3D parton imaging of the nucleon in high–energy \( pp \) and \( pA \) collisions

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Dedicated to Klaus Goehrke on the occasion of his 60th birthday

We discuss several examples of how the transverse spatial distribution of partons in the nucleon, as well as multiparton correlations, can be probed by observing hard processes (dijets) in high–energy \( pp \) (\( \bar{p}p \)) and \( pA \) (\( dA \)) collisions. Such studies can complement the information gained from measurements of hard exclusive processes in \( ep \) scattering. The transverse spatial distribution of partons determines the distribution over \( pp \) impact parameters of events with hard dijet production. Correlations in the transverse positions of partons can be studied in multiple dijet production. We find that the correlation cross section measured by the CDF Collaboration, \( \sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb} \), can be explained by “constituent quark” type quark–gluon correlations with \( r_q \approx r_N/3 \), as suggested by the instanton liquid model of the QCD vacuum. Longitudinal and transverse multiparton correlations can be separated in a model–independent way by comparing multiple dijet production in \( pp \) and \( pA \) collisions. Finally, we estimate the cross section for exclusive diffractive Higgs production in \( pp \) collisions at LHC (rapidity gap survival probability), by combining the impact parameter distribution implied by the hard partonic process with information about soft interactions gained in \( pp \) elastic scattering.

1 Introduction

Hard exclusive processes in \( ep \) scattering allow to probe not only the distribution of partons with respect to longitudinal momentum, but also their spatial distribution in the transverse plane. Examples include the hard electroproduction of light mesons and real photons (deeply virtual Compton scattering), as well as the photoproduction of heavy quarkonia \( (J/\psi, \psi', \Upsilon) \). Thanks to QCD factorization theorems the amplitudes for these processes can be separated into a “hard” part, calculable in perturbative QCD, and “soft” parts characterizing the non-perturbative structure of the involved hadrons. The information about the nucleon is contained in so-called generalized parton distributions (GPD’s). These are functions of the parton momentum fractions, \( x \) and \( x' \), as well as of the invariant momentum transfer to the nucleon, \( t \), and thus combine aspects of the usual parton distributions, measured in inclusive deep–inelastic scattering, with those of the elastic nucleon form factors. For \( x = x' \), their Fourier transform with respect to \( t \) describes the spatial distribution of partons in the transverse plane. The GPD’s thus, in a sense, provide us with a “3D parton image” of the nucleon.

Exclusive processes in \( ep \) scattering, however, are not the only reactions which probe the “3D” parton distributions. In fact, a lot more information about the longitudinal momentum and transverse spatial distribution of partons, as well as about multiparton correlations, can be obtained from the study of selected hard processes in (not necessarily exclusive) \( pp \) and \( pA \) scattering. Comparative studies of \( ep \) and \( pp/pA \) scattering.
induced hard processes will help to improve the quantitative description of both classes of processes and offer many new, fascinating insights into the partonic structure of the nucleon. The conceptual basis for such a program is the combination of Gribov’s space–time picture of hadron interactions at high energies, which allows for a unified description of ep and pp scattering, and the QCD factorization theorems for hard exclusive processes. In view of the planned new experiments in both ep scattering (Jefferson Lab at 12 GeV, EIC/eRHIC) and pp/pA scattering (RHIC, LHC), such studies are very timely.

In this paper we study various examples of hard processes in pp and pA scattering which probe the “3D distribution” of partons and their correlations in the nucleon. First, we show how the production of multiple dijets in pp events can resolve spatial correlations of partons in the transverse plane. These correlations provide interesting new information about the long–wavelength structure of the nucleon, which can e.g. be related to the role of instanton–type vacuum fluctuations in generating constituent quark masses (spontaneous breaking of chiral symmetry), which is the basis for the chiral quark–soliton model of the nucleon. Second, we discuss how similar experiments with nuclear targets (pA,dA) can help to separate between longitudinal and transverse parton–parton correlations in the nucleon wave function. Third, we discuss the role of the transverse spatial distribution of partons in the diffractive production of heavy particles (Higgs bosons) in pp collisions at LHC energies. Such diffractive events involve a delicate interplay of “hard” and “soft” processes (survival of the rapidity gaps), and the latter can be modeled relying on information gained from pp elastic scattering.

