**J/ψ transport in QGP and \( p_t \) distribution at SPS and RHIC**

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Combining the hydrodynamic equations for the QGP evolution and the transport equation for the primordially produced \( J/ψ \) in the QGP, we investigate the \( J/ψ \) transverse momentum distribution as well as its suppression in the \( \sqrt{s} = 17.3 \) GeV Pb-Pb collisions at SPS and \( \sqrt{s} = 200 \) GeV Au-Au collisions at RHIC. The two sets of equations are connected by the \( J/ψ \) anomalous suppression induced by its inelastic scattering with gluons in the QGP. The calculated centrality dependence of \( J/ψ \) suppression and average transverse momentum square agree well with the SPS data. From the comparison with the coalescence model where charm quark is fully thermalized, our calculated elliptical flow of the primordially produced \( J/ψ \) is much smaller. This may be helpful to differentiate the \( J/ψ \) production mechanisms in relativistic heavy ion collisions.

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I. INTRODUCTION

Many mechanisms are proposed to explain the phenomenon of charmonium suppression\(^1\) in relativistic heavy ion collisions.\(^2\) In models\(^3\, 4\, 5\, 6\, 7\, 8\, 10\, 11\), the charmonium is supposed to be created by the hard processes in the initial state, on the way out, it firstly collides inelastically with spectators which leads to the normal suppression.\(^12\, 13\). The anomalous suppression\(^2\) is attributed to different mechanisms: collisions with spectators,\(^3\) instantly melting in Quark-Gluon Plasma (QGP)\(^4\), collisions with gluons in QGP\(^5\, 6\, 7\) or collisions with hadronic comovers\(^8\, 10\, 11\). Recently, there appeared another kind of models\(^14\, 15\) based on the assumption that charmonium is created at QCD hadronization according to statistical law, which is the extrapolation of the well-tested thermal model\(^10\) for light hadrons. There is also a two component model\(^17\) which mixes these two kinds of mechanisms. With some adjustable parameters in each mechanism, almost all models can describe the \( J/ψ \) anomalous suppression data at SPS very well. In order to differentiate these models, we need more information such as open charm, \( \chi_c \) and \( \psi' \), and transverse momentum (\( p_t \)) distribution. In this Letter, we concentrate on the observables related to the \( p_t \) distribution of \( J/ψ \), the average transverse momentum (\( \langle p_T^2 \rangle \)) and the anisotropic asymmetry parameter (\( \nu_2 \)) of \( J/ψ \) as well as the \( p_t \) integrated anomalous suppression in the heavy ion collisions at SPS and RHIC.

We follow the method in Refs.\(^2\, 4\, 7\) to treat the anomalous suppression. We neglect the formation time and assume that the charmonium is created instantaneously after the binary collisions. On the way out, it collides firstly with spectators and then with gluons in the QGP. The former and the later are, respectively, the origin of normal and anomalous suppression in our approach. The QGP evolution is calculated with a \( 2 + 1 \) dimensional boost invariant relativistic hydrodynamics\(^19\). Since \( J/ψ \) is heavy enough, a classical Boltzmann-type transport equation in the transverse phase space\(^18\) is used to describe the evolution of its transverse distribution function. We assume that the local equilibrium is reached at a proper time \( \tau_0 \) when the normal suppression has ceased and the anomalous suppression starts. Therefore, the initial condition of the transport equation is determined by the normal suppression. We neglect the elastic collisions between charmonium and particles in the medium, for the much larger charmonium mass than the typical temperature of the medium.

The paper is organized as follows. We describe the evolution of the medium in Section 2, give the details of the cross sections between charmonium and particles in Section 3, discuss the transverse transport equation in Section 4, and show the numerical results in Section 5. Finally we conclude in Section 6.

