Some forgotten features of the Bose-Einstein Correlations∗

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Notwithstanding the visible maturity of the subject of Bose-Einstein Correlations (BEC), as witnessed nowadays, we would like to bring to ones attention two points, which apparently did not received attention they deserve: the problem of the choice of the form of $C_2(Q)$ correlation function when effects of partial coherence of the hadronizing source are to be included and the feasibility to model effects of Bose-Einstein statistics, in particular the BEC, by direct numerical simulations.

I. INTRODUCTION

The subject of Bose-Einstein Correlations (BEC) is so matured nowadays that (almost) everything seems to be already answered and/or understood and what remains is just to systematically deduce from the experimental data information on the spatio-temporal structure of the hadronizing source. That is the main reason of interest in BEC. Nevertheless, we would like to bring attention to two points which, in our opinion, are still worth of debate or, at least, worth to be remembered. They are:

• What is the proper form of the two-particle correlation function $C_2(Q)$ in the case when one wants to account for the effects of the possible partial coherence of the hadronizing source [1]?

• Is it possible to model numerically effects of Bose-Einstein statistics (BE), in particular BEC, and in what way [2]?

Since we address mainly readers already acquainted with the subject of BEC, no special introduction is offered. All necessary material can be found in [1, 2] and references therein.

The above mentioned two points will be addressed in two Sections that follow. We close with short summary.

II. WHICH FORM OF CORRELATION FUNCTION IS THE CORRECT ONE?

In most cases, when analyzing experimental data on BEC and discussing phenomenological models, one uses the following form for the two body correlation function ($Q = \sqrt{-(p_1 - p_2)^2}$ and $p_{1,2}$ are four-momenta of the observed identical bosons):

$$C_2(Q) = 1 + \lambda \cdot \Omega(Q \cdot r).$$

(II.1)

Here $r_\mu$ is a 4-vector parameter, such that $\sqrt{(r_\mu)^2}$ has dimension of length making the product $Q \cdot r = Q_\mu r_\mu = q$ dimensionless. It is usually regarded as representing dimension of the hadronizing source, but in reality it represents the mean distance between the emission points of the two particles considered [3]. Parameter $\lambda \in (0, 1)$, from the theoretical point of view, is usually understood as the degree of chaoticity of the source: totally chaotic source has $\lambda = 1$ and totally coherent $\lambda = 0$. Different shapes of the source were investigated in the literature: $\Omega(Qr) = e^{-Qr^2}; e^{-Q^2r^2}; 1/(1 + Qr)^2; [J_1(Qr)/(Qr)]^2$, to name a few [4]. The most advanced and complete discussion advocating this type of $C_2(Q)$ and justifying its structure can be found in [5].
However, in a number of works it was strongly suggested that in the case when a hadronizing source is partially coherent the proper form of $C_2(Q)$ is the following one \cite{6}:

$$C_2(Q) = 1 + 2p(1 - p)\sqrt{\Omega(Q \cdot r)} + p^2\Omega(Q \cdot r)^2,$$

(II.2)

with $r$ and $\Omega$ defined as above, where $p \in (0, 1)$ replaces $\lambda$ (retaining, however, essentially its meaning).

The imminent question arises then: which expression is the proper one? The answer was proposed in our paper \cite{1} were we have shown that both expressions are correct in its own way, i.e., their form encodes information on the specific features of hadronizing source which they describe; this point was not made apparent in the previous works \cite{5, 6}. And so:

- Eq. (II.1) describes situation in which hadronizing source can be regarded as consisting from the coherent and chaotic subsources acting independently in the proportion given by the chaoticity parameter $\lambda$ \cite{5}.

- Eq. (II.2) describes situation in which there is only one hadronizing source but, for some reason, the phases of all particles are partially aligned. This can happen, for example, when hadronizing source is located in some constant external field, as was the case considered in \cite{1}.

To summarize: the choice of one or other form of $C_2(Q)$ presented here amounts to making a nontrivial assumption concerning the nature of the hadronizing source under investigation. This should be at least remembered and acknowledged, even if in practice only the first choice is nowadays used \cite{2}.

### III. HOW TO MODEL BEC NUMERICALLY

The question of the numerical modelling of BEC is more important than usually anticipated. The point is that to study multiparticle production processes one uses numerical simulations, the Monte Carlo event generators (MCEG) of different kinds. MCEG are based on classical probabilistic schemes whereas BEC is, by definition, quantum phenomenon and as such cannot be incorporated straightforwardly into a MCEG. The suggested cure was the use of the so called *afterburners*: one takes outcome of a given MCEG and changes accordingly momenta of the selected identical particles in such way as to fit the observed data on $C_2(Q)$. However, it must be realized that by doing so one changes not only the original energy-momenta and/or multiplicities (for which one can correct later) but also (and usually unknowingly and in an unknown way) the physics of the model used as the basis of the MCEG chosen \cite{8}.

