CONSTRATNOS ON $Z_1 - Z_2$ MIXING FROM THE DECAY $Z_1 \rightarrow e^-e^+$ IN THE LEFT-RIGHT SYMMETRIC MODEL

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Abstract

We examine the decay of $Z_1$ in electrons with recent data from LEP. The partial width $\Gamma(Z_1 \rightarrow e^-e^+)$ is studied in the framework of a left-right symmetric model with standard electroweak corrections. Processes measured near the resonance has served to measure the neutral coupling constants very precisely, which is useful to set bounds on the parameters of the model. This partial decay occurs in the resonance zone. As a consequence the process is independent of the mass of the additional $Z_2$ heavy gauge boson which appears in this kind of models and so we have the mixing angle $\phi$ between the left and the right bosons as the only additional parameter. In this paper we take advantage of this fact to set a bound for $\phi$: $-9 \times 10^{-3} \leq \phi \leq 4 \times 10^{-3}$, which is in agreement with other constraints previously reported.

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1 Introduction

The Standard Model (SM) [1] of the electroweak interaction between the fermions has resisted all tests within the limits of the experimental errors. On the other hand there are several questions for which the SM have not answer. One of these is the origin of the parity violation at the present energies. The Left-Right symmetric models (LR) based on the $SU(2)_R \times SU(2)_L \times U(1)$ group [2] give an answer to that problem, since restore the parity symmetry at high energies and give their violation at low energies as a result of the breaking of gauge symmetry. At present the experiments are finest and we have excellent measures of neutral processes so we can put better restrictions to the parameters of the LR model. In the present work we consider the partial decay width for the neutral boson in an electron-positron pair, as is measured at LEP, in the framework of the LR model with two neutral bosons: $Z_1$ which is predominantly left and $Z_2$ which is predominantly right and heavier. The partial width $\Gamma(Z_1 \rightarrow e^- e^+)$ can be written as a function of the mixing angle $\phi$ between $W^3_L$, $W^3_R$ and $B$ gauge bosons of the model to give the physical bosons $Z_1$, $Z_2$ and the photon, being $\phi$ the only extra parameter besides the SM parameters. We use the recent LEP results [3] for the neutral coupling constant $g_A$ for constraining the mixing angle $\phi$ and found that $g_A$ is a good place to look for constraints on new physics. The fact that the decay $Z_1 \rightarrow e^- e^+$ occurs at the energies near the resonance of
the $Z_1$ gives a good place for looking for new physics.

In the Sec. 2 we describe the model with the Higgs sector having two doublets and one bidoublet and we find the masses of the physical bosons. In Sec. 3 we calculate the decay rate $\Gamma(Z_1 \to e^-e^+)$ including radiative corrections and using the LEP data we find the constraint for $\phi$ and in Sec. 4 we summarizes the results.

2 The LR model

We consider a LR model having one bidoublet $\Phi$ and two doublets $\chi_L, \chi_R$ whose vacuum expectation values break the gauge symmetry to give a mass to the right gauge bosons heavier than the mass of the left ones. This is the origin of the parity violation at low energies [4], that is, at energies available at actual accelerators and reactors. The lagrangian for the Higgs sector of the model is given by [5]

$$\mathcal{L}_{LR} = (D_\mu \chi_L)^\dagger (D^\mu \chi_L) + (D_\mu \chi_R)^\dagger (D^\mu \chi_R) + Tr(D_\mu \Phi)^\dagger (D^\mu \Phi). \quad (1)$$

In this lagrangian appears the covariant derivatives

$$D_\mu \chi_L = \partial_\mu \chi_L - \frac{1}{2} i g \vec{\tau} \cdot \vec{W}_L \chi_L - \frac{1}{2} i g' B \chi_L, \quad (2)$$

$$D_\mu \chi_R = \partial_\mu \chi_R - \frac{1}{2} i g \vec{\tau} \cdot \vec{W}_R \chi_R - \frac{1}{2} i g' B \chi_R,$$

$$D_\mu \Phi = \partial_\mu \Phi - \frac{1}{2} i g (\vec{\tau} \cdot \vec{W}_L \Phi - \Phi \vec{\tau} \cdot \vec{W}_R).$$
Then in this model there are seven gauge bosons: $W_{L,R}^1$ and $W_{L,R}^2$ that are charged and $W_{L,R}^3$ and $B$ that are neutral. The coupling constants for left and right sector are equal: $g_L = g_R$, since we assume manifiest left-right symmetry [6].

