Vanishing Effective Mass of the Neutrinoless Double Beta Decay?

Zhi-zhong Xing

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China and Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918 (4), Beijing 100039, China *
(Electronic address: xingzz@mail.ihep.ac.cn)

Abstract

We stress the point that massive neutrinos may be Majorana particles even if the effective mass of the neutrinoless double beta decay $\langle m \rangle_{ee}$ vanishes. We show that current neutrino oscillation data do allow $\langle m \rangle_{ee} = 0$ to hold, if the Majorana CP-violating phases lie in two specific regions. Strong constraints on the neutrino mass spectrum can then be obtained. A possible texture of the neutrino mass matrix is also illustrated under the $\langle m \rangle_{ee} = 0$ condition.

PACS number(s): 14.60.Pq, 13.10.+q, 25.30.Pt
The recent SK [1], SNO [2], KamLAND [3] and K2K [4] experiments have provided us with very convincing evidence that the solar and atmospheric neutrino anomalies are both due to neutrino oscillations. The occurrence of neutrino oscillations implies that neutrinos are massive and lepton flavors are mixed. If neutrinos are Majorana particles, a complete parametrization of the $3 \times 3$ lepton flavor mixing matrix requires three mixing angles, one Dirac-type CP-violating phase and two Majorana-type CP-violating phases [5]. While three mixing angles have been determined or constrained by current neutrino oscillation data to an acceptable degree of accuracy, three CP-violating phases are entirely unrestricted. It is expected that the Dirac phase can be measured from CP- or T-violating effects in the long-baseline neutrino oscillation experiments [6]. To measure two Majorana phases is extremely difficult, because all possible lepton-number-nonconserving processes induced by light Majorana neutrinos are strongly suppressed in magnitude [7].

The most sensitive way to get some information on two Majorana CP-violating phases is to detect the neutrinoless double beta decay of some even-even nuclei,

$$A(Z, N) \rightarrow A(Z + 2, N - 2) + 2e^-, \quad (1)$$

which can occur through the exchange of a Majorana neutrino between two decaying neutrons inside a nucleus, as illustrated in Fig. 1. It would be forbidden, however, if neutrinos were Dirac particles. Thus the neutrinoless double beta decay provides us with a unique opportunity to identify the Majorana nature of massive neutrinos. The rate of the neutrinoless double beta decay is proportional to an effective neutrino mass term, defined as

$$\langle m \rangle_{ee} = \left| m_1 V_{e1}^2 + m_2 V_{e2}^2 + m_3 V_{e3}^2 \right|, \quad (2)$$

where $m_i$ (for $i = 1, 2, 3$) denote the physical masses of three neutrinos, and $V_{ei}$ stand for the elements in the first row of the $3 \times 3$ lepton flavor mixing matrix $V$. It is obvious that $\langle m \rangle_{ee} = 0$ would trivially hold, if $m_i = 0$ were taken.

While $\langle m \rangle_{ee} \neq 0$ must imply that neutrinos are Majorana particles, $\langle m \rangle_{ee} = 0$ does not necessarily imply that neutrinos are Dirac particles. The reason is simply that the Majorana phases hidden in $V_{ei}$ may lead to significant cancellations on the right-hand side of Eq. (2), making $\langle m \rangle_{ee}$ vanishing or too small to be detectable [8]. Hence much care has to be taken, if no convincing signal of the neutrinoless double beta decay can be experimentally established 1: it may imply that (1) the experimental sensitivity is not high enough; (2) the massive neutrinos are Dirac particles; or (3) the vanishing or suppression of $\langle m \rangle_{ee}$ is due to large cancellations induced by the Majorana CP-violating phases. The third possibility is certainly interesting and important [12], and it deserves to be carefully examined from a model-independent point of view and with the help of the latest experimental data.

