EVIDENCE OF COSMIC EVOLUTION OF THE STELLAR INITIAL MASS FUNCTION

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ABSTRACT

Theoretical arguments and indirect observational evidence suggest that the stellar IMF may evolve with time, such that it is more weighted toward high-mass stars at higher redshift. Here we test this idea by comparing the rate of luminosity evolution of massive early-type galaxies in clusters at 0.02 \( \leq z \leq 0.83 \) to the rate of their color evolution. A combined fit to the rest-frame \( U - V \) color evolution and the previously measured evolution of the \( M/L_v \) ratio gives \( x = -0.3 \pm 0.2 \) for the logarithmic slope of the IMF in the region around 1 \( M_\odot \), significantly flatter than the present-day value in the Milky Way disk of \( x = 1.3 \pm 0.3 \). The best-fitting luminosity-weighted formation redshift of the stars in massive cluster galaxies is \( 3.7^{+2.3}_{-0.8} \), and a possible interpretation is that the characteristic mass \( m_\star \) had a value of \( \sim 2 M_\odot \) at \( z \sim 4 \) (compared to \( m_\star \sim 0.1 M_\odot \) today), in qualitative agreement with models in which the characteristic mass is a function of the Jeans mass in molecular clouds. Such a “bottom-light” IMF for massive cluster galaxies has significant implications for the interpretation of measurements of galaxy formation and evolution. Applying a simple form of IMF evolution to literature data, we find that the volume-averaged SFR at high redshift may have been overestimated (by a factor of \( 3-4 \) at \( z > 4 \)), and the cosmic star formation history may have a fairly well defined peak at \( z \sim 1.5 \). The \( M/L_v \) ratios of galaxies are less affected than their SFRs, and future data on the stellar mass density at \( z > 3 \) will provide further constraints on IMF evolution. The formal errors likely underestimate the uncertainties, and confirmation of these results requires a larger sample of clusters and the inclusion of redder rest-frame colors in the analysis.

Subject headings: cosmology: observations — galaxies: evolution — galaxies: formation

1. INTRODUCTION

The form of the stellar initial mass function (IMF) is of fundamental importance for many areas of astrophysics and a topic of considerable debate (for reviews see, e.g., Schmidt 1959; Miller & Scalo 1979; Scalo 1986; Larson 1998, 2003; Kroupa 2002; Chabrier 2003). Measurements of the IMF are difficult and somewhat model dependent as they require the conversion of the observed present-day luminosity function of a stellar population to its mass function at birth. Best estimates for the Galactic disk suggest that the IMF has a power-law slope at \( m \gtrsim 1 M_\odot \) and turns over at lower masses (Kroupa 2001; Chabrier 2003). This turnover can be modeled by a broken power law (Kroupa 2001) or by a lognormal distribution with a characteristic mass \( m_\star \) (Chabrier 2003). The value of \( m_\star \) is \( \sim 0.1 M_\odot \) in the disk of the Milky Way, with considerable uncertainty. The power-law slope at high masses is probably close to the Salpeter (1955) value of \( x = 1.35 \), with an uncertainty of \( \sim 0.3 \) (Scalo 1986; Chabrier 2003).

Although there is no direct evidence for dramatic variations of the IMF within the present-day Milky Way disk (e.g., Kroupa 2001; Chabrier 2003), this does not preclude variations with time, metallicity, and/or environment. In particular, Larson (1998, 2003) has argued that the characteristic turnover mass may be largely determined by the thermal Jeans mass, which strongly depends on temperature (\( \propto T^{3/2} \) at fixed density). In the context of this model one might expect that heating by ambient far-infrared radiation would disfavor the formation of low-mass stars in extreme environments, such as in super star clusters and in the center of the Milky Way. Other models emphasize the role of turbulence as opposed to temperature in determining the distribution of proto-stellar clumps (e.g., Padoan & Nordlund 2002), and in these models the role of the environment may be less direct (for a recent review of various models to explain the characteristics of the IMF see McKee & Ostriker 2007).

Observations may support the notion of a top-heavy (or “bottom-light”) IMF in extreme environments. Some young super star clusters in M82 appear to have a top-heavy mass function (e.g., Rieke et al. 1993; McCrady et al. 2003), as do clusters in the Galactic center region (e.g., Figer et al. 1999; Stolte et al. 2005; Maness et al. 2007). The interpretation of observed mass functions is complicated by dynamical effects, which tend to make the mass function more top-heavy over time, in particular in the central regions of star clusters (see, e.g., McCrady et al. 2003, 2005; Kim et al. 2006). Recently Harayama et al. (2008) studied the IMF of NGC 3603, one of the most massive Galactic star-forming regions, out to large radii and conclude that its IMF is substantially flatter than Salpeter for masses 0.4–20 \( M_\odot \).

The IMF may also depend on redshift. At earlier times star formation presumably occurred more often in a burst mode than in a relatively gradual “disk” mode (e.g., Steidel et al. 1996; Blain et al. 1999b; Lacey et al. 2007), which means that the IMF could generally be more skewed toward high-mass stars at redshifts 1–3 and beyond. Furthermore, the average metallicity in star-forming clouds was lower at higher redshift, which may have led to an extremely top-heavy IMF for the first generation of stars (e.g., Abel et al. 2002; Bromm et al. 2002). Finally, the cosmic microwave background (CMB) radiation sets a floor to the ambient temperature, and hence the Jeans mass, which scales with \( (1 + z) \).

Beyond \( z \sim 2 \) the CMB temperature exceeds the typical temperatures of dense prestellar cores in Galactic molecular clouds (e.g., Evans et al. 2001; Tafalla et al. 2004). Therefore, at sufficiently high redshift the characteristic mass may be expected to evolve roughly as \( m_\star \propto (1 + z)^{3/2} \), leading to IMFs that have a reduced fraction of low-mass stars (Larson 1998). The effects of the CMB are even more pronounced when its influence on the pressure in star-forming clouds is taken into account, and Larson (2005)
suggests that at \( z = 5 \) the characteristic mass may be higher than today’s value by as much as an order of magnitude. Such rapid evolution of the IMF would have important consequences for determinations of masses and star formation rates of distant galaxies and for measurements of evolution in these properties.

It is very difficult to constrain the IMF at early times directly, as the light of high-redshift galaxies is completely dominated by massive stars. The extremely blue rest-frame UV colors of galaxies and for measurements of evolution in these properties. The extremely blue rest-frame UV colors of galaxies and for measurements of evolution in these properties.

Fortunately, the form of the high-redshift IMF has implications for the properties of galaxies at much lower redshift, as all stars with masses \( \leq 0.8 \, M_\odot \) that formed in the history of the universe are still with us today. Tumlinson (2007) finds that the properties of carbon-enhanced metal-poor stars in our Galaxy are best explained with a relatively high number of stars in the mass range 1–8 \( M_\odot \) at high redshift. Various other constraints obtained from galaxies at low redshift (including our own) are reviewed in Chabrier (2003). Of particular interest are the stellar populations of massive early-type galaxies, as they are very homogeneous and should reflect conditions in star-forming regions at \( z > 2 \). Recently, Cappellari et al. (2006) used the kinematics of elliptical galaxies to constrain the IMF, as the dynamical \( M/L \) ratio provides an upper limit to the amount of mass that can be locked up in low-mass stars. Current data appear to rule out a Salpeter (1955) (or steeper) IMF but are consistent with Kroupa (2001) and Chabrier (2003) IMFs (Cappellari et al. 2006).

In this paper we provide new constraints on the IMF at high redshift by comparing the evolution of the \( M/L \) ratios of early-type galaxies to their color evolution. This method was first suggested by Tinsley (1980), but data of sufficient accuracy are only now becoming available. The method is sensitive to the IMF in the important mass range around 1 \( M_\odot \), where the effects of an evolving characteristic mass might be expected to manifest themselves.

A plan of the paper follows. In § 2 a relation between color evolution, luminosity evolution, and the logarithmic slope of the IMF \( x \) is derived using stellar population synthesis models. In § 3 published data and archival Hubble Space Telescope (HST) images of galaxy clusters are used to construct the redshift evolution of the \( U - V \) color-mass relation. In § 4 the color evolution from § 3 is combined with the previously measured evolution of the \( M/L_B \) ratio. The relations from § 2 are then used to derive constraints on the IMF slope \( x \) from the combined color and luminosity evolution. Section 5 is devoted to the (many) systematic uncertainties in the methodology and in the data, and § 6 asks whether our results are consistent with other constraints on the stellar populations of massive early-type galaxies. Although the constraints we derive in this paper are subject to many uncertainties, it is interesting to explore their consequences. In § 7 the fitting results of § 4 are interpreted in the context of an evolving characteristic mass \( m_c \). The data on cluster galaxies are combined with previous constraints on \( m_c \) for globular clusters and sub-millimeter galaxies, and a simple form of IMF evolution is proposed. This evolution is then applied to literature data on the evolution of the volume-averaged star formation rate and stellar mass density. The key results are summarized in § 8. We assume \( \Omega_m = 0.3, \Omega_\Lambda = 0.7, \) and \( H_0 = 71 \, \text{km s}^{-1} \, \text{Mpc}^{-1} \) where needed.

2. METHODOLOGY

2.1. Effects of the IMF on Luminosity and Color Evolution

The form of the IMF has a strong effect on the evolution of the \( M/L \) ratio of galaxies. The luminosity of a stellar population is expected to evolve even in the absence of star formation or mergers, due to the fact that stars turn off the main sequence and die. The rate of luminosity evolution in a given passband is determined by a combination of the lifetime of stars near the turnoff and the number of these stars. The lifetime of turnoff stars is directly determined by the age of the stellar population (with the lifetime approximately equal to the age), and the number of turnoff stars is directly determined by the IMF. A flat, top-heavy IMF implies a relatively large number of short-lived massive stars and rapid luminosity evolution, whereas a steep IMF implies a relatively large number of long-lived low-mass stars and slow evolution.

The color evolution behaves differently, as it is insensitive to the number of stars that evolve off the main sequence over time. Instead, it is mainly driven by the location of the turnoff in the Hertzsprung-Russell (H-R) diagram, which does not depend on the IMF but is determined by the age of the stellar population. The IMF does influence the color evolution to some extent, as a top-heavy IMF reduces the number of turnoff stars with respect to the more luminous red giants. The location of the giant branch evolves much less than the location of the turnoff, and a flat IMF therefore implies weaker color evolution than a steep IMF. As first pointed out by Tinsley (1980), the IMF thus has opposite effects on the luminosity and color evolution: a more top-heavy IMF leads to stronger luminosity evolution and weaker color evolution.

2.2. Stellar Population Synthesis Models

Following Tinsley (1980) and many later studies, we parameterize luminosity and color evolution by power laws. Power-law approximations render the predicted evolution independent of the Hubble constant and the absolute age calibration of the model and smooth out artifacts in the evolution of single-age stellar populations caused by numerical effects. In our passbands (see § 3) luminosity evolution takes the form

\[
\log(M/L) = \kappa_B \log(t - t_{\text{form}}) + C_1
\]

and color evolution takes the form

\[
U - V = 2.5\kappa_{U-V} \log(t - t_{\text{form}}) + C_2,
\]

with \( t_{\text{form}} \) the luminosity-weighted mean star formation epoch and \( C_1 \) and \( C_2 \) constants.

We use the Maraston (2005) stellar population synthesis models to determine \( \kappa_B \) and \( \kappa_{U-V} \). Figures 1a and 1b show the evolution of the \( M/L_B \) ratio and \( U - V \) color in this model for three different values of \( \chi (1.3, 0.7, 0) \). These IMFs are power laws with a fixed slope over the mass range 0.1–100 \( M_\odot \). The predictions are for a metallicity \( [Z/H] = 0.35 \). This relatively high value may be more appropriate for massive early-type galaxies than the solar value (e.g., Worthey et al. 1992), although solar models are much better calibrated. As expected, luminosity evolution has a strong IMF dependence, but color evolution only has a weak (and opposite) IMF dependence. Figure 1c directly compares the evolution of the \( M/L \) ratio to the color evolution. The IMF dependence is relatively strong in this panel, due to the opposite effects of the IMF on luminosity and color evolution.
Dashed lines in Figures 1a–1c are power-law fits to the model predictions. These fits generally provide good descriptions of the behavior of the models. The largest deviations are for the \( U/V \) colors at 9 and 12 Gyr, where the fits are off by +0.02 and -0.02 mag, respectively. Figures 1d–1f show the relation between the power-law coefficients and the IMF slope. In each case there is a well-defined relation that can be characterized by a simple linear fit. These fits take the form

\[
\kappa_B = \frac{d[\log(M/L_B)]}{d[\log(t - t_{\text{form}})]} = 1.30 - 0.25x, \tag{3}
\]

\[
\kappa_{U-V} = 0.4 \frac{d(U-V)}{d[\log(t - t_{\text{form}})]} = 0.23 + 0.016x, \tag{4}
\]

and

\[
\frac{\kappa_B}{\kappa_{U-V}} = 2.5 \frac{d[\log(M/L_B)]}{d(U-V)} = 5.44 - 1.23x. \tag{5}
\]

The fits are shown by the solid lines in Figures 1d–1f. Although the fits are excellent for 0 < x < 1.3, the extrapolations outside that range are obviously uncertain. Note that equation (5) is a fit and is not identical to the combination of equations (3) and (4). The dotted line in Figure 1f shows the relation implied by equations (3) and (4); this relation is consistent with equation (5) to a few percent. Open symbols and dashed lines are predictions for models with \([Fe/H] = 0\). Luminosity evolution is insensitive to \([Fe/H]\), but color evolution has a strong metallicity dependence. Therefore, the relation in Figure 1f also has a strong metallicity dependence and takes the form

\[
\frac{\kappa_B}{\kappa_{U-V}} = 6.93 - 1.81x \tag{6}
\]

for solar metallicity. When determining the IMF slope in \( x \) and \( t_{\text{form}} \), results are given for both supersolar and solar models (which are better calibrated).

