Experimental and Numerical Comparison of Prestressed Perforated Concrete Rafters of Different Configurations

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Abstract. This paper demonstrates an experimental and numerical study aimed to compare the influence of openings of different configurations on the flexural behavior of prestressed concrete rafters. The experimental program consisted of testing six simply supported prestressed concrete rafters; 5 rafters are perforated, and the other one is solid as a reference. All rafters were tested under monotonic midpoint load. The variable which has been investigated in this work was the opening's configuration (quadrilateral or circular) with the same upper and lower chords depths. The results indicate improvement in the beam flexural behavior using the circular openings compared to the quadrilateral openings, represented by increase the ultimate load capacity and decrease in deflection at the service limit. The experimental results of the tested rafters were confirmed by the non-linear finite element software (ABAQUS13.6.1). Comparisons are offered and the good agreement appears between the experimental results and the predictions of finite element analysis in terms of load-deflection and failure loads relationships.

1. Introduction

Prestressing can be defined in general terms as the preloading of a structure, before application of the service loads, to improve its performance in specific ways [1]. Extensive experimental and theoretical studies of prestressed concrete beams over many years have led to very well established methods for serviceability and strength design under static loads [2-3]. The insertion of openings into the solid beam will essentially affect the performance of the beam resulting in more complex behavior because the openings lead to a sudden change (decrease) in the dimensions of the beam cross-section. At the same time, the corners of opening would be subjected to a high concentration of stresses that could result in extensive cracking which is inadmissible from an esthetic and durable point of view. Also, the presence of the openings would reduce the overall stiffness of the beam, which may result in extreme deflections under service loads. The ultimate strength of such a beam might be reduced to critical grade except if special reinforcement is providing in adequate quantities. The design of these beams; therefore, requires special treatment to control the crack width and to avoid probable early failure of the beam [4-6]. The comparison to the reinforced concrete beam with the rectangular and circular-shaped openings would reduce the ultimate capacity of the beam by 4% and 8%, respectively, furthermore, the investigation showed that the minimum reduction in the ultimate load of the RC beam was observed when the opening was created within the pure bending zone and the maximum reduction of the capacity was monitored.
when the opening was inserted outside the mentioned zone [7]. For the experimental and numerical comparison, the influence of openings of different configurations on the flexural behavior of reinforced concrete rafters subjected to midpoint concentrated load the results indicate, improvement in the beams’ flexural behavior when circular openings were used compared with that of quadrilateral openings, represented by an increase in ultimate load capacity and a decrease in deflection at the service limit [8-9]. The link structural elements (Posts) between upper and lower chords of the rafter beam have many advantages such that, avoiding Vierendeel truss prevents shear failure and allows a beam to enhance its bending capacity and ductility [10]. The fact that, concrete does not efficient in resisting tensile stress makes it very difficult to reach a long span beam in design. So the addition of the prestressing reinforcement has become necessary to overcome this problem and reach span lengths cannot be reached by using ordinary reinforcements [11-12]. Five critical positions for possible cracking of prestressed perforated concrete rafters are classified in figure 1. These are: (a) at the opening edges due to the prestressed force; (b) in the opening corners due to framing the action in the opening region; (c) at the chord members due to shear; (d) in the chord members due to flexural stresses that arise from secondary moments; and (e) at the lower chord member due to normal tensile stresses. The tension cracks and shear cracks can trigger the complete collapse of the rafter's beam [13]. Two types of failure for reinforced concrete beams with a circular opening, the first was beam-type failure represented by diagonal cracks pass through the centre of the opening, and the second was a frame-type failure formed by diagonal cracks top and bottom of the opening [14]. The typical crack pattern for T-beams with multi openings at initial cracking, near the service load stage (70% of $M_u$), and at the ultimate stage ($M_u$) is illustrated in figure 2. The initial cracks are caused by localized stresses figure (2a) at the openings. The cracking also indicated Vierendeel truss-like end forces on the chords below some of the openings. As the load on the beam increased, the crack pattern changed from localized cracking to a more uniform caused by the flexure of the overall T-beam. At failure, there are flexural cracks across the middle half of the T-beam. In beams, the shear reinforcement adjacent to the openings are not bent into the flange, shear cracks observed to extend from the top corner of the opening to the underside of the flange, which then extends horizontally the flange-web interface [15]. To represent the structural behavior of prestressed perforated concrete beams under midpoint load, a non-linear finite element analysis was performed to analyze the experimentally tested rafters. The analysis was performed using a widespread (ABAQUS13.6.1) program, which is a comprehensive, general-purpose finite element computer program. [16].

