Constraints on the high-density nuclear equation of state from the phenomenology of compact stars and heavy-ion collisions

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Abstract

A new scheme for testing nuclear matter equations of state (EoS) at high densities using constraints from neutron star (NS) phenomenology and a flow data analysis of heavy-ion collisions is suggested. An acceptable EoS shall not allow the direct Urca process to occur in NSs with masses below 1.5 M⊙, and also shall not contradict flow and kaon production data of heavy-ion collisions. Compact star constraints include the mass measurements of 2.1 ± 0.2 M⊙ (1σ level) for PSR J0751+1807 and of 2.0 ± 0.1 M⊙ from the innermost stable circular orbit for 4U 1636-536, the baryon mass - gravitational mass relationships from Pulsar B in J0737-3039 and the mass-radius relationships from quasiperiodic brightness oscillations in 4U 0614+09 and from the thermal emission of RX J1856-3754. This scheme is applied to a set of relativistic EoS constrained otherwise from nuclear matter saturation properties with the result that no EoS can satisfy all constraints, but those with density-dependent masses and coupling constants appear most promising.

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I. INTRODUCTION

The investigation of constraints for the high-density behavior of nuclear matter (NM) has recently received new impetus when the plans to construct a new accelerator facility (FAIR) at GSI Darmstadt were published. Among others a dedicated experiment for the investigation of the phase transition from hadronic matter to the quark-gluon plasma (QGP) in compressed baryon matter (CBM) shall be hosted, which will study the phenomena of chiral symmetry restoration and quark (gluon) deconfinement accompanying the transition to the QGP. A firm theoretical prediction for the critical baryon densities and temperatures of this transition in the QCD phase diagram as well as the existence and the position of a critical point depends sensitively on both the properties of NM at high densities and the model descriptions of quark-gluon matter in the nonperturbative regime close to the hadronization transition.

In the present work we apply recently discovered astrophysical bounds on the high-density behavior of NM in β-equilibrium, i.e. neutron star matter (NSM), from compact star cooling phenomenology and neutron star mass measurements together with information about the elliptical flow in heavy-ion collisions (HICs) in order to suggest a scheme for testing NM models. This new test scheme will be applied to candidates for the NM EoS which describe properties at the saturation density nₛ ≈ 0.14 − 0.18 fm⁻³ such as the binding energy per particle in symmetric nuclear matter (SNM) aᵥ, the compressibility K and the asymmetry energy J and characteristics of large nuclei, such as the neutron skin, surface thickness and spin-orbit splitting probing the domain of subsaturation densities. In this paper we do not discuss the possibilities of various phase transitions, like hyperonization, pion and kaon condensations, quark matter etc. Corresponding comments on how their inclusion could affect our results are added at the appropriate places.

While there are several NM models giving a rather similar description of the saturation and subsaturation behavior they differ considerably in their extrapolations to densities above ∼ 2 nₛ, the regime which is relevant
for NS physics and heavy-ion collisions. Recent progress in astrophysical observations and new insights into the compact star cooling phenomenology allow us to suggest in this paper a test scheme for the high density EoS which consists of five elements.

The first one demands that any reliable nuclear EoS should be able to reproduce the recently reported high pulsar mass of $2.1 \pm 0.2 M_\odot$ for PSR J0751+1807, a millisecond pulsar in a binary system with a helium white dwarf secondary \cite{1}. Extending this value even to $2\sigma$ confidence level ($1.0\pm0.3 M_\odot$) means that masses of at least 1.6 $M_\odot$ have to be allowed. Thus the EoS should be rather stiff to satisfy this constraint.

The second constraint has recently been suggested in Ref. \cite{2} and concerns pulsar B in the double pulsar system J0737–3039 which has the lowest reliably measured mass for any NS to date, namely $M = 1.249 \pm 0.001 M_\odot$ \cite{3}. If this star originates from the collapse of an ONeMg white dwarf \cite{4} and the loss of matter during the formation of the NS is negligible, the baryon number, or equivalently the corresponding free baryon mass for the NS, has been determined to $1.366 M_\odot \leq M_N \leq 1.375 M_\odot$. It turns out that this constraint requires a rather strong binding of the compact star. A possible baryon loss of up to 1% of $M_\odot$ during the formation of the compact star broadens the corresponding baryon mass interval to $1.356 M_\odot \leq M_N \leq 1.375 M_\odot$.

The next constraint emerges from recent results of NS cooling calculations \cite{5} and population synthesis models for young, nearby NSs \cite{6}. Following the arguments in Refs. \cite{3, 5}, direct Urca (DU) processes, e.g., the neutron $\beta$-decay $n \rightarrow p + e^- + \bar{\nu}_e$, produce neutrinos very efficiently. The neutrino emissivities for these processes even with inclusion of nucleon superfluidity effects are large enough that their occurrence would lead to an unacceptably fast cooling of NSs in disagreement with modern observational soft X-ray data in the temperature - age diagram. According to these recent analyses, the DU process shall not occur in typical NSs which have masses in the range of $M_{\text{upp}} \sim 1.0 \pm 1.5 M_\odot$, obtained from population synthesis scenarios, see \cite{5} and references therein. This constrains the density dependence of the nuclear asymmetry energy which should not be too strong.

The fourth constraint defines an upper bound in the mass-radius plane for NSs, derived from quasiperiodic oscillations (QPOs) at high frequencies in the low-mass X-ray binary (LMXB) 4U 0614+09 \cite{7}. For some LMXBs there is evidence for the innermost stable circular orbit, which if confirmed suggests that the masses of the NSs in many of these systems is between 1.8 $M_\odot$ and 2.1 $M_\odot$ \cite{7, 8}.

The fifth constraint comes from a recent analysis of the thermal radiation of the isolated pulsar RX J1856 which determines a lower bound for its mass-radius relation that implies a rather stiff EoS \cite{9}.

Finally, we include into the scheme constraints that are derived from analyses of elliptic flow data and from kaon production in heavy ion collisions. Nuclear collusions have been described within a kinetic theory approach and the results have been compared to experimental data for the nucleon flow for densities up to $4.5 \times n_s$ \cite{10}. From this a region in the pressure-density diagram for SNM has been given which defines upper (UB) and lower (LB) bounds to the high density EoS and which is in accordance with measurements of the elliptic flow.

The outline of this work is the following. In section \textbf{III} we describe a set of modern relativistic nuclear EoS obtained within different approaches. In the section \textbf{III} the test scheme sketched above will be discussed in detail. This includes the astrophysical constraints from the determination of (maximum) NS masses in \textbf{III} as the new mass-baryon number test in \textbf{III} as constraints for DU-cooling in \textbf{III} and for the mass-radius relations of LMXBs in \textbf{III} as well as the mass-radius relation from thermal emission of the isolated NS RX J1856 in \textbf{III}.

The EoS for SNM at supernuclear densities is constrained by HIC experiments from flow data analysis in \textbf{IV} and kaon production in \textbf{IV}. In Section \textbf{IV} we derive two immediate consequences of this scheme: a conjecture about a universal symmetry energy contribution to the EoS in $\beta$-equilibrium and a sharpening of the flow constraint from HICs using new information about the masses of compact stars. A summary of the results of this work is given in section \textbf{V} together with the conclusions to be drawn from them.

