Scale dependence of the UHECR neutrino flux in extra-dimension models

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Ultra high energy cosmic ray (UHECR) neutrino fluxes measured in a fixed target detector can have a scale dependence. In the usual standard model or any extensions of this model (which are renormalizable), the effect is observationally very small. However, this need not be the case in models with extra-spatial dimensions, where the neutrino mass parameter can receive large corrections due to a power-law running. Hence, the scale dependence may lead to a measurable deviation from the standard prediction for the neutrino flux ratio.

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I. INTRODUCTION

In the last two decades, one of the significant success of particle physics has been the confirmation of a neutrino anomaly, both in the solar and atmospheric sector [1]. The most likely solution to the anomaly is to introduce a small neutrino mass and hence the notion of neutrino oscillation [2] which is similar in spirit to quark sector. Recent atmospheric neutrino data has indeed shown an observable dip in its zenith angle spectra as expected for massive neutrino oscillation [3]. In a realistic three flavor analysis, an important part of the solution amounts to finding the allowed parameter space for the the mixing angles; the solar ($\theta_S$), the atmospheric ($\theta_A$) and a reactor angle ($\theta_R$). The best fit values for these three mixing parameters seem to indicate a pattern spanning from almost negligible to moderate to maximal [4]. In the case of solar neutrinos, the mixing of $\nu_e$ with active neutrinos has the central value such that $\tan \theta_S \approx 0.7$ (moderate). In contrast, the atmospheric mixings involving largely of $\nu_\mu$ prefers $\tan \theta_A \approx 1$ (maximal) while the reactor angle, which determines the relative proportion of $\nu_e$ in the heaviest mass eigenstate, is consistent with zero mixings, $\tan \theta_R \ll 1$ [5].

The fact that $\tan \theta_A \approx 1$ has an important consequence for the neutrino fluxes which are ultra-relativistic in energies. It has long been realized that UHECR neutrinos (which are expected to be sourced by cosmic objects such as AGNs) when measured by ground based detectors, the expected flavor ratio \( \phi_e : \phi_\mu : \phi_\tau \approx 1 : 1 : 1 \) [6]. Henceforth, we shall call this expectation as the \textit{benchmark} value. This prediction is important for at least three fundamental reasons: (i) it forms an independent verification of the neutrino parameters which are phenomenologically extracted from solar, atmospheric and reactor data, (ii) it has been realized to be a test bed for some interesting new physics predictions (decay, pseudo-Dirac splittings, active-sterile mixings) which are not yet resolved [7] and (iii) it could provide further opportunity in our understanding of fermion mixings and masses; for instance, are there any fundamental symmetries in the $\mu - \tau$ block which leads to maximal mixings. It is expected that several of the upcoming neutrino telescopes [8] will be tuned to verify the \textit{benchmark} value besides looking for many of the new signatures mentioned here.

In the present analysis, we point out that the scale dependence of the neutrino parameters can also be a source which alters the \textit{benchmark} expectations. In scattering processes involving UHECR neutrinos, the momentum transfer square $\mu$ is expected to saturate at $10^4$ GeV$^2$ beyond which point there is a strong energy suppression [9]. It is well known that at this scale, the effects of running on neutrino mixings are very small [10]. However, this need not be the case in models with extra-spatial dimensions, thereby, leading to modifications to the \textit{benchmark} values. This forms the main theme of our analysis.

II. STANDARD LORE

It is instructive to first review the standard \textit{benchmark} expectations. Massive neutrinos, similar to quarks, have two eigenbasis, the flavor ($\nu_\alpha$) and mass eigenbasis ($\nu_i$) with corresponding mass eigenvalues, $m_i$. A unitary matrix relates the two basis, such that $u_{\alpha i} = U_{\alpha i} \nu_i$ where, the summation over the mass eigenstates is assumed. In the limit of small mixings, one could define the angles in the following manner. $\theta_\alpha$ mixes states $\nu_1$ and $\nu_2$, $\theta_\lambda$ mixes $\nu_2$ and $\nu_3$ and $\theta_R$ mixes $\nu_1$ and $\nu_3$. In this notation, without loss of generality, we can assume a hierarchy of states, where, $m_1 < m_2 < m_3$ and the relevant solar and atmospheric splittings, are $\Delta_S = m_3^2 - m_1^2$ and $\Delta_A = m_3^2 - m_2^2$ respectively. In the case of UHECR neutrinos which travel astronomical distances, the coherence between the various mass eigenstates is averaged out. As a result, once these neutrinos are produced, they essentially travel (galactic distances) as individual mass eigenstate, until at the point of detection. In a ground based detector, the probability of measuring a UHECR neutrino of a given flavor is then given as

\begin{align}
\phi_e &= 1 + 2x(2c_A^2 - 1) : x = (s_SC)^2, \\
\phi_\mu &= 2x c_A^2 + 2(c_A^2(1 - 2x) + s_A^4), \\
\phi_\tau &= s_A^2(1 - c_A^2),
\end{align}

(2.1)
where $s$ and $c$ denote sine and cosine, respectively. It is clear from the above expression, that maximal atmospheric mixing leads to the conclusion that all neutrino flavors must be detected with the same weight factor. In deriving this result, we have disregarded the mixing corresponding to reactor experiments, which is consistent with zero [5]. Given this result, we shall consider the modifications that may alter the prediction in (2.1) for $\theta_A \neq \pi/4$.

