Optical and Electro-Optical Properties of YbFe$_2$O$_4$ Thin Films

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Abstract. We present the optical and electro-optical properties of YbFe$_2$O$_4$ thin films, deposited on (0001) sapphire substrates. The optical spectra of YbFe$_2$O$_4$ show several electronic peaks arising from Fe$^{2+}$-$d$ to $d$ on-site and O $2p$ to Fe $3d$, Yb $6s$, and Yb $5d$ charge-transfer transitions. The temperature dependence of the $d$ to $d$ electronic transition displays an anomaly around 150 K, which could be associated with a structural instability. Moreover, the YbFe$_2$O$_4$ thin film show strong electric-field-induced changes in optical properties in the energy range of 1-3 eV for applied electric fields less than 1 kV/cm. These electro-optical effects, which vary almost linearly with applied electric fields, are up to 14% in magnitude at low temperatures and the effects completely disappear above 120 K. The observed electro-optical effects could be associated with the effects of external electric fields on the orbital-charge ordering of Fe ions.

INTRODUCTION

Materials possessing both magnetic and ferroelectric properties are referred to as multiferroics. These two properties could inter-couple through a unique magneto-electric coupling, making it possible to manipulate the magnetization by electric fields and electric polarization by magnetic fields. Such coupling of the two order parameters is intriguing from the points of view of fundamental physics and potential applications. [1, 2] For example, magnetoelectric coupling is potentially useful for many device applications including magnetic sensors and actuators, storage devices, energy harvesting devices, and spintronic devices. [3, 4, 5, 6, 7, 8] In general, multiferroics are classified into type-I and type-II multiferroics depending on what causes ferroelectricity in the system. In type-I multiferroics, ferroelectric polarization is caused by one of the physical mechanisms, such as lone-pair electrons, structural distortion, and charge-ordering. [2, 8, 9] The magnetic and ferroelectric orderings occur at two different temperatures and the magnetoelectric coupling is usually weaker. Most of multiferroics discovered so far fall into this category. In type-II multiferroics, a ferroelectric polarization is directly associated with the spin structures that break the inversion symmetry. [2, 8, 9, 10] Because the magnetism induces ferroelectricity, the magnetoelectric coupling is relatively strong in type-II multiferroics but the magnetic transition temperature is well below room temperature. While a tremendous progress has been made in sample synthesis and film fabrication processes, the discovery of room temperature multiferroics with a strong magnetoelectric coupling still remains elusive.

YbFe$_2$O$_4$ belongs to the RFe$_2$O$_4$ (R = Y, Dy to Lu) rare-earth ferrite family which is considered as a multiferroic. [11] This ferrite family has the rhombohedral crystal structure (R3m) [12] characterized by an alternating stacking of a hexagonal double layer of FeO$_6$ bipyramids and RO$_6$ octahedra. Similarly, this family has mixed-valence ions with an equal number of Fe$^{2+}$ and Fe$^{3+}$ ions in a triangular lattice, resulting in both charge and spin frustrations in the system. As a result of the spin and charge frustrations, these ferrites exhibit unique magnetic properties, charge-ordered state, and spin-lattice coupling. [9, 11, 13, 14] The magnetic transition due to the ordering of the Fe$^{2+}$ and Fe$^{3+}$ moments occurs around 240 K in this family. Similarly, the Fe$^{2+}$ - Fe$^{3+}$ charge-ordered state has been reported in LuFe$_2$O$_4$ [15, 16, 17] and in YbFe$_2$O$_4$ [18, 19, 20]. Such charge ordering is believed to break the space-inversion symmetry, leading to macroscopic ferroelectric polarization. [9] This type of ferroelectricity is referred to as electronic ferroelectricity. Furthermore, the structural instability has been reported for LuFe$_2$O$_4$ and YFe$_2$O$_4$. In particular,
the structural distortions from a hexagonal structure to a monoclinic structure and then to a triclinic structure have been reported below the magnetic transition temperature. [21, 22, 23]