When we introduce the vacuum expectation values of the multiplets of Higgs, i. e.

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}, \quad \langle \chi_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle \chi_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_R \end{pmatrix},$$

(3)
in the lagrangian (1), the interaction bosons get their masses. The part of the lagrangian that contains the mass terms for the charged bosons is

$$L_{\text{mass}}^C = \left( W_L^+ \ W_R^+ \right) M_C^C \left( W_L^- \ W_R^- \right),$$

(4)

where $W^\pm$ are the linear combinations

$$W^\pm = \frac{1}{\sqrt{2}} (W^1 \mp W^2).$$

The mass matrix $M_C^C$ is

$$M_C^C = \frac{g^2}{4} \begin{pmatrix} v_L^2 + k^2 + k'^2 & -2kk' \\ -2kk' & v_R^2 + k^2 + k'^2 \end{pmatrix}.$$ 

(5)

Since the process $Z \rightarrow e^-e^+$ is neutral, we fix our attention to the mass lagrangian for the neutral sector:

$$L_{\text{mass}}^N = \frac{1}{8} \left( W_L^3 \ W_R^3 \ B \right) M_N^N \left( W_L^3 \ W_R^3 \ B \right),$$

(6)
where the mass matrix is given by

$$M^N = \frac{1}{4} \begin{pmatrix} g^2(v_L^2 + k^2 + k'^2) & -g^2(k^2 + k'^2) & -gg'v_L^2 \\ -g^2(k^2 + k'^2) & g^2(v_R^2 + k^2 + k'^2) & -gg'v_R^2 \\ -gg'v_L^2 & -gg'v_R^2 & g'^2(v_L^2 + v_R^2) \end{pmatrix}. \tag{7}$$

The mass matrices (5) and (7) are diagonalized by orthogonal transformations. The charged mass matrix (5) is diagonalized with a rotation which is parametrized \[6\] by an angle $\zeta$ which is severely restringed \[7\]. The matrix that diagonalize the neutral mass matrix $M^N$ is \[8\]

$$U^N = \begin{pmatrix} c_Wc_\phi & -swt_Wc_\phi - rwsc_\phi/c_W & t_W(s_\phi - rwc_\phi) \\ c_ws_\phi & -swt_ws_\phi + rwc_\phi/c_W & -t_W(c_\phi + rwc_\phi) \\ sw & sw & r_W \end{pmatrix}, \tag{8}$$

with the definitions $c_W = \cos \theta_W$, $s_W = \sin \theta_W$, $t_W = \tan \theta_W$ and $r_W = \sqrt{\cos 2\theta_W}$, where $\theta_W$ is the electroweak mixing angle. Also $c_\phi = \cos \phi$ and $s_\phi = \sin \phi$. Here $\phi$ can be considered as the angle that mix the left and right handed neutral gauge bosons $W^3_{L,R}$ respectively, and $B$ to give the physical bosons $Z_1, Z_2$ and the photon:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix} = U^N \begin{pmatrix} W^3_L \\ W^3_R \\ B \end{pmatrix}. \tag{9}$$

The diagonalization of (5) and (7) gives the mass of the charged $W_{1,2}^\pm$ and neutral $Z_{1,2}$ physical fields:

$$M^2_{W_{1,2}} = \frac{g^2}{8} \left[ v_L^2 + v_R^2 + 2(k^2 + k'^2) \mp \sqrt{(v_R^2 - v_L^2)^2 + 16(kk')^2} \right], \tag{10}$$

$$M^2_{Z_{1,2}} = B \mp \sqrt{B^2 - 4C}, \tag{11}$$
respectively, with

\[ B = \frac{1}{8} [(g^2 + g'^2)(v_L^2 + v_R^2) + 2g^2(k^2 + k'^2)], \]

\[ C = \frac{1}{64} g^2 (g^2 + 2g'^2)[v_L^2 v_R^2 + (k^2 + k'^2)(v_L^2 + v_R^2)]. \]

Taking into account that \( M_{W_2}^2 \gg M_{W_1}^2 \), from the expressions for the masses of \( M_{Z_1} \) and \( M_{Z_2} \) we conclude that the relation \( M_{W_1}^2 = M_{Z_1}^2 \cos^2 \theta_W \) still holds in this model.

