The Single State Dominance Hypothesis and the Two-Neutrino Double Beta Decay of $^{100}$Mo

F. Šimkovic$,^1$ P. Domin$,^1$ and S.V. Semenov$^2$

$^1$Department of Nuclear Physics, Comenius University, Bratislava, Slovakia

$^2$Russian Research Center "Kurchatov Institute", Moscow, Russia

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Abstract

The hypothesis of the single state dominance (SSD) in the calculation of the two-neutrino double beta decay (2$\nu\beta\beta$-decay) of $^{100}$Mo is tested by exact consideration of the energy denominators of the perturbation theory. Both transitions to the ground state as well as to the $0^+$ and $2^+$ excited states of the final nucleus $^{100}$Ru are considered. We demonstrate, that by experimental investigation of the single electron energy distribution and the angular correlation of the outgoing electrons, the SSD hypothesis can be confirmed or ruled out by a precise 2$\nu\beta\beta$-decay measurement (e.g. by NEMO III collaboration).

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I. INTRODUCTION

The two-neutrino double beta decay (2$\nu\beta\beta$-decay) $^{100}$Mo, which is allowed by the Standard model, has been observed in direct counter experiments for a couple of isotopes during the last 15 years (see e.g. the recent review articles [4,5]). As the decay rate of this process is free of unknown parameters from the particle physics side this very rare process with a typical half-life above 10$^{18}$ years can be used to test the nuclear structure.

The 2$\nu\beta\beta$-decay is a second order process in perturbation theory. Thus the calculation
of the $2\nu\beta\beta$-decay matrix element is a complex task mainly due to the fact that it involves a summation over a full set of virtual intermediate nuclear states of the double–odd nucleus. Essentially there exist two different approaches to evaluate the $2\nu\beta\beta$-decay rate including an explicit summation over these states [4,5]: The shell model approach has been found successful in describing satisfactory the lowest excited states, however, it can not reliably describe the states in the giant Gamow-Teller resonance region for open shell medium heavy nuclei. The proton-neutron QRPA (pn-QRPA) and its extensions avoid this drawback but on other hand their predictions are very dependent upon model assumptions.

The crucial problem of the theoretical $2\nu\beta\beta$-decay studies is the question whether the contribution of the higher-lying states to the $2\nu\beta\beta$-decay amplitude, which is apparently disfavored by the large energy denominator, plays an important role. In Ref. [6] it was suggested that $2\nu\beta\beta$-decay transitions, where the first $1^+_1$ state of the intermediate nucleus $(A, Z \pm 1)$ is the ground state, are governed only by the following two beta transitions: i) The first one connecting the ground state of the initial nucleus $(A, Z)$ with $1^+_1$ intermediate state of $(A, Z \pm 1)$ nucleus. ii) The second one proceeding from the $1^+_1$ state to the final ground state $(A, Z \pm 2)$. This assumption is known as the single state dominance(SSD) hypothesis.

We note that the dominance of the ground state of intermediate nucleus in particular case of $2\nu\beta\beta$-decay of $^{100}$Mo was pointed out by A. Griffiths and P. Vogel, who analysed this decay in details [7].

The SSD hypothesis has been studied both experimentally [6,10–12] and theoretically [7–9]. The required beta transition amplitudes to the $1^+_1$ state have been deduced from the measured log$ft$ values, i.e., in the model independent way, or have been calculated e.g. within the pn-QRPA [10]. The obtained results indicate that the SSD hypothesis can be realized in the case of several $2\nu\beta\beta$-decay emitters through a true dominance of the $1^+_1$ state or by cancelations among the higher lying $1^+$ state of the intermediate nucleus. Till now the study has been concentrated mostly on the determination of the $2\nu\beta\beta$-decay half-life for the transition to the ground state. Recently the $2\nu\beta\beta$-decay transitions to excited $0^+$ states of the final nucleus has gained much attention [9]. It is worthwhile to notice that there is
now a first positive evidence for such nuclear $2\nu\beta\beta$-decay transition [13].

