Imprint of a Steep Equation of State in the growth of structure

Mariana Jaber-Bravo, Erick Almaraz and Axel de la Macorra

Instituto de Física, Universidad Nacional Autónoma de México, A.P. 20-364, CDMX 01000, México.

(Dated: March 9, 2022)

We study the cosmological properties of a dynamical of dark energy (DE) component determined by a Steep Equation of State (SEoS) \( w(z) = w_0 + w_a \left( \frac{z}{z_T} \right)^q \). The SEoS has a transition at \( z_T \) between two pivotal values \((w_i, w_0)\) which can be taken as an early time and present day values of \( w \) and the steepness is given by \( q \). We describe the impact of this dynamical DE at background and perturbative level. The steepness of the transition has a better cosmological fit than a conventional CPL model with \( w = w_0 + w_a (1 - a) \). Furthermore, we analyze the impact of steepness of the transition in the growth of matter perturbations and structure formation. This is manifest in the linear matter power spectrum, \( P(k) \), the logarithmic growth function, \( f\sigma_8(z) \), and the differential mass function \( dn/d\log M(z = 0) \). The differences in these last three quantities is at a percent-level using the same cosmological baseline parameters in our SEoS and a \( \Lambda \)CDM model. However, we find an increase in the power spectrum, producing a bump at \( k \approx k_T \) with \( k_T \equiv \alpha T H(\alpha T) \) the mode associated to the time of the steep transition \((\alpha T = 1/(1+z_T)) \). Different dynamics of DE lead to a different amount of DM at present time which has an impact in Power Spectrum and accordingly in structure formation.

I. INTRODUCTION

The standard \( \Lambda \)CDM paradigm is based on the assumptions of homogeneity and isotropy of the Universe at large scales, the validity of General Relativity and the cosmological constant term as cause of the accelerated cosmic expansion. Although it has been proven successful when tested against observations it faces some major theoretical issues such as the extreme fine-tuning problem known as the cosmological constant problem [1] which leads to the necessity of extending it. Some candidates include scalar field models or modifications of General Relativity

Observational probes coming from different physical phenomena such as the temperature and polarization of cosmic microwave background (CMB) [2], the luminosity distance of supernovae [3] or the statistical signature of the baryonic acoustic oscillations (BAO) from galaxy surveys [4–8], quasars [9, 10] or voids [11], have improved significantly over the years.

In this work we choose to focus on an effective model of a fluid with free parameters. In this, we consider the dark energy (DE) contribution, \( \rho_{DE} \) to be a perfect fluid so dissipative terms will not be present. In this situation we describe the dynamics of this component through its equation of state, \( w(z) \), defined by:

\[
\rho_{DE}(w) = w(z)\rho_{DE}(z)
\]  

which can be parameterized to match observations.

Several proposals for \( w(z) \) can be found in the literature [12–20]. These proposals attempt to describe the dynamics of dark energy without assuming a particular theoretical model, but providing practical parametrizations that can be readily confronted against observations. In this approach, a cosmological constant solution can be modelled as a fluid with pressure \( p_\Lambda = -\rho_\Lambda \), which implies an equation of state \( w = -1 \). This landscape has recently been extended to cover the background expansion rate as prescribed by \( f(R) \) theories [27].

The study of the perturbative regime could potentially be used to discriminate between a cosmological constant and models with a negative pressure component from Modified gravity. In this pursuit, ongoing and upcoming surveys such as eBOSS [28], DESI [29], LSST [30] and EUCLID [31] will provide extremely precise measurements of the growth of structure in the Universe, which in turn, will allow to probe the nature of the cosmic acceleration mechanism.

Studying the effect of dynamical DE into the clustering at large scales is thus a relevant task for the cosmological community. In this work we present the implications that a steep transition in the DE EoS, \( w(z) \) has in the growth of structure.

