Abstract

The problem of wave disturbance propagation in rarefied gas in gravity field is explored. The system of hydrodynamic-type equations for a stratified gas in gravity field is derived from BGK equation by method of piecewise continuous partition function. The obtained system of the equations generalizes the Navier-Stokes at arbitrary density (Knudsen numbers). The verification of the model is made for a limiting case of a homogeneous medium. Results are in the good agreement with experiment and former theories at arbitrary Knudsen numbers.

Introduction

There is a significant number of problems of gas dynamics at which it is necessary to use the mathematical apparatus beyond the limits of traditional hydrodynamics of Navier-Stokes. The hydrodynamics is valid under the condition for the Knudsen number \( Kn = l/L \ll 1 \), where \( l \) is a mean free path, and \( L \) is a characteristic scale of inhomogeneity of a problem under consideration. The first work, in which wave perturbations of a gas were investigated from the point of view of more general kinetic approach, perhaps, is the paper of Wang of Chang and Uhlenbeck [1]. Most consistently these ideas are formulated in the work of Foch and Ford [2]. Such general theory could be based on some kinetic approach, i.e. Boltzmann equation.

Numerous researches on a sound propagation in a homogeneous gas at arbitrary Knudsen numbers were made [4] - [12]. The investigations have shown, that at arbitrary Knudsen numbers the behaviour of a wave differs considerably from ones predicted on a basis of hydrodynamical equations of Navier - Stokes. These researches have revealed two essential features: first, propagating perturbations keep wave properties at larger values of \( Kn \), than it could be assumed on the basis of a hydrodynamical description. Secondly, at \( Kn \geq 1 \) such concepts as a wave vector and frequency of a wave become ill-determined.

The case, when Knudsen number \( Kn \) is non-uniform in space or in time is more difficult for investigation and hence need more simplifications in kinetic equations or their model analogues. A constructions of such approaches for analytical solutions based on kinetic equation Bhatnagar – Gross – Krook (BGK) of Gross-Jackson [3] in a case of exponentially stratified gas were considered at [28, 29].

In this paper we would develop and generalize the method of a piecewise continuous partition functions [28, 29] to take into account the complete set of nonlinearities (\(?\)). We consider the example of wave perturbations theory for a gas stratified in gravity field so that the Knudsen number exponentially depends on the (vertical) coordinate.
1 Piecewise continuous partition function method

The kinetic equation with the model integral of collisions in BGK form looks like:

\[
\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{r}} - q \frac{\partial f}{\partial v_z} = \nu (f_l - f) \tag{1}
\]

here \( f \) is the distribution function of a gas, \( t \) is time, \( \vec{v} \) is velocity of a particle of a gas, \( \vec{r} \) is coordinate,

\[
f_l(\vec{r}, \vec{v}, t) = \frac{n}{\pi^{3/2} v_T^3} \exp \left( -\frac{(\vec{v} - \vec{U})^2}{v_T^2} \right)
\]

is the local-equilibrium distribution function, \( H = kT/mg \) is the so-called height of a homogeneous atmosphere - a parameter of the gas stratification, \( v_T = \sqrt{2kT/m} \) is the average thermal speed of movement of particles of gas, \( \nu = \nu_0 \exp(-z/H) \) is the effective frequency of collisions between particles of gas at height \( z \). It is supposed, that density of gas \( n \), its average speed \( \vec{U} = (u_x, u_y, u_z) \) and temperature \( T \) are functions of time and coordinates.

Following the idea of the method of piecewise continuous distribution functions let’s search for the solution \( f \) of the equations (2) as combinations of two locally equilibrium distribution functions, each of which gives the contribution in its own area of velocities space:

\[
f(t, \vec{r}, \vec{V}) = \begin{cases} 
  f^+ = n^+ \left( \frac{m}{2\pi k T^+} \right)^{3/2} \exp \left( -\frac{m(\vec{V} - \vec{U}^+)^2}{2kT^+} \right), & v_z \geq 0 \\
  f^- = n^- \left( \frac{m}{2\pi k T^-} \right)^{3/2} \exp \left( -\frac{m(\vec{V} - \vec{U}^-)^2}{2kT^-} \right), & v_z < 0
\end{cases} \tag{2}
\]

here \( n^\pm, U^\pm, T^\pm \) are parameters of locally equilibrium distributions functions. Geometry of break, that is the area, in which various functions operate, is determined by geometry of a problem.

