Nuclear Magnetic Moments – 50 years of the Arima-Horie Paper

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Magnetic moments of nuclei attracted the attention of nuclear physicists since the early days. Magnetic moments of atoms played an important role in the development of quantum theory of the atom. They were easy to measure and their values posed puzzles which were solved only when the spin of the electron was discovered. Magnetic moments of nuclei were more difficult to measure but they also showed promise to lead to deeper understanding of nuclear structure. Of particular interest were magnetic moments of odd mass nuclei. They are the only ones to be discussed in this paper.

It is not surprising that Lande, who did the seminal work on atomic moments, wrote as early as 1934 a detailed paper[1] on nuclear magnetic moments. He applied to odd-even nuclei his formula for magnetic moments of spin 1/2 particles

\[ \mu = \frac{g_L [J(J+1)+L(L+1)-3/4] + g_S [J(J+1)-L(L+1)+3/4]}{2(J+1)} \]

and considered a very special model to obtain \( L \) and \( g_L \). There were only preliminary results of the value of \( g_S \) for protons and the value of the proton moment adopted by Lande is 2 nuclear magnetons \( (g_S=4) \). Using this value and the few measured moments of nuclei, he extracted for the neutron magnetic moment which was not measured yet, the value -0.6 n.m. \( (g_S=-1.2) \).

The model used by Lande is very simple. "One particle only, one proton or one neutron, is responsible for the total spin and the magnetic properties of the whole nucleus, the rest of it forming closed shells in general". The word "spin" for the total angular momentum, which is now in common use, appeared in that paper as a reminiscent of the days when a nucleus was considered as a single particle. The term "closed shells" actually means just spin 0 as evident from his statement concerning even-even nuclei: "we suppose them to consist of closed shells". This statement is probably based on the fact that in ground states of atoms, only in closed shells do the total \( L \) and \( S \) vanish. In this model, the values of \( g_L \) are well defined, i.e. 2 for an odd proton nucleus and 0 for an odd neutron one. There were about 20 measured magnetic moments and the agreement between results of the Lande model with the \( g \)-values he adopted is very good indeed.
By 1937, magnetic moments of about 50 nuclei have been measured. They were considered by Schmidt[2] who applied to them the Lande model with better values of $g_S$. Better measurements were available and he used the values of the proton magnetic moment of 2.85 n.m. and the neutron magnetic moment of -2.0 n.m. He plotted the measured moments as a function of $j$ for the various cases and compared them with the values calculated according to Lande. He connected by lines the latter points which became known as "Schmidt lines". He concluded by stating that the model of Lande can be only approximately correct. Still, it seems that it is a good approximation. He expressed the opinion that the nuclear coupling scheme lies between the "single particle model" and a more complicated situation.

In 1940, Margenau and Wigner published a paper[3] where to "understand the grouping of nuclear magnetic moments a generalization of Schmidt's single-particle model is considered." It is interesting to note that no reference is made to the paper of Lande, although "the customary Lande-Goudsmit formula" for magnetic moments is mentioned. They used the SU(4) scheme and showed that $g_t$ of the lowest super-multiplet, is related to the magnetic moment of the single nucleon in the same way as assumed by Schmidt ($g_S=2\mu$). The $g_t$ on the other hand, could not be determined in a simple way. They assumed that the contributions of protons and neutrons to the orbital angular momentum are comparable. This led them to adopt for $g_L$ the value $Z/A$ (approximately 1/2) for both odd proton and odd neutron nuclei. The lines they obtained are different from those of Schmidt and the available experimental data favored the latter only slightly. The authors do not believe that $L$ is a good quantum number and thus, both $L=J+1/2$ and $L'=J-1/2$ may contribute to the magnetic moments. As a consequence they expect that the latter will lie between the Margenau-Wigner lines. They were worried, however, by some measured magnetic moments of neighboring isotopes which are almost equal. In their picture "magnetic moments are expected to be distributed between" the Margenau Wigner lines "in a rather random fashion". Again, they ignored completely the Lande (or Schmidt) model and wrote "In fact we fail to see any model which would explain such a result ... "

Most physicists in those years believed that nuclear states are given by the LS-coupling scheme (Russel-Saunders scheme) in which the total $L$ and $S$ have definite values. When M.G.Mayer published her 1948 paper[4] in which she presented decisive evidence for the existence of magic numbers, there were attempts to arrange single nucleon orbits, characterized by $l$, into major shells[5,6]. Nordheim[6] based his considerations also on measured magnetic moments. He obtained at shell closures the magic numbers 50, 82, 126 by putting orbits of odd and even $l$ in the same major shell. Between 20 and 50 he put the 1f, 2p and 1d orbits, between 50 and 82, the 1g and 2f orbits. Beyond 82 he assumed that the 1h, 3s orbits etc. are being filled. Nordheim wrote that, there "is no definite trend discernible as to the relative alignment of intrinsic spin and orbital momentum." In view of this, it is not surprising that he was happy to see that $^{209}$Bi "gives beautiful evidence for a 6h orbit". As we know now, in $^{209}$Bi there are already 12 1h$_{11/2}$ neutrons in the closed shells of $^{208}$Pb.

