Transition from vibrational to rotational characters in low-lying states of hypernuclei

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In order to clarify the nature of hypernuclear low-lying states, we carry out a comprehensive study for the structure of 144–154 Λ C, which has provided a quantitative information on the spin-orbit splittings. In this experiment, the energy difference between the 1/2− and 3/2− states was determined to be 152 ± 54 ± 36 keV, which has been interpreted as the spin-orbit splitting between 1p1/2 and 1p3/2 hyperon states in 13 Λ C. This measurement, together with other measurements, thus has provided a solid evidence for that the spin-orbit splitting of hyperon states is smaller than that of nucleons, by more than an order of magnitude, which had been explained theoretically in terms of several different mechanisms.

A similar interpretation in other hypernuclei may need a caution, however. That is, the previous studies have demonstrated that most Λ hypernuclei are different from the 13 Λ C, in a sense that the lowest 1/2− and 3/2− states cannot be naively interpreted as pure 1p1/2 and 1p3/2 hyperon states, respectively, due to a large effect of configuration mixings. This perturbs an interpretation of their energy difference as the spin-orbit splitting for the p orbits. It has been shown that the amplitudes of the configuration mixing depend much on the the collective properties of the core nuclei.

In this paper, we investigate systematically the nature of configuration mixing in several hypernuclei which exhibit different collective properties. The Sm isotopes around N ~ 90 provide an ideal playground for this purpose, even though the production of these hypernuclei may still be difficult at this moment, since it is well known that they exhibit a shape phase transition from vibrational to rotational characters as the number of neutron increases. To this end, we shall use the microscopic particle-core coupling scheme, based on the covariant density functional theory, which uses results of the multi-reference covariant density functional theory for nuclear core excitations. In Sec. III we apply this method to the low-lying states of the Sm isotopes as well as the Sm Λ hypernuclei, and discuss the nature of low-lying collective states in these hypernuclei. We particularly discuss how the configuration mixing alters as the shape of a core nucleus changes from spherical to deformed. We then summarize the paper in Sec. IV.

II. METHOD

A. Multi-reference covariant density functional theory for nuclear core excitations

We describe low-lying states of hypernuclei in the particle-core coupling scheme. The first step in this
method is to construct the low-lying states of the core nuclei. To this end, we adopt the multi-reference covariant density functional theory (MR-CDFT) \cite{18,19}. In the MR-CDFT, the wave function of each nuclear core state is obtained as a superposition of a set of quantum-number projected mean-field reference states, $|\beta\rangle$. Here, the reference states $|\beta\rangle$ are obtained with deformation constrained relativistic mean-field plus BCS calculations with the quadrupole deformation parameter $\beta$. These states are then projected onto states with a good quantum number of angular momentum and particle number as,

$$|\Phi_{IM}(\beta)\rangle = \hat{P}_MK\hat{P}_N\hat{P}_Z|\beta\rangle,$$

(1)

where $\hat{P}_MK$ is the angular momentum projection operator, and $\hat{P}_N$ and $\hat{P}_Z$ are the particle number projection operators for neutron and proton, respectively. For simplicity, we impose axial symmetry on the reference wave functions on a harmonic oscillator basis with 12 major shells. See Refs. \cite{8–10} for more details.

Here, the weight function $F_{nI}(\beta)$ and the energy for the state $|\Phi_{nI}\rangle$ are obtained by solving the Hill-Wheeler-Griffin (HWG) equation, which is derived from the variational principle \cite{21}.

In this paper, we calculate the Hamiltonian kernel in the HWG equation with the mixed-density prescription. That is, we assume the same functional form for the off-diagonal elements of the energy overlap (sandwiched by two different reference states) as that for the diagonal elements, by replacing all the densities and currents with the mixed ones \cite{18,21}. In the calculations shown below, we adopt the PC-F1 parametrization \cite{22} for the relativistic point-coupling energy functional.

B. Microscopic particle-core coupling scheme for $\Lambda$ hypernuclei

We next construct the hypernuclear wave functions by expanding them on the nuclear core states, Eq. (2), which provide a set of basis. The resultant wave functions read,

$$\Psi_{JM}(r, \{r_i\}) = \sum_{n, I, j, l} \mathcal{R}_{j\ell nI}(r)|\Psi_{j\ell}(\vec{r}) \otimes \Phi_{nI}(\{r_i\})\rangle^{(JM)},$$

(3)

where $\vec{r}$ and $\vec{r}_i$ are the coordinates of the $\Lambda$ hyperon and the nucleons, respectively. $\mathcal{R}_{j\ell nI}(r)$ and $\Psi_{j\ell}(\vec{r})$ are the radial wave function and the spin-angular wave function for the $\Lambda$-particle, respectively. The index $n = 1, 2, \ldots$ distinguishes different core states with the same angular momentum $I$. In our previous publications \cite{8,10}, we have called this method “the microscopic particle-rotor model”, since we have mainly considered couplings of a $\Lambda$ particle to rotational states of deformed core nuclei. We instead call it “the microscopic particle-core coupling scheme” in this paper, as we deal with both spherical and deformed core nuclei.

