Fixed-Order H-infinity Optimization of Time-Delay Systems

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Summary. H-infinity controllers are frequently used in control theory due to their robust performance and stabilization. Classical H-infinity controller synthesis methods for finite dimensional LTI MIMO plants result in high-order controllers for high-order plants whereas low-order controllers are desired in practice. We design fixed-order H-infinity controllers for a class of time-delay systems based on a non-smooth, non-convex optimization method and a recently developed numerical method for H-infinity norm computations.

Robust control techniques are effective to achieve stability and performance requirements under model uncertainties and exogenous disturbances [16]. In robust control of linear systems, stability and performance criteria are often expressed by H-infinity norms of appropriately defined closed-loop functions including the plant, the controller and weights for uncertainties and disturbances. The optimal H-infinity controller minimizing the H-infinity norm of the closed-loop functions for finite dimensional multi-input-multi-output (MIMO) systems is computed by Riccati and linear matrix inequality (LMI) based methods [8, 9]. The order of the resulting controller is equal to the order of the plant and this is a restrictive condition for high-order plants. In practical implementations, fixed-order controllers are desired since they are cheap and easy to implement in hardware and non-restrictive in sampling rate and bandwidth. The fixed-order optimal H-infinity controller synthesis problem leads to a non-convex optimization problem. For certain closed-loop functions, this problem is converted to an interpolation problem and the interpolation function is computed based on continuation methods [1]. Recently fixed-order H-infinity controllers are successfully designed for finite dimensional LTI MIMO plants using a non-smooth, non-convex optimization method [10]. This approach allows the user to choose the controller order and tunes the parameters of the controller to minimize the H-infinity norm of the objective function using the norm value and its derivatives with respect to the controller parameters. In our work, we design fixed-order H-infinity controllers for a class of...
time-delay systems based on a non-smooth, non-convex optimization method and a recently developed H-infinity norm computation method [13].

1 Problem Formulation

We consider time-delay plant $G$ determined by equations of the form,

\[
\begin{align*}
\dot{x}(t) &= A_0 x(t) + \sum_{i=1}^{m} A_i x(t - \tau_i) + B_1 w(t) + B_2 u(t - \tau_{m+1}) \\
z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\
y(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t - \tau_{m+2}).
\end{align*}
\]

where all system matrices are real with compatible dimensions and $A_0 \in \mathbb{R}^{n \times n}$. The input signals are the exogenous disturbances $w$ and the control signals $u$. The output signals are the controlled signals $z$ and the measured signals $y$. All system matrices are real and the time-delays are positive real numbers. In robust control design, many design objectives can be expressed in terms of norms of closed-loop transfer functions between appropriately chosen signals $w$ to $z$.

The controller $K$ has a fixed-structure and its order $n_K$ is chosen by the user *apriori* depending on design requirements,

\[
\begin{align*}
\dot{x}_K(t) &= A_K x_K(t) + B_K y(t) \\
u(t) &= C_K x_K(t)
\end{align*}
\]

where all controller matrices are real with compatible dimensions and $A_K \in \mathbb{R}^{n_K \times n_K}$.

By connecting the plant $G$ and the controller $K$, the equations of the closed-loop system from $w$ to $z$ are written as,

\[
\begin{align*}
\dot{x}_{cl}(t) &= A_{cl,0} x_{cl}(t) + \sum_{i=1}^{m+2} A_{cl,i} x_{cl}(t - \tau_i) + B_{cl} w(t) \\
z(t) &= C_{cl} x_{cl}(t) + D_{cl} w(t)
\end{align*}
\]

where

\[
\begin{align*}
A_{cl,0} &= \begin{pmatrix} A_0 \\ B_K C_2 A_K \end{pmatrix}, & A_{cl,i} &= \begin{pmatrix} A_i \\ 0 \end{pmatrix} & \text{for } i = 1, \ldots, m, \\
A_{cl,m+1} &= \begin{pmatrix} 0 & B_2 C_K \\ 0 & 0 \end{pmatrix}, & A_{cl,m+2} &= \begin{pmatrix} 0 \\ 0 & B_K D_{22} C_K \end{pmatrix}, \\
B_{cl} &= \begin{pmatrix} B_1 \\ B_K D_{21} \end{pmatrix}, & C_{cl} &= \begin{pmatrix} C_1 \\ D_{12} C_K \end{pmatrix}, & D_{cl} &= D_{21}.
\end{align*}
\]

The closed-loop matrices contain the controller matrices $(A_K, B_K, C_K)$ and these matrices can be tuned to achieve desired closed-loop characteristics.

