On leptonic width of $X(4260)$

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QIN-FANG CAO†, HONG-RONG QI‡, GUANG-YI TANG,∗ YUN-FENG XUE,† HAN-QING ZHENG†,∗

† Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People’s Republic of China
‡ Department of engineering physics, Tsinghua University, Beijing 100084, People’s Republic of China
∗ Institute of High Energy Physics, Beijing 100049, People’s Republic of China
∗ Collaborative Innovation Center of Quantum Matter, Beijing 100871, People’s Republic of China

Abstract

New measurements on cross sections in $e^-e^+\rightarrow J/\psi\pi^+\pi^-$, $h_c\pi^+\pi^-$, $D_0 D^*-\pi^+ + c.c.$, $\psi(2S)\pi^+\pi^-$, $\omega\chi_c$ and $J/\psi\eta$ channels have been carried out by BESIII, Belle and BABAR collaborations, and also in the $D\bar{D}^*$ channel. We perform extensive numerical analyses by combining all these data available, together with those in $D\bar{D}$ and $D^*\bar{D}^*$ channels. Though the latter show no evident peak around $\sqrt{s} = 4.230$ GeV, the missing $X(4260)$ is explained as that it is concealed by the interference effects of the well established charmonia $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$. Our analyses reveal that the leptonic decay width of $X(4260)$ ranges from $O(10^{-2})$ eV to $O(1)$ keV, and hence may be explained in the conventional quark model picture. That is, the $X(4260)$ may well be interpreted as a mixture of $4^3S_1$ and $3^3D_1$ states.

1 Introduction

The $X(4260)$ resonance established by BABAR Collaboration in initial-state radiation (ISR) process, $e^-e^+\rightarrow \gamma_{ISR}J/\psi\pi^+\pi^-$ in year 2005 [1] (see also CLEO [2] and Belle [3]), has attracted much attention since then. The mass and width of this resonance are given with $M = 4230 \pm 8$ MeV and $\Gamma = 55 \pm 19$ MeV [4], respectively, and $\Gamma_{ee} \times Br(J/\psi\pi\pi) = 9.7 \pm 1.1$ eV [3] or $9.2 \pm 1.5$ eV [5].

The property of $X(4260)$ becomes a very interesting topic since its discovery, because it is generally thought that there are not enough unassigned vector states in charmonium spectrum (taking into account the recently reported $X(4360)$, $X(4630)/X(4660)$ states), according to the naive quark model predictions [6]. The only such $1^{--}$ states expected up to 4.4 GeV are $1S$, $2S$, $1D$, $3S$, $2D$ and $4S$, and they seem to be well established [7] – the situation is depicted in Figure 1. It is noticed that above $D\bar{D}$ threshold the number of $1^{--}$ states given by quark model prediction is inconsistent with that given by experiments. It is generally considered that the discrepancy between the naive quark model prediction and the observed spectrum is ascribed, at least partially, to the existence of many open charm thresholds, since the latter will distort the spectrum (see related discussions in, for example, Refs. [8, 9]). The situation is depicted in Figure 2.

Because of the situation as described above many studies have been devoted to the investigation of $X(4260)$ [1]. The suggestions given by these studies may be classified into three types: hadronic molecule [10, 15]; $c\bar{c}$ state [16, 20]; hybrid state [21, 22]; or non-resonant enhancement [23].

The open charm channels such as $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$ do not seem to be found in the final states of $X(4260)$ decays [21, 26]. If this is indeed the case, then it would make $X(4260)$ even more mysterious, since the $J/\psi\pi\pi$ channel would become a very important, if not the dominant one.

1 Here we apologize for only able to provide an incomplete list of references.
Hence the leptonic width $\Gamma_{e^+e^-}$ of $X(4260)$ would become very small, making it even harder to be understood as a conventional $1^{-+}$ $c\bar{c}$ state. For example, in a previous publication, we have studied the $X(4260)$ issue and suggested that there could be a sizable $\omega \chi_{c0}$ coupling [27], later confirmed by experimental researches [28]. At the same time, a very small $\Gamma_{e^+e^-}$ is found to be $\simeq 25\text{eV}$. However the nearby $4^3S^1$ state is expected to have a leptonic width $\simeq 1\text{keV}$, whereas for a pure $3^3D^1$ state $\Gamma_{e^+e^-} \simeq 44\text{eV}$ [19].

Many new experimental results have appeared since the work of Ref. [27], measured by BESIII, Belle and BABAR collaborations, such as $e^+e^- \rightarrow J/\psi\pi^+\pi^- [29, 31], h_c\pi^+\pi^- [32], D^0D^{*-}\pi^+ + \text{c.c.} [33], \psi(2S)\pi^+\pi^- [34, 36], \omega\chi_{c0} [37, 38], J/\psi\eta [39, 40] and D^*_sD^*_{1(2420)} [41].$ Hence the analyses of Ref. [27] urgently need to be upgraded. Among all it is worthwhile mentioning the $D^*_sD^*_s$ data near the $X(4260)$ region [41], which indicates a strong enhancement of events above the $D^*_sD^*_s$ threshold. If this is true, our analyses show that it decisively changes our previous
understandings on \(X(4260)\) resonance: It could probably be described by the conventional \(4^3S_1\) state heavily renormalized by the \(D_s^*D_s^*\) continuum (maybe a small mixing with the \(3^3D_1\) state as well). If the \(D_s^*D_s^*\) data are excluded from the fit, however, the final result on \(\Gamma_{cc}\) can still be \(O(10^6)\)eV, i.e., much larger comparing with that of Ref. [27], owing to other new data available as mentioned above. As a consequence, the \(X(4260)\) resonance may still be considered as a mixture of \(3^3D_1\) and \(4^3S_1\) states, i.e., a conventional \(cc\) resonance.