In previous SSD hypothesis studies, calculations have been performed with approximated energy denominators of the perturbation theory by ignoring their dependence on lepton energies [9]. However, this approximation can lead to significant overestimation of the $2\nu\beta\beta$-decay half-life as it was shown in Ref. [14]. Therefore, it is necessary to reconsider the SSD predictions without the above approximation. It is also supposed that exact calculations of the SSD hypothesis with unfactorized nuclear part and integration over the phase space of the outgoing leptons can strongly influence the behavior of some of the differential decay rates. Previously, they have not been analyzed in the framework of the SSD hypothesis. However, they are of current interest due to the prepared NEMO III experiment, which will allow to perform a precise measurement of the energy and angular distributions of the outgoing electrons [19].

In this paper we perform exact calculations of the SSD hypothesis of $2\nu\beta\beta$-decay of $^{100}\text{Mo}$ for the transitions to the $0^+$ ground state as well as to excited $0^+$ and $2^+$ states of the final nucleus $^{100}\text{Ru}$. In addition, we will discuss a possible signal in favor of the SSD hypothesis from the differential decay rates.

II. THEORY

The inverse half-life of the $2\nu\beta\beta$-decay transition to the $0^+$ and $2^+$ states of the final nucleus is usually presented in the following form [1–3]:

$$[T_{1/2}^{2\nu}(0^+ \rightarrow J^+)]^{-1} = G^{2\nu}(J^+)|M_{GT}(J^+)|^2,$$  

(2.1)

where $G^{2\nu}(J^+)$ is the kinematical factor. The nuclear matrix element $M_{GT}^{2\nu}(J^+)$ can be written as sum of two matrix elements $M_{GT}^{SS}(J^+)$ and $M_{GT}^{HS}(J^+)$ including the transitions through the lowest and higher lying states of the intermediate nucleus, respectively. We have

$$M_{GT}(J^+) = M_{GT}^{SS}(J^+) + M_{GT}^{HS}(J^+),$$  

(2.2)
where

\[ M^{SS}_{GT}(J^+) = \frac{1}{\sqrt{s}} \frac{M_f^j(J^+)M_i^j(0^+)}{|E_1 - E_i + \Delta|^\alpha}, \]

\[ M^{HS}_{GT}(J^+) = \frac{1}{\sqrt{s}} \sum_{n=2} \frac{M_n^f(J^+)M_n^i(0^+)}{|E_n - E_i + \Delta|^\alpha} \]  \tag{2.3}

with

\[ M_f^j(J^+) = < J_f^+ \parallel \sum \tau_m^+ \sigma_m \parallel 1_i^+ >, \]

\[ M_i^i(0^+) = < 1_n^+ \parallel \sum \tau_m^+ \sigma_m \parallel 0_i^+ >. \]  \tag{2.4}

Here, \( s = 1 \) for \( J = 0 \) and \( s = 3 \) for \( J = 2 \). \( |0_i^+ >, |0_f^+ > \) and \( |1_i^+ > \) are respectively the wave functions of the initial, final and intermediate nuclei with corresponding energies \( E_i, E_f \) and \( E_n \). \( \Delta \) denotes the average energy \( \Delta = (E_i - E_f)/2 \).

The SSD hypothesis assumes that the nuclear matrix element \( M^{HS}_{GT}(J^+) \) is negligible in comparison with \( M^{SS}_{GT}(J^+) \), which can be determined in phenomenological (with help of \( logft \) values) or nuclear model dependent way. Knowing the value of \( M^{SS}_{GT}(J^+) \) one can predict the \( 2\nu\beta\beta \)-decay half-life with help of Eq. (2.1) and compare it with the measured one. Henceforth we shall denote this approach as SSD1.

The SSD1 \( 2\nu\beta\beta \)-decay half-life is derived in the approximation in which the sum of two lepton energies in the denominator of the \( 2\nu\beta\beta \)-decay nuclear matrix element is replaced with their average value \( \Delta \):

\[ D(\varepsilon_i, \omega_j) \equiv E_1 - E_i + \varepsilon_i + \nu_j, \approx E_1 - E_i + \Delta, \]  \tag{2.5}

\((i, j = 1, 2)\). Here, \( \varepsilon_i = \sqrt{k_i^2 + m_e^2} \) (\( m_e \) is the mass of electron) and \( \nu_j \) are energies of electrons and antineutrinos, respectively. The main purpose of this approximation is to factorize the lepton and nuclear parts in the calculation of \([T_{1/2}^{2\nu}(0^+ \rightarrow J^+)]\). However, it is not necessary to do it within the SSD hypothesis. We note that in the particular case of \( 2\nu\beta\beta \)-decay of \(^{100}Mo\) the value \( E_1 - E_i \) is negative (-0.343 MeV) and that there is large difference between the minimal (0.168 MeV) and maximal (3.202 MeV) values of \( D(\varepsilon_i, \omega_k) \).

