Robust Output-Feedback Control in a Dynamic Positioning System via High Order Sliding Modes: Theoretical Framework and Experimental Evaluation

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ABSTRACT
Despite being an established and mature technology, Vessel Dynamic Positioning Systems are always in focus for the development of novel control applications in order to increase their operational capacities. Rapid and robust transient response without loss of control accuracy is, for instance, a requirement for autonomous operations. Standard technologies currently applied by the industry lack such properties. Emerging methods, such as nonlinear robust control techniques, may help to achieve this goal. This paper proposes using a Super-Twisting Controller, allied with a High Order Sliding Mode Observer to perform the Dynamic Positioning Task. This method provides the robustness of standard Sliding Modes Controllers while keeping accuracy and chattering suppression. A novel nonlinear sliding manifold and wave-frequency filtering is proposed for the Dynamic Positioning problem. The efficacy of this control structure is demonstrated in a series of experimental tests, whereby the subject vessel is controlled while disturbed by wave action.

INDEX TERMS
Dynamic positioning systems, super-twisting control, high order sliding modes, robust nonlinear control.

I. INTRODUCTION
A Dynamic Positioning (DP) System is a set of hardware and software components through which a Ship or Platform can keep its horizontal position and heading fixed by automatically modulating its propulsion system thrust, a vessel behavior referred to as stationkeeping. Since its first development in the 1960s, the DP system has shown to be a well-proven concept and one of the first and most successful examples of industrial cybernetics applications. It is now a fundamental technology prerequisite for offshore operations, such as oil offloading and extraction, cable launching and subsea equipment installations.

In a scenario of ever-growing complexity in marine operations, increasing levels of automation and, consequently, requirements for enhanced flexibility in tasks performed by DP vessels are demanded. In recent years, for example, much attention has been given to the development of fully autonomous ships, which rely on both autonomous navigation and positioning behaviors to perform any mission that a crewed ship is able to perform. Autonomous navigation and maneuvering in restricted waters, basic tasks required for automatic docking, may be extremely challenging operations due to the presence of strong disturbances, such as wind and current drag. These high precision tracking tasks require robust and accurate transient performance.

Industry-standard control methods applied to DP systems, such as gain scheduled PID or LQR algorithms along with state estimators, such as Kalman Filters (KF) are mostly based on linearized systems and assume slow speed or pure stationkeeping. The disturbances are often considered as slowly varying fields, restrictions which are not always true in the missions above. Gain adjustment for both controllers and filters requires time-consuming tests at sea during commissioning. It does not usually guarantee performance when
nonlinear inertial and environmental loads are present, limiting the accuracy and flexibility of the applications.

To achieve enhanced performances in DP, linear robust control methods have been proposed, such as $H_{\infty}$ [1], [2] and mixed-$\mu$ synthesis [3]. Despite providing robust behavior, such controllers are synthesized over linearized plants around a set of operational points, imposing a large amount of workload when determining tuning parameters. Usually $H_{\infty}$ methods also generate very high order controllers, which, even after model reduction techniques, can be cumbersome to implement.

Several classes of nonlinear controllers can be applied to systems with external disturbances and unmeasured states, including techniques based on robust adaptive control. These have already been successfully applied to classes of mechanical systems [4] and, more specifically, to the problem of vessel dynamic positioning [5].

Another approach is to use the model knowledge and apply nonlinear controllers based on Lyapunov analysis, as seen in [6]–[8], whereby Vectorial Backstepping methods for DP were studied. Despite being able to provide exponential stable systems for a nominal plant, deviations in the model are not usually taken into account, and robust performance is not guaranteed.

Due to their inherent robustness and ease of tuning, Sliding Mode Controllers (SMC) have also been extensively studied in stationkeeping control of ships. In [9] and in [10], numerical and experimental results for SMCs with notch wave filtering have been produced and discussed. An approach for multivariable SMC control of a Surface Vessel was presented in [11], not addressing, however, the state estimation problem and high-frequency wave filtering. In [12], the SM controller with feedback from a Nonlinear Passive Observer was proposed. Similarly, different versions of SM controllers have been applied to the trajectory control of autonomous vessels, such as [13]–[15]. In the latter, the SM control is associated with a finite-time disturbance observer.

The robustness of the SM controller comes with a price. Due to its discontinuous nature, they introduce motion chattering and consequent high control activity. This undesired and possibly destructive effect may be reduced by the introduction of a smoothing boundary layer [16]. The cost of this modification is a restriction of accuracy and robustness, with no achievement of real sliding motion. In practice, when the system states lie inside the boundary layer, the actual controller behavior is of a proportional-derivative controller.

In parallel to the mentioned advances, a new generation of SM controllers and observers based on High Order Sliding Modes (HOSM) was proposed by [17]. These controllers can perform robust Sliding Motion, with minimal loss of accuracy and reduced chattering. Several mechanical systems have been tested with HOSM controllers, such as Container Cranes [18], quad-rotors [19], wind energy conversion turbines [20], autonomous vessel trajectory controller [21], among others. For the particular case of Dynamic Positioning Systems, an Arbitrary Order Sliding Mode Algorithm [22], which required observation of high order derivatives of the states performed through an exact robust differentiator [23] was tested experimentally in [24]. Despite demonstrating the feasibility of the method, wave filtering was not addressed and, therefore, the overall performance was limited.

HOSM Controllers (and observers) are a set of algorithms that introduce control discontinuities only in the second or higher-order derivatives of the sliding manifold. This means that observing high order derivatives of states may be required for most of its implementation solutions. A simple but effective second-order sliding mode algorithm that does not require such observations is the so-called Super Twisting Algorithm (STA), [17]. The STA works in the second-order derivatives of the sliding manifold, but it does not need information on the derivative of the sliding variable. Due to its simplicity, it has received considerable attention in recent years, in the form of controllers and observers. The Super-Twisting Controller has been tried in several mechanical systems, as described in [25]. It should be noted that the STA can usually suppress chattering in systems with sliding variables restricted to the relative degree one. For higher relative degrees, its control signal is discontinuous and fails to eliminate chattering if not properly coupled to state observers [17]. In [26], for instance, an adaptive gains STA with states feedback from a HOSM differentiator were used for arbitrary order sliding variable systems.

A DP vessel does not rely purely on direct measurements of sensors. It incorporates signal processing and filtering methods to reject high-frequency motion disturbances introduced by waves or to estimate non-measured states. Standard techniques rely on notch filtering, a method that has been dropped later in favor of the more refined Kalman Filter (KF) and its nonlinear counterpart, the Extended Kalman Filter (EKF). Alternatives to these standard filters have been proposed and studied in the academy, such as the Nonlinear Passive Observer [27], which provided a simpler tuning framework, less online calculation requirements. Despite its lack of complexity, it could be proven Lyapunov Stable by passivity analysis. Variations in the original KF and EKF were also proposed for this task, such as the Unscented Kalman Filter (UKF), proposed by [28], which introduced the concept of particle filtering enhancing the results for nonlinear systems obtained by the EKF. The UKF was formulated and simulated for a DP system in [29] and in [30]. Yet these extensions keep some of the drawbacks of the KF, such as the intense computational requirements and the arduous task of tuning the filter covariance matrices.

HOSM estimators for robust exact differentiation were proposed in [22] as a method for feeding back higher-order derivatives of sliding variables. Since then, several works proposed observers based in high order sliding modes, such as [31], where the HOSM estimator was proposed for linear systems with unknown inputs and [32], where a generalization for the design method of HOSM observers was proposed for nonlinear systems. The application of such observers to a variety of mechanical systems can be found in [33]–[35].
This work proposes a framework for applying a Super Twisting Controller with state feedback from a High Order Sliding Mode Observer to perform the Dynamic Positioning of a supply vessel subjected to environmental disturbances. Unlike most sliding mode applications in DP systems, the proposed method supplies a complete framework for output-feedback control of the Dynamic Positioned Vessel, addressing both the wave-filtering and external disturbance problems. Controller and observer tuning are simple and only require information on the extent of modelling errors and disturbances to retain accuracy and dynamic performance robustly. This paper is an extension of a previous conference paper [36], in which the method was briefly introduced, and high-fidelity simulation results were presented. In the present text, extended attention is paid to the formulation of the control architecture, providing the requirements for finite-time stability. The experimental results from a reduced scale model running the controller in the Numerical Offshore Tank (TPN-USP) wave basin are then presented and analyzed. The originality of the present paper can be summarized as follows:

- The High Order Sliding Modes theory is applied to the development of a complete framework for output-feedback control of a DP vessel, preserving the robustness properties of the first order sliding mode controllers, without the high control activity (chattering) associated to them;
- The wave filter and external disturbance are incorporated in the design, with a simple tuning procedure, that is a major advantage since the tuning of conventional controllers in a real DP vessel is an expensive and time-consuming process;
- The system is robust to modelling errors and disturbances, guaranteeing the performance and stability under a large range of environmental conditions. This property is demonstrated by theoretical analysis and experimental tests in a model scale DP vessel.