The best solution would be to perform direct numerical simulations in which MCEG would start from the input containing already effects of BE statistics. What such input should be? The obvious suggestion is: the one possessing property that particles satisfying BE statistics tend to occupy in a maximal way the same state, i.e., they exhibit a *bunching property*. This property can be (at least in principle) modelled \cite{9}. There exist already some examples of such effort. In \cite{10} Metropolis method was used with fully symmetrized wave function to convert the set of $N$ uniformly distributed identical particles into the set of $N$ particles exhibiting the effect of BEC. Closer inspection shows that it happened because in this way particles were effectively bunched in the phase space forming what \cite{10} called *speckles*. In \cite{11} one starts with single particle and, using rejection method, adds to the $N$-th particle the $(N + 1)$-th one following the updated probability as given by the fully symmetrized wave function for $(N + 1)$ particles. Again: the final distribution is characterized by bunches of identical particles in the phase space. In \cite{12} the main point was to account for the Negative Binomial (NB) character of the observed multiplicity distributions $P(N)$ by assuming that particles of the same charge are most likely being produced in the same cells into which the phase space has been divided (it is rapidity in this work). All presented algorithms are very time consuming (especially the first two) and only the last one has been successfully applied to analysis of $e^+e^-$ data \cite{12} (and never again).
In [2] we have summarized our effort aimed at improving this approach. From the examples mentioned it is clear that the procedure of symmetrization of the initial set of identical particles distributed somehow in the phase space takes too much time to be of any practical use. On the other hand, it leads to very interesting result, namely it shows that this procedure results in the effect of bunching of particles in some regions of phase space. Because bunching is easier to simulate than symmetrization, it is this phenomenon which we propose to use as the cornerstone of the algorithm modelling BEC. Therefore we form bunches (called by us Elementary Emitting Cells - EECs) of particles in energy. It can be shown that using Bose-Einstein (or geometrical) form of distribution of particles in a single EEC one gets the characteristic proper BE form of $\langle N(E) \rangle$ together with the characteristic shape of the $C_2(Q)$ function. When the original energy distribution is thermal-like (exponential with scale parameter $T$) then $T$ is temperature seen in $\langle N(E) \rangle$, whereas chemical potential present there is the main parameter describing BE distribution of particles in EEC [2]. The picture proposed resembles closely a quantum version of the clan model [13] (in which all particles in a clan are identical bosons). We call it therefore Quantum Clan Model (QCM) [2]. In this case one gets final multiplicity distribution in the form of Pólya-Aeppli (geometric-Poisson) distribution [14], which differs from the NB distribution of [13] only for very small multiplicities. The strength of BEC, as given by parameter $\lambda$ in (II.1), is very sensitive to the maximal allowed number of particles in the single EEC. Therefore, for extremely high multiplicity cases, $\lambda$ exceeds 2 (even for $C_2(Q)$), a fact not noticed before.

IV. SUMMARY AND CONCLUSIONS

Let us summarize points raised here.

- The first is that deciding, as it is usually done, on the eq. (II.1) when addressing problem of BEC one tacitly assumes that hadronizing source is not influenced by any external field which could make it partially coherent (depending on its strength). Therefore in situations where this cannot be assured its better to use eq. [II.2]. However, the trouble is that so far its form is elaborated only for two-particle BEC, multiparticle case still awaits its proper treatment.

- The second point is that using one of the afterburners proposed in the literature in order to change the outcome of the MCEG actually used, one accepts also (most times unknowingly!) all changes in the physical picture underlying this MCEG. The only way out would be to build a MCEG using the principle of BE statistics as its cornerstone. It can be done by endowing MCEG from the very beginning of the numerical simulation process with property of bunching of identical particles (via geometrical, or Bose-Einstein, particle distribution in each bunch assured numerically). Effects of resonances, final state interaction and the like, can be (in principle) accounted for. They always reduce signal of BEC. The most difficult problem one encounters is the implementation of corrections for nonconservation of energy-momenta and charge introduced during the Monte Carlo selection procedure used. We demonstrate that such program is possible but, at the moment, still far from the completion.

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In [2] it was demonstrated using simple version of cascade model that forcing BEC in this way results in the multicharged vertices to be formed in the cascade process whereas the original cascade had only single valued vertices. Effects of this kind should be therefore expected when using MCEG based on different physical principles. To the best of our knowledge this problem has never been investigated.

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