Method of defining the crack formation moment in the bendable reinforced concrete components in the non-linear deformation model

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Abstract. The article proposes applying energy from law deformation mechanics of the rigid body to the development and improvement of the deformation method for calculating the strength of reinforced concrete constructions with the use of diagrams illustrating the deformation of concrete and reinforcement. In terms of the strength energy theory, concrete and rebar accumulate potential energy in the section of the construction component under stress; based on the contours of the diagram used in calculation it is possible to distinguish a stress diagram for concrete of the compressed zone. The value of the stress in the section is equivalent to the force used for the deformation of the concrete specimen under stress (prism- or cylinder-shaped); the force is equal to the area used in the calculation of the normable diagram. The calculations of the resolving equations are performed with the hypothesis of flat sections. The conditions of the stress balance in the section of the construction component are tested with the method of the successive approximation. The moment of cracking is calculated in reference to the neutral axis after defining the stress in concrete of the strained zone with the preconditioned accuracy.

1. Introduction
The basic distinctive feature of construction standards and regulations, which are being currently developed, is the transition from simple dependencies offered by the method of limit states [1], which is based mainly on empirical methods of calculation, to deformation methods of calculating the strength of the reinforced concrete structures with use of deformation diagrams for concrete and reinforcement [2, 3, 4]. A criterion of cracking in the construction component is achieving maximum relative values of deformation on the extreme fiber of the stretched zone, which exceed the normative limit stress values during axis. Several types of diagram presentation, different values for limit deformations during compression in piecewise linear and curvilinear diagrams of concrete cause confusion about the right choice of the initial diagram, which can be used for calculating crack formation. Establishing correlation of the calculation results and experimental data is one of the most acute issues in designing practice. For further improvement of calculation deformation methods, for bringing them in accordance with Russian and international norms, it is necessary to apply the laws of rigid bodies deformation mechanics [5-12]. In the preliminary calculations of the presented energy model, the strains in the concrete are equivalent to the force applied to produce deformation in a concrete specimen (prism, cylinder) under stress, which is equal to the area used in calculation of the
normative diagram. Energy models are important for the transition to the analysis of structures in complex stress modes, since they take into account the dissipation of energy on the relief lines [13-17].

2. Materials and methods
The research uses dual-line diagrams, trilinear diagrams (Figure 1 (a)) and a curvilinear diagram with a descending branch (Figure 1 (b)) for calculating the concrete stretching; bilinear diagram of concrete compression (Figure 1 (c)) for calculating deformation of the nonprestressed rebar (including class A-500) is: a bilinear Prandtl diagram, where the boundary of the elastic zone is limited by the deformations $\varepsilon_2 = \sigma_2 / E$. The rules of conditioning the diagram parameters and the main calculation standards are presented in [2, 12,13,14].

Figure 1. Design diagrams of deforming concrete: (a) – piecewise linear (dual-linear and trilinear), (b) – curvilinear stretch diagram; (c) – bilinear compression diagram.

For the rectangular section reinforce in the lower zone by the rebar with the area $A_x$ and in the upper zone, accordingly, with the area $A_y$ (Figure 2 (a)) taking into account the distribution of relative deformation of the concrete and the rebar according to the linear law (Figure 2 (b)) the diagrams of stress are presented in Figures 2 (c), (d), (e).

Figure 2. Charts of stress, tension and deformation in the cross-section of the bendable nonprestressed component in calculations of crack formation with the use of deformation diagrams for concrete and rebar.

On the basis of the linear law of distributing relative transformations according to the height of the element leads to the following equations
\[
\frac{1}{\rho} = \chi = \frac{\varepsilon_{bt2}}{h-x} = \frac{\varepsilon_{bn}}{x} = \frac{\varepsilon_{bn} + \varepsilon_{bt2}}{h},
\]

where \( h \) is the height of section; \( x \) is the height of the compressed zone; \( \varepsilon_{bn} \) is relative deformations on the compressed zone concrete \( \chi \) is curvature the component \( \rho \) is radius of the curvature.

