Decoupling property of the supersymmetric Higgs sector with four doublets

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ABSTRACT: In supersymmetric standard models with multi Higgs doublet fields, self-coupling constants in the Higgs potential come only from the D-terms at the tree level. We investigate the decoupling property of additional two heavier Higgs doublet fields in the supersymmetric standard model with four Higgs doublets. In particular, we study how they can modify the predictions on the quantities well predicted in the minimal supersymmetric standard model (MSSM), when the extra doublet fields are rather heavy to be measured at collider experiments. The B-term mixing between these extra heavy Higgs bosons and the relatively light MSSM-like Higgs bosons can significantly change the predictions in the MSSM such as on the masses of MSSM-like Higgs bosons as well as the mixing angle for the two light CP-even scalar states. We first give formulae for deviations in the observables of the MSSM in the decoupling region for the extra two doublet fields. We then examine possible deviations in the Higgs sector numerically, and discuss their phenomenological implications.

KEYWORDS: Supersymmetric Standard Model, Higgs Physics
1 Introduction

Although the standard model (SM) has enjoyed a marvelous success in explaining phenomena observed at collider experiments, the physics of electroweak symmetry breaking remains unknown. The experimental detection of the Higgs boson is the most important issue to confirm the standard picture for the origin of mass of particles, and the Higgs boson search is underway at the Fermilab Tevatron and the CERN LHC. On the other hand, the Higgs sector of the SM is known to cause the hierarchy problem [1] due to the quadratic divergence in the one-loop correction to the mass of the Higgs boson. In addition, there are several phenomena confirmed by the experiments which cannot be understood within the SM, such as the neutrino oscillation, the existence of dark matter and the baryon asymmetry of the Universe. Therefore, the SM must be extended to solve these problems.

Supersymmetry (SUSY) is expected to be a good candidate of new physics. It can solve the hierarchy problem by the consequence of the nonrenormalization theorem [2]. The stabilized Higgs boson mass makes it possible to directly connect the electroweak scale with very high scales such as the Planck scale or that of grand unification. Supersymmetric extensions of the SM with the R parity also provide dark matter candidates [3]. In addition, various mechanisms of generating tiny neutrino masses [4–8] as well as those of baryogenesis [9–11] may also be compatible to supersymmetric models.

The minimal supersymmetric standard model (MSSM) is a SUSY extension of the SM with the minimal number of particle content. It requires two Higgs doublet fields for cancellation of anomaly. The most striking phenomenological prediction of the model is that on the mass ($m_h$) of the lightest CP-even Higgs boson $h$. It can be calculated to be less than the mass of the Z boson at the tree level. Such an upper bound on $m_h$ comes from the fact that the interaction terms in the Higgs potential are given only by D-term
contributions which are determined by the gauge coupling constants. At the one-loop level
the trilinear top-Yukawa term in the superpotential gives a significant F-term contribution
to $m_h$ [12–14], by which $m_h$ can be above the lower bound from the direct search results
at the CERN LEP experiment [15]. The calculation has been improved with higher order
corrections [16–18]. Apart from $m_h$, the masses of $H$, $H^\pm$ and the mixing angle $\alpha$ are a
function of only two input parameters at the tree level; i.e., $m_A$ and $\tan \beta$, where $m_A$ is
the mass of CP-odd Higgs boson $A$, $H$ is the heavier CP-even Higgs boson, $H^\pm$ are the
charged Higgs bosons, $\tan \beta$ is the ratio of vacuum expectation values (VEVs) of the two
Higgs bosons and $\alpha$ is the angle which diagonalizes the CP-even scalar states. In particular,
there is a simple tree-level relationship among the masses of $H^\pm$, $A$ and the W boson $W^\pm$
as $m_{H^\pm}^2 = m_A^2 + m_W^2$, where $m_{H^\pm}$ and $m_W$ are respectively the masses of $H^\pm$ and $W^\pm$.

These characteristic predictions can be used to confirm the MSSM.

If by experiments the Higgs boson is found and its mass turns out to be higher than
the upper bound predicted in the MSSM, we must abandon the model. It however does not
necessarily imply that the SUSY itself is excluded. For example, in the next-to-minimal
supersymmetric standard model (NMSSM), where a gauge singlet chiral superfield $S$ is
added to the MSSM, the upper bound on $m_h$ is greater than that in the MSSM due to the
F-term contribution from the trilinear $\lambda H_1 H_2 S$ interaction in the superpotential [19, 20],
where $H_1$ and $H_2$ are two chiral superfields for the Higgs doublets. The effect of this
term can gain the mass upper bound, so that $m_h$ can be as large as 140 GeV (400 GeV)
by assuming that the running coupling constant $\lambda$ does not blow up below the grand
unification scale $\sim 10^{16}$ GeV (the TeV scale) around $\tan \beta \sim 2$ [20]. The similar effect on
$m_h$ also appears in the model with additional triplet chiral superfields with the hypercharge
of $Y = 0$ or $Y = \pm 1$ [21, 22]. The other possibility to change the MSSM prediction on
$m_h$ is to introduce extra gauge symmetries which are broken spontaneously at the TeV
scale. The mixing of gauge bosons of these new symmetries with those of the SM gauge
symmetries yields the new D-term contribution to the Higgs potential, which can change
for instance $m_h$ [23, 24].

In general, interaction terms of the Higgs potential in extended SUSY standard models
are composed of the D-term and the F-term as well as the trilinear soft-breaking term, so
that extended SUSY Higgs sectors predict different masses and coupling constants from
those in the MSSM. If the extra scalar fields are sufficiently light, they largely mix with
the MSSM-like Higgs bosons, and consequently predictions on the MSSM observables are
modified from the MSSM values at the tree level. In addition, such light extra scalars may
directly be detected at the experiments. In this case the model can be easily distinguished
from the MSSM. On the other hand, extra scalar bosons may be too heavy to be measured
directly. Even in such a case the effect of these extra fields can change predictions on masses
and coupling constants for the MSSM-like scalar bosons significantly at the low energy.
This is not contradict with the decoupling theorem by Appelquist and Carazzone [25]. In
fact, in the large mass limit of all the fields other than those in the SM, the model can be
continuously reduced to the SM.

In this paper, we consider the model (4HDSSM) where two extra doublet superfields
are added into the MSSM in order to examine the decoupling property of extended Higgs

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sectors without interactions from the F-term at the tree level. In this model, if there are mixings between the MSSM-like doublet fields and the extra doublet fields due to a large soft-breaking B-term mixing, nonvanishing effects can appear in the MSSM observables such as $m_h$, $m_H$, $m_{H^\pm}$ and $\alpha$. Notice that this nonvanishing effect is not the so-called nondecoupling effect. The usual nondecoupling effect, which for example appears in the radiative corrections to the gauge boson two-point functions, does not vanish in the SM-like limit with taking the extra particles to be heavy. On the other hand, the present effect due to the large soft-breaking B-term mixing on MSSM observables in the 4HDSSM can be significant in the MSSM-like limit but does decouple in the SM-like limit by taking $m_A \to \infty$ according to the decoupling theorem \cite{25}. Therefore, we call such a nonvanishing effect on the MSSM observables as the quasi-nondecoupling effect. We deduce analytic expressions of such quasi-nondecoupling effects on the MSSM observables when the extra doublet fields are heavy. Modifications from the MSSM predictions by these effects are then studied numerically. We find that the quasi-nondecoupling effect due to the B-term mixing can be significant. For example, $m_h$ can be larger than about 10% as compared to that of the MSSM prediction with the same MSSM input parameters, while $m_{H^\pm}$ can be about 20% (10%) smaller than the MSSM prediction when $m_A = 150$ GeV (200 GeV). The mixing angle $\sin(\beta - \alpha)$ also receives a significant deviation from the MSSM value. The Higgs potential and the mass matrices in the 4HDSSM have been analyzed by Gupta and Wells \cite{26}, but they have not included these quasi-nondecoupling effects discussed here.

Suppose that the candidates of SUSY partner particles and extra Higgs bosons in the MSSM ($H$, $A$ and $H^\pm$) are found at the LHC or the International Linear Collider (ILC) in future and that their properties look like consistent with the MSSM. In such a case, to confirm whether it is really of the MSSM or not, the MSSM relations are tested as accurately as possible by experiments. Our study can be particularly important in this case.

In general, in extended Higgs sectors with multi-doublets, there necessarily appears the flavor changing neutral current (FCNC), which is strictly constrained by experiments. A simple prescription to avoid it would be imposing the (softly-broken) discrete $Z_2$ symmetry to the model \cite{27}. We classify the type of Yukawa interaction under the discrete symmetry according to the assignment of the $Z_2$ charges. Phenomenology of the 4HDSSM has been discussed by Marshall and Sher in the case with the lepton specific Yukawa interaction \cite{28}. We here do not discuss influence of the heavy extra two doublet fields on the flavor physics, just assuming that the tree-level FCNC is forbidden by a $Z_2$ symmetry. We then concentrate on the deviations in the Higgs sector due to the quasi-nondecoupling effects of these extra doublets.

This paper is organized as follows. In Sec. II, the 4HDSSM is defined and the mass matrices for the Higgs bosons are obtained. In Sec. III, we give a short description of the decoupling property in extended SUSY Higgs sectors, where the (quasi-)nondecoupling effect is discussed. We then present formulae for the quasi-nondecoupling effects on the MSSM observables. Numerical studies are also shown, and the results are discussed. Conclusions are given in Sec. IV. We show the method of obtaining the Higgs potential in the useful basis from the general basis in Appendix A. The flavor structure in the 4HDSSM is
shortly discussed in Appendix B.

