Entanglement and quantum teleportation under superposed gravitational fields

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Abstract

The influence of gravitational field on entanglement of bipartite states is investigated based on the recent idea of superposition states of gravitational field. Different from earlier considerations, we study the case where the gravitational field cannot be separated unitarily from the bipartite system in the final stage of the interaction. When the different gravitational field states are orthogonal, entanglement cannot be generated for an initial product state. If the different gravitational field states are non-orthogonal, entanglement can be generated and the amount of generated entanglement depends on an overlap parameter between different gravitational field states. The influence of gravitational field on the transfer of the state through quantum teleportation is also studied, which might lead to an observable effect since the quantum teleportation can be performed using macroscopic object.

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I. INTRODUCTION

Entanglement is regarded as the most nonclassical manifestation of quantum formalism and presents the essential resources in quantum information processing tasks (e.g. super-dense coding, quantum teleportation, etc.) [1]. Meanwhile, quantum entanglement is fragile and is easily influenced by the outer environment. Classical gravity is such a general environment and always leads to the decoherence of quantum states, which also caused the loss of entanglement for initial entangled quantum states [2]. It is unclear whether gravity as a quantum entity leads always to the loss of entanglement. This is worthwhile to be studied since the description of the early universe and black holes implies gravity should have one quantum theory [3–7]. For our aim to study the influence of quantum gravitational field on entanglement, the crucial task is how to describe the gravitational interaction using the quantum theory.

In recent years, Bose et al. [8] and Marletto and Vedral [9] have proposed an effective and novel method to detect quantum properties of gravity by considering entanglement generation through gravitational interaction between two massive particles initially in a product state. Following the Bose-Marletto-Vedral (BMV) proposal, extensive discussions [10–18] have been carried out and feasible methods [19–22] have been proposed in different physical systems. Although the BMV proposal has not been confirmed experimentally, it provides an effective approach to analyze the influence of gravitational field on entanglement between particles [16, 17]. In this work, we explore the influence of gravitational field on entanglement for a general bipartite state further, and focus on the states of gravitational field accompanying different components of superposed particle’s states being not completely orthogonal. We also consider the case of an initial Bell state [23, 24] of maximal entanglement, and show that gravitational field would not affect entanglement for some situations. The Bell state will evolve, so does the quantum process of teleportation [25]. In this paper, we will also study the influence of gravitational field on quantum teleportation and reveal a noise-like entangled quantum channel in gravitational field.

This paper is organized as follows. In the second section, we introduce the description for the change of coherence and entanglement, by which the influence of gravitational field on a general bipartite quantum state along the spirit of the BMV proposal is analyzed. In the third section, we study the situation where gravitational field cannot be separated unitarily.
FIG. 1: The setting for a two-particle system with respective masses $m_A$ and $m_B$, placed at distance $d$ from each other. Each particle has two orthogonal eigenstates, $|0\rangle$ and $|1\rangle$. It is required that the distance of the particle at the state $|0\rangle$ is $L$ from that in the state $|1\rangle$, irrespective of whether there exists entanglement between the two particles or not.

from the two-particle system in the final stage of their interaction. Mutual information for estimating the change of entanglement between two particles is then calculated for two different cases: one concerns the orthogonal states of gravitational field, and the other when they are not orthogonal. Meanwhile, the coherence is also discussed for every cases. Subsequently, quantum teleportation under the influence of gravity is studied in the fourth section. Finally, conclusions and discussions are given in the fifth section.

II. GENERAL DESCRIPTION

In BMW proposal, they described the quantum properties of gravitational field using the quantum superposition of spacetime instead of giving the quantum states for gravitational field unambiguously. For indicating the quantum gravitational field, however, we would use the form of quantum states to sign it, i.e. $|g\rangle$ for gravitational field in this paper. The only quantum property is the superposition, i.e. for the same gravitational field, it could have different quantum states, dependent on the quantum states of the source. In particular, the BMV proposal used the first order perturbative quantum gravity, so the interaction
implemented by the gravitational field between two massive particles can be approximated by the Newtonian form, i.e. the interaction part of the Hamiltonian is $$G \frac{m_A m_B}{d}$$ for the two massive particles with the mass $$m_A, m_B$$, and the distance $$d$$ between them. In particular, the two particles do not interact with each other directly but through the gravitational field, which leads to the generation of entanglement for the two particles.

When the two particles interact with the gravitational field, the whole two-particle state would be changed and its coherence might also be changed. It is known that the change of coherence is closely related to the change of entanglement, but they have also the differences. The coherence derives from the superposition of the whole quantum state and depends on the choices of bases when measuring its change, while entanglement describes the relation between two particles. Besides studying the change of entanglement, we expect to discuss the change of coherence using a resource-related definition for quantum coherence [27, 28],

$$C(\rho) \equiv \min \|\rho - \sigma\|$$ where the minimization is made for the diagonal matrix $$\sigma$$ in the set of incoherent states. If the norm is taken as $$l_1$$-norm, the measure of the coherence is obtained as

$$C(\rho) = \sum_{i \neq j} |\rho_{ij}|. \tag{1}$$

It is shown that the $$l_1$$-norm-measure for the coherence is connected with the success probability of unambiguous state discrimination in interference experiments in some works [29, 30].

