Cyclic Changes of the Sun’s Seismic Radius

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Received 2018 January 6; revised 2018 May 7; accepted 2018 May 23; published 2018 July 9

Abstract

The questions asking whether the Sun shrinks with the solar activity and what causes this have been a subject of debate. Helioseismology provides a means to measure with high precision the radial displacement of subsurface layers, the so-called “seismic radius,” through the analysis of oscillation frequencies of surface gravity (f) modes. Here, we present results of a new analysis of 21 years of helioseismology data from two space missions, the Solar and Heliospheric Observatory and the Solar Dynamics Observatory, which allow us to resolve previous uncertainties and compare variations of the seismic radius in two solar cycles. After removing the f-mode frequency changes associated with the surface activity, we find that the mean seismic radius is reduced by 1–2 km during the solar maxima and that most significant variations of the solar radius occur beneath the visible surface of the Sun at a depth of about 5 ± 2 Mm, where the radius is reduced by 5–8 km. These variations can be interpreted as changes in the solar subsurface structure caused by the predominately vertical ~10 kG magnetic field.

Key words: Sun: activity – Sun: fundamental parameters – Sun: helioseismology – Sun: interior – Sun: magnetic fields – Sun: oscillations

1. Introduction

Accurate measurement of the solar radius and its variations is one of the oldest astronomical problems. It is important for two primary reasons. First, the radius serves as an astronomical standard. Second, the solar radius and, more generally, shape variations reflect still poorly understood physical processes associated with the cyclic magnetic activity, which occur in the Sun’s interior and affect the surface. The first accurate measurements of the size of the Sun, performed in the 18th and 19th centuries, indicated that “the systematically larger diameters correspond to the time when the number of spots and protuberances is lower” (Secchi 1872; Auwers 1873). Secchi (1872) conjectured that “the effect of the active forces in the Sun, which are made known to us by the variable formations on its surface, may produce changes of volume in the masses of luminous gas, perhaps perceptible in accurate observations of the Sun’s diameter.” Modern theories motivated by measurements of irradiance variations and shifts in solar oscillation frequencies during the solar cycles attempt to explain these measurements in terms of changes in the structure of the Sun caused by large-scale and turbulent magnetic fields (Goldreich et al. 1991; Dziembowski & Goode 2004; Mullan et al. 2007). The long standing question of whether the solar radius is constant or not is still debated. There have been numerous studies using spacecraft and ground-based instruments that led in the past to conflicting results (for a review, see Rozelot et al. 2016). The solar radius is determined by the position of the inflection point of the limb brightness. Thus, measuring the diameter of the Sun with high accuracy is a challenge at the cutting edge of modern techniques (Bush et al. 2010). Most recent measurements of the solar limb from the PICARD satellite put an upper limit of 14.5 km (20 parts in a million) on the solar radius changes during the rising phase of the current sunspot cycle (Mfeftah et al. 2015).

Helioseismology provides an alternative measure of the solar radius, the so-called “seismic radius” (Schou et al. 1997). The seismic radius is determined from frequencies of surface gravity waves (f-modes). The f-mode frequencies depend on the local gravity acceleration and the oscillation wavelength, which in turn depends on the solar radius and the mode spherical harmonic angular degree. The oscillation frequencies are measured to a very high precision (~10⁻⁶), and provide an accurate measure of the seismic radius. The surface gravity waves travel beneath the visible surface of the Sun, and their frequencies are sensitive to the sharp density gradient in the near-surface layers. Therefore, the seismic radius is an attribute of the subsurface stratification of the Sun. It is different from the visual radius; the relationship between them can be made only through modeling (Sofia et al. 2005). The first helioseismology measurements (Schou et al. 1997) showed that the Sun’s seismic radius is about 300 km smaller than the predictions of the standard evolutionary solar model calibrated to the visual radius. The discrepancy was explained by an apparent shift of the limb inflection point due to light absorption in the solar atmosphere (Haberreiter et al. 2008). This led to a revision of the standard solar radius to the value of 695,700 km by the International Astronomical Union in 2015. Initial measurements of the seismic radius from the Solar and Heliospheric Observatory (SoHO) showed that its changes do not exceed 1–2 km per year (Dziembowski et al. 2001; Antia 2003). It was found that the helioseismic measurements have to take into account sensitivity of the f-mode frequencies to surface perturbations (Dziembowski et al. 2001), and also dependence of the radial displacement on depth (Lefebvre & Kosovichev 2005). In addition, the measurements are affected by periodic annual variations due to orbital motion of the Earth and spacecraft orbit around the Sun, and also potential changes in the instrumental sensitivity (Antia 2003). Our new analysis, which includes observational data from two missions and covers almost two solar cycles, accounts for all of these factors.

