Experimental Investigation of the Change of Elastic Moduli of Clastic Rocks under Nonlinear Loading

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1. INTRODUCTION

Elastic moduli of rocks are not constant and have nonlinear nature under the influence of the dynamic loading [1]. The phenomenon of the dispersion of elastic moduli under vibrations is considered while geotechnical engineering in such fields as mining [2], modelling the tunnels constructed in liquefiable soil rock [3], cutting and blasting [4, 5], construction in limestone sands [6], well drilling and completion [7] and reservoir engineering [8]. Experimental studies of elastic rocks elastic moduli dispersion can be differentiated in two groups in accordance with rock saturation during the investigation. In the first group, dynamic elastic moduli are studied on rocks saturated with a fluid (e.g., brine). A study of Opalinus clay in literature [9, 10] and Mancos

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shale by Szewczyk et al. [11], showed an increase in the Young’s modulus and a decrease in Poisson’s ratio with an increase in frequency of the dynamic loading from 1 to 100 Hz at the strain of $10^{-6}$. Similar dispersion of the Young’s modulus was also reported in literature [12, 13] for Fontainebleau sandstone for frequency ranging from 1 to 100 Hz at the strain lower than $10^{-3}$. An increase in Young’s modulus was also reported in literature [14] while studying the Berea sandstone in the range of frequencies from 1 to 100 Hz. A schematic nonlinear relation among clastic rock elastic moduli and load frequency is given by Batzle et al. [15]. A strong frequency and rate amplitude dependent elastic behavior over variety of sandstones was reported in literature [16]. Based on these and other works, a number of poroelastic mechanisms have been proposed and modelled on the nonlinear nature of the Young’s modulus while studying saturated rocks. Those models characterize in saturated porous media global inertia mechanisms [17], local mechanisms [18, 19], both global and local mechanisms [20] and other (see literature [21]).

In the second group, the studies are performed on dry rocks. It was exhibited in literature [22] that with an increase of frequency from 1 to 100 Hz for samples with a strain range between $10^{-8}$ and $10^{-6}$, the Young’s modulus and Poisson’s ratio of Donnybrook sandstone are nearly constant, i.e. no dispersion of the Young’s modulus is observed. Earlier, it was reported in literature [23] that Young’s moduli of Navajo sandstone, Sperry limestone and Oklahoma granite do not depend on frequency of the dynamic loading at the strain of $10^{-7}$ in the frequency range from 4 to 400 Hz. Similarly, studies conducted by Winkler [24] on vacuum dry sandstones Massillon, Berea and Boise at frequencies up to 0.1 MHz did not reveal any relationship between attenuation and frequency at small strain, which is also relevant for man-made rocks with no intergranular cement. However, the phenomenon of dispersion of elastic moduli in dry clastic rocks is still exists and can be captured by hysteresis loop on the strain–stress diagram as shown in literature [16]. Manifestation of dispersion in dry clastic rocks is accompanied by the phenomenon when the dynamic Young’s modulus exceeds the static one which is directly related to a strain rate and a loading path [25, 26]. There is a number of models designed to describe the nature of dynamic elastic characteristics observed under high strain by, for example, inertial approach (see literature [27]). Nevertheless, the mechanism of the dynamic dispersion of elastic moduli under vibrations is still open for research.

Since the standard loading rigs allow us to perform linear loading of rocks at frequency ranging from 0.1 to 2 Hz and strain up to $10^{-3}$ [26-28], it is practically not possible to use such the rigs to study the effect of nonlinear dynamic loads (vibrations) on rocks. As is shown in literature [4, 29] studying of the dynamic elastic characteristics of rocks at high strain is limited by the complexity of experiments and data processing. It is possible to study the effect of nonlinear dynamic loads on rocks in the range of seismic frequencies from 1 to 100 Hz at the strain of $10^{-3}$-$10^{-5}$ using custom-built experimental set-ups, described in literature [10, 12, 15, 30-32]. Recently, Borgomano et al. [33] have built the experimental apparatus able to measure the dynamic rock elastic characteristics at frequencies up to 1 kHz and strain up to $10^{-3}$. However, considering that during mining (e.g., well drilling and perforation [34]) the rocks experience much higher strain ($\geq 10^{-2}$) the capacity expansion of existing rigs is required. Studying of the effect of significant dynamic loads up to $10^{-1}$-$10^{+4}$ is possible using the Kolsky-Hopkinson bar machines [35, 36], but those machines do not allow to create cyclic (harmonic) loads on rock samples. Therefore, to study the elastic moduli under high strain nonlinear dynamic loads new experimental techniques are necessary. The aim of this paper is to study the nonlinear nature of the dynamic elastic moduli of a dry clastic rock being in the zone of the linear elasticity using a small custom-built experimental rig.