The paper is organized as follows. First, the mathematical model of the low-frequency motion of the vessel and the high-frequency motion from first-order wave excitation, both suitable for the control and observer design, are reviewed. Then, the Super-Twisting Sliding Mode Control and the High Order Sliding Mode Observer, both adequately suited for the Dynamic Positioning Problem, are presented. Finally, the experimental data from wave basin tests in four different scenarios are presented and discussed in comparison with standard PID+EKF techniques.

II. MATHEMATICAL MODEL

The reference frames used throughout the text are illustrated in Fig. 1. Positions for Easting, Northing, and Attitude Angle referenced to an Earth Fixed coordinate system represented by \( oxyz \), hereafter called Inertial or Global Reference Frame (GRF). The moving frame \( o_{LxLyLz} \), attached to the vessel and centered in its midship position will be called Local Reference Frame (LRF).

In the GRF, the location of a vessel midship and attitude will be given by vector \( \eta = [x \ y \ \psi]^{T} \). In this representation, \( x \) is the easting, \( y \) is the northing and \( \psi \) is the yaw angle. A vector \( p \in \mathbb{R}^{3} \) represented in the LRF (noted as \( p^{L} \)) can be represented in the GRF (noted as \( p^{G} \)) through a rotation matrix transformation described by (1).

\[
p^{G} = \begin{bmatrix}
\cos(\psi) & -\sin(\psi) & 0 \\
\sin(\psi) & \cos(\psi) & 0 \\
0 & 0 & 1
\end{bmatrix} p^{L} = R_{L}^{G}(\psi) p^{L}
\]

Subscript \( L \) and superscript \( G \) indicate that this is a transformation of a vector described in the Local Reference Frame to the description in the Global Reference Frame. The inverse transform is equivalent to the transpose of the \( R_{L}^{G}(\psi) \) matrix and is given by (2).

\[
p^{L} = \begin{bmatrix}
\cos(\psi) & \sin(\psi) & 0 \\
-\sin(\psi) & \cos(\psi) & 0 \\
0 & 0 & 1
\end{bmatrix} p^{G} = R_{G}^{L}(\psi) p^{G}
\]

Subscript \( G \) and superscript \( L \) indicate that this is a transformation of a vector described in the Global Reference Frame to a representation in the Local Reference Frame. To simplify the notation, from now on, heading variable \( \psi \) will be omitted from the transformations but may appear in the required contexts. Note that \( R_{L}^{G} = R_{L}^{G-1} = R_{L}^{G^T} \).

Assumption 1: For filtering purposes transformation matrices \( R_{L}^{G} \) and \( R_{G}^{L} \) are calculated over the direct measurement of the heading state, that is, \( y \) (3). This assumption introduces errors of less than 5° in extreme sea situations and less than 1 degree in normal conditions [27].

The following nonlinear state-space model will be used to express the low-frequency horizontal motions of the vessel.

\[
\begin{align*}
\dot{\eta} &= R_{V}^{G} \dot{v} \\
\dot{v} &= M^{-1} [D + C_{cor}(v)] v + M^{-1} F_{thrust} + M^{-1} F_{env}
\end{align*}
\]

In which \( \dot{\eta} \) is the velocity of the vessel midship position expressed in the GRF, \( v = [u \ v \ r]^{T} \in \mathbb{R}^{3} \) is the velocity of the vessel midship position expressed in the LRF and \( \dot{v} \) its accelerations in the same frame. Vectors \( F_{thrust} \in \mathbb{R}^{3} \) and \( F_{env} \in \mathbb{R}^{3} \) are respectively the propeller thrust forces.
and the environmental force vector is composed by

\[ \mathbf{F}_{\text{env}} = \mathbf{F}_{\text{drift}} + \mathbf{F}_{\text{wind}}. \]  (4)

In which \( \mathbf{F}_{\text{wind}} \) are the wind forces, estimated through wind velocity measurement and \( \mathbf{F}_{\text{drift}} \) lumps second-order wave drift effects, current drag and its related inertial effects. The current and wave drift forces are modeled as a constant bias disturbance \( \mathbf{b} \) in the global reference frame [27], such that

\[ \mathbf{b} = 0. \]  (5)

And the drift forces are related to the bias by the following transformation:

\[ \mathbf{b} = R_{L}^{G} \mathbf{M}^{-1} \mathbf{F}_{\text{drift}} \]  (6)

Positive-definite real matrix \( \mathbf{M} \) and negative definite real matrix \( \mathbf{D} \) are given by

\[
\mathbf{M} = \begin{bmatrix}
    m + X_u & 0 & 0 \\
    0 & m + Y_v & mx_G + Y_I \\
    0 & mx_G + Y_I & I_z + N_r
\end{bmatrix}
\]

\[
\mathbf{C}_{\text{cor}}(\mathbf{v}) = \begin{bmatrix}
    0 & -mr - mx_G \omega - Y_{v}v - Y_{I}r \\
    mr & 0 & X_{I}u \\
    0 & -X_{I}u & 0
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
    -X_{I}u & 0 & 0 \\
    0 & -Y_{v} & -Y_{I}r \\
    0 & -N_r & -N_r
\end{bmatrix}
\]  (7)

where \( m \) is the vessel mass, \( I_z \) its inertia moment around the vertical axis, \( x_G \) its center of mass position related to the midship in the LRF. Added masses are given by \( X_u, Y_v, Y_I \) and \( N_r \) and the positive damping parameters are \( X_{I}, Y_{v}, Y_{I}, N_{r} \). The Coriolis Matrix \( \mathbf{C}_{\text{cor}}(\mathbf{v}) \) is usually discarded in DP modeling, as the slow speed motion assumption makes the magnitudes of its components small when compared to other effects. It was maintained here to have it accounted as a source of disturbance in the gain setting of the High Order Sliding Modes Algorithm.

**Assumption 2:** For the sake of simplicity, parametric and model matrices \( \mathbf{M}, \mathbf{D} \) and \( \mathbf{C}_{\text{cor}}(\mathbf{v}) \) are considered to be known. These assumptions facilitate the definition of boundaries in the modelling error and disturbances. Still, due to the robustness of the controller, they do not restrict the application of the algorithm. They can be discarded, if necessary, at the cost of extended algebraic computations and a possible increase in the error boundaries.

**Assumption 3:** For control design purposes, the manipulated vector \( \mathbf{F}_{\text{thrust}} \) is considered to carry no dynamics. This is a reasonable assumption, as the actuation time constants of thruster rotation commands are orders of magnitude faster than the system’s overall dynamics.

**Assumption 4:** Wind velocity measurements are available and may be used in feed-forward term estimation \( \mathbf{F}_{\text{wind}} \) to cancel out mean wind loads. The vessel is, though, subjected to wind gusts that may instantaneously deviate from the average. Such deviations may be introduced in the control design by as a lumped effect with \( \mathbf{F}_{\text{drift}} \), as long as that the boundaries of the rate of change of this vector are increased accordingly. An estimate of such boundaries can be obtained from the statistics of a standard wind-spectrum. See [37] for a list of the most used wind spectra.

