Accessing the degree of Majorana nonlocality with photons

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We explore the tunneling transport properties of a quantum dot embedded in an optical microcavity and coupled to a semiconductor-superconductor one-dimensional nanowire (Majorana nanowire) hosting Majorana zero modes (MZMs) at their edges. Conductance profiles reveal that strong light-matter coupling can be employed to distinguish between the cases of highly nonlocal MZMs, overlapped MZMs and quasi-MZMs. Moreover, we show that it is possible to access the degree of Majorana nonlocality (topological quality factor) by changing the dot spectrum through photon-induced transitions tuned by an external pump applied to the microcavity.

Introduction.— Over the past decade, a huge effort in both theoretical and experimental fields has been performing in the quest for an unquestionable signature of exotic ‘half-fermionic’ states, the so-called Majorana zero-modes (MZMs) in quasi-one dimensional hybrid semiconductor-superconductor nanowires with strong spin-orbit coupling subject to external magnetic field, termed as Majorana nanowires [1–4]. In such devices, each isolated ‘half-fermionic’ MZM appears at one of the opposite nanowire ends when the bulk of the system undergoes a topological phase transition [5–7].

In tunneling spectroscopy measurements performed through a Majorana nanowire [4, 7–14], the emergence of a quantized zero-bias peak (ZBP) robust to changing of relevant system parameters such as magnetic field and gate voltages is considered as a strong evidence supporting the emergence of the isolated MZMs. However, it has been extensively demonstrated that other physical mechanisms such as the formation of trivial zero-energy Andreev bound state (ABS) at wire ends due to inhomogeneous smooth confining potentials [15–24] and disorder-induced bound states [21, 24–26] can produce a robust quantized ZBP, leading to an ambiguity of the MZM signature.

The emergence of a trivial zero-energy ABS can be mathematically described as resulting from two half-fermionic states with some spatial separation between them. In this scenario, a quantized ZBP may arise if one of these half-fermionic states couples to a tunneling spectroscopy probe stronger than the other [14, 19, 20]. This kind of trivial ABS formed by two half-fermionic states with small spatial separation between them has been dubbed as partially separated ABS (ps-ABS) [19] or quasi-Majoranas (quasi-MZM) [20].

Thus, despite some recent advances, distinguishing ZBP’s resulting from genuine topological MZMs, trivial quasi-MZMs and disorder still remains a challenge [14, 21–24]. One possible way to experimentally distinguish between the ambiguous signatures of MZMs and quasi-MZMs is to measure the degree of Majorana nonlocality as proposed by Prada et al. [27, 28], which provides an estimate of the localization of the MZMs wavefunctions along the nanowire. This quantity can be accessed by means of a quantum dot (QD) working as a local tunneling spectroscopy probe at one of the ends of a Majorana nanowire [11, 12, 29, 30]. The degree of Majorana nonlocality is defined as $\eta^2 = (\lambda_{c,R}/\lambda_{c,L})$, where $\lambda_{c,L}$ ($\lambda_{c,R}$) is the coupling between the dot and the left (right) MZM [see Fig. 1(a)]. Equivalently, this quantity provides a topological quality factor $q = 1 - (\lambda_{c,R}/\lambda_{c,L})$ of the nanowire [31, 32]. Highly nonlocal MZMs are characterized by $\eta \rightarrow 0$ ($q \rightarrow 1$), while $\eta \rightarrow 1$ ($q \rightarrow 0$) for a trivial quasi-MZM. Experimentally [12, 27], the degree of Majorana nonlocality (quality factor) can be accessed by measuring the corresponding energies of anticrossing patterns appearing in tunneling conductance profiles as functions of both applied bias-voltage through the QD-nanowire and QD gate-voltage.

In the present work, we show that light-matter coupling can be employed as an alternative to differentiate between topological MZMs and a quasi-MZMs (ps-ABS) in tunneling conductance experiments. We consider the system schematically shown in Fig. 1(a): a QD placed inside a single-mode optical cavity with frequency $\omega_0$, which is tuned in resonance with valence-to-conduction band optical transition, i.e. $\Omega = \omega_0$, see Fig. 1(b). The conductance through the dot can be accessed by metallic source-drain leads. Moreover, the QD couples with both left and right MZMs hosted at the edges of the Majorana nanowire. We demonstrate, that conductance profiles through the QD as functions of both bias voltage and mean photon occupation adjusted by external pump reveal distinct patterns for the cases of highly nonlocal MZMs, overlapped MZMs and quasi-MZMs. We also show that it is possible to access the degree of Majorana nonlocality (topological quality factor) in the conductance profile by tuning the mean photon occupation instead of the QD gate-voltage as in the Prada’s original
and the QD is given by [33] between these levels. The interaction between the cavity where \( \omega_c \) and conduction levels of the QD with energy \( c, \) where \( \Omega \) eV QD is defined as chemical potential \( \mu \) and optical cavity, with \( \omega \) operators of the cavity photons with energy \( \omega \). An electron in the conduction level of the tunnelling dot (QD) embedded inside a single-mode optical cavity can be probed by means of source and drain metallic leads symmetrically coupled \( (V) \) to the dot. The energy of the cavity photons is brought in resonance with interband transition in the dot \( \Omega \). (b) The scheme of a QD energy levels: the valence level \( \omega_v \) is far below the Fermi level while the conduction level \( \omega_c \) is above \( E_F \). Strong coupling with cavity photons results in a Rabi splitting of the levels \( \Omega_R \), which is determined by the oscillator strength of the optical transition and the geometry of the cavity.

