Robust backstepping control of uncertain nonlinear systems with unknown time-varying input delay

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Abstract: A robust compensator is developed for a class of strict-feedback uncertain nonlinear systems with additive disturbance and unknown time-varying input delay. The compensator is composed of a Proportional-Integral (PI) control and delay compensation term based on a finite integral of the past control values. The sufficient inequality conditions on controller gains and upper bound of input delay are derived using a Lyapunov-based stability analysis by choosing suitable L-K functionals, which guarantee a global uniformly ultimately bounded (GUUB) tracking result. Simulation results show the performance and robustness of controller for different values of time-varying input delay.

Keywords: Unknown input delay, Lyapunov-Krasovskii functionals, Strict-feedback system.

1. INTRODUCTION

Early techniques of solving the problems of linear systems with known, input delays are available in Artstein (1982), Bahill (1983), Nihtilä (1989), Krstic (2010). Motivated by the fact that, most of the applications, where exact knowledge of delay is not available, the authors in Bresch-Pietri and Krstic (2009), Bresch-Pietri et al. (2012) proposed the adaptive control based design for the problems of uncertain dynamics with unknown delay. The stabilization and control design problem of nonlinear systems with constant input delays is discussed in Krstic (2008), Krstic (2009), Mazenc and Bliman (2006), Mazenc et al. (2012), Sharma et al. (2011), Karafyllis (2011), Krstic and Smyshlyaev (2008), Mazenc et al. (2004), Mazenc et al. (2013), Pepe et al. (2008). Adaptive control law based result of nonlinear systems with constant, unknown delay is addressed in Bresch-Pietri and Krstic (2014), while the dynamics is considered as known.

The literature on nonlinear systems with time-varying input delay is available in Choi and Lim (2010), Koo et al. (2012), Merad et al. (2016), Kamalapurkar et al. (2016), Obuz et al. (2017). The authors in Choi and Lim (2010), Koo et al. (2012) proposed output feedback regulation for chain of integrator system with unknown time-varying delays. A robust control law is designed for the class of nonlinear systems with known time-varying input delay in Merad et al. (2016), Kamalapurkar et al. (2016), while, the authors in Obuz et al. (2017), presented the robust control design method for uncertain dynamics with small unknown input delay.

For nonlinear systems in strict-feedback form with arbitrarily large input delay Mazenc and Bliman (2006), used backstepping control design to prove global asymptotic stability by compensating for known constant input delay. This work is extended for the known pointwise delay in the input in Mazenc et al. (2011). The authors in Zhou et al. (2009) proposed standard backstepping design to develop an adaptive controller for a non-minimum phase system with unknown input delay and unmodeled dynamics. The compensation of unknown input delay using predictor feedback techniques, which requires exact knowledge of delay, becomes a challenge. Still fewer results exist which solve the problem of nonlinear systems with unknown actuator delay. A recent result in Bresch-Pietri and Krstic (2014) utilized an adaptive control scheme for adaptation of the unknown constant actuator delay for unstable nonlinear systems, however, it requires exact knowledge of system dynamics. The authors in Jain and Bhasin (2020) developed a robust control law which includes a Proportional-Integral (PI) control action and delay compensator for a class of uncertain nonlinear systems with unknown constant input delay. The work in Jain and Bhasin (2020) is extended for the compensation of time-varying input delay for uncertain nonlinear system (Brunowski form) in Jain and Bhasin (2019).

