The Imprint of the Baryon Acoustic Oscillations (BAO) in the Cross-correlation of the Redshifted HI 21-cm Signal and the Ly-α Forest

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ABSTRACT
The cross-correlation of the Ly-α forest and redshifted 21-cm emission has recently been proposed as an observational tool for mapping out the large-scale structures in the post-reionization era $z \leq 6$. This has a significant advantage as the problems of continuum subtraction and foreground removal are expected to be considerably less severe in comparison to the respective auto-correlation signals. Further, the effect of discrete quasar sampling is considerably less severe for the cross-correlation in comparison to the Ly-α forest auto-correlation signal. In this paper we explore the possibility of using the cross-correlation signal to detect the baryon acoustic oscillation (BAO). To this end, we have developed a theoretical formalism to calculate the expected cross-correlation signal and its variance. We have used this to predict the expected signal, and estimate the range of observational parameters where a detection is possible.

For the Ly-α forest, we have considered BOSS and BIGBOSS which are expected have a quasar density of $16 \text{ deg}^{-2}$ and $64 \text{ deg}^{-2}$ respectively. A radio interferometric array that covers the redshift range $z = 2$ to $3$ using antennas of size $2 \text{ m} \times 2 \text{ m}$, which have a $20^\circ \times 20^\circ$ field of view, is well suited for the redshifted 21-cm observations. It is required to observe 25 independent fields of view, which corresponds to the entire angular extent of BOSS. We find that it is necessary to achieve a noise level of $1.1 \times 10^{-5} \text{ mK}^2$ and $6.25 \times$
10^{-6} \text{mK}^2 \text{ per field of view in the redshifted 21-cm observations to detect the angular and radial BAO respectively with BOSS. The corresponding figures are } 3.3 \times 10^{-5} \text{ mK}^2 \text{ and } 1.7 \times 10^{-5} \text{ mK}^2 \text{ for BIGBOSS. We also discuss possible observational strategies for detecting the BAO signal. Four to five independent radio interferometric arrays, each containing 400 antennas uniformly sampling all the baselines within 50 m will be able to carry out these observations in the span of a few years.}

**Key words:** cosmology: theory - large-scale structure of Universe - cosmology: diffuse radiation

# 1 INTRODUCTION

Neutral hydrogen (HI) in the post-reionization epoch \((z < 6)\) is known to be an important cosmological probe seen in both emission and absorption. Here, the redshifted 21-cm emission (Furlanetto et al., 2006; Morales & Wyithe, 2009) and the transmitted QSO flux through the Ly-\(\alpha\) forest (Weinberg et al., 1998; Mandelbaum et al., 2003) are both of utmost observational interest. In a recent paper Guha Sarkar et al. (2011) have proposed the cross-correlation of the 21-cm signal with the Lyman-\(\alpha\) forest as a new probe of the post-reionization era. While it is true that the emission and the absorption signals both originate from neutral hydrogen (HI) at the same redshift (or epoch), these two signals, however, do not originate from the same set of astrophysical sources. The 21-cm emission originates from the HI housed in the Damped Lyman-\(\alpha\) systems (DLAs) which are known to contain the bulk of the HI at low redshifts (Prochaska et al., 2005). The collective emission from the individual clouds appears as a diffuse background in low frequency radio observations (Bharadwaj et al., 2001). On the contrary, the Ly-\(\alpha\) forest consists of a large number of Ly-\(\alpha\) absorption lines seen in the spectra of distant background quasars. These absorption features arises due to small density fluctuations in the predominantly ionized diffuse IGM. On large scales, however, the fluctuations in the 21-cm signal and the the Ly-\(\alpha\) forest transmitted flux are both believed to be excellent tracers of the underlying dark matter distribution. Guha Sarkar et al. (2011) have proposed that the cross-correlation between these two signals can be used to probe the power spectrum during the post-reionization era.
The HI power spectrum can be determined separately from observations of the Ly-\(\alpha\) forest (Croft et al., 1998) and the redshifted 21-cm emission (Bharadwaj & Sethi, 2001). The detection of the individual signals, however, face severe observational challenges. The Ly-\(\alpha\) auto-correlation power spectrum is expected to be dominated by the Poisson noise arising from the discrete sampling of QSO lines of sight. The cross-correlation signal has the advantage that it is not affected by the Poisson noise which affects only its variance. Uncertainties in fitting the QSO continuum (Croft et al., 2002; Kim et al., 2004, 2007) pose another challenge for using Ly-\(\alpha\) observations to determine the power spectrum. Astronomical sources like the galactic synchrotron emission, extra-galactic point sources, etc. appear as foregrounds for the redshifted 21-cm signal. These foregrounds, which are several orders of magnitude larger than the signal, pose a great challenge for detecting the post-reionization 21-cm signal (Ghosh et al., 2010, 2011). The foregrounds and systematics in the Ly-\(\alpha\) forest and the redshifted 21-cm emission are, however expected to be uncorrelated and we therefore expect the problem to be much less severe for the cross-correlation signal. A detection of a cross-correlation signal shall, hence, conclusively ascertain its cosmological origin. Apart from being an independent probe of the large scale matter distribution, the cross-correlation signal can potentially unveil the same astrophysical and cosmological information as the individual auto-correlations.

Cosmological density perturbations drive acoustic waves in the primordial baryon-photon plasma which are frozen once recombination takes place at \(z \sim 1000\), leaving a distinct oscillatory signature on the CMBR anisotropy power spectrum (Peebles & Yu, 1970). The sound horizon at recombination sets a standard ruler that maybe used to calibrate cosmological distances. Baryons contribute to 15% of the total matter density, and the baryon acoustic oscillations are imprinted in the late time clustering of non-relativistic matter. The signal, here, is however suppressed by a factor \(\sim \Omega_b/\Omega_m \sim 0.1\), unlike the CMBR temperature anisotropies where it is an order unity effect (Komatsu et al., 2009). The baryon acoustic oscillation (BAO) is a powerful probe of cosmological parameters (Seo & Eisenstein, 2003; White, 2005). This is particularly useful since the effect occurs on large scales (\(\sim 150\) Mpc), where the fluctuations are still in the linear regime. It is possible to measure the angular diameter distance and the Hubble parameter as functions of redshift using the the transverse and the longitudinal oscillations respectively. These provide means for estimating cosmological parameters and placing stringent constraints on dark energy models. Nonlinear effect of gravitational clustering tend to wipe out the BAO signal, and it is preferable to avoid
very low redshifts where this is a potential problem. However, very high redshifts too are not very useful for constraining dark energy models. Several authors have reported a 2 to 3σ detection of the BAO in low redshift galaxy surveys (Eisenstein et al., 2005; Percival et al., 2007, 2010). The possibility of detecting the BAO signal in the Ly-α forest has been extensively studied by McDonald & Eisenstein (2007). Several groups have also considered the possibility of detecting the BAO signal using redshift 21-cm emission (Chang et al., 2008; Mao & Wu, 2008; Masui et al., 2010).

Several QSO surveys are now being considered with the intent of measuring the BAO using the Ly-α forest (eg. BOSS (McDonald et al., 2005) and BIGBOSS (Schlegel et al., 2009)). The possibility of a wide field redshifted 21-cm survey to detect the BAO is also under serious consideration. In this paper, we consider the possibility of studying the BAO using the cross-correlation signal. In Section 2. of this paper we quantify the cross-correlation between the Ly-α forest and the 21-cm emission, and present theoretical predictions of the expected signal. We have used the multi-frequency angular power spectrum $C_\ell(\Delta z)$ (MAPS, Datta et al. 2007) in preference to the more commonly used three dimensional power spectrum $P(k)$ to quantify the cross-correlation signal. This has several advantages which we briefly discuss here. MAPS refers to the angular multipole $\ell$ (or equivalently angle) and redshift interval $\Delta z$ which are the directly relevant observational quantities for both Ly-α surveys and redshifted 21-cm observations. This is particularly important if we wish to determine the angular scales and redshift intervals which need to be covered in order to detect a particular feature in the signal. The foregrounds in the redshifted 21-cm observations and the continuum in the Ly-α forest are both expected to have a smooth, slow variation along the frequency axis, and this plays a crucial role in removing these from the respective data. It is therefore advantageous to use MAPS which retains the distinction between the frequency (redshift) and angular information, unlike $P(k)$ which mixes these up.

Ali et al. (2008) have analyzed GMRT observations using MAPS to jointly characterize the angular and frequency dependence of the foregrounds at 150 MHz. Ghosh et al. (2010) and Ghosh et al. (2011) have applied MAPS to Analyze 610 MHz GMRT observations, and have used this to characterize the foregrounds for the post-reionization 21-cm signal. In fact, they show that it is possible to completely remove the foregrounds from the measured $C_\ell(\Delta \nu)$ by subtracting out polynomials in $\Delta \nu$. Finally, the signal could have a significant contribution from the light cone effect, particularly if the observations span a large redshift interval. It is, in principle, possible to account for this in the MAPS, though we have not done this here. It
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is, however, not possible to account for this effect in the three dimensional power spectrum which mixes up the information from different epochs through a Fourier transform along the radial direction.

In Section 3 of this paper we quantify the imprint of the BAO feature on the cross-correlation signal. Here MAPS has the advantage that it allows us to separately study the radial and the transverse oscillations through the $\Delta \nu$ and the $\ell$ dependence respectively. In Section 4 we introduce an estimator for the MAPS of the cross-correlation signal and derive its statistical properties. In particular, we present a detailed analysis of the noise for the cross-correlation estimator. In Section 5 we present several observational considerations which are relevant for the cross-correlation signal, some pertaining to the Ly-α forest and others to the redshifted 21-cm signal or both. Finally, in Section 6 we discuss the detectability of the cross-correlation signal and the BAO features. We have used the cosmological parameters $(\Omega_m h^2, \Omega_b h^2, \Omega_\Lambda, h, n_s, \sigma_8) = (0.136, 0.023, 0.726, 0.71, 0.97, 0.83)$ from Komatsu et al. (2009) throughout this paper.