The effective Hamiltonian which describes the 'half-fermionic' states corresponding to MZMs at the ends of the Majorana nanowire coupled to the QD reads [27, 31]:

\[
H_{MZMs} = \varepsilon_M \gamma_L \gamma_R + \lambda_c \gamma_L (d_e - d_o^\dagger) \gamma_L + \lambda_c \gamma_R (d_e + d_o^\dagger) \gamma_R, \tag{5}
\]

where \( \gamma_L, \gamma_R = \gamma_o \gamma_e \) represent the MZMs at the opposite ends of the Majorana nanowire with \( \varepsilon_M \) being the overlap strength between them [3]. The hybridization between the left and right MZMs with the conduction level of the dot is given by \( \lambda_c, \lambda_c \), respectively, and one can safely assume, that \( \lambda_c, \lambda_c = 0 \) for the situation of \( \omega_n \ll E_F \) considered by us.

The Hamiltonian 5 can be rewritten in terms of a fermionic operator \( f [1, 3] \) by considering the fermionic representation of Majorana operators \( \gamma_L = (f^\dagger + f)/\sqrt{2} \) and \( \gamma_R = (f - f^\dagger)/\sqrt{2} \), where \( f \) obeys the standard fermionic anticommutation relations. Hence, Eq. (5) becomes into \( H_{MZMs} = \varepsilon_M f^\dagger f + \lambda_c (d_e f^\dagger + d_o f + h.c.) \), with \( \lambda_c = (\lambda_c - \lambda_c)/\sqrt{2} \) and \( \Delta_e = (\lambda_c + \lambda_c)/\sqrt{2} \).

Conductance through the QD.— The application of a bias-voltage \( eV \) between source and drain leads, leads to the onset of the current through the system, and, according to the Landauer-type formula \([35, 36]\), at low temperatures \( T \rightarrow 0 \) the conductance through the QD reads:

\[
G(eV) = \left( \frac{e^2}{\hbar} \right) \pi \Gamma \rho_{1c}(eV) \tag{6}
\]

where \( e^2/\hbar \) is the quantum of conductance and \( \Gamma = 2\pi V^2\rho \) represents the effective broadening introduced by the coupling between the QD and the even conduction operator of the leads \([Eq. (4)]\), with a constant density of states \( \rho [36, 37] \), valid within the the wide-band limit approximation. The local density of states of the dot is given by \([36]\):

\[
\rho_{1c}(\omega) = -\frac{1}{\pi} \text{Im} \langle d_e^\dagger d_e \rangle |_{\omega}, \tag{7}
\]

where \( \langle d_e^\dagger d_e \rangle |_{\omega} \equiv G^* (\omega) \) is the retarded Green’s function in the spectral domain \( \omega \) \([38]\), which can be calculated by successive applications of equation-of-motion proposal \([27, 28, 31]\).

The model.— The full Hamiltonian which described the system of Fig. 1 reads \( \hbar = 1 \):

\[
H = H_{ph} + H_{QD} + H_{int} + H_{lead} + H_{MZMs}, \tag{1}
\]

where \( H_{ph} = \omega_0 c^\dagger c \) is the Hamiltonian of a single-mode optical cavity, with \( c^\dagger \) being creation and annihilation operators of the cavity photons with energy \( \omega_0 \). The Hamiltonian of the QD reads \( H_{QD} = \omega_d d_L^\dagger d_L + \omega_c d_o^\dagger d_o \), where \( d_c \) and \( d_e \) describe the electrons in the valence and conduction levels of the QD with energy \( \omega_v \) and \( \omega_c \), respectively, and \( \Omega = \omega_v - \omega_c \) is the energy difference between these levels. The interaction between the cavity and the QD is given by \([33]\)

\[
H_{int} = -\Omega_R (d_L^\dagger d_c + d_o^\dagger d_e), \tag{2}
\]

where \( \Omega_R \) is the Rabi splitting strength.

