Magnetic fields in the disk and halo of M 51

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Abstract. We propose and discuss a method to analyze the structures of regular magnetic fields in external spiral galaxies with allowance for a multi-layer distribution of magnetic field and thermal electron density in the source of polarized radio emission. Our method allows for both horizontal and vertical components of the regular magnetic field.

This approach we applied to the analysis of polarization observations of M 51 at the wavelengths λλ 2.8, 6.2, 18.0 and 20.5 cm smoothed to a resolution of ≃ 3.5 kpc. We fitted the observed azimuthal distributions of the polarization angle within rings of 3 kpc width for the radial range between 3 and 15 kpc in M 51.

We found a magneto-ionic halo in M 51 with a radial extent of about 10 kpc. The regular magnetic fields in the disk and the halo have different structures. The regular magnetic field in the halo is axisymmetric and horizontal. Its field lines are spirals pointing inwards and generally opposite to those in the disk.

The azimuthal structure of the magnetic field in the disk is fairly complicated; it is neither axisymmetric nor bisymmetric but can be satisfactorily represented by a superposition of these two basic harmonics with about equal weights. Magnetic lines of the regular field in the disk are spirals generally directed outwards.

We compare the magnetic field parameters deduced from our analysis with those implied by the observed total and polarized intensities and equipartition arguments. Using also data on the thermal radio emission from the M 51 disk, we show that all these results can be combined into a coherent picture of the global magnetic pattern in M 51 which includes a thermal disk and an extended gaseous halo.

The regular magnetic field strength averaged in 3 kpc wide rings is about 5-10 µG in the disk and reaches about 3 µG in the radial range 3-6 kpc in the halo. With the available resolution, the vertical component of the magnetic field is negligible inside the galactocentric radius of 12 kpc.

The general features of the magnetic patterns revealed in the disk and the halo (e.g., a reversal between the disk and the halo and the azimuthal structures of the field in these two regions) seem to be in agreement with predictions of dynamo theory, but detailed modelling of a dynamo in M 51 is required to reach definite conclusions.

Key words: spiral galaxies – interstellar magnetic field – radio continuum – polarized radio emission – Faraday rotation – M 51 – depolarization

1. Introduction

M 51 (NGC 5194) is the first external spiral galaxy from which linearly polarized radio emission was detected (at λ 20 cm by Mathewson et al. 1972, and at λλ 6.0 and 21.2 cm by Segalovitz et al. 1976), and for which the global structure of the regular magnetic field highlighted by these observations was investigated (Tosa & Fujimoto 1978). It is also the first galaxy for which the discovery of the so-called bisymmetric magnetic structure was claimed (Tosa & Fujimoto 1978), a configuration of magnetic field which was later suspected for some other nearby spiral galaxies (see Krause 1990, Beck 1993 and Beck et al. 1996 for a review). The explanation of the origin of the bisymmetric magnetic fields has become one of the major challenges to the theory of galactic magnetic fields of the last decade.

Later observations of M 51 with better sensitivity and resolution (Neininger 1992a; Horellou et al. 1992; Neininger et al. 1993a; Neininger & Horellou 1996), and also a more careful analysis of the observational data, revealed a magnetic pattern which is considerably more complicated than a simple bisymmetric structure. This is true also for some other galaxies, e.g. M83 (Neininger et al. 1991, 1993b).

One of the goals of the present paper is to introduce and test a general way to represent the complicated magnetic patterns observed in galaxies in terms of a tractably small number of parameters. Hopefully, this will facilitate a fruitful confrontation of theory with observations. Such a parametrization can be conveniently performed in terms of the Fourier expansion of the magnetic field in azimuthal angle. The lowest Fourier harmonic corresponds to the axisymmetric magnetic field, the
next higher one to the bisymmetric mode, etc. However, we emphasize that the Fourier harmonics thus derived are not necessarily connected with the dynamo modes (see Sect. 4.1) and their physical meaning should be established using models of the magnetic field generation and evolution in a given galaxy.

We attempted to build a coherent, self-consistent picture of the global magnetic structure based not only on the Faraday rotation analysis, but also on other available information coming from, e.g., intrinsic polarization angles, depolarization data, total synchrotron emission, thermal radio emission, the morphology of the galaxy, etc.

On pursuing our goals we used recent multifrequency observations of M 51, which allowed us to distinguish two magnetionic layers along the line of sight (the disk and halo) with significantly different magnetic fields.

We adopted the following parameters of M 51: centre coordinates $\alpha_{50} = 13^h 27^m 46.327$, $\delta_{50} = +47^\circ 27' 10'' 25$ (Ford et al. 1985), a position angle of the major axis of $-10^\circ$ measured counterclockwise from north, an inclination angle $i = -20^\circ$ ($i = 0^\circ$ is face-on - see also Appendix A) (Tully 1974), and a distance to M 51 of 9.7 Mpc (Sandage & Tammann 1974).

The structure of the paper is as follows. In Sect. 2 we briefly discuss the observational data used. In Sect. 3 the number density, scale height and filling factor of the thermal electrons, as well as the scale height of the synchrotron disk and the equipartition strength of the regular and turbulent magnetic fields are estimated for M 51. We also clarify the importance of depolarization effects and estimate the depth within the galaxy visible in polarized radio emission. Section 3 is concluded by an analysis of Faraday rotation measure produced in a partially transparent layer. In Sect. 4 we propose a method of interpretation of multi-frequency polarization observations of spiral galaxies aimed at the determination of the three-dimensional structure of the galactic magnetic field. In this section we also present the results of our fits to the observed polarization angles in M 51. We estimate the global properties of the magnetic fields in Sect. 5 and discuss our results in Sect. 6. Conclusions are briefly summarized in Sect. 7. The statistical tests employed and the method for estimating uncertainties in our fitted parameters are discussed in Appendix B. For the reader’s convenience the basic notation is compiled in Appendix C.

2. The observational database

2.1. The data

The observations of the spiral galaxy M 51 which we discuss below were obtained at the wavelengths 2.8 cm (Neininger 1992a), 6.2 cm (Neininger et al. 1993a), 18.0 cm and 20.5 cm (Horellou et al. 1992). In contrast to earlier discussions of these observations, here we analyze the observations for all four wavelengths simultaneously.

The 2.8 cm data are single-dish measurements obtained with the 100-m Effelsberg radio telescope. The other data sets were obtained with the VLA in its D-array. This imposes some restrictions for the use of the 6.2 cm data since at this wavelength the diameter of the primary beam of the VLA is 9 arcmin, which corresponds to a diameter of 25 kpc in the plane of M 51. Therefore, we considered the measurements at 6.2 cm to be reliable up to a radius of 9 kpc chosen to be somewhat smaller than the radius of the primary beam, and we did not use them at larger radii.

At all wavelengths the data were smoothed to a final resolution of 75” corresponding to $3.5 \times 3.8$ kpc in the plane of M 51. Fig. 1 shows the observed E-vectors rotated by $90^\circ$, superimposed onto an optical picture of M 51.

2.2. Data averaging in sectors

Following an usual procedure in the studies of regular magnetic fields in external galaxies, the galaxy was divided into several rings and we considered the values of polarization angle averaged in sectors in each ring, $\psi_{ni}$, where the subscript $n$ refers to the wavelength and $i$ to the sector. Here we chose the rings between the galactocentric distances $r = 3, 6, 9, 12$ and 15 kpc; within each ring sectors of an opening angle of $20^\circ$ were used. The azimuthal angle $\theta$ was measured counterclockwise from the northern major axis. Throughout the paper we specify sectors by their median value of $\theta$.

The input values from the observations were obtained by calculating separately the averages of the Stokes parameters $Q$ and $U$ over all the points of the regular data grid that lie in the specified sector. All data are slightly oversampled with a gridding interval of one third of the full width at half maximum (FWHM) of the Gaussian beam. After the averaging the polarized intensity and the polarization angle of the observed $E$-field, $\psi_{ni}$, were calculated using the mean $Q$ and $U$ values for each sector. The zero level of the polarized intensity was corrected for polarized noise (Wardle & Kronberg 1974). The $\pm \pi$ ambiguity in polarization angles was resolved by requiring that the difference in the values of $\psi_{ni}$ in neighboring sectors is minimum.

A lower estimate of the uncertainty of $\psi_{ni}$ is provided by the statistical noise in the $Q$ and $U$ maps. In addition an independent error calculation was used: in each sector the standard deviation, denoted as $\sigma_{ni}$, of the data points around the sector average was determined and used as an estimate of the uncertainty of the corresponding mean value, $\psi_{ni}$. Only if its value was less than the statistical noise value (which is usually not the case), the latter was adopted. For sectors with less than five data points these were used only to calculate the mean value. The standard deviation was then computed after combining the adjacent sectors within a ring until the total number of the data points was at least five. The resulting error was calculated as $\sigma_{ni}^2 = K^{-1} \sum_{j=1}^{K} \sigma_{nj}^2$, where summation was carried out over the sectors involved, whose number is K, and $\sigma_{nj}$ are undersampled standard deviations for individual sectors. This error was then assigned to all the sectors involved.

Earlier analyses by Ruzmaikin et al. (1990) and Sokoloff et al. (1992), based on a similar approach, used model values of the data errors. Here we use the errors obtained from observational data which makes our results more reliable. (We note, however,
that the model calculations of $\sigma_{ni}$ of the earlier analyses proved to be reasonable.)

3. The magneto-ionic medium in M 51

In order to interpret a polarization pattern in a galaxy, one should know certain parameters of the interstellar medium such as the scale heights of the thermal and synchrotron disks, electron volume density and filling factor. Furthermore, we should assess in advance the importance of depolarization effects in order to estimate the depth visible in polarized emission. In this section we discuss how this information can be extracted from data on synchrotron and thermal radio emission.
3.1. The nonthermal disk in M 51

3.1.1. The scale height of the nonthermal emission

The exponential scale heights of the synchrotron emission given in Table 1 were estimated from those in the Milky Way by scaling the latter values obtained at 408 MHz (Beuermann et al. 1985) with frequency as $\nu^{-0.25}$, as observed for NGC 891 (Hummel et al. 1991a) and M 31 (Berkhuijsen et al. 1991). As M 51 and the Milky Way are galaxies of a similar type (Sc and Sbc, respectively) and have about the same linear dimensions, the scale height in the Solar neighbourhood ($r_\odot = 8.5$ kpc) was assumed to apply at $r = 9$ kpc in M 51.

3.1.2. The magnetic field strength

The total field strength $B$ (including the regular and turbulent components) can be evaluated from the intensity of the nonthermal emission using, for example, the standard assumption of equipartition of energy density between the magnetic field and the cosmic rays (see, e.g., Krause et al. 1984). The nonthermal emission was obtained from the total emission by subtracting the thermal component derived by Klein et al. (1984).

Using the total nonthermal intensity one first estimates the strength of the transverse magnetic field (i.e. the projection of $B$ on the plane of the sky). From the polarized intensity one obtains the strength of the transverse regular magnetic field $B_{\text{reg}}$, from which one gets $B_{\text{reg}}$ by deprojection assuming that the field lies in the galaxy’s plane. The turbulent magnetic field strength is found from its transverse component by multiplication by $\sqrt{3}$ assuming statistical isotropy. As Faraday effects are negligible at $\lambda 2.8$ cm, the degree of polarization at this wavelength yields the best estimate for the strength of the regular magnetic field. Therefore we evaluated $B$, $B_{\text{reg}}$ and $B_{\text{tur}}$ from the $\lambda 2.8$ cm data.

In Table 1 we show the averaged strengths of regular and turbulent magnetic field inferred from the observed intensity of the total nonthermal emission at $\lambda 2.8$ cm, $P_{2.8}$, and the degree of polarization at $\lambda 2.8$ cm, $P_{2.8}$, using the assumption of energy equipartition for a nonthermal spectral index $\alpha_n = 1$ (Klein et al. 1984; $S \propto \nu^{-\alpha_n}$) and the standard ratio of relativistic proton to electron energy density of 100. The full thickness of the emission layer was adopted as $2h_{2.8}$.

We may note that the magnetic field strengths derived only weakly depend on both the scale height and the ratio of relativistic proton to electron energy density (as the power $1/(3 + \alpha_n)$). For example, a 50% increase in $h_{2.8}$ would lower the field strengths by 10%. However, the choice of pressure equilibrium between cosmic ray particles and magnetic field instead of energy equipartition would lower the field strengths by 25%. As $I_{2.8} \propto B^{3+\alpha_n}$ the measurement errors in $I_{2.8}$ and $P_{2.8}$ have a negligible effect on the derived field strengths. Therefore the uncertainty in the derived field strengths given in Table 1 is about 30%.

We stress that the estimates of $B_{\text{reg}}$ and $B_{\text{tur}}$ from the synchrotron emission are used only to assess the role of Faraday depolarization effects in Sect. 3.3. The analysis of the polarization pattern performed in Sect. 4 yields independent estimates of $B_{\text{reg}}$ which are in agreement with those given in Table 1.

3.2. The thermal disk in M 51

In this Section we estimate parameters of the disk of thermal electrons in M 51. For this purpose we used the distribution of the thermal flux density $S_{2.8}$ of M 51 (Klein et al. 1984). At $r = 9$ kpc in M 51 the thermal flux density is $S_{2.8} = 0.47 \pm 0.15$ mJy/beam. This is close to the value of $0.35 \pm 0.10$ mJy/beam for the Solar neighbourhood (see Sect. 3.2.3), which supports our use of an analogy with the Milky Way, where necessary.

