Simulation of Two-Dimensional Temperature Field via Kriging Method Based on Limited Conditioning Points in an Arc Dam

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Abstract. Temperature in an arc dam is a critical quantity of interest in safety monitoring. How to estimate a temperature field for an arc dam has been a challenging task, particularly when only a limited number of thermometers are arranged. This paper proposes to use the ordinary Kriging to estimate the two-dimensional temperature fields based on limited monitored temperature data from thermometers in an arc dam. The Kriging method interpolates the temperature field between spatially distributed values of temperature. The effects of scale of fluctuation (δ) and correlation structure on the estimated temperature fields are examined. The results indicate that the Kriging method can estimate the temperatures at locations where the thermometers are not arranged. The estimated temperatures near the measuring points have higher accuracy. The estimated variances near the thermometers are lower than the variances far away from the thermometers regardless of the δ. When the δ is high, the data from conditioning (known) points can reduce the uncertainties of estimated temperatures near the known points. The distributions of temperatures using different correlation structures are quite similar. The δ has obvious effects on the smoothness of the temperature fields, contours and variances.

1. Introduction
To be informed of working and safety state of an arc dam, temperature is a critical quantity in arc dam safety monitoring. Temperature can affect stress and strain in the concrete, and may further lead to large deformations and even cracks of an arc dam. During dam construction, the temperature in concrete is usually monitored and controlled to prevent the potential cracks of concrete. In this way, the concrete quality and dam safety can be guaranteed.

In practice, the temperatures in arc dams are usually monitored by thermometers and the measuring (conditioning) points are usually limited. It is desirable to obtain the two-dimensional (2D) temperature field to grasp the working state of all parts in an arc dam. This poses a challenge in the estimation of a 2D temperature field based on limited temperature data.

In contrast to such conventional interpolation techniques as moving least squares and spline interpolation, Kriging [1] incorporates the covariance into the interpolation. Specifically, information on the spatial correlation of the measured (conditioning) points is used. Moreover, the errors in the estimation can also be obtained, indicating the reliability of the estimation and the accuracy of the
prediction. Kriging provides a best linear unbiased prediction of the temperature of unknown point based on known data by assuming the stationarity of the mean and of the spatial covariance.

This paper proposes to use the Kriging to estimate the 2D temperature fields based on limited monitored temperature data from thermometers in an arc dam. The effects of scale of fluctuation (δ) and correlation structure on the estimated temperature fields are examined. Some conclusions are drawn.

2. Theory and Method

Given a set of spatially distributed values of temperature (T), Kriging method interpolates the temperature field between these values so that the expected error of the generated points in the field is minimized. The aim of Kriging is to give a best estimate of a random field between known values. In essence, the method estimates \( T \) at a desired location \( x_0 \), from a linear combination of the known values of \( T \) at various observation points \( x_a \) (\( \alpha = 1, 2, \ldots, n \), and \( n \) is the number of observations). The Kriged interpolation of \( T \) at \( x_0 \) (i.e. \( \hat{T}(x_0) \)) can be written as a linear combination of the known (conditioning) points \( T(x_a) \) [1, 2]:

\[
\hat{T}(x_0) = \sum_{\alpha=1}^{n} \lambda_\alpha T(x_\alpha), \quad \sum_{\alpha=1}^{n} \lambda_\alpha = 1
\]  

(1)

where \( \lambda_\alpha \) (\( \alpha = 1, 2, \ldots, n \)) is the unknown weight. The \( \lambda_\alpha \) needs to be determined to give the best estimate for \( T \) at a desired location \( x_0 \).

The family of Kriging includes ordinary Kriging, simple Kriging, universal Kriging, etc. Ordinary Kriging is adopted here because it is simple and convenient. The ordinary Kriging method assumes that the mean and standard deviation are constant and known across the entire region of interest. The estimated variance \( \sigma_E^2 \) at location \( x_0 \) can be written as:

\[
\sigma_E^2 = \gamma(0) - \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} \lambda_\alpha \lambda_\beta \gamma(x_\alpha - x_\beta) + 2 \sum_{\alpha=1}^{n} \lambda_\alpha \gamma(x_\alpha - x_0)
\]  

(2)

The unknown weights in Eq. (1) are estimated by minimising the Eq. (2). The kriging method uses the variogram or the covariance structure. In minimising the Eq. (2), the following system of equations is obtained:

\[
\begin{bmatrix}
\gamma(x_1 - x_1) & \ldots & \gamma(x_1 - x_n) & 1 \\
\vdots & \ddots & \vdots & \vdots \\
\gamma(x_n - x_1) & \ldots & \gamma(x_n - x_n) & 1 \\
1 & \ldots & 1 & 0
\end{bmatrix} \begin{bmatrix}
\lambda_1 \\
\vdots \\
\lambda_n \\
\mu^k
\end{bmatrix} = \begin{bmatrix}
\gamma(x_1 - x_0) \\
\vdots \\
\gamma(x_n - x_0)
\end{bmatrix}
\]  

(3)

where \( \gamma(x_\alpha - x_\beta) \) is the covariance of the data from positions of \( x_\alpha \) and \( x_\beta \) (\( \alpha, \beta = 1, 2, \ldots, n \)), \( \mu^k \) is the Lagrange parameter and \( \lambda_\alpha \) is the ordinary Kriging weight. The left-hand side of Eq. (2) contains information between data of known points, whereas the right-hand side contains information between each data point and the estimation point.