2 The model

We here discuss the 4HDSSM, in which two extra isospin-doublet chiral superfields \( H_3 \) \((Y = -1/2)\) and \( H_4 \) \((Y = 1/2)\) are introduced to the MSSM in addition to the Higgs doublets \( H_1 \) \((Y = -1/2)\) and \( H_2 \) \((Y = 1/2)\). The general expression for the superpotential with the R parity is given in terms of chiral superfields as

\[
W = - (\bar{Y}_u)_{ij} U_{Ri}^c H_2 \cdot Q_{Lj} + (\bar{Y}_d)_{ij} D_{Ri}^c H_1 \cdot Q_{Lj} + (\bar{Y}_e)_{ij} E_{Ri}^c H_1 \cdot L_{Lj} - (\bar{Y}_u')_{ij} U_{Ri}^c H_4 \cdot Q_{Lj} + (\bar{Y}_d')_{ij} D_{Ri}^c H_3 \cdot Q_{Lj} + (\bar{Y}_e')_{ij} E_{Ri}^c H_3 \cdot L_{Lj} - \mu_{12} H_1 \cdot H_2 - \mu_{14} H_1 \cdot H_4 - \mu_{32} H_3 \cdot H_2 - \mu_{34} H_3 \cdot H_4 ,
\]

where \( Q_{Li}, U_{Ri}^c (D_{Ri}^c) \) are chiral superfields for the \( i \)-th generation left-handed quark doublet and right-handed up-type (down-type) quark singlet while \( L_{Li} \) and \( E_{Ri}^c \) are those for the \( i \)-th generation left-handed lepton doublet and right-handed charged lepton singlet, respectively. The most general holomorphic soft-SUSY-breaking terms with the R parity is

\[
\mathcal{L}_{\text{soft}} = - \frac{1}{2} (M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{G} \tilde{G})
- \left( (\tilde{M}_d^2)_{ij} \tilde{q}_{Li} \tilde{q}_{Lj} + (\tilde{M}_d^2)_{ij} \tilde{u}_{Ri} \tilde{u}_{Rj} + (\tilde{M}_d^2)_{ij} \tilde{d}_{Ri} \tilde{d}_{Rj} + (\tilde{M}_d^2)_{ij} \tilde{\ell}_{Li} \tilde{\ell}_{Lj} + (\tilde{M}_d^2)_{ij} \tilde{e}_{Ri} \tilde{e}_{Rj} \right)
- \left( (\tilde{M}_u^2)_{11} \tilde{\Phi}_1^+ \tilde{\Phi}_1 + (\tilde{M}_u^2)_{13} \tilde{\Phi}_3^+ \tilde{\Phi}_3 + (\tilde{M}_u^2)_{13} \tilde{\Phi}_4^+ \tilde{\Phi}_4 + (\tilde{M}_u^2)_{33} \tilde{\Phi}_3^+ \tilde{\Phi}_3 \right.
+ (\tilde{M}_u^2)_{22} \tilde{\Phi}_2^+ \tilde{\Phi}_2 + (\tilde{M}_u^2)_{24} \tilde{\Phi}_2^+ \tilde{\Phi}_4 + (\tilde{M}_u^2)_{44} \tilde{\Phi}_4^+ \tilde{\Phi}_4
- \left( (A_u)^{ij} \tilde{u}_{Ri} \tilde{\Phi}_2 \cdot \tilde{q}_{Lj} - (A'_u)^{ij} \tilde{u}_{Ri} \tilde{\Phi}_1 \cdot \tilde{q}_{Lj} + (A_d)^{ij} \tilde{d}_{Ri} \tilde{\Phi}_1 \cdot \tilde{\ell}_{Lj} + (A'_d)^{ij} \tilde{d}_{Ri} \tilde{\Phi}_3 \cdot \tilde{\ell}_{Lj} + \tilde{\phi}_{Ri} \tilde{\phi}_{Rj} \cdot \tilde{\phi}_{Ri} \tilde{\phi}_{Rj} \cdot h.c. \right)
- \left( B_{12} \mu_{12} \tilde{\Phi}_1 \cdot \tilde{\Phi}_2 + B_{33} \mu_{34} \tilde{\Phi}_3 \cdot \tilde{\Phi}_4 + B_{14} \mu_{14} \tilde{\Phi}_1 \cdot \tilde{\Phi}_4 + B_{32} \mu_{32} \tilde{\Phi}_3 \cdot \tilde{\Phi}_2 + h.c. \right) ,
\]

where \( \tilde{B}, \tilde{W} \) and \( \tilde{G} \) are gauginos corresponding to the SM gauge symmetries of \( U(1), SU(2) \) and \( SU(3) \), respectively, \( \tilde{q}_{Li}, \tilde{u}_{Ri}^*, \tilde{d}_{Ri}^*, \tilde{\ell}_{Li} \) and \( \tilde{e}_{Ri}^* \) are respectively the scalar component fields of \( Q_{Li}, U_{Ri}^c, D_{Ri}^c, L_{Li} \) and \( E_{Ri}^c \), and \( \tilde{\Phi}_j (j = 1-4) \) are the scalar component doublet fields of the chiral superfields \( H_j \). From \( W \) and \( \mathcal{L}_{\text{soft}} \), the Lagrangian is constructed as

\[
\mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{gauge–matter}} - \left( \frac{1}{2} \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \psi_{Li} \cdot \psi_{Lj} + h.c. \right) - \frac{1}{2} (q_a)^2 (\varphi_i^* T_{ij}^a \varphi_j)^2 - \left| \frac{\partial W}{\partial \varphi_i} \right|^2
+ \mathcal{L}_{\text{soft}},
\]

where \( \psi_{Li} \) and \( \varphi_j \) represent fermion and scalar component fields of chiral superfields in the model.

There are two Higgs doublets for each quantum number, so that they can mix with each other. The Yukawa sector then produces a dangerous FCNC via the scalar boson exchange
Table 1. Classification for the charge assignment for the $Z_2$ symmetry in the 4HDSSM. Type C and Type D are introduced only when $N^c_{R \ell}$ are added to the model.
without loss of generality we can rewrite the Higgs potential as
\[
V_{11} = \left( \Phi_1^\dagger \Phi_1 \right) \left( \begin{array}{c} (M_1^2)_{11} & (M_1^2)_{12} \\ (M_1^2)_{21} & (M_1^2)_{22} \end{array} \right) \left( \Phi_1 \right) + \left( \Phi_2^\dagger \Phi_2 \right) \left( \begin{array}{c} (M_2^2)_{11} & (M_2^2)_{12} \\ (M_2^2)_{21} & (M_2^2)_{22} \end{array} \right) \left( \Phi_2 \right) \\
- \left( \Phi_1^\dagger \Phi_1 \right) \left( \begin{array}{c} (M_1^2)_{11} & (M_1^2)_{12} \\ (M_1^2)_{21} & (M_1^2)_{22} \end{array} \right) \left( \begin{array}{c} (M_2^2)_{11} & (M_2^2)_{12} \\ (M_2^2)_{21} & (M_2^2)_{22} \end{array} \right) \left( \Phi_2 \right) + \text{h.c.} \\
+ \frac{g^2 + g^2}{8} \left( \Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 - \Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2 \right)^2 \\
+ \frac{g^2}{2} \left\{ \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_1^\dagger \Phi_1 \right) + \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \left( \Phi_2^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_2 \right) - \left( \Phi_2^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_2 \right) \right\},
\]
where $\Phi_1$ ($Y = -1/2$) and $\Phi_2$ ($Y = 1/2$) have VEVs, while $\Phi_1'$ ($Y = -1/2$) and $\Phi_2'$ ($Y = 1/2$) do not. Throughout this paper, we restrict ourselves in the CP invariant case. We thus hereafter neglect all CP violating phases in the dimensionful parameters.

The rotated Higgs doublet fields $\Phi_1$, $\Phi_1'$, $\Phi_2$ and $\Phi_2'$ are expressed as
\[
\Phi_1 = \begin{bmatrix} \varphi_1^0 - \varphi_1^- \\ \varphi_1^- \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} \varphi_2^0 \\ \varphi_2^- \end{bmatrix}, \quad \Phi_1' = \begin{bmatrix} \varphi_1'^0 - \varphi_1'^- \\ \varphi_1'^- \end{bmatrix}, \quad \Phi_2' = \begin{bmatrix} \varphi_2'^0 \\ \varphi_2'^- \end{bmatrix},
\]
where the neutral scalar fields can be parameterized as
\[
\varphi_1^0 = \frac{1}{\sqrt{2}} (v_1 + \phi_1 + i\chi_1), \quad \varphi_2^0 = \frac{1}{\sqrt{2}} (v_2 + \phi_2 + i\chi_2), \\
\varphi_1'^0 = \frac{1}{\sqrt{2}} (\phi_1' + i\chi_1'), \quad \varphi_2'^0 = \frac{1}{\sqrt{2}} (\phi_2' + i\chi_2'),
\]
where the VEVs of these neutral components are given by $\langle \varphi_1^0 \rangle = v_1/\sqrt{2}$, $\langle \varphi_2^0 \rangle = v_2/\sqrt{2}$, $\langle \varphi_1'^0 \rangle = 0$ and $\langle \varphi_2'^0 \rangle = 0$. Introducing
\[
v = (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV},
\]
and the mixing angle $\beta$, we express $v_1$ and $v_2$ as $v_1 = v \cos \beta$ and $v_2 = v \sin \beta$. The vacuum conditions for the Higgs potential are given by
\[
\frac{1}{v} \frac{\partial V_{11}}{\partial \phi_1} \bigg|_{\phi_1 = 0} = c_\beta \left( (M_1^2)_{11} + \frac{m_2^2}{2} c_{2\beta} \right) - s_\beta (M_1^2)_{11} = 0, \\
\frac{1}{v} \frac{\partial V_{11}}{\partial \phi_2} \bigg|_{\phi_2 = 0} = s_\beta \left( (M_1^2)_{11} - \frac{m_2^2}{2} c_{2\beta} \right) - c_\beta (M_1^2)_{11} = 0, \\
\frac{1}{v} \frac{\partial V_{11}}{\partial \phi_1'} \bigg|_{\phi_1' = 0} = c_\beta (M_1^2)_{12} - s_\beta (M_1^2)_{21} = 0, \\
\frac{1}{v} \frac{\partial V_{11}}{\partial \phi_2'} \bigg|_{\phi_2' = 0} = s_\beta (M_1^2)_{12} - c_\beta (M_1^2)_{12} = 0.
\]
Solving this set of conditions, one can eliminate \((M^2_H)_{11}, (M^2_H)_{11}, (M^2_H)_{12},\) and \((M^2_H)_{12}\).