3.2.1. Volume density of the thermal electrons

Klein et al. (1984) derived the radial dependence of the thermal radio emission in M 51 at $\lambda 2.8$ cm. For each ring the average thermal flux density $S_{2.8}$ is given in Table 2. To calculate the average electron density we used the formulae for the thermal flux density (in terms of emission measure) of H II regions of Mezger & Henderson (1967) in the form given by Israel et al. (1973) for an unresolved source. In our case we have

$$\langle n_e \rangle = \frac{C f^{1/2} S_{2.8}^{1/2}}{m \text{Jy/beam} \left( \frac{2 h_{\text{th}} f}{\text{pc}} \right)^{-1/2}},$$

(1)

where $\langle n_e \rangle$ is the average electron density in the thermal ionized gas layer of exponential scale height $h_{\text{th}}$, $f$ is the volume filling
factor of the electron density defined by \( \langle n_e \rangle^2 = f \langle n_e^2 \rangle \), and \( C \) is a certain constant depending on distance, resolution and electron temperature. For a distance of 9.7 Mpc, a resolution of 75\(^s\), and a temperature of \( 10^4 \) K we have \( C = 7.4 \).

Using the values of \( f \) and \( h_{th} \) derived below we found the average electron densities presented in Table 2.

### 3.2.2. The scale height of the thermal electrons

The scale height \( h_{th} \) is not known for M 51. Again we can make an estimate using the values known for the Milky Way. Observations indicate that \( h_{th} \) varies with radius.

For the inner Galactic region we used the observation of Reich & Reich (1988) that the full halfwidth of the thermal emission at \( r < 7 \) kpc is 2\(^\circ\)-2\(^\circ\), which yielded a scale height of about 400 pc at a mean radius of \( r = 4 \) kpc. This scale height was adopted for the ring 3-6 kpc in M 51.

For the Solar neighbourhood Reynolds (1991a) found that the distribution of the electron density perpendicular to the Galactic plane could be described by two components: a thin disk of scale height 70 pc containing the giant and classical H\(\pi\) regions, and a thick disk of diffuse gas with a scale height of 900 pc. However, for our present purpose it is sufficient to consider one thermal disk described by a single exponential fitted to the sum of the thin and the thick disk (see Sect. 3.2.4). After scaling to \( r_\odot = 8.5 \) kpc Reynolds’ distribution becomes \( \langle n_e \rangle = 0.035 \exp(-|z|/600) \) cm\(^{-3}\). This scale height, \( h_{th} = 600 \) pc, was taken for 6-9 kpc in M 51.

For the outer regions of M 51 we scaled the values derived from a comparison of H\(\pi\) and H\(I\) observations of the Milky Way. The H\(I\) layer in the Milky Way becomes thicker at larger radii, and the same may hold for the ionized gas. Dickey & Lockman (1990) derived a constant H\(I\) scale height of 165 pc (the cool plus warm components) between 4 and 8 kpc, and it increases considerably beyond the Solar circle (Henderson et al. 1982). Near the Sun the H\(I\) scale height is about 200 pc, a factor of 3 smaller than the scale height of the ionized gas. Assuming this ratio to be constant for \( r \geq r_\odot \), and taking the solar-neighbourhood values at a radius of 9 kpc in M 51, we found the scale heights of the thermal gas for the outer rings as given in Table 2.

We refer to Fig. 2 for a sketch of M 51 illustrating the spatial distribution of the various components.

### 3.2.3. The filling factor

The filling factor of the free electrons in the thermal disk of M 51 was also adopted from the Solar neighbourhood. It was calculated by comparing the flux density of the thermal radio emission with that expected from the disk of thermal electrons (Berkhuijsen, in preparation).

The thermal flux density at 2.8 cm of the Solar neighbourhood, scaled to the distance of M 51 and seen with a beam width of 75\(^s\), is \( S_{2.8} = 0.35 \pm 0.10 \) mJy/beam. Using Eq. (1), with \( C \) taken for the distance of M 51, \( \langle n_e \rangle = 0.035 \pm 0.005 \) cm\(^{-3}\) and \( h_{th} = 600 \) pc (as applicable near the Sun) we obtained \( f = 0.075 \pm 0.030 \). This value is in good agreement with other estimates, being halfway between the filling factor of the diffuse warm gas (\( f > 0.2 \), Reynolds 1991b) and that of giant H\(\pi\) regions (\( f \approx 0.01 \), Güsten & Mezger 1983).

As no information is available on how \( f \) varies with \( r \) in the Milky Way, we adopted \( f = 0.075 \) for all 4 rings in M 51.

### 3.2.4. Thin and thick thermal disk

In order to check how the approximation of one thermal disk influences the results in Table 2, we also considered a thermal disk in M 51 consisting of two separate components: a thin disk containing the discrete H\(\pi\) regions and a thick disk of diffuse emission, as observed in the solar neighbourhood. An estimate of the relative contributions of the two disks to the thermal emission and the rotation measures is then possible.

Using the distribution of electron density of Reynolds (1991a) scaled to \( r_\odot = 8.5 \) kpc Berkhuijsen (in prep.) estimated the filling factors of the thin and thick disk to be \( f_1 = 0.0025 \) and \( f_2 = 0.3 \), respectively. In this case 20% of the thermal emission is coming from the thick disk.
Assuming the same values of \( f_1 \) and \( f_2 \) for M 51 we can now use Eq. (1) for each of the components separately. Compared to the results in Table 2 we then find \( h_1 \simeq 0.11h_h \), \( h_2 \simeq 1.15h_h \), \( \langle n_e \rangle_1 \simeq 0.5\langle n_e \rangle \) and \( \langle n_e \rangle_2 \simeq 0.85\langle n_e \rangle \), where \( h_1 \) and \( h_2 \) are the scale heights of the thin and thick disk, respectively.

As we have seen above the thin disk produces 80% of the thermal emission (\( \propto f_1^{-1}\langle n_e \rangle_1^2h_1 \)). The thick disk, however, causes 95% of the rotation measure (\( \propto \langle n_e \rangle_2^2h_2 \)). Therefore it is reassuring that the scale height and the electron density of the thick disk differ only 15% from the values in Table 2 which we used to calculate model magnetic field strengths (see Sect. 5). Clearly in the present context the approximation of the two-component thermal disk by a single exponential disk is fully acceptable.

### 3.3. Depolarization

In order to estimate the depth in the disk of M 51 visible in polarized emission, we discuss the various depolarization mechanisms and their significance in M 51.

#### 3.3.1. Wavelength-independent depolarization

At a wavelength as short as \( \lambda 2.8 \) cm Faraday rotation and Faraday depolarization effects are negligible. Hence only the wavelength-independent depolarization can reduce the degree of linear polarization. This depolarization is caused by a tangling of the magnetic field lines in the emission region both across the beam and along the line of sight. The data at \( \lambda 2.8 \) cm enables to distinguish between a magnetic field component which is uniform in the beam cylinder (yielding the observed polarized emission) and a nonuniform component (that significantly reduces the degree of polarization, see Table 1).

Observations of various spiral galaxies with different beam sizes (ranging from about 3 kpc to 250 pc) show a regular magnetic field \( B_{\text{reg}} \) on scales exceeding 1-3 kpc. But even polarized emission observed at a resolution as high as 250 pc at short wavelengths at intermediate radii is significantly depolarized (e.g., in IC 342 - Krause 1993) and the degree of polarization is about the same for different beam sizes in the above range. The latter is evidence for a turbulent magnetic field component \( B_{\text{tur}} \) with a correlation length \( d \) significantly smaller than 250 pc. This also implies that the galactic magnetic fields have a two-scale structure with the regular and turbulent magnetic fields being separated from each other by a gap in the wavenumber space. (The term “turbulent” is loosely applied here to any magnetic field tangled on scales smaller than the beamwidth without any reference to the turbulent cascade.) Also in the Milky Way the magnetic field has a regular component with a scale exceeding about 1 kpc and a random field at significantly smaller scales (see Rickett 1990; Ohno & Shibata 1993).

Another source of beam depolarization is unresolved curvature of the regular magnetic field. However, with our beam size of about 3 kpc in M 51 the curvature of, e.g., a circular regular magnetic field would reduce the degree of polarization by only about 10% in the inner part of the galaxy and even less at larger radii, so that this effect can be neglected. Therefore, the observed degree of linear polarization at \( \lambda 2.8 \) cm provides a good measure of the ratio \( B_{\text{tur}}/B_{\text{reg}} \).

#### 3.3.2. Wavelength-dependent depolarization (Faraday depolarization)

Both the regular and the turbulent magnetic field cause significant Faraday depolarization at \( \lambda \lambda 18.0 \) and 20.5 cm. The regular field component along the line of sight inside the source causes depolarization by differential Faraday rotation, \( DP_{\text{reg}} \). The turbulent field inside the source causes depolarization \( DP_{\text{in}} \) due to dispersion in Faraday depth both along and perpendicular to the line of sight. Furthermore, also a turbulent field in a layer with thermal electrons in front of the source depolarizes due to dispersion in Faraday depth across the beam, \( DP_{\text{ex}} \). Below
we discuss how significant each of these depolarization mechanisms is in M 51, and we estimate from which layer in the disk the observed polarization emission is coming.

First we discuss the external depolarization. Because of the high Galactic latitude of M 51 (b = 69°) DP_{ex} caused in the Milky Way appears to be negligible. Horellou et al. (1992) showed that the structures of RM and depolarization across M 51 are closely related to features in the disk. This implies that depolarization in the halo of M 51 must be small.

We may estimate DP_{ex} using the results of Burn (1966) or Tribble (1991), depending on the correlation length of the turbulent cells, d. Burn’s formula applies if d is much smaller than the beamwidth of 3.5 kpc, thus if, say, d ≤ 350 pc. Then DP_{ex} = \exp(-\frac{\sigma_{RM}}{\lambda^2}) where \sigma_{RM} is the observed dispersion in RM which we estimated from our maps as about 20 rad m^{-2} between 18.0 cm and 20.5 cm. But this value includes the contribution from both the disk and the halo. We assume that the halo contribution is smaller than that of the disk, i.e. \sigma_{RM} < 10 rad m^{-2}. For λ = 20.5 cm this yields DP_{ex} > 0.7. However, Dunke et al. (1995) found correlation lengths ≥ 1 kpc at z ≥ 2 kpc in spiral galaxies seen edge-on. In this case we may estimate the standard deviation of DP_{ex} from Eq. (20) of Tribble (1991) as 4d/kpc(\sigma_{RM}/\lambda^2)^{-1} at 20.5 cm. Using d = 2 kpc we find that it exceeds 0.8, which will be the typical value of DP_{ex}. We conclude that DP_{ex} is insignificant compared with the depolarization caused within the disk.

We now discuss the depolarization mechanisms within the synchrotron disk as described by Burn (1966).

The depolarization by differential Faraday rotation in a slab with uniform magnetic field and electron density is given by the well-known expression

\[ DP_{\text{reg}}(\lambda) = \frac{P_\lambda}{P_0} = \frac{\sin 2\text{RM} \lambda^2}{2\text{RM} \lambda^2}, \]

(2)

where \(P_\lambda\) is the degree of polarization at wavelength \(\lambda\), \(P_0 \approx 0.75\) the intrinsic degree of polarization and RM the observed rotation measure \(= 0.81 \langle n_e \rangle B_{\text{reg}} L / 2\) with RM measured in rad m^{-2}, \(\langle n_e \rangle\) in cm^{-3}, \(B_{\text{reg}}\) the line-of-sight regular magnetic field, \(\mu\), and \(L\), the line-of-sight in the Faraday active and emitting region, in pc).

Burn gives the expression for the internal Faraday dispersion as

\[ DP_{\text{in}}(\lambda) = 1 - \exp\left(-\frac{\sigma_{\text{RM}}^2}{\sigma_{R M }^2} \lambda^4 \right), \]

(3)

where \(\sigma_{R M }^2 = \langle \sigma_{\text{RM}} \rangle^2 L d\) with \(d\) being the correlation scale of \(B_{\text{in}}\).

In the lower part of the halo, in the region \(h_{\text{th}} \leq z \leq h_{\text{syn}}\), where synchrotron emission and thermal halo gas occur together, differential Faraday depolarization (2) and internal Faraday dispersion (3) may play a role. However, with the values for \(\langle n_e \rangle\), \(B_{\text{reg}}\) and \(B_{\text{tur}}\) in Tables 1, 2 and 5 these mechanisms appear to be negligible (DP_{in}, DP_{reg} > 0.98).

In what follows we assume that the Faraday depolarization occurs entirely in the thermal disk of M 51.

In order to take out the effect of the wavelength-independent depolarization at \(\lambda = 20.5\) cm we shall use the relative depolarization between \(\lambda = 20.5\) cm and \(\lambda = 2.8\) cm, denoted as DP_{20.5/2.8} = \(P_{20.5}/P_{2.8}\). As Faraday effects at \(\lambda = 2.8\) cm are negligible, this ratio is essentially a measure of the Faraday depolarization at the longer wavelengths. The observed values are given in Table 3.

The Faraday depolarization in the disk is caused by both differential Faraday rotation and internal Faraday dispersion. Using equations (2) and (3) with the values in Tables 1, 2 and 3 we find that in the disk of M 51 each of these effects is strong enough to significantly depolarize the emission at \(\lambda = 20/18\) cm. Due to internal Faraday dispersion, only an upper layer of the disk is visible. As we estimate below, this layer is only about 200-300 pc deep at \(r = 3-9\) kpc. It can be easily seen that depolarization due to differential Faraday rotation across this depth is relatively weak.

