Axial anomaly and energy dependence of hyperon polarization in Heavy-Ion Collisions

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Abstract

We address the issue of energy and charge dependence of global polarization of $\Lambda$ hyperons in peripheral $Au-Au$ collisions recently observed by the STAR collaboration at RHIC. We compare different contributions to the anomalous mechanism relating polarization to vorticity and hydrodynamic helicity in QCD matter. We stress that the suppression of the gravitational anomaly contribution in strongly correlated matter observed in lattice simulations confirms our earlier prediction of rapid decrease of polarization with increasing collision energy. Our mechanism leads to polarization of $\bar{\Lambda}$ of the same sign and larger magnitude than the polarization of $\Lambda$. The energy and charge dependence of polarization is suggested as a sensitive probe of fine details of QCD matter structure.

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I. INTRODUCTION

The local violation of discrete symmetries in strongly interacting QCD matter is entering a new important phase of its investigation. It started from Chiral Magnetic Effect (CME) which uses the (C)P-violating (electro)magnetic field generated in heavy ion collisions in order to probe the (C)P-odd effects in QCD matter.

Current development is related to counterpart of this effect, Chiral Vortical Effect (CVE) due to coupling to P-odd medium vorticity leading to the induced electromagnetic and all conserved-charge currents, in particular the baryonic one.

What became most important now is that P-odd effects might be observable as baryon polarization. A mechanism analogous to CVE (known as axial vortical effect, see and references therein) leads to an induced axial current of strange quarks which may be converted to polarization of Λ-hyperons. Another mechanism of this polarization is provided by so-called thermal vorticity in the hydrodynamical approach.

Recently pioneering preliminary results on global polarization of Λ and ¯Λ hyperons in peripheral Au − Au collisions in the RHIC Beam Energy Scan were released showing a decrease of polarization with increasing energy in qualitative agreement with the prediction of ref. Here, we address this issue and explore the relevant details of theoretical description. The decrease with energy is shown to be related to the suppression of the Axial Magnetic effect contribution in strongly correlated QCD matter found in lattice simulations. Consequently, accurate measurements of polarization energy dependence may serve as a sensitive probe of strongly correlated QCD matter.

II. AXIAL ANOMALY AND HYPERON POLARIZATION

We consider hyperon polarization as the observable related to vorticity and helicity. We shall concentrate mostly on Λ(¯Λ) hyperons, which are produced in large numbers and their polarization may be easily recovered from the angular distributions of their weak decays products. These advantages are important in the current STAR measurements.

We explore the mechanism of generation of axial current similar to the famous axial anomaly. In the medium described by a chemical potential $\mu(x)$ there is a contribution to the interaction Lagrangian proportional to the appropriate conserved charge density in
the medium rest frame $\rho(x) = j_0(x)$:

$$\Delta L(x) = \mu(x)\rho(x).$$

The Lorentz covariance allows one to transform this expression using the hydrodynamical four-velocity $u_\alpha = \gamma(1, \vec{v})$ where $\gamma$ is the Lorentz factor:

$$\Delta L(x) = \mu(x)u_\alpha(x)j_\alpha(x).$$

Here, the velocity $u_\alpha(x)$ and the chemical potential $\mu(x)$ play the role of the gauge field $A(x)$ and the corresponding coupling $g$, respectively:

$$gA_\beta(x)j_\beta(x) \rightarrow \mu(x)u_\alpha(x)j_\alpha(x) \quad (1)$$

This substitution can be applied to any diagram with the lines of external (classical) gauge fields leading to various medium effects. In the case of the famous anomalous triangle diagram (Fig.1) it leads to the induced (classical) axial current.

This effect is quite similar to the anomalous gluon contribution to the nucleon spin (see e.g. [10, 11]). The role of the gauge gluon field is played by the velocity field while the longitudinal polarization (helicity) of gluons corresponds, as we will see below, to the hydrodynamic helicity.

Note that for massive quarks the anomalous contribution is partially compensated by the "normal" one (see e.g. [10]). For a heavy quark axial current $\bar{Q}\gamma_\mu\gamma_5Q$ the resulting matrix element between momentum eigenstates may be expanded to inverse powers of the quark mass $m_Q$ [12]:

$$\langle p|\bar{Q}\gamma_\mu\gamma_5Q|p\rangle = i\frac{N_c\alpha_s}{2\pi}\epsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\left\{1 - \int_0^1 dx \frac{2m_Q^2(1-x)}{m_Q^2 - p^2x(1-x)}\right\}$$

$$= -i\frac{N_c\alpha_s}{12\pi}\epsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\frac{p^2}{m_Q^2} + O\left(\frac{1}{m_Q^4}\right), \quad (2)$$

where $e^\nu$, $e^{*\rho}$ are polarization vectors of these eigenstates and $N_c$ is the number of colors. As far as the $SU(3)$ wave function of $u$ and $d$ quarks form the spin singlet, we assume that $\Lambda$ spin is carried predominantly by the strange quark. The strange quark may be considered as both light (with respect to the nucleon mass) and heavy (with respect to intrinsic higher twist scale) [13]. When the transition to coordinate space is performed, the Fourier transform of the corresponding matrix elements (requiring, generally speaking, the
knowledge of the anomaly graph at arbitrary external momenta [14] should contribute to the classical axial current. We do not expect that these corrections could change the scale of the effect substantially.