This paper is organized as follows. In Section [IV] we describe our model for dark energy, the cosmology chosen and the analytical treatment used for the perturbations. Section [III] comprises our main results in the particular case of a smooth dark energy component and its impact on linear observables in the perturbative regime. Our conclusions and outlook are covered in section [IV].
II. METHODS

A. Steep Equation of State

In a previous work [32] we presented a parametric form for \(w(z)\) inspired in quintessence fields and tested its free parameters with observations such as the Baryon Acoustic Oscillations (BAO) peak measured in galaxies or in the Lymann-\(\alpha\) forest, as well as the compressed Cosmic Microwave Background likelihood [33, 34], and the local determination of \(H_0\) included in [35].

Our form for the equation of state is:

\[
w(z) = w_0 + (w_i - w_0) \frac{(z/z_T)^q}{1 + (z/z_T)^q}
\]  

(2)

which allows for a steep transition to take place at a pivotal redshift \(z = z_T\) with a steepness modulated by the exponent \(q\). For this reason we dubbed this equation “SEoS” (from “Steep Equation of State”) in this work.

We notice that in the case where the transition is smooth and occurs at a particular redshift: \(z_T = q = 1\), we recover a form for the equation of state known as the Chevallier-Polarski-Linder parametrization (CPL) [12, 13] which has been widely used in the literature,

\[
w(z; z_T = 1, q = 1) = w_0 + (w_i - w_0) \frac{z}{1 + z} = w_0 + w_a (1 - a),
\]  

(3)

where \(w_a \equiv w_i - w_0\) and we keep the convention \(a_0 = 1\).

In this work we will refer to the particular case of having arbitrary \(w_0\) and \(w_i\) but taking \(z_T = q = 1\), as the “CPL limit” of the SEoS [2].

We notice that the parameter \(q\) modulates the steepness of the transition: the greater the value for \(q\), the more abrupt is the transition, as figure 1 shows.

B. Background models

Once we have specified the equation for the dynamics of DE, the expansion rate (for a flat Universe) is given by:

\[
\frac{H(z)}{H_0} = \sqrt{\Omega_r^{(0)}(1 + z)^4 + \Omega_m^{(0)}(1 + z)^3 + \Omega_{DE}^{(0)} F(z)}
\]  

(4)

where \(H \equiv \left( \frac{da}{dt} \right) \frac{1}{a} \) is the Hubble parameter, \(t\) the cosmic time, \(a = (1 + z)^{-1}\) the scale factor of the Universe and \(H_0 = 100 \cdot h \ km \cdot s^{-1} Mpc^{-1}\) the Hubble constant at present time. The fractional densities of matter, radiation and dark energy at \(z = 0\), are given by \(\Omega_m^{(0)}, \Omega_r^{(0)}, \Omega_{DE}^{(0)}\), respectively, which follow the flatness constraint \(\Omega_m + \Omega_r + \Omega_{DE} = 1\).

The function \(F(z)\) in equation (4) encodes the evolution of the DE component in terms of its equation of state:

\[
F(z) = \frac{\rho_{DE}(z)}{\rho_{DE}(0)}
\]

(5)

\[
F(z) = \exp \left( -3 \int_0^z dz' \frac{1 + w(z')}{1 + z'} \right)
\]

For the free parameters in equation (2) we have chosen the best fit values obtained in [32] from the combination of BAO measurements [1, 7, 9, 10] and the local determination of \(H_0\) [35]. This corresponds to: \(w_0 = -0.92, w_i = -0.99, q = 9.97\) and \(z_T = 0.28\). The cosmological parameters, \(\Omega_m\) and \(H_0\), were set equal to those reported by the Planck collaboration [36], so we can compare the discrepancy arising only from the different dynamics of DE (table I models I-III). This is \(\Omega_m = 0.3089\) and \(H_0 = 67.74 \ (\omega_c = 0.1198)\).

However, to take into account the full result obtained in [32], we also set the values for \(\Omega_m\) and \(H_0\) to those obtained with the constraining procedure reported previously. This corresponds to the model IV from table I and values \(\Omega_m = 0.3340\) and \(H_0 = 73.22 \ (\omega_c = 0.1568)\). This value for \(H_0\) corresponds to the one reported in [37], which is known.