2.3. Implementation

As demonstrated in Figure 1, the IMF mostly influences the luminosity evolution, and the color evolution can be viewed as an IMF-insensitive “clock” constraining the age. By combining color and luminosity evolution for a well-chosen set of galaxies over a sufficiently large redshift range, the slope of the IMF \( x \) and the formation time of the stars \( t_{\text{form}} \) can, in principle, be constrained simultaneously (although there are many caveats; see Section 5).

The method requires very accurate rest-frame color and \( M/L \) measurements for a passively evolving sample of galaxies. This is not an easy task, as galaxy evolution is in general a complex process involving starbursts, mergers, and morphological changes. The most massive early-type galaxies in clusters at modest redshifts (\( z < 1 \)) come closest to a “purely passive” sample. Their colors and \( M/L \) ratios have very small scatter (e.g., Bower et al. 1992b; Jørgensen et al. 1996), their luminosity evolution and color evolution are consistent with passive evolution of stellar
populations formed at redshifts $z \gtrsim 2$ (e.g., van Dokkum et al. 1998a; Holden et al. 2004), and they appear to have been largely in place by $z \approx 0.8$ (Holden et al. 2006, 2007).

Recently a large number of measurements of the $M/L_B$ evolution of early-type galaxies with masses $M > 10^{11} M_\odot$ were compiled from the literature and placed on a consistent system (van Dokkum & van der Marel 2007, hereafter vv07). Color evolution has also been studied by many authors (e.g., Stanford et al. 1995; Ellis et al. 1997; Holden et al. 2006), but not yet for mass-limited samples and with the required accuracy. With the availability of archival multicolor HST WFPC2 and Advanced Camera for Surveys (ACS) data of a subset of the clusters in vv07, it is now possible to determine color and luminosity evolution of massive early-type galaxies in clusters simultaneously and begin to constrain the form of the IMF at the mean star formation epoch of massive early-type galaxies.

3. DATA

In this section, data from the literature and the HST Archive are used to determine the evolution of rest-frame $U-V$ colors of galaxies in clusters from the vv07 sample. Because mass measurements of individual galaxies are available, the color evolution can be determined from the zero point of the color-mass (rather than the color-magnitude) relation. Also, the same mass limit can be applied as was applied to the $M/L$ data. We focus on $U-V$ because redder rest-frame colors are not available for clusters beyond $z \sim 0.5$.

3.1. The Color-Mass Relation in Coma

The color-magnitude relation in the Coma Cluster was studied by Bower et al. (1992a, 1992b, hereafter collectively BLE92). An important goal of their study was to determine the relative distance between the Virgo Cluster and Coma Cluster, and this required very accurate absolute color measurements. The data from BLE92 are combined with velocity dispersions and effective radii from Jørgensen et al. (1995a, 1995b). Masses are calculated using

$$
\log M = 2 \log \sigma + \log r_e + 6.07,
$$

with $\sigma$ the velocity dispersion corrected to a 3.4′′ diameter aperture and $r_e$ the effective radius in kpc. Radii were converted from arcsec to kpc assuming a Hubble flow velocity of $v_{\text{flow}} = 7376 \pm 223 \text{ km s}^{-1}$ (see vv07).

Figure 2 shows the color-mass relation of galaxies in common between the BLE92 color-magnitude sample and the Jørgensen et al. (1995a, 1995b) fundamental plane (FP) sample. There is a very clear relation, of the form

$$
U - V = 0.113 (\log M - 11) + 1.460.
$$

The slope of the relation is determined by fitting a linear function to all galaxies with masses $M > 10^{11} M_\odot$, and the offset is determined with the biweight estimator (Beers et al. 1990), which gives low weight to outliers. This relation is indicated by the solid line in Figure 2. We note that the data in Figure 2 are not entirely self-consistent, as the colors were determined in a different aperture (11′′) than the velocity dispersions (3.4″). This has implications for the interpretation of equation (8), but it has no effect on our analysis as the distant galaxies are treated in the same way as the Coma galaxies.

The observed scatter in the relation is very small at $\sigma_{\text{bi}} = 0.029$ mag for $M > 10^{11} M_\odot$. This low scatter is remarkable: from repeat measurements of the same galaxies BLE92 estimate that the observational uncertainty for individual galaxies is 0.037, which would imply that the intrinsic scatter in $U - V$ is well below 0.03 for the most massive Coma galaxies. However, we note that the biweight estimator gives very little weight to the blue outlier at $\log M = 11.1$, and this is somewhat arbitrary given the small sample size. The rms scatter is 0.05, similar to values measured for clusters at redshift (see § 3.4).

In this study we are not concerned with the scatter or slope of the color-mass relation but only with its zero-point evolution. For each cluster in the sample equation (8) is subtracted from the rest-frame $U-V$ colors of early-type galaxies with $M > 10^{11} M_\odot$, and the offset $\Delta(U - V)$ is determined with the biweight estimator. By construction, $\Delta(U - V) \equiv 0$ for the Coma Cluster. The random uncertainty in the offset due to galaxy-galaxy color variations is 0.009. BLE92 give a systematic error of 0.02 in the $U-V$ colors, reflecting uncertainties in the $U$ and $V$ zero points and small errors due to reddening and aperture corrections. Adding the random and systematic errors in quadrature gives a combined uncertainty of 0.022. We note that the BLE92 data are still the most accurate available for Coma, even though they are now more than 15 years old. In fact, the comprehensive photometric studies by Terlevich et al. (2001) and Eisenhardt et al. (2007) calibrate their photometry by matching the magnitudes of galaxies to the BLE92 values.

3.2. Other Nearby Clusters

No other nearby clusters are currently available with high-quality $U-V$ colors, effective radii, and velocity dispersions for their constituent galaxies. As an example, effective radii and velocity dispersions are available for the clusters in the Jørgensen et al. (1996) sample, but accurate $U-V$ colors have not been measured. Conversely, McIntosh et al. (2005) analyzed the color-magnitude relation in the three nearby clusters Abell 85, Abell 496, and Abell 754 using the same apertures and methods as BLE92, but those clusters have not been the subject of FP studies.
Although the offsets $\Delta(U - V)$ for the three clusters of McIntosh et al. (2005) cannot be determined directly, they can be estimated by making use of the tight relation between rest-frame $V$-band luminosity and mass in nearby clusters (e.g., Jørgensen et al. 1996). Fitting directly to the BLE92 $V$-band magnitudes (in our cosmology) and Jørgensen et al. (1996) masses gives

$$\log M = -0.57 M_V - 0.78,$$

with an rms scatter of 0.14 in $\log M$. Applying this relation to the full BLE92 sample (i.e., not just to galaxies with measured masses) produces an offset in the color-“pseudo” mass relation of $-0.003$, almost identical to the offset derived from the actual color-mass relation.

Absolute $V$-band magnitudes and $U - V$ colors were taken from Table 6 in McIntosh et al. (2005). The $(U - V)_{134}$ colors are used as they were measured using the same physical aperture size as used by BLE92. The $V$ magnitudes are corrected to our cosmology and converted into pseudomasses using equation (9). Offsets are determined by subtracting equation (8) from the observed colors and determining the biweight mean residual. The offsets are $-0.050 \pm 0.042$, $-0.061 \pm 0.061$, and $-0.002 \pm 0.051$ for Abell 85, Abell 496, and Abell 754, respectively. The uncertainties are a combination of random and systematic errors. The systematic errors were calculated from the listed uncertainties in the zero points and $k$-corrections and a 16% uncertainty in the listed reddening (Schlegel et al. 1998). All three offsets are consistent with zero, in agreement with the analysis of McIntosh et al. (2005) and with earlier work by Andreon (2003), who studied a large sample of clusters in the Sloan Digital Sky Survey (SDSS). The uncertainties in $\Delta(U - V)$ are considerably larger than for Coma due to a combination of larger photometric zero-point uncertainties and larger Galactic reddening toward the McIntosh et al. (2005) clusters.

### 3.3. Data for Distant Clusters

#### 3.3.1. Available HST Imaging and Observed Colors

For each of the 14 distant clusters described in vv07 we determined whether HST data exist in the appropriate filters (i.e., roughly corresponding to rest-frame $U$ and $V$). HST data are crucial because of their photometric stability and because the combination of seeing and color gradients makes aperture effects difficult to control in ground-based data. Appropriate data are available for the seven clusters listed in Table 1. Clusters from vv07 that are not included in the analysis are Abell 665 ($z = 0.183$), as the bluest available HST filter corresponds to rest-frame $V$; Abell 2390 ($z = 0.228$), as the HST filters correspond to rest-frame $B - R$; 3C 295 ($z = 0.456$), as the vv07 sample only has one galaxy more massive than $10^{11} M_\odot$, except 3C 295 itself; CL 1601+42 ($z = 0.539$), as HST observations were obtained in a single filter only; and all three clusters with $z > 0.85$, as the reddest available HST colors are much bluer than rest-frame $U - V$.

For two of the seven clusters in Table 1 accurate colors have been published: Blakeslee et al. (2006) provide ACS measurements of galaxies in MS 1054−03 ($z = 0.831$) and RX J0152−13 ($z = 0.837$). For the five remaining clusters the HST data were obtained from the HST archive and reduced using standard techniques. The WFPC2 images were processed using a combination of STSDAS tasks and custom scripts (see, e.g., van Dokkum et al. 1998b). For ACS the photometrically and astrometrically corrected “drz” files were obtained, and further processing was limited to the removal of remaining cosmic rays and some other defects. Galaxies in common with the vv07 sample were deconvolved with the $\text{clean}$ task in the IRAF STSDAS package, using synthetic point-spread functions generated with Tiny Tim (Kirst 1995). The effect of deconvolution on the measured colors is typically $\sim 0.01$ at $z \sim 0.2$ and $\sim 0.02$ at $z \sim 0.5$.

Colors were measured in apertures scaled to match an 11′′ diameter aperture at the distance of the Coma Cluster, to allow a direct comparison with BLE92. The measurements were converted from instrumental counts to magnitudes on the Vega system using the WFPC2 and ACS zero points. For WFPC2 the zero points were taken from § 28.1 of the WFPC2 Data Handbook, which lists values for the PC chip and each of the wide-field detectors separately. For ACS the zero points were taken from the tables on the STScI Web site, which include small corrections made after publication of Sirianni et al. (2005). For the two $z \approx 0.83$ clusters the colors were taken from Tables 1 and 2 of Blakeslee et al. (2006). A small aperture correction was applied to each galaxy, as Blakeslee et al. (2006) used apertures containing 50% of the light rather than an aperture of fixed size (see the Appendix). The colors were transformed from the AB to the Vega system using the AB offsets listed in Sirianni et al. (2005).

#### 3.3.2. Rest-Frame Colors and Systematic Errors

A key step in the analysis is the transformation of the observed colors in the various HST filters to rest-frame $U - V$. Following van Dokkum & Franx (1996) and many later studies (e.g., Blakeslee et al. 2006), we derive transformations of the form

$$U_z = F_2 + \alpha_1 (F_1 - F_2) + \beta_1 + 2.5 \log (1 + z),$$

$$V_z = F_2 + \alpha_2 (F_1 - F_2) + \beta_2 + 2.5 \log (1 + z),$$

Note that McIntosh et al. (2005) list larger systematic errors; they do not add the individual error contributions in quadrature and also assume a slightly larger error in the extinction than given by Schlegel et al. (1998).