\section{HADRONIC EOS}

\subsection{Model independent description}

There are numerous comparative studies of NM approaches for HIC and NS physics applications in which a representation of the NM EoS has been employed which is based on the nucleonic part of the binding energy per particle given in the form

$$E(n, \beta) = E_0(n) + \beta^2 E_S(n),$$

where $\beta = 1 - 2x$ is the asymmetry parameter depending on the proton fraction $x = n_p/n$ with the total baryon density $n = n_n + n_p$. In Eq. (1) the function $E_0(n)$ is the binding energy in SNM, and $E_S(n)$ is the (a)symmetry energy, i.e. the energy difference between pure neutron matter and SNM. Both contributions $E_0(n)$ and $E_S(n)$ are easily extracted from a given EoS for the cases $\beta = 0$ and $\beta = 1$, respectively. The parabolic interpolation has been widely used in the literature, see e.g. \cite{12}. It proves to be an excellent parameterization of the asymmetry dependence for the purpose of the present study and we will not go beyond it here. Nevertheless, it should be mentioned in this context that an \textit{exact reproduction} of a given EoS might require higher order terms than $\beta^2$
which have been neglected here. From Eq. (1) all zero temperature EoS of NM can be derived by applying simple thermodynamic identities [16]. In particular, we obtain

\[
\varepsilon_B(n, \beta) = n E(n, \beta),
\]

(2)

\[
P_B(n, \beta) = n^2 \frac{\partial}{\partial n} E(n, \beta),
\]

(3)

\[
\mu_{n, p}(n, \beta) = \left(1 + n \frac{\partial}{\partial n}\right) E_0(n) - \left(\beta^2 \mp 2\beta - \beta^2 n \frac{\partial}{\partial n}\right) E_S(n)
\]

(4)

for the baryonic energy density \(\varepsilon(n)\) and pressure \(P(n)\) as well as the chemical potentials of neutron \(\mu_n\) (upper sign) and proton \(\mu_p\) (lower sign), respectively.

NSM has to fulfill the two essential conditions of \(\beta\)-equilibrium

\[
\mu_n = \mu_p + \mu_e = \mu_p + \mu_e,
\]

(5)

and charge neutrality

\[
n_p - n_e - n_\mu = 0,
\]

(6)

where \(\mu_e\) and \(\mu_\mu\) are the electron and muon chemical potentials, conjugate to the corresponding densities \(n_e\) and \(n_\mu\). In this paper we do not consider phase transitions to a deconfined phase at \(n > n_s\). If a first order phase transition were allowed a mixed phase could arise in some density interval, see [17]. In general, the local charge neutrality condition could be replaced by the global one. However, due to the charge screening this density interval is essentially narrowed [18, 19]. The effect of the mixed phase on the EoS is also minor.

Due to Eq. (5) the chemical potentials for muons and electrons are equal, \(\mu_\mu = \mu_e\) so that muons appear in the system, once their chemical potential exceeds their mass. The EoS for NSM is considered as an ideal mixture of a baryonic and a leptonic part,

\[
\varepsilon(n, \beta) = \varepsilon_B(n, \beta) + \varepsilon_e(n, \beta) + \varepsilon_\mu(n, \beta),
\]

(7)

\[
P(n, \beta) = P_B(n, \beta) + P_e(n, \beta) + P_\mu(n, \beta).
\]

(8)

Under NS conditions one parameter is sufficient for a complete description, e.g. the baryochemical potential \(\mu_b\) which is conjugate to the conserved baryonic charge. In \(\beta\)-equilibrated NSM and in SNM it is simply equivalent to the neutron chemical potential, \(\mu_b = \mu_n\). Applying Eq. (4) and Eq. (5) shows that the electron and muon chemical potential can be written as an explicit function of baryon density and asymmetry parameter,

\[
\mu_e(n, \beta) = 4\beta E_S(n).
\]

(9)

Both electrons and muons are described as a massive, relativistic ideal Fermi gas.

With the above relations only one degree of freedom, namely the baryon density, remains in charge neutral and \(\beta\)-equilibrated NSM at zero temperature. Within this comfortable description actual properties of NM depend on the behavior of \(E_0(n)\) and \(E_S(n)\) only. Both can be deduced easily from any EoS introduced in the following section.

B. Equations of state applied in this paper

A wide range of densities up to and above ten times the saturation density of NM is explored in the description of NSs and HICs. It is obvious that relativistic effects are important under these conditions. Consequently, we study only nuclear EoS that originate from relativistic descriptions of NM. There are a number of different approaches.

Phenomenological models are based on a relativistic mean-field (RMF) description of NM with nucleons and mesons as degrees of freedom [20, 21, 22, 23]. The mesons couple minimally to the nucleons. The coupling strengths are adjusted to properties of NM or atomic nuclei. A scalar meson (\(\sigma\)) and a vector meson (\(\omega\)) are treated as classical fields generating scalar and vector interactions. The isovector contribution is generally represented by a vector meson \(\rho\). In order to improve the description of experimental data, a medium dependence of the effective interaction has to be incorporated into the model. In many applications of the RMF model, non-linear (NL) self-interactions of the \(\sigma\) meson were introduced with considerable success [24, 25, 26, 27, 28, 29, 30, 31, 32, 33]. This approach was later extended to other meson fields [32]. As an alternative, RMF models with density-dependent nucleon-meson couplings were developed [32, 34, 35, 36, 37, 38]. They allow for a more flexible description of the medium dependence and several parameterizations were introduced recently. In our study we choose two versions of the NL models with self-couplings of the \(\sigma\) meson field that were used in the simulation of HICs [39]. In the parameter set NL\(\rho\) the isovector part of the interaction is described, as usual, only by a \(\rho\) meson. The set NL\(\rho\delta\) also includes a scalar isovector meson \(\delta\) that is usually neglected in RMF models [40]. It leads to an increased stiffness of the neutron matter EoS and the symmetry energy at high densities. These particular NL models were mainly constructed to explore qualitatively this scalar-isovector contribution in the symmetry energy. However, they have no non-linearity or density dependence in the isovector sector and lead to very, perhaps too, stiff symmetry energies at high densities. The density dependent RMF models are also represented here by two parameter sets [41]. They are obtained from a fit to properties of finite nuclei (binding energies, radii, surface thicknesses, neutron skins and spin-orbit splittings). The parameterization DD is the standard approach with constrained rational functions for the density dependence of the isoscalar meson couplings and an exponential function for the \(\rho\) meson coupling [35]. In the D\(^4\)C model
additional couplings of the isoscalar mesons to derivatives of the nucleon field are introduced that lead to a momentum dependence of the nucleon self-energies that is absent in conventional RMF model [41].