![Figure 1. Types of Cracking Around Opening [13].](image1)

![Figure 2. Crack Development in T-Beam with Multiple Openings [15].](image2)

2. Experimental Study

This experimental study was conducted to evaluate the behavior of simply supported prestressed perforated concrete rafters which exposed to monotonic static load during a wide spectrum of loading up to failure. The studied variable was the opening's configuration (quadrilateral or circular) with the same upper and lower chords depth.

2.1. Specimen Details

The experimental program consisted, casting and testing 6 rafter beams, including 5 beams with openings (perforated), and reference solid beam without openings. All the tested beams had the same
rafter geometry, it is, a rectangular cross-section of 100 mm width and height of 400 mm at centre tapered to 250 mm at the two ends, the overall length was 3000 mm with a clear span of 2800 mm, figure 3 shows the geometrical details of the tested beams. Figures 4 and 5 exhibits the details of solid prestressed rafter concrete beams and these with opening, respectively. Mild steel reinforcement of 4, 6, and 12 mm bar diameters were used while seven-wire low-relaxation strands, Grade 270 with Ø12.7 mm diameter was used as prestressing steel. The rafters were classified into three groups (A, B, and C). These groups were divided according to the variable used in this study as listed in table 1.

2.2. Material Properties
Normal concrete has been used to cast the rafters. The properties of the materials (cement, coarse aggregate, fine aggregate, and steel) were determined also, the mechanical properties of hardened concrete (compressive strength, splitting tensile strength, and modulus of elasticity) were tested using cylindrical steel molds (with 150 and 300mm diameter and height, respectively). The properties of normal concrete and steel reinforcements (rebar and tendons) used in this work are given in table 2. Whilst the prestressing force (110kN) from one end according to the ACI-318M-14[17] limitations.

![Figure 3. The experimental details of tested rafters (all dimensions in mm).](image)

![Figure 4. Details of reinforcement for solid rafter PGB (the dimensions are in mm).](image)
Figure 5. Details of reinforcement for rafter with openings PGT8 (all dimensions are in mm).

Table 1. Details of tested rafters.

| Group | Rafter’s label | Shape of openings | Number of opening | Total area of opening (mm²) | Width of opening (mm) | Height of upper chord (mm) | Height of lower chord (mm) |
|-------|----------------|-------------------|------------------|-----------------------------|----------------------|--------------------------|--------------------------|
| Control | PGB  | ------ | ------ | 0 | ------ | ------ | ------ |
| A | PGTH8 | Trapezoidal | 8 | 234000 | 150 | 75 | 75 |
|   | PGC1 | Circular | 8 | 184200 | D | 75 | 75 |
| B | PGT8 | Trapezoidal | 8 | 174000 | 150 | 100 | 100 |
|   | PGC2 | Circular | 8 | 128000 | 0.83D | 100 | 100 |
| C | PGP8 | Trapezoidal with inclined posts | 8 | 151000 | 150 | 100 | 100 |
|   | PGC2 | Circular | 8 | 128000 | 0.83D | 100 | 100 |

P: Prestressed, G: Gable, B: Beam, T: Trapezoidal, H: Height, C: Circular, P: Parallelogram, 8: Number of openings, 1 and 2: Number.

