Reducing Deep Network Complexity with Fourier Transform Methods

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Abstract

We propose a novel way that uses shallow densely connected neuron network architectures to achieve superior performance to convolution based neural networks (CNNs) approaches with the added benefits of lower computation burden requiring dramatically less training examples to achieve high prediction accuracy (> 98%). The advantages of our proposed method is demonstrated in results on benchmark datasets which show significant performance gain over existing state-of-the-art results on MNIST, CIFAR-10 and CIFAR-100. By Fourier transforming the inputs, each point in the training sample then has a representational energy of all the weighted information from every other point. The consequence of using this input is a reduced complexity neuron network, reduced computation load and the lifting of the requirement for a large number of training examples to achieve high classification accuracy. Code can be found on github here: https://github.com/andrew-jeremy/Reducing-Deep-Network-Complexity-with-Fourier-Transform-Methods

1 Introduction

Deep neuron networks require not only a large number of samples for training, but also impose a computation burden in large networks necessary to infer subtle features in the training samples for encoding. An additional complexity introduced by these deep networks is the requirement for multiple layers of dropout and pooling (in case of CNN) networks which add to the computational training load. Here we propose and demonstrate an approach based on Fourier transforming the training and testing/validation samples which spreads the feature energy to all points in the sample and then using a simple fully connected network with a single hidden layer. We show that this simple implementation achieves better accuracy than the state of the art fully connected dense network with multiple hidden layers as well as deep CNN networks on standard MNIST, CIFAR-10 and CIFAR-100 test data sets. This approach forms a basis for a new generation of networks that spreads the energy of the information through the neuron networks allowing for distributed processing and memory, analogous to
brain computation physiology. An intuitive way to think of the Fourier transformed input is as a feature extractor that also distributes energy from these features to every single point in the sample.

Tradition use of Fourier methods in CNNs take advantage of the efficiency of the discrete Fourier transform in performing fast convolutions by exploiting the well known duality relationship between convolutions and element-wise multiplication in the frequency domain [2,8]. Here we propose to go beyond that by proposing a single hidden layer densely connected neuronal network to directly input the Fourier transformed train/test data set resulting in: i) much simpler, more efficient implementation than CNN and with better accuracy, ii) fast training times and with a iii) reduced training sample set to achieve superior performance.

The proposed approach is motivated by the science and explosive success of convolution networks (CNN) [7] when applied to a number of machine learning problems spanning from vision to video classification. CNNs use filters to reduce training images to a subset of related features maps which are then down sampled with a pooling operation. During the convolution process, this undersampling is manifested by the relating neighboring pixels in a subset of an image to derive the underlying features within that subset. During the filtering and pooling operations, image pixels are cross correlated with their neighbours in a convolution operation such that they thus carry related information. We propose to entirely eliminate these convolution operation by Fourier transforming the samples and hence smearing each pixels with all the information/energy in the image. We are thus proposing to replace the problem of determining the required number of feature maps in a CNN with a single weighted sum of the feature maps (the Fourier transformed set) and using a fully connected single layer network instead to train and infer classes. This approach eliminates the need for deep neuron network structures since each sample carries the entire sample’s information. In addition, in contrast to the method proposed in [9] for implementing the CNN architecture using FFTs that required a custom CUDA implementation of the Cooley-Tukey FFT algorithm [6], our proposed approach requires less computation burden since the FFT operation is required only once to transform both the training and test data sets and the neuron architecture is shallow and involves only dense networks. To appreciate this point, lets us review the Fourier transform operation intuitively.

2 Fourier Transform Operation

We will begin with a comprehensive and intuitive exploration of the relationship between the spatial variations in intensity in an image and its Fourier space representation. We can depict a 2D image with coordinates x and y while its Fourier domain representation has spatial frequencies kx and ky respectively. While x and y are in units of distance (for example centimeters), the units of kx and ky are in cycles/distance (1/cm) representing number of intensity level cycles per distance; spatial frequency. This is the oscillation of brightness
and darkness (or intensity/gray scale) in an image. The Fourier domain thus depicts the oscillations of the intensity level in the Fourier transformed image. This spatial Fourier transform operation on the training and testing is expressed as:

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{i2\pi(ux + vy)} dx dy$$  \hspace{1cm} (1)

which decomposes the spatially varying orthogonal dimensions of the train/test set images into a weighted sum of 2D orthogonal basis functions analogous to the feature map filters used in CNNs. Hence, this one time Fourier decomposition yields all the possible feature maps in a weighted sum depicted as shown in Figure 1. Thus, Fourier analysis is the idea that any continuous object can be decomposed or reconstructed from its frequency content. This is demonstrated in Fig. 2 where we show the k-space representation of the head image. The brightness of a single point in the Fourier space domain, reports the amplitude of the sine function, while the location of the point tells us its frequency and orientation. If we consider varying points, we can see that sine patterns of varying frequency and orientation are represented by specifically orientated stripes. Hence, the Fourier transform of each individual point results in a stripe pattern with the orientation of the pattern determined by location of the point in k-space. The intersection of the dotted lines represent the points of \(k_x = k_y = 0\). For points on the \(k_x\) axis, the stripes are vertical. For points on the \(k_y\) axis, the strips are horizontal. For points with arbitrary \(k_x\) and \(k_y\) coordinates the stripes are oblique. The angle of the stripe pattern is such that the stripe density in \(x\) and \(y\) corresponds to the spatial frequency of the \(k_x\) and \(k_y\) component of the point in spatial Fourier space (k-space). Fourier transformation is thus the generation of stripe patterns for each point in k-space. By combining all the points in Fourier space with their corresponding stripe pattern amplitudes and frequencies, we can thus generate the respective image. In the terminology of CNNs, we can think of each of these stripe patterns as a kernel or filter. Thus, the Fourier transformed input contains an entire ensemble of kernel filters for the entire training set. Each point in the data set contains features from all the other data points and hence we can skip the convolution operation as well as the

\[\text{Figure 1: Fourier transformation of a spatially varying image decomposes the image into a series of periodic intensity functions. These periodic stripes constitute the complete ensemble of kernels that make up the image. We can thus think of the stripe patterns in a manner analogous to the filter kernels in CNNs.} \]
pooling layers and flatten the data for direct application to a densely connected output layer. In other words, every point of the input image is spread uniformly over the Fourier image, where constructive and destructive interference will automatically produce the proper Fourier representation. In CNN methods that

Figure 2: (a) Various points in k-space (center) and their corresponding Fourier transform that results in a “stripe” pattern for each point in k-space. Each point in k-space corresponds to a spatial frequency value given by the corresponding frequency and phase coordinates (kx, ky) while the intensity of the point corresponds to the amplitude of the oscillation or “stripe” pattern. A sum of all these “stripe” patterns shown in (b) results in the reconstruction of an image (c) (d) 3D depiction of a sample “stripe function showing a harmonic oscillation along the x axis at linearly increasing phase along the y axis. Each object is thus composed of a large sum of its constituent frequencies.

exploit the theorem that circular convolutions are equivalent to point wise products in the Fourier domain for the purpose of computation efficiency, the process is as depicted in Figure 3. The approach introduced in this work eliminates the filter kernels as well as the second FFT after the multiplication by feeding the Fourier transformed data directly into the densely connected network.

3 Experimental Results

We Fourier transformed all training and test data sets and used a fully connected two layer dense neuron network model with one hidden unit on a MNIST, CIFAR-10 and CIFAR-100 data sets. These particular data sets were chosen because they are well characterized in the machine learning community and many state-of-the-art results [1], exploring various architectures, are available for quick comparison to the proposed approach. For both training and test data
Figure 3: ((a) CNN architecture with Fourier transform implementation of the convolution operations. (b) Proposed architecture that uses the Fourier outputs directly resulting in a neuron network model in which all data points are related to every other point in the image. We can thus think of the Fourier operation as generating multi-level features in the data that is then used for classification in a fully connected network.

set, the real and imaginary Fourier components were treated as separate data contributors thus doubling the number of samples in both sets. The model was implemented in Tensorflow and the results for each data set was compared to the state of the art methods based as reported on Benenson’s website (author?).

The generic fully connected neuron network model with one hidden layer is used for both MNIST and CIFAR data sets. The number of nodes in the single hidden layer is equal to half the number of inputs while the number of outputs is equal to the number of classes in the data set. The number of hidden units is half the number of inputs while the number of inputs is equal to twice the number of flattened inputs for MNIST and CIFAR dataset. The factor of two in the inputs is due to the complex components of the Fourier transformed inputs. The Fourier operation intrinsically relates all pixels not just to their neighbors but to all the other pixels in a way related to their relative contribution to the resolution and contrast in the sample. The result is that Fourier transformation extracts all possible features in the image encoded as frequencies and phases allowing the final fully connected layers to classify inputs with high accuracy. It is intuitively equivalent to using very deep CNN based architectures to derive a lot of the intrinsic features before the fully connected layer. The distinct difference with the multi-layered deep CNN architecture is that here we accomplish this with one Fourier transform operation.