II. MEDIUM EVOLUTION

As in Ref.\(^20\), we assume that the produced medium reaches local equilibrium at a proper time \( \tau_0 = 0.8 \) fm/c for \( \sqrt{s} = 17.3 \) GeV Pb-Pb collisions at SPS and 0.6 fm/c for \( \sqrt{s} = 200 \) GeV Au-Au collisions at RHIC. The consequent evolution is described with relativistic hydrodynamics,

\[
\partial_\mu T^{\mu \nu} = 0, \quad \partial_\mu N^\nu = 0, \tag{1}
\]

where \( T^{\mu \nu} = (\epsilon + p) u^\mu u^\nu - g^{\mu \nu} p \) is the energy-momentum tensor and \( N^\mu = u^\mu \) the baryon current with four-velocity \( u^\mu \) of the fluid cell, energy density \( \epsilon \), pressure \( p \) and baryon density \( n \). In our calculation, we use Bjorken’s hydrodynamical model\(^19\). With the Hubble-like longitudinal expansion and boost invariant initial condition, the hydrodynamical quantities are functions of the proper time \( \tau = \sqrt{t^2 - z^2} \) and transverse coordinates only, and the equations can be simplified as

\[
\begin{align*}
\partial_\tau E + \nabla \cdot \vec{M} &= -(E + p)/\tau, \\
\partial_\tau M_x + \nabla \cdot (M_x \vec{v}) &= -M_x/\tau - \partial_\tau p, \\
\partial_\tau M_y + \nabla \cdot (M_y \vec{v}) &= -M_y/\tau - \partial_\tau p, \\
\partial_\tau M_t + \nabla \cdot (M_t \vec{v}) &= -M_t/\tau - \partial_\tau p, \\
\partial_\tau R + \nabla \cdot (R \vec{v}) &= -R/\tau \tag{2}
\end{align*}
\]

with the definitions \( E = (\epsilon + p) \gamma^2 - p, \vec{M} = (\epsilon + p) \gamma^2 \vec{v} \) and \( R = \gamma n \), where \( \gamma \) is the Lorentz factor.
To close the hydrodynamical equations we need to know the equation of state of the medium. We follow Ref. [21] where the deconfined phase at high temperature is an ideal gas of massless $u$, $d$ quarks, 150 MeV massed $s$ quarks and gluons, and the hadron phase at low temperature is an ideal gas of all known hadrons and resonances with mass up to 2 GeV [22]. There is a first order phase transition between these two phases. In the mixed phase, the Maxwell construction is used. The mean field repulsion parameter and the bag parameter are chosen as $K=450$ MeV fm$^{-3}$ [21] and $B^{1/4}=236$ MeV to obtain the critical temperature $T_c=165$ MeV at vanishing baryon number density.

We assume that the medium maintains chemical and thermal equilibrium until the energy density of the system drops to a value of 60 MeV fm$^{-3}$, when the hadrons decouple and their momentum distributions are fixed. We have tested that a slight modification on the freeze-out condition will not change our conclusions.

At SPS energy, we follow Ref. [22] to set the initial energy density and baryon number density with the wounded nucleon number density,

$$n_{wn}(\vec{b}, \vec{x}_i) = T_A(\vec{x}_i)(1 - e^{-\sigma_{NN}T_B(\vec{x}_i - \vec{b})}) + T_B(\vec{x}_i - \vec{b})(1 - e^{-\sigma_{NN}T_A(\vec{x}_i)}) , \tag{3}$$

where $T_A(B)(\vec{x}_i) = \int_0^\infty dz \rho_A(B)(\vec{x}_i, z)$ is the thickness function of nuclear A(B) with nuclear density profile $\rho_A(B)$. We choose Woods-Saxon profile, $\rho(r) = \frac{\rho_0}{e^{(r/R_0)/\xi}+1}$. The parameters $R$ and $\xi$ for $^{208}Pb$ and $^{197}Au$ are from Ref. [24]. At RHIC energy, we follow Refs. [22, 26] and assign 75% of the entropy density to soft contribution which is proportional to the wounded nucleon number density and 25% to hard contribution which is proportional to the binary collision number density $n_{NN}(\vec{b}, \vec{x}_i) = T_A(\vec{x}_i)T_B(\vec{x}_i - \vec{b})\sigma_{NN}$. The baryon density is obtained by adjusting the entropy per baryon to 250 [24]. The nucleon-nucleon inelastic cross section $\sigma_{NN}$ is 32 mb at SPS and increases to 41 mb at RHIC.