After imposing the vacuum conditions, the mass matrices \(M^2_\Lambda, M^2_{H^\pm}\) and \(M^2_H\) for the CP-odd, charged and CP-even scalar component states are respectively obtained in the basis of \((\Phi_1, \Phi_2, \Phi'_1, \Phi'_2)\). It is however more useful to work the mass matrices of the CP-odd scalar bosons and the charged Higgs bosons in the gauge eigenstate basis (the so-called Georgi basis) as [36]

\[
M^2_\Lambda = O_0^T M^2_\Lambda O_0 = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \frac{2(M^2_H)_{11}}{c^2_\beta} & \frac{(M^2_H)_{12}}{c^2_\beta} & \frac{(M^2_H)_{12}}{s^2_\beta} \\
0 & \frac{(M^2_H)_{12}}{c^2_\beta} & \frac{\text{m}^2_Z}{2} c^2_\beta + (M^2_H)_{22} & (M^2_H)_{22} - \frac{\text{m}^2_Z}{2} c^2_\beta \\
0 & \frac{(M^2_H)_{12}}{s^2_\beta} & (M^2_H)_{22} & s^2_\beta
\end{pmatrix},
\]

(2.9)

\[
M^2_{H^\pm} = O_0^T M^2_{H^\pm} O_0 = M^2_\Lambda + \text{m}^2_W \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -c^2_\beta & 0 \\
0 & 0 & 0 & c^2_\beta
\end{pmatrix},
\]

(2.10)

with the orthogonal matrix

\[
O_0 = \begin{pmatrix}
c_\beta & s_\beta & 0 & 0 \\
-s_\beta & c_\beta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
\]

(2.11)

where we used the abbreviation such as \(\sin \theta = s_\theta\) and \(\cos \theta = c_\theta\). In this basis the massless modes, which are Nambu-Goldstone bosons to be absorbed by the longitudinal modes of the weak gauge bosons, are separated in the mass matrices. The basis taken here is essentially the same as that discussed in Ref. [26]. It is also useful to rotate the mass matrix for the CP-even scalar bosons as

\[
M^2_H = O_0 M^2_H O_0^T
\]

\[
= \begin{pmatrix}
\text{m}^2_Z c^2_\beta & -\text{m}^2_Z s_\beta c^2_\beta & 0 & 0 \\
-\text{m}^2_Z s_\beta c^2_\beta & \text{m}^2_Z s_\beta + \frac{2(M^2_H)_{11}}{s^2_\beta} & \frac{(M^2_H)_{12}}{c^2_\beta} & \frac{(M^2_H)_{12}}{s^2_\beta} \\
0 & \frac{(M^2_H)_{12}}{c^2_\beta} & (M^2_H)_{22} + \frac{\text{m}^2_Z}{2} c^2_\beta & -(M^2_H)_{22} \\
0 & \frac{(M^2_H)_{12}}{s^2_\beta} & -(M^2_H)_{22} & (M^2_H)_{22} - \frac{\text{m}^2_Z}{2} c^2_\beta
\end{pmatrix}.
\]

(2.12)

3 Decoupling property of the extra doublet fields

3.1 Nondecoupling effects in the large mass regime

In general, new physics can be tested not only by direct searches but also by indirect searches. The indirect searches are performed by precise experiments to find effects of a heavy new physics particle on the observables which are well predicted in the low energy theory such as the SM. Such new particle effects on the low energy observables usually
decouple in the large mass limit after the renormalization calculation is completed. This is known as the decoupling theorem proposed by Appelquist and Carazzone [25]. It is also known that the decoupling theorem does not hold when the new particles receive their masses from the VEV of the Higgs boson. In fact, there is a class of the new physics models where nondecoupling effects of heavy particles can appear on the low energy observables. For example, chiral fermions such as quarks and charged leptons cannot have the mass term because of the chiral symmetry, so that their masses are generated after the chiral symmetry is spontaneously broken by the VEV. Therefore, the effect of a heavy chiral fermion does not decouple, and it appears as powerlike or logarithmic contributions of the mass in the predictions for the low energy observables. Another example is the additional scalar fields in extended Higgs sectors whose masses are typically expressed as

\[ m_{\varphi}^2 \sim \tilde{M}^2 + \lambda' v^2, \]

where \( \lambda' \) is the coupling constant with the SM-like Higgs boson and \( \tilde{M} \) is the invariant mass parameter which is irrelevant to the VEV. When \( \lambda' v^2 \gg \tilde{M}^2 \), the situation is similar to the case of chiral fermions, so that the effect of these scalar bosons do not decouple in the large \( m_{\varphi} \) regime.

The indirect effects of these nondecoupling particles appear in the low energy observables at the tree level, or at loop levels such as the electroweak S, T and U parameters [37] and vertex corrections to the SM coupling constants like the \( \gamma \gamma h \) vertex [38] and the \( hhh \) vertex [39, 40]. In particular, the nondecoupling effects of chiral fermions and additional scalar bosons to the triple Higgs boson coupling are known to give quartic powerlike contributions for the heavy particle masses in the radiative corrections, so that the effect can be very significant. As shown in Ref. [40], in the two Higgs doublet model, the deviation from the SM prediction can be of order 100% without contradiction with perturbative unitarity [41]. This nondecoupling effect on the triple Higgs boson coupling would be applied to realize the strong first-order phase transition [42] which is required for successful electroweak baryogenesis [11, 43, 44]. It is obvious that an excessive nondecoupling effect is bounded by the theoretical constraints such as perturbative unitarity [45] and triviality [46], because the large mass of new particles directly means a large coupling constant.

Let us consider the effect of the heavy particles in supersymmetric standard models. In general, a SUSY Higgs potential is composed of the D-term, the F-term and the soft-breaking term;

\[ V = |D|^2 + |F|^2 + \text{(soft-breaking term)}. \]

Quartic coupling constants in the potential can come from both the D-term and the F-term. In the MSSM, however, because of the multi-doublet structure only D-terms contribute to them, which are given by gauge coupling constants. Consequently, the mass of the lightest CP-even Higgs boson is determined by the gauge coupling constants and the VEVs at the tree level, which is less than \( m_Z \). A substantial F-term contribution enters into the Higgs potential at the one-loop level via the superpotential \( W \approx -y_t U_{R3} H_2 Q_{L3} + \ldots \), where \( y_t = \sqrt{2m_t/v} \sin \beta \approx (Y_u)_{33} \) is the top Yukawa coupling constant. The corrected mass of \( h \)
is described by

$$m_h^2 \sim m_Z^2 \cos^2 2\beta + \frac{3}{2\pi^2} \frac{m_t^4}{v^2} \ln \frac{m_{\text{stop}}^2}{m_t^2}, \quad (3.3)$$

This one-loop correction shows a nondecoupling property in the large mass limit of stops. Consequently $m_h$ can be above the LEP bound at least when one of the stops is heavy enough. There are also contributions from the $\mu$ parameter and the soft-breaking $A_t$ ($A_b$) parameter when there is the left-right mixing in the stop (sbottom) sector. Their one-loop effects can also be nondecoupling and then can be significant to some extent when they are taken to be as large as the scale of the soft-SUSY-breaking mass $m_{\text{SUSY}}$. In the NMSSM and the MSSM with triplets, $m_h$ can be significantly enhanced by the F-term contribution [19–21]. Notice that these F-term contributions should vanish in the SUSY limit due to the nonrenormalization theorem. These F-term contributions also affect the prediction on the other SM observables such as the triple Higgs boson coupling constants $\lambda_{hhh}$ similarly to the case of non-SUSY extended Higgs models which we already discussed above [22].

On the other hand, a typical example for extended SUSY Higgs sectors without interactions from the tree-level F-term is that with only multi-doublet structures, such as the 4HDSSM. In this class of models, if there is no mixing between the light two doublet fields and the additional ones, the effects of the extra fields on the MSSM observables become suppressed due to the decoupling theorem when the extra doublet fields are heavy, and the model behaves like the MSSM. However, nonvanishing effects can appear through the B-term mixing when $(M_3^2)_{12}$ or $(M_3^2)_{21}$ grows with $(M_1^2)_{22}$ or $(M_2^2)_{22}$. These effects appear at the tree level, so that they would give substantial modifications in the predictions in the MSSM for the low energy observables. We stress that these nonvanishing effects due to the B-term mixing are not the nondecoupling effects which appear in the large mass limit for the new particles when $X v^2 \gtrsim \tilde{M}^2$ as a consequence of violation of the decoupling theorem. In this sense, we call the nonvanishing B-term mixing effect as the quasi-nondecoupling effect. Notice that the quasi-nondecoupling effect only appears in the predicted values in the MSSM. It gives modifications in the MSSM predictions such as the masses of $h$, $H$ and $H^\pm$ and the mixing angle $\alpha$ as well as coupling constants for the MSSM-like Higgs bosons. Such an effect, however, does disappear in the predictable SM coupling constants of $h\gamma\gamma$, $hWW$, $hZZ$ and $hhh$ in the SM-like limit ($m_A \to \infty$) according to the decoupling theorem.