The paper is organized as follows: This section 1 is the introduction. A detailed description of the numerical fit program will be given in section 2. In section 3 combined fits to the hidden charm and open charm decay channels are performed, with two scenarios: one includes the \(D_s^*D_s^*\) cross section data and the other does not. We leave physical discussions and conclusions in section 4.

2 Theoretical Discussions

To begin with, the \(X(4260)\) propagator is written in the following form:

\[
D_X(s) = s - M_X^2 + i\sqrt{s}\Gamma_{tot}(s),
\]

(1)

where \(\Gamma_{tot}(s)\) is the total momentum dependent width comprising of all partial ones:

\[
\Gamma_{tot}(s) = \Gamma_{J/\psi\pi\pi}(s) + \Gamma_{\eta\pi\pi}(s) + \Gamma_{DD^*\pi}(s) + \Gamma_{\psi(2S)\pi\pi}(s) + \Gamma_{\chi_{c0}} + \Gamma_{J/\psi}\chi + \Gamma_{D_s^*D_s^*} + \Gamma_{DD}\chi + \Gamma_{D_sD_s}\chi + \Gamma_6.
\]

(2)

Considering the isospin symmetry, it is noticed that \(\Gamma_{J/\psi\pi\pi}(s) = 2\Gamma_{J/\psi\pi\pi-\pi}(s), \Gamma_{\eta\pi\pi}(s) = 2\Gamma_{h\pi\pi-\pi}(s), \Gamma_{DD^*\pi}(s) = 3\Gamma_{DD^*\pi-\pi(c.c.)}, \Gamma_{\psi(2S)\pi\pi}(s) = 2\Gamma_{\psi(2S)\pi-\pi}(s), \Gamma_{DD} = 2\Gamma_{D_sD_s-\pi(c.c.)}\)

As for the \(J/\psi\pi^+\pi^-, h\pi^+\pi^-, D^0D^-\pi^+\) and \(\psi(2S)\pi^+\pi^-\) channels, the three body partial decay width takes the standard form [4],

\[
d\Gamma = \frac{1}{(2\pi)^6} \frac{1}{16M^2} |\mathcal{M}|^2|p_1||p_2|dm_{12}d\Omega_1d\Omega_3,
\]

(3)

where \(p_{1,2} = p_i + p_j, m_{12}^2 = p_{12}^2, |p_1|, |p_2|\) are the momentum and angle of particle 1 in the rest frame of the system of particle 1 and 2, respectively; \(\Omega_3\) is the angle of particle 3 in the rest frame of particle \(M\); and \(|p_1|, |p_2|\) are defined as

\[
|p_1|^2 = \frac{[(m_{12}^2 - (m_1 + m_2)^2)(m_{12}^2 - (m_1 - m_2)^2)]^{1/2}}{2m_{12}},
\]

(4)

\[
|p_2|^2 = \frac{[(M^2 - (m_{12} + m_3)^2)(M^2 - (m_1 - m_3)^2)]^{1/2}}{2M}.
\]

(5)

In addition, the two body decay widths take the following simple forms:

\[
\Gamma_{\omega\chi_{c0}} = g_{\omega\chi_{c0}}k_{\omega\chi_{c0}}, \quad \Gamma_{J/\psi\chi} = g_{J/\psi\chi}k_{J/\psi\chi}^3, \quad \Gamma_{D_s^*D_s^*} = g_{D_s^*D_s^*}k_{D_s^*D_s^*}^3, \\
\Gamma_{D^+D^-} = g_{D^+D^-}k_{D^+D^-}^3, \quad \Gamma_{D^+D^-} = g_{D^+D^-}k_{D^+D^-}^3.
\]

(6)

In above \(k_{\omega\chi_{c0}}, k_{J/\psi\chi}, k_{D_s^*D_s^*}, k_{D^+D^-}\) and \(k_{D^+D^-}\) are three momentums of \(\omega\chi_{c0}, J/\psi\chi, D_s^*D_s^*, D^+D^-\) and \(D^+D^-\) in \(X(4260)\) rest frame, respectively. Further, except for the channels just discussed, there could be other channels with lower thresholds, for these channels we use a constant width \(\Gamma_0\) to describe the overall effects.

Because the quantum number of \(X(4260)\) is \(J^{PC} = 1^{-+}\), the interaction between \(X(4260)\) and photon can be written as

\[
\mathcal{L}_{\gamma X} = \frac{g_0}{M_X} X_{\mu\nu} F^{\mu\nu},
\]

(7)

---

2 We do not consider a term \(\propto \Gamma_{DD}\chi\) here, as it is found vanishing in Ref. [27]. Also the experimental data in \(Z_\pi\) channel is absent. Since the \(Z_0(3900)\) state is identified as a \(DD^*\) molecule [12][13], the experimental branching ratio is naturally expected to be small.