Let us therefore consider the classical strange axial charge [5]

$$Q_s^5 = \frac{N_c}{2\pi^2} \int d^3 x \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$ (3)

induced by the anomalous triangle diagram (Fig. 1). Here, we keep actually only the first term in the chiral vorticity coefficient

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_s^5 = N_c \int d^3 x c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$ (4)

since we assume that the chiral chemical potential $\mu_A$ is much smaller than the strange one $\mu_s$. The temperature dependent term in eq. (4), related to the gravitational anomaly [15], can naively be considered to be quite substantial. However, lattice simulations [16] lead to a zero result in the confined phase and to suppression by one order of magnitude at high
temperatures. As far as for free fermion gas the $T^2/6$ term is recovered\cite{17} for large lattice volume at fixed temperature, the above-mentioned suppression should be attributed to the correlation effects.

In order to relate the strange axial charge $Q_5^s$\cite{3} to hydrodynamical quantities one can use the mean-value theorem to evaluate it\cite{5}:

$$Q_5^s = \frac{<\mu^2 \gamma^2 >}{2\pi^2} N_c H,$$

where hydrodynamical helicity

$$H \equiv \int d^3x (\vec{v} \cdot \vec{w})$$

is the integrated projection of the velocity $\vec{v}$ to the vorticity $\vec{w} = \text{curl} \vec{v}$.

Note that the hydrodynamic helicity is related to the zeroth component ($\mu = 0$) of the four-current

$$K^\mu(x) = \epsilon^{\mu\nu\rho\gamma} u_\nu(x) \partial_\rho u_\gamma(x), \quad H = \int d^3x \frac{K^0(x)}{\gamma^2}.$$  \hspace{1cm} (6)

Coming back to the similarity with the spin crisis, this is the analog of the topological current, related\cite{10} (in the axial gauge) to the gluon polarization.

The space components of (6) for slow enough fields in the non-relativistic approximation are related to the vorticity vector:

$$K^i(x)|_{\gamma \rightarrow 1} = 2\epsilon^{ijk} \partial_j v_k(x) = 2\omega^i,$$

while the exact relation involves relativistic corrections. To avoid such complications, here we discuss the complementary approach\cite{18} relying on the $K^0(x)$, helicity and axial charge.

To pass from the classical charge (we are always dealing with) to the quantum matrix elements we will use the analogy with the conserved current case when the conserved charge $Q$ appears in the symmetric one-particle matrix elements of the current

$$< p_n | j^0(0) | p_n > = 2p_n^0 Q_n.$$ \hspace{1cm} (7)

To calculate the (quantum) average charge per particle resulting from the latter equation it is sufficient to divide the total classical charge by particles’ number $N$:

$$< Q > \equiv \frac{\sum_{n=1}^{N} Q_n}{N} = \frac{\int d^3x j^0_{\text{class}}(x)}{N}.$$ \hspace{1cm} (8)
Passing now to the axial charge case, let us note that the symmetric matrix element of quark axial current of the flavor $i$ is related to the fraction $a_i$ of a hadron covariant polarization $\Pi^\mu (p_\mu \Pi^\mu = 0)$ carried by that quark:

$$<p_n, \Pi_n|j_{5,i}^0(0)|p_n, \Pi_n>= 2a_{i,n}m_n\Pi_n^0,$$

where $m_i$ is a hadron $i$ mass. By analogy with [7, 8], the axial charge should correspond to

$$Q_{5,i,n} \rightarrow \frac{m_{n}a_{i,n}\Pi_{n}^{0}}{p_{0}^{n}}.$$

The average polarization is then

$$<\frac{a_{i}m_{i}\Pi_{0}^{0}}{p_{0}}>|\equiv \sum_{n=1}^{N} \frac{m_{n}a_{i,n}\Pi_{n}^{0}}{p_{0}^{n}}N = \int d^{3}x j_{5,\text{class}}^{0}(x).$$

In what follows we will consider all $N$ particles to be $\Lambda$ ($\bar{\Lambda}$) and put $a_{i}m/p_0 = 1$ assuming (following the above-mentioned $SU(3)$ based arguments) that strange (anti)quarks carry all the polarization and considering the $\Lambda$ mass is large enough with respect to its momentum. The latter assumption is a reasonable approximation at the NICA energies ($\sqrt{s_{NN}} = 4 - 11 GeV$) and will provide a lower bound estimate of the polarization.