Therefore, we would like to address the question of how the extra doublet fields in the extended SUSY model can affect the observables which appear in the MSSM, such as the mass $m_\phi$ ($\phi$ represent $h$, $H$ and $H^\pm$), the mixing angle $\alpha$, the vertex $F_{\phi'VV}$ ($V = W^\pm$ and $Z$; $\phi' = h$ and $H$) as well as quarks and leptons $Y_{\phi'^ff'}$ ($f, f'$ represent fermions and $\phi'' = h$, $H$, $A$ and $H^\pm$). Deviations from the renormalized MSSM observable parameters
may be expressed as

\[ m_\phi \simeq m_\phi^{\text{MSSM}} (1 + \delta_\phi), \]  
\[ \sin^2(\beta - \alpha_{\text{eff}}) \simeq [\sin^2(\beta - \alpha)]^{\text{MSSM}} (1 + \delta_s), \]  
\[ F_{\phi V V} \simeq F_{\phi V V}^{\text{MSSM}} (1 + \delta_{\phi V V}), \]  
\[ Y_{\phi' \ell f f'} \simeq Y_{\phi' \ell f f'}^{\text{MSSM}} (1 + \delta_{\phi' \ell f f'}), \]

where \( \delta_\phi, \delta_s, \delta_{\phi V V} \) or \( \delta_{\phi' \ell f f'} \) represent the effect of the extra heavy scalar fields on each observable in the extended SUSY models. In this paper, we study \( \delta_\phi, \delta_s, \delta_{\phi V V} \) and \( \delta_s \) in the 4HDSSM. The MSSM predictions are evaluated at the one-loop level using the on-shell renormalization scheme in Ref. [14]. We do not discuss the effect on the Yukawa coupling constants in this paper, which will be studied in details elsewhere [35].

### 3.2 Definition of the large mass limit and the decoupling property in the 4HDSSM

The soft-breaking mass parameters \( (M_3^2)_{ij} \) come from the B-terms in Eq. (2.2). When we consider the case with \( (M_3^2)_{12} = (M_3^2)_{21} = 0 \), the mass matrices \( \tilde{M}_A^2, \tilde{M}_{H^\pm}^2 \) and \( \tilde{M}_{H}^2 \) are block diagonal. The upper \( 2 \times 2 \) submatrix in each mass matrix corresponds to that in the MSSM; i.e., \( 2(M_3^2)_{11}/s_{23} \rightarrow m_\tau^2 \), and the other \( 2 \times 2 \) submatrix corresponds to that for the extra two scalar bosons. They are separated completely in this case. The model effectively becomes the MSSM in the large mass limit of the extra scalar bosons. On the other hand, in the case with nonzero \( (M_3^2)_{12} \) or \( (M_3^2)_{21} \), the masses of the light scalars \( h, H \) and \( H^\pm \) are modified from the MSSM predictions by the mixing via the B-terms between \( \Phi_1 \) and \( \Phi_1' \) or between \( \Phi_2 \) and \( \Phi_2' \). These effects are expected to be nonvanishing when \( (M_3^2)_{12} \) or \( (M_3^2)_{21} \) grows with taking a similar value to the 3-3 or 4-4 component in the mass matrices such as \( (M_3^2)_{22} \) or \( (M_3^2)_{11} \). We here discuss these effects in details in the following.

We start from discussing the CP-odd scalar mass matrix. In order to examine the decoupling property of the mass matrices, we further rotate \( \tilde{M}_A^2 \) as

\[ \tilde{M}_A^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_\tilde{\theta} & s_\tilde{\theta} \\ 0 & 0 & -s_\tilde{\theta} & c_\tilde{\theta} \end{pmatrix}, \]

\[ \tilde{M}_A^2 \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_\tilde{\theta} & -s_\tilde{\theta} \\ 0 & 0 & s_\tilde{\theta} & c_\tilde{\theta} \end{pmatrix}, \]

\[ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & k'M^2 & kM^2 \\ 0 & k'M^2 & M^2 & 0 \\ 0 & kM^2 & 0 & rM^2 \end{pmatrix}, \]  

where

\[ \tan 2\tilde{\theta} = \frac{2(M_3^2)_{22}}{(M_3^2)_{22} - (M_2^2)_{22} + m_Z^2 c_{2\tilde{\theta}}'}, \]  

and \( M^2, k, k' \) and \( r \) are defined such that

\[ (M_3^2)_{21} = \kappa_{21} M^2, \quad (M_3^2)_{12} = \kappa_{12} M^2, \]  
\[ (M_1^2)_{22} c_\tilde{\theta}^2 + (M_2^2)_{22} s_\tilde{\theta}^2 + (M_3^2)_{22} s_{2\tilde{\theta}} + \frac{m_Z^2}{2} c_{23} c_{2\tilde{\theta}} = M^2, \]  
\[ (M_1^2)_{22} s_\tilde{\theta}^2 + (M_2^2)_{22} c_\tilde{\theta}^2 - (M_3^2)_{22} s_{2\tilde{\theta}} - \frac{m_Z^2}{2} c_{23} c_{2\tilde{\theta}} = r M^2, \]  

(3.10)
and

\[ k' = \frac{c_\beta}{s_\beta} \kappa_{21} + \frac{s_\beta}{c_\beta} \kappa_{12}, \quad k = -\frac{s_\beta}{c_\beta} \kappa_{21} + \frac{c_\beta}{s_\beta} \kappa_{12}. \]  

(3.11)

These parameters are relevant to the extra doublets, then the decoupling limit is taken as \( M^2 \to \infty \). Here we assume that \( m_A^2 \ll M^2 \) and we treat \( m_A^2/M^2 \) as an expansion parameter. One of the eigenvalues of \( m_H \), the mass eigenvalues for heavier states \( A_1 \) and \( A_2 \) as

\[ \frac{2(M^2_{11})}{s_{2\beta}} = m_A^2 \left\{ \frac{k^2 + (1 + k'^2)r^2}{r^2} + O\left( \frac{m_A^2}{M^2} \right) \right\} + M^2 \frac{k^2 + rk'^2}{r}. \]  

(3.12)

The mass eigenvalues for heavier states \( A_1 \) and \( A_2 \) are

\[ m_{A_1}^2 \simeq a_1 M^2, \quad m_{A_2}^2 \simeq a_2 M^2, \]  

(3.13)

where \( a_1 \) and \( a_2 \) are given by

\[ a_1 = \frac{k^2 + rk'^2 + r(1 + r) - \sqrt{(k^2 + rk'^2 - r(r - 1))^2 + 4k^2 r(r - 1)}}{2r}, \]

\[ a_2 = \frac{k^2 + rk'^2 + r(1 + r) + \sqrt{(k^2 + rk'^2 - r(r - 1))^2 + 4k^2 r(r - 1)}}{2r}. \]  

(3.14)

We note that \( a_1 \to 1 \) and \( a_2 \to r \) for \( k = k' \to 0 \).

For the charged Higgs mass matrix, via the similar procedure to the case of the CP-odd Higgs bosons, we obtain the deviation in \( m_{H^\pm} \), the mass eigenvalue for the lightest charged scalar \( H^\pm \), from the MSSM prediction as

\[ m_{H^\pm} = \sqrt{(m_{H^\pm}^{MSSM})^2 (1 + \delta_{H^\pm})}, \]  

(3.15)

where

\[ \delta_{H^\pm} = -\frac{1}{2} m_W^2 \frac{k^2 + k'^2 r^2 - c_{2\beta} \left\{ (k^2 - k'^2 r^2) c_{2\beta} + 2kk'rs_{2\beta} \right\}}{\left\{ k^2 + (1 + k'^2) r^2 \right\}} + O\left( \frac{m_A^2}{M^2} \right), \]  

(3.16)

and \( (m_{H^\pm}^{MSSM}) \) is the prediction in the MSSM renormalized in the on-shell scheme [14], which is simply given by [47]

\[ (m_{H^\pm}^{MSSM}) = m_A^2 + m_W^2 - \Pi_{H^\pm H^\pm}^{PI}(m_A^2 + m_W^2) + \Pi_{AA}(m_A^2) + \Pi_{WW}(m_W^2), \]  

(3.17)

where \( \Pi_{\phi\phi}(p^2) \) represent the one particle irreducible diagram contributions to the two point function of the field \( \phi \) at the squared momentum \( p^2 \). Masses of the heavier charged scalar bosons \( H_1^+ \) and \( H_2^\pm \) are obtained as

\[ m_{H_1^+}^2 \simeq a_1 M^2, \quad m_{H_2^\pm}^2 \simeq a_2 M^2. \]  

(3.18)
Figure 1. The deviation $\delta_{H^\pm}$ defined in Eq. (3.15) due to the quasi-nondecoupling effect of the B-term mixing parameterized by $k$ and $k'$ in the 4HDSSM. We here take $M = 500$ GeV, $r = 1$ and $\bar{\theta} = 0$. The SUSY breaking scale for the MSSM particles is taken to be 1 TeV, and the trilinear soft-breaking parameters $A_t$ and $A_b$ as well as the $\mu$ parameter are taken to be zero. The upper figures: $\delta_{H^\pm}$ as a function of $m_A$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $k = 1.0, 2.0$ and $5.0$ with fixed $k'(=0.0)$. The lower figures: $\delta_{H^\pm}$ as a function of $k$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $k' = 0.0, 2.0$ and $5.0$ with the fixed $m_A (=150$ GeV). In all figures, the solid curves are the results from the full numerical calculation, while the dotted curves are those by using the approximated formula in Eq. (3.16).

