Spin Correlation in Top-Quark Production at Hadron Colliders.

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Abstract

We propose techniques to observe the correlation of the spins of top quarks and antiquarks at the Tevatron and the LHC. Observation of the spin correlation would confirm that the top quark decays before its spin flips, and would place a lower bound on the top-quark width and $V_{tb}$. The spin correlation may also be a useful tool to study the weak decay amplitude of the top quark.
The discovery of the top quark at the Fermilab Tevatron by the CDF and D0 collaborations [1] has ushered in the era of top-quark physics. The large mass of the top quark, \(m_t = 180 \pm 12\) GeV, in comparison with the five lighter quarks, suggests it may play a special role in particle physics. It is thus imperative that we examine the physics of the top quark in detail.

Run II at the Tevatron, beginning in 1999, will provide each experiment with approximately 1000 fully-reconstructed, \(b\)-tagged \(t\bar{t}\) events [2]. Even higher yields will become available from the CERN Large Hadron Collider (LHC) [3] and possible upgrades of the Tevatron [2]. These large statistics should allow a detailed study of the properties of the top quark.

The standard model predicts that the top quark decays before its spin flips [4]. This is in contrast with the lighter quarks, which are depolarized by QCD interactions long before they decay [5]. The spin of the top quark is therefore reflected by its decay products.

While the top quarks and antiquarks produced at hadron colliders are unpolarized, their spins are correlated [7]. Figure 1 shows the cross sections for the production of \(t\bar{t}\) pairs with the same and opposite helicities at the Tevatron and the LHC, as a function of the \(t\bar{t}\) invariant mass, for \(m_t = 175\) GeV. The helicity is the spin component along the particle’s momentum, in the \(t\bar{t}\) center-of-mass frame. At the Tevatron, 70% of the pairs have the opposite helicity, while 30% have the same helicity. Defining the correlation as

\[
C \equiv \frac{\sigma(t_R\bar{t}_R + t_L\bar{t}_L) - \sigma(t_R\bar{t}_L + t_L\bar{t}_R)}{\sigma(t_R\bar{t}_R + t_L\bar{t}_L) + \sigma(t_R\bar{t}_L + t_L\bar{t}_R)}
\]

we find that the helicities of the top quarks and antiquarks have a correlation of \(-40\%\). At the LHC the correlation is \(+31\%\). Figure 1 also shows that placing cuts on the invariant mass of the \(t\bar{t}\) system can enhance the correlation at both the Tevatron and the LHC.

In this paper we propose techniques for observing the correlation of the top-quark and

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1 An exception is the production of heavy baryons [6].

2 The spin correlation is easier to observe the larger the top-quark mass, so we use \(m_t = 175\) GeV throughout, to be conservative.

3 Parity conservation in QCD implies \(\sigma(t_R\bar{t}_L) = \sigma(t_L\bar{t}_R)\), and CP conservation implies \(\sigma(t_R\bar{t}_R) = \sigma(t_L\bar{t}_L)\).
-antiquark helicities experimentally. There are two motivations for doing so. First, observation of the spin correlation would confirm that the top quark does indeed decay before its spin flips, thereby setting an upper bound on the top-quark lifetime. This would in turn place a lower bound on the top-quark width, which is proportional to the combination of Cabbibo-Kobayashi-Maskawa (CKM) matrix elements $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2$. If there are just three generations of quarks this quantity equals unity, but it can be almost zero if there are more than three generations.

The spin of a heavy quark is flipped by its chromomagnetic moment, which is inversely proportional to its mass, $m_Q$. The spin-flip time is therefore proportional to $m_Q/\Lambda_{QCD}^2$. The chromomagnetic moment is also responsible for the hyperfine splitting in heavy mesons, so this splitting can be used to estimate the spin-flip time \[3\]. Scaling from the $D-D^*$ and $B-B^*$ mass splittings, we estimate the spin-flip time of the top-quark to be $(1.3 \text{ MeV})^{-1}$. This is much longer than the anticipated top-quark lifetime, $\Gamma^{-1} \approx (1.5 \text{ GeV})^{-1}$, assuming three generations.

Assuming more than three generations, observation of the spin correlation would imply $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 > (0.03)^2$. If we assume $|V_{tb}|$ is much larger than $|V_{td}|$ and $|V_{ts}|$, this yields $|V_{tb}| > 0.03$. If $|V_{tb}|$ proves to be less than this bound, it would mean that the recently-discovered “top” quark is not the SU(2) partner of the bottom quark, and that the real top quark is still at large.