3. Example

3.1. Description of arc dam and arrangement of thermometers

As shown in Fig. 1, the real arc dam consists of 14 dam sections. The arc dam is currently being constructed at Yichang, Yunnan, China. The thermometers are arranged in the 4\(^{th}\), 8\(^{th}\) and 12\(^{th}\) dam sections to monitor the temperatures of the concrete during construction and maintenance.
To clarify the performance of the Kriging method in the simulation of the temperature field, only the monitored data in the 4# section are used. The cross section the 4# dam section is shown in Figure 2, and currently 23 thermometers (T01DB04-T23DB04) are buried in the concrete at different elevations. The water flow direction is also denoted in Figure 2.

Figure 1. Arc dam from the upstream view.

Figure 2. Arrangement plan of thermometers in 4# dam section.
3.2. Estimated temperature field, contour and variance

Here, the monitored temperatures from the 23 thermometers on January 1, 2019 are used. The data are summarized in Table 1. For the correlation structure of the temperature field, the scale of fluctuation $\delta = 35 \text{ m}$ is used together with the assumption of a squared exponential correlation structure. The variance scaling parameter is taken from the 23 monitored data as $\gamma(0)=\sigma^2=5.43$. This leads to a correlation structure

$$\gamma(x_\alpha - x_\beta) = 5.43 \exp \left[ -\pi \left( \frac{(x_{1,\alpha} - x_{1,\beta})^2}{\delta_h^2} + \frac{(x_{2,\alpha} - x_{2,\beta})^2}{\delta_v^2} \right) \right]$$

(4)

where $\delta_h$ and $\delta_v$ are the horizontal and vertical scales of fluctuation, respectively, and $\delta_h = \delta_v = 35 \text{ m}$. $x_{1,\alpha}$, $x_{1,\beta}$ are the coordinates in the horizontal direction, and $x_{2,\alpha}$, $x_{2,\beta}$ are the coordinates in the vertical direction.

The estimated temperature field is shown in Figure 3(a), the contour of temperature is given in Figure 3(b), and the corresponding estimated variance is present in Figure 3(c).

Table 1. Monitored temperature from 23 thermometers (January 1, 2019).

| No.       | $x$ (m) | $y$ (m) | Elevation (m) | Temperature (°C) |
|-----------|---------|---------|---------------|------------------|
| T01DB04   | 1.20    | 31.78   | 800           | 14.55            |
| T02DB04   | 13.77   | 31.78   | 800           | 14.2             |
| T03DB04   | 26.30   | 31.78   | 800           | 14.35            |
| T04DB04   | 38.80   | 31.78   | 800           | 14               |
| T05DB04   | 52.04   | 31.78   | 800           | 12.8             |
| T06DB04   | 0.27    | 46.78   | 815           | 13.3             |
| T07DB04   | 0.00    | 61.78   | 830           | 13.3             |
| T08DB04   | 12.60   | 61.78   | 830           | 13.15            |
| T09DB04   | 25.06   | 61.78   | 830           | 13.2             |
| T10DB04   | 37.62   | 61.78   | 830           | 12.75            |
| T11DB04   | 50.51   | 61.75   | 830           | 14.3             |
| T12DB04   | 0.24    | 76.78   | 845           | 13.25            |
| T13DB04   | 0.84    | 91.78   | 860           | 16.4             |
| T14DB04   | 13.44   | 91.78   | 860           | 19.45            |
| T15DB04   | 26.04   | 91.78   | 860           | 17.55            |
| T16DB04   | 38.44   | 91.80   | 860           | 17.35            |
| T17DB04   | 49.51   | 91.80   | 860           | 15.9             |
| T18DB04   | 2.25    | 106.78  | 875           | 18.4             |
| T19DB04   | 4.24    | 123.78  | 892           | 16.15            |
| T20DB04   | 14.84   | 123.78  | 892           | 19.75            |
| T21DB04   | 25.32   | 123.78  | 892           | 18.6             |
| T22DB04   | 35.84   | 123.78  | 892           | 17.65            |
| T23DB04   | 46.41   | 123.78  | 892           | 18               |

The mean ensemble in Figure 3(a) and 3(b) provides a comprehensive estimate of temperature in terms of trend and details in the 4th dam section. The Kriging method can estimate the temperatures at locations where the thermometers are not arranged. The simulated temperature values at the measuring points have good agreement with the monitored data.