Table 2. Properties of material.

| Material | Dia. (mm) | Yield Strength (MPa) | Average Cylinder Compressive Strength \( f_c' \) (MPa) | Average Ultimate Tensile Strength (MPa) | Average Modulus of Elasticity (GPa) |
|----------|-----------|----------------------|---------------------------------|-------------------------------------|----------------------------------|
| Concrete | ------ | 40                   | 4.5                             | 31.5                                |
| rebar    | 4        | 420                  | ------                          | 650                                 | 200                             |
|          | 6        | 555                  | ------                          | 670                                 | 200                             |
|          | 12       | 597                  | ------                          | 695                                 | 200                             |
| strand   | 12.7     | 1674                 | ------                          | 1860                                | 197.5                           |

2.3. Setup and Testing Procedure

The experiments were executed in load control as shown in figure 6. All rafters were tested under monotonic midpoint load up to failure. The load was gradually increased with a load rate of 4 kN/min until failure. Vertical displacement readings (deflections) were recorded in all load intervals. Furthermore, the crack formation and propagation were examined at each loading step, as well as the first crack, and the failure loads were recorded. At the end of testing, the crack pattern was studied and highlighted.
3. Experimental Results and Discussion

3.1. Load-Deflection Response

Three loading stages have been chosen to study the behavior of the prestressed rafters within the elastic uncracked (40kN), service cracked (80kN) and ultimate loading stages. The experimental results of load deflection curves are shown in figure 7. The corresponding mid-span deflections for the loading stages of 40, 80 kN, and ultimate load with the comparison of these results with that of the control rafter (solid) are summarized in table 3. While, the comparison of the deflection for the different opening configuration of these loading limits are listed in table 4.

![Figure 7](image_url)

**Figure 7.** Experimental results of load-deflection curves.
Whereas, after cracking, the behavior developed gradually from linear to nonlinear, and the slope of the curves are considerable deviated in each group. The corresponding mid-span deflections for the two chosen opening shapes of the tested rafters are summarized in Table 4.

**Table 4.** Ultimate load and deflection at various loading stages of the perforated tested rafters.

| Group | Rafter's labeling | Deflection (mm) | Increasing ratio of Deflection % (a) | Deflection (mm) | Increasing ratio of Deflection % (a) | Deflection (mm) | Increasing ratio of Deflection % (a) | Failure load Pult (KN) | Decreasing ratio of Pult % (b) |
|-------|------------------|-----------------|-------------------------------------|-----------------|-------------------------------------|-----------------|-------------------------------------|----------------------|-------------------------------|
| A     | PGTH8 (Ref.)     | 4.054           | 70.5                                | 11.954          | 51.7                                | 26              | 28.4                                | 104.6                | 29.5                          |
|       | PGC1             | 3.366           | 41.5                                | 9.227           | 17.1                                | 23.65           | 16.8                                | 137.4                | 7.4                           |
| B     | PGTH8 (Ref.)     | 3.429           | 44.2                                | 9.001           | 14.2                                | 23.75           | 17.3                                | 140                  | 5.7                           |
|       | PGC2             | 2.898           | 21.8                                | 8.485           | 7.7                                 | 22.35           | 10.4                                | 143.8                | 3.1                           |
| C     | PGP8 (Ref.)      | 3.09            | 29.9                                | 8.869           | 12.5                                | 22.85           | 12.8                                | 141.7                | 4.5                           |

From Table 3 and Figure 7, the following results can be drawn:

The presence of openings in rafter reduces its ultimate load capacity. It can be noticed that the decreasing ratio of ultimate strength ranging between (3.1 % ~ 29.5%) for perforated rafters relatively to the solid one.

- The openings increase the deflection in rafter. It can be noticed that the increasing ratio of mid-span deflection at ultimate load ranging between (10.4 % ~ 28.4%) for perforated rafters relatively to the solid rafter.
- Prior to cracking, all rafters behaved in a linear elastic manner, and the slope of linear portions were slightly difference for the rafters of the same group. Whereas, after cracking, the behavior developed gradually from linear to nonlinear, and the slope of the curves are considerable deviated in each group.