Finally, we present a new parameterization of the RMF model with density-dependent couplings that is fitted to properties of finite nuclei (binding energies, charge and diffraction radii, surface thicknesses, neutron skin in 208Pb, spin-orbit splittings) as in Ref. [41] with an additional flow constraint (see below) by fixing the pressure of SNM to $P = 50$ MeV fm$^{-3}$ at a density of $n = 0.48$ fm$^{-3}$. The density dependence of the $\sigma$ and $\omega$ meson coupling functions is written as

$$\Gamma_i(n) = a_i \frac{1 + b_i (x + d_i)^2}{1 + c_i (x + d_i)^2} \Gamma_i(n_{\text{ref}}),$$

where for the $\rho$ meson a simple exponential law

$$\Gamma_\rho(n) = \Gamma_\rho(n_{\text{ref}}) \exp[-a_\rho(x - 1)]$$

is assumed. The coupling constants $\Gamma_i(n_{\text{ref}})$ have been fixed at a reference density $n_{\text{ref}}$. The density dependent couplings are functions of the ratio $x = n/n_{\text{ref}}$ with the vector density $n$. The parameters of this parameterization called DD-F are specified in Table I. For a more detailed description of these type of models see Ref. [41].

More microscopic approaches start from a given free nucleon-nucleon interaction that is fitted to nucleon-nucleon scattering data and deuteron properties. In these ab initio calculations based on many-body techniques one derives the nuclear energy functional from first principles, i.e., treating short-range and many-body correlations explicitly. A successful approach to the nuclear many-body problem is the Brueckner hole-line expansion. In the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) approach the nucleon inside the medium is dressed by the self-energy $\Sigma$ based on a T-matrix. The in-medium T-matrix which is obtained from the Bethe-Salpeter equation plays the role of an effective two-body interaction which contains all short-range and many-body correlations in the ladder approximation. Solving the Bethe-Salpeter equation the Pauli principle is respected and intermediate scattering states are projected out of the Fermi sea. The summation of the antisymmetrized T-matrix interactions with the occupied states inside the Fermi sphere yields finally the self-energy in Hartree-Fock approximation. This coupled set of equations constitutes a self-consistency problem which has to be solved by iteration. It is possible to extract the nucleon self-energies from DBHF calculations which can be compared with the corresponding quantities in phenomenological RMF models, but this is not completely unambiguous as discussed in Ref. [44]. Here, we use recent results of (asymmetric) NM calculations in the DBHF approach with the relativistic Bonn A potential in the subtracted T-matrix representation [43, 44, 45, 46].

In order to bridge the gap between fully microscopic and more phenomenological descriptions that can be applied more easily to various systems, it is often useful to adjust the parameters of the latter model to results extracted from the former method. As an example of this approach, we use a nonlinear RMF model (KVR) with couplings and meson masses depending on the $\sigma$- meson field [3]. The parameters were adjusted to describe the SNM and NSM EoS of the Urbana-Argonne group [47] at densities below four times the saturation density. Additionally, we study also a slightly modified parameter set (KVOR) of this RMF model that allows higher maximum NS masses. KVR and KVOR models elaborate the fact that not only the nucleon but also the meson masses should decrease with increasing NM density. Being motivated by the Brown-Rho scaling assumption, see [48], and the equivalence theorem between different RMF schemes, these models use only one extra parameter compared to the standard NL RMF model (NL model).

The nuclear EoS of these various models can be characterized by comparing the parameters in the approximation of the binding energy per nucleon

$$E = a_V + \frac{K}{18} \epsilon^2 - \frac{K'}{102} \epsilon^3 + \ldots + \beta^2 \left( J + \frac{L}{3} \epsilon + \ldots \right)$$

around saturation as a function of the density deviation $\epsilon = (n - n_s)/n_s$ and the asymmetry $\beta$. In this form the EoS is characterized at saturation by the binding energy $a_V$, the incompressibility $K$ and its derivative, the skewness parameter $K'$ and by the symmetry energy $J$ and the symmetry energy derivative or symmetry pressure $L$ for asymmetric NM. In Table I these parameters are given for the models employed in this study. Additionally, we give the Dirac effective mass $m_D = m - \Sigma$ (at the Fermi momentum) in units of the free nucleon mass $m$ depending on the scalar self-energy $\Sigma$ of the nucleon.

There are significant differences between the models. The saturation density $n_s$ in phenomenological models fitted to describe atomic nuclei (DD, D3C, DD-F) is in the range 0.147–0.15 fm$^{-3}$. The models NLρ, NLρ6 aiming at a description of low-energy HIC data use the still smaller value $n_s \approx 0.146$ fm$^{-3}$. In contrast to that, the “ab-initio” approach (DBHF) shows a saturation density that is considerably larger (0.181 fm$^{-3}$). Approximations of the Urbana-Argonne type EoS (KVR, KVOR) use 0.16 fm$^{-3}$. The binding energy per nucleon is very similar in all models. The incompressibility $K$ spans a

| Meson | $m_i$ | $\Gamma_i(n_{\text{ref}})$ | $a_i$ | $b_i$ | $c_i$ | $d_i$ |
|-------|------|-----------------|-----|-----|-----|-----|
| $\sigma$ | 555 | 11.024 | 1.4867 | 0.19560 | 0.42817 | 0.88233 |
| $\omega$ | 783 | 13.575 | 1.5449 | 0.18381 | 0.43969 | 0.87070 |
| $\rho$ | 763 | 3.6450 | 0.44793 |
The energy at high densities. In order to describe the exper-
quantity is closely related to the stiffness of the symmetry
[31, 41].

lead to a negative value of $K'$ with a rather stiff EoS
at higher densities. It is well known that the ratio of
the parameter $K'$ is rather large and, correspondingly, the
EoS of symmetric matter is softer at high densities. The
DD-F model constructed here is an exception. In this
parametrization we wanted to satisfy simultaneously the
description of finite nuclei and the flow constraint that
requires a soft EoS at high densities leading to a very
large $K'$. Correspondingly, the surface properties are not
optimally well described by the DD-F model with clear
systematic trends (radii too small for light nuclei and
too large for heavy nuclei, too small surface thicknesses
as compared to experimental data). We also remark that
the parameters of the nonlinear models NLρ and NLρδ
are not representative for conventional NL models that
are fitted to properties of finite nuclei, e.g., NL3, for
which one finds $K = 271.5$ MeV, $K' = -203.0$ MeV,
$J = 37.4$ MeV, $L = 100.9$ MeV and $m_D = 0.596$ m
[31, 41].

The symmetry energy $J$ is very similar for all models
with the exception of a slightly larger value in the DBHF
calculation. Here one has, however, to keep in mind that
this value is read off at a correspondingly larger density.
At $n = 0.16$ fm$^{-3}$ DBHF gives a value of $J = 31.5$ MeV,
which is in good agreement with the empirical models
and also with the variational approach of [47].

In contrast, the derivatives $L$ of the symmetry energy
of the various models are spread over a large range. This
quantity is closely related to the stiffness of the symmetry
energy at high densities. In order to describe the experi-
mental neutron skin thicknesses in atomic nuclei a small
slope of the neutron matter EoS is required [51, 52, 53].
Models with $L < 60$ MeV (DD, D3C, KVR, DD-F) fulfill
this requirement by introducing an effective density de-
pendence of the $\rho$ meson coupling to the nucleon which
goes beyond conventional NL RMF models. A too small
value for $L$ on the other hand seems to be in contrast with
data from isospin diffusion in heavy-ion collisions [54],
so that recently from a combination of these data the limits
$62$ MeV $< L < 107$ MeV have been suggested, see [55]
and Refs. therein. Only the models NLρ, NLρδ, DBHF
and KVOR satisfy this requirement. However, as our em-
phasis is on high density constraints of the EoS we will
not elaborate further on this interesting point here but
remark that it deserves a proper treatment.