The helicity shows the phenomenon of separation [5, 19] so that its sign is changed at the two sides of the reaction plane. As a result, the axial charge and the zeroth component of the hyperon polarization also manifest such a phenomenon of sign change.

As the axial charge is related to the zeroth component of the hyperon polarization in the laboratory frame $\Pi_{0}^{\Lambda,\text{lab}}$, the transformation to hyperon rest frame must be performed [18]. Taking into account that the polarization pseudovector should be directed along the $y-$axis transverse to the reaction plane (as it has to be collinear to the angular momentum pseudovector), one gets for the components of the laboratory frame polarization

$$\Pi^{\Lambda,\text{lab}} = (\Pi_{0}^{\Lambda,\text{lab}}, \Pi_{x}^{\Lambda,\text{lab}}, \Pi_{y}^{\Lambda,\text{lab}}, \Pi_{z}^{\Lambda,\text{lab}}) = \frac{\Pi_{0}^{\Lambda}}{m_{\Lambda}}(p_{y}, 0, p_{0}, 0).$$

One can use both its time $\Pi_{0}^{\Lambda,\text{lab}}$ and space $\Pi_{y}^{\Lambda,\text{lab}}$ components to recover the polarization in the hyperon rest frame. Note that for the time component the factor $p_y$ changes sign at two sides of reaction plane compensating the sign change due to helicity separation. At the same time, that factor is absent for space component. As we discussed above, this component is related to the vorticity rather than helicity and should not show the sign change. Therefore,
the use of either time or space component of $\Pi^\Lambda_{0,lab}$ should lead to equal polarization at both sides of the reaction plane. Leaving the exploration of space component for future work we use the time component and get:

$$< \Pi_0^\Lambda > = \frac{m_\Lambda \Pi^\Lambda_{0,lab}}{p_y} = < \frac{m_\Lambda}{N_\Lambda p_y} > Q_5^z \equiv < \frac{m_\Lambda}{N_\Lambda p_y} > \frac{N_c}{2\pi^2} \int d^3x \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k. \quad (10)$$

The appearance of $p_y$ in the denominator is not dangerous, as the particles with zero transverse momentum do not have also the time component of polarization.

The average polarization was firstly roughly estimated just by dividing $Q_5^z$ \cite{5} by the number of $\Lambda$ leading \cite{5} to a value of about 1%, later confirmed \cite{18} by more detailed simulations, which are compatible with the current STAR data \cite{8}.

The appearance of $\mu^2$ in eqs. \cite{3} and \cite{10}, related to the positive C-parity of axial current, immediately leads to the same expressions for axial charge of strange quarks and antiquarks. As far as there is a smaller number of $\bar{\Lambda}$s than of $\Lambda$s, so that the same axial charge should be distributed among smaller number of antiquarks comparing with the number of quarks, the corresponding factor in the denominator in eq. \cite{10} is smaller for $\Lambda$'s, which results in an increase of the effect for the latter. Thus, one could expect that the polarization of $\Lambda$ has to be of the same sign but of a larger magnitude than the polarization of $\Lambda$, which is compatible with the quite recent STAR data \cite{8}. This effect might be partly compensated by the fact, that a larger amount of axial charge in the case of strange antiquarks might be carried by more numerous $K^*$-mesons.

As far as the strange chemical potential is rapidly decreasing with energy, this provides a natural explanation of the observed hint \cite{8} for decrease of polarization with energy. More accurate measurements of $\Lambda$ and $\bar{\Lambda}$ polarization at RHIC, and at NICA and FAIR in future, might allow one to test the suppression of $T^2$ term and, at best, even to check experimentally the magnitude of its theoretically predicted coefficient and search for possible extra \cite{7} contributions.

III. CONCLUSIONS AND OUTLOOK

The generation of polarization by the anomalous mechanism (Axial Vortical Effect) naturally explains several features of the observed data.
• The decrease of chemical potential with energy leads to the decrease of polarization. An additional source of decrease is provided by the energy dependence of the helicity which was earlier found [5] to be maximal in the NICA energy range. The contribution related to the gravitational anomaly proportional to $T^2$ may be suppressed in strongly correlated matter. Moreover, the accurate measurements of the energy dependence of polarization should allow to separate the gravitational anomaly contribution and test the degree of its suppression in strongly correlated QCD matter.

• The proportionality of the polarization to the square of the chemical potential related to C-even parity of axial current leads to the same sign of polarization of $\Lambda$ and $\bar{\Lambda}$ hyperons. The smaller number of the latter should result in a larger fraction of the axial charge corresponding to each anti-hyperon and to a larger absolute value of polarization. Detailed numerical simulations may allow to quantify this prediction. Accurate measurements of $\Lambda$ and $\bar{\Lambda}$ polarization should allow to check these predictions and provide an additional check of the gravitational anomaly related contribution.

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