The fact that only polarized emission from an upper layer is observed is evident from the much smaller rotation measures observed between \(\lambda = 20.5\) and 2.8 cm (Horellou 1990) than between \(\lambda = 6.3\) and 2.8 cm (Neininger 1992b), at which wavelengths Faraday rotation is negligible. For the two inner rings RM(20.5/18.0) ≤ 0.25 × RM(6.3/2.8). As the disk is transparent at short wavelengths, this indicates directly that only a part of it is seen in polarized emission at \(\lambda = 20\) cm. The visible depth in the disk is then estimated as \(\Delta z \approx 0.25 \times 2h_{\text{th}} = 200-300\) pc for \(r = 3-9\) kpc. In the case of field reversals in the part of the disk invisible at \(\lambda = 20.5, 18.5\) cm this value is an upper limit to \(\Delta z\). Horellou et al. (1992), using different arguments, also concluded that at \(\lambda = 20.5, 18.0\) only the upper part of the polarized disk is observed, and Beck (1991) found the same for NGC 6946.

As the data at \(\lambda = 6.3, 2.8\) cm are not complete for the two outer rings, we cannot make the above comparison for \(R = 9-15\) kpc. Instead we propose the following estimate of the minimum visible depth. Let us define \(\Delta z\) as the depth in the thermal disk from which polarized emission is observed (see Fig. 2). Then the layer in the synchrotron disk, which produces the observed polarized emission at \(\lambda = 20.5\) cm, has the thickness \((h_{20.5} - h_{\text{th}}) + \Delta z\). If no Faraday depolarization occurred in the
visible layer, then the fraction $DP_{20.5/2.8}$ of the polarized emission at $\lambda 20.0$ cm would come from a layer $DP_{20.5/2.8} \times 2h_{20.5}$ deep. Since this depth must be equal to the former value, this yields
\[ \Delta z = h_{20.5}(2 DP_{20.5/2.8} - 1) + h_{th}. \] (4)

As some depolarization actually occurs within $\Delta z$, the true visible depth must be larger than this. The values of $\Delta z$ thus calculated are given in Table 3. These values are remarkably close to the upper limits derived above from the RMs observed in the two wavelength ranges. We note that in the radial range 12-15 kpc the thermal disk is completely transparent to polarized emission at $\lambda \lambda 18.0/20.5$ cm.

3.3.3. Qualitative analysis of Faraday rotation in a two-layer system

The polarization angle of the polarized emission is given by
\[ \psi = RM_{fg} \lambda^2 + RM \lambda^2 + \psi_0, \] (5)
where $RM_{fg}$ is the foreground Faraday rotation measure produced mainly within the Milky Way. RM is the intrinsic Faraday rotation measure produced by the magnetic field within the galaxy considered; $\lambda$ is the wavelength and $\psi_0$ is the intrinsic polarization angle.

At the longer wavelengths a complete depolarization occurs in certain localized regions in M 51 (Horellou et al. 1992), and then Eq. (5) no longer holds. However, this hardly affects the averages over the sectors, so that Eq. (5) is well applicable to sector averages; we directly checked this by fitting Eq. (5) to the available values of $\psi_{nl}$ for each sector (see Sect. 2.2).

In order to distinguish between the contributions of the disk and the halo (see Sect. 4.3), we write
\[ RM = \xi^{(D)} RM^{(D)} + \xi^{(H)} RM^{(H)}, \] (6)
where $RM^{(D)}$ and $RM^{(H)}$ are the Faraday rotation measures produced across the disk and the halo, respectively, if both are fully transparent to polarized emission. $RM^{(D)} = 0.81(n_e)^{(D)} B_{||}^{(D)} h_{th}$ is defined here in terms of the disk scale height. (The full thickness of the disk is $2h_{th}.$) However, it is more convenient to define $RM^{(H)} = 0.81(n_e)^{(H)} B_{||}^{(H)} Z$ in terms of the vertical extent of the halo, $Z$, i.e., the distance along $z$ between $z = h_{th}$ and the upper boundary of the halo (see Fig. 2). $B_{||}$ is the line-of-sight component of the regular magnetic field. Since not the whole disk (or even halo) may be visible in polarized emission at a given wavelength, we introduce factors $\xi^{(D)}$ and $\xi^{(H)}$. As follows from above $\xi^{(D)}$ and $\xi^{(H)}$ depend on the wavelength. We assume that both $\xi^{(D)}$ and $\xi^{(H)}$ are the same at $\lambda 2.8$ and 6.2 cm, and also at $\lambda 18.0$ and 20.5 cm.

We can see from Tables 1 and 2 that the synchrotron disk at $\lambda 2.8$ and 6.2 cm is about as thick as or thinner than the thermal one. Thus there is only little synchrotron emission originating in the halo and the halo magnetic field can be detected mainly via Faraday rotation in the near half. At $\lambda 18.0$ and 20.5 cm, where $h_{syn} > h_{th}$, the disk is not transparent to polarized emission at $r < 10$ kpc where the halo is present. As a result, it is impossible to determine the structure of the magnetic field in the part of the halo lying beyond the thermal disk from observations of the intrinsic polarized emission.

Now we express $\xi^{(D)}$ and $\xi^{(H)}$ in terms of the scale heights of the thermal and synchrotron disk, $h_{th}$ and $h_{syn}$, and $\Delta z$ in a given wavelength range. One should take into account that, if synchrotron emission and Faraday rotation occur in the same region, the observed Faraday rotation rotation measure of a transparent layer is equal to $\int \frac{1}{2} B_{l} n_{e} dL$, whereas that produced in a foreground Faraday screen (i.e., a magneto-ionic layer devoid of relativistic electrons) is $\int B_{l} n_{e} dL$. Assume that $h_{syn} \geq h_{th}$, which inequality is true in the case of M 51 for $\lambda \approx 20$ cm. The Faraday rotation measure observed from the disk is given by $\frac{1}{2} RM^{(D)} \Delta z / h_{th}$.

The contribution of the halo to the observed Faraday rotation measure is $\frac{1}{2} RM^{(H)}(h_{syn} - h_{th})/Z + RM^{(H)}[Z - (h_{syn} - h_{th})]/Z$, where the first term is due to the synchrotron-emitting region, and the second one is the contribution of the rest of the halo which acts as a foreground Faraday screen. Thus, we have
\[ \xi^{(D)} = \frac{1}{2} \frac{\Delta z}{h_{th}}, \quad \xi^{(H)} = 1 - \frac{h_{syn} - h_{th}}{Z} \] for $Z > 0$ ;
\[ \xi^{(H)} \] is undefined for $Z = 0$ whereas $RM^{(H)} = 0$ in this case.

At $\lambda = 2.8/6.2$ cm the galaxy is transparent, so that $\Delta z = 2h_{th}$ and we obtain $\xi^{(D)} = 1$; furthermore, $\xi^{(H)} \approx 1$ because at these wavelengths $h_{syn}$ differs insignificantly from $h_{th}$ for $r = 3-9$ kpc and $h_{syn} < h_{th}$ for $r = 9-12$ kpc. However, $\xi^{(D)}$ strongly differs from unity at $\lambda = 18.0/20.5$ cm. The dependence of $\xi^{(D)}$ and $\xi^{(H)}$ on $\lambda$ is due to the $\lambda$-dependence of $\Delta z$ and $h_{syn}$. The values of $\Delta z$, $\xi^{(D)}$ and $\xi^{(H)}$ given in Table 3 refer to $\lambda 18.0/20.5$ cm. The halo is transparent for polarized emission at all the wavelengths considered, and $\xi^{(H)}$ differs from unity only because some synchrotron emission originates within the halo (at $h_{syn} \leq z \leq h_{th}$), whereas the remaining part of the halo acts as a foreground screen.

For the sake of completeness and having in mind possible applications to other galaxies, we also give expressions for $\xi^{(D)}$ and $\xi^{(H)}$ for the case that $h_{syn} \leq h_{th}$. When calculating $\xi^{(D)}$, it is useful to distinguish two physically different cases: $\Delta z \geq h_{th} - h_{syn}$ (i.e., the synchrotron disk is visible at the longer wavelengths) and $\Delta z < h_{th} - h_{syn}$. In the former case, the total Faraday rotation measure produced in the disk is given by $\frac{1}{2} RM^{(D)}[\Delta z - (h_{th} - h_{syn})]/h_{th} + RM^{(H)}(h_{th} - h_{syn})/h_{th}$, where the first term is due to the synchrotron-emitting region, and the second one is the contribution of the rest of the thermal disk which acts as a foreground Faraday screen.

For $\Delta z < h_{th} - h_{syn}$, when the synchrotron disk in not visible at a given wavelength, we have the RM produced in the disk as $\frac{1}{2} RM^{(D)} \Delta z / h_{th}$. Thus,
\[ \xi^{(D)} = \begin{cases} \frac{1}{2} \left( 1 + \frac{\Delta z - h_{syn}}{h_{th}} \right) & \text{for } \Delta z \geq h_{th} - h_{syn}, \\ \frac{\Delta z}{h_{th}} & \text{for } \Delta z < h_{th} - h_{syn}. \end{cases} \]

We have $\xi^{(H)} = 1$ in this case.
4. Recognition of magnetic field patterns

4.1. The parametrization of the magnetic structure

When fitting the observed distribution of the polarization angle $\psi$ at a given wavelength $\lambda$ we adopt for each layer the following truncated Fourier representation for the cylindrical components of the regular magnetic field $B_{\text{reg}} = (B_r, B_\theta, B_z)$:

$$
\begin{align*}
B_r &= B_0 \sin p_0 + B_1 \sin p_1 \cos(\theta - \beta), \\
B_\theta &= B_0 \cos p_0 + B_1 \cos p_1 \cos(\theta - \beta), \\
B_z &= B_{z0} + B_{z1} \cos(\theta - \beta_z),
\end{align*}
$$

where $B_0$ and $B_{z0}$ are the strengths of the horizontal (parallel to the galactic plane) and vertical (perpendicular to the plane) components of the $m = 0$ mode, respectively, $B_1$ and $B_{z1}$ are those of the $m = 1$ mode, $p_0$ and $p_1$ are the pitch angles, and $\beta$ and $\beta_z$ are the azimuthal angles at which the corresponding non-axisymmetric components are maximum. In Eq. (8) only the two lowest modes have been retained; this proves to be sufficient to fit the available data. The magnetic pitch angle is the small angle measured from the magnetic field vector to the tangent of the local circumference. It is positive (negative) if the magnetic field direction can be either inwards or outwards along the magnetic spiral. In the case of M 51, a negative pitch angle corresponds to a trailing spiral.

The intrinsic polarization angle $\psi_0$ in Eq. (5) is determined by the transverse component of the magnetic field. A suitable expression relating $\psi_0$ to the magnetic field of the form given in Eq. (8) was given by Sokoloff et al. (1992); its detailed derivation can be found in Appendix A - see Eq. (A3). Insofar as the synchrotron emissivity in the halo is significantly weaker than in the disk, we assume that the intrinsic polarization angle depends solely on the field in the disk.

We should emphasize that we analyze simultaneously and consistently the longitudinal and transverse (with respect to the line of sight) components of the magnetic field which manifest themselves through RM and $\psi_0$, respectively. Previous work either considered solely the Faraday rotation measures between pairs of wavelengths or simplified the model by supposing that $\psi_0 = \text{const}$ (see Sokoloff et al. 1992). In both cases only the line-of-sight magnetic field could be recovered from observations. Attempts to extract additional information about the transverse component of the magnetic field from an independent analysis of “magnetic pitch angles,” or $\psi_0$, from total and polarized intensity, etc. often led to results that were inconsistent with those obtained from the RM analysis. Here for the first time we propose a consistent three-dimensional model of the regular magnetic field observed in a galaxy.

4.2. The fitting procedure

Having represented the galactic magnetic field in terms of the Fourier series (8), we calculate the corresponding intrinsic polarization angle using Eq. (A3) and the model Faraday rotation measure using Eqs. (6) and (A2) with $\zeta^{(D)}$ and $\zeta^{(H)}$ given in Table 3. This leads to the model azimuthal distributions of the polarization angle via Eq. (5). The model parameters are then determined from fits to the observed azimuthal distributions of polarization angles at the four wavelengths for each ring as described in Appendix B. Since the galactic magnetic field reveals itself mainly through Faraday rotation, this analysis yields an estimate of the products $B_i \langle n_e \rangle L$ rather than $B_i$. Therefore, we have to use independent information on the volume density of thermal electrons $\langle n_e \rangle$ and its distribution, and on the length of the line-of-sight through the magneto-ionic region $L$ in order to estimate $B_i$; these parameters are discussed in Sect. 3.2.

The results of the fits discussed below are given in terms of the Fourier coefficients $\mathcal{R}_i$ defined by

$$
\mathcal{R}_i \equiv 0.81 B_i \langle n_e \rangle h
= 24 \text{ rad m}^{-2} \left( \frac{B_i}{1 \mu G} \right) \left( \frac{\langle n_e \rangle}{0.03 \text{ cm}^{-3}} \right) \left( \frac{h_{\text{th}}}{1 \text{ kpc}} \right),
$$

for each part of the regular magnetic field $B_i = (B_0, B_1, B_{z0}, B_{z1})$ in the disk; for the halo, we adopt a similar definition with $h_{\text{th}}$ replaced by $Z$.