**Assumption 5:** Drag forces \( \mathbf{F}_{\text{drift}} \) are unmeasured but may be estimated. Variation in drag forces depends on changes in the environmental condition. Sea state contributions may be considered approximately stationary for periods of at least 20 minutes [38]; current field drag depends not only on the overall environment but may also change due to the vessel position and heading, especially in restricted waters. Yet, neither effect presents discontinuities, such that the rate of change in the drift vector is considered bounded by a real vector \( \Delta \in \mathbb{R}^3 \):

\[ |\dot{\mathbf{F}}_{\text{drift}}| \leq \Delta. \]  (8)

Equation (3) describes the 3 degrees-of-freedom low-frequency motion of a floating vessel subject to external forces. In order to account for higher frequency wave-driven motions, which need to be filtered from the measurement inputs to the controller, the second set of damped oscillator equations is introduced as a parcel of the total vessel motion:

\[
\begin{bmatrix}
    \dot{\xi}_w \\
    \dot{\xi}_w
\end{bmatrix} = \mathbf{A}_w \begin{bmatrix}
    \xi_u \\
    \xi_w
\end{bmatrix} + \mathbf{E}_w \mathbf{w}_w
\]  (9)

In which

\[
\mathbf{A}_w = \begin{bmatrix}
    0_{3 \times 3} & I_{3 \times 3} \\
    -\omega_0^2 I_{3 \times 3} & -2\xi\omega_0 I_{3 \times 3}
\end{bmatrix}, \quad \mathbf{E}_w = \begin{bmatrix}
    0_{3 \times 3} \\
    \omega_0^2 I_{3 \times 3}
\end{bmatrix}
\]  (10)

And \( \xi_u = [\xi_1 \xi_2 \xi_3]^T \) and \( \xi_w = [\xi_4 \xi_5 \xi_6]^T \) are the state vectors that account for the high-frequency wave excited motion in surge, sway and yaw. \( \mathbf{w}_w = [w_1 w_2 w_3]^T \) is a white noise excitation. The model parameters are the wave frequency corresponding to the peak of the wave spectrum \( \omega_0 \), the relative damping factor \( \xi \), usually chosen between 0.01 and 0.1 [39].

The total motion of a floating vessel is a composition of the low-frequency motion given by (3) and the high-frequency motion of (9), with the system output given by

\[ \mathbf{y} = \eta + \xi_w. \]  (11)

In which \( \xi_w = [\xi_4 \xi_5 \xi_6]^T \) is the first-order wave motion component.

**Assumption 6:** The only directly measured state is the vessel 2D position and attitude composed of both the low-frequency and wave-frequency motions (11). The filter estimates other system states required for control feedback.

The system composed of (3) and (9) is the subject of the output-feedback controller proposed next, based on a super-twisting controller fed by a third-order sliding mode observer. The objective is to control the low-frequency 2D position of the vessel, given by vector \( \eta \). The manipulated variable of the system is the thruster force vector \( \mathbf{F}_{\text{thrust}} \).
III. CONTROL DESIGN

A. DEFINITIONS

Some of the required vector functions shall be defined as the application of the operator to each element of the vector. Despite acknowledging these definitions are slight abuse of notation, they facilitate the understanding and increase conciseness. For a vector \( p \in \mathbb{R}^3 \), the modulus operator will be defined as the absolute value of each element of the vector:

\[
|p| = [\left| p_1 \right| \left| p_2 \right| \left| p_3 \right|]^T
\]

Similarly, the vector power function will be given by

\[
(p)^m = [(p_1)^m \ (p_2)^m \ (p_3)^m]^T
\]

The vector signum function will be defined as

\[
\text{sign}(p) = [\text{sign}(p_1) \ \text{sign}(p_2) \ \text{sign}(p_3)]^T
\]

B. SUPER TWISTING CONTROL

The standard Sliding Modes Control technique consists in forcing all the trajectories of the system states \( x(t) \in \mathbb{R}^n \) to be driven to and kept in a manifold \( S(t) \), whose dynamics are designed to meet the performance requirements. For an output vector \( y(t) \in \mathbb{R}^m \), this is accomplished by defining the manifold through a sliding vector \( \sigma(x,t) \in \mathbb{R}^m \) and respecting the sliding condition, that is, for each degree of freedom \( \sigma_i \) [16]:

\[
\frac{1}{2} \frac{d}{dt} \sigma_i^2 \leq \Delta_i |\sigma_i|, \quad i = 1, 2, \ldots, m
\]

In practice, this is done by guaranteeing that the sliding manifold vector converges to zero in finite time, that is, \( \sigma(x,t) = 0 \). This is achieved by means of discontinuous functions \( k_i \text{sign}(\sigma_i) \) operating in the first derivative of the sliding vector \( \dot{\sigma}(x,t) \). To fulfill the sliding condition (15), the extent of the positive gains \( k_i \) should be larger than the boundaries of the modelling error of the system by a strictly positive constant \( \Delta_i \), that is, \( k_i \geq \Delta_i + F_i \), where \( F_i \) are the bounds of the modelling errors.

The discontinuity introduced in the first derivative of the sliding vector introduces undesired high-frequency activity in the actuator, especially when implemented via digital control. To reduce these effects, the control system proposed for the DP vessel is a Second Order SM algorithm, that is, discontinuities are introduced in the second-order derivative of the sliding vector, enforcing \( \sigma(x,t) = \dot{\sigma}(x,t) = 0 \). Note, however, that the sliding condition is only relevant for first-order SMC, and it is not satisfied in higher-order methods.

Start by defining an appropriate sliding vector for the vessel model (Fig. 2):

\[
\sigma = v - R^G_G \dot{\eta}_d + \Lambda R^L_G \tilde{\eta} = \tilde{v} + \Lambda R^L_G \tilde{\eta}
\]

In which \( \tilde{\eta} = \eta - \eta_d \) is the error signal, \( \eta_d \) is the desired position of the vessel and \( \eta_d \) is its desired velocity. Note that this implies that the guidance system must generate smooth references to the control input. Unlike most of the sliding variables for the DP systems proposed in the literature (e.g.,

\[
\sigma = \dot{\eta} + \Lambda \ddot{\eta}, \quad \text{(16)}\]

is defined in the LRF, introducing a geometric nonlinearity from the transformation matrix. This approach allows decoupling the control action, facilitating the application and the tuning of the controller. An alternate form may be considered by defining \( \nu_d = R^L_G \ddot{\eta}_d \) and the local velocity error \( \nu = v - \nu d \). Matrix \( \Lambda \) is a positive diagonal gain matrix that shapes the dynamics of the sliding manifold. The proposed nonlinear sliding vector (16) retains the stability of the usual linear sliding vector.

Proposition 1: Consider the sliding vector given in (16). The origin of the system in \( \sigma = 0 \) is globally asymptotically stable, guaranteeing, in this condition, asymptotic convergence of the tracking error.

Proof: First, observe that if \( \sigma = 0 \), transformation \( R^G_L \sigma \) is also equal to 0. This means that, when sliding in the manifold, the system behavior from (16) may be written as

\[
R^G_L \dot{v} - \dot{\eta}_d + R^G_L \Lambda R^L_G \tilde{\eta} = \ddot{\eta} + R^G_L \Lambda R^L_G \tilde{\eta} = 0
\]

Consider then the Lyapunov Function candidate \( V = 1/2 \tilde{\eta}^T \tilde{\eta} \). For \( \sigma = 0 \), its derivative will be given by \( \dot{V} = \tilde{v}^T \tilde{\eta} = -\tilde{\eta}^T R^G_G \Lambda R^L_G \tilde{\eta} \). It is straightforward to show that all the eigenvalues of \( R^G_L \Lambda R^L_G \) are positive for positive values of \( \Lambda \) diagonal entries, that is, \( R^G_L \Lambda R^L_G \) is positive definite, which is a sufficient condition for (17) to be globally asymptotically stable.

Remark 1: From the performance point of view, it may seem interesting to make \( \Lambda \) high, but it cannot be enhanced indefinitely, as it is related to the passing band of the system. The following criteria should be applied when choosing each component of its diagonal \( \lambda_i \) [16]:

\[
\lambda_i \text{ must be smaller than the first unmodeled resonance frequency of the system (referred to as } \nu_r) \text{, according to the relation } \lambda_i < 2 \pi / 3 \nu_r;
\]

\[
\lambda_i \text{ must be smaller than the inverse of the largest delay time of system } T_d \text{ such that } \lambda_i < 1 / 3 T_d;
\]

\[
\lambda_i \text{ must be smaller than the digital sampling rate } \nu_s, \text{ such that } \lambda_i < 1 / 5 \nu_s.
\]

Then, \( \lambda_i \) is chosen as the smallest of the three criteria above. For a DP Vessel, \( \nu_s \) can be related to the heave (vertical motion) Response Amplitude Operator peak frequency,
which can be obtained via Hydrodynamic Engineering Software and $T_d$ to the time constant of the actuators.

Remark 2: As the relative degree of the system is 1, the compensation of modelling errors may be performed via super-twisting control, which does not require measurements of estimation of high order derivatives of the states.