In peripheral collisions or in the peripheral region of central collisions, the system can not reach thermalization even if the energy density is larger than the critical value for phase transition. In order to describe this effect, we incorporated a cut in the initial condition. We assume that the medium will hadronize and decouple at the initial time if the local entropy density is less than a critical value $s_c$. This critical value will be an adjustable parameter of our approach, and will be fixed by fitting the $J/\psi$ anomalous suppression data at SPS. The same value will be used in the calculation of the hydrodynamics at RHIC energy.

We use the well tested RHLLE algorithm [24, 25] to solve the hydrodynamical equations numerically. The simple first order operator splitting method is used to extrapolate the original one dimensional RHLLE algorithm to two dimensions.

### III. DISSOCIATION CROSS-SECTIONS

In the QGP phase, we consider gluon dissociation process ($g + \Psi \rightarrow e + \bar{e}$) only. We will use $\Psi$ as the shorthand notation of charmonium in this Letter. The gluon-$J/\psi$ dissociation cross section can be obtained from the perturbative calculation with non-relativistic and Coulomb potential approximation for the $c\bar{c}$ system in the vacuum [20],

$$\sigma_g^{\psi}(\omega) = A_0 \left(\frac{\omega}{\epsilon_\psi} - 1\right)^{3/2}\left(\frac{\omega}{\epsilon_\psi}\right)^2, \tag{4}$$

with $A_0 = (2^{11/27}/27)(m_0^2\epsilon_\psi)^{-1/2}$, where $\omega = p^2\epsilon/m_\psi = (s - m_0^2)/(2m_\psi)$ is the gluon energy in the rest frame of $\Psi$, $\epsilon_\psi = m_D - m_\Psi$ the binding energy of $\Psi$, $m_c = m_D = 1.87$ GeV and $m_\psi = 3.1$ GeV.

In a similar way, the gluon dissociation cross section for $\psi'$ [23] and $\chi_c$ [3] are

$$\sigma_g^{\psi'}(\omega) = 16A_0\left(\frac{\omega}{\epsilon_{\psi'}} - 1\right)^{3/2}\left(\frac{\omega}{\epsilon_{\psi'}}\right)^2 - 2\left(\frac{\omega}{\epsilon_{\psi'}}\right)^2 \left\{ 2\left(\frac{\omega}{\epsilon_{\psi'}}\right)^2 - 20\left(\frac{\omega}{\epsilon_{\psi'}}\right) + 12 \right\}, \tag{5}$$

The masses of $\psi'$ and $\chi_c$ are 3.7 GeV and 3.5 GeV, respectively. The $J/\psi$ from the feed-down of $\psi'$ is about 10% of the total final $J/\psi$’s in pp collisions. For simplicity, we neglect the contribution of $\psi'$ to $J/\psi$ in our calculation. Therefore, 40% of the final state $J/\psi$’s are from the feed-down of $\chi_c$ and others are created directly [3].

We have neglected the medium effect on the dissociation cross sections. In relativistic heavy ion collisions, the medium produced is quite finite and evolves very fast, perhaps the medium effect is not so important as that in a static and infinite medium.

In the hadron phase, the most populated hadrons are pions. There are many effective models that can calculate the inelastic cross sections between charmonium and hadrons [31]. For $J/\psi$, the dissociation cross section is about a few mb which is comparable to the gluon dissociation cross section. If the QGP is created in relativistic heavy ion collisions, the thermalized hadron phase is at the later time of the evolution, and at that time the number density of hadrons is very small compared with the number density of gluons at the early time. Although the hadron dissociation may be important to the consideration of $\chi_c$ and $\psi'$ suppression, we neglect the contribution of hadrons in the calculation for $J/\psi$.