We emphasize that the quantities $\mathcal{R}_i$, albeit having the dimension of Faraday rotation measure, have an entirely different physical meaning: if $\langle n_e \rangle$ and $h$ are constants, they characterize the amplitudes of the individual Fourier components of the magnetic field in a given layer. Unlike Faraday rotation measure, the coefficients $\mathcal{R}_i$ do not add algebraically in a system consisting of several layers with distinct magnetic structures. The relation of the coefficients $B_i$, or $\mathcal{R}_i$, to the line-of-sight and transverse components of the magnetic field (and thus to the Faraday rotation measure and the intrinsic polarization angle) is provided in Appendix A. We also use the quantity $\mathcal{R}_i$ which characterizes in a similar manner the total regular magnetic field, $B_{\text{reg}} = (B_0^2 + B_1^2 + B_{z0}^2 + B_{z1}^2)^{1/2}$.

Representations of the form (9) are introduced for the disk and the halo separately. For example, $\mathcal{R}_i^{(D)}$ denotes the amplitude of the $m = 0$ mode in the disk, $p_1^{(H)}$ is the pitch angle of the $m = 1$ mode in the halo, etc. For the radial range 9-15 kpc (see Sect. 4.4.3 and 4.4.4) the fits were performed for a one-layer magnetic field model without distinguishing the disk and halo contributions to RM (that is, we put $\mathcal{R}_i^{(H)} = 0$ there).

In order to obtain satisfactory fits to the data, we calculated the residual $S$ defined in Eq. (B1), that characterizes the deviation of the fit from the measured points, found its minimum with respect to the above fit parameters, and then employed the $\chi^2$ and Fisher statistical tests briefly discussed in Appendix B to assess the reliability of the fit. The $\chi^2$ statistical test, Eq. (B2), ensures that the fit is close enough to the measured points with allowance for their weights equal to $\sigma_{ni}^{-2}$. The Fisher test, Eq. (B3), was then applied to verify that the quality of the fit is the same at all individual wavelengths (see Sokoloff et al. 1992 for details).
In earlier studies $ψ_0$, and hence the pitch angle of the magnetic field, were assumed to be independent of $θ$ so that the residual $S$ defined by Eq. (B1) had a unique minimum in the parameter space (Ruzmaikin et al. 1990; Sokoloff et al. 1992). It was then possible to apply a linear least-squares method to find this minimum. The corresponding values of the parameters were considered the most probable ones. In our model $S$ is a strongly nonlinear function of the parameters of the model, mainly because of the nonlinear nature of $ψ_0$. Therefore, $S$ can have several local minima and, generally, many of them may satisfy the $χ^2$ test. In order to choose the best fit one should apply (i) the Fisher test and (ii) use all a priori information about the galaxy itself and the structure of the magnetic field. For instance, it is obvious that the resulting values of $RM_{tg}$ in different rings must agree within the errors. However, for $r = 12-15$ kpc this does not occur unless we apply special restrictions. Then we have to consider a conditional minimum of $S$, restricted by the requirement that $RM_{tg}$ resides within a certain range obtained for other rings.

A method of estimating the errors of the model parameters is also discussed in Appendix B. The errors given in Table 4 below are the estimates of the uncertainties corresponding to a $2\sigma$ Gaussian deviation.

Even though our model of magnetic field is advanced enough to provide generally good fits, it can happen that a few measurements deviate very strongly from both the fit curves and the neighboring measured points. This can be due to many various reasons, including strong local distortions of the magnetic field (that are beyond the scope of the present analysis focussed on a large-scale structure), underestimated errors of these measurements or systematic errors. In such cases the only reasonable remedy is to exclude the strongly deviating measurements from the analysis. Of course, this is the last resort and we did this only when other ways to reach a good fit failed. In our analysis below only one measurement in the ring 9-12 kpc was omitted.

4.3. Statistical evidence for a magneto-ionic halo in M 51

Fits for all wavelengths obtained with one transparent magneto-ionic layer were generally inconsistent with the Fisher test at $r < 9$ kpc. Therefore, we tried fits to the polarization angle distributions for $λλ 2.8/6.2$ cm and $λλ 18.0/20.5$ cm separately. The magnetic patterns revealed in the two wavelength ranges in the inner two rings turned out to be very different from each other. For example, the values of $R_{i0}$ obtained at the shorter and longer wavelengths had different signs. Apparently the only plausible explanation for this is that different regions are sampled in the two wavelength ranges and that the magnetic field has completely different configurations in them. Moreover, $B_{i0}$ changes sign either in the upper part of the disk $Δz$ or in a layer above $z = h_0$. Since it is very unlikely that the regular magnetic field may have such a complicated vertical structure within the upper part of the disk, we concluded that the change of direction of $B_{i0}$ suggests the existence of one more extended component in the galaxy. Thus, at $r \leq 9$ kpc, at least two extended components of the magneto-ionic medium in M 51 are present, namely the disk and the halo.

The measurements of depolarization discussed above also indicate that the galaxy is not transparent to polarized emission in the wavelength range $λ \approx 20$ cm. This again implies that different regions are sampled at the two wavelength ranges: at $λλ 2.8/6.2$ cm the whole galaxy is transparent to polarized radio waves, whereas at $λλ 18.0/20.5$ cm polarized emission originating from only the upper part of the galaxy is visible. Then the observed Faraday rotation at the short wavelengths is dominated by the magnetic field in the disk whereas at the long wavelengths is mainly determined by the field in the halo.

Interestingly, recent observations of M 51 in X-rays (Ehle et al. 1995) provide an independent confirmation of the presence of a hot thermal halo of about 10 kpc in radius. We also note that the distribution of thermal emission at $λλ 2.8$ cm (Klein et al. 1984) implies that the thermal electron density in the disk abruptly decreases at $r \approx 9$ kpc (see Table 2) indicating that the star formation rate decreases steeply at this radius. We adopted an elliptical shape for the halo with a height of 6 kpc above the midplane near the center (see Fig. 2) and assumed the Milky Way value for the electron density of $3 \times 10^{-3}$ cm$^{-3}$.

It is clear that separate fits for different wavelength ranges cannot be physically satisfactory. Therefore all the fits discussed below were obtained from all four wavelengths simultaneously using the values of $Δz$ in Table 3. Thus we used a two-layer model of the magneto-ionic medium for the interval 3-9 kpc and a one-layer model for 9-15 kpc.

4.4. Results of the fitting

In this Section we discuss the results of the fitting procedure described in Sect. 4.2. In Figs. 3-6 we present the variations of the measured polarization angles with the azimuthal angle $θ$ and the fits. The results of the fitting are given in Table 4. For each ring we compile the model parameters and their uncertainties at the significance level 95% as well as the value of the residual $S$ with the contributions from individual wavelengths and the value of the $χ^2$ test.

4.4.1. The radial range $r = 3-6$ kpc

A satisfactory fit for 3-6 kpc (i.e., with both the $χ^2$ and Fisher tests satisfied) is achieved without a vertical component of the magnetic field. As shown by the fits given in Table 4 and in Fig. 3, the magnetic field in this ring represents a superposition of horizontal axisymmetric ($\mathcal{H}^{(H)}_0$) and bisymmetric ($\mathcal{H}^{(B)}_0$) components in the disk and an axisymmetric field ($\mathcal{H}^{(D)}_0$) in the halo. The axisymmetric and bisymmetric components in the disk have comparable amplitudes with a moderate dominance of the latter. Since the axisymmetric components in the disk and halo have opposite signs, the fields in the disk and the halo are oppositely directed in most of the ring. In both the disk and the halo the pitch angles are negative, therefore the magnetic pattern is trailing like the optical spiral arms.
Table 4. Model for the global magnetic field in M 51 (a)

| r [kpc] | 3-6 | 6-9 | 9-12 | 12-15 |
|---------|-----|-----|------|-------|
| Disk    | Halo| Disk| Halo| Disk  |
| RM$_{lg}$ [rad m$^{-2}$] | +3 ±2 | +8 ±5 | +9 ±4 | +5 (b) |
| $\mathcal{H}_0$ [rad m$^{-2}$] | −170$^{+16}_{-28}$ $^{+41}_{-10}$ | −147$^{+69}_{-94}$ $^{+8}_{-21}$ | −22$^{+41}_{-8}$ $^{−}$ | −21$^{+4}_{-7}$ |
| $p_0$ [deg] | −12 ±1 | −23 ±13 | −12 ±2 | −23 ±180 | −6 ±3 | +18$^{+4}_{-7}$ |
| $\mathcal{H}_1$ [rad m$^{-2}$] | −245$^{+33}_{-73}$ $^{...}$ | −138$^{+136}_{-16}$ $^{...}$ | −21$^{+41}_{-7}$ | −21 ±5 |
| $p_1$ [deg] | −11 ±2 | −10$^{+10}_{-3}$ $^{...}$ | −3 ±3 | +32$^{+5}_{-13}$ |
| $\beta$ [deg] | +176 ±6 $^{...}$ | +68$^{+34}_{-68}$ $^{...}$ | +58 ±6 | −63$^{+7}_{-10}$ |
| $\mathcal{H}_{01}$ [rad m$^{-2}$] | $^{...}$ | $^{...}$ | $^{...}$ | $^{...}$ | +12 ±3 |
| $\mathcal{H}_{02}$ [rad m$^{-2}$] | $^{...}$ | $^{...}$ | $^{...}$ | $^{...}$ | +3 ±3 |
| $\beta_z$ [deg] | $^{...}$ | $^{...}$ | $^{...}$ | $^{...}$ | +186 ±8 |

$S^{(c)}$ | 20 $^{+18}_{-16}$ $^{+20}_{-16}$ | 20 $^{+17}_{-8}$ $^{+8}_{-9}$ | 20 $^{+24}_{-21}$ $^{+21}_{-21}$ | 11 $^{+12}_{-12}$ |

$\chi^2$ | 88 | 88 | 68 | 39 |

Notes:
(a) Dots mean that the corresponding parameter is insignificant and not included in the fit.
(b) RM$_{lg}$ for 12-15 kpc was fixed at the given value – see Sect. 4.4.4.
(c) $S$ is given as a sum of the respective contributions at individual wavelengths, $\lambda\lambda 2.8, 6.2, 18.0$ and 20.5 cm except for the ring 9-12 kpc, where the wavelength 6.2 cm is omitted, and the ring 12-15 kpc, where only the data at the two longer wavelengths were used.

There is one more fit which satisfies the statistical criteria equally well and has the same number of parameters. This is a fit with an axisymmetric field in the disk and a combination of axisymmetric and bisymmetric fields in the halo. However, in the next outer ring there is only one fit satisfying the statistical criteria (discussed in Sect. 4.4.2), which has a combination of an axisymmetric and a bisymmetric field in the disk and an axisymmetric field in the halo. Since it should be expected that the global configuration of the magnetic field in the galaxy cannot change strongly over the radial distance of about 3 kpc, we consider the fit shown in Fig. 3 and Table 4 as the more plausible one.

4.4.2. The radial range r = 6-9 kpc

In Table 4 and in Fig. 4 we show the results of the fit for this ring. The modes $m = 0$ and $m = 1$ are of almost equal amplitudes in the disk and the $m = 0$ mode dominates in the halo. The direction of the magnetic field in the halo is opposite to that in the disk as in the ring 3-6 kpc. The field is predominantly horizontal and the field pattern represents a trailing spiral.

Fig. 3a–d. The polarization angles (dots with error bars, measured from the local radial direction in the plane of M 51) as a function of azimuthal angle in the galactic plane and fits (solid lines) for the ring $3 \leq r \leq 6$ kpc at (a) $\lambda 2.8$, (b) $\lambda 6.2$, (c) $\lambda 18.0$ and (d) $\lambda 20.5$ cm. Error bars show the 1$\sigma$ errors of the measurements. Dashed lines indicate the foreground levels, RM$_{lg}\lambda^2$.

The configurations of magnetic field in the two inner rings are very similar to each other as evidenced by the similar combinations of azimuthal modes, close values of the pitch angles,
and the decrease of $\beta$ with radius as expected for a spiral of this pitch angle (cf. Appendix in Krause et al. 1989b). The amplitude of each mode decreases with radius between 3.6 and 6–9 kpc.

The very large positive error of $\mathcal{A}_1^{(D)}$ as given in Table 4 can be explained as follows. There is another fit (corresponding to shallower minima in the $S$ criterion but fails to meet the Fisher criterion. It has an axisymmetric field in the disk and a combination of $m = 0$ and 1 modes in the halo with the ratio on the left-hand side of Eq. (B3) being 3.13 and $\mathcal{F}(10, 10, 0.95) \approx 2.97$. The minimum in $S$ corresponding to the latter fit and that given in Table 4 are connected by a long, narrow ‘valley’ extended along the $\mathcal{A}_1^{(D)}$ axis which is mostly below the $\chi^2$ level. Since our estimate of errors is sensitive only to the relative values of $S$ and $\chi^2$ (see Appendix B) the resulting error in $\mathcal{A}_1^{(D)}$ turns out to be that large. In other words, the admissible region in the parameter space corresponding to the above fit is more like a dumbbell than an ellipsoid; the error of $\mathcal{A}_1^{(D)}$ given corresponds to the larger dimension of the region.\(^2\)

4.4.3. The radial range $r = 9.12$ kpc

As the measurements at $\lambda 6.2$ cm are unreliable beyond $r \simeq 10$ kpc (see Sect. 2) we consider only the wavelengths $\lambda 2.8$, $18.0$ and $20.5$ cm in this ring.

