A Compressive Method for Centralized PSD Map Construction with Imperfect Reporting Channel

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Abstract—Spectrum resources management of growing demands is a challenging problem and Cognitive Radio (CR) known to be capable of improving the spectrum utilization. Recently, Power Spectral Density (PSD) map is defined to enable the CR to reuse the frequency resources regarding to the area. For this reason, the sensed PSDs are collected by the distributed sensors in the area and fused by a Fusion Center (FC). But, for a given zone, the sensed PSDs by neighbor CR sensors may contain a shared common component for a while. This component can be exploited in the theory of the Distributed Source Coding (DSC) to make the sensors transmission data more compressed. However, channel imperfection transmission would lead to varying signal strength at different CRs, even placed close to each other. Hence, existence of some perturbations in the transmission procedure yields to some imperfection in the reporting channel and as a result it degrades the performance remarkably. The main focus of this paper is to be able to reconstruct the PSDs of sensors robustly based on the Distributed Compressive Sensing (DCS) when the data transmission is slightly imperfect. Simulation results verify the robustness of the proposed scheme.

Index Terms—Cognitive Radio, Spectrum Sensing, PSD Map, Distributed Compressive Sensing, Joint Sparsity Model.

I. INTRODUCTION

Nowadays, spectrum resources are facing huge demands by introduction of new emerging communication devices. Cognitive Radio (CR) can tackle with the spectrum scarcity challenge by allowing the secondary users (SUs) to opportunistically access a licensed band while the primary user (PU) is absent. To this end, SUs have to sense the spectrum constantly in order to detect the presence of a primary transmitter signal [1]. Furthermore, the spectrum sensing has been identified as a key enabling functionality to ensure that cognitive radios would not interfere with PUs, by reliably detecting PU signal. Recently, cooperative spectrum sensing algorithms are proposed to increase the reliability of the spectrum sensing. Correct PU detection probability can be greatly increased by allowing different SUs to share their information and to create collaboration through distributed transmission/processing, in which each user’s information is sent out not only by the user, but also by the collaborating users [2].

Creating an interference map of the operational region at arbitrary locations or frequencies enables the CR to reuse the frequency resources. Hence, it allows a dynamic spectrum allocation scheme [3]. The SUs can keep their transmitted power at the occupied frequency band as low as possible to reuse the frequencies without suffering from or causing harmful interference to the primary system. These reusable zones may be estimated by means of a collaborative scheme whereby receiving the CR Sensors (CRSs) cooperate to estimate the distribution of power in space and frequency as well as localize the positions of the transmitting PUs. In [4], sparsity assumption is used in CRs in order to distribute spectrum sensing process and PSD map construction by using Lasso and D-Lasso [5] algorithms.

It is known that spectrum sensing needs an accurate and fast decision. Existing papers in the literature mostly focus on the collaborative spectrum sensing performance examination when the fusion center (FC) receives and combines all CR reports [6], [7]. The FC has to deal with many reports from sensors and combine them wisely in order to form a PSD map for arbitrary locations in space and frequency domains. Conventional cooperative sensing can incur significant switching delay and synchronization overhead while wide-band spectrum sensing is of great interest since it can reduce these overheads. Moreover, it is known that the wireless channels are subject to fading and shadowing. When SUs experience multi-path fading or happen to be shadowed, the reports transmitted by CR sensors and users are subject to transmission loss. Besides, when the number of sensors is large, this data transmission which is required for PSD map construction can be challenging. Therefore, a novel method is needed to reduce the amount of data transmitted from the sensors to the FC and also deal with the channel imperfection transmission.

Further benefits such as reducing sensing time and number of sensing measurements with the sub-Nyquist sampling rate can be achieved by employing compressive wide-band spectrum sensing [6]. Compressive Sensing (CS) provides a simultaneous sensing and compression framework [8], enabling a potentially significant reduction in the sampling and computation costs at a sensor with limited capabilities. CS technique as the data acquisition approach in a WSN can significantly reduce the energy consumed in the process of sampling and transmission through the network, and also lower the wireless bandwidth required for communication. CS scheme also is exploited in PSD map construction [9] as well as many other research fields.