2 THE CROSS-CORRELATION SIGNAL

We quantify the fluctuations in the transmitted flux $F(\hat{n}, z)$ along a line of sight $\hat{n}$ to a quasar through the Ly-α forest using $\delta_F(\hat{n}, z) = F(\hat{n}, z)/\bar{F} - 1$. For the purpose of this paper we are interested in large scales where it is reasonable to adopt the fluctuating Gunn-Peterson approximation (Gunn & Peterson, 1965; Bi & Davidsen, 1997; Croft et al., 1998, 1999) which relates the transmitted flux to the matter density contrast $\delta$ as $F = \exp[-A(1 + \delta)^\kappa]$ where $A$ and $\kappa$ are two redshift dependent quantities. The function $A$ is of order unity (Kim et al., 2007) and depends on the mean flux level, IGM temperature, photo-ionization rate and cosmological parameters (Croft et al., 1999), while $\kappa$ depends on the IGM temperature density relation (McDonald et al., 2001; Choudhury et al., 2001). For our analytic treatment of the Ly-α signal, we assume that the measured fluctuations $\delta_F$ have been smoothed over a sufficiently large length scale such that it is adequate to retain only the linear term $\delta_F \propto \delta$ (Bi & Davidsen, 1997; Croft et al., 1998, 1999; Viel et al., 2002; Saitta et al., 2008; Slosar et al., 2009). The terms of higher order in $\delta$ are expected to be important at small length scales which have not been considered here.

We use $\delta_T(\hat{n}, z)$ to quantify the fluctuations in $T(\hat{n}, z)$ the brightness temperature of redshifted 21-cm radiation. In the redshift range of our interest ($z < 3.5$), it is reasonable
to assume that $\delta_T(\hat{n}, z)$ traces $\delta$ with a possible bias (Bharadwaj & Sethi, 2001; Bharadwaj et al., 2001). The bias is expected to be scale dependent below the Jeans length-scale (Fang et al., 1993). Fluctuations in the ionizing background also give rise to a scale dependent bias (Wyithe & Loeb, 2007, 2009). Moreover, this bias is found to grow monotonically with $z$ (Marin et al., 2009). However, numerical simulations (Bagla et al., 2009; Guha Sarkar et al., 2011), indicate that it is adequate to use a constant, scale independent bias at the large scales of our interest.

With the above mentioned assumptions and incorporating redshift space distortions we may express both $\delta_F$ and $\delta_T$ as

$$\delta_\alpha(\hat{n}, z) = C_\alpha \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k} \cdot \hat{n} r} [1 + \beta_\alpha \mu^2] \Delta(k).$$  

(1)

where $\alpha = F$ and $T$ refer to the Ly-\(\alpha\) forest and 21-cm signal respectively. Here $r$ is the comoving distance corresponding to $z$, $\Delta(k)$ is the matter density contrast in Fourier space and $\mu = \hat{k} \cdot \hat{n}$. We adopt the values $C_F = -0.13$ and $\beta_F = 1.58$ from Ly-\(\alpha\) forest simulations of (McDonald, 2003), and $C_T = \bar{T} \bar{x}_{\text{HI}} h$ and $\beta_T = f/b$ for the 21-cm signal (Guha Sarkar et al., 2011). These values are for $z = 2.5$ which is the fiducial redshift in our analysis. We note that there are large uncertainties in the values of all the four parameters $C_T, C_F, \beta_T$ and $\beta_F$ arising from our poor knowledge of the state of the diffuse IGM and the systems that harbour bulk of the neutral hydrogen at $z \sim 2.5$.

A QSO survey (eg. SDSS\(^1\)), typically, covers a large fraction of the entire sky. In contrast, a radio interferometric array (eg. GMRT\(^2\) usually has a much smaller field of view ($\sim 1^\circ$). Only the overlapping region common to both these observations provides an estimate of the cross-correlation signal. We therefore use the limited field of view $L \times L$ ($L$ in radians) of the radio telescope to estimate the cross-correlation signal. Given this constraint, it is a reasonable observational strategy to use several pointings of the radio telescope to cover the entire region of the QSO survey. Each pointing of the radio telescope provides an independent estimate of the cross-correlation signal, which can be combined to reduce the cosmic variance.

We have assumed that the field of view is sufficiently small ($L \ll 1$) so that curvature of the sky may be ignored. In the flat sky approximation the unit vector $\hat{n}$ along any line of sight can be expressed as $\hat{n} = \hat{m} + \bar{\theta}$, where $\hat{m}$ is the line of sight to the centre of the field of view and $\bar{\theta}$ is a two-dimensional (2D) vector on the plane of the sky. In this approximation

\(^1\) http://www.sdss.org
\(^2\) http://www.ncra.tifr.res.in (check this)
it is convenient to decompose $\delta_F(\vec{\theta}, z)$ and $\delta_T(\vec{\theta}, z)$ into Fourier modes instead of spherical harmonics. We then have the Fourier components

$$\Delta_\alpha(U, z) = \int_{-L/2}^{L/2} d^2\vec{\theta} e^{-2\pi i \vec{U} \cdot \vec{\theta}} \delta_\alpha(\vec{\theta}, z)$$  \hspace{1cm} (2)$$

where $U$ is a two dimensional vector conjugate to $\vec{\theta}$. Vector $U$ which represents an inverse angular scale, and also $\vec{\theta}$ are both perpendicular to the line of sight to the centre of the field of view $\hat{m}$. It is useful to visualize $\Delta_\alpha(U, z)$ as Fourier components of the signals on a plane perpendicular to the line of sight $\hat{m}$ located at a comoving distance $r$ corresponding to the redshift $z$. The redshift $z$, here conveys two different pieces of information, namely the distance along the line of sight and also the epoch where the HI signals originated.

We introduce the multi-frequency angular power spectrum (MAPS, Datta et al. 2007)

$$\langle \Delta_\alpha(U, z)\Delta^*_\gamma(U', z + \Delta z) \rangle = L^2 \delta_{UU'} P_{\alpha\gamma}(U, \Delta z)$$  \hspace{1cm} (3)$$

where the indices $\alpha$ and $\gamma$ have the possible values $\alpha = T, F$ and $\gamma = T, F$. The multi-frequency angular power spectrum $P_{TT}(U, \Delta z) \equiv P_{TT}(U, \Delta z)$ refers to the cross-correlation signal, while $P_{TF}(U, \Delta z)$ and $P_{TT}(U, \Delta z)$ refer to the respective auto-correlation signals. It should be noted that the multi-frequency angular power spectra refer to the signals at two different redshifts $z$ and $z + \Delta z$. It is useful to visualize these power spectra as the correlation of angular modes which are defined on two different planes at comoving distances $r$ and $r + \Delta r$ corresponding to $z$ and $z + \Delta z$ respectively. Our entire analysis is performed in a narrow range of redshifts $\Delta z \ll z$. The power spectrum also depends on $z$, however the variation with $\Delta z$ is much faster as compared to the $z$ dependence and the latter is not explicitly shown. The value of $z$ for which the results have been shown is mentioned separately wherever required.

We emphasize here that the power spectra $P_{\alpha\gamma}(U, \Delta z)$ are directly related to observable quantities. In particular, the visibilities measured in radio interferometric observations of the redshifted 21-cm signal (Bharadwaj & Sethi, 2001; Bharadwaj & Ali, 2005) are directly related to $\Delta_T(U, z)$ defined here. The multi-frequency angular power spectra $P_{\alpha\gamma}(U, \Delta z)$ contain the entire three dimensional information through the $U$ and $\Delta z$ dependence. The fluctuations in the signal in the transverse directions are analyzed using Fourier modes $U$ which are equivalent to the angular multipoles in the flat sky approximation, whereas $\Delta z$ corresponds to a comoving separation along the radial direction.

The statistical homogeneity and isotropy of the cosmological density fluctuations ensures that the power spectra defined above are real, though it is not apparent from equation (3).
It also follows that the power spectra are isotropic in $U$ and depend only on $U = |U|$. The assumption that both $\delta_F$ and $\delta_T$ are related to the matter fluctuations $\delta$ allows us to express all the angular power spectra considered here in terms of the three dimensional matter power spectrum $P(k)$. Following Datta et al. (2007), we have

$$P_{\alpha\gamma}(U, \Delta z) = \frac{1}{\pi v^2} \int_0^\infty dk_\parallel \cos(k_\parallel \Delta r) F_{\alpha\gamma}(\mu) P(k).$$

where $\Delta r = c\Delta z/H(z)$ is the radial comoving separation corresponding to the redshift separation $\Delta z$, $k = \sqrt{k_\parallel^2 + (\frac{2\pi U}{r})^2}$, $\mu = k_\parallel/k$, and

$$F_{\alpha\gamma}(\mu) = C_\alpha C_\gamma [1 + \beta_\alpha \mu^2] [1 + \beta_\gamma \mu^2]$$

We may identify the angular mode $U$ with the angular multipole $\ell$ as $2\pi U = \ell$ whereby the power spectrum $P_a(U, \Delta z)$ can be identified with the multi-frequency angular power spectrum (MAPS) $C_\ell^a(\Delta z) = P_a(U, \Delta z)$ under the flat-sky approximation (Datta et al., 2007). The approximation is known to be reasonably good on scales $\ell > 10$ considered in our subsequent analysis. We shall use $\ell$ and $U$ interchangeably to represent the inverse angular scale and similarly one may think of $P_a(U, \Delta z)$ and $C_\ell^a(\Delta z)$ to both equivalently represent the angular power spectrum.

Quasar surveys indicate that the redshift distribution of the quasars peaks in the range $z = 2$ to 3 (Schneider et al., 2005). For any given quasar, it is possible to estimate $\delta_F$ in only a small redshift range which is governed by the redshift of the quasar. The quasar’s proximity effect excludes the region very close to the quasar. Large redshift separations are excluded to avoid contamination from other spectral lines. Given these considerations, we have chosen $z = 2.5$ as the fiducial redshift for our analysis. For comparison, we have also shown several of the results at the neighbouring redshifts $z = 1.5$ and 3.5. In the subsequent discussion we shall interchangeably use the three dimensional wave number $k$, the two dimensional Fourier mode $U$ and the angular mode $\ell$ to refer to inverse angular scales on the sky. Similarly, it may

| $z$ | $r$ (Mpc) | $\nu_{21}$ (MHz) | $\lambda_{Ly}$ (Å) | $\ell$ | $\Delta r$ (Mpc) | $\Delta \nu$ (MHz) | $\Delta v_{\parallel}$ (Km/s) |
|-----|-----------|------------------|------------------|-------|-----------------|-----------------|------------------|
| 1.5 | 4435      | 568              | 3040             | 44    | 7               | 19              | 2.27             |
| 2.5 | 5944      | 406              | 4256             | 59    | 9.4             | 12              | 1.16             |
| 3.5 | 6945      | 316              | 5472             | 69    | 11              | 8.4             | 0.70             |

Table 1. The conversions between length scales expressed in various units.
Figure 1. The LCDM linear power spectrum normalised to $z = 0$. The correspondence between $\ell$ and $k$ is shown for three different redshifts. For a fixed $\ell$, all the Fourier modes to the right of the corresponding $k$ value contribute to $P_c(\ell, \Delta z)$.