To compare with experimental results [9], we introduce here a parametrization for matrix (8) used frequently [10, 11]. In this parametrization the mixing angle \( \theta_M \) is obtained as follows: first, the gauge fields \( W_{3L}, W_{3R} \) and \( B \) are transformed to interaction fields \( Z_L, Z_R \) and \( A \). The field \( Z_R \) does not couple with left-handed currents whereas the photon \( A \) interact only with the electromagnetic current. With these conditions, the relation between both sets of intermediate bosons is

\[
\begin{pmatrix}
Z_L \\
Z_R \\
A
\end{pmatrix} = U
\begin{pmatrix}
W_{3L}^3 \\
W_{3R}^3 \\
B
\end{pmatrix},
\]  

(12)

where

\[
U = \begin{pmatrix}
c_W & -s_W t_W & -t_W r_W \\
0 & r_W/c_W & -t_W \\
s_W & s_W & r_W
\end{pmatrix}.
\]  

(13)

Second, the interacting fields \( Z_L, Z_R \) and \( A \) are transformed to mass eigenstates \( Z_1, Z_2 \) and \( A \). The photon do not mix at this stage. The trans-
formation is realized with a matrix $U'$

$$
\begin{pmatrix}
Z_1 \\
Z_2 \\
A
\end{pmatrix}
= U'
\begin{pmatrix}
Z_L \\
Z_R \\
A
\end{pmatrix}.
\tag{14}
$$

The matrix $U'$ is a rotation that leave $A$ invariant:

$$
U' = 
\begin{pmatrix}
\cos \theta_M & \sin \theta_M & 0 \\
-\sin \theta_M & \cos \theta_M & 0 \\
0 & 0 & 1
\end{pmatrix},
\tag{15}
$$

then the complete transformation is

$$
\begin{pmatrix}
Z_1 \\
Z_2 \\
A
\end{pmatrix}
= U'U
\begin{pmatrix}
W_{3L} \\
W_{3R} \\
B
\end{pmatrix}.
\tag{16}
$$

We see that the matrix (8) is related to $U'U$ by

$$
U^N = U'U,
\tag{17}
$$

if we set $\phi = -\theta_M$.

3 The decay $Z_1 \rightarrow e^-e^+$

From the general lagrangian of the LR model we extract the terms for the neutral interaction of a fermion with the gauge bosons $W_{3L,R}$ and $B$:

$$
\mathcal{L}_{int}^N = g (J^3_L W^3_L + J^3_R W^3_R) + \frac{g'}{2} J_Y B.
\tag{18}
$$

Inverting the Eq. (9) for the fields $W^3_L, W^3_R$ and $B$ and inserting in (12) we find for $Z_1 \rightarrow e^-e^+$ [12]:

$$
\mathcal{L}_{int}^N = \frac{g}{c_W} Z_1 \left[ \left( c_\phi - \frac{s^2_W}{r_W s_\phi} \right) J_L - \frac{c^2_W}{r_W s_\phi} J_R \right],
\tag{19}
$$
where the left (right) current for the electrons are

\[ J_{L,R} = J_{L,R}^3 - \sin^2 \theta_W J_{em}, \]

and

\[ J_{em} = J_{L}^3 + J_{R}^3 + \frac{1}{2} J_Y, \]

is the electromagnetic current. From Eq. (19) we can find the amplitude \( M \) for the decay of the \( Z_1 \) boson with polarization \( e^\lambda \) into an electron-positron pair:

\[ M = \frac{g}{c_W} \left[ \bar{u} \gamma^\mu \frac{1}{2} (\alpha g_V - \beta g_A \gamma_5)v \right] e^\lambda_\mu, \quad (20) \]

with

\[ \alpha = c_\phi - \frac{1}{r_W} s_\phi, \quad (21) \]

\[ \beta = c_\phi + r_W s_\phi. \quad (22) \]

If we consider radiative corrections for the standard model then we will have

\[ M = \frac{g}{c_W} \left[ \bar{u} \gamma^\mu (g_{VLR} - g_{ALR} \gamma_5)v \right] e^\lambda_\mu, \quad (23) \]

with

\[ g_{VLR} = \left[ c_\phi - \frac{s_W^2}{r_W} s_\phi \right] \bar{g}_V + \frac{e^2_W}{r_W} s_\phi g_{VR}, \quad (24) \]

\[ g_{ALR} = \left[ c_\phi - \frac{s_W^2}{r_W} s_\phi \right] \bar{g}_A + \frac{e^2_W}{r_W} s_\phi g_{AR}, \quad (25) \]

here \( \bar{g}_V \) (\( \bar{g}_A \)) is the value for \( g_V \) (\( g_A \)), but including radiative corrections whereas \( g_{VR} \) (\( g_{AR} \)) is the value for \( g_V \) (\( g_A \)) but free of radiative corrections.
This is because in this kind of models only standard model radiative corrections are taking into account [11].

