Revisiting the active power direction method

Polina Shcherbakova1 | Gennady Senderovych2 | Alexander Abramovitz1

1 Faculty of Engineering, Holon Institute of Technology, Holon, Israel
2 Department of Automation and Cybersecurity of Power Systems, National Technical University Kharkov Polytechnic Institute, Kharkiv, Ukraine

Abstract
This paper presents a re-examination of the active power direction method for locating the distortion sources in the power distribution network. The inaccuracies in the earlier analysis challenging the active power direction method are identified, and the claim of the inconsistency of the method is re-examined. The article shows the derivation of the expression for the complex power, from which the equation for the active power flow through the point of measurement was derived in an explicit and in a normalised form. The derivations allow establishing the consumer dominance range at any harmonic frequency easily. Simulation results decisively support the theoretical findings. It is suggested that the main cause of uncertainty associated with the active power direction method is the stochastic nature of the amplitude and, especially, the phase between the harmonic current components.

1 | INTRODUCTION

In practice, the electric power quality (PQ) is usually regulated using two main schemes. The first approach, which can be characterised as “the last one pays”, suits well-established networks with customers highly committed to PQ norms. In case PQ indices are violated by a newly installed equipment, the owner is obligated to take measures to restore the PQ. Thus, to ensure the desired level of PQ, such an approach has to rely on the development and enforcement of appropriate PQ standards [1, 2].

The most common type of PQ violation is the generation of harmonic distortion (HD). Hence, the problem of precise identification of the source of HD arises. Several approaches to identify the source of HD have been reported in the literature. The reported methods for the detection of HD sources are based on the following principles: analysis of reactive power [10], the principle of superposition [11, 12], the concept of critical impedance [9, 13], as well as voltage-to-current ratio [14–17].

Recent literature reviews on HD generation [18, 19] present a rather extensive summary, generalization, and analysis of studies performed up to date. Experiments were carried out to provide a numerical comparison of the errors produced by each method under conditions of inaccurate data, resonance, and voltage unbalance. The analysis of [18, 19] showed that in the case of a dominant source of distortion (i.e. when only one HD sources can be neglected), the method of directed active power [3–6] can be appropriate, whereas in the case of multiple HD sources, the method using reactive power [10] and the method based on the voltage–current ratio [14–17] can perform better. Yet, [10, 14–17] show higher sensitivity to the input data errors than [3–6].

Methods requiring synchronous multi-point measurements [20] are prone to several disadvantages [21]. Implementation of such methods requires a complex measurement setup, higher initial financial investment, as well as additional operational and maintenance expenses.

Further discussion of the existing methods for determining the HD generation by consumers is given in [22]. The study showed that the accuracy of the final assessment strongly depends on the quality of the available data. In practice, the uncertainty of the system's parameters often compromises the accuracy of even the most sophisticated methods. One example of such an elusive parameter is the value of the harmonic impedance. A method to calculate the utility harmonic impedance based on FastICA was developed by Zhao and Yang [23]. Although the obtained results were satisfactory, work in [23] implies that to attain better accuracy, further improvement of the method is needed.

Another detailed analysis of the methods to identify the HD sources, their advantages, and disadvantages is found in [24]. The study recommends a method based on the direction of active power flow due to its ease of use, low cost, and minimum required input data. Moreover, the errors of the advocated
approach are only due to the meter’s errors. Thus, Sinvula et al. [24] suggest developing this approach further to resolve the problems related to the presence of distortion sources on both sides of the point of common coupling.

The literature survey above implies that a truly reliable method of identifying HD sources under any possible network scenarios is yet to be developed. Such a method is, in fact, highly desirable as it can be helpful to properly assess compliance to the PQ standards and, consequently, facilitate fair billing agreements between suppliers and consumers of electrical energy.

Another possible approach is the harmonic active power direction (APD) method. This is one of the most popular approaches used in practice [3–6]. In [4], it was proposed to limit the APD method. However, such an approach requires the input of additional parameters, which greatly complicates the process. In the past, however, the APD approach was heavily criticised in [7–9] that pointed out apparent inconsistencies between the APD method and the superposition method.

The rest of this paper is organised as follows. The earlier findings are re-examined in Section 2. The general case for the APD method is analysed in Section 3, which is then validated and discussed in Section 4. Section 5 derives the conditions for customer’s dominancy over the power flow at the point of metering. To further demonstrate the analytical findings, simulation example is presented in Section 6. Concluding remarks are given in Section 7. Details of the derivations can be found in the Appendix.

## 2 | RE-EXAMINATION OF THE EARLIER FINDINGS

A basic principle of electric network analysis states that the circuit solution is unique and invariant of the calculation method used. Hence, the statements made by earlier counterparts [7, 8] that the harmonic APD method is inconsistent with the results obtained by the superposition method seems questionable. The argument above prompted a re-examination of the earlier findings.

Following the approach of [7] and [8], a DC benchmark circuit in Figure 1 was studied with the idea that “although the actual harmonic source detection problem involves AC circuits, the DC circuit case can review key characteristics of the power direction method. A DC circuit is much simpler to analyse since there is no phasor involved.”

In the benchmark circuit in Figure 1, $I_u$ and $R_u$ comprise the utility, whereas $I_c$ and $R_c$ represent the customer side. The voltage and current reference directions at the node are also shown in Figure 1.

Starting by the method of superposition, the voltage at the node can be found as the sum of contributions of each current source $I_u$ and $I_c$, and considering the parallel equivalent resistance

$$V’ = \left(\frac{R_u R_c}{R_u + R_c}I_u\right) + \left(\frac{R_u R_c}{R_u + R_c}I_c\right). \quad (1)$$

Similarly, the current $I$ can be obtained as the sum of the contributions of each of the current sources and considering the current divider rule

$$I = \left(\frac{R_u}{R_u + R_c}I_u\right) - \left(\frac{R_c}{R_u + R_c}I_c\right) \quad (2)$$

whereas by the APD method, the voltage and current equations of the circuit are, respectively:

$$V = \frac{R_u R_c}{R_u + R_c}(I_u + I_c) \quad (3)$$

$$I = \frac{V}{R_c} - I_c = \frac{R_u}{R_u + R_c} I_u - \frac{R_c}{R_u + R_c} I_c. \quad (4)$$

Both sets of equations are identical and, therefore, are expected to yield the same result when used for power calculations.