Figure 2. The transverse angular power spectra $P_{FT}(\ell)$ and $P_T(\ell)$ of the cross-correlation and 21-cm auto-correlation signals respectively shown at $z = 1.5$, 2.5 and 3.5.

sometimes be convenient to express the redshift separation $\Delta z$ in terms of a radial separation $\Delta r$, or a frequency separation $\Delta \nu = 1420\Delta z/(1 + z)^2$ MHz (for 21-cm observations), or a velocity separation $\Delta v_\parallel = c\Delta z/(1 + z)$ (for Ly-$\alpha$ forest). We have indicated the conversion between these various possibilities in Table 1.

2.1 Transverse Angular Power Spectrum

We first consider $P_{FT}(\ell) \equiv P_{FT}(U, \Delta z)$ for $\Delta z = 0$. The Ly-$\alpha$ forest and the 21-cm signals, here, both lie on the same plane transverse to the line of sight $\hat{m}$. We refer to $P_{FT}(\ell)$, which only contains 2D information, as the transverse angular power spectrum of the cross-correlation signal. This has contribution from all the three dimensional Fourier modes $\mathbf{k}$ whose projection on the transverse plane matches $\ell/r \equiv 2\pi U/r$. Thus, all 3D Fourier modes
$k > \ell/r$ contribute to $P_{FT}(\ell)$. Figure 1 shows the 3D matter power spectrum $P(k)$ for the LCDM model with WMAP 5 cosmological parameters Komatsu et al. (2009). The correspondence between $k$ and $\ell$ is shown for redshifts $z = 1.5, 2.5$ and $3.5$. For a fixed $\ell$, all the Fourier modes to the right of the corresponding $k$ value contribute to $P_{FT}(\ell)$.

The detectability of the cross-correlation signal is crucially dependent on the amplitude of $P_{FT}(\ell)$. This amplitude depends on $C_T$ and $C_F$, whose values are highly uncertain. Given these uncertainties it would be very difficult to interpret the amplitude if measured, quantitatively in terms of the physical properties of the IGM. On the contrary it is easier to relate the shape of $P_{FT}(\ell)$ to the matter power spectrum and the comoving distance $r$. We elucidate this by considering a simple toy model for the matter power spectrum. In this model we have $P(k) = Ak$ for $k \leq k_{eq}$, and $P(k) = Ak_{eq}(k/k_{eq})^{-3}$, for $k > k_{eq}$. where $k_{eq}$ is the Fourier mode that enters the horizon at the epoch of matter-radiation equality. Simplifying eq. (4) by ignoring the effect of redshift space distortion, $P_{FT}(\ell)$ is the 2D projection of the 3D matter power spectrum $P(k)$ onto the plane transverse to the line of sight. For large values of $\ell$ ($> k_{eq}r$) we have a power law $P_{FT}(\ell) \approx AC_T C_F / \pi \ell^2$. The angular power spectrum flattens out around $\ell = k_{eq}r$, which corresponds to the peak in the matter power spectrum. The position of this feature scales with redshift as $\ell \propto r$. At the low $\ell$ ($< k_{eq}r$), we have $P_{FT} \approx AC_T C_F / 2 \pi r^2$ which is independent of $\ell$. We see that there is an enhancement of power at low $\ell$ (large angular scales ). This is a generic feature of projection from 3D to 2D (Kaiser & Peacock, 1991).

Figure 2 shows $P_{FT}(\ell)$ for the LCDM model. For comparison we have also shown the corresponding 21-cm auto-correlation angular power spectrum at the same probing redshifts. The Ly-\(\alpha\) auto-correlation power spectrum is expected to be dominated by the Poisson noise arising due to the discrete QSO sampling, and this is not shown here. In addition to the fiducial redshift 2.5 we have also shown $z = 1.5$ and 3.5 with the intent of displaying how the shape of the angular power spectrum changes with $z$. As mentioned earlier, the amplitude and also its $z$ dependence are relatively uncertain. To estimate the amplitude we have assumed that $C_F$ and $\beta_F$ do not change with $z$ while we have calculated $C_T$ and $\beta_T$ using the $z$ dependence from Guha Sarkar et al. (2011). We see that the shape of $P_{FT}(\ell)$ predicted for the LCDM model is in reasonable agreement with our simple toy model. At large $\ell$ ($> 1000$) we have $P_{FT}(\ell) \propto \ell^{-1.76}$ as compared to $P_{FT}(\ell) \propto \ell^{-2}$ for the toy model. This discrepancy arises because the actual matter power spectrum is shallower than $k^{-3}$ for $k > k_{eq}$. The LCDM predictions and the toy model are in close agreement at small $\ell$ where
$P_{FT}(\ell)$ is nearly independent of $\ell$. The $\ell$ value where the flattening occurs is also found to be consistent with the predictions of the toy model. The behaviour of the 21-cm auto-correlation angular power spectrum is very similar to that of the cross-correlation signal.

2.2 Radial decorrelation function

Here we fix $\ell$ (or equivalently $U$) and study $P_{FT}(U, \Delta z)$ as a function of $\Delta z$. This considers the correlation between the signals $\Delta F(U, z)$ and $\Delta T(U, z + \Delta z)$ which refer to the same angular mode $U$ but are respectively located on two different planes that are separated by a comoving distance $\Delta r$. The cosine term in eq. (4) arises from the fact that a single 3D mode projects onto the two different planes with a phase difference of $k_{\parallel} \Delta r$. This oscillatory cosine
term ensures that the radial correlation $P_{FT}(\ell, \Delta z)$ decreases with increasing $\Delta z$. Ignoring redshift space distortion, we have $P_{FT}(\ell, \Delta z) \propto \int_0^\infty dk_\parallel \cos(k_\parallel \Delta r) P(\sqrt{k^2_\parallel + (\ell/r)^2})$. This integral is very close to zero if $P(\sqrt{k^2_\parallel + (\ell/r)^2})$ is nearly constant over the range $k_\parallel = 0$ to $k_\parallel = 2\pi/\Delta r$ which corresponds to one oscillation of the cosine term. We thus expect $P_{FT}(\ell, \Delta z)$ to be maximum at $\Delta r = 0$, decrease with increasing $\Delta r$ and remain close to zero for $\Delta r$ larger than $\Delta r \sim 2\pi r/\ell$ (Bharadwaj & Pandey, 2003). We quantify this behaviour using the decorrelation function (Datta et al., 2007)

$$\kappa_\ell(\Delta z) = \frac{P_{FT}(\ell, \Delta z)}{P_{FT}(\ell)}$$

which has value $\kappa_\ell(\Delta z) = 1$ at $\Delta z = 0$, and varies in the range $0 \leq |\kappa_\ell(\Delta z)| \leq 1$. Figure 3 shows $\kappa_\ell(\Delta z)$ for different $\ell$ values at the fiducial redshift $z = 2.5$. We see that $\kappa_\ell(\Delta z)$ falls with increasing $\Delta z$ and crosses zero beyond which the signal is anti-correlated ($\kappa_\ell(\Delta z) < 0$).

We define the “decorrelation length” $\Delta z_{0.1}$ as the redshift separation where the decorrelation function falls to 10% of its peak value (i.e. $\kappa_\ell(\Delta z_{0.1}) = 0.1$). The signals $\Delta_F(U, z)$ and $\Delta_T(U, z + \Delta z)$ are weakly correlated beyond the corresponding radial separation $\Delta r_{0.1}$. The decorrelation length, we find, decreases with increasing $\ell$ i.e. the signal decorrelates faster at smaller angular scales. This relation is well fitted by the relation $\Delta z_{0.1} \propto \ell^{-0.76}$ for the entire $\ell$ and $z$ range considered here. A similar behaviour has been reported earlier for the 21-cm auto-correlation signal (Bharadwaj & Pandey, 2003).

The signal crosses zero at $\Delta z > \Delta z_{0.1}$, beyond which it is anti-correlated. For the particular value $\ell = 300$, Figure 4 shows a magnified view of the region where the signal is anti-correlated. The behaviour is similar for other values of $\ell$ where the value of $\kappa_\ell(\Delta z)$
Figure 6. The linear matter power spectrum $P(k)$, which has the BAO features, has been divided by $P(k)^{nw}$ the “no wriggles” power spectrum (Eisenstein & Hu, 1998). The corresponding $\ell$ values have been shown for $z = 1.5, 2.5$ and $3.5$. For a fixed $\ell$, all the Fourier modes to the right of the corresponding $k$ value contribute to $P_{FT}(\ell, \Delta z)$.

The decorrelation function is sensitive to the redshift being probed. Figure 5 shows $\kappa_{\ell}(\Delta z)$ for three different redshifts. We find that though the generic features remain the same, the decorrelation length $\Delta z_{0.1}$ increases with increasing redshift for all values of $\ell$.

We note that the $\kappa_{\ell}(\Delta z)$ for the 21-cm auto-correlation has a behaviour very similar to $\kappa_{\ell}(\Delta z)$ of the cross-correlation signal discussed here. In fact, the difference between these two is less than 1%. This small difference arises because of the difference in the values of $\beta_F$ and $\beta_T$. The behaviour of the decorrelation function $\kappa_{\ell}(\Delta z)$ is sensitive to the cosmological parameters and this can be used to observationally determine the cosmological parameters (Bharadwaj et al., 2009), however we do not discuss this here. We note here that the distinct behaviour of $\kappa_{\ell}(\Delta z)$ for the 21-cm auto-correlation allows one to distinguish the cosmological signal from astrophysical foregrounds (Ali et al., 2008; Ghosh et al., 2010, 2011).