The data at the short and long wavelengths do not exhibit the strong difference typical of the two inner rings (see Sect. 4.3) and a one-layer model is consistent with observations. We therefore conclude that there is no halo visible in this ring.

The $\chi^2$ and Fisher tests cannot be satisfied if all the measurements are included into the fit. In Fig. 5 and Table 4 we give the fit parameters with a combination of the $m = 0$ and 1 modes obtained after one measurement at $\lambda 18.0$ cm, $\theta = 180^\circ$, was omitted from the analysis. This point strongly deviates from both the general trend and the corresponding measurement at the close wavelength $\lambda 20.5$ cm. We should note that one might prefer to include this point and to employ a more complicated model with a vertical magnetic field in order to meet the $\chi^2$ test. However, then the result that there is a vertical magnetic field in this ring would rely on only a single measurement.

The magnetic field inferred for this ring also represents a superposition of $m = 0$ and 1 modes with almost equal weights. Magnetic lines again form a spiral opening clockwise, even though the spiral is more tightly wound than in the inner rings. The amplitudes of the two modes are smaller than in the inner regions, thus following the trend of the two inner rings. However, the decrease of $\beta$ with radius by about $110^\circ$ between two adjacent rings is not continued between the rings at $r = 6.9$ and $9–12$ kpc.

We should note that if we omit two more measurements strongly deviating from other points, at $\theta = 0^\circ$ at $\lambda \lambda 18.0$ and $20.5$ cm, then a model with a purely axisymmetric field provides a good fit.

4.4.4. The radial range $r = 12.15$ kpc

Results of the fit to the data at $\lambda \lambda 18.0$ and $20.5$ cm are shown in Table 4 and Fig. 6. Again measurements at $\lambda 6.2$ cm are not

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\(^2\) We note parenthetically one more fit (that does not satisfy the Fisher criterion even though it meets the $\chi^2$ test) that has axisymmetric fields in both the disk and the halo with the maximum value of the ratio on the left-hand side of Eq. (B3) equal to 2.89, whereas $\mathcal{F}(13, 13, 0.95) \approx 2.58$. 

---

Fig. 4a–d. The polarization angles (dots with error bars, measured from the local radial direction in the plane of M 51) as a function of azimuth and fits (solid lines) for the ring $6 \leq r \leq 9$ kpc at $a$ $\lambda 2.8$, $b$ $\lambda 6.2$, $c$ $\lambda 18.0$ and $d$ $\lambda 20.5$ cm. Error bars show the 1σ errors of the measurements. Dashed lines indicate the foreground levels, RMfg $\lambda^2$.

Fig. 5a–c. The polarization angles (dots, measured from the local radial direction in the plane of M 51) as a function of azimuth and fits (solid lines) for the ring $9 \leq r \leq 12$ kpc at $a$ $\lambda 2.8$ cm, $b$ $\lambda 18.0$ cm and $c$ $\lambda 20.5$ cm. Error bars show the 1σ errors of the measurements. Dashed lines indicate the foreground levels, RMfg $\lambda^2$. The point which is omitted from the analysis is shown without error bar.
available for this ring. In addition, in this ring the galaxy is completely transparent for polarized emission at all the wavelengths used (that is, $\Delta \zeta \approx 2h_{\text{th}}$ - see Tables 2 and 3), and we applied a one-layer model.

Only a few measurements are available at $\lambda 2.8$ cm which were also neglected; we stress, however, that the available points at $\lambda 2.8$ cm agree very well with the fit obtained. The data at $\theta = 300^\circ$ and $320^\circ$, $\lambda = 18.0$ cm, are not available because the polarized intensity in these sectors is too low.

A model with a purely horizontal magnetic field does not satisfy the $\chi^2$ test and we included the vertical field into the model. This leads to the following difficulty. An axisymmetric vertical magnetic field affects the polarization angles in almost the same way as the foreground Faraday rotation with the only difference that it not only produces a uniform Faraday rotation but also affects the intrinsic polarization angle, $\psi_0$. If $B_{\theta 0}$ is weak compared with horizontal components its effect on $\psi_0$ is similarly weak, so it is difficult to separate its effect from the foreground Faraday rotation. Thus, we present in Table 4 and Fig. 6 the fit obtained with $\mathcal{R}_0 = 5$ rad m$^{-2}$, approximatly the median value obtained for the other rings. Horellou et al. (1992) estimated $\mathcal{R}_0 = -5 \pm 12$ rad m$^{-2}$ using 9 sources located within $20^\circ$ from M 51, which agrees within errors with our estimates.

As a result the relief of $S$ in the parameter space has a narrow valley extended below the $\chi^2$ level along the surface $\mathcal{R}_{\theta 0} + \mathcal{R}_{\phi 0} \approx 19$ rad m$^{-2}$. Therefore, we cannot estimate $\mathcal{R}_{\theta 0}$ and $\mathcal{R}_{\phi 0}$ separately without additional constraints. We cannot put $\mathcal{R}_{\phi 0} = 0$ with $\mathcal{R}_{\phi 1} \neq 0$ because then $\mathcal{R}_{\theta 0} = 15 \pm 2$ rad m$^{-2}$, a value inconsistent with the values of $\mathcal{R}_{\theta 0}$ obtained for the other rings. Furthermore, this fit has $S = 31$, whereas all the fits with $\mathcal{R}_{\phi 0} \neq 0$ have considerably smaller $S$ of about 23 (even though all the fits discussed meet both the $\chi^2$ and Fisher tests).

Fig. 6a and b. The polarization angles (dots with error bars, measured from the local radial direction in the plane of M 51) as a function of azimuth and fits (solid lines) for the ring $12 \leq r \leq 15$ kpc at $\lambda 18.0$ cm and $b \lambda 20.5$ cm. Error bars show the $1\sigma$ errors of the measurements. Dashed lines indicate the foreground levels, $\mathcal{R}_{\theta 0} = \mathcal{R}_{\phi 0}$. Observations at $\lambda 18.0$ cm, $\theta = 300^\circ$ and $320^\circ$ are not available.

4.4.5. Sensitivity to the model parameters

The adopted values of $\Delta \zeta$ (and, hence, $\xi(D)$) are the lower estimates (see Sect. 3.3.2). Therefore, we checked how sensitive our results are to this parameter. The fits turned out to be quite stable under the variation of $\xi(D)$ showing an ordered, slow variation of the fit parameters. For example, in the ring $3-6$ kpc $\mathcal{R}_0(D) (\mathcal{R}_{\phi 0}(D))$ varies between $-201$ and $-54$ rad m$^{-2}$ ($-290$ and $-77$ rad m$^{-2}$) for $\xi(D)$ varying between $0.16$ and $1.00$. The other parameters vary very weakly: for instance, the corresponding range for $\mathcal{R}_0(H)$ is $37-47$ rad m$^{-2}$, and those for the pitch angles and $\beta(D)$ are as narrow as $4^\circ$ and $10^\circ$, respectively. For $\xi(D) < 0.16$ the fit fails to satisfy the $\chi^2$ test.4 Since our rings were chosen more or less arbitrarily with the only requirement that they should be wide enough to match the resolution of the observations, we also tried other rings to test the stability of the results. In particular, we considered the rings $r = 4-6$ kpc and $r = 10-14$ kpc. The results are only weakly sensitive to this change.

Figures 3 and 4 show that our fits do not exactly follow the variations in the polarization angles at $\lambda \lambda 2.8$ and $6.2$ cm. Although at these wavelengths the errors in $\psi$ are larger than at $\lambda 18.0$ and $20.5$ cm, we stress that the quality of the fit is uniformly good at all the wavelengths as ensured by the Fisher test.

5. Global structure of the regular magnetic field

In this section we derive the strength and the direction of the regular magnetic field, and we discuss the global properties of these parameters.

Results of the fits presented in Table 4 can be converted into the amplitudes of the regular magnetic field (see Eq. (7)) using the estimates of $\langle n_e \rangle$ and $h_{\text{th}}$ given in Table 2. The results are compiled in Table 5 (of course, the pitch angles and phases of individual modes remain the same as given in Table 4). A note of caution is appropriate here: the resulting amplitudes $B_{\theta}$ were obtained assuming that $\langle n_e \rangle$ is independent of azimuthal angle. For each ring we also give the strength of the regular magnetic field averaged over the azimuth, $B_{\text{reg}} = \langle \mathcal{R} \rangle h_{\text{th}}$, where $\mathcal{R} = 0.81 \langle n_e \rangle h (B^2_\theta + B^2_\phi + B^2_z)^{1/2}$ with $h = h_{\text{th}}$ for the disk and $h = Z$ for the halo.

In Fig. 7 we show the radial variation of $B_{\text{reg}}$ and that of the total and regular magnetic field strengths, $B$ and $B_{\text{reg}}$, obtained from the total nonthermal emission and the observed degree of polarization as described in Sect. 3.1 and given in Table 1. We note that for each ring there is a close agreement between the two

As a result the relief of $S$ in the parameter space has a narrow valley extended below the $\chi^2$ level along the surface $\mathcal{R}_{\theta 0} + \mathcal{R}_{\phi 0} \approx 19$ rad m$^{-2}$. Therefore, we cannot estimate $\mathcal{R}_{\theta 0}$ and $\mathcal{R}_{\phi 0}$ separately without additional constraints. We cannot put $\mathcal{R}_{\phi 0} = 0$ with $\mathcal{R}_{\phi 1} \neq 0$ because then $\mathcal{R}_{\theta 0} = 15 \pm 2$ rad m$^{-2}$, a value inconsistent with the values of $\mathcal{R}_{\theta 0}$ obtained for the other rings. Furthermore, this fit has $S = 31$, whereas all the fits with $\mathcal{R}_{\phi 0} \neq 0$ have considerably smaller $S$ of about 23 (even though all the fits discussed meet both the $\chi^2$ and Fisher tests).

The situation is qualitatively the same for 6-9 kpc; the only difference is that for $\xi(D) < 0.15$ the fits remain under the $\chi^2$ level, but the values of $\mathcal{R}_0(H)$ and $p_0^{(H)}$ become unstable and change their signs. The effect of varying $\xi(D)$ is trivial for 9-15 kpc, where a one-layer model was applied: all the angles remain the same, whereas $\mathcal{R}_{\phi 0}(D)$ satisfy the relation $\xi(D), \mathcal{R}_{\phi 0}(D) = \text{const.}$
Table 5. Amplitudes of magnetic modes and the regular magnetic field averaged along azimuth in M 51 \(^{(a)}\)

| \(r\) [kpc] | 3-6 | 6-9 | 9-12 | 12-15 |
|-------------|-----|-----|-----|-------|
|             | Disk | Halo| Disk | Halo  | Disk | Disk  |
| \(B_0\) [\(\mu G\)] | \(-4.8^{+1.3}_{-2.6}\) & \(+3.4 \pm 0.8\) & \(-5.2^{+3.0}_{-5.8}\) & \(+1.0 \pm 2.6\) & \(-1.0^{+1.1}_{-7.6}\) & \(-3.2^{+2.4}_{-31.3}\) |
| \(B_1\) [\(\mu G\)] | \(-6.9^{+2.1}_{-3.2}\) & \(\ldots\) & \(-4.9^{+4.8}_{-2.2}\) & \(\ldots\) & \(-1.4^{+1.0}_{-6.8}\) & \(-3.2^{+2.4}_{-28.9}\) |
| \(B_0\) [\(\mu G\)] | \(\ldots\) & \(\ldots\) & \(\ldots\) & \(\ldots\) & \(\ldots\) & \(+1.9^{+14.2}_{-1.4}\) |
| \(B_1\) [\(\mu G\)] | \(\ldots\) & \(\ldots\) & \(\ldots\) & \(\ldots\) & \(\ldots\) & \(+0.5^{+5.7}_{-0.5}\) |
| \(B_{\text{reg}}\) [\(\mu G\)] | \(6.9^{+3.4}_{-2.0}\) & \(3.4 \pm 0.8\) & \(6.3^{+5.8}_{-4.0}\) & \(1.0^{+2.6}_{-1.0}\) & \(1.9^{+9.0}_{-1.3}\) & \(4.4^{+40.2}_{-3.2}\) |

Notes:
\(^{(a)}\) Dots mean that the corresponding parameter is insignificant and thus not included in the fit.

![Fig. 7. The radial variation of the strength of the regular magnetic field obtained from our fits and averaged in the rings (circles with error bars), the total magnetic field obtained from the total intensity of the nonthermal emission assuming energy equipartition between cosmic-ray particles and magnetic field (dashed) and its regular component obtained using the observed degree of polarization (solid).](image)

values of the regular magnetic field which were obtained from completely independent physical parameters and methods. Between \(r = 3\) and 15 kpc the exponential radial scale length of the regular magnetic field obtained from the synchrotron emission is 14.3 \pm 6.0\ kpc.

Apart from the comparison of the ring averages in Fig. 7 one could also compare the azimuthal variation of the transverse magnetic field as given by the fits with the azimuthal distribution of the polarized intensity in each ring. However, as this would require the knowledge of \(\langle n_e \rangle\) and \(h_{\text{in}}\) as functions of azimuthal angle, we postpone this to a later paper.

