Evolution of Universe to the present inert phase

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Abstract

We assume that current state of the Universe can be described by the Inert Doublet Model, containing two scalar doublets, one of which is responsible for EWSB and masses of particles and the second one having no couplings to fermions and being responsible for dark matter. We consider possible evolutions of the Universe to this state during cooling down of the Universe after inflation. We found that in the past Universe could pass through phase states having no DM candidate. In the evolution via such states in addition to a possible EWSB phase transition (2-nd order) the Universe sustained one 1-st order phase transition or two phase transitions of the 2-nd order.

1 Introduction

According to the standard cosmological model about 25% of the Universe is made of Dark Matter (DM). Different candidates for DM particle are now discussed in the literature. One of the widely discussed models is the Inert Doublet Model (IDM) \[ \mathbb{Z}_2 \] – a $Z_2$ symmetric 2HDM with a suitable set of parameters. The model contains one "standard" scalar (Higgs) doublet $\phi_S$, responsible for electroweak symmetry breaking and masses of fermions and gauge bosons as in the Standard Model (SM), and one scalar doublet, $\phi_D$, which doesn’t receive vacuum expectation value (v.e.v.) and doesn’t couple to fermions\(^1\).

In this model four degrees of freedom of the Higgs doublet $\phi_S$ are as in the SM: three Goldstone modes become longitudinal components of the EW gauge bosons and one mode becomes the Higgs boson (here denoted as $h_S$). All the components of the scalar doublet $\phi_D$ are realized as massive scalar $D$-particles: two charged $D^\pm$ and two neutral ones $D_H$ and $D_A$. By construction, they

\(^1\)Our notations are similar to those in the general 2HDM with the change $\phi_1 \to \phi_S, \phi_2 \to \phi_D$. 


possess a conserved multiplicative quantum number and therefore the lightest particle among them can be considered as a candidate for DM particle.

Assuming, as usual, that DM particles are neutral, we consider such variant of IDM, in which masses of $D$-particles are

$$ M_{D^z}, M_{D^\pm} \gtrsim M_{D^H} \quad \text{or} \quad M_{D^z}, M_{D^H} \gtrsim M_{D^\pm}. $$

(1)

Possible masses of these $D$-particles are constrained by the present accelerator and astrophysical data (see e.g. [2, 3]).

In this paper we assume that the current state of the Universe is described by IDM. We discuss possible variants of the history of the phase state $s$ of Universe during its cooling down after inflation. In some respects, this analysis can be considered as particular case of analysis [4], [5]. We use below some results and notations from [4]-[7].

2 The Lagrangian

In this paper we consider an electroweak symmetry breaking (EWSB) via the Brout-Englert-Higgs-Kibble (BEHK) mechanism, as described by the Lagrangian

$$ \mathcal{L} = \mathcal{L}_{gfv}^{SM} + \mathcal{L}_H + \mathcal{L}_Y(\psi_f, \phi_S), \quad \mathcal{L}_H = T - V. $$

(2)

Here, $\mathcal{L}_{gfv}^{SM}$ describes the $SU(2) \times U(1)$ Standard Model interaction of gauge bosons and fermions, which is independent on the realization of the BEHK mechanism. In the considered case the Higgs scalar Lagrangian $\mathcal{L}_H$ contains the standard kinetic term $T$ and the potential $V$ with two scalar doublets $\phi_S$ and $\phi_D$. The $\mathcal{L}_Y$ describes the Yukawa interaction of fermions $\psi_f$ with only one scalar doublet $\phi_S$, having the same form as in the SM with the change $\phi \rightarrow \phi_S$.

**Potential.** The potential must be $Z_2$ symmetric in order to describe IDM. Without loss of generality it can be written in the following form

$$ V = -\frac{1}{2} \left[ m_{11}^2 (\phi_S^\dagger \phi_S) + m_{22}^2 (\phi_D^\dagger \phi_D) \right] + \frac{1}{2} \left[ \lambda_1 (\phi_S^\dagger \phi_S)^2 + \lambda_2 (\phi_D^\dagger \phi_D)^2 \right] + \lambda_3 (\phi_S^\dagger \phi_S) (\phi_D^\dagger \phi_D) + \lambda_4 (\phi_S^\dagger \phi_D) (\phi_D^\dagger \phi_S) + \frac{\lambda_5}{2} \left[ (\phi_S^\dagger \phi_D)^2 + (\phi_D^\dagger \phi_S)^2 \right], $$

(3a)

with all parameters real and with additional condition$^2$

$$ \lambda_5 < 0. $$

(3b)

$^2$In the general $Z_2$ symmetric potential the last term has a form $\lambda_5 (\phi_S^\dagger \phi_D)^2 + \lambda_5^* (\phi_D^\dagger \phi_S)^2$.

The physical content of theory cannot be changed by the global phase rotation $\phi_a \rightarrow e^{i \alpha_a} \phi_a$ ($a = S, D$). Starting with an arbitrary complex $\lambda_5 = |\lambda_5| e^{i \phi}$ we select $\alpha_S - \alpha_D = \rho/2 + \pi/2$, to get (3) with negative $\lambda_5 = -|\lambda_5|$. 
The IDM is realized in some regions of parameters of this potential. To study thermal evolution, we will consider also other possible vacuum states of such potential, at another values of parameters.

