig. 2, with a symmetric scissor-like linkage mechanism of different layers[40–43]. The linkages are connected with rotation joints, and the overall height of the structure can be designed with different rod lengths and layers. To employ the geometric nonlinearity, the springs and/or dampers should be connected with two horizontal joints in each layer, which is of course not the only method for introducing the geometric nonlinearity but must be the most convenient way since there is no need of additional connection points and rods. It can be seen that the rod length \( l \), assembly angle \( \theta \), spring and damping coefficients \((k, c)\), and layer number \( n \) can all be design parameters. Without further clarification, \( M \) (or \( m \)), \( K \) (or \( k \)), \( C \) (or \( c \)), \( L \) (or \( l \)), and \( \theta \) refer to mass, stiffness coefficient, damping coefficient, rod length, and associated angle, respectively; and \( y \) is the absolute displacement of the top mass, \( x \) is the displacement of a joint horizontally, and \( z \) is the absolute displacement in the base, throughout the paper.

Fig. 2 Basic X-structure/mechanism, taking symmetric 1, 2, or \( n \) layers with horizontal springs and/or dampers installed in a layer to employ the geometric nonlinearity[41–42]: (a) an \( n \)-layer X-structure/mechanism; (b) a 2-layer mechanism prototype; (c) a 2-layer X-structure/mechanism; (d) a 2-layer mechanism prototype (color online)
To achieve onsite adjustability, besides that the spring/damper can be directly changed, the spring/damper connection can be designed with an adjustable pre-extension/compression mechanism, and the rod length may also be tuned with a stretchable rod. The materials could be metal, stiff plastic, carbon fiber, etc. Elastic rods can also be considered for special performance in further studies.

### 3.2 Asymmetric X-structure

Based on the basic X-structure/mechanism, the rod length can be carefully designed to achieve different asymmetric forms as a new design factor, as shown in Fig. 3. The biological basis for asymmetrical structures comes from that the femur and tibia of animal legs or insect limbs usually take different lengths and inclination angles. The significance of studying asymmetric structures lies in meeting special space requirements in applications without sacrificing performance\(^{[15,44]}\), improvement by tuning structural asymmetry\(^{[15,45–46]}\), and potential applications in periodic structures\(^{[47]}\). It is shown in Refs. \([15, 45], [46]\) that the asymmetric factor is useful for achieving much better performance and obtaining the minimum resonant frequency in some special application situations (e.g., low gravity).

\[\text{Fig. 3} \quad \text{Asymmetric X-structure/mechanism, in horizontal, vertical, or both, regular or irregular change}^{[15, 45–46]}, \text{bringing different nonlinear properties and thus nonlinear responses consequently: (a) a horizontal asymmetric X-structure easily obtained from bird legs due to different inclination angles in each articulation; (b) a horizontal asymmetric structure; (c) a vertical asymmetric structure; (d) a different vertical asymmetric structure; (e) a freely connected vertical asymmetric structure (color online)}\]

#### 3.3 Half-X or other X-variants

With the structures shown in Figs. 2 and 3, some other variants can be developed, e.g., polygon-shaped structures (the X-structure can be regarded as a quadrilateral structure)\(^{[43,48]}\) and half layer structures like a V or K with/without linkages\(^{[49–50]}\). In modeling, the geometrical nonlinearity would be similar and not focused in our discussions. Importantly, multi-degrees-of-freedom (DOF) systems can also be obtained with independent layers, demonstrating much more flexibility in performance design and applications\(^{[51]}\) (see Fig. 4).

Noticeably, the X-structures/mechanisms above provide a very tunable and flexible tool to achieve nonlinear stiffness/damping properties. With this powerful tool, more additional design factors can thus be further considered to obtain some amazing performance. This motivation leads to the following structural designs and associated studies.

#### 3.4 Inertia-coupled X-structure

As mentioned before, arm swing takes an important role in stability control during human body walking and jumping. Inspired by the arm swing, rotation units can thus be introduced
into the X-structure to create an alternative and critical design factor for performance improvement. This can be easily done with different manners, as shown in Fig. 5\cite{52–54}. It is shown that the X-structure/mechanism reinforces the commonly used rotation units into a powerful and adjustable nonlinear inertia system with much better performance.

