Enhancement of Superconductivity in Disordered Films by Parallel Magnetic Field

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We show that the superconducting transition temperature \( T_c(H) \) of a very thin highly disordered film with strong spin-orbital scattering can be increased by parallel magnetic field \( H \). This effect is due to polarization of magnetic impurity spins which reduces the full exchange scattering rate of electrons; the largest effect is predicted for spin-\( \frac{1}{2} \) impurities. Moreover, for some range of magnetic impurity concentrations the phenomenon of superconductivity induced by magnetic field is predicted: superconducting transition temperature \( T_c(H) \) is found to be nonzero in the range of magnetic fields \( 0 < H^* \leq H \leq H_c \).

The problem of superconducting alloys with magnetic impurities was addressed long ago by Abrikosov and Gor’kov (AG) [1]. They have shown that superconductivity (SC) is suppressed due to exchange scattering (ES) of electrons on magnetic impurities, the transition temperature \( T \) determined from the equation (hereafter, we employ units, in which \( \hbar = 1 \)):

\[
\ln \frac{T_c}{T} = \pi T \sum_\varepsilon \left( \frac{1}{|\varepsilon|} - \frac{1}{|\varepsilon| + \nu_S} \right) \tag{1}
\]

Here \( \varepsilon = 2\pi T(m + 1/2) \) is the fermionic Matsubara frequency \((m \text{ is integer})\), \( T_c \) is the transition temperature of clean sample, and \( \nu_S = 2\pi N_F n_S J^2 S(S+1) \) is the ES rate of electrons on magnetic impurities \((N_F \text{ is the normal metal density of states per single spin state, } n_S \text{ is the concentration of magnetic impurities, } J \text{ is the exchange coupling constant, and } S \text{ is the impurity spin length})\). The solution of (1) yields the function \( T = T_{AG}(\nu_S) \). There exists a critical point at which the transition temperature is suppressed down to zero, the critical scattering rate being \( \nu_S^* = \pi/(2e^C) \). Setting \( \nu_S = 0.882 T_{AG} \), where \( C = 0.577 \) is the Euler constant. The critical concentration, corresponding to \( \nu_S^* \), is further denoted by \( n_S^* \). We emphasize that \( \nu_S \) is the full ES rate, i.e. the sum of the spin flip scattering rate \( 2\pi N_F n_S J^2 \left( \langle S_f^z \rangle + \langle S_i^z \rangle \right) = 2/3 \nu_S \) and the rate of scattering without spin flip \( 2\pi N_F n_S J^2 \langle S_f^z \rangle = 1/3 \nu_S \).

The AG’s results were derived for unpolarized magnetic impurity spins. In this Letter we investigate how the polarization of impurity spins affects the ES mechanism of SC suppression. We show that polarization of magnetic impurity spins by external magnetic field reduces the full ES rate \( I_\varepsilon \). It reaches its minimal value \( \nu_\infty = \nu_S S/(S+1) < \nu_S \) at the infinite field, when the impurity spins are completely polarized and spin flip processes have frozen out. This reduction is due to quantum fluctuations of impurity spins, thus it is strongest for \( S = 1/2 \) and vanishes in the limit \( S \gg 1 \).

If ES was the only mechanism of SC suppression in nonzero magnetic field \( h = \mu_B H \), the transition temperature \( T_c^0(h) \) would always be higher than \( T_c(h = 0) = T_{AG}(\nu_S) \), determined by AG’s result. \( T_c^0(h) \) is a growing function, approaching the value \( T_\infty = T_{AG}(\nu_\infty) \) at very high fields \( h \to \infty \). The transition temperature increases \( T_c^0(h) \), comparable to \( T_\infty - T_c(0) \), is attained at the field range \( h \gtrsim T_c^0(h) \). However, apart from ES, there are other mechanisms of SC suppression by magnetic field, namely, paramagnetic effect (PE) and orbital effect (OE). Thus, to observe an increase \( T_c(h) > T_c(0) \) of the actual transition temperature, PE and OE should be small compared to ES in the field range \( h \sim T_c(h) \). Strong reduction of PE is achieved in presence of high spin-orbital scattering rate \( \nu_\infty \gg T_{AG} \). OE is suppressed for a thin-film (thickness \( d \) shorter than the magnetic length \( l_H = \sqrt{\pi/e^C H} \) with parallel orientation of external magnetic field [2].

