CORRECTION TO: REPRESENTATION GROWTH
AND RATIONAL SINGULARITIES OF THE
MODULI SPACE OF LOCAL SYSTEMS

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ABSTRACT. We explain and correct a mistake in Section 2.6 and Appendix C of the first and second author’s paper “Representation Growth and Rational Singularities of the Moduli Space of Local Systems”[AA16].

We use throughout the notation and conventions of [AA16]. The source of the mistake is the description of the set $S_2$ in page 272. The elements

$(1)$ $\{(d, d-1), (d-1, d-1)\}, \{(d-1, d-1), (d-1, d)\}, (d-1, d)$

are not considered. This spoils the proof of Lemma 2.40 (the blue and green subgraphs are no longer trees), which is used to prove Theorem 2.1.

1. A STRAIGHTFORWARD CORRECTION

The right description of $S_2$ is

$$S_2 = \left\{ \{(i, j), (j, l)\}, (i, l) \in I^{(2)} \times J \mid \{i, l\} = \{d-1, d\} \text{ and } j = \left\lfloor \frac{i+l}{2} \right\rfloor , \right.$$ 

or \( \{i, l\} \neq \{d-1, d\} \text{ and } j = \left\lfloor \frac{i+l}{2} \right\rfloor + \delta_{i,l} \right\} .$$

Then, $\Gamma_2$ is the polygraph attached to the graph $\Gamma_3 = (I, E)$, with

$$E = \left\{ (i, j), (j, l) \in I^{(2)} \mid \{i, l\} = \{d-1, d\} \text{ and } j = \left\lfloor \frac{i+l}{2} \right\rfloor , \right.$$ 

or \( \{i, l\} \neq \{d-1, d\} \text{ and } j = \left\lfloor \frac{i+l}{2} \right\rfloor + \delta_{i,l} \right\} .$$

As for Fig. 1, 2 and 3 in [AA16], the two edges corresponding to $(1)$ are missing.

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Additionally, there are some mismatches concerning Fig. 2 and $\omega_3$ in [AA16]. The simplest way to make the labels of Fig. 2 match is first to multiply $\omega_3$ in page 273 by 5, obtaining

\begin{equation}
\omega_3((i, j))(m) = \begin{cases} 
5^{|i-j|+1} & \text{if } m \equiv i - j \pmod{3} \\
3 \cdot 5^{|i-j|} & \text{if } m \equiv (i - j - \text{sign}(i-j+1/2)) \pmod{3} \\
0 & \text{if } m \equiv (i - j + \text{sign}(i-j+1/2)) \pmod{3},
\end{cases}
\end{equation}

and then consider the colouring [red is $m = 0$, green is $m = 2$, blue is $m = 1$], so that we just have to swap the values of the blue and green labels along the diagonal of Fig. 2 in page 314.

Finally, the key to prove now Lemma 2.40 in the closest way to that of [AA16] is redefining $\omega_3$ at the nodes $(d-1, d-1)$, $(d-1, d)$ and $(d, d-1)$:

- $\omega_3(d-1, d-1)(0) = 3 \cdot 5^1$, $\omega_3(d-1, d)(0) = 5^2$, $\omega_3(d, d-1)(0) = 3 \cdot 5^0$,
- $\omega_3(d-1, d-1)(2) = 3 \cdot 5^1$, $\omega_3(d-1, d)(2) = 4 \cdot 5^1$, $\omega_3(d, d-1)(2) = 0$,
- $\omega_3(d-1, d-1)(1) = 0$, $\omega_3(d-1, d)(1) = 0$, $\omega_3(d, d-1)(1) = 5^2$,

so that the resulting diagram for $d = 6$ (Fig. 2 in [AA16]) is given by Image 1.

\begin{center}
\textbf{Image 1.} Graph $\Gamma_3$ with the weights $\omega_3$ for the case $d = 6$.
\end{center}

With the redefinition of $\omega_3$ above, we only need to add to the proof of Lemma 2.40 in [AA16] an analysis of the edges around $(d-1, d-1)$. They look, for $d \geq 3$, like the ones in Image 1. We thus have forests with maximal degree $\leq 3$, as we need. Finally, the cases $d = 1, 2$ are straightforward.
2. An alternative solution

We indicate here how to get an alternative solution that, although requires more changes, would keep better the original intuition for the proof.

At the beginning of Section 2.6 in [AA16], recall that $L = \{1, \ldots, d\}$. Stay with $J = L \times L \setminus \{(d, d)\}$ and replace $I$ by $I = L \times L \setminus \{(1, 1)\}$. This entails changes in $S_j$ and $\Gamma_j$, which we omit here for the sake of brevity. With the definition of $\omega_3$ as in (2) (without any redefinition) and the same conventions for the colours as described in Section 1, the corresponding $\Gamma_3$ for $d = 6$ is given by Image 2.

![Image 2](image.png)

**Image 2.** Graph $\Gamma_3$ with $I \neq J$ for the case $d = 6$.

The general proof then follows along the same lines as the original one.

REFERENCES

[AA16] Avraham Aizenbud and Nir Avni. Representation growth and rational singularities of the moduli space of local systems. *Invent. Math.*, 204(1):245–316, 2016.