Birefringence lens effects of atom ensemble enhanced by electromagnetically induced transparency

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We study the optic control for birefringence of a polarized light by an atomic ensemble with a tripod configuration, which is mediated by the electromagnetically induced transparency with a spatially inhomogeneous laser. The atom ensemble splits the linearly polarized light ray into two orthogonally-polarized components, whose polarizations depend on quantum superposition of the initial states of the atom ensemble. Accompanied with this splitting, the atom ensemble behaves as a birefringent lens, which allows one polarized light ray passing through straightly while focus another orthogonal to this polarization with finite aberration of focus.

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Introduction. An atom ensemble, manipulated by electromagnetic field, exhibits various quantum coherent properties, such as, extremely slow group velocities [1, 2, 3], large refractive indexes [4], giant nonlinearities [3, 4, 7], laser-induced birefringence [8], and electromagnetically induced transparency (EIT) [9, 10, 11, 12, 13]. Recently, an enhanced deflection for an unpolarized light beam is observed in an EIT medium with an external gradient field [16, 17]. Different from conventional EIT studies, the external fields, used to control the light propagation, are transversely-inhomogeneous [14, 15, 16, 17, 18, 19, 20, 21].

The EIT-induced light deflection has been explained using quantum approach with dark state polariton possessing an effective magnetic moment [16, 20], or the semiclassical approach [18] with the gradient-index medium, caused by the external fields with inhomogeneous profiles. The quantum approach in Ref. 20 exhibits the wave-particle duality of the dark polaritons, where an effective Schrödinger equation is derived to describe the EIT-enhanced spatial motion of the probe field, similar to spinless particle in an inhomogeneous field.

Most recently, we visualized a polarized light ray as a spin and study an optical analog of the Stern-Gerlach effect for this ray. Here, the atom medium becomes anisotropic with respect to the light polarization when a transversely inhomogeneous field [21] is applied. Therefore, the linearly polarized probe light splits into two with opposite circular-polarization. In this case the optical split can not be simply compared with the original Stern-Gerlach effect since the incoherent atomic population has been assumed and thus the split is not a superposition of two polarization exactly. Actually, such field-induced birefringence have been studied extensively [22, 23, 24, 25], but most of them can refer to the incoherent optical Stern-Gerlach effect.

In this paper, we consider what would happen if the atoms are initially superposed with submanifold states of the atoms. This consideration for atomic coherence offers the possibility to change the polarization of the outgoing wave from linear to any desired polarization state. It will be showed that, for the atoms with a tripod configuration, our optically control of polarization is a direct result using the superpositions of submanifold states with the intrinsic double-A-type EIT structure. As a laser drives the atom ensemble to become an anisotropic medium, the EIT assisted spatial motion of a linearly-polarized probe beam exhibits a birefringent phenomena with the polarization focusing (defocusing). Namely, a linearly-polarized light propagates along a straight line in the medium, another orthogonal-polarized beam comes to a focus at different positions along the z-axis. Such phenomenon – light rays parallel to a lens axis fail to converge to the same point, is called aberration. It is more interesting that, by changing the frequency of the control light from “blue detuning” to “red detuning”, the lens-like object can be adjusted optically form negative (or diverging ) to positive (or converging)cases. This divergence-to-convergence transition of the lens like effect is only enhanced in the double EIT configuration.

**FIG. 1:** (Color online)Energy level scheme (a) for the tripod atoms interacting with a coupling field (indicated by Rabi frequency Ω) and a linear-polarized probe field. Such atom ensemble confined in a gas cell behaves as a converging (or diverging) lens.

Setup. We consider an ensemble of $2N$ identical and
noninteracting atoms, which is confined in a gas cell of length \( L \) along \( z \)-axis in Fig. \( \Pi \). The atoms possess four levels in a tripod configuration as shown in Fig. \( \Pi \). The submanifold of ground state are spanned by two degenerate Zeeman sublevels \(| g \rangle \) and \(| h \rangle \). The atoms are initially prepared in the superposition \(| \phi \rangle = \alpha | g \rangle + \beta | h \rangle \) of the Zeeman sublevels. The linearly-polarized probe light is characterized by field operators \( \hat{E}_1 \) and \( \hat{E}_2 \). Due to some selection rule, the \( \sigma^+ - (\sigma^-) \) component \( \hat{E}_1 (\hat{E}_2) \) only couples the Zeeman sublevel \(| g \rangle (| h \rangle) \) to excited state \(| u \rangle \), while the transition \(| f \rangle - | u \rangle \) is driven by an intense, classical laser field with a Rabi frequency \( \Omega = \Omega(\delta) \) by applied along the \( z \)-direction. Obviously, there exist two \( \Lambda \) configurations consisting of energy levels \((g, f, u)\) and \((h, f, u)\), thus comprises the double-EIT set-up.

