On Constraining the Mesoscale Eddy Energy Dissipation Time-Scale

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Abstract  A physically plausible lower bound on the spatially varying geostrophic mesoscale eddy energy dissipation time-scale within the ocean, related to the geographical energy transfer rate out of the geostrophic mesoscales, is provided by means of a simple and computationally inexpensive inverse calculation. Data diagnosed from a high resolution global configuration ocean simulation is supplied to a parameterized model of the geostrophic mesoscale eddy energy, from which the dissipation time-scale results as a solution to an optimization calculation. We find that the dissipation time-scale is shortest in the Southern Ocean, in the Western Boundary Currents, and on the western boundaries, consistent with the expectation that these regions are notable sites of baroclinic activity with processes leading to energy transfer out of the geostrophic mesoscales. Although our solution should be interpreted as a lower bound given the assumptions going into the calculation, it serves as an important physically consistent base line reference for further investigations into ocean energetics, as well as for an intended inference calculation that is more complete but also much more complex.

Plain Language Summary  Energy plays an important role in quantifying the magnitude of motions at different time and spatial scales. Many different dynamical processes contribute to energy transfers within the ocean, and constraining the rate of transfer remains a formidable challenge. This work provides a bulk constraint on the overall magnitude and spatial variation of an eddy energy dissipation time-scale, which relates to the rate of energy transfer out of the motions at 10–100 km in the ocean where rotation and density stratification play a leading order role in the dynamics. A time-scale is “backed out” from a model via an inverse approach; given a model for the eddy energy evolution and what we should end up with (the eddy energy signature), what should we have started off with in the first place (the dissipation time-scale)? Although our solution should be interpreted as a lower estimate given the assumptions going into the calculation, it serves as an important physically consistent base line reference for further investigations into ocean energetics, as well as for an intended inverse calculation that is more complete but also much more complex.

1. Introduction

The ocean, being a key component of the Earth system, plays a central role in the Earth's energy and biogeochemical cycles through its ability to store and transport large amounts of tracers (e.g., Adkins, 2013; Bopp et al., 2017; Burke et al., 2015; Ferrari et al., 2014; Galbraith & de Lavergne, 2019; Jansen, 2017; Takano et al., 2018; Zhang & Vallis, 2013). A key component is the transport provided by the ocean circulation, taken here to mean both the large-scale mean circulation, as well as the smaller-scale eddy motions. The large-scale mean motions tend to generate smaller-scale eddy motions via instabilities, but the smaller-scale eddy motions also interact and feedback onto the large-scale mean. The ability to represent the multi-scale interaction faithfully (via explicit or sub-grid modeling means), besides the theoretical interest, is of central importance to ocean model performance, which impacts our ability to predict and assess impacts within the ocean component as well as the wider Earth system (e.g., Fox-Kemper et al., 2019; Hewitt et al., 2017).

An important quantity relating to dynamics at the relevant space and/or time-scales is the energy, and there has been ongoing theoretical, numerical and observational research into quantifying and constraining ocean energy content and pathways. Ferrari and Wunsch (2009, 2010) provide one of the more recent reviews of the global energy content and pathways of the overall ocean energetic cycles, though the authors note that there are still relatively
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Large uncertainties associated with the magnitudes as well as spatial distributions in the energy fluxes. From an ocean modeling point of view, energetically constrained parameterizations have been increasingly proposed and investigated, and such parameterizations have led to model improvements (e.g., Bachman, 2019; Cessi, 2008; Eden & GrebTac, 2008; Eden et al., 2014; Gaspar et al., 1990; Jansen & Held, 2014; Jansen et al., 2019; D. P. Marshall & Adcroft, 2010; D. P. Marshall et al., 2012; Mak et al., 2018, 2022; Nielsen et al., 2018; Olbers & Eden, 2013). Beyond improving our understanding of the dynamics within the ocean, constraints for the energy pathways play an important role in limiting the magnitude and form of energy transfers between different dynamical components, which is also expected to lead to improved performance of numerical ocean models via improvements to sub-grid parameterizations of dynamical processes.

The focus of the present work is on providing a leading order constraint on the energy pathway associated with the ocean geostrophic mesoscales, where the dynamics are strongly constrained by rotation and stratification. Analogous to processes such as internal waves where the impact of energy supply and removal will affect quantities such as diapycnal mixing, the energy content and pathways in the geostrophic mesoscale is expected to impact eddy induced advection and isoneutral diffusion, usually quantified by an eddy induced velocity coefficient or the Gent–McWilliams coefficient (Gent & McWilliams, 1990; Gent et al., 1995) and an isoneutral diffusion coefficient (Griffies, 1998; Redi, 1982), respectively. Eddy energy content and removal rates will affect the associated eddy–mean-flow feedbacks, with consequences for the global overturning circulation and stratification profile. Such an influence was demonstrated, for example, in the work of Mak et al. (2022) for prognostic calculations within a global ocean circulation model via an eddy energy constrained mesoscale eddy parameterization scheme (GEOMETRIC; Mak et al., 2018; D. P. Marshall et al., 2012). In that work, the model global ocean circulation and stratification were found to be acutely sensitive to modest changes in a linear eddy energy dissipation time-scale $\lambda^{-1}$, and were comparable to sensitivities with respect to significant variations (halving and doubling) in the Southern Ocean wind forcing magnitude. While we do not expect such significant changes in the Southern Ocean wind forcing (e.g., Lin et al., 2018), analogous constraints for the eddy energy dissipation time-scale, related in turn to the eddy energy flux out of the geostrophic mesoscales, are lacking.

While the transfer of energy into the mesoscale is known to be primarily via baroclinic instability accessing the available potential energy at the large planetary scales arising from large-scale wind and thermodynamic forcing (Ferrari & Wunsch, 2009, 2010), there are many dynamical processes that can lead to energy fluxes out of the mesoscale. These include but are not limited to: direct return to the mean flow, via an inverse cascade in rotationally dominant quasi-two-dimensional systems (e.g., Bachman, 2019; Jansen et al., 2019; Salmon, 1980); relative wind stress effects, whereby the atmospheric wind forcing can spindown baroclinic eddies (e.g., Rai et al., 2021; Xu et al., 2016; Zhai et al., 2012); bottom drag (e.g., Sen et al., 2008); non-propagating form drag arising from bathymetric form stress (e.g., Klymak, 2018; Klymak et al., 2021); scattering into lee waves by geostrophic flow interaction with the bottom topography (e.g., Melet et al., 2014, 2015; Nikurashin & Ferrari, 2011; Nikurashin et al., 2013; Yang, Nikurashin, et al., 2021; L. Yang et al., 2018); loss of balance, from secondary fluid instabilities of the mesoscale eddy motions themselves (e.g., Barkan et al., 2017; Chouksey et al., 2018; Molemaker et al., 2005; Rocha et al., 2018). While there are suggestions that some processes are more efficient at transferring energy out of the mesoscale (e.g., non-propagating form drag; Klymak et al., 2021), an overall quantification on the geographical distribution of the energy transfer rate is absent.

Given the preceding discussion, we are primarily interested in providing an overall estimate for the spatial distribution of the linear eddy energy dissipation time-scale $\lambda^{-1}$, to serve as a leading order constraint for (a) the energy fluxes out of the geostrophic mesoscales, (b) the various dynamical components leading to the aforementioned energy flux, (c) the development of further parameterizations of the energy fluxes, and (d) further theoretical, numerical and observation works into the ocean energetic cycles. In this work, we perform an inference calculation for a spatially varying $\lambda^{-1}$ via an inverse approach, and illustrate the approach via the GEOMETRIC description for the geostrophic mesoscale eddy energy evolution (although other choices are possible, e.g., MEKE from Jansen et al., 2019). This work additionally details the use of inference calculations as a way to tune for uncertain parameters for use in prognostic numerical models, and some methodology and associated computational tools that are perhaps less well-known in the field of Earth system science.

As will be detailed and argued later, while one would ideally like to carry out a dynamically constrained inference calculation, the associated calculation is numerically complex and computationally intensive (e.g., requiring derivation of adjoint models given the expected large amount of degrees of freedom to maintain computational
feasibility; Gunzburger, 2003; Kalnay, 2002; Wunsch, 1996). As a precursor to the proposed difficult (but more complete) dynamically constrained inference calculation, we report here a kinematic/diagnostic-type inference calculation that is much simpler and computationally inexpensive, with the caveat that there are simplifying assumptions involved, resulting in a spatially varying $\lambda^{-1}$ that will be argued to be a physically plausible lower bound. The present calculation thus provides an invaluable prior estimate for $\lambda^{-1}$ that can be utilized in an intended dynamically constrained inference calculation, to be reported elsewhere.

