Is X(3872) Really a Molecular State?

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After taking into account both the pion and sigma meson exchange potential, we have performed a dynamical calculation of the $D^0\bar{D}^{*0}$ system. The $\sigma$ meson exchange potential is repulsive from heavy quark symmetry and numerically important for a loosely bound system. Our analysis disfavors the interpretation of $X(3872)$ as a loosely bound molecular state if we use the experimental $D^*\bar{D}\pi$ coupling constant $g = 0.59$ and a reasonable cutoff around 1 GeV, which is the typical hadronic scale. Bound state solutions with negative eigenvalues for the $D\bar{D}^*$ system exist only with either a very large coupling constant (two times of the experimental value) or a large cutoff ($\Lambda \sim 6$ GeV). In contrast, there probably exists a loosely bound S-wave $BB^*$ molecular state. Once produced, such a molecular state would be rather stable since its dominant decay mode is the radiative decay through $B^* \rightarrow B\gamma$. Experimental search of these states will be very interesting.

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I. INTRODUCTION

Since the observation of the charmonium-like state $X(3872)$ in the $J/\psi\pi^+\pi^-$ channel by Belle collaboration in 2003 \cite{1}, $X(3872)$ has been confirmed by CDF \cite{2}, D0 \cite{3} and Babar collaborations \cite{4}. In the past three years, there have accumulated abundant experimental information of $X(3872)$, which is collected in Table I.

| $X(3872)$ | 
| --- | --- |
| Mass (MeV) | 3872.0 ± 0.6 ± 0.5 \cite{1} | 3871.3 ± 0.7 ± 0.4 \cite{2} | 3871.8 ± 3.1 ± 3.0 \cite{3} | 3873.4 ± 1.4 \cite{4} | 3875.4 ± 0.7 ± 1.3 \cite{5} | 3875.6 ± 0.7 ± 1.3 \cite{6} |
| Width | < 2.3 MeV \cite{1} |

Decay channels

$X(3872) \rightarrow J/\psi\pi^+\pi^-$ \cite{1, 2, 3, 4};
$X(3872) \rightarrow \gamma J/\psi, \omega J/\psi$ \cite{9, 10};
$X(3872) \rightarrow \rho J/\psi$ \cite{11};
$X(3872) \rightarrow D^*\bar{D}^*\pi^0$ \cite{5};
$X(3872) \rightarrow D^0\bar{D}^{*0} + h.c.$ \cite{6}.

Branching fractions

$BR[X(3872) \rightarrow J/\psi\pi^+\pi^-] = 0.14 \pm 0.05$ \cite{9};
$BR[X(3872) \rightarrow \gamma J/\psi] = 0.25$ \cite{10};
$BR[X(3872) \rightarrow D^*\bar{D}^*\pi^0] = 9.4^{+3.6}_{-4.3}$ \cite{5}.

TABLE I: A review of the experimental status of $X(3872)$.

Quark model calculation indicates that a $2I^G_P1^G$ $cc$ state $X^+_c(3872)$ lies $50 \sim 200$ MeV above $X(3872)$. Moreover a charmonium state with isospin $I = 0$ does not decay into $J/\psi\rho$ easily. Thus there is some difficulty of the charmonium assignment of $X(3872)$. The possible theoretical explanations of $X(3872)$ include a molecule state \cite{12, 13, 14, 15, 16}, a $1^+$ cusp \cite{17}, the S-wave threshold effect due to the $D^0\bar{D}^{*0}$ threshold \cite{18}, a hybrid charmonium \cite{19}, a diquark anti-diquark bound state \cite{20}, a tetraquark state \cite{21} and a dynamically generated resonance \cite{22}.

Among these theoretical schemes, the molecule picture is the most popular one due to the following reasons. The molecular picture naturally explains both the proximity of $X(3872)$ to the $D^0\bar{D}^{*0}$ threshold and the isospin violating $J/\psi\rho$ decay mode. It predicted the decay width of the $J/\psi\pi^+\pi^-\pi^0$ mode to be comparable with that of $J/\psi\rho$, which was confirmed by Belle collaboration \cite{9}. Within the same picture, Braaten and Kusunoki predicted that the branching ratio of $B^0 \rightarrow X(3872)K^0$ is suppressed by more than one order of magnitude compared to that of $B^+ \rightarrow X(3872)K^+$ \cite{22}.

Later both Belle and Babar collaborations observed the radiative decay mode. Belle’s measurement found \cite{9}

$$BR[X(3872) \rightarrow \gamma J/\psi] = 0.14 \pm 0.05$$ \hspace{1cm} (1)

while Babar collaboration got \cite{10}

$$BR[X(3872) \rightarrow \gamma J/\psi] \approx 0.25$$ , \hspace{1cm} (2)

which are against the prediction from the molecular picture $7 \times 10^{-3}$.

Recently Belle collaboration measured the ratio \cite{9}

$$BR[X(3872) \rightarrow D^0\bar{D}^{*0}] = 9.4^{+3.6}_{-4.3}$$ \hspace{1cm} (3)

which is much larger than the theoretical value 0.054 from the molecular assumption. From Ref. \cite{9}, one can also extract

$$BR[B^0 \rightarrow X(3872)K^0] \approx 1.62$$ \hspace{1cm} (4)
which is also much larger than the molecule prediction.