In order to investigate the change of entanglement between two particles before and after the interaction of gravitational field with the particles, we adopt mutual information [24] as a measure. The mutual information $$I(\rho_{AB})$$ is a well-known physical quantity in quantum information theory. It can measure the degree of bipartite entanglement according to the definition,

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \tag{2}$$

where $$\rho_{AB}$$ is the density matrix of the two particles, $$\rho_A = \text{Tr}_B(\rho_{AB})$$ and $$\rho_B = \text{Tr}_A(\rho_{AB})$$ are the reduced density matrices of $$\rho_{AB}(t)$$, and $$S(\rho)$$ denotes the von Neumann entropy $$S(\rho) = -\text{Tr}(\rho \log_2 \rho) = -\sum_i \lambda_i \log_2 \lambda_i$$, given $$\lambda_i$$ as the eigenvalues of the density matrix $$\rho$$. So in the following, the crucial task is to obtain the results for the change of quantum entangled states under the influence of gravitational field between two particles.

According to the covariant analysis of Ref. [15], the process of interaction between gravitational field and particles are described explicitly. We start with the product between
a general initial state for two particles and the gravitational field,

$$|\psi_1\rangle = (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle) |g\rangle,$$

where $|\psi(0)\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$ describes the initial state for the two particles, and the coefficients satisfy the normalization relation, $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. The two particles of masses $m_A$ and $m_B$, are separated by a distance $d$. Each particle has two orthogonal states, denoted by $|0\rangle$ and $|1\rangle$, arising from its intrinsic properties (e.g. the spin or internal state levels). When state $|\psi(0)\rangle$ is prepared, state $|0\rangle$ is displaced from the state $|1\rangle$ by a distance $L$, as presented in Fig. 1. An implicit assumption behind the state (3) is that it has to be prepared quickly during which cumulative interaction effect with gravity is negligible. It evolves under the influence of gravity for a while, and the state becomes,

$$|\psi_2\rangle = \alpha e^{-iH_{00}t} |00\rangle |g_{00}\rangle + \beta e^{-iH_{01}t} |01\rangle |g_{01}\rangle + \gamma e^{-iH_{10}t} |10\rangle |g_{10}\rangle + \delta e^{-iH_{11}t} |11\rangle |g_{11}\rangle,$$

where the Newtonian gravitational potential is considered as an interaction potential between the two massive particles with $H_{00} = H_{11} = -\frac{G m_A m_B}{d}$, $H_{01} = -\frac{G m_A m_B}{d+L}$, $H_{10} = -\frac{G m_A m_B}{d-L}$ according to the proposed configuration of Fig. 1. The two-particle state becomes entangled with gravitational field. This implies the coherence of the particle state could be lost, although this loss of coherence is not necessarily real for some situations, as discussed in Ref. [20] where coherence can be restored by some unitary operations. Whether the coherence of two-particle state changes depends on the orthogonality of the states of gravitational field and will be discussed in the next section.

Finally, the two components of each particle are brought back together, and the position distribution for the two particles returns to one location, which transforms all four states of gravitational field into the same one, ending up in the state

$$|\psi_3\rangle = (\alpha e^{-iH_{00}t} |00\rangle + \beta e^{-iH_{01}t} |01\rangle + \gamma e^{-iH_{10}t} |10\rangle + \delta e^{-iH_{11}t} |11\rangle) |g\rangle.$$

The final state is a product state between the particles and gravitational field, and the density operator of the particles is expressed as

$$\rho(t) = \begin{bmatrix} |\alpha|^2 & \alpha \beta^* e^{\frac{i\Delta}{\hbar} t} & \alpha \gamma^* e^{\frac{i\Delta}{\hbar} t} & \alpha \delta^* \\ \alpha^* \beta e^{-\frac{i\Delta}{\hbar} t} & |\beta|^2 & \beta \gamma e^{\frac{i\Delta}{\hbar} t} & \beta \delta e^{-\frac{i\Delta}{\hbar} t} \\ \alpha^* \gamma e^{-\frac{i\Delta}{\hbar} t} & \beta^* \gamma e^{-\frac{i\Delta}{\hbar} t} & |\gamma|^2 & \gamma \delta^* e^{-\frac{i\Delta}{\hbar} t} \\ \alpha^* \delta & \beta^* \delta e^{\frac{i\Delta}{\hbar} t} & \gamma^* \delta e^{\frac{i\Delta}{\hbar} t} & |\delta|^2 \end{bmatrix},$$