The DCS concept was merely used for wideband spectrum sensing in a reliable state of the art works. A distributed
Compressive wide-band spectrum sensing is proposed in [10] and shows that the performance gains arising from the use of spatial diversity as well as joint sparsity. A novel Analog-to-Information Converters (AIC) structure of CR front-end integrating low rate Analog-to-Digital Converters (ADC) and few storage units is proposed in [11] while authors also explore a new joint sparsity model in CR networks and provide a solution algorithm to perform joint spectrum reconstruction. To reduce the transmitted data, our proposed framework in [12] is different from the state-of-the-art papers in [10], [11], [13], [14] in some aspects. They suppose an special form of DCS (JSM-2) and developed their algorithms with respect to it. In addition some of them needs some other information such as the sparsity level of the spectrum too. While the JSM-2 is not necessarily the best form to deal and exploit in wideband spectrum sensing. The proposed framework in [12] is independent form these assumptions.

It is important to notice that while joint spectrum recovery requires much less samples for each CR, uncertain channel fading and random shadowing would lead to varying signal strength at different CRs, even placed close to each other. Different from [12], the main focus of this paper is to be able to reconstruct the PSDs of sensors robustly if some perturbations are occurred in the transmission procedure which yields some imperfection in the reporting channel. Therefore, we model the perturbation of the transmission system by using the disturbance filters between each sensor node and the FC. The reconstruction formula of the proposed method includes the estimated version of these filters and it brings some compensations against the occurred errors. In addition, by assuming that the common component is known with both FC and sensors, we propose an scheme to estimate the disturbance filters. We will show that we can greatly improve the performance by considering the reporting channel imperfection $^1$.

The rest of the paper is organized as follows: Section II describes the system model of the cognitive network. Section III presents the proposed method. Simulation results and discussions are given in section IV. Finally, section V concludes the paper and some future directions are presented.

II. Problem Formulation

In compressive spectrum sensing [16], the sensed PSD can be performed by

\[ s = WFa \]

where an smoothing operator $W$ followed by a Fourier transform $F$ are exploited on the auto-correlation vector $a$ of the sensed signals. Since using smoothing operator, the PSD is almost piecewise constant generally. Therefore the number of significant non-zero values in the edge vector of PSD $z$ is so sparse. The edge vector can be found by using $z = \Gamma s$ where in the most simple case $\Gamma$ is a differential operator as expressed in [16]. The dictionary of the sparse representation ($D$) can be found as

\[ z = \Gamma W Fa = D^{-1} a \]

Now, reconstructing the sparse edge vector of the PSD is possible from the sensed measurements $y = \Phi a$. Finally, the estimated PSD, $\hat{s}$, can be achieved by

\[ \hat{s} = G\hat{z} \]

where $\hat{z}$ is the estimated edge vector and $G$ is the cumulative sum matrix (i.e. a lower triangular matrix with +1 elements).

Suppose that $M^2$ sensors should capture a PSD $s_j \in R^N (j \in 1, 2, \ldots, M^2)$ and are distributed in the area. It can be expected that there is a shared common component $s_c \in R^N$ between the $J$ neighbor sensors which constitute a Group of Sensors (GoS), such that

\[ s_j = s_c + s_{inn_j} \]

where $s_{inn_j} \in R^N$ is the innovation part of each PSD $s_j$. PSDs can be represented as a cumulative sum of their edge vector as

\[ s_j = Gz_j \]

by matrix $G \in R^{N \times N}$. Obviously, we have sparse $z_c$ and $z_{inn_j}$s which belong to space $R^N$ with different sparsity levels. Therefore,

\[ s_j = Gz_j = G(z_c + z_{inn_j}) \]

and

\[ s_c = Gz_c, s_{inn_j} = Gz_{inn_j} \]

$\Phi_j \in R^{m_j \times N}$ is an individual measurement matrix for the $j$th sensor and its sensed measurements $y_j = \Phi_ja_j$ should be sent to the FC. $r_j \in R^{m_j}$ is the received signal by the FC according to the sent $y_j$. Notice that in the rest of the paper the $\hat{\cdot}$ denotes the reconstructed vectors or signals.

