Design of the Unsteady Vibrating Signals Order Analysis System for the Rotating Machinery

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Abstract: To eliminate the spectral ambiguity generated by FFT analysis method, the computational order tracking (COT) method is adopted to process the unsteady vibrating signals acquired from the rotating machinery. In this paper, the Lagrange 7-point interpolation algorithm is proposed to calculate the speed profile, the subsection interpolation method is developed based on the given angular acceleration threshold to obtain resampling time, and the sliding low-pass filtering interpolation method is adopted for angular resampling. According to these algorithms, the order analysis system is developed based on LabVIEW, and it is verified by the tests of simulation signals.

1. Introduction
The vibrating signals analysis is an important part for the rotating machinery fault diagnosis. The traditional spectrum analysis method is Fourier transform based on the equal time interval, and which has an obvious effect on the steady signals. However, due to the continuous movement of spectral components, serious frequency ambiguity will occur in some frequency bands, which is not suitable for processing non-stationary signals, especially in the start stop and variable speed rotation. For rotating machinery, the main influencing factors of the unsteady state are the changes of fundamental frequency and harmonics caused by start, stop and variation speed, and the applicable methods of the spectrum analysis includes SFFT three-dimensional spectrum method, Kalman time domain filtering method, ARX method of adaptive recursive filtering, and computational order tracking (COT) method, etc[1].

The COT method is to resample the vibrating signals at equal angle intervals at first, then convert the unsteady time domain signals to the steady angular domain signals, and then carry out the Fourier transform with the angle as variable. The steady fundamental frequency and harmonic order can be obtained, and which can also avoid the frequency ambiguity.

Many papers relating the COT analysis mainly used commercial software, however, only few papers related to algorithms and software design. In this paper, the algorithm of order analysis is analyzed, on the basis of it, the order analysis software is designed by the simulation signals, and then the real vibrating signals of rotating machinery are analyzed by the designed software.

The symbols used in this paper are as follows: \(X[N]\) is the vibrating signal sampling sequence, \(v_k\) is the key phasor pulse threshold, \(T[M]\) is the key phasor time scale vector, \(t = f(\theta)\) is the function between rotating time and angle, \(M\) is the number of key phasor pulse, \(S\) is the interpolation coefficient (the maximum order), \(TR[M][S]\) is the resampling time interpolation auxiliary array, \(t_r[W]\) \(W = M \times S\) is the resampling time vector, \(\theta\) is the rotation angle of the shaft, \(m[M]\) is the
array for storing the twice differential value of cubic spline function \( m_i \), and \( x_r[W] \) is the array for storing the resampled data vectors.

2. Order analysis process

Sampling vibrating signals and speed key phasor signals based on equal time interval \( T \) impulse; separating vibrating signals and speed key phasor signals and processing sampling impulse sequence of the speed key phasor signals to obtain key phasor time scale vector \( T[M] \); calculating speed profile based on key phasor time scale vector; defining self-adapting order based on speed profile; obtaining the resampling time scale array at the equal angle interval interpolated by highest order as the interpolation coefficient according to the key phasor time scale vector and the rotating angle \( (2\pi) \); and obtaining the vibrating signals of equal angle interval resampled by the filtering interpolation method combining the vibrating signals impulse sampling sequence and resampling time scale array. On the basis of these vibrating signals, the order analysis can be carried out.

The key elements of the design of the order analysis system are the calculation of resampling time and resampling of equal angle intervals. The flowchart is shown in Figure 1.

![Fig. 1 Design flowchart of order analysis system](image)

3. Speed profile calculation

3.1 Calculation of the key phasor time scale vector

The speed profile calculation process is as follows: ① Acquiring signals; ② Obtaining key phasor pulse signal waveform sampling data by separating the key phasor pulse signals and vibrating signals; ③ Automatically calculation the key phasor pulse threshold \( v_k \) according to the speed key phasor pulse sampling sequence; ④ Wave-shaping according to \( v_k \); ⑤ Detection key phasor pulse signals along the edge; ⑥ Calculating the time scale vector \( T[M] \) of the rising edge of the key phasor pulse signals.