Thus, a set of the parameters determining a state of the perturbed gas is increased twice. The increase of the number of parameters of distribution function (2) results in that the distribution function generally differs from a local-equilibrium one and describes deviations from hydrodynamical regime. In the range of small Knudsen numbers \( l \ll L \) we have \( n^+ = n^-, U^+ = U^-, T^+ = T^- \) and distribution function (2) tends to local-equilibrium one, reproducing exactly the hydrodynamics of Navier-Stokes. In the range of big Knudsen numbers the formula (2) gives solutions of collisionless problems. Similar ideas have resulted successfully in a series of problems. For example, in papers [18] - [20] a method of piecewise continuous partition function was demonstrated for the description of flat and cylindrical (neutral and plasma) Kuette flows [18] - [20]. Thus for a flat problem the surface of break in the velocity space was determined by a natural condition \( V_z = 0 \), and in a cylindrical case \( V_r = 0 \), where \( V_z \) and \( V_r \) are, accordingly, vertical and radial components of velocity of particles. Similar problem was solved by perturbations caused by pulse movement of plane [20, 21]. Solving a problem of of a shock wave structure [20, 22, 23] the solution was represented as a combination of two locally equilibrium functions, one of which determines the solution before front of a wave, and another - after. In the
problem of condensation/evaporation of drops of a given size [24, 25] a surface break was determined by so-called "cone of influence", thus all particles were divided into two types: flying "from a drop" and flying "not from a drop".

The similar approach was developed for a description of a nonlinear sound in stratified gas [26, 29].

The idea of a method of two-fold distribution functions given by (2) is realized as follows. Let's multiply equation BGK (1) on a set of linearly independent functions. In the one-dimensional case $\bar{U} = (0, 0, U_z)$ the following set is used:

$$\varphi_1 = m, \quad \varphi_4 = m(V_z - U_z)^2, \quad \varphi_2 = mV_z, \quad \varphi_5 = \frac{1}{2}m(V_z - U_z)|\bar{V} - \bar{U}|^2, \quad \varphi_3 = \frac{1}{2}m(V_z - U_z)^3. \tag{3}$$

Let's define a scalar product:

$$< \varphi_n, f > = \int d\vec{v} \varphi_n(t, z, \bar{V})f(t, z, \bar{V}). \tag{4}$$

$$< \varphi_1 > = m < 1 > = \rho(t, z), \quad < \varphi_4 > = m < (V_z - U_z)^2 > = P_{zz}, \quad < \varphi_2 > = m < V_z > = \rho U_z, \quad < \varphi_5 > = \frac{1}{2}m < (V_z - U_z)|\bar{V} - \bar{U}|^2 > = q_z, \quad < \varphi_3 > = \frac{1}{2}m < (V_z - U_z)^3 > = \bar{q}_z. \tag{5}$$

Here $\rho$ is density, $\rho U_z$ is specific momentum, $e$ is internal energy per unit mass of the gas, $P_{zz} = P + \pi_{zz}$ is the diagonal component of the strain tensor ($P$ is pressure, $\pi_{zz}$ is component of strain tensor), $q_z$ is a vertical component of a heat flow, $\bar{q}_z$ is the new parameter having meaning of a heat flow.

Multiplying equation (1) on eigen functions (3) we obtain the system of differential equations:

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial z}(\rho U_z) = 0, \quad \frac{\partial}{\partial t}U_z + U_z \frac{\partial}{\partial z}U_z + \frac{1}{\rho} \frac{\partial}{\partial z}(P + \Pi_{zz}) + g = 0, \quad \frac{\partial}{\partial t}e + U_z \frac{\partial}{\partial z}e + (e + P + \Pi_{zz}) \frac{\partial}{\partial z}U_z + \frac{\partial}{\partial z}q_z = 0, \quad \frac{\partial}{\partial t}(P + \Pi_{zz}) + U_z \frac{\partial}{\partial z}(P + \Pi_{zz}) + 3(P + \Pi_{zz}) \frac{\partial}{\partial z}U_z + 2 \frac{\partial}{\partial z}\bar{q}_z = -\nu(\Pi_{zz} + P - \rho \theta), \quad \frac{\partial}{\partial t}q_z + U_z \frac{\partial}{\partial z}q_z + 2(q_z + \bar{q}_z) \frac{\partial}{\partial z}U_z - \frac{1}{\rho}(e + P + \Pi_{zz}) \frac{\partial}{\partial z}(P + \Pi_{zz}) + \frac{\partial}{\partial z}J_1 = -\nu q_z, \quad \frac{\partial}{\partial t}\bar{q}_z + U_z \frac{\partial}{\partial z}\bar{q}_z + 4\bar{q}_z \frac{\partial}{\partial z}U_z - \frac{3}{2\rho}(P + \Pi_{zz}) \frac{\partial}{\partial z}(P + \Pi_{zz}) + \frac{\partial}{\partial z}J_2 = -\nu \bar{q}_z, \tag{6}$$

