Criteria of Motion Without Slipping for an Omnidirectional Mobile Robot

I. S. Mamaev, A. A. Kilin, Yu. L. Karavaev, V. A. Shestakov

In this paper we present a study of the dynamics of a mobile robot with omnidirectional wheels taking into account the reaction forces acting from the plane. The dynamical equations are obtained in the form of Newton–Euler equations. In the course of the study, we formulate structural restrictions on the position and orientation of the omnidirectional wheels and their rollers taking into account the possibility of implementing the omnidirectional motion. We obtain the dependence of reaction forces acting on the wheel from the supporting surface on the parameters defining the trajectory of motion: linear and angular velocities and accelerations, and the curvature of the trajectory of motion. A striking feature of the system considered is that the results obtained can be formulated in terms of elementary geometry.

Keywords: omnidirectional mobile robot, reaction force, simulation, nonholonomic model

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Ivan S. Mamaev
mamaev@rcd.r
Vyacheslav A. Shestakov
v.a.shestakov95@gmail.com
Kalashnikov Izhevsk State Technical University
ul. Studencheskaya 7, Izhevsk, 426069 Russia

Alexander A. Kilin
aka@rcd.ru
Ural Mathematical Center, Udmurt State University
ul. Universitetskaya 1, Izhevsk, 426034 Russia
Institute of Mathematics and Mechanics of the Ural Branch of RAS
ul. S. Kovalevskoi 16, Ekaterinburg, 620990 Russia

Yury L. Karavaev
karavaev_yury@istu.ru
Kalashnikov Izhevsk State Technical University
ul. Studencheskaya 7, Izhevsk, 426069 Russia
I. N. Ulianov Chuvash State University
Moskovskii prosp. 15, Cheboksary, 428015 Russia
1. Introduction

The development of transportation and logistic robots requires research aimed both at designing the most efficient robot configurations and at developing algorithms for controlling them. The most promising platforms that can be used as transportation robots are mobile omniwheel robots because they are capable of performing omnidirectional motion [1, 2], which makes it possible to use the surrounding space in the most efficient way while excluding places for U-turns and reparking, which are necessary for other types of wheeled robots [3].

A special feature of omnidirectional robots is that the wheel rims have rollers freely rotating relative to their axes. Depending on the orientation of the rotational axis of the roller relative to the plane of the wheel rim, one distinguishes between two types of wheels: omnidirectional wheels with the rotational roller axis parallel to the plane of the wheel, and Mecanum wheels whose rotational roller axis is at an angle of 45 degrees to the plane of the wheel rim [4–6].

We note that a wheel roller can be oriented at a different angle. Then it is necessary to give a special form to its surface to ensure constancy of contact as the contact of the rotating wheel with the ground changes from one roller to the next [7]. Review of relevant publications suggests that in practice the omnidirectional robots whose rotational roller axis lies in the plane of the wheel rim are usually used as educational complexes and for various robotic competitions [8–10], whereas robots with Mecanum wheels are used as transportation and manipulation mobile robots, for example, for handling large-size objects [11].

Algorithms and models for controlling omnidirectional robots are based on the assumption that there is no slipping along the rotational roller axis [12–14]. Such an approach provides a framework for investigating the stability of motion [15] and for path planning [10, 16, 17]. However, the question of what special features are involved in imposing the no-slip constraint on a roller remains either untouched or it is noted that the constraint is effected by the friction force directed along the rotational roller axis. This force is ensured by the large size of the robot and by the rubberlike material of which the rollers are made and which guarantees that the rollers have a high sliding friction coefficient when in contact with most materials. But it should be pointed out that a number of experimental studies in which the accuracy of the trajectory of omnidirectional robots [18] is estimated state that it is exactly the onset of slipping that is the main reason for deviation and that the slipping is primarily due to the fact that the underlying surface is not horizontal.

Recently, attempts have been made to take possible slipping into account [19] and to describe the processes occurring as the wheel’s contact with the ground changes from one roller to the next. The authors of [20] considered a mathematical model of motion taking into account slipping for a mobile robot with three omnidirectional wheels whose rotational axes lay in the plane of the wheel rim. Slipping was modeled by dry friction forces directed along and perpendicular to the rotational roller axis. The values of the friction coefficients were restored from experiments, but in spite of that a considerable deviation of modeling results from experiments was obtained. The authors proposed an improved model in which the value of the coefficients changed depending on the wheel turning angle, which determined indirectly what part of the roller had contacted the surface or whether there had been two of them simultaneously. The modified model considerably improved the agreement between the experimental and modeled trajectories of motion of the robot, but the question of the reasons for slipping remained unconsidered.

In [21], for a similar design of the mobile robot with three omnidirectional wheels, diagrams were constructed for possible relationships between the angular and linear velocities of the robot taking into account the technical limitations of the maximum angular velocities of rotation of
the omnidirectional wheels and the requirement not to exceed the ultimate sliding friction force acting along the rotational axis of the roller that is in contact with the surface. The diagrams allowed conclusions only on the limitations of the accelerations of motion depending on the direction of motion.