The paper is organized as follows. In Section 2, the experimental methodology, details on samples preparation procedure, quasistatic and nonlinear dynamic experiments are described. The obtained experimental results and its discussion are given in Section 3, followed by the conclusion in Section 4.

2. MATERIALS AND METHODS

2.1. Description of Rock Samples

A clastic Permian age rock (sandstone) was used for this study. The properties of the rock are given in Table 1 and Figure 1. Each sample was 7.5 mm in diameter $d$ and 15.6 mm in length $l$. The samples were prepared in accordance with industrial standards [37, 38].

| Property                        | Value       | Study rig                  |
|---------------------------------|-------------|----------------------------|
| Porosity, fr. units             | 11.7        | Helium porosimeter PHI-220 |
| Gas permeability, $\mu m^2$     | 0.353       | Benchtop Permeability     |
| Pore size, mm                   | 0.02-0.24   | Sistem BPS                 |
| Pore distribution               | uniform     |                            |
| Fracturing                      | no          | Microfocus X-ray          |
| X-ray density composition       | nonuniform  | tomography rig, Nikon      |
|                                | (0.05 mm high-density inclusions) | Metrology XT H 225        |

TABLE 1. Properties of the studied rock
Using diamond drill bits Tacklife AHS02C the samples were cored form rock blocks with water flush, then cut, washed and dried. In total there were 18 samples prepared. For both quasistatic tests and dynamic experiments there were 9 samples used in order to ensure reliable results.

2.2. Methodology of Nonlinear Loading Since the elastic moduli, and the dynamic ones in particular, have to be studied in the state when a rock experiences elastic deformation (the Young’s modulus can be determined as \( E = \sigma / \varepsilon \)) the limits of the linear elasticity zone should be defined prior to the dynamic experiments. In order to obtain the repeatable dependencies of elastic moduli the experiments on nonlinear dynamic loading were performed on 9 samples (Figure 2).

Elastic limits of the studied clastic rock were defined during quasistatic loading of prepared samples at the Tinius Hounsfild rig. Quasistatic loading was performed at the loading rate of displacement plate of 0.05 mm/min. As a result, the standard diagram of loading–displacement was obtained for all the samples tested which comprised compaction, elastic deformations, irreversible deformations and fracture. Figure 3 shows the limits of the linear elasticity zone equal to \( \sigma_{\text{lim,low}} = 10.2 \) MPa and \( \sigma_{\text{lim,up}} = 31.7 \) MPa defined as average values of limits of the linear section according to literature [39].

After the elasticity zone being determined in order to study the nonlinear behaviour of elastic moduli under dynamic loads at the strain up to 10-3 the small experimental rig was built. In the rig (see Figure 4) the rock sample was installed at a load table, below which load sensor Kistler 9027C was mounted. There was a piezoelectric actuator Extrema Model 250µ located on top of the sample through the metal dist. Above the actuator there was a static load ensuring sample’s preloaded (compacted) state. The actuator was powered with a signal generator Manual MOS-01 able to set the amplitude and frequency of the actuator operation and produce axial-stress oscillations.

During the dynamic experiment the sample was experiencing stationary load \( F_s \) equal to 700 N and periodic dynamic load \( F_{dyn} \) the amplitude of which was varied between 50 N and 250 N at two frequencies \( f \) such as 25 Hz to 40 Hz. At stationary load \( F_s \) the sample’s length was equal to \( l_0 \) and sample’s diameter was equal to \( d_0 \) (see Figure 5). The largest \( F_{\text{max}} = F_s + F_{dyn,max} \) and the smallest \( F_{\text{min}} = F_s - F_{dyn,min} \) load applied to a sample corresponded to maximum distance between the sensors and samples top and side surfaces (see Figure 4) and consequently the sample’s strain. The radial displacement...
of the sample’s side surface was recorded by Microepsilon. The axial displacement was recorded by an eddy current probe (ECP).

The data form the sensors went to a PC. After the signals of three sensors were acquired and processed in LabView and Matlab the data then was used to calculate the dynamic Young’s modulus and Poisson’s ratio by using the Equations (1) and (2):

\[
E_{\text{dyn}} = \frac{\Delta \sigma}{\varepsilon_{\text{ax}}} \quad \Delta \sigma = \frac{F_{\text{max}} - F_{\text{min}}}{X} \quad \varepsilon_{\text{ax}} = \frac{l_{\text{max}} - l_{\text{min}}}{l}
\]

\[
\nu = \frac{\varepsilon_{\text{rad}}}{\varepsilon_{\text{ax}}} = \frac{d_{\text{max}} - d_{\text{min}}}{d}
\]

3. RESULTS AND DISCUSSIONS

It was obtained during experiments that the elastic moduli of the elastic rock change under dynamic loading depending on frequency and amplitude of the load. Figure 6a demonstrates that at frequency of \( f = 25 \text{Hz} \) the Poisson’s ratio \( \nu \) increases from 0.4 to 0.58 when dynamic load \( F_{\text{dyn}} \) on piezoelectric actuator is increased from 100 N to 250 N. At the same time the dynamic Young’s modulus \( E_{\text{dyn}} \) increases up to 2.57 GPa (see Figure 6c). It is also noted that if frequency of dynamic loading is increased up to 40 Hz the Poisson’s ratio \( \nu \) increases from 0.52 to 0.6 (see Figure 6b), while the dynamic Young’s modulus \( E_{\text{dyn}} \) increases up to 3.0 GPa (see Figure 6c).