Up to now, several groups carried out the dynamical study of the molecular assignment of $X(3872)$. Swanson proposed that $X(3872)$ was mainly a $D^0\bar{D}^{*0}$ molecule bound by both the pion exchange and quark exchange. To obtain the potential between $D^0\bar{D}^{*0}$ through exchanging single pion, he followed the method proposed by Törnqvist. The formalism is based on a microscopic quark-pion interaction. Swanson indicated that one pion exchange alone can not bind $D$ and $D^*$. He also included the short-range quark-gluon force. In Ref. [15], Swanson indicated that $X(3872)$, effective Lagrangian and coupling constants were performed a detailed study of this state in the molecular picture [46]. A short review of the current theoretical status of $Z^+(4430)$ [47, 48, 49, 50, 51, 52, 53, 54, 55] was also given in Ref. [47].

III. FLAVOR WAVE FUNCTION, EFFECTIVE LAGRANGIAN AND COUPLING CONSTANTS

In the following, we will study whether $X(3872)$ is a bound state of the $DD^*$ meson pair. Before deriving the meson exchange potential, we first briefly discuss the convention of the flavor wave function of the molecular state $X(3872)$. In the previous literature [12, 13, 14, 15, 16], it was defined as

$$|X(3872)\rangle = \frac{1}{\sqrt{2}} [D^0\bar{D}^{*0} + c|D^{*0}\bar{D}^0\rangle]$$  \hspace{1cm} (5)

with $c = +1$. However, this definition does not reflect the positive C-parity of $X(3872)$ naturally [68]. According to the same approach in our previous paper [15], we reanalyze the flavor wave function of $X(3872)$.

The interpolating current of $X(3872)$ corresponding to $J_{X}(3872)$ in the quantum field theory reads

$$J_{X}(3872) = \frac{1}{\sqrt{2}} (J_1 + cJ_2)$$  \hspace{1cm} (6)
with

\[ J_1 = (\bar{u}^a \gamma_5 c^a)(e^b \gamma_\mu u^b), \quad J_2 = (\bar{e}^a \gamma_\mu u^a)(\bar{u}^b \gamma_\mu c^b), \]

where \(a, b\) denotes the color indices. Under the charge conjugate transformation, one gets

\[ \hat{C} J_1 \hat{C}^{-1} = -J_2 \quad \text{and} \quad \hat{C} J_2 \hat{C}^{-1} = -J_1. \]

We want to emphasize that there exists no arbitrary phase because the charm and anti-charm quark and the up and anti-up quark appear simultaneously. Therefore we obtain

\[ \hat{C} J_{X(3872)} \hat{C}^{-1} = \frac{1}{\sqrt{2}} (-J_2 - cJ_1). \]

Because the charge parity of \(X(3872)\) is +1, we have \(c = -1\). In other words, the natural definition of the flavor wave function of \(X(3872)\) should be

\[ \langle \bar{X} | \langle \bar{u} | \gamma_\mu | \gamma_5 \bar{u} | \gamma_\mu \bar{c} | X \rangle \rangle = \frac{1}{\sqrt{2}} (\bar{u} \gamma_\mu \gamma_5 \bar{u} | \gamma_\mu \bar{c} | X \rangle \rangle. \]

The above value was extracted by fitting the precise experimental width of \(D^*\) \[65\]. In order to estimate the values of the coupling constant \(g_\sigma\), we compare the Lagrangian with that in Ref. \[58\] and get

\[ g_\sigma = \frac{g_\pi}{2\sqrt{6}} \]

with \(g_\pi = 3.73\). Unlike the case of \(Z^+(4430)\) \[43\], it is unnecessary to care about the phases of the coupling constants in the present case. We will turn to this point later.

\[ IV. \quad \text{THE DERIVATION OF THE ONE PION AND SIGMA EXCHANGE POTENTIAL} \]

To derive the effective potential, we follow the same procedure in Ref. \[43\]. Firstly we derive the elastic scattering amplitudes of both the direct process and crossed channel. Secondly, we get the potential in the momentum space for a special component (e.g. \(J_z = 0\)) with the Breit approximation. Then we average the potential in the momentum space. Finally we make Fourier transformation to derive the potential in the coordinate space.

In the present case, the parity and angular momentum conservation ensures that the \(\pi\) exchange occurs only in the crossed channel while the \(\sigma\) exchange only in the direct channel (see Fig. 1). The zeroth component of exchange meson momentum is \(q_0 \approx M_1 - M_f\). For the direct scattering diagram, \(M_{i,f}\) denotes the mass of \(D^0\). Thus we can approximately take \(q_0 = 0\) and \(q^2 = -q^2\).

However, \(q_0\) could not be ignored because \(M_i\) and \(M_f\) denote respectively the masses of \(D^0\) and \(D^{*0}\) for the crossed diagram. \(q_0 = M_{D^{*0}} - M_{D^0}\) is larger than pion mass \(m_\pi\) which indicates that the exchanged pion can be on-shell. In this case, one can deal with the potential in the coordinate space by the principal integration as in Eq. \[14\] below.

\[ \begin{aligned}
  &D^0 &D^* \\
  &D^* &D^0 \\
  &\sigma &\sigma
\end{aligned} \]

FIG. 1: The scattering of \(D^0 - D^{*0}\) by exchanging the \(\pi\) and \(\sigma\) mesons.