While the ferroelectricity driven by the charge ordering mechanism has been proposed for the RFe$_2$O$_4$ family, the experimental evidence of a spontaneous polarization in this family is not convincing. For instance, the colossal dielectric properties of LuFe$_2$O$_4$ and other RFe$_2$O$_4$ members have been associated with ferroelectricity. [11, 24, 25, 26] In contrary, the colossal dielectric property of LuFe$_2$O$_4$ is controversial because the dielectric data can be described by the extrinsic effects, such as the Maxwell-Wagner model. [27, 28, 29, 30, 31] On the other hand, recently Nagata et. al have reported the spontaneous electric polarization in YbFe$_2$O$_4$. [32] Similarly, Fujiwara et. al have used second harmonic generation and neutron diffraction to confirm the existence of electronic ferroelectricity in YbFe$_2$O$_4$. [33] These new reports certainly bring attention to the electronic ferroelectricity in YbFe$_2$O$_4$. Although there has been a decent amount of work on YbFe$_2$O$_4$, most of the published articles have been on the bulk samples and there is only a small number of studies on YbFe$_2$O$_4$ thin films. [34, 35, 36, 37] Additionally, the systematic studies on the optical properties and electronic excitations of YbFe$_2$O$_4$ are very limited. Since both ferroelectric and magnetic properties of YbFe$_2$O$_4$ come from the Fe $d$ electrons, understanding of the electronic transitions and their temperature dependence is very important. And, for such studies, YbFe$_2$O$_4$ thin films are well suited than the bulk samples. Moreover, the electro-optical effects of the ferrite family could offer insight into the charge-ordered state and possible existence of ferroelectricity. We can utilize optical spectroscopy to investigate the optical and electro-optical properties of YbFe$_2$O$_4$.

In this article, we report on the optical and electro-optical properties of YbFe$_2$O$_4$ thin films, deposited on (0001) sapphire substrates by a reactive electron beam deposition system. We measured the temperature-dependent optical spectra in the temperature range of 10 - 300 K using optical spectroscopy. We show that the optical spectra of YbFe$_2$O$_4$ manifest several electronic transitions attributed to Fe$^{2+}$-$d$ to $d$ on-site and O 2$p$ to Fe 3$d$, Yb 6$s$, and Yb 5$d$ charge-transfer transitions. The temperature dependent absorption data shows a subtle change around $\sim$150 K. The electro-optical properties of YbFe$_2$O$_4$ have been measured and discussed in the context of a ferroelectric state in the system.

**EXPERIMENTAL METHODS**

YbFe$_2$O$_4$ was prepared by a conventional solid state reaction. The powder samples (≈99.9% purity from Alfa Aesar Inc.) of Yb$_2$O$_3$, Fe$_2$O$_3$, and FeO were mixed in a stoichiometric ratio. The mixture was thoroughly ground in mortar and pestle and then pressed into pellets and sintered at 1100 °C for 15 hours in a tube furnace. After sintering, the process was repeated two more times: ground, pressed into pellets, and sintered. Finally, the prepared pellets (13 mm x 5 mm) were used as a target material for the electron beam deposition. Before film deposition, we preheated substrates to 600 °C and maintained at this temperature for 30 min to remove any residues from the substrates. Then, we slowly increased the substrate temperature to the final temperature (830 °C). As for the source target, we slowly and carefully increased the electron beam power so that the pellet surface started melting without spattering of the material. Once the target surface started melting, the film deposition rate increased significantly. Controlling of the power is the key to the quality of the thin films. We deposited thicker than 100 nm YbFe$_2$O$_4$ films on single crystal (0001) sapphire and (111) yttrium-stabilized zirconia substrates, maintained at 830 °C with the film deposition rate of 90 Å/min. The film thickness was measured by a quartz crystal monitor during the deposition process and later by an optical method. The chamber base pressure and the oxygen partial pressure were 2.5 x 10$^{-4}$ Pa and 5 x 10$^{-2}$ Pa, respectively. All deposited thin films were annealed at 600 °C in the mixture of oxygen and air for about 3 hours.