In Section 2 we provide details to the mesoscale version of the GEOMETRIC parameterization, an overview of the inference and inverse approach, and the implementation details of the diagnostic-type inference problem for the geostrophic mesoscale eddy energy dissipation time-scale $\lambda^{-1}$. Analyses of the characteristics of the resulting solutions are detailed in Section 3. In Section 4 we investigate the consequences of utilizing the inferred solution in a prognostic model, providing some support that the reported dissipation time-scale is a physically plausible lower bound. In Section 5 we provide outlooks on the proposed dynamically constrained inference calculation to improve our understanding of ocean energy pathways, parameterization of energy pathways, and for inferring uncertain system parameters.

2. Methodology and Implementation

2.1. GEOMETRIC

For this work, we consider the GEOMETRIC description (D. P. Marshall et al., 2012) as a model for the evolution of mesoscale eddy energy. The choice of GEOMETRIC is made for its theoretical foundations (D. P. Marshall et al., 2012; Maddison & Marshall, 2013) and its demonstrated capabilities in recovering some key ocean sensitivities possessed by high resolution numerical models that permit and/or resolve mesoscale eddies in prognostic coarse resolution models (Mak et al., 2018, 2022); however we stress again that other choices (e.g., such as MEKE from Jansen et al., 2019) would be possible with suitable adaptations of the methodology detailed below. Denoting $\bar{E} = \int E \, dz$ as the depth-integrated total eddy energy, the mesoscale version of GEOMETRIC suggests we take (cf. Equation 1 of Mak et al., 2022)

$$\kappa_{gm} = \alpha \frac{\bar{E}}{M^2/N \, dz},$$

where $\kappa_{gm}$ is the eddy induced velocity coefficient (cf. Gent & McWilliams, 1990; Gent et al., 1995), $\alpha$ is a non-dimensional tuning parameter, and $M^2 = \nabla^2 \bar{b}$ and $N^2 = \partial \bar{b}/\partial z$ are the mean horizontal and vertical buoyancy gradients respectively. In a prognostic calculation, $\bar{E}$ evolves according to the parameterized eddy energy budget (cf. Equation 2 of Mak et al., 2022)

$$\frac{d\bar{E}}{dt} + \nabla_H \cdot \left( \bar{u}^z - |e| e_k \right) \bar{E} = \int \kappa_{gm} \frac{M^4}{N^2} \, dz \left( \text{source} \right) - \lambda \bar{E} \left( \text{dissipation} \right) + \eta_E \nabla_H^2 \bar{E} \left( \text{diffusion} \right),$$

where $\bar{u}^z$ is the depth-averaged mean flow, $\eta$ is the long Rossby wave phase speed, $e_k$ is the unit zonal vector pointing east, $\lambda$ is a linear eddy energy dissipation rate (so that $\lambda^{-1}$ is the linear eddy energy dissipation time-scale of principal focus in this work), and $\eta_E$ is an eddy energy diffusion coefficient. From prognostic calculations reported in the work of Mak et al. (2022), the dominant contributions in Equation 2 were found to be from the source and dissipation, with secondary contributions from diffusion and advection, though the latter two terms are important for the resulting eddy energy spatial distribution.

The principal aim of this work will be on constraining the spatial distribution of $\lambda^{-1}(\phi, \theta)$ (where $\phi$ and $\theta$ denote the longitude and latitude respectively), here taken to encapsulate all the aforementioned dynamical processes that leads to an energy flux out of the mesoscale. This is of course a rather drastic approximation, although there is some suggestion that a dominant source of eddy energy removal from the mesoscale could be from non-propagating form drag (Klymak, 2018; Klymak et al., 2021), and the energy transfer arising from non-propagating form drag is better represented as a linear drag and expected to largely depend on the bathymetry. We also make the simplifying assumptions of taking both $\eta_E$ and $\alpha$ to be prescribed constants. For $\eta_E$, this is partially justified a priori, where we expect the diffusion term to be of secondary importance in the evolution
of the parameterized eddy energy in prognostic calculations, and a posteriori, where the inferred solutions were found not to be overly sensitive to the choice of $\eta$. For $\alpha$, this is mainly for simplicity, and it is known $\alpha$ does vary in space (Poulsen et al., 2019). The inferred solution will be seen to display some sensitivity to the value of $\alpha$ through its role in the source term. The possibility of a joint inference calculation for $\lambda^{-1}$ and $\alpha$ is discussed in Section 5.

### 2.2. Parameter Inference Problem

The inference calculation for $\lambda^{-1}(\phi, \theta)$ here utilizes the variational approach (e.g., Kalnay, 2002). In the general case, we have state variables $w$ that depend on control variable $\lambda^{-1}(\phi, \theta)$ via some model $F(w; \lambda^{-1}) = 0$, and the aim is to seek $\lambda^{-1}(\phi, \theta)$ such that the mismatch between some target data $w_{data}$ and $w$ is minimized, possibly subject to some regularization $R(w; \lambda)$ that encapsulates our prior expectations for the state and/or control variables. A variational approach is for example utilized in the Estimating the Circulation and Climate of the Ocean framework (ECCO; e.g., Forget et al., 2015; Fukumori et al., 2018): the ocean state variables (e.g., temperature, salinity) are the state variables in ECCO, and the ECCO framework seeks to adjust the control variables (e.g., wind forcing, initial ocean state, parameterization parameters) with the aim to minimize the mismatch to observational data (e.g., sea surface height, hydrographic sections, currents strengths) over time, subject to regularizations (e.g., the wind forcing not deviating too far from climatology) and the constraint that the calculated state satisfies the dynamical equations as implemented in MITgcm (Marshall, Adcroft, et al., 1997; Marshall, Hill, et al., 1997). Within ECCO, the optimization problem is solved through an adjoint method (e.g., Gunzburger, 2003; Kalnay, 2002; Wunsch, 1996).

Ultimately the aim would be to carry out a dynamically constrained inference problem for the spatially varying $\lambda^{-1}(\phi, \theta)$ or other choices of control variables, where the parameterized eddy energy Equation 2 would be coupled to an ocean global general circulation model, and the evolution of the parameterized eddy energy profile is then dynamically and self-consistently interacting with the evolving the ocean state (e.g., $M^2$, $N^2$, $\hat{u}$ and $\kappa$ etc.). Such an inference calculation could leverage the ECCO framework with appropriate modifications of the constraining equations. However, such an endeavor will require a significant investment in human development time and computational resources, exacerbated by the fact that we have no leading order constraints of $\lambda^{-1}(\phi, \theta)$ to serve as a prior for regularizing the inverse problem. Thus, with the dynamically constrained inference as the eventual goal, in this work we present a useful complementary calculation that aims to provide a first leading order estimate for $\lambda^{-1}(\phi, \theta)$. We note that we can in principle diagnose the mean state variables required for the eddy energy Equation 2 from a high resolution global circulation model, and supply the inference calculation with prescribed physical state variables. The result is that we substantially reduce the complexity of the constraining model $F(w; \lambda^{-1}) = 0$ since we no longer need the calculation to be coupled to a dynamical model. A drawback however is that dynamical feedbacks are removed, and consequences are discussed at the end of the present subsection.

With sufficiently long time-averaging and with the assumption that the evolution of the eddy energy has reached a statistically steady state, the constraining model becomes an elliptic problem given by

$$
F\left(\dot{E}; \lambda^{-1}\right) \equiv \eta_k \nabla^2 E - \lambda \dot{E} + \alpha \int \frac{M^2/N^2}{M^2/N^2} E - \nabla_H \cdot \left((\hat{u}^2 - |\kappa|\hat{e}) \dot{E}\right) = 0, \quad (3)
$$

where we have substituted for the GEOMETRIC prescription of $\kappa_{pm}$ given by Equation 1 into the eddy energy source term. The control variable for the present problem will be $\lambda^{-1}(\phi, \theta)$, and the state variable will be $\dot{E}(\phi, \theta)$. With a constraining model such as Equation 3, we seek the optimal $\lambda^{-1}(\phi, \theta)$ that minimizes the cost functional

$$
J \equiv J_1 + J_2 = \frac{1}{A} \left\| \dot{E} - \dot{E}_{data} \right\|_{L_2}^2 + \epsilon \|\nabla_H \lambda\|_{L_2}^2, \quad (4)
$$