**Fig. 4** Multi-DOF X-structures/mechanisms with asymmetric units, introducing even more flexibility in performance adjustability and practical applications\cite{51}: (a) a human leg; (b) an \( n \)-layer multi-DOF human leg inspired mechanism; (c) a human leg inspired mechanism in extension (color online)

**Fig. 5** Several X-structures/mechanisms with inertial units to create beneficial nonlinear inertia inspired by arm swing of human body during walking\cite{53–54}, where the inertia can be created with commonly-used mechanical components such as leverages, rotation units/discs, and fly-wheels, for achieving beneficial nonlinear inertia (color online)
3.5 X-structure/mechanism with special spring arrangements

For vibration isolation, an enlarged quasi-zero-stiffness (QZS) area can ensure a superior isolation performance, while multi-stable nonlinearities are well employed for improving energy harvesting efficiency. The desired nonlinear stiffness can be easily obtained with the X-structure/mechanism via some special spring arrangements as shown in Fig. 6. For example, additional horizontal springs applied with a controlled contact with the X-structure can achieve specially improved QZS property, loading capacity, and multi-stable nonlinear stiffness. This spring arrangement method with the X-structure opens a larger potential for applying the X-structure/mechanism in various engineering systems.

From Figs. 2–6, it can be seen that the X-structure/mechanism not only is flexible in design and implementation, but also provides a reliable and adjustable manner of great potentials for realizing desired nonlinearities for different engineering requirements. The detailed advantageous nonlinear characteristics will be discussed in the following sections.

![Fig. 6](image)

**Fig. 6** X-structures/mechanisms combined with special arranged springs for achieving more flexibility in stiffness manipulation including enlarged QZS area, enhanced loading capacity, and improved multi-stable nonlinearities: (a) an X-structure/mechanism combined with an additional horizontal spring with/without preloading in springs; (b) another X-structure/mechanism with the same setting as (a) but with additional horizontal springs and pre-defined loading points. Both cases in (a) and (b) can create prolonged QZS area, higher loading capacity, and multi-stable stiffness (color online)

4 Beneficial nonlinear stiffness

The essential nonlinear stiffness created by the X-structures/mechanisms can be summarized in what follows of this section.

A typical reactive force between the loading force and the compression displacement of a 2-layer basic symmetric X-structure/mechanism displayed in Fig. 2 is shown in Fig. 7, with respect to different values of the assembly angle $\theta$ and/or applied spring stiffness $k$. From Fig. 7, it can be seen that the stiffness can be well changed from positive stiffness (red), via QZS (green), to negative stiffness (blue). One of the important nonlinear features is the degressive stiffness, decreasing with the increase in the loading force before it goes to the negative area, which is completely different from commonly used coil springs in engineering. The positive and negative stiffness can both be tuned for its amplitude, and the loading capacity can be easily changed with different values of the assembly angle $\theta$ and/or applied spring stiffness $k$. For
Fig. 7  A typical nonlinear stiffness response between the loading force and the compression displacement with different values of the assembly angle $\theta$ and/or the applied spring stiffness $k$\textsuperscript{[55]}: (a) typical nonlinear stiffness characteristics of the basic X-structure/mechanism; (b) a typical curve of a traditional coil spring with a progressive stiffness, indicating that the X-structure/mechanism presents a nonlinear degressive stiffness until reaching a zero stiffness point and then goes to negative stiffness (color online).

onsite adjustability, this can be done with a pre-extension mechanism which can be used to tune the assembly angle $\theta$ or change the spring directly. Importantly, it should be emphasized that for a rod length with 5 cm, the 2-layer X-structure can bear with a displacement range of several centimeters in the QZS area. This is particularly important for vibration isolation subject to large-stroke vibration excitation. Moreover, the negative stiffness can be well circumvented in practices with simple motion restriction mechanisms.

The detailed modeling and analysis can be referred to Refs. [15] and [41]–[55].