The transfer function from $w$ to $z$ is,

\[
T_{zw}(s) = C_{cl} \left( sI - A_{cl,0} - \sum_{i=1}^{m+2} A_{cl,i} e^{-\tau_i s} \right)^{-1} B_{cl} + D_{cl}
\]
and we define fixed-order H-infinity optimization problem as the following.

**Problem** Given a controller order $n_K$, find the controller matrices $(A_K, B_K, C_K)$ stabilizing the system and minimizing the H-infinity norm of the transfer function $T_{zw}$.

## 2 Optimization Problem

### 2.1 Algorithm

The optimization algorithm consists of two steps:

1. **Stabilization**: minimizing the spectral abscissa, the maximum real part of the characteristic roots of the closed-loop system. The optimization process can be stopped when the controller parameters are found that stabilizes $T_{zw}$ and these parameters are the feasible points for the H-infinity optimization of $T_{zw}$.

2. **H-infinity optimization**: minimizing the H-infinity norm of $T_{zw}$ using the starting points from the stabilization step.

If the first step is successful, then a feasible point for the H-infinity optimization is found, i.e., a point where the closed-loop system is stable. If in the second step the H-infinity norm is reduced in a quasi-continuous way, then the feasible set cannot be left under mild controllability/observability conditions.

Both objective functions, the spectral abscissa and the H-infinity norm, are non-convex and not everywhere differentiable but smooth almost everywhere [15]. Therefore we choose a hybrid optimization method to solve a non-smooth and non-convex optimization problem, which has been successfully applied to design fixed-order controllers for the finite dimensional MIMO systems [10].

The optimization algorithm searches for the local minimizer of the objective function in three steps [5]:

1. A quasi-Newton algorithm (in particular, BFGS) provides a fast way to approximate a local minimizer [12],
2. A local bundle method attempts to verify local optimality for the best point found by BFGS,
3. If this does not succeed, gradient sampling [6] attempts to refine the approximation of the local minimizer, returning a rough optimality measure.

The non-smooth, non-convex optimization method requires the evaluation of the objective function, in the second step this is the H-infinity norm of $T_{zw}$ and the gradient of the objective function with respect to controller parameters where it exists. Recently a predictor-corrector algorithm has been developed to compute the H-infinity norm of time-delay systems [13]. We computed the gradients using the derivatives of singular values at frequencies where the H-infinity norm is achieved. Based on the evaluation of the objective function and its gradients, we apply the optimization method to compute fixed-order controllers. The computation of H-infinity norm of time-delay systems (8) is discussed in the following section.
2.2 Computation of the H-infinity Norm

We implemented a predictor-corrector type method to evaluate H-infinity norm of $T_{zw}$ in two steps (for details we refer to [13]):

- **Prediction step**: we calculate the approximate H-infinity norm and corresponding frequencies where the highest peak values in the singular value plot occur.
- **Correction step**: we correct the approximate results from the predicted step.

### Theoretical Foundation

The following theorem generalizes the well-known relation between the existence of the singular values of the transfer function equal to the fixed value and the existence of the imaginary axis eigenvalues of a corresponding Hamiltonian matrix [7] to the time-delay systems:

**Theorem 1.** [13] Let $\xi > 0$ be such that the matrix 

$$ D_\xi := D_\xi^T D_{cl} - \xi^2 I $$

is non-singular and define $\tau_{\text{max}}$ as the maximum of the delays $(\tau_1, \ldots, \tau_{m+2})$. For $\omega \geq 0$, the matrix $T_{zw}(j\omega)$ has a singular value equal to $\xi > 0$ if and only if $\lambda = j\omega$ is an eigenvalue of the linear infinite dimensional operator $L_\xi$ on $X := C([-\tau_{\text{max}}, \tau_{\text{max}}], \mathbb{C}^{2n})$ which is defined by

