The axion flavour connection

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Outline

• Strong CP problem in one slide

• Benchmark axion solutions:
  o Focus on the origin and quality (phew...see Giacomo's talk) issues

• The Axion-Flavour connection
  o Cook our recipe: a simple (simplistic?) realisation
Strong CP problem

• The QCD Lagrangian contains a CP-violating term

\[ \mathcal{L}_{QCD} = \sum_f \bar{\psi}_f(i\not{D} - m_f)\psi_f - \frac{1}{4} (G_{\mu\nu}, G^{\mu\nu}) + \theta \frac{1}{32\pi^2} (G_{\mu\nu}, \tilde{G}^{\mu\nu}) \]

• However, experimental bounds indicate that, in strong interactions, \[ \theta \leq 10^{-10} \]

• How can we explain this? Anthropic reasoning? No..
  o Mechanism that constrains or drives it to 0.
Benchmark Axion Models

• Different ways of implementing the PQ mechanism can be organised in three large classes:
  
  o The Peccei-Quinn-Weinberg-Wilczek (PQWW) model.
  o The Dine-Fishler-Srednicki-Zhitnitsky (DFSZ) model;
  o The Kim-Shifman-Vainshtein-Zakharov (KSVZ) model.
PQWW Model

• In the PQWW model, the SM is extended by adding a new complex scalar, namely a second Higgs doublet. The Lagrangian contains:

\[ \mathcal{L} = -y_u \bar{q}_L H_u u_R - y_d \bar{q}_L H_d d_R - V(H_u, H_d) + \text{h.c.} \]

\[ V = \frac{\lambda_u}{4} \left( |H_u|^2 - \frac{v_u^2}{2} \right)^2 + \frac{\lambda_d}{4} \left( |H_d|^2 - \frac{v_d^2}{2} \right)^2 + \lambda_{ud} (H_u^\dagger H_d) (H_d^\dagger H_u) + \ldots \]

• One could naively say that, since there are two Higgs fields, there are two independent symmetries, which can be redefined to obtain the hypercharge and an orthogonal accidental Peccei-Quinn symmetry. Is it true? NO!
The general two Higgs doublet model potential, in fact, can be written as

\[
V(H_u, H_d) = m_u^2 H_u^\dagger H_u + m_d^2 H_d^\dagger H_d - \left[m_{ud}^2 H_u^\dagger H_d + \text{h.c.}\right] + \frac{1}{2} \lambda_1 \left(H_u^\dagger H_u\right)^2 + \\
+ \frac{1}{2} \lambda_2 \left(H_d^\dagger H_d\right)^2 + \lambda_3 \left(H_u^\dagger H_u\right) \left(H_d^\dagger H_d\right) + \lambda_4 \left(H_u^\dagger H_d\right) \left(H_d^\dagger H_u\right) + \\
+ \left[\frac{1}{2} \lambda_5 \left(H_u^\dagger H_d\right)^2 + \lambda_6 \left(H_u^\dagger H_u\right) \left(H_u^\dagger H_d\right) + \lambda_7 \left(H_d^\dagger H_d\right) \left(H_u^\dagger H_d\right) + \text{h.c.}\right],
\]

Therefore, if we want a PQ symmetry, we must impose that some terms are absent. **No accidental PQ symmetry**
In general, all the afore-mentioned axion models DO NOT feature an accidental PQ symmetry.

But why do we focus on the fact that the PQ symmetry is not accidental? For two reasons:

- Global symmetries are widely believed not to be fundamental in QFT;
- Being anomalous, $U(1)_{PQ}$ is not a symmetry of the quantum world.
Origin and quality of the PQ symmetry

Therefore, we should account for the origin of the PQ symmetry. It would be desirable that it came accidentally, as a result of imposing “sacred” principles: Lorentz and gauge invariance.

Moreover, experimental bounds constrain $\theta_{12} \lesssim 10^{-10}$, so the symmetry must be highly protected. This is commonly referred to as the PQ quality issue.
Origin and quality of the PQ symmetry

Various constructions that enforce a high quality, accidental PQ symmetry have been proposed, but they all rely on imposing the dimension of the first PQ breaking operator by hand. Not satisfying...