The source (S) and drain (D) metallic leads and their coupling with the QD are described by

\[
H_{lead} = \sum_{k, \alpha} \varepsilon_k d_k^\dagger c_{k, \alpha} + \sum_{k, \alpha} V_f (c_{k, \alpha}^\dagger d_L + h.c.), \tag{3}
\]

where \( c_{k, \alpha} \) creates (annihilates) an electron in the lead \( \alpha = S/D, \) with wave-number \( k, \) energy \( \varepsilon_k = \epsilon_k - \mu_\alpha \) and chemical potential \( \mu_\alpha \). The bias-voltage through the QD is defined as \( eV = \mu_S - \mu_D \). The parameter \( V_f \) represents the coupling strength of the conduction \( l = c \) and valence \( l = v \) levels of the QD with the leads. Once we are considering the situation in which the valence level is far below the Fermi level \( \omega_v \ll E_F, E_F = 0 \) \([Fig. 1(b)]\), one can assume \( V_v = 0 \) and \( V_c = V \) in Eq. (3) without loss of generality. Introducing even and odd linear combinations of the states of the leads \( c_{k, e}, c_{k, o} \), i.e. performing the unitary transformation according to \( c_{k, e} = (c_{k, e} + c_{k, o})/\sqrt{2} \) and \( c_{k, D} = (c_{k, e} - c_{k, o})/\sqrt{2} \), the odd states become decoupled from the dot, and the Hamiltonian of Eq. (3) transforms into \([34]\):

\[
H_{lead} = \sum_{k, \alpha} c_{k, \alpha}^\dagger c_{k, \alpha} + \sqrt{2} V \sum_k (c_{k, e} d_c + h.c.). \tag{4}
\]
40Γ, corresponding to the situation of highly nonlocal presence of the cavity photons ⟨ν⟩ expected for clean disorder-free nanowires [24, 26], long when the QD is coupled only with the left MZM (⟨ν⟩), taking ω ≈ 1/(ω + iδ − ωc) the term vΓ describes the corresponding broadening introduced by the coupling with metallic leads and

\[
\Sigma^c_{ph}(ω) = \frac{Ω_R^c(ν)}{(ω + iδ − ω_0 − ω_c)}
\]

is the self-energy associated to the valence-to-conduction band transition induced by the photonic field of the optical cavity, with ⟨ν⟩ = ⟨nc⟩ + Nph being the mean photon occupation, which depends on the number of excitations (⟨nc⟩) in the QD and photons (Nph) in the microcavity, which can be tuned by an external optical pump.

The part of the self-energy associated with the direct coupling between the QD conduction level and MZMs reads:

\[
\Sigma^c_{MZMs}(ω) = \kappa_1(ω) + (t_cΔc)^2κ_0(ω)K(ω),
\]

where

\[
\kappa_0(ω) = [(ω + iδ + ε_M)^{-1} + (ω + iδ − ε_M)^{-1}], \kappa_1(ω) = [t_c^2(ω + iδ − ε_M)^{-1} + Δc^2(ω + iδ + ε_M)^{-1}], \kappa_1(ω) = (Δc^2(ω + iδ − ε_M)^{-1} + t_c^2(ω + iδ + ε_M)^{-1}], K(ω) = κ_0(ω)/(ω + iΓ + ωc − Σ^c_{ph}(ω) − κ_1(ω)).
\]

If a QD is decoupled from the optical cavity (ΩR = 0), Σ^c_{ph}(ω) = 0 and Eq. (10) is reduced to the well-known expression for the self-energy associated to the leaking of a single MZM into the QD for λc,R = 0 [11, 29, 30, 43].

The presence of Σ^c_{ph}(ω) in the expression for K(ω) in the self-energy defined in Eq. (10) means that the MZMs somehow ‘feel’ the photonic field, although there is no direct coupling between cavity photons and the Majorana nanowire.

**Results and Discussion.** — In what follows, we analyze the conductance through the QD [Eq. (6)] as a function of bias-voltage eV, in presence of a photonic field in the cavity [Fig. 1(a)]. We take the effective broadening Γ = 40 meV as the energy unit [8, 30, 44, 45] and use typical parameters for QDs embedded in microcavities [46], taking ωc ≪ E_F, ωc ≈ 100Γ ≈ 968 GHz, ΩR = 2.5Γ = 0.1 meV (≈ 24 GHz). The energy of a cavity mode is taken in resonance with the excitonic transition in the QD, ω0 = Ω = 30.1 × 10^11Γ ≈ 1.4 eV (338 THz).

Fig. 2 describes the conductance as a function of eV when the QD is coupled only with the left MZM (λc,L = 40Γ), corresponding to the situation of highly nonlocal MZMs (λc,R = ε_M = 0, η = 0), for several values of ⟨ν⟩. This ideal condition of true topological MZMs is expected for clean disorder-free nanowires [24, 26], long enough to avoid the overlap between the Majorana wavefunctions located at the opposite ends [22]. In the absence of the cavity photons ⟨ν⟩ = 0, a robust ZBP with G(0) = 0.5 e^2/h appears, as it is shown by the solid red lines in Figs. 2(a) and (b), characterizing the ‘half-fermionic’ nature of an isolated MZM leaking into the QD [11, 30, 45, 47]. The conductance peak localized at eV ≈ 100Γ corresponds to the conduction level of the dot renormalized by the finite QD-left MZM coupling.