FIG. 1: [Color online] Evolution of equation (2) (“SEoS”) for different values of the transition redshift, \(z_T\), and the steepness parameter, \(q\). The parameters \(w_0\) and \(w_i\) were fixed to \(-0.92\) and \(-0.99\), respectively, and \(q\) was varied from \(q = 1\) (blue) to \(q = 10\) (red). Solid lines represent the evolution of \(w(z)\) with \(z_T = 0.28\), and dot-dashed lines indicate \(z_T = 1\). The CPL case is explicitly labeled and corresponds to \(z_T = q = 1\).
| Alias     | $w_0$ | $w_i$ | $q$ | $z_T$ | $H_0 [\text{km/s Mpc}^{-1}]$ | $c_{\text{s}}^2 \equiv \Omega_c h^2$ | $\Omega_m^{(0)}$ |
|-----------|-------|-------|-----|-------|-------------------------------|---------------------------------|----------------|
| (I) $\Lambda$CDM-P | -1    | -1    | 1   | 1     | 67.27                         | 0.1198                          | 0.3156         |
| (II) SEoS-P    | -0.92 | -0.99 | 9.97| 0.28  | 67.27                         | 0.1198                          | 0.3156         |
| (III) CPL-P    | -0.92 | -0.99 | 1   | 1     | 67.27                         | 0.1198                          | 0.3156         |
| (IV) SEoS-bf   | -0.92 | -0.99 | 9.97| 0.28  | 73.22                         | 0.1568                          | 0.3340         |

TABLE I: Models used in this work. The cosmological parameters in models I-III correspond to Planck TT, TE, EE+lowP [36], whereas model IV has the best fit values obtained in [32]. The rest of the parameters were kept fixed for all cases: $\Omega_b h^2 = 0.0225$, $\ln(10^{10} A_s) = 3.094$, $n_s = 0.9645$, also corresponding to those reported by Planck.

FIG. 2: [Color online] (Upper panel) Hubble expansion rate normalized to $H_0$ for the models described in Table I. (Bottom panel) Ratio of solutions for equation (2) to a cosmological constant: $\Delta H / H_0 \equiv (H - H_0) / H_0$, where $H_0$ refers to the solution $\Lambda$CDM-P. The inset plot shows only the solutions SEoS-P and CPL-P compared to $\Lambda$CDM-P. The vertical lines represent the transition redshift for each model.

C. Perturbative regime

We examine the growth of perturbations during the matter-DE domination era using “SEoS” (equation (2)) as the model for DE.

For a late time Universe we have a mixture of matter and DE and we know radiation to be sub-dominant. In that case the growth of over-densities can be studied in the Newtonian limit of the formalism this is, considering non-relativistic components that are well inside the horizon. For coupled fluids we have:

$$a^2 \frac{d^2 \delta_i(a)}{da^2} + a \left( 3 + \frac{\dot{H}}{H^2} \right) \frac{d \delta_i(a)}{da} = \frac{3}{2} \sum_j (\Omega_j \delta_j) - \left( \frac{c_{s}^2}{a^2 H^2} \right) \delta_i(a) = 0, \quad i, j = \text{matter, dark energy}. \quad (6)$$

where we have used $H^2 = \frac{8\pi G}{3} \rho$. The density contrast of the $i$-th fluid is represented by $\delta_i \equiv (\rho_i - \bar{\rho}) / \bar{\rho}$, where $\bar{\rho}$ is the background density, and $(c_{s}^2)_i$ represents the corresponding speed of sound, defined by $(c_{s}^2)_i \equiv \frac{\delta P_i}{\delta \rho_i}$.

We find the solutions for equation (6) in the particular case of $\delta_{DE} = 0$, this is, when DE does not cluster, since the spatial fluctuations of typical dark energy models are very much suppressed with respect to those of dark matter.

