Abstract

Differential Evolution (DE) is a population-based stochastic search algorithm for optimization. DE has three main control parameters, Crossover (cr), Mutation factor (F) and Population size (NP). These control parameters play a vital and crucial role in improving the performance of search process in DE. This paper introduces a brief review for control parameters in Differential evolution (DE).

Keywords: Differential evolution; Population size; Global optimization; Control parameters

Abbreviations: DE: Differential Evolution; Cr: Crossover; NP: Population Size; F: Mutation Factor

Introduction

Differential Evolution (DE) is a population-based heuristic algorithm proposed by Storn & Price [1] to solve global optimization problems with different characteristics over continuous space. Despite its simplicity, it proved a great performance in solving non-differentiable, non-continuous and multi-modal optimization problems [2]. DE has three main control parameters which are the crossover (CR), mutation factor (F) and population size (NP). The values of the control parameters affect significantly on the performance of DE. Therefore, the tuning of those control parameters is considered a challenging task. DE has a great performance in exploring the solution space and this is considered as the main advantage, on the other side, an obvious weak point is its poor performance in exploitation phase which may cause a stagnation and/or premature convergence.

The next section introduces differential evolution. Section 3 introduces a short review for control parameters in DE. And finally, the paper is concluded in section 4.

Differential Evolution

In simple DE, DE/rand/1/bin [1,2], an initial population of NP individuals, \( j = 1, 2, ..., NP \), is generated at random according to a uniform distribution within lower and upper boundaries \((L_j, U_j)\). Individuals are evolved by the means of crossover and mutation to generate a trial vector. The trial vector competes with his parent in order to select the fittest to the next generation. The steps of DE are:

\[
\begin{align*}
\text{Initialization of a population} \\
\text{Mutation} \\
\text{Recombination (crossover)}
\end{align*}
\]

Initialization of a population

Initial population in DE, as the starting point for the process of optimization, is created by assigning a random chosen value for each decision variable in every vector, as indicated in equation (1).

\[
x_i^0 = L_j + \text{rand}_j \times (U_j - L_j)
\]

(1)

Where \( L_j, U_j \) the lower and upper boundaries for \( x_j \), \( \text{rand}_j \): a random number uniform \([0, 1]\).

Mutation

A mutant vector \( v_{i+1}^G \) is generated for each target vector \( x_i^G \) at generation \( G \) according to equation (2).

\[
v_{i+1}^G = x_i^G + F \times (x_{r1}^G - x_{r2}^G), \quad r_1 \neq r_2 \neq r_3 \neq i
\]

(2)

Where \( r_1, r_2, r_3 \) are randomly chosen from the population. The mutation factor \( F \in [0,2] \). A new value for the component of mutant vector is generated using (1) if it violates the boundary constraints.

Recombination (crossover)

Crossover is the process of swapping information between the target and the mutated individuals using (3), to yield the trial vector \( x_{i+1}^G \). Two types of crossover can be used, binomial crossover or exponential crossover.

\[
x_{i+1}^G \left\{ \begin{array}{ll}
v_{i+1}^G, \text{rand}(j) \leq \text{CR} \quad \text{or} \quad j = \text{rand}(i), \\
x_i^G, \text{rand}(j) > \text{CR} \quad \text{and} \quad j \neq \text{rand}(i),
\end{array} \right.
\]

(3)
where \( j = 1, 2, ..., D \) and \( \text{rand}(j) \in [0,1] \) is the \( j \)th evaluation of a uniform random number. Crossover rate (CR) is between 0 and 1, \( r \) and \( (i) \) is a random index between 1 and \( D \) to ensure that \( u_{i}^{G+1} \) gets at least one element from \( v_{r}^{G} \); otherwise, the population remains without change.

In the exponential crossover, a starting index \( l \) and a number of components \( w \) are chosen randomly from the ranges \([l, D]\) and \([l, D-l]\) respectively. The values of variables in locations \( l \) to \( l+w \) from \( v_{r}^{G} \) and the remaining locations from the \( X_{i}^{G} \) are used to produce the trial vector \( u_{i}^{G+1} \).

