ON THE PRODUCTION OF FLUX VORTICES AND MAGNETIC MONOPOLES IN PHASE TRANSITIONS

Serge Rudaz

School of Physics and Astronomy, University of Minnesota
Minneapolis, Minnesota 55455, USA

Ajit Mohan Srivastava

Theoretical Physics Institute, University of Minnesota
Minneapolis, Minnesota 55455, USA

ABSTRACT

We examine the basic assumptions underlying a scenario due to Kibble that is widely used to estimate the production of topological defects. We argue that one of the crucial assumptions, namely the geodesic rule, although completely valid for global defects, becomes ill defined for the case of gauged defects. We address the issues involved in formulating a suitable geodesic rule for this case and argue that the dynamics plays an important role in the production of gauge defects.

1Present address: Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA.
1. Introduction

The existence of superheavy magnetic monopoles [1] and other types of topologically non-trivial field configurations is an almost inevitable consequence of unified gauge theories of elementary particle interactions. A classical argument due to Kibble [2] (hereinafter referred to as “the Kibble mechanism”) combined with a requirement of causality [3] leads to an estimated lower limit on the number of magnetic monopoles produced in a phase transition at Grand Unified Theory scale (typically $10^{16}$ GeV) in the early Universe that is much too large to be compatible with the standard Big Bang cosmology.

Various methods have been proposed to solve this “cosmological monopole problem”, of which inflation is the best known [4]. In this letter, we will reexamine the assumptions underlying the Kibble mechanism in some detail. These assumptions are, first, the existence of uncorrelated regions of space in which the vacuum degrees of freedom of scalar fields fluctuate randomly and second, what is usually called the geodesic rule, namely, that in between any two such uncorrelated regions of space, the scalar field traces the shortest path on the vacuum manifold.

We will argue in what follows that the geodesic rule in its conventional form becomes ambiguous when applied to the production of the topological defects in theories with spontaneously broken (local) gauge symmetries. This will lead us to suggest alternate physical criteria to modify the Kibble argument for this case. The original Kibble argument remains applicable without modification to the case of defects arising from the spontaneous breaking of global symmetries.

2. Formation of Global Defects

We start by examining the global case. For concreteness we will consider the case of strings arising due to the spontaneous breaking of a global U(1) symmetry in a first order phase transition where the transition proceeds through nucleation of vacuum bubbles [5]. We consider first order transition as in this case certain assumptions such as the existence of uncorrelated regions can be clearly formulated. The considerations are easily extended later to second order phase transition and to other topological defects. Let us assume that bubbles of a critical size randomly nucleate in space and collide with each other as they continue to expand. The strings form at the intersection of three (or more) bubbles if the Higgs phases in the three bubbles have appropriate values: Note that string production due to two bubbles (where the Kibble argument is in any case not applicable) has been shown to be extremely unlikely by numerical simulations for the 2+1 dimensional case.
Due to the minimization of gradient energy, the Higgs phase in a given bubble will be expected to be uniform [6]. The three bubbles here represent three uncorrelated regions where the Higgs phase varies randomly from one region to the other. The existence of such uncorrelated regions is one of the crucial elements in the Kibble mechanism and, at least for the global case, is well justified by physical considerations. However, this alone is not enough to determine the production of strings and one needs to make the following additional crucial assumption regarding the spatial variation of the Higgs phase.

Consider the collision of two bubbles in the above picture. As the bubbles coalesce, some intermediate value of the Higgs phase will arise in the region where the bubbles coalesced. It is immediately clear that the gradient energy will force the Higgs phase to smoothly interpolate between the two bubbles such that it traces the shortest path on $U(1)$ (this is also verified in the numerical simulations, see [6]). It is important to realize that this intermediate value of the Higgs phase is completely determined by the values of Higgs phase in the two (initially separated) bubbles and follows from the minimization of gradient energy. This is the geodesic rule crucial to the application of the Kibble mechanism.