$$ D(L_\xi) = \{ \phi \in X : \phi' \in X, \phi'(0) = M_0 \phi(0) + \sum_{i=1}^{m+2} (M_i \phi(-\tau_i) + M_{-i} \phi(\tau_i)) \}, \quad (9) $$

$$ L_\xi \phi = \phi', \phi \in D(L_\xi) \quad (10) $$

with

$$ M_0 = \begin{bmatrix} A_{cl,0} - B_{cl} D_\xi^{-1} D_{cl}^T C_{cl} & -B_{cl} D_\xi^{-1} B_{cl}^T \\ \xi^2 C_{cl}^T D_\xi^{-1} C_{cl} & -A_{cl,0}^T C_{cl}^T D_\xi^{-1} B_{cl}^T \end{bmatrix}, $$

$$ M_i = \begin{bmatrix} A_{cl,i} & 0 \\ 0 & 0 \end{bmatrix}, \quad M_{-i} = \begin{bmatrix} 0 & 0 \\ 0 & -A_{cl,i}^T \end{bmatrix}, \quad 1 \leq i \leq m+2. $$

By Theorem 1, the computation of H-infinity norm of $T_{zw}$ can be formulated as an eigenvalue problem for the linear operator $L_\xi$.

**Corollary 1.**

$$ \|T_{zw}\|_\infty = \sup \{ \xi > 0 : \text{operator } L_\xi \text{ has an eigenvalue on the imaginary axis} \} $$

Conceptually Theorem 1 allows the computation of H-infinity norm via the well-known level set method [2, 4]. However, $L_\xi$ is an infinite dimensional operator. Therefore, we compute the H-infinity norm of the transfer function $T_{zw}$ in two steps:

1) The prediction step is based on a matrix approximation of $L_\xi$. 

2) The correction step is based on reformulation of the eigenvalue problem of \( L_\xi \) as a nonlinear eigenvalue problem of a finite dimension.

The approximation of the linear operator \( L_\xi \) and the corresponding standard eigenvalue problem for Corollary 1 is given in Section 2.3. The correction algorithm of the approximate results in the second step is explained in Section 2.4.

### 2.3 Prediction Step

The infinite dimensional operator \( L_\xi \) is approximated by a matrix \( L^N_\xi \). Based on the numerical methods for finite dimensional systems [2, 4], the \( H\)-infinity norm of the transfer function \( T_{zw} \) can be computed approximately as

**Corollary 2.**

\[
\|T_{zw}\|_\infty \approx \sup\{\xi > 0 : \text{operator } L^N_\xi \text{ has an eigenvalue on the imaginary axis}\}.
\]

The infinite-dimensional operator \( L_\xi \) is approximated by a matrix using a spectral method (see, e.g. [3]). Given a positive integer \( N \), we consider a mesh \( \Omega_N \) of \( 2N + 1 \) distinct points in the interval \([-\tau_{\max}, \tau_{\max}]\):

\[
\Omega_N = \{\theta_{N,i}, \ i = -N, \ldots, N\}, \quad (11)
\]

where

\[-\tau_{\max} \leq \theta_{N,-N} < \ldots < \theta_{N,0} = 0 < \ldots < \theta_{N,N} \leq \tau_{\max}.
\]

This allows to replace the continuous space \( X \) with the space \( X_N \) of discrete functions defined over the mesh \( \Omega_N \), i.e. any function \( \phi \in X \) is discretized into a block vector

\[
x = [x^T_{-N} \cdots x^T_N]^T \in X_N
\]

with components

\[
x_i = \phi(\theta_{N,i}) \in \mathbb{C}^{2n}, \quad i = -N, \ldots, N.
\]

Let \( P_Nx, \ x \in X_N \) be the unique \( \mathbb{C}^{2n} \) valued interpolating polynomial of degree \( \leq 2N \) satisfying

\[
P_Nx(\theta_{N,i}) = x_i, \quad i = -N, \ldots, N.
\]

In this way, the operator \( L_\xi \) over \( X \) can be approximated with the matrix \( L^N_\xi : X_N \to X_N \), defined as

\[
\left(L^N_\xi x\right)_i = (P_Nx)'(\theta_{N,i}), \quad i = -N, \ldots, -1, 1, \ldots, N,
\]

\[
\left(L^N_\xi x\right)_0 = M_0P_Nx(0) + \sum_{i=1}^{m+2} (M_iP_Nx(-\tau_i) + M_{-i}P_Nx(\tau_i)).
\]