where

$$
\|f\|_{L_2}^2 = \int_A f^2 dA \quad (5)
$$
is the $L^2$ norm (so that $J_{1}$ is related to the root-mean-squared mismatch), $A$ is the two-dimensional domain and $dA$ is the area element. The optimization problem seeks for $\hat{E}(\lambda^{-1})$ that is close to the diagnosed $\hat{E}_{\text{data}}$ in the $L^2$ norm (the spatial integral of the square of the mismatches), measured here by the cost functional $J_{1}$. Generally, optimization problems without a regularization term are ill-posed, leading to numerical non-convergence. For this work, we employ a Tikanov-type regularization, introduced via a penalization term on the gradients of the control variable, given here by the cost functional $J_{2}$, with strength measured by some parameter $\epsilon$. Given that we are employing high resolution global circulation model data to infer for $\lambda^{-1}(\phi, \theta)$ (which is a representation of the processes on a coarse resolution grid), the penalization term could be thought as our prior belief that our control variable should have broad spatial structures, or as a coarse-graining or averaging operation for the control variable $\lambda^{-1}(\phi, \theta)$. If $\epsilon$ is too small, the optimization problem is trying to match per grid point, leading to extreme values in $\lambda^{-1}(\phi, \theta)$ and sometimes numerical non-convergence. The range of choices for $\epsilon$ is rather difficult to ascertain a priori, since we essentially want to choose it so that $J_{1}$ is at a similar size to $J_{2}$ for it to have any notable influence, but $J_{1}$ is unknown until the end of the calculation. The choice of $\epsilon$ however can be rationalized post-calculation as an implied length-scale for which the solution $\lambda^{-1}$ varies over (the length-scale $\ell$ associated with $\nabla_{\phi} \lambda \sim \lambda/\ell$). This length-scale is discussed at the start of Section 3 once we have carried out the calculations, and the supporting arguments can be found in Appendix A.

Before we move on, we caveat that, by prescribing the dynamical variables, the constraining Equation 3 is formally linear in the state variable $\hat{E}$. The resulting inference calculation then becomes relatively easy to implement and computationally inexpensive to perform, since this is essentially an optimization calculation subject to an elliptic partial differential equation under the present kinematic/diagnostic approximation. However, in a fully dynamical setting, the isopycnal slopes $M^2/N^2$ for example would be regarded as an implicit function of $k_{\text{grav}}$, which itself would be an explicit function of $\hat{E}$ through the mesoscale eddy parameterization. The growth of $\hat{E}$ will lead to an increase in $k_{\text{grav}}$ which would lead to a flattening of the isopycnals, and the growth rate of the eddy energy would thus normally be self-limiting. Without dynamical feedbacks, the growth rate is expected to be over-estimated. Since we expect the dominant balance in Equation 2 to be between the source and dissipation term, for a fixed target $\hat{E}_{\text{data}}$, the present inference procedure is expected to return a $\lambda^{-1}(\phi, \theta)$ with values that are too small (i.e., a dissipation time-scale that is too short). The results presented here should thus be viewed as a lower bound for $\lambda^{-1}(\phi, \theta)$. Some additional evidence in support of the proposed interpretation of the solutions as a lower bound is given via prognostic calculations with full dynamical feedbacks in Section 4.

### 2.3. Implementation of the Inference Problem

For the proposed inference calculation, the forcing data we require are depth-integrated values of $M^2, N^2, \mathbf{u}^2, |c|$, and the target $\hat{E}_{\text{data}}$. For this work, all the aforementioned variables were diagnosed from the nominally 1/12° horizontal resolution Nucleus for European Modeling of the Ocean (NEMO; Madec, 2008) ORCA0083-N01 hindcast outputs (see Data Availability). Data used here were calculated from the 5-day averaged outputs between and inclusive of the simulation years 2006–2010 (the last 5 years of the ORCA0083-N01 calculation), as depth-integrals on the native NEMO tri-polar grid (Madec & Imbard, 1996), and then time-averaged over the 5 year period. The time-averaging operation removes a substantial portion of the fluctuations below the 1° ($\approx$100 km) horizontal resolution for the variables of interest here (cf. Rai et al., 2021), consistent with our approach here that $\lambda^{-1}(\theta, \phi)$ is to represent a large-scale process that arises from the collective feedback of smaller-scale dynamical processes. We refer the reader to Appendix A for the technical details of the relevant data diagnoses for the inverse method.

We leverage the Firedrake software (Rathgeber et al., 2017), an automatic code generation framework with high level specification in Python that utilizes the finite element formalism (e.g., Durran, 2010, Chapter 6). The procedure is that, given a finite element mesh, we specify the function space on which we seek our solution, taken to be continuous Galerkin with first-order Lagrange polynomials as the basis here. We then implement our constraining model in the weak form (e.g., Evans, 1998, Chapter 1), which in this case involves multiplying by a test function with sufficient differentiability such that integration by parts may be performed, and implementing any natural or imposed boundaries on the problem accordingly. For a scalar test function $\psi$, the weak form $F(\hat{E}; \lambda) = 0$ associated with the constraining model Equation 3 is given by

\[ F(\hat{E}; \lambda) = 0 \]
\[ P(\hat{E}; \lambda) \equiv \int_{\hat{\Omega}} \left[ \eta \nabla_{\hat{\Omega}} \hat{E} \cdot \nabla_{\hat{\Omega}} \psi + \left( \lambda \hat{E} - a \int \frac{M^4/N^2}{M^2/N} \, d\hat{z} \right) \psi \right. \\
\left. - \left( (\hat{u}^2 - |c|e_x) \hat{E} \cdot \nabla_{\hat{\Omega}} \psi \right) \right] \, d\hat{A} = 0. \tag{6} \]

The boundary terms arising from integration by parts are identically zero from the boundary conditions for this problem, namely, \( \nabla_{\hat{\Omega}} \hat{E} \cdot \mathbf{n} = 0 \) and no-normal flow conditions \( (\hat{u}^2 - |c|e_x) \cdot \mathbf{n} = 0 \), where \( \mathbf{n} \) is the unit vector perpendicular to the domain boundary \( d\hat{A} \). As opposed to the more conventional strong formulation, where we seek a solution that satisfies Equation 3 point-wise and the solution \( \hat{E} \) is required to be twice differentiable, the weak form formalism only requires a weak form solution \( \hat{E} \) to satisfy Equation 6 in an integral sense, has weaker assumptions on differentiability, and the resulting problem is readily solved numerically within Firedrake.

Since Firedrake employs the finite element framework, we need a finite element mesh. For this work a choice was made to solve the problem on a two-dimensional spherical mesh embedded into the standard three-dimensional Euclidean space \( \mathbb{R}^3 \), where the relevant periodicities and land boundaries are built into the mesh itself (as opposed to constructing a longitude-latitude grid with North Pole folding, which leads to a point singularity). An unstructured mesh with triangular elements with characteristic length-scale 100 km was created using the Qmesh package (Avdis et al., 2018), with the land shape generated from the NEMO ORCA1 configuration (a global ocean configuration with a nominal horizontal resolution of 1°). We refer the reader to Appendix A for technical details regarding moving data between the mesh and grids.

Once we have data on the mesh, and upon specification of the parameters, we can proceed to build the constraint model, couple the model to an optimizer in Firedrake, and solve the resulting optimization problem. For numerical stability reasons, in order to obtain physical solutions, we modify Equation 6 by adding a term \( \hat{E}_0 \) that maintains a minimum energy value, and replacing \( \hat{E} \) by \( \hat{E}_{\text{data}} \) in the source term so that the source term becomes a diagnostic variable, that is, we solve

\[ P'(\hat{E}; \lambda) \equiv \int_{\hat{\Omega}} \left[ \eta \nabla_{\hat{\Omega}} \hat{E} \cdot \nabla_{\hat{\Omega}} \psi + \left( \lambda (\hat{E} - \hat{E}_0) - a \int \frac{M^4/N^2}{M^2/N} \, d\hat{z} \hat{E}_{\text{data}} \right) \psi \right. \\
\left. - \left( (\hat{u}^2 - |c|e_x) \hat{E} \cdot \nabla_{\hat{\Omega}} \psi \right) \right] \, d\hat{A}. \tag{7} \]

For the first modification, adding a \( \hat{E}_0 > 0 \) term is necessary for the numerical solver to converge to a non-zero solution in the absence of the second modification, otherwise its role is to ensure the solution \( \hat{E} \) has a minimum background value, consistent with how GEOMETRIC is currently implemented into NEMO (Mak et al., 2022). The second modification is perhaps a more severe one, and arises because we are forcing the present inference problem with prescribed data and removing dynamical feedbacks. With full dynamical coupling, the stratification responds to changing \( \hat{E} \) via the GEOMETRIC parameterization, and is a self-limiting process that arrests the growth of \( \hat{E} \). In the present setup, this is not possible since the dynamical variables are prescribed, and the form of the growth term implies exponential growth of \( \hat{E} \) during the iterations (through time-stepping or an iterative solver). Any imbalance in the initialization of the data will be amplified exponentially during the iterations. The imbalance will generically occur in the present setup, since the data diagnosed from ORCA0083-N01 will certainly not be in exact steady state, and we do not have the perfectly matched \( \lambda^{-1}(\phi, \theta) \) and \( \hat{E} \) at the initialization stage since that necessarily implies we have the solution before even solving the problem. Solving Equation 6 as is leads to significant over/undershoots, with very large positive and negative values in \( \hat{E} \); during the development of the present work, a channel model simulation could be integrated to equilibrium, and the described problem was lessened though not completely circumvented. Replacing \( \hat{E} \) by \( \hat{E}_{\text{data}} \) in Equation 7 acts as a numerical stabilizer that limits the amount of growth possible. The consequences of the approximation and how to alleviate it will be discussed in Section 5.

