A Multi-reference Extended Variable Step-size LMS Algorithm for Solving Dynamic Weighting Coefficients of the Satellite Beam-forming Networks

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Abstract. A Multi-beam satellite communication system can form beam by coherently superimposing the beams generated by multiple feeds in space. In order to form the beam based on dynamic user, a multi-reference extended variable step-size LMS algorithm is introduced in this paper to solve the dynamic weighting coefficients. According to the simulation result, when the mean square error of the synthesized beam is small, this algorithm can effectively improve the beamforming convergence rate.

1. Introduction

Multi-beam mobile communication system is widely used in satellite communication. Its synthesized beam is similar to the wireless cell on the ground, and it can seamlessly cover the region by overlapping beam. The enhanced beamforming method can form the required beam with the coherent superposition of beams from multiple feeds. It can adjust the weighting coefficients of each feed to control the gain and beam shape and finally improve the communication quality and anti-interference performance.

The algorithm in this paper is intended to find optimal weighting coefficients $w$ that can achieve the requirement of beamforming, gain, main-lobe width and side-lobe suppression.

In order to improve the beam service quality, a dynamic user-centered synthesis system is adopted by the multi-beam system to synthesize flexible and variable beams. The corresponding beamforming network not only needs to cover the full region, but also decrease the width of the main-lobe and suppress the side-lobe. For achieving real-time tracking effect and improving the information transfer rate, it is supposed to dynamically adjust the beam according to the user geographical position and channel or generate new beams to provide services. During the beamforming, the algorithm solving dynamic weighting coefficients plays an important role.

SMI, LCMV are the widely used beamforming algorithms. However, due to the complexity of the calculation and the accuracy of beamforming, they can't meet the system demand. Least mean square (LMS) algorithm is a self-adaptive beamforming algorithm based on Minimum Mean Square Error (MMSE) criterion and it is the most widely used beamforming algorithm currently. LMS can get optimal weighting coefficients and higher accuracy with lower complexity by iterative computation. Beams formed with LMS algorithm have better center directivity, stronger main-lobe constraint and greater side-lobe suppression than SMI, LCMV.

The dynamic beamforming network requires higher convergence speed, better precision and more flexibility of algorithm to satisfy dynamic user demands. Intended for this issue, a multi-reference
extended variable step-size LMS algorithm is designed to effectively improve the convergence rate when the beamforming accuracy is satisfied.

This paper consists of five parts. First, chapter 1 briefly introduces the background of the research. Then, chapter 2 describes the system model the proposed algorithm is based on. The multi-reference extended variable step-size LMS algorithm is elaborated in Chapter 3. Finally, chapter 4 shows the verification simulation result of the algorithm and the last chapter is a full-text summary.

2. System Model

Consider about the situation dynamically forming beams based on users shown in Figure 1. When the user’s position changes, especially in high-speed, it requires a faster tracking capability of the beam to ensure that the user is always in the center of the beam. This makes higher requirements on the convergence, precision and complexity of beamforming networks.

![Figure 1. Application Model of Dynamic Multi-beamforming](image)

![Figure 2. Dynamic Beamforming Network Framework](image)
The framework of dynamic beamforming network shows in Figure 2. If the beam k serves a user, it will select user’s position as the center of the beam at first, then choses corresponding feed combination and its weighting coefficients. Finally form the beam by changing the switch matrix to open or close a feed branch.

The beam needs to follow the user’s moving so a new feed combination and its corresponding weighting coefficient need to be calculated. Different with the method finding new feed combination to form fixed beam in traditional beamforming network, the scheme in this paper multiplexes the weighting coefficient network, namely, updates the weighting coefficient network of the original beam \(k\) and selects the new feed combination by switch matrix to make the center of the beam move arbitrarily.

3. Multi-reference Extended Variable Step-size LMS Algorithm

3.1 Variable Step-size LMS Algorithm

For the beamforming algorithm, the convergence performance of the traditional LMS algorithm makes great difference on the speed of the dynamic beamforming system to generate adaptive beam [1]. The early analysis about the convergence performance of LMS is studied by Widrow[2][3] and other researchers, they mainly make the theoretical derivation and verification on the convergence performance of the weight vector error. In paper [4] a variable step-size LMS algorithm based on sigmoid function is proposed which can effectively improve the convergence rate when ensuring the convergence.

\[
\mu(n) = \beta \left( \frac{1}{1 + \exp(-\alpha |e(n)|)} - \frac{1}{2} \right)
\]

\(e(n)\) is the multi-reference error vector, \(\alpha\) and \(\beta\) are function parameters.

\[
e(n) = d(n) - X^T(n)W(n)
\]

\(X(n)\) and \(d(n)\) respectively represent the reference field strength of the current synthesized beam and the target beam.

In the \(n+1\) times iteration, the weighting coefficients are calculated as follow,

\[
W(n+1) = W(n) + 2\mu(n)e(n)X(n)
\]

In this paper, the algorithm is extended to a multi-reference-point variable step-size dynamic beamforming algorithm which can improve the convergence performance and ensure the accuracy of beamforming at the same time.

3.2 Multi-reference Extended Variable Step-size LMS Algorithm

As a dynamic beamforming system, the requirement of synthetic beam quality is relatively high, especially the main-lobe width and side-lobe suppression. In order to improve the accuracy of beamforming, the variable step-size LMS beamforming algorithm proposed in the previous section is extended to an algorithm with more reference points and less error between the target beam and the actual synthesized beam. At the same time, considering the central position of the synthesized beam needs to follow the user, the beam must can track, so this algorithm is extended to a real-time algorithm the so-called dynamic beamforming algorithm [5].