Figure 3(c) shows that the estimated variances near the thermometers are quite lower that the variances far away from the thermometers. This means that estimated temperatures near the measuring points are more accurate and have higher confidence level.
3.3. Effect of scale of fluctuation on estimated temperature field

It is noted that in Section 3.2 the scale of fluctuation $\delta$ of the temperature field is assumed to be 35 m. The uncertain $\delta$ may affect the updated temperature field using Kriging. Here this effect is examined in terms of the estimated temperature field and variance. Figures 4 and 5 present the estimated temperature fields and variances under the assumption that the $\delta = 25$ m and 45 m, respectively.
The $\delta$ has some effects on the estimated temperature field and variance. Comparing Figures 3(a), 4(a) and 5(a), when the $\delta$ varies, the ensemble of temperature filed changes. From Figures 3(b), 4(b) and 5(b), whatever the scale of fluctuation is, the estimated variances near the thermometers are lower than the variances far away from the thermometers. When the $\delta$ is high (e.g., 45 m), the conditioning (known) points can reduce the uncertainties of estimated temperatures near the measuring points, and vice versa. This is embodied by the reduced variances in the same locations near the measuring points in Figure 5(b).

### 3.4. Effect of correlation structure on estimated temperature field

Several theoretical correlation models have been used in the literature [3, 4, 5], such as single exponential (SNX), squared exponential (SQX), second-order Markov (SMK) and cosine exponential (CSX). A suitable correlation model can be selected by fitting these theoretical correlation models to the sample autocorrelation function estimated from monitored data and comparing the goodness-of-fitting of different correlation models.

The theoretical correlation models are summarized in Table 2. The variogram $\gamma(x_a - x_b) = \sigma^2 \rho(\tau_x, \tau_y)$, in which $\tau_x$ and $\tau_y$ are the separation distance between two points (i.e., $x_a$ and $x_b$) in horizontal and vertical directions, respectively. Here, the $\delta$ is assumed to be 35 m. The effect of correlation structure on estimated temperature field is examined.

The simulated temperature fields and variances using four different correlation models (i.e., SNX, SQX, SMK and CSX) are given in Figure 6. Generally, the distributions of temperatures in the 4th section using SNX, SQX, SMK and CSX models are quite similar.

The $\delta$ has obvious effects on the temperature fields and variances in terms of details. When using the SQX and SMK models, the realized temperature fields and contours are smoother. While, when the SNX and CSX models are adopted, there are some turning points in the fields and contours. The variances in Figures 6(b) and 6(c) are more smooth than the variances in Figures 6(a) and (d), and there are more extreme points in Figures 6(a) and (d).
Table 2. Theoretical correlation models.

| Type                  | Theoretical correlation model                                      |
|-----------------------|-------------------------------------------------------------------|
| Single exponential    | \( \rho(\tau_x, \tau_y) = \exp \left[ -2 \frac{\tau_x + \tau_y}{\delta_h + \delta_v} \right] \) |
| Squared exponential   | \( \rho(\tau_x, \tau_y) = \exp \left[ -\pi \left( \frac{\tau_x}{\delta_s} + \frac{\tau_y}{\delta_v} \right) \right] \) |
| Second-order Markov   | \( \rho(\tau_x, \tau_y) = \exp \left[ -4 \left( \frac{\tau_x}{\delta_h} + \frac{\tau_y}{\delta_v} \right) \right] \left( 1 + \frac{4\tau_x}{\delta_h} \right) \left( 1 + \frac{4\tau_y}{\delta_v} \right) \) |
| Cosine exponential    | \( \rho(\tau_x, \tau_y) = \exp \left[ -\left( \frac{\tau_x}{\delta_h} + \frac{\tau_y}{\delta_v} \right) \right] \cos \left( \frac{\tau_x}{\delta_h} \right) \cos \left( \frac{\tau_y}{\delta_v} \right) \) |

(a) SNX
4. Conclusions and remarks
This paper proposes to use the ordinary Kriging to estimate the two-dimensional temperature fields based on limited monitored temperature data from thermometers in arc dam.

1. The Kriging method can estimate the temperatures at locations where the thermometers are not arranged. The estimated temperatures near the measuring points have higher accuracy.
2. The estimated variances near the thermometers are lower than the ones far away from the thermometers regardless of the scale of fluctuation $\delta$. When the $\delta$ is high, the conditioning (known) points can reduce the uncertainties of estimated temperatures near the measuring points.
3. The distributions of temperatures using SNX, SQX, SMK and CSX models are quite similar. The $\delta$ has obvious effects on the smoothness of the temperature fields, contours and variances.

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