The main purpose of this paper is to find out the parameter space of two Majorana phases in the case of $\langle m \rangle_{ee} = 0$. We demonstrate that current neutrino oscillation data do allow

---

1Klapdor-Kleingrothaus et al. have recently reported the first evidence for the neutrinoless double beta decay [9]. However, their result was criticized by some authors [10]. Future experiments will have sufficiently high sensitivity to clarify the present debates [11], to confirm or to disprove the alleged result in Ref. [9].
Our numerical analysis will show that the possibility \( m_{ee} = 0 \) to hold, if the Majorana phases lie in two specific regions. Very strong constraints on three neutrino masses can then be obtained. We find that the neutrino mass spectrum performs a normal hierarchy: \( m_1 < m_2 < m_3 \). Finally, we present some brief discussions about how to recast the texture of the neutrino mass matrix under the \( m_{ee} = 0 \) condition.

It is clear in Eq. (2) that only the flavor mixing matrix elements \( V_{e1}, V_{e2} \) and \( V_{e3} \) are relevant to the effective mass of the neutrinoless double beta decay. Without loss of generality, one may redefine the phases of three charged lepton fields in an appropriate way such that the phases of \( V_{e1} \) and \( V_{e2} \) are purely of the Majorana type and \( V_{e3} \) is real [13]. In other words,

\[
\arg(V_{e1}) = \rho, \quad \arg(V_{e2}) = \sigma, \quad \arg(V_{e3}) = 0. \tag{3}
\]

Note that \( \rho \) and \( \sigma \) have nothing to do with CP and T violation in normal neutrino oscillations. Taking account of Eqs. (2) and (3), we find that \( m_{ee} = 0 \) requires

\[
\begin{align*}
&m_1|V_{e1}|^2 \sin 2\rho + m_2|V_{e2}|^2 \sin 2\sigma = 0, \\
&m_1|V_{e1}|^2 \cos 2\rho + m_2|V_{e2}|^2 \cos 2\sigma + m_3|V_{e3}|^2 = 0.
\end{align*} \tag{4}
\]

These two conditions, together with current experimental data on the flavor mixing matrix elements (\(|V_{e1}|, |V_{e2}| \) and \(|V_{e3}|\)) and the mass-squared differences of solar and atmospheric neutrino oscillations (\( \Delta m^2_{\text{sun}} \equiv |m_2^2 - m_1^2| \) and \( \Delta m^2_{\text{atm}} \equiv |m_3^2 - m_2^2| \)), allow us to determine or constrain both the masses of three neutrinos (\( m_1, m_2 \) and \( m_3 \)) and the Majorana phases of CP violation (\( \rho \) and \( \sigma \)). More specific discussions about the consequences of Eq. (4) are in order.

(a) Note that the present solar neutrino data support \( 0 \leq m_1 < m_2 \) [2,3]. If \( m_1 = 0 \) holds, then Eq. (4) requires \( \sigma = (2n + 1)\pi/2 \) with \( n = 0, 1, 2 \cdots \) for arbitrary values of \( \rho \). In this case, we immediately obtain

\[
\frac{m_2}{m_3} = \frac{|V_{e3}|^2}{|V_{e2}|^2}, \quad \frac{\Delta m^2_{\text{atm}}}{\Delta m^2_{\text{sun}}} = \frac{|V_{e3}|^4}{|V_{e2}|^4 - |V_{e3}|^4}. \tag{5}
\]

Because of \( |V_{e3}| < |V_{e2}| \), the neutrino mass spectrum performs a normal hierarchy (i.e., \( m_3 > m_2 > m_1 = 0 \)). The absolute values of \( m_2 \) and \( m_3 \) are given by

\[
m_2 = \frac{|V_{e3}|^2 \sqrt{\Delta m^2_{\text{atm}}}}{|V_{e2}|^4 - |V_{e3}|^4}, \quad m_3 = \frac{|V_{e2}|^2 \sqrt{\Delta m^2_{\text{atm}}}}{|V_{e2}|^4 - |V_{e3}|^4}. \tag{6}
\]

Our numerical analysis will show that the possibility \( m_1 = m_{ee} = 0 \) is actually allowed by current neutrino oscillation data.