As can be seen from Table 4 the magnetic fields in the halo and in the disk inside \(r = 12\) kpc are horizontal. In the halo we have \(\mathcal{K}^{(H)}_0 > 0\), \(p_0^{(H)} < 0\) and \(\mathcal{K}^{(H)}_1 \simeq 0\), which means that the radial component of the regular magnetic field is directed \textit{inwards}, with the azimuthal component directed counterclockwise; that is, \(B_r^{(H)} < 0\) and \(B_\theta^{(H)} > 0\) - see Eq. (8). Meanwhile, in the disk we have \(\mathcal{K}^{(D)}_0 \approx \mathcal{K}^{(D)}_1\) and both are negative together with the pitch angles. Therefore, for \(r = 3-9\) kpc the radial field in the disk is directed \textit{outwards}, with \(B_\theta\) directed clockwise at almost all \(\theta\). We conclude that the regular magnetic fields in the disk and the halo have almost opposite directions everywhere within \(r = 9\) kpc except in the northwestern part of the ring at \(r = 3-6\) kpc.

In Fig. 8 the direction of \(B_{\text{reg}}\) in each sector is shown for the disk and the halo separately. The length of the vectors is proportional to \(\mathcal{K}\) with scaling factors specified in the caption.

In the outer ring, 12-15 kpc, the magnetic field structure is distorted. The values of \(p_0\) and \(p_1\) differ considerably from those for \(r \leq 9\) kpc and are even positive. Inspection of the polarization map in Fig. 1a confirms that in the northern part the magnetic pattern at these radii is plagued by strong distortions still having a rather large spatial scale.

6. Discussion

6.1. The magnetic field in the halo

The available polarization measurements performed at the two pairs of widely separated wavelengths allowed us to determine the magnetic field structure in two regions along the line of sight. In the text above we called these regions the disk and the halo. This usage was justified in Sect. 4.3.

Our results represent the first indication of a magneto-ionic halo in a galaxy seen nearly \textit{face-on}. The detection of a radio halo in an edge-on galaxy is a difficult observational problem, even more so the determination of the magnetic field structure. In the galaxies seen nearly face-on some of the difficulties are alleviated. First, the halo and its magnetic field are illuminated by a strong background source of polarized emission, the disk. Second, the polarization measurements over the entire disk can be used to reveal the global azimuthal structure of the field in the halo as it was done in the present paper. It is important to note that the magnetic field in M 51 has different structures in the disk and the halo. If the field structures were similar to each other, the detection of the magneto-ionic halo might be difficult.
Fig. 8a and b. Directions of the horizontal regular magnetic field in the disk (a, left) and halo (b, right) of M 51 according to the fits presented in Table 4. For clarity we scaled the vectors as follows: they are proportional to $R$ in the inner two rings in the disk, to $3R$ in the two outer rings in the disk, to $2.5R$ for 3-6 kpc in the halo, and to $5R$ for 6-9 kpc in the halo. The vertical component at $r = 12-15$ kpc was not included. The vectors are shown superimposed onto an optical picture of M 51. The sectors and rings used are indicated.

We showed that the field in the halo of M 51 is predominantly horizontal as in the halos of NGC 891 and NGC 253 (Hummel et al. 1991b; Sukumar & Allen 1991; Beck et al. 1994).

According to our fits we estimate the halo radius to be about 10 kpc. This estimate agrees with the data on X-ray emission from M 51 which also indicate a halo radius of about 10 kpc (Ehle et al. 1995).

With the values of $\langle n_e \rangle$ and $Z$ from Table 2, the estimated strength of the regular magnetic field in the halo decreases from about 3 $\mu$G at the radial distance of 3-6 kpc to about 1 $\mu$G at $r = 6-9$ kpc. The field is basically axisymmetric. The upper limits on the $m = 1$ mode in the halo are estimated from our fits as $|B_1^0| \lesssim 1 \mu$G and $\lesssim 3 \mu$G for $r = 3-6$ and 6–9 kpc, respectively.

It is interesting to compare the values of the regular magnetic field in the halo with the upper limit on the total magnetic field strength estimated from the equilibrium between thermal and magnetic energy densities in the X-ray emitting gas (Ehle et al. 1995). With $\langle n_e \rangle = 0.003$ cm$^{-3}$ and a volume filling factor of 0.8, their results yield $B < 7 \mu$G. Our results are consistent with this limit. If the true total field strength is close to the above upper limit, the turbulent field in the halo has a strength of about $6 \mu$G exceeding that of the regular magnetic field.

The global field directions are in general opposite in the disk and the halo of M 51. This implies that the regular magnetic field in the halo cannot be simply advected from the disk. Such reversals appear in the dynamo theory for galactic halos (Ruzmaikin et al. 1988, Sect. VIII.1; Sokoloff & Shukurov 1990; Brandenburg et al. 1992) and could be due to the topological pumping of magnetic field by a galactic fountain flow (Brandenburg et al. 1995). Moreover, the dominance of the axisymmetric field in the halo is also consistent with the mean-field dynamo theory which predicts that non-axisymmetric magnetic modes can be maintained only in a thin galactic disk and most likely decay in a quasi-spherical halo (see Ruzmaikin et al. 1988). We cannot say anything about the parity of the halo field with respect to the midplane because the galaxy is not transparent at $\lambda \approx 20$ cm in the rings where the halo is present and, in addition, the synchrotron emission from the halo is negligible.

6.2. The azimuthal structure of the field

The azimuthal distributions of polarization angle in M 51 seen over the radial range $r = 3-15$ kpc are successfully represented by a superposition of only two azimuthal modes of the large-scale magnetic field, $m = 0$ and $m = 1$ in the disk.

Even though we restrain ourselves from identifying these magnetic harmonics with dynamo-generated axisymmetric and bisymmetric modes before a more careful theoretical analysis has been made, we mention that dynamo theory also predicts that...
the two leading azimuthal modes \( m = 0 \) and \( m = 1 \) typically dominate in spiral galaxies. Furthermore, it follows from the dynamo theory that non-axisymmetric magnetic structures should be more often a superposition of the two azimuthal modes than a purely bisymmetric mode (Ruzmaikin et al. 1988, p. 231; Beck et al. 1996). A similar superposition of modes, but with a dominance of the bisymmetric mode, was found earlier in M81 by Sokoloff et al. (1992) (see also Krause et al. 1989b). In M31 the axisymmetric magnetic mode is dominant (Ruzmaikin et al. 1990), and for NGC 6946 a superposition of \( m = 0 \) and \( m = 2 \) magnetic modes was suggested by Beck & Hoernes (1996).

Of course our results do not imply that higher azimuthal magnetic modes are not present in M 51, but only that the accuracy of the available observations is insufficient to reveal them. One can expect that the amplitudes of the harmonics with \( m \geq 2 \) are considerably smaller than those of the modes \( m = 0 \) and \( 1 \).

Since the theory of the galactic mean field dynamo predicts an efficient generation of the bisymmetric mode in M 51 with the maximum of the \( m = 1 \) eigenmode at \( r \approx 2 \) kpc (Baryshnikova et al. 1987; Krasheninnikova et al. 1989), we are tempted to identify the \( m = 1 \) mode revealed for \( 3 \leq r \leq 9 \) kpc with a bisymmetric field generated by the dynamo. This suggestion is confirmed by the closeness of the pitch angles \( p_0 \) and \( p_1 \) of the \( m = 0 \) and \( 1 \) modes to each other in the two innermost rings: this is typical of the dynamo-generated fields in a thin disk (Ruzmaikin et al. 1988). This conclusion is also plausible for \( r = 9-12 \) kpc. We also note that a nonlinear dynamo model of Bykov et al. (1996) predicts a mixture of magnetic modes, which is roughly similar to that detected here, to be found in some vicinity of the corotation radius, i.e. just near 6 kpc in M 51.

The azimuthal modes inferred for the outermost ring can be hardly identified directly with the dynamo modes because the pitch angles of individual modes are positive. These modes rather arise due to distortions imposed by non-axisymmetric density and velocity distributions possibly caused by the encounter with the companion galaxy NGC 5195 (Howard & Byrd 1990). Concerning the total horizontal regular magnetic field, its pitch angle is negative for \( 70^\circ \leq \theta \leq 170^\circ \) and positive in the rest of the sectors. Inspection of polarization maps in Fig. 1 confirms that the pattern of polarization angles at these radii in the northern part is strongly distorted on a rather large scale.

We note that the regular magnetic field in the disk of M 51 is directed outwards, whereas those in IC 342 (Krause et al. 1989a), M31 and NGC 6946 (Beck et al. 1996) are directed inwards.

### 6.3. Inner and outer spiral structure

Inspection of Table 4 shows that for the disk most of the fitted parameters of the inner rings \( r < 9 \) kpc differ systematically from those of the outer rings. The phase angle \( \beta \) varies by about \( 110^\circ \) between the rings at 3-6 kpc and 6-9 kpc, and also between 9–12 kpc and 12–15 kpc, but not between 6-9 and 9–12 kpc. The inner pattern is more coherent and has stronger magnetic field than the outer pattern. Thus it seems that the magnetic field structure in the outer rings is not a smooth continuation of the structure in the inner rings, but that rather two distinctly different magnetic field structures are present in M 51.

This result is very interesting as Elmegreen et al. (1989) showed, using optical plates, that M 51 contains an inner and an outer spiral structure which are overlapping between \( r = 6 \) kpc and \( r = 8 \) kpc. The inner Lindblad resonance, corotation radius and outer Lindblad resonance of the inner structure are \( r_{\text{ILR}}(1) = 1.6 \) kpc, \( r_{\text{CR}}(1) = 6.2 \) kpc, and \( r_{\text{OLR}}(1) = 8.0 \) kpc, respectively. The outer spiral structure is dynamically coupled to the inner structure as \( r_{\text{ILR}}(2) = r_{\text{CR}}(1) \) with \( r_{\text{CR}}(2) = 10.5 \) kpc. Elmegreen et al. also found a phase jump in the \( m = 2 \) spiral mode at \( r = 8.0 \) kpc, the boundary between the two spiral structures. This is consistent with the inner and outer spirals being two separate features with different pattern speeds, which are \( 90 \text{ km s}^{-1} \) and \( 22 \text{ km s}^{-1} \), respectively. The outer spiral arms are thought to be material arms driven by the companion, whereas the inner spiral arms are due to density waves caused by the outer arms.

A discontinuity in the magnetic field pattern indicates that different physical effects contribute to the field structure at \( r \lesssim 9 \) kpc and \( r \gtrsim 9 \) kpc. The discontinuity occurs at the radius where the inner spiral structure ends and the outer spiral structure becomes dominant. Therefore, the two magnetic field structures can be physically connected with the inner and outer spiral patterns proposed by Elmegreen et al. (1989).

The relatively strong magnetic field and its regular pattern in the inner region are compatible with the idea of a dynamo acting under more or less steady conditions. In the outer regions, where the spiral arms are produced by a recent encounter with NGC 5195 about \( 10^8 \) years ago (Howard & Byrd 1990), the magnetic field may be the remnant of an older one disrupted by the velocity perturbation. Therefore it is understandable that the pitch angles are irregular and the values of \( \mathcal{R}_0 \) and \( \mathcal{R}_1 \) are small.

We note that at \( r < 9 \) kpc the maxima in polarized intensity are not located on the spiral arms but in between the arms (see Fig. 1a) as is also the case in M81, IC 342, NGC 1566 and NGC 6946 (Krause et al. 1989a,b, Ehle et al. 1996, Beck & Hoernes 1996). This suggests that the interaction between the density-wave spiral arms and the magnetic fields is more complicated than simple compression by shock waves. As the inner and outer spiral structures are different in physical nature, it is interesting to know whether or not the polarized intensity is enhanced in the interarm regions of the outer structure as well. A close inspection of Figs. 1a and 1b yields that at \( r > 9 \) kpc in the sectors \( 20^\circ \) to \( 80^\circ \) the polarized emission is at maximum in between the arms, whereas in the southwest in the sectors \( 220^\circ \) to \( 280^\circ \) there are maxima on as well as in between the arms. The tongue in polarized emission at \( 
\lambda = \text{2.8 cm} \) running south along \( \alpha = 13^h \text{27}^m \text{30}^s \) seems to be located on the arm. However, higher resolution is required to confirm this coincidence.

We conclude that the magnetic field pattern in the disk of M 51 appears to be not one global structure, but consists of an inner pattern associated with the inner spiral structure of density wave arms and an outer pattern related to the outer spiral.

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To display the equations and symbols, please refer to the original text as they are not visible in the image.
structure of material arms. The interaction between magnetic fields and the spiral patterns is not yet understood.

6.4. Pitch angles of the magnetic field and of the spiral arms

The pitch angle of the fitted regular magnetic field is given by

\[ p = \arctan \frac{B_r}{B_\phi} = \arctan \left( \frac{\mathcal{A}_0 \sin p_0 + \mathcal{A}_1 \sin p_1 \cos (\theta - \beta)}{\mathcal{A}_0 \cos p_0 + \mathcal{A}_1 \cos p_1 \cos (\theta - \beta)} \right) \]  \hspace{1cm} (10)

The pitch angle is constant with azimuth only when the field is represented by a single mode (that is, either \( \mathcal{A}_0 \) or \( \mathcal{A}_1 \) vanishes) or when the two modes have equal pitch angles, \( p_0 = p_1 \). The former is the case for the halo, where \( \mathcal{A}_1^{(H)} \approx 0 \). However, the field in the disk has a varying pitch angle because \( p_0 \) and \( p_1 \) slightly differ from each other at \( r = 3-12 \) kpc (if the median values are considered) and exhibit a stronger difference in the outermost ring.