The radial wave functions, $\mathcal{R}_{j\ell nI}(r)$, in Eq. (3) and the energy of the state, $E_J$, are obtained by solving the equation $\hat{H}|\Psi_{JM}\rangle = E_J|\Psi_{JM}\rangle$. We assume that the Hamiltonian $\hat{H}$ for the whole $\Lambda$ hypernucleus is given by \cite{10},

$$\hat{H} = \hat{T}_\Lambda + \hat{H}_c + \sum_{i=1}^{A_c} \hat{V}^{NA}(r, r_i),$$

(4)

where $\hat{T}_\Lambda$ is the relativistic kinetic energy for the $\Lambda$ particle and $\hat{H}_c$ is the many-body Hamiltonian for the core nucleus, satisfying $\hat{H}_c|\Phi_{nI}\rangle = E_{nI}|\Phi_{nI}\rangle$. The last term on the right side of Eq. (4) represents the interaction term between the $\Lambda$ particle and the nucleons in the core nucleus, where $A_c$ is the mass number of the core nucleus. We here use the $NA$ interaction derived from the point-coupling energy functional with the PCY-S4 parametrization \cite{23},

$$\hat{V}^{NA}_S(r, r_i) = \alpha_S^{NA} \gamma^0 \delta(r - r_i) \gamma^0_N + \gamma^{\Lambda}_S \gamma^0 \left[ \nabla^2 \delta(r - r_i) + 2 \nabla \cdot \delta(r - r_i) \right] \gamma^0_N,$$

(5)

$$\hat{V}^{NA}_V(r, r_i) = \alpha_V^{NA} \delta(r - r_i) + \gamma^{\Lambda}_V \left[ \nabla^2 \delta(r - r_i) + 2 \nabla \cdot \delta(r - r_i) \right],$$

(6)

$$\hat{V}^{NA}_{\text{ten}}(r, r_i) = i \alpha_T^{NA} \gamma^0 \left[ \nabla \delta(r - r_i) + \delta(r - r_i) \nabla \right] \cdot \alpha.$$

(7)

In practice, the equation $\hat{H}|\Psi_{JM}\rangle = E_J|\Psi_{JM}\rangle$ is transformed into coupled-channels equations in the relativistic framework, in which all the diagonal and off-diagonal potentials are determined from the MR-CDFT calculation. We solve the coupled-channels equations by expanding the four-component radial wave function $\mathcal{R}_{j\ell nI}(r)$ on a spherical harmonic oscillator basis. See Refs. \cite{8,10} for more details on the framework.

III. RESULTS AND DISCUSSION

Let us now apply the microscopic particle-core coupling scheme to $\text{Sm}$ hypernuclei and discuss their low-lying collective states. To this end, we generate the reference states $|\beta\rangle$ by expanding the single-particle wave functions on a harmonic oscillator basis with 12 major shells. In the particle-number and angular-momentum projection calculations, we choose the number of mesh points to be 9 for the gauge angle in $[0, \pi]$, and 16 for the Euler angle $\theta$ in the interval $[0, \pi]$. In the coupled-channels calculations, we include up to $l_{\text{max}} = 19, j_{\text{max}} = \ldots$
Wave functions, the spherical harmonic oscillator basis. The true energy minimum is thus the oblate minimum is actually a saddle point on the triaxiality (that is, a \( \gamma \) deformation). As the neutron number increases, the energy curves gradually present two prolate deformed ones in the Sm isotopes in Fig. 2 shows calculated energy spectra for the lowest \( 0^+, 2^+, 4^+, 6^+ \), and \( 8^+ \) states in the Sm isotopes, in comparison to the corresponding data. As one can see, the main characters of the energy spectra are reasonably reproduced in this calculation, although the excitation energies are somewhat overestimated. In fact, the energy spectra become close to the data, as shown in the figure, after all the excitation energies are scaled so that the experimental energy is reproduced for the first \( 2^+ \) state.

In literature, the energy ratio, \( \frac{E_{2^+}^{MR-CDFT}}{E_{2^+}^{N-CDFT}} \), of the excitation energy for the \( 4^+_1 \) state to that for the \( 2^+_1 \) state has often been adopted to characterize nuclear collective excitations. This value for \( 144\text{Sm} \), \( 146\text{Sm} \) and \( 148\text{Sm} \) is \( R_{4/2} = 1.98, 2.01 \) and \( 1.98 \), respectively, all of which are close to the value in the harmonic oscillator limit, \( R_{4/2} = 2.0 \). With the increase of the neutron number, the value of \( R_{4/2} \) increases up to 3.29 for \( 154\text{Sm} \), that is close to the value in the rigid rotor limit, \( R_{4/2} = 3.33 \). Figure 2 shows calculated energy spectra of the Sm isotopes, in comparison to the experimental data (the black lines) taken from Ref. [2]. The figure also shows the scaled levels (the blue lines) with a multiplicative factor of \( f = E_{2^+}^{MR-CDFT} / E_{2^+}^{N-CDFT} \), that is \( E_{2^+}^{scaled} = f \cdot E_{2^+}^{MR-CDFT} \).

