On Stability of the Linearized Spacecraft Attitude Control System

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Abstract

This note is concerned with the stability and stabilization of the linearized spacecraft attitude control system. Necessary and sufficient conditions are respectively provided to guarantee that the considered systems are polynomially stable and stable in the Lyapunov sense. These two classes of conditions guarantee that the linearized spacecraft attitude control system can be respectively stabilized semi-globally and globally by saturated linear state feedback.

Keywords: Spacecraft Attitude control; Stability and stabilization; Lyapunov stable; Saturated feedback.

1 Introduction

Attitude stabilization of spacecraft is a very important problem since it helps the spacecraft to maintain a certain prescribed attitude in the space during their useful life [1], [3], [4], [5], [7], [8]. Attitude stabilization controllers design can be based on either the nonlinear dynamics or the linearized dynamics. If the controllers are designed based on the nonlinear model, the design procedure can be quite complex while the claimed (global) stability can be ensured. If the controllers are designed based on the linear model, the design procedure can be much more simpler since a lot of linear design techniques are available, however, the claimed (global) stability of the practical closed-loop system may not be maintained. As usual, a trade-off thus exists.

If we are interested in the design of spacecraft attitude stabilizing controllers based on the linearized model, it would be very helpful if we can know some specific properties of the linearized model. This is particularly the case if we are considering the constraints existing in the actuators since the existence of a controller is highly dependent on the stability properties of the open-loop system (see, for example, [2], [6], [11], and [12]). Hence, in this note, we are interested in the stability and stabilization of the linearized spacecraft attitude control system. Particularly, we will provide conditions on the spacecraft parameters under which the linearized model is respectively polynomially stable and is stable in the Lyapunov sense. We notice that in these two cases the linearized spacecraft attitude control system can be respectively stabilized semi-globally and globally by saturated linear feedback (see [11] and [12]).

2 Main Results

We consider the following linear system

\[ \dot{\chi} = A\chi + Bu, \quad (1) \]

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where \( A \) and \( B \) are given by \([14], [5], [8]\)

\[
A = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-4\omega_0^2\sigma_1 & 0 & 0 & 0 & 0 & \omega_0(1 - \sigma_1) \\
0 & -3\omega_0^2\sigma_2 & 0 & 0 & 0 & 0 \\
0 & 0 & -\omega_0^2\sigma_3 & \omega_0(\sigma_3 - 1) & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{1}{j_x} & 0 & 0 \\
0 & \frac{1}{j_y} & 0 \\
0 & 0 & \frac{1}{j_z}
\end{bmatrix}.
\]

This linear system is the linearized model of the spacecraft attitude control system. For detailed derivation of this equation, see \([4], [5], [8], [12]\), and the references therein. In the above equation, the symbol \( \omega_0 = \sqrt{\mu r} \) denotes the orbital rate, in which \( \mu = 3.986 \times 10^{14} \text{m}^3/\text{s}^2 \) is the gravity constant and \( r \) is the semimajor axis of the orbit, \( J_x, J_y \) and \( J_z \) are the inertia of the spacecraft, \( \chi \) contains the attitude angles and velocities, \( u \) is the forces applied on the spacecraft, and

\[
\sigma_1 = \frac{J_y - J_z}{J_x}, \quad \sigma_2 = \frac{J_x - J_z}{J_y}, \quad \sigma_3 = \frac{J_y - J_x}{J_z}.
\]

In this note, we are interested in the stability properties of system \([1]\).

**Remark 1** We notice that \( \sigma_i, i = 1, 2, 3 \) are dependent. In fact, by letting \( \frac{j_y}{J_y} = \beta_1, \frac{j_z}{J_z} = \beta_2 \), then

\[
\sigma_1 = \frac{1}{\beta_1} - \frac{1}{\beta_1 \beta_2}, \quad \sigma_2 = \beta_1 - \frac{1}{\beta_2}, \quad \sigma_3 = \beta_2 - \beta_1 \beta_2,
\]

which implies that \( \sigma_i, i = 1, 2, 3 \) can be characterized by the two scalars \( \beta_i, i = 1, 2 \).