To make some equations shorter, we use the following abbreviations:

$$
\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad R = \frac{\lambda_{345}}{\sqrt{\lambda_1 \lambda_2}}.
$$

(4)

**Discrete symmetries.** This potential (3) is invariant under two discrete symmetry transformations of a $Z_2$ type:

$$
S: \quad \phi_S \overset{S}{\to} -\phi_S, \quad \phi_D \overset{S}{\to} \phi_D, \quad SM \overset{S}{\to} SM,
$$

(5)

$$
D: \quad \phi_S \overset{D}{\to} \phi_S, \quad \phi_D \overset{D}{\to} -\phi_D, \quad SM \overset{D}{\to} SM,
$$

(6)

where SM denote the SM fermions and gauge bosons.

We call these transformations $S$-transformation and $D$-transformation, respectively. In the case when vacuum has vanishing vacuum expectation values $\langle \phi_S \rangle = \langle \phi_D \rangle = 0$ the mentioned above invariance of $V$ results in the $D$-parity and $S$-parity conservation in the processes involving only scalars (or scalars and gauge bosons). The Yukawa term violates $S$-symmetry even if $\langle \phi_S \rangle = \langle \phi_D \rangle = 0$, while it respects $D$-symmetry in any order of perturbation theory.

**Positivity constraints.** To have a stable vacuum, the potential must be positive at large quasi–classical values of fields $|\phi_i|$ (positivity constraints), for an arbitrary direction in the $(\phi_S, \phi_D)$ plane. These conditions limit possible values of $\lambda_i$ (see e.g. [8]). In terms of parameters (4) positivity constraints which are needed in our analysis, can be written as

$$
\lambda_1 > 0, \quad \lambda_2 > 0, \quad R + 1 > 0.
$$

(7)

3 Thermal evolution

Main goal of this paper is to consider an evolution of the Universe during its cooling down to the present inert phase. For this purpose we consider thermal evolution of the Lagrangian, following the approach presented in [9, 5].

**Potential.** Since the Hubble constant is small, we assume a statistical equilibrium at every temperature $T$. In this approximation, at the finite temperature, the ground state of system is given by a minimum of the Gibbs potential

$$
V_G = \text{Tr} \left( V e^{-H/T} \right) / \text{Tr} \left( e^{-H/T} \right).
$$

(8)

In the first nontrivial approximation and high enough temperature the obtained Gibbs potential has the same form as the basic potential $V(3)$, i.e. as the potential at zero temperature. The coefficients $\lambda$’s of the quartic terms in the
potential $V_G$ and $V$ coincide, while the mass terms vary with temperature $T$, as follows

$$m_{11}^2(T) = m_{11}^2 - c_1 T^2, \quad m_{22}^2(T) = m_{22}^2 - c_2 T^2,$$

$$c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4 + 3g^2 + g'^2}{12} + \frac{g_t^2 + g_b^2}{8}, \quad (9)$$

$$c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4 + 3g^2 + g'^2}{12} + \frac{g_t^2 + g_b^2}{32}.$$  

Here $g$ and $g'$ are the EW gauge couplings, $g_t \approx 1$ and $g_b \approx 0.03$ are values of the SM Yukawa couplings for $t$ and $b$ quarks, respectively.

Generally each of coefficients $c_1$ and $c_2$ can be either positive or negative. However, in virtue of positivity conditions (7) their sum is positive,

$$c_2 + c_1 > 0,$$  

(10)

even neglecting (positive) contributions from gauge bosons $W/Z$ and fermions.

We will show later on that for a realization of the present inert vacuum with neutral dark matter particle one needs $\lambda_4 + \lambda_5 < 0$. Therefore, at $R > 0$ we have $\lambda_3 > 0$. Taking into account that $\lambda_5 < 0$ (3b), we obtain that $c_1 > 0$, $c_2 > 0$. At $R < 0$ there are no constraints on signs of $c_{1,2}$:

$$R > 0: \quad c_1 > 0, \quad c_2 > 0; \quad R < 0: \quad \text{arbitrary signs of } c_{1,2}. \quad (11)$$

**Yukawa interaction.** The form of Yukawa interaction and values of Yukawa couplings don’t vary during thermal evolution.

### 4 Extrema of the potential

Following [4] we first consider extrema of the potential (3) at arbitrary values of parameters. The extremum conditions:

$$\partial V/\partial \phi_i|_{\phi_i = \langle \phi_i \rangle} = 0, \quad \partial V/\partial \phi_i^\dagger|_{\phi_i = \langle \phi_i \rangle} = 0, \quad (i = S, D) \quad (12)$$

define the extremum values $\langle \phi_S \rangle$ and $\langle \phi_D \rangle$ of the fields $\phi_S$ and $\phi_D$, respectively. The extremum with the lowest energy (the global minimum of the potential) realizes the vacuum state of the system. Other extrema are saddle points, maxima or local minima of the potential.

The most general solution of (12) can be written in a following form:

$$\langle \phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_D \end{pmatrix}, \quad (v^2 = v_S^2 + |v_D|^2 + u^2) \quad (13)$$

since for each electroweak symmetry violating extremum (EWv) with $\langle \phi_S \rangle \neq 0$, one can choose the $z$ axis in the weak isospin space so that $\langle \phi_S \rangle \sim \begin{pmatrix} 0 \\ v_S \end{pmatrix}$, with real, nonnegative $v_S$ (choosing a ”neutral direction” in the weak isospin space).
Neutral extrema. The solutions of (12) with \( u = 0 \) are called neutral extrema, as they respect \( U(1) \) symmetry of electromagnetism. For these extrema the conditions (12) can be written as a system of two degenerate cubic equations:

\[
v_S(-m_{11}^2 + \lambda_1 v_S^2 + \lambda_{345} v_D^2) = 0, \quad v_D(-m_{22}^2 + \lambda_2 v_D^2 + \lambda_{345} v_S^2) = 0, \quad v_S^2 \geq 0, \quad v_D^2 \geq 0.
\] (14)