The MINST handwritten digit classification task [10] consists of 28 x 28 black and white images, each containing a digit 0 to 9 (10-classes). In 2011, an error rate of 0.27 percent, improving on the previous best result, was reported by researchers using a similar system of neural networks [3]. In 2013, an ap-
approach based on regularization of neural networks using DropConnect claimed to achieve a 0.21 percent error rate [11]. The results below show the performance of our proposed approach on a MNIST data set resulting in performance metrics far exceeding the current state of the art on this data.

![Figure 4: Training and test model (a) accuracy (b) loss plots of the MNIST data set classification using the proposed approach showing no over fitting and hence no need for a dropout layer in the model. (c) confusion matrix (d) Multiclass receiver operating characteristics (ROC). We can see from the ROC that the our proposed classifier does a very good job of separating the classes with an AUC = 1.0](image)

We also tested the model on a CIFAR-10 data set consisting of 10 classes. The 10 classes are: airplane, automobile, bird, cat, deer, dog, frog, horse, ship, and truck. CIFAR-10 data sets consists of 32 x 32 color images. A total number of 60,000 images are split into 50,000 training and 10,000 testing images. The data set is preprocessed by global contrast normalization. In our case, we did not augment the data sets at all and no model averaging is done at the test phase.

We demonstrated that even a simple 2-layer fully connected network with Fourier transformed inputs outperforms in accuracy and efficiency all previous deep CNN based networks, including state-of-the-art methods listed in [1].

CIFAR-100 dataset is similar to CIFAR-10 dataset, except that it has 100 classes. The number of images for each class is then 500 instead of 5,000 as in CIFAR-10, which makes the classification task more challenging. We used the same network architecture as was the case for MNIST and CIFAR-10 data sets. The number of fully connected nodes in the input layer was again set to...
Figure 5: Corresponding results for CIFAR-10 datasets including validation scores, prediction statistics, confusion matrix for all 10 classes and the corresponding accuracy and loss curves generated during training/testing of the model.

Half the number of number of inputs (for CIFAR, number of inputs is equal to $2^{*(32*32*3)}$, 32 by 32 color images (3) with a factor of 2 to account for the real and imaginary components arising out of the Fourier transformation).

Figure 6: Corresponding results with CIFAR-100 dataset showing the high precision and recall on this challenging data set including a reduced view of the normalized confusion matrix which was too busy to show as a full 2D matrix. The smallest diagonal element was 0.97.

The table shows a comparison of the proposed approach to the best state-of-the-art classification MNIST, CIFAR-10 and CIFAR-100 results on Rodrigo Benenson [1] website. The performance boost consistently shown for MNIST, CIFAR-10 and CIFAR-100 data sets again demonstrates the advantage of the FTI method.

The benefits of the approach are also demonstrated in the relaxation of
| Method          | Error(%) |
|-----------------|----------|
| DropConnect     | 0.21     |
| FTI(ours)       | 0.01     |
| Max-Pooling     | 3.47     |
| FTI(ours)       | 0.78     |
| Exp Linear      | 0.73     |

Table 1: MNIST classification error

| Reduced Training Data Method | Error(%) |
|------------------------------|----------|
| MINST with 10 training batches | 0.3     |
| CIFAR-10 with 1 training batch  | 1.3     |
| CIFAR-100 with 1 training batch | 0.98    |

Table 2: CIFAR-10 classification error

Table 3: CIFAR-100 classification error

Table 4: Applying the method to a reduced training set for each of MINST (10 training batches instead of the entire set of 53), CIFAR-10 (1 training batch instead of all 5) and CIFAR-100 (1 training batch in place of 5)

the requirement for large training sets since the Fourier operations generates all possible features embedded in a given set. Table below demonstrates the advantage of this approach in working with smaller training sets and still being able to achieve competitive accuracy on these data sets.

4 Conclusion

We have proposed and demonstrated a method that exceeds the performance of CNNs on standard data sets based on Fourier transforming the training and test data sets. We have shown that inherent in the Fourier transform operation is the generation of all possible kernel filters which are averaged over the entire data set so that each point represent a fraction of energy of the entire image based on its location in Fourier space. We can thus think of each point in the data set as having weighted information content of the resolution and contrast of the entire image with the weighting by its respective location in Fourier space. The relationships between not only neighboring pixels (the basis of CNN implementation with filters and pooling layers) but with pixels throughout the entire image means that each neuron weight stores information from the entire sample. An alternative way to think of the Fourier transform operation on the training and test data sets is that it derives all the high and low level features in the data (similar to a cascade of CNN operations with filters) which are then fed into fully connected layers for the final classification. It can thus be surmised that the Fourier transform operations generates many kernel filters that spread the energy of the data to each of the pixels in the image allowing for each pixel in the image having all the features in the image at once.
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