Performance Comparison of Probabilistic Amplitude Shaping and Multidimensional Modulation

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Abstract:
We numerically evaluate Achievable Information Rate (AIR) and Bit Error Rate (BER) performances of Probabilistic Amplitude Shaping (PAS), eight-dimensional modulation with Bit-Interleaved Coded Modulation Iterative Detection (BICM-ID), and conventional two-dimensional 16QAM (Quadrature Amplitude Modulation) for future high-speed optical communication systems. We confirm that end-to-end BER performances of three modulation formats are almost identical at a same transmission rate when the error correction is used once, and the iterative detection makes the performance of eight-dimensional modulation format better. Further, we verify the BER error-free conditions can be estimated by Normalized General Mutual Information (NGMI) for each modulation format.

Keywords: probabilistic amplitude shaping, multi-dimensional modulation, BICM-ID

Classification: Fiber-optic transmission for communications

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1 Introduction

PAS modulation and multi-dimensional modulation with BICM-ID, which are ones of coded modulation where encoders are combined with modulator, are promising to construct high-speed optical systems with flexible transmission capacity. PAS has been massively investigated in recent years, which uses the non-uniformly distributed symbols on a conventional m-QAM constellation with a distribution matcher (DM) to overcome a shaping gap of Shannon limit [1]-[3]. DM enables PAS systems to have flexible transmission rate with high signal-to-noise ratio (SNR) sensitivity characteristics.

Multi-dimensional modulation has been also actively studied as a power-efficient modulation formats [4] and been demonstrated to provide variable capacity with set-partitioning technique [5]. While multi-dimensional modulation formats suffer from performance degradation due to non-Gray code mapping resulting from multiple adjacent symbols, BICM-ID recovers the degradation [6].

In this paper, AIRs of the above two coded modulations, specifically, PAS on 64-QAMs having two types of DM with each Look-Up Table (LUT), and eight-dimensional (8D) modulation based on 16 QAM are evaluated by numerical calculation together with AIR evaluation of a conventional 16 QAM for comparison. Further, BER performances of end-to-end section over DM and inverse DM (DM⁻¹) are quantitatively evaluated at several coding rates of FEC as well as the BER performances over forward error correction (FEC) encoder/decoder section. This allows us to compare the performance of each format at the same transmission rates or net bitrates excluding FEC overhead and bitrate increase due to DM. And finally, the obtained BER performances are analyzed with NGMI [3] derived from the calculated AIR and FEC decoder characteristics.

2 Calculation model

2.1 Transmitter and Receiver

Transmitter generates two types of PAS constellations on two-dimensional 64-QAM with two types of DM, namely \((k, n) = (12, 10)\) and \((10, 10)\), which respectively transform uniformly distributed binary data blocks of length \(k\) into Maxwell-Boltzmann distributed amplitude data blocks of length \(n\) with each respective single LUT. The DM with a single LUT is a practical solution for high-throughput optical fiber communication systems, though Constant Composition Distribution Matching (CCDM) based on arithmetic coding [1] can achieve more
ideal distribution with a long coding block. At a receiver, a soft-demapper firstly calculates log-likelihood rations (LLRs) using bit-metric decoding (BMD) technique for demodulation, where the supposed probability density of each symbol is weighted with the probability of occurrence of the modulation symbol determined by the DM. The demodulated LLRs estimated by BMD are finally input to DM\(^4\), which has an inverse function of DM. Detailed configuration including encoder/decoder is described in [1]–[3].

