Disappearance of the Abrikosov vortex above the deconfining phase transition in SU(2) lattice gauge theory

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We calculate the solenoidal magnetic monopole current and electric flux distributions at finite temperature in the presence of a static quark-antiquark pair. The simulation was performed using SU(2) lattice gauge theory in the maximal Abelian gauge. We find that the monopole current and electric flux distributions are quite different below and above the finite temperature deconfining phase transition point and agree with predictions of the Ginzburg-Landau theory.

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I. INTRODUCTION

It is widely accepted that quark confinement is due to color-electric flux tube formation. In this scenario the QCD vacuum behaves like a dual superconductor. That is, magnetic monopoles in the QCD vacuum respond to a color-electric field by generating screening currents to confine the electric flux into a narrow tube dual to the Abrikosov vortex arising from the Meissner effect. Some direct evidence for this scenario has been reported for $U(1)$ LGT and SU(N) lattice gauge theories (LGT) simulations [1–5]. It is found that monopoles in $U(1)$ and the $U(1)^{N-1}$ abelian projection of SU(N) LGT in the maximal Abelian gauge [6] react to the electric flux from a static $q \bar{q}$ pair by producing a solenoidal current distribution which confines the flux to a narrow tube.

It is clear that the solenoidal magnetic monopole currents are associated with the confined flux tube. At finite temperatures the flux tube is expected to exist only in the low-temperature confined phase, but not in the high-temperature unconfined phase. Previous studies [1–5] addressed the issues of monopole current distributions and the structures of the electric flux tube at zero temperature. Questions remain as to the monopole current distributions at finite temperature, especially in the unconfined phase. In this paper we shall present our observations of monopole current and electric flux distributions at finite temperature. This study is a direct extension of the earlier work [2,3] in SU(2) LGT.

II. SIMULATIONS

Our simulations were performed on a lattice of the size $4 \times 17^2 \times 19$ with skew-periodic boundary conditions. Each link from point $s$ in the $\mu$ direction carries an $SU(2)$ element $U(s, \mu)$, and the plaquette operator $U_P(s)$ is formed in the usual fashion as a directed product of link variables, where the label $P$ represents the orientation of the plaquette, $(\mu, \nu)$. The standard Wilson action [7] is used in our calculations

$$S(U) = \beta \sum_{s, P} (1 - \frac{1}{2} \text{Tr} U_P).$$ (1)
We generated lattice configurations with the distribution \( \exp(-S) \) using a combination of the standard Metropolis algorithm \cite{8} and overrelaxation \cite{9}. Simulations were performed for \( \beta = 2.25, 2.28 \) and 2.40. In each case the initial 3000 sweeps were used to thermalize the system, then one measurement was made every 50 sweeps.

Each measurement was performed on a lattice configuration in the maximal Abelian gauge \cite{6}, which can be achieved by finding the gauge transformation that maximized the quantity \cite{10}

\[
R = \sum_{s,\mu} \text{Tr}[\sigma_3 U(s, \mu) \sigma_3 U^\dagger(s, \mu)].
\]

This is equivalent to diagonalizing

\[
X(s) = \sum_{\mu} [U(s, \mu) \sigma_3 U^\dagger(s, \mu) + U^\dagger(s - \mu, \mu) \sigma_3 U(s - \mu, \mu)].
\]

at each site \( s \). To measure the efficacy of gauge fixing in the simulations we used the lattice sum of the magnitude of the off-diagonal component of \( X(s) \), \( |Z|^2 = \sum_s |X(s)_{12}|^2 \). Typically we needed about 620 gauge fixing sweeps to attain \( |Z|^2 \approx 10^{-5}/\text{site} \) for each \( \beta \) value. Three gauge fixing methods were used: (1) generating and accepting random local changes only if \( R \) increased, (2) locally maximizing \( R \) exactly at alternate sites, (3) applying overrelaxation using the square of the gauge transformation of method (2) to sample configurations better.

**III. MEASUREMENTS**

After gauge fixing, the Abelian \( U(1) \) link variable is given \cite{10} by the phase of the diagonal component of the \( SU(2) \) link variable, \( u(s, \mu) = U(s, \mu)_{11}/|U(s, \mu)_{11}| \). We construct the Abelian plaquette variable \( p_{\mu\nu}(s) \) from \( u(s, \mu) \) in the usual fashion. In this study we use Abelian Polyakov loops to replace Abelian Wilson loop used in ref. \cite{2,3}, in order to study the finite temperature case. The Abelian Polyakov loop closed in the time direction can be constructed as

\[
P_{ab}(s) = \prod_{\tau=1}^{N_t} u(s, \tau),
\]

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where $N_t$ is the temporal size of the lattice, $N_t \times N_s^2 \times N_z$, specifically $N_t = 4$ for the lattice $4 \times 17^2 \times 19$.

