Joint target detection and tracking with heavy-tailed noises

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Abstract. This paper investigates the problem of simultaneous target detection and tracking with heavy-tailed noises under Bernoulli filtering framework. By exploiting the Student-t distributions to formulate the outlier-corrupted process noise and measurement noise, a novel target tracking algorithm named the Student-t mixture Bernoulli filter is proposed. Under linear assumptions on target dynamic model, sensor measurement model and birth process, closed-form recursions that propagate the parameters of Bernoulli random finite set are derived. The validity of the proposed algorithm is verified by numerical simulations.

1. Introduction
In practical tracking scenarios, the target can appear in and disappear from the scene randomly. Often, exact knowledge of target existence is unavailable due to false alarms. Furthermore, spurious measurements are received by the sensor when the target does not present in the scene. Again, when the target enters the surveillance region, it is difficult to detect and track it rapidly. Hence, there is an urgent need to design a filter that possesses the ability to detect and track the target, simultaneously. Originated from random finite set (RFS) theory, the Bernoulli filter [1] provides an exact Bayesian solution to the problem of simultaneous detecting and tracking a single target in the complex environment involving measurement uncertainty and false alarms. In the Bernoulli filter, system noises, i.e., process noise and measurement noise, are commonly supposed to obey the Gaussian distributions. However, in practical applications, e.g., tracking an agile target with unstable sensor, outliers may simultaneously occur in the target dynamic model and sensor measurement model due to target abrupt manoeuvring and sensor failure. In such cases, the standard Bernoulli filter demonstrates unsatisfactory performance. Various methods [2-3] have been provided to describe the outlier-corrupted noise in the literature. Among them, the heavy-tailed Student-t distribution [4] is considered as the most effective means. In [5-7], several robust estimators have been developed to estimate the state as well as noise parameters. However, these estimators are based on a premise that the process noise is well-behaved. In [8], a novel Student-t filter was proposed for single target tracking under heavy-tailed system noises. However, this approach does not take random appearance/disappearance switching of target into consideration. To the best of the authors’ knowledge, the Student-t distribution based Bernoulli filter has not been reported in the literature to solve the problem of joint target detection and tracking, which is the motivation of this work.

This paper proposes a novel tracking algorithm named the Student-t mixture Bernoulli (STMB) filter where the Student-t distributions are employed to describe the statistical characteristics of heavy-tailed process noise and measurement noise. Under linear Student-t assumptions on target dynamic model, sensor measurement model and birth process, closed-form recursions that propagate the parameter set of Bernoulli RFS are derived. Since the degree of freedom (DOF) parameter of Student-t
component increases as time progresses, we make use of the moment matching method to solve this issue. Moreover, a modified pruning and merging strategy is adopted to further improve the computational efficiency. Simulation experiments are provided to illustrate the capability of the STMB filter. The rest of this paper is outlined as follows. Section 2 formulates the system models and reviews the Bernoulli filter. Section 3 details the proposed STMB filter. Numerical studies are provided in Section 4. Conclusions are given in Section 5.

2. Problem formulation

2.1 Tracking model
Consider the following time-evolution models of target and measurement

\[
x_k = F_k x_{k-1} + q_k
\]

\[
z_k = H_k x_k + r_k
\]

where \( x_k \in \mathbb{R}^d \) represents the state vector, \( z_k \in \mathbb{R}^d \) represents the measurement vector, \( F_k \) denotes the state transition matrix, \( H_k \) denotes the observation matrix, \( q_k \) indicates the process noise vector and \( r_k \) indicates the measurement noise vector. It is assumed that \( q_k \) and \( r_k \) are independent of each other and satisfy

\[
p(q_k) = S(q_k; 0, Q_k, \eta_1)
\]

\[
p(r_k) = S(r_k; 0, R_k, \eta_2)
\]

where \( S(\cdot; \mu, \Sigma, \eta) \) denotes a Student-t probability density function (PDF) with mean \( \mu \), scale matrix \( \Sigma \) and DOF parameter \( \eta \). The tail characteristic of a Student-t PDF is mainly determined by its DOF parameter \( \eta \). When \( \eta \rightarrow \infty \), the Student-t PDF \( S(\cdot; \mu, \Sigma, \eta) \) to the Gaussian PDF \( N(\cdot; \mu, \Sigma) \).

2.2 The Bernoulli filter
In the Bernoulli filter, the Bernoulli RFS is a fundamental element. Mathematically, a Bernoulli RFS \( X \) is characterized by two parameters, i.e., the target existence probability \( r \) and the spatial density \( p \).