This paper mainly contributes the development of a robust compensator for a class of uncertain nonlinear systems in strict-feedback form with additive disturbance and unknown time-varying input delay. This design requires knowledge of upper bound of unknown time-varying delay. The controller is composed of a PI control and delay compensator term comprising a filtered tracking error signal which includes an integral of past values of control signal where the limits of integration are based on known bound of delay. The delay terms are cancelled out in stability analysis by choosing of suitable Lyapunov-Krasovskii functionals, and global uniformly ultimately bounded tracking result is obtained. Two illustrative examples are considered to demonstrate the robustness and performance of the controller.
2. PROBLEM FORMULATION & ASSUMPTIONS
Consider a nonlinear system described as
\[
\begin{align*}
\dot{x}_i(t) &= x_{i+1}(t) + f_i(x_1, \ldots, x_i, t), \\
\dot{x}_n(t) &= f_n(x(t), t) + g(x(t), t) u(t - \tau(t)) + d(t) \\
y(t) &= x_1(t)
\end{align*}
\]  
(1)
where \( x(t) \in \mathbb{R}^n \) is a vector of measurable state variables defined as \( x(t) \triangleq [x_1^T(t), x_2^T(t), \ldots, x_n^T(t)]^T \), and \( u(t) \in \mathbb{R}^n \) represents the control input vector, \( \tau(t) \in \mathbb{R}^+ \) is an unknown time-varying input delay, \( f_j(x_1(t), \ldots, x_j, t) \in \mathbb{R}^n, j = 1, \ldots, n \) and \( g(x(t), t) \in \mathbb{R}^{n \times n} \) are unknown smooth functions and \( d(t) \in \mathbb{R}^n \) represents disturbances.

The objective is to develop a controller \( u(t) \) such that the system output trajectory \( y(t) \) tracks the desired trajectory \( y_d(t) \). The following assumptions are considered for subsequent development.

Assumption 1. The desired trajectory \( y_d(t) \) and its derivatives are bounded by known positive constants.

Assumption 2. The delay \( \tau(t) \) is upper bounded as \( \tau(t) \leq \bar{\tau} \), where \( \bar{\tau} \in \mathbb{R}^+ \) is a known positive constant. The derivative of \( \tau(t) \) is bounded such that \( |\dot{\tau}(t)| \leq \bar{\gamma} \leq 1 \), where \( \bar{\gamma} \in \mathbb{R}^+ \) is a known constant.

Assumption 3. The unknown functions \( f_j(x_1, \ldots, x_j, t) \), \( 1 \leq j \leq n \) satisfy the following growth condition:
\[
\|f_j(x_1, \ldots, x_j, t)\| \leq \zeta_{j1} \|x_j\| + \zeta_{j2}
\]
where \( \dot{x}_j = [x_1(t), x_2(t), \ldots, x_j(t)]^T \) and \( \zeta_{j1}, \zeta_{j2} \) are known positive constants.

Assumption 4. The function \( g(X, t) \) is lower and upper bounded as \( \underline{g} \leq g(X, t) \leq \bar{g} \), where \( \underline{g}, \bar{g} \in \mathbb{R}^+ \) are known constants.

Assumption 5. \( d(t) \) is a known function.

Assumption 6. The system dynamics in (1) do not exhibit the finite escape time phenomenon for \( t \in [0, \bar{\tau}(t)]. \)

3. CONTROL DEVELOPMENT AND STABILITY ANALYSIS
This section presents the control design procedure by using the backstepping procedure:

Step 1: The position tracking error denoted by \( e_1(t) \in \mathbb{R}^n \), is defined as
\[
e_1(t) \triangleq x_1(t) - x_{id}(t)
\]  
(2)
and
\[
e_2(t) \triangleq x_2(t) - u_1(t),
\]  
(3)
where, \( u_1(t) \) denotes the virtual control input of state \( x_2(t) \). Using (1), (3), and differentiating (2), yields
\[
\dot{e}_1 = e_2 + u_1 + \Delta_1 - \dot{x}_{id},
\]  
(4)
where the function \( \Delta_1(f_1, t) \in \mathbb{R}^n \) is defined as
\[
\Delta_1(f_1(t), t) \triangleq f_1(x_1(t)).
\]  
(5)
The virtual control law can be designed as
\[
 u_1 = \dot{x}_{id} - k_1 e_1
\]  
(6)
Consider a Lyapunov function candidate \( V_1 \) defined as
\[
V_1 \triangleq \frac{1}{2} e_1^T e_1
\]  
(7)
Using (4), (6) and differentiating (7), yields
\[
\dot{V}_1 = -k_1 \|e_1\|^2 + e_1^T e_2 + e_1^T \Delta_1
\]  
(8)
Step 2: The error signal \( e_3 \in \mathbb{R}^n \) for the state \( x_3 \) is defined as
\[
e_3 \triangleq x_3 - u_2
\]  
(9)
where \( u_2 \) is virtual control input of state \( x_3 \), defined as
\[
u_2 \triangleq \dot{x}_{id} - k_2 e_2 + k_2 \Delta_2
\]  
(10)
Differentiating (3), using (9) and (10), the derivative of \( e_2 \) is obtained as
\[
\dot{e}_2 = \dot{e}_3 - k_2 e_2 + \Delta_2
\]  
(11)
where, the function \( \Delta_2(k_1, f_1, f_2, t) \in \mathbb{R}^n \) is defined as
\[
\Delta_2(k_1, f_1, f_2, t) \triangleq k_1 f_1(x_1(t)) + f_2(x_1(t), x_2(t))
\]  
(12)
Consider a Lyapunov function candidate \( V_2 \) defined as
\[
V_2 \triangleq V_1 + \frac{1}{2} e_2^T e_2
\]  
(13)
Differentiating (13), using (8) and (11), the following equation is obtained as
\[
\dot{V}_2 = -k_1 e_2^T e_2 + e_1^T e_2 + e_1^T e_3 + e_1^T \Delta_1 + e_2^T \Delta_2
\]  
(14)
Step 3 (3 ≤ i ≤ n - 2): The error signal is defined as
\[
e_i \triangleq x_{i+1} + u_i + f_i(x_i, x_{i+2}, \ldots, x_{i+n_1}),
\]  
(15)
where the virtual control law \( u_i \) can be designed as
\[
u_i \triangleq \dot{x}_{id}^i + \sum_{j=1}^{i-1} (-1)^j \sum_{j=1}^{i-t_e} k_j^i e_{i+1-j}
\]  
(16)
where the function \( \Psi_i(k_1, \ldots, k_{i-1}, e_2, \ldots, e_{i-1}) \) contains the terms utilized to cancel out the terms in expression of (16), and \( k_i \in \mathbb{R}^+ \) are control gains. Substituting the expression of \( u_i \) given in (17), in (16), the closed-loop dynamics of \( e_i \) is obtained as
\[
\dot{e}_i = e_{i+1} - k_i e_i + \Delta_i(k_1, \ldots, k_{i-1}, f_1, f_2, \ldots, f_i).
\]  
(18)
Consider the Lyapunov function candidate
\[
V_i = V_{i-1} + \frac{1}{2} e_i^T e_i
\]  
(19)
Differentiating (19), using (18), the derivative of \( V_i \) is obtained as
\[
\dot{V}_i = -\sum_{j=1}^{i} k_j \|e_j\|^2 + \sum_{j=1}^{i} e_j^T e_{j+1} + \sum_{j=1}^{i} e_j^T \Delta_j (f_1, \ldots, f_i)
\]  
(20)
Step n-1: Defining following error variables
\[
e_{n-1} \triangleq x_{n-1} - u_{n-2}
\]  
(21)
and
\[
e_n \triangleq x_n - u_{n-1} - e_\phi + u
\]  
(22)
where \( e_\phi \) is an auxiliary error signal, defined as
\[
e_\phi \triangleq \mu \int_{t-\tau}^{t} u(\phi) d\phi,
\]  
(23)
where \( \mu \in \mathbb{R}^+ \) is a known control gain. Using (1), (2) and (22), the error dynamics is given by differentiating (21) as
\[
\dot{e}_{n-1} = e_{n-1} + f_{n-1}(x_1, x_2, \ldots, x_{n-1}) - \dot{u}_{n-2} + u_{n-1} - e_\phi - u.
\]  
(24)
The virtual control input $u_{n-1}$ is defined as

$$u_{n-1} \triangleq x_{(n-1)}^1d + \sum_{l=1}^{n-1} (-1)^l \left( \sum_{j=1}^{n-l} k_j e_{n-l} \right) + \Delta e_{n-1} = e_n - e_{\phi} - u - k_{n-1}e_{n-1} + \Delta e_{n-1} (k_1, \ldots, k_{n-2}, f_1, f_2, \ldots, f_{n-1})$$

(25)