The power flow through the node is given by

$$P = V’ \cdot I = \left(\frac{R_u R_c}{R_u + R_c}I_u\right) \left(\frac{R_u}{R_u + R_c}I_u - \frac{R_c}{R_u + R_c}I_c\right)$$

$$= \frac{R_u R_c}{(R_u + R_c)^2} \left( R_u I_u^2 - R_c I_c^2 + I_u I_c (R_u - R_c) \right) \quad (5)$$

Introducing the normalised variables $I_n = \frac{I}{I_c}$, $r = \frac{R_u}{R_c}$ and the base power, $P_b = R_u I_n^2$, the expression for the normalised power can be obtained as

$$P_n = \frac{P}{P_b} = \frac{r}{(1 + r)^2} \left( I_n^2 - (r - 1) I_n - r \right) = \frac{r}{(1 + r)} \left( I_n + (1/r) \right). \quad (6)$$

The normalised power (6) represents an upright parabola, which has a pair of roots located at $I_{n1} = -1$ and $I_{n2} = r$. (In
The negative power flow, that is, from the customer towards the utility occurs within the $I_{c1} < I_u < I_{c2}$ range, that is for

$$-1 < \frac{I_u}{I_c} < \frac{R_c}{R_u}.$$  \hspace{1cm} (7)

To confirm the findings, the benchmark circuit in Figure 1 was simulated in PSIM v. 9.4. A comparison of the simulated normalised power vs the normalised power calculated by (6) is shown in Figure 2. Network parameters were $R_c = 10 \Omega$ and $R_u = 2 \Omega$. During the software experiments, the customer’s source was fixed, $I_c = 10$ A, while the utility source, $I_u$, was swept from negative to positive values. Excellent agreement of the theoretical and simulated results was found.

In consequence, it is possible to conclude that the theoretical analysis by the APD approach stands in agreement with the obtained by the superposition method, and, as expected, no contradiction between the two was found.

Yet, this does not mean that the APD method is free of other problems or limitations.

## 3 | THE GENERAL CASE FOR THE APD METHOD

To facilitate the analysis approach, simplified models of an AC network are considered next.

Simplified circuit models of the network to the $b$-th harmonic component are illustrated in Figure 3. Henceforth, the part of the network downstream of the point of measurement (PM) is termed load, whereas the part upstream of PM as the utility. In Figure 3(a), the PM is located on the supply input so that the load is comprised of all the consumers connected to the busbars. In Figure 3(b), the PM is located at the input of a specified busbar; therefore, the load is comprised of a single feeder, and the rest of the network, including other feeders, is considered as the utility.

These simple circuit models are adequate to analyse the harmonic components between the utility and a group of consumers, as well as for an individual feeder (consumer).

The direction of the power flow of the first harmonic is considered independent of the presence and location of sources of higher harmonics. In case the consumer has no power source of his own, the active power, $P_{(1)}$, flows from the utility towards the consumer; see Figure 4 for the equivalent circuit at the fundamental frequency. Here, the value of $P_{(1)}$ is determined by the equivalent electromotive force of the utility, $E_{(1)u}$, the system’s impedance, $Z_{(1)u}$, and the load impedance, $Z_{(1)l}$.

In general, both the system and the load contain non-linear elements. Therefore, harmonic currents and voltages are gener-
The model in Figure 5(b) represents a consumer operating a \textit{linear load}, which generates no harmonics \((I'_{(b)c} = 0)\). Here, the \(b\)-th harmonic current component in the circuit is entirely due to utility’s harmonic source \((I'_{(b)a} \neq 0)\). In such a case, the power of the \(b\)-th harmonic component \(P_{(b)}\) flows from the utility towards the consumer, matching the power flow of the fundamental harmonic, \(P_{(1)}\).

The model in Figure 5(c) represents the case of a \textit{linear system} that generates no harmonics \((I'_{(b)a} = 0)\). Here, the \(b\)-th harmonic current appears as a result of the consumer’s activity \((I'_{(b)c} \neq 0)\). In such a case, the power of the \(b\)-th harmonic component, \(P_{(b)}\), flows from the load side towards the system.

From Figure 5, the following relationships can be obtained.

- The phase voltage of the \(b\)-th harmonic at PM can be found as

\[
\begin{align*}
U'_{(b)} &= \frac{Z_{(b)a} Z_{(b)c}}{Z_{(b)a} + Z_{(b)c}} \left( I'_{(b)a} + I'_{(b)c} \right) \\
\end{align*}
\]

whereas the current at the PM is

\[
I_{(b)} = \frac{1}{Z_{(b)a} + Z_{(b)c}} \left( I'_{(b)a} Z_{(b)a} - I'_{(b)c} Z_{(b)c} \right) \\
\]

Here, \(Z_{(b)a} = R_a + jX_{(b)a}\) and \(Z_{(b)c} = R_c + jX_{(b)c}\) are the systems impedance and customer's impedances at the \(b\)-th harmonic frequency, respectively, which combined impedance is \(Z_{(b)aT} = Z_{(b)a} + Z_{(b)c}\).

The (single-phase) complex harmonic power at the PM is

\[
S_{(b)} = \overline{I'_{(b)} \cdot U_{(b)}}
\]

Here, \(\overline{I'_{(b)}}\) is the complex conjugate of the \(b\)-th harmonic current.

Substituting (8) and (9) into (10), and considering \(Z_{(b)aT}^2 = (R_a + R_c)^2 + (X_{(b)a} + X_{(b)c})^2\), the complex power at the PM can be obtained as (11)

\[
S_{(b)} = \frac{Z_{(b)a} Z_{(b)c}}{Z_{(b)aT}^2} \left( \left( I'_{(b)a} \right)^2 Z_{(b)c} - \left( I'_{(b)c} \right)^2 Z_{(b)a} \right)
\]

\[
+ \overline{\left( I'_{(b)a} \cdot I'_{(b)c} \right)} \left( Z_{(b)a} Z_{(b)c} - Z_{(b)a} Z_{(b)c} \right) \cos \delta_{(b)}
\]

Note that the equivalent harmonic current sources \(I'_{(b)a}\) and \(I'_{(b)c}\) in Figure 5(a) represent all of the non-linear phenomena of the network so that the rest of the network (i.e. the impedances \(Z_{(b)a}\) and \(Z_{(b)c}\)) is linear. Therefore, the equivalent circuit in Figure 5(a) can be considered as a linearised model of the network. Note that in the equivalent circuit in Figure 5(g), the phase of the harmonic current component, \(I_{(b)}\), and the direction, i.e. sign, of the active power, \(P_{(b)}\), measured at the PM both depend on the location of the generating source. The mentioned conditions correspond to the following cases.
Transforming the complex power (11), as shown in the Appendix, allows obtaining the expression for the active power of the $h$-th harmonic [see (12)], where $\delta_{(b)}$ is the angle between the phasors $I_{(b)u}$ and $I_{(b)v}$.

