Potential distribution around the charged particle in the collisional weakly ionized plasma in an external electric field

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Abstract. A point-like absorbing charged particle in the uniform anisotropic plasma with an external uniform static electric field is under consideration. The ion and electron motion is described in the drift-diffusion approach with a variable mobility. Small perturbations for plasma parameters are considered. The asymptotic expression for electric potential perturbation at large distances is derived and discussed. The analytical approximation for potential distribution around particle is proposed.

The interaction of the charged dust particles with an ion flow is responsible for formation of string-like and anisotropic crystalline structures in complex plasmas. Anisotropic or wake potential around a charged particle in plasma with ion drift is the most thoroughly studied problem for collisionless or rare collisions conditions [1–4]. But, string-like structures of dust particles have been observed in the discharge plasma where a mean free path of ions is three times smaller than interparticle distance [5]. Under such conditions the rare collisions approach is failed, but the hydrodynamic approach is valid.

A point-like charged particle in the uniform anisotropic plasma with an external field \( \mathbf{E}_0 \) is under consideration. The ion and electron motion is described by the following equations:

\[
\begin{align*}
\mathbf{j}_e &= -\mu_e \left( \frac{k_B T_e}{e} \right) \nabla n_e - \nabla \varphi n_e, \\
\mathbf{j}_i &= -\mu_i \left( \frac{k_B T_i}{e} \right) \nabla n_i + \nabla \varphi n_i, \\
\nabla \mathbf{j}_e &= \nabla \mathbf{j}_i = -J_d \delta(\mathbf{r}),
\end{align*}
\]

where \( \mathbf{j}_e, \mathbf{j}_i \) are electron and ion flux densities, \( \mu_e, \mu_i \) are the mobilities of the electrons and ions, which can depend on an electric field strength, \( T_e \) and \( T_i \) are the electron and ion temperatures, defined as ratios of diffusion coefficients to mobilities, \( k_B \) is the Boltzmann constant, \( e \) is the elementary charge, \( n_i, n_e \) are the ion and electron number densities, \( \varphi \) is the electric potential, \( J_d \) is the plasma flux on the particle, and \( \delta(\mathbf{r}) \) is the 3-dimensional delta-function. Small perturbations for plasma parameters are under consideration:

\[
n_e = n_0 + n_{\alpha 1}, \quad \nabla \varphi = -\mathbf{E}_0 + \nabla \varphi_1, \quad \mu_\alpha = \mu_{\alpha 0} \left( 1 + \mu_{\alpha 1} \right),
\]

were \( \alpha = e \) or \( i \) denotes electron and ion species. The flow continuity equations, linearized
Figure 1. Distributions of the potential additional to the external field around a charged particle for $Q_\infty/Q = 0.2$ (a) and 0.8 (b). The left sides of the pictures correspond to the fixed ion mobility ($\eta = 0$) and different normalized external electric fields, right sides correspond to the fixed external field and different values of the parameter $\eta$.

relative to the perturbations, as well as the Poison equation are:

\[
\begin{align*}
(k_B T_e/e) \Delta n_{e1} - \Delta \varphi_1 n_0 + (E_0 \nabla n_{e1}) + n_0 (E_0 \nabla \mu_{e1}) &= \delta(r) J_d/\mu_{e0}, \\
(k_B T_i/e) \Delta n_{i1} + \Delta \varphi_1 n_0 - (E_0 \nabla n_{i1}) - n_0 (E_0 \nabla \mu_{i1}) &= \delta(r) J_d/\mu_{i0}, \\
-\varepsilon_0 \Delta \varphi_1 &= e (n_{i1} - n_{e1}) + Q \delta(r),
\end{align*}
\]  

(5) \hfill (6) \hfill (7)

where $Q$ is the particle charge and $\varepsilon_0$ is the electric constant. Perturbations of the electric filed by a point-like particle gives a small effect on mobility of electrons because of the large energy relaxation length for electrons, so $\nabla \mu_{e1} = 0$. A dependence of the ion mobility on the electric field can be described as

\[
\mu_i = \frac{c}{\sqrt{1 + a |E_0|}},
\]  