3.2 Calculation of the speed vector

According to the acquisition process of key phasor pulses, the arrival time of the rotation speed pulse is a function of rotating angle, \( t = f(\theta) \), and the rotation interval of two adjacent pulses is the same. For each key phasor pulse, the angular step \( h = 2\pi \).

According to the circular kinematics, the angular velocity is:
\[ \omega(t) = \frac{d\theta}{dt} = \left( \frac{dt}{d\theta} \right)^{-1} = (f'_{\theta})^{-1} \] (1)

The corresponding relationship among the key phasor time scale \( T[M] \), function \( t = f(\theta) \) and the rotation angle is shown in Table 1.

| Table 1 Corresponding relationship between the rotation angle and the key phasor time scale |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| \( T[0] \) | \( \cdots \) | \( T[i] \) | \( \cdots \) | \( T[M - 1] \) |
| \( t_0 = f(\theta_0) \) | \( \cdots \) | \( t_i = f(\theta_i) \) | \( \cdots \) | \( t_{M-1} = f(\theta_{M-1}) \) |
| \( \theta_0 = 0 \) | \( \cdots \) | \( \theta_i = 2\pi(i - 1) \) | \( \cdots \) | \( \theta_{M-1} = 2\pi(M - 2) \) |

It can be seen from Eq.(1) that the interpolation polynomial \( t = L_m(\theta) \) can be established by \( t = f(\theta) \) and table 1, and the value of \( L_{\theta}^{\prime}(\theta) \) is taken as the approximate value of \( f_{\theta}^{\prime}(\theta) \), then the corresponding \( \omega_{[i]} \) of \( T[M] \) can be calculated. In order to ensure the accuracy, Lagrange 7-point interpolation algorithm is used. The algorithm model is \( L_7(\theta) = \sum_{j=0}^{6} f(\theta_j) \prod_{j=0, j \neq i}^{6} \frac{\theta - \theta_j}{\theta_i - \theta_j} \).

Assuming that \( \theta_i = \theta_0 + i\Delta \) (\( i = 0, 1, 2, \cdots, 6; \Delta = 2\pi \)), and the Lagrange 7-point interpolation formula is

\[
\begin{bmatrix}
    L_7(\theta_{i-3}) \\
    L_7(\theta_{i-2}) \\
    L_7(\theta_{i-1}) \\
    L_7(\theta_i) \\
    L_7(\theta_{i+1}) \\
    L_7(\theta_{i+2}) \\
    L_7(\theta_{i+3})
\end{bmatrix} = \frac{1}{180\Delta} \begin{bmatrix}
    -441 & 1080 & -1350 & -1200 & -675 & 216 & -30 \\
    -30 & -231 & 450 & 300 & 150 & -45 & 6 \\
    6 & -72 & -105 & -240 & -90 & 24 & -3 \\
    -3 & 27 & -135 & 0 & 135 & -27 & 3 \\
    3 & -24 & 90 & 240 & 105 & 72 & -6 \\
    -6 & 45 & -150 & -300 & -450 & 231 & 30 \\
    30 & -216 & 675 & 1200 & 1350 & -1080 & 441
\end{bmatrix} \begin{bmatrix}
    t_{i-3} \\
    t_{i-2} \\
    t_{i-1} \\
    t_i \\
    t_{i+1} \\
    t_{i+2} \\
    t_{i+3}
\end{bmatrix}.
\]

Calculating \( \omega_i = f^{\prime}(\theta_i)^{-1} = \left( \frac{1}{180\Delta} \begin{bmatrix}
    -3 & 27 & -135 & 0 & 135 & -27 & 3
\end{bmatrix} \right)^{-1} \)

When \( 3 \leq i \leq M - 4 \), the numerical differential formula of the time of the center point is:

\[
\omega_i = f^{\prime}(\theta_i)^{-1} = \left( \frac{1}{180\Delta} \begin{bmatrix}
    t_{i-3} \\
    t_{i-2} \\
    t_{i-1} \\
    t_i \\
    t_{i+1} \\
    t_{i+2} \\
    t_{i+3}
\end{bmatrix}\right)^{-1}
\]

3.3 Processing the boundary problem

In order to make both the starting point and the last point of the time scale vector be the center point of 7-point interpolation, three points must be inserted in front of \( t_0 \) and back of \( t_{M-1} \) before the calculation respectively, and the interpolations of them are linear fitting.

