On the evolution of relative permeability of two-phase flow in rock fractures: the effect of aperture distribution

Chen Wang¹, Yujing Jiang¹, Renzhe Gao², Xiaoshan Wang³

¹ Graduate School of Engineering, Nagasaki University, Nagasaki 8528521, Japan
² State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China
³ School of Science, Qingdao University of Technology, Qingdao 266525, China

Abstract. Two-phase flow in natural fractures is of great importance in many engineering applications, such as geothermal energy utilization and natural gas recovery. Multiple models of relative permeability were proposed, such as the X model, the viscous coupling model, the Corey model and the v-type model. The diversity of models indicates that the relative permeability is influenced by multiple factors, and the applicability of each model is limited to some specific conditions. In conventional models, relative permeability is expressed as the function of the saturation of a certain phase. However, relative permeability is also influenced by the aperture distribution of the fracture. In this study, visualized two-phase (water-nitrogen) flow experiments were conducted in one pair of fracture replicas, which includes a smooth replica and a plaster replica so that the flow structures can be recorded. These two replicas form a rough fracture, in which the two-phase flow experiment was conducted. In addition, the aperture data of the fracture was imported into the numerical model for simulation. The flow structures in the experiment are reproduced by the simulation, and the numerically obtained relative permeability also has similar features to that of experimentally obtain relative permeability. Then the two-phase flow simulation was conducted in two fractures, whose apertures are in normal distribution. The results show that the relative permeability in both fractures follow the Corey model approximately, but influenced by the aperture distribution obviously, especially the relative permeability of the wetting phase.

1. Introduction

Subsurface two-phase flow is of great importance in engineering, such as the gas and oil exploitation, CO₂ sequestration and geothermal energy utilization [¹,²,³,⁴]. Relative permeability is the key parameter to describe the conductivity of two-phase flow in the fracture, which is generalized from Darcy’s law. The evolution of relative permeability is critical in describing the hydraulic characteristics of two-phase flow. Several models are proposed in order to describe the evolution of relative permeability, including the X-model, the viscous coupling model and the Corey model. The X model indicates that each phase flows independently with no interference, so the relative permeability equals the saturation of each phase [⁵], as indicated by Equations 1 and 2, in which \( k_{rw} \) and \( S_w \) are the relative permeability and saturation of the wetting phase, \( k_{rn} \) and \( S_n \) are the relative permeability and saturation of the non-wetting phase.

\[
\begin{align*}
    k_{rw} &= S_w \\
    k_{rn} &= S_n
\end{align*}
\]
Viscous coupling model indicates the interference between two phases due to the difference of viscosity \[6\], which is expressed as Equations 3 and 4, in which \(\mu_w\) and \(\mu_{nw}\) are the viscosity of wetting phase and non-wetting phase, respectively.

\[
k_{rw} = \frac{S_w^2}{2}(3 - S_w)
\]

\[
k_{rn} = (1 - S_w)^3 + \frac{3}{2}\frac{\mu_n}{\mu_w}S_w(1 - S_w)(2 - S_w)
\]

In the cases where the capillary pressure is significant, the Corey model is supposed to be effective \[7\], as suggested by Equations 5 and 6, in which \(S_{wr}\) and \(S_{nr}\) are the residual saturation of wetting phase and non-wetting phase due the capillary pressure.

\[
k_{wr} = \left(\frac{S_w - S_{wr}}{1 - S_{wr} - S_{wr}}\right)^4
\]

\[
k_{rn} = (1 - S_w - S_{wr})^3[1 - \left(\frac{S_w - S_{wr}}{1 - S_{wr} - S_{wr}}\right)^2]
\]

Besides the above-mentioned models, there are other models to describe the evolution of relative permeability. In addition, it is found that the relative permeability is not only dependent on the saturation, but also on the aperture distribution of the fracture \[8,9,10\]. In order to clarify the correlation between the relative permeability and aperture distribution, a two-phase flow experiment was conducted in a fracture to obtain the flow structures and hydraulic characteristics. The flow structures and relative permeability were also obtained by simulation. Finally, the two-phase flow simulation was conducted in two fractures with the aperture of normal distribution. This study provides a first step to establish the relationship between two-phase hydraulic properties of a fracture and the aperture distribution.

2. Two-phase Flow Experiment and Simulation

**Figure 1.** The experiment system \[11\].
Figure 1 shows the experiment system, which is composed of the water injection subsystem (the peristaltic pump, the pulse damper), the gas injection subsystem (the gas cylinder, the pressure regulator, the mass flow controller), the measurement system (the camera, the pressure sensors, the electronic balance), and the flow box with specimens. Four kinds of data are collected during the experiment: the flow structures (by the camera), the pressure (by the pressure sensors), the flow rate of water (by the electronic balance) and the flow rate of gas (by the gas flow controller). There are pulses in the flow of water injected by the peristaltic pump. Consequently, the pressure and flow rate of water is always fluctuating. In order to decrease such fluctuation (pulse) and inject water to the fracture uniformly, the pulse damper is connected to the pump.