where $$J_1 = < (V_z - U_z)^2(\bar{V} - \bar{U})^2 >, \quad J_2 = < (V_z - U_z)^4 >. \tag{7}$$
The obtained system of the equations according to the derivation scheme is valid at all frequencies of collisions and within the limits of high frequencies should transform to the hydrodynamic equations. It is a system of hydrodynamical type and generalizes the classical equations of a viscous fluid on any density, down to a free-molecule flow. However, the system is not closed yet. It is necessary to add equations of state \( P = P(\rho, T) \) and \( e = e(\rho, T) \). Except for that it is necessary to present values of two integrals \( J_1 \) and \( J_2 \) as functions of thermodynamic parameters of the system.

Let's evaluate integrals directly, plugging the function (2). We estimate the functions as small, that corresponds to small Mach numbers \( M = max|\vec{v}|/v_T \). Values of integrals \( J_1 \) and \( J_2 \) within the specified approximation looks as

\[
J_1 = \frac{5}{16} (n^+V_T^4 + n^-V_T^{-4}) + \frac{3}{2\sqrt{\pi}} [n^+V_T^{-3}(U^+ - U) - n^-V_T^{-3}(U^- - U)], \\
J_2 = \frac{3}{16} (n^+V_T^4 + n^-V_T^{-4}) + \frac{1}{\sqrt{\pi}} [n^+V_T^{-3}(U^+ - U) - n^-V_T^{-3}(U^- - U)].
\]

(8)

Let's express parameters of the two-fold distribution function through the thermodynamic ones and substitute the result into the expression (8). To solve the specified problem we shall use a method of perturbations with the small parameter \( \max U \), expressing

\[
U^+ = -\frac{\sqrt{2\pi}}{4} \sqrt{e}(P_{zz} - \frac{2}{3}e) + \frac{1}{10\rho}(5ue + 3q), \\
\rho^+ = \rho + \frac{3e}{4\rho}(P_{zz} - \frac{2}{3}e) + \frac{3}{20\rho}\sqrt{3\pi}(\overline{e})^{3/2}q, \\
U^- = \frac{\sqrt{2\pi}}{4} \sqrt{e}(P_{zz} - \frac{2}{3}e) + \frac{1}{10\rho}(5ue + 3q), \\
\rho^- = \rho + \frac{3e}{4\rho}(P_{zz} - \frac{2}{3}e) - \frac{3}{20\rho}\sqrt{3\pi}(\overline{e})^{3/2}q, \\
V_T^+ = 2\sqrt{\frac{3e}{4\rho}} - \frac{1}{12}\sqrt{\frac{3}{\rho e}}(P_{zz} - \frac{2}{3}e) - \frac{3q}{10e}\sqrt{\pi}, \\
V_T^- = 2\sqrt{\frac{3e}{4\rho}} + \frac{1}{12}\sqrt{\frac{3}{\rho e}}(P_{zz} - \frac{2}{3}e) + \frac{3q}{10e}\sqrt{\pi}.
\]

(9)

Plugging the values of (9) into (8) one obtain the values of \( J_{1,2} \) in the first order:

\[
J_1 = \frac{10}{9} \frac{e^2}{\rho} + \frac{61e}{18\rho} \left( P_{zz} - \frac{2}{3}e \right), \\
J_2 = \frac{2}{3} \frac{e^2}{\rho} + \frac{13e}{6\rho} \left( P_{zz} - \frac{2}{3}e \right).
\]

(10)