2 The transverse spatial distribution of partons and hard processes in pp scattering

In order to define the transverse spatial distribution of gluons in the nucleon, it is convenient to write the gluon GPD in the form (analogous expressions apply to the quark flavor singlet and non-singlet distributions)

\[ H_g(x,t,Q^2) = g(x,Q^2) F_g(x,t,Q^2), \]

where \( g(x,Q^2) = H_g(x,t = 0,Q^2) \) is the usual gluon density, and \( F_g(x,t,Q^2) \) the “two-gluon form factor” of the nucleon, \( F_g(x,t = 0,Q^2) = 1 \). In the following, the \( Q^2 \)–dependence of the form factor and related quantities will not be indicated explicitly. One can represent this form factor as the Fourier transform of a function of a transverse coordinate variable, \( \rho \),

\[ F_g(x,t) = \int d^2 \rho \, e^{i(\Delta x \cdot \rho)} F_g(x,\rho), \quad (t = -\Delta^2). \]

On general grounds, the gluonic transverse size is expected to grow with decreasing \( x \). Different physical mechanisms are responsible for this growth in different regions of \( x \). Near \( x = 1 \), the growth of \( \langle \rho^2 \rangle \) is governed by the Feynman mechanism — the \( t \)–dependence of the two–gluon form factor disappears if the active gluon carries the entire longitudinal momentum of the nucleon, \( x \rightarrow 1 \), causing \( \langle \rho^2 \rangle \) to vanishes at \( x = 1 \). Note that this behavior is a trivial consequence of the relativistic kinematics of the form factor

\[ \langle \rho^2 \rangle \equiv \int d^2 \rho \, \rho^2 \, F_g(x,\rho) = 4 \frac{\partial}{\partial t} F_g(x,t)|_{t=0}. \]

1 We are considering here the GPD for zero longitudinal momentum transfer; in the general case the GPD would depend on the momentum fractions of the two gluons separately, \( x' \neq x \).
The scale of the parton distributions here is $m^2_q$. One can easily generalize this to the production of multiple dijets. Neglecting possible correlations in the transverse positions of two partons in the same proton (this question will be discussed in detail in Section 3), the $b$–dependent probability for the production of a double
dijet would be given by
\[ P_4(b) \equiv \frac{[P_2(b)]^2}{f d^2b [P_2(b)]^2}. \] (7)

While exclusive processes in ep scattering provide in principle the cleanest way to access the transverse spatial distribution of partons, there are several instances in which pp scattering is more effective. One is the study of large x, where the cross sections for exclusive processes in ep are small. Besides, in this kinematics, when x \sim 1 are probed, one is mostly sensitive to the Feynman mechanism; see the discussion above. In this case the global transverse distribution of matter can be measured more directly using various reactions combining a soft and hard trigger, in particular in connection with pA collisions [6]. New opportunities for such studies will emerge at LHC, where the high luminosity will allow, for example, to compare the characteristics of W+ and W− production at the same forward rapidities, corresponding to relatively high x where the d/u ratio deviates strongly from the naive value 1/2. By studying the accompanying production of hadrons one can learn which configurations in the nucleon have larger transverse size — those with a leading u–quark or with a leading d-quark. One suitable observable is, for example, the distribution of the number of events over the number of the produced soft particles. A larger transverse size corresponds to a larger probability of soft interactions, and hence to a larger probability of events with large multiplicity. It is interesting to note that the studies of the associated soft hadron multiplicity in the production of W± and Z bosons in \( \bar{p}p \) collisions by the CDF collaboration at Fermilab find an increase of this multiplicity by a factor of two as compared to generic inelastic events [7]. This appears natural if one takes into account that the hard quarks producing the weak bosons have a narrower transverse spatial distribution than the soft partons. As a result, the average impact parameters in events with weak boson production are much smaller than in generic inelastic collisions, leading to an enhancement of multiple soft and semi-hard interactions [4].

