Shape and pairing fluctuations effects on neutrinoless double beta decay nuclear matrix elements

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Nuclear matrix elements (NME) for the most promising candidates to detect neutrinoless double beta decay have been computed with energy density functional methods including deformation and pairing fluctuations explicitly on the same footing. The method preserves particle number and angular momentum symmetries and can be applied to any decay without additional fine tunings.

The finite range density dependent Gogny force is used in the calculations. An increase of 10%-40% in the NME with respect to the ones found without the inclusion of pairing fluctuations is obtained, reducing the predicted half-lives of these isotopes.

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The possible detection of lepton number violating processes such as neutrinoless double beta decay ($0\nu\beta\beta$) is one of the current main goals for particle and nuclear physics research. In this process, an atomic nucleus decays into its neighbor with two neutron less and two proton more emitting only two electrons. Fundamental questions about the nature of the neutrino such as its Dirac or Majorana character, its absolute mass scale as well as its mass hierarchy can be determined if this process is eventually measured [1]. On the one hand, searching for $0\nu\beta\beta$ decays represents an extremely difficult experimental task because an ultra low background is required to distinguish the predicted scarce events from the noise.

Recently, the controversial claim of detection in Ge by the Heidelberg-Moscow (HdM) collaboration [2] has been overruled by the latest data released by EXO-200, KamLAND-Zen and GERDA collaborations [3–5]. Nevertheless, these results are challenging the experiments that are already running or in an advanced stage of development to detect directly this process [3, 6–14]. On the other hand, in the most probable electroweak mechanism to produce $0\nu\beta\beta$ states for a given angular momentum, the half-life of this process is understood within these different frameworks. In particular, the decay is favored when the initial and final nuclear states have similar intrinsic deformation [28, 30, 31]. Indications [18, 21, 23, 28, 30] about the strong sensitivity of the transition operator to pairing correlations suggest that fluctuations in this degree of freedom will play a relevant role in the description of this process.

The purpose of this Letter is to report the first calculations of $0\nu\beta\beta$ NMEs including self-consistently shape and pairing fluctuations on the same footing within the EDF method. The finite range of the interaction used in the calculations (Gogny [32]), with a common source for the long and short range parts of the force, guarantees a self-consistent interplay of the shape and pairing fluctuations. In this framework, following the generator coordinate method (GCM) [33, 34], the many body nuclear states are described as a linear combination (mixing) of particle number and angular momentum projected Hartree-Fock-Bogoliubov (HFB) wave functions with different shapes and pairing content [35]:

$$|I_{i/f}^{\pm}\rangle = \sum_{\beta_2,\delta} g_{i/f}^{I}\langle\beta_2,\delta|\Psi_{i/f}^{I}\langle\beta_2,\delta\rangle$$

(2)

where $I$ is the angular momentum, $\sigma$ labels the different states for a given angular momentum, $\beta_2$ and $\delta$ are the intrinsic axial quadrupole and pairing degrees of freedom respectively, $g_{i/f}^{I}\langle\beta_2,\delta\rangle$ are the coefficients found by solving the Hill-Wheeler-Griffin (HWG) equations [33, 35] and the projected wave functions are defined as:

$$|\Psi_{i/f}^{I}\beta_2,\delta\rangle = P_{i/f}^{N(Z)} P^{2/I} P^I_{\phi}\langle\beta_2,\delta\rangle$$

(3)

with $P_{N(Z)}$ and $P^I$ being the neutron (proton) number and angular momentum projection operators respectively.

$$\left[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)\right]^{-1} = G_{01} |M^{0\nu}|^2 \left(\frac{m_\nu}{m_e}\right)^2$$