Nordheim used those orbits to explain also deviations of magnetic moments from the Schmidt lines. Since experimental moments lie between the two lines, the simplest explanation is by considering linear combinations of the states whose moments lie on the two lines. The parities of two states of a single nucleon with the same value of $j$ are opposite. Hence, Nordheim assumed that in the main component of the wave function, the single nucleon is coupled to the normal $J=0$ ground state with even parity. In the other, smaller component, the nucleon is coupled to a 0$^+$ state of the even-even core.
In 1949 the modern version of the shell model including a strong spin-orbit interaction was introduced by M.G.Mayer[7] and independently by Jensen, Haxel and Suess[8]. Single nucleon orbits in this model are characterized by the total spin $j$ of the single nucleon. These are obtained by coupling its orbital angular momentum $l$ to its intrinsic spin. In configurations of several nucleons in such an orbit, states are naturally characterized by $jj$-coupling wave functions. Such states have the same energy in the central potential well. Determination of ground states, as well as excitation energies of other states, may be obtained by introducing mutual interactions between nucleons. This was done by Mayer who postulated that the ground state spin $J$ of an even-even nucleus is 0, whereas that of an odd-even nucleus is equal to the spin $j$ of the orbit. These rules are simpler and different from the situation in atoms. There, Hund's rule is based on the maximum spatial antisymmetry of the wave functions of the electrons. This feature is a result of the mutual repulsion between electrons and leads to ground states with maximum possible value of the spin $S$. Also the orbital angular momentum $L$ of the ground state usually varies from one atom to another.

Mayer carried out some calculations in simple configurations[9] to "obtain theoretical reasons for these empirical rules". She calculated energy levels of $j^3$ configurations up to $j=7/2$. Such calculations could be carried out by elementary methods. She "assumed that an attractive potential acts between identical nucleons. For reasons of definite and easy evaluation this was assumed to have the shape of a $\delta$-function". The results of her calculations gave some support for the coupling rules which she suggested. In this way the $\delta$-interaction entered nuclear structure physics. Since then this interaction has been used by many authors, and also in the important paper we honor today. For a while, this interaction was even considered by some as an integral part of the shell model. It is worth while to point out that in prior work, the interaction was usually assumed to have rather long range. Inside nuclear dimensions it was often replaced by a constant.

The success of the shell model brought into focus the single nucleon degrees of freedom. The fact that the measured magnetic moments follow roughly the Schmidt lines was easy to accept. The problem turned out to be the origin of the deviations from the Schmidt lines and the reason for them being so regular. Bloch[10], de-Shalit[11] and Miyazawa[12] noted independently that all these deviations can be attributed to a change of $g_S$ of protons and neutrons. They adopted rather pure single nucleon wave functions and suggested that magnetic moments should be calculated using reduced $g_s$ factors. The reduction is due to the nucleons being imbedded in the nuclear medium. If it is only the anomalous moment which is reduced, magnetic moments should lie between the Schmidt lines and "Dirac lines" calculated by using for protons $g_S=2$ and for neutrons $g_S=0$. There were only a few measured moments of odd proton nuclei which lie outside that region. Other general prescriptions were also suggested. Davidson published a paper[13] assuming the main component of the wave function to be a single nucleon wave function $a la$ Lande. A smaller component, however, is a rather complicated nuclear state with $S=1/2$ as in the SU(4) theory. The value of $L$ is determined by the single nucleon orbit. If $j=L+1/2$, then $L=j-1/2$ but if $j=L-1/2$, then $L=j+1/2$. For $g_L$ he adopted the value $Z/A$ like Wigner and Margenau.