### IV. CHARMONIUM TRANSPORT

We define the full phase space distribution function of charmonium at a global time t as $f_\Psi(\vec{x}, \vec{p}, t)$. The transport equation describing the evolution of $f_\Psi$ is

$$p^\mu \partial_\mu f_\Psi(\vec{x}, \vec{p}, t) = -C_D^\Psi(\vec{x}, \vec{p}, t) f_\Psi(\vec{x}, \vec{p}, t). \tag{6}$$

The l.h.s is the drift term and the r.h.s is the dissociation term due to the inelastic collisions between charmonium and constituents of the medium. The coefficient $C_D^\Psi$ of the dissociation term in Eq. (6) is

$$C_D^\Psi(\vec{x}, \vec{p}, t) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} 2E_k \sigma_g^\Psi(s)4F_{g\Psi}(\vec{x}, \vec{k}, t) \tag{7}$$
with \( E_k = \sqrt{m_k^2 + k^2} \) and flux factor \( F_{y\Psi} = \sqrt{(p^\mu_\Psi p^\mu_\bar{\Psi})^2 - m_{\Psi}^2 m_{\bar{\Psi}}^2} \), where \( f_\Psi \) is the Lorentz invariant distribution function of the constituents, and \( \sigma_D^y(s) \) is the dissociation cross section for the charmonium in the medium. It is obvious that \( C_D^y \) is Lorentz invariant.

Considering the boost invariant initial condition at the proper time \( \tau_0 \), it is convenient to use momentum rapidity \( y = 1/2 \ln \left( (E + p_z)/(E - p_z) \right) \) and space-time rapidity \( \eta = 1/2 \ln \left( (t + z)/(t - z) \right) \) as longitudinal variables. With the assumption that the charmonium is produced at the point \( t = 0 \) and \( z = 0 \) due to the large Lorentz contraction of nucleus and short duration time of the initial hard processes in high energy heavy ion collisions, the charmonium momentum rapidity is equal to its space-time rapidity, \( y = \eta \). The distribution function can then be rewritten as

\[
 f_\Psi(x_1, \eta, \vec{p}_1, y, \tau) = \frac{\tau_0}{\tau} f_\Psi(x_1, \vec{p}_1, y, \tau) \delta(\eta - y) . \tag{8}
\]

After integration over \( \eta \), the l.h.s. of the transport equation \( \Box \) is simplified as

\[
 \frac{\tau_0}{\tau} (m_\Psi \frac{\partial}{\partial \tau} + \vec{p}_1 \cdot \frac{\partial}{\partial x_1^i} ) f_\Psi(x_1, \vec{p}_1, y, \tau) \tag{9}
\]

and the r.h.s becomes

\[
 - \frac{\tau_0}{\tau} C_D^\Psi(x_1, \vec{p}_1, \tau) f_\Psi(x_1, \vec{p}_1, y, \tau) \tag{10}
\]

In Bjorken’s hydrodynamics, the longitudinal rapidity of medium flow equals its space-time rapidity. Therefore, \( \Psi \)'s rapidity is zero in the longitudinal rest frame of the medium flow. Due to the Lorentz invariance of \( C_D^\Psi \), we can calculate it in this rest frame. Furthermore, we assume that the temperature and baryon chemical potential in the rest frame is \( \eta \) independent, since the rapidity region of the detected \( \Psi \)'s is very narrow and near the central rapidity region. With this assumption, \( C_D^\Psi \) becomes independent of the \( \Psi \) rapidity, the transport equation can then be simplified as

\[
 (m_\Psi \frac{\partial}{\partial \tau} + \vec{p}_1 \cdot \frac{\partial}{\partial x_1^i} ) f_\Psi(x_1, \vec{p}_1, y, \tau) \tag{11}
\]