For a finite photon occupation ⟨ν⟩ = 500 [solid blue line, Fig. 2(a)], optically-induced transitions between valence and conduction levels of the QD come into play, splitting the single peak associated with the QD conduction level into two polariton peaks located at eV ≈ 50Γ and eV ≈ 150Γ. The higher is the value of ⟨ν⟩, the bigger is the distance between the polariton peaks [Fig. 2(a), dash-dotted green and dotted black lines]. This behavior resembles the Rabi splitting for an individual dot inside a cavity, for which 2ΩRν[⟨ν⟩] [48] [Fig. 1(b)]. Note, however, that for sufficiently high values of ⟨ν⟩ additional polariton peaks stemming from indirect MZM to photon coupling of lower amplitude appear (see black and magenta curves).

The amplitude of the peak at zero-bias remains unchanged with increase of the number of the photons [Figs. 2(b)]. This robustness is characteristic for the situation of highly-nonlocal MZMs [21, 22, 24, 27]. In the same time, the width of the ZBP is monotonously increasing with increase of the number of the cavity photons, which means that the latter affect the effective lifetime of the electronic states of the dot and MZMs.

In Fig. 2(c), the conductance behavior at zero-bias as a function of ⟨ν⟩ is shown for several values of λc,L. A well-defined plateau of 0.5 e^2/h, independent on the mean photon occupation, characterizes the robustness of the ZBP for the cases of strong QD-nanowire coupling λc,L ≥ Γ [magenta stars and red circles]. For smaller values of λc,L, the plateau is destroyed, as can be seen in the green line with triangles of Fig. 2(c), corresponding to λc,L = 10^−2Γ. When the QD is totally decoupled from the Majorana nanowire [solid purple line, λc,L = 0], the conductance at eV = 0 exhibits a single resonance at ⟨ν⟩ = 1600, which corresponds to the crossing between the photon-induced lower polariton peak and Fermi energy.

The colormap of Fig. 2(d) summarizes the conductance behavior as a function of both eV and ⟨ν⟩, corresponding to a range of bias-voltage within the gray region of Fig. 2(a). One can notice that the photon-induced peak above eV = 0 have its amplitude reduced with the co-emergence of a conductance peak symmetrically localized below eV = 0 and a ZBP with unchanged 0.5 e^2/h height pinned at zero-bias. Direct comparison between profile of Fig. 2(d) and the corresponding results for highly nonlocal MZMs found by Prada et al. [27] and others [11, 12, 31, 49] shows that the increasing of the mean photon occupation (i.e. increasing the pump intensity) is a fast indirect way of tuning the QD energy without applying any gate-voltage due to renormalization
FIG. 2. (a) Conductance through the QD [Eq. (6)] as a function of bias-voltage $eV$ describing the case of highly nonlocal MZMs ($\varepsilon_M = \lambda_{c,R} = 0, \eta = 0$) for increasing number of the excitations in the system $\langle \nu \rangle$, which can be tuned by external pump (the curves are offset along the $y$ axis for the better viewing). One clearly sees the appearance of additional peaks due to the Rabi splitting. (b) Zoom of the conductance around $eV = 0$. (c) Conductance through the QD at zero-bias $eV = 0$ for increasing $\langle \nu \rangle$. (d) Colormap showing the conductance behavior as a function of both the $eV$ and $\langle \nu \rangle$, for a range of $eV$ corresponding to gray region of panel (a).

FIG. 3. (a) Conductance through the QD [Eq. (6)] as a function of bias-voltage $eV$ describing the case of overlapped MZMs localized at the nanowire ends ($\varepsilon_M = 10\Gamma, \lambda_{c,R} = 0$) for increasing numbers of photon occupation $\langle \nu \rangle$. For a better viewing, the curves are offset along the $y$ axis. (b) Same curves depicted in panel (a), but considering $eV$ only near zero-bias without offset along $y$ axis. (c) Conductance through the QD at zero-bias $eV = 0$ for increasing number of $\langle \nu \rangle$. (d) Colormap showing the conductance behavior as a function of both the $eV$ and $\langle \nu \rangle$, for a range of $eV$ corresponding to gray region of panel (a).

of the QD spectrum in the strong light-matter coupling regime, which bridges Quantum Optics and the physics of MZMs.