The typical reactive force of the X-structure can be shown in Eq. (1)\textsuperscript{[15]} with parameters shown in Fig. 8.

$$f = \frac{k_h}{2} \left( L_1 \cos \theta_1 + L_2 \cos \theta_2 - \sqrt{L_1^2 - \left( L_1 \sin \theta_1 + \frac{y}{2n} \right)^2} - \sqrt{L_2^2 - \left( L_2 \sin \theta_2 + \frac{y}{2n} \right)^2} \right)$$

$$\cdot \left( \frac{L_1 \sin \theta_1 + \frac{y}{2n}}{\sqrt{L_1^2 - \left( L_1 \sin \theta_1 + \frac{y}{2n} \right)^2}} + \frac{L_2 \sin \theta_2 + \frac{y}{2n}}{\sqrt{L_2^2 - \left( L_2 \sin \theta_2 + \frac{y}{2n} \right)^2}} \right) + k_v \frac{y}{n}. \quad (1)$$

Equation (1) clearly shows that the stiffness is a very complicated nonlinear function of the structural parameters and the compression displacement $y$. The structural asymmetry and ratio of the vertical and horizontal spring stiffness play an important role in tuning the nonlinear property. This understanding can help a lot in structural design and parameter selection in engineering practice.

With the QZS achieved through the X-structure, a resonant frequency around 1 Hz can be easily obtained in the prototype testing as shown in Refs. [15] and [55]. Compared with traditional QZS system, the resulting QZS is larger with much more flexible loading capacity, and bears with larger excitation stroke.

With the stiffness property, the negative stiffness can be readily exploited with positive stiffness components such that an enlarged QZS can be obtained (see Fig. 9)\textsuperscript{[55]}. There are several different designs which can be used for this purpose. For example, a vertical spring can be installed with the X-structure\textsuperscript{[15]}, and an additional horizontal spring with additional fixed support can be employed as those demonstrated in Fig. 6\textsuperscript{[55]}.

Similarly, multi-stable equilibria can be obtained with additional springs as shown in Fig. 6 or others demonstrated in Ref. [55]. The resulting nonlinear stiffness properties can be seen
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Fig. 8 X-structure in modeling [15], where both horizontal and vertical springs can be applied for a better adjustability of the nonlinear stiffness property: (a) an n-layer asymmetric X-mechanism with both horizontal and vertical springs applied, where the asymmetric ratio and the vertical/horizontal spring ratio can all be used to better tune the stiffness property; (b) a 2-layer prototype with only horizontal spring applied (color online).

Fig. 9 Enlarged QZS area together with different nonlinear properties [55]: (a) tunable nonlinear stiffness and bistable stiffness; (b) tunable multi-stable nonlinear stiffness, where \( \beta = \frac{h_2 - h_1}{2nL \sin \theta} \) in Fig. 9. Such nonlinear properties could be very beneficial in vibration based energy harvesting, etc. More different methods for tuning nonlinear stiffness properties based on the X-structure/mechanism can be further referred to Ref. [55]. It should be the first time for these nonlinear properties and associated adjustability to be demonstrated and obtained in such a flexible and convenient way.

It should also be noted that, for vibration control, the X-structure/mechanism should be tuned to be QZS with only one stable equilibrium in most cases. The multi-stable stiffness of the X-structure/mechanism will be used for other purposes, e.g., energy harvesting or sensors.

5 Beneficial nonlinear damping

The other advantageous feature of the X-structure/mechanism lies in the beneficial nonlinear damping characteristic that could be created simply with such a simple design. The beneficial nonlinear damping comes from the rotation joints or dampers installed horizontally within a
layer of the X-structure due to the geometric nonlinearity (see Fig. 10). A simple modeling for the model in Fig. 10 is given as follows (without detailed explanation about its notations)\cite{56}:

$$\ddot{y}_1 + \frac{k}{M} y_1 + \left( \frac{c_1}{M} + \frac{c_2 n_x}{M} \left( \frac{\partial \phi}{\partial y_1} \right)^2 + \frac{c_3}{M} \left( \frac{\partial x}{\partial y_1} \right)^2 \right) \dot{y}_1 + z = 0,$$

(2)

where $c_1$ is the air damping vertically, $c_2$ is the rotational friction parameter, and $c_3$ is the horizontal damper parameter. The damping effects incurred by $c_2$ and $c_3$ are both nonlinear and dependent on the relative vibration displacement.