Consider the following nonsingular matrices

\[
H = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\omega_0} & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\omega_0} & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\omega_0}
\end{bmatrix}, \quad L = \begin{bmatrix}
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \omega_0^2
\]

then it can be verified that

\[
\begin{align*}
H A H^{-1} &= \omega_0 \begin{bmatrix}
A_2 & 0 & 0 \\
0 & 0 & A_1 \\
0 & A_3 & 0
\end{bmatrix} \triangleq \omega_0 \begin{bmatrix}
A_2 & 0 & 0 \\
0 & 0 & A_1 \\
0 & A_3 & 0
\end{bmatrix} \triangleq \omega_0 A_0, \\
H B L &= \omega_0 \begin{bmatrix}
B_2 & 0 & 0 \\
0 & B_1 & 0 \\
0 & 0 & B_3
\end{bmatrix} \triangleq \omega_0 \begin{bmatrix}
B_2 & 0 & 0 \\
0 & B_1 & 0 \\
0 & 0 & B_3
\end{bmatrix} \triangleq \omega_0 B_0,
\end{align*}
\]

where \((A_1, A_2, A_3)\) and \((B_1, B_2, B_3)\) are independent of \(\omega_0\) and are given by

\[
\begin{align*}
A_1 &= \begin{bmatrix}
0 & 1 \\
-4\sigma_1 & 1 - \sigma_1
\end{bmatrix}, \quad B_1 &= \begin{bmatrix}
0 \\
\frac{1}{j_x}
\end{bmatrix}, \\
A_2 &= \begin{bmatrix}
0 & 1 \\
-3\sigma_2 & 0
\end{bmatrix}, \quad B_2 &= \begin{bmatrix}
0 \\
\frac{1}{j_y}
\end{bmatrix}, \\
A_3 &= \begin{bmatrix}
0 & 1 \\
-\sigma_3 & \sigma_3 - 1
\end{bmatrix}, \quad B_3 &= \begin{bmatrix}
0 \\
\frac{1}{j_z}
\end{bmatrix}.
\end{align*}
\]

Our first result provides necessary and sufficient conditions to guarantee that \(A\) is polynomially stable, namely, \(\text{Re}\{s\} \leq 0, \forall s \in \lambda(A)\), the eigenvalue set of \(A\).
Proposition 1 All the eigenvalues of $A$ have non-positive real parts if and only if

$$
\begin{aligned}
0 \leq \sigma_2, \\
0 \leq \sigma_1\sigma_3 \triangleq \phi_1, \\
0 \leq 3\sigma_1 + \sigma_3\sigma_1 + 1 \triangleq \phi_2, \\
0 \leq (3\sigma_1 + 1 + \sigma_3\sigma_1)^2 - 16\sigma_1\sigma_3 = \phi_2^2 - 16\phi_1.
\end{aligned}
$$

Moreover, if all the above conditions are satisfied, the eigenvalues $s_i, i \in [1, 6]$ of $A$ are given by

$$
\begin{aligned}
s_{1,2} = \pm \sqrt{3\sigma_2}\omega_0i, \\
s_{3,4} = \pm \sqrt{\frac{\phi_2 + \sqrt{\phi_2^2 - 16\phi_1}}{2}}\omega_0i, \\
s_{5,6} = \pm \sqrt{\frac{\phi_2 - \sqrt{\phi_2^2 - 16\phi_1}}{2}}\omega_0i.
\end{aligned}
$$

Proof. Clearly, any $s \in \lambda(A_2)$ has non-positive real parts if and only if $\sigma_2 \geq 0$. On the other hand, if $s \in \lambda(A_{13})$, then it follows from

$$
{\det}(sI_4 - A_{13}) = {\det}\begin{bmatrix} sI_2 & -A_1 \\ -A_3 & sI_2 \end{bmatrix} = {\det}(s^2I_2 - A_1A_3),
$$

that $-s \in \lambda(A_{13})$. Hence $\text{Re}\{s\} \leq 0$ if and only if $\text{Re}\{s\} = 0$. This is equivalent to that all the eigenvalues of $A_1A_3$ are non-positive real numbers. Let

$$
f(\lambda) = \det(\lambda I_2 - A_1A_3) = \lambda^2 + \phi_2\lambda + 4\phi_1.
$$