In addition to finding the numerical solutions of equation (6), we also used a modified version of the Boltzmann solver CAMB [37] in which we introduced “SEoS” as the background model.
III. RESULTS

Regarding the solution for $w_{DE}(a)$ we choose to explore the different scenarios which are referenced in Table II and were chosen as explained below:

- Model “$\Lambda$CDM-P” refers to a cosmological constant scenario with $\Omega_m$ and $h$ fixed to Planck cosmology [36].
- Model “$SEoS$-P” refers to the best fit values for the parameters in equation (2) as obtained in [32] while maintaining $\Omega_m$ and $h$ to a Planck cosmology [36].
- Model “CPL-P” refers to the scenario where we adopt the CPL limit of the above solution, meaning we keep $\{w_0, w_i\} = \{-0.92, -0.99\}$, as obtained in [32] and $\{\Omega_m, h\}$ fixed to a Planck cosmology [36], but we make $z_T = q = 1$.
- Finally, Model “$SEoS$-bf” refers to the best fit values for the parameters in equation (2) (i.e. $\{w_0, w_i, q, z_T\} = \{-0.92, -0.99, 9.97, 0.28\}$) with $\Omega_m$ and $h$ also fixed to the best fit values obtained in [32].

The corresponding expansion histories for those models are shown in figure 2 where we plot $H(a)/H_0$ and the relative ratio from models $SEoS$-P, CPL-P and $SEoS$-bf to $\Lambda$CDM-P in the bottom panel: $\Delta H/H_\Lambda \equiv (H - H_\Lambda)/H_\Lambda$.

A. Growth function

In the case where $\delta_{DE} = 0$, equation (6) reduces to:

$$a^2 \frac{d^2 \delta_m(a)}{da^2} + a \left( 3 + \frac{\dot{H}}{H^2} \right) \frac{d \delta_m(a)}{da} - \frac{3}{2} \Omega_m \delta_m(a) = 0.$$  (7)

This can be solved by setting initial conditions in the matter dominated era, $a_{ini} = 10^{-3}$, since we know that during this epoch, the solution for the growth function is $\delta_m(a) = a$, we have $\delta_m(a_{ini}) = a_{ini} = 10^{-3}$ and $\delta'_m(a_{ini}) = 1$.

A solution for equation (7) can be given up to a normalization. We choose to normalize it such that $D^{(+)}_m(a) = 1$ at $a = a_{ini}$, so we enhance the differences arising at present time. This is shown in Figure 3a for the models under consideration. Once we have the solution to equation (7), we can also find the logarithmic growth function, $f(a) \equiv \frac{d \log \delta_m(a)}{d \log a}$ (see fig. 3b).

To get a better idea of the effect of different dark energy models in the growth functions $D^{(+)}_m(a) \equiv \frac{\delta_m(a)}{\delta_{m(0)}}$, $f(a)$, we take the relative difference to a $\Lambda$CDM - P scenario: $\Delta F = \frac{F - F_\Lambda}{F_\Lambda}$ with $F = \{D^{(+)}_m, f(a)\}$ and $F_\Lambda$ the solution assuming $\Lambda$CDM-P as background model. This is shown, for instance, in the bottom panel of plots 3a and 3b respectively.

![FIG. 3: (Color online)](a) Evolution of matter overdensities normalized to the initial time, $D^{(+)}_m(a)$, and (b) logarithmic growth function, $f(a)$ for the models in table II. The bottom panel displays the relative difference to $\Lambda$CDM-P solution: $\Delta F/F_\Lambda \equiv F - F_\Lambda$ with $F$ representing $D^{(+)}_m(a)$ or $f(a)$, respectively.

Regarding the results for $D^{(+)}_m$ we find deviations from $\Lambda$CDM that are of order:

- of 1% at $z = 0$ if we assume model $SEoS$-P as our DE component,
- of order 1.5%, for CPL-P,
- and of order of 6% at $z = 0$ taking $SEoS$-bf.

The differences in $\Delta f(a)/f_\Lambda$ are consistent, showing deviations at percent level: the fastest expan-
sion rate corresponds to $SEoS$-bf model (as indicated in figure 2), followed by CPL-P and $SEoS$-P. Hence, we obtained a slower growth of structure and a slower logarithmic growth rate in $SEoS$-bf model (followed by CPL-P and $SEoS$-P).