Fig. 10 Beneficial passive nonlinear damping property of the X-structure/mechanism, obtained due to the geometric nonlinearity\cite{56}, which is determined by the structure parameters, and very flexible in design: (a) a 2-layer X-mechanism with a horizontal damper; (b) displacement relationship during the mechanism motion.

It is shown that the introduced damping through the X-mechanism depends on the vertical compression displacement of the structure/mechanism, i.e., the nonlinear damping effect is bigger around the resonant frequencies (see Fig. 11(a)), and can be well tuned with the structure parameters\cite{56}. Such displacement-dependent nonlinear property has been theoretically shown as beneficial\cite{18-21} but never been achieved in practice with such a simple passive manner before. It can reduce the vibration peak, but will not deteriorate high-frequency vibration. Importantly, when vibration excitation is stronger, the nonlinear damping effect becomes more robust to the parameters and excitation changes. All these demonstrate superior properties over other linear or nonlinear counterparts (see Fig. 11(b))\cite{56}, and show that the X-structure (or limb-like structure, LLS) has a much better damping effect, dependent on the displacement and velocity in a beneficial nonlinear way.

More detailed discussions about the damping effect of the X-structures can be referred to Ref. [56]. Because of such a beneficial nonlinear damping property, together with the flexibility in design and implementation, nonlinear tuned mass dampers can be designed in a more powerful way. This will be discussed in the potential application of Section 7.

6 Beneficial nonlinear inertia (or equivalent mass)

With rotation units deliberately introduced into the X-structure/mechanism as discussed before (see Fig. 5), some more interesting nonlinear effects in terms of equivalent mass or inertia can therefore be obtained. A critical question is about the equivalent mass or inertia of an engineering system, i.e., whether the equivalent mass should be maintained to be constant or changed in a beneficial way. Considering vibration control, the idea comes from biological motion, e.g., human body walking or animal jumping. Arm swing or tail swing provides new motivations to proceed with such a new topic as studied in Refs. [52], [53], [54], and [57].
Fig. 11 Beneficial nonlinear damping effect\textsuperscript{[56]}: (a) measured equivalent damping coefficient at different frequencies; (b) performance comparison with other linear or nonlinear damping, showing that the X-structure (or LLS) has a much better damping effect which is dependent on the displacement and velocity in a beneficial nonlinear way (color online).

It is for the first time shown that a unique and beneficial nonlinear inertia characteristic can be obtained through a simple leverage mechanism coupled with the X-structure/mechanism\textsuperscript{[53–54]} (see Fig. 12, showing a human body inspired anti-vibration structure with nonlinear inertia, referred to as HBIAS-NI\textsuperscript{[54]}), which can significantly reduce the resonant frequency due to its very special nonlinear characteristic, i.e., smaller inertia (equivalent mass) for smaller vibration displacement but much bigger inertia (equivalent mass) for larger vibration displacement; and has obvious advantages over its linear counterparts (see Fig. 13).

In modeling, it can be seen that the nonlinear inertia effects are reflected not only in the equivalent mass but also in a special term — inertia incurred conservative force (see Eq. (3) with parameters referring to Fig. 12), which can act as a very excellent storage pool for excessive interactive force of the excitation input and output platform (see Fig. 14). The latter implies that this nonlinear force reduces the interactive force in compression but increases the interactive

Fig. 12 An arm-swing inspired anti-vibration system based on the X-mechanism\textsuperscript{[54]}, revealing a unique nonlinear inertia property very beneficial to vibration isolation: (a) an arm-swing inspired nonlinear inertia mechanism combined with a small embedded X-mechanism with spring and damping components applied for mimicking muscles and tendons; (b) the corresponding prototype (color online)
force in extension such that this reactive force can be maintained relatively stable compared with its linear counterparts, and thus help reduce vibration.

\[ M_1 \ddot{y} + M_2 \gamma^2 \left( \frac{\partial \theta}{\partial y} \right)^2 \dddot{y} = -M_2 \gamma^2 \frac{\partial \theta}{\partial y} \frac{\partial^2 \theta}{\partial y^2} \dddot{y}^2 + k(x + l_s) \frac{\partial x}{\partial y} \frac{\partial \ddot{y}}{\partial y} + (M_1 + M_2)g = -c \frac{\partial x}{\partial y} \frac{\partial \dot{y}}{\partial t}. \] (3)