We use the following definitions in the potentials after Fourier transformation

\[ Y_\sigma(r) = \int \frac{1}{q^2 + m_\sigma^2} e^{iqr} \frac{dq}{(2\pi)^3}, \quad \text{(13)} \]

\[ Y_\pi(r) = \int \frac{q^2}{q^2 - m_\pi^2} e^{iqr} \frac{dq}{(2\pi)^3}. \quad \text{(14)} \]
Writing them explicitly, we have

$$
Y_\sigma(r) = \frac{1}{4\pi r} e^{-m_\sigma r},
$$

$$
Y_\pi(r) = -\delta(r) - \frac{\mu^2}{4\pi r} \cos(\mu r),
$$

(15)

where $\mu = \sqrt{m_0^2 - m_\pi^2}$. Except the relative sign, $Y_\pi(r)$ is similar to the expression derived in Ref. [31] by using the polarization vectors $e^{\pm 1} = \frac{1}{\sqrt{2}} (0, \pm 1, i, 0)$ and $e^0 = (0, 0, 0, -1)$ [69].

With the convention of the $X(3872)$ flavor wave function in Eq. (7), the potential in the study of the molecular picture finally reads as

$$
V(r) = g_\sigma^2 Y_\sigma(r) + \frac{g_\pi^2}{6f_\pi^2} Y_\pi(r).
$$

(16)

Here the sign between one sigma exchange potential (OSEP) and OPEP is determined by the relative sign of $[D^0 \bar{D}^{*0}]$ and $[D^{*0} \bar{D}^0]$ in the wave function in Eq. (7).

It’s important to note that the signs in the potential are completely fixed. The heavy quark spin-flavor symmetry ensures that the $D$ and $D^*$ mesons possess the same coupling constants. The resulting potential in Eq. (16) does not change with the phases of coupling constants.

Especially, we find that $\sigma$ exchange potential is repulsive, which differs from that in the nuclear forces. Because of this unique feature, one just needs to study whether the one-pion exchange can bind $D$ and $D^*$ mesons to form $X(3872)$. Only when the answer is positive, should we consider the effect from the $\sigma$ exchange.

We note that the potential in Eq. (16) is derived with the implicit assumption that all the mesons are point-like particles. Such an assumption is not fully reasonable due to the structure effect in every interaction vertex depicted in Fig. 1. Thus in the following we will introduce the cutoff to regulate the potential and further study whether it is possible to find a loosely bound molecular state using the realistic potential.

We will modify the potential through two approaches: (1) considering the form factor (FF) contribution; (2) smearing the potential. Although these two approaches look different, they are essentially the same, i.e. imposing a short-distance cutoff to improve the singularity of the effective potential.

### A. Introducing form factors in the potential

Before making a Fourier transformation, we introduce a form factor in the interaction vertex to compensate the off-shell effects of the exchanged mesons. The adopted FF is of the monopole type [24, 64]

$$
F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2},
$$

(17)

where $\Lambda \sim 1$ GeV denotes a phenomenological cutoff. $m$ and $q$ are the mass and the four-momentum of the exchanged meson respectively. As $q^2 \to 0$, FF becomes a constant. With $\Lambda \gg m$, it approaches unity. In other words, as the distance is infinitely large, the vertex looks like a perfect point. So the form factor is simply unity. On the other hand, as $q^2 \to \infty$, the form factor approaches to zero. In this situation, as the distance becomes very small, the inner structure (quark, gluon degrees of freedom) would manifest itself and the whole picture of hadron interaction is no longer valid.

The explicit expressions of the modified potentials are

$$
Y_\sigma(r) = \frac{1}{4\pi r} (e^{-m_\sigma r} - e^{-\Lambda r}) - \frac{\eta^2}{8\pi \Lambda} e^{-\Lambda r},
$$

(18)

$$
Y_\pi(r) = -\frac{\mu^2}{4\pi r} \cos(\mu r) - \frac{\eta^2 \alpha}{8\pi} e^{-\alpha r},
$$

(19)

where $\eta = \sqrt{\Lambda^2 - m_\pi^2}$, $\eta' = \sqrt{\Lambda^2 - m_\sigma^2}$ and $\alpha = \sqrt{\Lambda^2 - q_0^2}$. Note we use the same $\Lambda$ for $\pi$ and $\sigma$ exchange. As an example, we have plotted the above regulated potential in Fig. 2.

### B. Regulating the potential with the smearing technique

The potential can be written as

$$
V(r) = \int V(r') \delta(r - r') dr'.
$$

(20)

To smear the potential, we employ the replacement

$$
\delta(r - r') \to \left( \frac{\beta}{\pi} \right)^{3/2} e^{-\beta (r-r')^2},
$$

(21)
To solve the Schrödinger equation, one needs the following parameters: \( m_\sigma = 134.98 \text{ MeV} \), \( m_\rho = 600 \text{ MeV} \), \( f_\rho = 132 \text{ MeV} \), \( m_{D^*} = 2006.7 \text{ MeV} \), \( m_{D^0} = 1864.6 \text{ MeV} \). In this section, we first consider whether the one pion exchange interaction alone can bind \( D\bar{D}^* \).

Now we explore at what condition \( D \) and \( \bar{D}^* \) can form a bound state through one pion exchange interaction with two approaches. Our procedure to collect the numerical values is: (1) we fix the coupling constant \( g = 0.59 \) and vary the cutoff (\( \Lambda \) or \( \beta \)) from a small value until we find a solution with a binding energy less than 5 MeV; and (2) we increase \( g \) to several larger numbers and tune the cutoff until a solution with a binding energy less than 5 MeV is found.