This system has four solutions, one solution defines electroweak symmetric extremum \( \text{EWs} \) and three solutions define EWSB extrema: inert extremum \( I_1 \), inert-like extremum \( I_2 \) and mixed extremum \( M \). Below we list their v.e.v.’s and extrema energies \( \mathcal{E}_a \):

- **EWs**: \( v_D = 0, \quad v_S = 0, \quad \mathcal{E}_{	ext{EWs}} = 0; \) (15)
- **I_1**: \( v_D = 0, \quad v_S^2 = v^2 = \frac{m_{11}^2}{\lambda_1}, \quad \mathcal{E}_{I_1} = \frac{m_{11}^4}{8 \lambda_1}; \) (16)
- **I_2**: \( v_S = 0, \quad v_D^2 = v^2 = \frac{m_{22}^2}{\lambda_2}, \quad \mathcal{E}_{I_2} = \frac{m_{22}^4}{8 \lambda_2}; \) (17)
- **M**: \( v_S^2 = \frac{m_{11}^2 \lambda_2 - \lambda_{345} m_{22}^2}{\lambda_1 \lambda_2 - \lambda_{345}^2}, \quad v_D^2 = \frac{m_{22}^2 \lambda_1 - \lambda_{345} m_{11}^2}{\lambda_1 \lambda_2 - \lambda_{345}^2}; \)
  \[ \mathcal{E}_M = \frac{m_{11}^4 \lambda_2 - 2 \lambda_{345} m_{11}^2 m_{22}^2 + m_{22}^4 \lambda_1}{8(\lambda_1 \lambda_2 - \lambda_{345}^2)}. \] (18)

Some of the equations (16)-(18) can give also negative values of \( v_S^2 \) or \( v_D^2 \), in contradiction with basic condition for the extremum (14). In such case the extremum, described by corresponding equations, is absent.

The energy differences between \( I_{1,2} \) and \( M \) extrema are as follows:

\[ \mathcal{E}_{I_1} - \mathcal{E}_M = \frac{(m_{11}^2 \lambda_{345} - m_{22}^2 \lambda_1)^2}{8 \lambda_1 \lambda_2 (1 - R^2)}; \quad \mathcal{E}_{I_2} - \mathcal{E}_M = \frac{(m_{22}^2 \lambda_{345} - m_{11}^2 \lambda_2)^2}{8 \lambda_1 \lambda_2^2 (1 - R^2)}. \] (19)

Charge breaking extremum. For \( u \neq 0 \) the extremum violates not only EW symmetry but also the \( U(1) \) electromagnetic symmetry, leading to the electric charge non-conservation. According to general analysis in [10, 11, 12, 4, 7] this extremum can realize vacuum state only if:

\[ \lambda_4 + \lambda_5 > 0. \] (20)

We will see later on that at this condition the DM particle can not be neutral, that contradicts [1].

### 5 Vacuum states

Below we describe briefly properties of neutral extrema, listed in the previous section, provided that they are realized as true vacua.
5.1 Electroweak symmetric vacuum \( E W s \)

The electroweak symmetric extremum with \( \langle \phi_S \rangle = \langle \phi_D \rangle = 0 \) exists for all values of parameters of the potential \( (3) \). It respects the \( D \) and \( S \)-symmetries of the potential. This extremum is a minimum, realizing vacuum state, at

\[
m_{11}^2 < 0, \quad m_{22}^2 < 0.
\]  

In this case, gauge bosons and fermions are massless, while scalar doublets \( \phi_S \) and \( \phi_D \) have masses equal to \( |m_{11}| \) and \( |m_{22}| \), respectively.

5.2 Inert vacuum \( I_1 \)

In the case when \( I_1 \) extremum realizes vacuum, the Inert Doublet Model describes reality. The standard field decomposition near \( I_1 \) extremum has a form

\[
\phi_S = \left( v + h_S + iG \right) \sqrt{2}, \quad \phi_D = \left( D_H + iDA \right) \sqrt{2}, \quad (22)
\]

where \( G^\pm \) and \( G \) are Goldstone modes, while \( h_S \) and \( D = D_H, DA, D^\pm \) are scalar particles. Here the Higgs particle \( h_S \) interacts with the fermions and gauge bosons just as the Higgs boson in the SM.

**Symmetry properties.** The inert vacuum state violates the \( S \)-symmetry \( (3) \). However, this state is invariant under the \( D \)-transformation \( (6) \) just as the whole basic Lagrangian \( (2) \). Therefore the \( D \)-parity is conserved, and due to this fact the lightest \( D \)-particle is stable, being a good DM candidate.

**Allowed region of parameters.** For the inert extremum to exists it is necessary that \( m_{11}^2 > 0 \) \( (16) \). In accordance with \((10)\) and \((17)\), the extremum \( I_1 \) can be a vacuum only if \( m_{11}^2/\sqrt{\lambda_1} > m_{22}^2/\sqrt{\lambda_2} \). Additional condition arises from a comparison of \( I_1 \) and \( M \) extrema. In virtue of \((19)\) at \( 1 - R^2 < 0 \) the extremum \( M \) can exist but its energy is larger than energy of \( I_1 \) extremum – so that the extremum \( I_1 \) realizes vacuum. At \( 1 - R^2 > 0 \) the inert extremum still can be a vacuum, in the case when the mixed extremum does not exist, i.e. if at least one of quantities \( v_S^2, v_D^2 \) defined by eq. \((18)\) is negative.