It should be emphasized that the nonlinear inertial characteristic is relatively a new topic which could be explored more in further studies for different design mechanisms and different nonlinear forms. Note also in Eq. (3) that there are 4 nonlinear terms including nonlinear inertia \( M_2 \gamma^2 \left( \frac{\partial \theta}{\partial y} \right)^2 \), nonlinear stiffness \( k(x + l_s) \frac{\partial x}{\partial y} \frac{\partial \ddot{y}}{\partial y} \), nonlinear damping \( c \frac{\partial x}{\partial y} \frac{\partial \dot{y}}{\partial t} \), and nonlinear
inertia-incurred conservative force \( M_2 \gamma^2 \frac{\partial^2 \theta}{\partial y \partial \gamma} \frac{\partial \theta}{\partial y} \frac{\partial^2 \theta}{\partial \gamma^2} \). For more discussions, one may refer to Refs. [52], [53], [54], and [57].

7 Potential applications

In this section, several benchmark applications of the X-structures/mechanisms are discussed.

7.1 Passive vibration isolation (X-mount)

The X-structures/mechanisms can be well applied to various engineering systems for passive vibration isolation with an adjustable property. A benchmark application is the anti-vibration mounting technology which is greatly demanded in engineering for safety, quality, precision, noise reduction, and vibration control\(^{[1–3]}\). One critical issue of such mounting technology is to achieve high-quality vibration isolation in a very compact design with tunable stiffness to meet the space and changing load requirements. For this purpose, an X-cube technology is proposed in Ref. [55], demonstrating supervisor performance (see Fig. 15).

It can be seen that a resonant frequency about 1 Hz can be easily obtained with such an X-mount with a rod length of 10 cm and can bear with a relatively large vibration excitation in displacement up to 3 cm–5 cm.

Moreover, multi-DOF vibration isolation can also be done with the flexible X-structure approach. A steward platform with purely passive X-structured legs is presented in Ref. [44], and 3-DOF vibration isolation platforms are shown in Refs. [52], [58], and [76] for different purposes. Clearly, much more applications with advantageous performance can be obtained with this powerful X-structure approach.

7.2 Passive vibration absorption (X-absorber)

Due to the superior passive nonlinear damping characteristics that can be achieved with the X-structure/mechanism, an X-structured tuned mass damper (X-absorber) is proposed and discussed in Ref. [59] (see also Fig. 16). It is shown that the X-absorber can have some advantageous properties, including flexible to adjust resonant frequency, wider frequency band coverage in vibration suppression, less sensitive to parameter changes and excitation changes, more robust to inherent nonlinearity of master structures, etc. The X-absorber can be applied in the vertical direction as shown in Fig. 16 or the horizontal direction\(^{[59]}\), very flexible up to practical requirements.

7.3 X-dynamics based active vibration control

Nonlinear benefits in active vibration control or others have been extensively explored in the past years. However, nonlinear benefits are not only demonstrated in a better vibration control performance but much more. Some studies have thus been conducted in Refs. [60]–[64], and it is systematically shown that the following advantages are very much preferable in practice.

(i) Sufficient consideration of inherent nonlinearities can considerably reduce the burden in active controller development, and a simple linear controller could achieve very competitive performance\(^{[60–61]}\);

(ii) Noticeably, intentionally introduced nonlinearity in a carefully designed control scheme can significantly save practical control efforts (e.g., save energy cost) but without sacrificing vibration control performance\(^{[62–64]}\).