3 Hereafter, the notation \(D^0D^-\pi^+\) and \(D^+D^-\pi^+\) indicate \(D^0D^-\pi^+ + c.c.\) and \(D^+D^-\pi^+ + c.c.\) for simplicity, respectively.
where \( X_{\mu \nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \) and \( V^\mu \) represents the \( X(4260) \) field. \( F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \) describes the photon field. So the decay width of \( X(4260) \to e^+e^- \) reads

\[
\Gamma_{e^+e^-} = \frac{4\alpha g_0^2}{3} M_X. \tag{8}
\]

In \( h_c \pi^+\pi^- \), \( D^0 D^{*-\pi^+} \), \( \psi(2S) \pi^+\pi^- \), \( \omega \chi_{c0} \), \( J/\psi \eta \), \( D_s^* \bar{D}_s^* \), \( D^+D^{*-} \) and \( D^{*-}D^{*-} \) channels, using narrow width approximation, the cross section formulae take the form

\[
\sigma_{e^+e^- \to X(4260) \to f} = \frac{3\pi}{k^2} \sqrt{s \Gamma_{ee} \Gamma_f} + \sum_i \frac{c_i e^{i\phi_i}}{s - M_i^2 + i\sqrt{s} \Gamma_i} + \hat{c}^2, \tag{9}
\]

where \( k \) is the 3-momentum of incoming electron in c.m. frame, \( \Gamma_{ee} \) follows Eq. (8) and \( \Gamma_f \) takes the form of Eq. (3) and Eq. (4). The term parameterized as a resonance propagator with a mass \( M_i \) and width \( \Gamma_i \) and the complex constant \( \hat{c} \) play the role of a background in each decay channel here, as will be declared in details in the forthcoming section 3.

3 Numerical Analyses and Discussions

In section 3.1, we will perform comprehensive fits to relevant data available in the vicinity of \( X(4260) \), which include the \( J/\psi \pi^+\pi^- \) process and the cross section data. Different results are carefully compared and discussed, and we believe that a clearer understanding on the nature of \( X(4260) \) emerges.

3.1 The fit with \( D_s^* \bar{D}_s^* \) cross section data

3.1.1 The \( J/\psi \pi^+\pi^- \) process

For the \( J/\psi \pi^+\pi^- \) channel, the experimental data sets come from: BESIII \( [29] \), \( \sqrt{s} \in [3.81,4.60] \) GeV, 121 data points; BABAR \( [30] \), \( \sqrt{s} \in [4.15,4.47] \) GeV, 17 data points; Belle \( [31] \), \( \sqrt{s} \in [4.15,4.47] \) GeV, 17 data points. For the fit to the above data, we adopt the amplitude as Eq. (14) in Ref. \( [27] \) to describe the \( e^+e^- \to X(4260) \to J/\psi \pi^+\pi^- \) process and the propagator \( \frac{1}{D_X(s)} \) of that equation is rewritten as the following form:

\[
\frac{1}{D_X(s)} \Rightarrow \frac{1}{s - M_{4415}^2} + \frac{c_1 e^{i\phi_1}}{s - M_{4415}^2 + i\sqrt{s} \Gamma_{4415}} + \frac{c_2 e^{-\phi_2(\sqrt{s} - m_{th})}}{s - M_{4415}^2 + i\sqrt{s} \Gamma_{4415}}, \tag{10}
\]

where \( m_{th} \) is the threshold of \( J/\psi \pi^+\pi^- \). \( M_{4415} \) and \( \Gamma_{4415} \) is introduced to represent the mass and width of \( \psi(4415) \), respectively. See Figure 3 and Table 1 for fit results.

| \( c_1 \) | \( \phi_1 \) | \( c_2 \) | \( \phi_2 \) |
|---|---|---|---|
| 0.137 ± 0.026 | 4.283 ± 0.157 | 304.96 ± 20.612 | 5.356 ± 0.093 |

Table 1: Fit parameters in \( J/\psi \pi^+\pi^- \) channel.

3.1.2 The \( D^0 D^{*-\pi^+} \), \( h_c \pi^+\pi^- \) and \( \psi(2S) \pi^+\pi^- \) processes

In \( D^0 D^{*-\pi^+} \) channel, the BESIII \( e^+e^- \to D^0 D^{*-\pi^+} \) data \( [33] \) ranging from \( \sqrt{s} \in [4.05,4.45] \) GeV are chosen with 51 points. In \( h_c \pi^+ \) channel BESIII data \( [32] \) in \( \sqrt{s} \in [4.08,4.32] \) GeV region with 45 data points are chosen. For the \( \psi(2S) \pi^+\pi^- \) process, the experimental data come from: BESIII \( [34] \), \( \sqrt{s} \in [4.085,4.308] \) GeV, 8 data points; Belle \( [35] \), \( \sqrt{s} \in [4.11,4.31] \) GeV, 11 data points; BABAR \( [36] \), \( \sqrt{s} \in [4.13,4.32] \) GeV, 5 data points. The fit formula is

\[
\sigma_f = \frac{3\pi}{k^2} \sqrt{s \Gamma_{ee} \Gamma_f} + \sum_i \frac{c_i e^{i\phi_i}}{s - M_i^2 + i\sqrt{s} \Gamma_i} + \hat{c}^2, \tag{11}
\]

where \( i = 3,4,5 \) represents the \( D^0 D^{*-\pi^+} \), \( h_c \pi^+\pi^- \) and \( \psi(2S) \pi^+\pi^- \) channels, respectively. See Figures 4(a), 4(b), 4(c) and Table 2 for fit quality and results.