(6)
where

\[ \Delta_1 = -(H_{00} - H_{01}) = G_A m_B \left( \frac{1}{d} - \frac{1}{d + L} \right), \]

\[ \Delta_2 = -(H_{00} - H_{10}) = G_A m_B \left( \frac{1}{d} - \frac{1}{d - L} \right), \]

\[ \Delta_3 = -(H_{01} - H_{10}) = G_A m_B \left( \frac{1}{d + L} - \frac{1}{d - L} \right). \] (7)

As seen from the quantum states (4) to (5), the coherence changes using the formula
in Eq. (1) which means that the unitary operations in the disentangled process are made
for the combination of quantum states for the particles and the gravitational field, and the
energy is cost to transfer the particles to the same location in the final step. But compared
with the initial state \( |\psi(0)\rangle \), the coherence of the two-particle state does not change, i.e.
\( C_i(\rho) = C_t(\rho) = 2(|\alpha\beta| + |\alpha\gamma| + |\alpha\delta| + |\beta\gamma| + |\beta\delta| + |\gamma\delta|) \) where the superscript \( i \) (t)
represents the coherence at the initial time (at any time \( t \)). This can be checked using
the initial maximal entangled state, e.g. \( \alpha = \delta = 0 \) and \( \beta = \gamma = \frac{1}{\sqrt{2}} \), which leads to
\( C_i(\rho) = C_t(\rho) = 1 \). This implies that the disentangled process is indeed realized using the
unitary operations if the interaction between the particles and gravitational field is assumed
to be unitary. Since the coherence is not changed, what about entanglement?

Using the definition Eq. (2), entanglement of the initial state \( |\psi(0)\rangle \) is calculated to be

\[ I_i = -2 \left( \lambda_1 \log_2 \lambda_1 + \lambda_2 \log_2 \lambda_2 \right), \] (8)

where \( \lambda_1 \) and \( \lambda_2 \) are the eigenvalues of the density operator \( \rho_{AB} = \text{Tr}_{B/A} (|\psi(0)\rangle\langle\psi(0)|) \)
given by the expressions,

\[ \lambda_1 = \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4K_0K_0^*}, \]

\[ \lambda_2 = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4K_0K_0^*}. \] (9)

with \( K_0 = \alpha\delta - \beta\gamma \) and star \( * \) denoting complex conjugation. The initial state \( |\psi(0)\rangle \) is a
product state when \( K_0 = 0 \) or \( \alpha\delta = \beta\gamma \), its mutual information then reduces to \( I = 0 \). For
\( |\psi(0)\rangle \) a maximal entangled state, e.g., a Bell state with \( \alpha = \delta = 0 \) and \( \beta = \gamma = \frac{1}{\sqrt{2}} \), the
result becomes \( I = 2 \), as expected.

For the state in Eq. (6) after interaction with gravitational field, its mutual information
reduces to

\[ I_g = -2 \left( \lambda_3 \log_2 \lambda_3 + \lambda_4 \log_2 \lambda_4 \right) \] (10)
where \( \lambda_3 \) and \( \lambda_4 \) are the eigenvalues of the density operator \( \rho_{A/B} = \text{Tr}_{B/A}(\rho_{AB}) \) given by,

\[
\begin{align*}
\lambda_3 &= \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4K_1K_1^*}, \\
\lambda_4 &= \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4K_1K_1^*},
\end{align*}
\]

(11)

where \( K_1 = (\alpha \delta e^{i(\Delta_1 + \Delta_2)} - \beta \gamma) \).

Comparing the mutual information \( I_i \) in Eq. (8) with \( I_g \) in Eq. (10), it is not hard to find that for an initial product state specified by \( K_0 = 0 \), the final state would not be a product state except at specific instants. Taking \( \alpha = \beta = \gamma = \delta = \frac{1}{2} \) for a product state, we calculate the corresponding mutual information as shown in Fig. 2, which represents entanglement generated and confirms for gravitational field in a quantum superposition of states, as suggested in the BMV proposal. Entanglement is changed with time, which is different from coherence. In particular, when the Bell state with \( \alpha = \delta = 0, \beta = \frac{1}{\sqrt{2}}, \) and \( \gamma = \pm \frac{1}{\sqrt{2}} \) is taken, the mutual information \( I_g = I_i = 2 \) is obtained, which means that gravitational field does not influence entanglement between two maximally entangled particles. Since entanglement is usually regarded as a resource in quantum information, we will investigate in later sections on whether this invariance could hold for operations like quantum teleportation. Moreover, other trivial situations like \( \beta = \gamma = 0, \alpha = \frac{1}{\sqrt{2}}, \delta = \pm \frac{1}{\sqrt{2}} \) are consistent with invariance of entanglement under gravity, which is related to the assumed form of the initial state, i.e. the distance between the components of \(|0\rangle\) is the same as that with \(|1\rangle\) for the two particles.