Without loss of the generality, we consider the case that both quantum and classical fields propagate parallel to the \( z \)-axis. In reality, both the probe and control field are characterized by wave packets with spatially-Gaussian profiles. Therefore, each component of the probe field can be interpreted by a plane wave with a slowly varying operator [26]:

\[
\hat{E}_j^+ (\mathbf{r}, t) = \sqrt{\frac{\nu}{2\varepsilon_0 V}} E_j (\mathbf{r}, t) e^{i(kz - \nu t)}, (j = 1, 2). \tag{1}
\]

We also introduce the following collective continuous operator \( \hat{\sigma}_{\mu \nu} (\mathbf{r}) = \sum_{\nu' \in N_c} \sigma_{\mu \nu'}/N_r \) for the collective excitations in the atomic medium. It actually describes the average of \( \sigma_{\mu \nu} = |\mu\rangle \langle \nu' | \) over \( N_c (= 2N/V) \) atoms in a small but macroscopic volume \( V \) around position \( \mathbf{r} \). The slowly varying operators \( \sigma_{\mu \nu} \) for the atomic transition operator are respectively defined as \( \sigma_{ug} = \sigma_{ug} \exp(-ikz), \sigma_{ug} = \sigma_{ug} \exp(-ikz) \) and \( \sigma_{uf} = \sigma_{uf} \exp(-ikz) \). Here, \( k \) and \( k_c \) are the wave numbers to the central frequencies \( \nu \) and \( \nu_c \) of the probe and control field respectively. For cold atoms, the kinetic energy could be neglected, so the total system can be modeled by the interaction Hamiltonian

\[
H_I = \frac{N}{V} \int d^3 \mathbf{r} \left[ \delta g \sigma_{gg} + \delta h \sigma_{hh} + \delta f \sigma_{ff} \right] - (\Omega \sigma_{uf} + g \hat{E}_1 \sigma_{ug} + g \hat{E}_2 \sigma_{uh} + H.c.) \tag{2}.
\]

Due to the symmetry of the states \(| g \rangle \) and \(| h \rangle \), the transition matrix element in the above equation is the same \( g_{ug} = g_{uh} = g = \langle u | d | g \rangle /\sqrt{\nu}/(2\varepsilon_0 V) \) for both circular components, where \( \langle u | d | g \rangle \) is the dipole matrix element.

Effective motion equation of light. We follow the effective equation approach in Ref. [21] to study the atomic response by assuming the atomic operators as their average in some initial state, e.g. \(| \phi \rangle \). To elucidate the induced-lens behavior of the atom ensemble, we consider the Heisenberg-Langevin equations for the atomic and field operators with the ground-state coherence relaxation rate \( \gamma \) and the decay rate \( \Gamma \) of the excited state \(| u \rangle \), which are introduced phenomenologically. Since the intensity of the quantum probe field is much weaker than that of the control field, and the number of photons in the signal pulse is much less than the number of atoms in the sample, we treat the atomic equations perturbatively with the perturbative parameters \( g \Omega, \beta \).

For the initially superposition \(| \phi (0) \rangle \) of Zeeman sublevels, the averages of atomic operators up to the zeroth order are given by

\[
\sigma_{g \phi}^{(0)} = Tr(| \phi \rangle \langle \phi | \sigma_{mn}), \tag{3a}
\]

\[
\sigma_{h \phi}^{(0)} = \langle \phi | \langle \phi | \sigma_{mm} \rangle, \tag{3b}
\]

where \( \Phi = (g \delta s - \delta f)/[2|\Omega(x, y)|^2 \] for \( s = g, h \). The above equation shows that when light travels through an atomic ensemble, the atom response produces collective electric-dipole moments. This atom response also gives a back-action on light, thus leads to the paraxial wave equation

\[
(i \partial_t + ic \partial_z + \frac{c}{2k} \nabla^2_T) \Phi = V(x, y) \sigma^{(0)} \Phi \tag{5}
\]

for the light field envelope \( \Phi = (E_1, E_2)^T \), which behave as a spinor moving in a spin-dependent effective potential

\[
V(x, y) \sigma^{(0)} = \frac{|\gamma|^2 N \Delta}{|\Omega(x, y)|^2} \sigma^{(0)} \tag{6}
\]