In this document, we seek to propose criteria for compressing the sensed PSDs of the sensors to reduce the amount of the transmitted data from sensors to FC and also deal with the channel imperfection transmission.

III. Proposed Approach

A. Perfect Reporting Channel ($r_j = y_j$)

The simplest idea to reconstruct the PSD $s_j$ is $\hat{s}_j = G\hat{z}_j$ where $\hat{z}_j$ is attained by solving the BPDN [17] inspired problem eq. (2). Also we can use the shared common component $s_c$ in a GoS based on the Joint Sparsity model (JSM) [18] and therefore, reconstruct the data in lower measuring (sensing) rate. Inspired from JSM, equations (3) to (5) are defined to model reconstruction for a $J$ neighbor sensors (a GoS). Therefore, the desired PSDs can be yielded by

\[ \hat{s}_j = G(\hat{z}_c + \hat{z}_{inn_j}) \]  

\[ \hat{s} = G(\hat{z}_c + \hat{z}_{inn_j}) \]
where $\hat{z}_c$ and $\hat{z}_{inn,j}$s are located in the found $\hat{z} = \left[ \hat{z}_c^T \hat{z}_{inn,1}^T \cdots \hat{z}_{inn,j}^T \right]^T$ vector. $\hat{z}$ is computed by solving the optimization problem in equation (6) where $r \in R^W$, $n \in R^W$, $z \in R^{N(J+1)}$, $\Psi \in R^{W \times N(J+1)}$ and $W = \sum_{j=1}^{J} w_j$.

$$\hat{z}_j = \min_{\hat{z}_j} \frac{1}{2} \| r_j - \Phi_j D \hat{z}_j \|_2^2 + \lambda \| \hat{z}_j \|_1. \quad (2)$$

$$r = \Psi z + n \quad (3)$$

$$r = \left[ r_1^T r_2^T \cdots r_J^T \right]^T \quad (4)$$

$$n = \left[ n_1^T n_2^T \cdots n_J^T \right]^T \quad (5)$$

$$z = \left[ z_c^T z_{inn,1}^T \cdots z_{inn,j}^T \right]^T \quad (6)$$

$$\psi = \begin{bmatrix} \Phi_1 D & \Phi_1 D & \cdots & 0 \\ \Phi_2 D & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_J D & 0 & \cdots & \Phi_J D \end{bmatrix} \quad (7)$$

But, assume a scenario in which the common part $z_c$ is known by the FC and fix for a while of time. For example, the holding time is 30 seconds while the network is sensed in each 0.3 seconds. Here, in order to enhance the model, we remove the common part from the reconstruction equation. Equation (3) can be rewritten in form of equation (8) and split by using a combination of two distinct parts: Common part $z_c$ and Innovation part $z_i$. The reconstruction formula can be modified to just find the innovation parts of the PSDs by (9) where $r_{inn} = r - Az_c$. Consequently, the PSD of each sensor will be found by

$$\hat{s}_j = G(\hat{z}_c + \hat{z}_{inn,j}) \quad (7)$$

where $\hat{z}_{inn,j}$s are located in the computed $\hat{z}_i$ vector. This modification brings faster solution and also better reconstruction accuracy.

$$r = \left[ A \right] [H] \left[ \begin{array}{c} z_c^T \\ z_{inn,1}^T \\ \vdots \\ z_{inn,j}^T \end{array} \right]^T + n \quad (8)$$

$$\hat{z}_i = \min_{\hat{z}_i} \frac{1}{2} \| r_{inn} - H \hat{z}_i \|_2^2 + \lambda \| \hat{z}_i \|_1 \quad (9)$$

Now we try to find the optimum common component $z_c$ labeled as $z_{opt}$. Since our proposed model is based on JSM, the first well known approach to find the optimum is solving the JSM based optimization problem (11) where $\overline{G}$ is a matrix constructed by arranging $G$s similar to eq. (5) ($\beta$'s are replaced by $G$s). But remember that the desired variables in the proposed eq. (9) are just innovation parts $z_i$ and the optimization constraint is just the maximum sparsity of the innovation parts. Therefore the problem to find the $z_{opt}$ can be exchanged to (12) where the $z_{opt}$ is a part of the found $z_{opt}$ vector. The more details and achievements of these proposed criteria are published in [12].