Further manipulation, see the Appendix, allows expressing the real power of the $h$-th harmonic at PM in the normalised form

$$P_{(b)u} = \frac{P_{(b)u}}{P_{(b)v}} = a_{(b)} \left[ I_{(b)u}^2 + b_{(b)} I_{(b)v} \sin (\beta_{(b)}) - c_{(b)} \right]. \quad (13)$$

Here, $I_{(b)u} = \frac{I_{(b)u}}{I_{(b)v}}$ is the rms value of the normalised $h$-th harmonic current, $P_{(b)u} = (I_{(b)u})^2 R_u$ is the $h$-th harmonic base power, and $\beta_{(b)} = \gamma_{(b)} + \delta_{(b)}$ is the cumulative angle, where

$$\gamma_{(b)} = \tan^{-1} \left( \frac{1 - c_{(b)}}{x_{(b)u} + c_{(b)} x_{(b)v}} \right) \quad (14)$$

is the system’s angle and $\delta_{(b)}$ is as mentioned above.

The coefficients in (13) and (14) are defined as follows:

$$a_{(b)} = \frac{r \left(1 + x_{(b)v}^2\right)}{(1 + r)^2 + \left(x_{(b)u} + r x_{(b)v}\right)^2} \quad (15)$$

$$b_{(b)} = \sqrt{\left(1 - c_{(b)}\right)^2 + \left(x_{(b)u} + c_{(b)} x_{(b)v}\right)^2} \quad (16)$$

$$c_{(b)} = \frac{r \left(1 + x_{(b)v}^2\right)}{(1 + x_{(b)v}^2)} \quad (17)$$

where $r = \frac{R_c}{R_u}$ is the customer’s to utility’s resistance ratio, and $x_{(b)u} = \frac{x_{(b)u}}{R_u}$ and $x_{(b)v} = \frac{x_{(b)v}}{R_u}$ are the normalised reactances of the utility and the customer, respectively, at the $h$-th harmonic frequency.

The normalised power (13) is illustrated in Figures 6 and 7. As can be seen, the normalised power (13) is a negatively shifted parabolic function of the normalised current, $I_{(b)u}$. Since the normalised current, $I_{(b)u}$, is defined as the ratio of the rms values and is always positive, only the right half-plane of, $P_{(b)u}(I_{(b)u} > 0)$, has any physical meaning.

Also, note that the linear term in (13) depends on the value of the angle, $\beta_{(b)}$. As revealed by Figures 6 and 7, for $\beta_{(b)} > 0$, the locus of the normalised power, $P_{(b)u}$, begins at a negative value at the origin and ramps up quickly. However, for negative angles, $\beta_{(b)} < 0$, $P_{(b)u}$ first decreases before becoming positive at much higher values of $I_{(b)u}$. Therefore, the variations of $\beta_{(b)}$, which originate with the variations of $\delta_{(b)}$, bear significantly on the power direction measured at the PM.

4 | VALIDATION AND DISCUSSION

As shown next, the simple DC case discussed above lends itself to the general expression (13). First, for the DC case $X_{(b)u} = X_{(b)v} = 0$. Under these conditions, the coefficients (14)–(16) tend to

$$a_{(b)} \xrightarrow{r \to 0} 0 \quad (18)$$

$$c_{(b)} \xrightarrow{r \to 0} r \quad (19)$$

$$b_{(b)} \xrightarrow{r \to 0} \left\{ \begin{array}{ll} r - 1, & r > 1 \\ 1 - r, & r < 1 \end{array} \right. \quad (20)$$
FIGURE 7 The normalised power of 3rd, 5th, 7th, 9th, and 11th harmonics as a function of the normalised current $I_n$ at fixed angle $\beta_3 = -45$. (a) Wide range view. (b) At the origin. (For $R_s = 0.3 \, \text{m} \Omega$, $R_c = 10 \, \Omega$, $L_u = 10 \, \mu \text{H}$, $L_w = 3 \, \text{mH}$.)

Thus, (13) is reduced to

$$ Y_{\beta} \xrightarrow{\gamma(\hbar) \rightarrow 0} -\frac{\pi}{2}. \quad (21) $$

Thus, (13) is reduced to

$$ P_{(\beta)1} = \frac{r}{1 + r} \left[ \frac{I_{(\beta),u}^2}{(r - 1)} I_{(\beta)u} \cos \left( \delta_{(\beta)} \right) - r \right]. \quad (22) $$

Here, $\sin \left( \delta_{(\beta)} - \frac{\pi}{2} \right) = -\cos(\delta_{(\beta)})$ was also used.

The case of the aiding current sources, illustrated in Figure 1(a), can be represented by $\delta = 0$, whereas the case of the differential current sources, shown in Figure 1(b), can be represented by $\delta = \pi$. It can be easily shown that in both cases, (5) and (12) are identical.

To further validate the theoretical predictions, the network in Figure 5 was simulated using PSIM 9.4. During the software experiments, the real power at the PM was recorded. Comparison plots of the calculated vs the simulated power are presented in Figure 8. The predicted and the simulated results stand in good agreement. The minor discrepancies between the simulated and the predicted results are
due to the transient process that is considered by the PSIM simulator.

The examples above confirm that the general expressions (12) and (13) correctly represent the physics of the power transfer in the network and, therefore, are valid. In other words, (12) and (13) faithfully describe the mechanism of active energy transfer between the power system and the consumer.

The $P_{(\beta)1}$ term in (12) [the $a_{(\beta)} I_{(\beta)u}^2$ term in (13)] represents the utility’s contribution to the active power flow at the $b$-th harmonic frequency. When $I'_{(\beta)u} \neq 0$ and $I'_{(\beta)c} = 0$, only the $P_{(\beta)1}$ term can exist. In such a case, the direction of active power as measured at the PM at the $b$-th harmonic is positive (i.e. from the utility to the consumer).

Similarly, the opposing active power flow, $P_{(\beta)2}$, in (12) [the $a_{(\beta)} I_{(\beta)c}^2$ term in (13)] is generated by the consumer. When the harmonic current of the $b$-th-order component is generated only on the load side, $I'_{(\beta)u} = 0$ and $I'_{(\beta)c} \neq 0$, only the $P_{(\beta)2}$ term exists in (12) and the direction of active power at the $b$-th harmonic is negative (i.e. the power flows from the consumer to the utility).

The $P_{(\beta)3}$ term in (12) [the $a_{(\beta)} b_{(\beta)} I_{(\beta)u} \sin(\beta_{(\beta)})$ term in (13)] represents the mutual (exchange) active power and a function of current magnitudes as well as of the phase differ-
ence, \(\delta_{(b)}\), between the current phasors \(I'_{(b)a}\) and \(I'_{(b)c}\). Note that in the presence of the \(b\)-th harmonic components on both the utility and the load, \(I'_{(b)a} \neq 0\) and \(I'_{(b)c} \neq 0\), all three active power terms appear. Here, according to (12) and (13), the power flow can be either positive or negative. Nevertheless, it describes a true physical process of the utility and the customer exchanging the active power at \(b\)-th harmonic frequency.