Fig. 3 shows the profiles of conductance through QD coupled to the Majorana nanowire for the case of MZMs still localized at opposite wire ends, but with finite overlap between them ($\varepsilon_M = 10\Gamma \gg \lambda_{c,R}$), which may describe the situation of shorter nanowires [22]. For a null photon occupation, $G(eV)$ [Eq. (6)] is characterized by two near zero-energy peaks at $eV \approx \pm \varepsilon_M$ [Figs. 3(a) and (b), dashed red lines] coming from the ZBP splitting due to the finite overlap [29] and a third peak at $eV \approx 100\Gamma$, corresponding to the QD conduction level renormalized by the QD-nanowire coupling [Figs. 3(a), dashed red line]. As the mean photon occupation is increased, in analogy to what happened for nonoverlapped MZMs, the peak corresponding to the QD conduction level splits into two polariton peaks [Fig. 3(a), solid blue line], with the value of this splitting depending on $\langle \nu \rangle$. It can be seen that the increase of the mean photon occupation strongly affects the pattern of near zero-energy peaks associated with overlapped MZMs [see Fig. 3(b)]. First,
they become closer to each other, and coalesce into a single peak at zero-bias with height of $e^2/h$ for $\langle \nu \rangle = 1600$. Further increase of the number of cavity photons leads to the reappearance of the splitting.

The conductance profile at $eV = 0$ as a function of the mean photon occupation is shown in Fig. 3(c). One can notice that the zero-bias conductance plateau corresponding to nonoverlapped and highly nonlocal MZMs [solid red line] is destroyed even by a small overlap [dashed blue line]. For bigger values of the overlap strength $\varepsilon_M$, $G(eV = 0)$ as a function of $\langle \nu \rangle$, exhibits a single peak at $\langle \nu \rangle = 1600$, showing the behavior similar to those corresponding to a QD embedded in an optical cavity and decoupled from the nanowire [Fig. 2(a), solid purple line]. This similarity characterizes the single-fermion nature of the state formed from the overlap between the two half-fermionic MZMs.

The conductance through the QD as a function of both $eV$ and photon occupation, corresponding to the $eV$-range within the gray area in Fig. 3(a), is shown in Fig. 3(d). One can see that the photon-induced transitions in the QD renormalize its spectrum, giving rise to a "bowtie-like" shape, which resembles those found by Prada et al. [27].

In Fig. 4 we display the case of quasi-MZMs (a ps-ABS), which corresponds to MZMs which have lost their nonlocal feature ($\eta \to 1$). This situation describes a scenario where the right MZM is displaced towards left edge of the nanowire, leading to a partial separation between the MZMs and hence a finite overlap between the wavefunction of the right MZM and the QD ($\lambda_{c,R} \gg \varepsilon_M$) [20, 27]. The emergence of this quasi-MZMs with partial separation can result from inhomogeneous confining potentials in the nanowire [21, 22, 24, 28]. Similarly to Fig. 3(a)-(b), we can notice two near-zero-energy conductance peaks at $eV \approx \pm \lambda_{c,R}$ for the case of zero photon occupation [Fig. 4(a)-(b), dashed red line], which points that the quasi-MZMs leak into the dot with different intensities ($\lambda_{c,L} \gg \lambda_{c,R}$) and form a state of a regular fermionic character.

The peak corresponding to the QD conduction level at $eV \approx 100\Gamma$ is again splitted into two polariton peaks in presence of cavity photons, which also affects the position of the near-zero-energy conductance peaks [Fig. 4(b)]. But distinct from the previous case of overlapped MZMs localized at the nanowire edges [Fig. 3(b)], there is no coalescence into a single peak at $eV = 0$ for certain value of $\langle \nu \rangle$.

Fig. 4(c) displays the evolution of zero-bias conductance as a function of $\langle \nu \rangle$ for increasing values of coupling between the right-MZM and the QD. For the ideal situation of highly nonlocal MZMs [solid red line], one can see a ZBP plateau associated to the isolated MZM which has leaked into the dot. A plateau is still present for a tiny value of $\lambda_{c,R}$ [dashed orange line], but with a reduction of the ZBP height ($G(eV = 0) < 0.5e^2/h$). However, a little enhancement of $\lambda_{c,R}$ [dotted green and dash-dotted blue lines] drops the zero-bias conductance to almost zero for any value of $\langle \nu \rangle$.

The behavior of the conductance through the QD as a function of $eV$ and photon occupation $\langle \nu \rangle$ is shown in Fig. 4(d). One can clearly notice that the near-zero-
energy peaks corresponding to quasi-MZMs slightly move away from each other as photon-induced peak [upper branch in Fig. 4(d)] is driven towards $eV = 0$ and then start to approach each other again as the photon occupation is increased. This behavior yields a pattern that resembles the ‘diamond-like’ profile reported by Prada et al. [27]. In this work, it was suggested, that the degree of Majorana nonlocality can be extracted from the anticrossing points between the QD level and the quasi-MZMs levels by application of a QD-gate voltage [12, 27].