### III. Renormalization Group Effects

UHECR neutrinos incident on a target material can undergo both charged and neutral current scattering processes. The individual neutrino flavor states $\nu_\alpha$ are derived by folding the matrix element $U_{\alpha i}$ corresponding to the incident mass eigenstate $\nu_i$. In a scattering process, which involves large momentum transfers, the mixing matrix element $U_{\alpha i}$ can pick up a scale dependence. However, in practice, the momentum transfer square ($\mu$) saturates at $\mu \sim 10^4$ GeV$^2$ beyond which the cross section is damped [9]. It is well known that for scales around this value, the effects of neutrino mass running is negligible [10]. As mentioned earlier, this need not be the case if we consider models with extra-space dimensions. Furthermore, in this case, depending on the the mass of KK excitation for the gauge boson $\mu$ can saturate at a much higher value. Present collider bounds suggest that the lowest KK excited state can have a mass $\sim$ few 100 GeV [12] leading to $\mu \sim 1$ TeV as the scale of extra-dimension.

In the following, we consider a class of models where the neutrinos are localized in the brane, such that for $\delta$ extra spatial dimensions and for scales $\Lambda > \tilde{\Lambda}$ (electroweak scale) we have the evolution equation for the mass parameter [13]

$$16\pi^2 \frac{d\kappa}{d\ln \Lambda} = (-3g_2^2 + 2\lambda + 2S)\kappa_{\delta} - \frac{3t_\delta\kappa}{2}[(Y_d^\dagger Y_i) + (Y_u^\dagger Y_i)]^\dagger,$$

$$S = \text{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_i^\dagger Y_i),$$

$$t_{\delta} = \left(\frac{\Lambda}{\tilde{\Lambda}}\right)^\delta X_\delta \cdot X_\delta = \frac{2}{\delta} \pi^{\delta/2} \Gamma(\delta/2).$$

(3.2)

In (3.2) $Y_{u,d,i}$ are the up-quark, down-quark and charged lepton Yukawa couplings. For our purposes, we will focus on the contributions due to the charged lepton Yukawa couplings such that integrating (3.2) yields

$$\ln(\frac{\kappa}{\hat{\kappa}}) = \frac{3Y^2}{16\pi^2} \left(1 - \left(\frac{\Lambda}{\tilde{\Lambda}}\right)^\delta\right) X_\delta \equiv \eta_l.$$

(3.3)

It is important to note that in (3.3) the nature of running depends strongly on the value for $\delta$. As a result, the mixings can run significantly even for a small variation in the scales. This arises from the power law running which can compensate for the energy suppression in the propagator for an off-shell neutrino. Alternatively, the energy enhancement is due to the multiplicity factor (which is $(\Lambda/\tilde{\Lambda})^\delta$) and arises from the number of Kaluza-Klein states, which for a given $\Lambda$ and $\delta$ can be large.

The running of the masses translates to a running of the neutrino mixings. We estimate the corrections to the leading order in the enhancement (essentially $t_\delta$) although this expansion need not be perturbative, especially for large $\mu$ and/or $\delta$. Also, we do not write down the corrections other than due to $Y_l$ since it is not relevant to our discussion. Following (3.3) up to $O(\eta_l)$ the change in mass matrix element as a function of scale is obtained to be

$$\kappa_{\alpha\beta}(\Lambda) \approx \tilde{\kappa}_{\alpha\beta}(\tilde{\Lambda})(1 + \eta_l) \equiv \tilde{\kappa}_{\alpha\beta}(1 + \eta_l).$$

(3.4)

In (3.4) $\tilde{\kappa}_{\alpha\beta}(\tilde{\Lambda})$ is taken to be the value of the element at the electroweak scale. In the limit of two flavor mixing (which can be arranged if $U_{23} = 0$) following (3.4) the mixing angle depends on scale in the form

$$\tan \theta_{\alpha\beta}(\Lambda) = \frac{2\kappa_{\alpha\beta}(\Lambda)}{\kappa_{\alpha\alpha}(\Lambda) - \kappa_{\beta\beta}(\Lambda)} \approx \tan \tilde{\theta}_{\alpha\beta}(\tilde{\Lambda})(1 - \eta_l).$$

(3.5)

Having obtained this change in the mixing angle (up to first order in $\eta_l$) we can consider the change in $\theta_A$ for which case, we identify $\alpha = \mu$ and $\beta = \tau$ and take $Y_l = Y_\tau$. We plot the modification to the flavor fluxes as shown in Fig.1 where we choose $\delta = 4$. To be specific, we have assumed MSSM Yukawa couplings for the tau lepton at $\tan \beta = 50$. In the plot, we show the variation for flux $R$ with scale and as $\Lambda$ increases, $\theta_A \rightarrow 0$. As we should expect, in this limit, the muon flux approaches a value which is consistent with no $\nu_\mu \leftrightarrow \nu_\tau$ mixing. In fact, at this energy scale, an observation (if done) of the muon flux will constitute a direct measurement of the flux at the point of production (modulo the small errors
due to $U_{e3} \neq 0$.) In this simple exercise, our choice for $\delta$ is purely for illustrative purposes since, it is a free parameter and can be fixed depending on the cross section strength required for an observable effect.

