Reduction Behaviour of Hysteresis Cycle by Pushover Analysis

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Abstract

Objectives: In this study, three reinforced concrete moment frames were designed conforming to the third edition of 2800 standard under valid accelerograms which were equal in accordance with available improvement instructions. Methods/Analysis: Nonlinear static or pushover analysis is a new method which is interested by researchers and engineers in the construction industry for its easy fast computing. This method provides the possibility to evaluate the performance of structures against earthquakes. Pushover analysis and comparison of its results with results of nonlinear dynamic analysis, the values obtained for reduced hardness and resistance (C2) were compared with suggested values of the improvement guideline. Findings: Increase in period of the structure decreases the C2 coefficient; thus, it can be considered equal to one in higher periods and ignored in calculations of target displacement. Conclusion/Application: Greatest effect of reduced resistance and hardness is experienced on damages to structures with lower period.

Keywords: Hysteresis Curve, Nonlinear Dynamic Analysis, Nonlinear Static Analysis, Reduction Behavior, Reinforced Concrete Moment Frame

1. Introduction

During the past decades, progress in earthquake engineering and structures has made many changes in the seismic design codes. Many researchers and research institutions have worked on seismic design of structures and published their results. One of the most famous of these institutions is the federal emergency management agency which publishes results of its studies in the form of FEMA reports. The first publication of this agency for vulnerability of buildings is Journal FEMA2731. Meanwhile, the ATC 40 guidelines were also published2. Both publications are essentially similar to each other and express the same concept of performance and evaluation methods.

Over the years, Researchers have taken effective steps to improve seismic structures, including shifting in parameters effective on the structural design from resistance to performance of the structure to provide a safe design. Some studies in the field of seismic analysis1–6. According to these theories, seismic improvement guidelines and regulations of FEMA356 set standards to design and improve the performance-based structure. In the pushover analysis, the goal is to obtain the target displacement to determine the structural performance and improvements. In general, the nonlinear dynamic analysis is the only method that presents seismic behaviour of the structure. The pushover analysis does not consider the effects of reduced hardness and reduced hysteresis loop resistance on the actual behaviour of structure; therefore, the coefficients of reduced hardness and resistance (C2) are used to approach the structural behaviour to the actual behaviour.

Pushover analysis is a simple practical method that can be used to estimate demand responses under earthquake stimulations. Pushover analysis is a nonlinear static analysis of structures under incremental lateral loads to
determine the load-displacement curve or curved structural capacity in which values of base shear and lateral displacement of the highest structural level are used.

Static pushover analysis is not based on a specific theory; instead, it is based on the principle that structural response can be simulated by a system respond with one degree of freedom and equivalent characteristics (Figure 1). This assumption causes the response of the structure only depends on the first mode of vibration and its shape remains constant over time. Although both above assumptions may seem improper, extensive research over several decades has shown that a good estimate of the maximum system reflects can be derived from this analysis for structures on which the first mode of vibration is dominant.

In the most general case, the behaviour and performance estimation is done by nonlinear dynamic analyses using certain accelerograms. However, dynamic nonlinear analysis requires adequate background knowledge which needs courses of higher education, which is not available for all those involved in the construction industry. At the same time, complexity, time consuming and high cost of these analyses as well as further problems in the analysis and interpretation of results limit this type of analysis to research projects and special occasions. Thus, a simple practical method such as pushover analysis can help to overcome many of the problems mentioned.

One of the major drawbacks of pushover analysis is that it does not include explicitly the effect of reduction behaviour of structural components under earthquake cycles in calculation related to structural behaviour curve. This study addresses the effect of this parameter on the nonlinear static analysis of reinforced concrete moment frame. Improved pushover analysis and reduced errors compared to nonlinear dynamic analysis can help to get more realistic responses. In the present study, three reinforced concrete moment frames with moderate ductility were designed by ETABS software, based on seismic regulations of Iran for a region with very high relative seismicity. As mentioned above, the reducing effect of hysteresis curves are not considered in typical pushover analyses; hence, several hysteresis models with reduced hardness and maximum, medium and minimum resistance were used to calculate the adjustment factor of the discarded reducing effects of hysteresis curves in the pushover analysis carried out. In addition, the hysteresis model was selected to describe the hysteretic behaviour of the shear wall. For nonlinear dynamic analysis, five different accelerograms which were equal in consistent with the range of soil type III in Standard 2800 were used.