With the desired sliding manifold dynamics defined, the action that enforces the sliding motion is designed as

$$F_{\text{thrust}} = \dot{u} + u_{st}.$$  \hspace{1cm} (18)

In which the term $\dot{u}$ is introduced to cancel out the known part of the dynamics of the sliding vector as described in [16]:

$$\dot{u} = -[D + C_{\text{cor}}(v)]\dot{v} - \hat{F}_{\text{wind}} - \hat{F}_{\text{drift}}$$

$$+ M \left[ -\Lambda \dot{v} + R_G^l \dot{\eta}_d + \left( \dot{R}_G^l + \Lambda R_G^l \right) \dot{\eta}_d - \Lambda \dot{R}_G^l \eta + \Lambda \dot{R}_G^l \eta_d \right]$$  \hspace{1cm} (19)

And $u_{st}$ is the super twisting sliding mode term [17] introduced to provide robustness (see Fig. 3 and Fig. 4): :

$$u_{st} = -K_1 |\sigma|^{1/2} \text{sign}(\sigma) + u_a$$

$$\dot{u}_a = -K_2 \text{sign}(\sigma)$$  \hspace{1cm} (20)

Remark 3: Control function $u_{st}$ is continuous for all the values of $\sigma$, as the discontinuities are introduced in the derivative of the input.

Remark 4: The estimate for the drift forces $\hat{F}_{\text{drift}}$ can be obtained through the bias error estimation (the so-called DP Current) from an observer as an Extended Kalman Filter, a Nonlinear Passive Observer or the HOSM observer.

Arguments and the extent of modelling errors $F$ can be found in [17]. A consistent Lyapunov approach to the calculation of the gain was presented in [40] based on which most applications of the Super Twisting Controller are proved stable. Yet, a popular, usual and straightforward tuning rule, given in [23] may be applied:

$$\begin{align*}
K_1 &= 1.5 |F|^{1/2} \\
K_2 &= 1.1 F
\end{align*}$$  \hspace{1cm} (21)

Proposition 2: Application of the control action composed by (18), (19) and (20) with gains defined by (21) in the disturbed system composed of (3) and (9) under assumptions 2, 4 and 5, lead to finite-time convergence to the sliding manifold defined by the vector (16).

Proof: As in the standard sliding mode theory, consider the first derivative of the sliding vector:

$$\dot{\sigma} = \dot{v} + \Lambda \dot{v} - R_G^l \dot{\eta}_d - \left( \dot{R}_G^l + \Lambda R_G^l \right) \dot{\eta}_d + \Lambda \dot{R}_G^l \dot{\eta}_d$$

$$\Lambda \dot{R}_G^l \eta + \Lambda \dot{R}_G^l \eta_d.$$  \hspace{1cm} (22)

If all states, disturbances and parameters of the system were accurately known, the control input $F_{\text{thrust}}$ that sets the sliding vector derivative to zero could be calculated by substitution of (3) in (22):

$$
F_{\text{thrust}} = -[D + C_{\text{cor}}(v)]\dot{v} - F_{\text{env}}$$

$$- M \Lambda \dot{v} + M \left[ R_G^l \dot{\eta}_d + \left( \dot{R}_G^l + \Lambda R_G^l \right) \dot{\eta}_d - \Lambda \dot{R}_G^l \dot{\eta}_d \right].$$

Note, however, that $F_{\text{thrust}}$ is composed of elements that are not accurately known and must be estimated, such as the environmental forces that are not directly measured.

Given assumptions 2, 4 and 5, the differences between (23) and the cancelling term (19) are bounded, differentiable and, may be reduced to

$$\delta_F = F_{\text{thrust}} - \dot{u} = F_{\text{drift}} - \hat{F}_{\text{drift}}.$$  \hspace{1cm} (24)

The resulting derivative of the sliding vector will then be

$$\dot{\sigma} = \delta_F + u_{st}.$$  \hspace{1cm} (25)

By defining $\omega = u_a + \delta_F$, the dynamics of the sliding variable in closed loop will be given by (26).

$$\begin{align*}
\dot{\sigma} &= -K_1 |\sigma|^{1/2} \text{sign}(\sigma) + \omega \\
\dot{\omega} &= -K_2 \text{sign}(\sigma) + \delta_F
\end{align*}$$

Application of gains calculated by (21) provides finite time stability to the STA as recently shown in [41] - Theorem 4 showed that this rule provides finite time stability to the STA. The sliding variable and its derivatives will then be driven and kept to zero.

Remark 3: Control function $u_{st}$ is continuous for all the values of $\sigma$, as the discontinuities are introduced in the derivative of the input.
proposed in the next section. Estimates of wind forces \( \hat{F}_{\text{wind}} \) are obtained via wind velocity vector measurements and assumed to compensate real wind loads with reasonable accuracy, according to hypothesis 0.

Remark 5: By noticing that \( \delta_F \) refers to the rate of change of the difference between the actual drift forces and the estimated ones, the magnitude of each element of \( \delta_F \) and its required derivatives may be obtained through assumption (5) and the maximum rate of change of \( \hat{F}_{\text{drift}} \), which can be tuned by the state estimator. In particular, if the drag disturbance is modeled as a first-order Markov process (see, e.g., [27]), the value of the bounds of its time derivative can be estimated as \( |\delta_F| = T^{-1} F_{\text{max drift}} \), in which matrix \( T \) is a diagonal time constant matrix and \( F_{\text{max drift}} \) is the maximum expected operational drift force.

Remark 6: Modelling errors and other disturbances may be introduced, at the cost of increased algebraic complexity of the function \( \delta_F \).

C. OBSERVER DESIGN

Wave-filtered position and velocity feedback required for the smooth application of the designed controller can be provided by any DP-suitable observer, such as an Extended Kalman Filter [42] and Nonlinear-Passive Observer [27]. To provide a complete solution for the output-feedback control of the DP system and to take advantage of the robustness and ease of implementation of the sliding mode algorithm, this section proposes a High Order Sliding Mode Observer.

Despite seeming reasonable to base the observer in the super twisting algorithm, its application does not cope well with a super-twisting controller, as it introduces chattering in the velocity feedback [43]. The adequate implementation thus requires a higher order SM algorithm, which is not a drawback in the observer implementation as it does not need high order derivatives of the feedback as in the control case. The following approach reconstructs the system dynamics ((3) and (9)), adding correcting terms from the exact differentiator from [22].

The observer is designed in two parts (summarized in Fig. 5), corresponding to the low-frequency (LF) motion observer and the Wave Motion observer. The main objective of the filter is to obtain a precise estimation of the Low-Frequency motion. Equation (27) presents the LF part of the proposed observer and (28) presents the Wave Motion part.

\[
\begin{align*}
\dot{\hat{\eta}} &= K_G^L \dot{\hat{v}} + K_{o1} |e|^{1/3} \text{sign}(e) \\
\hat{\dot{v}} &= M^{-1} \left[ (D + C_{cor} (\hat{v})) \hat{v} + F_{\text{thrust}} + \hat{F}_{wind} \right] \\
+ K_G^L \left[ \hat{b} + K_{o2} |e|^{1/3} \text{sign}(e) \right] \\
\hat{\dot{b}} &= K_{o3} \text{sign}(e) \\
\hat{\xi}_u &= \hat{\xi}_w \\
\hat{\xi}_w &= \hat{A}_{wu} \hat{\xi}_u + \hat{A}_{ww} \hat{\xi}_w + K_{o4} |e|^{2/3} \text{sign}(e) + \hat{\dot{c}} \\
\hat{\dot{c}} &= \hat{\dot{d}} + K_{o5} |e|^{1/3} \text{sign}(e) \\
\hat{\dot{d}} &= K_{o6} \text{sign}(e)
\end{align*}
\]

Vectors \( \hat{\eta} \) and \( \hat{v} \) are the state estimates of \( \eta \) and \( v \). \( \hat{\xi}_u \) and \( \hat{\xi}_w \) are the estimates of the first-order wave motion components, modelled as a second order dampened oscillator (9) with the lower elements of system matrix \( \hat{A}_{wu}[6 \times 6] \) composed by \( \hat{A}_{wu} = -\hat{\omega}_0^2 I_{3 \times 3} \) and \( \hat{A}_{ww} = -2 \hat{\zeta} \hat{\omega}_0 I_{3 \times 3} \). The estimated dominant wave frequency \( \hat{\omega}_0 \) is inaccurate from the actual value by a factor of \( \alpha \), such that \( \hat{\omega}_0 = \alpha \omega_0 \). System matrices will then be given by

\[
\begin{align*}
\hat{A}_{wu} &= \alpha^2 A_{wu} \\
\hat{A}_{ww} &= \alpha A_{ww}
\end{align*}
\]

where \( A_{wu} \) and \( A_{ww} \) are the actual wave motion matrices. Damping factor \( \zeta \) is considered fixed. Vector \( \hat{b} \) is the slow frequency drift estimate in the GRF (referred to in the literature and in the industry as a DP Current). States \( \hat{c} \) and \( \hat{d} \) are introduced as the high order error estimate of the wave motion. \( K_{oi}, i = 1, \ldots, 6 \) are 3 \times 3 positive diagonal gain matrices.