To solve the optimization problem where we minimize the cost functional Equation 4 subject to the model given by Equation 7, we employ the tlm_adjoint library (Maddison et al., 2019) with the Firedrake wrapper, which allows us to build the constrained optimization problem in Firedrake. The optimization problem is solved using a L-BFGS (Limited memory Broyden–Fletcher–Goldfarb–Shanno) algorithm, which is a quasi-Newton method that looks for descents through an estimate of the inverse Hessian matrix (e.g., Byrd et al., 1995). As
demonstration, the actual production code used for implementing the elliptic solve for Equation 7, forming the cost functional Equation 4 and wrapping to the optimizer is given in Figure 1. Aside from the fact this code is incredibly short and required minimal time to write, the optimization and inference calculations become flexible, as changes in the problem statement are automatically propagated to the optimizer routines via automatic code generation capabilities. For example, if we want to modify the cost functional (e.g., changing the measure of the mismatch, changing the existing Tikanhov-type regularization, adding extra penalization terms), we simply modify the relevant lines defining $A_A A_A$. If we want to modify the constraining model (e.g., remove advection, employ quadratic dissipation, spatially varying $\alpha$, increasing the number of control variables, use another choice of eddy energy equation), we re-define the weak form $A_F$ and/or the control variables accordingly, as long as if we have relevant diagnostic inputs to force the resulting equations.

3. Analysis of Inferred Dissipation Time-Scale $\lambda^{-1}(\phi, \theta)$

Inference calculations for $\lambda^{-1}(\phi, \theta)$ with the cost functional Equation 4 subject to Equation 7 were performed using the parameters in Table 1, with values chosen partly to coincide with those used in the prognostic calculations documented in the work of Mak et al. (2022). All optimization calculations were initialized with a spatially uniform initial condition of $A_A A_A^{-1} = 365$ days and $A_E = 0$. Of principal interest here is the control calculation utilizing $\epsilon = 5 \times 10^{15}$.

For the optimization calculations, the default convergence criterion based on projected gradients or the differences in function values were not triggered, though it is clear that the values of the cost functional are converging to some asymptotic value (albeit rather slowly); see Figure 2a for the control calculation as a function of the iteration number. For consistency reasons, all results relating to the optimization calculation reported in this work were taken to be the solution at the 100th iteration. The corresponding run time for each calculation was around 3 mins on a laptop (the one utilized for this work has Intel i7 CPUs and a RAM capacity of 8 GB). Figure 2b shows the dependence of the cost functional $J$ and its components on the value of $\epsilon$. As expected, with increasing $\epsilon$, gradients in the optimized solution, proportional to the numerical value of $J_2$ are penalized at the expense of

```python
def forward(lam):
Edz = Function(P, name = "Edz")
F = ( + Constant(alp) * Etot_zint * N_over_M2 * M4_over_N2 * test * dx # source
- lam * ( Edz - Constant(E0) ) * test * dx # dissipation
- Constant(nu) * dot( grad(Edz), grad(test) ) * dx # diffusion
+ dot(Edz * u_zavg, grad(test)) * dx # advection)

solve(F == 0, Edz, solver_parameters = sp) # magic inherited from Patrick F.
J = Functional(name="J") # cost functional: mismatch + regularization
J1 = (1.0 / domain_area) * inner(Edz - Etot_zint, Edz - Etot_zint) * dx
J2 = Constant(eps) * inner(grad(lam), grad(lam)) * dx
J.assign(J1 + J2)
return Edz, J

lam = Function(P, name = "lam"); lam.assign(Constant(lam_z)) # initial guess
start_manager(); _, J = forward(lam); stop_manager() # start tape
def forward_J(lam):
# define function to optimise: J = forward(lam)
return forward_J(lam)[1]
lam_opt, result = minimize_scipy(forward_, lam,
method="L-BFGS-B",
options={"disp": True, "maxiter": 100})
```

Figure 1. Firedrake driver code for solving the optimization problem that minimizes the cost functional Equation 4 subject to Equation 7. The code defines the weak form, solves it with some specified solver parameters, and defines the cost functional. The tlm_adjacent equation manager is then called, and passes the defined cost functional $J = \mathcal{F}(|\lambda^{-1}|)$ to the optimization algorithm.
having larger mismatches between the target $\tilde{E}_{\text{data}}$ and the state variable $\tilde{E}(\lambda^{-1})$, encoded by $J_1$. Relating to $J_2$ is the length-scale $\epsilon$ for which the associated $\lambda^{-1}(\theta, \phi)$ varies over, and via the estimation procedure detailed in Appendix A $\epsilon$ can be estimated to be on the order of 100 km, proportional to the value of $J_2/\epsilon$ (see Figure 2b). The analysis is also suggestive of the fact that the solutions associated with smaller $\epsilon$ (associated with $J_2/\epsilon \leq 10^{-12}$) might actually be “under-resolved”, in that the implied length-scale of the solution displays more variation that should be allowed by the characteristic scale of the numerical mesh at 100 km, consistent with some of the solution behavior observed (e.g., large grid scale fluctuations, isolated negative values that are deemed unphysical). Solutions with shorter variation length-scales are also inconsistent with our underlying assumption that $\lambda^{-1}(\theta, \phi)$ represents a large-scale feedback arising from the collection of smaller-scale dynamical processes.

Figure 3 shows the target $\tilde{E}_{\text{data}}$ and the signed mismatch $\tilde{E} - \tilde{E}_{\text{data}}$ of the control calculation as a function of longitude and latitude. The target $\tilde{E}_{\text{data}}$ is that diagnosed from the eddy resolving calculation ORCA0083-N01 (cf. Figure 5a of Mak et al., 2022), showing a large eddy energy signature in the Southern Ocean and Western Boundary Currents, and lower eddy activity in the Arctic and the ocean basins. From the signed mismatch, the eddy energy signature associated with the inferred solution is generally larger than the target. If $\epsilon$ is decreased in magnitude, the local signed mismatches shown in Figure 3b decreases in magnitude, with negligible changes in the position; however, that results in larger gradients and more extreme values in $\lambda^{-1}(\phi, \theta)$, with the local signed mismatches still largely skewed toward positive values. Recalling from Section 2 that the stratification is prescribed in this work, thus forbidding dynamical adjustments, the positive skew observed here indicates that the now prescribed eddy energy growth rate is still too large, requiring the $\lambda^{-1}(\phi, \theta)$ to take smaller values to increase the dissipation to balance the growth rate. As argued in Section 2, in a dynamically constrained inference, the eddy energy growth rate will be arrested by the dynamical slumping of isopycnals, so that the resulting values of $\lambda^{-1}(\phi, \theta)$ would not be as small, and reinforces the interpretation that our inferred solution should be seen as a lower bound.

The eddy dissipation time-scale $\lambda^{-1}(\phi, \theta)$ for the control calculation is shown in Figure 4. With regards to spatial distribution, the dissipation time-scale is short within the Southern Ocean (particularly in the Atlantic and Indian sectors), around the Western Boundary Currents, and on the western ocean-land boundaries. The geographical locations of short eddy energy dissipation time-scale are perhaps not surprising, given that these are regions with strong flows and vigorous baroclinic eddy activity (cf. Figure 3a in the depth-integrated total eddy energy signature). The Southern Ocean and the Western Boundary Currents are strongly turbulent regions, with significant mean flows in the presence of rough bathymetry, leading to large eddy energy dissipation via the multitude of dynamical processes given in Section 1. The western boundary intensification of eddy energy dissipation is consistent with the findings of Zhai et al. (2010), resulting from eddy energy

| Table 1 | Parameter Values Employed in the Inference Problem |
|---------|---------------------------------------------------|
| Parameter | Values | Units |
| $\alpha$ | 0.04 | – |
| $\eta_E$ | 500 | m$^2$ s$^{-1}$ |
| $\tilde{E}_{\text{data}}$ | 4.0 | m$^3$ s$^{-2}$ |
| $\lambda^{-1}$ | 365 | Days |
| $\epsilon$ | $a \times 10^b$ | m$^8$ s$^{-2}$ |

Note: The boldfaced values denote the control calculation.

