Low frequency absolute calibration of complex sensitivity of vector receivers in free-field

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Abstract: Three-transducers spherical wave reciprocity method in free-field is demonstrated effectively for absolute calibration of the complex sensitivity of underwater sound pressure gradient of vector receiver in the frequency range 250 Hz to 4 kHz. The regularity of underwater sound pressure gradient distribution in the spherical wave, the theory of three-transducers reciprocity calibration method and the technique of complex moving weighted average (CMWA) are studied and reviewed. The VHS90 vector receiver manufactured by Hangzhou Applied Acoustics Research Institute (HAARI) is calibrated using underwater sound pressure gradient calibration facility in a 50 m×15 m×10 m anechoic tank. To verify the results of measurements, the VHS90 vector receiver is also calibrated using low frequency vector receiver calibration facility and the underwater sound pressure calibration facility. The calibration results and the comparisons with these facilities prove the accuracy of the calibration method and facilities described in this paper. The max deviation of modulus of complex sensitivity is 0.7 dB and max deviation of phase congruency of three channels is 1.6°.

1 Introduction

Vector receiver is a new kind of underwater sound receiver invented in the 20th century, and it can measure the underwater vector parameters (velocity, acceleration and sound pressure gradient) as well as scalar parameter (sound pressure) simultaneously. Besides sound pressure, vector receiver can receive more physical information and it would improve the signal processing capability of SONAR system. Great attentions had been paid to the research and development of vector receivers, and many kinds of vector receivers were designed and manufactured, such as MEMS vector receiver, fibre-optical vector receiver, bionics vector receivers [1-3]. These vector receivers are widely applied in noise measurement and SONAR system.

Before use of them, vector receivers need to be calibrated. From 1960s to 1970s, the MARK series calibration facilities are designed and improved by Bauer for the measurement of vector receiver in low frequency [4,5]. The method of vibrating liquid column can also be used for calibration of the receiver. This method is usually used for calibration of the sensitivity of hydrophone, and it can also be used for calibration of vector receiver [6]. Anther widely used calibration method is standing wave comparison method, which calibrate the vector receiver using a reference hydrophone in a standing wave tube. Except the sensitivity, the directivity of vector receiver can also be calibrated [7-9]. In free-field, three-transducers reciprocity method and comparison method are usually used for the calibration of vector receiver, and the frequency range of them is 1 kHz to 10 kHz [10,11].

However, the sensitivity of vector receiver calibrated using the method above could only measure the modulus of the complex sensitivity. In this paper, the complex sensitivity of vector receiver is calibrated using the reciprocity method in free field, and the low frequency is expanded to 250 Hz using the technique of CMWA [12].

2 Calibration principles

2.1 Distribution of sound pressure gradient in free-field

When the projector transmits sound wave in free-field, it can be considered as a point source in the far field and the sound pressure can be expressed as

$$p = \frac{P_0}{r} \exp[\text{jkr}(r - r_0)] \quad (1)$$

where, $P_0$ is the sound pressure at the distance of 1 meter from the point sound source, $r$ is the distance from sound source, $k$ is equal to $2\pi/c$, $f$ is frequency, $c$ is the sound velocity in the water.

The sound pressure gradient can be calculated according to the relationship between sound pressure and sound pressure gradient by following equation (2).

$$\nabla p = \frac{P_0}{r} \left[ -\text{jkr}(1 + \frac{1}{ikr}) \exp[\text{jkr}(r - r_0)] \right] \quad (2)$$

The sound pressure in the spherical wave can be considered as the plane wave in the far field, and the amplitude of sound pressure is reversed to the distance between projector and receiver. However, there is a deviation when measurement of the parameter is sound
pressure gradient (velocity or acceleration), and the spherical wave could not be considered as plane wave. The deviations between spherical wave and plane wave are shown in Fig.1. It can be seen that if the distance is far and the frequency is high, the deviation would be very small and the spherical wave could consider as the plane wave. Otherwise, a modification is necessary.

![Phase modification](image1)

(a) Phase modification

![Amplitude modification](image2)

(b) Amplitude modification

Fig. 1. The sound field distribution modification of sound pressure gradient in spherical wave.

### 2.2 Three-transducers spherical wave reciprocity method in free field

Different from the sound pressure receiver, the parameters measured by vector receiver depend on its internal sensor type. There is a relationship between sensitivities of sound pressure, sound pressure gradient, particle velocity and particle acceleration, which can be shown as

$$M_p = \frac{\omega}{c} M_{vp} = \frac{1}{\rho c} M_v = \frac{\omega}{\rho c} M_a$$

where, $M_p$ is the sensitivity of sound pressure, $M_{vp}$ is the sensitivity of sound pressure gradient, $M_v$ is the sensitivity of oscillation velocity of particle and $M_a$ is the oscillation acceleration of particle.