### A. Results for the case of FF

If the coupling constant \( g \) is fixed to be the experimental value \( g = 0.59 \), the possible bound state solution with a negative eigenvalue can only be found when \( \Lambda > 5.6 \text{ GeV} \). The larger the cutoff \( \Lambda \) is, the closer the regulated potential is to the delta function, hence the larger the binding energy. The binding energy is very sensitive to \( \Lambda \). This result is consistent with the behavior that \( F(q^2) \to 1 \) when \( \Lambda \to \infty \). It’s known that the three-dimensional \( -\delta(r) \) function alone does not generate a bound state. The requirement \( \Lambda > 5.6 \text{ GeV} \) is much much larger than the commonly used reasonable value \( \sim 1.0 \text{ GeV} \). In other words, the one pion exchange potential alone does NOT bind the \( D^0\bar{D}^0 \) pair into a molecular state with the physical values of \( g \) and \( \Lambda \)!

| \( g \) (0.59) | \( \Lambda \) (GeV) | \( E_0 \) (MeV) | \( r_{\text{rms}} \) (fm) | \( r_{\text{max}} \) (fm) |
|---|---|---|---|---|
| 5.7 | 5.8 | -0.3 | 5.8 | 0.2 |
| 4.1 | 4.2 | -0.8 | 3.7 | 0.2 |
| 3.1 | 3.2 | -0.1 | 8.7 | 0.4 |
| 3.2 | 3.3 | -1.6 | 2.6 | 0.3 |
| 2.5 | 2.6 | -0.6 | 4.2 | 0.4 |
| 2.0 | 2.1 | -0.2 | 7.2 | 0.5 |
| 2.5 | 2.6 | -2.9 | 2.0 | 0.3 |
| 2.0 | 2.1 | -1.8 | 2.5 | 0.4 |

\( \Lambda \) is our first important observation.

TABLE II: Solutions for various \( g \) and \( \Lambda \) in the case of FF with OPEP. Lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected.

We consider only the solutions with the eigenvalues between -0.1 MeV and -5.0 MeV corresponding \( \Lambda = 5.7 \) and \( \Lambda = 5.8 \). To understand the solutions more clearly, we present the numerical results in Table II. \( E_0 \) is the lowest eigenvalue of the system, \( r_{\text{rms}} \) is the root-mean-square radius, and \( r_{\text{max}} \) is the radius corresponding to the maximum of the wave function \( \chi(r) \). In Fig. 4 and 5, we present the radial wave functions \( R(r) \) and \( \chi(r) = rR(r) \).
respectively. According to the figures, as $\Lambda$ increases, the probability for a bound state appearing near the origin becomes larger. The large value of $r_{\text{rms}}$ indicates this possible bound state is very extended, which can be illustrated with the figures.

Secondly, we enlarge $g$ arbitrarily until $g = 1.0$ and perform a similar evaluation. The results are also presented in Table III. When $g$ becomes larger, the critical point for $\Lambda$ to generate a $DD^*$ bound state becomes smaller. With a reasonable cutoff $\Lambda \sim 1.0$ GeV, a bound state exists only when the coupling is very strong ($g > 1.0$), which is nearly two times of the experimental value. The wave functions corresponding to the solutions in Table III have similar shapes with those in Figs. 4 and 5.

Now we come back to discuss the partner state of $X(3872)$. We denote it as $\tilde{X}$. The $C$ parity of $\tilde{X}$ is negative.

$$|\tilde{X}\rangle = \frac{1}{\sqrt{2}} \left[ |D^0\bar{D}^{*0}\rangle + |D^{*0}\bar{D}^0\rangle \right].$$

With this convention, the signs in the OPEP are reversed while the sigma meson exchange is still repulsive. Therefore the attractive force is much weaker. We find that the potential is not attractive enough to bind $D$ and $\bar{D}^*$ even with $g = 1.0$. If we arbitrarily use $g = 5.0$ and $\Lambda = 1.0$ GeV, one finds a negative eigenvalue about -0.1 MeV. The value is not sensitive to $\Lambda$. In this case, $r_{\text{rms}} \approx 19$ fm and $r_{\text{max}}$ is about 14 fm. From these values, one concludes that this convention does not lead to a $DD^*$ bound state with the realistic coupling constant. It is not difficult to understand the results with the potential in Eq. [E] due to the part which could provide some attraction is $g^2 \mu^2 \cos(\mu r) \frac{1}{r}$. Since $\mu = 0.044$ GeV is small, a possible bound state exists only if $g$ is a very large number. The consideration of FF improves mainly the behavior of the most singular part. Thus the binding energy is insensitive to the cutoff.

From the above analysis, we conclude that $DD^*$ interaction through one pion exchange is not attractive enough to form a bound state with $g = 0.59$ and $\Lambda \sim 1.0$ GeV.

**B. Results for the case of smearing**

In the case of the smeared potential, one fails to find a bound state solution with negative eigenvalue for $\beta \leq 5.3$ GeV$^2$ if we fix $g = 0.59$. The binding energy is very sensitive to and increases with $\beta$. With a reasonable cutoff $\beta \sim 1$ GeV$^2$, there exists no loosely bound molecular state using the realistic coupling constant $g = 0.59$.

When we vary $g$ from 0.59 to 1.0 and select the solutions with $-5.0$ MeV $< E_0 < -0.1$ MeV, we obtain the
results in Table III. One gets similar conclusion as in the form factor case. The critical point for $\beta$ to generate a bound state is lowered as $g$ becomes larger. For example, a bound state can be obtained with $g = 0.9$ and $\beta \sim 1.0$ GeV$^2$. The shapes of the wave functions corresponding to these solutions are also similar to those in Figs. 4 and 5.