Thus, we need a mechanism that enforces a high quality, accidental PQ symmetry without imposing any condition by hand.
The Flavour puzzle

On a completely different note, the fermion mass hierarchy problem represents one among the most puzzling features of the Standard Model. In two lines, there is a 5 order of magnitude difference between the Yukawa couplings of the top quark and of the up quark.
The Axion Flavour connection: a top-down approach

• We will assume that the SM flavour pattern is generated by a SB flavour symmetry, which will be identified requiring that
  o It must automatically enforce an accidental global anomalous PQ symmetry
  o It must protect U(1)_{PQ} up to a sufficiently large operator dimension

• Then, we will analyse whether it reproduces, upon SSB, the observed pattern of quark mass hierarchies.
The Axion Flavour Connection: Rectangular symmetries

• Let us consider the following examples of flavour symmetries:
  o \( Y \) in the bifundamental, \( det(Y) \) is PQ-violating;
  o \( SU(N)_L \times SU(N)_R \) : \( Y \) in the bifundamental, no \( det(Y) \)

• Therefore, rectangular symmetries are more effective, as for the quality issue
The Axion Flavour Connection: Model Building

• **Simplicity:** simplest consistent gauge group and smallest number of fermions;

• **Phenomenology:**
  - Masses: top mass at tree level from renormalizable coupling to Higgs. Up and charm from effective operators
  - Mixings: field content sufficiently rich to generate all the masses and mixings of the quarks

• **Gauge anomalies:** gauge symmetries must be anomaly free

• **PQ origin and quality:** accidental and highly protected PQ symmetry
Cook our recipe: a simple realisation

• The “simplest” model complying with all these requirements is a 2HDM containing 6 scalars (X, Y, Z, K, Hu, Hd) and a 7x7 fermion mass matrix, transforming under the $G_{SM} \times G_F \times U(1)_F$ gauge group, where $G_F = SU(3) \times SU(2)$

• The most general $G_F$ invariant mass matrix is

$$
M_u = \begin{pmatrix}
0 & 0 & 0 & v & 0 & 0 & z_1 \\
0 & 0 & 0 & v & 0 & z_2 \\
0 & 0 & 0 & 0 & v & z_3 \\
0 & 0 & v & 0 & 0 & M \\
\Lambda_u & x_1^* & y_1^* & 0 & 0 & 0 \\
0 & \Lambda_u & x_2^* & 0 & y_2^* & 0 \\
x_1 & x_2 & \Lambda_i & z_1^* & z_2^* & z_3^* & v
\end{pmatrix}
$$
How to construct a (failing) model

• Under $G_F = SU(3) \times SU(2)$ we consider:

$$q_L \sim (3, 1), \quad u_R \sim (1, 2), \quad t_R \sim (1, 1),$$

$$U_R \sim (3, 1), \quad U_L \sim (1, 2), \quad T_L \sim (1, 1).$$

$$Y \sim (3, 2), \quad Z \sim (3, 1), \quad X \sim (1, 2)$$

seesaw structure
3 light and 3 heavy eigenstates
non-vanishing determinant

$$\mathcal{M} = \begin{pmatrix}
    u_R & u_R & t_R & U_R & U_R & U_R \\
    0 & 0 & 0 & v & 0 & 0 \\
    0 & 0 & 0 & 0 & v & 0 \\
    0 & 0 & 0 & 0 & 0 & v \\
    \Lambda_u & 0 & x_1^* & y_1^* & 0 & 0 \\
    0 & \Lambda_u & x_2^* & 0 & y_2^* & 0 \\
    x_1 & x_2 & \Lambda_t & z_1^* & z_2^* & z_3^* \\
\end{pmatrix}$$

$$\text{det} (\mathcal{M}) = v^3 \Lambda_u \left[ |X|^2 - \Lambda_t \Lambda_u \right]$$
How to construct a (failing) model

• From the determinant we can read off which operators must be allowed by $U(1)_F$

\[
\mathcal{L} \supset q_L H_u U_R + \Lambda_u \bar{U}_L u_R + \begin{cases} \Lambda_t \bar{T}_L t_R \\
\bar{T}_L X u_R + \bar{U}_L X^+ t_R \end{cases}
\]

• We can read off th

\[
q_L - U_R = H_u, \quad U_L - u_R = 0, \quad \begin{cases} T_L - t_R = 0 \\
T_L - u_R = t_R - U_L = X \end{cases}
\]

• and compute the

\[
\mathcal{A} = 3q_L - 2u_R - t_R + 2U_L + T_L - 3U_R
\]
How to construct a (failing) model

• Substituting the charges, the anomaly yields

\[ \mathcal{A} = \begin{cases} 
3H_u + (T_L - t_R) = 3H_u \\
3H_u + (T_L - u_R) + (U_L - t_R) = 3H_u 
\end{cases} \quad \text{or} \]

• If down-sector replicates the same structure, then

\[ \mathcal{A} = 3(H_u + H_d) \]

• \( U(1)_F \):

\[ A_F = 0 \quad F_{H_u} = -F_{H_d} \]