These two ingredients are now enough to determine the string production. As the three bubbles coalesce, one can trace out a closed path in the region of the true vacuum formed by these bubbles. If the Higgs phase changes by $2n\pi$ (where $n$ is a non-zero integer) around this path, one can conclude that eventually a string must form inside. The expected string number per correlation volume within this scenario can be calculated to be $1/4$ for 2 space dimensions and about 0.88 for 3 space dimensions. Again we emphasize the important aspects of this approach: First, the intermediate values of Higgs phase were determined by the local physics following energy considerations, and second, the signal for the formation of string came from the outer region by considering a large closed path along which the Higgs phase changed by $2n\pi$.

All of what has been said so far is valid for the case of second order phase transition as well. For example, the bubbles can represent various correlation volumes (or, as a limiting case, horizon volumes) and then the Higgs phase in the intermediate region of any two such volumes will be completely determined by the gradient energy considerations which will again enforce the geodesic rule. Also, all these considerations can be clearly generalized for other global defects such as monopoles, by considering appropriate number of bubbles etc. [7]. We remark here that, due to strong attractive forces, global monopole-antimonopole pairs annihilate very efficiently and their number density is quite consistent with observations [8].
3. Gauged Defects and the Geodesic Rule

Let us now consider the case of gauged strings. The Kibble argument can only be formulated once a non-unitary gauge choice has been made. Clearly, the fact that the phase of a Higgs field is not a gauge-invariant quantity should be of no consequence, provided the argument leading to an estimate of a gauge-invariant quantity like the number of topological defects, is properly carried out. Consider the case when a U(1) local gauge symmetry is spontaneously broken, and again at first that the phase transition is of first order. The first assumption of the Kibble mechanism concerning the existence of uncorrelated regions still seems reasonable as this is consistent with the spirit of local physics. Thus for the above case of three colliding bubbles we will assume that the value of the Higgs phase varies randomly from one bubble to another (which essentially amounts to a specific non-unitary gauge choice). As the inside of these bubbles are superconducting regions, the magnetic field must be zero inside and the vector potential there will be pure gauge.

Now let us examine the collision of two bubbles; further, again assume that the Higgs phase is uniform within a given bubble, which can be achieved by a suitable gauge choice and the condition that the covariant derivative $D_\mu \phi = 0$ inside a bubble. In the global case, this uniformity followed from the local minimization of $\partial_\mu \phi$ (where $\phi$ is the Higgs field) due to considerations of gradient energy which also determined the value of the Higgs phase in the intermediate region as the bubbles coalesced. However, in the presence of gauge fields, the gradient energy to be minimized involves $D_\mu \phi$, rather than just $\partial_\mu \phi$: Clearly, this is not sufficient to uniquely determine the value of the Higgs phase itself that arises as the bubbles coalesce, let alone to justify the statement that this quantity traces the shortest path on the vacuum manifold.

Here we may mention that there have been several numerical studies of the formation of gauged vortices [9,10] where it has been found that large number of vortices are produced. However, in these studies, the assumption of geodesic rule in one form or another is always present. For example, in [9], random values of the Higgs phase are assumed from one horizon volume to another while the values of the Higgs phase for points within a horizon volume are determined by using smooth interpolation. Similarly, in [10], initial vortices are identified by implicitly using the geodesic rule for the Higgs phase. Clearly, in such schemes the gauge field becomes irrelevant for determining the initial number density of defects (even though it affects the evolution of strings, see [10] for example) and one would expect the same initial number of defects as in the global case. It is this application of
geodesic rule for the Higgs phase which is the subject of our analysis.

One may argue that in the absence of any magnetic fields one can choose a gauge such that the vector potential $A_\mu$ is zero everywhere so that $D_\mu \phi$ reduces to $\partial_\mu \phi$ and one may recover the geodesic rule for the Higgs phase. One can certainly choose such a gauge before the bubbles collide. But then one can not guarantee that $A_\mu$ will remain zero at the time when the bubbles coalesce. On the other hand, if one wanted to make such a gauge choice (so that $A_\mu = 0$) at the time when the bubbles collide, then the assumption of different values of $\theta$ in the two bubbles may prohibit one from doing so. For colliding bubbles, such a gauge choice seems possible only for the case when $D_\mu \phi$ is identically zero at the junction (which would mean that with the gauge choice in which $A_\mu = 0$, $\theta$ will be uniform in the two bubbles). This is because if $D_\mu \phi$ is non-zero then there will be currents at the junction when the bubbles touch leading to magnetic fields and hence non-zero $A_\mu$. The geodesic rule is relevant only when the bubbles touch each other and as we have just argued, at that moment it is generally not possible to choose a gauge in which $A_\mu$ is zero. This again shows that the geodesic rule for $\theta$ can not be applied here (except for the trivial case when $\theta$ is uniform in the two bubbles, with $A_\mu = 0$).