3 THE BARYON ACOUSTIC OSCILLATIONS

The characteristic scale of the BAO is set by the sound horizon $s$ at the epoch of recombination given by

$$s = \int_{a_r}^1 \frac{c_s(a)}{a^2 H(a)} da$$

where $a_r$ is the scale factor at the epoch of recombination and $c_s$ is the sound speed given by $c_s(a) = c/\sqrt{3(1 + 3\rho_b/4\rho_\gamma)}$, where $\rho_b$ and $\rho_\gamma$ denotes the photon and baryonic den-
Figure 7. This shows the imprint of the BAO on the transverse angular power spectrum $P_{FT}(\ell)$ for the cross-correlation signal. To highlight the BAO we have divided $P_{FT}(\ell)$ by $P_{nw}^{FT}(\ell)$ which corresponds to $P(k)^{nw}$. This is shown for three redshifts $z = 1.5, 2.5$ and $3.5$.

The comoving length-scale $s$ defines a transverse angular scale $\theta_s = s[(1 + z)D_A(z)]^{-1}$ and a radial redshift interval $\Delta z_s = s H(z)/c$, where $D_A(z)$ and $H(z)$ are the angular diameter distance and Hubble parameter respectively. The comoving length-scale $s = 143$ Mpc corresponds to $\theta_s = 1.38^\circ$ and $\Delta z_s = 0.07$ at $z = 2.5$. Measurement of $\theta_s$ and $\Delta z_s$ separately, allows the independent determination of $D_A(z)$ and $H(z)$. The CMBR angular power spectra maps the density fluctuations on the plane of the sky at $z \sim 1000$. Hence, the imprint of BAO in the form of acoustic peaks in the CMBR anisotropy angular power spectrum is only in the transverse direction and constrains $\theta_s$. Low redshift galaxy surveys contain both the transverse and the radial BAO peaks. However, the difficulties in probing large radial distances leads to small survey depths. Moreover, a large shot noise contribution degrades the SNR making it very difficult to independently measure $D_A(z)$ and $H(z)$. Typically, the combination $[(1 + z)^2 D_A^2(z)c \Delta z/H(z)]^{1/3}$ is measured instead in galaxy redshift surveys (Eisenstein et al., 2005; Percival et al., 2007). The multi-frequency angular power spectrum discussed here allows us, in principle, to measure the BAO imprint in the both transverse and the radial directions. Large bandwidth radio observations covering a large portion of the sky, along with high density quasar surveys with spectra measured at high SNR would allow a detection of the BAO feature in both radial and transverse directions. The different sensitivities of the measured quantities $D_A(z)$ and $H(z)$ on the cosmological parameters would ensure breaking of degeneracies in the parameter space. For example $D_A(z)$ shall constrain curvature more efficiently than $H(z)$. Further, independent
The BAO manifests itself as a series of oscillations in the linear matter power spectrum (Eisenstein & Hu, 1998). We focus on the first BAO peak which has the largest amplitude. This is a $\sim 10\%$ feature in $P(k)$ at $k \approx 0.045$ (Figure 6) which corresponds to $\ell \approx 270$ at $z = 2.5$. We also see that the first peak will shift to lower $\ell$ at smaller redshifts.

Figure 7 shows the BAO feature in the transverse angular power spectrum of the cross-correlation signal $P_{FT}(\ell)$. The BAO, here, is seen projected onto a plane. The BAO appears as a series of oscillations in $P_{FT}(\ell)$, the positions of the peaks being consistent with $\ell \sim k/r$. The amplitude of the first oscillation in $P_{FT}(\ell)$ is around 1%, in contrast to the $\sim 10\%$ feature seen in $P(k)$. This reduction in amplitude arises due to the projection to a plane whereby several 3D Fourier modes which do not have the BAO feature also contribute to the $\ell$ where the first BAO peak is seen. At $z = 2.5$ the first peak occurs at $\ell \sim 270$ and it has a full width of $\Delta \ell \sim 200$. The position $\ell$ and width $\Delta \ell$ of the peak both scale as $r$ if the redshift is changed. It is, in principle, possible to determine $D_A(z)$ by measuring the $\ell$ position of the first BAO peak.
We next consider the imprint of the BAO on the cross-correlation signal along the radial direction. This is quantified, as before, through the radial decorrelation function $\kappa_\ell(\Delta z)$. We expect the first BAO peak to have an imprint on $\kappa_\ell(\Delta z)$ at only the angular modes $\ell$ which are less than $k_1 r$, where $k_1$ refers to the position of the first peak in the 3D matter power spectrum. Thus, for $z = 2.5$ we do not expect the first peak to have any impact on $\kappa_\ell(\Delta z)$ at $\ell > 500$. We expect the first BAO peak to have an impact at all angular modes $\ell \leq 500$. The relative contribution is expected to be maximum around $\ell \sim 270$, and then fall off at $\ell < 100$ where it will be diluted by the $k$ modes which are smaller than $k_1$ and hence do not contain the BAO signal. Figure 8 shows the radial decorrelation function $\kappa_\ell(\Delta z)$ for four different values of $\ell (= 100, 250, 300, 500)$. For comparison, we also show $\kappa_\ell(\Delta z)$ calculated using $P_{nw}(k)$ which does not have the BAO features.

We recollect that $\kappa_\ell(\Delta z)$ is maximum at $\Delta z = 0$ where $\kappa_\ell(\Delta z) = 1$. In the absence of the BAO features the value of $\kappa_\ell(\Delta z)$ falls rapidly with increasing $\Delta z$. This is followed by a $\Delta z$ range where $\kappa_\ell(\Delta z)$ is negative, beyond which it oscillates around 0. The BAO, we find, has little impact on the $\Delta z$ range near $\Delta z = 0$ where $\kappa_\ell(\Delta z)$ is positive. The BAO features are found to have a very significant effect at large $\Delta z$, typically in the range $\Delta z = 0.04$ to 0.16, where $\kappa_\ell(\Delta z)$ is mainly negative. The corresponding 21-cm frequency and radial velocity intervals are $\Delta \nu = 4.64 - 13.9$ MHz and $\Delta v_\parallel = 3.5 \times 10^3 - 10.3 \times 10^3$ km/s respectively. The imprint of the BAO appears as a ringing feature around the smooth ‘no wriggles’ $\kappa_\ell(\Delta z)$. The ringing feature due to the BAO is quite distinct from the slow oscillation of $\kappa_\ell(\Delta z)$ which is also present in the no-wiggles model. The value of $\kappa_\ell(\Delta z)$ is quite small ($\sim -0.01$) in the $\Delta z$ range where the BAO features occur. The deviation due to the BAO, however, could be as large as 40% to $\sim 100\%$ relative to the no-wiggles model.

The position of the BAO feature shifts to smaller $\Delta z$ values if $\ell$ is increased. For a fixed $\ell$, we use $\Delta z_{BAO}$ to denote the position corresponding to the maximum deviation from the no-wiggles model. The value $\Delta z_s = H(z)s/c$, introduced earlier, corresponds to $\Delta z_{BAO}$ at $\ell = 0$. It is not possible to directly determine $\Delta z_s$ from $\kappa_\ell(\Delta z)$. It is however, it is in principle possible to determine $H(z)$ by measuring $\Delta z_{BAO}$ at different $\ell$ values.

4 THE CROSS-CORRELATION ESTIMATOR

In this section we construct an estimator for $P_{FT}(U, \Delta z)$, and consider the statistical properties of this estimator. First, we assume that both the Ly-\(\alpha\) forest and the 21-cm observations
are pixelized along the $z$ axis into pixels or channels of width $\Delta z_c$ such that both $\delta \mathbf{F}(\bar{\theta}, z)$ and $\delta T(\bar{\theta}, z)$ are measured only at discrete redshifts $z_n = z_0 + n\Delta z_c$ with $n = 1, 2, ..., N_c$. Here $z_0$ is a reference redshift, $N_c$ is the total number of channels and $N_c\Delta z_c = B$ is the total redshift interval or the bandwidth spanned by the observations.

Considering first the Ly-\(\alpha\) forest, we have, till now, considered $\delta \mathbf{F}(\bar{\theta}, z_n)$ as a continuous field defined at all points on the sky. In reality, it is possible to measure this only along a few, discrete lines of sight where there are background quasars. We account for this by defining $\delta \mathbf{F}_o(\bar{\theta}, n)$. the observed fluctuation in the transmitted Ly-\(\alpha\) flux, as

$$\delta \mathbf{F}_o(\bar{\theta}, n) = \rho(\bar{\theta}) \left[ \delta \mathbf{F}(\bar{\theta}, z_n) + \delta \mathbf{F}_N(\bar{\theta}, n) \right] \tag{8}$$

where $\delta \mathbf{F}_N(\bar{\theta}, z)$ is the contribution from the pixel noise in the quasar spectra and

$$\rho(\bar{\theta}) = \frac{\sum_a w_a \delta^2 D(\bar{\theta} - \bar{\theta}_a)}{\sum_a w_a} \tag{9}$$

is the quasar sampling function. Here $a = 1, 2, ..., N$ refers to the different quasars in the $L \times L$ field of view, $\bar{\theta}_a$ and $w_a$ respectively refer to the angular positions and weights of the individual quasars. We have the freedom of adjusting the weights to suit our convenience. It is possible to change the relative contribution from the individual quasars by adjusting the weights $w_a$.