As in the form factor case, if the flavor wave function is used, no bound states can be found with $g = 1.0$. If $g = 5.0$, a bound state exists and the eigenvalue is insensitive to the cutoff. The numerical results are very close to those in the form factor case, which also indicates the insensitivity of the results to the cutoff. Therefore, it is also difficult to find a $D\bar{D}^*$ bound state by one pion exchange interaction with the realistic coupling constant $g = 0.59$ in the smearing case.

From the above analysis within two approaches, we find that the molecular interpretation of $X(3872)$ through one pion exchange interaction may be problematic. The regulated OPEP may generate bound states either with an unphysically large coupling constant $g \geq 1.0$ or an unreasonably large cutoff. The bound state solution with the realistic coupling constant does not exist if the value of the cutoff is around 1 GeV. The two approaches agree with each other and lead to the same conclusion. As a by-product, we point out that our sign convention for the flavor wave function of $X(3872)$ is much more helpful to form a bound state than the old convention used in the literature.

VI. NUMERICAL RESULTS WITH BOTH THE PION AND SIGMA MESON EXCHANGE INTERACTION

Now we move on to include the one $\sigma$ exchange interaction. The $\sigma$ contribution reinforces the above conclusion in the previous section due to the repulsive nature of OSEP. We will study carefully the variation of the numerical results and see how much it affects the conclusion when OSEP is considered. The procedure is similar to the OPEP case.

A. Results for the case of FF

We first take a look at the potentials plotted in Fig. 2. The curves are obtained with $g = 0.59$, $g_\sigma = 0.76$, and $\Lambda = 1.0$ GeV. From this figure, one notes that OSEP is small compared with OPEP. Thus one expects one sigma exchange interaction has small contributions to the binding energy. However, since a very loosely molecular state is expected, a small variation of the potential may lead to relatively big change of the eigenvalue.

By adding OSEP in the Schrödinger equation, one gets numerical solutions listed in Table IV. We only use the coupling constant $g_\sigma = 0.76$ to illustrate the results. Again, we chose the solutions with $-$5.0 MeV $< E_0 < -0.1$ MeV.

| $g$ (MeV) | $E_0$ (MeV) | $r_{\text{rms}}$ (fm) | $r_{\text{max}}$ (fm) |
|-----------|-------------|----------------|----------------|
| 0.59      | 6.0         | -1.3           | 2.8            |
| 6.1       | -4.9        | 1.5            | 0.1            |
| 0.7       | 4.3         | -1.1           | 3.1            |
| 4.4       | -4.5        | 1.5            | 0.2            |
| 0.8       | 3.3         | -0.7           | 3.8            |
| 3.4       | -3.7        | 1.7            | 0.2            |
| 0.9       | 2.6         | -0.4           | 5.0            |
| 2.7       | -2.8        | 2.0            | 0.3            |
| 1.0       | 2.1         | -0.3           | 5.9            |
| 2.2       | -2.4        | 2.2            | 0.5            |

TABLE IV: Solutions for various $g$ and $\Lambda$ in the case of FF with total potential. Lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected. Here $g_\sigma = 0.76$ is used.

By comparing the data in Tables III and IV one finds that many bound state solutions with negative eigenvalues for certain pairs of $g$ and $\Lambda$ disappear after we include the repulsive sigma meson exchange force. Only three solutions survive with $-5.0$ MeV $< E_0 < -0.1$ MeV. But their binding energy decreases by at least 83%, which clearly indicates that the sigma exchange force are numerically very important for a loosely bound molecular state.

B. Results for the case of smearing

The smeared potentials is plotted in Fig. 3 where we use $g = 0.59$, $\beta = 1$ GeV$^2$ and $g_\sigma = 0.76$. By using $g_\sigma = 0.76$ and selecting solutions for $E_0$ between -5.0 MeV and -0.1 MeV, we get the results given in Table V. Comparing data in this table with those in Table III, only two solutions (when $g = 0.9, \beta = 1.2$ GeV$^2$ and $g = 1.0, \beta = 0.8$ GeV$^2$) still satisfy our requirement. The binding energy decreases by at least 74%.
VII. NUMERICAL RESULTS FOR $B\bar{B}^*$ SYSTEM

Finally we apply the formalism to $B\bar{B}^*$ system.

$$|X_B| = \frac{1}{\sqrt{2}} \left[ |B^+ B^{-*}| - |B^{+*} B^-| \right].$$ \hspace{1cm} (24)

Because of the heavier masses of the B mesons, the kinematic term has relative small contribution. The possibility of forming a bound state is larger than that in the $D\bar{D}^*$ system. OSEP remains the same. But the expression of the OPEP is different now because $q^2_B = m_{B^*} - m_B < m_\pi$. Therefore the potential can be strictly derived and does not have an imaginary part. Now we have

$$Y_\pi(r) = -\delta(r) + \frac{\mu_B^2}{4\pi r} e^{-\mu_B r},$$ \hspace{1cm} (25)

where $\mu_B = \sqrt{m_\pi^2 - (q^2_B)}$.

If a form factor is introduced before the Fourier transformation, this function becomes

$$Y_{\pi}(r) = \frac{\mu_B^2}{4\pi r} \left[ e^{-\mu_B r} - e^{-\alpha_B r} \right] - \frac{\eta^2 \alpha_B}{8\pi} e^{-\alpha_B r},$$ \hspace{1cm} (26)

where $\alpha_B = \sqrt{\Lambda^2 - (q^2_B)}$ and $\eta = \sqrt{\Lambda^2 - m_\pi^2}$.