First, the number of target beam reference points is \(K\) which covers the main-lobe and side-lobe. \(E_m(\Phi,\Psi,t)\) represents the secondary radiation field intensity of the synthetic beam \(m\) at time \(t\), then we get

\[
\hat{d} = [E_m(\Phi_1,\Psi_1,t) \quad E_m(\Phi_2,\Psi_2,t) \quad \ldots \quad E_m(\Phi_K,\Psi_K,t)]
\]
Vector $\hat{d}$ represents sampling value of the target beam reference points, $\Phi, \Psi$ is the azimuth angle. In order to simplify the ratiocination, we set all feed weighting coefficient to be 1 at time $t$ so that all reference point amplitude and phase can be expressed as

$$
X(\text{t}) = \begin{bmatrix}
\varepsilon_1(\Phi_1, \Psi_1, t) & \cdots & \varepsilon_k(\Phi_k, \Psi_k, t)
\end{bmatrix}
$$

After the weighting operation, the synthesized beam at reference point is

$$
\hat{y}(\text{t}) = \hat{w}(\text{t})X(\text{t})
$$

$\hat{w}(\text{t}) = [w_1(\text{t}), w_2(\text{t}), ..., w_N(\text{t})]$ is the weighting coefficient vector of the feed array at time $t$, then the synthetic beam error is

$$
\hat{e}(\text{t}) = \hat{d}(\text{t}) - \hat{y}(\text{t})
$$

Mean square error is

$$
\xi(\text{t}) = \left| \hat{e}(\text{t}) \right|^2
$$

Because of multiple reference points and each point has relative error with target beam, it is impossible to make all the errors reach the minimum value at the same time. In this paper, use the synthetic beam optimization criteria which means that the optimization weighting coefficient vector needs to make the mean square error of all reference points minimum at time $t$.

$$
\min \left\{ \xi(\text{t}) \right\} = \sum_{k=1}^{K} \xi_k(\text{t})
$$

Among them

$$
\xi_k(\text{t}) = \left| d_k(\text{t}) - \hat{w}(\text{t})X(k, t, :) \right|^2
$$

When optimizing the weighting coefficients some reference point error may be relatively large, but the criterion can make sure the overall result so a few reference with large error almost make no difference on beamforming. At the same time, time $t$ is extended to the algorithm which makes the algorithm satisfy the real-time requirement. According to the steepest descent theory, the fastest changing direction for the function is the direction of the gradient, which means the direction of the negative gradient is the fastest decreasing direction of the function. Therefore, in each iteration, the iteration coefficients change along the negative gradient to get the weighting coefficient finally. The weighting coefficients iteration formula can be expressed as

$$
w_{k+1}(\text{t}) = w_k(\text{t}) + \mu(n)[\nabla_w f(w_k(\text{t}))]
$$

$f(w(\text{t}))$ is the function of $w(\text{t})$, $\nabla_w f(w_k(\text{t}))$ represents the gradient of the function at $w_k(\text{t})$. $\mu(n)$ is step size which will decrease when iteration increases. After several iterations, the steepest descent method reaches a steady state and the final weighting coefficients can get minimum mean square error. For its gradient
\[
\n\n\frac{\partial^2}{\partial t^2} \xi_k(t) = \frac{\partial}{\partial t} \left[ d_k(t) - \mathbf{w}^T X_k \right] - w X_k \mathbf{d}_k^T - w X_k \mathbf{d}_k^T \mathbf{d}_k^T - w X_k \mathbf{d}_k^T \mathbf{d}_k^T. 
\]

(12)

\[
\n\n\n\frac{\partial}{\partial t} \xi_k(t) = \sum_{k=1}^{K} \xi_k(t) = \sum_{k=1}^{K} \nabla \xi_k(t) = \sum_{k=1}^{K} -2\epsilon_k^* X(k,t,:)^T = -2\epsilon^* (n) X(t)^T 
\]

(13)

Finally get

\[
\hat{\mathbf{w}}(n+1,t) = \hat{\mathbf{w}}(n,t) + 2\mu(n)\epsilon^* (n) X(t)^T
\]

(14)

The above equation is the multi-reference extended variable step-size LMS algorithm.

4. Simulation Results

In general, choose \(\alpha=0.2, \beta=0.4\) for the simulation to compare the convergence and synthesized beam performance between LMS algorithm with variable step-size and fixed step-size LMS.

![Figure 3](image)

**Figure 3.** Convergence Performance Comparison of Variable Step-size and Fixed Step-size LMS Algorithm

In Figure 3, the fixed step-size LMS algorithm needs about 40 iterations to achieve mean square error stability, but the multi-reference extended variable step-size LMS algorithm only needs about 20 iterations. The beam convergence rate approximately doubled.
Then compare the beamforming performance between the fixed step-size LMS algorithm with the variable step-size LMS algorithm.

![Figure 4. Comparison of Beamforming Performance of Variable Step-size and Fixed Step-size LMS](image)

The simulation results show that the LMS algorithm with the fixed step-size $u=0.003$ and the variable step-size LMS algorithm with $\alpha=0.2$, $\beta=0.4$ are almost identical in the performance beamforming. Their graph almost coincident and these two algorithm both can achieve the main-lobe width and side-lobe level suppression requirement of dynamic beamforming system. The error vector $e(n)$ and corresponding value $\mu(n)$ is larger in the initial iteration but $e(n)$ and $\mu(n)$ will decrease over each iteration which make the weighting coefficients more accurate and ensure the performance of the synthesized beam.

5. Conclusion
This paper proposes a multi-reference extended variable step-size LMS algorithm for dynamic beamforming network. The simulation result shows that the algorithm can effectively improve the convergence rate of weighting coefficients and ensure the beamforming performance at the same time. It can effectively track users and meet the requirements of dynamic beamforming.

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