It is important to note that the discrepancy between $ΛCDM$ and a dynamic form of dark energy is bigger for the CPL scenario that the case \(\{w_0, w_i, q, z_T\} = \{-0.92, -0.99, 9.97, 0.28\}\) (Model CPL-P versus Model $SEoS$-P in figure 3). This is due to the fact that the CPL limit has $z_T = 1$, which implies that for this case $w(z) \to -0.92$ for $z \leq 1$, whereas Model $SEoS$-P has a later transition redshift, implying that $w(z) \to -0.92$ for $z \leq 0.28$.

B. Linear matter power spectrum

By means of a modified version of CAMB [37] in which we incorporated “$SEoS$” as expansion model and considered negligible DE perturbations, we computed the linear matter power spectrum, $P(k)$, which is calculated in the synchronous gauge, used internally by the code.

1. $SEoS$: DE dynamics only

Our results are shown in Figure 4. In this we show the linear matter spectrum for $ΛCDM$ and $SEoS$-P (figure 1a and their ratio $\Delta P(k)/P_Λ \equiv (P(k) - P_{ΛCDM}(k))/P_{ΛCDM}(k)$ for different redshift values ($z = 0$, $z = z_T = 0.28$, $z = 2z_T = 0.56$, and $z = 1$).

We notice a decrease in amplitude for all Fourier modes, of order $0.5\% (1.7\%)$ for redshift values $z = 1$ ($z = 0$), respectively (see figure 4a). This is to be expected since we have seen that a consequence of the dynamics of $SEoS$-P is an Universe that expands more rapidly as compared to one dominated by a cosmological constant. The effect appears after the transition has occurred, since for $z > z_T$, our EoS behaves as a cosmological constant term ($w_i \approx -1$).

In addition to this decrease, we notice a bump in $k \approx 6 \times 10^{-4}h^{-1}/\text{Mpc}$ for $\Delta P(k)/P_Λ|_{z=0}$. This is better depicted in figure 4b where we show the ratio between $SEoS$-P and $ΛCDM$-P for power spectra after the transition has occurred ($z < z_T$). From the bottom panel of figure 4a we notice the bump appears only after the transition has occurred, and in figure 4b we see it increases as $z \to 0$. We can know which modes are entering to the horizon during and after the transition epoch of $z_T = 0.28$.

Using $h = 0.6774$ we have $k_T \equiv a_T H(a_T) = 1.403 \times 10^{-4}h^{-1}/\text{Mpc}$ (shown as a red dotted vertical line in figure 4b) with $a_T = 1/(1 + z_T)$. Which means that modes $k < k_T$ enter into the horizon after the abrupt transition took place.

FIG. 4: [Color online] (a) Linear matter power spectra for Models I and II (Table I) at different redshift values ($z = 0$, $z = z_T = 0.28$, $z = 2z_T = 0.56$, and $z = 1$) and the ratio from “$SEoS$-P” to $ΛCDM$. (b) Zoom-in of the previous plot showing $\Delta P(k)/P_Λ(k)$ for $z < z_T$: $z = 0, 0.1, 0.2$. The (red) vertical line at $k = 1.403 \times 10^{-4}h^{-1}/\text{Mpc}$ indicates the mode associated to the transition redshift ($z_T = 0.28$), $k_T = a_T H(a_T)$, whereas the (black) vertical line in $k \approx 6 \times 10^{-4}h^{-1}/\text{Mpc}$ indicates the maximum of the bump in $\Delta P(k)/P_Λ(k)$. 

\[ \Delta P(k)/P_Λ \]
and the transfer function is roughly the same ($\approx 9/10$) for small modes, we can estimate the amount of deviation for small modes from $P_{SEoS}(k)/P_\Lambda$ to be of the order $(D_{1, SEoS}/D_{1, \Lambda})^2$. From results in figures 2 and 3a we get $(D_{1, SEoS}/D_{1, \Lambda})^2 = (0.95)^2 = 0.9$, which in turns means $\Delta P(k) \approx -33\%$, in agreement with figure 4.