### TABLE 1

| Cluster | $z$ | Camera | $F_1$ | $F_2$ | $\alpha_{1,-2}$ | $\beta_{1,-2}$ | $\sigma_{\text{sys}}$ | $\Delta(U - V)$ | $\sigma_{\text{int}}$ |
|---------|-----|--------|-------|-------|-----------------|-----------------|----------------|----------------|----------------|
| Coma... | 0.024 | ... | $U$ | $V$ | ... | ... | 0.036 | 0.000 | 0.037 |
| Abell 2218... | 0.176 | WFPC2 | F450W | F702W | 1.230 | -1.217 | 0.021 | -0.082 | 0.028 |
| CL 1358+62... | 0.327 | ACS | F475W | F775W | 0.955 | -1.253 | 0.022 | -0.101 | 0.033 |
| CL 0024+16... | 0.391 | WFPC2 | F450W | F814W | 0.793 | -1.181 | 0.024 | -0.122 | 0.031 |
| CL 0016+16... | 0.546 | WFPC2 | F555W | F814W | 1.065 | -1.227 | 0.023 | -0.165 | 0.035 |
| MS 2053−04... | 0.583 | WFPC2 | F606W | F814W | 1.503 | -1.243 | 0.035 | -0.167 | 0.039 |
| MS 1054−03... | 0.831 | ACS | F606W | F850LP | 0.881 | -1.176 | 0.033 | -0.200 | 0.038 |
| RX J0152−13... | 0.837 | ACS | F625W | F850LP | 1.035 | -1.164 | 0.021 | -0.185 | 0.024 |

2 Note that McIntosh et al. (2005) list larger systematic errors; they do not add the individual error contributions in quadrature and also assume a slightly larger error in the extinction than given by Schlegel et al. (1998).

3 See http://www.stsci.edu/hst/acs/analysis/zeropoints/.
with $U_{z}$ the rest-frame $U$ band, $V_{z}$ the rest-frame $V$, $F_{1}$ and $F_{2}$ the observed $HST$ filters (listed in Table 1), and $\alpha$ and $\beta$ determined from template spectra. The rest-frame color is given by

$$
(U - V)_{z} = (\alpha_{1} - \alpha_{2})(F_{1} - F_{2}) + (\beta_{1} - \beta_{2}) = \alpha_{1-2}(F_{1} - F_{2}) + \beta_{1-2}. \tag{12}
$$

Unless $F_{1}$, $F_{2}$ correspond exactly to $U_{z}$, $V_{z}$, the values of $\alpha$ and $\beta$ have a (modest) dependence on the assumed spectral type. For each cluster the appropriate values were determined in the following way. We used synthetic templates from Yi et al. (2003) with a large range of ages to establish the relation between the coefficients and the observed color in the $HST$ filters. The Yi et al. (2003) templates were chosen because they may be better calibrated in the rest-frame ultraviolet than the Bruzual & Charlot (2003) models; at rest-frame wavelengths $\lambda \gtrsim 3600$ Å the two sets of models are identical. Typically, $\alpha$ and $\beta$ vary by less than a few percent for ages varying from 1 to 12 Gyr; the amount of variation depends largely on how well the observed $HST$ filters match the rest-frame $U$ and $V$ bands. The relations between $\alpha$, $\beta$ and the observed colors $F_{1} - F_{2}$ were fitted with smooth functions, and the values corresponding to the median observed color of galaxies in the sample were adopted. Values of $\alpha_{1-2} \equiv \alpha_{1} - \alpha_{2}$ and $\beta_{1-2} \equiv \beta_{1} - \beta_{2}$ are listed in Table 1.

The rest-frame $U - V$ colors are subject to several sources of systematic uncertainty. The zero points of the $HST$ filters are accurate to $0.02$ mag. The dominant source of zero-point error is thought to be the absolute calibration of Vega; e.g., updates to the ACS zero points in 2006 due to a recalibration of Vega were $\leq 0.017$. As we are always dividing the flux in two ACS or WFPC2 passbands, such errors (nearly) cancel. Other effects that plague absolute calibration of (particularly) WFPC2 photometry, such as variations in charge transfer efficiency and the long-short anomaly, also cancel when measuring colors. Therefore, we estimate that the systematic uncertainty of the measured colors (before correcting for reddening) is $0.010$ for WFPC2 and $0.015$ for ACS (as it has a shorter calibration history). Uncertainties in the Galactic reddening corrections are $\approx 16\%$ of the applied values (Schlegel et al. 1998), which is typically $\approx 0.01$ mag for our clusters. This error is largest for CL 0024+16, at 0.021.

The transformations to rest-frame colors also have some uncertainty, particularly if the observed $HST$ filters are not well matched to rest-frame $U - V$. This uncertainty was assessed by varying the templates used for the transformation, comparing results for the Yi et al. (2003) templates, Bruzual & Charlot (2003) templates, and empirical templates from Coleman et al. (1980). For most clusters the error is $\approx 0.015$ mag. For MS 2053-04 and MS 1054-03, which have the worst match of observed and rest-frame filters among the seven clusters, the uncertainties are $0.025$ and $0.030$, respectively. We note that, once again, a potentially important systematic effect cancels: the form of the synthetic rest-frame $U$ and $V$ filters, as well as their AB zero points, is the same for all distant clusters, which means that the distant clusters are all on the exact same synthetic photometric system. There may be a systematic offset between this system and Coma, although we use the same Bessell (1990) passbands as Landolt (1992). This possible offset was taken into account by adding a 0.03 mag error in quadrature to the systematic uncertainty of the Coma offset. Assuming that the sources of systematic error are independent, the combined error is given by

$$
\sigma_{sys}^{2} = \alpha_{1-2}^{2}(\sigma_{zero}^{2} + \sigma_{red}^{2}) + \sigma_{trafo}^{2}. \tag{13}
$$

The values of $\sigma_{sys}$ are listed in Table 1. They are generally small, of order $2\%$–$3\%$. The reliability of these numbers is empirically assessed below.

### 3.4. Evolution of the Color-Mass Relation

For each of the distant clusters in the sample the color-mass relation is constructed by combining the rest-frame $U - V$ colors with masses determined using equation (7). The required effective radii and velocity dispersions are taken from the literature and corrected to a consistent system (see Appendix A of vv07). The color-mass relations are shown in Figure 3. Galaxies at higher redshift have bluer rest-frame $U - V$ colors, as expected from stellar evolution and from many previous studies of color evolution (e.g., Stanford et al. 1995, 1998; Kodama et al. 1998; Holden et al. 2004).

The evolution of the zero point is quantified by subtracting equation (8) from the data points and determining the center of the distribution of residuals with the biweight statistic. Choosing the average or the median rather than the biweight slightly changes the offset of each cluster (always well within its error) but does not have an effect on our conclusions. We verified that the slopes for the high-redshift clusters are consistent with the Coma slope by fitting the color-mass relation of all galaxies in the seven distant clusters simultaneously, after removing the zero-point evolution derived below. The difference between this slope and the adopted Coma slope is only $0.01 \pm 0.02$. Varying the adopted slope within these limits does not change the derived offsets. The analysis relies on the assumption that the slopes do not vary appreciably from cluster to cluster. The samples of galaxies with mass measurements are too small to determine the slopes of the color-mass relations in individual clusters, but we note that Holden et al. (2004) find no evidence for either evolution or cluster-to-cluster variation in the slope of the color-magnitude relation.

The offsets are listed in Table 1. For each cluster the uncertainty in the offset $\sigma_{tot}$ is the quadramic sum of the systematic error and the uncertainty in the biweight mean (which is caused by the galaxy-galaxy scatter within each cluster). For most clusters the random uncertainty is of the same order as the systematic uncertainty. The evolution of the zero point of the color-mass relation is shown in Figure 4. The evolution is very regular for these clusters, which is perhaps not surprising given previous studies (e.g., Kodama et al. 1998; Andreon 2003; Holden et al. 2006) and the small cluster-to-cluster scatter in the evolution of $M/L$ ratios (see, e.g., vv07).

The solid line in Figure 4 is the best-fitting linear function to the data (including the three McIntosh et al. [2005] clusters):

$$
\Delta(U - V) = (-0.028 \pm 0.018) - (0.214 \pm 0.041)z. \tag{14}
$$

The observed cluster-to-cluster scatter around this line is very small at $0.022 \pm 0.005$ for all 11 clusters or $0.019 \pm 0.005$ for the eight clusters with direct mass measurements. This remarkably small cluster-to-cluster scatter is qualitatively consistent with previous work by Andreon (2003), although we note that our study uses a much smaller number of clusters.

The small measured scatter provides an external check on the reliability of the systematic errors derived in § 3. Focusing on the eight clusters with direct mass measurements, the expected scatter from measurement errors alone is $0.032$, higher than the observed scatter of $0.019$. Assuming that the errors are correct, the probability of measuring a scatter $\leq 0.019$ is only $7\%$, which means that the systematic errors are much more likely to be underestimated than overestimated. A formal limit on the systematic error can
be derived by requiring that the observed scatter of 0.019 has a one-sided probability >5% and parameterizing the total error for each cluster by \( \sigma_{\text{tot}}^2 = \sigma_{\text{random}}^2 + F^2 \sigma_{\text{sys}}^2 \) with \( F \) the same for all clusters. From Monte Carlo simulations we find that \( 0 < F < 1.12 \), i.e., the systematic errors are consistent with zero and underestimated by at most 12%.

Most studies of the color-magnitude relation at high redshift have focused on the galaxy-galaxy scatter in the relation. Although this is not a focus of the present paper, we can robustly measure the scatter for galaxies with \( M > 10^{11} M_\odot \) by removing the average trend (eq. [14]) and combining the galaxies in all distant clusters. The observed biweight scatter in the full sample \( \sigma_{U-V} = 0.056 \pm 0.005 \). The scatter expected from observational error is less than \( \sim 0.02 \), and we estimate that the intrinsic scatter is \( \approx 0.052 \). Averaging the three clusters at \( 0.17 < z < 0.40 \), the two clusters at \( 0.54 < z < 0.59 \), and the two clusters at \( 0.83 < z < 0.84 \), we...
find no evidence for evolution with redshift: the scatter in each redshift bin is consistent with the value for the full sample. The value for the scatter that we find is significantly lower than results from previous studies (e.g., Holden et al. 2004; Blakeslee et al. 2006). This is most likely due to a dependence of the scatter on galaxy mass. We also note that the color-mass relation is expected to have a smaller scatter than the color-luminosity relation even within the same sample of galaxies, if the scatter is due to age variations (see van Dokkum et al. 1998b).

4. FITTING

4.1. Constraints on the IMF

As discussed in § 2, luminosity and color evolution each depend on the IMF and on the age of the stellar population, but the age dependence drops out when comparing the amount of luminosity evolution to the amount of color evolution. Figure 5 shows the measured evolution in \( \log(M/L_B) \) as a function of the evolution in \( U - V \). This figure is the equivalent of Figure 1c. The values for \( \Delta \log(M/L_B) \) are taken directly from vv07, and the values for \( \Delta (U - V) \) are listed in Table 1.

As expected, there is a clear relation, with galaxies becoming both bluer and more luminous at earlier times. The solid line is a linear fit to the data (taking the errors in both parameters into account) of the form

\[
\Delta \log(M/L_B) = 2.8^{+0.7}_{-0.5} \Delta (U - V).
\]

(15)

The residuals from this fit are consistent with expectations from the uncertainties in the data points. The solid and dashed red lines in Figure 5 are predictions for a Salpeter-like\(^4\) IMF with \( x = 1.3 \) and two different metallicities. Remarkably, the observed relation is much steeper than expected from a standard IMF, even for solar metallicity. For a given color evolution the luminosity evolves faster than expected, indicating a top-heavy IMF with a relatively large fraction of rapidly evolving massive stars.

The logarithmic slope of the IMF follows directly from equations (5), (6), and (15). We find \( x = -1.4^{+1.1}_{-1.3} \) for supersolar metallicity and \( x = -0.1^{+0.3}_{-0.1} \) for solar metallicity, where the uncertainties reflect the 68% confidence interval. Both values are well below the canonical Salpeter value of \( x = 1.3 \). Negative values of \( x \) have a large systematic uncertainty, as they require significant extrapolation of the Maraston (2005) models. Therefore, the results can best be expressed as upper limits, particularly for the supersolar metallicity model. The 90% confidence upper limits are \( x < 0.1 \) for the supersolar model and \( x < 0.9 \) for solar metallicity. The Salpeter value can be ruled out at >98% confidence.

4.2. Joint Constraints on the IMF and Formation Epoch

Directly fitting \( \Delta \log(M/L_B) \) as a function of \( \Delta (U - V) \), as done in § 4.1, has several advantages: the same galaxies are used to measure the relevant parameters, limiting selection effects; redshift-dependent selection effects such as progenitor bias cancel; and the IMF is the only free parameter (at fixed metallicity), as the formation epoch of the stars \( z_{\text{form}} \) also cancels. However, constraining \( z_{\text{form}} \) better is of great interest in its own right and allows us to associate a particular time in the history of the universe with the IMF result. Here we fit the redshift evolution of the \( M/L_B \) ratio and the \( U - V \) color simultaneously and derive joint constraints on the slope of the IMF \( x \) and the star formation epoch \( z_{\text{form}} \). The goals are to determine the luminosity-weighted star formation epoch of massive cluster galaxies in a self-consistent way and to verify the results of § 4.1. Although the

---

\(^4\) The observations constrain the IMF in the mass range around 1 \( M_\odot \), and at those masses differences between the Salpeter (1955), Kroupa (2001), and Chabrier (2003) IMFs are very small. In vv07 it is shown explicitly that these various IMFs all predict very similar \( M/L \) evolution over the relevant range of ages, hence the term “Salpeter-like” or “standard” to denote IMFs with slope \( x \approx 1.3 \) at \( m \geq 1 \ M_\odot \).
additional parameter implies more freedom in the fits, this is compensated by the fact that more data can be used, as we are no longer restricted to the sample of eight clusters that have both $M/L$ and color information.