Using the Lagrange representation of \( P_Nx \),

\[
P_Nx = \sum_{k=-N}^N l_{N,k} \ x_k,
\]
where the Lagrange polynomials $l_{N,k}$ are real valued polynomials of degree $2N$ satisfying

$$l_{N,k}(\theta_{N,i}) = \begin{cases} 1 & i = k, \\ 0 & i \neq k, \end{cases}$$

we obtain the explicit form

$$L^N_{\xi} = \begin{bmatrix} d_{-N,-N} & \ldots & d_{-N,N} \\ \vdots & & \vdots \\ d_{1,-N} & \ldots & d_{1,N} \\ a_{-N} & \ldots & a_N \\ d_{N,-N} & \ldots & d_{N,N} \end{bmatrix} \in \mathbb{R}^{(2N+1)(2n)\times(2N+1)2n},$$

where

$$d_{i,k} = l'_{N,k}(\theta_{N,i})I, \quad i, k \in \{-N, \ldots, N\}, i \neq 0$$

$$a_0 = M_0 x_0 + \sum_{k=1}^{m+2} (M_k l_{N,0}(-\tau_k) + M_{-k} l_{N,0}(\tau_k)),$$

$$a_i = \sum_{k=1}^{m+2} (M_k l_{N,i}(-\tau_k) + M_{-k} l_{N,i}(\tau_k)), \quad k \in \{-N, \ldots, N\}, k \neq 0.$$

### 2.4 Correction Step

By using the finite dimensional level set methods, the largest level set $\xi$ where $L^N_{\xi}$ has imaginary axis eigenvalues and their corresponding frequencies are computed. In the correction step, these approximate results are corrected by using the property that the eigenvalues of the $L^N_{\xi}$ appear as solutions of a finite dimensional nonlinear eigenvalue problem. The following theorem establishes the link between the linear infinite dimensional eigenvalue problem for $L^N_{\xi}$ and the nonlinear eigenvalue problem.

**Theorem 2.** [13] Let $\xi > 0$ be such that the matrix

$$D_{\xi} := D_{cl}^T D_{cl} - \xi^2 I$$

is non-singular. Then, $\lambda$ is an eigenvalue of linear operator $L^N_{\xi}$ if and only if

$$\det H_{\xi}(\lambda) = 0, \quad (12)$$

where

$$H_{\xi}(\lambda) := \lambda I - M_0 - \sum_{i=1}^{m+2} (M_i e^{-\lambda \tau_i} + M_{-i} e^{\lambda \tau_i}) \quad (13)$$

and the matrices $M_0, M_i, M_{-i}$ are defined in Theorem 1.

The correction method is based on the property that if $\hat{\xi} = \|T_{zw}(j\omega)\|_{\infty}$, then (13) has a multiple non-semisimple eigenvalue. If $\hat{\omega} \geq 0$ and $\hat{\xi} \geq 0$ are such that

$$\|T_{zw}(j\omega)\|_{H_{\infty}} = \hat{\xi} = \sigma_1(T_{zw}(j\hat{\omega})),$$

then setting

$$h_{\xi}(\lambda) = \det H_{\xi}(\lambda),$$

the pair $(\hat{\omega}, \hat{\xi})$ satisfies
Fig. 1. (left) Intersections of the singular value plot of \( T_{zw}(j\omega) \) with the horizontal line \( \xi = c \), for \( c < \hat{\xi} \) (top), \( c = \hat{\xi} \) (middle) and \( c > \hat{\xi} \) (bottom). (right) Corresponding eigenvalues of \( H_\xi(\lambda) \) (13).

\[
h_\xi(j\omega) = 0, \quad h_\xi'(j\omega) = 0. \tag{15}
\]

This property is clarified in Figure 1.