The Dirac effective mass $m_D$ of the nucleon that ap-
pears in the relativistic dispersion relation of the nucleons
also shows a large variation in the comparison. In order
to describe the spin-orbit splitting in atomic nuclei, a
small value, typically below or around 0.6 m is required.
Parameter sets with larger values (KVR, KVOR) might
have a problem in this respect with the construction of a
proper spin-orbit potential. Larger values of the effective
Dirac nucleon mass are motivated by fitting the single
nucleon spectra in nuclei [56] with a large Landau mass
$m_L^* \approx 0.9 - 1.0$ m. The works [57] find $m_L^* \approx 0.74 - 0.82$ m
from the analysis of neutron scattering off lead nuclei.
The latter values relate to $m_D \approx 0.7 - 0.8$ m [55]. For
a recent discussion of the momentum and isospin depen-
dence of the in-medium nucleon mass, see e.g. Ref. [14].

| Model  | $n_s$ [fm$^{-3}$] | $a_V$ [MeV] | $K$ [MeV] | $K'$ [MeV] | $J$ [MeV] | $L$ [MeV] | $m_D$ [MeV] |
|--------|-----------------|-------------|-----------|------------|----------|----------|-------------|
| NLρ   | 0.1459 $-16.062$ | 203.3       | 576.5     | 30.8       | 83.1     | 0.603    |
| NLρδ  | 0.1459 $-16.062$ | 203.3       | 576.5     | 31.0       | 92.3     | 0.603    |
| DBHF  | 0.1810 $-16.150$ | 230.0       | 507.9     | 34.4       | 69.4     | 0.678    |
| DD    | 0.1487 $-16.021$ | 240.0       | $-134.6$  | 32.0       | 56.0     | 0.565    |
| D3C   | 0.1510 $-15.981$ | 232.5       | $-716.8$  | 31.9       | 59.3     | 0.541    |
| KVR   | 0.1600 $-15.800$ | 250.0       | 528.8     | 28.8       | 55.8     | 0.805    |
| KVOR  | 0.1600 $-16.000$ | 275.0       | 422.8     | 32.9       | 73.6     | 0.800    |
| DD-F  | 0.1469 $-16.024$ | 223.1       | 757.8     | 31.6       | 56.0     | 0.556    |

TABLE II: Parameters of NM at saturation for various EoS
(see text).
above saturation as shown in Fig. III. The various models of this study predict considerably different values for $E_0(n)$ and $E_S(n)$ at high densities. Under the condition of $\beta$-equilibrium, however, the range of binding energy per nucleon $E(n, \beta)$ shows a much smaller variation than expected from $E_0(n)$ and $E_S(n)$. This is shown in the right panel of Fig. III and discussed further in Sect. IV.

III. CONSTRAINTS ON THE EOS AT HIGH DENSITIES

In this section we will investigate to what extent the different EsoS introduced in Sect. III fulfill the various constraints. We postpone the discussion of the results of these tests to section V after two new consequences from our analysis are presented in Sect. IV.

A. Constraints from compact stars

1. Maximum mass constraint

Measurements of “extreme” values, like large masses or radii, huge luminosities etc. as provided by compact stars offer good opportunities to gain deeper insight into the physics of matter under extreme conditions as provided by compact stars. Recent measurements on PSR J0751+1807 imply a pulsar mass of $2.1 \pm 0.2 (^{+0.4}_{-0.5}) M_\odot$ (first error estimate with 1$\sigma$ confidence, second in brackets with 2$\sigma$ confidence) \cite{4} which is remarkably heavy in comparison to common values for binary radio pulsars ($M_{BRP} = 1.35 \pm 0.04 M_\odot$ \cite{52}). This special result constrains NS masses to at least 1.6 $M_\odot$ (2$\sigma$ confidence level) or even 1.9 $M_\odot$ within the 1$\sigma$ confidence level.

The mass and structure of spherical, nonrotating stars, to which we limit ourselves in this paper, is calculated by solving the Tolman-Oppenheimer-Volkov (TOV)-equation, which reads as

$$\frac{dP(r)}{dr} = -\frac{G[\varepsilon(r) + P(r)][m(r) + 4\pi r^3 P(r)]}{r[r - 2Gm(r)]}$$

where the gravitational mass $m(r)$ inside a sphere of radius $r$ is given by

$$m(r) = 4\pi \int_0^r dr' r'^2 \varepsilon(r')$$

which includes the effects of the gravitational binding energy. The baryon number enclosed by that sphere is given by

$$N(r) = 4\pi \int_0^r dr' r'^2 n(r') \sqrt{1 - \frac{2Gm(r')}{r}}$$

with $n(r)$ the baryon density profile of the star. Eq. 13 describes the gradient of the pressure $P$ and implicitly the radial distribution of the energy density $\varepsilon$ inside the star. In order to solve this set of differential equations, one has to specify the EoS, i.e., the relation between $P$ and $\varepsilon$ for which we take the EsoS introduced in the previous Section III. We supplement our EsoS describing the NSs interior by an EoS for the crust. For that we use a simple BPS model \cite{60}. Due to uncertainties with different crust models one may obtain slightly different mass-radius relations.

The stellar radius $R$ is defined by zero pressure at the stellar surface, $P(R) = 0$. The star’s cumulative gravitational mass is given then by $M = m(R)$ and its total baryon number is $N = N(R)$. In order to solve the TOV equations the radial change of the pressure $P$ starting with a given central value at radius $r = 0$ has to be calculated applying, e.g., an adaptive Runge-Kutta algorithm.

![FIG. 2: Mass versus central density for compact star configurations obtained by solving the TOV equations \cite{13} and \cite{14} for all EsoS introduced in Subsect. 2.2. Crosses denote the maximum mass configurations, filled dots mark the critical mass and central density values where the DU cooling process becomes possible. According to the DU constraint, it should not occur in “typical NSs” for which masses are expected from population synthesis \cite{5} to lie in the lower grey horizontal band. The dark and light grey horizontal bands around 2.1 $M_\odot$ denote the 1$\sigma$ and 2$\sigma$ confidence levels, respectively, for the mass measurement of PSR J0751+1807 \cite{4}.

The resulting NS masses as a function of their central density for the different EsoS are given in Fig. II together with the mass range of typical NSs and the limits from PSR J0751+1807. Also shown in this figure are the points on the respective curves where the DU process becomes possible, as further discussed in subsection III A 3.

The maxima of the mass-central density relations are easily determined then and summarized in Table III for the EsoS investigated in this work. As can be seen none of these values falls below the 2$\sigma$ mass limit of 1.6 $M_\odot$.\]
whereas the 1σ mass limit of 1.9 $M_\odot$ would exclude NLρ and NLρδ, while marginally excluding KVR. Thus the ability of this first and rather trivial test to exclude a given EoS demands a high accuracy of observations. A more stringent test could be achieved with decreasing error estimates or the observation of at least one pulsar that is still more massive than PSR J0751+1807. We point out that if a pulsar with a mass $M > 2.1 M_\odot$ is observed in the future, this will imply serious restrictions on the viable EoS, see Fig. 4. Within the set of EoSs tested by us, only DD, D$^3$C and DBHF would survive. Moreover, the maximum mass constraint is closely related to the flow constraint. This point will be further investigated within subsection IV.B.