The corresponding mid-span deflections for the two chosen opening shapes of the loading stages of 40, 80 kN, and ultimate load for the perforated rafters are summarized in table 4.
Table 4 and figure 7 reveal, the decreasing ratio of ultimate strength ranging between (1.5 % ~ 31.4%) for rafters with quadratic openings comparing with that consist of circular openings. On the other hand, the deflection at different loading stages for rafters with circular openings decreases comparing with those consist of quadratic openings. The above results clarified that the circular openings gave an improvement in ultimate loading carrying capacity and enhancement in rafter’s stiffness, because of the nature of the circular configuration which reduces the possible stress concentration around the opening rather than quadratic opening.

3.2. Cracking and Ultimate Load Carrying Capacities

3.2.1. Effect of Opening Shape (Quadratic and Circular)

Three of the five critical locations of the cracking appearance possibility in beams with quadrilateral openings which were noticed by Abdalla and Kennedy (1995) [13] are initiated in the prestressed perforated concrete rafters in present experimental work, these are: cracking at opening corners of the quadrilateral ones (figure 1-b), flexural cracking in lower chords (Figure 1-d), and cracking of tension chord (figure 1-e). But it should be noted that in beams with quadrilateral openings the corner cracks were not observed due to the absence of the sharp corners, another type of cracks as described by Mansur and Tan 1999[14] were initiated; it is the independent diagonal inclined cracks traversed through the center of the openings. All these types were detailed in section one.

Generally, the First appearance of crack was at the opening corner (type – b) for all rafters with quadrilateral openings and the diagonal crack transverse through the circular opening center, followed by cracking at the tension chord (type – e) and finally the flexural cracking in the chord (lower chord) (figure 1-d). According to this sequence, the first cracking load (Pcr) is the lowest of the three cracking types and mostly is due to (the corner cracks for quadrilateral openings or diagonal inclined cracks traversed through the center of the openings), as demonstrated in table 5 for all groups.

| Group | Rafter’s labeling | First cracking load at opening corner (type – b) kN | Increasing ratio % (a) | First cracking load of tension chord (type – e) kN | Decreasing ratio % (a) | First flexural cracking load in cord (type – d) kN | Increasing ratio % (a) | Failure load Pult (KN) | Pcr /Pu % | Max. crack width at service load (mm) | Number of cracks |
|-------|------------------|-----------------------------------------------|----------------------|-----------------------------------------------|----------------------|-----------------------------------------------|----------------------|-----------------|---------|-------------------------------|-----------------|
| control | PGB             | -                                     | -                   | 65                                         | -                   | -                                             | -                   | 148.4 | 43.8  | 0.13                           |                  |
| A     | PGTH8 (Ref.)     | 40                                    | (Ref.)             | 45                                         | (Ref.)             | 50                                        | (Ref.)             | 104.6 | 38.2  | 0.28                           | 32               |
|       | PGC1             | 40                                    | 0                  | 45                                         | 0                  | 50                                          | 0                  | 137.4 | 29.1  | 0.25                           | 47               |
| B     | PGTH8 (Ref.)     | 55                                    | (Ref.)             | 65                                         | (Ref.)             | 60                                        | (Ref.)             | 140.0 | 39.3  | 0.27                           | 53               |
|       | PGC2             | 55                                    | 0                  | 65                                         | 0                  | 60                                          | 0                  | 143.8 | 38.2  | 0.22                           | 49               |
| C     | PGTH8 (Ref.)     | 45                                    | (Ref.)             | 50                                         | (Ref.)             | 60                                        | (Ref.)             | 141.7 | 31.8  | 0.3                            | 48               |
|       | PGC2             | 55                                    | 22.2               | 65                                         | 30                 | 60                                          | 20                 | 143.8 | 38.2  | 0.22                           | 49               |