(b) Whether \( 0 < m_2 < m_3 \) or \( 0 \leq m_3 < m_2 \) holds remains an open question. If \( m_3 = 0 \) held, then Eq. (4) would require \( |\rho - \sigma| = (2n + 1)\pi/2 \) with \( n \) being an arbitrary integer. In this case, we would be led to

\[
\frac{m_1}{m_2} = \frac{|V_{e2}|^2}{|V_{e1}|^2}, \quad \frac{\Delta m^2_{\text{atm}}}{\Delta m^2_{\text{sun}}} = \frac{|V_{e1}|^4 - |V_{e2}|^4}{|V_{e1}|^4}. \tag{7}
\]

as well as
\[ m_1 = \frac{|V_{e2}|^2 \sqrt{\Delta m_{\text{sun}}^2}}{\sqrt{|V_{e1}|^4 - |V_{e2}|^4}}, \quad m_2 = \frac{|V_{e1}|^2 \sqrt{\Delta m_{\text{sun}}^2}}{\sqrt{|V_{e1}|^4 - |V_{e2}|^4}}. \] (8)

Our numerical analysis will show that the possibility \( m_3 = \langle m \rangle_{ee} = 0 \) has definitely been ruled out by current neutrino oscillation data.

(c) If \( m_1 \neq 0 \) but \( \sin 2\rho = 0 \) or \( \sin 2\sigma = 0 \) holds, then Eq. (4) requires \( \rho = n\pi \) and \( \sigma = (2n' + 1)\pi/2 \) with

\[ m_1 |V_{e1}|^2 + m_3 |V_{e3}|^2 = m_2 |V_{e2}|^2; \] (9)

or \( \rho = (2n + 1)\pi/2 \) and \( \sigma = n'\pi \) with

\[ m_2 |V_{e2}|^2 + m_3 |V_{e3}|^2 = m_1 |V_{e1}|^2; \] (10)

or \( \rho = (2n + 1)\pi/2 \) and \( \sigma = (2n' + 1)\pi/2 \) with

\[ m_1 |V_{e1}|^2 + m_2 |V_{e2}|^2 = m_3 |V_{e3}|^2, \] (11)

where \( n \) and \( n' \) are arbitrary integers. With the help of

\[ m_1 = \sqrt{m_3^2 \pm \Delta m_{\text{atm}}^2 + \Delta m_{\text{sun}}^2}, \]
\[ m_2 = \sqrt{m_3^2 \pm \Delta m_{\text{atm}}^2}, \] (12)

\( m_3 \) can be solved from Eq. (9), (10) or (11). Thus the spectrum of three neutrino masses is fully determinable. Our numerical analysis will show that Eqs. (9) and (10) are consistent quite well with current neutrino oscillation data. In comparison, Eq. (11) is also allowed but it is less favored.

(d) Besides the special cases considered above, more general results for \((m_1, m_2, m_3)\) and \((\rho, \sigma)\) can be obtained from Eq. (4). Indeed,

\[ \frac{m_1}{m_2} = \frac{|V_{e2}|^2}{|V_{e1}|^2} \cdot \frac{\sin 2\sigma}{\sin 2\rho}, \]
\[ \frac{m_2}{m_3} = \frac{|V_{e3}|^2}{|V_{e2}|^2} \cdot \frac{\sin 2\rho}{\sin 2(\sigma - \rho)}. \] (13)

Since \( 0 \leq m_1/m_2 < 1 \) and \( 0 < m_2/m_3 \) hold, part of the \((\rho, \sigma)\) parameter space must be excluded. Furthermore, we arrive at

\[ \frac{\Delta m_{\text{sun}}^2}{\Delta m_{\text{atm}}^2} = \frac{|V_{e3}|^4}{|V_{e1}|^4} \cdot \frac{||V_{e1}|^4 \sin^2 2\rho - |V_{e2}|^4 \sin^2 2\sigma||}{||V_{e2}|^4 \sin^2 2(\sigma - \rho) - |V_{e3}|^4 \sin^2 2\rho||}. \] (14)

Note that the lepton flavor mixing matrix elements \(|V_{e1}|^2, |V_{e2}|^2, |V_{e3}|^2, |V_{\mu 3}|^2\) and \(|V_{\tau 3}|^2\) are associated respectively with the mixing factors of solar, atmospheric and CHOOZ [14] reactor neutrino oscillations,
\[\sin^2 2\theta_{\text{sun}} = 4|V_{e1}|^2|V_{e2}|^2,\]
\[\sin^2 2\theta_{\text{atm}} = 4|V_{\mu 3}|^2 \left(1 - |V_{\mu 3}|^2\right),\]
\[\sin^2 2\theta_{\text{chz}} = 4|V_{e3}|^2 \left(1 - |V_{e3}|^2\right).\] (15)