The quasar sampling function $\rho(\bar{\theta})$ is zero everywhere except the angular position of the different quasars. It is sometimes convenient to express the noise contribution in eq. (8) as

$$\rho(\bar{\theta}) \delta \mathbf{F}_N(\bar{\theta}, n) = \frac{\sum_a w_a \delta^2 D(\bar{\theta} - \bar{\theta}_a) \delta \mathbf{F}_N(\bar{\theta}_a, n)}{\sum_a w_a} \tag{10}$$

where $\delta \mathbf{F}_N(\bar{\theta}_a, n)$ refers to the pixel noise contribution for the different quasars. The faint quasars typically have noisy spectra in comparison to the bright ones. We can take this into account and choose the weights $w_a$ so as to increase the contribution from the bright quasars relative to the faint ones, thereby maximizing the SNR for the signal estimator. For the present analysis we have made the simplifying assumption that the magnitude of $\delta \mathbf{F}_N(\bar{\theta}_a, n)$ is the same across all the quasars irrespective of the quasar flux. We have modelled $\delta \mathbf{F}_N(\bar{\theta}_a, n)$ as Gaussian random variables with the noise in the different pixels being uncorrelated i.e.

$$\langle \delta \mathbf{F}_N(\bar{\theta}_a, n) \delta \mathbf{F}_N(\bar{\theta}_b, m) \rangle = \delta_{a,b}\delta_{n,m} \sigma^2_{\mathbf{F}_N} \tag{11}$$

where $\sigma^2_{\mathbf{F}_N}$ is the variance of the pixel noise contribution. It is appropriate to use uniform weights $w_a = 1$ in this situation. Such an assumption is justified in the situation where there exist high SNR measurements of the transmitted flux for all the quasars.

Considering next the sampling function $\rho(\bar{\theta})$, we assume that the quasars are randomly
distributed with no correlation amongst their angular position, and the positions also being unrelated to \( \delta_F \) and \( \delta_T \). In reality, the quasars do exhibit clustering (Myers et al., 2007), however the contribution from the Poisson fluctuation is considerably more significant here and it is quite justified to ignore the effect of quasar clustering for the present purpose. The Fourier transform of \( \rho(\theta) \) then has the properties that

\[
\langle \tilde{\rho}(U) \rangle = \delta_{U,0} \tag{12}
\]

and

\[
\langle \tilde{\rho}(U_1)\tilde{\rho}(U_2) \rangle = \frac{1}{N} \delta_{U_1,U_2} + \left(1 - \frac{1}{N}\right) \delta_{U_1,0} \delta_{U_2,0} \tag{13}
\]

which we shall use later. In the subsequent analysis, we also assume that \( N \gg 1 \) whereby \( (1 - 1/N) \approx 1 \).

We use \( \Delta F_0(U, z) \) to denote the Fourier of \( \delta F_0(\theta, z) \). Using eq. (8) we have

\[
\Delta F_0(U, z) = \tilde{\rho}(U) \otimes [\Delta F(U, z) + \Delta N_F(U, z)] \tag{14}
\]

where \( \otimes \) denotes a convolution defined as

\[
\tilde{\rho}(U) \otimes \Delta F(U, z) = \frac{1}{L^2} \sum_{U'} \tilde{\rho}(U - U') \Delta F(U', z) \tag{15}
\]

Using eqs. (3), (10), (11), (12), (13) and (14) we calculate the following statistical properties of \( \Delta F_0 \),

\[
\langle \Delta F_0(U, n) \rangle = 0 \tag{16}
\]

and

\[
\langle \Delta F_0(U_1, n)\Delta F_0(U_2, m) \rangle = \delta_{U_1,U_2} \left[ \frac{1}{L^2} P_{FF}(U_1, (n - m)\Delta z_c) \right. \\
+ \left. \frac{1}{N L^2} \sum_U P_{FF}(U, (n - m)\Delta z_c) + \delta_{n,m} \frac{\sigma_{F2}^2}{N} \right] \tag{17}
\]

It is possible to simplify the sum over \( U \) using Parseval’s theorem whereby

\[
\frac{1}{L^2} \sum_U P_{FF}(U, (n - m)\Delta z_c) = \xi_F((n - m)\Delta z_c), \tag{18}
\]

where \( \xi_F(\Delta z) = \langle \delta_F(\theta_a, z)\delta_F(\theta_a, z + \Delta z) \rangle \) is the one dimensional (1D) correlation function of the fluctuations in the transmitted flux along individual quasar spectra. The 1D correlation \( \xi_F(\Delta z) \), or equivalently \( \xi_F(v_{\|}) \), is traditionally used to quantify the Ly-\( \alpha \) forest along quasar spectra, and this has been quite extensively studied (Becker et al., 2004; Coppolani et al., 2006; D’Odorico et al., 2006).
Imprint of BAO on the Cross-correlation of Redshifted 21-cm Signal and Ly-α Forest

Using $\bar{n}_Q = N/L^2$ to denote the quasar density on the sky, we have

$$\langle \Delta_{F_o}(U_1, n) \Delta^*_{F_o}(U_2, m) \rangle = \delta_{U_1, U_2} L^{-2} P_{FF_o}(U_1, n - m)$$  \hspace{1cm} (19)

where

$$P_{FF_o}(U, p) = P_{FF}(U, p \Delta z_c) + \frac{1}{\bar{n}_Q} \left[ \xi_F(p \Delta z_c) + \delta_{p,0} \sigma^2_{F_N} \right]$$  \hspace{1cm} (20)

Ideally, we would expect $P_{FF_o}(U, p)$ to provide an unbiased estimate of $P_F(U, p \Delta z_c)$. The discrete quasar sampling, however, introduces an extra term $\xi_F(p \Delta z_c)/\bar{n}_Q$. The other term $\delta_{p,0} \sigma^2_{F_N}/\bar{n}_Q$ arises due to the pixel noise, and it contributes only when we correlate a channel with itself. Though assumed to be a constant across the redshift range considered here, we note that $\bar{n}_Q$ is in fact a function of the magnitude limit of the survey through the luminosity function $\frac{dn_Q}{dz}(z)$ (Jiang et al., 2006) which accounts for the variability of quasar luminosities.

Radio interferometric observations directly measure $\Delta_T(U, z_n)$. We consider a radio-interferometric array with several antennas, each of diameter $D$. The antenna diameter and the field of view $L$ are related as $\lambda/D \approx L$, where $\lambda$ is the observing wavelength. Each pair of antennas measures $\Delta_T(U, z_n)$ at a particular $U$ mode corresponding to $U = d/\lambda$, where $d$ is the antenna separation projected perpendicular to the line of sight. The baselines $U$ corresponding to the different antenna pairs are, in general, arbitrarily distributed depending on the array configuration. The observed HI fluctuation $\Delta_{T_o}(U, n)$ at two different $U$ values are correlated if $|U_1 - U_2| \leq 1/L$. It is possible to combine the baselines where the signal is correlated by binning the $U$ values using cells of size $L^{-1} \times L^{-1}$. We then have the binned baselines at $U = (n_x \hat{i} + n_y \hat{j})/L$ ($n_x, n_y$ are integers) which exactly match the Fourier modes of the Ly-α signal. The HI signal at different $U$ values are now uncorrelated. We then have

$$\Delta_{T_o}(U, n) = \Delta_T(U, z_n) + \Delta_{T_N}(U, n)$$  \hspace{1cm} (21)

where $\Delta_{T_N}$ is the corresponding noise contribution. The noise in different channels and baselines is uncorrelated, and

$$\langle \Delta_{T_o}(U_1, n) \Delta^*_{T_o}(U_2, m) \rangle = \delta_{U_1, U_2} L^2 P_{TT_o}(U_1, n - m)$$  \hspace{1cm} (22)

where

$$P_{TT_o}(U, p) = P_{TT}(U, p \Delta z_c) + \delta_{p,0} N_T(U).$$  \hspace{1cm} (23)

Here $N_T(U)$ is the noise power spectrum. For a single polarization and a single baseline, this is given by

$$N_T(U) = \left( \frac{T^2_{sys}}{2 \Delta \nu_c \Delta t} \right) \frac{[\int d\Omega \mathcal{P}(\hat{\theta})]^2}{[\int d\Omega \mathcal{P}^2(\hat{\theta})]}$$  \hspace{1cm} (24)
where $T_{\text{sys}}$ is the system temperature, $\Delta \nu_c$ the frequency interval corresponding to $\Delta z_c$, $\Delta t$ the integration time and $\mathcal{P}(\vec{\theta})$ is the normalised power pattern of the individual antennas (Chengalur et al., 2007). The exact value of the ratio of the two integrals in eq. (24) depend on the antenna design. It is convenient here to express eq. (24) as

$$N_T(U) = \frac{T_{\text{sys}}^2 L^2}{\chi N_{\text{pol}} M(U) \Delta \nu_c \Delta t}.$$  

where $N_{\text{pol}}$ is the number of polarizations being used, $M(U)$ the number of baselines in the particular cell corresponding to $U$, and $\chi$ is a factor whose value depends on the antenna beam pattern $\mathcal{P}(\vec{\theta})$. For the purpose of this paper it is reasonable to assume a value $\chi = 0.5$.

We have

$$\langle \frac{1}{2} [\Delta F_o(U_1, n) \Delta T_o^*(U_2, m) + \Delta T_o^*(U_1, n) \Delta T_o(U_2, m)] \rangle = \delta_{U_1, U_2} P_{FTo}(U_1, n - m)$$  

(26)

where

$$P_{FTo}(U, p) = P_{FT}(U, p \Delta z_c)$$  

(27)

which can serve as an estimator for the cross-correlation signal. It is, however, possible to increase the signal to noise ratio by averaging over the entire redshift interval (or frequency band). We therefore define the estimator $\hat{E}(U, p)$ as

$$E(p) = \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{1}{2} [\Delta F_o(n) \Delta T_o^*(m) + \Delta T_o^*(n) \Delta T_o(m)] \delta_{|n-m|, p}}$$  

(28)

The various terms in eq. (28) all refer to the same $U$ value which is not explicitly shown for convenience of notation. Note that we shall adopt this convention of not explicitly showing $U$ in several of the subsequent equations.

The estimator has the property that

$$\langle E(U, p) \rangle = P_{FT}(U, p \Delta z_c)$$  

(29)

ie. it is an unbiased estimator for the cross-correlation signal. We next consider the covariance

$$\langle E(U_1, p) E(U_2, q) \rangle - \langle E(U_1, p) \rangle \langle E(U_2, q) \rangle$$  

which, we find, is zero if $U_1 \neq U_2$. We therefore only consider

$$\text{Cov}(p, q) = \langle \Delta E(p) \Delta E(q) \rangle = \langle E(p) E(q) \rangle - \langle E(p) \rangle \langle E(q) \rangle$$  

(30)

where all the terms refer to the same $U$.

