Iterative learning PID Controller for Permanent Magnet Linear Synchronous Motor

Hongxin Li¹, Hu Sheng¹,*, Liang Shen¹

¹School of Electrical and Information Engineering, Dalian Jiaotong University, Dalian, China

*Corresponding author e-mail: hu.sheng@djtu.edu.cn

Abstract: At present, PID controllers have been widely used to control permanent magnet linear synchronous motor (PMLSM), but PID controllers are difficult to be suitable for the occasions with high precision requirements. In order to solve the problem of high precision control, the position-speed-current three-loop control structure is adopted in this study. In the design process of the position controller, the method of combining PID and Iterative learning controller is adopted. Iterative learning control (ILC) can deal with problems in a very simple way. A dynamic system with a fairly high degree of certainty, at the same time, is highly adaptable and easy to implement. It does not rely on the precise mathematical model of the dynamic system, and is an iterative generation of optimized input signals. The Matlab simulation results show that the PMLSM under the iterative learning PID control has accurate position tracking ability. The applicable object of this control system is the controlled system with repetitive motion of industrial robot, which can effectively realize the complete tracking task in a limited time interval.

Keywords: Permanent Magnet Linear Synchronous Motor, Pid Control, Iterative Learning Control

1. Introduction

PMLSM have been widely used in high-end manufacturing industries such as numerical control equipment, chip manufacturing and packaging, because of the advantages of high speed, high precision and high acceleration [1]. The PMLSM servo system eliminates the traditional intermediate transmission link of "ball screw". Compared with the servo system composed of rotating motors, the linear feed servo has the obvious advantages of accuracy and rapidity [2,3]. Besides, the linear servo cancels the intermediate transmission link, uncertain disturbances such as load changes and external disturbances will directly act on the linear motor mover without attenuation. Therefore, the control algorithm of the system should have high requirements and robustness [4].

ILC, a new type of learning control strategy, is an important branch of learning control. Uses information obtained from previous experiments repeatedly to obtain the control input that can produce the desired output trajectory to improve control quality. Unlike traditional control methods, ILC can deal with dynamic systems with high uncertainties in a very simple way and requires less prior knowledge and calculations, while being customizable and easy to implement. Most importantly,
it does not depend on the exact mathematical model of the dynamic system. It is an algorithm that repeatedly generates an optimized input signal to make the system output as close as possible to the ideal value. ILC is suitable for control systems with repetitive motion properties and aims to achieve integrated monitoring tasks in a limited time. ILC research is of great importance for dynamic systems with strong non-linear connectivity, high position repeatability, difficult modeling, and high-precision track monitoring control requirements.

2. Mathematical modelling of PMLSM

2.1. Mathematical model of PMLSM

Vector control is also called field-oriented control, which achieves the purpose of controlling three-phase alternating current (AC) motors by adjusting the output frequency of the frequency converter and the output voltage vector. The idea of vector control is to convert the current control method of the AC motor to the current control mode of the linear motor. Through vector coordinate transformation, the stator current is divided into the d-axis excitation component and the q-axis torque component perpendicular to the d-axis. Phase AC motors are decoupled to make the control convenient and effective. The mathematical model of PMLSM is established on the basis of vector control, and the numerical relationship between various quantities can be formed through the following coordinate transformation.

According to the vector control principle, the three-phase current of the stator is transformed by Clark and then transformed into the current of the rotating coordinate system by Park. Under the d-q axis coordinate, the voltage equation can be obtained as

\[
\begin{align*}
    u_d &= R_i d + d(\lambda_d) / dt - \pi v \hat{\lambda}_d / \tau \\
    u_q &= R_i q + d(\lambda_q) / dt - \pi v \hat{\lambda}_q / \tau
\end{align*}
\]

The flux linkage equation is

\[
\begin{align*}
    \lambda_d &= L_d i_d + \lambda_{PM} \\
    \lambda_q &= L_q i_q
\end{align*}
\]

The electromagnetic thrust can be derived as

\[
F = 3\pi[\lambda_{PM} i_q + (L_d - L_q) i_d i_q] / 2\tau
\]

In the formula: F is the electromagnetic thrust; \( \tau \) is the pole pitch; \( \lambda_{PM} \) is the excitation flux linkage; \( L_d, L_q \) is the mover d and q axis inductance respectively; \( i_d \) and \( i_q \) is the mover d and q axis current respectively.