Diagrams of concrete stretching are limited by the upper and lower values of relative deformations \( \varepsilon_{bt2} \). The equations of the stress balance in the section of the component is the following

\[ N_{bt} + N_s - N_{bl} - N'_s = 0. \]  

(2)

The stress of the concrete element is presented through parameters of the diagrams. In the general case the areas of the concrete diagrams boarded by the lines are divided along the deformation axis into separate small parts \( \Delta\varepsilon_{bi} \), \( \Delta\varepsilon_{bj} \) in the diagrams of stretching and compression accordingly \((i, j)\) the numbers of the parts). For each part of the diagram \((i, j)\) there are definitions of: \( \sigma_{bi} \) \((\sigma_{bj})\) – the values of stress, \( \varepsilon_{bi} \) \((\varepsilon_{bj})\) – coordinates of the gravity centre of small parts in the system of coordinates \( \varepsilon_{bi} \) \(c\sigma_{bi} \), \( A_{bi} = \Delta\varepsilon_{bi}/\varepsilon_{bi} \) \((A_{bj} = \Delta\varepsilon_{bj}/\varepsilon_{bj})\) – the areas of the parts. Relative deformations in the diagrams in parts \( \Delta\varepsilon_{bi} \), \( \Delta\varepsilon_{bj} \) in the normative section of the component correspond to the height of the component section zone \( \Delta h_{bi} = \Delta\varepsilon_{bi}/\chi \) \((\Delta h_{bj} = \Delta\varepsilon_{bj}/\chi)\) with the value of stress \( \sigma_{bi} \) \((\sigma_{bj})\).

The stress’ values experienced by the concrete of the stretched \( N_{bt} \) and compressed \( N_{bl} \) zones just before crack formation for the strip of unit value \((b=1)\), is calculated in the equation

\[ N_{bi} = \frac{N_{bi.d}}{\chi}; \quad N_{bj} = \frac{N_{bj.d}}{\chi}, \]

(3)

where \( N_{bi.d} = \sum_{i=1}^{n} \sigma_{bi} \Delta\varepsilon_{bi}, \quad N_{bj.d} = \sum_{j=1}^{m} \sigma_{bj} \Delta\varepsilon_{bj} \) in general case define the force used for the deformation of the sample (Prism, cylinder) with the stress: \( N_{bi.d} \) – stretching to the limit values of deformations \( \varepsilon_{bt2} \); \( N_{bj.d} \) – compression within the boundaries of a certain deformation value \( c_{bn} \).

The force is equal to the sum of the elementary parts on the concrete diagrams of stretching and compression correspondingly. The force \( N_{bi.d} \) – taking into account elastic deformations \( c_{bn} \) on the extreme fibre of the concrete in the compressed zone and the bilinear concrete diagram use of compression (Figure 1 (c)) is calculated according to the formula

\[ N_{bi.d} = \frac{(\chi h - \varepsilon_{bt2})^2}{2} E_{b,red} = \frac{R_b (\chi h - \varepsilon_{bt2})^2}{2E_{b,red}}. \]  

(4)

The values of the stress experienced by the rebar \( N_s, N'_s \) just before cracking are calculated with the formulas: \( N_s = \varepsilon_s E_s v_s A_s, \quad N'_s = \varepsilon'_s E_s v'_s A'_s \) where \( v_s, v'_s \) is the co-efficient of the changes of the section module (in the bendable zone \( v_s = v'_s = 1 \)); the deformation of the rebar is defined by the equations:

\[ \varepsilon_s = \varepsilon_{bt2} - \xi x, \quad \varepsilon'_s = \chi h - \varepsilon_{bt2} - \xi x'. \]  

(5)