The square of the collective wave function, \( |g_{nI}(\beta)|^2 \), for the ground state (\( 0^+_1 \)) of each isotope is shown by the dashed lines in the figure. Here, the collective wave function is defined as [20],

\[
g_{nI}(\beta) \equiv \sum_{\beta'} \left[ N^I(\beta, \beta') \right]^{1/2} F_{nI}(\beta'),
\]

where the norm kernel is given as \( N^I(\beta, \beta') = \langle \beta | \hat{P}_N^I \hat{P}_Z | \beta' \rangle \). Notice that the weight function \( F_{nI}(\beta) \) in Eq. (2) cannot be interpreted as a probability amplitude due to the non-orthgonality of the reference wave functions. The figure clearly indicates that the predominant component in the ground state changes gradually from the spherical configuration in \( 144\text{Sm} \) to the prolate deformed one in \( 154\text{Sm} \).

A. Shape transition in Sm isotopes

Before we discuss the structure of the hypernuclei, we first discuss the structure of the core nuclei. The solid lines in Fig. 1 show the total mean-field energy for the \( 144-154\text{Sm} \) nuclei as a function of the quadrupole deformation parameter, \( \beta \). For the \( 144\text{Sm} \) nucleus, one can see that the potential energy curve is almost parabolic centered at the spherical shape. As the neutron number increases, the energy curve gradually presents two pronounced minima, located on the oblate and the prolate sides, respectively. A previous study [17] has shown that these two minima are connected with a tunneling along the triaxiality (that is, a \( \gamma \) deformation). In other words, the oblate minimum is actually a saddle point on the energy surface. The true energy minimum is thus the prolate one, that shifts gradually towards a large \( \beta \) as the neutron number increases, from \( \beta = 0.08 \) for \( 148\text{Sm} \) to \( \beta = 0.32 \) for \( 154\text{Sm} \).
transition from spherical to deformed in the Sm isotopes around $N = 90$, which is in good agreement with the experimental data. This picture is verified also from the mass number dependence of the electric quadrupole transition strength from the first $2^+$ state to the ground state, $B(E2; 2^+_1 \to 0^+_1)$, as shown in Fig. 3(b).

B. Low-lying spectrum of $^{\Lambda}$Sm isotopes

We now discuss the low-lying states in the $^{\Lambda}$Sm hypernuclei, in which a $\Lambda$ particle couples to the core states presented in the previous subsection. Figure 3 shows the calculated yrast positive-parity states in the $^{\Lambda}$Sm isotopes. The probability for the dominant configuration in the wave function for the $1/2^+$, $3/2^+$, $5/2^+$, and $7/2^+$ states is also presented in Table I. These positive-parity states are dominated by the configuration of $[\Lambda s_{1/2} \otimes I_1^+]$ with the weight around 99%, where $\Lambda s_{1/2}$ denotes the $\Lambda$ particle in the $s_{1/2}$ configuration. They are nontrivial results obtained for all the $^{\Lambda}$Sm isotopes. These states have a similar excitation energy to that of the nuclear core state with $I_1^+$, and are nearly two-fold degenerate except for the $1/2^+_1$ state. These characters are similar to hypernuclei in the light-mass region, and are also consistent with our previous calculation for $^{155}_{\Lambda}$Sm with a simplified $\Lambda N$ interaction.

In the negative parity states of $^{\Lambda}$Sm isotopes, novel and interesting features are disclosed. The low-lying negative-parity states are shown in Fig. 4. We summarize in Table II the dominant components in the wave functions for a few selected levels. One can see that these negative-parity states are formed mainly from a $\Lambda$-particle in $p$ orbitals coupled to core states, and are nearly two-fold degenerate. The $E2$ transition strengths between the negative parity states are also shown in the figure. The ratio of the $E2$ transition strength for the transition $5/2^+_1 \to 1/2^+_1$ to that for $9/2^+_1 \to 5/2^+_2$ is 0.641 and 0.695 in $^{153}_{\Lambda}$Sm and $^{155}_{\Lambda}$Sm, respectively. These values are both close to 0.7, that is the ratio of the $E2$ transition strength for $2^+ \to 0^+$ to that for $4^+ \to 2^+$ in the $K = 0$ ground-state rotational band of well-deformed nuclei. A simple relation among the $B(E2)$ values for the negative-parity bands in a well deformed hypernucleus is further discussed in Appendix A.