### Selection

Greedy scheme for fast convergence of DE. The child \( u_{i}^{G+1} \) is compared with its parent \( X_{i}^{G} \) to select the better for the next generation according to the selection scheme in equation (4).

\[
X_{i}^{G+1} = \begin{cases} 
    u_{i}^{G+1}, & f(u_{i}^{G+1}) \leq f(X_{i}^{G}) \\
    X_{i}^{G}, & \text{otherwise}
\end{cases}
\]  

A detailed description of standard DE algorithm is given in Figure 1.

#### Short Review

During the last two decades, the problem of finding the balance between the exploration and exploitation has attracted many researchers in order to improve the performance of DE by developing new mutation strategies or hybridizing promising mutation strategies.

Das et al. [3] proposed an improved variant of DE/target-to-best/1/bin based on the concept of population members’ neighborhood. Zhang & Sanderson [4] proposed a new mutation strategy “DE/current-to-pbest” with an optional external archive that utilizes the historical data in order to progress towards the promising direction and called it JADE. Qin, Huang & Suganthan [5] proposed SaDE, in which a self-adaptive mechanism for trial vector generation is presented, that is based on the idea of learning from the past experience in generating promising solutions. Mohamed et al. [6-8] proposed a novel mutation strategy which is based on the weighted difference between the best and the worst individual during a specific generation, the new mutation strategy is combined with the basic mutation DE/rand/1/bin with equal probability for selecting each of them. Li & Yin [9] used two mutation strategies based on the best and random vectors. Mohamed [10] proposed IDE, in which a new triangular mutation rule that selects three random vectors and adding the difference vector between the best and worst to the better vector. The new mutation rule is combined with the basic mutation rule through a non-linear decreasing probability rule. And a restart mechanism to avoid the premature convergence is presented. Recently,
triangular mutation has been also used to solve IEEE CEC 2013 unconstrained problems [11], constrained non-linear integer and mixed-integer global optimization problems [12], IEEE CEC2006 constrained optimization problems [13], CEC 2010 large-scale optimization problems [14], and stochastic programming problems [15].

Extensive research was presented for controlling the parameters, as control parameters play a vital role in the evolution process. Brest et al. [16] presented a new self-adaptive technique for controlling the parameters. Noman & Iba [17] proposed an adaptive crossover based on local search and the length of the search was adjusted using hill-climbing. Peng et al. [18] proposed rJADE, in which a weighting strategy is added to JADE, with a “restart with knowledge transfer” method in order to benefit from the knowledge obtained from the previous failure. Montgomery & Chen [19] presented a complete analysis of how much the evolution process affected by the value of CR. Mallipeddi et al. [20] proposed a pool of values for each control parameter to select the appropriate value during the evolution process. Wang, Cai & Zhang [21] proposed a new method that randomly chooses from a pool that contains three strategies in order to generate the trial vector and three control parameter settings, they called it CoDE. Yong et al. [22] presented CoBiDE, in which a covariance matrix learning for the crossover operator and a bimodal distribution parameter to control the parameters are introduced. Draa, Bouzoubia & Boukhalfa [23] introduced a new sinusoidal formula in order to adjust the values of crossover and the scaling factor, they called it SinDE. A complete review could be found in [24,25].

DE mechanism depends on selecting three random individuals from the population to perform the mutation process. Therefore, the population size must be greater than the selected vectors. Large population size increases the diversity but consumes more resources (function calls), while small population size may cause stagnation or tripping in local optima. Thus, the choosing of the population size is considered a very critical aspect. From the literature, it has been found that researchers choose the population size in four different ways.

i. Choosing the population size for each problem separately based on the experience or previous knowledge and keep it constant during all runs [26,27].

ii. Relate the population size to the problem dimensionality [6,28,29].

iii. Setting the population size fixed during all runs and independent of the dimension of the problems [22,30].

iv. Allowing the population size to vary during the runs using adaptation rule [31-33]. A complete review of population size could be found in [34,35].

Conclusion

Control parameters plays a vital role in the evolution process of the DE. Over the last decades, many EAs have been proposed to solve optimization problems. However, all these algorithms including DE have the same shortcomings in solving optimization problems. One of them is the choice of the control parameters which are difficult to adjust for different problems with different characteristics. This paper introduced a brief review for a considerable number of research studies that have been proposed to enhance the performance of DE.

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