Substituting the expression of $u_{n-1}$ given in (25), in (24), the closed-loop dynamics of $e_{n-1}$ is obtained as

$$\dot{e}_{n-1} = e_n - e_{\phi} - u - k_{n-1}e_{n-1} + \Delta e_{n-1} (k_1, \ldots, k_{n-2}, f_1, f_2, \ldots, f_{n-1})$$

(26)

Consider the Lyapunov function candidate

$$V_{n-1} = V_{n-2} + \frac{1}{2} e_{n-1}^T e_{n-1}.$$  

(27)

Differentiating (27), using (26), the time derivative of $V_{n-1}$ is obtained as

$$\dot{V}_{n-1} = - \sum_{j=1}^{n-1} k_j e_j^2 + \sum_{j=1}^{n-1} e_{j+1}^T e_{j+1} + \sum_{j=1}^{n-1} \sum_{j=1}^{n} k_{j+1} e_j e_{n-j} - e_{n-1}^T e_{\phi} - e_{n-1}^T u$$

(28)

Step n: Differentiating (22), using (1), (23), the following equation is obtained as

$$\dot{e}_n = f_n (x (t), t) + g (x (t), t) u (t - \tau (t)) + d (t) + \mu u - \mu u (t - \bar{t}) + \dot{u} - u_{n-1}$$

(29)

where $\dot{u} \in \mathbb{R}^n$ is defined as

$$\dot{u} \triangleq - \mu u - ke_n$$

(30)

where $k \in \mathbb{R}^+$ is a known control gain. Adding and subtracting the term $g (x (t), t) u (t - \bar{t})$ in (29), using (25) and (30), cancelling the common terms, the closed-loop equation of error signal $e_n (t)$ is obtained as

$$\dot{e}_n = -(\mu - g (x (t), t)) u (t - \bar{t}) + g (x (t), t) \{ u (t - \tau (t)) - u (t - \bar{t}) \} + d (t) - \left( k - \sum_{j=1}^{n-1} k_j \right) e_n - \sum_{j=1}^{n-1} k_j e_\phi + \sum_{j=1}^{n-1} k_j u + \sigma_1 e_1 + \sigma_2 e_2 + \cdots + \sigma_{n-1} e_{n-1} + \Delta e_{n-1} (k_1, \ldots, k_{n-1}, f_1, \ldots, f_{n-1})$$

(31)

where $\sigma_1, \sigma_2, \ldots, \sigma_{n-1}$ are constant functions of virtual control gains $k_1, k_2, \ldots, k_{n-1}$. The corollary of the Mean Value Theorem (Theorem 5.19 in Rudin et al. (1964)), is utilized on $u (t - \bar{t}) - u (t - \tau (t))$ given in (31), yielding

$$\| u (t - \bar{t}) - u (t - \tau (t)) \| \leq | (\tau (t) - \bar{t}) | \| \dot{u} (t - \bar{t}) \|$$

(32)

where $\bar{t} (t) \in (\tau (t), \bar{t})$ is an auxiliary delay, defined as

$$\bar{t} (t) = \theta (t) + (1 - \theta) \bar{t}$$

(33)

where $\theta \in (0, 1)$ is a constant. Consider a Lyapunov-Krasovskii (LK) functional $V_n$ defined as

$$V_n = V_{n-1} + \frac{1}{2} e_{n-1}^T e_{n-1} + \frac{1}{2} u^T u + P + Q + R + S$$

(34)

where, $P, Q, R$ and $S$ are LK functionals defined as

$$P (t) \triangleq \xi \int_{t-\bar{t}(t)}^t \| \dot{u} (\phi) \|^2 d\phi$$

(35)

$$Q (t) \triangleq \frac{\omega_1}{2 r \mu} \int_{t-\bar{t}(t)}^t \int_{s}^t \| \dot{u} (\phi) \|^2 d\phi ds$$

(36)

$$R (t) \triangleq \frac{n + 6}{2 k^2} (\mu - g)^2 \int_{t-\bar{t}}^t \| u (\phi) \|^2 d\phi$$

(37)

$$S (t) \triangleq \frac{3}{2} \left( \sum_{j=1}^{n-1} k_j \right)^2 (\mu - g)^2 \int_{t-\bar{t}}^t \| u (\phi) \|^2 d\phi ds.$$  