(1)

where $m_e$ is the electron mass and $\langle m_\nu \rangle = |\sum_k U^2_{ek} m_k|$ is the combination of the neutrino masses $m_k$ provided by the neutrino mixing matrix $U$. The kinematic phase space factor can be determined precisely from the charge, mass and the energy available in the decay [16] while the nuclear matrix elements must be calculated using nuclear structure methods. The most commonly used ones are the quasiparticle random phase approximation [17–21] (QRPA), large scale shell model [22–24] (LSSM), interacting boson model [25, 26] (IBM), projected Hartree-Fock-Bogoliubov [27] (PHFB) and energy density functional [28–30] (EDF). In recent years, most of the basic nuclear structure aspects of the NMEs have been understood within these different frameworks.
Octupolarity, isospin restoration or explicit quasiparticle excitations are missing in this approach and their influence on the NMEs (or any other observable) is beyond the scope of this work. Concerning the specific details about the NMEs, these quantities are computed as the sum of Fermi (F) and Gamow-Teller (GT) terms [1] (tensor contribution is neglected in this work [20, 23]):

\[ M^{0\nu} = - \left( \frac{g_V}{g_A} \right)^2 M^{0\nu}_F + M^{0\nu}_{GT} \]

with \( g_V = 1 \) and \( g_A = 1.25 \) being the vector and axial coupling constants. In addition, the closure approximation [1, 47] is used due to the impossibility of calculating at the same level of accuracy the odd-odd intermediate nucleus. The neutrino potentials include finite size, higher order currents and short range correlations corrections and their parameters are the same as in Refs. [23, 28].

We now discuss in detail the decay of the 136Xe \( \rightarrow 136\text{Ba} \) to illustrate the method. The starting point is the determination of the mixing weights of the initial and final states (Eq. 2). To shed light on the physical insight of these states we analyze first the potential energy surfaces (PES) computed with the wave functions given in Fig. 3 with \( I = 0 \) (see Fig. 1(a)-(b)). In 136Xe we obtain a rather symmetric PES around \( \beta_2 = 0 \), with two degenerated minima at \( (\beta_2 = \pm 0.05, \delta = 3) \). The energy increases significantly by increasing the deformation from \( \beta_2 = \pm 0.15 \) and also by enlarging the pairing content from \( \delta \approx 4 \). On the other hand, a wider PES (both in \( \beta_2 \) and \( \delta \)) with two minima at \( (\beta_2 = 0.15, \delta = 3) \) -the absolute one- and \( (\beta_2 = -0.10, \delta = 3.5) \) are obtained for 136Ba. The absolute minimum in this case is softer in the \( \delta \) direction than the second one and the energy also rises considerably for \( \beta_2 > \pm 0.2 \) and \( \delta > 5 \). More interestingly, the softness of the PESs at \( \delta \in [1, 4] \) in the interval of shapes ranging from \( \beta_2 \in [-0.2, 0.2] \) is ignored in one
diagonal matrix elements have a significant value around

In addition, spherical shapes are also preferred and non-

initial and final states -diagonal part of the Fig. 2(a).

We obtain that the strength of the transition is larger

and represent the NME as a function of the quadrupole

-chosen to be inside the relevant part in Fig. 1(c)-(d) -

δ

presents a similar behavior and it is not shown here). To

to take into account the wave function shapes and looking at

Fig. 2(b) we find that the relevant part is the square de-

fined by the intersection of the horizontal and vertical

lines. Here we see that the pairing fluctuations allow

a large richness of values of the nuclear matrix element

(from zero up to approximately 5) which definitively con-

tribute to the final value.

The results for the most probable candidates to detect

νββ decays are summarized in Table I. We find in the

136Xe decay discussed above a 14% larger NME when

the pairing degree of freedom is explicitly included which

leads to a reduction of the half-life in a factor 0.77. This

result is consistent with exploring regions with larger val-

ues of the NME in the pairing degree of freedom thanks

to the fluctuations in δ included in the collective wave

functions. The same effect happens for the rest of can-

didates where the NME obtained including both deforma-

tion and pairing fluctuations are increased from 10% to

40% with respect to the values found by considering

only shape mixings. The 48Ca is the only particular case

where, due to its double magic character, the initial wave

function is significantly moved towards less pairing cor-

relations, thus giving a slightly smaller NME. Except for

this decay, the updated NMEs lead to a reduction of the

predicted half-lives up to factors from 0.81 (82Se) to 0.52

(128Te). Furthermore, a shorter 76Ge half-life as a func-

tion of the 136Xe one is predicted in the region allowed by

HdM, IGEX [49], GERDA, EXO-200 and KamLAND-Zen

experiments. However, the HdM claim is incompatible

both with the previous and these new values of the

NMEs.