2. Observational Data

Here, we use helioseismology data obtained in 1996–2017 from two NASA missions: the Solar and Heliospheric Observatory.
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Observatory SoHO (1996–2010) and the Solar Dynamics Observatory (SDO; 2010–2017). The data from the helioseismology instruments, the Michelson Doppler Imager (MDI; Scherrer et al. 1995) and the Helioseismic and Magnetic Imager (HMI; Scherrer et al. 2012), are available online from the SDO JSOC (Joint Science Operations Center) archive: http://jsoc.stanford.edu. The mode frequency analysis is performed using a 72 day series of full-disk Dopplergrams. The HMI high-resolution data are specially prepared to match the spatial resolution of the MDI Medium-\(\ell\) Structure Program (Kosovichev et al. 1997), and perform uniform data processing from the two instruments (Larson 2016). The frequency fitting of the oscillation power spectra was performed by using a symmetrical Gaussian line profile. Compared to the frequencies fitted with an asymmetrical profile, these frequencies have a systematic shift of about 0.006 \(\mu\)Hz, which remains constant during the solar cycle. We use the frequency data determined from the symmetrical fits because these data are less noisy and more complete. Additional systematic errors can be due to various instrumental effects, such as image-scale errors, cubic distortion from the instrument optics, misalignment of the CCD, an error in the inclination of the Sun’s rotation axis, and a potential tilt of the CCD. These errors and their corrections in the mode fitting procedure were described by Larson & Schou (2018). Even after the corrections, these factors cause systematic 6 month variations of the \(f\)-mode frequencies. These variations are most likely caused by an error in the inclination of the Sun’s rotation axis. In our analysis, they cause 6 month or annual variations of the seismic radius. To eliminate these systematic errors, we applied a smoothing Gaussian filter with the standard deviation of one year.

We study the whole time span ranging from 1996 April 30 to 2017 June 4. The MDI data cover the initial period until 03/20/2011, and the HMI data cover the rest. The total number of the frequency data sets combined from the two instruments for our analysis was 105. For all of these periods, we selected a common subset of \(f\)-modes. It includes 152 modes in the range of angular degree \(\ell\) from 139 to 299. Thus, the mode sets used in our analysis are identical in all 105 data sets. In addition to the mean \(f\)-mode frequencies that are sensitive to variations of the radial structure, the MDI and HMI observations provide data on frequency splitting, which allow us to investigate variations of the solar differential rotation and asphericity with the solar cycle (Kosovichev & Rozelot 2018).

Figure 1 shows the \(f\)-mode frequency differences observed during the maximum of Solar Cycle 23 (observing interval with the mid date of 2001 November 2), the solar minimum between Cycles 23 and 24 (2009 June 10), and during the Cycle 24 maximum (2014 May 15), relative to the first observing 72 day interval with the mid date of 1996 June 6. The relative frequency difference reached about \(7 \times 10^{-5}\) in Cycle 23 and \(5 \times 10^{-5}\) in Cycle 24. During the activity minimum of 2009–2010 the frequencies were lower than in the previous minimum of 1996–97, probably reflecting the lower solar magnetic activity during the last minimum.