(8)

with some constants $c$ and $a$, so

\[
\mu_{i1} \approx \frac{a (E_0 \nabla \varphi_1)}{2 |E_0| (1 + a |E_0|)} = \frac{(E_0 \nabla \varphi_1)}{|E_0|^2} \eta,
\]  

(9)

where

\[
\eta = \frac{a |E_0|}{2 + 2a |E_0|}.
\]  

(10)

The parameter $\eta$ can change from 0 to 0.5. The equation for the Fourier transformed additional potential is

\[
\varphi_1(k) = \frac{1}{(2\pi)^{3/2} \varepsilon_0 k^2} (1 + \chi(k))^{-1} \left[ Q - \frac{J_d}{n_0} \left( \frac{k_{D_i}^2/\mu_{i0}}{k^2 + 1 (kk_E)^2} + \frac{k_D^2/\mu_{e0}}{k^2 - 1 (kk_E)^2} \right) \right],
\]  

(11)

where

\[
\chi(k) = \frac{k_{D_i}^2 \left[ 1 - \eta (kk_E)^2/(k^2 k_E^2) \right]}{k^2 + 1 (kk_E)^2} + \frac{k_D^2}{k^2 - 1 (kk_E)^2}.
\]  

(12)
The ion mobility under the model of the energy independent mean free path length is 

\[ k_0^2 = n_0 e^2 (\varepsilon_0 k_B T_a) = 1/\lambda_D^2 (\alpha = e \text{ or } i), \quad k_E = eE_0/(k_B T_0), \text{ and } \tau = T_i/T_e. \]

In the important case of \( \tau \approx 0 \) and \( \mu_e \gg \mu_i \),

\[ \varphi(k) = \left( \frac{Q}{(2\pi)^{3/2}} \sum_{k} \left\{ k^4 + i(kk_E)k^2 + k^2_0 \left[ k^2 - \eta(kk_E)^2/(k_E)^2 \right] \right\} \right), \]

(13)

where \( Q_\infty = -J_1 \varepsilon_0/(\mu_0 \mu_i) \) and \( k_D \approx k_D i \). The spatial potential distributions corresponding to equation (13) are presented in figure 1, where profiles of the additional potential are shown for two ratios \( Q_\infty/Q \) of 0.2 and 0.5. The potential is normalized on \( Q/(4\pi \varepsilon_0 \lambda_D) \), and levels of the normalized potential are indicated in figure 1. The axis \( z \) is directed along the vector \( E_0 \) and distances are normalized on \( \lambda_D \).

For large distances (small \( |k| \)), equation (13) gives

\[ \varphi(r) \approx \frac{Q_\infty}{4\pi \varepsilon_0 r^7} \left\{ \left( Q - Q_\infty \right)(rk_E) \right\} + \frac{3\eta Q_\infty (rk_E)}{8\pi \varepsilon_0 r^5} \frac{(r^2 - (rk_E)^2)/(k_E^2)}{r^3}, \]

(14)

where

\[ r' = \left[ r^2 (1 - \eta) + \eta \frac{(rk_E)^2}{k_E^2} \right]^{1/2}. \]

(15)

The first term corresponds to the Coulomb-like potential in the isotropic plasma \([6, 7]\) which is well-known in the probe theory. The second term describes polarization of the ion cloud which partially screens the particle charge. In the weakly collisional conditions \( Q_\infty \) comes to zero, \( \eta \approx 0.5 \), and second term consists with the result of the kinetic approach \([4]\).