The key phasor pulse time scale \( T[M] \) and the corresponding rotation speed array \( \omega[M] \) are bound into a cluster, and which is the rotation speed profile.

4. Adaptive order determination

Assuming that the highest frequency of vibrating signals is \( f_{\text{max}} \), the time domain sampling frequency is \( f_s \), the minimum angular velocity obtained by searching in \( \omega[M] \) is \( \omega_{\text{min}} \), so the maximum order of angular domain signal is \( O_{\text{max}} = \frac{2\pi f_{\text{max}}}{\omega_{\text{min}}} \). According to the Nyquist's theory, there is \( f_{\text{max}} \leq f_s / 2 \), and \( O_{\text{max}} \leq \frac{\pi f_s}{\omega_{\text{min}}} \). Similar to the time domain sampling theory, in order to avoid order domain aliasing, the angular domain resampling rate \( O_x \geq 2O_{\text{max}} \), so the angular domain resampling rate is \( O_x \geq \frac{2\pi f_s}{\omega_{\text{min}}} \). In order to facilitate the FFT transform of resampling signals in the latter, \( O_x \) should be nth-power of 2 and greater than \( \frac{2\pi f_s}{\omega_{\text{min}}} \), it is set as \( S \), and which is the interpolation coefficient of resampling time.
5. Resampling time calculation
The COT is essentially to calculate the resampling time. At present, the representative methods include the linear method, circular motion equation method, conic fitting method and cubic spline interpolation method. In this paper, the circular motion equation method and the cubic spline function method are selected, and the linear method is selected in the steady stage.

5.1 Calculation of order tracking based on circular motion equation
In the speed profile, the corresponding angular velocity of the adjacent key phasor pulse time scale \([T[i], T[i + 1]]\) is \([\omega[i], \omega[i + 1]]\), and the corresponding angle \(\theta \in [0,2\pi]\). Assuming that the acceleration \(a\) is constant, and \(a = \frac{d\omega}{dt}\). Because \(\int_0^T d\omega = \int_0^T adt\), so \(\omega = \omega[i] + at\). Because \(\int_0^T d\theta = \int_0^T \omega dt = \omega[i]t + \frac{1}{2}at^2\), so \(\theta(t) = \theta_i + \omega_it + \frac{1}{2}at^2\), which is the relationship between the angle and time.

5.2 Calculation of order tracking based on cubic spline
For the situation that the rotation speed changes violently, because the rotation axis does not rotate with uniform angular acceleration in a turn, so it is not suitable to use the circular motion equation to calculate the resampling time, and the cubic spline interpolation can be used to calculate it. The rotation time of a particle on the shaft is a function of the rotation angle, and \(t = f(\theta)\), it is monotonically increasing and continuous, and which satisfies the condition of the cubic spline interpolation.

The corresponding relationship among the key phasor time scale \(T[i]\), function \(t = f(\theta)\) and rotation angle can be seen in Table 1.

Assuming that \(\theta_0 = 0\), and \(a = \theta_1 < \theta_2 < \ldots < \theta_{M-2} < \theta_{M-1} = b\), because one key phasor pulse is generated in one revolution, so the step length is \(h = \theta_{i+1} - \theta_i = 2\pi\).

Assuming that the cubic spline interpolation function in \(\theta \in [\theta_i, \theta_{i+1}]\) is \(S_i(\theta) = a_i + b_i(\theta - \theta_i) + c_i(\theta - \theta_i)^2 + d_i(\theta - \theta_i)^3\), \(i \in [0, M - 2]\), which satisfy the following conditions.

(1) \(S_i(\theta_i) = f(\theta_i), S_i(\theta_{i+1}) = f(\theta_{i+1})\) \((i \in [1, M - 2])\);

(2) \(S'_i(\theta_i) = S'_{i+1}(\theta_i), S''_i(\theta_i) = S''_{i+1}(\theta_i)\) \((i \in [1, M - 2])\);

(3) \(S'_0(\theta_0) = f'(a), S'_{M-1}(\theta_{M-1}) = f'(b)\):

Assuming that \(m_i = S''_i(\theta_i)\), and \(S\) is completely determined by \(\{m_i\}_{i=0}^{i=M-1}\) and \(\{f(\theta_i)\}_{i=0}^{i=M-1}\).