7.4 X-structured energy harvesting systems

Because of the advantages of the X-structures/mechanisms as discussed before, X-structured energy harvesting systems can then be developed, demonstrating much better energy harvesting efficiency. The nonlinear damping effect\(^{[65–66]}\), ultra-low and adjustable resonant frequency\(^{[67]}\), flexibility in structural design\(^{[68]}\), and multi-stable stiffness property\(^{[69]}\) can all be employed for improving harvesting performance. Prototypes and experimental validation demonstrate that these results can lead to a series of innovative technologies of great application potential\(^{[67,69]}\).
Fig. 15 Experimental results of the X-cube\textsuperscript{[55]}: (a) prototype manufactured with lightweight aluminum material with rotation joints and 2-layer X-mechanisms in parallel, where there is a pre-extension tuning mechanism at the bottom for adjusting the extension state of the horizontal spring $k_h = 6.680 \text{ N} \cdot \text{m}^{-1}$, and an additional horizontal spring $k_h = 160 \text{ N} \cdot \text{m}^{-1}$ supported by a U-shaped frame is used to construct the X-mechanism; (b) experimental and theoretical force-displacement curves of the X-cube, indicating a large QZS area; (c) vibration transmissibility for $M = 1 \text{ kg}$ exactly located in the QZS area, showing an excellent vibration isolation performance; (d), (e) vibration transmissibility for $M = 0.6 \text{ kg}$ and 1.1 kg when the system is not working in the QZS state before stiffness tuning; (g), (h) vibration transmissibility for $M = 0.6 \text{ kg}$ and 1.1 kg after stiffness tuning, indicating excellent performance; (f) experimental time response of the upper platform and the base excitation at different frequencies (color online).
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Fig. 16 X-absorber, demonstrating more advantageous performance over its counterparts: (a) prototype design; (b) prototype in testing; (c) comparison of different nonlinear absorbers, demonstrating that the X-absorber is less sensitive to parameter changes (color online)

Fig. 17 Control efforts in an active vehicle suspension system compared with a traditional controller[63], indicating an advantageous energy saving performance: (a) control force of the new method, significantly smaller than the traditional one; (b) control efforts in the frequency domain (color online)

7.5 X-structured sensor systems

Another innovative application of the X-structure approach is the design of X-structured sensor systems for measuring absolute vibration displacement or others. Due to the ultra-low resonant frequency that can be achieved, a fixed vibration-free point could be obtained at least for a wider frequency range above the resonant frequency and in single or multi-DOF. In terms of this advantage, innovative sensor systems for measuring absolute vibration displacement can therefore be developed with the X-structure/mechanism, and several prototypes were manufactured, for the first time validating the feasibility and sensitivity of this novel concept[30–31,70–72]. It is shown that, with appropriate signal processing methods, the measurement accuracy can be up to millimeter level for a very broad frequency range with pretty low implementation cost[70]. This series of studies present another unique insight into vibration displacement sensor design.

7.6 Others, discussions, and further topics

Vibration energy sinks, structural design of materials, noise absorption, and robotic design can all benefit from the X-structure/mechanism approach. An innovative anti-vibration exoskeleton for heavy-duty hand-held jackhammers is developed in Ref. [73], demonstrating superior vibration suppression performance over other methods including active control ones. A mobile robot with a novel suspension structure presents advantageous motion control stability
running on the rough ground\cite{32}. Some other recent development and applications can also be referred to Refs. \cite{47}, \cite{74}, and \cite{75}.

The X-structure/mechanism approach potentially presents one of the most effective designs and one of the most efficient ways in practice for exploiting nonlinearities in engineering, and some variants can be easily developed with a similar geometric nonlinearity such as X-shape, Z-shape, S-shape, L-shape, V-shape, polygon, and square. Therefore, more innovative results can be further developed with an inspiration from the X-structure/mechanism approach, especially for those which may not take a full X-shape but a variant.

The flexibility, adjustability, efficiency, and easiness in nonlinear stiffness manipulation are several noticeable and essential features of the X-structure/mechanism and its variants, which can well address the aforementioned issues existing in the literature and engineering practices. Compared with several other existing nonlinear methods, the nonlinearity provided by the X-structure/mechanism is completely controllable and designable not only because of available accurate mathematical models but also with respect to practical implementation (e.g., manufacturing, assembly, material selection, parameter selection, and fatigue issues). For some detailed comparisons with other literature results, it can be referred to those published re-
sults in Refs. [15] and [41]–[59]. Besides the benchmark case studies in this paper, more useful applications can definitely be developed in the future.

Although noticeable milestones have been achieved for this approach, there are still some open problems existing for further investigation. Several of those are listed here, which can be regarded as a reference in future research and development efforts.

(i) Micro/nano-level X-structures and their potential beneficial nonlinearity and applications could present new nonlinear features owing to couplings with material nonlinearities, and may bring new materials or structural designs for sound and vibration control and sensor systems.