### TABLE III: Maximum star masses, corresponding central densities and the fulfillment of the strong (1σ) and weak (2σ) maximum mass constraint, as well as the gravitational mass - baryon number constraint for Pulsar B in J0737-3039

| Model  | $M_{\text{max}}$ ([$M_\odot$]) | $n_{\text{max}}$ (10$^{14}$ cm$^{-3}$) | $\sigma$ | $\tau$ (107-s) |
|--------|---------------------------|-----------------|---------|-------------|
| NLρ   | 1.83 1.22                | $-$              | $+$     | $-$         |
| NLρδ  | 1.87 1.15                | $-$              | $+$     | $-$         |
| DBHF  | 2.33 0.94                | $+$              | $-$     | $+$         |
| DD    | 2.42 0.86                | $+$              | $-$     | $-$         |
| D$^3$C| 2.42 0.82                | $+$              | $-$     | $-$         |
| KVR   | 1.89 1.24                | $-$              | $+$     | $-$         |
| KVOR  | 2.01 1.12                | $-$              | $-$     | $-$         |
| DD-F  | 1.96 1.22                | $+$              | $-$     | $-$         |

2. Gravitational mass – baryon number constraint

Recently, it has been suggested in [2] that pulsar B in the double pulsar system J0737–3039 may serve to test models proposed for the EoS of superdense nuclear matter. The system J0737–3039 consists of a 22.7 ms pulsars J0737–3039A (pulsar A) [61], and a 2.77 ms pulsar companion J0737–3039B (pulsar B) [62], orbiting the common center of mass in a slightly eccentric orbit of 2.4 hours duration. One of the interesting characteristics of this system is that the mass of pulsar B is merely $1.249 \pm 0.001 M_\odot$ [1], which is the lowest reliably measured mass for any NS to date. Such a low mass could be an indication that pulsar B did not form in a type-II supernova, triggered by a collapsing iron core, but in a type-I supernova of an ONeMg white dwarf [3] driven hydrostatically unstable by electron captures onto Mg and Ne. The well-established critical density at which the collapse of such stars sets in is $4.5 \times 10^9$ g/cm$^3$ corresponding to an ONeMg core whose critical baryon mass is $M_B = N u / 1.37 M_\odot$, where the atomic mass unit $u = 931.5$ MeV has been used [3] to convert the baryon number to baryon mass. Assuming that the loss of matter during the formation of the NS is negligible, a predicted baryon mass for the NS of $M_N = 1.366 - 1.375 M_\odot$ was derived in [3]. This theoretically inferred baryon number range together with the star’s observed gravitational mass of $M = 1.249 \pm 0.001 M_\odot$ may represent a most valuable constraint on the EoS [61], provided the above key assumption for the formation mechanism of the pulsar B is correct. Then any viable EoS proposed for NSM must predict a baryon number in the range $1.366 \lesssim M_N \lesssim 1.375 M_\odot$ for a NS whose gravitational mass is in the range $M = 1.249 \pm 0.001 M_\odot$. None of the EoS tested in this work satisfies this strong constraint. The authors of [3] discussed caveats such as baryon loss and variations of the critical mass due to carbon flashes during the collapse. This constraint requires a very precise calculation of the baryon number, e.g. a lowering of $M_N$ by 1% changes the outcome of this test significantly. Since the simulation of e-capture supernovae and the evolution of their progenitors is still a work in progress, more interesting results are expected in the near future. The final value and accuracy of the baryon number of J0737-3039 are therefore highly important. The result of such calculations is shown in Fig. 3 and summarized in Table III. Finally we point out that this constraint is critically based on the assumption of the formation scenario for pulsar B. If this turns out to be incorrect the constraint has to be abandoned.

3. Direct Urca constraint

The maximum mass constraint seems to have, at least for the EoS investigated in this paper, a rather small exclusion potential. The $M - M_N$ criterion, however, would provide more stringent limits only if the assumed formation mechanism of pulsar B in RX J0737-3039 and the neglect of mass loss prove to be valid. Adding the DU criterion will be seen to improve this scheme.

If the proton fraction $x = n_p/(n_p + n_n)$ exceeds a critical value $x_{DU}$ the DU process $n \rightarrow p + e^- + \bar{\nu}_e$ becomes operative. An estimate of this DU-threshold follows from the triangle inequality for momentum conservation where the moduli of the momenta are given by the neutron, proton and electron Fermi momenta $p_F$. The typical neutrino energy of the order of the temperature $T$ is small and can be neglected. In quasi-equilibrium $n \rightarrow p + e^- + \bar{\nu}_e$ implies that $p_F \lesssim p_F + p_F$. From the charge neutrality condition $n_p = n_e + n_n$ one easily finds the DU-threshold $x_{DU}$ as

$$x_{DU} = \frac{1}{1 + (1 + x_e^{1/3})^3},$$

(16)
where $x_e = n_e/(n_n + n_p)$ is the leptonic electron fraction. Since this depends on the symmetry energy, the DU-threshold is model dependent. For $x_e = 1$ (no muons) this formula reproduces the muon-free threshold value of 11.1% \(\frac{M_\odot}{M_\odot}\). In the limit of massless muons, which is applicable for high densities ($x_e = 1/2$) one finds an upper limit of $x_{DU} = 14.8\%$. In Fig. 1, the proton fraction as a function of density is shown for the different models, together with the DU-threshold value $x_{DU}$, given as a band for all the models. As can be seen this threshold can be reached for a wide range of densities depending on the EoS. For some models (DD, D³C, DD-F) it does not take place at all. The model-dependent DU-threshold occurs at a corresponding critical baryon density. Setting this as a star’s central density results in a DU-critical star mass $M_{DU}$. These critical densities and DU-masses are marked in Fig. 1 as dots on those model curves, where the limit is reached. Every star with a mass only slightly above $M_{DU}$ will be efficiently cooled by DU-processes and very quickly becomes almost invisible for thermal detection. Nucleon superfluidity which suppresses the cooling rates has been included. Values of the pairing gaps used in the literature have been used and then varied to check the model dependence of the result. Table IV summarizes these DU critical masses for all models. The DU constraint is fulfilled by the DD, D³C and DD-F EoS models which are not affected by the DU process at all and by KVR, KVOR which are affected for masses higher than the limit for “typical NS” of 1.5 $M_\odot$ obtained from population synthesis models. As a weaker constraint we also use $M_{DU} > 1.35 M_\odot$. This follows from both the population synthesis and the mass measurement of binary radio pulsars. If the DU process were allowed for $M_{DU} < 1.35 M_\odot$ it would affect most of the NS population. It should, however, not be expected that the objects observed in X-rays were some exotic family of NSs rather than typical NSs. DBHF and both NL models do not pass the DU test. They have a DU threshold mass $M_{DU} < 1.1 M_\odot$. Note also that only NSs with $M \gtrsim 1.1 M_\odot$ are produced in the standard scenario of NS formation in type II supernova explosions.