According to the above three conditions, the equation of \(m_i\) can be derived as

\[
\begin{bmatrix}
2 & 1 & 0 & 0 & \ldots & 0 \\
1 & 4 & 1 & 0 & \ldots & 0 \\
0 & 1 & 4 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 1 & 4 & \ldots & 0 \\
0 & \ldots & 0 & 1 & 2 & \ldots & 0 \\
\end{bmatrix}
\begin{bmatrix}
m_0 \\
m_1 \\
m_2 \\
\vdots \\
m_{M-2} \\
m_{M-1} \\
\end{bmatrix}
= 6h^2
\begin{bmatrix}
\Delta f(\theta_0) - hf'(a) \\
\Delta^2 f(\theta_0) \\
\Delta^2 f(\theta_1) \\
\vdots \\
\Delta^2 f(\theta_{M-2}) \\
hf'(b) - \Delta f(\theta_{M-1}) \\
\end{bmatrix}
\]

where \(\Delta f(\theta_i) = f(\theta_{i+1}) - f(\theta_i), \Delta^2 f(\theta_i) = f(\theta_{i+2}) - 2f(\theta_{i+1}) + f(\theta_i)\).

Before solving the Eq.(2), the \(f'(a)\) and \(f'(b)\) must be determined. For the first four points \(\{\theta_i\}_{i=0}^{i=3}\) and the last four points \(\{\theta_i\}_{i=M-4}^{i=M-1}\) of \(t = f(\theta)\), the cubic polynomials \(P_0(\theta)\) and \(P_0(\theta)\) can be fitted by Newton interpolation formula, so that \(f'(a) \approx P'_0(\theta_0)\) and \(f'(b) \approx P'_0(\theta_{M-1})\). Eqs.(3) and (4) are the Newton interpolation polynomials of the first four points and the last four points respectively, and
\[ P_{a}(\theta) = f[\theta_{0}] + \sum_{i=1}^{3} f[\theta_{i}, \cdots, \theta_{i}] \prod_{j=0}^{i-1} (\theta - \theta_{j}); \quad (3) \]

\[ P_{b}(\theta) = f[\theta_{M-4}] + \sum_{i=1}^{3} f[\theta_{M-4}, \cdots, \theta_{M-4+i}] \prod_{j=0}^{i-1} (\theta - \theta_{M-4+j}) \quad (4) \]

where \( f[\theta_{0}, \cdots, \theta_{i}] \) is the mean deviation.

Finding the first derivative of Eq.(3) and (4), substituting \( \theta_{0} \), \( \theta_{M-1} \) into them respectively, and the boundary conditions can be obtained

\[ f'(a) = P'_{a}(\theta_{0}) = -12f(\theta_{0}) + 18f(\theta_{4}) - 9f(\theta_{2}) + 2f(\theta_{3}) \]

\[ f'(b) = P'_{b}(\theta_{M-1}) = -2f(\theta_{M-4}) + 9f(\theta_{M-3}) - 18f(\theta_{M-2}) + 11f(\theta_{M-1}) \]

Eq.(2) is a tridiagonal equation group. The algorithm for solving it is as follows: assuming that the tridiagonal matrix equation is \( Ax = b \), decomposing \( A \) into the upper diagonal matrix \( L \) and the lower diagonal matrix \( U \), and there is \( LUx = b \). Assuming that \( Ux = y \), and \( Ly = b \). So \( y \) can be calculated firstly, and then \( x \) can be determined.

The above algorithm is used to solve the equations, and the results are stored in the quadratic differential array \( m[M] \). According to the calculated quadratic differential array \( m[M] \) and the key phasor time scale array \( T[M] \), the coefficients of the cubic spline interpolation function \( S_{i}(\theta) \) can be calculated as follows:

\[ a_{i} = T[i]; \quad b_{i} = \frac{T[i+1]-T[i]}{h} - \frac{h}{6} (m[i] + 2m[i+1])c_{i} = \frac{m[i]}{2}; \quad d_{i} = \frac{m[i+1]-m[i]}{6h} \]

where \( i \in [0, M-2] \).