The specimens used in the experiment are shown in Figure 2(a). The upper specimen is transparent with a rough fracture so that the flow structures can be recorded by the camera; the lower specimen is made of plaster with a smooth surface. The roughness data of the fracture was obtained by scanning the height of the surface of the transparent specimen since the surface of the plaster specimen is smooth. With this data, the fracture aperture distribution can be obtained, as indicated by Figure 2(b).

The experiment was conducted in the following steps: (1) assemble the specimens in the two-phase flow box; (2) conduct the leakage test; (3) conduct the single-phase flow experiment with water in order to obtain the hydraulic properties of single-phase flow in this fracture; (4) conduct the two-phase flow experiment.

![Image](image_url)

(a) The specimens used in experiment

(b) The aperture distribution

**Figure 2.** The rock specimens used in the experiment

Equation 7 is the transport and continuity equation, which is derived by assuming the validity of the local cubic law and introducing the effect of capillary pressure\(^{[8,12,13,14]}\). In equation 7, \(b\) is the fracture aperture, \(P\) is the pressure, \(\mu\) is dynamic viscosity, \(F_a\) is the capillary pressure. This equation assumes that the flow of each phase follows Darcy’s law and the cubic law at each element. The values of \(b\) shown in Figure 2(b) is imported into the numerical model. As shown in Equation 8, the capillary pressure \(F_a\) is set as a volumetric force, in which \(\lambda\) and \(\theta\) are the surface tension and the contact angle, respectively; \(n\) is the unit vector that indicates the direction of \(F_a\); \(G\) is a variable which ensures that \(F_a\) is not zero only at the interface between two phases. Equation 9 is the level-set equation, and the fluid phase is tracked with the critical variable \(\phi\)-the level set variable. \(\phi\) ranges from 0 to 1. The fluid will be water when \(\phi\) is 0; the fluid will be gas when \(\phi\) is 1; it is the interface of two fluids when \(0<\phi<1\). Consequently, \(\phi\) is identical to the fraction of gas, namely \(\phi = V_g / V_{f}\). \(\gamma\) is the initialization parameter, which determines the reinitialization of the level set function. \(\epsilon\) is a controlling parameter which regulates the thickness of the region where \(\phi\) varies from 0 to 1, namely it is the interface thickness.

\[
\nabla \cdot \left( \frac{b^3}{12\mu} (\nabla P + F_a) \right) = 0 \tag{7}
\n\]
3. Results and Discussion

Figure 3 shows the flow structures obtained by both experiment and simulation, at the condition that the water flow rate (Wr) is 300 mL/min and gas flow rate (Gr) is 2000 mL/min. Three groups of simulation and experiment photos are given, which indicate three different moments. It shows that the flow structures obtained from the experiment has the following features: gas tends to bypass the area A and area B, and flows in the two channels as indicated by the two arrows in Figure 3; most of the gas flow out of the fracture from the area C. These features in the flow structures are reproduced by the simulation. Even though that there is some difference in the saturation, the main features are identical. This is because that the area A and area B has a small aperture, as shown in Figure 2(b), and in these areas the capillary pressure is significant. The difference in the saturation and flow structures between the experiment and simulation may be induced by the following reason: in the experiment gas is compressible and kept swelling and shrinking; however, in the simulation gas is simplified as an incompressible fluid since Level-set method cannot calculate the flow of compressible fluid.

\[
F_u = \frac{2\lambda \cdot \cos \theta}{b} G \cdot n \quad (8)
\]

\[
\frac{\partial \varphi}{\partial t} + \nabla (\varphi \cdot u) = \gamma \nabla \cdot \left[ \sigma \nabla \varphi - \varphi (1 - \varphi) \frac{\nabla \varphi}{|\nabla \varphi|} \right] \quad (9)
\]

Figure 3. The experimentally and numerically obtained flow structures at Wr = 300 mL/min, Gr = 2000 mL/min.
Figure 4 shows the relative permeability obtained from the experiment and simulation. The X-model, viscous coupling model and Corey model (with three different values of residual saturation) are also plotted for reference. It shows that both the simulation and experiment results are close to the Corey model, but the relative permeability obtained from the experiment tends to be smaller. This is because the fracture surface is rough, and the relationship between pressure drop and flow rate is generally nonlinear [15], which leads to more pressure drop than linear flow. However, in the simulation the nonlinear regime is not included in the equations, since the flow of each phase is assumed to be Darcy flow in each element.

Figure 4. The experimentally and numerically obtained relative permeability.