Each observer part reconstructs a high order Sliding Mode Observer, driven by a total observer error \( e \):

\[
e = y - \hat{\eta} - \hat{\xi}_w
\]

The estimation error \( e \) will be decomposed into two parts, \( e_1 \) and \( e_4 \), representing the LF and the Wave Motion estimation errors, respectively:

\[
\begin{align*}
e_1 &= \eta - \hat{\eta} \\
e_4 &= \xi_w - \hat{\xi}_w
\end{align*}
\]

Note that \( e = e_1 + e_4 \). Taking the time derivatives of (31):

\[
\begin{align*}
\dot{e}_1 &= \dot{\eta} - \dot{\hat{\eta}} = e_2 - K_{o1} |e|^{2/3} \text{sign}(e) \\
\dot{e}_4 &= \dot{\xi}_w - \dot{\hat{\xi}}_w = e_5 - K_{o4} |e|^{2/3} \text{sign}(e)
\end{align*}
\]
With velocity errors $e_2$ and $e_5$ defined as:

$$
\begin{align*}
\dot{e}_2 &= R^G_L (v - \hat{v}) \\
\dot{e}_5 &= \omega^2 \alpha^2 e_u - 2 \xi \omega_0 \alpha e_4 + \left( \alpha^2 - 1 \right) \omega^2 \xi_u \\
+ 2 \xi \omega_0 (\alpha - 1) \xi_w + \omega^2 \xi w_w - \dot{c}
\end{align*}
$$

The time derivatives of (33) will be:

$$
\begin{align*}
\dot{\dot{e}}_2 &= \ddot{R}^G_L (v - \hat{v}) + R^G_L \left( \dot{v} - \dot{\hat{v}} \right) \\
\dot{\dot{e}}_5 &= \omega^2 \alpha^2 \dot{e}_u - 2 \xi \omega_0 \alpha \dot{e}_4 + \left( \alpha^2 - 1 \right) \omega^2 \dot{\xi}_u \\
+ 2 \xi \omega_0 (\alpha - 1) \dot{\xi}_w + \omega^2 \dot{\xi} w_w - \dot{c}
\end{align*}
$$

If $C_{cor} (v) \equiv C_{cor} (\hat{v})$, and by applying (3), (9), (27) and (28):

$$
\begin{align*}
\dot{\dot{e}}_2 &= e_3 - K_{\alpha 2} |e|^{1/3} \text{sign} (e) \\
\dot{\dot{e}}_5 &= e_6 - K_{\alpha 5} |e|^{1/3} \text{sign} (e)
\end{align*}
$$

With acceleration errors $e_3$ and $e_6$ are given by

$$
\begin{align*}
e_3 &= A_1 (v) e_2 + b - \dot{b} \\
e_6 &= -\omega^2 \alpha^2 e_4 - 2 \xi \omega_0 \alpha e_5 + \left( \alpha^2 - 1 \right) \omega^2 \dot{\xi}_w \\
+ 2 \xi \omega_0 (\alpha - 1) \dot{\xi}_w + \omega^2 \dot{\xi} w_w - \dot{\delta}
+ 2 \xi \omega_0 \alpha K_{\alpha 4} |e|^{1/3} \text{sign} (e)
\end{align*}
$$

With

$$A_1 (v) = S_G + R^G_L M^{-1} \left( D + C_{cor} (v) \right) R^G_L$$

$$S_G = R^G_L R^G_G = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The time derivatives of (36) will then be

$$\begin{align*}
\dot{\dot{e}}_3 &= \dot{\delta}_1 - K_{\alpha 3} \text{sign} (e) \\
\dot{\dot{e}}_6 &= \dot{\delta}_2 - K_{\alpha 5} \text{sign} (e)
\end{align*}
$$

The lumped derivatives of disturbance $\delta_1$ can be written as

$$\begin{align*}
\dot{\delta}_1 &= \dot{A}_1 (v, \hat{v}) e_2 + A_1 (v) \dot{\hat{e}}_2 \\
\dot{A}_1 (v, \hat{v}) &= \dot{S}_G + \ddot{R}^G_L M^{-1} \left[ D + C_{cor} (v) \right] R^G_L \\
+ R^G_L M^{-1} \left[ D + C_{cor} (v) \right] \dot{R}_G^L + R^G_L M^{-1} C_{cor} (v) R^G_L
\end{align*}
$$

The lumped derivatives $\dot{\delta}_2$ are given by

$$\begin{align*}
\dot{\delta}_2 &= -\omega^2 \alpha^2 \dot{e}_4 - 2 \xi \omega_0 \alpha \dot{e}_5 + \omega^2 \dot{\xi}_w + \frac{4}{3} \xi \omega_0 \alpha K_{\alpha 4} \frac{\dot{\delta}}{|\dot{\delta}|^{1/3}} \\
+ \left( \alpha^2 - 1 \right) \omega^2 \dot{\xi}_u + 2 \xi \omega_0 (\alpha - 1) \dot{\xi}_w + 2 \xi \omega_0^3 (\alpha - 1) \dot{w}_w
\end{align*}
$$

Given the error boundaries lumped derivatives $\delta = \dot{\delta}_1 + \dot{\delta}_2$, appropriate observer gains may be calculated as [22]:

$$
\begin{align*}
K_{\alpha 3} + K_{\alpha 6} &= 1.1 \text{diag} (L) \\
K_{\alpha 1} &= 3 \text{diag} (L)^{1/3} \\
K_{\alpha 2} &= 1.5 \text{diag} (L)^{1/2} \\
K_{\alpha 3} &= 1.1 \text{diag} (L) \\
K_{\alpha 4} &= \sqrt{9 \text{diag} (L)^{1/3}} K_{\alpha 4} (I_{3 \times 3} + K_{\alpha 4}) \\
+ K_{\alpha 4}^3 - 2 \text{diag} (L) \right) \\
K_{\alpha 5} &= 1.5 \left[ \text{diag} (L_1 + L_2)^{1/2} - \text{diag} (L_1)^{1/2} \right] \\
K_{\alpha 6} &= 1.1 \text{diag} (L)
\end{align*}
$$

Inaccuracy in the wave frequency estimation $\alpha$ introduces high-frequency oscillations in the joint error output. Even
though this leads to overall reduction of accuracy, the convergence of the low-frequency estimator is retained, as the system model low-pass characteristic rejects the high-frequency disturbance from sign(e), responding only to sign(e1). Other parametric or even structural modeling errors may be incorporated in the values of δ1 and δ2, maintaining observer convergence if gains are set appropriately.

The stability of system (44) guarantees that e, eν, and eα are driven to zero. The convergence for e1, e2, ..., e6 cannot be claimed purely based on these results, as the actual stable system is a combination of (32), (35) and (39). That is, if e = 0, eν = 0 and eα = 0 then e1 = e4, e2 = e5 and e3 = e6. Simulation and processing of real data, though, indicate that individual errors are bounded to the wave motion amplitudes, an acceptable accuracy for the DP application. Formal solution for individual convergence is under research. Disturbance rejection for the wave observer can be verified at least locally (under linearizing conditions) and for each degree-of-freedom as follows. Consider first the derivatives of the acceleration errors e3 and e6:

$$\dot{e}_3 = \dot{A}_1(v, \dot{v}) e_2 + A_1(v) \dot{e}_3 - K_{\alpha 3} \text{sign}(e)$$

And

$$\dot{e}_6 = -\omega_0^2 \dot{e}_4 - 2\xi \omega_0 \omega_0 \dot{e}_5 + (\alpha^2 - 1) \omega_0^2 \dot{\xi}_w + 2\xi \omega_0 (\alpha - 1) \dot{\xi}_w$$

$$+ \omega_0^2 \dot{w}_w + 2\xi \omega_0 (\alpha - 1) \dot{w}_w - K_{\alpha 6} \text{sign}(e)$$

Replace (49) in (48), with adequate substitutions in the derivatives:

$$\dot{e}_3 = \dot{A}_1(v, \dot{v}) e_2 + A_1(v) \dot{e}_3 - K_{\alpha 3} \text{sign}(e)$$

$$\dot{e}_6 = -\omega_0^2 \dot{e}_4 - 2\xi \omega_0 \omega_0 \dot{e}_5 + (\alpha^2 - 1) \omega_0^2 \dot{\xi}_w + 2\xi \omega_0 (\alpha - 1) \dot{\xi}_w$$

$$+ \omega_0^2 \dot{w}_w + 2\xi \omega_0 (\alpha - 1) \dot{w}_w - K_{\alpha 6} \text{sign}(e)$$

By applying the “equivalent control” method as described in [45] and acknowledging that once e has converged and is in steady state condition, e2 = −e5 and e3 = −e6. Then, from (47):

$$\text{sign}(e) = K_{\alpha 3}^{-1} (-A_1(v) e_6 - \dot{A}_1(v, \dot{v}) e_5 + \dot{e}_6)$$

In which $G_{4w}$ is the transfer function between the error $e_4$ and wave disturbance $w_w$ and $G_{4\xi}$ is the transfer function between the error $e_4$ and $\xi_w$. The steady-state gain for each...
transfer function is given by $2\omega_0(\alpha - 1)\alpha^{-2}$ and $1 - \alpha^{-2}$, respectively, rendering the accuracy of the estimations dependent on $\alpha$. Observe also that the parameter $\gamma = K_o b^{-1}$ shapes the response of the system.