Note, that due to the positivity constraint \( 1 + R > 0 \) \( (7)\) in the case when \( 1 - R^2 < 0 \) we have \( R > 1 \). For the opposite case, with \( 1 - R^2 > 0 \), the quantity \( R \) can be either positive or negative.

**Particle properties.** The quadratic part of the potential written in terms of physical fields \( h_S, D_H, DA \) and \( D^\pm \) \((22)\) gives the following masses of scalars:

\[
M_{h_S}^2 = \lambda_1 v^2 - m_{11}^2, \quad M_{D^\pm}^2 = \frac{\lambda_3 v^2 - m_{22}^2}{2},
\]

\[
M_{D_A}^2 = M_{D^\pm}^2 + \frac{\lambda_4 - \lambda_5}{2} v^2, \quad M_{D_H}^2 = M_{D^\pm}^2 + \frac{\lambda_4 + \lambda_5}{2} v^2.
\]  

(23)
The requirement that lightest $D$-particle is a neutral one\(^1\) results in the condition
\[ \lambda_4 + \lambda_5 < 0. \quad (24) \]
Since $\lambda_5 < 0$\(^2\) the ”scalar” $D_H$ is lighter than ”pseudoscalar”$D_A$.
As in the standard 2HDM, scalars $D_H$ and $D_A$ have opposite $P$-parities but since they don’t couple to fermions, there is no way to assign to them a definite value of $P$-parity. However, their relative parity does matter and for example, vertex $Z D_H D_A$ is allowed while vertices $Z D_H D_H$ and $Z D_A D_A$ are forbidden. According to our basic assumption on the Yukawa interaction\(^2\) $D$-particles don’t interact with fermions. Neither there are interactions of $D$-particles with gauge bosons $V$ of the type $D_i V_1 V_2$.

5.3 Inert-like vacuum $I_2$

The inert-like vacuum $I_2$ is ”mirror-symmetric” to the inert vacuum $I_1$, compare\(^{16}\) and \(^{17}\). The interaction among scalars and between scalars and gauge bosons are mirror-symmetric as well, so the only difference between $I_2$ and $I_1$ arises from the Yukawa interaction.

Main formulae for this state are similar to those for the vacuum $I_1$ with obvious replacements. The corresponding field decomposition is given by
\[ \phi_S = \left( \frac{S^+}{\sqrt{2}}, \frac{S_H + iS_A}{\sqrt{2}} \right), \quad \phi_D = \left( \frac{G^+}{\sqrt{2}}, \frac{v + h_D + iG}{\sqrt{2}} \right), \quad (25) \]
with one Higgs particle $h_D$ and four $S$-particles: $S_H$, $S_A$, $S^\pm$.

Symmetry properties. The inert-like vacuum $I_2$ violates $D$-symmetry\(^8\). This state as well as the Higgs potential is invariant under the $S$-transformation\(^4\). However, in contrast to the inert vacuum, here $S$-parity is not conserved by the whole Lagrangian because of the form of Yukawa interaction.

Allowed regions of parameters. The inert-like extremum exists for $m_{22}^2 > 0$. In order to have an inert-like vacuum it is necessary that $m_{11}^2/\sqrt{\lambda_1} < m_{22}^2/\sqrt{\lambda_2}$. For $1 - R^2 < 0$ there are no additional demands. If $1 - R^2 > 0$ inert-like extremum can be a vacuum only if at least one of quantities $v_S^2$, $v_D^2$, defined by eq.\(^{18}\), appears to be negative. Both these conditions are similar to those for the inert vacuum $I_1$.

\(^3\)Note that the rephasing transformation $\phi_1 \rightarrow \phi_1$, $\phi_2 \rightarrow i\phi_2$, changing sign of $\lambda_5$, results in change $D_H \leftrightarrow D_A$ in $I_1$ state.
Particle properties. The masses of the Higgs boson $h_D$ and $S$-scalars are given by (cf. (23))

$$M_{h_D}^2 = \lambda_2 v^2 = m_{22}^2,$$
$$M_{S\pm}^2 = \frac{\lambda_3 v^2 - m_{11}^2}{2},$$
$$M_{S_A}^2 = M_{S\pm}^2 + \frac{\lambda_4 - \lambda_5}{2} v^2,$$
$$M_{S_H}^2 = M_{S\pm}^2 + \frac{\lambda_4 + \lambda_5}{2} v^2.$$  \tag{26}

The Higgs boson $h_D$ couples to gauge bosons just as the Higgs boson of the SM, however it does not couple to fermions at the tree level. The $S$-scalars do interact with fermions. Therefore, here there are no candidates for dark matter particles. That is the reason to call this vacuum inert-like vacuum.

Note that all fermions, by definition interacting only with $\phi_S$ with vanishing v.e.v. $\langle \phi_S \rangle = 0$, are massless. (Small mass can appear only as a loop effect.) In such vacuum state particles form roughly uniform plasma with massless fermions and heavy gauge bosons and scalars.