The data are shown in Figure 6. Figure 6a shows offsets in $\Delta \log (M/L_B)$ from vv07, and Figure 6b shows the offsets in $U-V$ determined in the present study. The data set is more extensive than used in the analysis of § 4.1. The $M/L$ sample includes data from the SDSS and for seven additional distant clusters. The color sample includes the three clusters from McIntosh et al. (2005), which help constrain the low-redshift end. Small corrections for progenitor bias have been applied, following vv07: $-0.05z$ for $\Delta \log (M/L_B)$ and $-0.023z$ for $\Delta (U-V)$. The color correction was chosen to be consistent with the correction for luminosity evolution and with equation (15). As shown in § 5.1, setting the progenitor bias to zero leads to slightly higher formation redshifts but has virtually no impact on the IMF constraints.

The data are fitted by creating models over a grid of $x$ and $t_{\text{form}}$, with $x$ the logarithmic slope of the IMF and $t_{\text{form}}$ the mean luminosity-weighted formation time of the stars. For each value of $x$ corresponding values of $\kappa_B$ and $\kappa_{U-V}$ are determined using equations (3) and (4). Next, equations (1) and (2) are used to determine the expected evolution with cosmic time for each combination of $x$ and $t_{\text{form}}$. The $\chi^2$ values of the fits are then calculated by

$$\chi^2_{x,t_{\text{form}}} = \sum_{i=1}^{16} \left[ \frac{(\Delta \log M/L_B)^i_{\text{obs}} - (\Delta \log M/L_B)^i_{x,t_{\text{form}}}}{\sigma^i_{\Delta \log M/L_B}} \right]^2 + \sum_{j=1}^{11} \left[ \frac{(\Delta (U-V))^j_{\text{obs}} - (\Delta (U-V))^j_{x,t_{\text{form}}}}{\sigma^j_{\Delta (U-V)}} \right]^2.$$  \hfill (16)

The fit has four free parameters: $x$, $t_{\text{form}}$ (or equivalently $z_{\text{form}}$), and the normalizations to the $M/L$ and color data. Figure 7 shows

**Fig. 6.—** Redshift evolution of (a) the rest-frame $M/L_B$ ratio of massive cluster galaxies from vv07 and (b) the rest-frame $U-V$ color. Corrections of $-0.05z$ and $-0.023z$ have been applied to the data in panels (a) and (b), respectively, to account for (mild) progenitor bias. The lines show models with different formation redshifts $z_{\text{form}}$ and IMF slopes $x$. The red model has a standard IMF and $x_{\text{form}} = 2$, the blue model has a standard IMF and $x_{\text{form}} = 6$, and the black model has a flat IMF and $x_{\text{form}} = 6$. Only the black model is a good fit to both data sets.

**Fig. 7.—** Results from simultaneous fits to the redshift evolution of $\log (M/L_B)$ and $U-V$ color. Contours indicate 68%, 95%, and 99% confidence limits, as determined from the $\chi^2(x, t_{\text{form}})$ distribution. Solid contours are for Maraston (2005) models with $[Fe/H] = 0.35$, and dashed contours are for $[Fe/H] = 0$. Early star formation and top-heavy IMFs are clearly preferred. Red, blue, and black filled circles indicate the locations of the models shown in Fig. 6.
the 68%, 95%, and 99% confidence intervals in the \((x, t_{\text{form}})\)-plane, as determined from the \(\Delta \chi^2\) values appropriate for two interesting parameters. Solid contours are for supersolar metallicity \((\text{[Fe/H]} = 0.35)\) Maraston (2005) models, and dashed contours are for solar metallicity models. Flat IMFs and high formation redshifts are clearly preferred, for both metallicities.

The red, blue, and black filled circles indicate examples of models with different parameters. The fits to the luminosity and color evolution for these three models are shown in Figure 6 and illustrate how the data simultaneously constrain \(x\) and \(t_{\text{form}}\). The red model has a standard Salpeter IMF and a low formation redshift \(z_{\text{form}} = 2\). This model provides an excellent fit to the evolution of the \(M/L_B\) ratio, as shown in Figure 6a and discussed in vv07. However, this model fits the data in Figure 6b poorly: the observed color evolution is much slower than predicted by this young model. The blue model is a Salpeter model with a high formation redshift \(z_{\text{form}} = 6\). This model fits the slow color evolution very well but underpredicts the \(M/L\) evolution in Figure 6a. The only models that fit the data in both panels simultaneously have high formation redshifts (to fit the colors) and a flat IMF (to fit the \(M/L\) ratios). The black model has \(z_{\text{form}} = 6\) and \(x = -0.9\). This model provides an excellent fit to the data in both panels.

The formal best-fitting values are \(z_{\text{form}} = 7.6, x = -1.2\) for supersolar metallicity and \(z_{\text{form}} = 3.7, x = -0.3\) for solar metallicity. The best-fit values for the IMF slope are in very good agreement with those derived in Section 4.1. As discussed earlier, negative values for \(x\) have large systematic uncertainties and may not be very meaningful. The supersolar solutions are therefore somewhat difficult to interpret. Formal 68% confidence intervals for the solar metallicity solution are \(-1.0 \leq x \leq 0.1\) and \(2.9 \leq z_{\text{form}} \leq 6.0\). These results are consistent with the analysis of the \(M/L\) evolution alone in vv07, where it was noted that the best-fitting \(z_{\text{form}} \approx 4\) for top-heavy IMFs with \(x = 0\).

5. Caveats and Uncertainties

Although straightforward in principle, the IMF test discussed in this paper has several associated uncertainties, some having to do with the data and some with the models that are used to fit the data.

5.1. Effects of Sample Selection and Progenitor Bias

Every cluster for which the required data were available in the literature and/or the HST archive was included in the analysis, and as they come from a wide range of projects by many different research groups, this may have introduced systematic errors. In particular, it may be that the sample of clusters with FP measurements is somehow different from the sample of clusters with color measurements. This potential bias is explicitly addressed in Section 4.1, where the analysis was limited to clusters in common between the two samples. As a further test, the analysis in Section 4.2 was repeated after removing the three clusters with \(z > 0.83\) from the sample. These clusters have a large weight in the \(M/L\) analysis as they are at the highest redshifts, and they have no counterparts in the color sample. The effects are small, but not negligible: for the supersolar model the best-fitting IMF slope changes from \(-1.2\) to \(-1.0\) and the best-fitting formation redshift changes from 7.6 to 5.9 when the highest redshift clusters are removed.

Redshift-dependent selection effects such as progenitor bias are not a major concern in this particular study. If we systematically miss the youngest progenitors of today’s early-type galaxies within clusters, the formation redshifts that we derive may be too high, but the IMF results should be robust. The IMF is constrained by the ratio of the amount of luminosity evolution to the amount of color evolution, and progenitor bias would (to first order) equally affect the colors and luminosities. This can be tested explicitly by varying the progenitor bias correction to the \(M/L\) ratios and colors and repeating the analysis. Increasing progenitor bias by a factor of 2 gives the same IMF slope \((-1.2)\) as the model adopted in Section 4.2 and a lower formation redshift (4.8 rather than 7.6). Setting the progenitor bias to zero again gives the same IMF slope \((-1.2)\) and a formal best-fitting formation redshift \(z_{\text{form}} = \infty\).

5.2. Systematic Errors in the Photometry

As discussed in Section 3.3.2, the low cluster-to-cluster scatter in the color evolution implies that the errors in the photometry are more likely overestimated than underestimated. Nevertheless, absolute photometry is challenging, and it is difficult to rule out subtle systematics in the various corrections and transformations that have been applied to the data. Increasing the error bars on the colors by 50% (much more than is plausible; see Section 3.3.2) obviously loosens the constraints on the IMF, but not by a large amount. Repeating the analysis of Figure 6a changes the upper limits on the IMF slope from \(x < 0.1\) (for supersolar metallicity) and \(x < 0.9\) (for solar metallicity) to \(x < 0.4\) and \(x < 1.1\), respectively.

Aside from general concern about the assumed errors, a specific worry is the tie between the low-redshift data and the (homogeneous) synthetic photometric system that is used for the distant clusters. Absolute photometry in the \(U\) band is particularly difficult: the bandpass is strongly influenced by the detector and the atmosphere, and the spectral energy distributions of old galaxies have a very steep slope around 3800 Å. We dealt with this issue in Section 3.3.2 by adding a 3% systematic uncertainty in the Coma offset. A more conservative approach is to simply discard Coma altogether. Interestingly, removing Coma has the effect of tightening the limits on the IMF slope. As an example, for solar metallicity the 90% upper limit changes from 0.9 to 0.3. The reason for this is readily apparent from Figure 5: Coma is the point with the largest deviation from the best-fitting line, and the relation becomes much steeper when it is removed. We note that the mean \(M/L_B\) ratio of Coma galaxies may also be somewhat anomalous, as it deviates from the best-fitting relation in Figure 6a. Adopting the \(M/L_B\) ratio from this fit rather than the actual value brings the cluster in close agreement with the solid line in Figure 5.

5.3. Interpretation of the Fundamental Plane and \(M/L\) Ratios

The evolution of the mean \(M/L\) ratio, shown in Figure 6a, is derived from the evolution of the FP relation (Djorgovski & Davis 1987). As discussed in detail in several papers (e.g., Franx 1993; van Dokkum & Franx 1996; Treu et al. 2001), the empirical FP relation can be rewritten as a relation between \(M/L\) ratio and mass, if it is assumed that \(M \propto \sigma^2 r_e\) (with \(\sigma\) the stellar velocity dispersion and \(r_e\) the effective radius) and early-type galaxies are a homologous family. If these assumptions are valid, the observed evolution of the FP should track the evolution of the zero point of the underlying \(M/L\)-mass relation. However, this may not be the case if galaxies undergo significant structural changes over time, due to merging or other processes (see, e.g., Almeida et al. 2007).

Strong gravitational lenses provide an independent check on the FP interpretation. Current results indicate that the evolution of the \(M/L\) ratio as derived from strong lenses is consistent with that derived for field early-type galaxies (Rusin & Kochanek 2005).

5 Abell 2218 is the point with the largest deviation in the opposite sense to Coma; arbitrarily removing this cluster (while retaining Coma) changes the 90% upper limit from 0.9 to 1.3 for solar metallicity.
although the systematic uncertainties in this comparison are still fairly substantial (see, e.g., Treu & Koopmans 2004; van der Wel et al. 2005; Rusin & Kochanek 2005). Recently van der Marel & van Dokkum (2007) used spatially resolved photometric and dynamical observations of cluster early-type galaxies at $z \approx 0.5$ to directly test the assumption that the FP tracks $M/L$. In the models identical to the stellar kinematical observations of cluster early-type galaxies at $z \approx 0.5$ by van Dokkum (2007) used spatially resolved photometric and dynamical observations of cluster early-type galaxies at $z \approx 0.5$ to directly test the assumption that the FP tracks $M/L$. Interestingly, the validity of the FP for determining $M/L$ ratios appears to depend on the mass: for galaxies with masses $\geq 10^{11} M_{\odot}$ or velocity dispersions $\geq 200$ km s$^{-1}$ the $M/L$ ratios are consistent with each other, but for low-mass galaxies the FP appears to systematically underestimate the $M/L$ ratio. Although these results suggest that the traditional interpretation of FP evolution is substantially correct at the high-mass end, further tests would be valuable, particularly at $z > 0.5$.

Another complication is that the measured $M/L$ ratio is not identical to the stellar $M/L$ ratio calculated in stellar population synthesis models. In the models $M_{\star}$ is the sum of the mass of all living stars and the mass of stellar remnants (black holes, neutron stars, and white dwarfs). By contrast, the measured mass includes all possible contributions, including dark matter and gas. Although the amount of dark matter in elliptical galaxies is still somewhat uncertain (e.g., Romanowsky et al. 2003; Mamon & Lokas 2005; Cappellari et al. 2006), its contribution within the effective radius is probably small (Mamon & Lokas 2005; Koopmans et al. 2006). The contribution of gas to the total mass is also uncertain. One would expect substantial amounts of gas and dust from stellar mass loss (see, e.g., Goudis & Goudfrooij et al. 1994), qualitatively consistent with Spitzer observations (Temi et al. 2007). However, the data appear to be inconsistent with simple expectations, and Temi et al. (2007) suggest that the gas is transported outward, possibly due to active galactic nucleus activity. Furthermore, in galaxy clusters the galaxies move through very hot, diffuse gas at great speed and it seems likely that any cold gas coming from stellar winds is fairly efficiently stripped or heated.