5 CUSTOMER’S RANGE OF DOMINANCE OVER THE ACTIVE POWER FLOW

The equation for the normalised power describes a negatively shifted upright parabola. This implies that the customer source dominates the power flow through the PM when the normalised power is negative \(P'_{(b)a} < 0\), whereas the utility dominates the power flow through the PM when \(P'_{(b)a} > 0\). The borderline of the power flow, that is \(P_{(b)a} = 0\), occurs at

\[
I_{(b)a1,2} = -\frac{1}{2}h_{(b)}\sin(\beta_{(b)}) \pm \sqrt{\frac{1}{4}h_{(b)}^2 \sin^2(\beta_{(b)}) + \epsilon_{(b)}}\; . \quad (23)
\]

The considerations above allow establishing the range of customer’s dominance over the power flow at the \(b\)-th harmonic frequency. The two limiting cases can be obtained substituting \(\sin(\beta_{(b)}) = \pm 1\). Since, \(I_{(b)a} > 0\), only the positive solutions of (23) should be considered.

For the \(\sin(\beta_{(b)}) = +1\) case, the positive solution is

\[
I_{(b)a1} = -\frac{1}{2}h_{(b)} + \sqrt{\frac{1}{4}h_{(b)}^2 + \epsilon_{(b)}}\; . \quad (24)
\]

Here, the customer dominates the power flow within a quite narrow range of the normalised values of \(I_a\):

\[
I_{(b)a1} > I_{(b)a} > 0\; . \quad (25)
\]

whereas for the \(\sin(\beta_{(b)}) = -1\) case, the solution is

\[
I_{(b)a2} = +\frac{1}{2}h_{(b)} + \sqrt{\frac{1}{4}h_{(b)}^2 + \epsilon_{(b)}}\; . \quad (26)
\]

Note that \(I_{(b)a2} \gg I_{(b)a1}\) and leads to a much wider range of customer’s dominance

\[
I_{(b)a2} > I_{(b)a} > 0. \quad (27)
\]

Therefore, the phase difference, \(\delta_{(b)}\), between the phasors \(I'_{(b)a}\) and \(I'_{(b)c}\) contributes the most to \(\beta_{(b)}\) and, hence, bears a significant impact on the power flow through the PM. Figures 6 and 7 illustrate the findings above.

6 SIMULATION EXAMPLE

The network model in Figure 5 was configured and simulated in PSIM v. 9.4. A single-phase benchmark network, illustrated in Figure 9, was used. The benchmark circuit is comprised of the HD current sources injecting the third, fifth, seventh, and ninth harmonics of the fundamental frequency of 50 Hz. The impedance \(R_{l,u}\) represents the customer’s load; \(R_{l,c}\) is the system’s impedance.

Based on the current and voltage data captured by \(I_{pm}\) and \(V_{pm}\) probes, respectively, the program calculated the average power flowing through the PM. The simulation results are summarised in Table 1 and are compared to the results calculated by (13). An excellent agreement of the result was found. The following cases were considered.

Row 1—the distorted utility. In this software experiment, the third harmonic is present only on the utility side. As expected, the resulting power flow is positive, i.e. from the utility towards the customer.

Row 2—the distorting customer. In this software experiment, the fifth harmonic is present only on the customer’s side. As expected, the resulting power flow is negative, i.e. from the customer towards the utility.

Row 3—both the customer and the utility introduce distortion. In this software experiment, the seventh harmonic is present on both sides, the normalised current is \(I_c = 30/20 = 1.5\), and the phase is \(\delta_{(b)} = -15^\circ\). The power flow is negative, i.e. from the customer towards the utility.

Row 4—both the customer and the utility introduce distortion. In this software experiment, the ninth harmonic is present on both the utility and the customer side. Here,

---

**FIGURE 9** Simulation set up (at seventh harmonic frequency)

**TABLE 1** Simulation example

| \(h\#\) | \(I'_{(b)a}\) [A] | \(I'_{(b)c}\) [A] | \(P_{(b)a} \text{ calc.}\) | \(P_{(b)a} \text{ sim.}\) |
|-------|----------------|----------------|-------------------|----------------|
| Third  | 50/15°         | 0              | 0.01              | 0.01           |
| Fifth  | 0              | 40/60°         | -0.24             | -0.24          |
| Seventh| 30/30°         | 20/45°         | -1.827            | -1.827         |
| Ninth  | 10/45°         | 5/30°          | 0.175             | 0.175          |
the normalised current is $I_n = 10/5 = 2$ and the phase shift, $\delta_n = +15^\circ$, which makes the power flow from the utility towards the customer.

In all cases, the calculated power, $P_{(h)}_{\text{calc}}$, and the simulated power, $P_{(h)}_{\text{sim}}$, stand in excellent agreement. This confirms the theoretical derivations in (12) and (13).

7 | EXPERIMENTAL RESULTS

An experiment was conducted to confirm the theoretical predictions. In the course of the experiment, the direction of flow of the spectrum of higher harmonics emitted by a distortion source was considered.

The experiment suggests three possible scenarios: a single distortion source on the system’s side, a single distortion source on the customer’s side, and two distortion sources—one on the system’s and another on the customer’s sides.

Since the test result is independent of the network parameters, a simple network shown in Figure 10 is quite adequate for the experiment. (No higher harmonics, capable of adversely affecting the experiment, were present in the mains.)

A single-phase low-power model of Figure 5 was implemented in the laboratory environment. The experimental setup included a power supply $E_u$, system’s impedance $Z_u$, serial impedances on both sides of PM, $Z_1$, and $Z_2$, were also included, as well as a linear load $Z_c$. Two adjustable non-linear loads NLL1 and NLL2 were incorporated to emulate the system’s and the consumer’s non-linearities, respectively. The parameters of the network elements were selected according to the acquisition capabilities of the metering device. The measurements were carried out by the SATEC PM130 PLUS installed on the PM (see Figure 10).

The non-linear loads NLL1 and NLL2 were activated to introduce HD at the appropriate location of the network. Figure 11 shows the typical harmonic profile of the nonlinear source. Accordingly, three experimental scenarios were examined.

**Case 1:** The distortion source is located on the system’s side. Here, the non-linear load NLL1 was activated, while NLL2 was idled. SATEC meter recorded the voltages and currents of harmonic components, as well as the angles between them at the PM. The experimental waveforms of the current and voltage at PM were obtained, as shown in Figure 12(a). The experimentally measured angles of harmonic components are shown in Figure 13(a). The experimental result reveals that for all higher harmonics, the phase shift between the current and voltage components does not exceed $90^\circ$. This confirms that when the source of distortion is located on the system side, the active power flows from the source towards the consumer.