If the smearing technique is applied, this function is regulated as

$$Y_{\pi}(r) = \left( \frac{\beta}{\pi} \right)^{3/2} e^{-\beta r^2} + \frac{\mu_B^2}{8\pi r} e^{-\beta r^2} \times \left[ e^{-\frac{(\mu_B - 2\beta r)^2}{4\pi}} \text{erfc} \left( \frac{\mu_B - 2\beta r}{2\sqrt{\beta}} \right) - e^{\frac{(\mu_B + 2\beta r)^2}{4\pi}} \text{erfc} \left( \frac{\mu_B + 2\beta r}{2\sqrt{\beta}} \right) \right].$$ \hspace{1cm} (27)

When performing numerical evaluations, $m_{B^*} = 5325$ MeV and $m_B = 5279$ MeV. For the coupling constants, we use the values in the heavy quark limit which are the same as in the $D\bar{D}^*$ case. With the same procedure as before, we obtain solutions in various cases. Results from the one pion exchange interaction for the case of FF (smearing) are presented in Table VII. After considering effects from the one sigma exchange interaction, the results corresponding to the case of FF (smearing) are collected in Table VIII. For comparison, we also present the radial wave function $R(r)$ and $\chi(r)$ in Figs. 6 and 7. From these tables, it’s very interesting to note that there probably exists a loosely bound S-wave $B\bar{B}^*$ molecular state. Once produced, such a molecular state would be rather stable since its dominant decay mode is the radiative decay through $B^* \to B\gamma$.

VIII. SUMMARY AND DISCUSSIONS

In this work we have studied whether $X(3872)$ is an S-wave $DD^*$ molecule state bound by the one pion and one sigma exchange interactions. We choose to work at

| $g$ (GeV) | $E_0$ (MeV) | $\tau_{\text{rms}}$ (fm) | $\tau_{\text{max}}$ (fm) |
|---------|-------------|-----------------|-----------------|
| 0.59    | 2.3         | 0.9             | 2.1             |
| 0.7     | 2.4         | -2.8            | 1.2             |
| 0.8     | 1.7         | -0.8            | 2.3             |
| 0.9     | 1.4         | -1.4            | 1.7             |
| 1.0     | 1.1         | -0.4            | 3.2             |
| 1.1     | 1.2         | -2.3            | 1.4             |
| 1.0     | 1.0         | -1.5            | 1.7             |
| 1.1     | 1.1         | -5.0            | 1.0             |

TABLE VI: Solutions for various $g$ and $\Lambda$ in the case of FF for the $B\bar{B}^*$ system with OPEP. The lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected.

FIG. 6: The radial wave functions $R(r)$ corresponding to $\Lambda = 2.3$ GeV and $\Lambda = 2.4$ GeV with $g = 0.59$ for the $B\bar{B}^*$ system.
the hadronic level and employ the effective Lagrangian incorporating both the heavy quark symmetry and chiral symmetry. We find the σ meson exchange potential is repulsive and numerically important for a loosely bound system.

Considering the internal structure and finite size of the hadrons, we have regulated the singular δ function in the potential using both the form factor and smearing technique. After solving the radial Schrödinger equation with regulated potentials, we find that there does NOT exist a \( D \bar{D}^{*} \) molecular state if we use the experimental value for the \( D \bar{D}^{*} \pi \) coupling constant and a reasonable value around 1 GeV for the cutoff (\( \Lambda \) or \( \sqrt{\beta} \)). The two approaches lead to the same conclusion. Bound state solutions with negative eigenvalues for the \( D \bar{D}^{*} \) system exist only with either a very large coupling constant (two times of experimental value) or a large cutoff (\( \Lambda \sim 6 \text{ GeV} \) or \( \beta \sim 6 \text{ GeV}^{2} \)).

Because B mesons are much heavier, hence their kinetic energy decreases which is helpful to the formation of the shallow \( B \bar{B} \) bound state. In fact, our analysis indicates that there probably exists a loosely bound S-wave \( B \bar{B}^{*} \) molecular state. Once produced, such a molecular state would be rather stable since its dominant decay mode is the radiative decay through \( B^{*} \rightarrow B \gamma \). Experimental search of these states will be very interesting.

In short summary, we have performed a dynamical calculation of the \( D^{0} \bar{D}^{*0} \) system in the mature meson exchange framework. Our analysis disfavors the interpretation of X(3872) as a loosely bound molecular state if we use the experimental coupling constant and a reason-

| \( g \) (GeV) | \( E_0 \) (MeV) | \( r_{\text{rms}} \) (fm) | \( r_{\text{max}} \) (fm) |
|-----------|----------|---------|---------|
| 0.59      | 0.9      | -0.9    | 2.0     | 0.3     |
|           | 1.0      | -3.2    | 1.2     | 0.3     |
| 0.7       | 0.5      | -1.0    | 2.0     | 0.4     |
|           | 0.6      | -4.7    | 1.0     | 0.3     |
| 0.8       | 0.3      | -0.5    | 2.9     | 0.5     |
|           | 0.4      | -5.0    | 1.0     | 0.4     |
| 0.9       | 0.2      | -0.4    | 3.3     | 0.6     |
| 1.0       | 0.2      | -4.1    | 1.1     | 0.5     |

TABLE VII: Solutions for various \( g \) and \( \beta \) in the case of smearing for the \( BB^{*} \) system with OPEP. The lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected. Here \( g_{s} = 0.76 \) is used.

