The Application of Polynomial Trend Term Removing in the Acoustic Wave Gauge Data Processing

Ying Yang¹,2,3,a, Li Hui¹,2,3,b, Xiangtao Ran¹,2,3,*, Zhi Wang¹,2,3, Yang Zhou¹,2,3 and Min Liu¹,2,3

¹Qilu University of Technology (Shandong Academy of Sciences), Institute of Oceanographic Instrumentation, Shandong Academy of Sciences, Qingdao 266000, China
²Shandong Provincial Key Laboratory of Marine monitoring instrument equipment technology, Qingdao 266000, China
³National Engineering and Technological Research Center of Marine Monitoring Equipment, Qingdao 266000, China

*Corresponding author e-mail: ranxiangtao2007@126.com, asiioi_yang@163.com, a11932807@qq.com

Abstract. In the practical measuring applications of acoustic wave gauge, the change of the tide will cause most of the measurements deviating from the center line of the data, result in the larger wave cycle and the inaccurate wave height measurements. The principle of polynomial trend term removing by the least square method in the acoustic wave gauge data processing is introduced in this paper. A great deal of practical measuring data shows that this method is effective for removing the interference of the tide variation.

1. Introduction

Acoustic wave gauge is a kind of automatic measuring system with high accuracy, high reliability and intelligence for measuring wave and water level in the sea, rivers and lakes. It is widely used in observation station, port, channel safety, oil platform and typhoon wave forecast.

The operational principle of the acoustic wave gauge is as follows. The underwater acoustic transducer fixed to the bottom of the sea launches a beam of ultrasonic pulses to sea surface vertically. The ultrasonic pulses reflect to the bottom of the sea when they reach the sea surface. The acoustic transducer is driven by the reflected ultrasonic pulses and output electronic signals. When the sound speed is constant, the time $t$ from launch to reception and the vertical distance $H$ from transducer to sea surface have the relation as follows,

$$H_i = V t / 2$$

Where $H_i$ is the distance from transducer to wave surface, $V$ is the disseminating speed of the sound in sea, when the water temperature is $13.5 \degree C$, $V=1500m/s$. Because $H_i$ is changing with the wave movement, time $t_1$, $t_2$, $t_3$ … from launch to reception of every pulses are changing simultaneously. If $t_1$, $t_2$, $t_3$ … subtract the disseminating time of the sound in basic water depth, $\Delta t_1$, $\Delta t_2$, $\Delta t_3$, ……
are gotten. The data of waves can be obtained after these time varieties collected and processed by the microcomputer.

According to the regulations of the National Standard of the People’s Republic of China "Seaside observation standard GB/T14914-2006", wave sample interval should be less than 0.5s, the number of waves recorded continuously is not less than 100, and the length of time recorded depends on the size of the average period. Sampling time usually takes 17mins~20mins. In these 17mins~20mins, the tidal level of seawater also varies. Therefore, the influence of the tide level in the measurement process becomes one of the interference signal sources of wave measurement.

After the 17min~20min data are collected, the actual wave fluctuation should be restored as much as possible in order to calculate the wave height and wave period more accurately. In this paper, the polynomial least square method is used to eliminate the tidal level noise in the signal.

2. Principle of eliminating trend term by least square method

The sampling data of the measured signal are \{x_k\} (k=1, 2, 3… n). Because the sampling data is equal time interval, for the sake of simplification, let the sampling time interval \(\Delta t = 1\), set up a polynomial function:

\[ \hat{x}_k = a_0 + a_1 k + a_2 k^2 + \ldots + a_m k^m \quad (k=1,2,3,\ldots,n) \]

Determine the undetermined coefficients \(a_j\) (j=0,1,…,m) of the function \(\hat{x}_k\). So that the sum of error squared between functions \(\hat{x}_k\) and discrete data \(x_k\) is minimized, that is,

\[ E = \sum_{k=1}^{n} (\hat{x}_k - x_k)^2 = \sum_{k=1}^{n} (\sum_{j=0}^{m} a_j k^j - x_k)^2 \]

The condition that the E value approaches the limit value is

\[ \frac{\partial E}{\partial a_i} = 2 \sum_{k=1}^{n} k^i (\sum_{j=0}^{m} a_j k^j - x_k) = 0 \quad (i = 0,1,2,\ldots,m) \]

The partial derivation of E pair \(a_i\) is obtained in turn. A system of linear equations with m+1 can be generated:

\[ \sum_{k=1}^{n} \sum_{j=0}^{m} a_j k^j + \sum_{k=1}^{n} x_k k^i = 0 \quad (i = 0,1,\ldots,m) \]

The equations are solved and the m+1 undetermined coefficient is obtained. In the above formulas, the order of m is a polynomial order, and the value range is 0 ≤ j ≤ m.