A dynamical model of the mobile robot, called Kuka youBot, with four Mecanum wheels was considered in [22] for a linear model and for the model of Coulomb friction at the point of contact of the roller with the surface. Results were presented for modeling within the framework of these friction models. These results were used as a basis for conclusions on a considerable influence of friction on the dynamics of motion. The results for the models considered were in qualitative agreement, but for the linear model of friction a sharp increase in the friction force occurred when the roller lost contact with the surface. This effect can perhaps be excluded in considering the forces taking into account the contact interaction of two rollers at the same time.

In [23], the process of change of contact of the wheel rollers with the supporting surface is discussed taking into account their impact interaction, equations of motion are obtained and examples of numerical solution are given for particular cases of the motion of the omnidirectional robot along a straight line and a spiral.

Despite a large amount of research in this field, the problem of the onset of slipping of the rollers of the omnidirectional wheels, which leads to a deviation from the prescribed trajectory of the mobile robot, remains open. Given that the maneuverability of omnidirectional robots makes it possible to achieve any trajectory of motion while combining the translational and rotational motion, there arises the possibility of choosing trajectories for which no slipping of the rollers [10] arises. In the future, the absence of slipping can become an important criterion to plan a trajectory for autonomous motion, including situations where the robots have to go around obstacles [17, 24].

This paper presents results obtained for the dynamical model of a mobile robot with omnidirectional wheels and aimed at analyzing the friction forces acting along the rotational axis of the wheel rollers. The structural restrictions on the position and orientation of the omnidirectional wheels and their rollers are formulated taking into account the possibility of implementing the omnidirectional motion. The influence of the parameters determining the trajectory (linear and angular velocities, accelerations, and curvature of the trajectory) on the possibility of slipping of the wheel rollers is revealed.

2. The robot’s scheme and control

Consider the motion of a mobile robot with \( n \) omnidirectional wheels in the plane \( OXY \) coinciding with the horizontal plane of the fixed coordinate system \( OXYZ \). A scheme of the omnidirectional mobile robot (OMR) is shown in Figure 1. The moving coordinate system \( O_1 x_1 y_1 z_1 \) is attached to the center of mass of the OMR. The coordinates of the center of mass \( C \) of the mobile robot relative to the fixed coordinate system \( OXYZ \) are denoted by \( X \) and \( Y \). The rotation of the moving coordinate system \( O_1 x_1 y_1 z_1 \) relative to the fixed axes is given by the angle \( \theta \). The vectors \( \mathbf{A}_k \), \( \mathbf{B}_k \) and \( \mathbf{C}_k \) characterize in the fixed coordinate system the direction of the axes of the rollers of the \( k \)th wheel at the point of contact with the surface, the orientation of the \( k \)th wheel and its position relative to the center of mass of the OMR, respectively:

\[
\mathbf{A}_k = \left( A_1^{(1)}, A_2^{(2)}, 0 \right), \quad \mathbf{B}_k = \left( B_1^{(1)}, B_2^{(2)}, 0 \right), \quad \mathbf{C}_k = \left( C_1^{(1)}, C_2^{(2)}, 0 \right), \quad k = 1, \ldots, n.
\]

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Fig. 1. A scheme of a mobile robot with omnidirectional wheels as projected onto the horizontal plane

To avoid uncertainty, we choose the directions of the vectors $A_k$ such that the condition $(C_k \times A_k, e_z) \geq 0$ is satisfied, where $e_z$ is the unit vector of the axis $Oz$ perpendicular to the horizontal plane in which the motion is considered.

In practice it is more convenient to refer these vectors, which define the configuration of the robot, to the moving coordinate system. Therefore, we write:

$$A_k = Q^{-1} \alpha_k, \quad B_k = Q^{-1} \beta_k, \quad C_k = Q^{-1} c_k,$$

where $\alpha_k$ and $\beta_k$ are the unit vectors defining the direction of the axes of the rollers of each $k$th wheel at the point of contact with the surface and the orientation of the $k$th wheel in the moving coordinate system $O_1x_1y_1$, $c_k$ is the vector defining the position of the $k$th wheel in the coordinate system $O_1x_1y_1$, and $Q$ is the matrix of rotation of the moving coordinate system relative to the fixed coordinated system. Next, we consider the motion of the mobile robot in the fixed coordinate system.

Let $V$ be the velocity of point $C$ in the fixed coordinate system $OXY$, and let $\Omega$ be the angular velocity of the platform. The condition that there be no slipping of the rollers at the point of contact of each of the wheels is given by the following expression [12]:

$$(V_k + h_k \dot{\psi}_k B_k, A_k) = 0,$$ (2.1)

where $h_k$ is the radius of the $k$th wheel, $\dot{\psi}_k$ is the angular velocity of rotation of the $k$th wheel, and $V_k$ is the velocity of the center of the $k$th wheel in the coordinate system $OXY$:

$$V_k = V + \Omega \times C_k,$$

where

$$V = \dot{R}_c, \quad R_c = (X, Y, 0), \quad \Omega = (0, 0, \dot{\theta}),$$

where $R_c$ is the radius vector defining the position of the center of mass $C$. Then the nonholonomic constraint (2.1), which guarantees that there is no slipping of the roller of the $k$th wheel of the OMR, can be written as

$$f_k = (V + \Omega \times C_k + h_k \dot{\psi}_k B_k, A_k) = 0.$$ (2.2)

Solving (2.2) for $\dot{\psi}_k$, we obtain an equation determining the velocity of rotation of the $k$th wheel for motion along a trajectory given by $V$ and $\Omega$:

$$\dot{\psi}_k = -\frac{1}{(A_k, B_k)h_k}(V + \Omega \times C_k, A_k).$$ (2.3)
Criteria of motion without slipping

Relations (2.3) represent the kinematic relation of the trajectory of the OMR to the angular velocities of rotation of the wheels, which can be used as control actions. It is assumed that no slipping arises along the axis of the roller. To estimate this assumption, we consider the dynamics of the OMR taking into account the forces acting on the omnidirectional wheels. But before doing so we present some conclusions on the structural features of the OMR for implementation of the omnidirectional motion, which we have drawn from analysis of the kinematic relations.