In Fig. 1, we show the numerical results for the deviation $\delta_{H^\pm}$ defined in Eq. (3.15) due to the quasi-nondecoupling effect of the B-term mixing parameterized by $k$ and $k'$ in our model. The solid curves in the figures represent the results from the full numerical calculation, while the dotted curves are those by using the approximated formula in Eq. (3.16). The deviation $\delta_{H^\pm}$ turns out to be negative, and amounts to $-20\%$ for a relatively small value of $m_A$. The magnitude of the deviation is smaller for a larger value of $m_A$, but still a few times $-1\%$ even for $m_A = 300$ GeV. On the other hand, the deviation is not very sensitive to $\tan \beta$. We note that the results are insensitive to the details of the MSSM parameters such as the soft-breaking mass parameters, the $\mu$ parameter and the trilinear $A_{t,b}$ parameters. In fact, when $\mu$ and $A_{t,b}$ are varied in the phenomenologically acceptable regions, the radiative corrections vary at most from $-2\%$ to $+2\%$. We have confirmed that our results on the one-loop correction in the MSSM agree with those given in Ref. [47].
\[ e^+e^- \rightarrow H^+H^- \] at the ILC [49]. The mass of \( A \) can also be determined with the resolution about 2% via the decays \( A \rightarrow \mu^+\mu^- \) at the LHC, while at the ILC it can be measured with the precision 0.2% via \( e^+e^- \rightarrow HA \) [49]. Therefore, the quasi-nondecoupling effect on \( m_{H^\pm} \) can be extracted when both \( m_{H^\pm} \) and \( m_A \) are measured at future collider experiments. The prediction on \( m_{H^\pm} \) (not on \( \delta_{H^\pm} \)) in the 4HDSSM is shown in Fig. 5 with the comparison of the result in the MSSM.

Next, the CP-even scalar mass matrix \( \hat{M}_H^2 \) can also be diagonalized. We first define \( \hat{M}_H^2 \) by

\[
\hat{M}_H^2 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & c_\theta & -s_\theta \\
0 & 0 & s_\theta & c_\theta
\end{pmatrix}
\begin{pmatrix}
\hat{M}_H^2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & c_\theta & s_\theta \\
0 & 0 & -s_\theta & c_\theta
\end{pmatrix},
\]

and according to the usual mathematical procedure \( \hat{M}_H^2 \) can be block-diagonalized by rotating the basis with an appropriate orthogonal matrix \( O \) as

\[
O_{MH}^T \hat{M}_H^2 O_{MH} = \begin{pmatrix}
-\frac{m^2_Z}{2} c^2_\theta & -m^2_Z c_\theta s_\theta R & 0 & 0 \\
-\frac{m^2_Z}{2} s^2_\theta R & \frac{m^2_A}{2} + m^2_Z s^2_\theta R^2 & 0 & 0 \\
0 & 0 & a_1 M^2 & 0 \\
0 & 0 & 0 & a_2 M^2
\end{pmatrix} + \mathcal{O}\left(\frac{m^2_A}{M^2}\right),
\]

where \( R \) is defined as

\[
R = \frac{1}{\sqrt{1 + \frac{k^2}{r^2} + k'^2}}.
\]

The upper \( 2 \times 2 \) submatrix coincides to the mass matrix of the two light scalar bosons \( H \) and \( h \) of the MSSM when \( M \rightarrow \infty \) if \( k = k' = 0 \). For the case with nonzero \( k \) and \( k' \), after diagonalizing the \( 2 \times 2 \) submatrix by the mixing angle \( \alpha_{\text{eff}} \) the mass eigenvalues of the CP-even Higgs bosons are obtained as

\[
m_h^2 = \frac{1}{2} m_A^2 + \frac{m^2_Z}{2} \left( c^2_\theta + R^2 s^2_\theta \right) - \left\{ \left( m_A^2 - m^2_Z \left( 1 - (1 - R^2) s^2_\theta \right) \right)^2 + 4 m^2_A m^2_Z s^2_\theta R^2 \right\} + m^2_A \mathcal{O}\left(\frac{m^2_A}{M^2}\right) + \Delta_{h}^\text{loop},
\]

\[
m_H^2 = \frac{1}{2} m_A^2 + \frac{m^2_Z}{2} \left( c^2_\theta + R^2 s^2_\theta \right) + \left\{ \left( m_A^2 - m^2_Z \left( 1 - (1 - R^2) s^2_\theta \right) \right)^2 + 4 m^2_A m^2_Z s^2_\theta R^2 \right\} + m^2_A \mathcal{O}\left(\frac{m^2_A}{M^2}\right) + \Delta_{H}^\text{loop},
\]

where \( \Delta_{h}^\text{loop} \) and \( \Delta_{H}^\text{loop} \) represent the one-loop corrections in the MSSM. The masses of heavier states \( H'_1 \) and \( H'_2 \) are given by \( m_{H'_1}^2 \simeq a_1 M^2 \{ 1 + \mathcal{O}(m^2_A/M^2) \} \) and \( m_{H'_2}^2 \simeq a_2 M^2 \{ 1 + \mathcal{O}(m^2_A/M^2) \} \). The mixing angle \( \alpha_{\text{eff}} \) satisfies the relation

\[
\tan(\beta - \alpha_{\text{eff}}) = \frac{m_h^2 - m_A^2 - m_Z^2 s^2_\theta R^2}{m_Z^2 c^2_\theta s^2_\theta R} \left\{ 1 + \mathcal{O}\left(\frac{m^2_A}{M^2}\right) + \Delta_{\tan(\beta-\alpha)}^\text{loop} \right\},
\]
Figure 2. The deviation $\delta_h$ in $m_h = m_{h}^{\text{MSSM}}(1 + \delta_h)$ due to the quasi-nondecoupling effect of the B-term mixing parameterized by $k$ and $k'$ in the 4HDSSM, where $m_{h}^{\text{MSSM}}$ is the renormalized mass of $h$. We here take $M = 500$ GeV, $r = 1$ and $\theta = 0$. The SUSY soft-breaking scale for the MSSM particles is taken to be 1TeV (solid curves) and 2 TeV (dotted curves), and the trilinear soft-breaking parameters $A_t$ and $A_t$ as well as the $\mu$ parameter are taken to be zero. The upper figures: $\delta_h$ as a function of $m_A$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $k = 1.0, 2.0$ and $5.0$ with fixed $k'(= 0.0)$. The lower figures: $\delta_h$ as a function of $k$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $k' = 0.0, 2.0$ and $5.0$ with the fixed $m_A (= 150$ GeV).

where $\Delta_{\tan(\beta-\alpha)}^{\text{loop}}$ is the one-loop correction in the MSSM. Notice that $m_h$ and $m_H$ given in Eq. (3.22) and $\tan(\beta - \alpha_{\text{eff}})$ in Eq. (3.23) do not depend on the sign of $k$ and $k'$. We note that the effective mixing angle $\alpha_{\text{eff}}$ contains information of the B-term quasi-nondecoupling effects between $\Phi_1$ and $\Phi'_1$ or between $\Phi_2$ and $\Phi'_2$ by $k$ and $k'$, but for $m_A^2 \ll M^2$ the tree level formula with the angle $\alpha$ in the MSSM can still hold by replacing $\alpha$ by $\alpha_{\text{eff}}$ in a good approximation. For example, the coupling constants of the two light CP-even Higgs bosons with the weak gauge bosons $V (V = W^\pm$ and $Z^0$) in the case with nonzero $k$ and $k'$ are given by

$$
\Gamma_{VVh} = -\frac{m_V^2}{v} \left(c_{\beta}(O_H)_{12} + s_{\beta}(O_H)_{22}\right) = \frac{m_V^2}{v} \sin(\beta - \alpha_{\text{eff}}) \left(1 + \Delta_{\text{loop}}^{\text{hVV}}\right),
$$

$$
\Gamma_{VVH} = -\frac{m_V^2}{v} \left(c_{\beta}(O_H)_{11} + s_{\beta}(O_H)_{21}\right) = \frac{m_V^2}{v} \cos(\beta - \alpha_{\text{eff}}) \left(1 + \Delta_{\text{loop}}^{\text{hVV}}\right),
$$

where the matrix $O_H$ is given in Eq. (B.5) in Appendix B, and $\Delta_{\text{loop}}^{\text{hVV}}$ and $\Delta_{\text{HVV}}^{\text{loop}}$ represent radiative corrections in the MSSM. Finally, in general, magnitudes of $k$ and $k'$ are not necessarily smaller than 1, still it is helpful to deduce the approximate formulae assuming
Figure 3. The deviation $\delta_H$ in $m_H = m_H^{\text{MSSM}} (1 + \delta_H)$ of the renormalized mass of the second lightest CP-even Higgs boson $H$ due to the quasi-nondecoupling effect of the B-term mixing parameterized by $k$ and $k'$ in the 4HDSSM. We here take $M = 500$ GeV, $r = 1$ and $\theta = 0$. The SUSY soft-breaking scale of the MSSM particles is taken to be 1 TeV (solid curves) and 2 TeV (dotted curves), and the trilinear soft-breaking parameters $A_t$ and $A_b$ as well as the $\mu$ parameter are taken to be zero. The upper figures: $\delta_H$ as a function of $m_A$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $k = 1.0$, 2.0 and 5.0 with fixed $k' (= 0.0)$. The lower figures: $\delta_H$ as a function of $k$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $k' = 0.0$, 2.0 and 5.0 with the fixed $m_A (= 150$ GeV), that they are small;