2. Theoretical Background

2.1 Three-Parameter Hysteresis Model of Park

The distinctive characteristics of concrete over steel should be considered in evaluating the dynamic response of reinforced concrete structures. Effects of reduced hardness and resistance as well as narrowing are required in such models. Park defined these properties by three parameters $\alpha$, $\beta$, and $\gamma$, which respectively represent the three attributes listed above.

2.2 Target Displacement

In different guidelines, there are two methods to calculate target displacement: target displacement coefficients and the capacity spectrum method.

The first method is used in improvement guidelines. In this procedure, the target displacement is calculated using analytical and statistical tests conducted on systems with one degree of freedom and non-reducing nonlinear behaviour (two-line or three-line) at 5% damping. Target displacement is estimated for structures with a rigid diaphragm considering the nonlinear behaviour of structures\(^8\). An approximate method for calculating the target displacement is:

$$\delta_r = C_0C_1C_2C_3S_a \frac{T^2}{4\pi^2} g$$

Where, $T_r$, main period of construction, is as follows:

$$T_r = \frac{1}{2\pi} \sqrt{\frac{k_r}{k_i}}$$

**Figure 1.** Conversion of a multi-degree of freedom system to an equivalent one-degree system.
2.3 Damage Index Model

For the first time in 1956, Hausner proposed an analysis of the extent design based on energy in which enough energy absorption capacity of structures against large earthquakes was considered as a factor of safety and health of structures. Bloom et al. presented the spectral sketch to estimate the potential range of damages to building or a group of buildings. By defining local, overall and collective fragility, Bertaut and Bresler (1977) determined the amount of damages to structures\textsuperscript{10}. Presenting a damage index (DI\textsubscript{P&A}), Park and his colleagues involved structural members in the incurred damage and practically strengthened the position of vulnerability\textsuperscript{10}. Rate of reduced resistance as the main characteristic of damage directly depends on parameter $\beta$; therefore, the Park's damage model is a part of integral of the three-parameter hysteresis model, defined as follows:

$$DI\textsubscript{P&A} = \frac{\delta_m - \delta_u}{\delta_u} + \frac{\beta}{P_y} \int dE_h$$ \hspace{1cm} (3)

Where, $\delta_m$ is the maximum deformation of the member, $\delta_u$ is final deformation of the member, $P_y$ is yield strength of the member, $\int dE_h$ is energy absorbed by the member during the time history analysis of response and $\beta$ is the parameter of reduced resistance. For the parameter $\beta$ in this equation, the value 0.01 was suggested as a nominal reduced resistance\textsuperscript{8}. This relationship is defined as a linear combination of damage from maximum deformation and absorbed hysteresis energy. The relation of this damage model depends on final displacements. Since the nonlinear behaviour of members is limited to the two end members, Kunnath and colleagues, University of Buffalo, modified the Park's relation as follows\textsuperscript{11}.

$$DI = \frac{\theta_m - \theta_u}{\theta_u - \theta_r} + \frac{\beta}{M_\theta \theta_u} E_h$$ \hspace{1cm} (4)

Where, $\theta_m$ is the maximum rotation created during the time history analysis, $\theta_u$ is the maximum capacity of the rotating member, $\theta_r$ is the rotating back after unloading, $M_\theta$ is the yield anchor of the member, $E_h$ is the wasted energy in the member. Based on the proposed indicators, story damage index and total structural damage index can be determined according to equations (5) and (6). In these relations, $\lambda$ is the weight ratio of energy, $\sum E_i$ is the absorbed energy by each member, $DI_i$ is the partial damage index of each member\textsuperscript{12}.

$$DI\textsubscript{story} = \sum (\lambda_i)\textsubscript{member} (DI_i)\textsubscript{member} (\lambda_i)\textsubscript{member}$$
$$= (\sum E_i)\textsubscript{member}$$ \hspace{1cm} (5)

$$DI\textsubscript{total} = \sum (\lambda_i)\textsubscript{story} (DI_i)\textsubscript{story} (\lambda_i)\textsubscript{story}$$
$$= (\sum E_i)\textsubscript{story}$$ \hspace{1cm} (6)

In other words, damages to beams and columns of each story are calculated as members of that story; then, damages of the stories and total frame can be calculated using weighted coefficients based on hysteresis energy depreciated in members and story levels. The amount of damage can be calibrated as shown in Table 1\textsuperscript{13}.