It indicates that one has to go beyond the above approximation. The SSD hypothesis
approach with exact consideration of the energy denominators will be denoted hereafter SSD2.

From the theoretical point of view one can discuss also an alternative assumption, which is the dominance of the contribution from higher order states of the intermediate nucleus to $2\nu\beta\beta$-decay rate. We shall denote it as higher order state dominance (HSD) hypothesis. We note that within this assumption one can factorize safely the nuclear part and the integration over the phase space. Thus it is expected that the behavior of the HSD hypothesis differential decay rates will differ considerably from those obtained within SSD2, if the value of the expression $E_1 - E_i + m_e$ is rather small. For such comparison of the SSD2 and HSD approaches we shall assume

$$M_{GT}^{HS} \approx M_{GT}^{exp} = [T_{1/2}^{2\nu - exp}(0^+ \to J^+) G^{2\nu}(J^+)]^{-1/2}. \quad (2.6)$$

Here, $T_{1/2}^{2\nu - exp}(0^+ \to J^+)$ is the measured $2\nu\beta\beta$-decay half-life.

The $2\nu\beta\beta$-decay half-life, the single electron and angular distribution differential decay rates within the SSD1, SSD2 and HSD approaches are given as follows:

$$[T_{1/2}^{2\nu - I}(0^+ \to J^+)]^{-1} = \frac{\omega}{\ln(2)} = \frac{c_{2\nu}}{\ln(2)} \int_{m_e}^{E_i - E_f - m_e} k_1 \varepsilon_1 F(Z_f, \varepsilon_1) d\varepsilon_1 \times \int_{m_e}^{E_i - E_f - \varepsilon_1} k_2 \varepsilon_2 F(Z_f, \varepsilon_2) d\varepsilon_2 \int_0^{E_i - E_f - \varepsilon_1 - \varepsilon_2} \nu_1^2 \nu_2^2 A_{f^\pi}^I d\nu_1, \quad (2.7)$$

$$\frac{d\omega^I(0^+ \to J^+)}{d\varepsilon_1} = c_{2\nu} k_1 \varepsilon_1 F(Z_f, \varepsilon_1) \times \int_{m_e}^{E_i - E_f - \varepsilon_1} k_2 \varepsilon_2 F(Z_f, \varepsilon_2) d\varepsilon_2 \int_0^{E_i - E_f - \varepsilon_1 - \varepsilon_2} \nu_1^2 \nu_2^2 A_{f^\pi}^I d\nu_1, \quad (2.8)$$

$$\frac{d\omega^I(0^+ \to J^+)}{d\cos \theta} = \frac{c_{2\nu}}{2} \int_{m_e}^{E_i - E_f - m_e} k_1 \varepsilon_1 F(Z_f, \varepsilon_1) d\varepsilon_1 \int_{m_e}^{E_i - E_f - \varepsilon_1} k_2 \varepsilon_2 F(Z_f, \varepsilon_2) d\varepsilon_2 \times \int_0^{E_i - E_f - \varepsilon_1 - \varepsilon_2} \nu_1^2 \nu_2^2 \left( A_{f^\pi}^I + B_{f^\pi}^I \frac{k_1 k_2}{\varepsilon_1 \varepsilon_2} \cos \theta \right) d\nu_1. \quad (2.9)$$
Here, \( c_{2\nu} = G_{\beta}^4 g_A^4 / 8\pi^7 \) and \( F(Z_f, \varepsilon) \) is the relativistic Coulomb factor \([1,2]\). The expressions for the factors \( A_{I}^{f,\pi} \) and \( B_{I}^{f,\pi} \) (\( I = SSD1, SSD2, HSD \) and \( J^\pi = 0^+, 2^+ \)) are presented in Table III.