Here \( \Delta = \delta_h - \delta_f \) is the two photon detuning. This visualized “spin” represents the polarization state of a probe light. Obviously, the coupling between atoms and light induces a spin-dependent potential \( V(x, y) \sigma^{(0)} \) to affect light propagation with opposite polarized-orientation. Consequently, a signal pulse parallel to the control beam may deviate from its original trajectory, when it travels across the medium. We also note that by applying position-dependent fields, an initially isotropic medium
becomes anisotropic [21]. However, the magnetic field is necessary for the system to display the circular birefringence. Here, we show that a linear birefringence may also occur.

To describe the propagation of the probe beam clearly, we introduce two polarized components $E_- \equiv \beta E_1 - \alpha E_2$ and $E_+ \equiv \alpha E_1 + \beta E_2$, which are the coherent superpositions of the left- and right-circular polarizations. In the following we only consider that case $\alpha$ and $\beta$ are real. In terms of $E_\pm$, the Schrödinger-like equation (5) becomes

$$\left( i \partial_t + \frac{c}{2k} \nabla^2 \right) E_\pm = \frac{1 \pm 1}{2} V(x, y)$$  \hfill (7)

The above equation indicates that, the $E_-$ ray propagates along a straight line (since no potential act on it), which means that the medium is homogenous for $E_-$. However, the component $E_+$, subject to an effective potential, experiences a declination.

In order to elucidate the spin-dependent lens behavior induced by the coupling laser, we assume that the coupling field has a Gaussian profile $\Omega(x) = \Omega_0 \exp(-x^2/2\sigma^2)$. Let the probe beam possess a Gaussian profile

$$E_\pm (0) = \frac{1}{\sqrt{\pi} b^2} e^{-\frac{(x-a^2)}{2\sigma^2}}$$  \hfill (8)

before it encounters the medium. Here, $\sigma$ is the width of the driving-field profile, $b$ is the width of the probe field, and $a$ is the initial location of the wave packet center of the probe field along the $x$ direction. When $b$ is much smaller than $\sigma$, $|\Omega|^2$ is expanded in Taylor series around $a$, and we retain up to the linear term. Equation (7) reads

$$i \partial_t E_\pm = -\left( i c \partial_x + \frac{c}{2k} \partial_x^2 \right) E_\pm - \frac{1 \pm 1}{2} \zeta (x - a + \eta/\zeta) E_+$$  \hfill (9)

where $\zeta = 2\eta/\sigma^2$ and $\eta = \Delta |g|^2 N \exp(\alpha^2/\sigma^2)/\Omega_0^2$. Here, we restrict our discussion to the 2D system in a $x-z$ plane. Equation (9) can be exactly solved by the Weix-Norman algebraic method [27] to give $E_-(t, x, t) = E(t, a)$ and

$$E_+ (x, t) = E(t, a + \frac{c}{2m} \zeta (x-a))$$  \hfill (10)

where

$$E(t, a) = \exp \left[ -\frac{(x-a)^2}{2b^2 + \frac{a}{m}} - \frac{(z-ct)^2}{2\sigma^2} \right] \sqrt{\pi} (b^2 + \frac{a}{m})$$

and we have defined the effective mass $m = k/c$.

**Polarization dependent deflection and birefringence lens effects.** Now we discuss the physics implied in Eq. (10). Equation (10) shows that, the profile center $(x, z) = (a, 0)$ of the linear-polarized component $E_+$ at time $t = 0$, is shifted to $(x = x_+, z = L)$ at time $t = L/c$, where

$$x_+ = a + \frac{g^2 N a \Delta L^2}{ck\Omega_0^2 \sigma^2 - c^2 \eta^2},$$  \hfill (11)

but nothing happens to the $E_-$-component. Therefore, if one tracks the center motion of the probe beam, the trajectory of the linear-polarized $E_-$-component is only a straight line along $z$-axis. However, its orthogonal-component is deflected by the atom ensemble. $E_+$ propagates either toward or away from the $z$-axis, which depends on the sign of the two-photon detuning $\Delta$ and the incident position $a$ of the probe field.