### Table 1. Calibration of damage index\textsuperscript{13}

| Situation       | Damage index | Appearance                              | Degree of damage |
|-----------------|--------------|-----------------------------------------|------------------|
| Destruction     | >1           | Localized or total building collapse     | Collapse         |
| Non-repairable  | 0.4-1        | Extensive smash of concrete, appearance of ricochet reinforcements | Severe           |
| Repairable      | 0.25-0.4     | Large and extensive cracks, concrete delamination in weaker members | Medium           |
| Repairable      | 0.1-0.4      | Small cracks, localized smash of concrete in columns | Low              |
| Repairable      | 0.1>         | Disperse cracks                          | Negligible       |

3. Methods of Analysis

The purpose of this research is to find adjustment factor of pushover analysis by target displacement coefficients. For this purpose, two approaches are considered below:

First approach: In this approach, reinforced concrete moment frames are analysed using nonlinear static analysis. The analysis does not take the factor $C_i$ into account; regardless of the reducing effect of hardness and resistance, analysis is conducted. After finding the value of target displacement and depicting the behavioural curves
of frames, nonlinear dynamic analysis is carried out again by taking into account the reduction using hysteresis curves. In this case, the ratio of maximum displacement of nonlinear dynamic analysis to target displacement of nonlinear static analysis can reflect the value of the coefficient \( C_0 \). However, the responses obtained may not be reliable because the coefficients used in the pushover analysis may not be accurate. Therefore, there is no guarantee that the ratio is exactly equal to the coefficient of reducing parameter.

Second approach: the second approach tries to calculate the coefficient \( C_0 \) completely independent of pushover analysis results. This time, it is required to conduct the nonlinear dynamic analysis without reducing effects. In this way, the coefficient \( C_0 \) is calculated from maximum displacement ratio considering the reducing effect. Acceptability of this practice is that this method obtains two values of the numerator and denominator displacement regarding common assumptions and a same process; the only difference is the shape of hysteric curve. Since the curves only consider the reduced resistance and hardness, the results may have less error than the first approach does. Finally, the results obtained for three concrete moment frames with different periods are compared with the recommendations of available codes and instructions.

3.1 Nonlinear Static Analysis

Initially, the nonlinear static or pushover analysis is used to acquire the target displacement. For non-linear static analysis of the studied structures, the nonlinear analysis of concrete damage IDARC-2D was used\(^{14}\). After performing a pushover analysis, the obtained results included the force-displacement curves of the frame. The next step is to calculate the target displacement. In this regard, \( K_s \) was calculated from the idealized two-line curve method. At first, a value is supposed for the yield force \( V_y \); then, \( K_s \) is calculated. Having \( K_s \), the effective period can be calculated by equation (2). Substituting \( T_s \) in (1), the target displacement is calculated. Here, values of other coefficients \( (C_0, C_1, C_2) \) are excluded in order to eliminate their effect on results (i.e., their value is considered equal to one). After calculating displacement, the area under the curve is compared to the area under the capacity by depicting a two-line curve. In case of non-compliance, another value is considered for \( V_y \) and previous steps are iterated.

Eventually, the final amount of target displacement can be obtained upon reaching to the acceptable accuracy.

3.2 Nonlinear Dynamic Analysis

The next step of this study is to perform nonlinear dynamic analysis by which values of displacement and reducing effect on structural behaviour can be compared. As mentioned above, the nonlinear dynamic analysis is the most accurate method for analysis of structural response to seismic loads. The results of nonlinear dynamic analysis can be misleading if the analyses are not done by correct assumptions. One assumption that can affect the results of this analysis is the nature of seismic force. If the accelerograms used in the analysis are not reliable or consistent with the analysed region, the responses can be misleading. This study used IDARC-2D software that can analyse concrete frames and damage.