The magnetic monopoles are identified using the DeGrand-Toussaint construction \[11\]. For each Abelian plaquette $p_{\mu\nu}(s)$ we decompose its phase angle $\theta_p = \arg(p_{\mu\nu}(s)) \ (-4\pi \leq \theta_p \leq 4\pi)$ into two parts,

$$\theta_p = \tilde{\theta}_p + 2\pi n_p, \quad (5)$$

with $\pi < \tilde{\theta}_p < \pi$, where $n_p$ describes the number of Dirac strings and $\tilde{\theta}_p$ the electromagnetic flux through the plaquette \[11\]. The magnetic monopole current density can be constructed as \[12\],

$$K_\mu(s) = \varepsilon_{\mu\nu\rho\sigma}[\tilde{\theta}_p(s + \nu, \rho\sigma) - \tilde{\theta}_p(s, \rho\sigma)]/4\pi. \quad (6)$$

It is convenient to associate the monopole current density $K_\mu(s)$ in each three-volume with a link on the dual lattice \[2\], making world lines that define a conserved current density $J_M$. To isolate the solenoidal monopole currents we construct the operator for the line integral of $J_M$ around a dual plaquette, $\nabla \times J_M$, as described in ref. \[2\].

In our measurements we chose two Abelian Polyakov loops correlated along the $N_z$ direction, $\langle P_{ab}(0)P_{ab}^\dagger(R) \rangle$, with the separation $R = 3a$, $4a$, $5a$ and $6a$, which represent the static $q\bar{q}$ pair. The electric field operator is given by $a^2 \mathcal{E}_i = \text{Im}p_{zi}$, where $a$ is the lattice spacing \[2\]. Then the electric field in the $N_z$ direction can be calculated as,

$$\langle E_z(R, x) \rangle = \frac{\langle P_{ab}(0)P_{ab}^\dagger(R)\text{Im}(p_{zt}(x)) \rangle}{a^2 e \langle P_{ab}(0)P_{ab}^\dagger(R) \rangle}, \quad (7)$$

with $e$ the Abelian electric charge and $p_{zt}$ the Abelian plaquette in the $z-t$ plane. The solenoidal monopole current distribution can be calculated from the correlation \[2\],

$$\langle \nabla \times J_M(R, x) \rangle_z = \frac{2\pi \langle P_{ab}(0)P_{ab}^\dagger(R)(\nabla \times J_M(x))_z \rangle}{a^4 e \langle P_{ab}(0)P_{ab}^\dagger(R) \rangle}, \quad (8)$$

where the solenoidal monopole current operator, $(\nabla \times J_M)_z$, is constructed as the dual plaquette in the $x-y$ plane. In practice, the expressions for $\langle E_z(R, x) \rangle$ and $\langle \nabla \times J_M(R, x) \rangle_z$
can be simplified further. If we denote $\theta_{PP^{\dagger}}$ as the phase angle of the product of two Abelian Polyakov loops, $(P_{ab}(0)P_{ab}^{\dagger}(R))$, Eq. (6) can be written as,

$$\langle E_z(R, x) \rangle = \frac{\langle \sin\theta_{PP^{\dagger}}\sin\theta_p \rangle}{a^2 e\langle \cos\theta_{PP^{\dagger}} \rangle},$$

(9)

where $\theta_p$ is the phase angle of the Abelian plaquette $p_{zt}$. Also, Eq. (8) becomes

$$\langle \nabla \times J_M(R, x) \rangle_z = \frac{2\pi \langle \sin\theta_{PP^{\dagger}}(\nabla \times J_M(x))_z \rangle}{a^4 e\langle \cos\theta_{PP^{\dagger}} \rangle}.$$

(10)

IV. NUMERICAL RESULTS

As we choose the lattice extent $N_t$ of the 4-dimensional lattice, $N_t \times N_s^2 \times N_z$, as the time direction, we can simulate the finite temperature case on the lattice with the temperature $T = 1/N_t a(\beta)$, where the lattice spacing $a$ is a function of $\beta$. For SU(2) lattice gauge theory the finite temperature deconfining phase transition point has been studied extensively. One recent simulation result [13] is, for $N_t = 4$ the transition point $\beta_c$ is given as,

$$\beta_c = 2.2986 \pm 0.0006.$$

(11)

Below $\beta_c$ ($\beta < \beta_c$) the system is in the confined phase, but above $\beta_c$ it is in the unconfined phase.