The current inner loop adopts the control strategy of the magnetic field component, so the magnetic field of the stator permanent magnet of the mover and the current vector are orthogonal in space. The thrust equation of PMLSM can be rewritten as

\[
F = 3\pi \lambda_{PM} i_q / 2\tau = K_F i_q
\]

The equation of mechanical motion is

\[
Md_j / d_i = F - Bv - F_L
\]

In the formula, B is the coefficient of viscous friction; M is the mass of the mover and \( K_F \) is the thrust coefficient; V is the speed of the mover, and \( F_L \) is the load disturbance. Load disturbance \( F \) includes friction disturbance \( F_{friction} \), end effect disturbance \( F_{end} \), cogging force disturbance \( F_{cog} \) and load resistance \( F_f \). When the running speed of the motor is low, the frictional force disturbance plays the main disturbing role, and when the running speed of the motor is high, the end effect disturbance
plays the main disturbing role.

In the PMLSM control system, friction can be regarded as a kind of external disturbance. It is necessary to pay attention to its influence when designing the controller, especially when the linear motor is running at low speed. If the friction is not eliminated, the error will increase. Affecting the tracking performance of the system. The friction force can be expressed as LuGre model[5-6].

\[
F_{\text{friction}}[y(t)] = [f_c + \left( f_m - f_c \right)e^{-\frac{t}{\tau}}] \text{sgn}[v(t)]
\]

In the formula: \( f_c \) is the Coulomb friction force; \( v_s \) is the lubrication coefficient; \( f_m \) is the maximum static friction force; \( \text{sgn}() \) is the symbolic function.

The end effect is an inevitable phenomenon of linear motors during operation, and it is also unique to linear motors. It is caused by the magnetic force of the two core ends of linear motors. Its fundamental wave has periodic characteristics and can generally be divided into transverse end effect and longitudinal end effect. The end effect expression is

\[
F_{\text{end}}(y) = A_1 \cos(2\pi / \tau + \theta_1)
\]

In the formula, \( F_{\text{end}} \) is the end effect; \( A_1 \) is the amplitude of the end effect; \( y \) is the position of the mover; \( \theta_1 \) is the initial phase electrical angle.

Due to the existence of cogging, the reluctance of the mover will change. When the motor is moving, cogging force will be generated, because the cogging force can be regarded as generated by the internal structure of the motor. When the motor is moving periodically, cogging force also has periodic characteristics. The expression of cogging force is

\[
F_{\text{cog}}(y) = A_2 \sin(2\pi y / \tau)
\]

In the formula, \( F_{\text{cog}} \) is the cogging force; \( A_2 \) is the amplitude of the cogging force.

If the state variables of the system are \( y \) and \( v \), and the state vector is \( x(t) = \begin{bmatrix} y(t) \\ v(t) \end{bmatrix} \), the state equation of PMLSM can be derived from equations (6) and (7) as

\[
\begin{align*}
\dot{x} &= Ax(t) + D_1u(t) + EF_L(t) \\
y(t) &= Cx(t)
\end{align*}
\]

Where

\[
A = \begin{bmatrix}
0 & 1 \\
-\frac{B}{M} & 0
\end{bmatrix}; \quad D_1 = \begin{bmatrix}
0 & K_F \\
0 & M
\end{bmatrix}; \quad E = \begin{bmatrix}
0 \\
\frac{K_F}{M}
\end{bmatrix}; \quad C = \begin{bmatrix}
1 & 0 \end{bmatrix}; \quad u(t) \text{ is the control signal of PMLSM, which is the q-axis current d in the actual control system. The mathematical model of PMLSM can be obtained from equations (6)–(10), as shown in Figure 1.}