(38)

Differentiating (34), using (28), (30) and (31), the derivative of $V_n$ is obtained as

$$\dot{V}_n = - \sum_{j=1}^{n-1} k_j e_j^2 + \sum_{j=1}^{n-1} e_{j+1}^T e_{j+1} + \sum_{j=1}^{n-1} \sum_{j=1}^{n} k_{j+1} e_j e_{n-j} - e_{n-1}^T e_{\phi} - e_{n-1}^T u$$

(39)

Using Assumption 3, $\Delta e_{n-1} (k_1, \ldots, k_{n-1}, f_1, \ldots, f_{n-1})$ in (39) can be upper bounded by the following inequality

$$\| \Delta e_{n-1} (k_1, \ldots, k_{n-1}, f_1, \ldots, f_{n-1}) \| \leq \gamma_{j_1} \| z \| + \gamma_{j_2}$$

(40)

where, $z \in \mathbb{R}^{n+2}$ is a vector, defined as

$$z \triangleq \left[ e_1^T e_2^T \cdots e_k^T u_n^T u_n^T \right]^T$$

(41)

and $\gamma_{j_1}, \gamma_{j_2}$ are known positive constants. Using the inequality given in (32) and (40), Assumption 2.4.5 in (39), the following inequality is obtained as

$$\dot{V}_n \leq - k_1 \| e_1 \|^2 - k_2 \| e_2 \|^2 - \cdots - k_{n-1} \| e_{n-1} \|^2 - \| e_{n-1} \|^2$$

(42)

Based on (30) and Young’s inequality i.e. $\| a \| \| b \| \leq \frac{1}{2} (\| a \|^2 + \| b \|^2)$, the following inequality is obtained as
\[ \| \dot{u}(t) \| \leq 2 \left( \mu^2 \| u(t) \| + k^2 \| e_n(t) \| \right). \tag{43} \]

Using (35)-(38), (43), applying Young’s inequality, the inequality \( f_{i-\tau} \left( \int_s^t |u(\phi)|^2 \, d\phi \right) \, ds \leq \tau \int_{i-\tau}^t |u(\phi)|^2 \, d\phi \), the Cauchy-Schwarz inequality \( \| e_\phi(t) \| \leq \mu^2 \| e_\phi(t) \| + \| \dot{e}(t) \| \), and square completion in certain terms of (42), yields

\[ \dot{V}_n \leq -\frac{1}{2} \left( k_1 - 1 - (n + 6) \frac{\sigma^2}{2k} \right) \| e_1 \|^2 + \frac{1}{2} \sum_{i=2}^{n-2} \left( k_i - 2 - (n + 6) \frac{\sigma^2}{2k} \right) \| e_i \|^2 \]

\[ - \frac{1}{2} \left( k_{n-1} - 3 - (n + 6) \frac{(\sigma_{n-1} + 1)^2}{2k} \right) \| e_{n-1} \|^2 \]

\[ - \left\{ \frac{k}{2(n + 6)} - \sum_{j=1}^{n-1} k_j - \frac{1}{2} \frac{n^2 + 2k^2}{2} \right\} \| e_n \|^2 \]

\[ -2 \left( \xi + \frac{\omega_1}{2k^2 \mu^2} \right) \| e_n \|^2 \]

\[ -\frac{1}{2} \left( k - 2 - \frac{n + 6}{2k} \right) \left( \sum_{j=1}^{n-1} k_j \right) - \frac{n + 6}{2k} \left( \mu - \frac{\bar{g}}{4} \right)^2 \]

\[ -3 \left( \sum_{j=1}^{n-1} k_j \right)^2 \left( \frac{n + 6}{2k} \right) \| e_n \|^2 \]

\[ -2 \left( \xi + \frac{\omega_1}{2k^2 \mu^2} \right) \| e_n \|^2 \]

\[ + \left( \sum_{i=1}^{n-1} \| \gamma_i \|^2 \right) \| z \|^2 + \frac{n + 6}{2k} \| \gamma_n \|^2 \| z \|^2 \]