The level structure of the negative-parity states shown in Fig. 4 can be understood in terms of the $LS$ coupling scheme. To demonstrate this, let us consider a simplified situation in which the core states with $K = 0$ shown in Fig. 4(a) are coupled to a $\Lambda$ particle in $p$ orbitals. We first couple the core angular momentum $I$ with the orbital angular momentum of the $\Lambda$ particle, $l_\Lambda = 1$. This results in the levels shown in Fig. 4(b). These levels may be categorized according to the projection of the total orbital angular momentum, $L = I + l_\Lambda$, on to the sym-

![FIG. 3.](image-url) The ratio $R_{4/2}$ of the excitation energy for the first $4^+$ state to that for the first $2^+$ state for the Sm isotopes as a function of the mass number. (b) The electromagnetic transition strength, in units $10^5 e^2$fm$^4$, from the first $2^+$ state to the ground state, $B(E2; 2^+_1 \to 0^+_1)$, as a function of the mass number for the Sm isotopes. The experimental data are taken from Ref. [24].

![FIG. 4.](image-url) The spectrum for the first positive parity states in the $^{\Lambda}$Sm isotopes. The location of the spin doublet states are arranged based on the yrast levels of the even Sm core nuclei.

![TABLE I.](image-url) The probability $P$ of the dominant components, defined as $P \equiv \int drr^2 |\Psi_{f_01}(r)|^2$, in the wave functions for the positive-parity states.
metric axis, that is, $m_L = K + m_A$. Since $K=0$, there are two possibilities for $|m_L|$, that is, $m_L = 0$ and $\pm 1$. The levels with $m_L = 0$ form a band with $1^+_1, 3^+_1, 5^+_1, 7^+_1, \cdots$, while the levels with $m_L = \pm 1$ form another band with $1^-_2, 2^-_2, 3^-_2, 4^-_2, \cdots$. If one further couples the spin 1/2 of the $\Lambda$ particle to these rotational bands, one obtains the levels in Fig. 6(c) for the $m_L = 0$ band, and the levels in Figs. 6(d) and (e) for the $m_L = \pm 1$ band. For a given $I$, the levels belonging to the $m_L = 0$ and the $m_L = \pm 1$ bands are degenerate in energy for spherical hypernuclei. For prolately deformed hypernuclei, on the other hand, the level in the $m_L = 0$ band is lower in energy than the levels in the $m_L = \pm 1$ band because

The solid line and the dashed line indicate the results of the coupled-channels and the single-channel calculations, respectively. (b) The energy splitting between the $1/2^-_1$ and $3/2^-_1$ states shown in the upper panel.

The former configuration gains more energy due to a better overlap with the core nucleus. This feature explains well the energy relation among the three doublet states of $(5/2^-_1, 7/2^-_1), (3/2^-_2, 5/2^-_2)$, and $(1/2^-_2, 3/2^-_2)$ in, e.g., $^{149}\Lambda$Sm and $^{151}\Lambda$Sm (see Fig. 5).

Figure 7(a) shows in details the excitation energy of

FIG. 5. The low-lying negative-parity states in the $\Lambda$Sm isotopes obtained with the microscopic particle-core coupling scheme based on the covariant density functional theory. The arrows indicate the E2 transition strengths, given in units of $e^2$ fm$^4$.

FIG. 6. A schematic picture for hypernuclear states based on the $LS$ coupling scheme. It is assumed that a $\Lambda$-particle is in a $p$-orbital and coupled to spherical core states.

FIG. 7. (a) The energy levels of the $1/2^-_1$ and $3/2^-_1$ states in the $\Lambda$Sm hypernuclei as a function of the mass number. The solid line and the dashed line indicate the results of the coupled-channels and the single-channel calculations, respectively. (b) The energy splitting between the $1/2^-_1$ and $3/2^-_1$ states shown in the upper panel.
the lowest 1/2\(^-\) and 3/2\(^-\) states in the \(\text{A}^{\text{Sm}}\) isotopes as a function of the neutron number. The dashed energy levels show the results of the single-channel calculations, for which the sum in Eq. (5) is restricted only to a single configuration. For the lowest 1/2\(^-\) and 3/2\(^-\) states, the configuration in the single-channel calculation is a pure configuration of \([\Lambda p_{1/2} \otimes 0_1^+]\) and \([\Lambda p_{3/2} \otimes 0_1^+]\), respectively. Their excitation energies are around 4.8 MeV for all the hypernuclei considered in this paper, which is close to the energy \(4/3 \times 41A^{-1/3} = 5.14\) MeV with \(A \sim 150\) for exciting one hyperon from \(s\) orbit to \(p\) orbit. The energy difference between these states remains around 70 keV, as shown by the open circles in Figure 2(b). In marked contrast, the energy of the 1/2\(^-\) and 3/2\(^-\) states obtained by including the configuration mixing effect decreases continuously from 4.7 MeV to 3.5 MeV as the neutron number increases from 82 to 92 (see the solid lines in Fig. 2(a)). The splitting of these two states also decreases from 68 keV to 4 keV, as shown in Fig. 2(b) by the filled circles. The deviation from the single-channel calculations increases as the core nucleus undergoes phase transition from a spherical vibrator to a well-deformed rotor, indicating a stronger configuration mixing effect in deformed hypernuclei.