(a) \( \frac{P_{cr \text{ (rafter)}} - P_{cr \text{ (Ref.rafter)}}}{P_{cr \text{ (Ref.rafter)}}} \times 100 \)

Pcr is the lesser first cracking load. Herein the type –b is the Pcr.
Figure 8 illustrates the propagation of crack width for the three types of cracking relative to the ultimate load (Pu). As illustrated in this figure, the cracking at opening corners for the quadrilateral openings and diagonal inclined cracks traversed through the circular openings almost are the higher curves due to the earlier initiation and propagation, then the cracking of tension chord which follow the flexural cracking in chords. While, table 5 demonstrates the effect of opening shape on the cracking load, for the specimens having the same openings number but different in configurations (circular openings and quadratic ones). It can be monitoring the following notes:

- The results indicate slight increase in cracking load for beams with circular openings than that with quadratic openings, and the cracking was initiated at range (29.1-43.8) % of the ultimate load.

- The maximum crack width propagates in opening corners at service load is not exceeded the serviceability requirements of concrete structures. The ACI 318M-1995 code [24], considered that the permissible maximum crack widths at service stages for exterior and interior exposure conditions are 0.3 and 0.4 mm, respectively.
3.2.2. Modes of Failure

The failure of the solid rafter was due to the formation of tension cracks at the soffit of the beam at the maximum bending moment, with attendant of steel yielding, these cracks propagated toward the upper zone (fibers) followed by compression failure near the load point. In contrast, three different patterns of failure were observed in the rafters with openings. The first, which is the most widespread mode, was diagonal splitting cracking at corners followed by compression failure, such in rafters PGC1, PGT8, and PGC2. While, the second failure mechanism occurred due to the formation of plastic hinges in several chords of rafters includes adjacent openings, these successive plastic hinges affect the curvature of the beam. Therefore, stresses will be distributed to larger parts of the beam, which have participated in flexural strength in Vierendeel action (PGTH8). Finally, the third failure mode was by the formation of diagonal cracks started from the corners of the opening and propagated towards the posts followed by shear failure in the top and bottom of the posts (PGP8). Figure 9 illustrates the failure mode and the crack patterns of each of the tested rafters.

![a- Failure and Crack patterns of PGB rafter. b- Failure and Crack patterns of PGH8 rafter. c- Failure and Crack patterns of PGT8 rafter. d- Failure and Crack patterns of PGP8 rafter. e- Failure and Crack patterns of PGC1 rafter. f- Failure and Crack patterns of PGC2 rafter. g- Failure mode of PGB rafter. h- Failure mode of PGT8 rafter. i- Failure mode of PGTH8 rafter. j- Failure mode of PGP8 rafter. k- Failure mode of PGC1 rafter. l- Failure mode of PGC2 rafter.](image)

Figure 9. Crack patterns and failure mode at the failure tested rafters.
4. Finite Element Modeling
To represent the structural behavior of prestressed perforated concrete beams under midpoint load, a non-linear finite element (FE) analysis was performed to analyze the experimentally tested rafters. The analysis was performed using a widespread (ABAQUS13.6.1) program. The first stage of modeling is creating the prestressed perforated concrete rafter to simulate the experimental work and to find out the values of first flexure crack, load capacity, deflection …etc. the result of applying the load.

4.1. Numerical Analysis Results and Discussion
The numerical analysis results for all prestressed perforated concrete rafters are presented and compared with experimental results in this section. The static analysis is used for all the modal in FEM analysis. Figure 10 explains deformed shape at ultimate stage for PGTH8 rafter.

Figure 10. Deformed shape at ultimate stage for PGTH8 rafter.