The drawback of working directly with (15) is that an explicit expression for the determinant of \( H_\xi \) is required. This scalar-valued condition can be equivalently expressed in a matrix-based formulation:

\[
\begin{align*}
H(j\omega, \xi) \begin{bmatrix} u \\ v \end{bmatrix} &= 0, \quad n(u, v) = 0, \\
3 \left\{ v^* \left( I + \sum_{i=1}^{m+2} A_{cl,i} e^{-j\omega\tau_i} \right) u \right\} &= 0
\end{align*}
\tag{16}
\]

where \( n(u, v) = 0 \) is a normalizing condition. The approximate H-infinity norm and its corresponding frequencies can be corrected by solving (16). For further details, see [13].

2.5 Computing the Gradients

The optimization algorithm requires the derivatives of H-infinity norm of the transfer function \( T_{zw} \) with respect to the controller matrices whenever it is differentiable. Define the H-infinity norm of the function \( T_{zw} \) as

\[
f(A_{cl,0}, \ldots, A_{cl,m+2}, B_{cl}, C_{cl}, D_{cl}) = \| T_{zw}(j\omega) \|_\infty.
\]

These derivatives exist whenever there is a unique frequency \( \hat{\omega} \) such that (14) holds, and, in addition, the largest singular value \( \hat{\xi} \) of \( T_{zw}(j\hat{\omega}) \) has multiplicity one. Let \( w_l \) and \( w_r \) be the corresponding left and right singular vector, i.e.
When defining \( \frac{\partial f}{\partial A_{cl,0}} \) as a \( n \times n \) matrix whose \((k, l)\)-th element is the derivative of \( f \) with respect to the \((k, l)\)-th element of \( A_{cl,0} \), and defining the other derivatives in a similar way, the following expressions are obtained [14]:

\[
\frac{\partial f}{\partial A_{cl,0}} = \Re (M(j\omega)^* C^T_d w_i w_r^* B_d^T M(j\omega)^*),
\]

\[
\frac{\partial f}{\partial A_{cl,i}} = \frac{\Re (M(j\omega)^* C^T_d w_i w_r^* B_d^T M(j\omega)^* e^{j\omega \tau_i})}{w_i^* w_r}
\text{for } i = 1, \ldots, m + 2,
\]

\[
\frac{\partial f}{\partial B_d} = \frac{\Re (w_i w_r^* B_d^T M(j\omega)^*)}{w_i^* w_r},
\]

\[
\frac{\partial f}{\partial C_d} = \frac{\Re (w_i w_r^* B_d^T M(j\omega)^*)}{w_i^* w_r},
\]

where \( M(j\omega) = \left(j \omega I - A_{cl,0} - \sum_{i=1}^{m+2} A_{cl,i} e^{-j \omega \tau_i}\right)^{-1} \).

We compute the gradients with respect to the controller matrices as

\[
\frac{\partial f}{\partial A_K} = \begin{bmatrix} 0_{n_K \times n} & I_{n_K} \end{bmatrix} \frac{\partial f}{\partial A_{cl,0}} \begin{bmatrix} 0_{n \times n_K} & I_{n_K} \end{bmatrix},
\]

\[
\frac{\partial f}{\partial B_d} = \begin{bmatrix} 0_{n_K \times n} & I_{n_K} \end{bmatrix} \frac{\partial f}{\partial A_{cl,0}} \begin{bmatrix} I_n & 0_{n \times n_K} \end{bmatrix} C^T_d
\]

\[
+ \begin{bmatrix} 0_{n_K \times n} & I_{n_K} \end{bmatrix} \frac{\partial f}{\partial A_{cl,1:m+2}} \begin{bmatrix} 0_{n \times n_K} & I_{n_K} \end{bmatrix} C^T_d D_{22} + \begin{bmatrix} 0_{n_K \times n} & I_{n_K} \end{bmatrix} \frac{\partial f}{\partial B_d} D_{21}^T,
\]

\[
\frac{\partial f}{\partial C_d} = \begin{bmatrix} B_d^T & 0_{n \times n_K} \end{bmatrix} \frac{\partial f}{\partial A_{cl,0}} \begin{bmatrix} 0_{n_K \times n} & I_{n_K} \end{bmatrix}
\]

\[
+ D_{22}^T B_d^T \begin{bmatrix} 0_{n_K \times n} & I_{n_K} \end{bmatrix} \frac{\partial f}{\partial A_{cl,1:m+2}} \begin{bmatrix} 0_{n \times n_K} & I_{n_K} \end{bmatrix} + D_{21}^2 \frac{\partial f}{\partial C_d} \begin{bmatrix} 0_{n \times n_K} & I_{n_K} \end{bmatrix}
\]

where the matrices \( I_n, I_{n_K} \) and \( 0_{n \times n_K}, 0_{n_K \times n} \) are identity and zero matrices.

