Polaron induced parametric interactions in magnetized semiconductors

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Abstract. In the framework of hydrodynamic model of semiconductor plasma, the parametric amplification due to polaron mode has been analytically investigated in a magnetized III–V semiconductor, viz. n-InSb. The origin of the phenomenon lies in the effective second-order optical susceptibility arising due to the induced nonlinear current density of the medium. Using the coupled-mode theory, the expressions for the parametric gain coefficient and the threshold amplitudes of the external electric field above which the parametric instability can occur are derived. The effect of the wave vector and the strength of the external magnetic field on the gain coefficient and the threshold amplitude of the pump electric field are analyzed. Reported results strongly suggest that at appropriate pump electric field, proper selection of the magnetic field not only lowers threshold pump field required for the onset of parametric excitation but also enhances gain effectively.

1. Introduction
The phenomena of parametric interaction exhibit a distinctive role in nonlinear optics. Parametric amplifiers, parametric oscillators, optical phase conjugation, pulse narrowing, squeezed state generation [1], etc. are some of the important devices and processes whose origins lie in the parametric interactions in a nonlinear medium. In optical parametric amplification, a weak signal is made to interact with a strong, higher frequency pump and both the generated difference frequency (known as the idler) and the original signal are amplified.

In a polar material, the electric field interacts with both optical phonons and electrons. The coupling of LO phonons and free carrier collective excitations by macroscopic longitudinal electric fields in polar semiconductors has been treated theoretically by a number of investigators [2, 3]. A polaron is a quasi-particle that arises due to the conduction electron (or hole) together with its self-induced polarization in an ionic crystal or in a polar semiconductor [4-6]. Physically the polaron is an electron in a localized state that induces a polarization of the medium. This polarization is local in character and is due to the displacements of ions from the equilibrium positions caused by the field produced by the electron density which gives rise to an electron density perturbation at the polaron frequency, which couples nonlinearly with the pump wave and drives the polaron wave at modulated frequency [7]. Thus in this paper using hydrodynamic model and coupled mode theory analytical expressions for the second order susceptibility arising from the nonlinear induced current density, gain coefficient of the amplified wave and threshold pump amplitude required to incite the parametric
interaction in a semiconductor are derived. Numerical estimations have been made with a set of data appropriate for a polar semiconductor crystal (n-InSb) duly irradiated by a frequency doubled CO2 laser to establish the validity of the present work.

2. Theoretical Formulation
We consider a spatially uniform pump electric field \( |k_p| \neq 0 \) \( \tilde{E}_0 = \tilde{x} \tilde{E}_0 \exp(-i\omega_{pt} t) \) applied in homogeneous semiconductor plasma. It restricts our analysis to be valid only in the limit \( kl << 1 \) (k is the wave number and l is the carrier mean free path).

2.1 Effective second-order susceptibility. The basic equations employed in the present analysis are:

\[
\frac{\delta^2 \tilde{R}}{\delta t^2} + (\delta \omega_0^2 + i \Gamma_p) \tilde{R} + 2 \Gamma_i \frac{\delta \tilde{R}}{\delta t} = -\frac{\epsilon}{m_e} \left[ \tilde{E}_0 + \frac{q}{M} \left( \tilde{E}_{el} + \frac{\partial \tilde{W}}{\partial t} \tilde{X} \right) \right],
\]

\[
\frac{\delta \tilde{S}}{\delta \tilde{r}} + (\delta \omega_0^2 + i \Gamma_p) \tilde{S} + 2 \Gamma_i \frac{\delta \tilde{S}}{\delta \tilde{r}} = -\frac{q}{M} \left[ \tilde{E}_0 + \frac{\partial \tilde{W}}{\partial \tilde{r}} \right],
\]

\[
\frac{\partial n_1}{\partial x} + n_0 \frac{\partial \varphi_0}{\partial x} + n_1 \frac{\partial \varphi_0}{\partial x} + g_0 \frac{\partial n_1}{\partial x} = 0
\]

\[
\frac{\partial \tilde{E}_{pl}}{\partial x} = -\frac{n_i e}{\epsilon_0} + \left( \frac{n_i e}{\epsilon_0} - \frac{N_q}{\epsilon_0} \right) \frac{dR}{dx}
\]

We assume that the electric fields associated with the electronic and lattice polarizations are parallel to each other and the polarizabilities of the electron and ion systems are additive. Thus, the simultaneous excitation of collective cyclotron vibrations and optical phonons results into coupling between them.