In this Letter we show that the increase in the transition temperature can be observed if two quite stringent conditions on the smallness of PE and OE are met. First, the spin-orbit scattering rate \( \nu_\infty \) must be suffi-
not exist below some critical field $h^*$. A nonzero transition temperature $T_c(h)$ (solid line in Fig. 2) exists in a range of fields starting from $h^*$ and terminating at some higher critical field $h_c$, when PE and OE dominate over ES. Such behavior is possible in the range of concentrations $n^*_S < n_S < n^*_S^{**}$, where $n^*_S$ is smaller than $n^*_S(S+1)/S$ and is determined by the parameters involved in PE and OE. The better the conditions (2), (3) are satisfied, the closer is $n^*_S$ to $n_S(S+1)/S$. The most favorable situation for the experimental observation of “magnetic-field-induced superconductivity” is realized when $n_S$ is only slightly larger than $n^*_S$. In this case $h^*$ is sufficiently small and the curve $T_c(h)$ produces a quite steep growth at the fields $h$ just above $h^*$ (see Fig. 2). Two specific examples of $T_c(h)$ behaviour are presented in Figs. 1 and 2 for $S = 1/2$, for the following set of parameters: $J < 0$ (ferromagnetic exchange), $\zeta = 5$, $\nu_0 = 10^3 T_c$, $\nu = 10^4 T_c$, $pFD = 30$. The similar set of parameters corresponds, for example, to the 3nm-thick PtSi film studied in [6], [7].

Below we briefly outline the method used to derive the announced results, details of our calculations will be presented in a separate publication [5].

The starting point of our problem is the following Hamiltonian:

$$\mathcal{H} = \mathcal{H}_{BCS} + \mathcal{H}_S + \mathcal{H}_{cS} + \mathcal{H}_{cU}. $$

Here,

$$\mathcal{H}_{BCS} = \int \left\{ \psi_\alpha^+ \left( \frac{1}{2m} (p - e/cA)^2 - \varepsilon_F \right) \psi_\alpha + \frac{\lambda}{2} \psi_\alpha^+ \psi_\beta^+ \psi_\beta \psi_\alpha - \psi_\alpha^+ \sigma^z \psi_\beta \right\} \, dr$$

is the BCS Hamiltonian which includes the orbital and paramagnetic effects of external magnetic field on conduction electrons;

$$\mathcal{H}_{cS} = \int \left\{ \psi_\alpha^+ \sum_a (u_S \delta_{a\beta} + J(S_\alpha, \sigma_{a\beta})) \delta(r - R_a) \psi_\beta \right\} \, dr$$

 describes the interaction with magnetic impurities and

$$\mathcal{H}_{cU} = - \sum_a \omega_S S_z^a$$

is the Hamiltonian of impurity spins in external magnetic field ($\omega_S = gS h = 2h$ is their Zeeman splitting). Finally,

$$\mathcal{H}_{cU} = \int \left\{ \psi_\alpha^+ (r) \sum_b v_{a\beta}(r - R_b, r' - R_b) \psi_\beta (r') \right\} \, dr \, dr'$$

describes the scattering of electrons on non-magnetic impurities, which includes both potential and spin-orbit parts. Here $v_{a\beta}(r, r')$ is the Born amplitude in coordinate representation; since we work in momentum space, we only need its Fourier transform $v_{a\beta}(p, p') = u_0 \delta_{a\beta} + i v_{a\beta}(p) p^2 (p, p', \sigma_{a\beta})$. Magnetic and non-magnetic impurities are uniformly distributed over the sample volume with concentrations $n_S$ and $n$ respectively.
We solve the problem using the standard diagrammatic technique for BCS theory and disordered metals \cite{1,4} and employing the following approximations: i) $pF \gg 1$, where $l = v_F/\nu$ is the mean free path for potential scattering; ii) Born approximation for impurity scattering; iii) “dirty limit”, i.e. $\nu \gg \nu_{\text{so}} \to T_c$.