| \( g \) (GeV) | \( E_0 \) (MeV) | \( r_{\text{rms}} \) (fm) | \( r_{\text{max}} \) (fm) |
|-----------|----------|---------|---------|
| 0.59      | 2.5      | -0.5    | 2.7     | 0.3     |
|           | 2.6      | -2.5    | 1.2     | 0.2     |
| 0.7       | 1.8      | -0.3    | 3.8     | 0.4     |
|           | 1.9      | -1.9    | 1.5     | 0.3     |
| 0.8       | 1.4      | -0.2    | 4.4     | 0.5     |
|           | 1.5      | -1.7    | 1.6     | 0.4     |
|           | 1.6      | -4.9    | 1.0     | 0.3     |
| 0.9       | 1.2      | -1.1    | 1.9     | 0.5     |
|           | 1.3      | -3.9    | 1.1     | 0.4     |
| 1.0       | 1.0      | -0.9    | 2.1     | 0.5     |
|           | 1.1      | -3.7    | 1.2     | 0.4     |

TABLE VIII: Solutions for various \( g \) and \( \Lambda \) in the case of FF for the \( BB^{*} \) system with total potential. The lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected. Here \( g_{s} = 0.76 \) is used.

| \( g \) (GeV) | \( E_0 \) (MeV) | \( r_{\text{rms}} \) (fm) | \( r_{\text{max}} \) (fm) |
|-----------|----------|---------|---------|
| 0.59      | 1.1      | -0.4    | 2.9     | 0.3     |
|           | 1.2      | -2.6    | 1.2     | 0.2     |
| 0.7       | 0.6      | -0.6    | 2.6     | 0.3     |
|           | 0.7      | -4.1    | 1.0     | 0.3     |
| 0.8       | 0.4      | -1.4    | 1.7     | 0.4     |
|           | 0.3      | -3.1    | 1.2     | 0.4     |
| 1.0       | 0.2      | -1.8    | 1.6     | 0.5     |

TABLE IX: Solutions for various \( g \) and \( \beta \) in the case of smearing for the \( BB^{*} \) system with total potential. The lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected. Here \( g_{s} = 0.76 \) is used.
able cutoff around 1 GeV, which is the typical hadronic scale. Clearly more theoretical and experimental efforts are require to understand the underlying structure of the charming and mysterious X(3872) state. Maybe one need consider some more exotic schemes like the admixture of a $c \bar{c}$ charmonium and a $D \bar{D}^*$ molecular state. Coupled channel effects will help further lower the energy of the system.