4. **Mass-Radius relation constraint from LMXBs**

The kilohertz quasi-periodic brightness oscillations (QPOs) seen from more than 25 NS X-ray binaries constrain candidate high-density EoSs because there are fundamental limits on how high-frequency such oscillations can be. A pair of such QPOs is often seen from these systems (see \[\text{[66]}\] for a general review of properties). In all currently viable models for these QPOs, the higher frequency of the QPOs is close to the orbital frequency at some special radius. For such a QPO to last the required many cycles (up to \(\sim 100\) in some sources), the orbit must obviously be outside the star. The orbit must also be outside the innermost stable circular orbit (ISCO), because according to the predictions of general relativity, inside the ISCO gas or particles would spiral rapidly into the star, preventing the production of sharp QPOs. This implies \[\text{[10, 67]}\] that observation of a source whose maximum QPO frequency is $\nu_{\text{max}}$ limits the stellar mass and...
radius to
\[ M < 2.2 \, M_\odot (1000 \, \text{Hz}/\nu_{\text{max}})(1 + 0.75 j) \]
\[ R < 19.5 \, \text{km} (1000 \, \text{Hz}/\nu_{\text{max}})(1 + 0.2 j) \].

Here \( j \equiv c J/G M^2 \) (where \( J \) is the stellar angular momentum) is the dimensionless spin parameter, which is typically 0.1-0.2 for these systems. There is also a limit on the radius for any given mass.

These limits imply that for any given source, the observed \( \nu_{\text{max}} \) means that the mass and radius must fall inside an allowed “wedge”. Therefore, any allowed EoS must have some portion of its corresponding mass-radius curve fall inside this wedge. The wedge becomes smaller for higher \( \nu_{\text{max}} \), therefore the highest frequency ever observed (1330 Hz, for 4U 0614+091; see [63]) places the strongest of such constraints on the EoS. Note, though, that another NS could in principle have a greater mass and thus be outside this wedge, but an EoS ruled out by one star is ruled out for all, since all NS have the same EoS. As can be seen from Fig. 5 the current constraints from this argument do not rule out any of the EoS we consider. However, because higher frequencies imply smaller wedges, future observation of a QPO with a frequency \( \sim 1500 \) - 1600 Hz would rule out the stiffest of our EoS. This would therefore be a complementary restriction to those posed by RX J1856.5-3754 (discussed below) and the implied high masses for some specific NSs, which both argue against the softest EoS.

If one has evidence for a particular source that a given frequency is actually close to the orbital frequency at the ISCO, then the mass is known (modulo slight uncertainty about the spin parameter). This was first claimed for 4U 1820-30 [69], but complexities in the source phenomenology have made this controversial. More recently, careful analysis of Rossi X-ray Timing Explorer data for 4U 1636-536 and other sources [11] has suggested that sharp and reproducible changes in QPO properties are strongest of such constraints on the EoS. Note, though, that another NS could in principle have a greater mass and thus be outside this wedge, but an EoS ruled out by one star is ruled out for all, since all NS have the same EoS. As can be seen from Fig. 5, the current constraints from this argument do not rule out any of the EoS we consider. However, because higher frequencies imply smaller wedges, future observation of a QPO with a frequency \( \sim 1500 \) - 1600 Hz would rule out the stiffest of our EoS. This would therefore be a complementary restriction to those posed by RX J1856.5-3754 (discussed below) and the implied high masses for some specific NSs, which both argue against the softest EoS.

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5. Mass-Radius relation constraint from RX J1856

After the discovery of the nearby isolated NS RX J1856.5-3754 (hereafter short: RX J1856) the analysis of its thermal radiation using the apparent blackbody spectrum with a temperature \( T_{\infty} = 57 \) eV [70] yielded a lower limit for the photospheric radius \( R_{\infty} \) of this object. The distance of RX J1856 was initially estimated to be 60 pc. Since \( R_{\infty} \) crucially depends on this quantity a very small value of \( R_{\infty} \approx 8 \) km was derived which could not have been explained even with RX J1856 being a self-bound strange quark star [70]. The true stellar radius \( R \) is given by \( R_{\infty} = R(1 - R/R_S)^{-1/2} \), with the Schwarzschild radius \( R_S = 2 GM/R \). New measurements predict a distance of at least 117 pc, which results in \( R_{\infty} = 16.8 \) km and turns RX J1856 from the formerly smallest known NS into the largest one [12]. The resulting lower bound in the mass radius plane is shown in Fig. 5. There are three ways to interpret this result:

A) RX J1856 belongs to compact stars with typical masses \( M \sim 1.4 M_\odot \) and would thus have to have a radius exceeding 14 km (see Fig. 2). None of the examined EoS can meet this requirement.

B) RX J1856 has a typical radius of \( R \sim 12 - 13 \) km, implying that the EoS has to be rather stiff at high density in order to allow for configurations with masses above \( \sim 2 M_\odot \). In the present work this condition would be fulfilled for DBHF, DD and D\( ^3 \)C. This \( M > 1.6 M_\odot \) explanation implies that the object is very massive and it is not a typical NS since most of NSs have \( M < 1.5 M_\odot \), as follows from population synthesis models.

C) RX J1856 is an exotic object with a small mass \( \sim 0.2 M_\odot \), which would be possible for all EoS considered here. No such object has been observed yet, but some mechanisms for their formation and properties have been discussed in the literature [71].
distance would turn out to be smaller than the present value, then this constraint would have no discriminative power any more since all EoS could possibly fulfill it. Should a revised distance value be larger than the present one, then only the exotic low-mass star interpretation would remain which again is possible for all (not self-bound) EoS but which would raise the question about the formation scenario for such a diffuse low-mass object. Certainly this explanation of the puzzling object would no longer qualify RX J1856 as an object to test the high-density nuclear EoS.

The flow data analysis of dense SNM probed in HICs [14] reveals a correlation to the stiffness of the EoS which can be formulated as a constraint to be fulfilled within the testing scheme introduced here.

### TABLE IV: Critical compact star mass for the occurrence of the DU constraint.

| Model  | $M_{\text{DU}}$ (Solar Mass) | $n_{\text{DU}}$ (n fm$^{-3}$) | $M_{\text{du}} \geq 1.35M_{\odot}$ | $M_{\text{du}} \geq 1.5M_{\odot}$ | $4U$ 1636-536 (UB) | $4U$ 1636-536 (LB) |
|--------|----------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| NLp    | 0.98                       | 0.32            | -               | -               | -               | -               |
| NLp\(\delta\) | 0.92                  | 0.28            | -               | -               | -               | -               |
| DBHF   | 1.06                       | 0.35            | -               | +               | -               | +               |
| DD     | -                          | -               | +               | +               | +               | +               |
| D\(^4\)C | -                            | -               | +               | +               | +               | +               |
| KVR    | 1.77                       | 0.81            | +               | -               | -               | -               |
| KVOR   | 1.73                       | 0.61            | +               | +               | -               | -               |
| DD-F   | -                          | -               | +               | +               | -               | -               |

Finally we want to emphasize another problem. Comparing Fig. 3 and Fig. 5 we observe that models producing a smaller radius (KVR, DD-F, DBHF) better accommodate the $M - M_N$ constraint and those having a larger radius (D\(^4\)C, DD) fulfill the RX J1856 constraints better. Out of the EoS tested in this work only DBHF could satisfy these constraints simultaneously and it would be a rather challenging task to resolve the problem of this EoS with the DU constraint. Further comments are given in the discussion below.