**Case 2:** The source of distortion is located on the consumer side. Here, the non-linear load NLL1 is idled, whereas the non-linear load NLL2 is activated. The captured by SATEC meter current and voltage waveforms shown in Figure 12(b). The SATEC meter was also used to record the phase of the higher harmonic components of voltage and currents [see Figure 13(b)]. The experiment shows that for all recorded higher harmonic components, the angle between the current and voltage exceeds $90^\circ$. This demonstrates that when the source of distortion is located on the consumer side, the active power, measured at the PM, flows from the consumer side towards the system side. Note that the direction of the active powers of the harmonic components is reversed as compared to Case 1.

**Case 3:** The sources of distortion are located on both system and consumer sides. Here, the non-linear loads NLL1 and NLL2 were activated simultaneously. The experimental waveforms of the current and voltage at PM were obtained as shown in Figure 12(c).

Same as above, the SATEC meter was used to acquire the voltage and currents of higher harmonic components, as well as the angles between them [see Figure 13(c)]. Examining Figure 13(c) reveals that for the 7th, 15th, 17th, 19th, 21st, 23rd, 25th, and 27th harmonic components, the angle between the
current and voltage does not exceed 90°, whereas, for the 3rd, 5th, 9th, 11th, and 13th harmonics, the angle between the current and voltage is greater than 90°.

8 | CONCLUSIONS

The advantage of the APD method is that it relies on minimal input data (instantaneous values of currents and voltages) acquired at a single measurement point and, thus, requires affordable metering equipment and simple installation. And, thus, it offers an attractive solution for PQ monitoring.

Yet, in the past, some researchers have criticised the APD approach as an inconsistent one. Indeed, in case the sources of HD are found on both sides of the measurement point, the ambiguous power flow readings may be observed for the same harmonic currents magnitude.

This paper revisits the APD method for locating the distortion sources in the power distribution network. The inaccuracies in the earlier analysis challenging the APD method were identified, and the claim of the inconsistency of the APD method was re-examined.

The article shows the derivation of the expression for the complex power, from which the equation for the active power flow through PM was derived in a normalised form, whence the consumer dominance range at any harmonic frequency was established. This paper also reports on simulation results that support the theoretical findings.

This paper discussed the problem of the power flow in the network as a deterministic one. That is, if the network parameters $Z_{\Omega}$ and $Z_{\Omega}$ and the harmonic current phasors $I_{\Omega}$ and $I_{\Omega}$ were precisely known, then the power flow could be correctly predicted by APD. Comparison with simulation results confirms that APD results are accurate, which implies that the theoretical foundations of the APD approach are solid.
In practice, however, the network parameters are unavailable. Moreover, both the magnitude and the phase of the harmonic sources represent the activities of a vast number of customers connected to the network, which, in reality, are unpredictable. Thus, the phasors $I^{h}\theta(x,y)$ and $I^{h}\phi(x,y)$ as well as the phase shift, $\delta^{h}(x,y)$, are, in fact, time-variable random quantities. The harmonic current magnitudes and the phase shift can change instantaneously affecting the direction of the harmonic power flow respectively. Thus, the problem of determining the direction of the power flow is a stochastic one.

In resume, the difficulties of locating the source of distortion should not be attributed to the theoretical inconsistency of the APD method per se, as some had claimed before, but rather to the random nature of consumers’ activities on the electrical network. The stochastic nature of the harmonic power flow process is the key difficulty one faces in establishing the location of the harmonic sources, which is yet to be resolved.

REFERENCES

1. IEEE Recommended Practice and Requirements for Harmonic Control in Electric Power Systems, IEEE Standard 519-2014 (Revision IEEE Standard 519–1992), 1–29 (2014)
2. Electromagnetic Compatibility (EMC)—Part 3–6: Limits—Assessment of Emission Limits for the Connection of Distorting Installations in MV, HV and EHV Power Systems, IEC Standard 61000-3-6, (2013)
3. Baki, M.E., Hocaoglu, M.H.: On the validity of harmonic source detection methods and indices. In Proc. 14th Int. Conf. Harmon. Qual. Power, Bergamo, Italy, 1–5 (2010)
4. Xu, F., et al.: Study on constraints for harmonic source determination using active power direction. IEEE Trans. Power Delivery 33(6), 2683–2692 (2018)
5. Azouaou, R., Rabahallah, S., Leulmi, S.: Study of the direction of the harmonic injections in the electrical power systems. In Proc. 39th Int. Univ. Power Eng. Conf., Bristol, UK, 944–947 (2004)
6. Wang, B., et al.: Several sufficient conditions for harmonic source identification in power systems. IEEE Trans. Power Delivery 3(3), 3105–3113 (2018)
7. Xu, W.: Power direction method cannot be used for harmonic source detection. In Proc. IEEE Power Eng. Soc. Summer Meeting 2, 873–876 (2000)
8. Xu, W., Liu, X., Liu, Y.: An investigation on the validity of power-direction method for harmonic source determination. IEEE Trans. Power Delivery 18(1), 214–219 (2003)
9. Li, C., Xu, W., Tayyasant, T.: A critical impedance-based method for identifying harmonic sources. IEEE Trans. Power Delivery 19(2), 671–678 (2004)
10. Cataliotti, A., Cosentino, V.: A new measurement method for the detection of harmonic sources in power systems based on the approach of the IEEE Std. 1459–2000. IEEE Trans. Power Delivery 25(1), 332–340 (2010)
11. Papić, L., et al.: A benchmark test system to evaluate methods of harmonic contribution determination. IEEE Trans. Power Delivery 34(1), 23–31 (2019)
12. Browne, T.J., Gosbell, V.J., Perera, S.: Allocated harmonic quantities as the basis for source detection. In Proc. IEEE Power Energy Soc. Gen. Meeting, 9, 1–6 (2009)
13. Vaid, K., Srikanth, P., Sood, Y.R.: Critical impedance based automatic identification of harmonic sources in deregulated power industry. In Proc. Int. Conf. Signal Process., Commun., Comput. Netw. Technol., 653–658 (2011)
14. Malekian, K.: A novel approach to analyze the harmonic behavior of customers at the point of common coupling. In Proc. 9th Int. Conf. Comput. Power Electron. 31–36 (2015)
15. Shojae, M., Mokhtari, H.: A method for determination of harmonics responsibilities at the point of common coupling using data correlation analysis. IET Gener. Transm. Distrib. 8(1), 142–150 (2014)
16. Karimzadeh, F., Esmaeili, S., Hosseinian, S.H.: Method for determining utility and consumer harmonic contributions based on complex independent component analysis. IET Gener. Transm. Distrib. 10(2), 526–534 (2016)
17. Tang, K., Shen, C.: Harmonic emission level assessment method based on parameter identification analysis. IET Gener. Transm. Distrib. 13(7), 976–983 (2019)
18. Safargholi, F., Malekian, K., Schufft, W.: On the dominant harmonic source identification—Part I: Review of methods. IEEE Trans. Power Delivery 33(3), 1268–1277 (2018)
19. Safargholi, F., Malekian, K., Schufft, W.: On the dominant harmonic source identification—Part II: Application and interpretation of methods. IEEE Trans. Power Delivery 33(3), 1278–1287 (2018)
20. Farhoodnea, M., Mohamed, A., Shareef, H.: A new method for determining multiple harmonic source locations in a power distribution system. In Proc. IEEE Int. Conf. Power Energy, 146–150 (2010)
21. Bazina, M., Tomisa, T.: Comparison of various methods for determining direction of harmonic distortion by measuring in point of common coupling. In Proc. IEEE Int. Energy Conf., 392–399 (2014)
22. Meyer, J., et al.: Survey on international practice of calculating harmonic current emission limits. In Proc. 17th Int. Conf. Harmon. Qual. Power, 539–544 (2016)
23. Zhao, X., Yang, H.: A new method to calculate the utility harmonic impedance based on FastICA. IEEE Trans. Power Delivery 31(1), 381–388 (2016)
24. Sinvula, R., et al.: Harmonic source detection methods: A systematic literature review. IEEE Access 7, 74283–74299 (2019)