Reversely, we have [15]
\[|V_{e1}|^2 = \frac{1}{2} \left(\cos^2 \theta_{\text{chz}} + \sqrt{\cos^4 \theta_{\text{chz}} - \sin^2 2\theta_{\text{sun}}}\right),\]
\[|V_{e2}|^2 = \frac{1}{2} \left(\cos^2 \theta_{\text{chz}} - \sqrt{\cos^4 \theta_{\text{chz}} - \sin^2 2\theta_{\text{sun}}}\right),\]
\[|V_{e3}|^2 = \sin^2 \theta_{\text{chz}},\]
\[|V_{\mu 3}|^2 = \sin^2 \theta_{\text{atm}},\]
\[|V_{\tau 3}|^2 = \cos^2 \theta_{\text{chz}} - \sin^2 \theta_{\text{atm}}.\] (16)

In view of the recent SK [1], SNO [2], KamLAND [3], K2K [4] and CHOOZ [14] data on neutrino oscillations, we have \(\Delta m^2_{\text{atm}} \in [5.9, 8.8] \times 10^{-5}\) eV\(^2\), \(\sin^2 2\theta_{\text{atm}} \in [0.25, 0.40]\) [16]; \(\Delta m^2_{\text{sun}} \in [1.65, 3.25] \times 10^{-3}\) eV\(^2\), \(\sin^2 2\theta_{\text{sun}} \in [0.88, 1.00]\) [17]; and \(\sin^2 2\theta_{\text{chz}} < 0.2\) at the 90\% confidence level. With the help of these experimental results, the allowed ranges of \(\rho\) and \(\sigma\) can then be obtained from Eqs. (14) and (16).

We plot the parameter space of two Majorana phases in Fig. 2(a). It becomes obvious that \(\rho\) and \(\sigma\) may take many nontrivial values, which guarantee \(\langle m\rangle_{ee} = 0\). This important point has not been observed before. Note that the \((\rho, \sigma)\) parameter space in Fig. 2(a) can be generalized to the larger \((\rho \pm n_1 \pi, \sigma \pm n_2 \pi)\) parameter space, where \(n_1\) and \(n_2\) are arbitrary integers. For illustration, we typically pick
\[(\rho, \sigma) = \left(\frac{\pi}{4}, \frac{2\pi}{3}\right) \quad \text{or} \quad \left(\frac{3\pi}{4}, \frac{\pi}{3}\right)\] (17)
from Fig. 2(a). Then we arrive at
\[\frac{m_1}{m_2} = \frac{\sqrt{3}|V_{e2}|^2}{2|V_{e1}|^2}, \quad \frac{m_2}{m_3} = \frac{2|V_{e3}|^2}{|V_{e2}|^2}.\] (18)

Given the ranges of \(\theta_{\text{sun}}\) and \(\theta_{\text{chz}}\) favored by current experimental data, \(m_1 < m_2\) and \(m_2 < m_3\) are found to hold from Eq. (18). Thus three neutrino masses perform a normal hierarchy in this specific but interesting case.

A detailed analysis of the \((m_1/m_2, m_2/m_3)\) parameter space is shown in Fig. 2(b). We find that two specific regions of \(\rho\) and \(\sigma\) in Fig. 2(a) correspond to a common region of \(m_1/m_2\) and \(m_2/m_3\) in Fig. 2(b), just like the specific case illustrated in Eqs. (17) and (18). One can see that both \(0 \leq m_1/m_2 < 1\) and \(0 < m_2/m_3 < 1\) hold, thus three neutrino masses have a normal hierarchy (i.e., \(m_1 < m_2 < m_3\)). When \(m_1/m_2\) approaches 1, \(m_2/m_3\) is somehow close to 1 too. In this case, which is not very likely, three neutrino masses are nearly degenerate (i.e., \(m_1 \approx m_2 \approx m_3\)). Note that \(m_2/m_3\) and \(|V_{e3}|\) have minimal values \((m_2/m_3)_{\text{min}} \approx 0.135\) and \(|V_{e3}|_{\text{min}} \approx 0.0695\), respectively. The latter, which corresponds to \((\theta_{\text{chz}})_{\text{min}} \approx 4^\circ\) or \((\sin^2 2\theta_{\text{chz}})_{\text{min}} \approx 0.02\), is associated with the prerequisite \(\langle m\rangle_{ee} = 0\). In
contrast, we do not find any restriction on the input and output values of $\theta_{\text{sun}}$ or $\theta_{\text{atm}}$ in our numerical calculation.