We measured variable-temperature (10 - 300 K) normal-incidence optical transmittance and reflectance in the wavelength range of 190 - 2500 nm, with a spectral resolution of 1 nm, using a dual-beam spectrophotometer (Lambda 950, Perkin Elmer) and in the wavelength range of 250 - 1000 nm, with a spectral resolution of 1 nm, using a fiber optic spectrometer (StellarNet) coupled with a closed-cycle helium refrigeration system. For the electro-optical measurements, gold films were evaporated on the two sides of the sample and the copper wires were attached with silver paste [as shown in the inset of Fig. 4(b)]. The thin film length was about 3-4 mm long between the electrodes. The electric fields were applied to the sample using a Keithley 2400 source meter. All electro-optical measurements were carried out by the fiber optic spectrometer with a charge-coupled detector (2048 pixel). The integration time for the detector was used between 1 and 2 seconds, thus minimizing any possible sample heating due to Joule heating as well as a signal drifting issues.

**RESULTS AND DISCUSSION**

We present below the optical and electro-optical data of the polycrystalline YbFe$_2$O$_4$ thin films, deposited by a reactive electron-beam evaporation. The thin film characterizations have been presented elsewhere. [38] We car-
carried out the transmittance and reflectance measurements on multiple YbFeO$_3$/sapphire thin films with zero field and with applied electric fields in order to investigate the absorption spectrum and electro-optical properties.

Figure 1(a) shows the room temperature reflectance (R) and transmittance (T) spectra for a YbFeO$_3$/sapphire thin film. The reflectance and transmittance spectra were simultaneously fitted to measure the thickness of the film. [39, 40, 41] We used the TFCompanion software (Semiconsoft Inc.) to fit the spectra, as shown in Fig. 1(b). The fits compare reasonably well with the experimental data. The solid lines represent the fittings, and the thickness of the film was measured to be $\sim$145 nm. Room temperature absorption coefficient ($\alpha$) versus photon energy for YbFeO$_3$ is shown in Fig. 1 (c). The absorption coefficient was calculated using $\alpha = -(1/d)\ln[T/(1 - R)]$, where $d$ is the film thickness, $R$ is the reflectance, and $T$ is the transmittance. The inset shows a close-up view of a very weak peak centered around $\sim$1.5 eV. (d) The peak fitting of the absorption spectrum. A solid line (red) represents the fitting for the absorption (solid circle) with 4 peaks as indicated by the colored solid lines. The Gaussian function was used for the peak fitting process.

The crystal-field splitting of (b) Fe$^{3+}$($d^7$) and (c) Fe$^{2+}$($d^8$) due to the oxygen ligands. The arrows represent the $d$-electrons with their spins in each state.

Figure 2(a) shows the two FeO$_5$ trigonal bipyramids are the building block of the YbFe$_2$O$_4$ structure. Two bipyramids with Fe$^{3+}$ and Fe$^{2+}$ ions, respectively, at the center of the bipyramids are shown. The crystal-field splitting of (b) Fe$^{3+}$($d^7$) and (c) Fe$^{2+}$($d^8$) due to the oxygen ligands. The arrows represent the $d$-electrons with their spins in each state.

The absorption spectrum of YbFe$_2$O$_4$ contains five electronic excitations, including the weak peak at $\sim$1.5 eV. The absorption spectrum is very similar to that of isostructural LuFe$_2$O$_4$ and YFe$_2$O$_4$ thin films. [42, 43] Based on the first principle calculations [44, 45] and the optical data [46], we assigned the peaks near $\sim$1.50 eV and $\sim$2.40 eV to the Fe$^{2+}$($3d^8$) d to d electronic transitions. Weak oscillator strengths of these transitions are consistent with the fact that these Fe d to d transitions are normally forbidden by the Laporte rule, but the p-d hybridization, the local distortion and/or the spin-lattice coupling in the system relax the selection rules. The broad transitions centered at 3.09 eV, 3.92 eV, and 5.11 eV are mainly due to the O 2p to Fe 3d, Yb 6s and 5d charge-transfer electronic transitions.