$$z_{opt} = \left[ z_{opt}^T z_{inn,1, opt}^T \cdots z_{inn,j, opt}^T \right]^T \quad (10)$$

$$s_{all} = [s_1^T s_2^T \cdots s_J^T]^T \quad (11)$$

$$z_{opt} = \min_{\hat{z}_{opt}} \| \hat{z}_{opt} \|_1 \text{ subject to } s_{all} = \overline{G} \hat{z}_{opt} \quad (12)$$

B. Imperfect Reporting Channel ($r_j \neq y_j$)

Now let us use the known and fixed for a while common part $z_{opt}$ more. The above mentioned criteria was based on our model in (3). But, it is clear that, recovering the PSDs by using eq. (2) and also eq. (6) only can be useful when there is no significant difference between $r_j$ and $y_j$. In order to embed the effect of this imperfection in the model, we have proposed to estimate a destructive filter $\beta_j \in R^{w_j}$ and exploit it with the model by circular convolution as

$$r_j = y_j \odot \beta_j + n_j \quad (13)$$

Similarly we can model the received signal of the common part $r_{c,j}$ as

$$r_{c,j} = y_{c,j} \odot \beta_j + \hat{n}_j$$

or in the matrix multiplication form as

$$r_{c,j} = Y_{c,j} \beta_j + \hat{n}_j$$

where $Y_{c,j} \in R^{w_j \times w_j}$ is the circulant matrix of $y_{c,j} \in R^{w_j}$. More clearly, if $y_{c,j}$ contains samples such that $y_{c,j} = [y_{c,j}(1), y_{c,j}(2), \cdots, y_{c,j}(w_j)]^T$ then the $\hat{Y}_{c,j}$ will be

$$\hat{Y}_{c,j} = \left[ \begin{array}{ccc} y_{c,j}(1) & y_{c,j}(w_j) & \cdots & y_{c,j}(2) \\
y_{c,j}(2) & y_{c,j}(1) & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
y_{c,j}(w_j) & \cdots & y_{c,j}(2) & y_{c,j}(1) \end{array} \right] \quad (14)$$

So if assume that $y_j$ signals are known by the FC or equivalently FC knows the $z_c$ and $\Phi_j$s. Therefore, estimated impulse response of the destructive filter can be achieved by solving the optimization problem

$$\hat{\beta}_j = \min_{\hat{\beta}_j} \| r_{c,j} - \hat{Y}_{c,j} \hat{\beta}_j \|_2 \quad (15)$$

Now, after estimating the destructive filters $\hat{\beta}_j$s, which are related to the communication paths between the $j$th sensor and the fusion center, we can construct the circulant matrix of the $\hat{\beta}_j$ as

$$\hat{B}_j = \left[ \begin{array}{cccc} \hat{\beta}_j(1) & \hat{\beta}_j(w_j) & \cdots & \hat{\beta}_j(2) \\
\hat{\beta}_j(2) & \hat{\beta}_j(1) & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\hat{\beta}_j(w_j) & \cdots & \hat{\beta}_j(2) & \hat{\beta}_j(1) \end{array} \right] \in R^{w_j \times w_j} \quad (16)$$
Reformulating eq. (3) brings new model as follows:

\[ r = \overline{B}\Phi\Psi z + n \]  

(16)

where \( \overline{B} \in \mathbb{R}^{W \times W} \) is defined as follows:

\[
\overline{B} = \begin{bmatrix}
\bar{B}_1 & 0 & \cdots & 0 \\
0 & \bar{B}_2 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & \bar{B}_J
\end{bmatrix}
\]  

(17)

The reconstruction criterion for this new model is similar to eq. (9) while the entities of matrix \( H \) are modified.

IV. EXPERIMENTS

In order to simulate the PSD map, the following constraints are considered:

- The zone consists of 122 \times 122 grids in which each PU or SU can exist.
- The zone consists of \( M \times M (M = 12) \) uniformly distributed sensors.
- The PSD of each sensor consists of 32 frequency sub-bands while each channel interval contains 8 samples (therefore, \( N = 256 \)).
- For each frequency sub-band, at most 50 PUs are active and located randomly permuted on the grids of the zone.
- The power of PUs signals vanishes with respect to the distance.
- We have selected the number of neighbor sensors in each GoS as \( J = 4 \).
- FC is located in the center of the zone.