**D. OUTPUT-FEEDBACK CONTROLLER**

The DP Super Twisting Controller based on the High Order Sliding Mode Observer will then be derived based on the estimated sliding vector $\hat{\sigma}$ of (53). The general structure of the controller + observer is shown in Fig. 6, with its step-by-step algorithm presented in Fig. 7.

$$\hat{\sigma} = \nu - R_G^d \hat{\eta}_d + \lambda R_G^d \hat{\eta} - \lambda R_G^d \eta_d$$  \hspace{1cm} (53)

Equation (53) may be written as a function of $\hat{\eta}$, $\nu$, $e_1$ and $e^2$:

$$\nu = \hat{\sigma} - \lambda R_G^d \hat{\eta} - R_G^d e_2 - \lambda R_G^d e_1$$

Or, as the actual sliding vector:

$$\sigma = \hat{\sigma} - R_G^d e_2 - \lambda R_G^d e_1$$  \hspace{1cm} (55)

For the controller design, the derivative of the observer sliding vector will be

$$\dot{\hat{\sigma}} = \dot{\nu} + \lambda R_G^d \dot{\hat{\eta}} + \lambda R_G^d (\hat{\eta} - \eta_d) - (R_G^d + \lambda R_G^d) \dot{\eta}_d$$  \hspace{1cm} (56)

The best approximation of the equivalent control that fulfills $\dot{\hat{\sigma}} = 0$ will then be given by

$$\hat{\nu} = -(M \lambda + D + C_{cor}(\hat{\eta})) \dot{\hat{\eta}} - F_{wind} - M R_G^d \dot{b}$$

$$\hat{\nu} = -M \lambda \dot{R}_G^d (\hat{\eta} - \eta_d) + M R_G^d \ddot{\eta}_d$$

$$\hat{\nu} = M \left( R_G^d + \lambda R_G^d \right) \ddot{\eta}_d - M R_G^d K_{o2} |e| \frac{\dot{\gamma}}{\gamma} \text{sign}(e) - M \lambda K_G e_1^2 \frac{\dot{\gamma}}{\gamma} \text{sign}(e)$$. \hspace{1cm} (57)

The best approximation of the equivalent control that fulfills $\dot{\hat{\sigma}} = 0$ will then be given by

$$\dot{\hat{\nu}} = -(M \lambda + D + C_{cor}(\hat{\eta})) \dot{\hat{\eta}} - F_{wind} - M R_G^d \dot{b}$$

$$\dot{\hat{\nu}} = -M \lambda \dot{R}_G^d (\hat{\eta} - \eta_d) + M R_G^d \ddot{\eta}_d$$

$$\dot{\hat{\nu}} = M \left( R_G^d + \lambda R_G^d \right) \ddot{\eta}_d - M R_G^d K_{o2} |e| \frac{\dot{\gamma}}{\gamma} \text{sign}(e) - M \lambda K_G e_1^2 \frac{\dot{\gamma}}{\gamma} \text{sign}(e)$$. \hspace{1cm} (57)

The following proposition states this work’s main result.

**Proposition 4:** Sliding vector $\sigma$ with dynamic behavior described by (58), estimated by $\hat{\sigma}$ given by (53) is finite time convergent to zero, consequently providing stable error tracking for the system composed of (3) and (9).

**Proof:** As discussed in [43], Proposition 2 in and the former section, the estimation error of the system $\Omega_2$ converges to zero in finite time. The dynamics of the observed sliding vector in $\Omega_1$ is ruled by the super-twisting algorithm and, therefore, $\hat{\sigma}$ is also robustly convergent to zero in finite time $[40], [41]$. Finally, for $e_1$, $e_2$ and $\hat{\sigma}$ substituted by zero, the system dynamics will be attained to the real sliding surface $\sigma$ in system $\Omega_0$, which, as discussed in section B, guarantees that error vector $\tilde{\eta}$ and velocity error $\tilde{\eta}$ are convergent to zero. An interesting property of (58) is that the only gain restrictions are related to the observer gains with respect to the bounds of $\delta_1$, which means that there are no direct gain conditions for the controller.

**IV. EXPERIMENTAL EVALUATION**

Simulation results using the proposed controller-observer can be found in [36]. To evaluate the system under a more
realistic environment, subjected to implementation issues, such as sampling, sensor noise and unmatched modelling errors, small scale model experimental tests were performed in the Wave Basin of the Numerical Offshore Tank (TPN-USP, Fig. 8). The Wave Basin is a 14m \times 14m square tank, with 148 active wave generation/absorption flaps, able to generate regular or irregular omnidirectional waves with tight parametric control [46]. For the case of Dynamic Positioning Operations, 3D motion tracking is performed through IR Qualisys® Cameras and passive markers installed on the ship and the actuation commands are performed via a radio link. Control and observer run in a computer located in the control room. Embedded software in the vessel model is responsible for commanding the motor based on the radio communication protocol. Fig. 9 presents the general schematic of the experimental setup. A detailed description of the hardware setup used for the DP tests is provided in [47].

The vessel under study is a 1:42 scale model of a typical Platform Supply Vessel (PSV, Fig. 11) with dimensional properties indicated in Table 1. The model was weighted and measured, with inertia moment about the vertical axis taken from a bifilar pendulum test and ballast position distribution. Added masses and RAOs were obtained through a hydrodynamic simulation software (Wamit®). All the damping was considered from the relative current drag, which translates to a zero linear damping matrix. Cross added mass $Y_F$ and the position of center of gravity $x_{CG}$ was considered negligible. It is interesting to observe that, although these assumptions are not perfectly accurate, the controller and the observer should be robust to such parametric uncertainties.

The vessels are equipped with two main thrusters, two tunnel thrusters (one at the bow and one at the stern), and a bow azimuth thruster fixed and pointing to the port side. The thruster calibration was performed via static Bollard Pull test, in which the vessel was attached to a load cell in a fixed structure and measured forces at discrete rotation commands were registered. Dimensional properties and thrust capacities are given in Table 2.

Real-time control and observation algorithms and the operation interface (Fig. 10) were implemented in Matlab®, providing flexibility for algorithm changes and ease of operation. This approach made the process of coding, testing and tuning of algorithms simple and straightforward. Controller and observer gains setting was previously performed via simulation, based on the extent of modelling error derivatives and using (21) and (46). As there are significant differences between the scaled model and a full-scale PSV simulation model, fine tuning of control had to be performed online. Given output sensor measurements for preliminary tests, offline tuning of the observer gains was then performed, using the same code blocks applied in the online interface. The gains are given in Table 4.

The sample time was fixed at 0.1s for all tests. Thrust allocation was performed via the simple analytical Pseudo-inverse allocation matrix, with saturation being handled algorithmically through thruster exclusion. For details on this approach, see [48]. For control purposes, saturation for each degree of freedom is presented in Table 3.

In order to provide means to compare to standard methods, maneuvers using a nonlinear PID and an Extended Kalman Filter were also performed. The nonlinear PID feedback equation is given by (59), and the formulation used for the EKF applied to a Dynamic Positioning System is given by [38].

$$u = -K_p R_G \ddot{\eta} - K_d \dot{\eta} - K_i \int_0^T R_G \ddot{\eta} dt \quad (59)$$

The PID gains were tuned using pole placement, by setting the system natural frequency to 0.17rad/s in all degrees.
of freedom and relative damping of 0.7 for linear motions and 1.2 for the heading motion. Kalman Filter Tuning was performed offline, by trial and error in order to find optimal values. Gains for the controller are given in Table 6 and filter design matrices are presented in Table 5.