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[1] Belle Collaboration, S.K. Choi et al., Phys. Rev. Lett. 91, 262001 (2003).
[2] CDF Collaboration, D. Acosta et al., Phys. Rev. Lett. 93, 072001 (2004).
[3] D0 Collaboration, V.M. Abazov et al., Phys. Rev. Lett. 93, 162002 (2003).
[4] BaBar Collaboration, B. Aubert et al., Phys. Rev. D 71, 071103 (2005).
[5] Belle Collaboration, G. Gokhroo et al., Phys. Rev. Lett. 97, 162002 (2006).
[6] Babar Collaboration, talk given by P. Grenier in Moriond QCD 2007, 17-24 March, http://moriond.in2p3.fr/QCD/2007/SundayAfternoon/Grenier.pdf.
[7] Belle Collaboration, K. Abe et al., arXiv: hep-ex/0505037.
[8] CDF Collaboration, A. Abulencia et al., Phys. Rev. Lett. 98, 132002 (2007).
[9] Belle Collaboration, K. Abe et al., arXiv: hep-ex/0505038.
[10] Babar Collaboration, B. Aubert et al., Phys. Rev. D 74, 071101(R) (2006).
[11] CDF Collaboration, A. Abulencia et al., Phys. Rev. Lett. 96, 102002 (2006).
[12] F.E. Close, P.R. Page, Phys. Lett. B 578, 119 (2004).
[13] M.B. Voloshin, Phys. Lett. B 579, 316 (2004).
[14] C.Y. Wong, Phys. Rev. C 69, 055202 (2004).
[15] E.S. Swanson, Phys. Lett. B 588, 189 (2004); ibid B 598, 197 (2004).
[16] N.A. Törnqvist, Phys. Lett. B 590, 209 (2004).
[17] D.V. Bugg, Phys. Lett. B 598, 8 (2004).
[18] J.L. Rosner, Phys. Rev. D 74, 076006 (2006).
[19] B.A. Li, Phys. Lett. B 605, 306 (2005).
[20] L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, Phys. Rev. D 71, 014028 (2005).
[21] H. Hogassen, J.M. Richard, P. Sorba, Phys. Rev. D 73, 054013 (2006); D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Lett. B 634, 214 (2006); N. Barnea, J. Vijande, A. Valcarce, Phys. Rev. D 73, 054004 (2006); Y. Cui, X.L. Chen, W.Z. Deng, S.L. Zhu, High Energy Phys. Nucl. Phys. 31, 7 (2007), arXiv: [hep-ph/0607220]; R.D. Matheus, S. Narison, M. Nielsen, J.M. Richard, Phys. Rev. D 75, 014005 (2007); T.W. Chiu, T.H. Hsieh, Phys. Lett. B 646, 95 (2007); Phys. Rev. D 73, 111503(R) (2006). Erratum-ibid. D 75, 019902 (2007); K. Terasaki, arXiv:0706.3944 [hep-ph], Prog. Theor. Phys. 118, 821 (2007).
[22] D. Gamermann and E. Oset, Eur. Phys. J. A 33, 119 (2007); arXiv: 0712.1758 [hep-ph].
[23] E. Braaten, M. Kusunoki, Phys. Rev. D. 71, 074005 (2005).
[24] N.A. Törnqvist, Nuovo Cim. A 107, 2471 (1994); Z. Phys. C 61, 525 (1994).
[25] M.T. AlFiky, F. Gabbiani and A.A. Petrov, Phys. Lett. B 640, 238 (2006).
[26] S. Fleming, M. Kusunoki, T. Mehen, and U. van Kolck, Phys. Rev. D 76, 034006 (2007).
[27] E. Braaten, Meng Lu and J. Lee, Phys. Rev. D 76, 054010 (2007).
[28] C. Hanhart, Y.S. Kalashnikova, A.E. Kudryavtsev, and A.V. Nefediev, Phys. Rev. D 76, 034007 (2007).
[29] M. B. Voloshin, Phys. Rev. D 76, 014007 (2007).
[30] P. Colangelo, F.De Fazio, S. Nicotri, Phys. Lett. B 650, 166 (2007).
[31] M. Suzuki, Phys. Rev. D 72, 114013 (2005).
[32] C. Meng, Y.J. Gao and K.T. Chao, arXiv: hep-ph/0506222.
[33] S.L. Zhu, Int. J. Mod. Phys. E 17, 283 (2008); arXiv:0707.4586 [hep-ph].
[34] M.B. Voloshin and L.B. Okun, JETP Lett. 23, 333 (1976).
[35] A.De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. Lett. 38, 317 (1977).
[36] M.B. Voloshin, arXiv:hep-ph/0602233.
[37] S. Dubynskiy and M.B. Voloshin, Mod. Phys. Lett. A 21, 2779 (2006).
[38] J. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659.
(1982); Phys. Rev. D 27, 588 (1983); Phys. Rev. D 41, 2363 (1990).
[39] X. Liu, X.Q. Zeng and X.Q. Li, Phys. Rev. D 72, 054023 (2005).
[40] C.Z. Yuan, P. Wang and X.H. Mo, Phys. Lett. B 634, 399 (2006).
[41] C.F. Qiao, Phys. Lett. B 639, 263 (2006).
[42] Belle Collaboration, S.K. Choi et al., arXiv:0708.1790 [hep-ex].
[43] J.L. Rosner, Phys. Rev. D 76, 114002 (2007).
[44] C. Meng and K.T. Chao, arXiv:0708.4222 [hep-ph].
[45] X. Liu, Y.R. Liu, W.Z. Deng and S.L. Zhu, arXiv:0711.0394.
[46] X. Liu, Y.R. Liu, W.Z. Deng and S.L. Zhu, arXiv:0803.1295 [hep-ph] (accepted by PRD); Xiang Liu, Bo Zhang, Shi-Lin Zhu, arXiv:0803.3270 [hep-ph].
[47] D.V. Bugg, arXiv:0802.0934 [hep-ph].
[48] K. Cheung, W.Y. Keung and T.C. Yuan, Phys. Rev. D 76, 117501 (2007).
[49] L. Maiani, A.D. Polosa and V. Riquer, arXiv:0708.3997 [hep-ph].
[50] S.S. Gershtein, A.K. Likhoded and G.P. Pronko, arXiv:0709.2058 [hep-ph].
[51] C.F. Qiao, arXiv:0709.4066 [hep-ph].
[52] S.H. Lee, A. Mihara, F.S. Navarra and M. Nielsen, arXiv:0710.0929 [hep-ph].
[53] G.J. Ding, arXiv:0711.1485 [hep-ph].
[54] E. Braaten and M. Lu, arXiv:0712.3885 [hep-ph].
[55] Xiang Liu, Yan-Rui Liu, Wei-Zhen Deng, arXiv:0802.3157 [hep-ph].
[56] A.F. Falk and M. Luke, Phys. Lett. B 292, 119 (1992).
[57] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. 281, 145 (1997).
[58] W.A. Bardeen, E.J. Eichten and C.T. Hill, Phys. Rev. D 68, 054024 (2003).
[59] V.M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. D 51, 6177 (1995).
[60] F.S. Navarra, M. Nielsen and M.E. Bracco, Phys. Rev. D 65, 037502 (2002).
[61] F.S. Navarra, M. Nielsen, M.E. Bracco, M. Chiapparini and C.L. Schat, Phys. Lett. B 489, 319 (2000).
[62] Y.B. Dai and S.L. Zhu, Eur. Phys. J. C 6, 307 (1999).
[63] CLEO Collaboration, S. Ahmed et al., Phys. Rev. Lett. 87, 251801 (2001); C. Isola, M. Ladisa, G. Nardulli and P. Santorelli, Phys. Rev. D 68, 114001 (2003).
[64] M.P. Locher, Y. Lu, and B.S. Zou, Z. Phys. A 347, 281 (1994); X.Q. Li, D.V. Bugg, and B.S. Zou, Phys. Rev. D 55, 1421 (1997); Y.B. Ding, X. Li, X.Q. Li, X. Liu, H. Shen, P.N. Shen, G.L. Wang and X.Q. Zeng, J. Phys. G 30, 841 (2004).
[65] S. Godfrey and N. Isgur, Phy. Rev. D 32, 189 (1985).
[66] V. LEDOUX, M.V. DAELE and G.V. BERGHE, Comp. Phys. Commun. 162, 151 (2004); V. LEDOUX, M.V. DAELE and G.V. BERGHE, ACM Transactions on Mathematical Software 31, 532 (2005); http://users.ugent.be/~vledoux/MATSLISE/.
[67] W.M. Yao et al., Particle Data Group, J. Phys. G 33, 1 (2006).
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