Similarly, in the geometry considered by us, one can also extract the information about the degree of Majorana nonlocality from the photon-induced conductance profile of Fig. 4(d). The dot spectrum now is optically, and not electrically, and the corresponding anticrossing points $\varepsilon_{QD}^{+}$ and $\varepsilon_{MZM}^{+}$ are indicated by dashed green lines. As originally proposed by Prada et al. [27], the degree of Majorana nonlocality in tunneling conductance profiles [12] is given by $\Omega_{M}^{2} \approx \eta^{2} = \varepsilon_{MZM}^{+}/\varepsilon_{QD}^{+}$, valid when $\varepsilon_{M} \ll \lambda_{c,L}, \lambda_{c,R}$. For the parameters corresponding to Fig. 4(d), $\Omega_{M} \approx 0.35$, or equivalently, a topological quality factor of $\approx 0.88$ [31, 32], thus indicating that the MZMs are not well-localized at the nanowire edges, giving rise to quasi-MZMs.

Conclusions.— In summary, we have theoretically explored the effects of strong light-matter coupling on transport properties of the hybrid device, consisting on quantum dot embedded inside a single-mode optical cavity and coupled to a Majorana nanowire. Conductance profiles as functions of the bias-voltage and mean photon occupation number controlled by an external pump revealed distinct shapes for the cases of highly nonlocal MZMs, overlapped MZMs localized at opposite nanowire edges and quasi-MZMs (ps-ABS). This makes possible to access the degree of Majorana nonlocality (topological quality factor) all optically, by means of the tuning of the polariton energies in the dot.

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CONDUCTION BAND GREEN’S FUNCTION DERIVATION

The quantum dot conduction band Green’s function \( G^c(\omega) \equiv \langle \langle d^i_c; d^j_c \rangle \rangle_\omega \) [Eq. (8) in the main text] can be derived through successive applications of the equation-of-motion (EOM) technique [S1, S2]. For retarded Green’s functions in the spectral domain, the EOM reads

\[
(\omega + i\delta)G_{A_i,B_j}(\omega) = [A_i, B_j^\dagger]_+ + \langle \langle [A_i, \mathcal{H}]; B_j^\dagger \rangle \rangle_\omega,
\]

where \( G_{A_i,B_j}(\omega) \equiv \langle \langle A_i; B_j^\dagger \rangle \rangle_\omega \) is the retarded Green’s function in the notation adapted from Zubarev [S2, S3], \( \mathcal{H} \) is the Hamiltonian of the system [Eq. (1) in the main text] and \([\ldots, \ldots]_+\) is the standard anticommutation relation for fermions [S1]. Considering \( A_i = B_j = d^\dagger_c \):

\[
(\omega - \omega_c + i\delta)G^c(\omega) = 1 + \sqrt{2V} \sum_k \langle \langle c_{k,c}; d^j_c \rangle \rangle - \Omega_R \langle \langle d^i_c, d^j_c \rangle \rangle_\omega - t_c \langle \langle f; d^j_c \rangle \rangle_\omega - \Delta_c \langle \langle f^\dagger; d^j_c \rangle \rangle_\omega,
\]

with \( t_c = (\lambda_L - \lambda_R)/\sqrt{2}, \) \( \Delta_c = (\lambda_L + \lambda_R)/\sqrt{2} \) and \( f = (\gamma_L + i\gamma_R)/\sqrt{2} \) is a complex fermionic operator built from combination of left \( (\gamma_L) \) and right \( (\gamma_R) \) MZMs [S4], and can acquire a nonlocal feature depending on the distance between these ‘half-fermionic’ Majorana states. As noticed in Eq. (S2), the high-order Green’s function \( \langle \langle d^i_c, d^j_c \rangle \rangle_\omega \) arises due to the light-matter coupling term given by \( \mathcal{H}_{int} \) [Eq. (2) in the main text]. According to the EOM:

\[
(\omega + i\delta)\langle \langle d^i_c; d^j_c \rangle \rangle_\omega = \omega_0 d^i_c d^j_c + \omega_v d^i_c d^j_c - \Omega_R \langle \langle d^i_c, d^j_c; d^j_c \rangle \rangle_\omega + \Omega_R \langle \langle c^\dagger d^j_c; d^j_c \rangle \rangle_\omega,
\]

once \( [d^i_c, d^j_c]_+ = 0 \) and the commutation relation

\[
[d^i_c, \mathcal{H}] = \omega_0 d^i_c d^j_c + \omega_v d^i_c d^j_c - \Omega_R \langle \langle d^i_c, d^j_c; d^j_c \rangle \rangle_\omega + \Omega_R \langle \langle c^\dagger d^j_c; d^j_c \rangle \rangle_\omega.
\]