![Figure 2](https://example.com/figure2.png)

**FIG. 2:** (Color online) Schematic of a probe ray passing through an EIT medium. The vertical line (black one) is the $z$-axis, the solid-yellow line indicates the profile of the coupling field, the green rectangle represents the atomic medium. The propagation of the $E_-$-component is illustrated by the blue-dashed line in (a). The red-dotted line in (a) indicates the trajectory of the $E_+$-component when $\Delta = 0$ or $a = 0$. The medium renders a lens-like effect in (b) and (c). The medium converges the $E_+$-component beam (the red-dotted line in (b)) when $\Delta < 0$, and diverges the $E_+$-component beam (the blue-dashed line in (c)) when $\Delta > 0$.

Figure 2 shows the different cases for the $E_+$-ray propagation. Panel (a) indicates the trajectory of the linear-polarized component $E_-$ (blue-dashed line) as well as its orthogonal component $E_+$ (red-dotted line) under the situation $\Delta = 0$ or $a = 0$. They all travels across the atomic medium straightly. Panel (b) corresponds to the case when $\Delta < 0$, where the medium acts as a lens causing focus of the $E_+$-component. Panel (c) describes the case when $\Delta > 0$, where the $E_+$-component experiences a defocusing.

The above analysis implies that the EIT medium behaves as a lens with a varying focus, which can be feasibly optical-controlled. Right after light leaves the medium, it obtains a transverse group velocity with magnitude

$$v_x = \frac{2La\Delta |g|^2 N}{k\sigma^2 \Omega_0^2} \exp(\alpha^2/\sigma^2) $$  \hfill (12)

In the case of “red detuning” $\Delta < 0$ [Fig. 2(a)], the EIT medium made $v_x$ toward the $z$-axis. Therefore, a collimated light ray, starting at points $z = \pm a$ parallel to the lens axis (i.e. $z$-axis), will meet the $z$-axis at

$$z = f_{\text{con}}(a) = \frac{L}{2} + F(a)$$  \hfill (13)
where $F(a) = \frac{k\alpha^2}{(2L/\Delta)[|q|^2N]} \exp(-a^2/\sigma^2)$. Hence the cuboid gas cell), this focal point runs from $f_{\text{con}}(h)$ to $f_{\text{con}}(0)$. This shows a typical aberration with a distortion length $l = F(0) - F(h)$. Obviously, such an EIT-based lens do not form perfect images, due to distortions or aberrations introduced by the $a$-dependent focal point and the widths spreading of the wave packet. Therefore, the atomic medium functions as a converging (or positive) lens to some extent. In the "blue detuning" case with $(\Delta > 0)$ in Fig.3,(b)), although the transverse velocity has the same magnitude $v_z$, its direction is opposite to the former case. Here, the light ray will not meet the z-axis in the propagating direction, but it virtually crosses the z-axis at $z = z_{\text{div}} = -[F(a) - L/2]$, which means that the atomic medium acts as a diverging lens. Thus the atom ensemble behaves as a negative or diverging lens, i.e., collimated light rays, passing through the medium, is diverged.

Therefore, changing the frequency of the control light from "blue detuning" to "red detuning", we can carry out an optically-controlled quantum manipulation based on EIT for the transition from the diverging lens effect to converging one. The light ray experiences either defocusing or focusing determined by the sign of the two-photon detuning $\Delta$. To realize an optically-controlled lens of divergence-to-convergence transition, we would like to consider some experimental data: $\nu = 3 \times 10^{13}\text{ rad/s}$, $N/V = 10^{13}\text{ cm}^{-3}$, $\Omega_0 = 0.6\Gamma$, $L = 10\text{ cm}$, $\sigma = L/4$. For a ray starting at $a = L/4$, the deflection angle $\alpha \simeq 1.9 \times 10^{-2}$ when two photon detuning $\Delta = 0.1\Gamma$. This EIT-gas lens seems different to put into service currently, but the further improvements with EIT enhancement can lead to the obviously-observable effect.

**Conclusion.** In this paper, we study the defocusing and focusing of probe light by an ensemble of four-level atoms in a tripod configuration driving by a control field with spatially inhomogeneous profile. We find the atomic medium serves as a polarization-selective lens for the probe beam. Different from previous proposal [21], the Zeeman splitting of magnetic sub-levels does not cause asymmetry atomic susceptibility for left- and right-circular polarization components of the optical field because no magnetic field is applied in our setup. The present investigation shows that the probe beam still splits into two, and each polarized component of the outgoing probe field contains the information of the atomic population. Therefore, preparation for the atomic internal state can also be used to control the deflection, focusing and defocusing effects.

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