A comparison of the pitch angles of the magnetic field and the spiral arms may provide important clues to their interaction, whose physical nature still remains unclear.

We compared the pitch angles of the magnetic field in the disk derived from Eq. (10) with the pitch angles of the dust lanes running along the inside of the optical spiral arms as tabulated by Howard & Byrd (1990). In each ring the optical pitch angles were averaged in the same sectors as were used for the model fits (see Fig. 8). The comparison was possible only for the inner two rings, as at larger radii the measured optical pitch angles and the magnetic model pitch angles have too few sectors in common.

Comparing the corresponding sectors we found general agreement between optical and magnetic model pitch angles. For the ring 3-6 kpc the mean of the optical pitch angles is \(-15^\circ \pm 8^\circ\) and that of the magnetic pitch angles is \(-11^\circ \pm 3^\circ\), whereas for the ring 6-9 kpc these values are \(-13^\circ \pm 12^\circ\) and \(-10^\circ \pm 8^\circ\), respectively. The errors are one standard deviation from the mean value and are due to intrinsic variation in pitch angle in each ring. Although the agreement is quite good, we note that the optical pitch angles show larger variations than those of the fitted magnetic field.

Altogether, we conclude that on average the magnetic field inferred from our fits is well aligned with the spiral arms, although local misalignments may be considerable (see Fig. 8).

6.5. The origin of the vertical field

Remarkably enough our analysis has revealed a vertical magnetic field only for \( r = 12-15 \) kpc even though the model has been tailored to detect this component.

For \( 3 \leq r \leq 12 \) kpc the upper limit of the vertical magnetic field can be obtained from fits with \( B_z \neq 0 \). For the sake of simplicity we used a one-component model of the magnetic field for this purpose. The resulting upper limit for a line-of-sight averaged vertical magnetic field is \( |R_z| \leq 10 \) rad m\(^{-2}\), or, taking \( \langle n_e \rangle h = 50 \) pc cm\(^{-3}\) for the total contribution of the disk and halo we have \( |B_z| \leq 0.3 \) \( \mu \)G assuming that \( B_z \) is uniformly directed at all positions along the line of sight. As all other results of this paper, this limit applies to the field averaged over the beam area of about 10 kpc\(^2\).

The vertical magnetic field detected in the outer ring can be either due to the flaring of the galactic disk with the regular magnetic field remaining parallel to the disk surface or simply represent a part of a general distorted magnetic pattern. In the former case the ratios \( B_{z0}/B_{00} \) and \( B_{z1}/B_{11} \) (vertical to radial fields for each mode) must be close to the tangent of the angle between the disk surface and the midplane, i.e., about 0.2 for \( r = 12-15 \) kpc (see Table 2). However, from Table 5 we obtain \( B_{z0}/B_{00} \approx 2 \) and \( B_{z1}/B_{11} \approx 0.3 \) for the mean values. Moreover, \( \beta \) and \( \beta_z \) must be equal to each other. This is not the case either. Therefore, we believe that the vertical magnetic field detected at \( r = 12-15 \) kpc is a result of strong three-dimensional distortions in the regular magnetic field in this ring. Such distortions seem to be natural, e.g. due to tidal effects since this ring is far from the galactic center and closest to the companion galaxy.

7. Summary and conclusions

We developed a method to determine a three-dimensional structure of the regular magnetic field from multi-frequency polarization observations of spiral galaxies. Using this method we analyzed polarization observations of the galaxy M 51 at \( \lambda \lambda 2.8, 6.2, 18.0 \) and 20.5 cm which have a resolution of about 3.5 kpc in the galactic plane. At each wavelength the observed polarization angles were averaged over sectors of 20° width in the rings at radial distances 3-6, 6-9, 9-12 and 12-15 kpc. For each ring the azimuthal distributions of polarization angles were fitted using a three-dimensional model of the magnetic field.

The galaxy is not completely transparent for polarized emission at \( \lambda \lambda 18 \) and 20 cm, which allowed us to analyze the line-of-sight structure of the magneto-ionic medium by considering simultaneously the data at the above four wavelengths.

We obtained the strengths of the total, regular and random magnetic fields in the disk for each ring from the total and polarized intensities of the nonthermal emission using equipartition arguments and synchrotron scale heights scaled from the Milky Way (see Sect. 3.1 and Table 1).

We also derived estimates of the volume density of thermal electrons, their scale height and filling factor for the disk based on the thermal radio emission from M 51 and an analogy with the Milky Way (see Sect. 3.2 and Table 2). Using these, we estimated the strength of the regular magnetic field in the disk and the halo of M 51 from our fits.

Our general conclusions are as follows:

1. The global magnetic pattern in M 51 at \( 3 \leq r \leq 15 \) kpc can be represented as a superposition of the two lowest azimuthal Fourier modes.

2. For the ring 3-6 kpc the mean of the optical pitch angles is \(-15^\circ \pm 8^\circ\) and that of the magnetic pitch angles is \(-11^\circ \pm 3^\circ\), whereas for the ring 6-9 kpc these values are \(-13^\circ \pm 12^\circ\) and \(-10^\circ \pm 8^\circ\), respectively.

3. The pitch angle of the fitted regular magnetic field is given by

\[ p = \arctan \frac{B_r}{B_\phi} = \arctan \left( \frac{\mathcal{A}_0 \sin p_0 + \mathcal{A}_1 \sin p_1 \cos (\theta - \beta)}{\mathcal{A}_0 \cos p_0 + \mathcal{A}_1 \cos p_1 \cos (\theta - \beta)} \right) \]  \hspace{1cm} (10)

4. The vertical magnetic field detected in the outer ring can be either due to the flaring of the galactic disk with the regular magnetic field remaining parallel to the disk surface or simply represent a part of a general distorted magnetic pattern.

5. The vertical magnetic field detected at \( r = 12-15 \) kpc is a result of strong three-dimensional distortions in the regular magnetic field in this ring. Such distortions seem to be natural, e.g. due to tidal effects since this ring is far from the galactic center and closest to the companion galaxy.
2. Our analysis indicates the existence of a magneto-ionic halo and shows that the magnetic fields in the disk and the halo have different configurations.

3. The radial extent of the halo is about 10 kpc, in agreement with X-ray data. The halo field is horizontal and axisymmetric. The regular magnetic fields in the halo and in the disk are spirals. The field directions along the spirals are generally opposite running inwards and outwards in the halo and the disk, respectively.

4. In the disk a superposition of axisymmetric (m = 0) and bisymmetric (m = 1) magnetic modes provides a satisfactory fit to the observations. The m = 1 mode slightly dominates at 3 < r < 6 kpc and the two modes have about equal amplitudes at 6 < r < 15 kpc. The field is predominantly horizontal between 3 and 12 kpc and has a weak vertical component at 12 < r < 15 kpc.

In the rings between 3 and 9 kpc the field structure in the disk is strongly non-axisymmetric with a field maximum in the eastern part of the galaxy and a weak field in the western part. Details are given in Table 4 and shown in Fig. 8.

5. The magnetic field pattern in the disk of M 51 shows a discontinuity at r ≃ 9 kpc, the position at which the inner and the outer spiral structure join (Elmegreen et al. 1989). The relatively strong, coherent magnetic field in the inner rings occurs in the system of spiral arms excited by density waves, whereas the weaker and partly distorted field in the outer rings exists in the area of the material spiral arms produced by the encounter with the companion.

6. The azimuthally averaged strength of the regular magnetic field obtained for the disk decreases from about 7 µG at radius 3-6 kpc to about 4 µG at 12–15 kpc (see Fig. 7). The strength of the regular field in the halo decreases from about 3 µG at 3-6 kpc to zero beyond 9 kpc. Details are given in Table 5.

7. The azimuthal averages of the regular magnetic field strength in the disk obtained from our fits are in good agreement with independent estimates from the total synchrotron emission and the degree of polarization. The radial scale length of the regular magnetic field is 14 ± 6 kpc.

8. We compared the pitch angles of the regular magnetic field obtained from the fits with those of the dust lanes delineating spiral arms for the sectors they have in common. Their mean values agree to within the errors.

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Appendix A: the intrinsic polarization angle

Below we give a detailed derivation of the intrinsic polarization angle ψ0 as a function of θ and parameters of the magnetic field. The magnetic field is specified by Eq. (1), the azimuthal angle θ is measured counterclockwise in the galaxy plane from the northern end of the major axis of the galaxy, ψ0 is measured in the sky plane from the local outward radial direction in the galaxy.

Let us introduce a Cartesian reference frame (x, y) lying in the galaxy plane with the origin at the galaxy’s center and another one, (x′, y′) lying in the sky plane; the x-axes of both frames point to the northern end of the major axis. Correspondingly, we introduce also the azimuthal angle in the sky plane, θ′.

It can be easily seen that θ and θ′ are related through tan θ′ = y′/x′ = tan θ cos i since y′ = y cos i and x′ = x, with i being the galaxy inclination angle (i = 0 corresponds to the face-on view). Insofar as the transverse component of the magnetic field (with respect to the line of sight) is represented by B⊥ = (Bx, By) and the intrinsic polarization angle ψ0 is orthogonal to B⊥, we have

\[ ψ_0 + θ' - \arctan B_y/B_x = \frac{1}{2} \pi, \]

or

\[ ψ_0 = \frac{1}{2} \pi - \arctan(\cos i \tan θ) + \arctan B_y/B_x. \] (A1)

Now we have

\[ B_x = B_x, \]

\[ = B_y \cos θ - B_θ \sin θ, \]

\[ B_y = B_y \sin i + B_z \cos i, \]

\[ = (B_y \sin θ + B_θ \cos θ) \cos i + B_z \sin i, \]

for the transverse field and

\[ B_\parallel = -B_y \sin i + B_z \cos i, \]

\[ = -(B_y \sin θ + B_θ \cos θ) \sin i + B_z \cos i \] (A2)

for the longitudinal field, where the direction to the observer is adopted as a positive direction of B_|| in accordance with the standard definition of the Faraday rotation measure. Here the inclination angle i is measured from the galaxy’s rotation axis to the line of sight. As the left-hand side of the image of M 51 is closer to the observer, this direction is clockwise when seen from the northern end of the major axis (i.e., the point from which θ is measured). We thus have i = −20° with our definition.

Using the above expressions for B_x and B_y in terms of the cylindrical polar components B_r, B_θ and B_z, we obtain from Eq. (A1):

\[ ψ_0 = \arctan \left\{ \frac{1}{2} \left( \frac{B [\sin(2θ - p) \sin^2 i - (1 + \cos^2 i) i \sin p]}{B \cos p \cos i + B_z \sin i \cos θ} - \frac{1}{2} B \cos p \cos i + B_z \sin i \cos θ \right) \right\}, \] (A3)
The criterion of quality, or reliability of a fit to the observational data is provided by the following dimensionless sum known as the residual:

\[ S = \sum_{n=1}^{N_N} S_n \]

\[ = \sum_{n=1}^{N_N} \sum_{i=1}^{N_1} \left[ \psi_{ni} - \psi_{ni}(\theta_i) \right]^2, \tag{B1} \]

where \( n \) enumerates the wavelengths at which observations have been carried out, of the total number \( N_N \), and \( i \) refers to individual sectors, \( N_1 \) of them per ring; \( \psi_{ni} \) is the set of polarization angles measured at the wavelength \( \lambda_n \), \( \psi_{ni}(\theta_i) \) is the value of the fitting function of the form specified by Eqs. (4)-(7), (A2) and (A3) for \( \theta = \theta_i \); and \( \sigma_{ni} \) are the uncertainties of the polarization angle values \( \psi_{ni} \) discussed in Sect. 2.

The representation (5) for the Faraday rotation measure was applied only for \( r = 3-12 \) kpc, where the disk is not transparent for the polarized emission at the longer wavelengths. For \( r = 9-15 \) kpc the fitting was performed for a one-layer model.

The fit is considered to be satisfactory if the following two conditions are fulfilled: the \( \chi^2 \) test

\[ S \leq \chi^2_{N-k}(\Psi), \tag{B2} \]

and, for any \( n \) and \( l \), the Fisher test

\[ S_n/(N_n - k) \leq F(N_n - k, N_l - k, \Psi), \tag{B3} \]

where \( \chi^2_{N-k}(\Psi) \) is the \( \chi^2 \) distribution with \( N = \sum_n N_n \) the total number of measurements, \( k \) the number of independent parameters of the model, \( \Psi \) the confidence level (\( \Psi = 0.95 \) corresponds to a 2\( \sigma \) error of a Gaussian random variable), and \( F(N_n - k, N_l - k, \Psi) \) is the Fisher distribution with \( N_n \) and \( N_l \) being the numbers of measurements at different wavelengths in a given ring.

Since the residual \( S \) is a strongly nonlinear function of its arguments, the estimation of the uncertainties of the magnetic field parameters resulting from the fit becomes also more complicated in comparison with the earlier linear models. The inequality Eq. (B2) defines a region in the \( k \)-dimensional parameter space where the values reside of the parameters that are considered admissible. For a quadratic function \( S \) typical of the linear models, this region is an ellipsoid and the uncertainties of the parameters are determined by the sizes of the ellipsoid along the corresponding axes. They can be expressed through the diagonal terms of the matrix \( \partial^2 S/\partial x_i \partial x_j \), with \( x_i \) the parameters of the model. On the contrary, for the present nonlinear model the above region has a very complicated shape that may differ drastically from an ellipsoid. Therefore, the above estimation of the uncertainties in terms of the second derivative matrix usually leads to strongly underestimated values. In the case presented in Fig. B1, this happens because the minimum point, marked by a cross, is far from the “centre” of the admissible region marked by the hatched line.