3. Application to the design of the OMR

The kinematic relations (2.3) are used to form the control of the OMR, and for a modification with \( n \) wheels they can be obtained in the following form:

\[
\dot{\psi} = Dx, \tag{3.1}
\]

where

\[
\dot{\psi} = \begin{bmatrix}
\dot{\psi}_1 \\
\vdots \\
\dot{\psi}_n
\end{bmatrix}, \quad D = -\frac{1}{(A_k, B_k)h} \begin{bmatrix}
A^{(1)}_1 & A^{(2)}_1(JC_1, A_1) \\
\vdots & \vdots \\
A^{(1)}_n & A^{(2)}_n(JC_n, A_n)
\end{bmatrix}, \quad x = \begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{\theta}
\end{bmatrix}, \quad J = \begin{bmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \\
\]

\( h = h_k = h_n, \quad k = 1, \ldots, n. \)

The expression (3.1) defines the dependence of the angular velocities of rotation of the wheels on the required velocities of motion along the trajectory and the structural characteristics of the OMR. Analysis of this expression reveals some structural features that make highly maneuverable motion impossible.

A. The rotational axis of the roller is parallel to the rotational axis of the wheel. Taking into account possible manufacturing errors of the rollers and the wheels, this condition can be written as

\[
(A_k, B_k) \rightarrow 0. \tag{3.2}
\]

As the wheel rotates, the rollers rotate without creating the driving force. In designing the wheels such an arrangement of the rollers is of no use (an example of the arrangement of the wheel rollers is shown in Fig. 2a). In the case of \( (A_k, B_k) = 0 \) the \( k \)th omnidirectional wheel can be modeled as a passive wheel or a skate.

B. Attachment of the wheels to the platform where the rotational roller axis is parallel to the vector specifying the position of the wheel relative to the platform:

\[
(A_k, JC_k) = 0. \tag{3.3}
\]

When condition (3.3) is satisfied for all omnidirectional wheels, the rotation of the mobile robot relative to the origin of the moving coordinate system becomes impossible. In designing the OMR it should be taken into account that, if the rotational axes of the rollers of all wheels intersect at the same point, then the rotation of the OMR relative to the vertical axis with a given angular velocity \( \omega \) is impossible (examples of the arrangement of the wheels are shown in Fig. 2b). A single wheel for which this condition is satisfied will not contribute to the formation of the rotational motion of the OMR (Fig. 2c).

Such an analysis is made in [25], where the authors conclude that the mobile robot is capable of performing the omnidirectional motion only if the rotational axes of the rollers of any wheel
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Fig. 2. Examples of the inadmissible arrangement (a) of the wheel rollers at a right angle to the rotational axes of the wheels and (b) of all wheels with parallel vectors $A_k$ and $C_k$ and (c) an arrangement where only one wheel with parallel vectors $A_k$ and $C_k$ does not contribute to motion

intersect at no less than two points, and the matrix $D$ is a full rank matrix for any three wheels. These conditions have been verified for three- and four-wheeled robots with different orientations of the rollers and different positions of the omnidirectional wheels.

A similar conclusion is drawn in [12] where an estimate of the controllability of the OMR for motion along a trajectory is given. It is shown that, if $\text{rank}(D) = 3$ one can always choose a control for implementation of motion for $n \geq 3$.

4. Constraint reaction forces for a three-wheeled robot

4.1. Equations of motion and the constraint reaction forces

The onset of slipping and the violation of the constraint (2.1) make a control using the kinematic model impossible. In other words, if the reaction force exerted on the wheel from the plane is smaller than the sliding friction force acting along the rotational roller axis, then the control obtained using the kinematic model (2.3) is possible.

To estimate the reaction force acting on the $i$th omnidirectional wheel from the horizontal plane, we consider the dynamics of the OMR. Complete equations of motion of the OMR which take into account the control torques applied to the wheel axes are examined in detail in [12, 26]. In a number of other papers, e.g., in [22, 23] and [27–30], an analysis has been made of the dynamics of the motion of the OMR, but only for specific wheel modifications and their specific arrangements, in particular, taking into account the rotation of rollers. However, there has been no estimate of the reaction force with emphasis on ensuring the constraint (2.1) (no-slip constraint on the wheels).