This feature can be seen also in the compositions of the wave functions listed in Table II. In \(\text{A}^{\text{Sm}}\) isotopes, the 1/2\(^-\) and 3/2\(^-\) states are almost pure configuration of \([\Lambda p_{1/2} \otimes 0_1^+]\) and \([\Lambda p_{3/2} \otimes 0_1^+]\), respectively. This is consistent with the fact that the single-channel calculation works well for this hypernucleus (see Fig. 7). With the increase of the neutron number, the mixing between the \([\Lambda p_{1/2} \otimes 0_1^+]\) and \([\Lambda p_{3/2} \otimes 2_1^+]\) configurations in the 1/2\(^-\) state becomes stronger and reaches the largest value in \(\text{A}^{\text{Sm}}\). This feature is shown clearly in Fig. 3(a). One can see that the mixing between 0\(^-\) and 2\(^-\) becomes almost half-and-half in \(\text{A}^{\text{Sm}}\). In the well-deformed \(\text{A}^{\text{Sm}}\), the weight for the \([\Lambda p_{1/2} \otimes 0_1^+]\) configuration becomes 32.2% while that for the \([\Lambda p_{3/2} \otimes 2_1^+]\) configuration becomes 63.9%. For the 3/2\(^-\) state, on the other hand, the wave function shows a mixture of the \([\Lambda p_{1/2} \otimes 0_1^+]\), \([\Lambda p_{1/2} \otimes 2_1^+]\) and \([\Lambda p_{3/2} \otimes 2_1^+]\) configurations. The mass number dependence of the weight factors is shown in Fig. 3(b), indicating a similar feature as in the 1/2\(^-\) state. That is, the configuration mixing becomes stronger as the core nucleus undergoes a transition from spherical to deformed. For the \(\text{A}^{\text{Sm}}\) hypernucleus, the weight factors are 36.3%, 28.1%, and 31.8%, for the \([\Lambda p_{1/2} \otimes 0_1^+]\), \([\Lambda p_{1/2} \otimes 2_1^+]\) and \([\Lambda p_{3/2} \otimes 2_1^+]\) configurations, respectively.

It is worth mentioning that for all the negative-parity states in the \(m_{LL} = 0\) band of \(\text{A}^{\text{Sm}}\) shown in Table II, the weight for the configurations with \(p_{1/2}\) is around 33%, while a sum of the weight factors for the configurations with \(p_{3/2}\) is close to 67%. To understand this, let us employ the Nilsson model for the hyperon with an axially deformed potential, \(V(r) = V_0(r) - \beta R_0 \frac{dV_0(r)}{dr} V_2(r)\). Notice that, with this deformed potential, several or-

| \(J^\pi\) | \((l_j) \otimes I_n\) | \(155\text{Sm}\) | \(149\text{Sm}\) | \(157\text{Sm}\) | \(147\text{Sm}\) | \(153\text{Sm}\) | \(159\text{Sm}\) |
|-------|-----------------|---------|---------|---------|---------|---------|---------|
| 1/2\(^-\) | \(p_{1/2} \otimes 0_1^+\) | 0.986 | 0.964 | 0.859 | 0.484 | 0.348 | 0.322 |
| \(p_{1/2} \otimes 1_1^+\) | 0.012 | 0.033 | 0.136 | 0.503 | 0.627 | 0.639 |
| 3/2\(^-\) | \(p_{1/2} \otimes 0_1^+\) | 0.006 | 0.015 | 0.054 | 0.204 | 0.271 | 0.281 |
| \(p_{1/2} \otimes 1_1^+\) | 0.986 | 0.965 | 0.876 | 0.545 | 0.395 | 0.363 |
| \(p_{1/2} \otimes 2_1^\pm\) | 0.006 | 0.017 | 0.064 | 0.238 | 0.309 | 0.318 |
| 5/2\(^-\) | \(p_{1/2} \otimes 0_1^+\) | 0.980 | 0.959 | 0.573 | 0.453 | 0.385 | 0.346 |
| \(p_{1/2} \otimes 1_1^+\) | 0.012 | 0.034 | 0.154 | 0.377 | 0.462 | 0.504 |
| \(p_{1/2} \otimes 2_1^\pm\) | 0.262 | 0.156 | 0.127 | 0.112 |
| 7/2\(^-\) | \(p_{1/2} \otimes 0_1^+\) | 0.008 | 0.022 | 0.074 | 0.183 | 0.232 | 0.258 |
| \(p_{1/2} \otimes 1_1^+\) | 0.980 | 0.954 | 0.854 | 0.653 | 0.554 | 0.497 |
| \(p_{1/2} \otimes 2_1^\pm\) | 0.006 | 0.018 | 0.062 | 0.150 | 0.188 | 0.207 |
| 9/2\(^-\) | \(p_{1/2} \otimes 0_1^+\) | 0.843 | 0.931 | 0.570 | 0.481 | 0.398 | 0.372 |
| \(p_{1/2} \otimes 1_1^+\) | 0.071 | 0.016 | 0.288 | 0.210 | 0.166 | 0.154 |
| \(p_{1/2} \otimes 2_1^\pm\) | 0.040 | 0.033 | 0.131 | 0.295 | 0.407 | 0.437 |