We note that

$$\langle [\Delta F_o(n) \Delta T_o^*(m) + \Delta T_o^*(n) \Delta T_o(m)] [\Delta F_o(s) \Delta T_o^*(t) + \Delta T_o^*(s) \Delta T_o(t)] \rangle =$$

$$4 P_{FTo}(n - m) P_{FTo}(s - t) + 2 P_{FTo}(n - t) P_{FTo}(s - m)$$

$$+ 2 P_{FT}(n - s) P_{TT}(t - m)$$  

(31)
whereby

\[
\text{Cov}(p, q) = \frac{\sum_{\alpha,\beta} \sum_{m,n,s,t} P_{\alpha\beta o}(m-s) P_{\alpha'\beta' o}(n-t) \delta_{m-n,p} \delta_{s-t,q}}{4 \sum_{\alpha,\beta} \sum_{m,n,s,t} \delta_{m-n,p} \delta_{s-t,q}} \tag{32}
\]

where the variable \(\alpha'\) has value \(F\) when \(\alpha = T\) and vice-versa, and \(\beta, \beta'\) are defined in a similar way. We simplify eq. (32) in two steps where we first have

\[
\text{Cov}(p, q) = \sum_{\alpha,\beta} \sum_{n=1}^{N_c-p} \sum_{m=1}^{N_c-q} P_{\alpha\beta o}(p+m-n) P_{\alpha'\beta' o}(m-n-q) \frac{8(N_c-p)(N_c-q)}{8(N_c-p)N_c-q} \tag{33}
\]

Next, assuming that \(p \geq q\), we have

\[
\text{Cov}(p, q) = \sum_{\alpha,\beta} \sum_{n=1}^{N_c-p} \sum_{m=1}^{N_c-q} P_{\alpha\beta o}(p+m-n) P_{\alpha'\beta' o}(m-n-q) \frac{8(N_c-p)(N_c-q)}{8(N_c-p)N_c-q} \tag{34}
\]

We use eq. (34) to calculate the covariance \(\text{Cov}(U, p, q)\) for the individual \(U\) values. The expected signal is statistically isotropic in \(U\), and it is useful to bin the estimates of the power spectrum. We have the 'binned' estimator

\[
E_B(U_B, p) = \frac{\sum_i W_i E(U_i, p)}{\sum_i W_i} \tag{35}
\]

where \(W_i\) refers to the weights assigned to the estimates at different \(U_i\), and \(U_B\) refers to the average baseline of the particular bin

\[
U_B = \frac{\sum_i W_i |U_i|}{\sum_i W_i} \tag{36}
\]

The binned estimator has a covariance

\[
\text{Cov}_B(U_B, p, q) = \frac{\sum_i W_i^2 \text{cov}(U_i, p, q)}{N_{\text{poin}} \sum_{i,j} W_i W_j} \tag{37}
\]

where the parameter \(N_{\text{poin}}\) refers to the number of independent pointings. This parameter is introduced to allow for the possibility that we are combining estimates of the power spectrum from observations in \(N_{\text{poin}}\) independent parts of the sky. It is quite clear from eq. (37) that this helps to increase the signal to noise ratio as \(1/\sqrt{N_{\text{poin}}}\). In subsequent parts of this paper, we have used eq. (37) to predict the noise in observations to measure \(P_{FT}(U, \Delta z)\).

5 OBSERVATIONAL CONSIDERATIONS

The quasar redshift distribution peaks in the range \(z = 2\) to 3 (Schneider et al., 2005), and for our analysis we only consider the quasars in this redshift range. For a quasar at a redshift \(z_Q\), the region 10,000 km s\(^{-1}\) blue-wards of the quasar’s Ly-\(\alpha\) emission is excluded from the
Ly-\(\alpha\) forest due to the quasar’s proximity effect. Further, only the pixels at least 1,000 km s\(^{-1}\) red-ward of the quasar’s Ly-\(\beta\) and O-VI lines are considered to avoid the possibility of confusing the Ly-\(\alpha\) forest with the Ly-\(\beta\) forest or the intrinsic O-VI absorption. For a quasar at the fiducial redshift \(z_Q = 2.5\), the above considerations would allow the Ly-\(\alpha\) forest to be measured in the redshift range \(1.96 \leq z \leq 2.39\) spanning an interval \(\Delta z = 0.43\).

We consider redshifted 21 cm observations of bandwidth \(B = 128\) MHz covering the frequency range 355 MHz to 483 MHz which corresponds to the redshift range 1.94 \(\leq z \leq 3\) with bandwidth \(B = 1\) in redshift units. The Ly-\(\alpha\) forest of any particular quasar will be measured in a smaller interval \(\Delta z \approx 0.4\) which is the deciding factor for the cross-correlation signal. Thus, only a fraction (approximately 40\%) of the total number of quasars in this redshift range 2 to 3 will contribute to the cross-correlation signal at any redshift \(z\). We incorporate this in our estimates by noting that \(\bar{n}_Q\) in eq. (20) refers to only 40\% of all the quasars in the entire \(z\) range 2 to 3.

Cosmic variance is a limiting factor for measuring the BAO signal. We first present a preliminary analysis to determine the observational considerations so that the first BAO peak is above the cosmic variance. This depends on \(N_k\) the number of independent Fourier modes in the \(k\) range \(k_a = 0.03\) Mpc\(^{-1}\) to \(k_b = 0.07\) Mpc\(^{-1}\) which covers the first BAO peak (Figure 6). We have

\[
N_k = \frac{V}{2\pi^2} \int_{k_a}^{k_b} k^2 \, dk
\]

where \(V\) is the observational volume. For observations covering the solid angle \(\Omega\) and the \(z\) range 1.94 to 3, we have \(N_k = 2.38 \times 10^5 \Omega\). The BAO peak is a 10\% feature in \(P(k)\) and it is necessary to divide the interval \(k_a\) to \(k_b\) into \(N_{\text{bin}}\) bins in order to identify this feature. An \(N_\sigma\) detection of the first BAO peak requires that the uncertainty in the observed power spectrum \(\Delta P(k)/P(k) = \sqrt{N_{\text{bin}}/N_k}\) should be less than \(1/(10N_\sigma)\). We then have an estimate of the solid angle that needs to be observed for a \(N_\sigma\) detection

\[
\Omega = 1.89 \times 10^{-2} \left(\frac{N_{\text{bin}}}{5}\right) \left(\frac{N_\sigma}{3}\right)^2 \text{sr}.
\]

We would like a single field of view of the radio-interferometer \(\Omega = L^2\) to be large enough so as to cover this solid angle, whereby we need

\[
L = 8^\circ \left(\frac{N_{\text{bin}}}{5}\right)^{0.5} \left(\frac{N_\sigma}{3}\right).
\]

A 3 – \(\sigma\) detection with \(N_{\text{bin}} = 5\) requires a field of view \(L = 8^\circ\) which can be achieved if we have a radio-interferometric array where the individual antennas are around \(D = 5\) m.
Imprint of BAO on the Cross-correlation of Redshifted 21-cm Signal and Ly-α Forest

Figure 9. The data points shows the binned transverse angular power spectrum \( P_{FT}(\ell) \) for the cross-correlation signal at \( z = 2.5 \) with 1 − \( \sigma \) error bars. The error bars correspond to the set of observational parameters (see text) that give a 5 − \( \sigma \) detection of \( P_{FT}(\ell) \).

In diameter. It is advantageous to have a larger field of view, and we consider antennas of diameter \( D = 2 \) m which gives a \( L = 20^\circ \) field of view for which a 7.7 − \( \sigma \) detection is possible if \( N_{\text{bin}} = 5 \). Note that this sets the upper limit for the signal to noise ratio (SNR) that can be achieved in a single pointing. In the next section we present more detailed estimates of the SNR, taking into account various factors like the discrete quasar sampling and the noise in the quasar spectra and the radio data.

We next discuss the array layout that would be required for these observations. Using \( d/\lambda = U = kr/2\pi \), we estimate that the Fourier modes \( k_a \) and \( k_b \) correspond to antenna separations \( d_a = 15m \) and \( d_b = 36m \) respectively. These figures roughly set the range of antenna separations that would be required in the radio-interferometric array. Based on these considerations we consider a radio interferometric array which has \( N_{\text{ann}} \) antennas distributed such that all the baselines \( \vec{d} \) within \( d_{\text{max}} = 50m \) are uniformly sampled, whereby \( M(U) \) is independent of \( U \) and we have \( M(U) \approx 4 (N_{\text{ann}}/100)^2 \). Using this in eq. (25), assuming \( T_{\text{sys}} = 100K \), \( N_{\text{pol}} = 2 \), \( \chi = 0.5 \) we have

\[
N_T = 1.0 \times 10^{-3} [mK]^2 \left( \frac{100}{N_{\text{ann}}} \right)^2 \left( \frac{100 \text{ KHz}}{\Delta \nu} \right) \left( \frac{1000 \text{ hrs}}{\Delta t} \right).
\]

6 DETECTABILITY

In the previous section we have discussed several observational considerations which essentially determine the redshift interval and angular scales that need to be covered in order
to detect the imprint of the BAO in the cross-correlation signal. We have seen that observations covering the redshift interval from $z = 2$ to 3 using a radio interferometric array with antennas of diameter 2 m each are well suited for this purpose. Further, for the purpose of this paper, we have made the simplifying assumption that the array layout is such that it uniformly samples all the baselines within 50 m. Given this observational framework, we now estimate the required observational sensitivity for a detection of the BAO signal.

The variance of the cross-correlation estimator $\text{Cov}(p, p) = \langle \Delta E(p) \Delta E(p) \rangle$ is a sum of several terms (eq. 34) of the form $P_{FT}(p + m) P_{FT}(p - m)$ and $P_{TT}(p + m) P_{FF}(p - m)$, where $P_{FT}$ refers to the cross-correlation signal that we are trying to detect. The terms
Imprint of BAO on the Cross-correlation of Redshifted 21-cm Signal and Ly-α Forest

Figure 12. This shows the first BAO peak in the transverse angular power spectrum $P_{TT}(\ell)$ for the cross-correlation signal. To highlight the BAO we have divided $P_{TT}(\ell)$ by $P_{TT}^{nw}(\ell)$ which has no-wiggles. The binned data points and error bars correspond to $1 - \sigma$ for the set of observational parameters (see text) which give a $5 - \sigma$ detection of the transverse BAO.

