The Establishment of Engineering Coordinate System and Its Coordinate Transformation by Changing the Geometric Parameters of Ellipsoid

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Abstract. We often encounter the problem of mutual transformation between the various coordinate expression in the work. Because the engineering ellipsoid is only for the country long ellipsoid half shaft made of change, and the two centre, flattening the same, so their space rectangular coordinate system of the three axis also overlap, namely national ellipsoid space rectangular coordinate with engineering ellipsoid space rectangular coordinate exactly equal. Based on this, this paper introduces some key factors that need to be considered in establishing engineering coordinates, national coordinates and urban coordinates based on different ellipsoid parameters is put forward. It provides useful reference for the practice of surveying and mapping engineering.

Keywords. Engineering coordinates, ellipsoid, coordinate transformation.

1. Introduction
In engineering, in order to meet the requirements of construction lofting and mapping control, the side length calculated inversely from the plane coordinates of control points should be as close as possible to the measured plane distance. That is, the relative deformation value of the total length caused by the elevation naturalization of the side length and the projection transformation should be less than 1/40000, which is the basic principle of choosing the plane coordinate system. When the national 3 degree band Gaussian plane cartesian coordinate system is used, the total length relative deformation value will be less than 1/40000 only if the central meridian of the national 3 degree band is near the survey area (the maximum distance from the most eastern (western) edge of the survey area is not more than 45KM), and the average altitude of the survey area is not too large (i.e., less than 100M). Obviously, in the plateau region, it is difficult to meet the above basic principles. At this time, it is necessary to adopt the engineering coordinate system different from the unified national coordinate system of 3 degrees, so as to make the measured side length as consistent as possible with the computed side length, so as to meet the requirements of construction control and lofting accuracy. Therefore, in the actual surveying and mapping work, we often encounter the problem of the conversion of various coordinate expressions, such as conversion between GS-84 world geodetic coordinate system, 1954 Beijing coordinate system, 1980 Xi’an coordinate system, 2000 National geodetic coordinate system (CGCS2000), local coordinate system, engineering coordinate system and other coordinate systems [1-5].
2. Engineering Coordinate System
In order to reduce the length deformation caused by naturalization and projection to a negligible degree, offset elevation coordinate system or custom projection band (arbitrary band) can be used. The normalized elevation surface of the offset elevation system is an ellipsoid which is tangent to a certain elevation surface and parallel to the reference ellipsoid. It makes use of the characteristics that the naturalized correction of the observed value of the side length is opposite in sign and approximately equal in size to that of the projection correction. The offset elevation coordinate system does not change the projection band, and the coordinate transformation relationship is simple. The self-defined projection zone is an ellipsoid that is tangent to the average elevation plane of the survey area and parallel to the reference ellipsoid. The central meridian of the survey area is taken as the central meridian. The coordinate system defined by this method is called the engineering coordinate system.

3. Establishment of Engineering Coordinate System
For the national coordinate system, the national coordinate system is determined after determining the key elements such as ellipsoid parameters, ellipsoid positioning, ellipsoid orientation, geodetic origin, coordinate axis orientation, and projection zoning. For the establishment of engineering coordinate system, it is also necessary to determine the key elements such as projection plane, projection belt, central meridian and starting point [6-10].

3.1. Position of the Central Meridian of the Coordinate System (Longitude)
In order to control the deformation value of gauss projection of side length, the position of the central meridian is generally taken as the central longitude value of the survey area, or the longitude value of the meridian at a designated project construction point, that is, the custom projection belt.

3.2. Starting Point Coordinates
In the engineering coordinate system, a certain point in the optional engineering control network is the starting point, and the coordinate of the starting point is usually a specific value. For example, it can be set as \( x = 0, y = 0 \), or other special value.

3.3. Azimuth from Start
In the engineering coordinate system, an important axis is usually taken as the X-axis. For example, in the construction project, the direction line parallel or perpendicular to the main axis of the construction area is usually taken as the X-axis. In the coordinate system of bridge and tunnel engineering, the axis of bridge or tunnel is usually the X-axis. At this time, the axis has a definite azimuth \( \alpha \) relative to the central meridian, which is one of the important parameters in coordinate transformation.