Mizushima and Umezawa[14] and independently Flowers[15] noticed that in certain states, good isospin contradicts the assumption that the spin of the nucleus is due to the "odd nucleon". Consider
the state with \( J=j,v=1 \) and \( T=1/2 \) of the \( j^3 \) configuration outside closed shells. If there are two neutrons and one proton, the magnetic moment is not that of a \( j \)-proton. It is given by

\[
\mu = \mu_p [1-(2j-1)/(6j+1)] + \mu_n (2j-1)/(6j+1)
\]

For very large values of \( j \), the moment is only 1/3 of the moment of the odd nucleon (the proton). This result was derived by Flowers and was generalized for any number of nucleons by Teitelbaum[16]. In ground states of most nuclei, however, valence protons and valence neutrons occupy different orbits. Isospin is a good quantum number for all states of these configurations. There is no definite structure of the wave functions which is prescribed by isospin.

Some of the states described above belong to mixed configurations. Mixing of configurations was considered in the shell model since the early days. It was natural to assume that the nuclear wave functions could not belong to pure shell model configurations. There was no way, however, to determine the amount of mixing in a systematic and consistent fashion. The admixtures were often obtained by using a certain, rather arbitrary, mutual interaction between nucleons. If a calculation assuming a certain configuration did not agree with experiment, the blame was put on configuration mixing. In the attempts, described above, to use it for reproducing measured magnetic moments, there was a common feature. The resulting magnetic moment was given by a linear combination of the moments of the mixed states. The coefficients were the probabilities of the corresponding states. No cross terms of the magnetic moment operator between the major component of the wave functions and other components were considered. As a result, to have a visible effect, the admixtures must have been rather large. One author who decided to ignore cross terms had to mix many states to obtain agreement with measured moments. He concluded by stating that the simple shell model states have only about 1% probability in the wave functions which he obtained.

The possible importance of cross terms was pointed out first by Ross[17] in 1952. He explained "that the expectation value of the moment may be linear in the amplitude of small admixtures to the wave function". Hence, "a very substantial modification of the moment can be associated with a very small modification of the wave function". He did not do much with this modification but the approach he advocated turned out to be very fruitful. Such admixtures were considered by Blin-Stoyle[18] for some special case. The complete treatment, both theoretical derivation of simple algebraic expressions and successful reproduction of measured magnetic moments, was carried out by Arima and Horie[19].

The title of the Arima-Horie paper is "Configuration Mixing and Magnetic Moments of Odd Nuclei". Of the configurations which mix into the simple shell model wave functions, only those that have non-vanishing cross terms with the former were considered. Only such admixtures contribute corrections to the magnetic moments which are linear in the mixing amplitude. Arima and Horie found that these amplitudes are of order 0.1 and they can bring the magnetic moments to their experimental values. The probabilities of the perturbing states are then only of order of 1% which is very reasonable. For the two-body interaction which mixes the configurations, they adopted the zero range potential, introduced by Mayer and since then used by Pryce[20] for the Pb region, by de-Shalit[21] for odd-odd nuclei as well as by others. Arima and Horie defined different strengths for spin singlet states \( (T=1) \) and for spin triplet states \( (T=0) \). The interaction is thus defined by:
\[ V = \frac{V_s(1 - 4s_1s_2)}{4} + \frac{V_t(3 + 4s_1s_2)}{4} \delta (r_1 - r_2) \]

As it turned out, the potentials multiplied by spin operators led to mixings which yield cross terms of the magnetic moment.

Arima and Horie considered all configurations which yield such cross terms. They started from rather simple shell model configurations where in ground states, the odd nucleons as well as the even ones are in states with lowest seniorities. In such states there are nucleons of both kinds in closed orbits with single nucleon spins \( j_1 = l_1 + 1/2 \) and \( j_2 = l_2 - 1/2 \). In the highest orbits, the even nucleons are in the \( j_1 = (l_1 + 1/2) \) configuration. "Usually, \( n_2 = 0 \) if \( n_1 < 2j_1 + 1 \) (unfilled) and \( n_1 = 2j_1 + 1 \) (closed) if \( n_2 > 0 \)." The odd group of nucleons is in the state \( j' (J = j, \nu = 1) \). One type of configuration mixing is obtained from the simple shell model state by raising a nucleon from the closed \( j_1 = l_1 + 1/2 \) into the \( j_2 = l_1 - 1/2 \) orbit. A state of this excited configuration with \( J = 1 \) has a non-vanishing matrix element of the magnetic moment operator of the even group with the lower state.