FIG. 8. The probability \(P_{20}\) for the dominant components in the wave function of (a) the 1/2\(^-\) state and (b) the 3/2\(^-\) state as a function of the mass number of the \(\text{A}^{\text{Sm}}\) isotopes.
hedral angular momenta $l$ and total angular momenta $j$ are mixed in the hyperon wave function. Treating the deformed part of the potential, $-\beta R_0 \frac{d\phi_0}{dr}Y_{20}(\hat{r})$, with the first order perturbation theory and neglecting the mixture across two major shells, one can write the wave function for the lowest negative parity state as $|\psi_\lambda\rangle = C_1|p3/2,1/2\rangle + C_2|p1/2,1/2\rangle$, where $|p3/2,1/2\rangle$ and $|p1/2,1/2\rangle$ are single-particle wave functions in the spherical limit with $j_\lambda = 1/2$. The coefficients $C_1$ and $C_2$ are simply determined by the following eigenvalue equation,

\[
\begin{pmatrix}
\langle \psi_{p3/2,1/2} | Y_{20} | \psi_{p3/2,1/2} \rangle \\
\langle \psi_{p1/2,1/2} | Y_{20} | \psi_{p3/2,1/2} \rangle \\
\langle \psi_{p1/2,1/2} | Y_{20} | \psi_{p1/2,1/2} \rangle \\
\end{pmatrix}
\times \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \lambda \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}.
\]

(9)

The value of the matrix elements

\[
\begin{align*}
\langle \psi_{p3/2,1/2} | Y_{20} | \psi_{p3/2,1/2} \rangle &= \frac{1}{\sqrt{20\pi}}, \\
\langle \psi_{p1/2,1/2} | Y_{20} | \psi_{p3/2,1/2} \rangle &= -\frac{2}{\sqrt{20\pi}}, \\
\langle \psi_{p1/2,1/2} | Y_{20} | \psi_{p1/2,1/2} \rangle &= -\frac{2}{\sqrt{20\pi}}, \\
\end{align*}
\]

is $-\frac{2}{\sqrt{20\pi}}$, $\frac{1}{\sqrt{20\pi}}$, and $0$, respectively. The solutions of Eq. (9) then give two eigenvectors, $(C_1, C_2)^T = (\sqrt{2/3}, -\sqrt{1/3})^T$ and $(\sqrt{1/3}, \sqrt{2/3})^T$, with the eigenvalues of $\lambda = 2/\sqrt{20\pi}$ and $-1/\sqrt{20\pi}$, respectively. For a positive value of $\beta$, the former state is lower in energy. For this state, the probability of the $p3/2$ component reads 66.7% and that of $p1/2$ component is 33.3%. This clearly implies that the weight factors shown in Table II are consistent with the Nilsson model and thus can be understood in terms of the strong coupling limit of the particle-rotor model.

In addition, we also carry out the coupled-channels calculation for $^{155}\Lambda$Sm by setting the excitation energies of the nuclear core states to be zero. In order to draw the energy curve as a function of the deformation parameter, we take $F_{nl}(\beta) = \delta_3 \beta^2$ in Eq. (2) and compute the total energy for $\beta = \beta'$. Notice that this is a reasonable approximation for well-deformed hypernuclei. The calculated energy for the $1/2_1^-, 1/2_2^-$ and $3/2_1^-$ states are shown in Fig. 9(a). The splitting of the single-particle states due to nuclear deformation is a well known feature of the Nilsson diagram. The main components of the wave function are shown in Figs. 9(b) and (c) for the $1/2_1^-$ and $1/2_2^-$ states, respectively. One can see that the $1/2_1^-$ state comprises mainly of the $[\Lambda p_{1/2} \otimes 0_1^-]$ and $[\Lambda p_{3/2} \otimes 2_1^-]$ configurations with a rather constant mixing weight of around 30% and 70%, respectively, on the prolate side. The mixing weights for these two configurations are exchanged on the oblate side. The mixing weights for the $1/2_2^-$ state are just opposite to those for the $1/2_1^-$ state. These findings confirm the analysis based on the simple Nilsson potential presented in the previous paragraph.