In the paper of Sukhinin et al \([8]\) the total dipole moment inside calculation region was calculated by the molecular dynamic for the mean free path length of ions \( l_i = 10 \lambda_D \) and \( l_i = 2 \lambda_D \). The last case can be compared with the present results taking into account finite size of the calculation region. The distribution of the space charge integrated over plane layers perpendicular to the direction of the external electric field

\[ h(z) = 2\pi \int_{-\infty}^{\infty} q(\rho, z) \rho d\rho, \]

(16)

where \( q \) is the space charge of the ion cloud around the particle, can be easy derived from equation (13). The result is

\[ h(z) = \frac{Q (1 - \eta)}{2k_1} \left( e^{-z(k_1 |k_E|/2)} \theta(z) + e^{z(k_1 |k_E|/2)} \theta(-z) \right), \]

(17)

where \( k_1 = \sqrt{k_E^2/4 + k_0^2} (1 - \eta) \). The dipole moment of the layer from \( z = -L/2 \) to \( z = L/2 \) is

\[ P = \int_{-L/2}^{L/2} h(z) zdz. \]

(18)

The ion mobility under the model of the energy independent mean free path length is

\[ \mu_i = \frac{\sqrt{\pi} e l_i}{4 \sqrt{k_B} i/m}, \]

(19)

where \( m \) is the mass of the ion, for the weak field and

\[ \mu_i = \sqrt{2e l_i}/(\pi |E|m) \]

(20)

for the strong field. Taking into account (8) and (10) we have

\[ \eta = \frac{E' l'}{16/\pi^2 + 2E' l'}. \]

(21)
Figure 2. The dependence of the reduced dipole moment on a size of the integration layer for $E' = 1$ (a) and on the reduced external electric field for $L = 10\lambda_D$ (b); line is hydrodynamic approach, triangles and squares are molecular dynamic results [8].

where $E' = |kE|\lambda_D/2$ is the reduced electric field (according [8]) and $l' = l_i/\lambda_D$. The dependence of the reduced dipole moment $P' = P/(Q\lambda_D)$ on a size of the integration layer is presented in figure 2(a) for $E' = 1$ and $l' = 2$. The figure 2(b) exhibits the dependence of the reduced dipole moment on the reduced external electric field calculated by equations (17), (18), and (21) for the calculation region length $L = 10\lambda_d$ and the results of [8]. The hydrodynamic approach underestimates a dipole moment for distances less or comparable with ion mean free path length, but gives good accordance with the molecular dynamic results at the distance from the particle as low as $L/2 = 2.5l_i$.

In conclusion, we can propose an analytical approximation of the potential perturbation more useful for simulation of the dust particle dynamics than Fourier transformed expression (13). The following approximation gives correct asymptote for long distances and is in the reasonable accordance at intermediate distances with the potential numerically deriving from equation (13):

$$\varphi_1 \approx \varphi_2 - \varphi_3 - \varphi_4,$$

where

$$\varphi_2 = \frac{1}{4\pi\varepsilon_0} \left\{ \frac{Q - Q_\infty}{|r|} \exp (-k_D|r|) + \frac{Q_\infty}{[r^2(1 - \eta) + \eta(r_0)^2]^{1/2}} \right\},$$

$$\varphi_3 = \frac{Q - Q_\infty^{1+\eta/2}}{4\pi\varepsilon_0} \left\{ \frac{(r - d_1)E_0}{[(r - d_1)^2(1 - \eta) + \eta((r - d_1)e_0)^2 + c_1]^{3/2}} + \frac{c_2((r - d_2)E_0)}{[(r - d_2)^2 + c_3]^{3/2}} \right\}^{1/2}.$$
Figure 3. Field distributions (solid lines) and analytical approximations (dashed lines) for $Q_\infty/Q = 0.2$ and $|E_0|e_\lambda_D/(k_B T_i) = 2$ (a); $Q_\infty/Q = 0.8$ and $|E_0|e_\lambda_D/(k_B T_i) = 1$ (b).

This approximation was validated for $|E_0|e_\lambda_D/(k_B T_i) \leq 2$.

The comparison of the fits by equation (22) with the numerical evaluations of equation (13) is shown in figure 3.

So, the anisotropic potential around the charged and absorbing small particle in the plasma with an external uniform field and variable ion mobility is calculated in the hydrodynamic and linear approach. The field dependence of the ion mobility with a plasma flux on the charged particle leads to the long-range anisotropic potential with asymptotic $\sim 1/r$. The analytic approximation is proposed for calculation of interaction between the charged particles placed at distances much more than the ion mean free path length.

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