The trend term obtained when m = 0 is constant, and

\[ \sum_{k=1}^{n} a_0 k^0 + \sum_{k=1}^{n} x_k k^0 = 0 \]

Solution of equation:

\[ a_0 = \frac{1}{n} \sum_{k=1}^{n} x_k \]

It can be seen that the trend term when m = 0 is the arithmetic average of the signal sampling data. The formula for eliminating the constant trend term is:
When \( m=1 \) is a linear trend term, there are:

\[
\begin{align*}
\sum_{k=1}^{n} a_0 k^0 + \sum_{k=1}^{n} a_1 k - \sum_{k=1}^{n} x_k k^0 &= 0 \\
\sum_{k=1}^{n} a_0 k + \sum_{k=1}^{n} a_1 k^2 - \sum_{k=1}^{n} x_k k &= 0
\end{align*}
\]

Solving the equation set,

\[
\begin{align*}
a_0 &= \frac{2(2n + 1)\sum_{k=1}^{n} x_k - 6\sum_{k=1}^{n} x_k k}{n(n - 1)} \\
a_1 &= \frac{12\sum_{k=1}^{n} x_k k - 6(n - 1)\sum_{k=1}^{n} x_k}{n(n - 1)(n + 1)}
\end{align*}
\]

The formula for eliminating the linear trend term is:

\[
y_k = x_k - \hat{x}_k = x_k - (a_0 - a_1k) \quad (k = 1,2,3, \ldots, n)
\]

\[M \geq 2\] is the trend term of the curve. In the actual vibration signal data processing, usually we take \( m=1\sim 3 \) to carry out the sampling data processing to remove the polynomial trend term.

### 3. Practical application in acoustic wave gauge

A lot of noise signals are superimposed on the data signals collected in the actual measurement of the acoustical wave gauge. It can be seen that the noise signals are not only periodic interference signals such as the power frequency of 50Hz and its frequency doubling, but also a lot of random interference signals that are induced by transducers and cables. In addition, the variation of tidal level is an interference signal which can not be ignored in the measurement process, especially when the wave height is small, most of the signals deviate from the data centerline because of the variation of the tidal level value, which results in the large period when calculating the wave characteristic value. The measurement of wave height is not accurate.

According to the principle of polynomial least square method, first through the implementation in MATLAB, the program is as follows.

```matlab
% get signal data length
n = length (x);
% Establish discrete time column vector
t = (0: 1 / sf: (n-1) / sf)';
% the polynomial undetermined coefficient vector a for the calculation of the trend term
a = polyfit (t, x, m);
% subtracting the trend term generated by the polynomial coefficient a by X
x1 = polyval(a, t);
y = x - x1;
% the first column, which is divided into 2 rows and 1 rows, is set as the current drawing area.
subplot (2, 1, 1);
% Draw the time - history curve graph of x pair t
xave = mean(x);
```
\[ x = x - x_{ave}; \]
\[ x_{1ave} = \text{mean}(x_1); \]
\[ x_1 = x_1 - x_{1ave}; \]

Fig. 1 and Fig. 2 is comparison of wave data with trend removal during ebb tide.

By processing wave data in MATLAB, it is proved that this principle is feasible to remove the trend term caused by tidal level change in acoustical wave gauge. The principle is brought into the acoustic wave meter data collector microcomputer to realize this application. Through this way, the measurement
data are smoothed many times, and a smooth curve is obtained, which is processed with the original signal and the trend item. The purpose of eliminating interference signals is achieved. Several wave curves are processed and compared with the measured data of the East China Sea, the North Sea and the South China Sea. The measured results are as follows:

Table 1. Comparison of wave characteristic values before and after tidal level removal in a certain sea area of the East China Sea

|                  | Hmax | T   | H1/10 | T   | Hs  | T   | HA  | T   | Tmax | H   |
|------------------|------|-----|-------|-----|-----|-----|-----|-----|------|-----|
|                  | (m)  | (s) | (m)   | (s) | (m) | (s) | (m) | (s) | (s)  | (m) |
| High tide Before | 0.45 | 13.00 | 0.38 | 5.78 | 0.31 | 5.84 | 0.24 | 5.23 | 20.5 | 0.24 |
| After            | 0.46 | 2.00  | 0.36 | 2.07 | 0.30 | 2.03 | 0.23 | 2.08 | 4.50 | 0.18 |
| Low tide Before  | 0.40 | 29.50 | 0.30 | 11.5 | 0.30 | 11.0 | 0.20 | 7.10 | 71.5 | 0.30 |
| After            | 0.40 | 0.30  | 0.30 | 3.40 | 0.30 | 3.30 | 0.20 | 3.40 | 5.00 | 0.20 |

Table 1 shows the comparison of data when the acoustic wave gauge is measured in a certain sea area of the East China Sea. The acoustic wave gauge is installed about 2km off the shore. The measuring point belongs to the open area and is a representative survey position. This sea area is a semi-diurnal tide, in the middle of a day.

4. Conclusion

From the trend eliminating effect diagram and eigenvalue data in high tide and low tide, it can be seen that the wave eigenvalue data are consistent with the actual wave data after the disturbance trend caused by the removal of tidal level value. In practical application, a large amount of data is removed by this method, and the comparison results show that it is feasible.

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