III. INFLUENCE FROM THE REDUCTION

There exist some situations where the two-particle state cannot be disentangled from its interaction with gravitational field, even though gravitational field is regarded as quantum. For example, with two two-level atoms trapped in an optical cavity [31], when a single photon with proper frequency is sent into the (running wave) cavity, the state of the two atoms would become a Bell state with the photon absorbed by either atom. Whichever atom absorbs the photon, its position would be changed slightly due to the backaction of photon momentum, which gives a concrete realization of the state illustrated in Fig. 1. On the other hand, the randomness associated with the absorption (or not absorbing) makes it impossible for the positions to return by unitary operations for the different components of
FIG. 2: Time evolution of mutual information under gravitational interaction when the two-particle system is initially in a product state.

each atom, so the different states $|g_{ij}\rangle$ ($i, j = 0, 1$) of gravitational field cannot become the same one, leading to a real loss of coherence for the two-particle state.

In this situation, the final state after gravitational interaction becomes $|\psi_2\rangle$ in Eq. (4).

In order to study change of entanglement for the two-particle state, we trace out the gravitational field part. The resulting density operator describing the two-particle state is given by

$$\rho_{AB} = \text{Tr}_g (|\psi_2\rangle \langle \psi_2|).$$

We will respectively evaluate for two cases for orthogonal or non-orthogonal quantum states of gravitational field. First when the different quantum states of gravitational field are orthogonal, $\langle g_{ij}|g_{kl}\rangle = 0$, iff the subscript $ij$ is not completely same with $kl$. The density operator describing the two-particle state reduces to

$$\rho_{AB} = \begin{pmatrix} |\alpha|^2 & 0 & 0 & 0 \\ 0 & |\beta|^2 & 0 & 0 \\ 0 & 0 & |\gamma|^2 & 0 \\ 0 & 0 & 0 & |\delta|^2 \end{pmatrix}$$

This is a diagonal matrix and its coherence loses completely according to Eq. (1). Its mutual information becomes

$$I_{ge} = S_A + S_B - S_{AB}$$
where $S_A = S(\rho_A) = S(\text{Tr}_B(\rho_{AB}))$, $S_B = S(\rho_B) = S(\text{Tr}_A(\rho_{AB}))$, and $S_{AB} = S(\rho_{AB})$ stand for von Neumann entropies for the different density operators $\rho_A$, $\rho_B$, and $\rho_{AB}$. Their calculated results are

$$S_A = -(|\alpha|^2 + |\beta|^2) \log_2 (|\alpha|^2 + |\beta|^2) - (|\gamma|^2 + |\delta|^2) \log_2 (|\gamma|^2 + |\delta|^2), \quad (14)$$

$$S_B = -(|\alpha|^2 + |\gamma|^2) \log_2 (|\alpha|^2 + |\gamma|^2) - (|\beta|^2 + |\delta|^2) \log_2 (|\beta|^2 + |\delta|^2), \quad (15)$$

$$S_{AB} = -|\alpha|^2 \log_2 (|\alpha|^2) - |\beta|^2 \log_2 (|\beta|^2) - |\gamma|^2 \log_2 (|\gamma|^2) - |\delta|^2 \log_2 (|\delta|^2). \quad (16)$$

It is surprising to note despite of interaction with quantum gravitational field, entanglement for the two-particle state does not change with time. For an initial product state with $K_0 = 0$, entanglement cannot be generated ($I_{ge} \equiv 0$) for a completely mixed final state $\rho_{AB}$. It is interesting to note that gravitational field can be superposed although entanglement is not generated if the states of gravitational field are orthogonal and cannot be separated unitarily from the two-particle state. Assuming an initial Bell state, the mutual information decreases from $I_i = 2$ to $I_{ge} = 1$ even for the trivial case discussed in the last section, which provides an obvious signal for the gravitational influence if other environmentally decoherent elements are absent.

Next for the second case where different quantum states of gravitational field are nonorthogonal. Without loss of generality, we assume that only $\langle g_{01} | g_{10} \rangle = k \neq 0$ where $k$ is a complex nonzero number while other ones are orthogonal. The same calculation can be extended to the more general case where none of the gravitational states are mutually orthogonal, although the calculations and presentations become more complex but adds no new results. The possible existence of these cases can be understood since the distance between two components of each particles is not large enough to make gravitational fields generated by the two components obviously different. So it is possible to have nonzero overlaps for the different states of gravitational field.