The finite set statistics (FISST) \(^{[9]}\) PDF of \( X \) is

\[
\pi(X) = \begin{cases} 
1 - r & X = \emptyset \\
0 & |X| \geq 2 \\
r \cdot p(x) & X = \{x\}.
\end{cases}
\]

In the following, we summarize the time prediction and measurement update stages of the Bernoulli filter. Let \( \pi_{k-1} = \{r_{k-1}, p_{k-1}\} \) denotes the posterior density at time \( k - 1 \), then the time predicted density at time \( k \) is \( \pi_{k|k-1} = \{r_{k|k-1}, p_{k|k-1}\} \) where

\[
r_{k|k-1} = p_{b,k}(1 - r_{k-1}) + r_{k-1}\int p_{S,k}(x)p_{k-1}(x)dx
\]

\[
p_{k|k-1}(x) = \frac{p_{b,k}(1 - r_{k-1})}{r_{k|k-1}}b_k(x) + \frac{r_{k-1}}{r_{k|k-1}}\int p_{S,k}(x')\phi_{k|k-1}(x|x')p_{k-1}(x')dx'
\]

\( p_{b,k} \) is the probability of spontaneous birth, \( b_k(\cdot) \) is the birth intensity, \( \phi_{k|k-1}(\cdot|\cdot) \) denotes the state transform density and \( p_{S,k}(\cdot) \) represents the target survival probability. Furthermore, the measurement update density at time \( k \) is \( \pi_k = \{r_k, p_k\} \), where

\[
r_k = \frac{1 - \Psi_k}{1 - r_{k|k-1}}r_{k|k-1}
\]
\[\Psi_k = \int p_{D,k}(x)p_{z,k-1}(x)dx - \sum_{k \in z_k} \int p_{D,k}(x)\phi_k(z|x)p_{z,k-1}(x)dx\]

(9)

\[p_k(x) = \frac{1 - p_{D,k}(x) + p_{D,k}(x)\sum_{k \in z_k} \phi_k(z|x)/\lambda c(z)}{1 - \Psi_k} p_{k,k-1}(x)\]

(10)

\[p_{D,k}(\cdot)\] indicates the detection probability, \(\phi_k(\cdot | \cdot)\) is the measurement likelihood, \(Z_k\) represents the collection of observations at time \(k\), \(\lambda\) is the clutter rate (expected number of clutter per scan), and \(c(z)\) is the spatial density of clutter.

3. Student-t mixture Bernoulli filter

In this section, we present the main results of this paper, i.e., the STMB filter. Analogous to the Gaussian mixture Bernoulli (GMB) filter \[1\], certain assumptions on target dynamic model, sensor measurement model and birth process are needed. These are summarized as follows:

A1. The time-evolution models for state and measurement are linear Student-t, i.e.,

\[\phi_{k,k-1}(x | x') = S(x; F_{k-1}, Q_{k-1}, \eta_{k-1})\]

(11)

\[\phi_k(z | x) = S(z; H_k, R_k, \eta_z)\]

(12)

A2. The target survival probability and sensor detection probability are state-independent, i.e.,

\[p_{S,k}(x) = p_{S,k} \cdot p_{D,k}(x) = p_{D,k}\]

(13)

A3. The birth intensity has a Student-t mixture form

\[b_k(x) = \sum_{i=1}^{J_k} w_{i,k}^{(i)} S(x; m_{i,k}^{(i)}, P_{i,k}, \eta_{i,k}^{(i)})\]

(16)

A4. The joint PDF of posterior state and process noise obeys a Student-t distribution, i.e.,

\[p(x_{k-1}, q_{k-1} | Z_{k-1}) = S\left(\begin{bmatrix} x_{k-1} \\ q_{k-1} \end{bmatrix}; \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} P_{k-1} & 0 \\ 0 & Q_{k-1} \end{bmatrix}, \eta_{k-1}\right)\]

(14)

A5. The joint PDF of predicted state and measurement noise obeys a Student-t distribution, i.e.,

\[p(x_k, q_k | Z_{k-1}) = S\left(\begin{bmatrix} x_k \\ r_k \end{bmatrix}; \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} P_{k|k-1} & 0 \\ 0 & R_k \end{bmatrix}, \eta_{k|k-1}\right)\]

(15)

Next, we details the recursions and implementation issues of the proposed STMB filter.