\[ + \left( \sum_{i=1}^{n-1} \frac{\| \gamma_i \|^2}{k_i} \right) \| z \|^2 + \frac{n + 6}{2k} \| \gamma_n \|^2 \| z \|^2 \]

\[ - \left( \xi (1 - \Gamma') - \frac{1}{2k^2 \mu^2} \right) \| \dot{u}(t - \dot{\tau}(t)) \|^2 \]

\[ - \frac{\omega_1 (1 - \Gamma')}{4k^2 \mu^2} P(t) - \frac{1}{2\tau} Q(t) \]

\[ - \dot{\bar{\tau}} \left( \sum_{j=1}^{n-1} k_j \right)^2 \left( \frac{\mu}{\mu - \frac{\bar{g}}{4}} \right)^2 R(t) - \frac{1}{3\tau} S(t). \tag{44} \]

Provided following sufficient gain conditions

\[ k_1 > 1 + (n + 6) \frac{\sigma^2}{2k} \]

\[ k_i > 2 + (n + 6) \frac{\sigma^2}{2k}, \quad 2 \leq i \leq n - 2 \]

\[ k_{n-1} > 3 + (n + 6) \frac{(\sigma_{n-1} + 1)^2}{2k} \]

\[ \xi > \frac{1}{2(1 - \Gamma')k^2 \mu^2} \tag{45} \]

Using (34), it can be shown that

\[ V_n \leq \lambda \| y(t) \|^2 \tag{53} \]

where, \( \lambda \in \mathbb{R}^+ \) is a known constant and

\[ y(t) \triangleq \left[ z^T(t) \sqrt{P(t)} \sqrt{Q(t)} \sqrt{R(t)} \sqrt{S(t)} \right]^T \tag{54} \]

Further, the inequality in (51), yields

\[ \dot{V}_n \leq -\frac{\beta}{\lambda} V_n + \epsilon \tag{55} \]

where,

\[ \epsilon = \sum_{i=1}^{n-1} \frac{\| \gamma_i \|^2}{k_i} + \left( \frac{n + 6}{2k} \right) \left( \| \gamma_n \|^2 + d^2 \right) \tag{56} \]
and
\[
\bar{\beta} = \min \left[ \left( \beta - \sum_{i=1}^{n-1} \frac{\| \gamma_{i1} \|^2}{k_i} - \frac{n+6}{2k} \| \gamma_{n1} \|^2 \right) , \frac{1}{3\tau} \right]
\]
provided
\[
\beta > \sum_{i=1}^{n-1} \frac{\| \gamma_{i1} \|^2}{k_i} + \frac{n+6}{2k} \| \gamma_{n1} \|^2 .
\]

The solution of differential inequality in (55) can be obtained as
\[
V_n(t) \leq \left( V_n(0) - \frac{\lambda \epsilon}{\beta} \right) \exp \left( -\frac{\beta}{\lambda} t \right) + \frac{\lambda \epsilon}{\beta} .
\]

Using (34) & (59), the error \( e(t) \) can be expressed as
\[
\| e_1(t) \| \leq \sqrt{2 \left( V(0) - \frac{\lambda \epsilon}{\beta} \right) \exp \left( -\frac{\beta}{\lambda} t \right) + 2 \frac{\lambda \epsilon}{\beta} .
\]

The expression in (60) can be used to prove GUUB result. Based on (54), it is concluded that \( e_1(t), \ldots, e_n(t), e_d(t), u(t), P(t), Q(t), R(t), S(t) \in \mathcal{L}_\infty \). Since \( e_1(t), \ldots, e_n(t), e_d(t), u(t), x_d(t), x_d^{(i)}(t) \in \mathcal{L}_\infty \), (2) indicates that \( x_1(t), \ldots, x_1(t) \in \mathcal{L}_\infty \).