$$m_h^2 = (m_h^2)^{\text{MSSM}} \left(1 + \frac{m_Z^2 s^2_2 (k^2 + k'^2)}{\sqrt{(m_A^2 - m_Z^2)^2 + 4m_Z^2 m_A^2 s^2_2}} + O(k^4, k'^4, k^2 k'^2) + O \left( \frac{m_A^2}{M^2} \right) \right),$$

$$m_H^2 = (m_H^2)^{\text{MSSM}} \left(1 - \frac{m_Z^2 s^2_2 (k^2 + k'^2)}{\sqrt{(m_A^2 - m_Z^2)^2 + 4m_Z^2 m_A^2 s^2_2}} + O(k^4, k'^4, k^2 k'^2) + O \left( \frac{m_A^2}{M^2} \right) \right),$$

$$\tan(\beta - \alpha_{\text{eff}}) = (\tan(\beta - \alpha))^{\text{MSSM}} \left(1 + \frac{(m_A^2 - 2m_h^2 - m_Z^2 s^2_2) (k^2 + k'^2)}{2(m_A^2 - m_h^2 + m_Z^2 s^2_2)} + O(k^4, k'^4, k^2 k'^2) + O \left( \frac{m_A^2}{M^2} \right) \right),$$

where $(m_h^2)^{\text{MSSM}}$, $(m_H^2)^{\text{MSSM}}$ and $(\tan(\beta - \alpha))^{\text{MSSM}}$ are the corresponding parameters eval-
Figure 4. The deviation $\delta_s$ in Eq. (3.5) due to the quasi-nondecoupling effect of extra doublet fields via the B-term mixing parameterized by $k$ and $k'$ in the 4HDSSM. We here take $M = 500$ GeV, $r = 1$ and $\bar{\theta} = 0$. The SUSY soft-breaking scale of the MSSM particles is taken to be 1 TeV (solid curves) and 2 TeV (dotted curves), and the trilinear soft-breaking parameters $A_t$ and $A_b$ as well as the $\mu$ parameter are taken to be zero. The upper figures: $\delta_s$ as a function of $m_A$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $k = 1.0, 2.0$ and $5.0$ with fixed $k'(=0.0)$. The lower figures: $\delta_s$ as a function of $k$ for $\tan \beta = 3$ (left) and $\tan \beta = 10$ (right) for $k' = 0.0, 2.0$ and $5.0$ with the fixed $m_A (= 150$ GeV).

In Fig. 2, we show the numerical results for the deviation $\delta_h$ in $m_h = m_h^{\text{MSSM}}(1 + \delta_h)$, where $m_h^{\text{MSSM}}$ is the one-loop corrected mass of $h$, due to the quasi-nondecoupling effect of the B-term mixing parameterized by $k$ and $k'$ in the 4HDSSM. The SUSY soft-breaking scale of the MSSM is taken to be 1 TeV and 2 TeV, and the trilinear soft-breaking parameters $A_t$ and $A_b$ and the $\mu$ are taken to be zero. It is found that $\delta_h$ is always positive. This is understood from Eq. (3.20). The parameter $R$ is unity for $k = k' = 0$, and is smaller for larger values of $k$ and $k'$. A smaller value of $R$ ($R < 1$) reduces the value of the off-diagonal term in Eq. (3.20), which makes the mixing between the first two CP-even states weaker. Consequently, the mass difference between $h$ and $H$ becomes smaller than the case with the MSSM case with the same value of $m_A$ and $\tan \beta$. The deviation takes its maximal values (6-20 % for $\tan \beta = 3$ and 2-5 % for $\tan \beta = 10$) around the crossing point ($m_A \sim 130-150$ GeV) where the role of $h$ and $H$ are exchanged. For larger values of $m_A$
the magnitude of \(\delta_h\) is smaller, but it can be still 3-6 % (about 1 %) at \(m_A = 200\) GeV for \(\tan\beta = 3\) (10). These values are substantial and can be tested by the precise measurement of \(m_h\) at the LHC (the ILC), where \(m_h\) is expected to be determined with about 0.1\% [50] accuracy at the LHC, while at the ILC it is expected to be measured within less than 70 MeV [51]) error. The prediction on \(m_h\) (not on \(\delta_h\)) in the 4HDSSM is shown in Fig. 5 with the comparison of the result in the MSSM. We can see that in the 4HDSSM \(m_h\) reaches its maximal value at a smaller \(m_A\) than that in the MSSM, although the predicted upper bound on the \(m_h\) is the same in both models.

In Fig. 3, we show the deviation \(\delta_H\) in \(m_H = m_H^{\text{MSSM}}(1 + \delta_H)\), where \(m_H^{\text{MSSM}}\) is the one-loop corrected mass of \(H\), due to the quasi-nondecoupling effect of the B-term mixing parameterized by \(k\) and \(k'\) in the 4HDSSM. The SUSY parameters are taken as in the same way as Fig. 2. As we discussed, the mixing of the light two CP-even states is weakened by non-zero values of \(k\) and \(k'\), so that \(m_H\) is smaller than the prediction in the MSSM. Therefore, \(\delta_H\) is negative as we expect. The behavior of \(\delta_H\) as a function of \(m_A\) and \(\tan\beta\) are similar to the case of \(\delta_h\) except for the sign. The magnitude is maximal around the crossing point (\(m_A = 130-150\) GeV), and amounts to \(-18\%\) (\(-5\%\)) for \(\tan\beta = 3\) (10). At the LHC and the ILC, the mass of \(H\) can be determined with the similar precision to that of \(A\) mentioned in the previous paragraph. The prediction on \(m_H\) (not on \(\delta_H\)) in the 4HDSSM is shown in Fig. 5 with the comparison with the result in the MSSM.

In Fig. 4, we show the numerical results for the deviation \(\delta_s\) defined in Eq. (3.15), in which \([\sin^2(\beta - \alpha)]^{\text{MSSM}}\) is the one-loop corrected mixing factor \(\sin^2(\beta - \alpha)\) evaluated in the MSSM. \(\delta_s\) is the net deviation from the MSSM prediction due to the quasi-nondecoupling effect of the B-term mixing parameterized by \(k\) and \(k'\) in the 4HDSSM. The SUSY soft-breaking scale of the MSSM is taken to be 1 TeV and 2 TeV, and the trilinear soft-breaking parameters \(A_t\) and \(A_b\) and the \(\mu\) parameter are taken to be zero. In the figures, we can see that \(\delta_s\) is negative when \(m_A\) is smaller than the crossing point at \(m_A \sim 130-150\) GeV, while it is positive for larger \(m_A\). The deviation can be as large as \(\mathcal{O}(10)\%\) (\(\tan\beta = 3\)) and \(\mathcal{O}(20)\%\) (\(\tan\beta = 10\)) just above the crossing point; i.e., at around \(m_A \sim 140-150\) GeV. It is rapidly close to unity for larger values of \(m_A\). Notice that for larger soft-SUSY-breaking scale, a larger \(\delta_s\) is possible. The prediction on \(\sin^2(\beta - \alpha_{\text{eff}})\) (not on \(\delta_s\)) in the 4HDSSM is shown in Fig. 5 with the comparison with the result in the MSSM.

4 Conclusions

We have investigated the decoupling property of extra heavy doublet fields in the 4HDSSM. Even without interaction terms from the tree-level F-term contribution such as in the NMSSM, significant quasi-nondecoupling effects of extra scalar fields can occur at the tree level due to the B-term mixing among the Higgs bosons. We have deduced formulae for deviations in the MSSM observables in the decoupling region for the extra heavy fields. The possible modifications in the Higgs sector from the MSSM predictions have been studied numerically.

From the results shown in Fig. 1 to Fig. 5, we have found that the quasi-nondecoupling effect from the B-term mixing can be significant in the 4HDSSM, which can change the
MSSM observables $m_{H^\pm}, m_h, m_H$ and $\sin^2(\beta - \alpha)$ to a considerable extent. When the Higgs boson $h$ is found via the processes of gluon fusion or vector boson fusion at the LHC, $m_h$ is expected to be measured very accurately. The correction $\delta_0$ due to the quasi-nondcoupling effect can be much larger than the expected error at the LHC and the ILC. Therefore, we conclude that the effect on $m_h$ can be measured. The other MSSM Higgs bosons $H, A$ and $H^\pm$ are expected to be discovered at the LHC as long as $m_A$ is not too large. The deviations due to the quasi-nondcoupling effect in these quantities may also be identified when they are precisely measured at the LHC or the ILC.

Detecting the deviations from the MSSM predictions on these MSSM observables, the MSSM Higgs sector can be tested, and at the same time the possibility of extended SUSY Higgs sectors including the 4HDSSM can be explored even when only the MSSM particles are discovered in near future at the LHC and at the ILC. A detailed discussion on the Yukawa sector is given elsewhere.