Figure 13. This shows $\Delta \kappa_{\ell}(\Delta z)$, the difference between the BAO and no-wiggle models for the cross-correlation signal at the $\ell$ bin centered at $\ell = 230$. The binned data points and error bars correspond to $1 - \sigma$ for the set of observational parameters (see text) which give a $5 - \sigma$ detection of the radial BAO.

$P_{FF_o}$ and $P_{TT_o}$ refer to the auto-correlation of the Ly-α and redshifted 21-cm observations respectively. Note that $P_{FF_o}$ and $P_{TT_o}$ both have contributions from observational noise, and hence they differ from the respective cosmological auto-correlations signals $P_{FF}$ and $P_{TT}$.

We first consider $P_{FF_o}$ which refers to the Ly-α forest. In addition to $P_{FF}$, this also has a contribution from noise that arises due to the discrete QSO sampling. This noise, which arises from the discrete sampling of quasars, is proportional to $\bar{n}_Q^{-1}$ (eq. 20). To recapitulate, $\bar{n}_Q$ refers to approximately 0.4 times the total angular density of quasars in the redshift range 2 to 3. This noise also depends on $\xi_F$ for which we have used the values from Becker
et al. (2004). Further, we have assumed that the Lyman-α forest spectra are all measured with a high sensitivity such that the pixel noise contribution has a value $\sigma_{F,N}^2 = 0.04$.

We next consider $P_{TT_o}$ which refers to the redshifted 21-cm observations. In addition to the cosmological signal $P_{TT}$, this also has a contribution $N_T$ from the system noise (eq. 23). The quantity $N_T$ refers to the noise power spectrum of the radio interferometric observations. The value of $N_T$ depends on several observational parameters. In the present context, for observations with a fixed channel width $\Delta \nu$, $N_T$ depends only on the number of antenna $N_{ann}$ and the observing time $\Delta t$ (eq. 41). We see that there are many different combinations of $N_{ann}$ and $\Delta t$ which will give the same value of $N_T$.

In the present analysis we have used $\bar{n}_Q$ and $N_T$ to parametrize the respective sensitivities of the QSO survey and the redshifted 21-cm observations. We have $P_{FF_o} \rightarrow P_{FF}$ and $P_{TT_o} \rightarrow P_{TT}$ in the limits $\bar{n}_Q \rightarrow \infty$ and $N_T \rightarrow 0$ respectively. The limit $\bar{n}_Q \rightarrow \infty$ and $N_T \rightarrow 0$ sets the upper bound for the SNR, which also corresponds to the cosmic variance. The additional noise contributions arising from $\bar{n}_Q$ and $N_T$ will degrade the SNR to a value which is lower than the cosmic variance. In this work we would like to determine the range of $\bar{n}_Q$ and $N_T$ where it will be possible to detect the BAO feature. The entire subsequent discussion refers to the fiducial redshift $z = 2.5$. Though we expect all the quantities to vary across the redshift range $z = 2$ to $z = 3$, we anticipate that the values at $z = 2.5$ will be representative of the entire $z$ range.

We first consider the transverse angular power spectrum $P_{TT}(\ell) \equiv P_{TT}(U, \Delta z)$ at $\Delta z = 0$. It is also possible to study the transverse angular power spectrum by considering the $U$ dependence of $P_{TT}(U, \Delta z)$ while holding $\Delta z$ fixed at some other value $\Delta z > 0$. The signal, however, is maximum for $\Delta z = 0$. Further, it is also possible to increase the SNR by suitably combining the estimates at different values of $\Delta z$. However, we do not consider these possibilities here, and we only consider the transverse angular power spectrum at $\Delta z = 0$. The $\ell$ range $125 \leq \ell \leq 330$, we have seen, is adequate for detecting the first BAO peak (Figure 7). We assume that the baselines $U$ within this $\ell = 2\pi |U|$ range have been divided into 5 bins of equal $\Delta U$, and we consider the results (eg. Figure 9) at the average $\ell$ values (eq. 36) corresponding to each of these bins. We have set $W_i = 1$ whereby all all the baselines within a bin contribute equally to the binned estimator (eq. 35). Here we have chosen a channel width of 100KHz for the 21-cm data, and we assume that the Ly-α forest is smoothed at the corresponding velocity width of $\approx 100\text{Kms}^{-1}$. The cross-correlations signal starts to decorrelate (Figure 3) if the frequency channel width is larger. We have used the
results for the central $\ell$ bin, which roughly corresponds to the center of the first BAO peak, to assess the overall SNR. Note that in the subsequent discussion we use $\text{SNR}_1$ to refer to the signal to noise ratio for a single field of view, and $\text{SNR}$ (or total SNR) to refer to the general situations where there are $N_{\text{poin}}(\geq 1)$ independent pointings of the $20^\circ \times 20^\circ$ field of view. Figure 10 shows $\text{SNR}_1$ contours as a function of $\bar{n}_Q$ and $N_T$. We see that the $\text{SNR}_1$ does not increase very significantly beyond $\bar{n}_Q > 60\,\text{deg}^{-2}$ or $N_T < 10^{-7}\text{mK}^2$. We have the maximum value of the $\text{SNR}_1$ at the bottom right corner of Figure 10 where both of these conditions are satisfied. This corresponds to the $\bar{n}_Q \to \infty$ and $N_T \to 0$ limit, and the maximum $\text{SNR}_1$, which corresponds to the cosmic variance, has a value $\text{SNR}_1 = 300$. Note that the $\text{SNR}_1$ contours in Figure 10 refer to the detection of the cross-correlation signal $P_{F_T}(\ell)$, and not the BAO which is just a 1% feature in $P_{F_T}(\ell)$. We find that a $5-\sigma$ detection of $P_{F_T}(\ell)$ is possible for $\bar{n}_Q \sim 0.1\,\text{deg}^{-2}$ and $N_T \sim 10^{-4}\text{mK}^2$. The required QSO density is well within the scope of present observational capabilities, for example the currently available SDSS (Schneider et al., 2005) has a total QSO number density of $\sim 1\,\text{deg}^{-2}$ implying $\bar{n}_Q \sim 0.4\,\text{deg}^{-2}$ which is in excess of the required QSO number density. The requirement that $N_T \sim 10^{-4}\text{mK}^2$ can be achieved in $1,000\text{hrs}$ if we have 300 antennas. A longer observation will be required if we have fewer antennas (eq: 41). Figure 7 shows the binned data points and the $1-\sigma$ error-bars that are expected in such an observation.

The first BAO peak is a 1% feature in $P_{F_T}(\ell)$, and a $5-\sigma$ detection of the BAO peak requires an SNR of 500 for $P_{F_T}(\ell)$. It is not possible to reach an SNR greater than 300 with a single field of view (Figure 10), and it is necessary to consider multiple pointings. The total signal to noise ratio increases as

$$\text{SNR} = \sqrt{N_{\text{poin}}\text{SNR}_1},$$

when the number of independent fields of view is increased. We require 100 independent fields of view to detect the BAO with the currently available SDSS. This exceeds the angular area of the SDSS, and is not viable. It is necessary to consider a survey with a higher quasar density. The upcoming BOSS (McDonald et al., 2005) is expected to have a QSO density of $16\,\text{deg}^{-2}$ which corresponds to $\bar{n}_Q = 6.4\,\text{deg}^{-2}$. The BOSS $^3$ survey is expected to cover $\sim 10,000\text{deg}^2$ of the sky, and we could ideally have $N_{\text{poin}} = 25$ independent pointings of the $20^\circ \times 20^\circ$ field of view. We see that, with BOSS, it is possible to achieve an SNR larger than 100 in a single field of view (Figure 10). Therefore, a $5-\sigma$ detection of the first

$^3$ http://cosmology.lbl.gov/BOSS/
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BAO peak is possible with \( N_{\text{poin}} \leq 25 \), which is within the total angular coverage of BOSS. Figure 11 shows how SNR\(_1\) varies with \( N_T \) for a single field of view of BOSS. We see that the SNR\(_1\) scales as \( N_T^{-1/2} \) for large \( N_T \) where the system noise in the radio observations is much larger than the HI signal. In this regime we have \( \text{SNR} \propto \sqrt{N_{\text{poin}}/N_T} \) i.e. relatively shallow observations of a large number of fields of view, or deep observations of a few fields of view would both give the same SNR provided that the total observing time \( N_{\text{poin}} \Delta t \) (eq. 41) is the same in both of these situations. Ideally, it is most advantageous to work in this region. However, we see that the SNR\(_1\) is pretty low (\( \sim 40 \)), and a \( 5 - \sigma \) detection of the BAO peak is not possible within the total angular coverage of BOSS. It is necessary to consider observations that go deeper than \( 10^{-4} \text{ mK}^2 \). The increases in SNR\(_1\) is slower than \( N_T^{-1/2} \) for \( N_T \leq 10^{-4} \text{ mK}^2 \), and the SNR\(_1\) saturates at \( N_T \leq 10^{-7} \text{ mK}^2 \) where \( N_T \) is much smaller than the HI signal. The range \( 10^{-5} \text{ mK}^2 \geq N_T \geq 10^{-6} \text{ mK}^2 \), where the SNR\(_1\) is in the range 100 to 200, is relevant for detecting the BAO with the cross-correlation signal using BOSS. In this region it is optimal to increase the total SNR by increasing the number of fields of view instead of increasing the depth of the individual observations. We find that we have SNR\(_1\) = 100 for \( N_T = 1.1 \times 10^{-5} \text{ mK}^2 \), and a \( 5 - \sigma \) detection of the first BAO peak is possible with \( N_{\text{poin}} = 25 \) fields of view. Alternatively, a \( 5.8 - \sigma \) detection is possible with \( N_{\text{poin}} = 16 \) and \( N_T = 6.25 \times 10^{-6} \text{ mK}^2 \). The scaling is approximately \( \text{SNR} \propto N_{\text{poin}}^{0.5} N_T^{-0.38} \) here, and the total observing time is smaller if we consider the shallower observations. Figure 12 shows the expected binned data points and error-bars for the detection of the first BAO peak.