Compared to other methods the new NMEs are get-

ting closer to QRPA/IBM results for 48Ca, 76Ge, 128Te

and 150Nd decays while they are the largest ones for the

other candidates -see Fig. 7 of Ref. [26] for updated val-

ues. However, neither QRPA nor IBM calculations have
TABLE I: Columns (2-7): theoretical and experimental binding energies [38] (in MeV), radii [39] (in fm) and total Gamow–Teller strength [40–44] $S_{(+)}$ for the initial (final) state- for the $0\nu\beta\beta$ candidates. Theoretical values for $S_{(+)}$ are quenched by a factor $(0.74)^2$. Columns (8-9): nuclear matrix elements for the most probable $0\nu\beta\beta$ emitters considering shape fluctuations $(\beta_2)$ and both shape and pairing fluctuations $(\beta_2, \delta)$ explicitly. Superscript and subscript values correspond to the Gamow-Teller $-M^{0\nu}_{GT}$- and Fermi $-(\frac{\alpha}{4\pi})^2 M^{\nu\nu}_F$- components respectively. The last two columns are the variation of the NME and half-lives when the additional pairing degree of freedom is included.

| Isotope | $(BE)^{th}$ | $(BE)^{exp}$ | $R^{th}$ | $R^{exp}$ | $S^{th}_{+/-}$ | $S^{exp}_{+/-}$ | $M^{0\nu}_{GT}(\beta_2)$ | $M^{0\nu}_{GT}(\beta_2, \delta)$ | Var (%) | $t^{1/2}(0\nu\beta\beta)$ |
|---------|-------------|--------------|----------|-----------|--------------|---------------|----------------------|---------------------|---------|---------------------|
| $^{48}$Ca | 420.919 | 415.991 | 3.467 | 3.473 | 13.48 | 14.4 $\pm$ 2.2 | 2.370$_{0.394}^{1.914}$ | 2.229$_{0.431}^{0.797}$ | -6 | 1.13 |
| $^{48}$Ti | 423.773 | 418.699 | 3.560 | 3.591 | 1.94 | 1.9 $\pm$ 0.5 | 4.60$_{0.886}^{3.715}$ | 5.55$_{1.082}^{4.707}$ | 21 | 0.69 |
| $^{76}$Ge | 664.604 | 661.598 | 4.025 | 4.081 | 20.96 | 19.89 | 4.218$_{0.837}^{3.381}$ | 4.67$_{0.931}^{3.743}$ | 11 | 0.81 |
| $^{76}$Se | 665.268 | 662.072 | 4.075 | 4.139 | 1.26 | 1.45 $\pm$ 0.07 | 4.194$_{0.936}^{3.341}$ | 4.67$_{0.931}^{3.743}$ | 11 | 0.81 |
| $^{82}$Se | 717.034 | 712.842 | 4.122 | 4.139 | 23.57 | 21.91 | 4.218$_{0.837}^{3.381}$ | 4.67$_{0.931}^{3.743}$ | 11 | 0.81 |
| $^{82}$Kr | 718.220 | 714.273 | 4.131 | 4.192 | 1.26 | 1.26 | 4.194$_{0.936}^{3.341}$ | 4.67$_{0.931}^{3.743}$ | 11 | 0.81 |
| $^{90}$Zr | 829.801 | 828.995 | 4.299 | 4.349 | 27.73 |  | 5.65$_{1.032}^{4.618}$ | 6.498$_{2.202}^{5.296}$ | 15 | 0.76 |
| $^{96}$Mo | 834.212 | 830.778 | 4.320 | 4.384 | 2.64 | 0.29 $\pm$ 0.08 | 5.58$_{0.935}^{4.219}$ | 6.58$_{1.227}^{5.961}$ | 30 | 0.60 |
| $^{100}$Ru | 862.003 | 860.457 | 4.373 | 4.445 | 28.04 | 26.69 | 5.08$_{0.935}^{4.149}$ | 5.58$_{1.227}^{5.961}$ | 30 | 0.60 |
| $^{110}$Cd | 998.809 | 997.440 | 4.567 | 4.628 | 34.40 | 32.70 | 4.79$_{0.964}^{4.002}$ | 5.34$_{0.937}^{3.762}$ | 12 | 0.80 |
| $^{116}$Sn | 991.990 | 988.684 | 4.569 | 4.626 | 2.61 | 1.09 $\pm$ 0.13 | 4.80$_{0.916}^{3.893}$ | 5.78$_{1.070}^{4.689}$ | 20 | 0.69 |
| $^{124}$Sn | 1051.981 | 1049.96 | 4.622 | 4.675 | 40.71 |  | 4.80$_{0.916}^{3.893}$ | 5.78$_{1.070}^{4.689}$ | 20 | 0.69 |
| $^{124}$Te | 1052.019 | 1050.69 | 4.664 | 4.717 | 1.63 |  | 4.10$_{1.027}^{0.759}$ | 5.68$_{1.432}^{4.255}$ | 38 | 0.52 |
| $^{128}$Te | 1082.541 | 1081.44 | 4.685 | 4.735 | 40.48 | 40.80 | 4.10$_{1.027}^{0.759}$ | 5.68$_{1.432}^{4.255}$ | 38 | 0.52 |
| $^{130}$Te | 1097.320 | 1095.94 | 4.695 | 4.742 | 43.69 | 45.90 | 5.13$_{0.899}^{4.141}$ | 6.40$_{0.838}^{5.161}$ | 25 | 0.64 |
| $^{130}$Xe | 1097.655 | 1096.91 | 4.733 | 4.783 | 1.33 | 1.33 | 4.19$_{0.526}^{0.673}$ | 4.77$_{0.604}^{3.170}$ | 14 | 0.77 |
| $^{136}$Ba | 1143.500 | 1141.88 | 4.757 | 4.799 | 46.77 | 46.77 | 4.19$_{0.526}^{0.673}$ | 4.77$_{0.604}^{3.170}$ | 14 | 0.77 |
| $^{150}$Nd | 1234.729 | 1237.45 | 5.033 | 5.041 | 50.35 | 50.35 | 1.70$_{0.429}^{1.278}$ | 2.190$_{0.551}^{1.639}$ | 29 | 0.61 |
| $^{150}$Sm | 1236.249 | 1239.25 | 4.987 | 5.040 | 1.54 | 1.54 | 1.70$_{0.429}^{1.278}$ | 2.190$_{0.551}^{1.639}$ | 29 | 0.61 |