Although dark matter and gas could contribute to the measured $M/L$ ratio, these contributions will to large extent cancel when comparing galaxies over a range of redshifts. In the Maraston (2005) models the amount of mass loss over the relevant age range from 5 to 12 Gyr is only $\approx 2\%$ for a Salpeter (1955) IMF and $\approx 3\%$ for a Kroupa (2001) IMF. Even if half of this mass was retained rather than lost to the intracluster medium, the $M/L$ evolution would change by only $1\%–2\%$.

5.4. Uncertainties in Stellar Population Synthesis Models

Uncertainties in the stellar population synthesis models are perhaps the largest source of error, and at the same time they are notoriously difficult to quantify. The fundamental problem is that the main direct tests and calibrations of the models are offered by Galactic open clusters and globular clusters, but that these may not be representative for the integrated stellar populations of massive early-type galaxies.

Recently the confidence in these models was somewhat shaken by the large differences that exist between predictions of Bruzual & Charlot (2003) and Maraston (2005). As discussed in detail by Maraston (2005) and others (e.g., van der Wel et al. 2006b; Maraston et al. 2006), the predicted rest-frame optical to near-infrared colors of simple stellar populations can deviate by $\geq 0.5$ mag for identical input metallicities and ages. The differences are in large part due to a different numerical implementation of the thermally pulsating asymptotic giant branch (TP-AGB) phase of stellar evolution and are therefore largest for ages $0.5–2$ Gyr and colors that include a near-infrared band (such as $B – K$).

Our analysis avoids the region of parameter space where the differences between the models are most pronounced (ages of $0.5–2$ Gyr and rest-frame near-infrared passbands). The models are better calibrated (and in much better agreement) for ages $> 3$ Gyr and rest-frame optical passbands, largely because the contribution of giants to the integrated light is significantly smaller. However, subtle differences between the models also exist at ages $> 3$ Gyr; as an example, the $B – K$ colors of the Maraston (2005) models are slightly bluer than those of Bruzual & Charlot (2003) (see van der Wel et al. 2006b).

To get some handle on the effects of the choice of model, the derivation of equation (5) was repeated for a solar metallicity Bruzual & Charlot (2003) model. By comparing the color evolution to the $M/L$ evolution for two different IMF slopes ($x = 1.35$ and $x = 0.35$), we find $\kappa_{B}/\kappa_{U-V} \approx 5.0–1.2 x$. The relation is less steep than that of the solar metallicity Maraston (2005) model and is close to Maraston’s supersolar metallicity model (Fig. 5, solid line). For a Salpeter IMF and solar metallicity the Bruzual & Charlot (2003) model provides an even worse fit to the data than the Maraston (2005) model, and the measured slope of the $M/L$-color relation implies $x \approx -0.4$ (compared to $x \approx -0.1$ for the solar metallicity Maraston model). We infer that our conclusions are robust in the context of presently popular models, but note that there are significant variations in the model predictions even in the rest-frame optical.

Finally, an important check on the entire enterprise is whether the models can match the absolute colors of the galaxies. So far, we have not used the absolute colors in any way and only considered the rate of color evolution, leaving the normalization a free parameter. The absolute colors are difficult to interpret, as they suffer from strong degeneracies between age, metallicity, and dust content. Nevertheless, the models should be able to reproduce the observed colors of massive galaxies in Coma for plausible combinations of these parameters. Figure 8 shows the location of galaxies in the Coma Cluster in a color-color diagram of $V - R$ versus $U - V$, along with model predictions of Maraston (2005). All models have an age of 12 Gyr, corresponding to $z_{\text{form}} \approx 4$. The models have approximately the correct $U - V$ colors at $z = 0$ for $z_{\text{form}} \approx 4$ and are therefore self-consistent. The fact that the agreement is particularly good for models with solar metallicity and $x = 0$ is probably coincidence, given the uncertainties.

5.5. Blue Stars in Old Stellar Populations

As is well known, old stellar populations can include very luminous hot stars. These stars are rare, but due to their high luminosity they can have a significant effect on the integrated luminosity of a stellar population, particularly in blue passbands. These hot stars fall in three broad categories: blue horizontal branch stars (e.g., Zinn et al. 1972; Rich et al. 1997), extreme horizontal branch stars (e.g., Dorman et al. 1993), and blue stragglers (BSs; e.g., Sandage 1953; Bailin 1995). Blue horizontal branch stars are generally included in current stellar population synthesis models. Extreme horizontal branch stars are thought to be responsible for the UV upturn in elliptical galaxies at $\lambda \leq 2000$ Å (Burstein et al. 1988) and should not affect the $U - V$ colors (see, e.g., Yi et al. 2003, 2005).

6 It should be noted that the treatment of TP-AGB stars is revised in the most recent incarnation of the Bruzual & Charlot (2003) models, which brings them more in line with the Maraston (2005) models (see Bruzual 2007).
Different colors indicate different IMF slopes: the question is not how much BSs contribute to the integrated colors, galaxies (see, e.g., Stryker 1993). Nevertheless, they probably have a priori reason to suppose that BSs do not exist in elliptical galaxies, larger than the IMF effects discussed in this paper, and there is no certain that in M67 would be larger than the IMF effects discussed in this paper, and there is no a priori reason to suppose that BSs do not exist in elliptical galaxies (see, e.g., Stryker 1993). Nevertheless, they probably do not have a very large effect on our results. First, the relevant question is not how much BSs contribute to the integrated colors, but to what extent they influence their evolution from $5$ Gyr (the age at $z = 0$) to $\sim 12$ Gyr (the age at $z = 0$). To have a significant impact on our results BSs would have to have been absent in elliptical galaxies for $5$ Gyr or more and then have "turned on." This seems rather contrived, particularly since the turnoff mass changes by only $\sim 0.4 M_\odot$ over this age range. If anything, one might expect BSs to be slightly more prevalent at younger ages. One of the preferred models for their formation is through the coalescence of binary stars (Mateo et al. 1990), and if binaries start out with a flat distribution of orbital parameters, the rate at which some of them turn into BSs will gradually decrease. Coupled with the fact that BS lifetimes are short compared to the main-sequence stars, this could lead to a gradual decline in the BS population. More to the point, Xin & Deng (2005) have determined the contribution of BSs to the integrated light of open clusters with a large range of ages. They find no correlation between the contribution of BSs to the integrated colors and the age of the cluster. As a result, the integrated colors of clusters with ages varying from $1$ to $8$ Gyr are reasonably well described by a single stellar population synthesis model with a constant offset.

Another reason why the effects of BSs are probably fairly mild is that it is difficult to hide them completely in nearby elliptical galaxies. If BSs are as prevalent in elliptical galaxies as they are in open clusters, they would lead to relatively blue near-UV colors and strong Balmer absorption lines. Population synthesis models would (correctly) infer the presence of A and F main-sequence stars and (incorrectly) attribute them to a secondary young stellar population and/or assign an overall young luminosity-weighted age to the galaxy. As pointed out by Schiavon et al. (2004), if a galaxy had the same stellar population as M67, it would be classified as "E+A" (Dressler & Gunn 1983) to signify the presence of a poststarburst component in addition to an older component. Xin & Deng (2005) quantify these effects and conclude that the ages of unresolved stellar populations will typically be underestimated by a factor of $\sim 2$ if BSs are present in similar numbers as in open clusters. Turning to many years of population synthesis modeling of massive elliptical galaxies in clusters, such dramatic effects can be safely ruled out. As an example, Thomas et al. (2005) infer ages of $\sim 12$ Gyr (i.e., close to the age of the universe) for cluster galaxies with $M \geq 10^{11} M_\odot$, not $6$ Gyr as one might expect from a BS-rich stellar population. Qualitatively, the red colors and weak Balmer lines of massive cluster ellipticals place strong limits on the relevance of BSs in these galaxies.

Finally, we note that Trager et al. (2000a) argue that BSs do not contribute significantly to the integrated light even in elliptical galaxies that do have enhanced Balmer lines and signs of young populations (typically relatively low mass field ellipticals). Rather than discuss their arguments, we note that the finding that the derived age of ellipticals appears to correlate with the presence of morphological fine structure (Schweizer & Seitzer 1992) provides additional support for the notion that late star formation, rather than a BS population, is generally responsible for the young appearance of some nearby ellipticals. We are left to wonder why BSs seem to be deficient in elliptical galaxies, as compared to old open clusters. No speculation is offered here, as this question is well outside the scope of the present study.

5.6. Complex Evolution and Dust

As argued in § 2.3, the population of massive early-type galaxies in clusters is not thought to have experienced significant star formation at redshifts $< 1$. Young stellar populations have been seen in some field ellipticals (e.g., Trager et al. 2000b; Thomas et al. 2005), but a large body of observational evidence supports the notion that massive cluster galaxies are remarkably homogeneous (e.g., Bower et al. 1992b; Stanford et al. 1995, 1998; Ellis et al. 1997; Kodama et al. 1998; van Dokkum et al. 2000; Holden et al. 2004; Thomas et al. 2005; Tran et al. 2007). This apparent homogeneity may be misleading and mask more complex evolution (van Dokkum & Franx 2001), but this is not likely for the most massive galaxies (Holden et al. 2007). It may well be that cluster early-type galaxies experienced more complex evolution at $z > 1$, in particular during and before virialization of the clusters in which they now live, but this does not affect the analysis. The parameter $z_{\text{form}}$ does not refer to a single star formation event, but reflects the luminosity-weighted mean formation epoch of the stars. As shown explicitly in van Dokkum et al. (1998b), color evolution and luminosity evolution are nearly...
identical for stellar populations with different star formation histories but the same luminosity-weighted age, as long as star formation terminated at least \( \sim 1 \) Gyr prior to the epoch of observation.

Dust needs to be treated separately, as significant amounts of gas and dust are expected to exist in early-type galaxies as a result of winds emanating from massive stars (see also § 5.3). Cluster galaxies at \( z \sim 0.8 \) may still have retained a significant fraction of the dust expelled by massive stars, whereas it may have been stripped, blown out, or obliterated at later times (see, e.g., Goudfrooij et al. 1994; Temi et al. 2007). Quantifying these effects is very difficult, as they depend not only on the amount of mass loss but also on the detailed processes and timescales for cooling and heating of the dust. Qualitatively, one might expect higher reddening at \( z \sim 0.8 \) and therefore a lower apparent color evolution from \( z \sim 0.8 \) to \( z \sim 0 \). The true color evolution would then be stronger, and the data might be consistent with a normal IMF in combination with a relatively low stellar age.

However, dust would affect not only the colors but also the luminosities. The relation between reddening and extinction in elliptical galaxies is not well known but is probably not very different from that in the Milky Way (e.g., Goudfrooij et al. 1994). For reference, arrows in Figure 5 show the effects of Milky Way–like dust and of the (relatively gray) Calzetti (1997) extinction curve. Calzetti-like dust would have essentially no effect on the analysis, as the dust vector is nearly parallel to the observed relation between \( M/L \) ratio and color. Milky Way–like dust would have an effect but would lead to unrealistically low ages for the galaxies. To change the observed relation to the one expected for a standard IMF, the \( z = 0.8 \) galaxies would have to be reddened by \( \sim 0.3 \) mag in \( U - V \), corresponding to a \( B \)-band extinction of \( \sim 0.6 \) mag. The implied luminosity evolution from \( z = 0.8 \) to the present would then be \( \sim 1.7 \) mag (instead of 1.1 mag; see vv07). This in turn would mean that massive cluster galaxies formed their stars at \( z \sim 1.3 \), which is ruled out by many independent observations (e.g., Kelson et al. 2001; Thomas et al. 2005; Kodama et al. 2007).

5.7. Summary of Fitting Results and Associated Uncertainties

The main result from the fits in § 4 is that stellar population synthesis models with a standard, Salpeter-like IMF in the region around \( 1 \) \( M_\odot \) are not able to simultaneously fit the luminosity and color evolution of massive cluster galaxies. By contrast, models with flat IMFs and high stellar formation redshifts provide excellent fits. The slow color evolution measured here is not easily explained by invoking sample selection effects, photometric errors, a BS population, or differential dust extinction.

The formal limits on the IMF slope and formation redshift are very difficult, as they depend not only on the amount of mass loss but also on the detailed processes and timescales for cooling and heating of the dust. Qualitatively, one might expect higher reddening at \( z \sim 0.8 \) and therefore a lower apparent color evolution from \( z \sim 0.8 \) to \( z \sim 0 \). The true color evolution would then be stronger, and the data might be consistent with a normal IMF in combination with a relatively low stellar age.