\(^4\)Here we use \( s = q^2 \).
Figure 3: The results of the fit with $D^*_s D^*_s$ cross section data: (a) Fit to the cross section of $J/\psi \pi^+ \pi^-$. The dots (blue) and squares (orange) are from BABAR [30] and Belle [31], respectively. (b) Fit to the cross section of $J/\psi \pi^+ \pi^-$ measured by BESIII [29]. The squares (orange) are from the XYZ data sample at BESIII [29] and the dots (blue) are from the $R$-scan data sample by BESIII [29]. The solid red curves show the best fit.

Figure 4: The results of the fit with $D^*_s D^*_s$ cross section data: (a) Fit to $D^0 D^{*-} \pi^+$ data by BESIII [33]. (b) Fit to $h_c \pi^+ \pi^-$ data by BESIII [32]. The squares (orange) are from the XYZ data sample at BESIII [32] and the dots (blue) are from the $R$-scan data sample by BESIII [32]. (c) Fit to $\psi(2S) \pi^+ \pi^-$ data. The orange dots come from BESIII [34], the green squares are from Belle [35] and the blue rhombuses come from BABAR [36]. The solid red curves show the best fit, and the dashed black ones represent $X(4260)$ components.

| $c_3$ | $\phi_3$ | $c_4$ | $\phi_4$ | $c_5$ | $\phi_5$ |
|-------|----------|-------|----------|-------|----------|
| $0.482 \pm 0.029$ | $4.225 \pm 0.143$ | $0.248 \pm 0.025$ | $0.137 \pm 0.242$ | $0.190 \pm 0.015$ | $5.535 \pm 0.355$ |

Table 2: Fit parameters in $D^0 D^{*-} \pi^+$, $h_c \pi^+ \pi^-$ and $\psi(2S) \pi^+ \pi^-$ channels with $c_i (i = 3, 4, 5)$ in unit of $\text{GeV}^2$. 
Figure 5: Fit to $\omega \chi_c^0$ cross section data under the fit with $D_s^* \bar{D}_s^*$ cross section data scheme. The blue dots and the orange squares come from BESIII experiments in Ref. [37] and Ref. [38], respectively. We do not fit the two data points (orange squares) on the left side since they are below the threshold.

Figure 6: Fit to $J/\psi \eta$ data under the fit with $D_s^* \bar{D}_s^*$ cross section data scheme. The blue dots represent the Belle results [40] and the orange squares indicate the BESIII data [39].

3.1.3 The $\omega \chi_c^0$ process

The $\omega \chi_c^0$ data comes from Ref. [37], $\sqrt{s} \in [4.21, 4.39]$ GeV, 8 data points and Ref. [38], $\sqrt{s} \in [4.199, 4.278]$ GeV, 7 data points. The fit formula can be parameterized as

$$
\sigma_{\omega \chi_c^0} = \frac{3\pi}{k^2} \left| \frac{\sqrt{\Gamma_{ee} \Gamma_{\omega \chi_c^0}}}{s - M_X^2 + i\sqrt{\Gamma_{tot}}} \right|^2,
$$

and the fit results are presented as Figure 5.

3.1.4 The $J/\psi \eta$ process

Concerning the $J/\psi \eta$ channel, 8 data points ranging from $\sqrt{s} \in [4.09, 4.31]$ GeV measured by BESIII [39] and 13 data points from $\sqrt{s} \in [4.07, 4.31]$ GeV measured by Belle [40] are adopted simultaneously. On account of the influence of $\psi(4160)$, which was taken into account in the fit by Belle in Ref. [40], the cross section is written as the following:

$$
\sigma_{J/\psi \eta} = \frac{3\pi}{k^2} \left| \frac{\sqrt{\Gamma_{ee} \Gamma_{J/\psi \eta}}}{s - M_X^2 + i\sqrt{\Gamma_{tot}}} \right|^2 + \frac{c_6 e^{i\phi_6}}{s - M_{4160}^2 + i\sqrt{\Gamma_{4160}}}.
$$

Besides, $M_{4160}$ and $\Gamma_{4160}$ are introduced to represent the mass and width of $\psi(4160)$. 
Table 3: Fit parameters in $J/\psi\eta$ channel and $c_6$ is in unit of GeV$^2$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\(c_6\) & \(\phi_6\) \\
\hline
\(-0.028 \pm 0.015\) & \(0.504 \pm 0.654\) \\
\hline
\end{tabular}
\caption{Fit parameters in $J/\psi\eta$ channel and $c_6$ is in unit of GeV$^2$.}
\end{table}

Figure 7: Fit to $D^*_s\bar{D}^*_s$ data from BESIII [41] under the fit with $D^*_s\bar{D}^*_s$ cross section data scheme.

### 3.1.5 The $D^*_s\bar{D}^*_s$ process

In $D^*_s\bar{D}^*_s$ channel we take the data from Ref. [41], $\sqrt{s} \in [4.23, 4.36]$ GeV, 5 data points. Then the cross section reads

$$
\sigma_{D^*_s\bar{D}^*_s} = \frac{3\pi}{k^2} \left| s - M^2_X + i\sqrt{s}\Gamma_{\text{tot}} \right|^2 + \frac{c_7 e^{i\phi_7}}{s - M^2_{4415} + i\sqrt{s}\Gamma_{4415}} \right|^2.
$$

(14)

See Figure 7 and Table 4 for fit results.