Let us consider the motion of the center of mass $C$ of the mobile robot in the fixed coordinate system $OXY$ (see Fig. 1).
We write the dynamical equations of the mobile robot using the theorems of the motion of the center of mass and of changes in the angular momentum [31, 32]:

\[
\begin{align*}
m \ddot{R}_c &= \sum_k F_k, \\
I_c \dot{\Omega} &= \sum_k \sum M_k = \sum C_k \times F_k,
\end{align*}
\]  

(4.1)

where \( I_c = \text{diag}(I_h, I_h, I_c) \) is the tensor of inertia of the mobile robot relative to the center of mass, \( F_k \) is the external force acting on the \( k \)th wheel of the OMR from the horizontal plane, \( m \) is the mass of the OMR, and \( M_k \) is the moment of the external force acting on the \( k \)th wheel of the OMR relative to the center of mass \( C \).

In the horizontal plane, it is the friction force that is the external force acting on the roller of each wheel of the OMR. In this paper, we neglect the rotation of the wheel rollers and consider only the friction force directed along the rotational axis of the wheel rollers. Then we write the system (4.1) as

\[
\begin{align*}
m \ddot{R}_c &= \sum_k F_k = \sum_k F_k A_k, \\
I_c \dot{\Omega} &= \sum_k C_k \times F_k = \sum C_k \times F_k A_k,
\end{align*}
\]  

(4.2)

where and \( F_k \) is the absolute value of the friction force.

The system (4.2) has a unique solution for \( F_k \) only for an OMR with three wheels, i.e., for \( n = 3 \). An OMR with the number of wheels \( n > 3 \) requires introducing additional conditions that exclude the uncertainty in finding forces (for example, the condition that the sums of the reactions of the wheels located on the diagonal be equal, etc.). Below we consider a mobile robot with three omnidirectional wheels (see Fig. 3).

Fig. 3. A scheme of a mobile robot with three omnidirectional wheels
Proposition 1. The solution of the system (4.2) for \( F_k \) for \( n = 3 \) \((k = 1, 2, 3)\) has the following form:

\[
F_k = \frac{\hat{t}_k}{2S}(1\,\Omega + \vec{C}_k \times m\hat{R}_c),
\]

where \( \hat{t}_k \) is the length of the \( k \)th side of the triangle formed by the intersection of the rotational axes of the wheel rollers, \( S \) is the area of this triangle, and \( \vec{C}_k \) is the vector drawn from the center of mass to the triangle’s vertex opposite to the \( k \)th side.

The expression (4.3) (a proof of it is given in Appendix A) shows that the absolute value of the friction force depends both on the trajectory given by the angular and linear accelerations of the center of mass and on the geometric parameters of the OMR: the area \( S \) of the triangle formed by the intersection of the rotational axes of the wheel rollers, the length \( \hat{l}_k \) of the triangle’s side coinciding with the rotational axis of the roller of the \( k \)th wheel and the vector \( \vec{C}_k \).

These parameters are easily expressed in terms of more natural characteristics which define the structure of the OMR, namely, the vectors \( \vec{A}_i \), which specify the orientation of the rollers of the omnidirectional wheels, and the vectors \( \vec{C}_i \), which specify the position of the omnidirectional wheels relative to the center of mass:

\[
\vec{C}_k = \frac{d_i\vec{A}_j - d_j\vec{A}_i}{(\vec{A}_i \times \vec{A}_j, e_z)}, \quad d_i = (\vec{C}_i \times \vec{A}_i, e_z), \quad \hat{t}_k = |\vec{C}_i - \vec{C}_j|, \quad S = \frac{\hat{t}_k(\vec{A}_i \times \vec{A}_j, e_z)}{2},
\]

where \( i, j, k \) take the values 1, 2, 3 and their cyclic permutations. We note that this formalism is applied to the case where the directions of the vectors ensure that the following condition is satisfied:

\[
(\vec{A}_i \times \vec{A}_j, e_z) > 0.
\]

The expression (4.3) provides insight into the nonobvious dependence of the reaction force \( F_k \), acting from the plane on the roller of the \( k \)th wheel, on the vector \( \vec{C}_k \), which is defined by the rotational axes of the rollers of the \( i \)th and \( j \)th omnidirectional wheels and by the position of these wheels relative to the center of mass of the OMR.

Next, we consider the influence of the trajectory of the OMR on the reaction forces from the plane.

4.2. Constraint reaction forces during motion along a prescribed trajectory

According to (4.3), the reaction force from the plane of motion, which is the friction force \( F_k \), depends on the trajectory and the geometric parameters of the mobile robot. Consider the influence of the parameters of the trajectory and the geometric parameters of the robot on the reaction force during motion along a curvilinear trajectory (see Fig. 4).

Let the trajectory of motion be specified in parametric form in the coordinate system \( Ox_1y_1 \) attached to the center of mass of the robot:

\[
x(s), \quad y(s), \quad \theta(s), \quad s(t),
\]

where \( s(t) \) is a natural parameter (the length of the trajectory).

Let us define the vector \( \vec{v} = (\dot{x}, \dot{y}) \), which characterizes the velocity of the center of mass of the OMR in the fixed coordinate system:

\[
\hat{R}_c = Q^{-1}\vec{v}.
\]
4.3. Dry friction for the modeling of reaction forces

The condition that there be no slipping of the wheel rollers can be represented as

$$F_k \leq F_k^s,$$  \hspace{1cm} (4.5)

where $F_k^s$ is the sliding friction force for the $k$th wheel.