### B. Constraints from heavy-ion collisions

#### 1. The flow constraint

The flow data analysis of dense SNM probed in HICs [14] reveals a correlation to the stiffness of the EoS which can be formulated as a constraint to be fulfilled within the testing scheme introduced here.
ties this constraint is based on flow data from the AGS energy regime \( (E_{\text{lab}} \sim 4 - 11 \text{ AGeV}) \). At these energies a large amount of the initial bombarding energy is converted into new degrees of freedom, i.e., excitations of higher lying baryon resonances and mesons, which makes conclusions on the nuclear EoS more ambiguous than at low energies. Nevertheless, the analysis of [80, 81, 82] provides a guideline for the high density regime which we believe to be reasonable.

As can be seen in Fig. 6, this last constraint is well fulfilled by the KVR, KVOR, NLρ and NLρδ models. For the latter two models this is rather obvious since they have already been tested to reproduce flow data. The constraint is satisfied for densities below 3 \( n_s \) by DBHF. When comparing our models with the flow constraint below, we separate regions of SIS \( (n \lesssim 3n_s) \) and AGS \( (n \gtrsim 3n_s) \) energies considering weak and strong flow constraints. DD and D^3C, which fulfilled the DU constraint well, are significantly above the demanded region. We want to emphasize that the DD-F model was constructed in this paper to pass this test. It is based on a reparametrization of the DD model, in order to satisfy the introduced test scheme in most points.

2. Constraints from subthreshold kaon production

\( K^+ \) mesons were suggested as promising tools to probe the nuclear EoS, almost 20 years ago [74]. The first channel to open in order to produce a \( K^+ \) meson is the reaction \( NN \rightarrow NAK^+ \) which has a threshold of \( E_{\text{lab}} = 1.58 \text{ GeV} \) kinetic energy for the incident nucleon. When the incident energy per nucleon in a heavy ion reaction is below this value one speaks of subthreshold kaon production. This process is particularly interesting since it ensures that the kaons originate from the high density phase of the reaction. The missing energy has to be provided either by the Fermi motion of the nucleons or by energy accumulating multi-step reactions. Both processes exclude significant distortions from surface effects if one goes sufficiently far below threshold. In combination with the long mean free path subthreshold \( K^+ \) production is an ideal tool to probe compressed NM in relativistic HICs, see Ref. [74] for a recent review.

Within the last decade the KaoS Collaboration at GSI has performed systematic measurements of the \( K^+ \) production far below threshold [75, 76, 77]. At subthreshold measurements which range from 0.6 to 1.5 AGeV laboratory energy per nucleon compressions of two to maximally three times \( n_s \) are reached. Transport calculations have demonstrated that subthreshold \( K^+ \) production provides a suitable tool to constrain the EoS of SNM at supersaturation densities [74, 78, 79]. The theoretical analysis of the data implies a soft behavior of the EoS in the considered density range consistent with the flow constraint at moderate densities \( (n \lesssim 3n_s) \) and supports DBHF, NLρ, NLρδ, KVR, KVOR and DD-F EoS in SNM [80, 81, 82].

IV. CONSEQUENCES

A. Universal symmetry energy conjecture

![FIG. 7: Density dependence of the asymmetry contribution to the energy per particle (left panel) and of the proton fraction (right panel) in SNM. Encircled curves correspond to EoS that violate the DU-constraint.](image)

Investigating the onset of DU processes in section III A 3 has shown that the DU threshold for the investigated models can be reached for rather small baryon densities slightly below \( 2n_s \) for NLρ, NLρδ, DBHF or, as the most extreme opposite, not at all for DD, D^3C and DD-F. Eq. (16) states that the threshold \( x_{DU} \) depends on the electron-muon ratio. The electron and muon densities are determined by their chemical potentials. In \( \beta \)-equilibrium \( \mu_e = \mu_\mu \) is given in turn by Eq. (9) as a function of the asymmetry parameter \( \beta \) and the symmetry energy \( E_S \). The resulting proton fraction \( x \), shown on the right hand side of Fig. 7, mainly maps the topological behavior of \( E_S(n) \), see Fig. 1. As a rule of thumb therefore a large proton fraction \( x \) is attained for stiff symmetry energies \( E_S(n) \). The NLρ, NLρδ and DBHF models confirm this rule well and the symmetry energy used by these models can be sorted out for contradicting the present cooling phenomenology, as described in section III A 3. Fig. 2 illustrates both DU- and maximum mass constraint.

The asymmetry contribution \( \beta^2 E_S(n) \) to the energy per nucleon (left panel of Fig. 7) only shows a marginal dependence for different EoS when compared to differences in the energy per nucleon of SNM. These form a narrow band which allows two important statements. First, the behavior of \( \beta^2 E_S(n) \) is to good approximation universal for all EoS which pass the DU-constraint. The second conclusion regards the influence of the symmetry energy on the mass of NS. Here we find that due to the
The task we intended with this work, developing a test scheme for the nuclear EoS by the present phenomenology of dense NM in compact stars and heavy-ion collisions, is satisfactorily completed at this point. Applying this scheme to specific EoS offers some interesting insights which indicate that astrophysical measurements might become more important for the interpretation of terrestrial measurements than presently accepted. We have summarized the results of all suggested tests performed on our choice of relativistic, high-density EoS in Table V which reveals the discriminative power of their combined application in a broad region of densities and isospin asymmetry. We want to point out here, however, that each model was derived to describe a restricted region in the \( (n, \beta) \)-plane and was not necessarily meant to describe a broader region. In Tab. V we rate the performance of the models when applied nevertheless in a very wide \( (n, \beta) \)-region.

Due to its sensitivity to different contributions to the energy this scheme motivates the necessity for changes in several EoS if one wants to apply them to the whole available \( (n, \beta) \) interval although they well describe prop-
Table V: Summary of results for the suggested scheme of tests. Non separated columns show the results for a strict (left) and weakened (right) interpretation of the corresponding constraint. The last column gives the total number of fulfilled tests in the suggested scheme. Symbols are defined in Table III.

| Model     | $M_{\text{min}}$ | $M_{\text{max}}$ | $M_{\text{DU}} \geq M_{\text{min}}$ | $M_{\text{DU}} \leq 1.5M_{\odot}$ | $M_{\text{DU}} \leq 3.5M_{\odot}$ | $4U\,1636-536\,(\text{u})$ | $4U\,1636-536\,(\text{h})$ | $\text{RX}\,J1856\,(\text{A})$ | $\text{RX}\,J1856\,(\text{B})$ | $\text{J0757}\,(\text{loss}1.5M_{\odot})$ | $\text{J0757}+\text{AGS}+\text{flow constr.}$ | $\text{SIS}+\text{AGS}+\text{flow constr.}$ | $\text{SIS}+\text{flow}+\text{K}^+\text{flow}$ | No. of passed tests (out of 6) |
|-----------|------------------|-----------------|-------------------------------------|---------------------------------|---------------------------------|-----------------------------|-----------------|-----------------------------|-----------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| NLρ       | +                | +               | +                                   | +                               | +                               | +                           | +               | +                           | +                           | +                               | +                               | +                               | +                               | +                               | 1 2                           |
| NLρδ      | +                | +               | +                                   | +                               | +                               | +                           | +               | +                           | +                           | +                               | +                               | +                               | +                               | +                               | 1 2                           |
| DBHF      | +                | +               | +                                   | +                               | +                               | +                           | +               | +                           | +                           | +                               | +                               | +                               | +                               | +                               | 2 5                           |
| DD        | +                | +               | +                                   | +                               | +                               | +                           | +               | +                           | +                           | +                               | +                               | +                               | +                               | +                               | 3 4                           |
| D3C       | +                | +               | +                                   | +                               | +                               | +                           | +               | +                           | +                           | +                               | +                               | +                               | +                               | +                               | 3 5                           |
| KVR       | +                | +               | +                                   | +                               | +                               | +                           | +               | +                           | +                           | +                               | +                               | +                               | +                               | +                               | 3 5                           |
| KVOR      | +                | +               | +                                   | +                               | +                               | +                           | +               | +                           | +                           | +                               | +                               | +                               | +                               | +                               | 3 5                           |
| DD-F      | +                | +               | +                                   | +                               | +                               | +                           | +               | +                           | +                           | +                               | +                               | +                               | +                               | +                               | 3 5                           |