After integration over the rapidity \( y \), it is reduced to

\[
 (m_\Psi \frac{\partial}{\partial \tau} + \vec{p}_1 \cdot \frac{\partial}{\partial x_1^i} ) f_\Psi(x_1, \vec{p}_1, \tau) \tag{12}
\]

With Cooper-Frye formula \( \left[ \frac{1}{2\pi} \right] \), the number of \( \Psi \)'s at proper time \( \tau \) is

\[
 N_\Psi(\tau) = \frac{1}{2\pi^3} \int d^2 \vec{p}_1^2 d^2 \vec{x}_1 d\eta \, \tau \, m_\Psi \cosh(y - \eta) \, f_\Psi(\vec{x}_1, \vec{p}_1, y, \tau) \tag{13}
\]

Since the normal suppression of \( \Psi \) has ceased before the starting time of the medium evolution, the number of \( \Psi \)'s at \( \tau_0 \) is

\[
 N_\Psi(\tau_0) = \int f_N(\vec{x}_1, \vec{p}_1^0) d^2 \vec{p}_1 d \vec{x}_1 . \tag{14}
\]

where \( f_N \) is the \( \Psi \) transverse distribution function at impact parameter \( \vec{b} \) after the normal suppression \[18\],

\[
 f_N(x_1, \vec{p}_1^0|\vec{b}) = \frac{\sigma^{\Psi}_{NN}}{\pi} \int dz_A dz_B \rho_A(x_1, z_A) \rho_B(x_1 - \vec{b}, z_B) \times e^{-\sigma_{abs}(T_A(x_1, z_A) + T_B(x_1 - \vec{b}, -z_B))} \times \frac{1}{\langle p^2_1 \rangle} e^{-p^2_1/\langle p^2_1 \rangle} \tag{15}
\]

with averaged transverse momentum square \[32\]

\[
 \langle p^2_1 \rangle = \langle \vec{b}^2 \rangle, \langle \vec{x}_1^2 \rangle, \langle \vec{p}_1 \rangle = \langle \vec{p}_1 \rangle_{NN} + \langle \vec{p}_1 \rangle_{\chi_c} \tag{16}
\]

where the thickness function \( T \) is defined as \( T(x_1, z_1, z_2) = \int_{z_1}^{z_2} dz \rho(x_1, z) \). The constant \( \sigma_{abs} \) is usually adjusted to the data from \( pA \) collisions where \( \Psi \)'s experience only normal suppression. From Ref. \[33\], we get the latest value, \( \sigma_{abs} = 4.3 \text{ mb} \) for \( J/\psi \) and \( \chi_c \) at SPS energy. At RHIC energy, the \( d+Au \) experiments show that it is little normal suppression \[34\].

When we know the time evolution of the produced \( \Psi \) distribution at freeze-out time \( \tau = \tau_0 \), the theoretical calculation of normal suppression can be achieved with averaged transverse momentum square \[35\]

\[
 \langle p^2_1 \rangle = \langle \vec{b}^2 \rangle, \langle \vec{x}_1^2 \rangle, \langle \vec{p}_1 \rangle = \langle \vec{p}_1 \rangle_{NN} + \langle \vec{p}_1 \rangle_{\chi_c} \tag{17}
\]

The transverse transport equation \( \Box \) can be solved analytically with the result \[15\]

\[
 f_N(x_1, \vec{p}_1, \tau) = e^{-\int_0^\tau d\tau' C_D^\Psi(\vec{x}_1 - \vec{v}_1(\tau' - \tau), \vec{p}_1, \tau')/m_\Psi} \times f_N(x_1 - \vec{v}_1(\tau - \tau_0), \vec{p}_1, \tau_0) \tag{18}
\]

It should be noted that the leakage term, that is the second term on the l.h.s. of Eq. \( \Box \), has been shown in Ref. \[15\] to be important in the consideration of charmonium \( p_t \) distribution.

When we know the time evolution of the produced medium and the \( \Psi \) dissociation cross section in the medium, we can calculate the number of \( \Psi \) and its \( p_t \) distribution at freeze-out time \( \tau_f \) with Eq. \( \Box \).