5.4 Mixed vacuum $M$

The mixed extremum $M$ violates both $D$- and $S$-symmetries, i.e. the full $Z_2$ symmetry of the potential. In this vacuum we have massive fermions and no candidates for DM particle, like in the SM. The decomposition around the mixed vacuum looks as follows:

$$\phi_S = \left( \frac{\rho^+_S + v_S + i \chi_S}{\sqrt{2}} \right), \quad \phi_D = \left( \frac{\rho^+_D + v_D + i \chi_D}{\sqrt{2}} \right),$$  \tag{27}

where the $\rho^+_S$ and $\rho^+_D$ lead to two orthogonal combinations $G^+$ and $H^+$, while $\rho_S$ and $\rho_D$ ($\chi_S$ and $\chi_D$) – to two orthogonal combinations $h$ and $H$ ($G$ and $A$), respectively. There are here five Higgs bosons - two charged $H^\pm$ and three neutral ones: the CP-even $h$ and $H$ and CP-odd $A$.

Allowed regions of parameters. In accordance with (18) and (19) the mixed extremum is global minimum of potential, i.e. vacuum, if and only if the following conditions hold: $v_S^2 > 0$, $v_D^2 > 0$ and $1 - R^2 > 0$. For v.e.v.’s squared given by eqs. (18) the latter conditions can be transformed to the relations between mass parameters $m_{11}^2$ and $m_{22}^2$:

at $1 > R > 0$ : $0 < R \frac{m_{11}^2}{\sqrt{\lambda_1}} < \frac{m_{22}^2}{\sqrt{\lambda_2}} < \frac{m_{22}^2}{R \sqrt{\lambda_1}}$;

at $0 > R > -1$ : $\frac{m_{22}^2}{\sqrt{\lambda_2}} > R \frac{m_{11}^2}{\sqrt{\lambda_1}}$, $\frac{m_{11}^2}{\sqrt{\lambda_1}} > \frac{m_{22}^2}{R \sqrt{\lambda_2}}$.  \tag{28}

\footnote{Sometimes called a normal extremum $N$, see e.g. [12]}
Particle properties.  Masses of scalars are as follows (see, e.g. [6, 4])

\[ M_{H^\pm}^2 = \frac{\lambda_4 + \lambda_5}{2} v^2, \quad M_A^2 = -v^2 \lambda_5, \quad (v^2 = v_S^2 + v_D^2). \]  

(29)

The neutral CP-even mass matrix is equal to

\[ M = \begin{pmatrix} \lambda_1 v_S^2 & \lambda_{345} v_S v_D \\ \lambda_{345} v_S v_D & \lambda_2 v_D^2 \end{pmatrix}. \]  

(30)

Note, that the extremum can be minimum only if both diagonal elements of mass matrix and its determinant are positive, i.e. \( \lambda_1 \lambda_2 v_S^2 v_D^2 (1 - R^2) > 0 \), in agreement with the above mentioned conditions. It means also that in the case if mixed extremum is minimum, it is global minimum – vacuum.

The mass matrix (30) gives masses of the neutral CP-even Higgs bosons:

\[ M_{h,H}^2 = \lambda_1 v_S^2 + \lambda_2 v_D^2 \pm \sqrt{(\lambda_1 v_S^2 + \lambda_2 v_D^2)^2 - 4 \text{det} M}, \]  

(31)

with sign + for the \( H \) and sign – for \( h \).

Couplings of the physical Higgs bosons to fermions and gauge bosons have standard forms as for the 2HDM, with the Model I Yukawa interaction.

6 Evolution of phase states of the Universe

In this section we consider possible phase history of the Universe, leading to the inert vacuum \( I_1 \) today, using the thermal evolution described in sec. 3.

To summarize properties of different vacua of the \( Z_2 \)-symmetric potential and to classify all possible ways of evolution of the Universe we will use phase diagrams in the \((\mu_1(T), \mu_2(T))\) plane, where

\[ \mu_1(T) = m_{11}^2(T)/\sqrt{\lambda_1}, \quad \mu_2(T) = m_{22}^2(T)/\sqrt{\lambda_2}. \]  

(32)

Let us remind (sect. 3) that in our approximation during cooling down of Universe parameters \( \lambda_i \) are fixed, while mass parameters \( m_{ii}^2 \) vary. These variations result in modification of vacuum state and a possible change of its nature. Possible types of evolution depend on value of parameter \( R \) and are depicted in the figures [6.1, 6.2 and 6.3] The possible current states of Universe are represented in these figures by small black dots \( P = (\mu_1, \mu_2) \). Since currently we are in the inert phase, we have \( \mu_1 > 0 \) for each point \( P \) (see sect. [5.2]). The parameter \( \mu_2 \) can be both positive (points P1 and P3) and negative (points P2, P4 and P5).

In accordance with ([6]) a particular evolution leading to a given physical vacuum state \( P \) is represented by a ray, that ends at a point \( P \). Arrows on

\footnote{In subsequent discussion we will distinguish present day values of parameters \( \mu_i \equiv \mu_i(0) \) and their values \( \mu_i(T) \) at some temperature \( T \).}
these rays are directed towards a growth of time (decreasing of temperature). The direction of the ray is determined by parameters (cf. (9))

\[ \tilde{c}_1 = c_1 / \sqrt{\lambda_1}, \quad \tilde{c}_2 = c_2 / \sqrt{\lambda_2}, \quad \tilde{c} = \tilde{c}_2 / \tilde{c}_1. \] (33)

For different possible positions of today’s point \( P \) we consider typical evolutions for different possible values of parameter \( \tilde{c} \). In figures below all representative rays are shown; they are labeled by two numbers, with the first one corresponding to the label of the final point \( P \).