4. Model of the Reinforced Concrete Moment Frames

Purpose of this study was to examine reduced resistance and hardness factor \( (C_0) \) or the hysteresis reducing behaviour using nonlinear static analysis. For this purpose, three reinforced concrete moment frames (5, 9 and 13 story) were used for analysis, as shown in Figure 2. All these frames had three 5m span and the height of stories was equal (3.2m). Their ductility was \( R = 7 \) (the proposed value of 2800 standard for intermediate moment frames). The design of these frames has tried to consider a symmetric shape and avoid the effects of twisting and irregularity on responses. Gravity loading of the frames was conducted on the basis of Regulations Section VI\(^{14}\) and seismic loading as well as regulations for design of structures against earthquake\(^{2}\).

Initial analysis of frames was done by the equivalent static analysis by ETABS software. The frames were designed by ETABS\(^{15}\) according to local regulations. To prevent failure of frames during dynamic analysis, however, the elements were typed in a way that the structure had an acceptable level of extra resistance for ductility. Natural period of three frames were calculated (0.397, 0.816 and 1.209, respectively). It was supposed that the frames were in an area with very high risk of seismicity and the base acceleration was 0.35g; the terrain was also the soil Type III, Standard 2800\(^{2}\).
5. The Used Accelerograms

This article tried to select records from earthquakes happened in Iran in order to consider the ruling tectonic conditions in the analysis. Therefore, seismic accelerograms for earthquakes of Ahar, Bam, Manjil, Tabas and El Centro were scaled in accordance with the standard 2800; then, they were inserted in IDARC-2D to perform analysis.

According to improvement guideline, at least three different accelerograms with different components and at least three different events are mandatory. All accelerograms are based on assumptions regarding the location of frames for soil type III of the standard. Improvement guideline of Iran requires that accelerograms be consistent and compatible in a period of 1.0 to 3 seconds. By calculate the response spectrum of an accelerogram, the area under the curve can be obtained and compared to the same value of the regional spectrum. It was tried to achieve a new accelerogram by selecting the proper coefficient multiplied by values of accelerogram in order to have a similar appearance for both spectra and the area under the curve.

6. Selection of the Used Hysteresis Curves

Hysteresis curves are chosen to meet the expected demands. For this purpose, the selected curves are required to do three types of analysis:

1. Nonlinear dynamic analysis without effects of reduced resistance and hardness: in this model, the selected
curves did not show any reduction during loading while presenting the concrete behaviour and separating it from other materials (such as steel); for this purpose, the Clough hysteresis curve was used without reducing effect.  

2. Nonlinear dynamic analysis with typical effects of reduction and hardness: in this case, the behaviour of concrete elements was modelled by selecting proper curves considering the reducing effects of resistance and hardness. For this purpose, the Takeda hysteresis curve with reduced hardness and resistance and source-oriented hysteresis curve were used for modelling the flexural behaviour of all elements and shear behaviour of shear walls, respectively.  

3. Nonlinear dynamic analysis with strongest reducing effects: In order to evaluate the worst case, a hysteresis curve was selected to consider the worst reducing conditions including resistance, hardness and narrowing. These values were selected according to the program guideline. Table 2 presents the main values of the three-parameter model for the three above-mentioned analyses.  

### 7. Calculations  

#### 7.1 Coefficient $C_2$  
The value of $C_2$ can be calculated after calculating the maximum roof displacement from results of dynamic analysis and target displacement. As mentioned above, the $C_2$ coefficient is first calculated from dynamic analysis results compared with the target displacement; then, the value of $C_2$ is obtained by comparing the dynamic results. Table 3 shows static and dynamic analysis results for all three frames. Coefficients $C_2$ calculated by both methods are greater than unit. In addition, the results show that $C_2$ values obtained from pushover analyses are reliable, because the values obtained in higher periods are lower than the dynamic analysis.  