Taking into account the calculated relations, the balance equation (2) for the symmetrical section with the width \( b \) can be formulated as

\[ \frac{N_{bt,d}b}{\chi} - \frac{N_{bd,d}b}{\chi} + \varepsilon_s E_s v_s A_s - \varepsilon'_s E_s v'_s A'_s = 0. \]  

(6)

Test of the balance equation (6) is performed by the method of successive approximations (iteration method). During the first approximation it is conditioned as \( \varepsilon_{bt2}^{(1)} = \varepsilon_{bt2} \), that is, the line of deformation
divides the section along height into two equal parts \((x = h/2)\). Taking into account the symbols on the scheme (negative values of force in the compressed zone and positive ones on the stretched zone) the results of the equations calculations \((6)\) can be presented in two cases: in the first case, the sum in the left part of the equation is less than zero; in the second case the left part is more than zero. In the first case it is necessary to apply the following calculations: during the second approximation, to diminish the deformation of the first approximation \(\varepsilon_b^{(1)}\) and define the new value of deformations \(\varepsilon_b^{(2)}\) deformation in the extreme fiber of the concrete compressed zone \(\varepsilon_b^{(2)} = \varepsilon_b^{(1)} - \Delta \varepsilon_b^{(1)}\) (if \(\Delta \varepsilon_b^{(1)} = 0, \varepsilon_b^{(2)} = \varepsilon_b^{(1)}\)), test the balance equation \((6)\). Gradual diminishing of deformations is performed until reaching the following condition

\[
\Delta \varepsilon_b^{(k)} \leq 0.01 \varepsilon_b^{(1)}.
\] (7)

In the second case, the algorithm of testing the balance equation \((6)\) follows the same steps when the left side of the equation is above zero. Nevertheless, the deformations in the extreme fiber of the compressed zone, set for the first approximation \(\varepsilon_b^{(1)} = \varepsilon_b^{(1)}\), will increase in the second cycle of iterations in the increased value \(\varepsilon_b^{(2)} = \varepsilon_b^{(1)} - \Delta \varepsilon_b^{(1)}\) if the fixed values of deformation in the extreme fiber of the stretched zone are \(\varepsilon_b^{(1)}\). The calculations are performed until the satisfactory (set) accuracy in providing the condition \((7)\). After finalizing the moment iterations of internal force in defined in reference to the zero line through the diagram parameters. Moments of force in the stretched \(M_{bt}\) and compressed \(M_b\) zones of concrete if \(b = 1\) are the following

\[
M_{bt} = \frac{N_{bt,d} z_{bt}}{\chi}, \quad M_b = \frac{N_{b,d} z_b}{\chi}.
\] (8)

In the equations \((8)\) the values in the numerator are: \(e_{bt,c}, e_{b,c}\) - the distances of the gravity centers of the diagram parts bordered by the branches from the stress axis correspondingly \(\sigma_{bt}\) and \(\sigma_b\); \(z_{bt} = e_{bt,c}/\chi\), \(z_b = e_{b,c}/\chi\) - the distances of the gravity centers of the tensile diagrams in the stretched and compressed zones of the component section from the neutral axis; \(N_{bt,d} e_{bt,c} = S_b\) - the static moment of the unit force in concrete diagrams of stretching in reference to the axis of stress \(\sigma_{bt}\), which is equal to the product of the concrete diagram whole area to the distances of their gravity centers \(O_1\) and \(O_2\) from that axis, \(N_{b,d} e_{b,c} = S_b\) - is the static moment of unit force in the concrete diagram of compression in reference to the axis of tension \(\sigma_t\), equal to the whole area production of the concrete diagram to the distance of its gravity centre \(O_4\) from that axis. The value of moment \(S_b\) is calculated according to the formula

\[
S_b = \frac{(zh - e_{bt2})^3}{3} E_{b,red} = \frac{R_h(zh - e_{bt2})}{2e_{bt,red}}.
\] (9)