2 Limiting case of gas oscillations at high frequencies of collisions (small Knudsen numbers).

Let us consider a system in the hydrodynamical limit (\( \nu \to \infty \)). It follows from the last three equations of the system (6) that the orders of values relate as \( \max \{\Pi_{zz}, q_z, \bar{q}_z\} \sim \nu^{-1} \max \{\rho, U_z, e, P\} \). Next assume \( \nu^{-1} = 0 \) in the zero order by the parameter \( \nu^{-1} \). One hence have \( \Pi_{zz} = 0, q_z = 0, \bar{q}_z = 0 \) at the l.h.s. and at the r.h.s. of forth equation of the system \( P = \rho \theta = 2e/3 \). Substituting mentioned limits in the first three equations of the
system (6) we obtain a system of Euler equations of a liquid in gravity field:

\[
\begin{align*}
\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial z}(\rho U_z) &= 0, \\
\frac{\partial}{\partial t}U_z + U_z \frac{\partial}{\partial z}U_z + 2 \frac{\partial}{\partial z}e + g &= 0, \\
\frac{\partial}{\partial t}e + U_z \frac{\partial}{\partial z}e + \frac{5}{3}e \frac{\partial}{\partial z}U_z &= 0.
\end{align*}
\]

(11)

The functions \(\{\Pi_{zz}, q_z, \bar{q}_z\} \sim \nu^{-1}\{\rho, U_z, e, P\}\) belong to the next order of the parameter \(\nu^{-1}\). Then from the last three equations of the system, (6) taking into account the equation of state \(P = \rho\theta\), one obtains following relations

\[
\begin{align*}
\pi_{zz} &= -\frac{8}{9\nu(z)}e \frac{\partial}{\partial z}U_z, \\
q_z &= -\frac{10}{9\nu(z)}\frac{\partial}{\partial z}\left(\frac{e}{\rho}\right), \\
\bar{q}_z &= -\frac{2}{3\nu(z)}\frac{\partial}{\partial z}\left(\frac{e}{\rho}\right).
\end{align*}
\]

(12)

Further substituting (12) in the first three equations of the system (6) we obtain

\[
\begin{align*}
\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial z}(\rho U_z) &= 0, \\
\frac{\partial}{\partial t}U_z + U_z \frac{\partial}{\partial z}U_z + 2 \frac{\partial}{\partial z}e + g - \frac{8}{9\nu(z)}\frac{\partial}{\partial z}\left(\frac{e}{\nu\frac{\partial}{\partial z}U_z}\right) &= 0, \\
\frac{\partial}{\partial t}e + U_z \frac{\partial}{\partial z}e + \frac{5}{3}e \frac{\partial}{\partial z}U_z - \frac{10}{9}\frac{\partial}{\partial z}\left(\frac{e}{\nu\frac{\partial}{\partial z}\rho}\right) - \frac{8}{9\nu}\left(\frac{\partial}{\partial z}U_z\right)^2 &= 0.
\end{align*}
\]

(13)

System (13) is the system of equation of a non-ideal liquid, to compare it with the Navier-Stokes equations, we continue the evaluation of viscosity factor and coefficient of heat conductivity. Expressions for strain tensor and heat flow tensor in one-dimensional hydrodynamics take a form

\[
\begin{align*}
\pi_{zz} &= -\frac{4}{3}\eta \frac{\partial}{\partial z}U_z, \\
q_z &= -\frac{2}{3}\kappa \frac{\partial}{\partial z}\theta,
\end{align*}
\]

where \(\eta\) is the viscosity factor, and \(\kappa\) is the coefficient of heat conductivity. Comparing mentioned expressions with corresponding items in equations (13) we obtain

\[
\begin{align*}
\eta &= -\frac{n_0 kT_0}{\nu}, \\
\kappa &= -\frac{5}{2}\frac{n_0 kT_0}{\nu},
\end{align*}
\]

(14)

that coincides with the well known relations, given, for example, in [30]. Finding the Prandtl number, taking into account of molecular thermal capacity of the ideal gas under constant pressure \(C_p = 5/2\), we obtain

\[
\text{Pr} = \frac{\eta C_p}{\kappa} = 1,
\]

that do not coincide with the Prandtl number of ideal gas (\(\text{Pr}_{id} = 2/3\)). The wrong Prandtl number is the main disadvantage of BGK model, that, however, can be removed by changing to the more exact models of collision integral, for example, of Gross-Jackson [3].

5
3 Linearized system of the equations. Dispersion relation.