As far as the spatial variation of the Higgs phase alone is concerned, any possible continuous variation of the Higgs phase is allowed on an open path in the region intermediate to the two bubbles, all such possibilities being related to each other by gauge transformations and hence physically equivalent. We would like to emphasize here that we are not saying that any variation of Higgs phase can be gauged away. In the broken phase the net change in the Higgs phase around a closed path is gauge invariant. Similarly the integral of the vector potential around a closed path (holonomy) is gauge invariant. What we are saying here is that the change in the Higgs phase (or the change in the vector potential in the superconducting region) from one point to another different point is completely gauge dependent. As the geodesic rule concerns the variation of Higgs phase from one point to another different point, we conclude that for gauge theories the geodesic rule for $\theta$ alone becomes unjustified. Without a criterion like the geodesic rule, one can not specify the configuration of $\theta$ on a closed path and hence can not determine whether the path encloses any string or not.

4. Possible Alternatives to the Use of the Geodesic Rule

Given the above discussion, the immediate natural thing to do would be to concentrate on $D_\mu \phi$ (or more appropriately, on the physically well defined gauge invariant quantity $\phi^\dagger D_\mu \phi$) in order to establish a geodesic like criteria. We now analyze this possibility. As
the energy minimization will imply that $D_\mu \phi = 0$ inside the two colliding bubbles (at least initially), we may work in a gauge where the Higgs phase is uniform inside each bubble with values $\theta_1$ and $\theta_2$ respectively. The vector potentials are then zero inside each of the two bubbles. In the absence of any magnetic fields, one can work in a gauge such that $A_\mu$ is zero initially in the entire region (assuming that the same gauge choice still allows for random variation of $\theta$ in the two bubbles). Note that, as we discussed above, this is possible in general only when the bubbles are separated so that we can assume that there are no currents anywhere. Now as the bubbles are brought together, the question arises that what will be the value of $D_\mu \phi$ at the junction of the bubbles. There are two possibilities and we consider them in the following.

One possibility is that $A_\mu$ evolves in the intermediate region so that so that $\int_1^2 A \cdot d\ell = \theta_1 - \theta_2$ leading to $D_\mu \phi = 0$ at the bubble junction. Indeed, this is what one will expect if the bubbles collide very slowly (for example when bubbles nucleate very rapidly so that they expand very little, with small velocity, before colliding with each other). However, as now a non-zero value of $A_\mu$ has been generated (say only inside the bubbles, near the collision region) one should properly account for any associated magnetic field. If we consider one junction at a time then it is clear that such a magnetic field will arise initially in the form of a closed ring around the perimeter of the collision area. Such a ring of magnetic field should quickly shrink down and disappear near the mid point of the collision, again if the dynamics is slow. For fast dynamics it is conceivable that such rings of magnetic fields will form at each of the junctions which will then quickly combine to give one vortex. It seems interesting to investigate the structure of this vortex comprising of three rings (at least initially) which should result in such type of formation process. However, for slow enough dynamics, each of these rings should shrink down at the associated junctions and no vortex formation should result. When these rings shrink down, wavefronts of $\theta$ and $A_\mu$ will emanate from that region to make the field distribution consistent with the absence of the vortex. For example, if the values of $\theta$ in the two bubbles were (say) 0 and $2\pi/3$ respectively, then after the magnetic field dissipates in the above manner it is completely possible that $\theta$ in the middle winds around the longer path on $U(1)$ rather than having value $2\pi/6$ in that region. Which direction the variation of $\theta$ should choose on $U(1)$ should be governed by the condition whether there is a vortex formation or not. Note that here we are reversing the standard sequence of arguments as applied to the case of global defects. For the global case, the winding of $\theta$ was first deduced and then vortex formation was concluded. Here we are saying that since open paths of $\theta$ on $U(1)$ do not have any physical meaning, they can not govern the production of vortex but rather the
production of vortex should decide the path of $\theta$ on $U(1)$.