The transmitter also modulates a binary sequence encoded by Low-Density Parity-check Code (LDPC) into 8D-SP4096-16QAM formats, where \(2^{16}\) symbols consisted of 16 binary digits on four two-dimensional (2D) Gray-mapped 16QAM planes are set-partitioned four times to form \(2^{12} (=4,096)\) symbols [7]. Noted that \(2^{12}\) symbols per 8D correspond to \(2^3 (=8)\) symbols per 2D. At a receiver BICM-ID is applied to the 8D-SP4096-16QAM symbols, where bit-metric LLRs are exchanged up to 10 round trips between a demodulator (soft-demapper) and a decoder via interleaver and de-interleaver, called external iterations. Every bit-metric LLR is repeatedly recalculated in the demapper, where the supposed probability density of each bit consisting one symbol is weighted with LLRs of other bits in the same symbol that are updated in the decoder every round.

In addition, the transmitter also modulates the LDPC encoded binary sequence into a conventional Gray-mapped 2D-16QAM formats for comparison purpose, which is detected at the receiver using bit-metric decoding technique again, but without any weighing before LDPC decoding.

### 2.3 LDPC encoder/decoder and Transmission Rate

Encoder and Decoder use Low-Density Parity-check Code (LDPC) code defined by Digital Video Broadcasting–Satellite–Second Generation (DVB-S2) with codeword length of 64,800 for every modulation format. Each LDPC code is assumed to have 20 Inner iterations.

Transmission rate per 2D of PAS on 64QAM is given by

\[
R = 2 \cdot \left\{ \frac{k}{n} + 1 - 3(1 - R_c) \right\}
\]

(1)

where \((k, n)\) is defined by a type of DM, \(R_c\) is code rate of LDPC. On the other hands, transmission rates of 8D-SP4096-16QAM and 2D-16QAM are respectively as follows;

\[
R = 3R_c
\]

(2)

\[
R = 4R_c
\]

(3)

A set of two LDPC code rates are deliberately assumed for each modulation format, which differs from one another, as described in Table 1. For example, \(R_c = 4/5\) and \(2/3\) are respectively assumed for \((k, n) = (12, 10)\) of PAS to form transmission rates of 3.2 and 2.4bit/symbol.

Table 1. Transmission Rate for each Modulation Format for Numerical Calculations

| Modulation format | \(R_c\) | Transmission Rate |
|------------------|--------|------------------|
| PAS \((k,n) = (12,10)\) | 4/5, 2/3 | 3.2, 2.4 |
| \((k,n) = (10,10)\) | 5/6, 2/3 | 3.0, 2.0 |
| 8D-SP4096-16QAM | 4/5, 2/3 | 2.4, 2.0 |
| 2D-16QAM | 4/5, 3/4 | 3.2, 3.0 |
2.3 Achievable Information Rate

AIR with BMD is derived for each modulation formats using a sufficiently large number of samples by

$$ R_{BMD} = \sum_{x \in X} H(x) - \sum_{x \in X} H(x|y) $$

$$ \approx -\sum_{x \in X} p_x \log_2 p_x - \frac{1}{n_s} \sum_{k=1}^{m} \sum_{n=1}^{n_s} \log_2 \left( 1 + e^{-(1)^{b_{k,n}}} \right) $$

(4)

where $p_x$ is a probability of each symbol $x$, $m$ is number of bits composing one symbol, $n_s$ is number of samples, $b$ is specific value of $B_i$ with a value of 0 or 1, $\lambda_{k,b}^{(n)}$ is LLR of $k$th bit in a symbol for binary value of $b$. To be noted, the value of the first term in Eq. (5) depends on a type of DM but not on FEC code rate of $R_c$ for PAS, while the values of the first term respectively equal to a fixed number of 3 and 4 for 8D-SP4096-16QAM and 2D-16QAM.