Thus, Eq. (S3) becomes

\[
(\omega - \omega_0 - \omega_v + i\delta)\langle \langle d^i_c; d^j_c \rangle \rangle_\omega = -\Omega_R \langle \langle d^i_c, d^j_c; d^j_c \rangle \rangle_\omega - \Omega_R \langle \langle c^\dagger d^j_c; d^j_c \rangle \rangle_\omega,
\]

where new high-order Green’s functions \( \langle \langle d^i_c, d^j_c; d^j_c \rangle \rangle_\omega \) and \( \langle \langle c^\dagger d^j_c; d^j_c \rangle \rangle_\omega \) arises. In order to find an analytical expression for \( \langle \langle d^i_c; d^j_c \rangle \rangle_\omega \) and avoid cumbersome calculations, we prevent the emergence of more high-order Green’s functions in the EOM process by considering the following truncation:

\[
\langle \langle d^i_c, d^j_c \rangle \rangle_\omega = \langle \langle d^i_c \rangle \rangle_\omega = \langle \langle d^i_c ; d^j_c \rangle \rangle_\omega = \langle \langle d^i_c, d^j_c \rangle \rangle_\omega = \langle \langle d^i_c, d^j_c \rangle \rangle_\omega,
\]

wherein we considered the one-electron assumption \( \langle \langle d^i_c \rangle \rangle_\omega = \langle \langle d^i_c, d^j_c \rangle \rangle_\omega = 1 \) valid for all times [S5] and

\[
\langle \langle c^\dagger d^j_c; d^j_c \rangle \rangle_\omega = \langle \langle c^\dagger c \rangle \rangle_\omega.
\]

Without loss of generality, in the truncation scheme of Eqs. (S6) and (S7) we turned the high-order Green’s functions of Eq. (S5) into the conduction level Green’s function \( \langle \langle d^i_c; d^j_c \rangle \rangle_\omega \) modulated by the associated mean number of excitations in the QD \( \langle \langle d^i_c \rangle \rangle_\omega = \langle n_c \rangle_\omega \) and the mean number of photons within the optical cavity \( \langle \langle c^\dagger c \rangle \rangle_\omega = N_{ph} \), respectively. Thus, Eq. (S5) reads

\[
(\omega - \omega_0 - \omega_v + i\delta)\langle \langle d^i_c; d^j_c \rangle \rangle_\omega = -\Omega_R (\langle n_c \rangle + N_{ph}) \langle \langle d^i_c, d^j_c \rangle \rangle_\omega.
\]

Considering that the \( \langle n_c \rangle + N_{ph} = \langle \nu \rangle \) gives the mean photon occupation in the cavity, Eq. (S8) turns into

\[
\langle \langle d^i_c; d^j_c \rangle \rangle_\omega = \frac{-\Omega_R (\langle \nu \rangle)}{\langle \omega - \omega_0 - \omega_v + i\delta \rangle} \langle \langle d^i_c, d^j_c \rangle \rangle_\omega.
\]

Substituting Eq. (S9) into (S2), we find

\[
(\omega - \omega_c + i\delta)G^c(\omega) = 1 + \sqrt{2V} \sum_k \langle \langle c_{k,c}; d^j_c \rangle \rangle + \sum_{ph} \langle \langle c^\dagger c \rangle \rangle_\omega - t_c \langle \langle f; d^j_c \rangle \rangle_\omega - \Delta_c \langle \langle f^\dagger; d^j_c \rangle \rangle_\omega,
\]

wherein we considered the one-electron assumption
where \( \Sigma_{ph}^c(\omega) = \frac{\Omega_{ph}^2(\omega)}{\omega - \omega_0 + i\delta} \) [Eq. (9) in the main text] is the self-energy associated to the valence-to-conduction level transition in the quantum dot induced by the single-mode photonic field of the cavity.

At this point, the Green’s function of Eq. (S10) that mixes the even conduction operator \( c_{k,c} \) from the leads with the operator \( d_c \) from the dot should be calculated. According to the EOM procedure, this Green’s function reads

\[
\langle\langle c_{k,c}; d_c^\dagger \rangle\rangle_\omega = \frac{\sqrt{2}V}{\omega + \mu - \epsilon_k} \langle\langle d_c; d_c^\dagger \rangle\rangle_\omega
\]

(S11)

Substitution of the expression above into Eq. (S10) yields

\[
(\omega - \omega_c + i\delta)G^c(\omega) = 1 + \Sigma_{lead}(\omega)G^c(\omega) + \Sigma_{ph}^c(\omega)G^c(\omega) - t_c\langle\langle f; d_c^\dagger \rangle\rangle_\omega - \Delta_c\langle\langle f^\dagger; d_c^\dagger \rangle\rangle_\omega,
\]

(S12)

where \( \Sigma_{lead}(\omega) = \sum_k \frac{2\nu^2}{\omega + \nu - \epsilon_k} \) is the self-energy due to the coupling between the QD and the even conduction operators of metallic leads [see Eq.(4) of main text]. In the wide-band limit, \( \text{Re}[\Sigma_{lead}(\omega)] \to 0 \) and this self-energy is reduced to \(-i\Gamma\), which is independent of \( \omega \), with \( \Gamma = 2\pi V^2 \rho \) [S6] as defined in the main text. Thus, Eq. (S12) reads

\[
(\omega - \omega_c + i\delta)G^c(\omega) = 1 - i\Gamma G^c(\omega) + \Sigma_{ph}^c(\omega)G^c(\omega) - t_c\Delta_c\kappa_0(\omega)\langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega
\]