Figure 2. Behavior of the cost functional Equation 4 as a function of (a) iterations for the $\epsilon = 5 \times 10^{15}$ calculation, and (b) as a function of $\epsilon$. The markers denote the parameters with calculations, and the solid circle marker denotes the control calculation.

Figure 3. Target $\tilde{E}_{\text{data}}$ and signed mismatch $\tilde{E} - \tilde{E}_{\text{data}}$ of the control calculation as a function of longitude and latitude.

Figure 4. Eddy dissipation time-scale $\lambda^{-1}(\phi, \theta)$ for the control calculation.
convergence at the western boundaries via propagation of eddies at the long Rossby wave phase speed (Chelton et al., 2011; Klocker & Marshall, 2014), with energy being transferred out of the geostrophic scales by processes such as loss of balance and non-propagating form drag (Yang, Zhai, et al., 2021). On the other hand, the dissipation time-scale is long generally in the equatorial regions, but more wide spread in the Eastern Pacific. The long dissipation time-scales in the Eastern parts of the basins are consistent with the aforementioned westward flux of eddy energy, leading to lower eddy energy signatures, and requiring a weaker eddy energy dissipation to balance the diagnosed growth term. The tropical regions have particularly long dissipation time-scales, consistent with the large westward advection of eddy energy and known weaker baroclinic eddy energy generation in the region. The region of long dissipation time-scale is most significant in the East Pacific, perhaps simply because it is furthest away from a Western Boundary Current region.

Figures 5a–5d show the representative features in the inferred $\lambda^{-1}(\phi, \theta)$ for varying $\epsilon$. Calculations at different values of $\epsilon$ seem to possess qualitatively similar spatial patterns as the control calculation, up to a shift by a constant. The reasons for the spatial patterns are as rationalized in the previous paragraph. With regards to the relatively uniform shift in magnitude, for lower values of $\epsilon$, the solution is allowed to take more extreme values given a weaker penalization on the gradient of the solution, and the resulting values of $\lambda^{-1}(\phi, \theta)$ are generally smaller, while the converse is true. Figures 5e and 5f show the result of a calculation where we take $\epsilon = 5 \times 10^{15}$ but exclude the advective contribution in the inference calculation (by commenting out the last line defining $F$ in Figure 1 when performing the optimization calculation). The resulting spatial and zonal distribution is largely similar to the solutions with advection, but with reduced western intensification, particularly in the Western Pacific, Indian subtropics and Eastern Australia, and with shorter time-scales in the tropics, both attributed to the lack of eddy energy advection westward at the long Rossby phase speed. The inclusion of advection leads to local differences in the distribution of the eddy energy and thus $\lambda^{-1}(\phi, \theta)$, but with only minor difference to the overall magnitudes in its zonal average.

The parameters chosen for the calculations given in Table 1 were motivated by the choices made in prognostic calculations of Mak et al. (2022) with NEMO ORCA1 employing GEOMETRIC, though we are free to explore the parameter space for the inference calculation. It is found that if we increase the horizontal eddy energy diffusion coefficient $\eta_e$ for fixed $\epsilon$, the model given by Equation 7 outputs an $\hat{E}$ that is weaker and more diffused, and the optimizer returns a $\lambda^{-1}(\phi, \theta)$ with similar spatial distributions but marginally larger values. Increasing $\hat{E}_0$ by no more than an order of magnitude for fixed $\epsilon$ leads to very mild effects in the basin regions where $\hat{E}$ is small. Beyond the eddy energy equation parameters, if $\epsilon \lesssim 10^{13}$, the optimization calculation starts to have large variations in $\lambda^{-1}(\phi, \theta)$ and can even return negative values of $\hat{E}$. Such results are somewhat consistent with the associated value of $J_z/\epsilon$ and the solution being “under-resolved” as noted earlier (see also Appendix A), were deemed unphysical and/or inconsistent with our underlying assumptions, and have been omitted from this work. For changing $\alpha$ at fixed $\epsilon$, the results of varying $\alpha = 0.04$ are largely similar to Figures 5a–5d in that the spatial patterns are largely similar to the control up to a shift by some constant (around $\pm 15$ days in the time-scale for every $\pm 0.01$ variation respectively; diagram omitted). The observation is physically consistent, since increasing $\alpha$ leads to a larger growth term, to be balanced by a large dissipation, leading to an overall decrease in the dissipation time-scale. The calculations with smaller $\alpha$ also lead to a smaller overall mismatch in the energy given by $J_z$, and can be attributed to an improvement in the parameterized eddy energy signature in the basin regions.
This seems to suggest that $\alpha$ should take smaller values in the basins, consistent with the physical expectation that baroclinic instability is most vigorous in the Southern Ocean but decreases somewhat in prominence in the basin regions, which may be verified with alternative diagnostics methods to the one detailed here (e.g., Poulsen et al., 2019).

4. Prognostic Model Calculations

The presence of feedback loops means there is no guarantee that a diagnostic result such as the one here will lead to improvements in a prognostic calculation. The inferred $\lambda^{-1}(\phi, \theta)$ as a prescribed input is utilized in an ocean global circulation model in prognostic mode, to assess the consequences on the model output and behavior compared with the case where a prescribed spatially constant $\lambda^{-1}$ field is used (cf. Mak et al., 2022). The principal hypothesis is that the use of $\lambda^{-1}(\phi, \theta)$ will improve on the resulting parameterized total eddy energy signature. A secondary aim is a demonstration that the inferred $\lambda^{-1}(\phi, \theta)$ is a physically plausible lower bound for the mesoscale eddy energy dissipation time-scale. The prognostic model and set up we employ are exactly the same as those reported in the work of Mak et al. (2022) and utilize NEMO ORCA1 (Madec, 2008) and parameters given in Table 1; we refer the reader to Appendix A for model details. Prognostic calculations with $\lambda^{-1}(\phi, \theta)$ for $\epsilon = 5 \times 10^{15}$ and $\epsilon = 5 \times 10^{16}$ (cf. Figures 4 and 5b, denoted calculation A and B) are reported here; the calculation with $\lambda^{-1}(\phi, \theta)$ for $\epsilon = 1 \times 10^{14}$ (cf. Figure 5a) was found to lead to numerically unstable evolution in the parameterized eddy energy equation, and has been omitted. The model is spun up from WOA13 climatology (Locarnini et al., 2013; Zweng et al., 2013) for 300 years.

Denoting $\langle \cdot \rangle$ to be the domain-integrated quantity, we first show in Figure 6 the time-series of the calculation A for the diagnosed domain-integrated eddy energy $\langle \rho E \rangle$, the total and thermal wind component of the Antarctic Circumpolar Current transport $T_{ACC}$ (where the total is the transport over the whole depth of the Drake passage, and the thermal wind component is calculated as the residual of the total and the depth-integrated bottom flow), and the total ocean heat content $\langle \rho c_p \Theta \rangle$, where $\rho$ is the locally referenced density, $c_p$ is the seawater heat capacity, and $\Theta$ is the conservative temperature. The main purpose here is to demonstrate the multiple adjustment time-scales inherent in the different diagnosed quantities. The parameterized eddy energy as represented through GEOMETRIC here adjusts on a fast time-scale (of around 5–10 years), while the $T_{ACC}$ might be argued to have reached a quasi-equilibrium over centennial time-scales but, like the ocean heat content, there is a much longer adjustment on millennium time-scales associated with the deep/abyssal stratification (e.g., Mak et al., 2022).

With these observations in mind, for simplicity and computation resource reasons, we will make a direct comparison of the present calculations employing $\lambda^{-1}(\phi, \theta)$ where diagnostics were averaged over the model years 291–300, with the previously reported results of Mak et al. (2022) employing a spatially constant $\lambda^{-1} = 100$ days that already exists, but where the data was diagnosed from averages over the model years 3001–3100 (denoted calculation C). For comparing the eddy energy signature this should not be an issue, given both calculations have...
For comparing circulation metrics such as $T_{\text{ACC}}$ and the Atlantic Meridional Overturning Circulation $T_{\text{AMOC}}$, the diagnosed values can be compared as long as we bear in mind that there will likely be 10%–20% upward drift of $T_{\text{ACC}}$ and $T_{\text{AMOC}}$ in the present results until equilibration occurs over the 500–1,000 years time-scale. We refrain from comparing the ocean heat content.