The second motivation is that we envision that the spin correlation can be used to help probe for non-standard interactions in the weak decay of the top quark \[10\]. The weak decay does not affect the correlation, which arises from QCD, but it does affect how the correlation manifests itself in the top-quark decay products. Non-standard weak interactions of the top quark could result from the mechanism which provides the top quark with its large mass \[11\]. In this paper we restrict our attention to the spin correlation in the standard model,

\[4\] A measurement of the invariant mass of the top-quark decay products places an upper bound on the width.

\[5\] This assumption can be tested by measuring the ratio $\frac{|V_{td}|^2}{|V_{td}|^2 + |V_{ts}|^2}$, which can be extracted by comparing the number of single- and double-$b$-tagged $t\bar{t}$ events \[2\].
since one must first establish that it is observable in that case.

The dominant production mechanism for $t\bar{t}$ pairs at the Tevatron is $q\bar{q} \rightarrow t\bar{t}$, which proceeds through a $J = 1$ s-channel gluon. Near threshold, the $t\bar{t}$ pair has zero orbital angular momentum, so the $t\bar{t}$ pair is in a $^3S_1$ state [13], with spin eigenstates

$$|++\rangle$$
$$\frac{1}{\sqrt{2}} [ |+-\rangle + |-+\rangle]$$
$$|--\rangle.$$ 

Since the $t$ and $\bar{t}$ move oppositely in the $t\bar{t}$ center-of-mass frame, they have the opposite helicity if they have the same spin, and the same helicity if they have the opposite spin. Two of the three states have the opposite helicity, hence the correlation near threshold is $C = +\frac{1}{3} - \frac{2}{3} = -33\%$. Far above threshold, helicity conservation at high energy ensures that the $t$ and $\bar{t}$ are produced with the opposite helicity, so $C = -100\%$. The formula that interpolates between the two extremes is

$$\frac{\sigma(t_R\bar{t}_L + t_L\bar{t}_R)}{\sigma(t_Rt_R + t_L\bar{t}_L)} = 2 \frac{M_{t\bar{t}}^2}{4m_t^2}. \quad (2)$$

Convoluting with parton distribution functions, and including the small contribution from $gg \rightarrow t\bar{t}$, yields the Tevatron curves in Fig. 1. Integrating over the $t\bar{t}$ invariant mass yields an average correlation of $-40\%$.

At the LHC the situation is reversed. The dominant contribution to the cross section comes from $gg \rightarrow t\bar{t}$. Near threshold, the $t\bar{t}$ pair is in a $^1S_0$ state [12] [13]

$$\frac{1}{\sqrt{2}} [ |+-\rangle - |-+\rangle] .$$

The $t$ and $\bar{t}$ therefore have the same helicity, with a correlation of $+100\%$. Far above threshold, helicity conservation again ensures that the $t$ and $\bar{t}$ are produced with the opposite helicity. Convoluting with parton distribution functions, and including the small contribution from $q\bar{q} \rightarrow t\bar{t}$, yields the LHC curves in Fig. 1. The average correlation, integrating over the $t\bar{t}$ invariant mass, is $+31\%$. 

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Since the ratio of the same- and opposite-helicity cross sections varies with the $t\bar{t}$ invariant mass, the correlation at both the Tevatron and the LHC can be enhanced by cutting on this quantity. A cut of $M_{t\bar{t}} > 415$ GeV increases the correlation to $-50\%$ at the Tevatron, with an acceptance of 55\%. A cut of $M_{t\bar{t}} < 475$ GeV increases the correlation to $+50\%$ at the LHC, with an acceptance of 45\%.

We now consider how the top quark’s helicity is reflected by its decay products, either $t \rightarrow b\ell^+\nu$ or $t \rightarrow bu\bar{d}$. In the rest frame of the parent top quark, the angular distribution of fermion $i$ with respect to the momentum of the top quark in the $t\bar{t}$ center-of-mass frame is

$$
\frac{dN_{R,L}}{d\cos\theta^*_i} = \frac{1}{2}(1 \pm h_i \cos \theta^*_i)
$$

where $h_i$ is a constant between $-1$ and 1. The ability to distinguish $t_R$ from $t_L$ evidently increases with $|h_i|$. For top antiquarks, the subscripts $R, L$ are interchanged in Eq. (3).

The most powerful spin analyzer is the charged lepton in semi-leptonic decay, for which $h_\ell = 1$ \cite{14,15}. Similarly, in hadronic decay, the $\bar{d}$ has $h_\bar{d} = 1$. Unfortunately, it is impossible to distinguish the $u$ and $\bar{d}$ jets. However, half of the hadronic decays are $t \rightarrow bc\bar{s}$, and one might be able to tag the charm quark. The high analyzing power of this decay might compensate the charm-tagging efficiency, which is unknown at present.

The $b$ quark has $h_b = -0.41$, which can be derived as follows. If the $W$ boson is longitudinal, $h_b = -1$; if it is transverse, $h_b = +1$. Since 70\% of the $W$ bosons in top decay are longitudinal, the net value of $h_b$ is approximately $-0.4$ \cite{14}.