The BIGBOSS (Schlegel et al., 2009) has been conceived as the successor to the upcoming BOSS QSO survey. BIGBOSS may achieve a QSO density of \( \sim 64 \text{ deg}^{-2} \) which corresponds to \( \bar{n}_Q = 25.6 \text{ deg}^{-2} \). This is very close to the region where we have the cosmic variance limit, and the limiting SNR\(_1\) of \( \sim 300 \) can be achieved if \( N_T \approx 2 \times 10^{-6} \text{ mK}^2 \) (Figure 10). For this value of \( N_T \), a \( 5 - \sigma \) detection of the first BAO peak is possible with only two fields of view. However this is not the optimal observational strategy. For BIGBOSS, the behaviour of the SNR\(_1\) as a function of \( N_T \) is very similar to that for BOSS, except that the SNR\(_1\) values are 1.5 times larger (Figure 11). The range \( 3.3 \times 10^{-5} \text{ mK}^2 \geq N_T \geq 10^{-6} \text{ mK}^2 \), where the SNR\(_1\) is in the range 100 to 300, is relevant for detecting the BAO with the cross-correlation signal using BIGBOSS. It is most advantageous to consider relatively shallow observations with \( N_T = 3.3 \times 10^{-5} \text{ mK}^2 \), for which a detection is possible with \( N_{\text{poin}} = 25 \). The scaling here is \( \text{SNR} \propto N_{\text{poin}}^{0.5} N_T^{-0.43} \), which is quite close to the \( N_T^{-0.5} \) behaviour.
We next consider the detection of the radial oscillations. We have seen (Figure 8) that the radial oscillations occur in the $\Delta z$ range $0.04$ to $0.14$. Where, in the absence of the BAO, the cross-correlation signal is anti-correlated \( \kappa_\ell(\Delta z) \leq 0 \), and relatively weak with \( |\kappa_\ell(\Delta z)| \sim 0.01 \). The BAO introduces a ringing feature which is highlighted in Figure 13 which shows $\Delta \kappa_\ell(\Delta z)$ which is the difference between the BAO and the no-wiggles model. We see that the deviation could be as large as $\Delta \kappa_\ell(\Delta z) \approx 0.01$. Therefore, the net effect of the radial oscillations is a $\sim 1\%$ deviation relative to $P_{FT}(\ell)$, which is comparable to the deviation introduced by the angular oscillations. Though the angular and the radial oscillations both introduce $\sim 1\%$ deviations relative to the no-wiggles model, we do not expect that every observation which is capable of detecting the angular oscillations will also be able to detect the radial oscillations. The radial oscillations occur at $\Delta z \approx 0.1$, and we have only $(B/\Delta z) \sim 10$ independent estimates in our observation. In contrast, the tangential feature occurs at $\theta_s \approx 1.38^\circ$ and we have $(20^\circ/20^\circ)^2 \approx 210$ independent estimates. We thus expect the radial oscillations to have a lower SNR in comparison to the tangential oscillations discussed earlier. The BAO signal is maximum in the vicinity of $\ell \approx 250$, and we have collapsed the three central $\ell$ (or $U$) bins in order to enhance the SNR for the radial oscillations. We find that this enhances the SNR by a factor of approximately $\sqrt{2.7}$. The frequency channel width is maintained at $\Delta \nu = 100$ KHz which is the same as the value used for the tangential oscillations. Note that the errors in the $P_{FT}(U, \Delta z)$ values estimated at different $\Delta z$ will, in general, be correlated and a rigorous error analysis would require us to calculate the full covariance matrix (eq. 34). We have not used the covariance matrix in the present analysis, Instead, we have assessed the SNR for a detection of the radial oscillations by using just the single value at $\Delta z = 0.1$ where the deviation from the no-wiggles model is maximum (Figure 13). We find that a $5 - \sigma$ detection is possible with BOSS if we observe 25 fields of view with $N_T = 6.25 \times 10^{-6}$ mK$^2$. We require $N_T = 1.7 \times 10^{-5}$ mK$^2$ for a similar detection with BIGBOSS.

7 SUMMARY AND DISCUSSION

In this paper we have developed a theoretical formalism for estimating the cross-correlation signal between the fluctuations in the Ly-$\alpha$ forest and the fluctuations in the redshifted 21-cm emission from neutral hydrogen. Both of these quantities are measured as functions of frequency (redshift) and angular scale. Consequently, we have used the Multi-frequency An-
Angular Power Spectrum (MAPS) to quantify the statistical properties of the cross-correlation signal. This deals directly with the observed quantities, and retains the distinction between the angular and the frequency information. This, as we shall elaborate shortly, is very important in the light of foreground removal and continuum subtraction.

Continuum fitting and subtraction is a very critical step in calculating $\delta_T$ for the Ly-\(\alpha\) forest and several different methods have been proposed for handling this (Croft et al., 2002; McDonald et al., 2006). Errors in continuum subtraction can be a serious problem for the Ly-\(\alpha\) forest auto-correlation signal (Kim et al., 2004). The redshifted 21-cm signal is buried deep under astrophysical foregrounds which are several orders of magnitude larger (Shaver et al., 1999; Di Matteo et al., 2002; Santos et al., 2005; Wang & Hu, 2006; Ali et al., 2008; Pen et al., 2009; Ghosh et al., 2010). Several different techniques have been proposed for separating the cosmological 21-cm signal from the foregrounds (Ali et al., 2008; Jelič et al., 2008; Bowman et al., 2009; Liu et al., 2009; Ghosh et al., 2010). Foreground removal is a rather severe problem for observations of the redshifted 21-cm auto-correlation signal. In addition to the 21-cm signal, the foregrounds make a very large contribution to the expectation value of the auto-correlation estimator. However, the foregrounds are believed to have a slowly varying, smooth frequency (or $\Delta z$) dependence which is quite distinct from the signal which decorrelates rapidly with increasing $\Delta z$ (Figure 3). It is therefore possible to remove the foregrounds from the measured MAPS ($P_{FT}(\ell, \Delta z)$) by subtracting out any component that varies slowly with $\Delta z$. In fact, Ghosh et al. (2011) have recently used MAPS to analyze 610-MHZ GMRT observations and show that it is possible to remove the foregrounds from the m auto-correlation by fitting and subtracting out slowly varying polynomials in $\Delta \nu$.

We do not expect the continuum in the Ly-\(\alpha\) forest to have any correlation with the foregrounds in the redshifted 21-cm observations, and consequently they will not contribute to the expected cross-correlation signal. The continuum and the foregrounds will, however, appear as extra contributions to the variance of the cross-correlation estimator. Since these contributions appear in the variance, it is possible to reduce these by combining different independent estimates of the cross-correlation. We can reduce the continuum and foreground contributions by combining estimates of the cross-correlation at different baselines $U$ and different fields of view. The problem, therefore, is much less severe in comparison to the auto-correlation. Further, the continuum and the foregrounds are both expected to have a slowly varying frequency (or $\Delta z$) dependence and it should be possible to remove these from
the measured MAPS by subtracting out the component that varies slowly with \( \Delta z \). We plan to perform a detailed analysis of these issues in future.

Our study shows that it is possible to have a \( 5 - \sigma \) detection of the imprint of the first BAO peak in the cross-correlation signal using BOSS, an upcoming QSO survey. For this, we have considered a radio interferometric array that covers the \( z \) range \( z = 2 \) to \( 3 \) using antennas of size \( 2 \text{ m} \times 2 \text{ m} \) which have a \( 20^\circ \times 20^\circ \) field of view. We find that we need to observe 25 fields of view, approximately the full angular coverage of BOSS, with a noise level of \( N_T = 1.1 \times 10^{-5} \text{mK}^2 \) and \( N_T = 6.25 \times 10^{-6} \text{mK}^2 \) in order to achieve a \( 5 - \sigma \) detection of the angular and radial oscillations respectively. The corresponding noise levels are \( N_T = 3.3 \times 10^{-5} \text{mK}^2 \) and \( N_T = 1.7 \times 10^{-5} \text{mK}^2 \) for BIGBOSS whose quasar density is expected to be four times larger than that of BOSS.

We now briefly discuss how it may be possible to carry out such observations. In our analysis we have made the simplifying assumption that the antennas are distributed such that all the baselines within 50 m are uniformly sampled. We consider an interferometric array with \( N_{ann} = 400 \) antennas which roughly corresponds to the maximum number of \( 2 \text{ m} \times 2 \text{ m} \) antennas that can fit in a \( 50 \text{ m} \times 50 \text{ m} \) region. We see that we need \( \Delta t = 5700 \text{ hr} \) and 10,000 hr of observation per field of view (41) to reach the noise levels required to detect the angular and radial oscillations respectively with BOSS. The corresponding figures are 1900 hr and 3700 hr for the BIGBOSS. Note that we it is required to observe a single field of view for 8 hr a day for a whole year in order to achieve 3000 hrs of observing time.

It is quite evident that we require to observe 25 fields of view, with 2 to 3 years of dedicated observations for each field, in order to detect the BAO. Operating sequentially, considering one field after the next, the required observations would possibly run over a period of 50 yr to a century, which raises the need to consider alternative observational strategies. It is a viable possibility to have antennas that can simultaneously observe several independent fields of view. However, it is unlikely (if not impossible) to have antennas that can simultaneously observe 16 or 25 such \( 20^\circ \times 20^\circ \) fields of view. For the purpose of this discussion, we assume that we have antennas that can simultaneously observe 4 fields of view. We then see that it would approximately require observations over a decade (or more) in order to detect the BAO. Another possibility is to have several radio interferometric arrays, each located at a different location and observing a different parts of the sky. Four to five separate arrays, each with 400 antennas, would be required to carry out these observations in the span of a few years. It is important to note that it may be possible to reduce the
observational requirements to some extent by optimally distributing the baselines instead of considering them to be uniformly distributed. We propose to investigate these issues in a future study.

Observations of the BAO can be used to constrain the values of various cosmological parameters. The equation of state of the Dark Energy is particularly important in this context. In this paper we have mainly estimated the range of observational parameters for which it will be possible to detect the BAO using the cross-correlation signal. We plan to study a variety of issues including the optimal array configuration and cosmological parameter estimation in future.

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