We measured the electric flux $\langle E_z(R, x) \rangle$ and the solenoidal monopole current $\langle \nabla \times J_M(R, x) \rangle_z$ on the transverse plane midway between the $q\bar{q}$ pair for several $\beta$ values, $\beta = 2.25, 2.28$ and 2.40. Two $\beta$ values (2.25 and 2.28) are below the transition point $\beta_c$ given by Eq. (11), one $\beta$ value (2.40) is above the $\beta_c$. In the following we proceed to discuss our results in the two cases, which correspond to the confined and the unconfined phases separately.

A. Results in the confined phase

We measured the electric flux and the solenoidal monopole current distributions of a $q\bar{q}$ pair in the confined phase: $\beta = 2.25$ and 2.28, In each case we accumulated about 800 measurements.
Fig. 1 shows the electric field $E_z$ distribution for $\beta = 2.28$, with the $q\bar{q}$ separation $R = 3a, 4a, 5a$ and $6a$ separately. This displays the strength of $E_z$ versus the distance $r$ from the $q\bar{q}$ axis on the transverse plane midway between the $q\bar{q}$ pair. For small quark separations, e.g. $R = 3a$ and $4a$, as shown in Fig. 1(a) and (b), our results are consistent with those of the earlier works, which were obtained from simulations using Abelian Wilson loops of sizes $3 \times 3$ in ref. 2 and $3 \times 3$ and $5 \times 5$ in ref. 3. In this study we observe clear signals for $E_z$ at $R = 5a, 6a$ as shown in Fig. 1(c) and (d). From Fig. 1 one can see that the electric flux $E_z$ has the peak value on the $q\bar{q}$ axis ($r = 0$), then it decreases with the off-axis distance $r$ rapidly. As the $q\bar{q}$ separation $R$ increases the $E_z$ values also decrease as we expected. Our data also shows that the electric flux $E_z$ basically vanishes within errors at the off-axis distance $r \approx 5a$ for all $R$ values in Fig. 1(a)-(d). This agrees with the expectation that electric flux is squeezed into a narrow tube in the confined phase.

In Fig. 2 we show the monopole current $-(\nabla \times J_M)_z$ as a function of the off-axis distance $r$ for $\beta = 2.28$, with $R = 3a$ and $4a$. Again our data shows that the curl of the monopole current $-(\nabla \times J_M)_z$ has the same functional form as those of the earlier works for SU(2) LGT [2, 3]. Fig. 2 only displays the results at small quark separations, e.g. $R = 3a$ and $4a$ since for large quark separations, e.g. $R = 5a$ and $6a$ the noise is too large. The large signals shown in Fig. 2 confirm that the solenoidal monopole current surrounding the electric flux tube is large in the finite temperature confined phase. For $\beta = 2.25$ the results are similar to the case of $\beta = 2.28$.

To clarify the significance of these profiles, we wish to summarize briefly the results of ref. 1,2. Picture for the moment the points in Fig. 2 at $r/a = 1, 1.4$ to be negative and further that the curve matches, up to a factor, the negative of the electric field profile for $r/a \neq 0$. This is precisely what we found for U(1), ref. 1, and is the prediction of the London theory. The linear combination of $E_z - \lambda^2(\nabla \times J_M)_z$ is defined as the fluxoid, where $\lambda$ is the London penetration depth. For this geometry the fluxoid gives a lattice delta function at the origin, with the coefficient equal to the total electric flux in the vortex.

The departure of Fig. 2 from this picture is fully explained by the more general Ginzburg-
Landau theory in which the order parameter for superconductivity turns on at a boundary over a finite distance (called the coherence length) rather than abruptly at the boundary as in the above case. The coherence length in the Fig. 2 is the distance at which the profile deviates from the London theory which is \( r/a = \approx 2.0 \). The London penetration depth is independently determined by the exponential fall-off of \( E_z \).

**B. Results in the unconfined phase**

We also studied the electric flux and monopole current distributions in the unconfined phase. The data were measured at \( \beta = 2.40 \), and 410 measurements were accumulated.

Fig. 3 shows the electric flux \( E_z \) as a function of the off-axis distance \( r \) for \( \beta = 2.40 \), with the \( q\bar{q} \) separation \( R = 3a, 4a, 5a \) and \( 6a \) separately. The significant difference between Fig. 3 and Fig. 1 is that the \( E_z \) flux values for \( \beta = 2.40 \) approach zero very slowly, even at large off-axis distances (e.g. \( r = 6a \) ) \( E_z \) flux values still do not vanish within errors, as shown clearly in Fig. 3(c) and (d). However, Fig. 1 shows that the \( E_z \) flux data for \( \beta = 2.28 \) decrease very quickly and vanish at \( r \approx 5a \) within errors. This implies that the electric flux spreads out from the \( q\bar{q} \) axis in the unconfined phase (\( \beta = 2.40 \)), it is more Coulomb-like than the flux distribution in the confined phase (e.g. \( \beta = 2.28 \)).