Another type of admixtures which contribute cross terms may be obtained if the odd nucleon is in a \( j = l - 1/2 \) orbit. It is obtained when a nucleon in the filled \( j' = l + 1/2 \) orbit is raised into the orbit of the odd nucleon. On the other hand, if the odd nucleons are in a \( j = l + 1/2 \) orbit and \( p > 1 \), one of them could be raised into the empty \( j = l - 1/2 \) orbit. All cross terms are proportional to the matrix element \( \langle j = l + 1/2 | \mu | j' = l - 1/2 \rangle \) of the magnetic moment operator for protons or for neutrons. Arima and Horie used in their derivations coefficients of fractional parentage and made very efficient use of formulae of tensor algebra. They obtained simple expressions for the deviations of magnetic moments from the Schmidt values. It is clear that such mechanism which they considered does not work in the case of a single nucleon (or hole) outside a core in which all \( l + 1/2 \) and \( l - 1/2 \) orbits are completely filled. This applies to \(^{17}\)O or \(^{41}\)Ca nuclei. The \(^{41}\)Ca moment was not measured yet and the \(^{17}\)O moment was known to lie right on the Schmidt line. Also, the only configuration which could contribute cross terms, to a state of an odd \( p_{1/2} \) nucleon cannot be excited by the \( \delta \) -potential. The authors note that in nuclei where the odd nucleon is in a \( p_{1/2} \) orbit, the deviations from the Schmidt lines are rather small.

Arima and Horie derived their simple and elegant formulae seperately for protons and neutrons. The assumption that the even group is in a \( J = 0 \) state even if the valence shell is not completely filled may be a reasonable approximation in most cases which they consider. This is probably the case in heavier nuclei. Only in some nuclei, where valence protons and neutrons occupy the same orbit, must the wave function of the even group have components with \( J > 0 \) in states with good isospin. Only in a few cases such isospin effects, considered by Mizushima-Umezawa[14] and Flowers[15], were ignored. One case in which such effects are important is \(^{35}\)Cl, where the two \( 1d_{5/2} \) neutrons have 83% probability to be in a \( J = 0 \) state and 17% probability to have \( J = 2 \). Thus, the simple shell model state has the moment \( (13 \mu_p + 2 \mu_n)/15 \) rather than \( \mu_p (.26 \text{ rather than } .13 \text{ n.m.}) \).

Arima and Horie calculated the magnetic moments of all measured nuclei. They assumed rather simple and pure shell model configurations and calculated the mixing due to the two-body interaction. They assumed certain strengths of the coefficients \( V_s \) and \( V_t \) based on energies of nuclear states. The
amount of mixing to first order in perturbation theory is inversely proportional to the spin-orbit splitting. They took it from available experimental information and in cases where it was not known, they made reasonable assumptions. In some of the nuclei which they considered, the configuration is determined directly by the shell model. In $^{209}$Bi, for instance, they obtained for the magnetic moment the value of 3.4 n.m. which reduces considerably the gap between the Schmidt value 1.565 n.m. and the measured value, 4.11 n.m.

Some of the simplifying approximations made in the paper could be challenged. Still, the paper of Arima and Horie was a very important landmark in understanding nuclear moments and nuclear structure. Arima and Horie demonstrated that the regular features of nuclear magnetic moments need not arise from a general *ad hoc* mechanism. Shell model states with lowest seniorities of protons and neutrons reproduced the Schmidt magnetic moments. Arima and Horie demonstrated that the systematic deviations from the Schmidt moments can be simply reconciled with the shell model description of nuclei. They showed how rather small deviations from pure $jj$-coupling wave functions yield magnetic moments in agreement with experiment. The corrections to the Schmidt moments were obtained by Arima and Horie in a systematic way as the result of the mutual interaction. A residual interaction must be introduced into the shell model to obtain a complete description of nuclear states. They used such an interaction to obtain configuration mixings yielding corrections to the simple shell model magnetic moments which are in good agreement with experiment.

Arima and Horie applied their idea also to quadrupole moments of odd-even nuclei[22]. They made similar assumptions on the two-body interaction and considered mixing of configurations whose contributions to quadrupole moments are linear in the mixing amplitudes. In this case, it is the quadrupole-quadrupole part of the interaction that leads to such configuration mixing. In the conclusion they summarize their results: "the quadrupole moments of odd-neutron nuclei can be explained as resulting from the excitation of the proton group" and "that the additional quadrupole moments of odd-proton nuclei due to the same cause are rather large". They explained that the latter "also contain the quadrupole moment of the initial configuration". "The agreement between the calculated and observed values of the quadrupole moments of odd nuclei is fairly good except for the nuclei with very large quadrupole moments". They are aware of the collective model explanation of those moments.

The Arima-Horie paper of 50 years ago attracted the attention of the world community of nuclear physicists to the excellent work on nuclear structure which had been going on in Japan. During the last 50 years, Japan has been one of the major centers of nuclear physics. All of us wish our Japanese colleagues many more years of activity and achievements.

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