All setpoint changes were filtered by a second-order guidance filter, responsible for smoothing the input and generating reference derivatives. The reference model had relative damping of 1.0 and natural frequency of 0.2 \( \text{rad/s} \). These produce a reference suitable to the vessel actuator responses, avoiding saturation. For the case without waves, in which a comparison with the PID controller response is performed, the natural frequency of the guidance filter was set to 0.1 \( \text{rad/s} \), respecting the designed closed-loop natural frequency of the PID.

To evaluate control dynamic properties and robustness, four situations were tested for the proposed controller. Three square maneuvers, referred to in the literature as 4-corner test [49], see Fig. 12 and Table 8, were performed under different wave conditions (no wave, regular wave excitation and irregular wave excitation from a JONSWAP spectrum, see Table 8), all coming from the East. The length traveled was kept short to keep the vessel target tracked by the cameras. A fourth condition, in which the vessel setpoint remains static pointing northwards (\( \psi = 90^\circ \)) and receiving a wave train coming from the East (therefore being hit by its starboard),
A. CASE 1: SQUARE MANEUVER WITHOUT WAVES

The first case concerns the square maneuver without any external environmental action over the vessel. This permits to evaluate the tracking performance under almost no disturbance. However, drag from the motion is still present and should be compensated by the bias. This test also permits to set a performance reference for the other cases under wave action. The maneuver is also performed by using the nonlinear PID controller for comparison purposes.

Fig. 13 and Fig. 14 display the resulting track and the motion footprint for this case, applied to the proposed controller and to the nonlinear PID. The positioning tracking accuracy is very high for the ST controller, with very small deviation verified throughout the whole maneuver. As for the PID, a larger error can be verified on the right side of the square (step 4-5) and on the lower side (step 5-6), in which there is a coupling effect caused by the drag and the misalignment of the vessel axis with the motion direction. The time series of the measured motion for all the degrees of freedom are shown in Fig. 15. The smoothed reference permits accurate tracking for the ST controller, even during the

### TABLE 7. Set Points Sequence.

| Step | $x$(m) | $y$(m) | $\psi$(°) | Time(s) |
|------|--------|--------|-----------|---------|
| 1    | -0.25  | -0.25  | 0         | 0       |
| 2    | -0.25  | 0.75   | 0         | 120     |
| 3    | 0.75   | 0.75   | 0         | 180     |
| 4    | 0.75   | 0.75   | 45        | 240     |
| 5    | 0.75   | -0.25  | 45        | 300     |
| 6    | -0.25  | -0.25  | 0         | 360     |

### TABLE 8. Tests performed (in real scale/model scale).

| Maneuver   | Wave Spectrum   | Sign. Wave Height (m) | Dom. Wave Period (s) |
|------------|-----------------|-----------------------|----------------------|
| 4-corner   | None            | 0.0                   | 0.0                  |
| 4-corner   | Regular         | 1.051/0.025           | 5.8100/0.8965        |
| 4-corner   | JONSWAP (y = 3.3) | 1.4005/0.033           | 6.15/0.9491         |
| Stationkeeping | Regular     | 0.4984/0.0119            | 4.0/ 0.6172         |

was tested to evaluate stationkeeping robustness under a change of drift conditions. In order to disturb the motion in the worst-case scenario, wave frequency was chosen near the Roll resonant frequency of the boat, taken from the RAO analysis.

The performance of each case study was evaluated for tracking, sliding accuracy, demanded thrust, observer accuracy and wave filtering (when applicable). For cases with waves, the filtering performance is compared to that of the Extended Kalman Filter applied to the same dataset. For the case without waves, plots of the nonlinear PID response are also presented, along with a normalized performance index comparison based on an optimality criterion as follows:

\[
J = J_\eta + J_u = \int_0^T \tilde{\eta}(t)^T Q \tilde{\eta}(t) dt + \int_0^T u(t)^2 R dt \quad (60)
\]

Variable $u(t)$ corresponds to the instantaneous energy usage and can be calculated by

\[
u(t) = \sum_j^n \tau_j(t)^3 \quad (61)
\]

In which $n$ is the number of thrusters and $\tau_j$ is the instantaneous thrust of a single propeller $j$. The exponent $3/2$ reflects the propeller’s relation of the thrust force to the power usage. Diagonal weighting matrix $Q$ correspond to the maximum position deviation observed in the maneuvers, with diagonal entries $q_i$ corresponding to each degree of freedom $i = x, y, \psi$ given by

\[
q_i = \frac{1}{6 \max\left(\max(\tilde{\eta}_{PID,i}), \max(\tilde{\eta}_{ST,i})\right)} \quad (62)
\]

Vectors $\tilde{\eta}_{PID,i}$ and $\tilde{\eta}_{ST,i}$ are the sequence of measured positioning errors in a time series for each degree of freedom $i$, in the PID and a Super-Twisting controller maneuvers evaluated.

The scalar energy weight $R$ is calculated by

\[
r_i = \frac{1}{2 \max\left(\max(u(t)_{PID}), \max(u(t)_{ST})\right)} \quad (63)
\]

In which $u(t)_{PID}$ and $u(t)_{ST}$ are the vectors of instantaneous energy usage throughout the maneuvers for each controller evaluated.

Smaller values of $J$ indicate better performance. To provide a balanced comparison, the weighting matrices coefficients are normalized for the same value for both controllers, and to ease the visualization, performance index $J$ is also normalized such that the worst performance has a maximum of 1 for the whole maneuver. For a better understanding of the factors influencing the performance, the individual parcels of index $J_\eta$ corresponding to accuracy and $J_u$ corresponding to the energy are also calculated.

The scalar energy weight $R$ is calculated by

\[
r_i = \frac{1}{2 \max\left(\max(u(t)_{PID}), \max(u(t)_{ST})\right)} \quad (63)
\]
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FIGURE 14. Motion Footprint for the ST controller.

FIGURE 15. Square Tracking time series for the case without waves using both ST and PID controllers.

FIGURE 16. Sliding Vector time series for the case without waves.

FIGURE 17. Performance Indices for PID and ST controllers

transient, while the PID produces a delayed response, even when considering that the input reference filter has slower response than that of the closed-loop response designed for the PID (filter natural frequency of 0.1 rad/s as opposed to the designed 0.17 rad/s of the closed loop system). Pure yaw response for uncoupled motion presents an accurate tracking behavior both for the ST and the PID. However, note that the ST controller presents a transient disturbance in the response during other motion modes, while the PID seems more robust to those. This merely indicates that “balancing” priorities of the tuning of the controllers may be a bit different, with the ST controller being more responsive to errors in the surge/sway and the PID being more responsive to errors in the yaw motion. Indeed, by looking at the designed damping ratio of the PID, the value for yaw is observed to be about 70% larger than the values for surge and sway. This reflects more of a design choice than necessarily a shortcoming in the controller. Anyway, an enhancement in the boundary values of the modelling error derivative used in the design of the ST controller should produce a better tracking of the yaw error even in this coupled condition, at the cost of increased control activity.

An important variable for evaluating tracking characteristics of sliding modes is the sliding vector. Fig. 16 shows the time series of the sliding vector in each motion mode. The first noticeable characteristic is that, despite presenting small limit cycle oscillations when in steady-state (especially in the sway mode), there is no visible discontinuity. The smooth input helps the controller to keep the sliding vector with almost no deviation from the origin in surge and sway. For pure yaw motion, the sliding variable takes about 20s to recover from tracking errors during the transient, while during the coupled motion, the time increases to about 50s.
There are no noticeable discontinuities in the demanded thruster (Fig. 18). Reflecting the sliding vector behavior, the control action presents small limit cycles oscillations over the sliding manifold (of under 0.75% of total available thrust for surge, 1% for sway and 2% for yaw). In turn, the PID controller thruster output is smooth, without any oscillations in either surge or sway motions, but has a larger activity in the yaw motion, presenting oscillations at higher frequencies. This is coherent with the accuracy results from each motion mode observed in the tracking time series.