For hadronic decay, one can use the least-energetic quark (in the top-quark rest frame) from the $W$ decay, which has $h_q = +0.51$. This follows from the fact that the $\bar{d}$ quark is the least-energetic quark 61\% of the time. Since the $\bar{d}$ quark has the greatest analyzing power, the least-energetic quark has significant analyzing power \cite{14}.

The angular distribution of fermion $i$ from the $t$ decay and fermion $j$ from the $\bar{t}$ decay in
$t\bar{t}$ events is

$$\frac{d^2N}{dz_idz_j} = (1 - z_iz_jh_ih_jC)$$ \hspace{1cm} (4)

where

$$z_i = \cos \theta_i^*$$ \hspace{1cm} (5)

and where $C$, defined in Eq. (1), is the degree of spin correlation. For uncorrelated events $C = 0$, and the distribution is flat in the $z_i z_j$ plane. A simple measure of the correlation is the asymmetry in the $z_i z_j$ plane. We find

$$A \equiv \frac{N_+ - N_-}{N_+ + N_-} = -\frac{1}{4}h_ih_jC$$ \hspace{1cm} (6)

where $N_+$ is the number of events with the product $z_i z_j > 0$ and $N_-$ is the number of events with $z_i z_j < 0$. The largest asymmetry is obtained by maximizing the product $|h_i h_j|$. Since the lepton in semi-leptonic decays has $h_\ell = 1$, we always use one semi-leptonic decay.

For dilepton events, evaluating Eq. (3) at the Tevatron ($C = -40\%$) yields $A = +10\%$. However, since the dilepton events have two neutrinos, the events are not fully reconstructable, and therefore are not amenable to our analysis. This asymmetry represents the theoretical upper bound. Other methods for observing spin correlation with dilepton events are discussed in Refs. [7, 12, 13, 16].

Let us concentrate on the fully-reconstructable $W + 4$ jet events, where the $W$ boson decays leptonically. An asymmetry of $A = +10\%$ is achievable via charm tagging, but the efficiency of this is unknown at present. If the efficiency is $\epsilon$, the number of charm-tagged events in 1000 $W + 4$ jet events is $500\epsilon$. The significance of the asymmetry (its difference from zero) is thus $10\% \times \sqrt{500\epsilon} = 2.2\sqrt{\epsilon\sigma}$.

The next most powerful spin analyzer is the least-energetic quark. This yields an asymmetry of $A = +5.1\%$, which has a significance of $1.6\sigma$ with 1000 events. Using the $b$ quark yields $A = -4.1\%$, which has a significance of $1.3\sigma$ with the same number of events. We have found that these measurements are uncorrelated, so we may combine the two to increase the

\footnote{This equation assumes the cross section factorizes into production times decay, maintaining the spin correlation, but neglecting interference effects. We have checked that the interference effects are indeed small.}
significance. This may be understood by recalling that the $b$-quark’s analyzing power arises from longitudinal $W$ bosons, and is degraded by the transverse $W$ bosons. On the other hand, the least-energetic quark’s analyzing power arises from both the longitudinal and the transverse $W$ bosons. Combining the two measurements, the significance of 1000 $W + 4$ jet events is $2\sigma$.

We conclude that the top-quark spin correlation is potentially observable, at the $2\sigma$ level, in Run II at the Tevatron. Increasing the integrated luminosity of the Tevatron by a factor of 10 increases the significance by a factor of three, ensuring observation of the correlation, and perhaps allowing the correlation to be used as a tool to study the weak decay amplitude of the top quark. At the LHC, the top-quark cross section is about 100 times greater than at the Tevatron. That, combined with the anticipated large integrated luminosity, will result in at least a million fully-reconstructed, $b$-tagged $t\bar{t}$ events. The spin correlation is potentially measurable to one percent, and it may be a powerful tool to study the weak decay amplitude of the top quark with high precision.

The results in this paper are based on leading-order parton-level calculations of the signal. Further work is needed to determine the effect of higher-order corrections, backgrounds, hadronization, and detector response on this analysis.

Note: A recent paper by Mahlon and Parke also studies top-quark spin correlation at hadron colliders [17]. Our work agrees with theirs where there is overlap.

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Figure Captions

Fig. 1 - Cross sections for $t\bar{t}$ with the same helicities ($t_R\bar{t}_R + t_L\bar{t}_L$) and the opposite helicities ($t_R\bar{t}_L + t_L\bar{t}_R$) at the Tevatron ($\sqrt{s} = 2$ TeV $p\bar{p}$ collider) and the LHC ($\sqrt{s} = 14$ TeV $pp$ collider), versus the $t\bar{t}$ invariant mass. The MRS(A') parton distribution functions were used. 


$d\sigma/dM(t\bar{t})$ (pb/GeV) vs $M(t\bar{t})$ (GeV)

$LHC$ and $Tevatron$

$m_t = 175 \text{ GeV}$