### V. NUMERICAL RESULTS

We first fix the parameter \( s_c \) in the initialization of hydrodynamics by fitting the \( J/\psi \) suppression data from SPS. The best fit is achieved with \( s_c = 31.7 \text{ fm}^{-3} \). Fig. \[1\] is our calculation compared with the NA50 data \[32\]. The thin dashed and solid lines indicate, respectively, the theoretical calculation of normal suppression with Drell-Yan cross section rescaled to MRS 43 and GRV LO parton distribution function \[33\]. The
thick dashed and solid lines are the corresponding results with anomalous suppression. The transverse energy $E_t = 0.274 N_p(b)$ GeV is the measure of centrality with $N_p(b)$ being the average number of participants in the collisions with impact parameter $b$. In very peripheral collisions with $E_t < 23$ GeV, the maximal initial entropy density is less than the critical value $s_c$, the medium will decouple at the initial time, and there is only normal suppression. It is obvious that due to the missing of $E_t$ fluctuation in our calculation, the anomalous suppression starts suddenly at $E_t = 23$ GeV. With increasing centrality, the volume and the life-time of the produced QGP will increase, and the anomalous suppression becomes more and more important. A good fit with the suppression data has been achieved in the whole $E_t$ range except in most central collisions where the fluctuations was shown to be important.[37].

After fitting the parameter $s_c$, we can calculate observables related to the $p_t$ broadening effect. In Fig.1 (B), we show the $\langle p_t^2 \rangle$ as a function of $E_t$. The dashed and solid lines are, respectively, $J/\psi$ $\langle p_t^2 \rangle$ calculated without and with anomalous suppression. The later agrees well with the experimental results from NA50.[34]. With increasing centrality, $\langle p_t^2 \rangle$ increases steadily till $E_t = 90$ GeV. After that, $\langle p_t^2 \rangle$ becomes saturated and finally drops slightly in most central collisions. There are two competing mechanisms that affect $J/\psi$ $\langle p_t^2 \rangle$: The $J/\psi$ with large $p_t$ is mostly produced in central collisions according to the Cronin effect. Since the matter produced in central collisions is denser and hotter than that in peripheral collisions, the $J/\psi$ with large $p_t$ has more chance to be absorbed by the QGP; On the other hand, the $J/\psi$ with large $p_t$ has more chance to escape the anomalous suppression region, when its duration time in the QGP is shorter than the anomalous suppression time. This is the leakage effect initially indicated by Matsui and Satz[1] and studied in detail in the transport approach in [15]. The former mechanism suppresses the $\langle p_t^2 \rangle$, while the later enhances the $\langle p_t^2 \rangle$.

With the fixed parameter $s_c$, we calculate now the $J/\psi$ suppression and $\langle p_t^2 \rangle$ at RHIC energy. Fig.2 (A) shows that our result on $J/\psi$ suppression agrees with the low statistical data from PHENIX[35]. The suppression at RHIC is much stronger than that at SPS, but does not approach zero. Since there is still no transverse momentum data from RHIC, we displayed only the theoretical results in Fig.2 (B). The behavior of the $\langle p_t^2 \rangle$ at RHIC is similar to the corresponding result at SPS energy. However, it is obvious that the leakage effect prevails even in very central collisions, and there is no clear decrease of $\langle p_t^2 \rangle$, very different from the predictions[39, 40] calculated with $p_t$ independent dissociations and without considering the leakage effect.