3 Calculation result and Discussion

3.1 Achievable Information Rate

Figure 1 shows the obtained results of AIRs per two-dimension obtained by Eq. (4) with numerical calculations for each modulation format as well as the AIRs calculated by theoretical analysis with numerical integrations for conventional BPSK, QPSK, 2D-16-QAM and PASs. Each dotted line of 2D-16QAM or PASs by the numerical integration respectively corresponds well to the solid blue, red or orange line obtained by numerical calculation, which proves the accuracy of the numerical calculations. A green solid line of 8D-SP4096-16QAM asymptotically approaches a value of 3 at large SNR as designed. Orange and red lines respectively get closer to a solid black line representing Shannon limit in the range of about 12 dB or less, compared to the blue and green lines. This shows that both types of PAS, which asymptotically approach 4.65 and 4.26bit/symbol respectively, can reduce the gap between Shannon limit and so-called constellation constrain capacity. The reason why the two lines are separated from the lines obtained by numerical integration in the range of 4 dB or less must be the performance loss of SD FEC due to the asymmetric distribution of LLRs [3].

Fig 1. Achievable Information Rate per 2D-Symbol as a function of Signal-to-Noise Ratio.

3.2 BER performance

Figure 2(a) shows BER performances obtained by numerical calculations for each modulation format at each LDPC code rate. Since the results described by circles
and squares in orange and red for PAS have almost the same characteristics as each other at the respective four transmission rates, the DMs and DM1’s are shown not to cause major BER deterioration. Further, the end-to-end BERs of PAS shown in orange and red closed circles and 2D-QAM shown in blue triangles have almost the same characteristics at transmission rate of 3.2 and 3.0bit/symbol. Similarly, the BERs of PAS shown in orange and red open circles are almost equal to the ones of 8D-DP4096-16QAM for initial output of the decoder without external iteration shown in green closed and open circles at transmission rate of 2.4 and 2.0bit/symbol. These results show that end-to-end BER performances of three modulation formats are almost identical at a same transmission rate when the error correction is used once. According to our results, the BER performances of PAS do not necessarily exceed those of other methods due to rate back-off that PAS essentially has [1], despite of the better AIRs of PAS than the others. In addition, the obtained results show that the BER performances improve as the number of external iterations increases for 8D-DP4096-16QAM.

We analyze the obtained BER performances using the NGMI [3] derived from the obtained AIR shown in Fig.1 as follows;

\[
R_M = 1 - \frac{(H(x) - R_{BD})}{m} \approx 1 - \frac{1}{m \cdot n_s} \sum_{k=1}^{m} \sum_{n=1}^{n_s} \log_2 \left( 1 + e^{-1(1.5)^2 n^2} \right) \tag{5}
\]

This parameter corresponds to mutual information (MI) for an LLR sequence [8].

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Fig. 2. BER performances and their analysis
(a) BER performances as a function of Signal-to-Noise Ratio per Symbol.
(b) Normalized GMI
(c) MI Input / Output characteristics of LDPC FEC
Solid lines in Fig.2(b) show the calculated NGMI rates at soft-demapper output or LDPC decoder input as a function of SNR.

On the other hand, Fig.2(c) shows MI input/output characteristics of the LDPC, which is estimated by the method shown in [8]. The dotted lines respectively show minimum values of MI input leading to error-free transmission under each LDPC code rate, whose MI output values approach 1. By comparing the solid line in Fig.2(b) and the dotted line in Fig.2(c) at each condition, SNR that makes error-free can be estimated, as shown in dotted lines in Fig.2(b).

The estimated SNRs in Fig.2(b) agree very well with the SNRs in Fig.2(a). For example, both BERs of PAS for \((k,n)= (10,10)\) at LDPC code rate of 5/6 and 2D-16QAM at LDPC code rate of 3/4 turn into error-free around SNR of 11.1dB. These results indicate correctness of numerical calculations for AIR and BER in this paper.

4 Conclusion

We numerically evaluated AIR and BER performances of PAS with a single LUT, 8D-SP4096-16QAM with BICM-ID, and 2D-16QAM for future high-speed optical communication systems. We confirmed that BER performances of three modulation formats are almost identical at a same transmission rate, when the error correction is used once. Further, the error-free SNR conditions agree very well with the values determined by the NGMI statically estimated from received LLR and MI input/output characteristics of the LDPC.

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