In the general case, the process of motion of the roller can be represented as a combination of rolling and sliding, and, as is well known from the studies concerned with the motion of the wheels of simple mechanical systems, the friction models featuring such motion can be fairly complicated [33–35] and require separate complex research.

As a first approximation we consider the Amonton–Coulomb dry friction model:

$$F_k^s = \mu N_k \frac{v_k}{|v_k|},$$  \hspace{1cm} (4.6)

where $\mu$ is the coefficient of sliding friction and $N_k$ is the absolute value of the normal pressure force (the support reaction) for the $k$th wheel.
According to (4.5), in the case where the reaction force acting on the wheel from the horizontal plane (the tangent reactions) is larger than the absolute value of the sliding friction force (4.6), there arises slipping along the rotational axis of the roller of the \( i \)th wheel.

To determine the support reaction, we will use the laws of quasi-statics, and to exclude static uncertainty, we consider a mobile robot with three omnidirectional wheels (see the schemes in Fig. 5).

![Fig. 5. (a) A scheme of the reaction forces acting on the wheel; (b) a geometric scheme of the OMR](image)

**Proposition 2.** The reaction force acting from the supporting plane on the roller of the \( k \)th wheel is defined by the following relation:

\[
N_k = \frac{1}{\Delta} \left( \Delta_{ij} m g e_z + \frac{h}{2} l_k \times F_f \right),
\]  

(4.7)

where \( \Delta_{ij} = \frac{1}{2} |C_i \times C_j| \) is the area of the triangle composed of the vectors \( C_i \) and \( C_j \) (see Fig. 5b), \( \Delta \) is the area of the triangle at the vertices of which the points of attachment of the wheels (\( \Delta EFG \)) lie, \( g \) is the free-fall acceleration, \( l_k = C_i - C_j \), \( F_f = \sum F_i A_i \), \( i = 1, 2, 3 \) is the resultant of the friction forces acting on the wheel rollers from the plane, and \( e_z \) is the unit vector of the vertical.

A proof of Proposition (4.7) is given in Appendix B. The first term in (4.7) is the reaction of the gravity force, and the second term is the reaction of the resultant of the friction forces acting along the rotational axes of the rollers.

It can be seen from the expression (4.7) that, when \( h \ll |l_k| \), i.e., when the radius of the wheel is much smaller than the distance between the wheels, the component of the reaction which depends on the friction force is small compared to the component of the gravity force, and in this case we can turn our attention to the quasi-static model of determination of the reaction

\[
N_k = \frac{\Delta_{ij} m g e_z}{\Delta}.
\]  

(4.8)

Thus, the expressions (4.6) and (4.8) allow the maximum value of the friction force to be determined for each of the wheels at which the wheel begins to slip.
5. Criteria of motion without slipping for a symmetric structure of the robot

We carry out a numerical study to determine the conditions of motion under which slipping of the wheel rollers may arise for a configuration of the mobile robot with the following mass-geometric characteristics:

\[
\begin{align*}
\alpha_1 &= \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right), & \alpha_2 &= \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right), & \alpha_3 &= (1, 0, 0), \\
\beta_1 &= \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right), & \beta_2 &= \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right), & \beta_3 &= (-1, 0, 0), \\
c_1 &= \left(L_1 \frac{\sqrt{3}}{2}, L_1, 0\right), & c_2 &= \left(-L_2 \frac{\sqrt{3}}{2}, L_2, 0\right), & c_3 &= (0, -L_3, 0),
\end{align*}
\]

\[L_1 = 0.4 \text{ m}, \quad L_2 = 0.4 \text{ m}, \quad L_3 = 0.4 \text{ m}, \quad m = 5.15 \text{ kg}, \quad I_c = 0.42 \text{ kg} \cdot \text{m}^2.\]

These values have been determined for the most commonly used modification of the OMR (see Fig. 6).

For the OMR model of interest the support reactions defined within the framework of the quasi-static model (4.6) take the following values:

\[N_1 = N_2 = N_3 = 16.4 \text{ H}.\]

Then the following limiting values of the static friction force above which slipping arises are defined for the friction couple rubber-concrete \( k = 0.6 \):

\[F_1^* = F_2^* = F_3^* = 9.8 \text{ H}. \quad (5.1)\]

The resulting values (5.1) are the maximum values of the reactions directed along the rotational axis of the wheel rollers for the above-mentioned parameters of the mobile robot. When these values are exceeded, the wheels of the mobile robot begin to slip.
Each of the terms of the equations of the system (4.4) corresponds to a particular type of motion. Consider the dependence of the forces $F_k$ for specific types of motion of the OMR, which are defined by its individual terms.

1. **Motion along a straight line with constant orientation.** The motion is described by the following condition:

$$\dot{\theta} = 0, \quad \rho \to \infty.$$  

The trajectory of motion is given as

$$x(s) = 0, \quad y(s) = s, \quad \theta(s) = \phi, \quad s(t) = at^2,$$

where $\phi = \text{const}$ defines the constant direction of the robot’s motion, and $a$ is a parameter.

According to (4.4), the absolute value of the force $F_k$ is directly proportional to the linear acceleration of the mobile robot. Consequently, for each modification of the OMR there exists a limiting value of the linear acceleration above which slipping arises:

$$\ddot{s}^*_{k} = \frac{2F^*_{k}S}{l_k \left( \hat{C}_k \times mQ^{-1} \tau \right)},$$

where $\ddot{s}^*_{k}$ is the linear acceleration limit for the $k$th wheel.

