Curvature-driven diffusive and dynamo actions in vortex unstretched filaments from solar prominences data

by

L.C. Garcia de Andrade

Departamento de Física Teórica – IF – Universidade do Estado do Rio de Janeiro-UERJ
Rua São Francisco Xavier, 524
Cep 20550-003, Maracanã, Rio de Janeiro, RJ, Brasil
Electronic mail address: garcia@dft.if.uerj.br

Abstract

Examples of the use of Frenet curvature-driven motion of vortex unstretched filamentary diffusion processes in plasmas are investigated. The first example addresses the unstretched filaments which are embedded in a steady plasma flow. The particle number density of a weakly ionized plasma is shown to be proportional to the total Frenet curvature. The particle number does not decays in plasma along the filaments and is maintained against diffusion losses. This relation is tested against the solar prominence data for particle density of $10^{19} cm^{-3}$ and height of the order of $10^{10} cm$ and non-thermal velocities of the order of $10^{5} cm.s^{-1}$. Making use of a molecular diffusion constant of $10^{6} cm^2 s^{-1}$ one obtains a Frenet curvature of the solar loops of the order of $10^{-8} cm^{-1}$ which coincides with the twist (torsion) of the helical model where torsion coincides with curvature. The second one handles a generalization of the Grunzig et al [Phys. Plasmas 1 (1),259 (1995)] to the case a nonvanishing plasma velocity besides the auxiliary medium velocity considered by them. It is shown that the filaments are necessary torsion-free for the condition that the plasma velocity has the same direction of the auxiliary velocity flow. In this last example no dynamo action is present and diffusion action is fully dominant despite of the difference in velocities of plasma and auxiliary flows. This can be better understood by the unstretched carachther of the filaments despite of its folding or curvature.
I Introduction

Filamentary structures have been present [1, 2] in many physical and biological phenomena as bio-actine, solar and plasma filaments and dynamo filaments [3]. Curvature-driven motion [4] along the vortex filament or thin tubes filled with plasma flow is found present in solar coronal regions and prominences [5, 6]. Diffusion processes were thoroughly investigated by S. Molchanov [7] in the seventies in Riemannian geometry and more recently diffusion processes have played a fundamental role in plasma physics, mainly in the case where plasma lifetime [8] is determined by diffusion rate which varies by an order of magnitude inside a hollow plasma pre-ionized by a laser beam. Another interesting plasma diffusion application has been recently given by Chen [8], by investigating the solutions of diffusion-convection equations which are important for magnetic plasma confinement. In this report one is able to investigate two new examples of the application of the vortex filaments in solar prominences and electrons density data is used to obtain the curvature of the solar loop which coincides with the twist obtained by Lopez-Fuentes et al [9] of the magnetic twist. Another example of diffusion in plasmas is given by the case of auxiliary velocity in semi-ideal magnetohydrodynamics which shows that as long as the constraints on the filament are given by a torsion-free or planar filaments which appears frequently in tokamaks. However the new feature here is that contrary to the work by Grunzig et al [10] is that here the plasma velocity is nonzero and diffusion is not a pure one. Therefore as a consequence of the filamentary [9] magnetic diffusion in solar loops, flow perturbation is fully determined from the magnetic twist. The paper is organized as follows: In section 2 a brief review on holonomic Frenet frame is presented. In section 3 the magnetic diffusion equation is solved in this frame for solar loops and data is seen to be compatible with the model. Section 4 presents the generalized Grunzig et al model for plasma filaments. Conclusions and future prospects are presented in section 5.
II Plasma vortex filaments in holonomic Frenet frame