### 3 Probing correlations in the proton parton wave function via multiple di-jet production

Single parton densities and GPDs do not carry information about longitudinal and transverse correlations of partons in the hadron wave function. Such information can be extracted from high energy pp and pA collisions where two (or more) pairs of partons can collide to produce multiple dijets, with a kinematics distinguishable from those produced in 2 \to 4 \text{ parton processes.} Since the momentum scale of the hard interaction, \( p_\perp \), corresponds to much smaller transverse distances in coordinate space than the hadronic radius, in a double parton collision the two interaction regions are well separated in transverse space. Experimentally, one measures the ratio
\[ \frac{d\sigma}{d\Omega_1 d\Omega_2 d\Omega_3 d\Omega_4 (p + \bar{p} \to \text{jet1 + jet2 + jet3 + } \gamma)} \frac{d\sigma}{d\Omega_1 d\Omega_2 (p + \bar{p} \to \text{jet1 + jet2})} \frac{d\sigma}{d\Omega_3 d\Omega_4 (p + \bar{p} \to \text{jet3 + } \gamma)} = \frac{f(x_1, x_3, \mu^2) f(x_2, x_4, \mu^2)}{\sigma_{\text{eff}} f(x_1, \mu^2) f(x_2, \mu^2) f(x_3, \mu^2) f(x_4, \mu^2)}, \] (8)

where \( f(x_1, x_3), f(x_2, x_4) \) are the longitudinal light-cone double parton densities at the hard scale \( \mu^2 \) (we assume for simplicity that the virtuality in both hard processes is comparable; in the following equations we suppress dependence on \( \mu^2 \)), and the quantity \( \sigma_{\text{eff}} \) can be interpreted as the “transverse correlation area”. The variables \( \Omega_i \) characterize the observed jets (or photons) — their transverse momenta, rapidities, cuts on the opening angle, etc.

Parton correlations can emerge due to nonperturbative effects at a low resolution scale, or due to the effects of QCD evolution. One possible nonperturbative mechanism is the existence of “constituent quarks”
within the nucleon, which appear due to the interaction of current quarks with localized non-perturbative gluon fields, resulting in local short–range correlations in the transverse spatial distribution of gluons. The instanton model of the QCD vacuum suggests a constituent quark radius of about $1/3$ the nucleon radius, $r_q \approx r_N/3$. Another nonperturbative mechanism, relevant at small $x$, are fluctuations of the color field in the nucleon due to the fluctuations of the transverse size of the quark distribution. Perturbative correlations emerge due to small transverse distances in the emission process in the perturbative partonic ladder in DGLAP evolution. Of all the mentioned mechanisms, only the first one is effective at $x \geq 0.05$, where the data of the CDF experiment were collected.

The CDF experiment observed correlation effects in a restricted $x$ range (two balanced jets, and jet plus photon) and found $\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3}$ mb. This value is significantly smaller than the naive estimate obtained by taking a uniform distribution of partons of a transverse size determined by the e.m. form factor of the nucleon, which gives $\sigma_{\text{eff}} \approx 53$ mb, indicating strong correlations between the transverse positions of partons in the transverse plane. The longitudinal correlation between partons in the measured kinematics due to energy conservation is likely to be small, as $x_1 + x_2$ and $x_3 + x_4$ are much smaller than 1. If this effect were important it would likely lead to a suppression of the double parton collision cross section, and hence to an increase of $\sigma_{\text{eff}}$. However, no dependence of $\sigma_{\text{eff}}$ on $x_i$ was observed in the experiment.

For a more quantitative analysis of the CDF data, we can make use of the information about the transverse spatial distribution of gluons gained from $J/\psi$ photoproduction, as summarized in Section 2. Since the $x$ values of the partons probed were reasonably small compared to 1, the simple “geometric” picture of the $\bar{p}p$ collision in transverse position in the spirit of Eqs. (6) and (7) is justified, and one has

$$\sigma_{\text{eff}} = \left[ \int d^2 b \, P_2^2(b) \right]^{-1}. \quad (9)$$

Evaluating this with the dipole parametrization of the two–gluon form factor (4), this comes to