Figure 7 shows graphs of dependences of the absolute values of the forces $F_k$ for the trajectory, which are given in the form (5.3) for different directions of motion $\phi \in [0, 360^\circ]$ and $s = 2 \frac{m}{s^2}$.

![Graph showing forces F_k on orientation angle θ]

Fig. 7. Dependences of the forces $F_i$ on the orientation of the robot $\theta$

The maximum values of the reaction forces for the symmetric structure of the OMR considered are observed for the following values of the orientation angle $\phi$ (the angle is measured counterclockwise relative to the positive direction of the axis $C_{x_1}$):

$$F_1: \phi = 150^\circ, \ 330^\circ, \quad F_2: \phi = 30^\circ, \ 210^\circ, \quad F_3: \phi = 90^\circ, \ 270^\circ.$$

Thus, the largest force $F_k$ will be applied to the $k$th wheel during motion in the direction perpendicular to the vector $\hat{C}_k$ (see Fig. 8a). For the special case at hand this direction coincides with the rotational axis of the roller of the $k$th wheel.
The dependence (4.4) allows the value of the linear acceleration limit to be determined for particular parameters of the mobile robot moving in a prescribed direction. Uniform distribution of reaction forces between all wheels can be achieved as the OMR moves with the orientation $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$.

2. **Turning in place.** The motion is described by the following condition:

$$\dot{s} = 0, \quad \ddot{s} = 0.$$ 

The trajectory of motion is represented as

$$x(s) = 0, \quad y(s) = 0, \quad \theta(s) = \omega s, \quad s(t) = t^2,$$

where $\omega$ is a parameter.

The reaction forces $F_k$ are directly proportional to the angular acceleration of the mobile robot. Consequently, for each modification of the OMR there exists a limiting value of the angular acceleration above which slipping arises:

$$\ddot{\theta}_k = \frac{2F_k^* S}{l_k I_c},$$

where $\ddot{\theta}_k^*$ is the angular acceleration limit for the $k$th wheel.

3. **The motion along a straight line with rotation** is defined by the following condition:

$$\dot{\theta} = 0, \quad \ddot{s} = 0, \quad \rho \to \infty.$$ 

An example of the trajectory of motion can be represented as

$$x(s) = \frac{\sqrt{2}s}{20}, \quad y(s) = \frac{\sqrt{2}s}{20}, \quad \theta(t) = \frac{\omega t \pi}{5}, \quad s(t) = at.$$

As the OMR moves, it rotates with the constant angular velocity $\omega = \frac{\Omega \pi}{5}$. Therefore, the dependences of the absolute values of the forces $F_k$ are periodic functions, and so we consider only the maximum values $F_k^*$. 

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A graph of the surface defining the dependence $F_1$ on the linear velocity $\dot{s} \in [0, 7] \frac{m}{s}$ and the angular velocity $\dot{\theta} \in [0, 7] \frac{\text{rad}}{s}$ for a typical trajectory given in the form (5.4) is shown in Fig. 9. Since the structure of the robot is symmetric, the graphs of $F_2$ and $F_3$ are similar. The limiting value of the static friction force $F_k^*$ is shown as a horizontal plane. The values of the angular and linear velocities which correspond to the region of the surface that lies above the plane of the ultimate friction force will cause the wheels to slip.

Figure 10 shows the region of admissible values of the linear and angular velocities of motion for the OMR modification at which no slipping arises.

4. Motion along a curvilinear trajectory with constant orientation. This motion is described by the following condition:

$$\dot{\theta} = 0, \quad \ddot{s} = 0.$$  

An example of the trajectory of motion can be given in the form of a circle of radius $R$:

$$x(s) = R \cos(s), \quad y(s) = R \sin(s), \quad \theta(s) = 0, \quad s(t) = \frac{at}{R}. \quad (5.5)$$

Given periodic changes in the direction of motion of the OMR relative to the trajectory considered here, it is appropriate to consider the maximum values of the reaction forces for determining the ultimate radius of curvature of the trajectory at which motion without slipping is possible. Let us define the maximum values of the reaction forces $F_k$ for the following values of the linear velocity $\dot{s}$ and the radius of curvature of the trajectory $R$:

$$\dot{s} \in [0, 1] \frac{m}{s}, \quad R \in [0.5, 1] \text{ m}.$$  

Figure 11 shows a graph of the surface defining the dependence of the reaction force $F_k$ on the linear velocity of motion $\dot{s} \in [0, 1] \frac{m}{s}$ and the radius of curvature of the trajectory $R \in [0.5, 1] \text{ m}$.
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Fig. 11. Dependence of the force $F_k$ on the linear velocity of motion $\dot{s}$ and the radius of curvature of the trajectory $R$

Fig. 12. The region of admissible values of the linear velocity of motion in a circle of radius $R$

for the trajectory given in the form (5.5), and a graph of the plane representing the limiting value of the static friction force $F^*_i$.

Figure 12 shows the region of admissible values of the linear velocity of the robot moving along a curvilinear trajectory of radius $R$ for the OMR design under consideration.