#### 7.2 Damage Assessment  
Calculated values of damage to frames were calculated for beams and column-wall elements. The damage to each story and the building, as a whole, was also obtained. Obviously, increase in reduction behaviour raises the amount of damage; however, this increase is different for each frame. In the 13-story frame, the difference between

| The model            | Parameter α | Parameter β | Parameter γ |
|----------------------|-------------|-------------|-------------|
| Clough               | 200         | 0.01        | 1           |
| Takeda               | 2           | 0.1         | 1           |
| Strong reduction     | 4           | 0.6         | 0.01        |
| Source-orienting     | 0           | 0           | 0.01        |

| Accelerogram | No. of story | AHAR | BAM | EL CENTRO | MANJIL | TABAS | Mean |
|--------------|--------------|------|-----|-----------|--------|-------|------|
|              |              |      |     | 181.07    | 101.36 | 53.85 |      |

| No. of story | C2 coefficient calculated using the Response Analysis |
|--------------|-----------------------------------------------------|
|              |                                                     |

| Maximum displacement of linear dynamic analysis by hysteresis curve | Target displacement of pushover analysis by modal adaptation model (mm) | Accelerogram |
|---------------------------------------------------------------|---------------------------------------------------------------|--------------|
| Strong reduction (mm) | Takeda-source oriented (mm) | Clough (mm) | 13 | 9 | 5 | 13 | 9 | 5 | 13 | 9 | 5 | 181.07 | 101.36 | 53.85 | No. of story |
| 13 | 9 | 5 | 13 | 9 | 5 | 13 | 9 | 5 |
| 175.2 | 112.2 | 62.3 | 1178.0 | 114.0 | 68.2 | 177.5 | 113.8 | 54.0 |
| 249.5 | 149.6 | 92.7 | 244.7 | 147.7 | 82.2 | 244.4 | 182.4 | 76.1 |
| 281.0 | 165.6 | 93.1 | 269.4 | 159.2 | 75.5 | 246.0 | 183.2 | 32.0 |
| 252.4 | 151.6 | 61.5 | 243.6 | 146.7 | 19.1 | 223.4 | 137.0 | 58.9 |
| 159.2 | 105.7 | 95.0 | 155.5 | 102.7 | 44.5 | 151.9 | 101.7 | 37.2 |
| 223.5 | 136.9 | 80.9 | 218.2 | 134.1 | 57.9 | 208.6 | 143.6 | 51.6 |
| 1.02 | 1.08 | 1.21 | 1.03 | 1.02 | 1.01 | 1.00 | 1.00 | - | - | - |
| 1.47 | 1.33 | 1.60 | 1.55 | 1.22 | 1.31 | 1.00 | 1.00 | 1.00 | - | - | - |
damage from non-reducing behaviour and the strongest reduction as well as the strongest reduction and the strongest assumed reduction was 51%; while, this difference was 87% in 5-story frame with lower period and ductility, which was more than the case without reduction.

8. Conclusions

1. For all three frames, the displacement from nonlinear dynamic analysis is beyond the solutions of nonlinear static analysis.

2. If the ratio of dynamic analysis respond to pushover analysis result is used in calculating $C_2$, the values 1.31 and 1.60 will be achieved for the 5-story frame with moderate reduction and strong reduction, respectively. If the same comparison is conducted with the result from dynamic analysis without reduction, the values 1.01 and 1.21 will be achieved for the model with moderate reduction and strong reduction, respectively. For the 9-story frame, the values 1.22 and 1.33 are achieved for the model with moderate reduction and strong reduction, respectively. If the same comparison is conducted with the result from dynamic analysis without reduction, the values 1.02 and 1.08 will be achieved for the model with moderate reduction and strong reduction, respectively. Finally, the values 1.45 and 1.47 are achieved for the model with moderate reduction and strong reduction, respectively, in a 13-story frame. If the same comparison is conducted with the result from dynamic analysis without reduction, the values 1.03 and 1.02 will be achieved for the model with moderate reduction and strong reduction, respectively.

3. results from analyses conducted in this study show that increase in period reduces the amount of $C_2$; so that, it can be considered equal to one in higher time periods and it can be discarded in calculations of target displacement. These results are consistent with recommendation of FEMA\textsuperscript{20} 440, although they are higher. This is because of the fact that FEMA 356\textsuperscript{21} recommends more conservative values for $C_2$.