(S13)

Also through application of EOM [Eq. (S1)], the Green’s functions of Eq. (S10) which mix the operator of the dot conduction level with the fermionic operator built from Majorana modes are given by

\[
\langle\langle f; d_c^\dagger \rangle\rangle_\omega = -\frac{t_c\langle\langle d_c; d_c^\dagger \rangle\rangle_\omega}{\omega - \varepsilon_M + i\delta} + \frac{\Delta_c^2}{\omega - \varepsilon_M + i\delta} \langle\langle d_c; d_c^\dagger \rangle\rangle_\omega
\]

(S14)

and

\[
\langle\langle f^\dagger; d_c^\dagger \rangle\rangle_\omega = \frac{t_c\langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega}{\omega - \varepsilon_M + i\delta} - \frac{\Delta_c^2}{\omega - \varepsilon_M + i\delta} \langle\langle d_c; d_c^\dagger \rangle\rangle_\omega.
\]

(S15)

Hence, Eq. (S13) reads

\[
(\omega - \omega_c + i\delta)G^c(\omega) = 1 - i\Gamma G^c(\omega) + \Sigma_{ph}^c(\omega)G^c(\omega) + \kappa_1(\omega)G^c(\omega) - t_c\Delta_c\kappa_0(\omega)\langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega
\]

(S16)

with

\[
\kappa_1(\omega) = \frac{t_c^2}{\omega - \varepsilon_M + i\delta} + \frac{\Delta_c^2}{\omega - \varepsilon_M + i\delta} \quad \text{and} \quad \kappa_0(\omega) = \frac{1}{\omega - \varepsilon_M + i\delta} + \frac{1}{\omega + \varepsilon_M + i\delta}.
\]

(S17)

In Eq. (S16), one can notice the presence of the retarded Green’s function \( \langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega \) associated to the superconductivity in the quantum dot induced by the coupling with the Majorana nanowire [S7–S10] and, according to the EOM technique, is given by

\[
(\omega + \omega_c + i\delta)\langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega = -i\Gamma\langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega + \tilde{\Sigma}_{ph}^c(\omega)\langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega + \Delta_c\langle\langle f^\dagger; d_c^\dagger \rangle\rangle_\omega + t_c\Delta_cK(\omega)\langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega,
\]

(S18)

where \( \tilde{\Sigma}_{ph}^c(\omega) = \frac{\Omega_{ph}^2(\omega)}{\omega + \omega_0 + i\delta} \) as defined in the main text. Substituting Eqs. (S14) and (S15) into Eq. (S18), we find

\[
\langle\langle d_c^\dagger; d_c^\dagger \rangle\rangle_\omega = -t_c\Delta_cK(\omega)G^c(\omega),
\]

(S19)

with

\[
K(\omega) = \frac{\kappa_0(\omega)}{\omega + \varepsilon_M + \tilde{\Sigma}_{ph}^c(\omega) - \tilde{\kappa}_1(\omega)} \quad \text{and} \quad \tilde{\kappa}_1(\omega) = \frac{t_c^2}{\omega + \varepsilon_M + i\delta} + \frac{\Delta_c^2}{\omega - \varepsilon_M + i\delta}.
\]

(S20)

Straightforward substitution of Eq. (S19) into Eq. (S16) yields

\[
G^c(\omega) = g_0(\omega) + g_0(\omega) [\Sigma_{ph}^c(\omega) + \Sigma_{MZMs}(\omega) - i\Gamma] G^c(\omega),
\]

(S21)

as the Dyson equation [S1, S11] for the quantum dot conduction band, with the self-energy [Eq. (10) in the main text]

\[
\Sigma_{MZMs}(\omega) = \kappa_1(\omega) + (t_c\Delta_c)^2\kappa_0(\omega)K(\omega)
\]

(S22)
responsible for renormalizing the dot energy spectrum due to the coupling with the MZMs located at the nanowire. The presence of $\Sigma^c_{\text{ph}}(\omega)$ within $K(\omega)$ [Eq. (S20)] in the resulting self-energy of Eq. (S22) reveals that although the Majorana nanowire is decoupled from the optical cavity, the MZMs are indirectly affected by the photon-induced transitions in the quantum dot.

Finally, by isolating $G^c(\omega)$ in Eq. (S21), we can write the expression for the Green’s function of the quantum dot conduction band as

$$G^c(\omega) = \frac{g_0(\omega)}{1 + g_0(\omega) \left[ i\Gamma - \Sigma^c_{\text{ph}}(\omega) - \Sigma_{\text{MZMs}}(\omega) \right]} \tag{S23},$$

which is the same relation of Eq. (8) in the main text.

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