This section addresses a small review of the holonomic frame [11] equations that are specially useful in the investigation of vortex filaments in MHD in homogeneous and isotropic flows as well as isotropic magnetic diffusion. Since the magnetic filaments, considered here, might possess Frenet torsion and curvature [11], which completely determine topologically the filaments, one needs some dynamical relations from vector analysis and differential geometry of curves such as the Frenet frame $(\vec{t}, \vec{n}, \vec{b})$ equations

\[
\begin{align*}
\vec{t}' &= \kappa \vec{n} \tag{II.1} \\
\vec{n}' &= -\kappa \vec{t} + \tau \vec{b} \tag{II.2} \\
\vec{b}' &= -\tau \vec{n} \tag{II.3}
\end{align*}
\]

The holonomic dynamical relations from vector analysis and differential geometry of curves by $(\vec{t}, \vec{n}, \vec{b})$ equations in terms of time

\[
\begin{align*}
\dot{\vec{t}} &= [\kappa' \vec{b} - \kappa \tau \vec{n}] \tag{II.4} \\
\dot{\vec{n}} &= \kappa \tau \vec{t} \tag{II.5} \\
\dot{\vec{b}} &= -\kappa \vec{t} \tag{II.6}
\end{align*}
\]

along with the flow derivative

\[
\dot{\vec{t}} = \partial_\vec{t} \vec{t} + (\vec{v}.\nabla)\vec{t} \tag{II.7}
\]

From these equations and the generic flow [12]

\[
\vec{X} = v_s \vec{t} + v_n \vec{n} + v_b \vec{b} \tag{II.8}
\]

one obtains

\[
\frac{\partial l}{\partial \vec{t}} = (-\kappa v_n + v_s') l \tag{II.9}
\]

where $l$ is given by

\[
l := (\vec{X}' . \vec{X}')^{\frac{1}{2}} \tag{II.10}
\]

which shows that if $v_s$ is constant, which fulfills the solenoidal incompressible flow

\[
\nabla . \vec{v} = 0 \tag{II.11}
\]
and \( v_n \) vanishes, one should have an unstretched plasma vortex filament. Is exactly this choice \( \vec{v} = v_s \vec{t} \), of steady flow one might choose here. Before solving the magnetic diffusive equation in the Frenet, let us consider the form of filaments

\[
\partial_t \vec{N} + (\vec{v} \cdot \nabla) \vec{N} = D \nabla^2 \vec{N} \tag{II.12}
\]

where the parameter \( D \) is the here considered constant diffusion coefficient since one is considering weakly ionized plasmas, and \( N \) is the particle number density of electrons or ionized particles in plasmas. The solution

\[
\vec{B} = B(s) \vec{t} \tag{II.13}
\]

shall be considered here. This definition of magnetic filaments is shows from the solenoidal character of the magnetic field

\[
\nabla \cdot \vec{B} = 0 \tag{II.14}
\]

and \( B_s \) is constant along the filaments. In the next section one shall solve the diffusion equation in the steady case in the non-holonomic Frenet frame as

\[
\frac{\partial}{\partial n} \vec{t} = \theta_{ns} \vec{n} + [\Omega_b + \tau] \vec{b} \tag{II.15}
\]

\[
\frac{\partial}{\partial n} \vec{n} = -\theta_{ns} \vec{t} - (\text{div} \vec{b}) \vec{n} \tag{II.16}
\]

\[
\frac{\partial}{\partial n} \vec{b} = -[\Omega_b + \tau] \vec{t} - (\text{div} \vec{b}) \vec{n} \tag{II.17}
\]

\[
\frac{\partial}{\partial b} \vec{t} = \theta_{bs} \vec{b} - [\Omega_n + \tau] \vec{n} \tag{II.18}
\]

\[
\frac{\partial}{\partial b} \vec{n} = [\Omega_n + \tau] \vec{t} - [\kappa + (\text{div} \vec{n})] \vec{b} \tag{II.19}
\]

\[
\frac{\partial}{\partial b} \vec{b} = -\theta_{bs} \vec{t} - [\kappa + (\text{div} \vec{n})] \vec{n} \tag{II.20}
\]