Hence we must have $f(0) \geq 0, -\frac{\phi_2}{2} \leq 0, \Delta = \phi_2^2 - 16\phi_1 \geq 0$, which are equivalent to (S). Finally, if (S) are satisfied, the eigenvalues of $A$ can be easily computed by noting (10). The proof is finished.

Under the condition that all the eigenvalues of $A$ have non-positive real parts, the linear system (11) can be semi-globally stabilized by linear feedback in the presence of actuator saturation (see, for example, [2] and [10]). Particularly, explicit semi-globally stabilizing controllers can be designed by the parametric Lyapunov equation approach in [11].

We next provide necessary and sufficient conditions to guarantee that $A$ is Lyapunov stable (or neutrally stable), namely, all the eigenvalues of $A$ have non-positive real parts and those eigenvalues on the imaginary axis are simple (or equivalently, any eigenvalue on the imaginary axis has the same algebraic and geometric multiplicities).

Theorem 1 Let $A$ be given by (3). Then $A$ is Lyapunov stable if and only if $(\sigma_1, \sigma_2, \sigma_3)$ satisfies

$$
\begin{aligned}
0 < \sigma_2, \\
0 < \sigma_1, \\
0 < \phi_2, \\
0 < \phi_2^2 - 16\phi_1.
\end{aligned}
$$

Proof. Clearly, $A$ is Lyapunov stable if and only if $A_2$ and $A_{13}$ are all Lyapunov stable. Moreover, the matrix $A_2$ is Lyapunov stable if and only if $\sigma_2 > 0$ since $A_2$ has repeated eigenvalues 0 and is nonzero when $\sigma_2 = 0$.

Obviously, if $\phi_i > 0, i = 1, 2, 3$, then $A_{12}$ has four different eigenvalues on the imaginary axis and is thus Lyapunov stable. Hence, we only need to show the converse, namely, if $A_{13}$ is Lyapunov stable, then $\phi_i > 0, i = 1, 2, 3$.

We first show that the Lyapunov stability of $A_{13}$ implies $\phi_1 > 0$. We show this by contradiction. Let

$$
\phi_1 = 0. \text{ If } \sigma_1 = \sigma_3 = 0, \text{ then we must have } \sigma_2 = 0 \text{ (see equation (3)), which is not allowed. Hence we have either } (\sigma_1 = 0, \sigma_3 \neq 0) \text{ or } (\sigma_3 = 0, \sigma_1 \neq 0). \text{ If } (\sigma_1 = 0, \sigma_3 \neq 0), \text{ then } A_{13} \text{ has repeated eigenvalues } 0 \text{ with geometric multiplicity } 1, \text{ and hence is not Lyapunov stable. If } (\sigma_3 = 0, \sigma_1 \neq 0), \text{ we consider two cases. Case } 1: \sigma_1 \neq -\frac{2}{3}. \text{ Then the algebraic and geometric multiplicities of } 0 \text{ are respectively } 2 \text{ and } 1, \text{ which implies that } A_{13} \text{ is not Lyapunov stable. Case } 2: \sigma_1 = -\frac{1}{3}. \text{ Then the algebraic and geometric multiplicities of } 0 \text{ are}
$$

3
respectively 4 and 1, which, again, implies that \( A_{13} \) is not Lyapunov stable. In conclusion, we must have \( \phi_1 > 0 \).

We next show that the Lyapunov stability of \( A_{13} \) implies \( \phi_2 > 0 \). Otherwise, if \( \phi_2 = 0 \), then \( \phi_2^2 - 16\phi_1 \geq 0 \) is equivalent to \( \phi_1 \leq 0 \), which contradicts with \( \phi_1 > 0 \). Hence we must have \( \phi_2 > 0 \).