We see that $m_2/m_3$ is most likely to be around 0.2, implying that $m_3 \approx \sqrt{\Delta m_{\text{atm}}^2} \sim 0.05$ eV should be a good approximation. In this case, the effective mass of the tritium beta decay $\langle m \rangle_e$ is too small to be detected by the KATRIN experiment \[18\]. If $m_2/m_3 \sim 0.9$ is taken, one will get $m_3 \sim 0.1$ eV and $m_1 + m_2 + m_3 \sim 0.3$ eV, consistent with the recent WMAP data $m_1 + m_2 + m_3 < 0.71$ eV \[19\].

Let us proceed to discuss how to recast the neutrino mass matrix under the condition $\langle m \rangle_{ee} = 0$. In the flavor basis where the charged lepton mass matrix is diagonal, the neutrino mass matrix $M$ can be written as

$$M = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T. \tag{19}$$

The lepton flavor mixing matrix $V$ consists of three nontrivial CP-violating phases: the Dirac phase $\delta$ and the Majorana phases $\rho$ and $\sigma$. For simplicity, we assume $\delta = 0$ to examine the dependence of $M$ on $\rho$ and $\sigma$. Then $V$ may take the form

$$V = \begin{pmatrix} |V_{e1}| & |V_{e2}| & |V_{e3}| \\ -|V_{\mu1}| & |V_{\mu2}| & |V_{\mu3}| \\ |V_{\tau1}| & -|V_{\tau2}| & |V_{\tau3}| \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{20}$$

in which $|V_{e1}|$, $|V_{e2}|$, $|V_{e3}|$, $|V_{\mu3}|$ and $|V_{\tau3}|$ have been given in Eq. (16) in terms of $\theta_{\text{sun}}$, $\theta_{\text{atm}}$ and $\theta_{\text{cha}}$, and $|V_{\mu1}|$, $|V_{\mu2}|$, $|V_{\tau1}|$ and $|V_{\tau2}|$ read as follows (in the assumption of $\delta = 0$ \[15\]):

$$|V_{\mu1}| = \frac{|V_{e2}| |V_{\tau3}| + |V_{e1}| |V_{e3}| |V_{\mu3}|}{1 - |V_{e3}|^2},$$

$$|V_{\mu2}| = \frac{|V_{e1}| |V_{\tau3}| - |V_{e2}| |V_{e3}| |V_{\mu3}|}{1 - |V_{e3}|^2},$$

$$|V_{\tau1}| = \frac{|V_{e2}| |V_{\mu3}| - |V_{e1}| |V_{e3}| |V_{\tau3}|}{1 - |V_{e3}|^2},$$

$$|V_{\tau2}| = \frac{|V_{e1}| |V_{\mu3}| + |V_{e2}| |V_{e3}| |V_{\tau3}|}{1 - |V_{e3}|^2}. \tag{21}$$

With the help of Eqs. (19) and (21), six independent elements of the symmetric neutrino mass matrix $M$ can be expressed as