• \( U(1)_{PQ} \):

\[ A_{PQ} \neq 0 \quad \chi_{H_u} + \chi_{H_d} \neq 0 \]

• Therefore, \( H_u H_d \) gauge-allowed, but PQ-violating at D=2!
Cook our recipe: a simple realization

- In the models analysed, we were able to retrieve compatible quark mass hierarchies from non-hierarchical (or mildly hierarchical) input parameters without imposing any number by hand.

|         | model (GeV) | experimental (GeV) |
|---------|-------------|-------------------|
| $m_b$   | 1.5         | 1.5               |
| $m_c$   | 0.5         | 0.4               |
| $m_s$   | 20          | 30                |
| $m_d$   | 0.5         | 1.5               |
| $m_u$   | 0.3         | 0.7               |

Numerical values of the quark masses and their experimental values evolved at the scale of $10^8$ GeV. $m_t = 102.5$ GeV
The Axion Flavour connection: conclusion and prospects

• As we have shown, the axion-flavour connection is a sensible ansatz to tackle the SM flavour puzzle and the strong CP problem in one fell swoop.
The Axion Flavour connection: conclusions and prospects

- I have a dream! Flavour + Strong CP problem + CDM
- Research strategies:
  - Implement powerful minimisation routines (any suggestion accepted!!!) and try to obtain CKM + Hierarchies simultaneously
  - Extend the gauge symmetry (trade off with simplicity though)
  - Extend to the lepton sector
BACKUP SLIDES
Hierarchical structure of the VEVs

• The minimisation of the scalar potential yields hierarchical entries in the VEVs of the scalar fields.

\[ |x_1|, |y_{1,2}|, |z_3|, |k_2| \sim O(\Lambda), \quad |x_2|, |z_{1,2}| \sim O(10^{-5}\Lambda), \quad |k_{1,3}| \sim O(10^{-6}\Lambda) \]

\[ \Lambda = 10^{11} \text{GeV} \]

• This intuitively explains how we can get hierarchies starting from non-hierarchical (or mildly hierarchical) input parameters.
PQ quality: the role of higher-dim operators

• In general, a PQ-breaking operator can be written as:

\[ \eta \frac{\Phi^D}{M_P^{D-4}} + \text{h.c.} \approx \left( \frac{f_a}{M_P} \right)^{D-4} f_a^4 \cos \left( \frac{a}{f_a} + \xi_\eta \right) \]

• The total potential (QCD + PQ-break) would then be

\[ V \supset -m_\pi^2 f_\pi^2 \cos \left( \frac{a}{f_a} \right) + \left( \frac{f_a}{M_P} \right)^{D-4} f_a^4 \cos \left( \frac{a}{f_a} + \xi_\eta \right) \]

• To comply with

\[ \theta \lesssim 10^{-10} \]

then

\[ \left( \frac{f_a}{M_P} \right)^{D-4} f_a^4 \lesssim 10^{-10} m_\pi^2 f_\pi^2 \rightarrow D \gtrsim 8 \ (13) \iff f_a \lesssim 10^8 \ (10^{13}) \]
You cannot have your cake and eat it too: a no-go theorem

• The Yukawa Lagrangian can be written as

\[ \mathcal{L}_Y = \sum_{i,j} \lambda_{ij} \bar{Q}_m Y_{m_in_j} q_{n_j} \]

For which the mass matrix is

\[
\mathcal{M} = \begin{pmatrix}
\lambda_{m_1n_1} Y_{m_1n_1} & \cdots & \lambda_{m_2n_3} Y_{m_2n_3} & \cdots & \lambda_{m_in_j} Y_{m_in_j} \\
\vdots & \ddots & \vdots & \ddots & \ddots \\
\vdots & \ddots & \vdots & \ddots & \ddots \\
\vdots & \ddots & \vdots & \ddots & \ddots \\
\vdots & \ddots & \vdots & \ddots & \ddots \\
\end{pmatrix}
\]
You cannot have your cake and eat it too: a no-go theorem

• In order to have massive quarks, we need to require

\[ \text{Det}(\mathcal{M}) = \varepsilon_{\alpha_1\alpha_2...\alpha_N} \mathcal{M}_{1\alpha_1} \mathcal{M}_{2\alpha_2}... \mathcal{M}_{N\alpha_N} \neq 0 \]

• Such an operator has a charge proportional to the anomaly

\[ \mathcal{A}_{PQ} \propto \sum_i m_i \chi_{Q_i} - \sum_j n_j \chi_{q_j} \neq 0 \]

and thus breaks PQ at dimension N. Thus, if we want massive quarks and a PQ colour anomaly, we cannot help but break PQ at dimension N.