The resulting coupled vibrations appear in the form of a new mode known as polaron mode. The equation of motion of a polaron mode is given by

\[
\frac{\delta^2 \tilde{R}}{\delta t^2} + \omega_{pl}^2 \tilde{R} + 2 \Gamma_i \frac{\delta \tilde{R}}{\delta t} = \left( NM \right)^2 \left( \frac{\epsilon}{m_e} + \frac{q}{M} \right) \left( \tilde{E}_{el} + \frac{\partial \tilde{W}}{\partial t} \tilde{X} \right)
\]

Meanings of all the symbols are given in the reference [8]. We also assume that the energy transfer between the pump, polaron and side band waves satisfy phase matching conditions which are \( \omega_p = \omega_{pl} \pm \delta \) and \( k_0 = k_p \pm k_s \); under spatially uniform laser irradiations \( |k_s| = 0 \) so that \( |k_p| = k_s \pm k_p, |k_p| = k_p \) (say).

Following the procedure adopted by Dubey and Ghosh [8] and using equations (1-4), we may obtain second order susceptibility via nonlinear polarization at amplified frequencies considered as

\[
P_{eff}^{(2)} = \epsilon_0 \chi_{eff}^{(2)} E_0 E_{pl}^{*}
\]

\[
\chi_{eff}^{(2)} = \frac{-i \kappa e}{m_e \omega_s} \Omega_p \omega_s^2 \omega_e A_1 A_2 X_1 \left[ \delta \omega_e + 2 i \Gamma_{pl} \omega_p \omega_e - \frac{k^2 |A_2|^2 |E|^2}{\delta \omega_e^2 - 2 i \Gamma_e \delta \omega_e} \right]^{-1}
\]

Where

\[
A_1 = \frac{\omega_i^2}{\omega_{pl}^2 - \omega_i^2} \quad A_2 = \frac{\omega_e^2}{\omega_{pl}^2 - \omega_e^2} \quad A_3 = 1 + \frac{n_{ei} e}{M} + \frac{N_{ei} e}{M} \quad A_4 = \frac{\omega_i^2}{4 \Gamma_{pl} \omega_p \omega_e}
\]

\[\delta \omega_e = A_1 \Omega_p \omega_e \quad \delta \omega_p = \omega_{pl}^2 \quad \delta \omega_e = \Omega_p \omega_e \quad \delta \omega_i = -\omega_{pl}^2 + \omega_{pl}^2 \quad \delta \omega_e = -\frac{e}{m_e} E_0 \omega_e^2 - \omega_i^2
\]

Equation (5) characterizes the steady state optical response of the medium and reveals that the total crystal susceptibility is influenced by the equilibrium carrier concentration through \( \Omega_p \neq 0 \) and external magnetostatic field \( \omega_e \neq 0 \).
2.2 Gain coefficient and Threshold. It can be observed from equation (5) that there is an intensity dependent refractive index \([\chi^{(3)}_{\text{eff}}]\) leading to the possibility of a focusing or defocusing effect of the propagating beam and positive dispersive characteristics of the dissipative medium is possible at \(\Omega_p \gg \Gamma_c\). As \(\chi^{(3)}_{\text{eff}}\) becomes more positive, one may expect more effective self-defocusing of the amplified polaron mode. In order to express the possibility of parametric amplification in semiconductor plasma, we employ the relation

\[
\alpha_{\text{eff}} = k \frac{\chi^{(3)}_{\text{eff}}}{2\epsilon_0}
\]

(6)

Here, \(\alpha_{\text{eff}}\) is the effective nonlinear absorption coefficient. The nonlinear growth of the amplified signal is possible only if \(\alpha_{\text{eff}}\) obtainable from equation (6) is positive.