With an increase in the amplitude of dynamic load from 100 N to 250 N the value of \( E_{\text{dyn}} \) increased from 0.75 GPa to 0.86 GPa. After normalization the strain cycles (shown on Figures 6a and 6b) narrowed to the dashed lines.

The dispersion of the dynamic Young’s modulus (see Figures 6c and 6d) corresponds to the varying in a loading cycle rock density. This fact may lead to a perception of rock hardening and softening under vibration. Another possible explanation for the phenomenon of rock elastic
moduli dispersion under high strain vibrations can be inertial resistance of the rock to the dynamic load applied.

However, the experimental results represent a basis for further elastic moduli mathematical models’ formulations. The phenomenon of dispersion of the dynamic elastic moduli under high strain vibrations has to be conceptualized, and further developed mathematical models needs to be verified.

4. CONCLUSION

In this work, we present the results of investigation of the nonlinear nature of elastic rock (sandstone) elastic moduli obtained during the series of experiments. The investigation was conducted in two phases. First, the limits of the zone of linear elasticity of samples with 7.5 mm in diameter were determined in quasistatic loading. Then, at the second stage, the experiments on dynamic loading were performed with help of a custom-built small experimental rig. During the course of dynamic loading experiments elastic properties were studied at two frequencies \( f \) such as 25 Hz and 40 Hz. At both these frequencies amplitude of nonstationary load varied from 50 N to 250 N. The nonlinear nature of the Poisson’s ratio and the dynamic Young’s modulus was revealed as a result of the study. The dispersion of elastic moduli values under dynamic loading was observed at each amplitude of the load for both frequencies at the strain up to \( 10^{-3} \). The obtained results confirm the nonlinear nature of the dynamic elastic moduli of a elastic rock shown in the similar mentioned researches on dry rocks but at smaller strain. The newly developed experimental rig allows to investigate elastic moduli dispersion of rocks at the strain up to \( 10^{-3} \) under vibrations with frequency up to 40 Hz.

5. ACKNOWLEDGEMENTS

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چکیده
در این مقاله یک تحقیق تجربی در مورد ماهیت غیرخطی ویژگی‌های فیزیکی سنگ‌های آواری (سارکس) ارائه شده است. این سنگ‌ها در شرایط خاصی شکل می‌گیرند و در زمینه‌های مختلفی به کار می‌رود. در این تحقیق به منظور تعریف و بهبود شکل‌دادن در محدوده منطقه کششی، نمونه‌های مورد نظر در دکل‌های مختلف بارگذاری شدند. این نمونه‌ها شامل سه دسته از سنگ‌های آواری مختلفی بودند که شامل سنگ‌های آواری با شکل‌های خاصی بودند. در این مقاله، نتایج تحقیق به گونه‌ای ارائه شده است که مهندسان و پژوهشگران می‌توانند به آنها ارجاع کنند.

در دکل نمونه از قبل ثابت مانده بود. بار دینامیک توسط یک محرک پیزوالکتریک که از تولید شد. جابجایی سطوح نمونه سنگ کند و تعیین مقاومت فشاری تک محور در منطقه کشش خطی طراحی و طول توسط تکنیک‌های مختلفی انجام شد.

در مرحله دوم، دکل کوچک سفارشی ساخته شده برای آزمایش ماهیت غیرخطی مدول اپنپ‌های بیوت و ثبات در شرایط مختلفی به کار می‌رود. در این مرحله، نمونه‌های مورد نظر در دکل‌های مختلف بارگذاری شدند.

در این مقاله، یک تحقیق تجربی در مورد ماهیت غیرخطی ویژگی‌های فیزیکی سنگ‌های آواری (سارکس) ارائه شده است. نمونه‌های مورد نظر در دکل‌های مختلف بارگذاری شدند. این نمونه‌ها شامل سه دسته از سنگ‌های آواری مختلفی بودند که شامل سنگ‌های آواری با شکل‌های خاصی بودند. در این مقاله، نتایج تحقیق به گونه‌ای ارائه شده است که مهندسان و پژوهشگران می‌توانند به آنها ارجاع کنند.

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