Table 4: Fit parameters in $D^*_s\bar{D}^*_s$ channel and $c_7$ is in unit of GeV$^2$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\(c_7\) & \(\phi_7\) \\
\hline
\(-0.941 \pm 0.022\) & \(4.896 \pm 0.031\) \\
\hline
\end{tabular}
\caption{Fit parameters in $D^*_s\bar{D}^*_s$ channel and $c_7$ is in unit of GeV$^2$.}
\end{table}

### 3.1.6 The $D^+D^{*-}$ and $D^{*+}D^{*-}$ processes

For the $D^+D^{*-}$ channel, see Figure 8(a) we take the Belle data [25], $\sqrt{s} \in [3.93, 4.37]$ GeV, with 23 data points. In the fit we add Breit-Wigner resonances and a complex constant to simulate the interference background contribution:

$$
\sigma_{D^+D^{*-} + \text{c.c.}} = \frac{3\pi}{k^2} \left| s - M^2_X + i\sqrt{s}\Gamma_{\text{tot}} \right|^2 + \frac{c_8 e^{i\phi_8}}{s - M^2_{4040} + i\sqrt{s}\Gamma_{4040}} + \frac{c_9 e^{i\phi_9}}{s - M^2_{4160} + i\sqrt{s}\Gamma_{4160}} + \frac{c_{10} e^{i\phi_{10}}}{s - M^2_{4415} + i\sqrt{s}\Gamma_{4415}} + \frac{c_{11} e^{i\phi_{11}}}{s - M^2_{4415} + i\sqrt{s}\Gamma_{4415}} + \frac{c_{12} e^{i\phi_{12}}}{s - M^2_{4415} + i\sqrt{s}\Gamma_{4415}} \right|^2.
$$

(15)

in which $M_{4040}$ and $\Gamma_{4040}$ are used to describe the mass and width of $\psi(4040)$.

Additionally, 18 data points released by Belle [25] from $\sqrt{s} \in [4.11, 4.45]$ are adopted in the $D^{*+}D^{*-}$ process and the cross section is written as

$$
\sigma_{D^{*+}D^{*-}} = \frac{3\pi}{k^2} \left| s - M^2_X + i\sqrt{s}\Gamma_{\text{tot}} \right|^2 + \frac{c_{10} e^{i\phi_{10}}}{s - M^2_{4040} + i\sqrt{s}\Gamma_{4040}} + \frac{c_{11} e^{i\phi_{11}}}{s - M^2_{4160} + i\sqrt{s}\Gamma_{4160}} + \frac{c_{12} e^{i\phi_{12}}}{s - M^2_{4415} + i\sqrt{s}\Gamma_{4415}} \right|^2.
$$

(16)
Figure 8: The results of the fit with \( D^*_s D^*_c \) cross section data: (a) \( D^+ D^{**} \) cross section. The blue dots come from Belle results \(^{25}\) and the solid curve is the projection from the best fit. The dashed curves show the individual components of \( \psi(4040) \), \( \psi(4160) \), \( X(4260) \), respectively. \( \) (b) \( D^+ D^{**} \) cross section. The blue dots come from Belle results \(^{25}\). The solid curve is the projection from the best fit. The dashed curves show the individual components of \( \psi(4040) \), \( \psi(4160) \), \( X(4260) \) and \( \psi(4415) \), respectively.

| \( c_8 \) | \( \phi_8 \) | \( c_9 \) | \( \phi_9 \) | \( c_{13} \) | \( c_{14} \) |
|---------|---------|---------|---------|---------|---------|
| 1.441 ± 0.040 | 4.481 ± 0.251 | -0.525 ± 0.195 | 0.803 ± 0.287 | 1.059 ± 0.137 | -0.667 ± 0.271 |

Table 5: Fit parameters in \( D^+ D^{**} \) channel with \( c_i (i = 8, 9) \) in unit of GeV\(^2\).

| \( c_{10} \) | \( \phi_{10} \) | \( c_{11} \) | \( \phi_{11} \) | \( c_{12} \) | \( \phi_{12} \) |
|---------|---------|---------|---------|---------|---------|
| 1.218 ± 0.133 | 1.471 ± 0.268 | 0.477 ± 0.120 | 3.756 ± 0.592 | 1.565 ± 0.066 | 5.760 ± 0.134 |

Table 6: Fit parameters in \( D^{**} D^{**} \) channel with \( c_i (i = 10, 11, 12) \) in GeV\(^2\).

### 3.1.7 Further discussions on the fit program with \( D^*_s D^*_c \) process

We have attempted to fit the experimental data with three well established charmonia, \( \psi(4040) \), \( \psi(4160) \) and \( \psi(4415) \), together with other coherent background contributions in the above decay channels. Since \( X(4260) \) is our only interest here, the mass of \( \psi(4040) \), \( \psi(4160) \) and \( \psi(4415) \) is fixed and the widths are fit parameters in this research. The parameters related to backgrounds in each process are listed above, and the widths of \( \psi(4040) \), \( \psi(4160) \) and \( \psi(4415) \) are listed in Table 7, which are found in reasonable agreement with the widths given by Particle Data Group \(^{14}\). The coupling coefficients between \( X(4260) \) and different final states are presented in the Table 7 as well. Especially, with heavy quark spin symmetry considered, the relationship between \( g_{D^+ D^-} \), \( g_{D^+ D^{**} +c.c.} \) and \( g_{D^+ D^{**}} \) can be calculated \(^{14}\) to be

\[
\begin{align*}
g_{D^+ D^-} & : g_{D^+ D^{**} +c.c.} : g_{D^+ D^{**}} = 1 : 4 : 7, \text{ which is used in our fit.} \\
\text{Therefore, there is only one parameter in need to describe the coupling coefficient in these three channels. The goodness of} \\
\text{the fit is} \chi^2/d.o.f. = 293.099/(357 - 42) = 0.930. \text{The value of} g_0 \text{corresponds to the leptonic decay width} \\
\Gamma_{e^+ e^-} = 1.302 \text{ keV, which gives a strong support for} X(4260) \text{ to be a} \ 3^3S_1 \text{ vector charmonium} \(^{19}\).
\end{align*}
\]