3. Theoretical Model

For the data analysis, we employ a theoretical model developed by Dziembowski et al. (Dziembowski et al. 2001; Dziembowski & Goode 2004, 2005). The model considers two primary contributions to the \(f\)-mode frequencies, arising from changes in the stratification of subsurface layers and also from surface effects caused by interaction of \(f\)-modes with surface magnetic fields and changes in the near-surface structure. Following this model, we represent the relative frequency variations, \(\delta \nu/\nu\), in the following form:

\[
\frac{\delta \nu}{\nu} = \frac{3}{2} \frac{\Delta R}{R} + \frac{\gamma_{\ell,t}}{I_{\ell}}
\]

(1)

where \(\Delta R/R\) are relative variations of the seismic radius determined by \(f\)-modes of angular degree \(\ell\), and \(\gamma_{\ell,t}\) is a parameter that describes variations of the surface effects with the solar cycle (the so-called “surface term”), and \(I_{\ell}\) is the mode inertia that takes into consideration the dependence of this term on the mode angular degree. The \(f\)-mode frequencies can be affected by a number other than the variations of the seismic radius, e.g., solar cycle changes in turbulent convection and surface magnetic fields. These effects are scaled with the inverse mode inertia and are taken into account by the second term of Equation (1) (Dziembowski et al. 2001).

The first right-hand side term is calculated using the variational principle for nonradial stellar oscillations (Chandrasekhar 1964), and provides a relationship between displacements of the subsurface layers, \(\delta r/r\), and the frequency variations in the following form:

\[
\left( \frac{\Delta R}{R} \right)_{\ell} = \frac{1}{I_{\ell}} \int_0^{R_s} S_{\ell}(r) \frac{\delta r}{r} dr
\]

(2)
where the kernel function, \( S_\ell(r) = \ell \rho(r) g(r) r^3 \xi_\ell^2 / (2\pi \nu I_r) \), describes the sensitivity of the \( f \)-mode frequencies to the radial displacement (Dziembowski & Goode 2004), \( I_r = \int_0^R \xi_\ell^2 \rho r^2 dr \) is the mode mass, \( \rho(r) \) is the mass density, \( g(r) \) is the gravity acceleration, \( R_o \) is the Sun’s photospheric radius, and \( \xi_\ell \) is the mode displacement eigenfunction. The accuracy of the sensitivity kernels was tested using pairs of different solar models by Chatterjee & Antia (2008), who found that for high \( \ell \) values additional terms that depend on variations of the sound speed near the surface may become significant, but for the medium-degree modes (\( \ell < 300 \)) the displacement sensitivity kernel is dominant. The primary difference in their test was due to large differences between the models close to the surface. In our model, the surface effects are taken into account by the second term in the RHS of Equation (1).

The surface term, \( \gamma_{\text{surf}} \), is determined empirically by fitting it to the observed variations \( \delta \nu/\nu_0 \). Then, the first term of Equation (1), \( -\frac{3}{2} \Delta R / R \), is determined by subtracting the surface term fit from the observed frequency variations, \( \delta \nu/\nu_0 \). To ensure that there is no residual correlation, the surface term is represented in the form: \( \gamma_{\text{surf}} = -\frac{3}{2} \gamma_0 S(R_o) \), where \( S(R) \) is the surface value of the \( f \)-mode sensitivity function (see Equation (2)) and \( \gamma_0 \) is the fitting coefficient. Examples of this fitting are shown in Figure 1 by solid curves. Apparently, the surface term fits the \( \ell \)-dependence quite well meaning that the main part of the frequency variations can be assigned to the “surface effects,” which can be due to both variations of the solar structure and interaction of \( f \)-modes with magnetic fields at the solar surface. The surface term is subtracted from the observed frequency variations, and the remaining difference represents changes in the seismic radius described by Equation (2) (Dziembowski et al. 2001).