Figure 2(b) and (c) show the five-coordinated oxygen ligand fields in FeO$_3$ split the originally degenerated Fe d states into three groups: (d$_{xy}$, d$_{xz}$, d$_{yz}$), (d$_{z2}$, d$_{x2-y2}$), and (d$_{x2}$, d$_{y2}$). Since the Fe$^{2+}$($d^8$) ions are in a high spin state, all five d-states are singly occupied except the lowest one which is doubly occupied by two electrons with anti-parallel spins (up and down arrows). For Fe$^{3+}$($d^7$)
ions, all five $d$-states are singly occupied. Therefore, with the given electronic configurations, there are two possible spin-allowed electronic transitions in Fe$^{2+}(d^8)$: $d_{xy} \rightarrow (d_{xx}, d_{yz})$, and $d_{xy} \rightarrow d_{zz}$, which are consistent with the observed electronic transitions at 1.50 and 2.40 eV in the absorption spectrum. On the other hand, there are no spin-allowed electronic transitions in Fe$^{3+}(d^7)$ because each $d$-state is only singly occupied by five electrons.

The energy bandgap of the YbFe$_2$O$_4$ thin film can be extracted by fitting the $(\alpha \cdot E)^2$ versus energy ($E$) data to the theoretical model, $\alpha(E) = E/E_g - 1/2$ [47], where $E_g$ is the direct energy band gap and $\alpha$ is a constant. Figure 3(a) shows the $(\alpha \cdot E)^2$ versus $E$ graphs for YbFe$_2$O$_4$ at 10 K and 300 K, respectively. The solid lines represent the fitting of the graph using the direct energy band gap model. The energy bandgaps were measured to be 2.20 $\pm$ 0.03 eV and 2.23 $\pm$ 0.03 eV at 300 K and 10 K, respectively. Figure 3(b) shows a close-up view of the low-energy region of the absorption spectra between 10 and 300 K. The band edge shifts to the higher energy region on cooling. To study the temperature dependence, we extracted the absorption edge of the 2.40 eV electronic transition for each temperature. We measured the intensity of the absorption edge at 2.20 eV, as shown in Fig. 3(c), which displays a non-linear behavior. The slope of the the curve changes below 150 K, suggesting that the electronic structures are different below and above 150 K. It is very likely that this non-linear behavior of the absorption edge is indicative of the low temperature structural changes below 170 K. We earlier reported the non-linear temperature dependence of the energy gap for YbFe$_2$O$_4$. [38] Figure 3(d) shows the reflectance ratio $|\Delta R(T)/R|$ as a function of photon energy between 10 K and 300 K. The reflectance at each temperature is divided by the 300 K reflectance data. We discuss this graph in relation to the electro-optical effects below.

In order to investigate the electric-field induced changes in the optical properties, we measured the electro-optical effects $|\Delta R/E| = R(E)/R(0)-1$ of a YbFe$_2$O$_4$ thin film. Here, $R$ is the reflectance of the sample and $E$ is the applied electric field on the sample. Figure 4 (a-d) shows the electro-optical (EO) response of the YbFe$_2$O$_4$ thin film as a function of photon energy at (a) 10 K, (b) 20 K, (c) 50 K, and (d) 70 K, respectively. The EO effects are up to $\pm 10\%$ at 10 K and $\pm 3\%$ at 70 K. The integrated area of the EO effects at 10 K and 50 K varies linearly with the applied electric fields, as shown in the insets of (a) and (c). The EO effects are localized in the 1.25 - 3.0 eV range. The inset of (b) shows the sample with copper wires mounted on the gold electrodes.

FIGURE 3. (a) The energy bandgap extraction from the $(\alpha \cdot E)^2$ versus photon energy for YbFe$_2$O$_4$ at 10 K and 300 K, respectively. A solid line represents the fitting of the graph using the direct energy band gap model. (b) A close-up view of the absorption spectra in the low-energy region between 10 and 300 K. (c) Temperature dependence of the absorption edge of the 2.40 eV electronic transition, indicating a non-linear behavior. (d) The reflectance ratio $|\Delta R(T)/R|$ as a function of photon energy between 10 K and 300 K. The reflectance at each temperature (in a temperature increment of 10 K) is divided by the 300 K reflectance data.