The structures \( \theta_{ns}, \Omega_n \) fulfills the following identities

\[
\Omega_n = \vec{b} \cdot \partial_b \vec{n} - \tau \tag{II.21}
\]

\[
\frac{\partial}{\partial b} \vec{n} = \tau \vec{t} - [\kappa + (\text{div} \vec{n})] \vec{b} \tag{II.22}
\]
III Steady Diffusion in Solar Prominences

To solve the diffusion equation one must express the gradient operator along the magnetic loop in the form of nonholonomic Frenet equations

$$\nabla = \tilde{t}\partial_{s} + \tilde{n}\partial_{n} + \tilde{b}\partial_{b}$$  \hspace{1cm} (III.23)

Substitution of these expressions in the magnetic diffusion equation yields

$$\nabla^{2} = \nabla^{2\parallel} - \kappa \partial_{n} + (\theta_{bs} + \theta_{ns})\partial_{s} + \text{div}\tilde{b}\partial_{b}$$  \hspace{1cm} (III.24)

where $\nabla^{2\parallel} := \partial_{s}^{2} + \partial_{n}^{2} + \partial_{b}^{2}$. To simplify matters one considers that $\theta_{bs} + \theta_{ns}$ and $\text{div}\tilde{b}$ both vanish. Substitution of these expression in the steady plasma flow where $\frac{\partial N}{\partial t}$ vanishes, by the method of separation of variables in PDE yields

$$N(s,n) = \psi(s)\phi(n)$$  \hspace{1cm} (III.25)

yields the following equations

$$\partial_{s}^{2}\psi + \frac{v_{0}}{D}\partial_{s}\psi + k^{2}\psi = 0$$  \hspace{1cm} (III.26)

where $v_{s} = v_{0} = \text{constant}$, and

$$\partial_{n}^{2}\phi - \tau_{0}\partial_{n}\phi - k^{2}\phi = 0$$  \hspace{1cm} (III.27)

where $\tau_{0}$ is the constant Frenet torsion and $k^{2}$ is the separation constant. Solution of these equations yields together the final solution

$$N(s,n) = k_{1}^{2}n + \frac{D}{v_{0}}\int \kappa(s)ds$$  \hspace{1cm} (III.28)

where the last term integral represents the total Frenet curvature along the solar loop.

To test this model one considers its application to a solar prominence where non-thermal electrons density of $10^{19}cm^{-13}$ with velocities averaged to the order of $2.6\times10^{5}cm$. With these data and the height of $10^{10}cm$ and molecular diffusivity of $10^{6}cm^{2}.s^{-1}$. By considering helical loops where the torsion equals the Frenet curvature and it is constant, the curvature can be computed from these data and expression (III.28) as of order $10^{-8}cm^{-1}$
which agrees with the twist or filament torsion of coronal loop recently computed by Lopez-Fuentes et al. The vortex equation

\[ \vec{\omega} = \nabla \times \vec{v} = v_0 [\kappa \vec{b} - (\tau + \Omega_b) \vec{t}] \] (III.29)

which along with solenoidal vorticity equation yields

\[ \nabla \cdot \vec{\omega} = v_0 \partial_s [\tau + \Omega_b] = 0 \] (III.30)

which yields

\[ \tau = -\Omega_b + c_1 \] (III.31)

where \( c_1 \) is an integration constant. Now let us consider [8] the weakly ionized plasma gas where instead as in previous solution, now one considers that These relations shall be useful now to apply them to the diffusive plasma particles equation

\[ D \nabla^2 N = -Q \] (III.32)

which considered then that in this case the source term \( Q = ZN \) where \( Z \) is the "ionization function" yields

\[ \nabla^2 N = -\frac{Z}{D} N \] (III.33)

which by assuming now that \( N \) depends only on coordinate-s yields

\[ \partial_s^2 N + \frac{Z}{D} N = 0 \] (III.34)

and final solution

\[ N = N_0 \sin[\sqrt{ZD} s] \] (III.35)

which shows that the solution is periodic. Thus plasma is maintained again diffusion losses along the plasma filament.
IV Magnetic field diffusion along plasma filaments and dynamo action