$$\sigma_{\text{eff}} = \frac{28\pi}{m_g^2} \approx 34 \text{ mb}. \quad (10)$$

Thus, about 50% of the enhancement compared to the naive estimate of the previous paragraph is due to smaller actual transverse radius of the gluon distribution. Still, our value indicates significant correlations in the transverse positions of the partons. In the kinematics discussed here the relevant partons are both quarks and gluons. We can estimate the effect of correlations assuming that most of the partons are concentrated in a small transverse area associated with the “constituent quarks”, as implied by the instanton liquid model of Diakonov and Petrov [8]. Assuming a constituent quark radius of $r_q \sim r_N/3$, we obtain an enhancement factor due to transverse spatial correlations of partons of

$$\frac{8}{9} + \frac{1}{9} \frac{r_N^2}{r_q^2} \sim 1.6 \div 2. \quad (11)$$

This is roughly the value needed to explain the remaining discrepancy with the CDF data. Thus, the combination of the relatively small transverse size of the distribution of large–$x$ gluons and the quark–gluon correlations implied by “constituent quarks” with $r_q \approx r_N/3$ is sufficient to explain the trend of the CDF data. Further studies of multijet events at hadron–hadron colliders, with a broader range of final states, would in principle allow to measure separately quark–quark, quark–gluon, and gluon–gluon correlations for different $x$.

However, studies based on $\bar{p}p$ or $pp$ collisions alone do not allow for a model–independent separation of transverse and longitudinal correlations. This is possible only in $pA$ collisions at RHIC and LHC. The reason is that the nucleus, having a thickness which practically does not change on the nucleon transverse scale, provides an important contribution which is sensitive only to the longitudinal correlations of hadrons.
This is the contribution when two partons of the incident nucleon interact with partons belonging to two different nucleons in the nucleus, $\sigma_2$,

$$\sigma_2 = \sigma_{NN}^{\text{double}} \frac{A - 1}{A} \int d^2b \frac{T^2(b)}{\frac{f(x_1)f(x_2)}{f(x_1, x_2)}}. \quad (12)$$

The other term is the impulse approximation — two partons of the incoming nucleon interact with two partons of the same nucleon in the nucleus, $\sigma_1$, which is simply equal to $A$ times the cross section of double scattering in $pp$ collisions. Thus, by measuring the ratio of $pA$ and $pp$ double scattering cross sections we can determine $1/\sigma_{\text{eff}}$ via the relation

$$\frac{1}{\sigma_{\text{eff}}} = \left[ \frac{\sigma_{\text{double}}^{pA}}{A\sigma_{\text{double}}^{pp}} - 1 \right] \left[ \frac{1 - 1/A}{\int d^2b T^2(b)} \right]. \quad (13)$$

This expression applies for $x_A \geq 0.03$, where nuclear effects in the structure functions are small. Note that the experimental measurement of the $A$–dependence will provide an independent test of this equation.

For the ratio of double to single scattering terms we find, for $A \geq 12$,

$$R = \frac{\sigma_2}{\sigma_1} \approx 0.68 \left( \frac{A}{12} \right)^{0.38} \frac{\sigma_{\text{eff}}}{14 \text{ mb}}. \quad (14)$$

Taking the CDF value of $\sigma_{\text{eff}} \sim 14 \text{ mb}$, we obtain $R \sim 3$ for $A \sim 200$. Thus, the separation of the two terms will be quite straightforward. Even in the case of deuteron–nucleus scattering, which was studied at RHIC recently, the contributions from two partons of one nucleon of the deuteron interacting with two different nucleons in the nucleus remains significant. It constitutes about 50% of the cross section for $A \sim 200$. Hence we conclude that both measurements of $pA$ and $dA$ collisions will allow to measure $\sigma_{\text{eff}}$ if it is $\geq 5 \text{ mb}$, with $pA$ being a better option. Finally, if $\sigma_{\text{eff}}$ will have been measured in $pA$ collisions, it will be possible to extract the longitudinal two–parton distributions in a model independent way.

To summarize, we have demonstrated that future experiments will be able to measure independently the longitudinal and transverse two–parton distributions in the nucleon. With a detector of sufficiently large acceptance it would be possible to extend these studies even to the case of three parton correlations.