To quantify whether our choice of $\lambda^{-1}(\phi, \theta)$ is able to improve the eddy energy signature, we take $\tilde{E}_{\text{data}}$ as diagnosed from ORCA0083-N01 (interpolated onto the ORCA1 grid) as the reference, and compute the area-averaged $L^2$ mismatch of the corresponding $\tilde{E}_{\text{ORCA1}}$ from the prognostic calculations, given by

$$J_3 = \frac{1}{A} \| \tilde{E}_{\text{data}} - \tilde{E}_{\text{ORCA1}} \|^2 = \frac{1}{A} \int_A (\tilde{E}_{\text{data}} - \tilde{E}_{\text{ORCA1}})^2 \, dA. \tag{8}$$

We additionally compute the domain-integrated total eddy energy value $\langle \rho E \rangle$, the total and thermal wind component of $T_{\text{ACC}}$ and the Atlantic Meridional Overturning Circulation $T_{\text{AMOC}}$ (diagnosed as the northward transport over the top 1,000 m at 26°N at the Western side of the Atlantic). The computed diagnostics are given in Table 2.
From the computed values of $J_3$, it may be seen that calculation A has a lower $L_2$ mismatch compared to calculations B and C, although arguably the improvements are somewhat modest. Figure 7 shows the spatial distribution of the signed mismatch, and the lower values of $J_3$ from calculation A compared to calculation C primarily come from improvements within the Southern Ocean, where the prognostic calculations using $\lambda^{-1}(\theta, \phi)$ has reduced coverage of positive biases in the depth-integrated eddy energy. The reduction in the average values of the eddy energy $\langle \rho E \rangle$ can also be seen to be arising from the reduced values of the eddy energy in the Southern Ocean. The negative biases however are generally large, as can be seen in Figure 7. Additionally, note that the domain-integrated eddy energy value of the reference ORCA0083-N01 calculation is $\langle \rho E_{\text{ori}} \rangle = 9.52$ EJ, and the corresponding values from the prognostic calculations given in Table 2 are a few factors lower. While the Southern Ocean biases have been reduced somewhat, over the rest of the globe the negative bias is prevalent, which is somewhat consistent with an eddy energy dissipation time-scale that is too short.

Beyond the response in the eddy energy signature, the calculations with $\lambda^{-1}(\theta, \phi)$ also results in a plausible $T_{\text{ACC}}$ as well as $T_{\text{AMOC}}$, although the latter is a little on the low side; see for example, Table 2 of Farneti et al. (2015) and Figure 1 of Danabasoglu et al. (2014) for a summary of model and observational estimates for $T_{\text{ACC}}$ and $T_{\text{AMOC}}$ respectively. It should be noted that calculation A has the highest $T_{\text{ACC}}$ (both total and thermal wind component) and $T_{\text{AMOC}}$. This is consistent with expectations, since the associated dissipation time-scale in the Southern Ocean is the shortest, leading to reduced flattening of isopycnals by mesoscale eddies, steeper stratification in the Southern Ocean (Mak et al., 2018; D. P. Marshall et al., 2017), and in turn a deepening of the global pycnocline and increased $T_{\text{AMOC}}$ via isopycnal connectivity (D. P. Marshall & Johnson, 2017), something that has been demonstrated in numerical models (Mak et al., 2018, 2022). The results here also provide some additional evidence for the interpretation that the $\lambda^{-1}(\theta, \phi)$ given here should be viewed as a lower bound, since $T_{\text{ACC}}$ is already on the

\begin{table}[h]
\centering
\caption{Computed Diagnostics of the Prognostic Calculations A, B, C Respectively}
\begin{tabular}{|c|c|c|c|c|}
\hline
& $J_3$ & $\langle \rho E \rangle$ & $T_{\text{ACC}}$ & $T_{\text{AMOC}}$ \\
& (m$^3$s$^{-2}$) & (EJ = 10$^{18}$ J) & (Sv) & (Sv) \\
\hline
A: $\lambda^{-1}(\theta, \phi)$ & 1,560.89 & 1.80 & 159.17 (Total) & 11.13 \\
& ($\epsilon = 5 \times 10^{15}$) & & 145.41 (Thermal) & \\
\hline
B: $\lambda^{-1}(\theta, \phi)$ & 1,954.70 & 2.98 & 143.27 (Total) & 9.05 \\
& ($\epsilon = 5 \times 10^{16}$) & & 99.29 (Thermal) & \\
\hline
C: $\lambda^{-1} = 100$ days & 1,922.22 & 3.33 & 138.10 (Total) & 10.38 \\
& (Mak et al., 2022) & & 121.23 (Thermal) & \\
\hline
\end{tabular}
\end{table}
rather high side. Some further analysis of the statistical distribution of the associated dissipation rates to calculation A (cf. Pearson & Fox-Kemper, 2018) are given in Appendix A.

5. Conclusions and Outlooks

In this work we provide a leading order constraint for the spatial distribution of eddy energy flux out of the geostrophic mesoscales, interpreted here as a geostrophic mesoscale eddy energy dissipation time-scale $\lambda^{-1}(\phi, \theta)$. The problem is viewed as one of parameter inference for $\lambda^{-1}(\phi, \theta)$, here inferred from high resolution numerical model data and constrained by a parameterized eddy energy equation, as a precursor for the more complete dynamically constrained inference problem utilizing ocean observational data, to be discussed below. A simple and computationally inexpensive optimization problem was performed, seeking an optimal $\lambda^{-1}(\phi, \theta)$ that minimizes the mismatch between the depth-integrated total eddy energy from the parameterized eddy energy equation $\hat{E}(\lambda^{-1})$ and the depth-integrated total eddy energy diagnosed from a high resolution numerical model $\hat{E}_{data}$. The present implementation utilizes the Firedrake software, leveraging the inbuilt solvers as well as the automatic code generation capabilities to solve and explore the associated optimization problem and its sensitivities to parameter choices. The inferred $\lambda^{-1}(\phi, \theta)$ has the smallest values within the Southern Ocean, Western Boundary Currents, and is western boundary intensified, regions where baroclinic turbulence is particularly dominant, and coinciding with where we expect the greatest energy flux out of the geostrophic mesoscales from dynamical considerations (e.g., Melet et al., 2015; Nikurashin & Ferrari, 2011; Rai et al., 2021; Rocha et al., 2018; Zhai et al., 2010). We caveat that while the inferred spatial distribution of $\lambda^{-1}(\phi, \theta)$ may be consistent with expectations, the resulting magnitudes should be viewed as a physically plausible lower bound.

Prognostic calculations utilizing the inferred $\lambda^{-1}(\phi, \theta)$ in the coarse resolution global configuration ocean model NEMO ORCA1 were performed. The coarse resolution calculations result in an improved mismatch in the parameterized depth-integrated eddy energy signature in the globally integrated $L^2$ sense compared to a previous work that employs a spatially constant $\lambda^{-1} = 100$ days also in NEMO ORCA1 (Mak et al., 2022), where the reference eddy energy signature is diagnosed from the high resolution calculation NEMO ORCA0083-N01. The use of $\lambda^{-1}(\phi, \theta)$ reduces positive biases in total eddy energy signature in the Southern Ocean, though negative biases remain prevalent throughout the globe, and the coarse resolution calculation at $1^\circ$ horizontal resolution possesses an average eddy energy value that is too low. The total and thermal wind component of the Antarctic Circumpolar Current from utilizing $\lambda^{-1}(\phi, \theta)$ inferred here is on the high side (total transport at around 160 Sv, cf. Table 2 of Farneti et al., 2015), which is in line with the dynamical arguments provided by D. P. Marshall et al. (2017) that a higher eddy energy dissipation rate (so a shorter eddy energy dissipation time-scale) leads to increased Antarctic Circumpolar Current transport and steepening of isopycnals. Together, there is evidence in support of the inferred $\lambda^{-1}(\phi, \theta)$ leading to an improved eddy energy signature in prognostic calculations, as well as being a physically plausible lower bound for the eddy energy dissipation time-scale.

Beyond providing a leading order constraint and estimate for eddy energy dissipation time-scale $\lambda^{-1}(\phi, \theta)$ as progress toward understanding ocean energetic pathways, this work also highlights and demonstrates some perhaps lesser known but really quite powerful machinery relating to inverse methods and calculations, such as automatic code generation software such as Firedrake (Rathgeber et al., 2017), automatic adjoint generation libraries (e.g., Farrell et al., 2013; Maddison et al., 2019) and mesh generating software (Avdis et al., 2018) that...
is expected to have applications in various branches of earth system modeling. Such tools have been applied to problems in global tidal modeling with uncertain bathymetry (David et al., 2019), uncertainty quantification associated with ice sheets (Kolzioz et al., 2021), Tsunami source inversion (Wallwork, 2021), and sediment transport modeling (Clare et al., 2022), to name a few examples.