IV. SUMMARY

We have systematically investigated the configuration mixing in low-lying states of Sm hypernuclei using the microscopic particle-core coupling scheme based on the covariant density functional theory. We emphasize that this is the first microscopic calculation for hypernuclear spectra in (medium-)heavy hypernuclei and can be achieved only with the mean-field based calculations, in which the beyond-mean-field correlations are also included. We have found that the positive-parity ground-state band shares a similar structure to that for the core nucleus. That is, the hypernuclear states with spin-parity of $(I \pm 1/2)^+$ are dominated by the configuration of $[\Lambda s_{1/2} \otimes I^+]$, where $\Lambda s_{1/2}$ denotes the $\Lambda$ particle in the $s_{1/2}$ state, regardless of whether the core nucleus is spherical or deformed. In contrast, the low-lying negative-parity states show an admixture of the
Ap_{1/2} and the Ap_{3/2} configurations coupled with nuclear core states having I and I ± 2. We have shown that the mixing amplitude is negligibly small in spherical and weakly-deformed nuclei, while it becomes increasingly stronger as the core nucleus undergoes a shape transition to a well-deformed shape. We have demonstrated that the energy spectra for low-lying negative parity states in Sm hypernuclei can be well understood with the LS coupling scheme with the orbital angular momentum of L = |I − 1|, I, I + 1 and the spin angular momentum of S = 1/2. For well-deformed hypernuclei, the spectra as well as the wave functions are also consistent with the Nilsson model.

The conclusion obtained in this paper can be applied to hypernuclei in any mass region, provided that the low-lying states of the core nucleus are dominated by quadrupole collective excitations. This indicates that the spin-orbit splitting for the hyperon p-orbital should be estimated from the energy difference between the first 1/2− and 3/2− states in hypernuclei with a nearly spherical nuclear core.

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Appendix A: E2 transition strengths in a well deformed hypernucleus

In this Appendix, we derive a simple expression for the B(E2) values for the E2 transition between the negative-parity states in a well deformed hypernucleus. To this end, we use the LS coupling scheme discussed in Sec. [III] and consider a transition from the initial state |J_iM_i⟩ to the final state |J_fM_f⟩, whose wave function is given by

\[ |J_iM_i⟩ = \left[ |I_i \otimes l_\Lambda|^l_{(I_i)} \otimes s_\Lambda \right]^{(J_i,M_i)}, \]  
\[ |J_fM_f⟩ = \left[ |I_f \otimes l_\Lambda|^l_{(I_f)} \otimes s_\Lambda \right]^{(J_f,M_f)}, \]  

respectively. Here, I_i and I_f are the initial and the final spin of the core nucleus, respectively, and s_\Lambda = 1/2 is the spin of the Λ particle. We assume that the initial and the final states are dominated by configurations with the Λ particle in the p orbits, and we take the orbital angular momentum of the Λ particle to be l_\Lambda = 1.

The E2 transition operator, \( \hat{T}_2 \), acts only on the core states. Using Eq. (7.1.7) in Ref. [20], the reduced matrix element of the E2 operator between the initial and the final states reads,

\[ \langle J_f||\hat{T}_2||J_i⟩ = (-1)^{J_f+1+J_i+2} J_f J_i L_f L_i \frac{1/2}{2} \times \langle J_f||\hat{T}_2||J_i⟩ . \]  

Here, we have used a shorthand notation of \( \hat{J} = \sqrt{2J + 1} \). The B(E2) value for the transition from the initial to the final states is then given as

\[ B(E2; J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \frac{\langle J_f||\hat{T}_2||J_i⟩^2}{\langle J_f||\hat{T}_2||J_i⟩^2}, \]  

\[ = (2J_f + 1)(2L_f + 1) \times \left\{ \frac{L_f}{J_f} \frac{J_f}{L_i} \frac{1/2}{2} \right\}^2 \times \left\{ \frac{L_i}{I_f} \frac{L_i}{I_i} \frac{1/2}{2} \right\}^2 . \]  