For this second case, tracing out gravitational field, the normalized density matrix can be obtained as

$$\rho'_{AB} = D \begin{bmatrix}
|\alpha|^2 & 0 & 0 & 0 \\
0 & |\beta|^2 (|k|^2 + 1) & 2\beta \gamma e^{\Delta \chi} k^* & 0 \\
0 & 2\beta^* \gamma e^{-\Delta \chi} k & |\gamma|^2 (|k|^2 + 1) & 0 \\
0 & 0 & 0 & |\delta|^2
\end{bmatrix, \quad (17)$$

where $D = \frac{1}{1 + |k|^2(|\beta|^2 + |\gamma|^2)}$. It shows that the coherence and entanglement for the two-particle
state remains independent of time although the state itself is time-dependent. The general
expression for mutual information becomes a little complicated to give out in this case, so
instead we will discuss several specific situations.

For a direct product state, \( K_0 = 0 \), we choose \( \alpha = \beta = \gamma = \delta = \frac{1}{2} \), and the result of
mutual information can be expressed as

\[
I_{gd} = 2 + 2 \left( \frac{1}{4 + 2|k|^2} \right) \log_2 \left( \frac{1}{4 + 2|k|^2} \right) + \left( \frac{|k| + 1}{4 + 2|k|^2} \right) \log_2 \left( \frac{|k| + 1}{4 + 2|k|^2} \right) \\
+ \left( \frac{|k| - 1}{4 + 2|k|^2} \right) \log_2 \left( \frac{|k| - 1}{4 + 2|k|^2} \right),
\]

which depends on \( k \) but is independent of time. If \( k \) is related to the difference between
different gravitational fields, with \( k = 0 \) denoting the maximal difference as presented in
first case where states of gravitational field are mutually orthogonal, and \( k = 1 \) for the
minimal difference as discussed in the last section when the states of gravitational field are
separable from the two-particle system. The dependence on parameter \( k \) is illustrated in
the upper plot of Fig. 3. It is seen that the mutual information is monotonically increasing
with parameter \( k \), which is consistent with the change of coherence calculated by Eq. (1),

\[
C = \frac{2k}{2 + |k|^2} \quad \text{and} \quad \frac{dC}{dk} = \frac{4 - 2|k|^2}{(2 + |k|^2)^2} > 0 \quad \text{for} \quad 0 \leq |k| \leq 1.
\]

For Bell states, two different situations arise. For the trivial situation discussed in the
last section, \( \beta = \gamma = 0, \alpha = \frac{1}{\sqrt{2}} \), and \( \delta = \pm \frac{1}{\sqrt{2}} \), the mutual information is independent of \( k \)
and is equal to 1 as in the first case. However, for situations with \( \alpha = \delta = 0, \beta = \frac{1}{\sqrt{2}}, \) and
\( \gamma = \pm \frac{1}{\sqrt{2}} \), the mutual information becomes

\[
I_{gb} = 2 + \left( \frac{1}{2} + \frac{|k|}{1 + |k|^2} \right) \log_2 \left( \frac{1}{2} + \frac{|k|}{1 + |k|^2} \right) + \left( \frac{1}{2} - \frac{|k|}{1 + |k|^2} \right) \log_2 \left( \frac{1}{2} - \frac{|k|}{1 + |k|^2} \right),
\]

which is shown in the lower plot of Fig. 3. When \( k = 0, I_{gb} = 1 \), as for the first case where
gravitational field states are mutually orthogonal, and when \( k = 1, I_{gb} = 2 \), as obtained in
the last section where entanglement is unchanged after interaction with gravitational field.
According to the formula (1), the coherence is obtained as \( C = \frac{2k}{1 + |k|^2} \) which is monotonically
increasing with parameter \( k \), i.e. \( \frac{dC}{dk} = \frac{2 - 2|k|^2}{(1 + |k|^2)^2} > 0 \) for \( 0 \leq |k| \leq 1 \), consistent with the change
of entanglement presented in the lower plot of Fig. 3.

An interesting phenomena has to be pointed out that the change trends for the coherence
and entanglement are the same for these different cases in this section. This distinguishes
from the situation that the two-particle state is disentangled from the interaction with
FIG. 3: Mutual information as a function of parameter $k$ when gravitational field states are non orthogonal. The upper plot describes the case of an initial two-particle product state with $\alpha = \beta = \gamma = \delta = \frac{1}{2}$. The lower plot describes the case of an initial two-particle Bell state with $\alpha = \delta = 0$ and $\beta = \gamma = \frac{1}{\sqrt{2}}$.

gravitational field, where the coherence remains unchanged but entanglement changes with time for an initially quantum entangled state.
FIG. 4: The process of quantum teleportation [24]. Top lines represent Alice’s system, while the bottom line represents Bob’s system. $|\phi\rangle$ is an unknown quantum state. After a series of operations including a CNOT gate, a Hadamard gate (H), quantum measurement (M) and other processes (X), the state $|\phi\rangle$ is transferred from Alice to Bob. For a more detailed description please refer to Ref. [24].