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6. COMPARISON TO PREVIOUS RESULTS

Many previous studies have determined or constrained the properties of the stellar populations of early-type galaxies. Although direct comparisons are difficult because of differences in sample selection and methodology, it is important to verify that our results are broadly consistent with the large body of literature that exists on this topic.

6.1. Other Studies of the Evolution of \( M/L \) Ratios and Colors

As discussed in detail in vv07, the values shown in Figure 6 show are mostly determined from literature data and therefore “automatically” consistent with a large body of previous work. The only exception is Jørgensen et al. (2006), who find substantially slower evolution in the first version of their published paper. However, this was due to a conversion error, and the vv07 results are fully consistent with the erratum to Jørgensen et al. (2006) published in 2007.

The evolution of the rest-frame \( U - V \) color cannot be directly compared to previous work, as most studies choose to compare the observed colors to redshifted model predictions rather than convert the data to the rest frame. Furthermore, color evolution has not been studied previously for mass-limited samples of cluster galaxies. Nevertheless, it is interesting to compare the constraints on the stellar ages that are reported in the literature, as they should reflect the measured rate of color evolution.

Most studies of the color evolution of early-type galaxies in clusters find very old ages, in qualitative agreement with the results found here (e.g., Stanford et al. 1995, 1998; Kodama et al. 1998; Koo et al. 2005). In particular, no studies have reported formation redshifts as low as \( z \sim 2 \) for the most massive galaxies in clusters, which is the value implied by the evolution of the \( M/L \) ratios for a standard IMF. The most comprehensive study to date is by Holden et al. (2004), who combined data from Stanford et al. (1998) with new measurements to obtain a combined sample of 31 clusters at \( 0.3 < z < 1.3 \) with multiband photometry. They find that the highest age at which the colors can be matched is \( (6.2 \pm 1.7) \), and this is clearly inconsistent with the data and can be rejected with high confidence. By contrast, the top-heavy/old model fits the data very well. This model is the formal best fit to the \( M/L \) and color evolution (as discussed in § 5.7), but other models with formation redshifts in the range 3–6 also fit well.

The evolution of the Balmer lines thus provides strong independent support for the analysis in § 4. We note that the inferred ages are consistent with the analysis in Kelson et al. (2001): they give 95% confidence lower limits to \( z_{\text{form}} \) of 2.4 and 2.9 for two different stellar population synthesis models.

6.2. Evolution of Balmer Absorption Lines

Kelson et al. (2001) study the evolution of the Balmer absorption line strengths of early-type galaxies in clusters at \( 0.6 \leq z \leq 0.83 \), and their data offer an independent check on the results derived here. The Balmer lines should behave like the \( U - V \) colors, in the sense that they are fairly insensitive to the IMF and their rate of evolution is mostly determined by the age of the stellar population. Importantly, the sources of systematic error (both in the data and in the models) are very different from those that affect the colors (see, e.g., Kelson et al. 2006).

The Kelson et al. (2001) data are shown in Figure 9. The data are compared to two solar metallicity Bruzual & Charlot (2003) models: a model with a standard IMF and a young age, and a model with a top-heavy IMF and an old age. As discussed in vv07 and in § 4.2, these are the only two classes of models that provide good fits to the observed evolution of the \( M/L \) ratio. The young model is clearly inconsistent with the data and can be rejected with high confidence. By contrast, the top-heavy/old model fits the data very well. This model is the formal best fit to the \( M/L \) and color evolution (as discussed in § 5.7), but other models with formation redshifts in the range 3–6 also fit well.

The evolution of the Balmer lines thus provides strong independent support for the analysis in § 4. We note that the inferred ages are consistent with the analysis in Kelson et al. (2001): they give 95% confidence lower limits to \( z_{\text{form}} \) of 2.4 and 2.9 for two different stellar population synthesis models.

6.3. Evolution of Field Early-Type Galaxies

Field early-type galaxies probably evolve in more complex ways than their counterparts in clusters. Galaxy-galaxy mergers
are expected to be rare in virialized clusters (e.g., Makino & Hut 1997) but could be common in groups (e.g., van Dokkum 2005; Bell et al. 2006), and the same is probably true for late star formation (e.g., Trager et al. 2000b). Field samples may therefore not be suitable for measuring the subtle effects discussed in this paper. Nevertheless, it is interesting to note that van der Wel et al. (2006a) find that Bruzual & Charlot (2003) models with a standard IMF do not fit the observed relation between $B - K$ color and $M/L_B$ ratio of field early-type galaxies at $0 < z < 1$. Models with top-heavy IMFs fit better, although Maraston (2005) models with a standard IMF also provide good fits.

### 6.4. “Red and Dead” Galaxies at High Redshift

Recent studies of faint $K$-selected samples have identified a population of apparently “red and dead” galaxies at $z \approx 2.5$ (Franx et al. 2003; Labbé et al. 2005; Daddi et al. 2005; Kriek et al. 2006). Deep rest-frame optical spectra demonstrate strong Balmer breaks and no line emission in a substantial fraction of massive galaxies at these redshifts, implying that star formation in these objects mostly terminated at $z \approx 3$ or earlier (Kriek et al. 2006). These objects are found in blind surveys of relatively small fields, and it seems likely that many galaxies in (proto)clusters terminated star formation even earlier, and/or that the fraction of (nearly) passive galaxies is higher in those environments (see, e.g., Thomas et al. 2005; Steidel et al. 2005; Quadri et al. 2007; Kodama et al. 2007). Assuming that these galaxies are not “rejuvenated” at later times (which is entirely possible), the presence of substantial numbers of apparently old galaxies at this redshift places an interesting constraint on the luminosity-weighted ages of massive early-type galaxies.

For any star formation history the time of last star formation will be more recent than the luminosity-weighted mean star formation epoch. After a $\delta$-function, the simplest star formation history is a top hat with a constant star formation rate from $t = t_{\text{start}}$ to $t = t_{\text{stop}}$. Such models may actually be fairly realistic (as compared to exponentially declining models, for instance), as they superficially resemble the star formation histories of massive galaxies in hydrodynamical simulations (Nagamine et al. 2005). As shown in van Dokkum et al. (1998b), there is a simple relation between the luminosity-weighted age $t - t_{\text{form}}$ and $t_{\text{stop}}$ in these models: $t - t_{\text{form}} \approx (t - t_{\text{start}})(t - t_{\text{stop}})^{-1/2}$. Assuming that $t_{\text{stop}} \approx 3$ (extrapolating from the Kriek et al. [2006] results for red field galaxies), the luminosity-weighted mean formation epoch $t_{\text{form}} \approx 3.7$ for $t_{\text{stop}} - t_{\text{start}} \geq 1$ Gyr.

Luminosity-weighted formation redshifts of $\sim 4$ are entirely consistent with the old/top-heavy model that is implied by the analysis in § 4. Formation redshifts of 2–2.5, which are implied by the $M/L$ ratio evolution for Salpeter-like IMFs (see vvd07 and § 4), may be difficult to reconcile with the presence of apparently old field galaxies at $z \approx 2.5$. We note, however, that this type of evidence should be treated with caution, as the old high-redshift objects could constitute only a small fraction of the progenitors of today’s early-type galaxies.

### 6.5. Line Strengths of Nearby Elliptical Galaxies

Many studies have constrained the properties of the stellar populations of nearby early-type galaxies using absorption-line strengths (e.g., Peletier 1989; Worthey et al. 1992; Trager et al. 2000b; Thomas et al. 2005). Line strengths are generally more helpful than colors, as they suffer slightly less from the strong degeneracies that exist between (particularly) age and metallicity (e.g., Worthey 1994). One of the difficulties in line strength studies is that the abundance ratios of early-type galaxies do not match those of solar abundance ratios can date the star formation epoch of early-type galaxies. These models indicate that cluster early-type galaxies formed their stars at $z = 3–5$, in good agreement with the ages derived here for top-heavy IMFs.

### 6.6. Absolute $M/L$ Ratios of Nearby Elliptical Galaxies

In § 4 we were only concerned with the evolution of the derived $M/L$ ratios, and not with their absolute values. The absolute $M/L$ ratios of nearby elliptical galaxies provide a powerful additional constraint on the IMF, as the stellar $M/L$ ratio implied by the combination of the IMF and the observed luminosity should not exceed the dynamical $M/L$ ratio. As discussed in § 1, the observations of Cappellari et al. (2006) are consistent with the stellar mass implied by a Chabrier (2003) IMF and rule out IMFs that are significantly steeper than the Salpeter IMF because they have too much mass locked up in low-mass stars. Perhaps surprisingly, the dynamical $M/L$ ratios also rule out top-heavy IMFs of the form fitted in § 4. These IMFs have less low-mass stars than a standard Salpeter (1955) IMF but many more high-mass stars. As a result, the stellar mass function at late times is “remnant heavy,” dominated as it is by white dwarfs, neutron stars, and black holes. Maraston (1998) shows that the $M/L$ ratio at old ages reaches a minimum for $x \approx 1.3$: for larger values the IMF is dwarf dominated and for smaller values the IMF is remnant dominated. Specifically, solar metallicity Maraston (2005) models predict $M/L_B = 9$ at 12 Gyr for $x = 1.35$ and $M/L_B = 87$ for $x = 0$. Such high $M/L$ ratios are an order of magnitude higher than measured (see, among many other studies, van der Marel 1991; Cappellari et al. 2006; Gavazzi et al. 2007), which means that IMFs with a constant logarithmic slope $x = 0$ over the entire mass range of $100 M_\odot$ can effectively be ruled

![Figure 9. Evolution of the Balmer absorption line strengths of early-type galaxies in clusters, taken from Kelson et al. (2001). The lines show the two classes of models that fit the evolution of the $M/L$ ratio of early-type galaxies: young models with a Salpeter IMF and old models with a top-heavy IMF. Only the old/top-heavy model fits the line strength data, providing independent support for our results.](image-url)
out. In the next section we explore IMFs that have a Salpeter-like slope at high masses and \( x \sim 0 \) in the region around \( 1 M_\odot \). Such IMFs are physically more plausible than IMFs with a constant slope and do not violate dynamical limits on the stellar \( M/L \) ratios of nearby galaxies (see § 7.4).

7. IMPLICATIONS

7.1. Implications for the Characteristic Mass at \( z \sim 4 \)

In the Maraston (2005) models described in § 2.2 the IMF is parameterized as a power law with constant slope \( x \) over the entire mass range \( 0.1 - 100 M_\odot \). The best-fitting slope of \( x = 0 \) implies an extremely top-heavy mass function, with a very large number of massive stars. However, other forms of the IMF are also consistent with the data, as the analysis of § 4 is completely insensitive to the IMF at masses exceeding the turnoff mass at \( \sim 5 \) Gyr (\( \approx 1.15 M_\odot \)). A physically more plausible IMF would have a Chabrier (2003) form, with a fixed high-mass slope of \( x \approx 1.3 \) and a varying characteristic mass \( m_c \). As quantified later, such IMFs are not only physically motivated but also entirely consistent with the dynamical \( M/L \) ratios measured for nearby ellipticals.

So far, no stellar population synthesis modeling has been done with Chabrier IMFs with varying \( m_c \). However, we can use the fact that our observations probe the IMF only over a limited stellar mass range to derive an approximate relation between \( m_c \) and the slope of the IMF as parameterized in the Maraston (2005) models. The Chabrier (2003) IMF has the form

\[
\xi = \begin{cases} 
A_1 \exp \left[ -(\log m - \log m_c)^2/2\sigma^2 \right], & m \leq 1 M_\odot, \\
A_b m^{-x}, & m > 1 M_\odot, 
\end{cases}
\]

(17)

with \( A_1 = 0.158 \), \( m_c = 0.079 M_\odot \), \( \sigma = 0.69 \), \( A_b = 0.0443 \), and \( x = 1.3 \), and is shown by the dashed lines in Figure 10. As shown in Chabrier (2003), this IMF is very similar to the Kroupa (2001) IMF, and both IMFs are nearly identical to the Salpeter (1955) IMF for \( m > 1 M_\odot \). Figure 10b shows the derivative \( d[\log \xi]/d\log m \), which can be thought of as the local logarithmic slope of the IMF. The slope gradually steepens until leveling off at the Salpeter value for masses \( m > 1 M_\odot \).

The derivative of the Chabrier (2003) IMF has a discontinuity at \( 1 M_\odot \), where the lognormal part of the IMF and the power-law part connect. This discontinuity is rather minor and has no practical implications, but it becomes more pronounced when considering higher values of \( m_c \). Therefore, we adopt an extension of the Chabrier (2003) disk IMF parameterization that does not produce discontinuities in the derivative and explicitly accommodates a varying \( m_c \):

\[
\xi = \begin{cases} 
A_1 (0.5 n_c m_c)^{-x} \exp \left[ -(\log m - \log m_c)^2/2\sigma^2 \right], & m \leq n_c m_c, \\
A_b m^{-x}, & m > n_c m_c, 
\end{cases}
\]

(18)

with \( A_1 = 0.140 \), \( n_c = 25 \), and \( \sigma, A_b, \) and \( x \) identical to equation (17) (for an alternative parameterization of IMFs with a high characteristic mass see Chabrier 2003). The behavior of this modified Chabrier IMF for \( m_c = 0.08, 0.4, \) and \( 2 M_\odot \) is shown in Figures 10a and 10b. For \( m_c = 0.08 \) this IMF is nearly identical to the Chabrier (2003) disk IMF.