Figure 1. The mathematical model of PMLSM

2.2. Vector control system of PMLSM

The principle diagram of PMLM vector control is shown in Figure 2. PMLSM system Includes clarke transformation, park transformation, park inverse transformation, Space Vector Pulse Width Modulation (SVPWM), inverter, PI controller, PID controller and controlled object PMLSM. Where
\( i_d \) is the given d-axis current, and \( y_d \) is the desired position signal. The position error is obtained by making a difference with the actual position signal. The position error passes through the PID controller and then the speed loop PI controller and then outputs the q-axis current, which is the control signal for controlling the operation of the motor. The detected three-phase current \( i_a, i_b, i_c \) is transformed into a two-phase current \( i_d, i_q \) after clarke transformation. The two-phase current is transformed into \( i_d \) and \( i_q \) in the rotating coordinate system by Park transformation, which is different from the output signal of the speed loop PI controller, and then input to the current Ring PI controller. The output \( u_d \) and \( u_q \) of PI undergo Park transformation to obtain \( u_a \) and \( u_\beta \). After SVPWM, a pulse width modulation (PWM) signal is generated. The PWM signal controls the interruption and conduction of the switch in the inverter, thereby controlling the operation of the linear motor. Vector control adopts SVPWM, which is the abbreviation of space vector pulse width modulation. It controls the turn-on and turn-off time of the rotating vector and the corresponding inverter switch. Its purpose is not to generate a sinusoidal voltage by PWM technology, but to directly generate a circular magnetic field to make the motor The electromagnetic thrust is constant. Compared with PWM technology, the use of SVPWM can make the tracking of the system more accurate, and has higher voltage utilization and smaller switching losses.

![Figure 2. Model diagram of PMLSM](image)

3. PMLSM type ILC

3.1. Principles of Iterative Learning
ILC uses the error signal and control signal generated by the previous iteration of the system, combined with the error signal of the current system operation, to generate a control signal that allows the system to track more accurately, thereby reducing or eliminating the tracking error of the system. The open-loop ILC system is shown in Figure 3.

![Figure 3. Open loop ILC system](image)
Since Arimoto et al. proposed the concept of ILC, people have conducted a lot of research on it. On this basis, open-loop and closed-loop ILC laws have been deeply studied. This paper proposes a PID-type iterative learning law that uses proportional, integral, and differential actions at the same time, and gives the conditions for the system to converge under the learning law.

A nonlinear time-varying system with the iterative PID repetitive motion of a synchronous motor is

\[
\begin{align*}
\dot{x}(t) &= f(t, x(t)) + B(t)u(t) \\
y(t) &= C(t)x(t)
\end{align*}
\]

(12)

Where \( x(t) \in \mathbb{R}^{n \times 1}, y(t) \in \mathbb{R}^{m \times 1} \) and \( u(t) \in \mathbb{R}^{p \times 1} \) are the state and output vector of the system respectively, \( t \in [0, T] \), \( f, B \) and \( C \) are vectors or matrices of appropriate dimensions. When the system can be run multiple times, the system output \( f \) is required to be accurate in the time interval \( t \in [0, T] \).

When tracking the desired output \( y(t) \) for the \( k \)th run, the dynamic equation of the system’s iterative learning is expressed as

\[
\begin{align*}
x_k(t) &= f(tx_k(t)) + B(t)u_k(t) \\
y_k(t) &= C(t)x_k(t)
\end{align*}
\]

(13)

The open and closed loop PID type ILC law proposed for the system shown in formulas (12) and (13) is

\[
u_{k+1}(t) = u_k(t) + L_p(t)\dot{e}_k(t) + L_i(t)\int_0^t \dot{e}_k(s)ds + L_d(t)(\dot{x}_k(t) + L_i(t)\int_0^t \dot{e}_k(s)ds + L_d(t)\dot{e}_k(t))
\]

(14)

Where \( L_p(t) \), \( L_i(t) \) and \( L_d(t) \) are P-type, I-type, and D-type learning coefficient matrices and bounded respectively, \( e(t) = y_d(t) - y_k(t) \) is the tracking error, and \( y_d(t) \) is the expected output. Obviously, depending on whether \( L_p(t) \), \( L_i(t) \) and \( L_d(t) \) takes a zero matrix, (14) can be changed to open and closed loop P-type, PI-type, PD-type, PID-type ILC law. \( \rho(\varphi) \) is the spectrum radius.