If the posterior density is given by \(\pi_{k-1} = \{r_{k-1}, p_{k-1}\}\) and \(p_{k-1}\) has a Student-t mixture form

\[p_{k-1}(x) = \sum_{i=1}^{J_k} w_{i,k}^{(i)} S(x; m_{i,k}^{(i)}, P_{i,k}, \eta_{i,k}^{(i)})\]

(16)

Then, the predicted density \(\pi_{k|k-1} = \{r_{k|k-1}, p_{k|k-1}\}\) is calculated by

\[r_{k|k-1} = p_{0,k}(1 - r_{k-1}) + r_{k-1} p_{S,k}\]

(17)

\[p_{k|k-1}(x) = \frac{p_{S,k}(1 - r_{k-1})}{r_{k|k-1}} b_k(x) + \frac{r_{k-1}}{r_{k|k-1}} \sum_{i=1}^{J_k} w_{i,k}^{(i)} S(x; m_{i,k-1}^{(i)}, P_{i,k-1}^{(i)}, \eta_{i,k-1}^{(i)})\]

(18)

where \(P_{k|k-1}^{(i)} = Q_{k-1} + F_{k-1} P_{k-1}^{(i)} F_{k-1}^T\), \(m_{k|k-1}^{(i)} = F_{k-1} m_{k-1}^{(i)}\) and \(\eta_{k|k-1}^{(i)} = \eta_{k-1}^{(i)}\).

If the predicted density is given by \(\pi_{k|k-1} = \{r_{k|k-1}, p_{k|k-1}\}\) and \(p_{k|k-1}\) has a Student-t mixture form

\[p_{k|k-1}(x) = \sum_{j=1}^{J_k} w_{j,k}^{(j)} S(x; m_{j,k-1}^{(j)}, P_{j,k-1}^{(j)}, \eta_{j,k-1}^{(j)})\]

(19)

Then, the update density \(\pi_k = \{r_k, p_k\}\) is calculated by
\[ r_k = \frac{1 - \Psi_k^{(j)}}{1 - \Psi_k^{(j)} - r_{k-1}} \]  

\[ p_k(x) = \frac{(1 - p_{D,k})}{1 - \Psi_k} p_{k-1}(x) + \frac{p_{D,k}}{1 - \Psi_k} \sum_{j=1}^{J_k} \sum_{i=1}^{J_{k-1}} \frac{v^{(j)}_k(z)}{\lambda(c)} S(x; m^{(j)}_k(z), P^{(j)}_k, \eta^{(j)}_k) \]  

\[ \Psi_k = p_{D,k} \left[ 1 - \frac{1}{1 - \Psi_k} \sum_{j=1}^{J_k} \sum_{i=1}^{J_{k-1}} \frac{v^{(j)}_k(z)}{\lambda(c)} \right] \]  

where \( z^{(j)}_k = S(z; z^{(j)}_{k-1}, S^{(j)}_{k-1}, \eta^{(j)}_{k-1}) \), \( \xi_k^{(j)} = H_k m^{(j)}_{k-1} - H_k P^{(j)}_{k-1} H_k^T + R_k \), \( \eta_k^{(j)} = \eta_{k-1}^{(j)} + d_z \),  

\[ (\Omega_k^{(j)})^2 = (z - z^{(j)}_{k-1})^T S^{(j)}_{k-1}^{-1} (z - z^{(j)}_{k-1}) \],  

\[ m_k^{(j)} = m^{(j)}_{k-1} + K_k^{(j)} (z - z^{(j)}_{k-1}) \],  

\[ K_k^{(j)} = P^{(j)}_{k-1} H_k^T S^{(j)}_{k-1}^{-1} \]  

and \( P_k^{(j)} = \eta_k^{(j)} + (\Omega_k^{(j)})^2 \left[ 1 - K_k^{(j)} H_k \right] P_k^{(j)} \)  

Since the DOF parameter \( \eta_k^{(j)} \) will increase infinitely as time progresses, we adopt the moment matching approach \[8\] to replace \( S(x; m_k^{(j)}(z), P_k^{(j)}, \eta_k^{(j)}) \) by \( S(x; \tilde{m}_k^{(j)}(z), \tilde{P}_k^{(j)}, \eta_k^{(j)}) \) where  

\[ \tilde{m}_k^{(j)} = m_k^{(j)}, \tilde{P}_k^{(j)} = \frac{(\eta_k^{(j)} - 2)\eta_k^{(j)}}{(\eta_k^{(j)} - 2)\eta_k^{(j)} - 1} P_k^{(j)} \]  

Like the GMB filter, pruning and merging are needed to restrict the quantity of Student-t components.

The difference lies in that the true covariance matrix \( \frac{\eta}{\eta - 2} P(\eta > 2) \) rather than the scale matrix \( P \) should be used in the merging step.

4. Simulation results

In this section, numerical experiments are designed to verify the capability of the proposed STMB filter. The GMB filter is selected as the benchmark. We exploit the Optimal Sub-pattern Assignment (OSPA) distance \[10\] to evaluate the performance of the two filters. The order parameter and cut-off parameter employed in the OSPA distance are respectively set as \( p = 1 \) and \( c = 100 \).