Figure 5 shows the aperture distribution of two generated fractures, which follow the normal distribution. The average aperture $\mu$ of both fractures is 1 mm. The standard deviation $\sigma$ of fracture 1 is 0.25 and that of fracture 2 is 0.64. The flow structures of both fractures are shown in Figure 6. The relative permeability is shown in Figure 7. It shows that the simulation data show some discreteness, and more data are needed for better evaluation, but some regularity can be obtained. It shows that compared with X model and VC model, the relative permeability of two-phase flow in both fractures...
is close to Corey model, but the interference between two phases seems to be stronger than that of Corey model. The relative permeability in fracture 1 lies between the Corey curve \( (S_{wr} = 0.01 \text{ and } S_{nr} = 0.22) \) and the Corey curve \( (S_{wr} = 0.01 \text{ and } S_{nr} = 0.12) \); the relative permeability in fracture 2 is close to the Corey curve at \( S_{nr} = 0.1 \text{ and } S_{wr} = 0.05 \). This is because in a fracture with larger standard deviation \( \sigma \) the flow tends to be more tortuous and forms more residual phase, and consequently leads to a lower relative permeability.

![Image](image1.png)

**Figure 6.** The flow structures of two-phase flow in the fractures with normal distribution.

![Image](image2.png)

**Figure 7.** The numerically obtained relative permeability of fractures with normal distribution.

4. Conclusions
(1) With an experiment system, two-phase flow experiment is conducted in a fracture with natural roughness, and the fracture aperture was imported into the numerical model for simulation with the level-set method. The flow structures obtained from the experiment were reproduced by simulation. Both the experimentally and numerically obtained relative permeability tend to follow the Corey model. However, the relative permeability obtained from the experiment is smaller due to the flow nonlinearity.

(2) Simulation was conducted in two generated fractures, which have an aperture of normal distribution. The relative permeability of two-phase flow in both fractures is close to the Corey model but shows stronger interference than Corey model. The two-phase conductivity decreases with the increase of standard deviation $\sigma$ due to the increase of tortuosity and residual phase. In future study, the relationship between the residual saturation and the fracture roughness should be further investigated.

5. References

[1] Kimura S 1997 Geothermal energy development and multiphase flows Japanese Journal of Multiphase Flow 11(3) pp 11-14
[2] Detwiler L Rajaram H and Glass J 2009 Interphase mass transfer in variable aperture fractures: Controlling parameters and proposed constitutive relationships Water Resources Research 45 W08436
[3] Persoff P and Pruess K 1995 Two-phase flow visualization and relative permeability measurement in natural rough-walled rock fractures Water resources research 31(5) pp 1175-1186
[4] Nuske P Faigle B Helmig R Niessner J and Neuweiler I 2010 Modeling gas-water processes in fractures with fracture flow properties obtained through upscaling Water Resources Research 46(9) pp 201-210
[5] Romm S 1966 Fluid flow in fractured rocks (in Russian) (Moscow: Nedra Publishing House)
[6] Fourar M and Lenormand R 1998 A viscous coupling model for relative permeabilities in a fracture SPE Annual Technical Conference and Exhibition (Louisiana: New Orleans)
[7] Corey T 1986 Mechanics of immiscible fluids in porous media water resources publications (Colorado: Littleton)
[8] Watanabe N Sakurai K Ishibashi T et al 2014 New v-type relative permeability curves for two-phase flows through subsurface fractures Water Resources Research 51(4) pp 2807-2824
[9] Pruess K and Tsang Y W 1990 On two-phase relative permeability and capillarity in rough-walled rock fractures Water resources research 26(9) pp 1915-1926
[10] Sheng J Han Y Ye Z et al 2020 The relative permeability model for water-air two-phase flow in rough-walled fractures and numerical analysis Rock and Soil Mechanics 41(3) pp 1048-1055
[11] Wang C 2019 Experimental and Numerical Investigations on the Hydraulic Characteristics of Two-phase Flow in Rock Fractures (Japan: Nagasaki University)
[12] Brown R 1987 Fluid flow through rock joints: The effect of surface roughness Journal of Geophysical Research 92(B2) pp 1337-1347
[13] Sauve J 2002 Hydromechanical properties and alteration of natural fracture surfaces in the Soulzu granite (Bas-Rhin, France) Tectonophysics 348(1) pp 169-185
[14] Watanabe N Hirano N and Tsuchiya N 2009 Diversity of channelling flow in heterogeneous aperture distribution inferred from integrated experimental-numerical analysis on flow through shear fracture in granite Journal of Geophysical Research 114 B04208.
[15] Zimmerman W Al-yaarub A Pain C et al 2004 Non-linear regimes of fluid flow in rock fractures International Journal of Rock Mechanics and Mining Science 41 pp 163-169