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A Rotation of the basis

The Higgs potential in our model is given from Lagrangian in Eq. (2.3) by

$$
V_H = (M^2)_{11} \hat{\Phi}_1^\dagger \hat{\Phi}_1 + (M^2)_{13} \hat{\Phi}_1^\dagger \tilde{\Phi}_3 + (M^2)_{13} ^* \tilde{\Phi}_3^\dagger \hat{\Phi}_1 + (M^2)_{33} \tilde{\Phi}_3^\dagger \tilde{\Phi}_3
$$

$$
+ (M^2)_{22} \hat{\Phi}_2^\dagger \hat{\Phi}_2 + (M^2)_{24} \hat{\Phi}_4^\dagger \hat{\Phi}_4 + (M^2)_{24} ^* \hat{\Phi}_4^\dagger \hat{\Phi}_2 + (M^2)_{44} \hat{\Phi}_4^\dagger \hat{\Phi}_4
$$

$$
+ \left( B_{12} \mu_{12} \hat{\Phi}_1 \cdot \hat{\Phi}_2 + B_{34} \mu_{34} \tilde{\Phi}_3 \cdot \hat{\Phi}_4 + B_{14} \mu_{14} \hat{\Phi}_1 \cdot \hat{\Phi}_4 + B_{32} \mu_{32} \tilde{\Phi}_3 \cdot \hat{\Phi}_2 + h.c. \right)
$$

$$
+ \frac{g^2 + g'^2}{8} \left( \tilde{\Phi}_1^\dagger \tilde{\Phi}_2 - \tilde{\Phi}_1^\dagger \tilde{\Phi}_4 - \hat{\Phi}_1^\dagger \hat{\Phi}_3 - \tilde{\Phi}_3^\dagger \tilde{\Phi}_3 \right)^2
$$

$$
+ \frac{g^2}{2} \left\{ (\tilde{\Phi}_1^\dagger \tilde{\Phi}_2)^2(\tilde{\Phi}_1^\dagger \tilde{\Phi}_2) + (\tilde{\Phi}_1^\dagger \tilde{\Phi}_4)^2(\tilde{\Phi}_1^\dagger \tilde{\Phi}_4) + (\tilde{\Phi}_3^\dagger \tilde{\Phi}_2)^2(\tilde{\Phi}_3^\dagger \tilde{\Phi}_2) + (\tilde{\Phi}_3^\dagger \tilde{\Phi}_4)^2(\tilde{\Phi}_3^\dagger \tilde{\Phi}_4) \\
+ (\hat{\Phi}_1^\dagger \hat{\Phi}_1)(\tilde{\Phi}_3^\dagger \tilde{\Phi}_3) - (\tilde{\Phi}_1^\dagger \tilde{\Phi}_1)(\hat{\Phi}_3^\dagger \hat{\Phi}_3) + (\hat{\Phi}_2^\dagger \hat{\Phi}_4)(\hat{\Phi}_4^\dagger \hat{\Phi}_2) - (\hat{\Phi}_2^\dagger \hat{\Phi}_2)(\hat{\Phi}_4^\dagger \hat{\Phi}_4) \right\},
$$

(A.1)

where $g$ and $g'$ are the SU(2)$_L$ and U(1)$_Y$ gauge couplings respectively, and

$$
(M^2)_{11} = (\tilde{M}^2)_{11} + |\mu_{12}|^2 + |\mu_{14}|^2,
$$

$$
(M^2)_{33} = (\tilde{M}^2)_{33} + |\mu_{32}|^2 + |\mu_{34}|^2,
$$

$$
(M^2)_{13} = (\tilde{M}^2)_{13} + \mu^*_{12} \mu_{32} + \mu^*_{14} \mu_{34},
$$

$$
(M^2)_{22} = (\tilde{M}^2)_{22} + |\mu_{12}|^2 + |\mu_{32}|^2,
$$

$$
(M^2)_{24} = (\tilde{M}^2)_{24} + |\mu_{14}|^2 + |\mu_{34}|^2,
$$

$$
(M^2)_{44} = (\tilde{M}^2)_{44} + |\mu_{12}|^2 + |\mu_{32}|^2,
$$

$$
(M^2)_{24} = (\tilde{M}^2)_{24} + \mu^*_{12} \mu_{14} + \mu^*_{32} \mu_{34}.
$$

(A.2)
Because $\hat{\Phi}_1$ and $\hat{\Phi}_3$, and $\hat{\Phi}_2$ and $\hat{\Phi}_4$ have respectively the same quantum numbers, we may rotate the basis by introducing the $2 \times 2$ unitary mixing matrices $U_+$ and $U_-$ as

$$
\begin{align*}
(\hat{\Phi}_1, \hat{\Phi}_3) &\rightarrow (\Phi_1, \Phi'_3) = U_- (\hat{\Phi}_1, \hat{\Phi}_3), \\
(\hat{\Phi}_2, \hat{\Phi}_4) &\rightarrow (\Phi_2, \Phi'_4) = U_+ (\hat{\Phi}_2, \hat{\Phi}_4).
\end{align*}
$$

(A.3)

By using this degrees of freedom, one may choose the basis where only $\Phi_1$ and $\Phi_2$ have VEV's while those of $\Phi'_1$ and $\Phi'_2$ are zero. In this basis, the Higgs potential is expressed by

$$
V_H = (\Phi_1, \Phi'_1) U_- \left( \begin{array}{cc} (M^2)_{11} & (M^2)_{13} \\ (M^2)_{13} & (M^2)_{33} \end{array} \right) U_+^\dagger (\Phi_1, \Phi'_1) + (\Phi_2, \Phi'_2) U_+ \left( \begin{array}{cc} (M^2)_{22} & (M^2)_{24} \\ (M^2)_{24} & (M^2)_{44} \end{array} \right) U_+^\dagger (\Phi_2, \Phi'_2)
+ \frac{g^2 + g'^2}{8} (\Phi_1^2 \Phi_2 + \Phi_2^2 \Phi_1 - \Phi_1^2 \Phi_1 - \Phi_2^2 \Phi'_1)^2
+ \frac{g^2}{2} \left((\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + (\Phi_1^\dagger \Phi_2')(\Phi_2^\dagger \Phi_1') + (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1') + (\Phi_1^\dagger \Phi_2')(\Phi_2^\dagger \Phi_2') \right).
$$

(A.4)

Hereafter, we reparameterize the parameters as

$$
U_- \left( \begin{array}{cc} (M^2)_{11} & (M^2)_{13} \\ (M^2)_{13} & (M^2)_{33} \end{array} \right) U_+^\dagger \rightarrow \left( \begin{array}{cc} (M^2)_{11} & (M^2)_{12} \\ (M^2)_{12} & (M^2)_{22} \end{array} \right),
$$

$$
U_+ \left( \begin{array}{cc} (M^2)_{22} & (M^2)_{24} \\ (M^2)_{24} & (M^2)_{44} \end{array} \right) U_+^\dagger \rightarrow \left( \begin{array}{cc} (M^2)_{11} & (M^2)_{12} \\ (M^2)_{12} & (M^2)_{22} \end{array} \right),
$$

$$
-U_* \left( \begin{array}{cc} B_{12} & B_{14} \\ B_{32} & B_{34} \end{array} \right) U_+^\dagger \rightarrow \left( \begin{array}{cc} M_{33} & M_{31} \\ M_{31} & M_{22} \end{array} \right).
$$

(A.5)

With the above notation, the Higgs potential can be rewritten as in Eq. (2.4).

**B Yukawa Interactions**

In the gauge eigenstate basis, the Yukawa coupling matrices $(Y_f)_{ij}$ and $(Y'_f)_{ij}$ ($f = u, d, e$) associated with $\Phi_k$ and $\Phi'_k$ ($k = 1$ for $f = d, e$ and $k = 2$ for $f = u$) are given as

$$
(\begin{array}{c}
Y_u \\
Y_d \\
Y_e
\end{array})_{ij} = (U_+)^{\dagger}_{11}(\tilde{Y}_u)_{ij} + (U_+)^{\dagger}_{12}(\tilde{Y}_d)_{ij},
\quad
(\begin{array}{c}
Y_u \\
Y_d \\
Y_e
\end{array})_{ij} = (U_-)^{\dagger}_{11}(\tilde{Y}_u)_{ij} + (U_-)^{\dagger}_{12}(\tilde{Y}_d)_{ij},
\quad
(\begin{array}{c}
Y'_u \\
Y'_d \\
Y'_e
\end{array})_{ij} = (U_+)^{\dagger}_{21}(\tilde{Y}_u)_{ij} + (U_+)^{\dagger}_{22}(\tilde{Y}_d)_{ij},
\quad
(\begin{array}{c}
Y'_u \\
Y'_d \\
Y'_e
\end{array})_{ij} = (U_-)^{\dagger}_{21}(\tilde{Y}_u)_{ij} + (U_-)^{\dagger}_{22}(\tilde{Y}_d)_{ij}.
$$

(B.1)

The formulae here are given in the basis where $Y_u$ and $Y_d$ are diagonal, i.e.,

$$
(\begin{array}{c}
Y_u \\
Y_d \\
Y_e
\end{array})_{ij} = \frac{\sqrt{2}m_{u_d}}{v \sin \beta} \delta_{ij},
\quad
(\begin{array}{c}
Y_u \\
Y_d \\
Y_e
\end{array})_{ij} = \frac{\sqrt{2}m_{d_e}}{v \cos \beta} \delta_{ij},
\quad
(\begin{array}{c}
Y'_u \\
Y'_d \\
Y'_e
\end{array})_{ij} = \frac{\sqrt{2}m_{u}}{v \cos \beta} \delta_{ij}.
$$

(B.2)
where $m_u$, $m_d$, and $m_e$ are the masses of up-type quarks, down-type quarks, and the charged leptons, respectively, and $V_{ij}$ is the Cabbibo-Kobayashi-Maskawa matrix. The fields $f_{Ri}$ and $f_{Li}$ ($f = u, d, e$) denote the mass eigenstates, and the left-handed component of the quark mass eigenstates are embedded into the SU(2) doublets as

$$q_{Li} = \begin{pmatrix} u_{Li} \\ V_{ij}d_{Lj} \end{pmatrix}.$$  

(B.3)