How to cite this article: Shcherbakova P, Senderovych G, Abramovitz A. Revisiting the active power direction method. IET Gener Transm Distrib, 2021;15:1056–1069. https://doi.org/10.1049/gtd2.12080
APPENDIX A

DERIVATION OF THE ACTIVE POWER FORMULA OF THE H-TH HARMONIC

In order to obtain the desired expressions for active power the complex equation of full power, equation (10) is manipulated as follows:

$$S_{(b)3} = \frac{Z_{(b)a} \cdot Z_{(b)c}}{Z_{(b)}^2} \cdot \left(-L'_{(b)a} \cdot Y'_{(b)c} \cdot Z_{(b)c} + L'_{(b)c} \cdot Y'_{(b)a} \cdot Z_{(b)a}\right)$$

$$= \frac{Z_{(b)a} \cdot Z_{(b)c}}{Z_{(b)}^2} \cdot \left[\left(ReL'_{(b)a} - jImL'_{(b)a}\right) \cdot \left(ReL'_{(b)c} + jImL'_{(b)c}\right) \cdot Z_{(b)a}\right]$$

$$= \frac{Z_{(b)a} \cdot Z_{(b)c}}{Z_{(b)}^2} \cdot \left(ReL'_{(b)a} \cdot ReL'_{(b)c} \cdot ImL'_{(b)a} \cdot ImL'_{(b)c} + jReL'_{(b)a} \cdot ImL'_{(b)c} - jImL'_{(b)a} \cdot ReL'_{(b)c}\right)$$

$$- \frac{Z_{(b)a} \cdot Z_{(b)c}}{Z_{(b)}^2} \cdot \left(ReL'_{(b)a} \cdot ReL'_{(b)c} \cdot ImL'_{(b)a} \cdot ImL'_{(b)c} - jReL'_{(b)a} \cdot ImL'_{(b)c} + jImL'_{(b)a} \cdot ReL'_{(b)c}\right).$$

(A1)

The product of complex numbers in the numerators of (A1) can be replaced by

$$Z_{(b)a} \cdot Z_{(b)c} = Z_{(b)a} < \varphi_a \cdot Z_{(b)c} < \varphi_c \cdot Z_{(b)a} < -\varphi_a = Z_{(b)a} \cdot Z_{(b)c} < (\varphi_a + \varphi_c) = Z_{(b)a} \cdot Z_{(b)c} \cdot Z_{(b)a} < \varphi_c.$$ Similarly,

$$Z_{(b)a} \cdot Z_{(b)c} \cdot Z_{(b)c} = Z_{(b)a} \cdot Z_{(b)c} \cdot Z_{(b)a} < \varphi_a.$$ The arguments for these complex numbers are known: \(\varphi_c = \arg Z_{(b)c}\) and \(\varphi_a = \arg Z_{(b)a}\).

Thus, a complex expression of the total power of the third component is obtained as

$$S_{(b)3} = \frac{Z_{(b)a} \cdot Z_{(b)c}}{Z_{(b)}^2} \cdot \left[\left(ReL'_{(b)a} \cdot ReL'_{(b)c} + ImL'_{(b)a} \cdot ImL'_{(b)c} + jReL'_{(b)a} \cdot ImL'_{(b)c} - jImL'_{(b)a} \cdot ReL'_{(b)c}\right) \cdot Z_{(b)a} \cdot \varphi_c\right]$$

$$- \left(ReL'_{(b)a} \cdot ReL'_{(b)c} + ImL'_{(b)a} \cdot ImL'_{(b)c} - jReL'_{(b)a} \cdot ImL'_{(b)c} + jImL'_{(b)a} \cdot ReL'_{(b)c}\right) \cdot Z_{(b)a} \cdot \varphi_a].$$

(A2)

Considering that \(Z_{(b)c} \leq \varphi_c = Z_{(b)c}(\cos \varphi_c + j \sin \varphi_c)\) and \(Z_{(b)a} \leq \varphi_a = Z_{(b)a}(\cos \varphi_a + j \sin \varphi_a)\), the real component of the complex expression in brackets above can be represented as

$$\text{Re} \left[\left(ReL'_{(b)a} \cdot ReL'_{(b)c} + ImL'_{(b)a} \cdot ImL'_{(b)c} + jReL'_{(b)a} \cdot ImL'_{(b)c} - jImL'_{(b)a} \cdot ReL'_{(b)c}\right) \cdot Z_{(b)a} \cdot \varphi_c\right]$$

$$= \left(ReL'_{(b)a} \cdot ReL'_{(b)c} + ImL'_{(b)a} \cdot ImL'_{(b)c} + jReL'_{(b)a} \cdot ImL'_{(b)c} - jImL'_{(b)a} \cdot ReL'_{(b)c}\right) \cdot Z_{(b)a} \cdot \cos \varphi_c$$

$$- \left(ReL'_{(b)a} \cdot ReL'_{(b)c} + ImL'_{(b)a} \cdot ImL'_{(b)c} - jReL'_{(b)a} \cdot ImL'_{(b)c} + jImL'_{(b)a} \cdot ReL'_{(b)c}\right) \cdot Z_{(b)c} \cdot \cos \varphi_a$$

$$= Z_{(b)a} \cdot \left[\left(ReL'_{(b)a} \cdot ReL'_{(b)c} + ImL'_{(b)a} \cdot ImL'_{(b)c}\right) \cos \varphi_c + \left(-ReL'_{(b)a} \cdot ImL'_{(b)c} + ImL'_{(b)a} \cdot ReL'_{(b)c}\right) \sin \varphi_c\right]$$

$$- Z_{(b)c} \cdot \left[\left(ReL'_{(b)a} \cdot ReL'_{(b)c} + ImL'_{(b)a} \cdot ImL'_{(b)c}\right) \cos \varphi_a + \left(ReL'_{(b)a} \cdot ImL'_{(b)c} - ImL'_{(b)a} \cdot ReL'_{(b)c}\right) \sin \varphi_a\right].$$