Thus, the largest force $F_k$ will be applied to the $k$th wheel during motion along a straight line parallel to the vector $\hat{C}_k$ (see Fig. 8b), with the maximum linear velocity $\dot{s}$.

The dependences $F_k$ considered in this section make it possible to determine the limiting values of the motion parameters of the OMR ($\dot{s}$, $\ddot{s}$, $\dot{\theta}$, $\ddot{\theta}$ and $R$) above which the wheel rollers will begin to slip at the points of contact with the surface.

6. Conclusion

In this paper, we have considered the dynamics of a mobile robot with omnidirectional wheels which moves on a plane. Using the equations obtained, we analyzed arising reaction forces acting on the wheels from the supporting surface. This allowed us to formulate conclusions on the influence of the configuration and the trajectory parameters on the onset of slipping of the wheel rollers.

The possibility of implementing the omnidirectional motion is guaranteed when the position and the orientation of the rollers and wheels of the mobile robot correspond to certain conditions, namely: in developing the design of the OMR one should avoid a position in which the rotational axes of the rollers coincide with the rotational axis of the wheel and in which the rotational axes of the rollers pass through the center of the platform.

In order to ensure the maximum grip of the wheels with the supporting surface, it is necessary to choose a direction of motion that ensures the maximum value of the reaction which is directed along the rotational axis of the roller of the omnidirectional wheel, but which does not exceed the sliding friction force.
The onset of slipping is influenced by linear and angular velocities and accelerations during motion along a prescribed trajectory, and by the radius of curvature of the trajectory. The motion of the mobile robot with omnidirectional wheels must be planned taking into account the limiting values of velocities, accelerations and the curvature of the trajectory for the robot’s configuration used. Given the design parameters of the mobile robot, one can determine the directions of motion in which it is possible to achieve the maximum and minimum linear accelerations by fulfilling the condition that there be no slipping of the wheel rollers.

Future research can be aimed at describing the dynamics of the spatial motion of omnidirectional robots taking into account surface irregularities.

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A. Appendix A

Proof of Proposition 1. Consider the dynamical equations of the mobile robot (4.2):

\[ m\ddot{R}_c = \sum_k F_k = \sum_k F_k A_k, \]

\[ \mathbf{I}_c \ddot{\Omega} = \sum_k C_k \times F_k = \sum_k C_k \times F_k A_k. \]  

We write the system (A.1) in matrix form for an OMR with three omnidirectional wheels, i.e., for \( n = 3 \):

\[ \mathbf{P} = \mathbf{U} \mathbf{F}, \]

where

\[ \mathbf{P} = \begin{bmatrix} m\ddot{R}_c^{(1)} \\ m\ddot{R}_c^{(2)} \\ M^{(3)} \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} A_1^{(1)} & A_2^{(1)} & A_3^{(1)} \\ A_1^{(2)} & A_2^{(2)} & A_3^{(2)} \\ d_1 & d_2 & d_3 \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}, \]

\[ M = \mathbf{I}_c \ddot{\Omega}, \quad d_k = (C_k \times A_k, e_z) > 0, \]

where \( e_z \) is the unit vector of the vertical.

The solution of the system (A.2) for \( \mathbf{F} \) has the form

\[ \mathbf{F} = \mathbf{U}^{-1} \mathbf{P}, \]

where the inverse matrix \( \mathbf{U}^{-1} \) with respect to the matrix \( \mathbf{U} \) has the form:

\[ \mathbf{U}^{-1} = \frac{1}{\det \mathbf{U}} \mathbf{U}_*^T, \]

where

\[ \det \mathbf{U} = d_1 (A_2^{(2)} A_3^{(1)} - A_2^{(1)} A_3^{(2)}) + d_2 (A_1^{(2)} A_3^{(1)} - A_1^{(1)} A_3^{(2)}) + d_3 (A_1^{(1)} A_2^{(2)} - A_1^{(2)} A_2^{(1)}) = \]

\[ = d_1 (A_2 \times A_3, e_z) + d_2 (A_3 \times A_1, e_z) + d_3 (A_1 \times A_2, e_z), \]

\[ \mathbf{U}_*^T = \begin{bmatrix} A_2^{(2)}d_3 - A_3^{(2)}d_2 & A_3^{(1)}d_2 - A_2^{(1)}d_3 & A_2^{(1)}d_3 - A_3^{(2)}d_2 \\ A_3^{(2)}d_1 - A_2^{(2)}d_1 & A_2^{(1)}d_1 - A_3^{(2)}d_2 & A_1^{(1)}d_1 - A_3^{(2)}d_2 \\ A_2^{(1)}d_1 - A_3^{(1)}d_1 & A_1^{(1)}d_1 - A_3^{(1)}d_2 & A_1^{(1)}d_1 - A_2^{(1)}d_2 \end{bmatrix}. \]

Let us express the area of a triangle formed by the intersection of the rotational axes of the wheel rollers of the OMR in the expression of the determinant of the matrix \( \mathbf{U} \) by multiplying and dividing it by the product \( \hat{l}_1 \hat{l}_2 \hat{l}_3 \):

\[ \det \mathbf{U} = (d_1 (A_2 \times A_3, e_z) + d_2 (A_3 \times A_1, e_z) + d_3 (A_1 \times A_2, e_z)) \frac{\hat{l}_1 \hat{l}_2 \hat{l}_3}{l_1 l_2 l_3} = \]

\[ = \frac{2S}{l_1 l_2 l_3} \left( d_1 \hat{l}_1 + d_2 \hat{l}_2 + d_3 \hat{l}_3 \right) = \frac{4S^2}{l_1 l_2 l_3}. \]
The expression (A.5) corresponds to the condition where the directions of the vectors ensure that the following condition is satisfied:

\[(\mathbf{A}_i \times \mathbf{A}_j, \epsilon_z) > 0.\]