For a closure of the system we use the equation of state of ideal gas. Linearized system of the equations \[ (11) \] is given by

\[
\begin{align*}
\frac{\partial}{\partial t} \rho + V_T \frac{\partial}{\partial z} U_z &= 0, \\
\frac{\partial}{\partial t} U_z + \frac{1}{2} V_T \frac{\partial}{\partial z} (\rho + T + \Pi_{zz}) &= 0, \\
\frac{\partial}{\partial t} T + \frac{1}{3} V_T \frac{\partial}{\partial z} (2U_z + 3q_z) &= 0, \\
\frac{\partial}{\partial t} \Pi_{zz} + \frac{1}{3} V_T \frac{\partial}{\partial z} (4U_z - 3q_z + 9\bar{q}_z) &= -\nu \Pi_{zz}, \\
\frac{\partial}{\partial t} q_z + \frac{1}{36} V_T \frac{\partial}{\partial z} (30T + 31\Pi_{zz}) &= -\nu q_z, \\
\frac{\partial}{\partial t} \bar{q}_z + \frac{1}{12} V_T \frac{\partial}{\partial z} (6T + 7\Pi_{zz}) &= -\nu \bar{q}_z.
\end{align*}
\]

(15)

For convenience we would introduce new notations \( n_i \) for hydrodynamical variables: \( n_1 = \rho, n_2 = U_z, n_3 = T, n_4 = \Pi_{zz}, n_5 = q_z, n_6 = \bar{q}_z \). The solution of system \[ (15) \] we search as

\[
n_i = a_i \exp(-iwt + ik_z z),
\]

(16)

where \( w \) is frequency of a wave, \( k_z \) - the vertical component of a wave vector.

Substituting \[ (16) \] in \[ (15) \], one obtains a system of the homogeneous algebraic equations with constant coefficients which solution exists if

\[
\frac{18}{125} \tilde{k}^6 + \left( \frac{3}{5} r^2 - \frac{39}{25} - \frac{48}{25} ir \right) \tilde{k}^4 + \left( -ir^3 - \frac{24}{5} r^2 + \frac{23}{3} ir + \frac{58}{15} \right) \tilde{k}^2 + ir^3 - 1 - 3ir + 3r^2 = 0
\]

(17)

Here the dimensionless wave number \( \tilde{k} = kC_0/w \) and the Reynolds number \( r = \nu/w \) are introduced, where \( C_0 = \sqrt{\frac{5}{6} V_T} \) - sound speed in Euler’s approximation. The Reynolds’ number \( r \) and the Knudsen number are obviously linked:

\[
Kn = \frac{\lambda}{\lambda_b} = \frac{w}{\nu} \frac{V_T}{2\pi C_0} = \sqrt{\frac{6}{5} \frac{1}{2\pi r}}.
\]

Let \( \tilde{k} = \beta + i\alpha \), then

\[
n_i = a_i \exp(-iwt - \beta \frac{z}{C_0}) \exp(-w \alpha \frac{z}{C_0})
\]

and the real part \( \beta = C_0/C, \alpha \) - the factor of attenuation.

4 The joint account of three modes.

The basic Fourier component solution of the system \[ (15) \] we shall search as a superposition of three plane waves

\[
n_i = A^1_i \exp(-iwt + ik_1 z) + A^2_i \exp(-iwt + ik_2 z) + A^3_i \exp(-iwt + ik_3 z),
\]

(18)
where \( k_j, \quad j = 1, 2, 3 \), are solutions of the dispersion equation (14) correspondent to the modes.

Substituting (18) into the linearized system, we express \( A_2^j, A_3^j, A_4^j, A_5^j, A_6^j \) through \( A_1^j \equiv A^j \). For \( A_2^j, A_3^j \) we have:

\[
A_2^j = \frac{w A^j}{k_j V_T}, \\
A_3^j = \frac{A^j (-31 V_T^2 k_j^2 + 24 i w^2 - 24 \nu w + 62 w^2)}{V_T^2 k_j^2 + 36 i w^2 - 36 \nu w}.
\] (19)

To determine the coefficients \( A^1, A^2, A^3 \) we should choose boundary conditions. We consider a problem in half-space and the reflection of molecules from a plane as a diffuse one [7]. The boundary condition for the distribution function looks as

\[
f(z = 0, \vec{V}, t) = \frac{n \pi^{3/2}}{3} \exp \left\{ -\frac{(\vec{V} - \vec{U}_0 e^{-i\nu t})^2}{V_T^2} \right\} \text{ by } V_z > 0.
\]

Here \( U_0 \) stands for an amplitude of the hydrodynamic velocity oscillations. For \( \frac{U_0}{V_T} \ll 1 \) we have:

\[
\varphi(z = 0, \vec{V}, t) = \frac{f - f(0)}{f(0)} \sim 2 \frac{U_0}{V_T} e^{-i\nu t} \text{ by } V_z > 0.
\]