3.2. ILC Design

Since the D-type learning law contains the differential value of the error, it will be affected by noise, so in practical applications, the P-type learning law is more robust and effective than the D-type learning law. This paper selects the P-type closed-loop iterative learning law, the discrete-time closed-loop P-type learning law iterative learning law can be expressed as

\[
u_{k+1} = u_k + \Gamma(t)e_{k+1}(t + 1)
\]

(15)

Among them, the learning law gain \( \Gamma(t) \) can be obtained by the convergence condition of the system.

3.3. Convergence analysis of ILC algorithm

Suppose the model of the nonlinear discrete system is:

\[
\begin{align*}
x(i+1) &= f(i, x(i)) + B(i)u(i)) \\
y(i+1) &= C(i)x(i)
\end{align*}
\]

(16)

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^p \) are the state vector and the input vector; output vector \( y \in \mathbb{R}^p \); \( f \) is Matrix function; \( B \) and \( C \) are Matrix with appropriate dimensions.

The expected output of the controlled object is \( y_d(t) \), The goal of ILC is to iterate the control
amount multiple times so that the output error $e_{k+1}(t) = y_d(t) - y_{k+1}(t)$ tends to zero. For the above system, we propose a closed-loop P-type iterative learning law can be expressed as

$$u_{k+1}(i) = u_k(i) + \Gamma(i)e_{k+1}(i+1)$$

(17)

For any given initial control $u_0(i)$ and the initial state $x_0(0)$ of each operation, the system convergence conditions are

$$\rho([1 + \Gamma(i)C(i+1)B(i)]^{-1}) < 1, \quad \forall i \in [0, T]$$

(18)

Necessary conditions $\rho([1 + \Gamma(i)C(i)B(0)]^{-1}) < 1$, where $\rho(\cdot)$ is the spectral radius.

4. Experimental results and analysis

The parameters for setting PMLSM are mover armature $d$ -axis, $q$ -axis inductance $L_d = L_q = 8.5\text{mH}$, mover armature resistance $R = 1\Omega$; mover mass $M = 0.66\text{kg}$; viscous friction coefficient $B = 0.2\text{N}\cdot\text{s}/\text{m}$; pole pitch $r = 60.96\text{mm}$; permanent magnet flux $\psi = 0.16\text{Wb}$. The target position is given as $x_d = 0.5\sin \pi t \text{m}$.

Select the optimal parameters through repeated debugging, and set the parameters of the speed loop PID type ILC as: $K_p = 40$, $K_I = 10$, $K_D = 10$. The parameter selection for setting the position loop PID type ILC is: $K_p = 30$, $K_I = 200$, $K_D = 0$.

Set the number of iterations to 25. After 25 iterations, the expected position output and actual output curves of PMLSM are shown in Figure 4, and the position error of PMLSM is shown in Figure 5. The position output of PMLSM approaches the expected output curve as the number of iterations increases.

![Figure 4. The desired position and the position output curve after 25 iterations](image)

![Figure 5. Position error after the 25th iteration](image)

5. Conclusion

This research takes PMLSM as the controlled object. In the design process of the position controller, first designs the vector controller, then designs the ILClar, and finally simulate in the Matlab/Simulink environment. Based on the principle of vector control based on PMLSM, PID closed-loop ILC
position controller is set up and simulated. After 25 iterations of learning, the actual output has basically followed the expected curve, and the magnitude of the position error is about 0.02m. The applicable object of iterative learning PID control is a controlled system with repetitive motion properties such as industrial robots. The goal is to achieve complete tracking tasks in a limited time interval. ILC adopts the learning strategy of "learning in repetition", with a memory and correction mechanism. The research of ILC is of great significance to dynamic systems that have strong nonlinear coupling, high position repeatability, difficult modeling and high-precision trajectory tracking control requirements.

Acknowledgments
The study was supported by “Natural science research project of Liaoning Provincial Department of Education, China (Grant No. JDL2019014)”.

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