The equation for the transition temperature $T_c$ can be obtained in the form

$$\ln \frac{T_c}{T} = \pi T \sum_{\varepsilon} \left( \frac{1}{|\varepsilon|} - C_0(\varepsilon) \right),$$

where $C_0(\varepsilon) = 1/2(C_{i+}^{\dagger} - C_{i+}^{\dagger} + C_{i+}^{\dagger} - C_{i+}^{\dagger})$ is the singlet Cooperon component. In the approximation $pF \gg 1$ the Cooperon is given by an infinite sum of ladder-type diagrams, each “ladder step” containing an impurity line and the product of two disorder-averaged normal state Green functions. The expression for the components of such Green function with electron spin directed along (↑) the external field $h$ and in the opposite direction (↓) reads:

$$G_{i,\downarrow}(\varepsilon, p) = i \varepsilon - \xi \pm h' + \frac{i}{2} (\nu + \nu_{\text{so}} + \Gamma(\varepsilon)) \text{sgn} \varepsilon \pm i \nu_{\text{so}} \text{sgn} \varepsilon.$$

Here, $\nu = 2\pi N_F (nS u_3^2 + n u_2^2)$ is the potential scattering rate, $\nu_{\text{so}} = 2\pi N_F n u_2^2/3$ is the spin-orbit scattering rate, $\nu_S = 2\pi N_F nS u_3^2 JS(J_S)$ is the interference contribution between potential and exchange scattering on magnetic impurities (however, this term is irrelevant and falls out of the final result), $S' = h - nS J(J_S)$ is the effective magnetic field acting on electron spins comprised of the external field $h$ and exchange field of polarized impurities $-nS J(J_S)$. Hereafter $\langle \ldots \rangle$ stands for thermodynamic average over the states of an isolated impurity spin, subjected to external magnetic field $h$:

$$\langle S_z \rangle = \langle S_z + 1 \rangle \coth \left( \frac{S + 1/2}{2} \frac{\omega_S}{T} \right) - \frac{1}{2} \coth \frac{\omega_S}{2T}.$$

Further, $\Gamma(\varepsilon) = \nu_{\text{orb}} + \Gamma(\varepsilon)$ is the full ES rate due to exchange interaction of electrons with polarized magnetic impurities. It is given by the sum of the rate of scattering without spin flip $\nu_{\text{orb}} = \nu_S \left( \frac{S_S}{2S(S+1)} \right)$ and the spin flip scattering rate

$$\Gamma(\varepsilon) = \nu_S \frac{\langle S_z \rangle}{S(S+1)} - \delta \Gamma(\varepsilon),$$

where

$$\delta \Gamma(\varepsilon) = \nu_S \frac{\langle S_z \rangle}{S(S+1)} - \frac{2\omega_S}{T} \sum_{|\omega| > |\varepsilon|} \frac{\omega_S}{\omega^2 + \omega_S^2}.$$

Here $\omega = 2\pi T \nu_S h$ is the bosonic Matsubara frequency ($n$ is integer) and $S^2_\perp = S_x^2 + S_y^2$.