$$\begin{pmatrix} M_{ee} \\ M_{e\mu} \\ M_{e\tau} \\ M_{\mu\mu} \\ M_{\mu\tau} \\ M_{\tau\tau} \end{pmatrix} = m_1 \begin{pmatrix} |V_{e1}|^2 \\ -|V_{e1}| |V_{\mu1}| \\ |V_{e1}| |V_{\tau1}| \\ -|V_{\mu1}| |V_{\tau1}| \\ |V_{\mu1}|^2 \\ |V_{\tau1}|^2 \end{pmatrix} e^{2i\rho} + m_2 \begin{pmatrix} |V_{e2}|^2 \\ -|V_{e2}| |V_{\mu2}| \\ |V_{e2}| |V_{\tau2}| \\ -|V_{\mu2}| |V_{\tau2}| \\ |V_{\mu2}|^2 \\ |V_{\tau2}|^2 \end{pmatrix} e^{2i\sigma} + m_3 \begin{pmatrix} |V_{e3}|^2 \\ |V_{e3}| |V_{\mu3}| \\ |V_{e3}| |V_{\tau3}| \\ |V_{\mu3}|^2 \\ |V_{\tau3}|^2 \end{pmatrix}. \tag{22}$$

As $M_{ee} = \langle m \rangle_{ee}$ holds, we have $M_{ee} = 0$ under the condition $\langle m \rangle_{ee} = 0$. For illustration, we calculate the other five matrix elements in Eq. (22) by using the typical values of $\rho$ and
σ chosen in Eq. (17) and taking $\theta_{\text{sun}} = 33^\circ$, $\theta_{\text{atm}} = 45^\circ$ and $\theta_{\text{chz}} = 9^\circ$. The numerical result are $m_1/m_2 \approx 0.42$, $m_2/m_3 \approx 0.15$, and

$$M \approx m_3 \begin{pmatrix} 0 & 0.11 e^{\pm i 36^\circ} & 0.15 e^{\pm i 26^\circ} \\ 0.11 e^{\mp i 36^\circ} & 0.48 e^{\pm i 2.3^\circ} & 0.51 e^{\pm i 3.8^\circ} \\ 0.15 e^{\pm i 26^\circ} & 0.51 e^{\pm i 3.8^\circ} & 0.45 e^{\mp i 6.5^\circ} \end{pmatrix},$$

where $m_3 \sim 0.05$ eV. We see that this one-zero texture of $M$ does not perform an apparent hierarchy and has little similarity with those two-zero textures of $M$ illustrated in Ref. [20]. Of course, the form of $M$ in Eq. (23) will get modified, if the Dirac CP-violating phase of $V$ is switched on. A more general and delicate analysis of possible patterns of $M$ under the $\langle m \rangle_{ee} = 0$ condition will be done elsewhere.

It is worth mentioning that the $\langle m \rangle_{ee} = 0$ condition may be taken as a prerequisite to build models for the Majorana neutrino mass matrix $M$, if the neutrinoless double beta decay is unable to be detected. Another two empirical conditions, $\text{Det} M = 0$ [21] and $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (where $|\lambda_i| = m_i$ [22]), have also been discussed to constrain the form of $M$ in a phenomenological way. Such “zero” conditions are similar to those taken for the matrix elements of $M$ [20,23], in order to reduce the number of free parameters in $M$. This is one of a few realistic approaches [24], towards some deeper understanding of lepton mass generation and flavor mixing.

In summary, we have stressed the point that $\langle m \rangle_{ee} = 0$ does not necessarily indicate the Dirac nature of light neutrinos. Current neutrino oscillation data do allow $\langle m \rangle_{ee} = 0$ to hold, if neutrinos are Majorana particles and their two Majorana CP-violating phases lie in two specific regions. This observation will be important for model building, in order to completely understand or interpret current and future experimental data on the neutrino mass spectrum and the neutrinoless double beta decay.

I am grateful to C. Giunti and F. Vissani for useful comments and discussions, and to F. Vissani and W. Rodejohann for bringing a few relevant references to my attention. I am also indebted to W.L. Guo for his help in dealing with the figures. This work was supported in part by the National Natural Science Foundation of China.
REFERENCES