3.4. Coordinate Projection Surface
The projective plane in the engineering coordinate system is generally selected as a specific elevation plane. According to the different engineering types, the projective plane can be selected as a different elevation plane. For the planned building area, its control network can be projected on the average elevation surface of the building area. For the bridge control network, the pier top plane is chosen as the projection plane. No matter which plane is projected to, it can be regarded as a projection to a specific ellipsoid. We can call the corresponding ellipsoid engineering ellipsoid, and the engineering ellipsoid and the national ellipsoid objectively have a specific relationship, and there must also be a corresponding transformation relationship.

3.5. Coordinates Plus Constants
Usually, to ensure that the coordinates of each point in the survey area are positive, a constant value is applied to the coordinate value in the engineering coordinate system.
3.6. Establishment of Engineering Ellipsoid

There are two ways to establish the engineering ellipsoid. One is to directly use the national ellipsoid, in which case the parameters of the engineering ellipsoid are the same as the national ellipsoid. The second is to make some changes to the national ellipsoid so that the ellipsoid coincides with the selected coordinate projection plane. After the engineering ellipsoid is established, the gaussian plane coordinate system corresponding to the engineering ellipsoid is established on the basis of the engineering ellipsoid.

The above elements are determined, and the engineering coordinate system is established.

The construction methods of engineering ellipsoid varies with the methods used. If the control network is established by the conventional measurement method, it is a change to the national ellipsoid. If GPS network is used, wGS-84 ellipsoid is changed. The concrete way of constructing engineering ellipsoid according to the second method mentioned above is as follows: to change the long half axis $a$ of the national ellipsoid, and keep the flattening of the ellipsoid unchanged to obtain the engineering ellipsoid, so that the engineering ellipsoid coincides with the determined coordinate projection plane. The following discussion is based on this.

4. Coordinate Transformation [1-5]

The coordinate transformation discussed in this paper includes the conversion between national coordinate, engineering coordinate and city coordinate. The three kinds of coordinates can be expressed in three different ways: spatial rectangular coordinates $(X, Y, Z)$, geodetic coordinates $(H, L, B)$ and Gaussian plane rectangular coordinates $(x, y)$.

4.1. Spatial Cartesian Coordinate Relationship between Engineering Ellipsoid and National Ellipsoid

Since the engineering ellipsoid is only a change of the long half axis $\Delta a$ of the national ellipsoid, and the center of the two ellipsoids coincides with each other, the three axes of their space cartesian coordinate system also coincide. That is, the space cartesian coordinates of the national ellipsoid and the space cartesian coordinates of the engineering ellipsoid are completely equal. As shown in equation (1).

\[
(X, Y, Z)_n = (X, Y, Z)_e
\] (1)

This equation will become an important bridge for coordinate transformation.

4.2. Geodetic Coordinate Transformation between Engineering Ellipsoid and National Ellipsoid

Because the space coordinates of the national ellipsoid and the space rectangular coordinates of the engineering ellipsoid are completely equal, the transformation of geodetic coordinates between two ellipsoids can be realized through the transition of spatial rectangular coordinates. The transformation relationship between spatial rectangular coordinates and geodetic coordinates is shown in Table 1.

| Space rectangular coordinates $(X, Y, Z)$ | Geodetic coordinates $(H, L, B)$ |
|----------------------------------------|---------------------------------|
| $B = tg^{-1} \left( \frac{1}{\sqrt{X^2 + Y^2}} (Z + e^2 \sin B) \right)$ | $X = (N + H) \cos B \cos L$ |
| $L = tg^{-1} \left( \frac{X}{Y} \right)$ | $Y = (N + H) \cos B \sin L$ |
| $H = \sqrt{\frac{X^2 + Y^2}{\cos B} - N}$ | $Z = (N(1 - e^2) + H) \sin B$ |
| $N = a/\sqrt{1 - e^2 \sin^2 B}$ | $e^2 = 2a - a^2$ |

The solution of $B$ in table is iterative method, and Its initial value can be evaluated according to equation (2).
\[ B_o = \arctg \left( \frac{Z}{\sqrt{X^2 + Y^2}} \right) \]  

(2)

The steps of transforming the geodetic coordinates \((B,L,H)_N\) of the national ellipsoid into the geodetic coordinates \((B,L,H)_E\) of the engineering ellipsoid are:

Firstly, the geodetic coordinates \((B,L,H)_N\) of the national ellipsoid are converted into the spatial cartesian coordinates \((X,Y,Z)_N\) of the national ellipsoid according to the equation in table 1. Since the spatial cartesian coordinates of the national ellipsoid are equal to the spatial cartesian coordinates of the engineering ellipsoid, the spatial cartesian coordinates of the engineering ellipsoid can be converted into the geodetic coordinates of the engineering ellipsoid according to the equation in the table. The above steps can be succinctly expressed by equation (3), equation (4) and equation (5).

\[(B,L,H)_N \rightarrow (X,Y,Z)_N \]  

(3)

\[(X,Y,Z)_N = (X,Y,Z)_E \]  

(4)

\[(B,L,H)_N \rightarrow (X,Y,Z)_E \rightarrow (B,L,H)_E \]  

(5)

Conversely, the geodetic coordinates \((B,L,H)_E\) of the engineering ellipsoid can also be transformed into the geodetic coordinates \((B,L,H)_N\) of the national ellipsoid. As shown in equation (6).

\[(B,L,H)_E \rightarrow (X,Y,Z)_E \rightarrow (B,L,H)_N \]  

(6)

4.3. Conversion between National GAUSS Plane Rectangular Coordinates \((x,y)_N\) and Engineering Gauss Plane Rectangular Coordinates \((x,y)_E\)

According to the forward calculation and inverse calculation equations of Gaussian projection coordinates in relevant literature and materials, the mutual transformation between earth coordinates and Gaussian plane coordinates can be realized. The transformation can be realized by taking spatial rectangular coordinates as a public bridge. The transformation idea can be expressed concisely as equation (7):

\[(x,y)_N \rightarrow (B,L,H)_N \rightarrow (X,Y,H)_N = (X,Y,Z)_E \rightarrow (B,L,H)_E \rightarrow (x,y)_E \]  

(7)

4.4. Similar Transformation of Plane Coordinates between Engineering Coordinates and Urban Coordinates

Similar to the engineering coordinate system, the urban coordinate system also implies a reference ellipsoid corresponding to the local average elevation. The center, axial direction and oblateness of the ellipsoid are the same as the national reference ellipsoid, but there is a correction \(\Delta a\) in its long radius, and the ellipsoid may be called the local ellipsoid.

For practical, convenient and scientific purposes, many cities and mining areas in China set up the urban control network on the average elevation surface of the local area, and take the local meridian as the central meridian to carry out gaussian projection to obtain plane coordinates. These urban networks have their own origin, their own orientation, and these control networks are referenced to the urban coordinate system.

If the accuracy allows, the transformation of engineering coordinates and urban coordinates can be carried out according to the plane similarity transformation method of different earth ellipsoid coordinate systems for the sake of simple calculation. The plane similarity transformation of coordinate system is actually a two - dimensional transformation. In general, two plane coordinate systems contain four original conversion factors, namely, two translation factors, one rotation factor and one scale factor. Set:

\[ [x,y]_E \]  

Is the coordinate of a point in the engineering coordinate system;

\[ [x,y]_U \]  

Is the coordinate of this point in the urban coordinate system;
\[ \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x_m \\ y_m \end{bmatrix} + (1 + m) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \]

is the translation parameter converted from engineering coordinate system to local coordinate system;

\( m \) is the scale parameter converted from engineering coordinate system to local coordinate system.

\( \alpha \) is the rotation parameter, that is, the azimuth angle of the X axis of the engineering coordinates in the urban coordinate system.

There are two basic methods for converting from engineering coordinate system to local coordinate system:
Method 1: first rotate, then translate, and finally unify the scale, as shown in equation (8):

\[ \begin{bmatrix} x' \\ y' \end{bmatrix} = (1 + m) \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \]

Method 2: first translation, then rotation, and finally unified scale, as shown in equation (9):

\[ \begin{bmatrix} x' \\ y' \end{bmatrix} = (1 + m) \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \]

In a certain range, the above method can also realize the similar transformation of plane coordinates between \((x,y)_e\), \((x,y)_h\) and \((x,y)_l\) if the accuracy allows.

5. Conclusion
The engineering coordinates and urban coordinates discussed in this paper are based on the national ellipsoid, which adopted by ellipsoid of positioning, orientation, flat rate unchanged. Only by changing the long axis of the national ellipsoid (changing the projection plane of elevation), the central meridian different from the national 3 degree band was selected to define the coordinates. Therefore, in the naturalization correction and projection correction of direction and side length observation, the corresponding ellipsoid long half axis and central meridian should be used for calculation.

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