In this paper, sound pressure gradient of vector receiver will be calibrated. The measurement procedure using reciprocity method is shown in Fig.2. In this figure, P is the projector, T is the reciprocity transducer and G is the vector receiver. They are laid in the far field of each other. The distance between the reference center of projector and reciprocity transducer is $r_{PG}$, the distance between the reference center of projector and vector receiver is $r_{TG}$, and the distance between the reference center of reciprocity transducer and the vector receiver is $r_{TG}$.

![Diagram of three-transducers reciprocity calibration of vector receiver in free field](image3)

**Fig. 2.** Diagram of three-transducers reciprocity calibration of vector receiver in free field.

The complex output voltage of the vector receiver $U_{PG}$ in the distance $r_{PG}$ from projector P in free field is proportional to the sound pressure gradient

$$U_{PG} = M_{vp} \cdot \nabla \vec{p}$$  \hspace{1cm} (4)

where, $M_{vp}$ is the complex sensitivity of sound pressure gradient; $\nabla \vec{p}$ is the complex sound pressure gradient in spherical wave of free-field.

Sound pressure gradient in spherical wave of free field may be calculated in equation (2). Here, we consider $\vec{\theta} = -jk \left(1 + \frac{1}{jk r} \right)$, the transfer impedance of the transducer and the vector receiver $Z_{PG}$ could be expressed as the ratio of the complex open circuit voltage $U_{PG}$ to complex transmitting current $I_p$

$$Z_{PG} = \frac{U_{PG}}{I_p} = -M_{vp} \frac{\vec{S}_p \cdot \vec{\theta}}{r_{PG}}$$ \hspace{1cm} (5)

where, $\vec{S}_p$ is the complex current transmitting response.

Similarly, the complex transfer impedance $Z_{TG}$ of the reciprocity transducer and the vector receiver could be expressed as

$$Z_{TG} = \frac{U_{TG}}{I_T} = -M_{vp} \frac{\vec{S}_T \cdot \vec{\theta}}{r_{TG}}$$ \hspace{1cm} (6)

The complex transfer impedance $Z_{PT}$ of the projector and the transducer could be expressed as:

$$Z_{PT} = \frac{M_T \vec{S}_p}{r_{PT}}$$ \hspace{1cm} (7)

The modulus and phase of sound pressure gradient sensitivity could be expressed:

$$|M| = \sqrt{\frac{2}{r^2} \left( \frac{Z_{PT} | \vec{Z}_{TG} | r_{TG}}{Z_{PG}} \right)} \frac{r_{PG}}{r_{PT}} \frac{r_{TG}}{\sqrt{k^2 r_{PG}^2 + 1}}$$ \hspace{1cm} (8)

$$\arg \theta = \frac{1}{2} \left[ \arg Z_{TG} + \arg Z_{PG} + \arg Z_{PT} + k(r_{TG} + r_{PG}) \right]$$  \hspace{1cm} (9)

### 2.3 Complex moving weighted average (CMWA) technique

Calibration of the vector receiver at low frequency is
limited by the reflecting wave from the boundary or water surface of water tank. In order to expand the low frequency limit of water tank, the technique of CWMA is used to calibrate the vector receivers. In this method, the orthogonal linear frequency sweeping signal to form the complex signal and the complex transfer impedance of projector and vector receiver is expressed:

$$Z(f) = \frac{U(f)}{I(f)}$$  \hfill (10)

where, $U(f)$ is the frequency response of open circuit voltage of vector receivers containing the directive sound wave and the reflecting sound wave, $I(f)$ is the frequency response of transmitting current, and $f = f_0 + st$, $s$ is the change rate of frequency in the linear sweeping signal.

The $Z'(f)$ is not the transfer impedance of projector and vector receiver in free-field, and it need to eliminate the influence of reflecting sound wave. Using the technique of CMWA, one could reduce the reflecting sound wave and get the transfer impedance in free-field, the using of CMWA could be expressed as:

$$\frac{1}{\Delta f_{mv}} \int \frac{h(f')}{Z'(f-f')}d f' = Z(f, \Delta f_{mv}) + \sum_{i=1}^{s} \xi_i(f, \Delta f_{mv})$$ \hfill (11)

where, the $\Delta f_{mv}$ is the width of windows of average; $h(f')$ is weighted function, and its size is depended on the kind of average window used in the measurement; $Z(f)$ is the frequency response of transfer impedance in free-field after using CMWA; $\xi_i(f)$ is remainder term of the $Z'(f)$ after using CMWA, and $\sum_{i=1}^{s} \xi_i(f) \approx 0$.

When using technique of CMWA, we could get the frequency response of transfer impedance $Z(f)$ without reflecting wave in free field.

### 3 Vector receiver and the calibration facility

#### 3.1 Vector receiver to be calibrated

Fig. 3. The structure diagram and photo of VHS90.