The above possibility was when the bubble collision is very slow. If this is not true then it is possible that $A_\mu$ remains zero until the walls collide (for example when bubble walls are extremely relativistic as in ref. [11]) and there may be non-zero $D_\mu \phi$ generated at the junction. This will then lead to transient currents at the junction which could add up and lead to the vortex formation.

The situation here is similar to that of the Josephson effect [12]. The two bubbles can thus be thought of as two pieces of superconductors with the region of false vacuum between the bubbles representing the Josephson junction. The net current across the Josephson junction is determined by external conditions which can be, either the presence of a potential difference across the junction, or the presence of a magnetic field at the junction, or the fact that there is supercurrent flowing through the circuit of which the Josephson junction is a part [12]. However, even if there are no external influences, there may still be a transient current across the junction which, for a single junction, will eventually relax to zero but may possibly add up in a closed array of more than two junctions. For example one may consider three Josephson junctions, with the value of $D_\mu \phi$ always having same sign across each of the junctions (which can be arranged by choosing $A_\mu = 0$ and monotonically increasing value of the Higgs phase as one goes around the shaded region), then the transient currents may add up to give a net circulating current with corresponding formation of vortex. This indeed suggests the intriguing possibility of carrying out such an experiment (for example in Josephson junction arrays) where the probability of spontaneous generation of magnetic flux can be experimentally determined.

These arguments can be extended to the case of a second order phase transition. Thus the bubbles can be taken to represent different correlation volumes as now the whole region is in the same phase (depending on the temperature). The essentials of our argument are still the same. As far as generation of non-zero $D_\mu \phi$ is concerned, in this case it should be even more natural to expect (due to more homogeneous nature of the phase transition) that, as different uncorrelated regions come into contact, the vector potential will develop such that $D_\mu \phi$ relaxes to zero. One has to then consider the magnetic fields generated by the evolution of $A_\mu$. Though it seems likely that here as well the magnetic field may arise in the form of a ring (at the junction of the two domains) which may then shrink down. This is especially likely when the vector boson in the broken phase is very heavy so that the time scale associated with it is very short. Extensions of our arguments to the case of magnetic monopoles can be carried out straightforwardly by considering the collision of four bubbles and seeing whether non-zero values of $D_\mu \phi$ can arise at the junctions. If it
does turn out that for certain types of dynamics the production of gauge defects through
the Kibble mechanism is suppressed then a competitive mechanism for their production
may be through thermal fluctuations [13].

5. Conclusions

We conclude by emphasizing the main points of this paper. What we have argued
is that it is not appropriate to treat gauge defects in exactly the same way as global
defects. The analysis of the formation of gauge defects requires much more detailed
considerations and it seems possible that the dynamics plays a very crucial role in this
case. This is because a simple criterion like the geodesic rule for the Higgs phase is not
available here which could be used to complete the specification of field configuration given
the values of Higgs phases in two (separated) uncorrelated regions. One therefore can not
trust the conventional estimates of the gauged monopole (or string) number density which
are based on the use of the geodesic rule for \( \theta \) alone. A more thorough analysis of the
dynamics at the phase transition is essential in the case of gauge theories, and the mainly
topological arguments appropriate to the global case are not sufficient. It is possible
that transient currents that fail to dissipate given sufficiently rapid dynamics could act
coherently and lead to the creation of the gauged topological defects. On the other hand
it also seems possible that the magnetic fields may arise only in the form of rings around
individual junctions of colliding bubbles which then shrink down. These issues can be
settled by extending the study of [11] by considering three-bubble collisions with varying
field configurations and bubble wall velocities. We are presently investigating this. As our
arguments suggest that there may be a difference in the production of global and gauge
defects, it is interesting to see if the formation of flux tubes in type II superconductors
can be experimentally tested along the same lines as the test for the global case in liquid
crystal systems [14]. We are presently investigating this and the results will be presented
in a subsequent paper [15].

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