We now discuss properties of the full exchange scattering rate $\Gamma(\varepsilon) = \nu_{\text{orb}} + \Gamma_{\text{sf}}(\varepsilon)$ and then use the knowledge of this function while determining $T_c(h)$. For $|\varepsilon| \gg \omega_S$ at any ratio $\omega_S/T$ we have $\Gamma_{\text{sf}}(\varepsilon) = \nu_S S^2(S+1)$ and $\Gamma(\varepsilon) = \nu_S$. At zero field $\Gamma_{\text{sf}}(0) = 2/3 \nu_S$, $\nu_{\text{orb}} = 1/3 \nu_S$, and $\Gamma(\varepsilon)$ is $\nu_{\text{so}}$ for any $\varepsilon$. The full ES rate $\Gamma(\varepsilon) \approx \nu_S$ for electrons with energies $|\varepsilon| \gg \omega_S$ is not modified by magnetic field, although $\Gamma_{\text{sf}}(\varepsilon)$ and $\nu_{\text{orb}}$ do depend on $h$.

Consider the limit of strong polarization $\omega_S \gg T$. In this case one can replace $\nu_S$ in the sum over $\omega$ by the integral obtain

$$\Gamma(\varepsilon) = \nu_S \frac{1}{S(S+1)} \frac{2}{\pi} \frac{|\varepsilon|}{\omega_S} \arctan \frac{|\varepsilon|}{\omega_S} \quad \text{and} \quad \nu_z = \nu_S S/(S+1).$$

(7)

For electron energies $|\varepsilon| \ll \omega_S$ less than Zeeman splitting $\Gamma_{\text{sf}}(\varepsilon) \approx \nu_S S/(S+1) \ll \nu_S$ reflecting the fact that spin flip processes freeze out for strongly polarized spins. Hence, the full ES rate $\Gamma(\varepsilon) \approx \nu_z = \nu_S S/(S+1) < \nu_S$ in a wide range of energies $|\varepsilon| \ll \omega_S$. At very strong field $\Gamma_{\text{sf}}(\varepsilon) \to 0$ and $\Gamma(\varepsilon) = \nu_{\text{orb}} = \nu_S S/(S+1)$ for all $\varepsilon$. Expressing $\Gamma(\varepsilon)$ in the form $\Gamma(\varepsilon) = \nu_z - \delta \Gamma(\varepsilon)$ we see that the full ES rate in nonzero field is always less than $\nu_z$, with $\delta \Gamma(\varepsilon, \omega_S)$ for a fixed $\varepsilon$ being a growing function of $\omega_S$ with limiting values $\delta \Gamma(\varepsilon, 0) = 0$, $\delta \Gamma(\varepsilon, \infty) = \nu_S S/(S+1)$.

The Cooperon can be shown to obey the following equation for $C_0(\varepsilon)$

$$\left( |\varepsilon| + \Gamma(\varepsilon) + \frac{1}{2} (\bar{L}_0 - \bar{\Gamma}_{\text{sf}}(\varepsilon)) + \frac{3h^2}{2 \nu_{\text{so}}} + \gamma_{\text{orb}} \right) C_0(\varepsilon) = 1.$$

(8)

Here $\gamma_{\text{orb}} = \frac{1}{2} D(\frac{d^2H}{d^2p_F})^2 \frac{2}{\omega_S^2} = \frac{2}{3} (p_F d)^2 \frac{2}{\omega_S^2}$ is the dephasing rate corresponding to OE of magnetic field ($D = \frac{1}{3} p_F l$ is the diffusion constant) and the operator $\bar{L}_0$ acts as

$$\bar{L}_0 C_0(\varepsilon) = \nu_S \frac{\langle S_z \rangle}{S(S+1)} T \sum_{\omega} \frac{2\omega_S}{\omega^2 + \omega_S^2} C_0(|\varepsilon| - \omega).$$

At zero field $h = 0$ it is straightforward to check that $\bar{L}_0 - \bar{\Gamma}_{\text{sf}}(0) = 0$ and $\Gamma(\varepsilon) = \nu_S$. Therefore the solution to Eq. (8) is $C_0(\varepsilon) = 1/(|\varepsilon| + \nu_S)$ and one recovers the AG's result \cite{3} for transition temperature.