IV. INFLUENCE ON QUANTUM TELEPORTATION

The influence of quantum gravitational field on entanglement of the two-particle system has been analyzed above, and it is found that there exist some situations where entanglement is invariant under gravity. In this section, we will investigate how gravitational field affect another quantum entanglement process according to the BMV proposal. Specifically, we consider the process of quantum teleportation, which describes a protocol for transferring an unknown quantum state from one location to another [25]. The two parties are commonly referred to as Alice and Bob, who share an entangled state such as a Bell state to start with. The general process is shown in Fig. 4.

Suppose the teleportation happens between Alice and Bob and they share a Bell state $\frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)$. According to the description in the above, the Bell state becomes $|\psi_B\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\frac{\hbar}{k}t}|01\rangle + e^{-i\frac{\hbar}{k}t}|10\rangle\right)$ under the influence of gravity. Now, an unknown quantum state

$$|\phi\rangle = \tilde{\alpha}|0\rangle + \tilde{\beta}|1\rangle,$$

(20)
with $|\tilde{\alpha}|^2 + |\tilde{\beta}|^2 = 1$, is to be transferred from Alice to Bob. The combined state becomes initially

$$|\phi_0\rangle = |\phi\rangle|\psi_B\rangle = \frac{1}{\sqrt{2}} (\tilde{\alpha}|0\rangle + \tilde{\beta}|1\rangle) \left( e^{-i\frac{H_{01}}{\hbar}} |0\rangle|1\rangle + e^{-i\frac{H_{10}}{\hbar}} |1\rangle|0\rangle \right).$$

(21)

After Alice implements a CNOT gate on the unknown state and her part of the Bell state, the whole state becomes

$$|\phi_1\rangle = \frac{1}{\sqrt{2}} (\tilde{\alpha}e^{-i\frac{H_{01}}{\hbar}} |00\rangle + \tilde{\alpha}e^{-i\frac{H_{10}}{\hbar}} |01\rangle + \tilde{\beta}e^{-i\frac{H_{01}}{\hbar}} |11\rangle + \tilde{\beta}e^{-i\frac{H_{10}}{\hbar}} |10\rangle).$$

(22)

Then, she sends the first qubit through a Hadamard gate, and the state evolves into

$$|\phi_2\rangle = \frac{1}{2} \left( |00\rangle|\phi\rangle_{00} + |01\rangle|\phi\rangle_{01} + |10\rangle|\phi\rangle_{10} + |11\rangle|\phi\rangle_{11} \right),$$

(23)

with

$$|\phi\rangle_{00} = \tilde{\alpha}e^{-i\frac{H_{01}}{\hbar}} |1\rangle + \tilde{\beta}e^{-i\frac{H_{10}}{\hbar}} |0\rangle,$$

(24)

$$|\phi\rangle_{01} = \tilde{\alpha}e^{-i\frac{H_{01}}{\hbar}} |0\rangle + \tilde{\beta}e^{-i\frac{H_{10}}{\hbar}} |1\rangle,$$

(25)

$$|\phi\rangle_{10} = \tilde{\alpha}e^{-i\frac{H_{01}}{\hbar}} |1\rangle - \tilde{\beta}e^{-i\frac{H_{10}}{\hbar}} |0\rangle,$$

(26)

$$|\phi\rangle_{11} = \tilde{\alpha}e^{-i\frac{H_{01}}{\hbar}} |0\rangle - \tilde{\beta}e^{-i\frac{H_{10}}{\hbar}} |1\rangle.$$  

(27)

If Alice measures the first and second qubits, she will find four different outcomes: 00, 01, 10, and 11. The corresponding quantum states obtained by Bob would be $|\phi\rangle_{00}$, $|\phi\rangle_{01}$, $|\phi\rangle_{10}$, and $|\phi\rangle_{11}$. After the measurement, Alice sends the measured results to Bob through a classical channel. Bob converts his quantum state using the associated unitary operations according to Alice’s message, and finally obtains the same quantum state as if there were no influence of gravity. In order to study the influence of gravity, we assume that the measured result by Alice is 00, and thus the Bob would obtain a state with the form $\tilde{\alpha}e^{-i\frac{H_{01}}{\hbar}} |0\rangle + \tilde{\beta}e^{-i\frac{H_{10}}{\hbar}} |1\rangle$ after an operation of X gate. After absorbing the factor of $e^{-i\frac{H_{10}}{\hbar}}$, the final quantum state obtained by Bob can be reduced to

$$|\phi'\rangle = \tilde{\alpha} |0\rangle + \tilde{\beta}e^{i\frac{H_{10}}{\hbar}} |1\rangle.$$  

(28)