The tracking scenario is configured as follows. A target is observed over the 2-dimensional plane area \([-1000,1000]m \times [-1000,1000]m \). The target appears in the scene at time \( k = 10s \) and disappears from the scenario at time \( k = 79s \). The state vector \( x_k = [p_x, p_y, \bar{p}_x, \bar{p}_y]^T \) consists of position information \([p_x, p_y]\) and velocity information \([\bar{p}_x, \bar{p}_y]\). The dynamic and sensor models are specified by  

\[ F_k = \begin{bmatrix} I_2 & \delta I_2 \\ 0_2 & I_2 \end{bmatrix}, \quad Q_0 = \sigma^2 \begin{bmatrix} \delta I_2/4 & \delta I_2/2 \\ \delta I_2/2 & \delta I_2 \end{bmatrix} \]  

\[ H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad R_0 = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \]  

where \( \delta = 1s, \sigma = 5m/s, I_2 \) and \( 0_2 \) respectively denote \( 2 \times 2 \) identity matrix and zero matrix. The outlier-contaminated process noise and measurement noise are generated by  

\[ q_k \sim N(0, Q_0) w.p. 1 - \tau, \quad r_k \sim N(0, R_0) w.p. 1 - \tau \]  

\[ q_k \sim N(0, 100Q_0) w.p. \tau, \quad r_k \sim N(0, 100R_0) w.p. \tau \]  

\[ (24) \]
where \( w.p. \) denotes "with probability" and \( \tau \) denotes the probability of outlier occurrence. The birth processes of the STMB filter and the GMB filter are respectively expressed as \( p_{b,k}^{\text{STMB}} = p_{b,k}^{\text{GMB}} = 0.02 \),
\[
b_k^{\text{STMB}}(x) = S(x; \mathbf{m}_{b,k}, \mathbf{P}_{b,k}, \eta_{b,k}),
b_k^{\text{GMB}}(x) = S(x; \mathbf{m}_{b,k}, \mathbf{P}_{b,k})
\]
where \( \mathbf{m}_{b,k} = [-100, 0, 100, 0] \), \( \eta_{b,k} = 8 \) and \( \mathbf{P}_{b,k} = \text{diag}([100, 100, 25, 25]^T) \). The detection probability and survival probability are set as 0.9 and 0.95, respectively. It is assumed that the clutter rate \( \lambda \) is 20. Moreover, the parameters used in pruning and merging scheme are \( T = 10^{-3}, U = 4 \) and \( J_{\max} = 100 \).

We begin with a comparison between the STMB filter and the GMB filter under heavy-tailed noises. To this end, \( \tau \) is set as 0.15 and 100 Monte Carlo (MC) trials are performed. At each trail, true trajectory of target is fixed while the measurements are randomly generated. The estimated target existence probability of the two filters is shown in figure 1. Figure 2 presents the mean OSPA distances of the two filters. It can be seen from figures 1 and 2 that the GMB filter exhibits significant performance degradation under heavy-tailed noises. The reason for this observation is the inaccurate estimation of target existence probability shown in figure 1, which is caused by the lightweight tail property of Gaussian distribution. Consequently, the GMB filter exhibits higher OSPA distance than the STMB filter. On the contrary, the proposed STMB filter has an advantage over the GMB filter due to the tail characteristic of Student-t distribution. When outliers exist, the Student-t based noise models

![Figure 1. Estimated existence probability of the GMB and STMB filters.](image1)

![Figure 2. Time-averaged OSPA distances of the GMB and STMB filters.](image2)

![Figure 3. OSPA distances of the GMB and STMB filters with different clutter rate.](image3)

![Figure 4. OSPA distances of the GMB and STMB filters with different outlier probability.](image4)
is capable of matching the non-Gaussian noises well, resulting in a more accurate target existence probability estimate and a lower OSPA distance.

For further verifying the capability of the proposed STMB filter, 100 MC trials are performed for the STMB and GMB filters with varying clutter rate $\lambda$ and outlier probability $\tau$. Firstly, $\tau$ is set as 0.15, while $\lambda$ varies from 10 to 50. The OSPA distances of the two filters with different $\lambda$ are illustrated in figure 3. Then, $\lambda$ is set as 20, while $\tau$ varies from 0.05 to 0.30. Figure 4 plots the OSPA distances of both filters with different $\tau$. The results in figures 3 and 4 indicate that the STMB filter has strong robustness to outliers, especially for high outlier probability.

5. Conclusion
Aiming at the problem of performance degradation of the Bernoulli filter under heavy-tailed noises, this paper proposes a novel target tracking algorithm, i.e., the STMB filter. In the STMB filter, the Student-t distributions are employed to formulate outlier-corrupted process and measurement noises. Closed-form recursions are derived under linear Student-t assumptions on target dynamics and birth process. Simulation results demonstrate that the STMB filter can effectively suppress the interference generated by outliers and achieve accurate tracking of the target.

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