Enhancement of $T_c$ by parallel field. We start our analysis from the case $\nu_S < \nu_{\text{orb}}$, when a nonzero transition temperature $T_c(0) = T_{\text{AG}}(\nu_S)$ exists at zero field. First we study the equation

$$\left( |\varepsilon| + \nu_S - \delta \Gamma(\varepsilon) + \frac{1}{2} \bar{L}_0 - \bar{\Gamma}_{\text{sf}}(\varepsilon) \right) C_0(\varepsilon) = 1$$

(9)

leaving in Eq. (8) the terms related to ES only and neglecting PE and OE. In the limit $h \to \infty$ we get: $\bar{L}_0 \to 0$, $\bar{\Gamma}_{\text{sf}}(\varepsilon) \to 0$, $\Gamma(\varepsilon) \to \nu_{\text{so}}$, and $C_0(\varepsilon) = 1/(|\varepsilon| + \nu_S)$. Thus in the strong-field limit and in the absence of PE and OE the transition temperature would be $T_{\infty} = T_{\text{AG}}(\nu_{\text{so}})$ (indicated by dotted line in Fig. 1), which is higher than the zero field value $T_{\text{AG}}(\nu_S)$ since $\nu_{\text{so}} < \nu_S$. For an arbitrary field solving Eqs. (8,9) together numerically, one
obtains the transition temperature curve \( T_c(h) \) with PE and OE disregarded (dashed line in Fig. 1). Formally, the enhancement of transition temperature compared to AG’s zero field result \( T_{AG}(\nu_S) \) is due to the term \(-\delta \ell(\varepsilon)\) in \( \delta \) that effect is always stronger than the (opposite-sign) effect from the term operator \( 1/2(\hat{L}_0 - \Gamma_{sf}(\varepsilon)) \) in the same equation.

We are now in position to derive the conditions \( \mathcal{L} \) and \( \mathcal{K} \) for the strengths of paramagnetic and orbital effects compatible with observation of an increase of the actual transition temperature \( T_c(h) \). Indeed, the terms in \( \delta \) related to PE and OE must be sufficiently smaller than the terms responsible for ES in the relevant fields \( h \sim T_{AG} \): \( |h'(h \sim T_{AG})|/\nu_{so} \ll \nu_S \) and \( \gamma_{orb}(h \sim T_{AG}) \ll \nu_S \). Since we are interested in \( \nu_S \sim T_{AG} \) the latter condition immediately leads to \( \mathcal{K} \). Due to Born approximation \( (\zeta \gg 1) \) for \( h \sim T_{AG} \) and \( T \lesssim T_{AG} \) the exchange field \( n_S J(S) \) dominates over \( h \) in the effective field \( h' \) and is of the order of its maximal value \( n_S J S \). Therefore, estimating \( h' \sim n_S J S \), we obtain \( \mathcal{L} \). Thus, provided the conditions \( \mathcal{L} \) and \( \mathcal{K} \) are satisfied, one observes an increase in the transition temperature \( T_c(h) \) (solid line in Fig. 1).

**Superconductivity induced by magnetic field.** Now we turn to the case \( \nu_S > \nu_S' > \nu_{\infty} \) or, expressed in terms of magnetic impurity concentrations, \( n_S^* < n_S < n_S(S+1)/S \). First we study the Eqs. \( \mathcal{L}, \mathcal{K} \) neglecting PE and OE. Since \( \nu_S > \nu_S' \), the SC is totally suppressed at \( h = 0 \), but at infinite field one obtains a finite transition temperature \( T_{AG} = T_{AG}(\nu_{\infty}) \) (indicated by dotted line in Fig. 2), because \( \nu_{\infty} < \nu_S^* \). This leads to the existence of critical field \( h_S^* \), below which SC does not exist at any temperature, but appears in greater fields \( h \geq h_S^* \). The field \( h_S^* \) is determined from the equation