Obviously, the state $|\phi'\rangle$ includes a gravitational field dependent phase and is different from the unknown quantum state $|\phi\rangle$. In principle, one can carry out unitary operations to transform the state $|\phi'\rangle$ to $|\phi\rangle$, but this is difficult because the influence of gravity cannot be controlled precisely.
Now we estimate the faithfulness or fidelity of the teleportation, which can be measured by the mean distance \[ F = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi F \sin \theta d\theta, \] between the transferred state \(|\phi\rangle\) and the received state \(|\phi'\rangle\), where \(\rho\) is the density matrix of state \(|\phi\rangle\), \(\rho'\) is the density matrix of state \(|\phi'\rangle\), \(p_k\) is the probability of obtaining the result \(k\) by Alice, and \(M(\phi)\) denotes uniform distribution on the Bloch sphere \(S\). It has been shown \[33\text{-}35\] that \(F = \frac{1}{2}\) if the received state is completely independent of the transferred state; \(F = \frac{2}{3}\) for the purely classical channel or the channel with the product state; \(F \simeq 0.87\) for the boundary for the local and non-local states in the sense of local hidden variables, i.e. the state of the channel reveals “nonclassical aspects” incompatible with local hidden variables only if \(F > 0.87\).

For the transferred state \(|\phi\rangle\), we take \(\tilde{\alpha} = \cos \left(\frac{\theta}{2}\right)\), \(\tilde{\beta} = \sin \left(\frac{\theta}{2}\right) e^{i\phi}\), where \(\theta \in [0, \pi]\) and \(\phi \in [0, 2\pi]\), and obtain the averaged fidelity as

\[
F = \sum_k p_k Tr(\rho' \rho) \quad \text{where} \quad F = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi F \sin \theta d\theta,
\]

This averaged fidelity is time-dependent and depends on gravitational field, as shown in the upper plot of Fig. 5. Note that the maximal value of the averaged fidelity is \(1\), which shows that at some instants the teleportation is perfect; the minimal value of the averaged fidelity is \(\frac{1}{3}\), which is hardly possible to teleport any unknown state. In the upper plot of Fig. 5, the black line represents \(F(\frac{\sqrt{3}}{2})\), above which non-product states exist; the red line represents \(F(\frac{\sqrt{3}}{2}) \simeq 0.87\), above which the non-local states exist, unexplainable by the local hidden variables. In particular, the limit for the averaged fidelity over time becomes

\[
limit_{t \to \infty} \frac{\int_0^t F dt}{t} = \frac{2}{3},
\]

which shows that the coherence and entanglement for the two-particle state disappear after long enough time.
FIG. 5: The averaged fidelity of an unknown quantum state transferred through quantum teleportation under the influence of gravitational field. The upper plot represents the change of averaged fidelity with time when entanglement is not changed by gravitational interaction. The lower plot represents the change of averaged fidelity with time and parameter $k$ when the gravitational field states are non-orthogonal.

In general, the gravitational field states cannot be disentangled unitarily in the final stage of the interaction, since during quantum teleportation the required operations are difficult to implement. The Bell state as an entanglement channel becomes

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\left( e^{-i\frac{H_{01}}{\hbar}t}|01\rangle|g_{01}\rangle + e^{-i\frac{H_{10}}{\hbar}t}|10\rangle|g_{10}\rangle \right).$$

(32)
The teleportation as implemented between Alice and Bob goes through the same procedures above for without gravity interaction. Before Alice performs the measurement, the state becomes

\[ |\phi'_2\rangle = \frac{1}{2} (|00\rangle |\phi'_{00}\rangle + |01\rangle |\phi'_{01}\rangle + |10\rangle |\phi'_{10}\rangle + |11\rangle |\phi'_{11}\rangle), \]

(33)

where

\[ |\phi'_{00}\rangle = \tilde{\alpha} e^{-i\frac{H_{00} t}{\hbar}} |g_0\rangle + \tilde{\beta} e^{-i\frac{H_{10} t}{\hbar}} |g_10\rangle, \]

(34)

\[ |\phi'_{01}\rangle = \tilde{\alpha} e^{-i\frac{H_{00} t}{\hbar}} |g_0\rangle + \tilde{\beta} e^{-i\frac{H_{10} t}{\hbar}} |g_10\rangle, \]

(35)

\[ |\phi'_{10}\rangle = \tilde{\alpha} e^{-i\frac{H_{00} t}{\hbar}} |g_0\rangle - \tilde{\beta} e^{-i\frac{H_{10} t}{\hbar}} |g_10\rangle, \]

(36)

\[ |\phi'_{11}\rangle = \tilde{\alpha} e^{-i\frac{H_{00} t}{\hbar}} |g_0\rangle - \tilde{\beta} e^{-i\frac{H_{10} t}{\hbar}} |g_10\rangle, \]

(37)

Analogously, we suppose the measured result by Alice is 00, so the final state obtained by Bob becomes

\[ |\phi'\rangle = \tilde{\alpha} e^{-i\frac{H_{00} t}{\hbar}} |g_0\rangle + \tilde{\beta} e^{-i\frac{H_{10} t}{\hbar}} |g_10\rangle, \]