Fig. 4 shows the corresponding monopole current distributions \( - (\nabla \times J_M)_z \) for \( \beta = 2.40 \), with \( R = 3a \) and \( 4a \). From this figure one can see that almost all data vanish within errors except one small value on the \( q\bar{q} \) axis \( (r = 0) \) for each case. Comparing with the monopole current distributions in the confined phase \( (\beta = 2.28) \), as shown in Fig. 2 it is clear that the solenoidal monopole current \( - (\nabla \times J_M)_z \) becomes very small in the unconfined phase, as expected.
V. CONCLUSIONS

We extend the earlier studies \cite{2,5} on solenoidal monopole current and electric flux distributions to the finite temperature case. We find that both the electric flux and the monopole current have quite different behaviors in the finite temperature confined and unconfined phases. In the confined phase our data are consistent with the results of earlier works \cite{2,5}, which correspond to the zero-temperature case. We find that in the confined phase the electric flux is confined in a narrow tube, and the solenoidal monopole currents are large. However, in the unconfined phase our data show clearly that the electric flux is not confined, and the solenoidal monopole current is vanishingly small. This presents the significant difference between the two phases. This study confirms the point that the electric flux is repelled by the surrounding solenoidal monopole currents, and is squeezed into a narrow tube. In the unconfined phase solenoidal monopole current vanishes, then the electric flux is unconfined.

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REFERENCES

[1] V. Singh, D.A. Browne and R.W. Haymaker, Phys. Rev. D 47, 1715 (1993).
[2] V. Singh, D.A. Browne and R.W. Haymaker, Phys. Lett. 306 B, 115 (1993).
[3] V. Singh, D.A. Browne and R.W. Haymaker, Nucl. Phys B (Proc. Suppl.) 30, 568 (1993).
[4] R. W. Haymaker, V. Singh, D.A. Browne and J. Wosiek, Proc. Workshop on QCD Vacuum Structure, American University, Paris, World Scientific, 1993
[5] Y. Matsubara, S. Ejiri and T. Suzuki, Nucl. Phys. B(Proc. Suppl.) 34, 176 (1994).
[6] G. ’t Hooft, Nucl. Phys. B190 [FS3], 455 (1981).
[7] M. Creutz, Quarks, Gluons and Lattices, Cambridge University Press, 1983.
[8] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller and E. Teller, J. Chem. Phys. 21, 1087(1953).
[9] M. Creutz, Phys. Rev. D 36, 515 (1987).
[10] A.S. Kronfeld, G. Schierholz and U.J. Wiese, Phys. Lett. 198 B, 516 (1987); A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.J. Wiese, Nucl. Phys. B293, 461 (1987).
[11] T.A. DeGrand and D. Toussaint, Phys. Rev. D 22, 2478 (1980).
[12] S. Hioki, et. al., Phys. Lett. 272 B, 326 (1991); V.G. Bornyakov, et. al., Phys. Lett. 261 B, 116 (1991).
[13] J. Engels, J. Fingberg and D.E. Miller, Nucl. Phys. B387, 501 (1992). J. Engels, J. Fingberg and M. Weber, Nucl. Phys. B332, 737 (1990). J. Engels and V.K. Mitrjushkin, Phys. Lett. 282 B, 415 (1992).
FIGURES

FIG. 1. Profile of the electric field $E_z$ as a function of the transverse distance $r$ from the $q\bar{q}$ axis, with the $q\bar{q}$ separation (a). $R = 3a$, (b) $R = 4a$, (c) $R = 5a$ and (d) $R = 6a$. The data were measured on the lattice $4 \times 17^2 \times 19$ with $\beta = 2.28$. The values of $E_z$ are measured in the lattice unit $1/a^2e$. The dashed lines are just to guide the eye.

FIG. 2. Profile of the solenoidal monopolar current $-(\nabla \times J_M)_z$ as a function of $r$, with the $q\bar{q}$ separation (a). $R = 3a$ and (b) $R = 4a$. The data were measured on the lattice with $\beta = 2.28$. The values of $-(\nabla \times J_M)_z$ are measured in the lattice unit $1/a^4e$.

FIG. 3. Profile of $E_z$ vs. $r$ in the unconfined phase with $\beta = 2.40$. The $q\bar{q}$ separation is (a). $R = 3a$, (b) $R = 4a$, (c) $R = 5a$ and (d) $R = 6a$. The values of $E_z$ are measured in the lattice unit $1/a^2e$.

FIG. 4. Profile of $(\nabla \times J_M)_z$ vs. $r$ in the unconfined phase with $\beta = 2.40$. The $q\bar{q}$ separation is (a). $R = 3a$ and (b) $R = 4a$. The values of $-(\nabla \times J_M)_z$ are measured in the lattice unit $1/a^4e$.  

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