The vector receiver to be calibrated in this paper is VHS90 which is a sphere of diameter 90 mm and manufactured by HAARI. There are three pairs of accelerators set in three orthogonal directions in VHS90. These sensors and their preamplifiers are fixed onto a base metal block and sealed into a polymer ball. Fig.3 shows the structure diagram and photo of the VHS90 vector receiver. The signals from three pairs of sensors will be output separately through a multi-core cable. To keep a vector receiver in suitable orientation in application, three pairs fixing ring are attached on the polymer ball in face of sensors.

#### 3.2 Calibration facility

The facility used for calibration of vector receiver in free-field is underwater sound pressure gradient calibration facility which consists of function generator, power amplifier, voltage & current sensor, preamplifier and PXI digitizer and controller. When conducting calibration, the PXI controller controls the function generator to drive the projector or reciprocity transducer to transmit a linear sweeping signal through power amplifier. Vector receiver or the reciprocity transducer receives the sound pressure gradient signals, and the signals are amplified and impedance matched by preamplifier, and the signal is collected by PXI digitizer and calculated. The calibration facility is linear system without phase shift. The schematic diagram of the calibration facility is shown in Fig.4.

![Fig. 4. Schematic diagram of free-field underwater sound pressure gradient calibration facility.](image)

Fig. 5. The calibration bracket of underwater sound pressure gradient calibration facility.
The calibration bracket which used for location of the projector, the reciprocity transducer and the vector receiver accurately is shown in Fig.5. The projector and transducer are mounted on a 2000 mm long carbon fibre pole with a 30 mm diameter. The vector receiver is fixed on a 300mm metal ring through four rubber bands, which leaves the vector receiver suspending in water. The projector, reciprocity transducer and vector receiver are in the same horizontal level and their reference sound center is in a line. The measured channel of vector receiver needs to be pointed to the sound source during the measurement. The three transducers and calibration bracket are put in water tank at the depth of 5 meter.

**4 Calibration results and discussion**

**4.1 Modulus of sound pressure gradient sensitivity**

The VHS 90 vector receiver is calibrated using underwater sound pressure gradient calibration facility in the frequency range of 250 Hz to 4 kHz. The modulus of sound pressure gradient sensitivity of three channels is shown in Fig.6. It can be seen, the results of three channels are very close, and max deviation of three channels is 0.3 dB.

To verify accuracy of the calibration results, these calibration results are further compared with the calibration results using low frequency vector receiver calibration facility using standing wave tube and underwater sound pressure calibration facility in free-field using tone-burst technique. The comparison results of channel X, Y and Z is shown in Fig.7.

![Fig.6](image-url)  
**Fig.6.** The amplitude of sound pressure gradient sensitivity of three channels of VHS90 vector receiver.

The measurement uncertainty of low frequency vector receiver calibration facility using comparison method in standing wave tube is 1.0 dB and the calibration frequency range in this paper is 250 Hz to 1.25 kHz. The measurement uncertainty of underwater sound pressure gradient calibration facility using reciprocity method and tone-burst technique in free-field is 0.7 dB, and the calibration frequency range in this paper is 1 kHz to 4 kHz. From the comparison results shown in Fig.7, the results of three calibration facilities are coincident. The deviation of calibration results is not larger than 0.7 dB in the frequency range of 250 Hz to 1.25 kHz. In the frequency range 1 kHz to 4 kHz, the max deviation is 0.6 dB, at the frequency of 1.25 kHz, and deviation in other frequency points is not larger than 0.3 dB. And in the common frequency range 1 kHz to 1.25 kHz, the deviation of three facilities is not larger than 0.7 dB, which further proves accuracy and stabilization of three facilities.

![Fig.7](image-url)  
**Fig.7.** The comparison between calibration results of three channel using three different facilities.

**4.2 Phase of sound pressure gradient sensitivity**

As shown in Fig.8, the broadband phase response of three channels of vector receiver is nearly consistent, however, only a deviation of 1.6° in the channel Z of the receiver is observed at the frequency of 4 kHz.

The reason of deviation of channel Z is that with the measurement frequency increase, the phase of vector receiver is more sensitive to distance. The reference center location deviation of three vector channels will lead to the phase shift of three channels. Besides, the vector receiver needs to suspend on the calibration bracket during measurement, and the cable or the rubber will disturb its condition of free suspension in water, which may have little effect in amplitude of sensitivity,
but will have a great impact on its phase measurement.

Fig. 8. Congruency of three channels of vector receiver.

5 Conclusions

The absolute calibration of complex sensitivity vector receiver is introduced in this paper, and a VHS90 vector receiver which manufactured by HAARI is calibrated in the frequency range of 250 Hz to 4 kHz.

The calibration results and their comparison with low frequency vector receiver calibration facility and underwater sound pressure calibration facility prove the accuracy of the calibration method and the facility described in this paper.

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