Notice that \( \langle J_f||\hat{T}_2||J_i⟩^2 \) in the last line is related to the B(E2) value for the core transition as \( \langle J_f||\hat{T}_2||J_i⟩^2 = (2I_i + 1)B(E2; I_i \rightarrow I_f) \). In order to evaluate it, we use the collective model [20], that is,

\[ B(E2; I + 2 \rightarrow I) = Q_0^2 \left( \frac{5}{16\pi} \frac{3}{2} \frac{(I + 1)(I + 2)}{(2I + 3)(2I + 5)} \right) . \]
The $E2$ transition strengths for transition between low-lying negative parity states in a well deformed hypernucleus. The $B(E2)$ values are given relative to the value for the transition from the first excited state with $(I, L, J) = (2, 3, 5/2)$ to the ground state with $(I, L, J) = (0, 1, 1/2)$, where $I$, $L$, and $J$ are the spin of the core nucleus, the total orbital angular momentum of the hypernucleus and the total spin of the hypernucleus, respectively. Only the $B(E2)$ values larger than 0.25 are listed.

| $I_f$ | $L_f$ | $J_f$ | $I_i$ | $L_i$ | $J_i$ | $B(E2; J_i \rightarrow J_f)$ |
|------|------|------|------|------|------|-----------------------------|
| 2    | 3    | 5/2  | 0    | 1    | 1/2  | 1.00                        |
| 2    | 3    | 5/2  | 0    | 1    | 3/2  | 0.286                       |
| 2    | 3    | 7/2  | 0    | 1    | 3/2  | 1.29                         |
| 2    | 2    | 3/2  | 0    | 1    | 1/2  | 0.643                        |
| 2    | 2    | 3/2  | 0    | 1    | 3/2  | 0.643                        |
| 2    | 2    | 5/2  | 0    | 1    | 1/2  | 0.286                        |
| 2    | 2    | 5/2  | 0    | 1    | 3/2  | 1.00                         |
| 2    | 1    | 1/2  | 0    | 1    | 1/2  | 1.29                         |
| 2    | 1    | 3/2  | 0    | 1    | 1/2  | 0.643                        |
| 2    | 1    | 3/2  | 0    | 1    | 3/2  | 0.643                        |
| 4    | 5    | 9/2  | 2    | 3    | 5/2  | 1.73                         |
| 4    | 5    | 11/2 | 2    | 3    | 7/2  | 1.84                         |
| 4    | 7    | 2/2  | 2    | 3    | 5/2  | 0.273                        |
| 4    | 9    | 2/2  | 2    | 3    | 7/2  | 0.289                        |
| 4    | 7    | 2/2  | 2    | 3    | 3/2  | 1.38                         |
| 4    | 9    | 2/2  | 2    | 2    | 5/2  | 1.53                         |
| 4    | 3    | 5/2  | 2    | 2    | 3/2  | 0.315                        |
| 4    | 3    | 7/2  | 2    | 2    | 5/2  | 0.354                        |
| 4    | 5    | 2/2  | 2    | 1    | 1/2  | 1.10                         |
| 4    | 3    | 5/2  | 2    | 1    | 3/2  | 0.315                        |
| 4    | 3    | 7/2  | 2    | 1    | 3/2  | 1.42                         |
| 6    | 7    | 13/2 | 4    | 5    | 9/2  | 1.97                         |
| 6    | 7    | 15/2 | 4    | 5    | 11/2 | 2.02                         |
| 6    | 6    | 11/2 | 4    | 4    | 7/2  | 1.82                         |
| 6    | 6    | 13/2 | 4    | 4    | 9/2  | 1.89                         |
| 6    | 5    | 9/2  | 4    | 3    | 5/2  | 1.75                         |
| 6    | 5    | 11/2 | 4    | 3    | 7/2  | 1.86                         |

for $K = 0$, where $Q_0$ is the intrinsic quadrupole moment of a deformed nucleus. One then obtains,

$$B(E2; J_i \rightarrow J_f) \propto (2J_f + 1)(2L_i + 1)(2I + 1) \times \left\{ \frac{L_f}{J_f} \frac{J_f}{1/2} \right\}^2 \left\{ \frac{I}{L_i} \frac{L_i}{2} \right\}^2 \times \frac{(I + 1)(I + 2)}{(2I + 3)},$$

for $I_i = I + 2$ and $I_f = I$.

Table III summarizes the $B(E2)$ values. Here, the $B(E2)$ values are given relative to the one for the transition from the first $5/2^-$ to the first $1/2^-$ states, which is $7/9$ times $B(E2; 2^+ \rightarrow 0^+)$ in the core nucleus. Only the values larger than 0.25 are listed. The transition strengths are also graphically shown in Fig. 10. The formation of the band structures can be clearly seen in the figure. The inter-band transitions are much stronger than the intra-band transitions, except for the low-lying states. That is, as expected, the $E2$ cascade transitions are exclusively strong within the stretched angular momentum states with spin-up (or spin-down), particularly because the $E2$ operator is spin-independent.

It should be pointed out that the $B(E2)$ values for the states shown in Fig. 10 are derived based on the simple picture that only one rotational band is taken into account for the core nuclei. In our actual calculations for the $\Lambda$Sm isotopes, three states ($n_{\text{max}} = 3$) for a given angular momentum $I$ are adopted, which is much more realistic.
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