In this section one considers the equations for the semi-ideal MHD given by

\[ \partial_t \vec{B} = \eta \nabla \times (\vec{u} \times \vec{B}) \]  

(IV.36)

which by considering the auxiliary flow velocity

\[ \vec{u} = \vec{v} - \eta \kappa \vec{n} \]  

(IV.37)

by which considering that the grad operator now is just \( \partial_s \) and the \( \vec{B} \) defined in the last section one obtains

\[ \vec{u} = \vec{v} - \eta \kappa \vec{n} \]  

(IV.38)

which by convenience one chooses to be \( \vec{v} = v_0 \vec{n} \) and with this choice the equations of ideal MHD reduce to

\[ \kappa \tau = 0 \]  

(IV.39)

\[ \kappa = v_0 \]  

(IV.40)

and

\[ \partial_t B_s = 0 \]  

(IV.41)

while the first two equations simply tells us that torsion vanishes the last one tells us that there is no dynamo action in the plasma despite the difference in velocities and that the diffusion is strongly predominant.

V Conclusions

A particular solution of magnetic resistivity MHD equations, is found representing a filamentary solar loop perturbed flow which is determined by the magnetic twist. The magnetic field is on a magnetic resistivity setting. From the mathematical point of view, the magnetic fields are given by holonomic filaments.Future prospects includes the filamentary dynamo generation solutions by the conformal geometrical technique [12] of stretch,twist
and fold dynamos [?] previously investigated. Speed flows involved in plasma loops [?] are extremely bigger than the perturbation computed here and thus it can be experimentally disregarded, nevertheless if \( v_0 \) vanishes it is easy to see from expression (II.11) that the magnetic diffusion coefficient is a complex number. More general framework of this work would includes the anisotropic magnetic diffusion case. This investigation may appear elsewhere. Curvature decay is well-within the diffusion limits of \( 10^8 \text{yrs} \) [?] of the solar corona.

VI Acknowledgements

I appreciate financial supports from Universidade do Estado do Rio de Janeiro (UERJ) and CNPq (Brazilian Ministry of Science and Technology). This paper is in memory to Professor Vladimir Tsypin, teacher, colleague and friend who taught us so much about applications of Riemannian geometry in plasma physics. Fractal geometry of filamentary dynamos can also be discussed in near future much in the same way was done earlier by Vainshtein et al [?].
References

[1] D. Kivotides, A. Mee and C. Barenghi, New J Phys. 9,291 (2007).

[2] C. Rogers, W. Schieff, Baecklund and Darboux transformations: Geometry and Modern Applications of Soliton Theory (2002) Cambridge University Press, UK.

[3] L.C. Garcia de Andrade, Non-holonomic dynamo filaments as Arnolds map in Riemannian space, Astronomical notes (2008) in press.

[4] R. Pismen, Vortices in nonlinear fields (2000) Oxford. M. Berger and C. Prior, J Phys A (2006).

[5] R. Bray, L.E. Cram, C. Durrant and R. Loughhead, Plasma loops in solar corona (1991), Cambridge University Press.

[6] M. Aschwanden, Physics of solar corona (2006) Springer.

[7] S. Molchanov, Russian Math surveys (1978).

[8] F. Chen, Plasma Physics (1976) Plenum.

[9] M.C. Lopez Fuentes, P. Demoulin, C.H. Mandrini, A.A. Pevtsov and L. van Driel-Gesztelyi, Astr and Astrophys. 397, 305 (2003). L. Tian, D Alexander, Y. Liu and J Yang, Solar Physics (2005) 229: 63.

[10] (1995).

[11] R. Ricca, Solar Physics 172 (1997),241. L. C. Garcia de Andrade, Physics of Plasmas 13, 022309 (2006). L. C. Garcia de Andrade, Phys Plasmas 14 (2007). L.C. Garcia de Andrade, Phys Scripta 13 (2006).

[12] P.K. Newton, The N-Vortex Problem (2001) Springer.