### 6.1 The case \( R > 1 \)

Phase diagram for this case is presented in Fig. 6.1. It contains one quadrant with EWs phase and two sectors, describing the \( I_1 \) and \( I_2 \) phases. These two phases \( I_1, I_2 \) are separated by the phase transition line \( \mu_1 = \mu_2 \) (thick black line). Two typical positions of today’s state are represented by points \( P1 (\mu_2 > 0) \) and \( P2 (\mu_2 < 0) \). Since (according to (11)), both \( \tilde{c}_1, \tilde{c}_2 > 0 (\tilde{c} > 0) \), all possible phase evolutions are represented by rays 11 and 12 for the today’s point \( P1 \) and by a ray 21 which leads to the today’s point \( P2 \).

![Figure 1: Phase diagram for \( R > 1 \) case.](image)

**Ray 11: \( \tilde{c} > \mu_2/\mu_1 > 0 \).** The Universe started from the EWs state and after the second-order EWSB transition at \( m^2_{11}(T) = 0 \), i.e. at the temperature

\[ T_{EWs,1} = \sqrt{m^2_{11}/c_1} = \sqrt{\mu_1/c_1}, \] (34)

has entered to the present inert phase \( I_1 \).
Ray 12: $0 < \tilde{c} < \mu_2/\mu_1$. The Universe started from the EWs state. Then it went through the EWSB second-order phase transition into the inert-like phase $I_2$ at $m_{22}^2(T) = 0$, i.e. at the temperature equal to

$$T_{EW,s,2} = \sqrt{m_{22}^2/c_2} = \sqrt{\mu_2/c_2}.$$  \hfill (35)

The next transition is the phase transition from the inert-like phase $I_2$ into the today’s inert phase $I_1$ at the point $R$, where $\mu_2(T) = \mu_1(T)$, i.e. at the temperature

$$T_{2,1} = \sqrt{\frac{\mu_1 - \mu_2}{c_1 - c_2}}.$$ \hfill (36)

That is the first-order phase transition with the latent heat given by

$$Q_{I_2 \rightarrow I_1} = T \left. \frac{\partial \mathcal{E}_{I_2}}{\partial T} - T \frac{\partial \mathcal{E}_{I_1}}{\partial T} \right|_{\mu_2(T) \rightarrow \mu_1(T)} = (\mu_2 \tilde{c}_1 - \mu_1 \tilde{c}_2) T_{2,1}^2/4.$$ \hfill (37)

Ray 21: $\mu_2 < 0$. The Universe started from EWs state and after the second-order EWSB transition at the temperature \[34\] has entered the today’s phase.

6.2 The case $1 > R > 0$

Figure 2: Phase diagram for $1 > R > 0$ case.

The phase diagram in Fig. 6.2 is modified in comparison with the Fig. 6.1 as according to \[28\] in the upper right quadrant of the considered $(\mu_1, \mu_2)$ plane the new sector – the mixed phase $M$ – occurs in the region

$$0 < R \mu_1 < \mu_2 < \mu_1/R.$$ \hfill (38)

As before, since $R > 0$ both $c_{1,2} > 0$ and consequently $\tilde{c} > 0$. 

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Since currently we are in the inert vacuum, the possible today’s states are of type of points $P_3$ and $P_4$, for which

$$\mu_2 < R\mu_1.$$  \hspace{1cm} (39)

All possible phase evolutions are represented by three rays in Fig. 6.2 with rays 31 and 32 having the today’s endpoint $P_3$ while the ray 41 is pointing $P_4$.

For the rays 31 and 41, phase evolutions are as for the rays 11 and 21, respectively. New situation appears for the ray 32.

**Ray 32: $0 < \tilde{c} < \mu_2/\mu_1$.** The Universe started from the EWs state. Then at the temperature given by (35) it went through the EWSB second-order phase transition into the inert-like phase $I_2$. At the subsequent cooling down the Universe goes through the mixed phase $M$ into the present inert phase $I_1$. The second-order phase transitions $I_2 \to M$ and $M \to I_1$ happened at the following temperatures

$$T_{phtr} : \quad T_{2,M} = \sqrt{\frac{\mu_1 - R\mu_2}{\tilde{c}_1 - \tilde{R}\tilde{c}_2}}, \quad T_{M,1} = \sqrt{\frac{R\mu_1 - \mu_2}{\tilde{R}\tilde{c}_1 - \tilde{c}_2}}.$$ \hspace{1cm} (40)

In accordance with equations in sect. 5, at the transition point $I_2 \to M$ masses of $S_H$ and $h$ vanish, while at the transition point $M \to I_1$ masses of $h$ and $D_H$ become 0. At small distance from the transition point with temperature $T_{phtr}$ these masses grow as a function of the temperature $T$ as $M_a^2 = A_a[T^2 - T_{phtr}^2]$, with different coefficients $A_a$.

### 6.3 The case $0 > R > -1$

Figure 3: Phase diagram for $0 > R > -1$ case.
The phase diagram is presented in Fig. 6.3. In this case, as follows from (28), the mixed phase $M$ region is realized in a wider region than in Fig. 6.2 (for $0 < R < 1$), even beyond an upper right quadrant of this plane, namely:

$$\mu_2 > \mu_1/R, \quad \mu_2 > \mu_1R.$$  \hspace{1cm} (41)

Since currently we are in the inert vacuum, we have

$$\mu_2 < R\mu_1 \quad (\mu_2 < 0).$$ \hspace{1cm} (42)

Therefore, in this case we have only one type of today’s point $P5$. However, new opportunities appear due to larger freedom for temperature coefficients $c_i$, as in accordance with (11) in this case $\tilde{c}_1$ and $\tilde{c}_2$ can be either positive or negative.