5.3 Calculation of resampling time

According to the two adjacent points in the speed profile, the angular acceleration \( \alpha_{i} = \frac{\omega[i+1]-\omega[i]}{T[i+1]-T[i]} \) of the rotation can be calculated. A threshold value \( \alpha_{k} = 10 \) is set, linear interpolation is used for \( |\alpha_{i}| \leq 0.01 \), circular equation of motion method is used for \( 0.01 \leq |\alpha_{i}| \leq \alpha_{k} \), cubic spline function is used for \( |\alpha_{i}| > \alpha_{k} \) The calculation flowchart of resampling time is shown in Figure 2.

6. Resampling

Since the time distribution of resampling is non-uniform, the usual up sampling or down sampling methods is not suitable. If there is an analog antialiasing filter at the input, the original sampling signal can be directly stored by spline interpolation or polynomial interpolation to obtain the corresponding \( t_{r}[i] \). Generally, the system does not install an analog antialiasing filter at the input, the \( II \) FIR low-pass filter interpolation should be used[5].

6.1 Approximation design of ideal causal low-pass filter

The frequency response of an ideal low-pass filter is

\[ H(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_{c} \\ 0 & \text{other} \end{cases} \quad (\omega_{c} = \frac{\omega_{p} + \omega_{s}}{2}) \]

where \( \omega_{c} \) is the cut-off frequency, \( \omega_{p} \) is the highest pass band frequency, and \( \omega_{s} \) is the start-stop frequency of stopband.

The unit impulse response of a finite length ideal filter is

\[ h_{d}(n) = \frac{1}{2\pi} \int_{-\omega_{c}}^{\omega_{c}} H(e^{j\omega}) \cdot e^{j\omega n} d\omega = \frac{\sin(n\omega_{c})}{n\pi} \left( -\frac{L-1}{2} \leq n \leq \frac{L-1}{2} \right) \]

where \( L \) is the order of filter, and which is an odd number.
The Kaiser window has the best effect in many window functions. The Kaiser window function can be described as

$$w(n) = \frac{I_0(\beta \sqrt{1 - [2n/(N-1)]^2})}{I_0(\beta)} - \frac{L - 1}{2} \leq n \leq \frac{L - 1}{2}$$

where $I_0(\alpha)$ is the zero order modified Bessel function, and $d_0(\alpha) = 1 + \sum_{k=1}^{\infty} \left[ \frac{(\alpha/2)^k}{k!} \right]^2$, and $\beta$ is the parameter required to realize attenuation $\alpha_s = -20 \log \delta$ in stopband, and it is given by the following empirical formula:

$$\beta = \begin{cases} 0.1102(\alpha_s - 8.7) & \alpha_s > 50 \\ 0.5842(\alpha_s - 21)^{0.4} + 0.07886(\alpha_s - 21) & 21 \leq \alpha_s \leq 50 \\ 0 & \alpha_s < 21 \end{cases}$$

The empirical formula of the order of the filter is

$$L = \left[ \frac{(\alpha_s - 7.95)}{2.285(\Delta \omega)} + 2 \right] \times 2 + 1$$

Fig. 2 Calculation flowchart of resampling time

The Kaiser window has the best effect in many window functions. Kaiser window function can be described as
where $\Delta \omega = \omega_s - \omega_p$ is the transition bandwidth.

The impulse response of the low-pass filter with window is as follows:

$$h_w(n) = h_d(n) \cdot w(n) - (L - 1)/2 \leq n \leq (L - 1)/2$$

The $h_w(n)$ is an non causal, and the causal filter is:

$$h(n) = h_w(n - (L - 1)/2) \quad 0 \leq n \leq L - 1$$

Unbinding the speed vector $\omega_M$ from the speed profile cluster, obtaining the maximum value $\omega_{max}$, and the normalized frequency is $\omega_{max}/f_s$. Assuming the maximum rotating speed is 6000r/s, the maximum shaft frequency is 100Hz, and the sampling rate is 10K, thus the corresponding normalized frequency $\omega_{max}$ is $\pi/100$. Generally, the vibrating signals frequency is a multiple of the shaft frequency, assuming that it is 10 times, so the highest pass band frequency $\omega_p = \pi/10$.