Finally we consider the elliptical flow defined by

$$v_2 = \left( \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right).$$

In high-energy nuclear collisions, the initial spatial anisotropy is transferred to final momentum anisotropy via partonic and/or hadronic interactions. Interesting information about the early stage of the hot and dense medium may be obtained from the study of this azimuthal anisotropy [11, 42, 43, 44, 45]. The measured
values of anisotropy \( v_2 \) of hadrons with light quarks (u,d,s) at RHIC are positive and large compared with lower energy results, and show a clear hadron mass dependence in the low transverse momentum region, \( p_t < 2 - 3 \) GeV/c \[13,17\]. Hydrodynamic calculations have been used to explain the observed collective expansion behavior \[15\]. The intrinsic charm mass is large compared with the initial temperature that might be reached at RHIC \[48\]. Therefore, any collective motion of charmed hadrons will be a useful tool for studying: (i) charm production mechanism in nuclear collisions: via direct early pQCD-type creation \[49\] or via later charm production mechanism in nuclear collisions: J/ψ's are created at hadronization, their minimum-bias \( v_2 \) could be large \[51\], while it is predicated that the direct pQCD produced charmed hadrons carry small or zero \( v_2 \). However, the leakage effect, discussed above, will lead to finite \( v_2 \) for J/ψ in non-central collisions \[6,54\]. This can be understood transparently: The anomalous suppression of the J/ψ depends on the length that the particle travels through the medium. Hence more suppression is expected in the out-of-plane direction (y-direction) which leads to finite value of \( v_2 \) with positive sign.

In Fig. 3 the solid and dashed lines are our results of J/ψ \( v_2 \) at fixed transverse momentum \( p_t = 3 \) GeV/c at SPS and RHIC. It is clear that \( v_2 \) is not zero but finite in mid-central collisions. For very central and very peripheral collisions \( v_2 \) approaches zero due to symmetric anomalous absorption and no anomalous absorption, respectively. The values of \( v_2 \) at RHIC are found to be larger than that at SPS due to the fact that the colored medium created at RHIC has larger volume and longer life-time which lead to the stronger leakage effect. If the J/ψ’s are created at hadronization, their minimum-bias \( v_2 \) has been evaluated in coalescence model with the assumption of complete thermalization of charm quark with the medium and the assumption of the same maximum \( v_2 \) (about 9\%) for charm and light quarks \[51\]. The result is shown in Fig. 4 as a function of transverse momentum \( p_t \), and compared with our calculation at fixed impact parameter \( b = 7.8 \) fm, at which the \( v_2 \) is found to have the maximum. The minimum-bias \( v_2 \) which is the average of \( v_2 \) over the impact parameter should be less than the maximum. It is clear to see that the maximal \( v_2 \) calculated in the frame of J/ψ transport is less than 10\% of that calculated in the approach with full charm quark thermalization \[51\].

VI. CONCLUSIONS

By combining the hydrodynamic equations for the QGP evolution and the transport equation for the primordially produced J/ψ in the QGP, and considering the anomalous charmonium suppression induced by the gluon dissociation process, we calculated the J/ψ suppression, averaged transverse momentum square, and elliptic flow at SPS and RHIC energies. It is found that the leakage effect reflected in the transport equation and the \( p_t \) dependence of the charmonium dissociation cross sections play an important rule in explaining the experimental \( p_t \) broadening effect at SPS.

In our transport approach, the main driving force of the elliptic flow is the leakage effect, the J/ψ’s moving in the direction of the long axis of the anisotropic medium are strongly suppressed, but those moving in the direction of the short axis are easy to escape the medium. The inelastic interaction with the medium will transfer collective flow to the charmonium, but the effect is small. Since the primordially produced J/ψ is not thermalized in our transport approach, the calculated \( v_2 \) is the low limit of the J/ψ elliptic flow. Any calculation with partial or full charm quark or charmonium thermalization should be larger than this low limit. From the comparison of theoretical calculations in different models with the experimental data, the elliptic flow may be a useful tool to determine which mechanism is the dominant one of J/ψ production, initial pQCD production, coalescence at hadronization, or the mixture of the both.

The possible gain term on the right hand side of the transport equation due to the recombinations of c and \( \bar{c} \) in the parton stage will probably enhance the charmonium production at RHIC energy and affect \( \langle p_t^2 \rangle \) and elliptic flow. These effects are now under consideration.

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