All possible phase evolutions leading to the point $P5$ are represented in Fig. 6.3 by four rays 51, 52, 53 and 54.

The ray 51 describes similar evolution as rays 21 and 41. New are rays 52, 53 and 54 with common feature, which is a lack of electroweak symmetry in very early stages of the Universe.

Ray 52: $\tilde{c}_1 > 0$, $\tilde{c}_2 < 0$, $\tilde{c} > \mu_2/\mu_1$. Here a high-temperature state of the Universe is the inert-like vacuum $I_2$. With cooling down the Universe goes through electroweak symmetric phase $EWs$ into the present $I_1$ phase. The second-order phase transitions $I_2 \rightarrow EWs$ and $EWs \rightarrow I_1$ happened, respectively, at the temperatures

$$T_{2,EWs} = \sqrt{\mu_2/\tilde{c}_2}, \quad T_{EWs,1} = \sqrt{\mu_1/\tilde{c}_1}.$$ \hspace{1cm} (43)

Ray 53: $\tilde{c}_1 > 0$, $\tilde{c}_2 < 0$, $\tilde{c} < \mu_2/\mu_1$. Here a high-temperature state of the Universe is an inert-like vacuum $I_2$. With cooling down the Universe passes through the mixed phase $M$ into the present $I_1$ phase. The phase transitions $I_2 \rightarrow M$ and $M \rightarrow I_1$ are of the second order; they happened at the temperatures given by eqs. (40).

Ray 54: $\tilde{c}_1 < 0$, $\tilde{c}_2 > 0$. For this ray the Universe stays in the inert vacuum $I_1$ during the whole evolution.

6.4 Summary of possible evolutions

We find that in the considered approximation the thermal evolution of Universe to the current inert phase can be studied effectively in the $(\mu_1, \mu_2)$ plane, at fixed values of quartic parameters $\lambda_i$. Different types of such evolution represented as directed rays depend crucially on two parameters: $R$ (4), describing the allowed for various vacua regions of the $(\mu_1, \mu_2)$ plane, and $\tilde{c}$ (33), determining the direction of rays. The first one depends only on the ratios between coefficients

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6This case was overlooked in the literature, see e.g. [1] and also [2]. We thank G. Gil and B. Gorczyca for discussion on this point.
of quartic part of potential \( \lambda_i \), while the second depends both on mentioned parameters \( \lambda_i \), which are unknown up to now, and on precisely known gauge and Yukawa couplings.

One can distinguish following types of evolution to the current inert phase:

- **I.** The Universe evolves from the initial electroweak symmetric state.
  - **Ia.** The simplest evolution to the inert phase is realized through a single EWSB phase transition of the second-order type (rays 11, 21, 31, 41, 51). Dark matter appears at this single transition simultaneously with EWSB.
  - **Ib.** After the first EWSB phase transition Universe passes into the inert-like phase \( I_2 \). Then it passes into the inert phase \( I_1 \) either directly (first-order phase transition, ray 12), or through the mixed phase \( M \) (two second-order phase transitions, ray 32). In both cases dark matter appears only after the last phase transition to the inert phase.

- **II.** The Universe evolves from the initial state having no electroweak symmetry.
  - **IIa.** The initial phase is the inert one \( I_1 \). Evolution contains no phase transitions. Dark matter existed always (ray 54).
  - **IIb.** The initial phase is the inert-like one \( I_2 \). It contains no dark matter. Evolution to the current inert phase \( I_1 \) undergoes through two second-order phase transitions either via the mixed phase \( M \) (ray 53), or via the EWs phase, i.e. with a temporary appearance of electroweak symmetry (ray 52). In both cases dark matter appears only after the last phase transition to the inert phase \( I_1 \).

Each of these evolutions can be realized in wide range of parameters.

### 7 If DM is charged

Model independent analysis shows that the case with charged DM particle is not ruled out absolutely, but charged DM particles must be heavier than 100\( q \) TeV, where \( q \) is electric charge of DM particle in units of electron charge \[14\]. For IDM it means \( M_{H^\pm} > 100 \) TeV. Such a heavy mass seems to be unnatural in the modern particle physics with natural energy scale \( \lesssim 1 \) TeV. In this case, our new model will include both new particles and new energy scale of phenomena. Obviously, such an opportunity cannot be tested at colliders in the estimable future.

\[14\] Such opportunity was discussed by a number of authors – see e.g. \[13\]. Certainly, it is not ruled out, but it contradicts a key idea of modern approach – the state at very high temperatures has high symmetry which is broken at cooling down of the Universe. In this sense this opportunity is unnatural.
The case with charged DM particle can be realized in IDM only if \( \lambda_4 + \lambda_5 > 0 \). Since unitarity and perturbativity constraints \( |\lambda_i| \lesssim 8\pi \) must hold (see for details [15, 16, 6]) very large \( D^\pm \) mass can arise only from very large negative \( m_{22}^2 \). The position on the \( (\mu_1, \mu_2) \) plane of the actual state of the Universe, for the anticipated in the SM value of the Higgs mass \( M_{hS} \lesssim 200 \) GeV and \( M_{D^\pm} \gtrsim 100 \) TeV, corresponds to \( \mu_1 > 0, \mu_2 < 0 \), with large ratio of their absolute values \( \gtrsim 10^5 - 10^6 \), if \( \lambda_1/\lambda_2 \sim 1 \).