Assuming that the star-stop band frequency of stopbond is $\omega_s = \pi/5$, and $\alpha_s = 125$, the $\beta$ and $L$ can be calculated according to Eqs.(5) and (6).

According to the above algorithm and given parameters, the coefficients of the causal low-pass filter can be stored in the array $h[L]$, which is used as the convolution kernel to participate in windowing filtering[4].

6.2 Low-pass filter interpolation resampling

Windowed filtering of sampling signals is to convolute the subset of sampling signal array whose length is equal to the order of filter. Its algorithm formula is:

$$y[n] = \sum_{k=0}^{L-1} h[k] \cdot x[n-k] = h[0] \cdot x[n] + h[1] \cdot x[n-1] + \cdots + h[L-1] \cdot x[n-L+1],$$

where $L$ is the filter order. To solve the above formula, the sample signal array subset $x[n-L+1] \sim x[n]$ should be intercepted firstly, and then the inverted array subset is multiplied by the filter coefficient array $h[L]$. There is a boundary filtering problems should be considered here, such as the resampling value corresponding to the resampling time $t_r[0]$. Assuming that $m = \text{int}(\frac{t_r[0]}{T_s})$, $y[m] = \sum_{k=0}^{L-1} h[k] \cdot x[m-k] = h[0] \cdot x[m] + h[1] \cdot x[m-1] + \cdots + h[L-1] \cdot x[m-L+1]$, if $m < L$, the $x[m-L+1] \sim x[-1]$ does not exist, thus $L$ array elements with 0 should be inserted before $x[0]$ of the original sampling signal array before the filtering, the $m = \text{int}(\frac{t_r[i]}{T_s})$ is used as the starting index of the sampling sequence array, and then a subset with length of $L$ is obtained to participate in the convolution operation[3].

According to the resampling time array $t_r[W]$, the sampling values at each resampling time are calculated by cyclic sliding filter interpolation. The algorithm steps of resampling value of resampling time $t_r[i]$ are as follows:

1. $m = \text{int}(\frac{t_r[i]}{T_s})$;
2. $y(m) = \sum_{k=0}^{L-1} h[k] \cdot x[(m-k)]$; $y(m+1) = \sum_{k=0}^{L-1} h[k] \cdot x[(m+1-k)]$;
3. The original sampling times corresponding to $y(m)$ and $y(m+1)$ are $mT_s$ and $(m+1)T_s$ respectively, and the linear interpolation is used to calculate $x_r[i]$.

The $x_r[0], x_r[1], x_r[2], \cdots, x_r[W-1]$ can be obtained by calculating from step(1) to (3) when $i = 0$ to $W - 1$, and which is the resampled sequence array.

If the quadratic polynomial interpolation is adopted, the $y(m-1)$, $y(m)$ and $y(m+1)$ should be calculated in step (2), and then the interpolation can be operated after fitting the quadratic polynomial. If the cubic spline interpolation is adopted, the $y(m-j)$, $\cdots, y(m)$, $\cdots, y(m+j)$ should be calculated in step (2), the cubic spline function can be fitted by combining the corresponding original sampling time $(m-j)T_s$, $\cdots, mT_s$, $\cdots, (m+j)T_s$, where $j = \text{min}(L, N - \text{int}(\frac{t_r[W]-1}{T_s}))$, and $N$ is the number of samples of the original signals, and the interpolation can be operated. In the process of sliding filter interpolation, the cubic spline fitting interpolation should be repeated $W$ times, but the calculation is too big, thus the linear interpolation can be used.

7. Simulation

According to the above algorithm, using LabVIEW software to programming, using the simulation
signals in Figure 3 as the speed and vibrating signals, the program is operated, and the speed profile is shown in Figure 4, the simulation results in the angular domain signals after filtering and interpolation are shown in Figure 5, and the order spectrum is shown in Figure 6.

![Fig. 3 simulation signals](image1)

![Fig. 4 speed profile](image2)

![Fig. 5 angle domain signals](image3)

![Fig. 6 order spectrum](image4)

8. Conclusions
For the unstable vibrating signals, the frequency ambiguity in FFT analysis can be effectively eliminated by the order analysis of the angle domain signal obtained by the algorithm. The angular domain signal and speed profile can be used for analyzing waterfall plot, cascade plot, bode plot, et al.

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