(A3)
Since the factor $\frac{Z_{(b)u}Z_{(b)c}}{Z_{(b)T}}$ is real, the expression for the active power of the third component can be derived as

$$P_{(b)3} = \frac{Z_{(b)u} \cdot Z_{(b)c}}{Z_{(b)T}^2} \left[ \begin{array}{c}
Z_{(b)u} \left[ \frac{\left( \text{Re}I_{(b)u}^t \cdot \text{Re}I_{(b)c}^t + \text{Im}I_{(b)u}^t \cdot \text{Im}I_{(b)c}^t \right) \cdot \cos \delta_{(b)} + \sin \delta_{(b)} \sin \delta_{(b)c}}{\left( Z_{(b)u} \cos \varphi_{(b)} - Z_{(b)c} \cos \varphi_{(b)c} \right) \cos \delta_{(b)}} \right] \\
+ \left( \text{Im}I_{(b)u}^t \cdot \text{Im}I_{(b)c}^t - \text{Re}I_{(b)u}^t \cdot \text{Re}I_{(b)c}^t \right) \cdot \sin \delta_{(b)} \\
- \left( \text{Re}I_{(b)u}^t \cdot \text{Re}I_{(b)c}^t - \text{Im}I_{(b)u}^t \cdot \text{Im}I_{(b)c}^t \right) \cdot \cos \delta_{(b)c} \end{array} \right] $$

whence

$$P_{(b)3} = \frac{Z_{(b)u} \cdot Z_{(b)c}}{Z_{(b)T}^2} \left[ \begin{array}{c}
\left( \text{Re}I_{(b)u}^t \cdot \text{Re}I_{(b)c}^t + \text{Im}I_{(b)u}^t \cdot \text{Im}I_{(b)c}^t \right) \cdot \left( Z_{(b)u} \cos \varphi_{(b)} - Z_{(b)c} \cos \varphi_{(b)c} \right) \\
+ \left( \text{Im}I_{(b)u}^t \cdot \text{Im}I_{(b)c}^t - \text{Re}I_{(b)u}^t \cdot \text{Re}I_{(b)c}^t \right) \cdot \left( Z_{(b)u} \sin \varphi_{(b)} + Z_{(b)c} \sin \varphi_{(b)c} \right) \end{array} \right] $$

The real and imaginary components of the higher harmonic currents can be written in the form of their projection onto the coordinates of the complex plane:

$$\left( \text{Re}I_{(b)u}^t \cdot \text{Re}I_{(b)c}^t + \text{Im}I_{(b)u}^t \cdot \text{Im}I_{(b)c}^t \right) = I_{(b)u}^t \cdot I_{(b)c}^t \cos \left( \delta_{(b)u} - \delta_{(b)c} \right) = I_{(b)u}^t \cdot I_{(b)c}^t \cos \delta_{(b)}$$

$$\left( \text{Im}I_{(b)u}^t \cdot \text{Im}I_{(b)c}^t - \text{Re}I_{(b)u}^t \cdot \text{Re}I_{(b)c}^t \right) = I_{(b)u}^t \cdot I_{(b)c}^t \sin \left( \delta_{(b)u} - \delta_{(b)c} \right) = I_{(b)u}^t \cdot I_{(b)c}^t \sin \delta_{(b)}$$

where $\delta_{(b)}$ is the angle between $I_{(b)u}^t$ and $I_{(b)c}^t: \delta_{(b)} = \delta_{(b)u} - \delta_{(b)c}$.

Hence, the active power can be expressed as

$$P_{(b)3} = I_{(b)u}^t \cdot I_{(b)c}^t \frac{Z_{(b)u} \cdot Z_{(b)c}}{Z_{(b)T}^2} \left( A_r \cos \delta_{(b)} + A_x \sin \delta_{(b)} \right)$$

where $A_r = Z_{(b)u} \cos \varphi_{(b)} - Z_{(b)c} \cos \varphi_{(b)c}$ and $A_x = Z_{(b)u} \sin \varphi_{(b)} + Z_{(b)c} \sin \varphi_{(b)c}$.

Also considering that

$$\frac{Z_{(b)u}Z_{(b)c}}{Z_{(b)T}^2} \left( Z_{(b)u} \cos \varphi_{(b)} - Z_{(b)c} \cos \varphi_{(b)c} \right) \cos \delta_{(b)} = \left( \frac{Z_{(b)u}^2}{Z_{(b)T}^2} R_r - \frac{Z_{(b)c}^2}{Z_{(b)T}^2} R_r \right) \cos \delta_{(b)}$$

$$\frac{Z_{(b)u}Z_{(b)c}}{Z_{(b)T}^2} \left( Z_{(b)u} \sin \varphi_{(b)} + Z_{(b)c} \sin \varphi_{(b)c} \right) \sin \delta_{(b)} = \left( \frac{Z_{(b)u}^2}{Z_{(b)T}^2} X_r - \frac{Z_{(b)c}^2}{Z_{(b)T}^2} X_r \right) \sin \delta_{(b)}$$

leads to the final expression for the third component of the active power of the $b$-th harmonics

$$P_{(b)3} = I_{(b)u}^t \cdot I_{(b)c}^t \frac{Z_{(b)u}^2 R_r - Z_{(b)c}^2 R_r}{Z_{(b)T}^2} \cos \delta_{(b)} + \frac{Z_{(b)u}^2 X_r + Z_{(b)c}^2 X_r}{Z_{(b)T}^2} \sin \delta_{(b)}$$
Hence, the active power at PM is given by

\[ P_{(b)} = \left( I'_{(b)u} \right)^2 \frac{Z_{(b)u}^2}{Z_{(b)u}^2} R_u - \left( I'_{(b)r} \right)^2 \frac{Z_{(b)r}^2}{Z_{(b)r}^2} R_u + \frac{I'_{(b)u} I'_{(b)r}}{Z_{(b)u}^2} \left( \left( Z_{(b)u}^2 R_u - Z_{(b)r}^2 R_u \right) \cos \delta_{(b)} + \left( Z_{(b)u}^2 X_{(b)u} + Z_{(b)r}^2 X_{(b)r} \right) \sin \delta_{(b)} \right). \]