In order to obtain the present lower bound for $\lambda^{-1}(\phi, \theta)$, some approximations were made in order to make the problem tractable. The approximations we made were (a) prescribing the stratification and thus removing dynamical feedbacks, and (b) the choice and assumption of data, cost functional, control variable and/or constraining model. As argued in the methodology section, the choice of prescribing the stratification, while reducing the complexity of the inference problem (e.g., removing coupling to an ocean global circulation model), has the consequence that the dynamical feedbacks are removed. As a result of the reduction in the complexity, we are somewhat restricted to the choice of target data, in this case to the total eddy energy signature as the state variable. Given the reduced amount of target data, it means we are somewhat limited to the choice and number of control variables we can take unless we impose severe and somewhat ad hoc regularizations.

Ultimately, those were choices we made in light of our primary objective, to provide a leading order reference constraint for which further investigations can be based on. A “simple” fix in principle is to dispense with prescribing the stratification parameters, and carry out a dynamically constrained inference calculation, which was part of the motivation behind the present work. The related machinery is already in place in the form of the ECCO framework within MITgcm (e.g., Forget et al., 2015; Fukumori et al., 2018), set up utilizing the inbuilt algorithmic differentiation capabilities (e.g., Giering & Kaminski, 1998) to deriving adjoints for performing state estimation using the variational or smoothing approach (e.g., Kalnay, 2002). There is already a form of GEOMETRIC in MITgcm (from Mak et al., 2018), and the principal modification that would be required for the intended dynamical inference is to couple the GEOMETRIC parameterization for $\kappa_{\text{eddy}}$ accordingly to the existing ECCO framework, and including $\lambda^{-1}(\phi, \theta)$ and possibly $\alpha$ as additional control variables. By increasing the complexity of the problem, we are now also able to utilize ocean observational data as the target instead of just relying on the eddy energy signature. Given the large amounts of degrees of freedom associated with the proposed control variables, the adjoint method, which has a linear scaling in the complexity and requires a smaller amount of model calculations (one forward and one backward for each iteration; Kalnay, 2002; Gunzburger, 2003), is a particularly well-suited computational method (cf. Green's function methods, while linear in complexity, requires ocean general circulation model runs scaling with the number of degrees of freedom, and is better when the number of control variables are low; e.g., Nguyen et al., 2011). While “simple” in principle, in practice the above proposal is still a formidable technical challenge and computationally expensive, and given there have been no strong constraints on $\lambda^{-1}(\phi, \theta)$ thus far, the present result serves as an important leading order prior for the proposed dynamical inference calculation. The proposed work is currently underway and will be reported in a future publication.

A simpler procedure we have also considered is to stick with the strategy in this work, but diagnose the time-varying stratification and eddy energy data from a high resolution model, and consider an adjoint-based calculation where the cost functional is taken to be the mismatch between the parameterized eddy energy and target eddy energy over time. The inference will still not be dynamically constrained in the sense that the dynamical parameters for the proposed calculation will not be functions of the state variable, that is, changes in $\lambda^{-1}(\phi, \theta)$ and thus $\dot{E}$ will have no bearing on the evolution of the stratification, given the latter is prescribed. While the calculation is certainly possible with the existing machinery since there are adjoint libraries that can be coupled to Firedrake (e.g., Farrell et al., 2013; Maddison et al., 2019), we are of the opinion that there is very little to be gained from that approach, given the theoretical and probably practical limitation is still that of prescribed stratification.

To close, we note that the variational methods considered here can be interpreted in the Bayesian formalism as a maximum likelihood approach (e.g., Bui-Thanh et al., 2013; Kalnay, 2002), which by itself does not provide estimates of the uncertainties, and can have issues with over-tuning in cases where multiple different parameters are tuned at the same time (Williamson et al., 2017). Being able to quantify uncertainties associated with inference calculations will also be a research focus in the planned investigations. As an aside, sample prognostic calculations varying the Southern Ocean wind stress (cf. Mak et al., 2022) employing the spatially varying eddy energy dissipation time-scale appear to reproduce the eddy saturation phenomenon in the Antarctic Circumpolar Current transport in the thermal wind component (e.g., Farneti et al., 2015; Munday et al., 2013), and shows hints of eddy compensation (e.g., Bishop et al., 2016; Gent & Danabasoglu, 2011; Viebahn & Eden, 2012), although limitation...
Appendix A: Technical Details

A1. Forcing and Target Data Diagnoses

Complementary details to Section 2. The forcing data we require are depth-integrated values of $M^2$, $N^2$, $\overline{u}^2$, $\zeta_l$, and the target $\tilde{E}_{\text{data}}$, and all the variables were diagnosed from the nominally 1/12° horizontal resolution Nucleus for European Modeling of the Ocean (NEMO) (Madec, 2008) ORCA0083-N01 hindcast outputs (see Data Availability).

The depth-integrated total eddy energy $\tilde{E}_{\text{data}}$ is computed as the sum of the depth-integrated eddy kinetic and potential energy. The depth-integrated (specific) eddy kinetic energy (with units of m$^2$ s$^{-2}$) is defined in the usual fashion as

$$EKE = \frac{1}{2} \int_{-H}^{0} (\overline{u} \cdot \overline{u} - \overline{u} \cdot \overline{u}) dz,$$  \hspace{1cm} (A1)

where $\overline{\cdot}$ is a time average. To obtain the analogous depth-integrated eddy potential energy, we take the 5-day averaged temperature and salinity outputs, convert into neutral density $\gamma^a$ co-ordinates via the McDougall and Jackett (2005) expression, re-bin the variables into an interval $[\gamma^a_1 = 1020\, \text{kgm}^{-3}, \gamma^a_2 = 1029\, \text{kgm}^{-3}]$ (with smaller bin widths toward $\gamma^a_2$), calculate the depth associated with the neutral density $\gamma^a$ every 5 days over the 5 year period, and compute

$$EPE = \frac{1}{2} \int_{\gamma^a_1}^{\gamma^a_2} \left( \overline{z^2} - \overline{z}^2 \right) d\gamma^a.$$  \hspace{1cm} (A2)

For the stratification parameters $M^2N$ and $M^4N^2$ as ratios, first we compute $N^2$ and $N$ as a three-dimensional field using the CDFTOOLS package from 5-day averaged temperature and salinity data (using cdfbn2; see Data Availability). A new program (cdfsn2) was created to mirror the NEMO computation of stratification gradient parameters, with code largely taken from the NEMO ldfs1p subroutine and dependencies. The new cdfsn2 routine computes the isopycnal slopes $M^2N^2$ (wslp in NEMO), appropriately modified by slope limiters, partial step corrections, and tapering as the lateral, mixed layer and surface boundaries are approached, and passed through a Shapiro filter in the horizontal. The intermediary $N$ and $N^2$ are then multiplied by the slope variables to obtain $M^2N^2$ and $M^4N^2$ as three-dimensional fields, which are then depth-integrated and time-averaged. The cdfsn2 routine is able to reproduce the NEMO outputs directly up to very minor discrepancies (code tested on a simple re-entrant channel model and outputting the wslp variable and its modifications directly).

The depth-averaged mean flow $\overline{u}^2$ is computed in the usual way from the available data. The long Rossby phase speed $\zeta_l$ is computed as (e.g., Gill, 1982, Equation 12.3.13)

$$|c| = \frac{c_1^2 \cos \theta}{2 \Omega R_a \sin \theta} \cdot \zeta_l = \frac{1}{\pi} \int_{-H}^{0} |N| dz,$$  \hspace{1cm} (A3)

where $\theta$ is the latitude, $\Omega$ is the Earth’s angular frequency, $R_a$ is the Earth’s radius, and $c_1$ is an approximation of the first baroclinic phase speed (e.g., Nurser & Bacon, 2014). This phase speed is computed in this case from the time-averaged $|N| dz$ field directly. One thing to note is that, because of the NEMO tri-polar grid, the $(u, v)$ components of the velocity only correspond exactly to the zonal and meridional velocities when south of around 20°N (Madec & Imbard, 1996). While the anisotropy is relatively small, there are minor inconsistencies when...
interpolating data to and from the tri-polar grid. The effect of this error is not expected to be significant, since the advective contributions are expected to be rather minimal (cf. Figures 5e and 5f).

All the high resolution processed data on the tri-polar grid were additionally passed through a diffusion-based filter (Grooms et al., 2021) to smooth out variations smaller than 12 grid points, that is, filtering to obtain the fields that are coherent over a nominally 1° horizontal resolution. This filtering step does not appear to be strictly necessary for the conclusions presented here, since the time-averaging operation already removes a substantial portion of the fluctuations below the 1° horizontal resolution for the variables of interest here (cf. Rai et al., 2021). Sample calculations without the filtering step led to no noticeable changes in the results presented.