Figure 2(a) shows the temporal variations of the seismic radius averaged over all \( \ell \) values, relative to the seismic radius in 2009, during the solar minimum between Solar Cycles 23 and 24. We adopt this value as a reference. Figure 2(b) shows the variations of the surface term coefficient, \( \gamma_0 \). It changes in phase with the solar activity and correlates quite well with the sunspot number averaged for the same time intervals as the helioseismology data. The arrow in panel (a) indicates the start of the HMI data set.

![Figure 2. Solar cycle variations of (a) the mean seismic radius, (b) the sunspot number, and (c) the surface perturbation of \( f \)-mode frequencies.](image)

4. Inversion Results

The decrease of \( \langle \Delta R / R \rangle \) does not represent simple “shrinking” of the Sun. In fact, the different subsurface layers are displaced by different amounts, and these variations are not necessarily homologous. To determine these variations, we adopt the helioseismology inversion approach (Lefebvre & Kosovichev 2005). According to this method the depth dependence of the radial displacements, \( \delta r/r \), is determined by solving the system of integral equations (Equation (2)). Localization of the integral kernels, \( K_\ell(r) = S_\ell(r) / I_r \), for different modes is different (Figure 3(a)), and this allows us to determine the depth dependence of \( \delta r/r \). Equation (2) is solved by applying the Tikhonov–Phillips (Phillips 1962; Tikhonov 1963) regularization method in the formulation of Twomey (1963).

Because the kernel functions are significantly different from zero only in the near-surface layers, we solve Equation (2) in the range of \([0.97 R_s, 1 R_s]\). In this interval, we introduce a uniform radial grid, \( r_i, i = 0, \ldots, N - 1 \), and approximate \( \delta r/r \) by a piece-wise linear function with unknown grid values \( \delta_\ell = (\delta r/r)_\ell \). Then, Equation (2) is reduced to a system of linear equations for \( f_\ell \):

\[
Af = g + \sigma, \tag{3}
\]

where elements of matrix \( A \): \( a_{i\ell} = [K_\ell^{(1)} - (K_\ell^{(2)} - K_\ell^{(3,-1,0)})/\sigma], \quad K_\ell^{(1)} = \int_{r_i}^{r_{i+1}} K_\ell(r) dr, \quad K_\ell^{(2)} = \int_{r_i}^{r_{i+1}} K_\ell(r) (r - r_i)/(r_{i+1} - r_i) dr, \quad \beta_\ell = (\delta_\ell^2)/\sigma^2, \quad \sigma_x \) are measurement error estimates of the \( f \)-mode frequencies, which are used as a weighting function.
Following Twomey (1963), Equation (3) is solved by minimizing the second derivative of $f$:

$$f = (A^*A + \gamma H)^{-1}A^*g,$$

where $A^*$ is transposed of $A$, matrix $H$ represents the smoothness constraint, and $\gamma$ is the regularization parameter. It was chosen by applying the L-curve method of Hansen (Hansen 1992). For the expression of $H$ and computational details, we refer to Twomey (1963). The spatial resolution of the inversion results is estimated as the half-width of averaging kernels given by matrix $(A^*A + \gamma H)^{-1}A^*$, which is displayed in Figure 3(b). It shows that the given set of $f$-modes provides a localized solution in the depth range from 2 to 10 Mm. The uncertainties in variations of the seismic radius were estimated using a Monte Carlo simulation: the inversion procedure was repeated 100 times by perturbing the $f$-mode frequencies with random Gaussian noise with the standard deviation corresponding to the observed error estimates of the SDO ISOC data archive.

The inversion results for $\delta r$ shown in Figure 4 for the three intervals corresponding to the beginning of Cycle 23 and the solar maxima of Cycles 23 and 24 reveal that most of the radius displacement was beneath the visible surface in the depth range of 3–8 Mm. The radius variations were not monotonic: above and below this range the radius increased by 1–2 km. This means that the deeper layers (6–10 Mm) were compressed while the subsurface layers (3–5 Mm) expanded. The most significant variations were at the depth of about 5 ± 2 Mm, and reached about 8 ± 3 km in Cycle 23 and 5 ± 3 km in Cycle 24 (Figure 5(a)). Figure 5(b) shows the variations of displacement $\delta r$ (smoothed in time using a Gaussian kernel with the standard deviation of one year to remove the annual instrumental variations) as a function of depth during the whole 21 year period of the helioseismology observations from SoHO and SDO, relative to the deep activity minimum between Cycles 23 and 24 in 2010. Apparently, in Solar Cycle 23 the radius changed substantially more than in Solar Cycle 24.