The equation of the inner force moment in reference to the zero line, with the value of it being equal to the external moment \(M_{crc}\), just before crack formation is the following

\[
M_{crc} = S_b z_b \frac{z^s}{\chi} = \frac{S_b z_b}{\chi} + e_s E_s v_s A_z z_s - e_s E_s v_s A^i z^i,
\] (10)

where the distances of the rebar gravity centers to the neutral axis are \(z_s = \frac{(e_{bt2} - zn)}{\chi}\). For defining the moment of crack formation \(M_{crc}\), formula \((10)\) uses the values \(\chi\), \(e_s, e^i_s, z_s, z^i\), received during the last cycle of reiteration after providing the condition \((7)\).
3. Results and discussions
Comparative analysis of the calculated $M_{crc}$ using normative diagrams with their experimental values was performed for the reinforced concrete samples of rectangular section bendable according to the beam scheme with dimensions of height $h=18$ cm, width $b=12$ cm, span $l=194$ cm. Samples were made from the same composition of concrete. Compression resistance of the concrete was $\sigma_b = 30.6$ MPa, resistance to stretching $\sigma_{bt}$ MPa, module of elasticity $E_b = 3.07 \times 10^4$ MPa. The values of the parameters were defined on the basis of testing the standard prism samples with the proportionate increase of the stretching stress and compression until destruction. In the compressed and stretched zone two reinforced bars of class $A400$ same diameter were placed. According to the intensity of the reinforcement the samples were divided into three groups with the percentage of reinforcement $\mu = \mu_t = 0.52\%$; $\mu = \mu_t = 0.82\%$; $\mu = \mu_t = 1.18\%$. The procedure of gradual approximation while balance equation testing (6) using piecewise linear and non-linear normable diagrams [2] was performed with the use of a specially developed computer program. Values of $M_{crc}$ are presented in the Table 1.

| $\mu$, % | Calculating diagrams of concrete stretching, C.R. [2] | C.R.R. [1] | Experiment |
|----------|--------------------------------------------------|-------------|------------|
|          | Calculating diagrams of concrete stretching, C.R. [2] | C.R.R. [1] | Experiment |
| $M_{crc}$, kN-m | $M_{crc}$, kN-m | $M_{crc}$, kN-m | $M_{crc}$, kN-m | $M_{crc}$, kN-m |
| 0.52     | 2.52                                            | 2.40        | 2.21        | 2.23        | 2.34        |
| 0.82     | 2.68                                            | 2.58        | 2.39        | 2.40        | 2.53        |
| 1.18     | 2.88                                            | 2.80        | 2.62        | 2.61        | 2.67        |

In the experimental research, the crack formation moment corresponded to such stress, when the cracks’ width rats the level of the rebar gravity center reached 0.1 mm. Substantial convergence and reliability of calculations of $M_{crc}$ in comparison with the experimental data, is provided by the limit state method [1] and the use of the curvilinear diagram for stretched concrete in the deformation model. Though, the results of calculation with the use of the normable piecewise linear diagrams are more than the experimental values no more 7%. The balance of the efforts in the section is achieved through diminishing the values of deformations $\epsilon_t^{(1)} = \epsilon_t^{(2)}$ (curvature of the component) in the extreme fiber of the compressed zone with the fixed deformations of the stretched zone equal to their limit stretch values. The calculation results correlation of the deformation model with the normable diagrams use of concrete and rebar is 8-10%.

4. Conclusion
- The research results in the development of the energy model and the method of defining stress in the standard section of the bendable reinforced concrete component just before crack formation through the concrete diagrams parameters of stretching, compression and the procedure of testing the force balance equation with the method of gradual approximation.
- The limit state method showed high reliability and practicability in calculations for crack formation, as it was also seen during its long history. In calculations of the deformation model the “standard” should be considered the curvilinear concrete diagram of stretching, which shows high convergence with the experimental data and, besides, can be used in the calculations of the constructions in the complex stress modes taking into account the dissipation of the accumulated energy in the relief lines.
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