In order to determine the \( 2\nu\beta\beta \)-decay half-life within the SSD hypothesis the matrix elements \( M_{i}^{f}(J^+) \) and \( M_{i}^{1}(0^+) \) have to be specified. They can be deduced from the \( logft \) values of electron capture and the single \( \beta \) decays as follows:

\[
M_{i}^{1}(0^+) = \frac{1}{g_A} \sqrt{\frac{3D}{f_{EC}}} \quad \text{and} \quad M_{i}^{f}(J^+) = \frac{1}{g_A} \sqrt{\frac{3D}{f_{\beta^e}}}.
\]  

(2.10)

Here, \( D = (2\pi^3 \ln 2) / (G_\beta^2 m_e^5) \) (\( G_\beta = 1.149 \times 10^{-5} \text{GeV}^{-2} \)) and \( g_A \) is the vector-axial coupling constant. The advantage of this phenomenological determination of the beta transition amplitudes \( M_{i}^{1}(0^+) \) and \( M_{i}^{f}(J^+) \) consists in their nuclear model independence and in the fact that the associated \( 2\nu\beta\beta \)-decay rate does not depend explicitly on \( g_A \) (\( g_A \) factors from beta amplitudes in Eq. (2.10) are canceled with \( g_A^4 \) from \( c_{2\nu} \) factor).

### III. CALCULATION AND DISCUSSION

In this paper we study the SSD hypothesis for \( 2\nu\beta\beta \)-decay of \(^{100}\text{Mo} \) to the ground state as well as to the \( 0^+ \) and \( 2^+ \) excited states of the final nucleus \(^{100}\text{Ru} \). The calculated \( 2\nu\beta\beta \)-decay half-lifes are presented in Table III and compared with the available experimental data. By comparing the results of the SSD1 and SSD2 approaches we see that the exact consideration of the energy denominators leads to a significant reduction of the half-lifes for all studied transitions and that this effect is especially large for transitions to the \( 2^+ \) states of the final nucleus. The obtained SSD2 values are close to the experimental ones both for the transition to the ground and excited \( 0^+_1 \) states of \(^{100}\text{Ru} \). However, it is not possible to draw a general conclusion with respect to the SSD approach as there is some disagreement between different experimental measurements (see Table III). We note also that the phenomenological predictions for the \( 2\nu\beta\beta \)-decay half-lifes (SSD1 and SSD2) have a big uncertainty too (\( \approx 50\% \)) due to inaccurate experimental determination of the \( logft_{EC} \) value.
for the electron capture [19]. It is expected that the above drawbacks will be eliminated by the future experimental measurements [19].

Till now the $2\nu\beta\beta$-decay transition to the $2^+$ state of the final nucleus has been not observed. The SSD2 results given in Table II suggest that this transition for the A=100 system can be detected on the level of $10^{23}$ years. In spite of the advantage of $2\nu\beta\beta$-decay measurement to $2^+$ excited state in coincidence with gamma transition to the $0^+$ ground state the detection of this $2\nu\beta\beta$-decay transition seems to be unreachable in near future.

We note that our calculation has been performed by considering that all outgoing leptons are emitted in the S-wave state. This case is favored from the viewpoint of lepton wave but suppressed due to the small factor $(K - L)$. However, there are other possibilities from the higher partial waves of leptons with favored additive combination $(K + L)$ of denominators. In Ref. [3] it was estimated that the suppression factor due to a P-wave to S-wave ratio is about $10^{-3}$ for the of $2\nu\beta\beta$-decay amplitude. We have found that the contribution from higher lepton partial wave can be comparable with the pure S-wave contribution only in the case in which the above ratio is about 50, something unexpected. Thus within the SSD hypothesis the $2\nu\beta\beta$-decay transition to the $2^+$ state is governed by the pure lepton S-wave contribution.

Further, we have found that it is possible experimentally decide whether one low-lying state dominates or not by precise measuring the single electron spectra and/or angular distributions. The single electron spectrum of the emitted electrons calculated within SSD (i.e., SSD2) and HSD approaches is shown in Fig. 1. The SSD and HSD distributions associated with the transitions to the $0^+$ ground (Fig. 1a) and excited (Fig. 1b) states of the final nucleus were normalized to the experimental half-lifes of Ref. [15] and Ref. [13] (see Table II), respectively. As there are no available $2\nu\beta\beta$-decay data for the transition to the $2^+_1$ excited state of the final nucleus, the distributions in Fig. 1c were normalized to the half-life predicted by the SSD2 approach (see Table II). By glancing Figs. 1 we see that there is different behavior of the single electron differential decay rate calculated within SSD (i.e., SSD2) and HSD approaches especially for small electron energy. It is supposed
that this SSD versus HSD effect is enough large to be studied by the NEMO III experiment, which is currently in preparation \[19\].