(38)

which depends on gravitational field. After tracing out gravitational field, the state obtained by Bob is described by the density operator

\[ \rho' = |\tilde{\alpha}|^2 |0\rangle\langle 0| + \frac{2k\tilde{\alpha}\tilde{\beta}^*}{|\tilde{k}|^2 + 1} e^{-i\frac{\Delta_k t}{\hbar}} |0\rangle\langle 1| + \frac{2k^*\tilde{\alpha}^*\tilde{\beta}}{|\tilde{k}|^2 + 1} e^{i\frac{\Delta_k t}{\hbar}} |1\rangle\langle 0| + |\tilde{\beta}|^2 |1\rangle\langle 1|, \]

(39)

assuming \( \langle g_{01}|g_{10}\rangle = k \).

The fidelity can be calculated as \( F(|\phi\rangle, \rho') = 4 \times \frac{1}{4} \langle \phi|\rho'|\phi \rangle = 1 - \frac{1}{2} Q \sin^2 \theta \) with \( Q = 1 - \frac{k^2 + k}{|k|^2 + 1} \cos(\frac{\Delta_k t}{\hbar}) + i \frac{k^2 - k}{|k|^2 + 1} \sin(\frac{\Delta_k t}{\hbar}) \). According to the definition of averaged fidelity (29), we obtained

\[ \overline{F}(|\phi\rangle, \rho') = 1 - \frac{1}{3} Q. \]

(40)

When the parameter \( k \) is taken as real, the fidelity takes the form \( \overline{F}(|\phi\rangle, \rho') = \frac{2}{3}(1 + \frac{k}{|k|^2 + 1} \cos(\frac{\Delta_k t}{\hbar})) \), which is presented in the lower plot of Fig. 5. It is seen from the figure that the fidelity of transferring an unknown state is closely dependent on the properties of gravitational field. When \( k \) is large, the oscillation of the averaged fidelity is obvious with time, which includes the states for the channel with their averaged fidelity below \( \frac{2}{3} \), and some other states that presented the non-locality above 0.87. When \( k \) decreases, the fluctuation becomes smaller and smaller. It is equal to \( \frac{2}{3} \) for \( k = 0 \), which shows that the state for the channel becomes a complete mixed state or a classical state, as discussed at the beginning of the second section, and such states have no coherence.
V. CONCLUSION AND DISCUSSION

In this work, we investigate the influence of gravitational field on quantum entanglement following the BMV proposal but for a general two-particle state. It is found again that entanglement can be generated for an initial product state inside gravitational field but the coherence is unchanged in the process. This entanglement can be invariant with time when the initial state is a Bell state. Both require a crucial operation to separate unitarily gravitational field from the two-particle state in the final stage of evolution.

In some cases, gravitational field is not easily or is impossible to be separated unitarily from the particles. We have analyzed the corresponding processes of interaction between a two-particle system and gravitational field. When the gravitational field states are orthogonal, the influence on entanglement for the two-particle state is time-independent. For an initial product state, entanglement cannot be generated through interaction with gravitational field, since the final two-particle state is a maximally mixed state. For a Bell state, its entanglement decreases as the mutual information is decreased from 2 to 1. This is an evident signal for experimental observation if other environmentally decoherent elements can be avoided. When the gravitational field states are not completely orthogonal, the influence of gravitational field on entanglement for the two-particle state depends on the degree of overlap (measured by the parameter $k$) among the different gravitational field states, for all the above cases considered with a tunable parameter $k$. Whether the states for gravitational field are orthogonal or under what conditions they could be orthogonal is unclear for the time being, it is therefore necessary and significant to analyze the case where the gravitational field states are non-orthogonal. Our results indeed present different behaviors which are helpful for possible future observations. In particular, the change of coherence has the same trend of increasing monotonically with the change of entanglement in this case that the two-particle state cannot be disentangled from gravitational field. While the two-particle state is disentangled from gravitational field, the coherence remains unchanged but entanglement presents a oscillating behaviors with time.

We have also discussed the influence of gravitational field on quantum teleportation, and we find that an exact transfer of an unknown quantum state is hardly possible and the relative phase containing information about gravitational field always appears in the transferred state. In particular, quantum teleportation can be broken for some evolved
states since they have lost the non-locality and even lost all entanglement and coherence.

Finally, we point out that the gravitational field effect discussed is very small for microscopic particles, but could be observable in the future for mesoscopic particles as analyzed in the initial BMV proposals. Since quantum teleportation can be performed using entangled channel between mesoscopic or even macroscopic physical systems [36–38], the influence of gravitational field might be observed in these experiments in the near future.

VI. ACKNOWLEDGEMENT

This work is supported by Grant No. 11654001 of the National Natural Science Foundation of China (NSFC).

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