[1] For a review, see: C.K. Jung, C. McGrew, T. Kajita, and T. Mann, Ann. Rev. Nucl. Part. Sci. 51, 451 (2001).
[2] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89, 011301 (2002); Phys. Rev. Lett. 89, 011302 (2002).
[3] KamLAND Collaboration, K. Eguchi et al., Phys. Rev. Lett. 90, 021802 (2003).
[4] K2K Collaboration, M.H. Ahn et al., Phys. Rev. Lett. 90, 041801 (2003).
[5] H. Fritzsch and Z.Z. Xing, Phys. Lett. B 517, 363 (2001); and references therein.
[6] See, e.g., A. Blondel et al., Nucl. Instrum. Meth. A 451, 102 (2000); C. Albright et al., hep-ex/0008064; H. Fritzsch and Z.Z. Xing, Phys. Rev. D 61, 073016 (2000); G. Barenboim et al., hep-ex/0304017.
[7] V. Barger, S.L. Glashow, P. Langacker, and D. Marfatia, Phys. Lett. B 540, 247 (2002); A. de Gouvea, B. Kayser, and R.N. Mohapatra, Phys. Rev. D 67, 053004 (2003).
[8] S.M. Bilenky, S. Pascoli, and S.T. Petcov, Phys. Rev. D 64, 053010 (2001); W. Rodejohann, Nucl. Phys. B 597, 110 (2001); hep-ph/0203214; and references therein.
[9] H.V. Klapdor-Kleingrothaus, A. Dietz, H.L. Harney, and I.V. Krivosheina, Mod. Phys. Lett. A 16, 2409 (2002).
[10] F. Feruglio, A. Strumia, and F. Vissani, Nucl. Phys. B 637, 345 (2002); C.E. Aalseth et al., Mod. Phys. Lett. A 17, 1475 (2002); Y.G. Zdesenko, F.A. Danevich, and V.I. Tretyak, Phys. Lett. B 546, 206 (2002).
[11] H.V. Klapdor-Kleingrothaus, hep-ph/0302237; hep-ph/0303217; and references therein.
[12] L. Wolfenstein, in Proc. of Neutrino 84 (1984), p. 730; F. Vissani, JHEP 9906, 022 (1999); F. Feruglio, A. Strumia, and F. Vissani, in Ref. [10].
[13] Z.Z. Xing, Phys. Rev. D 65, 077302 (2002); Phys. Rev. D 65, 113010 (2002).
[14] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B 420, 397 (1998); Palo Verde Collaboration, F. Boehm et al., Phys. Rev. Lett. 84, 3764 (2000).
[15] W.L. Guo and Z.Z. Xing, Phys. Rev. D 67, 053002 (2003).
[16] V. Barger and D. Marfatia, hep-ph/0212126; G.L. Fogli et al., hep-ph/0212127; M. Maltoni, T. Schwetz, and J.W.F. Valle, hep-ph/0212129; J.N. Bahcall, M.C. Gonzalez-Garcia, and C. Pena-Garay, hep-ph/0212147; P. Aliani, V. Antonelli, M. Picariello, and E. Torrente-Lujan, hep-ph/0212212; P.C. de Holanda and A.Yu. Smirnov, hep-ph/0212270.
[17] G.L. Fogli, E. Lisi, A. Marrone, and D. Montanino, hep-ph/0303064.
[18] Z.Z. Xing, hep-ph/0303178.
[19] C.L. Bennett et al., astro-ph/0302207; D.N. Spergel et al., astro-ph/0302209.
[20] Z.Z. Xing, Phys. Lett. B 530, 159 (2002); Phys. Lett. B 539, 85 (2002).
[21] G.C. Branco, R. Felipe, F. Joaquim, and T. Yanagida, hep-ph/0212341.
[22] X.G. He and A. Zee, hep-ph/0302201.
[23] P.H. Frampton, S.L. Glashow, and D. Marfatia, Phys. Lett. B 536, 79 (2002); Z.Z. Xing, Phys. Lett. B 550, 178 (2002).
[24] For recent reviews with extensive references, see: H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. 45, 1 (2000); G. Altarelli and F. Feruglio, hep-ph/0206077, to appear in Neutrino Mass - Springer Tracts in Modern Physics, edited by G. Altarelli and K. Winter (2002).

8
FIG. 1. Illustrative plot for the neutrinoless double beta decay of some even-even nuclei via the exchange of a virtual Majorana neutrino.
FIG. 2. Implications of $\langle m \rangle_{ee} = 0$: the $(\rho, \sigma)$ and $(m_1/m_2, m_2/m_3)$ regions allowed by current neutrino oscillation data.