The above conclusions are rather stable against variations of background parameterizations. For example, note that other effects except the well established charmonia can contribute to the distortion of line-shapes, the complex constant coherent background can be employed in the Eq. \(^{16}\) in \( D^+ D^{**} \) channel, and the expression is

\[
\sigma_{D^+ D^{**}} = \frac{3\pi}{K^2} \left| \frac{\sqrt{s} e_1 \Gamma_{D^+ D^{**}}}{s - M_X^2 + i\sqrt{s} \Gamma_{tot}} + \frac{c_{10} e^{i\phi_{10}}}{s - M_{4040}^2 + i\sqrt{s} \Gamma_{4040}} + \frac{c_{11} e^{i\phi_{11}}}{s - M_{4160}^2 + i\sqrt{s} \Gamma_{4160}} \right|^2 + \frac{c_{12} e^{i\phi_{12}}}{s - M_{4415}^2 + i\sqrt{s} \Gamma_{4415}} + c_{15} + |c_{16}|^2.
\]

It turns out that the fit quality is \( \chi^2/d.o.f. = 282.636/(357 - 44) = 0.903 \) and the fit results for each decay channel make practically no difference.
Table 7: The coupling coefficients between $X(4260)$ and different final states, the widths of the background resonance and other parameters involved in the fit with the $D_s^* D_s^*$ cross section data.

| parameters | value |
|------------|-------|
| $g_0$      | $23.751 \pm 0.567$ MeV |
| $M_X$      | $4.220 \pm 7.982 \times 10^{-4}$ GeV |
| $\Gamma_0$| $2.722 \times 10^{-5} \pm 7.982 \times 10^{-4}$ GeV |
| $h_1$      | $-2.610 \times 10^{-4} \pm 4.743 \times 10^{-4}$ |
| $h_2$      | $-6.248 \times 10^{-4} \pm 4.793 \times 10^{-4}$ |
| $h_3$      | $1.624 \times 10^{-2} \pm 9.658 \times 10^{-4}$ |
| $g_{D^+ D^*- \pi^+}$ | $239.48 \pm 34.977$ |
| $g_{h_0, \pi^+ \pi^-}$ | $70.151 \pm 23.236$ |
| $g_{\omega X_{\pi \pi}}$ | $12.765 \pm 8.7416$ |
| $g_{J/\psi \eta}$ | $0.743 \times 10^{-4} \pm 7.203 \times 10^{-5}$ |
| $g_{D_s^* D_s^*}$ | $1.694 \times 10^{-4} \times 5.811 \times 10^{-5}$ GeV$^{-2}$ |
| $\Gamma_{4040}$ | $1.103 \times 2.259 \times 10^{-4}$ GeV$^{-2}$ |
| $\Gamma_{4160}$ | $7.211 \times 9.722 \times 10^{-4}$ GeV$^{-2}$ |
| $\Gamma_{4415}$ | $0.090 \pm 0.016$ GeV |
| $\Gamma_{4160}$ | $0.080 \pm 0.020$ GeV |
| $\Gamma_{4415}$ | $0.082 \pm 0.002$ GeV |

3.2 The fit without $D_s^* D_s^*$ cross section data

Since the $D_s^* D_s^*$ cross section data from BESIII are preliminary, the program without fitting the $D_s^* D_s^*$ has also been carried out. In this subsection, the total width of the $X(4260)$ propagator is also Eq. (2), which includes the $D_s^* D_s^*$ decay width, even though the data are not fitted. It is noticed that the branching ratios of each decay channel remain similar whether the program includes fitting the $D_s^* D_s^*$ cross section data or not. However, the leptonic decay width $\Gamma_{\pi^+ \pi^-}$ turns out to be distinct from that in the fit with $D_s^* D_s^*$ cross section data, as will be seen below.

3.2.1 The $J/\psi \pi^+ \pi^-$ process

The fit formula for the $J/\psi \pi^+ \pi^-$ process is also used as the form in the section 3.1. The fit results are displayed in Figure 9 and Table 7.

Figure 9: The results of the fit without $D_s^* D_s^*$ cross section data: (a) Fits to the cross section of $J/\psi \pi^+ \pi^-$. The dots(blue) and squares(orange) are from BABAR and Belle, respectively. (b) Fits to the cross section of $J/\psi \pi^+ \pi^-$. The dots(measured by BESIII) and the squares(orange) are from the XYZ data sample at BESIII and the dots(blue) are from the R-scan data sample by BESIII. The solid red curves show the best fit.
### 3.2.2 The $D^0D^*-\pi^+$, $h_c\pi^+\pi^-$ and $\psi(2S)\pi^+\pi^-$ processes

As for the $D^0D^*-\pi^+$, $h_c\pi^+\pi^-$ and $\psi(2S)\pi^+\pi^-$ channels, the cross section formula is also in the same form as the Eq. (11). See Figures 10(a), 10(b), 10(c) and Table 9 for fit results.