From the superpotential given in Eq. (2.1), the Yukawa interactions between the matter fields and the neutral Higgs scalar fields are read as follows:

$$\mathcal{L} = - \left( \frac{m_u \delta_{ij}}{v s_{\beta}} (O_H)_{2a} + \frac{(Y_u')_{ij}}{\sqrt{2}} (O_H)_{4a} \right) \bar{u}_{Ri} u_{Li} \phi^\text{even}_a$$

$$- \left( \frac{m_d \delta_{ij}}{v c_{\beta}} (O_H)_{1a} + \frac{(Y_d')_{ij}}{\sqrt{2}} (O_H)_{3a} \right) \bar{d}_{Ri} d_{Li} \phi^\text{even}_a$$

$$- \left( \frac{m_e \delta_{ij}}{v c_{\beta}} (O_H)_{1a} + \frac{(Y_e')_{ij}}{\sqrt{2}} (O_H)_{3a} \right) \bar{\nu}_{Ri} \nu_{Li} \phi^\text{odd}_a$$

$$- i \left( \frac{m_u \delta_{ij}}{v} (\cot \beta (\bar{O}_A)_{2a} - (\bar{O}_A)_{1a}) + \frac{(Y_u')_{ij}}{v} (\bar{O}_A)_{4a} \right) \bar{u}_{Ri} u_{Li} \phi^\text{odd}_a$$

$$- i \left( \frac{m_d \delta_{ij}}{v} (\tan \beta (\bar{O}_A)_{2a} + (\bar{O}_A)_{1a}) + \frac{(Y_d')_{ij}}{v} (\bar{O}_H)_{3a} \right) \bar{d}_{Ri} d_{Li} \phi^\text{even}_a$$

$$- i \left( \frac{m_e \delta_{ij}}{v} (\tan \beta (\bar{O}_A)_{2a} + (\bar{O}_A)_{1a}) + \frac{(Y_e')_{ij}}{v} (\bar{O}_H)_{3a} \right) \bar{\nu}_{Ri} \nu_{Li} \phi^\text{even}_a + h.c.,$$  

(B.4)

where we parameterize the extra down-type Yukawa couplings as $(Y_d')_{ij} = (\tilde{Y}_d')_{ik} V_{jk}$, CP-even and CP-odd Higgs scalar bosons are written as $\phi^\text{even}_a = (H, h, H_1', H_2')$ and $\phi^\text{odd}_a = (z^0, A, A_1, A_2)$ with $z^0$ denoting a Nambu-Goldstone mode, and the mixing matrix $O_H$ and $\bar{O}_A$ are defined as

$$O_H^T M_H^2 O_H = \begin{pmatrix} m_H^2 & 0 & 0 & 0 \\ 0 & m_h^2 & 0 & 0 \\ 0 & 0 & m_{H_1'}^2 & 0 \\ 0 & 0 & 0 & m_{H_2'}^2 \end{pmatrix}, \quad \bar{O}_A^T \bar{M}_A^2 \bar{O}_A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m_A^2 & 0 & 0 \\ 0 & 0 & m_{A_1}^2 & 0 \\ 0 & 0 & 0 & m_{A_2}^2 \end{pmatrix},$$  

(B.5)

with $(\bar{O}_A)_{1a} = (\bar{O}_A)_{a1} = \delta_{1a}$.

For the Yukawa interactions with a charged scalar fields, they are found as

$$\mathcal{L} = \left( \frac{\sqrt{2} m_u \delta_{ik}}{v} (\cot \beta (\bar{O}_\pm)_{2a} - (\bar{O}_\pm)_{1a}) + (Y_u'^*)_{ik} (\bar{O}_\pm)_{4a} \right) V_{kj} \bar{u}_{Ri} u_{Li} \phi^+_a$$

$$+ \left( \frac{\sqrt{2} m_d \delta_{kj}}{v} (\tan \beta (\bar{O}_\pm)_{2a} + (\bar{O}_\pm)_{1a}) + (Y_d'^*)_{jk} (\bar{O}_\pm)_{3a} \right) V_{ik} \bar{d}_{Ri} d_{Li} \phi^+_a$$

$$+ \left( \frac{\sqrt{2} m_e \delta_{ij}}{v} (\tan \beta (\bar{O}_\pm)_{2a} + (\bar{O}_\pm)_{1a}) + (Y_e'^*)_{ij} (\bar{O}_\pm)_{3a} \right) \bar{\nu}_{Ri} \nu_{Li} \phi^+_a + h.c.,$$  

(B.6)
where the charged scalar bosons are written as \( \varphi^+_i = (w^+, H^+, H_1^+, H_2^+) \) with \( w^+ \) being a Nambu-Goldstone boson, and the mixing matrix \( \tilde{O}_\pm \) is defined as

\[
\tilde{O}_\pm^T M_{H_\pm}^2 \tilde{O}_\pm = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & m_{H_\pm}^2 & 0 & 0 \\
0 & 0 & m_{H_\pm}^2 & 0 \\
0 & 0 & 0 & m_{H_\pm}^2 \\
\end{pmatrix}, \tag{B.7}
\]

with \((\tilde{O}_\pm)_{1\alpha} = (\tilde{O}_\pm)_{1\alpha} = \delta_{1\alpha}\).

In the decoupling region \( m_A/M \ll 1 \), the Yukawa interactions with the MSSM Higgs bosons are given as

\[
\mathcal{L} = - \left\{ \frac{m_u}{v} \delta_{ij} \left( \sin(\beta - \alpha_{\text{eff}}) + R \cot \beta \cos(\beta - \alpha_{\text{eff}}) \right) \right.
\]
\[
- \left. \left( \frac{Y_u}{r} \right)_{ij} |k| R \cos(\beta - \alpha_{\text{eff}}) + \mathcal{O} \left( \frac{m_A^2}{M^2} \right) \right\} \bar{u}_{Ri} u_{Lj} h
\]
\[
- \left\{ \frac{m_d}{v} \delta_{ij} \left( \sin(\beta - \alpha_{\text{eff}}) - R \tan \beta \cos(\beta - \alpha_{\text{eff}}) \right) \right.
\]
\[
- \left. \left( \frac{Y_d}{r} \right)_{ij} |k'| R \cos(\beta - \alpha_{\text{eff}}) + \mathcal{O} \left( \frac{m_A^2}{M^2} \right) \right\} \bar{d}_{Ri} d_{Lj} h
\]
\[
- \left\{ \frac{m_u}{v} \delta_{ij} \left( \cos(\beta - \alpha_{\text{eff}}) - R \cot \beta \sin(\beta - \alpha_{\text{eff}}) \right) \right.
\]
\[
+ \left. \left( \frac{Y_u}{r} \right)_{ij} |k| R \sin(\beta - \alpha_{\text{eff}}) + \mathcal{O} \left( \frac{m_A^2}{M^2} \right) \right\} \bar{u}_{Ri} u_{Lj} H
\]
\[
- \left\{ \frac{m_d}{v} \delta_{ij} \left( \cos(\beta - \alpha_{\text{eff}}) + R \tan \beta \sin(\beta - \alpha_{\text{eff}}) \right) \right.
\]
\[
+ \left. \left( \frac{Y_d}{r} \right)_{ij} |k'| R \sin(\beta - \alpha_{\text{eff}}) + \mathcal{O} \left( \frac{m_A^2}{M^2} \right) \right\} \bar{d}_{Ri} d_{Lj} H
\]
\[
- \left\{ \frac{m_u}{v} \delta_{ij} \left( \cos(\beta - \alpha_{\text{eff}}) + R \tan \beta \sin(\beta - \alpha_{\text{eff}}) \right) \right.
\]
\[
+ \left. \left( \frac{Y_u}{r} \right)_{ij} |k| R \cot \beta + \mathcal{O} \left( \frac{m_A^2}{M^2} \right) \right\} \bar{u}_{Ri} u_{Lj} A
\]
\[
- \left\{ \frac{m_d}{v} \delta_{ij} \left( \cos(\beta - \alpha_{\text{eff}}) - R \tan \beta + \mathcal{O} \left( \frac{m_A^2}{M^2} \right) \right\} \bar{d}_{Ri} d_{Lj} A
\]
\[
- \left\{ \frac{m_e}{v} \delta_{ij} \left( \cos(\beta - \alpha_{\text{eff}}) - R \tan \beta + \mathcal{O} \left( \frac{m_A^2}{M^2} \right) \right\} \bar{e}_{Ri} e_{Lj} A
\]
In the SM limit where both $M$ and $m_A$ are enough heavy compared to $m_Z$, the contributions from $Y'_f$ disappear. On the other hand, effects of extra Yukawa contribution can remain in the Yukawa couplings with $H$, $A$, and $H^\pm$ as quasi-nondecoupling effects, even if $M$ is much larger than $m_A$. In general, $Y'_f$ has non-trivial flavor structure and the flavor changing processes are enhanced a lot, if there are off-diagonal elements in $(Y'_u)_{ij}$, $(\tilde{Y}'_d)_{ij}$ and $(Y'_e)_{ij}$. In order to suppress a dangerous contributions to the flavor changing processes, some mechanism to forbid such the off-diagonal elements is necessary unless $m_A$ is very large. Discrete symmetries such as $Z_2$ symmetry are often considered. The way of the $Z_2$ parity assignment are discussed in Sec. II, and the possible types are listed in Table 1. It is interesting to study how the extra Yukawa interaction contribute to flavor measurements in these types of Yukawa interaction in the 4HDSSM. Further discussions on flavor physics are given elsewhere [35].

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Figure 5. The values of $m_{H^\pm}, m_h, m_H$ and $\sin^2(\beta - \alpha_{\text{eff}})$ in the 4HDSSM and the MSSM as a function of $m_A$ for $k = 1.0$, 2.0 and 5.0. The soft-SUSY-breaking scale of the MSSM is set to be 1 TeV (solid curves) and 2 TeV (dotted curves). The trilinear soft-breaking parameters $A_t$ and $A_b$ as well as the $\mu$ parameter are taken to be zero. The other parameters are taken as $M = 500$ GeV, $r = 1$, $\theta = 0$ and $k' = 0$. Figures in the left column are for $\tan \beta = 3$ and those in the right are for $\tan \beta = 10$. 