5. Discussion

The presented analysis resolves the previous uncertainties in the determination of variations of the seismic radius, caused by annual variations of observational data and potential instrumental effects. It takes into account variations of the $f$-mode frequencies, associated with the surface activity. The results show that the mean seismic radius estimated from the $f$-mode frequencies in the angular degree range of 130–300 decreased during the solar maxima by ~3 km in Cycle 23 and ~2 km in Cycle 24. The frequency inversion showed that the most significant changes occurred in the depth range of 3–8 Mm beneath the photosphere.
Because $f$-mode frequencies are not sensitive to temperature or sound-speed variations the physical mechanism of the inferred displacements is probably associated with magnetic fields accumulating in the subsurface layers during the solar maxima. Currently, there is no theory to relate the inferred displacements to changes in properties of the solar magneto-convection. Nevertheless, simple relations obtained by Goldreich et al. (Goldreich et al. 1991) provide an interesting insight into properties of the subsurface fields (Dziembowski et al. 2001). According to this theory, the Lagrangian change of the local radius can be expressed in terms of the averaged temperature and magnetic field changes by using the hydrostatic equilibrium condition and thermodynamic relations:

$$\Delta r \sim L \left[ \frac{\Delta (\beta p_m)}{p_s} + \frac{\Delta T}{T} \right], \tag{5}$$

where $\Delta r$ is the radius change over distance $L$, $p_m = (B^2_m + B^2_\perp)/8\pi$ is the magnetic pressure, $\beta = (B^2_m - B^2_\perp)/(8\pi p_s)$ is a measure of the anisotropy of the field, $p_s$ is the gas pressure, and $\Delta T/T$ is the relative temperature change. For a pure radial magnetic field $\beta = -1$, and for an isotropic magnetic field $\beta = 1/3$. Therefore, the radius may decrease due to a local decrease of temperature or due to a predominantly radial magnetic field. To explain the radius decrease by 5–8 km in a 10 Mm subsurface layer, the relative temperature change should be $\sim -5 \times 10^{-4}$. This could result in an increase in the superadiabatic gradient and a corresponding change in the energy flux and the solar irradiance. Alternatively, the radius change can be explained if the magnetic field is predominantly radial in the subsurface layer located at the depth of 5–10 Mm beneath the solar surface. The pressure in this layer, $p_s \approx 2 \times 10^9$ dyn cm$^{-2}$. Hence, the average magnetic field strength $\sqrt{B^2_\perp \approx \sqrt{8\pi p_s}} \Delta r/L \approx 10$ kG.

This should be considered as an upper limit on the field strength. As shown by numerical simulations (e.g., Kitiashvili et al. 2010; Chen et al. 2017) the subsurface magnetic field becomes predominantly vertical due to convective downdrafts that cause concentration of the magnetic field into vertical structures of several kG in strength. This is significantly greater than the strength of the magnetic field emerging on the solar surface.

More studies based on realistic numerical simulations of solar magnetoconvection and oscillations are needed for a better understanding of the observed variations of the $f$-mode frequencies, and for a more accurate interpretation of variations of the Sun’s seismic radius with the activity cycle.

The work was performed with the support of the International Space Science Institute (ISSI) in Bern (CH), the VarSITI (Variability of the Sun and Its Terrestrial Impact) Program of the Scientific Committee On Solar-Terrestrial Physics (SCO-STEP). The authors thank ISSI for holding a scientific meeting on solar variability, which was organized by K. Georgiev. The work was partially supported by NASA grants NNX14AB70G and NNX17AE76A.

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