#### Table 8: Fit parameters in $J/\psi\pi^+\pi^-$ channel.

| $c_1$  | $\phi_1$  | $c_2$  | $\phi_2$  |
|-------|----------|-------|----------|
| 0.153 ± 0.033 | 4.263 ± 0.153 | 312.140 ± 25.059 | 5.278 ± 0.124 |

Table 8: Fit parameters in $J/\psi\pi^+\pi^-$ channel.

### 3.2.3 The $\omega\chi_{c0}$ process

The fit formula can be parameterized as the Eq. (12) and the fit result is presented as Figure 11.

### 3.2.4 The $J/\psi\eta$ process

The fit equation used in the $J/\psi\eta$ process is denoted as Eq. (13), too. So the fit results are displayed in Figure 12 and Table 10.

#### Table 9: Fit parameters in $D^0D^*-\pi^+$, $h_c\pi^+\pi^-$ and $\psi(2S)\pi^+\pi^-$ channels with $c_i (i = 3, 4, 5)$ in unit of GeV$^2$.

| $c_3$  | $\phi_3$  | $c_4$  | $\phi_4$  | $c_5$  | $\phi_5$  |
|-------|----------|-------|----------|-------|----------|
| 0.501 ± 0.031 | 4.202 ± 0.150 | 0.245 ± 0.025 | 0.130 ± 0.270 | 0.192 ± 0.015 | 5.524 ± 0.361 |

Table 9: Fit parameters in $D^0D^*-\pi^+$, $h_c\pi^+\pi^-$ and $\psi(2S)\pi^+\pi^-$ channels with $c_i (i = 3, 4, 5)$ in unit of GeV$^2$. 
Figure 11: Fit to $\omega \chi_{c0}$ cross section under the fit without $D_s^*\bar{D}_s^*$ cross section data scheme. The blue dots and the orange squares come from BESIII experiments in Ref. [37] and Ref. [38], respectively. We do not fit the two data points (orange squares) on the left side since they are below the threshold.

Figure 12: Fit to $J/\psi \eta$ data under the fit without $D_s^*\bar{D}_s^*$ cross section data scheme. The blue dots represent the Belle results [40] and the orange squares come from BESIII [39].

| $c_6$   | $\phi_6$ |
|---------|---------|
| -0.038 ± 0.016 | 0.322 ± 0.599 |

Table 10: Fit parameters in $J/\psi \eta$ channel and $c_6$ is in unit of GeV$^2$.

### 3.2.5 The $D^+D^{*-}$ and $D^{**}D^{*-}$ processes

The cross section expressions for the $D^+D^{*-}$ and $D^{**}D^{*-}$ processes are also represented as Eq. (15) and Eq. (16), respectively, and the results are shown in Figure 13 and Tables 11 and 12. The widths of $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$ as well as the coupling coefficients between $X(4260)$ and different final states are presented in Table 13. The fit quality is $\chi^2/d.o.f. = 292.575/(352−40) = 0.938$. The value of $g_0$ corresponds to a leptonic decay width $\Gamma_{e^+e^-} = 0.466$

| $c_8$   | $\phi_8$ | $c_9$   | $\phi_9$ | $c_{10}$ | $\phi_{10}$ | $c_{11}$ | $\phi_{11}$ | $c_{12}$ | $\phi_{12}$ |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1.432 ± 0.041 | 4.690 ± 0.700 | -0.791 ± 0.331 | 0.725 ± 0.788 | 1.143 ± 0.191 | -0.396 ± 0.790 |

Table 11: Fit parameters in $D^+D^{*-}$ channel with $c_i(i = 8, 9)$ in unit of GeV$^2$.

| $c_{10}$ | $\phi_{10}$ | $c_{11}$ | $\phi_{11}$ | $c_{12}$ | $\phi_{12}$ |
|---------|---------|---------|---------|---------|---------|
| -1.053 ± 0.141 | 1.764 ± 0.385 | 0.440 ± 0.069 | 2.179 ± 0.515 | 0.556 ± 0.137 | 6.112 ± 0.177 |

Table 12: Fit parameters in $D^{**}D^{*-}$ channel with $c_i(i = 10, 11, 12)$ in unit of GeV$^2$. 
3.3 Summary and discussions on numerical fits

To compare with the fits discussed above and to further test the stability of the whole fit program, here we also test the fit without including the $D^+D^{**}$ and $D^{++}D^{*-}$ cross sec-

4.3. Further discussions on the $D^+D^{*-}$ and $D^{**}D^{*}$ processes

Now following the strategy of section 3.1.7 to test the stability of outputs against the variation of backgrounds, we add a complex coherent background in $D^{**}D^{*-}$ channel. The fit is plotted in Figure 14. The fit quality is $\chi^2/d.o.f. = 277.047/(352 - 42) = 0.894$. It is found, however, unlike the result of section 3.1.7, the fit is not quite stable here. The difference is clearly seen when comparing Figure 13 and Figure 14, the destructive interference between different resonances are done in rather different manner. The leptonic width behaves quite differently, with a value of $\Gamma_{e^+e^-} = 0.157$ keV, comparing with the result of section 2.2.5 $\Gamma_{e^+e^-} = 0.466$ keV.