explored explicitly this degree of freedom so far. On the other hand, these values move away from the LSSM ones and some work is in progress to study the NMEs along isotopic chains to disentangle the similarities/differences between both methods [30, 50].

Part of this disagreement could be produced by the large values of the Fermi part obtained within QRPA, IBM and EDF methods compared to the LSSM ones that has been recently discussed in terms of isospin symmetry violation. Hence, spurious contributions to Fermi -and possibly GT- matrix elements exist in those cases where the initial and final states are not isospin eigenstates. In Ref. [51] is shown in the QRPA framework that correcting the parameters to have the Fermi part of the $2\nu\beta\beta$ decay equal to zero, the $M^{\nu\nu}_F$ is reduced but $M^{0\nu}_{GT}$ is barely affected. In Table I we show separately the GT and F components of the NME and we see that the gain including pairing fluctuations is similar in both channels. This fact could indicate that the observed increase is not produced by a stronger isospin symmetry violation.

In summary, we have presented calculations for $0\nu\beta\beta$ matrix elements within the EDF framework, including for the first time pairing and quadrupole axial deformation fluctuations together. We have confirmed that NMEs between states with similar quadrupole deformation are largest. Concerning the pairing degree of freedom we found the following characteristics of the the NMEs: 1.- They are zero for weakly correlated states, $\delta$ and $\delta'$ < 2, 2.- They grow considerably for increasing pairing correlations and 3.- There exists a set of states belonging to a band along the main diagonal, defined by $\delta' = \pm 3$, with large NMEs. This effect and the allowance of having pairing fluctuations in the initial and final wave functions produce a rise in the NMEs from 10% to 40% with respect to the values obtained without including them. The updated values reduces correspondingly the expected half-lives for the most probable candidates.

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