We finally show that the Lyapunov stability of \( A_{13} \) implies \( \phi_2^2 - 16\phi_1 > 0 \). Otherwise, if \( \phi_2^2 - 16\phi_1 = 0 \), then \( A_{13} \) has repeated eigenvalues \( \lambda_{3,4} = \pm \sqrt{\phi_2/2} \). Notice that, for any \( \lambda = \lambda_i \neq 0 \), \( i = 3, 4 \), we have

\[
\text{rank} \begin{bmatrix} \lambda_2 - A_1 & \lambda_2 \\ -A_3 & \lambda_2 \end{bmatrix} = \text{rank} \begin{bmatrix} I_2 & 0 \\ A_3 & \lambda_2 \end{bmatrix} \begin{bmatrix} \lambda_2 & -A_1 \\ 0 & \lambda_2 I_2 - A_3 A_1 \end{bmatrix} \\
= \text{rank} \begin{bmatrix} \lambda_2 & -A_1 \\ 0 & \lambda_2 I_2 - A_3 A_1 \end{bmatrix} \\
= 2 + \text{rank} [\lambda^2 I_2 - A_3 A_1] \\
\geq 3, \quad (13)
\]

where we have noticed that the matrix

\[
\lambda^2 I_2 - A_3 A_1 = \begin{bmatrix} \left( \frac{\sigma_3}{4} - \frac{1}{4} \sigma_3 \right) \sigma_1 - \frac{1}{2} & \frac{\sigma_1 - 1}{2} \\ 4 (\sigma_3 - 1) \sigma_1 & \frac{1}{2} + \left( \frac{1}{2} \sigma_3 - \frac{1}{2} \right) \sigma_1 \end{bmatrix}, \quad (14)
\]

cannot be zero. Hence the geometric multiplicities of \( \lambda_i, i = 3, 4 \), is 1 and \( A_{13} \) cannot be Lyapunov stable. The proof is finished. \( \blacksquare \)

Consider the following Lyapunov equation

\[
A^T P + PA = -D^T D, \quad (15)
\]

where \( D \) is any matrix with proper dimensions. It is well known that the above Lyapunov equation has a solution \( P > 0 \) for some \( D \) if and only if \( A \) is neutrally stable (Lyapunov stable). Then we have the following corollary.

**Corollary 1** There exists a \( P > 0 \) such that the Lyapunov equation (15) is satisfied for some \( D \) if and only if \( D = 0 \) and (12) are satisfied.

**Proof.** It follows from Theorem 1 that \( A \) is neutrally stable if and only if (12) are satisfied and, in this case, all the eigenvalues of \( A \) are on the imaginary axis. We next show that \( D \) must be zero. Let \( y_i, i \in [1,6] \) be the eigenvectors of \( A \) corresponding to the eigenvalue \( s_i \). Assume that \( D \) is not zero. Then there exists a \( y_j \) such that \( Dy_j \neq 0 \) (otherwise, we must have \( D = 0 \)). Consequently, it follows from (15) that

\[
-\|Dy_j\|^2 = -y_j^H D^T D y_j \\
= y_j^H (A^T P + PA) y_j \\
= 2 \text{Re} (s_j) y_j^H P y_j \\
= 0, \quad (16)
\]

which contradicts with \( Dy_j \neq 0 \). The proof is finished. \( \blacksquare \)

We can actually give explicit solutions to the Lyapunov equation (15), as shown in the following theorem.