| parameters | value |
|------------|-------|
| $g_0$      | 14.221 ± 3.876 |
| $M_X$      | 4.220 ± 1.066 \times 10^{-3} \text{ GeV} |
| $\Gamma_0$ | 2.602 \times 10^{-4} ± 1.000 \times 10^{-2} \text{ GeV} |
| $b_1$      | 4.065 \times 10^{-4} ± 2.564 \times 10^{-4} |
| $b_2$      | 9.676 \times 10^{-2} ± 2.504 \times 10^{-2} |
| $b_3$      | -2.501 \times 10^{-2} ± 6.256 \times 10^{-3} |
| $g_{D^0D^{*-}\pi^+}$ | 524.35 ± 268.29 |
| $g_{h_{\pi^+}\pi^-}$ | 164.39 ± 101.55 |
| $g_{\omega(2S)\pi^+\pi^-}$ | 27.493 ± 23.279 |
| $g_{\omega X_{0}}$ | 1.812 \times 10^{-3} ± 9.401 \times 10^{-4} |
| $g_{J/\psi}$ | 2.959 \times 10^{-4} ± 1.881 \times 10^{-4} \text{ GeV}^{-2} |
| $g_{D^*D^*}$ | 1.109 \times 10^{-4} ± 4.439 \times 10^{-4} \text{ GeV}^{-2} |
| $g_{D^{**}D^*}$ | 4.143 \times 10^{-3} ± 4.541 \times 10^{-3} \text{ GeV}^{-2} |
| $\Gamma_{4040}$ | 0.090 ± 0.014 \text{ GeV} |
| $\Gamma_{4160}$ | 0.080 ± 0.013 \text{ GeV} |
| $\Gamma_{4415}$ | 0.082 ± 0.004 \text{ GeV} |

Table 13: The coupling coefficients between $X(4260)$ and different final states, and the widths of the background resonances in the fit without the $D^*_0D^*_0$ cross section data.
Figure 14: Given the complex coherent background in $D^{+}D^{-}$ process, the fit result of the $D^{++}D^{*-}$ cross section. The blue dots come from Belle results. The solid curve is the projection from the best fit. The dashed curves show the individual components of $\psi(4040)$, $\psi(4160)$, $X(4260)$ and $\psi(4415)$, respectively.

tion, with and without the $D_s^* D_s^*$ data included. For the former, the fit quality is $\chi^2/d.o.f. = 272.890/(316 - 28) = 0.948$, with the leptonic width 1.069 keV. Besides, the constant width $\Gamma_0$ describing other decay channel is raised up to 48.567 MeV, which indicates that there are other wide decay widths except those of $J/\psi\pi^+\pi^-$, $h_c\pi^+\pi^-$, $D^0 D^{*+} \pi^+$, $\psi(2S)\pi^+\pi^-$, $\omega_{X(0)}$, $J/\psi\eta$ and $D_s^* D_s^*$. As is mentioned above, the decay widths of $D^{+}D^{-}$, $D^{+}D^{*-}$ and $D^{+}D^{*-}$ are indeed about 40 MeV, which implies that the fit programs are self-consistent.

For the fit excluding both $D^{+}D^{-}$, $D^{*+}D^{*-}$ and $D_s^* D_s^*$ cross sections, the parameter $g_0$ is 4.584 MeV with $\Gamma_{e^+e^-} = 0.48$ keV, which is still twice as large as the value estimated in Ref. [27]. However, we believe that this scenario does not have much chance to be physically correct, since there is no reason a priori to exclude the couplings between $X(4260)$ and these states. We may conclude, in the most conservative situation, one still get a leptonic width well above $10^2$ eV. If taking the $D_s^* D_s^*$ data into account, the leptonic width will easily exceed 1 keV.

Even though there is a variation on the value of $\Gamma_{e^+e^-}$, in any case, however, the $X(4260)$ persists in a pole structure on Riemann sheets like an “elementary” particle, i.e., a confining state, according to the pole counting criteria.\footnote{The pole counting criteria is originally proposed in Ref. [15], and has been applied to the discussions of the $X, Y, Z$ states in, for example, Refs. [22,16,48].}

4 Conclusions

Studies on $X(4260)$ resonance play an important role in deepening our understandings on exotic particles and strong interactions. Ref. [27] pointed out that $X(4260)$ is a confining state with a very small leptonic decay width which is hard to be understood by a simple quark model calculation. Thanks to the new experimental data available, a correct understanding gradually emerges, as we believe: A combined fit with the “old” $D^{+}D^{-}$ and $D^{*+}D^{*-}$ data – even though there is no apparent $X(4260)$ peak showing up in these channels – reveals that the $X(4260)$ can have a sizable leptonic width up to $O(10^2)$ eV. Further the fit including the $D_s^* D_s^*$ data can raise the value up to 1 keV. In Ref. [19], which use a screening potential instead of a linear confining potential to calculate the spectrum, it is estimated that a $4^3S_1$ state has a leptonic width $\sim 1$ keV, whereas a $3^3D_1$ state has a leptonic width $\sim 50$ eV. Hence the smaller $\Gamma_{e^+e^-}$ obtained in this paper may be provided by a $3^3D_1$ state (maybe a small portion of $2^3D_1$ state as well) mixed with certain portion of $4^3S_1$ state, and the larger value estimated in this paper may corresponds to a $4^3S_1$ state, and is probably largely renormalized by the $D_s^* D_s^*$ continuum. To further determine the accurate portion of these mixing is still an open question awaiting more fine studies both theoretically and experimentally.
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