Properties at the saturation density and a specific region of the $(n, \beta)$ plane.

In particular $E_S(n)$, the contribution of the symmetry energy to the total energy, is probed by the DU-constraint. It states that the DU process would cool NSs much too fast, so that it should occur for stars with masses greater than $\approx 1.35 - 1.5M_{\odot}$ only. If it would affect stars with masses below this limit the DU-process would affect most of the known NS population. We have shown that only EoS with a rather soft symmetry energy at high density fulfill the DU-constraint.

The symmetric matter contribution to the energy per nucleon $E_0(n)$ should sufficiently describe the elliptic flow. Adding the results of Ref. [14] to the test scheme, very soft EoS are allowed. The latter, however, can be sorted out by the maximum mass constraint. As a result, these two combined constraints limit the stiffness of a reliable EoS to be rather low (flow), but not too low at $n \leq 4n_s$ and rather high for higher densities (maximum mass).

We want to stress that out of the tested EoS only DBHF passes simultaneously the gravitational binding $(M - M_N)$ as well as both mass-radius $(M - R)$ tests. The $M - M_N$ constraint to be fulfilled requires a smaller radius of the $M \approx 1.25M_{\odot}$ star, whereas the $M - R$ test from RX J1856 favors substantially larger star radii, at least for $M \leq 1.4M_{\odot}$. This contradictory situation would be resolved when RX J1856 is a star with a large mass or when an EoS would fulfill the $M - M_N$ constraint and nevertheless assign large radii to NS with typical masses. Such an EoS would be qualitatively different from the ones we investigate here.

The whole scheme left three of eight model EoS, namely DD-F, KVR and KVOR as most effective within a broad $(n, \beta)$ region under consideration. The DD-F model explicitly fits properties of finite nuclei, especially the neutron skin thickness of $^{208}\text{Pb}$ that implies a small value of $L$. The KVR model yields a similar value of $L$. It might, however, have problems with respect to isospin diffusion in heavy ion collisions since this small $L$ does not fit the constraint deduced in Ref. [58] and [59]. The KVOR model fulfills this latter constraint. Here we point out that both KVR and KVOR were not applied to finite nuclei. Thus it would be a challenge to apply such models to finite nuclei in the future. All the models except DD-F demonstrate their predictive power within a broad $(n, \beta)$ region whereas the DD-F model (as a modified DD model) has been constructed in the present work in order to fulfill the flow constraint in addition to constraints from saturation and finite nuclei common to DD models. In contrast to the phenomenological RMF models, DBHF is an ab initio approach without room left for the readjustment of free parameters. But correlations beyond the ladder approximation are not taken into account. In a certain sense, they are included, although hidden in the fitted parameters, in the phenomenological approaches. However, an interesting aspect would be to perform calculations for different types of free space nucleon-nucleon interactions. In particular the CD-Bonn potential [80] which accounts more precisely for the isospin dependence of the nuclear forces than Bonn A (used here) would be appropriate for future investigations. Another point would be the explicit inclusion of hyperonic degrees of freedom which may have a significant impact on the NS matter EoS at high densities (depending on yet badly known nucleon-hyperon interaction). This could open a possibility for the DBHF and other EoS to satisfy appropriately the DU constraint.

Beside the scheme’s good overall selectivity the joint application of different constraints might give new interesting insights. One of these is the universal behavior of the contribution $\beta^2E_S(n)$ to the binding energy in NSM we observed for all EoS that fulfill the DU-constraint. Then it seems to us that the flow constraint limits the maximum mass of NSs to values around or not much about the expected mass of PSR J0751+1807 with $M = 2.1M_{\odot}$, which also coincides with the upper mass limit for 4U 1636-536. To verify this suggestion, a more detailed analysis, similar to that shown in Ref. [57], has to be performed.

Next we want to emphasize that the maximum mass constraint as a result of astrophysical measurements further limits the pressure-density-region which results from analysis of elliptic flow data governed in terrestrial HIC experiments [11]. Although the introduced scheme would not change, it seems useful to us to repeat these calculations under this point of view. It would be interesting too, to examine the agreement of experimental flow data with numerically calculated values explicitly applying the KVOR and DD-F models that have passed above constraints.
We have used here models which do not allow for phase transitions. Any possible phase transition that may appear in the NSs interior results in a decrease of the maximum NS mass. All the models may well pass the constraint $M_{\text{max}} \geq 1.6 M_{\odot}$ ($2\sigma$ uncertainty for PSR J0737-3039) but KVR, and even DD-F and KVOR which successfully have passed most of our tests might get problems with the restriction $M_{\text{max}} \geq 1.9 M_{\odot}$ ($1\sigma$ level for PSR J0737-3039 and lower limit for 4U 1636-536) if the phase transition is sufficiently strong.

If the phase transition would occur in SNM it would also soften the EoS thus modifying the flow constraint depicted in Fig. 4. Nevertheless, if the energy gain due to the phase transition is not too large the band of the flow constraint \(^{14}\) is rather broad and all the common “flow+maximum mass” constraint as indicated by Fig. 6, the NL\(\rho\) and NL\(\rho\delta\) models could get a problem crossing the lower boundary of the thus obtained new band, again if the phase transition would be sufficiently strong.

The charged pion, kaon and the hyperonization transition in NSM change the proton and electron concentrations thus affecting the DU threshold. This threshold is then pushed up to higher densities. Simultaneously, these transitions open new reaction channels, allowing for DU reactions which involve new particles: $\pi^-$, $K^-$, hyperons and quarks. The appearance of the condensates and/or filling of the new Fermi seas also affects the values of the pairing gaps which are not well known. With small gaps the new reaction channels lead to very rapid cooling of NS raising the problem to appropriately describe the NS cooling. However, if gaps are large these reactions might be non-operative and the DU constraint may become softer or even non-effective. The threshold densities for hyperonization strongly depend on poorly known baryon-baryon interactions. In case of repulsion the threshold density is pushed up \(^{15}\) and the situation becomes more cumbersome. We avoided studying such models here.

We postpone the analysis of EoS allowing for phase transitions to future work. Besides hyperonization, the possibility of a quark matter phase transition should be studied. This will be important for both NSs and for the planned HICs in the planned CBM experiment at FAIR where large baryon densities are to be created.

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