A. An example of a $2 \rightarrow 3$ scattering process

We now consider a physical process where it might be possible to have a measurement of the scale dependence along with a unique signature. Essentially, we are considering a $2 \rightarrow 3$ tree level scattering process whose Feynman graph is shown in Fig.2. In this process, a deeply virtual neutrino ($\nu^*$) eventually fragments to a gauge boson ($G$) and an accompanying lepton: $\nu^* \rightarrow G+\text{leptons}$. The final state gauge bosons can be identified via their decay jets. This process is very similar to the electroproduction of heavy Majorana neutrinos considered earlier by Buchm"uller and Greub [14]. We remind that in our case, the state $G$ can also include KK excitations, hence, unlike in the standard model case, $\mu$ can saturate at values larger than $10^4$ GeV$^2$.

In the following, we outline the feasibility of measuring the process, while, a detailed calculation is beyond the scope of this paper. In principle, we wish to show that the process may not encounter the usual propagator suppression for $\mu \gg 1$. Let us examine the off-shell neutrino propagator in this energy regime. The relevant part of interest in the propagator reads as

$$D(\mu) = \frac{\kappa_{\alpha\beta}}{\mu - \kappa^2_{\alpha\beta}} \simeq \tilde{\kappa}_{\alpha\beta} \left( \frac{\Lambda}{\mu} \right)^{\delta} \frac{1}{\mu - \kappa^2_{\alpha\beta}} + \ldots ,$$

$$\simeq \tilde{\kappa}_{\alpha\beta} \left( \frac{\Lambda}{\mu} \right)^{\delta} \frac{\tilde{\Lambda}^\delta}{\mu \Lambda^\delta - \Lambda^2 \tilde{\kappa}^2_{\alpha\beta}} + \ldots (3.6)$$

where $\ldots$ denote higher order corrections to $\kappa_{\alpha\beta}$. We consider the possibility where the scale of extra-dimensions is within the range of experimental reach such that for some allowed $\mu$ we have $\Lambda \sim \sqrt{|\mu|}$. In this case, depending on the value of $\delta$ we should expect the cross section to grow with energy. Clearly, from (3.6) we find

$$D(\mu \gg 1) \sim \frac{\tilde{\kappa}_{\alpha\beta}}{\Lambda^\delta} \mu^{3/2 - 1} . \quad (3.7)$$

Note that from (3.7) for $\delta = 0$ we reproduce the expected energy suppression as in conventional non-extra-dimensional models. Thus, in all such theories, neutrinos which are emitted off the gauge boson vertex will always prefer to be on-shell. For $\delta \neq 0$ we find that the theory shows the usual pathology of cross section growing with energy [11]. This becomes severe as $\delta$ increases. Therefore, as $\delta$ increases, even for scales not too far from $\Lambda$ the cross section can grow significantly with energy. This also reflects the fact that the theory is unitarity violating. However, we also need to ensure that there are no low-energy anomalous processes which might be in conflict with the standard model results [15]. For instance, neutrino-nucleon cross sections which violate unitarity can have observable anomalous cross sections in the corresponding low-energy elastic processes [16]. Alternatively, one can examine the effects of new physics on final state interactions for a given process. If new physics occurs at the TeV scale, then an observable deviation of $\sim (0.01\%)$ is expected for scattering processes at the electroweak scale [17]. Currently, this small deviation is consistent with the LEP limits. However, we note that isolating any anomalous events may be an experimental challenge, especially, due to a lack of knowledge on the parton distribution functions involving states in the continuum.

In conclusion, the present analysis does demonstrate a possible window to observe the scale dependence of UHECR neutrino fluxes. We have taken a representative set of low energy neutrino parameters and analyzed the evolution of the mixing with scale. It might be of interest perform a more general analysis where we also consider the running of the CP phases and the solar mixing as well. An important ingredient in estimating the running is the value of neutrino parameters at the electroweak scale. Fortunately, we already have a good idea about the neutrino parameters ($\theta_{S,A}$, $\Delta_{S,A}$) from some very accurate phenomenological analysis of the solar and atmospheric data [4]. Contrary to non-extra dimensional models where neutrino mass degeneracy is an important ingredient; extra-dimensional models may relax this requirement since power-law running can account for large radiative corrections. As we have shown, the variation to the bench mark values could already occur for scales not too far from the electroweak scale. This implies that if the scale of extra-dimensions is within the reach of the neutrino telescopes, (then independent of the nature of the neutrino spectra), the effect which we predict should be observed. In addition, a measurement of scale dependence can also carry some unique and interesting signals, like the one described in the $2 \rightarrow 3$ scattering process.
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