We thus used two additional estimates to characterize the uncertainties (see Fig. B1). The first one is the distance \( \Delta_T \), from the minimum point, to the border of the region defined by Eq. (B2) as measured along the axis corresponding to a given parameter. The second estimate of the uncertainty has been obtained as follows. When searching for the minimum of \( S \), we apply an iterative procedure starting with certain initial conditions which results in a sequence of parameter values, or a trajectory that converges to the final estimate corresponding to the minimum of \( S \). At a certain step of the iterations, the trajectory intersects the border of the region defined by Eq. (B2); after that, the trajectory can be quite complicated and tangled within the region but never leaves it. Thus the second estimate of the uncertainty of the final fit results, \( \Delta_T \), is provided by the lengths of the projections of the trajectory segment within the admissible region onto the corresponding axes (see Fig. B1). In Table 4 we adopted for the uncertainties the maximum of these three estimates. Insofar as the confidence level of the \( \chi^2 \) test was adopted as 95\%, the resulting uncertainties correspond, in a certain restricted sense, to a 2\( \sigma \) deviation of a Gaussian random variable.
Appendix C: basic notation

\begin{align*}
B & \quad \text{total strength of magnetic field, including both regular and turbulent components} \\
B_{\text{reg}} & \quad = (B_x^2 + B_y^2 + B_z^2)^{1/2}, \text{ strength of the regular magnetic field} \\
B_{\theta \phi z} & \quad \text{radial, azimuthal and vertical components of the regular magnetic field} \\
B_{\text{tur}} & \quad \text{turbulent (random) magnetic field} \\
B_0 & \quad \text{the amplitude of the horizontal component of the axisymmetric (} m = 0 \text{) mode} \\
B_1 & \quad \text{the amplitude of the horizontal component of the bisymmetric (} m = 1 \text{) mode} \\
B_{00} & \quad \text{the amplitude of the vertical component of the axisymmetric (} m = 0 \text{) mode} \\
B_{11} & \quad \text{the amplitude of the vertical component of the bisymmetric (} m = 1 \text{) mode} \\
B_i & \quad \text{line-of-sight component of the regular magnetic field} \\
B_{\perp} & \quad \text{transverse (with respect to the line-of-sight) component of the regular magnetic field} \\
d & \quad \text{correlation length of } B_{\text{tur}} \\
\lambda \text{ (D)} & \quad \text{superscript referring to quantities in the thermal disk} \\
\lambda \text{ (H)} & \quad \text{superscript referring to quantities in the halo} \\
\lambda_{\text{sym}} & \quad \text{exponential scale height of the synchrotron disk} \\
\lambda_{\text{th}} & \quad \text{exponential scale height of the disk of thermal electrons} \\
\lambda_{2.8} & \quad \text{exponential scale height of the synchrotron disk at } \lambda_{2.8}\text{ cm} \\
\lambda_{20.5} & \quad \text{exponential scale height of the synchrotron disk at } \lambda_{20.5}\text{ cm} \\
i & \quad \text{galaxy’s inclination angle (} i = 0^\circ \text{ corresponds to the face-on view)} \\
I & \quad \text{nonthermal flux density per beam area} \\
k & \quad \text{number of independent parameters of the model} \\
L & \quad \text{the line-of-sight depth of a region where both Faraday rotation and synchrotron emission originate} \\
m & \quad \text{azimuthal wave number} \\
n_e & \quad \text{volume density of thermal electrons} \\
N & \quad \text{the total number of measurements per ring} \\
N_n & \quad \text{the number of measurements at the } n^{\text{th}} \text{ wavelength} \\
N_\lambda & \quad \text{number of wavelengths involved in analysis} \\
p & \quad \text{pitch angle of the total horizontal magnetic field, } \arctan(B_x/B_y) \\
p_0 & \quad \text{pitch angle of the axisymmetric horizontal magnetic field} \\
p_1 & \quad \text{pitch angle of the bisymmetric horizontal magnetic field} \\
P & \quad \text{degree of polarization at given } \lambda \\
P_0 & \quad \approx 0.75 \text{ intrinsic degree of polarization} \\
r & \quad \text{cylindrical radius} \\
\mathcal{R} & \quad = (2\pi)^{-1} \int_0^{2\pi} \mathcal{R}_{\theta i} d\theta, \text{ the strength of the regular magnetic field averaged over azimuth in a ring} \\
\mathcal{R}_{\theta i} & \quad \text{azimuthal Fourier coefficients defined for each Fourier component of the magnetic field, } B_i = (B_0, B_1, B_{01}, B_{11}), \text{ with } h = h_{\text{th}} \text{ in the disk and } h = Z \text{ in the halo} \\
\mathcal{R}_{\text{in}} & \quad \text{depolarization due to internal Faraday dispersion} \\
\mathcal{R}_{\text{ex}} & \quad \text{depolarization due to external Faraday dispersion} \\
\mathcal{R}_{\text{reg}} & \quad \text{depolarization caused by differential Faraday rotation due to the regular magnetic field} \\
\mathcal{F} & \quad \text{the Fisher test - see Eq. (B3)} \\
\mathcal{T} & \quad \text{the residual - see Eq. (B1)} \\
\sigma_{\text{RM}} & \quad \text{fraction of RM(D) actually produced within a layer visible in polarized emission at a given wavelength} \\
\sigma_{\text{RM}} & \quad \text{fraction of RM(H) actually produced in the halo} \\
\sigma_{\text{RM}} & \quad \text{standard deviation of polarization angle in the } i^{\text{th}} \text{ sector at the } n^{\text{th}} \text{ wavelength} \\
\xi & \quad \text{azimuthal angle measured counterclockwise from the northern major axis in the plane of the galaxy} \\
\xi_0 & \quad \text{azimuthal angle measured in the plane of the sky} \\
\xi_{\text{sym}} & \quad \text{phase of the azimuthal distribution of thermal electrons} \\
\xi_{\text{th}} & \quad \text{phase of the horizontal bisymmetric field} \\
\Delta \psi & \quad \text{phase of the azimuthal distribution of thermal electrons} \\
\Delta \zeta & \quad \text{phase of the vertical bisymmetric field} \\
\psi & \quad \text{azimuthal angle measured counterclockwise from the northern major axis in the plane of the galaxy} \\
\psi_i & \quad \text{value of the fitting function for the } i^{\text{th}} \text{ sector at the } n^{\text{th}} \text{ wavelength} \\
\psi_0 & \quad \text{intrinsic polarization angle measured counterclockwise from the local radial direction} \\
\chi^2 & \quad \text{the } \chi^2 \text{ test - see Eq. (B2)} \\
\langle \ldots \rangle & \quad \text{volume averaging} \\
\langle \ldots \rangle & \quad \text{observed dispersion in RM} \\
\langle \ldots \rangle & \quad \text{polarization angle of the polarized emission, i.e. the position of the electric vector, measured counterclockwise from the local radial direction in the galaxy’s plane} \\
\langle \ldots \rangle & \quad \text{averaged polarization angle in the } i^{\text{th}} \text{ sector at the } n^{\text{th}} \text{ wavelength, measured counterclockwise from the local radial direction in the galaxy’s plane} \\
\langle \ldots \rangle & \quad \text{volumer averaging}
\end{align*}
References

Baryshnikova Y., Ruzmaikin A., Sokoloff D., Shukurov A., 1987, A&A 177, 27
Beck R., 1991, A&A 251, 15
Beck R., 1993. In: Krause F., Rädler K.-H., Rüdiger G. (eds.) Proc. IAU Symp. 157, The Cosmic Dynamo. Kluwer, Dordrecht, p. 283
Beck R., Hoernes P., 1996, Nat 397, 47
Beck R., Carilli C.L., Holdaway M.A., Klein U., 1994, A&A 292, 409
Beck R., Brandenburg A., Moss D., Shukurov A., Sokoloff D., 1996, ARAA 34, 153
Berkhuijsen E.M., Golla G., Beck R., 1991. In: Bloemen H. (ed.) Proc. IAU Symp. 144, The Interstellar Disk-Halo Connection in Galaxies. Kluwer, Dordrecht, p. 233
Beuermann K., Kanbach G., Berkhuijsen E.M., 1985, A&A 153, 17
Brandenburg A., Donner K.J., Moss D., Shukurov A., Sokoloff D.D., Tuominen I., 1992, A&A 259, 453
Brandenburg A., Moss D., Shukurov A., 1995, MNRAS 276, 651
Burn B.J., 1966, MNRAS 133, 67
Bykov A.A., Popov V.Yu., Shukurov A., Sokoloff D.D., 1997, MNRAS (submitted)
Dickey J.M., Lockman F.J., 1990, ARA&A 28, 215
Dumke M., Krause M., Wielebinski R., Klein U., 1995, A&A 302, 691
Ehle M., Pietsch W., Beck R., 1995, A&A 295, 289
Ehle M., Beck R., Haynes R.F., Vogler A., Pietsch W., Elmouttie M., Ryder S., 1996, A&A 306, 73
Elmegreen B.G., Elmegreen D.M., Seiden P.E., 1989, ApJ 343, 602
Ford H., Crane P., Jacoby G., Laurie D., 1985, ApJ 293, 132
Güsten R., Mezger, P.G., 1983, Vistas in Astronomy 26, 159
Henderson A.P., Jackson P.D., Kerr F.J., 1982, ApJ 263, 116
Horellou C., 1990, Diploma Thesis, University of Bonn
Horellou C., Beck R., Berkhuijsen E.M., Krause M., Klein U., 1992, A&A 265, 417
Howard S., Byrd G.G., 1990, AJ 99, 1798
Hummel E., Dahlem M., van der Hulst J.M., Sukumar S., 1991a, A&A 246, 10
Hummel E., Beck R., Dahlem M., 1991b, A&A 248, 23
Israel F.P., Habing H., de Jong T., 1973, A&A 27, 143
Klein U., Wielebinski R., Beck R., 1984, A&A 135, 213
Krasheninnikova Y., Ruzmaikin A., Sokoloff D., Shukurov A., 1989, A&A 213, 19
Krause M., 1990. In: Beck R., Kronberg P.P., Wielebinski R. (eds.) Proc. IAU Symp. 140, Galactic and Intergalactic Magnetic Fields. Kluwer, Dordrecht, p. 187
Krause M., 1993. In: Krause F., Rädler K.-H., Rüdiger G. (eds.) Proc. IAU Symp. 157, The Cosmic Dynamo. Kluwer, Dordrecht, p. 305
Krause M., Beck R., Klein U., 1984, A&A 138, 385
Krause M., Hummel E., Beck R., 1989a, A&A 217, 4
Krause M., Beck R., Hummel E., 1989b, A&A 217, 17
Mathewson D.S., van der Kruit P.C., Brouw W.N., 1972, A&A 17, 408
Mezger P.G., Henderson A.P., 1967, ApJ 147, 471
Neininger N., 1992a, A&A 263, 30
Neininger N., 1992b, PhD Thesis, University of Bonn
Neininger N., Horellou C., 1996. In: Roberge W.G., Whittet D.C.B. (eds.) Polarimetry of the Interstellar Medium. ASP Conf. Ser. Vol. 97, 592
Neininger N., Klein U., Beck R., Wielebinski R., 1991, Nat 352, 781
Neininger N., Horellou, C., Beck, R., Berkhuijsen, E., Krause, M., Klein, U., 1993a. In: Krause F., Rädler K.-H., Rüdiger G. (eds.) Proc. IAU Symp. 157, The Cosmic Dynamo. Kluwer, Dordrecht, p. 313
Neininger N., Beck R., Sukumar S., Allen R.J., 1993b, A&A 274, 687
Ohno H., Shibata S., 1993, MNRAS 262, 953
Reich P., Reich W., 1988, A&A 196, 211
Reynolds R.J., 1991a. In: Bloemen H. (ed.) Proc. IAU Symp. 144, The Interstellar Disk-Halo Connection in Galaxies. Kluwer, Dordrecht, p. 67
Reynolds R.J., 1991b, ApJ 372, L17
Rickett B.J., 1990, ARA&A 28, 561
Ruzmaikin A.A., Shukurov A.M., Sokoloff D.D., 1988, Magnetic Fields of Galaxies. Kluwer Acad. Publ., Dordrecht
Ruzmaikin A., Sokoloff D., Shukurov A., Beck R., 1990, A&A 230, 284
Sandage A., Tammann G.A., 1974, ApJ 194, 559
Segalovitz A., Shane W.W., de Bruyn A.G., 1976, Nat 264, 222
Sokoloff D., Shukurov A., 1990, Nat 347, 51
Sokoloff D., Shukurov A., Krause M., 1992, A&A 264, 396
Sukumar S, Allen R.J., 1991, ApJ 382, 100
Tosa M., Fujimoto M., 1978, PASJ 30, 315
Tribble P.C., 1991, MNRAS 250, 726
Tully R.B., 1974, ApJS 27, 415
Wardle J.F.C., Kronberg P.P., 1974, ApJ 194, 249

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