In order to provide a measurement of comparison between the controllers, the normalized performance indices $J$, $J_\eta$ and $J_u$ evolution during the maneuver for both controllers are presented in Fig. 17. The main index, $J$, which accounts for both energy and accuracy, shows a rather large advantage of the Super Twisting controller over the PID, at least for this metric. This is explained by the higher accuracy of this controller, as can be seen by the results of the accuracy performance index $J_\eta$. The larger accuracy observed in the ST tests is not equally counterbalanced by the increased usage of energy (summarized in $J_u$), which justifies the difference.

Finally, observer performance can be seen in Fig. 19. Tracking accuracy is kept under 5mm for Easting and Northing and under 0.3 degree for yaw (Fig. 20). Despite some oscillations in the steady-state error output of about 1mm in easting and 0.1 degree in yaw, the observer output is smooth when compared with the measurement signal, indicating that the observer also filters measurement noise.

B. CASE 2: SQUARE MANEUVER UNDER REGULAR WAVES

This case concerns the square maneuver while regular waves of equivalent 1.05m height and 5.81s period (real scale values) come from the East. This period corresponds, approximately, to the resonant frequency of the vessel Roll RAO.
In this situation, disturbance values are not only due to the relative current (drag), but also from slow and medium wave drift effects.

The resulting track and the motion footprint for the regular wave-disturbed motion is shown in Fig. 21 and Fig. 22. Once again, the main source of loss of accuracy is during the coupled motion on the low side of the square, which is a bit more prominent in this case. During this phase of the motion, the vessel center deviates up to 0.07m in the northing direction before hitting the target value.

Overshoot is once again present in the pure motion in surge (Fig. 23) but, in this case, an overshoot in the pure sway motion is also present. Finally, it is evident that the yaw motion is the most affected by the disturbance, presenting a rather large overshoot in its individual step. The subsequent motion, in which the northing is changed while the waves hit the vessel shoulder, starts before the heading is settled, causing an oscillatory behavior in this state. The corresponding sliding variable, displayed in the bottom graph of Fig. 24, shows that this larger error comes from deviations from the sliding manifold reference, which, despite not being considerably larger than that observed in the undisturbed case, takes longer to settle after the setpoint change, taking up to 50s to reach the sliding manifold. The northing setpoint change before the heading is settled produces an even longer time to the overall convergence of the yaw sliding variable. For the surge and sway motions, the time to settling is once again under 50s. Small amplitude oscillatory behavior in the settled sliding variables is once again present, but these may not only be from the inner limit cycle behavior but also a response of partially unfiltered wave motion.

These small oscillations are noticeable in the thruster output (Fig. 25) but seem to affect the sway motion more than the surge and the yaw. The amplitudes of the oscillations are a bit larger than those seen in the undisturbed test, and the allocated force power spectrum (Fig. 26) indicate that there is indeed a small response due to unfiltered wave motion in the corresponding frequency (around 1.01Hz for the 1:42 model scale).

Although some of the wave motion seems to be unfiltered, the observer output shows that most of it is cut from the estimated low-frequency positions. Fig. 27 displays the tracking performance of these observed variables, as compared to an “Exact Filter” output, a post-processed signal obtained from the offline Inverse Fourier Transform of the modified motion spectrum, without components in the wave frequency range. The plot also shows the output from an extended Kalman Filter. Note that the output of the HOSM has comparable accuracy and wave filtering performance to the EKF, having the Exact Filter as a reference. Deviations from the mean value observed in the detail figures are due to transients. The low-frequency error $e_1$, calculated by subtracting the Exact
Filter output from the estimated low-frequency value, has zero mean and is shown in Fig. 28. For the surge and sway motion, the accuracy and convergence rate are even better than those observed in the Kalman Filter. The maximum error amplitude is of about 20mm in positioning and 1.4 degree in heading for the HOSM, while it reaches 90mm and 1 degree, respectively, for the EKF.

C. CASE 3: SQUARE MANEUVER UNDER JONSWAP SPECTRUM WAVES

The third case evaluates the square maneuver in a more realistic (however, less critical) scenario, in which irregular waves with JONSWAP spectrum and equivalent 1.4005 m
significant height and 6.15s (real scale values) mean period come from the East. As in the case of regular waves, this period corresponds to the resonant frequency of the vessel Roll RAO and slow and medium wave drift effects arise.

Fig. 29 and Fig. 30 show the track the footprint for the disturbed motion. The accuracy is better than that observed with regular waves. The largest deviation during the coupled motion is kept under 0.04m in the northing direction.

The time series of the tracking performance is displayed in Fig. 31. The characteristic overshoot in the pure motion in surge has lower amplitude, and the sway motion presents a performance similar to that in the case without waves. The yaw motion, which was largely affected by the regular wave disturbances, presents a much more accurate response and almost no overshoot, being settled before the next set-point change and only disturbed by the coupled subsequent motion. The filtering action of the observer seems to be more effective, as the small amplitude oscillations observed in the sliding vector (Fig. 32) during steady state have amplitudes and frequency similar to that in the case without waves. Deviations from $\sigma = 0$ take less than 50s for the surge to converge back to steady state; for sway and yaw, the time is of approximately 30s.

The steady thruster output (Fig. 33) presents oscillations comparable to those verified in the case without waves,
indicating that the wave motion is filtered out. Indeed, the allocated force power spectrum, shown in Fig. 34, shows no significant power increase in the wave frequency.

The observer output time series (Fig. 35) confirms the wave filtering performance. The filtered motion most of the time corresponds to that of the “Exact Filter”, despite, once again, with comparable performance to that of an Extended Kalman Filter. The low-frequency errors are kept under 20mm and 1 degree during transients and under 5mm and 0.2 degree in
steady-state for the HOSM, as opposed to 80mm and 1 degree during transients and a similar performance for steady state of the EKF. The low-frequency observation error is shown in Fig. 36.

**D. CASE 4: STATION KEEPING**

In the latter case, the vessel is kept in a steady setpoint pointing Northwards, and regular waves with equivalent 0.498 m height and 4.0 s period come from the East, hitting the side of the vessel through the starboard. At this higher frequency, chosen near the maximum wave generator response when scaled, wave drift effects are larger.

The following results are presented from the moment the waves start hitting the vessel. Fig. 38 displays the motion footprints once the waves started hitting the vessel. The track (Fig. 37) shows there is a slight deviation (of about 0.035m) from the target in the x direction. The heading is also affected, as evidenced by the time series displayed in Fig. 39. The yaw value was not in the setpoint when the waves started hitting the vessel, and a transient oscillation, aggravated by the wave drift forces, took place at the beginning of the test. The deviations from the sliding manifold can be verified in Fig. 40.

There is no observable deviation in the surge mode, which is coherent as there is no disturbance in this direction. The transient deviation in the sway variable is rapidly corrected, and for the yaw motion, the larger oscillation from the beginning is also noticeable. When in steady-state, no discontinuity is present in any of the motion modes.

The thruster output (Fig. 41) presents a small bias in the negative direction, counteracting the wave drift. The allocated force power spectrum in Fig. 42 shows a very small peak in the corresponding wave frequency, but its amplitude is rather
small when compared to the main response from the control action.

Finally, the observer time series from Fig. 43 reinforces that the wave is filtered, and once again the resulting response is very close to that of the “Exact Filter” and to that of the EKF. The total observer error (Fig. 44) is kept under 6mm in the Easting direction and under 1.5mm in the Northing.
direction. In yaw, it reaches at most 0.2 degrees. It should be noticed that the Kalman Filter suffers a little performance degradation due to the “detuning” of the wave natural frequency set to the model as opposed to the actual wave frequency. To account for these differences, the EKF is usually fed with an estimation of the sea state, either external or via internal parameter estimation. The HOSM observer used the same wave model natural frequency for all the test cases.

V. CONCLUSION

The floating structure Dynamic Positioning problem has shown to be an interesting problem to the application of the Super-Twisting controller along with the High Order Sliding Mode Observer. Its unique requirements of wave motion rejection from the measurements introduce an extra challenge to the sliding mode observer. With adequate tuning, the wave filtering performance matches that of an “Exact Filter” and of an EKF, with little computing power requirements when compared to the former. The performance index comparison indicates that the method yields high accuracy when compared to a nonlinear PID controller, at the cost of slightly higher energy usage. The method is easy to implement and tune, and experimental results show that performance is robust to environmental loads, even during transient motion, making it suitable to highly dynamical operations.

Tuning depends on running simulated or, in the case of the filter, past collected data in order to define the extent of modelling errors, values that are not always easily found theoretically. Further work should focus on finding other methodologies to find such error extents, including neural-network and machine learning algorithms.

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CONFLICTS OF INTEREST

The authors declare no conflict of interest in this work.

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