To move the diagnosed data from the tri-polar NEMO grid onto the mesh, a simple change of co-ordinates from \((\phi, \theta)\) to \((x, y, z)\) and linear interpolation (through the Python scipy package) was performed, with extrapolations where necessary (e.g., near the geographical locations associated with the edges of the tri-polar grid). Note that while interpolating scalars is straightforward, interpolating vectors require preserving both the direction and magnitude, which can be achieved via a multiplication by a rotation matrix. To move data off the computational mesh onto the a regular longitude-latitude grid for further analyses or the NEMO ORCA1 tri-polar grid for utilizing \(\lambda^{-1}(\phi, \theta)\) in prognostic calculations is more complicated. A technical complication arises here that the spherical mesh is an immersed manifold (a sub-manifold of the 2-sphere \(S^2\) embedded in \(\mathbb{R}^3\), which has zero measure in the embedding space (\(S^2\) has no “volume”). The standard procedure of probing the function values on the immersed manifold will fail as it is mathematically ill-defined: the probability of a given co-ordinate point being on the manifold is related to the measure, and an arbitrary co-ordinate point we wish to query the function value at will almost surely not be on the zero-measure manifold.

One way round this technical issue is to probe the finite element output using the \(\texttt{vtk}\) package directly, locating the cell closest to the co-ordinate of the point being queried, project that point onto the cell, which is now regarded as a sub-manifold of \(\mathbb{R}^2\) (the measure of interest is now “area”), and the function can then be queried accordingly. For the analyses presented in Section 3, all relevant mesh data was interpolated onto a regular longitude-latitude grid with 1/4° spacing for ease of visualization. For the prognostic calculations in Section 4, the inferred \(\lambda^{-1}(\phi, \theta)\) was interpolated directly from the spherical mesh onto the NEMO ORCA1 tri-polar grid.

**A2. Post-Calculation Analysis for Choice of Regularization Parameter \(\epsilon\)**

Complementary details to Section 3. One way to measure the degree of fluctuations of a function is via the Sobolev semi-norm \(H^1_\lambda\) (e.g., Evans, 1998)

\[
\|\lambda\|_{H^1_\lambda}^2 = \|\nabla \lambda\|_{L^2_A}^2 = \int_A |\nabla \lambda|^2 \, dA, \tag{A4}
\]

which in our case is precisely the definition of \(J_2\) in Equation 4 up to a factor of \(\epsilon\). Note that

\[
\frac{J_2}{\epsilon} \sim \|\nabla \lambda\|_{L^2_A}^2 \sim A \frac{\lambda^2}{\epsilon^2}, \tag{A5}
\]

assuming \(\lambda\) is some representative value of \(\lambda\) over the ocean area \(A\), and \(\epsilon\) is a length-scale for which the function \(\lambda\) varies over. With 100 days\(^{-1} \approx 10^{-7}\) s\(^{-1}\),

\[
\epsilon^2 \sim \frac{A \lambda^2}{J_2/\epsilon} \sim \frac{10^{11} (10^{-7})^2}{J_2/\epsilon} = \frac{10^{-3}}{J_2/\epsilon}. \tag{A6}
\]

For \(\epsilon \gtrsim 10^{15}\), we obtain \(J_2/\epsilon \sim 10^{13}\) from Figure 2b, which implies a length-scale of \(\epsilon \sim (10^{10})^{1/2} = 10^5 m = 100 km\). The analysis somewhat indicates that the \(\lambda^{-1}(\theta, \phi)\) for the ones with smaller \(\epsilon\) might be “under-resolved,” in that the implied length-scale is actually below the length-scale allowed by the numerical mesh, but also that it is inconsistent with our working assumption that the processes represented via \(\lambda^{-1}\) is supposed to a large-scale manifestation of the collective effects of smaller-scale dynamical processes.
A3. Prognostic Model Details, and Further Analysis

Complementary details to Section 4. An ocean only global configuration model employing NEMO (Madec, 2008, v3.7dev r8666) using a tri-polar grid ORCA1 grid (Madec & Imbard, 1996) with nominally 1° horizontal resolution (ORCA1) with the LIM3 ice model (Rousset et al., 2015) is utilized. With 46 uneven vertical levels, placing more resolution near the ocean surface, the model employs the TEOS-10 equation of state (Roquet et al., 2015). Forcing by the atmosphere is modeled by the NCAR bulk formulas with normal year forcing (Large & Yeager, 2009). Sea surface salinity but not temperature restoration is included to reduce model drift. The GEOMETRIC implementation in NEMO for this model is described in the work of Mak et al. (2022), and is essentially given by Equation 2 here, with the addition of the $E_0$ term (cf. Equation 7). The model is spun up from WOA13 climatology (Locarnini et al., 2013; Zweng et al., 2013) for 300 years.

It may be of interest to consider the statistical distribution in addition to the spatial distribution of the eddy energy dissipation arising from the prognostic calculation. We note that the work of Pearson and Fox-Kemper (2018) reports that the dissipation of eddy kinetic energy in a high resolution global ocean model follows a log-normal distribution on horizontal slices, with slightly varying statistics in depth, and makes the point that mesoscale parameterizations should be such that the energy flux statistics are consistent with the reported diagnostics. We show in Figure A1 the probability distribution function (PDF) of the depth-averaged eddy energy dissipation as parameterized, given by

$$\tilde{\Lambda} = \frac{1}{H} \left( \frac{1}{\lambda} \int_0^H (E - E_0) dz \right),$$

which has units of $m^2 s^{-3}$ like the turbulent energy flux in the work of Pearson and Fox-Kemper (2018), for the prognostic calculation using $\lambda^{-1}(\phi, \theta)$ with $\epsilon = 5 \times 10^{15}$ as well as for the case with uniform $\lambda^{-1} = 100$ days.

A direct comparison of the results here with the work of Pearson and Fox-Kemper (2018) is slightly problematic in the sense that the quantities to be compared are different (depth-integrated dissipation of eddy kinetic and potential energy here as compared to horizontal slices of eddy kinetic energy dissipation), but nevertheless the values of the energy dissipation obtained here are not unreasonable compared to Figure 1 of Pearson and Fox-Kemper (2018), although our dissipation is on the larger side. While there are hints of log-Gaussianity in our PDFs, there is also notable skewness to the larger values, as well and kurtosis in the data reflected by the heavy tails, as can be seen in Figure A1b with the distribution of data points relative to the best fit log-Gaussian PDF. The use of $\lambda^{-1}(\phi, \theta)$ with $\epsilon = 5 \times 10^{15}$ additionally leads to a reduction in the variance and larger dissipation values compared to the case with spatially uniform $\lambda^{-1}$, as seen by the relatively more narrow best-fit PDF, and a noticeable dip in the values of the data points at the higher end of the distribution. The observed relative

![Figure A1.](image-url) The probability distribution function (PDF) of the depth-averaged total eddy energy dissipation from the calculation utilizing $\lambda^{-1}(\phi, \theta)$ with $\epsilon = 5 \times 10^{15}$ (red) and $\lambda^{-1} = 100$ days everywhere (orange), for diagnosed data (markers) and the best-fit Gaussian PDF (lines). The PDF shows one particular snapshot from 5 day averaged data within the averaging period of the respective prognostic calculation, but is representative of other sampled time snapshots as well as similar to the analogous PDF from time-mean data. (a) The PDF of the $log_{10}(\tilde{\Lambda})$ given in Equation A7. (b) Same as (a) but on a base 10 logarithmic scale.
differences may be attributed to a significant decrease in the positive biases of eddy energy signatures in the Southern Ocean where dissipation is strong. Our interpretation that $\lambda^{-1}(\phi, \theta)$ is expected to be a lower bound in the time-scale is somewhat consistent with the observation that our dissipation rate is possibly on the larger side compared to the values reported in Pearson and Fox-Kemper (2018).

Data Availability Statement

For the pre-processing, NEMO ORCA0083-N01 data was obtained from http://gws-access.jasmin.ac.uk/public/nemo/, provided by UK National Oceanography Center through the JASMIN service. The base version of CDFTOOLS was taken from https://github.com/meom-group/CDFTOOLS. For the optimization calculations, this work uses a natively installed version of Firedrake, with a mesh generated via the Qmesh package (https://www.qmesh.org/) via a Docker image, with the Firedrake wrapper of tlm_adjoint (https://github.com/jrmadison/tlm_adjoint). The post-processing analysis uses standard Python packages. Modifications of CDFTOOLS, the relevant Python scripts, the processed data from this work (in both the native HDF5 finite element format as well as the various gridded versions), and documentation of software, versions and its dependencies are available on http://dx.doi.org/10.5281/zenodo.6559892.

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