Define

\[ \sin (\alpha_{(b)}) = \frac{\left( Z_{(b)u}^2 R_u - Z_{(b)r}^2 R_u \right)}{\sqrt{\left( Z_{(b)u}^2 R_u - Z_{(b)r}^2 R_u \right)^2 + \left( Z_{(b)u}^2 X_{(b)u} + Z_{(b)r}^2 X_{(b)r} \right)^2}} \]

and

\[ \cos (\alpha_{(b)}) = \frac{\left( Z_{(b)u}^2 X_{(b)u} + Z_{(b)r}^2 X_{(b)r} \right)}{\sqrt{\left( Z_{(b)u}^2 R_u - Z_{(b)r}^2 R_u \right)^2 + \left( Z_{(b)u}^2 X_{(b)u} + Z_{(b)r}^2 X_{(b)r} \right)^2}}. \]

Applying the trigonometric identity \( \sin(\alpha + \delta) = \sin(\alpha) \cos(\delta) + \cos(\alpha) \sin(\delta) \), we attain

\[ P_{(b)} = \left( I'_{(b)u} \right)^2 \frac{Z_{(b)u}^2}{Z_{(b)u}^2} R_u - \left( I'_{(b)r} \right)^2 \frac{Z_{(b)r}^2}{Z_{(b)r}^2} R_u + \frac{I'_{(b)u} I'_{(b)r}}{Z_{(b)u}^2} \sqrt{\left( Z_{(b)u}^2 R_u - Z_{(b)r}^2 R_u \right)^2 + \left( Z_{(b)u}^2 X_{(b)u} + Z_{(b)r}^2 X_{(b)r} \right)^2} \sin (\alpha_{(b)} + \delta_{(b)}). \]

Define \( \beta_{(b)} = \alpha_{(b)} + \delta_{(b)}, r = \frac{R_{(b)u}}{R_{(b)r}}, \frac{X_{(b)u}}{R_{(b)r}}, \frac{X_{(b)r}}{R_{(b)r}} \), and further manipulate

\[
\frac{P_{(b)}}{\left( I'_{(b)u} \right)^2} = \frac{1}{Z_{(b)u}^2} \left[ I'^2 \left( Z_{(b)u}^2 R_u - Z_{(b)r}^2 R_u + I \sqrt{\left( Z_{(b)u}^2 R_u - Z_{(b)r}^2 R_u \right)^2 + \left( Z_{(b)u}^2 X_{(b)u} + Z_{(b)r}^2 X_{(b)r} \right)^2} \sin (\beta_{(b)}) \right) \right]
\]

\[
\frac{P_{(b)}}{\left( I'_{(b)r} \right)^2} = \frac{1}{\left( R_u + R_r \right)^2 + \left( X_{(b)u} + X_{(b)r} \right)^2} \]

\[
\times \left[ I'^2 \left( 1 + X_{(b)u}^2 \right) R_u - R_r^2 \left( 1 + X_{(b)r}^2 \right) R_u + I \sqrt{\left( R_u^2 \left( 1 + X_{(b)u}^2 \right) R_u - R_r^2 \left( 1 + X_{(b)r}^2 \right) R_u \right)^2 + \left( R_u^2 \left( 1 + X_{(b)u}^2 \right) R_r X_{(b)u} + R_r^2 \left( 1 + X_{(b)r}^2 \right) R_r X_{(b)r} \right)^2} \sin (\beta_{(b)}) \right] \]

\[
\frac{P_{(b)}}{\left( I'_{(b)r} \right)^2} = \frac{R_u R_r}{R_u^2 \left( 1 + r \right)^2 + R_r^2 \left( X_{(b)u} + r X_{(b)r} \right)^2} \times \left[ I'^2 \left( 1 + X_{(b)u}^2 \right) R_u - \left( 1 + X_{(b)r}^2 \right) R_u + I \sqrt{\left( \left( 1 + X_{(b)u}^2 \right) R_u - \left( 1 + X_{(b)r}^2 \right) R_u \right)^2 + \left( 1 + X_{(b)u}^2 \right) R_r X_{(b)u} + \left( 1 + X_{(b)r}^2 \right) R_r X_{(b)r} \right)^2} \sin (\beta_{(b)}) \right] \]
\[ \frac{P_{(b)}}{\left( I_{(b)\mu}^r \right)^2 R_y} = \frac{r}{(1 + r)^2 + (x_{(b)\mu} + rx_{(b)\nu})^2} \]

\times \left[ I_\nu^2 \left( 1 + x_{(b)\mu}^2 \right) - r \left( 1 + x_{(b)\mu}^2 \right) + I_\mu \sqrt{\left( 1 + x_{(b)\mu}^2 \right)^2 - r \left( 1 + x_{(b)\mu}^2 \right)^2 + \left( 1 + x_{(b)\mu}^2 \right) x_{(b)\nu} + r \left( 1 + x_{(b)\mu}^2 \right) x_{(b)\nu}^2 \sin (\beta_{(b)})} \right] \]

\[ \frac{P_{(b)}}{\left( I_{(b)\nu}^r \right)^2 R_y} \]

\[ = \frac{r \left( 1 + x_{(b)\mu}^2 \right)}{(1 + r)^2 + (x_{(b)\mu} + rx_{(b)\nu})^2} \times \left[ I_\nu^2 - r \left( 1 + x_{(b)\mu}^2 \right) + I_\mu \sqrt{\left( 1 - r \left( 1 + x_{(b)\mu}^2 \right) \right)^2 + \left( x_{(b)\nu} + \frac{r \left( 1 + x_{(b)\mu}^2 \right)}{1 + x_{(b)\mu}^2} \right)x_{(b)\nu} \sin (\beta_{(b)})} \right] \]

\[ \frac{P_{(b)}}{\left( I_{(b)\nu}^r \right)^2 R_y} \]

\[ = \frac{r \left( 1 + x_{(b)\mu}^2 \right)}{(1 + r)^2 + (x_{(b)\mu} + rx_{(b)\nu})^2} \times \left[ I_\nu^2 - r \left( 1 + x_{(b)\mu}^2 \right) + I_\mu \sqrt{\left( 1 - r \left( 1 + x_{(b)\mu}^2 \right) \right)^2 + \left( x_{(b)\nu} + \frac{r \left( 1 + x_{(b)\mu}^2 \right)}{1 + x_{(b)\mu}^2} \right)x_{(b)\nu} \sin (\beta_{(b)})} \right] \]

whence expression (13) follows.