\[
\int_{\nu_S}^{\infty} d\varepsilon \left( C_0(\varepsilon, h) - 1/(\varepsilon + \nu_S^*) \right) = 0 \tag{10}
\]

where \( C_0(\varepsilon, h) \) is the solution to \( \mathcal{K} \) in zero temperature limit, and depends on only one parameter \( \nu_S \). The transition temperature \( T_c^*(h) \) in the absense of PE and OE (dashed line in Fig. 2) is a growing function of \( h \), starting from the zero value \( T_c^*(h_\nu^*) = 0 \) at \( h_\nu^* \) and tending to \( T_{\infty} \) as \( h \to \infty \). The critical field \( h_S^* \) as a function of \( n_S \) has the following limiting values: \( h_S^* \to 0 \) as \( n_S \to n_S^* = 0 \), \( h_S^* \to \infty \) as \( n_S \to n_S^*(S+1)/S \to 0 \); and \( h_S^* \to T_{AG} \) when \( n_S \) is close neither to \( n_S^*(S+1)/S \) nor to \( n_S^*(S+1)/S \).

For magnetic impurity concentrations \( n_S \) not very close to \( n_S^*(S+1)/S \), the field \( h_S^* \leq T_{AG} \). Then, provided the conditions \( \mathcal{L} \) and \( \mathcal{K} \) are met, the described behavior of transition temperature in the fields \( h \sim h_S^* \) survives under the action of orbital and paramagnetic effects. PE and OE slightly change \( h_S^* \), making the actual critical field \( h_S^* \) greater than \( h_S^* \). The actual transition temperature curve \( T_c(h) \) (solid line in Fig. 2) is close to \( T_c^*(h) \) at fields \( h \sim h_S^* \) and deviates sufficiently only at higher fields when PE and OE dominate over ES. We found the critical field \( \omega_S^* = g_S h_S^* \) analytically (with logarithmic accuracy in \( \omega_S/\nu_S^* \)) for the case when \( n_S \) is slightly greater than critical \( n_S^* \), i.e. \( \delta \nu_S = \nu_S - \nu_S^* \ll \nu_S^* \):

\[
\frac{\omega_S^*}{\nu_S^*} \approx \frac{n_S(S+1)}{2\nu_{so} \nu_S^*} \ln \frac{\nu_S^*}{\nu_S} = \pi(S + 1) \left[ \frac{\delta \nu_S}{\nu_S^*} + \frac{3 (n_S JS)^2}{2 \nu_{so} \nu_S^*} \right] \tag{11}
\]

If \( h_S^* \geq T_{AG} \), i.e. \( n_S \) is close to \( n_S^*(S+1)/S \), accounting for PE and OE, even with conditions \( \mathcal{L} \) and \( \mathcal{K} \) fulfilled, destroys SC in such a high field. Thus, for such \( n_S \) SC is totally suppressed at any field. This yields that the regime of “magnetic-field-induced SC” actually exists in a more narrow (than in the absence of PE and OE) range of concentrations \( n_S^* < n_S < n_S^* \), where \( n_S^* \) is smaller than \( n_S^*(S+1)/S \) and is determined by the values of parameters involved in PE and OE.

In conclusion, we have predicted the mechanism of superconductivity enchancement in thin films by external parallel magnetic field. The effect is due to the polarization of magnetic impurity spins, which reduces the full rate of electron exchange scattering. In some range of magnetic impurity concentrations the phenomenon of magnetic-field-induced superconductivity is predicted. The predicted effect is expected to be observable in very thin disordered superconductive films containing heavy metals leading to high spin-orbital scattering rate. We expect that similar effect may exist in superconductive-ferromagnet thin-film bilayers with spontaneous magnetization parallel to the surface.

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