4.1.1. Load and deflection at failure stage
Table 6 shows a comparison of the mid-span deflection and failure loads obtained from experimental work and the finite element models at the failure stage of all rafters under static test. A good agreement was obtained between deflection and ultimate loads of numerical models and the experimentally tested rafters, with a mean value and the coefficient of variation \((Pu)_{FE}/(Pu)_{Exp}\) was 0.97 and 1.62, respectively, for the ultimate loads, while the mean value and coefficient of variance for \(W_{FE}/ W_{Exp}\) 0.99 and 6.22 respectively.

| Group | Rafter's labeling | Failure Load kN \((Pu)_{Exp}\) | \((Pu)_{FE}/(Pu)_{Exp}\) | Mid-span deflection( mm) \(W_{Exp}\) | \(W_{FE}\) | \(W_{FE}/ W_{Exp}\) |
|-------|------------------|-------------------------------|---------------------------|-----------------------------|----------|----------------|
| Control | PGB        | 148.4                          | 0.98                      | 20.25                        | 20.98    | 1.04                  |
| A      | PGTH8      | 104.6                          | 0.99                      | 26                           | 24.952   | 0.96                  |
|        | PGC1       | 137.4                          | 0.99                      | 23.65                        | 22.573   | 0.95                  |
| B      | PGT8       | 140                            | 0.98                      | 23.75                        | 25.713   | 1.08                  |
|        | PGC2       | 143.8                          | 0.95                      | 22.35                        | 20.531   | 0.92                  |
|        | PGP8       | 141.7                          | 0.97                      | 22.85                        | 23.925   | 1.05                  |
|        | PGC2       | 143.8                          | 0.95                      | 22.35                        | 20.531   | 0.92                  |
|        | Mean       | 0.97                           | Mean                      | Mean                         | 0.99     | 6.22                  |

\(c.o.v\) 1.62  \(c.o.v\) 6.22
4.1.2. Load - Deflection Relation

The numerical results (FE) of load versus deflection of the static analysis models were compared with the experimental results in figure 11. It can be noticed that, the numerical models were stiffer and stronger than the experimental data in both linear and nonlinear regions of the behavior this may due to, the finite element method assumes perfect bond between concrete and steel reinforcement and ideal material properties, but there was a good agreement between them.

\[ \text{Load} - \text{Deflection Relation} \]

The experimental data in figure 11. It can be noticed that, the numerical models were stiffer and stronger than the experimental data in both linear and nonlinear regions of the behavior this may due to, the finite element method assumes perfect bond between concrete and steel reinforcement and ideal material properties, but there was a good agreement between them.

**Figure 11.** Predicted and experimental load-mid span deflection relations for Group C.

5. Conclusions

Providing openings in a prestressed rafter reduce its weight, which can be cast in place or manufactured in precast plants and transported for use as supporting members for long span roofs. So, providing openings in prestressed rafters offer many advantages such as lightweight, handling, erection, and geometric flexibility. Based on the current study, it can be concluded that:

- Insertion openings in a prestressed rafter lead to reduce the ultimate load-carrying capacity by (3.1%-29.5%).
- Insertion openings in a prestressed rafter lead to increase in mid-span deflection (i.e. stiffness decrease) between 21.8%-70.4%, 7.6% - 51.6%, and 10.3% - 28.3% through all loading stages of elastic, service, and ultimate limits, respectively.
- Using circular openings instead of quadrilateral ones improve the flexural behavior of the rafters represented by an increase in ultimate load capacity (31.4-1.5%) and a decrease in deflection (22.8-4.3%) at service limit.
A slight increase in cracking load for beams with circular openings than that with quadratic openings by 22.2%.

- Finite element analysis using the ABAQUS 13.6.1 software program was used to validate the results of the tested rafters. Comparisons were presented and good agreement emerged between experimental results and predictions of finite element analysis in terms of load-deflection relationships and failure loads.
- The mean ratio of deflection and ultimate loads of numerical models to the experimentally tested rafters was 0.99 and 0.97, respectively. Since finite element analysis can be considered good.

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