The NEMO III experiment is supposed to achieve precise measurement of the angular correlation of outgoing electrons as well. The curves representing the SSD (i.e., SSD2) and HSD approaches for this observable characteristic are just lines with different asymptotic behavior [see Eq. 2.9]. We have

$$\frac{d\omega^I(0^+ \rightarrow J^\pi)}{d\cos \theta} = \frac{1}{2} \omega^I(0^+ \rightarrow J^\pi) [1 + \kappa^I(0^+ \rightarrow J^\pi) \cos \theta].$$

In the case of $2\nu\beta\beta$-decay of $^{100}Mo$ one obtains

$$\kappa^I(0^+ \rightarrow 0^+_g.s.) = -0.627 \ (I = SSD), \ -0.646 \ (I = HSD)$$

$$\kappa^I(0^+ \rightarrow 0^+_1) = -0.487 \ (I = SSD), \ -0.450 \ (I = HSD)$$

$$\kappa^I(0^+ \rightarrow 2^+_1) = 0.153 \ (I = SSD), \ 0.149 \ (I = HSD)$$

We see that there is only a small difference between the SSD and HSD values of $\kappa^I$. Nevertheless, it is expected that this effect can be tested by the NEMO III experiment too \[19\].

We maintain that the study of $2\nu\beta\beta$-decay differential characteristics offers a new possibility to decide whether one low-lying state dominates or not. It is more reliable way as a simple comparison of calculated and measured half–lifes.

**IV. SUMMARY AND CONCLUSIONS**

We have studied the $2\nu\beta\beta$-decay of $^{100}Mo$ in the context of the SSD hypothesis. To our knowledge, the validity of the separation of the lepton and nuclear parts has been discussed for the first time. The transitions to the ground ($0^+$) and excited ($0^+$ and $2^+$) states of the final nucleus has been considered. We have shown that by exact treatment of the lowest state of the intermediate nucleus the $2\nu\beta\beta$-decay half–life is reduced by factor of 20 percent for transitions to $0^+$ states. However, much larger reduction appears for $2\nu\beta\beta$-decay transitions
to $2^+$ states amounting to 300 percent (see Table I). In addition, we have found that the emitted electrons in these $2\nu\beta\beta$-decay transitions are predominantly in the S-wave states in the case the SSD dominance is realized.

Further, we have shown that one can learn more about details of the $2\nu\beta\beta$-decay nuclear transition by measuring the single electron spectra and/or angular distributions of the emitted electrons. We have found that the SSD and the HSD differential decay–rates exhibit different behaviour (see Fig. I). It is expected that this SSD versus HSD effect can be studied experimentally, e.g. by the NEMO III collaboration [19], which has the chance to confirm or rule out the SSD hypothesis in the near future. This kind of information is expected to be very helpful in understanding the details of nuclear structure.

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FIG. 1. Single electron differential decay rate $dw/d\epsilon$ for the $2\nu \beta \beta$-decay of $^{100}Mo$ to the $0^+$ ground (a), the first $0^+_1$ excited (b) and the first $2^+$ excited (c) states in $^{100}Ru$. $\epsilon$ and $m_e$ represent the energy and mass of the electron, respectively. The calculations have been performed within the single-state dominance hypothesis (SSD2-exact calculation) and by assuming the dominance of higher lying states (HSD).
TABLE I. The functions $A^I_{\nu\nu}$ and $B^I_{\nu\nu}$ entering the expressions for the $2\nu\beta\beta$-decay rate in Eqs. (7), (8) and (9) within the SSD1, SSD2 and HSD hypothesis. Here, $K = 1/D(\varepsilon_1, \omega_1) + 1/D(\varepsilon_2, \omega_2)$ and $L = 1/D(\varepsilon_1, \omega_2) + 1/D(\varepsilon_2, \omega_1)$ with $D(\varepsilon_i, \omega_j) = E_1 - E_i + \varepsilon_i + \nu_j$ (i,j = 1,2).