By using this mentioned scenario, each sensor captures its individual PSD and therefore the occupancy and un-occupancy of each sub-band is modeled randomly. Figures 1 and 2 show a PSD map of a sub-band and its corresponding occupancy set of projections. The sensed samples \( y_j \in \mathbb{R}^{w_j}, \quad j = \{1,2,...,M^2\} \) are sent to the FC through a digital transceiver system. BPSK modulation, 1/2 channel encoding, and DS-CDMA with 4-chip’s length are the specifications of the used transceiver system. Simulations are experimented for 100 snapshots of time with random behavior PSD maps and the achieved mean results are reported. We use the SparseLab [19] Matlab toolbox to run the BPDN algorithm.

By using above mentioned scenario, each sensor capture its individual PSD and therefore the occupancy and un-occupancy of each sub-band is modeled randomly. Figures 1 and 2 show a PSD map of a sub-band and its corresponding occupancy.

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Fig. 1: A representation of a PSD Map in a given frequency sub-band. 122 \times 122 grid points with 12 \times 12 uniformly distributed sensors.

Fig. 2: The representation of the occupancy map regarding to the PSD Map of Fig. 1 for its 12 \times 12 uniformly distributed sensors.

Fig. 3: Comparison between compression ability and reconstruction time of the mentioned models. a) Individual reconstruction, eq. (1). b) JSM reconstruction, eq. (5). c) Proposed reconstruction eq. (7) with \( z_{c_{o_p_t}} \) found by eq. (9). d) Proposed reconstruction eq. (7) with \( z_{c_{o_p_t}} \) found by eq. (10) [12].
In order to consider the effect of imperfect channel, another experimental results are simulated by using a Rician channel with 3 paths by different path-gains and delays depending on the relative locations of the sensors with respect to the FC. The spatial distribution of Rician channel’s path gains (dB) and the normalized Rician channel’s path delay for each CR sensor are depicted in Fig. 4, respectively. The maximum of each path-gains are $[10 \times 10^{-1}, \ 9.0 \times 10^{-1}, \ 8.0 \times 10^{-1}]$ (dB), the minimum of each path delays are $[10^{-015}, \ 10^{-017}, \ 10^{-019}]$ seconds. It is clearly shown that when one CR sensor is far away from the FC, the received signal is very week, therefore the farthest CR sensors’ data over the channel experiences the maximum delay and minimum path-gain and vice versa. In addition, we assume an additive white Gaussian noise with different noise levels.

The impact of using destructive filters in the proposed method can be more inferred by considering Figs. 5 and 6. Figure 5 shows the boxplot of the fail-rates of the reconstruction equations of the different methods in all of the investigated BERs and the sensing rates. It can be seen in Fig. 5 that using the destructive filters make the compressive sensing based reconstruction optimization problem be more feasible and solvable. Alternatively, Fig. 6 compares the ROC curves of the detection performance between the proposed method with and without using the destructive filters for three different SNRs. Figure 6 implies that using the destructive filters in the proposed framework, brings more accuracy of the spectrum sensing especially in higher bit error rates (lower SNRs).

V. DISCUSSION AND CONCLUSIONS
Since the CR sensors are distributed in the region of support, in order to construct the PSD maps, the sensed PSD by each sensor should be transmitted to a FC. Therefore, when the number of sensors is large, transmitting this type of overhead data can be challenging. In this paper some criteria are proposed based on using shared part of the signals to compress the PSDs and reconstruct them robustly when the data transmissions to the FC is imperfect. Therefore, the perturbation of the transmission system are modeled in the
framework by using disturbance filters between each sensor node and the FC. Moreover, the estimation approach for mentioned disturbance filters is proposed. In addition, due to compressing the PSDs of the sensors and therefore the lower transmitted data to send, the proposed method can be used in lower bandwidth usage too. Furthermore, since the compressive sensing sampling method is used to compress the signals of the sensors, using the proposed framework brings lower computational cost and therefore more lifetime in the sensor side too.

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