3 Examples

We consider the time-delay system with the following state-space representation,

\[
\dot{x}(t) = -x(t) - 0.5x(t-1) + w(t) + u(t),
\]

\[
z(t) = x(t) + u(t),
\]

\[
g(t) = x(t) + w(t).
\]

We designed the first-order controller, \( n_K = 1 \),

\[
\dot{x}_K(t) = 3.61 x_K(t) + 1.39 y(t),
\]

\[
u(t) = -0.83 x_K(t)
\]
achieving the closed-loop H-infinity norm 0.064. The closed-loop H-infinity norms of fixed-order controllers for \( n_K = 2 \) and \( n_K = 3 \) are 0.021 and 0.020 respectively.

Our second example is a 4th-order time-delay system. The system contains 4 delays and has the following state-space representation,

\[
\begin{align*}
\dot{x}(t) &= \begin{pmatrix} -4.4656 & -0.4271 & 0.4427 & -0.1854 \\ -0.8601 & -5.6257 & 0.8577 & -0.5210 \\ 0.9001 & -0.7177 & -6.5358 & 0.0417 \\ -0.0836 & 0.0242 & 0.4997 & -3.5618 \end{pmatrix} x(t) + \begin{pmatrix} 0.6848 & -0.0618 & 0.5399 & 0.5057 \\ 0.3259 & -0.3810 & 0.6592 & -0.0066 \\ 0.6325 & 0.3752 & 0.4122 & 0.7303 \\ 0.5878 & 0.9737 & 0.1907 & -0.8639 \end{pmatrix} x(t-3.2) \\
&+ \begin{pmatrix} 0.9371 & -0.7859 & 0.1332 & 0.7429 \\ -0.8025 & 0.4483 & 0.6226 & 0.0152 \\ 0.0940 & 0.2274 & 0.1536 & 0.5776 \\ -0.1941 & 0.5659 & 0.8881 & -0.0539 \end{pmatrix} x(t-3.4) + \begin{pmatrix} 0.6576 & -0.8543 & -0.3460 & 0.6415 \\ -0.3550 & 0.5024 & 0.6081 & 0.9038 \\ 0.9523 & 0.6624 & 0.0765 & -0.8475 \\ -0.4436 & 0.8447 & -0.0734 & 0.4173 \end{pmatrix} x(t-3.9) \\
&+ \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} w(t) + \begin{pmatrix} 0.2 \\ -1 \\ 0.1 \\ -0.4 \end{pmatrix} u(t - 0.2) \\
z(t) &= \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0.1 & 1 \\ -1 & 0.2 \end{pmatrix} w(t) + \begin{pmatrix} 1 \\ -1 \end{pmatrix} u(t) \\
y(t) &= \begin{pmatrix} 1 & 0 & -1 & 0 \end{pmatrix} x(t) + (-2 & 0.1) w(t) + 0.4u(t - 0.2)
\end{align*}
\]

When \( n_K = 1 \), our method finds the controller achieving the closed-loop H-infinity norm 1.2606,

\[
\dot{x}_K(t) = -0.712x_K(t) - 0.1639y(t),
\]
\[
u(t) = -0.2858x_K(t)
\]
and the results for \( n_K = 2 \) and \( n_K = 3 \) are 1.2573 and 1.2505 respectively.

4 Concluding Remarks

We successfully designed fixed-order H-infinity controllers for a class of time-delay systems. The method is based on non-smooth, non-convex optimization techniques and allows the user to choose the controller order as desired. Our approach can be extended to general time-delay systems. Although we illustrated our method for a dynamic controller, it can be applied to more general controller structures. The only requirement is that the closed-loop matrices should depend smoothly on the controller parameters. On the contrary, the existing controller design methods optimizing the closed-loop H-infinity norm are based on Lyapunov theory and linear matrix inequalities, which are conservative if the form of the Lyapunov functions are restricted and requires full state information.

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