| I | SSD1 | SSD2 | HSD |
|---|------|------|-----|
| $A^I_{0+}$ | $|M^I_{(0+)}M^I_{(0+)}|^2$ | $|M^I_{(0+)}M^I_{(0+)}|^2 \frac{K^2 + L^2 + KL}{12}$ | $|M^\text{exp}_{GT}(0^+)|^2$ |
| $B^I_{0+}$ | $-|M^I_{(0+)}M^I_{(0+)}|^2$ | $-|M^I_{(0+)}M^I_{(0+)}|^2 \frac{2K^2 + 2L^2 + 5KL}{36}$ | $-|M^\text{exp}_{GT}(0^+)|^2$ |
| $A^I_{2+}$ | $|M^I_{(2+)}M^I_{(0+)}|^2 \frac{3}{E_1 - E_i + \Delta}$ | $|M^I_{(2+)}M^I_{(0+)}|^2 \frac{(K-L)^2}{4}$ | $|M^\text{exp}_{GT}(2^+)|^2 \times \frac{(\varepsilon_1 - \varepsilon_2)^2(\nu_1 - \nu_2)^2}{3}$ |
| $B^I_{2+}$ | $|M^I_{(2+)}M^I_{(0+)}|^2 \frac{3}{E_1 - E_i + \Delta}$ | $|M^I_{(2+)}M^I_{(0+)}|^2 \frac{(K-L)^2}{12}$ | $|M^\text{exp}_{GT}(2^+)|^2 \times \frac{(\varepsilon_1 - \varepsilon_2)^2(\nu_1 - \nu_2)^2}{3}$ |
TABLE II. Calculated half-lifes for the $2\nu\beta\beta$-decay transitions from the ground state of $^{100}Mo$ to the ground state ($0^+_{g.s.}$) and excited states ($0^+_1$ and $2^+_k$, $k=1,2$) of $^{100}Ru$ within the single-state dominance hypothesis with approximated (SSD1) and exact (SSD2) $K$ and $L$ factors. $T^{2\nu-\text{exp}}_{1/2}$ is the experimental half-life and $W_{if} = E_i - E_f$ is the energy difference of initial and final nuclei. We considered $\log f_{tEC}$ to be 4.45 [10]. The product of matrix elements $M^I_1M^F_1$ is calculated for $g_A = 1.25$.

| Transition          | $W_{if}$ (MeV) | $\log f_{t\beta^-}$ | $M^I_1M^F_1$ | $T^{2\nu-\text{SSD1}}_{1/2}$ (y) | $T^{2\nu-\text{SSD2}}_{1/2}$ (y) | $T^{2\nu-\text{exp}}_{1/2}$ [Ref.] |
|---------------------|----------------|----------------------|--------------|-------------------------------|-------------------------------|----------------------------------|
| $0^+_{g.s.} \rightarrow 0^+_{g.s.}$ | 4.057          | 4.6                  | 0.352        | $8.97 \times 10^{18}$         | $7.15 \times 10^{18}$         | $(6.82^{+0.38}_{-0.53} \pm 0.68) \times 10^{18}$ [13] |
|                     |                |                      |              |                               |                               | $(9.5 \pm 0.4 \pm 0.9) \times 10^{18}$ [16] |
|                     |                |                      |              |                               |                               | $(11.5^{+3.0}_{-2.0}) \times 10^{18}$ [17] |
|                     |                |                      |              |                               |                               | $(7.6^{+2.2}_{-1.4}) \times 10^{18}$ [18] |
| $0^+_{g.s.} \rightarrow 0^+_1$ | 2.926          | 5.0                  | 0.222        | $5.44 \times 10^{20}$         | $4.45 \times 10^{20}$         | $(6.1^{+1.8}_{-1.1}) \times 10^{20}$ [13] |
| $0^+_{g.s.} \rightarrow 2^+_1$ | 3.517          | 6.5                  | 0.0395       | $4.66 \times 10^{23}$         | $1.73 \times 10^{23}$         | $> 16 \times 10^{20}$ [13] |
| $0^+_{g.s.} \rightarrow 2^+_2$ | 2.694          | 7.1                  | 0.0198       | $3.34 \times 10^{25}$         | $1.45 \times 10^{25}$         | $> 13 \times 10^{20}$ [13] |