Characterising the chaotic nature of ocean ventilation
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Eddy stirring and filamentation

Lagrangian tracing of filaments

Manucharyan and Thompson (2017)

Plumb et al. (1994)
In nonlinear dynamical systems, \textit{filament width} characterises the chaotic nature of trajectories by establishing \textit{sensitivity to initial conditions}.

\textbf{Forced double-well oscillator}

\begin{align*}
  x \quad \text{frequency, } \omega \\
  \begin{array}{c}
    \text{Initial acceleration, } x_0 \\
    \text{Final position, } x_N
  \end{array}
\end{align*}

\textit{Strogatz (1994)}
The thinning of filaments in dynamical systems is analogous to stretching and folding of puff pastry, at a rate defined by the *strain*

\[
\frac{d\Delta x}{dt} = -\gamma \Delta x
\]

\[
\Delta x(t) = \Delta x(0) e^{-\int_0^t \gamma dt} = \Delta x(0) e^{-\bar{\gamma} t}
\]

\(\bar{\gamma}\) is the (average) vigour with which the baker rolls the pastry

\(t\) is the time they’ve been working for
In the ocean, the role of the baker is played by the circulation, with the strain rate set by local velocity gradients.

\[
4\gamma^2 = \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2
\]

\[
\Delta x(t) = \Delta x(0)e^{-\bar{\gamma}L t}
\]

\(\bar{\gamma}L\) is the average strain rate following a Lagrangian trajectory.
For a *ventilated* fluid parcel, the ‘time that the baker has been working for’ is the time since ventilation, allowing the definition of a *filamentation number, $F$*

\[ \sqrt{\gamma} L t = \frac{\tau_{\text{vent}}}{\tau_{\text{strain}}} = F \]

\[ \Delta x(t) = \Delta x(0) e^{-F} \]

*In a region with $F = 4$, we would expect typically a 50-fold reduction in filament width since ventilation*
We calculated F in the subtropical thermocline of a 1/4° ocean model, using backwards-in-time Lagrangian trajectories.

\[ \tau_{\text{vent}} \text{ (years)} \]

\[ \sigma_\theta = 26 \text{ kgm}^{-3} \]
\[ \sim 100 - 300 \text{ m} \]

\[ \sigma_\theta = 26.5 \text{ kgm}^{-3} \]
\[ \sim 300 - 500 \text{ m} \]

\[ \sigma_\theta = 27 \text{ kgm}^{-3} \]
\[ \sim 400 - 700 \text{ m} \]
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The filament width of the Lagrangian maps exhibits the expected behaviour: smaller filaments for larger $F$.

Power spectra of ventilation longitude  

PDFs of ventilation longitude gradients
**Summary**

- By analogy to dynamical systems, the chaotic nature of ocean ventilation can be characterised by a reduction in filament width since subduction.
- This is quantified by the non-dimensional number $F$, a ratio of ventilation and strain timescales.
- $F$ is large across three density surfaces in the subtropical North Atlantic thermocline.
- Resolving filament width directly (through backwards-in-time Lagrangian maps) shows the expected relationship with $F$.

MacGilchrist *et al.* (2017) Characterizing the chaotic nature of ocean ventilation, *JGR Oceans*, 122.