(ii) Different variants of the X-structure/mechanism can be combined together such that a very compact vibration control unit of multi-directions/DOF can be obtained [44, 58, 76].

(iii) The X-structure/mechanism can be combined with some other useful linear or nonlinear stiffness/damping components/materials (e.g., metal-materials, origami structures, etc.) to achieve much more advantageous nonlinear stiffness or damping systems. This can partially be referred to the X-mount/absorber or others as shown in Refs. [55] and [59] and those references included there.

(iv) Active control components with electromagnetic or other methods can be introduced into the X-structure/mechanism to obtain much better vibration control performance for a better balance among structure stability in different directions.

(v) Various potential applications in real engineering systems can always be explored as those examples demonstrated in this paper. Except vibration control, robotic systems would definitely benefit from these mechanic studies [32]. Active vibration control or other robust control with these nonlinear dynamics would be a very promising topic for energy saving and thus carbon neutralization [60–63].

In manufacturing, the X-mechanisms would involve some rotational joints, bearings, or sliders for ensuring the desired motion of the designed mechanism. For example, in Fig. 8 or Fig. 12, a vertical slider for ensuring vertical motion is simply applied, which would bring a friction to the system. This friction can be reduced with linear bearing (or others) instead of sliding on smooth surface (the latter is a typical Coulomb friction). When the payload is bigger, the damping effect due to such a slider would become smaller, and sometimes more friction effects are expected to increase the overall damping of the system. A slider can also be introduced horizontally as those in Fig. 2 and Fig. 15. However, the significance is that the friction in this way will be introduced into the system and become a beneficial nonlinear damping effect owing to the X-structure/mechanism. These can be clearly seen in the modeling and analysis results such as Refs. [53] and [56]. From Fig. 10 and Fig. 12, it can be seen that more damping effects can be introduced into a beneficial way with a traditional damper. Traditional dampers are always limited by its high friction force to overcome because a lower payload vibration isolation system might not work for low-frequency excitation. Through the X-structure/mechanism with a horizontal damper, the working frequency could be effectively reduced without sacrificing a desired damping effect [77]. More beneficial and nonlinear damping effects due to the X-mechanisms with special design features, actively controlled components, or surface/material treatments can be further explored as another open problem.

8 Conclusions

The X-structure/mechanism discussed in the present paper is generally referred to as a class of scissor-like, quadrilateral or polygon-like structures, or several-bar linkage mechanisms, which basically form different layers (half-layer, 1-layer, or n-layer) with basic elements taking X-shape, quadrilateral, V-shape (a half of the X-shape or quadrilateral), polygon, Z-shape, etc. and appropriately coupled with some other linear/nonlinear active/passive mechanical components, e.g., stiffness and/or damping elements. These basic elements of the
X-structure/mechanism are all featured with a similar geometrical nonlinearity in deformation/motion modeling, thus leading to the stiffness/damping/inertial nonlinearities as shown. They can be used repetitively to form an n-layer structure/mechanism in one or several directions or combined together to form some other variant structures or mechanisms.

The X-structure/mechanism approach has been studied in recent years for the purpose of passive employment of nonlinearity being extensively demanded in engineering systems for achieving advantageous performance with the minimum implementation cost and complexity. Since nonlinearity is ubiquitous, working with it in a proactive and reciprocal way is the best philosophy. The X-structure/mechanism is shown to be capable to provide beneficial and designable nonlinear properties required for stiffness (positive, QZS, zero, negative, multi-stable, high-loading), damping (dependent on vibration displacement), and/or inertia designs (dependent on vibration displacement) in various engineering applications. Therefore, those results introduced in this paper serve as an up-to-date summary of those recent advances of this topic, and present a relatively comprehensive perspective on this new methodology and its various potentials. It is believed that the X-structure/mechanism approach presents a novel insight into modeling, understanding, and design of beneficial nonlinearities in engineering systems, and will definitely inspire a series of innovative research and development for meeting challenging requirements from various engineering fields. Although this series of studies have been done for about 10 years ever since 2010, potential research and development topics together with new findings and new understandings could be explored more, targeting a much greener, safer, more reliable, and more sustainable future of engineering fields.

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