C. Large scale structure

Galaxy redshift maps constrain the combination $f\sigma_8(z)$ using measurements of the redshift space distortions (RSD). So, in order to have an insight on the possible implications of the model into the growth of structure at large scales, we consider the $f\sigma_8(z)$ function and compare with some of the current observational constraints reported in the literature and listed below. This is shown in figure 6. We added the observational points reported by the following surveys: 6dFG [39], SDSS MGS [40], SDSS-LRG [41], BOSS-LOWZ and BOSS-CMASS [42], WIGGLE-z [43], and the VIPERS [44]. As previously mentioned, we show the relative difference $\Delta F/F_\Lambda \equiv \Delta F/F_\Lambda$ with $F = \{f\sigma_8(z), d\sigma/d\log M(z = 0)\}$ and $F_\Lambda$ the solution assuming $\Lambda CDM$ as background model.

From this result we see that model $SEoS$-bf predicts a larger value for $f\sigma_8(z)$ at all redshift values $z \in [0, 1.5]$. This increase (of order $\Delta f\sigma_8 \sim 20\%$) makes model $SEoS$-bf in discordance with the current observations of $f\sigma_8(z)$, whereas $SEoS$-P and its CPL limit are within observational error bars and deviate from $\Lambda CDM$-P by less of 3%, in conformity with our previous results, in particular, we see that the difference in $\Delta f\sigma_8(z)$ is in agreement with the result shown in figure 6.

Additionally we consider the fractional number of collapsed structures by means of the Press-Schechter formalism, which describes the matter over-density field in real space by a smooth gaussian field whose variance on a sphere of radius $R$ is $\sigma^2_R$. In this formalism, the number of collapsed objects per unit volume with mass between $M$ and $M + dM$ is given by:

$$dn = -\sqrt{\frac{2}{\pi}} \frac{d\sigma_R}{dM} \left( \frac{\rho_m \delta_c}{M \sigma^2_R} \right) \exp \left( -\frac{\delta^2}{2\sigma^2_R} \right) dM \quad (9)$$

where $\delta_c = 1.686$, the linear over-density at collapse is set to the value for $\Lambda CDM$ since the dependence on cosmology is not strong [46]. In figure 7 we show the differential mass function for the models considered and their relative ratio to $\Lambda CDM$-P model.

In this case we notice that $SEoS$-bf model predicts a decrease in the number of smalls structures
(masses \(M \sim \mathcal{O}(10^{10}M_\odot)\)) by 10\% compared to \(\Lambda CDM\) scenario, and an increase of 30\% (masses \(M \sim \mathcal{O}(10^{14}M_\odot)\)) and as big as 50\% for the biggest collapsed structures \((M \geq 4 \times 10^{14}M_\odot)\).

For \(SEoS\)-P and \(CPL\)-P models, the behavior is the opposite: we find an increase in the number of small objects (masses \(M \leq 6 \times 10^{12}M_\odot\)) of order 1−2\% and a decrease in the number of big structures \((M \geq 5 \times 10^{14}M_\odot)\) of \sim 3\% for \(SEoS\)-P and 6\% for its \(CPL\) limit.

We recall that the mass \(M\) is inversely proportional to the wave-number since \(M \propto R^3\) and \(R = \pi/k\), indicating that large masses correspond to small modes (large scales) and vice versa. In \(SEoS\)-bf model, additionally to the DE dynamics we have a different value for \(\rho_m\) than in \(\Lambda CDM\) (see equation 9), which impacts importantly the mass function, as we have just discussed. For models \(SEoS\)-P and \(CPL\)-P, however, the matter content is the same and hence when we compare the differential mass function at a particular mass scale we are also comparing that function at the same mode. Moreover, since the age of the Universe is practically the same in models I-III (table I), the resulting discrepancies previously discussed mean that large structures take more time to form in a \(SEoS\) model, while small objects form more quickly.