4 The transverse spatial distribution of gluons and diffractive Higgs production at LHC

Exclusive diffractive production of Higgs bosons,

$$p + p \rightarrow p + (\text{gap}) + H + (\text{gap}) + p, \quad (15)$$

seems to be one of the promising candidates for the Higgs search at LHC; see Ref. [10] and references therein. From the point of view of strong interactions, such processes involve a delicate interplay between “hard” and “soft” interactions, which leads to a characteristic dependence of the cross section on the impact parameter of the $pp$ system, $b$. The heavy particle is produced in a hard partonic process (virtualities $\sim M_H^2$) involving the exchange of two gluons between the nucleons — one for the gluon–gluon fusion making the Higgs, the other for color neutralization. The impact parameter distribution for this process is described by the function $P_3(b)$, see Eq. (7). In addition, the soft interactions between the two nucleons (viz. the spectator systems) have to conspire in such a way as not to fill the rapidity gaps left open by the hard process. The probability for this can be inferred from the amplitude of $pp$ elastic scattering in the impact parameter representation, $\Gamma(s, b)$. Namely,

$$|1 - \Gamma(s, b)|^2 \quad (16)$$
is the probability for having no inelastic interaction in a $pp$ collision with impact parameter $b$. This function can be evaluated using available phenomenological parametrizations, which can be extrapolated to the LHC energy, $\sqrt{s} = 14$ TeV. Fig. 1a shows $|1 - \Gamma(s, b)|^2$ for the parametrization of Ref. [11] and the multipomeron model of Ref. [12]. Both parametrizations indicate that at this energy the nucleon is “black” [$|\Gamma(s, b)| \approx 1$] for $b \leq 1$ fm. Fig. 1b shows the product

$$|1 - \Gamma(s, b)|^2 P_4(b),$$

which governs the $b$–distribution of the cross section. The distribution is suppressed at small $b$ (because the probability for no inelastic interaction is small) as well as at large $b$ (because the overlap of the gluon distributions, $P_4(b)$, vanishes), and is thus concentrated at intermediate values of the impact parameter, $b \sim 1$ fm.

**Fig. 1** (a) The probability distribution for no inelastic interaction, Eq. (16), for $\sqrt{s} = 14$ TeV, as calculated with the parametrizations of the $pp$ elastic amplitude of Islam et al. [11], and Khoze et al. [12]. (b) The $b$–distribution for diffractive exclusive Higgs production, Eq. (17), as obtained with the dipole–type two–gluon form factor with $m_g^2 = 1$ GeV$^2$. Shown are the “radial” distributions including a factor $2\pi b$. 

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| $b$ [fm] | $|1 - \Gamma(b)|^2$ |
|---|---|
| 0 | 0 |
| 1 | 0.5 |
| 2 | 0.9 |
| 3 | 1.0 |

| $b$ [fm] | $2\pi b \cdot |1 - \Gamma(b)|^2 P_4(b)$ [fm$^{-1}$] |
|---|---|
| 0 | 0 |
| 1 | 0.02 |
| 2 | 0.04 |
| 3 | 0.06 |
Fig. 2 The rapidity gap survival probability, $S^2$, Eq. (18), obtained by integrating the product $|1 - \Gamma(s, b)|^2 P_4(b)$ shown in Fig. 1 over impact parameters. Shown is the result as a function of $s$, for various values of the mass parameter in the two–gluon form factor, $m_g^2$. The Tevatron and LHC energies are marked by arrows.

The integral of Eq. (17) defines the so-called rapidity gap survival probability,

$$S^2 = \int d^2 b |1 - \Gamma(s, b)|^2 P_4(b).$$  

(18)

Fig. 2 shows the variation of this quantity with $s$ between Tevatron and LHC energies, for various values of the dipole mass in the two–gluon form factor of the nucleon, Eq. (4). The gap survival probability decreases with $s$ because the “black” region in the proton grows with the collision energy. Note that our results for $S^2$ are in agreement with those obtained by Khoze et al. [12] in a multi–pomeron model, given the uncertainty in the basic nucleon size parameter (denoted by $b$) in that approach, as well as with those reported by Maor et al. [13]. In view of the different theoretical input to these approaches this is very encouraging. It is worth noting that, if we consider the production of an object of fixed mass at different energies, the values of $x_i$ decrease with the energy, corresponding to a smaller effective value of $m_g^2$ in Fig. 2. When this is taken into account, the actual drop of the survival probability with energy become much more modest.

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