The observations discussed in this paper are sensitive to the IMF in a limited stellar mass range near \( 1 M_\odot \). The B-band luminosity-weighted mean stellar mass of a stellar population is typically slightly smaller than the turnoff mass and ranges from 1.0 to 0.8 \( M_\odot \) for ages of \( 5 - 12 \) Gyr. As is evident from Figure 10, the IMF can be approximated by a power law over this small mass range, and the slope of this power law depends on \( m_c \). We find

\[
\log m_c \approx -0.05 - 1.1 x_1 \quad (x_1 < 1.3),
\]

(19)

with \( x_1 = -d [\log \xi/d \log m] \) the logarithmic slope of the IMF near \( m \approx 1 M_\odot \). This expression relates Salpeter-like IMFs with varying slope \( x \) as used in the Maraston (2005) models and equations (3) and (4), to physically more plausible Chabrier-like IMFs with fixed high-mass slope and varying characteristic mass.

The best-fitting slope of the IMF as derived from solar metallicity Maraston (2005) models is \( x = -0.3^{+0.4}_{-0.7} \) (see § 5.7). In the context of a Chabrier-like IMF this slope implies a characteristic mass \( m_c = 1.9^{+0.3}_{-0.5} M_\odot \) at \( z = 3.7^{+2.3}_{-0.4} \). For supersolar metallicity the best-fitting values of \( x \) are strongly negative and may imply \( m_c \sim 10 M_\odot \), although this represents a substantial extrapolation of the models.

We conclude that the color evolution and luminosity evolution of early-type galaxies are consistent with IMFs that have a Salpeter slope at high masses but turn over near \( \sim 1 M_\odot \). Such an IMF is perhaps best described as “bottom-light” rather than top-heavy; it does not have a larger number of massive stars than a standard Chabrier (2003) IMF, but has a deficit of low-mass stars. The form of the IMF at masses \(< 1 M_\odot \) is not constrained by the data presented here, and it is an open question whether the slope of the IMF actually becomes negative at low masses or stays constant at \( x = 0 \).

7.2. Evolution of the Characteristic Mass

The evolution of the characteristic mass is shown in Figure 11. The results obtained in this study are represented by the formal
best fit for solar metallicity, as discussed in § 4.2. The point labeled “SMGs” reflects results of Blain et al. (1999a) for submillimeter galaxies (SMGs). Blain et al. (1999a) find that a standard Salpeter (1955) IMF has too many low-mass stars, which would produce too much K-band light at $z = 0$ (a similar argument was made by Dwek et al. 1998, based on COBE data). Blain et al. (1999a) find that a top-heavy IMF with a simple cutoff at $1 M_\odot$ resolves this discrepancy. No error bar is given, but we can assume that the uncertainty in the amount of “missing mass” is at least 50%. The value we adopt is $m_c = 0.34$, to match the total stellar mass of the truncated Salpeter (1955) IMF invoked by Blain et al. (1999a). The redshift comes from Chapman et al. (2003).

The figure also includes the IMFs of the Milky Way disk (from Chabrier 2003) and of globular clusters. The age for globular clusters reflects the results of Gratton et al. (2003), who find that globular clusters in the inner halo have ages ranging from $\sim 13.4$ to $\sim 10.8$ Gyr with a systematic uncertainty of $\pm 0.6$ Gyr. The characteristic mass was determined by Paresce & De Marchi (2000), who infer a typical characteristic mass $m_c = 0.33$, with formal uncertainty $\pm 0.03$. The true uncertainty is probably somewhat larger, as dynamical effects may play a role even at low masses (see, e.g., de Marchi et al. 2005). The Galactic bulge is not included; no estimates have been made for the characteristic mass of the bulge IMF, but we note that there is evidence that the IMF in the range $0.15 - 1 M_\odot$ is flatter than the Salpeter IMF (Zoccali et al. 2000).

The dashed and dotted lines in Figure 11 are examples of models in which the temperature of the CMB effectively sets the characteristic mass scale of star formation at high redshift (see Larson 1998, 2005; Jappsen et al. 2005). The CMB temperature exceeds the typical temperatures in prestellar cores in molecular clouds of $\sim 8$ K (e.g., Evans et al. 2001) beyond $z \sim 2$. As discussed in Larson (2005), the exact temperature dependence is very uncertain and can vary from $\propto T^{1.7}$ (Jappsen et al. 2005; dotted line) to $\propto T^{3.35}$ (Larson 1985; dashed line). The normalization of these models is also very uncertain. The value of $\sim 0.3 M_\odot$ is the approximate value of the Jeans mass in present-day cores (Larson 2005), but other aspects may play a role in setting the characteristic mass at low redshift (see McKee & Ostriker 2007). The gray solid line is a nonphysical “toy model” that fits the Milky Way disk, submillimeter galaxies, and cluster galaxies and is intermediate between the two physically motivated models at high redshift. This model has the form

$$m_c^2 = 0.08 \left[ 1 + \left( T_{\text{CMB}} / 6 \right)^6 \right]$$

and is used to explore the effects of an evolving characteristic mass in the following sections.

It is interesting that the IMF of old globular clusters appears to have a lower characteristic mass than the IMF of massive cluster galaxies, even though their stars probably have similar mean ages. Determining the IMF in globular clusters is notoriously difficult, as the mass function as observed today is heavily influenced by dynamical effects in combination with stellar evolution (see, e.g., Chabrier 2003). In particular, globular clusters with a very top-heavy IMF would not be expected to survive in the tidal field of our Galaxy for a Hubble time (e.g., Joshi et al. 2001), and the clusters that are surviving today might not be representative for the original population. Nevertheless, taking the existing constraints at face value, it seems that other parameters than the CMB temperature (e.g., metallicity, star formation rate, or mass of the star-forming complex) may play a role in determining $m_c$ at high redshift.

7.3. Cosmic Star Formation History

As pointed out by others (e.g., Larson 2005; Hopkins & Beacom 2006; Tumlinson 2007; Fardal et al. 2007), an evolving IMF has direct implications for the derived cosmic star formation history. Essentially all indicators of the star formation rate (such as ultraviolet luminosity, H$_\alpha$ line luminosity, and infrared luminosity) measure the effects of luminous O and B stars, which have masses $\geq 10 M_\odot$. Inferred total star formation rates therefore rely on an extrapolation of the IMF from $\sim 10$ down to $\sim 0.1 M_\odot$ (see, e.g., Lilly et al. 1996; Madau et al. 1996, 1998). As most of the total mass is in low-mass stars, changes to the IMF in the mass range $0.1 - 1 M_\odot$ will have virtually no effect on the observed star formation indicators but potentially large effects on the actual total stellar mass that is produced.

The effects of an evolving IMF of the form of equation (20) are shown in Figure 12a. The dashed line shows the integral of equation (18), i.e., the integrated stellar mass for an increasing value of $m_c$ with redshift. The solid line shows the integrated mass relative to that in stars with masses $> 10 M_\odot$, as the energy output of stars in this mass range drives the star formation measurements. The effects are substantial at high redshift, and at $z \sim 6$ the star formation rate could be overestimated by a factor of 3–4 relative to $z = 0$.

In Figure 12b the correction of Figure 12a is applied to measurements of the star formation rate at a range of redshifts. The data are a combination of the extensive compilation of Hopkins (2004) and recent measurements at high redshift by Bouwens et al. (2007). Extinction-corrected values were used from both studies. The data as reported in the literature are shown in gray,
and the gray line shows a simple fit to the points. Black points are the same data, corrected to the modified Chabrier (2003) IMF proposed in this paper. This correction is a combination of two effects: a constant offset to account for the difference between a Salpeter (1955) IMF (which is assumed in the quoted literature) and a Chabrier (2003) IMF, and the redshift-dependent effect shown in Figure 12a. As is clear from the black line, the cosmic star formation history has a fairly well defined peak at $z \sim 1.5$ if the IMF depends on redshift.

7.4. $M/L$ Ratios

The influence of an evolving IMF on the $M/L$ ratios of galaxies is complex, as several competing effects play a role. The number of low-mass stars with respect to high-mass stars is reduced, which lowers the $M/L$ ratio as these stars contribute little to the integrated light. However, for $m_c \approx 0.4 \, M_\odot$ the number of turnoff stars is also reduced (see Fig. 10), and these stars dominate the light at rest-frame optical wavelengths. As a result, the net effect of an increased $m_c$ on the $M/L$ ratio is generally smaller than on the star formation rate. A further complication is that the turnoff mass can be similar to the characteristic mass (e.g., in elliptical galaxies at low redshift where both values are $\sim 1 \, M_\odot$). This means that the effect on the $M/L$ ratio is not a constant, but depends on the age of the population. A final complication is that the mass in stellar remnants, which is a larger fraction of the total stellar mass for more top-heavy IMFs. As discussed by Maraston (2005) and in § 6.6, IMFs that have a power-law slope of zero from 0.1 to 100 $M_\odot$ imply completely remnant-dominated mass functions at old ages, with unrealistically high $M/L$ ratios.

A correct treatment of these issues requires full stellar population synthesis modeling, which is beyond the scope of the present paper. Instead, we used simple stellar evolutionary tracks to estimate what the net outcome is of the various competing mechanisms. The Yale-Yonsei isochrones were used (Yi et al. 2003; Demarque et al. 2004). Monte Carlo simulations of 100,000 stars were generated, with solar metallicity, a range of ages, and a Salpeter (1955) IMF. Next, the mass function was resampled to match equation (18) for a grid of values of $m_c$. The total $I$-band luminosity was determined by linearly adding the light of individual stars. The mass in living stars was determined by integrating equation (18) up to the turnoff mass. The mass in remnants was calculated by integrating equation (18) from the turnoff mass to $100 \, M_\odot$ and multiplying by the fraction of the initial mass that is retained in the form of black holes, neutron stars, and white dwarfs. This (age-dependent) fraction was determined in the same way as described in Bruzual & Charlot (2003).

The results are shown in Figure 13. At young ages the light is dominated by relatively massive stars, and the behavior is similar to that of the implied star formation rate (see Fig. 12a). At intermediate ages and high values of $m_c$, the turnoff mass is similar to the characteristic mass, and the effect on the luminosity is similar to the effect on the mass. The net effect is a roughly constant $M/L_V$ ratio as a function of $m_c$. At the oldest ages the IMF becomes remnant dominated, with $\sim 80\%$ of the mass in remnants for an age of 10 Gyr and $m_c = 1.5 \, M_\odot$. Therefore, the $M/L_V$ ratios are actually higher than for a standard Chabrier (2003) IMF with $m_c = 0.08 \, M_\odot$, approaching or even exceeding those implied by a Salpeter (1955) IMF. Although these results are somewhat uncertain due to the crude nature of our modeling, the qualitative conclusion is that the $M/L_V$ ratios of galaxies are not necessarily strongly reduced by an evolving IMF, in contrast to their star formation rates.
The dashed line in (a) shows the effect on the $M/L_V$ ratio with respect to a standard Chabrier (2003) IMF. Discontinuities are due to the discrete grid of ages used in the computation; the solid line is a smoothed version of the dotted line. (b) Measurements of the mass density from Cole et al. (2001), Dickinson et al. (2003), Drory et al. (2005), Rudnick et al. (2006), and Fontana et al. (2006). Gray symbols are for a Salpeter (1955) IMF, and black symbols are for the evolving IMF proposed in this paper. The gray and black lines show the evolution implied by the evolution of the star formation rate shown in Fig. 12. The black line is somewhat better fit to the black points than the gray line is to the gray points.

7.5. Evolution of the Stellar Mass Density

In Figure 14 we show the effects of an evolving IMF on the cosmic stellar mass density, in a similar way as was done in Figure 12 for the cosmic star formation history. Figure 14a shows the redshift dependence of the $M/L_V$ ratio, with respect to the $M/L_V$ ratio implied by a standard Chabrier (2003) IMF. To generate this relation, it was assumed that all galaxies start forming stars at $z = 10$ and have a constant star formation rate until the epoch of observation. The implied mean age of galaxies at $z = 6$ is $\sim$0.2 Gyr, and the mean age of galaxies at $z = 0$ is $\sim$6.5 Gyr. These values are reasonable, and we note that the results are not very sensitive to the details of the star formation history of the galaxies. This assumption provides an estimate of the mean mass-weighted formation time of the stars, and hence through equation (20) the appropriate value of $m_c$. The relations between age, $m_c$, and $M/L_V$ ratio shown in Figure 13 are then used to estimate the change in $M/L_V$ as a function of redshift.