In order to determine the threshold value of the pump amplitude required for the onset of the parametric interaction, we set \(\alpha_{\text{eff}} = 0\) and obtain,

\[
E_{\text{th}} = \frac{m_i \omega_i \phi_i^2 (a_{\phi i}) \int (a_{\phi i}^2 - a_i^2)}{\epsilon_0 k \alpha_{\text{eff}}}
\]

(7)

It is observed from equation (7) that the parametric instability of the signal wave has a nonzero intensity threshold, even in the absence of damping. The threshold field \(E_{\text{th}}\) is found to have complex characteristics and is strongly dependent on the external magnetostatic field.

3. Results and Discussion

The above theory is now applied to study the threshold and amplification characteristics of polaron induced parametric interaction in magnetised semiconductor. The relevant parameters chosen for compound semiconductor InSb are listed in [8].

Figures 1 to 3 are plotted to have qualitative appreciation of threshold and amplification characteristics with respect to wave vector and cyclotron frequency. In figure 1 initially threshold pump amplitude shows a slow declination with increasing \(\omega_c\) till \(\omega_c\) approaches polaron wave frequency, at \(\omega_{b_{\text{pl}}} \approx \omega_{b_c}\), threshold field achieves a minimum value \((\approx 3 \times 10^7 \text{V/m})\). Further increasing \(\omega_c\) results into rise in \(E_{\text{th}}\) but again at \(\omega_c \approx \omega_{b_c}, E_{\text{th}}\) decreases. However this dip corresponds to

![Figure 1](image1.png)

**Figure 1.** Variation of the threshold electric field \(E_{\text{th}}\) with cyclotron frequency \(\omega_c\).

![Figure 2](image2.png)

**Figure 2.** Variation of threshold and absorption coefficient with wave number \(k\).
\[ \omega_c \approx \omega_0 \text{ and we cannot increase the value of } \omega_c \text{ indefinitely otherwise cyclotron absorption has to be taken into consideration. Figure 2 displays the variation of pump amplitude and absorption coefficient } \alpha \text{ Vs wave vector } k \text{ and threshold pump amplitude } E_{th} \text{ Vs cyclotron frequency } \omega_c \text{. It is observed that threshold pump amplitude decreases whereas gain coefficient increases on increasing wave vector. Therefore lower threshold pump field amplitude and higher gain of the polaron mode is obtained at polaron wave number } k > 10^7 \text{ m}^{-1}. \]

The figure 3 displays the parametric absorption coefficient \( \alpha \) as a function of cyclotron frequency \( \omega_c \) with pump electric field \( E_0 \) as a parameter. Here it is worth mentioning that absorption coefficient \( \alpha \) can be both positive and negative under the anomalous regime. Resonance between polaron frequency and cyclotron frequency gives rise to amplification. A slight variation in \( \omega_c \) beyond polaron wave frequency abruptly decreases the absorption coefficient resulting into absorption or loss of the polaron mode. Hence polaron frequency is a critical frequency beyond which amplification cannot be achieved at any of the pump field amplitudes. Application of higher pump field results into less gain at a lower critical magnetic field. Increment in the pump amplitude shifts the critical frequency towards the higher magnitudes of cyclotron frequency.

The magnetic field is found to augment the growth rate of the polaron mode. Considerable reduction in the threshold pump amplitude \( \left(10^3 \text{ to } 10^4 \text{ Vm}^{-1}\right) \) and gain \( \left(10^4 \text{ m}^{-1}\right) \) can be achieved by proper selection of pump amplitude and magnetic field. Hence, compound magnetized semiconductor namely InSb establishes its potential as a candidate material for the fabrication of optical parametric amplifiers.

4. References

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**Figure 3.** Variation of the absorption coefficient \( \alpha \) with cyclotron frequency \( \omega_c \) with the pump electric field \( E_0 \) as a parameter.