As we can see, in Eq. (23) we have made definitions for the vector and axial-vector constants as effective coupling constants in the LR model. The plots for $g_{v_{LR}}$ and $g_{A_{LR}}$ (Eqs. (24) and (25) respectively) are shown in Figures 1 and 2 as functions of the left-right mixing angle $\phi$ and the electroweak mixing angle $s_{W}^{2}$. We can see how $g_{v_{LR}}$ has a stronger dependence on $s_{W}^{2}$ than on $\phi$, while $g_{A_{LR}}$ presents the opposite situation, a stronger dependence on $\phi$ than on $s_{W}^{2}$. This is not a surprise because in the SM at tree level $g_{A}$ is independent on $s_{W}^{2}$, so the only dependence on this angle in $g_{A}$ is through radiative corrections and through the LR correction, meanwhile in $g_{V}$ the $s_{W}^{2}$ dependence is presented already at tree level what makes it more important.

It has been noted [12] that $g_{A}$ is a good place to looking for deviations from the standard model at low energies. As we can see from discussion above and from Figs. 1 and 2 this is also valid for high energy experiments.

In Fig. 3 we have plotted $g_{A_{LR}}$ as a function of $\phi$, where we put $s_{W}^{2} = 0.2247$ which is a result which comes from the $M_{Z}$ measure in the On-Shell scheme [13]. The values for $\bar{g}_{V}$ and $\bar{g}_{A}$ are given by [13]

$$\bar{g}_{V} = \sqrt{\rho_{f}} \left( -\frac{1}{2} + 2\kappa_{f} \sin^{2} \theta_{W} \right), \quad (26)$$

$$\bar{g}_{A} = \sqrt{\rho_{f}} \left( -\frac{1}{2} \right), \quad (27)$$
with $\rho_f = 1.0031$ and $\kappa_f = 1 + 0.0031/\tan^2 \theta_W$. The values for $g_{V_R}$ and $g_{A_R}$ are those with $\rho_f = \kappa_f = 1$. The horizontal lines in the plot give us the experimental region of Ref. [3]:

$$g_{A_{\text{exp}}}^{\text{exp}} = -0.4998 \pm 0.0014,$$

with a 90% C. L. With this experimental data from LEP for $g_A$ we found for the mixing angle between $Z_1$ and $Z_2$ the constraint

$$-9 \times 10^{-3} \leq \phi \leq 4 \times 10^{-3},$$

(28)

with a 90% C. L.. This limit is in good agreement with theoretical results [14] previously reported. In our computation the advantage is that the fit is independent on the mass of the $Z_2$ heavy gauge boson because we are in the $Z_1$ resonance zone. Our result also agrees with the experimental estimation of Ref. [9], if we take the angle $\theta_M$ of that reference as the negative of $\phi$ as was explained in Sec. 2.

4 Summary

As a conclusion we can say that $g_A$ is a good place for looking for constraints on the mixing angle $\phi$ and in general for new physics because it has not a strong dependence on the electroweak mixing angle $\sin^2 \theta_W$ which has different values depending on the experiment and on the renormalization scheme [13]. Besides, studing $g_A$ in the resonance zone has the extra advantage that
the mass of an extra neutral heavy boson does not appear in the computation, leaving only an extra parameter in the case of the LR model and in any other model with only one additional neutral gauge boson as in the case, for example, of the $SU(2)_L \times U(1) \times U(1)$ coming from $E_6$ models. Further, this computation has the virtue that is necessary only one experimental quantity, that is, the axial coupling constant $g_A$.

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Figure Captions

Fig. 1 Plot of $g_{vLR}$ as a function of $s_{W}^{2}$ and $\phi$. $g_{vLR}$ is highly dependent on $s_{W}^{2}$.

Fig. 2 Plot of $g_{ALR}$ as a function of $s_{W}^{2}$ and $\phi$. $g_{ALR}$ is almost independent on $s_{W}^{2}$ compared with the dependence on $\phi$.

Fig. 3 Plot of $g_{ALR}$ as a function of the LR parameter $\phi$ with the value $s_{W}^{2} = 0.2247$. The horizontal lines give the experimental region with a 90 % C. L.