Introduce the vector \(\mathbf{\hat{C}}_k\) drawn from point \(C\) to the vertex opposite to the \(k\)th side of the triangle (see Fig. 3) for which the following equation holds:

\[
\mathbf{\hat{C}}_k \times \mathbf{A}_i = \mathbf{C}_i \times \mathbf{A}_i.
\]

(A.6)

Consider the dependence of the absolute value of the force \(F_k\):

\[
F_k = \frac{\ell_k l_k}{4S^2} \left( m \ddot{R}_c^{(1)} (\mathbf{A}_i^{(2)} d_j - \mathbf{A}_j^{(2)} d_i) + m \ddot{R}_c^{(2)} (\mathbf{A}_j^{(1)} d_i - \mathbf{A}_i^{(1)} d_j) + M^{(3)} (\mathbf{A}_i^{(1)} \mathbf{A}_j^{(2)} - \mathbf{A}_j^{(1)} \mathbf{A}_i^{(2)}) \right) = \frac{\ell_k l_k}{4S^2} \left( d_j (m \ddot{R}_c \times \mathbf{A}_i, \epsilon_z) - d_i (m \ddot{R}_c \times \mathbf{A}_j, \epsilon_z) + M^{(3)} (\mathbf{A}_i \times \mathbf{A}_j, \epsilon_z) \right).
\]

Then, taking into account relations (A.6) of the vectors \(\mathbf{\hat{C}}_k\) and \(\mathbf{\hat{C}}_i\) and expressing the area of the triangle \(S\), as in (A.5), the dependence of the absolute value of the force \(F_k\) takes the following form:

\[
F_k = \frac{\ell_k}{2S} (\mathbf{I}_k \dot{\Omega} + \mathbf{\hat{C}}_k \times m \ddot{R}_c, \epsilon_z).
\]

(A.7)

The expression (A.7) represents the dependence of the absolute value of the friction force, acting along the axis of the roller of the \(k\)th wheel, on the linear and angular accelerations of the center of mass of the OMR and on the geometric relations defining the configuration of the mobile robot.

B. Appendix B

We write the equilibrium condition for the OMR in the form of an equation of the sum of torques relative to the axis \(l_k\) of the triangle \(\triangle EFG\) opposite to the \(k\)th wheel (see Fig. B.1):

\[
N_k B_k - m g b_k - h F_j^{(2)} = 0,
\]

(B.1)

where \(m\) is the mass of the robot, \(g\) is the absolute value of the free-fall acceleration, and \(F_j^{(2)}\) is the projection of the resultant of the friction force \(F_j = \sum F_i \mathbf{A}_i, i = 1, 2, 3\) onto the axis \(B_k\) perpendicular to the side \(l_k\).

We write equation (B.1) taking into account the geometric dependences in the triangle \(EFG\):

\[
\frac{N_k}{|l_k|} (l_k \times l_i, \epsilon_z) - \frac{m g}{|l_k|} (l_k \times \mathbf{C}_j, \epsilon_z) - \frac{h}{|l_k|} (l_k \times F_f, \epsilon_z) = 0,
\]

where \(l_k = \mathbf{C}_i - \mathbf{C}_j\).

After expressing the area \(\Delta\) of the triangle composed of the vectors \(\mathbf{C}_i, \mathbf{C}_j\) and \(l_k\) (see Fig. B.1) and the area \(\Delta_{ij}\) of the triangle \(EFG\), we obtain the following expression:

\[
2N_k \Delta - 2 m g \Delta_{ij} - h (l_k \times F_f, \epsilon_z) = 0,
\]

(B.2)
where
\[ \Delta = \frac{1}{2}(l_i \times l_k, e_z), \quad \Delta_{ij} = \frac{1}{2}(C_i \times C_j, e_z). \]

Expressing the absolute value of the reaction \( N_k \) in equation (B.2), we obtain the following relation determining the absolute value of the reaction force acting from the supporting plane on the roller of the \( k \)th wheel:
\[ N_k = \frac{mg\Delta_{ij}}{\Delta} + \frac{h(l_k \times F_f, e_z)}{2\Delta}. \] (B.3)

The dependence (B.3) in vector form can be written as
\[ N_k = \frac{1}{\Delta} \left( \Delta_{ij}mge_z + \frac{h}{2}l_k \times F_f \right). \] (B.4)

The expression (B.4) represents the dependence of the reaction force, acting on the wheel from the surface of motion, on the ratio of the area of the triangle composed of the vectors \( C_i, C_j \) and \( l_k \) to the area of the triangle \( \Delta EFG \) composed of the vectors \( l_i, l_j, l_k \), and on the ratio of the product of the vectors \( l_k \) and \( F_f \) to the area of the triangle \( \Delta EFG \).

A similar approach can be applied on other directions of vectors \( A_i \).