As a consequence we can say that we would expect to observe less massive galaxy clusters and more light structures (isolated galaxies and poorly populated clusters) in \(SEoS\)-P or \(CPL\)-P universe.

We studied a DE model with the characteristic of a steep transition between two pivotal values. This model was previously analyzed at background level and its free parameters were tested against observations such as the latest local determination of \(H_0\), the BAO peak and the angular distance to the CMB [32], and constrained its free parameters to: \((w_0 = -0.92, w_i = -0.99, q = 9.97, z_T = 0.28)\). This work investigates how a steep transition in the DE EoS can affect the growth of structure, and we restricted ourselves to the case of a smooth DE component.

We find that the effect of a \(SEoS\) for DE in structure formation can basically be separated into two phenomena: 1) On one hand the presence of a dynamical dark energy changes the expansion of the background, leading to different growth rates and affecting the matter fluctuations 2) While on the other hand, the change in \(\Omega_m\) as well the Hubble rate (according to the BFV obtained previously) has a bigger impact than just the DE dynamics, modifying the observable quantities such as \(P(k)\), \(f\sigma_8(z)\), and \(dn/d\log M\) beyond the current observational constraints.

In the fist case we find that the change in the Hubble expansion \(2\) of 1.5\% percent at the transition epoch \((z_T = 0.28\) or \(z_T = 1\) in “\(SEoS\)-P” or “\(CPL\)-P”, respectively), impacts the growth functions in an equivalent amount, diminishing the growth of struc-
tecture at linear order by 1.5% − 2% (figure 3). This consequently imprints into $f \sigma_8(z)$ as a decrease of $\sim 2 − 3\%$ and lies in agreement with RSD observational measurements from surveys [39–44]. As for the differential mass function, $dn/d\log M(z=0)$, we find as a prediction, a slight increment in the number of small collapsed objects of order 1% (2%) and a decrement in the number of large structures or order 3% (6%) for SEoS-P (CPL-P) model.

The CPL limit of SEoS-P model (which means taking $z_T = q = 1$ in equation (1)), consistently shows bigger differences from $\Lambda CDM$ model than SEoS-P, as a result of an earlier (yet smooth) transition from $w_i \approx -1$ to a bigger value $w_0 = -0.92$, which implies the DE dilutes first in CPL-P model. As for the matter power spectrum, we notice the appearance of a bump in the modes close to those entering near the steep transition, in the linear regime ($k \sim 10^{-4}h^{-1}/\text{Mpc}$), which appears only after the transition took place ($z < z_T$) and increases amplitude as $z \rightarrow 0$.

In the second case, this is, for SEoS-P model, we find an interplay between having an Universe with $\Delta \omega_c = 0.1568/0.1198 \sim 30\%$ bigger than in a $\Lambda CDM$ scenario, with the change in the expansion rate, such that the clustering is prevented at large scales (small Fourier modes or large masses) and enhanced at enhanced at small scales (large Fourier modes or small masses). See for instance figures 5 and 7. From the differential mass function, for instance, the prediction is that the number of collapsed objects decreases (increases) by approximately 10% (50 – 60%) for light (the largest) structures. Lastly, the effect on $f \sigma_8(z)$, however implies that model SEoS-bf is not in agreement with RSD observational constraints.

To summarize, the study of dynamics of Dark Energy is a matter of profound implications for our understanding of the Universe and its physical laws. Studying the behavior of a model beyond background level is nowadays required given the important amount of data coming from redshift galaxy surveys and its potential to test discrepancies among a cosmological constant, fluids with negative pressure or modifications to the gravity sector. In this paper we have contributed towards that direction showing that the evolution of matter over-densities is sensitive to the parameters in equation (2), and a model with a steep transition such as the one probed in this paper can lead to interesting features in the growth of structure.

ACKNOWLEDGEMENTS

This project was done with funding from the CONACYT grant Fronteras de la Ciencia 000281 and PASPA-DGAPA UNAM. M.J. thanks Omar A. Rodríguez L. for computational help and the group of Extragalactic Astronomy and Cosmology of Institute of Astronomy UNAM for fruitful discussions.
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