**Theorem 2** Assume that \( \sigma_1 \sigma_2 \sigma_3 \neq 0 \). Then all the solutions to the Lyapunov equation (15) with \( D = 0 \) are given by

\[
P = H^T P_0 H, \quad P_0 = \begin{bmatrix} P_2 \\ P_1 \\ P_3 \end{bmatrix}, \quad (17)
\]

in which \( P_2 = \text{diag} \{ 3\sigma_2 \alpha_2, \alpha_2 \} \) with \( \alpha_2 \) being any constants, and

\[
\begin{cases}
P_1 = \begin{bmatrix} \sigma_3 (\alpha_3 + (1 - \sigma_1) \alpha_{13}) & -\sigma_3 \alpha_{13} \\ -\sigma_3 \alpha_{13} & \alpha_1 \end{bmatrix}, \\
P_3 = \begin{bmatrix} 4 \sigma_1 (\alpha_1 + (1 - \sigma_3) \alpha_{13}) & 4 \sigma_1 \alpha_{13} \\ 4 \sigma_1 \alpha_{13} & \alpha_3 \end{bmatrix}.
\end{cases} \quad (18)
\]
where $\alpha_1, \alpha_3$ and $\alpha_{13}$ are any scalars and such that

$$
(1 - \sigma_1) \left( \alpha_1 - \frac{J_x}{J_z} \alpha_3 \right) + (4 \sigma_1 - \sigma_3) \alpha_{13} = 0.
$$

(19)

Particularly, if we choose $\alpha_{13} = 0$ and $\alpha_1 = \frac{J_x}{J_z} \alpha_3$, then we have

$$
P_1 = \text{diag} \left\{ \sigma_3 \alpha_3, \frac{J_x}{J_z} \alpha_3 \right\}, \quad P_3 = \text{diag} \left\{ 4 \frac{J_x}{J_z} \sigma_1 \alpha_3, \alpha_3 \right\}.
$$

(20)

**Proof.** We only need to show that all the solutions $P_0$ to the equation $A_0^T P_0 + P_0 A_0 = 0$ are in the form of (17). Since $A_i, i = 1, 2, 3$, are all nonsingular, by noting the structure of $A_0$, we know that $P_0$ takes the following form

$$
P_0 = \begin{bmatrix}
P_2 & 0 & 0 \\
0 & P_1 & P_{13} \\
0 & P_{13} & P_3
\end{bmatrix},
$$

(21)

which gives the following five equations

$$
\begin{cases}
0 = A_2^T P_2 + P_2 A_2, \\
0 = A_3^T P_3 + P_3 A_3, \\
0 = A_{13}^T P_{13} + P_{13} A_{13}, \\
0 = A_1^T P_1 + P_1 A_1, \\
0 = A_3^T P_3 + P_3 A_3.
\end{cases}
$$

(22)

We first consider the first equation. Let

$$
P_2 = \begin{bmatrix}
p_{21} & p_{22} \\
p_{22} & p_{23}
\end{bmatrix},
$$

(23)

where the parameters are to be specified. Then

$$
A_2^T P_2 + P_2 A_2 = \begin{bmatrix}
-6 \sigma_2 p_{22} & p_{21} - 3 \sigma_2 p_{23} \\
p_{21} - 3 \sigma_2 p_{23} & 2 p_{22}
\end{bmatrix}.
$$

(24)

Hence we must have $p_{22} = 0$ and $p_{21} = 3 \sigma_2 p_{23}$. This is just the desired result if we set $p_{23} = \alpha_2$ with $a_2$ being any constants.

We assume that $P_{13}$ takes the form

$$
P_{13} = \begin{bmatrix}
p_{131} & p_{132} \\
p_{133} & p_{134}
\end{bmatrix},
$$

(25)

where the parameters are to be specified. Then we can compute

$$
A_{13}^T P_{13} + P_{13} A_{13} = \begin{bmatrix}
-2 \sigma_3 p_{132} & p_{131} + (\sigma_3 - 1) p_{132} - \sigma_3 p_{134} \\
p_{131} + (\sigma_3 - 1) p_{132} - \sigma_3 p_{134} & 2 p_{133} + 2 (\sigma_3 - 1) p_{134}
\end{bmatrix},
$$

(26)

$$
A_1^T P_{13} + P_{13} A_1 = \begin{bmatrix}
-8 \sigma_1 p_{133} & p_{131} + (1 - \sigma_1) p_{133} - 4 \sigma_1 p_{134} \\
p_{131} + (1 - \sigma_1) p_{133} - 4 \sigma_1 p_{134} & 2 p_{132} + 2 (1 - \sigma_1) p_{134}
\end{bmatrix}.
$$