4. Increase in reduction adds to structural vulnerability. Nevertheless, the greatest effect of reduced resistance and hardness on damages is observed in structures with lower periods.

5. Finally, averaging the results suggests the value 1.1 for $C_2$ (value of the improvement guideline) in reinforced concrete moment structures with special ductility considering the soil type III in the regional equivalent period (0.8s). Increase in period gradually reduces the value of $C_2$; therefore, it is constantly considered 1 after 1.5s.

9. References

1. FEMA 273. NEHRP Guidelines for the seismic rehabilitation of buildings. Federal Emergency Management Agency; 1996.

2. ATC 40. Seismic evaluation and retrofit of concrete buildings. 1997.

3. Pouramini M, Hosseini M. Seismic safety evaluation of Tabriz historical citadel using finite element and simplified kinematic limit analyses. Indian Journal of Science and Technology. 2014; 7(4):409–17.

4. Nikkhou A, Amankhani M, Ghafari H. Vibration suppression in smart thin beams with piezoelectric actuators under a moving load/mass accounting for large deflections of the base structure. Indian Journal of Science and Technology. 2014; 7(2):211–20.

5. Jayaramappa N, Krishna A, Annpurna BP, Kiran T. Prediction of base shear for three dimensional RC frame subjected to lateral load using artificial neural network. Indian Journal of Science and Technology. 2014; 7(6):729–33.

6. Hosseini M, Fanaie N, Yousefi AM. Studying the vulnerability of steel moment resistant frames subjected to progressive collapse. Indian Journal of Science and Technology. 2014; 7(3):335–42.

7. Standard 84-2800. Earthquake resistant design of buildings regulations. Standing Committee for revising the regulations of Earthquake Resistant Design of Buildings, Building and Housing Research Center. 2011.

8. Park YJ, Reinhorn AM, Kunnath SK. IDARC: Inelastic damage analysis of reinforced concrete frame-shear-wall structures. N.Y.: State University of New York at Buffalo; 1987. Technical report NCEER-87-0008.

9. Office of Engineering and Planning Management. Seismic rehabilitation of existing buildings. International Institute of Seismology and Earthquake Engineering; 2011.

10. Park YJ, Ang A, Wen YK. Seismic damage analysis and damage limiting design of reinforced concrete building. University of Illinois at Urbana-Champaign; 1984.

11. Valles RE, Reinhorn AM, Kunnath SK, Mandan A. IDARC V6.1 a program for the inelastic damage analysis of reinforced concrete structure. National Centre for Earthquake Engineering Research, State University of New York at Buffalo; 1996. Technical report NCEER-96-0010.

12. Valles RE, Reinhorn AM, Kunnath SK, Mandan A. A program for the inelastic damage analysis of buildings. NCEER Task Numbers 943103A and 94310 A. 1996.
13. Ghobarah A, Biddah A. Dynamic analysis of reinforced concrete frames including joint shear deformation. Journal of Engineering Structures. 1999; 21:971–87.
14. Office of formulation and promotion of the National Building Regulations. Section VI: loads on buildings. Department of Housing and Urban Development; 2011.
15. ETABS. Extended three dimensional analysis and design of building systems. Berkeley, California: Computers & Structures, Inc; 2004.
16. Naghie M. Regulations AC I318-02 concrete structures and commentary ACI 318R-02. Publication of Arkan. 2004.
17. Clough RW. Effect of stiffness degradation on earthquake ductility requirements. Berkeley, CA: Structural and Materials Research, Structural Engineering Laboratory, University of California; 1966. Report 66-16.
18. Takeda T, Sozen MA, Nielson NN. Reinforced concrete response to simulated earthquakes. J Struct Div, ASCE. 1970; 96:ST12.
19. Kabeyasawa T, Shiohara H, Otani S, Aoyama H. Analysis of the full-scale seven-story reinforced concrete test structure. Faculty of Engineering, University of Tokyo. 1983; XXXVII(2):432–78.
20. FEMA 440. NEHRP Improvement of nonlinear static seismic analysis procedures. Federal Emergency management Agency; 2005.
21. FEMA 356. NEHRP Recommended provisions for the seismic rehabilitation of buildings. Federal Emergency management Agency; 2000.