Going up in redshift, the $M/L_V$ ratio first declines due to the increase in $m_c$ from $\sim$0.1 to $\sim$0.4 $M_\odot$. The $M/L_V$ ratio reaches a minimum at $z \sim 2$ and then increases as the characteristic mass becomes similar to the turnover mass. Beyond $z \sim 4$ the $M/L$ ratio once again decreases, as the galaxies become younger and the turnover mass increases rapidly. Measurements of the evolution of the mass density from Cole et al. (2001), Dickinson et al. (2003), Drory et al. (2005), Rudnick et al. (2006), and Fontana et al. (2006) are shown in Figure 14b. Error bars are taken from the literature sources and typically do not include systematic uncertainties. Gray points show the literature values, determined assuming a Salpeter (1955) IMF, and black points show the corrected values. The correction is a constant offset (to account for the difference between a Salpeter [1955] IMF and a standard Chabrier [2003] IMF) in addition to the relation shown in Figure 14a.

The gray and black lines show the evolution of the stellar mass density as implied by the observed evolution of the star formation rate shown in Figure 12. Mass loss was taken into account using the same scheme as employed by Bruzual & Charlot (2003). Note that mass loss is a larger effect for Chabrier-like IMFs than for a Salpeter (1955) IMF, as a larger fraction of the total mass is in high-mass stars.

The median difference between the data points and the curve is a factor of 2.3 for a nonevolving IMF and a factor of 1.7 for the evolving Chabrier-like IMF. Considering that this type of comparison has many systematic uncertainties quite independent of the IMF (Hopkins & Beacom 2006; Fardal et al. 2007), the fact that the two independent measures of the buildup of stellar mass in the universe agree to within a factor of $\sim$2 can be considered a success. In any case, the discrepancy is smaller for the evolving IMF than for a nonevolving IMF because the star formation rate is reduced by a larger fraction than the mass density. Our evolving IMF has a qualitatively similar effect as the “paunchy” IMF with an increased contribution from stars around $1.5-4 M_\odot$ proposed by Fardal et al. (2007), which was specifically designed to give better agreement between the star formation history of the universe (as implied by the extragalactic background radiation) and the observed evolution of the mass density.

8. SUMMARY AND CONCLUSIONS

This paper compares the color evolution of massive cluster galaxies to their luminosity evolution, with the aim of constraining the form of the IMF at the time when the stars in these galaxies were formed. It is found that the evolution of the rest-frame $U - V$ color is not consistent with the previously determined evolution of the rest-frame $M/L_B$ ratio, unless the IMF slope is significantly flatter than the Salpeter value around $1 M_\odot$. For standard IMFs with a slope of 1.3 at $m \geq 1 M_\odot$, the luminosity evolution is too fast for the measured color evolution, and the implied stellar ages derived from $M/L$ evolution and color evolution are not consistent with each other. The only models that are able to fit the color evolution and the luminosity evolution simultaneously have IMF slopes of $\sim$0 to $\sim 1 M_\odot$ and mean luminosity-weighted stellar formation redshifts of $\sim$4 (for solar metallicities).

This result is somewhat uncertain, as the currently available sample of cluster galaxies with accurate rest-frame $U - V$ colors and dynamical masses is somewhat limited and there are many systematic effects that may play a role. In particular, it is an open question whether stellar population synthesis models are able to predict color evolution with the required accuracy. The commonly used Bruzual & Charlot (2003) and Maraston (2005) models give broadly similar answers, but that may be because they share many of the same assumptions.

As discussed in 6, the higher stellar ages implied by a flat IMF are consistent with many other studies, which lends some credibility to the results presented here. Of particular importance is the agreement with the data on Balmer line strengths of Kelson et al. (2001), as they do not suffer from the same systematic
uncertainties as the color data. Formation redshifts substantially larger than two also fit more comfortably with the direct detection of old galaxies at high redshifts. A firm independent measurement of the star formation epoch of massive cluster galaxies, combined with a better understanding of selection effects at high redshift, would leave the IMF as the only free parameter and greatly simplify the problem.

The implications discussed in § 7 are obviously somewhat speculative. Although the interpretation in terms of an evolving characteristic mass is physically plausible according to some models (e.g., Larson 2005), many other forms of the IMF are consistent with the data. The observations described in this paper are only sensitive to a narrow mass range near 1 $M_\odot$, and the IMF proposed in equation (18) represents a very substantial extrapolation. This is illustrated in Figure 15: both the top-heavy IMF (red line) and the “bottom-light” IMF (green line) are consistent with the data presented in this paper. The main reason for preferring the bottom-light IMF over a top-heavy form is that the absolute $M/L$ ratios of galaxies are within a reasonable range. As shown in Figure 13, the $M/L_V$ ratios are similar to those implied by a standard Chabrier (2003) IMF, which means that they are consistent with dynamical measurements at $z = 0$ (Cappellari et al. 2006).

An “unintended” effect of an evolving IMF of the form proposed here is that it reduces the discrepancy between the observed stellar mass density and the density implied by the cosmic star formation history. This result is in excellent (albeit qualitative) agreement with several other recent studies (e.g., Hopkins & Beacom 2006; Fardal et al. 2007; Pérez-González et al. 2008; Wilkins et al. 2007; Davé 2007; see also, e.g., Fields 1999). The differences between a nonevolving IMF and an evolving IMF are fairly large at $z \sim 4$ (as the effect on the star formation rate is strong and the effect on $M/L$ ratios is weak at that redshift), and it will be interesting to see where future measurements of the mass density at high redshift will fall in Figure 14. We note that the effects on the $M/L_V$ ratios are somewhat uncertain, as they rely on rather rudimentary stellar population synthesis modeling. The effects on star formation rates are more robust and suggest that the cosmic star formation rate peaked at $z \sim 1.5$.

This paper adds to previous theoretically and observationally motivated suggestions that the IMF may evolve with redshift (e.g., Worthey et al. 1992; Larson 1998, 2005; Fields 1999; Blain et al. 1999a; Baugh et al. 2005; Stanway et al. 2005; Hopkins & Beacom 2006; Tumlinson 2007; Lacey et al. 2007; Fardal et al. 2007; P�rez-González et al. 2008). Although these studies vary greatly in their parameterization of IMF evolution and the range of stellar masses that are considered, they all suggest that the ratio of high-mass stars to low-mass stars was higher in the past. It should be pointed out that most of these papers invoke a change in the IMF as a “last resort” possibility, to explain data that are otherwise difficult to interpret. In the present study a different approach was followed, in that we set out with the specific purpose of constraining the slope of the IMF. An advantage of the applied method is that it is fairly direct, as the rate of luminosity evolution is determined by the number of stars as a function of mass. Disadvantages are that it is only sensitive to a very limited mass range (see Fig. 15); that it relies on stellar population synthesis models, which are not well calibrated in the relevant parameter range; that the progenitors of early-type galaxies may not be representative for the general population of high-redshift galaxies; and that the currently available data are somewhat limited.

Accepting the possibility of an evolving characteristic mass, it is interesting to speculate what could be the cause or causes. The proximate cause may well be a higher temperature in molecular clouds at high redshift, which would raise the Jeans mass and could inhibit the formation of low-mass stars (Larson 1998, 2005). The ultimate cause could be the higher temperature of the CMB, the fact that star formation tends to proceed in more extreme environments at high redshift, or a combination. Available information on IR-bright galaxies suggests that dust temperatures in starburst galaxies are of order 30–40 K (Dunne et al. 2000; Chapman et al. 2005) and hence exceed the CMB temperature for all relevant redshifts. However, it is as yet unclear what fraction of the total star formation has taken place in these extreme environments (see, e.g., Reddy et al. 2008).

The analysis presented here can be improved in various ways. The number of clusters with accurate rest-frame $U - V$ is currently smaller than the number of clusters with accurate $M/L_B$ measurements, and this can be remedied by obtaining accurate (space-based) photometry in well-chosen filters of the remaining clusters in the vv07 sample. It is also important to measure the evolution in a redder rest-frame color, such as $V - I$. Redder colors suffer less from possible contributions of hot stars, and their evolution is probably somewhat better calibrated in stellar population synthesis models. This requires very accurate photometry in the near-infrared, which should be possible with WFC3 on HST. On the modeling side, it would be helpful to implement more variations of the IMF in stellar population synthesis codes than the standard Salpeter, Kroupa, and Chabrier forms. Ultimately it may be fruitful (or prove necessary) to have the characteristic mass, or some other parameter describing the form of the IMF, as one of the “standard” parameters in these models, on a par with the age and metallicity.

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Figure 15: Illustration of the key results of this paper. In § 4 the slope of the IMF of massive cluster galaxies $\alpha$ is estimated to be approximately $-0.3$ in a narrow region around 1 $M_\odot$ (thick black line). The colored lines show two possible interpretations: a global change of the slope of the IMF (with respect to a standard Chabrier or Salpeter IMF) at all masses (red dashed line), or a change in the characteristic mass (solid green line). The “top-heavy” interpretation is inconsistent with the dynamical $M/L$ ratios of nearby elliptical galaxies (see § 6.6), whereas the “bottom-light” interpretation is consistent with all data that we are aware of. The blue and yellow areas illustrate that stars with masses $> 10^{-2} M_\odot$ drive star formation measurements, whereas stars with masses $1 - 5 M_\odot$ drive $M/L$ measurements (see §§ 7.3–7.5).
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APPENDIX

APERTURE CORRECTIONS

For five of the seven distant clusters discussed in this paper colors were measured directly from HST images in apertures of fixed physical size. For each cluster this size was chosen to match an 11” diameter aperture at the distance of the Coma Cluster, allowing a direct comparison to data from BLE92 and to McIntosh et al. (2005). For the clusters MS 1054–03 (z = 0.831) and RX J0152–13 (z = 0.837) color measurements were taken from Blakeslee et al. (2006), who used apertures scaled to contain 50% of each galaxy’s light. In this section we give the corrections that were applied to bring the Blakeslee et al. (2006) data onto our system and test these corrections using data from McIntosh et al. (2005).

The reason why aperture corrections need to be applied even when data are corrected for seeing effects is that early-type galaxies have color gradients. These gradients do not vary much from galaxy to galaxy, when expressed as a color gradient in mag dex. The measured color within an aperture $r_{ap}$ is not simply the integral of the color gradient, as the radial dependence of the luminosity has to be taken into account as well.

We determine the expected dependence of color on aperture size numerically, assuming an $r^{1/4}$ law for the radial luminosity dependence. The solid line in Figure 16 shows the expected relation for a color gradient of −0.15, appropriate for $U - V$ (e.g., Peletier et al. 1990). The long-dashed line is a polynomial fit to the numerically derived relation, of the form

$$ (F_1 - F_2)_{ap} = (F_1 - F_2)_{eff} + 6.67 \frac{\Delta(F_1 - F_2)}{\Delta \log r} \left\{ -0.118 \log \left( \frac{2r_c}{D_{ap}} \right) - 0.042 \left[ \log \left( \frac{2r_c}{D_{ap}} \right) \right]^2 \right\}, \quad (A1) $$

with $(F_1 - F_2)_{ap}$ the color measured through an aperture of diameter $D_{ap}$, $(F_1 - F_2)_{eff}$ the color measured through an aperture containing 50% of the galaxy’s luminosity, $r_c$ the half-light radius, and $\Delta(F_1 - F_2)/\Delta \log r = −0.15$ the color gradient in mag dex$^{-1}$. The long-dashed line is a very good match to the (exact) solid line. For reference, short-dashed lines show the behavior of equation (A1) for color gradients of −0.10 and −0.20. The data points show binned data from McIntosh et al. (2005), who measured $U - V$ colors of three nearby clusters in apertures of fixed size and in apertures of diameter $2r_c$. The data follow the predicted relation very closely, with an rms of $\sim 0.01$ and no apparent systematic deviations.

The data points from Blakeslee et al. (2006) were corrected to our fiducial aperture size by calculating $r_c/D_{ap}$ for each galaxy (using the information supplied in Blakeslee et al. 2006) and applying equation (A1). As on average $D_{ap} \sim 2r_c$ for our aperture size, the correction is only $\sim 0.015$ in the mean for both clusters, with an object-to-object scatter of $\sim 0.03$. Galaxies requiring a correction $>0.05$ or $<−0.05$ were removed from the sample; this step slightly reduces the scatter in the color-mass relations of the clusters but otherwise has a negligible effect on our results.

Fig. 16.—Effect of color gradients on the measured color within an aperture. The solid line shows the expected difference between the color measured within aperture $D_{ap}$ and the color measured within an aperture containing 50% of the galaxy’s light, assuming an $r^{1/4}$ law and a color gradient of $−0.15$ mag dex$^{-1}$. Data points are from McIntosh et al. (2005); they follow the predicted curve very well. Dashed lines are simple polynomial fits to the predicted evolution for color gradients of $−0.10, −0.15,$ and $−0.20$. 
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