(27)

Hence we must have $p_{132} = 0, p_{133} = 0$ and $p_{131} = \sigma_3 p_{134}$. Consequently, we get

$$
A_{13}^T P_{13} + P_{13} A_{13} = \begin{bmatrix}
0 & 0 \\
0 & 2 (\sigma_3 - 1)
\end{bmatrix} p_{134}.
$$

(28)

$$
A_1^T P_{13} + P_{13} A_1 = \begin{bmatrix}
0 & \sigma_3 - 4 \sigma_1 \\
\sigma_3 - 4 \sigma_1 & 2 (1 - \sigma_1)
\end{bmatrix} p_{134}.
$$

(29)

We consider two cases. Case 1: $\sigma_1 \neq 1$. In this case we must have $p_{134} = 0$. Case 2: $\sigma_1 = 1$. Then we have $(1 - \sigma_3) p_{134} = 0$ and $(\sigma_3 - 4) p_{134} = 0$. Since $\sigma_3$ can not equal to 1 and 4 simultaneously, we also must have $p_{134} = 0$. As a result, we have $P_{13} = 0$. 

5
We next let $P_1$ and $P_3$ take the form

$$ P_1 = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{13} \end{bmatrix}, \quad P_3 = \begin{bmatrix} p_{31} & p_{32} \\ p_{32} & p_{33} \end{bmatrix}, $$

where the parameters are to be specified. Then

$$ A_3^T P_3 + P_1 A_1 = \begin{bmatrix} -\sigma_3 p_{32} - 4\sigma_1 p_{12} & 0 \\ 0 & -\sigma_3 + p_{11} + p_{12} - \sigma_1 p_{12} \end{bmatrix}, $$

$$ A_3^T P_1 + P_3 A_3 = \begin{bmatrix} -\sigma_3 p_{32} - 4\sigma_1 p_{12} & 0 \\ 0 & p_{31} + p_{32} + \sigma_3 p_{32} - 4\sigma_1 p_{12} \end{bmatrix}. $$

Hence we must have

$$ \begin{cases} p_{12} = -\sigma_3 p_{32}/4/\sigma_1, \\ p_{11} = -(\sigma_3 p_{33} + p_{12} - \sigma_1 p_{12}), \\ p_{31} = -(p_{32} + \sigma_3 p_{32} - 4\sigma_1 p_{12}). \end{cases} $$

Consequently, the above two equations are satisfied if and only if

$$ (1 - \sigma_1) p_{13} + \left(1 - \frac{\sigma_3}{4\sigma_1}\right) p_{32} + (\sigma_3 - 1) p_{33} = 0, $$

which is

$$ (1 - \sigma_1) p_{13} + (4\sigma_1 - \sigma_3) p_{32} + (\sigma_3 - 1) p_{33} = 0. $$

by setting $p_{32} \rightarrow 4\sigma_3 p_{12}$. This is just [19] since

$$ \sigma_3 - 1 = \frac{J_z}{J_z}(\sigma_1 - 1). $$

and we have denoted $p_{13} = \alpha_1, p_{33} = \alpha_3$ and $p_{32} = \alpha_1$. The proof is finished by noting that $P_1$ and $P_3$ are now exactly in the form of [18].

Of course, the parameters $\alpha_1, \alpha_2, \alpha_3$ and $\alpha_1$ can be well chosen such that $P_i > 0, i = 1, 2, 3$, if the conditions in [12] are satisfied.

The positive definite solutions to the Lyapunov equation [15] can be utilized to stabilize system [1] globally in the presence of actuator saturation even when $B(t)$ is a time-varying periodic matrix. Interested readers may refer to [12] for details.

3 Conclusion

This note has studied the stability properties of the linearized spacecraft attitude control system. Necessary and sufficient conditions are respectively provided to guarantee that the considered systems are Lyapunov stable (neutrally stable) and polynomially stable.

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