For hydrodynamical variables on the bound we obtain

\[
\rho(z = 0, t) = \langle \varphi(z = 0, \vec{V}, t) \rangle = \frac{1}{\pi^{3/2} V_T^3} \int d\vec{V} \varphi(z = 0, \vec{V}, t) e^{-V^2/V_T^2} = \frac{U_0}{\sqrt{\pi}} e^{-i\nu t},
\]

\[
U_z(z = 0, t) = \langle V_z \varphi(z = 0, \vec{V}, t) \rangle = \frac{U_0}{2} e^{-i\nu t},
\]

\[
T(z = 0, t) = \langle V_z^2 \varphi(z = 0, \vec{V}, t) \rangle = \frac{U_0}{\sqrt{\pi}} e^{-i\nu t}.
\] (20)

Substituting the values of (19) into (18) and comparing right-hand sides of expression (18) and (20) we obtain the system of equations in variables \( A^j \). Solving given system of equations we obtain variables \( A^1, A^2, A^3 \).

In experiment acoustic pressure perturbation amplitude is measured. Appropriate combination of the basic variables for the pressure it is given by the formula

\[
P(z, t) e^{i\nu t} = \rho' + T' = (A^1 + A_3^j) e^{i k_1 z} + (A^2 + A_3^j) e^{i k_2 z} + (A^3 + A_3^j) e^{i k_3 z}
\] (21)

The real part of this expression relates to experiment. In fig. 1a) the real part of this expression is represented at \( r = 0.2 \), where \( \tilde{Z} = zw/C_0 \) - dimensionless coordinate. The attenuation factor \( \alpha \) is determined as a slope ratio of the diagram of the logarithm of amplitude of pressure depending upon distance between oscillator and the receiver. It is illustrated in fig. 1 b).
5 Comparison with with experiment and results of other evaluations.

In figures 2, 3 a comparison of theoretical results of the sound propagation parameters with experimental data [4, 5] is made.

The dispersion relation (17) represents the binary cubic equation with variable coefficients. The exact analytical solution by the formula Cardano is very huge and therefore we do not show it in this paper. At \( r \to 0 \) (free molecule flow) we start from the propagation velocity by the formula

\[
\frac{C_0}{C} = 0.54 + 0.15r^2 + 0(r^4)
\]
The attenuation factor is determined graphically as shown in the fig. 1. Therefore we cannot introduce analytical expression.

Results for phase speed give the good consistency with the experiments. As we see, the account of three modes allows us to enter further the area of intermediate Knudsen numbers.

In figures 4, 5 a comparison of our results of numerical calculation of dimensionless sound speed and attenuation factor depending on $r$ is carried out with the results of the other authors.
Lacks of the BGK model used in this article is that it gives correct value of viscosity factor, but wrong value of coefficient of heat conductivity. To the superior models of the Gross - Jackson this lack can be eliminated by transition.

At the solution of Boltzman equation the method of the Gross - Jackson revealed sudden disappearance of discrete modes at some values $r_c$ ([6], [7], [9]), and with increase of number of the moments $r_c$ decreased.

For example, Buckner and Ferziger in the paper [7] have shown, that for $r > 1$ the solution is determined mainly by the discrete sound mode and the dispersion relation may be used in calculating the sound parameters. For $r < 1$, the continuous modes are important. The solution remains "wavelike", but it is no longer a classical plane wave. In fact, the sound parameters are depend on the position of the receiver.

Below $r_c$ the solution is represented as superposition of a continuous spectrum of eigen functions, therefore the classical understanding of a sound should be changed. The concept of a dispersion relation is not applicable more.

6 Conclusion

The attenuation of sound at big Knudsen numbers is not "damping" (due to intermolecular collisions), but rather "phase mixing" (due to molecules which left the oscillator at different
phases arriving at the receiver at the same time)

The attenuation factor at big Knudsen numbers $Kn > 1$ is modelled by the account of effects of a relaxation in integral of collisions. The model of the Gross - Jackson at given $N$ limits an opportunity of the account external times of a relaxation (fast attenuation) as essentially bases on a condition:

$$\lambda_i = \lambda_{N+1}, \quad i > N + 1$$

Supreme times of a relaxation are assumed identical. It means, that the inclusion of the supreme eigen functions $\chi_i, \quad i \geq N + 1$ is necessary, that would allow to move in the range of higher Knudsen numbers.

In piecewise continuous partition function method the number of waves is twice more, but restrictions on attenuation factor remain.

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