4. SIMULATION

4.1 Example 1

The dynamics of an input-delayed second-order nonlinear system is considered as
\[
\begin{align*}
\dot{x}_1 &= \theta_1 x_1 \sin(x_1) + x_2 \\
\dot{x}_2 &= \theta_2 x_1 x_2 + \theta_3 x_2 + 10 \left( 1 + 0.5 \cos^2(x_1) \right) u(t - \tau(t)) \\
&\quad + d
\end{align*}
\]
where \( \theta_1 = -1, \theta_2 = 0.1 \) and \( \theta_3 = -0.2, \tau(t) \in \mathbb{R}^+ \) is unknown input delay, \( u(t) \) is the control input and \( d(t) \) denotes the external disturbance given by \( d(t) = 0.1 \sin(t) \). The desired trajectory is considered as \( x_d(t) = 5 \sin(t/4) \).

Table 1. Tracking errors for different values of delay.

| Case | Input delay \( \tau \) (in sec.) | \( k \) | \( \epsilon_1 \) | \( \mu \) | RMS value of error \( |x_1 - x_{1,d}| \) |
|------|-------------------------------|-------|--------|--------|----------------|
| 1    | 20 sin(2\pi t)+22              | 200   | 120    | 30     | 0.0584 |
| 2    | 40 sin(2\pi t)+44              | 30    | 6      | 0.1    | 0.4398 |

The simulation shows the performance of controller given in (30) for the different values of sinusoidal delay i.e. Case 1 and 2 given in Table 1. The delay bound of 100 msec is fixed to obtain the errors and control response shown in Fig.1 for the dynamics given in (61). It is observed that the tracking performance depends upon the value of input delay. As the delay increases, the errors increase, however, remains bounded. The control gains \( k, k_1, \mu \) and RMS value of errors are mentioned in the Table 1. The initial values of states are chosen in simulation as \( [0.3, 0.1] \).

4.2 Example 2

The mathematical model of the motion control of a one link manipulator with time-varying input delay is given by Gao et al. (2016)
\[
\begin{align*}
\left\{ \begin{array}{l}
\dot{q} + b q + N \sin(q) = T + T_d \\
L \ddot{q} + R \dot{q} = u(t - \tau(t) - k_m q)
\end{array} \right.
\end{align*}
\]
where, \( q, \dot{q} \) and \( \ddot{q} \) denote the link position, velocity and acceleration, respectively, \( J = 1Kg.m^2 \) is the mechanical inertia, \( b = 1Nm.sec/rad \) is the viscous friction coefficient, \( N = 10 \) is a positive constant, \( T \) is the torque, \( T_d \) represents the additive disturbance, \( L = 1H \) and \( R = 1 \) are the armature inductance and resistance of the motor, \( K_m = 0.2Nm/A \) is the back electromotive force coefficient, \( u(t - \tau(t)) \) is the delayed control input which denotes the electromechanical torque, and \( \tau(t) \) is the time-varying input delay. The desired trajectory \( q_d(t) = \sin(t) + \sin(0.5t) \) rad. The time-varying input delay and the disturbance term are chosen as \( \tau(t) = 10 \sin(0.5t) + 30ms \) and \( T_d = \sin(t) Nm \), respectively.

The dynamics given in (62) can be rewritten in following form as
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
J \ddot{x}_2 &= x_3 - bx_2 - N \sin(x_1) + T_d \\
L \ddot{x}_3 &= u(t - \tau(t) - k_m x_2 - R x_3
\end{align*}
\]
where, state variables are defined as \( [x_1 \ x_2 \ x_3]^T \triangleq [q \ \dot{q} \ \ddot{q}]^T \). The Simulation is performed on (63), choosing initial states as \( [x_1(0) \ x_2(0) \ x_3(0)] = [0.3 \ 0.1 \ 0.5] \) and control gains \( k = 50, k_1 = 20, k_2 = 6 \) and \( \mu = 1 \) in (30), the tracking error and control input responses are obtained as shown in Fig. 2.

5. CONCLUSION

This paper extends the results of Jain and Bhasin (2019) for a class of uncertain nonlinear systems in strict-feedback form with unknown time-varying input delay. The upper bound on unknown time-varying input delay and lower bounds on control gains are determined in stability analysis by choosing suitable L-K functionals to prove global uniformly ultimately bounded tracking. Simulation results with different values of the input delay demonstrate the effectiveness of the proposed controller.
Fig. 2. Tracking error and control input response for the delay bound of 100 msec.

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