FIGURE 4. The electro-optical response $|\Delta R(E)/R|$ of a YbFe$_2$O$_4$ thin film as a function of photon energy at (a) 10 K, (b) 20 K, (c) 50 K, and (d) 70 K, respectively. The EO effects are localized in the 1.25 - 3.0 eV range. The inset of (b) shows the sample with copper wires mounted on the gold electrodes.
if a sudden change in the sample temperature contributes to the observed EO effects. It is noted that a typical resistance of a YbFe$_2$O$_4$ thin film is on the order of $10^{12} \ \Omega$ below 100 K, which means that the Joule heating effect on the sample for $E = 400 \ \text{V/cm}$ would be very small. Further, we note that the sample is being continuously cooled while taking the data and the data acquisition time was less than 2 seconds. One of the main reasons we carried out the EO measurements using a fiber optic spectrometer was that the spectrometer has a charge-coupled detector which enables us to capture the whole spectrum (250 - 1080 nm) within a few seconds, thus minimizing any sample heating issues. As shown in Fig. 3d, the temperature-induced effects ($\Delta R/R$) are 1-2 % for a temperature change of $\sim 10 \ \text{K}$ and about 15 % for a temperature change of $\sim 290 \ \text{K}$. Even if the sample temperature was increased due to Joule heating, it would be on the order of 10 K. Consequently, the contributions of the heating effects on the observed EO effects are insignificant. Moreover, the temperature-induced $\Delta R/R$ is mostly localized between 2 eV and 2.5 eV whereas the EO effects are spread over the 1.25 - 3 eV range, indicating that the EO effects are indeed the field-induced effects.

For the given temperature, the EO effect increases with the applied field. To understand the field dependence of the effects, we integrated the area under the EO curve for each field. The insets of Fig. 4(a),(c) show the integrated area of the EO effects as a function of $E$ for 10 and 50 K. As shown, the EO effects vary almost linearly with the applied electric fields, confirming that the EO effects are linear in YbFe$_2$O$_4$. Note that the EO effects are mostly observed between $\sim 1.25$ and 3.0 eV where Fe$^2+$ to d on-site electronic transitions take place. Moreover, the Fe$^{2+}$ and Fe$^{3+}$ ions are reported to have the orbital-charge ordering in YbFe$_2$O$_4$. [9, 11] Therefore, the observed EO effects can be linked with the effects of external electric fields on the orbital-charge ordering of the Fe ions, consequently affecting the Fe$d$ to $d$ transitions. We have observed similar but weaker EO effects in isostructural YFe$_2$O$_4$ and LuFe$_2$O$_4$ thin films. [42, 43] It is likely that the EO effects in the RFe$_2$O$_4$ family are the results of the Fe$^{2+}$-Fe$^{3+}$ orbital-charge orderings and therefore, the EO effects could have a direct link with ferroelectricity in RFe$_2$O$_4$. Our future work will focus on understanding the charge-ordering induced ferroelectricity and its connections with the EO effects in the RFe$_2$O$_4$ system.

**CONCLUSION**

In summary, we investigated the optical absorption, electronic excitations, and EO effects in the polycrystalline YbFe$_2$O$_4$ thin films. The absorption spectrum of YbFe$_2$O$_4$ show two Fe$d$ to $d$ on-site transitions at 1.50 eV and 2.40 eV, while the charge-transfer transitions from O 2p to Fe 3d, Yb 6s, and Yb 5d are observed at 3.09 eV, 3.92 eV, and 5.11 eV, respectively. And, these electronic transitions display strong temperature dependence with an anomaly at $\sim 150 $ K, indicating a structural instability. More importantly, YbFe$_2$O$_4$ exhibits strong electric-field-induced changes in optical properties in the 1-3 eV energy range for applied electric fields up to 400 V/cm. The linear EO effects are greater than 10 % at low temperatures which completely disappear at $\sim 120 $ K. The observed EO effects are caused by the effects of external electric fields on the Fe$^{2+}$-Fe$^{3+}$ orbital-charge ordering. Finally, the EO effects and ferroelectricity could possibly be directly related in YbFe$_2$O$_4$.

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