In accordance with results of refs. [4, 10, 7], if \( \lambda_4 + \lambda_5 > 0 \) then the mixed phase cannot exist (see e.g. (29)) while the charge breaking phase can. The charge breaking vacuum can be realized if in addition \( |R_3| < 1 \), where \( R_3 = \lambda_3/\sqrt{\lambda_1\lambda_2} \). Simple analysis shows that the phase diagrams for this case are similar to those in Figs. 6.2, 6.3 with the replacement \( R \rightarrow R_3 \). Rays similar to the rays 41, 51, 52 and 54 give nothing new in comparison with the cases discussed in sect. 3.

The really new opportunity could appear for the ray similar to ray 53 in Fig. 6.3 (with rays going through the charge breaking vacuum). However, this opportunity is ruled out. Indeed, to realize it one needs \( c_2 < 0 \) and \( |c_2|/c_1 > |m_{22}^2|/m_{11}^2 \gtrsim (10^5 \div 10^6) \). The latter inequality contradicts the \( c_1 > -c_2 \) relation based on the positivity condition. It means that in our simple model Universe evolution to the current inert phase cannot pass through the charge breaking phase.

8 Results and discussion

Main results. The most important observation we made in this paper is as follows: if current state of the Universe is described by IDM, then during the thermal evolution the Universe can pass through various intermediate phases, different from the inert one. These possible intermediate phases contain no dark matter, which appears only at the relatively late stage of cooling down of the Universe.

A complete set of possible ways of evolution of the Universe, including both EW symmetric (\( EWs \)) and EW non-symmetric (\( EWv \)) initial states, can be summarized as follows:

\[
\begin{align*}
EWs: I_1 \xrightarrow{H} & I_2 \xrightarrow{H} M \xrightarrow{H} I_1 \quad \text{ray 11, 21, 31, 41, 51} \\
\quad & \xrightarrow{I} I_1 \quad \text{ray 12}
\end{align*}
\]

\[
\begin{align*}
EWv: I_2 \xrightarrow{H} & \begin{cases} I_1 \xrightarrow{H} \text{EWs } \xrightarrow{H} I_1 \quad \text{ray 52} \\
M \xrightarrow{H} I_1 \quad \text{ray 53} \end{cases} \\
I_1 \rightarrow & I_1 \quad \text{ray 54}
\end{align*}
\]

\(^8\)Symbol I or H over arrows corresponds to the type of phase transition.
We see that both a simple EWSB with a direct transition to the inert phase, as well as sequences of two or three transitions from EWs to the inert phase are possible. We found also that the current inert state of Universe can be obtained from both initial high temperature state with EW symmetry and from the initial state without this symmetry.

As far the charged DM is concerned, still in principle allowed by the data, if heavy enough, it seems to be excluded in IDM.

To find what scenario of evolution is realized in nature, one should measure all parameters of potential. The program how to measure these parameters at LHC and ILC is under preparation.

**Outlook.** In contrast to the standard picture, these scenarios allow for the phase transition to the current inert phase at relatively low temperature, giving new starting point for calculation of a today’s abundance of the neutral DM components of the Universe.

In this paper we calculated thermal evolution of the Universe in the very high temperature approximation, i.e. for $T^2 \gg |m^2_{ii}|$. The most interesting effects are expected at lower temperatures, where more precise calculations are necessary. The simplest expected modifications of the presented description are:

1. Appearance of cubic terms like $\phi^3 T$ [17]. These terms are important near phase transition point, as they can transform some second-order phase transitions into the first-order transitions.

2. The parameters become depend on temperature in more complicated way than that given by [10]. Therefore, the rays, depicted thermal evolutions in Figs. 6.1, 6.2 and 6.3 can become non-straight. The bending of these rays can be different in different points of our plots and at different $\lambda_i$. It can give possible spectrum of phase evolutions even reacher that discussed above.

However, we expect that the general picture will not change too much.

**Possible extensions of model.** The model we focused on in this paper contains two doublets. One can consider similar model with three doublets (particular case of 3HDM), as it is discussed in ref. [19]. It contains two standard Higgs doublets of 2HDM $\phi_{S1}$ and $\phi_{S2}$, coupled to fermions, and one Higgs doublet $\phi_D$ (having no coupling to fermions). The potential is invariant under $S$, $D$ transformations:

$$\{\phi_{S1}, \phi_{S2}\} \xrightarrow{S} -\{\phi_{S1}, \phi_{S2}\}, \quad \phi_D \xrightarrow{S} \phi_D, \quad SM \xrightarrow{S} SM;$$

$$\{\phi_{S1}, \phi_{S2}\} \xrightarrow{D} \{\phi_{S1}, \phi_{S2}\}, \quad \phi_D \xrightarrow{D} -\phi_D, SM \xrightarrow{D} SM. \quad (45)$$

This model can incorporate all phenomena which appear in the standard 2HDM, at the same time it contains DM particles as in the IDM. Thermal evolution of its parameters gives very diverse phase diagram which contain in
addition to the phases discussed above other phases, discussed in refs. [4, 5, 7] and their mixtures. However, even for this model our main conclusion about possible transformation of the Universe through the phase without DM holds.

Reacher variants of both $S$- and $D$-sectors can be considered similarly. For example, one widely discussed model of this type (see e.g. [20]) contains the same $S$-sector as in our paper, but $D$-sector consists of one doublet and one singlet scalars, non-interacting with fermions. This model has phenomenology similar to that discussed above but one hope to derive more or less natural values of couplings starting from SO(10) universality at the GUT scale. The biography of Universe in this model can be studied as above – obviously it will give more diverse phase story.

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